New World of Gossamer Superconductivity

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Since the discovery of the high-Tc cuprate superconductor La₂₋ₓBaCuO₄ in 1986 by Bednorz and Müller, controversy regarding the nature or origin of this remarkable superconductivity has continued. However, d-wave superconductivity in the hole-doped cuprates, arising due to the anti-paramagnon exchange, was established around 1994. More recently we have shown that the mean field theory, like the BCS theory of superconductivity and Landau’s Fermi liquid theory are adequate to describe the cuprates. The keys for this development are the facts that a) the pseudogap phase is d-wave density wave (dDW) and that the high-Tc cuprate superconductivity is gossamer (i.e. it exists in the presence of dDW).

1 Introduction

The epoch-making discovery of high-Tc cuprate superconductivity by Bednorz and Müller [1] put the entire superconductivity community in exaltation and confusion. This situation is nicely described by Enz [2]. As to the theoretical modeling of high-Tc cuprates the most influential proposal was the two-dimensional one band Hubbard model and related resonance valence band state proposed by P.W. Anderson [3]. In particular, Anderson gave the ground state wave function as

\[ |\psi_{BCS}\rangle = \prod_i (1 - d_i) |\psi_{BCS}\rangle \]

(1)

where \(\psi_{BCS}\rangle\) is the BCS wave function for s-wave superconductors [4] and \(\prod_i (1 - d_i)\) is the Gutzwiller projector where \(d_i = n_i^+n_i\). This Gutzwiller operator annihilates all the doubly occupied states. Then in 1994 the d-wave symmetry of high-Tc cuprate superconductivity was established through Josephson interferometry [5, 6] and powerful angle-resolved photoemission spectroscopy (ARPES) [7]. Therefore, at a minimum \(\psi_{BCS}\rangle\) in Eq.(1) has to be replaced by the corresponding one for d-wave superconductors [8, 9].

In 2002 R.B. Laughlin proposed that the Gutzwiller operator used by Anderson should be replaced by the more general Jastrow function, since the Gutzwiller projector is intractable [10]. But it is well known that both the d-wave superconducting wave function and the d-wave density wave (dDW) wave functions can be recast in the Jastrow form [11].

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We show in Fig. 1 the schematic phase diagram of the hole-doped high-Tc cuprates. The antiferromagnetic state at zero doping (\(x=0\)) vanishes around \(x=3\%\). Also the d-wave superconducting dome appears for \(5 < x < 25\). Under \(T^*\) there is the pseudogap phase. Recently a few people proposed that the pseudogap phase is d-wave density wave (dDW) [12, 13, 14, 15]. Indeed the giant Nernst effect observed in...
the underdoped region in LSCO, YBCO and Bi-2212 \cite{16,17,18,19,20} and the angle-dependent magneto-resistance in Y$_{1-x}$Pr$_x$Ba$_2$Cu$_3$O$_y$ can be described consistently in terms of dDW \cite{21,22}. If we accept that the pseudogap phase is dDW, we expect that d-wave superconductivity coexists with dDW. In other words the superconductivity in high-T$_c$ cuprates is gossamer superconductivity \cite{23,24}.

In 1993 Volovik \cite{25} showed how to calculate the quasiparticle density of states in the vortex state in d-wave superconductors. The striking $H^p$ dependence of the specific heat has been observed in single crystals of YBCO \cite{26,27}, LSCO \cite{28}, and Sr$_2$RuO$_4$ \cite{29,30}, where $H$ is the magnetic field strength. This analysis has been extended into several directions: a) thermodynamic functions; b) thermal conductivity; c) scaling relations; and d) for a variety of gap functions ($\kappa$)\textsuperscript{z} \cite{31,32,33,34,35,36}. Unfortunately the literature on Volovik’s effect is rather confused. We recommend that readers study Ref. \cite{36} for a brief summary. Since 2001 Izawa et al have succeeded in determining the superconducting gap functions ($\kappa$)\textsuperscript{z} in Sr$_2$RuO$_4$ \cite{37}, CeCoIn$_5$ \cite{38}, -$\text{(ET)}$_2$Cu(NCS)$_2$ \cite{39}, PrOs$_4$Sb$_{12}$ \cite{41,42} and UPd$_2$Al$_3$ \cite{43,44} through the angle-dependent magneto-thermal conductivity (ADMTC). These are shown in Fig. 2. These experiments are only possible due to a) availability of high-quality single crystals with $R\, R\, R > 100$ b) low-temperature facility operating at 1000 - 10 mK, and c) the recent theoretical development \cite{36}.

In Fig. 3 we show the phase diagram of the hole-doped high-T$_c$ cuprates \cite{24}. As you may recognize, we have replaced the pseudogap phase with d-wave density wave (dDW). However, instead of our usual phase diagram, we take the chemical potential $\mu$ as a control parameter and show that dDW exists in three varieties.

Also the $T_c$ of dDW $T_{c1} (= T^*)$ is determined by

$$\ln \left( \frac{T_{c1}}{T_{c10}} \right) = \Re e \left( \frac{1}{2} - \frac{1}{2} \ln \left( \frac{1}{2} \frac{1}{T_c} \right) \right)$$

(2)

where $T_{c10} \approx 800K$ is the $T_{c1}$ in the limit $\mu = 0$. Here $\mu$ is the chemical potential and $\Re e$ is the di-gamma function. Eq (1) is the same as for the s-wave or d-wave superconductors in the presence of the Pauli paramagnetic term \cite{45,46}. Also as shown by the broken curve $T_{c1}$ in Eq. (1) bends back for $T_{c1} = T_{c10}$ 0.55. However, if you allow a spatial variation for dDW like $\cos(\theta) \cos(r) \cos(q \cdot r)$ we have to solve a new equation

$$\ln \left( \frac{T_{c1}}{T_{c10}} \right) = \Re e \left( \frac{1}{2} - \frac{1}{2} \ln \left( \frac{1}{2} \frac{1}{T_c} \right) \right) \cos(q \cdot r)$$

(3)
where \( p = \sqrt{\frac{1}{2}} \) and is determined to optimize \( T_{c1} \). Actually Eq. (2) is the same as for the Fulde-Ferrell-Larkin-Ovchinnikov state in d-wave superconductors, as discussed in [47], and its solution is known. According to [47] dDW splits further into dDW II and dDW III. In the region dDW II and dDW III we find \( \mathbf{q} = [110] \) for the - sign in Eq. (3) and \( \mathbf{k} = [100] \) for the + sign, respectively. Here \( \langle \cdots \rangle \) means the average over . Therefore it appears that together with periodic dDW's we can reproduce the observed \( T_{c1} \) for dDW. Also the theoretical study of dDW II and dDW III will be of great interest.

## 2 D-wave density waves

As already mentioned many people have proposed d-wave density wave (dDW) [12, 13, 14, 15] for the pseudogap phase of high-\( T_c \) cuprate superconductors. But until recently no quantitative test of these proposals was available. Recently we have shown that the giant Nernst effect observed in the pseudogap phase in YBCO, LSCO and Bi-2212 [16, 17, 18, 19] can be described in terms of dDW [20]. First of all, we stress that the dDW in the pseudogap phase is very different from that proposed in [12, 13, 14, 15]. The present dDW is incommensurate and possesses the U(1) gauge symmetry while the earlier proposal is the descendant of the flux phase or the staggered phase [48, 49] and carries miniscule loop currents which have not been observed. Furthermore, it is clear that such commensurate dDW with \( Z_2 \) symmetry are unstable in the 3D environment, and cannot have the chemical potential as a control parameter.
The quasiparticle energy of dDW is given by \[ E(\mathbf{k}) = q \left( 2\left(\mathbf{k}\right) + 2\sin^2(2\theta) \right) \] (4)

with

\[ \theta(\mathbf{k}) = v/0_k + v_0 \cos(2k_z) \] (5)

where \( k_z \) is the radial component in the x-y plane and \( v \) and \( v_0 \) are the Fermi velocities and \( \tan(\phi) = k_x/k_y \). As we shall see the chemical potential plays the crucial role in the construction of the phase diagram of high-\( T_c \) cuprate superconductors. But the chemical potential is absent in the descendant of the staggered phase as in [50] for example. Indeed as already discussed in [24] such a model lacks physical relevance. Then the quasiparticle density of states is given by

\[ N(E) = N_0 = G(x, y) \] (6)

where

\[ G(x) = \begin{cases} 2\pi K(x) & \text{for } x \leq 1 \\ 2K(x^{-1}) & \text{for } x > 1 \end{cases} \] (7)

and \( x = E = \), \( y = = \) and \( K(x) \) is the complete elliptic integral. \( N(E) = N_0 \) for a few \( \phi \)'s is shown in Fig. 3. As we shall see later \( \phi \) is essential for the presence of gossamer superconductivity. In addition, the presence of nonzero \( \phi \) is required to account for the Fermi arcs seen in ARPES [51].

Therefore how to identify d-DW or more generally unconventional density wave (UDW) is the central issue [52]. As is well known [53][54], in a magnetic field the quasiparticle energy in UDW is quantized à la Landau. In other words, except for the \( n=0 \) Landau level, the quasiparticle energy gap \( \Delta_n \) opens up, where \( B \) is the strength of the magnetic field. This leads to the giant Nernst effect [20], the angle-dependent magnetoresistance (ADMR) [55][56] and the nonlinear Hall effect [22].
Fig. 4  The quasiparticle density of states for a dDW superconductor

From the experimental data of ADMR we have identified UDW in the low temperature phase (LTP) in -(ET)$_2$KHg(SCN)$_4$, in the metallic phase of (TMTSF)$_2$X with X= PF$_6$ and ReO$_4$, both under pressure and magnetic fields. More recently we have identified dDW in the pseudogap phases in the underdoped high-$T_c$ cuprates $Y_{1-2x}Pr_{0.8}CuO$_4$ with $T_c = 55$ K, and the heavy-fermion system CeCoIn$_5$. Also it will be of great help to explore both the giant Nernst effect and the nonlinear Hall effect in these systems.

3 Concluding Remarks

Earlier we have seen that most of the metallic ground states in high-$T_c$ cuprates, heavy-fermion conductors and organic conductors belong to one of the mean field ground states: a) unconventional superconductivity, b) unconventional density wave; or c) the coexistence of both unconventional superconductivity and UDW. The present analysis suggests that a) most of the the so-called “non-Fermi liquid” state is in fact the Fermi liquid à la Landau and UDW, b) the superconductivity in both high-$T_c$ cuprates and CeCoIn$_5$ are gossamer; and c) the superconductivity in -(ET)$_2$ salts and in Bechgaard salts (TMTSF)$_2$PF$_6$ and URu$_2$Si$_2$ are also gossamer. This suggests a vast forest of gossamer superconductors are awaiting our exploration.

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References
[1] J.G. Bednorz and K.A. Müller, Z. Phys. B 64, 180 (1986).
[2] C.P. Enz, A Course in Many-Body Theory Applied to Solid State Physics (World Scientific, Singapore, 1992).
[3] P.W. Anderson, Science 235, 1196 (1987); The Theory of High-$T_c$ Superconductivity (Princeton, 1998).
[4] J. Bardeen, L.N. Cooper and J.R. Schrieffer, Phys. Rev. 108, 1185 (1957).
[5] D.J. Van Harlingen, Rev. Mod. Phys. 67, 515 (1995).
[6] C.C. Tsuei and J.R. Kirtley, Rev. Mod. Phys. 72, 969 (2000).
[7] A. Damascelli, Z. Houssain and Z.X. Shen, Rev. Mod. Phys. 72, 969 (2000).
[8] H. Won and K. Maki, Phys. Rev. B 49, 1397 (1994).
[9] K. Maki and H. Won, Phys. Rev. Lett. 72, 1758 (1994).
[10] R.B. Laughlin, cond-mat/0209269.
[11] P.G. de Gennes, “Superconductivity in Metals and Alloys”, Benjamin, New York (1966).
[12] E. Capelluti and R. Zeyher, Phys. Rev. B 59, 6475 (1999).
[13] L. Benfatto, S. Caprara and C. Di Castro, Eur. Phys. J. B 17, 95 (2000).
[14] S. Chakraverty, R.B Laughlin, D.K. Morr and C. Nayak, Phys. Rev B 63, 094503 (2001).
[15] R. Zeyher and A. Greco, Phys. Stat. Sol. (b) 242, 356 (2005).
[16] Z.A. Hu, N.P. Ong, Y. Wang, T. Kakeshita and S. Uchida, Nature 87, 486 (1998).
[17] Y. Wang, Z.A. Xu, T. Kakeshita, S. Uchida, S. Ono, Y. Ando and N.P. Ong, Phys. Rev. B 64, 224519 (2001).
[18] Y. Wang, N.P. Ong, Z.A. Xu, T. Kakeshita, S. Uchida, D.A. Bonn, R. Liang and W.N. Hardy, Phys. Rev. Lett. 87, 257003 (2002).
[19] C. Capan, K. Behnia, J. Hinderer, A.G.M. Jansen, W. Lang, C. Marcenat, C. Marin and J. Flouquet, Phys. Rev. Lett. 88, 056601 (2003).
[20] K. Maki, B. Dora, A. Vanyolos and A. Virosztek, Curr. Appl. Phys. 4, 693 (2004).
[21] V. Sandu, E. Cimpoiasu, T. Katuwai, Shi Li, M.B. Maple and C.C. Almason, Phys. Rev. Lett. 89, 177005 (2004).
[22] B. Dora, K. Maki, A. Virosztek, Europhys. Lett. 72, 1 (2005).
[23] H. Won, S. Haas, D. Parker and K. Maki, Phys. Stat. Sol. (b) 242, 363 (2005).
[24] H. Won, S. Haas, K. Maki, D. Parker, B. Dora and A. Virosztek, cond-mat/0508234.
[25] G.E. Volovik, JETP Lett. 58, 496 (1993).
[26] A. Molter et al., Phys. Rev. Lett. 73, 2744 (1994).
[27] B. Revaz et al., Phys. Rev. Lett. 80, 3364 (1998).
[28] S.J. Chen, C.F. Chang, H.L. Tsay, H.D. Yang and J.-Y. Lin Phys. Rev. B 58, 14753(R), (1998).
[29] S. Nishizaki, Y. Maeno and Z. Mao, J. Phys. Soc. Jpn. 69, 573 (2000).
[30] H. Won and K. Maki, Europhys. Lett. 52, 427 (2000).
[31] C. Kübert and P. Hirschfeld, Solid Stat. Comm. 105, 459 (1998).
[32] C. Kübert and J.P. Hirschfeld, Phys. Rev. Lett. 80, 4963 (1998).
[33] I. Vehkijärvi, J.P. Tarjanne, E.J. Nicol, Phys. Rev. B 59, 7123 (1999).
[34] H. Won and K. Maki, cond-mat/0004105.
[35] T. Dahm, K. Maki and H. Won, cond-mat/0006301.
[36] H. Won, S. Haas, D. Parker, S. Telang, A. Vanyolos and K. Maki, in Lectures on the Physics of Highly Correlated Electron Systems IX, AIP proceedings 789 (Melville 2005), pp. 3-43.
[37] K. Izawa, H. Takahashi, H. Yamaguchi, Yuji Matsuda, M. Suzuki, T. Sasaki, T. Fukase, Y. Yoshida, R. Settai and Y. Onuki, Phys. Rev. Lett. 86, 2653 (2001).
[38] K. Izawa, H. Yamaguchi, Yuji Matsuda, H. Shishido, R. Settai and Y. Onuki, Phys. Rev. Lett. 87, 57002 (2001).
[39] K. Izawa, H. Yamaguchi, T. Sasaki and Yuji Matsuda, Phys. Rev. Lett. 88, 027002 (2002).
[40] K. Izawa, K. Kamata, Y. Nakajima, Y. Matsuda, T. Watanabe, M. Nohara, H. Takagi, P. Thalmeier and K. Maki, Phys. Rev. Lett. 89, 137006 (2002).
[41] K. Izawa, Y. Nakajima, J. Goryo, Y. Matsuda, S. Osaki, H. Sugawara, H. Sato, P. Thalmeier and K. Maki, Phys. Rev. Lett. 90, 117001 (2003).
[42] K. Maki, S. Haas, D. Parker, H. Won, K. Izawa and Y. Matsuda, Europhys. Lett. 65, 720 (2004).
[43] T. Watanabe et al, Phys. Rev. B 70, 184502 (2004).
[44] H. Won, D. Parker, K. Maki, T. Watanabe, K. Izawa and Y. Matsuda, Phys. Rev. B 70, 140509 (2004).
[45] G. Sarma, J. Phys. Chem. Solids 24, 1629 (1963).
[46] H. Won, H. Jang and K. Maki, cond-mat/9901252.
[47] K. Maki and H. Won, Czech J. Phys. 46, 1033 (1996); Physica B 322, 315 (2002).
[48] I. Affleck and J. B. Marston, Phys. Rev. B 37, 3744 (1988).
[49] C. Wu, S. Capponi, S.C. Zhang, Phys. Rev. B 70, 220505(R), (2004).
[50] P. Thalmeier, Z. Phys. B 95, 39 (1994).
[51] J.C. Campuzano, H. Ding, M.R. Norman, M. Randeira, Physica B 259-261, 517 (1999).
[52] B. Dora, K. Maki and A. Virosztek, Mod. Phys. Lett. B 18, 327 (2004).
[53] A.A. Nersesyan and G.I. Vachnadze, J. Low Temp. Phys. 77, 293 (1989).
[54] A.A. Nersesyan, G.I. Japaridze and I.G. Kimeridze, J. Phys. Cond. Matt. 3, 3353 (1991).
[55] K. Maki, B. Dora, M. Kartsovnik, A. Virosztek, B. Korin-Hamzic and M. Basletic, Phys. Rev. Lett. 90, 256402 (2003).
[56] B. Dora, K. Maki, A. Vanyolos and A. Virosztek, Europhys. Lett. 67, 1024 (2004).
[57] W. Kang, H.Y. Kang, Y.J. Jo and S. Uji, Synth. Metals 133-134, 13 (2003).
[58] T. Hu, M. Xiao, V. Sandu, C.C. Almason, K. Maki, B. Dora, T.A. Sayles and M.B. Maple, preprint.
[59] M. Pinteric, S. Tomic and K. Maki, Physica C 408-410, 75 (2004).
[60] I.J. Lee, S.E. Brown, W. Yu, M.J. Naughton and P. Chaikin, Phys. Rev. Lett. 94, 197001 (2005).