Hadronic spectra in AdS / QCD correspondence

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Abstract. We present an holographical soft wall model which is able to reproduce Regge spectra for hadrons with an arbitrary number of constituents. The model includes the anomalous dimension of operators that create hadrons, together with a dilaton, whose form is suggested by Einstein’s equations and the AdS metric.

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INTRODUCTION

From its beginnings, progress in QCD at low energies has been impeded because there are no good analytical tools available in order to work with strongly coupled Yang Mills theories. Nevertheless, in the last few years the AdS / CFT ideas has provided a new approach that could improve this situation.

At present a dual to QCD is unknown, but a simple approach known as Bottom - Up has been quite successful in several concrete QCD applications, such as in hadronic scattering processes [1], hadronic spectra [2, 3, 4, 5], hadronic couplings and chiral symmetry breaking [6], mesonic wave function [7], among other applications.

Here we summarize the main ideas developed in [4, 5], where a soft wall holographical model that describes hadronic spectra with an arbitrary number of constituents was proposed.

The present work has been structured as follow. Section II is a summary of the model considered. In section III we give some examples, and finally in IV we present some conclusions.

SPECTRA OF HADRONS IN ADS / QCD.

We begin by considering an asymptotically AdS space defined by

\[ ds^2 = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu), \] (1)

and an action for arbitrary spin modes (which depends on the spin of the hadron described).

For modes with integer arbitrary spin the corresponding equation of motion is [5]

\[ \partial_\zeta^2 \phi - \left[ \partial_\zeta (\Phi(z) - \beta A(z)) \right] \partial_\zeta \phi + [M^2 - m_\zeta^2 e^{2A(z)}] \phi = 0, \] (2)
where $\beta = k(2S - 1)$, $k$ is a constant and $S$ corresponds to the spin of the mode considered and $\Phi(z)$ is a dilaton field. We fix $\beta$ for each spin mode using experimental data.

Consider an AdS space, i.e. $e^{2A(z)} = R^2 z^2$, with a quadratic dilaton ($\Phi(z) = \kappa^2 z^2$, and with $\kappa = \text{constant}$). This allows us to obtain spectra with Regge behavior.

Nevertheless, according to [8], in the second order Dirac equation the dilaton in AdS can be factorized, and the equation looks like

$$\partial_z^2 f_{\pm} - \frac{4}{z} \partial_z f_{\pm} + \left[ M^2 + \frac{6}{z^2} - \frac{m_5^2 R^2}{z^2} + \gamma m_5 R \right] f_{\pm} = 0,$$

where $\gamma = \pm 1$, depending on whether we are considering the left or right part.

When $m_5 R$ is constant, the spectra obtained don’t have Regge behavior.

Since in this case the dilaton field cannot improve the situation with respect to hard wall models, it is necessary to try other possibilities, for example a trivial dilaton but with a family of metrics [3, 9], or consider an AdS metric with different dilaton and include anomalous dimension for the operators that create the hadrons, as considered in [5].

Einstein’s equations determine the dilaton directly from the metric ($\Phi’ = \sqrt{3A'^2 - 3A''}$)[9]. So, our model is defined by $A(z) = \rho \ln(R/z)$, then $\Phi(z) = \lambda \ln(z)$, where $\lambda$ depends on $\rho$, although in order to get equations with exact solutions, we will use $\lambda = 2$.

The AdS / CFT dictionary tell us that the twist dimension of operators on the CFT side and the conformal dimension of the AdS modes must be equal, and this establish possible values for $m_5^2 R^2$, that correspond to [5]

$$m_5^2 R^2 = \begin{cases} (\Delta_0 + L - S + \omega z^2)(\Delta_0 + L - S - 3 + \beta + \omega z^2) ; \text{Integer spin} \\ \Delta_0 + L - S - 3 + \omega z^2 ; \text{Spin 1/2}. \end{cases}$$

where in each case an anomalous dimension was considered for the operators.

**SOME HADRONIC SPECTRUM.**

The spectrum for all cases considered, is [5]

$$M^2 = A[n + L + v],$$

(4)
FIGURE 2. Nucleons and Δ resonances spectra. The continuous line is the model prediction using an universal value of \( A = 1.1 GeV^2 \), while the dashed line was obtained using Regge slopes adjusted to each case, with values \( A = 0.9 GeV^2 \) for nucleons and \( A = 1.01 GeV^2 \) for Δ resonances [3].

where \( A = 4\omega \) is the Regge slope, and \( v \) is given by

\[
v = \begin{cases} \\
\Delta_0 + \frac{\beta}{2} - 1 - S & \text{If } S \text{ is integer} \\
\Delta_0 - \frac{S}{2} & \text{spin } 1/2.
\end{cases}
\]

Notice that the model gives us the Regge slope in terms of \( \omega \), which constitutes a phenomenological input in our model. Equation (4) can be applied to different kinds of hadrons with an arbitrary number of constituents, considering different values for \( \Delta_0 \) and the right value for \( \beta \) depending on the spin, and in general we take \( A \sim 1.1 GeV^2 \), which can be considered approximately universal for all trajectories [10].

The spectrum for some scalar mesons is shown in Fig 1, while some examples about model predictions for scalar exotic hadrons appear in Table 1.

In the scalar case a universal Regge slope, with value \( 1.1 GeV^2 \) and \( \beta = -3 \), was used. \( \Delta_0 \) was calculated considering that each quark and / or antiquarks contribute with \( 3/2 \) to \( \Delta_0 \), and that gluons contribute with \( 2 \) to \( \Delta_0 \).

TABLE 1. Scalar exotic hadron masses, with \( n = L = 0 \). We consider hadrons with \( n \) quarks (and / or antiquarks) and \( m \) gluons.

| \( \Delta_0 \) | \( nQ(mG) \) | \( M \) [GeV] |
|-------------|-------------|--------------|
| 4           | (2G)        | 1.28         |
| 5           | (2Q)(1G)    | 1.66         |
| 6           | (4Q)        | 1.96         |

For other integer spin hadrons it is necessary to do a similar analysis, but take s different value for \( \beta \). For example, in the vector case \( \beta = -1 \) was used in [5].

For the spin 1/2 case, as one can see from Fig 2, using an universal Regge slope gives results somewhat higher than the experimental data, but using a value of 0.9 \([GeV^2]\) [3], adjusted to baryonic data, the results are better. Both values are used in Table 2, where model predictions for some spin 1/2 exotic hadrons are shown. On the other side, solutions to Rarita - Schwinger equation in AdS space are more difficult to get, but its spectrum is similar to the Dirac case, as you can see for example in Ref. [3]. As is possible to see in Fig 2, again the results are somewhat high, but using \( A = 1.01 GeV^2 \), adjusted to Δ resonances gives better results.
TABLE 2. Spin 1/2 exotic hadron masses with \( n = L = 0 \). We consider hadrons with \( n \) quarks (and/or antiquarks) and \( m \) gluons. Column \( M_U \) was calculated using \( A = 1.1 \text{GeV}^2 \), the universal Regge slope used in this work, while \( M \) contains the results obtained using \( A = 0.9 \text{GeV}^2 \), a value fixed from nucleon data [3].

| \( \Delta_0 \) | (nQ)(mG)     | \( M_U \) [GeV] | \( M \) [GeV] |
|----------------|--------------|-----------------|----------------|
| 13/2           | (1Q)(3G)     | 2.10            | 2.01           |
| 15/2           | (5Q)         | 2.35            | 2.25           |
| 17/2           | (3Q)(2G)     | 2.57            | 2.46           |

CONCLUSIONS.

The holographical model discussed here allowed us to obtain hadronic spectra with Regge behavior, not only for the integer spin case, but also for spins 1/2 and 3/2, and also to calculate masses for exotics. In order to do this we considered anomalous dimensions for operators that create hadrons, and the dilaton that was used has a form suggested by Einstein’s equations, corresponding to the AdS metric. This two traits allowed the model to reproduce Regge spectra in all cases considered, and therefore the model can describe hadronic masses in a unified phenomenological model.

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