Quantum Effects on Higgs Winding Configurations

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Abstract

We examine the quantum corrections to the static energy for Higgs winding configurations. We evaluate the effective action for such configurations in Weinberg-Salam theory without $U(1)$-gauge fields or fermions. For a configuration whose size is much smaller than the inverse $W$-mass, quantum contributions to the energy are comparable to the classical energy. Moreover, it is insufficient to consider only one-loop corrections, even as $\hbar \to 0$. Indeed, all loop orders contribute equally to the static energy. Nevertheless, quantum fluctuations do not stabilize winding configurations.
I. INTRODUCTION

The Higgs sector in the standard model is a linear sigma model. Such a theory exhibits configurations of nontrivial winding, though they are not stable. Winding configurations in the standard model shrink to some small size and then unwind via a Higgs zero when allowed to evolve by the Euler-Lagrange equations. These winding configurations can be stabilized if one introduces four-derivative Higgs self-interaction terms which are not present in the standard model [1–3]. The motivation typically cited for introducing such terms is that one may treat the Higgs sector of the Lagrangian as an effective field theory of some more fundamental theory which only manifests itself explicitly at some high energy scale. The stabilized configurations have phenomenological consequences in electroweak processes and provide an arena for testing nonperturbative aspects of field theory and the standard model.

Because the procedure just described for stabilization is inconsistent, we will take a different approach; we wish to see whether just the quantum fluctuations of a renormalizable $SU(2)$-Higgs theory can stabilize winding configurations. We will take the Higgs sector to be that found in the standard model. In this paper, we identify the quantum effects on the energy of static winding configurations by evaluating the effective action. If quantum effects stabilize solitons, that effect should be reflected by some extremum in the effective action. If we take the weak gauge-coupling limit, $g^2 \to 0$, an analytic expression is available for the effective action. The weak coupling limit is equivalent to the semiclassical limit when fields are scaled properly. When Planck’s constant is small, we need only focus on small field configurations. It is only for such configurations that quantum corrections are important, and thus have the possibility of stabilizing configurations which are unstable classically.

II. ASYMPTOTIC BEHAVIOR OF THE EFFECTIVE ACTION

Consider the Weinberg-Salam theory of electroweak interactions, neglecting the $U(1)$-gauge fields and fermions. Our field variables form the set $\{A_\mu(x), \phi(x)\}$ where the gauge
field \( A_\mu(x) = \sigma^a A_{\mu a}(x)/2 \) is in the adjoint representation of \( SU(2) \) (\( \{\sigma^a\} \) are the Pauli matrices), and the Higgs field \( \phi(x) \) is in the fundamental representation of \( SU(2) \). We choose the \( R_c \)-gauge to properly quantize this theory. In the following treatment, the parameter \( m \) is the mass of gauge field (the W-particle) and \( m_H \) is the physical Higgs mass. The Feynman rules derived from the specified action are familiar. In this analysis, we restrict ourselves to the semiclassical limit, which is equivalent to taking \( g^2 \to 0 \) while holding \( m, m_H \) fixed.

We wish to determine the effects of quantum fluctuations on Higgs winding configurations. We will evaluate the effective action, \( \Gamma[A_\mu^a, \phi] \), where \( A_\mu^a(x) = 0 \) and \( \phi(x) = [U(x) - 1] \phi_0 \). Here, \( \phi_0 \) is some constant field such that \( \phi_0^\dagger \phi_0 = m^2/g^2 \) and \( U(x) \in SU(2) \) is a static configuration such that \( U(x) \to 1 \) as \( |x| \to \infty \) with characteristic size, \( a \). We require the field \( U(x) \) to be a configuration of unit winding number.

The static energy for the state whose expectation value of the operator associated with the Higgs field is \( \phi(x) \) will be the quantity \( E \) in the expression \( \Gamma[\phi] = -\int dt E \). The effective action \( \Gamma[\phi] \) is the generating functional for the one-particle irreducible green’s functions with \( n \) external \( \phi \)’s, \( \Gamma^{(n)} \). Normally, the effective action would not be solvable exactly. However, because we are investigating the semiclassical limit, we are only interested in configurations whose size, \( a \), is small.

We find that under such a circumstance, we may use the Callan-Symanzik equation for our theory to evaluate the leading-order size dependence of the one-particle irreducible green’s functions and thus evaluate the quantum corrections to the static energy. We implement the condition of small, static background configurations by requiring the field \( \hat{\phi}(p) \), the Fourier transform of the field \( \phi(x) \), to have support only for \( p_0 = 0 \) and \( |p| \gg m^{-1}, m_H^{-1} \) which implies \( 0 < m^{-2}, m_H^{-2} \ll -p^2 \). Under this circumstance, the asymptotic dependence of \( \Gamma^{(n)} \) will be determined by the Callan-Symanzik equation. The one-loop beta functions may be easily obtained from the literature \[4\]. The one-loop anomalous dimension is also easy to evaluate.

So long as \( m_H/m \) is not too large, we find that the leading-order size dependence of the effective action comes from the two-point one-particle irreducible green’s function. All other
terms are suppressed by powers of $a$ and other factors. The leading-order contribution to the effective action from quantum fluctuations yields

$$\Gamma[\phi] = \int \frac{d^4p}{(2\pi)^4} \phi^\dagger(p)\phi(p)p^2 \left[ 1 + \frac{1}{2} b_0 g^2 \ln \left( \frac{-p^2}{m^2} \right) \right]^{\frac{c_0}{b_0}}. \quad (2.1)$$

Here $b_0 = 43/48\pi^2$ and $c_0 = 3[1 + (\xi - 1)/4]/16\pi^2$, where $\xi$ is the gauge parameter ($\xi > 0$).

The scale dependence of the above expression is

$$\Gamma[\phi] \sim -\int dt \frac{m^2a}{g^2} \left[ 1 + b_0 g^2 \ln \left( \frac{1}{ma} \right) \right]^{\frac{c_0}{b_0}}. \quad (2.2)$$

One can recover the classical result from (2.2) by setting the $g^2$ inside the brackets to zero. Note that when $b_0 g^2 \ln(\frac{1}{ma}) \sim 1$, the quantum corrections to the energy are as significant as the classical contribution. Nevertheless, the static energy that corresponds to this effective action is a monotonically increasing function of the size, $a$, such that $E(a = 0) = 0$. This would imply that Higgs winding configurations would shrink to zero size and unwind via a Higgs zero, just as in the classical scenario.

### III. DISCUSSION

Let us take a closer look at our expression for the leading contribution to the effective action (2.1). Expanding in powers of $g^2$ we get

$$\Gamma[\phi] = \int \frac{d^4p}{(2\pi)^4} \phi^\dagger(p)\phi(p)p^2 + \frac{c_0}{2} g^2 \int \frac{d^4p}{(2\pi)^4} \phi^\dagger(p)\phi(p)p^2 \ln \left( \frac{-p^2}{m^2} \right) + \cdots$$

The first term is the contribution from the classical action. The next term is the leading order contribution from one-loop one-particle irreducible graphs. The scale dependence of the static energy goes like

$$E = \frac{m^2a}{g^2} \left[ A + B g^2 \ln \frac{1}{ma} + C g^4 \left( \ln \frac{1}{ma} \right)^2 + \cdots \right] \quad (3.1)$$

where $A, B, C$ are numbers. Again the first term is the classical energy, the second is the one-loop energy, and the rest of the terms in the expansion (3.1) correspond to higher-loop energies order by order. We can see by comparing (2.2) with (3.1) that loop contributions
to the effective action beyond one loop can only be neglected when \( b_0 g^2 \ln(1/ma) \ll 1 \). However, that is precisely the condition where the one-loop contribution can be neglected relative to the classical action. Thus, drawing conclusions concerning solitons based on one-loop results may be difficult. When dealing with small configurations, one still needs to include higher-loop contributions, even in the semiclassical limit.

There are limitations to (2.1) which we will not discuss here. Complications occur from \( m_H/m \) dependence and the running of the Higgs self-coupling. For a more complete discussion, please refer to [5].

ACKNOWLEDGMENTS

The author wishes to express gratitude for the help of E. Farhi in this work. The author also wishes to acknowledge helpful conversations with J. Goldstone, K. Rajagopal, K. Huang, K. Johnson, L. Randall, M. Trodden, and T. Schaefer. Moreover, I would also express gratitude to the organizers of this workshop. This work was supported by funds provided by the U.S. Department of Energy.
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