The Longitudinal Structure Function at HERA

R.S. Thorne*

Department of Physics and Astronomy, University College London, WC1E 6BT, UK

I investigate the theoretical uncertainties on the predictions for the longitudinal structure function \(F_L(x, Q^2)\). I compare the predictions using fixed-order perturbative QCD, higher twist corrections, small-\(x\) resummations and the dipole picture. I compare the various predictions to the recent HERA measurements and examine how the data still to be analysed may discriminate between the approaches.

1 Introduction

I consider the impact of a measurement of \(F_L(x, Q^2)\) [1]. Until recently we have been limited to consistency checks on the relationship between \(F_2(x, Q^2)\) and \(F_L(x, Q^2)\) at high \(y\), where both contribute to the total cross-section \(\tilde{\sigma}(x, Q^2) = F_2(x, Q^2) - y^2/(1 + (1 - y)^2) F_L(x, Q^2)\). Extracting \(F_L\) data requires an extrapolation in \(y\) making some theory assumptions. It is more useful to fit directly to \(\tilde{\sigma}(x, Q^2)\) where there is a turn-over at the highest \(y\). Previous studies have sometimes found that at NLO the turn-over is too small, but better at NNLO due to large corrections to \(F_L(x, Q^2)\) [2]. However, the precision of such studies is limited, and can be affected by systematics, e.g. the photo-production background uncertainty. Using the final low-energy run at HERA, a direct measurement of \(F_L(x, Q^2)\) at HERA is possible [3, 4].

Here I outline the implications and the predictions. The measurement of \(F_L(x, Q^2)\) gives an independent test of the gluon distribution at low \(x\) to accompany that determined from \(dF_2(x, Q^2)/d\ln Q^2\). However, it is also a direct test of success of alternative theories in QCD. These are slightly different issues. It is not obvious for which \(F_L(x, Q^2)\) has more discriminating power. First I will discuss the predictions at LO, NLO and NNLO perturbation theory. I briefly highlight the issue of heavy flavours, since this is more important for \(F_L(x, Q^2)\) than for \(F_2(x, Q^2)\) at low orders. The total structure function is dominated by \(C_{Lg}(\alpha_S, x) \otimes g(x, Q^2)\) contributions. In the massless quark approximation charge weighting means \(F_L^c(x, Q^2)\) is nearly 40% of the total. However, there is a large massive quark suppression in heavy flavour coefficient functions. \(F_L^c(x, Q^2)\) is suppressed by a factor of \(v^3\) where \(v = 1 - 4m_z^2 \frac{Q^2}{(1 - z)}\) is the velocity of the heavy quark in the centre-of-mass frame. \(Q^2 >> m_z^2\) before the massless limit starts to apply, as shown in Fig. 1.

\[\text{Figure 1: An illustration of the charm contribution to } F_L(x, Q^2) \text{ in a general-mass variable flavour number scheme.}\]

*Royal Society University Research Fellow

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At small $x$ the large order-by-order change in the splitting functions, particularly $P_{qg}$, leads to a large variation in the gluon extracted from a global fit, shown in the left of Fig. 2. However, the NNLO $O(\alpha_s^3)$ longitudinal coefficient function $C^3_{Lg}(x)$ has a large positive contribution at small $x$, and this counters the decrease in the small-$x$ gluon. The predictions for $F_L(x, Q^2)$ at LO, NLO and NNLO are shown in the right of Fig. 2. The $F_L(x, Q^2)$ prediction is more stable than the gluon at small $x$. The uncertainty (shown only for NNLO) becomes enormous as $x$ decreases below $0.0001$, but at the lowest $Q^2$ the NLO and NNLO predictions are discrepant in some regions. The LO prediction is far larger than either, reflecting the huge correction in the small-$x$ gluon going to NLO.

2 Beyond fixed order

There are various potentially large corrections beyond fixed-order perturbation theory. It is possible there is a large higher twist contribution from renormalons in the quark sector. For $F_2(x,Q^2)$ the renormalon calculation of higher twist dies away at small $x$ (due to satisfying the Adler sum rule). It is a completely different picture for $F_L(x,Q^2)$. At small $x$ the contribution is proportional to the quark distributions, i.e. $F^{HT}_L(x,Q^2) \propto F_2(x,Q^2)$. The explicit renormalon calculation [7] gives

$$F^{HT}_L(x,Q^2) = \frac{A}{Q^2} (\delta(1-x) - 2x^3) \otimes \sum_f Q_f^2 q_f(x,Q^2),$$

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where $A \approx 4C_f \exp(5/3) \Lambda_{QCD}^2 \approx 0.4 - 0.8 \text{GeV}^2$. It should be stressed that this effect is nothing to do with the gluon distribution, and is not part of the higher twist contribution included in the dipole approach. At small $x$ the correction becomes effectively

$$F_L^{HT}(x, Q^2) = \frac{A}{3Q^2} \delta(1-x) \otimes \sum_f Q_f^2 q_f(x, Q^2) \approx \frac{A}{3Q^2} F_2(x, Q^2),$$

and is $\sim 0.1$ for $x = 0.0001$ and $Q^2 = 2 \text{GeV}^2$.

If the small-$x$ NNLO correction is itself rather large, might not higher orders still be important? There are leading $\ln(1/x)$ terms of the form $P_{gg}(x) \sim \frac{\alpha_s}{x} \ln^n(1/x)$, $P_{qg}(x) \sim \frac{\alpha_s}{x} \ln^{n-2}(1/x)$ and $C_{Lg}(x) \sim \frac{\alpha_s}{x} \ln^{n-2}(1/x)$. A fit which performs a double resummation of leading $\ln(1/x)$ and $\beta_0$ terms leads to a better fit to small-$x$ data than a conventional perturbative fit [8]. The gluon distribution from this resummed fit is larger at small $x$ and $Q^2$ than NLO or NNLO, and this is reflected also in the prediction for $F_L(x, Q^2)$. Similar approaches [9, 10] all lead to rather comparable results for the calculated splitting functions, but only in [8] has detailed phenomenological studies taken place. A comparison of the longitudinal coefficient functions from two approaches is shown in Fig. 3. The two results are clearly of the same form, so it is expected that a prediction for $F_L(x, Q^2)$ using the approach in [9] should be similar to that produced in [8].

![Figure 3: The longitudinal coefficient function $C_{L,g}$ calculated using the approaches in [8] (left, solid line), and [9] (right, blue line).](image)

Finally I consider the dipole picture [11]. As with small-$x$ resummations this can be cast in the language of $f(x, k^2)$ – the unintegrated gluon distribution – which is directly related to the dipole-proton cross-section. The structure functions are obtained by convoluting this dipole cross-section with the wave-functions for the photon to fluctuate into a quark-antiquark pair. This picture includes some of the resummation effects, and also higher twist contributions, and is designed to approach $Q^2 = 0$ smoothly. However, it misses quark and higher-$x$ contributions. Overall $F_L(x, Q^2)$ predicted in this approach is steeper at small $x$ than fixed order, and automaticallystable at lowest $Q^2$, see e.g. [12]. The general features are rather insensitive to whether saturation effects are included in the dipole cross-section.

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3 Predictions

I present the various predictions for $F_L(x, Q^2)$ along a line of $x = Q^2/35420$, which corresponds well to the current HERA measurements [13], and to those yet to appear. The data are in good agreement with all the predictions. Along this line the NLO and NNLO predictions [6] are very similar, and the higher twist corrections are slightly smaller than the pdf uncertainties at NLO and NNLO. The resummation prediction [8] and dipole model prediction [14] (very similar in [15]) have a different shape, and it is perhaps possible to distinguish the former at lower $Q^2$. At $Q^2 \geq 10\text{ GeV}^2$ the uncertainty on fixed order predictions is a few percent. A measurement of $F_L(x, Q^2)$ will not add much to the direct constraint on the gluon. However, there may be deviations from NLO/NNLO predictions of $20 – 30\%$ due to e.g. resummations or dipole models. For $Q^2 \leq 10\text{ GeV}^2$ the uncertainty in NLO/NNLO predictions for $F_L(x, Q^2)$ due to the gluon uncertainty increases to $> 20\%$. A good measurement of $F_L(x, Q^2)$ here will automatically improve the gluon determination. Resummations/dipole models suggest a higher low-$Q^2 F_L(x, Q^2)$ by an absolute value of up to $0.15 – 0.16$, well outside the fixed-order uncertainties. A good measurement of $F_L(x, Q^2)$ will start to discriminate between theories.

References

[1] Slides: http://indico.cern.ch/contributionDisplay.py?contribId=93&sessionId=17&confId=24657
[2] A. D. Martin, W. J. Stirling and R. S. Thorne, Phys. Lett. B 635 (2006) 305 [arXiv:hep-ph/0601247].
[3] B. Antunovic, these proceedings; V. Chekelian, these proceedings.
[4] D. Kollar, these proceedings.
[5] S. Moch, J. A. M. Vermaseren and A. Vogt, Phys. Lett. B 606 (2005) 123 [arXiv:hep-ph/0411112]; J. A. M. Vermaseren, A. Vogt and S. Moch, Nucl. Phys. B 724 (2005) 3 [arXiv:hep-ph/0504242].
[6] G. Watt, A. D. Martin, W. J. Stirling and R. S. Thorne, [arXiv:0806.4890 [hep-ph]].
[7] E. Stein, et al., Phys. Lett. B 376 (1996) 177 [arXiv:hep-ph/9601356].
[8] C. D. White and R. S. Thorne, Phys. Rev. D 75 (2007) 034005 [arXiv:hep-ph/0611204].
[9] G. Altarelli, R. D. Ball and S. Forte, Nucl. Phys. B 799 (2008) 199 [arXiv:0802.0032 [hep-ph]].
[10] M. Ciafaloni, et al., JHEP 0708 (2007) 046 [arXiv:0707.1453 [hep-ph]].
[11] L. L. Frankfurt and M. I. Strikman, Phys. Rept. 160, (1988) 235; A. H. Mueller, Nucl. Phys. B 335, (1990) 115; N. N. Nikolaev and B. G. Zakharov, Z. Phys. C 49, (1991) 607.
[12] K. Golec-Biernat and M. Wusthoff, Phys. Rev. D 59, (1999) 014017.
[13] F. D. Aaron et al. [H1 Collaboration], Phys. Lett. B 665 (2008) 139 [arXiv:0805.2809 [hep-ex]].
[14] R. S. Thorne, Phys. Rev. D 71 (2005) 054024 [arXiv:hep-ph/0501124].
[15] G. Watt and H. Kowalski, Phys. Rev. D 78 (2008) 014016 [arXiv:0712.2670 [hep-ph]].

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