KiloHertz QPO and Gravitational Wave Emission as the Signature of the Rotation and Precession of a LMXB Neutron Star Near Breakup

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Received _______________; accepted _______________

submitted to Ap.J (August 15, 2000)

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ABSTRACT

The basic theory of torque free precession (TFP) of the outer crust of a neutron star (NS) as the signature of the approach to NS breakup is a viable explanation of the uniform properties of kHz Quasi-periodic Oscillations (QPO) observed in X-rays emitted by Low Mass X-ray Binary (LMXB) sources. The theory outlined in this paper relates the intrinsic properties of NS structure to the observed kHz frequencies. The range of kHz frequencies and the observed quality factors (Qs) are also explained by this simple dynamical model. A scenario that begins with the melting of the inner crust of an LMXB NS creates the conditions necessary for the generation of kHz QPO. We suggest that a mechanism analogous to that proposed to explain giant glitches in radio pulsars drives the dynamics of variations of the kHz QPO. Furthermore, the theory provides a simple explanation for the high Q of the unique millisecond X-ray pulsar SAX J1808.4-3658, and also explains why it does not exhibit kHz QPO. The theory relates the ratio of the observed kHz frequencies to the ratios of the components of the moments of inertia of the NS, thereby tightly constraining the equation of state (EOS) of NS matter (polytrope index \( n \approx 1.0 \)). The TFP model is in strong contrast to existing models which primarily relate the kHz QPO phenomenon to the physics of gas dynamics near the inner edge of the accretion disk and the transition flow onto the surface of the NS. The TFP theory is consistent with the presence of Lense-Thirring precession of matter orbiting the NS. We suggest the possibility of the direct detection of very low frequency (\( \sim 1 \) kHz) radio waves from magnetic dipole radiation and also predict kHz gravitational wave emission from the LMXB
Sco X-1 that may be detectable by LIGO. The high accretion rates consistent with the predicted GW emission indicate the likely conversion of some LMXBs to maximally rotating Kerr black holes (BH) and further suggest that these systems are progenitors of some gamma-ray bursts (GRB).

Subject headings: QPO: LMXB: NS: EOS: Gravitational Wave: LIGO: GRB: Sco X-1: SAX J1808.4-3658
1. Introduction

KiloHertz (kHz) oscillations in low-mass X-ray binaries (LMXB) were first discovered in observations with the Proportional Counter Array (PCA) onboard the *Rossi X-ray Timing Explorer* \cite{Bradt_1993, Swank_1998}. The kHz Quasi-Periodic Oscillations (QPO) have now been observed in the persistent flux of well over a dozen sources \cite{van_der_Klis_2000} and references therein. These kHz QPO sometimes occur in pairs whose difference frequency is not constant \cite{van_der_Klis_1997, Mendez_1998, Ford_1998} and show properties that are remarkably uniform over a wide range of accretion rates \cite{Psaltis_1998}. Some of these sources also show oscillations in the $\sim 300 - 600$ Hz range that occur during Type I (thermonuclear) bursts \cite{Strohmayer_1998}. Current models for kHz QPOs have explanations related to the inner portions of the accretion disk and the transition flow of matter onto the NS surface \cite{Miller_1998, Cui_2000}. Other alternatives focus on the motions of blobs of matter near the last stable orbit of a NS \cite{Zhang_1997} and propose evidence for Lense-Thirring precession of orbiting matter \cite{Stella_1998}. The unique millisecond X-ray pulsar, SAX J1808.4-3658, shows high Q millisecond pulsations (401 Hz) and a lack of kHz QPOs \cite{Wijnands_1998}. Also, pairs of kHz QPO are not observed in sources with black holes \cite{van_der_Klis_2000} indicating that the generation of kHz QPO is likely related to a mechanism near the surface of the NS.

In this paper, we present the torque free precession (TFP) model for kHz QPO observed in LMXB sources which relates the properties of the observed X-ray emissions from the surface of the NS over a wide range of accretion rates to the intrinsic properties of neutron star (NS).
structure. This model is in strong contrast to existing models which primarily relate the kHz QPO phenomenon to the physics of gas dynamics external to the NS surface.

2. Torque Free Precession Model

KiloHertz oscillations are explained as a natural consequence of the spinup of a neutron star to near breakup due to sustained high rates of accretion. In this model, the fluid core of the NS spins up due to accretion torques (∼300 – 800 Hz). Further transfer of angular momentum to the NS causes the “crust” of the NS to partially decouple from the core and rotate at a higher rate (up to ∼1100 Hz) than the fluid core. The crust of the NS both rotates and undergoes torque free precession (TFP) as a rigid body: this causes a modulation of the X-ray emission from this observable outer region at a pair of frequencies in the kHz band. We designate these frequencies as $\nu_l$ and $\nu_u$, the lower and upper kHz QPO. These oscillations are quasi-periodic because of fluctuations in the moments of inertia of the outer region of the NS and possible exchange of angular momentum between the rigid crust and the fluid core. Although the kHz QPOs observed in X-rays are the most well studied predictions of this model, most of the energy released is likely in the form of kHz gravitational waves (GW).

2.1. Theoretical Overview

The crust of the NS will undergo classical TFP in the absence of any accretion. Torques due to the accretion flow or changes in the internal structure of the NS will modify the precession and rotation of the NS on a timescale which is slow compared to either the spin of the crust
at frequency $\nu_{cs}$, the precession of the crust at frequency $\nu_{cp}$, or the spin of the fluid core at a frequency $\nu_{fs}$. For the purposes of the model proposed here, we identify the “rigid” crust as that portion of the outer part of the NS which undergoes rotation and precession independently of the fluid core of the NS. The structure of the outer layers of a NS is divided into a surface layer at low density ($< 10^6$ gm cm$^{-3}$), an outer crust ($\sim 10^{11}$ gm cm$^{-3}$), and an inner crust ($\sim 10^{14}$ gm cm$^{-3}$). These layers are outside of a neutron fluid core at nuclear density ($\sim 10^{15}$ gm cm$^{-3}$). There may be an interior core region that is solid or some other form of matter distinct from a neutron liquid at even higher densities (Shapiro, & Teukolsky 1983). The exact details of the NS structure are not important for the simple application of the TFP model.

Simplifying these constructs, we refer to only two components, the “crust” and the “core” of the NS. In a latter section, we identify the outer crust as the portion of the NS that undergoes TFP because the inner crust is likely melted in an accreting LMXB NS. We further assume that the crust is rigid and undergoes rotation and precession while the core is a fluid and only undergoes rotation. Clearly there is a complex interface between the crust and core as well as time dependent deformations of the crust and core, and perhaps non-rotational differential flows in the fluid interior. All of these details are ignored in modeling the basic response of the NS to increasing angular momentum and approach to breakup conditions.

Let $I_{c\parallel}$ and $I_{c\perp}$ be the parallel and perpendicular components of the moment of inertia of the crust of the NS as related to the rotational axis of the NS. We approximate the response of the crust of the NS to a variation in rotation rate by the following pair of equations where $\nu_{cs}$ is the NS crust spin frequency and $I_{c0}$ is the moment of inertia of the non-rotating crust. Clearly, $I_{c\parallel}$ must equal $I_{c\perp}$ in the limit of no rotation:
\[ I_{c\parallel} = I_{c0} \left[ 1 + \frac{\nu_{cs}}{\nu_{c\parallel}} \right] \]  

\[ I_{c\perp} = I_{c0} \left[ 1 + \frac{\nu_{cs}}{\nu_{c\perp}} \right] \]  

where \( \nu_{c\parallel} \) and \( \nu_{c\perp} \) are constants that characterize the response of the crust to rotation, in the directions parallel and perpendicular to the NS spin axis.

We also define a dimensionless ratio ellipticity \( \epsilon_c \):

\[ \epsilon_c = \frac{I_{c\parallel} - I_{c\perp}}{I_{c\perp}} \]  

The ellipticity \( \epsilon_c \) is related to the eccentricity \( e_c \) by

\[ \epsilon_c = \frac{\nu_{cp}}{\nu_{cs}} = \frac{e_c^2}{2 - e_c^2} = \frac{\nu_l}{\nu_u} \]  

where the eccentricity \( e_c \) is defined as

\[ e_c^2 = 1 - \left( \frac{r_p}{r_e} \right)^2 \]  

Here \( r_e \) indicates the equatorial radius of the NS and \( r_p \) indicates the polar radius. The NS is assumed to have the shape of a oblate spheroid which is induced by rotation of the crust at frequency \( \nu_{cs} \). Note that in equation 4 we have made the interpretation that the observed lower kHz QPO frequency is the crust’s precession frequency (\( \nu_{cp} = \nu_l \)) while the upper kHz QPO frequency is the crust’s spin frequency (\( \nu_{cs} = \nu_u \)). Equations 3 and 4 are consistent with TFP of a rigid body (Goldstein 1950).
From the Maclaurin sequence of the equilibrium figures of uniform rotation and density (Chandrasekhar 1969) critical values of the ellipticity $\epsilon$ are determined as purely geometrical functions of the ratios of moments of inertia. In the following discussion $\epsilon$ and $e$ (no subscripts) refer to the ellipticity and eccentricity respectively of a MacLaurin spheroid defined analogously as the equations 3, 4 and 5 for the crust. Generalizations (Ostriker, & Bodenheimer 1973) of the Maclaurin sequence indicate that the critical values of $\epsilon_c$ are approximately the same as those for $\epsilon$, even in cases of non-uniform rotation and density. Even if the surface shape of the NS does not match an ideal oblate spheroid, $\epsilon_c$ is still well defined as a function of the ratios of the moments of inertia. The critical values of ellipticity ($\epsilon$) and also the eccentricity ($e$) are often expressed as corresponding critical values of the ratio of kinetic energy ($T$) to the absolute value of the potential energy ($|W|$) designated $\frac{T}{|W|}$.

Considering a MacLaurin sequence of increasing angular momentum, we first encounter a secular instability at $\epsilon = 0.4930$; $e = 0.8127$; $\frac{T}{|W|} = 0.1375$. This first instability determines the maximum rotation of a NS for an assumed equation of state (EOS) and mass. Note that the values of eccentricity (and also $\frac{T}{|W|}$) are slightly less than the critical value for the “normal” sequence of maximally rotating NS computed by Cook (see table 7 of Cook, Shapiro, & Teukolsky 1994b). This result is robust for all EOS for which Cook computed models. The next important critical value ($\epsilon = 0.7618$; $e = 0.9300$; $\frac{T}{|W|} = 0.2379$) occurs at the state with the maximum possible rotation rate. Beyond this point, increasing the angular momentum will not increase the rotation rate. This state does not indicate the onset of any instability, but rather reflects the fact that the system has reached its maximum possible spin for a rigid internal structure. The next critical value ($\epsilon = 0.8315$; $e = 0.9529$; $\frac{T}{|W|} = 0.2738$) occurs when the rotating rigid body becomes dynamically
unstable because of the rapid growth of nonradial toroidal modes. Any rotating system, including a rigid body, is unlikely to maintain a value of eccentricity greater than 0.9529.

Recall that we have previously associated the precession frequency of the NS crust ($\nu_{cp}$) with the observed lower kHz QPO frequency ($\nu_\ell$), and the spin frequency of the crust ($\nu_{cs}$) with the upper kHz QPO frequency ($\nu_u$). For the conditions considered here the accretion is likely higher onto a large equatorial region of the crust. The polar regions of the crust likely rotate with interior fluid which is observed in Type I bursts from LMXB NSs (see a later section for more discussion of this possibility). At low angular momentum, the fluid and crust rotate together as a single rigid body at the spin frequency $\nu_{fs}$ ($= \nu_{cs}$). As the angular momentum is increased, the NS reaches the critical internal state which results in the partial decoupling of an equatorial portion of the crust from the interior of the NS. The details of this internal change in the NS structure are likely due to heating of the inner crust due to nuclear burning associated with the accretion of matter onto the NS surface and subsequent migration to deeper layers of the NS. This catastrophic event will occur at a unique spin for a particular NS among an uncertain range of spin frequencies depending on the details of the history of accretion. Since we have no quantitative theoretical model we identify a plausible range as 300 – 800 Hz for consistency with observations of NCO in Type I bursts and the possibility that Z-sources have spins as high as 800 Hz. The details of this event are discussed in a later section. For the discussion here, we only need assume that a portion of the crust has been reduced in mass ($\sim 10^{-4}M_\odot$) and is less coupled to the NS fluid interior. We will show later that the EOS of the NS is such that at the time just prior to the catastrophic event, the shape of the NS has an eccentricity $e \sim 0.2$ and is a stable. Subsequent to the decoupling of the outer crust from the interior of the NS, this part of the crust begins to rotate at a spin frequency
different and higher than the fluid core. In this context, the descriptive term decouple means that the interaction of the crust (reduced in mass) with the NS interior is sufficiently weak that an equatorial portion of the crust can rotate and precess independently from the NS fluid core. Clearly there is still a significant interaction between the crust and core. The crust will experience torques due to both accretion and internal changes in the NS. Since the crust is reduced in mass, its spin can more easily be changed. The torques due to accretion are not large enough to change the spin of the crust by a large amount (\(\sim 300\) to \(\sim 1200\) Hz). However, internal changes in the NS will produce torques on the crust that can significantly spin the crust up or down. The crust will always spin faster than the fluid core. Therefore the spin frequency of the fluid core \(\nu_{fs}\) is less than the minimum value of the frequency of the upper kHz QPO \(\nu_u\) in each particular source. As the crust spins up its shape will change due to a continuous elastic response. As the crust reaches a spin frequency of \(\sim 500\) Hz and an eccentricity \(e \sim 0.8127\) it approaches the onset of a secular instability. The response of the crust to this secular instability is the likely initiation of TFP. This transition near \(\sim 500\) Hz is supported by observations of 4U1728-34 and 4U1608-52 that show both kHz QPO for \(\nu_u >\sim 500\) Hz but only the upper kHz QPO for \(\nu_u <\sim 500\) Hz (see figure 1 of Mendez, van der Klis, & Ford 2000). The crust now both spins and precesses over a range of spin frequencies \(\nu_{cs} \approx 500 - 1100\) Hz \((e_c \approx 0.8 - 0.93)\), while the fluid core continues to rotate near the same rate that it had just prior to decoupling of the equatorial crust.

The NS does not “break up” as a single body but rather breaks into two separate bodies. The core is a fluid which rotates but does not precess, while the “rigid” crust rotates and precesses. This motion of the crust leads to the appearance of the pair of kHz QPO at the rotation frequency of the crust \(\nu_{cs} (= \nu_u)\) and the precession frequency of the crust \(\nu_{cp} (= \nu_\ell)\). As \(\nu_{cs}\) increases to
\( \sim 1100 \text{ Hz}, \) the crust reaches the next critical value of eccentricity \( e \sim 0.9300 \) which corresponds to the maximum rotation rate of the rigid crust that will allow both rotation and precession to occur. As the rotation rate increases higher than \( \sim 1100 \text{ Hz} \) toward slightly greater values (\( \sim 1200 \text{ Hz} \) which corresponds to \( e \sim 0.9529 \)), the crust approaches dynamical instability which quenches its precession. An alternative and perhaps more likely scenario is that for spins above \( \sim 1100 \text{ Hz} \) the equatorial crustal material and the plasma atmosphere above the crust are moving very close to escape velocity of the NS and may merge with orbiting material at the inner edge of the accretion disk. The upper kHz QPO in the frequency range \( \sim 1100 – 1200 \text{ Hz} \) may be caused by orbital motion hence the lack of TFP and a missing lower kHz QPO. When this spin/orbital frequency reaches \( \sim 1200 \text{ Hz} \) the accretion disk and crust come into contact which results in the cessation of the upper kHz QPO.

At this state of maximum angular momentum, the fluid core is still rotating at frequency \( \nu_{fs} \) (some value in the range \( \sim 300 – 800 \text{ Hz} \)) while the crust is rotating at \( \nu_{cs} (\sim 1100 \text{ Hz}) \).

For greater angular momenta, the entire NS system (crust and core) can no longer rotate as two bodies. Also the surface of the crust is moving at a speed near the escape velocity. Further increases in angular momentum are likely to lead to mass ejection or the real “breakup” of the NS. The decoupling of the crust from the core at spin frequencies \( \sim 300 – 800 \text{ Hz} \) stalls the further spinup of the NS while the equatorial crust spins up to \( \sim 1100 \text{ Hz} \). At this point the NS (crust and core) has reached the breakup state.
2.2. TFP Model Calculation

The successful quantification of the TFP model is confirmed by fitting the observed frequencies of the kHz QPO from Sco X-1. We can approximately model the moments of inertia of the crust \( (I_{c\parallel} \text{ and } I_{c\bot}) \) as a response to rotation at frequency \( \nu_{cs} \) as a linear increase from an initial value at zero rotation \( (I_{c0}) \) as shown in equations 1 and 2. The fixed free parameters in this model are \( \nu_{c\parallel}, \nu_{c\bot} \text{ and } I_{c0} \). The value of \( \epsilon_{c} \) defined by equation 3 is independent of the value of \( I_{c0} \) since it only depends on ratios of moments of inertia. Equation 4 relates the moments of inertia to the ratio of the lower and upper frequencies of the kHz QPO with the interpretation that the upper frequency is the spin frequency of the crust \( (\nu_{u} = \nu_{cs}) \) and the lower frequency is the precession frequency of the crust \( (\nu_{l} = \nu_{cp}) \). We use this model to fit the data for ScoX-1 and thereby empirically determine the values for \( \nu_{c\parallel} = 1009.4 \pm 1.0 \) Hz and \( \nu_{c\bot} = 6435 \pm 22 \) Hz. These values are computed by a \( \chi^2 \) fit to the 38 pairs of measurements of \( \nu_{l} \) and \( \nu_{u} \) for Sco X-1 with two free parameters and 50% error bars \( (\Delta \chi^2 = 2.37) \) derived according to Lampton, Margon, & Bowyer 1976. The minimum value of \( \chi^2 \) is 92.4 for 34 degrees of freedom. This fit is of similar quality to that from Psaltis et al. 1998 and implies systematic errors \( \sim 2 \) times larger than the statistical errors. The value of \( I_{c0} \) is not determined by the fit.

This fit is shown in figure 1 (a) and (b). Panel (a) shows the fit to the data by the TFP model (solid line) as well as the phenomenological equation of Psaltis et al. 1998 (dashed line). The fits are similar; however for the TFP model we are fitting a dynamical model rather than a phenomenological function. In panel (a), the data point near the abscissa value of 500 Hz is obtained from observations of GX 5-1 (Wijnands, & van der Klis 1998). It has not been included in the fit for \( \nu_{c\parallel} \) and \( \nu_{c\bot} \), but is included in the figure to demonstrate that the TFP model appears
to fit the data over the full observed range of $\nu_u$ (500 – 1100 Hz) with $\nu_\eta$ present. In panel (b) we show an expanded view of the Sco X-1 data and the models. Panels (a) and (b) show the ordinate as the difference frequency of the kHz QPO as usually shown in prior work (Psaltis et al. 1998 and references therein).

In panels (c) and (d) we show the same data and the TFP model fit with the ordinate as the ratio $\nu_\ell/\nu_u$ which is equal to the ellipticity of the crust $\epsilon_c$. Again the TFP model is the solid curve which is shown to match the Sco X-1 data and the single point from GX5-1. Panel (d) is the same as panel (c) at an expanded scale showing a better view of the Sco X-1 data. Panel (c) also shows horizontal dotted lines for the critical values of the eccentricity of the crust. Also shown in panel (c) is one curve that correspond to a particular model of a rotating NS. The model uses the specific EOS FPS (Lorenz, Ravenhill, & Pethick 1993) for a NS of mass $1.4 \, M_\odot$ taken from a computation in Cook, Shapiro, & Teukolsky 1994b. Given the interpretation of the model proposed here, the curve for the real EOS lies below the solid curve for all values of $\nu_u$. By inspection of table 15 from Cook, Shapiro, & Teukolsky 1994b, it can be shown that the ratio of the radius of a NS at maximum spin is close to $\sqrt{2}$ times the radius at zero spin. Similarly the decoupled crust also increases its equatorial radius by $\sim \sqrt{2}$ as it approaches maximum spin. This result is consistent with the condition $\nu_u \sim \nu_{c\parallel}$ which from equation [1] means that the moment of inertia of the crust has doubled corresponding to an increase in the equatorial radius by a $\sim \sqrt{2}$. In other words, the radius of the decoupled crust at maximum spin is nearly the same as the radius that the NS would have at maximum spin if the crust did not decouple. The condition that the rotating and precessing crust must be outside the radius of the fluid core mean that the EOS of the NS must have a maximum spin greater than $\sim 1100$ Hz. Since observations support the contact on the inner
edge of the accretion disk with the equatorial crust at maximum spin of the crust 1100 – 1200 Hz therefore the EOS of the NS must have a maximum spin less than \( \sim 1200 \) Hz and likely close to \( \sim 1100 \) Hz. Of all the EOS considered by Cook, Shapiro, & Teukolsky 1994b, EOS FPS is closest to matching this requirement. Of the polytropes considered in Cook, Shapiro, & Teukolsky 1994a only the index \( n \sim 1.0 \) matches this requirement. If the TFP model is correct, then the EOS of a NS is tightly constrained within a range that is comparable to the systematic errors in the computation of the structure of a rotating NS. The best fit polytrope for the interior of a NS has an index \( n \) likely within \( \pm 0.1 \) of 1.0. The empirical relationship in equation 6 relates the value of the polytropic index \( n \) to the maximum spin frequency \( \nu_{\text{max}} \) of a NS with a mass \( M_{\text{ns}} \). This equation is derived from equation 31 and the data in table 3 from Cook, Shapiro, & Teukolsky 1994a. For a 1.4\( M_{\odot} \) NS and assuming \( \nu_{\text{max}} \) is equal to the maximum value of \( \nu_{u} \sim 1100 \) Hz with a simultaneous measurement of \( \nu_{l} \) yields a value \( n \sim 0.94 \). If the masses of the NS in an LMXB are as high as \( \sim 2 \) \( M_{\odot} \) then \( n \sim 1.16 \). The basic conclusion of a tightly constrained EOS with \( n \sim 1 \) still holds.

\[
  n = 1.0 + 1.4 \left[ \log_{10} \left( \frac{M_{\text{ns}}}{1.4 \ M_{\odot}} \right) - \log_{10} \left( \frac{\nu_{\text{max}}}{1004 \ \text{Hz}} \right) \right] 
\]  

(6)

In panel (d) we show an expanded view of the Sco X-1 data. Note that the maximum value of \( \nu_{u} \) at \( \sim 1070 \) Hz corresponds to the critical value of \( e \sim 0.9300 \). The dashed curve in panels (c) and (d) shows the locus for a Maclaurin spheroid that just passes through these points at maximum spin with precession. The mass (1.4 \( M_{\odot} \)) and radius (\( \sim 17 \) km) of the spheroid is approximately correct for nominal values of a NS. The upper kHz QPO from Sco X-1 has been observed at values greater than 1100 Hz but without the presence of the lower kHz QPO. The model suggests that
the maximum spin rate without precession present (missing lower kHz QPO) would occur at a frequency \( \sim 1180 \) Hz which is determined by the intersection of the \( e = 0.9529 \) line with the solid curve. This point marks an upper limit for the breakup frequency of the NS in the context of the TFP model. An alternative, and more likely interpretation, is that the true breakup frequency of full NS (core and crust) is closer to 1100 Hz. The observed upper kHz QPO in the frequency range \( \sim 1100 \) – 1200 Hz (without the lower kHz QPO) marks a transition region in which the outer crust and the NS plasma atmosphere are moving near escape velocity and are merging with the inner edge of the accretion which has a matching Keplerian velocity. Full merger of the outer crust with the inner edge of the accretion disk marks the cessation of the upper kHz QPO.

In panel (c) a vertical dot-dash line is plotted at a frequency \( \nu_u = 401 \) Hz, which is the spin frequency of SAX J1808.4-3658, the millisecond X-ray pulsar discovered by Wijnands et al. 1998a. The locus for EOS FPS intersects the line for this pulsar near \( \epsilon_c \sim 0.03 \). For the EOS FPS the radius of the NS varies from 10.85 – 15.45 km (see table 12 in Cook, Shapiro, & Teukolsky 1994b). For a \( \sim 1.4 \ M_\odot \) NS with \( \epsilon_c = 0.03 \) (or \( e = 0.17 \)) the radius as set by EOS FPS is \( \sim 11 \) km in agreement with the upper limit \( \sim 16 \) km suggested by Burderi, & King 1998. A high Q pulsar such as SAX J1808.4-3658 has not reached the condition for melting the inner crust. In the context of the TFP model such a high Q pulsar would never show kHz QPO. Also the millisecond radio pulsar PSR 1937+214 discovered by Backer et al. 1982 has a spin frequency of \( \sim 640 \) Hz. The high Q of this pulsar is consistent with the lack of accretion and a solid inner crust. Perhaps this radio pulsar once had a melted inner crust and significant accretion rate and has since reformed a solid inner crust after cooling subsequent to a cessation of accretion.
3. Scenario for Initiation and Maintenance of TFP

The correctness of the TFP model is strongly supported by the consistent prediction of the sequence of kHz QPO observed in LMXB sources. The angular momentum transfer via accretion is too small to drive a large variation in the spin of the outer crust (300 – 1200 Hz) over a short period ($\sim$ day). Any realistic and complete TFP model requires a mechanism that taps a small fraction of the large reservoir of both rotational energy and angular momentum of a rapidly spinning NS that drives the outer crust to vary in spin frequency. We suggest that the controlling mechanism is related to giant glitches observed in radio pulsars (Alpar et al. 1981).

Initially the NS is spinning up due to the accretion of material onto the surface of the outer crust. Over a long accretion timescale the outer and inner crust are repeatedly replenished. Brown, E. F. 2000 has calculated detailed models for the equilibrium structure of the crust in such a accreting source. For a sufficiently high accretion rates ($10^{-9}M_{\odot}\text{yr}^{-1}$), Brown’s model predicts that the temperature in the inner crust is independent of the surface temperature of the NS. Just below the layer beginning at onset of neutron drip, the boundary between the outer and inner crust, the nuclear reactions heat the interior of the inner crust. Most of this heat is conducted into the core of the NS. Brown’s model suggests that the inner crust will melt for a sufficiently high accretion rate. The fall of the average value of Z in the inner crust also lowers the melting temperature of this layer as well. Also the presence of any r-mode driven by the rapid spin of the NS will heat the boundary-layer between the fluid core and inner crust (Bildsten, & Ushomirsky 2000) and further raise the temperature of the inner crust. This scenario leads to the possibility of melting the inner crust. However for rates as high as 5 times Eddington the outer crust may not melt (see figure 10 of Brown, E. F. 2000). The conductivity of the outer crust is low compared
to that for the inner crust (see figure 5 of Brown, E. F. 2000). We suggest that the onset of TFP occurs when the inner crust melts leaving the outer crust as a solid. The spin of the NS star at the time when the inner crust melts will vary depending on the history of the accretion rate, the detailed structure of the inner and outer crust, and the presence of any additional non-nuclear heating sources (for example an r-mode). The onset of melting of the inner crust is consistent with a range of spin rates of the fluid core seen in Type I bursts \((300 – 800 \text{ Hz})\). In the TFP model only the equatorial portion of the outer crust of the NS spins up and down \((300 – 1200 \text{ Hz})\). Large polar regions of the NS outer crust continue to spin with the interior fluid core. A later section contains a more detailed discussion of the implications of the TFP model for observations of Type I bursts in LMXB sources.

Once the inner crust melts then the outer crust can undergo TFP driven by a mechanism that taps internal NS rotational energy. We suggest that a likely candidate for this mechanism is similar to that proposed to explain giant glitches in radio pulsars \((\text{Alpar et al. 1981})\). These giant glitches are observed to cause discrete jumps in the spin period of the outer crust that correspond to a changes in angular momentum of the a large portion of the NS. The size of the giant glitch in Vela is \(\sim 10^{-6}\) expressed as the fractional change in the spin frequency before and after the glitch. The energy source for the giant glitches is likely a internal superfluid component which has a moment of inertia, \(\sim 10^{-2}I_{f||}\), as measured by the fractional change of the spin’s first derivative before and after the giant glitch, where \(I_{f||}\) is nearly the total moment of inertia of the NS. Both of these parameters are determined by radio observations of giant glitches in the spin of a NS and are not based on any theoretical models of NS interiors \((\text{Alpar et al. 1981})\). The energetics of this ”glitch” in the spin rate of an LMXB NS is somewhat larger than a giant glitch in a radio
pulsar. This possibility is not that extreme since a LMXB NS is spinning ~ 50 times faster than the radio pulsars (Vela) for which the giant glitches are measured. We suggest that the "glitch" in an LMXB likely scales as the period of rotation yielding an estimate of the fractional change in the total angular momentum ~ 10^{-4}. If the superfluid transfers a large fraction of this stored angular momentum to the solid outer crust (I_{c||} \sim 10^{-4} I_{f||}) then a small differential in the spin of the superfluid (~ 5 Hz) compared to the spin of the core of the NS (300 – 800 Hz) would cause a large change in the spin of the outer crust (from 300 – 800 Hz to ~ 1100 Hz) since the ratio of the moment of inertia of the outer crust to that of the superfluid layer is ~ 10^{-2}. Also the timescale for recovery from a giant glitch in radio pulsars is order ~ 100 days. This long timescale is likely due to low viscosity in the superfluid layer. Theoretical estimates for this timescale are uncertain but may also scale with the period of rotation (see equation 36 in Sauls, J. A. 1988). Therefore in an LMXB NS spinning ~ 50 times faster than the Vela pulsar, the glitch recovery time would become ~ 1 day. We suggest that the outer crust spins up and down rapidly (300 – 1100 Hz) on a timescale of days due to the pinning and unpinning of vortices in a superfluid layer within the NS. This superfluid layer (which includes the melted inner crust) likely has an interface at the base of the outer crust where it interacts via slow viscous effects. This allows the outer crust to undergo TFP on a fast timescale (1 ms) while exchanging angular momentum with the superfluid reservoir on a slow timescale (~hours). This exchange of angular momentum proceeds in a limit-cycle that is observed in the spin and precession of outer crust. This process results in a rapid changes in the spin and shape of the outer crust and is consistent with the observed Qs of the kHz QPO. The random meander of this limit-cycle (see figure 2 in Zhang et al. 1996) is associated with the pinning and unpinning of vortices in the superfluid with resulting discrete interaction with
the outer crust. These interactions induce changes in the both the parallel and perpendicular components of the moments of inertia of the outer crust and excite TFP. Some of the unpinned superfluid likely rotates with the outer crust up to $\sim 1100$ Hz. This superfluid then repins with the pinned superfluid causing the spin down of the outer crust. Since the outer crust in an LXMB NS is weakly coupled to the interior of the NS as compared to the crust of a NS in a radio pulsar, the manifestation of the observed "glitch" is slow both going up and down in an LMXB NS.

In addition the accretion onto the surface of the outer crust also causes torques which excite TFP. The outer crust is also continuously elastically responding to the changes in the internal strain. These internal adjustments also contribute to fluctuations in the components of the moments of inertia and the excitation of TFP. Simultaneous with all of these effects, GWs are emitted due to TFP. Using equation 2.13 from Cutler, C., & Jones, D. I. 2000 in an entirely different context, we estimate the exponential timescale for GW damping of the TFP ($\sim 500$ s).

Equation 7 is a recasting of this equation assuming that ($\Delta I/I_c$) is of order unity rather than $\sim 10^{-7}$. In the context of the TFP model the outer crust responds elastically to changes in spin rate completely independent of the shape of the fluid core of the NS which is spinning slower than the outer crust.

$$\tau_{th} = 5.4 \times 10^2 \left[ \frac{1 \text{ kHz}}{\nu_{cs}} \right] \left[ \frac{10^{41} \text{ g cm}^2}{I_{c||}} \right] s \quad (7)$$

The slower positive and negative variations ($\sim 100$s) in the spin of the outer crust seen in figure 2 of Zhang et al. 1996 may be due to the varying back reaction of the emission of GWs due to TFP.
4. Implications of the Torque Free Precession Model

In the following sections, we discuss a few of the implications of the TFP model including GW emission, LT precession of material orbiting a NS, periodicities in the X-ray emission from Type I bursts, magnetic dipole radiation emitted at the spin frequency of the crust, and possible models for GRB based on accretion driven conversion of a LMXB NS to a BH.

4.1. Generation of Gravitational Waves

The accretion in the brighter LMXB sources may exceed the Eddington rate. Examples of mass transfer in LMXB that result in accretion rates as high as $10^{-5} \text{M}_\odot \text{yr}^{-1}$ have been modeled by Harpaz, & Rappaport 1994 and Iben, Tutukov, & Fedorova 1997. Companion stars with masses greater than the NS ($\sim 3\text{M}_\odot$) have stable accretion rates due to Roche lobe overflow on a thermal time scale that can reach super Eddington levels (Pilyser and Savonije 1988; Rappaport 2000). The full range of conditions of possible donor stars in accreting binary systems allows a wide variation in the accretion rate. As the rate approaches or exceeds the Eddington limit for a portion of the lifetime of the X-ray source, the X-ray emitting NS may decrease the accretion onto the NS surface by radiatively ejecting material from the system. Even though such complex radiative driven processes may occur, we suggest that in some cases higher accretion rates are sustained in a steady state.

In particular, we propose that in systems with high accretion rates, the NS spins up to near breakup and emits most of the accretion derived energy in GWs produced by the TFP of the NS crust. The validity of the concept of classical TFP in general relativity (GR) has been
demonstrated by Thorne 1983. TFP implies the possibility that the unmeasured emission of GWs may exceed the X-ray emission. Therefore in the subsequent discussion we consider that some LMXB NSs may have extreme super Eddington accretion rates only limited by realistic models for flow of material from donor stars. GWs emitted due to TFP of sufficient amplitude may cool the accretion flow allowing a significant fraction of the accreting material to reach the surface of the NS. Also the rapid rotation of the outer crust as it approaches contact with the inner edge of the accretion disk at a matching velocity reduces the conversion of kinetic energy at the surface of the NS. The X-ray emission from LMXB may be poor indicator of the accretion rate if significant TFP is present. However, it may be true that on average the LMXBs with lower X-ray emission (atoll sources) and those with higher X-ray emission (Z-sources) still have a range of accretion rates both below and above the Eddington limit. Even sources with current low accretion rates may undergo large amplitude TFP driven by stored internal NS energy. We have little evidence concerning the relationship between the long term average accretion rate and current observed X-ray emission. The likelihood of an unmeasured emission of GWs from LMXB should caution any firm interpretations about the total accretion onto the surface of the NS.

The following estimation of the emission of GWs via NS precession is derived entirely from the work of Zimmermann, & Szenes 1979 and Zimmermann 1980. The power at the inertial precession frequency \( \nu_{cs} + \nu_{cp} \) emitted by the precessing crust of the NS in the form of gravitational waves \( L_{GW}(\nu_{cs} + \nu_{cp}) \) is given by equation 8 derived from Zimmermann, & Szenes 1979. We assume that the precession angle \( \theta_{cp} \) is small and neglect emission at the second harmonic of the inertial precession frequency.
\[ L_{GW}(\nu_{cs} + \nu_{cp}) = \frac{64\pi^6 G}{5} \frac{\theta^2_{cp}}{c^2} \nu_{cs}^2 I_c \nu_{cp}^2 \theta^2_{cp} \] (8)

We define \( \alpha \) in equation 9 where \( I_c \parallel \) is the moment of inertia of part of the crust that is precessing, and \( I_f \parallel \) is the moment of inertia of the fluid interior that spins without precessing (as discussed in the previous section). For estimation of GW emission, we ignore the frequency dependence of \( \alpha \) implied by equation 9. The parameter \( \theta_{cp} \) is the angular amplitude of the precession in radians.

\[ \alpha = \frac{I_c \parallel}{I_f \parallel} \] (9)

Assuming \( \alpha \) and \( \theta_{cp} \ll 1 \), we then estimate \( L_{GW}(\nu_{cs} + \nu_{cp}) \) with selected values of variables in equation 10. The value of \( I_f \parallel \) is from a NS model with an EOS FPS. The value of \( \nu_{cs} + \nu_{cp} \) is set for the condition of maximal rotation of the crust near NS breakup. The values of \( \alpha \) and \( \theta_{cp} \) are not well determined and would be best constrained by future direct observation of the GWs. Note that the modest values of these two parameters imply that crust TFP is a very efficient emitter of GWs. Theoretical NS models for the outer crust suggest that \( \alpha \sim 10^{-4} \) (Mochizuki, Oyamatsu, & Izuyama 1997; Shapiro, & Teukolsky 1983). We assume that the inner crust has melted as discussed previously. The values for \( \alpha \) and \( \theta_{cp} \) are uncertain but are estimated based on the specific scenario of TFP of the outer crust. Alternative variations of the scenario would still allow TFP with different values for \( \alpha \) and \( \theta_{cp} \). For TFP in an LMXB NS we select a value of \( \theta_{cp} \sim 0.02 - 0.1 \) based on the observations of the amplitude of the lower kHz QPO (RMS of the mean X-ray flux; van der Klis et al. 1996). The ease of detection of the kHz QPO in LMXB
suggests significant amplitudes for $\alpha$ and $\theta_{cp}$. The lower values of $\theta_{cp}$ are likely associated with higher values of $\nu_a$ because of the increasing GW emission at higher spin rates.

The modulation of the X-ray emission at the precession frequency $\nu_{cp} (= \nu_\ell)$ could be caused either by geometrical effects similar to those for X-ray pulsations from a spinning NS or because the velocity and acceleration vary at the surface of the NS modulated by TFP with an amplitude proportional to $\theta_{cp}$. These variations on the surface of the NS would modulate the accretion energy and thereby be tracked by modulation in the X-ray emission on timescales of order $\sim 1$ ms. Justification for the rapid tracking of these modulations is implied by the radiation dominated conditions on the surface on the NS (Arons 1992; Klein et al. 1996; Jernigan, Klein, & Arons 2000).

The energy emitted in GW at the inertial crust precession frequency seen by a distant observer is given by

$$
L_{GW}(\nu_{cs} + \nu_{cp}) = 2.3 \times 10^{42} \left[ \frac{\epsilon_c}{0.7} \right]^2 \left[ \frac{I_{f\parallel}}{1.2 \times 10^{45} \text{ g cm}^2} \right] \left[ \frac{\nu_{cs} + \nu_{cp}}{1700 \text{ Hz}} \right]^6 \left[ \frac{\alpha}{10^{-4}} \right]^2 \left[ \frac{\theta_{cp}}{0.02 \text{ radians}} \right]^2 \text{ergs s}^{-1}
$$

Equation (10) (derived from the first term of equation 14 in Zimmermann 1980) is an estimate of the gravitational power emitted at the second harmonic of the spin frequency of the crust due to fluctuations in the components of the moments of inertia of the NS. Here we estimate the fluctuations of $I_{c\perp}$ as $\sim I_{c\perp}/Q$ where Q is a typical quality factor of the upper kHz QPO. Previously we simplified the TFP model by assuming that both components of $I_{c\perp}$ were equal. Here we must assume that the two orthogonal components of $I_{c\perp}$ can fluctuate independently.
\[ L_{GW}(2\nu_{cs}) = 1.6 \times 10^{42} \left[ \frac{I_{f\parallel}}{1.2 \times 10^{45} \text{ g cm}^2} \right]^2 \left[ \frac{\nu_{cs}}{1000 \text{ Hz}} \right]^6 \left[ \frac{Q}{100} \right]^{-2} \left[ \frac{\alpha}{10^{-4}} \right]^2 \text{ergs s}^{-1} \] (11)

For Sco X-1 at a distance of 2.8 kpc (Bradshaw, Geldzahler, & Fomalont 1999) and with the selected values for \( \theta_p \) and \( \alpha \), the amplitude of the GWs emitted at frequency \( \nu_{cs} + \nu_{cp} \) is given in equation (12) and numerically estimated in (13).

\[ h_{GW}(\nu_{cs} + \nu_{cp}) = 7.5 \times 10^{-24} \left[ \frac{\nu_{cs} + \nu_{cp}}{1700 \text{ Hz}} \right]^2 \left[ \frac{\alpha}{10^{-4}} \right] \left[ \frac{\theta_{cp}}{0.02} \right] \] (13)

Such an amplitude would indicate that the advanced LIGO system (\( h_{3/yr} \sim 10^{-26} \) at 1700 Hz, Brady et al. 1998) could detect this GW emission within a single day of integration. Such an attempt would require simultaneous tracking of the kHz QPO in X-rays so that power density spectra of the GW emission could be coherently coadded with appropriate frequency shifts. Equation (14) is an estimate of the amplitude of the GW emission at the sensitivity threshold for an observation with the advanced LIGO of duration \( T_{\text{obs}} \) where \( Q \) is the quality factor of the lower kHz QPO.

\[ h_{eff}(\nu_{cs} + \nu_{cp}) = 3.4 \times 10^{-24} \left[ \frac{h_{3/yr}}{10^{-26}} \right] \left[ \frac{Q}{100} \right]^{-\frac{1}{4}} \left[ \frac{\nu_{cs} + \nu_{cp}}{1000 \text{ Hz}} \right]^{\frac{1}{4}} \left[ \frac{T_{\text{obs}}}{1 \text{ day}} \right]^{-\frac{1}{4}} \] (14)

Equation (14) shows the weak dependence on \( T_{\text{obs}} \) expected for the addition of many power density spectra for periodic GW emission with a finite \( Q \). X-ray detectors with apertures much
larger than the PCA onboard RXTE (Bradt, Rothschild, & Swank 1993; Swank 1998) could track individual kHz oscillations. Such X-ray data could be combined with LIGO data to achieve much better sensitivity. Even without such detailed X-ray data, the prospects for LIGO detection of GWs from LMXB sources and Sco X-1 in particular are possible though uncertain.

4.2. Gravitational Waves at Other Frequencies

The dominant frequencies of expected GWs ($\nu_{cs} + \nu_{cp} = \nu_u + \nu_t$ and $2\nu_{cs} = 2\nu_u$) are generated from precession and rotation of the outer crust respectively. Measurements of GWs at these frequencies would determine the magnitude of $\alpha$ and $\theta_{cp}$ and confirm the estimates for these parameters based on the X-ray emission and thereby constrain the details of the precessional motion.

This motion will likely excite distinct modes corresponding to spherical harmonics of the NS (Ostriker, & Bodenheimer 1973). Some of these modes can induce a triaxial form of either the crust or fluid interior of the NS. This would cause the emission of GWs at frequency $2\nu_u$ from the outer crust (see equation $\square$) and at frequency $2\nu_{fs}$ from the fluid interior (nearly constant value for a particular LMXB NS in the range $300 - 800$ Hz). These motions could also drive the g-mode ocean waves on the surface of the crust (Bildsten, Ushomirsky, & Cutler 1996) or modify r-mode oscillations present in the fluid interior (Owen et al. 1998; Bildsten, & Ushomirsky 2000). In both of these cases the influence of near breakup rotation and the precession of the NS may significantly alter predictions from previous work. Since the precession and spin frequencies vary over a wide range in a single LMXB, there is the possibility that some of these modes will resonantly match the precession or spin frequencies or their harmonics. Future observations of a
full GW spectrum with many frequencies present, including the effects of temporal evolution of these modes in response to changes in the precession frequency, will constrain many details of NS structure beyond just the EOS, including fluid properties, vortex pinning, viscosity, internal flows and crust and core structure.

4.3. Lense-Thirring Precession

Following the work of Stella, & Vietri 1998, we revise the possibility of the presence of Lense-Thirring (LT) and classical precession of material orbiting near the NS in the context of the TFP model. The total precession frequency can be evaluated as a sum of two terms $\nu_{LT}$, the LT precession, and $\nu_{cl}$, the classical precession. These two terms have contributions from the rotating fluid core and the decoupled crust which are each spinning at different rates. The total LT effect of the rotating crust is small since the moment of inertia $I_{c\parallel} (= \alpha I_{f\parallel})$ is small ($\alpha << 1$). We estimate $I_{f\parallel} = 1.3 \times 10^{45}$ g cm$^2$ at rotation rate $\nu_{fs} \sim 740$ Hz using the NS model based on EOS FPS from Cook, Shapiro, & Teukolsky 1994b. The LT term for the NS fluid core is given by equation (15) and numerically estimated by equation (16).

$$\nu_{LT} = \frac{8\pi^2 I_{f\parallel} \nu_{k}^2 \nu_{fs}}{c^2 M}$$ (15)

$$\nu_{LT} = 45 \left[\frac{I_{f\parallel}}{1.3 \times 10^{45} \text{ g cm}^2}\right] \left[\frac{M}{1.4 \text{ M}_\odot}\right]^{-1} \left[\frac{\nu_{k}}{1200 \text{ Hz}}\right]^2 \left[\frac{\nu_{fs}}{740 \text{ Hz}}\right] \text{ Hz}$$ (16)

where $M$ is the mass of the NS and $\nu_{k}$ is the Keplerian orbital frequency of the material that is precessing.
The classical precession effect is given by equation (17) and numerically estimated by equation (18).

\[
\nu_{cl} = \frac{3}{8\pi^2} \frac{G \cos \beta}{r^5 \nu_k} (I_{f\perp} - I_{f\parallel}) = -\frac{3}{16\pi^2} \frac{G \cos \beta}{r^5 \nu_k} \frac{e_f^2}{1 - e_f^2} I_{f\parallel}
\]

\[
\nu_{cl} = -2 \times 10^{-2} \left[ \frac{I_{f\parallel}}{1.3 \times 10^{45} \text{ g cm}^2} \right] \left[ \frac{M}{1.4 M_\odot} \right]^{-\frac{5}{7}} \left[ \frac{\nu_k}{1000 \text{ Hz}} \right]^{\frac{7}{3}} \cos \beta \text{ Hz} \quad (18)
\]

In the above equation for \( \nu_{cl} \) we set the value of \( e_f = 0.4 \) from model FPS for a spin rate \( \nu_{fs} \sim 740 \text{ Hz} \) (see table 15 in Cook, Shapiro, & Teukolsky 1994I). The ellipticity \( \epsilon_f \) and eccentricity \( e_f \) are defined for the fluid core in analogy to equation (3) for the crust, where \( I_{f\parallel} \) and \( I_{f\perp} \) are the moments of inertia of the fluid core of the NS.

\[
\epsilon_f = \frac{e_f^2}{2 - e_f^2} = \frac{[I_{f\parallel} - I_{f\perp}]}{I_{f\perp}} \quad (19)
\]

Equation (18) shows that classical precession negligible compared to LT precession.

These estimates are consistent with the basic LT conjecture of Stella, & Vietri 1998 except that their assumption that \( \nu_k = \nu_u \) must be based on a indirect correlation. We suggest that both \( \nu_k \) and \( \nu_u (= \nu_{cs}) \) are independently proportional to the accretion rate, \( \dot{M} \). Another possibility is that the precession of the crust at frequency \( \nu_{cp} (= \nu_{\ell}) \) induces a tilt in the accretion disk at the location where material orbits the NS at the TFP frequency (not to be confused with the precession of orbiting material). This in turn leads to a modulation of the accretion rate by material in orbit at the frequency \( \nu_k = \nu_{cp} \). This would provide a reason for identifying \( \nu_k \) with \( \nu_{\ell} \) yielding a similar result to that in Stella, & Vietri 1998 with a somewhat different interpretation.
The prior difficulty in Stella, & Vietri 1998 of a factor of ∼ 4 discrepancy for the $I_M$ for a NS may be removed if NS spin frequency $\nu_{fs}$ is as high as ∼ 800 Hz. In the context of the TFP model, the spin of the fluid core $\nu_{fs}$ is not related to the difference frequency of the kHz QPO. For example in the source Sco X-1, $\nu_{fs}$ may be as high as the minimum observed value of $\nu_u$ (∼ 845 Hz; van der Klis 2000). Setting the value of $\nu_{fs}$ ∼ 740 Hz in the equation 16 yields a $\nu_{LT}$ ∼ 45 Hz in agreement with observations. We are not proposing a specific detailed version of the effects of LT precession but only caution that both $\nu_{fs}$ and $\nu_k$ are uncertain.

4.4. Periodicity Observed in Type I Bursts

Observations indicate a near coincidence of $\Delta \nu$ (= $\nu_u - \nu_l$) with $\nu_{burst}$ or $\nu_{burst}/2$. The conclusion that $\nu_{burst}$ is near the actual spin rate of the surface of the NS is inconsistent with the simple version of the TFP model since $\nu_{cs}$, the spin frequency of the NS surface, is significantly larger than $\Delta \nu$. The TFP model is consistent with an rapidly rotating outer crust (up to 1100 Hz with eccentricity ∼ 0.93) which is flatten in form compared to the nearly spherical rotating fluid core (300 − 800 Hz with eccentricity ∼ 0.2). The NS in Sco X-1 may possibly be rotating as fast as ∼ 800 Hz. In such a configuration the accretion rate onto the equatorial region may be significantly higher than onto the polar regions. This possibility is consistent with the lack of any detection of oscillations at the frequency $\nu_{fs}$ in the X-ray emission due to steady accretion. We suggest the following possibility that the outer crust is rotating with the fluid core in the polar regions (300 − 800 Hz) and is rotating independently and more rapidly in the equatorial region (up to 1100 Hz). Further in the mid-latitude regions there is the possibility that the outer crust rotates with an internal superfluid layer at a rate ∼ 5 Hz faster than the fluid core. Unstable
thermonuclear burning would occur for each latitude region depending on the level of the local accretion rate. The progression of observed frequencies in a Type I burst would depend on the particular pattern of the burning front. The scenario is consistent with the frequencies observed in Type I bursts (see \cite{Strohmayer1998}). The fractional variation in the spin rate of the fluid core \( \nu_{fs} \) as the crust spins up and down is \( \sim 10^{-4} \) assuming full transfer of the angular momentum between the outer crust and the fluid core of the NS. This result is consistent with the repeatability of the asymptotic NCO frequency observed in several independent Type I bursts from 4U1728-34 and 4U1636-53 (\( 1.5 \times 10^{-4} \) and \( 2.3 \times 10^{-4} \) respectively; \cite{Strohmayer1998}). In SAX J1808.4-3658, we expect higher Q NCO at the spin frequency (401 Hz) which is the same for both the crust and core of the NS.

Recently \cite{Wijnands2000} discovered nearly coherent oscillations (NCO) in Type I bursts from X1658-298. These NCO occurred at frequencies \( \sim 567 \) Hz during the main part of the burst and at frequency \( \sim 572 \) Hz during the tail of the burst. This pattern of frequencies is consistent with a burning front which migrates from a polar region to a mid-latitude regions. The discrete jump in frequency matches that expected for the superfluid layer that spins differentially from the NS core as discussed previously. The burning front may continue into the equatorial region which might produce a feature in the power density spectrum similar to the upper kHz QPO. The higher frequency of such a peak and its finite Q may decrease the likelihood of detection. Alternately, higher accretion rates in the equatorial region may suppress unstable thermonuclear burning.
4.5. Magnetic Dipole Radiation

The maximum spin rate of the surface of a NS is the maximum observed value of the upper kHz QPO. Such high spin rates suggest that LMXB NSs are candidates for the direct detection of magnetic dipole radiation since the power emitted is strongly dependent on spin rate. The following equation is an estimate of the power emitted from Sco X-1 where $B_0$ is the surface magnetic field, $r_e$ is the equatorial radius of the NS, and $\theta_d$ is the viewing angle of a distant observer relative to the spin axis. The numerical estimate of the power emitted given by equation (20) assumes a viewing angle normal to the rotation axis ($\theta_d = \frac{\pi}{2}$)

$$P_{\text{dipole}} = \frac{r_e^6 B_0^2 \sin^2 \theta_d}{6c^3} = 6.2 \times 10^{38} \left[ \frac{r_e}{2 \times 10^6 \text{ cm}} \right]^6 \left[ \frac{B_0}{10^9 \text{ Gauss}} \right]^2 \left[ \frac{\nu_{cs}}{1000 \text{ Hz}} \right]^4 \text{ ergs s}^{-1} \quad (20)$$

This radiation, if present, is energetically of the order of the X-ray portion of the accretion energy but likely less than the energy emitted in GWs. The plasma near the NS might convert much of this energy into local plasma modes; however the wavelength of the radiation ($\sim 10^7 \text{ cm}$) exceeds the size of the NS by a large factor, therefore some of this power may leave the system.

The plasma dynamics near the NS and interaction with the interstellar medium along the line of sight will likely decrease the chances of detection of this radiation. However since Sco X-1 is nearby on a galactic scale (2.8 kpc), perhaps detection is possible by spacecraft such as the Voyagers (Gurnett, & Kurth 1996), now located in the outer solar system where the plasma frequency is sufficiently low. The plasma frequency $\nu_{pl}$ is related to the local electron density by:
\[ \nu_{pl} = \left[ \frac{e^2 N_e}{\pi m_e} \right]^{\frac{1}{2}} = 8.97 \times 10^3 N_e^{\frac{1}{2}} \text{ Hz} \]  

(21)

If at any point along the line of sight the electron density rises above \( \sim 10^{-2} \text{ cm}^{-3} \) then such plasma will damp out the direct dipole radiation at frequency \( \nu_u \) (\( \sim 1000 \text{ Hz} \)). The solar wind drops below this density in the outer regions of the solar system. Also the Voyagers are within about a decade of reaching the heliopause (see figure 7 of Gurnett, & Kurth 1996). There is also the possibility of electromagnetic radiation at somewhat higher frequency due to TFP (\( \nu_{cs} + \nu_{cp} \sim 1700 \text{ Hz} \)).

4.6. Progenitors of Gamma Ray Burst Sources

Neutron stars in LMXB spinning near breakup speed with high accretion rates would likely emit most of their energy in the form of GWs while the X-ray emission is maintained at levels influenced by the Eddington limit. The LMXB sources with low mass donor stars may be the progenitors of millisecond radio pulsars (Radhakrishnan, & Srinivasan 1982; Alpar et al. 1982). If the donor companion star is massive enough (few M\(_\odot\)) to provide \( \sim 0.5 \text{ M}_{\odot} \) of material, then the NS might approach the maximum stable mass while rotating near the breakup rate. The GW cooling of the accretion flow near the surface of a NS may allow a large fraction of the material that leaves the companion star to reach the surface of the NS. The X-ray emission is still restricted by the Eddington limit but at much higher mass accretion rates onto the NS surface. The fluid core would collapse into a maximally rotating Kerr black hole (\( \sim 2 \text{ M}_{\odot} \)) and perhaps release energy of a fraction of \( 10^{54} \text{ ergs} \) in the form of jets of material, GWs, neutrinos, and gamma-rays.
An LMXB NS could be a progenitor of a GRB which leaves behind a maximally spinning black hole (Cui, Zhang, & Chen 1998). Subsequent continued accretion from the donor companion star would increase the BH mass. To explain the observed properties of the brighter GRBs would likely require conversion of not just the kinetic energy of the NS material as it forms the black hole (BH) but conversion of some of the rest mass of the NS into gamma-rays.

Ruffini 1999 has suggested that temperatures in the dyadosphere of the BH at the time of formation may rise enough to cause copious $e^\pm$ pair creation and subsequent annihilation with conversion to gamma-rays. As much as 50% of the mass-energy of the NS might be released at the time of formation of a maximally spinning Kerr BH (Ruffini 1999). After the BH forms, the crust of the NS ($\sim 10^{-4} M_\odot$) may still be present and spinning at frequency $\nu_{cs} \sim 1000$ Hz. The remnant crust will no longer have pressure support and will likely transform to a triaxial form which will quickly form two blobs of rotating hot plasma around the BH at a radius of the former NS. This material ($\sim 10^{-4} M_\odot$) will fall onto the BH releasing its kinetic energy in the form of GWs, neutrinos, electromagnetic waves and jets of material. We crudely estimate the time for this remnant NS crust to accrete onto the BH due to emission of GWs as the ratio of the sum of kinetic ($E_k$) and potential ($E_p$) energies to the luminosity emitted in GWs ($L_{GW}$). This ratio is $\left( \frac{E_k + E_p}{L_{GW}} \right) \sim 1$ s from equations 22, 23 and 24. This estimate is likely too fast since $L_{GW}$ is probably an upper limit and release of other forms of energy will likely retard the flow towards the BH event horizon. This model for a GRB might have a unique signature of nuclear composition since the accreting material is from the breakup of the remnant crust of a NS (Lattimer et al. 1977). The spectrum of the gamma-ray emission could in principal reveal the details of the nuclear composition (Hailey, Harrison, & Mori 1999).
\[
L_{GW} = \frac{512\pi^6 G}{5e^4} I_{c||} \nu_{cs}^6 = 5.2 \times 10^{49} \left[ \frac{I_{c||}}{2.4 \times 10^{14} \text{ g cm}^2} \right] \left[ \frac{\nu_{cs}}{1000 \text{ Hz}} \right]^6 \text{ ergs s}^{-1} \tag{22}
\]

\[
E_k = 2\pi^2 I_{c||} \nu_{cs}^2 = 4.7 \times 10^{49} \left[ \frac{I_{c||}}{2.4 \times 10^{11} \text{ g cm}^2} \right] \left[ \frac{\nu_{cs}}{1000 \text{ Hz}} \right]^2 \text{ ergs} \tag{23}
\]

\[
E_p = \frac{GM_M c}{r_e} = 3.3 \times 10^{49} \left[ \frac{M}{1.4 M_\odot} \right] \left[ \frac{M_c}{10^{-4} M_\odot} \right] \left[ \frac{r_e}{10 \text{ km}} \right]^{-1} \text{ ergs} \tag{24}
\]

In rough numbers we can estimate the rate of GRBs that would be produced by accreting \(\sim 0.5M_\odot\) of matter onto the NS in an LMXB and thereby initiating its conversion to a BH. Assuming a long term average accretion rate of \(10^{-7} M_\odot \text{ yr}^{-1}\) yields a rough lifetime for each LMXB, \(t_{LMXB} \sim 10^7 \text{ yr}\). If the universe contains \(10^{11}\) galaxies equivalent to the Milky Way, and each galaxy contains a few LMXB \(N_{LMXB}\) at any time which are accreting at high rates from donor companion stars of a few solar masses, and the radiation is beamed by a fraction \(f_b\) then the total rate of observed GRB \((R_{GRB})\) is estimated in equation 25.

\[
R_{GRB} \sim 1 \left[ \frac{f_b}{10^{-2}} \right] \left[ \frac{N_{gal}}{10^{11}} \right] \left[ \frac{N_{LMXB}}{3} \right] \left[ \frac{t_{LMXB}}{10^7 \text{ yr}} \right]^{-1} \text{ day}^{-1} \tag{25}
\]

5. Conclusions

The basic theory of TFP of a NS as the signature of the approach to NS breakup is a viable explanation of the kHz QPO observed in X-rays emitted by LMXB sources. A simple version of the theory relates the intrinsic properties of NS structure with the observed kHz frequencies.
The range of kHz frequencies observed is also explained by this simple dynamical model. Furthermore, the TFP theory provides a simple explanation for the high Q of the millisecond pulsar SAX J1808.4-3658 including the reason why it does not exhibit kHz QPO. Since the theory relates the ratio of the observed kHz frequencies to the ratios of the moments of inertia of the NS, it is not surprising that the EOS of NS matter is so tightly constrained by this interpretation.

The primary issue not fully addressed by the TFP model is the complex interaction of the crust and core of the NS. On a short timescale (∼seconds) the fluctuations in the kHz QPO frequencies are explained by likely fluctuations in the moments of inertia of the NS crust. On a longer timescale (∼hours) internal torques drive the system into a limit-cycle with a range of crust spin frequencies. Angular momentum may be conserved since the three dominant and independent reservoirs of angular momentum are the rotation of the fluid core, the rotation of the crust, and precession of the crust. These components can exchange angular momentum and vary their moments of inertia. The simple TFP model constrains the locus of the ratios of the kHz QPO frequencies but does not explain all the details of the dynamics. We have also outlined a plausible mechanism for a limit-cycle of exchange of angular momentum between the outer crust and a superfluid layer internal to the NS that is analogous to giant glitches in radio pulsars. The TFP theory predicts strong kHz GW emission and magnetic dipole radiation as very low frequency radio waves (∼1000 Hz).

A LIGO detection of GWs from an LMXB NS source due to TFP would be the definitive confirmation of the TFP theory. The high sustained accretion rates that such GW emission would require suggests that some LMXB NSs will collapse into BHs. This possibility suggests that NSs in LMXB are progenitors of some GRBs and leave behind maximally rotating Kerr BHs as the
final successors to these GRBs.

6. Acknowledgments

The author acknowledges Lynn Cominsky for helpful conversations and a careful reading of the manuscript. The author also thanks Michiel van der Klis for providing the previously published Sco X-1 data on kHz QPO in numerical form. Walter Lewin, Saul Rappaport, Wei Cui and Ben Owen are acknowledged for their generous efforts reading and commenting on the manuscript. NASA provided the primary support for the author who is a Co-I on the MIT portion of the RXTE project (PI: Hale Bradt) under a subcontract to the University of California, Berkeley (NAG 5-30612). The author thanks Hale Bradt for his sustained support of this subcontract. The author also thanks Bill Mayer and George Clark, the project scientist and principal investigator for SAS-3 respectively, and the APL engineers that build the spacecraft for SAS-3 for providing the opportunity to study torque free precession of coupled rigid bodies. In particular, the nutation damper crisis that occurred just after the launch of SAS-3 seeded the concept for this paper. Additional support was also provided by Eureka Scientific (President: John Vallerga). The work was carried out at the Space Sciences Laboratory of the University of California, Berkeley, and at the Little H-Bar Ranch located in Petaluma, California.
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Fig. 1.— (a) the difference frequency $\nu_u - \nu_\ell$ versus the Upper kHz QPO frequency $\nu_u$ for Sco X-1; The single point for GX 5-1 is the observation with the lowest value of $\nu_u$. The solid curve is the best fit TFP model. The dashed curve is the phenomenological function derived by Psaltis et al. 1998; (b) an expanded version of (a) showing the Sco X-1 data with error bars and the two model fits; (c) the ratio $\nu_\ell/\nu_u$ versus $\nu_u$. The TFP model is the solid curve; The dashed curve is the locus for a MacLaurin oblate spheroid. The locus is shown a model of a NS based EOS FPS from Cook, Shapiro, & Teukolsky 1994b. A vertical dot-dash line (label J1808) marks the spin frequency (401 Hz) of SAX J1808.4-3658. The three critical values of eccentricity $e$ are marked by horizontal dotted lines; (d) an expanded version of (c) showing the Sco X-1 data in detail.
