Production of charged $\pi$-mesons in exclusive $B_c \rightarrow V(P) + n\pi$ decays

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The paper is devoted to vector or pseudoscalar heavy quarkonia production $\psi^{(*)}$, $B^{(*)}$ in association with five charged $\pi$-mesons in exclusive $B_c$-meson decays. Using available formfactors parameterizations and spectral functions of $W \rightarrow 5\pi$ transition we obtain branching fractions of these decays and distributions over invariant mass of $(5\pi)$-system.

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I. INTRODUCTION

Heavy quarkonia (for example charmonium mesons $\eta_c$, $J/\psi$, $\psi(2S)$, etc, or bottomonia $\eta_b$, $\Upsilon$, $\chi_{bJ}$, . . . ) always played a special role in elementary particle physics. Due to the presence of a heavy quark with mass $m_Q \gg \Lambda_{QCD}$ the scales of quark-antiquark annihilation ($\sim 1/m_Q$) and hadronization into experimentally observed meson ($\sim 1/\Lambda_{QCD}$) differ significantly and these two processes are separated. As a result the reactions of heavy quarkonia production and decays can be used for analysis of strong interaction both in perturbative and nonperturbative regimes. Over the last years significant theoretical and experimental results were obtained in this field.

Heavy quarkonia with open flavor ($bc$), i.e. $B_c$-meson and its excitations, take intermediate place between charmonium and bottomonium mesons. So, they provide a possibility for independent test of models used in analysis of states with hidden flavor. Theoretical predictions for the width and lifetime of $B_c$-meson

$$M_{B_c} = 6.25 \text{ GeV}, \quad \tau_{B_c} = 0.46 \text{ ps}$$

are in good agreement with experimental values. Predictions for partial widths, on the other hand, differ significantly from experimental results. For example, ratios presented in ref. [3]

$$\frac{\sigma_B \cdot Br (B_c \rightarrow J/\psi e^+\nu_e)}{\sigma_B Br (B \rightarrow J/\psi K)} = 0.282 \pm 0.038 \pm 0.074$$

and

$$\frac{\sigma_B \cdot Br (B_c \rightarrow J/\psi \mu^+\nu_\mu)}{\sigma_B Br (B \rightarrow J/\psi K)} = 0.249 \pm 0.045 \pm 0.107$$

are approximately an order of magnitude higher than theoretical expectations based on current estimates for $B_c$-meson production cross section and the branching fraction of its semileptonic decay [1].

In the present paper we consider exclusive decays $B_c \rightarrow V(P) + \mathcal{R}$, where $\mathcal{R}$ is a set of light mesons (e.g. $5\pi$). According to factorization theorem the widths of these decays are connected directly with the widths of $\tau$-lepton decays $\tau \rightarrow \nu_\tau + \mathcal{R}$. In both cases the final state $\mathcal{R}$ is controlled by virtual $W$-boson that is produced in heavy quark weak decays for hadronic processes or $\tau \rightarrow \nu_\tau W$ in leptonic ones. In contrast to $\tau$-lepton decays, $B_c \rightarrow V(P)W$ transition is described by formfactors, so it is possible to determine these formfactors using available experimental data about $\tau \rightarrow \nu_\tau \mathcal{R}$ and $B_c \rightarrow V(P)\mathcal{R}$ decays and compare them with theoretical predictions based on QCD sum rules and different potential models. Analysis of $B_c \rightarrow V(P)\mathcal{R}$ decays can also be used to describe $W \rightarrow \mathcal{R}$ transitions in energy regions that cannot be achieved in $\tau$-lepton decays.

In the next section of our paper we present analytical expressions for $B_c \rightarrow V(P)\mathcal{R}$ widths and distributions over invariant mass of light meson system $\mathcal{R}$. Technique of spectral functions used in our paper is also described briefly in that section. In section [1] we give numerical expressions for $B_c \rightarrow V(P)$ formfactors, $W \rightarrow \mathcal{R}$ spectral functions and present our predictions for branching fractions of decays and $q^2$ distributions considered in our article. Brief discussion of the results is given in the last section.

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II. ANALYTICAL RESULTS

It is well known that in the valence approximation $B_c$-mesons consist of $b$- and $c$-quarks, so their decays into heavy quarkonia are caused by weak decays of one of the constituent quarks: $b \rightarrow cW^*$ for charmonium meson in the final state or $c \rightarrow sW^*$ in the case of $B_c^{(*)}$ production. Produced in this decay heavy quark in combination with other $B_c$ constituent forms final heavy quarkonium, while virtual $W$-boson hadronizes into light mesons system $\mathcal{R}$. Typical diagram of this process is shown in fig.1.

In our paper we consider decays $B_c \rightarrow V(P) + n\pi$, (4) where $V(P)$ is vector (pseudoscalar) heavy quarkonium $J/\psi, \psi(2S)$ or $B_c^{(*)}$, and $\mathcal{R} = (5\pi)_{c1}$ is a set of five charged $\pi$-mesons (production of smaller number of $\pi$’s was considered in previous articles [4–6]). The amplitude of the reaction (4) in the framework of factorization theorem can be expressed through formfactors of $B_c \rightarrow V(P)$ transition and is equal to

$$\mathcal{M}_{V(P)} = \frac{G_F V_{ij}}{\sqrt{2}} a_1 H^\mu_{V(P)} \varepsilon^{(W)}_\mu,$$  

where $V_{ij}$ is the element of CKM mixing matrix, $\varepsilon^{(W)}$ is the effective polarization vector of virtual $W$-boson, coefficient $a_1$ describes the effect of soft gluon rescattering [6], and $H_{V(P)}$ is the amplitude of $B_c \rightarrow V(P)$ transition. This amplitude can be written in the form

$$H^\mu_v = 2M_2 A_0 \left(q^2 \frac{q^\mu (q^\nu q_2)}{q^2} + (M_1 + M_2) A_1 \left(q^2 \right) \left(\varepsilon^\mu - \frac{q^\mu (q^\nu q_2)}{q^2}\right)\right) -$$

$$- A_2 \left(q^2 \right) \frac{(q^\nu q_2)}{M_1 + M_2} \left(p_1^\mu + p_2^\mu - \frac{q^\mu (q^\nu q_2)}{q^2} q^\mu\right) - \frac{2iV (q^2)}{M_1 + M_2} \varepsilon^\mu \varepsilon^\nu \varepsilon^\alpha p_1^\alpha p_2^\beta,$$  

for vector meson production and

$$H^\mu_P = F_+ \left(q^2 \right) \left(p_1^\mu + p_2^\mu - \frac{M_1^2 - M_2^2}{q^2} q^\mu\right) + F_0 \left(q^2 \right) \frac{M_1^2 - M_2^2}{q^2} q^\mu$$  

in the case of pseudoscalar meson in the final state. In the above expressions $p_{1,2}$ and $M_{1,2}$ are momenta and masses of initial and final heavy quarkonia, $\varepsilon^\mu$ is the polarization vector of final vector meson, $q = p_1 - p_2$ is the momentum.
of virtual $W$-boson, $F_{0,+} (q^2)$, $A_{0,1,2} (q^2)$ and $V (q^2)$ are formfactors. It is clear that these formfactors cannot be determined from perturbative QCD, so one should apply some nonperturbative methods. In our paper we use the results based on QCD sum rules \cite{8} and solution of different potential models \cite{8, 10, 11}. Further these formfactor sets will be denoted as SR, PM1, and PM2 respectively.

If we are interested only in partial widths of the considered decays and $q^2$-distributions, it is convenient to integrate over the phase space of light mesons system and use the technique of spectral functions (see \cite{12} for detailed description). In the framework of this approach the differential widths of $B_c$-meson decays into vector or pseudoscalar quarkonium have the form

$$
\frac{d\Gamma [B_c \to V \mathcal{R}]}{dq^2} = \frac{G_F^2 V_{cd}^2 a_2^2 M_1^2 |V_{ij}|^2}{128\pi M_2^2 (M_1 + M_2)^2} \beta^3 \times
\left\{ \rho_T^V \left[ (M_1 + M_2)^2 \left( 1 + \frac{12M_2^2q^2}{M_1^2\beta^2} \right) A_1^2 + M_1^2 \beta^2 A_2^2 + 8M_2^2q^2V^2 - 2(M_1 + M_2)^2 (M_1^2 - M_2^2 - q^2) A_1 A_2 \right] + 4A_0^2 (M_1 + M_2)^2 \rho_T^V \right\},
$$

and

$$
\frac{d\Gamma [B_c \to P \mathcal{R}]}{dq} = \frac{G_F^2 a_2^2 |V_{ij}|^2}{32\pi M_1} \beta \left\{ (M_1^2 - M_2^2)^2 F_0^2 \rho_T^P + M_1^2 \beta^2 F_+ \rho_T^P \right\},
$$

respectively. In these expressions $\beta$ is the velocity of final quarkonium in $B_c$-meson rest frame

$$
\beta = \sqrt{1 - \left( \frac{M_2 + \sqrt{q^2}}{M_1} \right)^2} \sqrt{1 - \left( \frac{M_2 - \sqrt{q^2}}{M_1} \right)^2},
$$

and $\rho_{L,T}^P (q^2)$ are longitudinal and transverse spectral functions defined according to

$$
(q_{\mu} q_{\nu} - q^2 g_{\mu\nu}) \rho_T (q^2) + q_{\mu} q_{\nu} \rho_L (q^2) = \frac{1}{2\pi} \int d\Phi_n (q \rightarrow \mathcal{R}) \epsilon^{(W)}_{\mu} (\epsilon^{(W)}_{\nu})^*,
$$

where

$$
d\Phi_n (q \rightarrow \mathcal{R}) = (2\pi)^4 \delta^4 (q - \sum k_i) \prod \frac{d^3 k_i}{(2\pi)^3 2E_i}
$$

is the Lorentz-invariant phase space of system $\mathcal{R}$.

### III. NUMERICAL RESULTS

Let us first consider explicit expressions for spectral functions of different final states that will be used in our article. The amplitude of $W \to \pi$ transition has the form

$$
\langle \pi (k) | J_\mu | W \rangle = f_\pi k_\mu,
$$

where $f_\pi \approx 130$ MeV. Resulting transverse and longitudinal spectral functions are equal to

$$
\rho_T^{(\pi)} (q^2) = 0, \quad \rho_L^{(\pi)} (q^2) = f_\pi^2.
$$

If we have five charged $\pi$-mesons in the final state (i.e. $\mathcal{R} = (5\pi)_{ch} = \pi^+ \pi^+ \pi^- \pi^- \pi^-\bar{\pi}$) the amplitude of $W \to (5\pi)_{ch}$ transition can be obtained from resonance model \cite{13} (see diagram shown in fig.2). Due to the partial conservation of vector current the longitudinal spectral function is equal to zero, while the transverse one can be approximated by the expression. 
\[
\rho_T^{(5\pi)_{ch}}(s) \approx 32 \left( \frac{s - 25m^2_{\pi}}{s} \right)^{10} \frac{1 - 1.65s + 0.69s^2}{(s + 2.21)^2 - 4.69}^{1.65},
\]

where \( s \) is measured in GeV\(^2\) (the spectral function itself is dimensionless). The dependence of this function on \( q^2 \) is shown in fig.2.

It should be noted that in addition to direct decays \( J/\psi \) and five charged \( \pi \)-mesons can be produced in the reaction \( B_c \to \psi(2S) + (3\pi)_{ch} \to J/\psi + (5\pi)_{ch} \), so the spectral function of \((3\pi)_{ch}\) state should also be considered. As in the previous case, this transition can be described in the framework of resonance model with diagrams shown \( \text{fig.3(a)} \) (see ref.\[14\]). The contribution of longitudinal spectral function can be neglected, and the transverse one is approximated by the expression \( \text{fig.3(b)} \)

\[
\rho_T^{(3\pi)_{ch}}(s) \approx 2.93 \times 10^{-5} \left( \frac{s - 9m^2_{\pi}}{s} \right)^{4} \frac{1 + 190s}{(s - 1.06)^2 + 0.48}^{1.2}. 
\]

Experimentally these two processes can be easily separated.

Formfactors of \( B_c \)-meson transition into vector or pseudoscalar quarkonium can be calculated, for example, in the framework of QCD sum rules or various potential models. In our work we use formfactor sets presented in papers \[8\] (denoted hereafter as SR), \[8,10\] (PM1) and \[11\] (PM2). The values of these formfactors at points \( q^2 = 0 \) and \( q^2 = q^2_{\text{max}} = (M_1^2 - M_2^2)/(2M_1) \) are given in tables \[8\] and \[14\]. It can be easily seen from these tables that for ground quarkonium states formfactors’ behaviour for different models is similar, while predictions for excited \( \psi(2S) \)-meson of PM2 model differ dramatically from others. For example, PM2 model formfactors decrease with the increase of the squared transferred momentum, while formfactors from SR and PM1 models increase. Such difference can be probably explained in the following way: in the framework of potential models \( B_c \)-meson formfactors are determined from the overlap of initial and final quarkonia wave functions; the wave function of \( \psi(2S) \)-meson (in contrast to ground states \( J/\psi \) and \( B_s^{(*)} \)) has a node that leads to such unusual behaviour of the formfactors. It will be shown later that
such effect leads to large difference between different models’ predictions for $q^2$-distributions in $B_c \to \psi(2S) + (5\pi)_{ch}$ decays.

Numerical values of $a_1$ coefficient for $B_c \to \psi^{(')} + \mathcal{R}$ and $B_c \to B_s^{(*)} + \mathcal{R}$ decays are

$$a_1(m_c) = 1.14$$

and

$$a_1(m_b) = 1.2$$

respectively.

Substituting these numbers into relations (8) and (9) it is easy to obtain numerical values of the branching fractions of the decays considered in our article (see table III). In order to compare them with the experimental results it is also useful to consider the ratio of $B_c \to V(P) + 5\pi$ and $B_c \to V(P) + \pi$ branching fractions, where the dependence on the choice of a formfactor model is partially canceled. In the case of $B_c \to J/\psi + \mathcal{R}$ decay these ratios for different models are equal to

$$\frac{\text{Br}[B_c \to J/\psi + (3\pi)_{ch}]}{\text{Br}[B_c \to J/\psi\pi]} = (2.3)_{SR}, \quad (2.2)_{PM1}, \quad (2.1)_{PM2},$$

that agrees well with experimental result [15]

$$\frac{\text{Br}_{exp}[B_c \to J/\psi + (3\pi)_{ch}]}{\text{Br}_{exp}[B_c \to J/\psi\pi]} = 2.4 \pm 0.3 \pm 0.3.$$

and

$$\frac{\text{Br}[B_c \to J/\psi + (5\pi)_{ch}]}{\text{Br}[B_c \to J/\psi\pi]} = (1.1)_{SR}, \quad (0.95)_{PM1}, \quad (1.1)_{PM2}.$$

For other decays we get

$$\frac{\text{Br}[B_c \to \psi(2S) + (5\pi)_{ch}]}{\text{Br}[B_c \to \psi(2S)\pi]} = (0.53)_{SR}, \quad (0.5)_{PM1}, \quad (0.05)_{PM2},$$

$$\frac{\text{Br}[B_c \to B_s^{(*)} + (5\pi)_{ch}]}{\text{Br}[B_c \to B_s^{(*)}\pi]} = (1.1 \times 10^{-6})_{SR}, \quad (1.2 \times 10^{-6})_{PM1}, \quad (2.1 \times 10^{-6})_{PM2}.$$
Table III: Branching fractions of $B_c \rightarrow V(P) + (5\pi)_{ch}$ decays

| mode                  | SR     | PM1     | PM2     |
|-----------------------|--------|---------|---------|
| $B_c \rightarrow J/\psi \pi$ | 0.17   | 0.21    | 0.064   |
| $B_c \rightarrow J/\psi + (3\pi)_{ch}$ | 0.39   | 0.45    | 0.14    |
| $B_c \rightarrow J/\psi + (5\pi)_{ch}$ | 0.18   | 0.2     | 0.066   |
| $B_c \rightarrow \psi(2S)\pi$       | $6.6 \times 10^{-3}$ | $7.8 \times 10^{-3}$ | 0.014 |
| $B_c \rightarrow \psi(2S) + (5\pi)_{ch}$ | $3.5 \times 10^{-3}$ | $3.9 \times 10^{-3}$ | $6.9 \times 10^{-4}$ |
| $B_c \rightarrow B_s\pi$          | 17.    | 12.     |         |
| $B_c \rightarrow B_s + (5\pi)_{ch}$ | $9.1 \times 10^{-6}$ | $6.5 \times 10^{-6}$ |         |
| $B_c \rightarrow B_s^*\pi$         | 7.7    | 9.6     |         |
| $B_c \rightarrow B_s^* + (5\pi)_{ch}$ | $1.9 \times 10^{-5}$ | $2. \times 10^{-5}$ |         |

Figure 4: Transferred momentum distributions for $B_c \rightarrow J/\psi + (3\pi)_{ch}$ and $B_c \rightarrow \psi(2S) + (3\pi)_{ch}$ decays (left and right figures respectively). Solid, dashed and dotted lines correspond to formfactor sets SR, PM1, and PM2.

where in the last case we take into account contributions from both vector and pseudoscalar $B_s$-mesons. One can notice some interesting properties of the ratios presented above. First of all, branching fractions of charmonium production in association with one and five $\pi$-mesons are comparable with each other, while for $B_s$-meson in the final state the production of large number of $\pi$-mesons is strongly suppressed. This effect was observed also for 4$\pi$ final state (see ref. [6]) and is caused by the small difference between $B_c$- and $B_s(\ast)$-meson masses. In addition, one can see that PM2 prediction for $B_c \rightarrow \psi(2S)\mathcal{R}$ decay is about an order of magnitude smaller, than SR and PM1 results. This difference can be explained by the behaviour of PM2 model formfactors for excited charmonium state mentioned above.

Using relations (8), (9) one can also obtain $q^2$-distributions for the branching fractions considered in our article. In the case of $B_c \rightarrow J/\psi + (3\pi)_{ch}$ and $B_c \rightarrow \psi(2S) + (3\pi)_{ch}$ decays these distributions are shown in fig.4. Available experimental results from [17] are shown in the left plot of this figure with dots. One can see that theoretical predictions based on SR formfactor model are in good agreement with the experimental data.

Similar distributions for $B_c \rightarrow \psi^{(*)} + (5\pi)_{ch}$ and $B_c \rightarrow B_s^{(*)} + (n\pi)_{ch}$ decays are shown in figs.5 and 6 respectively. One can easily see that for $B_c \rightarrow J/\psi + (5\pi)_{ch}$ the results of different formfactor models agree with each other up to overall normalization. In the case of $B_c \rightarrow \psi(2S) + (5\pi)_{ch}$ decay, on the other hand, PM2 model prediction differs strongly from SR and PM1 results. This difference is caused by the behaviour of the formfactors mentioned above.

IV. CONCLUSION

$B_c$-meson, i.e. heavy quarkonium build from $b$- and $c$-quarks, was discovered by ALEPH collaboration in 1997 [16]. Reported values of its mass and lifetime are in excellent agreement with theoretical predictions [1]. The situation with branching fractions of different decays (e.g $B_c \rightarrow J/\psi\ell\nu$ or $B_c \rightarrow J/\psi\pi$) is much worth. It is clear that additional investigation of this question is required.

In our paper $B_c$-meson decays into heavy quarkonium ($J/\psi$, $\psi(2S)$ or $B_s^{(*)}$) and a set of charged $\pi$-mesons ($n\pi)_{ch}$ are considered. In the framework of QCD factorization theorem the amplitude of these processes can be splitted into two independent parts that describe $B_c \rightarrow V(P)W$ vertex and $W^* \rightarrow n\pi$ transition. The first part is expressed
through $B_c$-meson formfactors, that can be calculated using different nonperturbative methods: QCD sum rules, various potential models, etc. Information about $W^* \to 5\pi$ transition amplitude, on the other hand, can be obtained from theoretical and experimental analysis of $\tau$-lepton decays $\tau \to \nu + n\pi$. Using this procedure we calculated branching fractions and distributions over invariant mass $q^2 = M_{(n\pi)}^2$ for $n = 3$ and 5. It is shown that in the case of $B_c \to J/\psi + (3\pi)_{ch}$ decay our predictions agree well with experimental results, presented in [15]. Other considered in our article decays were not observed yet, but their experimental investigation can be expected in the nearest future.

It should be also noted, that experimental analysis of considered in our article decays could be useful for studying hadronization of virtual $W$-boson into a set of light mesons. In the ratio

$$\rho_T(q^2) \sim \left( \frac{d\Br[B_c \to V(P) + n\pi]}{dq^2} / \frac{d\Br[B_c \to V(P) + \ell\nu]}{dq^2} \right)$$

(22)

all dependence on $B_c$-meson formfactors is canceled, so it describes only $W \to n\pi$ transition. Previously this function was studied only in $\tau$-lepton decays, but in $B_c$-meson decays one can probe higher values of transferred momentum.

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