Underdetermined Blind Source Separation for Sparse Signals Based on the Law of Large Numbers and Minimum Intersection Angle Rule

Pengfei Xu · Yinjie Jia · Zhijian Wang · Mingxin Jiang

Received: 8 October 2018 / Revised: 10 September 2019 / Accepted: 14 September 2019 / Published online: 18 September 2019
© Springer Science+Business Media, LLC, part of Springer Nature 2019

Abstract

Underdetermined blind source separation (UBSS) is an important issue for sparse signals, and a novel two-step approach for UBSS based on the law of large numbers and minimum intersection angle rule (LM method) is presented. In the first step, an estimation of the mixed matrix is obtained by using the law of large numbers, and the number of source signals is displayed graphically. In the second step, a method of estimating the source signals by the minimum intersection angle rule is proposed. The significance of this step is that the minimum intersection rule is better than the shortest path method, and the decomposition components can be found optimally by the former. The simulation results illustrate the effectiveness of the LM method. It has a simple principle, has good transplantation capability and may be widely applied in various fields of digital signal processing.

Keywords Blind source separation · Underdetermined · Sparse signal · Large numbers · Minimum intersection angle
1 Introduction

Blind source separation (BSS) is a research hot spot in the field of signal processing because it aims to separate unknown source signals from observed mixtures through an unknown transmission channel. The BSS technique is widely applied in many fields, such as speech signal processing, image processing, radar signal processing, communication systems and data mining [2, 8, 11]. Underdetermined blind source separation (UBSS) is one case of BSS in which the number of observed signals is less than the number of source signals. Currently, many researchers have proposed several improved algorithms related to this issue. Van Vaerenbergh [12] aimed to invert different nonlinearities, thus reducing the problem to linear UBSS. To this end, a spectral clustering technique is first applied that clusters the mixture samples into different sets corresponding to the different sources. Kim [5] proposed a novel algorithm based on a single-source detection algorithm that detects time–frequency regions of single-source occupancy. Xie [13] presented a new time–frequency (TF) UBSS approach based on the Wigner–Ville distribution (WVD) and Khatri–Rao product to separate N nonstationary sources from M (M < N) mixtures. Liu [7] exploited a source temporal structure and proposed a linear source recovery solution for the UBSS problem that does not require the source signals to be sparse. Zhang [15] developed an effective mechanism based on compressed sensing (CS) from an experimental viewpoint to prove that the accuracy of CS when retrieving sources is guaranteed. Zhen [14] used an effective approach to discover some 1-D subspaces from a set consisting of all time–frequency (TF) representation vectors of observed mixture signals. Based on a sparse reconstruction model, a single-layer perceptron artificial neural network was introduced into the proposed algorithm, and the optimal learning factor was calculated, which improved the precision of recovery [3].

The lack of prior knowledge and the irreversibility of the system in an undetermined case have brought great difficulties to research. However, the undetermined case is transformed into a determined case under certain circumstances if the source has some sparsity, the number of sources not exceeding the number of observed signals is not zero at a certain time or in a certain period of time, and the others are all zero or very small, and then the problem becomes relatively simple. For this reason, sparse signal processing has become a means to study underdetermined separation.

In this paper, we researched and analyzed UBSS for sparse signals based on the law of large numbers and the minimum intersection angle rule. The paper is organized as follows. In Sect. 2, we introduce the UBSS model. A new two-step approach for UBSS is introduced and deduced in Sect. 3. In Sect. 4, the simulation experiments that indicate the effectiveness of this method are presented. The final section is a summary of the content of this paper and provides some questions that need further study.

2 UBSS Model

Linear, noiseless and memoryless models of blind source separation can be described by the equation $x(t) = A \ast s(t)$. 
\[
\begin{pmatrix}
    x_1(t) \\
    x_2(t) \\
    \vdots \\
    x_m(t)
\end{pmatrix}
= \begin{pmatrix}
    a_{1,1} \\
    a_{2,1} \\
    \vdots \\
    a_{m,1}
\end{pmatrix}
\begin{pmatrix}
    s_1(t) \\
    s_2(t) \\
    \vdots \\
    s_n(t)
\end{pmatrix}
+ \begin{pmatrix}
    a_{1,2} \\
    a_{2,2} \\
    \vdots \\
    a_{m,2}
\end{pmatrix}
\begin{pmatrix}
    s_1(t) \\
    s_2(t) \\
    \vdots \\
    s_n(t)
\end{pmatrix}
+ \cdots + \begin{pmatrix}
    a_{1,n} \\
    a_{2,n} \\
    \vdots \\
    a_{m,n}
\end{pmatrix}
\begin{pmatrix}
    s_1(t) \\
    s_2(t) \\
    \vdots \\
    s_n(t)
\end{pmatrix} 
\]  

(1)

In time \( t \), \( s(t) = [s_1(t), s_2(t), \ldots, s_n(t)]^T \) represents the \( n \)-dimensional source signal vector and \( x(t) = [x_1(t), x_2(t), \ldots, x_m(t)]^T \) represents the \( m \)-dimensional mixed signal vector. The mixed matrix \( A = (a_1, a_2, \ldots, a_n) \), \( a_i = [a_{1,i}, a_{2,i}, \ldots, a_{m,i}]^T \) is the \( i \)th column vector of \( A \). When \( M > N \), the number of mixed signals is greater than that of source signals, which is called overdetermined blind source separation. When \( M = N \), the number of mixed signals is equal to the number of source signals, which is called well-posed blind source separation. When \( M < N \), the number of mixed signals is less than that of source signals, which is called UBSS. UBSS is a more realistic and challenging problem. When the problem becomes underdetermined, the mixed system is no longer invertible, so the estimation of the source signal cannot be obtained simply by inverting the mixed matrix. Equation (1) can be considered an underdetermined mixture, as shown in Fig. 1.

The UBSS for sparse signals is discussed in this paper. If a signal is sparse, the value at most times is zero or close to zero. A signal can be considered sparse in a certain representation domain if the number of nonzero values in that domain is much smaller than the total signal length. A common situation in the case of real-world signals is that the number of significant coefficients is small compared to the number of other components [10]. The sparsity of signals brings great convenience to signal processing.

Currently, the two-step method [6] is a common method used to solve the blind separation of sparse signals; that is, the mixed matrix \( A \) is first estimated, and then the source signal is estimated by the reconstruction algorithm. In this paper, a new two-step UBSS method based on the law of large numbers and the minimum intersection

Fig. 1 Underdetermined mixture
angle rule (LM method) is proposed. In the first step of estimating the matrix, cluster calculations are performed on a uniform sample of mixed data points by using the law of large numbers; thus, one engineering model for estimating the mixed matrix has been established that can directly estimate the number of source signals. In the second step, the source signals are estimated by way of the minimum intersection angle rule for better separation performance.

### 3 LM Method

In general, the only data that we can analyze are the received mixed data. When a source signal is assumed to be sparse, we can separate the source signal by sparsity. The following is an example.

Suppose that there are three sparse source signals $s_1$, $s_2$ and $s_3$ that are combined into two mixed signals $x_1$ and $x_2$ according to Eq. (2):

$$
\begin{pmatrix}
  x_1 \\
  x_2
\end{pmatrix} =
\begin{pmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23}
\end{pmatrix}
\begin{pmatrix}
  s_1 \\
  s_2 \\
  s_3
\end{pmatrix}
$$

Because the source signals $s_1$, $s_2$ and $s_3$ are sparse, the following situation must occur in $x_1$ and $x_2$.

- In time $t = t_1$, only $s_1$ has value. At this time, $x_1^t = a_{11}s_1^t$, $x_2^t = a_{21}s_2^t$ and then $x_2^t/x_1^t = a_{21}/a_{11} = a_1$.
- In time $t = t_2$, only $s_2$ has value. At this time, $x_1^t = a_{12}s_1^t$, $x_2^t = a_{22}s_2^t$ and then $x_2^t/x_1^t = a_{22}/a_{12} = a_2$.
- In time $t = t_3$, only $s_3$ has value. At this time, $x_1^t = a_{13}s_1^t$, $x_2^t = a_{23}s_3^t$ and then $x_2^t/x_1^t = a_{23}/a_{13} = a_3$.

#### 3.1 Law of Large Numbers

As long as a signal is sparse, the values of $a_1$, $a_2$ and $a_3$ exist. When the signal sparsity is relatively large, $a_1$, $a_2$ and $a_3$ are the three largest frequencies of occurrence by using statistical analysis. According to Bernoulli’s law of large numbers, frequency is close to probability in a certain sense. Therefore, these values constitute an estimation matrix $A' = \begin{pmatrix} 1 & 1 & 1 \\ a_1 & a_2 & a_3 \end{pmatrix}$ that can replace the mixed matrix. These data $a_1$, $a_2$ and $a_3$ can be represented by scatter plots, bar graphs and so on. Here, they are represented by a bar graph, as shown in Fig. 5.

First, we use a bar graph to estimate the mixed matrix $A$ and the number of source signals. The number of sparse source signals and the parameters of the estimation matrix can be observed from the bar graph, which provides a good basis for the subsequent linear programming.

The number of bars in a bar graph reflects the number of sparse source signals, and the corresponding values are $a_1$, $a_2$ and $a_3$. When the number of sources is changed,
the number of bars varies accordingly. To a certain extent, a bar graph is more intuitive and precise for describing the sparsity of mixed data than a scatter plot.

Here, we use three cases to illustrate and then perform experiments on these three situations.

If there is only one source signal at any time in $x_1$ and $x_2$, the estimation matrix of Eq. (2) is a column in $A'$ at a certain time. The source signals can be restored normally.

If there are two source signals at any time in $x_1$ and $x_2$, the estimation matrix of Eq. (2) is two columns in $A'$ at a certain time. The source signals can again be restored normally.

If there are three source signals at any time in $x_1$ and $x_2$, the estimation matrix of Eq. (2) is $A'$ at a certain time. At these moments, the equation is underdetermined, and there are infinitely many solutions to the underdetermined equation. The source signals are not well recovered under normal circumstances.

### 3.2 Minimum Intersection Angle Rule

The minimum intersection angle rule is used to extract a source signal. Each mixed signal $x$ is the sum of all vectors $a_i s_i$ and can be described by Eq. (3):

$$x = \sum_{i=1}^{N} a_i s_i$$  \hspace{1cm} (3)

At a certain time $t$, the angle of $x_t$ is $\theta_t = \arctan(x_t^1/x_t^2)$. The three angles corresponding to $a_1, a_2$ and $a_3$ are $\theta_1, \theta_2$ and $\theta_3$. A sketch map of the minimum intersection angle is shown in Fig. 2.

The intersection angles between these angles and $\theta_t$ are $\theta_{t1}, \theta_{t2}$ and $\theta_{t3}$, respectively, as follows: $\theta_{t1} = |\theta_t - \theta_1|$, $\theta_{t2} = |\theta_t - \theta_2|$ and $\theta_{t3} = |\theta_t - \theta_3|$.

**Fig. 2** Sketch map of the minimum intersection angle
The two angles closest to $\theta_t$ are selected as the base vectors of time $t$. For example, at time $t$, if inequality (4) is satisfied, the vectors with the minimum intersection angles to vector $\theta_t$ are $\theta_1$ and $\theta_2$.

$$\theta_{12} \leq \theta_{13} \leq \theta_{11}$$

Accordingly, $a_1$ and $a_2$ are the base vectors of time $t$, which can be described by Eq. (5):

$$x_t = a_1 s_1 + a_2 s_2$$

Therefore, we decompose the mixed value $x_t$ of time $t$ into a sparse combination of only two source signals and assume that the third source signal has no value at that time. In this way, the optimal solution of Eq. (2) is obtained, which can be described by Eq. (6):

$$A_m^{-1} x = \left( \begin{array}{c} \mathbf{s}_1(t) \\ \mathbf{s}_2(t) \\ s_3(t) = 0 \end{array} \right) = \left( \begin{array}{cc} 1 & 1 \\ a_1 & a_2 \end{array} \right)^{-1} \left( \begin{array}{c} x_1(t) \\ x_2(t) \end{array} \right)$$

The sampling point of mixed signals is set to $T, t = 1, \ldots, T$. The mixed signals are decomposed into a combination of several sparse source signals when time $t$ is calculated one by one. The source signals at each time $t$ are obtained by solving Eq. (6), and then, the source signals of the whole observation period are obtained.

In the process of computation, at each time $t$, $A_m$ is a square matrix composed of any two columns of the estimation matrix $A' = \left( \begin{array}{ccc} 1 & 1 & 1 \\ a_1 & a_2 & a_3 \end{array} \right)$. In Eq. (6), the number of inverse square matrix operations is the same as the number of sampling points of mixed signals. This operation is needed for all $t$ and facilitates the decomposition of mixed signals into linear combinations of sparse sources.

Notably, the difference between this algorithm and the shortest path algorithm [1] is that the latter selects the two vectors closest to it on both sides of $\theta_t$ as the base vectors. ($x_t$ decomposes as $a_2$ and $a_3$.) Therefore, this example shows that the decomposition signal obtained by the shortest path method may not be optimal. The minimum intersection angle rule avoids this point.

4 Application and Simulation

There are few sparse signals in reality, but ordinary signals can be transformed into sparse signals by various transformations (such as a Fourier transform or wavelet transform). For the convenience of illustration, we experimented with a typical time-domain sparse signal (UWB signal). Ultrawideband (UWB) is generally based on narrow pulses with extremely low duty cycles that are sparsely distributed on the time axis. A source signal is generated by a time-hopping ultrawideband (TH-UWB) system. The number of source signals is set to 3. The pulses used are a Gaussian pulse,
first-order Gaussian pulse and second-order Gaussian pulse. The pulse width is \( T_c = 161 \), and the frame length is \( T_f = 4T_c \). The number of mixed signals is set to 2, and the data length is set to 2898. The exact bit is set to four bits after the decimal point.

The mixed matrix \( A \) is generated randomly. By evaluating the performance of blind source separation, a correlation coefficient \( C \) is introduced as a performance index as follows [9]:

\[
C(x, y) = \frac{\text{cov}(x, y)}{\sqrt{\text{cov}(x, x)\text{cov}(y, y)}}
\]

\( C(x, y) = 0 \) means that \( x \) and \( y \) are uncorrelated, the signal correlation increases as \( C(x, y) \) approaches unity, and the signals become fully correlated when \( C(x, y) \) becomes unity.

In the first experiment, we verify the results of blind separation under the same conditions according to the shortest path and the minimum intersection angle. We adopt the ideal sparse source signal for the convenience of verification; there is only one source signal at any time in mixed signals \( x_1 \) and \( x_2 \). The mixed matrix is \( A = \begin{pmatrix} 0.5000 & 0.7000 & 0.3000 \\ 0.8000 & 0.2000 & 0.6000 \end{pmatrix} \). The time-domain waveforms of the three source signals are shown in Fig. 3. The time-domain waveforms of the two mixed signals are shown in Fig. 4.

The first step is to determine the estimation matrix; the \( a_1 \), \( a_2 \) and \( a_3 \) values in the estimation matrix \( A' \) are 2.0000, 0.1667 and 1.1667, respectively. The corresponding bar graph is shown in Fig. 5.

---

**Fig. 3** Waveforms of the three source signals
Fig. 4 Waveforms of the two mixed signals

Fig. 5 Bar graph of the number of sources and the estimation matrix
The number of source signals and the values of the estimation matrix can be intuitively obtained from the diagram.

The second step is to separate the source signals from the mixed signals. The separated signals obtained from the shortest path method are shown in Fig. 6.

As shown in Figs. 3 and 6, the proposed algorithm can effectively estimate (separate) the source signals from the mixed signals. The correlation coefficients $C$ between them are 0.9997, 0.9997 and 0.9997, respectively, showing that the separation effect is good.

The separated signals obtained from the shortest path method are shown in Fig. 7. The corresponding correlation coefficients $C$ are 0.9868, 0.8559 and 0.7600, respectively, showing that the separation performance calculated by the shortest path method is not good enough here.

In the second experiment, we choose another UBSS algorithm for sparse signals based on a subspace method for comparison with the proposed method. This algorithm demonstrates that a single source is detected for the TF points when both the real and the imaginary parts of the STFT coefficients of the mixtures have sufficient energies [5]. The experiment is conducted with the same conditions as the first experiment (same source signals and mixed matrix) to make the comparison as fair as possible. Therefore, the time-domain waveforms of the three source signals and the two mixed signals are also shown in Figs. 3 and 4, respectively. Because the separation effect of the selected algorithm depends to some extent on the setting of the DFT points, we display the correlation coefficients (separation performance) between the separated signals and the source signals at different DFT points in Table 1.

In Table 1, $cc_{11}$ represents the correlation coefficient between the first source signal and the first separated signal, $cc_{22}$ represents the correlation coefficient between the second source signal and the second separated signal, and $cc_{33}$ represents the correla-
Fig. 7 Waveforms of the three separated signals obtained from the shortest path method

Table 1 Separation performance calculated by the subspace method

| Correlation coefficients | DFT points |
|--------------------------|------------|
|                          | 8          | 16         | 32 | 64 | 128 | 256 | 512 |
| cc11                     | 0.9972     | 0.9987     | 0.9991 | **0.9992** | 0.9989 | 0.7849 | 0.6470 |
| cc22                     | 0.9988     | 0.9993     | 0.9995 | **0.9996** | 0.9996 | 0.9854 | 0.9848 |
| cc33                     | 0.9787     | 0.9926     | 0.9969 | **0.9980** | 0.9974 | 0.6015 | 0.6222 |

Table 2 Separation performance of the three algorithms

| Correlation coefficients | Shortest path method [1] | Subspace method (DFT points = 64) [5] | Proposed method in this paper |
|--------------------------|--------------------------|----------------------------------------|-------------------------------|
| cc11                     | 0.9868                   | 0.9992                                 | **0.9997**                   |
| cc22                     | 0.8559                   | 0.9996                                 | **0.9997**                   |
| cc33                     | 0.7600                   | 0.9980                                 | **0.9997**                   |

The correlation coefficient between the third source signal and the third separated signal. As given in Table 1, with an increasing number of DFT points, each correlation coefficient first increases and then decreases. The three correlation coefficients (in bold text) are the highest when the number of DFT points reaches 64, and we provide the best separation results in Table 2 for further comparison.

For convenience, we summarize the results of the above two experiments in Table 2. As given in Table 2, under the same experimental conditions, the correlation coefficient (separation performance) of the proposed method (in bold text) is higher than that of
the other two methods (shortest path method and subspace method). Table 2 shows the superiority of the proposed method in this paper.

Therefore, the next two experiments use the proposed method to achieve blind source separation.

In the third experiment, there are two source signals \([4]\) at any time in mixed signals \(x_1\) and \(x_2\). The mixed matrix is 
\[
A = \begin{pmatrix}
0.4000 & 0.6000 & 0.3000 \\
0.8000 & 0.1000 & 0.5000
\end{pmatrix}
\]. The time-domain waveforms of the three source signals are shown in Fig. 8. The dotted line is used to mark two signals at a certain time. The time-domain waveforms of the two mixed signals are shown in Fig. 9.

The first step is to determine the estimation matrix; \(a_1, a_2\) and \(a_3\) in the estimation matrix \(A'\) are 0.1667, 1.1667 and 2.0000, respectively. The corresponding bar graph is similar to Fig. 5 and is not repeated here.

The second step is to separate the source signals from the mixed signals. The time-domain waveforms of the three separated signals are shown in Fig. 10.

As shown in Figs. 8 and 10, the proposed algorithm can effectively estimate (separate) the source signals from the mixed signals. The correlation coefficients \(C\) between them are 0.9996, 0.9946 and 0.9965, respectively, showing that the separation effect is good.

In the fourth experiment, there are three source signals at any time in mixed signals \(x_1\) and \(x_2\), as shown by the dotted line boxes. The mixed matrix is 
\[
A = \begin{pmatrix}
0.5000 & 0.4000 & 0.3000 \\
0.9000 & 0.2000 & 0.6000
\end{pmatrix}
\]. The time-domain waveforms of the three source signals are shown in Fig. 11. The time-domain waveforms of the two mixed signals are shown in Fig. 12.
The first step is to determine the estimation matrix; $a_1$, $a_2$ and $a_3$ in the estimation matrix $A'$ are 0.5000, 2.0000 and 1.8000, respectively. The corresponding bar graph is similar to Fig. 5 and is not repeated here.
The second step is to separate the source signals from the mixed signals. The time-domain waveforms of the three separated signals are shown in Fig. 13.
As shown intuitively in Figs. 11 and 13, the source waveforms can be separated normally except in three overlapping regions. Although we estimate the mixed matrix, the corresponding waveforms cannot be separated at these times.

Two schemes are proposed to solve this problem: (1) Using a signal reconstruction technique, the known sequence length is used to “truncate” the sequence in the time domain, and the signal is reconstructed from the undistorted part of the data. Signal reconstruction technology is widely used in voice, communication, control and so on. (2) Each separated signal is regarded as a signal after passing through a certain channel. In both wired and wireless channels, there will be interference or fast fading, which makes the signal at the receiver have an error code or distortion. Thus, it can be corrected or deinterleaved to recover this part of the error code or distortion. Then, the whole signal is restored.

In the last part of this section, we analyze the influence of noise on the algorithm for the stability of the proposed method. Considering noise, Eq. (1) can be rewritten as \( x(t) = A \ast (s(t) + v(t)) \), where \( v(t) \) is additive white Gaussian noise; this is the most basic noise and interference model, its amplitude distribution obeys the Gaussian distribution, and the power spectral density is uniformly distributed. The experiment is conducted with the same source signals and mixed matrix as the first experiment for the sake of convenience. The three source signals pass through the noise channel (the signal-to-noise ratio is assumed to increase from 10 dB to 90 dB) and arrive at the receiving end to form the two mixed signals. Because the noise is random and the influence on the source signal and the mixed signal is uncertain, the separation results of each operation of the algorithm will be different. Therefore, in the case of a fixed SNR, the program runs repeatedly (set to 100 times here) to check its
stability. For statistical convenience, we define the average correlation coefficient as
\[ \text{CC}_{\text{ave}} = \frac{(cc_{11} + cc_{22} + cc_{33})}{3} \]
and count the number of occurrences under the condition \( \text{CC}_{\text{ave}} \geq 0.9 \) (fine separation effect). Figure 14 shows the influence of different signal-to-noise ratios on the stability of the proposed algorithm obtained by this method.

In Fig. 14, the abscissa represents the signal-to-noise ratio (SNR) of the source signal passing through the AWGN channel (from 10 to 90 dB), and the ordinate represents the number of occurrences under the condition \( \text{CC}_{\text{ave}} \geq 0.9 \). With increasing SNR, the number of occurrences with an average correlation coefficient greater than 0.9 also increases. In other words, the higher the signal-to-noise ratio, the higher the probability of the algorithm successfully separating the source signal. When SNR = 90, the average correlation coefficient exceeds 0.9 100% of the time. The lower the signal-to-noise ratio, the lower the probability that the algorithm can successfully separate the source signal. When SNR = 10, the average correlation coefficients are all lower than 0.9. To improve the stability (probability of success) of the proposed method, it is necessary to denoise the noisy mixed signal first and then run the proposed algorithm.

5 Conclusion

For UBSS problems, sparse component analysis (SCA) is a good choice. The two-step method is the most representative method of SCA. It includes estimation of the mixed
matrix and the source signal recovery. A novel two-step approach for UBSS based on the LM method is presented in this paper. In the first stage, according to the law of large numbers, each data point is clustered, and a bar graph is proposed to represent the values of the estimation matrix and the number of source signals. In the second stage, the source signals are estimated by the minimum intersection angle rule. This method is not only suitable for sparse ultrawideband communication signals but also suitable for other types of sparse signals. Because a source signal is not ideally sparse resulting in local distortion of the separated waveform, if the signal waveform has some regular repetition, signal reconstruction technology can be used to restore the original waveform, or through channel error correction, deinterleaving technology can be used to restore the source signal. At present, only two receiving channel results have been determined by the proposed method. If we extend this method to multidimensional space, this algorithm may be suitable for multichannel reception scenarios. In addition, the LM method is limited to instantaneous mixing, and the case of underdetermined convolutional mixing needs further study and discussion.

Acknowledgements The authors would like to thank the anonymous reviewers for their insightful comments and helpful critiques of the manuscript that helped improve this paper. This work was partially supported by the Natural Science Foundation of Jiangsu Province (No. BK20171267) and the Major Program of the Natural Science Research of Jiangsu Higher Education Institutions of China (No. 18KJA520002).

References

1. P. Bofill, M. Zibulevsky, Underdetermined source separation using sparse representation. Signal Process. 81(11), 2353–2362 (2001)
2. C. Deng, Y. Wei, Y. Shen, Q. Su, J. Xu, An improved approach of blind source separation for delayed sources using Taylor series expansion, in 3rd IEEE International Conference on Computer and Communications (2017), pp. 284–288
3. W. Fu, B. Nong, X. Zhou, J. Liu, C. Li, Source recovery in underdetermined blind source separation based on artificial neural network. China Commun. 15(1), 140–154 (2018)
4. Y. Jia, P. Xu, Underdetermined blind source separation applied to the ultra-wideband communication signals. J. Nanjing Univ. Posts Telecomun. 31(1), 23–28 (2011)
5. S. Kim, C.D. Yoo, Underdetermined blind source separation based on subspace representation. IEEE Trans. Signal Process. 57(7), 2604–2614 (2009)
6. Y. Li, S. Amari, A. Cichocki, D.W.C. Ho, S. Xie, Underdetermined blind source separation based on sparse representation. IEEE Trans. Signal Process. 54(2), 423–437 (2006)
7. B. Liu, V.G. Reju, A.W.H. Khong, A linear source recovery method for underdetermined mixtures of uncorrelated AR-model signals without sparseness. IEEE Trans. Signal Process. 62(19), 4947–4958 (2014)
8. S. Mirzaei, H. Van, Y. Norouzi, Blind audio source counting and separation of anechoic mixtures using the multichannel complex NMF framework. Signal Process. 115, 27–37 (2015)
9. R. Peled, S. Braun, M. Zackenhouse, A blind deconvolution separation of multiple sources, with application to bearing diagnostics. Mech. Syst. Signal Process. 19(6), 1181–1195 (2005)
10. L. Stanković, E. Sejić, S. Stanković, M. Đaković, I. Orović, A tutorial on sparse signal reconstruction and its applications in signal processing. Circuits Syst. Signal Process. 38(3), 1206–1263 (2019)
11. O. Tichý, V. Šmídl, Bayesian blind separation and deconvolution of dynamic image sequences using sparsity priors. IEEE Trans. Med. Imaging 34(1), 258–266 (2015)
12. S. Van Vaerenbergh, I. Santamaría, Spectral clustering approach to underdetermined postnonlinear blind source separation of sparse sources. IEEE Trans. Neural Netw. 17(3), 811–814 (2006)
13. S. Xie, L. Yang, J. Yang, G. Zhou, Y. Xiang, Time–frequency approach to underdetermined blind source separation. IEEE Trans. Neural Netw. Learn. Syst. 23(2), 306–316 (2012)
14. L. Zhen, D. Peng, Z. Yi, Y. Xiang, P. Chen, Underdetermined blind source separation using sparse coding. IEEE Trans. Neural Netw. Learn. Syst. 28(12), 3102–3108 (2017)
15. Y. Zhang, S. Zhang, R. Qi, Compressed sensing construction for underdetermined source separation. Circuits Syst. Signal Process. 36(11), 4741–4755 (2017)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.