Transient Hydrodynamic Analysis of Ball Mill Multi-Pocket Pivoted Pad Hydro-static Bearing

Miaomiao Li\(^1\)\(^*\) and Xizhi Ma\(^1\)

\(^1\)National Key Laboratory of Science and Technology on Helicopter Transmission, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
Email: limiaomiao@nuaa.edu.cn

Abstract. huge ball mill bearings experience a transient period when they are fed with raw material or their basis is impacted. The behavior of such bearings during the transient period is significant to the safety of the machine. In the present paper, the step load is used to simulate this process. Transient hydrodynamic lubricating models of a multi-pocket pivoted pad hydro-static bearing used in ball mill are established. The hydrodynamic and squeeze effects are taken into account, and the models are solved using nonlinear numerical methods. The journal center trajectory, time response of the pad tilting, oil pressure in the pockets, and minimum pad film thickness during the transient period are presented. The oil flow rate evolutions out of the main pockets in all pads are exhibited. The effects of step load on bearing performance are discussed. It was found that the decrease in minimum film thickness can be quite large under the step load impact, which may lead to the damage of the bearing.

Keywords: hydro-static bearing; Ball mill; Lubrication; Multi-pocket pivoted pad; Transient response

1. Introduction

Ball mills are a type of typical non-metallic mining grinding devices that have become the main tool for ore crushing. In modern huge ball mills, the dimensions become larger and larger, with their girth ring extending to several meters in diameter and a meter in width. A single bearing can be imposed exert a total load of thousands of tons, however it can only spin at 10 rpm or slightly higher. Bearings guarantee the correct positioning and stable rotation of trunnions, therefore, the lubrication performance of the bearing is of great significance for the normal operation of a ball mill. Thus, large ball mills adopt widely oil hydro-static bearings, which are characterized by high load capacity and low speed.

Hydro-static bearings were first constructed and applied in 1865. Nowadays, due to the characteristics including load-carrying capacity, large fluid film stiffness and low friction, hey can be found everywhere [1-2].

Hydro-static bearings have been widely used in the different industry. In addition, they have been used in analytical magnets at the nuclear facilities of the SERC Daresbury Laboratory, while other applications include the Halle optical telescope. Over the past four decades, a large number of researchers have conducted extensive research on hydro-static bearing systems, and their research efforts have involved various aspects, including different recess shapes [3-5], pad deformation (Nagaraju et al. [6]), and restrictor types (Kumar et al. [7]; Sinhasan et al. [8]).

Compared with other bearings, multi-recess hydro-static/hybrid journal bearings have excellent performance characteristics such as minimum friction, greater fluid film stiffness, better damping characteristics, and relatively stable motion, therefore, the multi-recess hydro-static pads have a good...
effect on improving the automatic adjustment of the tilted hydro-static bearing [9-12].

Pivot pads can be used to compensate for different errors. Some researchers investigated and analyzed the effect of tilt about the performance of hydro-static bearings [13-15].

Pivoted multi-recess pad hydro-static bearings have been used as ball mill bearings, since their performance characteristics are superior to other bearing types.

Most previous studies have focused on the steady state of fluid dynamic bearings, and only very few studies are available on the transient period of multi-recess hydro-static journal bearing systems.

In ball milling, the performance of the ball mill is inevitably affected by many factors, such as impact load, vibration, and alignment faults [16-17]. Therefore, a steady state condition does not actually exist. When these operating conditions change suddenly, it takes time for the main parameters to stabilize. Consequently, it is of great significance to have a clear understanding on the time required for stabilization and on how do the parameters change during the transient period. Changes in film thickness can be used as a sign to determine faults or failures. Since changes in operating conditions, such as speed or load, are potential triggers for equipment damage, it is of vital importance to know whether the observed changes are within acceptable limits or indicate a problem. Most studies on transient properties concern hydrodynamic bearings [18-21].

To the best of our knowledge, until today, the only study concerning the transient properties of hydro-static bearings has been conducted by Pang et al. [22]. In that study, they theoretically and experimentally considered the compressibility of oil, including bubbles, and then investigated the transient characteristics of hydro-static bearings under a step load. The duration of the transition phase was obtained by establishing a characteristic equation of the bearing in the transitional phase. Then, the effect of the main parameters of the bearing system on transient characteristics was analyzed.

In this paper, we presented a new fundamental basis for enhancing the safety of large hydro-static bearings used in ball milling, and reported the rapid changes in key operating parameters of hydro-static pivoted-pad journal bearings. Non-dimensional equations that define the transient problems were established, while the evolution of film thickness, pressure profile, and bearing friction torque following sudden changes in load were analyzed. The transient properties of five-pocket flexible pivoted pad journal bearings, used in huge ball mills, were numerically investigated in order to provide decision basis for the correct selection of parameters in the design process of bearings under load.

2. Model Establishment

Figure 1 and figure 2 illustrate the schematic of the hydro-static multi-pocket pivoted oil bearing and its arrangement. Each bearing pad is placed on a spherical pivot, which is supported by a hydraulic cylinder, and can tilt relatively to the pivot center. The pivot position can be adjusted along the radial direction of the bearing, and maintain the load equal on every pad. The oil is supplied to the pad using a constant speed pump and a flow divider is used to distribute it evenly to each pad. Five pockets are distributed on each pad surface as shown in figure 2. The rectangular pocket in the center is the main pocket, and the four triangular pockets at the four corners of the rectangle are balanced pockets. The each balance pocket are connected together with ducts. The function of the balance pockets is to adjust the attitude of the pad relative to the girth ring. Based on the bearing principle, the following equations can be established:
2.1. Transient Reynolds Equation
Based on the conventional assumptions of lubrication theory, the distribution of oil film pressure can be expressed by the Reynolds equation [23]. Its form is given as follows:

\[
\frac{\partial}{\partial \varphi} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial \varphi} \right) + \lambda^2 \frac{\partial}{\partial y} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial y} \right) = 3\omega \frac{\partial h}{\partial \varphi} + 6 \frac{\partial h}{\partial t}
\]

(1)

where
\[
\lambda = D/B
\]

2.2. Boundary Conditions
Since the depth of the chamber far exceeds the film thickness at the edge of the oil seal, the pressure distribution in the chambers is assumed to be uniform, and the pressure in the main pocket is the same to the supply pressure. The pressure at the external boundary is equal to the atmospheric pressure p0.

\[
p \bigg|_{r=r_1} = p_s \\
p \bigg|_{r=r_2} = p_a
\]

where \(\Gamma_1\) is the boundary of the main pocket and \(\Gamma_2\) is the boundary of the pad.
2.3. Film Thickness
The oil film thickness of the pad \( i \) can be calculated according to the following equation:

\[
h_i = c - (c - c') \cos(\beta_i - \varphi_i) + e \cos(\varphi_i - \theta) - R y_i \sin(\beta_i - \varphi_i) + H
\]  

(2)

2.4. Bearing Motion Equations
It is assumed that there are two degrees of freedom in the motion of the trunnion center in the \( x \) and \( y \) directions. Only one degree of freedom is assumed in the \( \phi \) direction of the angular rotation about the pivot of each pad, under the external load and the film force. The equations are given as follows:

\[
\begin{align*}
    m_r \ddot{x}_i &= F_{xi} - W_x \\
    m_r \ddot{y}_i &= F_{yi} - W_y \\
    M_i - F_i \dot{x}_i &= I_i \ddot{y}_i
\end{align*}
\]

where \( i = 1, 2, \ldots, n_p \) is the pad number.

The film force components of the bearing are the sum of the pad film forces in the \( x \) and \( y \) directions. Therefore, the equations can be given as follows:

\[
\begin{align*}
    m_r \ddot{x}_i &= F_{xi} - W_x \\
    m_r \ddot{y}_i &= F_{yi} - W_y \\
    M_i - F_i \dot{x}_i &= I_i \ddot{y}_i
\end{align*}
\]

where \( F_{xi} \) and \( F_{yi} \) are the film force components of the \( i^{th} \) pad in the \( x \) and \( y \) direction, respectively.

\[
\begin{align*}
    F_i &= \int_0^1 \int_{\beta_i-\alpha}^{\beta_i+\alpha} p d\varphi dy \\
    F_{xi} &= \int_0^1 \int_{\beta_i-\alpha}^{\beta_i+\alpha} p \cos \varphi d\varphi dy \\
    F_{yi} &= \int_0^1 \int_{\beta_i-\alpha}^{\beta_i+\alpha} p \sin \varphi d\varphi dy \\
    M_i &= \int_0^1 \int_{\beta_i-\alpha}^{\beta_i+\alpha} p \sin(\varphi - \beta_i) d\varphi dy
\end{align*}
\]

(5)

2.5. Oil Flow in the Pad
For a single pad, a balance between the oil flowing out and the supplied oil through the oil hole is required. If the total oil supplied by the pump is equal to the sum of the oil flowing out of all pads, the equation can be given as follows:

\[
Q_s = 4Q_{si} \\
Q_{si} = Q_{mi} + \sum_{j=1}^{4} Q_{bij}
\]

(6)

(7)

where \( Q_{si} \) is the oil flow rate from the pump to pad \( i \), \( Q_{mi} \) is the oil flow rate into the main pocket of pad \( i \), and \( Q_{bij} \) is the flow rate of the oil flowing out of the balance pocket \( j \) in pad \( i \). The equations (Eqs. (8) and (9)) can be used to calculate the oil flowing out of the pocket \([8,9]\). The oil flow rate at every point along the film thickness is given as Eq. (8):

\[
q\varphi = -\frac{h^3}{12\mu} \frac{\partial p}{\partial \varphi} + \frac{\omega h}{2}
\]

\[
q_y = -\frac{h^3}{12\mu} \frac{\partial p}{\partial y}
\]

(8)

The following function defines the amount of oil flowing from the closed boundary of the pocket:

\[
Q = \oint (q\varphi + q_y) d\Gamma
\]

(9)
2.6. Restrictor Flow Equation

Since the main and each balance pocket are interconnected with ducts with a capillary current limiter, the capillary of each balance pocket limits the oil flow. Assuming that the flow in the capillary is laminar, the pressure difference between the main pocket side and the equilibrium pocket side can be obtained by the following formula:

\[ p_{bi} = p_s = \frac{128\mu d_e Q_{bij}}{\pi d_e^4} \]  \hspace{1cm} (10)

Where

\[ Re = \frac{ud_e}{v} < 2000, \quad \frac{l_e}{d_e} > 20, \]
\[ l_e > 0.65d_e Re, \quad v = \frac{\mu}{\rho} \]

3. Results and Discussion

3.1. Numerical Procedure

The main solution scheme adopted during the research process is as follows: firstly, the steady journal center position and pad attitudes are calculated, and then, the transient behavior is obtained by numerically integrating the motion (Eq. (3)) using, in the following time step, the Runge-Kutta methods. The finite difference method is employed to solve the constitutive equations. For the transient Reynolds equation, the Gauss-Seidel iterative scheme with excessive relaxation is used.

To calculate the steady position of the journal and pads, initial values of the film pressure, pad tilted angle, eccentricity, and pocket pressure need to be provided. The steady position of each pad is determined by solving the momentum equations.

Approximately 101 nodes are distributed in both the circumferential and width direction of the pad. The oil flow is calculated by numerical integral methods. The flow, load, and pad momentum equations (Eqs. (4), (5), and (7), respectively) can be solved by adopting the Newton method.

The oil flow through every pocket is obtained by numerical integration along the pocket edges. The general process is as follows:

(a) Assign the initial pressure at the pockets. Solve the Reynolds equation (Eq. (1)) to obtain the oil flow of each pocket. Then, use Eqs. (8)–(10) to calculate the pressure of each balance pocket, until convergence for the pad pressure is achieved.

(b) Adjust the supply oil pressure, and then repeat step (a) until convergence for the pad oil flow is achieved. If the oil flow of pads 1 and 4 do not meet the load equation condition, adjust the pivot position along the radius direction.

(c) Adjust the title angle of the pad, until convergence for the pad attitude is achieved.

(d) Calculate the value of the film force in the vertical direction. If the value of the film force and the value of the bearing load are not equal, adjust the eccentricity of the journal until convergence for the bearing load is achieved.

(e) Integrate the motion using Eq. (3). The steady results are used as initial values, and then step by step until the last steady state is reached under step load.

3.2. Calculation Results

3.2.1. Steady State Results. Table 1 shows the parameters of the bearing. Tables 2 and 3 show the characteristics of lubricants and solids, respectively.
Table 1. Bearing parameters.

| Bearing parameters | Notation | Value | Unit |
|--------------------|----------|-------|------|
| Pad number         | \( n_p \) | 4     |      |
| Girth ring         | D        | 4000  | mm   |
| Bearing width      | B        | 900   | mm   |
| Radius clearance   | c        | 0.25  | mm   |
| Capillary diameter | \( d_c \) | 2.4   | mm   |
| Capillary length   | \( l_c \) | 80    | mm   |
| Pivot ratio        | \( \theta_p / \theta \) | 0.5   |      |
| Assemble radius    | \( c' \) | 0.25  | mm   |
| Load of bearing    | \( W \) | \( 1.415 \times 10^7 \) | N     |
| Rotational speed of bearing | n | 11.5 | r/min |
| Pad oil flow       | Q        | 280   | l/min|

Table 2. Lubricant characteristics.

| Bearing parameters | Notation | Value | Unit |
|--------------------|----------|-------|------|
| Feeding and bath temperature | \( T_{in} \) | 3.5 | ℃ |
| Dynamic viscosity at 30℃ | \( \mu_0 \) | 0.195 | Pa.s |
| Oil density at 30℃ | \( \rho_0 \) | 886.0 | Kg.m\(^{-3}\) |

Table 3. Ambient and material characteristics.

| Bearing parameters | Notation | Value | Unit |
|--------------------|----------|-------|------|
| Ambient temperature | \( T_0 \) | 30 | ℃ |
| Ambient pressure   | \( P_a \) | 0.1 | MPa |
| Density of the solid at 30℃ | \( \rho_s \) | 7800 | kg/m\(^3\) |
| Young’s modulus    | \( E \) | \( 2.1 \times 105 \) | MPa |
| Poisson’s ratio    | \( v \) | 0.30 |      |

The pressure profile and contour of the oil are shown in figure 3. Figure 3 (a) shows the oil pressure profile on one pad, where, due to that the main pocket and each balance pocket were connected together with ducts with a capillary current limiter, the pressure value of the main pocket was the highest and the pressure of the balance pocket was the lowest. In figure 3 (b), the pressure distribution can be clearly observed. At the pad edges, an oil seal edge around the pad was observed, and the oil pressure gradient was large. The exact oil pressures of the pockets in each pad are listed in table 4 together with other parameters.
Figure 3. Pressure profiles on the pad surface.

Table 4. Bearing calculation results.

| Pad number | ps (MPa) | pb1 (MPa) | pb4 (MPa) | hmini (mm) | \( \gamma_i \) (\( \times 10^{-5}\)rad) | em (mm) | Eccentricity |
|------------|----------|-----------|-----------|------------|---------------------------------|---------|-------------|
| 1          | 9.28     | 7.16      | 7.14      | 0.144      | -2.27                           | 0.0     | 0.368       |
| 2          | 9.28     | 7.04      | 7.02      | 0.145      | -0.94                           | -0.111  | -0.111      |
| 3          | 9.28     | 7.04      | 7.02      | 0.146      | 0.66                            | -0.111  | -0.111      |
| 4          | 9.28     | 7.16      | 7.14      | 0.146      | 1.91                            | 0.0     | 0.0         |

3.3. Transient Behavior of Journal and Pads

After the steady performance was calculated in the last section, the performance of the hydro-static pivoted bearing during the transient period following a sudden change with applied load in the y direction was investigated as follows:

The evolution of trunnion center position with time is presented in figure 4. Figure 4(a) displays the center position in the x direction and figure 4(b) in the y direction under different step load. The displacement in the x direction was no more than 0.8 \( \mu \)m and about 30-50 \( \mu \)m in the y direction for different step load. The transient period was quite short, about 0.03 s, in the y direction and 1 s in the x direction, since the applied external load was in the y direction. In the x direction, the reaction film force was almost zero due to the symmetric arrangement of the pads. Thus, the acceleration was very small.
Figure 4. Journal center position versus time.

Figure 5 demonstrates the time responses of the pad tilting angle. Figure 5 (a) shows the tilting angle evolution of all four pads under the 0.25 MPa step load, where it can be observed that the change processes of all pads were similar. Figure 5 (b) shows the tilting angle evolution of pad 1 under different step load. It can be seen that the pad tilting angle was negative, and its value decreased with time. Its steady value tended to -5.6e-5 ~ -5.65e-5 under the different given step loads, and the period elapsed from the beginning to the steady state was shorter than 0.2 s.

(a) Pad angle of all four pads.
Figure 5. Pad transient motion versus time.

The pressure evolution of the main pocket is illustrated in figure 6 (a). It can be seen that, at the beginning, the pressure decreased quickly, and then it began to increase until it reached a steady state. This transient period was very short. The oil pressure evolution in balance pocket 1 and 4 are shown in figures 6 (b) and (c), respectively. Their variation trends were similar to that of the main pocket, while there was some overshoot for the pressure evolution of balance pocket 4. The pressure variation in the main pocket under different step loads is presented in figure 6 (d). The steady pressures of the main pocket was about 0.4-0.8MPa for different step load.
Figure 6. Pressure evolution of the elastic model.

In the beginning, the pocket pressure decreased sharply, and then, at the next period, it increased. The duration of pressure decrease was very short, thus it is difficult to be clearly observed in the demonstrated curves. This can be explained as follows:

At the beginning, after a step load has been applied, the journal begins to move down and the oil film gets squeezed, which leads to an increase in oil pressure at the sealing edge. Therefore, for the corresponding load, due to the increase in oil pressure at the sealing edge, the pressure in the pocket decreases at the beginning.

The oil flow rate evolutions out of the main pockets in all pads are exhibited in figure 7. More specifically, the oil flow rate evolution of the main pockets after a step load has been applied is shown in figure 7 (a), where the oil flow rate out of the main pockets increased rapidly and then decreased until a steady state was reached. Figure 7 (b) shows the oil flow rate of balance pocket 1 in all four pads, where the oil flow rate decreased quickly and then increased slightly. At last, the oil flow rate decreased again until it reached a steady state.

The evolution of oil flow rate of balance pocket 4 is presented in figure 7 (c). It can be seen that the oil flow rate decreased sharply at the beginning, and then, it increased until its steady state was reached. The differences in oil flow rate evolution between different pockets are apparent, with the main reason being the swing of the pads. Figure 7 (d) demonstrates the oil flow rate variation with time of the main pocket under different step loads. It can be seen that the larger the step load, the less the increment of the oil flow rate out of the main pocket. On the contrary, the oil flow rate of the balance pocket decreased with the step load. Thus, it can be deduced that the larger the step load, the larger the increment of the oil flow rate.
During the transient period, after an impacted load was applied, the journal and pads moved with time. The minimum film thickness in every pad is shown in figure 8. The evolution of minimum film thickness on each pad under the given step load are shown in figure 8 (a), where the variation of minimum film thickness was about 0.016 mm under a step load of 0.25 MPa. This was about 11% the steady value. The evolution of film thickness of pad 2 under different step loads is shown in figure 8 (b). According to the obtained curves, the larger the step load, the larger the variation of minimum film thickness.

(a) Minimum pad film thickness.  
(b) Minimum bearing film thickness.

Figure 8. Minimum film thickness versus time.
4 Conclusions
In this paper, a numerical scheme was described, which was used to analyze the transient lubricating process of a ball mill bearing. The employed bearing was a multi-pocket pivoted hydro-static journal bearing.

A nonlinear analysis on the bearing properties under the step load has been performed. The trajectory of the journal center, the transient behavior of all pads, and the minimum film thickness variation with time were obtained. The above results can be summarized as follows:

The minimum film thickness of the bearing decreased from 0.146 mm to 0.130 mm under a step load 0.25 MPa, which was decreased by 11% compared to the steady minimum film thickness. The response time of the film thickness was longer than that of film pressure, and the period was about 0.2s. The pressure of the main pocket increased from 10.1 MPa to 10.8 MPa, and the pressure in the balance pocket increases as well.

During the transient period, the oil flow rate of the main bag was positively correlated with time, and there was an overshoot. The larger the step load, the smaller the increment of oil flow rate out of the main pocket. On the contrary, the oil flow rate of the balance pocket decreased with step load. Thus, the larger the step load, the larger the increment of oil flow rate.

At last, the step load may lead to film thickness reduction and then induce bearing damage. Consequently, during the operation of ball mills, attention should be paid to the change of working conditions.

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