Hydraulic jump on smooth and uneven bottom

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Abstract—The mechanism of absorption of excess power of the flow within hydraulic jump has been studied in the article based on theoretical manner. Mathematical model of hydraulic jump has been investigated by taking additional water body (mass) in hydraulic jump zone as basic. Theoretical research has shown that main part of excess power is discharge for rotation of additional water mass and a formula has been obtained to make calculation thereof. The article also has provided a formula for calculating the portion of flow energy needed for overcoming friction resistance emerged in bed bottom. Because of conducted studies, formulas have been suggested for calculating hydraulic jump length occurred in flat and uneven beds. Obtained formulas have been mutually analyzed with results found by other researchers.

Keywords—hydraulic jump, variable mass, motion quantity, additional discharge, critical section, jump length.

I. INTRODUCTION

It is obvious that process of absorption of excess flow energy in tailraces of hydro-technical facilities happen by hydraulic jump. One of the main parameters when designing tailraces of hydro-technical facilities is accurate and proper calculation of length of emerged hydraulic jump. Study of energy losses in the jump zone bears great importance when analyzing flow structure. Number of studies have been devoted to the given problem.

Results of these studies indicate that intensive turbulent agitation takes place in the area of hydraulic jumping, which causes penetration of large vortex structures in the form of additional discrete liquid masses from the turbulent (stormy) zone into transit (tranquil) one.

An analysis of the existing assignments on this issue suggests that most of them are devoted to a flow with a constant mass. Currently, several empirical formulas are used in practice to determine the length of hydraulic jump.

a) The formula of N.N.Pavlovsky:

\[ l = 2.5 \left(1.9 \frac{h_2}{h_1}\right) \]

b) The formula of M.O. Chertausov:

\[ l = 10.3 h_1 \left(\sqrt{Fr} - 1\right)^{0.81} \]

c) The formula of Safrenech:

\[ L = 4.5 \ h_2 \]

d) The formula of Bakhmetov-Matchick:

\[ L = 5 \ (h_2 - h_1) \]

Great majority of these formulas have been proposed considering analysis of the results of studies conducted in various laboratories globally that carry out hydraulic investigations. Results derived from the calculation formulas sometimes vary from each other up to 50-80%. Proper design of water stilling wells constructed in tailraces of hydro-technical facilities depends on accurate calculation of hydraulic jump length. Studies regarding hydromechanical analysis of energy absorption within hydraulic jump have not been conducted in known formulas. All calculation formulas have been empirically suggested based on the results of laboratory tests performed within a given range.

Theoretical research.

Unlike existing tasks and works, we consider a hydraulic jump in which the motion of a liquid occurs with a variable mass with decreasing number of motions. With a sudden transition of the flow from a turbulent state to a calm one between sections I-I and II-II, a hydraulic jump is generated within which highly complex hydrodynamic process takes place (Fig. 1).

Both connection to the main flow (between sections I-I and K-K) and separation of the additional flow (between sections K-K and II-II) from it happens within the limits of hydraulic jump. In this case, specific water discharge in the range of I-I will be \( q_0 \), it will increase in the range of K-K and become \( (q_0 + q_1) \), where \( q_0 \) is the specific flow rate of the connected flow. Separation of connected discharge \( q_0 \), from main stream takes place in the area between the ranges of K-K and II-II.

As a result, specific discharges of main stream in sections of I-I and II-II are the same and equal to \(-q_0\).
It has been established according to available studies that energy of the stream within the hydraulic jump is getting decreased. However, according to results of the studies we have conducted it becomes obvious that the nature of the change in energy of the stream between sections I-I and II-II differs to some extent from existing similar studies. When processing the data of numerous studies it is confirmed that specific energy of the stream decreases to a minimum - $E_2$ in section K-K along the length of the hydraulic jump, and then, it increases somewhat due to restoration of the pressure and equals to $E_2$ in section II-II.

We reviewed G.A. Petrov equation in the following form (10,11,12,13,14...), to study pressure loss in hydraulic jump:

\[ d\left(\frac{\alpha_0 v^2}{2g}\right) + \frac{dP}{\gamma} + dz + i_f dx + \frac{\alpha_0 (v - \theta)}{g} \cdot \frac{vdQ}{Q} = 0 \]

(1)

where: $Q$ - is discharge along the flow; $\bar{v}$ - average flow velocity along the flow; $P$ - hydrodynamic pressure; $i_f$ - hydraulic slope of the friction; $\theta$ - projection of velocities of attached particles on the direction of main stream velocity. Accepting bed bottom as horizontal and integrating equation (1) between the sections I-I and II-II along the flow length and after not complicated transformation, we get:

\[ \frac{\alpha_0 v_1^2}{2g} + h_1 = \frac{\alpha_0 v_2^2}{2g} + h_2 + \int_i \frac{\alpha_0 (v - \theta)}{g} \cdot \frac{v}{Q} \cdot dQ \]

(2)

Supposing that energy loss during the jump is equal to difference in energy of $E_1$ and $E_2$ in sections I-I and II-II, instead of (2) we find out:

\[ \frac{\alpha_0 v_1^2}{2g} + h_1 = \frac{\alpha_0 v_2^2}{2g} + h_2 + \int_i \frac{\alpha_0 (v - \theta)}{g} \cdot \frac{v}{Q} \cdot dQ \]

(3)

Equation (3) reminds the Bernoulli equation, but with new term on the right side. During flow movement with variable discharge along the path, we determine the pressure loss on the friction against the bottom and the side walls of the channel bed by the formula of the same kind as with constant discharge, i.e.:

\[ h_f = \frac{\alpha_0}{g} \int_i \frac{v^2}{Q} \cdot dQ = \frac{\alpha_0}{g} \cdot \frac{v_{av}^2}{C_{av}^2 \cdot R_{av}} \]

(4)

where $C_{med}$ is average value of Chezy coefficient between the sections; $R_{med}$ average value of hydraulic radius between the sections.

Another integral in (3) expresses mainly the pressure loss caused by the variability of the flow discharge:

\[ h_{mix} = \frac{\alpha_0}{g} \int_i \frac{v^2 (v - \theta)}{Q} \cdot dQ \]

(5)

where: $\bar{v}$ means the ratio of the projection of the velocity of the attached flow to the main one.

We will assume in future studies $\bar{v} = \bar{\theta} = 1$ and $\bar{v} = 0$ (meeting of two streams happens at an angle of 90°).

Taking into account assumptions and considering that $b = 1.0 \text{ m}$, $dQ = qdx$, we determine the value of the pressure loss by formula (5) as follows:

\[ h_{mix} = \frac{\alpha_0}{g} \int_i \frac{v^2 (v - \theta)}{Q} \cdot dQ = \frac{1}{g} \int_i \frac{2v^2q_x}{bh_v} \cdot dx = \frac{1}{g} \int_i \frac{2\bar{v}^2q_x}{\bar{h}_x} \cdot dx \]

(6)

$\bar{v}_x$ and $\bar{h}_x$ in expression (6) are variable values and depend on the length of the jump.
We determine the value of \( q_x \), for running value of separable or connected value from the expression:

\[
q_x = \frac{q_d}{l_x}
\]  

(7)

Where: \( l_x \) is the length of connection and separation section; \( q_x \) is additional discharge.

By integrating he dependence (6) within the boundaries of I-I and K-K sections, we determine pressure loss for mixing additional discharge with main discharge:

\[
\Delta h_{mix} = \frac{1}{g} \int \frac{\vec{h}_{s_x}}{h_{s_x} - h_1} \frac{q_d}{l_1} dx
\]  

(8)

We apply change of \( \vec{v} \) and \( h_1 \) in these sections according to straightforward principle:

\[
\vec{v}_{s_x} = v_1 - \left( v_1 - v_{cr} \right) \frac{x}{l_1}
\]

\[
\vec{h}_{s_x} = h_1 + \left( h_{cr} - h_1 \right) \frac{x}{l_1}
\]

(9)

Considering (9) and (10), equation (8) obtains the following form:

\[
\Delta h_{mix} = \frac{1}{g} \int \frac{v_1 - v_{cr}}{h_{cr} - h_1} \frac{x}{l_1} l_1 \frac{q_d}{g l_1} dx
\]

(11)

For hydraulic jump area between I-I and K-K sections with water depth \( h_1 \) and \( h_{cr} \), expression (11) can be presented in the following form after minor transformations:

\[
dh_{mix} = \frac{\alpha q_d}{g l_1} \int \frac{v_1 - v_{cr}}{h_{cr} - h_1} \frac{x}{l_1} l_1 \frac{q_d}{g l_1} dx
\]

(12)

By integrating and transforming expression (12), we find out dependence of pressure loss on the mixing of the additional discharge with the primary one during the jump in the form below:

\[
\Delta h_{mix} = \frac{\alpha q_d}{g l_1} \left[ \frac{\left( v_1 \right) h_{cr} - v_{cr} h_1}{h_{cr} - h_1} \ln h_{cr} - h_1 + \frac{v_1 - v_{cr}}{h_{cr} - h_1} \right]
\]

(13)

By integrating the dependence (8) within the limits of K-K – II-II sections, we determine pressure loss on separation of the additional discharge from the primary one:

\[
\Delta h_{sec} = \frac{1}{g} \int \frac{\vec{h}_{s_x}}{h_{s_x}} \frac{q_d}{l_{s_x}} dx
\]

(14)

We accept the change of \( \vec{v} \) and \( \vec{h} \) according to straightforward principle in the following manner:

\[
\vec{v}_{s_x} = v_{cr} - \left( v_{cr} - v_2 \right) \frac{x}{l_2}
\]

\[
\vec{h}_{s_x} = h_{cr} + \left( h_2 - h_{cr} \right) \frac{x}{l_2}
\]

(15)

(16)

Considering (15) and (16), equation (14) obtains the following form:

\[
\Delta h_{sec} = \int \frac{v_{cr} - v_2}{h_{cr} - h_2} \frac{x}{l_2} \frac{q_d}{g l_2} dx
\]

(17)

Expanding the integral in expression (17) and conducting certain transformations, we get the formula for pressure loss on separation of additional discharge during the jump:

\[
\Delta h_{sec} = \frac{\alpha q_d}{g l_1} \left[ \frac{\left( v_{cr} \right) h_{cr} - v_{cr} h_1}{h_{cr} - h_1} \ln h_{cr} - h_1 + \left( v_2 - v_{cr} \right) \right]
\]

(18)

Thus, two expressions (13) and (18) were obtained for determining pressure loss during hydraulic jump on connection and separation of additional discharge.

It should be mentioned that, according to the adopted scheme, the energy of the stream gets decreased before the critical section. Arriving at minimum value in the critical section, and further due to separation of additional discharge, the flow partially recovers its energy. This condition indicates that pressure restoration happens in the section K-K – II-II. Pressure restoration value is determined by the expression (18).

The values \( q_d \) and \( h_{cr} \) can be determined from the following expression \( \square 13 \square \) with known magnitudes of hydraulic parameters of the jump:

\[
h_{cr} = \sqrt{\frac{1}{3} \left[ \frac{h_2 \left( h_1 + h_2 \right) + h_1^2}{h_2 - h_{cr}} \right]}
\]

(19)

\[
q_d = \sqrt{\alpha_0} \left[ \frac{h_{cr}^3}{2 \alpha_0} - \frac{h_{cr}^2 \left( h_1 + h_2 \right)}{2 \alpha_0} \right]
\]

(20)

Being aware of parameters on hydraulic jump elements, it is possible to determine pressure loss due to the connection and separation of the additional discharge along the length of the jump according to formulas (13) and (18) with great accuracy.

Proposed dependencies (13) and (18) enables for determining energy loss in hydraulic jumping. These energy losses are formed under the influence of a surface roller. On the other hand, part of the flow energy is spent
on overcoming the resistance of the bed bottom. Specific energies $E_1$, $E_2$ and $E_3$ in sections I-I, II-II and K-K of the hydraulic jump are identified by using hydraulic flow parameters. At the same time, energy losses between the sections I-I and K-K constitute \( E_1 = E_1 - E_2 \), and between the sections of K-K and II-II they become \( E_2 = E_2 - E_3 \). In all cases, terms of \( E_1 \) and \( E_2 \) are respectively satisfied. Hence it is obvious that during flow movement within the boundaries between the sections K-K and II-II, the specific flow energy increases additionally from the minimum (section K-K) to \( E_2 \) (section II-II) by an amount of \( E_2 \). While calculating for (13) and (18) of values \( h_1 \) and \( h_2 \) you can determine energy loss to overcome resistance along a segment of length \( L_1 \) from the expression \( H_L = E_2 - h_1 \) and along the length of section \( L_2 \) from the expression \( H_L = h_2 - E_2 \). It becomes obvious from the presented material that, in hydraulic jump, energy loss necessary for overcoming bottom resistance of the bed will be equal to \( h_1 + h_2 = E_2 \).

It should be mentioned that in order to determine hydraulic parameters of the stream and to find the magnitude of value of the pressure loss in the hydraulic jump according to (13), (18) and other expressions we recommended, the data of laboratory studies by D.A. Akhutin were processed, the results of which are presented in Table 1. It is obvious from this table that experiments were carried out at the values of the conjugate depths \( h_1 = 1.30 \ldots 7.35 \) cm and \( h_2 = 10.60 \ldots 32.80 \) cm, and the length of the hydraulic jump fluctuated within the range of 35 ... 185 cm.

Pressure losses in the hydraulic jump calculated by (13) and (18) are respectively \( h_2 = 1.01 \ldots 36.0 \) cm and \( h_2 = 0.85 \ldots 3.87 \) cm. The total pressure loss along the length of the hydraulic jump varied from 0.15 to 11.81 cm. Furthermore, according to the data in the table, it is obvious that for all the experiments \( E_2 \) is negative and the conditions of \( E_2 - h_1 \) and \( E_2 - h_2 \) are satisfied. This statement fully confirms the recommendations we made on the assessment of hydraulic jumps.

To determine the magnitude of pressure loss for overcoming the bottom resistance of the channel, bypassing formulas (13) and (18), after processing numerous data, a dependence was obtained in the form:

\[
\Delta E = E_1 \left[ 0.78 - 0.89 \left( \frac{h_1}{h_2} \right)^{0.15} \right],
\]

(21)

where \( h_1, h_2 \) – are first and second conjugate depths in hydraulic jump; \( E_1 \) – is specific flow energy within initial section of hydraulic jump.

| Water depth in section I-I, \( h_1, \) sm | Water depth in section II-II, \( h_2, \) sm | Length of hydraulic jump, \( L_1, \) sm | Froude number | Flow velocity in section I-I, \( q_{1, r}, \) m/s | Flow energy in section I-I, \( E_1, \) sm | Flow energy in section II-II, \( E_2, \) sm | Critical depth, \( h_{cr}, \) sm | Critical flow discharge, \( q_{cr}, \) sm$^3$/s | Critical flow energy, \( E_{cr}, \) sm | \( h_1 = E_1 - E_2 \) | \( h_2 = E_2 - E_3 \) | \( c_{H_1} \) for (4.27), m | \( c_{H_2} \) for (4.32), m | \( E = H_{h_1} + H_{h_2} \) | \( E = H_1 + H_2 \) |
|-----------------|-----------------|-----------------|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1.3             | 10.9            | 55              | 39.3          | 224             | 26.7            | 26.9            | 11.26           | 6.7             | 251.99          | 81.07           | 10.05           | 16.82           | -1.21           | 13.36           | 1.3             | 3.47            | 0.08            |
| 1.3             | 16.0            | 82              | 81.9          | 323.2           | 26.3            | 54.5            | 16.35           | 9.63            | 516.58          | 97.22           | 14.45           | 40.08           | -1.90           | 31.08           | 2.07            | 9.0             | 0.17            |
| 1.9             | 8.08            | 35              | 13.0          | 155.9           | 33.7            | 14.3            | 9.38            | 5.71            | 130.99          | 74.84           | 8.56            | 5.73            | -0.81           | 4.73            | 0.85            | 1.0             | 0.03            |
| 1.9             | 14.7            | 72              | 33.8          | 251.0           | 32.4            | 34.0            | 15.24           | 9.09            | 380.84          | 94.41           | 13.63           | 20.38           | -1.61           | 16.26           | 1.71            | 4.12            | 0.10            |

Elements of hydraulic jump and energy loss in it

Table 1.
| 1.9 | 21.0 | 107 | 66.6 | 352.3 | 31.9 | 65.2 | 21.52 | 12.71 | 749.50 | 111.66 | 19.06 | 46.11 | -2.46 | 36.00 | 2.67 | 10.1 | 0.21 | 10.31 |
|-----|------|-----|------|------|-----|------|-------|-------|--------|--------|------|-------|-------|--------|------|------|-----|------|
| 2.52 | 6.20 | 15 | 4.30 | 102.6 | 41.7 | 7.9 | 7.090 | 4.49 | 39.240 | 66.35 | 6.730 | 1.15 | -0.35 | 1.01 | 0.36 | 0.14 | 0.00 | 0.15 |
| 2.52 | 13.7 | 63 | 17.5 | 208.0 | 38.3 | 24.6 | 14.45 | 8.73 | 283.61 | 92.54 | 13.09 | 11.47 | -1.35 | 9.37 | 1.42 | 2.11 | 0.06 | 2.17 |
| 2.52 | 19.1 | 104 | 32.5 | 283.5 | 37.4 | 43.5 | 19.81 | 11.82 | 558.72 | 107.69 | 17.73 | 25.75 | -2.08 | 20.57 | 2.21 | 5.18 | 0.13 | 5.31 |
| 2.52 | 24.4 | 127 | 51.7 | 357.6 | 36.9 | 67.7 | 25.10 | 14.87 | 894.61 | 120.77 | 22.30 | 45.38 | -2.79 | 35.72 | 3.01 | 9.66 | 0.22 | 9.88 |
| 3.11 | 12.6 | 48 | 10.2 | 176.7 | 43.6 | 19.0 | 13.57 | 8.32 | 202.02 | 90.34 | 12.48 | 6.54 | -1.09 | 5.46 | 1.13 | 1.08 | 0.03 | 1.11 |
| 3.08 | 18.5 | 90 | 21.0 | 252.1 | 42.0 | 35.5 | 19.4 | 11.7 | 472.40 | 107.0 | 17.5 | 18.0 | -1.89 | 14.58 | 1.99 | 3.40 | 0.10 | 3.50 |
| 3.06 | 23.0 | 120 | 32.0 | 310.0 | 41.2 | 52.0 | 23.9 | 14.2 | 735.40 | 118.2 | 21.4 | 30.7 | -2.50 | 24.51 | 2.66 | 6.15 | 0.16 | 6.31 |
| 3.07 | 29.5 | 160 | 51.0 | 391.8 | 40.8 | 81.3 | 30.3 | 18.0 | 1185.6 | 132.8 | 27.0 | 54.3 | -3.37 | 42.79 | 3.63 | 11.6 | 0.26 | 11.86 |
| 3.72 | 16.0 | 65 | 11.4 | 204.0 | 47.4 | 24.9 | 17.1 | 10.5 | 303.50 | 101.4 | 15.7 | 9.2 | -1.43 | 7.65 | 1.48 | 1.56 | 0.05 | 1.61 |
| 3.68 | 23.1 | 123 | 22.8 | 287.1 | 45.7 | 45.7 | 24.2 | 14.5 | 675.6 | 119.3 | 21.8 | 23.9 | -2.39 | 19.35 | 2.52 | 4.58 | 0.13 | 4.71 |
| 3.70 | 26.7 | 140 | 29.6 | 328.0 | 45.5 | 58.5 | 27.8 | 16.6 | 902.1 | 127.6 | 24.9 | 33.7 | -2.87 | 26.98 | 3.05 | 6.69 | 0.18 | 6.86 |
| 3.72 | 32.2 | 170 | 41.8 | 390.5 | 45.1 | 81.5 | 33.2 | 19.8 | 1296.8 | 139.2 | 29.6 | 51.8 | -3.61 | 41.06 | 3.87 | 10.76 | 0.26 | 11.02 |
| 4.42 | 15.4 | 48 | 7.8 | 184.0 | 52.8 | 21.7 | 16.8 | 10.4 | 237.7 | 101.0 | 15.6 | 6.1 | -1.22 | 5.14 | 1.25 | 0.93 | 0.03 | 0.96 |
| 4.40 | 20.1 | 95 | 12.7 | 234.3 | 51.3 | 32.4 | 21.4 | 13.1 | 447.6 | 113.2 | 19.6 | 12.8 | -1.85 | 10.58 | 1.92 | 2.21 | 0.07 | 2.28 |
| 4.40 | 24.2 | 128 | 17.9 | 277.8 | 50.5 | 43.7 | 25.5 | 15.4 | 670.7 | 122.9 | 23.1 | 20.6 | -2.40 | 16.83 | 2.51 | 3.80 | 0.11 | 3.91 |
| 4.42 | 30.5 | 160 | 27.3 | 343.8 | 49.8 | 64.7 | 31.8 | 19.0 | 1077.3 | 136.6 | 28.5 | 36.1 | -3.24 | 29.04 | 3.43 | 7.10 | 0.19 | 7.29 |
| 4.40 | 33.7 | 185 | 33.2 | 378.3 | 49.4 | 77.4 | 34.9 | 20.8 | 1315.8 | 143.0 | 31.3 | 46.1 | -3.68 | 36.80 | 3.92 | 9.29 | 0.24 | 9.53 |
| 4.93 | 12.8 | 30 | 4.7 | 150.3 | 57.9 | 16.4 | 14.5 | 9.2 | 126.3 | 94.8 | 13.7 | 2.7 | -0.78 | 2.36 | 0.79 | 0.35 | 0.01 | 0.36 |
| 4.93 | 18.0 | 70 | 8.5 | 202.6 | 55.5 | 25.9 | 19.6 | 12.1 | 314.3 | 108.8 | 18.1 | 7.8 | -1.47 | 6.54 | 1.51 | 1.22 | 0.04 | 1.26 |
| 4.94 | 23.2 | 110 | 13.4 | 254.6 | 54.2 | 38.0 | 24.7 | 15.0 | 566.4 | 121.4 | 22.5 | 15.4 | -2.16 | 12.74 | 2.24 | 2.70 | 0.08 | 2.78 |
| 4.97 | 27.4 | 135 | 18.0 | 295.9 | 53.7 | 49.6 | 28.9 | 17.4 | 809.2 | 130.8 | 26.1 | 23.4 | -2.72 | 19.12 | 2.85 | 4.32 | 0.13 | 4.45 |
Hence, energy loss in in hydraulic jumps will be:

\[ E_1 - E_2 = \Delta h_f + \Delta h_{mix} + \Delta h_{sec}. \]  

(22)

It is obvious from the equation (22) that for hydraulic jumps, following conditions must be necessarily fulfilled:

\[ \Delta E = E_1 - E_2 - (\Delta h_{mix} - \Delta h_{sec}). \]  

(23)

We use Darcy-Weisbach formula for critical section form to determine length of hydraulic jump:

\[ \Delta E = (\lambda_p + \lambda_f) \frac{L_{jump}}{h_{cr}} \frac{v^2}{2g}. \]  

(24)

where: \[ E \] - is the energy for overcoming frictional resistance;  
\[ \lambda_p \] - coefficient of hydraulic friction from the slopeof the gradient of pressure;  
\[ \lambda_f \] - coefficient of hydraulic friction from the slopeof the friction of rough sections;  
\[ L_{jump} \] - length of jump; \[ v_{cr} \] - critical flow velocity; \[ h_{cr} \] - critical flow depth.

We define the following from (24) for length of hydraulic jump:

\[ L_{jump} = \frac{2g \Delta E h_{cr}}{v_{cr}^2 \lambda_p + \lambda_f}. \]  

(25)

For determining \[ \lambda_p \] and \[ \lambda_f \] we processed data of laboratory studies by several authors for smooth and uneven bed separately. Coefficient of hydraulic friction \[ \lambda_f \] for smooth bed is determined by the formula:

\[ \lambda_f,smo. = 0,08 \cdot \lambda^{0.42}_{0} \left( \frac{h_{cr}}{h_{l}} \right)^{1.6}. \]  

(27)
\( \phi \) is determined by formula (27).

\[
\lambda_\phi = \frac{0.035}{\text{Re}^{0.25}}.
\]

where: \( \text{Re} \) is Reynolds number and determined by the formula:

\[
\text{Re} = \frac{u_1 \cdot h_1}{v} \quad (28)
\]

For uneven bed, hydraulic friction coefficient is determined from the expression:

\[
\lambda_{f,\text{fric.}} = 0.033 \lambda_f \left( \frac{h_1}{h_1} \right)^{1.6} \quad (29)
\]

In dependency (29) the magnitude of \( \lambda_f \) value is determined from (30).

\[
\lambda_f = \frac{1}{\left( 3.9 \log \frac{h_1}{h_1} + 4 \right)^2} \quad (30)
\]

Results of the comparison of length of the hydraulic jump according to the experimental data of M.D. Chertausov

| \( h_1 \)-first conjugate depth, cm | \( h_2 \)-second conjugate depth, cm | \( q \)-specific discharge, \( \text{cm}^3/\text{sec} \) on 1 m | Experimental length of hydraulic jump, \( L_0 \), cm | Jump length according to the author, \( L_{\text{sm}} \), cm | Difference between \( L_0 \) and \( L_{\text{sm}} \) | Jump length according to N.N. Pavlovsky, \( L_{\text{nc}} \), cm | Difference between \( L_0 \) and \( L_{\text{nc}} \) | Jump length according to M.D. Chertausov, \( L_{\text{sm}} \), cm | Difference between \( L_0 \) and \( L_{\text{sm}} \) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0.24 | 2.92 | 33.0 | 11.5 | 11.6 | -0.4 | 13 | -15 | 13 | -16.9 |
| 3.75 | 33.8 | 1527.9 | 180.0 | 178.9 | 0.6 | 151 | 16 | 161 | 10.4 |
| 2.20 | 15.5 | 544.1 | 70.0 | 74.7 | -6.7 | 68 | 3 | 77 | -10.1 |
| 1.95 | 15.02 | 493.8 | 60.0 | 73.2 | -22.0 | 66 | -11 | 73 | -21.0 |
| 1.96 | 14.93 | 492.4 | 67.0 | 72.6 | -8.2 | 66 | 1 | 73 | -9.0 |
| 2.25 | 17.64 | 622.3 | 87.0 | 87.7 | -0.9 | 78 | 10 | 85 | 2.1 |
| 1.94 | 15.85 | 518.0 | 70.0 | 78.1 | -11.5 | 70 | -1 | 76 | -8.9 |
| 0.34 | 2.980 | 40.6 | 13.0 | 12.2 | 6.1 | 13 | -2 | 14 | -10.1 |
| 1.01 | 9.22 | 216.2 | 41.5 | 42.6 | -2.6 | 41 | 1 | 44 | -6.5 |
| 0.24 | 2.54 | 28.8 | 10.5 | 10.1 | 3.8 | 11 | -9 | 12 | -13.1 |
| 0.22 | 2.67 | 28.9 | 11.5 | 10.5 | 9.1 | 12 | -6 | 12 | -7.0 |
| 0.24 | 2.92 | 33.0 | 11.5 | 11.6 | -0.4 | 13 | -15 | 13 | -16.9 |
|   |   |   | 0.17 | 0.21 | 0.25 | 0.21 | 0.23 | 0.18 | 0.11 | 0.19 | 0.18 | 0.10 | 3.35 | 3.23 | 3.29 | 3.14 | 3.03 | 2.68 | 2.68 | 2.16 | 2.50 | 0.40 | 2.10 | 0.51 |
|---|---|---|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
|   |   |   | 2.18 | 2.72 | 3.55 | 3.11 | 3.79 | 3.34 | 2.45 | 4.16 | 4.10 | 2.63 | 13.64 | 13.88 | 14.42 | 15.27 | 14.60 | 15.5 | 15.89 | 14.35 | 16.62 | 2.750 | 14.51 | 3.68 |
|   |   |   | 20.7 | 28.7 | 40.7 | 32.6 | 41.5 | 32.2 | 18.4 | 41.1 | 39.40 | 18.80 | 617.1 | 613.4 | 642.0 | 658.0 | 618.5 | 608.6 | 622.8 | 501.0 | 624.2 | 41.20 | 498.3 | 62.1 |
|   |   |   | 10.0 | 12.0 | 14.0 | 12.0 | 17.0 | 13.0 | 10.0 | 19.0 | 14.0 | 9.0  | 61.5 | 59.0  | 62.0  | 79.5  | 65.0  | 65.0  | 78.0 | 67.0 | 98.5 | 14.0 | 16.5 | 16.5 | 15.5 |
|   |   |   | 8.3 | 10.5 | 13.8 | 11.9 | 14.6 | 12.1 | 8.3  | 14.9 | 14.2 | 8.5  | 54.9 | 56.9  | 60.3  | 68.7  | 64.2 | 72.4 | 75.1 | 69.4 | 81.3 | 14.0 | 70.4 | 5.9  |
|   |   |   | 17.1 | 12.2 | 1.1 | 1.1 | 14.4 | 7.1  | 17.0 | 21.6 | -1.3 | 5.8  | 10.8 | 3.5  | 2.7  | 13.5 | 1.3  | -11.4 | 3.7 | -3.6 | 17.5 | 20.2 | 62.0 | -6.6 | 5.9  |
|   |   |   | 10 | -3 | 15 | -19 | 17 | -2 | 19 | -36 | -36 | 12 | 56 | 58 | 60 | 65 | 2 | -3 | 14 | 63 | 73 | 18 | 16 | 2 |
|   |   |   | 1.0 | -8 | -16 | -19 | 14 | -14 | 19 | -36 | 11 | 8 | 65 | 2 | 3 | 19 | 5 | -3 | 12 | 6 | 78 | 70 | 2 |
|   |   |   | 0.8 | 21 | 16 | 14 | 17 | 15 | 10 | 18 | 11 | 8 | 65 | 2 | 3 | 19 | 5 | -3 | 12 | 6 | 78 | 70 | 2 |

Extensive research activities have been carried out under the leadership of academician M. Vyzgo regarding the impact of bed roughness on hydraulic jump length \( L \). The length of hydraulic jump has been studied under laboratory conditions within same hydraulic parameters in smooth and uneven beds. Results of carried studies are summarized and presented in Figure 2. As it is obvious from this graph, obtained results are subject to parabolic functioning appropriateness.

The results of the calculation done by formulas (25), (29), (29) and (30) that we obtained through theoretical method for the length of hydraulic jump occurring in uneven beds have been compared to research results by academician M. Vyzgo (Figure 2).

When comparing the parameters of the hydraulic jump according to the recommendations developed by us for the uneven bottom, the data of M.S. Vyzgo and Y.A. Kuzminova were used, results of which are presented in Figure 2. According to data of this figure, hydraulic jump parameters determined according to our recommendations, as well as by M.S. Vyzgo expression \( L_{\text{fik}} = L_0 \frac{\Delta}{h_2} \), almost coincide, which is confirmed by graphics \( L_{\text{fik}} = f \left( \frac{\Delta}{h_2} \right) \).

Furthermore, according to recommendations made by us, a schedule of changes was developed

\[
\frac{L}{(h_2 - h_1)} = f \left( \frac{\nu_1}{\sqrt{gh_1}} \right)
\]

for hydraulic jump and (figure 3) where it where it was given together with similar graphs of US Bureau of Reclamation, N.N. Pavlovsky and M.D. Chertausov [14]. The analysis of materials of figure 3 indicate that schedule of change for length of hydraulic jump developed according to the methodology suggested by us is satisfactorily compliant with the graph of US Bureau of Reclamation \( 18,19 \) and well confirmed by experimental data of A.N.Akhutin and
A.N. Rakhmanov. When determining the length of hydraulic jump according to the schedules of the US Bureau of Reclamation, the discrepancy does not exceed 9.5...15.6%. However, in case of these graphs significantly differ from the graphs of N.N. Pavlovsky and M.D. Chertausov. Significant deviations of all these graphs descend from the experimental data of Pistrovsky and Bradley. It should be noted that at a value of \[ \frac{u_1}{\sqrt{gh_1}} > 4 \]
the convergence of the lengths of the hydraulic jump determined by different, including dependencies recommended by us are satisfactory.

Fig. 2: Diagram

\[ \frac{L_{jih}}{L_0} = f \left( \frac{\Delta}{h_2} \right) \]

1- K.A. Suleymanova; 2; 3; 4 – Y.M. Kuzminova

Fig. 3: Jump length: 1 - according to experiments of Safranch; 2 - according to experiments of Einwachter; 3 - according to experiments of Voychiska; 4 - according to experiments of Smetana; 5 - according to experiments of Pistrovsky; 6 - according to experiments of Bakhmeteyev and Matchsko; 7 - according to experiments of Akhutin; 8 - according to experiments of Rakhmanov; 9 - according to experiments of Mur; 10 - according to experiments of Bradley and Peterky.
Main outcomes

1. The mechanism of flow energy absorption within hydraulic jump zone has been explained by the model for the rotation of $q_d$ - additional water body on the mainstream and overcoming the resistance emerged in bed bottom against the flow. According to this selected mathematical model, the maximum flow energy - $E_i$ within I-I section in the hydraulic jump zone falls to minimum - $E_2$ value while decreasing to the critical depth located in K-K cross section and then increases to - $E_3$ value within section of II-II.

2. Within the hydraulic jump zone, $q_d$ - additional water body is connected to mainstream between the section of I-I and K-K and the process of energy loss occurs, the formula (13) is developed to calculate the same. Between the cross sections K-K and II-II the process of separation of $q_d$ - additional water body from the mainstream happens; the formula f to computed. Among the K-K and II-II fragments, the main stream is the $q_d$ - separation of the water mass, which is composed of the formula (18) has been developed to calculate the additional energy generated at that time.

3. A proposal has been made for using the formula (19) to calculate the critical depth in the hydraulic jump zone and using the formula (20) to calculate the discharge of $q_d$ - additional water body.

4. Formulas (21) and (23) have been developed to calculate the energy used by the flow bed for overcoming the bed bottom resistance.

5. It is recommended to use formulas of (25), (26), (27), (28), (29) and (30) obtained by theoretical method for calculating the length of hydraulic jump formed in smooth and rough beds.

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