The possible new force in nature

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Abstract

The zitterbewegung region is studied in this paper and we investigate the possible presence of a new force in this region. This force was first proposed by Sidharth in [1]. This article intends to study the existence of a fifth force. Here, essentially, equations that characterize new particles and their antiparticles are obtained. In this manner a pedestal is built for a field theoretic approach that will be the subject of the second part of this theory.

1 Introduction

As is known, zitterbewegung is a rapid oscillatory motion with regards to the wavefunction of elementary particles obeying relativistic wave equations. The existence of such phenomenon was first proposed by Erwin Schrodinger [2] and then studied by Dirac [3]. Many authors [4, 5, 6] including the authors of this paper [7, 8, 9, 10, 11, 12] have studied such a phenomenon extensively. Particularly, the affinity of the zitterbewegung and the Compton scale has been studied by Sidharth [9, 10], where the Compton scale has been considered to be elementary in nature. In some of our previous papers [11, 12], we have substantiated that in the vicinity of the Compton scale the entire space is comprised of an interior (I-region) and an exterior region (E-region) separated by a Jordan curve. Our endeavours were related to the I-region which had been treated in a meticulous and elaborate manner. The methodology was primarily based on the stochastic methods introduced by Kiyosi Ito [13, 14]. In a nutshell, it had been shown that the region below the Compton scale, namely the I-region is characterized by a different type of physics, in the sense that stochastic effects or zitterbewegung fluctuations are embedded intrinsically.

As mentioned in the abstract, a mysterious fifth force had been speculated few years ago [15, 16, 17]. Our attempt in this paper would be to derive a platform on which we can investigate such a force along with it’s characteristic attributes. The inception begins as we revisit and reinvestigate the I-region or the zitterbewegung region below the Compton

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scale to extract the dynamics and some fundamental properties inherent of it. The reader might be surprised to find out that our considerations bring forth some very radical results that can indeed extend the boundaries of known Physics.

The paper is organized as follows: in the second section we modify the Schrodinger and Dirac equations apposite to the stochastic nature of the I-region, resorting to Ito’s lemma; in the third section we use Einstein’s methodology for Brownian like motion to obtain new equations that describe new types of particles and their antiparticles; in the fourth section we discuss the results of our paper and some other new aspects that can be attributed to the I-region.

2 The modified Schrodinger and Dirac equations

In a previous paper [12] the Schrodinger equation was modified for the I-region, with respect to Ito’s lemma. Let us begin with the initial equations that lead to the aforementioned modification. Suppose, the state of a particle is defined by a wavefunction $\psi(x)$ in the interior region below the Compton scale. Now, the wavefunction $\psi(x)$ will be written as $\psi(x_t)$ in the following differential form

$$d\psi(x_t) = \psi'(x_t)dx_t + \frac{1}{2}\psi''(x_t)\sigma_t^2 dt$$

where, the primes denote differentiation with respect to $x_t$, $t$ being the instant of time and $\sigma_t$ is the standard deviation of a random variable which we will choose later on. Now, the above equation can also be written as

$$\frac{d\psi}{dt} = \frac{dx_t}{dt} \frac{d\psi}{dx_t} + \frac{1}{2} \frac{d^2\psi}{dx_t^2} \sigma_t^2$$

Again, the Schrodinger equation is given by

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) = i\hbar \frac{\partial \psi(x)}{\partial t}$$

Thus, using partial derivatives for the previous equation and employing the Schrodinger equation we have

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x_t)}{\partial x_t^2} + V(x_t)\psi(x_t) = i\hbar \frac{\partial \psi(x_t)}{\partial t} + \frac{i\hbar}{2} \frac{\partial^2 \psi(x_t)}{\partial x_t^2} \sigma_t^2$$

which is the modified Schrodinger equation in the I-region. Specifically, this means that this region has different characteristic attributes compared to the E-region outside. However, let’s rewrite this in a more subtle way.

$$-\left(\frac{\hbar^2}{2m} + \frac{i\hbar}{2} \sigma_t^2\right) \frac{\partial^2 \psi}{\partial x_t^2} + V \psi = i\hbar \frac{\partial \psi}{\partial t}$$

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Since the Laplacian has an extra complex term multiplied to it, this transformation can be looked upon as the passing of $SU(n)$ to the complex region. This means that $SU(n)$ gets complexified. One can invoke the idea that the modified Schrodinger equation gives a subtle description of the Compton region in terms of new physics. Now, considering the radial form of the Schrodinger equation we have the modified radial potential in the three-dimensional zitterbewegung region as

$$V'(r_t) = V(r_t) + \frac{k^2 l(l+1)}{2m r_t^2} + \phi_{3D}(r_t)$$

where, the first term is the usual Coulomb part, the second term is the centrifugal part and $\phi_{3D}(r_t)$ is given by

$$\phi_{3D}(r_t) = 4i\hbar\pi l(l+1)\Delta r_t$$

Or simply,

$$\phi(r) = 4i\hbar\pi l(l+1)\Delta r$$

Incidentally, this potential has the azimuthal quantum number that is the measure of the orbital angular momentum. This feature could bring forth interesting explanations in future research. Now, we are in a position to make the ansatz that this extra potential term gives rise to a new force. For now, let us call this force as the zitterbewegung force that gives rise to a new field which shall be the subject of a different paper.

We had hypothesized previously that this extra part of the potential gives rise to confinement due to its complex nature. Now, we would like to elaborate on this matter. This complex potential can be looked upon as the basis of a new force that confines the properties of the region within itself. This could also be the reason as to why the physics inside the interior region is in stark contrast with the physics outside the said region. The complexification of the Schrodinger equation can be considered as the passage from real to complexified Lie algebra as

$$su(n) + isu(n) = sl(n, \mathbb{C})$$

where, $sl(n, \mathbb{C})$ denotes the Lie algebra of the special linear group of ‘n’ degree over $\mathbb{C}$. This complexification could be the reason for the confinement potential arising in the modified Schrodinger equation. Interestingly, the Dirac equation also gets modified with the advent of the stochastic considerations. By Ito’s lemma and using the same methodology for the Schrodinger equation we have for the Dirac equation

$$i\hbar\gamma_k \partial_k \psi - mc\psi = -i\hbar\gamma_0 \partial_0 \psi$$

$$i\hbar\gamma_k \partial_k \psi - mc\psi = -i\hbar\gamma_0 [\partial_0 \psi + \frac{\sigma_t^2}{2} \partial_t^2 \psi]$$

3
where, the indices $k = 1, 2, 3$. This finally gives us

$$i \hbar \gamma_\mu \partial_\mu \psi + \frac{i \hbar \gamma_0}{2} \sigma_t^2 \partial_k^2 \psi - m c \psi = 0$$  \hspace{1cm} (6)$$

This is the modified version of the Dirac equation in the I-region, below the Compton scale. As usual, we shall identify $\sigma_t$ in a latter paper. Thus, we have a modified Schrodinger equation which gives the quantum mechanical description of the I-region of zitterbewegung, a modified Dirac equation that describes all the spin-$1/2$ particles and a complexification of the real Lie algebra, in this particular region. On the other hand, the modified Schrodinger equation (3) brings about an extra force that could altogether be a new force that was hitherto unknown.

It is worth mentioning that in a previous paper \[1\] the authors had used a similar type of modified Dirac equation as that of (6) and construed the Schwinger terms of the gyromagnetic ratio of the electron. Therein, a constant $\lambda = -\frac{\alpha}{2\pi}$ ($\alpha$ being the fine structure constant) had been obtained by means of which the Chandrasekhar limit for white dwarfs had been modified \[19\]. So, it is evident that equation (6) is physically quite significant when we consider such effects or results as mentioned above.

Now, in many of his previous papers Sidharth \[20\] \[21\] has argued that noncommutativity is a characteristic attribute of the zitterbewegung region. Also, we have shown that from noncommutativity one can obtain the basic results of special relativity \[22\], feasibly derive superluminal velocities \[23\] or the noncommutativity of momenta and the Einstein-De Haas effect \[24\]. One can say that the noncommutative nature of the zitterbewegung region gives rise to these unorthodox conclusions and results owing to the continual fluctuations in the I-region. We would like to elaborate on this matter in a latter paper concerned with the field of the zitterbewegung region, as mentioned in the abstract of this paper.

However, now we shall see that the equation (6) can be modified with the following transformations of the time derivatives and the space derivatives respectively

$$\partial_k \rightarrow \mathcal{D}_k$$  \hspace{1cm} (7a)$$

$$\partial_0 \rightarrow \partial_0 + \frac{\sigma_t^2}{2} \partial_k^2 = \partial_0 + \frac{\sigma_t^2}{2} \mathcal{D}_k = \mathcal{D}_0$$  \hspace{1cm} (7b)$$

With these transformations we have the modified Dirac equation from (6) as

$$i \hbar \gamma_\mu \mathcal{D}_\mu \psi - m c \psi = 0$$  \hspace{1cm} (8)$$

It is evident from the transformations (7a) and (7b) that outside the I-region and above the Compton scale when the randomness parameter ($\sigma_t$) tends to zero, we get back the Dirac equation with the ordinary partial derivatives $\partial_\mu$. In a similar manner we will get back the Schrodinger equation. Essentially, inside the I-region the partial time derivative intrinsically introduces the partial space derivatives. This corresponds with Ito’s lemma.
So, we can argue that below the Compton scale the basic equations change according to Ito’s lemma and consequently they are modified with a new form of derivative - the $\mathcal{D}_\mu$’s.

3 Particle-antiparticle equations from Brownian motion like stochastic considerations

In this section we shall deal with the derivation of a wave equation from the considerations of Brownian like motion, as done by Einstein [25]. However, in contradistinction to Einstein’s approach, we assume that the movement of the particles may or may not be mutually independent of one another. The reason can be attributed to the new force field due to the zitterbewegung field (Z-field).

We will consider a time-interval $\tau$ that is infinitesimal. Suppose, there are $n$ number of particles under observation in the inner region (I-region), i.e. below the Compton scale. In that interval of time $\tau$ the $x$-Co-ordinates of the single particles will increase by some amount say $x_\delta$, where $x_\delta$ will be of different values (positive or negative) for each particle. It can be said that $x_\delta$ is the randomness in the $x$-coordinate. Now, we make a fundamental assumption here that on account of the noncommutative nature of spacetime and the randomness of the Z-field, space symmetry ceases to exist. This assumption is crucial to our work.

Now, the number of particles (say, $dn$) which experience in the time interval $\tau$ a displacement which lies between $x_\delta$ and $d + dx_\delta$, will be given by

$$dn = n\rho(x_\delta)dx_\delta$$

where, $\rho(x_\delta)$ denotes the probability density function and $\int_{-\infty}^{+\infty} \rho(x_\delta)dx_\delta = 1$. Now, let us consider that a particular particle has a total wavefunction by which it can be characterized inside the I-region. Therefore, the wavefunction for the $n$-th particle in the said region will be

$$\psi_n(x, t) \rightarrow \psi_n(x, t + \tau)$$

Expanding this in a Taylor series we have

$$\psi_n(x, t + \tau) = \psi_n(x, t) + \tau \frac{\partial \psi_n}{\partial t} + \cdots$$

(9)

Here, we have neglected the second and other higher order derivatives. Again, from Einstein’s considerations we can write

$$\psi_n(x, t + \tau) = \int_{-\infty}^{+\infty} \psi_n(x - x_\delta, t) \rho(x_\delta)dx_\delta$$
which gives us

\[
\psi_n(x, t + \tau) = \psi_n(x, t) \int_{-\infty}^{+\infty} \rho(x_{\delta}) dx_{\delta} - \frac{\partial \psi}{\partial x} \int_{-\infty}^{+\infty} x_{\delta} \rho(x_{\delta}) dx_{\delta} + \cdots \tag{10}
\]

where, we have considered only up to the first order derivative since in \((9)\) we have done the same. Another rationale for not taking the second order derivative into account is that in the I-region the transformations \((7a)\) and \((7b)\) intrinsically introduce the second order derivatives with the randomness parameter \(\sigma_t\), which would suffice as a start. However, now, in light of our assumption that space-symmetry is broken due to the noncommutative nature of spacetime and the randomness of the \(Z\)-field, we find that the second and the other even terms (i.e. the first and other odd moments) will remain. We shall consider only the first odd moment for now. Now, remembering that \(\int_{-\infty}^{+\infty} x_{\delta} \rho(x_{\delta}) dx_{\delta} = 1\), and using \((9)\) we derive

\[
\tau \frac{\partial \psi}{\partial t} = -\frac{\partial \psi}{\partial x} \alpha \tag{11}
\]

where, \(\alpha = \int_{-\infty}^{+\infty} x_{\delta} \rho(x_{\delta})\). Since, we have established earlier in the preceding section that inside the I-region the transformations \((7a)\) and \((7b)\) are essential, we use it in the above equation to obtain

\[
\tau \frac{\partial \psi}{\partial t} + \frac{\tau \sigma_t^2}{2} \frac{\partial^2 \psi}{\partial x^2} = -\frac{\partial \psi}{\partial x} \alpha
\]

Therefore, for three dimensions we finally derive

\[
\xi_\mu \partial_\mu \psi + \beta \partial_k^2 \psi = 0 \tag{12}
\]

where, the indices have their usual meanings, \(\beta = \frac{\tau \sigma_t^2}{2}\) and

\[
\xi_0 = \tau
\]

\[
\xi_k = \alpha_k = \int_{-\infty}^{+\infty} x_{\delta}^k \rho(x_{\delta}) dx_{\delta}^k
\]

Now, it is interesting to note that equation \((11)\) can also be written as

\[
\xi_\mu \partial_\mu \psi = 0 \tag{13}
\]

which resembles the relativistic Weyl equation of quantum field theory. This can be looked upon as an analogous form of the Weyl equation in the I-region. However, now we would like to derive a more compact form of equation \((12)\) resorting to the modified Dirac equation
We can write the aforesaid equations in the following manner respectively as

\[ \xi_k \partial_k \psi + \beta \partial_k^2 \psi = -\xi_0 \partial_0 \psi \]

and

\[ i\hbar \gamma_k \partial_k \psi + \frac{i\hbar \gamma^0}{2} \sigma^2 \partial^2_k \psi - mc\psi = -i\hbar \gamma_0 \partial_0 \psi \]

Eliminating \( \partial_0 \psi \) from the last two equations and remembering that \( \xi_0 = \tau \), we obtain

\[ \tau^{-1} \xi_k \partial_k \psi + \tau^{-1} \beta \partial_k^2 \psi = \gamma_0^{-1} \gamma_k \partial_k \psi + \frac{\sigma^2}{2} \partial^2_k \psi - \frac{mc\gamma_0^{-1}}{i\hbar} \psi \]

Again, since \( \tau^{-1} \beta = \frac{\sigma^2}{2} \), we have

\[ i\hbar (\gamma_0 \tau^{-1} \xi_k + \gamma_k) \partial_k \psi + mc\psi = 0 \]

Thus, writing \( \Gamma_k = \gamma_0 \tau^{-1} \xi_k + \gamma_k \), we finally obtain

\[ i\hbar \Gamma_k \partial_k \psi + mc\psi = 0 \]

This is a fundamental equation for a particle in the I-region. Interestingly, there are two stark differences between the Dirac equation and equation (11). The first is that the new equation for the I-region doesn’t have the time derivative included in it. Instead, there is an infinitesimal time (\( \tau \)) intrinsically involved in the \( \Gamma \) matrices. Secondly, the mass term has become positive. Essentially, with the use of the modified Dirac equation in the I-region we have obtained a new equation that describes the particles in the I-region in a novel manner altogether.

Now, once again, let us consider the equations (6) and (12)

\[ i\hbar \gamma_\mu \partial_\mu \psi + \frac{i\hbar \gamma^0}{2} \sigma^2 \partial^2_\mu \psi - mc\psi = 0 \]

and

\[ \xi_\mu \partial_\mu \psi + \frac{\tau \sigma^2}{2} \partial^2_\mu \psi = 0 \]

From these two, if we eliminate \( \frac{\sigma^2}{2} \partial^2_\mu \psi \), then we get

\[ i\hbar \gamma_\mu \partial_\mu \psi - mc\psi = i\hbar \gamma_0 \tau^{-1} \xi_\mu \partial_\mu \psi \]

Therefore, we derive the equation
\[ i\hbar \Gamma'_\mu \partial_\mu \psi - mc\psi = 0 \]  
where, \( \Gamma'_\mu = \gamma_\mu - \gamma_0 \tau^{-1} \xi_\mu \). This can also be looked upon as a new form of the Dirac equation inside the I-region. Essentially, equations (14) and (15) are both fundamental for the I-region, where the stochastic nature of spacetime has to be taken into consideration in order to describe particles and the system in a fundamental and precise manner. It is interesting to put forward the question that what do equations (14) and (15) describe? It is possible that the said equations describe new types of particles that are borne out of the stochastic considerations and the fuzzy nature of the \textit{zitterbewegung region}. Next, we would like to investigate the properties that entail from the equations (14) and (15) and we will find that indeed we have new particles emerging in the I-region.

Now, it is known that [26] considering a free electron at rest the solutions are given by the equation

\[ i\hbar \frac{\partial \psi}{\partial t} = \gamma_0 mc^2 \psi \]

in the form of spinors. Thus, resorting to the same methodology we find that for a particle at rest equation (15) gives

\[ i\hbar \Gamma'_0 \partial_0 \psi - mc\psi = 0 \]

Now, \( \Gamma'_0 = \gamma_0 - \gamma_0 \tau^{-1} \xi_0 \). Again, since \( \xi_0 = \tau \) we have the first term in the above equation to be zero. Thus

\[ m = 0 \]

which implies that the mass of the particle is zero when at rest. This is a new characteristic which is unlike a Dirac particle. However, we can make the ansatz that while in motion the particles and antiparticle acquire mass due to the breakdown of space symmetry on account of the noncommutative nature of spacetime and the stochastic nature of the I-region. Essentially, for a particle in motion we have equation (14) and the following equation

\[ i\hbar \Gamma'_{k} \partial_{k} \psi - mc\psi = 0 \]  
(16)

The equations (14) and (16) can be looked upon as the equations for a particle and it’s antiparticle, with mass \( m \). Therefore, without considering Dirac’s approach of electron-hole theory we have a particle-antiparticle interpretation emerging from our theory quite inherently.
4 Discussions

1) In the current paper, we have investigated the \textit{zitterbewegun region} (I-region) below the Compton scale in a quantitative manner. Thereby we have found some very interesting results that are epitomized by equations that take after the Dirac equation. Essentially, the equations derived from our considerations describe new particles and their antiparticles that emerge due to the stochastic movements in the I-region. These particles are incidentally massless, when at rest. This feature is a novel trait that is not given by the original Dirac equation. It is only when we consider Ito’s stochastic methodology and the randomness of the I-region that we obtain such results.

We have also proposed a new field, namely the \textit{zitterbewegung field}, that engenders in the I-region on account of it’s nature. This force is a confinement like force that traps all the characteristics of the I-region from within. The potential has been seen to arise from the modified Schrodinger equation and will be investigated and delineated in a latter paper.

2) Now, in the context of complexification that was discussed in the second section we would like to interpret it in terms of ‘boosts’ and ‘rotations’. For a Lorentz boost we have the transformations

\[ ct' = x^0 = x^0 \cosh \omega - x^1 \sinh \omega \]

\[ x^1' = x \cosh \omega - x^0 \sinh \omega \]

\[ x^2' = x^2 \]

\[ x^3' = x^3 \]

where, \( \omega \) is the hyperbolic angle. In matrix form this is written as

\[
\begin{bmatrix}
x^0' \\
x^1' \\
x^2' \\
x^3'
\end{bmatrix} =
\begin{bmatrix}
\cosh \omega & -\sinh \omega & 0 & 0 \\
\sinh \omega & \cosh \omega & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x^0 \\
x^1 \\
x^2 \\
x^3
\end{bmatrix}
\]

(17)

Thus, when one delves into the I-region below the Compton scale and we have a complexified spacetime, then the transformations will be like: \( x \rightarrow ix \). So, we will have

\[ \sinh x = \frac{e^x - e^{-x}}{2} \rightarrow \frac{e^{ix} - e^{-ix}}{2} = \sin x \]
\[
\cosh x = \frac{e^x + e^{-x}}{2} \rightarrow \frac{e^{ix} + e^{-ix}}{2} = \cos x
\]

Therefore, the boost in the previous matrix equation is modified as

\[
\begin{bmatrix}
  x' \ 0 \\
  x' \ 1 \\
  x' \ 2 \\
  x' \ 3
\end{bmatrix} =
\begin{bmatrix}
  \cos \omega & -\sin \omega & 0 & 0 \\
  \sin \omega & \cos \omega & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x' \ 0 \\
  x' \ 1 \\
  x' \ 2 \\
  x' \ 3
\end{bmatrix}
\tag{18}
\]

which essentially epitomizes a pure rotation. This transformation can be discerned as the inherent presence of spin in the I-region that emerges due to the stochastic nature of the region. Now, when we generalize the coordinates \(x + iy\) to 3D, we actually end up with the 4D quaternion expression. Furthermore, there is the invariant, namely

\[
x'^2_0 + x'^2_1 + x'^2_2 - x'^2_3
\]

which resembles the Minkowski metric and the whole theory parallels the theory of the Dirac equation with spin.

It is interesting to note that if the coordinates are complexified as above then the parameters \(\xi_k\)'s become complex and immediately we can deduce that the matrices \(\Gamma_k\) and \(\Gamma'_k\) become conjugates of each other. Incidentally, we then have

\[
\Gamma_k^* = \Gamma'_k
\]

and the particle-antiparticles equations \((14)\) and \((16)\) can be respectively written as

\[
\begin{align*}
i\hbar \Gamma \partial_k \psi + mc\psi &= 0 \tag{19} \\
i\hbar \Gamma'^* \partial_k \psi - mc\psi &= 0 \tag{20}
\end{align*}
\]

Essentially, this validates our proposition furthermore that the equations derived for the I-region represent particles and antiparticles with their respective gamma matrices.

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