Investigation on fluid added mass effect in the modal response of a pump-turbine runner

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Abstract. With the improvement of technology and manufacturing level of hydraulic turbine, there is a trend to increase the power concentration of the units. As a consequence, heads, fluid velocities and rotational speeds are higher which lead to larger hydraulic excitation forces on the structures. Accordingly, vibration and high stress levels will arise, which may cause fatigue damage. Therefore, how to predict the natural frequencies and mode shapes of the runner during the design stage is of paramount importance. In this paper, numerical simulation to analyze the influence of the surrounding water in a pump-turbine runner modal has been carried out by using finite element method. The modal behavior of the runner in air and in water has been calculated. In addition, the added mass effect by comparing the natural frequencies and mode shapes in both cases has been determined. The results show that, due to the added mass effect of the surrounding water, natural frequencies are considerably reduced. The frequency reduction ratio (FRR) varies in a range of 0.06–0.43, depending on the mode shapes. Vibration amplitude and complexity of modes are the two main factors that affect the reduction ratio of runner natural frequencies. For the FRR of in-phase (IP) mode shapes, the vibration amplitude of each mode is considered to be the dominant reason. It is clear that the FRR decreases as the nodal diameter (ND) increases (except for 0ND). While for counter-phase (CP) mode shapes, with the frequency increases, the runner modes will become more and more complex, as a result of which, the FRR increases.

1. Introduction
With the improvement of technology and manufacturing level of hydraulic turbine, there is a trend to increase the power concentration of the units. As a consequence, heads and fluid velocities are higher and the hydraulic excitation forces on the turbine runners increase. Furthermore, the operation range of hydraulic turbines is widened in order to satisfy the end-users’ demand of larger regulation capacity. For the pump-turbines, pressure fluctuation induced by the rotor-stator interaction are the major excitation forces [1,2,3]. High excitation forces may lead to vibration and high stress levels which can produce damage by fatigue [4,5]. Therefore, It is important to predict the natural frequencies and mode shapes of the runner during the design stage.

At present, many researchers have done a lot of work about modal analysis, including experiments and numerical simulation. With a large number of high-head pump-turbine units being put into operation, resonance problems have arisen. In Toshiba Hydraulic Research Laboratory, extensive studies on the modal and dynamic response of runner have been carried out by means of prototype head model tests [1]. In order to analyse the influence of the surrounding water in a turbine runner, a
detailed experimental investigation on a reduced scale model of a Francis turbine runner has been carried out [6]. The corresponding simulation using the same model runner and compatible conditions has also been published [7]. The numerical method and simulation results have been well validated by experimental modal test results. Some results can be extrapolated to other geometrically similar runners. The added mass effect of the surrounding water has been evaluated by comparing the frequencies in air and in water. Stefan Laisse et al. [8] experimentally and numerically investigated the influences of different geometries on modal parameters and frequency reduction ratio (FRR), on two different runners, one prototype and one model runner. In order to identify sensitivity of the structural properties, several parameters such as material properties, different model scale and different hub geometries were investigated. The study shows that the FRR of a Francis runner is mainly determined by the blade geometry. It is nearly independent of the crown geometry and fully independent of material and scale.

However, a pump-turbine impeller has a quite different design and a more complex geometry than the Francis turbine runner. Modal test of a prototype pump-turbine runner was carried out by E.Egusquiza et al. [9]. Due to the difficulty of the test conditions, only the 2ND mode was clearly identified. In order to make a deeper research on the modal characteristics of the pump-turbine runner, the research group investigated the modal of a pump turbine model runner in air and in water by two methods [10,11].

The above literatures show that due to the fluid added mass effect, the natural frequencies of runner in water have a significant decrease. The frequency reduction ratio does not remain constant for all modes, but varies depending on the corresponding mode shape. In addition, the modal analysis in water was carried out by submerging the runner in an open tank full of water. However, as the runner is confined in a narrow space during operation, the water around the runner moves with much larger amplitude than the runner itself, which causes very high added mass effect [1]. For high-head pump turbine, the dimensions of the gap outside the runner crown and bend as well as the seal part are smaller, which may have a greater impact on the modal characteristics of runner.

In this paper, numerical simulation to analyse the influence of the surrounding water on modal behaviour of a pump-turbine runner during operation has been carried out by using finite element method. By comparing the natural frequencies and mode shapes in both cases, the added mass effect has been determined. The relationship between frequency reduction ratio (FRR) and runner mode shapes was discussed in detail. Furthermore, the possibility of resonance of runner was checked.

2. Methodology

2.1. Classification of pump turbine vibration modes
Runner is a rotational periodical structure which is constructed by repeating one basic sector around an axis. Due to the structural characteristics of pump-turbine runner, the crown and band have much lower stiffness in the axial direction compared with the blades, especially near the external diameter of the impeller. As a result, the mode shapes of runner can be divided into two groups [1]. One group is in-phase (IP) modes with both crown and band deflects in same direction, while the other is counter-phase (CP) modes which deflect in opposite direction. Besides, these modes can be classified according to the numbers of harmonic index \(k\) and nodal diameters (ND) [9]. In a simple geometry vibrating (e.g., a circular disk), most mode shapes contain lines of zero out-of-plane displacement which cross the entire structure (Figure 1) [9].

2.2. Resonance theory of runner
The excitation frequencies and mode shapes of a pump-turbine runner depends significantly on the combination of the number of runner blades \(Z_r\) and guide vanes \(Z_g\), which satisfies the following formula [1]:

\[
 nZ_r \pm k = mZ_g
\]
Where \( k \) is the number of diametrical nodes (ND); \( m \) and \( n \) are integers. The plus sign in front of \( k \) indicates the mode turning in the same direction of runner rotation and the minus sign the opposite. The runner and the stationary parts suffered different excitation frequencies induced by the rotor-stator interference effect.

The excitation frequencies on the runner and stationary components are expressed as:

\[
f_r = nZ_n \omega_r / 60 (Hz)
\]

(2)

and

\[
f_s = nZ_n \omega_s / 60 (Hz)
\]

(3)

Where \( \omega_r \) is the rotational speed of runner.

Although there are a lot of mode shapes meet the above formula, most of the natural vibration frequencies vary greatly with the excitation frequency, actually only a few modes of the runner and stationary parts are excited, with small value of \( k \).

The response of the structure depends on the mode of the excitation and the natural mode shape as well as the natural frequency excited. If the mode shape and the frequency are different, the response is small; otherwise, the response is very high, and resonance occurs.

![Figure 1. Mode-shapes in a disk (From left to right: Zero, one and two nodal diameters).](image)

3. Simulation

3.1. Calculation model

The runner rotates at 500 rpm with 7 blades and an external diameter of 3930 mm. The number of guide vanes is 20. The material property of Elastic modulus is 207GPa, with Poisson's ratio 0.3 and density 7.75g/cm\(^3\). The finite element models of modal analysis in air and in water are shown in Figure 2.

![Figure 2. Finite element model.](image)

3.2. Mesh sensitivity analysis

At first, a sensitivity analysis was carried out to determine the influence of the mesh density. Several meshes with different element densities were simulated. In Figure 3, the natural frequency values of the runner in air were plotted. It was clear that the natural frequencies of each mode converged to a
constant. By comparing the results, finally, a mesh model with 236046 elements was selected for the simulation.

![Graph showing mesh sensitivity analysis results.](image)

**Figure 3.** Results of the mesh sensitivity analysis.

### 4. Results and Analysis

#### 4.1. Modal analysis results

Figure 4 presented the mode shapes of runner in water. A comparison of natural frequencies of the two sets was listed in Table 1. The frequency reduction ratio (FRR) of each mode was shown in Figure 5.

![Mode shapes of runner in water.](image)

**(1) IP-1ND.** *(2) IP-2ND.* *(3) IP-0ND.* *(4) CP-1ND.* *(5) IP-3ND.* *(6) CP-2ND.*

**Figure 4.** Mode shapes of runner in water.
Table 1. Comparison of natural frequencies of runner in air and in water.

| Mode shape | IP-1ND | IP-2ND | IP-0ND | CP-1ND | IP-3ND | IP-2ND | CP-3ND | IP-4ND | CP-4ND |
|------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| \( f_a \)  | 160.6  | 222.3  | 233.3  | 388.3  | 382.6  | 477.7  | 519.5  | 520.6  | 541.7  |
| \( f_w \)  | 144.8  | 202.0  | 220.6  | 282.5  | 356.7  | 299.2  | 296.1  | 483.7  | 324.7  |
| FRR(%)     | 9.86   | 9.14   | 5.58   | 27.3   | 9.15   | 37.3   | 43.1   | 7.1    | 40.1   |

Figure 5. Frequency reduction ratio (FRR) of each mode.

4.2. Fluid added mass effect analysis
The results show that, due to the added mass effect of the surrounding water, natural frequencies are considerably reduced. The frequency reduction ratio varies in a range of 5.6%~43.1%, depending on the mode shapes. The FRR increases as the fluid added mass increases.

It is obvious in Figure 5 that the frequency reduction ratio of IP mode is smaller while the values of CP mode are larger. As in the case of IP modes, the displacements of runner crown and band are mostly in-phase, so the added mass of surrounding water has little influence on the natural frequency. However, the CP modes with a relative motion between crown and band have much larger added mass effect.

It can be concluded from the results that vibration amplitude, complexity of modes are the two main factors that affect the reduction ratio of runner natural frequencies, which indicates that fluid added mass effect has a close relationship with the mode shapes. To sum up, the more complex mode shapes, the greater fluid added mass effect; the larger vibration amplitude, the higher frequency reduction ratio.

Table 1 shows that the IP-0ND has the smallest fluid added mass and the FRR is 5.6%. As can be seen from Figure 4, IP-0ND vibration mode is an axial or (and) circumferential motion which was restricted by the boundary. Thus there is little possibility to occur large movement in the axial or radial direction. Besides, 0ND belongs to runner overall vibration, and the runner displacements at different locations do not make much difference, so the FRR of 0ND mode shape is very small.

For the IP mode shapes, it is clear that the FRR decreases as the ND increases (except for 0ND). The added mass effect can be explained by mode-shapes and energy theory. As the nodal diameters increases, the frequency becomes higher. Under the same excitation force, the higher order mode presents smaller vibration amplitude due to its high rigidity. All the IP mode shapes seems to be regular, so the vibration amplitude of each mode is considered to be the main reason.

While for CP mode shapes, the added mass effect is different. The runner modes will be more and more complex with increase of the frequencies. The FRR of high order mode will become larger despite there is a reduction with the vibration amplitude. It can be concluded that, the fluid added mass effect of CP modes increases with increase of the natural frequency.
4.3. Resonance analysis of runner

The number of runner blades $Z_r$ is 7 and the number of guide vanes $Z_g$ is 20, so the excitation frequencies of runner suffered consist of various harmonics with a principal frequency of $(20N)$ Hz and its high order harmonics of $(n\cdot20N)$ Hz, where $N$ is the rotational frequency runner and $n$ is an arbitrary integer.

Excitation mode is determined by $k$ which can be obtained by Equation(1). According to the combination of $Z_r$ and $Z_g$, the smallest absolute value of $k$ can be found, which represents the number of diametrical nodes of excitation mode. Therefore, the critical excitation mode of the runner is 1ND, and the corresponding frequency in water is 144.76Hz. At the same time the excitation frequency from RSI is 167 Hz. The deviation between them is only 13.3% which is not safe enough. As the excitation frequency is higher than the natural frequency of runner, resonance phenomenon will occur during the start-up and close-off conditions of the pump-turbine unit with a rotational speed of 434.28 rpm.

5. Conclusion

In this paper, numerical simulation has been carried out to analyze the fluid added mass effect in the modal response of a pump-turbine runner during operation. Furthermore, the possibility of runner resonance was discussed.

Natural frequencies and mode shapes of runner were obtained in air and in water. Due to the added mass effect of the surrounding water, natural frequencies are considerably reduced. The frequency reduction ratio varies in a range of 0.06–0.43, depending on the mode shapes.

Vibration amplitude and complexity of modes were confirmed to be the two main factors that affect the reduction ratio of runner natural frequencies. The frequency reduction ratio of CP modes was much larger than the IP modes. For IP mode shapes, the vibration amplitude of each mode is considered to be the main reason. It is clear that the FRR decreases as the ND increases (except for 0ND). While for CP mode shapes, with increase of the frequency, the runner modes will become more and more complex, as a result of which, the FRR increases.

By comparing the critical excitation mode (1ND) and the natural mode of the runner, it was found that the excitation frequency is higher than the natural frequency. Therefore, resonance will occur during the start-up and close-off conditions.

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