Teaching kinematics in two dimensions

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Abstract. Kinematics has almost always been introduced in one dimension. This seems to have been more or less universally assumed to be the simplest way, with little explicit deliberation. This paper presents a suggested ‘teaching experiment’, as a contribution to the didactical debate, starting in two dimensions, arguing that this might well provide a more fruitful route into the tautological complexities of kinematics. Some of the affordances exploited will be relatively recent, due to the current culture and availability of mapping and tracking technologies. Other suggested affordances will be conceptual and more evolutionary, as the barriers to thinking with vectors and about accumulations are now much lower if the potential for pedagogical support in representational and measurement technology is exploited fully. But perhaps the most significant reason to consider a change is that current practices and explanations could be improved. The spatialising of duration in Cartesian graphs may present further tripwires for learning. There are no guarantees here, and practice is rather resistant to change, but perhaps new thinking and new tools might make the topic more accessible: more children could then reason more fruitfully if given access to new tools for representing, reasoning and predicting.

1. Starting out
I am going to suggest that it's at least plausible that by rethinking how we represent and reason about kinematics that we can achieve significant learning gains for students in secondary schools with no subsequent losses for those who move on to higher education.

There are five supporting lines of argument for this assertion, each of which is, in effect a pedagogic suggestion:

- That taking a point of view is essential for recording or noticing a motion or change in motion.
- That starting with accumulations, and reasoning about accumulations, is a much simpler approach to looking at the kinematic relations that thinking about operational definitions based on rates which rely on calculations of differences and then divisions by infinitesimal times.
- That we might find imagined learning journeys to have fewer trip-wires en route if we started in two dimensions rather than in one dimension: here an imagined learning journey is the research aware didactical path that is designed for a learner, so an intended curriculum in all of its richness.
- That we make use of vectors as our primary tool for representing motion, and delay representing motion on Cartesian graphs until kinematic ideas are well-established.
- That the widespread availability of motion sensors, in particular for position and acceleration, together with significant computational power, allows more freedom of choice about which quantities might form the basis for kinematics than in the era where only metre rulers and stopwatches were available.
These five suggestions are now developed and supported.

2. A line of reasoning
Each of these suggestions is at least not widespread in current practice in the UK and probably more widely, and in some cases it's clear that these run counter to the conventional sequence for learning about kinematics.

So there will need to be good reasons for each of these five suggestions. These reasons need to be persuasive in order to think that such an approach could be successful, and therefore is worth persevering with.

3. Starting with two dimensions
Let's start with perhaps the most controversial suggestion which is that we begin by reasoning about motions in two dimensions. I suppose it's obvious but worth noting that almost all representational surfaces on which we inscribe records of journeys, or which we use to make plans for journeys, are two-dimensional: from formal paper maps, through GPS trails, to sketch maps of how to get from a home to the local shops. This is, I think, not accidental: stories about journeys involve relating ourselves to the local geometry and often involve talking about, for example: 'going left', 'going right', 'going straight on', and 'turning half left' by some landmark or after traveling for some distance or duration. We should seek to co-opt these patterns of thinking and talking in the students' experiences: not something that can so readily be done if we start along the conventional route of teaching about kinematics in one dimension. The phrases that naturally involve two dimensions are a widely-used way to give directions and to talk about paths that are followed. It's possible to see connections here with the revolution in geometrical understanding derived with computational approaches embedded in Logo-like languages. These appear often in many early introductions to computational activity, in languages such as Logo itself or Scratch. The widespread opportunities to 'program' in a set of instructions, much like offering guidance in getting from A to B when 'programming' a human, provides additional support for thinking in two dimensions, often using relative positioning, (implicitly)relative velocities and durations. The availability of more-or-less educational computational toys such as Sphero™ further widen access to this 'two-dimensions first' approach to kinematics.

4. Taking a point of view
'Taking a point of view' is neither more nor less than establishing a frame of reference, but it is perhaps a more accessible formulation as a starting point (for a development of the idea see, for example, the web resources for Supporting Physics Teaching). It is essential, but often not explicit, to take a point of view in order to record a velocity or a displacement. Starting with "what Charlie notices" or "what Alice records", rather than with somewhat formal arrays of measuring rods and clocks is a better way to make it more likely that this essential step of taking a point of view and incorporating that into the records of motion will become more widespread. Such an approach can make an approach to relative velocities, which often appears as a later, somewhat optional complication in current curricula, appear both more natural and more essential. You can start somewhat informally by exploring what Charlie or Alice (in an example where this pair are not co-moving) notice, then what they record, and how that move to formalisation can handled by either character.
Figure 1. What Charlie notices - all movement is relative movement.
This actor, or observer-based approach also aligns well with the stories-of-journeys approach promoted in the discussion about starting with two dimensions.

5. Accumulations
Velocity is often defined as the rate of change of displacement, and acceleration as the rate of change of velocity, and there comes a time along the learning pathway when this conceptually charged formalisation, with its infinitesimals, is an essential part of the physicists toolkit. But perhaps not when starting out. Here integration, as simple repetitive addition, seems much simpler than subtraction and division. 'Acceleration accumulates velocity' (and the more demotic version, 'acceleration tells velocity how to change' is more approachable still, but remains operationally precise [1]). 'Acceleration accumulates velocity' is intrinsically a simpler formulation than 'acceleration is the rate of change of velocity', as assayed by the number and subtlety of mathematical operations required. Indeed, numerical methods (see [2] for an account of how these can be made accessible to younger children) use just such repetitive addition to predict the velocity from a given acceleration, and the displacement from a given velocity. Of course, such predictions are risky: here the velocity as reported by the GPS-enabled device does not guarantee that I'll end up in the sea and miss Edinburgh, or that I'll travel directly from the current location, shown by the cross-hairs, to Edinburgh. (For more on repetitive addition, and predictions using vectors see [3]).

Figure 2. A velocity vector, as reported by a GPS-enabled device held by a passenger in an Edinburgh-bound train
Either approach must be sensitive to the physical situation: in the case of modeling with either rates or with accumulations, the situation determines the interval used for calculations.
This consideration becomes much more persuasive when considering vectors as a part of the didactic mix, as adding vectors can be made rather straightforward (see, for example, [3], chapters 6 and 8).

6. Vectors as a primary representation

Velocity, acceleration and displacement are of course vector quantities, and therefore both magnitude and direction is essential to their nature. 'Simplifying' such quantities by restricting them to a single dimension entangles teachers in verbiage, as evidenced by this series of quotations from experienced teachers: 'signed quantities, but not like power', 'the deceleration is negative', 'these are one-dimensional vectors, but not just like numbers on the number-line', 'acceleration happens when the speed is not in the same direction', 'directed speed'. All of these, which cause significant cognitive loading for teachers and children, could be sidestepped, I think, by using vectors in their natural introductory environment of two dimensions. This may be one place where the affordances of the computer as a (two-dimensional!) representation machine can kickstart a reformulation of our customs and rituals. Adding vectors is now easy: 'just place the arrows tip to tail'. With computational assistance the arrows become objects-to-reason with, tools for thought. Later, perhaps much later, one can burrow down into components and algebraic representations. But in introductory teaching we might try and get the physics established with the most approachable representations that we share with mathematics: just arrows. In fact arrows such as those which appear on sketch maps, on GPS displays, on diagrams to show velocity from a point of view.

7. Motion sensors enable acceleration to be a primitive

From the point of view of didactics there is no need to introduce acceleration or velocity as a derived quantity, whatever the status as an SI unit, or the desired final definition. Tools to measure acceleration more or less directly are widely available, and not so expensive, even if the ability to represent it as a vector is often sidestepped. There is a gap in the market for a good velocity measuring tool, but it's over 20 years since I saw an affordable doppler-based velocity meter working in a school laboratory (in Auckland, as it happens). Alternatively, at a larger scale, that GPS technology is widely embedded now presents many possibilities for representing the quantities, in didactically appropriate ways. Melding such sensors effectively to computational power and portable high-resolution screens offers genuine choices about how to present the data. Dials, gauges, data tables and cartesian graphs may be comfortingly familiar to those inducted into physics when the current representational flexibility was not available, but it does not follow that this is the best or only pedagogic path. I think we could make more of vectors, particularly in teaching about kinematics. The idea of starting with acceleration was explored in an earlier GIREP conference; for more see [4]), at a time when such sensors were becoming more commonplace.

8. But what about graphs and algebra?

All in good time. These are highly abstract representations of motion, embedding many conventions far removed from our starting point of describing journeys. (There is little less removed from the lived-in world than graphing my bicycle journey to the shops in one dimension. Yes, it can have value, but the greatest value may lie some way down the didactical path.) It seems at least likely that the best place to learn about the intricacies of constructing and interpreting graphs is not (as so often happens at the moment) in studies of motion, for however many carefully engineered sensors are employed in automatically generating the graph, time remains spatialised. Two axes extended in space (measured in metres) must represent seconds, metres, metres second$^{-1}$ and metres second$^{-2}$: this is not obviously or immediately a well-formed tactic if clarity of representation is the aim. It is also the case that algebraic representations, manipulations and conventions can serve physics well, but cannot replace a physical understanding. There is more to understanding kinematics than achieving algebraic and arithmetic fluency, and the approach suggested here ensures that these fluencies are not even a necessary precursor.
9. A possible sequence

- Start with lots of stories of motion in a horizontal plane, mostly qualitative and semi-quantitative. These could be a 'drone's-eye' (perhaps even live) or 'bird's-eye' view: a two-dimensional view of a tracked motion. These have exploitable resonances in lived-in world representations, such as GPS-enabled mappings of personal journeys (Strava, MapmyRide), maybe aerial traffic cams or city mapping of traffic flows.

- In the school laboratory this could be coupled with strobed long exposure images of polished steel ball bearing: records of 'natural motions' on different sculpted surfaces. Or put a suitably mounted 'flasher' to work in similar environments[5]).

- This line of development could be technologically supported by using toy robots that can be programmed to traverse two-dimensional paths, perhaps introducing challenges to explore or reinforce an understanding of sequences of track segments, beginning to formalise the idea of a motion as a sequence of track segments.

- On the screen, sequences of displacements could render a track visible, track segment by track segment.

- On paper, introduce motion diagrams, moving from qualitative to semi-quantitative and then fully qualitative interpretations, again utilising the idea of a track segment.

The first formalisation is a plan view of a motion diagram which is intrinsically two-dimensional, and composed of a sequence of track segments.
Figure 3. Track segments on a motion diagram
From these can emerge the idea of taking a point of view and recording when something was where, so relating a sequence of displacements to the motion diagram. This sequence of displacements can then be simply represented by a sequence of arrows, from the chosen point of view.

- Then introduce accumulations, relating the velocity (just shown by an arrow, perhaps on even a map) to predictions about where the actor will end up.

Figure 4. Velocity accumulating displacement as you predict the motions of a hedgehog
- This segues into a series of predictive stories, starting with a sequence of velocities and predicting the displacements. So you can bring together the experimental records of motion with a predictive model: manipulating the representations as here enables children to reason about what will happen.
These predictive stories rely on 'velocity telling displacement how to change' (less demotically: velocity accumulates displacement), so introducing the first tautology of the pair that comprise kinematics.

- Extend the idea of accumulations to acceleration accumulating velocity: it may be better to resist the temptation to place the velocity vectors tip to tail at this stage to avoid any suggestion of representing a track that is followed.
- Link both accumulations, one after another, to predict a path.
- This introduces the second tautology of kinematics ('acceleration tells velocity how to change', again in its natural two-dimensional environment).

**Figure 5.** Making predictions: from velocity accumulating displacements to a diagram of track segments (a motion diagram)
After completing the two-dimensional sequence, the one-dimensional case might be introduced as useful in some special cases.

For some, now might be the time to formalise the vector representation as an extension to their understanding of kinematics, as an ordered set of numbers.

Graphs, as frozen accumulations, might well be introduced last (preferably, for the reasons outlined above, leveraging representational competences introduced in another topic).

10. Thinking about designing a sequence
The difference between vectors and scalar quantities is important in physics, and so vectors deserve to be introduced so that they do real enabling work, allowing for learners to see how they are a powerful tool with which to think. The motivation behind this stricture is that an idea only becomes meaningful when we reason with it: it is incorporated into a mental model[6].

Connections between kinematics and the children's lived-in world can be strengthened by spending much more time developing qualitative descriptions before formalising kinematics. This can focus on stories of motion, to lay the groundwork for seeing motion diagrams as depictions and even records of movement, with the potential for insightful interpretation.

Working with local geometries has supported significant gains in mathematical understanding, and such gains should also help us here. Focusing on motion in a plane exploits the sweet spot, where manipulated simplicity meets representational adequacy. There is no easy access to three-dimensional spaces in which to draw, manipulate and represent. And one-dimensional spaces do not contain enough information to easily develop the idea of vector quantities, which will be essential to kinematics.

Such two-dimensional motion diagrams serve to connect the vector accumulations, which are the core manipulated representation, to the lived-in world. As such, they provide a transitional tool for both noticing and recording: two-dimensional representations are what one might expect children to reason with. Reasoning fluently with tautologies is essential: kinematics is not a set of empirical relationships (the relationships between acceleration, velocity and displacement are essential to the meaning of the terms, and could not be otherwise: they are a toolkit with which to apprehend the world, not something we learn about the world). Therefore, introducing the quantities simply as empirically calculated is a mis-step. As a consequence it is probably better to start with some predictive narratives as these mimic the predictions that show understanding, in exemplifying mental models in action, rather than a set of calculations from data. The central resources from which these mental models are constructed are accumulations and vector quantities.

One can accumulate vectors graphically because the grouping of magnitude and direction into a single graphical object allows you to accumulate unified entities, while working in more than one dimension, which turns out to be crucial to avoiding difficulties in constructing an idealised learning journey for the topic. In constructing such a journey, making early use of these operationalised measures is crucial: allowing time for these to become tools to think with.

It seems to me more than likely that kinematics is a really bad place to learn about graphs, perhaps because it relies on relating changes in space to changes in time, whereas scatter graphs spatialise both variables. This is probably an issue to be concerned about in any proposal to learn about graphs with all data based on a time series([7][8][9]).

Above all in implementing a sequence, avoid a dash to formalism, especially one well-abstracted from the lived-in world.

11. Significant teaching and learning challenges
- The ideas of time, space and motion may be inextricably tangled: to notice and record the passage of time you need a clock, which will involve regular changes over time, and these are otherwise known as motions. Any approach should respect this potential difficulty, and not use any representations that might conflate duration and distance, for example.
Any representational path may impede learning, or may be helpful in developing the idea. The facility of the path will be connected to the way children can reason with the representations employed([10][11][12][13][14]).

Current practice commits to a selection of operationalised and customary measures and definitions, which are strongly linked to 'teacher rituals'[8]. These are rather formal, complex and forbidding, and current practice does not seem to focus on providing tools for thought.

Using scatter or line graphs as the primary representation in motion, often as a time series, causes didactical and pedagogic difficulties due to unavoidable reduction to one dimension. As a result at best you're dealing with signed quantities, and teacher talk often seems rather knotty (e.g. 'one dimensional vectors' seems not to be a very simple way to introduce the idea of a vector), as the resources to hand do not seem to easily support the kinds of distinctions they want to make.

12. Concluding
Visitors to 'physics land' have wondered, with some justification, about our current practices:
"While it is easy for pupils to see that 'speed' is a very useful concept with everyday applications, it is not so easy to see what the 'motivation' is for introducing one-dimensional velocities...I found it difficult to get a sense of how pupils were persuaded that this opened up new areas of understanding of motion."

I'd hope that this observation would not apply to the current scheme, where the vectors do real work in the mental model that the scheme promotes, and form a necessary part of a coherent explanatory narrative (for more on the idea of explanations as narratives, see[15]. There will, of course, as with any new approach, be some transitional pain, but I think that it's at least plausible that such an approach could be fruitful and cause significantly fewer difficulties for both teachers and learners([16]) in being able to reason with the tautologies of kinematics, so providing a sound picture of physics ([17][18][19][20]).

13. Appendix A: a note on sources and references
This whole paper is an attempt to show that there is a reasonable change that such a line of story telling about kinematics could work for the good of many in perfectly ordinary classrooms, whilst being accessible to supporting the development of reasoning in physics for children 8-16 years old. That's why I have preferred sources that emphasise practice and references to published schemes over references that elaborate theoretical positions. Nevertheless, many of the cited practices are more than research-aware, and there are sound reasons, here often indicated to rather than rigorously supported by argument (both for reasons of space, and because I think many are rather well known). There is certainly a lot of evidence that the current widespread approach based on graphs, algebra and one-dimension leaves room for improvement.

14. Appendix B: Sources
MapmyRide is at https://www.mapmyride.com [access date Thursday, 7 June 2018]
Strava is at https://www.strava.com [access date Thursday, 7 June 2018]
A (cheapish) programmable, steerable robot is Sphero™, at https://www.sphero.com [access date Thursday, 7 June 2018]
More on taking a point of view at http://supportingphysicsteaching.net/Mo, right at the start of the physics narrative [access date Thursday, 14 June 2018] and, in more depth, http://supportingphysicsteaching.net/Fm02PNnugget04.html [access date Thursday, 14 June 2018].
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