One-loop Correction and the Dilaton Runaway Problem
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Abstract

We examine the one-loop vacuum structure of an effective theory of gaugino condensation coupled to the dilaton for string models in which the gauge coupling constant does not receive string threshold corrections. The new ingredients in our treatment are that we take into account the one-loop correction to the dilaton Kähler potential and we use a formulation which includes a chiral field $H$ corresponding to the gaugino bilinear. We find through explicit calculation that supersymmetry in the Yang-Mills sector is broken by gaugino condensation. The dilaton and $H$ field have masses on the order of the gaugino condensation scale independently of the dilaton VEV. Although the calculation performed here is at best a model of the full gaugino condensation dynamics, the result shows that the one-loop correction to the dilaton Kähler potential as well as the detailed dynamics at the gaugino condensation scale may play an important role in solving the dilaton runaway problem.
1 Introduction

The determination of the dilaton ($S$) and moduli VEVs is an important problem in string phenomenology because it is directly related to the predictions of the string models [1]. The dilaton and moduli are flat directions to all orders in string perturbation theory, and it is hoped that some non-perturbative effects will lift these flat directions and determine these VEVs dynamically. Non-perturbative gaugino condensation seems to be a ready candidate for doing this, but so far much effort has failed to achieve convincing success. A major reason is that the determination of dilaton VEV through gaugino condensation suffers from the dilaton runaway problem [2], i.e. the potential energy is minimized at either $S \to 0$ or $S \to \infty$. There are several proposals to solve the problem at tree-level in the observable sector fields. One is c-number solution [2, 3, 4], another is multiple gaugino condensation [5, 6, 7, 8]. In ref. [9] it is pointed out that another necessary condition for preventing the dilaton from running away at tree-level is that the potential energy is positive semi-definite. This is because the Kähler potential for the dilaton is usually assumed to be $K(S, \bar{S}) = -\log(S + \bar{S})$ and the potential energy is proportional to $1/(S + \bar{S})$. If the potential energy is negative, it would always have a minimum at $S = 0$ and the dilaton still runs away. The no-scale structure, which was proposed to naturally suppress the cosmological constant [10], is so far the most natural way to yield the positive semi-definite potential energy. We therefore focus on the models with the no-scale structure in this paper.

In no-scale models, the dilaton corresponds to a flat direction at tree level; one-loop effects will lift this degeneracy. But for many string models, the dilaton VEV determined by the one-loop potential energy is rather small. The dilaton VEV consistent with the weak scale measurement is $\langle S \rangle \sim 2$, which corresponds to a (model-dependent) hierarchy of at least an order of magnitude between the Planck scale and the gaugino condensation. It appears difficult in the present formulation to dynamically generate such
a hierarchy since the only scale in the theory is the Planck scale or the string scale. Because of this, in many models it is found that the gaugino condensation scale as determined by the dilaton VEV is again on the Planck scale, i.e. $\langle e^{-S/2b_0} \rangle \sim 1$.

These features are illustrated in the simplest c-number gaugino condensation model

\begin{align*}
K &= - \log(S + \bar{S}) - 3 \log(T + \bar{T} - |\phi|^2), \quad (1) \\
W &= c_0 + ae^{-3S/2b_0}. \quad (2)
\end{align*}

At tree level, dilaton potential is flat and the one-loop corrections give a potential which is minimized at $\langle e^{-S/2b_0} \rangle \sim 1$. In this model, although one avoid the dilaton runaway problem at the tree-level, but it “runs too little” at the one-loop level. In this sense, the dilaton runaway appears to be a generic problem in many models.

In this work we examine the one-loop structure of the formulation for the dynamics of gaugino condensation coupled to the dilaton first proposed in ref. [11], in which a chiral field $H$ corresponding to the gaugino bilinear is introduced. The new ingredient of our analysis is that we take into account the one-loop corrections to the dilaton Kähler potential. The inclusion of these effects leads to conclusions that are quite different from the usual scenario of gaugino condensation. We find that the dilaton mass as well as that of the $H$ field are on the same order as the gaugino condensation scale. This indicates that the usual approach to determining the dilaton VEV which treats the dilaton as a light field below the gaugino condensation scale may be incorrect. We also find that the supersymmetry in the Yang-Mills sector is broken by gaugino condensation, unlike the pure Yang-Mills theory first discussed in [12]. These results may shed some lights on the dilaton runaway problem. The computations also show that the loop-correction to the dilaton Kähler potential may lead to determining the dilaton VEV at a value consistent with weak scale measurements.
This paper is organized as follows: In section 2, we will derive in more detail the no-scale formulation of gaugino condensation proposed in ref. [11]. We show that the inclusion of the one-loop correction to the dilaton Kähler potential and the introduction of a chiral field related to the gaugino bilinear are crucial for making the model of the no-scale type. In section 3, we give the necessary formulas for the analysis of the one-loop effective theory which is developed in [13]. In section 4 and 5, we will analyze the one-loop vacuum structure for our model. We will do this in two different ways. In section 4, we treat the gaugino-bilinear chiral fields as heavy field and we integrate it out by solving its equation of motion. In section 5, we treat it as a dynamical field with mass comparable to the dilaton. We will find that the second analysis yields a minimum at the finite dilaton VEV, giving a new scenario for gaugino condensation. Section 6 contains our conclusions.

2 No-Scale Formulation

In this section, we discuss the no-scale formulation of gaugino condensation for the string models in which the gauge coupling constant does not receive string threshold corrections. We are particularly interested in the models with the no-scale structure because they naturally suppress the cosmological constant [10], prevent the dilaton from running-away at tree level [1], and also because they might be a natural symmetry for generating the large mass hierarchy between $M_{\text{GUT}} = 10^{16}$ GeV and $M_{\text{SUSY}} = 1$ TeV [14, 4]. In [4] it is shown that in a modified c-number model, the induced constant term in the superpotential is not quantized; furthermore this supersymmetry breaking scheme makes it possible to construct affine level one $SU(5)$ or $SO(10)$ string models with the intermediate gauge symmetry breaking scale $M_{\text{GUT}} = 10^{16}$ GeV. We therefore restrict attention to the c-number supersymmetry breaking schemes.

The modular-invariant formulation of effective action with gaugino con-
The gaugino condensation coupled to the dilaton field has been discussed in [15, 16, 17] for string models in which the gauge coupling constant receives string threshold corrections. The no-scale formulation for this type of string models is given in [9]. The one-loop analysis of these models are given in [13]. In [11], the no-scale formulation of the supersymmetry breaking dynamics is given for the string models in which the gauge coupling constant does not receive string threshold corrections. The one-loop analysis of these models has not been carried out so far; this is the subject of this work.

In the following, we will give a more detailed derivation of the no-scale formulation of gaugino condensation for the string models that do not receive string threshold corrections. We will show that the one-loop correction to the dilaton Kähler potential and the introduction of the $H$ field play a crucial role in the no-scale structure, and in preventing the dilaton from running away at tree level.

It has been shown in ref. [18] that the one-loop contribution to the gauge kinetic terms should be viewed as a field-dependent wave-function renormalization of the dilaton field rather than a renormalization of gauge coupling function $f$. For example, to cancel the modular anomaly the Kähler potential is modified to be

$$K = -\log \left( S + \bar{S} - \frac{2b_0}{3} k \right) - k, \quad (3)$$

$$k = -3 \log (T + \bar{T} - |\phi|^2), \quad (4)$$

while the gauge coupling function remain unchanged:

$$f = S. \quad (5)$$

This is usually called Green–Schwarz mechanism [13, 18, 20, 21]. The above Kähler potential is obviously not of the no-scale type. Now one takes into account the full one-loop contribution to the dilaton Kähler potential which has the form:

$$K = -\log \left( S + \bar{S} + 2b_0 \log \Lambda^2 - \frac{2b_0}{3} k \right) - k, \quad (6)$$
where $\Lambda$ is the renormalization scale. The gaugino condensation scale corresponds to $\Lambda^2 = e^{k/3}|H|^2$, where $H$ is the chiral field relating to the gaugino bilinear $\langle \lambda \bar{\lambda} \rangle$ (the factor $e^{k/3}$ is included to make $\Lambda$ modular invariant). We then obtain

$$K = -\log \left( S + \bar{S} + 2b_0 \log |H|^2 \right) - k,.$$  

This model is of no-scale type provided that the superpotential is independent of the moduli fields.

The superpotential for the $H$ field can be obtained from symmetry arguments \cite{12, 22, 23, 24}:

$$W = d \left[ \frac{1}{4} SH^3 + \frac{b_0}{2} H^3 \log(\eta H) \right] + c_0 + W_0. \tag{8}$$

Here $c_0$ and $W_0$ are the contribution from the charged background VEVs and matter fields. The parameters $d$ and $\eta$ are not fixed by symmetry requirements, and specified by the underlying gaugino condensation dynamics. Under modular transformations,

$$T \mapsto T' = \frac{aT - ib}{icT + d}, \tag{9}$$
$$\Phi^i \mapsto \Phi'^i = \frac{\Phi^i}{icT + d}, \tag{10}$$
$$S \mapsto S' = S + 2b_0(icT + d), \tag{11}$$
$$H \mapsto H' = \frac{H}{icT + d}, \tag{12}$$
$$c_0 \mapsto c'_0 = \frac{c_0}{(icT + d)^3}, \quad ad - bc = 1, \tag{13}$$

so that

$$K' = K + F + \bar{F}, \tag{14}$$
$$W' = We^{-F}, \tag{15}$$
$$F = 3 \log(icT + d). \tag{16}$$
For the supergravity theory, the lagrangian depends on the combination of the Kähler potential and superpotential \( G = K + \log |W|^2 \), the above theory is invariant under the modular transformation and also has the desired no-scale structure.

Although the modification of the dilaton kinetic energy by loop effects has been known for several years, its consequences for gaugino condensation have not been fully explored. We see from the above derivation that the inclusion of the one-loop effects to the dilaton Kähler potential is crucial for the no-scale formulation of the gaugino condensation. In our following analysis, we find that it has nontrivial effects on the one-loop vacuum structure and the dynamical determination of dilaton VEV.

### 3 Formalism

In this section, we write down the necessary formulas for our analysis. Our calculation largely follows ref. [13], although (as we explain below) some aspects of the analysis are quite different.

At tree level, the potential energy can be written as [13]

\[
V = e^K \left( K^{-1} W^{a \bar{a}} \tilde{W}^{\bar{a} \\ b} \right),
\]

(17)

where \( z_a = (S, \phi, H) \) and

\[
\tilde{W}_a \equiv \frac{\partial W}{\partial z_a} + K_a W - 3 W \frac{K_a \bar{T}}{K_T},
\]

(18)

which manifestly has the no-scale structure. The tree-level vacuum conditions are

\[
\langle \tilde{W}_a \rangle = 0.
\]

(19)

To calculate the one-loop effective potential, one must first calculate the mass matrices of the chiral fields. The scalar squared mass matrix is given
by
\[
M^2_S = \begin{pmatrix}
  v_{ab} & v_{ac} \\
v_{db} & v_{db}
\end{pmatrix},
\]  
(20)

\[
v_{ab} \equiv \frac{\partial^2 V}{\partial z_a \partial z_b},
\]  
(21)

\[
v_{ab} \equiv \frac{\partial^2 V}{\partial z_a \partial z_b} - G_{cd}(G^{-1})^e_f V_f.
\]  
(22)

The normalized scalar masses are
\[
M^2_S = \begin{pmatrix}
  G^{-1/2} & 0 \\
0 & (G^{-1/2})^T
\end{pmatrix} \begin{pmatrix}
v_{ab} & v_{ac} \\
v_{db} & v_{db}
\end{pmatrix} \begin{pmatrix}
  G^{-1/2} & 0 \\
0 & (G^{-1/2})^T
\end{pmatrix},
\]  
(23)

which has the same eigenvalues as
\[
\tilde{M}^2_s = \begin{pmatrix}
v_{ab} & v_{ac} \\
v_{db} & v_{db}
\end{pmatrix} \begin{pmatrix}
  G^{-1} & 0 \\
0 & (G^{-1})^T
\end{pmatrix}.
\]  
(24)

Under the tree-level vacuum condition above, one obtains
\[
v_{ab} = e^K \left[ \tilde{W}_{ac}(G^{-1})^{cd}\tilde{W}_{db} + \tilde{W}_{ac}(G^{-1})^{cd}\tilde{W}_{db} \right],
\]  
(25)

\[
v_{ab} = e^K \left[ \tilde{W}_{ac}(G^{-1})^{cd}\tilde{W}_{db} + \tilde{W}_{ac}(G^{-1})^{cd}\tilde{W}_{db} \right].
\]  
(26)

The gaugino mass parameter is given by
\[
(M_{1/2})_{\alpha \beta} = \frac{1}{2} e^{G/2} (G^{-1})^{ab} G^{\dagger} f_{a \beta, a},
\]  
(27)

while the normalized gaugino mass-squared is
\[
(M_{1/2}M_{1/2}^+)_{\alpha \beta} \equiv (M_{1/2})_{\alpha \gamma} [(Re f)^{-1}]^{\gamma \delta} (M_{1/2}^+)_{\delta \beta}.
\]  
(28)

The fermion masses are given by
\[
\mu_{ab} = e^{G/2} \left[ G_{ab} + G_a G_b - \frac{1}{3} G_{c}(G^{-1})^{cd} G_{dab} \right]
\]  
(29)

The normalized fermion mass matrix is,
\[
m_{ab} = (G^{-1})^c_a \mu_{cd} (G^{-1})^d_b,
\]  
(30)
which has the same eigenvalue as

\[ \tilde{m}_{ab} \equiv m_{ac}(G^{-1})^c_b. \]  

(31)

In the above,

\[ G_a = K_a + \frac{W_a}{W}, \]  

(32)

\[ G_{ab} = K_{ab} + \frac{W_{ab}}{W} - \frac{W_a W_b}{W^2}, \]  

(33)

\[ G_{\bar{a}b} = K_{\bar{a}b}, \]  

(34)

\[ (G^{-1})^{\bar{a}\bar{b}} G_{\bar{c}b} = \delta_{\bar{b}}^a. \]  

(35)

The one-loop potential is

\[ V^{1-\text{loop}} = \frac{1}{4(4\pi)^2} [2\text{Str}(M^2 \Lambda^2) + \text{Str}(M^4 \log(M^2/\Lambda^2))], \]  

(36)

where \text{Str} is the supertrace. In our one-loop analysis, we use the tree-level vacuum conditions to calculate the one-loop potential energy, with which we determine the rest of the VEVs of the scalar fields. This approximation corresponds to determining the VEVs to order \( \hbar \).

4 Gaugino Bilinear as a Heavy Field

In this section, we will carry out the one-loop analysis for the models formulated in Section 2. Here we treat the gaugino bilinear as a heavy field (as is usually assumed), i.e. we integrate out the \( H \) field using its equation of motion.

From the tree-level vacuum condition, we get the classical equation of motion for \( H \) field,

\[ \tilde{W}_H \equiv W_H - \frac{2b_0}{H(S + S + 2b_0 \log |H|^2)} W = 0. \]  

(37)
To solve this equation, we write

$$H = h(S)e^{S/2b_0}.$$  \tag{38}$$

The function $h$ is determined by

$$dh^3((3L - 2b_0) \log(h\eta) + L) = 4c_0 e^{3S/2b_0},$$  \tag{39}$$

where

$$L = S + \bar{S} + 2b_0 \log |H|^2 = 2b_0 \log |h|^2. \tag{40}$$

It is easy to see that gaugino condensation occurs ($h \neq 0$) if and only if the constant part of the superpotential $c_0$ is nonzero. This is consistent with our assumption that the $c_0$ is induced by gaugino condensation. After solving for $H$, we obtain the effective theory below the gaugino condensation scale:

$$K = -\log L + k, \quad k = -3 \log \left[T + \bar{T} - |h|^2 e^{(S + \bar{S})/2b_0}\right],$$  \tag{41}$$

$$W = \frac{d b_0}{2} e^{-3S/2b_0} h^3 \log \eta h + c_0 + W_0. \tag{42}$$

The tree level potential energy is

$$V = e^K \left(|\tilde{W}_S|^2 + |W_i|^2\right), \tag{43}$$

which is minimized at

$$\langle W_i \rangle = 0,$$  \tag{44}$$

$$\langle \tilde{W}_S \rangle \equiv \langle W_S + K_S W - \frac{3K_{ST}}{K_T} W \rangle = \langle W_S - \frac{L_S}{L} W \rangle = 0. \tag{45}$$

Note that in the tree-level potential energy, the $A$ term and scalar masses of the matter sector remain zero although local supersymmetry is broken by $\langle W \rangle \neq 0$. We find that at the tree-level minimum

$$G^S = (G^{-1})^{S\bar{a}} G_{\bar{a}} = 0, \quad G^T = (G^{-1})^{T\bar{a}} G_{\bar{a}} = -e^{k/3}, \tag{46}$$

so the gauginos in the matter sector remain massless if the $f$ function does not depend on the moduli; these are the models in which the gauge coupling...
constant does not receive moduli-dependent string threshold corrections. We see that at tree-level, global supersymmetry is broken in the dilaton sector but not in the matter sector for the models we are considering. In this approximation, all the moduli masses vanish.

We now solve the tree-level vacuum conditions eqs. (44) and (45). Eq. (44) is automatically satisfied if the VEVs of the matter fields are zero for the trilinear superpotential. Eq. (45) imposes one relation between $c_0$ and $S$; the dilaton and moduli fields correspond to flat directions which might be lifted by loop effects. Solving eq. (45), we obtain

$$\langle h \rangle = \eta^{-1}. \quad (47)$$

The parameter $\eta$ is determined by the gaugino condensation dynamics and is expected to be of order 1. Now we can also determine $c_0$ in terms of $S$,

$$\langle W \rangle = c_0 = \frac{1}{4} \langle dLh^3e^{-3S/2b_0} \rangle. \quad (48)$$

We obtain the dilaton scalar squared masses

$$M_S^2 = m_{3/2}^2 \left[ (G^{-1})^{SS} \frac{1 \pm 6L/(2b_0)}{4L^2} \right]^2$$

$$= \frac{(1 \pm 6L/(2b_0))^2}{(1 + 3xL^2/(4b_0^2))^4}, \quad (49)$$

and the dilaton fermion mass

$$M_f = m_{3/2}^2 (G^{-1})^{SS} \frac{3}{4b_0 L}$$

$$= m_{3/2}^2 \frac{3L/b_0}{(1 + 3L^2x/(4b_0^2))^2}, \quad (50)$$

with

$$(G^{-1})^{SS} = \frac{4L^2}{(1 + 3L^2x/(4b_0^2))^2}, \quad (52)$$

$$m_{3/2}^2 = e^K |W|^2 = \frac{1}{16} d^2 L x^3, \quad (53)$$

$$x = \frac{|h|^2}{k} e^{-(S+\bar{S})/2b_0}. \quad (54)$$
The cutoff is
\[ \Lambda^2 = M_S^2 |H|^2 e^{K/3} \]
\[ = \frac{2}{S + S + 2b_0 \log(T + T)} |H|^2 e^{K/3} \]
\[ = \frac{2x}{L + 2b_0 \log(x + 1)}, \quad (55) \]

where
\[ M_S^2 = \frac{2M_p^2}{S + S + 2b_0 \log(T + T)} \quad (56) \]
is the string scale, which is also the scale at which the string gauge coupling constants “unify” [25, 26]. We can now obtain the one-loop potential energy, which depends only on the moduli-invariant dilaton and moduli combination \( x \):

\[ V = 2[-4 + \frac{2}{(1 + 3L2x/4b_0)^2}] \frac{Ld^2}{16} x^4 \]
\[ + (\frac{Ld^2}{16})^2 x^6 \{-4 + \frac{2 + 12(6L/2b_0)^2}{(1 + 3L^2x)^4} \log \left[ \frac{d^2L}{32} (L + 2b_0 \log(x + 1)) x^2 \right] + g} \]
\[ g = \frac{1}{(1 + 3L^2x/4b_0)^4} \{ (1 + 6L/2b_0)^4 \log(1 + 6L/2b_0)^2 \]
\[ + (1 - 6L/2b_0)^4 \log(1 - 6L/2b_0)^2 - 2(6L/2b_0)^4 \log(6L/2b_0)^2 \}. \quad (58) \]

We find that the minimum of the above one-loop potential energy is reached at either \( x \to \infty \) or \( x = 0 \) depending the different ways of minimizing the potential. In any case, the model suffers the dilaton runaway problem.

In the next section, we consider the possibility that the dilaton mass is on the same order as the gaugino condensation scale, and find by an explicit calculation that this is a real possibility.

5 Gaugino Bilinear As A Dynamical Field

In this Section, we study the one-loop vacuum structure of our model treating the \( H \) field as a dynamical field. As already pointed out, this model cannot
be viewed as anything more than a model of the dynamics at the gaugino condensation scale, similar in spirit to the linear sigma model as a model of QCD. Still, it is a reasonable model, and the qualitative conclusions may be correct.

In this case, the dilaton field and the $H$ field mix, and the inverse of the Kähler metric is

$$(G^{-1})^{i\bar{j}} = \begin{pmatrix} L^2 + 4b^2 C/(3|H|^2) & -2b C/(3H) & -\frac{2}{3}b C \\ -2b C/3\bar{H} & \frac{1}{3}C & \frac{1}{3}HC \\ -\frac{2}{3}b C & \frac{1}{3}HC & \frac{1}{3}C(|H|^2 + C) \end{pmatrix}, \quad (59)$$

where

$L = S + \tilde{S} + 2b_0 \log |H|^2, \quad C = T + \bar{T} - |H|^2. \quad (60)$

Solving the tree-level vacuum condition

$$\langle \tilde{W}_H \rangle \equiv \langle W_H - \frac{L_H}{L} W \rangle = 0, \quad (61)$$

$$\langle \tilde{W}_S \rangle \equiv \langle W_S + K_S W \rangle = 0, \quad (62)$$

we get

$$H = \frac{1}{\eta} e^{-s/(2b_0)} = he^{-s/(2b_0)}, \quad (63)$$

$$\langle W \rangle = c_0 = \frac{1}{4} \langle dLH^3 \rangle. \quad (64)$$

The orders of magnitude of the solutions are

$$H \sim e^{-s/(2b_0)}, \quad (65)$$

$$\langle W \rangle \sim e^{-3s/(2b_0)}, \quad (66)$$

so $h \sim 1, d_0 \equiv \frac{1}{4}dL = \frac{1}{4}d\log(2b_0|h|^2) \sim 1$. We take the point of view that $h$ and $d_0$ are constants to be determined from a better understanding of the gaugino condensation dynamics. This is different from the point of view of ref. [13], where these parameters are taken to be dynamical variables to be determined by minimization of the effective potential.
From the vacuum conditions eqs. (61) and (62) we obtain
\[ \langle G^i \rangle = \langle G^{ik} G_k \rangle = (0\ 0\ -C), \] (67)
so the tree-level gaugino mass is zero in the models we are discussing. With
the above relations, we obtain the fermion mass matrix
\[ (\mu_{1/2})_{IJ} = e^{G/2} \begin{pmatrix} 0 & 3/(LH) & 0 \\ 3/(LH) & 12b_0/(H^2L) & 0 \\ 0 & 0 & 0 \end{pmatrix}. \] (68)
The normalized fermion mass matrix is
\[ (m_{1/2})_{IJ} = (\mu_{1/2})_{IJ}(G^{-1})^{JK} \]
\[ = e^{G/2} \begin{pmatrix} -2bC/(|H|^2) & c/(LH) & C/L \\ 3L/H - 4b_0^2C/(|H|^2HL) & 2b_0C/(H^2L) & 2b_0C/(HL) \\ 0 & 0 & 0 \end{pmatrix}. \] (69)
Assuming that \( S = \bar{S}, H = \bar{H} \) (i.e. CP violation is highly suppressed) we
obtain the fermion mass eigenvalues
\[ m_1^{1/2} = m_2^{1/2} = e^{G/2} \sqrt{3C/HL}, \quad m_3^{1/2} = 0. \] (70)
The scalar masses are computed from
\[ \tilde{W}_{ik} = \begin{pmatrix} 1 & 2b_0/(\bar{H}) & 0 \\ 2b_0/H & 4b_0^2/|H|^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} W/L^2, \] (71)
\[ \tilde{W}_{ik} = \frac{3}{4} H^2 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 4b_0/H & 0 \\ 0 & 0 & 0 \end{pmatrix}, \] (72)
\[ v_{ab} = e^{G} \begin{pmatrix} \frac{3C+|H|^2}{|H|^2L^2} & \frac{2b_0(3C+\bar{H}^2)}{|H|^2HL^2} & 0 \\ \frac{2b_0(3C+\bar{H}^2)}{|H|^2L^2} & 4b_0^2(3C+|H|^2)+9|H|^2L^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \] (73)
\[ v_{ab} = e^{G} \begin{pmatrix} \frac{3}{HL} & \frac{12b_0}{HL^2} & 0 \\ \frac{12b_0}{HL^2} & \frac{3}{HL^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \] (74)
Again assuming \( S = \bar{S}, H = \bar{H} \), the normalized mass matrix has the same eigenvalues as

\[
\tilde{M}_s^2 \equiv \frac{1}{2} e^G \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \nu_{ab}G^{-1} & \nu_{ac}G^{-1} \\ \nu_{db}G^{-1} & \nu_{dc}G^{-1} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.
\]  
(75)

With this trick, the scalar masses can be easily obtained:

\[
M_{s_{1,3}}^2 = e^G \frac{|H|^2(6C + |H|^2) + H^3 \sqrt{12C + |H|^2}}{2|H|^4},
\]  
(76)

\[
M_{s_{2,4}}^2 = e^G \frac{|H|^2(6C + |H|^2) - H^3 \sqrt{12C + |H|^2}}{2|H|^4},
\]  
(77)

\[
M_{s_{5,6}}^2 = 0.
\]  
(78)

Here the gravitino mass is

\[
m_{3/2}^2 = e^G = e^K |W|^2 = \frac{d_0^2|H|^6}{LC^3}.
\]  
(79)

The moduli scalar masses are also zero as expected.

The above result indicates that the inclusion of one-loop correction to the dilaton Kähler potential yields several interesting features. The dilaton mass is equal to the \( H \) mass (to be identified with the scale of gaugino condensation) independently of the value of the dilaton VEV. Also, supersymmetry is broken in the hidden sector. We will discuss the possibility of obtaining a hierarchy between the gaugino condensation scale and the Planck (or string) scale below.

We also calculate the \( H \) field and dilaton masses in a more general class of models which have the same Kähler potential but different superpotential:

\[
W = d e^{-3S/2b_0} Y^n \log(\eta Y), \quad Y = e^{S/2b_0} H.
\]  
(80)

(The case \( n = 3 \) corresponds to the model discussed above.) In this class of models, we find the fermion masses are

\[
m_f = e^{G/2} \left\{ \frac{(n - 3)L}{2b_0} \pm \sqrt{\left[ \frac{(n - 3)L}{2b_0} \right]^2 + 3z} \right\}
\]  
(81)
and the scalar masses are

\[ M_{1,3}^s = e^G \left\{ 3z + \frac{1}{2} \left[ 1 + (3 - n) \frac{L}{b_0} \right] \right\} \]

\[ M_{2,4}^s = e^G \left\{ 3z + \frac{1}{2} \left[ 1 - (3 - n) \frac{L}{b_0} \right] \right\} \]

\[ M_{5,6}^s = 0, \]

(82)

(83)

(84)

here \( z = C/|H|^2 \). For \( n \neq 3 \), the dilaton and the \( H \) field masses are no longer equal, but they are still the same order of magnitude for any value of the dilaton VEV. Furthermore, supersymmetry is still broken in the Yang-Mills sector.

With the above mass matrixes calculated, we can easily write down the one-loop vacuum potential energy. The potential energy only depends on the modular invariant function \( z = C/|H|^2 \) so the moduli and dilaton VEVs are not uniquely determined in this model. We take the cut-off scale to be

\[ \Lambda^2 = M_S^2 |H|^2 e^{K/3} \]

\[ = \frac{2}{S + S + 2b_0 \log(T + T)} |H|^2 e^{K/3} \]

\[ = \frac{2z^{-1}}{L + 2b_0 \log(z + 1)}, \]

(85)

where

\[ M_S^2 = \frac{2M_p^2}{S + S + 2b_0 \log(T + T)} \]

(86)

is the string scale, which is also the scale at which the string gauge coupling constants “unify” \[ 25, 26 \]. The one-loop effective potential is

\[ v_1 = 64\pi^2 V^{1\text{-loop}} \]
The minimum condition of the potential energy determine $z$ as a function of $d_0$ and $h$. The result is that for fixed $d_0 \sim 1$, $z$ is a smooth function of $h$ which diverges at $h = 1$ as

$$\frac{z}{\log z} \propto \frac{b_0 d_0}{L} = \frac{d_0}{4 \log h}. \quad (88)$$

We know that $L$ is related to the one-loop gauge coupling constant $g^2$ at the gaugino condensation scale through: $2/L = g^2$. One can see that the large $g^2$ might yield to the large $z$, thus the hierarchy between the string scale and the gaugino condensation scale. Since the gauge coupling constant at the gaugino condensation scale is very large, the higher-loop correction becomes very important. The conclusive result will depend on the inclusion of the higher-loop corrections to the dilaton Kähler potential. We will defer this discussion to the future work.

6 Conclusion

We have shown that the one-loop correction to the dilaton Kähler potential may significantly change the dynamics of gaugino condensation coupled to the dilaton. In a specific model including a dynamical field $H$ for the gaugino bilinear, we find that the supersymmetry is broken by gaugino condensation in the Yang-Mills sector. We also find that the dilaton and the $H$ field
have masses on the same order of magnitude as the gaugino condensation scale. We thus propose that the determination of the dilaton VEV through gaugino condensation may depend sensitively on the dynamics at the gaugino condensation scale. This is very different from the usual scenario in which the dilaton is lighter than the gaugino condensation scale, and is treated as a light field in an effective theory below the gaugino condensation scale.

We also find that the large value of the gauge coupling at the gaugino condensation scale might lead to a hierarchy between the string scale and the gaugino condensation scale, and fix the dilaton VEV at a realistic value. The higher loop corrections to the dilaton Kähler potential may also be important and we defer the investigation to the future work.

The main conclusion of this work is that the inclusion of the loop correction to the dilaton Kähler potential may dramatically change the scenario of determining the dilaton VEV through the gaugino condensation and may lead to the solution of the dilaton runaway problem.

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