Relevance of pseudospin symmetry in proton-nucleus scattering

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Abstract

The manifestation of pseudospin-symmetry in proton-nucleus scattering is discussed. Constraints on the pseudospin-symmetry violating scattering amplitude are given which require as input cross section and polarization data, but no measurements of the spin rotation function. Application of these constraints to \( p^{58}\text{Ni} \) and \( p^{208}\text{Pb} \) scattering data in the laboratory energy range of 200 MeV to 800 MeV, reveals a significant violation of the symmetry at lower energies and a weak one at higher energies. Using a schematic model within the Dirac phenomenology, the role of the Coulomb potential in proton-nucleus scattering with regard to pseudospin symmetry is studied. Our results indicate that the existence of pseudospin-symmetry in proton-nucleus scattering is questionable in the whole energy region considered and that the violation of this symmetry stems from the long range nature of the Coulomb interaction.

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I. INTRODUCTION

Originally the concept of pseudospin was introduced on an observational basis \[1, 2\] to explain the quasi-degeneracy of spherical shell orbitals with non relativistic quantum numbers \((n_r, \ell, j = \ell + \frac{1}{2})\) and \((n_r - 1, \ell + 2, j = \ell + \frac{3}{2})\), where \(n_r, \ell\) and \(j\) are the single-nucleon radial, orbital angular momentum, and total angular momentum quantum numbers, respectively. This symmetry approximately persists also for deformed nuclei \[3, 4, 5\] and even for the case of triaxiality \[6, 7\]. The origin of pseudospin-symmetry was not well understood for a long time. Only in the nineties its relation to the invariance of the Dirac Hamiltonian with \(V_V = -V_S\) under specific \(SU(2)\) transformations \[8\] has been pointed out \[9, 10, 11, 12, 13\]. Here, \(V_S\) and \(V_V\) are the scalar and vector potentials, respectively. In the non relativistic limit this leads to a Hamiltonian which conserves pseudospin,

\[
\tilde{s} = \frac{2\mathbf{s} \cdot \mathbf{p}}{p^2} \mathbf{p} - \mathbf{s},
\]

where \(\mathbf{s}\) is the spin and \(\mathbf{p}\) is the momentum operator of the nucleon.

A Dirac Hamiltonian with \(V_V = -V_S\) does not sustain any Dirac valence bound state \[9\]. Therefore, realistic mean fields used in nuclear structure physics must exhibit at least a weak pseudospin-symmetry violation. Actually the violation is smaller than anticipated from realistic relativistic mean field calculations \[10\].

The question then arises whether this symmetry, associated with \(V_V = -V_S\), manifests itself also in proton-nucleus scattering. From studies within Dirac phenomenology, the proton-nucleus scattering is described quite well by complex Dirac potentials with \(V_V \approx -V_S\) \[14\]. In 1988 Bowlin et al. \[15\] evaluated the analyzing power \(P(\theta)\) and the spin rotation function \(Q(\theta)\) under the assumption that \(V_V = -V_S\), where \(\theta\) is the scattering angle. They found a significant deviation of the experimental polarization and spin-rotation data from the predicted ones. Based on an algebraic estimate, they concluded that the symmetry is destroyed for low-energy proton scattering and that at high energies only some remnants might survive.

Recently, Ginocchio \[16\] revisited this question and evaluated, in a first order approximation, the ratio \(R_{ps}\) between the pseudospin symmetry breaking and the non-breaking part of the scattering amplitude. By considering experimental p-\(^{208}\)Pb elastic scattering data at \(E_{Lab} = 800\) MeV \[17\], he obtained a relatively small pseudospin dependent part of the
scattering amplitude at all scattering angles $\theta$ at which data were available ($\theta < 18^\circ$). This result, confirmed also by exact calculations\textsuperscript{18}, has been interpreted as an indication for the relevance of pseudospin-symmetry for proton-nucleus scattering – at least at medium energies. However, to make more conclusive statements about this point systematic studies of proton-nucleus scattering data covering a range of nuclei and energies are required. At present, such studies are hampered by the limited availability of complete data sets. This is due to the fact that measurements of the spin rotation function are difficult to obtain and therefore data are and will be very scarce.

In the present work we investigate whether conclusive statements on the size of the pseudospin-symmetry breaking part of the proton-nucleus scattering amplitude can be made from polarization and cross section data alone. Based on the exact relations derived in Ref.\textsuperscript{18}, we formulate constraints on the polarization $P(\theta)$ and spin rotation $Q(\theta)$ in terms of the aforementioned ratio $R_{ps}$. It turns out that the polarization data provide a lower bound for $|R_{ps}(\theta)|$. In addition, we show that a lower bound of the absolute value of the pseudospin-symmetry breaking part of the proton-nucleus scattering amplitude can be extracted from polarization and cross section data and thus a systematic study over a range of nuclei and energies can be made.

In Sec. II we briefly outline the scattering formalism in terms of the pseudospin-independent and pseudospin-dependent scattering amplitudes. Based on exact relations for the observables, we formulate constraints on the pseudospin-symmetry breaking scattering amplitude which require only the knowledge of the cross section and polarization. In Sec. III we present a systematic study of elastic proton scattering data concerning the size of the pseudospin symmetry breaking term. In Sect. IV we discuss the role played by the Coulomb potential in the pseudospin symmetry breaking. Finally, we summarize our concluding remarks in Sec. V.

**II. DERIVATION OF CONSTRAINTS**

For the derivation of the constraints we must briefly recall the formalism for the elastic scattering of a nucleon on a spin zero target. The scattering amplitude $f(k, \theta)$, according to the standard notation of the literature (see, for example, Ref.\textsuperscript{19}), is given by

$$f(k, \theta) = A(k, \theta) + B(k, \theta)\vec{\sigma} \cdot \hat{n},$$

(2)
where \( k \) is the momentum of the nucleon, \( \sigma \) is the vector formed by the Pauli matrices, \( \hat{n} \) is the unit vector perpendicular to the scattering plane, and \( \theta \) is the scattering angle. The complex-valued functions \( A(k, \theta) \) and \( B(k, \theta) \) are the spin-independent and spin-dependent parts of the scattering amplitude which are not fully accessible to experiment.

As shown by Ginocchio \[16\], one can determine from the standard representation of the scattering amplitudes, Eq. (2), the pseudospin-independent \( \tilde{A} \) and pseudospin-dependent \( \tilde{B} \) scattering amplitudes via a unitary transformation

\[
\begin{pmatrix}
\tilde{A} \\
\tilde{B}
\end{pmatrix} =
\begin{pmatrix}
\cos(\theta) & i\sin(\theta) \\
i\sin(\theta) & \cos(\theta)
\end{pmatrix}
\begin{pmatrix}
A \\
B
\end{pmatrix}.
\]

Partial wave expansions of \( \tilde{A} \) and \( \tilde{B} \) must be performed in terms of the pseudo-orbital angular momentum,

\[
\tilde{\ell} = \ell + 1, \text{ for } j = \ell + 1/2 = \ell - 1/2,
\]

\[
\tilde{\ell} = \ell - 1, \text{ for } j = \ell - 1/2 = \ell + 1/2,
\]

using the partial-wave S-matrix elements, \( \tilde{S}_{\tilde{\ell},j} \), defined for pseudo-orbital angular momentum

\[
\tilde{S}_{\tilde{\ell} = \ell - 1/2, j = \ell - 1/2} = S_{\ell - 1, j = \ell - 1/2}, \quad \tilde{S}_{\tilde{\ell} = \ell + 1/2, j = \ell + 1/2} = S_{\ell + 1, j = \ell + 1/2}.
\]

For details of this transformation we refer to the original work of Ginocchio \[16\].

The observables in nucleon-nucleus scattering are usually described in terms of the amplitudes \( A(k, \theta) \) and \( B(k, \theta) \). Because of the unitary transformation, Eq. (3), they can be equally described in terms of \( \tilde{A}(k, \theta) \) and \( \tilde{B}(k, \theta) \). For the scattering by a spinless target the observables are the differential cross section,

\[
\frac{d\sigma}{d\Omega}(k, \theta) = |\tilde{A}(k, \theta)|^2 + |\tilde{B}(k, \theta)|^2,
\]

the polarization,

\[
P(k, \theta) = \frac{\tilde{B}(k, \theta)\tilde{A}^*(k, \theta) + \tilde{B}^*(k, \theta)\tilde{A}(k, \theta)}{|\tilde{A}(k, \theta)|^2 + |\tilde{B}(k, \theta)|^2},
\]

and the spin rotation function

\[
Q(k, \theta) = \frac{\sin(2\theta) \left[ |\tilde{A}(k, \theta)|^2 - |\tilde{B}(k, \theta)|^2 \right] + i\cos(2\theta) \left[ \tilde{B}(k, \theta)\tilde{A}^*(k, \theta) - \tilde{B}^*(k, \theta)\tilde{A}(k, \theta) \right]}{|\tilde{A}(k, \theta)|^2 + |\tilde{B}(k, \theta)|^2}.
\]
As can be shown, e.g. from Eqs. (7) and (8), \( P^2 + Q^2 \leq 1 \).

The extraction of the full scattering amplitude (moduli and phases of \( \hat{A}(k, \theta) \) and \( \hat{B}(k, \theta) \)) from measurements is a very challenging task in quantum mechanics intimately related to the longstanding *phase problem* in diffraction analyses (see, for instance, Refs. [20, 21, 22]). Here, however, we are interested only in the formulation of constraints on \( R_{ps}(k, \theta) = \frac{\hat{B}(k, \theta)}{\hat{A}(k, \theta)} \) which yields a measure of the strength of the pseudospin-dependent part of the scattering. From Eqs. (6) to (8) it is straightforward to write the observables in terms of the ratio \( R_{ps} \) (we suppress from now on the \( k \)-dependence)

\[
\frac{d\sigma(\theta)}{d\Omega} = |\hat{A}(\theta)|^2 (1 + |R_{ps}(\theta)|^2),
\]

\[
P(\theta) = \frac{2 \text{Re}(R_{ps}(\theta))}{1 + |R_{ps}(\theta)|^2},
\]

and

\[
Q(\theta) = \frac{1 - |R_{ps}(\theta)|^2}{1 + |R_{ps}(\theta)|^2} \sin(2\theta) - 2 i \text{Im}(R_{ps}(\theta)) \cos(2\theta) \]

For pseudospin symmetry the ratio \( R_{ps} \) vanishes and consequently \( P = 0 \) and \( Q = \sin(2\theta) \) [15], independent of \( k \). One may also express the ratio \( R_{ps} \) in terms of the polarization \( P \) and the spin rotation \( Q \) [18].

From Eq. (10) it is obvious that for a given value of \( R_{ps} \) the polarization is a constant, independent of the scattering angle \( \theta \). Specifically, if we assume an upper admissible limit for the pseudospin symmetry breaking, i.e. if we assume \( |R_{ps}|^2 \leq \Gamma \), we obtain the following bound for the polarization

\[
|P(\theta)| \leq \frac{2\sqrt{\Gamma(\theta)}}{1 + \Gamma(\theta)}. \tag{12}
\]

The angle independence of the upper bound in \( P(\theta) \) makes it an ideal criterion to estimate from the polarization the pseudospin symmetry breaking term in nucleon-nucleus scattering. In Fig. 1 we show the upper bound \( |P_{\text{max}}| \) for a given \( |R_{ps}|^2 \)-value.

From this figure one can immediately extract the corresponding minimum and maximum values \( \Gamma_{\text{min}} \) and \( \Gamma_{\text{max}} \) for the ratio \( |R_{ps}|^2 \) at each angle,

\[
\sqrt{\Gamma_{\text{min}}} = \frac{1}{|P|} [1 - \sqrt{1 - P^2}], \quad \tag{13}
\]

\[
\sqrt{\Gamma_{\text{max}}} = \frac{1}{|P|} [1 + \sqrt{1 - P^2}], \quad \tag{14}
\]
Thus one can judge whether the pseudospin-symmetry breaking scattering amplitude yields an important contribution in a certain angular range or not.

The ratio $R_{ps}$ is not perhaps the best choice for an overall judgment of the pseudospin symmetry breaking term, because it will exhibit rather high values, when the pseudospin independent amplitude $\tilde{A}$ goes through minima in the angular range. Therefore, for a more general consideration, an estimate of the absolute value of the pseudospin dependent scattering amplitude $\tilde{B}$ should accompany the analysis because it directly refers to the size of the contribution of the pseudospin dependent part. This amplitude can be expressed in terms of the differential cross section \( \frac{d\sigma}{d\Omega} \) and the ratio $|R_{ps}|^2$,

$$|\tilde{B}(\theta)|^2 = \frac{|R_{ps}(\theta)|^2}{1 + |R_{ps}(\theta)|^2} \frac{d\sigma(\theta)}{d\Omega}. \quad (15)$$

It was shown in Ref. [18], that $R_{ps}$ can be fully determined from experiment if the polarization and the spin-rotation function are measured. Since, however, in most cases the spin-rotation data are not available, we must look for an estimate of $|\tilde{B}|$ using differential cross section and polarization data alone. From Eq. (15) we obtain

$$\frac{\Gamma_{\text{min}}}{\Gamma_{\text{max}}} \frac{d\sigma}{d\Omega} \leq |\tilde{B}|^2 \leq \frac{\Gamma_{\text{max}}}{\Gamma_{\text{min}}} \frac{d\sigma}{d\Omega}, \quad (16)$$

where we have used the admissible range of $|R_{ps}|$ determined from the polarization via Eq. (13) and (14). The ratio $|\tilde{B}|^2/d\sigma/d\Omega$, given by Eq. (16), as a function of the polarization $P$ is plotted in Fig. 2. As can be seen from this figure, the boundaries are useful when $P$ is in the vicinity of one. Specifically, for $|P| \leq 0.972$ the upper bound is better estimated by $d\sigma/d\Omega$.

### III. ANALYSIS OF EXPERIMENTAL DATA

The exact relation for $R_{ps}$, derived in Ref. [18], can be directly applied to elastic proton-nucleus scattering data where measurements of the spin-rotation function $Q(\theta)$, analyzing power $P(\theta)$, and differential cross section $d\sigma(\theta)/d\Omega$ are available. As already mentioned, due to the difficulty in measuring the spin-rotation function, such measurements are scarce. Complete sets of measurements, however, are available e.g. for $^{58}\text{Ni}$ at 295 MeV \[23, 24\] and for $^{208}\text{Pb}$ at 200 MeV \[25, 26\], and 800 MeV \[17, 27, 28\]. The latter data have already been analyzed with respect to $|R_{ps}|$ in Ref. [18] and no significant violation of the pseudospin
symmetry at this energy was found. A similar analysis for the two data sets at lower energies is shown in Fig. 3. It is clear that at these energies the modulus of the ratio $R_{ps}$ exhibits values which indicate a significant contribution of the pseudospin-dependent scattering amplitude within the range of the measured angles. This finding of a stronger violation of the pseudospin-symmetry at lower energies is in qualitative agreement with the estimate of Bowlin et al. [15].

In order to investigate further this finding, we studied the energy dependence of the ratio $R_{ps}$ by considering experimental p-$^{58}$Ni analyzing powers at $E_{Lab} = 192$ MeV, 295 MeV, 400 MeV [23], and 800 MeV [28]. Since spin-rotation data are not available for all data sets, we have applied the estimate $\Gamma_{\min} \leq |R_{ps}|^2 \leq \Gamma_{\max}$, given in Eqs. 13 and 14, and the results obtained are displayed in Fig. 4. At low energies the estimate indicates again significant contributions to pseudospin-symmetry violation stemming from the pseudospin-dependent scattering amplitude at specific angles, in contrast to the small violation observed at $E_{Lab} = 800$ MeV. This systematics for the p-$^{58}$Ni scattering confirms once more the increased effect of pseudospin-symmetry violating contributions at lower energies.

As pointed out in section II, we consider also the absolute value of $\tilde{B}$ gives a more direct measure for the size of the pseudospin-symmetry violation. For a complete data set, including $d\sigma/d\Omega(\theta)$, $P(\theta)$, and $Q(\theta)$, the amplitude $|\tilde{B}|^2$ can be evaluated from Eq. 15. In Fig. 5 estimates for $|\tilde{B}|^2$ for p-$^{58}$Ni scattering at $E_{Lab} = 192$ MeV and 400 MeV from polarization and cross section data are given which demonstrate the feasibility of the procedure by means of Eq. 16. To get a feeling about the relative size of $\tilde{B}$, the cross section data are also shown for the purpose of comparison. In all cases considered, the admissible values for $|\tilde{B}|^2$ are of the same order as those of the cross sections. This indirectly corroborates the significant pseudospin-symmetry violation in low energy proton-nucleus scattering.

IV. THE ROLE OF COULOMB POTENTIAL

The observation of weakly broken pseudospin-symmetry in proton and neutron shell orbit states is well established. As already mentioned above, it is related to a symmetry of the Dirac Hamiltonian with $V_V = -V_S$, which is almost satisfied in relativistic mean field calculations. Such studies yield a small pseudospin-symmetry breaking term which is necessary to explain the nuclear spectra. They lead to a splitting of quasi-degenerated
states with a given $\ell$. The experimentally observed splittings are smaller than the theoretical ones, thus indicating that the actual pseudospin-symmetry breaking contributions are even weaker than expected from theory \[29\]. The role played by the Coulomb potential with regard to pseudospin-symmetry breaking has recently been addressed by Lisboa and Malheiro \[30\]. They found only weak pseudospin-symmetry violation because significant cancellations between nuclear and Coulombic terms occur.

At a first glance one would not expect drastic changes when going above threshold to the scattering region. However, the long range nature of the Coulomb interaction requires a special treatment. Albeit it is included in all scattering calculations, its role with regard to the pseudospin-symmetry has not been considered so far. In what follows we shall report some considerations on the Coulomb interaction in proton-nucleus scattering and its consequences with regard to pseudospin-symmetry.

First we consider the pure Coulomb scattering problem within the Dirac equation for which closed-form expressions for the phase shifts $\sigma^{(C)}_\ell$ are known \[31\],

$$\exp(2i\sigma^\pm_\ell) = \frac{\gamma - i\eta \Gamma(\gamma + 1 + i\eta)}{\lambda - i\bar{\eta} \Gamma(\gamma + 1 - i\eta)} \exp(i\pi(\ell - \gamma)) \quad (17)$$

with

$$\gamma = \sqrt{\lambda^2 - Z^2\alpha_j^2}, \quad \eta = Z\alpha_f \frac{E}{\hbar k c}, \quad \bar{\eta} = Z\alpha_j \frac{mc^2}{\hbar k c} \quad (18)$$

and

$$\lambda = \begin{cases} 
-(\ell + 1) & \text{for } j = \ell + \frac{1}{2} \\
\ell & \text{for } j = \ell - \frac{1}{2}
\end{cases} \quad (19)$$

Here, $Z$ is the charge of the nucleus, $\ell$ the orbital angular momentum quantum number, $E$ the energy, and $k$ the wave number of the proton, respectively. The upper index $\pm$ refers to the angular momentum quantum number $j = \ell \pm \frac{1}{2}$. With these phase shifts the Coulomb scattering amplitudes $A_C(\theta)$ and $B_C(\theta)$ can be evaluated. The corresponding amplitudes in pseudospin representation $\tilde{A}_C(\theta)$ and $\tilde{B}_C(\theta)$ are obtained via Eq. \[3\] and yield the ratio $R^{C}_{\text{ps}} = \tilde{B}_C(\theta)/\tilde{A}_C(\theta)$ which is a measure for the breaking of pseudospin-symmetry by the Coulomb interaction. In Fig. \[4\] we show, as an example, this ratio for proton-Pb scattering at different energies. It is seen that pseudospin breaking due to the Coulomb potential is largest at 90 degrees and that it decreases with energy.

A satisfactory description of proton-nucleus scattering is usually obtained within the Dirac phenomenology using a scalar potential $V_S(r)$ and a fourth component of a vector potential
$V(r)$ which is composed of a nuclear part $V_N(r)$ and the Coulomb potential $V_C(r)$. Both, $V_S(r)$ and $V_N(r)$ are complex potentials and are frequently taken to be of Woods-Saxon shape (see e.g. [14]). The associated scattering amplitudes consist of three contributions,

\[ A(\theta) = A_N(\theta) + A_C(\theta) + A_I(\theta), \]

\[ B(\theta) = B_N(\theta) + B_C(\theta) + B_I(\theta). \]

Here, the indices N, C, and I denote the nuclear, the Coulomb, and the interference contributions, respectively. The latter takes into account non-linear modifications of the scattering amplitudes due to the superposition of the interactions $V_N + V_C$. Via Eq. (3) one obtains also the corresponding pseudospin representations of the amplitudes $\tilde{A}(\theta)$ and $\tilde{B}(\theta)$. Due to the short range nature of the nuclear interaction, the nuclear parts of the scattering amplitudes decrease more rapidly than the Coulomb parts with increasing momentum transfer. Therefore, the ratio $R_{ps}(\theta)$ at increasing scattering angle will be dominated by the Coulomb contribution.

In order to demonstrate this characteristic behavior of the scattering amplitudes we study a schematic example of proton-$^{208}$Pb scattering assuming $V_S(r)$ and $V_N(r)$ to be real and of Woods-Saxon shape

\[ V_S(r) = W_0 \left[ 1 + \exp\left(\frac{r - R_0}{a}\right)\right]^{-1}, \]

\[ V_N(r) = V_0 \left[ 1 + \exp\left(\frac{r - R_0}{a}\right)\right]^{-1}, \]

with the half density radius $R_0 = 1.25 \text{ fm } A^{1/3}$ and diffuseness $a = 0.6 \text{ fm}$. The Coulomb potential $V_C(r)$ is that of a homogeneously charged sphere with radius $R_C = 1.25 \text{ A}^{1/3}$. In the first example we consider the nuclear strengths $V_0 = -W_0 = 300 \text{ MeV}$ which implies exact pseudospin-symmetry in the nuclear part. The corresponding scattering amplitudes have been evaluated for several energies between $E_{\text{Lab}} = 200 \text{ MeV}$ and $800 \text{ MeV}$.

In Fig. 7 the angular dependence of the absolute values of the ratio $R_{ps}(\theta)$ is compared with that of the pure Coulomb interaction. This comparison clearly indicates that the ratio $R_{ps}$ approaches at backward angles the values of the pure Coulomb interaction. Hence, at all energies considered, there exists an angular range with $|R_{ps}|$-values not compatible with pseudospin-symmetry. The non-vanishing $|R_{ps}|$-values are solely caused by the presence of the Coulomb potential since the nuclear part alone satisfies, because of $V_N(r) = -V_S(r)$, an exact pseudospin-symmetry.
Relativistic mean field calculations reveal a small but necessary pseudospin-symmetry breaking nuclear part. In order to simulate this effect we consider the same scattering system as before but with strengths \( V_0 = -W_0 + \Delta = 300 \) MeV and thus the pseudospin-symmetry is broken by the extra strength \( \Delta \). In Fig. 8 the moduli of the corresponding ratio \( R_{ps}(\theta) \) are shown for different \( \Delta \)-values at \( E_{Lab} = 200 \) MeV.

Qualitatively we observe the same behavior as in the case of \( \Delta = 0 \). There are again significant values of \( |R_{ps}| \) at backward angles which are not in agreement with pseudospin-symmetry. It should be emphasized that in all cases and at all angles considered the \( |R_{ps}| \)-values associated with the nuclear part alone do not exceed 0.15 thus indicating a small pseudospin-symmetry violating contribution.

Unfortunately it is not possible to extract the nuclear contribution from the experimental proton-nucleus scattering data, the main reason being the non-linear relationship between potential and scattering amplitudes in the energy region considered. The importance of higher order Born terms is best reflected in the importance of the amplitude \( |A_I(\theta)| \). In Fig. 9 the modulus of \( A_I(\theta) \) is compared with that of \( A_N(\theta) \) and \( A_C(\theta) \) which are of the same size. It is clear that it is not possible at present to separate the nuclear term without further model assumptions.

V. SUMMARY

We have derived boundaries for the ratio of the pseudospin-dependent to the pseudospin-independent scattering amplitude requiring only the knowledge of polarization data. In addition, we also derived boundaries for the absolute size of the pseudospin dependent scattering amplitude. These boundaries are based on the differential cross section and polarization data alone and, thus, one can avoid measurements of the spin-rotation function as required by the methods of Refs. [16, 18]. Because of the difficulties in measuring \( Q(\theta) \), these constraints could be very useful and represent an improvement with regard to the previous situation. Their use together with the exact relationships derived in [18] allow us to assess the relevance of pseudospin symmetry in proton-nucleus scattering at various energies. Furthermore, by considering p-\( ^{58}\text{Ni} \) and p-\( ^{208}\text{Pb} \) scattering data the mass number dependence is also shown up.

The results for the ratio \( |R_{ps}|^2 \) exhibit a systematic decrease with increasing energy. At
lower proton-nucleus scattering energies, up to about 400 MeV, the results obtained indicate that there is a significant symmetry breaking term present which confirms the conjecture of Bowlin et al. \cite{15} based on analytical estimates. At these energies and at certain scattering angles the extracted $|R_{ps}|^2$-values approach 1, implying that the pseudospin dependent and independent scattering amplitudes are of comparable size. It should be emphasized here that estimates of $|\tilde{B}|^2$ exhibit an angular dependence which is quite similar in form and size to that of the differential cross section. In contrast, at 800 MeV there is only a weak violation of pseudospin symmetry which confirms the finding of \cite{16,18}.

To investigate the origin of this behavior, we have studied the role played by the Coulomb interaction. Considering the Coulomb scattering problem within the Dirac equation, leads to scattering amplitudes $\tilde{A}(\theta)$ and $\tilde{B}(\theta)$ whose ratio $|R_{ps}|$ shows a clear peak, with values greater than 1 at 90 degrees, which decreases with energy. Assuming typical nuclear interactions of the Dirac phenomenology, we have evaluated the ratio $R_{ps}$ in the presence of the Coulomb interaction. The results clearly indicate that the modulus of the ratio $|R_{ps}|$, at all energies, exceeds 1 and approach at backward angles that of the Coulomb problem.

Smaller $|R_{ps}|$-values are found at small scattering angles which can be attributed to highly non-linear superposition of nuclear and Coulomb effects. One might conjecture that this phenomenon is of the same nature to the cancellation effects observed in the bound state regime \cite{29}. Anyway it is limited to a small and energy dependent angular region reflecting the finite range of the nuclear interaction. Because of the importance of nonlinearities between potential and scattering amplitude (higher order Born terms) it is impossible to separate from experimental data the nuclear components unambiguously without further model assumptions. In addition, we didn’t find any characteristic behavior of $R_{ps}(\theta)$ which would give a clear indication of a weakly pseudospin-symmetry violating nuclear term.

In short, proton-nucleus scattering does not exhibit the features of pseudospin-symmetry. The violation of the symmetry stems from the long range Coulomb interaction and shows up in the values of the ratio $R_{ps}$ at large scattering angles. The previous finding of small values of the moduli of $R_{ps}$ at 800 MeV proton-$^{208}$Pb scattering, can be attributed to the limited angular range of the experimental data and cannot be considered as a clear sign for the relevance of pseudospin-symmetry in proton-nucleus scattering.
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Figure Captions

Fig. 1: The upper limit of the polarization for a given pseudospin breaking ratio $|R_{ps}|^2$.

Fig. 2: The lower and upper bounds (solid lines) of the ratio $|\tilde{B}|^2/\frac{d\sigma}{d\Omega}$ given by Eq. (16) as a function of the polarization $P$. In addition the upper bound due to the cross section is shown by the dotted line. The shaded area shows the admissible range.

Fig. 3: Angle dependence of the ratio $|R_{ps}|^2$ extracted from complete data sets.

Fig. 4: The range of $|R_{ps}|^2$ for p-$^{58}$Ni scattering extracted from polarization data at different energies $E_{Lab}$. For a better estimate of the size, the values $|R_{ps}|^2 = 0.3$ and $|R_{ps}|^2 = 0.6$ are shown by a dotted and a dashed line, respectively.

Fig. 5: The values $|\tilde{B}|^2$ extracted via Eq. (16) from experimental proton-$^{58}$Ni polarization and scattering cross section data at $E_{Lab} = 192$ MeV and at 400 MeV. For comparison, the cross sections are also shown by a dotted line.

Fig. 6: The modulus of the ratio $R_{ps}(\theta)$ for pure Coulomb scattering of a proton by a Pb-nucleus at $E_{Lab} = 100$ MeV (solid line), $E_{Lab} = 200$ MeV (dotted line), $E_{Lab} = 400$ MeV (dashed line), and $E_{Lab} = 800$ MeV (long dashed line).

Fig. 7: The modulus of the ratio $R_{ps}(\theta)$ evaluated with a schematic Dirac potential for proton-$^{208}$Pb scattering at several energies. The corresponding ratio obtained for pure Coulomb scattering is also shown for comparison by dashed line. The nuclear part of potential satisfies pseudospin-symmetry. See text for more details.

Fig. 8: The modulus of $R_{ps}(\theta)$ obtained from model calculations for proton-$^{208}$Pb scattering at $E = 200$ MeV assuming different strength $\Delta$ of the nuclear pseudospin-symmetry breaking term. The results for $\Delta = 0$ MeV (solid line), $\Delta = -50$ MeV (dotted line), and $\Delta = +50$ MeV are shown. For comparison also the absolute values of $R_{ps}$ for pure Coulomb scattering are given by long dashed line.

Fig. 9: The relative contributions of $\tilde{A}_N(\theta), \tilde{A}_C(\theta)$, and $\tilde{A}_I(\theta)$ to the scattering amplitude $\tilde{A}(\theta)$ for proton-$^{208}$Pb scattering at $E = 200$ MeV. The values are evaluated with the Dirac
potentials given in Fig. 7. The quantities $|\tilde{A}_N(\theta)/\tilde{A}(\theta)|$ (solid line), $|\tilde{A}_C(\theta)/\tilde{A}(\theta)|$ (dotted line), and $|\tilde{A}_I(\theta)/\tilde{A}(\theta)|$ (dashed line) are displayed.
FIG. 1:
FIG. 2:
FIG. 3:
FIG. 4:
FIG. 5:

FIG. 6:
FIG. 7:
FIG. 8:

$E = 200 \text{ MeV}$

FIG. 9: