Compensator models based on block-oriented neural networks

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Abstract. Block-oriented neural networks represented as a cascade structure of linear dynamic circuits and inertialess nonlinearities are used for the non-linear compensator synthesis. The behavioral models of these neural networks describe the mapping of the set of input signals into the set of output signals. A block-oriented architecture is convenient for building the compensator structure inverse to the structure of the distorting system. Under the stated approach, the compensator model is simpler than universal models, for instance, the Volterra series. This fact is demonstrated when modelling a non-linear compensator based on the neural Hammerstein network. This compensator is applied for the suppression of non-linear distortion in the digital communication channel described by the Wiener model.

1. Introduction

In cases of complex devices and the absence of information sufficient for device modelling at the component level, an operator approach to mathematical modelling is used. This approach consists in establishing a relationship between the sets of input and output signals. The object of modelling is imagined as a “black box” with an available input and output. A mathematical model uniquely maps input signals into output ones.

There are different universal forms of models such as functional polynomials, regression models and neural networks [1, 2]. Functional polynomials and regression models are linear-in-the-parameter forms that is why their estimated parameters are globally optimal. It is the essential advantage of these models. However, the universality of polynomial and regression forms results in their complexity. Neural networks can overcome this disadvantage due to the wide variety of the network structures. Nevertheless, it is worth noting many local optimums resulting from evaluating the network parameters. The reason of this fact is the nonlinearity of neural networks with respect to their parameters. In spite of this disadvantage, neural networks succeed in non-linear devices modelling [1, 2].

2. Approach to behavioural modelling

The behavioural modelling of a device is based on building a mathematical model whose parameters are determined with the sets of input and output signals. Under this approach the device operator \( F \), that maps the set \( X \) of input signals into the set \( Y^o \) of output signals:

\[
Y^o = F[X],
\]
is described by another operator $F_e$, establishing a relationship between the sets of input and output signals with some error. This can be written as:

$$y(n) = F_e[x(n)],$$

where $x(n)$ is the input signal from set $X$; $n$ is the normalized discrete time; $y(n)$ is the output signal of a mathematical model embodying the operator $F_e$. The model parameters result from solving the approximation problem written as:

$$\|y''(n) - F_e[x(n)]\| \rightarrow \min_C,$$

where $y''(n)$ is the output signal from the set $Y''$, $C$ is the parameters vector of the model.

The norm in expression (1) is considered to be uniform or mean-square.

Among neural networks acting as the part of mathematical models, recurrent neural networks can be distinguished. In this group of neural networks one can highlight the following main types of structures [3]:

– fully recurrent networks (the Real Time Recurrent Network (RTRN));
– partially recurrent networks (the Elman structure, the Jordan structure, the Recurrent Multi-Layer Perceptron (RMLP));
– state-space networks;
– cellular neural networks;
– networks with dynamic neurons and static feedforward;
– block-oriented neural networks.

The last mentioned type of recurrent neural networks is known for the separation of a total structure into units represented, for instance, as linear dynamic circuits and memoryless nonlinearities [2]. This property is convenient for the separated adjustment of the memory length and the nonlinearity power on designing a non-linear dynamic model.

Block-oriented neural networks are useful when modelling non-linear compensators and filters when knowing the block structures of non-linearly distorting systems. In other words, if we know the block-oriented model of a non-linearly distorting system, we can build the inverse block-oriented model of a compensator for suppressing interfering nonlinearity.

3. Block-oriented compensator models for digital communication channels

The communication channels can be described by non-linear block-oriented models since every channel includes linear dynamic filters and the power amplifier producing non-linear dynamic distortion. This distortion means the amplitude and phase fluctuations of the transformed signal envelope, as well as spectral extension beyond the current channel passband to the adjacent channel passband. The reason of the power amplifier nonlinearity is processing of broadband signals in regime next to saturation to gain the maximum performance factor and the output power [4, 5].

Let us consider the non-linear dynamic model of a digital communication channel in the form of the Wiener structure depicted in Figure 1 [4, 5]. The block of memoryless nonlinearity (N) is followed by the block of a linear dynamic circuit (LDC). The investigated Wiener structure is shown in Figure 2 with denoted parameters $b_1 = 1.0119 - 0.7589j$, $b_2 = -0.3796 + 0.5059j$, $d_1 = 1$, $d_2 = 0.2$, $d_3 = 0.1$, the delay element $T$, as well as different signals including input $\hat{z}(n)$ and output $x(n)$ signals, where ",," is the sign of complexity. All the signals are the complex envelopes of corresponding modulated signals. The Wiener model of the communication channel is subjected to 8PSK- and 4QAM-signals.

As follows from the analysis of Figure 1, the compensator model is desirable to design as the Hammerstein structure that comprises the N block and the LDC block arranged in order inverse to the Wiener structure. The Hammerstein structure is shown in Figure 3. The compensator model is implemented as the neural Hammerstein network depicted in Figure 4. Nonlinearity is represented by a
perceptron network, the linear dynamic circuit is a discrete recurrent system.

Figure 1. The Wiener structure

Figure 2. The Wiener structure of the investigated digital communication channel

Figure 3. The Hammerstein structure

Figure 4. The neural Hammerstein structure of the designed compensator.

The input signal of this structure is a vector:

\[ \begin{bmatrix} \dot{x}_0(n), \dot{x}_1(n), \ldots, \dot{x}_m(n) \end{bmatrix} = \begin{bmatrix} 1, \dot{x}(n), \dot{x}(n-1), \ldots, \dot{x}(n-(m-1)) \end{bmatrix}. \]  \hspace{1cm} (2)

The mathematical model of the represented neural Hammerstein network is given by equation:
\[ \hat{y}(n) = \sum_{b=0}^{n_b} b \, n_{\varepsilon}^{(2)}(n - r_b) - \sum_{r_{\varepsilon} = 1}^{n_{\varepsilon}} a_{\varepsilon} \hat{y}(n - r_{\varepsilon}), \]

where:

\[ n_{\varepsilon}^{(2)}(n) = \sum_{k=0}^{I} c_k n_{\varepsilon}^{(0)}(n), \quad n_{\varepsilon}^{(0)}(n) = 1, \quad n_{\varepsilon}^{(1)}(n) = G(\hat{u}_{\varepsilon}^{(1)}(n)), \]
\[ \hat{u}_{\varepsilon}^{(1)}(n) = \sum_{l=0}^{m} w_{\varepsilon l} \hat{x}(n), \quad k = 1, 2, ..., I, \quad \hat{x}_0(n) = 1, \]

\( G \) is a sigmoid function (hyperbolic tangent); \( I \) is the number of neurons \( (I = 3) \); \( (m-1) \) is the memory length \( (m = 2) \), \( R_b = 1 \), \( R_{\varepsilon} = 1 \).

The suppression quality of the non-linear communication channel distortion is evaluated by means of the root-mean-square error \( \varepsilon \) calculated from equation:

\[ \varepsilon = \frac{1}{998} \sqrt{\frac{\sum_{n=1}^{998} [\hat{y}(n) - \hat{\xi}(n)]^2}{}}. \]

The results of non-linear compensation gained by the neural Hammerstein network are compared with the appropriate results obtained in using the following compensator models:

- the two-layer perceptron network:

\[ \hat{y}(n) = G \left( c_0 + \sum_{i=1}^{I} c_i G \left( \sum_{j=0}^{m} w_{ij} \hat{x}(n) \right) \right), \]

where the input signal is a vector \( (2) \); \( G \) is a sigmoid function (hyperbolic tangent); \( I \) is the number of neurons \( (I = 5) \); \( (m-1) \) is the memory length \( (m = 5) \);

- the Volterra polynomial:

\[ \hat{y}(n) = \sum_{I}^{I_1} \sum_{I_2}^{I_2} ... \sum_{I_{m+1}}^{I_{m+1}} \sum_{I_{m+2}}^{I_{m+2}} ... \sum_{I_m}^{I_m} \hat{\xi}_{I}(n) \hat{x}^{(n)}(n-1) ... \]
\[ ... \hat{x}_{I_{m+1}}^{\left(n - \frac{m}{2}\right)} \hat{x}_{I_m}^{\left(n - \frac{m}{2}\right)} ... \hat{x}_{I_{m+1}}^{\left(n - \frac{m}{2}\right)} \]

where \( * \) is the sign of complex conjugation; \( I = I_1 + I_2 + ... + I_{m+2} + I_{m+3} + ... + I_m \) is the polynomial degree \( (I = 3) \); \( m/2 \) is the memory length \( (m = 10) \).

Error \( \varepsilon \) obtained under the effect of 4QAM-signal on the communication channel and the number \( Q \) of parameters in the compensator models are represented in table 1.

**Table 1.** The error of compensation and the number of parameters in the compensator models.

| Model                  | \( \varepsilon \)     | \( Q \)  |
|------------------------|------------------------|--------|
| Hammerstein network    | \( 0.2*10^{-3} \)      | 16     |
| Two-layer perceptron   | \( 0.6*10^{-3} \)      | 36     |
| Volterra polynomial    | \( 0.2*10^{-3} \)      | 286    |
For the clarity of presentation, the output signal constellations of the different compensator models, such as the neural Hammerstein network \((I = 3, R = R_v = 1)\), the two-layer perceptron network \((I = 5, m = 5)\) and the Volterra polynomial \((I = 3, m = 10)\) are shown in Figure 5, a, b, c, respectively. Uniform errors \(\delta(n)\), calculated from equation \(\delta(n) = y(n) - \hat{y}(n), n \in [3, 1000]\) under the different compensator models are depicted in the centre of the complex planes in Figure 5, a, b, c.

![Figure 5](image-url)

**Figure 5.** The output signal constellations of the different compensator models: a – the neural Hammerstein network; b – the two-layer perceptron network; c – the Volterra polynomial.

As it is seen from table 1 and Figure 5, the neural Hammerstein model of a digital compensator ensures the highest accuracy of the non-linear distortion compensation for the digital communication channel with the Wiener model. This result is obtained when comparing the offered compensator model with other models such as the perceptron network and the Volterra polynomial. Moreover, the neural Hammerstein model proves to be the simplest among represented ones.

The separation of nonlinearity and dynamics is convenient for the adjustment of the nonlinearity power and the memory length of a block-oriented neural model when reaching the assigned accuracy of modelling. This separation facilitates building the mathematical model as being simpler than universal models.

**4. Conclusion**

An approach to modelling of non-linear dynamic devices based on establishing the unique mapping between the sets of input and output signals is evolved by means of expending the application of various neural networks, for instance, block-oriented neural networks.

This type of networks is convenient for the synthesis of non-linear filters and compensators since the block-oriented structures of these devices are arranged inversely to the structures of distorting systems. This aspect allows a priori to set the block schemes of synthesised behavioural models. The separation of nonlinearity and dynamics contributes to creating models simpler than universal ones. It is demonstrated based on modelling non-linear compensators used for the suppression of non-linear distortion in the communication channel.

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