Analytic solutions to the Maxwell–London equations and levitation force for a superconducting sphere in a quadrupole field

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Abstract

Recent proposals suggest using magnetically trapped superconducting spheres in the Meissner state to create low-loss mechanical oscillators with long coherence times. In these proposals the derivation of the force on the superconducting sphere and the coupling to the sphere typically relies on a vanishing penetration depth $\lambda$ as well as a specific symmetry (i.e. restricting the position of the sphere to one axis) or heuristic methods (e.g. assigning an equivalent point magnetic dipole moment to the sphere). In this paper we analytically solve the Maxwell–London equations with appropriate boundary conditions for a superconducting sphere in a quadrupole field. The analytic solutions provide the full field distribution for arbitrary $\lambda$ and for an arbitrary sphere position as well as the distribution of shielding currents within the sphere. We furthermore calculate the force acting on the sphere and the maximum field over the volume of the sphere. We show that for a certain range of $\lambda$ the maximum field experienced by the superconducting sphere is actually lower than it is for a non-magnetic sphere.

Keywords: magnetic traps, magnetic levitation, quadrupole field

(Some figures may appear in colour only in the online journal)

1. Introduction

The last decade has seen significant progress in achieving quantum control over solid state mechanical devices [1]. One key idea is to exploit the available toolbox of quantum optics in both the optical and microwave domain by coupling mechanical motion to optical cavities or superconducting circuits [2, 3]. Recent examples include the generation of non-classical states of motion [4–6] and even of quantum entanglement involving micro-mechanical systems [7–10]. Several proposals have suggested that quasi-static magnetic levitation of superconductors in the Meissner state allows to further increase both system size and coherence time in such experiments, thereby not only improving the system performance but also enabling access to a completely new parameter regime of macroscopic quantum physics [11–13]. The requirements on the magnetic traps are similar to those of atom traps [14], as in both cases a minimum in the magnetic field norm is necessary for levitation. Several trap configurations, such as the Anti-Helmholtz setup suggested in [11], produce a (local) quadrupole field, i.e. a magnetic field of the form $\frac{1}{2} \mathbf{b} \times \mathbf{r}$, where $\mathbf{b}$ denotes the magnetic gradient along the $z$-axis. Coupling to the motion of the sphere is facilitated by placing a pickup loop in the proximity of the sphere. The flux through the pickup loop then depends on the position of the sphere. A detailed knowledge of the magnetic field distribution for arbitrary sphere positions is essential for a good understanding of both the trap dynamics and the...
coupling strength. However, a full analysis of the magnetic field distribution of such a configuration has not been carried out yet. The proposals referenced above provide results only for vanishing penetration depth ($\lambda = 0$) and rely either on symmetry features, where the sphere is restricted to the $z$-axis or use heuristic methods such as approximating the sphere as a point dipole. The aim of this paper is to avoid these restrictions and provide analytic expressions for the magnetic field for arbitrary $\lambda$ and arbitrary sphere positions.

In section 2 we derive the magnetic field and the super-current on the surface of the sphere for the special case $\lambda = 0$ by solving the Maxwell equation with the appropriate boundary condition; in section 3 we generalize these results for arbitrary $\lambda$ by solving the Maxwell–London equations with the appropriate boundary condition.

In section 4 we derive the force acting on the sphere and the maximum field over the volume of the sphere when it is located at the origin of the quadrupole field. The latter is important because if the maximum field seen by the sphere surpasses a critical field the superconductor will no longer be in the Meissner state.

Section 5 provides a brief summary and a discussion of the results.

In the appendix we apply our results to the specific case of a lead sphere and demonstrate the applicability of this work to experiments.

For mathematical simplicity we choose the coordinate system such that the superconducting sphere of radius $R$ is at the origin and the center of the quadrupole field is displaced relative to the origin by $-\text{d}x = [-\text{d}x, \text{d}y, \text{d}z]$. We use $x = \{x, y, z\}$ for the position vector in Cartesian coordinates. A sketch of the geometry is shown in figure 1. As is conventionally done, we refer to the magnetic flux density $B$ as the magnetic field. The applied quadrupole field thus takes the form

$$B_0 = \frac{1}{2} b_z \{x + \text{d}x, y + \text{d}y, -2(z + \text{d}z)\}. \quad (1)$$

We will also use spherical coordinates $(r, \theta, \phi)$ and the corresponding basis vectors $\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi$. Spherical harmonics $Y_n^m$ are understood to have the normalization

$$Y_n^m = \left[\frac{2n + 1}{4\pi} \left(\frac{n - m}{n + m}\right)^{\frac{1}{2}}\right] P_n^m(\cos \theta) \exp(i\phi),$$

where $P_n^m$ stands for the associated Legendre polynomials [15]. The vector potential and magnetic field inside the sphere are denoted by $A_{in}$ and $B_{in}$, respectively, while $B_{out}$ is used for the field outside the sphere. There is no current outside the sphere, so we can use a scalar potential $\Phi$ such that $B_{out} = B_0 - \nabla \Phi$. Since physical solutions for the induced field density $-\nabla \Phi$ vanish at infinity, it follows from $\nabla \Phi = 0$ that

$$\Phi = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_{n,m} Y_n^m, \quad (2)$$

where the coefficients $a_{n,m}$ will be determined below. Components of a vector will be denoted by a superscript rather than a subscript, e.g. $B_i^i = B_{in} \cdot e_i$. We will use the Coulomb gauge $\nabla A = 0$ for any vector potential $A$ throughout this paper.

### 2. Magnetic field for $\lambda = 0$

For vanishing penetration depth there is no magnetic field inside the superconductor and the normal component of the magnetic field vanishes at the surface of the superconductor [16], which in our case corresponds to

$$B_{in} = 0, \quad B_{out}^i |_{r=R} = 0. \quad (3)$$

It follows from (1) and (2) that the radial part of the applied field and the induced field are given by

$$B_i^0 = b_i \left(\sqrt{4\pi/3} \int \right) Y_i^1 + \frac{\sqrt{\pi/6}}{2} (i \text{d}y - \text{d}x) Y_i^1 \right)$$

$$+ \frac{\sqrt{\pi/6}}{2} (i \text{d}y - \text{d}x) Y_i^1 - \frac{\sqrt{4\pi/5}}{2} r Y_i^2,$$

and

$$\left(\nabla \Phi\right)^r = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_{n,m} Y_n^m,$$

respectively. The boundary condition (3) then readily yields the coefficients as

$$a_{1,0} = b_1 \sqrt{\pi/3} R^3 \text{d}z,$$

$$a_{1,-1} = -b_1 \sqrt{\pi/24} R^3 (\text{d}x + i \text{d}y),$$

$$a_{1,1} = b_1 \sqrt{\pi/24} R^3 (\text{d}x - i \text{d}y),$$

$$a_{2,0} = b_2 \sqrt{\pi/45} R^5,$$

all other coefficients being zero. We also introduce normalized quantities by measuring length in units of $R$ and the magnetic field in units of $b_1$, i.e.

$$\tilde{x} = x/R, \quad \tilde{\text{d}x} = \text{d}x/R, \quad \tilde{B}(\tilde{x}) = B(\tilde{x}R)/(b_1 R).$$
is thus obtained as
\[
\begin{align*}
\vec{K}^\theta &= \frac{3}{4} (d\vec{x} \sin \phi - d\vec{y} \cos \phi), \\
\vec{K}^\phi &= \frac{3}{4} \cos \theta (d\vec{x} \cos \phi + d\vec{y} \sin \phi) \\
&\quad + \frac{3}{2} d\vec{z} \sin \theta + \frac{5}{4} \sin(2\theta).
\end{align*}
\] (5)

3. Magnetic field for finite \( \lambda \)

For finite \( \lambda \) the magnetic field inside the sphere is finite and determined by the London equation, i.e.
\[
\Delta A_{in} = 1/\lambda^2 A_{in}, \quad \vec{B}_{in} = \nabla \times \vec{A}_{in}.
\] (6)

The boundary condition that takes the place of (3) is simply
\[
\vec{B}_{in}|_{r=\infty} = \vec{B}_{out}|_{r=\infty}.
\] (7)

To find the solution we first introduce the vector spherical harmonics [17]
\[
\vec{Y}^m_n = Y^m_n e_\theta, \quad \vec{\Phi}^m_n = r \nabla Y^m_n, \quad \vec{\Phi}^m_n = e_\theta \times \vec{\Phi}^m_n,
\]
and make the ansatz
\[
\vec{A}_{in} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} C_{n,m}(r) \vec{\Phi}^m_n(\theta, \phi).
\]

Note that if a solution of this form with continuously differentiable \( C_{n,m}(r) \) exists, then
\[
\nabla (C_{n,m}(r) \vec{\Phi}^m_n(\theta, \phi)) = 0,
\]
and the Coulomb gauge condition is fulfilled. The London equation (6) leads to
\[
r \frac{\partial^2}{\partial r^2} (r C_{n,m}) - (r^2 / \lambda^2 + n(n+1)) C_{n,m} = 0,
\]
which is the modified spherical Bessel equation. The only solution (convergent at the origin) for the inside vector potential is therefore given by
\[
\vec{A}_{in} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} i_n \left( \frac{\lambda}{r} \right) \sum_{m=-n}^{n} C_{n,m} \vec{\Phi}^m_n(\theta, \phi),
\]
where the \( i_n \) are first order modified spherical Bessel functions [18] and the coefficients \( C_{n,m} \) are yet to be determined. The fields can be expressed in vector spherical harmonics as
\[
\begin{align*}
\vec{B}_{in} &= -\frac{1}{r} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} C_{n,m} \left[ n(n+1) i_n \left( \frac{r}{\lambda} \right) Y^m_n + \partial_r \left( r i_n \left( \frac{r}{\lambda} \right) \right) \Phi^m_n \right], \\
\vec{B}_{out} &= \vec{B}_0 + \sum_{n=0}^{\infty} r^{-(\nu+2)} \sum_{m=-n}^{n} a^\nu_{n,m} ((n+1) Y^m_n - \Phi^m_n),
\end{align*}
\] (8)

\[
\begin{align*}
\vec{B}_0 &= b_i [ -x Y^3_\nu + \Psi^3_i ] \\
&\quad + \sqrt{\pi/6} (dx + i dy)(Y^3_\nu + \Psi^3_i) \\
&\quad - \sqrt{\pi/6} (dx - i dy)(Y^3_\nu - \Psi^3_i) \\
&\quad - \sqrt{\pi/3} r (2 Y^3_\nu + \Psi^3_i).
\end{align*}
\]
The boundary condition (7) then determines the coefficients as

\[ a_{i,m}^l = f_i(\lambda/R) a_{i,m}, \]
\[ a_{2,0}^l = f_2(\lambda/R) a_{2,0,} \]
\[ c_{1,m} = (3 \lambda) \left( R^2 i_2 \left( \frac{R}{\lambda} \right) \right) a_{1,m}, \]
\[ c_{2,0} = (5 \lambda) \left( 2 R^4 i_2 \left( \frac{R}{\lambda} \right) \right) a_{2,0}, \]

with

\[ f_i(\lambda/R) = 1 - \frac{3 \lambda}{R} \left( \coth \frac{R}{\lambda} - \frac{\lambda}{R} \right), \]
\[ f_2(\lambda/R) = 1 - \frac{5 \lambda}{R} \left( \coth \frac{R}{\lambda} - \frac{\lambda}{R} \right)^{-1} - 3 \frac{\lambda}{R}. \]

Here the \( a_{i,m}^l \) denote the coefficients of the scalar potential for finite \( \lambda \), while the \( a_{i,m} \) still refer to the coefficients for \( \lambda = 0 \) as given in (4). Note that \( a_{i,m}^l \) is related to \( a_{i,m} \) by a scaling function that depends only on the ratio \( \lambda/R \). For \( \lambda/R \to 0 \) we get \( a_{i,m}^l \to a_{i,m} \) and \( c_{i,m} \to 0 \) and the solution for finite \( \lambda \) thus converges to the solution for \( \lambda = 0 \) determined in the last section. Normalized field components are plotted in figure 2, the scaling functions are plotted in figure 3. The supercurrent distribution \( j \) inside the sphere can now be simply obtained from the London equation (6) and \( \nabla \times B_m = \mu_0 j \) as \( j = -\frac{\mu_0 \lambda}{\mu_0 \lambda} A_m \).

4. Levitation force and maximum field

The force on the sphere can be written in terms of the magnetic field on the surface of the sphere as

\[ F = \frac{R^2}{\mu_0} \int_0^\pi d\theta \int_0^{2\pi} d\phi \sin \theta \left[ (e, B) B - \frac{1}{2} e, B^2 \right]_{r=R}. \]

Here \( B \) denotes either \( B_{in} \) or \( B_{out} \), as they coincide on the surface of the sphere. Carrying out the integration we find

\[ F_i = -\frac{3V}{2\mu_0} b_i^2 f_i(\lambda/R) \left( \frac{\lambda}{R} \right) dz, \quad \frac{1}{d} F_i = \frac{1}{4} d F_i, \quad (9) \]

where \( V = \frac{4}{3} \pi R^3 \) is the volume of the sphere. For \( \lambda \to 0 \) we have \( f_i(\lambda/R) \to 1 \) and we recover the expressions given in [11].

We now determine the maximum field strength \( B_{max} = \max(\{ B_{in} \}^2) \), where the maximum is evaluated over the surface of the sphere, for \( dx = 0 \). In this case the squared magnetic field inside the sphere reduces to

\[ B_m^2 = \left( 45 e_2^2 / (4\pi^2) \right) \left[ (3 \cos^2 \theta - 1)^2 i_3 \left( \frac{R}{\lambda} \right) \right] + \cos^2 \theta \sin^2 \theta \left( \frac{R}{\lambda} i_3 \left( \frac{R}{\lambda} \right) + 3 i_3 \left( \frac{R}{\lambda} \right) \right)^2. \]

Evaluating the partial derivatives with respect to \( r \) and \( \theta \) it follows that \( \partial \lambda B_m^2 \geq 0 \) \( \forall (r, \theta) \), i.e. the maximum lies on the surface of the sphere, and that the maximum occurs for \( \theta = \theta_{max} \) with \( \theta_{max} \) determined by

\[ \cos^2 \theta_{max} = \left( \frac{R}{\lambda} i_3 \left( \frac{R}{\lambda} \right) / i_2 \left( \frac{R}{\lambda} \right) - 3 \right)^2 - 6 \]
\[ \left( i_3 \left( \frac{R}{\lambda} \right) / i_2 \left( \frac{R}{\lambda} \right) - 3 \right)^2 - 9, \quad (10) \]

for \( \lambda/R \leq 0.54 \) and \( \theta_{max} \in \{ 0, \pi \} \) otherwise. We then get \( B_{max} = |B_m(\lambda/R, \theta_{max})| \). Note that the expression on the right hand side of (10) converges to \( 1^2 \) for \( \lambda/R \to 0 \), which corresponds to \( \theta_{max} \to \pi/2 \) and

\[ B_{max} \to \frac{5}{4} b_0 R, \quad (11) \]

(this result can of course also directly be read off equation (5) derived in section 2 for \( \lambda = 0 \)). On the other hand, for \( \lambda/R \to \infty \), i.e. a non-magnetic sphere, we simply have \( \theta_{max} \in \{ 0, \pi \} \) and \( B_{max} \to b_0 R \). In figure 4 we plot \( \theta_{max} \) as well as \( B_{max} \) against the ratio \( \lambda/R \). It is interesting to note that for \( \lambda/R \geq 0.14 \) the maximum field strength is smaller than \( b_0 R \), i.e. in that range \( B_{max} \) is actually smaller for a superconducting sphere than it would be for a non-magnetic sphere. This result is counter-intuitive at first glance and in stark contrast to the case of a superconducting sphere in a homogeneous field [19] \( B_{hom} \) where the maximum field strength for any value of \( \lambda/R \) will always be higher than \( B_{hom} \). One can understand this behavior qualitatively by looking at the absolute fields at \( \theta = 0 \) and \( \theta = \pi/2 \) for increasing values of \( \lambda/R \). In the former case we have \( |B_m(\lambda/R, 0, \phi)| \to 0 \) monotonically increasing to \( |B_m(\lambda/R, 0, \phi)| \to b_0 R \), while in the latter case we have \( |B_m(\lambda/R, \pi/4, \phi)| \to b_0 R \) monotonically decreasing to \( |B_m(\lambda/R, \pi/4, \phi)| \to b_0 R \). Thus, when \( \theta_{max} \) shifts towards \( \theta = 0 \) with increasing \( \lambda/R \) we will, at some point, have \( |B_m(\lambda/R, \theta_{max}, \phi)| < b_0 R \). Analytic solutions for the maximum field strength for \( |dx| > 0 \) can be found as well, but the resulting expressions are bulky and do not serve to further enhance understanding of the physics. For anyone interested in these results we recommend starting with (8) and using a
computer algebra system to derive the expressions for the maximum.

5. Summary and discussion

In the previous sections we have derived analytical solutions for the magnetic field distribution for a superconducting sphere in the Meissner state placed in an applied quadrupole field by analytically solving the Maxwell–London equations with appropriate boundary conditions. The solutions are obtained by expanding the fields in terms of vector spherical harmonics. We then derived the force acting on the sphere. The results are valid as long as the maximum field strength on the surface of the sphere is below a critical strength $B_{c1}$, the exact value of which depends on the superconducting material. Above $B_{c1}$ the superconductor will enter an intermediate state (type-I) or mixed state (type-II) [16], respectively, and the analysis provided here can no longer be applied. We also calculated the maximum field strength seen by the superconducting sphere when it is located at the center of the quadrupole field, and demonstrated that for a certain range of $\lambda/R$ the maximum field strength is lower than it is for a non-magnetic sphere.

We expect these results to be applied in the context of quasi-static magnetic traps for superconductors and to greatly enhance understanding of these traps. From the analytic solutions for the force and the field distribution one can directly obtain analytic results for trapping frequencies and coupling strengths. Previous analysis was limited to spheres that are large compared to their penetration depth ($\lambda/R \to 0$), while our results are valid for arbitrarily sized spheres. Our results furthermore show a way to connect dynamical parameters of the magnetic trap (e.g. frequency) to material constants (e.g. penetration depth), opening up new ways to measure these quantities.

The solution can easily be extended to magnetic fields of various forms, as long as they possess an expansion in vector spherical harmonics.

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Appendix

To show the practicality of this analysis with regard to magnetic levitation, we consider a lead sphere with density $\rho = 11340 \text{ kg m}^{-3}$ at millikelvin temperatures, s.t. the London penetration depth $\lambda \approx 30 \text{ nm}$ and the critical field $B_c \approx 80 \text{ mT}$. To successfully levitate the sphere we need the magnetic force (9) to counteract the gravitational force (which we take to act along the $z$-axis) $F_{\text{grav}} = \{0, 0, -V \rho g\}$, where $g \approx 10 \text{ m s}^{-2}$. This results in an equilibrium position of the sphere at $dz = -2 \mu_0 \rho g \sqrt{3 b_z^2 f_0}$. Self-consistency requires that the maximum field over the sphere at this position is below $B_c$. As shown in figure A1, this condition is indeed fulfilled for a wide range of particle radii $R$ and field gradients $b_z$. Quadrupole fields with gradients of up to $500 \text{ T m}^{-1}$ have already been demonstrated [20, 21]. Specifically, taking $R = 1 \mu \text{m}$ and $b_z = 500 \text{ T m}^{-1}$, the rest position of the sphere is $dz = -0.4 \mu \text{m}$ and the maximum field strength on its surface is approximately $0.8 \text{ mT}$, two orders of magnitude below the critical field.

It is noteworthy that the minimum gradient $b_{z,\text{min}}$ that is necessary for levitation stays approximately constant over a wide range of radii, in this case $b_{z,\text{min}} \approx 1.78 \text{ T m}^{-1}$ for
particle radii between approximately 1 \( \mu \)m and 1 mm. This is due to the fact that for \( \lambda \ll R \ll |dz| \) the maximum field on the particle surface is approximately given by \( \frac{3}{2} b_0 |dz| \approx \mu_0 \rho \frac{g}{b_0} b_2^{-1} \) and it follows that
\[
b_{2,\text{min}} \approx \mu_0 \rho g B_c^{-1}.
\]

For radii below 1 \( \mu \)m one has to take into account the penetration depth, while for radii above 1 mm one has to consider the radius of the sphere. For \( |dz| \ll R \), the maximum radius \( R_{\text{max}} \) follows directly from (11) as
\[
R_{\text{max}} \approx 4 B_c / (5 b_2).
\]

These two limiting cases correspond to the red dotted lines in figure A1.

Our analysis does not predict a lower limit to the radius as long as one can produce a sufficiently high magnetic gradient, but the London equations (and therefore this analysis) are only valid for materials with dimensions well above the atomic length scale.

This analysis can easily be adapted to any specific superconductor and temperature, as long as the material parameters are known.

Figure A1. Plot of the maximum field strength on the surface of the sphere at its rest position in a gravitational field. The solid line corresponds to a field strength of 80 mT, while the dashed lines correspond to sphere displacements of 1 \( \mu \)m resp. 1 mm. The dotted red lines are the limiting cases as described in the text. The units on the color bar are Tesla.

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