Higher Dimensional Operators and Low Energy Left-Right Symmetry

Alakabha Datta\textsuperscript{a)}, Sandip Pakvasa\textsuperscript{a)} and Utpal Sarkar\textsuperscript{b)}

\textsuperscript{a}) Physics Department, University of Hawaii at Manoa, 2505 Correa Road, Honolulu, HI 96822, USA.
\textsuperscript{b}) Theory Group, Physical Research Laboratory, Ahmedabad - 380009, India.

Abstract

We consider higher dimensional operators due to quantum gravity or spontaneous compactification of extra dimensions in Kaluza-Klein type theory and their effect in the $SO(10)$ Lagrangian. These operators change the boundary conditions at the unification scale. As a result one can allow left-right symmetry to survive till very low energy (as low as $\sim$ TeV) for a wide range of values for the coupling of these higher dimensional operators and still make the theory compatible with the latest values of $\sin^2 \theta_W$ and $\alpha_s$ derived from LEP. We consider both non-supersymmetric and supersymmetric cases with standard higgses. Proton lifetime is very large in these theories.

The precision measurements\textsuperscript{[1]} at LEP has put constraints\textsuperscript{[2, 3, 4, 5]} on many extensions of the standard model. Grand Unified Theories also fall under these constrained category\textsuperscript{[4, 5]}.

The measurements of the $Z$ mass and width and also the jet cross-sections and the various asymmetries provide very accurate values for the $\sin^2 \theta_W$ and $\alpha_s$ at the scale $M_Z$. Using these experimental values\textsuperscript{[4]},

\begin{equation}
\sin^2 \theta_W = 0.2333 \pm 0.0008 \\
\alpha_s = 0.113 \pm 0.005
\end{equation}

and taking the fine structure constant at the electroweak scale to be, $\alpha_{em}(M_Z) = 1/127.9$, one can write down the values\textsuperscript{[9]} of the three coupling constants at the electroweak scale ($M_Z$). Then using the evolution of these coupling constants with energy it is possible to see if the coupling constants meet at a point giving rise to grand unification of all the three forces\textsuperscript{[9]}.

It was found that any GUT without supersymmetry with no intermediate mass scale are ruled out by this analysis, whereas theories with intermediate mass scale\textsuperscript{[9]} get constraints on the allowed values for the intermediate mass scale.

An important class of GUTs with intermediate symmetry breaking scale is the one containing those with left-right symmetry\textsuperscript{[8]}. In these theories the standard electroweak model $SU(2)_L \otimes U(1)_Y$ emerge at low energy as a result of symmetry breaking of a larger left-right symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ group. The strongest bound on this left-right symmetry breaking scale $M_R$ comes from an analysis of the LEP data and that is only about a TeV\textsuperscript{[8]}. However, if this theory is embedded in a GUT, then the constraint becomes\textsuperscript{[8]}

\begin{equation}
M_R \geq 10^9 \text{GeV}.
\end{equation}

Here we address the question if the right handed gauge bosons are seen in the near future in SSC or LHC, then will that rule out the possibility of GUT?

Attempts have been made to understand this problem\textsuperscript{[6, 7]}. It was found that if one breaks the left-right D parity spontaneously, then one can have $g_L \neq g_R$. In this case the higgs sector is left-right asymmetric and one can have low $M_R$. But for $M_R \sim$ TeV, it is required to break the $SU(2)_R \rightarrow U(1)_R$ at a high scale and as a result one can have a light $Z_R$ but not light right handed charged gauge bosons\textsuperscript{[6]} naturally. To get all the right handed gauge bosons light one requires large numbers of artificial higgs scalars, which are left-right asymmetric. In the supersymmetric $SO(10)$ theory it is possible to have low $M_R$ for a very specific choice of higgs scalars\textsuperscript{[7]}.

In the case of minimal $SU(5)$ GUT it was argued that the addition of nonrenormalizable terms induced by quantum gravity or by the compactification of the higher dimensions (in some theories like Kaluza-Klein theories) can allow a range of parameters\textsuperscript{[9, 10, 11]} for which the correct values for the $\sin^2 \theta_W$ and $\alpha_s$ are reproduced and which remain consistent with proton decay. We shall consider the higher dimensional operators in a $SO(10)$ version of the GUT and see if for some range of values for the coupling constants of the higher dimensional operators we can have low value of the $M_R$ consistent with $\sin^2 \theta_W$, $\alpha_s$ and proton decay.
In the case of $SU(5)$ it was found that by inclusion of the dimension five operators alone it is not possible to get correct value of the $\sin^2 \theta_W$ and $\alpha_s$, but by including the dimension six operators along with the dimension five operators it is possible to solve the problem. In the case of $SO(10)$ GUT we find that the dimension six operators do not play any role and if we have only two stages of symmetry breaking then we can not have low energy left-right symmetry breaking, but if one breaks $SO(10)$ in two stages near the Planck scale to the left-right symmetric model then it is possible to have low energy left-right symmetry breaking with a wide range of the values of the coupling constants.

We consider the symmetry breaking chain to be,

$$SO(10) \rightarrow SU(4) \times SU(2)_L \times SU(2)_R \equiv G_{PS}$$

$$M_U \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \equiv G_{LR}$$

$$M_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \equiv G_{std}$$

$$M_W \rightarrow SU(3)_c \times U(1)_{em}.$$

The $SO(10)$ invariant lagrangian, which allows the above symmetry breaking chain, in the domain of energies near the Planck scale is given as a combination of the usual four dimensional terms and the new induced higher dimensional nonrenormalizable terms. These higher dimensional terms will be suppressed by the Planck scale (or by the compactification scale which can even be two orders of magnitude below the Planck scale [11]). The lagrangian can then be written as,

$$L = -\frac{1}{2} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \sum_{n=1} L^{(n)}$$

(3)

and the sum in Eq. 3 runs over all possible higher dimensional operators. We write down the five- and six-dimensional operators explicitly as

$$L^{(1)} = -\frac{1}{2} \frac{\eta^{(1)}}{M_{Pl}} \text{Tr}(F_{\mu\nu}\phi F^{\mu\nu})$$

(4)

$$L^{(2)} = -\frac{1}{2} \frac{1}{M_{Pl}^2} \left[ \eta_a^{(2)} \text{Tr}(F_{\mu\nu}\phi^2 F^{\mu\nu}) + \eta_b^{(2)} \text{Tr}(\phi^2) \text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \eta_c^{(2)} \text{Tr}(F^{\mu\nu}\phi) \text{Tr}(F_{\mu\nu}\phi) \right]$$

(5)

In the above equations $\eta^{(n)}$ specify the couplings of the higher dimensional operators.

Let us first consider the case when only the scale $M_U$ is large and $M_I$ is some intermediate symmetry breaking scale, which does not receive any contribution from the higher dimensional operators. Suppose the $SO(10)$ is broken by a 54-plet of higgs field $\Sigma$. $\Sigma$ is a traceless symmetric field of the $SO(10)$ and the vectors of $\Sigma$ which can mediate this symmetry breaking to the group $G_{PS}$, is given by,

$$\langle \Sigma \rangle = \frac{1}{\sqrt{30}} \sum_n \text{diag}(1,1,1,1,1,1,-3/2,-3/2,-3/2).$$

(6)

where, $\Sigma_n = \sqrt{\frac{6}{5\pi \alpha_G}} M_U$ and $\alpha_G = g_0^2/4\pi$ is the GUT coupling. Note that this has the larger symmetry $O(6) \otimes O(4)$.

If we now consider terms of order dimension five only, then the $G_{PS}$ invariant lagrangian will be given by,

$$-\frac{1}{2} (1 + \epsilon_4) \text{Tr}(F^{(4)}_{\mu\nu} F^{(4)\mu\nu}) - \frac{1}{2} (1 + \epsilon_2) \text{Tr}(F^{(2L)}_{\mu\nu} F^{(2L)\mu\nu})$$

$$-\frac{1}{2} (1 + \epsilon_2) \text{Tr}(F^{(2R)}_{\mu\nu} F^{(2R)\mu\nu})$$

(7)

Now defining the physical gauge fields below the unification scale to be $A'_i = A_i \sqrt{1 + \epsilon_i}$, we recover the usual $G_{PS}$ lagrangian with modified coupling constants

$$g^2_A(M_U) = \tilde{g}^2_A(M_U)(1 + \epsilon_4)^{-1}$$

$$g^2_{2L}(M_U) = \tilde{g}^2_{2L}(M_U)(1 + \epsilon_2)^{-1}$$

$$g^2_{2R}(M_U) = \tilde{g}^2_{2R}(M_U)(1 + \epsilon_2)^{-1}$$

(8)
The couplings \( \tilde{g}_i \) are the couplings that would have appeared in the absence of the higher dimensional operators, whereas the \( g_i \) are the physical couplings which are evolved down to lower scales.

We introduce the parameter \( \epsilon^{(n)} \) associated with a given operator of dimension \( n + 4 \) in the following way:

\[
\epsilon^{(n)} = \left[ \frac{1}{\sqrt{4\pi}} \frac{\phi_0}{MP_t} \right]^n \eta^{(n)}
\]

(9)

We then have

\[
\epsilon^{(n)} = \left[ \left\{ \frac{2}{25\pi\alpha_G} \right\} \frac{1}{2} \frac{M_U}{MP_t} \right]^n \eta^{(n)}
\]

(10)

The change in the coupling constants are, \( \epsilon_4 = \epsilon^{(1)} \) and \( \epsilon_2 = -\frac{3}{4}(\epsilon^{(1)}) \). It is to be noted that the \( SU(2)_L \) and \( SU(2)_R \) always receive equal contributions. As a result the effect of the higher dimensional terms will be to shift the relative contributions between the \( SU(2)_s \) and the \( SU(4) \) gauge coupling constants (overall shifts of all the coupling constants do not contribute to the predictions of \( \sin^2 \theta_W \) and \( \alpha_s \)). This relative shift can be parametrized by only one parameter, which is related to \( \epsilon^{(1)} \). Thus if by considering only dimension five terms we cannot allow low \( M_R \), then dimension six or other higher dimensional operators will also not allow low \( M_R \). Hence it suffices to consider only dimension 5 terms.

As discussed above, the effect of higher dimensional terms will change the boundary conditions and at the unification scale \( M_U \) we have,

\[
\tilde{g}_4^2 = g_{2L}^2 = g_{2R}^2 = g_0^2
\]

or equivalently,

\[
g_4^2(M_U)(1 + \epsilon_4) = g_{2L}^2(M_U)(1 + \epsilon_2) = g_{2R}^2(M_U)(1 + \epsilon_2) = g_0^2.
\]

(11)

Using the matching conditions at the scales \( M_I \),

\[
g_{3c}^{-2}(M_I) = g_{1(B-L)}^{-2}(M_I) = g_4^{-2}(M_I)
\]

(12)

and \( M_R \),

\[
g_{3Y}^{-2}(M_R) = \frac{3}{5} g_{2R}^{-2}(M_R) + \frac{2}{5} g_{1(B-L)}^{-2}(M_R)
\]

\[
g_{2L}^{-2}(M_R) = g_{2R}^{-2}(M_R)
\]

(13)

and the evolution of the coupling constants [12],

\[
\mu \frac{d\alpha_i(\mu)}{d\mu} = 2\beta_i \alpha_i^2(\mu)
\]

(14)

where \( \alpha_i = \frac{g_i^2}{4\pi} \), and \( \beta_i = -\frac{1}{4(\pi)} \left( \frac{11}{3} T_g[i] - \frac{4}{3} T_f[i] - \frac{1}{6} T_s[i] \right) \), we can now write the following relations between the standard model coupling constants and the unification coupling constant:

\[
\alpha_Y^{-1}(M_Z) = \alpha_G^{-1}(1 + \frac{3}{5} \epsilon_2 + \frac{2}{5} \epsilon_4) + \left( \frac{6}{5} b_{2R} + \frac{4}{5} b_4 \right) M_{UI}
\]

\[
+ \left( \frac{6}{5} b_{2R} + \frac{4}{5} b_{1(B-L)} \right) M_{IR} + 2b_{1Y} M_{Rw}
\]

\[
\alpha_{2L}^{-1}(M_Z) = (1 + \epsilon_2) \alpha_G^{-1} + 2b_{2L}(M_{UI} + M_{IR} + M_{Rw})
\]

\[
\alpha_{3c}^{-1}(M_Z) = (1 + \epsilon_4) \alpha_G^{-1} + 2b_4 M_{UI} + 2b_{3c}(M_{IR} + M_{Rw})
\]

(15)

where, \( M_{ij} = \ln \frac{M_{i}}{M_{j}} \).

We define two quantities \( A \) and \( B \) by the following relations,

\[
A = \alpha_Y^{-1} - \alpha_{2L}^{-1}
\]

\[
B = \alpha_{2L}^{-1} + \frac{5}{3} \alpha_Y^{-1} - \frac{8}{3} \alpha_{3c}^{-1}
\]

(16)

\( A \) and \( B \) are related to \( \sin^2 \theta_W \), \( \alpha_s \), and \( \alpha_s \) by

\[
\sin^2 \theta_W = \frac{3}{8} - \frac{5}{8} A
\]

\[
1 - \frac{8}{3} \alpha_s = \alpha B
\]

(17)
Equations [15] may be solved for $M_{U1}$ and $M_{IR}$ in terms of $A$ and $B$ to obtain

$$
M_{U1} = \frac{X}{2} + \frac{M_{Rw}}{2} - \frac{\epsilon}{2b_2} \frac{\alpha_s}{b_2}
$$

$$
M_{IR} = \frac{Y}{2} - 2M_{Rw}
$$

where, $X = -\frac{5A-B}{2b_2}$; $Y = -\frac{5A+B}{2b_2}$; $b_2 = -\frac{11}{6}$, and $\epsilon = \epsilon_0 - \epsilon_2$. Using the measured values of $\sin^2 \theta_w$ and $\alpha_s$ from equation 1 one gets $M_{IR} \approx 2 \times 10^{22} M_R$ for $M_R \sim 10 M_w$.

Thus we see that when the $SO(10)$ GUT breaks to the $G_{PS}$ at the unification scale it is impossible to have low $M_R$. Let us now consider the symmetry breaking chain in which $SO(10)$ breaks down to $G_{LR}$ at the unification scale $M_U$ directly, when the component of a 45-plet of higgs $H$ which is invariant under $G_{LR}$ acquires v.e.v. This can be written as,

$$
\langle H \rangle = \frac{1}{\sqrt{12}} H_0 \left( \begin{array}{ccc}
0_{33} & 1_{33} & 0_{34} \\
-1_{33} & 0_{33} & 0_{34} \\
0_{43} & 0_{43} & 0_{44}
\end{array} \right)
$$

where, $0_{mm}$ is a $m \times n$ null matrix and $1_{mm}$ is a $m \times m$ unit matrix. The antisymmetry of the matrix will imply that to dimension five operators there is no contribution from this higgs. The lowest order contribution comes from the dimension six operators. To order six the $G_{LR}$ invariant lagrangian is written as,

$$
-\frac{1}{2}(1 + \epsilon_3) \text{Tr}(F^{(3)}(\mu \nu) F^{(3)\mu \nu}) - \frac{1}{2}(1 + \epsilon_2) \text{Tr}(F^{(2L)}(\mu \nu) F^{(2L)\mu \nu})
$$

$$
-\frac{1}{2}(1 + \epsilon_2) \text{Tr}(F^{(2R)}(\mu \nu) F^{(2R)\mu \nu}) - \frac{1}{2}(1 + \epsilon_1) \text{Tr}(F^{(1)}(\mu \nu) F^{(1)\mu \nu})
$$

(20)

where, $\epsilon_3 = \epsilon_1$ and $\epsilon_2 = 0$. As argued earlier, in this case also there is no relative shift in the boundary condition between the $SU(3)$ and the $U(1)$ coupling constants and as a result it will not be possible to have low $M_R$.

We shall now discuss the situation when the $SO(10)$ group is broken to the $G_{LR}$ subgroup in two stages, but we now assume that the higher dimensional operators contribute to both the scales $M_U$ and $M_I$. Consider the five dimensional operator,

$$
L^{(1)} = -\frac{1}{2 M_{Pl}} \text{Tr}([F F]\phi_{45} F^{\mu \nu})
$$

. This operator contains the $SU(4)$ invariant piece

$$
L''^{(1)} = -\frac{1}{2 M_{Pl}} \text{Tr}([F_{45} F^{(4)}] F^{(4)\mu \nu})
$$

where, $\phi_{45}$ transforms as $(15, 1, 1)$ under $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$. At the scale $M_I$ one can now break $SU(4) \rightarrow SU(3)_c \otimes U(1)_{(B-L)}$ by giving a v.e.v to $\phi_{15}'$ of the form,

$$
\phi_{15}' = \frac{1}{\sqrt{24}} \phi_{15}' \text{diag}[1, 1, 1, -3].
$$

(21)

The $SU(3)_c \otimes U(1)_{(B-L)}$ invariant kinetic energy term for the gauge bosons will now be

$$
-\frac{1}{2}(1 + \epsilon'_3) \text{Tr}(F^{(3)}(\mu \nu) F^{(3)\mu \nu}) - \frac{1}{2}(1 + \epsilon'_1) \text{Tr}(F^{(1)}(\mu \nu) F^{(1)\mu \nu})
$$

(22)

where,

$$
\epsilon'_3 = \left[ \frac{1}{\sqrt{24}} \phi_{15}' \right]^{(1)} \phi_{15}' \left[ \frac{2}{25 \pi \alpha_4} \right] \frac{1}{2} M_I
$$

$$
\epsilon'_1 = -2 \epsilon'_3
$$

and $\epsilon'_1$ (for dimension 5 operators only).

The one loop matching conditions at the scale $M_U$ is same as in equation [24] while that at the scale $M_I$ is modified to,

$$
g^2_{1(B-L)}(M_I)(1 + \epsilon'_1) = g^2_{3c}(M_I)(1 + \epsilon'_3) = g^2_{4}(M_I).
$$

(23)

The other matching conditions remain unchanged.
Table 1: Allowed ranges of $\epsilon$ and $\epsilon'$ for various $M_U$ and $M_I$.

| $\epsilon'$ | $\epsilon$ | $M_I$ | $M_U$ | $\alpha_{iI}^{-1}$ |
|------------|------------|-------|-------|-------------------|
| -.119 - .141 | .16 - .163 | $10^{18}$ | $10^{18}$ | 55.253 - 54.211 |
| -.153 - .176 | .204 - .208 | $10^{17}$ | $10^{18}$ | 56.493 - 55.462 |
| -.177 - .201 | .244 - .248 | $10^{16}$ | $10^{18}$ | 58.252 - 57.218 |
| -.153 - .176 | .165 - .168 | $10^{17}$ | $10^{17}$ | 53.658 - 52.628 |
| -.177 - .201 | .208 - .212 | $10^{16}$ | $10^{17}$ | 55.418 - 54.383 |

Table 2: Allowed ranges of $\epsilon$ and $\epsilon'$ for various $M_U$ and $M_I$ in the supersymmetric version.

| $\epsilon'$ | $\epsilon$ | $M_I$ | $M_U$ | $\alpha_{iI}^{-1}$ |
|------------|------------|-------|-------|-------------------|
| -.178 - .210 | .246 - .251 | $10^{18}$ | $10^{18}$ | 29.383 - 28.771 |
| -.229 - .264 | .317 - .324 | $10^{17}$ | $10^{18}$ | 29.741 - 29.123 |
| -.265 - .301 | .382 - .389 | $10^{16}$ | $10^{18}$ | 30.425 - 29.283 |
| -.229 - .264 | .254 - .260 | $10^{17}$ | $10^{17}$ | 28.421 - 27.804 |
| -.265 - .301 | .324 - .331 | $10^{16}$ | $10^{17}$ | 29.105 - 28.503 |

The equations now modify to

$$
\alpha^{-1}_Y(M_Z) = \alpha^{-1}_G(1 + \frac{3}{5}\epsilon_2 + \frac{2}{5}\epsilon_4 + \frac{2}{5}\epsilon'_1(1 + \epsilon_4))
$$

$$
+ \frac{4}{5}b_{2R} + \frac{4}{5}(1 + \epsilon'_1)b_{4U} + \frac{4}{5}(b_{Y(B-L)})\alpha^{-1}_{IR} + 2b_1Y M_{Rw}
$$

$$
\alpha^{-1}_{2L}(M_Z) = (1 + \epsilon_2)\alpha^{-1}_{G} + 2b_{2L}(M_{U1} + M_{IR} + M_{Rw})
$$

$$
\alpha^{-1}_{3c}(M_Z) = (1 + \epsilon_4)(1 + \epsilon'_3)\alpha^{-1}_{G} + 2b_4(1 + \epsilon'_3)M_{U1} + 2b_{3c}(M_{IR} + M_{Rw})
$$

One can now solve for $M_{U1}$ and $M_{IR}$ to obtain

$$
M_{U1} = \frac{-X}{2} + \frac{M_{Rw}}{2} - \frac{\epsilon\alpha^{-1}_{G}}{2b_2}
$$

$$
M_{IR} = \frac{Y}{2} + \frac{yX}{2M_{Rw}}(1 + \frac{y}{2}) - \frac{\epsilon\alpha^{-1}_{G}}{b_2}\sigma
$$

where, $\epsilon = \epsilon_4 - \epsilon_2 = \frac{5}{6}\epsilon^{(1)}_c$; $x = \frac{5}{6}\epsilon^{(1)}_c(1 + \frac{2}{3}\epsilon)$; $y = \epsilon^{'1}_3 b_3 - \epsilon^{'1}_3 b_2$; $\sigma = x - y$ and $b_4 f$ and $b_2$ are the beta functions with and without fermionic contributions for the appropriate gauge groups. With $\frac{M_{Rw}}{M_{U1}} = 10$ we give in table 1 the allowed ranges of $\epsilon'_3$ and $\epsilon$ for some choices of $M_U$ and $M_I$. Note that given $\epsilon'_3$ and $M_{U1}$ one can use Eq.25 and any one of the relations in Eq.24 to solve for $\epsilon$ and $\alpha^{-1}_G$. The parameters $\eta^{(1)}$ and $\eta^{(1)}$ appearing in the Lagrangian can be obtained from Eq.10 and Eq.22 respectively.

The analysis can be extended in a straightforward way to supersymmetric $SO(10)$ by making appropriate changes in the $\beta$-functions, $\beta_i = -\frac{1}{(4\pi)}(3T_y[i] - T_f[i] - T_s[i])$. For this case the form of Eq.24 and Eq.25 remain unchanged though the values for the beta functions change. The results for the supersymmetric $SO(10)$ neglecting the higgs contributions to the $b$'s are shown in table 2. The effect of higgs will change the result in the sense that the allowed ranges for the $\epsilon$ will get slightly modified. Similarly the higher loop contributions are also likely to change the allowed ranges by small amount [13]. However, none of these smaller effects can, in practice, change the conclusion.

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In summary, we have pointed out that the low energy restoration of parity is not completely ruled out by the LEP data. In principle the effect of quantum gravity can allow low energy left-right symmetry in Grand Unified Theories with intermediate symmetry breaking mass scales.

0.1 Acknowledgement

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0.2 Note Added:

For an earlier discussion on the possibility of low $M_R$ with gravitational corrections see T. Rizzo, Phys. Lett. B142, 163 (1984).

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[13] Two loop calculations will be reported elsewhere.