Strategic delegation and second mover advantage in duopoly

Jeong-Yoo Kim\textsuperscript{a} and Joon Yeop Kwon\textsuperscript{b}

\textsuperscript{a}Economics, Kyung Hee University, Seoul, The Republic of Korea; \textsuperscript{b}Division of Humanities and Social Sciences, POSTECH, Pohang-si, The Republic of Korea

\textbf{ABSTRACT}

We consider a duopoly in which each firm has one owner and one manager playing a multi-stage delegation game. The decision of each firm consists of two stages. In the first stage, the owner offers his manager a contract based on profits and sales. In the second stage, the manager chooses its output or price. Several possible sequential games will be analysed, depending on the sequence of the strategic variables. In the first scenario in which firm 1 makes a contract decision and a producing decision sequentially, and firm 2 follows in the same fashion, we show that any delegation equilibrium in which both owners commit their managers to profit-maximising behaviour disappears. In the second scenario in which the firms first enter into the contract stage and then Stackelberg competition follows in the second stage, sales-based delegation occurs. If firms compete in quantities, second mover advantage appears if firms make simultaneous delegation contracts, while first mover advantage is recovered if they make sequential contracts. If firms compete in prices, the results are reversed.

\textbf{1. Introduction}

Do firms really maximise profits? Is it the case that profit maximisation increases the long-run viability of firms? Alchian (1950) advocated the profit maximisation hypothesis on the grounds that natural selection results in the survival of only the profit maximisers in the long run. Friedman (1953) also argued that surviving firms are those that attained the highest profits. These arguments led to the wide acceptance of the profit maximisation hypothesis.

On the other hand, the managerial theory of the firm has proposed that firms in reality do not necessarily behave like profit maximisers. Given the separation of management from ownership, the firm’s manager maximises his own utility function rather than the profit of the firm. Also, Baumol (1958) suggested sales maximisation as an alternative objective function of firms.

The profit maximisation hypothesis and the managerial theory of the firm seemed to have been irreconcilable until Vickers (1985), Sklivas (1987) and Fershtman and Judd (1987) presented a two-stage strategic delegation game in which profit-maximising owners in the
first stage offer a contract to their managers and managers in the second stage compete
given the contracts. They showed paradoxically that firms can maximise profits by dele-
gating to non-profit-maximising managers. More specifically, they proved that in a two-
stage Cournot game, each firm’s owner will twist his manager’s incentive away from profit
maximisation in order to commit them to more aggressive behaviour during the output
competition stage, thereby becoming a Stackelberg leader. Thus, both firms’ owners have
incentives to delegate decisions to their managers. Then, in equilibrium, they end up with
higher output and lower profits than in the Cournot Nash equilibrium, which is exactly
like a prisoner’s dilemma situation.

The literature on strategic delegation hitherto nonetheless neglects the possibility that a
firm can be a Stackelberg leader in delegating decisions without paying much attention to
the sequence of delegations. A Stackelberg leader in the output market competition does
not necessarily imply a Stackelberg leader in delegation contracts. A Stackelberg leader
in the output market may offer a contract earlier than the rival firm or later or simultane-
ously without observing the contracts of the other. That is, Stackelberg leadership in two
stages (delegation stage and producing stage) should be distinguished. This distinction is
important, especially when one firm plays as a Stackelberg leader in the output market. One
of the most interesting issues that arise in this situation is whether one firm or the other
firm can strengthen its Stackelberg leadership or countervail the rival firm’s leadership by
sequential strategic delegation.

In this article, we will consider a duopoly in which each firm has one owner and one
manager playing a multi-stage delegation game. The decision of each firm consists of two
stages. In the first stage, the owner offers his manager a contract based on profits and sales.
In the second stage, the manager chooses its output or price. Depending on the sequence of
the strategic variables of the firms, we can think of several possible sequential games. The
first scenario is a model in which firm 1 makes a contract decision and a producing decision
sequentially, and then firm 2 follows the decisions in the same fashion. We show that any
sales-based delegation equilibrium disappears in this model, in other words, both owners
commit their managers to profit-maximising behaviour. The second scenario involves a
situation in which the firms first enter into the contract stage and then Stackelberg com-
petition follows in the second stage. Two cases will be considered. In the first case, firms
make contracts simultaneously, and in the second case, they make contracts sequentially.
The outcome will turn out to be very sensitive to the nature of strategies, whether they are
strategic substitutes or strategic complements. If firms engage in quantity competition, we
will show that there is a second mover advantage in the first case in the sense that firm 2
choosing its quantity level later than firm 1 enjoys higher profit, and that first mover advan-
tage reappears in the second case. The intuition for the first result is that firm 2 is provided
a new opportunity to commit to more aggressive behaviour by the delegation contract prior
to quantity competition, and the intuition for the second result is that firm 1 can circumvent
firm 2’s strategic delegation by his additional upfront contract. If firms compete in prices,
the advantages are reversed. In the first case, there is first mover advantage, while there is
second mover advantage in the second case.

The Korean entertainment market can serve as an interesting example. Among three big
record companies in Korea, YG Entertainment, which successfully released rapper Psy’s
music video ‘Gangnam Style’, is known to often use a ‘running guarantee’, a contract form
based on sales rather than profits, whereas its competitor JYP Entertainment does not often
use a running guarantee. It is also known that its share prices saw a year-over-year increase of more than 60% and were currently higher than the share prices of the other two competing entertainment companies. Considering that YG Entertainment is the last entrant, this may suggest the importance of the entry sequence and the delegation contract, although it is not clear due to the inherent complexity of the entertainment industry whether or not the current market shares are a direct consequence of the running guarantees.

There are several papers asserting that firms can have second mover advantages. Gal-Or (1985) showed that the second mover has the advantage if the reaction curves of the firms are upward sloping, as in the case of price competition. Amir and Stepanova (2006) strengthened the result of Gal-Or (1985) with the general demand. Rasmusen and Yoon (2012) argued that when one firm has superior information, there is a second mover advantage as the informed player’s information is more accurate, because delay can prevent the spillover of this information. Bonatti and Martina (2004) and Kopel and Löffler (2008) also showed second mover advantages in a similar setting of quantity competition rather than price competition. Our article is distinguished from these two papers in the sense that it covers more general cases. The second mover advantage they identified falls under only a special case in our model.

The article is organised as follows. In Section 2, we describe the model. In Sections 3 and 4, we analyse the case of quantity competition and the case of price competition respectively. Concluding remarks follow in Section 5. All the proofs are contained in the appendix.

2. Model

We consider a duopoly in which each firm has one owner and one manager playing a multi-stage delegation game. The decision of each firm consists of two stages. In the first stage, the owner offers his manager a contract. In the second stage, the manager chooses its output or price given the contract.

It is assumed that contracts take a particular form: managers are paid in proportion to some linear combination of profits and revenues (sales). Formally, firm $i$’s manager is offered the following form of contract,

$$O_i = \alpha_i \pi_i + (1 - \alpha_i)R_i,$$  \hspace{1cm} (1)

where $\pi_i$ and $R_i$ are firm $i$’s profits and revenues. Here, $\alpha_i$ is chosen by owner $i$ strategically. Since two firms are involved in our model, there are four strategic variables, $\alpha_1$, $\alpha_2$, $x_1$ and $x_2$, where $x_i$ can be either $q_i$ (output) or $p_i$ (price). We will consider several sequential games, depending on the sequence of the four strategic variables.

The first scenario we consider is that firm 1 makes sequential decisions $\alpha_1$ and $x_1$, and then firm 2 follows the decisions in the same fashion. This corresponds to the case in which firm 1 is a market-dominating incumbent whose decisions are well known to other firms. We will call this a model of sequential firm decisions. In the second scenario, we consider the situation in which the firms first enter into the contract stage and then Stackelberg competition follows in the second stage. This setup is consistent with the reality that contracts are often made on a long-term basis, whereas output or price decisions are more flexible. We will also divide the second scenario into two cases. In the first case, firms make contracts simultaneously, and in the second case, they make contracts sequentially. The former corresponds to the case that both firms are incumbents despite a difference in
size, and the latter corresponds to the case that firm 1 is an incumbent and firm 2 is a new entrant. We will call the former a model of simultaneous delegation, and the latter a model of sequential delegation.

Due to the sequential nature of the games we consider, we will use the subgame perfect equilibrium as our main solution concept. In characterising equilibria, we will assume throughout the article that all the previous moves of the players are common knowledge. It will turn out that differences in information in alternative models lead to significant differences in the outcome.

3. Quantity competition

We examine three models of duopoly Cournot competition in a homogeneous good market. For tractability, we assume that demand is linear:

$$P = a - bQ, \quad a, b > 0,$$

where $Q = q_1 + q_2$ is total output or market demand and $P$ is market price. We also assume that firms have common marginal cost $c$ ($> 0$) which is constant. Then, we have $\pi_i = (P - c)q_i$ and $R_i = Pq_i$. The values of parameters $a$, $b$ and $c$ are all common knowledge. To ensure $q_i > 0$, we assume that $a > c$.

3.1. Model of sequential firm decisions (Model Q - I)

In this model, firm 1 (or its owner) makes a contract with its manager by choosing $\alpha_1$ and then the manager chooses its output $q_1$ in the first stage, and in the second stage, firm 2 and its manager pick $\alpha_2$ and $q_2$ sequentially. We will use backward induction to find the subgame perfect equilibrium.

Given $\alpha_2$ and $q_1$, manager 2’s best-response function is given by

$$q_2 = \frac{a - bq_1 - \alpha_2c}{2b}. \quad (2)$$

It directly follows from maximising $O_2 = (P - \alpha_2q_2)(2 - a - b(q_1 + q_2) - \alpha_2c)q_2$. By substituting equation (2) into the profit function, we obtain the indirect profit function for the owner 2:

$$\pi_2 = (P - c)q_2 = \left[ a - b\left( q_1 + \frac{a - bq_1 - \alpha_2c}{2b} \right) - c \right] \left( \frac{a - bq_1 - \alpha_2c}{2b} \right). \quad (3)$$

Note that the profit (the owner’s payoff) of firm 2 depends on $\alpha_2$ as well as $q_1$. Also, note that the contract of the other firm (firm 1), which can be represented by $\alpha_1$, does not enter the indirect profit function, because it can only affect the profit indirectly through affecting $q_1$. Surprisingly enough, however, simple algebra shows that the best response of owner 2 is neutral to $q_1$ and consequently to $\alpha_1$, i.e., $\alpha_2 = 1$. This is surprising, because it is usual that the optimal decision is contingent on the previous moves ($\alpha_1$ and $q_1$ in this case). Also, one could conjecture that owner 2 may be able to nullify the Stackelberg leadership of firm 1 by a credible threat of offering a contract inducing his manager to produce more aggressively. The threat is, however, never credible, because manager 1’s choice of Stackelberg output has a commitment value and so the subsequent contract of firm 2 cannot affect the choice...
of manager 1, and would leave owner 2 worse off with the aggressive contract \( \alpha_2 < 1 \) than with \( \alpha_2 = 1 \).

Next, let us consider the decisions of firm 1 in the first stage. Given \( \alpha_1 \), the best response of manager 1 is given by

\[
q_1 = \frac{a + c - 2c\alpha_1}{2b}.
\]

Again, this follows from maximising \( O_1 = (P - \alpha_1c)q_1 = \left(a - b\left(\frac{a+bq_1-c}{2b}\right) - \alpha_1c\right)q_1 \). Substituting equations (2) and (4) together with \( \alpha_2 = 1 \), we can compute the indirect profit function of firm 1 as follows:

\[
\pi_1 = (P - c)q_1 = \left(\frac{a + (2\alpha_1 - 3)c}{4}\right)\left(\frac{a + (1 - 2\alpha_1)c}{2b}\right),
\]

and the first-order condition completes our analysis for this model.

Proposition 1. Let \( \alpha_{Q1}^i \) and \( \pi_{Q1}^i \) be the equilibrium contract and the equilibrium profit of firm \( i \) of Model Q - I. Then, (i) \( \alpha_{Q1}^i = 1 \) for all \( i = 1, 2 \), (ii) \( \pi_{Q1}^1 = \frac{(a-c)^2}{8b} \) (>) \( \pi_{Q1}^2 = \frac{(a-c)^2}{16b} \).

In equilibrium, both owners commit their managers to profit-maximising behaviour, that is, no sales-based delegation equilibrium occurs in the model of sequential firm decisions. Although firm 2 seems to be able to get out of the position of Stackelberg follower by offering a more aggressive contract to his manager, the owner does not offer such an incentive scheme. The intuition goes as follows. The owner of firm 2 is well aware that he cannot affect the output decision of firm 1, and that there is no better response to the Stackelberg quantity of firm 1 than the Stackelberg follower’s output (right on his original reaction curve associated with \( \alpha_2 = 1 \)) that he can induce.\(^3\) Firm 1 does not offer a sales-based contract either. The owner of firm 1 who wants \( \pi_1 \) to be maximised knows that it is possible when his manager produces the quantity of a Stackelberg leader and in turn this is achieved only when his manager behaves as a profit maximiser himself. Thus, the owner must offer a contract that matches their interests exactly in order to implement the Stackelberg output, meaning that \( \alpha_1 = 1 \). Proposition 1-(ii) simply says that there is a first mover advantage, as is well known in the literature (see Figure 1).

3.2. Model of simultaneous delegation (Model Q - II)

In this model, the game begins with two firms’ simultaneous contracting with their managers. Both contracts are assumed to be observable by two managers. Then, in the second stage, the managers of firm 1 and firm 2 choose their outputs \( q_1 \) and \( q_2 \) sequentially. Firm 1 is still a Stackelberg leader in the second stage of output competition, although the two firms are in a symmetric position in the first stage of contract competition.

Given \( \alpha_1 \) and \( \alpha_2 \), the Stackelberg outcome in the second stage can be computed as a standard solution as follows:

\[
q_1 = \frac{a + (\alpha_2 - 2\alpha_1)c}{2b},
\]

\[
q_2 = \frac{a + (2\alpha_1 - 3\alpha_2)c}{4b}.
\]
These two equations suggest that if \( \alpha_2 < \alpha_1 \), i.e., firm 2 makes a more aggressive contract than firm 1, it is possible that \( q_2 > q_1 \), i.e., firm 1 may lose the advantage of a Stackelberg leader. Substituting equations (6) and (7) into profit functions, we obtain

\[
\pi_1 = (P - c)q_1 = a - \left( \frac{3a - (2\alpha_1 + \alpha_2)c}{4} \right) - c \left( \frac{a + (\alpha_2 - 2\alpha_1)c}{2b} \right), \quad (8)
\]

\[
\pi_2 = (P - c)q_2 = a - \left( \frac{3a - (2\alpha_1 + \alpha_2)c}{4} \right) - c \left( \frac{a + (2\alpha_1 - 3\alpha_2)c}{4b} \right). \quad (9)
\]

From these two profit functions, we can find the best-response functions of owners given in equations (10) and (11), respectively:

\[
\alpha_1 = 1, \quad (10)
\]

\[
\alpha_2 = \frac{2(3 - \alpha_1)c - a}{3c}. \quad (11)
\]

Interestingly, it is optimal for the owner of firm 1 to commit his manager to profit maximising behaviour regardless of the rival firm’s contract (i.e., \( \alpha_1 = 1 \) is the dominant strategy for the owner of firm 1), whereas the optimal contract of firm 2 depends on the contract of firm 1. This is because the owner of firm 1 knows from the argument provided in the previous subsection that he can implement the output of a Stackelberg leader without resorting to any sales-based delegation contract. Now, we have

**Proposition 2.** Let \( \alpha_i^{Q_2} \) and \( \pi_i^{Q_2} \) be the equilibrium contract and the equilibrium profit of firm \( i \) of Model Q - II. Then, (i) \( \alpha_1^{Q_2} = 1, \alpha_2^{Q_2} = \frac{4c-a}{3c} < 1 \), and (ii) \( \pi_1^{Q_2} = \frac{(a-\alpha_1^2)}{18b} < \) and \( \pi_2^{Q_2} = \frac{(a-\alpha_2^2)}{12b} \).
Surprisingly, there is a second mover advantage even if firm 2 has no strategic advantage (in the sense that it is a symmetric mover in the first stage and a Stackelberg follower in the second stage). The intuition goes as follows. Firm 2 knows that firm 1 will choose $\alpha_1 = 1$ (since it is the dominant strategy for firm 1) and that he can be made better off by using an aggressive contract. By such a contract, firm 2 can increase its output, while firm 1 has to reduce its output, because the outputs are strategic substitutes. As a result, firm 1’s profit is reduced and firm 2’s profit is increased, and this change leads to profit reversal $\pi_{Q2} > \pi_{Q1}$ in this model with linear demand. The main difference from Model Q - I is that the manager of firm 1 is aware that the reaction curve of firm 2 shifts outwards due to a more aggressive contract of firm 2, and so he must respond to the rival firm’s contract. Thus, firm 1 cannot behave as a Stackelberg leader any longer and firm 2 enjoys the second mover advantage by taking advantage of firm 2’s output response to firm 1’s contract (see Figure 2).

3.3. Model of sequential delegation (Model Q - III)

In this model, firm 1 behaves as a Stackelberg leader both in the first-stage contract competition and in the second-stage output competition. In the first stage, firm 1 first chooses its optimal contract and then firm 2 follows. In the second stage, too, firm 1 makes its output decision and then firm 2 follows.

The decisions in the second stage remain unaffected by the first-period sequential contracting. Thus, we can focus only on the first-stage competition. As shown in equation (11), $\alpha_2$ is a strategic substitute to $\alpha_1$, implying that firm 2 responds optimally to a high value of $\alpha_1$ by a low value of $\alpha_2$. Therefore, by choosing a high value of $\alpha_1$, the owner of firm 1 regains the first mover advantage but does not fully recover the profit of a Stackelberg leader. Formally, by substituting equations (6), (7) and (11) into (8), we can compute the indirect profit function of firm 1 as follows:

Figure 2. $E^{Q2}$ is the Equilibrium in Model Q – II. Source: Author.
By using the first-order condition and equation (11), we can find the equilibrium contracts $\alpha_{Q3}^1$ and $\alpha_{Q3}^2$ as follows. Note that both firms use sales-based delegation contracts $\alpha_{Q3}^1$, $\alpha_{Q3}^2 < 1$ in equilibrium.

**Proposition 3.** Let $\alpha_{Q3}^i$ and $\pi_{Q3}^i$ be the equilibrium contract and the equilibrium profit of firm $i$ of Model Q - III. Then, (i) $\alpha_{Q3}^1 = \frac{(a-c)^2}{8b}$, $\alpha_{Q3}^2 = \frac{5c-a}{4c}$, and (ii) $\pi_{Q3}^1 = \frac{(a-c)^2}{16b}$, $\pi_{Q3}^2 = \frac{3(a-c)^2}{64b}$.

As we expected, it follows that $\pi_{Q3}^1 > \pi_{Q3}^2$ and $\pi_{Q3}^2 < \pi_{Q2}^2$, which is due to firm 1’s strategic pre-commitment in its delegation contract. Also, in this equilibrium, both firms are made worse off than in Model Q - I in which both firms commit their managers to profit-maximising behaviour in equilibrium. This confirms the intuition of Fershtman and Judd (1987) and Sklivas (1987) that strategic delegations of both firms make both profits lower than in the Nash equilibrium profits of no delegation. Finally, the proof of this proposition provided in the appendix shows that $q_{Q3}^i > q_{Q3}^j$ where $q_{Q3}^i$ is the equilibrium output of firm $i$ in Model Q - III. This suggests that firm 1 partially recovers the first mover advantage. (Note that $\alpha_{Q3}^1 > \alpha_{Q3}^2$, meaning that the manager of firm 1 is less aggressive.)

We summarise delegation contracts and profits of firms under quantity competition in Table 1. For firm 1, profits are higher in the order of Model Q - I to Model Q - III to Model Q - II. For firm 2, profits are higher in the order of Model Q - II to Model Q - I to Model Q - III.

| Model | Delegation Contract | Profit |
|-------|---------------------|--------|
| $Q$ - I | $\alpha_1 = 1$, $\alpha_2 = 1$ | $\pi_{Q1} = \frac{(a+3c-4ca_1)(a-3c+2ca_1)}{18b}$ |
| $Q$ - II | $\alpha_1 < 1$, $\alpha_2 = 1$ | $\pi_{Q2} = \frac{(a-c)^2}{16b}$ |
| $Q$ - III | $\alpha_1 < 1$, $\alpha_2 < 1$ | $\pi_{Q3} = \frac{3(a-c)^2}{64b}$ |

Source: Author.

Table 1. Delegation contracts and profits of each firm under quantity competition.

By using the first-order condition and equation (11), we can find the equilibrium contracts $\alpha_{Q3}^1$ and $\alpha_{Q3}^2$ as follows. Note that both firms use sales-based delegation contracts $\alpha_{Q3}^1$, $\alpha_{Q3}^2 < 1$ in equilibrium.

**4. Price competition**

In this section, we consider a price competition in a differentiated good market. Again, we assume linear demand:

$$q_i = a - p_i + bp_j,$$

where $0 < b < 1$ and $0 < c < \frac{a}{1-b}$. The first condition ($0 < b < 1$) implies that the demand for a product is more sensitive to a change in its own price than to a change in the competitor’s price. The second condition ($0 < c < \frac{a}{1-b}$) is just to ensure positive sales ($q_i > 0$).

**4.1. Model of sequential firm decisions (Model P - I)**

As in quantity competition, we first consider the decision problems that firm 2 faces. Given $p_1$ and $\alpha_2$, we can derive the best response function of firm 2 as
by maximising \( O_2 = (p_2 - \alpha_2)q_2 = (p_2 - \alpha_2)(a - p_2 - bp_1) \). This yields the indirect profit function of firm 2 as follows:

\[
\pi_2 = (p_2 - c)(a - p_2 + bp_1) = \frac{(a + bp_1 - c\alpha_2)(a - 2c + bp_1 + c\alpha_2)}{4}.
\]

Again, \( \alpha_1 \) can affect the profit of firm 2 only indirectly through \( p_1 \). The first-order condition of this profit maximisation implies that the optimal contract for firm 2 is \( \alpha_2 = 1 \) as before. The intuition is clear. As in Section 3, the decision of firm 2 is neutral to the decision of firm 1 in the sense that it cannot affect any foregone decision of firm 1. Therefore, it is optimal for the second mover to choose its best response to \( p_1 \) which is on its reaction curve. This can be achieved by no sales-based delegation contract.

Now, consider the decision of firm 1. The objective function of the manager for firm 1 is given by

\[
O_1 = (p_1 - \alpha_1)c\left[a - p_1 + \frac{b(a + c + bp_1)}{2}\right].
\]

From the first-order condition, it follows that the best response function of firm 1 is

\[
p_1 = \frac{2a + ab + bc + c\alpha_1(2 - b^2)}{2(2 - b^2)}.
\]

Deriving the indirect profit function of firm 1 by substituting this into its profit function is tedious, but after straightforward calculations, we obtain

**Proposition 4.** Let \( \alpha_{i1} \) and \( \pi_{i1} \) be the equilibrium contract and the equilibrium profit of firm \( i \) of Model P - I. Then, (i) \( \alpha_{i1} = 1 \) for all \( i = 1, 2 \), (ii) \( \pi_{i1} = \frac{(2 + b)^2(a - (1-b))}{2(2 - b^2)} \) \( (<) \pi_{21} = \frac{(4 + 2b - b^2)^2(a - (1-b))}{16(2 - b^2)^2} \).

This proposition implies that both firms commit their managers to behaviour in accord with profit maximisation, just as in quantity competition. Also, note that there is a second mover advantage since prices are strategic complements, as Gal-Or (1985) showed. The intuition for second mover advantage goes as follows. Firm 1 (first mover) raises its price very much to attain the highest profit possible (on the reaction curve of firm 2), which benefits firm 2 (second mover) more, since firm 2 only needs to undercut the rival price slightly.

### 4.2. Model of simultaneous delegation (Model P - II)

In this model, two firms set their prices sequentially after observing delegation contracts of both firms. Given \( \alpha_1 \) and \( \alpha_2 \), in the second stage, the Stackelberg prices are

\[
p_1 = \frac{a(2 + b) + (2 - b^2)c\alpha_1 + bc\alpha_2}{2(2 - b^2)},
\]

\[
p_2 = \frac{a(4 + 2b - b^2) + b(2 - b^2)c\alpha_1 + (4 - b^2)c\alpha_2}{4(2 - b^2)}.
\]
Reaction curves of the owners can be found from the indirect profit functions obtained by substituting (17) and (18) into profit functions as follows:

\[ \alpha_1 = 1, \]  
\[ \alpha_2 = \frac{ab^2(4 + 2b - b^2) + 2(8 - 10b^2 + 3b^4)c + b^3(2 - b^2)c\alpha_1}{(16 - 16b^2 + 3b^4)c}. \]  

Note that \( \alpha_2 \) is a strategic complement to \( \alpha_1 \) unlike in the case of quantity competition. Now, we have

**Proposition 5.** Let \( \alpha_i^{P_2} \) and \( \pi_i^{P_2} \) be the equilibrium contract and the equilibrium profit of firm \( i \) of Model \( P - II \). Then, (i) \( \alpha_1^{P_2} = 1, \alpha_2^{P_2} > 1 \), and (ii) \( \pi_1^{P_2} > \pi_2^{P_2} \).

As in Model \( Q - II \), \( \alpha_1 = 1 \) is the dominant strategy for firm 1. Firm 2 commits the manager to less aggressive behaviour by a contract \( \alpha_2 > 1 \) to induce a higher price. It is also interesting that the first mover advantage appears in this model. Analogously to Model \( P - I \), the second mover (firm 2) wants to raise its price (to earn more profit) by signing a contract \( \alpha_2 > 1 \) committing the manager to less aggressive behaviour, but the first mover (firm 1) gains more by this price increase.

### 4.3. Model of sequential delegation (Model \( P - III \))

The main issue in this model is whether the owner of firm 1 can increase its profit by pre-committing to a sales-based delegation contract (inducing the less aggressive behaviour of his manager) rather than passively playing the dominant strategy of the static delegation game (\( \alpha_1 = 1 \)).

Given \( \alpha_1 \) and \( \alpha_2 \), Stackelberg prices remain the same as equations (17) and (18). Let us consider the first stage. Our intuition goes as follows. The owner of firm 1, who is a Stackelberg leader in contract competition, offers a contract \( \alpha_2 > 1 \) to induce less aggressive behaviour of his manager, and subsequently the owner of firm 2 also increases \( \alpha_2 \) in a way that \( \alpha_2 > \alpha_2^{P_2} \), since \( \alpha_2 \) is a strategic complement to \( \alpha_1 \). The following proposition shows that, as a result, the profit of firm 1 is increased, but interestingly the profit of firm 2 is increased more.

**Proposition 6.** Let \( \alpha_i^{P_3} \) and \( \pi_i^{P_3} \) be the equilibrium contract and the equilibrium profit of firm \( i \) of Model \( P - III \). Then, (i) \( \alpha_1^{P_3}, \alpha_2^{P_3} > 1 \), and (ii) \( \pi_1^{P_3} < \pi_2^{P_3} \).

This proposition says that the second mover advantage reappears when Stackelberg leadership is allowed to firm 1 in contract competition as well.

| Table 2. Delegation contracts and profits of each firm under price competition. |
|--------------------------------------|-----------------|-----------------|-----------------|
| Model     | \( a_1 \) | \( a_2 \) | \( \pi_1 \) | \( \pi_2 \) |
| \( P - I \) | 1 | 1 | \( \frac{2b^2(2a-1-bc)^2}{8(2-b^2)} \) | \( \frac{4+b^2(2a-1-bc)^2}{16(2-b^2)} \) |
| \( P - II \) | 1 | \( > 1 \) | \( \frac{(2b^2(2a-1-bc)^2}{8(2b^2-4a+6b^2-8b^3+3b^4)}}{2(16+16b^2+3b^4)} \) | \( \frac{(4+2b^2(2a-1-bc)^2}{4(4-4b^2+3b^4)} \) |
| \( P - III \) | \( > 1 \) | \( > 1 \) | \( \frac{(8+4b^2-6b^4+2b^6+(a-1-bc)^2)(2a-1-bc)^2}{16(2-b^2)(2b^2-4a+6b^2+8b^3+3b^4)} \) | \( \frac{(4-3b^2(2b^2-4a+6b^2-8b^3+3b^4)}{64(2-b^2)(2b^2-4a+6b^2+8b^3+3b^4)} \) |

Source: Author.
In Table 2, we summarise each firm’s delegation contracts and profits under price competition. It is noted that preferences of both firms coincide in the sense that \( \pi_{i}^{P-III} > \pi_{i}^{P-II} > \pi_{i}^{P-I} \) for \( i = 1, 2 \).

5. Conclusion

In this article, we have considered various models involving a Stackelberg leader and a Stackelberg follower in the output market, and examined under what circumstances the second mover advantage rather than the first mover advantage appears or disappears. It is also interesting to observe from the analysis of three models described in Section 2 that \( \pi_{i}^{P3} > \pi_{i}^{P2} > \pi_{i}^{P1} \) for \( i = 1, 2 \) regardless of first mover advantage or second mover advantage, while two firms’ preferences for sequences do not coincide under quantity competition. This raises the possibility of endogenising the sequence of firms’ decisions as interesting future research.

Notes

1. The term ‘strategic delegation’, which was introduced by Schelling (1960) in a situation where delegation is used as a ‘self commitment device’, has received great attention in the industrial organisation literature since then.
2. We do not lose generality by assuming linear contracts because Holmstrom and Milgrom (1987) showed that linear contracts are optimal under some realistic conditions.
3. In Figure 1, it does not help to induce, for example, \( E' \) or \( E'' \) by a contract \( \alpha_2 < 1 \) or \( \alpha_2 > 1 \). For it cannot change firm 1’s output.
4. Bonatti and Martina (2004) and Kopel and Löffler (2008) also obtained similar findings.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Alchian, A. A. (1950). Uncertainty, evolution, and economic theory. *Journal of Political Economy*, 58, 211–221. Retrieved from [http://www.jstor.org/stable/1827159](http://www.jstor.org/stable/1827159)

Amir, R., & Stepanova, A. (2006). Second-mover advantage and priceleadership in bertrand duopoly. *Games and Economic Behavior*, 55, 1–20. doi:10.1016/j.geb.2005.03.004

Baumol, W. (1958). On the theory of oligopoly. *Economica*, 25, 187–198. Retrieved from [http://www.jstor.org/stable/2550723](http://www.jstor.org/stable/2550723)

Bonatti, A., & Martina, R. (2004, September). *Multi-stage games with sequential choices and the second-mover advantage*. Paper presented at the 31st conference of the European Association for Research in Industrial Economics (E.A.R.I.E.), Berlin.

Fershtman, C., & Judd, K. (1987). Equilibrium incentives in oligopoly. *American Economic Review*, 77, 927–940. Retrieved from [http://www.jstor.org/stable/1810218](http://www.jstor.org/stable/1810218)

Friedman, M. (1953). The methodology of positive economics. In *Essay in positive economics* (pp. 3–43). Chicago, IL: Chicago University Press.

Gal-Or, E. (1985). First mover and second mover advantages. *International Economic Review*, 26, 649–653. Retrieved from [http://www.jstor.org/stable/2526710](http://www.jstor.org/stable/2526710)

Holmstrom, B., & Milgrom, P. (1987). Aggregation and linearity in the provision of intertemporal incentives. *Econometrica*, 55, 303–328. Retrieved from [http://www.jstor.org/stable/1913238](http://www.jstor.org/stable/1913238)

Kopel, M., & Löffler, C. (2008). Commitment, first-mover-, and second-mover advantage. *Journal of Economics*, 94, 143–166. doi:10.1007/s00712-008-0004-4
Appendix

Proof of Proposition 1: (i) Differentiating the indirect profit function of firm 2 given by equation (3) with respect to \( \alpha_2 \) leads to

\[
\frac{\partial \pi_2}{\partial \alpha_2} = \frac{c}{2} \left( \frac{a - bq_1 - \alpha_2 c}{2b} \right) + \left[ a - b \left( q_1 + \frac{a - bq_1 - \alpha_2 c}{2b} \right) - c \right] \left( \frac{-c}{2b} \right)
\]

\[
= \frac{a - bq_1 - \alpha_2 c - (a - bq_1 + \alpha_2 c)}{4b} + \frac{c^2}{2b}
\]

\[
= \frac{c^2(1 - \alpha_2)}{2b}.
\]

Therefore, we obtain \( \alpha_2 Q_1^* = 1 \). Similarly, differentiating equation (5) with respect to \( \alpha_1 \) yields

\[
\frac{\partial \pi_1}{\partial \alpha_1} = \frac{1}{8b} \left( 2c(a + (1 - 2\alpha_1)c) - 2c(a + 2\alpha_1 - 3c) \right) = \frac{c^2(1 - \alpha_1)}{b},
\]

implying that \( \alpha_1 Q_1^* = 1 \). (ii) Let \( q_i Q_1^* \) be the equilibrium quantity of firm \( i \) in Model Q - I. We obtain \( q_1 Q_1^* = \frac{a - c}{2b} \) and \( q_2 Q_1^* = \frac{a - c}{4b} \) by substituting \( \alpha_1 = \alpha_2 = 1 \) into equations (2) and (4), and in turn \( \pi_1 Q_1^* = \frac{(a - c)^2}{8b} \) and \( \pi_2 Q_1^* = \frac{(a - c)^2}{16b} \) by substituting \( \alpha_i Q_1^* \) and \( q_i Q_1^* \) (for \( i = 1, 2 \)) into equations (3) and (4). Therefore, we have \( \pi_1 Q_1^* > \pi_2 Q_1^* \).

Proof of Proposition 2: (i) We have \( \alpha_2 Q_2^* = \frac{2(a - 3\alpha_1 c - a)}{3c} = \frac{4(a - 3\alpha_1 c - a)}{6c} \) by substituting (10) into (11). (ii) Substituting (10) and (11) into (8) and (9), we obtain \( \pi_1 = \frac{3(a - c)^2}{16b} \) and \( \pi_2 Q_2^* = \frac{(a - c)^2}{12b} \). Therefore, we have \( \pi_1 Q_2^* < \pi_2 Q_2^* \).

Proof of Proposition 3: (i) The first-order condition of (12) requires that

\[
\frac{\partial \pi_1}{\partial \alpha_1} = \frac{c(a - 9c + 8\alpha_1 c)}{9b} = 0,
\]

implying that \( \alpha_1 Q_3^* = \frac{9c - a}{c} \), and consequently \( \alpha_2 Q_3^* = \frac{5c - a}{c} \) from equation (11). (ii) Let \( q_i Q_3^* \) be the equilibrium quantity of firm \( i \) in Model Q - III. By substituting \( \alpha_i Q_3^* \) and \( q_i Q_3^* \) into equations (6) and (7), we obtain \( q_1 Q_3^* = \frac{a - c}{2b} \) and \( q_2 Q_3^* = \frac{3(a - c)^2}{8b} \). Therefore, \( \pi_1 Q_3^* = \frac{(a - c)^2}{16b} \) and \( \pi_2 Q_3^* = \frac{3(a - c)^2}{64b} \), hence \( \pi_1 Q_3^* > \pi_2 Q_3^* \).

Proof of Proposition 4: (i) It is trivial to see that \( \alpha_1 P_1^* = \alpha_2 P_2^* = 1 \) from the first-order conditions of problems maximising (14) and (21). (ii) We have

\[
\pi_1 = (p_1 - c)q_1
\]

\[
= \frac{[a(2 + b) - (4 - 2b - 2b^2)c + (2 - b)^2 c(a_1)](a(2 + b) + bc - (2 - b^2)c a_1)}{8(2 - b^2)}.
\]

(21)
Substituting (13), (16) and $\pi'_1 = \pi'_2 = 1$ into (14) and (21) yields
\[
\pi'_1 = \frac{(2 + b)^2(a - (1 - b)c)^2}{8(2 - b^2)},
\]
\[
\pi'_2 = \frac{(4 + 2b - b^2)^2(a - (1 - b)c)^2}{16(2 - b^2)}.
\]
It follows that $\pi'_1 < \pi'_2$. ■

**Proof of Proposition 5:** (i) We have
\[
\pi'_2 = 1
\]
\[
\pi'_2 = \frac{ab^2(4 + 2b - b^2) + (16 - 20b^2 + 2b^3 + 6b^4 - b^5)c}{(16 - 16b^2 + 3b^4)c}.
\]
Since $\pi'^2_2 - 1 = \frac{b^2(4 + 2b - b^2)(a - (1 - b)c)}{(16 - 16b^2 + 3b^4)c} > 0$, $\pi'^2_2 > 1$. Also, we have
\[
\pi'_2 = \frac{(2 - b^2)(8 + 4b - 4b^2 - b^3)(a - (1 - b)c)^2}{2(16 - 16b^2 + 3b^4)^2},
\]
\[
\pi'_2 = \frac{(4 + 2b - b^2)^2(a - (1 - b)c)^2}{4(4 - b^2)(4 - 3b^2)}.
\]
Therefore, $\pi'_1 - \pi'_2 = \frac{b^2(16 + 16b - 4b^2 - 3b^3)(a - (1 - b)c)^2}{4(4 - 3b^2)(4 - b^4)} > 0$. ■

**Proof of Proposition 6:** (i) We have
\[
\pi'_3 - 1 = \frac{b^4(8 + 4b - 4b^2 - b^3)(a - (1 - b)c)}{4(2 - b^2)^2(8 - 8b^2 + b^4)c} > 0,
\]
\[
\pi'_3 - 1 = \frac{b^2(16 + 8b - 12b^2 - 4b^3 + b^4)(a - (1 - b)c)}{4(2 - b^2)(8 - 8b^2 + b^4)c} > 0.
\]
(ii) Also, we have
\[
\pi'_3 = \frac{(8 + 4b - 4b^2 - b^3)(a - (1 - b)c)^2}{16(2 - b^2)(8 - 8b^2 + b^4)},
\]
\[
\pi'_2 = \frac{(2 - b)(2 + b)^3(4 - 3b^2)(8 - 6b^2 + b^3)(a - (1 - b)c)^2}{64(2 - b^2)^2(8 - 8b^2 + b^4)^2}.
\]
Therefore, it follows that
\[
\pi'_1 - \pi'_2 = \frac{-b^2(64 + 64b - 48b^2 - 48b^3 + 8b^4 + 7b^5)(a - (1 - b)c)^2}{64(2 - b^2)^3(8 - 8b^2 + b^4)^2} < 0
\]