No Spontaneous CP Violation at Finite Temperature in the MSSM?\textsuperscript{a}

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Abstract

In order to generate the baryon asymmetry of the Universe sufficiently strong CP violation is needed. It was therefore proposed that at finite temperature there might be spontaneous (transitional) CP violation within the bubble walls at the electroweak phase transition in supersymmetric models. We investigate this question in the MSSM.

1 Introduction

For producing the baryon asymmetry of the Universe, we need extensions to the Standard Model. One of the Zakharov conditions requires nonequilibrium. In the MSSM this can be fulfilled by a strong enough first order phase transition with a light scalar top.

Also a small bubble wall velocity seems to support baryogenesis. But Zakharov’s conditions also require CP violation and in the MSSM there are several mechanisms known to generate it. Explicit CP violating operators might conflict to experimental EDM bounds. It were interesting to have a mechanism generating enough CP violation for baryogenesis without any conflict to experiments. While spontaneous CP violation is excluded at $T = 0$ for the experimentally allowed parameter values, there is a suggestion that it might be more easily realized at finite temperatures.

Previously, the moduli of the two Higgs doublets around the phase boundary have been determined from the 2-loop effective potential. The CP violating phase between the two Higgs doublets has been addressed perturbatively and nonperturbatively.

We present the first complete solution of the equations of motion for the phase between the two Higgs doublets within the MSSM, utilizing a perturbative effective potential, but without restricting it to the effective quartic couplings. Our conclusions differ from those obtained earlier on.

2 Searching for CP violating phases

We parameterize the two Higgs doublets of the MSSM as

\[ H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} h_1 e^{i\theta_1} \\ 0 \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h_2 e^{i\theta_2} \end{pmatrix}. \]

In addition, because of gauge invariance, the effective Higgs potential depends on the phases only via $\theta = \theta_1 + \theta_2$, and we have an additional constraint $h_1^2 \partial_\mu \theta_1 = h_2^2 \partial_\mu \theta_2$. We can then concentrate on $\theta$. Assuming tree-level kinetic terms and

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moving to a frame where the bubble wall is static and planar, the action to be minimized is

$$S \propto \int dz \left[ \frac{1}{2} (\partial_z h_1)^2 + \frac{1}{2} (\partial_z h_2)^2 + \frac{1}{2} \frac{h_1^2 h_2^2}{h_1^2 + h_2^2} (\partial_z \theta)^2 + V_T(h_1, h_2, \theta) \right], \quad (2)$$

where $V_T(h_1, h_2, \theta)$ is the finite temperature effective potential for $h_1, h_2, \theta$. In general, we are solving the equations of motion for $h_1, h_2, \theta$ following from this action. In the numerical solution we use the method outlined in [4] which deals with the minimization of a functional of the squared equations of motion.

At the first stage, we consider the case with no explicit CP phases, and ask whether a particular solution without CP violation ($\theta = 0, \pi$), is in fact a local minimum of the action or not. Clearly, it is not if

$$m^2_3(h_1, h_2) = \frac{1}{|h_1 h_2|} \left. \frac{\partial^2 V_T(h_1, h_2, \theta)}{\partial \theta^2} \right|_{\theta=0} < 0, \quad (3)$$

where we have divided by $|h_1 h_2|$, assuming that this is non-zero. Eq. (3) is to be evaluated along the path found by solving the equations of motion for $h_1, h_2$. We have chosen the convention that $h_1$ can have either sign, allowing us to consider only $\theta = 0$. For the case of the most general quartic two Higgs doublet potential, Eq. (3) agrees with the constraint on which most of the investigations of spontaneous CP violation are based. However, Eq. (3) is true more generally, independent of the form of the potential $V_T(h_1, h_2, \theta)$.

The tree-level potential of the theory is

$$V_{\text{tree}} = \frac{1}{2} m_1^2 h_1^2 + \frac{1}{2} m_2^2 h_2^2 + m_{12}^2 h_1 h_2 \cos \theta + \frac{1}{32} (g^2 + g'^2) (h_1^2 - h_2^2)^2, \quad (4)$$

where $g, g'$ are the SU(2) and U(1) gauge couplings, and at tree-level

$$m_{12}^2 = -\frac{1}{2} m_A^2 \sin 2\beta. \quad (5)$$

It follows that $m_3^2(h_1, h_2) = (1/2) m_A^2 \sin 2\beta > 0$, so that the minimum of the potential in the $\theta$ direction is at $\theta = 0$. Thus, in order to get spontaneous CP violation one needs radiative corrections which can overcome the tree-level term.

Older considerations are based on the approximation to the effective potential where only the quadratic and quartic operators are considered. At finite temperatures around the electroweak phase transition, important contributions come from infrared sensitive non-analytic contributions which are not of this form, and can affect spontaneous CP violation. Thus, it is important to solve the equations of motion more generally for the full effective potential.

Here we consider the full finite temperature 1-loop effective potential of the MSSM. It is known that 2-loop corrections are very important in the MSSM, allowing for larger values of $h_1, h_2$ in the broken phase. Nevertheless, for the present problem we find that even 1-loop effects are in most cases very small, so we do not expect qualitative changes from the 2-loop effects.

3 A scan for spontaneous transitional CP violation.

The tree-level part of the effective potential $V_T(h_1, h_2, \theta)$ is in Eq. (4). In the resummed 1-loop contribution to $V_T(h_1, h_2, \theta)$, we include gauge bosons, stops, charginos and neutralinos. This introduces dependences on the trilinear squark
phenomenon requires very small values of \( m \) and large of \( \tan \beta \), such that \( m_\text{eff} \) is small. (This is in contrast to the requirements of a strong phase transition at \( T \).) There is also a relatively strong dependence on \( A_\mu \) and \( \mu \): the region favoured is shown in Fig. 3. The dependences on the other parameters are less significant; for \( m_U \) and \( m_Q \), small values are preferred. The region found is in rough agreement with those found in other studies.

At the second stage, we make further restrictions. For instance, we exclude the cases leading to non-physical negative mass parameters. We also exclude cases leading to \( T = 0 \) spontaneous CP violation in the broken phase: this phenomenon requires very small values of \( m_A \). We also discard phase transitions which are exceedingly weak, \( v/T \ll 0.1 \).
Figure 1: The average value of $m_3^2$ versus $\mu$ and $A_t$. We observe that small values of $m_3^2$ are not typical in any part of the plane but are on the average more likely for small $\mu$, $A_t$, and that the distribution is wider (and thus more favourable) for like signs of $\mu$, $A_t$, as shown by the noisy contours obtained with a finite amount of statistics.

In the special point $m_U^2 \approx 0$ was considered. Since the thermal mass corrections were neglected this corresponds in the physical MSSM to a case where $m_U^2 + \#T^2 \sim 0$. Expanding the 1-loop cubic term from the stops to a finite order in $v_1/v_2$, it was suggested that transitional spontaneous CP violation can take place. This region is quite dangerous due to the vicinity of a charge and colour breaking minimum. Without expanding the 1-loop contribution in $v_1/v_2$, we cannot reproduce the behaviour proposed there. In any case, even before taking into account the experimental lower limits on the Higgs masses, we cannot find any promising case in the sample of $\sim 2 \times 10^6$ configurations of stage 2.

We conclude that after taking into account the infrared sensitive effects inherent in the 1-loop effective potential, coming from a light stop and gauge bosons, and solving for the wall profile from the equations of motion, spontaneous CP violation does not take place in the physical MSSM bubble wall.

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