Prediction of Shear Contribution for the FRP Strengthening Systems in RC Beams: A Simple Bonding-based Approach

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Abstract
This paper introduces a novel and simple model for estimation of the shear contribution of the fiber-reinforced polymer (FRP) strengthening system in the FRP-strengthened beams. The model utilizes the bonding-based approach, which considers the shear resisting mechanism of FRP-strengthened beam via the bond behavior between FRP strengthening system and concrete. Herein, the beams strengthened in shear with near-surface mounting (NSM) rods or laminates and embedded through-section (ETS) bars are examined. By utilizing only mechanical consideration, the shear resistances of the NSM-strengthening or ETS-strengthening laminates or bars in the beams are simply derived when several bond factors (i.e. maximum bond stress and slip at peak bond stress) are known without using any empirical coefficients. The reliability of the proposed model is first validated against the test results available in the open literature. The extensive investigation to complement the model validation is then carried out through comparison of the results produced by the experiments and the proposed approach as well as the existing methods. The analyses demonstrate that the bonding-based approach is greatly effective to predict the shear contribution of the FRP strengthening system in the beam. Two examples for calculation of the shear resisting forces of the ETS-FRP and NSM-FRP bars in the FRP-strengthened beams are provided to depict the use of the model.

1. Introduction

Various methods, such as externally bonding (EB), near surface mounting (NSM) and embedded through-section (ETS) methods, for strengthening and rehabilitation of the reinforced concrete (RC) beams in shear have been widely created in both laboratory research and real applications; including those by Dai et al. (2005, 2006), Coelho et al. (2015), Dias and Barros (2011), Challal et al. (2011), Mofidi et al. (2012), Godat et al. (2012), Breveglieri et al. (2014, 2015, 2016), Bui et al. (2017, 2018a, 2018b, 2020a, 2020b), and Zou et al. (2019). As reported in most of their studies, those retrofitting methods illustrated great efficiency in the enhancement of the shear strength and performance of the strengthened RC beams. Furthermore, Bui et al. (2020b) indicated that the use of the ETS strengthening system in the RC beams improved member ductility, which the brittle shear failure was converted into the ductile flexural failure. A crucial problem that induces negative influences on the strengthened members was the inefficiency bonding between the FRP retrofitting systems and concrete. If the FRP-concrete interfaces are not assured, the composite action between the FRP-strengthening materials and the members is not fully triggered, resulting in the low effectiveness in the adoption of FRP.

To enhance the bond performance between the FRP intervening systems and concrete in the beams, the studies in the literature revealed several solutions as follows. The increase of the bond length of the retrofitting configuration caused its beneficial bond response by a slow strain development in the shear transfer mechanism. In addition, as studied by Bui et al. (2020b), a mechanical anchorage system could be inserted to the intervening configuration in the appropriate zone where the strengthening materials were placed. The mechanical anchorage system could compensate for the bonding deficiency. Currently, there are a number of studies investigating the bond behavior of the ETS or NSM retrofitting system to concrete through pullout tests and bending tests (Nakaba et al. 2001; Lorenzis and Nanni 2002; Lorenzis et al. 2002; Teng et al. 2006; Soliman et al. 2011; Coelho et al. 2015; Godat et al. 2012; Caro et al. 2017; Bui et al. 2020a, 2020c). Those researches assessed the bond stress-slip relationship, modes of failure and bond strength under various effects such as the bonded length, concrete strength, FRP size and property. The obtained results presented the feasibility of the ETS and NSM methods for structural intervention. Additionally, the bond models represented the interfacial mechanism of the ETS and NSM retrofitting systems to concrete have been proposed and developed. Some models were applied in the literature, for example the bond laws provided by Cosenza et al. (1997) and Dai et al. (2005),

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which exhibited acceptable efficiency. However, an issue that made the researchers concern about is how the bond model is used in the analysis for the real structures strengthened with the ETS-FRP or NSM-FRP system. Although several studies have been carried out on this issue, they mainly focused on the numerical analyses, in which the bond model was utilized. Particularly, the shear contributions of the FRP strengthening systems (such as ETS-FRP and NSM-FRP) in the RC beams have not been interpreted through the bonding-based approach yet.

Based on truss analogy, the American Concrete Institute method ACI 440.1R-15 (ACI 2015), and the method of the Japan Society of Civil Engineers (JSCE 1997) have been offered to predict the shear resisting forces of the FRP reinforcement in RC beams. The shear resistance of RC beams with internally ordinary FRP shear reinforcement was examined in many studies. Various models were effectively developed to derive the shear contribution of internally embedded FRP shear reinforcement in RC members. However, the predictions for shear contribution of a FRP strengthening system in RC beams made by ACI method (ACI 2015) and the JSCE method (JSCE 1997) underestimate the experimental measurement (Bui et al. 2020b). Kamiharako et al. (2001) proposed a model for estimating the shear contribution of externally bonded FRP sheets via the bond constitutive law. In their model, they used the numerical analysis associating with the iteration procedure to calculate the shear resistance of the FRP sheets via the stress distribution in the FRP, which was derived by the interfacial constitutive law. Additionally, they employed the linear and bilinear constitutive laws for defining the bond response of EB FRP sheets to concrete, which was a crucial part of their proposed shear strength model. However, the linear and bilinear relationships have not gained the reasonable reliability for expressing the interfacial mechanism between ETS/NSM FRP and concrete (Lorenzis and Nanni 2002; Bui et al. 2020c).

Owing to the adoption of many variables and procedures, the model of Kamiharako et al. (2001) has not widely been used in the open literature.

To achieve the simplicity for using the model, this study examines estimation of the shear contribution of a FRP strengthening system in RC beams with a new approach based on a proposed bond model, which was conveniently used to simulate interfacial mechanism of ETS/NSM FRP strengthening attached to concrete by adhesive. Only a study by Bui et al. (2020a) showed the bonding-based approach to derive the shear contribution of the ETS-strengthening bars. The comparison between their calculated results and the test results was also made to show the potential of the prediction method based on the interfacial mechanism. In the above study, however, the formulations for estimation of the shear resistance of the ETS-FRP bars were developed based on the empirical and regressing procedures, which resulted in the low reliability and narrow usable range for the proposed model. The purpose of this study is therefore to provide a newly comprehensive bonding-based approach for predicting the shear contribution of ETS and NSM strengthening systems in RC beams without using any empirical coefficients and regressing procedures. The proposed approach, which considers only mechanical and mathematical aspects, can be applied for all beams strengthened in shear with NSM or ETS composite system. Only two bond factors (maximum bond stress, \(\tau_m\) and slip at peak bond stress, \(s_m\)), which are available and open in the existing sources, are needed. The key novel feature of the proposed shear resisting model is an accurate and simply applicable formulation based on a careful interpretation of the bond mechanism between the ETS-FRP and NSM-FRP strengthening systems and concrete in the intervened beams. The organization of this paper is as follows: (1) The bond model developed in this study is validated with the bond test results for the interfacial mechanism between the NSM-FRP systems and concrete; (2) The assessment of the bond performance of the ETS-concrete and NSM-concrete interfaces using the proposed bond model is carried out; (3) The bonding-based approach to interpret the behavior of the ETS-FRP and NSM-FRP strengthening systems in the RC beams is proposed; (4) Through the bonding-based method, the validation for the \(\tau-s\) relationship of both ETS-FRP bars and NSM-FRP laminates embedded directly in the beams employing the proposed approach is presented; (5) The ETS-FRP and NSM-FRP shear contribution predictions made by the bonding-based approach are validated to the test data as well as the existing shear models; (6) In addition, based on the calculation procedure, the examples for application of the proposed method for deriving the ETS-FRP and NSM-FRP shear resisting forces in the retrofitted RC beams are demonstrated to show the usability and simplicity of the model. The scheme for summarizing the primary contents of the study is shown in Fig. 1.

2. Analysis of bond behavior between FRP strengthening systems and concrete

The bond model proposed by Dai et al. (2005) was originally used for interpreting the bond behavior between EB-FRP sheets and concrete, which represented by a governing equation in Eq. (1a). Based on this approach, Bui et al. (2020a) developed a model for simulating the bond behavior of ETS-FRP bars to concrete, which represented with a governing equation in Eq. (1b). The main difference of these two models is in term of geometry properties, \(E_{df}\) for EB-FRP sheets and \(E_{A_{d}p}\) for ETS-FRP bars. Many subsequent works have shown that the \(\tau-s\) models proposed by Dai et al. (2005) and Bui et al. (2020a) could be adopted reliably to represent the bond mechanism between the FRP strengthening system and concrete for both EB technique and ETS method. To the best of the authors’ knowledge, there is
currently no clear consideration on the use of approaches proposed Dai et al. (2005) or Bui et al. (2020a) for investigating the bond characteristics between NSM strengthening system and concrete. Because this study focuses on the laminate and rod shapes for NSM and ETS strengthening systems, this section uses the FRP-concrete interfacial model established by Bui et al. (2020a) for interpreting the bond mechanism of the NSM system to concrete.

Figures 2(a) and 2(b) illustrate the free-body diagrams for analyzing the interfacial profiles of the NSM and ETS composites to concrete. Therefore, following Bui et al. (2020a), it is easily realized that the governing equation for the ETS and NSM methods can be expressed by Eq. (1b).

\[
\tau = \frac{E_A s}{p_r} \left(1 - e^{-B s} \right) \quad (1b)
\]

where \(\tau\) is the bond stress, \(s\) is the slip, \(E_f\) is the Young’s modulus of FRP sheet, \(t_f\) is the thickness of FRP sheet, \(E_r\) is the Young’s modulus of FRP bar/laminate, \(A_r\) is the cross section area of NSM-FRP, \(p_r\) is the perimeter of FRP and \(\varepsilon\) is the strain. For ETS technique, \(p_r\) is the full perimeter of FRP bars. For NSM method, \(p_r\) is the contact perimeter of failure surface contacted to concrete via adhesive.

According to the approach of Bui et al. (2020a), the strain-slip (\(\varepsilon\)-\(s\)) relationship at a point on the FRP composite embedded in the concrete could be expressed as an exponential function in Eq. (2).

\[
\varepsilon = f(s) = A \left(1 - e^{-B s} \right) \quad (2)
\]

By substituting Eq. (2) into the governing equation Eq. (1b), the bond stress-slip relationship can be described as Eq. (3). Eq. (3) indicates that the bond stress-slip (\(\tau\)-\(s\)) relationship can be determined if the local strain-slip relationship is defined.

\[
\tau = \frac{E_A A^* B e^{-B s}}{p_r} \left(1 - e^{-B s} \right) \quad (3)
\]

\[
\frac{d\tau}{ds} = 0 : s_m = \ln \left(\frac{2}{B} \right) \quad (4)
\]

where \(A = \varepsilon_m\) is the maximum strain corresponding to the maximum pullout force (\(\varepsilon\)). As shown in Eq. (4), \(B = \ln 2/s_m\) is defined as bond ductility of the interfacial response between NSM-FRP and concrete (1/mm) and it is derived by solving \(d\tau/ds = 0\). Note that \(\tau_m\) = maximum bond stress (MPa), \(s_m\) = slip corresponding to maximum bond stress (mm), \(s_f\) = slip at onset of debonding (mm), \(\tau_f\) = bond stress at onset of debonding (MPa).

To validate the reliability of the aforementioned bond model regarding to the NSM method, the test data in previous works of Lorenzis and Nanni (2002), Teng et al. (2006) and Soliman et al. (2010) are summarized in Table 1. In the study by Lorenzis and Nanni (2002), a specimen consisting of the bond stress and slip curve was used to discuss the model reliability. The material and geometry properties of that specimen could be easily found by searching the specimen name “C3D18a” in their study. The “C3D18a” specimen employed carbon FRP (CFRP) rod with 9.5 mm diameter, which was bonded to concrete prism with concrete strength of 27.6 MPa through the near-surface groove longed 171 mm. Based on the \(\varepsilon\)-\(s\) data recorded from the test, the curve fitting was created through regressing Eq. (2). As demonstrated in Fig. 2(c), the curve fitting by regressing technique produced bond parameters \(A\) and \(B\), as displayed in Table 1. Figure 3(a) presents a comparison between test measurement and curve fitting with Eq. (2) on the strain-slip relationship. As a result, the \(\varepsilon\)-\(s\) curve analyzed by the curve fitting model agrees well with the \(\varepsilon\)-\(s\) relationship drawn from the test. The initial behavior is identical for both curves. This is because, at initial stage, the interfacial-profiled performance was assured to transfer the stress fully between the concrete and the NSM-CFRP rod for developing strong composite action. Figure 3(b) shows the validation of the bond model by

![Fig. 1 Scheme for the study.](image-url)
comparing the $\tau$-$s$ curve drawn by Eq. (3) to the experimental data measured from the test. The model prediction curve is similar to experimental dots at low load level but not close enough to the experimental dots at high load level and descending branch. This is because that the model parameters ($A$, $B$) were obtained as the curve fitting of the $\varepsilon$-$s$ diagram, which were shown the slight discrepancy to experimental data [Fig. 3(a)]. Therefore, this results in the deviation of the $\tau$-$s$ model to the $\tau$-$s$ measurement. Additionally, in the tests, the local effects such as local bending, improper epoxy injection might exert on the reading and recording of strain gauges, therefore the experimental $\tau$-$s$ diagrams were irregular leading to existence in deviation. Although there is some deviation, Fig. 3(b) depicts obviously the considerable efficiency of the proposed bond model for prediction of the $\tau$-$s$ interfacial relationship between the NSM-CFRP rod and concrete.

For investigation purpose on the interfacial behavior of the NSM systems to concrete, Fig. 4 presents the calibration using the bond responses for the experiments in terms of the $\tau$-$s$ curves. The specimens “CS-200” and

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**Fig. 2 Bond behavior:** (a) NSM system and concrete; (b) ETS system and concrete; (c) scheme to plot $\tau$-$s$ curves using bond model.
“CS-250” in the study by Teng et al. (2006) and the specimen “N/C-10-E-1.5-48” in the study by Soliman et al. (2010) are employed to solve the interfacial mechanism between the NSM strips or bars and concrete. The geometry and material characteristics of those specimens can be found in Teng et al. (2006) and Soliman et al. (2010); only maximum slip and strain are used in calibrating the τ-s relations. The comparison between the test curves and the analytical curves in Figs. 4(a) and 4(b) indicates the efficiency of the NSM systems-concrete interfacial model proposed in Eq. (3) for calibrating and fitting the bond test results. Although there was discrepancy in descending branch, the ascending trend derived from the bond model agreed well with the ascending branch obtained from the experiments. This phenomenon implies the acceptable accuracy of the model in predicting of the interfacial ductility (B) and the debonding fracture energy (Gf), which are deemed to affect strongly the performance of the concrete members retrofitted with the NSM-FRP system. On the other hand, in Fig. 4(a), the main difference of those two specimens is the bond length, in which CS-200 was 200 mm and CS-250 was 250 mm. Obviously, in both test and model results, the specimen CS-250 with longer embedment length offered higher bond stress and lower slip than those achieved from the specimen CS-200.

The validated bond model between FRP strengthening and concrete is then used to assess the bonding efficiencies of the ETS and NSM techniques. The interfacial performances in the fracture energy (Gf) defined by the area underneath the τ-s curve and the interfacial ductility (B) are examined. Thereby, a concrete specimen with ETS embedded bar was designed and created to have the pullout strength, which is similar to the pullout force of 37.0 kN measured in the NSM-retrofitted specimen tested by Lorensiz and Nanni (2002). To have same pullout force, the specimen with NSM system required longer bond length than that with ETS system. The configuration and material properties of the design specimens in the present study and previous work are shown in Table 2.

**Table 2 Specimen configurations of NSM-bond tests for model validation.**

| Specimen          | $f'_c$ (MPa) | $E_r$ (GPa) | $f_t$ (MPa) | $E_a$ (GPa) | $f_{ta}$ (MPa) | $L_e$ (mm) | $d_b$ (mm) | $b_f$ (mm) | $A$ (ε) | $B$ (1/mm) |
|-------------------|--------------|-------------|-------------|-------------|--------------|------------|------------|------------|---------|------------|
| C3D18a            | 28.2         | 164.7       | 1550        | Not available | 13.8        | 171        | 9.5        | CFRP     | 0.00516  | 2.71       |
| CS-200            | 44.0         | 151         | 2068        | 2.62        | 42.6        | 200        | 2 × 16     | CFRP     | 0.00564  | 2.29       |
| CS-250            | 44.0         | 151         | 2068        | 2.62        | 42.6        | 250        | 2 × 16     | CFRP     | 0.00643  | 4.78       |
| N/C-10-E-1.5-48   | 44.0         | 128         | 1546        | 1.49        | 43.5        | 456        | 9.5        | CFRP     | 0.01511  | 0.97       |

Note: $f'_c$ = concrete compressive strength (MPa); $E_r$ = Elastic modulus of FRP (GPa); $f_t$ = tensile strength of FRP (MPa); $E_a$ = Young’s modulus of adhesive (GPa); $f_{ta}$ = tensile strength of adhesive (MPa); $L_e$ = embedment length (mm); $d_b$ = bar diameter (mm); $b_f$ = width of laminate (mm); $A$, $B$ are bond parameters obtained from the tests based on regression of Eq. (2).

**Fig. 3 Validation of proposed bond model against the experimental results of the specimens embedded by NSM-CFRP rod:** (a) strain-slip curve; (b) bond stress-slip response.
with confinement guaranteed, resulting in the favorable force transfer and the hybrid stiffness between the ETS bar and concrete. Meanwhile, the NSM bar was inserted to the surface of the concrete specimen; therefore, the confinement and the interfacial stiffness of the concrete to the NSM rod are not effective. This leads to the low force transfer between the NSM system and concrete, resulting in lower bond performance than that of the ETS-embedded specimen.

Figure 5 reveals that the average maximum slips, approximately 0.22 mm, are similar for the specimens embedded with the NSM and ETS bars. Thereby, the average bond ductility defined by $B = \ln 2 / \delta_m$ of the ETS intervened block is equal to the ductility of the NSM intervened specimen. However, the interfacial fracture energy of the ETS specimen is significantly greater than that of the NSM specimen.

### 3. Analytical model for ETS-FRP and NSM-FRP in strengthened beams by means of bonding-based approach

This section introduces the bonding-based approach for predicting the shear resistance of the FRP strengthening system in the RC beams.

#### 3.1 Bond mechanism for FRP-strengthening composites in RC beams

Figures 6(a) and 6(b) describe the approach to convert the behavior of the influenced NSM or ETS elements, which passed the main crack plane, embedded in shear zones of the RC beams to the bond behavior of the average NSM or ETS configurations to equivalent concrete blocks. As demonstrated in Figs. 6(a) and 6(b), the ETS and NSM systems that have passed critical cracks contributed to shear resistance of the strengthening system through bonding between the ETS or NSM systems, adhesive, and concrete.

To determine the characteristics of the corresponding bonding-based specimens, the number of the ETS or NSM composites ($N_f$) that contributed to shear resistance is first defined by the number of the strengthening systems crossing the main crack plane, as displayed by Eq. (5). Then, the bond length ($L_e$) of an influenced strengthening element in the whole strengthening system in a RC beam is expressed by Eq. (6). The sizes of the equivalent blocks are shown in Figs. 6(a) and 6(b).

| Specimen | $f'_c$ (MPa) | $E_c$ (GPa) | $f_t$ (MPa) | $E_a$ (GPa) | $L_e$ (mm) | $d_b$ (mm) | ETS/NSM material | Ultimate pullout force (kN) | Maximum slip (mm) |
|----------|--------------|-------------|-------------|-------------|------------|-----------|------------------|----------------------------|------------------|
| ETS in this study | 38 | 50 | 1076 | 3.1 | 21 | 120 | 10 | GFRP | 37.9 | 0.64 |
| NSM in the work of Lorenz and Nanni (2002) | 28.2 | 41.3 | 799 | Not available | 156 | 13 | GFRP | 37.0 | Not available |

Note: $f'_c$ = concrete compressive strength (MPa); $f_t$ = tensile strength of FRP (MPa); $E_a$ = Young’s modulus of adhesive (GPa); $f_{tu}$ = tensile strength of adhesive (MPa); $L_e$ = embedment length (mm); $d_b$ = bar diameter (mm).

Table 2 Summary of specimen details and experimental results.
\[ N_j = \text{round off} \left( h_w \frac{\cot \theta + \cot \alpha}{s_{fj}} \right) \]  
\[ L_{\beta} = \begin{cases} 
\frac{i s_{fj} \sin \theta}{\sin (\theta + \alpha)} & \text{for } x_{fj} < \frac{h_w}{2} (\cot \theta + \cot \alpha) \\
L_f - \frac{i s_{fj} \sin \theta}{\sin (\theta + \alpha)} & \text{for } x_{fj} \geq \frac{h_w}{2} (\cot \theta + \cot \alpha)
\end{cases} \]  

where \( x_{fj} \) is the distance from the end of main crack plane to the end of the \( j \)th FRP single bar/laminate passed the critical crack plane [as described in Figs. 6(a) and 6(b)], \( s_{fj} \) (mm) is the ETS or NSM spacing, and \( h_w \) (mm) is the beam height for beams with the ETS bars, while \( h_w \) (mm) is the height of the groove for beams with the NSM system; \( \theta \) (°) and \( \alpha \) (°) are the crack angle and the inclination of strengthening systems, respectively.

\[
\tau = \frac{P s_w}{s_{fj}} \text{ Bond model in Eq. (3)}
\]

For ETS-anchored bars, it can be assumed that slip at anchored end and slip at starting point of anchorage are negligible, because strains at those points are very small.
The average bond length of the influenced ETS or NSM strengthening elements in the shear zone of a beam is calculated as Eq. (7). As reported in Bui et al. (2020c)’s study, when the embedment lengths of ETS bars are shorter than 180 mm, it affects the bond strength. In this study, the average bond lengths derived by Eq. (7) for all investigated specimens, which will be shown in Section 4, are in range of 33 to 168.5 mm. Therefore, the average bond lengths of the influenced strengthening elements are considered in the calculation using the bonding-based approach. In addition, Fig. 6(c) illustrates the assumption of the interfacial response of the ETS and NSM intervening systems to concrete in the shear zones of RC beams. Note that this interfacial bond response is the model proposed in previous sections. In this section, the bond model proposed in Section 2 (for bond behavior investigation) is applied for creating the bonding-based approach of FRP strengthening systems to concrete. Herein, the validation by comparison between the bond model and the beams’ test results is first presented in the following section.

\[ L_{\bar{p}} = \frac{1}{N_f} \sum_{i=1}^{N_f} L_{\bar{p}} \]  

(7)

Adopting Section 2 and the approach assumption in Figs. 6(a) to 6(c), the interfacial fracture energy \( (G_i) \) and the theoretical maximum tensile force in FRP \( (P_{\text{max}}) \) of the converting bond specimens can be defined by Eqs. (8) and (9), respectively. The maximum tensile force in FRP \( (P_{\text{max}}) \), which is equated to the bond force by considering equilibrium equation, in Eq. (9) is determined based on the material properties \( (E_r, A_r) \) of FRP and maximum strain measured in FRP. It is noted that the bond properties illustrated by Eqs. (8) and (9) contain the bond parameters \( A \) and \( B \), which are dependent on the bond length.

\[ G_i = \int_0^s \tau ds = \frac{E_r A_r}{p_r} \times A_b \times \left( \frac{1}{2} e^{-\frac{s}{\varepsilon_{\text{sf}}} A_r} - e^{-\frac{s}{\varepsilon_{\text{sf}}} A_r} + \frac{1}{2} \right) \]  

(8)

\[ P_{\text{max}} = E_r A_r \varepsilon_{\text{max}} = E_r A_r A = E_r \frac{2G_i}{E_r A_r} \]  

(9)

where \( \tau \) is the bond stress, \( s \) is the slip, \( E_r \) is the Young’s modulus of FRP bar/laminate, \( A_r \) is the cross section area of NSM-FRP, \( p_r \) is the perimeter of FRP, \( \varepsilon \) is the strain, \( G_i \) is the interfacial fracture energy, \( s_j \) is the slip at onset of debonding and \( P_{\text{max}} \) is the maximum tensile force in FRP. Note that the notations are consistent throughout the paper and the units are exchanged to be consistent in the calculation. From the equilibrium described in Fig. 6(c), the force equilibrium can be expressed as Eq. (10a). By substituting Eq. (2) into Eq. (10a), the bond parameter \( A \) can be obtained as Eq. (10b). It can be recognized that the slip \( s_j \) will be eliminated when substituting \( A \) [in Eq. (10b)] into Eq. (8).

\[ p_r \tau_m \overline{L_{\bar{p}}} = A_r E_r \varepsilon \]  

(10a)

\[ A = \frac{p_r \tau_m \overline{L_{\bar{p}}}}{A_r E_r \left( 1 - e^{-s_j A_r} \right)} \]  

(10b)

According to Eq. (4), the bond ductility \( B \) (1/mm) can be derived as follows.

\[ B = \frac{\ln 2}{s_m} \]  

(11)

For model validation purpose, two test beams in previous studies by Bui et al. (2020b) (beam name: B4) and Dias and Barros (2008) (beam name: 2S-3L160) for the ETS and NSM methods are utilized. The inclinations for the ETS and NSM systems in their studies are 45º and 60º, respectively. The strain values in the ETS bars and NSM rods recorded respectively at strain gauges SG7 in ETS-beam B4 and SG_L2 in NSM-beam 2S-3L160, which were located at the main shear cracking planes in the investigated beams as demonstrated in Fig. 7(a), are used. The properties and test data of those specimens can be found in their works. In this investigation, two techniques to derive the coefficients \( A \) and \( B \) for bond model from the beams’ tests are examined. The descriptions for two techniques are as follows. One is the monitored \( A \) and \( B \) technique, i.e. using maximum strain \( (\varepsilon_{\text{max}}) \) measured in FRP and then utilizing Eqs. (9) and (11) to compute the \( A \), \( B \) factors directly. It means that from Eq. (9) the factor \( A \) equaled to maximum strain in FRP \( (\varepsilon_{\text{max}}) \). Furthermore, the slip \( s_m \) at peak bond stress \( \tau_m \) is obtained based on Eqs. (12) and (13); therefore, the factor \( B \) is calculated by Eq. (11). Another is the regressing \( A \) and \( B \) technique, i.e. using Eq. (2) to obtain the \( A \) and \( B \) to curve fitting with experimental results obtained from the relationship between slip [computed by Eq. (12)] and measured strain in FRP. To examine the local bond behavior at the interfacial locations employing the strain values recorded in the strengthening systems, the \( \tau - s \) curves are achieved by Eqs. (12) and (13). For the beams with anchored ETS bars, as described in Fig. 6(d), the slip at the anchored end was assumed to be zero as a boundary condition. Actually, it is explained by Bui et al. (2020a, 2020c), the anchored end at \( x = 0 \) (mm) was strongly constrained by the anchoring nuts [as demonstrated in Fig. 6(d)]. This leads the strain at anchored end to be extremely small in anchorage, therefore the slip at the starting point of anchorage is negligible.

\[ s_i = \frac{Ax}{2} \left( \varepsilon_{\text{sf}} + 2 \sum_{j=1}^{i-1} \varepsilon_j + \varepsilon_i \right) \]  

(12)

\[ \tau_r = \frac{E_r A_r \varepsilon_{\text{sf}} - \varepsilon_{\text{si}}}{p_r} \Delta x \]  

(13)

Figures 7(b) and 7(c) present the comparison between the bond model curves and the test curves in terms of the bond stress and slip relationship. In general, the bond model established in Sections 2 and 3 could
well perform for the prediction of the real interfacial behaviors for both ETS and NSM cases in the strengthened members. The values of bond factors \((A, B)\) for two techniques are also provided in Figs. 7(b) and 7(c). Particularly, in Fig. 7(b) for ETS method, the bond response of ETS bars to concrete using the interfacial model accurately agreed with the bond response obtained directly from the beams’ tests. Furthermore, the

![Fig. 7 Validation of a local bond-slip curves in the beam tests: (a) positions to measure strain in strengthening system; (b) at SG7 in ETS FRP strengthening bars; (c) at SGL2 in NSM FRP strengthening bars.](image-url)
values of $A$, $B$ derived from two techniques are similar. Meanwhile, the bond behavior between the NSM bars and concrete in the intervened beams defined by the model is distinct with that achieved from the beams’ tests. Especially, the bond response resulted by the regressing $A$, $B$ offers the overestimation in comparison with the bond response found from the experiments. The values of $A$, $B$ determined from two techniques are remarkably different.

These aforementioned findings are because in the ETS-strengthened beam, the ETS bars inclined at 45º made the interfacial behavior of the ETS bars to concrete similar to the direct pullout (i.e. the direction of the forces impacted the ETS 45º-inclined bars is perpendicular to the shear cracking plane). Whereas, in the NSM-strengthened beam, the NSM system inclined at 60º might cause the NSM-concrete interfacial behavior to be different to the direct pullout (i.e. the direction of the forces induced the NSM 60º-inclined bars is not perpendicular to the shear cracking plane). This results in the existence of the bending effect to the NSM bars, leading to the discrepancy between the $r$-$s$ curves derived from the model and the $r$-$s$ relationship defined from the beams’ tests. Although there was a slight difference in the results obtained from the bond model and the actual experiments, the proposed interfacial model can be used to analyze the bond performance of the FRP strengthening systems to concrete in the FRP-intervened members.

3.2 Shear resistance-related bonding

Sections 2 and 3.1 indicate that the proposed bond approach can represent the interfacial profiles of the ETS and NSM systems to concrete in the shear retrofitted beams. If a beam failed without FRP rupture, the shear resistance of the ETS-FRP or NSM- FRP system is mainly governed by the bond force of that ETS-FRP or NSM-FRP system embedded in concrete. The assumptions for the proposed model are as follows. The FRP strengthening bars (or laminates) intersected by main shear crack in a shear region of a RC beam was converted to be an equivalent concrete block embedded by an equivalent FRP element, which the average bond lengths of influenced strengthening elements associating with the dimensions and geometries are described in Figs. 6(a) and 6(b). Therefore, the bond force of FRP to concrete, which governed the shear contribution of FRP strengthening system, could be derived in the equivalent concrete block via the validated bond law. Then, the bond force would be exchanged to the shear contribution of the FRP strengthening system in a RC beam through mathematical expressions. Thereby, the theoretical approach presented in the above sections can be used to derive the bond-based shear resisting forces of the FRP strengthening systems.

Through the experimental parameters together with Eqs. (5), (8), (9), (10) and (11), the bond resisting forces (i.e. the shear contribution) of the ETS and NSM strengthening systems are expressed as Eqs. (14) and (15), respectively. The $P_{\text{max}}$ in Eqs. (14) and (15) is the maximum bond force obtained from the equivalent concrete block embedded by the average bond length of the influenced strengthening elements via the validated bond model. Then, the shear resisting forces ($V_{b}^p$) will be derived by the product of $P_{\text{max}}$ and number of the influenced strengthening elements. The magnitude of $P_{\text{max}}$ depends on the bond factors, the crack angle and the strengthening inclination. Note that the factors $A$, $B$, $G_f$ are determined from the existing bond tests for all techniques by using Eqs. (8), (10a), (10b) and (11). It means that if the bond behaviors ($\tau_{\text{m}}$, $s_{\text{r}}$, and $s_{\text{i}}$) of the FRP strengthening system to concrete are known, the shear contribution of that FRP system in a strengthened beam can be derived. With this method, there is neither empirical nor regressing parameter needed for computing the shear contribution of the FRP retrofitting systems.

In the ETS method, single FRP bar is embedded through the beams’ sections, thereby the shear contribution of ETS-FRP strengthening system in ETS-FRP strengthened beams is written as follows.

$$V_{b_E T S}^p = N_f P_{\text{max}} = N_f E_f A_f \sqrt{2G_f \frac{P_f}{E_f A_f}}$$  \hspace{1cm} (14)

In the NSM method, two FRP laminates (or rods) are placed at two outer sides of the beams’ sections, thereby the shear contribution of NSM-FRP strengthening system in NSM-FRP strengthened beams is written as follows.

$$V_{b_N S M}^p = 2N_f P_{\text{max}} = 2N_f E_f A_f \sqrt{2G_f \frac{P_f}{E_f A_f}}$$  \hspace{1cm} (15)

On the other hand, in the ACI 440.1R-15 (ACI 2015), the shear contribution of the ETS-strengthening and NSM-strengthening bars using truss analogy is computed as follows.

$$V_{f} = A_f f_f \frac{z (\cot \theta + \cot \alpha)}{s} \sin \alpha = A_f E_f \varepsilon_{\mu} \frac{z (\cot \theta + \cot \alpha)}{s} \sin \alpha$$  \hspace{1cm} (16a)

$$f_f = \min \left[ 0.004 E_f, f_{f,u}, f_{f,bond} = \left( \frac{0.05 \sigma_y + 0.30}{d_{\sigma}} \right) f_{f,u} \right]$$  \hspace{1cm} (16b)

From Eqs. (14), (15), (16a), the effective strain of the ETS-FRP and NSM FRP systems can be derived as below.

$$\varepsilon_{\mu} = \frac{V_{f}^p \times s}{A_f E_f z (\cot \theta + \cot \alpha) \sin \alpha}$$  \hspace{1cm} (17)

where $z$ = effective depth of section (mm); $A_f$ = cross section area of ETS-FRP or NSM-FRP (mm²); $f_f$ = effective stress in FRP (MPa); $f_{f,u}$ = the ultimate strength of the FRP bar (MPa); $f_{f,bond}$ = the tensile strength of the
FRP bent bar (MPa); \( r_b \) = the bending radius of the FRP bar (mm); \( d_b \) = the diameter of the FRP bar in the bent portion (mm).

4. Validation of the model

In this section, the accuracy of the model for estimating the shear contribution \( V_f \) is evaluated by comparing the experimental and analytical results of the beams intervened in shear with ETS-FRP and NSM-FRP systems. Because the proposed model needs the values (\( \tau_{en}, s_{ef} \)) in the ETS and NSM bond response, the beams in previous studies for which the corresponding interfacial mechanism was known are considered. All beams strengthened with ETS bars are considered for the model verification. Although the experimental tests on the beams retrofitted with NSM composites are also available in the literature, they are not necessarily considered. This is because (1) the bond behavior in those studies was not clear. (2) In general, the proposed model is considering mainly the mechanical and mathematical aspects.

Fourteen beams with ETS-strengthening bars tested by Bui et al. (2020b), Breveglieri et al. (2015) and Mofidi et al. (2012), and 48 beams with NSM-strengthening laminates tested by Dias and Barros (2013) are utilized. The bond behaviors to determine the \( \tau_{en}, s_{ef}, s_{fy} \) values are collected from the studies by Lorenzis and Nanni (2002), Godat et al. (2012), Caro et al. (2017) and Bui et al. (2020a) considering the correlations in the configuration and material properties between the bond test specimens and the NSM-FRP-retrofitted or ETS-FRP-strengthened beams.

All investigated beams failed by concrete fractures in the shear zones of the beams or debonding of FRP strengthening system to concrete. Ruptures of FRP-strengthening bars and laminates were not observed in the test beams. These findings in failure mode of the examined beams match completely to the assumptions of bonding-based approach. The details of the test beams such as beams’ configurations and dimensions, material properties, and tested results of the ETS and NSM strengthening systems are shown in Table 3. Figure 8 and Table 3 depict the comparison on the ETS-FRP or NSM-FRP shear contribution between the experiment results and the bonding-based model values. As a result, the shear contributions of the FRP strengthening systems in the RC beams predicted by the bond-based approach agreed well with the shear resisting forces of the FRP retrofitting systems determined from the tests. In addition, the mean value of the ratio of the calculated results to the measured results is 1.06 and the coefficient of variation (COV) of the mean by 26.5% is found. Owing to the scatter of the experimental shear contribution of some specimens, the COV is slightly high. Indeed, the test value for shear resistance of the ETS-strengthening bars in the beam C3 with ETS method was significantly lower than the predicted value. The fact is that the beam C3 failed due to shear deformation that resulted from premature debonding between the ETS-FRP-strengthening bars and concrete. Actually, by ignoring the data of specimen C3, the new mean value for the ratio between shear contribution values of analytical model to experimental measurement is 1.02 and the COV of the mean is 7.45%. This finding indicates that the proposed model for estimation of FRP strengthening systems in RC beams achieves acceptable reliability. Furthermore, Fig. 9 presents the relationship between the effective strain in the FRP strengthening systems calculated by Eq. (17) and the total axial stiffness of both FRP strengthening materials and existing steel stirrups. This relationship indicates that the FRP effective strain decreased as the total rigidity of shear reinforcement increased. This phenomenon completely matched with the results reported in the literature, such as Sato et al. (1994) and Bui et al. (2020b). Therefore, the accuracy of the proposed method for the shear contribution prediction of the FRP retrofitting systems in the RC beams is attributed to be acceptable.

![Fig. 8 Validation in shear contribution of FRP strengthening systems.](image-url)

![Fig. 9 Relationship between effective strain in FRP strengthening systems and total axial stiffness of combined FRP-strengthening systems and existing steel stirrups.](image-url)
Table 3 ETS and NSM strengthening configurations and test results.

| Beam ID | $f'_c$ (MPa) | $\rho_{ws}$ (%) | Number of ETS/NSM bars | $E_r$ (MPa) | $\epsilon$ (µ) | $A_r$ (mm²) | $h_s$ (mm) | $\theta$ (º) | $\alpha$ (º) | $V_{/Model}$ (kN) | $V_{/Exp.}$ (kN) |
|---------|--------------|-----------------|-------------------------|-----------|-------------|------------|--------|--------|--------|----------------|----------------|
| ETS method | | | | | | | | | | | | |
| B1 | 38 | 0.11 | 5 | 50000 | 21520 | 78.5 | 31.4 | 400 | 180 | 180 | 45 | 90 | 124.8 | 138.2 |
| B2 | 38 | 0.11 | 5 | 50000 | 21520 | 78.5 | 31.4 | 400 | 180 | 180 | 45 | 45 | 154.8 | 154.8 |
| B3 | 38 | 0.24 | 5 | 50000 | 21520 | 78.5 | 31.4 | 400 | 180 | 180 | 45 | 90 | 96.6 | 101.9 |
| B4 | 38 | 0.24 | 5 | 50000 | 21520 | 78.5 | 31.4 | 400 | 180 | 180 | 45 | 45 | 151.0 | 146.7 |
| C1 | 29.7 | 0.11 | 5 | 160000 | NA | 50.3 | 25.1 | 400 | 180 | 180 | 45 | 90 | 84.8 | 77.1 |
| C2 | 29.7 | 0.11 | 5 | 160000 | NA | 50.3 | 25.1 | 400 | 180 | 180 | 45 | 45 | 178.1 | 175.6 |
| C3 | 29.7 | 0.24 | 5 | 160000 | NA | 50.3 | 25.1 | 400 | 180 | 180 | 45 | 90 | 15.7 | 13.9 |
| C4 | 29.7 | 0.24 | 5 | 160000 | NA | 50.3 | 25.1 | 400 | 180 | 180 | 45 | 45 | 147.3 | 157.8 |
| D1 | 25 | NA | 8 | 148000 | 12736 | 126.7 | 39.9 | 406 | 142 | 130 | 45 | 90 | 100.2 | 99.5 |
| D2 | 29.6 | 0.38 | 4 | 148000 | 12736 | 70.9 | 29.8 | 406 | 142 | 260 | 45 | 90 | 28.6 | 28.1 |
| D3 | 29.6 | 0.38 | 4 | 148000 | 12736 | 70.9 | 29.8 | 406 | 142 | 260 | 45 | 90 | 34.1 | 34.4 |
| D4 | 25 | NA | 8 | 148000 | 12736 | 126.7 | 39.9 | 406 | 142 | 260 | 45 | 90 | 37.5 | 31.4 |
| D5 | 29.6 | 0.38 | 4 | 155000 | 12736 | 70.9 | 29.8 | 406 | 142 | 260 | 45 | 90 | 45.7 | 48.5 |
| D6 | 29.6 | 0.38 | 8 | 148000 | 12736 | 126.7 | 39.9 | 406 | 142 | 130 | 45 | 90 | 87.5 | 87.1 |
| NSM method | | | | | | | | | | | | |
| 2S-3LV-A | 31.1 | 0.1 | 3 | 166000 | 17700 | 14.0 | 22.8 | 300 | 180 | 267 | 45 | 90 | 14.1 | 15.7 |
| 2S-5LV-A | 31.1 | 0.1 | 5 | 166000 | 17700 | 14.0 | 22.8 | 300 | 180 | 160 | 45 | 90 | 39.7 | 40.3 |
| 2S-8LV-A | 31.1 | 0.1 | 8 | 166000 | 17700 | 14.0 | 22.8 | 300 | 180 | 100 | 45 | 90 | 63.4 | 63.7 |
| 2S-3LI45-A | 31.1 | 0.1 | 3 | 166000 | 17700 | 14.0 | 22.8 | 300 | 180 | 367 | 45 | 45 | 38.6 | 37.9 |
| 2S-5LI45-A | 31.1 | 0.1 | 5 | 166000 | 17700 | 14.0 | 22.8 | 300 | 180 | 220 | 45 | 45 | 57.9 | 56.5 |
| 2S-8LI45-A | 31.1 | 0.1 | 8 | 166000 | 17700 | 14.0 | 22.8 | 300 | 180 | 138 | 45 | 45 | 74.0 | 70.3 |
| 2S-3LI60-A | 31.1 | 0.1 | 3 | 166000 | 17700 | 14.0 | 22.8 | 300 | 180 | 325 | 45 | 60 | 35.9 | 35.4 |
| 2S-5LI60-A | 31.1 | 0.1 | 5 | 166000 | 17700 | 14.0 | 22.8 | 300 | 180 | 195 | 45 | 60 | 58.2 | 61.3 |
| 2S-8LI60-A | 31.1 | 0.1 | 8 | 166000 | 17700 | 14.0 | 22.8 | 300 | 180 | 139 | 45 | 60 | 68.3 | 69.7 |
| 2S-4LV-B | 39.7 | 0.1 | 4 | 170900 | 16000 | 13.3 | 21.8 | 300 | 180 | 180 | 45 | 90 | 23.0 | 20.2 |
| 2S-7LV-B | 39.7 | 0.1 | 7 | 170900 | 16000 | 13.3 | 21.8 | 300 | 180 | 114 | 45 | 90 | 58.5 | 57.5 |
| 2S-10LV-B | 39.7 | 0.1 | 10 | 170900 | 16000 | 13.3 | 21.8 | 300 | 180 | 80 | 45 | 90 | 72.7 | 71.5 |
| 2S-4LI45-B | 39.7 | 0.1 | 4 | 170900 | 16000 | 13.3 | 21.8 | 300 | 180 | 275 | 45 | 45 | 51.4 | 53.4 |
| 2S-7LI45-B | 39.7 | 0.1 | 7 | 170900 | 16000 | 13.3 | 21.8 | 300 | 180 | 157 | 45 | 45 | 71.7 | 70.7 |
| 2S-10LI45-B | 39.7 | 0.1 | 10 | 170900 | 16000 | 13.3 | 21.8 | 300 | 180 | 110 | 45 | 45 | 88.4 | 85.6 |
| 2S-4LI60-B | 39.7 | 0.1 | 4 | 170900 | 16000 | 13.3 | 21.8 | 300 | 180 | 243 | 45 | 60 | 50.4 | 49.6 |
| 2S-6LI60-B | 39.7 | 0.1 | 6 | 170900 | 16000 | 13.3 | 21.8 | 300 | 180 | 162 | 45 | 60 | 52.3 | 54.4 |
| 2S-9LI60-B | 39.7 | 0.1 | 9 | 170900 | 16000 | 13.3 | 21.8 | 300 | 180 | 108 | 45 | 60 | 69.6 | 65.3 |

(continued in next page)
Table 3 ETS and NSM strengthening configurations and test results (continued from previous page).

| Beam ID       | $f'_c$ (MPa) | $\rho_{re}$ (%) | Number of ETS/NSM bars | $E_r$ (GPa) | $c$ (µ) | $A_r$ (mm²) | $P_r$ (mm) | $h_n$ (mm) | $\varepsilon_n$ (µ) | $hw$ (mm) | $bw$ (mm) | $sfw$ (mm) | $\theta$ (º) | $\alpha$ (º) | $V_f$ (kN) | $V_f$ (kN) |
|---------------|--------------|-----------------|------------------------|-------------|---------|------------|------------|------------|-----------------|-----------|-----------|------------|-------------|-------------|-----------|-----------|
| 4S-4LV-B      | 39.7         | 0.17            | 4                      | 170900      | 16000   | 13.3       | 21.8       | 300        | 180             | 45         | 90        | 30.9       | 31.9        |             |           |           |
| 4S-7LV-B      | 39.7         | 0.17            | 7                      | 170900      | 16000   | 13.3       | 21.8       | 300        | 180             | 45         | 90        | 34.2       | 33.6        |             |           |           |
| 4S-4LI45-B    | 39.7         | 0.17            | 4                      | 170900      | 16000   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 41.4       | 42.7        |             |           |           |
| 4S-7LI45-B    | 39.7         | 0.17            | 7                      | 170900      | 16000   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 62.9       | 64          |             |           |           |
| 4S-4LI60-B    | 39.7         | 0.17            | 4                      | 170900      | 16000   | 13.3       | 21.8       | 300        | 180             | 45         | 60        | 43.0       | 43.5        |             |           |           |
| 4S-6LI60-B    | 39.7         | 0.17            | 6                      | 170900      | 16000   | 13.3       | 21.8       | 300        | 180             | 45         | 60        | 52.3       | 51.7        |             |           |           |
| 2S-7LV-C      | 18.6         | 0.1             | 7                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 90        | 43.7       | 43.6        |             |           |           |
| 2S-4LI45-C    | 18.6         | 0.1             | 4                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 32.2       | 33.9        |             |           |           |
| 2S-7LI45-C    | 18.6         | 0.1             | 7                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 52.1       | 48          |             |           |           |
| 2S-4LI60-C    | 18.6         | 0.1             | 4                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 60        | 35.3       | 33.1        |             |           |           |
| 2S-6LI60-C    | 18.6         | 0.1             | 6                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 60        | 41.4       | 42.7        |             |           |           |
| 4S-4LI45-C    | 18.6         | 0.1             | 7                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 90        | 8.1        | 6.8         |             |           |           |
| 4S-7LI45-C    | 18.6         | 0.1             | 4                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 28.6       | 26          |             |           |           |
| 4S-4LI60-C    | 18.6         | 0.1             | 7                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 40.0       | 31.6        |             |           |           |
| 4S-6LI60-C    | 18.6         | 0.1             | 4                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 60        | 28.7       | 25.1        |             |           |           |
| 3S-6LV-D      | 59.4         | 0.1             | 6                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 90        | 41.3       | 44.7        |             |           |           |
| 3S-10LV-D     | 59.4         | 0.1             | 10                     | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 90        | 82.7       | 81.5        |             |           |           |
| 3S-5LI45-D    | 59.4         | 0.1             | 5                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 80.3       | 81.7        |             |           |           |
| 3S-9LI45-D    | 59.4         | 0.1             | 9                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 119.9      | 117.4       |             |           |           |
| 3S-5LI60-D    | 59.4         | 0.1             | 5                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 60        | 125.2      | 127.9       |             |           |           |
| 3S-5LI45F-D   | 59.4         | 0.1             | 5                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 88.9       | 85.8        |             |           |           |
| 3S-5LI45F2-D  | 59.4         | 0.1             | 5                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 81.7       | 80.9        |             |           |           |
| 3S-5LI45-D    | 59.4         | 0.1             | 5                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 45        | 72.2       | 74.9        |             |           |           |
| 3S-5LI60-D    | 59.4         | 0.1             | 5                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 60        | 124.2      | 122.5       |             |           |           |
| 3S-5LI60F-D   | 59.4         | 0.1             | 5                      | 174300      | 16300   | 13.3       | 21.8       | 300        | 180             | 45         | 60        | 102.2      | 101.1       |             |           |           |
In the case of not knowing the bond behavior of the FRP strengthening system to concrete, the values of $\tau_m$, $s_m$, $s_f$ can be estimated by the available formulations in the existing guidelines and codes, such as the ACI 440.1R-15 (ACI 2015). Obviously, the values for the ETS or NSM FRP-concrete interfacial profiles derived from the guidelines and codes do not gain high accuracy in its applications, which have been reported by many researchers. However, it is simply used when all necessary factors are not available. Thereby, Fig. 10 shows the model validations, in which the bond behavior parameters with maximum bond stress $\tau_m$ (MPa) expressed as Eq. (18) and slip at peak bond stress $s_m$ = 1 (mm) were respectively utilized from the ACI 440.1R-15 (ACI 2015) and Harajli and Abouniaj (2010), in terms of the shear contributions of the ETS and NSM intervening systems by comparison with those in the experiments.

$$\tau_m = 0.083f'_c \sqrt{4 + 0.3c/d_b + 100d_s/l_e}$$

(18)

where $f'_c$ = concrete compressive strength (MPa); $d_b$ = bar diameter (mm); $c$ = the lesser of the cover to the center of the bar (mm); $l_e$ = embedded length of reinforcing bar (mm).

Figure 10 also presents the comparisons in the FRP-strengthening shear contributions between the results obtained by the developed approach and the ACI 440.1R-15 guide (ACI 2015). Although the insufficiency of the actual bond behavior data exists, the predictions made by the proposed model are found to be effective as follows. The results calculated by the bonding-based approach with available bond parameters in the ACI guide provide considerable validation to the test data compared to the results derived from the model of the same guide. In fact, for the proposed model, the mean of the ratio of the calculated values to measured values and the coefficient of variation (COV) of the mean are respectively 1.1 and 65.4%. While, for the results predicted by the ACI guide, the mean of the ratio of the calculated values to measured values and the coefficient of variation (COV) of the mean are respectively 0.81 and 155.8%. By adopting the model in the ACI guide, the shear contributions of the FRP strengthening systems significantly underestimated the actual values. It is mainly due to the underestimation of the FRP effective stress equation [Eq. (16b)], which was offered by the ACI guide, compared to the real data.

Because the proposed analytical model is governed by the interfacial behavior between FRP strengthening system and concrete, the bond parameters affected substantially the calculated results. Thereby, as observed in Fig. 10, although most of the calculated values underestimated the tested values, the validation reveals the existence of overestimation of some predicted results compared to the experimental data. This is mainly because the bond responses for the FRP-concrete interface defined from the codes and guidelines are differed from its actual interfacial mechanism. The overestimation may pose a threat to the safety of structures, therefore the bond parameters derived from the existing codes or guidelines should be refined to improve their accuracy. Additionally, the scatter of the experimental results and the assumption of the shear cracking plane inclined at 45º may also affect the results computed by the model.

Based on the above discussions and interpretations, the proposed bonding-based approach considering mainly the mechanical and mathematical aspects can be a predictive model to estimate the shear contributions of the FRP retrofitting systems in the RC beams. With the accurate bond behavior obtained from the tests, the shear resisting forces of the FRP strengthening composites are well fitted to the experimental results. However, the data for the bond behavior may actually be limited. Therefore, the lack of the bond behavior parameters in using of the proposed method can be overcome by utilizing the bond data available in the existing guidelines.

![Fig. 10 Comparisons between tests, ACI 440.1R-15, and proposed model using bond behavior from guideline in ACI 440.1R-15.](image-url)
and codes. In spite of the fact that the adoption of the interfacial parameters in the current codes and guidelines decreases the accuracy of the model, the prediction created by the bonding-based approach is acceptable to avoid the very underestimation in the prediction made by ACI 440.1R-15. In future research, the bonding-based approach should be further investigated to enhance the accuracy of the model in cases of using bond parameters from the current codes and guidelines.

5. Conclusions

This study assesses the shear resisting forces of the ETS and NSM strengthening systems in the RC beams through the bonding-based method. This method considers mainly the mechanical and mathematical aspects, in which the few bond factors are required. In addition, this is a very promising study since the results achieved illustrated the efficiency of the bonding-based approach for estimation of the shear contribution of the FRP strengthening systems via the ETS and NSM methods. The primary findings are explicit in this study as follows. Based on the validation against the experimental data, the bond model proposed by Dai et al. (2005) and Bui et al. (2020a) could represent for the interfacial mechanism between the ETS-FRP and NSM-FRP systems (rod, strip, and laminate) and concrete with high accuracy. The model results indicated that, with the same pullout force, the concrete prism embedded by ETS bar offered the bond efficiency significantly higher than that embedded with NSM rod.

In this study, the proposed theoretical method, which was to convert the shear resistance of the complicated arrangements of the FRP strengthening systems in the RC beams into the bond behavior of the FRP elements to concrete, gained great reliability. Therein, the validated bond model could effectively interpret the $\tau_s$ characteristics of the ETS-FRP or NSM-FRP retrofitting systems-concrete interfaces in the strengthened beams. Furthermore, when exact interfacial factors, $f_m$ and $s_m$, were known from the bond tests, the proposed method for estimation of the shear contribution of the ETS-FRP or NSM-FRP strengthening system in the FRP-strengthened RC beams made good appraisal against the experimental results. In cases of the deficiency of exact interfacial parameters, the proposed model combined with the bond factors furnished in the available codes is sufficiently effective to predict the ETS-FRP or NSM-FRP shear resistance rather than the existing shear models. On the other hand, the examples made the application of the bonding-based model for calculation of the shear contributions of the ETS-FRP-strengthening and NSM-FRP-strengthening bars more explicit. In practical use, when the prediction of the shear contribution for the FRP retrofitting in the beams requires high accuracy, the bond tests of the FRP strengthening systems to concrete should be carried out to obtain accurate bond factors $f_m$, $s_m$, $S_Y$. Further investigation on the estimation of shear contribution for FRP strengthening system in the beams utilizing the bonding-based approach is broadly needed to enhance the applicability and accuracy of the model. In addition, the validation of the proposed model to the other models, which used the approach based on the interfacial constitutive law (for example, the model of Kamiharako et al. (2001)), for estimation of the FRP strengthening shear contribution should be made to assess the simplicity of the model for the practical application.

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Appendix

Examples for using proposed method

These examples show the computation of the contributions of the shear ETS and NSM FRP strengthening bars in the RC beams. As presented in Fig. 11(a), the beams investigated are in T-shaped cross-sections containing the tension reinforcement and steel stirrups. The FRP rods are inserted in the shear zone of those beams through the ETS and NSM methods. The following sections present the shear contribution prediction of the FRP-retrofitting bars in the RC beams.

A.1 RC beam properties
(1) Flange width \( B = 480 \text{ mm} \);
(2) Web width \( b_w = 200 \text{ mm} \);
(3) Height of beams \( h = 450 \text{ mm} \);
(4) Height of web \( h_w = 350 \text{ mm} \);
(5) Effective depth considering the distance from the extreme top to the centroid of the tension bars \( d = 420 \text{ mm} \);
(6) Shear span length \( a = 900 \text{ mm} \);
(7) Tension reinforcement percentage \( \rho_s = 2.80 \% \);
(8) Existing steel stirrups percentage \( \rho_{sw} = 0.14 \% \);
(9) Assumption of the shear cracking angle \( \theta = 45^\circ \).

A.2 ETS and NSM FRP properties
(1) Bar diameter \( d_b = 10 \text{ mm} \);
(2) Tensile strength of FRP bars \( f_t = 2600 \text{ MPa} \);
(3) Elastic modulus of FRP bars \( E_f = 160 \text{ GPa} \);
(4) Amount of FRP-strengthening bars in the shear zone is four bars spaced 225 mm;
(5) ETS and NSM bars are inclined at 45° for arrangement with length of each ETS bar is 637 mm and length of each NSM bar is 495 mm;
(6) Bond behavior for ETS method \( \tau_m = 6.3 \text{ MPa}, s_f = 0.25 \text{ mm}, s_f = 2.5 \text{ mm} \);
(7) Bond behavior for NSM method \( \tau_m = 4.8 \text{ MPa}, s_m = 0.25 \text{ mm}, s_f = 2.5 \text{ mm} \).

A.3 Shear contribution of ETS or NSM intervening system in the strengthened RC beams

Following the procedure summarized in Fig. 11(b), the effective length of the effective strengthening systems carrying the shear resistance can be calculated as Eqs. (6), (7) and (8). Therefore, the results are shown as follows.

For ETS method:
\[
\bar{L}_h = \frac{1}{N_f} \sum_{i=1}^{N_f} L_k = 159.1 \text{ mm}
\]  
(19)

For NSM method:
\[
\bar{L}_h = \frac{1}{N_f} \sum_{i=1}^{N_f} L_k = 117.9 \text{ mm}
\]  
(20)

Using Eqs. (8), (10b) and (11), the values of \( G_f, A, B \) can be obtained as below by Eqs. (21), (22) and (23) for the ETS method and by Eqs. (24), (25) and (26) for the NSM method.

\[
\text{Beam configurations and material properties}
\]
\[
\text{Converting shear configurations to an equivalent block subjected to pullout force}
\]
\[
\text{Calculate shear contribution of strengthening system in a beam}
\]
\[
\text{Collect the available test data or use guidelines or conduct pullout tests}
\]

Fig. 11 (a) Configurations of beams for examples; (b) procedure for calculation of shear contribution of FRP strengthening system.
Then, adopting Eqs. (14) and (15), the shear contributions of the ETS and NSM retrofitting systems can be derived from Eqs. (27) and (28), respectively.

\[ V_f^\text{ET} = N_f P_{\text{max}} = N_f E_i A_i \sqrt{2G_i \frac{P_i}{E_i A_i}} = 126.0 \text{ kN} \]  
\[ V_f^\text{NSM} = 2N_f P_{\text{max}} = 2N_f E_i A_i \sqrt{2G_i \frac{P_i}{E_i A_i}} = 106.6 \text{ kN} \]