BOSONIC QUADRATIC ACTIONS FOR 11D SUPERGRAVITY ON $\text{AdS}_{7/4} \times S^{4/7}$

by

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Abstract

We determine from 11$d$ supergravity the quadratic bulk action for the physical bosonic fields relevant for the computation of correlation functions of normalized chiral operators in $D = 6$, $\mathcal{N} = (0,2)$ and $D = 3$, $\mathcal{N} = 8$ supersymmetric CFT in the large $N$ limit, as dictated by the AdS/CFT duality conjecture.

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0. Introduction

The duality [1–3] between string/M theory on Anti-de Sitter space (AdS) times a compact manifold and conformal field theory (CFT) living on the boundary of AdS is quite useful in the program of extracting non-trivial information for various strongly coupled quantum field theories. The most studied duality relates type IIB superstring theory on $\text{AdS}_5 \times S_5$ to $D=4$, $\mathcal{N}=4$ SU($N$) super Yang-Mills (YM) theory. In the large $N$ limit, the AdS side of the duality can be approximated by type IIB supergravity. In one of the many applications [4], the latter has been employed to compute 2 and 3 point functions for chiral primary operators of the super YM theory, whose conformal dimensions had already been identified in [3] by relating them to the masses of the corresponding Kaluza–Klein scalars on $\text{AdS}_5 \times S_5$. Various efforts are being made also to investigate the structure and compute the more complex 4 point functions (see [5–8] and references therein).

However, other interesting dualities with maximal supersymmetry (i.e. 16 supersymmetry charges on the CFT side) have been proposed in [1]. According to this proposal, M–theory on $\text{AdS}_7 \times S_4$ is dual to the $D=6$, $\mathcal{N}=(0,2)$ supersymmetric CFT (to be denoted by SCFT$_6$). Similarly, M–theory on $\text{AdS}_4 \times S_7$ is dual to the $D=3$, $\mathcal{N}=8$ supersymmetric CFT (to be denoted by SCFT$_3$). These SCFTs are realized, for example, on the world-volume of $N$ coincident M5 and M2 branes, respectively. Yet, little is known about them, and the AdS/CFT duality offers a useful tool to learn more about these remarkable theories. Again, in the large $N$ limit, one can approximate M–theory by its low energy limit, 11$d$ supergravity. By using that, some groups have identified the conformal dimensions of the corresponding chiral operators [9–12]. Quite recently the calculation of 3 point functions of chiral operators in the large $N$ limit of the SCFT$_6$ theory has appeared in [13].

In this paper, we determine the 11$d$ supergravity quadratic bulk action of the physical bosonic fields relevant for the computation of the correlation functions of normalized chiral operators of both SCFT$_6$ and SCFT$_3$. In particular, we carefully compute the normalization constants of the kinetic terms of the scalar fields, as these numbers are a crucial input in such calculations.

We have found that the method originally described by Lee, Minwalla, Rangamani and Seiberg in [4] does not yield any sensible result, though we have verified the correctness of their final answer. This fact has recently been pointed out also Corrado, Florea and McNees [13]. Therefore, we carry out our analysis by extending the method of Arutyunov and Frolov [14], originally worked out for type IIB 10$d$ supergravity on $\text{AdS}_5 \times S_5$, to 11$d$ supergravity on $\text{AdS}_7 \times S_4$ and $\text{AdS}_4 \times S_7$. Contrary to what claimed in [13], we find
that this approach provides the correct quadratic action. The method is completely self contained in the sense that it does not require the knowledge of the action beyond the quadratic approximation. As it consists in isolating the scalar fields of interest by simply performing field redefinitions, one does not need to put on-shell the scalar fluctuations nor study complicated boundary terms in the supergravity action. Moreover, it does not lead to either non local or higher derivative terms in AdS space.

From our final results, one can read off the masses of the scalar fluctuations together with the searched for normalization factors. Needless to say that we reproduce the values of the masses as originally worked out in [15] and in [16,17] for the AdS$_7 \times $ S$_4$ and AdS$_4 \times $ S$_7$ cases, respectively. As for the normalization factors, we confirm the result recently obtained for the AdS$_7 \times $ S$_4$ case in [13] by using the same method that, as we learn from that reference, was originally employed in [4].

In sect. 1, we present the AdS$_7 \times $ S$_4$ compactification relevant for SCFT$_6$. Then, in sect. 2, we describe the AdS$_4 \times $ S$_7$ compactification related to SCFT$_3$. This case presents a technical complication which we avoid by resorting to a dual formulation. In sect. 3, we illustrate our conclusions.

1. The AdS$_7 \times $ S$_4$ model

The model considered here is the usual 3–form formulation of 11d supergravity. The basic fields of the bosonic sector are the metric $g$ and the 3–form field $A$. The bosonic part of the action is

$$ I = \frac{1}{4\kappa^2} \int_{M_{11}} \left[ R(g) * g 1 - F(A) \wedge * g F(A) + \frac{2}{3} A \wedge F(A) \wedge F(A) \right]. $$

(1.1)

Here, $R(g)$ is the Ricci scalar of the metric $g$. $F(A)$ is the 4–form field strength of the 3–form $A$ and is given by

$$ F(A) = dA. $$

(1.2)

The space time $M_{11}$ is taken to have the topology AdS$_7 \times $ S$_4$.

Then, one can easily verify that the field equations admit the standard solution $\bar{g}_{ij}$, $\bar{A}_{ijk}$ generated by the Freund Rubin ansatz [18]. For this, the only non vanishing components of the Riemann tensor and field strength are given by

$$ R(\bar{g})_{\alpha \beta \gamma \delta} = -\frac{1}{18} e^2 (\bar{g}_{\alpha \gamma} \bar{g}_{\beta \delta} - \bar{g}_{\alpha \delta} \bar{g}_{\beta \gamma}) , \quad R(\bar{g})_{\kappa \lambda \mu \nu} = \frac{2}{3} e^2 (\bar{g}_{\kappa \mu} \bar{g}_{\lambda \nu} - \bar{g}_{\kappa \nu} \bar{g}_{\lambda \mu}) , \quad (1.3a) - (1.3b) $$

$$ F(\bar{A})_{\kappa \lambda \mu \nu} = e \delta_{\kappa \lambda \mu \nu} , \quad (1.4) $$
where $e$ is the compactification scale $^1$.

We expand the action in fluctuations around the background $\bar{g}_{ij}, \bar{A}_{ijk}$. We parametrize the fluctuations $\delta g_{ij}, \delta A_{ijk}$ of the fields $g_{ij}, A_{ijk}$ around the background as in [15]

$$
\delta g_{\kappa\lambda} = m_{\kappa\lambda} + \nabla_\kappa n_\lambda + \nabla_\lambda n_\kappa + (\nabla_\kappa \nabla_\lambda - \frac{1}{4} \bar{g}_{\kappa\lambda} \nabla^\rho \nabla_\rho) p + \frac{1}{4} \bar{g}_{\kappa\lambda} \pi, \quad (1.5a)
$$

$$
m^\rho_{\rho} = 0, \quad \nabla^\rho m_{\rho\kappa} = 0, \quad \nabla^\rho n_{\rho} = 0,
$$

$$
\delta g_{\kappa\alpha} = k_{\kappa\alpha} + \nabla_\kappa l_\alpha, \quad (1.5b)
$$

$$
\nabla^\rho k_{\rho\alpha} = 0,
$$

$$
\delta g_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{5} \bar{g}_{\alpha\beta} \pi, \quad (1.5c)
$$

and

$$
\delta A_{\kappa\lambda\mu} = 3 \nabla_\kappa [a_{\lambda\mu}] + \bar{e}_{\kappa\lambda\mu} \nabla^\rho b, \quad (1.6)
$$

$$
\nabla^\rho a_{\rho\kappa} = 0.
$$

We find that the quadratic part of the action is given by the following expression

$$
I_{[2]} = \frac{1}{4\kappa^2} \int_{\text{AdS}_7} d^7y (-\bar{g}_7)^{\frac{3}{2}} \int_{S^4_{4}} d^4x (\bar{g}_4)^{\frac{1}{2}}
$$

$$
\left\{ \frac{1}{2} \bar{\nabla}^\alpha h_{\alpha\gamma} \bar{\nabla}^\beta h_{\beta\gamma} - \frac{1}{2} \bar{\nabla}^\alpha h_{\alpha\beta} \bar{\nabla}^\gamma h_{\beta\gamma} + \frac{1}{4} \bar{\nabla}^\alpha h_{\beta} \bar{\nabla}^\beta h_{\alpha} \bar{\nabla}^\gamma h_{\beta} \bar{\nabla}^\gamma h_{\alpha}
$$

$$
+ \frac{1}{4} \bar{\nabla}^\alpha h_{\alpha} \bar{\nabla}^\beta h_{\beta} \bar{\nabla}^\gamma h_{\gamma} - \frac{1}{4} \bar{\nabla}^\alpha h_{\beta} \bar{\nabla}^\beta h_{\alpha} \bar{\nabla}^\gamma h_{\gamma} + \frac{1}{18} e^2 h_{\alpha} h_{\beta} + \frac{1}{36} e^2 h_{\alpha} h_{\beta}
$$

$$
- \frac{2}{90} \bar{\nabla}^\alpha \pi \bar{\nabla}^\alpha \pi - \frac{2}{90} \bar{\nabla}^\alpha \pi \bar{\nabla}^\alpha \pi - \frac{9}{20} e^2 \pi \pi
$$

$$
- \bar{\nabla}^\alpha \bar{\nabla}^\kappa b \bar{\nabla}^\alpha \bar{\nabla}^\kappa b - \bar{\nabla}^\kappa \bar{\nabla}^\kappa b \bar{\nabla}^\lambda \bar{\nabla}^\lambda b - \frac{12}{5} e \bar{\nabla}^\kappa \bar{\nabla}^\kappa b \pi
$$

$$
+ h_{\alpha} \left( \frac{3}{10} \bar{\nabla}^\kappa \bar{\nabla}^\kappa \pi + e h_{\alpha} \bar{\nabla}^\kappa b \right) + \frac{1}{2} (\bar{\nabla}^\kappa \bar{\nabla}^\lambda - \frac{1}{4} \bar{g}_{\kappa\lambda} \bar{\nabla}^\rho \bar{\nabla}_\rho) p \bar{\nabla}^\kappa \bar{\nabla}^\lambda (h_{\alpha} - \frac{2}{15} \pi)
$$

$$
- \bar{\nabla}^\kappa l^\alpha \bar{\nabla}_\kappa \left( \frac{2}{20} \bar{\nabla}_\alpha \pi + 2 e \bar{\nabla}_\alpha \bar{\nabla}^\kappa b + \bar{\nabla}^\beta h_{\beta\alpha} - \bar{\nabla}_\alpha h_{\beta\beta} \right) + \ldots \right\}. \quad (1.7)
$$

Above, the ellipses denote terms not containing the fields $h_{\alpha\beta}, \pi, b$. Note that the field components $m_{\kappa\lambda}, n_\kappa, k_{\kappa\alpha}, a_{\alpha\lambda}, \delta A_{\alpha ij}$ are decoupled at quadratic level from $h_{\alpha\beta}, \pi, b$. Also, $p$ and $l_\alpha$ act as Lagrange multipliers enforcing certain constraints. We have focused

$^1$ In this paper, we adopt the following conventions. Latin lower case letters $i, j, k, l, \ldots$ denote $M_{11}$ indices. Early Greek lower case letters $\alpha, \beta, \gamma, \delta, \ldots$ denote $\text{AdS}_7/4$ indices. Late Greek lower case letters $\kappa, \lambda, \mu, \nu \ldots$ denote $S_{4/7}$ indices.
on the fields $\pi, b$ and those mixing with them, since they contain the scalar fluctuations which couple to the chiral operator of the boundary conformal field theory of our interest.

At this point, one might partially fix the gauge by imposing

$$\bar{\nabla}^\rho (\delta g_{\rho \kappa} - \frac{1}{4} \bar{g}_{\rho \kappa} \delta g^\sigma \sigma) = 0, \quad \bar{\nabla}^\rho \delta g_{\rho \alpha} = 0; \quad (1.8a) - (1.8b)$$

$$\bar{\nabla}^\rho \delta A_{ij\rho} = 0, \quad (1.9)$$

as shown in [15]. These conditions imply in particular that $p = 0$ and $l_{\alpha} = 0$ and so, upon gauge fixing, the last two terms of $I[2]$ vanish. The corresponding constraints must then be enforced by hand.

However, instead of fixing the gauge at this stage, we prefer first to isolate the relevant scalar degrees of freedom by performing field redefinitions, as done in [14]. We write the AdS metric fluctuations as

$$h_{\alpha \beta} = \phi_{\alpha \beta} + \eta \bar{g}_{\alpha \beta} + \bar{\nabla}_\alpha \bar{\nabla}_\beta \zeta$$

with $\eta$ and $\zeta$ chosen in such a way to decouple at the quadratic level the field $\phi_{\alpha \beta}$ from the $\pi-b$ sector. We find

$$h_{\alpha \beta} = \phi_{\alpha \beta} + (\bar{\nabla}_\alpha \bar{\nabla}_\beta - \frac{1}{5} \bar{g}_{\alpha \beta} \bar{\nabla}^\kappa \bar{\nabla}_\kappa) \varphi,$$

where

$$\varphi = (- \bar{\nabla}^\lambda \bar{\nabla}_\lambda + \frac{5}{18} e^2)^{-1} (\frac{3}{8} \pi + \frac{5}{3} eb). \quad (1.11b)$$

The action $I[2]$ then becomes

$$I[2] = \frac{1}{4 \kappa^2} \int_{\text{AdS}_2} d^7 y (-g_7)^{\frac{3}{2}} \int_{S_4} d^4 x (\bar{g}_4)^{\frac{1}{2}}$$

$$\left\{ \frac{1}{2} \bar{\nabla}^\alpha \phi_{\alpha \gamma} \bar{\nabla}^\beta \phi_{\beta \gamma} - \frac{1}{2} \bar{\nabla}^\alpha \phi_{\alpha \beta} \bar{\nabla}^\beta \phi_{\gamma \gamma} + \frac{1}{4} \bar{\nabla}^\alpha \phi_{\beta \gamma} \bar{\nabla}_\alpha \phi_{\beta \gamma} - \frac{1}{4} \bar{\nabla}^\alpha \phi_{\beta \gamma} \bar{\nabla}_\alpha \phi_{\beta \gamma} + \frac{1}{4} \bar{\nabla}^\kappa \phi_{\beta \gamma} \bar{\nabla}_\kappa \phi_{\beta \gamma} - \frac{1}{4} \bar{\nabla}^\kappa \phi_{\beta \gamma} \bar{\nabla}_\kappa \phi_{\beta \gamma} + \frac{1}{18} e^2 \phi_{\alpha \beta} + \frac{1}{36} e^2 \phi_{\alpha \beta} + \frac{1}{36} e^2 \phi_{\alpha \beta} \right\}$$

$$- \frac{3}{80} \bar{\nabla}^\alpha \bar{\nabla}^\kappa \bar{\nabla}_\kappa \bar{\nabla}_{\lambda} \bar{\nabla}_\lambda - \frac{1}{8} e^2 (\frac{3}{8} \pi + \frac{5}{3} eb) \bar{\nabla}_\alpha (\frac{3}{8} \pi + \frac{5}{3} eb)$$

$$- \frac{21}{50} \bar{\nabla}^\kappa \bar{\nabla}^\lambda \bar{\nabla}_\lambda \bar{\nabla}^\mu \bar{\nabla}_\mu - \frac{1}{18} e^2 (\frac{2}{8} \pi + \frac{5}{3} eb) (\frac{2}{8} \pi + \frac{5}{3} eb)$$

$$- \frac{21}{50} \bar{\nabla}^\kappa \bar{\nabla}^\lambda \bar{\nabla}_\lambda \bar{\nabla}_\mu + \frac{1}{18} e^2 (\frac{2}{8} \pi + \frac{5}{3} eb) (\frac{2}{8} \pi + \frac{5}{3} eb)$$

$$- \frac{9}{80} \bar{\nabla}^\alpha \bar{\nabla}^\beta \bar{\nabla}_{\alpha \beta} \bar{\nabla}_{\alpha \beta} - \frac{9}{800} \bar{\nabla}^\alpha \bar{\nabla}^\kappa \bar{\nabla}_\kappa \bar{\nabla}_\beta \bar{\nabla}_\beta - \frac{2}{8} e^2 \bar{\nabla}^\alpha \bar{\nabla}^\kappa \bar{\nabla}_\kappa \bar{\nabla}_\beta \bar{\nabla}_\beta$$

$$+ \frac{1}{2} \left( \bar{\nabla}^\kappa \bar{\nabla}_\kappa - \frac{1}{4} \bar{g}_{\kappa \lambda} \bar{\nabla}_\rho \bar{\nabla}_\rho \right) \bar{p} \bar{\nabla}^\kappa \bar{\nabla}^\lambda \left[ \phi_{\alpha \beta} - \frac{9}{10} \pi \right]$$

$$+ \left( \bar{\nabla}^\alpha \bar{\nabla}_\alpha - \frac{5}{8} \bar{\nabla}^\kappa \bar{\nabla}_\kappa \right) \left( - \bar{\nabla}^\kappa \bar{\nabla}_\kappa + \frac{5}{18} e^2 \right) (\frac{2}{8} \pi + \frac{5}{3} eb)$$

$$- \bar{\nabla}^\kappa \bar{\nabla}_\kappa \left( \bar{\nabla}^\beta \phi_{\beta \alpha} - \bar{\nabla}_\alpha \phi_{\beta \beta} \right) + \ldots \right\}.$$  \quad (1.12)
Let us concentrate on the scalar $\pi-b$ sector of the above action. We expand the fields $\pi, b$ in harmonics $Y_I^{(4)}$ of the $S_4$ d’Alembertian $\nabla^\kappa \nabla_\kappa$

$$\nabla^\kappa \nabla_\kappa Y_I^{(4)} = -\frac{2}{3} e^2 k(k+3) Y_I^{(4)},$$

where $k = 0, 1, 2, \ldots$ depends on $I$. The $Y_I^{(4)}$ are conventionally normalized as

$$\int_{S_4} (g_4)^{\frac{1}{2}} Y_{I}^{(4)} Y_{J}^{(4)} = (\frac{2}{3} e^2)^{-\frac{1}{2}} z^{(4)}(k) \delta_{IJ},$$

$$z^{(4)}(k) = \frac{8\pi^2 k!}{(2k+3)!}.$$  

The expansions read

$$\phi_{\alpha\beta} = \sum_I \phi_{I\alpha\beta} Y_I^{(4)}, \quad \pi = \sum_I \pi_I Y_I^{(4)}, \quad b = \sum_I b_I Y_I^{(4)},$$

$$p = \sum_I p_I Y_I^{(4)}, \quad l_\alpha = \sum_I l_{I\alpha} Y_I^{(4)}.$$  

(1.15a) – (1.15e)

Now, to diagonalize the kinetic terms, one introduces new scalars $s_I, t_I$ such that

$$\pi_I = \frac{4}{3} k s_I + \frac{4}{3} (k+3) t_I,$$

$$b_I = \frac{3}{4} e^{-1} (s_I - t_I).$$

(1.16a)  

(1.16b)

In terms of $s_I, t_I, I_{[2]}$ takes the simple form

$$I_{[2]} = \frac{1}{4\kappa^2} (\frac{2}{3} e^2)^{-2} \int_{\text{AdS}_7} d^7y (-g_7)^{\frac{1}{2}} \sum_I z^{(4)}(k)$$

$$\left\{ \frac{1}{2} \nabla^\alpha \phi_{I\alpha\gamma} \nabla^\beta \phi_{I\beta\gamma} - \frac{1}{2} \nabla^\alpha \phi_{I\alpha\beta} \nabla^\beta \phi_{I\gamma\gamma} + \frac{1}{4} \nabla^\alpha \phi_{I\alpha\beta} \nabla^\gamma \phi_{I\phi\gamma} - \frac{1}{4} \nabla^\alpha \phi_{I\beta\gamma} \nabla^\alpha \phi_{I\beta\gamma} + \frac{1}{4} m_\phi I^{\alpha\beta} \phi_{I\alpha\beta} - \frac{1}{4} m_\phi'' I^{\alpha\beta} \phi_{I\alpha\beta} + A_{sI} \left[ - \frac{1}{2} \nabla^\alpha s_I \nabla_\alpha s_I - \frac{1}{2} m_{sI}^2 s_I^2 s_I \right] + A_{tI} \left[ - \frac{1}{2} \nabla^\alpha t_I \nabla_\alpha t_I - \frac{1}{2} m_{tI}^2 t_I^2 t_I \right] + u_{I} p_{I} \left[ \phi_{I\alpha\alpha} + v_{sI} \nabla^\alpha \nabla_\alpha s_I - m_{sI}^2 s_I \right] + v_{tI} \nabla^\alpha \nabla_\alpha t_I - m_{tI}^2 t_I \right\},$$

(1.17)

where

$$A_{sI} = \frac{(2k+3)k(k-1)}{2(2k+1)},$$

(1.18a)
\[ A_{\ell I} = \frac{(2k + 3)(k + 3)(k + 4)}{2(2k + 5)}, \quad (1.18b) \]

\[ m'^2_{\phi I} = \frac{2}{9} e^2 (k^2 + 3k + 1), \quad (1.19a) \]

\[ m''^2_{\phi I} = \frac{2}{9} e^2 (k^2 + 3k - \frac{1}{2}), \quad (1.19b) \]

\[ m_{sI}^2 = \frac{2}{9} e^2 k(k - 3), \quad (1.20a) \]

\[ m_{tI}^2 = \frac{2}{9} e^2 (k + 3)(k + 6), \quad (1.20b) \]

\[ u_I = \left(\frac{2}{9} e^2\right)^2 \frac{2}{9}(k - 1)k(k + 3)(k + 4), \quad (1.21) \]

\[ v_{sI} = \left(\frac{2}{9} e^2\right)^{-1} \frac{1}{(2k + 1)}, \quad (1.22a) \]

\[ v_{tI} = \left(\frac{2}{9} e^2\right)^{-1} \frac{1}{(2k + 5)}, \quad (1.22b) \]

\[ w_I = \frac{2}{9} e^2 k(k + 3). \quad (1.23) \]

Note that the modes \( s_I \) with \( k = 0, 1 \) do not appear, so these are gauge degrees of freedom.

Fixing the gauge involves setting \( p_I = 0, l_{I\alpha} = 0 \). Thus, after gauge fixing, \( s_I, t_I \) are free fields. On the \( s_I, t_I \) mass shell the constraints are simply

\[ \phi_I^\alpha_{\alpha} = 0, \quad k \geq 2, \quad (1.24a) \]

\[ \nabla^\beta \phi_{I\beta\alpha} = 0, \quad k \geq 1. \quad (1.24b) \]

As a check of our results we note that we have reproduced the masses worked out in [15]. The particular coefficient \( A_{sI} \) has also been deduced recently in [13] by using a consistency requirement involving other normalizations and the values of certain three point couplings. We confirm that result as well.

2. The \( \text{AdS}_4 \times S_7 \) model

We now analyze the 11d supergravity compactification on \( \text{AdS}_4 \times S_7 \). The relevant action is again (1.1), where now \( M_{11} \) has the topology \( \text{AdS}_4 \times S_7 \). The field equations admit the standard solution \( \bar{g}_{ij}, \bar{A}_{ijk} \) generated by the Freund Rubin type ansatz [18]. On this background, the only non vanishing components of the Riemann tensor and field strength are given by

\[ R(\bar{g})_{\alpha\beta\gamma\delta} = -\frac{2}{9} e^2 (\bar{g}_{\alpha\gamma} \bar{g}_{\beta\delta} - \bar{g}_{\alpha\delta} \bar{g}_{\beta\gamma}), \quad R(\bar{g})_{\kappa\lambda\mu\nu} = \frac{1}{18} e^2 (\bar{g}_{\kappa\mu} \bar{g}_{\lambda\nu} - \bar{g}_{\kappa\nu} \bar{g}_{\lambda\mu}), \quad (2.1a) - (2.1b) \]
\[ F(\bar{A})_{\alpha\beta\gamma\delta} = -e\epsilon_{\alpha\beta\gamma\delta}, \]  

(2.2)

where \( e \) is the compactification scale.

We parametrize the metric fluctuations \( \delta g_{ij} \) around the background as follows [17]

\[
\begin{align*}
\delta g_{\kappa\lambda} &= m_{\kappa\lambda} + \nabla_\kappa n_\lambda + \nabla_\lambda n_\kappa + (\nabla_\kappa \nabla_\lambda - \frac{1}{2} \bar{g}_{\kappa\lambda} \nabla^\rho \nabla_\rho) p + \frac{1}{2} \bar{g}_{\kappa\lambda} \pi, \\
m^\rho_{\rho} &= 0, \quad \nabla^\rho m_{\rho\kappa} = 0, \quad \nabla^\rho n_\rho = 0, \\
\delta g_{\kappa\alpha} &= k_{\kappa\alpha} + \nabla_\kappa l_\alpha, \quad (2.3b) \\
\nabla^\rho k_{\rho \alpha} &= 0, \\
\delta g_{\alpha\beta} &= h_{\alpha\beta} - \frac{1}{2} \bar{g}_{\alpha\beta} \pi. \quad (2.3c)
\end{align*}
\]

As to the 3-form fluctuations, it is sufficient for our purposes to use

\[ b^\alpha = -\frac{1}{3!} \epsilon^{\alpha\beta\gamma\delta} \delta A_{\beta\gamma\delta} \]  

(2.4)

and denote by \( a_{\kappa\gamma\delta} \) the longitudinal (in \( S_7 \)) components of \( \delta A_{\kappa\gamma\delta} \). Then, the part of the action quadratic in fluctuations is given by the following expression

\[
\begin{align*}
I_{[2]} &= \frac{1}{4\kappa^2} \int_{\text{AdS}_4} d^4y(-\bar{g}_4)^{\frac{3}{2}} \int_{S_7} d^7x(\bar{g}_7)^{\frac{1}{2}} \\
&\quad \times \left\{ \frac{1}{2} \nabla^\alpha h_{\alpha\gamma} \nabla^\beta h_{\beta\gamma} - \frac{1}{4} \nabla^\alpha h_{\alpha\beta} \nabla^\beta h_{\gamma\gamma} + \frac{1}{4} \nabla^\alpha h_{\alpha\beta} \nabla^\beta h_{\gamma\gamma} - \frac{1}{4} \nabla^\alpha h_{\alpha\gamma} \nabla^\beta h_{\beta\gamma} \\
&\quad + \frac{1}{4} \nabla^\kappa h_{\beta\gamma} \nabla^\kappa h_{\beta\gamma} - \frac{1}{4} \nabla^\kappa h_{\beta\gamma} \nabla^\kappa h_{\beta\gamma} + \frac{1}{36} e^2 h_{\alpha\alpha} h_{\beta\beta} + \frac{1}{9} e^2 h_{\alpha\beta} h_{\alpha\beta} \\
&\quad - \frac{9}{56} \nabla^\alpha \pi \nabla_\alpha \pi + \frac{9}{196} \nabla^\alpha \pi \nabla_\pi \pi + \frac{45}{28} e^2 \pi \pi \\
&\quad + \nabla^\alpha b_\alpha \nabla^\beta b_\beta + \nabla^\kappa b^\alpha \nabla_\kappa b_\alpha - 3e \nabla^\alpha b_\alpha \\
&\quad + h_{\alpha\alpha} \left( \frac{9}{28} \nabla^\kappa \nabla_\kappa \pi - \frac{3}{2} e^2 \pi + e \nabla^\beta b_\beta \right) + h_{\alpha\alpha} \left( \frac{9}{28} \nabla^\kappa \nabla_\kappa \pi - \frac{3}{2} e^2 \pi \right) \\
&\quad - \nabla^\kappa l_{\alpha\gamma} \nabla_\kappa \left( \frac{1}{14} \nabla^\alpha \pi - 2e b_\alpha + \nabla^\beta h_{\beta\alpha} - \nabla_\alpha h_{\beta\beta} \right) + \nabla^\kappa a_{\kappa\gamma\delta} \epsilon^{\alpha\beta\gamma\delta} \nabla_\alpha b_\beta + \ldots \right\}, \quad (2.5)
\end{align*}
\]

where the dots indicate a totally decoupled sector which is not relevant for our purposes. In fact, the scalar fields of interest are contained only in \( \pi \) and \( b_\alpha \) and those fields mixing with them. One can check this statement by analyzing the corresponding field equations

\[
\begin{align*}
\frac{9}{28} \nabla^\alpha \nabla_\alpha \pi - \frac{9}{98} \nabla^\kappa \nabla_\kappa \pi + \frac{45}{14} e^2 \pi \pi - 3e \nabla^\alpha b_\alpha + \left( \frac{9}{28} \nabla^\kappa \nabla_\kappa \pi - \frac{3}{2} e^2 \pi \right) h_{\alpha\alpha} &= 0, \\
2 \nabla^\alpha \nabla^\beta b_\beta + 2 \nabla^\kappa \nabla_\kappa b_\alpha - 3e \nabla^\alpha \pi + e \nabla^\alpha h_{\beta\beta} &= 0, \\
h_{\alpha\alpha} - \frac{9}{7} \pi &= 0, \\
\nabla^\alpha b_\beta - \nabla^\beta b_\alpha &= 0. \quad (2.6d)
\end{align*}
\]
The last two equations are constraints arising from imposing gauge fixing conditions analogous to (1.8a)–(1.8b) and (1.9), which imply in particular $p = 0$, $a_{\kappa\gamma\delta} = 0$ \(^2\). Substituting the solution of the constraints in the first two equations, one gets

\[
\bar{\nabla}^\alpha \nabla_\alpha \pi - \frac{28}{3} e \bar{\nabla}^\alpha \nabla_\alpha b + \bar{\nabla}^\kappa \nabla_\kappa \pi + 4e^2 \pi = 0, \quad (2.7a)
\]
\[
\bar{\nabla}^\alpha \nabla_\alpha b + \bar{\nabla}^\kappa \nabla_\kappa b - \frac{6}{7} e \pi = 0, \quad (2.7b)
\]

where $b$ is defined by the relation $b^\alpha = \bar{\nabla}^\alpha b$, which solves (2.6d). Thus, the above action describes a symmetric rank 2 tensor $h_{\alpha\beta}$ and two scalar fields $\pi$, $b$. One can expand them in spherical harmonics and diagonalize the field equations (2.7a)–(2.7b) to obtain the massive Kaluza–Klein towers of scalars of interest \([17]\).

However, we now face a technical difficulty. It is apparently hard to diagonalize the action (2.5) by performing field redefinitions. The reason for this is that, as shown above, the vector field $b_\alpha$ describes simply a scalar field $b$. A field redefinition replacing the vector field by the scalar one is a form of duality transformation. Typically, such transformations exchange field equations and Bianchi identities and are not easily done at the level of action in a consistent way by simply substituting in some of the field equations. In our case, however, this goal can be achieved by rewriting the original 11d supergravity action introducing appropriate auxiliary fields. These, when eliminated in a judicious way, realize the desired duality transformation.

Explicitly, instead of the customary 3–form formulation of 11d supergravity, we shall adopt the dual 3/6–form formulation worked out in \([19–20]\). In this approach, the basic fields of the bosonic sector are the metric $g$, the 3–form field $A$ and the 6–form field $C$. The bosonic part of the action now reads

\[
I = \frac{1}{4\kappa^2} \int_{M_{11}} \left[ R(g) \ast_g 1 - H(A,C) \wedge \ast_g H(A,C) - \frac{2\frac{1}{3}}{3} A \wedge F(A) \wedge F(A) \right]. \quad (2.8)
\]

$R(g)$ is the Ricci scalar of the metric $g$. $F(A)$, $H(A,C)$ are the 4–form field strength of the 3–form $A$ and the 7–form field strength of the 6–form $C$ and are given by

\[
F(A) = dA, \quad H(A,C) = dC + \frac{1}{2\sqrt{2}} A \wedge dA. \quad (2.9)
\]

\(^2\) To be precise, (2.6c)–(2.6d) holds only up to zero modes of the operators $\bar{\nabla}_\kappa \bar{\nabla}_\lambda - \frac{1}{7} g_{\kappa\lambda} \bar{\nabla}_\rho \bar{\nabla}_\rho$ and $\bar{\nabla}_\kappa$, respectively \([15]\). We will neglect this technical complications in the following discussion for the sake of simplicity.
The model must be supplemented with the duality constraint

\[ \ast_g H(A, C) + F(A) = 0. \]  \hfill (2.10)

Using additional auxiliary/gauge fields, one can incorporate this constraint directly at the level of the action [20], but this will not be necessary for our purposes.

One can easily verify that the field equations admit a standard solution \( \bar{g}_{ij}, \bar{A}_{ijk}, \bar{C}_{ijklmn} \) generated by a Freund Rubin type ansatz [18]. For this, the only non-vanishing components of the Riemann tensor and the 4–form field strength are as in (2.1a)–(2.1b), (2.2), while the 7–form field strength is given by

\[ H(\bar{A}, \bar{C})_{\kappa\lambda\mu\nu\rho\sigma} = e\epsilon_{\kappa\lambda\mu\nu\rho\sigma}. \]  \hfill (2.11)

We expand the action (2.8) in fluctuations around the above background. We parametrize the fluctuations \( \delta g_{ij} \) as in (2.3a)–(2.3c) and write the relevant fluctuations \( \delta C_{ijklmn} \) as

\[ \delta C_{\kappa\lambda\mu\nu\rho\sigma} = 6\bar{\nabla}_{[\kappa a} a_{\lambda\mu\nu\rho\sigma]} + \bar{\epsilon}_{\kappa\lambda\mu\nu\rho\sigma}^\tau \bar{\nabla}_\tau b, \]  \hfill (2.12)

\[ \bar{\nabla}^\rho a_{\rho\kappa\lambda\mu\nu} = 0. \]

The fluctuations \( \delta A_{ijk} \) are not independent from \( \delta g_{ij}, \delta C_{ijklmn} \) because of the constraint (2.10). We have explicitly checked, however, that the components of \( \delta A_{ijk} \) that couple in the action and in the constraint to the fields \( h_{\alpha\beta}, \pi, b \) give a vanishing contribution to the quadratic action of these latter fields, once the constraint is taken into account.

Therefore, the quadratic action of the fields \( h_{\alpha\beta}, \pi, b \) could be evaluated by setting formally \( \delta A_{ijk} = 0 \) from the beginning. The expression we obtain is

\[ I_{[2]} = \frac{1}{4\kappa^2} \int_{\text{AdS}_4} d^4y(-\bar{g}_4)^{\frac{1}{2}} \int_{S^7} d^7x(\bar{g}_7)^{\frac{1}{2}} \frac{1}{2} \left\{ \frac{1}{2} \bar{\nabla}^\alpha h_{\alpha\gamma} \bar{\nabla}^\beta h_{\beta\gamma} - \frac{1}{2} \bar{\nabla}^\alpha h_{\alpha\beta} \bar{\nabla}^\beta h_{\gamma\gamma} + \frac{1}{4} \bar{\nabla}^\alpha h_{\beta\gamma} \bar{\nabla}_\alpha h_{\beta\gamma} - \frac{1}{4} \bar{\nabla}^\alpha h_{\beta\gamma} \bar{\nabla}_\alpha h_{\beta\gamma} \\
+ \frac{1}{4} \bar{\nabla}^\kappa h_{\beta\gamma} \bar{\nabla}_\kappa h_{\beta\gamma} - \frac{1}{4} \bar{\nabla}^\kappa h_{\beta\gamma} \bar{\nabla}_\kappa h_{\beta\gamma} + \frac{1}{18} e^2 h_{\alpha\beta} h_{\beta\gamma} + \frac{1}{9} e^2 h_{\alpha\beta} h_{\alpha\beta} \right. \\
- \frac{9}{56} \bar{\nabla}^\alpha \pi \bar{\nabla}_\alpha \pi + \frac{9}{196} \bar{\nabla}^\kappa \pi \bar{\nabla}_\kappa \pi - \frac{9}{14} e^2 \pi \pi \\
- \bar{\nabla}^\alpha \bar{\nabla}^\kappa b \bar{\nabla}_\alpha \bar{\nabla}_\kappa b - \bar{\nabla}^\kappa \bar{\nabla}_\kappa b \bar{\nabla}^\lambda \bar{\nabla}_\lambda b + 3e \bar{\nabla}^\kappa \bar{\nabla}_\kappa b \pi \\
+ h_{\alpha\beta} \left( \frac{9}{28} \bar{\nabla}^\kappa \bar{\nabla}_\kappa \pi - e \bar{\nabla}^\kappa \bar{\nabla}_\kappa b \right) + \frac{1}{2}(\bar{\nabla}^\kappa \bar{\nabla}_\kappa - \frac{1}{7} \bar{g}_{\kappa\lambda} \bar{\nabla}^\rho \bar{\nabla}_\rho) p \bar{\nabla}^\kappa \bar{\nabla}^\lambda \left( h_{\alpha\beta} - \frac{9}{7} \pi \right) \\
- \bar{\nabla}^\kappa l^\alpha \bar{\nabla}_\kappa \left( \frac{9}{14} \bar{\nabla}^\alpha \pi - 2e \bar{\nabla}^\alpha b + \bar{\nabla}^\beta h_{\beta\alpha} - \bar{\nabla}^\alpha h_{\beta\beta} \right) + \ldots \right\}. \]  \hfill (2.13)
Above, the ellipses denote terms not containing the fields $h_{\alpha\beta}$, $\pi$, $b$. Note that the field components $m_{\kappa\lambda}$, $n_{\kappa}$, $k_{\kappa\alpha}$, $a_{\kappa\lambda\mu\nu}$, $\delta C_{ijklm}$ are decoupled at quadratic level from $h_{\alpha\beta}$, $\pi$, $b$. Also, $p$ and $l_{\alpha}$ act as Lagrange multipliers enforcing certain constraints.

The gauge is partially fixed by imposing

\[ \nabla^\rho (\delta g_{\rho\kappa} - \frac{1}{4} \bar{g}_{\rho\kappa} \delta g^\sigma) = 0, \quad \nabla^\rho \delta g_{\rho\alpha} = 0; \quad (2.14a) - (2.14b) \]

\[ \nabla^\rho \delta C_{ijklm\rho} = 0, \quad (2.15) \]

as in [17]. These imply in particular that $p = 0$, $l_{\alpha} = 0$. Fixing the gauge as indicated, the last two terms of $I[2]$ vanish. The corresponding constraints must then be enforced by hand.

However, instead of fixing the gauge at this stage, we prefer first to isolate the relevant scalar degrees of freedom by performing field redefinitions, as done in the previous section. Following again [14], we perform a field redefinition of the form (1.12) with $\eta$ and $\zeta$ chosen in such a way to decouple at the quadratic level the field $\phi_{\alpha\beta}$ from the $\pi-b$ sector. One finds in this way that

\[ h_{\alpha\beta} = \phi_{\alpha\beta} + (\nabla_\alpha \nabla_\beta - \frac{1}{2} g_{\alpha\beta} \nabla^\kappa \nabla_\kappa) \varphi, \quad (2.16a) \]

where

\[ \varphi = (- \nabla^\lambda \nabla_\lambda + \frac{4}{9} e^2)^{-1} \left( \frac{3}{7} \pi - \frac{4}{3} eb \right). \quad (2.16b) \]

The action $I[2]$ then becomes

\[
I[2] = \frac{1}{4 \kappa^2} \int_{\text{AdS}_4} d^4 y (- \bar{g}_4) \frac{1}{2} \int_{S^7} d^7 x \bar{g}_7 \frac{1}{2} \left\{ \frac{1}{2} \nabla^\alpha \phi_{\alpha\gamma} \nabla^\beta \phi_{\beta\gamma} - \frac{1}{2} \nabla^\alpha \phi_{\alpha\beta} \nabla^\gamma \phi_{\beta\gamma} + \frac{1}{4} \nabla^\alpha \phi_{\beta\gamma} \nabla_\alpha \phi_{\beta\gamma} - \frac{1}{4} \nabla^\alpha \phi_{\alpha\beta} \nabla_\alpha \phi_{\beta\gamma} + \frac{1}{4} \nabla^\alpha \phi_{\beta\gamma} \nabla_\alpha \phi_{\beta\gamma} + \frac{1}{18} e^2 \phi_{\alpha\beta} \phi_{\alpha\beta} + \frac{1}{9} e^2 \phi_{\alpha\beta} \phi_{\alpha\beta} \\
- \frac{3}{4} \nabla^\kappa \nabla_\kappa \left( - \nabla^\lambda \nabla_\lambda + \frac{4}{9} e^2 \right)^{-1} \nabla^\alpha \left( \frac{3}{7} \pi - \frac{4}{3} eb \right) \nabla_\alpha \left( \frac{3}{7} \pi - \frac{4}{3} eb \right) \\
- \frac{3}{4} \nabla^\kappa \nabla_\kappa \nabla^\lambda \nabla_\lambda \left( - \nabla^\mu \nabla_\mu + \frac{4}{9} e^2 \right)^{-1} \left( \frac{2}{7} \pi - \frac{4}{3} eb \right) \\
- \frac{3}{3} \nabla^\alpha \nabla_\alpha \nabla^\alpha \nabla_\alpha + \frac{9}{16} \nabla^\alpha \nabla_\alpha \nabla^\kappa \nabla_\kappa - \frac{9}{4} e^2 \nabla^\alpha \nabla_\alpha \\
- \nabla^\alpha \nabla_\alpha \nabla^\alpha \nabla_\alpha b - \nabla^\kappa \nabla_\kappa \nabla^\lambda \nabla_\lambda b + 3 e \nabla^\kappa \nabla_\kappa b \\
+ \frac{1}{2} \left( \nabla^\kappa \nabla_\kappa - \frac{1}{4} g_{\kappa\lambda} \nabla^\mu \nabla_\mu \right) \nabla^\kappa \nabla_\kappa [\phi_{\alpha\beta} - \frac{9}{7} \pi] \\
+ (\nabla^\alpha \nabla_\alpha - 2 \nabla^\kappa \nabla_\kappa) \left( - \nabla^\lambda \nabla_\lambda + \frac{4}{9} e^2 \right)^{-1} \left( \frac{3}{7} \pi - \frac{4}{3} eb \right) \right\} \right\}. \quad (2.17)
\]
Let us concentrate on the scalar $\pi$–$b$ sector of the above action. Proceeding as in the previous section, we expand the fields $\pi$, $b$ in harmonics $Y^{(7)}_I$ of the S$_7$ d’Alembertian $\nabla^\kappa \nabla_\kappa$

$$\nabla^\kappa \nabla_\kappa Y^{(7)}_I = -\frac{1}{18}e^2k(k + 6)Y^{(7)}_I,$$

where $k = 0, 1, 2, \ldots$ depends on $I$. The $Y^{(7)}_I$ are conventionally normalized as

$$\int_{S_7} (g_7)\frac{1}{2} Y^{(7)}_I Y^{(7)}_J = (\frac{1}{18}e^2)^{-\frac{7}{2}z^{(7)}(k)}\delta_{IJ}, \quad z^{(7)}(k) = \frac{\pi^4}{2k-1(k + 1)(k + 2)(k + 3)}.$$

The expansions read

$$\phi_{\alpha\beta} = \sum_I \phi_{I\alpha\beta}Y^{(7)}_I, \quad \pi = \sum_I \pi_I Y^{(7)}_I, \quad b = \sum_I b_I Y^{(7)}_I,$$

$$p = \sum_I p_I Y^{(7)}_I, \quad l_\alpha = \sum_I l_{I\alpha} Y^{(7)}_I. \quad (2.20a) - (2.20e)$$

One introduces new scalars $s_I$, $t_I$ such that

$$\pi_I = -\frac{7}{3}ks_I + \frac{7}{3}(k + 6)t_I, \quad (2.21a)$$

$$b_I = 3e^{-1}(-s_I + t_I), \quad (2.21b)$$

In terms of $s_I$, $t_I$, the scalar $\pi$–$b$ sector of $I_{[2]}$ takes the simple form

$$I_{[2]} = \frac{1}{4\kappa^2}(\frac{1}{18}e^2)^{-\frac{7}{2}} \int_{\text{AdS}_4} d^4y(-g_4)^{\frac{1}{2}}\sum_I z^{(7)}(k)$$

$$\left\{ \frac{1}{2}\nabla^\alpha \phi_{I\alpha\gamma} \nabla^\beta \phi_{I\beta\gamma} - \frac{1}{2}\nabla^\alpha \phi_{I\alpha\beta} \nabla^\beta \phi_{I\beta\gamma} + \frac{3}{4}\nabla^\alpha \phi_{I\beta\beta} \nabla^\beta \phi_{I\gamma\gamma} - \frac{3}{4}\nabla^\alpha \phi_{I\gamma\beta} \nabla^\beta \phi_{I\gamma\gamma} \right.$$ 

$$+ \frac{1}{4}m_{I\alpha}^2 \phi_{I\alpha\beta} \phi_{I\beta\beta} - \frac{1}{4}m_{I\beta}^2 \phi_{I\alpha\beta} \phi_{I\alpha\beta}$$

$$+ A_{sI} \left[ -\frac{3}{2}\nabla^\alpha s_I \nabla_\alpha s_I - \frac{3}{2}m_{sI}^2 s_I s_I \right] + A_{tI} \left[ -\frac{3}{2}\nabla^\alpha t_I \nabla_\alpha t_I - \frac{3}{2}m_{tI}^2 t_I t_I \right]$$

$$+ u_{JI} \left[ \phi_{I\alpha} \phi_{J\beta} - \phi_{I\alpha} \phi_{J\beta} \right] + v_{IJ} \left( \nabla^\alpha \nabla_\alpha s_I - m_{sI}^2 s_I \right) + v_{IJ} \left( \nabla^\alpha \nabla_\alpha t_I - m_{tI}^2 t_I \right)$$

$$- w_{JI} \left[ \nabla^\alpha \phi_{I\beta} - \nabla^\alpha \phi_{I\beta} \right] + \ldots \right\}, \quad (2.22)$$

where

$$A_{sI} = \frac{2(k + 3)k(k - 1)}{(k + 2)}, \quad (2.23a)$$
\[ A_{tI} = \frac{2(k + 3)(k + 6)(k + 7)}{(k + 4)}, \tag{2.23b} \]
\[ m''_{\phi I} = \frac{1}{18} e^2 (k^2 + 6k + 4), \tag{2.24a} \]
\[ m'''_{\phi I} = \frac{1}{18} e^2 (k^2 + 6k - 8), \tag{2.24b} \]
\[ m_{sI}^2 = \frac{1}{18} e^2 k(k - 6), \tag{2.25a} \]
\[ m_{tI}^2 = \frac{1}{18} e^2 (k + 6)(k + 12), \tag{2.25b} \]
\[ u_I = \left( \frac{1}{18} e^2 \right)^\frac{2}{3} \frac{1}{(k - 1)k(k + 6)(k + 7)}, \tag{2.26} \]
\[ v_{sI} = \left( \frac{1}{18} e^2 \right)^{-1} \frac{1}{(k + 2)}, \tag{2.27a} \]
\[ v_{tI} = \left( \frac{1}{18} e^2 \right)^{-1} \frac{1}{(k + 4)}, \tag{2.27b} \]
\[ w_I = \frac{1}{18} e^2 k(k + 6). \tag{2.28} \]

Note that, also in this case, the modes \( s_I \) with \( k = 0, 1 \) do not appear so that these are gauge degrees of freedom. Fixing the gauge involves setting \( p_I = 0, l_{I\alpha} = 0 \). Thus, after gauge fixing, \( s_I, t_I \) are free fields. On the \( s_I, t_I \) mass shell the constraints are again of the form (1.24a)–(1.24b). The mass spectrum coincides with that found in [17], as expected.

### 3. Conclusions and outlook

We have determined from the 11d supergravity action compactified on \( \text{AdS}_7 \times \text{S}_4 \) and \( \text{AdS}_4 \times \text{S}_7 \) the quadratic AdS bulk action for two Kaluza–Klein towers of scalar excitations described by the fields \( s_I, t_I \). In the AdS/CFT correspondence these fields are sources for the boundary SCFT operators whose dimensions have been worked out in [9–12] from the knowledge of their masses reported in [15–17]. Apart from reobtaining those masses, we have computed the normalization factors \( A_{sI}, A_{tI} \) which fix the normalization of the 2 point functions. We have obtained this result by isolating the leading quadratic term of the relevant actions through field redefinitions. This has allowed us to remain off–shell for the \( s, t \) scalar fluctuations, thus avoiding the delicate task of keeping track of all possible boundary terms. On the other hand, we have found that the on–shell method proposed in [4] is not complete and does not yield the correct result, as noticed also in [13]. Presumably certain boundary terms have been missed in [4].

In the \( \text{AdS}_7 \times \text{S}_4 \) case, the coefficient \( A_{sI} \) was also recently obtained in [13] using a different method involving the calculation of certain three point couplings. We agree with their result.
In the $\text{AdS}_4 \times S_7$ compactification, the identification of the normalizations is more laborious. In this case the standard 3–form formulation of 11d supergravity gives the off-shell description of a linear combination of the scalars $s, t$ in terms of a vector fields $b_\alpha$. Therefore, we have used a dual description which produces directly both scalars and computed the relevant normalizations.

With these normalizations at hand, one can proceed to compute the cubic $s$–$t$ self-coupling to obtain the large $N$ limit of the three point correlation functions of the corresponding operators belonging to chiral representations of the maximally supersymmetric $SCFT_3$ and $SCFT_6$ theories, thus adding extra pieces of information on these somewhat mysterious theories.
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