Transient bubble oscillations near an elastic membrane in water

C K Turangan¹, B C Khoo²

¹ Fluid Dynamics, IHPC, 1 Fusionopolis Way, #16-16, Connexis, Singapore 138632
² Department of Mechanical Engineering, National University of Singapore, Singapore 119260

E-mail: cary@ihpc.a-star.edu.sg

Abstract. We present a study of transient oscillating bubble-elastic membrane interaction by means of an experiment and a numerical simulation to study the dynamics of bubble’s inertial collapse near an elastic interface. The bubble is generated very close to a thin elastic membrane using an electric spark, and their interaction is observed using high speed photography. The high pressure and temperature plasma from the dielectric breakdown precedes the bubble formation. The bubble then expands and creates a dimple on the membrane. After reaching its maximum size, the bubble begins to collapse. The membrane retracts back, transmitting a perturbation on the bubble surface. The coupling between bubble contraction and this perturbation strengthens the collapse and leads to the formation of a mushroom-shaped bubble, bubble pinching and splitting. Towards the end of the collapse, the water inertia surrounding the bubble pulls the membrane upwards forming a relatively sharp conical hump. The dynamics of this interaction is well predicted by the boundary element method (BEM) simulation.

1. Introduction

Studies of shock-wave lithotripsy (for kidney stone fragmentation) has shown that cavitation bubbles are generated in-vivo near the lithotripter shock focal point [1], and their collapse (either inertial or forced) are correlated to both stone fragmentation and tissue injury [2]. When bubbles oscillate near boundaries that exhibit elastic properties, they form mushroom-shapes that lead to bubble pinching and splitting [3, 4, 5]. Understanding this behaviour is important particularly in applications where bubbles may be induced near biological tissues e.g. in laser angioplasty, arthroscopic cartilage ablation and laser thrombolysis [6]. The experiment of a laser-generated bubble interaction with a thick elastic boundary made of polyacrylamide gel of 80% water concentration shows the formation of a mushroom-shape bubble, bubble pinching and splitting [7]. Bubble oscillation near an elastic membrane may also produce similar behaviour but only limited work have ever been reported in the literature. Turangan et al. [6] studied the behaviour of an electric-spark generated bubble near a thin elastic membrane and compared the observation with numerical simulations using the boundary element method (BEM) [8, 5]. The results show a perturbation on the bubble surface created by the membrane’s repulsion, which strengthens the bubble collapse and forms the mushroom-shaped bubble that leads to bubble pinching and splitting. The impetus of the work presented here are to extend the methodologies presented by Turangan et al. [6] to study the dynamics of a single bubble-elastic membrane interaction where the bubble is generated very close to the membrane (small standoff distance) and to simulate the interaction using BEM to better understand the interaction behaviour.
2. An experimental technique of electric spark-generated bubble and the boundary element method for a transient bubble-elastic membrane interaction

2.1. An electric spark-generated bubble

We have developed a low-voltage experimental technique to generate a single transient bubble in water using an electric spark via touching electrodes [6]. The set-up has 4 main components: an electric circuitry, a water tank, a high speed camera and a light source (figure 1). A thin elastic membrane (0.11 mm thickness) is clamped horizontally and stretched in one direction to have a tension of $\sigma = 43.6$ N/m. It is placed about 50 mm below the water surface to eliminate the free surface influence on the bubble dynamics. The copper-alloy electrodes of 0.11 mm in diameter are crossed manually. The experiment is started by charging the capacitors. The electricity is then discharged with the switch via the electrodes. This generates a single bubble with maximum radius $R_m$ of 2.8-4.0 mm range. To record the bubble-elastic membrane interaction, the electrical discharge is synchronized with a 12,500 frames/s ($\Delta t = 80 \mu$s) high speed camera.

![Figure 1. (a) The experimental setup. (b) The water tank (17 × 17 × 17 cm³). A 55 V DC supply is connected to 5300 µF capacitors and 1 kΩ resistor. The water tank is equipped with an adjustable fixture and filled with preboiled water. Two 0.11 mm-dia copper-alloy electrode wires are placed above the membrane that is stretched in one direction and clamped horizontally.](image)

2.2. The boundary element model for an oscillating bubble-elastic membrane interaction

We consider a scenario where two fluids are separated by a thin elastic membrane (see ref. [8, 5, 6] for full detail). The model assumes that the fluids are inviscid and incompressible. The flow field satisfies the Laplace’s equation ($\nabla^2 \phi = 0$, where $\phi$ is the velocity potential), and the flow is driven by the fluid inertia. Assuming the fluid densities are the same (i.e. $\rho_1 = \rho_2 = \rho$, where 1 and 2 refer to fluid 1 and fluid 2, respectively), the fluid pressures on both sides of the membrane ($p_1$ and $p_2$) and on the bubble surface $p_b$ can be obtained using the Bernoulli equation, i.e.

$$
p_1 = p_\infty - \rho \frac{D\phi_1}{Dt} + \frac{1}{2}\rho |\vec{v}_1|^2, \quad p_2 = p_\infty - \rho \frac{D\phi_2}{Dt} + \rho \vec{v}_2 \cdot (\vec{v}_1 - \frac{1}{2} \vec{v}_2), \quad p_b = p_\infty - \rho \frac{D\phi_b}{Dt} + \frac{1}{2}\rho |\vec{v}_b|^2 \quad (1)
$$

where $t$ is time, $p_\infty$ is ambient pressure, $\vec{v}_1$ and $\vec{v}_2$ are velocity vectors of fluids 1 and 2 at the membrane surface and $\vec{v}_b$ is the velocity vector on the bubble surface. For a thin elastic membrane, the membrane does not support bending but acts like a surface tension. Here, it is only stretched in one direction. Therefore, the pressure difference between the fluids separated by the membrane is $p_1 - p_2 = \sigma K$, where $\sigma$ is the interfacial tension and $K$ is the membrane local curvature. We assume that the bubble internal pressure satisfies the adiabatic conditions. Therefore, bubble pressure $p_b = p_b(\gamma V/V_0)^{\gamma - 1} + p_v$, where $p_b$ is the bubble pressure at the initial volume $V_0$, $V$ is bubble volume, $p_v$ is vapour pressure and $\gamma$ is the specific heat ratio.

Using the scaling factors ($R_m$ for lengths, $p_r = p_\infty - p_v$ for pressures, $t_0 = R_m \sqrt{T_r/T_v}$ for time, $\phi_0 = R_m \sqrt{T_r/T_v}$ for potentials, $1/R_m$ for curvature and $v_0 = \sqrt{T_r/T_v}$ for velocities), the
pressure difference \( p_1 - p_2 = \sigma K \) and the bubble surface pressure \( p_b \) in dimensionless forms are

\[
\frac{D}{Dt'} (\phi'_{i2} - \phi'_{i1}) = \hat{v}_2' \cdot \left( \frac{1}{2} \hat{v}'_2 - \hat{v}'_1 \right) - \frac{1}{2} |\hat{v}'_1|^2 + \kappa_m K', \quad \text{and} \quad \frac{D\phi'_b}{Dt'} = 1 - p'_b + \frac{1}{2} |\hat{v}'_b|^2 \quad (2)
\]

where \( \kappa_m \) is the elastic parameter of the membrane and is given by \( \kappa_m = \sigma/(R_m p_r) \).

Boundary integral equations are used to calculate the normal component of velocity potential \( \partial\phi/\partial n \). For fluid 1 that contains the bubble \( b \) and elastic membrane interface \( i \), and for fluid 2 that contains only the membrane interface \( i \), they are respectively given by

\[
c' \phi'_1 = \int_{b+i} G \frac{\partial\phi'_1}{\partial n} dS - \int_{b+i} \phi'_1 \frac{\partial G}{\partial n} dS, \quad \text{and} \quad (4\pi - c) \phi'_2 = \int_i G \frac{\partial\phi'_2}{\partial n} dS + \int_i \phi'_2 \frac{\partial G}{\partial n} dS. \quad (3)
\]

The variable \( c \) denotes a solid angle that is a function of location of the computational nodes that represent the bubble and membrane surface and \( G \) is the Green function (see ref. [8, 5]).

3. The growth and collapse of a single bubble near a thin elastic membrane

In the experiment, the electrodes are connected manually. Their contact point is placed about \( d \approx 1.60 \text{ mm} \) above the membrane. The plots of the interaction is shown in figure 2 (frame-by-frame plots). Due to the limitation in temporal resolution, the exact time of bubble inception can only be estimated. After the inception, the bubble expands rapidly, and the flow is eventually dominated by the water inertia. The bubble’s top hemisphere expands freely as it is not obstructed but the expansion of the bottom hemisphere is impeded by the membrane that forms a dimple as the bubble pushes it downwards. This exerts a localised tension in the membrane.

\[ R_m \approx 3.84 \text{ mm}, \quad d \approx 1.60 \text{ mm}, \quad \xi = 0.42 \quad \text{and} \quad \kappa'_m = 0.114. \]

The perturbation on bubble surface couples with bubble’s contraction to form a mushroom-shape that leads to pinching and splitting. Dimensionless time \( t' \) from the BEM simulation and the equivalent time \( t = t' \cdot t_0 \) are given.

Figure 2. Frame-by-frame comparison of the interaction between experiment and simulation. The bubble maximum radius of \( R_m \approx 3.84 \text{ mm} \) is estimated from figure 2(f1), and at this moment, its pressure has fallen well below the pressure of the surrounding water. The expansion is halted and the bubble starts to contract. The membrane also begins to retract upwards, and the dimple will then flatten. The bubble’s bottom surface is so close to the membrane that they move upwards together. As the membrane’s retraction accelerates, a perturbation is imparted to the collapsing bubble’s surface. This upwards propagating perturbation couples with the bubble’s contraction to form the unique mushroom-shaped bubble (\( t \approx 0.96 \text{ ms} \)). The thin film
separating the bubble from the membrane seen in figure 2(k1) is likely an artefact associated with the rapid movement of the membrane that the camera is unable to capture. At \( t \approx 1.04\) ms, the collapse intensifies. The perturbation eventually pinches the bubble, splitting it into top and bottom fractions with a slender ‘bubble tubulus’ connecting them. The top fraction forms a ‘mushroom cap’. The bottom fraction is attached to the membrane’s surface (figure 2(11)) that forms an upwards facing conical shape more profound than what was observed before [6].

In this experiment, the bubble collapse time (\( t_c \approx 0.48\) ms) is considerably higher than the Rayleigh collapse time (\( t_{rc} \approx 0.35\) ms) of a spherical bubble with \( R_m = 3.84\) mm. By comparing the radius-time histories of a spherical bubble generated using the same method and from computations based on the Rayleigh-Plesset equation, Turangan et al. [6] found that the one with \( p_r = 50\) kPa matched well with the experiment. They argued that there might be a multiple dielectric breakdown (owing to the conventional way of connecting the electrodes) that leads to an excessive amount of hot metal vapour in the bubble and the prolonged heating of the bubble surface, and hence a higher vapour pressure and longer collapse time. The bright light that lasts up to \( t \approx 0.56\) ms (see figure 2) may indicate so but further investigation is required to prove it.

A BEM simulation is carried out to complement the experimental observation. Figure 2 shows the comparison. The code solves the governing equations in an axisymmetric form. The bubble and the elastic membrane are represented using 51 computational nodes each. The bubble is assumed to initially contain a mixture of high pressured non-condensable gas and vapour, and thus \( \gamma = 1.25\). The bubble initial radius \( R_0 = 0.1458 R_m\) at initial time \( t' = 0\) is iterated using the Rayleigh-Plesset equation. This corresponds to the bubble’s initial pressure of \( p_{g0} = 10\) MPa. For \( d \approx 1.60\) mm and \( R_m \approx 3.84\) mm, the bubble’s stand-off distance is \( \xi = d/R_m = 0.42\). The membrane is only stretched in one direction, and therefore the elastic parameter is redefined to be \( \kappa_m^* = \kappa/2 = 0.114\) (for \( \sigma = 43.6\) N/m). The plots for comparison are obtained by estimating the bubble’s oscillation period and dividing it with the number of frames to be compared. The dimensionless time \( t'\) of each plot is then multiplied with the scaling factor for time (\( t_0\)) to give the dimensional time \( t\) (i.e. \( t = t' \cdot t_0\), where \( t_0 = R_m \sqrt{\rho/(p_\infty - p_r)}\), \( \rho = 1000\) kg/m\(^3\), \( p_\infty = 100\) kPa and \( p_r = 50\) kPa). The comparison shows that the simulation is able to model the interaction well including the membrane’s response, the perturbation on the bubble surface, the mushroom shape and the pinching. However, there are several disparities that can be observed including the difference in bubble size in the early stage and the way the bubble splits.

4. Conclusions
The comparison between the experiment and simulation for a transient bubble generated very near to an elastic membrane matches well, showing a profound membrane’s conical shape when the bubble splits. The experiment is easy to implement but manually connecting the electrodes is likely to lead to multiple electric breakdowns that cause high vapour pressures and longer collapse times. Nevertheless, the work demonstrates the efficacy of the method that can be easily applied to study the dynamics of bubbles near membrane materials with different elasticities, including those of biological materials. The numerical technique can be developed further to be a valuable tool to predict the dynamics of the interaction when parametric studies are required.

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