Constraining SUSY GUTs and Inflation with Cosmology

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Abstract. In the framework of Supersymmetric Grand Unified Theories (SUSY GUTs), the universe undergoes a cascade of symmetry breakings, during which topological defects can be formed. We address the question of the probability of cosmic string formation after a phase of hybrid inflation within a large number of models of SUSY GUTs in agreement with particle and cosmological data. We show that cosmic strings are extremely generic and should be used to relate cosmology and high energy physics. This conclusion is employed together with the WMAP CMB data to strongly constrain SUSY hybrid inflation models. F-term and D-term inflation are studied in the SUSY and minimal SUGRA framework. They are both found to agree with data but suffer from fine tuning of their superpotential coupling ($\lambda \lesssim 3 \times 10^{-5}$ or less). Mass scales of inflation are also constrained to be less than $M \lesssim 3 \times 10^{15}$ GeV.

Keywords: SUSY GUTs, Supersymmetric hybrid inflation, Cosmic strings, CMB.

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INTRODUCTION

The Supersymmetric Grand Unified Theories (SUSY GUTs) are a motivated framework in which to describe the early universe physics. The symmetries of the Standard Model (SM) $G_{SM} \equiv SU(3)C \times SU(2)L \times U(1)Y$ are then supposed to be obtained at low energies from a larger gauge group $G_{GUT}$ through a cascade of Spontaneous Symmetry Breaking (SSB). According to the Kibble mechanism, topological defects should form during the phase transition associated with these SSBs. However, it is well known that the formation of domain walls or monopoles would rapidly dominate the dynamics of the universe, in contradiction with observations. The situation for Cosmic Strings (CS) however is very different since a network of such objects reaches a phase of scaling where their relative energy stays constant with time. With the current observational precision however, they have not yet been detected, for example in the WMAP CMB data.

In this review, we will first quantify how probable is the cosmic string formation within the SUSY GUT framework in a large number of motivated and realistic models. This class of models is chosen to be in agreement with proton decay data, observed neutrino oscillations, baryon asymmetry and CMB anisotropies. Any realistic model must include a phase of inflation and we need to know whether the formation of strings occurs before or after the last inflation phase. Indeed, in the last case only, they can have an observable effect on cosmology. This analysis allows us to conclude that cosmic string are a highly generic prediction of SUSY GUTs.

In a second section, we confront this prediction with the WMAP CMB data (first year release). The low allowed contribution from strings to CMB anisotropies is employed to constrain strongly the parameters of the two motivated inflationary models: F-term and D-term SUSY hybrid inflation. We will show that all parameters can be constrained, including superpotential coupling constants and mass scale of inflation. This allows us to re-analyse the naturalness of these models.

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GENERICITY OF COSMIC STRING FORMATION IN SUSY GUTS

Model building

A model for the early universe, to be realistic, should explain \ref{6,7} the observed oscillation of neutrinos, be in agreement with lower bounds on proton decay, explain the baryon/anti-baryon asymmetry in the universe, and be consistent with the most recent CMB data. SUSY GUTs offer a coherent framework for constructing such models, the unification being suggested by the running of SM coupling constants and allowing one to reduce the number of free parameters of the SM. Supersymmetry is motivated since it offers an elegant solution to the hierarchy problem and it allows a unification at sufficiently high energy ($M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV) for the dimension 6 operators of proton decay to be suppressed. Within SUSY GUTs, the most economical way to protect the proton from operators of dimension 4 (exchange of sfermions) is by keeping unbroken at low energy the R-parity. This R-parity is a $Z_2$ symmetry defined by

$$R = (-1)^{2S + 3(B-L)},$$

where $S$ is the spin, $B$ is the baryon number and $L$ is the lepton number of a particle. We can easily check that if this parity is conserved, the decay of the Lightest Supersymmetric Particle (LSP) is forbidden, introducing a good candidate for dark matter without any new assumption. This R-parity can be preserved by breaking a GUT group containing the $U(1)_{B-L}$ symmetry with an even-charged representation \ref{8}. It will be necessary to identify how this abelian symmetry is embedded in $G_{\text{GUT}}$ to make sure no SSB patterns incompatible with proton decay are considered, i.e. R-parity is kept unbroken. For model builders, this also means that the choice of Higgs representations employed has to be done carefully.

The observed neutrino oscillations \ref{10} require that these particles are massive and thus that we go beyond the SM. We then need to introduce a right-handed (RH) neutrino, singlet under the SM gauge group. The most economical way to generate small mass neutrinos is via a see-saw mechanism, i.e. generate a large Majorana mass $M_N$ of the GUT scale and a Dirac mass of the electroweak scale $M_Y$. The mass eigenstates are then of the order of $M_N$ and $M_Y^2/M_N$ which allows one to explain current limits on light neutrino masses. The Majorana mass term is usually generated through the breaking of the $U(1)_{B-L}$ symmetry within GUTs. This will induce some constraints on the GUT groups considered in our study since we will assume that they contain the $U(1)_{B-L}$ symmetry.

The current CMB data clearly favor an inflationary origin for generation of the primordial anisotropies (even if a subdominant contribution from cosmic strings is not excluded, see next section). A phase of inflation in the early universe history is thus almost necessary to explain these data, or to solve the monopole problem and other problems of the standard model of cosmology. Within our framework, the most natural model of inflation is the one of hybrid inflation. It is based on the superpotential \ref{11}

$$W_F = \kappa S (\Phi_+ \Phi_- - M^2).$$

(1)

It couples an inflaton field $S$ to a pair of GUT Higgs fields $\Phi_+$ and $\Phi_-$ non trivially charged under a symmetry $G$. The model possesses only two parameters : a coupling constant $\kappa$ and a mass scale $M$. The fields $\Phi_{\pm}$ must belong to non trivial complex conjugate representations of $G$ and the dynamics of the system induce a Vacuum Expectation Value (VEV) for these fields, which breaks the symmetry $G$. This induces a waterfallow that ends the inflationary phase. In F-term hybrid inflation, this symmetry $G$ can be part of $G_{\text{GUT}}$ but in the case of D-term inflation, $G = U(1)$ and this abelian factor cannot be a subgroup of a non-abelian group, since the model requires a non-vanishing Fayet-Iliopoulos term $\xi$. In this section we will assume a GUT model based on a simple group $G_{\text{GUT}}$ to unify the coupling constant at high energy and thus restrict ourselves to F-term inflation. This model is motivated in our framework since

- no new fields have to be introduced (except the inflaton singlet) since GUT Higgs fields are already present. Note that this singlet can also be a Higgs singlet already present in the theory.
- the form of Eq \ref{1} is the most general that is renormalizable, invariant under $G$ and under an R-symmetry.
- the potential is naturally flat (even perfectly at tree level) and the radiative corrections don’t spoil the model but introduce a lift in the potential that helps the slow roll.
- F-term inflation is known to become unstable under Supergravity (SUGRA) corrections; but in this model, the dangerous terms happen to cancel in a minimal SUGRA. Moreover, the value of the inflaton field in the observable range stays far below the Planck scale \ref{12}. This shows that the SUGRA corrections may be safely neglected.
- the model has been introduced to explain the level of CMB anisotropies with a non fine tuned coupling $\kappa (10^{-2})$. This major motivation will however be re-analyzed in the next section.

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The consequences of this choice of inflation model on our study are important: inflation is then assumed to occur before one of the SSB of GUT patterns. We remind the reader that this inflation has to occur after the last formation of monopoles to avoid contradiction with cosmology.

The baryon/anti-baryon asymmetry can naturally be explained through a first stage of leptogenesis in our framework. This model is able to explain the observed level of asymmetry and does not require any new ingredient, if we have already introduced RH neutrinos for see-saw. The lepton asymmetry is then generated by the decay of RH neutrinos. The baryon asymmetry is generated subsequently through sphaleron transitions. Since a phase of inflation would wash out any existing asymmetry, this phase has to occur after the end of inflation. The standard model of leptogenesis is called thermal, which means that the decay of RH neutrinos occurs when the temperature of the universe drops below their mass. However, RH neutrino masses are constrained by the light neutrino masses and the reheating temperature is constrained by nucleosynthesis and gravitino production. This induces a tension in the parameter space for the thermal model to take place. One way to get out is to consider a small extension of the model: non thermal leptogenesis.

In this model to take place. One way to get out is to consider a small extension of the model: non thermal leptogenesis. We will assume it in this study since it requires two ingredients we already assumed: an F-term hybrid inflation and the see-saw mechanism. This model assumes that the right-handed neutrinos get a mass through a coupling with one of the \( \Phi \) Higgs fields. At the end of inflation, the inflaton and Higgs fields start to oscillate and disintegrate into right-handed neutrinos \( S \rightarrow N_R N_R \). Then, for this to be allowed, we just need the mass of the inflaton field to be higher that twice the mass of one right-handed neutrino and no tension exists on the reheating temperature. The consequence of this assumption on our study is to impose the see-saw mechanism and thus the breaking of \( U(1)_{B-L} \) (or a group that contains it) after inflation.

### GUT gauge groups and SSB patterns

We consider in this study only simple GUT gauge groups. There are several constraints on them to be eligible (minimal rank of 4, complex and anomaly free fermionic representations). Only simple groups of rank lower than 8 are considered: \( SU(n) \), with \( n \leq 9 \), \( SO(10) \), \( SO(14) \), and \( E_6 \). Larger groups would mainly generate a lot of similar or identical SSB patterns, and lose a lot of predictability. Moreover, if we require that \( U(1)_{B-L} \subseteq G_{GUT} \), then \( n \geq 7 \).

To write down all possible SSB pattern from \( G_{GUT} \) down to \( G_{SM} \), we must know how the SM gauge group \( G_{SM} \) is embedded inside \( G_{GUT} \) in order not to break part of \( G_{SM} \). We also need to know the possible embeddings of \( U(1)_{B-L} \) in order to know when the R-parity can appear. For example, if we consider \( SO(10) \) broken through the Pati-Salam group \( G_{PS} \equiv SU(4)_C \times SU(2)_L \times SU(2)_R \), the SM can be embedded following

- \( SU(4)_C \supset SU(3)_C \times U(1)_{B-L} \),
- \( Y/2 = I_3 \pm (B - L)/2 \),

where \( Y \), \( (B - L) \) and \( I_3 \) denote respectively the generator of hypercharge, of \( U(1)_{B-L} \) and the diagonal generator of \( SU(2)_R \). As a consequence, the only possible SSB schemes from \( SO(10) \) through \( G_{PS} \) are \[ (2) \]

\[
SO(10) \rightarrow _4 C_2 \rightarrow _R_2 \rightarrow _L_2_\rightarrow _L_2_\rightarrow _R_2 \rightarrow _L_2_\rightarrow _R_2
\]

where all notations are the same as in \[ (3) \]. These schemes must be read from left to right, each arrow being a SSB and each line being a different model. The number over the arrows gives the nature of topological defects (if any) formed during the SSB: 0 for no topological defects, 1 for monopoles, 2 for cosmic strings, and \( \pi' \) for embedded strings. We remind the reader that homotopy group analysis permit us to study the formation of topological defect \[ (5) \]. For a given SSB \( G \rightarrow H \), the vacuum manifold of interest is \( M = G/H \). The topological properties of \( M \) control the formation of topological defects: \( \pi_0(M) \neq 1 \) is the condition for domain wall formation, \( \pi_1(M) \neq 1 \) is the condition for cosmic string formation and \( \pi_2(M) \neq 1 \) is the condition for monopole formation. The condition for embedded string formation \[ (13) \] is similar but apply on \( M_{emb} = G_{emb}/H_{emb} \) where \( G_{emb} \subseteq G \) and \( H_{emb} \subseteq H \). There are additional conditions for their formation, concerning the properties of the defect solution.
The other possible schemes for $SO(10)$ are obtained by breaking $SO(10)$ down to each of the groups that appear in Eq. 2, since we want to consider minimal schemes (only one group is broken at each step) and more direct schemes.

We can see on Eq. 2 that if one now wants to couple the hybrid inflation phase to one of the SSB for each model, only one choice is possible. It must occur at the last SSB, and we can read that this SSB that ends the inflation always produces cosmic strings. This was to illustrate the result of 3 where all groups of rank less than 8 are considered and all known embeddings of the SM have been used to write every possible SSB pattern. Based on $SO(10)$, we find 34 viable models that keep R-parity unbroken at low energy and all of them produce cosmic strings at the end of inflation. Because of the smallness of this group, the mass per unit length of the strings is directly related to the inflation energy scale. When considering larger GUT groups, the number of possibilities increases dramatically but the conclusion remains identical. The formation of topological cosmic strings is unavoidable within our assumptions. If we relax the assumption of thermal leptogenesis, then 98% of $E_8$ based models produce topological strings. We then conclude 3 that cosmic strings are generic and that their mass scale is generically proportional to the energy scale of hybrid inflation. We study in the next section the implication of this result for hybrid inflationary models.

**CONSTRAINTS ON INFLATIONARY PHYSICS**

This second part is motivated by the results of the previous section. We wish to study the consequences of cosmic string formation, generically at the end of the hybrid inflation phase. We will first consider the F-term SUSY hybrid model and study the constraints coming from cosmology on its parameters, namely the coupling constant and the energy scale of inflation. These constraints will come from the fact that a highly subdominant contribution of cosmic strings to the CMB anisotropies is imposed by the current CMB data. We will also consider the D-term SUSY/SUGRA hybrid inflation model since, by construction, the breaking of $G = U(1)$ at the end of inflation must generate cosmic strings. This property of D-term inflation is totally independent of the GUT framework or any other assumption.

**F-term Supersymmetric hybrid inflation**

As introduced in the previous section, F-term hybrid inflation is based on the superpotential

$$W_F = \kappa S (\Phi_+ \Phi_- - M^2),$$

(3)

where $\kappa$ is the coupling constant between the inflaton and the Higgs fields and $M$ is the mass scale parameter. $S$, $\Phi_\pm$ are chiral superfields, and $S$ is a singlet under a group $G$ while $\Phi_\pm$ belong to non trivial conjugate representations of $G$. In the global SUSY framework, we can derive the tree level scalar potential

$$V_F(\Phi_+, \Phi_-, S) = \kappa^2 |M^2 - \Phi_+ \Phi_-|^2 + \kappa^2 |S|^2 (|\Phi_-|^2 + |\Phi_+|^2) + D - \text{terms}.$$  

(4)

At high inflaton values $S \gg S_c = M$ (we assume that chaotic initial conditions for inflation will impose large values for the inflaton field initially), the potential is minimized by the configuration $\langle \Phi_\pm \rangle = 0$ ($\Phi_\pm$ correspond to the scalar components of the chiral superfields $\Phi_\pm$). Thus we obtain a perfectly flat potential which is non vanishing $V_0 = \kappa^2 M^4$.

As a result, SUSY is temporarily broken, and this induces radiative corrections to the tree level potential. The 1-loop contribution can be calculated by the Coleman-Weinberg formulae leading to an effective inflation potential 12

$$V_{\text{eff}}(|S|) = V_0 + [\Delta V(|S|)]_{1\text{-loop}},$$

$$= \kappa^2 M^4 \left\{ 1 + \frac{\kappa^2 N}{32 \pi^2} \left[ 2 \ln \frac{|S|^2 \kappa^2}{\Lambda^2} + (z + 1)^2 \ln (1 + z^{-1}) + (z - 1)^2 \ln (1 - z^{-1}) \right] \right\},$$

(5)

where $N$ is the dimensionality of the representation of $G$ to which $\Phi_\pm$ belongs and $\Lambda$ is a renormalization scale. We have also defined $z \equiv S^2 / S_c^2$. Note that, in the literature, papers often consider only the first term in the radiative correction, corresponding to the limit $z \gg 1$. This turns out to be not consistent since the last 60 e-folds of inflation can occur for values of the inflaton field close to its critical value $z = 1$. What are typical values for $N$? Interestingly enough, this integer is related to the choice of GUT group such that $G \subset G_{\text{GUT}}$ and the SSB pattern. For example, if we consider models based on $SO(10)$, the inflation phase often occurs at the breaking of the $U(1)_{B-L}$ symmetry. This
can be seen on SSB patterns through the Pati-Salam group where \( U(1)_{B-L} \) is readily identified [see Eq. 2]. To protect the R-parity subgroup of \( U(1)_{B-L} \), one needs a pair of \( \mathbf{126} \) and \( \overline{\mathbf{126}} \) to break \( U(1)_{B-L} \). Thus, typically for \( SO(10) \) GUT, \( \mathcal{N} = 126 \). Typical values for \( E_6 \) Higgs representations are \( \mathbf{27} \) and \( \mathbf{351} \).

The end of the inflation phase is triggered by the first of the two following mechanisms:

- the Higgs fields become massive, and \( \langle \phi_+ \rangle = \langle \phi_- \rangle = 0 \) is not stable anymore. This condition is reached at \( z = 1 \).
- the end of the slow roll conditions: \( \epsilon = \mathcal{O}(1) \) or \( \eta = \mathcal{O}(1) \). We can show numerically [12] that this condition is also reached for \( z \approx 1 \).

Once slow roll inflation ends, the dynamics will set the system to the global minimum of the potential of Eq. 4, namely \( \langle S \rangle = 0 \), \( \langle \Phi \pm \rangle = M \). The symmetry \( G \) under which \( \Phi \pm \) are charged is broken. Note that the VEV that is responsible for cosmic string formation is thus given by \( M \).

The contribution from inflation to the CMB temperature anisotropies can be divided into two parts: the scalars and the tensors. They are functions of the inflation potential and the value of the inflaton fields at the considered scale. Hereafter we will focus on the quadrupole anisotropy. At this scale, the contributions are given by the Sachs-Wolfe effect and read

\[
\left( \frac{\delta T}{T} \right)_{Q-\text{scal}} = \frac{1}{4\sqrt{45\pi}} \frac{V^{3/2}(S_Q)}{M_{Pl}^4} \, V'(S_Q),
\]

and

\[
\left( \frac{\delta T}{T} \right)_{Q-\text{tens}} \sim \frac{0.77}{8\pi} \frac{V^{1/2}(S_Q)}{M_{Pl}^2},
\]

where \( M_{Pl} = (8\pi G)^{-1/2} \approx 2.43 \times 10^{18} \) GeV is the reduced Planck mass and \( S_Q \) is the value of the inflaton field corresponding to the quadrupole. This value is another unknown as well as the parameters \( \kappa \) and \( M \).

A first relation between these unknowns comes from the number of e-folds between the quadrupole scale and the end of inflation. This number reads

\[
N_Q = N(S_Q \to S_c) = -\frac{1}{M_{Pl}^2} \int_{S_Q}^{S_c=M} \frac{V(S)}{V'(S)} \, dS.
\]

To solve the horizon problem, we require this number to be \( N_Q \approx 60 \). For a given value of \( \mathcal{N} \) and \( \kappa \), this equation can give a relation between the inflaton value \( S_Q \) and the energy scale \( M \). To completely fix this scale, we need to normalize the total contribution to the COBE measurement. The important point here to understand is that since cosmic strings are formed at the end of inflation, we need to take their contribution into account before normalizing. The contribution of a local Nambu-Goto string network at the quadrupole scale is proportional the mass per unit length \( \mu \),

\[
\left( \frac{\delta T}{T} \right)_{cs} = \alpha G \mu.
\]

In first approximation, \( \mu = 2\pi(h)^2 \) where \( h \) is the VEV of the Higgs field responsible for cosmic string formation. In our case, \( h = M \) and the numerical factor \( \alpha \) is computed through heavy numerical simulations. Recent results obtained in realistic cosmologies give \( \alpha = 9 - 10 \), and thus [12]

\[
\left( \frac{\delta T}{T} \right)_{cs} \sim \frac{9}{4} \frac{M^2}{M_{Pl}^2}.
\]

Therefore, the total quadrupole anisotropy is given by

\[
\left[ \left( \frac{\delta T}{T} \right)_{Q-\text{tot}} \right]^2 = \left[ \left( \frac{\delta T}{T} \right)_{\text{scal}} \right]^2 + \left[ \left( \frac{\delta T}{T} \right)_{\text{tens}} \right]^2 + \left[ \left( \frac{\delta T}{T} \right)_{cs} \right]^2.
\]

The normalization to the COBE measurement must then be done by setting the left-handed side of this equation to \( \langle \delta T/T \rangle_{Q, \text{COBE}} \approx 6.3 \times 10^{-6} \). This allows us to calculate the energy scale \( M(\kappa) \) as a function of the coupling constant and for a given value of \( \mathcal{N} \). This is shown on Fig. [1]
We can also define the fraction of anisotropies generated by cosmic strings

\[ \mathcal{A}_{cs} \sim \left[ \frac{\delta T}{T} \right]_{cs}^2 \left( \frac{\delta T}{T} \right)_{COBE}^2, \]  

which is also represented in Fig. 1.

A recent Bayesian analysis in a 6+2 dimensional parameter space [14] has shown that a cosmic string contribution to the CMB fluctuations \( \mathcal{A}_{cs} \) higher than 14% is excluded by the WMAP 1st year data up to 99% confidence level. This can be translated into a constraint on \( \kappa \) or \( M \),

\[ \kappa \lesssim 1 \times 10^{-6} \times \frac{126}{\mathcal{N}}, \quad M \lesssim 2.5 \times 10^{15} \text{ GeV}. \]  

These relations hold for \( \mathcal{N} = 1, 16, 27, 126, 351 \). The first conclusion [12] is that contrary to what was previously thought, F-term hybrid inflation suffer from fine tuning of its superpotential coupling constant. We are also able to constrain the inflation energy scale, the constraint being robust and independent of the GUT group considered. Interestingly, this work also shows a relation between a measurable quantity (the cosmic string contribution to CMB) and a parameter that is directly related to the fundamental GUT group of the model (the parameter \( \mathcal{N} \)). Future improvement in CMB experiments and the detection of a string contribution could bring new information on high energy physics.

### D-term inflation

Let’s now turn to the D-term inflation model [15, 16] and the constraints on the parameter space imposed by the formation of cosmic strings. Here this formation is not linked to the embedding of the inflation model inside any framework. It is simply due to the fact that D-term inflation is necessarily ended by the breaking of a \( U(1) \) symmetry.

The procedure here is similar to the one of the previous subsection for F-term inflation and we will just give here the guidelines and the results for D-term inflation.

In this model, \( \Phi_{\pm} \) must be charged under a \( U(1) \) symmetry with, let’s say, charges \( \pm 1 \). The inflaton superfields \( S \) remain trivially charged and the model requires also a non-vanishing Fayet-Iliopoulos term \( \xi \). The model is finally based on a superpotential of the form

\[ W_D = \lambda S \Phi_+ \Phi_- \]  

where \( \lambda \) denotes the superpotential coupling. We can show [17, 12] however that the global SUSY analysis for this model is not consistent, since the inflaton field \( S_Q \) can reach plankian values even if the energy of the inflaton potential
stays below the Planck scale. This means that the analysis should be performed in SUGRA. We will hereafter consider a minimal SUGRA, i.e. the minimal structure for gauge kinetic function \( f_i(\Phi) = \delta_{ij} \) and a Kähler potential given by
\[
K = |\phi_-|^2 + |\phi_+|^2 + |S|^2.
\] (14)

Therefore, as it was found in [15], the scalar potential reads
\[
V_{\text{SUGRA}}^D = \lambda^2 \exp \left( \frac{|\phi_-|^2 + |\phi_+|^2 + |S|^2}{M_{Pl}^2} \right) \left[ |\phi_+ \phi_-|^2 \left( 1 + \frac{|S|^4}{M_{Pl}^4} \right) + |\phi_+ S|^2 \left( 1 + \frac{|\phi_+|^4}{M_{Pl}^4} + 3 \frac{|\phi_+ S|^2}{M_{Pl}^4} \right) + \frac{g^2}{2} \left( |\phi_+|^2 - |\phi_-|^2 + \xi \right)^2 \right],
\] (15)

where \( g \) is the gauge coupling of the \( U(1) \) symmetry. Once again, for large values of the inflaton field \( S \gg S_c \) the potential is minimized by \( |\phi_+| = |\phi_-| = 0 \) and we get a perfectly flat potential at tree-level \( V_0 = g^2 \xi^2/2 \). At one loop, during the inflation phase, the effective scalar potential is given by [17]
\[
V_{\text{eff}}^D = \frac{g^2 \pi^2}{2} \left\{ 1 + \frac{g^2}{16 \pi^2} \left[ 2 \ln \frac{|S|^2 \lambda^2}{\Lambda^2} \exp \left( \frac{|S|^2}{M_{Pl}^2} \right) + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1}) \right] \right\},
\] (16)

where \( z = \left[ \lambda^2 |S|^2/(g^2 \xi) \right] \exp(|S|^2/M_{Pl}^2) \). We can show that the end of inflation is reached when \( z \simeq 1 \) and then the breaking of \( U(1) \) and the formation of strings are generated by \( \langle \phi_+ \rangle = 0 \) and \( \langle \phi_- \rangle = \sqrt{\xi} \).

The procedure to compute the mass scale \( M \equiv \sqrt{\xi} \) as a function of \( \lambda \) and \( g \) is similar to the case of F-term inflation and the results are shown in Fig. 2.

**FIGURE 2.** On the left, the cosmic string contribution as a function of the mass scale \( \sqrt{\xi} \). This holds for all studied values of \( g \). On the right, the cosmic string contribution to the CMB temperature anisotropies, as a function of the superpotential coupling \( \lambda \), for different values of the gauge coupling \( g \). The maximal contribution allowed by WMAP is represented by a dotted line. The plots are derived in the framework of SUGRA.

The first conclusion is that D-term inflation can be in agreement with CMB data even if cosmic strings of the GUT scale are formed [17]. We can also see that all parameters of the model are constrained by imposing an upper limit on the cosmic string contribution to the CMB anisotropies. The mass scale possesses the same limit as in the F-term case, at 99% CL,
\[
\sqrt{\xi} \lesssim 2.5 \times 10^{15} \text{ GeV}.
\] (17)

which can be understood by the fact that this limit is equivalent to the limit on the string contribution. The coupling constants of the model are also constrained at 99% of CL,
\[
\lambda \lesssim 3 \times 10^{-5}, \quad g \lesssim 2 \times 10^{-2}.
\] (18)

These results allow us to conclude that as for the F-term case, D-term inflation can be in agreement with CMB data, but the price is a fine tuning on its superpotential coupling constant. Note that contrary to what has been previously
found \([12,17]\), the inflaton field, in SUGRA, stays in the range \(S_Q \in [10^{-3} - 10]\) for typical values of \(g\) and \(\lambda\). Note also that D-term inflation has been recently revisited \([16]\). However, our results remain valid since the modification of the model is of amplitude \(\xi / M^2_{Pl} \lesssim 10^{-6}\).

**CONCLUSION**

In this paper, we have reviewed the formation of topological defects within a large number of SUSY GUT models that are in agreement with major observational constraints from particle physics experiments and cosmology observations. Our goal is to get a probability of cosmic string formation in this framework. The conclusion is that cosmic strings are highly generic objects and their energy is generically of the same order of magnitude as the energy scale of inflation. As a consequence, they are cosmologically relevant and we should continue searching for them in various cosmological observations.

We analyzed in the second section the consequences of this conclusion on two of the main inflationary models: F- and D-term SUSY hybrid inflation. The generic formation of cosmic strings at the end adds an additional contribution to the generation of CMB anisotropies. This un-observed contribution is constrained by CMB observations to be subdominant (\(\lesssim 14\%\)), and is employed to constrain the parameters of the models. We have shown that they can both be in agreement with most recent CMB data but face a fine tuning of their superpotential coupling constant. We also show that while the global SUSY description of F-term inflation can be sufficient, this is not the case of D-term inflation, that must be studied in SUGRA.

Several interesting questions suggested by these results remain open: What is the generic micro-structure of cosmic strings formed in our framework? How could this affect their impact on CMB and the constraint on their contribution? In the case of F-term inflation, is it possible to find a lower limit on the coupling constant to really test the model with future experiments? Is our conclusion for D-term inflation modified if we embed it in a non minimal SUGRA description? What kind of SUGRA superpotentials and Kähler potentials are motivated from high energy physics?

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