Time varying $\alpha$ in $N = 8$ extended Supergravity

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Abstract

There has been some evidence that the fine structure “constant” $\alpha$ may vary with time. We point out that this variation can be described by a scalar field in some supergravity theory in our toy model, for instance, the $N = 8$ extended supergravity in four dimensions which can be accommodated in M-theory.
There exists a lot of analysis about the observational constraints on possible variation of the fine structure constant $\alpha$ \cite{1} in time. A number of absorption systems in the spectra of distant quasars suggests a smaller value of $\alpha$ in the past, with a favored value of the change $\frac{\delta \alpha}{\alpha} \approx (-0.72 \pm 0.18) \times 10^{-5}$ over the red-shift range $0.5 \leq z \leq 3.5$ \cite{2}. And the analysis of the isotope abundances in the Oklo natural reactor operated 1.8 Gyr ago gives a new constraint, $|\frac{\delta \alpha}{\alpha}| \sim 10^{-7}$ \cite{3}. Some theoretical issues of a varying $\alpha$ are discussed in \cite{4, 5, 6}.

There are two scenarios to explain the variation of $\alpha$. One possibility is that there was a first order phase transition between the time at which the quasar light was emitted and the present. But if the Oklo natural reactor confirms the above data which indicates that $\alpha$ at the red-shift $z \sim 0.13$ is different from its present value, it is most possible that $\alpha$ varied with time continuously. On the other hand, in string or M-theory, the couplings in the effective field theory depend on the expectation values of some dynamical scalar fields such as the dilation and other string moduli, then the coupling “constants” in general vary with time, if they are not trapped in the minimum of a potential in an early time. Naturally the fine structure constant varies continuously in this scenario.

In a four dimensional effective field theory, the change of the fine structure constant is controlled by a dynamical scalar field $\phi$, which may be a combination of several canonically normalized moduli scalars \cite{4, 5, 6}. The change in $\alpha$ during the last Hubble time requires that the scalar field $\phi$ should be extraordinarily light, with a mass comparable to the present Hubble scale $H_0 \sim 10^{-33}$ eV \cite{4}. It is usually difficult to find a candidate for this scalar field in a fundamental theory.

However it was found that one can describe the present state of quasi-exponential expansion of the universe in a broad class of models based on four dimensional $N = 8$ extended supergravity \cite{7, 8, 9}. And it is important that there are scalars whose masses in $N \geq 2$ supergravity are quantized in units of the Hubble constant $H_0$ corresponding to DS solutions: $m^2 = -n H_0^2$, where $n$ are some positive integers of the order 1. The minus sign says that the scalar fields are tachyonic, consistent with the fact that $\alpha$ becomes larger and larger. In particular, there is a scalar whose mass square of the scalar field $m^2 = -6H^2$ in $N = 8$ supergravity \cite{7}. And we know N=8 supergravity with de Sitter maximim and one scalar field has only non-Abelian gauge fields $SO(3) \times SO(5)$ or $SO(4) \times SO(4)$. But in our real world the supersymmetry must be broken and then the Abelian gauge field will appear in our toy model.

In an effective field theory or M-theory, the photon kinetic term reads

$$f(\phi)F_{\mu\nu}F^{\mu\nu}$$

where $f(\phi)$ is a function of $\phi$. The most general expansion of the function $\alpha(\phi)$ about its
present day value $\alpha_0 = \alpha(\phi_0)$ is

$$\alpha(\phi) = \alpha_0 + \lambda_\phi \frac{\delta \phi}{M} + ...$$

(2)

where M is a typical scale over which $\phi$ varies. The variation of $\alpha$ with $\phi$ is generally written to the leading order in $\phi$ as

$$\frac{\delta \alpha}{\alpha} \approx \frac{\lambda_\phi \delta \phi}{\alpha_0 M}$$

(3)

where $\delta \alpha = \alpha(z) - \alpha_0$, $\delta \phi = \phi(z) - \phi_0$ and $z$ is cosmological red-shift.

For simplicity, we take $\phi$ to be governed by the Lagrangian $L = (\partial \phi)^2 - m^2 \phi^2 + ...$, namely $\phi$ is a canonical scalar field. As usual, we assume the scalar field $\phi$ be homogeneous and the equation of motion for $\phi$ reads

$$\ddot{\phi} + 3H_0 \dot{\phi} + m^2 \phi^2 + ... = 0$$

(4)

We take $m^2$ to be negative, or simply switch to the notion $m^2 \rightarrow -m^2$ with positive $m^2$.

We consider in general the case when $m^2$ is the same order as $H^2$, which means the scalar field is not slow rolling. Firstly we assume the Hubble constant is a constant and assume $\phi$ has the form as $\phi(t) = \phi_0 e^{i\omega(t-t_0)}$. Equation (4) leads to

$$\omega = iH \left( \frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{m^2}{H^2}} \right)$$

(5)

Choose the solution with the minus sign corresponding to the exponentially growing solution

$$\phi(z) = \phi(t) = \phi_0 e^{i\omega(t-t_0)} = \phi_0 e^{i\omega(t-H_0(t-t_0))} = \phi_0 e^{i\omega(zH_0)}$$

with

$$F\left( \frac{m^2}{H^2} \right) = \sqrt{\frac{9}{4} + \frac{m^2}{H^2} - \frac{3}{2}}.$$  

(7)

Since we are interested only in the range $0 \leq z \leq 1$, we can set $H(t_0 - t) = z$. Substitute this result to the equation (3), we find

$$\frac{\delta \alpha(z)}{\alpha} \approx \Lambda e^{\exp[-zF\left( \frac{m^2}{H^2} \right)] - 1}$$

(8)

here $\Lambda$ is a constant which can be fixed by experiment. As an example, in $N = 8$ supergravity, the mass square of the scalar field is $-6H^2$ in order to get a DS solution, so $m^2 = 6H^2$ in equation (8) and $F\left( \frac{m^2}{H^2} \right) = 1.37$. Using the data of [2] $\frac{\delta \alpha(z=1)}{\alpha} = -0.7 \times 10^{-5}$ fixes $\Lambda = 9.38 \times 10^{-6}$. Thus

$$\frac{\delta \alpha(z)}{\alpha} \approx 9.38 \times 10^{-6} (e^{-1.37z} - 1)$$

(9)
Putting \( z = 0.13 \) into formula (9), we get \( \delta \alpha(z=0.13) \approx -1.5 \times 10^{-6} \sim -10^{-7} \). It is slightly off the constraints of the Oklo experiment. To better satisfy all constraints (assuming they are all reliable), we can increase \( m^2 \) and decrease \( \Lambda \), and still keep \( m^2 \) reasonably close to \( H^2 \).

In fact the Hubble parameter is not a constant during this stage. To be more precise, we can numerically solve the equations which Linde et. suggested in [9]. We also assume that the universe is spatially flat. Thus we have

\[
\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0
\]  

\[
H^2 = \frac{1}{3}(\rho_M + \frac{1}{2}\dot{\phi}^2 + V(\phi))
\]  

\[
\rho_{\text{total}} = \rho_M + \frac{1}{2}\dot{\phi}^2 + V(\phi)
\]

where we consider the extended \( N = 8 \) supergravity which tells us \( V(\phi) = 3H_0^2(2 - \cosh(\sqrt{2}\phi)) \) and we use the method of [9] (in units \( M_p = 1 \) and the present Hubble parameter \( H_0 = 1 \)). Here we assume that the Hubble parameter in de Sitter regime approximately equals the present Hubble parameter \( H_0 \). In our calculations we shall also assume that initially the matter energy density \( \rho_M \) is much greater than the energy density of the scalar field \( \phi \). Thus the scalar field \( \phi \) freezes and its initial velocity can be set to zero resonably, since at that moment the friction term in (10) is very large. According to equation (10) the scalar field \( \phi \) will not change if its initial value is zero. In our case we choose the initial value of \( \phi \) leading to the dark energy \( \Omega_D = 0.73 \) today [10]. We show the numerical solution in Figure 1 (here we take equation (3) into account and describe the variation of \( \alpha \) with time directly).

In Figure 1 we fit the data \( \frac{\delta \alpha(z=1)}{\alpha} \approx -0.7 \times 10^{-5} \). According to this figure we can read out \( \frac{\delta \alpha(z)}{\alpha} \sim -1 \times 10^{-5} \), which is consistent with [2]. And [2] also requires \( \frac{\dot{\alpha}}{\alpha} = (-2.2 \pm 5.1) \times 10^{-16} \text{yr}^{-1} \) over the red-shift range \( 0.6 \leq z \leq 1.0 \). Using \( H_0 = 71 \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \) we predict \( \frac{\dot{\alpha}}{\alpha} = -5 \times 10^{-16} \text{yr}^{-1} \) around \( z = 1 \).

In short, in this paper we suggest that scalar fields in some supergravity theory with a mass comparable to the Hubble scale can be a candidate for describing the variation of the fine structure constant. Linde et al. have discussed some possibilities for describing our universe with a tiny positive cosmological constant in extended \( N = 2, 4, 8 \) supergravity theories, and suggested that some scalar fields in these supergravity theories can be used to describe the present stage of our universe, but not the early universe.

The authors of [6] pointed out that in an effective field theory with a cut-off, such as a theory accommodating SUSY breaking, a varying fine structure constant is always
accompanied by a variation in the vacuum energy or the cosmological constant, which in general is too large. We do not know how the resolve this problem.

Other theoretical attempts on explaining a varying fine structure constant can be found in [11].

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References

[1] T. Damour, F. Dyson, Nucl. Phys.B 480, 37 (1996);
    P. Sisterna, H. Vucetich, Phys. Rev. D41, 1034 (1990);
    D. Prestage et al., Phys. Rev. Lett. 74, 3511 (1995);
    J. Bernstein, L. S. Brown, G. Feinberg, Rev. Mod. Phys. 61, 25 (1989);
    P. P. Avelino et al., Phys. Rev. D62, 123508 (2000);
    R. A. Battye et al., Phys. Rev. D63, 044505 (2001);
    S. Landau, D. H. Harari, M. Zaldarriaga, Phys. Rev. D63, 083505 (2001);
    J. P. Uzan, Rev. Mod. Phys. 75 (2003) 403.

[2] J. K. Webb et al., Phys. Rev. Lett. 87, 091301 (2001), astro-ph/0012539
    J.K.Webb et al.,Phys.Rev.Lett.82,884(1999).

[3] Y. Fujii, A. Iwamoto, T. Fukahori, T. Ohnuki, M. Nakagawa, H. Hikida, Y. Oura, P.
    Moller, Nucl. Phys. B573, 377 (2000),
    Y.Fujii,Astrophys.Space Sci,283(2003)559-564(qr-gc/0212017).
[4] G. Dvali, M. Zaldarriaga, Phys. Rev. Lett. 88, 091303 (2002).

[5] T. Chiba, K. Kohri, Prog. Theor. Phys. 107 (2002) 631-636.

[6] T. Banks, M. Dine, M. Douglas, Phys. Rev. Lett. 88, 131301 (2002).

[7] R. Kallosh, A. Linde, S. Prokushkin, M. Shmakova, Phys. Rev. D65, 105016 (2002).

[8] R. Kallosh, A. Linde, astro-ph/0301087

[9] R. Kallosh, hep-th/0205315
R. Kallosh, A. Linde, S. Prokushkin, M. Shmakova, hep-th/0208156
R. Kallosh, A. Linde, hep-th/0208157
P. Fre, M. Trigiante, A. Van Proeyen, hep-th/0205119
M. Gutperle, R. Kallosh, A. Linde hep-th/0304225

[10] C. L. Bennett et al. astro-ph/0302207

[11] S. Das, G. Kunstatter, hep-th/0212334
J. D. Barrow, D. F. Mota, gr-qc/0212032
J. D. Barrow, gr-qc/0211074
F. P. Correia, M. G. Schmidt, Z. Tavartkiladze, hep-ph/0211122
J. D. Bekenstein, Phys. Rev. D66, 123514 (2002).