Charged current weak production of the $\Delta$ resonance

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The reactions $e^- p \rightarrow \Delta^0 \nu_e$ and $e^+ p \rightarrow \Delta^{++} \bar{\nu}_e$ are considered as a possible source of information about the weak $N\Delta$ transition form factors. The low $q^2$ BNL data on $\nu_\mu$ production of $\Delta$ are used to extract the axial vector $N\Delta$ coupling, taking into account the deuteron structure and the $\Delta$ width. Finally, pion production induced by neutrinos in $^{16}\text{O}$ in the $\Delta$ region, relevant to atmospheric $\nu$ experiments, is investigated.

1. INTRODUCTION

The nucleon excitation spectrum is a valuable source of information about baryon structure. The $N\Delta$ transition presents clear advantages from the experimental point of view since the $\Delta$ is separated from the rest of resonances. The bulk of the existing information on the weak $N\Delta$ transition form factors (FF) comes from the analysis of the ANL [1] and BNL [2] experiments, performed with $\nu_\mu$ beams, whose energies span from 0.5 to 6.0 GeV with poorly known distributions. Nowadays, with the advent of the new generation of electron accelerators in the GeV region and achieving high luminosities, it is possible to perform electron scattering experiments in the resonance region. We have considered the possibility to extend these studies to the weak charged current physics. For this reason, we have studied the reactions $e^- p \rightarrow \Delta^0 \nu_e$ and $e^+ p \rightarrow \Delta^{++} \bar{\nu}_e$ at the typical energies of MAMI and TJNAF, and using the available information about the FF [3].

Since the vector $N\Delta$ FF are related to the isovector electromagnetic ones, which can be obtained from electroproduction data, these experiments would allow to study the axial FF and, in particular, the dominant $C_A^A$. The determination of its value at $q^2 = 0$ is important in view of the discrepancies between the PCAC prediction and theoretical estimates obtained in most quark models [4]. We have used the low $q^2$ BNL data on the ratio of $\mu^- \Delta^{++}$ and $\mu^- p$ events from $\nu_\mu d$ collisions to extract the value of the axial vector coupling $C_A^A(0)$, taking into account the deuteron structure and the $\Delta$ width [5].

The study of weak $N\Delta$ transitions in nuclei is relevant for the analysis of atmospheric neutrino experiments. In fact, the energy distribution of the part of the atmospheric $\nu$ flux producing fully contained events at Kamiokande is such that $< E_\nu > \approx 700$ MeV, well above the $\Delta$ production threshold. These $\Delta$’s decay into pions and photons (through $\pi^0$ decay), that are a source of background. For this reason, we have studied the impact of nuclear effects in $\nu_{e(\mu)}$ production of $\Delta$ in $^{16}\text{O}$ [6].
2. WEAK ELECTROPRODUCTION CROSS SECTION

The matrix element for the process $e^-(k) + p(p) \rightarrow \Delta^0(p') + \nu_e(k')$ is proportional to the product of the leptonic and hadronic currents. The hadronic current is expressed in terms of vector and axial vector FF $C_i^V$ and $C_i^A$ ($i = 3, 4, 5, 6$) \[3\]. The imposition of the CVC hypothesis $q_\mu J_{\mu}^V = 0$ implies $C_i^V = 0$. The other three vector FF are obtained from the isovector electromagnetic ones. Assuming $M1$ dominance, one gets $C_5^V = 0$ and $C_4^V = -(M/M_\Delta)C_3^V$. $C_i^V$ is determined from electroproduction experiments \[4\] and from a quark model \[8\]

\[
C_3^V(q^2) = 2.05 \left(1 - q^2/0.54\text{GeV}^2\right)^{-2},
\]

\[
C_3^V(q^2) = \frac{M}{(\sqrt{3}m)} e^{-q^2/6},
\]

where $m = 330$ MeV is the quark mass and $\bar{q} = |q|/\alpha_{ho}$, with $\alpha_{ho} = 320$ MeV. Concerning the axial FF, $C_6^A$ can be related to $C_5^A$ using pion pole dominance and PCAC, then $C_6^A(q^2) = C_5^A(q^2)M^2/(m_\pi^2 - q^2)$. The value of $C_5^A(0)$ can be taken from the off-diagonal Goldberger-Treiman relation \[6\], $C_5^A(0) = g_{\Delta N\pi} f_{\pi}/(\sqrt{6}M) = 1.15$, where $f_{\pi} = 92.4$ MeV, $g_{\Delta N\pi} = 28.6; C_3^A(q^2), C_4^A(q^2)$ and $C_5^A(q^2)/C_5^A(0)$ are given by the Adler model \[9\]

\[
C_{i=3,4,5}(q^2) = C_i(0) \left[1 - \frac{a_i q^2}{b_i - q^2}\right] \left(1 - \frac{q^2}{M_\Delta^2}\right)^{-2}.
\]

with $C_3^A(0) = 0, C_4^A(0) = -0.3, a_4 = a_5 = -1.21, b_4 = b_5 = 2$ GeV$^2$ and $M_A = 1.28$ GeV. The value of $M_A$ comes from a best fit to the $\mu^-\Delta^{++}$ events at BNL \[3\]. For a comparison, we also use a non-relativistic quark model calculation \[8\]

\[
C_5^A(q^2) = \left(\frac{2}{\sqrt{3}} + \frac{1}{3\sqrt{3}m}\right) e^{-q^2/6}, C_4^A(q^2) = -\frac{1}{3\sqrt{3}M_\Delta m} e^{-q^2/6}, C_3^A(q^2) = 0.
\]

From the amplitude given above, the differential cross section $d\sigma/d\Omega_\Delta$ can be obtained in the standard way. The $\Delta$ width has been accounted for by means of the substitution

\[
\delta(p^2 - M_\Delta^2) \rightarrow \frac{1}{\pi} \frac{1}{2M_\Delta} \text{Im} \left[\frac{1}{W - M_\Delta + \frac{1}{2}i\Gamma_\Delta}\right], \Gamma_\Delta = \Gamma_0 \frac{M_\Delta}{W} \frac{q_{c.m.}(W)}{W} \frac{q_{c.m.}(M_\Delta)}{W} , W = \sqrt{p^2}
\]

with $q_{c.m.}$ being the pion momentum in the $\Delta$ rest frame and $\Gamma_0 = 120$ MeV. The angular distribution is shown in Fig. \[1\] for two different sets of FF: I, phenomenological [Eqs. \[10\], \[11\]], solid line; II, quark model, [Eqs. \[2\], \[4\]], dashed line. The invariant mass has been restricted to $W < 1.4$ GeV to select $\Delta$ events. The differential cross section is found to be high enough in a large angular region to consider the possibility of measuring them.

3. DETERMINATION OF THE AXIAL VECTOR COUPLING

In order to obtain $C^A_5(0)$ we have evaluated the ratio

\[
R(Q^2) = \frac{(d\sigma/dq^2)(\nu d \rightarrow \mu^- \Delta^{++} n)}{(d\sigma/dq^2)(\nu d \rightarrow \mu^- pp)}, \quad Q^2 = -q^2
\]

at $E_\nu = 1.6$ GeV, which is the mean energy of the BNL $\nu_\mu$ spectrum; the $\Delta$ production cross section has been calculated in the impulse approximation, and using the deuteron
wave function of the Paris potential. The quasielastic cross section, in the same approxi-
mation, is taken from Ref. [10]. We found that, in the data region i.e. at $Q^2 \leq 0.1 \text{ GeV}^2$,
deuteron effects are negligible and, hence, one can treat the BNL data as if they were
data on the ratio of the free reactions

$$R(Q^2) \approx R_0(Q^2) = \frac{\langle d\sigma/dq^2 \rangle (\nu p \rightarrow \mu^- \Delta^{++})}{\langle d\sigma/dq^2 \rangle (\nu n \rightarrow \mu^- p)}.$$  \hspace{1cm} (7)

At $Q^2 = 0$, $R_0(Q^2)$ is given by the quotient of

$$\frac{d\sigma}{dq^2} = \left( C_5^A \right)^2 \frac{1}{24\pi^2} G^2 \cos^2 \theta_c \frac{\sqrt{s}(M + M_{\Delta})^2(s - M_{\Delta}^2)}{(s - M^2)M_{\Delta}^3} \int dk^0 \frac{\Gamma_{\Delta}(W)}{(W - M_{\Delta})^2 + \Gamma_{\Delta}^2(W)/4}.$$  \hspace{1cm} (8)

and the well known expression for the forward quasielastic cross section. Equating this
eratio to the experimental value $0.55 \pm 0.05$ [1], we obtain $C_5^A = 1.22 \pm 0.06$; this result
is consistent with the value given by the off-diagonal Goldberger-Treiman relation. The
proper inclusion of the $\Delta$ width causes a 30 % reduction of the cross section and cannot
be neglected in the extraction of $C_5^A(0)$.

4. NEUTRINO PRODUCTION OF $\Delta$ IN $^{16}\text{O}$

When the reactions $\nu_t p(n) \rightarrow \ell^- \Delta^{++}(\Delta^+)$ and $\bar{\nu}_t p(n) \rightarrow \ell^+ \Delta^0(\Delta^-)$ take place in the
nucleus, the nucleon momentum is constrained within a density dependent Fermi sea.
The produced $\Delta$ does not have this constraint, but its decay is inhibited by the Pauli
blocking of the final nucleon. On the other side, there are other disappearance channels
open through particle-hole excitations. The situation is well described if one replaces in
the $\Delta$ propagator $\Gamma_{\Delta} \rightarrow \bar{\Gamma}_{\Delta} - 2\text{Im}\Sigma_{\Delta}$ and $M_{\Delta} \rightarrow M_{\Delta} + \text{Re}\Sigma_{\Delta}$, where $\bar{\Gamma}_{\Delta}$ is the Pauli
blocked decay width and $\Sigma_{\Delta}$ is the $\Delta$ selfenergy in the nuclear medium [1]. The pions
produced inside the nucleus are rescattered and absorbed in their propagation through
the nucleus. The absorption coefficient required to estimate the produced pion flux has
been calculated in the eikonal approximation, taking the pion energy dependent mean
free path from Ref. [2]. For the $N\Delta$ transition FF, the phenomenological set I described
above has been taken; possible medium modification of the FF has not been considered.
In Fig. 2 a) $d\sigma/dE_{k'}$ ($k'$ being the momentum of the outgoing electron) is shown for $E_\nu = 750$ MeV. The medium modification effects cause an overall reduction of about 40 %. Therefore, the Kamiokande analysis, which makes use of free $\Delta$ production cross sections, overestimates one pion production. However, as can be seen in Fig. 2 b), the ratio of total pion production cross sections induced by electron and muon type neutrinos and antineutrinos $R(E_\nu) = \sigma_\Delta(\mu)/\sigma_\Delta(e)$ is not affected by these modifications.

Figure 2. (a) $\nu_e$ induced $\Delta$ excitation in $^{16}O$ without (long-dashed line) and with medium effects (solid line); pion production with medium effects, without (short-dashed line) and with absorption (dotted line). (b) $R(E_\nu) = \sigma_\Delta(\mu)/\sigma_\Delta(e)$ with (solid line) and without medium effects (dotted line).

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