Analysis of students’ mathematical reasoning ability in geometry through distance learning

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Abstract. This study investigates the mathematical reasoning abilities of middle school students in geometry during the implementation of distance learning. This research is qualitative with a case study design by analyzing students' mathematical reasoning abilities. The subjects in this study were 64 tenth grade senior high school students. The instrument in this study was a written test consisting of 5 questions. The questions were distributed to students by using google form due to the COVID-19 pandemic. Based on the research, it was obtained that the average mathematical reasoning ability of students was 64.5, which was categorized as a moderate category. Furthermore, the indicators of mathematical reasoning abilities that are mastered by students are (1) mathematical manipulation and (2) draw conclusions and compile evidence, while indicators of reasoning abilities that are less mastered by students are (1) draw conclusions from a statement, (2) assess argument validation, and (3) perform the generalization. These results indicate that the three indicators need to be improved in optimize students’ mathematical reasoning abilities.

1. Introduction

Mathematical reasoning is an essential aspect of learning mathematics. As stated by The National Council of Teachers of Mathematics (NCTM), [1] reasoning in finding observed patterns and procedures used in mathematics will assist students in developing their deeper understanding of mathematics. Furthermore, the importance of reasoning and proof is also expressed in NCTM [1] no matter what topics are studied in mathematics; students will be required to argue and defend their arguments through reasoning and proof.

The reasoning is an ability that must be mastered by students in learning mathematics [1]. However, the current reality, the mathematical ability of Indonesian students is still low, students’ score of reasoning ability can be seen from the TIMSS (Trends in International Mathematics and Science Study) survey results [2]. The average score of high school students’ mathematics ability in Indonesia is lower than the international average score. Because mathematical reasoning ability is one of the topics assessed in the TIMSS survey, it shows that the mathematical reasoning abilities of middle school students in Indonesia are unsatisfactory. In general, students’ mathematical reasoning abilities are still dominated by imitative ability, students can only do something they do routinely, and they have difficulty dealing with the reasoning [3]. Students also have difficulty in using their
mathematical reasoning skills to solve problems, because, in the learning process, they are given procedural learning which results different than developing their thinking and reasoning skills [4].

In addition, students’ mathematical reasoning abilities, together with geometry, are also one of the topics tested in the TIMSS survey [2]. Studying geometry topic requires mastery of good mathematical reasoning abilities because in studying geometry, students are required to investigate, measure, visualize, compare, change, and classify objects [1]. Meanwhile, from several studies that have been carried out, the mathematical reasoning abilities of middle school students in Indonesia are still poor. This fact can have an impact on students who have difficulty learning geometric topics that require reasoning skills. Students still have difficulty to perform their mathematical reasoning skills in studying the topic of polyhedron volume [5]. Another finding shows that mathematical reasoning abilities in the topic of spatial geometry were still relatively low. The indicator of mathematical reasoning abilities that were less mastered by students in this topic was to draw conclusions from a statement [6].

Based on the mentioned description, it can be concluded that the mathematical reasoning ability of high school students on geometry topic in Indonesia through direct learning in class is still relatively low. Moreover, since the beginning of 2020, Indonesia has been applying distance learning in the education system due to the COVID-19 pandemic. Because this distance learning is unprecedented and new in the education system in Indonesia, it requires adaptation for teachers and students. The most significant learning barrier during the implementation of distance learning in Indonesia comes from students [7]. Therefore, this study aims to determine high school students’ mathematical reasoning abilities in geometry, especially spatial geometry, during distance learning.

2. Methods
The method used in this research is qualitative research with a case study design which aims to determine the mathematical reasoning abilities of high school students on geometry topic, particularly spatial geometry during the implementation of distance learning. This research was conducted to the first-grade students of the academic year 2020/2021 at one of the high schools in Jambi, Indonesia. The research was conducted in high schools because, at the time of the research, students who had studied the topic of polyhedron had already graduated from middle school. The subjects in this study were 64 tenth grade senior high school students in the city of Jambi.

This research commenced by preparing a mathematical reasoning test instrument for students with the topic of polyhedron volume, which is adjusted to the mathematical reasoning ability indicator. This test instrument went through several stages of the validity of the research instrument. Then the students’ mathematical reasoning abilities measured in this study were (1) mathematical manipulation, (2) draw conclusions and compile evidence (3) draw from a statement, (4) assess argument validation, and (5) perform the generalization. This reasoning ability can be seen from the scores obtained by the students in solving geometry problems with the polyhedron as many as five items in the form of descriptions. Sample of test question are stated in table 1

| No | Problem |
|----|---------|
| 1  | A prism has an extensive board 14 cm² and has a high 9 cm, if it wants to build a pyramid by volume, form the base, and the area of the base that equal to the prism, how high is the pyramid to be made? Show how to find it! |
| 2  | “The pyramid and the hexagonal prism have the same base area size, then the height of the hexagon pyramid is three times longer than the hexagon prism”. From this statement, how is the relationship between a hexagon pyramid and a hexagon prism? Are the volumes of the two the same? Is there a bigger volume than others? |
In data collection, questions were given to students through an online questionnaire, because there were still no learning activities at school. This test was done by students individually. Furthermore, the data analysis technique in the study was calculated based on the scoring rubric and checking the score of each answer to the question. Table 2 is a rubric for assessing mathematical reasoning abilities used in this study developed by Sulistiawati, Suryadi, and Fatimah [5]:

**Table 2. Rubric for Assessment of Mathematical Reasoning Ability.**

| Score | Criteria |
|-------|----------|
| 4     | A correct and complete answer |
| 3     | Some of the answers are correct, but there is one significant error |
| 2     | Some of the answers are correct, but there is more than one significant error |
| 1     | The answer is incomplete but contains at least one correct argument |
| 0     | The answer is not correct based on the process, or there is no response at all |

From the calculation results, the average score obtained the average score of mathematical reasoning ability of students. The data were categorized using references adapted from Sulistiawati et al. in [8] as shown in table 3:

**Table 3. Categories of students’ mathematical reasoning abilities.**

| Category | Criteria |
|----------|----------|
| High     | \( \bar{X} \geq 70 \) |
| Moderate | \( 55 < \bar{X} < 70 \) |
| Low      | \( \bar{X} \leq 55 \) |

3. Results and Discussion

After conducting a scoring step, the results of the research showed that the mathematical reasoning ability of high school students on the geometry topic of polyhedron, as shown in table 4:

**Table 4. The score of high school students’ mathematical reasoning ability.**

| Score  | Category  | Frequency (Student) | Percentage |
|--------|-----------|---------------------|------------|
| 86-100 | Very high | 0                   | 0%         |
| 71-100 | High      | 15                  | 24%        |
| 56-70  | Moderate  | 38                  | 59%        |
| 41-55  | Low       | 7                   | 11%        |
| 0-40   | Very low  | 4                   | 6%         |
| Total Student | 64 | 100% |
| Average Score | 64.45 |

Based on table 4, it can be seen that the average score of mathematical reasoning abilities obtained by high school students is 64.45. By referring to table 2, it can be concluded that high school students’ mathematical reasoning abilities during distance learning in schools classified as a moderate category. The following will discuss how high school students’ mathematical reasoning abilities per indicator of their reasoning abilities.
Problem number 1 aims to measure the mathematical reasoning abilities of high school students in performing mathematical manipulation. In general, most students could perform mathematical manipulations well, students could use the data that is known in the questions to obtain the appropriate height of the pyramid to get the volume of the prism with the same volume. However, several students had difficulty answering question number 1 due to their lack of understanding of pyramid volume and prism. Other students also had difficulty answering those questions because they did not understand how to construct questions correctly. In Figure 1, students had difficulty finding the height of the pyramid correctly; it appears that the students determined any number on the height of the pyramid so that the volume of the prism is the same as the volume of the pyramid, this solution was not the way students are expected to do it.

Problem number 2 aims to see high school students' mathematical reasoning abilities in drawing conclusions and compiling evidence. In this study, students could answer questions well and draw conclusions by providing evidence that showed the correctness of their conclusions. However, several students had difficulty answering question number 2 because of their lack of prerequisite/preliminary knowledge to answer question number 2. As stated in Figure 2, students did not know how to calculate the area of a right triangle so that students had difficulty in calculating the volume of a triangular prism with right angles.

Figure 1. Students’ errors in answering question number 1.

Figure 2. Students’ errors in answering question number 2.
These two findings are consistent with the previous research in which students have no difficulty in performing mathematical manipulation, and drawing conclusions and compiling evidence indirect learning in the class [6]. This means that for these two indicators of mathematical reasoning, students did not face difficulties either through direct learning in class or through distance learning.

![Figure 3](image1.jpg)

**Figure 3.** Students’ error in question number 3 (i).

Problem number 3 aims to see the mathematical reasoning abilities of high school students in drawing conclusions from a statement. Figure 3 is one of the mistakes that many students made, where they only looked for the volume of the hexagon pyramid, but they did not compare it with the volume of the hexagon prism as directed by the problem. Then the other mistake made by many students as they did not have sufficient understanding of the concept of polyhedron volume. In Figure 4, the students were still confused in finding the volume of the pyramid so that they had difficulty in answering question number 3. This finding also reinforced by another research that shows students still have an error in drawing conclusions [3], particularly students with the low mathematical ability [9] In general, almost all students had difficulties answering this question. Consequently, Students still have the inadequate ability in drawing conclusions from a given statement.

Problem number 4 aims to see the mathematical reasoning abilities of high school students in assess argument validation. Figure 5 is the most frequent error that many students did; namely, students had difficulty in drawing a conclusion even though they were correct in doing the calculations. Moreover, students could not use the calculations they obtained to prove the truth of the statements in the question. Another mistake that was made by students was the lack of preliminary knowledge. To answer this question, students must have preliminary knowledge about the volume of...
polyhedron, as in Figure 6 it can be seen that they experienced an error in calculating the area of the prism which triangular as its base. Another research also proofs that students have insufficient knowledge to assess the validity of arguments, so they were unable to arrange the desired conclusion[10]. Students with low mathematical ability were struggling to achieve this indicator[11]. In general, almost all the students had difficulty answering this question. So, it can be concluded that there is still a lack of students’ ability to check the validity of an argument.

Figure 5. Students’ error in question number 4 (i).

Figure 6. Students’ error in question

Problem number 5 aims to see the mathematical reasoning abilities of high school students in perform the generalization. In Figure 7, students were not able to formulate the general form of the data they get, even though they could find the pattern, they could not operate it. Another case faced by students stated in Figure 8, students could not find the pattern that they should get from the steps they were working on, it can be seen in the figure that they did not know what the pattern after "n" is. Meanwhile, there is also a research which proof student generalization in mathematical reasoning ability classified as a good category[12]. In general, almost all students had indeed been able to follow the steps of the process. However, they were not able to formulate what they were doing in general, or it can be said that students are still less capable in making mathematical generalizations. Based on result study, it can be seen that the average score of mathematical reasoning abilities obtained by high school students is 64.45, this indicates that high school students’ mathematical reasoning abilities after
the implementing of distance learning during the Covid-19 Pandemic are classified as a moderate category.

![Figure 7. Students’ error in question number 5 (i).](image)

![Figure 8. Students’ error in question number 5 (ii).](image)

From the discussion per indicator of mathematical reasoning ability, it can be inferred that several indicators of mathematical reasoning ability that are the focus of this study are still not mastered by students such as drawing conclusions from a statement, checking the validity of an argument and making mathematical generalizations. The same problem was also expressed by [13] with the results of his research, namely the students’ mathematical conjecture ability during the implementation of distance learning was still inadequate, where this ability was needed to solve problems in mathematics to draw conclusions. Then the low ability of students’ mathematical reasoning during distance learning can be caused by constraints in the learning process of students at school because it requires students to carry out a learning process that they do not normally do, this is in accordance with the results of research [14] student learning difficulties during distance learning, namely difficulties in learning. In
terms of devices, difficulties in adapting students such as not being used to learning online, and the unpreparedness of educators in teaching online. So, distance learning carried out in Indonesia affects the mathematical reasoning abilities of high school students.

4. Conclusion
Results of research at a high school in Jambi city, it was found that the average score of students’ mathematical reasoning ability was 64.45, which was categorized as moderate. In general, students still had problems using their mathematical reasoning abilities, there were several indicators of mathematical reasoning that has been mastered by students such as mathematical manipulation, and drawing conclusions and compiling evidence, but most students have difficulty in drawing conclusions from a statement, assess argument validation, and perform the generalization. This study can be used as a reference that measures students’ mathematical reasoning ability. However, this study has some limitations, including no similar research conducted offline before the pandemic. It is difficult to control students working on their assignment test, which occurs online.

References
[1] NCTM 2000 Principles and Standards for School Mathematics (United States of America)
[2] TIMSS 2012 Timss 2011 International Results in Mathematics 2012 136
[3] Sukirwan S, Darhim D and Herman T 2018 Analysis of students’ mathematical reasoning J. Phys.: Conf. Ser. 948 012036
[4] Payadnya I P A A 2019 Investigation of students’ mathematical reasoning ability in solving open-ended problems J. Phys.: Conf. Ser. 1200 012016
[5] Sulistiawati, Suryadi D and Fatimah S 2015 Desain Didaktis Penalaran Matematis untuk Mengatasi Kesulitan Belajar Siswa SMP pada Luas dan Volume Limas Kreano, J. Mat. Kreat. 6 135
[6] Muslimin and Sunardi 2019 Analisis Kemampuan Penalaran Matematika Siswa Kelas X pada Materi Dimensi Tiga J. Ilm. Pendidik. Mat. AL-QALASADI 3 81–92
[7] Mailizar, Almanthari A, Maulana S and Bruce S 2020 Secondary school mathematics teachers' views on e-learning implementation barriers during the COVID-19 pandemic: The case of Indonesia Eurasia J. Math. Sci. Technol. Educ. 16 7
[8] Yusdiana B I and Hidayat W 2018 Analisis Kemampuan Penalaran Matematis Siswa Sma Pada Materi Limit Fungsi JPMI (Jurnal Pembelajaran Mat. Inov.) 1 409
[9] Hasanah S I, Tafriyanto C F and Aini Y 2019 Mathematical Reasoning: The characteristics of students' mathematical abilities in problem-solving J. Phys.: Conf. Ser. 1188 0–8
[10] Cahya I M and Warmi A 2019 Analisis Tingkat Kemampuan Penalaran Matematis Siswa SMP pada Materi Relasi dan Fungsi Pros. Semin. Nas. Mat. dan Pendidik. Mat. Sesiomadika pp 602–09
[11] Wahyuni E S, Susanto and Hadi A F 2019 Profile of the student’s mathematical reasoning ability in solving geometry problem J. Phys.: Conf. Ser. 1211 012079
[12] Suprihatin T R, Maya R and Senjayawati E 2018 Analisi Kemampuan Penalaran Matematis Siswa SMP pada Materi Segiempat dan Segiempat J. Kaji. Pendidikan 2 09
[13] Supriani Y, Giyanti and Hadi T S 2020 Conjecturing Ability Dalam Pembelajaran Daring Masa Pandemi Covid-19 Inomatika 2 69–77
[14] Annur M F and Hermansyah 2020 Analisis kesulitan mahasiswa pendidikan matematika J. kajian, Penelitian dan Pengemb. Kependidikan 6356 195–201