Collision Drag Effect on Two-Fluid Hydrodynamics of Super $^3$He in Aerogel

M. Miura, S. Hijashitani, M. Yamamoto and K. Nagai

Faculty of Integrated Arts and Sciences,
Hiroshima University, Kagamiyama 1-7-1,
Higashi-Hiroshima 739-8521, Japan

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Abstract

Sound propagation in super $^3$He in aerogel is studied on the basis of a two-fluid model taking into account the effect by the drag force due to collisions between $^3$He-quasiparticles and aerogel molecules. The drag force plays a role of frictional force between the aerogel and the normal-fluid component. In local equilibrium, they move together in accordance with McKenna et al’s model. The deviation from the local equilibrium leads to the damping of sound. We give explicit expressions for the attenuation of longitudinal sounds in this system. We also discuss the sound propagation in a super $^3$He-aerogel system embedded in a narrow pore. It is shown that the forth sound propagates in such a system because of the clamping of the normal-fluid by the aerogel.

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I. INTRODUCTION

It is well known that there exists a variety of sound modes in liquid $^3$He. In the normal phase, in addition to the usual hydrodynamic sound mode (the first sound), the so-called zero sound propagates as a well-defined sound mode in the collisionless regime at low temperatures. In the superfluid phase, other two kinds of sound, the second and forth sound, can propagate which reflects the presence of the normal and the superfluid component of liquid. The second sound is the out-of-phase oscillation of the two fluids. The forth sound is the oscillation of only the superfluid component when the normal fluid is clamped owing to its viscosity in a narrow channel.

Recently, there has been increasing interest in the sounds in liquid $^3$He confined in highly porous aerogels. The aerogel consists of a random structure of silica strands of approximately 3 nm in diameter. The $^3$He-aerogel system has been regarded as a model system for studying the impurity scattering effects on the Fermi liquid which undergoes a transition to an unconventional Cooper pairing state. Nomura et al. and Gervais et al. have observed that the longitudinal sound attenuation in normal liquid $^3$He in aerogel does not show the characteristic temperature dependence of the Fermi liquid, i.e., the first-to-zero sound transition. In addition, the normal phase attenuation is of the same order of magnitude as the pure $^3$He case in spite of the presence of random scatterers. They also found the absence of the attenuation peak due to the order parameter collective excitation in the superfluid phase of $^3$He in aerogel. To study the attenuation theoretically, the homogeneous scattering model can be a good starting point, because the sound wavelength (20 nm) is much longer than the inter-strands spacing (30 nm) so that the detailed structure of aerogel is expected not to play a significant role. The homogeneous scattering model, however, predicts a significantly large attenuation (which is two order of magnitude larger than the observed value) though it can reproduce the qualitative temperature dependence.

To resolve the discrepancy between the theory and the experiment, Ichikawa et al. have studied the effect of simultaneous aerogel motion caused by collisions with $^3$He-quasiparticles. Such a collision drag effect reduces drastically the longitudinal sound attenuation and the experimental results can be reproduced by the collision drag model. This model has recently been applied to the transverse sound in the $^3$He-aerogel system. It was shown that
the collision drag effect modifies strongly the nature of the transverse sound propagation; for example, the transverse sound in the hydrodynamic regime, which is an overdamped mode in pure $^3$He, can propagate over a long distance.

In this paper, we discuss the collision drag effect in super uid $^3$He-aerogel system. McKenna et al. [3] have already proposed a theory of the sound in super uid-aerogel system. They modified a set of two-uid hydrodynamic equations to take into account the interlocking motion of the normal component and the aerogel. This theory was used by Golov et al. [4] to extract the super uid density $s$ in the $^3$He-aerogel system from the sound velocity measurements.

In deriving the modified two-uid hydrodynamic equations, McKenna et al. [3] assumed the normal uid to be completely locked to the aerogel matrix, i.e., the system is assumed to be in local equilibrium. The resulting equations can determine only the sound velocity and cannot give any information about the attenuation. In this paper, we discuss the longitudinal sound attenuation in super uid $^3$He in aerogel when the system deviates slightly from local equilibrium. In addition to the bulk $^3$He-aerogel system, we shall consider a restricted geometry such that the aerogel is built in a pore formed by sintered silver powders, as used in the fourth sound experiment [9].

II. MODIFICATION OF TWO-FLUID HYDRODYNAMIC EQUATIONS

Now we consider the drag force exerted on aerogel molecules. The drag force can be defined as the net momentum transfer per unit time from super uid $^3$He to the aerogel during impurity scattering processes. Thus the drag force can be determined from the collision integral in the quasiparticle transport equation [6,7,8]. In the normal state, the drag force density $\mathbf{F}$ is given by $\mathbf{F} = \frac{1}{\tau_{tr}}(1 + F_{1}^s=3) (v - v_a)$, where $\tau_{tr}$ is the transport relaxation time for the impurity scattering, the mass density of liquid $^3$He, $F_{1}^s$ the Landau parameter and $v$ and $v_a$ the local velocities of the liquid and the aerogel, respectively. Since $\mathbf{F}$ is proportional to the relative velocity $v - v_a$, it can be also interpreted as friction between the liquid and the aerogel. In the super uid phase, the motion of the super uid component is expected to uncouple with the aerogel. It is, therefore, plausible to generalize
the normal state of $F$ into

$$ F = \frac{1}{r} n (v_n - v_a); \quad (1) $$

where $n$ is the normal uid density, $v_n$ is the normal uid velocity and $r$ denotes a relaxation time characterizing the frictional effect between the normal uid and the aerogel. The relaxation time $r$ is expected to be of the same order as the impurity scattering time $\tau_i$. In fact in the normal phase it is given by $\tau_r = (1 + F_i^2)^{-3}$. In super uid phase, involved microscopic calculations are necessary to obtain an explicit expression for $r$. We shall, therefore, treat $r$ as a phenomenological parameter in this paper.

We can now obtain two-uid hydrodynamic equations taking into account the collision drag effect. In the set of the conventional two-uid equations, the momentum conservation law is modified because of the momentum loss due to the drag force, while the equation of motion of the super uid velocity $v_a$ remains unchanged. The two-uid equations for longitudinal sound are thus given by

$$ \frac{\partial}{\partial t} + \text{div} J = 0; \quad (2) $$
$$ \frac{\partial \text{div} J}{\partial t} = r^2 P + \frac{4}{3} r^2 \text{div} v_n \text{ div} F; \quad (3) $$
$$ \frac{\partial v_a}{\partial t} = \frac{1}{r} P + S T; \quad (4) $$
$$ \frac{\partial S}{\partial t} + \text{div} ( S v_n ) = 0; \quad (5) $$

where $J = n v_n + s v_s$, is the viscosity and $P$, $S$ and $T$ are pressure, entropy and temperature, respectively. The above equations are different from those by McKenna et al. [3] in that the drag force $F$ and the viscosity are included. The former enables us to study the case when $v_n \neq v_a$. The latter is necessary to reproduce the normal state dispersion relation derived previously [6, 8]. The two-uid equations must be supplemented by the equations of motion of aerogel:

$$ \frac{\partial v_a}{\partial t} = \frac{1}{r} P_a + F; \quad (6) $$
$$ \frac{\partial S_a}{\partial t} + \text{div} ( S_a v_a ) = 0; \quad (7) $$

where $a$ is the aerogel mass density and the restoring force due to the elasticity of the aerogel is denoted as $r P_a$. The pressure $P_a$ is related to the longitudinal sound velocity $C_{al}$ of the skeleton aerogel by [3]

$$ C_{al}^2 = \frac{\partial P_a}{\partial a}; \quad (8) $$
III. SOUND DISPERSION RELATION

From the above set of equations, we can derive the dispersion relation of the longitudinal sound in super uid $^3$He-aerogel system. To do that, we shall assume, as usual, that all the non-equilibrium quantities have the space and time dependence of $\exp(iq \cdot r \cdot \text{time})$. Then, using eq. (1) and the equations of motion of aerogel, we obtain the following relation between $v_n$ and $v_a$:

$$ (1 + \frac{i2}{l^2}) v_a = \frac{i}{\epsilon} n (v_n - v_a) \tag{9} $$

where $\omega_q = \omega_{a1}$ is the frequency of skeleton aerogel sound. It follows that the local equilibrium ($v_n = v_a$) is achieved by taking the limit $\omega_q \to 0$. Combining eq. (3) with our two-uid equations, we find that the dispersion relation can be written as

$$ (z^2 C_1^2)(z^2 C_2^2) + \left(\frac{1}{3} \frac{\omega_q^2}{\omega_1} + \frac{1}{\epsilon^2} \frac{z^2}{\omega_a} \right) (z^2 C_q^2) = 0 \tag{10} $$

with $z = \omega_q$ and

$$ a = \frac{12}{l^2 \lambda^2 q^2} = \frac{z^2 C_{a1}^2}{C_{q}}. \tag{11} $$

In eq. (10), $C_1, C_2$ and $C_4 = [(s = \omega_{s1}^2 + (n = \omega_{n1}^2)]^{-2}$ are the first, second and fourth sound velocities as defined in the conventional manner. It is easy to show that eq. (10) is reduced to the previous results [6,8] in the normal manner. Moreover, eq. (10) with $\omega_q = 0$ and in the limit $\omega_q \to 0$ coincides, as expected, with the dispersion relation of McKenna et al. [3]

$$ (z^2 C_1^2)(z^2 C_2^2) + \frac{a}{n} (z^2 C_{a1}^2) (z^2 C_q^2) = 0 \tag{12} $$

Equation (12) determines the velocities of two sound modes (fast and slow modes) in local equilibrium [3,4]. The two sound modes in the $^3$He-aerogel system have been observed by Golov et al. [4]. According to this experiment, the velocities of the fast and slow modes, $C_f$ and $C_s$, satisfy the conditions such that $C_f^2 = C_{a1}^2, C_{2f}^2, C_{1f}^2$ and $C_s^2 = C_{a1}^2$. Taking into these conditions, we find from eq. (12) that $C_f$ and $C_s$ are approximately given by

$$ C_f^2 = \frac{C_{a1}^2 + (n = \omega_{n1}^2) C_q^2}{1 + \frac{\omega_q}{\omega_{a1}}} \tag{13} $$

$$ C_s^2 = \frac{C_{a1}^2}{1 + \frac{\omega_q}{\omega_{a1}}} \tag{14} $$

When $\omega_q$ and $\omega_q$ are finite, the complex velocities $z's$ have corrections which give the attenuation $= \text{Im } q$ of the two hydrodynamic sound modes. Using eq. (10) and keeping
only the leading order correction term $s$, we obtain the attenuations of the fast and slow modes, $\tau$ and $s$, as

$$\tau = \frac{!}{2C_\tau^3 (1 + \frac{a}{n})^2} \left( \frac{4}{3} \frac{C_\tau^2}{n C_\tau^2} + \frac{2^2}{n} ! \right)$$

(15)

$$s = \frac{!}{2C_s (1 + \frac{a}{n})^2} \left( \frac{4}{3} \frac{C_s^2}{n C_s^2} + \frac{2^2}{n} ! \right)$$

(16)

The expressions for the attenuations include terms $s$ and $\tau$, reflecting the viscous effect and the frictional effect, respectively. The information on the phenomenological parameters and $s$ can thus be obtained from attenuation measurements. We shall show in the next section that a more useful and direct information on the frictional relaxation time $\tau$ can be provided by the forth sound attenuation measurement.

IV. FOURTH SOUND IN NARROW PORE

Now we discuss the sound propagation in a super uid $^3$He-aerogel system built in a narrow pore. Such a system has recently been prepared by Kotera et al. using sintered silver powders of 70 m in diameter to study the fourth sound propagation in aerogel. The sintered silver powders provide pores with the average diameter of $L = 10$ m.

In such a system, the sintered silver will play a role to clamp the aerogel strand ends. The longitudinal wave of the aerogel along the pore channel is, therefore, accompanied by a transverse standing wave so that the fixed end boundary condition should be satisfied. It follows that the eigenfrequency ! of the aerogel is replaced by

$$!^2 = C_{at}^2 q^2 + \frac{C_{at}^2}{L}$$

(17)

where $C_{at}$ is the transverse sound velocity of aerogel. The second term gives an energy gap due to the standing wave motion. Using $C_{at} = 1785$ m/s for 91% porous aerogel, the gap at $q = 0$ is estimated to be 56 MHz for $L = 10$ m. We see that a condition $!^2 = 0$ is fully satisfied in low-frequency acoustic experiments in which the frequencies are typically about 10 kHz or less.

This observation shows that the aerogel motion in the pore is hardly excited by the low frequency sounds. In the limit $!^2 = 0$, the aerogel cannot move and then the dispersion relation (10) takes the form

$$\left( z^2 C_1^2 \right) \left( \frac{C_2}{C_1} \right) + \left( \frac{4}{3} \frac{C_2^2}{n} + \frac{2}{n} ! \right) z^2 = 0$$

(18)

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The fourth sound can propagate in the limit \( f_1 \rightarrow 0 \) because the normal fluid is clamped by the "hard" aerogel. The attenuation \( \sigma_4 \) of the fourth sound is obtained from eq. (13). The result is given, up to leading order in \( f_1 \), by

\[
\sigma_4 = \frac{1}{2C_4^2} (C_2^4 - C_1^4)! f_1 \quad (19)
\]

Note that eq. (19) does not involve the viscosity because its contribution to \( \sigma_4 \) occurs only in higher order terms in the \( f_1 \) expansion. The attenuation of the fourth sound in the \( ^3\)He-aerogel system is thus dominated by the frictional relaxation time \( f_1 \). Using eq. (13), the parameter \( f_1 \) can be determined from simultaneous measurements of \( C_4 \) and \( \sigma_4 \).

The reason why the frictional effect dominates the fourth sound damping is the following. In the pore channel with the aerogel, the normal flow no longer obeys the Hagen-Poiseuille law but is governed by the Druyvesteyn law. In the presence of aerogel, the viscous penetration depth for \( f_1 \gg 1 \) is given by

\[
f = \frac{r}{n} \quad (20)
\]

and the viscosity in the superfluid phase is

\[
n v F^2 \quad ;
\]

where \( v_F \) is an appropriate average of the quasi-particle group velocity. At low temperatures, the viscous relaxation time is also limited by the impurity scattering time \( \tau_1 \). In fact, in case of the sound experiment by Nomura et al. (98% aerogel, 16 bar) one can see that \( \nu_1 \) happens at 10 mK. We find therefore that the viscous penetration depth in the presence of the aerogel is given by

\[
y_F \quad (22)
\]

and is as small as the mean free path.

V. CONCLUDING REMARKS

We have discussed hydrodynamic sound propagation in superfluid \(^3\)He-aerogel system. In a bulk system, the aerogel oscillates together with the normal liquid by the collision drag effect. As a result, two kinds of sound (the fast mode and the slow mode) can propagate as was observed by Golov et al. (3). The deviation in the local velocity between the normal component and the aerogel leads to the damping of the sounds. We have given explicit
expressions for the sound absorption. On the contrary in a narrow pore, the aerogel cannot move because the aerogel is clamped by the pore surface. In that case, the fourth sound can propagate along the pore channel. It is to be emphasized that the inter-locking between the normal component and the aerogel occurs not by the viscous effect as so far considered but by the collision drag effect. In other words, the normal component is in the Drude regime.[14]

The modified two-uid theory presented in this paper is still a phenomenological theory and includes many unknown parameters. In addition to the super uid mass density, microscopic information on the relaxation times in dirty super uid $^3$He is necessary for more detailed comparison with experiments. This problem and also the justification of our two-uid model shall be discussed elsewhere.

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