Investigating Lorentz and CPT Symmetry with Antihydrogen

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This talk discusses theoretical aspects of tests of CPT and Lorentz Symmetry that will in principle be possible with trapped antihydrogen. The framework is the standard-model extension, which admits minuscule violations of CPT and Lorentz symmetry in a general manner without giving up other features of the standard model of particle physics. Spectroscopic transitions in hydrogen and antihydrogen that exhibit leading-order effects are identified. Such comparisons of spectral frequencies in antimatter with the corresponding frequencies in regular matter will bound parameter combinations that are not accessible with regular-matter atoms alone.

1. The Standard-Model Extension

The laws of physics appear to be unchanged under the Lorentz and CPT transformations. These symmetries are built into the standard model of particle physics, and so calculations in the usual standard-model context cannot predict how violations of these symmetries might occur. However, the standard-model extension [1] allows symmetry-violating effects to be investigated in an explicit microscopic framework. In this proceedings, I give an overview of results in this context, mention the experimental findings, and then focus on hydrogen-antihydrogen symmetry tests investigated in collaboration with R. Bluhm and V.A. Kostelecky [2].

It is possible that a Lorentz- and CPT-symmetric fundamental theory underlying the standard model could violate these symmetries [3], perhaps through spontaneous symmetry breaking at lower energies [4]. No satisfactory fundamental theory is known, but a candidate is string theory. A practical way to proceed is to perform calculations in the context of the standard-model extension [1]. The standard-model extension adds to the conventional standard model all possible terms built from conventional fields that violate Lorentz symmetry while preserving observer coordinate independence. This involves a large set of parameters, which can be reduced by making simplifying assumptions. For example, it is often useful to require that the terms be gauge invariant, position independent, and renormalizable. Since the standard-model extension is expected to be the low-energy limit of an underlying theory, the symmetry violations would most likely be suppressed by ratios involving the low-energy mass and the Planck mass. Other theoretical issues about the standard-model extension include supersymmetry [5], noncommutative field theory [6], and anomaly cancellations [7].

The standard-model extension is applicable to all areas of physics. Applications of this framework include a possible mechanism for generating the baryon asymmetry in the universe [8]. For the neutral mesons, the prediction of sidereal dependencies [9] in transition probabilities have been tested in the neutral kaons [10] and in the neutral-B systems [11].
Other relevant aspects of neutral-meson physics include efforts to bound symmetry parameters in the neutral-D system [12] and studies of analogue models [13]. In the photon sector, excellent bounds on Lorentz symmetry have been placed through analysis of light from distant cosmological sources [1, 14]. In the lepton sector, recent results on muonium and on anomaly-frequency comparisons of oppositely-charged muons at CERN and BNL have been obtained [15]. Electron-positron comparisons using Penning traps have placed several bounds on standard-model extension parameters [16]. An experiment with a spin-polarized torsion pendulum has obtained impressive results [17].

Clock-comparison experiments [18], comparing spectral lines of atomic transitions, are capable of Planck-scale tests of Lorentz symmetry [19]. In the context of the standard-model extension, tests have been performed with Hg and Cs magnetometers [20], a noble-gas maser [21], and with an H maser [22]. In such experiments, rotational symmetry is tested by monitoring frequency variations of a Zeeman hyperfine transition as the quantization axis of the clock changes direction with the Earth’s rotation. A practical way to do this is to operate two atomic clocks of different atomic species in the same laboratory, so as to avoid issues with signal propagation. To increase the sensitivity of clock-comparison experiments to other components of the standard-model extension couplings, and to address various other limitations, there is considerable interest in mounting experiments of this type on the International Space Station or other spacecraft [23].

Calculations in the standard-model extension context indicate that comparisons of matter and antimatter counterparts provide clean tests of CPT symmetry. It would be ideal to compare the narrow spectral lines of an atomic clock with the spectral lines of the corresponding antimatter clock. However, high-Z atoms like Rb or Cs used in current clocks are unlikely to be produced in antimatter form. For atoms of low atomic number, however, developments indicate that it may soon be possible to investigate the spectral lines of cold antimatter.

2. Antihydrogen

After experiments at CERN and Fermilab produced hot free antihydrogen (\(\bar{\text{H}}\)) [24], the ATHENA and ATRAP collaborations [25] at CERN have worked towards the goals of obtaining cold and trapped antihydrogen [26]. Numerous technological advances have been made along the way [27]. The reported production of cold \(\bar{\text{H}}\) atoms [28] by the ATHENA collaboration marks a milestone on this path. When spectroscopy of \(\bar{\text{H}}\) atoms is performed, it will mark the first direct comparisons of neutral atoms with the corresponding antimatter clock, and clean tests of CPT symmetry are likely to result.

From the experimental perspective, a comparison of the 1S to 2S transition in the two systems would be natural because of the parts in \(10^{14}\) and \(10^{12}\) precisions possible with hydrogen (H) beams [29] and with trapped H [30]. In principle, the maximum possible precision is about a part in \(10^{18}\), or about a 1-mHz resolution [31]. With the same precision in \(\bar{\text{H}}\), an excellent test of CPT would be achieved.

From the theoretical perspective, there are many possible transitions in H and \(\bar{\text{H}}\) atoms, and the challenge is to identify those that are most likely to show CPT-violating differences.
The point is that some will be suppressed by powers of the fine structure constant, or perhaps by the speed of the atoms in the reference frame. The standard-model extension provides a calculational framework of powerful generality for analyzing the various transitions in $H$ and $\bar{H}$. In the following sections, I consider the spectroscopy of free atoms, and then atoms confined in magnetic traps. The 1S to 2S transitions and the hyperfine transitions are considered.

### 3. Free Hydrogen and Antihydrogen

The effect of the standard-model extension on free atoms of $H$ or $\bar{H}$ can be studied by calculating the perturbative shifts in energy levels of the quantum stationary states, starting from the modified Dirac equation,

$$
(i\gamma^\mu D_\mu - m_e - a^e_\mu \gamma^\mu - b^e_\mu \gamma_5 \gamma^\mu - \frac{1}{2} H^e_{\mu\nu} \sigma^{\mu\nu} + ic^e_{\mu\nu} \gamma^\mu D^\nu + id^e_{\mu\nu} \gamma_5 \gamma^\mu D^\nu) \psi = 0 \quad .
$$

In this expression, $\psi$ is a four-component electron field describing an electron of mass $m_e$ and charge $q = -|e|$. We define $iD_\mu \equiv i\partial_\mu - qA_\mu$, and use units with $\hbar = c = 1$. The field interacts with the proton Coulomb potential $A^\mu = (|e|/4\pi r, 0)$. The coefficients $a^e_\mu$, $b^e_\mu$, $H^e_{\mu\nu}$, $c^e_{\mu\nu}$ and $d^e_{\mu\nu}$ parameterize Lorentz violation, and the coefficients $a^p_\mu$ and $b^p_\mu$ also parameterize CPT violation. All are expected to be minuscule \cite{1}. Energy shifts in the $H$ and $\bar{H}$ systems could also be due to perturbative couplings $a^p_\mu$, $b^p_\mu$, $H^p_{\mu\nu}$, $c^p_{\mu\nu}$, and $d^p_{\mu\nu}$ in a modified Dirac equation for free protons.

In conventional relativistic quantum mechanics, the $H$ and $\bar{H}$ hamiltonians are identical: reversing the charges of the particles leaves the interaction potential unchanged, and the reduced mass is also unaffected. Thus, the eigenfunctions and associated energies are the same for $H$ and $\bar{H}$, and within conventional physics there should be no observable differences between the spectra of $H$ and $\bar{H}$. This argument carries over to all perturbative effects from conventional quantum electrodynamics.

If the Lorentz- and CPT-violating couplings are included in a perturbative calculation, the energy shifts in the spectrum of the $H$ electron can differ from those for the $\bar{H}$ positron. To make the comparison, the perturbative effect in $\bar{H}$ is found from Eq. (1) by a method involving charge conjugation and appropriate field redefinitions \cite{32}. The energy perturbations arising from the proton and antiproton may also be included by using relativistic two-fermion techniques \cite{33}. In the uncoupled basis with electronic angular momentum $J$ and nuclear angular momentum $I$, and corresponding third components $m_J$, $m_I$, the perturbative Lorentz-violating energy corrections for the basis states $|m_J, m_I\rangle$ may be found. With this approach, the energy corrections for protons or antiprotons have the same mathematical form as those for electrons or positrons, except for the replacement of superscripts $e$ with $p$. The shifts in the 1S and 2S energy levels for free $H$ are found to be matched at leading order:

$$
\Delta E^H(m_J, m_I) \approx (a^e_0 + a^p_0 - c^e_{00} m_e - c^p_{00} m_p) \\
+ (-b^e_3 + d^e_{30} m_e + H^e_{12}) m_J / |m_J| \\
+ (-b^p_3 + d^p_{30} m_p + H^p_{12}) m_I / |m_I| ,
$$

(2)
where $m_p$ is the proton mass. The same is true for $\overline{\text{H}}$; the leading-order energy shifts $\Delta E^{\overline{\text{H}}}$ in the 1S and 2S levels are identical. They are obtained from the expression (2) with the substitutions $a_e^\mu \to -a_e^\mu$, $d_{e\mu\nu} \to -d_{e\mu\nu}^\prime$, $a_p^\mu \to -a_p^\mu$, $d_{p\mu\nu} \to -d_{p\mu\nu}^\prime$, $H_{e\mu\nu} \to -H_{e\mu\nu}^\prime$.

To find the effect of the Lorentz-violation on the frequency spectrum of $\text{H}$ and $\overline{\text{H}}$, we must calculate appropriate energy differences. Since the electron and proton spins in $\text{H}$ interact through the hyperfine interaction, the appropriate basis is the coupled basis $|F,m_F\rangle$, where $F$ is the total angular momentum, and $m_F$ is the third component. Similarly, this is true for the positron and antiproton spins in $\overline{\text{H}}$. There are four allowed 1S–2S two-photon transitions in $\text{H}$ and $\overline{\text{H}}$, determined by the selection rules $\Delta F = 0$ and $\Delta m_F = 0$ [34]. In each, the spins remain unchanged. It follows from (2) that the frequencies are unshifted at leading order since both energy levels in the transition are increased or decreased by identical amounts. So, the standard-model extension shows no leading-order evidence of Lorentz or CPT violation in 1S–2S spectroscopy of free $\text{H}$ or $\overline{\text{H}}$.

One way to proceed is to consider the next-largest energy-level shifts in the Lorentz-violating couplings for $\text{H}$ and $\overline{\text{H}}$. They are suppressed by a factor of $\alpha^2$. However, the shifts differ in some cases, and so effects observable in principle could occur. One possibility is the $m_F = 1 \to m_F' = 1$ line, which is shifted by

$$\delta \nu_{1S-2S}^{\text{H}} \approx -\alpha^2 b_3'/8\pi$$

relative to the unshifted $m_F = 0 \to m_F' = 0$ line. Suppression by a factor at least of order $\alpha^2 \approx 5 \times 10^{-5}$ would also occur in the proton-antiproton corrections to the 1S to 2S lines.

Because of these suppression effects in free $\text{H}$ and $\overline{\text{H}}$ lines, it is important to note that electron-position anomaly-frequency comparisons in Penning traps [32] are capable of producing tighter bounds on similar combinations of Lorentz and CPT violating parameters. Similar g-2 experiments with protons and antiprotons could in principle also place tighter bounds on proton-antiproton parameters than would be possible with 1S to 2S spectroscopy.

The advantages of Penning-trap experiments over spectroscopy in free $\text{H}$ and antihydrogen may seem counterintuitive given that g-2 experiments compare gyromagnetic ratios with fractional resolutions of about $2 \times 10^{-12}$, as compared to the ultimate fractional resolution of about $10^{-18}$ in $\text{H}$, and perhaps $\overline{\text{H}}$, experiments. However, bounds on the coefficients of the Lorentz- and CPT-violating terms are determined by absolute, not relative, resolutions. In principle, 1S to 2S transitions could reach absolute resolutions of about one mHz, as compared with Penning-trap frequencies measured with about one-Hz resolution. However, the suppression in the former by about five orders of magnitude would effectively give the Penning-trap experiments a 100-fold advantage in placing bounds. These limitations of $\text{H}$ and antihydrogen in testing Lorentz and CPT symmetry apply only to the free case where the transitions maintain the spin direction. Other considerations apply to $\text{H}$ and antihydrogen confined by external fields.

4. Trapped hydrogen and antihydrogen

Given the technical difficulties of creating antihydrogen atoms, the advantages of trapping them appear clear. From the perspective of testing Lorentz and CPT symmetry, the external
trapping fields offer the possibility of separating different spin states and hence of avoiding the suppression effects seen in the previous case of free H and \( \overline{\text{H}} \). We limit our discussion to the case of H or antihydrogen in a constant uniform magnetic field. This provides an approximation to the physical environment of the Ioffe-Pritchard trap [35], relevant in the CERN antihydrogen experiments.

In a uniform magnetic field \( B \), the four eigenstates corresponding to any given principal quantum number \( n \) are dependent on the magnetic field. In increasing energy, we denote each of the 1S and 2S hyperfine Zeeman levels by \( |a\rangle_n, |b\rangle_n, |c\rangle_n, |d\rangle_n \), with \( n = 1 \) or 2, for both H and \( \overline{\text{H}} \). The states for H, expressed in terms of the uncoupled basis \( |m_J, m_I\rangle \) are

\[
|d\rangle_n = |\frac{1}{2}, \frac{1}{2}\rangle , \\
|c\rangle_n = \sin \theta_n |\frac{1}{2}, -\frac{1}{2}\rangle + \cos \theta_n |\frac{1}{2}, \frac{1}{2}\rangle , \\
|b\rangle_n = |\frac{1}{2}, -\frac{1}{2}\rangle , \\
|a\rangle_n = \cos \theta_n |\frac{1}{2}, -\frac{1}{2}\rangle - \sin \theta_n |\frac{1}{2}, \frac{1}{2}\rangle .
\]

The mixing angles \( \theta_n \) are magnetic-field dependent, and differ for the 1S and 2S states:

\[
\tan 2\theta_n \approx \frac{(51 \text{ mT})}{n^3 B} .
\]  

The energies of the \( |c\rangle_1 \) and \( |d\rangle_1 \) states increase with increasing \( |B| \), and so are low-field seekers. In principle, they remain confined near the region of minimum field in the trap. Collisions within the trap, however, lead to spin exchange collisions \( |c\rangle_1 + |c\rangle_1 \rightarrow |b\rangle_1 + |d\rangle_1 \) that tend to decrease the \( |c\rangle_1 \) population, and create a predominance of confined \( |d\rangle_1 \) states.

The transition between these unmixed-spin states would appear attractive since it avoids broadening due to magnetic-field dependence. A Lorentz or CPT test could be envisaged involving comparison of the 1S-2S transition \( |d\rangle_1 \rightarrow |d\rangle_2 \) in H and in \( \overline{\text{H}} \). However, in both H and \( \overline{\text{H}} \), the spin configurations of the \( |d\rangle_1 \) and \( |d\rangle_2 \) states are the same, so there are again no leading-order shifts in these frequencies.

A comparison between the \( |c\rangle_1 \rightarrow |c\rangle_2 \) transitions in H and antihydrogen is of more interest for testing Lorentz or CPT symmetry. The difference here is that the spin-mixing coefficients \( \sin \theta_n \) and \( \cos \theta_n \) depend on the principal quantum number \( n \), and lead to an unsuppressed shift in the corresponding frequency. For H:

\[
\delta \nu_c^H \approx -\kappa (b_3^e - b_3^p - d_{30}^em_e + d_{30}^pm_p - H_{12}^e + H_{12}^p)/2\pi ,
\]

where the spin-mixing function \( \kappa \) is

\[
\kappa \equiv \cos 2\theta_2 - \cos 2\theta_1 .
\]

It is positive, less than one, and has maximum value \( \kappa \approx 0.67 \) occurring at \( B \approx 0.011 \text{ T} \). At this field value the atoms are optimally sensitive to any Lorentz- or CPT-violating effects.

The frequency \( \nu_c^H \) depends on spatial components of Lorentz-violating couplings and would therefore vary sidereally due to the rotation of the Earth. This effect means it is
possible in principle to detect Lorentz-violating signals with H alone. Such clock-comparison tests \[13\] involve comparing two different transitions, and have been done with a H maser \[21\] and other atomic systems \[24\, 22\]. A limitation of these experiments is that they bound combinations of couplings that violate Lorentz symmetry, but do not isolate CPT-violating couplings alone. The analysis can be complicated because of the geometry of motion of the laboratory relative to a suitable inertial reference frame.

In the case of $\overline{H}$, the calculation of $\delta \nu^\overline{H}$ for the same magnetic field direction and magnitude involves using the opposite positron and antiproton spins compared to the electron and proton spins in H. The result is identical to $\delta \nu^H$ in Eq. (3) except that the signs of $b^e_3$ and $b^p_3$ are reversed:

$$\delta \nu^\overline{H}_c \approx -\kappa (b^3_3 - b^3_3 - d_{30}^e m_e + d_{30}^p m_p - H_{12}^e + H_{12}^p)/2\pi .$$

The result (8) can be compared with (4), to give the instantaneous difference

$$\Delta \nu_{1S-2S,c} \equiv \nu^H - \nu^\overline{H}_c \approx -\kappa (b^e_3 - b^p_3)/\pi .$$

This expression involves couplings that are both CPT violating, and so provides a clean test of CPT symmetry. It should be noted that the H and antihydrogen measurements need to be made at effectively equal times to ensure that the magnetic field has the same orientation for both cases.

5. Hyperfine Transitions

Hyperfine transitions within the 1S level of H can be measured with accuracies exceeding 1 mHz in masers \[30\]. So transitions of this type in trapped H and antihydrogen are interesting candidates for performing tests of Lorentz or CPT symmetry.

To find the energy shifts in the possible transitions within the 1S level of H or $\overline{H}$, we may ignore the constant shift of $a_0^e + a_0^p - c_{90}^em_e - c_{90}^pm_p$ occurring in each of them, since this cannot contribute to the difference in energy levels. The terms of relevance are

$$\Delta E^H_a \approx \hat{\kappa} (b^3_3 - b^3_3 - d_{30}^e m_e + d_{30}^p m_p - H_{12}^e + H_{12}^p) ,$$

$$\Delta E^H_b \approx b^3_3 + b^3_3 - d_{30}^e m_e - d_{30}^p m_p - H_{12}^e - H_{12}^p ,$$

$$\Delta E^H_c \approx -\Delta E^H_a ,$$

$$\Delta E^H_d \approx -\Delta E^H_b ,$$

where $\hat{\kappa} \equiv \cos 2\theta_1$. Conventionally, the triplet $|b\rangle_1, |c\rangle_1$, and $|d\rangle_1$ is degenerate at zero field. However, with the presence of Lorentz- and CPT-violating couplings, this degeneracy is broken even at zero field: equal and opposite energy shifts occur for $|b\rangle_1$ and $|d\rangle_1$. In fact, the difference in the $|d\rangle_1 \rightarrow |a\rangle_1$ and $|b\rangle_1 \rightarrow |a\rangle_1$ transitions is unsuppressed and varies sidereally,

$$|\Delta \nu^H_{d-b}| \approx |b^e_3 + b^p_3 - d_{30}^e m_e - d_{30}^p m_p - H_{12}^e - H_{12}^p|/\pi .$$

In a nonzero magnetic field, energies are shifted in all four hyperfine Zeeman levels. The function $\hat{\kappa} \equiv \cos 2\theta_1$ controls the shifts in $|a\rangle_1$ and $|c\rangle_1$. As $B$ increases, so does $\hat{\kappa}$, reaching $\hat{\kappa} \approx 1$ when $B \approx 0.3$ T.
The leading-order effects from CPT and Lorentz violation in the standard maser line, $|c\rangle_1 \rightarrow |a\rangle_1$, are suppressed. This is because at the magnetic field of about $B \approx 10^{-6}$ Tesla, the function $\hat{\kappa}$ is approximately $\hat{\kappa} \lessapprox 10^{-4}$. Two other candidate transitions for detecting unsuppressed Lorentz or CPT violation are in the field-dependent transitions $|d\rangle_1 \rightarrow |a\rangle_1$ and $|b\rangle_1 \rightarrow |a\rangle_1$. The difference between these two frequencies is $\Delta \nu^{H}_{d\rightarrow b}$, and could be used to study unsuppressed sidereal variations. One might, however, expect experimental limitations such as broadening of the line due to field dependence.

Another possibility would be to measure radio-frequency transitions between states within the triplet of hyperfine levels in H and $\overline{H}$. This would also offer the possibility of working at a field-independent point where the limitations of magnetic-field inhomogeneities can be eliminated while maintaining sensitivity to Lorentz- and CPT-violating effects. An interesting option would be spectroscopy of the $|d\rangle_1 \rightarrow |c\rangle_1$ transition at the field-independent point $B \approx 0.65$ Tesla. One might expect various experimental challenges requiring high homogeneity of the field and extremely low temperatures. Assuming these issues can be handled, frequency resolutions of order $1 \text{ mHz}$ might be envisaged.

For this $|d\rangle_1 \rightarrow |c\rangle_1$ transition at this field of $B \approx 0.65$ Tesla, the electron and proton spins in state $|c\rangle_1$ interact more strongly with the field than with each other; they are highly polarized with $m_J = 1/2$ and $m_I = -1/2$. So, the transition $|d\rangle_1 \rightarrow |c\rangle_1$ is basically a proton spin-flip, and the frequency shifts are found to be

$$\delta \nu^{H}_{c\rightarrow d} \approx ( - b^p_3 + d^p_{30} m_p + H^p_{12})/\pi, \quad \delta \nu^{\overline{H}}_{c\rightarrow d} \approx ( b^p_3 + d^p_{30} m_p + H^p_{12})/\pi$$

for H and $\overline{H}$. One would thus expect these frequencies to exhibit sidereal variations. Another option would be to consider the instantaneous difference in these variations,

$$\Delta \nu_{c\rightarrow d} \equiv \nu^{H}_{c\rightarrow d} - \nu^{\overline{H}}_{c\rightarrow d} \approx -2b^p_3 / \pi.$$  \hspace{1cm} (14)

This comparison could provide a clean and unsuppressed test of the CPT-violating coupling $b^p_3$ for the proton. If, for example, a frequency resolution of $1 \text{ mHz}$ were attained, this would correspond to an upper bound of about

$$|b^p_3| \lessapprox 10^{-27} \text{GeV}$$

Compared with estimated bounds in Penning traps, this is an improvement of about 1000 [32]. It is also more than 10,000 times better than estimated bounds attainable from $1S-2S$ transitions.

Clock-comparison experiments are able to resolve spectral lines to about $1 \mu\text{Hz}$ [18]. However, the coupling expressions they bound [19] are complex and isolation of $b^p_3$ is difficult, due to the structure of the nuclei involved. We also note that the experiments discussed here are sensitive only to spatial components of Lorentz-violating couplings. Sensitivity to timelike couplings, like $b^e_0$, would require a boost, but would magnify CPT- and Lorentz-violating effects [4].
6. Conclusion

The standard-model extension has been used to analyze various transitions in the H and antihydrogen systems, with the aim of understanding ways in which Lorentz and CPT symmetry violations may occur. In free H and $\bar{H}$, signals for Lorentz and CPT violation are suppressed by at least one power of the fine-structure constant. For trapped H or $\bar{H}$ atoms, 1S-2S transitions involving the mixed-spin $|c\rangle$ states, or the spin-flip $|d\rangle_1 \rightarrow |c\rangle_1$ hyperfine transition could give rise to unsuppressed signals of Lorentz and CPT violation. Such signals would indicate qualitatively new physics existing in a fundamental theory at the Planck scale.

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