Efficient Transmit Antenna Selection for Receive Spatial Modulation-Based Massive MIMO

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ABSTRACT In this article, two efficient transmit antenna subset (TAS) selection schemes are proposed for receive spatial modulation (RSM)-based massive multiple-input multiple-output. First, an incremental TAS selection algorithm based on the maximization of the received signal-to-noise ratio is presented to select N\text{S} active transmit antennas effectively among the available N\text{T} transmit antennas. Then, to reduce complexity further, the modified TAS selection algorithm performs two consecutive selection stages. The pre-processing stage selects active transmit antennas whose number N\text{P} is less than the number of N\text{S} of the total transmit antennas to be selected and is equal to or greater than the number N\text{R} of the receive antennas. Then the post-processing stage chooses the remaining N\text{S} − N\text{P} active antennas. In the first stage, a simple norm-based algorithm is employed to reduce the complexity significantly. In the second stage, an incremental selection strategy is performed to find additional transmit antennas. It is demonstrated that the bit error rate and achievable rate of the proposed TAS selection algorithms are close to those of the decremental algorithm. Further, the simulation results show that the proposed TAS selection schemes offer significantly reduced complexity compared to the decremental TAS selection when the difference between the number of selected transmit antennas and the number of total available transmit antennas available is large. Furthermore, the impacts of the channel estimation error on the performance of TAS selection-based RSM systems are examined.

INDEX TERMS Transmit antenna selection, multiple-input multiple-output (MIMO), precoding, zero-forcing (ZF), receive spatial modulation.

I. INTRODUCTION Spatial modulation (SM) has been considered as a promising multiple-input multiple-output (MIMO) transmission technique for devices with low complexity and low power consumption [1]–[4]. It exploits the transmit antenna index as an additional means to convey information. In an SM scheme, only a single transmit antenna is activated during transmission, thereby removing the problem of inter-channel interference and employing only a single radio frequency (RF) chain at the transmitter. In [5], a generalized version of SM called GSM has been introduced to increase the system spectral efficiency; in GSM, more than one transmit antenna is active at the same time. Recently, precoding-aided spatial modulation (PSM), also called receive SM [6]–[8] has been developed to enhance MIMO spectral efficiency by utilizing the indices of the receive antennas to transmit more information. It can also facilitate the designing of a simplified receiver structure for downlink MIMO transmission. In [9] and [10], the original PSM has been extended to the generalized PSM scheme, which targets the exploiting of more than one receive antenna to achieve higher spectral efficiency.

Antenna selection can be performed to enhance the reliability of the SM systems [11]–[15]. Euclidean distance optimized antenna selection (EDAS) schemes with low-complexity have been considered in [11] and [12]. They have obtained less computational complexity while achieving the same symbol error rate performance as an optimal exhaustive search-based EDAS algorithm. In [13], the achievable transmit diversity order of SM systems using EDAS has been analytically quantified. In [14], various algorithms for transmit antenna subset (TAS) selection for SM systems have been examined in terms of the bit error rate (BER) performance versus the complexity tradeoff.
For conventional PSM, receive antenna subset (RAS) selection has been introduced in [16]–[18]. In [16] and [17], exhaustive search-based optimal and suboptimal RAS selection algorithms have been presented. A tradeoff exists between the BER performance and the computational complexity. In [18], the author derives the achievable diversity order of the zero-forcing (ZF)-based PSM system with RAS selection. The diversity order is the product of the respective diversity orders offered by ZF-based precoding and RAS selection. This means that, in the presence of $N_T$ transmit antennas and $N_R$ receive antennas, the overall diversity gain can be achieved by using $(N_T - N_S + 1)(N_R - N_S + 1)$, where $N_S$ denotes the number of selected receive antennas.

By contrast, TAS selection can be performed to reduce the number of RF units at the transmitter. The effects of TAS selection on the performance of PSM systems have been investigated in [19] and [20]. It examines the tradeoff between system performance and the number of RF channels when the TAS selection is applied to PSM. It has been demonstrated that decreasing the number of activated RF units through TAS selection in ZF-PSM systems always degrades the performance. In addition, to reduce the computational complexity of an exhaustive search-based optimal TAS selection algorithm considerably, a decremental TAS selection algorithm is presented. However, when the number of activated RF units is significantly smaller than that of total transmit antennas especially in massive MIMO systems, the computational complexity of the decremental TAS selection remains high even though it achieves reduced complexity compared to the optimal search.

In this article, we propose two efficient TAS selection schemes to provide a better tradeoff between system performance and computational complexity when the number of selected transmit antennas is significantly smaller than that of total transmit antennas. First, an incremental strategy is considered to achieve the low-complexity. Next, a two-stage TAS selection scheme similar to that in [21] is proposed to reduce the computational complexity of the first TAS selection algorithm further. In the first stage, a transmitter selects $N_P(N_R \leq N_P < N_S)$ transmit antennas from the total $N_T$ transmit antennas using a low-complexity algorithm. In the second stage, $N_S - N_P$ transmit antennas are incrementally selected among the $N_T - N_P$ unselected transmit antennas. We demonstrate that the TAS selection algorithm developed in [21] for massive MIMO is unsuitable for the PSM systems; this will be shown later in this article. We also demonstrate that the proposed TAS selection algorithms can achieve significantly reduced complexity compared to the decremental algorithm in [19] and [20], especially for $N_S \ll N_T$, at the cost of slight performance degradation. Further, we compare the proposed algorithms with the TAS selection methods introduced in [21] with respect to performance and complexity. Moreover, we demonstrate it is possible to adjust the tradeoff between performance and complexity by introducing a new design parameter $N_P$. Finally, in the presence of channel estimation errors, we show that the proposed TAS selection algorithms can provide more robustness than other low-complexity algorithms.

The remainder of this article is organized as follows. In Section II, a system model of the PSM system with TAS selection, based on the ZF precoder is briefly presented. In Section III, the two suboptimal TAS selection algorithms with low-complexity are proposed. The computational complexity is derived in Section IV. The simulation results are presented in Section V. Finally, some conclusions are drawn in Section VI.

Notation: Throughout this article, the boldface lower-case and upper-case letters represent the vectors and matrices, respectively. We use the superscript $*$ to denote the conjugate of a complex number. Further, we use the superscript $H$ to denote the Hermitian transpose of a matrix or a vector. $Tr(\cdot)$ and $(\cdot)^{-1}$ represent the trace operation and inverse operation, respectively. $E[\cdot]$ and $\| \cdot \|$ denote the expectation and the Euclidean norm, respectively. $I_n$ and $Q(t)$ are the $n \times n$ identity matrix and the Q function, respectively. $X(:, k)$ denotes the $k$-th column vector of matrix $X$. $X(:, [1 : (k-1) (k+1) : end])$ represents the remaining submatrix obtained by deleting the $k$-th column vector in matrix $X$.

II. SYSTEM MODEL OF TAS-PSM

We consider a MIMO system with an $N_T$ antenna transmitter and an $N_R(\geq N_T)$ antenna receiver. The transmitter is equipped with only $N_S(N_R \leq N_S < N_T)$ RF transmission units. Thus, we assume that $N_S$ antennas are selected out of the $N_T$ transmit antennas. The full channel matrix is represented as a quasi-static channel matrix of $H \in \mathbb{C}^{N_R \times N_T}$, whose elements are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero mean and unit variance denoted by $\mathcal{CN}(0, 1)$. The spatial modulated super-symbol vector is given as $x_m \in \mathbb{C}^{N_x \times 1}$, which can be described as $x_m = s_m e_r$ where $s_m$ and $E[s_m^* s_m] = 1$ is the $m$-th symbol generated from the $M$-ary quadrature amplitude modulation or phase-shift keying (PSK) constellation set and $e_r$ denotes the $r$-th column of the $N_R$-dimensional unit matrix, thereby indicating that the $r$-th receive antenna is activated. The super-symbol $x_m$ is first precoded before transmission. Then the transmit signal vector is given by $\beta(P x_m^r)$ where $P \in \mathbb{C}^{N_S \times N_P}$ is a precoding matrix and $\beta$ is a power normalization factor that is used to ensure $E[\| \beta(P x_m^r) \|^2] = 1$.

A. PERFECT CHANNEL ESTIMATION CASE

We assume that the transmitter of the PSM system has perfect knowledge of the channel side information (CSI). Then the ZF precoder can be given by [22]

$$P_{ZF} = H_S^H (H_S H_S^H)^{-1}$$

where $H_S \in \mathbb{C}^{N_R \times N_S}$ denotes the channel submatrix selected by a TAS selection algorithm.
The received block signal at the receiver can be represented as
\[ y = β_S H_S P x_m^r + n = β_S x_m^r + n \] (2)
where the power normalization factor related to the selected TAS can be expressed as
\[ β_S = \frac{N_R}{\sqrt{Tr \left( H_S H_S^H \right)^{-1}}} \] (3)
and \( n \in C^{N_R \times 1} \) is an i.i.d. additive white Gaussian noise vector whose elements are the zero-mean circular complex white Gaussian noise component of a variance of \( σ_n^2 \). Hence, the optimal maximum likelihood (ML) detector for the ZF-PSM can be given by
\[ < \hat{r}, \hat{s}_m > = \hat{x}_m^r = \arg \min \| y - β_S x_m^r \|^2 \] (4)
Then, the average BER (ABER) for the PSM systems can be obtained by using the union bounding technique [23]. An upper bound on the ABER can be expressed as
\[ \text{ABER} \leq \frac{1}{2^L} \sum_{i=1}^{2^L} \sum_{j=1}^{2^L} \frac{N}{L} \left( x_i \rightarrow x_j \right) E_{H_S} \left\{ \text{PEP}_S(x_i \rightarrow x_j) \right\} \] (5)
where \( L \) is the total number of bits conveyed in each transmission, \( N(x_i \rightarrow x_j) \) is the number of bits in error between \( x_i \) and \( x_j \) with \( x_i \) and \( x_j \) denoting two possible super-symbols, and \( \text{PEP}_S(x_i \rightarrow x_j) \) is the pairwise error probability (PEP) for a given \( H_S \) when \( x_i \) is transmitted but \( x_j \) is detected. From [19], the PEP for a given \( H_S \), i.e., \( β_S \) can be expressed as
\[ \text{PEP}_S(x_i \rightarrow x_j) = \Pr \left\{ \| y - β_S x_i \|^2 > \| y - β_S x_j \|^2 \right\} = Q \left( \frac{β_S^2}{2σ_n^2} \| z_m \|_2^2 \right) \] (6)
Minimizing the ABER of the ZF-PSM systems is equivalent to maximizing the term \( β_S^2 \). Hence, the TAS problem for the ZF-PSM systems can be described as
\[ S_{opt} = \arg \max_{S \in \{ S_k, k=1,2,..,C(N_T,N_S) \}} β_S^2 \] (7)
where \( S_k \) is the \( k \)-th enumeration of the set of all available \( C(N_T,N_S) \) TASs. Here \( C(N_T,N_S) \) is the total number of combinations for selecting \( N_S \) antennas among the \( N_T \) transmit antennas. Hence, the optimal TAS selection algorithm for the PSM system can be expressed as [19]
\[ S_{opt} = \arg \min_{S \in \{ S_k, k=1,2,..,C(N_T,N_S) \}} \text{Tr} \left( H_S H_S^H \right)^{-1} \] (8)
It should be noted that the computational complexity required for an optimal selection in (8) is prohibitive owing to an exhaustive search, especially when the number of all the possible TASs is large.

### B. IMPERFECT CHANNEL ESTIMATION CASE

Under the assumption of imperfect CSI at the transmitter, the ZF precoder can be given by
\[ P_{ZF, err} = H_S^H \left( H_{S, err} H_{S, err}^H \right)^{-1} \] (9)
where \( H_{S, err} \in C^{N_R \times N_S} \) is the channel submatrix with CSI errors, which is chosen by a TAS selection algorithm from the full channel matrix, \( H_{err} \in C^{N_R \times N_T} \), with channel estimation errors. Here, the estimated channel coefficient from the \( a \)-th transmit antenna to the \( b \)-th receive antenna, which is an element corresponding to the \( b \)-th row and \( a \)-th column of \( H_{err} \), is given by \( h_{ba} = h_{ba} + e_{ba} \), where \( h_{ba} \) is the \( (a,b) \)-th element of \( H \) distributed with \( CN(0,1) \) and \( e_{ba} \) indicates the error component caused by the imperfect channel estimation and is modeled as an i.i.d. circular complex Gaussian random variable with zero mean and a variance of \( σ_e^2 \).

The received block signal at the receiver can be written as
\[ y = β_{S, err} H_S P_{ZF, err} x_m^r + n \] (10)
where the power normalization factor associated with the selected TAS is given by
\[ β_{S, err} = \frac{N_R}{\sqrt{\text{Tr} \left( H_{S, err} H_{S, err}^H \right)^{-1}}} \] (11)
Then the ML detection at the ZF-PSM receiver is given by
\[ < \hat{r}, \hat{s}_m > = \hat{x}_m^r = \arg \min \| y - β_{S, err} x_m^r \|^2 \] (12)
It should be noted that the optimal TAS selection is performed using \( H_{S, err} \) instead of the \( H_S \) in (8).

### III. PROPOSED INCREMENTAL TAS SELECTION ALGORITHMS

It should be noted that the computational complexity of (8) while finding an optimal TAS is very high owing to an exhaustive search with the computation of \( \left( H_S H_S^H \right)^{-1} \). To obtain efficient TAS selection algorithms with a significantly reduced-complexity, we adopt an incremental selection strategy. Here we present two types of incremental TAS selection algorithms. The first is based on the minimization of the trace of (8) at each intermediate step for incremental selection. The second is a more efficient version of the first one and consists of two distinct consecutive processing stages. In the first pre-processing stage, a simple norm-based algorithm is used to select a TAS subset. Then the second post-processing stage employs an incremental algorithm to select the remaining antennas.

#### A. INCREMENTAL TAS SELECTION ALGORITHM

The first TAS selection algorithm begins with an empty set of a selected TAS and selects one transmit antenna at each incremental step. After taking \( n \) incremental steps, \( n \) transmit antennas are selected; then the corresponding channel sub-matrix is denoted by \( H_n \in C^{N_R \times n} \), where \( 1 \leq n \leq N_S \). After
selecting \( (n + 1) \) antennas, the selected submatrix of \( H \) can be expressed as
\[
H_{n+1} = [H_n, h_{n+1}]
\]
where \( h_{n+1} \) is the column vector of \( H \) corresponding to the \((n + 1)\)-th selected antenna. Based on the pre-determined channel \( H_n \), the \((n + 1)\)-th selected antenna can be obtained using the following optimization.

\[
S_{n+1} = \arg \min_{(n+1) \in R_n} Tr \left[ \left( H_{n+1}H_{n+1}^H \right)^{-1} \right]
\]

where \( S_{n+1} \) denotes the TAS selected at the \((n+1)\)-th selection step and \( R_n \) is the TAS unselected at the \(n\)-th selection step.

In each step of the greedy procedure, the expensive computational burden is due to the calculation of the matrix product \( H_{n+1}H_{n+1}^H \) and the matrix inversion in \( (H_{n+1}H_{n+1}^H)^{-1} \). To reduce the computational complexity further, the popular Sherman-Morrison formula is employed. Then \( (H_{n+1}H_{n+1}^H)^{-1} \) can be re-expressed as
\[
\left( H_{n+1}H_{n+1}^H \right)^{-1} = \left( H_nH_n^H + h_{n+1}h_{n+1}^H \right)^{-1}
= \left( H_nH_n^H \right)^{-1} - \frac{(H_nH_n^H)^{-1}h_{n+1}h_{n+1}^H(H_nH_n^H)^{-1}}{1 + h_{n+1}^H(H_nH_n^H)^{-1}h_{n+1}}
\]

Thus (14) can be written as
\[
S_{n+1} = \arg \min_{(n+1) \in R_n} Tr \left[ \left( H_nH_n^H \right)^{-1} - \frac{(H_nH_n^H)^{-1}h_{n+1}h_{n+1}^H(H_nH_n^H)^{-1}}{1 + h_{n+1}^H(H_nH_n^H)^{-1}h_{n+1}} \right]
\]

where the inverse of \( H_nH_n^H \) is iteratively updated using the previous inverse result of \( H_{n-1}H_{n-1}^H \) and can be expressed as
\[
\Xi_n = \Xi_{n-1} - \frac{\Xi_{n-1}h_{n+1}h_{n+1}^H\Xi_{n-1}}{1 + h_{n+1}^H\Xi_{n-1}h_{n+1}}
\]
where \( \Xi_n = (H_nH_n^H)^{-1} \) and \( \Xi_{n-1} = (H_{n-1}H_{n-1}^H)^{-1} \). Further, (12) can be re-expressed as
\[
S_{n+1} = \arg \min_{(n+1) \in R_n} Tr \left[ \Xi_n - \frac{\Xi_{n-1}h_{n+1}h_{n+1}^H\Xi_{n-1}}{1 + h_{n+1}^H\Xi_{n-1}h_{n+1}} \right]
\]

Based on the above analysis, the procedure of the proposed incremental TAS selection algorithm is summarized in Table 1. Here \( H_{n-1}(:, q) \) denotes the \(q\)-th column vector of the channel submatrix \( H_{n-1} \), which is associated with the transmit antennas that are unselected after completing the \((n - 1)\)-th step. It should be noted that under the knowledge of imperfect CSI, \( H_{err} \) is used as an input of the proposed TAS selection algorithm.

| TABLE 1. Proposed incremental TAS selection algorithm. |
|-----------------------------------------------|
| **Inputs:** & \( H, N_T, N_S \) |
| **Procedure** |
| 1: & \( \tilde{H}_2 = H \) |
| 2: & \( \Xi_0 = I_{N_T} \) |
| 3: & for \( n = 1, 2, \ldots, N_S \) |
| 4: & for \( q = 1, 2, \ldots, (N_T - n + 1) \) |
| 5: & \( \lambda_{n, q} = \Xi_{n-1} + \frac{\Xi_{n-1}h_{n+1}h_{n+1}^H\Xi_{n-1}}{1 + h_{n+1}^H\Xi_{n-1}h_{n+1}} \) |
| 6: & \( \lambda_{n, q} = Tr \left[ \Xi_{n-1} - \lambda_{n, q} \right] \) |
| 7: & end |
| 8: & \( \hat{q} = \arg \min_q \lambda_{n, q} \) |
| 9: & \( H_{\hat{q}}(:, n) = \tilde{H}_{n+1}(:, \hat{q}) \) |
| 10: & \( \tilde{H}_n = \tilde{H}_{n+1}(:, [1:(\hat{q} - 1) (\hat{q} + 1):N_T]) \) |
| 11: & \( \Xi_n = \Xi_{n-1} - \lambda_{n, \hat{q}} \) |
| 12: & end |
| **Output:** & \( H_S \) |

**B. TWO-STAGE TAS SELECTION ALGORITHM**

To decrease the complexity of the first incremental TAS selection algorithm further, we propose a two-stage TAS selection algorithm in which the first stage is a pre-selection stage and the second is a post-processing stage. In the pre-selection stage, \( N_P(N_R = N_P < N_S) \) transmit antennas from the total \( N_T \) transmit antennas are selected by employing a simple norm-based algorithm [19], which is able to contribute to the effective reduction of the number of candidate combinations for the subsequent second-stage selection; therefore, it can contribute to the reduction of the overall complexity with marginal performance degradation. The resulting submatrix can be represented as \( H_{n_P} \in C^{N_R \times N_P} \). In the proposed two-stage algorithm, the second-stage selection operates under the condition of \( N_S > N_P \). In the post-processing stage, \( N_S - N_P \) transmit antennas from the \( N_T - N_P \) unselected transmit antennas are incrementally selected. Here, the \( N_S - N_P \) transmit antennas that can offer the best incremental signal-to-noise ratio (SNR) of the ZF-PSM MIMO systems are selected sequentially. The optimization criterion for the second-stage selection is equivalent to (14). Thus the second stage employs the incremental optimization method of (18). We can anticipate that the two-stage TAS selection algorithm depends on the result of the pre-selection method. It should be noted that in the case of \( N_S \leq N_P \), the proposed two-stage algorithm performs only the first-stage processing and, therefore, is equivalent to the norm-based algorithm.

The proposed TAS selection algorithm is described in Table 2. It begins with an \( N_R \times N_T \) full channel matrix \( H \). First, \( N_P \) antennas is selected from \( N_T \) transmit antennas by computing the Frobenius norms, given by \( C_n = \| H(:, n) \|^2 \), where \( n = 1, 2, \ldots, N_T \), of each column vector \( H(:, n) \) of
the full channel matrix $\mathbf{H}$ and then obtaining the antenna indices, $\{u(1), u(2), \cdots, u(N_P)\}$, corresponding to the $N_P$ largest values. After determining the $N_P$ transmit antennas, the resulting submatrix can be formed as $\mathbf{H}_{S} = \mathbf{H}(\cdot, u(1) : u(N_P)) \in \mathbb{C}^{N_S \times N_P}$. After completing a pre-selection, a post-selection based on the optimization, as shown in (18), is conducted for the unselected columns of $\mathbf{H}$ to find the $N_S - N_P$ antennas among the remaining transmit antennas. In this case, $\mathbf{X}_n = (\mathbf{H} \mathbf{H}_n^H)^{-1}$ is calculated with $\mathbf{X}_{n-1}$ and $\mathbf{H}_{n-1}(\cdot, q)$ by the matrix inversion lemma and then $\mathbf{X}_n$ is updated using $\mathbf{X}_{n-1}$ and $\mathbf{H}_{n-1}(\cdot, q)$, where $\hat{q}$ is the selected column vector in the channel submatrix $\mathbf{H}_{n-1}$, until the $N_S - N_P$ transmit antennas are selected. Finally, a selected channel matrix is given as $\mathbf{H}_S = \mathbf{H}_S \in \mathbb{C}^{N_S \times N_S}$ by adding the channel column vectors corresponding to the transmit antennas that were selected during the $N_S - N_P$ incremental steps to $\mathbf{H}_S$ by columns, which is pre-determined in the pre-selection stage.

### IV. COMPUTATIONAL COMPLEXITY ANALYSIS

Considering the number of real multiplications (RMs) and the number of real summations (RSs) [24], the computational complexity of various TAS selection algorithms such as the two proposed schemes, the incremental method [19], and the two-stage algorithm of [21], are analytically evaluated and compared. Here a complex multiplication requires 4 RMs and 2 RSs whereas a complex summation uses 2 RSs. From (8), the computational complexities of the exhaustive search-based TAS selection algorithm in terms of the RMs and RSs, respectively, can be evaluated as follows

$$N_{\text{RM}}^{\text{exhaustive}} = C(N_T, N_S) \left( 2N_SN_R^2 + 2N_SN_R + 2N_R^2 + 6N_R^2 \right)$$

$$N_{\text{RS}}^{\text{exhaustive}} = C(N_T, N_S) \left( 2N_SN_R + 2N_SN_R - N_R + 2N_R^2 + 2N_R^2 \right) \tag{19}$$

### A. COMPLEXITY OF PROPOSED TAS SELECTION ALGORITHMS

From Table 1, the computational complexities of the proposed incremental TAS selection algorithm in terms of the RMs and RSs, respectively, can be analyzed line by line as follows: Line 5:

- RM in $\mu = \mathbf{H}_n^H \Rightarrow 4N_R$
- RS in $\mu = \mathbf{H}_n^H \Rightarrow 4N_R + 2N_R$
- RM in $\mu \mu^H \Rightarrow 2N_R^2 + 2N_R$
- RS in $\mu \mu^H \Rightarrow N_R^2 + N_R$
- RS in $\mathbf{H}_n^H(\cdot, q) \mu \Rightarrow 4N_R - 2$

Thus the total complexities of the proposed incremental TAS selection algorithm in terms of the RMs and RSs, respectively, are given as

$$N_{\text{RM}}^{\text{proposed incremental}} = \sum_{n=1}^{N_S} (N_T + 1 - n) \left( 6N_R^2 + 6N_R + 1 \right) \tag{21}$$

$$N_{\text{RS}}^{\text{proposed incremental}} = \sum_{n=1}^{N_S} (N_T + 1 - n) \left( 5N_R^2 + 5N_R - 1 \right) \tag{22}$$

From Table 2, the computational complexities of the proposed two-stage TAS selection algorithm in terms of the RMs and RSs, respectively, can be summarized as follows: Line 2:

- RM in $\| \mathbf{H}(\cdot, n) \|^2 \Rightarrow 4N_R$
- RS in $\| \mathbf{H}(\cdot, n) \|^2 \Rightarrow 4N_R - 2$

Line 10:

- RM in $\mathbf{H}_S \mathbf{H}_S^H \Rightarrow 2N_PN_R^2 + 2N_PN_R$
- RS in $\mathbf{H}_S \mathbf{H}_S^H \Rightarrow 2N_PN_R^2 + 2N_PN_R - N_R^2 - N_R$
- RM in $(\cdot)^{-1} \Rightarrow 2N_R^2 + 6N_R^2$
- RS in $(\cdot)^{-1} \Rightarrow 2N_R^2 + 2N_R^2$

### TABLE 2. Proposed two-stage TAS selection algorithm.

| Procedure |
|-----------------|
| Inputs: $\mathbf{H}, N_T, N_P, N_S$ |
| 1. Define initially selected and unselected TASs by $S_0 = \{\text{empty set}\}$ and $R_0 = \{1, 2, \cdots, N_T\}$ |
| 2. $C_n = \|\mathbf{H}(\cdot, n)\|^2$, $n = 1, 2, \cdots, N_T$ |
| 3. $[V, u] = \text{sort}\{C_1, C_2, \cdots, C_N\}$ in descending order |
| 4. $S_{N_P} \leftarrow S_0 \cup \{u(1), u(2), \cdots, u(N_P)\}$, $R_{N_P} \leftarrow R_0 - \{u(1), u(2), \cdots, u(N_P)\}$ |
| 5. $\mathbf{H}_S = \mathbf{H}(\cdot, S_{N_P})$ |
| 6. if $N_S \leq N_P$ |
| 7. $\mathbf{H}_S = \mathbf{H}(\cdot, S_{N_S})$ |
| else |
| 9. $\mathbf{H}_{N_P} = \mathbf{H}(\cdot, R_{N_P})$ |
| 10. compute $\mathbf{X}_{N_P} = (\mathbf{H}_S \mathbf{H}_S^H)^{-1}$ |
| 11. for $n = (N_P + 1), (N_P + 2), \cdots, N_S$ |
| 12. for $q = 1, 2, \cdots, (N_T - n + 1)$ |
| 13. $\lambda_{n+q} = \mathbf{X}_{n+q} \mathbf{H}_{n+q}(\cdot, q) \mathbf{H}_{n+q}(\cdot, q) \mathbf{X}_{n+q}$ |
| 14. $\lambda_{n+q} \leftarrow \text{Tr}[\mathbf{X}_{n+q} - \lambda_{n+q}]$ |
| 15. end |
| 16. $\hat{q} = \text{arg \; min}_q \lambda_{n+q}$ |
| 17. $\mathbf{H}_S(\cdot, n) = \mathbf{H}_{n+\hat{q}}(\cdot, q)$ |
| 18. $\mathbf{H}_S = \mathbf{H}_{n+\hat{q}}(\cdot, [1: \hat{q} - 1]) (\hat{q} + 1)$ |
| 19. $\mathbf{X}_n = \mathbf{X}_{n-1} - \lambda_{n+q}$ |
| 20. end |
| 21. end |

| Output: $\mathbf{H}_S$ |
Thus the total complexities of the proposed two-stage TAS selection algorithm in terms of the RM and RS, respectively, are given by

\[
N^{\text{RM}}_{\text{proposed two-stage}} = 4N_TR + 2NP_R^2 + 2NP + 2N_R^2 + 6N_R + 1
\]
\[+ 6N_R^2 + \sum_{n=1}^{N_S - NP} (N_T - N_P + 1 - n) \left( 6N_R^2 + 6N_R + 1 \right) \quad (23)
\]

\[
N^{\text{RS}}_{\text{proposed two-stage}} = 4N_TR - 2N_T + 2NP_R^2 + 2NP + 2N_R^2 + 2N + 2N =
\[+ N_R^2 - N_R + \sum_{n=1}^{N_P - N_T} (N_T - N_P + 1 - n) \left( 5N_R^2 + 5N_R - 1 \right) \quad (24)
\]

where the complexities of lines 13 and 14 in Table 2 are identical to those in lines 5 and 6 in Table 1.

B. COMPLEXITY OF DECREMENTAL TAS SELECTION ALGORITHM

The decremental TAS selection procedure in [19], in which a criterion for minimizing the received SNR loss is presented, is based on the successive elimination of transmit antennas until \(N_S\) columns of matrix remain [19]. It begins with the entire set of \(N_T\) transmit antennas and successively removes \((N_T - N_S)\) transmit antennas. At each iteration, one antenna with the largest received SNR loss is deleted. Its complexity is given by [20]

\[
N^{\text{RM}}_{\text{decremental}} = 2N_TN_R^2 + 2N_TN_R + 4N_R^3 + 8N_R^2
\]
\[+ \sum_{n=1}^{N_T - N_S} (N_T - n + 1) \left( 8N_R^2 + 8N_R \right) \quad (25)
\]

\[
N^{\text{RS}}_{\text{decremental}} = 2N_TN_R^2 + 2N_TN_R + 4N_R^3 + 2N^2 - 2N_R
\]
\[+ \sum_{n=1}^{N_T - N_S} \left( (N_T - n + 1) \left( 8N_R^2 + 4N_R - 4 \right) + 2N^2 \right) \quad (26)
\]

C. COMPLEXITY OF PREVIOUS TWO-STAGE TAS SELECTION ALGORITHM

In [21], the rectangular maximum-volume (RMV) theory is introduced as an effective method to reduce the number of RF chains in massive MIMO systems. The TAS selection scheme based on the RMV method is given in [21], in which various previous algorithms could be employed to obtain a square maximum-volume submatrix in the preselection stage. To achieve this, this study considers a simple norm-based approach as in the proposed two-stage algorithm shown in Table 2 and an incremental strategy, which is identical to the proposed incremental algorithm shown in Table 1. Thus the first one is termed a norm-RMV TAS selection algorithm, and the second is called an incremental-RMV selection method. Taking the same complexity analysis as in Subsection IIIA, the complexities of the norm-RMV-based and incremental-RMV-based TAS selection algorithms in terms of the RM and RS, respectively, are evaluated as

\[
N^{\text{RM}}_{\text{norm-RMV}} = 4N_TR + 4N_R^3 + 8N_R^2
\]
\[+ \sum_{n=1}^{N_T - N_S} (N_T - n + 1) \left( 6N_R^2 + 6N_R + 1 \right) \quad (27)
\]

\[
N^{\text{RS}}_{\text{norm-RMV}} = 4N_TR + 4N_R^3 + 3N_R^3 - 2N_T - N_R
\]
\[+ \sum_{n=1}^{N_T - N_S} ((N_T - n + 1) \left( 6N_R^2 + 6N_R + 1 \right)
\]

\[
N^{\text{RM}}_{\text{incremental-RMV}} = \sum_{n=1}^{N_T} (N_T + 1 - n) \left( 6N_R^2 + 6N_R + 1 \right)
\]
\[+ \sum_{n=1}^{N_T - N_S} ((N_T + 1 - n) \left( 6N_R^2 + 6N_R + 1 \right)
\]

\[
N^{\text{RS}}_{\text{incremental-RMV}} = \sum_{n=1}^{N_T} (N_T + 1 - n) \left( 5N_R^2 + 5N_R - 1 \right)
\]
\[+ \sum_{n=1}^{N_T - N_S} ((N_T + 1 - n) \left( 5N_R^2 + 5N_R - 1 \right)
\]

V. SIMULATION RESULTS

In this section, several TAS selection algorithms for the ZF-based PSM system, which has \(N_T\) transmit antennas and \(N_R\) receive antennas are evaluated through Monte Carlo simulations over Raleigh flat-fading channels. The SNR is defined by the symbol energy to noise power spectral density ratio, i.e., \(\eta = 1/\sigma^2\). In the plots, \((N_T, N_S, N_R)\) represents that \(N_T\) transmit antennas, \(N_S\) selected transmit antennas, and \(N_R\) receive antennas are employed as the system parameters, whereas \((N_T, N_R)\) with two parameters represents no TAS selection (named no-TAS). We assume that the CSI is completely known at the transmitter of the ZF-PSM system unless otherwise mentioned. The quadrature PSK modulation is assumed and the receiver is based on ML detection. In the simulations, we compare the BER and achievable rate performance of the ZF-based PSM system using the following TAS selection algorithms.

- proposed incremental TAS selection
- proposed two-stage TAS selection for various \(N_P\) values where \(N_P = N_R + K\) and \(K (\geq 0)\) is an integer
the performance of the proposed incremental TAS selection algorithm is close to that of the decremental algorithm and better than that of the incremental-RMV algorithm. It can be observed that the BER performance of the proposed incremental TAS selection algorithm is almost similar to that of the optimal selection. Note that the simulation result of the optimal TAS selection is almost similar to that of the optimal-RMV algorithm. In this scenario, system parameters such as \((N_T, N_S, N_R) = (16, 8, 4)\) and \((N_T, N_R) = (N_S, N_R) = (8, 4)\) are employed. We demonstrate that the performance of the proposed incremental TAS selection algorithm is close to that of the optimal TAS selection algorithm under two SNR values, namely 2 dB and 4 dB, respectively, when \(N_T = 32\) and \(N_R = 4\) are given in ZF-PSM system without TAS selection or with \(K = 0\), which corresponds to \(N_P = N_R\), and the decremental algorithm is relatively small. As \(K\) increases to two, the proposed two-stage TAS selection algorithm displays slightly degraded performance, but outperforms the norm-RMV algorithm and the norm-based algorithm. It is further observed from [19] that the transmit diversity order of ZF-PSM with optimal TAS selection is achieved as \(G = (N_T - N_R + 1)\). On the other hand, the \((N_S, N_R)\) ZF-PSM system without TAS selection has no selection diversity, but if \(N_S > N_R\), the transmit diversity is always available and then the diversity gain is given by \(G = (N_S - N_R + 1)\), which is smaller than that of optimal TAS selection if \(N_T > N_S\). Thus the ZF-PSM scheme using the proposed TAS selection algorithms can achieve the diversity order of 13 for the \((16, 8, 4)\) system at high SNR ranges. It is obvious from Fig. 1 that TAS selection achieves better diversity gain than the no-TAS case with \(G = 5\). It can be noticed that if an optimal search is used instead of the norm-based algorithm in the first stage of the proposed two-stage algorithm while maintaining the same algorithm in the second stage, the improvement of the BER performance could be minor, but its complexity becomes much higher, even if the simulation results are not included here.

Meanwhile, Fig. 2 shows that the computational complexity of the proposed incremental algorithms is lower than that of the decremental algorithm for the \((16, 8, 4)\) system. Particularly, the computational complexities of the proposed incremental TAS selection algorithm in terms of summation of the RM and RS are equal to approximately 70.7% and 64.8%, respectively, of those of the decremental algorithm for the \((16, 8, 4)\) systems. In addition, by increasing a new design parameter value of \(N_P(= N_R + K)\) in the proposed two-stage TAS selection algorithm, the complexity decreases significantly as shown in Fig. 2, whereas the BER performance degrades slightly, as shown in Fig. 1. Particularly, the proposed two-stage TAS selection algorithm with \(K = 2\) can achieve approximately 17.6% and 16.1% of the complexities of the decremental algorithm in terms of the summation of RM and RS, respectively. It should be noted that the proposed two-stage algorithm with \(K = 2\) for \((16, 8, 4)\) systems outperforms the norm-RMV TAS selection algorithm with slightly higher complexity. It should be also noted that RM and RS of the optimal selection algorithm are specified as 7,001,280 and 5,920,200, respectively, which are not plotted in Fig. 2 because of significant gaps from those of the other selection algorithms.

In Figs. 3 and 4, the BER results of various TAS selection algorithms together with no-TAS for \((N_T, N_R) = (N_S, 4)\) are plotted as a function of the number of selected transmit antennas under two SNR values, namely 2 dB and 4 dB, respectively, when \(N_T = 32\) and \(N_R = 4\) are given in ZF-PSM system. It is obvious that all the TAS selection algorithms outperform no-TAS owing to selection diversity. As the number of selected transmit antennas increases, their performance improves. It is shown that the proposed incremental algorithm can achieve a BER performance close to that of the decremental algorithm and much better than those of the incremental-RMV and norm-RMV algorithm. Furthermore, the performance of the proposed two-stage algorithm approaches that of the proposed incremental algorithm as
as that of the decremental algorithm as the number of selected transmit antennas increases. However, increasing $K$ degrades the performance of the proposed two-stage algorithm for a small number of selected transmit antennas. This occurs because the pre-selection algorithm in the proposed two-stage algorithm with larger $N_P$ dominates the overall performance for a small number of selected transmit antennas. It should be noted that when $N_S \leq N_P(= N_S + K)$, the proposed two-stage algorithm works as the norm-based scheme. It is observed that if the number of selected transmit antennas satisfies $N_S > N_P + 2$ under the simulation scenario given as $N_T = 32$ and $N_R = 4$, the proposed two-stage algorithm attains a performance similar to that of the proposed incremental algorithm. Under the above-mentioned simulation condition, the incremental algorithm employed for the second post-selection stage in the proposed two-stage algorithm appears to affect the good BER result considerably. Hence, we believe that the proposed two-stage algorithm is suitable when the number of selected transmit antennas is greater than $N_R + K + 2$, where $N_R \ll N_T$.

Figs. 5 and 6 present the achievable rate performance of the above-mentioned TAS selection algorithms as a function of the number of total available transmit antennas when $N_S = 8$ and $N_R = 4$ are fixed in the ZF-PSM system under two SNR values, namely 4 dB and 8 dB, respectively. Fig. 7 shows the achievable rate under the simulation setup with $N_S = 10$ and $N_R = 4$ under SNR = 8 dB. Here the achievable rate [25] is given as

$$R_S = \log_2 \left( 1 + \frac{\beta^2}{2\sigma_n^2} \right)$$

(31)

The results indicate that the achievable rate of all the TAS selection algorithms increases as the number of available transmit antennas increases. It is shown that the proposed
incremental TAS selection algorithm exhibits slightly worse performance than the decremental TAS selection scheme and its performance gap is insignificant. It is also observed that the decrease in the achievable rate of the proposed two-stage TAS selection algorithm for a fixed $N_T$ is coupled with the increase in the $N_P$ value. Nevertheless, the proposed two-stage TAS selection algorithm with $K = 3$ outperforms the norm-RMV TAS selection algorithm. Particularly, Fig. 7 shows that the proposed two-stage TAS selection algorithm with $K = 3$ outperforms the incremental-RMV for $N_S = 10$, which is larger than that in Figs. 5 and 6.

Thus far, it has been assumed that the transmitter in the TAS selection and precoding scheme has perfect knowledge of the channel fading coefficients. However, presently the effects of imperfect CSI on the BER performance of various TAS selection algorithms are being investigated. Under the same simulation conditions as shown in Fig. 1 and without any channel estimation error, Figs. 8 and 9 comparatively show the impacts of a channel estimation error of $\sigma^2_e = 0.04$ and $\sigma^2_e = 0.08$, respectively, on the BER performance of various TAS selection algorithms. It is observed that in comparison with the results of Fig. 1, the given channel estimation errors cause degradation of the BER performance in all TAS selection algorithms. Additionally, an error floor begins at high SNR values owing to the channel estimation error. It is evident that the proposed incremental selection algorithm achieves BER results close to the decremental one and better performance than the incremental-RMV algorithm. Moreover, as $\sigma^2_e$ increases, the error floor becomes higher. Fig. 9 shows that the proposed incremental selection algorithm outperforms the incremental-RMV by approximately more than 5 dB prior
to an error floor of the incremental-RMV. The proposed two-stage TAS selection algorithm offers a worse BER performance than the decremental algorithm at high SNR values, whereas its BER results are lower than those of the incremental-RMV algorithm. This means that the incremental-RMV is more affected by channel estimation errors than the proposed selection algorithms. Thus, the proposed selection algorithms are more robust to channel estimation errors than the incremental-RMV. It should be noted that norm and norm-RMV algorithms still perform worse than the proposed selection algorithms even under channel estimation errors.

Fig. 10 compares the complexity of the proposed incremental algorithm and the proposed two-stage scheme of $K = 0$ with that of the optimal and decremental approaches in log scale under a simulation setup such as $N_T = 32$ and $N_R = 4$, which is the same situation as shown in Figs. 3 and 4. It is found that the complexity of the suboptimal algorithms is far less than that of the optimal algorithm. Fig. 11 exhibits the complexity of various TAS selection algorithms as a function of the number of selected transmit antennas under the same simulation setup as that shown in Fig. 10. It can be observed that as the number of selected transmit antennas increases, the complexity of the proposed TAS selection algorithms including the norm-RMV and incremental-RMV becomes larger, whereas the decremental algorithm offers a decreasing complexity. However, the proposed TAS selection algorithms are capable of achieving much lower complexity than the decremental algorithm, especially when the difference between $N_T$ and $N_S$ is sufficiently large. It is also observed that the complexity of the proposed two-stage TAS selection algorithm depends on the new parameter $N_P$. As $N_P$ increases, the complexity decreases. Thus the proposed two-stage TAS selection algorithm with $K = 4$ provides a lower complexity than the incremental-RMV selection algorithm. Especially when $N_S \leq N_P (= N_R + K)$, its complexity is the same as the norm-based case. Figs. 3 and 4 show that the former outperforms the latter in terms of BER for values of $N_S$ greater than approximately nine.

Fig. 12 shows the complexity curves of the proposed TAS selection algorithms in log scale as a function of $N_T$. It becomes obvious that the growth rate of complexity in the optimal algorithm is significantly greater than the speed in the suboptimal algorithms as $N_T$ becomes increasingly larger. Fig. 13 presents the complexity of various TAS selection algorithms as a function of $N_T$ for the same simulation condition as that shown in Fig. 12. It is shown that the rate of increase for the proposed algorithms is significantly smaller than that for the decremental algorithm. The complexity of the proposed two-stage algorithm with $K = 3$ is slightly larger than that of norm-RMV, even for a large $N_T$. 

**FIGURE 11.** Complexity comparison of the proposed TAS selection algorithm for various $K$ values as a function of $N_S$.

**FIGURE 12.** Complexity comparison of the proposed TAS selection algorithms in log scale as a function of $N_T$.

**FIGURE 13.** Complexity comparison of the proposed TAS selection algorithm for various $K$ values as a function of $N_T$. 

$N_R = 4$, $N_S = 8$, $N_T = 32$, and $K = 4$.
VI. CONCLUSION
This article presents an incremental TAS selection scheme to reduce the number of RF chains for ZF-PSM massive MIMO systems effectively. When the number of selected transmit antennas is sufficiently smaller than the number of the total transmit antennas available, its complexity is much less than that of the decremental algorithm, along with the marginal BER performance degradation. In addition, an efficient two-stage TAS selection algorithm consisting of pre-selection and post-selection stages is proposed for massive MIMO systems. In the pre-selection stage, norm operations with simple computations are conducted to select the \(N_P\) transmit antennas effectively. In a follow-up stage, the \(N_S - N_P\) transmit antennas are incrementally selected so that the received SNR of the ZF-PSM systems is maximized. The proposed two-stage TAS selection algorithm with \(N_P = N_g\) achieves significantly better performance, which is close to that of the decremental algorithm, compared to that of the norm-RMV and incremental-RMV TAS selection algorithms. Furthermore, it is able to achieve a tradeoff between performance and complexity through a new parameter \(N_P\). The simulation results show that the maximum transmit antenna diversity gain can be obtained by the proposed TAS selection algorithms. The proposed algorithms are well suited for the diversity gain can be obtained by the proposed TAS selection algorithms. The proposed algorithms are well suited for the case of \(N_S < N_T\). Even under imperfect channel estimation, the proposed incremental and two-stage selection algorithms offer significantly better performance than incremental-RMV and norm-RMV, respectively. Particularly near the error floor of the incremental-RMV and norm-RMV, they demonstrate a significant advantage in SNR values.

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