Qubit gates with simultaneous transport in double quantum dots

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Abstract
A single electron spin in a double quantum dot in a magnetic field is considered in terms of a four-level system. By describing the electron motion between the potential minima via spin-conserving tunneling and spin flip caused by a spin–orbit coupling, we inversely engineer faster-than-adiabatic state manipulation operations based on the geometry of four-dimensional rotations. In particular, we show how to transport a qubit among the quantum dots performing simultaneously required spin rotations.

1. Introduction
Device architecture based on electrons confined in coupled quantum dots [1–4] is considered as a potential and significant candidate for quantum computing and quantum information processing. The advantages of this architecture rely on the facts that electron spin is a natural qubit with spin-up and spin-down states, mature semiconductor technology may be used, and long coherence times on the scale of microseconds have been achieved in these systems [5, 6]. Laboratories use electric, microwave or magnetic fields to manipulate spin states, performing $10^3$–$10^5$ operations in the spin dephasing time [5–10].

Scalability of quantum information devices is a major challenge for any architecture, and it is associated with the capability to transport qubits. In this paper we theoretically explore a four-level model for a spin in a double quantum dot (DQD) aiming at the possibilities to implement fast qubit transport with simultaneous qubit rotations. We achieve this goal for arbitrary rotations by controlling the synchronized time dependences of interdot tunneling and spin–orbit coupling (SOC). We inverse-engineer these time dependences based on our recent work [11] on the control of four-level systems. The method separates population control from control of the phases of the bare state basis [12]. Populations can be mapped onto a four-dimensional (4D) sphere such that their evolution amounts to 4D transformation controlled by the (4D-)rotation Hamiltonian that may be engineered from the target state (in our case via isoclinic rotations and quaternions). A full Hamiltonian can then be constructed from the rotation Hamiltonian to realize the desired phase changes. Arbitrary state manipulations require full flexibility in the Hamiltonian, i.e. the possibility to implement the different Hamiltonian matrix elements with specific time-dependences. In the systems of interest, however, there are constraints that hinder certain manipulations and transitions. In particular, in this paper we examine the Hamiltonian structure that corresponds to combined tunneling and SOC controllable couplings, and deduce the possible transformations.

Spin–orbit coupling in semiconductors consists of two main contributions due to the Dresselhaus- and the Bychkov–Rashba-effect. The former is due to the bulk inversion asymmetry of the material and the latter results from the structure inversion asymmetry, produced, e.g. by the confining potential or an external electric field [13]. The practical advantage of the Rashba coupling is the ability to manipulate it by an external electric field applied across the semiconductor structure [14, 15]. The Rashba coupling controlled by a high-frequency ac gate voltage [16] provides an effective method to control the spin states in a short time [17–20] and produce exact nonadiabatic transformations for the electron in a parabolic potential [21].
This paper is organized as follows. In section 2, we introduce first the method that parameterizes the time-dependent Hamiltonian and time evolution operator of a four-level system by using isoclinic rotations and quaternions [11]. Then we map the Hamiltonian of the spin in a DQD coupled by SOC and tunneling onto this scheme. In section 3, we apply the method developed in section 2 to design the synchronized time dependences of the control parameters to perform different qubit operations, such as the interdot transports combined with spin rotations. In section 4 we analyze the stability of our protocol with respect to systematic errors and noise. Section 5 provides a discussion of the results and their relation to other systems. Some details on the structure of the Hamiltonian are presented in the appendix.

2. Electron in a DQD: a 4D approach

2.1. 4D Hamiltonians and evolution operators

The state vector of a four-level system

\[ |\psi(t)\rangle = \sum_{n=1}^{n=4} c_n(t) e^{i\phi_n} |n\rangle, \]  

where \( c_n, \phi_n \) are real amplitudes and phases (we set \( \phi_1 = 0 \)), and \( \sum_{i=1}^{4} c_i^2 = 1 \), can be decomposed as

\[ |\psi(t)\rangle = K(t) |\psi_f(t)\rangle, \]  

where

\[ |\psi_f(t)\rangle = \sum_{n=1}^{n=4} c_n(t) |n\rangle \]  

is a vector on the surface of a 4D sphere, and the phase information is contained in

\[ K(t) = \sum_{n=1}^{n=4} e^{i\phi_n} |n\rangle \langle n| \]  

The states \( |\psi(t)\rangle \) and \( |\psi_f(t)\rangle \) evolve via evolution operators \( U(t) \) and \( U_f(t) \) related by \( U_f(t) = K^{-1}(t) U(t) K(0) \),

\[ |\psi(t)\rangle = U(t)|\psi(0)\rangle, \]

\[ |\psi_f(t)\rangle = U_f(t)|\psi_f(0)\rangle, \]

where we set the initial time as 0. We may set from \( U_f(t) \) a rotation Hamiltonian as

\[ H_f(t) = i\hbar U_f(t) U_f^+(t), \]

whereas the total Hamiltonian is

\[ H(t) = i\hbar U(t) U^+(t) = K(t) H_f(t) K^{-1}(t) + i\hbar K(t) K^{-1}(t). \]

To engineer \( H_f(t) \) for a specific 4D rotation, it is convenient to express first a general 4D rotation matrix as a product of two isoclinic rotation matrices [22, 23]:

\[ U_f(t) = \begin{bmatrix} q_w & -q_x & -q_y & -q_z \\ q_x & q_w & -q_z & q_y \\ q_y & q_z & q_w & -q_x \\ q_z & -q_y & q_x & q_w \end{bmatrix} \begin{bmatrix} P_w & -P_x & -P_y & -P_z \\ P_x & P_w & -P_z & -P_y \\ P_y & P_z & P_w & -P_x \\ P_z & -P_y & P_x & P_w \end{bmatrix}, \]

where \( q_i \) and \( p_i \) are components of two unit quaternions \( q = q_w + q_x i + q_y j + q_z k \) and \( p = p_w + p_x i + p_y j + p_z k \). We shall parameterize them in terms of generalized 4D spherical angles [24, 25],

\[ q_w(t) = \cos \gamma_1(t), \]

\[ q_x(t) = \sin \gamma_1(t) \cos \theta_x(t), \]

\[ q_y(t) = \sin \gamma_1(t) \sin \theta_x(t) \cos \phi(t), \]

\[ q_z(t) = \sin \gamma_1(t) \sin \theta_x(t) \sin \phi(t), \]

\[ p_w(t) = \cos \gamma_2(t), \]

\[ p_x(t) = \sin \gamma_2(t) \cos \theta_2(t), \]

\[ p_y(t) = \sin \gamma_2(t) \sin \theta_2(t) \cos \phi(t), \]

\[ p_z(t) = \sin \gamma_2(t) \sin \theta_2(t) \sin \phi(t), \]

where \( 0 \leq \phi \leq 2\pi, 0 \leq \theta, \gamma_1, \gamma_2 \leq \pi \). Thus by using \( U(t) = K(t) U_f(t) K^{-1}(0) \) and \( H(t) = K(t) H_f(t) K^{-1}(t) + i\hbar K(t) K^{-1}(t) \), we find the parameterized forms for the evolution operator \( U(t) \) and the Hamiltonian \( H(t) \). The explicit expressions are lengthy, and will not be reported here.
2.2. Single electron in a DQD

Consider a single electron in a semiconductor DQD, for example made of Si or GaAs, with tunneling and Rashba SOC, as shown in Figure 1. We use a bare basis of spin up and down states localized in each well, numbered as $|y\downarrow\rangle = |\downarrow\rangle_1$, $|y\uparrow\rangle = |\uparrow\rangle_3$, $|y\uparrow\rangle = |\uparrow\rangle_4$. Following the derivation in appendix, and after a diagonal energy shift of $\Delta/2$, the Hamiltonian of this system (see (A.7)) can be written as

$$H_{t}(t) = \hbar \begin{pmatrix} 0 & \tau(t) & \alpha(t) & 0 \\ \tau(t) & 0 & 0 & -\alpha(t) \\ \alpha^*(t) & 0 & \Delta & \tau(t) \\ 0 & -\alpha^*(t) & \tau(t) & \Delta \end{pmatrix}$$

Here $\tau(t)$ represents the tunneling coupling between the two quantum dots, $\alpha(t)$ is the Rashba coupling, and $\Delta$ is the Zeeman splitting. All these quantities have dimensions of frequency. Following the approach of Mal’shukov et al [16], we consider the time-dependent Rashba coupling in the complex form $\alpha(t) = \alpha_0 + \alpha_1(t)e^{i\omega t}$. Below we put the constant $\alpha_0 = 0$ as it can be compensated by applying the proper electric field across the nanostructure [14, 26], and only the time-dependent term has an effect on the spin and charge motion as a result.

The Hamiltonian structure corresponds topologically to a ‘diamond’ configuration [11], which, in the parametric expression of $H(t)$ we may impose with the conditions

$$\dot{\theta}_1 = \dot{\theta}_2 = \dot{\phi}_1 = \dot{\phi}_2 = 0, \; \phi_1 = \phi_2 = 0,$$

see figure 2. Specifically, after substituting (10) in the parameterized form of (6), the Hamiltonian $H(t)$ becomes

$$H(t) = \hbar \{ -\phi_2(t)\hat{2}\} \{2\} - \phi_2(t)\hat{3}\langle 3\} - \phi_2(t)\hat{4}\langle 4\}
- i\{ e^{-i\phi_2(t)}(\gamma_1(t)\cos \theta_1 + \gamma_2(t)\cos \theta_2)\} [\hat{1}\langle 2\}
+ e^{-i\phi_2(t)}(\gamma_1(t)\sin \theta_1 + \gamma_2(t)\sin \theta_2)\} [\hat{1}\langle 3\}
+ e^{i\phi_2(t)-\gamma_2(t)}(\hat{1}\langle 1\})\hat{2}\sin \theta_1 + \hat{2}\langle 1\} \Theta_2)\} [\hat{3}\langle 3\}
+ e^{i\phi_2(t)-\gamma_2(t)}(\hat{1}\langle 1\})\hat{2}\sin \theta_1 + \hat{2}\langle 1\} \Theta_2)\} [\hat{3}\langle 4\}]
+ h.c.$$
To make $H_0(t)$ and $H(t)$ fully consistent, we further fix the angles as
\[ \hat{\theta}_2 = \theta_2 = \gamma_2 = 0, \]
\[ \phi_2(t) = -\frac{\pi}{2}, \quad \phi_3(t) = -\Delta t + \frac{\pi}{2} \phi_4(t) = -\Delta t. \] (12)

Then equation (11) gives
\[ H(t) = \hbar \{ \hat{\gamma}(t) [\cos \theta |1] \langle 2 | + | 3 \rangle \langle 4 |] - e^{i\Delta t} \sin \theta |1 \rangle \langle 2 | + | 2 \rangle \langle 4 |] \}
\[ + \Delta (| 3 \rangle \langle 3 | + | 4 \rangle \langle 4 |] + \text{h.c.}, \]
where we have simplified the notation as $\gamma(t) = \gamma_3(t), \theta = \theta_1$. Now we may impose $H_0(t) = H(t)$, as they have the same structure, to find the following relations between control functions and auxiliary angles,
\[ \tau(t) = \gamma(t) \cos \theta, \]
\[ \alpha(t) = -e^{i\Delta t} \gamma(t) \sin \theta, \] (14)
which implies $\alpha_0 = 0, \alpha_1(t) = -\gamma_3(t), \omega = \Delta$ (i.e. the external bias is in resonance with the Zeeman frequency), and $\theta$ can be considered as a coupling mixing angle. Under the conditions stated in equations (12), the parameterized time-evolution operator becomes
\[ U(t) = \begin{bmatrix}
\cos \gamma(t) & -i \cos \theta \sin \gamma(t) & i \sin \gamma(t) \sin \theta & 0 \\
-i \cos \theta \sin \gamma(t) & \cos \gamma(t) & 0 & -i \sin \gamma(t) \sin \theta \\
ie^{-i\Delta t} \sin \gamma(t) \sin \theta & 0 & e^{-i\Delta t} \cos \gamma(t) & -ie^{-i\Delta t} \cos \theta \sin \gamma(t) \\
0 & -ie^{-i\Delta t} \sin \gamma(t) \sin \theta & -ie^{-i\Delta t} \cos \theta \sin \gamma(t) & e^{-i\Delta t} \cos \gamma(t)
\end{bmatrix}. \] (15)

We impose the boundary condition $\gamma(0) = 2n\pi, n = \ldots, -2, -1, 0, 1, 2, \ldots$, to guarantee $U(0) = 1$ at the initial time.

3. Applications

3.1. Qubit preparation

Assume that the four-level system is initialized in state $|\psi(0)\rangle = |1\rangle$ on the left well and the objective is to prepare from it an arbitrary qubit in the right well encoded in levels $|2\rangle$ and $|3\rangle$. We write a generic final state as $|\psi(T)\rangle = b_1|1\rangle + b_2|2\rangle + b_3|3\rangle + b_4|4\rangle$, where $T$ is the duration time and $b_n$ are final complex amplitudes which satisfy $\sum_n |b_n|^2 = 1$. By using $|\psi(T)\rangle = U(T)|\psi(0)\rangle$, we have
\[ b_1 = \cos \gamma(T), \]
\[ b_2 = -i \cos \theta \sin \gamma(T), \]
\[ b_3 = ie^{-i\Delta T} \sin \gamma(T) \sin \theta, \]
\[ b_4 = 0. \] (16)

We can transfer $|1\rangle$ to any bare state except $|4\rangle$, or to arbitrary superpositions of $|2\rangle$ and $|3\rangle$ (i.e. any qubit on the right well) by imposing $\gamma(T) = (2n + 1)\pi/2, n = 0, \pm 1, \ldots$

As an example we shall perform a state transfer to $b_1 = 0, b_2 = 1/2, b_3 = e^{\pi i/2 \sqrt{3}}/2, b_4 = 0$. Equations (16) with
\[ \theta = -\frac{\pi}{3}, \quad T = \frac{3\pi}{2\Delta}, \]
\[ \gamma(0) = 0, \quad \gamma(T) = \frac{\pi}{2}, \]
\[ \gamma(0) = 0, \quad \gamma(T) = 0, \] (17)
correspond to the desired final state $|\psi(T)\rangle = -i(|2\rangle + e^{\pi i/2 \sqrt{3}} |3\rangle)/2$ within an irrelevant global phase factor. An Ansatz for $\gamma(t)$ consistent with the above boundary conditions is
\[ \gamma(t) = \frac{\pi}{4} \left[ 1 - \cos \left( \frac{\pi t}{T} \right) \right]. \] (18)
The resulting tunneling and Rashba SOC are calculated from (14) as
\[ \tau(t) = \frac{\pi^2}{4T} \sin \left( \frac{\pi t}{T} \right) \sin \theta, \]
\[ \alpha(t) = -e^{i\Delta t} \frac{\pi^2}{4T} \sin \left( \frac{\pi t}{T} \right) \sin \theta, \] (19)
with the characteristic values $\tau, \alpha \propto 1/T$. We can prepare a qubit with an arbitrary relative phase by adjusting $\Delta$ and the operation time $T$ as long as the tunneling and SOC are experimentally feasible. We plot the time dependences of the tunneling matrix elements, Rashba SOC, and populations evolution of all bare states in figure 3 with parameters corresponding to the electron $g^*$-factor in GaAs ($g^* = -0.44$), with $B = 100$ mT, $\Delta \approx 2\pi \times 0.5$ GHz, and $T = 1.5$ ns [27].

### 3.2. Qubit transport and rotation

Our method may be applied to transport the qubit from one dot to the other applying simultaneously some qubit rotation, i.e. to produce an arbitrary gate. Suppose we have already prepared a qubit in the left dot in an arbitrary superposition of $|1\rangle$ and $|4\rangle$ as $|\psi(0)\rangle = \cos \chi |1\rangle + e^{i\mu} \sin \chi |4\rangle$, where $\chi$ is the initial amplitude mixing angle and $\mu$ is the initial relative phase. The corresponding general final state with the unitary evolution operator (15) is given by the amplitudes

\[
\begin{align*}
  b_1 &= \cos \chi \cos \gamma(T), \\
  b_2 &= A e^{i\lambda}, \\
  b_3 &= B e^{i(\lambda - \Delta T)}, \\
  b_4 &= \sin \chi \cos \gamma(T) e^{i(\mu - \Delta T)},
\end{align*}
\]

where

\[
\begin{align*}
  A &= \frac{\sin \gamma(T)}{\sqrt{2}} \sqrt{1 + \cos 2\chi \cos 2\theta + \cos \mu \sin 2\chi \sin 2\theta}, \\
  B &= \frac{\sin \gamma(T)}{\sqrt{2}} \sqrt{1 - \cos 2\chi \cos 2\theta - \cos \mu \sin 2\chi \sin 2\theta}, \\
  \zeta_A &= -\arctan \left( \cot \mu + \cot \theta \frac{\cos \chi}{\sin \mu} \right), \\
  \zeta_B &= -\arctan \left( \cot \mu - \tan \theta \frac{\cos \chi}{\sin \mu} \right).
\end{align*}
\]

We can inversely calculate the coupling mixing angle $\theta$ under the condition that $\gamma(T) = n\pi/2, n = 1, 3, 5, \cdots$, so that the amplitudes $b_1$ and $b_4$ vanish, and for given desired final real amplitudes $A$ and $B$ we find

\[
\theta = \pm \arccos \left( \frac{A^2 \cos 2\chi + \sin^2 \chi (1 - 2 \cos^2 \chi \sin^2 \mu) + 2 S \cos \mu \sin 2\chi}{1 - \sin^2 2\chi \sin^2 \mu} \right),
\]

where

\[
S = \sqrt{4A^2B^2 - \sin^2 \mu \sin^2 2\chi}.
\]

Notice that there is still a degree of freedom to control the final relative phase of the qubit on the right. Suppose our target relative phase is $\lambda = \zeta_B - \zeta_A - \Delta T$. By adjusting the operation time as $T = (\zeta - \lambda)/\Delta$, where $\zeta = \zeta_B - \zeta_A$, the process produces the desired relative phase $\lambda$. Now we can consider two examples of application of (22).

**Example 1: transport and phase gate.** We assume that $A = \cos \chi$ and $B = \sin \chi$ and substitute them into (22) to get $\theta = 0$, which means that $\tau = \gamma(t)$ and $\alpha = 0$. The final state is calculated as

![Figure 3. (a) $T\gamma(t)$ (dotted—dashed blue line) and $T\alpha(t)$ (solid red line). (b) Populations of $|1\rangle$ (dotted—dashed blue line), $|2\rangle$ (dashed purple line), $|3\rangle$ (green triangle) and $|4\rangle$ (solid red line) for the transfer from $|1\rangle$ to $-i(|2\rangle + e^{i\pi/\sqrt{2}} |3\rangle)/\sqrt{2}$. Parameters: $B = 100$ mT, $\Delta \approx 2\pi \times 0.5$ GHz, and $T = 1.5$ ns.](image)
By letting \( γ(t) \) evolve from 0 to \( π/2 \), the qubit is transported from left to right and rotated by a relative phase factor \( e^{−i\Delta T} \).

**Example 2: transport and NOT gate.** The NOT gate with transport swaps the amplitudes between up and down states, so we set \( a = \sin \chi \) and \( b = \cos \chi \) in (22) to get

\[
\begin{align*}
    b_1 &= \cos \chi \cos γ(T), \\
    b_2 &= −i \sin γ(T) \cos \chi, \\
    b_3 &= −ie^{i(μ−ΔT)} \sin γ(T) \sin \chi, \\
    b_4 &= i \sin \chi \cos γ(T)e^{−iΔT}.
\end{align*}
\]

(24)

We also impose \( λ = −μ \), so we fix the operation time as \( T = (ζ + μ)/Δ \).

The envelopes of tunneling rate and SOC of the (NOT gate) + (transport) operation are depicted in figure 4(a). We use the same Ansatz for \( γ(t) \) in (18) to keep \( γ(T) = π/2 \) and choose for the example \( μ = π/4 \), \( χ = π/3 \). The population is successfully swapped as seen in figure 4(b), and figure 4(c) shows the fidelity

\[
F(t) = |⟨\psi_{\text{trag}}|\psi(t)⟩|^2,
\]

where the target state is \( |\psi_{\text{trag}}⟩ = \sin \chi e^{iθ}|2⟩ + \cos \chi |3⟩ \).

### 4. Robustness of the approach

In practice, the fidelity of experimental devices, including DQDs, is sensitive to the operational noise and systematic device and system-dependent errors. In this section, we discuss the robustness of our approach with respect to these effects. Without loss of generality, we analyze the stability of two different initializations corresponding to the two cases in section 3. For convenience, we define the transition from \( |1⟩ \) to

\[
−i(2) + e^{iπ/2}\sqrt{3} \ |3⟩)/2
\]
as Case 1, corresponding to figure 3, and the transition from

\[
(|1⟩ + e^{iπ/4}\sqrt{3} \ |4⟩)/2
\]
to \( (|1⟩ + e^{iπ/4}\sqrt{3} \ |2⟩ + |3⟩)/2 \) as Case 2, see figure 4.

#### 4.1. Zeeman energy bias

Due to hyperfine interaction with nuclear spins [28, 29] and the local variability in the g-factor [30], Zeeman energies of two spatially separated quantum dots can be somewhat different. At the system parameters of our interest, the variation in the g-factor is negligibly small, while the hyperfine coupling can lead to a difference of the order of 0.1 \( Δ \). This imbalance can be presented using a time-independent sum.
because the nuclear dynamics is much slower than electron dynamics. We plot the final time fidelity \( F \) versus bias parameter \( \delta \) in Figure 5. This figure confirms that even a relatively large Zeeman bias decreases the fidelity by less than two percent.

4.2. Noise in the driving fields

Here we analyze the effect of the noise in the driving field. As a generic model of the noise, we introduce non-synchronized fluctuations in the tunneling rate and in the SOC as

\[
r'(t) = \gamma (t) \cos \theta + \epsilon_r(t), \quad \alpha'(t) = e^{i \omega t} \sin \theta + \epsilon_s(t),
\]

where \( \epsilon_r(t) \) and \( \epsilon_s(t) \) are different realizations of a real time-dependent stochastic function

\[
\epsilon(t) = \epsilon_0 \sum_{j=1}^{N} s_j f(t - t_j),
\]

where \( f(t) = \exp(-t^2 / \eta^2) \) is a Gaussian function with time width \( \eta \). The factor \( s_j = \pm 1 \) is a random function of \( j \) with \( \Sigma_{j=1}^{N} s_j = 0 \), making the mean value of \( \epsilon(t) \) close to zero. The mean density of the \( t_j \) points can be described by \( d = N / T \) and the overall noise is determined by both the \( d \) and \( \eta \) with the results weakly dependent on the details of the shape of the \( f(t - t_j) \)-function. Figure 6 demonstrates the fidelities versus \( \epsilon_0 \). The insets show the corresponding noisy amplitudes of couplings at \( \epsilon_0 \approx 0.1 \max(|\alpha(t)|) \). We observe that the system remains stable having the fidelity \( F > 0.97 \) since the transitions depend largely on the areas of envelope functions \( \alpha(t) \) and \( \tau(t) \). However, with increase in the noise correlation time \( \eta \), the fidelity slightly decreases due to the stronger effect of the fluctuations demonstrating a weaker self-averaging over the driving time \( T \).
To complete this brief discussion, we mention the effect of the charge noise on the spin dephasing. Recent theoretical analysis (e.g. in [31]) demonstrates that the corresponding dephasing times are longer than $10^4$ ns, greatly exceeding the time scale of our problem.

5. Discussion

By applying an approach based on 4D rotations, we studied electron charge and spin motion in tunneling- and spin–orbit coupled quantum dots. Through a proper synchronization of their time-dependences, we inversely engineered the tunneling and SOC matrix elements to achieve robust spin transport with simultaneous single qubit rotations in quantum information transformations such as the qubit preparation, $U_{\text{phase}}$ and $U_{\text{NOT}}$ gates. After establishing the transition protocols, we demonstrated its robustness against a Zeeman energy imbalance and a high-frequency noise.

In a chain of quantum dots, these transport + rotation operations may be applied sequentially for a long-distance qubit transfer in a multi-dot architecture, where the ability of a coherent spin transfer has been recently demonstrated [32, 33]. Figure 7 demonstrates these processes for a particular sequence starting in Dot 1 and ending in Dot 4. We point out that this technique can also be applied to heavy-hole systems, where the control of the hole spin via the tunneling and strong SOC has been demonstrated for silicon-based DQDs [34]. Recent publications presenting theoretical proposals for DQD with strong SOC [19, 20] and transverse magnetic field [35] are potentially consistent with our method and can be compared to other protocols based on the approach proposed here. Finally, a similar approach can be used to design the spin and mass transport of cold atoms in optically produced potentials [36].

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Appendix. Four-level Hamiltonian of a DQD

We consider a single electron in a DQD modeled by a one-dimensional Hamiltonian, as can be realized in nanowire-based systems [37], where the electron is tightly confined in the perpendicular directions, as

\[ H_{\text{DQD}}(t) = \frac{\hat{p}^2}{2m} + V(x). \]  

(A.1)

Here the first term is the kinetic energy with \( \hat{p} = -i\hbar \partial / \partial x \) and \( m \) is the electron effective mass (e.g. in GaAs \( m \approx 0.067 \) of the free electron mass). We assume that \( V(x) \) is a spatially symmetric potential \( V(x) = V(-x) \) with two equivalent minima at points \( x_0 > 0 \) and \( -x_0 \) and choose the basis for the tunneling-related Hamiltonian as two approximate states of this potential, localized in the vicinity of the points \( -x_0 \) and \( x_0 \), which we will denote as \( \psi_{L(R)}(x) \) [38, 39], respectively. The Hamiltonian in this basis becomes

\[ H_{\text{nn}} = \hbar \begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix}, \]

(A.2)

where \( \tau \) is the tunneling rate between the two quantum dots determined by deviation of the total potential \( V(x) \) from its shape in the vicinity of the minima [40]. Thus, by modifying \( V(x) \) by a time-dependent external field, one can produce time-dependent \( \tau(t) \).

A magnetic field \( \mathbf{B} = (0, 0, -B) \) along the \( z \)-axis causes the Zeeman spin splitting corresponding to the Hamiltonian \( H_z = -g \mu_B \sigma_z B/2 \), where \( g \) is the conduction band Landé factor, \( \mu_B \) is the Bohr magneton, and the level splitting is \( \Delta = \mu_B g B \). In the basis of \( \sigma_z \)-representation, the eigenstates of \( H_z \) are given by: \( |\psi_1\rangle = (1, 0)^T \) and \( |\psi_2\rangle = (0, 1)^T \).

The one-dimensional Rashba SOC is represented as

\[ H_{\text{SOC}} = \frac{i\tilde{\alpha}}{\hbar} \hat{p} \sigma_x, \]

(A.3)

where \( \tilde{\alpha} \) is the corresponding coupling parameter.

We define the full four-state basis of a single electron in the DQD as

\[ |L(R)_{\uparrow/\downarrow}\rangle = |\psi_{L(R)}\rangle \otimes |\psi_{\uparrow/\downarrow}\rangle \]

(A.4)

and get nonzero coupling Rashba terms as (with \( \hat{k} \equiv \hat{p} / \hbar \))

\[ \langle L | H_{\text{SOC}} | R \rangle = (\downarrow \sigma_y | \downarrow \sigma_y) \langle L | \hat{k} | R \rangle = -i\tilde{\alpha} \langle L | \hat{k} | R \rangle, \]

\[ \langle R | H_{\text{SOC}} | L \rangle = (\downarrow \sigma_y | \downarrow \sigma_y) \langle R | \hat{k} | L \rangle = -i\tilde{\alpha} \langle R | \hat{k} | L \rangle, \]

(A.5)

and the Hermitian conjugate terms. The diagonal elements are all zeros because \( \langle L | \hat{k} | L \rangle = \langle R | \hat{k} | R \rangle = 0 \) and \( \langle L | \hat{k} | R \rangle = -\langle R | \hat{k} | L \rangle \). We finally find in the basis \{\( |L_1\rangle, |R_1\rangle, |R_2\rangle, |L_2\rangle \}\, corresponding to the \{\( |1\rangle, |2\rangle, |3\rangle, |4\rangle \}\ basis of the main text

\[ H_{\tilde{\alpha}} = \hbar \begin{bmatrix} 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & -\alpha \\ \alpha^* & 0 & 0 & 0 \\ 0 & -\alpha^* & 0 & 0 \end{bmatrix}, \]

(A.6)

where \( \alpha \equiv -i\tilde{\alpha} \langle L | \hat{k} | R \rangle \).

For symmetric \( V(x) \), the full Hamiltonian in the \{\( |1\rangle, |2\rangle, |3\rangle, |4\rangle \}\ basis of the main text, taking into account spin-conserving and spin-flip tunnelings, takes the form

\[ H_0 = \hbar \begin{bmatrix} -\Delta/2 & \tau(t) & \alpha(t) & 0 \\ \tau(t) & -\Delta/2 & 0 & -\alpha(t) \\ \alpha^*(t) & 0 & \Delta/2 & \tau(t) \\ 0 & -\alpha^*(t) & \tau(t) & \Delta/2 \end{bmatrix}. \]

(A.7)

The time dependence \( \alpha(t) \) in (A.7) comes from two main sources: time-dependent \( \tilde{\alpha} \) due to ac external bias and time-dependent overlap of the wave functions localized near the left \((-x_0)\) and right \(x_0\) minimum of the potential \( V(x) \).

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5 Since we consider a system with \( g' < 0 \), we take \( \mathbf{B} \) antiparallel to the \( z \)-axis for consistency with the main text.
References

[1] Loss D and DiVincenzo D P 1998 Phys. Rev. A 57 120
[2] Oosterkamp T H, Fujisawa T, van der Wiel W G, Ishibashi K, Hijman R V, Tarucha S and Kouwenhoven L P 1998 Nature 395 873
[3] Hu X and Sarma S D 2000 Phys. Rev. A 61 062301
[4] Hu X and Sarma S D 2001 Phys. Rev. A 64 042312
[5] Bluhm H, Foletti S, Neder I, Rudner M, Mahalu D, Umansky V and Yacoby A 2011 Nat. Phys. 7 109
[6] Veldhorst M et al 2014 Nat. Nanotechnol. 9 981
[7] Nowack K C, Koppens F H L, Nazarov Y V and Vandersypen L M K 2007 Science 318 1430
[8] McNeil R, Kataoka M, Ford C, Barnes C, Anderson D, Jones G, Farrier I and Ritchie D 2011 Nature 477 439
[9] Baart T A, Shafiei M, Fujita T, Reichl C, Wegscheider W and Vandersypen L M K 2016 Nat. Nanotechnol. 11 330
[10] Baart T A, Iovanovic N, Reichl C, Wegscheider W and Vandersypen L M K 2016 Appl. Phys. Lett. 109 043101
[11] Li Y C, Martínez-Cercoño D, Martínez-Garaot S, Chen X and Muga J G 2018 Phys. Rev. B 97 013320
[12] Kang Y H, Huang B H, Lu P M and Xian Y 2017 Laser. Phys. Lett. 14 025201
[13] Fabian J, Matos-Abiague A, Ertler C, Stano P and Žutić I 2007 Acta Phys. Slovaca 57 565
[14] Nitta J, Akazaki T, Takayanagi H and Enoki T 1997 Phys. Rev. Lett. 78 1335
[15] Sawada A, Faniel S, Mineshige S, Kawabata S, Saiito K, Kobayashi K, Sekine Y, Sugiyama H and Koga T 2018 Phys. Rev. B 97 195303
[16] Mal’shukov A G, Tang C S, Chu C S and Chao K A 2003 Phys. Rev. B 68 233307
[17] Sadreev A F and Sherman E Y 2013 Phys. Rev. B 88 115302
[18] Echeverría-Arrondo C and Sherman E Y 2013 Phys. Rev. B 88 155328
[19] Liu Z H, Li R, Hu X D and You J Q 2018 Sci. Rep. 8 2302
[20] Zhao X Y and Hu X D 2018 Sci. Rep. 8 13968
[21] Cadiz, T, Jefferson J H and Ramík A 2014 Phys. Rev. Lett. 112 150402
[22] Thomas F 2014 IEEE. Trans. Robot. 30 1037
[23] Pérez-Gracia A and Thomas F 2017 Adv. Appl. Clifford Algebras 27 523
[24] Sommerfeld A 1949 Partial Differential Equations in Physics (New York: Academic) p 227
[25] Muga J G and Wardlaw D M 1995 Phys. Rev. E 51 5377
[26] Faniel S, Matsusra T, Mineshige S, Sekine Y and Koga T 2011 Phys. Rev. B 83 115309
[27] Petta J R, Johnson A C, Taylor J M, Laird E A, Yacoby A, Lukin M D, Marcus C M, Hanson M P and Gossard A C 2005 Science 309 2180
[28] Merkulov I A, Efros A L and Rosen M 2002 Phys. Rev. B 65 205309
[29] Koppens F H L, Buizert C, Tielrooij K J, Vink J T, Nowack K C, Meunier T, Kouwenhoven L P and Vandersypen L M K 2006 Nature 442 766
[30] Hwang J C C, Yang C H, Veldhorst M, Hendrickx N, Fogarty M A, Huang W, Hudson F E, Morello A and Dzurak A S 2017 Phys. Rev. B 96 045302
[31] Li R 2018 J. Phys.: Condens. Matter 30 395304
[32] Flentje H, Mortemousque P A, Thalineau R, Ludwig A, Wieck A D, Bäuerle C and Meunier T 2017 Nat. Commun. 8 501
[33] Fujita T, Baart T A, Reichl C, Wegscheider W and Vandersypen L M K 2017 NPJ Quantum Inf. 3 22
[34] Bogun A, Studenikin S, Korkusinski M, Gaudreau L, Zawadzki P and Sachrajda A S 2018 Phys. Rev. Lett. 120 207701
[35] Güngördü U and Kestner J P 2018 Phys. Rev. B 98 165306
[36] Kartashov Y V, Konotop V and Vysloukh V A 2018 Phys. Rev. A 97 063609
[37] Nadi-Perge S, Frolkov S M, Bakkers E P A M and Kouwenhoven L P 2010 Nature 468 1084
[38] Li X, Barnes E, Kestner J P and Das Sarma S 2017 Phys. Rev. A 96 012309
[39] Burkard G, Loss D and DiVincenzo D P 1999 Phys. Rev. B 59 2070
[40] Ashcroft N W and Mermin N D 1976 Solid State Physics (Philadelphia, PA: Saunders)