Introduction

Harmonic instabilities have been reported in practical installations such as BorWin1, which was the first HVDC-connected Offshore Wind Power Plant (OWPP) [1], [2]. More recently, electrical interactions between offshore HVDC converters and series resonances have been identified in DolWin1 and highlighted by CIGRE Working Groups as potential causes of instability during the energization of the offshore ac grid [3], [4], [5]. Such interactions are known as electrical resonance instabilities [6]. In HVDC-connected OWPPs, the long export ac cables and the power transformers located on the offshore HVDC substations can cause series resonances at low frequencies in the range of 100 Hz to 500 Hz [3] - [5], [7]. Moreover, the offshore grid is a poorly damped system directly connected without a rotating mass or resistive loads [3], [2]. The control of the offshore HVDC converter can further reduce the total damping at the resonant frequencies until the system becomes unstable.

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Criterion for the Electrical Resonance Stability of Offshore Wind Power Plants Connected through HVDC Links

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Abstract—Electrical resonances may compromise the stability of HVDC-connected Offshore Wind Power Plants (OWPPs). In particular, an offshore HVDC converter can reduce the damping of an OWPP at low frequency series resonances, leading to system instability. The interaction between offshore HVDC converter control and electrical resonances of offshore grids is analyzed in this paper. An impedance-based representation of an OWPP is used to analyze the effect that offshore converters have on the resonant frequency of the offshore grid and on system stability. The positive-net-damping criterion, originally proposed for subsynchronous analysis, has been adapted to determine the stability of the HVDC-connected OWPP. The reformulated criterion enables the net-damping of the electrical series resonance to be evaluated and establishes a clear relationship between electrical resonances of the HVDC-connected OWPPs and stability. The criterion is theoretically justified, with analytical expressions for low frequency series resonances being obtained and stability conditions derived based on the total damping of the OWPP. Examples are used to show the influence that HVDC converter control parameters and the OWPP configuration have on stability. A root locus analysis and time-domain simulations in PSCAD/EMTDC are presented to verify the stability conditions.

Index Terms—electrical resonance, offshore wind power plant, HVDC converter, positive-net-damping stability criterion.

In this paper, the impact that low frequency series resonances have on the voltage stability of HVDC-connected OWPPs is analyzed and discussed. Preliminary work was reported in [19], where the stability criterion presented in this paper was assessed with examples. This paper furthers the initial contributions of [19] by providing a formal framework for the analysis of electrical resonance stability in HVDC-connected OWPPs. An impedance-based representation is used to identify resonances and to assess stability considering the effect of the offshore converters. The resonance stability of an OWPP is determined using an alternative approach to the positive-net-damping criterion [13]. This has been reformulated to evaluate the net-damping for electrical series resonances and to provide a clear relationship between electrical resonances of the OWPP and stability. The main contributions of this paper are summarized as follows:

- The alternative approach to the positive-net-damping criterion is demonstrated using the phase margin condition. This criterion defines the relation between the damping at electrical series resonances and system stability.
- The relationship between the total damping and resonant frequencies with the poles of the system is demonstrated. This relationship shows that the pole analysis and the positive-net-damping criterion provide the same information about resonance stability.

Electrical resonance instability has been studied with an impedance-based representation by several authors. In [8], [9] Voltage Source Converters (VSCs) are modeled as Thevenin or Norton equivalents with a frequency-dependant characteristic. Nyquist and Bode criteria are used to analyze electrical resonance stability [2], [10]. In [9], a clear relationship between electrical resonances and the phase margin condition is established, but the complexity of the loop transfer function limits further analysis of the elements that cause instability. Alternative approaches, the passivity conditions of the system [11], [12] and the positive-net-damping criterion [13], [14], have been used to define stability conditions.

A number of studies on electrical resonance instability in OWPPs have been reported in the literature. In [15] and [16], the impact of electrical resonances on Wind Turbine (WT) converters is investigated. However, few studies are focused on the interactions between resonances and the offshore HVDC converter. In [17], a modal analysis in a HVDC-connected OWPP is used to characterize possible resonances and to assess the stability of the offshore converters. Also, in [9] and [18] the impact of resonances on offshore HVDC converters is analyzed using an impedance-based representation.
- Analytical expressions of the low frequency series resonances are proposed considering the effect of VSC controllers. These expressions are employed to calculate the resonant frequencies where the total damping is evaluated to determine system stability.
- A stable area of an OWPP is defined as a function of the HVDC converter control parameters and the OWPP configuration. Such an area is obtained from the damping of the OWPP and indicates conditions of stability.

The effect that the HVDC converter control parameters and the OWPP configuration have on stability is shown using examples. For completeness, root locus analysis and time-domain simulations in PSCAD/EMTDC are used to validate the stability conditions. The examples presented in this paper are complementary to those included in [19].

II. IMPEANCE-BASED REPRESENTATION OF AN HVDC-CONNECTED OWPP

An impedance-based representation is suitable for the modeling of converters of an HVDC-connected OWPP whenever detailed design information is not available. Such a converter representation offers advantages as it can easily be combined with the equivalent impedance of the offshore ac grid to characterize resonant frequencies. It is also possible to consider the effect of the converter controllers. Moreover, the stability assessment methods for impedance-based representations are simple and less computational intensive compared to other traditional methods such as eigenvalue analysis [2], [10].

The configuration of an HVDC-connected OWPP is shown in Fig. 1. Type 4 WTs are connected to strings of the collector system through step-up transformers from low to medium voltage. Each WT grid side VSC has a coupling reactor and a high frequency filter represented as an equivalent capacitor. The strings are connected to a collector substation, where transformers step-up from medium to high voltage. The collector transformer in Fig. 1 is an equivalent representation of 4 transformers that are connected in parallel [3]. Export cables send the generated power to an offshore HVDC substation, where a VSC based Modular Multilevel Converter (MMC) operates as a rectifier and delivers the power to the dc transmission system. The dc transmission system and the onshore HVDC converters are not represented in this study.

Fig. 2 shows an impedance-based model of the HVDC-connected OWPP suitable for the analysis of electrical resonances and stability. The ac cables of the export and collector system are modeled as single \( \pi \) sections with lumped parameters and the transformers are modeled as RL equivalents. These models are accurate enough to characterize the low frequency resonances that are responsible for stability issues [3]. The VSCs are represented by equivalent circuits, which include the frequency response of the controller. The offshore VSC is represented by a Thévenin equivalent as it controls the ac voltage of the offshore grid [9], [20]; however, Norton equivalents are used to represent the WT VSCs since they control current [8], [20].

III. IMPEDANCE-BASED MODEL OF VSCs

The VSC models are represented in a synchronous \( dq \) frame and the Laplace \( s \) domain, where complex space vectors are denoted with boldface letters for voltages and currents as \( \mathbf{v} = v_d + jv_q \) and \( \mathbf{i} = i_d + ji_q \).

A. Offshore VSC model

The offshore VSC controls the ac voltage of the offshore grid. Fig. 3a describes the control structure of this converter. If the VSC uses a MMC topology, high frequency filters are not required and only a voltage control loop is considered [18], [21]. Additionally, the internal MMC dynamics can be neglected if a circulating current control is implemented [18]. A control action based on a PI controller is expressed as:

\[
\begin{align*}
\mathbf{v}_{\text{h}} & = \mathbf{F}_{\text{PV}}(\mathbf{v}_r - \mathbf{v}_{\text{POC}}) \quad (1) \\
F_{\text{PV}} & = k_{p,v} + \frac{k_{i,v}}{s} \quad (2)
\end{align*}
\]

where \( \mathbf{v}_{\text{h}} \) is the reference voltage for the offshore converter, \( \mathbf{V}_r \) is the control reference voltage at the Point of Connection (POC), \( \mathbf{v}_{\text{POC}} \) is the voltage measured at the POC and \( F_{\text{PV}} \) is the PI controller for the voltage control loop. The dynamics across the equivalent coupling inductance of the offshore converter are expressed as:

\[
\begin{align*}
\mathbf{v}_{\text{h}} & = \mathbf{v}_{\text{POC}} + i_{c}(R_{f}^{h} + sL_{c}^{h} + j\omega_{1}L_{f}^{h}) \quad (3) \\
\end{align*}
\]

where \( i_{c} \) is the current from the HVDC converter, \( L_{c}^{h} \) is the coupling inductance, \( R_{f}^{h} \) is the equivalent resistance of \( L_{f}^{h} \) and \( \omega_{1} = 2\pi f_{1} \text{ rad/s} \) (\( f_{1} = 50 \text{ Hz} \)). The coupling inductance is equal to \( L_{c}^{h} = L_{\text{arm}}/2 + L_{f}^{h} \), where \( L_{\text{arm}} \) is the arm
inductance of the MMC and $L_{th}^h$ is the equivalent inductance of the offshore HVDC transformers.

A Thévenin equivalent of the offshore VSC (see Fig. 2) is obtained by combining (1) and (3):

$$v_{poc} = v_{r} \cdot G_{v}^w - i_{e} \cdot Z_{c}^w \quad (4)$$

$$G_{c}^w = \frac{F_{Pl.c}}{1 + F_{Pl.v}}; \quad Z_{c}^w = \frac{R_{f}^c + sL_{f}^c + j\omega L_{f}^c}{1 + F_{Pl.v}} \quad (5)$$

where $G_{c}^w$ is the voltage source transfer function and $Z_{c}^w$ is the input-impedance of the converter.

**B. Wind Turbine VSC model**

Each WT is equipped with a back-to-back converter, but only the grid side VSC is represented in this study. Its control is based on an ac current loop employing a PI controller as shown in Fig. 3b. The dc voltage outer loop is not represented in the WT VSC model since its dynamic response is slow; i.e. there is sufficient bandwidth separation with the inner current loop [2], [22]. This ensures that there are no interactions between harmonic resonances and the outer loops, which are not of interest in this paper. A Norton equivalent of the WT converter (see Fig. 2) is obtained as:

$$i_{wt} = i_{r} \cdot G_{w}^w - v_{wt} \cdot Y_{w}^w \quad (6)$$

where $i_{wt}$ is the current from the WT VSC, $i_{r}$ is the control reference current, $G_{w}^w$ is the current source transfer function, $v_{wt}$ is the voltage after the coupling filter and $Y_{w}^w$ is the input-admittance of the VSC. $G_{w}^w$ and $Y_{w}^w$ are expressed as [8]:

$$G_{c}^w = \frac{F_{Pl.c}}{R_{f}^w + sL_{f}^w + F_{Pl.c}}; \quad Y_{c}^w = \frac{1 - H_{c}}{R_{f}^w + sL_{f}^w + F_{Pl.c}} \quad (7)$$

$$F_{Pl.c} = k_{p,c} + k_{c,c}; \quad H_{v} = \frac{\alpha f}{\alpha f + \omega} \quad (8)$$

where $F_{Pl.c}$ is the PI controller of the current loop, $L_{f}^w$ the coupling inductance, $R_{f}^w$ the equivalent resistance of $L_{f}^w$, $H_{v}$ the low pass filter of the voltage feed-forward term [8] and $\alpha f$ the bandwidth of $H_{v}$. The PI design is based on [8], [23], with proportional and integral gains given as $k_{p,c} = \alpha c L_{f}^w$ and $k_{c,c} = \alpha c R_{f}^w$ and the bandwidth of the current control by $\alpha c$.

**IV. STABILITY ANALYSIS OF HVDC-CONNECTED OWPPS**

The stability analysis considers the impedance-based circuit presented in Fig. 4, where the offshore grid is modeled with an equivalent circuit (further explained in Section V). A similar representation can be found in [9].

The impedances were expressed in the stationary $\alpha\beta$ frame [6], [11], which is denoted in boldface letters for voltages and currents as $v^w = v_{a} + jv_{b}$ and $i^w = i_{a} + ji_{b}$. The current in the stationary $\alpha\beta$ frame and the Laplace s-domain is given as:

$$i^w = (v_{r}^w G_{c}^w - i_{e}^w Z_{c}^w) \frac{T^h}{1/Z_{g} + 1/Z_{c}^w} \quad (9)$$

where $Z_{g} = Z_{g}^{grid} + 1/Y_{w}^w$ is the equivalent impedance of the OWPP from the offshore VSC and $T^h$ is the OWPP closed loop transfer function, which can be also expressed as:

$$T^h(s) = \frac{M(s)}{1 + M(s)N(s)} = \frac{M(s)}{1 + L(s)} \quad (10)$$

where $M(s) = 1/Z_{g}$ is the open loop transfer function, $N(s) = Z_{c}^w$ is the feedback transfer function and $L(s)$ is the loop transfer function.

Assuming that the voltage and current sources in Fig. 4 are stable when they are not connected to any load [10], the stability of the OWPP can be studied in the following ways:

- By analyzing the poles of $T^h$ or the roots of $Z_{g} + Z_{c}^w = 0$.
- By applying the Nyquist stability criterion of $Z_{c}^w/Z_{g}$ [10].
- By considering the passivity of $T^h$ [6], [11].

In addition to the previous alternatives, a variation to the positive-net-damping criterion given in [13], [14] is here employed instead to analyze system stability. The criterion has been reformulated to evaluate electrical resonance stability as explained in section IV-B.

**A. Passivity**

A linear and continuous-time system $F(s)$ is passive if [11]:

- $F(s)$ is stable and,
- $\text{Re}\{F(j\omega)\} > 0 \quad \forall \omega$, which is expressed in terms of the phase as $-\pi < \text{arg}\{F(j\omega)\} < \pi$. This condition corresponds to a non-negative equivalent resistance in electrical circuits.

Passivity can be applied to determine the stability of closed loop systems [6], [11]. A system represented by the closed loop transfer function in (10) is stable if $M(s)$ and $N(s)$ are passive since $-\pi < \text{arg}\{L(j\omega)\} < \pi \quad \forall \omega$. This implies that the Nyquist stability criterion for $L(s)$ is satisfied. Therefore, the OWPP is stable if $Z_{g}$ and $Z_{c}^w$ are passive. When the HVDC converter is connected to a passive offshore grid, $Z_{g}$ is passive.
the stability only depends on the passivity conditions of the converter input-impedance, \( Z_h \).

In no-load operation (i.e., when only the passive elements of the OWPP are energized), the passivity of \( Z_d \) is ensured as the WTs are assumed to be disconnected from the offshore grid. However, the WTs represent active elements when they are connected to the offshore grid (i.e., \( Z_d \) can have a negative resistance), which may compromise the OWPP stability.

**B. Positive-net-damping stability criterion**

The criterion states that a closed loop system is stable if the total damping of the OWPP is positive at the following frequencies: (i) open loop resonant frequencies and (ii) low frequencies where the loop gain is greater than 1 [13]. However, it does not provide a clear relation between electrical resonances of the OWPP and system stability. This increases the complexity of analyzing the impact that system parameters have on resonance stability.

The criterion presented in [13] has been reformulated to evaluate the net-damping for electrical series resonances. The approach proposed in this paper is developed from the phase margin condition [9]. If stability is evaluated in terms of the phase margin, \( L(j\omega) = M(j\omega)N(j\omega) \) must satisfy the following conditions at angular frequency \( \omega \):

\[
|M(j\omega)N(j\omega)| = 1, \quad -\pi < \arg\{M(j\omega)N(j\omega)\} < \pi \quad \forall \omega.
\]

The criterion presented in [13] has been reformulated to evaluate the net-damping for electrical series resonances. The approach proposed in this paper is developed from the phase margin condition [9]. If stability is evaluated in terms of the phase margin, \( L(j\omega) = M(j\omega)N(j\omega) \) must satisfy the following conditions at angular frequency \( \omega \):

\[
|M(j\omega)N(j\omega)| = 1, \quad -\pi < \arg\{M(j\omega)N(j\omega)\} < \pi \quad \forall \omega.
\]

The phase margin condition (11) is equivalent to:

\[
R_h^c(\omega)^2 + X_h^c(\omega)^2 = R_g(\omega)^2 + X_g(\omega)^2
\]

(16)

The resistive components in ac grids and VSCs may be usually neglected compared to the reactive components. Therefore, \( R_g \ll X_g, R_h^c \ll X_h^c \) and (16) is simplified to:

\[
X_h^c(\omega) = \pm X_g(\omega)
\]

(17)

The electrical series resonances observed from the voltage source \( v^c \) in Fig. 4 correspond to frequencies where \( Z_d^c \) in (15) has a dip or a local minimum. If the resistive components are neglected, the series resonance condition is reduced to:

\[
\text{Im}\{Z_d^c(j\omega_{\text{res}})\} \approx 0 \Rightarrow X_h^c(\omega_{\text{res}}) = -X_g(\omega_{\text{res}})
\]

(18)

It can be observed that (18) is a particular case of (17); i.e., the series resonance condition of \( Z_d^c \) coincides with the stability condition \( |M(j\omega)N(j\omega)| = 1 \) given by (11).

Phase margin condition (12) can be expressed in terms of the imaginary part of \( L(j\omega) \) as follows:

\[
\begin{align*}
\text{If } \frac{dL(j\omega)}{d\omega} > 0 & : 0 < \arg\{L(j\omega)\} < \pi \Rightarrow R_g(\omega)X_h^c(\omega) - R_h^c(\omega)X_g(\omega) > 0 \\
\text{If } \frac{dL(j\omega)}{d\omega} < 0 & : -\pi < \arg\{L(j\omega)\} < 0 \Rightarrow R_g(\omega)X_h^c(\omega) - R_h^c(\omega)X_g(\omega) < 0
\end{align*}
\]

If the resonance condition in (18) is combined with (19):\n
\[
\begin{align*}
& \text{If } \frac{dL(j\omega)}{d\omega} > 0 : X_h^c(\omega_{\text{res}})[R_g(\omega_{\text{res}}) + R_h^c(\omega_{\text{res}})] > 0 \\
& \text{If } \frac{dL(j\omega)}{d\omega} < 0 : X_h^c(\omega_{\text{res}})[R_g(\omega_{\text{res}}) + R_h^c(\omega_{\text{res}})] < 0
\end{align*}
\]

(20)

It can be shown (see Appendix A) that if the offshore grid is capacitive (i.e., \( X_h > 0 \)) and the HVDC converter is inductive (i.e., \( X_h^c > 0 \)), then \( \frac{dL(j\omega)}{d\omega} > 0 \). On the other hand, if the offshore grid is inductive (i.e., \( X_h > 0 \)) and the HVDC converter is capacitive (i.e., \( X_h^c > 0 \)), then \( \frac{dL(j\omega)}{d\omega} < 0 \). Considering the previous conditions, (20) is simplified to:

\[
R_T(\omega_{\text{res}}) = R_g(\omega_{\text{res}}) + R_h^c(\omega_{\text{res}}) > 0
\]

(21)

where resistance \( R_T \) represents the total damping of the system, resistance \( R_h^c \) the HVDC converter damping and resistance \( R_g \) the offshore grid damping.

It can be observed that (21) is equivalent to the positive-net-damping criterion in [13], but evaluated for the series resonances of \( Z_h \). Therefore, the offshore HVDC VSC is asymptotically stable if the total damping of the system, \( R_T \), is positive in the neighborhood of an electrical series resonance. The advantage of this criterion with respect to the passivity approach is that the stability can be ensured even if \( Z_d \) and \( Z_h \) are not passive because it considers the contribution of both terms in the closed loop system.

It should be noted that if the resistive components of the offshore grid and HVDC VSC are large compared to the reactive elements (e.g., \( X_g/R_g < 10 \) and \( X_h^c/R_h^c < 10 \)), the approximations in (17) and (18) are not valid and this criterion cannot be used.

**C. Relation between total damping and poles of the system**

The HVDC-connected OWPP is a high order system with several poles. However, the system response is governed by a dominant poorly-damped pole pair. If this pole pair is related to the electrical series resonance, impedances \( Z_h \) and \( Z_g \) around this resonance can be approximated as:

\[
Z_h(\omega) = Z_h^c(\omega) + Z_g(\omega)
\]

(23)

where \( C_g \) is the equivalent capacitor of the offshore grid impedance when the frequency is close the resonance. Using (18), the series resonance reduces to \( \omega_{\text{res}} = 1/\sqrt{L_h^c C_g} \).

The poles related to the series resonance are obtained from \( 1 + Z_h(\omega) \), equation (23) is approximated to:

\[
s = -(R_h^c + R_g)C_g \pm \sqrt{(R_h^c + R_g)^2C_g^2 - 4L_h^c C_g}
\]

(24)

The imaginary part of the closed loop system poles corresponds to the resonant frequency. Also, the real part of the poles is correlated to the total damping, \( R_h^c + R_g \), as mentioned in [14]. Therefore, there is a pair of poles that represent the series resonance and can be used to identify instabilities.
V. RESONANCE CHARACTERIZATION

In this section, the low frequency series resonances of an OWPP are characterized. It is useful to identify resonant frequencies in an OWPP since they can destabilize an offshore HVDC converter. To this end, the frequency response of $Z_{eq}^h(j\omega)$ is here used to identify electrical resonances. Due to the complexity of the VSC and offshore grid equations, simplifications are used to obtain analytical expressions of the resonant frequencies.

A. Simplifications of the OWPP impedance model

Fig. 5 shows that the frequency response of a VSC impedance can be simplified to RL equivalents above 100 Hz. The input-impedance of the VSCs was represented in an $\alpha\beta$ frame (see Fig. 4). To achieve this, a reference frame transformation from $dq$ to $\alpha\beta$ was performed using the rotation $s \rightarrow s - j\omega_1$ [6], [15]. For frequencies higher than $\omega_1$, the offshore VSC impedance, $Z_c^h(s - j\omega_1)$, is approximated to:

$$R_c^h = \frac{R_f^h}{1 + k_{p,v}}; \quad L_c^h = \frac{L_f^h}{1 + k_{p,v}} \quad (25)$$

Similarly, the WT VSC impedance, $Z_c^w(s - j\omega_1) = 1/Y_c^w(s - j\omega_1)$, is approximated to:

$$R_c^w = R_f^w + (\alpha_f + \alpha_c)L_f^w; \quad L_c^w = L_f^w \quad (26)$$

The previous simplifications do not consider the VSCs as active elements since $R_c^h$ and $R_c^w$ are positive for all frequencies.

Fig. 6 shows the equivalent model of the HVDC-connected OWPP with the simplified VSC and cable models. The capacitor $C_{ec}$ represents the export cable capacitance. The inductive and resistive components of the export cable are small enough to be combined with the RL equivalent of the transformers and the HVDC converter. Also, the collector cables are removed because their equivalent inductance and capacitance are small and only affect the response at high frequencies, which are not considered in this study.

When the collector cables are removed, the aggregation of WTs is reduced to a combination of parallel circuits independent to the collector system topology. Fig. 7 shows the OWPP model under this scenario, which is equivalent to the model in Fig. 4. The parameters of the aggregated model are defined as follows:

- $R_t^h$ and $L_t^h$ are the RL values of the collector transformers.
- $R_{tr,a}^w$ and $L_{tr,a}^w$ are the RL values of the aggregated WT transformers:

$$R_{tr,a}^w = R_{tr}^w/N; \quad L_{tr,a}^w = L_{tr}^w/N \quad (27)$$

where $N$ is the number of WTs and $R_{tr}^w$ and $L_{tr}^w$ are the RL values of one WT transformer.
- $R_{c,a}^w$ and $L_{c,a}^w$ are the RL values of the aggregated WT converters:

$$R_{c,a}^w = R_c^w/N; \quad L_{c,a}^w = L_c^w/N \quad (28)$$

- $C_{f,a}^w$ is the equivalent capacitance of the aggregated WT low pass filters:

$$C_{f,a}^w = C_f^w \cdot N \quad (29)$$

where $C_f^w$ is the capacitance of one WT low pass filter.
Additionally, the simplification of the collector cables slightly shifts the series resonance from 459 Hz to 497 Hz. In light of these results, it can be concluded that the simplified frequency response represents a good approximation for low frequency resonances in the range of 200 ~ 1000 Hz.

B. Analytical expression for the series resonant frequency

The expression of the lowest series resonant frequency of $Z_{eq}^h$ is obtained for no-load operation and when WTs are connected. The resistances are neglected as they only have a damping effect on resonance (i.e. they barely modify the resonant frequency).

In no-load operation, the WTs are not connected and the contribution of the collector system at low frequencies is negligible. Therefore, the OWPP impedance $Z_{eq}^h$ in (15) is equivalent to an LC circuit with a resonant frequency:

$$f_{res}^{load} = \frac{1}{2\pi \sqrt{L_c^h C_{cc}^h}}$$  \hspace{1cm} (30)

The lowest series resonant frequency when WTs are connected has been obtained following an algebraic calculation using Fig. 7:

$$\begin{align*}
  f_{res}^{cut} &= \frac{1}{2\pi} \sqrt{b - \sqrt{b^2 - 4ad}} \\
  a &= C_{cc}^h L_{c,a}^h L_{tr,a}^h + L_{c,a}^w L_{tr,a}^w C_{cc}^w \\
  b &= C_{cc}^h L_{c,a}^h (L_{c,a}^h + L_{tr,a}^h + L_{tr,a}^w) + \frac{C_{cc}^h L_{c,a}^w L_{tr,a}^h (L_{c,a}^h + L_{tr,a}^h)}{L_{c,a}^w L_{tr,a}^w} \\
  d &= L_c^h + L_c^w + L_{tr,a}^h + L_{tr,a}^w
\end{align*}$$  \hspace{1cm} (31)

Expressions (30) and (31) are employed to calculate the frequencies where the total system damping is evaluated to determine stability.

VI. VOLTAGE STABILITY ANALYSIS

The modified positive-net-damping criterion was applied to analyze the impact of electrical series resonances in the voltage stability of an HVDC-connected OWPP. The effects of the offshore HVDC converter control and the OWPP configuration are considered in the study. For completeness, the root locus of the system and time-domain simulations in PSCAD/EMTDC are used to confirm the results.

The cable model simplifications considered in the resonance characterization are used in the stability analysis given that the low frequency response is well-represented and the damping contribution from the cable resistances can be neglected. However, the VSC simplifications in (25) and (26) are not considered, because the converters are not represented as active elements. The system is analyzed in no-load operation and when WTs are connected based on the OWPP described in Appendix B.

A. No-load operation

In no-load operation, the positive-net-damping stability criterion only includes the damping contribution of the offshore converter, $R_c^h$, because the export and collector cables are passive elements with a small resistance and thus can be neglected (i.e. $R_c = 0$). Therefore, condition (21) is reduced to $R_c^h (\omega_{res}) > 0$, which is equivalent to analyzing the passivity of the HVDC converter control at a resonant frequency.

Stability is ensured if the electrical series resonance is located in a frequency region with positive resistance. This region is determined using the zero-crossing frequencies of $R_c^h$ (i.e. $R_c^h(\omega) = \text{Re}\{Z_c^h(\omega)\} = 0$) in (5). The two following solutions are obtained:

$$\begin{align*}
  \omega_{cut1} &= \omega_1 = 2\pi(50) \\
  \omega_{cut2} &= \frac{\omega_c}{1 - k_{i,v} L_f^h} \\
  k_{i,v} L_f^h &= \frac{1}{R_c^h(1 + k_{i,v})}
\end{align*}$$  \hspace{1cm} (32)

When $\omega_{cut2} < 0$, the only zero-crossing frequency considered is 50 Hz and $R_c^h$ is negative for $\omega > 2\pi(50)$. Therefore, the converter is always unstable for resonant frequencies above 50 Hz. If $\omega_{cut2} > 0$, then $R_c^h$ is negative for $2\pi(50) < \omega < \omega_{cut2}$ and positive for $\omega > \omega_{cut2}$. In this case, the converter is stable for frequencies higher than $\omega_{cut2}$ since the resonance is located in a positive-resistance region. Thus, the offshore HVDC converter is stable when $R_c^h$ has two zero-crossing frequencies ($\omega_{cut2} < 0$ and $\omega_{res} > \omega_{cut2}$). The following inequalities are obtained by combining (30) and (32):

$$\begin{align*}
  \omega_{cut2} > 0 &\Rightarrow R_c^h(1 + k_{i,v}) - k_{i,v} L_f^h > 0 \\
  \omega_{res} > \omega_{cut2} &\Rightarrow \omega_c^h(1 + k_{i,v})^2 - 2R_c^h L_f^h(1 + k_{i,v})k_{i,v} = -\omega_c^h + L_f^h L_c^h C_{cc}^h(1 + k_{i,v}) + k_{i,v} L_f^h > 0
\end{align*}$$  \hspace{1cm} (33)

Fig. 9 shows the stability area ($R_c^h(\omega_{res}) > 0$) defined by (33) as a function of the control parameters of the offshore HVDC converter, $k_{p,v}$ and $k_{i,v}$, and the export cable length, $l_c$. It is observed that when the cable length increases the stable area is reduced.

Fig. 10 shows the root locus of the low frequency resonant poles for parametric variations of $k_{p,v}$, $k_{i,v}$, and $l_c$. It should be emphasized that these poles are not complex conjugate due to the transformation of the VSC input impedance from a synchronous dq to a stationary $\alpha\beta$ reference frame, which introduces complex components. The increase of cable length moves the resonance to lower frequencies since $C_{cc}$ increases. As $k_{p,v}$ increases, the resonance shifts to higher frequencies given that $L_c^h$ in (25) decreases. Changes in $k_{i,v}$ do not affect the resonant frequency. The system becomes unstable when one of the resonant poles moves to the positive side of the real axis; this is equivalent to have a negative damping. It can be observed that the stability conditions of the resonant poles agree with the stable areas shown in Fig. 9.

Figs. 11 and 12 show examples of stable and unstable cases when $k_{i,v}$ is modified. The intersection between $Z_c^h$
and \( Z_g \) (i.e. \( 1/|M(j\omega)| = |N(j\omega)| \)) approximately determines the series resonant frequency, as defined in (18). When the system is stable the resonant frequency is located in a positive-resistance region of \( Z_{hc} \), as shown in Fig. 11a. Also, following the Nyquist criterion, the Nyquist curve encircles \((-1, 0)\) in anti-clockwise direction and the open loop system does not have unstable poles. Therefore, the system is stable as it does not have zeros with positive real part. Although the ac voltage control can be designed to ensure stability, all the poles have a low damping. This slows down the dynamic response, as shown in Fig. 11c, which is not acceptable for the operation of the offshore converter.

When the system is unstable the resonant frequency is located in the negative-resistance region of \( Z_{hc} \), as shown in Fig. 12a. Following the Nyquist criterion, the Nyquist curve encircles \((-1, 0)\) in clockwise direction and the open loop system does not have unstable poles. Therefore, the system is unstable because the total number of zeros with positive real part is 1. In Fig. 12c, the voltage at POC shows oscillations at 309 Hz due to the resonance instability identified in Fig. 12a.

### B. Connection of Wind Turbines

When the WTs are connected to the offshore ac grid, the WT converters modify the low frequency resonance location and the total damping. The stability conditions are discussed,
but the expressions for the zero-crossing frequencies of $R_T$ are not obtained analytically due to the complexity of the system.

Fig. 13 shows the stable area defined by $R_T(\omega_{res}) > 0$. There is a significant increase of the stable region when the WTs are connected. Therefore, the ac control parameters can be modified for a larger range of values to improve the dynamic response without compromising stability.

Fig. 13. Stable area of offshore HVDC converter as a function of $k_{p,v}$ and $k_{i,v}$ and the number of connected WTs (the stable and unstable examples of Fig. 15 and 16 are marked with a circle).

Fig. 14 shows the root locus of the low frequency resonant poles for different ac voltage control parameters and number of WTs ($N = 80$). The connection of WTs improves the resonance stability because the associated poles move to the left hand side of the real axis and increase the damping of those low frequency modes. This damping contribution of the WTs is also mentioned in [2]. The stability conditions of the resonant poles agree with the stable area shown in Fig. 13. Also, the resonance moves to higher frequencies when $k_{p,v}$ and the number of WTs increases, as shown in Fig. 14.

Figs. 15-17 describe two situations where the ac voltage control is designed to have a fast dynamic response (e.g. $k_{p,v} = 1$ and $k_{i,v} = 500$) and the number of WTs decreases from 40 to 20. When all the WTs are connected, the offshore converter is stable because the resonance is located in a positive-resistance region, as shown in Fig. 15a. The converter introduces a negative resistance at the resonant frequency, but

Fig. 15. Stable example when 40 WTs are connected, $k_{p,v} = 1$ and $k_{i,v} = 500$.

Fig. 16. Unstable example when 20 WTs are connected, $k_{p,v} = 1$ and $k_{i,v} = 500$. 
the total damping is compensated by \( R_g \), as shown in Fig. 15b. When the number of WTs reduces to 20 the offshore converter becomes unstable since the resonance lies in the negative-resistance region, as shown in Fig. 16a. In this case, \( R_g \) cannot compensate \( R_{es} \), as shown in Fig. 16b. Also, the Nyquist curve agrees with the positive-net-damping criterion in both situations (Figs. 15c and 16c). In Fig. 17, the instantaneous voltages at POC show oscillations at 444 Hz when the number of WTs is reduced at 1 s; this is due to the resonance instability identified in Fig. 16a.

The variation of connected WTs can be caused by switching configurations during commissioning phases or during outages due to maintenance or contingencies [3]. As shown by the previous examples, a sudden reduction in the number of WTs should be carried out with care as this can lead to instability. Active damping can be implemented as a virtual resistor in the offshore HVDC converter to compensate the negative resistance introduced by the ac voltage control for all operational states. This will allow a design of the ac voltage control to have a fast dynamic response without compromising the stability.

![Instantaneous and RMS voltages at POC](image)

**VII. CONCLUSION**

Instabilities in BorWin1 have increased interest in electrical resonance interactions in HVDC-connected OWPPs. The CIGRE Working Groups suggest that series resonances can be found in the range of a few hundred Hz. These resonances can interact with the offshore HVDC converter control leading to system instability.

This paper has reformulated the positive-net-damping criterion to define the conditions of stability of an HVDC-connected OWPP as a function of the ac voltage control parameters of the HVDC converter and the configuration of the OWPP. The modified criterion is evaluated for electrical series resonances based on the phase margin condition. This reduces the complexity of the stability analysis. In addition, expressions for the low frequency resonance are obtained from simplified VSC and cable models.

Risk of detrimental resonance interaction increases in no-load operation and when a limited number of WTs are connected. This is due to the poor damping exhibited by the series resonance of the offshore grid and the resonance location at the lowest frequencies. The HVDC converter reduces the total damping at the resonant frequency if the control is designed to have a fast dynamic response. Resistive elements or active damping are necessary to compensate the negative resistance of the converter control for all possible operational states and to allow a fast dynamic response.

**APPENDIX A**

If \( R_{es}^2(\omega) \ll X_{es}^2(\omega) \) and \( R_g(\omega) \ll X_g(\omega) \), the loop transfer function is approximated as \( L(j\omega) \approx X_g^2(\omega)/X_{es}^2(\omega) \) and its derivative as a function of \( \omega \) is:

\[
\frac{d[L(j\omega)]}{d\omega} \approx \frac{1}{|X_g(\omega)|^3} \left| \frac{d[X_g^2(\omega)]}{d\omega} \right| \frac{|X_g^2(\omega)|}{|X_{es}^2(\omega)|^2} \left| \frac{d[X_{es}^2(\omega)]}{d\omega} \right| \frac{|X_{es}^2(\omega)|}{|X_g^2(\omega)|} \left| \frac{d[X_g^2(\omega)]}{d\omega} \right|
\]

(34)

Specific conditions for \( \frac{d[L(j\omega)]}{d\omega} \) can be defined depending on the offshore grid and HVDC converter impedances:

- If the offshore grid impedance is inductive, \( X_g > 0 \), and the HVDC converter impedance is capacitive, \( X_{es}^c < 0 \):
  \[
  \left| \frac{d[X_g^2(\omega)]}{d\omega} \right| > 0 \quad \text{and} \quad \left| \frac{d[X_{es}^2(\omega)]}{d\omega} \right| > 0
  \]

- If the offshore grid impedance is capacitive, \( X_g < 0 \), and the HVDC converter impedance is inductive, \( X_{es}^c > 0 \):
  \[
  \left| \frac{d[X_g^2(\omega)]}{d\omega} \right| < 0 \quad \text{and} \quad \left| \frac{d[X_{es}^2(\omega)]}{d\omega} \right| < 0
  \]

**APPENDIX B**

**OWPP DESCRIPTION**

A 480 MW OWPP is considered in this study. A total number of 80 WTs is distributed in 16 strings of 5 units.

**Offshore HVDC VSC:** MMC-VSC; rated power, 560 MVA; rated voltage, 320 kV; arm inductance, \( L_{arm} = 183.7 \) mH.

**Offshore HVDC transformer:** 2 units in parallel; rated power, 280 MVA; rated voltages, 350 kV/220 kV; equivalent inductance and resistance at 220 kV, \( L_{tr}^2 = 99.03 \) mH, \( R_{tr}^2 = 0.86 \) Ω.

**Export cables:** 2 cables in parallel; length, \( l_{ec} = 10 \) km; equivalent lumped parameters per cable, \( L_{ec} = 4 \) mH, \( R_{ec} = 0.32 \) Ω, \( C_{ec} = 1.7 \) μF.

**Collector transformers:** 4 units in parallel; rated power of each unit, 140 MVA; rated voltages, 220 kV/33 kV; equivalent inductance and resistance of the transformers at 220 kV, \( L_{tr}^2 = 20.63 \) mH, \( R_{tr}^2 = 0.22 \) Ω.

**WT transformers:** rated power, 6.5 MVA; rated voltages, 33 kV/0.9 kV; equivalent impedance and resistance at 33 kV, \( L_{tr}^2 = 31 \) mH, \( R_{tr}^2 = 1.46 \) Ω.

**WT grid side VSC:** 2-level VSC; rated power, 6.5 MVA; rated voltage, 0.9 kV; coupling inductance, \( L_{tr}^2 = 50 \) μH; coupling resistance, \( R_{tr}^2 = 0.02 \) mΩ; equivalent capacitance of high frequency filter, \( C_T = 1 \) μF; low pass filter bandwidth, \( \alpha_f = 50 \); current control bandwidth, \( \alpha_C = 1000 \).

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