Favored Variants of Cold Dark Matter Cosmologies *

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Abstract

We discuss variants of Cold Dark Matter (CDM) that give good agreement with a range of observations. We consider models with hot dark matter, tilt, $\Omega < 1$, or a cosmological constant. We also discuss the sensitivity of the results to other parameters, such as the Hubble parameter and the baryon fraction. We obtain constraints by combining the COBE data, cluster abundances, abundance of damped Lyman-α systems at $z \sim 3$, the small-angle Cosmic Microwave Background anisotropy, and the small-scale non-linear power spectrum. We present non-linear power spectra from a new suite of N-body simulations for the “best-bet” models from each category.

1 Introduction

Since CDM comes close to explaining many fundamental observations, it has now become popular to consider models that are mostly CDM, with a little bit of something else. There is no shortage of possibilities for the “something else,” and at present neither the total mass density $\Omega_0$, the baryonic density $\Omega_b$, the Hubble parameter $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$, nor the power-law index $n$ of the spectrum of primordial fluctuations ($P(k) \propto k^n$) are very well constrained. However, it is an interesting exercise to see what corner of parameter space large scale structure observations alone would push us into, for various classes of models. This will be the primary focus of this discussion. We will present “best” versions of the four most popular classes of CDM variants, based on a comparison of semi-analytic calculations and N-body simulations with various observations. The four classes of variants considered are Cold plus Hot Dark

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Matter (CHDM)\(^1\) and tilted CDM (TCDM)\(^2\), both with \(\Omega = 1\), and low-\(\Omega\) CDM models in either a flat cosmology with a cosmological constant \(\Lambda\) such that \(\Omega_\Lambda = \Lambda / (3H_0^2) = 1 - \Omega_0\) (LCDM)\(^3\) or an open cosmology with \(\Lambda = 0\) (OCDM).\(^4\) As this exercise is to some degree a matter of taste, we will discuss the reasoning behind our choice of each parameter set. For more on the current observational constraints on the fundamental cosmological parameters, see e.g. Primack.\(^5\)

2 Constraints from Large Scale Structure

2.1 Cluster Abundance

The abundance of clusters \(N(> M)\) combined with the COBE normalization provides a strong constraint on cosmological models. The number densities of clusters with X-ray temperatures exceeding \(k_B T = 3.7\) keV and 7 keV were measured by Henry & Arnaud.\(^6\) The main uncertainty in \(N(> M)\) lies in the translation from velocity dispersion or X-ray luminosity to mass. The X-ray mass estimates could be affected by temperature gradients, substructure, or ellipticity.\(^7\) We have taken the mass range for the \(k_B T > 3\) keV clusters from White et al.\(^8\) and the mass range for the \(k_B T > 7\) keV clusters from Liddle et al.\(^3\).

Figure 1 shows the cluster abundances obtained from the Press-Schechter approximation\(^9\) for a variety of models (see the figure caption). The CHDM model with \(\Omega_\nu = 0.2\) in one species of massive neutrinos (\(N_\nu = 1\)) overproduces clusters by a factor of about two. The lower neutrino mass that arises with the same \(\Omega_\nu\) but \(N_\nu = 2\) species of neutrinos results in a longer free-streaming length, which decreases the power by about 20% on cluster scales without significantly affecting larger or smaller scales.\(^1\)\(^1\)\(^0\)\(^1\)\(^1\)\(^1\) The CHDM model with \(N_\nu = 2\) is in better agreement, but still too high on the basis of this analysis. However, moderate over-prediction of the cluster abundance is probably not too worrisome, since cluster masses derived from gravitational lensing tend to be systematically higher than the X-ray masses by as much as a factor of two.\(^1\)\(^1\)\(^1\)\(^1\)\(^1\) Also, preliminary work with simulations suggests that the Press-Schechter approximation is systematically high on these scales.\(^1\)\(^2\) Introducing a very moderate tilt \((n = 0.95)\) would bring this model into agreement with the cluster data in Figure 1, but this would reduce the power on small scales, leading to less early structure formation.

For the TCDM models, the best agreement is obtained with either \(n = 0.8\) and \(h = 0.5\) or \(n = 0.9\) and \(h \sim 0.45\). For OCDM, one can see that there is no hope of producing enough clusters with \(\Omega < 0.5\). The only way to improve the very low-\(\Omega\) models would be to introduce a “positive tilt” \((n > 1)\). One can see that there is an almost perfect degeneracy in \(N(> M)\) between \(\Omega_0\) and
Figure 1: Number density of halos with mass greater than $M$ from a Press-Schechter approximation with $\delta_c = 1.69$ and a top-hat filter. Points show the number density of clusters with X-ray temperature exceeding 3 keV (upper left) and 7 keV (lower right) [6]. In order of decreasing number density, the models shown are: CHDM $N_\nu = 1$ and $\Omega_\nu = 0.1, 0.2,$ and 0.3; $N_\nu = 2$ and $\Omega_\nu = 0.2; N_\nu = 1, \Omega_\nu = 0.2,$ and $n = 0.95; and N_\nu = 1, \Omega_\nu = 0.2,$ and $n = 0.9.$ TCDM: $n = 0.9$ and $h = 0.5; n = 0.9$ and $h = 0.45; n = 0.8$ and $h = 0.5; n = 0.9$ and $h = 0.42; n = 0.8$ and $h = 0.45.$ OCDM: $\Omega_0 = 0.7$ and $h = 0.5; \Omega_0 = 0.6$ and $h = 0.5; \Omega_0 = 0.5$ and $h = 0.6; \Omega_0 = 0.4$ and $h = 0.65.$ $\Lambda$CDM: $\Omega_0 = 0.4, h = 0.65; \Omega_0 = 0.4, h = 0.60; \Omega_0 = 0.3, h = 0.70; \Omega_0 = 0.4, h = 0.60, n = 0.9, \sigma_8 = 0.88; \Omega_0 = 0.4, h = 0.65, n = 0.9; \Omega_0 = 0.4, h = 0.60, n = 0.9.$ Bold lines indicate the “favored” models discussed in the text. Parameters not specified are as given in Table 1.
ΛCDM models with \( \Omega_0 = 0.3 \) or \( \Omega_0 = 0.4 \) with some tilt \( (n = 0.9) \) agree reasonably well with the cluster data.

2.2 Early Structure Formation

Observations of quasar absorption systems (clumps of gas or proto-galaxies that produce absorption features in quasar spectra) place constraints on structure formation up to \( z \sim 4 \). We will focus on observations of high-density absorption systems called damped Lyman-\( \alpha \) systems (DLAS). The Press-Schechter approximation can again be used to provide a lower limit on the abundance of collapsed gas at a given redshift — see Figure 2. Some of the gas may be ionized or consumed by star formation, so a viable model should predict at least as much collapsed gas as the observations. A more detailed analysis including physical modeling of gas cooling, star formation, and supernova feedback is necessary in order to calculate the actual amount of gas in absorption systems.\[13\] It should be noted that the measurement of \( \Omega_{\text{gas}} \) at \( z = 3–3.5 \) has come down by nearly a factor of three due to new observations,\[15\] easing the constraint on CHDM models. The new observations also suggest that \( \Omega_{\text{gas}} \) may peak at \( z \sim 3 \) and fall off at higher redshifts, which could be explained very naturally by the decreased supply of collapsed gas predicted in CHDM models.

If the DLAS have relatively small masses (\( \sim 10^{10–10^{11}} M_\odot \)), then CHDM models with \( \Omega_\nu \sim 0.2 \) produce enough collapsed gas to be compatible with these observations, even with a small tilt \( (n = 0.95) \). Models with higher \( \Omega_\nu \) or larger tilt probably will not produce enough collapsed gas. The models with \( N_\nu = 2 \) would produce very similar results because changing the number of neutrino species does not affect the power on small scales. Figure 2 (lower panel) shows the predicted \( \Omega_{\text{gas}} \) for tilted models with pure CDM. These models have no difficulty producing enough collapsed gas. The OCDM and LCDM models have even earlier structure formation and produce plenty of collapsed gas at high redshift — but they might not easily account for a fall-off of \( \Omega_{\text{gas}} \) with \( z \).

Recent observations of “normal” (i.e. non-AGN) galaxies at high redshift\[16\] may provide another constraint on early structure formation. The masses of these objects are uncertain, however, so a comparison with theory requires a fairly detailed treatment of galaxy formation\[17\].

3 Summary of “Best” CDM Variants

Taking all the constraints into account, we can arrive at “best” choices for each of the classes of models. The parameters for these models are summarized in Table 1. We have run a suite of large, high-resolution Particle Mesh (PM) N-body simulations\[12\] of these models, in order to study their non-linear clustering and large scale structure properties in more detail. We will now summarize the considerations that led to our choices of parameters for each model.
Figure 2: Collapsed gas as a function of redshift, from a Press-Schechter analysis with $\delta_c = 1.4$ and top-hat smoothing (cf. Klypin et al. [14] and references therein). Long dashed, short dashed, and dotted lines correspond to halos with masses of $10^{12}$, $10^{11}$, and $10^{10}h^{-1}M_\odot$ respectively. Data points are from Storrie-Lombardi et al. [15]. Models shown are CHDM $\Omega_{\nu} = 0.2$, $n = 1$, $N_{\nu} = 1$ and TCDM $n = 0.9$, $h = 0.45$.

| Model       | $t_0$ | $h$ | $\Omega_0$ | $\Omega_b$ | $\Omega_{\nu}$ | $\Omega_{\Lambda}$ | $n$ | $N_{\nu}$ | $\sigma_8$ |
|-------------|-------|-----|-------------|-------------|------------------|---------------------|-----|-----------|------------|
| SCDM        | 13.0  | 0.50| 1.0         | 0.0         | 0.0              | 0.0                 | 1.0 | 0         | 0.667      |
| TCDM        | 14.5  | 0.45| 1.0         | 0.1         | 0.0              | 0.0                 | 0.9 | 0         | 0.732      |
| CHDM-$2\nu$ | 13.0  | 0.50| 1.0         | 0.1         | 0.2              | 0.0                 | 1.0 | 2         | 0.719      |
| tCHDM-$2\nu$| 10.9  | 0.60| 1.0         | 0.069       | 0.2              | 0.0                 | 0.9 | 2         | 0.677      |
| OCDM        | 14.7  | 0.60| 0.5         | 0.069       | 0.0              | 0.0                 | 1.0 | 0         | 0.773      |
| tACDM       | 14.5  | 0.60| 0.4         | 0.035       | 0.0              | 0.6                 | 0.9 | 0         | 0.878      |

Table 1: Model Parameters

3.1 CHDM

We chose two CHDM models to investigate. The $\Omega_{\nu} = 0.2$ and $N_{\nu} = 2$ model with $n = 1$ gives reasonable agreement with the cluster data (see the caveat in section 2.1) and has ample collapsed gas at $z \sim 3$ to be compatible with the DLAS data. We also consider a tilted CHDM model with a high Hubble parameter ($h = 0.6$). The tilt reduces the power on small scales, but the increased
Hubble parameter partially compensates for this, so this model still gives good agreement with both the cluster data and the DLAS data. However, the age of the universe is only 11 Gyr, several Gyr younger than most estimates of globular cluster ages. But if the evidence for a large Hubble parameter becomes more certain, this model might be worth taking seriously.

3.2 TCDM

We chose to investigate a tilted model with $n = 0.9$ and $h = 0.45$. A model with $n = 0.8$ and $h = 0.5$ gives very similar results for the cluster abundance, but is disfavored by the Saskatoon data on the CMB anisotropy at small angular scales. As for the CHDM models, we have chosen a rather high $\Omega_b$, consistent with the low D/H measured by Tytler et al., to lessen the conflict with cluster baryons for these $\Omega = 1$ models.

3.3 OCDM

Several independent analyses of galaxy peculiar velocities lead to strong lower limits on $\Omega_0$ in all models with Gaussian primordial fluctuations. It is interesting that one would reach similar conclusions based only on the cluster data — open models with $\Omega_0 \leq 0.4$ are very strongly ruled out because they simply do not produce nearly enough clusters. Even open models with $\Omega_0 = 0.5 - 0.6$ will probably underproduce clusters. We have chosen a model with $\Omega_0 = 0.5$ and $h = 0.6$. This model is almost completely degenerate with a model with higher $\Omega_0$ and lower $h$. We chose this model because of the interest in models with higher values of the Hubble Parameter.

3.4 $\Lambda$CDM

Previously favored $\Lambda$ models with $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$, and $h = 0.7$ give good agreement with cluster abundance data but produce far too much power on small scales. This is difficult to reconcile with physical models of galaxy formation, which predict that the galaxy power spectrum should if anything be larger than the matter power spectrum. If one is unwilling to accept a scale-dependent anti-bias of galaxies with respect to dark matter, the only way to save this class of models is to go to higher $\Omega_0$ (thus smaller $\Omega_\Lambda$, also favored by recent data on quasar lensing, HST galaxy counts, and $z \sim 0.5$ supernovae), smaller $h$, and to add a tilt to reduce the small scale power. We have chosen a model with $\Omega_0 = 0.4$, $h = 0.60$, and a tilt of $n = 0.9$. As we saw in Figure 1 when normalized to the central COBE value this model underproduces clusters. We have therefore normalized the model to an amplitude about 10% higher than the central value, but still within the 1σ uncertainty. The same model normalized to the central COBE value but with a slightly higher Hubble parameter $h = 0.65$ would also fit the cluster data, but may have too much power on small scales.
3.5 Non-linear Power Spectra

Figure 3 shows the linear power spectra for our “favorite” models and the non-linear power spectra from the N-body simulations. All of the models appear to agree reasonably well with the APM real space power spectrum (plotted with triangles), however one should keep in mind that there is still the issue of galaxy bias to contend with. The OCDM and tΛCDM models in particular may still have too much power on small scales. This needs to be investigated using a physical model for galaxy bias, which will also permit investigation of many other potentially powerful statistics of the galaxy distribution on nonlinear scales, such as the pairwise velocity, group velocity dispersion, filament statistics, or the void probability function.

![Image of power spectra](https://example.com/image)

Figure 3: Power Spectra. Solid lines are the linear power and dotted lines are the non-linear power from N-body simulations. Triangles are the galaxy power spectrum from the APM redshift survey. Model parameters are given in Table 1.
4 Conclusions

Although we have argued that the parameters summarized in Table 1 are the “best” choices for these classes of models, each of these models is potentially inconsistent with at least one observation. The CHDM models that we have discussed may have great difficulty explaining high redshift galaxies and other data on structure formation at high redshift. In addition, any $\Omega = 1$ model may be difficult to reconcile with measurements of the baryon fraction in clusters, and will correspond to a worryingly young universe if the Hubble parameter is large. The TCDM model has $h < 0.5$ and is barely consistent with the small-angle CMB data. The OCDM and $\Lambda$CDM models may produce too much power on galaxy scales relative to cluster scales.

Our knowledge of the fundamental cosmological parameters is likely to improve dramatically in the next decade with the launch of the next generation of CMB satellites (MAP and COBRAS/SAMBA). In the meantime, a “phenomenological” approach to cosmology may enable us to make progress towards understanding galaxy formation and evolution as well as large scale structure, all of which may remain difficult problems even after the cosmological parameters are determined. In addition, the efforts to identify and directly detect the dark matter are extremely important in the effort to form a firmer theoretical foundation for Cold Dark Matter dominated cosmology.

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