Bottomonium spectroscopy with mixing of $\eta_b$ states and a light CP-odd Higgs

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The mass of the $\eta_b(1S)$, measured recently by BABAR, is significantly lower than expected from QCD predictions for the $T(1S) - \eta_b(1S)$ hyperfine splitting. We suggest that the observed $\eta_b(1S)$ mass is shifted downwards due to a mixing with a CP-odd Higgs scalar $A$ with a mass $m_A$ in the range 9.4 – 10.5 GeV compatible with LEP, CLEO and BABAR constraints. We determine the resulting predictions for the spectrum of the $\eta_b(nS) - A$ system and the branching ratios into $\tau^+ \tau^-$ as functions of $m_A$.

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The BABAR collaboration has recently determined the $m_{\eta_b(1S)}$ mass with an error of only a few MeV in radiative decays $\Upsilon \to \gamma \eta_b$ of excited $\Upsilon$ states and the observation of peaks in the photon energy spectrum. The result from $\Upsilon(3S)$ decays is $m_{\eta_b(1S)} = 9388.9_{-2.3}^{+3.1}$ (stat)$\pm$2.7 (syst) MeV [1], and the result from $\Upsilon(2S)$ decays is $m_{\eta_b(1S)} = 9392.9_{-6.8}^{+4.6}$ (stat)$\pm$1.9 (syst) MeV [2]. The average gives

$$m_{\eta_b(1S)} = 9390.9 \pm 3.1 \text{ MeV}
$$

(1)

implying a hyperfine splitting $E_{hfs}(1S) = m_{T(1S)} - m_{\eta_b(1S)}$ of

$$E_{hfs}^{exp}(1S) = 69.9 \pm 3.1 \text{ MeV}
$$

(2)

This result can be compared to predictions from QCD. Recent results based on perturbative QCD are in good agreement with each other and give $E_{hfs}(1S) = 44 \pm 11$ MeV [8] and $E_{hfs}(1S) = 39 \pm 14$ MeV [4] (whereas $E_{hfs}(1S)$ varies over a wider range in phenomenological models [5]): in the following we will use tentatively an average value

$$E_{hfs}^{QCD}(1S) = 42 \pm 13 \text{ MeV}
$$

(3)

which is about two standard deviations away from the experimental result [2]. The most recent result from (unquenched) lattice QCD is

$$E_{hfs}^{latQCD}(1S) = 61 \pm 14 \text{ MeV}
$$

(4)

which is within 1 $\sigma$ of [2]. However, the hyperfine splitting is quite sensitive to short distances or hard quark momenta. It has been argued in [7] that the perturbative results in [4] can be used for short distance corrections of the Wilson coefficient of the corresponding spin-flip operator measured on the lattice. The additional contribution $\delta^h d E_{hfs}(1S)$ to $E_{hfs}^{QCD}(1S)$ has been estimated as $\delta^h d E_{hfs}(1S) \sim -20$ MeV [7], which brings the lattice result [4] in good agreement with the perturbative result [4]. Although this conclusion needs to be checked within perturbation theory with lattice regularization, we consider it as a support for the perturbative QCD result [4].

Whereas an explanation of the discrepancy between [2] and [4] within QCD is not excluded at present, we will elaborate below the consequences of an explanation of this discrepancy due to new physics in the form of a mixing of the $\eta_b$ states with a CP-odd Higgs scalar $A$ with a mass $m_A$ in the range 9.4 – 10.5 GeV: as a result of such a mixing, the masses of the $\eta_b$ -like eigenstates of the full mass matrix can differ considerably from their values in pure QCD without the presence of the CP-odd Higgs $A$ [3, 9, 10], and the mass of the state interpreted as the $\eta_b(1S)$ can be smaller than expected if $m_A$ is somewhat above 9.4 GeV.

In such a scenario, the masses of the states interpreted as $\eta_b(2S)$ and $\eta_b(3S)$ can also be affected, and all states can have non-negligible branching ratios into $\tau^+ \tau^-$ due to their mixing with $A$. According to recent results of BABAR [11], the corresponding branching ratio of the observed state is below 8% at 90% confidence level. The branching ratio into $\mu^+ \mu^-$ would be smaller by a factor $m_A^2/m_\tau^2$, and well below the present upper limit [12]. The investigation of these phenomena is the purpose of the present paper.

A relatively light CP-odd Higgs scalar can appear, e. g., in non-minimal supersymmetric extensions of the Standard Model (SM) as the NMSSM (Next-to-Minimal Supersymmetric Standard Model) [10, 13, 14, 15, 16, 17]. Its mass has to satisfy constraints from LEP, where it could have been produced in $e^+ e^- \to Z^* \to Z H$ and $H \to AA$ (where $H$ is a CP-even Higgs scalar). For $m_A > 10.5$ GeV – where $A$ would decay dominantly into $b \bar{b}$ and $m_H < 90$ GeV, corresponding LEP constraints are quite strong [18]. For $2 m_\tau < m_A < 10.5$ GeV, $A$ would decay dominantly into $\tau^+ \tau^-$ and values for $m_H$ down to $\sim 86$ GeV are allowed [18] even if $H$ couples to the $Z$ boson with the strength of a SM Higgs boson.

In fact, searches for $e^+ e^- \to Z^* \to Z H$ with $H \to b \bar{b}$ indicate a light excess of events (of $\sim 2.3 \sigma$ significance) for $m_H \sim 95 – 100$ GeV [15], which could be explained by a reduced branching ratio $BR(H \to b \bar{b}) \sim 0.1$ and a dominant branching ratio $BR(H \to AA) \sim 0.9$ [16, 17] if $A$ decays dominantly into $\tau^+ \tau^-$. The possible explanation of this excess of events at LEP is an additional motivation for a CP-odd Higgs scalar with a mass below 10.5 GeV. Allowing for $m_H$ somewhat below 100 GeV, such a scenario would also alleviate the “little fine tuning problem” of supersymmetric extensions of the SM [15, 16, 17].
Finally CLEO [13] and BABAR [11] have searched for a light CP-odd scalar $A$ with $A \rightarrow \tau^+ \tau^-$ in radiative $\Upsilon(1S)$ and $\Upsilon(3S)$ decays respectively, which mainly constrains the range $m_A \lesssim 9.4$ GeV. In this work we focus on the range $9.4 \text{ GeV} \lesssim m_A < 10.5 \text{ GeV}$, which would be the most relevant for strong $\eta_b - A$ mixing effects as advocated in $^{10}$.

Apart from $m_A$, such mixing effects depend in a calculable way on the model-dependent coupling of $A$ to $b$ quarks. The corresponding coupling, normalized with respect to the coupling of the SM Higgs scalar to $b$ quarks, will be denoted by $X_d$. In models with two Higgs doublets $H_u$ and $H_d$, where $H_u$ couples to up quarks and $H_d$ to down quarks and leptons, one has $^{10, 13, 14, 15, 16, 17}$

$$X_d = \cos \theta_A \tan \beta$$

where $\cos \theta_A$ denotes the SU(2) doublet component of the CP-odd scalar $A$, and $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$. For $\tan \beta \gg 1$, $X_d$ can equally satisfy $X_d \gg 1$. $^{10}$

Below we proceed as follows: i) First we assume that, in the absence of a CP-odd Higgs scalar, $m_{\eta_b(1S)}$ would have a value compatible with $^{13}$, i.e.

$$m_{\eta_b(1S)} \sim 9418 \pm 13 \text{ MeV}.$$  \hspace{1cm} (5)

ii) We diagonalise the $\eta_b(nS) - A$ mass matrix $(n = 1, 2, 3)$ and require that one eigenvalue coincides with the mass measured by BABAR within errors $^{11}$; this condition gives us an allowed strip in the $X_d - m_A$ plane, which depends only weakly on the assumed masses of $\eta_b^0(2S)$ and $\eta_b^0(3S)$.

The resulting values of $X_d$ as function of $m_A$ allow to determine the remaining eigenvalues of the mass matrix, and the decompositions of the eigenvectors in terms of $A$ and $\eta_b^0(nS)$. Finally the $A$ components of the eigenstates allow us to determine their partial widths and to estimate the branching ratios into $\tau^+ \tau^-$. For the $\eta_b^0(1S) - \eta_b^0(2S) - \eta_b^0(3S) - A$ mass matrix we make the ansatz

$$\mathcal{M}^2 = \begin{pmatrix}
  m_{\eta_b(1S)}^2 & 0 & 0 & \delta m_1^2 \\
  0 & m_{\eta_b(2S)}^2 & 0 & \delta m_2^2 \\
  0 & 0 & m_{\eta_b(3S)}^2 & \delta m_3^2 \\
  \delta m_1^2 & \delta m_2^2 & \delta m_3^2 & m_A^2
\end{pmatrix}. \hspace{1cm} (7)$$

The diagonal elements $m_{\eta_b(nS)}^2$ are assumed to be known (within errors) from QCD with $^{13}$ for $m_{\eta_b(1S)}$. The results depend only weakly on $m_{\eta_b(2S)}$ and $m_{\eta_b(3S)}$ masses, for which we take $^{13}$ $m_{\eta_b(2S)} = 10002$ MeV, $m_{\eta_b(3S)} = 10343$ MeV. (We neglect finite width effects in $\mathcal{M}^2$, and take real matrix elements.)

The off-diagonal elements $\delta m_n^2$ can be computed in the framework of a non-relativistic quark potential model in terms of the radial wave functions at the origin $^{13, 10}$. These can be considered as identical for vector and pseudoscalar states, and be determined from the measured $\Upsilon \rightarrow e^+e^-$ decay widths. Substituting recent values for these widths (see $^{10}$ for details) we find

$$\delta m_1^2 \simeq (0.14 \pm 10\%) \text{ GeV}^2 \times X_d,$$  \hspace{1cm} (8a)

$$\delta m_2^2 \simeq (0.11 \pm 10\%) \text{ GeV}^2 \times X_d,$$  \hspace{1cm} (8b)

$$\delta m_3^2 \simeq (0.10 \pm 10\%) \text{ GeV}^2 \times X_d. \hspace{1cm} (8c)$$

We estimated the errors from higher order QCD corrections to the relation between the radial wave functions at the origin and $\Gamma(\Upsilon(nS) \rightarrow e^+e^-)$ $^{8, 13}$ to be $\sim 10\%$. (These errors play only a minor role for our results.)

In order that one eigenvalue of $\mathcal{M}^2$ coincides with the BABAR result $^{11}$ subsequently denoted as $m_{\text{obs}}, m_A$ in $^{11}$ has to satisfy

$$m_A^2 = m_{\text{obs}}^2 + \frac{\delta m_1^4}{m_{\eta_b(1S)}^2 - m_{\text{obs}}^2} + \frac{\delta m_2^4}{m_{\eta_b(2S)}^2 - m_{\text{obs}}^2} + \frac{\delta m_3^4}{m_{\eta_b(3S)}^2 - m_{\text{obs}}^2}. \hspace{1cm} (9)$$

Once $m_A$ is expressed in terms of $X_d$, $X_d$ remains the only unknown parameter in $\mathcal{M}^2$. Varying $m_{\text{obs}}$ within the errors in $^{11}$, $m_{\eta_b(1S)}$ within the errors in $^{13}$ and $\delta m_n^2$ within the errors in $^{13}$, we obtain for $X_d$ as a function of $m_A$ the result shown in Fig. 1. (Recall that $m_A - \pi$, the heaviest eigenstate of $\mathcal{M}^2$ with a large $A$ component -- has to be below 10.5 GeV in order to satisfy LEP constraints in the presence of a Higgs scalar with a mass around 100 GeV.)

Now the masses of all 4 eigenstates of $\mathcal{M}^2$ can be computed, which are shown together with the error bands from $m_{\text{obs}}, m_{\eta_b(nS)}$ and $\delta m_n^2$ (in orange/grey) in Fig. 2 as functions of $m_A$. Henceforth we denote the 4 eigenstates of $\mathcal{M}^2$ by $\eta_i, i = 1 \ldots 4$ where, by construction, $m_{\eta_i} \equiv m_{\text{obs}}$. For clarity we have indicated in Fig. 2 our assumed values for $m_{\eta_b(nS)}$ as horizontal dashed lines. For $m_A$ not far above 9.4 GeV (where $X_d$ is relatively small) the effects of the mixing on the states $\eta_b^0(2S)$ and $\eta_b^0(3S)$ are negligible, but for larger $m_A$ the spectrum can differ considerably from the one expected without the presence of $A$.

Now we consider the branching ratios of the eigenstates into $\tau^+ \tau^-$, which are induced by their $A$-components.
The decomposition of the eigenstates into the states before mixing can be written as

$$\eta = P_{1,1} \eta^0_{(1S)} + P_{1,2} \eta^0_{(2S)} + P_{1,3} \eta^0_{(3S)} + P_{1,4} A .$$

(10)

It turns out that the coefficients $P_{i,j}$ can be expressed analytically in terms of the eigenvalues $m^2_{\eta_i}$ of $M^2$:

$$P_{i,4} = \left[1 + \frac{\delta m_i^4}{(m^2_{\eta^0_i(1S)} - m^2_{\eta_i})^2} + \frac{\delta m_j^4}{(m^2_{\eta^0_j(2S)} - m^2_{\eta_i})^2} + \frac{\delta m_j^4}{(m^2_{\eta^0_j(3S)} - m^2_{\eta_i})^2}\right]^{-1/2} ;$$

$$P_{i,j} = \frac{-\delta m_i^2}{m^2_{\eta^0_i(1S)} - m^2_{\eta_i}} P_{i,4} \text{ for } j = 1, 2, 3 .$$

(11)

In Fig. 3 we show our results for the $A$-components $P_{i,4}$ for all 4 eigenstates together with the error bands from $m_{\text{obs}}, m^2_{\eta^0_i(1S)}$ and $\delta m^2_i$.

In the case of $\eta_i \equiv \eta_{\text{obs}}$, only the coefficients $P_{1,1}$ and $P_{1,4}$ differ significantly from 0. This allows to express the branching ratio $BR(\eta_i \rightarrow \tau^+ \tau^\mp)$ as

$$BR(\eta_i \rightarrow \tau^+ \tau^-) = \frac{P_{1,4}^2 \Gamma^{\pm\pm}_A}{P_{1,1}^2 \Gamma^0_{\eta^0_i(1S)} + P_{1,4}^2 \Gamma^0_{\eta^0_i(1S)}}$$

(12)

where $\Gamma^{\pm\pm}_A$ is the partial width for $A \rightarrow \tau^+ \tau^-$, $\Gamma^0_{\eta^0_i(1S)}$ the width of the state $\eta^0_i(1S)$ without mixing with $A$, and $\Gamma^0_{\eta^0_i(1S)}$ the total width of $A$ (without mixing). For $\Gamma^{\pm\pm}_A$ we have

$$\Gamma^{\pm\pm}_A = X^2 A \frac{G_F m_A^2 M}{4 \sqrt{2} \pi} \sqrt{1 - \frac{m_A^2}{M^2}}$$

(13)

where we would have to identify $M$ with $m_A$ if $A$ decays on its mass shell. Since the dependence on $M$ originates from phase space integrals, $M$ has to be identified with the mass of the decaying state (which is actually always close to $m_A$) in our case. Since the $BR(A \rightarrow \tau^+ \tau^-)$ is typically ~90% (for large tan $\beta$), we take $\Gamma^{\pm\pm}_A \sim 1.1 \Gamma^{\pm\pm}_A$ in [12].

It is remarkable that, once we insert eqs. (11) and (13) into the expression [12] for $BR(\eta_i \rightarrow \tau^+ \tau^-)$, all dependence on $X_\beta$ cancels; even for $m_A \rightarrow 9.4$ GeV where $P_{1,4}^2 \rightarrow 1$, one has $\Gamma^{\pm\pm}_A \rightarrow 0$ from $X_\beta \rightarrow 0$, and hence the first term in the denominator in [12] always dominates. The numerical result is

$$BR(\eta_i \rightarrow \tau^+ \tau^-) = (2.4_{-2.0}^{+3.2}) \times 10^{-2} \times \left(\frac{10 \text{ MeV}}{\Gamma^0_{\eta^0_i(1S)}}\right) .$$

(14)

Since the analytic expression for this $BR$ is proportional to $(m_{\text{obs}}^2 - m^2_{\eta^0_i(1S)})^2$ in our approach, its lowest possible value is quite sensitive to the smallest allowed value for this difference.

Using $\Gamma^0_{\eta^0_i(nS)} / \Gamma_{\eta^0_i}(nS) \simeq (m_{\eta_i}/m_A) [\alpha_s(m_{\eta_i})/\alpha_s(m_A)]^5 \simeq 0.25 - 0.75$ [20] and $\Gamma_{\eta_i}(1S) = 26.7 \pm 3$ MeV [21] we estimate $\Gamma^0_{\eta^0_i(1S)} \sim 5 - 20$ MeV. Hence the predicted branching ratio is compatible with BABAR upper limit of 8% [11].

Turning to the remaining heavier eigenstates, expressions similar to [12] are always very good approximations for the branching ratios into $\tau^+ \tau^-$, since the eigenstates consist essentially of just one $\eta^0_i(nS)$ state and the CP-odd Higgs $A$. However, the branching ratios vary now with $m_{\eta_i}$ and hence with $m_A$; the results are shown graphically in Fig. 3 assuming $\Gamma^0_{\eta^0_i(1S)} \sim 10$ MeV and $\Gamma^0_{\eta^0_i(2S)} \sim \Gamma_{\eta^0_i}(3S) \sim 5$ MeV. For larger (smaller) total widths these branching ratios would be somewhat smaller (larger).

An important issue is the production rate of the eigenstates $\eta_i$ in radiative decays of excited $\Upsilon$ states, notably in $\Upsilon(3S) \rightarrow \gamma \eta_i$. For a pure CP-odd scalar $A$, the $BR(\Upsilon(3S) \rightarrow \gamma A)$ is given by the Wilczek formula [22]

$$\frac{BR(\Upsilon(nS) \rightarrow \gamma A)}{BR(\Upsilon(nS) \rightarrow \mu^+ \mu^-)} = \frac{G_F m^4_A X^2 A \left(1 - \frac{m_A^2}{m_{\Upsilon(nS)}^2}\right)}{\sqrt{2} \pi \alpha} \times F .$$

(15)

$F$ is an $m_A$ dependent correction factor, which includes three kinds of corrections (the relevant formulas are summarized in [23]): bound state, QCD, and relativistic corrections. Unfortunately these corrections become unreliable for $m_A \gtrsim 8$ GeV; a naïve extrapolation of the known
corrections leads to a vanishing correction factor $F$ for $m_A \gtrsim 8.8 \text{ GeV}$ \cite{9} as relevant here. Thus it is difficult to predict the branching ratios $BR(\Upsilon(3S) \to \gamma \eta_i)$: if the $BR(\Upsilon(3S) \to \gamma A)$ is assumed to vanish, the production of the states $\eta_i$ has to rely on their $\eta_i^0(nS)$ components. Otherwise, negative interference effects could appear leading to suppressed branching ratios. Hence it cannot be guaranteed that the (kinematically accessible) part of the spectrum shown in Fig. 2 is actually visible in radiative $\Upsilon(3S)$ decays in the form of a peak in the photon energy spectrum \cite{11}.

In view of the possibly quite low photon energies and/or large backgrounds, the photons can well escape undetected even if the process $\Upsilon(3S) \to \gamma \eta_i$ occurs with a non-negligible rate. In this case, as advocated in \cite{24}, the $A$ components of $\eta_i$ can still manifest themselves through a breakdown of lepton universality in the form of an excess of $\tau^+ \tau^-$ final states in $\Upsilon(3S) \to l^+ l^- \eta_i$ \cite{9,10}. However, in the case of a vanishing correction factor $F$ in \cite{15}, this phenomenon would disappear as well.

On the other hand, if the existence of a CP-odd scalar $A$ and a CP-even scalar $H$ with $m_H \sim 95 - 100 \text{ GeV}$ (and a dominant $H \to AA$ branching ratio of $\sim 90\%$) is responsible for the excess of events at LEP as noted above, it becomes important to test this scenario at the LHC: The standard search channels for a SM-like CP-even scalar $H$ would fail, and the final states from $H \to AA$ with $m_A$ below the $b\bar{b}$ threshold would be difficult to detect. Proposals for a verification of this scenario at the LHC have been made recently in \cite{23,26,27}.

To conclude, if the mixing with a CP-odd Higgs scalar is responsible for the discrepancy between the BABAR measurement of $m_{\eta_i(1S)}$ and the expectations from QCD, it can manifest itself in the form of a completely distorted spectrum of states as shown in Fig. 2. The branching ratios into $\tau^+ \tau^-$ would be non-vanishing, albeit below the present experimental upper limit (for the lowest lying state). These manifestations of a light CP-odd Higgs scalar in $\Upsilon$ physics at (Super) B factories \cite{28} would be complementary to its possible discovery at the LHC.

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