Abstract

We review two different theoretical approaches to the strong interaction dynamics at the early times immediately following heavy ion collisions. One approach is based on small-coupling physics of the Color Glass Condensate (CGC). The other approach is based on Anti-de Sitter space/Conformal Field Theory (AdS/CFT) correspondence and may be applicable to describing large-coupling QCD interactions. We point out that in terms of theoretical tools the two approaches are somewhat similar: in CGC one deals with classical gluon fields produced in a nuclear shock wave collision, while in AdS/CFT one studies classical gravity in a gravitational shock wave collision. We stress, however, that the resulting physics is different: the classical gluon fields in CGC lead to a free-streaming medium produced in heavy ion collisions, while the classical gravity in the 5-dimensional AdS bulk is likely to lead to ideal hydrodynamics description of the produced medium. Also, the valence quarks in colliding nuclei in CGC continue along their light cone trajectories after the collision with very little recoil, while we show that in AdS the colliding nuclei are likely to lose most of their energy in the collision and stop.

1. Classical Gluon Fields in CGC

We begin by considering a high energy heavy ion collision. The basic premise of the CGC physics is that for each of the colliding ultrarelativistic nuclei the small-x wave function is characterized by the large transverse momentum scale called the saturation scale and denoted by $Q_s$. At high energy and/or for large nuclei the saturation scale is large, $Q_s \gg \Lambda_{QCD}$, making the strong coupling constant small, $\alpha_s \ll 1$, thus allowing for a small-coupling description of the small-x nuclear wave functions. Collisions of two nuclei with such wave functions would lead to interactions characterized by perturbatively large saturation scales as well, allowing for a small-coupling description of the early stages of heavy ion collisions. We refer the reader to [1] for a review of CGC physics in nuclear collisions.

If one is interested in the dynamics of the medium produced in the collisions over a not very broad rapidity interval ($\Delta y \leq 1/\alpha_s$), then the relevant collision dynamics is described in the framework of the McLerran–Venugopalan (MV) model [2]. The MV model states that, due to the high parton density in the colliding nuclei, the dominant gluon fields produced in heavy ion collisions are classical, and are described by the classical Yang-Mills equations

$$D_\mu F^{\mu
\nu} = J^\nu$$

with the source current $J^\mu$ given by the color charges in the colliding nuclei. This setup is
illustrated in Fig. 1 where the two nuclei moving away from each other provide the source current, and the classical gluon field is left behind the two nuclei.

The exact analytical solution of Yang–Mills equations (1) providing gluon field generated in a heavy ion collision does not exist due to the complexity of the problem. In the diagrammatic language to find the classical gluon field one has to resum an infinite set of Feynman diagrams, and example of which is shown in Fig. 2. There exist however perturbative solutions [3, 4, 5], an analytic solution for the gluon production cross section in proton–nucleus (pA) collisions [6] and a numerical solution of the full nucleus–nucleus (AA) problem [7]. There is also an analytical ansatz for the full solution for the gluon production cross section in AA collisions [8].

To understand the matter produced by the classical gluon fields in AA collisions let us first note that the distribution of this classical matter is rapidity-independent. In the MV model the nuclei have a very large transverse extent and are translationally invariant in the transverse direction. The matter distribution thus does not depend on the transverse coordinate \( x_\perp \) and on the space-time rapidity \( \eta = (1/2) \ln[(x^0 + x^3)/(x^0 - x^3)] \), and depends only on the proper time \( \tau = \sqrt{(x^0)^2 - (x^3)^2} \). (Here \( x^0 \) is time and \( x^3 \) is the collision axis: see Fig. 3 for the explanation of

Figure 1: Heavy ion collision in McLerran–Venugopalan model.
Figure 3: The space-time picture of the ultrarelativistic heavy ion collision in the center-of-mass frame. The collision axis is labeled $x^3$, the time is $x^0$.

The most general energy-momentum tensor of a matter distribution dependent only on $\tau$ can be shown to be of the following form at mid-rapidity ($x^3 = 0$) [9]

$$T^{\mu\nu} = \begin{pmatrix}
\epsilon(\tau) & 0 & 0 & 0 \\
0 & p(\tau) & 0 & 0 \\
0 & 0 & p(\tau) & 0 \\
0 & 0 & 0 & p_3(\tau)
\end{pmatrix}$$ (2)

in the $x^0, x^1, x^2, x^3$ coordinates. In general the transverse pressure $p(\tau)$ is not equal to the longitudinal pressure $p_3(\tau)$. Imposing the energy-momentum conservation condition $\partial_\mu T^{\mu\nu} = 0$ on Eq. (2) yields

$$\frac{d\epsilon}{d\tau} = -\frac{\epsilon + p_3}{\tau}.$$ (3)

Classical gluon field dynamics is conformal: therefore $T^{\mu\mu} = 0$ which implies $\epsilon = 2p + p_3$.

At early proper times $\tau \ll 1/Q_s$ the classical gluon fields lead to the following scaling of energy density $\epsilon(\tau)$ [10, 11]

$$\left.\frac{d\epsilon}{d\tau}\right|_{\tau \approx 1/Q_s} \sim \ln^2 \frac{1}{\tau Q_s}.$$ (4)

With the help of Eq. (3) this leads to the following early-time energy-momentum tensor of the produced matter

$$T^{\mu\nu}\left|_{\tau \approx 1/Q_s}\right. = \begin{pmatrix}
\epsilon(\tau) & 0 & 0 & 0 \\
0 & \epsilon(\tau) & 0 & 0 \\
0 & 0 & \epsilon(\tau) & 0 \\
0 & 0 & 0 & -\epsilon(\tau)
\end{pmatrix}.$$ (5)

Note that the longitudinal pressure at early times is negative $p_3 = -\epsilon$. This result is true for any rapidity-independent medium distribution at early time which has a finite total energy.
At late proper times $\tau \gg 1/Q_s$, both the analytical perturbative approaches [9] and the full numerical simulations [7] lead to the energy density scaling as

$$\epsilon(\tau) \Big|_{\tau \gg 1/Q_s} \sim \frac{1}{\tau}.$$  (6)

This gives the following energy-momentum tensor

$$T^{\mu\nu} \Big|_{\tau \gg 1/Q_s} = \begin{pmatrix}
\epsilon(\tau) & 0 & 0 & 0 \\
0 & \epsilon(\tau)/2 & 0 & 0 \\
0 & 0 & \epsilon(\tau)/2 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}.$$  (7)

This late-time energy-momentum tensor has zero longitudinal pressure and corresponds to free-streaming of the produced medium. One can see that by arguing that the net energy of the produced medium is $E \sim \epsilon \tau$. Hence $E$ is constant for the energy density from Eq. (6), which can be understood as being due to non-interacting particles free-streaming away from the collision point.

2. Classical Gravity in AdS

The classical CGC picture of heavy ion collisions is self-consistent, in the sense that it assumes that $\alpha_s \ll 1$ and then justifies the assumption by generating a large momentum scale $Q_s$. However, it lacks an essential ingredient needed to describe heavy ion collisions: it does not lead to ideal hydrodynamics, which is known to describe RHIC data on particle spectra and elliptic flow rather well [12, 13]. In the rapidity-independent case considered above, ideal hydrodynamics was first considered by Bjorken [14]. It has

$$T^{\mu\nu} = \begin{pmatrix}
\epsilon(\tau) & 0 & 0 & 0 \\
0 & p(\tau) & 0 & 0 \\
0 & 0 & p(\tau) & 0 \\
0 & 0 & 0 & p(\tau)
\end{pmatrix}.$$  (8)

For the ideal gas equation of state $\epsilon = 3p$ it has

$$\epsilon(\tau) \sim \frac{1}{\tau^{4/3}}.$$  (9)

Obtaining (ideal) hydrodynamics in a first-principles calculation for heavy ion collisions is the important problem of thermalization and/or isotropization of the produced medium. Below we discuss this problem assuming that strong-coupling non-perturbative QCD effects are responsible for the onset of the hydrodynamic behavior. Since it is unknown how to consistently study QCD at strong coupling using analytic methods we will instead use AdS/CFT correspondence, which would allow us to study a distant cousin of QCD — the $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory.

AdS/CFT correspondence conjectures that the dynamics of $\mathcal{N} = 4$ SU($N_c$) SYM theory in four space-time dimensions is dual to the type IIB superstring theory on AdS$_5 \times S^5$ [15]. In the limit of large number of colors $N_c$ and large ’t Hooft coupling $\lambda = g^2 N_c$ (with $g$ the gauge coupling constant) such that $N_c \gg \lambda \gg 1$, AdS/CFT correspondence reduced to the gauge-gravity duality: $\mathcal{N} = 4$ SU($N_c$) SYM theory at $N_c \gg \lambda \gg 1$ is dual to (weakly coupled) classical
supergravity in AdS$_5$. Hence the gauge dynamics at strong coupling, which includes all-orders quantum effects, is equivalent to the classical dynamics of supergravity. Instead of summing infinite classes of Feynman diagrams in the gauge theory or using other non-perturbative methods, one can simply study classical supergravity in 5 dimensions. For a review of AdS/CFT correspondence see [16].

In a real-life heavy ion collision at RHIC the early-time dynamics is likely dominated by the weak-coupling CGC physics. Strongly-coupled dynamics may set in only at later times $\tau \sim 1/\Lambda_{QCD}$, though even this time estimate is very crude. As can be shown using the techniques of [17], matching of perturbative CGC physics onto AdS/CFT dynamics at later times is not unique, and does not lead to a single uniquely defined dual geometry in AdS$_5$, allowing instead for a variety of possible metrics. Indeed the expectation value of a single local operator (the energy momentum tensor) coming from CGC can not uniquely constrain the full quantum state of the field theory. To keep our calculations under theoretical control we will consider the whole collision of two nuclei in the strong coupling AdS/CFT framework, understanding that this is simply a rough approximation of the real heavy ion collision.

Our goal is to describe the isotropization (and thermalization) of the medium created in heavy ion collisions assuming that the medium is strongly coupled and using AdS/CFT correspondence to study its dynamics. We want to construct a metric in AdS$_5$ which is dual to an ultrarelativistic heavy ion collision as pictured in Fig. 3.

We start with a metric for a single shock wave moving along a light cone [18]:

$$ds^2 = \frac{L^2}{z^2}\left\{-2dx^+dx^- + \frac{2\pi^2}{N_c^2} \langle T_{--} (x^-) \rangle z^4 d\tau^2 + d\tau_\perp^2 + dz^2\right\}. \quad (10)$$

Here $x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$, $z$ is the coordinate describing the 5th dimension such that the boundary of the AdS space is at $z = 0$, and $L$ is the curvature radius of the AdS space. According to holographic renormalization [19], $\langle T_{--} (x^-) \rangle$ is the expectation value of the energy-momentum tensor for a single ultrarelativistic nucleus moving along the light-cone in $x^+$-direction in the gauge theory.

![Figure 4: A representation of the metric (10) as a graviton (wavy line) exchange between the nucleus at the boundary of AdS space (the solid line) and the point in the bulk where the metric is measured (denoted by a cross).]

The metric in Eq. (10) is an exact solution of Einstein equations in AdS$_5$: $R_{\mu\nu} + \frac{1}{L^2} g_{\mu\nu} = 0$. It can also be represented perturbatively as a single graviton exchange between the source nucleus near the AdS boundary and the location in the bulk where we measure the metric/graviton field. This is shown in Fig. 4 where the solid line represents the nucleus and the wavy line is the graviton propagator. Incidentally a single graviton exchange, while being a first-order perturbation of the empty AdS space, is also an exact solution of Einstein equations. This means higher order tree-level graviton diagrams are zero (cf. classical gluon field of a single nucleus in covariant gauge in the CGC formalism [20]).
The metric of Eq. (11) is illustrated in Fig. 5. The first two terms in Fig. 5 (diagrams A and B) correspond to one-graviton exchanges which constitute the individual metrics of each of the nuclei, as shown in Eq. (10). We need to calculate the next order correction to these terms, which is shown in the diagram C in Fig. 5.

Fig. 5 illustrates that construction of dual geometry to a shock wave collision in AdS_5 consists of summing up all tree-level graviton exchange diagrams. It is similar diagrammatically to the classical gluon field formed by heavy ion collisions in CGC [3, 4]. The diagram for the gluon field shown in Fig. 2 can also be interpreted as the diagram for gravitons in AdS_5 bulk leading to the metric produced in a collision of two gravitational shock waves.

Without going into details of the calculation which can be found in [21] one can calculate the graph in Fig. 5C as follows. Let us take delta-function shapes for the shock waves: \( t_1(x^-) = \mu_1 \delta(x^-) \) and \( t_2(x^+) = \mu_2 \delta(x^+) \). The contribution of diagram C in Fig. 5 to the energy density of the produced medium \( \epsilon \) should therefore be proportional to \( \mu_1 \mu_2 \). As \( \mu_1 \) and \( \mu_2 \) have dimensions of mass cubed each \( (M^3) \), and dimension of \( \epsilon \) is \( M^4 \), we need to multiply \( \mu_1 \mu_2 \) by the square of
proper time, \( \tau^2 \), as this is the only dimensionful variable left. One obtains \(^{22, 21}\)

\[
\epsilon(\tau) \bigg|_{\tau = \frac{\mu_1}{\mu_2^{1/3}}} \sim \mu_1 \mu_2 \tau^2. \tag{12}
\]

A more detailed calculation fixes the prefactor in Eq. (12) \(^{22, 21}\) and also specifies the region of validity of this result. Energy density is rapidity-independent, because of the cancellation of rapidity factors due to graviton exchanges in Fig. 5C. The energy-momentum tensor corresponding to the energy density from Eq. (12) is

\[
T^{\mu\nu} = \begin{pmatrix}
\epsilon(\tau) & 0 & 0 & 0 \\
0 & 2\epsilon(\tau) & 0 & 0 \\
0 & 0 & 2\epsilon(\tau) & 0 \\
0 & 0 & 0 & -3\epsilon(\tau)
\end{pmatrix} \tag{13}
\]

and also has large negative longitudinal pressure, similar to the CGC one \(^5\). There is a problem with energy density scaling as shown in Eq. (12) \(^{22, 21}\) (or, equivalently, with the energy-momentum tensor in Eq. (13)): as was shown in \(^18\), in this case there is a frame characterized by time 4-direction \( t^\mu \) in which the energy density is negative \( T^{t\nu} t^\nu < 0 \). Hence the scaling of energy density in Eq. (12) with proper time leads to negativity of energy density in other (boosted) frames. At this point it is not clear whether this result presents a problem, as there may be nothing wrong with energy density becoming negative for a short period of time \(^22\).

To better understand dynamics of the shock wave collisions let us follow one of the shock waves after the interaction. First we “smear” the delta-function profile of that shock wave:

\[
l_1(x^-) = \frac{\mu_1}{a_1} \theta(x^-) \theta(a_1 - x^-). \tag{14}
\]

Here \( \mu_1 \propto p^+ \Lambda^2 A^{1/3} \) and \( a_1 \propto R \frac{A}{p^+} \propto \frac{A^{1/3}}{p^+} \), where the nucleus of radius \( R \) has \( A \) nucleons in it with \( N_2^2 \) valence gluons each. \( p^+ \) is the light cone momentum of each nucleon and \( \Lambda_1 \) is the typical transverse momentum scale. The “++” component of the energy-momentum tensor of a shock wave after the collision at \( x^- = a_1/2 \) is \(^{21}\)

\[
\langle T^{++}(x^+, x^- = a_1/2) \rangle = \frac{N_2^2}{2 \pi^2 a_1} \frac{\mu_1}{a_1} \left[ 1 - 2 \mu_2 x^+ a_1 \right]. \tag{15}
\]
The first term on the right of Eq. (15) is due to the original shock wave while the second term describes energy loss due to graviton emission. Eq. (15) shows that \( \langle T^{++} \rangle \) of a nucleus becomes zero at light-cone times (as \( \mu_2 \propto p^2 \Lambda^2 A^{1/3} \approx \mu_1 \) in the center-of-mass frame)

\[
x^+ \sim \frac{1}{\sqrt{\mu_2 a_1}} \sim \frac{1}{\Lambda A^{1/3}}.
\]

Zero \( \langle T^{++} \rangle \) would mean stopping of the shock wave and the corresponding nucleus. The result can be better understood by doing all-order resummation of graviton exchanges with one shock wave, which is needed for modeling proton-nucleus collisions [23]. The full result for the proton’s “++” component of the energy-momentum tensor is

\[
\langle T^{++} \rangle = \frac{N_c^2 \mu_1}{2 \pi^2 a_1} \frac{1}{\sqrt{1 + 8 \mu_2 (x^+)^2}} \frac{1}{x^-}, \text{ for } 0 < x^- < a_1.
\]

Eq. (17) is illustrated in Fig. 6 in which one can see that the proton loses all of its light cone momentum over a rather short time.

We thus conclude that the collision of two nuclei at strong coupling leads to a necessary stopping of the two nuclei shortly after the collision. If the nuclei stop completely in the collision, the strong interactions between them are almost certain to thermalize the system, probably leading to Landau hydrodynamics [24]. It is possible that the mid-rapidity region of such collision may be well-described by Bjorken hydrodynamics [14], but this still remains to be shown.

Acknowledgments

This work is sponsored in part by the U.S. Department of Energy under Grant No. DE-FG02-05ER41377.

References

[1] J. Jalilian-Marian and Y. V. Kovchegov, Prog. Part. Nucl. Phys. 56, 104 (2006), hep-ph/0505052
[2] L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 2233 (1994), hep-ph/9309289
[3] A. Kovner, L. D. McLerran, and H. Weigert, Phys. Rev. D52, 3809 (1995), hep-ph/9505320
[4] Y. V. Kovchegov and D. H. Rischke, Phys. Rev. C56, 1084 (1997), hep-ph/9704201
[5] I. Balitsky, Phys. Rev. D70, 114030 (2004), hep-ph/0409314
[6] Y. V. Kovchegov and A. H. Mueller, Nucl. Phys. B529, 451 (1998), hep-ph/9802440
[7] A. Krasnitz, Y. Nara, and R. Venugopalan, Nucl. Phys. A717, 268 (2003), hep-ph/0209269
[8] Y. V. Kovchegov, Nucl. Phys. A692, 557 (2001), hep-ph/0011252
[9] Y. V. Kovchegov, Nucl. Phys. A762, 298 (2005), hep-ph/0503038
[10] T. Lappi, Phys. Lett. B643, 11 (2006), hep-ph/0606207
[11] K. Fukushima, Phys. Rev. C76, 021902 (2007), 0704.3626
[12] U. W. Heinz and F. R. Kohl, Nucl. Phys. A702, 269 (2002), hep-ph/0111075
[13] D. Teaney, J. Lauret, and E. V. Shuryak, Phys. Rev. Lett. 86, 4783 (2001), nucl-th/0110058
[14] J. D. Bjorken, Phys. Rev. D27, 140 (1983).
[15] J. M. Almada, Adv. Theor. Math. Phys. 2, 231 (1998), hep-th/9711200
[16] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, Phys. Rept. 323, 183 (2000), hep-th/9905111
[17] Y. V. Kovchegov and A. Taliotis, Phys. Rev. C76, 014905 (2007), 0705.1234
[18] R. A. Janik and R. Peschanski, Phys. Rev. D73, 045013 (2006), hep-th/0512162
[19] S. de Haro, S. N. Solodukhin, and K. Skenderis, Commun. Math. Phys. 217, 395 (2001), hep-th/0002230
[20] Y. V. Kovchegov, Phys. Rev. D55, 5445 (1997), hep-ph/9701228
[21] J. L. Albacete, Y. V. Kovchegov, and A. Taliotis, JHEP 07, 004 (2008), 0805.2927
[22] D. Gruenwald and P. Romatschke, JHEP 08, 027 (2008), 0803.3226
[23] J. L. Albacete, Y. V. Kovchegov, and A. Taliotis, JHEP 05, 060 (2009), 0902.3046
[24] L. D. Landau, Izv. Akad. Nauk SSSR Ser. Fiz. 17, 51 (1953).