Partial Field Coverage Based on Two Path Planning Patterns

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Abstract

This paper presents a path planning method for partial field coverage. We therefore propose to use a specific path planning pattern. Guiding notion is that lighter machinery with smaller storage tanks can alleviate soil compaction, but does not permit to cover a given field in a single run, for example, during a spraying application. Instead, multiple returns to a mobile or stationary depot located outside of the field are required for storage tank refilling. We therefore suggest a suitable path planning method that accounts for the limited turning radii of agricultural vehicles, satisfies repressed area minimisation constraints, and aims at overall path length minimisation. The benefits of the proposed method are illustrated by means of a comparison to a method that is based on a S-shaped planning motif. It is illustrated how the proposed path planning pattern can also be employed efficiently for single-run field coverage.

Keywords: Partial Field Coverage; Path Planning; Shortest Paths; Patterns; Optimisation; Decision Support System.

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| Symbols | Definition |
|---------|------------|
| $(x, y)$ | Position in the global coordinate system |
| $(\xi, \eta)$ | Position in the normalised coordinate system |
| $Z$ | Position $Z = (\xi, \eta)$ |
| $N$ | Number of interior lanes |
| $e_{i,j}$ | Edge connecting nodes $i$ and $j$ |
| $T$ | Transition graph |
| $Z_0$ | Start position $Z_0 = (\xi_0, \eta_0)$ |
| $Z_0^{(l)}$ | Two sets of start positions with $l = 1, 2$ |
| $Z_i$ | Position of node $i$ |
| $Z(t)$ | Position of agricultural vehicle at time $t$ |
| $Z(\tau_{\text{last}})$ | Position for resuming field coverage |
| $\gamma(t)$ | Vehicle state (resume, coverage, return) |
| $f(t)$ | Storage tank fill-level |
| $F^{\text{trigger}}(t)$ | Binary indicator of active return state |

| Abbreviations |
|----------------|
| MEAND | Path Planning Method 1 (meandering) |
| CIRC | Path Planning Method 2 (circular) |
| CIRC* | Path Planning Method 3 (circular) |

Tab. 1. Nomenclature table. All position coordinates are expressed in unit meters. The storage tank fill level is measured in percent of the maximum storage tank capacity. All other symbols are without units.
1. Introduction

According to Ahumada & Villalobos (2009) and Bochtis (2010), the agri-food supply chain can be decomposed into four main functional areas. These are in order of the yearly work cycle: production, harvesting, storage and distribution. For improved supply chain efficiency, logistical optimisation and route planning play an important role in all of the four functional areas. Regarding production, for example, by means of minimisation of the non-working distance travelled by machines operating in the headland field as presented in Bochtis & Vougioukas (2008), optimal route planning based on B-patterns according to Bochtis et al. (2013), or route planning for the coordination of fleets of autonomous vehicles as discussed in Conesa-Muñoz et al. (2016a) and Seyyedhasani & Dvorak (2017). See also Day (2011) for an overview of means for efficiency improvements, Bochtis (2013) for the importance of satellite-based navigation systems in modern agriculture, and Sørensen & Bochtis (2010) for a distinction between in-field, inter-field, inter-sector and inter-regional logistics. The path planning method for partial field coverage presented in this paper can be attributed to the first functional area of the agri-food supply chain.

The last decades have witnessed a trend towards the employment of larger and more powerful machines in agriculture. This trend is expected to further continue in the near future as discussed in Kutzbach (2000) and Dain-Owens et al. (2013). Among the main benefits are higher work rates. The drawbacks include increased soil compaction due to higher machinery weights that typically coincide with larger machinery; see Raper (2005) and Hamza & Anderson (2005). See also Antille et al. (2013) for the influence of tyre sizes on
 Concurrently to this ongoing trend in industry towards larger machinery, there are alternative considerations about the replacement of heavy machinery by the coordinated control of teams of smaller and lighter autonomous robots to mitigate soil compaction, see Blackmore et al. (2008), Bochtis & Sørensen (2010), Bochtis et al. (2012), Bochtis (2013), Gonzalez-de Santos et al. (2016) and Seyyedhasani & Dvorak (2017). This paper is motivated thereby and continues previous work that was presented in Graf Plessen & Bemporad (2016).

In this paper, a suitable path planning method for partial field coverage is characterised by a) the minimisation of traveled non-working path length, and b) the compliance with repressed area minimisation constraints. This implies driving along unique and established transitions between headland path and interior lanes, thereby avoiding the creation of any additional tyre traces. Within this paper, two different path planning patterns are discussed.

In contrast to route planning methods such as in Conesa-Muñoz et al. (2016b) for the in-field operation of a fleet of vehicles, the presented method focuses on the in-field operation of a single vehicle that is repeatedly returning to the field entrance for refilling. This is primarily motivated by the targeted crops (wheat, rapeseed and barley) and the expensive costs of corresponding agricultural vehicles. A support unit, acting as a mobile depot, is assumed to be waiting at the field entrance at scheduled times for refilling.

In contrast to route planning methods that do not follow any predetermined pattern motif and instead freely optimise the field coverage path according to one or more optimisation criteria as outlined in Bochtis et al.
the presented method is pattern-based. This is done for practical reasons. Freely optimised route plans for non-rectangularly shaped field contours typically result in unintuitive path plans and irregular sequences of lane traversals. While this does not matter for a fully autonomous robot, it is relevant for the case of vehicles driven by human operators. Namely, a navigation guidance application is required. According to the experience of the author, some practitioners prefer to drive according to well-defined and repeatable patterns, even if this may incur some additional (limitedly small) detour, rather than following a complex route planning that in general also requires accurate and accordingly expensive GPS-equipment as well as a navigation application. For field shapes that do not include large indents, bays or tree islands, detours typically are very small and at most in the tens of meters. This motivated us to use path planning patterns as building blocks. Note that for the field shapes considered in this paper, the presented pattern-based path planning is also optimal.

This paper is organised as follows. The problem and nomenclature are formulated in Section 2. The main contribution is given in Section 3. An illustrative example is discussed in Section 4 before concluding with Section 5.

2. Problem Formulation and Notation

This paper addresses path planning for partial field coverage. Throughout, path planning must account for repressed area minimisation constraints. These imply a) unique transitions between headland and interior lanes, and b) accounting for vehicle agility capabilities in form of limited turning radii.
Fig. 1. Visualisation of the repressed area minimisation constraint. Any agricultural vehicle that is traveling along lanes and the headland path must respect tractor traces established upon first field coverage. For above illustrative example, transitions A-B and A-D are admissible. In contrast, transition A-C is not admissible. Such a transition would deviate from established tyre traces when accounting for the limited turning radius of the vehicle, and would therefore repress and destroy precious crop.

![Image of real-world transitions between headland path and interior lanes.](image)

Fig. 2. Visualisation of real-world transitions between headland path and interior lanes. In the satellite image, the effects of a limited turning radius of the employed agricultural vehicle are clearly visible.

For visualisation and further explanation, see Fig. 1 and 2.

While the most general in-field path planning method under repressed area minimisation constraints is briefly discussed in Section 3.1, the main focus of this paper is on field shapes that permit path planning based on patterns. Relevant components for planning include a headland path and multiple interior lanes. In combination, they enable field coverage. The headland path is generated from an erosion (mathematical operation) of the
Fig. 3. Exemplatory illustration of two types of field shapes. The path planning method for partial field coverage developed in this paper is based on the left type. See Section 2 for further discussion.

field contour to the field interior. All position coordinates are initially expressed in the global $(x, y)$-coordinate system. Then, all coordinates can be transformed by a rotation of angle $\theta$ to a rotated $(x_\theta, y_\theta)$-coordinate system such that interior lanes are aligned with the vertical $y_\theta$-axis. Thus, we have

$$(x_\theta, y_\theta) = R(\theta)(x, y),$$

where $R(\theta)$ denotes a standard rotation matrix with rotation angle $\theta$. In addition, we employ at most two coordinate reflections (Householder transformation in two dimensions) in order to normalise the path planning problem with respect to the field entrance position. Thus, the normalised coordinate system in which paths are ultimately planned is described by coordinates $(\xi, \eta)$. Mathematical details about the coordinate reflections follow in Section 3.2. All three transformation steps (rotation and at most two reflections) are linear. Ultimately, paths planned in the $(\xi, \eta)$-system are recovered in the $(x, y)$-plane by inversion of the linear transformations.

In the following, a position within the normalised coordinate system is
Fig. 4. Illustration of notation and problem formulation. The headland path is denoted by the solid line. Interior lanes are here indicated by the dashed lines. The arrow indicates the headland traversal direction. Headland and lanes are expressed in the \((\xi, \eta)\)-coordinate system such that lanes are aligned to the \(\eta\)-axis. The field entrance (start position) is denoted by \(Z_0\). Only for simplicity and spatial reasons, the field contour is plotted in rectangular shape.

abbreviated by

\[
Z = (\xi, \eta).
\]

With the exception of Section 3.1, field contours are assumed such that any rotated interior lane is \textit{uninterrupted}. Thus, it can be represented by one continuous lane segment for a given \(\xi\)-coordinate. This assumption is made to enable path planning based on \textit{patterns}. See Fig. 3 for illustration. In general, aforementioned \textit{interruptions} may result from strong field indents, bays or tree islands that are prohibited from trespassing.

Note that the entire concept and discussion of the previous paragraph can be generalised to interior lanes that are \textit{curvedly aligned} to a particular part of the field contour.

We make the following additional assumptions. First, it is assumed that
there exists one headland path that has to be traversed entirely, before starting coverage of the interior lanes. Second, we assume that the orientation of the interior lanes within the global coordinate system is given. This orientation consequently determines transformation parameter \( \theta \). The precise transitions between headland path and interior lanes are a priori not yet specified. These transitions result from the methods presented in this paper.

Third, besides the headland path and interior lanes, we assume a field entry located along the headland path. We additionally assume that a designated field exit exists, which may be identical to the field entrance. Alternatively, it may be positioned along the headland path. The field exit may suitably be selected as a location close to the field entrance of the field to be covered next.

See Fig. 4 for illustration of notation and the labeling of transition points between headland path and interior lanes. Not displayed are a) the field exit, which we label by \( Z_{2N+3} \) and interchangedly abbreviate by \( Z_e \) for brevity, and which frequently is identical to \( Z_0 \), and b) the agricultural vehicle position at time \( t \) which we label by \( Z_{2N+4} \) or interchangedly by \( Z(t) \), and which may be located anywhere along the headland path or along any interior lane. Note that the precise transitions between headland and interior lanes that account for limited turning radii are initially not yet specified. These transitions will be apparent in subsequent figures. We interpret locations \( Z_i, \forall i = 0, \ldots, 2N+4 \) as nodes and their path connections as edges denoted by \( e_{i,j}, \forall i, j = 0, \ldots, 2N+4 \).
Combining edges and nodes, we define a transition graph $T$ with

$$T_{i,j} = \begin{cases} d_{i,j}, & \text{if } \exists \text{ a direct admissible path } i \rightarrow j, \\ \infty, & \text{otherwise}, \end{cases}$$

where $d_{i,j} = \sqrt{(\xi_i - \xi_j)^2 + (\eta_i - \eta_j)^2}$ denotes the path length from node $i$ to $j$, $\forall i, j \in \{0, \ldots, 2N + 4\}$. Nodes indexed by $i = 2N + 1$ and $i = 2N + 2$ are introduced to ensure unique edge connections between any two nodes. Based on the transition graph $T$, shortest paths can be determined according to Bertsekas et al. (1995). For shortest path computations within agricultural fields under trajectory constraints, see Graf Plessen & Bemporad (2016).

The objectives of this paper can be summarised as follows. First, a suitable path planning pattern is sought. Second, accounting for the specific field entry and exit positions, the pattern must be applied appropriately. Third, the following tasks arise in online operation: 1) path following according to the field coverage plan, 2) navigation from a position along the path network to the field entrance for refilling of storage tanks, and 3) navigation from the field entrance after refilling back to the position along the field coverage path for the resumption of work. Refilling may be conducted at a stationary depot located outside of the field. Alternatively, a mobile depot (typically a truck) may be waiting at the field entrance. The design of the path planning pattern must account for the completion of these tasks. A discussion is sought about when to optimally trigger the signal for a return. This may not necessarily be upon complete emptying of the storage tanks.
3. Path Planning for Partial Field Coverage

The main part of this section discusses three path planning methods for partial field coverage. They are labeled as MEAND, CIRC and CIRC*. Before their presentation, the general case for shortest path in-field navigation is briefly discussed.

3.1. The general case for shortest path in-field navigation

The reason for us focusing on field shapes according to Fig. 3 is that they permit to work with patterns and to employ analytically derived and optimised rules for navigation. For arbitrarily shaped field contours, and including areas prohibited from trespassing within field contours such as, for example, tree islands, areas around power pole masts, and for given transitions between headland path and interior lanes, a customised shortest path algorithm must be employed that also accounts for repressed area minimisation constraints. Such a method was presented in Graf Plessen & Bemporad...
3.2. Lane traversal in S-forms – MEAND

In current working practice, the overwhelming majority of field coverage paths is planned based on sequential concatenation of lanes such that a meandering path results, see Palmer et al. (2003). It can also be described as a path resembling S-forms and is often referred to as A-B pattern. The fact that it is so widespread can be observed from satellite images and their display of tyre traces. Field coverage can be decomposed into, first, the traveling along the headland path, and, second, the subsequent following of the meandering path until completion of the field coverage. Under the assumptions of a) a continuous and thorough initial headland path traversal before starting interior lane coverage, b) field shapes according to the description of Section 2, and c) field coverage in one single traversal without requiring at least one intermediate return to a stationary or mobile depot for refilling (or a similar task), the path planning method based on meandering patterns is the optimal strategy, even under repressed area minimisation constraints. This is since the non-working distance is minimised. It is constrained to the headland path segments that were already covered during the initial headland path traversal. However, as will be shown, this method is not optimal for partial field coverage. Because of its widespread usage and its importance for path planning in agriculture, this method will serve as a comparison to the methods proposed in Section 3.3.

In the following, we refer to the path planning method based on the meandering pattern as MEAND. In the remainder of this section, we discuss how to optimally employ it and thereby illustrate its intrinsic disadvantages.
Fig. 6. MEAND. Distinction between four cases of combinations of uneven and even $N$ and the set of start positions, $Z_0^{(1)}$ and $Z_0^{(2)}$. The blue dotted lines indicate the $\frac{\xi_1 + \xi_2}{2}$ coordinate, respectively. Note that only for better visualisation of the route planning logic, the meandering path is not displayed as coinciding with the headland path. See Fig. [1] for comparison.
Fig. 7. MEAND. Visualisation of a path planning example that required a normalisation step with coordinate reflection (3)-(4). The result is displayed after the inversion of the coordinate reflection.

(a) The path planning pattern for MEAND.

(b) Concatenation of two path planning patterns for MEAND.

Fig. 8. MEAND. (Top plot) Illustration of the path planning pattern. The red bar indicates the area that cannot be reached by neither traversal of the path planning pattern nor traversal of the headland segments in the directions as indicated by the two arrows. (Bottom plot) Concatenation of two pattern elements. The traversal along the “upper” and “lower” headland path is emphasised in blue. Importantly, the area indicated by the red bar can still not be reached, see Proposition 1.
for partial field coverage.

Let us first outline the framework for MEAND, before generalizing and interpreting it. We distinguish between a) an even or an uneven number of interior lanes $N$, and b) between two sets of possible start positions expressed within the normalised coordinate framework. Consequently, we distinguish between four cases of combinations of $N$ and the set of start positions $Z_0$. They are: (uneven $N$, $Z_0^{(1)}$), (even $N$, $Z_0^{(1)}$), (uneven $N$, $Z_0^{(2)}$) and (even $N$, $Z_0^{(2)}$). These four distinctions entail path planning as displayed exemplatorily in Fig. 6 for 7 lanes (uneven $N$) and 8 lanes (even $N$). The mathematical description of $Z_0^{(1)}$ and $Z_0^{(2)}$ is derived as follows. Based on the definitions in Fig. 4 we define $H = \{ (\xi, \eta) : (\xi, \eta) \in \text{headland path} \}$, i.e., as the set of $(\xi, \eta)$-coordinates along the headland path. We further define the auxiliary location $Z_M = (\xi_M, \eta_M)$ with $\xi_M = (\xi_1 + \xi_N)/2$ and $\eta_M = \max \{ \eta : (\xi, \eta) \in H, \xi = \xi_M \}$. Then, we initialise the path length coordinate $s$ along the headland path at $Z_M$ with $s_M = 0$. This permits us to define two sets of path coordinates for field entrance positions, i.e., $S_0^{(1)} = \{ s : 0 \leq s \leq s_{N+1} \}$ and $S_0^{(2)} = \{ s : s_{N+1} \leq s \leq s_1 \}$, whereby $s_{N+1}$ and $s_1$ denote the path length coordinates at location $Z_{N+1}$ and $Z_1$, respectively. Consequently, the two sets of possible start positions expressed within the normalised coordinate framework can be defined as $Z_0^{(l)} = \{ Z(s) : s \in S_0^{(l)} \}$ for $l = 1, 2$, and where $Z(s)$ denotes a location at path length coordinate $s$ along the headland path.

Let us elaborate on the employed coordinate system transformations. The rotation step transforms coordinates from the global $(x, y)$-description to the $(x_\theta, y_\theta)$-coordinate system such that interior lanes are aligned to the $y_\theta$-axis. Then, we apply at most two additional coordinate reflections. We therefore
first define \( x_{\theta,M} = (\max_{x_{\theta} \in X_{\theta}} x_{\theta} - \min_{x_{\theta} \in X_{\theta}} x_{\theta})/2 \) with \( X_{\theta} \) denoting the set of all \( x_{\theta} \)-coordinates defining the headland path, before applying the first coordinate reflection by the linear mapping:

\[
\begin{align*}
x_{\theta}^x &= x_{\theta,M} - (x_{\theta} - x_{\theta,M}), \\
y_{\theta}^y &= y_{\theta}.
\end{align*}
\]

If the transformed coordinates are not yet sufficiently normalised such that the starting position falls into above framework and according to Fig. 6, then the following second coordinate reflection is applied:

\[
\begin{align*}
x_{\theta}^{xy} &= x_{\theta}^x, \\
y_{\theta}^{xy} &= y_{\theta,M} - (y_{\theta} - y_{\theta,M}),
\end{align*}
\]

with \( y_{\theta,M}^\theta = (\max_{y_{\theta} \in Y_{\theta}} y_{\theta}^\theta - \min_{y_{\theta} \in Y_{\theta}} y_{\theta}^\theta)/2 \), and where \( Y_{\theta} \) describes the set of \( y_{\theta} \)-coordinates defining the headland path. At the latest after this second transformation, coordinates are normalised such that the starting position falls into above framework and according to Fig. 6. Thus, dependent on which coordinate transformations are required, \((\xi, \eta)\) represents the \((x_{\theta}, y_{\theta})\)-, \((x_{\theta}^x, y_{\theta}^y)\)-, or \((x_{\theta}^{xy}, y_{\theta}^{xy})\)-coordinate system.

After path planning in the normalised coordinate system, all linear mappings required for normalisation must be inverted to obtain the result within the \((x, y)\)-plane. For an illustrative example requiring a coordinate reflection for path planning in the normalised coordinate frame, see Fig. 7.

The path planning method for MEAND is summarised in the following Algorithm. It describes the offline fitting of a traversable path to a given
field of interest. The logic for online navigation for partial field coverage is described further below.

Algorithm 1: MEAND (offline)

1. Normalisation of the coordinate system description:
   - one rotation, and at most two reflection steps.
   - description in the $(\xi, \eta)$-plane.

2. Distinction between four cases:
   - four combinations of even/uneven $N$ and $Z_0^{(1)}/Z_0^{(2)}$.

3. Path planning according to Fig. 6.

4. Retransformation of coordinates to the $(x, y)$-plane.

Let us interpret characteristics of the method. The pattern on which MEAND is founded is displayed in Fig. 8. We define “lower” and “upper” headland segments as the set of edges $E^{\text{lower}} = \{e_{i,j} : i, j = 1, \ldots, N, 2N + 1\}$ and $E^{\text{upper}} = \{e_{i,j} : i, j = N + 1, \ldots, 2N, 0, 2N + 2\}$, respectively. It is distinguished between two possible methods for the transition from lane $N$ towards the headland path; see the labeling “towards $Z_e$” in Fig. 6.

For all four cases, (a)-(d), the direction pointing towards $Z_0$, which is also the method implicitly assumed for the remainder of Section 3.2, is more favourable than its alternative with regard of path length minimisation from any given position back towards $Z_0$. This is easy to see from the fact that a transition from interior lane to headland path is created, which must be respected as a repressed area minimisation constraint. Even if $Z_e$ is located such that $\xi_e > \xi_N$, for overall path length minimisation, the method with a
final transition pointing towards smaller \( \xi_0 \) may typically still be preferably. This holds especially when frequent returns to a mobile depot are required. Importantly, it also guarantees that after traversal of the \( N \)th lane, one can then move along the “upper” headland path heading towards \( Z_0 \).

**Proposition 1.** Assume a normalised coordinate system description with \( Z_0 = (\xi_0, \eta_0) \) according to Fig. 6, in which we account for repressed area minimisation constraints, and in which we aim at finding the shortest path from position \( Z(t) = (\xi(t), \eta(t)) \) at time \( t \) to \( Z_0 \). Then, if \( \xi(t) \geq \xi_0 \), a corresponding agricultural vehicle has to always traverse the ultimate lane \( N \) as part of the path to reach \( Z_0 \), unless it already has covered all interior lanes and is heading back towards \( Z_0 \) along the “upper” headland path, or unless it is heading towards \( Z_0 \) along the “upper” headland path as part of the initial headland path traversal.

**Proof.** The proof is by construction and follows directly from the meandering path motif in Fig. 8 and the assumption of complying with repressed area minimisation constraints. See Fig. 6 for visualisation.

Proposition 1 is particularly relevant for fields with many lanes \( N \) (“fat” fields). For \( \xi(t) < \xi_0 \), no such generalizing statement can be made without making further differentiations between even/uneven \( N \) and \( Z_0^{(l)} \) for \( l = 1, 2 \). However, Proposition 1 can be generalised to alternative locations different from \( Z_0 \). See Fig. 8 for visualisation of areas that cannot be reached without reaching the final lane after the concatenation of multiple pattern elements.

**Remark 1.** While Proposition 1 guarantees that the last lane \( N \) must be reached, no generalizing statement can be made with respect to the short-
est path for reaching it. The $\xi$-coordinate must be monotonously increasing throughout the process of reaching it. However, it does not necessarily have to be strictly monotonously increasing. For example, dependent on the field contour and orientation of interior lanes, the shortest path may involve transitions along interior lanes from “upper” to “lower” headland segments, vice versa, and even multiple times during the process of reaching lane $N$. After traversal of the $N$th lane, the shortest path to $Z_0$ is sought. Here, the same concepts apply. Monotonous, but not strictly monotonous, movement towards $Z_0$ is required with potential transitions between “upper” and “lower” headland path segments. In practice, a variation of a label correcting algorithm, such as $A^*$ discussed in Bertsekas et al. (1995), can suitably be employed for the finding of shortest paths for both, a) the reaching of the $N$th lane starting from location $Z(t)$ at time $t$, and b) the reaching of $Z_0$ after the traversal of the $N$th lane.

**Remark 2.** For the resumption of work at location $Z(\tau_{\text{last}}) = (\xi(\tau_{\text{last}}), \eta(\tau_{\text{last}}))$, the shortest possible path from $Z_0$ to that location can be selected if $\xi(\tau_{\text{last}}) > \xi_0$ and $Z(\tau_{\text{last}})$ is not located along the “upper” headland. This can be seen from Fig. 8: there is no transition from headland to interior lane or vice versa that is prohibiting such shortest path. In contrast, for alternative locations of $Z(\tau_{\text{last}})$, the heading direction along the field coverage path plays an important role. To resume a specific heading orientation, a path may have to be taken that is deviating from the shortest path connecting $Z_0$ and $Z(\tau_{\text{last}})$.

At any time $t$, the agricultural vehicle can be in any of three states denoted by $\gamma(t) \in \{0, 1, 2\}$. The case $\gamma(t) = 0$ corresponds to a mode in which the vehicle is on its way back to the resuming location $Z(\tau_{\text{last}})$ at which the
field coverage was terminated last at time $\tau^{\text{last}}$. The case $\gamma(t) = 1$ corresponds to a mode in which the vehicle is following the field coverage path plan according to Fig. 6. Finally, $\gamma(t) = 2$ indicates the mode in which the vehicle is in the process of returning to $Z_0$ for refilling. The predicted fill-level at time $t + \Delta t$ is denoted by $\hat{f}(t + \Delta t) \in [0, f^{\text{max}}]$, where $f^{\text{max}}$ denotes the maximum fill capacity and $\Delta t$ the time discretisation (sampling time). We denote $F^{\text{trigger}}(t) = 1$ if the return command is triggered or active at time $t$. We have $F^{\text{trigger}}(t) = 0$ otherwise. In online operation and dependent on the location of $Z_0$, it often is favourable to trigger $F^{\text{trigger}}(t) = 1$ on the last lane with heading in negative $\eta$-direction before the fill-level is about to reach zero. This is since the resulting path to $Z_0$ typically involves the following steps: a) completion of the current interior lane, b) traveling along the “lower” headland path, c) a transition to the “upper” headland path via interior lane $N$, and d) traveling along the “upper” headland path until $Z_0$. The examples of Section 4 will further illustrate this consideration. In general, the decision upon when to trigger the return command must trade-off current fill-level $f(t) \geq 0$, the shortest path length $P(Z(t), Z_0)$ from the current location $Z(t)$ to the depot $Z_0$, and the path length $P(\hat{Z}_{f=0}, Z_0)$ from the predicted location $\hat{Z}_{f=0}$ at which the fill-level is expected to reach zero to the depot $Z_0$. A return command may be triggered if $f(t) \leq f^{\text{thres}}$ and $P(Z(t), Z_0) < P(\hat{Z}_{f=0}, Z_0)$, where $f^{\text{thres}} \ll f^{\text{max}}$ denotes a small threshold fill-level above which a return to $Z_0$ is undesired. Alternatively, it must be triggered if $f(t) = 0$. For the prediction of $\hat{f}(t)$ and $\hat{Z}_{f=0}$, two linear model descriptions can be employed, assuming a field coverage path traversal at constant velocity and a constant emptying rate, i.e., $f(t + \Delta t) = f(t) - a_f \Delta t$ where $a_f$ denotes the emptying
rate. For adaptive online estimation, a Kalman Filter according to Anderson & Moore (1979) can be employed to better predict $\hat{Z}_{f=0}$. Since we employ a linear model and under the assumption of Gaussian and additive noise disturbances, the Kalman Filter is the optimal estimator. To summarise, we state the following Algorithm for \textit{online} operation of MEAND:
Algorithm 2: MEAND (online)

1. Initialisation: $t = 0, \tau^{\text{last}} = \infty, \gamma(t) = 1$.

2. While the field is not yet covered:

3. Read current location $Z(t)$ and fill-level $f(t)$.

4. Determine $F^{\text{trigger}}(t)$.

5. if ($\gamma(t) == 1$) and ($F^{\text{trigger}}(t) == 0$):
   $\gamma(t + \Delta t) = 1$;
   Path following according to Fig. 6.

6. else if ($\gamma(t) == 1$) and ($F^{\text{trigger}}(t) == 1$):
   $\gamma(t + \Delta t) = 2$ and $\tau^{\text{last}} = t$;
   Shortest path from $Z(\tau^{\text{last}})$ to $Z_0$;
   According to Proposition 1 and Remark 1

7. else if ($\gamma(t) == 2$) and ($Z(t) \neq Z_0$):
   $\gamma(t + \Delta t) = 2$;
   Shortest path from $Z(t)$ to $Z_0$;
   According to Proposition 1 and Remark 1

8. else if ($\gamma(t) == 2$) and ($Z(t) == Z_0$):
   $\gamma(t + \Delta t) = 0$;
   Shortest path from $Z_0$ to $Z(\tau^{\text{last}})$;
   According to Remark 2

9. else if ($\gamma(t) == 0$) and ($Z(t) \neq Z(\tau^{\text{last}})$):
   $\gamma(t + \Delta t) = 0$;
   Shortest path from $Z(t)$ to $Z(\tau^{\text{last}})$;
   According to Remark 2

10. else if ($\gamma(t) == 0$) and ($Z(t) == Z(\tau^{\text{last}})$):
    $\gamma(t + \Delta t) = 1$ and $\tau^{\text{last}} = \infty$;
    Path following according to Fig. 6

11. $t = t + \Delta t$. 
3.3. Lane traversal in circular shapes – CIRC and CIRC

The path planning method for partial field coverage labeled CIRC is visualised in Fig. 9. It is referred to as CIRC because of its circular path planning pattern shown in Fig. 10. It is summarised in the following Algorithm:

Algorithm 3: CIRC (offline)

1. Normalisation of the coordinate system description:
   - one rotation, at most two reflection steps.
   - description in the \((\xi, \eta)\)-plane.
2. Distinction between four cases:
   - four combinations of even/uneven \(N\) and \(Z_0^{(1)}/Z_0^{(2)}\).
3. Path planning according to Fig. 9.
4. Retransformation of coordinates to the \((x, y)\)-plane.

**Proposition 2.** Assume a normalised coordinate system description with \(Z_0 = (\xi_0, \eta_0)\) according to Fig. 9, in which we account for repressed area minimisation constraints, and in which we aim at finding the shortest path from position \(Z(t) = (\xi(t), \eta(t))\) at time \(t\) to \(Z_0\). Then, for \(\xi_1 \leq \xi(t) < \xi_{N-1}\), an agricultural vehicle can always traverse the latest the second next interior lane such that afterwards it can travel along the “upper” headland path in direction of \(Z_0\). For \(\xi_1 \leq \xi(t) < \xi_{N-1}\), either lane \(n\) or \(n + 1\) permit such traversal, whereby \(n\) is such that \(\xi_{n-1} \leq \xi(t) < \xi_n\). For \(\xi(t) < \xi_1\), in general, either lane \(n = 2\) or \(n = 3\) permit such traversal, see Fig. 9. For \(\xi(t) \geq \xi_{N-1}\), lane \(n = N\) or a path through edges \(e_{N,2N+2}\) and \(e_{2N+2,2N}\) permit the traversal.
Fig. 9. CIRC. Distinction between four cases of combinations of uneven and even $N$ and the set of start positions, $Z_0^{(1)}$ and $Z_0^{(2)}$. The blue dotted lines indicate the $\frac{\xi_1 + \xi_2}{2}$ coordinate, respectively.
Fig. 10. CIRC and CIRC*. (Top plot) Illustration of a path planning pattern element. The red bar indicates the area that cannot be reached by neither traversal of the path planning pattern element nor traversal of the headland segments in the directions of the two arrows. (Bottom plot) Concatenation of two patterns. Importantly, the area indicated by the red bar in the top plot can now be reached. See the green paths for emphasis of the traversal along the “lower” headland path, and a transition via an interior lane to the “upper” headland path.
Proof. The proof is by construction and follows directly from the circular path motif in Fig. 10 and the assumption of complying with repressed area minimisation constraints. See also Fig. 9 for visualisation.

Remark 3. An implication of the proposed path planning method is that once the “upper” headland path is reached (assuming a normalised coordinate system description), the vehicle is constrained to travel along it until reaching $Z_0$. This is because of the repressed area minimisation constraint and the characteristic pattern according to Fig. 10. This is also in contrast to the MEAND-method and implies that no additional invoking of an $A^*$-algorithm is required. While for MEAND the shortest path to $Z_0$ after reaching of the $N$th lane may, in general, involve traversals of interior lanes and thus switching between “upper” and “lower” headland paths, this is not the case for CIRC. This distinction is the reason that no guarantee can be given about a shorter path length for CIRC. Consider an extremely large bulb-like headland segment located between two interior lanes. While MEAND can avoid this by a traversal to the “lower” headland path, the method according to CIRC is enforced to traverse it. Note that such (theoretical) scenarios are unlikely in practice. In Section 4, a quantitative comparison for a rectangular field is given as a function of $N$, the length of interior lanes and the machine operating width.

The importance of a normalised coordinate system is stressed in which a start position $Z_0$ is located as shown in Fig. 9. In fact, the path planning method CIRC is tailored to such coordinate system representation.

Fig. 11 illustrates two possible scenarios for resuming work after refilling at the depot. In these two scenarios, $Z(t_{\text{last}})$ is situated either along a lane
Fig. 11. CIRC. (Top plot) Resuming work at a position $Z(\tau_{\text{last}})$ located along a lane with heading direction towards positive $\eta$. (Bottom plot) Resuming work at a position $Z(\tau_{\text{last}})$ located along a lane with heading direction towards negative $\eta$. The path length is much shorter for the first scenario.
with heading direction towards positive or negative $\eta$. As indicated, the first method is preferable. This is because it avoids the traversal of an entire interior lane without performing actual application work.

**Remark 4.** If the field entrance is located such that $Z_0 \in \mathbb{Z}_0^{(1)}$ with $\xi_1 < \xi_0 \leq \xi_2$, and $Z(\tau^{\text{last}})$ for the resumption of work is located such the $\xi(\tau^{\text{last}}) > \xi_0$, then lane $\tilde{k} = 1$ must be traversed in order to reach the “lower” headland path, before proceeding to $Z(\tau^{\text{last}})$. Instead, if $\xi_{\tilde{k}} < \xi_0 \leq \xi_{\tilde{k}+1}$ for $\tilde{k} \in \{2, \ldots, \lfloor \frac{N}{2} \rfloor \}$, then either lane $\tilde{k}$ or $\tilde{k} - 1$ must be traversed. All these scenarios imply an initial movement towards negative $\xi$-direction despite $\xi(\tau^{\text{last}}) > \xi_0$. However, since lane $\tilde{k}$ or $\tilde{k} - 1$ are the immediate next and the second next lane to $Z_0$, the corresponding detour with respect to a movement monotonously increasing from $\xi_0$ towards $\xi(\tau^{\text{last}})$ is always very small. For $Z_0 \in \mathbb{Z}_0^{(2)}$, such a detour does also not occur.

*Online* operation of CIRC can be summarised as follows:
Algorithm 4: CIRC (online)

1. Initialisation: \( t = 0, \tau_{\text{last}} = \infty, \gamma(t) = 1 \).

2. While the field is not yet covered:

3. Read current location \( Z(t) \) and fill-level \( f(t) \).

4. Determine \( F^\text{trigger}(t) \).

5. if \((\gamma(t) == 1) \text{ and } (F^\text{trigger}(t) == 0)\):
   \( \gamma(t + \Delta t) = 1 \);
   Path following according to Fig. 9.

6. else if \((\gamma(t) == 1) \text{ and } (F^\text{trigger}(t) == 1)\):
   \( \gamma(t + \Delta t) = 2 \) and \( \tau_{\text{last}} = t \);
   Shortest path from \( Z(\tau_{\text{last}}) \) to \( Z_0 \);
   According to Proposition 2 and Remark 3.

7. else if \((\gamma(t) == 2) \text{ and } (Z(t) \neq Z_0)\):
   \( \gamma(t + \Delta t) = 2 \);
   Shortest path from \( Z(t) \) to \( Z_0 \);
   According to Proposition 2 and Remark 3.

8. else if \((\gamma(t) == 2) \text{ and } (Z(t) == Z_0)\):
   \( \gamma(t + \Delta t) = 0 \);
   Shortest path from \( Z_0 \) to \( Z(\tau_{\text{last}}) \);
   According to Remark 4.

9. else if \((\gamma(t) == 0) \text{ and } (Z(t) \neq Z(\tau_{\text{last}}))\):
   \( \gamma(t + \Delta t) = 0 \);
   Shortest path from \( Z(t) \) to \( Z(\tau_{\text{last}}) \);
   According to Remark 4.

10. else if \((\gamma(t) == 0) \text{ and } (Z(t) == Z(\tau_{\text{last}}))\):
    \( \gamma(t + \Delta t) = 1 \) and \( \tau_{\text{last}} = \infty \);
    Path following according to Fig. 9.

11. \( t = t + \Delta t \).
Let us discuss CIRC\(^*\), a variation of CIRC when modifying the method for headland path coverage. It is our preferred method for both single-run and partial field coverage. First, a remark is made about path planning for single-run field coverage based on the methods according to Fig. 6 and 9. Under the assumption of an initial uninterrupted headland path traversal, MEAND is preferred over CIRC with respect to path length minimisation. This is since some headland edges are traversed less frequently. Specifically, for CIRC every second “lower” headland edge is traversed three times: once along the headland path traversal, and two times according to the path planning pattern of Fig. 10. Similarly, also every second “upper” headland edge is traversed three times: once along the headland path, once according to the pattern of Fig. 10 and once after traversal of the \(N\)th lane when returning to \(Z_0\). This frequent traversal of the same edges is suboptimal. However, when dropping the assumption of an initial uninterrupted headland path traversal, an optimal field coverage path can be constructed based on the pattern of Fig. 10. Specifically, the headland path is covered as a byproduct of concatenations of the proposed circular pattern. Traversing these concatenations, every second “upper” headland edge is not yet covered. However, after traversal of the final lane, all these edges can be covered when returning to \(Z_0\) along the “upper” headland path. This method is referred to as CIRC\(^*\) and represents the optimal field coverage method since every edge is covered at most twice. For an even \(N\), the set of edges that are traversed twice is confined to headland segments. For an uneven \(N\), the two edges \(e_{N,2N+2}\) and \(e_{2N+2,2N}\) are additionally traversed twice. See Fig. 12 for illustration. This discussion is likewise relevant for partial field coverage. This is since its
overall path length is just composed of the field coverage path length plus
the summed distances from returning to the depot and resuming work in
the field. The concepts for returning to the depot and for resuming of work
in the field are identical for CIRC and CIRC*. This is since they are both
based on the same path planning pattern displayed in Fig. 10. However,
they differ in their method of how to cover the headland path. This implies
different on/off switching sequences for the nozzles of an automatic section
control (ASC) system for spraying applications. For CIRC*, more switchings
are required. This is to be regarded as its main disadvantage. See Batte &
Ehsani (2006) and Luck et al. (2010) for the discussion of ASC. Algorithms
3 and 4 similarly apply for CIRC*. To summarise, for overall path length
minimisation and in case of admittance of the modified method for headland
traversal, we propose to a) use the method according to CIRC* for generation
of the field coverage path plan, and b) conduct the returns to the depot for
refilling as discussed. For the field shapes under consideration, this method
is optimal for both single-run field coverage and partial field coverage.

Two options of application are envisioned. The first option includes
model-based a priori planning. This admits to preplan the entire field cov-
erage by dividing it into partial field coverages. This method is particularly
useful for the scheduling of support units (mobile depots). It therefore re-
quires an accurate model for storage tank fill-level dynamics. The second
option is less model-dependent. It does not preplan how to partition the
entire field coverage. This method is particularly useful if support units can
be summoned quickly, for example, because of a short traveling distance be-
tween a stationary depot and the field entrance. In practice, the mobile depot
Fig. 12. CIRC*. There is only a distinction between two cases: uneven and even $N$. For CIRC*, the path planning is identical for $Z_0^{(1)}$ and $Z_0^{(2)}$ as defined in Fig. 6. Thus, for CIRC* we have the set of admissible field entrances within the normalised coordinate system as $Z_0 = Z_0^{(1)} \cup Z_0^{(2)}$. An exemplary $Z_0$ is shown. The field coverage path is displayed as not closed in order to visualise the manner in which the headland path is traversed. The blue dotted lines indicate the $\xi_N$-coordinate, respectively.

is summoned once it is foreseeable that the fill-level of the in-field working agricultural vehicle will be zero soon. In both scenarios, the agricultural vehicle is preferably summoned to $Z_0$ for refilling when it is currently traveling along a lane with heading direction towards positive $\eta$, see Fig. 11.

3.4. Navigation in practice

In practice, the proposed path planning methods can be employed in form of a navigation application serving the human operator. Alternatively, they may provide the reference paths in a two-layered auto-steering framework with reference tracking as the second layer. A possible control method for
Fig. 13. Example 1. Comparison of the planned path for a return from $Z(t)$ to $Z_0$ according to MEAND and CIRC. Location $Z(t)$ is identical for both plots (a) and (b). However, because of the planned paths according MEAND and CIRC, the initial heading direction along the initial interior lane is different.

reference path tracking is described, for example, in Backman et al. (2012) or Graf Plessen & Bemporad (2017).

4. Illustrative Example

For illustration, we consider a rectangular field shape with an uneven number of lanes $N$ and a field entrance according to the $Z_0^{(1)}$-type. $Z_0$ is located between the first and second interior lane. See also Fig. 13 (which illustrates one specific scenario discussed further below). The path length differences between MEAND and CIRC, which both assume an initial headland path traversal, are reported in Table 2. All locations along the path network for both the returning to $Z_0$ and the resumption of work at $Z(\tau_{\text{last}})$

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Fig. 14. Example 2. Comparison of the planned path for a return from $Z(t)$ to $Z_0$ according to MEAND and CIRC. Location $Z(t)$ is identical for both plots (a) and (b). However, because of the planned paths according MEAND and CIRC, the initial heading direction along the initial interior lane is different.

are considered. All locations can be considered since results coincide for locations along the same edges. The distance between “upper” and “lower” headland path and inter-lane spacing (the machine operating width) are denoted $H_0$ and $W_0$, respectively. Accounting for the turning radius $R$, we obtain $H = H_0 - 2R + 2C$ with quarter circle path length $C = \frac{R\pi}{2}$, and $W = W_0 - 2R$. The fraction along an interior lane is indicated by $p \in [0, 1]$.

For example, in Fig. 13 and 14, we have $p = 0.25$. The difference in path lengths for MEAND and CIRC is abbreviated by $d_{\text{MEAND}} - d_{\text{CIRC}}$. The lane immediately neighboring $Z_0$ is indicated by $j_t$ with $\xi_{j_t} \leq \xi_0 < \xi_{j_t+1}$. Accordingly, we denote lane $j_p$ when $\xi_{j_p} = \xi_{j_t+1}$. We further define $q_t = \xi_0 - \xi_{j_t}$ and $q_p = \xi_{j_p} - \xi_0$ such that $q_t + q_p = W_0$. 

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Several interpretations can be drawn from Table 2. First, the largest spread $d_{\text{MEAND}} - d_{\text{CIRC}}$ can be observed for the case of returning to $Z_0$ from an uneven lane with $j = 3$. This results in $d_{\text{MEAND}} - d_{\text{CIRC}} = 2(N-3)W_0 + 2pH - 2R$, which scales linearly with the number of lanes $N$. Second, the smallest spread is achieved along even lanes when resuming work with $d_{\text{MEAND}} - d_{\text{CIRC}} = -2q_l - 2(1-p)H - 2W_0$. This is negative and therefore indicates a shorter path for MEAND. The equivalent setting for the resumption of work along an uneven lane index $j$ is $d_{\text{MEAND}} - d_{\text{CIRC}} = -2q_l - 2pH$. Third, this motivates guidelines for optimal operation of CIRC with respect to MEAND for path length minimisation. Ideally, we have $q_l \to 0$, indicating $Z_0$ to be located very close to the first interior lane. We here denote the limit-operator by ‘$\to$’. Ideally, a return-command is triggered along a lane with heading towards positive $\eta$ according to the path plan for CIRC and $p \to 0$. Ideally, the agricultural vehicle permits a small turning radius $R$ and an operating width $W_0 > R$. Then, the only setting for which the path length for CIRC is worse than for MEAND is along lane $N$ (the ultimate lane) with $d_{\text{MEAND}} - d_{\text{CIRC}} \to -2R$. This is a very small shortcoming, since turning radii for agricultural vehicles typically are in the single-digit meter-range (e.g., $R = 6m$). In all other settings, i.e., along the remaining entire path network, the method according to CIRC yields shorter paths for the return to $Z_0$ and the resumption of work in the field.

In two different scenarios, Fig. 13 and 14 visualise the planned paths for MEAND and CIRC for a return from $Z(t)$ to $Z_0$. The two scenarios differ in that $Z(t)$ is located along an uneven and even lane, respectively. The assumed vehicle locations upon triggering the return command are identical.
for MEAND and CIRC. However, because of the characteristic path planning for both methods, the vehicle is heading towards different directions along the lanes, respectively. The two scenarios are meant to illustrate the following. First, for MEAND the disadvantage of always having to travel until the $N$th lane is made apparent. Second, comparing Fig. 13 and 14 the benefit of triggering the return command for CIRC on a lane with heading direction towards positive $\eta$ can be observed. The non-working distance, which represents the entire path from $Z(t)$ to $Z_0$, is much shorter for the former scenario. See also Fig. 11 for the corresponding resumption of application work.

Table 3 indicates the path length differences between the methods MEAND, CIRC and CIRC* for single-run field coverage. CIRC* differs from CIRC in its method to handle the headland coverage. MEAND and CIRC proceed sequentially. Before covering any interior lane, they first cover the headland path entirely. In contrast, CIRC* combines lane and headland coverage simultaneously. Concatenations of patterns according to Fig. 10 admit simultaneous coverage of the “lower” headland path. After completion of the last pattern, the “upper” headland path is followed until $Z_0$, thereby covering the last not-yet covered headland edges. Even if partial field coverage requires frequent returns to the depot, essentially, the same field coverage path plan is traversed. Thus, the overall path length is affected, which is composed of the single-run field coverage path length plus the summed distances for returning to the depot and for resuming of work in the field.

Here, a crucial remark is made. As Table 3 illustrates, MEAND is preferable over CIRC with respect to single-run field coverage path length. On
the other hand, CIRC is preferable with respect to path length minimisation for returning to $Z_0$ and resuming of work within the field. Thus, dependent on the frequency of such return and resume states, the overall path length for MEAND may still be shorter than for CIRC. This, however, changes drastically when employing the method according to CIRC*. Not only does CIRC* enjoy the benefits of the path planning pattern in Fig. 10 for partial field coverage, it also significantly lowers (linear scaling in $N$) the single-run path length, see Table 3. Even if neglecting the repressed area minimisation constraints, CIRC* yields consistently shorter path lengths than MEAND. These results encourage to replace the currently predominant working practice of in-field path planning based on the meandering pattern in Fig. 8 by path planning according to CIRC*.

While the discussion of this section is based on rectangular fields, the results are transferable to alternative field shapes that comply with the assumptions of Section 2 (no interruptions of interior lanes within the normalised coordinate system). For CIRC and CIRC* this holds directly without modification because of the characteristic path planning pattern. For MEAND it holds especially if $W \ll H$. A typical difference for MEAND and alternative field shapes is the selection of the interior lane for the transition from “lower” to “upper” headland path. In Fig. 14, this interior lane is located close to $Z_0$. For an alternative field shape, it may be located much closer to the ultimate lane $N$. 

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5. Conclusion

We discussed three path planning methods for partial field coverage, which we labeled MEAND, CIRC and CIRC*. Repressed area minimisation constraints and field contours that permit path planning based on concatenations of patterns were assumed. We discussed a normalised coordinate system description that is derived from at most three linear transformations, a rotation and at most two coordinate reflections.

For CIRC, it was distinguished between a total of four combinations of an even and uneven number of lanes \( N \), and two distinct sets of field entrance positions. The field entrance is relevant for partial field coverage planning in that agricultural vehicles are meant to return to it for refilling of application material. The path planning pattern for CIRC was discussed as well as its application based on the normalised coordinate system description. It was discussed how characteristics of the proposed path planning pattern can be exploited in a suitable online implementation transitioning between three modes: 1) path following according to the offline planned field coverage route, 2) navigation from a position along the field coverage path back to the field entrance for refilling, and 3) navigation from the field entrance after refilling back to the position along the field coverage path for the resumption of application work.

A second method for partial field coverage was considered, which we labeled MEAND. The choice of its path planning pattern was motivated by its widespread usage in practice for in-field field coverage path planning. Its characteristic disadvantages for partial field coverage were discussed by means of a comparison to CIRC. Under repressed area minimisation con-
straints, for many scenarios these include the obligatory traversal of the ultimate lane $N$ as part of the path when returning to $Z_0$ for storage tank refilling.

Finally, we proposed CIRC*. It was illustrated how the overall field coverage path length can be reduced significantly when covering the headland path as a byproduct of concatenations of the proposed path planning pattern in Fig. 10. It was stressed how this is relevant for both partial and single-run field coverage. We emphasised how CIRC* outperforms MEAND for single-run field coverage due to its efficient way of covering the headland, thereby minimising non-working distance. At the same time, CIRC* enjoys the favourable properties of the proposed circular pattern for partial field coverage. For the assumed field shapes, CIRC* is the optimal path planning method for both single-run and partial field coverage and therefore our preferred method. The findings suggest to replace the currently widespread practice of in-field path planning based on a meandering pattern by the method according to CIRC*.

In practice, the presented methods can be employed either in form of a navigation application for a human operator, or in form of an auto-steering system. For the latter application, localisation and fill-level sensors are required in addition to a control system for reference path tracking. The reference path is provided by the method presented in this paper.

Subject of future work may include the optimized filling of storage tanks for weight minimisation and avoidance of soil compaction subject to optimized partial field coverage routes. In such a setting, an accurate predictive model for storage tank fill-level dynamics must first be identified from data,
before it can be recursively updated online.

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| Headland path | from $Z(t) \in e_{j,k}$ to $Z_0$ | $d_{MEAND} - d_{CIRC}$ |
|--------------|---------------------------------|---------------------|
| $j = 1, 2N + 1$ | $2(N - 3)W_0$                  |                     |
| $1 < j < N$   | $2(N - j - 1)W_0$               |                     |
| otherwise     | 0                               |                     |

| from $Z_0$ to $Z(\tau_{\text{last}}) \in e_{j,k}$ | $d_{MEAND} - d_{CIRC}$ |
|--------------------------------------------------|---------------------|
| $3 \leq j \leq N$, $j = 2N + 2$                  | $-2ql$              |
| $j = 2N + 1$                                     | 0                   |
| $j \geq N + 2$, $j$ uneven                       | $-2ql + (4N - 2j - 2)W_0$ |
| $j \geq N + 2$, $j$ even                         | $-2ql + (4N - 2j)W_0$  |
| otherwise                                        | 0                   |

| Interior lane | from $Z(t) \in e_{j,N+j}$ to $Z_0$ | $d_{MEAND} - d_{CIRC}$ |
|---------------|---------------------------------|---------------------|
| even lane $j \geq 2$ | $2(1 - p)H + (2N - 2j - 3)W_0 + W$ |                     |
| uneven lane $j \geq 3$ | $2(N - j)W_0 + 2pH - 2R$ |                     |
| lane $j = 1$     | 0                               |                     |

| from $Z_0$ to $Z(\tau_{\text{last}}) \in e_{j,N+j}$ | $d_{MEAND} - d_{CIRC}$ |
|--------------------------------------------------|---------------------|
| even lane $j \geq 2$ | $-2ql - 2(1 - p)H - 2W_0$ |                     |
| uneven lane $j \geq 3$ | $-2ql - 2pH$ |                     |
| lane $j = 1$     | 0                               |                     |

Tab. 2. Path length differences for the example of Section 4 with uneven $N$. Field entrance $Z_0$ is located between the first and second interior lane. For illustration, see also Fig. 13. It is distinguished between edges along the headland path and edges representing interior lanes. For the former case, index $k$ is determined by the headland path, see Fig. 1 for indexing. For the latter case, the heading direction of a vehicle along an edge $e_{j,N+j}$ varies for MEAND and CIRC according to their field coverage path plans, see Fig. 6 and 9. The results for all edges of the entire path network are reported.
Tab. 3. Single-run field coverage path lengths for the illustrative example in Section 4 with uneven $N$. The method according to CIRC has a path length that is proportional to the total number of lanes $N$ longer than MEAND. In contrast, the method according to CIRC* is similarly shorter and also scaling linearly with $N$. CIRC* and CIRC are both based on the same path planning pattern. However, their method of covering the headland path differs.

| Method $m$ | $d_{\text{MEAND}} - d_m$ |
|------------|--------------------------|
| CIRC       | $-(N - 1)W_0$            |
| CIRC*      | $(N - 3)W_0$             |