Measurement of the strontium triplet Rydberg series by depletion spectroscopy of ultracold atoms

Luc Couturier, Ingo Nosske, Fachao Hu, Canzhu Tan, Chang Qiao, Y. H. Jiang, Peng Chen and Matthias Weidemüller

1Shanghai Branch, National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China
2CAS Center for Excellence and Synergetic Innovation Center in Quantum Information and Quantum Physics, University of Science and Technology of China, Shanghai 201315, China
3Shanghai Advanced Research Institute, Chinese Academy of Sciences, Shanghai 201210, China
4Physikalisches Institut, Universität Heidelberg, Im Neuenheimer Feld 226, 69120 Heidelberg, Germany

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We report on the atom loss spectroscopy of strontium Rydberg atoms in a magneto-optical trap, using a two-photon excitation scheme through the intermediate state 5s5p1P1. Energies of the 5s5p1S1 and 5snd3D1,2 Rydberg series of 88Sr in the range 13 ≤ n ≤ 50 are determined with an absolute accuracy of 10 MHz, including the perturbed region where the 5snd 3D2 series couples to the 5snd 1D2 series. This represents an improvement by more than two orders of magnitude compared to previously published data. The quantum defects for each series are determined using the extended Rydberg-Ritz formula in the range where there is no strong perturbation. A value of 1.377012721(10) MHz for the first ionization limit of 88Sr is extracted.

I. INTRODUCTION

Spectroscopy of the intricate electronic level structure of two-electron Rydberg atoms has stimulated the development of theoretical models for the description of correlated electrons, in particular multi-channel quantum defect theory (MQDT) [1, 2]. The interaction between the electrons leads to phenomena such as, e.g., autoionization of Rydberg states [3, 4]. With the advent of laser cooling techniques, new opportunities for applications of two-electron Rydberg atoms were identified [5]. The optical transition of the ion core of the Rydberg atoms allows for dipolar trapping of Rydberg atoms in lattices [6] but also new techniques such as spatially selective Rydberg atom detection through autoionization [7]. Alkaline-earth like atoms feature narrow intercombination lines, which, in combination with strong Rydberg-Rydberg interaction, make them a good candidate for quantum simulation via Rydberg dressing [8, 9] and for the generation of spin-squeezed states with applications in metrology [10].

In the case of strontium, the singlet Rydberg series are well known since the early days of laser spectroscopy [11] and have been further studied over the past years [12, 13]. Rydberg excitation of the triplet Rydberg states in an ultracold atomic gas has been realized only recently [9, 14–16]. Surprisingly, available spectroscopic data about the triplet series date back to the late 70’s [17, 19], where the measurements were performed with hot atoms at pressures on the order of ~ 10^{-2} mbar, leading to a significant line broadening. Accuracies of the absolute transition frequencies are on the order of few GHz. Such a large uncertainty has been the limiting factor in predicting properties of the strontium triplet series [20, 21].

In this paper we present spectroscopic data with an improved accuracy of the triplet Rydberg series 5s5p1S1, 5s5d 3D, and 5s5d 1D in the range n = 13...50. The data include the strongly perturbed region of the 5s5d 1D2 series which couples to the singlet series. The measurement of the triplet series is performed on an ultracold gas of 88Sr atoms using standard spectroscopic techniques [22, 23]. The Rydberg transitions are detected through atom-loss spectroscopy in a magneto-optical trap operated on the Sr intercombination line. The spectroscopic data are fitted to the extended Rydberg-Ritz formula far away from perturbations to extract reliable quantum defects for energy level prediction. We also extract an updated value for the first ionization limit of 88Sr [11].

II. EXPERIMENTAL METHODS

A. Rydberg excitation

A strontium magneto-optical trap (MOT), operated on the 5s2 1S0 → 5s5p 1P1 transition, is loaded from a strontium two-dimensional MOT as described in Ref. [24]. Atoms are then transferred to a MOT operated on the narrow transition 5s2 1S0 → 5s5p 3P1 transition (see, e.g., [25]), at a magnetic field gradient of 5 G/cm, as shown schematically in Fig. 1(b). We obtain about 10^6 atoms at a temperature of 1 μK. Due to the narrow linewidth of the transition, the atoms accumulate in the lower shell of an ellipsoid, as shown in the inset of Fig. 1(b), with a peak atomic density about 2 × 10^{10} atoms/cm^3 [26]. The shape of the atomic cloud reflects the fact that the Zeeman shift compensates the MOT beam detuning, as

*jiangyh@sari.ac.cn
†peng07@ustc.edu.cn
‡weidemueller@uni-heidelberg.de
described in Refs. 26, 27, at a finite magnetic field offset, essentially pointing along the vertical axis.

We excite the atoms to Rydberg states with two photons using the transitions \(5s^2 \, ^1S_0 \rightarrow 5p5p \, ^3P_1 \rightarrow 5nld^3L_J\), as shown in Fig. 1 (a). Starting from the \(5p5p \, ^3P_1\) state, we can access the \(5ns^3S_1\), \(5nd^3D_1\) and \(5nd^3D_2\) Rydberg series (abbreviated \(^3S_1\), \(^3D_{1,2}\) in the rest of the paper). These states decay back to \(5p5p \, ^3P_1\) but also \(5s5p \, ^3P_2\) and \(5s5p \, ^3P_0\) which are long-lived metastable states. The first photon is provided by the MOT laser field (see Fig. 1 (a)), which is generated by a 689 nm diode laser. The MOT is operated at a saturation parameter \(s \approx 20\). The laser is stabilized to an ultrastable cavity reducing its linewidth to less than 10 kHz. The cavity drift amounts to a 8 kHz/day laser frequency deviation which is compensated by using saturated absorption spectroscopy in a strontium heat pipe as a reference.

We excite the atoms in the \(5s5p \, ^3P_1\), \(m_J = +1\) state to a Rydberg state with a UV beam of 1.4 mm \(1/e^2\) diameter, larger than the size of the MOT. The UV beam is linearly polarized along the vertical direction, which drives \(\pi\) transitions due to the magnetic alignment of the atoms in the narrow-band MOT. 26, 27. We use a frequency doubled dye laser which can be tuned from \(\lambda\) in the narrow-band MOT 26, 27. We use a frequency doubled dye laser which can be tuned from \(\lambda\) in the narrow-band MOT 26, 27. The UV laser has a linewidth below 200 kHz to few mW, adjusted to keep a reasonable contrast as the loss is observed to increase at lower \(n\).

The UV laser frequency is scanned over the transition twice in each direction at a scan speed of \(\sim 200\) kHz/s. Atoms decaying to the metastable states, through direct or cascade decay induced by blackbody radiation, do not participate to the cooling cycle any longer and result in atom loss when performing absorption imaging at 461 nm on the \(5s^2 \, ^1S_0 \rightarrow 5s5p \, ^3P_1\) transition. We determine the number of remaining atoms before and after the Rydberg excitation. The overall repetition rate for the detection of Rydberg atoms is 0.5 Hz. There is a finite loss of \(\sim 30\%\) occurring without Rydberg excitation due to the finite MOT storage time. We plot the rescaled atom number versus the total energy, as shown in Fig. 2 for each Rydberg line.

B. Determination of the energy levels

The total energy is deduced from the sum of the two photon energies, at 689 nm and 318...331 nm. The energy of the first photon corresponds to the literature value of the transition for the \(5s5p \, ^3P_1\) state 29 plus a finite detuning of \(\Delta f_{\text{MOT}}\) = -600 kHz corresponding to the MOT laser detuning. This detuning is known on a 10 kHz level through absorption spectroscopy in a heat pipe.

As shown in Fig. 1 (b), we determine the frequency of the Rydberg excitation beam by measuring the frequency \(f_{\text{dye}}^{\text{WLM}}\) of the Rydberg excitation laser with a commercial wavelength meter (HighFinesse WSU-10). The wavelength meter has a specified accuracy of 10 MHz at three standard deviation for a range of \(\pm 200\) nm around the calibration wavelength. We calibrate the wavelength meter with the 689 nm laser, for which the corresponding strontium resonance frequency \(f_{\text{Ry}}^{\text{fit}}\) is known to an accuracy of 10 kHz.

As an additional frequency calibration close to the respective Rydberg lines, part of the light of the dye laser is sent to a saturated absorption spectroscopy of iodine. As described in Appendix A we find a systematic frequency shift of \(\delta f_{\text{sys}} = 16.8\) MHz with a statistical error of \(\pm 9.4\) MHz (at 1 \(\sigma\)) on the frequency reading. To determine the Rydberg state energy, we also include the Zeeman shift \(\delta f_{\text{Zeeman}}\) of the Rydberg states, which is series dependent and typically \(|\delta f_{\text{Zeeman}}| < 500\) kHz.

To determine the center of the Rydberg lines, we fit the spectrum obtained by scanning the UV light frequency with a Lorentzian function, as exemplarily shown in Fig. 2. The full-width half-maximum is typically 1 MHz and the fit error on the center determination from the fit is typically 10 kHz.

The uncertainty from the wavelength meter reading as described above is by far the major contribution to the uncertainty. Due to the small Rabi frequencies (below 500 kHz), AC Stark shifts are negligible on this scale. The frequency shifts due to interactions are limited by the excitation linewidth, i.e. the laser linewidth of 200 kHz or the Rabi frequency, which is much smaller than the wavelength meter uncertainty. To our knowl-
edge, the only published measurements of the DC polarizabilities for the triplet Rydberg states are found in Refs. 30, 31. Using the rescaled polarizability, a DC Stark shift of 100 kHz would correspond to a stray field of 30 mV/cm for the 5s50d 3D1 state, which is the measured state with one of the highest polarizabilities. Considering that there is no electrode inside or outside the steel vacuum chamber, these residual electric field values are assumed to be small and we therefore neglect a residual DC Stark shift. This is confirmed by the fact that there is no observable contribution in the Rydberg state energies which scales as $\propto n^4$ like the polarizability. When added in quadrature, we obtain a total uncertainty of 10 MHz (rounded to a 1 MHz precision).

III. RESULTS AND DISCUSSION

Unlike the singlet Rydberg series which have been determined with a an accuracy of 30 MHz 11, 32, the triplet series 3S1 and 3D1,2,3 have only been measured previously with an accuracy on the order of few gigahertz 17, 19. With our setup we have improved the accuracy on the transition frequencies to 10 MHz, i.e. two orders of magnitude, for the 3S1 and 3D1,2 triplet Rydberg series which are accessible through dipole transitions. We have mapped out all transition energies from $n = 13$ to $n = 50$ for these series; they are plotted in Fig. 3(a). The measured values are given by the Tables II and III in Appendix B.

The energies can be described by the Rydberg-Ritz formula:

$$E_n = I_s - \frac{\tilde{R}}{(n - \delta(n))^2}.$$  

where $I_s$ is the first ionization threshold, $\tilde{R}$ is the mass-corrected Rydberg constant for 88Sr, $n$ is the principal quantum number and $\delta(n)$ is the quantum defect, which is specific to each Rydberg series. $\tilde{R}$ is taken as 109 736.631 cm$^{-1}$ using the latest values of the fundamental constants 33 and of the strontium mass 34, 35. An accurate prediction of $\delta(n)$ using a model allows to reproduce and predict the Rydberg energies.

In Fig. 3(b) we show a plot of the experimental quantum defects versus the binding energy of the Rydberg states of the different series. It shows the energy dependence of the quantum defect and perturbations of the series, as described in detail in previous works 21, 36. Proper description of these energies would require MQDT 1, 2, which is beyond the scope of this paper. We can however extract some qualitative features. For small binding energies, the energy levels converge to the ionization energy, as expected from Eq. 1.

In the case of the 3S1 series, the quantum defect is

FIG. 2. Atom loss spectroscopy of the Rydberg line 5s17s 3S1. The blue circles is the fraction of atom number measured by absorption imaging remaining after Rydberg excitation in the MOT. The red dashed line is a Lorentzian fit. The frequency axis is the relative UV laser frequency with the origin set at the fitted center frequency. The error bars are estimated by analyzing the noise on the area of the imaging pictures where no atom is present.

FIG. 3. Experimental states energies and quantum defects of the 5sns 3S1 and 5snd 3D1,2 Rydberg series. (a) Measured binding energies of the triplet Rydberg series. The exact values can be found in the tables II and III in the Appendix B. (b) Quantum defects deduced from the Rydberg state energies by the Rydberg-Ritz formula versus its binding energy. The perturbation around $n = 15$ creates a coupling between 5snd 3D2 series with the singlet series 5snd 3D2. The solid lines are guides to the eye. The inset is a zoom on the 5sns 3S1 Rydberg series which exhibit a slight energy dependence. The error bars are much smaller than the symbols.
nearly independent of the binding energy, indicating a small influence of the ionic core polarizability. The small residual energy dependence is depicted by the inset in Fig. 3(b), which was not resolved in previous work [36]. The $^3D_1$ series is strongly perturbed near $n = 15$, that is attributed to a coupling to the $^3D_3$ series [21]. These perturbations for two-electron Rydberg atoms are essentially due to admixtures of doubly-excited states which shift the position of the Rydberg level. The $^3D_2$ series also exhibits a similar perturbation around $n = 15$. The behavior was explained by a six-channel MQDT [21], which includes a coupling to the $^1D_2$ series through a doubly excited state. Though the admixture, the transition from the 5s5p $^3P_1$ to the $^1D_2$ series becomes dipole allowed. As a consequence, we can observe three states of this series for $n = 14, 15, 16$. We assign the lines according to Refs. [11, 19], even though they are not pure states due to the strong mixing described in Ref. [21].

For practical purposes, we perform an analysis far away from the perturbation of the Rydberg series and describe the quantum defect using the extended Rydberg-Ritz formula:

$$\delta(n) = \delta_0 + \frac{\delta_2}{(n-\delta_0)^2} + \frac{\delta_4}{(n-\delta_0)^4} + ... \quad (2)$$

with $\delta_i$ ($i = 0, 2, 4,...$) parameters that have to be extracted from a fit to the experimental data. We perform such a fit of Eq. 1 combined with Eq. 2 with $\delta_i$ and the ionization limit $J_s$ as free parameters. We choose the fitting range such that the standard error on the fitted parameters is minimized. The fit results are shown in Table I and the residuals of the fits are depicted in Fig. 4 for the series $^3S_1$, $^3D_1$ and $^3D_2$. All three series can be well reproduced within the selected fitting range by including up to $\delta_4$. Higher orders do not improve the quality of the fit. Our findings represent a one to three order of magnitude improvement on the evaluation of the first term of the quantum defect $\delta_0$, as compared to Ref. [20]. The improved coefficients can be used to extrapolate the Rydberg state energies at higher $n$. However at lower principal quantum number, there are significant deviations due to Rydberg series perturbations and would require a MQDT model for an accurate description.

The ionization limit is determined from independent fits of the three Rydberg series (see Table I). All three values agree with each other within the error bar. We calculate a mean value weighted by the inverse of the square of the errors. The error on the ionization limit is taken as the uncertainty on the experimental data. The ionization limit for strontium $^{88}$Sr is thus $1377.012721(10)$ MHz. This value is $62$ MHz higher than the value from Ref. [11]. The discrepancy can be explained by the lower $n$ range used in the original work to extract the ionization limit, which is subject to Rydberg series perturbation.

IV. CONCLUSION

We have measured the total energy of the strontium Rydberg states for the $^3S_1$, $^3D_1$ and $^3D_2$ Rydberg series over the range of $n = 13$ to $n = 50$ by depletion spectroscopy in a magneto-optical trap operated on the narrow intercombination line, yielding spectral lines with a linewidth around $1$ MHz. Using the precision of a high precision wavelength meter combined with the absolute accuracy of an iodine absorption spectroscopy, we have achieved a $10$ MHz accuracy on the determination of the Rydberg energy levels and of the ionization limit. The improvement of the accuracy on these energies, in particular in the strongly perturbed region of the spectra, will be useful for improved theoretical predictions of the energy level positions [21] and of the Rydberg-Rydberg interactions [20], which can be in turn used to predict more accurately more complex effects such as Rydberg dressing [5]. The existence of a considerable mixing between the $^3D_2$ and $^1D_2$ Rydberg series around $n = 15$ has been confirmed through the direct observation of singlet states, which might offer interesting perspectives for optical multiwave mixing via Rydberg singlet-triplet coupling.

To further improve on the energy determination, one would need major improvements on the experimental setup. So far, the determination of the energy levels is mainly limited by the absolute accuracy of the standard wavelength meter combined with a simple iodine spectroscopy, but could be greatly reduce by the use of a frequency comb [37]. At this level, one would need a trap-free measurement with an accurate electric and magnetic field control by having electrodes and additional coils, that would result in even narrower Rydberg lines.
Appendix A: Wavelength meter calibration by iodine spectroscopy

To have a reliable estimate of the uncertainty of the frequency measurement by the wavelength meter, we employ Doppler-free spectroscopy of iodine, which has a well-known spectrum [39]. The Doppler-free hyperfine lines have a finite width and partially overlap with each other, as shown by two typical spectra depicted in Fig. 5(a) and (b) for a more accurate frequency determination, we chose groups of hyperfine lines with a comparatively small width.

To determine the frequency, we simulate the spectrum $S_{I2,sim}(f)$ with the software IodineSpec [40, 41] which provides an absolute accuracy about ±1.5 MHz (at 1σ). We adjust the entire spectrum by fitting the parameters $a$ and $\delta_f$ such that the experimental data overlap with $S_{I2,adj}(f) = a \times S_{I2,sim}(f + \delta_f)$, as plotted in Fig. 5(a) and (b) with a red solid line.

We choose iodine lines close to Rydberg resonances. We thus obtain 91 absolute frequencies whose position in frequency follows the Rydberg spectrum. Fig. 6 shows the deviation of the iodine line position of the spectra, for which the frequency is acquired by the wavelength meter, with the value from the IodineSpec software. As there is no obvious trend in this shift as a function of the transition frequency, we estimate the shift by the statistical mean of all measurements weighted by their respective error bar. We find a mean value of $\delta_f$ of +8.4 MHz.

| Series | $\delta_0$ | $\delta_2$ | $\delta_4$ | $I_S$ (MHz) | Fitted range |
|--------|------------|------------|------------|-------------|--------------|
| $5s_{rat}^3S_1$ | 3.370 778(4) | 0.418(1) | -0.3(1) | 1 377 012 720.6(7) | 15 $\leq n \leq 50$ |
| $5s_{nd}^3D_1$ | 2.675 17(20) | -13.15(26) | -4.444(91) $\times 10^3$ | 1 377 012 718(8) | 28 $\leq n \leq 50$ |
| $5s_{nd}^3D_2$ | 2.661 42(30) | -16.77(38) | -6.656(134) $\times 10^3$ | 1 377 012 718(12) | 28 $\leq n \leq 50$ |

TABLE I. Fitted quantum defects parameters $\delta_k$ ($k = 0, 2, 4$) and the ionization limit $I_S$ according to Eq. 1 and 2. The fitted range has been optimized to minimize the residual at high $n$, even though the series cannot be described fully by the Rydberg-Ritz formula. The uncertainties are obtained from the fit and are larger than the precision needed to reproduce the experimental data on a MHz level.
with a standard deviation of 4.7 MHz, which have to be multiplied by two for the UV frequency after frequency doubling. The standard deviation is used as a statistical error of the wavelength meter, even though the distribution is not Gaussian. Following the statistical analysis, 85% of the data points fall into one standard deviation, which indicates that the standard deviation might actually overestimate the real error.

The statistical uncertainty includes three sources of error: 1) The fitting error of the calculated spectrum to the data, 2) the statistical uncertainties of the spectrum predicted by the calculation, 3) the statistical error of wavelength measurement itself, which includes a possible long-term drift of the wavelength meter as the lines have been measured over a ten day period. The first source of error is the largest as our experimental data for the iodine spectrum have large error bars due to the electronic noise and distortion of the absorption signals which affects the fitting procedure. The second source of uncertainty is expected to be around 1.5 MHz, that is, the statistical error of the original data used by the software. As for the last source of error, from a previous work in Ref. [42], we have shown that this wavelength meter has a relative reading stability of 1.4 MHz at a 1 σ level over 10 hours (taking the frequency doubling into account). All three sources of error contribute to the statistical error that we provide as the error bar of the wavelength meter.

Appendix B: Experimental energies of the Rydberg states

The experimental energies of the Rydberg states for the 5ns \(^3\)S\(_1\) and 5snrd \(^3\)D\(_{1,2}\) Rydberg series are presented in Tables [I] and [II]. The total energies are referenced to the ground state 5s\(^2\) \(^1\)S\(_0\) and corrected for the systematic frequency shifts as presented in the main text.
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| n  | Series | $E_{exp}$ (MHz) | n  | Series | $E_{exp}$ (MHz) |
|----|--------|----------------|----|--------|----------------|
| 13 | $^3S_1$ | 1341 500 517   | 32 | $^3S_1$ | 1372 998 803   |
| 14 | $^3S_1$ | 1347 874 127   | 33 | $^3S_1$ | 1373 265 188   |
| 15 | $^3S_1$ | 1352 673 833   | 34 | $^3S_1$ | 1373 505 903   |
| 16 | $^3S_1$ | 1356 377 995   | 35 | $^3S_1$ | 1373 724 155   |
| 17 | $^3S_1$ | 1359 296 416   | 36 | $^3S_1$ | 1373 922 642   |
| 18 | $^3S_1$ | 1361 636 650   | 37 | $^3S_1$ | 1374 103 691   |
| 19 | $^3S_1$ | 1363 541 952   | 38 | $^3S_1$ | 1374 269 280   |
| 20 | $^3S_1$ | 1365 113 813   | 39 | $^3S_1$ | 1374 421 124   |
| 21 | $^3S_1$ | 1366 425 741   | 40 | $^3S_1$ | 1374 560 698   |
| 22 | $^3S_1$ | 1367 532 054   | 41 | $^3S_1$ | 1374 689 300   |
| 23 | $^3S_1$ | 1368 473 584   | 42 | $^3S_1$ | 1374 808 037   |
| 24 | $^3S_1$ | 1369 281 502   | 43 | $^3S_1$ | 1374 917 901   |
| 25 | $^3S_1$ | 1369 979 949   | 44 | $^3S_1$ | 1375 019 753   |
| 26 | $^3S_1$ | 1370 587 852   | 45 | $^3S_1$ | 1375 114 353   |
| 27 | $^3S_1$ | 1371 120 204   | 46 | $^3S_1$ | 1375 202 375   |
| 28 | $^3S_1$ | 1371 589 028   | 47 | $^3S_1$ | 1375 284 413   |
| 29 | $^3S_1$ | 1372 004 044   | 48 | $^3S_1$ | 1375 360 997   |
| 30 | $^3S_1$ | 1372 373 187   | 49 | $^3S_1$ | 1375 432 602   |
| 31 | $^3S_1$ | 1372 702 970   | 50 | $^3S_1$ | 1375 499 653   |

**TABLE II.** Experimental value of the Rydberg state energies for the 5s$n^3S_1$ series. Listed here are the principal quantum number $n$, the Rydberg series and the experimental Rydberg state energy $E_{exp}$ expressed in MHz. The uncertainty on these value is 10 MHz, see in the text.
| $n$ Series $E_{exp}$ (MHz) | $n$ Series $E_{exp}$ (MHz) |
|---------------------------|---------------------------|
| $3^2D_1$ 1 344 688 300 | $3^2D_1$ 1 372 918 946 |
| $3^2D_2$ 1 344 870 880 | $3^2D_2$ 1 372 925 192 |
| $3^2D_1$ 1 349 925 421 | $3^2D_1$ 1 373 192 657 |
| $3^2D_2$ 1 350 174 107 | $3^2D_2$ 1 373 198 097 |
| $3^2D_1$ 1 353 661 576 | $3^2D_1$ 1 373 439 862 |
| $3^2D_2$ 1 353 850 897 | $3^2D_2$ 1 373 444 629 |
| $3^2D_2$ 1 354 207 372 | $3^2D_1$ 1 373 663 874 |
| $3^2D_1$ 1 356 885 051 | $3^2D_2$ 1 373 668 084 |
| $3^2D_2$ 1 356 969 526 | $3^2D_1$ 1 373 867 493 |
| $3^2D_2$ 1 357 360 134 | $3^2D_2$ 1 373 871 228 |
| $3^2D_1$ 1 359 296 415 | $3^2D_1$ 1 374 053 114 |
| $3^2D_2$ 1 359 574 504 | $3^2D_2$ 1 374 056 446 |
| $3^2D_2$ 1 359 922 783 | $3^2D_1$ 1 374 222 798 |
| $3^2D_1$ 1 361 493 566 | $3^2D_2$ 1 374 225 784 |
| $3^2D_2$ 1 361 682 770 | $3^2D_1$ 1 374 378 312 |
| $3^2D_1$ 1 363 313 169 | $3^2D_2$ 1 374 381 902 |
| $3^2D_2$ 1 363 452 466 | $3^2D_1$ 1 374 521 191 |
| $3^2D_1$ 1 364 863 538 | $3^2D_2$ 1 374 523 622 |
| $3^2D_2$ 1 364 960 612 | $3^2D_1$ 1 374 652 762 |
| $3^2D_1$ 1 366 181 658 | $3^2D_2$ 1 374 654 968 |
| $3^2D_2$ 1 366 249 880 | $3^2D_1$ 1 374 774 191 |
| $3^2D_1$ 1 367 305 009 | $3^2D_2$ 1 374 777 201 |
| $3^2D_2$ 1 367 354 291 | $3^2D_1$ 1 374 886 489 |
| $3^2D_1$ 1 368 266 930 | $3^2D_2$ 1 374 888 324 |
| $3^2D_2$ 1 368 303 621 | $3^2D_1$ 1 374 990 551 |
| $3^2D_1$ 1 369 095 323 | $3^2D_2$ 1 374 992 230 |
| $3^2D_2$ 1 369 123 407 | $3^2D_1$ 1 375 087 158 |
| $3^2D_1$ 1 369 812 958 | $3^2D_2$ 1 375 088 706 |
| $3^2D_2$ 1 369 834 979 | $3^2D_1$ 1 375 177 012 |
| $3^2D_1$ 1 370 438 257 | $3^2D_2$ 1 375 178 436 |
| $3^2D_2$ 1 370 455 884 | $3^2D_1$ 1 375 260 722 |
| $3^2D_1$ 1 370 986 135 | $3^2D_2$ 1 375 262 039 |
| $3^2D_2$ 1 371 000 499 | $3^2D_1$ 1 375 338 834 |
| $3^2D_1$ 1 371 468 700 | $3^2D_2$ 1 375 340 055 |
| $3^2D_2$ 1 371 480 583 | $3^2D_1$ 1 375 411 845 |
| $3^2D_1$ 1 371 895 834 | $3^2D_2$ 1 375 412 978 |
| $3^2D_2$ 1 371 905 792 | $3^2D_1$ 1 375 480 182 |
| $3^2D_1$ 1 372 275 642 | $3^2D_2$ 1 375 481 234 |
| $3^2D_2$ 1 372 284 085 | $3^2D_1$ 1 375 544 238 |
| $3^2D_1$ 1 372 614 826 | $3^2D_2$ 1 375 545 219 |
| $3^2D_2$ 1 372 622 055 | $3^2D_1$ 1 375 545 219 |

**TABLE III.** Experimental value of the Rydberg state energies for the 5s$^2$D$_{1,2}$ and 5s$^2$D$^2$ Rydberg series. Listed here are the principal quantum number $n$, the Rydberg series and the experimental Rydberg state energy $E_{exp}$ expressed in MHz. The uncertainty on these values is 10 MHz, see in the text.