Study of Fractal and Multifractal Features of Pore Structure in Tight Sandstone Reservoirs of the Permian Lucaogou Formation, Jimsar Sag, Junggar Basin, Northwest China

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ABSTRACT: The primary factor impacting the tight sandstone reservoirs and fluid flow capacity represents the pore-throat structure. On the basis of petrophysical characteristics test, scanning electron microscopy (SEM), and casting thin-section examination of tight sandstone reservoir specimens from the Permian Lucaogou Formation in Jimsar Sag, Junggar Basin., the full-size pore-throat parameters and distribution characteristics were determined by constant-rate mercury injection (CRMI) analysis. Using fractal theory and multifractal theory, the pore architectures of sandstone pores are analyzed. Mercury intrusion capillary pressure (MICP) is used to compute the dimensions of fractals using various fractal models and multifractal characteristics. Analysis is done on the relationships between tight sandstone pore architectures and fractal and multifractal characteristics. According to the research, a network of tightly packed sandstone pores may be assessed using the dimensions of fractals computed from a 3D capillary model. When displacement pressure is increased, the dimensions of fractals rise; when permeability, pore-throat diameter, and variable coefficient are increased, it falls. Tight sandstone pores exhibit multifractal features, according to the multifractal analysis, and multifractal parameters may depict the size, concentration, and asymmetry of the pore size distribution (PSD). Sandstone’s PSD is comparable when its multifractal parameters ($\Delta \alpha$, $\Delta f$, $\alpha_0$, $\alpha_1$, $\alpha_2$) are identical. Pore diameters of tight sandstone are positively connected with information dimensions $D_1$ and correlation dimensions $D_2$, and information dimensions $D_1$ have a greater impact on PSD than correlation dimensions $D_2$. Additionally, the 3D capillary model’s dimensions of fractals and $D_1$ exhibit a substantial negative association. These findings play a significant guiding role in the quantitative characterization of unconventional reservoir pore structures. The multifractal technique is effective to define the heterogeneity of the sandstone pore system and to differentiate between distinct PSD in heterogeneity.

INTRODUCTION

As the theory and exploration of oil and gas advance and energy demand rises, unconventional oil and gas exploration and production are becoming increasingly important. Tarim Basin, Junggar Basin, Ordos Basin, and Bohai Bay Basin are among the basins where domestic employees will have finished exploring and exploiting tight oil and gas reserves by 2021.¹,² In Jimsar Sag, Junggar Basin, the Permian Lucaogou Formation has high-quality tight reservoirs with low porosity and low permeability, typically 10−20% and permeability greater than 1 mD.

The pore-throat structure of tight sandstone is complicated, there are few connections between reservoirs, and mining it is difficult. Pore-throat shape also affects reservoir quality.³ The pore-throat connections in tight sandstone reservoirs are complicated, and they differ greatly from one another.⁴ When diagenesis occurs, compaction and cementation mostly control permeability and pore-throat microstructures. It is possible for pore-throat radius distribution and permeability of a rock to be very different, even if the porosity of the rock represents the same. As a result of these facts, tight sandstone reservoirs’ permeability and pore-throat microstructure are the most important factors in determining how much oil and gas can leak out of the reservoir, how easily oil and gas can move through the reservoir, and how much oil and gas can be stored.⁵ The development of fractal theory has made measuring how pore-throat microstructures are formed easier using statistics to measure the complexity and irregularity of fractal objects in nature.⁶

It is possible to deal with the complexity of narrow sandstone pores in porous media using fractal theory. To

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study the fractal properties of rock pores, Pfeifer et al. used molecular adsorption technology. They examined the relationship between the different dimensions of fractals of rock in oil and gas reservoirs and their relationship with the permeability coefficient and porosity using a three-dimensional sphere model, a three-dimensional capillary model, a geometric model, and a volumetric dimensions of fractals model. In the Guizhou Longtan Formation, Huang et al. examined the relationship between permeability and pores in marine and continental transitional shales. To examine how different shale is from one another, Wang et al. used SEM digital images, fractal and multifractal theory, and shale samples. Using field emission scanning electron microscopy (FE-SEM), gas adsorption, and helium porosity measurements, Yang et al. studied the pore structure of organic-rich shale. Shale micropores were found to be primarily responsible for the dimensions of fractals. Mercury intrusion capillary pressure (MICP) results were influenced by the submicron pore network and the density of the rock, as stated by Elgmati et al. A lattice Boltzmann method (LBM) can be utilized to examine porosity and pore-throat distributions in porous media. It is also possible to determine the relationship between permeability and structural properties such as particle radius, tortuosity, and porosity using LBM. In addition, we examined a number of single fractal models and selected the one with the strongest correlation to study the pore-throat space and microscopic features of these middle-deep tight reservoirs of high quality. The pore-throat space of tight oil reservoirs can also be described using multifractal theory. To verify the accuracy of the pore-throat space multifractal spectrum, the single fractal model with the highest correlation is used. Pore-throat space can also be described in this manner.

**Geological Setting.** Junggar Area in Northwest China is a substantial hydrocarbon-rich basin with considerable oil and gas resources and great development potential (Figure 1A). Jimsar Sag is a 1200 km² oil deposit located in the eastern Junggar Basin (Figure 1B). The Santai fault surrounds it in the south, the Jimsar fault in the north, the Shaqi uplift in the east, Xidi fault in the west, Laozhuangwan fault in the south, and the Qitai uplift in the south. Santai, Jimsar, Shaqi, Xidi, and Laozhuangwan are the faults and uplifts found in this order: Santai, Jimsar, Shaqi, Xidi, and Laozhuangwan. The Jimsar Sag has been through many stages of tectonic activity ever since the Hercynian orogeny. These stages have had an effect on the Carboniferous igneous rock that serves as its foundation as well as the sedimentation of the Permian, Triassic, Jurassic, Cretaceous, Paleogene, and Neogene series. In the area under study, the thickness of the strata gradually decreases as one travels from west to east (Figure 1C). Most of the source rock for the sag may be found in the semideep to deep lake
components. In addition, a Zeiss Merlin FE-SEM was used to observe the pore structure. A polarized microscope was used to determine modal compositions for each slice. More than 300 points were analyzed on each slice using a Zeiss Axioskop40 polarized microscope to determine modal compositions for each slice.

Deposits of the Permian Lucaogou Formation. A substantial area of the surface is covered by these deposits. There should be at least 200 m of thickness in the ideal area, which covers 806 km². Tight oil is extracted from the Lucaogou Formation, which has a thickness between 200 and 300 m. Its intricate architecture, size, connectivity, and mineral distribution can explain the preponderance of fine-grained mixed sedimentary rock in the Lucaogou Formation (Figure 1D).

**RESEARCH METHODS**

All core samples were divided into tiny, cylindrical columns with a diameter of around 2.5 cm and a length of roughly 5 cm prior to the experiment. All tiny columns were then dried at 100 °C for 24 h after being extracted with a solution of methanol and dichloromethane in a Soxhlet extractor to remove the oil. Prior to cleaning, the porosity and permeability of all cylindrical samples were measured using a porosity meter (HZ-K16) with a relative error of less than 2%. Permeability under nitrogen stability was measured using a gas permeability meter, which has excellent data collection accuracy of less than 0.1 L. To conduct the experiment, the samples were polished with argon ions, then placed in the core chamber, and vacuum-pumped for 1 h. Mercury was poured into the core chamber after the two measuring and balancing valves were opened. Afterward, mercury was injected with steady increasing pressure and 24 points were recorded. Vacuum extraction, mercury injection, and pressure readings were all managed automatically by the AutoPore iv 9505 software. People’s Republic of China (SY/T) 5346−2005 Oil and Gas Industry Standard was followed during the test.

**Thin-Section Examination.** Depending on the kind of study they were intended for, all samples were divided into sections of 1 or 2.5 cm. We prepared and examined materials for casting thin sections, FE-SEM, and high-pressure mercury intrusion (HPMI). To directly analyze the materials’ tiny pore-throat structural features, two-dimensional imaging of the samples was acquired.

Blue epoxy resin was thinly cast in all cylindrical samples to emphasize the pores. An optical acquisition system was used to count more than 300 points on each slice using a Zeiss Axioskop40 polarized microscope to determine modal components. In addition, a Zeiss Merlin FE-SEM was used for ultrahigh resolution analysis of samples ranging from 0.1 nm to 1 m in size, providing a better understanding of pore architecture, size, connectivity, and mineral distribution.

**Rinse-Flow Testing.** To examine the possible impact of injected brine on pore-throat structure, 24 dry core samples with various permeabilities were injected with 100 pore volume (PV) brine. In accordance with reservoir conditions, the synthetic brine was injected into the dry core sample at a pressure of 20 MPa (2900.75 psi) and a temperature of 65 °C. The experiment came to an end when the injection volume reached 100 PV. To determine the sample’s porosity and permeability, it was dried once more after being injected. To lessen the inherent uncertainty, samples were obtained at the same depth and drilled in the same direction, and then variations in pore-throat structure were compared to neighboring samples using HPMI and CRMI assays.

**Experiment Using High-Pressure Mercury Intrusion.** All samples were tested using an AutoPore iv 9505 automated mercury porosity meter, which has excellent data collection accuracy and a mercury intrusion and extrusion volume accuracy of less than 0.1 L. To conduct the experiment, the samples were polished with argon ions, then placed in the core chamber, and vacuum-pumped for 1 h. Mercury was poured into the core chamber after the two measuring and balancing valves were opened. Afterward, mercury was injected with steady increasing pressure and 24 points were recorded. Mercury intrusion, mercury injection, and pressure readings were all managed automatically by the AutoPore iv 9505 software. People’s Republic of China (SY/T) 5346−2005 Oil and Gas Industry Standard was followed during the test.

**Constant-Rate Mercury Injection Experiment.** Using the CRMI test, pores and pore throats can be distinguished efficiently. Generally, the CRMI technique injects mercury into rock samples at modest, constant flow rates without varying contact angles or interfacial tensions. Mercury enters the hole throat, causing pressure to rise. If mercury enters the pores, pressure drops. By capturing mercury pressure−volume curves, rock samples can be analyzed for pore and throat information. The ASPE-730 facility (Core test System, Inc.) conducted 20 CRMI tests on 24 core samples before and after the brine-flow test. Mercury injection pressures of 4.552 MPa (660.21 psi) to 6.205 MPa (899.96 psi) were observed for various samples with the mercury injection rate remaining constant at 0.1 mL/min. The throat radius should be between 0.12 and 0.16 mm.

**Analysis of X-ray Diffraction.** A helpful technique for determining the mineralogical makeup of rock samples,
including clay minerals, is X-ray diffraction. The samples of tight sandstone are first pulverized into 200 meshes and formed into glass slides for necessary XRD investigation. To avoid carbonization of the sample during the grinding process, anhydrous ethanol was added to 5 g of samples that were chosen at random and ground in the grinder. The samples were dried at 40 °C to a consistent weight. XRD was used to determine the mass fraction of each phase in Table 1. For whole-rock (bulk) mineralogical analysis, an XRD experiment is performed using an X’Pert MPD PRO according to Chinese Oil and Gas Industry Standard SY/T5983–1994 and SY/T S163-1995. For the quantitative analysis, Jade was used. The results of the study are shown in Figure 2.

Figure 2. Folk categorization triangle (1980) illustrates the detrital content of tight sandstones. The F/R ratio can be defined as the ratio of feldspar to rock fragments.

An Overview of the Experiment’s Results and Materials. This study examined samples from the Jimsar Sag sandstone reservoir, part of the Lu 2 Member of the Lucaogou Formation. A total of 24 cast thin sections and 24 core samples of typical sandstone were selected from various wells between 2717.93 and 3760.548 m in depth (Figure 3). Table 2 shows sample features. Most of the samples are composed of siltstone. Pore-throat structures can be identified using direct and indirect techniques. A direct extraction of pore bodies and throats, pore-throat connectivity, and clay mineral content may be achieved using SEM and XRD in the first case. Examples of indirect detection techniques are HPMI and CRM2 investigations, which monitor and record mercury injection saturation and measure pore-throat radius. CRM1 may also be used to separate pore bodies and throats. To accurately identify the pore-throat structure, a comprehensive technique was employed since each pore-throat structure detection technique has technological limitations.

Fractal Theory. 3D Capillary Model. The pore system’s fractal character has been validated by earlier investigations on sedimentary rocks. On the basis of SEM, thin sections, and mercury intrusions, many approaches have been put forward to compute the dimensions of fractals of pore structure. But in this investigation, only the mercury intrusion method was taken into account. To obtain dimensions of fractals from capillary pressure curves, there have primarily been three methods developed. Fractals are imaginary, self-similar geometric objects that seem the same regardless of the magnification size. An essential characteristic of a fractal item in nature is self-affinity with a dimension, and this characteristic may be mathematically expressed by a power-law function (eq 1).

\[ N(r) \propto r^{-D_f} \]  

where \( r \) represents a unit’s characteristic length, \( \alpha \) represents “proportional to”, \( N(r) \) represents the number of objects having a characteristic linear dimension greater than \( r \), and \( D_f \) represents the dimensions of fractals. \( D_f \) lies in the range 0 < \( D_f < 2 \) in two-dimensional space and 0 < \( D_f < 3 \) in three-dimensional space.

It is assumed that the pores in sandstone are composed of capillary bundles. With the fractal power-law function, the mercury saturation can be calculated as follows (eq 2). The Younge–Laplace law can be utilized to determine the relationship between the pore radius \( r \) and capillary pressure \( \sigma \).

\[ S_{Hg} = \frac{V_{Hg}}{V_p} = \frac{N(r) \cdot V_c \cdot r^{D_f - 2}}{V_p} = \frac{r^{-D_f} \cdot V_c \cdot r^{D_f - 2}}{V_p} = \frac{2 \pi \cos \theta}{\sigma} \]  

where \( V_{Hg} \) represents the volume of mercury in total; \( S_{Hg} \) represents the saturation level of mercury (%); \( r \) represents the radius of the pores; \( I \) represents the length of the core; \( V_p \) represents the pore volume of the core samples (%); \( V_c \) represents the pressure in capillaries (MPa); \( D_f \) represents the dimensions of fractals of the capillary model in 3D; \( \sigma \) represents surface tension; \( \theta \) represents the contact angle (deg); and \( a \) represents the coefficient for eq 2.

Geometry Model. Yu and Luo proposed a formula to calculate the dimensions of fractals from MICP. Taking the logarithm on both sides of the above equation

\[ \frac{d \ln S_{Hg}}{d P} = (D_f - 4) \ln P_c + \ln C \]  

where \( D_f \) represents the geometry model’s dimensions of fractals and \( C \) represents eq 3 of parameters.

Wetting Phase Model. There have been a number of models proposed for calculating the dimensions of fractals using mercury intrusion porosimetry (MIP). This study used the capillary tube model to calculate the dimensions of...
fractals. This simplified mathematical relationship can be expressed as follows:

$$S_{tg} = \frac{V_{tg}}{V_0} = \frac{N(r) \cdot r^2}{N_{\infty} \cdot r^2} = \int_{r_{\min}}^{r_{\max}} P(r) dr \cdot r^2 / \int_{r_{\min}}^{r_{\max}} P(r) dr \cdot r^2$$

$$= \frac{D_{f_{max}}}{D_{f_{min}} - 3} \left( r_{\max}^{3 - D_{f_{max}}} - r_{\min}^{3 - D_{f_{min}}} \right) \left( 1 - \left( \frac{r_{\min}}{r_{\max}} \right)^{3 - D_{f_{min}}} \right)$$

$$= \frac{D_{f_{max}}}{D_{f_{min}} - 3} \left( r_{\max}^{3 - D_{f_{max}}} - r_{\min}^{3 - D_{f_{min}}} \right) = 1 - \left( \frac{r_{\min}}{r_{\max}} \right)^{D_{f_{min}} - 3}$$

where $D_{f_{max}}$ represents the dimensions of fractals of the wetting phase model, $r_{\max}$ represents the maximum pore radius, and $r_{\min}$ represents the minimum pore radius. Because $r_{\max}$ is far larger than $r_{\min}$ in the samples, the fractal formula can be obtained assuming that the wetting contact angle is unaffected by the pore-throat size.

$$1 - S_{tg} = \left( \frac{r}{r_{\max}} \right)^{D_{f_{min}} - 3} \left( \frac{P_{\infty}}{P_{\min}} \right)^{D_{f_{max}} / 3}$$

$$\log(1 - S_{tg}) = (D_{f_{max}} - 3) \log P_{\infty} - (D_{f_{min}} - 3) \log P_{\min}$$

**Small Island Model.** Zhao states the following equation based on the dimensions of fractals theory of the “small island method”

$$Z = \frac{1}{n} A^{D_{f_{island}} / 2}$$

where $Z$ represents the pore circumference ($\mu m$), $A$ represents the pore area ($\mu m^2$), $D_{f_{island}}$ represents the dimensions of fractals of the small island model, and $n$ represents the fractal coefficient. $D_{f_{island}}$ represents only related to the selected scale, but not to the size of the selected graph. According to the fitting formula, pore circumference and area were fitted using a power function

$$y = ax^b$$

where $y$ and $x$ represent the perimeter ($\mu m$) and area ($\mu m^2$), respectively. As a function of the fractal coefficient $n$ and the dimensions of fractals, the fitting coefficient $b$ can be calculated $D_{f_{island}}$.

**Multifractal Theory.** Multifractal theory is an extension of traditional dimensions of fractals, which can describe the hidden information ignored by traditional dimensions of fractals. It is assumed that boxes with different sizes of $\varepsilon$ cover the investigation objects, and the number $N(\varepsilon)$ of boxes with a given size of $\varepsilon$ is

$$N(\varepsilon) \sim \varepsilon^{-D_0}$$

Taking logarithm of both sides of eq 9, the dimensions of fractals $D_0$ can be expressed as

$$D_0 = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log \varepsilon}$$

The probability of pore volume of mercury intrusion satisfies

$$P(\varepsilon) = \frac{N(\varepsilon)}{N_i}$$

where $N_i(\varepsilon)$ represents the injection pore volume in the $i$-th box and $N_i$ represents the total pore volume. According to multifractal theory, the probability will scale with different exponents for each box of size $\varepsilon$ unit as

$$P_i \propto \varepsilon^{\alpha_i}, \quad i = 1, 2, ..., N$$

where $\alpha_i$ is the singularity exponent, also known as Lipschitz–Holder index, and it represents the exponent for boxes, which

### Table 2. Characteristics of the Tight Sandstone Samples

| sample | well number | depth (m) | porosity (%) | permeability (mD) | range of grain size (mm) | rock |
|--------|-------------|-----------|--------------|------------------|-------------------------|------|
| 1      | J174        | 3143.30   | 16.20        | 0.25             | 0.063–0.25              | silstone in gray |
| 2      | J174        | 3144.84   | 14.50        | 0.18             | 0.010–0.13              | silstone in gray |
| 3      | J174        | 3274.65   | 13.90        | 0.024            | 0.031–0.13              | silstone in gray |
| 4      | J174        | 3275.43   | 15.50        | 0.047            | 0.031–0.13              | silstone in gray |
| 5      | J174        | 3276.61   | 16.20        | 0.21             | 0.031–0.13              | silstone in gray |
| 6      | J174        | 3277.50   | 13.40        | 0.038            | 0.031–0.13              | silstone in gray |
| 7      | J251        | 3750.70   | 15.20        | 0.020            | 0.010–0.060             | gray fine sandstone |
| 8      | J251        | 3754.45   | 12.60        | 0.010            | 0.030–0.060             | gray fine sandstone |
| 9      | J251        | 3760.55   | 18.50        | 0.13             | 0.010–0.060             | silstone in gray |
| 10     | J31         | 2717.93   | 17.40        | 0.045            | 0.063–0.13              | gray argillaceous siltstone |
| 11     | J31         | 2718.15   | 18.60        | 0.074            | 0.010–0.060             | gray argillaceous siltstone |
| 12     | J31         | 2718.52   | 15.80        | 0.043            | 0.060–0.18              | gray dolomitic siltstone |
| 13     | J31         | 2722.00   | 14.50        | 0.074            | 0.063–0.13              | gray argillaceous siltstone |
| 14     | J31         | 2722.40   | 19.00        | 0.53             | 0.063–0.13              | gray argillaceous siltstone |
| 15     | J31         | 2722.73   | 16.90        | 0.55             | 0.063–0.13              | gray argillaceous siltstone |
| 16     | J31         | 2723.73   | 18.80        | 0.95             | 0.063–0.13              | gray argillaceous siltstone |
| 17     | J31         | 2723.84   | 17.40        | 0.14             | 0.063–0.13              | gray argillaceous siltstone |
| 18     | J31         | 2725.10   | 13.70        | 0.63             | 0.063–0.13              | gray argillaceous siltstone |
| 19     | J31         | 2725.73   | 19.70        | 0.41             | 0.063–0.13              | gray argillaceous siltstone |
| 20     | J31         | 2725.90   | 17.20        | 0.14             | 0.063–0.13              | gray argillaceous siltstone |
| 21     | J31         | 2859.84   | 15.00        | 0.039            | 0.063–0.13              | gray argillaceous siltstone |
| 22     | J31         | 2896.77   | 12.60        | 0.24             | 0.063–0.13              | gray argillaceous siltstone |
| 23     | J31         | 2897.20   | 19.70        | 0.68             | 0.063–0.13              | gray argillaceous siltstone |
| 24     | J31         | 2897.75   | 14.00        | 0.016            | 0.063–0.13              | gray argillaceous siltstone |
theoretically represents how singularities of the systems. When \( \varepsilon \to 0 \), eq 12 can be expressed as

\[
\alpha_\varepsilon = \lim_{\varepsilon \to 0} \frac{\ln \mathcal{N}_\varepsilon}{\ln \varepsilon}
\]

where \( \alpha_\varepsilon \) represents the magnitude of probability in the region where it is located. The object is then divided into a series of subsets and every unit within each subset has the same singularity index \( \alpha \). We describe the relationship between the number of units \( \mathcal{N}_\varepsilon \) and \( \varepsilon \) as follows

\[
\mathcal{N}_\varepsilon \sim \varepsilon^{-\alpha(\varepsilon)} \quad \varepsilon \to 0
\]

where \( f(\alpha) \) represents the fractal spectrum, which indicates the dimensions of fractals of a subset with the same value \( \alpha \). \( f(\alpha) \) represents a fixed value if the object can be described by one dimension of fractals. Multifractal objects generally present a curve with a peak. Equation 14 can be expressed as follows

\[
f(\alpha) = -\lim_{\varepsilon \to 0} \frac{\ln \mathcal{N}_\varepsilon}{\ln \varepsilon}
\]

Information-theoretically, multifractal measures can be described by \( \alpha \sim f(\alpha) \) and by the set of index \( q \sim D_q \) (the generalized dimensions of fractals). The first step is to define the partition function \( \chi(q, \varepsilon) \) of moment of order \( q \). \( \chi(q, \varepsilon) \) represents the weighted sum of the probability, which interconnects macroscopic physical states and microphysical quantities.

\[
\chi(q, \varepsilon) = \sum_{i=1}^{N(\varepsilon)} P_i^q(\varepsilon) \propto \varepsilon^{\tau(q)}
\]

where \( q \in (-\infty, \infty) \) means of the scaling of the \( q \)th moments of \( \{P_i\} \) distributions. For \( q > 0 \), regions in the distribution having a high degree of a measure are magnified, whereas the opposite is true for \( q < 0 \). The \( \tau(q) \) is called the mass index, which can be expressed as

\[
\tau(q) = \lim_{\varepsilon \to 0} \frac{\log \chi(q, \varepsilon)}{\log \varepsilon}
\]

The generalized dimensions of fractals \( D_q \) is defined as

\[
D_q = \frac{\tau(q)}{q - 1} = \frac{1}{q - 1} \lim_{\varepsilon \to 0} \frac{\log \sum_{i=1}^{N(\varepsilon)} P_i^q(\varepsilon) \log P_i(\varepsilon)}{\log \varepsilon}
\]

For \( q = 1 \), the generalized dimension of fractals is defined as

\[
D_1 = \lim_{\varepsilon \to 0} \frac{\log \sum_{i=1}^{N(\varepsilon)} P_i(\varepsilon) \log P_i(\varepsilon)}{\log \varepsilon}
\]

The generalized dimension of fractals \( D_q \) is the constant function of \( q \) when the objective represents a monofractal. In generalized dimensions of fractals, the capacity dimensions \( D_q \) at \( q = 0 \), the information dimensions \( D_1 \) at \( q = 1 \), and the correlation dimensions \( D_2 \) at \( q = 2 \) refer to the global information of structure, the information (Shannon) entropy, and the correlation function, respectively. Furthermore, singularity spectra can be computed using

\[
\alpha(q) = \frac{d \tau(q)}{dq}
\]

Scaling exponents \( f(\alpha) \) and \( \tau(q) \) are connected by the Legendre transformation

\[
f(\alpha) = q^* \alpha(q) - \tau(q)
\]

In conclusion, it is possible to calculate the width to the left and right of the multifractal spectra \( \alpha_+(q) \) and \( \alpha_-(q) \) as follows

\[
\alpha_+(q) = \alpha_0(q) - \alpha(q^+)
\]

\[
\alpha_-(q) = \alpha(q^-) - \alpha_0(q)
\]

As a result of the inhomogeneity in the distribution of probabilities for physical quantities over the entire fractal structure, the width of multifractal spectra \( \Delta \alpha(q) \) can be calculated as follows

\[
\Delta \alpha(q) = \alpha_+(q) + \alpha_-(q)
\]

Multifractal spectra can be deviated by calculating

\[
R_d = \alpha_0(q) - \alpha_+(q)
\]

Figure 4 illustrates the flowchart of the parameters that need to be calculated to understand multifractal theory.

**RESULTS AND DISCUSSION**

**Characteristics of Reservoir Pore Structure.** Casting thin-section observations can be utilized to classify pores into three groups: dissolved pores (Figure 5B,D,H), dissolved pores within grains (Figure 5E), and pores remaining within the granules (Figure 5C,G,I). Almost all of the primary pores are pores remaining within the granules and primary intergranular pores due to the materials’ severe compaction, dissolution, and cementation. As a result of diagenesis, primary intergranular pores were less developed. These intergranular holes, typically having a diameter of 20 \( \mu \)m and a triangular or polygonal shape, became the predominant pore types. Dissolution of zeolite, rock fragments, and well-connected feldspar particles creates these pores (Figure 5E) (Figure 5B,D,H). Secondary pores make up the primary reservoir of the Lucaogou Formation, including intergranular pores, secondary dissolved pores, and microfractures (Figure 5A,C). In addition to diagenetic microfractures and biologic cavity pores, these two types of pores make up the majority of pore space and have the biggest impact on sample permeability.

Feldspar particles and certain rock pieces dissolve at cleavage fissures, creating dissolved inlets. Pore sizes range from 1 to 10 \( \mu \)m, and connections are poor. A chlorite layer (Figure 5C), illite layer (Figure 5D,F), and mixed illite/montmorillonite layer are examples of legitimately formed clay minerals that
contain micropores. It is likely that these pores are relatively small (often less than 2 m), have a complicated form, and contribute little to pore seepage. The most prevalent forms of pore-throats in the research region were tubular throats, curved and flaked throats, or a mix of both (Figure 5h). Fluid movement is restricted by the diverse distribution and poor connectivity of various throat types. Mica chips and cuts are distorted by strong compaction (Figure 5), and the majority of particle interactions are linear, concave, and convex. The pores are filled by calcite cementation and metasomatism, making the reservoir thick and pore less.

CRMI can provide MICP curves as shown in Figure 6. MICP curves reflect the structural properties of reservoir pores, and mercury injection curves can be loosely classified into four groups based on their morphology. The type I curve has a concave shape due to its high mercury removal efficiency and low displacement pressure. Intergranular pores and dissolution pores, both of which have large throats and excellent separation, have evolved. Due to the broad distribution range

Figure 5. Thin sections and scanning electron microscope micrographs of the casting show the pore types of the sample. The pore spaces are indicated in blue. (a) Intercrystalline calcite pore, from sample 1, depth of 3114.86 m, light with a plane polarization. (b) Dissolution pores of zeolite minerals, calcite and quartz microcrystals, from sample 1, depth of 3114.86 m, light with a plane polarization. (c) Intergranular dissolution pores of lamellar chlorite and zeolite minerals with irregular grain surface from sample 3, depth of 3144.84 m, light with a plane polarization. (d) Dissolution pores abounding with zeolite minerals and irregular illite, montmorillonite mixed layer minerals, from sample 5, depth of 3275.43 m, light with a plane polarization. (e) The dissolved pores in the grain contain feldspar fragments, the sample depth is 3291.24 m, and the light with a plane polarization. (f) Irregular illite, montmorillonite mixed layer minerals, and calcite grains and granular quartz in the matrix, sample 5 depth of 3210.69 m, light with a plane polarization. (g) Calcite crystals abounding with quartz detritus and intergranular solution pores, from sample 2, depth of 3143.3 m, light with a plane polarization. (h) Zeolite minerals, calcite, and quartz microcrystals filled in dissolution pores. The dissolution of feldspar clasts was observed. The pores are not developed, there are corrosion pores and intergranular pores, poor connectivity. From sample 1, with a depth of 3114.86 m. (i) Zeolite minerals, calcite microcrystals, and dolomite abounding with intergranular pores. The pores were not developed, intergranular pores and intergranular dissolved pores were found, and the connectivity was poor. From sample 2 at a depth of 3143.3 m.

Figure 6. Cumulative curves of mercury intrusion (a) and extrusion (b) were measured for the samples.
of pore-throat separation differences and the coexistence of many pore-throat types, the type II curve may have a steep straight line shape, poor mercury removal efficiency, and low drainage pressure. Type III curves roughly resemble straight lines and indicate high drainage pressure, excellent pore-throat sorting, a single pore-throat type, and dissolution and intergranular pores. Additionally, it indicates a moderate level of mercury removal. Intergranular pores with narrow throats and weak connections may cause the type IV curve to have an upward convex form, low mercury removal efficiency, and high drainage pressure. In Figure 6, mercury injection and withdrawal curves for some rock samples are categorized. Table 3 shows the results of the experiment. Mercury saturation corresponded to the maximum MICP at 163.84 MPa, the highest incursion pressure approaches 3, the average pore size and sorting coefficient of the 3D capillary model $D_f$ shows a strong correlation between the dimensions of fractals of cores with higher permeability is less than 2.5, $D_f > 2.5$ for very low permeability cores. A strong correlation exists between $D_f$ and pore parameters of tight sandstone, indicating that $D_f$ can be utilized to evaluate tight sandstone petrophysical properties.

The dimension of fractals of geometric model $D_f$ ranges from 2.72 to 3.29. As can be seen from Table 4, tight sandstone permeability and porosity decrease with the increase of $D_f$, although in comparison to $D_f$, the link is modest. Fractal dimensions in three-dimensional space should also not exceed three, according to fractal theory. $D_f$ is not appropriate for pore investigation of tight sandstone due to the low connection between these parameters and $D_f$.

Table 4 demonstrates that the correlation between $D_f$ and pore parameters of tight sandstone is weak and poor, which is contrary to the dimensions of fractals analysis results calculated by other models. This may be due to mercury intrusion not being suitable for wet phase models.

The dimension of fractals of the small island model $D_f$ ranges from 1.4 to 1.68. As can be seen from Table 4, the correlation between $D_f$ and pore parameters of tight sandstone is weak. In conclusion, the dimensions of fractals calculated by the 3D capillary model is most appropriate for the analysis of tight sandstone pores.

### Fractal Analysis of Tight Sandstone Pores

Figure 7 shows a strong correlation between the dimensions of fractals of the 3D capillary model $D_f$ and the tight sandstone pore parameters. The mean correlation coefficient is greater than 0.6, and some coefficients are greater than 0.8. $D_f$ values varied from 2.24 to 2.83, with an average value of 2.57. As $D_f$ approaches 3, the average pore size and sorting coefficient of tight sandstone decrease and the core permeability decreases exponentially. It can be clearly seen from Figure 7(a) that the dimensions of fractals of cores with higher permeability is smaller, with $D_f$ less than 2.5. $D_f > 2.5$ for very low permeability cores. A strong correlation exists between $D_f$ and pore parameters of tight sandstone, indicating that $D_f$ can be utilized to evaluate tight sandstone petrophysical properties.

**Table 3. Mercury Intrusion Data of Samples**

| sample | displacement pressure (MPa) | the median radius (μm) | max throat radius (μm) | mean capillary radius (μm) | sorting coefficient | variable coefficient | pore types | reservoir types |
|--------|-----------------------------|------------------------|-----------------------|---------------------------|---------------------|---------------------|-----------|----------------|
| 1      | 0.83                        | 0.12                   | 0.88                  | 0.26                      | 2.18                | 0.18                | intergranular pore and intergranular pore | I          |
| 2      | 0.85                        | 0.030                  | 0.87                  | 0.20                      | 2.20                | 0.17                | solution pores | II         |
| 3      | 3.19                        | 0.050                  | 0.23                  | 0.070                     | 1.47                | 0.11                | solution pores | III        |
| 4      | 2.35                        | 0.080                  | 0.31                  | 0.10                      | 1.30                | 0.10                | solution pores | III        |
| 5      | 1.19                        | 0.11                   | 0.62                  | 0.19                      | 1.53                | 0.13                | solution pores | II         |
| 6      | 3.38                        | 0.050                  | 0.22                  | 0.070                     | 1.36                | 0.10                | solution pores | IV         |
| 7      | 2.36                        | 0.070                  | 0.31                  | 0.10                      | 2.00                | 0.16                | solution pores | III        |
| 8      | 1.35                        | 0.040                  | 0.54                  | 0.14                      | 2.37                | 0.18                | fissure and solution pores | II         |
| 9      | 2.45                        | 0.080                  | 0.30                  | 0.13                      | 2.04                | 0.16                | solution pores | III        |
| 10     | 1.37                        | 0.020                  | 0.54                  | 0.14                      | 2.21                | 0.16                | solution pores | II         |
| 11     | 1.13                        | 0.030                  | 0.65                  | 0.16                      | 2.07                | 0.16                | solution pores | II         |
| 12     | 2.20                        | 0.030                  | 0.33                  | 0.090                     | 1.83                | 0.13                | pores remaining within the granules and intergranular pores | III        |
| 13     | 2.60                        | 0.060                  | 0.28                  | 0.13                      | 2.28                | 0.18                | pores remaining within the granules, dissolved pores | III        |
| 14     | 0.62                        | 0.15                   | 1.19                  | 0.33                      | 2.36                | 0.20                | solution pores | I          |
| 15     | 0.56                        | 0.13                   | 1.32                  | 0.42                      | 2.89                | 0.24                | solution pores | I          |
| 16     | 0.61                        | 0.10                   | 1.21                  | 0.43                      | 2.79                | 0.23                | solution pores | I          |
| 17     | 2.77                        | 0.060                  | 0.26                  | 0.10                      | 1.91                | 0.15                | pores remaining within the granules | III        |
| 18     | 0.66                        | 0.090                  | 1.11                  | 0.34                      | 2.65                | 0.22                | pores remaining within the granules | I          |
| 19     | 1.34                        | 0.080                  | 0.55                  | 0.18                      | 2.01                | 0.16                | solution pores | I          |
| 20     | 1.13                        | 0.060                  | 0.65                  | 0.18                      | 2.16                | 0.17                | pores remaining within the granules, dissolved pores | II         |
| 21     | 1.36                        | 0.040                  | 0.54                  | 0.15                      | 2.05                | 0.16                | solution pores | II         |
| 22     | 2.74                        | 0.020                  | 0.27                  | 0.080                     | 1.84                | 0.13                | solution pores | III        |
| 23     | 0.14                        | 0.17                   | 5.13                  | 1.27                      | 3.28                | 0.29                | solution pores | I          |
| 24     | 2.62                        | 0.030                  | 0.28                  | 0.080                     | 1.77                | 0.13                | solution pores | III        |
By comparing the generalized dimensions of fractal curves from different samples of tight sandstone, it can be determined that the tight sandstone has a more complicated pore structure as $D_q$ increases. The highest $D_q$ value in core no. 6 indicates an intricate pore structure. Based on the complexity of the pore structure, Figure 8 can be categorized into four types: first group of cores 1, 14, 15, 16, 18, 19, and 23; second group of cores 2, 5, 8, 10, 11, 20, and 21; third group of cores 4, 7, 9, 12, 13, 17, 22, and 24; and fourth set of cores 3 and 6. The partitioning results are similar to the partitioning structure of mercury injection curve in Table 3. The $D_q$ curves of cores 5, 11, and 21 coincide, suggesting that the pore structures of

Figure 7. Correlations between $D_f$ and different tight sandstone pores parameters ((a) dimensions of fractals $D_f$ versus sorting coefficient of pores; (b) $D_f$ versus variable coefficient; (c) $D_f$ versus porosity (%); (d) $D_f$ versus permeability (mD); (e) $D_f$ versus displacement pressure (MPa); (f) $D_f$ versus mean capillary radius (μm); (g) $D_f$ versus median radius (μm); (h) $D_f$ versus max throat radius (μm)).
these cores are similar, as well as the pore structures of cores 4, 17, 22, and 24, and cores 10, 19, and 20 have similarities.

By determining whether the partition function scales are single fractals or multifractals, we can further characterize the scaling properties. Figure 9 shows the scaling function relation between the mass exponential function $\tau_q$ of all experimental MIP curves and the simulated single fractal distribution. Additionally, the slope of $\tau_q$ function is different when $q = 0$ than when $q > 0$. Accordingly, a multiscale nature in which the variable scales of low-density and high-density regions differ.

The mass exponential function $\tau_q$ (Figure 9) also shows multiscale properties of multifractal behavior. In general, the closer the $\tau_q$ function on the MIP curve is to the straight line $\tau_q = q$, the closer it is to the monofractal model.

Multifractal spectra may be displayed to depict the pore size distribution of tight sandstone samples, as illustrated in Figure 10. The asymmetry of multifractal spectra may be shown using the formula

$$\Delta f(\Delta f = f(\alpha_{\min}) - f(\alpha_{\max}))$$

The multifractal spectrum curve is slanted to the right when $\Delta f$ is less than 0 and to the left when $\Delta f$ is more than 0.$^{35}$ As they reveal the variety of scale indices required to define these spectra, the curvature and symmetry of $f(\alpha)$ spectra are also signs of heterogeneity.$^{40}$ The left side of the $f(\alpha)$ spectrum is, on average, smaller than the right. In PSD, bigger values predominate or have very large values, according to the asymmetry of the $f(\alpha)$ spectra on the left, while smaller values predominate or have extremely tiny values, according to the asymmetry on the right.

In Table 5, we show the multifractal parameters calculated by MICP for different tight sandstone core samples. $\alpha_0(q)$ represents the concentration of PSD, which ranges from 0.39 to 0.77. The smaller the $\alpha_0(q)$ value is, the more concentrated the distribution is. As shown in Table 6, some multifractal parameters, especially $\alpha_0$ and $\alpha_2$, are strongly correlated with other fractal parameters. $\Delta \alpha$ is positively correlated with $\alpha_0$ and $\alpha_2$, $f(\alpha)_{min}$ is positively correlated with $\Delta f$. $\alpha_0(q)$ is positively correlated with $\alpha_2$. $R_d$ was positively correlated with $\alpha_0(q)$ and $\alpha_2$.
The multifractal spectrum is left-biased, high-value information has a significant impact on the multifractal spectrum, the $R_d$ deviation of the fractal spectrum spans from 0.0076 to 0.297, and the $R_d$ of 24 cores is more than 0. The remaining tight sandstone samples’ multifractal spectra are left-biased, and high-value information significantly affects the multifractal spectra. The pore size distribution is also reflected in the multifractal spectrum’s $R_d$ deviation. The multifractal parameter values of the four tight sandstone samples are comparable, as shown in Figure 11. The findings demonstrate that tight sandstone PSD values with comparable multifractal characteristics ($\Delta \alpha$, $\Delta f$, $\alpha_0$, $\alpha_1$, $\alpha_2$) are also likely to be similar.

All core samples have a volume dimension of $D_0$ of 0.23, which is less than 1. The PSD disorder degree may be represented by the dimensions of information $D_1$, and the higher the $D_1$ value, the greater the fraction of macropores. Figure 12 demonstrates that $D_1$ has a positive correlation with both the sorting coefficient and the variable coefficient. $D_1$ may reflect the concentration of pore size distribution because sorting coefficient and variable coefficient are physical characteristics that indicate the degree of porous flow concentration. $D_1$’s pore distribution is more tightly confined as it becomes smaller. $D_1$ may also represent the pore size of tight sandstone, as shown by the high association between it and the displacement pressure of median radius. The pore size of tight sandstone increases as $D_1$ increases. Figure 13

| core no. | $\alpha_2$ | $R_d$ | $\Delta \alpha$ | $f_{min}$ | $\Delta f$ | $\alpha_0$ | $D_0$ | $D_1$ | $D_2$ | $\alpha_1$ |
|---------|------------|------|-----------------|----------|-----------|-----------|------|------|------|----------|
| 1       | 0.077      | 0.12 | -0.022          | 0.032    | -0.10     | 0.25      | 0.23 | 0.22 | 0.21 | 0.056    |
| 2       | 0.096      | 0.053| 0.028           | 0.017    | -0.046    | 0.28      | 0.23 | 0.21 | 0.19 | 0.12     |
| 3       | 0.16       | 0.11 | 0.056           | 0.00050  | -0.076    | 0.37      | 0.23 | 0.19 | 0.18 | 0.22     |
| 4       | 0.15       | 0.23 | 0.027           | 0.0019   | -0.094    | 0.34      | 0.23 | 0.20 | 0.19 | 0.18     |
| 5       | 0.11       | 0.30 | -0.030          | 0.010    | -0.11     | 0.27      | 0.23 | 0.22 | 0.21 | 0.085    |
| 6       | 0.17       | 0.14 | 0.060           | 0.00010  | -0.081    | 0.38      | 0.23 | 0.19 | 0.18 | 0.23     |
| 7       | 0.15       | 0.053| -0.0035         | 0.0021   | -0.10     | 0.31      | 0.23 | 0.21 | 0.20 | 0.14     |
| 8       | 0.10       | 0.0076| 0.0070        | 0.0082   | -0.055    | 0.27      | 0.23 | 0.21 | 0.20 | 0.11     |
| 9       | 0.14       | 0.062| -0.018          | 0.0044   | -0.090    | 0.30      | 0.23 | 0.21 | 0.20 | 0.12     |
| 10      | 0.097      | 0.012| 0.029           | 0.0063   | -0.033    | 0.27      | 0.23 | 0.21 | 0.19 | 0.13     |
| 11      | 0.11       | 0.051| 0.019           | 0.0037   | -0.051    | 0.28      | 0.23 | 0.21 | 0.19 | 0.13     |
| 12      | 0.14       | 0.023| 0.035           | 0.00040  | -0.047    | 0.31      | 0.23 | 0.20 | 0.18 | 0.17     |
| 13      | 0.055      | 0.11 | -0.012          | 0.063    | -0.069    | 0.24      | 0.23 | 0.22 | 0.22 | 0.044    |
| 14      | 0.039      | 0.044| -0.0010         | 0.090    | -0.030    | 0.24      | 0.23 | 0.22 | 0.22 | 0.038    |
| 15      | 0.047      | 0.064| -0.0011         | 0.078    | -0.040    | 0.24      | 0.23 | 0.22 | 0.22 | 0.046    |
| 16      | 0.16       | 0.056| 0.0082          | 0.0080   | -0.080    | 0.33      | 0.23 | 0.21 | 0.20 | 0.17     |
| 17      | 0.050      | 0.058| 0.0023          | 0.066    | -0.032    | 0.24      | 0.23 | 0.22 | 0.22 | 0.052    |
| 18      | 0.098      | 0.10 | -0.015          | 0.0082   | -0.091    | 0.26      | 0.23 | 0.22 | 0.21 | 0.083    |
| 19      | 0.098      | 0.067| -0.0077         | 0.0058   | -0.077    | 0.26      | 0.23 | 0.21 | 0.21 | 0.090    |
| 20      | 0.11       | 0.059| 0.0079          | 0.0036   | -0.066    | 0.27      | 0.23 | 0.21 | 0.20 | 0.12     |
| 21      | 0.15       | 0.0066| 0.036       | 0        | -0.045    | 0.33      | 0.23 | 0.20 | 0.18 | 0.19     |
| 22      | 0.036      | 0.049| 0.00010        | 0.097    | -0.024    | 0.23      | 0.23 | 0.22 | 0.22 | 0.036    |
| 23      | 0.15       | 0.042| 0.034           | 0.00010  | -0.062    | 0.33      | 0.23 | 0.20 | 0.18 | 0.19     |

Table 5. Multifractal Parameters of Different Tight Sandstone Samples Calculated from Mercury Intrusion Capillary Pressure (MICP)

Table 6. Correlations among Different Multifractal Parameters

The multifractal spectrum is left-biased, high-value information has a significant impact on the multifractal spectrum, the $R_d$ deviation of the fractal spectrum spans from 0.0076 to 0.297, and the $R_d$ of 24 cores is more than 0. The remaining tight sandstone samples’ multifractal spectra are left-biased, and high-value information significantly affects the multifractal spectra. The pore size distribution is also reflected in the multifractal spectrum’s $R_d$ deviation. The multifractal parameter values of the four tight sandstone samples are comparable,
demonstrates that $D_2$ has a weaker connection with tight sandstone pore characteristics than $D_1$ does.

Figure 12. Correlations between dimensions of information $D_1$ and parameters of pore structure ((a) dimensions of information $D_1$ versus sorting coefficient of pores; (b) dimensions of information $D_1$ versus variable coefficient; (c) dimensions of information $D_1$ versus displacement pressure (MPa); (d) dimensions of information $D_1$ versus the median radius (μm)).

Figure 13. Correlations between dimensions of correlation $D_2$ and parameters of pore structure ((a) dimensions of correlation $D_2$ versus sorting coefficient of pores; (b) dimensions of correlation $D_2$ versus variable coefficient; (c) dimensions of correlation $D_2$ versus displacement pressure (MPa); (d) dimensions of correlation $D_2$ versus the median radius (μm)).

Lower $D_2$ values and the long-range correlation are both influenced by the location of the vast quantity of pore space.
The characteristics of the multifractal spectra, such as the dimensions of information $D_1$, the dimensions of correlation $D_2$, and the multifractal spectra $(\alpha_1, \alpha_2, \Delta\alpha, \alpha_0)$, may be beneficial as predictive metrics that capture part of the inner intricacies of the PSDs, according to our findings. A potent tool for differentiating between the local scaling characteristics of the various PSDs is the multifractal spectrum.

**CONCLUSIONS**

Through the use of multifractal theory and fractal theory, HPMI examined the pore structure of tight sandstone. The fractal properties of tight sandstone pores are examined using four distinct dimensions of fractals computation techniques. Analysis is done on the relationship between tight sandstone pore characteristics and fractal dimensions. The findings indicate that the greatest association exists between the tight sandstone’s pore characteristics and the diameters of the fractals in the 3D capillary model. The tight sandstone pores' computed fractal diameters vary from 2.24 to 2.83. The average pore size and permeability of tight sandstone drop exponentially as fractal dimensions grow.

The tight sandstone’s pore distribution exhibits multifractal properties, according to multifractal research. For the sandstones with similar multifractal parameters $(\Delta\alpha, \Delta\beta, \alpha_0, \alpha_1, \alpha_2)$, their pore size distributions are very close to each other. Due to its association with PSD disorder, the information dimension $D_1$ is more closely related to tight sandstone pore characteristics than the correlation dimension $D_2$. There is a negative correlation between the information dimension $D_1$ and the fractal dimension of the 3D capillary model.

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**Author Contributions**

C.H. and G.L. contributed equally to this manuscript. C.H. and G.L. contributed to conceptualization and writing—original draft. S.D. contributed to project administration, data curation, investigation, and resources. Y.Y. and G.L. involved in writing—review & editing, methodology, and validation. X.H. contributed to supervision and resources. G.L. performed formal analysis. M.Q. carried out visualization.

**Notes**

The authors declare no competing financial interest.

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