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Stability of de Sitter solution in mimetic $f(R)$ gravity

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Abstract. The mimetic $f(R)$ gravity is revisited from the stability of de Sitter point of view. We show that how de Sitter evolves stable in the cosmological era. Our investigation can be used as a potentially interesting idea for early inflation as well as late time acceleration expansion.

1. Introduction
Recently, observational dates such as supernovae Ia (SN Ia)[1], large-scale structure (LSS)[2], baryon acoustic oscillations (BAO) [3] and cosmic microwave background (CMB) [4] indicated accelerating expansion of our universe. Observational data shows that the universe has undergone two phases of cosmic acceleration. One of phase which is believed to have occurred prior to the radiation domination is called inflation. This phase can explain a flat spectrum of temperature anisotropies which is observed in CMB. The another phase of accelerating is unknown phenomena dark energy which has started after the matter domination.

The generalization of general relativity is considered as alternative for a unified description of the inflation and dark energy. The properties and structure of modified gravities such as $f(R)$ gravity, $f(T)$ gravity, scalar-tensor theory, Gauss - Bonnet gravity, non-local gravity, string inspired, non-minimally coupled models, dilaton gravity are considered. The cosmological reconstruction of these modified theories is investigated in great detail.

One of simplest approach in scalar field models of dark energy and inflation correspond to a modification of the energy-momentum tensor. Second approach to explain the acceleration of the universe is modification of gravitational part. Generally, to modify gravitational part, adds higher-order corrections to the Einstein-Hilbert action. One of simplest model that is $f(R)$ gravity. Its perform by replace Ricci scalar $R$ by an arbitrary function $f(R)$ in the action [5]. The modified $f(R)$ gravity studied also in Ref. [6, 7, 8, 9, 10, 11, 12, 13].

Recently has been proposed a new type modified gravities as titled mimetic model [14]. Modification of mimetic model was investigated in [15, 16, 17]. The idea of model proposed scalar theory of gravitation, conformally-invariant in which ghosts does not cause any problem with scalar degree of freedom. The approach is to parametrize metric of an auxiliary metric as a conformal transformation. Unification $f(R)$ gravity with mimetic gravity as mimetic $f(R)$ gravity proposed in [17]. This new modified mimetic theory interesting, because of its has more physical solutions and complexity. The purpose of our work is study stability of de Sitter solution in mimetic $f(R)$ gravity.
The structure of paper is organized as following. In Sect. II we briefly review modified theory of gravity such as $f(R)$ gravity. In Sect. III we analyzed equations of motion for mimetic $f(R)$ gravity in FRW spacetime. Also investigated de-Sitter solution and stability.

2. Preliminaries of $f(R)$ gravity
We start with the action in $f(R)$ gravity
\[ S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + \mathcal{L}_m, \]
where $\kappa^2 = 8\pi G$, $g$ is the determinant of the metric $g_{\mu\nu}$ and $\mathcal{L}_m$ is a matter Lagrangian that depends on $g_{\mu\nu}$ and matter fields $\phi_m$. The $f(R)$ is an arbitrary function $f$ of Ricci scalar $R$.

The Ricci scalar $R$ is defined by
\[ R = g^\alpha_\mu g^\lambda_\nu \Gamma^\rho_\mu_\lambda - \Gamma^\rho_\mu_\lambda \Gamma^\rho_\nu_\gamma . \]

In the metric formalism, the connections $\Gamma^\alpha_\beta_\gamma$ are the usual metric connections defined by metric tensor $g_{\mu\nu}$ as
\[ \Gamma^\alpha_\beta_\gamma = \frac{1}{2} g^{\alpha\lambda} \left( \frac{\partial g_{\gamma\lambda}}{\partial x^\beta} + \frac{\partial g_{\lambda\beta}}{\partial x^\gamma} - \frac{\partial g_{\beta\gamma}}{\partial x^\lambda} \right) . \]

This can be obtained from the metricity relation
\[ \nabla_\lambda g_{\mu\nu} = \partial g_{\mu\nu}/\partial x^\lambda - g_{\rho\delta} \Gamma^\delta_\mu_\lambda - g_{\mu\rho} \Gamma^\rho_\nu_\lambda = 0. \]

The field equation can be obtained by varying the action (1) with respect to $g_{\mu\nu}$:
\[ f(R)R_{\mu\nu}(g) - \frac{1}{2} f(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu f(R) + g_{\mu\nu} f(R) = \kappa^2 T_{\mu\nu} \]
where $f(R) \equiv \partial f/\partial R$.

Energy-momentum tensor $T_{\mu\nu}^{(m)}$ of the matter fields is the defined by
\[ T_{\mu\nu}^{(m)} = - \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_M}{\delta g^{\mu\nu}} . \]

Continuity equation satisfied
\[ \nabla^\mu T_{\mu\nu}^{(m)} = 0 , \]

From equation (6) the trace gives
\[ 3 \Box f(R) + f(R)R - 2f(R) = \kappa^2 T , \]
where
\[ T = g^{\mu\nu} T_{\mu\nu}^{(m)} \]
and
\[ \Box f = \left( \frac{1}{\sqrt{-g}} \right) \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu f \right) \]

We now assume the Friedmann-Lemetre-Robertson-Walker (FLRW) space-time
\[ ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} dx_i^2 , \]
where $t$ is cosmic time.

For FLRW metric the Ricci scalar $R$ is given by

$$R = 6 \frac{\dot{a}}{a} + 6 \frac{\dot{a}^2}{a^2}$$

(13)

Then, we can rewritten as

$$\frac{\dot{a}}{a} = \frac{1}{6}(R - 6(\frac{\dot{a}}{a})^2)$$

(14)

where $H \equiv \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter.

The energy-momentum tensor of matter is given by

$$T^{\mu(m)}_{\nu} = \text{diag}(-\rho_m, p_m, p_m, p_m)$$

(15)

where $\rho_m$ is the energy density and $P_m$ is the pressure. The field equations (6) in the flat FLRW background give

$$3fH^2 = \frac{fR - f}{2} - 3H \dot{f} + \kappa^2 \rho_m,$$

(16)

$$-2FH = \dot{f} - H \ddot{f} + \kappa^2 (\rho_m + p_m),$$

(17)

where the perfect fluid satisfies the conservation equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. $$

(18)

3. Mimetic $f(R)$ gravity

In mimetic theory, we will parametrize the metric tensor as the following

$$g_{\mu\nu} = -\hat{g}^{\rho\sigma} \partial_{\rho} \phi \partial_{\sigma} \phi \hat{g}_{\mu\nu},$$

(19)

In this case we consider the variation with respect to $\hat{g}$ and $\phi$. According (19), we can rewrite action (1) following form as

$$S = \int d^4x \sqrt{-g(\hat{g}_{\mu\nu}, \phi)} \left[ \frac{f(R(\hat{g}_{\mu\nu}, \phi))}{2\kappa^2} + \mathcal{L}_m \right].$$

(20)

So, if we perform variation with respect to metric tensor $\hat{g}$, we obtain

$$\frac{1}{2} g_{\mu\nu} f(R(\hat{g}_{\mu\nu}, \phi)) - R(\hat{g}_{\mu\nu}, \phi) f_R(R(\hat{g}_{\mu\nu}, \phi))$$

$$+ \nabla \left( g(\hat{g}_{\mu\nu}, \phi) \right)_{\mu\nu} \nabla \left( g(\hat{g}_{\mu\nu}, \phi) \right)_{\mu\nu} f_R(R(\hat{g}_{\mu\nu}, \phi))$$

$$- g(\hat{g}_{\mu\nu}, \phi) \Box (\hat{g}_{\mu\nu}, \phi) f_R(R(\hat{g}_{\mu\nu}, \phi)) + \kappa^2 T_{\mu\nu}$$

$$+ \partial_{\mu} \phi \partial_{\nu} \phi \left[ 2f(R(\hat{g}_{\mu\nu}, \phi)) - R(\hat{g}_{\mu\nu}, \phi) f_R(R(\hat{g}_{\mu\nu}, \phi)) \right]$$

$$- 3 \Box \left( g(\hat{g}_{\mu\nu}, \phi) \right) f_R(R(\hat{g}_{\mu\nu}, \phi)) + \kappa^2 T = 0,$$

(21)

Also, if we take variation respect to the scalar field $\phi$, the field equation written as

$$\nabla \left( g(\hat{g}_{\mu\nu}, \phi) \right) \mu \left\{ \partial_{\mu} \phi \left[ 2f(R(\hat{g}_{\mu\nu}, \phi)) - R(\hat{g}_{\mu\nu}, \phi) f_R(R(\hat{g}_{\mu\nu}, \phi)) \right]$$

$$- 3 \Box \left( g(\hat{g}_{\mu\nu}, \phi) \right) f_R(R(\hat{g}_{\mu\nu}, \phi)) + \kappa^2 T \right\} = 0,$$

(22)
Here, in our case we define
\[ T = g (\tilde{g}_{\mu\nu}, \phi)^{\mu\nu} T_{\mu\nu} \]  
(23)
Then, from (19) we have
\[ g (\tilde{g}_{\mu\nu}, \phi)^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = -1 \]  
(24)
Now, we will slightly modify action (20) for mimetic \( f(R) \) gravity by including a Lagrange multiplier \( \lambda \) and potential \( V(\phi) \) as the following
\[ S = \int d^4x \sqrt{-g} (\tilde{g}_{\mu\nu}, \phi) \left[ \frac{f(R(\tilde{g}_{\mu\nu}, \phi))}{2\kappa^2} - \lambda (\partial_\mu \phi \partial^\mu \phi + 1) - V(\phi) + \mathcal{L}_m \right]. \]  
(25)
From action (25) find point-like Lagrangian \( \mathcal{L}(a, R, \dot{R}, \dot{\phi}) \) as
\[ \mathcal{L} = a^3 f - a^3 R f_R - 6 \left( a\dot{a} f_a + a^2 \dot{a} f_{RR} \right) - p a^3 - \lambda ( 1 - \dot{\phi}^2 ) a^3 - V a^3 \]  
(26)
Note that \( p \) depends on \( a, \dot{a} \) and \( \phi \). The Euler-Lagrange equations are
\[ \frac{\partial \mathcal{L}}{\partial a} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{a}} = 0 \]  
(27)
\[ \frac{\partial \mathcal{L}}{\partial \phi} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = 0 \]  
(28)
By using Euler-Lagrange equations in (27) and (28) we obtain equation of motion for \( a \) and \( \phi \)
\[ 6 f_{RR} \ddot{R} + 6 \dot{R}^2 f_{RRR} + a \ddot{a} p_a a + a p_{aat} + a p_t \dot{a} + 3 a \ddot{a} - a p_a = 0 \]  
(29)
\[ \dot{\phi} + \dot{\phi} \left( \frac{\dot{\phi}}{\lambda} + 3H \right) + \frac{V_\phi}{2\lambda} = 0. \]  
(30)
If we set \( p = p(a) \), \( \lambda = 1 \), \( \phi(t) = t \), \( V(\phi) = 0 \), we becomes simple equation of motion for \( f(R) \) gravity.

Then, we consider de Sitter solution, where \( H = H_0 \) we find
\[ R = R_0 = 12 H_0^2 \]  
(31)
\[ \dot{R} = \ddot{R} = 0 \]  
(32)
Now we perform small perturbations for \( H \) and \( R \) as
\[ H = H_0 + \delta H_0, \quad R = R_0 + \delta R_0, \quad \delta \ll 1, \]  
(33)
By substituting this perturbation into (29) and taking only linear terms of \( \delta(H, R) \), we obtain
\[ \frac{6 f_{RR} R_0 \ddot{a}}{a^2} + \left( \frac{12 H_0^2 R_0}{a} + 12 H_0 f_R - 18 H_0 f_R \right) \delta + (36 f_R + 18 f_R + 3 p_a) H_0^2 - \]  
(34)
\[ - p_a H_0 - \frac{3 f_R \dot{a}}{a^2} + \frac{3 \lambda f_R \ddot{a}}{a^2} - \frac{3 \lambda f_R \dot{a}}{a^2} + 3 V f_R \dot{a} + \frac{3 p f_R \dot{a}}{a^2} = 0 \]
We take form of $\delta(H, R) = \exp(\eta t)$, where $\eta$ is a constant. In our case, if $\eta > 0$, then amplitude of value $\eta$ grows as the cosmic time passes, the de Sitter solution is unstable. So that the universe can exit from the inflationary stage. By plugging $\delta = \exp(\eta t)$, we have

$$\frac{6f_{RR}\dot{\alpha}}{a^2} \eta^2 + \left(\frac{12H_0}{a} + 12f_R - 18f_R\right) \eta +$$

$$\left(36f_R + 18f_R\right) H_0 - p_a -$$

$$- \frac{3f_{RR}\dot{\alpha}}{a^2} + \frac{3\lambda f_{R}\dot{\alpha}}{a^2} - \frac{3\lambda \dot{\phi}^2 f_{R}\dot{\alpha}}{a^2} + \frac{3V f_{R}\dot{\alpha}}{a^2} + \frac{3p f_{R}\dot{\alpha}}{a^2} = 0\quad (35)$$

Then, solve it approximately, we obtain

$$\eta_\pm = \left(\frac{-12H_0}{a} - 12f_R + 18f_R\right) \pm \sqrt{\mathcal{D}}\quad (36)$$

$$\mathcal{D} = \left(\frac{12H_0}{a} + 12f_R - 18f_R\right)^2 - 4\mathcal{M}\quad (37)$$

$$\mathcal{M} = \frac{6f_{RR}\dot{\alpha}}{a^2} \times \left(36f_R + 18f_R\right) H_0 - p_a -$$

$$- \frac{3f_{RR}\dot{\alpha}}{a^2} + \frac{3\lambda f_{R}\dot{\alpha}}{a^2} - \frac{3\lambda \dot{\phi}^2 f_{R}\dot{\alpha}}{a^2} + \frac{3V f_{R}\dot{\alpha}}{a^2} + \frac{3p f_{R}\dot{\alpha}}{a^2}\quad (38)$$

The de Sitter solutions are stable only if the value of $\eta_-$ is a real and positive number.

4. Conclusion

In this paper we investigated stability of de Sitter solution in mimetic $f(R)$ gravity. Also, analyzed our results for explanation early and late time acceleration expansion.

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5