Distributed Detection over Fading MACs with Multiple Antennas at the Fusion Center

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Abstract

A distributed detection problem over fading Gaussian multiple-access channels is considered. Sensors observe a phenomenon and transmit their observations to a fusion center using the amplify and forward scheme. The fusion center has multiple antennas with different channel models considered between the sensors and the fusion center, and different cases of channel state information are assumed at the sensors. The performance is evaluated in terms of the error exponent for each of these cases, where the effect of multiple antennas at the fusion center is studied. It is shown that for zero-mean channels between the sensors and the fusion center when there is no channel information at the sensors, arbitrarily large gains in the error exponent can be obtained with sufficient increase in the number of antennas at the fusion center. In stark contrast, when there is channel information at the sensors, the gain in error exponent due to having multiple antennas at the fusion center is shown to be no more than a factor of $8/\pi$ for Rayleigh fading channels between the sensors and the fusion center, independent of the number of antennas at the fusion center, or correlation among noise samples across sensors. Scaling laws for such gains are also provided when both sensors and antennas are increased simultaneously. Simple practical schemes and a numerical method using semidefinite relaxation techniques are presented that utilize the limited possible gains available. Simulations are used to establish the accuracy of the results.

I. INTRODUCTION

Sensors are becoming commonplace in factories, environmental, and home appliance monitoring, as well as in scientific study. In many such applications, a number of independent sensors each make a local observation, which are transmitted to a fusion center (FC) after limited initial processing at the sensors, and combined at the FC to calculate a global decision [1]. Sensors may adopt either a digital or an analog method for relaying the sensed information to the FC. The digital method consists of quantizing the sensed data and transmitting the digital data over a rate-constrained channel [2]. In these cases, the required channel bandwidth is quantified by the number of bits being transmitted between the sensors and the FC. In contrast, the analog method consists of amplifying and then forwarding the sensed data to the FC, while respecting a power constraint [3], [4]. The transmissions can be appropriately pulse-shaped and amplitude modulated to consume finite bandwidth. The channels between the sensors and the FC can be orthogonal, in which case, the transmissions from each sensor are separately received at the FC [2]. On the other hand, with multiple-access channels between the sensors and the FC, the noisy sum of all the transmissions are received at the FC to make a decision [3], [5]–[8]. The bandwidth requirements of sensor networks with orthogonal channels scale linearly with the number of sensors, whereas, when the channels are multiple-access, transmissions are simultaneous and in the same frequency band, keeping the utilized bandwidth independent of the number of sensors in the network.

Distributed detection problems have been mainly studied assuming a single receive antenna at the FC. It is possible that introducing multiple antennas at a receiver may overcome the degradations caused by multi-path fading and noise. Inspired by conventional MIMO systems, a natural question is how much performance gain can be expected from adding multiple antennas at the FC in a distributed detection

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Adding multiple antennas to the FC for distributed detection problems is different when compared to the analysis of conventional MIMO systems for two reasons: (i) the presence of sensing noise (the parameter of interest is corrupted before transmission); and (ii) a large number of sensors enable asymptotic analysis. To the best of our knowledge, references [9], [10] are the only works that consider multiple antennas at the FC. In [9], a decision fusion problem with binary symmetric channels between the users and the FC is considered where the data are quantized at the sensors, transmitted over parallel channels, and processed after being received by three antennas. In [10], the authors consider multiple antennas at the FC. However, they consider a set of deterministic gains for the orthogonal channels, known at the sensors. They do not consider multiple-access channels, or characterize the performance benefits of adding antennas at the FC in the presence of fading. The system models in [11]–[16] are similar to adding multiple antennas at the FC, where the authors consider other forms of diversity, such as independent frequencies, CDMA codewords or several time intervals over fast-time-varying channels. The main difference between these papers and our results is the fact that we use asymptotic techniques to investigate the benefits of adding multiple antennas at the fusion center, when the number of sensors grows large. We show that the gain on the error exponent by adding antennas to the FC when there is no CSI at the sensors grows linearly with the number of antennas. In stark contrast, when there is CSI at the sensors, only limited gains are possible by adding antennas at the FC. This is unlike what we see in traditional MIMO wireless communications, where adding antennas at the FC will result either in diversity gain or array gain, for asymptotically large SNRs.

In this paper, a distributed detection problem over a multiple access channel, where the FC has multiple antennas is considered (Figure 1). The data collected by the sensors are transmitted to the FC using the amplify and forward scheme, with a total power constraint on the sensor gains. Performance is evaluated when the sensors have no channel information, have full channel information and partial channel information in the presence of fading, both with zero and non-zero mean. Analysis is performed for two cases: (a) large number of sensors and a fixed number of antennas, and (b) large number of antennas and sensors with a fixed ratio. In each case, the error exponent is used as the metric to quantify performance through the effect of channel statistics and the number of antennas. It is shown that the system performance depends on the channel distribution through its first and second order moments. This information is used to address our main objective, which is to quantify the gain possible by adding multiple antennas at the FC over fading multiple-access channels for distributed detection problems.

II. SYSTEM MODEL

A sensor network, illustrated in Figure 1 consisting of $L$ sensors and a fusion center with $N$ antennas is considered. The sensors are used to observe a parameter $\Theta \in \{0, \theta\}$. The value, $x_l$, observed at the $l^{th}$
sensor is
\[
x_l = \begin{cases} 
\eta_l & \text{under } H_0 \\
\theta + \eta_l & \text{under } H_1 
\end{cases}
\tag{1}
\]
for \(l = 1, \ldots, L\). It is assumed that \(\eta_l \sim \mathcal{CN}(0, \sigma^2_\eta)\) are iid, the hypothesis \(H_1\) occurs with a priori probability, \(0 < p_1 < 1\), and the hypothesis \(H_0\) with probability \(p_0 = 1 - p_1\). The \(l^{th}\) sensor applies a complex gain, \(\alpha_l\), to the observed value, \(x_l\). This amplified signal is transmitted from sensor \(l\) to antenna \(n\) over a fading channel, \(h_{nl}, n = 1, \ldots, N,\) and \(l = 1, \ldots, L\), which are iid and satisfy \(E[|h_{nl}|^2] = 1\). Unless otherwise specified, no other assumptions are made on the channel distribution. The \(n^{th}\) antenna receives a superposition of all sensor transmissions in the presence of iid channel noise, \(\nu_n \sim \mathcal{CN}(0, \sigma^2_\nu)\), such that
\[
y_n = \sum_{i=1}^{L} h_{in} \alpha_i (\Theta + \eta_i) + \nu_n,
\tag{2}
\]
where \(\{\eta_i\}_{i=1}^{L}\) and \(\{\nu_n\}_{n=1}^{N}\) are independent.

Defining \(\alpha\) as an \(L \times 1\) vector containing \(\{\alpha_i\}_{i=1}^{L}\), \(D(\alpha)\) an \(L \times L\) diagonal matrix with the components of \(\alpha\) along the diagonal, the received signal is expressed in vector form as
\[
y = H\alpha\Theta + HD(\alpha)\eta + \nu,
\tag{3}
\]
where \(H\) is an \(N \times L\) matrix containing the elements \(h_{nl}\) in the \(n^{th}\) row and \(l^{th}\) column, \(\eta\) is an \(L \times 1\) vector containing \(\{\eta_i\}_{i=1}^{L}\), and \(\nu\) is an \(N \times 1\) vector containing \(\{\nu_n\}_{n=1}^{N}\). Based on the received signal, \(y\) (from (3)), the FC decides on one of the two hypotheses \(H_0\) or \(H_1\). Since the FC has full knowledge of \(H\) and \(\alpha\), \(y\) is Gaussian distributed under both hypotheses:
\[
H_0 : y \sim \mathcal{CN}(0_N, R(\alpha)) \\
H_1 : y \sim \mathcal{CN}(\theta H\alpha, R(\alpha))
\tag{4}
\]
where \(0_N\) is an \(N \times 1\) vector of zeros and \(R(\alpha)\) is the \(N \times N\) covariance matrix of the received signal given by
\[
R(\alpha) = \sigma^2_\eta H D(\alpha) D(\alpha)^H H^H + \sigma^2_\nu I_N.
\tag{5}
\]
We consider detection at a single snapshot in time, and therefore, we do not have a time index.

A. Power Constraint

The \(i^{th}\) sensor transmits \(\alpha_i (\Theta + \eta_i)\). The total transmitted power is given by
\[
P_T = E \left[ \sum_{i=1}^{L} |\alpha_i (\Theta + \eta_i)|^2 \right] = (p_1 \theta^2 + \sigma^2_\eta) \sum_{i=1}^{L} |\alpha_i|^2.
\tag{6}
\]
It should also be noted here that the instantaneous transmit power from the sensors is \(|\alpha_i (\theta + \eta_i)|^2\). This is a function of the actual realizations of sensing noise, making it difficult to predict and constrain. Therefore, we constrain \(\alpha_i\)’s, which allows imposing an average (over sensing noise) power constraint. The sensor gains, \(\{\alpha_i\}\), are constrained by
\[
P := \sum_{i=1}^{L} |\alpha_i|^2 = \frac{P_T}{p_1 \theta^2 + \sigma^2_\eta}.
\tag{7}
\]
B. The Detection Algorithm and its Performance

Given the received data, \( y \), the FC selects the appropriate hypothesis according to

\[
\Re\{\theta y^H R(\alpha)^{-1} H \alpha \} \geq \frac{1}{R_0} \theta^2 \alpha^H H^H R(\alpha)^{-1} H \alpha + \tau,
\]

where \( \tau \) is a threshold that can be selected using the Neyman-Pearson or the Bayesian approach. Using (4) and (5), and the Bayesian test with the detection threshold, \( \tau = (1/2) \ln(p_0/p_1) \), the probability of error conditioned on the channel can be calculated as

\[
P_{e|H}(N) = p_0 Q(\omega + \tau/\omega) + p_1 Q(\omega - \tau/\omega),
\]

where \( \omega := \theta \sqrt{\alpha^H H^H R(\alpha)^{-1} H \alpha} / 2 \) for brevity, \( N \) is the number of antennas at the FC and \( Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \). The error exponent is defined in terms of the conditional error probability for the FC with \( N \) antennas as [15], [17], [18]

\[
\mathcal{E}(N) = \lim_{L \to \infty} -\frac{1}{L} \log P_{e|H}(N).
\]

Note that even though \( P_{e|H}(N) \) in (9) is a channel-dependent random variable, we will show that the limit in (10) converges in probability to a deterministic constant for the cases of interest to us. Substituting (9) into (10), using L'Hôpital’s rule, and the Leibniz Integral rule for differentiating under the integral sign,

\[
\mathcal{E}(N) = \lim_{L \to \infty} -\frac{1}{8L} \frac{1}{\tau/\omega} \theta^2 \alpha^H H^H R(\alpha)^{-1} H \alpha
\]

in probability, which does not depend on \( p_0 \) and \( p_1 \). Since \( \mathcal{E}(N) \) is the negative exponent of the probability of error, a larger value represents better performance. The error exponent in (11) is a deterministic performance metric over fading channels and depends on fading statistics. It can also be viewed as a “generalized SNR” expression in this system with multiple sensor and channel noise sources. We follow [15], [17], [18] in our definition of the error exponent in (10). Alternatively, one can consider the unconditional error exponent, \( E_H[P_{e|H}(N)] \), which would depend on the distribution of \( H \) in (10), in place of \( P_{e|H}(N) \). We will not pursue this approach herein.

Our primary focus throughout this paper is the dependence of (11) on

(i) the number of antennas, \( N \), for different fading-channel distributions;
(ii) different assumptions about the dependence of the sensor gains, \( \alpha \), on the channel, \( H \).

With the Neyman-Pearson test, rather than the Bayesian test, it can be shown that the error exponent is given by \( \lim_{L \to \infty} 0.5L^{-1} \theta^2 \alpha^H H^H R(\alpha)^{-1} H \alpha \), which does not depend on the false alarm probability and is a factor of four greater than the error exponent derived in the Bayesian case. Since the two cases differ only by a fixed constant, the Bayesian approach will be used throughout.

III. Performance over AWGN Channels

The error exponent with AWGN channels is computed to establish a benchmark for the fading case of the next section, which is our main focus. For AWGN channels, \( h_{nl} = 1 \). Due to symmetry and to respect the power constraint, \( \alpha_i = \sqrt{P/L} \forall i \). Defining \( 1_L \) as an \( L \times 1 \) vector of ones, and \( 1_{N \times L} \) as an \( N \times L \) matrix of ones, we have \( \alpha = \sqrt{P/L} 1_L \) and \( H = 1_{N \times L} \). Substituting these in (5),

\[
R := R(\sqrt{P/L} 1_L) = \sigma_n^2 P 1_{N \times N} + \sigma_c^2 1_N.
\]

The inverse of (12) can be expressed using the Sherman-Morrison-Woodbury formula for matrix inversion and substituted into (11) to yield

\[
\mathcal{E}_{AWGN}(N) := \frac{1}{8N \gamma_c} \frac{N \gamma_s \gamma_c}{p_1 \gamma_s + 1}.
\]
where the sensing SNR is defined as \( \gamma_s := \theta^2 / \sigma_n^2 \), and the channel SNR, \( \gamma_c := P_T / \sigma_p^2 \). Since the partial derivative \( \partial \mathcal{E}_{AWGN}(N) / \partial N > 0 \), for the AWGN case, having multiple antennas improves the error exponent which can be interpreted as array gain on the channel SNR \( \gamma_c \). As a special case, consider \( N = 1 \), to get the result for the single antenna case:

\[
\mathcal{E}_{AWGN}(1) = \frac{1}{8} \frac{\gamma_c \gamma_s}{\gamma_c + p_1 \gamma_s + 1}.
\] (14)

With \( p_1 = 0.5 \), \( \gamma_s = 1 \) and \( \gamma_c = 1 \), adding a second antenna at the FC provides a gain of 3.1dB. Adding a third antenna provides a further gain of 1.34dB, indicating diminishing returns. To study the benefits of having multiple antennas, we compare the error exponent in each case with \( \mathcal{E}_{AWGN}(1) \). The multiple antenna gain for the AWGN case is given by

\[
G_{AWGN}(N) := \frac{\mathcal{E}_{AWGN}(N)}{\mathcal{E}_{AWGN}(1)} = \frac{N \gamma_c + N p_1 \gamma_s + N}{N \gamma_c + p_1 \gamma_s + 1}.
\] (15)

It can be seen from (15) that by making \( N \) sufficiently large, and \( \gamma_c \) sufficiently small, (15) can be made arbitrarily large. In contrast, it will be seen in Section IV-B that when the channels are fading and known at the sensors, the corresponding gain expression will be bounded for all parameter values, indicating limited gains due to antennas.

### IV. Performance over Fading Channels

Suppose that the elements of \( \mathbf{H} \) are non-zero-mean: \( h_{nl} = \sqrt{K / (K + 1)} + (1 / \sqrt{K + 1}) h_{nl}^{diff} \), where the first term is the line-of-sight (LOS) component, \( h_{nl}^{diff} \) is the zero-mean diffuse component, and the parameter \( K \) is the ratio of the LOS power to the power of the diffuse component, chosen so that the channel satisfies \( E[|h_{nl}|^2] = E[|h_{nl}^{diff}|^2] = 1 \).

In what follows, different cases of channel state information at the sensors (CSIS) are considered.

#### A. No Channel State Information at the Sensors

When the sensors have no channel knowledge, then the sensor gains are set to \( \alpha = \sqrt{P / L} \mathbf{1}_L \) due to the i.i.d. nature of the channels and to respect the power constraint in (7). Substituting in (5),

\[
\mathbf{R} := \mathbf{R}(\sqrt{P / L} \mathbf{1}_L) = \sigma_n^2 P \frac{1}{L} \mathbf{H}^H \mathbf{H} + \sigma_p^2 \mathbf{I}_N.
\] (16)

Since the elements of \( \mathbf{H} \) are i.i.d., from the weak law of large numbers,

\[
\lim_{L \to \infty} \mathbf{R} = \sigma_n^2 P \frac{K}{K + 1} \mathbf{1}_{N \times N} + \frac{\sigma_p^2 P + \sigma_p^2 (K + 1)}{K + 1} \mathbf{I}_N,
\] (17)

in probability. Since the right-hand-side of (17) is non-singular, \( \lim_{L \to \infty} \mathbf{R}^{-1} = (\lim_{L \to \infty} \mathbf{R})^{-1} \) [19] Thm. 2.3.4. Using the matrix inversion lemma on (17) and substituting into (11),

\[
\mathcal{E}_{NoCSIS}(N, K) = \frac{\theta^2}{8} \frac{P(K + 1)}{\sigma_n^2 P + \sigma_p^2 (K + 1)} \left( \lim_{L \to \infty} \sum_{n=1}^{N} \left| \frac{1}{L} \sum_{l=1}^{L} h_{nl} \right|^2 \right)
- \frac{\theta^2}{8} \frac{\sigma_p^2 P^2 K (K + 1)}{\left[ \sigma_n^2 P + \sigma_p^2 (K + 1) \right] \left[ \sigma_n^2 P N K + \sigma_p^2 P + \sigma_p^2 (K + 1) \right]} \lim_{L \to \infty} \left| \frac{1}{L} \sum_{n=1}^{N} \sum_{l=1}^{L} h_{nl} \right|^2.
\] (18)

Using the weak law of large numbers and (7), the error exponent can be expressed in terms of \( \gamma_c \) and \( \gamma_s \) as

\[
\mathcal{E}_{NoCSIS}(N, K) := \frac{1}{8} \frac{NK \gamma_c \gamma_s}{\gamma_c (NK + 1) + (p_1 \gamma_s + 1) (K + 1)},
\] (19)
which can be shown to be a monotonically increasing function of \( N, K, \gamma_s \) and \( \gamma_c \), as expected. For the single antenna case, using (14) we have \( E_{\text{NoCSIS}}(1, K) = E_{\text{AWGN}}(1)K/(K + 1) \), which is a factor \( K/(K + 1) \) worse than \( E_{\text{AWGN}}(1) \).

As the antennas increase, \( \lim_{N \to \infty} E_{\text{NoCSIS}}(N, K) = \gamma_s/8 \), which is the same as \( \lim_{N \to \infty} E_{\text{AWGN}}(N) \). That is, so long as there is some non-zero LOS component, as the number of antennas at the FC increases, the performance approaches the AWGN performance even in the absence of CSI at the sensors. Furthermore, it can be seen that \( \lim_{K \to \infty} E_{\text{NoCSIS}}(N, K) = E_{\text{AWGN}}(N) \), which matches the AWGN result, as expected.

To characterize the gain due to having multiple antennas at the FC, we define

\[
G_{\text{NoCSIS}}(N, K) := \frac{E_{\text{NoCSIS}}(N, K)}{E_{\text{NoCSIS}}(1, K)} = \frac{N(K + 1)(\gamma_c + \gamma_s + 1)}{\gamma_c(NK + 1) + (\gamma_s + 1)(K + 1)};
\]  

(20)

When the channel noise is large, \( (\gamma_c \to 0) \), we have \( G_{\text{NoCSIS}}(N, K) = N \) and the gain increases with the number of antennas at the FC. However, when \( \gamma_c \to 0 \), the absolute performance of the system is poor, as can be verified by substituting in (19). Conversely, when the channel SNR grows, the maximum gain in (20) is given by \( (K + 1)/K \). This leads to the conclusion that when the channels between the sensors and the FC are relatively noise-free, there is little advantage in having multiple antennas at the FC when \( K \) is large. When the channel is zero-mean \( (K = 0) \), the error exponent in (19) is zero for any \( N \), indicating that the probability of error does not decrease exponentially with \( L \) for any \( N \), confirming results from [3], [20], [21]. However, from (20), it is clear that the gain satisfies \( \lim_{K \to 0} G_{\text{NoCSIS}}(N, K) = N \), which shows that when the channel is zero-mean, gain in the error exponent due to antennas is linear and can be made arbitrarily large. We have thus established the following:

**Theorem 1:** For zero-mean channels, with no CSI at the sensors, the error exponent in (19) is zero and therefore, the error probability does not decrease exponentially with \( L \) for any \( N \), confirming results from [3], [20], [21]. However, from (20), it is clear that the gain satisfies \( \lim_{K \to 0} G_{\text{NoCSIS}}(N, K) = N \), implying unlimited gains from multiple antennas for zero-mean channels when CSI is unavailable at the sensors.

In what follows, it will be seen that when CSI is available at the sensors, the antenna gain is bounded over all parameter values for zero-mean channels.

**B. Channel State Information at the Sensors**

We have just seen that when the non-zero-mean channel assumption does not hold, the incoherent sum of signals at each antenna leads to poor performance at the FC, which results in a zero error exponent. If channel information is available at the sensors, the sensor gains can be adjusted in such a way that the signals are combined coherently. It should be noted here that full CSI at the sensors implies full CSI of the network, \( \mathbf{H} \), at the sensors. In such a case, \( \alpha \) is chosen as a function of the channels, \( \mathbf{H} \).

As a benchmark result for fading channels, the sensor gains are selected in such a way as to maximize the error exponent of the system given in (11), subject to the power constraint in (7):

\[
\alpha_{\text{OPT}} = \arg \max_{\alpha} [\alpha^H \mathbf{H}^H \mathbf{R}(\mathbf{\alpha})^{-1} \mathbf{H} \alpha] \quad \text{subject to} \quad \|\alpha\|^2 \leq P,
\]  

(21)

to obtain the error exponent in the presence of CSIS,

\[
E_{\text{CSIS}}(N) = \lim_{L \to \infty} \frac{\theta^2}{8} \frac{1}{L} \mathbf{\alpha}_{\text{OPT}}^H \mathbf{H}^H \left( \sigma_v^2 \mathbf{H} \mathbf{D}(\mathbf{\alpha}_{\text{OPT}})^H \mathbf{H} + \sigma_p^2 \mathbf{I}_N \right)^{-1} \mathbf{H} \mathbf{\alpha}_{\text{OPT}}.
\]  

(22)

The optimization problem in (21) is not tractable when \( N > 1 \) since \( \mathbf{R}(\mathbf{\alpha}) \) depends on \( \mathbf{H} \) and \( \mathbf{\alpha} \). In order to assess the effect of number of antennas, the solution for (22) with \( N = 1 \), and two upper bounds on (22) are derived for \( N > 1 \).
1) Solution for Single Antenna at the FC: When \( N = 1 \), the channel matrix reduces to a column vector, given by \([h_1 h_2 \ldots h_L]^T\), where \( h_i \) is the channel between the \( i \)-th sensor and the FC. The maximization problem in (21) reduces to

\[
\alpha_{\text{OPT}} = \arg \max_\alpha \frac{\left| \sum_{i=1}^{L} \alpha_i h_i \right|^2}{\sigma_\eta^2 \sum_{i=1}^{L} |\alpha_i|^2 |h_i|^2 + \sigma_\nu^2}
\] subject to \( \sum_{i=1}^{L} |\alpha_i|^2 \leq P \). (23)

A similar problem was formulated in [7] and in a distributed estimation framework in [3], [22]. We recognize that the best value for the phase of the sensor gain is \( \angle \alpha_l = -\psi_l \) where \( \psi_l = \angle h_l \). Therefore, we set \( \angle \alpha_l = -\psi_l, \forall l \). We then define \( s := \sum_{i=1}^{L} \alpha_i h_i \) and swap the objective function with the constraint so we can rewrite the optimization problem as

\[
\alpha_{\text{OPT}} = \arg \min_{\{\alpha_i\}, s} \sum_{k=1}^{L} |\alpha_k|^2 \quad \text{subject to} \quad \sigma_\eta^2 \sum_{i=1}^{L} |\alpha_i|^2 |h_i|^2 + 1 \leq v_i s^2
\] subject to \( \sum_{l=1}^{L} |(\alpha_l|h_l)| - s = 0, \) (24)

where \( v_i \) is an auxiliary variable. The optimization problem in (24) is now a (convex) second-order-cone problem [23]. Using the Karush-Kuhn-Tucker conditions [23], the optimal solution is given by

\[
\alpha_i = \left[ \frac{P}{\sum_{i=1}^{L} \left( \frac{|h_i|}{P|h_i|^2 \sigma_\eta^2 + \sigma_\nu^2} \right)^2 \left( \frac{|h_i|^2}{\sigma_\eta^2 P|h_i|^2 + \sigma_\nu^2} \right) e^{-j\angle h_i}} \right]^{1/2}
\] (25)

The error exponent can be obtained by substituting (25) in (22) with \( N = 1 \):

\[
\mathcal{E}_{\text{CSIS}}(1) = \lim_{L \to \infty} \frac{\theta^2}{8} \frac{1}{L} \sum_{l=1}^{L} \frac{\sigma_\eta^2 + \sigma_\nu^2}{\sigma_\eta^2 |h_l|^2 + \sigma_\nu^2} = \frac{\theta^2}{8} E \left[ \frac{1}{\sigma_\eta^2 + \sigma_\nu^2} \right]
\] (26)

from the weak law of large numbers, where the expectation is with respect to \( \{h_l\} \). As an example, for Rayleigh fading channels (26) yields [24, §3.353]

\[
\mathcal{E}_{\text{CSIS}}(1) = \frac{1}{32} \gamma_8 \left[ 2 - \frac{p_1 \gamma_8 + 1}{\gamma_c} \exp \left( \frac{p_1 \gamma_8 + 1}{2 \gamma_c} \right) E_1 \left( \frac{p_1 \gamma_8 + 1}{2 \gamma_c} \right) \right]
\] (27)

where \( E_1(\cdot) \) is an exponential integral function [25, pp. 228]. The expression for \( \mathcal{E}_{\text{CSIS}}(1) \) is obtained when the channels between the sensors and the FC are fading. To compare with the AWGN case, note that \( Px/(\sigma_\eta^2 P x + \sigma_\nu^2) \) in (26) is a concave function of \( x \), and from Jensen’s inequality, \( \mathcal{E}_{\text{AWGN}}(1) \geq \mathcal{E}_{\text{CSIS}}(1) \), as expected.

Since (26) is rather complicated, it is desirable to find a simpler expression as a lower bound to (26). Any choice of \( ||\alpha||^2 = P \) will yield such a lower bound, since \( \alpha_{\text{OPT}} \) is optimal. Considering phase-only correction at the sensors, \( \alpha_i = \sqrt{P/L} \exp(-j\angle h_i) \) is substituted in (11) with \( N = 1 \) to yield the error exponent for phase-only CSIS for \( N = 1 \):

\[
\mathcal{E}_{\text{PO}}(1) = \lim_{L \to \infty} \frac{\theta^2}{8} \frac{P \left[ \frac{1}{L} \sum_{l=1}^{L} |h_l|^2 \right]^2}{\sigma_\eta^2 P \sum_{l=1}^{L} |h_l|^2 + \sigma_\nu^2}
\] (28)
From the weak law of large numbers, the random sequences in the numerator and denominator converge separately. However, since the expression for \( \mathcal{E}_{PO}(1) \) is a continuous function of these sequences, the value of \( \mathcal{E}_{PO}(1) \) converges to [26, Thm. C.1]

\[
\mathcal{E}_{PO}(1) = (E[|h|])^2 \mathcal{E}_{AWGN}(1)
\]

(29)
in probability, since \( E[|h|^2] = 1 \). The expression in (29) serves as a lower bound to \( \mathcal{E}_{CSIS}(1) \) as follows:

\[
\frac{1}{\zeta} \mathcal{E}_{AWGN}(1) \leq \mathcal{E}_{CSIS}(1) \leq \mathcal{E}_{AWGN}(1),
\]

(30)
where \( \zeta = (E[|h|])^{-2} \).

2) Upper Bound (AWGN channels): Since \( \gamma_c \) cannot be solved in closed form when \( N > 1 \), one cannot evaluate the error exponent in (22) by substitution as it was done for \( N = 1 \). Two upper bounds on (22) will be convenient at this stage. Since the AWGN performance is a benchmark for fading channels, the error exponent of the system over AWGN channels is an upper bound on that of fading channels, even in the case of full CSIS. Therefore, the first upper bound to (22) is given in (13):

\[
\mathcal{E}_{CSIS}(N) \leq \mathcal{E}_{AWGN}(N) = \frac{1}{8} N \gamma_c + \frac{p_1 \gamma_s}{N} + 1.
\]

(31)

3) Upper Bound (No Sensing Noise): Clearly, (22) is a monotonically decreasing function of the sensing noise variance, \( \sigma^2_{\eta} \). The second benchmark is obtained by setting \( \sigma^2_{\eta} = 0 \), which also affects \( \alpha_{OPT} \) in (21), since \( R(\alpha) \) no longer depends on \( \alpha \) when \( \sigma^2_{\eta} = 0 \). Substituting this in (21), the optimal value of \( \alpha \) when \( \sigma^2_{\eta} = 0 \) is

\[
\arg \max_{\alpha} \left( \alpha^H H^H H \alpha \right) \text{ subject to } ||\alpha||^2 \leq P.
\]

(32)
The solution to (32) is the eigenvector corresponding to the maximum eigenvalue of \( H^H H \), scaled in a way to satisfy the constraint with equality. Substituting into (22) with \( \sigma^2_{\eta} = 0 \), we have the second upper bound to \( \mathcal{E}_{CSIS}(N) \):

\[
B(N, K) = \frac{\theta^2 P}{8} \frac{\lim_{L \to \infty}}{\sigma^2_\nu} \lambda_{max} \left( \frac{1}{L} H^H H \right),
\]

(33)
where \( \lambda_{max}(\cdot) \) denotes the maximum eigenvalue function. Using the fact that \( \lambda_{max}(H^H H) = \lambda_{max}(HH^H) \), and that \( \lambda_{max}(\cdot) \) is a continuous function of the matrix elements [19, Thm. 8.1.5], one can interchange the limit with the maximum eigenvalue function [26, pp. 422, Thm. C.1] to yield

\[
B(N, K) = \frac{\theta^2 P}{8} \frac{\lim_{L \to \infty}}{\sigma^2_\nu} \lambda_{max} \left( \lim_{L \to \infty} \frac{1}{L} HH^H \right).
\]

(34)
From the weak law of large numbers,

\[
\lim_{L \to \infty} \frac{1}{L} HH^H = \frac{K}{K+1} I_{N \times N} + \frac{1}{K+1} N_{N \times N},
\]

(35)
in probability, so that with the substitutions \( \sigma^2_{\eta} = 0 \) and \( \theta^2 P/\sigma^2_\nu = \gamma_c/p_1 \), we have the bound:

\[
\mathcal{E}_{CSIS}(N) \leq B(N, K) = \frac{1}{8} \frac{\gamma_c N K + 1}{p_1} \frac{K+1}{K+1}.
\]

(36)
In (36), \( B(N, K) \) is an upper bound when there is sensing noise in the system. When there is no sensing noise, it is the actual error exponent of the system with full CSIS. Furthermore, \( \lim_{K \to \infty} B(N, K) = \lim_{\gamma_s \to \infty} \mathcal{E}_{AWGN}(N) \), verifying that as \( K \to \infty \), \( B(N, K) \) converges to the AWGN error exponent with no sensing noise. In addition, if \( K = 0 \), there is no advantage to having multiple antennas at the FC, for asymptotically large number of sensors, since the right hand side of (36) is independent of \( N \) in that case.
Since both $\mathcal{E}_{\text{AWGN}}(N)$ and $B(N, K)$ are upper bounds to $\mathcal{E}_{\text{CSIS}}(N)$, a combination of the two bounds, $\min[\mathcal{E}_{\text{AWGN}}(N), B(N, K)]$, provides a single, tighter upper bound. Equating the right hand sides of (31) and (36), it can be shown that this combined upper bound is given by
\[
C(N, K) = \begin{cases} \mathcal{E}_{\text{AWGN}}(N) & \text{if } \sigma^2_\eta \geq \frac{N^{-1}}{N(NK+1)}, \\ B(N, K) & \text{if } \sigma^2_\eta \leq \frac{N^{-1}}{N(NK+1)}. \end{cases} \tag{37}
\]
Combining the upper and lower bounds,
\[
\frac{1}{\zeta} \mathcal{E}_{\text{AWGN}}(1) \leq \mathcal{E}_{\text{CSIS}}(1) \leq \mathcal{E}_{\text{CSIS}}(N) \leq C(N, K), \tag{38}
\]
obtained from (27), (30) and (37). The bounds in (38) will be used to further examine the effect of $N$ on $\mathcal{E}_{\text{CSIS}}(N)$.

The value of $\mathcal{E}_{\text{CSIS}}(N)$ from (22) is the best achievable performance for fading channels. Defining the gain due to multiple antennas as $G_{\text{CSIS}}(N) := \mathcal{E}_{\text{CSIS}}(N)/\mathcal{E}_{\text{CSIS}}(1)$, the following theorem can be stated:

**Theorem 2:** When the channels have full CSI at the sensors, the gain due to multiple antennas at the FC can be upper bounded as
\[
G_{\text{CSIS}}(N) \leq \zeta \frac{\mathcal{E}_{\text{CSIS}}(N)}{\mathcal{E}_{\text{AWGN}}(1)} \leq \zeta \min \left[ \frac{N(z+1)}{Nz+1}, (z+1) \frac{NK+1}{K+1} \right], \tag{39}
\]
where $z := \gamma_c/(p_1 \gamma_s + 1)$.

**Proof:** The first inequality in (39) follows from the first inequality in (38). The second inequality in (39) follows from the last inequality in (38) and dividing the terms of (37) by (14).

With $p_1 = 0.5$, $K = 1$, $\gamma_c = 1$ and $\gamma_s = 1$, for $N = 2$, $G_{\text{CSIS}}(2) \leq 1.4286\zeta$. For $N = 3$, $G_{\text{CSIS}}(3) \leq 1.6667\zeta$ and for $N = 4$, $G_{\text{CSIS}}(4) \leq 1.8182\zeta$. These results indicate that there is diminishing returns in the multiple antenna gain.

**Corollary 1:** $G_{\text{CSIS}}(N)$ can be bounded by an expression depending on $N$ and $K$ only:
\[
G_{\text{CSIS}}(N) \leq \zeta \frac{N^2K + 2N - 1}{N(K+1)} \tag{40}
\]

**Proof:** The first argument of the $\min[\cdot, \cdot]$ function of the right hand side of (39) is a decreasing function in $z$ and the second argument is an increasing function in $z$. Therefore, when the arguments are equal for fixed values of $N$ and $K$, the maximum value of the $\min[\cdot, \cdot]$ function is obtained. This occurs when $z = N^{-1}(NK+1)^{-1}(N-1)$, allowing us to upper bound the $\min[\cdot, \cdot]$ function by the value in (40).

**Corollary 2:** When the channels have zero-mean, the maximum gain due to having multiple antennas at the FC is bounded by a constant independent of $N$ and only dependent on $\zeta = (E[|h_l|])^{-2}$:
\[
G_{\text{CSIS}}(N) \leq 2\zeta. \tag{41}
\]

**Proof:** Substituting $K = 0$, it is clear that (40) is monotonically increasing in $N$. Taking the limit as $N \to \infty$ yields the proof.

As an example, in the case of Rayleigh fading, when full channel information is available at the sensors, the maximum gain that can be obtained by adding any number of antennas at the FC for any channel or sensing SNR is at most $2\zeta = 8/\pi$, which is less than $3$.

The results in (39)-(41) have been derived for the case of iid sensing noise. We now address the correlated sensing noise case. To this end, we define $R_{\eta}$ as the $L \times L$ covariance matrix of the sensing noise samples, $\{\eta_l\}_{l=1}^L$. 
Theorem 3: Suppose that the sensing noise samples are correlated and let $\lambda_{\min}$ be the minimum eigenvalue of $R_{\eta}$. The gain due to multiple antennas in (39) holds with the change $z = \gamma_c / (p_1 \tilde{\gamma}_s + 1)$, where $\tilde{\gamma}_s := \theta^2 / \lambda_{\min}$.

Proof: The proof is shown in Appendix A.

Theorem 3 shows that any full-rank sensing noise covariance matrix changes the conclusion in (39) only through a redefinition of $z$. By maximizing over $z$, the same upper-bound in (40) is obtained, and for zero-mean channels, the bound in (41) remains valid. This shows that the bounds in (40) and (41) are general, and hold even when the iid condition is relaxed to any arbitrary full-rank covariance matrix, $R_{\eta}$. The gain due to adding multiple antennas is still upper-bounded by a factor of $2\zeta$, for zero-mean channels, when there is full CSI at the sensors.

C. Phase-only CSIS

One simplification to the full CSIS case is to provide only channel phase information to the sensors. For the single antenna case, and when the channels between the sensors and the FC have zero-mean, the phase-only results have been presented in (29) and (30). What follows is an extension of those results to the multiple antenna case when $K = 0$.

Since there is only phase information at the sensors, the amplitudes of the sensor gains are selected such that $|\alpha_i| = \sqrt{P/L}, \forall l$, so that $D(\alpha)D(\alpha)^H = (P/L)I_L$ and $R(\alpha)$ is given by (16).

With phase-only information, one can constrain $|\alpha_i|$ to be constant to reformulate (21) as the following:

$$\alpha_{PO} = \arg\max_{\alpha} \alpha^H H^H H \alpha \quad \text{subject to} \quad |\alpha_i|^2 = P/L, i = 1, 2, \ldots, L. \quad (42)$$

In Section IV-E4, a semidefinite relaxation approach will be presented to solve (42).

D. Asymptotically large sensors and antennas

When CSIS is available, (39 - 41) shows that only limited multiple antenna gains are available. It is interesting to see whether such limits would still be present if $N \to \infty$ simultaneously with $L$. A similar problem was considered, but in the context of CDMA transmissions in [16]. Note that this will in general yield results different than first sending $L \to \infty$ and then $N \to \infty$ as was done in Section IV-B. Such a situation can be interpreted as a case where a group of sensors is transmitting to another group, functioning as a virtual antenna array [27]. For such a system the scaling laws when $L$ and $N$ simultaneously increase [28, pp. 7], in such a way that

$$\lim_{L,N \to \infty} \frac{L}{N} = \beta, \quad (43)$$

are of interest. It should be noted that in spite of scaling the number of sensors and antennas, the power constraint is still maintained.

In this case, the error exponent is redefined as

$$\mathcal{E}^\infty(\beta) = \lim_{L,N \to \infty} -\frac{1}{L} \log P_e|_{H}(N), \quad (44)$$

with (43) satisfied. Similar to the upper bounds in (31) and (36), upper bounds on (44) are now derived. For the AWGN case,

$$\mathcal{E}^\infty(\beta) \leq \mathcal{E}_{AWGN}^\infty := \lim_{L,N \to \infty} \mathcal{E}_{AWGN}(N) = \lim_{L,N \to \infty} \frac{1}{8} \frac{N \gamma_s \gamma_c}{N \gamma_c + p_1 \gamma_s + 1} = \frac{1}{8} \gamma_s. \quad (45)$$

When there is no sensing noise, with $\sigma^2_\eta = 0$, the second bound can be calculated as

$$\mathcal{E}^\infty(\beta) \leq B^\infty(\beta) := \lim_{L,N \to \infty} \frac{\theta^2 P}{8 \sigma^2_\nu} \lambda_{\max}\left(\frac{1}{L} H^H H\right). \quad (46)$$
For fading channels with $K > 0$, it can be shown that the error exponent in (46) goes to infinity. Therefore, with any line-of-sight (LOS) and no sensing noise, increasing the number of sensors and the number of antennas to infinity provides very good performance. When $K = 0$, the Marčenko-Pastur Law [28, pp. 56] provides an empirical distribution of the eigenvalues of $N^{-1}HH^T$. From [29], [30], the maximum eigenvalue of $N^{-1}HH^T$ is shown to converge in such a way that

$$\lim_{L,N \to \infty} \lambda_{\text{max}} \left[ \left( \frac{1}{\sqrt{N}}H \right)^H \left( \frac{1}{\sqrt{N}}H \right) \right] = \frac{(1 + \sqrt{\beta})^2}{\beta},$$

in probability, which yields

$$B^\infty(\beta) = \frac{1}{8} \frac{\gamma_c (1 + \sqrt{\beta})^2}{\beta},$$

which is the optimum performance of the system in the absence of sensing noise. Similar to (37), the minimum of (45) and (48) yields

$$E^\infty(\beta) \leq \min \left[ E^\infty_{\text{AWGN}}, B^\infty(\beta) \right] = \begin{cases} \frac{1}{8} \gamma_s (1 + \sqrt{\beta})^2 & \text{if } P \sigma_n^2 \geq \frac{\beta}{(1 + \sqrt{\beta})^2} \\ \frac{1}{8} \frac{\gamma_c (1 + \sqrt{\beta})^2}{\beta} & \text{if } P \sigma_n^2 \leq \frac{\beta}{(1 + \sqrt{\beta})^2}. \end{cases} \tag{49}$$

The gain due to antennas is expressed in terms of the ratio $\beta$ in (43) as $G^\infty(\beta) := E^\infty(\beta)/E_{\text{CSIS}}(1)$. Using the bounds, we have the following:

**Theorem 4:** With asymptotically large number of sensors and antennas, the gain due to having multiple antennas at the FC is bounded by

$$G^\infty(\beta) \leq \zeta \left( 1 + \frac{(1 + \sqrt{\beta})^2}{\beta} \right). \tag{50}$$

**Proof:** The relationship between $E_{\text{AWGN}}(1)$ and $E_{\text{CSIS}}(1)$ from (30) provides a lower bound on $E_{\text{CSIS}}(1)$, and consequently an upper bound on $G^\infty(\beta)$, to yield the first inequality in (51) below. The expression in (49) provides an upper bound on $E^\infty(\beta)$, and dividing by (14) yields the second inequality in

$$G^\infty(\beta) \leq \frac{E^\infty(\beta)}{E_{\text{AWGN}}(1)} \leq \zeta \min \left[ 1 + \frac{1}{w}, \frac{1}{1 + w}, \frac{1 + \sqrt{\beta})^2}{\beta} \right], \tag{51}$$

where $w := \gamma_c/(p_1 \gamma_s + 1)$. The first argument in the min[$\cdot$, $\cdot$] function decreases as $w$ increases, while the second argument is an increasing function of $w$. Therefore, the min[$\cdot$, $\cdot$] function is maximized when arguments of the min[$\cdot$, $\cdot$] function are equal for a fixed value of $\beta$. This result is obtained when $w = (1 + \sqrt{\beta})^{-2} \beta$, to yield (50) and the proof.

To interpret (50), cases corresponding to three values of $\beta$, are considered:

(i) $\beta \ll 1$ ($N$ scales faster than $L$): When the number of antennas increases at a faster rate than the number of sensors, it can be seen that $B^\infty(\beta)$ is large. When there is no sensing noise, the performance obtained is exactly $B^\infty(\beta)$ as seen in (48). In this case, arbitrarily large gains are achievable. In case there is sensing noise in the system, $E^\infty_{\text{AWGN}}$ and $B^\infty(\beta)$ become bounds, and the gain is bounded as shown in (50). As $\beta \to 0$ in this case, the bound goes to infinity, which indicates that there could be large gains possible.

(ii) $\beta = 1$ ($N$ scales as fast as $L$): The number of antennas at the FC and the number of sensors scale at the same rate, the maximum possible gain can be calculated from (50) to yield $G^\infty(1) \leq 5\zeta$.

(iii) $\beta \gg 1$ ($N$ scales slower than $L$): When the number of sensors scales much faster than the number of antennas at the FC, it resembles the previous setting where $L \to \infty$, first, and $N$ was scaled. Not surprisingly, when $\beta$ is large in this case, $G^\infty(\beta) \leq 2\zeta$, same as in Section IV-B.
It should be noted here that in cases (ii) and (iii), where both the number of sensors and antennas are scaled to infinity simultaneously, only limited gain is achievable, when the sensors have complete channel knowledge.

In Table I we summarize the rate at which the gain due to number of antennas increases, both when CSI is available and unavailable at the sensor side. Recalling that the gain is defined in terms of the ratio of error exponents relative to the single antenna case, all the results in the table apply when \( L \) is large, which is a major distinguishing factor between this study and standard analysis of multi-antenna systems. It is seen that when \( K > 0 \) the gain in error exponent grows like \( O(N) \) depending on whether \( \gamma_c = 0 \). More interestingly, when the channel is zero-mean \( (K \to 0) \), adding antennas improves the error exponent linearly when CSI is not available. In stark contrast, when CSI is available, the gain is bounded \( (O(1)) \) by \( 2 \zeta \). Finally, the column on the right of Table I illustrates how the gain depends on the ratio \( \beta = L/N \) as both \( N \) and \( L \) increase. The error exponents for \( K > 0 \) are infinite, yielding an undefined gain. For zero-mean channels, the dependence on \( \beta \) indicates an increasing gain when \( \beta \) is small \((L \ll N)\), and bounded gain when \( \beta \) is large \((L \gg N)\).

E. Realizable Schemes

So far, we have provided bounds on the achievable gains due to antennas when CSI is available at the sensors, without providing a realizable scheme. This is because the calculation of \( \alpha_{\text{OPT}} \) in (21) in closed form is intractable. Moreover, it is not clear how \( \alpha \) should be chosen as a function of \( H \) when \( N > 1 \) to achieve a multiple-antenna gain. This is because each sensor sees \( N \) channel coefficients, corresponding to \( N \) antennas, and each channel coefficient has a different phase making the choices of \( \angle \alpha_i \) non-trivial.

We now present two sub-optimal schemes for the full CSIS case that are shown to provide gains over the single antenna case.

1) Method I: Optimizing Gains to Match the Best Antenna: In this method, the sensor gains, \( \alpha \), are selected in order to target the best receive antenna. However, the received signals at all of the other antennas are also combined at the FC, which uses the detection rule defined in (8).

Since \( L \) is finite for any practical scheme, (25) will be used to select \( \alpha \) and (26) without the limit can be used to assess which antenna has the “best” channel coefficients. Therefore, using the channels from the sensors to all of the receive antennas,

\[
n^* = \arg\max_n \frac{\theta^2}{8 \sum_{l=1}^{L} \frac{1}{\sigma_n^2 + \frac{1}{P|h_n|^2}}},
\]

is calculated and the sensor gains are set to (25) computed for the channels \( \{h_{n^*}\}_{i=1}^{L} \). The FC then uses all of the receive antennas for detection using (8). Since there are multiple antennas at the FC, for any realization of the channels between the sensors and the FC, the error exponent of this scheme is at least as good as the single antenna case.

Such an approach requires the calculation of (52) and the corresponding \( \alpha \) from (25). Since these calculations require the complete knowledge of \( H \), they can be calculated at the FC, and fed back to the sensors.
2) Method II: Maximum Singular Value of the Channel Matrix: It was shown in Section IV-B3 that when \( \sigma_n^2 = 0 \), the bound obtained in (36) is achievable. In this method, the values of \( \alpha \) are selected as though there is no sensing noise. The sensor gains, \( \alpha \), are selected in such a way that they are a scaled version of the eigenvector corresponding to \( \lambda_{\text{max}}(HH^H) \), such that \( ||\alpha||^2 = P \). In most practical cases, sensing noise is non-zero, and therefore, this method is sub-optimal. Similar to Method I, \( \alpha \) can be calculated at the FC and fed back to the sensors.

3) Hybrid of Methods I and II: Since Method II is tuned to perform optimally when there is no sensing noise, it outperforms Method I when the sensing SNR, \( \gamma_s \), is high. As the sensing SNR reduces, Method I begins to outperform Method II. These observations are illustrated and elaborated on in the simulations section (Section V, Figure 8).

Since one of the schemes performs better than the other based on the value of \( \gamma_s \), a hybrid scheme can be used: Method I for low values of \( \gamma_s \), and Method II for high values. The exact value where the cross-over occurs depends on the parameters of the system, and can determined empirically. An example is shown in the simulation section in Figure 8, where it is also argued that an underestimation of the value of \( \gamma_s \) is tolerable, while an overestimation is not.

4) Semidefinite Relaxation: Following [22, 31] a semidefinite relaxation of the problem in (42) is obtained as follows:

\[
X_{\text{PO}} = \arg\max_{X} \text{trace}(H^HX) \quad \text{subject to} \quad X \succeq 0,
\]

\[
X_{ii} = \frac{P}{L}, \quad i = 1, 2, \ldots, L,
\]

where \( X \) is an \( L \times L \) matrix. If \( X \) has a rank-1 decomposition, \( X := \alpha \alpha^H \), then \( \alpha \) is a solution to (42) [22, 31]. In the more likely case where \( X \) does not have rank-1, then an approximation to the solution of (42) is obtained by choosing \( \alpha \) as the vector consisting of the phases of the eigenvector corresponding to the maximum eigenvalue of \( X \). The semidefinite relaxation in (53) causes a loss of upt to a factor of \( \pi/4 \) in the final answer of (42) [31]. The phases of eigenvector corresponding to the maximum eigenvalue of \( X_{\text{PO}} \) are extracted to constitute a possible set of values of \( \alpha \). In order to obtain the solution to the SDR problem, an eigenvalue decomposition of \( X_{\text{OPT}} \) is required, which is an \( O(L^3) \) operation [19]. It is argued with the help of simulations (Figure 9) that the SDR outperforms the hybrid scheme when \( \gamma_s \) is small, at the expense of increased complexity.

V. SIMULATION RESULTS

The theoretical results obtained are verified using simulations. The channels are generated as complex Gaussian (Rayleigh or Ricean) for the purposes of simulation, even though the results only depend on the first and second order moments of the channels.

In Figure 2 it is verified that increasing the number of sensors improves the performance except when the channels are Rayleigh fading and there is no CSIS. Since the error exponent is zero for the Rayleigh fading case with no CSIS, the asymptotic average probability of error is computed and plotted. The Ricean case outperforms the Rayleigh fading case, and the AWGN channels provide the best performance. It can also be seen that the decay in probability of error is exponential in \( L \), when the channels between the sensors and the FC are AWGN or Ricean fading. The decay is slower than exponential when the channels are Rayleigh fading. This confirms the observations in Section IV-A. In all cases, the performance improves as the number of antennas increases.

In Figure 3 the expression of error exponent is compared against the value of \( L^{-1} \log P_e[H(5)] \) for increasing \( L \), with AWGN channels and Ricean fading channels between the sensors and the FC. It can be seen that fewer than 200 sensors are required for the asymptotic results to hold. Therefore, in subsequent simulations, \( L = 200 \) sensors have been used.

The effect of increasing the number of antennas on the error exponent for the AWGN case and Ricean fading case with no CSIS is seen in Figure 4. As expected, increasing \( \gamma_c \) improves performance and there
is an improvement in performance as the number of antennas at the FC increases. As predicted in Section IV-A, with an increase in $N$, the performance of $E_{\text{AWGN}}(N)$ and $E_{\text{NoCSIS}}(N, K)$ get closer to each other. There is a large performance gain between the $N = 1$ case and the $N = 2$ case, and almost the same gain between the $N = 2$ case and the $N = 10$ case, indicating diminishing returns, corroborating the results in Section III.

In Figure 5, the error exponent is evaluated when there is a single antenna at the FC. The cases of AWGN channels, Ricean channels with no CSIS, Rayleigh fading channels with full CSIS and Rayleigh fading channels with phase-only CSIS are compared in Figure 5. It is seen that the AWGN performance is the best, and when the Ricean channels have larger line of sight, the performance improves, as expected.
In fact, by increasing the amount of LOS, the no-CSIS Ricean case performs better than the full CSIS Rayleigh channel case, when $\gamma_c$ is large. The performance of the Ricean no CSIS case is a constant factor $K/(K+1)$ worse than the AWGN case, corroborating the result of $\mathcal{E}_{\text{NoCSIS}}(1, K)$. Similarly, the performance of the phase-only CSIS case confirms the result in (30). For Rayleigh fading channels, the phase-only CSIS case performs a constant $\pi/4$ worse than the AWGN case.

For the case of full CSIS, but with multiple antennas at the FC, bounds were derived on the error exponent of the system in Section IV-B2 and Section IV-B3 and combined to provide a single bound in (37). The value of $\mathcal{E}_{\text{CSIS}}(1)$ is set as a lower-bound on $\mathcal{E}_{\text{CSIS}}(N)$. In Figure 6 with $N = 1$, the upper bound can be seen to be about 0.76 dB (in terms of error exponent) away from the actual value at $\gamma_c = 8$.
Fig. 6. For a single antenna, optimal performance and performance bounds.

Fig. 7. Comparison of antenna gains vs $N$.

dB. For small values of $\gamma_c$, the AWGN bound is better, and as $\gamma_c$ increases, the bound with the no sensing noise assumption is better, as expected.

Figure 7 shows the effect of increasing the number of antennas at the FC on the antenna gains of the different systems. Also, for the cases of partial CSIS and full CSIS, the upper bounds on the antenna gains are plotted. The actual error exponent for the AWGN case is larger than for the Ricean no-CSIS case. However, as seen in Figure 7, the gain for the Ricean no-CSIS case is larger than the gain for the AWGN channel case. The bound on the Ricean CSIS antenna gain grows rapidly with $N$, as predicted by (40). The maximum gains possible for the Rayleigh CSIS case and the Rayleigh no CSIS cases are also plotted. These results indicate that with full CSIS, there is not much to be gained by adding antennas at the FC, corroborating our results in Section IV-B.

The schemes introduced in Section IV-E for the known CSIS case are simulated in Figure 8. The performance of these schemes are evaluated for $N = 5$ and $N = 50$. The performance of these systems is compared against a lower bound given by $\mathcal{E}_{\text{CSI-S}}(1)$ from (27) and an upper-bound, $C(5, K)$ from (37).
The hybrid scheme from Section [IV-E3] selects the better of the two practical methods depending on the value of $\gamma_s$. It can be seen that even with these simple sub-optimal practical schemes, the hybrid scheme is always better than $\mathcal{E}_{\text{CSIS}}(1)$, indicating that it is possible to obtain multiple antenna gain. However, for each $N$, the hybrid scheme does not approach the upper-bound of $C(5, K)$. When $N = 5$, this is an expected result, since firstly, $C(N, K)$ is a bound that is not necessarily achievable, and secondly, the practical schemes are obtained as sub-optimal approximations to the optimal scheme with full CSIS. The hybrid scheme for $N = 50$ provides more gain over $\mathcal{E}_{\text{CSIS}}(1)$ than the hybrid scheme for $N = 5$, but does not beat $C(5, K)$. This means that although gains are possible with the practical schemes, large gains are not possible, as predicted by the bounds in Section [IV-B]. For the hybrid scheme, Method I is better at low values of $\gamma_s$ and Method II is better at high values of $\gamma_s$. The value of $\gamma_s$ at which the hybrid scheme changes methods can also be seen in the simulations. In Figure 8 the system has a channel SNR, $\gamma_c = 10$ dB, $p_1 = 0.5$ and the Ricean-$K$ parameter is one. When there are five antennas at the FC, the hybrid scheme changes from Method I to Method II at $\gamma_s \approx 3$ dB, and when $N = 50$, the change occurs at $\gamma_s \approx 8.25$ dB. It can be seen that the hybrid scheme changes from Method I to Method II at different values of $\gamma_s$ based on the system parameters. It can also be seen that when Method I is selected by the hybrid scheme, the error in performance between Method I and Method II is small. However, when Method II is selected by the hybrid scheme, the performance gap between Method I and Method II increases rapidly as $\gamma_s$ increases. Therefore, an underestimation of the value of $\gamma_s$ is tolerable, while an overestimation is not.

The semidefinite relaxation (SDR) approach in Section [IV-E4] is compared against the hybrid scheme (Section [IV-E3] in Fig 9. For the SDR solution, the value of $X_{\text{OPT}}$ from (53) is calculated using CVX, a package for specifying and solving convex programs in MATLAB [32]. It can be seen from these simulations that for low values of sensing SNR, $\gamma_s$, the SDR solution outperforms the hybrid scheme. However, as the value of $\gamma_s$ begins to increase, the hybrid scheme (which is designed to be optimal as $\gamma_s \to \infty$) outperforms the SDR solution. The comparison with the upper-bound on the optimal error exponent, $C(N, K)$ is tight with respect to the better of the hybrid and SDR approaches. In order to obtain the solution to the SDR problem, an eigenvalue decomposition of $X_{\text{OPT}}$ is required, which is an $O(L^3)$ operation [19]. The SDR outperforms the hybrid scheme when $\gamma_s$ is small, at the expense of increased complexity.
A distributed detection system with sensors transmitting observations to a fusion center with multiple antennas is considered. The error exponent is derived from the conditional probability of error. AWGN and fading channels are considered between the sensors and the fusion center with varying amounts of CSI at the sensors. A large number of sensors and a finite number of antennas, or a large number of sensors and antennas are considered. The gain due to having multiple antennas, defined in terms of the ratio of error exponents relative to the single antenna case for large $L$ is quantified. The asymptotic nature of the sensors and the corruption of the sensed data with noise are the two major distinguishing factors in this work, compared to the conventional analysis of point-to-point MIMO systems.

The gains due to the number of antennas at the FC, both when CSI is available and unavailable at the sensor side, are now summarized. It is seen that when $K > 0$, when CSIS is unavailable, and when $\gamma_c = 0$, the gain in error exponent can grow unbounded with $N$. When the channel is zero-mean (such as the Rayleigh case with $K \rightarrow 0$), and when CSIS is not available, adding antennas improves the error exponent linearly. However, when CSIS is available, the gain is bounded by $2(E[|h_l|])^{-2}$. In the special case of Rayleigh fading channels, when CSIS is available, the gain due to multiple antennas at the FC is bounded by $8/\pi$. As both $N$ and $L$ increase, the error exponents for $K > 0$ are infinite, yielding an undefined gain. For zero-mean channels the dependence on $\beta$ from (43) indicates an increasing gain when $\beta$ is small ($L \ll N$), and bounded gain when $\beta$ is large ($L \gg N$). When the sensors use only channel phase information, the gain due to multiple antennas at the FC is upper bounded by $(E[|h_l|])^{-2}$, irrespective of the number of antennas at the FC.

In the case when full CSIS is available, it has been argued that having multiple antennas at the FC provides limited gains, which can be exploited by sub-optimal schemes. In one approach, the system is configured to beamform to the antenna that provides the best performance, where the FC still uses the data gathered at the other antennas. On an average, this is shown to perform better than in the single antenna case. Another approach is to assume there is no sensing noise, and set sensor gains tuned for such a system even when sensing noise is present. In this case, the system performs optimally when the sensing noise in the system is low. A hybrid scheme is proposed which selects the better of these two methods depending on the sensing SNR, $\gamma_s$.

Depending on the number of sensors and antennas at the FC, and their rates of growth, the following system design recommendation can be made. If CSIS is available and $N \ll L$, then for better performance, it is recommended to increase the number of sensors, rather than the number of antennas at the FC.
However, if $N$ can be increased at a much faster rate than $L$, it is possible to achieve greater gains due to adding antennas at the FC.

**Appendix A**

**Proof of Theorem 3**

We begin by noting that the presence of correlation in $\eta_l$ affects the total average transmit power. Therefore, to prove Theorem 3, we need to reconsider the following in presence of correlation: (i) the power constraint; (ii) the AWGN upper-bound in (31); (iii) the “no sensing noise” upper-bound in (36), which will then be used to redefine the combined upper-bound in (37).

(i) Power constraint: The total transmitted power is given by

\[ P_T = E \left[ \sum_{l=1}^{L} |\alpha_l (\Theta + \eta_l)|^2 \right] = \alpha^H (p_1 \theta^2 I_L + R_\eta) \alpha, \tag{54} \]

and constrained as

\[ \alpha^H (p_1 \theta^2 I_L + R_\eta) \alpha \leq P_T. \tag{55} \]

If (55) holds, then

\[ \|\alpha\|^2 \leq \frac{P_T}{p_1 \theta^2 + \lambda_{\min}} := P, \tag{56} \]

also holds. Since (56) is less stringent than (55), if (56) is used instead of the original power constraint in (55), an upper-bound will be obtained in the subsequent derivation of the error exponent.

(ii) Upper-bound (AWGN channels): Recall that in this case, $H = 1_{N \times L}$. Since the sensing noise is not iid, $\alpha$ has to be selected in such a way that the error exponent is maximized:

\[ \text{maximize} \quad \alpha^H 1_{L \times N} R(\alpha)^{-1} 1_{N \times L} \alpha \quad \text{subject to} \quad \alpha^H \alpha \leq P, \tag{57} \]

to yield the error exponent in the AWGN case with correlated sensing noise:

\[ E_{\text{AWGN}}(N) \leq \frac{1}{L} \frac{\theta^2}{8} (\alpha_{\text{opt}}^\text{AWGN})^H 1_{L \times N} R(\alpha_{\text{opt}}^\text{AWGN})^{-1} 1_{N \times L} \alpha_{\text{opt}}^\text{AWGN}, \tag{58} \]

where $\alpha_{\text{opt}}^\text{AWGN}$ provides to solution to (57) and the inequality in (58) is due to (11) and the modified power constraint in (56). To fully compute an upper bound on the right hand side of (58), first, $R(\alpha)$ is inverted and simplified. For the case of correlated noise, $R(\alpha)$ is given by

\[ R(\alpha) = 1_{N \times L} D(\alpha) R_\eta D(\alpha)^H 1_{L \times N} + \sigma_\nu^2 I_N \]
\[ \geq \lambda_{\min} 1_{N \times L} D(\alpha)^H D(\alpha) 1_{L \times N} + \sigma_\nu^2 I_N, \tag{59} \]

where $A \succeq B$ indicates that the matrix $(A - B)$ is positive semi-definite. Using the Sherman-Morrison-Woodbury formula for matrix inversion,

\[ R(\alpha)^{-1} \preceq \frac{1}{\sigma_\nu^2} I_N - \frac{1}{\sigma_\nu^2} 1_{N \times L} \left[ \text{diag} \left( \frac{1}{\lambda_{\min} |\alpha_i|^2} \right) + \frac{1_{L \times N} 1_{N \times L}}{\sigma_\nu^2} \right]^{-1} 1_{L \times N} \frac{1}{\sigma_\nu^2}. \tag{60} \]

Invoking the Sherman-Morrison-Woodbury formula for matrix inversion once again,

\[ R(\alpha)^{-1} \preceq \frac{1}{\sigma_\nu^2} I_N - \frac{1}{\sigma_\nu^2} M 1_{N \times N}, \tag{61} \]

where

\[ M := \frac{\sum_{l=1}^{L} \lambda_{\min} |\alpha_l|^2}{1 + N \sum_{l=1}^{L} \lambda_{\min} |\alpha_l|^2} \leq \frac{\lambda_{\min} P}{N \lambda_{\min} P + \sigma_\nu^2}, \tag{62} \]
due to the fact that $\alpha^H \alpha \leq P$ from (56).

By substituting (61) in (57), the solution to (57) is upper-bounded by the solution to

$$\text{maximize } \alpha^H \mathbf{1}_{L \times L} \alpha \text{ subject to } \alpha^H \alpha \leq P. \quad (63)$$

The value of $\alpha$ that maximizes (63) is the eigenvector corresponding to the maximum eigenvalue of $\mathbf{1}_{L \times L}$, scaled to satisfy the constraint with equality. Substituting this in (58), the bound in (31) obtained, with the substitution, $\gamma_s = \tilde{\gamma}_s$, where $\tilde{\gamma}_s = \theta^2 / \lambda_{\min}$ and $P_T \leq P/(p_1 \theta^2 + \lambda_{\min})$.

(iii) Upper-bound (no sensing noise): With no sensing noise, $R_\eta = 0_{L \times L}$. The optimization problem to obtain the best error exponent is the same as in (32), to yield (36).

Combining the modified AWGN upper-bound and the no sensing noise upper-bound in (56), a joint upper-bound is obtained, which is identical to (57), except for the substitution $\sigma^2_\eta = \lambda_{\min}$ and $\tilde{\gamma}_s = \theta^2 / \lambda_{\min}$. It follows that (39) holds with $z = \gamma_c / (p_1 \tilde{\gamma}_s + 1)$, to provide the proof.

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