Photon mass and quantum effects of the Aharonov-Bohm type

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The magnetic field due to the photon rest mass $m_{ph}$ modifies the standard results of the Aharonov-Bohm effect for electrons, and of other recent quantum effects. For the effect involving a coherent superposition of beams of particles with opposite electromagnetic properties, by means of a table-top experiment, the limit $m_{ph} \approx 10^{-51} g$ is achievable, improving by 6 orders of magnitude that derived by Boulware and Deser for the Aharonov-Bohm effect.

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INTRODUCTION

The possibility that the photon possesses a finite mass and its physical implications have been discussed theoretically and investigated experimentally by several researchers[1]-[7]. Originally, the finite photon mass $m_{\gamma}$ (measured in centimeters$^{-1}$) has been related to the range of validity of the Coulomb law[1], [2]. If $m_{\gamma} \neq 0$ this law is modified by the Yukawa potential $U(r) = e^{-m_{\gamma}r}/r$, with $m_{\gamma}^{-1} = \hbar/m_{ph}c = \lambda_{C}/2\pi$ where $m_{ph}$ is expressed in grams and $\lambda_{C}$ is the Compton wavelength of the photon.

There are direct and indirect tests for the photon mass. The best result obtained so far through developments of the original Cavendish technique for direct tests of Coulomb’s law is still that from 1971 by Williams, Faller and Hill[2]. The null result of this experiment expressed in the form of range of the photon rest mass is $m_{\gamma}^{-1} > 3 \times 10^{9} cm$. Indirect methods are provided by geomagnetic and astronomical tests. By studying the behavior of the magnetic field of planets, Davis, Goldhaber and Nieto[3] were able to determine that $m_{\gamma}^{-1} > 5 \times 10^{10} cm$.

Other indirect verifications are related to lumped circuit tests[4], cryogenic experiments[5], and the ambient cosmic vector potential developed by Lakes[6]. If $m_{\gamma} \neq 0$ the ambient cosmic vector potential acquires physical significance and may interact with the dipole field of a magnetized toroid. The related experiment by Luo, Tu, Hu, and Luan[7] yielded the range $m_{\gamma}^{-1} > 1.66 \times 10^{13} cm$ and corresponding photon mass $m_{ph} < 2.1 \times 10^{-51} g$.

Several conjectures related to the Aharonov-Bohm (AB) effect[8] have been developed assuming electromagnetic interaction of fields of infinite range, i.e., zero photon mass. The possibility that any associated effects become manifest within the context of finite-range electrodynamics has been discussed by Boulware and Deser (BD)[9]. In this paper we consider and extend BD’s approach, and evaluate the limits of the photon mass that can be determined by means of recent, new quantum effects of the Aharonov-Bohm type.

PHOTON MASS AND NEW EFFECTS OF THE AHARONOV-BOHM TYPE

In their approach, BD consider the coupling of the photon mass $m_{\gamma}$, as predicted by the Proca equation, and calculate the resulting magnetic field $\mathbf{B}$

$$\partial_{\nu} F^{\mu\nu} + m_{\gamma}^{2} A^{\mu} = J^{\mu}, \quad \mathbf{B} = \mathbf{B}_{0} + \hat{\mathbf{k}} m_{\gamma}^{2} \Pi(\rho), \quad (1)$$

that might be used in a test of the AB effect. The first term, $\mathbf{B}_{0}$, is the standard magnetic field for zero photon mass — the field confined inside a long solenoid of radius $a$ and carrying the current $j$ — and the second term $\Delta \mathbf{B} = \hat{\mathbf{k}} m_{\gamma}^{2} \Pi(\rho)$ represents a correction due to the photon nonvanishing rest mass $m_{\gamma}$. Because of the extra mass-dependent term, BD obtained a nontrivial limit on the range of the transverse photon from a table-top experiment: $m_{\gamma}^{-1} > 1.4 \times 10^{7} cm$. In the case of the standard Aharonov-Casher (AC) effect[10], this analysis has been performed by Fuchs[11] who points out that, for a neutral particle with a magnetic dipole moment that couples to nongauge fields, no observable corrections are expected.

After the AB and AC effects, other quantum effects of this type have been developed. Thus, it would be interesting to consider other effects of the AB type, such as those associated with neutral particles that have an intrinsic magnetic[12] or electric dipole moment[13]-[15], and those with particles possessing opposite electromagnetic properties, such as opposite dipole moments or charges[12], [16]-[18]. In the next Sections we consider the impact of some of these new effects on the photon mass. The goal would be to see if they provide similar correction terms that might be suitable for setting more precise limits on the range of $m_{\gamma}^{-1}$.

Before dealing with effects for electric dipoles, we recall that in Eq. (1) the quantity $\Pi(\rho)$ can be expressed in terms of the Bessel functions $I_{0}(m_{\gamma}\rho)$ and $K_{0}(m_{\gamma}\rho)$, which are regular at the origin and infinity respectively.
and reads
\[
\Pi(\rho) = j\theta(a - \rho)[K_0(m_\gamma \rho) \int_0^\rho I_0(m_\gamma \rho') \rho' d\rho' \\
+ I_0(m_\gamma \rho) \int_0^a K_0(m_\gamma \rho') \rho' d\rho'] \\
- j\theta(\rho - a)K_0(m_\gamma \rho) \int_0^a I_0(m_\gamma \rho') \rho' d\rho'.
\]

**EFFECTS FOR ELECTRIC DIPOLES**

The interaction term of all the effects for electric dipoles has the same strength $\mu I_0$ so that, for the purpose of performing a table-top experiment, we find it convenient to analyze the Tkachuk effect [15] because the resulting equations for the mass correction possess a symmetry analogous to that of the AB effect.

For the Tkachuk effect we can consider a long solenoid with the magnetization linear density $\mu = \mu z$ and a magnetic flux $\Phi = BS = 4\pi \mu z = \pi_0^2 a^2$, where $a$ is the radius of the solenoid and $\pi_0$ its current density. The resulting vector potential reads $A = A_{AB} z$, where $A_{AB}$ is the vector potential of the AB effect with $\mu_{AB}$ substituted by $\mu$.

The starting equation is $-(\nabla^2 + m_\gamma^2)A = J$ with $J = (4\pi/\alpha^2)z\delta(\rho - a)$. The only difference with the AB effect is that the current depends on $z$. Separation of variables with $A = z A_T(x, y)$ yields $-(\nabla^2 + m_\gamma^2)A_T(\rho, \phi) = 4(\pi/\alpha^2)\rho \delta(\rho - a)$, which is the same equation of BD. Thus, the mass treatment for the Tkachuk effect for the electric dipole $d = d \hat{k}$ can be reduced to that of BD.

The magnetic field is $B = \nabla \times A = z \nabla \times A_T - A_T \times \nabla z$. In the plane of motion of the dipole, $z = 0$, and the Tkachuk phase shift is [15]
\[
\Delta \varphi \propto \oint B \cdot d\ell = -\oint (A_T \times \hat{k}) \times d\ell \\
= d \oint (A_T) \cdot d\ell = d \oint S \nabla \times A_T \cdot dS
\]
where the last integral is the flux through the surface as in the AB effect and BD approach. From Eq. [9], we write for the photon mass contribution $\Delta B(\gamma, \rho) = \nabla \times A_{Tm0}(\gamma, \rho) = \hat{k} m_\gamma^2 \Pi(\gamma, \rho)$ so that the mass correction to the phase reads
\[
\Delta \varphi = 2\pi(d/4\pi c) \int_0^{\rho} \left[m_\gamma^2 \Pi(\gamma, \rho)\right] \rho d\rho.
\]
In the exterior ($\rho > a$) region, $\Delta B = m_\gamma^2 \Pi(\gamma, \rho) = \gamma(\gamma/2)(m_\gamma a)^2 \ln(2/m_\gamma a)$. With $4\pi = \gamma a^2$, and the Tkachuk phase $\varphi_0 = 4\pi a/4\pi c$, the relative variation of the phase due to the photon mass is
\[
\frac{\Delta \varphi}{\varphi_0} = \frac{7\alpha^2}{4\pi^2} \int_0^{\rho} m_\gamma^2 \ln \left(\frac{2}{m_\gamma \rho}\right) \rho d\rho \sim \frac{1}{2} (m_\gamma \rho)^2 \ln \left(\frac{2}{m_\gamma \rho}\right).
\]

Following BD [9] we set $\Delta \varphi \geq 2\pi \varepsilon = 2\pi \times 10^{-3}$, where $\varepsilon$ is the precision of the measurement, and write Eq. [2] as
\[
2\pi \varepsilon/\varphi_0 = (1/2)(m_\gamma \rho)^2 \ln(2/m_\gamma \rho).
\]
This result, valid for the Tkachuk effect, can be compared with that of BD derived for the AB effect,
\[
2\pi \varepsilon/\varphi_{AB} = (1/2)(m_{\gamma BD} \rho)^2 \ln(2/m_{\gamma BD} \rho),
\]
where $m_{\gamma BD}$ is the value of the photon mass obtained by BD and $\varphi_{AB}$ is the value of the AB phase shift when $m_\gamma = 0$. In this case, the contribution due to the logarithmic terms is not relevant and can be neglected. For the comparison, we use $d = e a_0$ for the dipole with $a_0$ the Bohr radius, $\mu = \mu_{AB}/l$ with $l \simeq 1 cm$ the realistic length of the solenoid in the Tkachuk effect [15], and obtain
\[
m_{\gamma}^{-1} = m_{\gamma BD}^{-1} \left[\frac{\varphi_0}{\varphi_{AB}}\right]^{1/2} = m_{\gamma BD}^{-1} \left[\frac{a_0}{l}\right]^{1/2} \sim 10^{-4} m_{\gamma BD}^{-1},
\]
which represents a range limit of the photon mass 4 orders of magnitude lower than that of BD.

As expected, no improvement for the range $m_{\gamma}^{-1}$ is achieved from a table-top experiment involving electric dipoles because of the lower strength of the em interaction.

**EFFECT FOR SUPERPOSITION OF $\pm$ CHARGED PARTICLES**

One of us [16] has pointed out that the observable quantity in the AB effect is actually the phase difference
\[
\Delta \varphi = \frac{e}{hc} \int A \cdot d\ell - \int A_0 \cdot d\ell
\]
where the integral can be taken over an open path integral. For the usual closed path $c$ encircling the solenoid and limiting the surface $S$, the observable quantity is the phase-shift variation, $\Delta \phi \propto \int c A \cdot d\ell$ or phase shift $\phi \propto \int c A \cdot d\ell$, impossible in principle without the comparison of the actual interference pattern with an interference reference pattern. Thus, $\varphi$ or $\phi$ are not observable, but the variations $\Delta \varphi$ and $\Delta \phi$ are both gauge-invariant observable quantities [16].

It follows that, in analogy with the AC effect for a coherent superposition of beams of magnetic dipoles of opposite magnetic moments $\pm \mu$ [17] and the effect for electric dipoles of opposite moments $\pm d$ [18], an effect of the AB type for a coherent superposition of beams of charged particles $\pm e$ is theoretically feasible [16]. In the mentioned cases, the beam...
of particles possessing opposite em properties do not en-
circle the singularity (e. g., solenoid for the AB effect, and line of charges for the AC effect) but travel at one side of it along a straight path C. Depending on the interferometric technique used [17], [18], the length of C can be of the order of a few cm up to a few m. In the experiment by Sangster et al. [17], the beam splitter of the magnetic dipoles \( \pm \mu \) is the magnetic field \( B \) of a Ramsey loop [17]. Similarly, in the experimental set up considered by Dowling et al. [18], the electric dipoles \( \pm d \) are split by an electric field \( E \). An external uniform electric potential \( V \) could act as a possible beam splitter for particles of opposite charge \( \pm q \).

Although the effect for \( \pm q \) charged particles is viable [10], the technology and interferometry for the test of this effect needs improvements. It is worth recalling that not long ago the technology and interferometry for beams of particles with opposite magnetic \( \pm \mu \) or electric \( \pm d \) dipole moments was likewise unavailable, but is today a reality [17], [18]. Discussions on this subject may act as a stimulating catalyst for further studies and technological advances that will lead to the experimental test of this quantum effect. An important step in this direction has already been made [10] by showing that, at least in principle and as far as gauge invariance requirements are concerned, this effect is physically feasible. Therefore, using this effect in a table-top experiment analogous to that of BD, one is entitled to ask what would be its relevance in eventually determining a bound for the photon mass \( m_{\gamma} \).

**Determining the mass correction \( \Delta \varphi \) in the effect for \( \pm q \) charged particles**

In the experimental set ups detecting the traditional AB effect there are limitations imposed by the suit-
able type of interferometer related to the electron wave-
length, the corresponding convenient size of the solenoid or toroid, and the maximum achievable size \( \rho \) of the co-
herent electron beam encircling the magnetic flux [9]. In
the analysis made by BD, the radius of the solenoid is
\( a = 0.1 \text{cm} \), and \( \rho \) is taken to be about 10 \( \text{cm} \), implying that the electron beam keeps its state of coherence up to a size \( \rho = 10^2 a \), i. e., fifty times the solenoid diameter. The advantage of the new approach for the \( \pm q \) beam of particles is that the dimension of the solenoid has no upper limits and is conditioned only by practical limits of the experimental set up, while the size of the coherent beam of particles plays no important role.

In order to calculate the line integral appearing in Eq. [3] we need the analytical expression of \( A(x) \). This can be obtained from Stokes’ theorem, \( \int \mathbf{B} \cdot d\mathbf{S} = \oint \mathbf{A} \cdot d\ell = 2\pi A_{y}\rho \) and solving for \( A_{x} \). Another approach consists of calculating \( A \) from the expression \( A = \hat{k} \times \nabla \Pi(\rho) \) in Ref. [3], using the recurrence relations for the modified Bessel functions. The same result can be obtained using the approach [13], [21] that consists of calculating the interaction electromagnetic momentum, which in the Coulomb gauge yields \( eA/c \). The result is

\[
A_{x} = j a^{2} \frac{1}{\rho} + \left( \frac{j}{2} \right) (m_{\gamma} a)^{2} \frac{\rho}{2} \ln \left( \frac{m_{\gamma} \rho}{2} \right).
\]

Taking the path \( C \) along the \( x \) axis for a path length \( 2x \) with \( x \gg y \) we find \( \varphi_{0} = \int_{C} A_{y} = \rho \cdot d\ell \simeq -(\pi/2)a^{2} j \). The contribution due to \( m_{\gamma} \) yields

\[
\int_{C} A \cdot d\ell = (j/2) (m_{\gamma} a)^{2} y x \ln \left( m_{\gamma} \sqrt{x^{2} + y^{2}/2} \right)
\]

for the same path length \( 2x \). Consequently, from Eq. [3] and Ref. [16] the observable phase shift variation is

\[
\Delta \varphi = 2 j (m_{\gamma} a)^{2} y x \ln \left( m_{\gamma} \sqrt{x^{2} + y^{2}/2} \right)
\]

and

\[
\frac{\Delta \varphi}{\varphi_{0}} = \frac{4}{\pi} m_{\gamma}^{2} xy \ln \left( m_{\gamma} \sqrt{x^{2} + y^{2}/2} \right).
\]

**Evaluating the photon mass limit**

Following BD [3] we set \( \Delta \varphi \geq 2\pi \varepsilon = 2\pi \times 10^{-3} \) where \( \varepsilon \) is the precision of the measurement. The value of \( m_{\gamma} \) at which the effect is just observable is

\[
\frac{2\pi \varepsilon}{\varphi_{0}} = \frac{4}{\pi} m_{\gamma}^{2} xy \ln \left( m_{\gamma} \sqrt{x^{2} + y^{2}/2} \right).
\]

This value can be compared with the corresponding one by BD [3], while, as done by BD, we neglect the small corrections due to the contribution of the logarithms.

The question is now: what would be the size of the solenoid in order to achieve a photon mass limit of the order of that of Ref. [3] found by Luo et al.? We estimate \( m_{\gamma} \) with respect to \( m_{\gamma BD} \) for an ideal experimental set up that, apart from considerations of cost, is realistically within reach of present technology. For a vector potential produced by the magnet of a huge cyclotron-type solenoid (radius \( a = 5m \) and length or height \( D \) several times the radius), we estimate \( \varphi_{0} / \varphi_{BD} \simeq a^{2} / (aBD)^{2} = 5^{2} / (10^{-3})^{2} \). For a path of \( x = 6a = 300\rho \) at the distance \( y = 80\rho \) we obtain

\[
m_{\gamma BD}^{-1} = m_{\gamma BD}^{-1} \left[ \frac{8 \varphi_{0} xy}{\pi \varphi_{BD} \rho^{2}} \right]^{1/2} \simeq 10^{6} m_{\gamma BD}^{-1} \quad (5)
\]

With their table-top experiment, BD obtained the value \( m_{\gamma BD}^{-1} \simeq 140Km \) that is equivalent to \( m_{\gamma BD} = 2.5 \times 10^{-45}g \). With our approach, the new limit [3] of the photon mass is \( m_{\gamma ph} \simeq 2 \times 10^{-51}g \) which is of the same order of magnitude of that found by Luo et al. [3].
Secondary effects

When the AB effect was tested for the first time \[19\], physicists were concerned about the effect of the stray fields just outside the solenoid on the phase shift of electrons that was going to be observed. The stray field \(\Delta B\) acts on the beam of charges, bends it, and displaces the interference pattern. Depending on the technique used for observing the AB phase shift, the effect of the stray field may mask, or not, the AB phase shift. With the approach used by Chambers \[19\], the interference pattern is shifted by a large amount but the figure of the pattern is left unaltered. However, the AB effect changes the figure of the pattern so that the AB phase shift is easily observable. In this case the two effects, that of the stray fields and of the AB phase shift, can be separated and observed. The field \(\Delta B = m_e^2 \Pi(\rho) \hat{k}\) produces a variation of the particle momentum \(\delta p_{\perp} \propto \int ev \Delta B dt\) in the direction perpendicular to the direction of motion \(v\). The corresponding angular deflection of the beam, \(\alpha \approx \delta p_{\perp}/p\), can be estimated and the equivalent shift of the interference pattern, considering a beam of particles through a double-slit interferometer, can be determined. The equivalent resulting phase shift \((\Delta \varphi)_{\Delta B}\) is smaller by about two orders of magnitude than the phase shift value \(\Delta \varphi_{BD}\) found by BD \[9\].

Although this \((\Delta \varphi)_{\Delta B}\) is small and does not lead to important phase shift variations with respect to \(\Delta \varphi_{BD}\), the fact that the leakage field \(\Delta B = m_e^2 \Pi(\rho) \hat{k}\) bends the electron beam suggests that, within a classical approach, \(m_e\) may be estimated by measuring directly the angular or linear deflection of the beam. In order to magnify the deflection and improve the \(m_e^{-1}\) range, it would be convenient to use the magnetic field generated by the cyclotron-type solenoid, as described above, supposing ideally that the other stray fields due to imperfections in the construction of the solenoid and to its finite length can be taken into account separately.

The effect of the leakage field \(\Delta B = m_e^2 \Pi(\rho) \hat{k}\) will not be considered in this paper and will be discussed in detail elsewhere. We simply mention that measurements of the linear displacement \(s_{\perp} \approx \delta s_{\perp}\) performed with this approach, that treats the electron as a classical particle, possesses quantum restrictions and can be meaningful only up to the value of the bound established by Heisenberg’s uncertainty principle \(\delta p_{\perp} \delta s_{\perp} \approx \hbar\).

CONCLUSIONS

We have considered the table-top approach of BD and extended it to several effects of the AB type. In discussing the quantum effects for electric dipoles and comparing them with the BD approach, we have found no improvement for the \(m_e^{-1}\) range in this case. For the case of the AB solenoid, improvements are possible by taking into account the effect of the leakage field \(\Delta B\) on the beam of particles, testing its effect on the bending of the beam in a classical approach.

Moreover, if a cyclotron-type solenoid is used for testing the \(\pm q\) quantum effect proposed by Spavieri \[16\], a photon mass bound of the value of \(m_{ph} \sim 10^{-51} g\) should be achievable. This result represents a lower limit improvement of 6 orders of magnitude with respect to the approach of BD with the standard AB effect. The latest results by Luo et al. \[6\] and the prospects of the AB type of quantum effect scenario here discussed are certainly remarkable if one considers that, according to the uncertainty principle, a purely theoretical estimate of the photon mass is given by \(m_{ph} = h/(\Delta t)^2\) which yields an order of magnitude number of \(m_{ph} = 10^{-65} g\), where the age of the universe is taken to be roughly \(10^{10}\) years.

In closing, advances in the area related to the AB type of effects indicate that the photon mass limit achievable with this quantum approach could compete with other methods. However, it is not only a question of improving the limits, but of extending the scenario where tests of the photon mass can be realized, as in the cryogenic photon-mass experiment performed by Ryan et al. \[5\] where the validity of the results is extended from the standard terrestrial (‘room’) temperatures to those of the galactic environment. Each approach is important in itself as it extends the range of validity of the Coulomb law and of the \(m_e^{-1}\) range as a function of the physical conditions of the measurement.

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