Quantizing remote sensing radiation field research based on J-C model

Ming ZHEN¹,² and Siwen BI¹
¹State Key Laboratory of Remote Sensing Science, Jointly Sponsored by the Institute of Remote Sensing and Digital Earth of Chinese Academy of Sciences and Beijing Normal University, Beijing, 100101, China
²University of Chinese Academy of Sciences, Beijing, 100049, China
E-mail:216hjk@163.com

Abstract. Remote sensing provides a powerful tool for human to explore the environment around us from multidimensional perspective and macroscopic view. As marrow of remote sensing, remote sensing information is about the message of light or electromagnetic wave obtained by remote sensing platform. Quantum remote sensing reveals remote sensing theories and methods in quantum level. Quantum remote sensing information is about how to express and transmit information by quantum state. Quantizing remote sensing radiation field is its main basis. Based on J-C model, which describes interaction between single mode light field and a two-level atom, expressions of operators correlated with light field can be obtained through state vector of atom-light field coupling system and Schrodinger equation. Both analysis and calculations show that quantum fluctuation of the light field can be squeezed. Numerical simulation is used to study the variation of quantum fluctuation, which deepens our understanding of quantum remote sensing information.

1. Introduction
With the development of science and technology, remote sensing is given a higher request by scientific problems in both theory and application. Such as information theory of remote sensing, spectral imaging mechanism, radiation transfer model of electromagnetic waves, quantitative inversion algorithms, all need to be expounded in a new level. And quantum remote sensing may be a new approach [1]. Quantum remote sensing, a new technology from quantum world, reveals remote sensing theories and methods in quantum level. Basis on quantum mechanics, the expression of quantum remote sensing is mainly Schrodinger equation and quantum state. Quantum remote sensing research includes: quantum remote sensing theory, quantum remote sensing information, quantum remote sensing imaging, quantum remote sensing calculation and quantum remote sensing measurement.

Quantum remote sensing information mainly focuses on how to express information by quantum state and solve the problem of remote sensing information loss. It includes sending, receiving, transmitting, losing, enhancing, processing and analyzing quantum remote sensing information [2]. And it is mainly based on quantizing remote sensing radiation field.

2. Quantizing remote sensing radiation field
Quantizing pure remote sensing radiation field [3-5] is as follows.
The basic idea of field quantization is: find a set of complete canonical coordinate and momentum to describe classic field, take them as corresponding operators that satisfy the commutator of canonical coordinate and momentum, and finally make them quantized. It is acknowledged that classic radiation field could be taken as a system consisting of infinite number of independent harmonic oscillators. The canonical coordinate and momentum of oscillator are written as \( Q, P \). According to canonical quantization scheme, there is

\[
[Q, P] = i\hbar \delta_{ij}
\]  

(2.1)

For the sake of convenience, dimensionless operators \( a, a^\dagger \) are introduced

\[
Q = \sqrt{\frac{\hbar}{2\omega_j}} (a + a^\dagger) \\
P = -i \sqrt{\frac{\hbar\omega_j}{2}} (a - a^\dagger)
\]  

(2.2)

Then, Eq. (2.3) can be proved by Eq. (2.1) and Eq. (2.2)

\[ [a, a^\dagger] = 0, \quad [a^\dagger, a^\dagger] = 0, \quad [a, a] = \delta_{ij} \]  

(2.3)

This is the very commutation relation between creation operator and annihilation operator. Thus, radiation field can be expressed as

\[
\hat{A}(\vec{r}, t) = \sum_n \sqrt{\frac{\hbar}{2\omega_j}} [a_n \hat{A}_j(\vec{r}) \exp(i\omega_j t) + a_n^\dagger \hat{A}_j(\vec{r}) \exp(-i\omega_j t)]
\]  

(2.4)

If appropriate phase regulation is taken, operations of \( a, a^\dagger \) in occupation number representation can be written as

\[
\begin{align*}
|a_n^\dagger |n\rangle &= \sqrt{n+1} |n+1\rangle \\
|a_n |n\rangle &= \sqrt{n} |n-1\rangle
\end{align*}
\]  

(2.5)

It is easy to verify

\[
[a_n^\dagger a_n |n\rangle = n |n\rangle
\]  

(2.6)

Positive definite hermitian operator \( a_n^\dagger a_n \) is just bosons operator or number operator on state of \( \lambda \), and its eigenvalue \( n \) equals 0, 1, 2... with corresponding eigenstate \( |n\rangle \). And then Hamiltonian of radiation field can be obtained through Eq. (2.2).

\[
H = \frac{1}{2} \sum_j (P_j^2 + \omega_j^2 Q_j^2) = \sum_j (a_j^\dagger a_j + \frac{1}{2} \hbar \omega_j)
\]  

(2.7)

The corresponding eigenvalue (energy) is

\[
E = \sum_j (n_j + \frac{1}{2}) \hbar \omega_j \quad \text{for} \quad n_j = 0, 1, 2, ..., 
\]  

(2.8)

And momentum operator of radiation field is

\[
\hat{P} = \sum_j \frac{\hbar \omega_j}{\omega_j} (P_j^2 + \omega_j^2 Q_j^2) = \sum_j (a_j^\dagger a_j + \frac{1}{2} \hbar \vec{k}_j)
\]  

(2.9)

Its eigenvalue is

\[
\hat{P} = \sum_j (n_j + \frac{1}{2}) \hbar \vec{k}_j
\]  

(2.10)

From Eq. (2.8) and Eq. (2.10), it can be found out that radiation field changes into a system consisting of photons after quantizing. And the number of photons on state of \( \lambda \) is \( n \), each with energy \( \hbar \omega_j \) and momentum \( \hbar \vec{k}_j \). Thus, quantum remote sensing information can be studied through quantizing light field and state vector.
3. Theoretical analysis
It is known that J-C model [4] [6] describes single photon interaction between one two-level atom and single mode light field. With rotating-wave approximation, its Hamiltonian can be expressed as

\[ H = H_0 + V \]  
\[ H_0 = \omega_0 S_z + \alpha a^\dagger a \]  
\[ V = u(a^\dagger S_z + a S_z) \]

(3.1) \hspace{1cm} (3.2) \hspace{1cm} (3.3)

Where, \( H_0 \) represents energy operator of system under the condition of bare atom without coupling with light field. The first and second term of \( H_0 \) shows bare atom energy and light field energy, respectively. \( V \) represents interaction energy between light field and atom. \( \omega_0 \) is atom intrinsic transition frequency, while \( \omega \) is the frequency of single mode light field. And \( u \) represents atom and light field coupling constant that describes coupling strength between them. \( a, a^\dagger \) are the same as \( a_g, a_g^\dagger \), which are mentioned before. Atom pseudo-spin operators \( S_z, S_\pm \) satisfy the following relationship:

\[ [S_z, S_\pm] = \pm S_\pm \]  
\[ [S_+, S_-] = 2S_z \]  

(3.4)

For the sake of easy calculation, resonance between atom and light field is assumed, which means \( \omega_0 = \omega \). Then the interaction Hamiltonian in interaction picture is

\[ V^I(t) = u(a^\dagger S_z + a S_z) \]

(3.5)

Assuming that there is no interaction between atom and light field at the moment of \( t = 0 \), light field is in vacuum state while two-level atom is in the superposition states of ground state \( |g\rangle \) and excited state \( |e\rangle \), then state vector in atom-light field coupling system can be written as,

\[ |\psi(0)\rangle = \cos(\theta/2)|e,0\rangle + \sin(\theta/2)\exp(-i\phi)|g,0\rangle \]

(3.6)

If single photon interaction between atom and light field happens at \( t = 0^+ \), due to atom transition only between \( |e,0\rangle \) and \( |g,1\rangle \), the atom-light field coupling system state vector in interaction picture at the moment of \( t \) can be written as:

\[ |\psi'(t)\rangle = C_g(t)|g,0\rangle + C_e(t)|g,1\rangle + C_3(t)|e,0\rangle \]

(3.7)

Substituting Eqs. (3.5), (3.7) into Schrodinger equation in interaction picture

\[ \text{i}h \frac{\partial}{\partial t} |\psi'(t)\rangle = V^I(t)|\psi'(t)\rangle \]

(3.8)

Using orthogonality of \( |g,0\rangle, |g,1\rangle, |e,0\rangle \), we can obtain

\[ \begin{align*}
C_g(t) = 0 \\
iC_e(t) &= uC_g(t) \\
iC_3(t) &= uC_e(t)
\end{align*} \]

(3.9)

According to the initial condition Eq. (3.6), Eq. (3.9) can be solved

\[ \begin{align*}
C_g(t) &= \sin(\theta/2)\exp(-i\phi) \\
C_e(t) &= -i\cos(\theta/2)\sin(ut) \\
C_3(t) &= \cos(\theta/2)\cos(ut)
\end{align*} \]

(3.10)

If Eq. (3.10) is substituted into Eq. (3.7), state vector of the system at the moment of \( t \) is obtained. With Eq. (3.7), it is easy to find out:

\[ \langle a \rangle = \langle \psi'(t) |a^\dagger(t)|\psi'(t)\rangle = -i\exp(i\phi)\sin(\theta/2)\cos(\theta/2)\sin(ut)\exp(-i\alpha t) \]

(3.11)

\[ \langle a^\dagger \rangle = 0 \]

(3.12)

\[ \langle a^\dagger a \rangle = \cos^2(\theta/2)\sin^2(ut) \]

(3.13)
Here, $a' (t) , a'' (t)$ are creation operator and annihilation operator of light field in interaction picture respectively. And there is a relationship:

$$a' (t) = a \exp(-i\omega t) = [a'' (t)]^*$$ \hspace{1cm} (3.14)

In actual measurement, detecting instruments can’t respond to the fast oscillation of the light field frequency, therefore, they can just respond to slowly varying envelope amplitude. With the above reason, here we only discuss about the slowly varying amplitude fluctuation of two orthogonal components of light field. In other words, we discuss

$$X_1 = \frac{1}{2} [a \exp(i\omega t) + a^* \exp(-i\omega t)]$$ \hspace{1cm} (3.15)
$$X_2 = \frac{1}{2i} [a \exp(i\omega t) - a^* \exp(-i\omega t)]$$

With formula $\langle \Delta X_1 \rangle = \{X_1^2\} - \{X_1\}^2$ and Eqs. (3.12), (3.13) and (3.14), fluctuations of $X_1, X_2$ are

$$\langle \Delta X_1 \rangle^2 = \cos^2 (\theta/2) \sin^2 (ut) [\frac{1}{2} - \sin^2 \phi \sin^2 (\theta/2)] + \frac{1}{4}$$ \hspace{1cm} (3.16)

$$\langle \Delta X_2 \rangle^2 = \cos^2 (\theta/2) \sin^2 (ut) [\frac{1}{2} - \cos^2 \phi \sin^2 (\theta/2)] + \frac{1}{4}$$ \hspace{1cm} (3.17)

According to quantum mechanics, if light field is in coherent state, fluctuations of its two orthogonal components have a minimum 1/4. Obviously, when $\theta = 2\pi/3, 4\pi/3$ and $\phi = \pi/2$, Eq. (3.16) changes into

$$\langle \Delta X_1 \rangle^2 = \frac{1}{4} + \frac{1}{16} \sin^2 (ut)$$ \hspace{1cm} (3.18)

This equation shows that fluctuation of $X_1$ component of light field can be squeezed, and it arrives at a minimum at the moment of $t = n\pi/(2u) \hspace{0.5cm} (n = 1,3,\ldots)$. When $\theta = 0$, atom initially is in excited state, and Eqs. (3.16), (3.17) are simplified into

$$\langle \Delta X_1 \rangle^2 = \frac{1}{4} + \frac{1}{2} \sin^2 (ut) \geq \frac{1}{4}$$ \hspace{1cm} (3.19)

So quantum noise of light field can’t be squeezed with influence of vacuum field and initially excited atom. Furthermore, atom spontaneous radiation results in light field quantum noise increase.

From the above discussion, it is clear that initial state of atom has obvious effect on squeezing property of light field quantum fluctuation in atom-light field coupling system. Next we still take advantage of J-C model to discuss whether initial state of light field has effect on light field operator quantum fluctuation in interacting with atom system.

Firstly, there is an assumption that light field is in coherent state at time $t=0$:

$$|\alpha\rangle = \sum_{n=0}^{\infty} C_n |n\rangle, C_n = \exp(-|\alpha|^2/2) \alpha^n / \sqrt{n!}$$ \hspace{1cm} (3.20)

Then, the initial state of atom is assumed in excited state $|e\rangle$. Thus, the state vector of atom-light field coupling system at initial time can be written as

$$|\psi(0)\rangle = \sum_n C_n |e,n\rangle$$ \hspace{1cm} (3.21)

As time evolves, the state vector of atom-light coupling system in interaction picture can be expanded as

$$|\psi'(t)\rangle = \sum_n a_n(t) |e,n\rangle + b_{n+1}(t) |g,n+1\rangle$$ \hspace{1cm} (3.22)

Substituting Eqs. (3.5) and (3.22) into Schrodinger equation Eq. (3.8) in interaction picture, we can get

$$\begin{cases} i a_n'(t) = u\sqrt{n+1} a_{n+1}(t) \\ i b_{n+1}(t) = u\sqrt{n+1} a_n(t) \end{cases}$$ \hspace{1cm} (3.23)
According to initial condition Eq. (3.21), the solution of above equation is as follows:

\[
\begin{align*}
    a_n(t) &= C_n \cos(u\sqrt{n} + It) \\
    b_{n+1}(t) &= -iC_n \sin(u\sqrt{n} + It)
\end{align*}
\]

(3.24)

Substitute Eq. (3.24) into Eq. (3.22), state vector of the system at time \( t \) is obtained. With Eq. (3.22), it is easy to find out:

\[
\langle a \exp(i\omega t) \rangle = \sum_{n=0}^{\infty} \frac{n^{1/2}}{n!} \exp[2(n+1)\eta] \cos(u\sqrt{n+1} + 2t) + \sqrt{\frac{n+2}{n+1}} \sin(u\sqrt{n+1} + 2t) \]

(3.25)

\[
\langle a'^*a \rangle = \sum_{n=0}^{\infty} \frac{n^{1/2}}{n!} \exp[2(n+1)\eta] + \exp[\alpha \eta] \sum_{n=0}^{\infty} \frac{n^{1/2}}{n!} \exp(2i\eta) \sin^2(u\sqrt{n} + 2t)
\]

(3.26)

\[
\langle a^2 \exp(2i\omega t) \rangle = \sum_{n=0}^{\infty} \frac{n^{1/2}}{n!} \exp[2(n+1)\eta] \cos(u\sqrt{n+1} + 3t) + \sqrt{\frac{n+3}{n+1}} \sin(u\sqrt{n+1} + 3t) \]

(3.27)

In the above calculation, \( \alpha = \bar{n}^{1/2} \exp(i\eta) \), \( \bar{n} \) is average number of photons in coherent light field. Just as the former case, here we just discuss slowly varying amplitude fluctuation of two orthogonal components of light field. Actually, \( X_1, X_2 \) are mutually orthogonal and the phase factor difference between them is \( \pi/2 \), so they can be converted to one another. Take \( X_1 \) component for example, and obviously, there is

\[
Q_t = \langle (\Delta X_1)^2 \rangle - \frac{1}{4} = \frac{1}{2} \left\{ \langle a'^*a \rangle + \text{Re}[\langle a^2 \exp(2i\omega t) \rangle] - \text{Re}[\langle a \exp(i\omega t) \rangle^2] - |\langle a \exp(i\omega t) \rangle|^2 \right\}
\]

(3.28)

Substituting Eqs. (3.25), (3.26) and (3.27) to the above expression, we can get \( Q_t(t) \) time evolution.

4. Numerical simulation

Due to complexity of Eq. (3.28), \( Q_t(t) \) time evolution can’t be immediately apparent. So it was calculated and analysed by MATLAB programming, and the result is just as figure 1 shows.
Here, for the sake of convenience, only $\eta = 0$ was considered. From figure 1, we can see that when average photons are separately 10, 20, 50, $Q_j(t) < 0$ happens at the initial short time domain. That is to say, for single photon J-C model, the quantum noise of two orthogonal components of light field can be squeezed with time evolution under condition of atom initially in excited state. In the former case, we can found that for single photon J-C model, light field will not be squeezed because of atom spontaneous radiation while initially atom is in the excited state and light field is in the vacuum state. So it can be concluded that both initial state of atom and initial state of light field can have much effect on the squeezing property of light field interacting with atom.

5. Conclusions
In this article, quantizing pure remote sensing radiation field is introduced; slowly varying amplitude fluctuations of two orthogonal components of light field are calculated and analyzed within J-C model and the result shows that both initial state of atom and initial state of light field can have much effect on the squeezing property of light field interacting with atom.

Note that in actual remote sensing radiation field, quantizing scheme the second part introduced doesn’t apply due to nonzero charge density and electric current density. But it doesn’t influence the quantizing intrinsic which has been proved by theories and experiments. Squeezing property of light field components reveals remote sensing information that classic theory can’t predict, so remote sensing information loss may be solved from the source by quantum remote sensing.

Acknowledgements
This work was supported by the Open Research Fund, State Key Laboratory of Remote Sensing Science, Institute of Remote Sensing and Digital Earth of Chinese Academy of Sciences and Beijing Normal University, under Grant NO.Y1Y00320KZ.

References
[1] Bi S W 2003 J. Infrared Millim. Waves. 22, Suppl 1-9
[2] Han J X 2006 Information Mechanism Study of Quantum Remote Sensing (Master of Science Dissertation)
[3] Bi S W 2003 J. Infrared Millim. Waves. 22, Suppl 92-96
[4] Peng J S and Li G X 1996 Introduction to Modern Quantum Optics (Beijing: Academic Press)
[5] Zeng J Y 2007 Quantum Mechanics (Beijing: Academic Press)
[6] Jaynes E T and Cummings F W 1963 Proceedings of the IEEE 51 89-109