Octupole bands in even isotopes of Ra, Th, U and Pu

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Abstract. Many heavy nuclei have low-lying $K^\pi = 0^-$ bands. Furthermore, several even isotopes of Ra, Th, U and Pu have $K^\pi = 1^-$ bands with bandhead excitation energies very close to 1 MeV. The E3 transitions from the $0^+$ ground state to the $3^-$ members of these bands have reduced strengths $B(E3; 0^+ \rightarrow 3^-)$ comparable to those measured in the isotopes of Pb. An exotic cluster model with a Pb core is proposed to account for these observations. A good account of the data is given, and higher lying $K^\pi = 2^-$ and $3^-$ bands are predicted. The intermediate strength quadrupole-quadrupole coupling between cluster and core leads to irregular, inverted doublets in the calculated $K^\pi = 1^-$ bands.

This study of negative parity bands in eight even isotopes of Ra, Th, U and Pu is motivated by the following experimental observations, summarised in Table 1. All eight nuclei have low-lying $K^\pi = 0^-$ and $1^-$ bands with closely comparable bandhead excitation energies at or below 1 MeV (although the $3^-$ member of the $K^\pi = 1^-$ band in $^{226}$Ra has not yet been seen). The $B(E3; 0^+ \rightarrow 3^-)$ strengths to populate the $3^-$ levels of these bands from the $0^+$ ground state are comparable in magnitude to those observed in the even isotopes of Pb, and are somewhat smaller for the $K^\pi = 1^-$ band than for the $K^\pi = 0^-$ band. At slightly higher excitation energies, four of the chosen nuclei are also found to have a $K^\pi = 2^-$ band.

This strongly suggests that the same mechanism may well be operating in all eight nuclei to produce these common features. A possible explanation for them is provided by an exotic cluster model in which an isotope of C, O, Ne or Mg orbits a Pb core which is excited into its low-lying $3^-$ state. Such a model is suggested by the phenomenon of exotic decay and by a variety of cluster selection techniques [1,2]. With cluster and core in their $0^+$ ground states the model has previously given a good account of excitation energies and enhanced E2 electromagnetic transition strengths for the $K^\pi = 0^+$ ground state bands of many heavy nuclei [3]. In these earlier calculations, the exotic cluster is taken to orbit the core in a deep, local potential. The cluster nucleons are excluded from the core, as required by the Pauli principle, by restricting the quantum numbers of relative motion to a large value of $G = 2n + L$ (n is the number of nodes in the radial wave function and $L$ is the orbital angular momentum). Thus, an even value of $G$ leads to even values of $L$ and hence to $J^\pi = 0^+, 2^+, 4^+, \ldots$ etc. when cluster and core are spinless entities, restricted to their ground states.

One way to obtain a single negative parity band in this model is to increase $G$ by 1. It will then be an odd integer, and so only odd values of $L$ will be produced, leading to a $K^\pi = 0^-$ band for spinless, unexcited cluster and core. This modification has previously given a good account of the $K^\pi = 0^-$ band in $^{226}$Ra [4], but it is not adequate for a unified description of the
To illustrate the situation more clearly we make the following simplifications. Contractions of tensors in the core intrinsic and radial coordinates up to and including rank 6. These simplifications lead to coupling matrix elements \( \xi \) labelled by \( \phi \) the core-cluster relative motion, and \( r \) where

\[ \text{The problem thus reduces to a diagonalization of } 4 \times 4 \text{ matrices for states of odd-} J \text{ (where mixtures of } L = J - 3, J - 1, J + 1, J + 3 \text{ occur) or of } 3 \times 3 \text{ matrices for states of even-} J \text{ (where mixtures of } L = J - 2, J, J + 2 \text{ occur) [5].}

Figure 1 shows, in a generic way, the effect on the energy levels of varying the value of \( \beta \), taken as negative for an attractive interaction. For \( \beta = 0 \) the degeneracy of the various \( I \otimes L \)

Table 1. Experimental excitation energies (MeV) and B(E3 ↑) strengths (\( e^3b^3 \)) of \( 3^- \) states in some even isotopes of Ra, Th, U and Pu.

| Nucleus | \( K^\pi = 0^- \) | \( K^\pi = 1^- \) | \( K^\pi = 2^- \) |
|---------|-----------------|-----------------|-----------------|
| \( ^{226}\text{Ra} \) | 0.321 | 1.10 ± 0.11 | — | — |
| \( ^{230}\text{Th} \) | 0.571 | 0.64 ± 0.06 | 1.01 | ≤ 0.57 | — | — |
| \( ^{234}\text{U} \) | 0.849 | ≤ 0.59 ± 0.07 | 1.02 | 0.22 ± 0.05 | 1.31 | 0.22 ± 0.07 |
| \( ^{236}\text{U} \) | 0.744 | 0.53 ± 0.07 | 1.04 | 0.31 ± 0.08 | 1.15 | 0.16 ± 0.06 |
| \( ^{238}\text{Pu} \) | 0.731 | 0.570 ± 0.036 | 1.00 | 0.184 ± 0.018 | 1.17 | 0.166 ± 0.022 |
| \( ^{238}\text{Pu} \) | 0.661 | 0.71 ± 0.12 | 1.02 | — | — | — |
| \( ^{240}\text{Pu} \) | 0.648 | 0.41 ± 0.06 | 1.00 | — | 1.24 | — |
| \( ^{242}\text{Pu} \) | 0.823 | 0.42 ± 0.07 | 1.02 | 0.45 ± 0.07 | — | — |
combinations is not lifted and we see a rotational spectrum with $E \propto L(L+1)$ where the energies are measured relative to the core $3^{-}$ excitation. At large negative $\beta$ the strong coupling limit is obtained, and the levels separate into four distinct bands which can be labelled $K^\pi = 0^{-}, 1^{-}, 2^{-}$ and $3^{-}$, with all the associated rotational model connotations [6]. By reducing the magnitude of $\beta$ in small increments from a large value the states can be tracked from their unambiguous strong coupling energies in towards the weak-coupling limit.

Although the $K$-labelling loses its significance for reduced $\beta$ it is still convenient to use it to facilitate eventual comparison with nuclear data tabulations. For a given $J$, the highest energy state is assigned to $K^\pi = 3^{-}$ and the next highest to $K^\pi = 2^{-}$ since these two bands remain clearly distinct for all $\beta$. The $K^\pi = 0^{-}$ and $1^{-}$ bands are nearly on top of one another, but we may assign the lowest energy odd-$J$ states to the $K^\pi = 0^{-}$ band, and whatever is left over is taken as belonging to the $K^\pi = 1^{-}$ band. It is worth noting that for very small $\beta$ the lowest state is $J^\pi = 3^{-}$ with predominant configuration $I^\pi \otimes L = 3^{-} \otimes 0$. To obtain a $1^{-}$ lowest state, as found in all the nuclei under study, we need an intermediate value of $\beta$ of $-0.3$ MeV or more (in magnitude) for the calculation displayed in Figure 1.

Figure 2 shows the associated spectrum for $\beta = -1.0$ MeV. The $K^\pi = 0^{-}, 2^{-}$ and $3^{-}$ bands (obtained from Figure 1) are fairly normally ordered, albeit with irregular energy separations. However, the $K^\pi = 1^{-}$ band shows a sequence of inverted doublets with $6^{-}$ below $5^{-}$, $8^{-}$ below $7^{-}$, $10^{-}$ below $9^{-}$ etc. From the strong-coupling viewpoint this can be seen as a cluster model manifestation of signature [6]. From the weak coupling viewpoint, the occurrence of doublets is due to two states having the same dominant $L$-value. For example, $J = 5, 6$ both have $L = 6$ as their main component, whereas $J = 7, 8$ both have $L = 8$ etc. The inversion, or not, is a more delicate feature dependent on the precise value of $\beta$.

To address real nuclei we need values for $\beta$ and the excitation energy $E(3^{-})$ of the core $3^{-}$ state. These can be systematically extracted from experiment by noting that the Hamiltonian

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Variation in energy of the first four states of each of the four negative parity bands in a generic nucleus as a function of the strength of the quadrupole-quadrupole coupling.}
\end{figure}
Table 2. Estimation of $E(3^-)$ and $\beta$ from experimental spectra

| Nucleus    | $E(1^-)$ (keV) | $E(1^-)$ (keV) | $E(2^-)$ (keV) | $E(2^-)$ (keV) | $\beta$ (MeV) | $E(3^-)$ (MeV) |
|------------|----------------|----------------|----------------|----------------|---------------|---------------|
| $^{226}$Ra | 253.73         | 1048.8         | 1070.5         | —              | —             | 1.13          |
| $^{230}$Th| 508.16         | 951.94         | 971.69         | 1079.26        | -0.4431       | 1.13          |
| $^{234}$U | 786.29         | —              | 989.45         | —              | —             | —             |
| $^{236}$U | 687.60         | 966.63         | 987.67         | 1110.67        | -0.3331       | 1.12          |
| $^{238}$U | 680.11         | 930.55         | 950.12         | 1128.84        | -0.3512       | 1.12          |
| $^{238}$Pu| 605.14         | 962.780        | 985.45         | —              | —             | —             |
| $^{240}$Pu| 597.34         | 938.06         | 958.85         | 1240.8         | -0.4982       | 1.26          |
| $^{242}$Pu| 780.45         | 956            | 992.5          | —              | —             | —             |

Figure 2. Generic negative parity spectra obtained from the calculation of Fig.1 for an interaction strength of $-1.0$ MeV. Energies are given relative to the $I^{\pi} = 3^- \text{ core state.}$

matrices for $J = 1$ and $J = 2$ are only $2 \times 2$. For $J = 1$ we have

$$H^{J=1} = \begin{pmatrix} E(3^-) + E(2) + \frac{8\beta}{7} & \frac{2\sqrt{3}\beta}{21} \\ \frac{2\sqrt{3}\beta}{21} & E(3^-) + E(4) + \frac{25\beta}{21} \end{pmatrix}$$

and for $J = 2$

$$H^{J=2} = \begin{pmatrix} E(3^-) + E(2) + \frac{2\beta}{7} & -\frac{\sqrt{10}\beta}{7} \\ -\frac{\sqrt{10}\beta}{7} & E(3^-) + E(4) + \frac{5\beta}{7} \end{pmatrix}.$$
Figure 3. Comparison of theoretical, T, and experimental, E, negative parity $K^\pi = 0^-, 1^-, 2^-$ and $3^-$ bands in $^{234}$U and $^{236}$U. Angular momenta are labelled on the left of the calculated levels and on the right of the experimental levels. Where no ambiguity occurs, only the highest and lowest levels of a sequence are labelled.

Their traces give the sum of the energies of the two states and so provide two simultaneous equations for $E(3^-)$ and $\beta$. Thus, whenever data exist for the excitation energies of both the $1^-$ and both the $2^-$ states, in a given nucleus, they imply values for the required parameters. We proceed with the values deduced from the spectrum of $^{238}$U [5].

\[ E(3^-) = 1.12 \text{ MeV}, \quad \beta = -0.3512 \text{ MeV}. \] (5)

A comparison of calculated energies with experiment for $^{238}$U, where inverted doublets in the $K^\pi = 1^-$ band are indeed seen, has already been published [5]. Figure 3 shows a similar comparison for the other even-even isotopes of U. A generally good account of the relative positions of the bandheads is given, although no inversions are seen in the $K^\pi = 1^-$ bands (remember, we are using $\beta$-values fitted to $^{238}$U). Many predicted higher-J states remain to be discovered in all four bands. Similar results are obtained for all the other nuclei and will be reported in greater detail at a later date.

Our model can also be used to calculate $B(E3; 0^+ \rightarrow 3^-)$ reduced transition strengths. In general, the appropriate operator contains three separate terms: two describing internal excitations of the cluster and core and a third describing the transition of their state of relative motion. Here, the cluster internal excitation does not occur (since the cluster is restricted to its ground state). Neither does the relative motion transition, because we only have states of even-L which cannot be linked by an octupole $r^3Y_3(\hat{r})$ operator. Thus, only an internal excitation of the core can contribute to the parent nucleus $B(E3 \uparrow)$. In fact, the L-value of the relative motion must remain unchanged at $L = 0$, and so the $B(E3 \uparrow)$ is simply the Pb-core value multiplied by the squared modulus of the $L = 0$ coefficient in the $J^\pi = 3^-$ state eigenfunction (the energy diagonalization produced coefficients for $L = 0, 2, 4, 6$ which can all be combined with $I^\pi = 3^-$ to
The excited $3^-$ states used are those listed in Table 1, plus the state in $^{242}$Pu at 1.650 MeV excitation energy, which we propose as a possible member of the $K^\pi = 2^-$ band.

obtain $J^\pi = 3^-\). This yields a sum rule for the B(E3 $\uparrow$) transition strengths to the four $J^\pi = 3^-$ states, meaning that together their values should add to that of the Pb-core. In particular, it turns out that the transition to the $K^\pi = 3^-$ bandhead should be very weak, and so Coulomb excitation will be a very poor mechanism for exciting these largely unknown bands. Using a fitted value of $Q_3 = 1.129$ e$^2$b$^3$ [5] for the Pb core, we obtain the results listed in Table 3. A generally good account of the data is given, although further analysis of the band assignments in $^{238}$U and $^{242}$Pu may be warranted.

Using a Pb core in a $3^-$ state and an orbiting exotic cluster we have been able to explain the gross features of negative parity spectra and E3 transitions to the $0^+$ ground state across a range of eight heavy nuclei using three adjustable parameters. We predict $K^\pi = 3^-$ bands with bandheads around 1.5–2.0 MeV which should have very weak transitions to and from the ground state band. To test the model further more high quality spectroscopic data and electromagnetic transition rates are needed. On the theoretical side, the free parameters of the model need to be justified on a more microscopic basis. Finally, we note that it might be possible to extend this model to use a $2^+$ cluster and/or core excitation to address the ubiquitous $\beta$ and $\gamma$ bands seen at low energy in so many heavy nuclei.

### Table 3. Octupole transition strengths

| Nucleus | $K^\pi$ | $B(E3 \uparrow; 0^+_{gs} \rightarrow 3^-)$ e$^2$b$^3$ |
|---------|---------|----------------------------------|
| $^{226}$Ra | 0$^-$ | $0.486Q_3=0.55$ | 1.10 $\pm$ 0.11 |
| $^{230}$Th | 0$^-$ | $0.459Q_3=0.52$ | 0.64 $\pm$ 0.06 |
| $^{234}$U | 1$^-$ | $0.383Q_3=0.43$ | $\leq 0.57$ |
| $^{236}$U | 0$^-$ | $0.433Q_3=0.49$ | $\leq 0.59$ $\pm$ 0.07 |
| $^{238}$U | 1$^-$ | $0.380Q_3=0.43$ | 0.22 $\pm$ 0.05 |
| $^{238}$Pu | 2$^-$ | $0.158Q_3=0.18$ | 0.22 $\pm$ 0.07 |
| $^{240}$Pu | 0$^-$ | $0.439Q_3=0.50$ | 0.53 $\pm$ 0.07 |
| $^{242}$Pu | 1$^-$ | $0.381Q_3=0.43$ | 0.31 $\pm$ 0.08 |
| $^{242}$Pu | 2$^-$ | $0.154Q_3=0.17$ | 0.16 $\pm$ 0.06 |

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