Magnetic Moment of the $\Omega^-$ in QCD sumrule (QCDSR)

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Abstract: The $\Omega^-$ magnetic moment was measured very accurately and experimentalists remarked that it differs from the theoretical estimates, thus posing a challenge to the latter. One such estimation uses QCDSR. We revisit this sumrule method, using condensate parameters which were obtained from fitting the differences ($\mu_p - \mu_n$), ($\mu_{\Sigma^+} - \mu_{\Sigma^-}$) and ($\mu_{\Xi^0} - \mu_{\Xi^-}$) [1] and confirm the experimental number. The $\mu_{\Delta^{++}}$ is also found to agree with the experimental estimate.

Keywords: QCD sumrules, magnetic moments of baryons.

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The $\Omega^-$ magnetic moment, $\mu_{\Omega^-}$, has been the subject of many studies \[2, 3, 4, 5, 6\]. The magnetic moment was unknown when \[2\] was published but on hindsight the value predicted there, within the acceptable parameter range, agrees with the present accurately determined experimental result \[7\]. The results of Lee \[3\] using QCD sumrules and those from the lattice calculation \[4\] underestimate it whereas the light-cone relativistic quark model \[5\] and the chiral quark soliton model \[6\] overestimate it. We re-investigate this intriguing situation by looking at the calculations of Lee using a slightly different point of view advocated in \[1\] and find that one indeed gets good agreement with experiment. Further, as pointed out by Lee, the $\mu_{\Omega^-}$ depend sensitively on the magnetic susceptibility so that we can pinpoint this parameter more effectively.

The QCD sumrule method is a very powerful tool in revealing a deep connection between hadron phenomenology and vacuum structure \[8\] via a few condensates like

\[ a = -2\pi^2 < \bar{q}q >, \quad b = < g^2 G^2 >, \quad (1) \]

related to the quark ($q$) and gluon ($G$) vacuum expectation values. This can be used for evaluating magnetic moments of hadrons \[9\] where some new parameters enter, for example, $\chi$, $\kappa$ and $\xi$, defined through the following equations:

\[ < \bar{q}\sigma_{\mu\nu}q > F = e_q \chi < \bar{q}q > F_{\mu\nu}, \quad (2) \]
\[ < \bar{q}G_{\mu\nu}q > F = e_q \kappa < \bar{q}q > F_{\mu\nu}, \quad (3) \]
\[ < \bar{q}\epsilon_{\mu\nu\rho\gamma}G^{\rho\gamma}q > = e_q \xi < \bar{q}q > F_{\mu\nu}. \quad (4) \]

where the $F$ denotes the usual external electromagnetic field tensor. Lee \[3\] very carefully evaluated the contributions of these operators to the magnetic moments of the $\Omega^-$ and $\Delta^{++}$, the latter emerging from the former when the quark mass $m_s$, is put equal to zero, the parameter $f$ and $\phi$ are put equal to 1 and the quark charge $e_s = -1/3$ is replaced by $e_u = 2/3$. The parameter $f$ and $\phi$ measure the values of quark condensates and quark spin-condensates with strange and (ud) quarks.

\[ f = \frac{< \bar{s}s >}{\bar{u}u}, \quad (5) \]
\[ \phi = \frac{< \bar{s}\sigma_{\mu\nu}s >}{\bar{u}\sigma_{\mu\nu}u}. \quad (6) \]

For the expression for the $\mu_{\Omega^-}$ sumrule we refer the reader to the paper by Lee \[3\] which we reproduce here for the sake of completeness, in terms of the Borel parameter $M$ and the intermediate state contribution $A$:
\[
\begin{align*}
\frac{9}{28} e_s L^{4/27} E_1 M^4 &- \frac{15}{7} e_s f \phi m_s \chi a L^{-12/27} E_0 M^2 + \frac{3}{56} e_s b L^{4/27} - \frac{18}{7} e_s f m_s a L^{4/27} \\
- \frac{9}{28} e_s f (2 \kappa + \xi) m_s a L^{4/27} &- \frac{6}{7} e_s f^2 \phi \chi a^2 L^{12/27} - \frac{4}{7} e_s f^2 \kappa \phi a^2 L^{28/27} \frac{1}{M^2} \\
- \frac{1}{14} e_s f^2 (4 \kappa + \xi) a^2 L^{28/27} &+ \frac{1}{4} e_s f^2 \phi \chi m_s^0 a^2 L^{-2/27} \frac{1}{M^2} \\
- \frac{9}{28} e_s f m_s m_0^2 a L^{-10/27} &+ \frac{1}{12} e_s f^2 m_s^0 a^2 L^{14/27} \frac{1}{M^4} \\
= \tilde{\lambda}_\Omega \left( \frac{\mu_\Omega}{M^2} + A \right) e^{-M_\Omega^2/M^2}.
\end{align*}
\]

Here

\[
E_n(x) = 1 - e^{-x} \sum_n \frac{x^n}{n!}, \quad x = w_B^2/M_B^2
\]

where \(w_B\) is the continuum, and

\[
L = \frac{\ln(M^2/\Lambda_{QCD}^2)}{\ln(\mu^2/\Lambda_{QCD}^2)}
\]

For evaluating the magnetic moment we use the above equation and divide by the equation for the mass sum rule given earlier by Lee [10]. Thus we eliminate the parameter \(\lambda_\Omega\) in the spirit of [1] and we get an excellent fit to the resulting numbers in the form \(\mu_\Omega - A/M^2\). We find that the results are not very sensitive to \(\kappa_v\), the so called factorization violation parameter, defined through

\[
<\bar{u}\bar{u}\bar{u}\bar{u}> = \kappa_v <\bar{u}^2>.
\]

Neither are the results very sensitive to the parameters \(\kappa\) and \(\xi\). We use the crucial parameters \(a\) and \(b\) from [1], since they must fit the octet baryon moment-differences \((\mu_p - \mu_n)\) and \((\mu_{\Sigma^+} - \mu_{\Sigma^-})\). It was shown in [1] that by using the empirical scaling of the \(\tilde{\lambda}\) with the (baryon mass)\(^3\) - these differences depend only of \(a\) and \(b\), and one gets \(a = 0.475 \text{ GeV}^3\) and \(b = 1.695 \text{ GeV}^4\). Further, to fit the difference \((\mu_{\Xi^0} - \mu_{\Xi^-})\), \(m_s\) was set to be 170 MeV in [1] and we use this value.

Table 1 shows the dependence of the magnetic moments on the parameters. Obviously \(\mu_{\Delta^{++}}\) does not depend on \(f\) and \(\phi\). It is clear that \(\mu_{\Omega^-}\) also does not depend so much on \(f\) but it is sensitive to both \(\phi\) and \(\chi\), and it appears that \((\chi = 6.5, \phi = 0.6)\) and \((\chi = 5.5, \phi = 0.7)\) are preferred values, close to the experimental number \(\mu_{\Omega^-} = 2.019 \pm 0.054 \mu_N\) [7]. The \(\mu_{\Delta^{++}}\) is known only approximately, \(4.52 \pm 0.95 \mu_N\) [11] and a better determination will enable us to pinpoint \(\chi\).

It is satisfactory to see that there is no conflict between experiment and QCDSR since sum rules are a ‘first principle method’, although it is based partly on phenomenology.
In summary we find that using the constrained values of the parameters $a$ and $b$ \[1\] one can get a good fit to the known decuplet magnetic moments. The moments do not depend sensitively on the factorization violation parameter but may be used to pinpoint the susceptibility $\chi$ and $\phi$, the ratio of the spin condensate for strange and ud quarks.

**Table 1.** The values of the parameters and the corresponding magnetic moments.

| $\kappa$ | $\xi$ | $\chi$ | $\kappa_V$ | $f$ | $\phi$ | $\mu_{\Omega^{-}}$ | $\mu_{\Delta^{++}}$ |
|-----------|-------|-------|------------|-----|-------|----------------|------------------|
| 0.70      | -1.5  | -6.5  | 1.0        | 0.83| 0.6   | -2.007        | 3.702            |
| 0.75      | -1.5  | -6.5  | 1.0        | 0.83| 0.6   | -2.005        | 3.697            |
| 0.80      | -1.5  | -6.5  | 1.0        | 0.83| 0.6   | -2.002        | 3.691            |
| 0.75      | -1.4  | -6.5  | 1.0        | 0.83| 0.6   | -1.983        | 3.670            |
| 0.75      | -1.6  | -6.5  | 1.0        | 0.83| 0.6   | -2.026        | 3.724            |
| 0.75      | -1.5  | -7.0  | 1.0        | 0.83| 0.6   | -2.146        | 3.964            |
| 0.75      | -1.5  | -6.0  | 1.0        | 0.83| 0.6   | -1.884        | 3.457            |
| 0.75      | -1.5  | -6.5  | 1.5        | 0.83| 0.6   | -1.928        | 3.588            |
| 0.75      | -1.5  | -6.5  | 1.0        | 0.83| 0.7   | -2.750        | 3.697            |
| 0.75      | -1.5  | -5.5  | 1.0        | 0.83| 0.7   | -2.011        | 3.217            |
| 0.75      | -1.5  | -6.5  | 1.0        | 0.88| 0.6   | -2.020        | 3.697            |

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