Static Output Feedback $l_2 - l_\infty$ Asynchronous Control of Markov Jump Systems Under Dynamic Event-Triggered Scheme

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ABSTRACT This article studies the control problem of the discrete-time Markov jump systems (MJSs). First, the asynchronous switching between the plant modes and the controller modes is taken into consideration. To handle this issue, the estimated modes are utilized in the controller design. Second, in order to reduce the frequency of data transmission in the control system, a dynamic event-triggered communication scheme is introduced into the controller-to-actuator channel. Third, the exogenous disturbance is considered and the $l_2 - l_\infty$ performance index is introduced. In addition, all the states are unavailable and only the output signals are used for the feedback design. The highlight of this study is that a more general case is considered, in which both the plant modes and the states are assumed to be unavailable. The fundamental issue is to determine the feedback gains and the event-triggering matrix simultaneously such that the closed-loop MJSs are stochastically stabilized with a certain level of $l_2 - l_\infty$ performance. By constructing a mode-dependent Lyapunov function, a set of sufficient conditions is derived and the control algorithm is developed. At last, a numerical example and a DC-DC switched boost converter circuit are given to show the effectiveness and practicability of the proposed control technique.

INDEX TERMS Markov jump systems, dynamic event-triggered mechanism, static output feedback.

I. INTRODUCTION
The past decades have witnessed significant advances on the studies of Markov jump systems (MJSs) for their capability of modeling the dynamic systems with sudden changes owing to component failures, exogenous disturbances, and so on. The MJSs have been widely applied in many fields involving economic systems, power systems, etc. A considerable amount of valuable results has been reported in the literature, see for instance, stability and stabilization [1], [2], model reduction [3], optimal control [4], receding horizon control [5], sliding mode control [6], [7], [8], $H_\infty$ control [9], [10], distributed state estimation [11], and on forth.

In the most existing results on the control problem of MJSs, the mode-dependent controllers are applied. In such case, it is generally assumed that the mode information of MJSs is completely available to the controllers [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]. The synchronization between the controller modes and the plant modes can thereby be achieved. In practical applications, however, it is unrealistic due to the network-induced reasons (such as data dropouts and connection failures). Under such situation, the asynchronous phenomenon inevitably appears between the plant modes and the controller modes. It should be pointed out that the control techniques developed for MJSs can not be directly applicable to the asynchronous case, which results in the asynchronous control issues for MJSs [12]. In the past few years, the asynchronous control for MJSs has gained increasingly attention and has been intensively studied, such as passivity control [12], $H_\infty$ control [13], stochastic control [14], model predictive control [15], sliding mode control [16], and so on. Despite many achievements, it is far from mature. Most of the results mentioned above are based on the assumption that the system states are available to the controllers. This
motivates us to consider a more general case when the states are unmeasurable.

In practical applications, the data transmission among different components of the control systems is accomplished through a shared network, which may cause heavy communication burden. In such situation, the event-triggered mechanism (ETM) has been employed to schedule the data transmission over network [17], [18], [19], [20], [21]. Due to its advantage of reducing communication burden, the ETM has also been widely utilized in the control of MJSs [22], [23], [24]. In [22], the guaranteed cost control was studied for MJSs under ETM scheme. In [23], the ETM based controller was designed such that the closed-loop MJSs are finite-time bounded with certain level of $H_{\infty}$ performance. Based on the event-triggered and self-triggered communication schemes, the $L_{\infty}$ control problem of MJSs under DoS attacks was investigated in [24]. Some recent studies can also be found in [25] and [26] and the references therein. It is worth mentioning that most of these results are concerned with the static ETM. Compared to the static ETM, the dynamic ETM can further reduce the communication burden due to its introduction of an additional dynamic variable [20], [21]. In [27], the dynamic ETM was introduced into the sliding mode control of MJSs. It makes sense to introduce the dynamic ETM into the control of MJSs. However, it has received little attention in the literature. This is another motivation of this work.

In response to the discussion above, it is imperative to introduce the asynchronous control and event-triggered control into MJSs. This work investigates the static output feedback asynchronous control of MJSs based on the dynamic event-triggered scheme. In addition, the external disturbance is taken into account and the $l_2 - l_\infty$ performance is considered in the controller design. This is the first attempt to study this control problem. Compared to the existing results, the main contributions of this work are summarized as follows:

1. Most results on MJSs such as [1], [2], [3], [4], [5], [6], [7], [8], [9], [10] are based on the fact that the controllers can fully access the mode information of the plant and that the controller is synchronous with the plant. While in our work, the asynchronous phenomenon is concerned with.

2. Many studies on the asynchronous control of MJSs such as [12], [13], [14], [15], [16] require that the states are measurable. While in our work, a more general case is considered, in which the states are assumed to be unavailable. Then the static output feedback controller is designed for the MJSs. Moreover, based on the feedback signal, the event-triggering condition is constructed.

3. In the literature on event-triggered asynchronous control of MJSs, such as [22], [23], [24], [25], [26], little attention is paid on the dynamic ETM. We will fill this gap in this study.

The remainder of this study is organized as follows. In Section 2, the dynamic ETM scheme together with some preliminaries is introduced, based on which the asynchronous control problem for MJSs is formulated. Section 3 presents the main results of this work, in which the controller design is derived in terms of a set of linear matrix inequalities (LMIs). A numerical example is provided to verify the theoretical findings in Section 4, and Section 5 summarizes this work.

Notation: Throughout this work, $\| \cdot \|$ denotes the Euclidean vector norm. $l_2([0, \infty); \mathbb{R}^n)$ is the space of square-summable of $n$-dimensional vector functions over $[0, \infty)$. For $w(k) \in l_2([0, \infty); \mathbb{R}^n)$, its norm is defined as $||w(k)||_2 = \sqrt{\sum_{k=0}^{\infty} ||w(k)||^2}$. $l_\infty([0, \infty); \mathbb{R}^n)$ is the space of all essentially bounded $n$-dimensional vector functions over $[0, \infty)$. For $z(k) \in l_\infty([0, \infty); \mathbb{R}^n)$, its norm is given by $||z(k)||_\infty = \max_{k \geq 0} ||z(k)||$. Prob${\{X\}}$, $E\{X\}$ and $E_k\{X\}$ refer to respectively the occurrence probability, the mathematical expectation and the expectation conditional on the information available at time $k$. $\| \cdot \|$ represents a block-diagonal matrix. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote, respectively, the maximal and minimal eigenvalue of a matrix. The symbol * means an ellipsis for terms induced by symmetry.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a discrete-time MJS of the form,

$$x(k + 1) = A(\theta(k))x(k) + B(\theta(k))u(k) + E(\theta(k))w(k),$$

$$y(k) = C(\theta(k))x(k) + D(\theta(k))w(k),$$

$$z(k) = F(\theta(k))x(k),$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^u$, $w(k) \in l_2([0, \infty); \mathbb{R}^w)$, $y(k) \in \mathbb{R}^y$ and $z(k) \in \mathbb{R}^z$ are the state, the input, the deterministic disturbance, the measured output and the controlled output respectively. The parameter $\theta(k) \in \mathcal{N} \triangleq \{1, 2, \ldots, N\}$ is a Markov chain subject to transition probability matrix $\Pi_\theta = [\pi_{is}]$, where

$$\pi_{is} \triangleq \text{Prob}\{\theta(k + 1) = s | \theta(k) = i\}, \quad \forall i, s \in \mathcal{N},$$

with $\pi_{is} \geq 0$ and $\sum_{i=1}^{N} \pi_{is} = 1$. $A(\theta(k))$, $B(\theta(k))$, $E(\theta(k))$, $C(\theta(k))$, $D(\theta(k))$ and $F(\theta(k))$ are known matrices with appropriate dimensions. In this work, all the system state values are supposed to be unavailable. The output $y(k)$ is used for the feedback design.

![System structure diagram.](image)

The considered MJS in this work is mainly composed of a physical plant, a controller and an actuator with zero-order hold (ZOH), etc., as shown in Fig.1. All are connected...
through shared communication networks. In order to save
the communication resource, a dynamic ETM is adopted
into the controller-to-actuator (C/A) channel to stochastically
stabilize the system. The controller receives the output signals
together with the mode information and calculates the control
inputs at every instant. Based on the control signals a dynamic
ETM is constructed and used to determine when the events
are triggered. Only when an event is triggered, the control
signals are transmitted to the actuator over the C/A channel.
Otherwise, the communication task is skipped.

In this work, the mode-dependent static output feedback
(SOF) controller is exploited,
\[ \hat{u}(k) = K(\sigma(k))y(k), \]  
where \( y(k) \) is the output sampled at instant \( k \), \( K(\sigma(k)) \) is
a mode-dependent gain to be determined. \( \sigma(k) \in \mathcal{M} \triangleq \{ 1, 2, \ldots, M \} \) is another
Markov chain and satisfies the given conditional probability matrix \( \Pi_\sigma = [\tau_{ij}] \) with
\[ \tau_{ij} \triangleq \text{Prob}(\sigma(k) = j|\theta(k) = i), \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{M}, \]
where \( \tau_{ij} \geq 0 \) and \( \sum_{j=1}^{M} \tau_{ij} = 1 \).

Remark 1: Note that there are two Markov chains, i.e.,
\( \theta(k) \) and \( \sigma(k) \). \( \theta(k) \) characterizes the mode switching in the
original plant (1), while \( \sigma(k) \) represents the received mode
information of (1) by the controller. In order to determine
a mode-dependent SOF controller, the mode information
should be detected by the controller. However, it is unreal-
estic to precisely measure the variable \( \theta(k) \) due to unreliable
networks. Inspired by [12], [13], [14], [15], and [16], another
variable \( \sigma(k) \) is used to provide the estimated values of \( \theta(k) \)
with some probability. In this case, the SOF controller (3)
is asynchronous with the original plant (1). It is worth men-
tioning that when \( M = N \) and \( \Pi_\sigma = I \), the SOF controller
(3) achieves perfect synchronous with the original plant (1).
In other words, the asynchronization disappears. Particularly,
the SOF controller (3) reduces to the mode-independent one
when \( M = \{ 1 \} \).

Let \( \{ k_t | t \in \mathbb{N} \} \) represents the triggering sequence, where \( k_t \)
means the \( t \)-th triggered instant. Define an error signal \( e(k) \) as below,
\[ e(k) \overset{\Delta}{=} \hat{u}(k_t) - \hat{u}(k), \quad k \in \{ k_t, \ldots, k_{t+1} - 1 \}. \]  
(5) Based on the error \( e(k) \), the feedback signal \( \hat{u}(k) \) and the mode
estimation \( \sigma(k) \), a dynamic ETM is adopted to determine the
triggering sequence \( \{ k_t | t \in \mathbb{N} \} \), which is given as follows,
\[ k_0 = 0, \quad k_{t+1} = \inf_{k_t \in \mathbb{N}} \{ k > k_t : \epsilon_1 \ell(k) > \eta(k) \}, \quad t \in \mathbb{N}, \]
(6a)
where \( \ell(k) = \| e(k) \|_\Phi^2 - \delta \| \hat{u}(k) \|_\Phi^2, \Phi \in \mathbb{R}^{n_u \times n_u} \) is a positive-
definite matrix to be designed later. \( \eta(k) \) is an internal auxiliary
whose dynamic characteristics are described by,
\[ \eta(k+1) = \epsilon_2 \eta(k) - \ell(k), \quad \eta(0) \geq 0, \]
(6b)
\( \delta, \epsilon_1 \) and \( \epsilon_2 \) are constant scalars and satisfy,
\[ \delta \in (0, 1), \quad \epsilon_1 \geq 1/\epsilon_2, \quad \epsilon_2 \in (0, 1). \]  
(6c)

Now, we are in a position to investigate an important
feature of the auxiliary variable \( \eta(k) \). Based on the event-
triggered control theorem [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], the dynamic ETM in (6a) guar-
tees that \( \epsilon_1 \ell(k) \leq \eta(k) \), \( \forall k \in \mathbb{N} \). This together with (6b)
results in \( \eta(k+1) \geq \epsilon_2 \eta(k) - 1/\epsilon_1 \eta(k) \), which further leads to
\[ \eta(k+1) \geq (\epsilon_2 - 1/\epsilon_1) \eta(k). \]
From \( \eta(0) \geq 0 \) and \( \epsilon_2 \geq 1/\epsilon_1 \), it follows that,
\[ \eta(k) \geq 0, \quad \forall k \in \mathbb{N}. \]  
(7)
It is worth mentioning that (7) is an important relationship
used to derive the main results of this work.

Remark 2: In this study, the ETM is only applied in the
C/A network. It is worth mentioning that the control tech-
nique can also be extended to the case where the ETM is used
in S/C channel, or even in both S/C and C/A channels.

Remark 3: In (5)-(6), \( e(k) \) denotes the difference between the
current control signal \( \hat{u}(k) \) and the one \( \hat{u}(k_t) \) computed
at the last triggering instant \( k_t \). Once the triggering condition
in (6a) is satisfied a new event is triggered. Then the control
signal is transmitted to the actuator over the C/A network.
Otherwise, the transmission task is skipped and the actuator
continues to implement the control signal received at the last
triggered instant.

Remark 4: In most of the existing results on ETM-based
control of MJSs, it is assumed that the states can be mea-
surable [22], [23], [24], [25], [26], [27]. Then the event-
triggering condition is constructed based on the system states.
It is not practical due to the unavailability of the states in
many cases. In this study, a more general case is considered,
in which the event-triggering condition is constructed based
on the output signal. Then the design of SOF is investigated.

Remark 5: Note that the triggering condition (6a) depends
not only on the current feedback signals \( \hat{u}(k) \) and the one
obtained at the last triggered instant but also on an internal
dynamic state \( \hat{u}(k) \), which is called dynamic ETM in the
literature [20], [21]. When \( \epsilon_1 \rightarrow \infty \), the triggering condition
(6a) reduces to
\[ k_{t+1} = \inf_{k \in \mathbb{N}} \{ k > k_t : \ell(k) > 0 \}, \]
(8)
which is called static ETM [17], [18]. Compared with static
ETM, dynamic ETM has a lower triggering rate and thus can
further save the network resources [20], [21].

Under dynamic ETM, only the feedback signal \( \hat{u}(k_t) \) com-
puted at triggered instant is available for the actuator. \( \hat{u}(k_t) \)
will be held until next event happens at instant \( k_{t+1} \). To keep
the control input signal continuous, a ZOH is embedded at
the actuator side. The real control signal implemented to the
plant is
\[ u(k) = \hat{u}(k_t), \quad k \in \{ k_t, \ldots, k_{t+1} - 1 \}, \quad t \in \mathbb{N}. \]  
(9a)
Based on (5), the controller (3) can be rewritten as
\[ u(k) = K(\sigma(k))(C(\theta(k))x(k) + D(\theta(k))w(k)) + e(k). \]  
(9b)
The closed-loop system is then obtained,

\[
x(k + 1) = (A(\theta(k)) + B(\theta(k))K(\sigma(k)))x(k) + B(\theta(k))w(k) + (E(\theta(k)) + B(\theta(k)) \times K(\sigma(k))D(\theta(k)))w(k),
\]

\[
z(k) = F(\theta(k))x(k).
\]

(10)

In the following, the matrices with argument \(\theta(k) = i\) are denoted by their subscript for simplicity. For instance, \(A(\theta(k) = i) = A_i, K(\sigma(k) = j) = K_j\), etc.

In this work, the aim is to determine the mode-dependent feedback gain \(K(\sigma(k))\) and the event-triggering matrix \(\Phi\) simultaneously by only using the estimated values of the mode information \(\theta(k)\) such that the closed-loop system (10) is stochastic stable.

**Definition 1 (Mean-Square Stability, MSS, [11]):** For any initial state \(x(0) = x_0\) and initial mode \(\theta(0) = \theta_0\), the closed-loop system (10) is said to be MSS if

\[
\mathbb{E}\left\{\|x(k)\|^2_2 | x_0, \theta_0\right\} \to 0, \quad \text{as } k \to \infty.
\]

(11)

**Definition 2 (MSS With an \(l_2 - l_\infty\) Performance \(\gamma\)):** Given a scalar \(\gamma > 0\), system (10) is said to be MSS with an \(l_2 - l_\infty\) performance \(\gamma\), if it is MSS with \(w(k) = 0\), and it holds that

\[
\mathbb{E}\left\{\|z(k)\|^2_\infty | x_0, \theta_0\right\} < \gamma^2 \|w(k)\|^2_2 + \zeta(x_0, \eta_0),
\]

(12)

for any \(w(k) \in l_2(0, \infty); \mathbb{R}^n\) and some nonnegative definite function \(\zeta(\cdot)\) satisfying \(\zeta(0, 0) = 0\).

**III. MAIN RESULTS**

In this section, we first analyze the MSS with a certain level of \(l_2 - l_\infty\) performance of the closed-loop system by utilizing a mode-dependent Lyapunov function, which is summarized as the following theorem.

**Theorem 1:** Consider the MJS (1). Given a scalar \(\gamma > 0\), if there exist positive definite matrices \(P_1 \in \mathbb{R}^{n_x \times n_x}, L_{ij} \in \mathbb{R}^{n_y \times n_y}, \Phi \in \mathbb{R}^{n_u \times n_u}\) satisfying the following conditions,

\[
\begin{bmatrix}
\Xi_1 & * & * \\
\alpha \otimes \Xi_2 & -\Xi_3^{-1} & * \\
\Xi_4 & 0 & -\Phi^{-1}
\end{bmatrix} < 0,
\]

(13)

\[
\sum_{j=1}^{M} \tau_{ij}L_{ij} < P_1, \quad (14)
\]

\[
\begin{bmatrix}
-L_{ij} & * & * \\
0 & -\Phi & * \\
0 & 0 & -I
\end{bmatrix} < 0,
\]

(15)

for all \(i \in \mathcal{N}, j \in \mathcal{M}\), where

\[
\Xi_1 = \begin{bmatrix}
-L_{ij} & * & * \\
0 & -\Phi & * \\
0 & 0 & -I
\end{bmatrix},
\]

\[
\Xi_2 = \begin{bmatrix}
A_i + B_iK Ci & B_i & E_i + B_iK Di
\end{bmatrix},
\]

\[
\Xi_3 = \text{diag}(P_1, P_2, \ldots, P_N),
\]

\[
\Xi_4 = \begin{bmatrix}
\delta^{1/2}K Ci & 0 \times \times \delta^{1/2}K Di
\end{bmatrix},
\]

\[
\alpha = \begin{bmatrix}
\sqrt{\tau_{1i}} & \ldots & \sqrt{\tau_{Ni}}
\end{bmatrix}^T,
\]

(16)

then the closed-loop system (10) is MSS with an \(l_2 - l_\infty\) performance \(\gamma\).

**Proof:** Define a mode-dependent Lyapunov function as follows

\[
V(k) \triangleq V_1(k) + \eta(k),
\]

\[
V_1(k) \triangleq x^T(k)P(\theta(k))x(k),
\]

(17)

where \(P(\theta(k)) \in \mathbb{R}^{n_u \times n_u}\) is a positive matrix to be determined. For \(\theta(k) = i\), taking the conditional expectation of the difference of \(V(k)\) along the trajectories of closed-loop system (10), one has

\[
\mathbb{E}\{\Delta V(k) | x(k), \theta(k) = i\}
\]

\[
= \mathbb{E}_k\{V_1(k + 1)\} + \mathbb{E}_k\{\eta(k + 1)\} - x^T(k)P_i x(k) - \eta(k)
\]

Define \(\xi(\cdot) = \begin{bmatrix} x^T(\cdot) & e^T(\cdot) & w^T(\cdot) \end{bmatrix}^T\). One can obtain the relations as follows,

\[
\mathbb{E}_k\{V_1(k + 1)\} = \sum_{j=1}^{M} \sum_{s=1}^{N} \tau_{ij}\xi_j^T(k)\Xi_2^T P_s \Xi_2 \xi(k).
\]

(18)

Note that \(\varepsilon_2 < 1\) and \(\eta(k) \geq 0\). It leads to \((\varepsilon_2 - 1)\eta(k) \leq 0\) and

\[
\mathbb{E}_k\{\eta(k + 1)\} - \eta(k)
\]

\[
= \mathbb{E}_k\{\varepsilon \eta(k) - \ell(k)\} - \eta(k)
\]

\[
= \varepsilon \eta(k) - \ell(k) + \delta \mathbb{E}_k\{K(\sigma(k))\eta(k)\} - \eta(k)
\]

\[
\leq -e^T(k)\Phi\eta(k) + \delta \sum_{j=1}^{M} \tau_{ij}(C_i x(k) + D_i w(k))^T \times K_j^T \Phi K_j (C_i x(k) + D_i w(k))
\]

\[
= -e^T(k)\Phi\eta(k) + \sum_{j=1}^{M} \tau_{ij}\xi_j^T(k)\Xi_2^T(\Phi \Xi_4 \xi(k)).
\]

(19)

Combining (18) and (19), one has

\[
\mathbb{E}_k\{\Delta V(k)\} - w^T(k)w(k)
\]

\[
\leq \sum_{j=1}^{M} \sum_{s=1}^{N} \tau_{ij}\pi_{is}\xi_j^T(k)\Xi_2^T P_s \Xi_2 \xi(k) - x^T(k)P_i x(k)
\]

\[
- e^T(k)\Phi\eta(k) + \sum_{j=1}^{M} \tau_{ij}\xi_j^T(k)\Xi_2^T(\Phi \Xi_4 \xi(k))
\]

\[
- w^T(k)w(k)
\]

\[
= \sum_{j=1}^{M} \sum_{s=1}^{N} \tau_{ij}\pi_{is}\xi_j^T(k)\Xi_2^T P_s \Xi_2 \xi(k)
\]

\[
+ \sum_{j=1}^{M} \tau_{ij}\xi_j^T(k)\Xi_2^T(\Phi \Xi_4 \xi(k)) + \xi^T(k)\Omega \xi(k),
\]

(20)
where the matrices $\Xi_2$, $\Xi_4$ are given in (16), and
$$
\Omega_1 = \begin{bmatrix}
-P_i & 0 & 0 \\
0 & -\Phi & 0 \\
0 & 0 & -I
\end{bmatrix}.
$$

Note that
$$
\sum_{s=1}^N \pi_{ss} P_s = (\alpha \otimes I_n)^T \Xi_3 (\alpha \otimes I_n).
$$

(21)

From (14), it follows $\Omega_1 < \sum_{j=1}^M t_j \Xi_1$. This together with (20) results in
$$
\mathbb{E}_k \{\Delta V(k)\} - w^T(k) w(k)
\leq \sum_{j=1}^M t_j \xi^T(k) (\Xi_2^T (\alpha \otimes I_n) ^T \Xi_3 (\alpha \otimes I_n) \Xi_2
$$
$$
+ \Xi_4 \Phi \Xi_4 + \Xi_1) \xi(k).
$$

(22)

Applying the Schur complement to (13), it gives
$$
\Gamma = \Xi_2^T (\alpha \otimes I_n)^T \Xi_3 (\alpha \otimes I_n) \Xi_2 + \Xi_4 \Phi \Xi_4 + \Xi_1 < 0.
$$

(23)

Let $\lambda_1 = \max_{i \in \mathcal{N},j \in \mathcal{M}} \{\lambda_{\max}(\Gamma)\}$. Clearly, $\lambda_1 < 0$. From (22), it follows that
$$
\mathbb{E}_k \{\Delta V(k)\} - w^T(k) w(k) \leq \lambda_1 \|\xi(k)\|^2.
$$

Taking the expected values conditional on the information available at time $0$ further results in
$$
\mathbb{E}_0 \{V(k+1)\} - \mathbb{E}_0 \{V(k)\} - w^T(k) w(k) \leq \lambda_1 \|\xi(k)\|^2.
$$

(25)

In the absence of the external disturbance, i.e., $w(k) = 0$, one has that $\|\xi(k)\| = \|\hat{\xi}(k)\|$ with $\hat{\xi}(k) = [x^T(k) \; \epsilon^T(k)]^T$. Then, (25) can be rearranged as
$$
\mathbb{E}_0 \{V(k+1)\} - \mathbb{E}_0 \{V(k)\} \leq \lambda_1 \|\hat{\xi}(k)\|^2.
$$

(26)

Let $\epsilon > 0$. From (26), it follows that
$$
\frac{1}{\epsilon^{k+1}} \mathbb{E}_0 \{V(k+1)\} - \frac{1}{\epsilon^k} \mathbb{E}_0 \{V(k)\}
$$
$$
= \frac{1}{\epsilon^{k+1}} \left(\mathbb{E}_0 \{V(k+1)\} - \mathbb{E}_0 \{V(k)\}\right)
$$
$$
+ \frac{1}{\epsilon^k} \left(\frac{1}{\epsilon} - 1\right) \mathbb{E}_0 \{V(k)\}
$$
$$
\leq \frac{\lambda_1}{\epsilon^{k+1}} \|\hat{\xi}(k)\|^2 + \frac{1}{\epsilon^k} \left(\frac{1}{\epsilon} - 1\right) \mathbb{E}_0 \{V(k)\}.
$$

(27)

Note that when $\epsilon = 1$, one has that
$$
\frac{\lambda_1}{\epsilon^{k+1}} \|\hat{\xi}(k)\|^2 + \frac{1}{\epsilon^k} \left(\frac{1}{\epsilon} - 1\right) \mathbb{E}_0 \{V(k)\} = \lambda_1 \|\hat{\xi}(k)\|^2 < 0.
$$

(28)

From the continuity, it follows that there exists a constant $\epsilon_0 \in (0, 1)$ such that
$$
\frac{\lambda_1}{\epsilon_0^{k+1}} \|\hat{\xi}(k)\|^2 + \frac{1}{\epsilon_0^k} \left(\frac{1}{\epsilon_0} - 1\right) \mathbb{E}_0 \{V(k)\} < 0.
$$

(29)

Substituting (29) into (27) yields,
$$
\frac{1}{\epsilon_0^{k+1}} \mathbb{E}_0 \{V(k+1)\} - \frac{1}{\epsilon_0^k} \mathbb{E}_0 \{V(k)\} < 0.
$$

(30)

Then, one has
$$
\frac{1}{\epsilon_0^k} \mathbb{E}_0 \{V(k)\} < \mathbb{E}_0 \{V(0)\} = x^T(0) P(\theta_0) x(0) + \eta(0)
$$
$$
\leq \lambda_{\max}(P(\theta_0)) \|x(0)\|^2 + \eta(0).
$$

(31)

When $\theta(k) = i \in \mathcal{N}$, it follows from (17) that
$$
\lambda_{\min}(P_i) \left\{\|x(i)\|^2\right\} + \eta(k) < V(k),
$$

which further leads to,
$$
\lambda_2 \mathbb{E}_0 \left\{\|x(k)\|^2\right\} + \eta(k) < \mathbb{E}_0 \{V(k)\},
$$

(32)

where $\lambda_2 = \max_{i \in \mathcal{N}} \{\lambda_{\min}(P_i)\}$. Note that $\eta(k) \geq 0$. This together with (31) further implies that
$$
\mathbb{E}_0 \{\|x(k)\|^2\} \leq \epsilon_0^k \left(\frac{1}{\lambda_2} \left(\lambda_{\max}(P(\theta_0)) \|x(0)\|^2 + \eta(0)\right)\right).
$$

(33)

By Definition 1, the closed-loop system (10) is MSS.
On the other hand, from (25) it infers to
$$
\mathbb{E}_0 \{V(k+1)\} - \mathbb{E}_0 \{V(k)\} \leq w^T(k) w(k).
$$

Then we have
$$
\mathbb{E}_0 \{V(k)\} \leq \mathbb{E}_0 \{V(0)\} + \sum_{v=0}^{v=k-1} w^T(v) w(v).
$$

(34)

From (15), it immediately leads to $\|z(k)\|^2 < \gamma^2 V(k)$. Then, taking the conditional expectation on both sides of the inequality one has
$$
\mathbb{E}_0 \{z^T(k) z(k)\} < \gamma^2 \mathbb{E}_0 \{V(k)\}.
$$

(35)

This together with (31) and (34) leads to
$$
\mathbb{E}_0 \left\{\|z(k)\|^2\right\} < \gamma^2 \mathbb{E}_0 \{V(k)\}
$$
$$
\leq \gamma^2 \lambda_{\max}(P(\theta_0)) \left(\|x_0\|^2 + \eta_0\right) + \gamma^2 \sum_{v=0}^{v=k-1} \|w(v)\|^2.
$$

(36)

Taking the supremum over $k > 0$ yields (12) with $\zeta(x_0, \eta_0) = \gamma^2 \lambda_{\max}(P(\theta_0)) \left(\|x_0\|^2 + \eta_0\)$. This completes the proof. ■

In Theorem 1, a set of conditions is obtained to ensure the MSS with an $l_2 - l_{\infty}$ performance $\gamma$ for the closed-loop system. However, it is difficult to directly solve the inequalities in Theorem 1 due to the coupling terms in (13)-(15). By taking into account the solvability, a set of sufficient conditions for (13)-(15) is derived in the below.

**Theorem 2:** Consider the MJSS (1). Given a scalar $\gamma > 0$ and ETM constants $\delta_1, \epsilon_1, \epsilon_2$ satisfying (6c), if there exist positive definite matrices $X_i \in \mathbb{R}^{n_i \times n_i}, H_{ij} \in \mathbb{R}^{n_i \times n_j}, \forall i, j \in \mathcal{N}$,
$Z \in \mathbb{R}^{n_x \times n_x}$, matrices $G_i \in \mathbb{R}^{n_x \times n_x}$, $U_j \in \mathbb{R}^{n_x \times n_x}$, $V_j \in \mathbb{R}^{n_y \times n_y}$ such that the following LMIs are feasible,

$$
\begin{bmatrix}
\Gamma_1 & \ast & \ast & \ast \\
\ast & \Gamma_2 & \ast & \ast \\
\ast & \ast & \Gamma_3 & \ast \\
\ast & \ast & \ast & \ast \\
\end{bmatrix} < 0,
$$

(37)

$$
\begin{bmatrix}
-X_i \\
\beta \otimes X_i \\
F_iX_i \\
\end{bmatrix} < 0,
$$

(38)

$$
\begin{bmatrix}
-\Psi_{ij} + B_iU_jV_j^{-1}\Psi_{2ij} \\
E_i + B_iK_iD_i \\
\delta^{1/2}K_iC_i \\
\end{bmatrix} < 0,
$$

(39)

where

$$
\Psi_{ij} = A_iG_i + B_iU_jC_i, \\
\Psi_{2ij} = C_iG_i - V_jC_i.
$$

Then (42) can be rearranged as

$$
\Omega_2 + \Lambda_7 V_j^{-1}\Lambda_8 + (\Lambda_7 V_j^{-1}\Lambda_8)^T < 0,
$$

(44)

where

$$
\begin{bmatrix}
\Lambda_4 & \ast & \ast \\
\ast & \Lambda_5 & \ast \\
\ast & \ast & \Lambda_6 \\
\end{bmatrix} < 0.
$$

(45)

To see this, pre- and post-multiplying $I (V_j^{-1}\Lambda_8)^T$ and its transpose on both sides of (45), the condition in (44) is then obtained. (45) can be rewritten as

$$
\begin{bmatrix}
\Lambda_4 & \ast & \ast & \ast \\
\ast & \Lambda_5 & \ast & \ast \\
\ast & \ast & \Lambda_6 & \ast \\
\ast & \ast & \ast & \ast \\
\end{bmatrix} < 0.
$$

Introducing the variable definition

$$
L_j^{-1} = H_j, \quad P_i^{-1} = X_i, \quad \Phi^{-1} = Z,
$$

produces the LMIs (37). Note that

$$
\sum_{j=1}^{M} v_{ij}L_{ij} = (\beta \otimes I_{n_c})^T \Omega_3 (\beta \otimes I_{n_c}),
$$

(47)

where $\Omega_3 = \text{diag}[L_{11}, L_{02}, \ldots, L_{MM}]$. Substituting (47) into (14), and applying the Schur complement, it results in

$$
\begin{bmatrix}
P_i & \ast \\
\ast & \ast \\
\end{bmatrix} < 0.
$$

(48)
Pre- and post-multiplying $\text{diag}\{P_1^{-1}, I, \ldots, I\}$ and its transpose on both sides of (48), and then introducing the variable definition in (46) leads to the LMIs in (38).

Pre- and post-multiplying $\text{diag}\{P_1^{-1}, I\}$ and its transpose on both sides of (15), it immediately results in the LMIs in (39).

The proof is completed.

Remark 6: In Theorem 1, the disturbance attenuation level $\gamma$ is a prescribed given scalar. In many practical applications, it is necessary to obtain the optimal $\gamma$. Replacing the term $\gamma^2$ in (39) by $\tilde{\gamma}$, the obtained inequalities are linear with respect to $\tilde{\gamma}$. Then finding the optimal disturbance attenuation level is transformed to the following optimization problem,

$$
\min_{x_h, h_j, z_t, u_t} \tilde{\gamma} \\
\text{s.t. (37) – (39)}.
$$

By solving the feasibility problem in Theorem 2 or the optimization problem in (49), a set of mode-dependent SOF controller together with the event-triggering matrix can be obtained ensuring the MSS with an $l_2 - l_\infty$ performance $\gamma$ of the closed-loop system. The whole design procedure is specified in Algorithm 1 as follows.

Algorithm 1: Asynchronous SOF Control Design

1. Initialization: Set the initial mode $\theta(0)$. Choose the ETM constants $\delta, \epsilon_1, \epsilon_2$ satisfying (6c). Solve the feasibility problem in Theorem 2 or the optimization problem in (49) to obtain the feedback gains $K_j, \forall j \in \mathcal{M}$, and event-triggering matrix $F$.
2. At time $t \geq 0$, sample the outputs $y(t)$, obtain the mode estimation $\hat{\sigma}(k)$ and determine the triggering instant $\hat{t}_k$.
3. If $k = 0$ or an event is triggered, go to step 3; otherwise go to step 4.
4. Calculate $u(k)$ by (3) and broadcast it to the actuator.
5. Go to step 2 at time $k + 1$.

IV. NUMERICAL EXAMPLE

In order to show the effectiveness and practicability of the proposed static output feedback $l_2 - l_\infty$ asynchronous control technique, a numerical example and a DC-DC switched boost converter circuit are presented in this section.

Example 1 (Numerical example): Consider a 2-mode MJS of the form (1) with the following parameters,

$A_1 = \begin{bmatrix} -0.05 & -0.55 \\ 0.05 & 0.85 \end{bmatrix}$, \quad $A_2 = \begin{bmatrix} -0.65 & -1.0 \\ -0.45 & -0.15 \end{bmatrix}$,

$B_1 = \begin{bmatrix} 1.0 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$, \quad $B_2 = \begin{bmatrix} 0.5 & 0.8 \\ 0.2 & 0.1 \end{bmatrix}$, \quad $E_1 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}$,

$E_2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, \quad $C_1 = \begin{bmatrix} 0.6 & 0.2 \end{bmatrix}$, \quad $C_2 = \begin{bmatrix} 0.8 & 0.6 \end{bmatrix}$,

$D_1 = D_2 = 0.2$, \quad $F_1 = \begin{bmatrix} 0.6 & 0.1 \end{bmatrix}$, \quad $F_2 = \begin{bmatrix} 0.9 & 0.6 \end{bmatrix}$.

The disturbance $w(k)$ is supposed to be $w(k) = 0.5 e^{-0.15k} \sin(0.5k)$. The transition probability matrices $\Pi_\theta$ and $\Pi_\sigma$ are respectively given as

$$
\Pi_\theta = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}, \quad \Pi_\sigma = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}.
$$

Let the scalars in (6c) be $\delta = 0.5, \epsilon_1 = 2.5$ and $\epsilon_2 = 0.5$. Select the $l_2 - l_\infty$ performance $\gamma = 5$. The initial conditions for (1) and (6b) are chosen as $x(0) = [5 - 5]^T$, $\theta(0) = 1$ and $\eta(0) = 5$.

Let us first show the stability of the closed-loop MJS. Calculating the LMIs in Theorem 2, the feedback gains $K_j, j = 1, 2$, and the ETM parameter $\Phi$ are obtained respectively as below,

$$
K_1 = \begin{bmatrix} 0.1503 \\ 0.5665 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.1368 \\ 0.6256 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 0.0302 & 0.0246 \\ 0.0246 & 0.0398 \end{bmatrix}.
$$

By 1000 repeated simulations using the same initial conditions given above, the state trajectories (in gray lines) and their mean values (in blue lines) are plotted in Fig.1. The corresponding control inputs (in gray lines) and their mean values (in blue lines) are shown in Fig.2. It is clear that the state trajectories converge to zero. In one of these 1000 simulations, the evolution of system mode $\theta(k)$ and controller mode $\sigma(k)$ are given in Fig.4. It can be seen that the controller mode is asynchronous with the system mode. From these figures, it is shown that the stability is guaranteed under the asynchronous switching.

Next, we will show the transmission rate measured by

$$
T = \frac{\text{Number of triggered instants}}{\text{System runtime}}.
$$

By 1000 calculations, the mean value of transmission rate is $T = 0.4876$. This means that only 48.76% packets are released and transmitted over the C/A channel. In other words, 51.24% network bandwidth is saved during the simulation. For comparison, the mean value of $T_\text{r}$ of the static ETM (8) is obtained as $T_\text{r} = 0.5376$. One control trajectory of these 1000 simulations and the corresponding triggered instances are shown in Fig.5, where 1 means that an event is triggered and 0 otherwise. From these results, it is clear that the event-triggered communication scheme can really reduce the frequency of data transmission. Compared to the static ETM, the dynamic ETM can further reduce the communication burden.

At last, let us show the level of $l_2 - l_\infty$ performance. Without of generality, the initial conditions are reset as $x(0) = [0 0]^T$ and $\eta(0) = 0$. By 1000 calculations the ration of $\|z(k)\|_{l_\infty}/\|w(k)\|_{l_2}$ (in gray lines) and their mean values (in blue lines) are given in Fig.6, from which it is clear that the ration is always less than 5. This implies that the predefined level of $l_2 - l_\infty$ performance $\gamma = 5$ is guaranteed. From the aforementioned simulation results, the effectiveness of the control technique presented in this work has been confirmed.

Example 2: (DC-DC switched boost converter circuit) Consider a DC-DC switched boost converter circuit [7]. As shown in Fig.7, the DC-DC switched boost converter circuit is composed of a power source $V_{in}(t)$, an inductor...
Define the system state \( x(t) = [i_L(t) V_c(t)]^T \) and the control input \( u(t) = V_{in}(t) \), then the dynamic behavior of the DC-DC switched boost converter circuit is given by,

**Model 1:**
\[
\dot{x}(t) = \begin{bmatrix}
\frac{R_1}{L} & -\frac{1}{L} \\
\frac{1}{C} & -\frac{1}{R_2 C}
\end{bmatrix} x(t) + \begin{bmatrix}
\frac{1}{L} \\
0
\end{bmatrix} u(t).
\]

**Model 2:**
\[
\dot{x}(t) = \begin{bmatrix}
-\frac{R_1}{L} & 0 \\
0 & -\frac{1}{R_2 C}
\end{bmatrix} x(t) + \begin{bmatrix}
\frac{1}{L} \\
0
\end{bmatrix} u(t).
\]
The constants are chosen as $R_1 = 5\Omega, R_2 = 10\Omega, L = 1H, \ C = 1F$. Considering the influence of disturbance $w(t)$, the model of the circuit is obtained as [7],

**Model 1:**

$$\dot{x}(t) = \begin{bmatrix} 5 & -1 \\ 1 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} -0.2 \\ -0.1 \end{bmatrix} w(t),$$

**Model 2:**

$$\dot{x}(t) = \begin{bmatrix} -5 & 0 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} -0.2 \\ -0.1 \end{bmatrix} w(t).$$

By setting the sampling time $T_s = 0.01$, the discrete-time model of the circuit with the form (1) is obtained, whose parameters are given as,

$$A_1 = \begin{bmatrix} 1.0512 & -0.0102 \\ 0.0102 & 0.9989 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0103 \\ 0.0001 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.9512 & 0 \\ 0 & 0.9990 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0098 \\ 0 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} -0.0020 \\ -0.0010 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.0020 \\ -0.0010 \end{bmatrix}.$$

Other system matrices are chosen as,

$$C_1 = \begin{bmatrix} 0.6 & -0.2 \\ 0.5 & 0.5 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.02 \\ 0.02 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 1 & 1 \end{bmatrix},$$

$$C_2 = C_1, \quad D_2 = D_1, \quad F_2 = F_1.$$

Set the initial condition $x(0)$ as $x(0) = [25 \ 25]^T$. The other parameters $\delta, \varepsilon_1, \varepsilon_2, \gamma, \theta(0), \eta(0)$, disturbance signal $w(k)$, transition probability matrices $\Pi_\theta$ and $\Pi_\sigma$, are chosen the same as those in Example 1. By applying Theorem 2, the feedback gains and the ETM parameter are computed respectively as follows,

$$K_1 = \begin{bmatrix} -21.5617 & -72.9727 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -19.9282 & -73.3891 \end{bmatrix},$$

$$\Phi = 1.5795 \times 10^{-5}.$$

By 1000 realizations, the state trajectories (in gray lines) and their mean values (in blue lines) are shown in Fig.9. In one of these 1000 realizations, the evolution of system mode $\theta(k)$ and controller mode $\sigma(k)$ are given in Fig.10. It can be seen that the controller mode is asynchronous with the system mode. From these figures, it is clear that the stability is ensured by the obtained asynchronous static output feedback controller.

By 1000 calculations, the mean value of transmission rate is $T_r = 0.2282$. One control trajectory of these
1000 simulations and the corresponding triggered instants are shown in Fig.11, where 1 means that an event is triggered and 0 otherwise. From these results, it is clear that the circuit works well with reduced communication burden. From these simulation results, the practical applications of the proposed control technique has been verified.

V. CONCLUSION
This study has investigated the control problem of MJSs. The dynamic ETM has been introduced to regulate the data transmission with the aim of reducing the communication burden. The asynchronous switching between the plant modes and the controller modes has been considered in the controller design. It has been shown that the MJSs can be stochastically stabilized with a certain level of \( \mathcal{I}_2 - \mathcal{I}_\infty \) performance. By employing a mode-dependent Lyapunov function, a set of LMI conditions has been established for the controller design. Finally, a numerical and a DC-DC switched boost converter circuit have been provided as examples to show the effectiveness and the practicability of the proposed control technique.

It is possible to extend the obtained control technique in this work to the case in which some other network phenomena are involved, such as packet dropout, dynamic quantization and cyber attacks. This will be one of our research directions in the future.

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