Accelerated expansion in modified gravity with a Yukawa-like term

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Abstract

We discuss the Palatini formulation of modified gravity including a Yukawa-like term. It is shown that in this formulation, the Yukawa term offers an explanation for the current exponential accelerated expansion of the universe and reduces to the standard Friedmann cosmology in the appropriate limit. We then discuss the scalar-tensor formulation of the model as a metric theory and show that the Yukawa term predicts a power-law acceleration at late-times. The Newtonian limit of the theory is also discussed in context of the Palatini formalism.

1 Introduction

The expansion of the universe is currently undergoing a period of acceleration which is directly measured from the light-curves of several hundred type Ia supernovae [1, 2] and independently from observations of the cosmic microwave background (CMB) by the WMAP satellite [3] and other CMB experiments [4, 5]. However, the mechanism responsible for this acceleration is not well understood and many authors introduce a mysterious cosmic fluid, the so called dark energy, to explain this effect [6]. Recently, within the framework of higher order gravity theories [10], it has been shown that such an accelerated expansion could be the result of a modification to the Einstein-Hilbert action [7]. One such modification has been proposed in [8] where a term of the form $R^{-1}$ was added to the usual Einstein-Hilbert action, whose origin can be traced back to the string/M-theory [9]. It was then shown that this term could give rise to accelerating solutions of the field equations without dark energy.

Based on this modified action, we first use the Palatini variational formalism to derive our field equations. In the Palatini formulation, instead of varying the action with respect to the metric, one views the metric and connection as independent field variables and vary the action with respect to both independently. In the original Einstein-Hilbert action, this approach gives the same field equations as the metric variation. However when nonlinear terms are added to the action, Palatini formalism leads to different dynamical equations. The importance of this formalism lies in the fact that, contrary to the theory resulting from the metric variation which explains the accelerated expansion of the universe but is in conflict with solar system experiments, it is free of such conflicts. Interestingly, the Palatini formalism has been shown to be equivalent to a scalar-tensor type theory [14] in which the scalar field kinetic term is absent from the action [11]. These results are important and fundamental.

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for this formalism. Also in [16], Nojiri and Odintsov have shown that a combination of the $R^{-1}$ and $R^2$ terms can drive both the current acceleration and inflation. The Palatini form of this theory is studied in [13] where the authors obtain the current acceleration of the cosmic which is compatible with observations. In [22] the authors propose a $\ln R$ term to account for the current acceleration and also inflation. The Palatini formulation of $\ln R$ gravity is studied in [15] where the acceleration of the cosmic in the limit of small curvature is predicted. For a more recent and comprehensive review, the reader should consult [17] for models with modified gravity and [18] for models with the Palatini approach.

In this paper, we have used the Palatini formalism in a theory where a Yukawa type term has been added to the action. Since the term $R^{-1}$, mentioned above, is similar to the Yukawa term presented below when $R$ is small, we also expect this theory to have its origins in string/M-theory, at least for small curvatures. We have shown that the resulting theory could account for the accelerated expansion of the universe. We also study our model in the context of what is known as the scalar-tensor theory [14] where the metric formalism is used, leading to a power-law acceleration in the limit of small curvature. The Newtonian limit of the theory is investigated and shown to be correctly satisfied.

2 Friedmann equation in Palatini formalism

To begin with, a quick review of the Palatini formalism [12, 13] would be in order. Consider the action

$$S = -\frac{1}{2\kappa} \int d^4x \sqrt{-g} \mathcal{L}(R) + \int d^4x \sqrt{-g} \mathcal{L}_M,$$

where $\kappa = 8\pi G$ and $\mathcal{L}_M$ is the Lagrangian density for matter. Variation with respect to $g_{\mu\nu}$ gives

$$\mathcal{L}'R_{\mu\nu} - \frac{1}{2}\mathcal{L}g_{\mu\nu} = -\kappa T_{\mu\nu},$$

where a prime denotes differentiation with respect to $R$ and $T_{\mu\nu}$ is the energy-momentum tensor in the matter frame. We assume that the universe is filled with dust and radiation, represented by energy densities $\rho_m$ and $\rho_r$ respectively, thus

$$T^\mu_\nu = \{-\rho_m - \rho_r, p_r, p_r, p_r\},$$

with $T = g^{\mu\nu}T_{\mu\nu} = -\rho_m$, on account of $p_r = \rho_r/3$. As is well known, in the Palatini formulation the connection is not associated with $g_{\mu\nu}$ but with $h_{\mu\nu} \equiv \mathcal{L}'g_{\mu\nu}$ which is known from varying the action with respect to $\Gamma^\lambda_{\mu\nu}$. The Christoffel symbols associated with $h_{\mu\nu}$ are given by

$$\Gamma^\lambda_{\mu\nu} = \{^\lambda_{\mu\nu}\}_g + \frac{1}{2\mathcal{L}'} \left[ 2\delta^\lambda_{(\mu}\partial_{\nu)}\mathcal{L}' - g_{\mu\nu}g^{\lambda\sigma}\partial_\sigma\mathcal{L}' \right],$$

where the subscript $g$ indicates association with metric $g_{\mu\nu}$. The Ricci curvature tensor is given by

$$R_{\mu\nu} = R_{\mu\nu}(g) - \frac{3}{2}\mathcal{L}'^{-2}\nabla_\mu\mathcal{L}'\nabla_\nu\mathcal{L}' + \mathcal{L}'^{-1}\nabla_\mu\nabla_\nu\mathcal{L}' + \frac{1}{2}\mathcal{L}'^{-1}g_{\mu\nu}\nabla_\sigma\nabla^\sigma\mathcal{L}' ,$$

with the curvature scalar written as

$$R = R(g) + 3\mathcal{L}'^{-1}\nabla_\mu\nabla^\mu\mathcal{L}' - \frac{3}{2}\mathcal{L}'^{-2}\nabla_\mu\mathcal{L}'\nabla^\mu\mathcal{L}' ,$$

where $R_{\mu\nu}(g)$ is the Ricci tensor associated with $g_{\mu\nu}$ and $R = g^{\mu\nu}R_{\mu\nu}$. Contracting (2), one gets

$$\mathcal{L}'R - 2\mathcal{L} = -\kappa T.$$
Equations (5) and (6) now define the Ricci tensor with respect to $h_{\mu\nu}$. To derive the Modified Friedmann (MF) equation we follow [13, 15] and consider the flat Robertson-Walker metric for the evolution of the cosmos

$$ds^2 = -dt^2 + a(t)^2dx^2.$$  

(8)

This choice is considered to be consistent with observations [2]. From equations (8) and (5) we get the non-vanishing components of the Ricci tensor

$$R_{00} = \frac{3}{a^2} - \frac{3}{2}L'^{-2}(\partial_0 L')^2 + \frac{3}{2}L'^{-1}\nabla_0 \nabla_0 L',$$  

(9)

$$R_{ij} = -\left(a\ddot{a} + 2\dot{a}^2 + L'^{-1}\Gamma_{ij}^0 \partial_0 L' + \frac{a^2}{2}L'^{-1}\nabla_0 \nabla_0 L'\right)\delta_{ij}.$$  

(10)

Substituting equations (9) and (10) into field equations (2), we find

$$6H^2 + 3HL'^{-1}\partial_0 L' + \frac{3}{2}L'^{-2}(\partial_0 L')^2 = \frac{\kappa(\rho + 3p) - \mathcal{L}}{L'},$$  

(11)

where $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter with $\rho$ and $p$ being the total energy density and total pressure respectively. If we solve $R$ in terms of $T$ from equation (7) and substitute it into the expressions for $L'$ and $\partial_0 L'$ in the above equation, we will get the MF equation.

3 Yukawa term

To continue, let us consider an action of the form

$$\mathcal{L}(R) = R - \frac{\beta}{R} \exp(-\alpha R),$$  

(12)

where $\alpha, \beta > 0$ and, as can clearly be seen, the Yukawa term dominates at small curvature. This feature is important since, as well shall see, it helps to explain the accelerated expansion together with the assumption that $R < 0$. The contracted field equation (7) now reads

$$f(R) \equiv -\frac{R}{\beta} + \alpha \exp(-\alpha R) + \frac{3}{R} \exp(-\alpha R) = -\kappa T/\beta = \frac{\kappa \rho_m}{\beta},$$  

(13)

where $f(R)$ is a monotonically decreasing function and we have that $\lim_{R \to -\infty} f(R) \to +\infty$ and $\lim_{R \to 0} f(R) \to -\infty$. Thus $R$ is uniquely determined for any value of $\kappa \rho_m/\beta \equiv x$ through equation (13). From the conservation equation

$$\dot{\rho}_m + 3H\rho_m = 0,$$  

(14)

we find

$$\partial_0 L' = \frac{3\beta (\alpha^2/R + 2\alpha/R^2 + 2/R^3)}{-[\alpha^2 + 3(\alpha/R + 1/R^2) + \exp(\alpha R)/\beta]} \left(\frac{\kappa \rho_m}{\beta}\right) H.$$  

(15)

Let us now define $\frac{\partial_0 L'}{H} \equiv F(x, \alpha, \beta)H$ such that in the limit $\beta \to 0$, $F(x, \alpha, \beta) = 0$, since from (15) one can see that $\partial_0 L'$ vanishes in this limit. Noting that $R = R(x)$ and substituting this into equation (11) we get the MF equation

$$H^2 = \frac{\kappa \rho_m + 2\kappa \rho_r - \beta [R/R - \exp(-\alpha R)/R]}{[1 + \frac{\alpha \beta \exp(-\alpha R)}{R} + \frac{\beta \exp(-\alpha R)}{R^2}] [6 + 3F(x, \alpha, \beta)(1 + 1/2F(x, \alpha, \beta))]}.$$  

(16)

It can be seen from equations (13), (15) and (16) that when $\beta \to 0$ or equivalently $F(x, \alpha, \beta) = 0$, the MF equation will reduce to the standard Friedmann equation

$$H^2 = \frac{\kappa (\rho_m + \rho_r)}{3}.$$  

(17)
Let us now consider the evolution of the universe in vacuum, i.e. \( \rho_m = 0 \) and \( \rho_r = 0 \). If one defines the parameter \( k > 1 \) according to the following relation

\[
R_0 = -\frac{1}{\alpha} \ln k, \tag{18}
\]

where the subscript zero represents association with vacuum and substitutes this into the vacuum field equation \( f(R) = 0 \), one gets

\[
\alpha = \pm \frac{\ln k}{\sqrt{k\beta(3 - \ln k)}}. \tag{19}
\]

Assuming \( \alpha > 0 \), we find

\[
R_0 = -\sqrt{k\beta(3 - \ln k)}. \tag{20}
\]

Substituting this into the the vacuum MF equation and setting \( x = 0 \) results in

\[
H_0^2 = \frac{\sqrt{\beta}}{12} \sqrt{k(3 - \ln k)}. \tag{21}
\]

Note that equation (16) reduces to the above equation in vacuum. This equation tells us that when \( \sqrt{\beta} \sim H_0^2 \sim (10^{-33}\text{eV})^2 \) and \( k < e^3 \), modified gravity with a Yukawa term points to an accelerated evolution of the universe. It is worth noting that the role of \( \sqrt{\beta} \) is similar to a cosmological constant in this theory. If the density of dust cannot be ignored, i.e. \( \kappa \rho_m/\beta \gg 1 \), then this will be equivalent to \( \beta \) being as small as the dust energy, so that \( F(x, \alpha, \beta) \sim 0 \) and from equation (13) we obtain \( R \approx -\kappa \rho_m \). Therefore, in the Big Bang Nucleosynthesis (BBN) epoch, the MF equation reduces to the standard Friedmann equation. Consequently, the Yukawa term would no longer be effective there. However, for the present epoch the condition \( \kappa \rho_m/\beta \gg 1 \) breaks down and the Yukawa term starts to dominate.

4 Scalar-tensor formulation

In this section we study the scalar-tensor formulation of our model along the lines introduced in [11]. In this theory, the Palatini form of the action is shown to be equivalent to a scalar-tensor type theory from which the the scalar field kinetic energy is absent. This is achieved by introducing a conformal transformation in which the conformal factor is taken as an auxiliary scalar field. We show that the resulting field equations in the small curvature regime predict a power-law type acceleration for the universe. Let us then first give a brief review of the formalism discussed in [14].

Introducing the auxiliary scalar field \( \phi \), one may write the action (1) as \([14, 16]\)

\[
S[g_{\mu\nu}, \phi, \psi_m] = -\frac{1}{2\kappa} \int d^4x \sqrt{-g} [\mathcal{L}(\phi) + (R - \phi)\mathcal{L}'(\phi)] + S_m[g_{\mu\nu}, \psi_m], \tag{22}
\]

where \( \psi_m \) represents the matter field. The equation of motion resulting from this action for the scalar field is \( \ddot{\phi} = R \), as long as \( \mathcal{L}''(\phi) \neq 0 \), making action (22) classically equivalent to the action (1). This action is sometimes referred to as the Jordan (physical) frame action. Under the conformal transformation

\[
\tilde{g}_{\mu\nu} = e^{\sqrt{\frac{3}{2\kappa}} \phi} g_{\mu\nu}, \tag{23}
\]

with

\[
\Phi = \sqrt{\frac{3}{2\kappa}} \ln \mathcal{L}'(\phi), \tag{24}
\]

action (22) can be written in the Einstein frame as \([14, 16]\)

\[
\tilde{S}[\tilde{g}_{\mu\nu}, \Phi, \psi_m] = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{\tilde{R}}{2\kappa} - \frac{1}{2} \left( \nabla \Phi \right)^2 - V(\Phi) \right] + S_m \left[ \exp \left( -\sqrt{2\kappa/3} \Phi \right) \tilde{g}_{\mu\nu}, \psi_m \right]. \tag{25}
\]
Here, $\tilde{g}_{\mu\nu}$ is the metric in the Einstein frame [14] and $\tilde{R}$ is the scalar curvature associated with $\tilde{g}_{\mu\nu}$. It may be shown that the potential is given by

$$V(\phi) = \frac{\phi \mathcal{L}'(\phi) - \mathcal{L}(\phi)}{2\kappa \mathcal{L}^2(\phi)}.$$  \hfill (26)

This is the standard form of the scalar-tensor type theories mentioned above. It is worth noting that since we are working with metric connections here, the kinetic term in action (25) is present, contrary to the case when one works within the framework of the Palatini formalism.

Let us now concentrate on the action represented by (12) for which the potential can be written as

$$V(\phi) = \frac{\alpha \beta e^{-\alpha \phi} + \frac{2\beta e^{-\alpha \phi}}{\phi} + \frac{\beta e^{-\alpha \phi}}{\phi^2} \phi^2}{2 \kappa (1 + \alpha \beta e^{-\alpha \phi} + \frac{\beta e^{-\alpha \phi}}{\phi^2} \phi^2)^2}.$$  \hfill (27)

When $\phi$ is small, equation (24) is approximated by $\phi \approx \sqrt{\beta} \exp \left(-\frac{1}{2} \sqrt{2\kappa} \Phi \right)$ and in this limit potential (27) takes the following form

$$V(\Phi) \approx \frac{\phi^3}{\kappa \beta} \approx \frac{1}{\kappa} \sqrt{\beta} \exp \left(-\sqrt{\frac{3\kappa}{2}} \Phi \right).$$  \hfill (28)

As will be shown below, this potential predicts a universe evolving with a power-law acceleration. Typical plots of $V(\phi)$ are given in figure 1. For $\alpha^2 \beta = 0.1$, the potential has a maximum at $\phi \approx 0.47$. Therefore, if the universe starts from the maximum, it either asymptotically evolves to a de Sitter solution, or undergoes a power-law acceleration. The minimum of the potential occurs at $\phi \approx -0.67$ and thus if the universe starts at this point, it is in an anti-de Sitter phase where the curvature is negative. Therefore, when the absolute value of $\phi$ becomes small, the universe evolves to a power-law acceleration at late-times and at $\phi \rightarrow -3.98$, $V(\phi)$ is unbounded from above. For $\alpha^2 \beta = 0.6$ the behavior of the potential for $\phi \geq 0$ is qualitatively the same as in the previous case. In the region $\phi < 0$, when $\phi \rightarrow -1.99$ the potential becomes unbounded from below and for $\phi \approx -2.1$ it has a maximum.

To proceed further, let us consider evolution of the scale factor with time. From action (25) one gets

$$3\tilde{H}^2 = \kappa (\rho_\Phi + \tilde{\rho}_m),$$  \hfill (29)

and

$$\Phi'' + 3\tilde{H} \Phi' + \frac{dV(\Phi)}{d\Phi} - \frac{(1 - 3w)}{\sqrt{6}} \tilde{\rho}_m = 0,$$  \hfill (30)

where a prime denotes $d/d\tilde{t}$, and

$$\rho_\Phi = \frac{1}{2} \Phi'^2 + V(\Phi).$$  \hfill (31)

These are cosmological equations of motion in the Einstein frame where $w$ is the usual equation of state parameter and $\tilde{\rho}_m$ is the matter density with $\tilde{H}$ being the Hubble parameter [8, 16]. We must now solve the system of equations (29) and (30). We first consider the case where $\tilde{\rho}_m = 0$. When the potential is given by (28) and $\phi = R$ is small, a solution is given by

$$\tilde{a}(\tilde{t}) \propto \tilde{t}^{4/3},$$  \hfill (32)

and

$$\Phi \propto -\frac{4}{3} \ln \tilde{t}. \hfill (33)$$

Here, $\tilde{t}$ is the time coordinate in the Einstein frame, which is related to the coordinate $t$ in the Jordan frame by $e^{-\frac{t}{\sqrt{2\kappa} \Phi}} d\tilde{t} = dt$. As a result

$$3\tilde{t}^{1/3} = t,$$  \hfill (34)
The power-law acceleration also occurs in the physical (Jordan) frame

\[ a(t) = e^{-\frac{1}{2}\sqrt{-\frac{2\kappa}{3}}\Phi} a \propto t^2 \] (35)

which is consistent with the result in [8]. Hence, at small curvatures, the cosmic acceleration is predicted by the Yukawa-like term.

Although there is no need to introduce dark energy in modified gravity, a better understanding of the equation of state in this theory is afforded by following [21] and define the effective Equation of State (EOS). Written in this form, it has the advantage that any parameterizations done within the context of EOS for dark energy can be compared with that of modified gravity. Let us then start from equation (35) and find the Hubble parameter as a function of the redshift \( z \) as

\[ H(z) = 2H_0(1 + z)^{1/2}, \] (36)

where \( a_0/a = 1 + z \) with \( a_0 \) and \( H_0 \) being the values of the parameter at the present epoch. It is worth noting that equation (36) is the same as that derived from the standard Friedmann equation with \( w = -2/3 \). Now, we may write the Friedmann equation in a formal fashion which would encapsulate any modification to the standard Friedmann equation in the last term regardless of its nature [21], that is

\[ H^2/H_0^2 = \Omega_m(1 + z)^3 + \delta H^2/H_0^2, \] (37)

where \( \Omega_m = \rho/\rho_{0c} \), \( \rho_{0c} = 3H_0^2/\kappa \). Also, defining the effective EOS, denoted by \( w_{eff}(z) \), as

\[ w_{eff}(z) = -1 + \frac{1}{3} \frac{d\ln\delta H^2}{d\ln(1 + z)}, \] (38)

we can calculate \( w_{eff}(z) \) using equations (36), (37) and (38) with the result

\[ w_{eff}(z) = -1 + \frac{1}{3} \frac{[4 - 3(1 + z)^2\Omega_m](1 + z)}{4(1 + z) - \Omega_m(1 + z)^3}. \] (39)

Figure 2 shows variations of \( w_{eff}(z) \) for various values of \( \Omega_m \). Equation (39) shows that \( -1 < w_{eff}(z) \leq -2/3 \), which is consistent with the observational data when the limits imposed by appropriate constraints are taken into account. It is therefore relevant at this point to have a brief look at these constraints and see how our results can be interpreted. Before doing so however, it should be mentioned that the model presented here is not expected to produce an effective phantom behavior, at least for small curvatures, since for small \( R \) it behaves like \( R^{-1} \) theories for which it is impossible to have phantom behavior, although an effective quintessence behavior is feasible [23].

The present observational data [24] suggest that our universe is dominated by a mysterious form of dark energy. As a result, the universe expansion is undergoing a period of acceleration. In terms of
the constant EOS parametrization, observational data provided by SDSS indicate that this constant is close to $-1$. In other words, the accelerating universe could be either caused by the cosmological constant $w = -1$, or quintessence, $-1 < w < -1/3$ or phantom era, $w < -1$. These constraints have been verified by the Supernovae, WMAP and cluster abundance observations [25]. The above results, derived within the framework of a scalar-tensor type model, seems to favor the quintessence which predicts values for $w$ in the range given above. However, in the Palatini formulation presented in section 3, we have obtained the cosmological constant represented by equation (21), which seems to be more in line with the cosmological constant type models.

5 Newtonian limit

The Newtonian limit of theories that predict cosmic acceleration, that is in the limit of weak field approximation, is naturally of interest and should be examined since such limits are expected to be compatible with the present cosmological data. It is known that the criteria for the correct Newtonian limit of $L(R)$ theories is provided by the Dick’s condition for fourth order theories [19], that is $L''(R)|_{R=R_0} = 0$. The same condition has also been obtained in the Palatini formalism when the equations of motion are of the second order [20]. With this criteria in mind, inspection of our Lagrangian, equation (12), suggests that it does not satisfy Dick’s condition. However, this should not be considered as a setback since with a small modification which does not affect our general results, we should be able to obtain the correct Newtonian limit.

Let us then modify Lagrangian (12) and write it as

$$L(R) = R - \frac{k\alpha^3}{(\ln k)^3} \beta R^2 - \beta \frac{e^{-\alpha R}}{R}. \quad (40)$$

Now, substitution of the above Lagrangian in equation (7) leads to $R_0 = \frac{1}{\alpha} \ln k$. Thus for $k = e^2$ we have that $L''(R_0) = 0$. This means that for the choice of $k$ made above, we recover the correct Newtonian limit. The interesting point is that if one applies the Palatini formalism to the above Lagrangian, the second term does not contribute in general and Lagrangian (40) behaves exactly the same as our original Lagrangian does so that the predictions for a universe with accelerated expansion are unaltered.

6 Conclusions

In this paper we have discussed the Palatini formulation of modified gravity with a Yukawa like term. This term may be used to explain the current exponential accelerated expansion and reduces to the standard Friedmann equation in vacuum. The scalar-tensor formulation of our model was discussed.
and shown to predict a power-law acceleration at late times. We have shown that in both formulations the accelerated expansion of the cosmic may be accounted for. However, we expect the universe to pass through a matter dominated era before the accelerated expansion phase is reached [26].

The Newtonian limit of this theory was also examined. It was shown that in the Palatini formulation our modified gravity action with a Yukawa term is equivalent to Lagrangian (40), with the latter satisfying Dick’s condition for the correct Newtonian limit.

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