An Evolutionary Game Model for Understanding Fraud in Consumption Taxes

Abstract

This paper presents a computational evolutionary game model to study and understand fraud dynamics in the consumption tax system. Players are cooperators if they correctly declare their value added tax (VAT), and are defectors otherwise. Each player’s payoff is influenced by the amount evaded and the subjective probability of being inspected by tax authorities. Since transactions between companies must be declared by both the buyer and seller, a strategy adopted by one influences the other’s payoff. We study the model with a well-mixed population and different scale-free networks. Model parameters were calibrated using real-world data of VAT declarations by businesses registered in the Canary Islands region of Spain. We analyzed several scenarios of audit probabilities for high and low transactions and their prevalence in the population, as well as social rewards and penalties to find the most efficient policy to increase the proportion of cooperators. Two major insights were found. First, increasing the subjective audit probability for low transactions is more efficient than increasing this probability for high transactions. Second, favoring social rewards for cooperators or alternative penalties for defectors can be effective policies, but their success depends on the distribution of the audit probability for low and high transactions.

I. Introduction

The value added tax (VAT) is the most common consumption tax worldwide. With extensive use since the 1960s, it has reached significant tax revenue capacity. In fact, consumption taxes demonstrated a similar collection capacity to income taxes during the 1990-2010 period [1]. All deliveries of goods and services by companies, professionals, and importers are subject to VAT. The tax base is the value added (value of production minus value of intermediate consumption) generated at each step of the production and distribution process. The tax debt is calculated by applying the tax rate to the value of the goods and services sold, and deducting the VAT attached to intermediate consumption. Companies and professionals declare the VAT passed on to the customers, while deducting the amount borne from purchases from their own suppliers. The resulting statement can be either positive or negative.

This refunding system potentially allows for significant levels of fraud in terms of undervaluation of sales and overvaluation of purchases [2]. However, the fact that both buyers and sellers record each transaction offers some self-enforcement capability to the VAT system. Das-Gupta and Gang [3] identified circumstances where the ability of tax administrators to match the sales and purchase invoices strengthens this self-reinforcement capacity. In any case, tracking the relationships between buyers and sellers is clearly of the utmost importance to detect and prevent fraud. For this purpose, tax administration normally requires the declaration of all purchase and sales transactions between pairs that exceed a certain threshold, with an explicit declaration of the counterparts.

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In this work, we propose a computational intelligence (CI) model, based on evolutionary game theory [4, 5], to study the VAT fraud dynamics of buyers and sellers in an economic system. The goal of the model is to represent a network of players—either cooperating (correctly declaring VAT) or defecting (incorrectly declaring VAT)—linked by their pairwise transactions. Evolutionary game theory has been applied to models of cooperation such as the well-known prisoner’s dilemma [6–8], snowdrift games [9, 10], or trust dilemmas [11, 12]. These game models represent one of the most prominent CI techniques for representing economic markets and designing economic policies [13]. The application of evolutionary game models to study tax fraud and evasion is, however, very limited [14–17], and nonexistent when focusing on consumption taxes such as VAT.

Our CI model is studied in a structured population where players are linked by means of a social network of transactions. Here, players are given two possible strategies: being a cooperator C or being a defector D (i.e., a free rider). The model considers the amount of tax accrued on transactions not declared by the free riders, a perceived probability of being inspected by the tax agency, and the corresponding fine when tax evasion is detected. Cooperators, who are players correctly paying their taxes, receive a recognition or social reward; however, they can also have their transactions inspected, with a certain probability, when their transaction records do not match those of their transaction partners in the network.

The combination of CI techniques with agent-based modeling (ABM) [18, 19] offers many opportunities for practitioners [20], and our work is a perfect example. Our model represents players of the tax system as agents on nodes of a heterogeneous social network [21]. The social network follows a power-law distribution, equivalent to the scale-free network topology used in previous studies for promoting cooperation in social dilemmas [22]. This social network is weighted, with weights of the edges representing values of the tax debt associated with the transactions between two linked nodes. These weights make the model a mixed game [23] where players have different payoff matrices depending on the volume of their transactions. The players, through a social evolutionary learning process, can imitate others’ strategies by using an evolutionary update rule and a mutation operator to randomly modify their own strategies.

We used real-world data from the Canary Islands tax agency to feed most of the parameters of the model and fit the power-law distribution of the scale-free social network [24]. After investigating the general dynamics of the model and effects of having well-mixed and structured populations on scale-free networks with different properties, we focused our experiments on determining policies to promote cooperation and reduce the number of players who do not correctly pay their consumption taxes. To achieve this goal, we defined different experimental scenarios that allow us to understand when the best cooperative behaviors occur. These scenarios include policies regarding the shared pressure to increase the perceived probability of being inspected for high and low transactions and how diversity in subjective probabilities affects the levels of cooperation in the population; the impact of modifying the reputational reward for cooperators; and a sensitivity analysis on different inspection fines for defectors or free riders.

In the next section (i.e., Section II), we discuss related work and the motivation of our study. Details of the CI agent-based model are then described in Section III. Section IV presents the analysis of real data from the tax agency and setup of the model. The results and model’s dynamics are discussed in Section V. Finally, Section VI summarizes the key contributions.

II. Background and Related Work

The neoclassical economic model on tax fraud by Allingham and Sandmo [25] is considered one of the cornerstones of the economic analysis of tax evasion. They represent how individual agents decide to evade taxes, while also considering how the government would eventually punish them. However, this model is unable to explain low levels of fraud under low penalty and detection rates. Bordignon [26] was one of the first to describe this problem and the need to explain tax fraud using explanation other than just selfishness. Subsequent models stress that tax compliance by agents is dependent on how they perceive unfairness in their relations with not only the administration, which is the vertical factor, but also the rest of the agents, which is the horizontal factor [27].

When analyzing the horizontal factor, the tax evasion literature tries to identify how the compliance level of an agent affects the compliance level observed by the rest of the agents. Traxler [28] attempted to model different levels of tax evasion within and between groups of agents. He was able to bring issues related to belief management into the discussion, extending the spectrum of policy instruments to the scope of changing individual beliefs, besides the economic incentives. Prichard et al. [1] reflected on the main reasons of the failure of mainstream neoclassical models in their survey. They identified two main lines of research that can address the limitations of the traditional models by including the relevance of behavioral aspects: experiments and ABM.

Experiments, as Alm [29] stated, are not without problems, but they overcome the simplicity of theoretical models of individual choice, since they can incorporate many explanatory factors suggested by theory. They also favor the combination of economic theory with other disciplines like psychology, increasing the realism of explanatory factors of tax fraud [30].

Bonein [31] has identified different levels of reciprocity between agents. Under “strong reciprocity”, taxpayers would tend to evade more (less) if they observe a more (less) disadvantageous, inequitable behavior by the remaining agents. This completely contradicts the predictions by self-interest models [32], where agents are only motivated by a future economic benefit. Frey and
There are several advantages of using ABM in the analysis of tax fraud. The ABM approach enables the inclusion of agents with very different response patterns, which allows for a dynamic bottom-up structure. On the other hand, the agents included in an agent-based model do not maintain static positions and reactions.
observed behavior of the relevant agents must always be an end to pursue. Given that the data necessary for this type of analysis is inexorably linked to individual data protection rights, collaboration between researchers and tax administration is a must. This is especially the case if we consider increasing our knowledge of taxpayer compliance behavior, as indicated by Bloomquist [49].

III. Model Description

A. Game Strategies and Payoff Matrix

Players of the game are a finite set of Z agents (companies) occupying the nodes of a social network, with edges denoting economic transactions between them. The network is undirected but weighted: a weight $d_{ij}$ means the accrued tax to be declared and paid in relation with all cumulative transactions between both players $i$ and $j$. Under a correct behavior, this value $d_{ij}$ is the consumption tax involved in the transactions between both companies $i$ and $j$. We differentiate between two quantities to be paid, high ($d_a$) and low ($d_l$), based on high and low transaction values, respectively. Therefore, $d_t = \{d_a, d_l\}$, $\forall (i, j)$ being an edge of the social network. In this sense, the game can be modeled as a mixed game [23,50] where two players play a different game depending on the type of transaction, given by parameter $d_t$.

Each player $i$ chooses a strategy $s$ from two possibilities at every time step ($s(i) = \{C, D\}$): being a cooperator or tax payer (C), or being a defector or tax evader (D). When being a tax evader, the player does not pay a fraction of the transaction value $d_{ij}$, saving this cost as a personal benefit (free rider). We model this fraudulent fraction by parameter $\alpha \in [0, 1]$, which also measures the difficulty of the social dilemma. Higher values of $\alpha$ correspond to higher economical benefits for free riders when declaring transactions.

In order to set the fitness of a player $i$ for a specific time step $t$ ($f_i^t$), the player accumulates all the payoffs $w_{ij}^t$ from the pairwise interactions over all its direct contacts in the social network (i.e., neighborhood in the network): $f_i^t = 1/(k) \sum w_{ij}^t$, where $k_i$ is the degree of player $i$. The payoff $w_{ij}$ of focal agent $i$ with respect to neighbor $j$ is obtained by considering their strategies in the previous time step and the specific payoff matrix played depending on the type of transaction (either low or high).

Table I shows the payoff matrix defining the mixed game. Parameter $R$ is the social and reputational reward for a player when acting in accordance to its tax duties. $\Gamma$ is the inspection cost a company should pay when a tax agency examines the company and its documentation regardless of its own behavior (i.e., playing strategy). $\phi$ is the fine multiplier a company must pay when the tax agency audits the company and discovers fraudulent behavior.

$\Theta()$ is a linear probability function to define a player’s perception of how probable a tax audit is. This probability is subjective, even if all the players have the same perception of this probability (we can say there is a “shared collective perception”), and depends on the taxpayer’s subjective probabilities about being audited, following previous studies such as that of Hashimzade et al. [43]. The function $\Theta()$ depends on the difference in the amount declared by the players, and is only applied when the tax agency discovers a transaction mismatch for a pair of players. The probability function is built from two values ($2 \alpha d_L$ and $2 \alpha d_H$), which set the probability of being inspected for both low and high transactions. For clarity, we have defined $\Theta_{R}, \Theta_{L}, \Theta’_{R},$ and $\Theta’_{L}$ as the values of the probability function based on its arguments. Table II shows a summary of all the model parameters.

The payoff matrix in Table I includes different two-strategy games depending on the values of the parameters. These games determine the decision of every player to either cooperate or defect, and therefore influence the final outcome of the evolutionary game. To characterize the included games we consider a general payoff matrix:

| NAME | DESCRIPTION |
|------|-------------|
| $R$  | REPUTATIONAL AND SOCIAL REWARD FOR CORRECTLY PAYING TAXES |
| $\Gamma$ | INSPECTION COST |
| $d = \{d_a, d_l\}$ | AMOUNT OF TAX DEBT TO BE PAID FOR THE INVOLVED TRANSACTIONS |
| $\alpha \in [0,1]$ | RATIO OF UNPAID TAX DEBT BY DEFFECTORS |
| $\phi$ | FINE MULTIPLIER TO BE PAID BY A DEFECTOR UPON INSPECTION |
| $\Theta = \{\Theta_R, \Theta_L\}$ | TAXPAYER’S SUBJECTIVE PROBABILITY OF BEING AUDITED WHEN ONE PLAYER DEFECTS |
| $\Theta_R = \Theta(a \alpha d_R)$ | SUBJECTIVE PROBABILITY FOR HIGH TRANSACTIONS WHEN ONE PLAYER DEFECTS |
| $\Theta_L = \Theta(a \alpha d_L)$ | SUBJECTIVE PROBABILITY FOR LOW TRANSACTIONS WHEN ONE PLAYER DEFECTS |
| $\Theta = \{\Theta_R, \Theta_L\}$ | TAXPAYER’S SUBJECTIVE PROBABILITY OF BEING AUDITED WHEN BOTH PLAYERS DEFECT |
| $\Theta_R = \Theta(2 \alpha d_R)$ | SUBJECTIVE PROBABILITY FOR HIGH TRANSACTIONS WHEN BOTH PLAYERS DEFECT |
| $\Theta_L = \Theta(2 \alpha d_L)$ | SUBJECTIVE PROBABILITY FOR LOW TRANSACTIONS WHEN BOTH PLAYERS DEFECT |

$$
\begin{bmatrix}
C & D \\
C & R & S \\
D & T & P
\end{bmatrix}
$$

where $S$, $T$, and $P$ represent the expressions of Table I. The definition of the
payoff matrix in Table I satisfies $R > S$ and $T > P$, facilitating the level of cooperation in the game. According to Allen and Nowak [51], a social dilemma occurs when $R > P$ (mutual cooperation benefits both players) and at least one of the following conditions is met to favor the adoption of defection: (D1): $T > R$, (D2): $P > S$, or (D1): $T > S$. Values of the parameters in Table II determine which of these conditions are satisfied.

Figure 1 shows possible games according to $ad$ and inspection cost $\Gamma$ values; and assuming a constant audit probability $\Theta() = \Theta$. We see three regions in the figure depending on the parameters’ values: there is no social dilemma in two of them and the third is associated with a classical game. Cooperation prevails if the non-declared amount $ad$ is below $R/(1 - \Theta \phi)$, whereas defection is the preferred option if $ad$ is high enough. A Stag Hunt game appears for intermediate values of $ad$, where conditions for cooperation $R > P$ and temptation to defect ($D_2$ and $D_1$) coexist.

In case of a non-constant audit probability $\Theta()$, the outcome is more complicated. The payoff matrix includes two possible values, $\Theta(ad) = \Theta$ and $\Theta(2ad) = \Theta'$. Figure 2 shows the set of games for different values of $\Theta$ and $ad$, assuming a fixed $\Theta'$. For large values of $ad$ (above the horizontal line defined by $(r + \Theta' \Gamma)/(1 - \Theta' \phi)$), there is no social dilemma as mutual defection is always preferred over mutual cooperation ($P > R$). Below the horizontal line, multiple games arise. In general, cooperation is expected when $ad$ is low and $\Theta$ is high. More specifically, when the audit probability is a decreasing function ($\Theta > \Theta'$), the most expected games are those favoring cooperation (coordination and harmony games). However, when the audit probability is increasing ($\Theta < \Theta'$), there is a significant region where games such as the prisoner’s dilemma or snowdrift prevail and defection is the expected outcome.

**B. Evolutionary Update Rule**

The players can change their strategies $s(i)$, $\forall i \in Z$ during the whole discrete-event simulation. These changes in strategies come from two evolutionary mechanisms. First, a player $i$ can imitate others in the population (generally, their direct contacts in the social network). Second, players can also change their strategies by adopting a strategy at random, following a random mutation mechanism with probability $\mu$. The mutation operator does not take into account if the new strategy was beneficial in the past in terms of the fitness values of the players. However, the social imitation update rule is a social learning process of the players in the game [52]. Social imitation update rules consider the fitness of direct neighbors on a network in the previous steps to make their decision, either in a deterministic or a probabilistic way. In our model, we use the Fermi function as the social imitation update rule.

Fermi’s rule is one of the most well-known imitation processes [53], [54] and is applied synchronously: for each step $t$, a focal agent $i$ compares its fitness value in the previous step $t - 1$, $f^{-1}$, with one of its direct neighbors in the social network, $j$, also in $t - 1$: $f^{-1}$. Therefore, the Fermi’s rule is a stochastic pairwise comparison rule, where players can also make mistakes during the imitation process (i.e., a player can imitate a neighbor with
worse fitness). Mathematically, agent $i$ with strategy $X$ adopts strategy $Y$ of agent $j$ (a randomly selected direct contact of $i$) with a probability given by Equation 1, where $\beta$ is the intensity of selection parameter, set to 0.5 in the model.

$$\text{prob}_{ij}^t = \frac{1}{1 + e^{-\beta(f_i^t - f_j^t)}}.$$ (1)

### IV. Data Analysis and Parameters of the Model

#### A. Data Description

Most of the parameters in the model were set using real data. The real data used for our study includes VAT declarations by businesses registered in the Canary Islands (Spain) in 2002 about transactions, with persons or firms, exceeding 3,005.06 euros. The anonymized data was accessible only within the tax administration under confidentiality agreement of not revealing any information that could be used to identify either the buyers or sellers, and under the commitment that the data was not to be passed on to third parties in any format.

Every taxpayer must independently declare purchases and sales. Therefore, all transactions between two firms should be declared twice—once each by the seller and the buyer. The original information is split into two files: one with the total sales in the economy and the other with total purchases. A database was built by merging both files and taking the transactions (sales and purchases in 2002) declared by the same firms in the two files into account. Transactions without a counterpart were eliminated, since in some cases (e.g., individual buyers) a counterpart is not obliged to declare. Transactions where both counterparts match were also removed. Finally, the number of firms in the database is $N = 32,886$ including $E = 197,791$ operations.

#### B. Generation of the Real Social Network of Transactions

In order to examine the structure of VAT declarations, we set up a network of firms, where two firms are linked when they declare transactions between them. The network is undirected, since we do not differentiate whether the firm acts as a buyer or a seller, although the amount declared by the seller is included as a link weight. Specifically, we use a scale-free network starting with the degree distribution of the VAT declaration network (Figure 3), which is a long-tailed distribution. Following the methodology in Clauset et al. [55], the network fits to a power law $k^{c}$ with $x_{\text{min}} = 88$ and $c = 3.04$. According to the taxonomy in Broido and Clauset [24], the VAT declaration network is a weak scale-free network, since its power law cannot be rejected and it includes more than 50 nodes. For the model, we have built a scale-free network of 10,000 nodes, with the same exponent $c$ and $x_{\text{min}}$.

#### C. Feeding Transaction Parameters from Data

The values and distribution of the quantities to be paid $\{d_{si}, d_{si}\}$ were set from the empirical data. Figure 4 shows the edge-weight distribution by considering sellers’ declarations (note that we obtained a similar distribution when we looked at buyers’ declarations). More than 95% of the transaction amounts declared are under 1 million euros, whereas 0.01% are over 100 million euros. On the other hand, according to official records, the percentage of large firms (those with more than 20 employees) in the Canary Islands is about 2% [56]. Therefore, we set the probability of an edge to have a high transaction to 0.02 (i.e., $\text{prob}_{d_{hi}} = 0.02$). Additionally, we obtained a ratio $r = 45.7589$ between the average values of the total volume of transactions of

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**FIGURE 3** Degree distribution of the VAT declaration network.

**FIGURE 4** Edge-weight distribution in the real VAT declaration network.
small and big firms (those transactions below and above quantile 2% in the edge-weight distribution, respectively). We assume that the amount to be paid is a constant fraction of the transaction value. Therefore, we set the value $d_0 = \{d_L,d_H\}$, $\forall (i,j) \in E$, to a fixed value of $d_L = 10$ and $d_H = n d_L = 457.59$. Edges’ weights $d_{ij}$ are randomly initialized, characterized by $\text{prob}_{d_{ij}} = 0.02$. Note that weights $d_{ij}$ do not change over time. They are static and therefore, the same payoff matrix of the mixed game is used for every pair of players $i$ and $j$ in this study.

We define the ratio of divergence in the tax declarations between seller $i$ and buyer $j$ ($\alpha_{i\rightarrow j}$) as the percentage of VAT declaration mismatch between the tax declared by seller $i$ and buyer $j$. We only take those mismatches benefiting the sellers (this is the case where the seller declares less amount than the buyer). The ratio is calculated as follows: Given $d_{ij}$ as the amount that firms $i$ and $j$ need to declare, let $\alpha_{i\rightarrow j}$ be the percentage of this amount that seller $i$ does not declare and $\alpha_{j\rightarrow i}$ the percentage that buyer $j$ declares in excess. Then, the ratio of undeclared accrued tax between $i$ and $j$ is:

$$\alpha_{i\rightarrow j} = \frac{(1 + \alpha_{j\rightarrow i}) d_{ij} - (1 - \alpha_{i\rightarrow j}) d_{ij}}{(1 + \alpha_{i\rightarrow j}) d_{ij} + (1 - \alpha_{i\rightarrow j}) d_{ij}}$$

$$= \frac{\alpha_{i\rightarrow j} + \alpha_{j\rightarrow i}}{2 + \alpha_{j\rightarrow i} - \alpha_{i\rightarrow j}}, \forall (i,j) \in E$$

This ratio is between 0 (when both counterparts declare exactly the same amount, $\alpha_{i\rightarrow j} = \alpha_{j\rightarrow i} = 0$) and 1 (when the seller does not declare any amount). Note that when the fraction of the amount that is incorrectly declared is constant and identical for any firm and transaction, we have $\alpha_{i\rightarrow j} = \alpha_{j\rightarrow i} = \alpha$, with $\alpha$ being the ratio of unpaid quantity for defectors (Table II). We also have $\alpha_{i\rightarrow j} = \alpha$, $\forall (i,j) \in E$. Then, the assigned values for parameter $\alpha$ in the model would be calibrated by the $\alpha_{i\rightarrow j}$ values in the real data.

Figure 5 represents the cumulative distribution of the ratio $\alpha_{i\rightarrow j}$ in the VAT declaration network. As can be observed, the ratio of undeclared transactions is almost zero for around 75% of operations and below 0.5 for 99.06% of them. Given this, in our simulations,
we consider the range of realistic values for parameter $\alpha$ between 0 and 0.5.

**D. Model Setup**

We set the model up for 50 Monte-Carlo runs, with 1,000 time-steps in each run, thereby ensuring that all the realizations reach a stationary stable state (as we can see in Section V-A). The simulation results were obtained by averaging the last 25% of the simulation time-steps in the independent Monte-Carlo runs. Source code and data files are available at https://bitbucket.org/mchserrano/evolutionary-game-tax-fraud.

We set the remaining parameters—when not explicitly specified—as follows: inspection cost $\Gamma = 1$ and reputational reward for cooperators $R = 1$. Fine value $\phi$ was set to 1.5 (50% fine plus the undeclared quantity) as per previous models [43]. The values used to generate the linear audit probability function were set to $\Theta_H = \Theta_L = 0.5$, when the analysis was not focused on these subjective probabilities (see Section V-C). The mutation probability of the evolutionary dynamics was always set to $\mu = 0.01$.

**V. Analysis of the Results**

**A. General Dynamics of the Model**

We first show the dynamics of the model for the base parameters. The upper plot in Figure 6 shows the evolution of the model over 1,000 time-steps for different values of $\alpha$. The plot also shows the max-min range of the simulations for the 50 Monte-Carlo realizations. The stationary state is quickly reached and even 500 time-steps are sufficient in this case. Additionally, the max-min ranges in the plot highlight that the deviation of the model is low. The lower plot in Figure 6 shows how the model dynamics is independent of the initial strategy settings of the population. By enabling a player to randomly change their strategy using the mutation operator, we also eliminate differences in the outputs in case of extreme conditions (e.g., the initial frequency of cooperators is either 0 or 1). Therefore, we fixed the initial frequency of cooperators to 0.5 for the rest of the analysis.

We also analyze the impact of changing the $\alpha$ values and inspection cost $\Gamma$ by running a sensitivity analysis. Results can be observed in the heatmaps of Figure 7. This graph is in agreement with the set of possible games analyzed in Figure 1. These results show how the model is sensitive to the values of $\alpha$, which smoothly regulates how the population converges to cooperation or defection and therefore, the difficulty of the game. The impact of inspection cost $\Gamma$ is less significant in the model dynamics when $\alpha$ has either high or low values. In fact, when $\alpha$ values are high (or low) and therefore, cooperation (or defection) is restricted,

![Figure 7](image-url)  
**FIGURE 7** Sensitivity analysis on $\alpha$ and $\Gamma$. $\alpha$ controls the difficulty of the game, directly affecting the cooperation level. $\Gamma$ is the inspection cost when a player is inspected due to mismatch declaration. We see how inspection cost $\Gamma$ is only significant for $\alpha$ values approximately between 0.2 and 0.6. If $\alpha$ is either lower or higher, $\Gamma$ has no impact on the cooperation level.

| **TABLE III** Main features of the social network topologies. |
|-------------------|-------------|---|---|
| **NETWORK** | **AVERAGE DEGREE** | **CC** | **D** |
| REAL NETWORK FROM DATA | 1.9952 | 0 | 111 |
| SF (BA WITH $m = 2$) | 3.0090 | 0.0035 | 12 |
| SF (BA WITH $m = 4$) | 5.0070 | 0.0044 | 9 |
| SF (BA WITH $m = 6$) | 7.0100 | 0.0065 | 9 |
| SF (BA WITH $m = 8$) | 9.0220 | 0.0082 | 8 |
| ASS. SF ($p = 0.5, m = 2$) | 2.9970 | 0.0029 | 20 |
| ASS. SF ($p = 1, m = 2$) | 2.9850 | 0.0120 | 283 |
| DISS. SF ($p = 0.5, M = 2$) | 2.9935 | 0.0016 | 18 |
| DISS. SF ($p = 1, m = 2$) | 2.9860 | 0.0002 | 15 |
| ASS. SF ($p = 0.5, m = 8$) | 9.0040 | 0.0069 | 11 |
| ASS. SF ($p = 1, m = 8$) | 9.0460 | 0.0325 | 53 |
| DISS. SF ($p = 0.5, m = 8$) | 9.0120 | 0.0097 | 8 |
| DISS. SF ($p = 1, m = 8$) | 8.9820 | 0.0005 | 19 |
the consequence of changing $\Gamma$ values is minimal. The role of the inspection cost is significant only when the game has an intermediate level of difficulty and, as expected, increasing inspection cost $\Gamma$ promotes higher cooperation. This means that relying on the inspection cost to promote tax compliance is more worthy when the population has a mixture of cooperators and defectors.

**B. Scale-free Networks Analysis**

Here, we compare the dynamics of different scale-free networks with respect to a well-mixed population. Apart from the fitted real scale-free network, we considered four networks generated by the Barabasi-Albert (BA) algorithm [57] for different values of parameter $m$, controlling the average degree of the networks. Additionally, we used the Xulvi-Brunet-Sokolov algorithm [58] to obtain assortative and disassortative scale-free networks. The assortativity property of networks denotes the preferences of highly connected nodes to be connected with other highly connected nodes [21]. On the other hand, the disassortativity property denotes the preference of highly connected nodes to connect with less connected nodes. Parameter $p$ of the Xulvi-Brunet-Sokolov algorithm is used to control the degree of assortativity and disassortativity of existing scale-free networks. In our case, we applied the algorithm to the most and least dense networks generated by the BA algorithm (i.e., $m = 2$ and $m = 8$). Thanks to these network generation algorithms, we employed 12 networks with diverse clustering coefficient (CC), diameter (D), and density. Table III lists the features of the networks.

Figure 8 shows the dynamics of the model with the networks and a well-mixed population. We can see in the upper plot of Figure 8 that cooperation is non-existent with a well-mixed population, except when $\alpha$ is lower than or equal to 0.2. Note that the expected level of coexistence in the well-mixed population can be analytically derived under some settings of the tax fraud game. In the upper plot of the figure, we can also see that the trends of the BA scale-free networks are similar. Networks with lower density ($m = 2$ and fitted network from data) are able to better promote cooperation when $\alpha$ is increasing (the game is harder). When the game is easy (low $\alpha$ values), higher density is better for achieving total cooperation because it increases the speed of diffusion through the network. These results are in line with the well-mixed population output, which jumps from total defection to total cooperation when the game is easy. This abrupt shift in the model results is in agreement with the observation in Figure 1, where we have two extreme cases (defection and harmony games).
as the most prevalent games for the parameter values.

The lower plot of Figure 8 shows the dynamics of the model with the above assortative and disassortative topologies and a well-mixed population. As observed with low density networks, disassortativity favors cooperation when the game is hard (high values of $\alpha$). Assortativity plays its role in promoting cooperation when the game is easy. We see from Table III that the real network has high diameter values and the clustering coefficient is 0. Therefore, the dynamics of the game with this network is equivalent to neither full assortative nor full disassortative networks (Table III). Instead, the low density and large diameter of the real network explain the slow decay of cooperation for large $\alpha$ values.

C. Balancing between the Subjective Audit Probability for High and Low Transactions

One of the main insights the analysis of real data from the Canarian tax agency revealed was the distinction between two types of transaction volumes: high and low. We would like to find the best policy to promote cooperation and correct tax paying behavior by determining the type of transaction the tax agency must focus on. In order to understand the impact of investigating these types of transactions, we use the evolutionary model to balance the focus on the subjective audit probability—which can be modulated differently depending on the transaction volume. Thus, we set different values for $\Theta_H$ and $\Theta_L$, which changes the construction of the probability linear function. We started from base values of $\Theta_H = 0.5$ and $\Theta_L = 0.5$ and considered a wide range of pairs for analysis, from 0 to 1 for both parameters. Figure 9 has three heatmaps showing the final frequency of cooperators for different subjective audit probabilities. The upper and middle plots show the results when $\alpha$ is equal to 0.2 and 0.4, respectively. The lower plot shows the dynamics when the numbers of high and low transactions are equal (i.e., $prob_{HL}$ is 0.5) and $\alpha = 0.4$.

**FIGURE 9** The upper and middle heatmaps show sensitivity analysis on $\Theta_H$ and $\Theta_L$ for $\alpha = 0.2$ and $\alpha = 0.4$ (real data scenario where $prob_{HL} = 0.02$). The bottom heatmap shows the sensitivity analysis when $prob_{HL}$ is 0.5 and $\alpha = 0.4$ for comparison. We observe that increasing the inspection probability for low transactions is preferable in the real world scenario where $prob_{HL} = 0.02$, but this conclusion does not apply when we have the same number of low and high transactions in the network (see the lower heatmap).
The analysis reflects important variations when modifying these subjective probabilities to favor a particular transaction type. We can see how tax fraud is limited when the subjective probability is higher for low transactions than high transactions. In fact, differences are not relevant when \( \Theta_i > 0.5 \). However, when \( \Theta_i \) decreases, the number of cooperators declines almost independently of \( \Theta_H \). The number of cooperators declines and differences are significant when \( \sigma \) values differ (e.g., cooperators are the dominant strategy in the final population only when \( \sigma = 0.05 \) and the game is easy). The results change when the probability of high and low transactions is equal (Figure 9, lower plot). In this case, there are no major differences in the frequency of cooperators when \( \Theta_L \) and \( \Theta_H \) change. These results show that the significant effect of subjective probability on tax fraud, for low transactions in the upper and middle heatmaps, is mainly due to the larger number of low transactions in the network.

D. Population Diversity in the Subjective Inspection Probabilities

Next, we analyze how diversity in the individuals of the population, with respect to their subjective probabilities, affects cooperation. In order to run this, we considered \( \Theta_H = \Theta_L = 0.5 \) as the mean \( \mu \) of the normal distribution \( N(\mu, \sigma) \) of the subjective probabilities of the whole population, and we modified the standard deviation \( \sigma \) of the distribution. Figure 10 shows the output of seven simulations with different standard deviation \( \sigma \) values: from 0, corresponding to the default configuration of the experiments in this work, to 0.4, where individuals are highly diverse.

Figure 10 shows how population diversity is beneficial for promoting cooperation when the game is hard (high values of \( \sigma \)), but cannot promote cooperation when the game is easy. Similar trends were observed when changing the density and other properties of the networks in Section V-B. This diversity changes the cooperation levels because of the polarization of the entire population, as observed by Antonioni et al. [59]. Figure 10 also reveals that diversity always induces a shift of the population to a 50% polarization (gray horizontal line).

E. Impact of Rewarding and Penalty Policies

In this final section of our model analysis, we focus on ascertaining if policies to increase the reward for cooperators are more efficient than those to increase the punishment for defectors via the fine values. Punishment versus reward has been studied in different public goods games and common pool resources [53], [60], [61]. For our analysis, we increased the values of reward \( R \) from 1 to 2 and fines from 1 (most liberal—defectors just have to return the unpaid tax) to 2 (the fine is double the unpaid quantity). Figure 11 shows the impact of different reward and fine values on cooperation under a sensitivity analysis of \( \alpha \) and for three different scenarios of subjective audit probability \( \Theta_L \) and \( \Theta_H \).

We first observe how the impacts of both reward and fine policies differ depending on the subjective audit probability. When high transactions have a higher subjective audit probability (the third scenario), increasing the reward, \( R \), is more efficient for promoting cooperation than increasing the fine, \( \phi \). The highest cooperation percentage is obtained with \( R = 2 \) while keeping the base fine of 1.5. However, for all the \( \sigma \) values, increasing the fine up to 2 does not generally induce as many new cooperators as rewarding policies do.

When the subjective audit probability for low transactions is higher than for high transactions, the output of the model changes dramatically. If we are in the balanced second scenario \( (\Theta_H = 0.5 \) and \( \Theta_L = 0.5 \)), penalizing defectors...
FIGURE 11 Analysis of the reward $R$ (upper plots) and fine $\phi$ (bottom plots) parameters based on a sensitivity analysis on $\alpha$ for three subjective audit probability scenarios ($\Theta_L = 0.8; \Theta_H = 0.2$ in the first column, $\Theta_L = \Theta_H = 0.5$ in the second column, and $\Theta_L = 0.2; \Theta_H = 0.8$ in the third column). We can see how the impacts of reward and punishment strategies depend on the scenario.
with high fines is the most convenient option for promoting cooperation. In the first scenario, rewarding cooperators is almost invariant for the model dynamics. Therefore, both reward and punishment strategies must be carried out together while balancing the focus on either high or low transactions or, at least, apply them depending on the current scenario.

VI. Final Discussion

We presented the first evolutionary game model for consumption taxes. This model represents cooperators and defectors and includes parameters to penalize tax evaders. It also considers the subjective probability of being inspected by the tax agency, which can be modulated with the size of the economic transaction. Players are linked through a scale-free network. Both the network topology and most of the model’s parameters are fed with real data from the Canarian tax agency.

The stability and robustness of the model were demonstrated by simulating the effects of the undetermined quantity (α parameter), initial distribution of the population, and convergence to a steady state. After illustrating the main dynamics, we evaluated the two main questions for the tax agencies. First, we explored whether the agencies must focus on high or low transactions. We found that it is better to increase pressure on low transactions rather than high transactions. This result is in line with previous work. But the level of this pressure on high or low transactions. We found that whether the agencies must focus on high or low transactions. For example, one can study how temporal changes on the type of transaction between players (i.e., $d_0$) can influence the game output as the employed payoff matrix would also change over time. A more comprehensive study about diversity in the subjective inspection probabilities of the individuals can be performed as well. Finally, a CI algorithm could identify the most influential companies (nodes) to be targeted with specific policies such as in Robles et al. [62].

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“Real-world optimization problems, such as aerodynamic design of turbine engines and automated trading, have been successfully solved by metaheuristics. However, practitioners are confronted with the challenge of how to choose an appropriate metaheuristic algorithm to solve a particular instance of these problems. This paper proposes a recommender system that can automatically select a best-suited metaheuristic algorithm without trial and error on a given problem. The proposed method develops a generic tree-like data structure for representing the difficulties of optimization problems and then trains a deep recurrent neural network to learn to choose the best metaheuristic algo-

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rium, making automated algorithm recommendation practical for real-world problem-solving. The method will make metaheuristic optimization techniques accessible to industrial practitioners, policy makers, and other stakeholders who have no knowledge in metaheuristic algorithms.”

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