Light tetraquark state at nonzero temperature

ACHIM HEINZ, STEFAN STRÜBER, FRANCESCO GIACOSA and DIRK H. RISCHKE

Institute for Theoretical Physics, Johann Wolfgang Goethe University, Max-von-Laue-Str. 1, D–60438 Frankfurt am Main, Germany

and

Frankfurt Institute for Advanced Studies, Johann Wolfgang Goethe University, Ruth-Moufang-Str. 1, D–60438 Frankfurt am Main, Germany

We study the implications of a light tetraquark on the chiral phase transition at nonzero temperature $T$. The behavior of the chiral and four-quark condensates and the meson masses are studied in the scenario in which the resonance $f_0(600)$ is described as a predominantly tetraquark state. It is shown that the critical temperature is lowered and the transition softened. Interesting mixing effects between tetraquark, and quarkonium configurations take place.

PACS numbers: 11.30.Rd, 11.30.Qc, 11.10.Wx, 12.39.Mk

1. Introduction

In the last decades theoretical and experimental work on light scalar mesons with masses below $\sim 1.8$ GeV [1] initiated an intense debate about their nature. Quarkonia, tetraquark and mesonic molecular assignments, together with the inclusion of a scalar glueball state around 1.5 GeV as suggested by lattice simulations, have been proposed and investigated in a variety of combinations and mixing patterns [2].

Nowadays evidence toward a full nonet of scalars below 1 GeV is mounting: $f_0(600)$, $f_0(980)$, $a_0(980)$, and $K_0^*(800)$. An elegant way to explain such resonances is the tetraquark assignment proposed by Jaffe long ago [3]. The reversed mass ordering is naturally explained in this way and also decays can be successfully reproduced [4]. Within this context the lightest

---

*Presented by A. Heinz at the Excited QCD Workshop, 31.1.-6.2.2010, in Tatranska Lomnica (Slovakia)
scalar resonance $f_0(600)$ is interpreted as a predominantly tetraquark state $1/2[u, d][\bar{u}, \bar{d}]$, where the commutator indicates an antisymmetric flavour configuration of the diquark.

The lightest quark-antiquark state, i.e., the chiral partner of the pion with flavor wavefunction $\bar{n}n = \sqrt{1/2}(\bar{u}u + \bar{d}d)$, is then predominantly identified with the broad resonance $f_0(1370)$. The fact that scalar quarkonia are $p$-wave states supports this choice. According to this picture quarkonia states, together with the scalar glueball, lie above 1 GeV, see Ref. [5] and refs. therein.

It is natural to ask how the scenario outlined here affects the physics at nonzero temperature $T$. It is in fact different from the usual assumptions made in hadronic models at $T > 0$, where the chiral partner of the pion has a mass of about 600 MeV. Moreover, besides the chiral condensate, new quantities emerge: a tetraquark condensate and the mixing of tetraquark and quarkonium states in the vacuum and at nonzero $T$. Remarkably, the mixing angle increases for increasing $T$ and the behavior of the chiral condensate is affected by the presence of the tetraquark field. Details can be found in Ref. [6], on which these proceedings are based.

2. The Model

We work with a simple chiral model with the following fields: the pion triplet $\vec{\pi}$, the bare quarkonium field $\phi \equiv \bar{n}n$, and bare tetraquark field $\chi \equiv \frac{1}{2}[u, d][\bar{u}, \bar{d}]$. The chiral potential was derived in Ref. [7]:

$$V = \frac{\lambda}{4} (\varphi^2 + \vec{\pi}^2 - F^2)^2 - \varepsilon \varphi + \frac{1}{2} M_\chi^2 \chi^2 - g \chi (\varphi^2 + \vec{\pi}^2),$$

(1)

where, besides the usual Mexican hat, the parameter $g$ describes the interaction strength between the quark-antiquark fields and the tetraquark field $\chi$. In the limit $g \rightarrow 0$ the field $\chi$ decouples, and a simple linear sigma model for $\varphi$ and $\vec{\pi}$ emerges. The minimum of the potential (1) is, to order $O(\varepsilon)$:

$$\varphi_0 \simeq \frac{F}{\sqrt{1 - 2g^2/(\lambda M_\chi^2)}} + \frac{\varepsilon}{2\lambda F^2}, \quad \chi_0 = \frac{g}{M_\chi^2} \varphi_0^2,$$

(2)

and $\vec{\pi}_0 = 0$. The condensate $\varphi_0$ is identified with the pion decay constant $f_\pi = 92.4$ MeV. Note that the tetraquark condensate $\chi_0$ is proportional to $\varphi_0^2$: it is induced by spontaneous symmetry breaking in the quarkonium sector. Shifting the fields by their vacuum expectation values (vev’s) $\varphi \rightarrow \varphi + \varphi_0$ and $\chi \rightarrow \chi + \chi_0$, and expanding around the minimum, we obtain up to second order in the fields

$$V = \frac{1}{2} (\chi, \varphi) \begin{pmatrix} M_\chi^2 & -2g\varphi_0 \\ -2g\varphi_0 & M_\varphi^2 \end{pmatrix} \begin{pmatrix} \chi \\ \varphi \end{pmatrix} + \frac{1}{2} M_\varphi^2 \vec{\pi}^2 + \ldots,$$

(3)
where
\[ M_\varphi^2 = \varphi_0^2 \left( 3\lambda - \frac{2g^2}{M_\chi^2} \right) - \lambda F^2, \quad M_\pi^2 = \frac{\epsilon}{\varphi_0}, \] (4)

Since the mass matrix has off-diagonal terms, the fields $\varphi$ and $\chi$ are not mass eigenstates. The mass eigenstates $H$ and $S$, identified with the resonances $f_0(600)$ and $f_0(1370)$, respectively, are obtained from an $SO(2)$ rotation of the fields $\varphi$ and $\chi$,
\[ \begin{pmatrix} H \\ S \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & \sin \theta_0 \\ -\sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} \chi \\ \varphi \end{pmatrix}, \quad \theta_0 = \frac{1}{2} \arctan \frac{4g\varphi_0}{M_\varphi^2 - M_\chi^2}. \] (5)

The tree-level masses of $H$ and $S$ are
\[ M_H^2 = M_\chi^2 \cos^2 \theta_0 + M_\varphi^2 \sin^2 \theta_0 - 2g\varphi_0 \sin(2\theta_0), \] (6)
\[ M_S^2 = M_\varphi^2 \cos^2 \theta_0 + M_\chi^2 \sin^2 \theta_0 + 2g\varphi_0 \sin(2\theta_0). \] (7)

For the reasons discussed in the Introduction, the bare tetraquark is chosen to be lighter than the bare quarkonium, thus: $M_S > M_\varphi > M_\chi > M_H$. The state $H \equiv f_0(600)$ is the predominantly tetraquark state, and the state $S \equiv f_0(1370)$ is the predominantly quarkonium state.

3. Results and discussions

In order to investigate the nonzero $T$ behavior, we employ the CJT formalism in the Hartree-Fock approximation [8]; for specification of the method in the case of mixing we refer to Ref. [9]. The CJT-formalism leads to temperature-dependent masses $M_S(T)$, $M_H(T)$, $M_\pi(T)$, and a temperature-dependent mixing angle $\theta(T)$. Moreover, both scalar-isoscalar fields have a $T$-dependent vev, for the quarkonium $\varphi_0 \to \varphi(T)$ and for the tetraquark $\chi_0 \to \chi(T)$. For both fields the zero-temperature limits are $\varphi(0) = \varphi_0$ and $\chi(0) = \chi_0$.

When the tetraquark decouples (limit $g \to 0$), $S$ is a pure quarkonium and $H$ is a pure tetraquark state. The transition is crossover for $M_S \leq 0.95$ GeV and first order above this value. This is a well-established result, see e.g. Ref. [10]. The fact that a heavy chiral partner (i.e., mass larger 1 GeV) of the pion leads to a first order phase transition disagrees with lattice QCD calculations [11].

The inclusion of the tetraquark state changes this conclusion as shown in Fig. 1: In Fig. 1.a $M_H = 0.4$ GeV is fixed and the parameters $M_S$ and $g$ are varied. In Fig. 1.b the behavior of the quark condensate for fixed $M_S = 1.0$ GeV and $M_H = 0.4$ GeV is shown for different values of the parameter $g$. One observes that for increasing values of the coupling $g$
Fig. 1. Panel (a): Order of the phase transition as a function of the parameters of the model. $M_H = 0.4$ GeV and $M_S$ and $g$ are varied. The forbidden area violates the constraint $|M_S^2 - M_H^2| \geq 4g\varphi_0$ [6]. On the border line between the first-order and the crossover transitions a second-order phase transition is realized. Panel (b): the chiral condensate is shown for $M_H = 0.4$ GeV and $M_S=1.0$ GeV for different values of $g$ (step of 0.5 GeV). The dots in panel (a) correspond to the curves in panel (b).

the critical temperature $T_c$ decreases: while $T_c = 250$ MeV for $g \to 0$, a value $T_c \simeq 200$ MeV is obtained for $g = 2.0$ GeV. Also the order of the phase transition is affected: when increasing the parameter $g$, the first-order transition is softened and, if the coupling is large enough, becomes a crossover.

We now turn to the explicit evaluation of masses, condensates, and the mixing angle at nonzero $T$. The masses are chosen to be in the range quoted by Refs. [1, 12]: $M_S = 1.2$ GeV and $M_H = 0.4$ GeV. The coupling strength is set to $g = 3.4$ GeV in order to obtain a crossover phase transition. Together with the pion mass $M_\pi = 139$ MeV and the pion decay constant $\varphi_0 = f_\pi = 92.4$ MeV the parameters are determined as: $\lambda = 52.85$, $M_\chi = 0.96$ GeV, and $F = 64.2$ MeV.

The behavior of the two condensates is shown in Fig. 2.a. At $T_c = 180$ MeV the quark condensate $\varphi(T)$ drops and approaches zero, thus restoring chiral symmetry. Below $T_c$ the tetraquark condensate $\chi(T)$ follows the quark condensate, but above $T_c$ the condensate starts to increase. (This result could be different if additional terms $\sim \chi^4$ in Eq. [1] were included.)

By increasing $T$ the function $M_S(T)$ first drops softly, but at a certain temperature $T_s \simeq 160$ MeV a step-like decrease occurs, while the function
$M_H(T)$ undergoes a step-like increase. The solid line in Fig. 2.b describes the state $S$ according to the following criterion: $S$ is the state containing the largest amount of the bare quarkonium state $\varphi$. For $T < T_s$ it corresponds to the heavier state, for $T > T_s$ to the lighter one. A similar analysis holds for the dashed line referring to $H$ as the state with the largest bare tetraquark amount.

The mixing angle $\theta(T)$ is shown in Fig. 3.c. At $T_s$ the mixing becomes maximal and the angle jumps suddenly from $\pi/4$ to $-\pi/4$, $\lim_{T \to T_s} \theta = \pm \pi/4$. At $T_s$ the two physical states $H$ and $S$ have the same amount (50%) of quarkonium and tetraquark.

4. Conclusions

We have shown that the interpretation of $f_0(600)$ as a predominantly tetraquark state sizably affects the thermodynamical properties of the chiral phase transition: the behavior of the quark condensate is softened rendering the order of the phase transition cross-over for a sufficiently large tetraquark-quarkonium interaction, and the value of the critical temperature $T_c$ is reduced, in agreement with recent Lattice simulations [11].

In future studies one should include a complete treatment of the other scalar-isoscalar states $f_0(980)$, $f_0(1500)$, and $f_0(1710)$ which appear in an $N_f = 3$ context (together with the inclusion of the scalar glueball). Also, (axial-)vector mesons shall be considered [13]. Nevertheless, the emergence of mixing of tetraquark and quarkonium states is general, and it is expected to play a relevant role at nonzero temperature also in this generalized context.
REFERENCES

[1] W. M. Yao et al. [Particle Data Group], J. Phys. G 33 (2006) 1.
[2] C. Amsler and N. A. Tornqvist, Phys. Rept. 389, 61 (2004); F. E. Close and N. A. Tornqvist, J. Phys. G 28, R249 (2002); E. Klempt and A. Zaitsev, Phys. Rept. 454 (2007) 1; F. Giacosa, Phys. Rev. D 80 (2009) 074028.
[3] R. L. Jaffe, Phys. Rev. D 15 (1977) 267. R. L. Jaffe, Phys. Rev. D 15 (1977) 281.
[4] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. Lett. 93 (2004) 212002; F. Giacosa, Phys. Rev. D 74 (2006) 014028; G. ’t Hooft, G. Isidori, L. Maiani, A. D. Polosa and V. Riquer, Phys. Lett. B 662 (2008) 424; F. Giacosa and G. Pagliara, Nucl. Phys. A 833 (2010) 138; A. H. Fariborz, R. Jora and J. Schechter, Phys. Rev. D 79 (2009) 074014; A. H. Fariborz, R. Jora and J. Schechter, Phys. Rev. D 72 (2005) 034001.
[5] C. Amsler and F. E. Close, Phys. Lett. B 353, 385 (1995); W. J. Lee and D. Weingarten, Phys. Rev. D 61, 014015 (1999); F. E. Close and A. Kirk, Eur. Phys. J. C 21, 531 (2001); F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Phys. Rev. D 72, 094006 (2005); F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Phys. Rev. D 72 (2005) 094006.
[6] A. Heinz, S. Struber, F. Giacosa and D. H. Rischke, Phys. Rev. D 79 (2009) 037502.
[7] F. Giacosa, Phys. Rev. D 75 (2007) 054007.
[8] J. M. Cornwall, R. Jackiw and E. Tomboulis, Phys. Rev. D 10 (1974) 2428.
[9] J. T. Lenaghan, D. H. Rischke and J. Schaffner-Bielich, Phys. Rev. D 62 (2000) 085008.
[10] S. Strüber and D. H. Rischke, Phys. Rev. D 77 (2008) 085004.
[11] F. Karsch, arXiv:hep-ph/0701210; Z. Fodor and S. D. Katz, JHEP 0404 (2004) 050.
[12] J. R. Pelaez, Phys. Rev. Lett. 92 (2004) 102001; I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001.
[13] S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. 41 (1969) 531; P. Ko and S. Rudaz, Phys. Rev. D 50 (1994) 6877; D. Parganlija, F. Giacosa and D. H. Rischke, arXiv:1003.4934 [hep-ph].
Forbidden area

First-order

Crossover

$M_H = 0.4 \text{ GeV (fixed)}$