Reconfigurable radiofrequency filters based on versatile soliton microcombs

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The rapidly maturing integrated Kerr microcombs show significant potential for microwave photonics. Yet, state-of-the-art microcomb based radiofrequency (RF) filters have required programmable pulse shapers1–3, which inevitably increase the system cost, footprint, and complexity. Here, by leveraging the smooth spectral envelope of single solitons, we demonstrate for the first time microcomb based RF filters free from any additional pulse shaping. More importantly, we achieve all-optical reconfiguration of the RF filters by exploiting the intrinsically rich soliton configurations. Specifically, we harness the perfect soliton crystals4 to multiply the comb spacing thereby dividing the filter passband frequencies. Also, a completely novel approach based on the versatile interference patterns of two solitons within one round-trip5, enables wide reconfigurability of RF passband frequencies according to their relative azimuthal angles. The proposed schemes demand neither an interferometric setup nor another pulse shaper for filter reconfiguration, providing a practical route towards chip-scale, widely reconfigurable microcomb based RF filters.

Thanks to the ever-maturing photonic integration, RF photonic systems and subsystems have been brought to new heights6,7, in terms of footprint, scalability, and potentially cost-effectiveness. Particularly, RF filtering towards chip-scale is a key enabling function1–3,8–13. Paradigm demonstrations include the integration of the basic filtering blocks, such as delay lines8, optical spectral shaper9, programmable mesh topologies10, ring resonators11, as well as the use of stimulated Brillouin scattering (SBS) in waveguides12. Recently, an all-integrated RF photonic filter has been shown in a monolithic platform13. Among others, RF filters constructed from tapped delay line (TDL) structures attract great attention. They can be classified into two types14, depending on whether the filtering profile is given by the physical path delays10,13 or the light source spectral1–3,8,9,15,16. While the former approach is straightforward, a multi-wavelength source combined with dispersive propagation also functions as a TDL filter. This greatly simplifies the structure complexity of a finite impulse response (FIR) filter, as only a single dispersive delay line is needed. Nevertheless, the main complexity is then shifted to the multi-wavelength source. Electro-optic combs9,15,17 or mode-locked lasers16 are generally adopted as light sources, which remain expensive and bulky options.

Integrated optical Kerr combs (microcombs) have appeared as an interesting alternative. Indeed, microcombs have already been applied not only to filter RF signals1–3, but also for various RF photonic processing, such as true-time delay beamforming18, RF channelization19, and analog computation20. The large comb spacing of microcombs also enhances RF filters with broader Nyquist zone (spur free range), lower latency1, less dispersion induced fading, as well as larger number counts of equivalent delay lines2, unparalleled by other approaches. However, so far all these microcomb based RF filters have been implemented on either dark pulses1,21 or complex soliton crystal states2,3. Additional programmable pulse shaping modules are inevitably required to equalize or smooth the comb spectral shape. Thus, the system complexity is significantly increased while the potential for low-cost and high-volume applications is compromised.

To date, harnessing the smooth sech² spectral envelope of single soliton22–24 or other regulated dissipative Kerr soliton (DKS) states for photonic RF filtering is yet to be investigated. Given the fact that single-soliton microcombs have facilitated a myriad of applications, ranging from Tb/s coherent communication25, ultrafast ranging26, dual-comb spectroscopy27, astronomical spectrometer calibration28, to microwave synthesis29, a great benefit for RF filtering can be expected.

In this paper, we demonstrate for the first time, to our knowledge, soliton microcomb based RF photonic filters without any external pulse shaping. In addition, the synthesized RF filters can be all-optically reconfigured through the internal versatile soliton states. Specifically, we trigger, in a deterministic fashion, the perfect soliton crystal states (PSC) to multiply the comb spacing4,30, thereby dividing the RF passband frequencies. Moreover, a completely new regime of filter reconfiguration is achieved based on versatile two-soliton microcombs (TSM). The spectral interference of two solitons is functionally equivalent to an interferometric setup, shifting the filter passband frequency via modification of the angle between them. A proof-of-concept filter reconfiguration experiment is also shown using TSM based RF filters. The internal exploitation of abundant and regulated soliton formats of microresonator effectively bypasses the need of another programmable pulse shaper and interferometric setup for RF filter tuning1,2. Thus, the proposed scheme dramatically reduces the system complexity and
Figure 1. Schematic diagram of reconfigurable soliton based RF photonic filters and their underlying microcomb generation. a, The conceptual setup consists of four parts: microcomb generation, RF signal upconversion, dispersive propagation, and photodetection. ECDL: external cavity diode laser; MZM: Mach-Zehnder modulator; SMF: single mode fiber; PD: photodiode; VNA: vector network analyzer. Various RF filters are synthesized based on versatile soliton microcombs: (1) single-soliton based RF filter with passband centered at \( f_{\text{FSR}} \) (blue); (2) \( N \)-PSC (perfect soliton crystals of \( N \) equally spaced solitons within one round-trip) based RF filters with passband centered at \( f_{\text{FSR}}/N \) (green, \( N=4 \) is shown); (3) Two-soliton microcomb (TSM) based RF filters with passband centered at \( f_{\text{FSR}} \alpha/360^\circ \) (orange), where \( \alpha \) is the relative azimuthal angle between two solitons (\( \alpha = 90^\circ \) is shown). b, Simulated stability diagram of the Lugiato–Lefever equation (LLE) involving the experimental avoided mode crossing (AMX) condition. Four different stability regions are listed: modulation instability (MI, blue), breathers (red), spatio-temporal and transient chaos (chaos, yellow), and stable dissipative Kerr soliton (DKS, green). PSC and TSM/single-soliton spectra are obtained by distinct approaches. PSC states are accessed under the threshold power to avoid the chaos region. Single-soliton or TSM states are accessed above the threshold power, by either directly falling to the states or backward tuning from higher number of solitons. c, Examples of experimentally generated spectra at resonance of 1555.1 nm: (1) single-soliton, (2) PSC \((N=4)\), and (3) TSM \((\alpha = 132.7^\circ)\). The corresponding pump power is also shown for each microcomb generation.
responds to a TDL filter, where the power of each comb line $p_k$ is the tap weight, and the delay is determined by the comb spacing $f_m$ and the accumulated dispersion $\phi_2 = -\beta_2 L \left( \text{the product of SMF second-order dispersion } \beta_2 \text{ and fiber length } L \right)$. When the filter tap weights take the $\text{sech}^2$ envelope of a single-soliton comb (case 1), the RF filter response is given as (see Supplementary Information):

$$H(f_{\text{RF}}) \sim \cos(2\pi^2 \phi_2 f_{\text{RF}}^2) \sum_{n=-\infty}^{\infty} G(f_{\text{RF}} - nf_{\text{FSR}})$$

(1)

where $f_{\text{FSR}}$ and $G(f_{\text{RF}})$ are respectively defined as $1/(2\pi \phi_2 f_m)$ and $2 T_0 f_{\text{RF}} \sinh^{-1}(T_0 f_{\text{RF}})$, with $T = 1/f_m$ the repetition period, and $T_0$ the soliton pulse width. $f_{\text{RF}}$ denotes the RF frequency. Notice that higher order dispersion of SMF is neglected here to give a more intuitive picture. The overall RF filter response can be seen as a periodic function of lineshape $G(f_{\text{RF}})$ with RF free spectral range (FSR) of $f_{\text{FSR}}$, modulated by a envelope due to the double-sideband (DSB) modulation scheme being used. As the tap weights are all-positive, the passband frequencies of the RF filters are at every multiples of $f_{\text{FSR}}$, including a DC response. Throughout this letter, we focus on the first passband.

Besides, by exploiting the rich soliton states of microresonator, the RF filters can be easily reconfigured at no additional cost nor complexity. Among soliton crystal structures, the deflector PSC is of particular interest, as $N(N \in \mathbb{N}_+ | N \geq 2)$ equally-spaced solitons (case 2) within one roundtrip time simply multiphase the initial comb spacing by $N$-times. This imparts $N$-times division of the filter passband frequencies, while preserving the filter bandwidth. The automatic PSC control is equivalent to the Talbot-based processor for discrete programming the RF filters in ref[16]. Less intuitively, all-optical reshaping of the RF filters can also be achieved via versatile TSM spectra (case 3). Two solitons residing in one period induce sinusoidal interference on the $\text{sech}^2$ spectral shape of a soliton, modulating the tap weights of the TDL filter. This rewrites the RF filter response as (see Supplementary Information):

$$H(f_{\text{RF}}) \sim \cos(2\pi^2 \phi_2 f_{\text{RF}}^2) \sum_{n=-\infty}^{\infty} 2G(f_{\text{RF}} - nf_{\text{FSR}})$$

$$G(f_{\text{RF}} - (n - \frac{\alpha}{2\pi})f_{\text{FSR}}) + G(f_{\text{RF}} - (n + \frac{\alpha}{2\pi})f_{\text{FSR}})$$

(2)

where $\alpha$ is the relative azimuthal angle between two solitons (expressed in radian for calculation). Clearly, new RF passbands of halved amplitude appear due to two-soliton interference, which are displaced at both sides from the initial response according to the azimuthal angle between them. Thus, the RF filter passbands can slide inside $f_{\text{FSR}}$ by modifying the relative soliton angles. Unlike ref[2] in which the authors artificially introduce the sinusoidal modulation via programming the spectral carving, we alleviate the need for a pulse shaper and realize sinusoidal modulation by directly generating a series of TSM spectra. This novel scheme achieves for the first time wideband reconfiguration of RF filters without either interferometric configuration or additional pulse shaping.

The soliton microcombs used for RF filtering are generated from an 104 GHz ultra-low loss integrated silicon nitride (Si$_3$N$_4$) microresonator ($Q \sim 1 \times 10^7$), fabricated by the photonic Damascene reflow process[33]. By employing the frequency-comb-assisted diode laser spectroscopy, the detailed properties of resonances and the integrated group velocity dispersion (GVD) of the microresonators are measured (see Supplementary Information). Strong avoided mode crossings (AMX) are observed around 1565 nm, which lead to the modulation of intracavity CW background, thereby resulting in the ordering of the DKS pulses[5] and the formation of soliton crystals[4,31,32]. Figure 1b shows the simulated stability diagram (see Methods), which consists of modulation instability (MI), breathers, chaos (spatio-temporal chaos and transient chaos), and stable DKS states. Additionally, it has been revealed that the pump power level is critical for whether the PSC or stochastic DKS states are formed[4]. In our case, the threshold pump power $P_{th}$ is found to be around 20 mW in the bus waveguide. When the laser scanning route is operated below threshold pump power, PSC states can be accessed without crossing the chaos region. Contrarily, DKS states with stochastic soliton number are accessed above the threshold power. Experimentally, the single soliton and TSM states are obtained by either directing falling to the states or backward tuning from the states with higher soliton number[34]. Thus, through controlling the pump power and resonance frequency, various soliton microcombs (single-soliton, PSC, and TSM) can be obtained on demand to produce the desired RF filter responses. For example, Figure 1c shows three distinct optical spectra obtained from the resonance of 1555.1 nm: single soliton, PSC ($N = 4$), and TSM ($\alpha = 132.7^\circ$), respectively.

Figure 2 depicts the RF photonic filters using single soliton and various PSC microcombs. The single-soliton based RF filter (Figure 2a) is centered at 16.24 GHz, with main-to-sidelobe suppression ratio (MSMR) of 23.2 dB. Further, various PSC states are deterministically obtained at different resonances under the threshold power, thereby all-optically reconfiguring the corresponding RF filters. The comb spacing multiplication via PSC results in the division of the corresponding RF passbands. RF filters centered at 8.12 GHz, 5.42 GHz, and 4.06 GHz (Figure 2b-d) are experimentally synthesized through 2, 3, and 4 equally spaced solitons, with MSMR of 22.6 dB, 25.6 dB, and 20.4 dB, respectively. All these RF filters achieve MSMR over 20 dB without additional programmable spectral shaping. The MSMR here are limited by the smoothness of the optical spectra[15], as several AMX can be seen in the microcombs. Nevertheless, all these microcombs preserve well the $\text{sech}^2$ envelope, and remained smooth after amplification. In addition, the
measured RF filter responses are in excellent agreement with simulations, by taking into account of third-order dispersion ($\beta_3$) of SMF (see Methods). The bandwidth of the RF filters are respectively 1060 MHz, 755 MHz, 690 MHz, and 645 MHz, which scale inversely with their center frequencies, also due to the third-order dispersion of SMF.

Figure 3a shows the TSM spectra and their corresponding RF filter responses, pumped at resonance of 1556.0 nm. According to Eq. (2), the first passband frequencies of RF filters scale linearly with the relative angles between two solitons, so that the filter reconfiguration is achieved. In the experiment, TSM spectra with relative angles of 19.7°, 43.0°, 68.1°, 94.6°, 117.0°, 142.5°, and 169.2° are obtained, where the angles are extracted from the fitting of the microcomb spectral envelope (see Methods). The measured RF filters are correspondingly centered at 0.85 GHz, 1.96 GHz, 3.05 GHz, 4.24 GHz, 5.26 GHz, 6.40 GHz, and 7.51 GHz, confirming the linear relation with the soliton angle. As in the case of PSC, a slight broadening of the filter passband width from 490 MHz to 620 MHz is attributed to the third-order dispersion of SMF. Overall, the RF filters obtained at resonance 1555.96 nm could vary from DC to 8.1 GHz ($f_{FSR}/2$) with maximum grid of 1.2 GHz, while roughly preserving the filter bandwidth in the meantime. The granularity of TSM based RF filters can be further reduced to less than 1 GHz by exploiting adjacent resonances of 1556.0 nm (see Supplementary Information).

Importantly, the possible angles between two solitons are determined by the overall AMX profile, and are rather robust to both laser power and frequency detuning, thereby deterministically dictating the filter passband frequencies to be either one of those shown in Figure 3a. To gain insights of the relative angles between two solitons, we also perform perturbed Lugnati–Lefever equation (LLE) simulation to investigate the TSM formations (see Methods). The blue curve in Figure 3b shows one example of the steady state two-soliton temporal intracavity profile. Due to the AMX effect, periodic intensity modulation is observed upon the CW background. It is clearly seen that the soliton can only be excited at $85\,\text{GHz}$ ($f_{FSR}/4$), $170\,\text{GHz}$ ($f_{FSR}/2$), and $255\,\text{GHz}$ ($3f_{FSR}/4$), as manifested by the green dashed lines which correspond to the stationary solutions obtained in simulation. These possible soliton angles are in good agreement with experimental results, indicated as red dashed lines. To further test the robustness of the angle between two solitons, an external perturbation is
Figure 3. **TSM spectra and their corresponding RF photonic filters, together with TSM simulation investigation.** By accessing different two-soliton states, the RF filters can be all-optically reconfigured. **a,** Left column: TSM spectra at resonance of 1555.96 nm (blue: experiment, red: envelope fitting). The insets illustrate two soliton distribution inside the microresonator: the angles between them are 19.7°, 43.0°, 68.1°, 94.6°, 117.0°, 142.5°, and 169.2°, respectively. Right column: corresponding normalized RF filter responses (blue: experiment, red: simulation) with passbands at 0.85, 1.96, 3.05, 4.24, 5.26, 6.40, and 7.51 GHz, respectively. **b,** Simulation of TSM relative azimuthal angles. One example of the simulated TSM intracavity intensity profile (blue), where AMX induced background modulation is observed. The red and green lines respectively indicate measured and simulated possible relative angles between two solitons. **c,** Simulation of the intracavity waveform evolution of TSM for robustness test. First, TSM state with relative angle of 168.0° is excited by scanning the pump over the resonance. Once the TSM becomes stable, a 10.0° perturbation is introduced to one of the solitons at white dashed line. The relative angle will re-stabilize to the original angle of 168.0° after a period of free running.

deliberately introduced on their relative angle. Figure 3c illustrates the dynamical evolution of the two-soliton formation. The simulation is initiated as a standard laser scanning scheme to kick out two solitons. Once the simulation reaches stable two-soliton solution (relative angle of 168.0°), one of the solitons is dragged from its original position by a 10.0° on purpose. After a period of free running, the two solitons converge back to their original relative positions, again at 168.0° apart. This confirms the regulation of two solitons under AMX background modulation.

A proof-of-concept RF filter reconfiguration experiment is also illustrated in Figure 4 using TSM based RF filters (see Methods). Two superimposed phase-shift
keying (PSK) signals in which a 40 Mb/s modulation at 1.96 GHz tone and a 20 Mb/s modulation at 3.05 GHz tone, are prepared as input test signals. The RF filters are then respectively reconfigured at 1.96 GHz and 3.05 GHz to filter the input signals, by triggering the TSM spectra of corresponding soliton angles. At the output of the RF filters, nearly complete rejection of either one of the PSK signals is observed on the electrical spectra (Figure 4a), where the extinction ratio exceeds 35 dB for both cases. Figure 4b-c show the filtered output RF waveforms. The periodicity of the output temporal traces corroborate the filtering of the original RF signals.

In conclusion, we demonstrate reconfigurable soliton based RF photonic filters using simple approaches. Contrary to previous demonstrations where pulse shapers are necessary to obtain descent passband responses\textsuperscript{1-3}, the proposed schemes are intrinsically well-shaped with the smooth spectral envelope of solitons. More importantly, we harness various intrinsic DKS states of microresonator, like PSC and TSM, for RF filter reconfiguration at no additional cost. The diversity and regularization of soliton formats in microresonator are investigated in the favor of RF photonic filters. To certain extent, these inherent soliton states could be in place of substantial efforts made in the past for reconfiguring the comb based RF filters, such as using interferometric architecture\textsuperscript{1,15}, programmable pulse shaping\textsuperscript{6,17}, or Talbot-based signal processor\textsuperscript{16}. Nevertheless, subjected to the same challenges of any other comb based RF filters, our current filters are not yet optimized in terms of the link performance. While the noise reduction and gain enhancement could be achieved using high power-handling balanced detectors\textsuperscript{35}. Besides, the recent advancement on the integration between laser chip and microresonator\textsuperscript{23,24}, as well as replacing the SMF with a highly dispersive integrated waveguide\textsuperscript{8}, can be further connected to the current work for miniaturization. To conclude, our work significantly reduces the system complexity, size, and cost of the microcomb based RF filters, while preserving their wide reconfigurability. The proposed schemes set as a stepping stone for chipscale, cost-effective, and widely reconfigurable microcomb based RF filters.

Methods

Experimental setup: A C-band tunable CW laser is amplified by an Erbium-doped fiber amplifier (EDFA) with amplified spontaneous emission (ASE) filtered, polarization aligned at the TE mode, and then coupled to the Si$_3$N$_4$ microresonator for soliton microcomb generation. The input and output coupling of the chip is achieved via lensed fibers of around 30% fiber-chip fiber coupling efficiency. The soliton microcombs are initiated by scanning the pump over the resonances, with the assistance of an arbitrary function generator (AFG)\textsuperscript{72}. The residual pump of generated microcombs are then filtered by a tunable fiber Bragg grating (FBG), while a circulator is inserted in between to avoid back-reflection. 10% of light is tapped to an optical spectrum analyzer (OSA) to record the microcomb spectra. The other 90% of the light is amplified, and polarization managed, before sending to a 30 GHz bandwidth MZM. RF signals from the VNA are applied to the MZM in DSB modulation format. The modulated spectra are then propagated through a spool of 4583 m of SMF to acquire dispersive delays, and finally beats at a 18 GHz PD to convert the signals back to the RF domain. The length of SMF is measured by a commercial optical time-domain reflectometer (OTDR).

For the system demonstration, a 12 Gs/s arbitrary waveform generator (AWG) is used to prepare the input RF signals. 40 Mb/s PSK signal modulated at 1.96 GHz tone and 20 Mb/s PSK signal modulated at 3.05 GHz tone, are generated separately from the two channels of the AWG. After adding the two streams of signals in a combiner, the composite signal is then sent through the TSM based RF filters, tuned at 1.96 GHz and 3.05 GHz, respectively. The output spectra are measured by a electrical spectrum analyzer (ESA), while the waveforms are measured using a high-speed real-time oscilloscope.

Si$_3$N$_4$ microresonator: The Si$_3$N$_4$ microresonator used in the experiment is a ring structure with radius of 217 μm. Its waveguide cross section (width × height), is made to be 1500 nm × 750 nm. The microresonator is coupled with a bus waveguide, which possesses the same cross section as the ring to realize high coupling ideality\textsuperscript{34}. To achieve critical coupling for the resonances, the gap distance between the ring and bus waveguide is designed to be 690 nm. In our experiment, the pumped resonances are around 1556 nm, where both the intrinsic linewidths and coupling strengths are approximately 20 MHz (see Supplementary Information). With respect to the reference resonance of $\omega_0/2\pi = 192.8$ THz, the dispersion parameters of microresonator are measured: FSR of microresonator $D_1/2\pi \approx 103.9$ GHz, second-order dispersion term $D_2/2\pi \approx 1.28$ MHz, and negligible third-order dispersion term $D_3/2\pi \sim O(1)$ kHz (see Supplementary Information).

LLE Simulation: The simulation performed in this work is based...
on the perturbed LLE model:
\[
\frac{\partial A(\phi,t)}{\partial t} = -\left(\frac{\kappa}{2} + j(\omega_0 - \omega_p)\right)A(\phi,t) + j\frac{D_2}{2} \frac{\partial^2 A(\phi,t)}{\partial \phi^2} + jg|A(\phi,t)|^2A(\phi,t) + \ldots
\]
with an ultrahigh peak rejection. Optics express 21, 23286–23294 (2013). URL https://doi.org/10.1364/oe.21.023286.

where \(A(\phi,t)\) is the temporal envelope of the intracavity field. \(\kappa = \kappa_{\text{ex}} + \kappa_0\) is the total cavity loss rate, where \(\kappa_{\text{ex}}\) is the coupling rate, and \(\kappa_0\) is the internal loss rate. \(\omega_0\) and \(\omega_p\) denote the angular frequencies of the pumped resonance and the CW pump laser, respectively.



\[
p_{\text{ex}} = \frac{\omega_0}{\omega_p}, \quad \frac{D}{\kappa} = \frac{\omega_0 - \omega_p}{\omega_0 - \omega_{\text{ex}}}, \quad \frac{\beta_1}{\kappa} = \frac{\omega_0 - \omega_{\text{ex}}}{\omega_0 - \omega_p}, \quad \frac{\beta_2}{\kappa} = \frac{\omega_0 - \omega_{\text{ex}}}{\omega_0 - \omega_{\text{eff}}},
\]

The parameters for the AMX in the simulation are set as dispersion measurement and the generated microcomb spectra, the dispersion is limited to the effective mode volume.

The strength of the AMX here is estimated to introduce the regularizability of solitons, but without disturbing their formations. Other parameters used in the simulation are retrieved from the characterization, that is, \(D_1/2\pi = 103.9\ \text{GHz}\), \(D_2/2\pi = 1.28\ \text{MHz}\), \(\kappa_{\text{ex}}/2\pi = \kappa_0/2\pi = \pi/4 = 20\ \text{MHz}\).

The simulation of stability chart is obtained by initializing the numerical model with single-soliton solution at various pump power and detuning conditions. Four different states are found: MI, breathers, spatiotemporal and transient chaos, and stable DKS states. The threshold pump power, separating the PSC and stochastic DKS formations, is estimated from both the simulation and experimental results. For the TSM simulation, the numerical model is operated under standard CW laser pump scanning from blue-detuned to red-detuned side, until it reaches the stable TSM states. All the possible angles of TSM are recorded. To test the robustness of the TSM azimuthal angle, the model is initialized with one of the exact two-soliton solution but deliberately perturbed by 10\(^\circ\) angle deviation. Two solitons are gradually re-stabilized at its original azimuthal angle after a period of free running.

RF filter response fitting: As the generated microcomb spectra are broader than the amplifying bandwidth of EDFA, we also measured the optical spectra after the EDFA, in order to extract the TDL filter tap weights. The third-order dispersion \(\beta_3\) of SMF is taken into account for the fitting of RF responses, which can be formulated as:
\[
H(f_{\text{RF}}) \sim \sum_k p_k \cos(2\pi f_{\text{RF}}^k f_m f_{\text{RF}}) + 4\pi^3 \phi_k k f_m^2 f_{\text{RF}}^k + \frac{4}{3} \pi^3 \phi_k f_m^3 f_{\text{RF}}^k
\]
with \(\phi_k = -\beta_3 L_k\). In accordance with typical values of SMF dispersion, \(\beta_3 = 20.2\ \text{ps}^2/\text{km}\) and \(\beta_3 = 0.117\ \text{ps}^3/\text{km}\) at 1550 nm are estimated for all the above fittings of RF filters. The simulation results are in excellent agreement with experimental RF filter responses.

TSM spectral fitting: First, we extract the power of each comb mode of experimental TSM spectra, and indexed them with respect to the pump mode. Power equation (see Supplementary Information), thereby retrieving the azimuthal angle \(\alpha\) between two solitons. Excellent match between simulations and experimental spectra are obtained.

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Data Availability Statement: The code and data used to produce the plots within this work will be released on the repository Zenodo upon publication of this preprint.

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Supplementary Information to:
Reconfigurable radiofrequency filters based on versatile soliton microcombs

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I. DERIVATION OF RF FILTER RESPONSES

The responses of comb based RF filters can be described by the discrete Fourier transform (FT) of their underlying frequency comb intensity¹:

\[ H(f_{RF}) \sim \cos(2\pi^2 \phi_2 f_{RF}^2) \sum_k p_k \exp(j4\pi^2 \phi_2 k f_m f_{RF}) \]  

(1)

where \( p_k \) denotes the power of each comb line, \( \phi_2 = -\beta_2 L \) is the product of the second-order dispersion \( \beta_2 \) of the dispersive element and its length \( L \), and \( f_m \) is the comb line spacing. \( f_{FSR} = 1/(2\pi\phi_2 f_m) \) is the FSR of the RF filters. The exact RF filter responses are obtained by substituting the microcomb spectral profile into Eq. (1). For single-soliton microcomb, the optical field is given by:

\[ E(t) \sim \text{sech}(\frac{t}{T_0}) \otimes \sum_{n=-\infty}^{\infty} \delta(t - nT) \]  

(2)

where \( T_0 \) and \( T = 1/f_m \) are the soliton pulse width and period, respectively. Taking the FT of Eq. (2), the single soliton spectrum is derived as:

\[ \tilde{E}(f) \sim \text{sech}(\pi^2 T_0 f) \sum_{k=-\infty}^{\infty} \delta(f - kf_m) \]  

(3)

This leads to the power of each comb line, or equivalently filter tap weights, being \( p_k \sim \text{sech}^2(\pi^2 kT_0/T) \), \( k \in \mathbb{Z} \) is the mode index with respect to the center comb line. Disregarding the envelope term of Eq. (1) for the moment, we can rewrite the summation part using Poisson summation formula:

\[ \sum_{k=-\infty}^{\infty} p_k e^{j2\pi k f_{FSR} f_{RF}} = f_{FSR} P(f_{RF}) \otimes \sum_{n=-\infty}^{\infty} \delta(f_{RF} - nf_{FSR}) \]  

(4)

where \( P(f_{RF}) \) is the FT of the generalized form of \( p_k \), at which the mode index \( k \) is substituted by an arbitrary variable \( x \), times a factor \( f_{FSR} \):

\[ P(f_{RF}) = \int \text{sech}^2\left(\frac{\pi^2 T_0 f_{FSR}}{T} x\right) e^{j2\pi f_{RF} x} dx \sim \frac{2 T_0 f_{FSR}}{\sinh(T f_{FSR})} \equiv G(f_{RF}) \]  

(5)

where the FT of sech-squared function can be found using the residue theorem², and is defined as \( G(f_{RF}) \). The single-soliton based RF filter response is derived by substituting \( G(f_{RF}) \) back to Eq. (4) and incorporating the envelope term.

Two-soliton microcomb (TSM) based RF filter responses can be obtained in a similar manner. Assume two solitons are of identical amplitude and pulse width:

\[ E(t) \sim [\text{sech}(\frac{t}{T_0}) + \text{sech}(\frac{t - \alpha T}{T_0})] \otimes \sum_{n=-\infty}^{\infty} \delta(t - nT) \]  

(6)

where \( \alpha \) is the azimuthal angle between two solitons, expressed in radian. The TSM spectrum can be found by FT of Eq. (6):

\[ \tilde{E}(f) \sim \text{sech}(\pi^2 T_0 f)(1 + e^{-j\alpha k}) \sum_{k=-\infty}^{\infty} \delta(f - kf_m) \]  

(7)

We then obtain the filter tap weights as:

\[ p_k \sim \text{sech}^2(\pi^2 k T_0/T)(2 + 2\cos(\alpha k)) \]  

(8)

where \( k \in \mathbb{Z} \) is the comb mode index relative to the center mode. Inserting Eq. (8) back to Eq. (4) derives:

\[ \sum_{k=-\infty}^{\infty} p_k e^{j2\pi k f_{FSR} f_{RF}} \sim (2G(f_{RF}) + G(f_{RF} - \frac{\alpha}{2\pi} f_{FSR}) + G(f_{RF} + \frac{\alpha}{2\pi} f_{FSR})) \otimes \sum_{n=-\infty}^{\infty} \delta(f_{RF} - nf_{FSR}) \]  

(9)

Thus, incorporating the envelope term would eventually lead to the RF filter responses of TSM spectra.
Si₃N₄ microresonator characterization. (a) Optical image of the Si₃N₄ microresonator chips. (b) Total linewidth, coupling strength, and intrinsic linewidth of each resonance in the TE₀₀ mode family. The shaded area corresponds to the resonances pumped in the experiment. (c) Top: Measured integrated GVD ($D_{\text{int}}$) of the TE₀₀ mode family in the microresonator, with respect to to the resonance of 1555.1 nm; Bottom: zoom-in of integrated GVD region between 1540 nm and 1580 nm. Dominant AMX is observed around wavelength region of 1565 nm.

II. Si₃N₄ MICRORESONATOR CHARACTERIZATION

Supplementary Figure 1a shows a picture of Si₃N₄ microresonator chips used in the experiment. Accurate calibrated transmission spectrum of the microresonator is obtained using frequency-comb-assisted diode laser spectroscopy³, covering the wavelength region from 1500 nm to 1630 nm. Supplementary Figure 1b illustrates the detailed properties of the resonances, i.e. intrinsic linewidths $\kappa_0/2\pi$, coupling strengths $\kappa_{\text{ex}}/2\pi$, as well as the total linewidths $\kappa/2\pi = (\kappa_0 + \kappa_{\text{ex}})/2\pi$ of the TE₀₀ mode family, which are extracted from the fittings of calibrated transmission spectrum. The shaded area of Supplementary Figure 1b denotes the experimentally accessed resonances. All these resonances show intrinsic linewidths $\kappa_0/2\pi \approx 20$ MHz, indicating the Q factors to be around $10^7$. Besides, the coupling strengths of the pumped resonances are similar to their intrinsic linewidths, implying these resonances near critical coupling condition.

Then, the integrated GVD of the TE₀₀ mode family are extracted by identifying the precise frequency of each TE₀₀ resonance, as shown in the upper part of Supplementary Figure 1c. It can be formulated as:

$$D_{\text{int}}(\mu) = \omega_\mu - (\omega_0 + D_1\mu) = D_2\mu^2/2 + D_3\mu^3/6 + ...$$  \hspace{1cm} (10)

where $D_1/2\pi$ is the FSR of microresonator, and $D_n (n \in N_+ \mid n \geq 2)$ correspond to the $n$-th order dispersion coefficients. $D_{\text{int}}(\mu)$ is defined as the deviation of the $\mu$-th resonance frequency $\omega_\mu/2\pi$ from the equidistant frequency grid, constructed from the FSR around the reference resonance frequency $\omega_0/2\pi$. Here, the reference resonance is chosen at $\omega_0/2\pi = 192.8$ THz (i.e. $\lambda_0 = 1555.1$ nm). The retrieved dispersion terms are $D_1/2\pi \approx 103.9$ GHz, $D_2/2\pi \approx 1.28$ MHz, and $D_3/2\pi \sim O(1)$ kHz. Note that operating at anomalous dispersion ($D_2 > 0$) is a prerequisite for soliton formation. The bottom part of Figure 1c shows the zoom-in view of the integrated GVD between 1540 nm
Supplementary Figure 2. **Soliton steps for PSC and TSM formations.** Transmission curves are obtained by scanning a laser across the resonance below (a) and above (b) the threshold pump power, at resonance of 1555.1 nm. The shaded areas in (a) and (b) correspond to 4-PSC step and TSM step, respectively.

III. PSC AND TSM SOLITON STEPS

Supplementary Figure 2 depicts different soliton step formations by pumping the resonance of 1555.1 nm under and above the threshold pump power level. The soliton step is manifested from the transmission of the microresonator by scanning the CW pump laser over the resonance. When the pump power is around 15 mW, only a single PSC step is formed, and a soliton number of 4 can be deduced from the depth of the step. However, if the power of scanning CW laser is increased to around 50 mW, both two-soliton and single-soliton steps would appear. The distinct soliton steps clearly indicate two different soliton generation regimes, and are consistent with experimentally generated microcomb spectra.

IV. TSM BASED RF FILTERS OF DIFFERENT RESONANCES

We also investigate TSM based RF filters from different pumped resonances. Supplementary Figure 3a shows the experimentally synthesized RF filters centered at 1.83 GHz, 2.83 GHz, 3.94 GHz, 4.95 GHz, 6.00 GHz, and 7.06 GHz from the resonance of 1555.1 nm. While pumped at resonance of 1556.8 nm, the filter passband frequencies are relocated at 0.90 GHz, 2.07 GHz, 3.23 GHz, 4.52 GHz, 5.55 GHz, and 6.90 GHz (Supplementary Figure 3b). By exploring adjacent resonances of 1556.0 nm, the maximum grid of TSM based RF filters is reduced to be less than 1 GHz.

The passband frequency shifts of RF filters arise from the variation of their underlying two-soliton azimuthal angles, which are experimentally retrieved from their TSM spectra and compared to numerical simulation (Supplementary Figure 4a). In the simulation, the AMX position is varied from 14-th to 16-th away from the pump of the same strength, to resemble the change of resonances in line with experimental condition. The background modulation period is then modified according to the relative distance between the pump mode and the AMX. Besides, towards large relative soliton angle, the modification of its value through pumped resonance also becomes more prominent. This effect is simply due to the accumulated periodicity difference, and is clearly observed in both measured and simulated results. Supplementary Figure 4b illustrates the relation between the experimental RF passband frequencies and their corresponding TSM azimuthal angles. A good linear approximation confirms well the operation principle of proposed TSM based RF filters.
Supplementary Figure 3. **TSM based RF filter responses at resonances of 1555.1 nm and 1556.8 nm.** (a) RF filters centered at 1.83 GHz, 2.83 GHz, 3.94 GHz, 4.95 GHz, 6.00 GHz, and 7.06 GHz are obtained at resonance of 1555.1 nm. (b) RF filters centered at 0.90 GHz, 2.07 GHz, 3.23 GHz, 4.52 GHz, 5.55 GHz, and 6.90 GHz are obtained at resonance of 1556.8 nm.

Supplementary Figure 4. **Experimental and simulated azimuthal angles of TSM spectra.** (a) Experimentally retrieved (solid lines, top) and simulated (dashed lines, bottom) two-soliton angles at resonances of 1555.1 nm, 1556.0 nm, and 1556.8 nm. (b) The synthesized RF filter frequencies versus their underlying two-soliton angles retrieved from TSM spectra.

### V. DISCUSSION ON THE ALL-OPTICAL RECONFIGURATION OF THE RF FILTERS

The modification of our RF filters rely fundamentally on the inherent stable and accessible DKS states. In terms of PSC microcombs, different PSC states are accessed by pumping at different resonances under the threshold power. The maximum soliton number that can be sustained in the microresonator is roughly estimated as $\sqrt{\kappa/D}$, where $\kappa$ is the total linewidths of the pumped resonances, the maximum PSC number in our chip is estimated around 5. This provides a good approximation as we can achieve PSC numbers from 2 to 4 experimentally. We also need to point out here that not every PSC state below this predicted value is easily generated, especially for a large maximum PSC number 4.

For the TSM spectra, the two solitons are locked to a few relative angles according to the AMX profile. Since the route to TSM undergoes chaotic region, the soliton number as well as relative distribution remain stochastic, unlike PSC formation. However, these different TSM seem to be roughly equiprobable, and all TSM states in this letter are obtained within 25 times trial in total for each resonance. A pulse triggering technique could be envisioned in the future to deterministically switch to the TSM with targeted azimuthal angle, as has been adopted in fiber cavity5. While the demand for continuous tuning of the RF passband frequency may be achieved via controlled mode interaction6,7.
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