Nuclear matrix elements in neutrinoless double beta decay: beyond mean-field covariant density functional theory

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Abstract

We report a systematic study of nuclear matrix elements (NMEs) in neutrinoless double-beta decays with state-of-the-art beyond mean-field covariant density functional theory. The dynamic effects of particle-number and angular-momentum conservations as well as quadrupole shape fluctuations are taken into account with projections and generator coordinate method for both initial and final nuclei. The full relativistic transition operator is adopted to calculate the NMEs which are found to be consistent with the results of previous beyond non-relativistic mean-field calculation based on a Gogny force with the exception of \(^{150}\text{Nd}\). Our study shows that the total NMEs can be well approximated by the pure axial-vector coupling term, the calculation of which is computationally much cheaper than that of full terms.

Introduction. – The neutrinoless double beta (\(0\nu\beta\beta\)) decay is a process where an even-even nucleus decays into the even-even neighbor with two neutrons less and two protons more emitting only two electrons. The search of this lepton-number-violating (LNV) process in atomic nuclei is one of the current main experimental goals in nuclear and particle physics. This LNV process occurs only if the neutrino is a Majorana particle. In particular, several fundamental questions about the nature of a neutrino, such as its absolute mass scale and mass hierarchy, are expected to be answered by combining the results from the measurements of this process and neutrino oscillations [1, 2, 3, 4]. To date, the \(0\nu\beta\beta\)-decay has not been detected except for the controversial claim of detection in \(^{76}\text{Ge}\) by the Heiderberg-Moscow collaboration [5] that has recently been overruled by the constraints from the cosmology observations [6, 7] and the latest data released by the EXO-200, KamLAND-Zen and GERDA collaborations [8, 9, 10].

According to the neutrino mass mechanism of exchange light Majorana neutrinos, the inverse of the half-life \(T^{\nu\beta\beta}_{1/2}\) of the \(0\nu\beta\beta\) decay process is directly related to the effective Majorana neutrino mass [11, 12, 13]

\[
[T^{\nu\beta\beta}_{1/2}]^{-1} = G_{\text{\beta\beta}} g_{A}(0) \frac{(m_{\nu})^2}{m_e} |\mathbf{M}^{\nu\beta\beta}(0_{f} \rightarrow 0_{f}^\prime)|^2,  \tag {1}
\]

where the axial-vector coupling constant \(g_{A}(0)\) and the electron mass \(m_e\) are constants, and the kinematic phase-space factor \(G_{\text{\beta\beta}}\) can be determined precisely [14]. An accurate value of the nuclear matrix element (NME) \(\mathbf{M}^{\nu\beta\beta}\) is essential for determining the effective neutrino mass \((m_{\nu})\) if the decay rate is eventually measured. Although the \(0\nu\beta\beta\) decay has not bee observed yet, the NME can provide a constraint on the upper limit for the effective neutrino mass based on the current data on the lower limit of \(T^{\nu\beta\beta}_{1/2}\). Inversely, together with the constraints on the neutrino mass from other measurements, the NME can provide a lower limit on the half-life of the \(0\nu\beta\beta\)-decay, which serves as a guideline for the development of “next-generation” experiments [15]. In any case, an accurate knowledge of the NME for the \(0\nu\beta\beta\)-decay is therefore very important in nuclear physics, particle physics and cosmology [16, 17, 18, 19, 20, 21].

The calculation of the NME requires two main ingredients. One is the wave functions of the initial and final states, which have been calculated based on different nuclear models, including configuration-interaction shell model (ISM) [22, 23, 24, 25], quasi-particle random phase approximation (QRPA) [26, 27, 28, 29, 31], interacting boson model (IBM) [32], angular momentum projected Hartree-Fock-Bogoliubov (PHFB) theory based on a schematic Hamiltonian [33], and the beyond mean-field density functional theory (BMF-DFT) based on a non-relativistic energy density functional (NREDF) Gogny force [34, 35]. Compared with the PHFB, the BMF-DFT includes additional correlations connected with particle number projection, as well as fluctuations in quadrupole shapes [34] and pairing gaps [35]. Another important ingredient is the decay operator, which reflects the mechanism of decay process. All the previous calculations of the NMEs are based on non-relativistic frameworks, which adopt the non-relativistic reduced transition operators derived from charge-changing nuclear current.

In recent years, we have established a beyond mean-field covariant density functional theory (BMF-CDFT), which has been successfully applied to study many interesting phenomena re-
lated to nuclear low-lying states [36]. In the present Letter, we report a systematic calculation of nuclear structural properties and the NMEs for the popularly studied 0νββ decay candidate nuclei within this relativistic framework. The full relativistic transition operators derived from the one-body charge-changing nuclear current, together with the ground-state wave functions from the BMF-CDFT [36] are adopted in the calculation of the NMEs. Detailed formalism and a proof-of-principle calculation for the 0νββ decay in 150Nd have been described in Ref. [37]. The aim of this Letter is to present the systematic calculated results for the NMEs based on a relativistic energy density functional (REDF) and to make a detailed comparison with the previous BMF calculations based on the NREDF of Gogny force [34]. This study sheds some light on the sensitivity of the NMEs to the underlying EDF adopted in the BMF calculations and provides a benchmark for future studies.

**Formalism.**— In our BMF-CDFT, nuclear many-body wave functions of low-lying states in initial (I) mother or final (F) daughter nuclei are given as linear combinations of particle number N, Z, and angular momentum J projected relativistic mean-field (RMF) plus BCS wave functions |ψ⟩ constrained to have different intrinsic axial deformation β,

\[ |JMNZ; α⟩ = \sum_β f_{α}^{INZ}(β) P_{MK=α}^{J} P_{α}^{N} P_{α}^{Z}(β), \]

where α is a label distinguishing the states with same quantum numbers J, N, and Z. The \( P_{MK}^{J} \) and \( P_{NZ}^{J} \) are projection operators onto angular momentum J and particle number of neutrons or protons, respectively. This method is also referred to as GCM+PNAMP based on CDFT or multi-reference CDFT. The weight function \( f_{α}^{INZ}(β) \) is determined from the minimization of the total energy of the state, which leads to Hill-Wheeler-Griffin (HWG) equation [38]. The solution of the HWG equation provides the energy spectra and all the information needed for calculating the electric multipole transition strengths in the initial or final nuclei. We note that all the observables are calculated in full model space of occupied single-particle states.

With the ground-state wave functions \( |0^+_I⟩ \) of the initial and final nuclei from the BMF-CDFT calculation, the NME \( M^{βν} \) for the 0νββ decay can be calculated straightforwardly as follows [18, 37]

\[ M^{βν} = \frac{4πR}{g_{χ}^{2}(0)} \int d^3x_1 \int d^3x_2 \int d^3q \frac{e^{iq(x_1-x_2)}}{(2π)^3} g(q + E_d) \]

\[ × \langle 0^+_F | J_{Lμ}^{α}(x_1) J_{Lμ}^{α†}(x_2) |0^+_I⟩, \]

with the nuclear radius \( R = 1.2α^{1/3} \) is introduced to make the NME dimensionless. \( E_d \) is the average energy of intermediate states. Substituting the standard expression \( J_{Lμ}^{α} \) for the one-body charge-changing nuclear current, one finds that the NME is composed of five terms: vector coupling (VV), axial-vector coupling (AA), interference of the axial-vector and induced pseudoscalar coupling (AP), the induced pseudoscalar coupling (PP), and weak-magnetism coupling (MM) terms, which are related to the products of two current operators \( J_{Lμ}^{α}(x_1) J_{Lμ}^{α†}(x_2) \) respectively, where \( q^μ \) is the momentum transferred from leptons to nucleons, \( τ_\pm \) is the isospin lowering operator that changes neutrons into protons, and \( σ^μν = \frac{i}{2} [γ^μ, γ^ν] \). Following Ref. [26], the form factors \( g_V(q^2), g_A(q^2), g_P(q^2), \) and \( g_P(q^2) \) for neutrons and protons, respectively, which were determined to the accuracy of 10% on the NME for light nuclei, were adopted in the BMF-CDFT calculation for the structural properties of the ten pairs of 0νββ decay candidate nuclei.

**Numerical details.**— The mean-field wave functions |ψ⟩ in Eq. (2) are generated by the RMF calculation based on the point-coupling EDF PC-PK1 [39]. Pairing correlations between nucleons are treated with the BCS approximation using a density-independent δ force \( V_δ(r_1, r_2) = V_δ δ(r_1 - r_2) \) supplemented with an energy-dependent cutoff function. The pairing strength parameter \( V_δ = -314.555 \text{ MeV fm}^3 \) and the masses \( m_π = 0.93827 \text{ GeV} \) and \( m_τ = 0.13957 \text{ GeV} \) for pions, respectively.

In the calculation of the NME \( M^{βν} \), closure approximation is adopted with the average energy of intermediate states given by \( E_d = 1.12α^{1/3} \) [11]. All the terms in Eq. (4) are fully incorporated in the relativistic framework. The finite-nucleon-size (FNS) correction is taken care of by the momentum dependent form factors in Eq. (4). According to the recent studies based on the unitary correlation operator method (UCOM) [41, 23, 42, 43], the short range correlation (SRC) has a marginal reduction effect (< 10%) on the NME for light neutrons. To include it would considerably complicate our computational procedure, and therefore we omit the SRC contribution in the present study. More numerical details about the calculation of the NME within the BMF-CDFT have been introduced in Ref. [37].

**Structural properties of low-lying states.**— We first examine the reliability of our BMF-CDFT calculation for the structural properties of the ten pairs of 0νββ decay candidate nuclei 86Ca, 76Ge-Se, 82Se-Kr, 86Zr-Mo, 100Mo-Ru, 116Cd-Sn, 128Sn-Tc, 138Te-Xe, 138Xe-Ba, and 150Nd-Sm, which are compared with the results of the BMF-CDFT calculations based on the non-relativistic Gogny D1S force [34], and with the data in Fig. 1.
A good agreement with the data is found in both relativistic and non-relativistic BMP calculations for all the candidate nuclei except for the doubly closed-subshell nucleus $^{96}\text{Zr}$. For this nucleus, the data of high $E_i(2^+_1)$ and weak $B(E2: 0^+_1 \rightarrow 2^+_1)$ indicate the pronounced proton $Z = 40$ and $N = 56$ subshells. Actually, the overestimation of the collectivity in $^{96}\text{Zr}$ is a common problem of most EDF based GCM or collective Hamiltonian calculations [47, 48]. The lowest excited states in $^{96}\text{Zr}$ are of particle-hole type [49], the description of which requires the inclusion of noncollective configurations.

The beyond mean-field effects on nuclear binding energy and charge radius can be learnt from the comparison of the BMF-CDFT and CDFT results in Fig. 1. We define the dynamic correlation energy (DCE) as $E_{\text{DCE}} = E(0^+_1) - E(0^+_0)$, where $E(0^+_1)$ and $E(0^+_0)$ are the total energies of mean-field ground state (global minimum of energy surface) and $0^+_0$ state (correlated ground state), respectively. The DCE ranges from 1.6 MeV to 6.0 MeV, and improves overall the description of binding energies and $Q_{\beta\beta}$ values as depicted in Fig. 2. The $Q_{\beta\beta}$ value of $^{48}\text{Ca}$ has been reproduced in the non-relativistic calculation but is overestimated in our calculation by about 2.0 MeV after taking into account the DCE, which is 1.6 MeV and 4.8 MeV for $^{48}\text{Ca}$ and $^{48}\text{Ti}$, respectively. We note that in our calculation pairing collapse happens around the spherical configuration of $^{48}\text{Ca}$, but not in $^{48}\text{Ti}$. Therefore, a smaller DCE is gained in $^{48}\text{Ca}$ than in $^{48}\text{Ti}$, which leads to the overestimation of its $Q_{\beta\beta}$. However, pairing collapse is avoided in the non-relativistic calculation where the particle number projection has also been carried out before variation in the mean-field calculation. For $^{76}\text{Ge}$, on the other hand, the underestimation of the $Q_{\beta\beta}$ might be due to the deficiency of the underlying EDF or the missing of triaxiality, which turns out to be important in the low-lying states [50, 51].

Figure 3 displays the distribution of collective wave function $g^2(\beta)$ for the ground states of initial and final nuclei as a function of the deformation parameter $\beta$, where the $g^2(\beta)$ are related to the weight function $f_\alpha^{JNZ}(\beta)$ in Eq. (2) by the following relation,

$$g^2(\beta) = \sum_{\beta'} |\langle \beta' | J_{\lambda}^{N\lambda}(\beta') \rangle |^2 f_\alpha^{JNZ}(\beta'),$$

with the overlap kernel, $N^{J}(\beta, \beta') = \langle \beta | \tilde{P}^{J}_{\lambda_0} \hat{P}^{J}_{\lambda}(\beta') \rangle$. Since $g^2(\beta)$s are orthonormal, they can reflect the dominant configuration of each state. It is seen that the deformations of dominant configurations in the ground state of mother and daughter nuclei are somewhat different, as already discussed in Refs. [34, 37]. This static deformation effect can quench the NME of $0\beta\beta$ decay significantly in particular for the case where the deformation...
tions of the mother and daughter nuclei differ considerably from each other, such as 76Ge-Se and 150Nd-Sm. Moreover, shape fluctuation is shown to be significant in the light 0νββ candidate nuclei, the description of which is impossible with the approaches based on single-reference state \cite{33, 28, 29}. This dynamic deformation effect (or shape mixing effect) could moderate the quenching effect from the static deformation on the NMEs \cite{37}, which is fully taken into account in the present multi-reference BMF-CDFT approach.

_Nuclear matrix elements for the 0νββ decay._—In order to show the deformation-dependence of the NME, Table 1 presents the normalized NME $\tilde{M}^{0\nu}(\beta_1, \beta_F)$ at spherical shape ($\beta_1 = \beta_F = 0$) for the 0νββ-decay obtained with both the relativistic and non-relativistic reduced transition operators, where $\tilde{M}^{0\nu}$ is defined as

$$\tilde{M}^{0\nu}(\beta_1, \beta_F) = N_F N_I (\beta_1 \beta_F) \tilde{P}^{\nu} \tilde{P}^{\nu} (\beta_1 \beta_F),$$

with $N_F^{-2} = (\beta_1 \beta_F P^{\nu} \tilde{P}^{\nu} (\beta_1 \beta_F))$ for $a = I, F$. It is seen that the error arisen from the first-order non-relativistic reduction is marginal, which can either increase or decrease the total NME by a factor within 2%. This value is modified only slightly in the full GCM calculation, for instance becoming $\sim 5\%$ for 150Nd \cite{37}. The one-body charge-changing nucleon current, Eq. (4), generates not only the Fermi and Gamow-Teller (GT) terms but also tensor terms that have been neglected in the non-relativistic study \cite{34}. With the help of non-relativistic approximation of the transition operator, one can isolate the contribution of the tensor part \cite{26, 37}, which is obtained by subtracting the contributions of Fermi and GT terms from the total NME. It is shown in Table 1 that the contribution of tensor terms is within 5% of the total NME.

Figure 4 displays the normalized NME $\tilde{M}^{0\nu}$ as a function of the intrinsic quadrupole deformation $\beta_1$ and $\beta_F$ of the mother and daughter nuclei, respectively. Similar to the behavior of the GT part shown in the MR-DFT (D1S) calculation \cite{34}, the normalized NME $\tilde{M}^{0\nu}$ is concentrated rather symmetrically along the diagonal line $\beta_1 = \beta_F$, implying that the decay between nuclei with different deformation is strongly hindered. Moreover, the $\tilde{M}^{0\nu}$ has the largest value at the spherical configuration for most candidate nuclei except for 48Ca-Ti, 96Zr-Mo, and 136Xe-Ba. It implies that generally the 0νββ-decay is favored if both nuclei are spherical. The largest $\tilde{M}^{0\nu}$ in 136Xe-Ba is found around the deformation region with $\beta_1 = \beta_F \approx 0.5$, at which deformed configuration, pairing energy is peaked in both nuclei due to the very high single-particle level density. However, this configuration ($\beta \approx 0.5$) has a negligible contribution to the final NME of 136Xe-Ba because its weight is almost zero in the ground-state wave function, cf. Fig. 3.

Figure 5(a) displays the contribution of each coupling term $(AA, VV, PP, MM, AP)$ in Eq. (4) to the total NMEs. It is shown that the weak-magnetism (MM) term is negligible ($\sim 4\%$). The interference term of the axial-vector (AA) and pseudoscalar coupling (AP) has an opposite contribution ($\sim 30\%$), which almost cancels out the sum of VV, PP, and MM terms. Of particular interest is that the total NME has a very similar behavior as that of the predominated AA term with the ratio $R_{AA} \approx 95\%$. Actually, we have found that the deformation-dependent NMEs shown in Fig. 4 are also very similar even if we include only the AA term. It indicates that the AA term provides a good approximation for the total NME, Eq. (3). In the non-relativistic approximation, the two-current operator with only the axial-vector coupling term is simplified as $J_{\nu \nu}^{A\nu}(x_1) J_{\nu \nu}^{A\nu}(x_2) = -g^2 (q^2) \sigma^{[1]} \cdot \sigma^{[2]} P^{\nu} P^{\nu}$, the calculation of which is much cheaper than computing the full terms, cf. (4). Similar conclusion can also be made based on the results of QRPA calculation \cite{26} using the non-relativistic reduced operators. Figure 5(b) displays the NMEs calculated either with pure spherical configuration or with full configurations in the GCM+PNAMP (PC-PK1), in comparison with those of the non-relativistic results \cite{34}. Before comparing the two results, we should point out that in the non-relativistic calculation \cite{34}, the SRC effect was taken into account with the UCOM, while

Table 1: The normalized NME $\tilde{M}^{0\nu}$ for the 0νββ-decay obtained with the particle number projected spherical mean-field configuration ($\beta_1 = \beta_F = 0$) by the PC-PK1 force using both the relativistic and non-relativistic reduced (first-order of $q/m_\pi$ in the one-body current) transition operators. The ratio of the AA term to the total NME, $R_{AA} = M^{0\nu}_{AA}/M^{0\nu}$, the relativistic effect $\Delta_{rel} = (M^{0\nu} - M^{0\nu}_{NN})/M^{0\nu}$ and the ratio of the tensor part to the total NME, $R_T = M^{0\nu}_{TT}/M^{0\nu}$, are also presented.

| Nucleus | 0νββ | M^{0\nu} | R_{AA} | M^{0\nu}_{NN} | $\Delta_{rel}$ | R_{TT} |
|---------|-------|----------|--------|----------------|-------------|--------|
| $^{48}$Ca $\rightarrow ^{48}$Ti | 3.66 | 81% | 3.74 | -2.1% | -2.4% | |
| $^{76}$Ge $\rightarrow ^{76}$Se | 7.59 | 94% | 7.71 | -1.6% | 3.5% | |
| $^{82}$Se $\rightarrow ^{82}$Kr | 7.58 | 93% | 7.68 | -1.4% | 2.9% | |
| $^{96}$Zr $\rightarrow ^{96}$Mo | 5.64 | 95% | 5.63 | 0.2% | 3.6% | |
| $^{100}$Mo $\rightarrow ^{100}$Ru | 10.92 | 95% | 10.91 | 0.1% | 3.5% | |
| $^{116}$Cd $\rightarrow ^{116}$Sn | 6.18 | 94% | 6.13 | 0.7% | 1.9% | |
| $^{124}$Sn $\rightarrow ^{124}$Te | 6.66 | 94% | 6.78 | -1.8% | 4.9% | |
| $^{130}$Te $\rightarrow ^{130}$Xe | 9.50 | 94% | 9.64 | -1.4% | 4.3% | |
| $^{136}$Xe $\rightarrow ^{136}$Ba | 6.59 | 94% | 6.70 | -1.7% | 4.1% | |
| $^{150}$Nd $\rightarrow ^{150}$Sm | 13.25 | 95% | 13.08 | 1.3% | 2.5% | |

Figure 4: (Color online) Normalized NME $\tilde{M}^{0\nu}$ as a function of the intrinsic deformation parameter $\beta$ of the initial $^4Z$ and final $^4(Z + 2)$ nuclei.
the tensor terms were neglected. These two effects can bring a difference up to \(\sim 15\%\) in the NMEs. By taking into account this point, one can draw the conclusion from Fig. 5(b) that these two calculations give consistent results for the total NMEs for all the candidate nuclei with the exception of \(^{150}\)Nd.

Moreover, we note that in the calculation with pure spherical configuration, PNP increases significantly the NMEs for the \(0\nu\beta\beta\)-decay evolved with the 0νββ-decay from the present GCM+PNAMP (PC-PK1) calculation, the lower limits of the half-life \(T_{0ν}^{\betaβ}\) for the 0νββ-decay in comparison with those of GCM+PNAMP (D1S) from Ref. [38]. The shaded area indicates the uncertainty of the SRC effect within 10%. See text for more details.

Figure 5: (Color online) (a) Decomposition of the total NMEs from the final GCM+PNAMP (PC-PK1) calculation; (b) the total NMEs calculated with either only spherical configuration or full configurations, in comparison with those of GCM+PNAMP (D1S) from Ref. [38]. The shaded area indicates the uncertainty of the SRC effect within 10%. See text for more details.

Figure 6: (Color online) Comparison of the NME \(M^{0ν}\) for the 0νββ-decay from different model calculations. The shaded area indicates the uncertainty of the SRC effect within 10%. The adopted values are available on the web site [52].

Table 2: The upper limits of the effective neutrino mass (\(m_{ββ}\) (eV)) based on the NMEs from the present GCM+PNAMP (PC-PK1) calculation, the lower limits of the half-life \(T_{0ν}^{\betaβ}\) (\(\times 10^{25}\) yr) for the 0νββ-decay from most recent measurements [56, 10, 57, 58, 8, 9, 59] and the phase-space factor \(G_{0ν}\) (\(\times 10^{-15}\) yr\(^{-1}\)) from Ref. [14].

| \(^{48}\)Ca | \(^{76}\)Ge | \(^{82}\)Se | \(^{106}\)Mo | \(^{136}\)Te | \(^{136}\)Xe | \(^{150}\)Nd |
|---|---|---|---|---|---|---|
| \(m_{ββ}\) | \(≤ 2.92\) | \(≤ 0.20\) | \(≤ 1.00\) | \(≤ 0.38\) | \(≤ 0.53\) | \(≤ 0.11\) | \(≤ 1.76\) |
| \(T_{0ν}^{\ββ}\) | \(≥ 0.058\) | \(≥ 30\) | \(≥ 0.36\) | \(≥ 1.1\) | \(≥ 2.8\) | \(≥ 34\) | \(≥ 0.018\) |
| \(G_{0ν}\) | 24.81 | 2.363 | 10.16 | 15.92 | 14.22 | 14.58 | 60.03 |

Summary and outlook.— In summary, we have reported a...
The latest data on the 0\alpha-decay candidate nuclei with our BMF-CDFT, where the NMEs have been calculated with the full relativistic transition operators derived from one-body charge-changing nuclear current. The effects of particle number and angular momentum projection as well as the static and dynamic deformations in the nuclear wave functions have been taken into account automatically. The reliability of the nuclear wave functions has been examined by comparing the calculated low-energy structural properties with the corresponding data. The conclusions of the present study are summarized as follows: 1) the NMEs are overall consistent with the non-relativistic calculation results except for the phase transition critical nucleus \(^{150}\)Nd, indicating that the NMEs are not much sensitive to the underlying energy density functional; 2) The axial-vector coupling (\(\varepsilon\)) term exhausts about 95% of the total NME, which provides an economical way to calculate the total NME in the future; 3) The relativistic effect turns out to be within 2% and the contribution of tensor terms to the NME is within 5%. The PNP increases significantly the NME for the 0\alpha-decay where magic or semi-magic nuclei are evolved. The net effects of static and dynamic deformation turn out to reduce significantly the NME for most candidate nuclei; 4) The smallest upper limit on \(m_{\text{ee}}\) (\(\lesssim 0.11 \) eV) has been found based on the latest data on the 0\alpha-decay of \(^{136}\)Xe.

Finally, we point out that the effect of higher-order deformation need to be studied in this framework. Moreover, the quenching effect of two-body currents and enhancement effect of pairing fluctuation on the NME are comparable in size, that is, 10% – 40%. These two effects may not cancel out exactly. The fluctuation effect in proton-neutron pairing amplitude also has influence on the value of NME. A more careful study within the BMF-CDFT by taking into account all these effects will be carried out in the near future.

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References

1. Y. Fukuda, et al. (Super-Kamiokande), Phys. Rev. Lett. 81 (1998) 1562.
2. Q. R. Ahmad, et al. (SNO), Phys. Rev. Lett. 89 (2002) 013001.
3. K. Eguchi, et al. (KamLAND), Phys. Rev. Lett. 90 (2003) 021802.
4. F. P. An, et al. (Daya Bay), Phys. Rev. Lett. 108 (2012) 171803.
5. H. V. Klapdor-Kleingrothaus, et al., Phys. Lett. B 586 (2004) 198.
6. S. Hannestad, et al., JCAP 08 (2010) 001.
7. E. Komatsu, et al., ApJS 192 (2011) 18.
8. M. Auger, et al., (EXO), Phys. Rev. Lett. 109 (2012) 032505.
9. A. Gando, et al., (KamLAND-Zen), Phys. Rev. Lett. 110 (2013) 062502.
10. M. Agostini et al. (GERDA), Phys. Rev. Lett. 111 (2013) 122503.
11. W. C. Haxton and G. J. Stephenson, Prog. Part. Nucl. Phys. 12 (1984) 409.