**Study of the $\psi(1S,2S)$ and $\eta_c(1S,2S)$ weak decays into DM**

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**Abstract**

Inspired by the recent measurements on the $J/\psi \rightarrow D_s \rho, D_u K^*$ weak decays at BESIII and the potential prospects of charmonium at high-luminosity heavy-flavor experiments, we study $\psi(1S,2S)$ and $\eta_c(1S,2S)$ weak decays into final states including one charmed meson plus one light meson, considering QCD corrections to hadronic matrix elements with QCD factorization approach. It is found that the Cabibbo-favored $\psi(1S,2S) \rightarrow D_s^- \rho^+, D_s^- \pi^+, D_s^0 \bar{K}^{*0}$ decays have large branching ratios $\gtrsim 10^{-10}$, which might be accessible at future experiments.

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More than forty years after the discovery of the $J/\psi$ meson, the properties of charmonium (bound state of $c\bar{c}$) continue to be the subject of intensive theoretical and experimental study. It is believed that charmonium, resembling bottomonium (bound state of $b\bar{b}$), plays the same role in exploring hadronic dynamics as positronium and/or the hydrogen atom plays in understanding the atomic physics. Charmonium and bottomonium are good objects to test the basic ideas of QCD [1]. There is a renewed interest in charmonium due to the plentiful dedicated investigation from BES, CLEO-c, LHCb and the studies via decays of the $B$ mesons at $B$ factories.

The $\psi(1S,2S)$ and $\eta_c(1S,2S)$ mesons are $S$-wave charmonium states below open-charm kinematic threshold, and have the well-established quantum numbers of $I^GJ^{PC} = 0^+1^{--}$ and $0^+0^{--}$, respectively. They decay mainly through the strong and electromagnetic interactions. Because the $G$-parity conserving hadronic decays $\psi(2S) \rightarrow \pi\pi J/\psi$, $\eta J/\psi$ and $\eta_c(2S) \rightarrow \pi\pi\eta_c(1S)$ are suppressed by the compact phase space of final states, and because the decays into light hadrons are suppressed by the phenomenological Okubo-Zweig-Iizuka (OZI) rules [2-4], the total widths of $\psi(1S,2S)$ and $\eta_c(1S,2S)$ are narrow (see Table I), which might render the charmonium weak decay as a necessary supplement. Here, we will concentrate on the $\psi(1S,2S)$ and $\eta_c(1S,2S)$ weak decays into $DM$ final states, where $M$ denotes the low-lying $SU(3)$ pseudoscalar and vector meson nonet. Our motivation is listed as follows.

| meson     | $I^GJ^{PC}$ | mass (MeV)       | width     |
|-----------|-------------|------------------|-----------|
| $\psi(1S)$ | $0^+1^{--}$ | $3096.916\pm0.011$ | $92.9\pm2.8\text{ keV}$ |
| $\psi(2S)$ | $0^+1^{--}$ | $3686.109^{+0.012}_{-0.014}$ | $299\pm8\text{ keV}$ |
| $\eta_c(1S)$ | $0^+0^{--}$ | $2983.6\pm0.7$ | $32.2\pm0.9\text{ MeV}$ |
| $\eta_c(2S)$ | $0^+0^{--}$ | $3639.4\pm1.3$ | $11.3^{+3.2}_{-2.9}\text{ MeV}$ |

From the experimental point of view: (1) some $10^9 \psi(1S,2S)$ data samples have been collected by BESIII since 2009 [6]. It is inspiringly expected to have about 10 billion $J/\psi$ and 3 billion $\psi(2S)$ events at BESIII experiment per year of data taking with the designed
luminosity \( \mathcal{L} \); over \( 10^{10} J/\psi \) at LHCb \([8]\), ATLAS \([9]\) and CMS \([10]\) per fb\(^{-1}\) data in \( pp \) collisions. A large amount of data sample offers a realistic possibility to explore experimentally the charmonium weak decays. Correspondingly, theoretical study is very necessary to provide a ready reference. (2) Identification of the single \textit{D} meson would provide an unambiguous signature of the charmonium weak decay into \textit{DM} states. With the improvements of experimental instrumentation and particle identification techniques, accurate measurements on the nonleptonic charmonium weak decay might be feasible. Recently, a search for the \( J/\psi \rightarrow D_s \rho, D_u K^* \) decays has been performed at BESIII, although signals are unseen for the moment \([11]\). Of course, the branching ratios for the inclusive charmonium weak decay is tiny within the standard model, about \( 2/(\tau_D \Gamma_\psi) \sim 10^{-8} \) and \( 2/(\tau_D \Gamma_\eta_c) \sim 10^{-10} \), where \textit{D} denotes the neutral charmed meson \([12]\), \( \Gamma_\psi \) and \( \Gamma_\eta_c \) stand for the total widths of the \( \psi(1S, 2S) \) and \( \eta_c(1S, 2S) \) resonances, respectively. Observation of an abnormally large production rate of single charmed mesons in the final state would be a hint of new physics beyond the standard model \([12]\).

From the theoretical point of view: (1) The charm quark weak decay is more favorable than the bottom quark weak decay, because the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements obey \( |V_{cb}| \ll |V_{cs}| \). Penguin and annihilation contributions to nonleptonic charm quark weak decay, being proportional to the CKM factor \( |V_{cb}V_{ub}| \sim \mathcal{O}(\lambda^5) \) with the Wolfenstein parameter \( \lambda \approx 0.22 \), are highly suppressed, and hence negligible relative to tree contributions. Both \textit{c} and \( \bar{c} \) quarks in charmonium can decay individually, which provides a good place to investigate the dynamical mechanism of heavy flavor weak decay and crosscheck model parameters obtained from the charmed hadron weak decays. (2) There are few works devoted to nonleptonic \( J/\psi \) weak decays in the past, such as Ref. \([13]\) with the covariant light-cone quark model, Ref. \([14]\) with QCD sum rules, and Refs. \([15–17]\) with the Wirbel-Stech-Bauer (WSB) model \([18]\). Moreover, previous works of Refs. \([13–17]\) concern mainly the weak transition form factors between the \( J/\psi \) and charmed mesons. Fewer papers have been devoted to nonleptonic \( \psi(2S) \) and \( \eta_c(1S, 2S) \) weak decays until now even though a rough estimate of branching ratios is unavailable. In this paper, we will estimate the branching ratios for nonleptonic two-body charmonium weak decay, taking the nonfactorizable contributions to hadronic matrix elements into account with the attractive QCD factorization (QCDF) approach \([19]\).

This paper is organized as follows. In section \([\text{II}]\), we will present the theoretical framework.
and the amplitudes for the $\psi(1S, 2S)$, $\eta_c(1S, 2S) \rightarrow DM$ decays. Section III is devoted to numerical results and discussion. Finally, section IV is our summation.

II. THEORETICAL FRAMEWORK

A. The effective Hamiltonian

Phenomenologically, the effective Hamiltonian responsible for charmonium weak decay into $DM$ final states can be written as [20]:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q_1, q_2} V_{cq_1}^* V_{uq_2} \left\{ C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu) \right\} + \text{H.c.},$$

where $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$ is the Fermi coupling constant; $V_{cq_1}^* V_{uq_2}$ is the CKM factor with $q_{1,2} = d, s$; The Wilson coefficients $C_{1,2}(\mu)$, which are independent of one particular process, summarize the physical contributions above the scale of $\mu$. The expressions of the local tree four-quark operators are

$$Q_1 = [\bar{q}_{1,\alpha} \gamma_\mu (1 - \gamma_5)c_\alpha][\bar{u}_\beta \gamma^\mu(1 - \gamma_5)q_{2,\beta}],$$

$$Q_2 = [\bar{q}_{1,\alpha} \gamma_\mu (1 - \gamma_5)c_\beta][\bar{u}_\beta \gamma^\mu(1 - \gamma_5)q_{2,\alpha}],$$

where $\alpha$ and $\beta$ are color indices.

It is well known that the Wilson coefficients $C_i$ could be systematically calculated with perturbation theory and have properly been evaluated to the next-to-leading order (NLO). Their values at the scale of $\mu \sim O(m_c)$ can be evaluated with the renormalization group (RG) equation [20]

$$C_{1,2}(\mu) = U_4(\mu, m_b)U_5(m_b, m_W)C_{1,2}(m_W),$$

where $U_f(\mu_f, \mu_i)$ is the RG evolution matrix which transforms the Wilson coefficients from scale of $\mu_i$ to $\mu_f$. The expression for $U_f(\mu_f, \mu_i)$ can be found in Ref. [20]. The numerical values of the leading-order (LO) and NLO $C_{1,2}$ in the naive dimensional regularization scheme are listed in Table II. The values of coefficients $C_{1,2}$ in Table II agree well with those obtained with “effective” number of active flavors $f = 4.15$ [20] rather than formula Eq.(4).

To obtain the decay amplitudes and branching ratios, the remaining works are to evaluate accurately the hadronic matrix elements (HME) where the local operators are sandwiched between the charmonium and final states, which is also the most intricate work in dealing with the weak decay of heavy hadrons by now.
B. Hadronic matrix elements

Analogous to the exclusive processes with perturbative QCD theory proposed by Lepage and Brodsky [21], the QCDF approach is developed by Beneke et al. [19] to deal with HME based on the collinear factorization approximation and power counting rules in the heavy quark limit, and has been extensively used for $B$ meson decays. Using the QCDF master formula, HME of nonleptonic decays could be written as the convolution integrals of the process-dependent hard scattering kernels and universal light-cone distribution amplitudes (LCDA) of participating hadrons.

The spectator quark is the heavy-flavor charm quark for charmonium weak decays into $DM$ final states. It is commonly assumed that the virtuality of the gluon connecting to the heavy spectator is of order $\Lambda_{QCD}^2$, where $\Lambda_{QCD}$ is the characteristic QCD scale. Hence, the transition form factors between charmonium and $D$ mesons are assumed to be dominated by the soft and nonperturbative contributions, and the amplitudes of the spectator rescattering subprocess are power-suppressed [19]. Taking the $\eta_c \to DM$ decays for example, HME can be written as

$$
\langle DM|Q_{1,2}|\eta_c\rangle = \sum_i F_{\eta_c \to D}^i f_M \int dx H_i(x) \Phi_M(x),
$$

(5)

where $F_{\eta_c \to D}^i$ is the weak transition form factor, $f_M$ and $\Phi_M(x)$ are the decay constant and LCDA of the meson $M$, respectively. The leading twist LCDA for the pseudoscalar and longitudinally polarized vector mesons can be expressed in terms of Gegenbauer polynomials [22, 23]:

$$
\Phi_M(x) = 6 \bar{x} \sum_{n=0}^{\infty} a_n^M C_n^{3/2}(x - \bar{x}),
$$

(6)

where $\bar{x} = 1 - x$; $C_n^{3/2}(z)$ is the Gegenbauer polynomial,

$$
C_0^{3/2}(z) = 1, \quad C_1^{3/2}(z) = 3 z, \quad C_2^{3/2}(z) = \frac{3}{2}(5 z^2 - 1), \quad \cdots
$$

(7)

$a_n^M$ is the Gegenbauer moment corresponding to the Gegenbauer polynomials $C_n^{3/2}(z)$; $a_0^M \equiv 1$ for the asymptotic form; $a_n = 0$ for $n = 1, 3, 5, \cdots$ because of the $G$-parity invariance of the $\pi, \eta(\prime)$, $\rho$, $\omega$, $\phi$ meson distribution amplitudes. In this paper, to give a rough estimation, the contributions from higher-order $n \geq 3$ Gegenbauer polynomials are not considered for the moment.

Hard scattering function $H_i(x)$ in Eq.(5) is, in principle, calculable order by order with the perturbative QCD theory. At the order of $\alpha_s^0$, $H_i(x) = 1$. This is the simplest scenario, and
one goes back to the naive factorization where there is no information about the strong phases and the renormalization scale hidden in the HME. At the order of $\alpha_s$ and higher orders, the renormalization scale dependence of hadronic matrix elements could be recuperated to partly cancel the $\mu$-dependence of the Wilson coefficients. In addition, part of the strong phases could be reproduced from nonfactorizable contributions.

Within the QCDF framework, amplitudes for $\eta_c \to DM$ decays can be expressed as:

$$\mathcal{A}(\eta_c \to DM) = \langle DM | \mathcal{H}_{\text{eff}} | \eta_c \rangle = \frac{G_F}{\sqrt{2}} V_{cq_1}^* V_{uq_2} a_i \langle M | J^\mu | 0 \rangle \langle D | J_\mu | \eta_c \rangle.$$ (8)

In addition, the HME for the $\psi(1S, 2S) \to DV$ decays are conventionally expressed as

$$H_\lambda = \langle V | J^\mu | 0 \rangle \langle D | J_\mu | \psi \rangle = \epsilon_\psi^* \epsilon_V \left\{ a g_{\mu\nu} + \frac{b}{m_\psi m_V} (p_\psi + p_\bar{D})^\mu p_\nu + \frac{ic}{m_\psi m_V} \epsilon_{\mu\nu\alpha\beta} p_\alpha^\psi (p_\psi + p_\bar{D})^\beta \right\}.$$ (9)

The relations among helicity amplitudes and invariant amplitudes $a, b, c$ are

$$H_0 = -a x - 2b (x^2 - 1),$$ (10)

$$H_\pm = a \pm 2c \sqrt{x^2 - 1},$$ (11)

$$x = \frac{p_\psi \cdot p_\bar{V}}{m_\psi m_V} = \frac{m_\psi^2 - m_D^2 + m_V^2}{2 m_\psi m_V},$$ (12)

where three scalar amplitudes $a, b, c$ describe the $s, d, p$ wave contributions, respectively.

The effective coefficient $a_i$ at the order of $\alpha_s$ can be expressed as [19]:

$$a_1 = C_1^{\text{NLO}} + \frac{1}{N_c} C_2^{\text{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_2^{\text{LO}} \mathcal{V},$$ (13)

$$a_2 = C_2^{\text{NLO}} + \frac{1}{N_c} C_1^{\text{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_1^{\text{LO}} \mathcal{V},$$ (14)

where the color factor $C_F = 4/3$; the color number $N_c = 3$. For the transversely polarized light vector meson, the factor $\mathcal{V} = 0$ in the helicity $H_\pm$ amplitudes beyond the leading twist contributions. With the leading twist LCDA for the pseudoscalar and longitudinally polarized vector mesons, the factor $\mathcal{V}$ is written as [19]:

$$\mathcal{V} = 6 \log \left( \frac{m_\psi^2}{\mu^2} \right) - 18 - \left( \frac{1}{2} + i3\pi \right) + \left( \frac{11}{2} - i3\pi \right) a_1^M - \frac{21}{20} a_2^M + \cdots.$$ (15)

From the numbers in Table. [11] it is found that (1) the values of coefficients $a_{1,2}$ agree generally with those used in previous works [14–17, 26]. (2) The strong phases appear by
taking nonfactorizable corrections into account, which is necessary for CP violation. The strong phase of $a_1$ is small due to the suppression of $\alpha_s$ and $1/N_c$. The strong phase of $a_2$ is large due to the enhancement from the large Wilson coefficients $C_1$.

TABLE II: Numerical values of the Wilson coefficients $C_{1,2}$ and parameters $a_{1,2}$ for the $\eta_c \to D\pi$ decay with $m_c = 1.275$ GeV [5], where $a_{1,2}$ in Ref. [26] is used in the $D$ meson weak decay.

| $\mu$ | LO  | NLO | QCDF | Previous works |
|-------|-----|-----|------|----------------|
|       | $C_1$ | $C_2$ | $C_1$ | $C_2$ | $a_1$    | $a_2$    | Ref.   | $a_1$    | $a_2$    |
| 0.8 $m_c$ | 1.335 | -0.589 | 1.275 | -0.504 | 1.275$e^{+14^\circ}$ | 0.503$e^{-i154^\circ}$ | [14, 16, 17] | 1.26 | -0.51 |
| $m_c$  | 1.276 | -0.505 | 1.222 | -0.425 | 1.219$e^{+13^\circ}$ | 0.402$e^{-i154^\circ}$ | [15] | 1.3±0.1 | -0.55±0.10 |
| 1.2 $m_c$ | 1.240 | -0.450 | 1.190 | -0.374 | 1.186$e^{+13^\circ}$ | 0.342$e^{-i154^\circ}$ | [26] | 1.274 | -0.529 |

C. Form factors

The weak transition form factors between charmonium and a charmed meson are defined as follows [18]:

$$\langle D(p_2)|V_\mu - A_\mu|\eta_c(p_1)\rangle = \left\{(p_1 + p_2)_\mu - \frac{m_\psi^2 - m_D^2}{q^2} q_\mu\right\} F_1(q^2) + \frac{m_\psi^2 - m_D^2}{q^2} q_\mu F_0(q^2),$$

(16)

$$\langle D(p_2)|V_\mu - A_\mu|\psi(p_1, \epsilon)\rangle = -\epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu}_\psi q^\alpha (p_1 + p_2)^{\beta} \frac{V(q^2)}{m_\psi + m_D} - i \frac{2 m_\psi \epsilon \cdot q}{q^2} q_\mu A_0(q^2) - i \epsilon_{\psi,\mu} (m_\psi + m_D) A_1(q^2) - i \frac{\epsilon \cdot q}{m_\psi + m_D} (p_1 + p_2)_\mu A_2(q^2) + i \frac{2 m_\psi \epsilon \cdot q}{q^2} q_\mu A_3(q^2),$$

(17)

where $q = p_1 - p_2$; $\epsilon_\psi$ denotes the $\psi$'s polarization vector. The form factors $F_0(0) = F_1(0)$ and $A_0(0) = A_3(0)$ are required compulsorily to cancel singularities at the pole of $q^2 = 0$. There is a relation among these form factors

$$2m_\psi A_3(q^2) = (m_\psi + m_D) A_1(q^2) + (m_\psi - m_D) A_2(q^2).$$

(18)
There are four independent transition form factors, \( F_0(0) \), \( A_{0,1}(0) \) and \( V(0) \), at the pole of \( q^2 = 0 \). They could be written as the overlap integrals of wave functions \[18\]:

\[
F_0(0) = \int d\vec{k}_\perp \int_0^1 dx \left\{ \Phi_{\eta_c}(\vec{k}_\perp, x, 0, 0) \Phi_D(\vec{k}_\perp, x, 0, 0) \right\}, \tag{19}
\]

\[
A_0(0) = \int d\vec{k}_\perp \int_0^1 dx \left\{ \Phi_\psi(\vec{k}_\perp, x, 1, 0) \sigma_z \Phi_D(\vec{k}_\perp, x, 0, 0) \right\}, \tag{20}
\]

\[
A_1(0) = \frac{m_c + m_q}{m_\psi + m_D} I, \tag{21}
\]

\[
V(0) = \frac{m_c - m_q}{m_\psi - m_D} I, \tag{22}
\]

\[
I = \sqrt{2} \int d\vec{k}_\perp \int_0^1 dx \frac{dx}{x} \left\{ \Phi_\psi(\vec{k}_\perp, x, 1, -1) i\sigma_y \Phi_D(\vec{k}_\perp, x, 0, 0) \right\}, \tag{23}
\]

where \( \sigma_{y,z} \) is the Pauli matrix acting on the spin indices of the decaying charm quark; \( x \) and \( \vec{k}_\perp \) denote the fraction of the longitudinal momentum and the transverse momentum of the nonspectator quark, respectively.

With the separation of the spin and spatial variables, wave functions can be written as

\[
\Phi(\vec{k}_\perp, x, j, j_z) = \phi(\vec{k}_\perp, x) |s, s_z, s_1, s_2\rangle, \tag{24}
\]

where the total angular momentum \( \vec{j} = \vec{L} + \vec{s}_1 + \vec{s}_2 = \vec{s}_1 + \vec{s}_2 = \vec{s} \) because the orbital angular momentum between the valence quarks in the \( \psi(1S, 2S), \eta_c(1S, 2S), D \) mesons in question have \( \vec{L} = 0 \); \( s_{1,2} \) denote the spins of valence quarks in meson; \( s = 1 \) and \( 0 \) for the \( \psi \) and \( \eta_c \) mesons, respectively.

The charm quark in the charmonium state is nearly nonrelativistic with an average velocity \( v \ll 1 \) based on arguments of nonrelativistic quantum chromodynamics (NRQCD) \[27,29\]. For the \( D \) meson, the valence quarks are also nonrelativistic due to \( m_D \approx m_c + m_q \), where the light quark mass \( m_u \approx m_d \approx 310 \text{ MeV} \) and \( m_s \approx 510 \text{ MeV} \) \[30\]. Here, we will take the solution of the Schrödinger equation with a scalar harmonic oscillator potential as the wave functions of the charmonium and \( D \) mesons:

\[
\phi_{1S}(\vec{k}) \sim e^{-\vec{k}^2/2\alpha^2}, \tag{25}
\]

\[
\phi_{2S}(\vec{k}) \sim e^{-\vec{k}^2/2\alpha^2}(2\vec{k}^2 - 3\alpha^2), \tag{26}
\]

where the parameter \( \alpha \) determines the average transverse quark momentum, \( \langle \phi_{1S}|\vec{k}_\perp^2|\phi_{1S}\rangle = \alpha^2 \). With the NRQCD power counting rules \[27\], \( |\vec{k}_\perp| \sim mv \sim m_\alpha s \) for heavy quarkonium. Hence, parameter \( \alpha \) is approximately taken as \( m_\alpha s \) in our calculation.
Using the substitution ansatz \[31\],
\[
\vec{k}^2 \to \frac{\vec{k}^2 + \bar{x} m_q^2 + x m_c^2}{4 x \bar{x}},
\]
(27)
one can obtain
\[
\phi_{1S}(\vec{k}_\perp, x) = A \exp\left\{\frac{\vec{k}^2}{8 \alpha^2 x \bar{x}} + \frac{\bar{x} m_q^2 + x m_c^2}{6 \alpha^2 x \bar{x}} - 1\right\},
\]
(28)
\[
\phi_{2S}(\vec{k}_\perp, x) = B \phi_{1S}(\vec{k}_\perp, x)\left\{\frac{\vec{k}^2}{6 \alpha^2 x \bar{x}} + \frac{\bar{x} m_q^2 + x m_c^2}{6 \alpha^2 x \bar{x}} - 1\right\},
\]
(29)
where the parameters \(A\) and \(B\) are the normalization coefficients satisfying with the normalization condition,
\[
\int d\vec{k}_\perp \int_0^1 dx |\phi(\vec{k}_\perp, x)|^2 = 1.
\]
(30)

The numerical values of transition form factors at \(q^2 = 0\) are listed in Table III. It is found that (1) the model dependence of form factors is large; (2) isospin-breaking effects are negligible and flavor breaking effects are small; (3) as stated in Ref. \[18\] that \(F_0 \simeq A_0\) holds within collinear symmetry.

### III. NUMERICAL RESULTS AND DISCUSSION

In the charmonium center-of-mass frame, the branching ratio for the charmonium weak decay can be written as

\[
\mathcal{B}(\eta_c \to DM) = \frac{p_{cm}}{4 \pi m_{\eta_c}^2 \Gamma_{\eta_c}} |\mathcal{A}(\eta_c \to DM)|^2,
\]
(31)
\[
\mathcal{B}(\psi \to DM) = \frac{p_{cm}}{12 \pi m_{\psi}^2 \Gamma_{\psi}} |\mathcal{A}(\psi \to DM)|^2,
\]
(32)

where the common momentum of final states is
\[
p_{cm} = \sqrt{m_{\eta_c,\psi}^2 - (m_D + m_M)^2} [m_{\eta_c,\psi}^2 - (m_D - m_M)^2]^{1/2};
\]
(33)
The decay amplitudes for \(\mathcal{A}(\psi \to DM)\) and \(\mathcal{A}(\eta_c \to DM)\) are collected in Appendix A and B respectively.

In our calculation, we assume that the light vector mesons are ideally mixed, i.e., \(\omega = (u\bar{u} + d\bar{d})/\sqrt{2}\) and \(\phi = s\bar{s}\). For the mixing of pseudoscalar \(\eta\) and \(\eta'\) meson, we will adopt the
TABLE III: The numerical values of transition form factors at \( q^2 = 0 \), where uncertainties of this work come from the charm quark mass.

| Transition                  | Reference | \( F_0(0) \)     | \( A_0(0) \)     | \( A_1(0) \)     | \( V(0) \)     |
|-----------------------------|-----------|------------------|------------------|------------------|----------------|
| \( \eta_c(1S), \psi(1S) \rightarrow D_{u,d} \) | This work | 0.85±0.01        | 0.85±0.01        | 0.72±0.01        | 1.76±0.03      |
|                            | Ref. [13]^a | ...              | 0.68±0.01        | 0.68±0.01        | 1.6±0.1        |
|                            | Ref. [32]^b | ...              | 0.27^{+0.02}_{-0.03} | 0.27^{+0.03}_{-0.02} | 0.81^{+0.12}_{-0.08} |
|                            | Ref. [15]^c | ...              | 0.40 (0.61)      | 0.44 (0.68)      | 1.17 (1.82)    |
|                            | Ref. [17]^d | ...              | 0.55±0.02        | 0.77^{+0.09}_{-0.07} | 2.14^{+0.15}_{-0.11} |
|                            | Ref. [17]^e | ...              | 0.54             | 0.80             | 2.21           |
| \( \eta_c(1S), \psi(1S) \rightarrow D_s \) | This work | 0.90±0.01        | 0.90±0.01        | 0.81±0.01        | 1.55±0.04      |
|                            | Ref. [13]^a | ...              | 0.68±0.01        | 0.68±0.01        | 1.8            |
|                            | Ref. [32]^b | ...              | 0.37±0.02        | 0.38^{+0.02}_{-0.01} | 1.07^{+0.05}_{-0.02} |
|                            | Ref. [15]^c | ...              | 0.47 (0.66)      | 0.55 (0.78)      | 1.25 (1.80)    |
|                            | Ref. [17]^d | ...              | 0.71^{+0.04}_{-0.02} | 0.94±0.07        | 2.30^{+0.09}_{-0.06} |
|                            | Ref. [17]^e | ...              | 0.69             | 0.96             | 2.36           |
| \( \eta_c(2S), \psi(2S) \rightarrow D_{u,d} \) | This work | 0.62±0.01        | 0.61±0.01        | 0.54±0.01        | 1.00±0.04      |
| \( \eta_c(2S), \psi(2S) \rightarrow D_s \) | This work | 0.65±0.01        | 0.64±0.01        | 0.59±0.02        | 0.83±0.04      |

^a\text{The form factors are computed with the covariant light-front quark model, where uncertainties come from the decay constant of charmed meson.}

^b\text{The form factors are computed with QCD sum rules, where uncertainties are from the Borel parameters.}

^c\text{The form factors are computed with parameter } \omega = 0.4 (0.5) \text{ GeV using the WSB model.}

^d\text{The form factors are computed with flavor dependent parameter } \omega \text{ using the WSB model.}

^e\text{The form factors are computed with parameter } \omega = m_{c_s} \text{ using the WSB model.}

quark-flavor basis description proposed in [33], and neglect the contributions from possible gluonium compositions, i.e.,

\[
\begin{pmatrix}
\eta \\
\eta' \\
\end{pmatrix}
= \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi \\
\end{pmatrix}
\begin{pmatrix}
\eta_q \\
\eta_s \\
\end{pmatrix},
\]

where \( \eta_q = (u\bar{u} + d\bar{d})/\sqrt{2} \) and \( \eta_s = s\bar{s} \); the mixing angle \( \phi = (39.3\pm1.0)^\circ \) [33]. The mass relations are

\[
m_{\eta_q}^2 = m_{\eta}^2 \cos^2 \phi + m_{\eta'}^2 \sin^2 \phi - \frac{\sqrt{2} f_{\eta'}}{f_{\eta q}} (m_{\eta q}^2 - m_{\eta}^2) \cos \phi \sin \phi, \quad (35)
\]
\[ m_{\eta_s}^2 = m_s^2 \sin^2 \phi + m_\eta^2 \cos^2 \phi - \frac{f_{\eta_s}}{\sqrt{2} f_\eta} (m_\eta^2 - m_{\eta_s}^2) \cos \phi \sin \phi. \] (36)

**TABLE IV:** Numerical values of input parameters.

| Parameter | Value |
|-----------|-------|
| \( \lambda \) | 0.22537±0.00061 [5] |
| \( \hat{\rho} \) | 0.117±0.021 [5] |
| \( m_c \) | 1.275±0.025 GeV [5] |
| \( m_{D_u} \) | 1864.84±0.07 MeV [5] |
| \( m_{D_d} \) | 1869.61±0.10 MeV [5] |
| \( m_{D_s} \) | 1968.30±0.11 MeV [5] |
| \( f_\pi \) | 130.41±0.20 MeV [5] |
| \( f_K \) | 156.2±0.7 MeV [5] |
| \( f_{\eta_s} \) | (1.07±0.02) \( f_\pi \) [33] |
| \( f_\rho \) | 216±3 MeV [23] |
| \( f_\omega \) | 187±5 MeV [23] |
| \( f_\phi \) | 215±5 MeV [23] |
| \( f_{K^*} \) | 220±5 MeV [23] |
| \( a_2^\rho = a_2^{\eta_s} = a_2^{\eta} = 0.25±0.15 \) [22] | \( a_2^\rho = a_2^{\eta_s} = 0.15±0.07 \) [23] |
| \( a_1^K = -a_1^{\eta_s} = 0.06±0.03 \) [22] | \( a_2^K = a_2^{\eta_s} = 0.25±0.15 \) [22] |
| \( a_1^{K^*} = -a_1^{\eta_s^*} = 0.03±0.02 \) [23] | \( a_2^{K^*} = a_2^{\eta_s^*} = 0.11±0.09 \) [23] |
| \( a_1^\sigma = a_1^\omega = a_1^\phi = 0 \) | \( a_2^\sigma = 0.18±0.08 \) [23] |

The input parameters including the CKM Wolfenstein parameters, decay constants, Gegenbauer moments and so on, are collected in Table IV. If not specified explicitly, we will take their central values as the default inputs. Our numerical results on branching ratios for the nonleptonic two-body \( \psi(1S, 2S) \rightarrow DM \) weak decays are displayed in Tables V and VI where the uncertainties of this work come from the CKM parameters, the renormalization scale \( \mu = (1±0.2)m_c \), hadronic parameters including decay constants and Gegenbauer moments, respectively. For comparison, previous results on the \( J/\psi \) weak decays with parameters \( a_1 = 1.26 \) and \( a_2 = -0.51 \) are also listed in Table VI.

The following are some comments.

1. There are some differences among the estimates of branching ratios for the \( J/\psi \rightarrow DM \) weak decays (see the numbers in Table V). These inconsistencies among previous works, although the same values of parameters \( a_{1,2} \) are used, come principally from different values of form factors. Our results are generally in line with the numbers in columns “A” and “B” which are favored by Ref. [17].

2. Branching ratios for the \( J/\psi \) weak decay are about two or more times as large as
those for $\psi(2S)$ decay into the same final states, because the decay width of $\psi(2S)$ is about three times as large as that of $J/\psi$.

(3) Due to the relatively small decay width and relatively large space phases for the $\eta_c(2S)$ decay, branching ratios for the $\eta_c(2S)$ weak decay are some five (ten) or more times as large as those for the $\eta_c(1S)$ weak decay into the same $DP$ ($DV$) final states.

(4) Among the $\psi(1S,2S)$ and $\eta_c(1S,2S)$ mesons, the $\eta_c(1S)$ has a maximal decay width and a minimal mass resulting in a small phase space, while $J/\psi$ has a minimal decay width. These facts lead to the smallest [or the largest] branching ratio for $\eta_c(1S)$ [or $J/\psi$] weak decay among $\psi(1S,2S)$, $\eta_c(1S,2S)$ weak decays into the same final states.

(5) Compared with the $\psi(1S,2S) \to DV$ decays, the corresponding $\psi(1S,2S) \to DP$ decays, where $P$ and $V$ have the same flavor structures, are suppressed by the orbital angular momentum, so have relatively small branching ratios. There are some approximative relations $B_{\psi(1S,2S) \to DV} \approx 3B_{\psi(1S,2S) \to DP}$ and $B_{\psi(2S) \to DV} \approx 3B_{\psi(2S) \to DP}$.

(6) According to the CKM factors and parameters $a_{1,2}$, nonleptonic charmonium weak decays could be subdivided into six cases (see Table VII). Case “i-a” is the Cabibbo-favored one, so it generally has large branching ratios relative to case “i-b” and “i-c”. The $a_2$-dominated charmonium weak decays are suppressed by a color factor relative to the $a_1$-dominated ones. Hence, the charmonium weak decays into the $D_s\rho$ and $D_s\pi$ final states belonging to case “1-a” usually have relatively large branching ratios; the charmonium weak decays into the $D_u^0K^{*0}$ final states belonging to case “2-c” usually have relatively small branching ratios. In addition, the branching ratio of case “2-a” (or “2-b”) is usually larger than that of case “1-b” (or “1-c”) due to $|a_2/a_1| \geq \lambda$.

(7) Branching ratios for the Cabibbo favored $\psi(1S,2S) \to D_s^\pm \rho^\mp$, $D_s^-\pi^+$, $\overline{D}_u^0K^{*0}$ decays can reach up to $10^{-10}$, which might be measurable in the forthcoming days. For example, the $J/\psi$ production cross section can reach up to a few $\mu b$ with the LHCb and ALICE detectors at LHC [8, 9]. Therefore, over $10^{12} J/\psi$ samples are in principle available per $100\, fb^{-1}$ data collected by LHCb and ALICE, corresponding to a few tens of $J/\psi \to D_s^-\rho^+$, $D_s^-\pi^+$, $\overline{D}_u^0K^{*0}$ events for about 10% reconstruction efficiency.

(8) There is a large cancellation between the CKM factor $V_{ud}V_{cd}^*$ and $V_{us}V_{cs}^*$, which results in a very small branching ratio for charmonium weak decays into $D_u\eta'$ state.

(9) There are many uncertainties on our results. The first uncertainty from the CKM factors is small due to high precision on the Wolfenstein parameter $\lambda$ with only 0.3% relative
errors now $\frac{\Delta}{\Delta}$. The second uncertainty from the renormalization scale $\mu$ could, in principle, be reduced by the inclusion of higher order $\alpha_s$ corrections. For example, it has been showed [34] that tree amplitudes incorporating with the NNLO corrections are relatively less sensitive to the renormalization scale than the NLO amplitudes. The third uncertainty comes from hadronic parameters, which is expected to be cancelled or reduced with the relative ratio of branching ratios.

(10) The numbers in Table V and VI just provide an order of magnitude estimate. Many other factors, such as the final state interactions, $q^2$ dependence of form factors and so on, which are not considered here, deserve many dedicated studies.

IV. SUMMARY

With the anticipation of abundant data samples on charmonium at high-luminosity heavy-flavor experiments, we studied the nonleptonic two-body $\psi(1S, 2S)$ and $\eta_c(1S, 2S)$ weak decays into one ground-state charmed meson plus one ground-state light meson based on the low energy effective Hamiltonian. By considering QCD radiative corrections to hadronic matrix elements of tree operators, we got the effective coefficients $a_{1,2}$ containing partial information of strong phases. The magnitude of $a_{1,2}$ agrees comfortably with those used in previous works [14–17]. The transition form factors between the charmonium and charmed meson are calculated by using the nonrelativistic wave functions with isotropic harmonic oscillator potential. Branching ratios for $\psi(1S, 2S)$, $\eta_c(1S, 2S) \to DM$ decays are estimated roughly. It is found that the Cabibbo favored $\psi(1S, 2S) \to D^-_s \rho^+$, $D^-_s \pi^+$, $D_u^0 K^-$ decays have large branching ratios $\gtrsim 10^{-10}$, which are promisingly detected in the forthcoming years.

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Appendix A: The amplitudes for $\psi \rightarrow DM$ decays

\begin{align}
A(\psi \rightarrow D_s^- \pi^+) &= \sqrt{2} G_F m_\psi (\epsilon_\psi \cdot p_\pi) f_\pi A_0^{\psi \rightarrow D_s} V_{cs}^* V_{ud} a_1, \\
A(\psi \rightarrow D_s^- K^+) &= \sqrt{2} G_F m_\psi (\epsilon_\psi \cdot p_K) f_K A_0^{\psi \rightarrow D_s} V_{cs}^* V_{us} a_1, \\
A(\psi \rightarrow D_d^- \pi^+) &= \sqrt{2} G_F m_\psi (\epsilon_\psi \cdot p_\pi) f_\pi A_0^{\psi \rightarrow D_d} V_{cd}^* V_{ud} a_1, \\
A(\psi \rightarrow D_d^- K^+) &= \sqrt{2} G_F m_\psi (\epsilon_\psi \cdot p_K) f_K A_0^{\psi \rightarrow D_d} V_{cd}^* V_{us} a_1, \\
A(\psi \rightarrow D_u^0 \pi^0) &= -G_F m_\psi (\epsilon_\psi \cdot p_\pi) f_\pi A_0^{\psi \rightarrow D_u} V_{cd}^* V_{ud} a_2, \\
A(\psi \rightarrow D_u^0 K^0) &= \sqrt{2} G_F m_\psi (\epsilon_\psi \cdot p_K) f_K A_0^{\psi \rightarrow D_u} V_{cd}^* V_{us} a_2, \\
A(\psi \rightarrow D_u^0 \eta) &= \cos \phi A(\psi \rightarrow D_u^0 \eta) - \sin \phi A(\psi \rightarrow D_u^0 \eta'), \\
A(\psi \rightarrow D_u^0 \eta') &= \sin \phi A(\psi \rightarrow D_u^0 \eta) + \cos \phi A(\psi \rightarrow D_u^0 \eta'). \\
A(\psi \rightarrow D_s^- \rho^+) &= -i \frac{G_F}{\sqrt{2}} f_\rho m_\rho V_{cs}^* V_{ud} a_1 \left\{ (\epsilon_\rho^* \cdot \epsilon_\psi) (m_\psi + m_{D_s}) A_1^{\psi \rightarrow D_s} \\
& \quad + (\epsilon_\rho^* \cdot p_\rho) (\epsilon_\psi \cdot p_\rho) \frac{2 A_2^{\psi \rightarrow D_s}}{m_\psi + m_{D_s}} - i \epsilon_{\mu \nu \alpha \beta} \epsilon_\rho^{* \mu} \epsilon_\nu^\alpha \epsilon_\psi^\beta p_\rho^\alpha p_\psi^\beta \frac{2 V^{\psi \rightarrow D_s}}{m_\psi + m_{D_s}} \right\}, \\
A(\psi \rightarrow D_s^- K^{*+}) &= -i \frac{G_F}{\sqrt{2}} f_K m_K V_{cs}^* V_{us} a_1 \left\{ (\epsilon_{K^*}^* \cdot \epsilon_\psi) (m_\psi + m_{D_s}) A_1^{\psi \rightarrow D_s} \\
& \quad + (\epsilon_{K^*}^* \cdot p_{K^*}) (\epsilon_\psi \cdot p_{K^*}) \frac{2 A_2^{\psi \rightarrow D_s}}{m_\psi + m_{D_s}} - i \epsilon_{\mu \nu \alpha \beta} \epsilon_{K^*}^{* \mu} \epsilon_\nu^\alpha \epsilon_\psi^\beta p_{K^*}^\alpha p_\psi^\beta \frac{2 V^{\psi \rightarrow D_s}}{m_\psi + m_{D_s}} \right\}, \\
A(\psi \rightarrow D_d^- \rho^+) &= -i \frac{G_F}{\sqrt{2}} f_\rho m_\rho V_{cd}^* V_{ud} a_1 \left\{ (\epsilon_\rho^* \cdot \epsilon_\psi) (m_\psi + m_{D_d}) A_1^{\psi \rightarrow D_d} \\
& \quad + (\epsilon_\rho^* \cdot p_\rho) (\epsilon_\psi \cdot p_\rho) \frac{2 A_2^{\psi \rightarrow D_d}}{m_\psi + m_{D_d}} - i \epsilon_{\mu \nu \alpha \beta} \epsilon_\rho^{* \mu} \epsilon_\nu^\alpha \epsilon_\psi^\beta p_\rho^\alpha p_\psi^\beta \frac{2 V^{\psi \rightarrow D_d}}{m_\psi + m_{D_d}} \right\}, \\
A(\psi \rightarrow D_d^- K^{*+}) &= -i \frac{G_F}{\sqrt{2}} f_K m_K V_{cd}^* V_{us} a_1 \left\{ (\epsilon_{K^*}^* \cdot \epsilon_\psi) (m_\psi + m_{D_d}) A_1^{\psi \rightarrow D_d} \\
& \quad + (\epsilon_{K^*}^* \cdot p_{K^*}) (\epsilon_\psi \cdot p_{K^*}) \frac{2 A_2^{\psi \rightarrow D_d}}{m_\psi + m_{D_d}} - i \epsilon_{\mu \nu \alpha \beta} \epsilon_{K^*}^{* \mu} \epsilon_\nu^\alpha \epsilon_\psi^\beta p_{K^*}^\alpha p_\psi^\beta \frac{2 V^{\psi \rightarrow D_d}}{m_\psi + m_{D_d}} \right\}, \\
A(\psi \rightarrow D_u^0 \rho^0) &= +i \frac{G_F}{2} f_\rho m_\rho V_{cd}^* V_{ud} a_2 \left\{ (\epsilon_\rho^* \cdot \epsilon_\psi) (m_\psi + m_{D_u}) A_1^{\psi \rightarrow D_u} \\
& \quad + (\epsilon_\rho^* \cdot p_\rho) (\epsilon_\psi \cdot p_\rho) \frac{2 A_2^{\psi \rightarrow D_u}}{m_\psi + m_{D_u}} - i \epsilon_{\mu \nu \alpha \beta} \epsilon_\rho^{* \mu} \epsilon_\nu^\alpha \epsilon_\psi^\beta p_\rho^\alpha p_\psi^\beta \frac{2 V^{\psi \rightarrow D_u}}{m_\psi + m_{D_u}} \right\}.
\end{align}
\begin{equation}
\mathcal{A}(\psi \to D_u^0 \omega) = -i \frac{G_F}{2} f_\omega m_\omega V_{cd} V_{ud} a_2 \left\{ \left( \epsilon_{\omega}^* \epsilon_{\psi}^* \right) (m_\psi + m_{D_u}) A_1^{\psi \to D_u} \right. \\
+ \left( \epsilon_{\omega}^* p_{\psi}^* \right) (\epsilon_{\psi}^* p_\omega) \frac{2 A_2^{\psi \to D_u}}{m_\psi + m_{D_u}} - i \epsilon_{\mu_\alpha \beta} \epsilon_{\omega}^* \epsilon_{\psi}^* \epsilon_\nu^\alpha \epsilon_\rho^\beta \frac{2 V_{\omega \psi}^{\psi \to D_u}}{m_\psi + m_{D_u}} \left\}, \tag{A17} \end{equation}

\begin{equation}
\mathcal{A}(\psi \to D_u^0 \phi) = -i \frac{G_F}{\sqrt{2}} f_\phi m_\phi V_{cs}^* V_{us} a_2 \left\{ \left( \epsilon_{\phi}^* \epsilon_{\psi}^* \right) (m_\psi + m_{D_u}) A_1^{\psi \to D_u} \right. \\
+ \left( \epsilon_{\phi}^* p_{\psi}^* \right) (\epsilon_{\psi}^* p_\phi) \frac{2 A_2^{\psi \to D_u}}{m_\psi + m_{D_u}} - i \epsilon_{\mu_\alpha \beta} \epsilon_{\phi}^* \epsilon_{\psi}^* \epsilon_\nu^\alpha \epsilon_\rho^\beta \frac{2 V_{\phi \psi}^{\psi \to D_u}}{m_\psi + m_{D_u}} \left\}. \tag{A18} \end{equation}

\begin{equation}
\mathcal{A}(\psi \to D_u^0 K^{-0}) = -i \frac{G_F}{\sqrt{2}} f_K m_K V_{cd}^* V_{us} a_2 \left\{ \left( \epsilon_{K^-}^* \epsilon_{\psi}^* \right) (m_\psi + m_{D_u}) A_1^{\psi \to D_u} \right. \\
+ \left( \epsilon_{K^-}^* p_{\psi}^* \right) (\epsilon_{\psi}^* p_K) \frac{2 A_2^{\psi \to D_u}}{m_\psi + m_{D_u}} - i \epsilon_{\mu_\alpha \beta} \epsilon_{K^-}^* \epsilon_{\psi}^* \epsilon_\nu^\alpha \epsilon_\rho^\beta \frac{2 V_{K^- \psi}^{\psi \to D_u}}{m_\psi + m_{D_u}} \left\}. \tag{A19} \end{equation}

\begin{equation}
\mathcal{A}(\psi \to D_u^0 \bar{K}^{-0}) = -i \frac{G_F}{\sqrt{2}} f_K m_K V_{us}^* V_{cd} a_2 \left\{ \left( \epsilon_{K^-}^* \epsilon_{\psi}^* \right) (m_\psi + m_{D_u}) A_1^{\psi \to D_u} \right. \\
+ \left( \epsilon_{K^-}^* p_{\psi}^* \right) (\epsilon_{\psi}^* p_{\bar{K}^-}) \frac{2 A_2^{\psi \to D_u}}{m_\psi + m_{D_u}} - i \epsilon_{\mu_\alpha \beta} \epsilon_{K^-}^* \epsilon_{\psi}^* \epsilon_\nu^\alpha \epsilon_\rho^\beta \frac{2 V_{K^- \psi}^{\psi \to D_u}}{m_\psi + m_{D_u}} \left\}. \tag{A20} \end{equation}

Appendix B: The amplitudes for the $\eta_c \to DM$ decays

\begin{equation}
\mathcal{A}(\eta_c \to D_s^- \pi^+) = i \frac{G_F}{\sqrt{2}} \left( m_{\eta_c}^2 - m_{D_s}^2 \right) f_\pi F_{0}^{\eta_c \to D_s} V_{ud} V_{cs}^* a_1, \tag{B1} \end{equation}

\begin{equation}
\mathcal{A}(\eta_c \to D_s^- K^+) = i \frac{G_F}{\sqrt{2}} \left( m_{\eta_c}^2 - m_{D_s}^2 \right) f_K F_{0}^{\eta_c \to D_s} V_{us} V_{cs}^* a_1, \tag{B2} \end{equation}

\begin{equation}
\mathcal{A}(\eta_c \to D_d^- \pi^+) = i \frac{G_F}{\sqrt{2}} \left( m_{\eta_c}^2 - m_{D_d}^2 \right) f_\pi F_{0}^{\eta_c \to D_d} V_{ud} V_{cd}^* a_1, \tag{B3} \end{equation}

\begin{equation}
\mathcal{A}(\eta_c \to D_d^- K^+) = i \frac{G_F}{\sqrt{2}} \left( m_{\eta_c}^2 - m_{D_d}^2 \right) f_K F_{0}^{\eta_c \to D_d} V_{us} V_{cd}^* a_1, \tag{B4} \end{equation}

\begin{equation}
\mathcal{A}(\eta_c \to D_u^0 \pi^0) = -i \frac{G_F}{2} \left( m_{\eta_c}^2 - m_{D_u}^2 \right) f_\pi F_{0}^{\eta_c \to D_u} V_{ud} V_{cs}^* a_2, \tag{B5} \end{equation}

\begin{equation}
\mathcal{A}(\eta_c \to D_u^0 K^0) = i \frac{G_F}{2} \left( m_{\eta_c}^2 - m_{D_u}^2 \right) f_K F_{0}^{\eta_c \to D_u} V_{us} V_{cs}^* a_2, \tag{B6} \end{equation}

\begin{equation}
\mathcal{A}(\eta_c \to D_u^0 \bar{K}^0) = i \frac{G_F}{2} \left( m_{\eta_c}^2 - m_{D_u}^2 \right) f_K F_{0}^{\eta_c \to D_u} V_{us} V_{cd}^* a_2, \tag{B7} \end{equation}

\begin{equation}
\mathcal{A}(\eta_c \to D_u^0 \eta) = i \frac{G_F}{2} \left( m_{\eta_c}^2 - m_{D_u}^2 \right) f_\eta F_{0}^{\eta_c \to D_u} V_{ud} V_{cd}^* a_2, \tag{B8} \end{equation}

\begin{equation}
\mathcal{A}(\eta_c \to D_u^0 \eta') = i \frac{G_F}{2} \left( m_{\eta_c}^2 - m_{D_u}^2 \right) f_\eta F_{0}^{\eta_c \to D_u} V_{us} V_{cs}^* a_2, \tag{B9} \end{equation}

\begin{equation}
\mathcal{A}(\eta_c \to D_u^0 \eta) = \cos \phi \mathcal{A}(\eta_c \to D_u^0 \eta) - \sin \phi \mathcal{A}(\eta_c \to D_u^0 \eta'), \tag{B10} \end{equation}

\begin{equation}
\mathcal{A}(\eta_c \to D_u^0 \eta') = \sin \phi \mathcal{A}(\eta_c \to D_u^0 \eta) + \cos \phi \mathcal{A}(\eta_c \to D_u^0 \eta), \tag{B11} \end{equation}

15
\[ A(\eta_c \rightarrow D_s^+ \rho^+) = \sqrt{2} G_F m_\rho (\epsilon_\rho^* p_\eta_c) f_\rho F_{1_2}^{\eta_c \rightarrow D_s^+ V_{ud} V_{cs}^* a_1}, \quad (B12) \]

\[ A(\eta_c \rightarrow D_s^- K^{*+}) = \sqrt{2} G_F m_{K^*} (\epsilon_{K^*}^* p_\eta_c) f_{K^*} F_{1_2}^{\eta_c \rightarrow D_s^- V_{us} V_{cs}^* a_1}, \quad (B13) \]

\[ A(\eta_c \rightarrow D_d^+ \rho^+) = \sqrt{2} G_F m_\rho (\epsilon_\rho^* p_\eta_c) f_\rho F_{1_2}^{\eta_c \rightarrow D_d^+ V_{ud} V_{cs}^* a_1}, \quad (B14) \]

\[ A(\eta_c \rightarrow D_d^- K^{*+}) = \sqrt{2} G_F m_{K^*} (\epsilon_{K^*}^* p_\eta_c) f_{K^*} F_{1_2}^{\eta_c \rightarrow D_d^- V_{us} V_{cs}^* a_1}, \quad (B15) \]

\[ A(\eta_c \rightarrow D_0^+(\rho^0)) = -G_F m_\rho (\epsilon_\rho^* p_\eta_c) f_\rho F_{1_2}^{\eta_c \rightarrow D_0^+ V_{ud} V_{cs}^* a_2}, \quad (B16) \]

\[ A(\eta_c \rightarrow D_0^- (\omega^0)) = G_F m_\omega (\epsilon_\omega^* p_\eta_c) f_\omega F_{1_2}^{\eta_c \rightarrow D_0^- V_{cd} V_{cs}^* a_2}, \quad (B17) \]

\[ A(\eta_c \rightarrow D_0^- (\phi^0)) = \sqrt{2} G_F m_\phi (\epsilon_\phi^* p_\eta_c) f_\phi F_{1_2}^{\eta_c \rightarrow D_0^- V_{cs} V_{cd}^* a_2}, \quad (B18) \]

\[ A(\eta_c \rightarrow D_1^- (\rho^0)) = \sqrt{2} G_F m_{K^*} (\epsilon_{K^*}^* p_\eta_c) f_{K^*} F_{1_2}^{\eta_c \rightarrow D_1^- V_{us} V_{cs}^* a_2}, \quad (B19) \]

\[ A(\eta_c \rightarrow D_1^- (\omega^0)) = \sqrt{2} G_F m_\omega (\epsilon_\omega^* p_\eta_c) f_\omega F_{1_2}^{\eta_c \rightarrow D_1^- V_{cs} V_{cd}^* a_2}. \quad (B20) \]

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TABLE V: Branching ratios for the nonleptonic two-body $J/\psi$ weak decays, where the uncertainties of this work come from the CKM parameters, the renormalization scale $\mu = (1\pm 0.2)m_c$, hadronic parameters including decay constants and Gegenbauer moments, respectively. The results of Refs. 14, 16, 17 are calculated with $a_1 = 1.26$ and $a_2 = -0.51$. The results of Ref. 14 are based on QCD sum rules. The numbers in columns of “A”, “B”, “C” and “D” are based on the WSB model with flavor dependent $\omega$, QCD inspired $\omega = \alpha_s m_s$, universal $\omega = 0.4$ GeV and 0.5 GeV, respectively.

| final states | Ref. [14] | Ref. [17] | Ref. [16] | This work |
|--------------|-----------|-----------|-----------|-----------|
| $D_s^{-}\pi^+$ | 1-a | $2.0 \times 10^{-10}$ | $7.41 \times 10^{-10}$ | $3.32 \times 10^{-10}$ | $8.74 \times 10^{-10}$ | $(1.09^{+0.01}_{-0.01}\times10-0.01)\times10^{-9}$ |
| $D_s^{-}K^+$ | 1-b | $1.6 \times 10^{-11}$ | $5.3 \times 10^{-11}$ | $2.4 \times 10^{-11}$ | $5.5 \times 10^{-11}$ | $(6.18^{+0.03}_{-0.03}\times10-0.03)\times10^{-11}$ |
| $D_d^{-}\pi^+$ | 1-b | $0.8 \times 10^{-11}$ | $2.9 \times 10^{-11}$ | $1.5 \times 10^{-11}$ | $5.5 \times 10^{-11}$ | $(6.37^{+0.03}_{-0.03}\times10-0.03)\times10^{-11}$ |
| $D_s^{-}K^+$ | 1-c | $2.3 \times 10^{-12}$ | $2.2 \times 10^{-12}$ | $1.2 \times 10^{-12}$ | $5.5 \times 10^{-12}$ | $(3.79^{+0.04}_{-0.04}\times10-0.04)\times10^{-12}$ |
| $D_s^0\pi^0$ | 2-b | $2.4 \times 10^{-12}$ | $2.3 \times 10^{-12}$ | $2.0 \times 10^{-13}$ | $3.56^{+0.02}_{-0.02}\times10-0.02)\times10^{-12}$ |
| $D_s^0K^0$ | 2-c | $4.0 \times 10^{-13}$ | $4.0 \times 10^{-13}$ | $2.8 \times 10^{-10}$ | $(1.44^{+0.01}_{-0.01}\times10-0.01)\times10^{-10}$ |
| $D_s^0K^0$ | 2-a | $3.6 \times 10^{-11}$ | $1.34 \times 10^{-10}$ | $3.6 \times 10^{-12}$ | $(1.03^{+0.01}_{-0.01}\times10-0.01)\times10^{-11}$ |
| $D_s^0\eta$ | ... | $7.0 \times 10^{-12}$ | $6.7 \times 10^{-12}$ | $2.0 \times 10^{-13}$ | $(5.82^{+0.03}_{-0.03}\times10-0.03)\times10^{-13}$ |
| $D_s^0\eta'$ | ... | $4.0 \times 10^{-13}$ | $4.0 \times 10^{-13}$ | $3.0 \times 10^{-13}$ | $3.63^{+0.01}_{-0.01}\times10-0.01)\times10^{-9}$ |
| $D_s^{-}\rho^+$ | 1-a | $1.26 \times 10^{-9}$ | $5.11 \times 10^{-9}$ | $1.77 \times 10^{-9}$ | $2.12 \times 10^{-10}$ | $(2.00^{+0.01}_{-0.01}\times10-0.01)\times10^{-10}$ |
| $D_s^{-}K^{*+}$ | 1-b | $0.82 \times 10^{-10}$ | $2.82 \times 10^{-10}$ | $0.97 \times 10^{-10}$ | $2.2 \times 10^{-10}$ | $(1.14^{+0.01}_{-0.01}\times10-0.01)\times10^{-11}$ |
| $D_s^{-}\rho^+$ | 1-b | $0.42 \times 10^{-10}$ | $2.16 \times 10^{-10}$ | $0.72 \times 10^{-10}$ | $2.2 \times 10^{-10}$ | $(1.08^{+0.01}_{-0.01}\times10-0.01)\times10^{-11}$ |
| $D_s^{-}K^{*+}$ | 1-c | $1.3 \times 10^{-11}$ | $1.3 \times 10^{-11}$ | $4.2 \times 10^{-12}$ | $2.2 \times 10^{-11}$ | $(8.10^{+0.04}_{-0.04}\times10-0.04)\times10^{-12}$ |
| $D_s^0\rho^0$ | 2-b | $1.8 \times 10^{-11}$ | $1.9 \times 10^{-11}$ | $6.0 \times 10^{-12}$ | $1.8 \times 10^{-11}$ | $(1.92^{+0.01}_{-0.01}\times10-0.01)\times10^{-11}$ |
| $D_s^0\omega$ | 2-b | $1.6 \times 10^{-11}$ | $1.7 \times 10^{-11}$ | $5.0 \times 10^{-12}$ | $6.5 \times 10^{-11}$ | $(1.19^{+0.01}_{-0.01}\times10-0.01)\times10^{-12}$ |
| $D_s^0\phi$ | 2-b | $4.2 \times 10^{-11}$ | $4.4 \times 10^{-11}$ | $1.4 \times 10^{-11}$ | $7.0 \times 10^{-13}$ | $(4.09^{+0.01}_{-0.01}\times10-0.01)\times10^{-10}$ |
| case | final states | $\psi(2S)$ decay | $\eta_c(1S)$ decay | $\eta_c(2S)$ decay |
|------|--------------|------------------|------------------|------------------|
| 1-a  | $D_s^- \pi^+$ | $(5.07^{+0.10+0.48+0.03}_{-0.01-0.27-0.02}) \times 10^{-10}$ | $(7.35^{+0.01+0.69+0.04}_{-0.01-0.39-0.04}) \times 10^{-12}$ | $(3.96^{+0.01+0.37+0.02}_{-0.01-0.21-0.02}) \times 10^{-11}$ |
| 1-b  | $D_s^- K^+$  | $(3.43^{+0.02+0.33+0.04}_{-0.02-0.18-0.04}) \times 10^{-11}$ | $(4.97^{+0.03+0.48+0.06}_{-0.03-0.27-0.06}) \times 10^{-13}$ | $(2.87^{+0.01+0.27+0.04}_{-0.01-0.15-0.04}) \times 10^{-12}$ |
| 1-b  | $D_s^- \pi^+$ | $(2.76^{+0.01+0.26+0.01}_{-0.01-0.15-0.01}) \times 10^{-11}$ | $(4.39^{+0.02+0.41+0.02}_{-0.02-0.23-0.02}) \times 10^{-13}$ | $(2.13^{+0.01+0.20+0.01}_{-0.01-0.11-0.01}) \times 10^{-12}$ |
| 1-c  | $D_s^- K^+$  | $(1.90^{+0.02+0.18+0.02}_{-0.02-0.10-0.02}) \times 10^{-12}$ | $(3.04^{+0.03+0.29+0.04}_{-0.03-0.16-0.04}) \times 10^{-14}$ | $(1.58^{+0.02+0.15+0.02}_{-0.02-0.08-0.02}) \times 10^{-13}$ |
| 2-b  | $\bar{D}_0^0 \pi^0$ | $(1.51^{+0.01+0.85+0.02}_{-0.01-0.42-0.02}) \times 10^{-12}$ | $(2.41^{+0.01+1.36+0.04}_{-0.01-0.67-0.04}) \times 10^{-14}$ | $(1.16^{+0.01+0.66+0.02}_{-0.01-0.32-0.02}) \times 10^{-13}$ |
| 2-c  | $\bar{D}_0^0 K^0$  | $(2.07^{+0.02+1.17+0.05}_{-0.02-0.57-0.05}) \times 10^{-13}$ | $(3.35^{+0.04+1.89+0.09}_{-0.04-0.93-0.08}) \times 10^{-15}$ | $(1.73^{+0.02+0.97+0.04}_{-0.02-0.48-0.04}) \times 10^{-14}$ |
| 2-a  | $\bar{D}_0^0 \rho^0$  | $(7.75^{+0.01+4.04+0.17}_{-0.01-1.98-0.16}) \times 10^{-11}$ | $(1.16^{+0.01+0.65+0.03}_{-0.01-0.32-0.03}) \times 10^{-12}$ | $(5.96^{+0.01+3.37+0.14}_{-0.01-1.65-0.14}) \times 10^{-12}$ |
| 1-a  | $D_s^- \rho^+$  | $(1.67^{+0.01+0.15+0.05}_{-0.01-0.09-0.05}) \times 10^{-9}$  | $(5.28^{+0.01+0.50+0.15}_{-0.01-0.28-0.15}) \times 10^{-12}$ | $(7.24^{+0.01+0.68+0.21}_{-0.01-0.38-0.21}) \times 10^{-11}$ |
| 1-b  | $D_s^- K^{**}$  | $(9.59^{+0.05+0.89+0.46}_{-0.05-0.50-0.45}) \times 10^{-11}$ | $(1.18^{+0.01+0.11+0.06}_{-0.01-0.06-0.06}) \times 10^{-13}$ | $(3.47^{+0.02+0.33+0.17}_{-0.02-0.18-0.16}) \times 10^{-12}$ |
| 1-b  | $D_s^- \rho^+$  | $(8.99^{+0.05+0.83+0.26}_{-0.05-0.47-0.26}) \times 10^{-11}$ | $(4.32^{+0.02+0.41+0.12}_{-0.02-0.23-0.12}) \times 10^{-13}$ | $(4.13^{+0.02+0.39+0.12}_{-0.02-0.22-0.12}) \times 10^{-12}$ |
| 1-c  | $D_s^- K^{**}$  | $(5.15^{+0.06+0.48+0.25}_{-0.05-0.27-0.24}) \times 10^{-12}$ | $(1.38^{+0.01+0.13+0.07}_{-0.01-0.07-0.07}) \times 10^{-14}$ | $(2.03^{+0.02+0.19+0.10}_{-0.02-0.11-0.10}) \times 10^{-13}$ |
| 2-b  | $\bar{D}_0^0 \rho^0$  | $(4.36^{+0.02+2.44+0.15}_{-0.02-1.21-0.15}) \times 10^{-12}$ | $(2.38^{+0.01+1.35+0.08}_{-0.01-0.66-0.08}) \times 10^{-14}$ | $(2.24^{+0.01+1.27+0.08}_{-0.01-0.62-0.08}) \times 10^{-13}$ |
| 2-b  | $\bar{D}_0^0 \omega$  | $(3.28^{+0.02+1.84+0.20}_{-0.02-0.91-0.19}) \times 10^{-12}$ | $(1.74^{+0.01+0.98+0.11}_{-0.01-0.48-0.10}) \times 10^{-14}$ | $(1.67^{+0.01+0.94+0.10}_{-0.01-0.46-0.10}) \times 10^{-13}$ |
| 2-b  | $\bar{D}_0^0 \phi$  | $(9.40^{+0.05+5.28+0.52}_{-0.05-2.61-0.50}) \times 10^{-12}$ | $(8.57^{+0.04+4.84+0.47}_{-0.04-2.38-0.45}) \times 10^{-15}$ | $(3.28^{+0.02+1.85+0.18}_{-0.02-0.91-0.17}) \times 10^{-13}$ |
| 2-c  | $\bar{D}_0^0 K^{*0}$  | $(5.09^{+0.05+2.86+0.31}_{-0.05-1.42-0.30}) \times 10^{-13}$ | $(1.50^{+0.02+0.85+0.08}_{-0.02-0.42-0.08}) \times 10^{-15}$ | $(2.18^{+0.02+1.25+0.12}_{-0.02-0.60-0.12}) \times 10^{-14}$ |
| 2-a  | $\bar{D}_0^0 K^{0}$  | $(1.74^{+0.01+0.98+0.11}_{-0.01-0.49-0.10}) \times 10^{-10}$ | $(5.20^{+0.01+2.94+0.29}_{-0.01-1.44-0.28}) \times 10^{-13}$ | $(7.57^{+0.01+4.27+0.42}_{-0.01-2.10-0.40}) \times 10^{-12}$ |
### TABLE VII: Classification of the nonleptonic charmonium weak decays.

| case | parameter | CKM factor |
|------|-----------|------------|
| 1-a  | $a_1$     | $|V_{ud}V^*_{cs}| \sim 1$ |
| 1-b  | $a_1$     | $|V_{ud}V^*_{cd}|, |V_{us}V^*_{cs}| \sim \lambda$ |
| 1-c  | $a_1$     | $|V_{us}V^*_{cd}| \sim \lambda^2$ |
| 2-a  | $a_2$     | $|V_{ud}V^*_{cs}| \sim 1$ |
| 2-b  | $a_2$     | $|V_{ud}V^*_{cd}|, |V_{us}V^*_{cs}| \sim \lambda$ |
| 2-c  | $a_2$     | $|V_{us}V^*_{cd}| \sim \lambda^2$ |