In this work we consider \(q\)-form fields in a \(p\)-brane embedded in a \(D = (p + 2)\) space-time. The membrane is generated by a domain wall in a Randall-Sundrum-like scenario. We study conditions for localization of zero modes of these fields. The expression agrees and generalizes the one found for the zero, one, two and three-forms in a 3-brane. By a generalization we mean that our expression is valid for any form in an arbitrary dimension with codimension one. We also point out that, even without the dilaton coupling, some form fields are localized in the membrane. The massive modes are considered and the resonances are calculated using a numerical method. We find that different spaces have identical resonance structures, which we call dual spaces of resonances (DSR).

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Duality is a sort of symmetry that exists between two or more very special theories. The remarkable fact about these kinds of symmetries is that when we make the duality transformations over the important fields, we change the regime of couplings in the theories involved. In other words, if we have a theory whose coupling constant is \( g \), it is possible to build another theory with coupling constant \( \frac{1}{g} \), i.e., we can do strong coupling calculations in one model in perturbation approach if we change it by its dual. The most basic example is the duality between the Sine-Gordon Model and the Thirring Model. In this case, the solitons of the first model are exchanged by the particles of the former \([1]\). Another examples are the network of dualities relating the five Superstring theories and eleven-dimensional M-Theory \([2]\). The most recent duality found, still in the Superstring context, is the AdS/CFT correspondence, where a 10-dimensional Type IIB superstring in the \( AdS_5 \times S^5 \) background is conjectured to be dual to a \( N = 4 \) Super-Yang Mills theory in \( D = 4 \) \([3]\). In this piece of work we study tensorial fields in extra-dimensional spaces and find an intriguing characteristic that is, in some sense, related to duality. The fact is that we find evidences of an structure of resonances that have the same characteristics but in different higher dimensional spaces with different regimens of coupling constants. This last fact is the one that made us call such a behavior as a "duality".

Antisymmetric tensor fields or simply differential forms arise naturally in string theory \([4, 5]\) and supergravity \([6]\) and play an important role in dualization \([7, 8]\). In particular they appear in the \( R - R \) sector of each of the type II string theories. These tensor fields couple naturally to higher-dimensional extended objects, the \( D \)-branes and are important for their stability. Besides this, they are related to the linking number of higher dimensional knots \([9]\). The rank of these antisymmetric tensors is defined by the dimension of the manifold \([10]\). Beyond this, these kind of fields play an important role in the solution of the moduli stabilization problem of string theory \([11, 13]\).

Because of these aspects, it is important to study higher rank tensor fields in membrane backgrounds. In this direction antisymmetric tensor fields have already been considered in models of extra dimension. Generally, the \( q \)-forms of highest rank do not have physical relevance. This is due to the fact that when the rank increases, it also increases the number of gauge freedom. Such fact can be used to cancel the dynamics of the field in the brane \([13]\).

The mass spectrum of the two and three-form have been studied, for example, in Refs. \([22, 23]\) in a context of five dimensions with codimension one. Later, the coupling between the two and three-forms with the dilaton was studied, in different contexts, in \([24, 26]\).

In another direction soliton-like solutions are studied with increasing interest in physics, not only in Condensed Matter, as in Particle Physics and Cosmology. In brane models, they are used as mechanism of field localization, avoiding the appearance of the troublesome infinities. Several kinds of defects in brane scenarios are considered in the literature \([27, 29]\). In these papers, the authors consider brane world models where the brane is supported by a soliton solution to the baby Skyrmme model or by topological defects available in some models. As an example, in a recent paper, a model is considered for coupling fermions to brane and/or antibrane modeled by a kink antikink system \([30]\).

The localization of fields in a framework that consider the brane as a kink has been studied for example in \([31, 38]\). In this context the present authors have studied the issue of localization and resonances of a three-form field in \([39]\) and in a separate paper \( q \)-form fields in a scenario of a \( p \)-brane with codimension two \([40]\).

In these scenarios, some facts about localization of fields are known: The scalar field\((0 \text{-form})\) is localizable, but the vector gauge fields \((1 \text{-form})\), the Kalb-Ramond field\((2 \text{-form})\) and the three-form field are not. The reason why this happens with the vector field is that, in four dimensions, it is conformal and all information coming from warp factors drops out, necessarily rendering a non-normalizable four dimensional effective action. However, in the work of Kehagias and Tamvakis \([41]\), it is shown that the coupling between the dilaton and the vector gauge field produces localization of the vector field. In analogy with the work of Kehagias and Tamvakis \([41]\), as cited above, the authors have also considered these coupling with a three-form field \([39]\), where a condition for localization is found.

The facts above are even clear if we consider, in a general way, a \( q \)-form field in an arbitrary dimension. This can be considered with and without the dilaton coupling. In this reasoning, general conditions for localization can be found in an elegant way. For example, for branes with space dimensions bigger than three, one should expect that the gauge vector field is localizable even without the dilaton coupling. This is due to the fact that this field is not conformal in these cases. In fact what we find is that, bigger is the space-time dimension, bigger will be the order of the \( q \)-form localized without the need of the dilaton. For the other cases the dilaton coupling is needed.

The main goal of this paper is to present the structure of dual resonance spaces. However, standard calculations related to aspects of \( q \)-form fields living in the background of a codimension one \( p \)-brane are necessary. Therefore, the paper is organized as follows. Section 1 is devoted to discuss a solution of Einstein equation with source given by a kink. In Section 2 we analyze how the gauge freedom can be used to cancel the angular component of an arbitrary \( q \)-form. Yet in this section we discuss aspects of localization of the fields involved. In the third section, we study
massive modes, resonances and show aspects of dual spaces containing same resonance structures. At the end, we discuss the conclusions and perspectives.

THE KINK AS A MEMBRANE

We start our analysis studying the space-time background. Before analyzing the coupling of the $q$–forms with the dilaton, it is necessary to obtain a solution of the equations of motion for the gravitational field in the background of the dilaton and the membrane. For such, we introduce the following action, similar of the one used in [44]:

$$S = \int d^Dx \sqrt{-G}[2M^3R - \frac{1}{2}(\partial \phi)^2 - \frac{1}{2}(\partial \pi)^2 - V(\phi, \pi)],$$

(1)

where $D = p + 2$, $G$ is the metric determinant and $R$ is the Ricci scalar. Note that we are working with a model containing two real scalar fields. The field $\phi$ plays the role of membrane generator of the model while the field $\pi$ represents the dilaton. The potential function depends on both scalar fields. It is assumed the following ansatz for the spacetime metric:

$$ds^2 = e^{2A_s(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2B_s(y)} dy^2,$$

(2)

where $\eta_{\mu\nu} = \text{diag}(-1, 1, \cdots, 1)$ is the metric of the $p$-brane, $y$ is the codimension coordinate, and $s$ is a deformation parameter. The deformation parameter controls the kind of deformed topological defect we want in order to mimic different classes of membranes. The deformation method is based in deformations of the potential of models containing solitons in order to produce new and unexpected solutions [45]. As usual, capital Latin index represent the coordinates in the bulk and Greek index, those on the $p$-brane. The equations of motion are given by

$$\frac{1}{2} (\phi')^2 + \frac{1}{2} (\pi')^2 - e^{2B_s(y)} V(\phi, \pi) = 24M^3(A_s')^2,$$

(3)

$$\frac{1}{2} (\phi')^2 + \frac{1}{2} (\pi')^2 + e^{2B_s(y)} V(\phi, \pi) = -12M^3A_s'' - 24M^3(A_s')^2 + 12M^3A_s'B_s',$$

(4)

$$\phi'' + (4A_s' - B_s')\phi' = \partial_\phi V,$$

(5)

and

$$\pi'' + (4A_s' - B_s')\pi' = \partial_\pi V.$$  

(6)

In order to solve this system, we use the so-called superpotential function $W_s(\phi)$, defined by $\phi' = \frac{\partial W_s}{\partial \phi}$, following the approach of Kehagias and Tamvakis [44]. The particular solution regarded follows from choosing the potential $V_s(\phi, \pi)$ and superpotential $W_s(\phi)$ as

$$V_s = \exp\left(\frac{\pi}{\sqrt{12M^3}}\right)\left\{\frac{1}{2} \left(\frac{\partial W_s}{\partial \phi}\right)^2 - \frac{5}{32M^2} W_s(\phi)^2\right\},$$

(7)

and

$$W_s(\phi) = a\phi^2 \left(\frac{s}{2s - 1} \left(\frac{\phi}{\phi}\right)^{1/s} - \frac{s}{2s + 1} \left(\frac{\phi}{\phi}\right)^{1/s}\right),$$

(8)

where $a$ and $v$ are parameters to adjust the dimensionality. As pointed out in [44], this potential give us the desired soliton-like solution. In this way it is easy to obtain first order differential equations whose solutions are solutions of the equations of motion above, namely

$$\pi_s = -\sqrt{3M^3}A_s,$$

(9)

$$B_s = \frac{A_s}{4} = -\frac{\pi_s}{4\sqrt{3M^3}}.$$  

(10)
The solutions for these new set of equations are the following:

For $s = 1$:

$$\phi(y) = v \tanh(ay),$$  \hspace{1cm} (12)

$$A(y) = -\frac{v^2}{72M^3} \left(4 \ln \cosh(ay) + \tanh^2(ay)\right)$$  \hspace{1cm} (13)

and for $s > 1$:

$$\phi(y) = v \tanh^s(ay/s),$$  \hspace{1cm} (14)

$$A_s(y) = -\frac{v^2}{12M^3} \frac{s}{2s + 1} \tanh^{2s} \left(\frac{ay}{s}\right)$$

$$-\frac{v^2}{6M^3} \left(\frac{s^2}{2s - 1} - \frac{s^2}{2s + 1}\right)$$

$$\times \left\{\ln \left[\cosh \left(\frac{ay}{s}\right)\right] - \sum_{n=1}^{s-1} \frac{1}{2n} \tanh^{2n} \left(\frac{ay}{s}\right)\right\}$$  \hspace{1cm} (15)

An important observation to be made here is concerned to the appearance of a spacetime singularity due to the dilaton field. The Ricci scalar is given by

$$R_s = -\left(8A''_s + 18A'^2_s\right) e^{\tilde{\phi}/\sqrt{3M^3}}$$

and the plot is given in the figure below.

![Ricci scalar plot](image)

**FIG. 1.** Ricci scalar for $s = 1$ (lined), $s = 3$ (dashed) and $s = 5$ (dotted).

This is a naked singularity emerging at the infinity and in principle could jeopardize the issue of fields localization. However this singularity can be understood from a higher dimensional viewpoint. As an example, a $D4$-brane in type II-A theory is singular because the presence of a diverging dilaton field. However, that singularity disappears when it is lifted to the eleventh dimensional supergravity viewpoint. Here, we changed our solution to six dimensions

$$ds^2 = e^{3A(y)/2} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dz^2) + dy^2$$

the dilaton turns out to be the radius of an extra $S^1$ direction (that is why it is called dilaton). The above higher dimensional metric is free of singularities. This interpretation also explain why, in this manuscript, we consider the same coupling between all $q$-forms and the dilaton. This kind of solutions have been studied, for example, in [17] and...
In fact, we used the string like solution of [11] to analyze the localization q-forms in co-dimension two brane word [16]. Although the objective of the present manuscript is not to deal with this singularity problem, this issue is widely discussed in the literature. It has been argued that the imposition of unitary boundary conditions renders these singularities harmless from the physical viewpoint [19], [20], [21], [16]. Besides this aspect, there are reasons to believe that considering these kind of spaces as physically meaningful. There are proposals [14, 15] that five-dimensional bulk gravity in the thin domain-wall case has an equivalent description in terms of a cutoff four-dimensional conformal field theory on the domain-wall, on the lines of the AdS/CFT correspondence [3]. From the viewpoint of AdS/CFT correspondence, some singularities may have physical significance and it is very interesting to see the role the dilaton field plays in this framework [16].

LOCALIZATION OF $q$–FORM FIELDS

In this section we study $q$–form fields in the gravitational background of the last section. We consider these fields with and without the dilaton coupling. It is important to note that, when we exclude this coupling, the metric do not has the $B$ factor. With these considerations, we must look for localization of forms in these frameworks. The first important thing to be understood is the gauge symmetry of these fields. The fact is that, when the number of dimension increases, the number of gauge freedom also increases, and this can be used to cancel the degrees of freedom in the visible brane. This analysis was performed in details by the authors in a recent paper [18]. Due the importance of this for our present work, we give here a sketch of it. First, note that the number of degrees of freedom (d.o.f.) of an antisymmetric tensor field with $q$ indices is given by

$$
\binom{D}{q} = \frac{D!}{q!(D-q)!}.
$$

For the one form we have

$$
\delta X_M = \partial_M \phi
$$

where $\phi$ is a scalar field. Therefore we have for the number of physical d.o.f.

$$
\binom{D-1}{1} - \binom{D-1}{0} = \binom{D-1}{1}.
$$

(16)

In the above expression we have used the Stiefel relation. After this we claim that the number of physical d.o.f. of a $q$–form is given by

$$
\binom{D-1}{q}.
$$

(17)

and proves that this is valid for a $(q+1)$–form. In this case the gauge parameter will be an $q$–form and therefore, using the Stiefel relation, we have for the physical d.o.f. of the $(q+1)$–form

$$
\binom{D}{q+1} - \binom{D-1}{q} = \binom{D-1}{q+1}.
$$

The D-form has no dynamics. Using gauge symmetries, from the above result, we can see that the d.o.f. of the $(D-1)$ form can be made all null at the visible brane. Therefore we must analyze only the cases $q = 0, 1, 2, 3...p$. The full action for this field is given by

$$
S_X = \int d^D x \sqrt{-G} [Y_{M_1 M_2...M_{q+1}} Y^{M_1 M_2...M_{q+1}}],
$$

(18)

where $Y_{M_1 M_2...M_{q+1}} = \partial[Y_{M_1 M_2...M_{q+1}}]$ is the field strength for the $q$-form $X$. We can use gauge freedom to fix $X_{\mu_1...\mu_{q+1}} = \partial_{\mu_1} X_{\mu_2...\mu_{q+1}} = 0$ and we are left with the following two type of terms

$$
Y_{\mu_1...\mu_{q+1}} = \partial_{\mu_1} X_{\mu_2...\mu_{q+1}},
$$

(19)

$$
Y_{\eta_1...\eta_{q'}} = \partial_{[\eta_1} X_{\eta_2...\eta_{q}]} = \partial_\eta X_{\eta_1...\eta_{q}} = X_{\mu_1...\mu_{q}}.
$$

(20)
Using the above facts we obtain for the action
\[ S_X = \int d^Dx \left\{ e^{((p-2q-1)A_s)Y_{\mu_1\cdots\mu_{q+1}}Y_{\mu_1\cdots\mu_{q+1}}} + (q+1)e^{((p+1-2q)A_s)X'_{\mu_1\cdots\mu_q}X'^{\mu_1\cdots\mu_q}} \right\}, \]
and the equation of motion are given by
\[ \partial_{\mu_1}Y_{\mu_1\cdots\mu_{q+1}} + e^{(2q+1-p)A_s}\partial_{\nu}e^{((p+1-2q)A_s)X'^{\mu_1\cdots\mu_q}} U^{\mu_2\cdots\mu_{q+1}} = 0 \tag{21} \]

We can now separate the \( y \) dependence of the field using
\[ X^{\mu_1\cdots\mu_q}(x^\alpha, y) = B^{\mu_1\cdots\mu_q}(x^\alpha)U(y). \tag{22} \]

Defining \( Y^{\mu_1\cdots\mu_{q+1}} = \tilde{Y}^{\mu_1\cdots\mu_{q+1}}U \), where \( \tilde{Y} \) stands for the four dimensional field strength, we get
\[ \partial_{\mu_1}\tilde{Y}^{\mu_1\cdots\mu_{q+1}} + m^2B^{\mu_2\cdots\mu_{q+1}} = 0 \tag{23} \]
and
\[ U''(y) - ((2q - p - 1)A_s')U'(y) = -m^2e^{-2A_s}U(y). \tag{24} \]

In the above expression \( m \) is a constant of separation of variable that represents the mass of \( q \)-form \( B \) in the \( p \)-brane. It is easy to see that \( U = U_0 \), with \( U_0 \) being a constant, solves the above equation for \( m = 0 \). In this particular case, the effective action can be found easily to give us
\[ S_X = \int dy e^{(p-2q-1)A_sU^2} \int d^qx [\tilde{Y}_{\alpha\mu\lambda\gamma}\tilde{Y}_{\alpha\mu\lambda\gamma}]. \tag{25} \]

Using now our solution for \( A_s \) we have that the integration in the extra dimension is finite for \( q < (p-1)/2 \). It becomes clear now that only the scalar field is localized for \( p = 3 \). We also see that for \( p = 4 \), for example, the vector field is localized. As commented in the introduction, this was expected because this field is not conformal in this space dimension. Despite of this, the dilaton is yet needed for the localization of higher order forms. The coupling is inspired in string theory and we have for the action
\[ S_X = \int d^Dx \sqrt{-G}e^{-\lambda}e^{Y_{M_1\cdots M_{q+1}}Y^{M_1\cdots M_{q+1}}}, \tag{26} \]
with the same definitions used above. Repeating the same steps as before we obtain
\[ S_X = \int d^Dx \left\{ e^{((p-2q-1)A_s+B_s-\lambda)Y_{\mu_1\cdots\mu_{q+1}}}Y^{\mu_1\cdots\mu_{q+1}} \right\} \]
\[ (q+1)\partial_{y}e^{((p+1-2q)A_s-B_s-\lambda)X'_{\mu_1\cdots\mu_q}X'^{\mu_1\cdots\mu_q}} \]
\[ U''(y) - ((2q - p - 1)A_s')U'(y) = -m^2e^{2(A_s-B_s)}U(y). \tag{28} \]

Again we can see that \( U = U_0 \), with \( U_0 \) being a constant, solves the above equation for \( m = 0 \). The effective action can again be found easily and is given by
\[ S_X = \int dy e^{((p-2q-1)A_s+B_s-\lambda)U^2} \int d^qx [\tilde{Y}_{\alpha\mu\lambda\gamma}\tilde{Y}_{\alpha\mu\lambda\gamma}]. \tag{29} \]

Using now our solution for \( A_s, B_s \) and \( \pi_s \) we have that, when \( \lambda > (8q - 4p + 3)/(4\sqrt{3m^2}) \), the integration in the extra dimension is finite for all \( q \)-forms in any \( p \)-brane. It is important to stress that for \( p = 3 \), the above expression reproduces the same conditions found for the one, two and three form found in the literature.
The Massive Modes

For the massive modes, the best way to make the analysis is to transform the equation (28) in a Schrödinger type equation. It is easy to see that an equation of the form

\[
\left( \frac{d^2}{dy^2} - P_s(y) \frac{d}{dy} \right) U(y) = -m^2 Q_s(y) U(y),
\]

(30)

can be put in a Schrödinger form

\[
\left( -\frac{d^2}{dz^2} + V_s(z) \right) \bar{U}(z) = m^2 \bar{U}(z),
\]

(31)

through the transformations

\[
\frac{dz}{dy} = f_s(y), \quad U(y) = \Omega_s(y) \bar{U}(z),
\]

(32)

with

\[
f_s(y) = \sqrt{Q_s(y)}, \quad \Omega_s(y) = \exp(P_s(y)/2)Q_s(y)^{-1/4},
\]

(33)

and

\[
V_s(z) = (P_s(y)\Omega_s'(y) - \Omega_s'(y))/\Omega_s f_s^2
\]

(34)

where the prime is a derivative with respect to \(y\). Now using the equation (28) we obtain

\[
V_s(z) = e^{3A_s/2} \left( \frac{\alpha^2}{4} - \frac{9}{64} A_s'(y)^2 - \frac{\alpha}{2} + \frac{3}{8} A_s''(y) \right),
\]

\[
f_s(y) = e^{-\frac{3A_s}{2}}, \quad \Omega(y) = e^{\left(\frac{\alpha}{2} + \frac{3}{8}\right)A_s},
\]

(35)

where \(\alpha = (8q - 4p - 3)/4 - \lambda \sqrt{3M^3}\), and we take \(y\) of function of \(z\) through \(z(y) = \int_0^y f(\eta)d\eta\). We show below the graphic of \(V_s(z)\) by considering \(v^2/72M^3 = 1\) and \(a = 1\) for various \(\alpha\) values.

In order to analyze resonances, we must compute transmission coefficients \(T\) numerically, which gives a clear physical interpretation about what happens to a free wave that interact with the membrane. The idea of the existence of a resonant mode is that for a given mass the potential barrier is transparent to the particle, i.e., the transmission coefficient has a peak at this mass value. That means that the amplitude of the wave-function has a maximum value at \(z = 0\) and the probability to find this KK mode inside the membrane is higher. In order to obtain these results numerically, we have developed a computational code to compute transmission coefficients for the above potential profile. A more extensive and detailed analysis of resonances with transmission coefficients will be given in a separate paper by the authors [10].

In FIG. 3 we give the plots of \(\log(T)\) for \(\alpha = -1.75\), and considering the case \(s = 1, 3, 5\) As mentioned before, we find resonances for this value of \(\alpha\) only for \(s = 1\). In FIG. 4 we show the plot for \(\alpha = -20\)

and we can see how this alter the existence of resonances. For this values we have an interesting structure of resonances, which is similar to that found in quantum mechanical problems. As we stress in our separate paper, the existence of resonances depends strongly of the shape of the potential. In our case, therefore, the resonance is driven by the choice of the parameter \(\alpha\).

The case \(p = 3\)

The case for \(p = 3\) is doubtless the more important one and several issues deserve a careful analysis. First, the generalization employed here simplifies the problem and clearly shows how the condition \(\lambda > (8q - 4p + 3)/4\sqrt{3M^3}\), necessary for localization of the zero mode, changes for different \(p\) and \(q\). For comparison, previous articles found this condition for \(q = 1, 2, 3\) separately [16] [17] [39]. Therefore, we gain a considerable simplification and understanding for all those cases with our generalization. Moreover, we found resonances that was never observed before in [10] and [17]. This is mainly due the fact that we realized that the resonances depends strongly of the parameter \(\alpha\). In [10]
the authors study a one form and found only one resonance peak. By changing the parameter we discovered much more resonant modes FIG. 3 indicating that the higher the parameter more resonances we get. The same study was performed by the same authors in ref. [47], but for the two form. They founded the localization condition for the zero mode, that agrees with our expression. After this, they studied the possibility of resonances and found just one resonance peak. Again, with our generalization, besides the peak found by them we found a lot of other peaks when we change the parameter $\alpha$ as in FIG. 4. The case $q = 3$ was presented in a recent paper by the present authors [39], where the rich structure of resonances was first found. It is important to note here that all analysis pointed above can be visualized in the two graphics given below FIG. 3 and FIG. 4. This was only possible due the general expression for $\alpha$ that we are presenting here.

**Dual Spaces of Resonance**

As a byproduct the present authors has observed that for a fixed value of $\alpha$ and $\lambda$, a $q$-form in a $p$-brane has the same resonance structure of a $q'$-from in a $p'$-brane if $2q - p = 2q' - p'$. All graphics above are plotted for $2q - p = 3$ and are the same as that with a three-form in 3-brane [39]. Also for a fixed $\lambda$ and $q$ the resonance peaks increase for larger values of the dimension of the brane. There are an interesting situation with $\alpha$ constant. We can have models with small values of $\lambda$ displaying the same resonance structure. We see in table bellow some values of $p$, $q$ and $\lambda \sqrt{3M^3}$ with $\alpha = -20$.

| $p$ | $q$ |
|-----|-----|
| 20  | 1   |
| 22  | 2   |
| 24  | 3   |

**TABLE I.** In the left $q = (p + 3)/2$, $\lambda \sqrt{3M^3} = 89/4$ and in the rigth $q = (p - 18)/2$, $\lambda \sqrt{3M^3} = 5/4$. 

FIG. 2. Potential of the Schroedinger like equation for $s = 1$ (lined) scaled by $1/10$, $s = 3$ (dashed), $s = 5$ (dotted) for $\alpha = -1.75$ (top) and $\alpha = -20$ (bottom).
As we can see, we have dual spaces of resonance (DSR): they have the same resonance structure. It is interesting to note that these DSR admits situations with large and low coupling constant. This is similar to the $S$ duality of string theory. The difference is that here we have an infinite class of spaces, with a certain $\lambda$, which has the same resonant mass peaks as another infinite class, with a different $\lambda$. It is not clear to the authors if there is some structure or symmetry behind these DSR that could explain why they appear. We let this question for a future work.

CONCLUSIONS AND PERSPECTIVES

In this paper we have studied the issue of localization of $q$–form fields in a $p$–brane embedded in a $D = (p + 2)$ space-time. The membrane was described by a kink, a codimension one topological defect, and related topological objects called deformed defects. Furthermore, we have considered a gravitational background where the dilaton field plays an important role. We have obtained the localization of the zero modes of several antisymmetric fields regarding specific peculiarities for each dimension. For example, for branes with space dimensions bigger than three, we found that the gauge vector field is localizable even without the dilaton coupling. This is due to the fact that this field is not conformal in these cases. For other types of forms, the dilaton coupling is still needed. We have calculated massive states through the related quantum mechanical problem and have analyzed the appearance of possible resonances. In this case we have made use of numerical computations of transmission coefficients through the membrane. The existence of resonances depends strongly of the shape of the potential, as is well known. We have been able to find several peaks of resonant modes for specific parameters that models the potential and for specific dimensionality of
the spacetime. In particular, as an important and intriguing result, we have found a class of resonances that have the same behavior, but living in spacetimes with different dimensions and with different dilaton coupling constant. Because of this last fact, we call this set of resonances in this model as dual spaces of resonances (DSR). However, it is not at all understood how this appears from the usual way to treat dualities in field theories. We let these sort of discussions for a future work.

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This paper is dedicated to the memory of my wife Isabel Mara (R. R. Landim)
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