An Optimized Design Method of PSWFs Pulse Group Based on DFT Precoding

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Abstract. Aiming at the problems of non-sinusoidal time-domain multiple quadrature modulation signals based on Prolate Spheroidal Wave Functions (PSWFs), as the number of pulses increases, the bandwidth efficiency of the modulation signal frequency band decreases, the Peak-to-Average Power Ratio (PAPR) and the implementation complexity are high, a method for optimal design of PSWFs pulse groups based on DFT precoding is proposed. The PSWFs pulse signal frequency domain generation method is adopted. By introducing the precoding matrix, the multi-channel PSWFs pulse group signals are optimized and designed, and then the input data is redistributed to obtain new PSWFs pulse group signals. Experimental results show that this method can further improve the power efficiency of the modulated signal and effectively reduce the peak-to-average power ratio of the modulated signal without changing the orthogonality of the original PSWFs subcarriers. As the number of carriers increases, the suppression effect on the peak-to-average power ratio becomes more obvious.

Keywords: PSWFs, Precoding, Power efficiency, PAPR

1. Introduction
Prolate Spheroidal Wave Functions (PSWFs) have excellent basic characteristics such as high energy aggregation in the time-frequency domain, biorthogonality in the time domain, approximate time band limitation, and controllable frequency spectrum [1-4], since its introduction, it has received extensive attention from academia, and it has been widely used in signal analysis and communication systems [5-7]. Non-sinusoidal orthogonal modulation system based on prolate spheroidal wave function [8] is a typical non-sinusoidal multi-carrier communication system. It uses the orthogonality between prolate spheroidal wave functions to transmit information in parallel, which can effectively improve the system's band utilization. However, with the increase of the number of PSWFs pulses, on the one hand, it will cause the main lobe expansion and side lobe of the power spectrum to be higher, and the power efficiency and frequency bandwidth efficiency of the system will be reduced. On the other hand, the peak-to-average power ratio of the PSWFs modulated signal is higher [9], which is susceptible to the nonlinear characteristics of the power amplifier, causing nonlinear distortion of the signal, thereby reducing system performance.
Discrete prolate spheroidal wave sequence (DPSS) [10] is a discrete representation of prolate spheroidal wave functions (PSWFs). DPSS is the sequence with the highest energy concentration and spectrum concentration, and has many application values in signal analysis. The precoding technology is a technology that uses channel state information to preprocess the signal to be transmitted when the transmitter knows or can obtain channel state information, this can further increase the throughput capacity of the system. In a multi-user MIMO system, the precoding technology can effectively suppress the multi-user interference of the MIMO channel, and can greatly reduce the complexity of the receiver [11]. In this paper, combined with the discrete Fourier transform (DFT) precoding technology, taking full advantage of the excellent basic characteristics of the prolate spheroidal wave function, using frequency domain modulation, a DFT precoding-based PSWFs pulse group optimization design method, that is, DFT extended PSWFs (which is ‘DFT-S-PSWFs’). DFT-S-PSWFs is a signal-based distortion-free processing method, which effectively reduces the instantaneous power fluctuation during multi-carrier transmission, making the peak-to-average power ratio (PAPR) of PSWFs modulated signals about 2dB lower. For the spectrum of PSWFs modulated signals, the bandwidth will not affect, and the optimized new PSWFs pulse group has the advantages of lower computational complexity, power spectral density to meet the requirements of signal transmission energy and lower PAPR compared to before optimization. Thanks to the high time-frequency energy aggregation of the PSWFs signal, the DFT-S-PSWFs system can have higher spectral efficiency and power efficiency than the OFDM system.

2. DFT-S-PSWFs system principle

The optimized design of the DFT-S-PSWFs pulse group proposed in this paper is based on the frequency domain generation method, generate baseband $m$-th order PSWFs pulse group through DPSS, load baseband mapping data information, and pre-code PSWFs pulse group sequence through DFT precoding matrix method[12] to obtain DFT extended PSWFs (DFT-S-PSWFs) optimized pulse group sequence, after inverse fast Fourier transform (IFFT) modulation, the system transmits the time domain signal, avoiding the problem of high PAPR caused by transmitting the frequency domain signal. The information received at the receiving end is converted to the frequency domain signal by Fast Fourier Transform (FFT), using a coherent demodulation method, multiplied with the loaded optimized pulse group, demodulating the precoded data information, and then obtaining the input data by baseband demapping. The principle block diagram of PSWFs system (DFT-S-PSWFs) based on DFT precoding is shown in Figure 1. The introduction of the precoding matrix is equivalent to further power allocation of the mapped data, thereby reducing the PAPR of the PSWFs signal. This method does not affect the spectral efficiency of the system, and does not require additional operations and handshake information at the transceiver end.

Like the OFDM modulation subcarrier, when the phase of the PSWFs modulation pulse group sequence is the same, the superposition of the modulation symbol peaks will produce higher peak power. Improving the correlation between $d_i$ of modulation symbols can reduce PAPR. Construct a precoding matrix consistent with the order of PSWF pulse generation, and introduce it into the correlation between subcarriers of PSWFs, this process is also equivalent to encoding the baseband binary data. The DFT precoding matrix acts on the PSWFs pulse group to form a new DFT-S-PSWFs optimized pulse group. The function of the precoding matrix is equivalent to shaping the subcarriers (pulses) of the original PSWFs in the frequency domain.

The DFT matrix can be expressed as:

$$
P = \frac{1}{\sqrt{M}} \begin{pmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & W_1 & W_2 & \cdots & W_{M-1} \\
1 & W_2 & W_4 & \cdots & W_{2(M-1)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & W_{M-1} & W_2(M-1) & \cdots & W_{(M-1)(M-1)}
\end{pmatrix}
$$
Among them, $M$ is the number of subcarriers, and the DFT precoding matrix itself has the characteristic that each row is orthogonal to each other, and the modulus of matrix elements is 1, that is, satisfy $P^*P=I$, only in this way can data be correctly restored when demodulating at the receiving end. Where matrix $I$ is the $M \times M$ identity matrix and $P^*$ is the conjugate to rank of the precoding matrix $P$, the characteristic of this matrix is to ensure that there is no (distortion) interference to the original PSWFs carrier signal. The DFT precoding matrix used in this paper is obtained by Fourier transform through the unit matrix of $M \times M$ diagonal of $I$[13], which is very simple, and the dimension of the precoding matrix is consistent with the number of PSWFs baseband carriers. The new PSWFs pulse group formed according to this method can directly modulate the data symbols.

3. Performance analysis of DFT-S-PSWFs

3.1. Modulation signal orthogonality

The time-bandwidth product $c$ of PSWFs is 70Hz·s, the number of subcarriers is 64, the precoding matrix is $64 \times 64$, the symbol sampling period $T_s$ is 1s, and the number of sampling points per symbol period is 1024. Table 1 shows the cross-correlation values of the first 6 orders of DFT-S-PSWFs (take the real part).

It can be seen from Table 1 that the DFT-S-PSWFs carrier pulse groups of each order basically maintain the orthogonality performance between the subcarrier pulse groups of the original PSWFs, Moreover, from the point of view of the correlation data magnitude of the PSWFs of each order after precoding (a difference of more than $10^3$), from the perspective of engineering practice, the carrier signal orthogonality remains good. The significance of carrier modulation is that it can load data information without distortion interference.

![Figure 1. Schematic diagram of PSWFs system based on DFT precoding](image-url)

| $u_0(t)$ | $u_1(t)$ | $u_2(t)$ | $u_3(t)$ | $u_4(t)$ | $u_5(t)$ |
|----------|----------|----------|----------|----------|----------|
| $u_0(t)$ | 1        | -4.8×10^{-16} | 2.6×10^{-16} | 1.9×10^{-15} | 3.4×10^{-16} | 4.1×10^{-16} |
| $u_1(t)$ | -4.8×10^{-16} | 1        | -2.3×10^{-16} | 1.1×10^{-16} | 1.8×10^{-15} | 3.4×10^{-16} |
| $u_2(t)$ | 2.6×10^{-16} | -2.3×10^{-16} | 1        | -3.6×10^{-16} | 1.2×10^{-15} | 1.8×10^{-15} |
| $u_3(t)$ | 1.9×10^{-15} | 1.1×10^{-16} | -3.6×10^{-16} | 1        | -3.7×10^{-16} | 3.2×10^{-16} |
| $u_4(t)$ | 3.4×10^{-16} | 1.8×10^{-15} | 1.2×10^{-16} | -3.7×10^{-16} | 1        | -4.8×10^{-16} |
| $u_5(t)$ | 4.1×10^{-16} | 3.4×10^{-16} | 1.8×10^{-15} | 3.2×10^{-16} | -4.8×10^{-16} | 1        |
3.2. Modulated signal power spectrum

One of the shortcomings of PSWFs as multi-carrier transmission is the large change in instantaneous transmission power. This means that the efficiency of the power amplifier is reduced, and it leads to higher power consumption of the mobile terminal. The conventional PSWFs system is combined with precoding based on discrete Fourier transform (DFT) to realize the optimal design of PSWFs pulse group. By adopting the method based on DFT precoding, the original PSWFs multi-carrier system greatly reduces the fluctuation of the transmission power, and the peak value of the modulated signal power is generally low.

![Figure 2. PSWFs modulated signal PSD before and after DFT precoding](image)

When the pulse group parameter of PSWFs is set to the time bandwidth product \( c \) is 70Hz·s, the pulse (carrier) symbol period \( T_s \) is 1s, and the sampling point of each symbol period is 1024, before and after the 64 baseband PSWFs carrier pulses are subjected to DFT precoding PAPR suppression, the power spectral density (PSD) of the modulated signal is shown in Figure 2. Through simulation analysis, it can be seen that the PSWFs modulated signal PSD before and after DFT precoding has basically not changed, the peak power after DFT precoding is lower than the peak power of the original modulated signal. Moreover, the first side lobe attenuation of the original PSWFs modulated signal is about -47.49dB, the DFT pre-coded PSWFs modulated signal out-of-band first side lobe attenuation is about -54.41dB. The energy concentration of the DFT-S-PSWFs modulated signal remains basically the same, and the attenuation of the first out-of-band sidelobe has decreased. The overall level of out-of-band attenuation is lower than before DFT precoding.

3.3. Modulated signal peak-to-average power ratio

The modulation signal PAPR of PSWFs is closely related to the number of channels. As the number of channels increases, the modulation signal PAPR will continue to increase. The PAPR performance of the pulse group under different time bandwidth products and different carrier numbers before and after the DFT precoding matrix is simulated and analyzed. Complementary cumulative distribution function (CCDF) is commonly used to measure the PAPR indicator of DFT-S-PSWFs signals.
For the time-bandwidth product $c$ is 36Hz·s, the pulse width $T_s$ is 1s, $M$ is the number of subcarriers, a total of 32-order baseband orthogonal PSWFs signals are generated, and the DFT precoding matrix is 32×32. When CCDF=10^{-4}, the system frequency band utilization rate is 1.77bit/s/Hz, using the PAM baseband mapping method, and before and after the DFT precoding matrix, the PAPR of the PSWFs modulated signal is 9.05dB and 7.23dB. Compared with the original modulated signal, its PAPR is reduced by about 1.82dB after DFT precoding. Here, set the PSWFs time-bandwidth product $c$ is 70Hz·s, the pulse (carrier) symbol period is $T_s$ is 1s, and generate $M$ is 64 order baseband PSWF signals as the carrier for PAPR suppression, each baseband pulse PSWFs samples 1024 points, discrete Fourier transform (DFT) precoding of the 64th order baseband PSWF carrier signal, a 64×64 DFT precoding matrix is generated, using the PAM mapping method, and the CCDF of the PSWFs modulated signal before and after DFT precoding is shown in Figure 4. When CCDF=10^{-4}, the system band utilization is 1.83bit/s/Hz, the PAPR of the PSWFs modulated signals before and after DFT precoding are 9.46dB and 7.50dB, respectively, and the PAPR is reduced by about 1.96dB.

![Figure 3](image3.png)

**Figure 3.** $c=36\text{Hz}\cdot\text{s}, M=32$ PAPR performance comparison of PSWFs modulated signal before and after DFT precoding

![Figure 4](image4.png)

**Figure 4.** $c=70\text{Hz}\cdot\text{s}, M=64$, PAPR performance comparison of PSWFs modulated signal before and after DFT precoding
As the number of subcarriers increases, the higher the modulation signal PAPR, the more effective the suppression effect of this method. At the same time, under the same system band efficiency conditions, the PAPR performance of the large time bandwidth product DFT-S-PSWFs signal is better than that of the small time bandwidth product DFT-S-PSWFs signal.

4. Conclusion
The optimized design method of PSWFs pulse group based on DFT precoding proposed in this paper is a distortion-free orthogonal transformation of PSWFs pulse group. And before and after the transformation, the orthogonality of PSWFs carrier pulse group, the power spectral density of the modulated signal and the energy aggregation have basically not changed, with this method, before and after DFT precoding, the effectiveness and reliability of the system are not affected. And the power spectrum and PAPR performance of the modulated signal are better than the original PSWFs modulated signal system, as the number of carriers increases, the PAPR value of the modulated signal will be further reduced. Combining the DPSS sequence with the highest energy concentration and spectrum concentration, the method of generating PSWFs signal pulse groups in the frequency domain further reduces the complexity of system implementation. It can effectively improve the efficiency of linear power amplifier in multi-carrier communication system, and provide useful reference value for engineering application based on PSWFs signal in multi-carrier mobile communication system.

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