Gamma Ray Bursters and Black Holes in Gravity’s Rainbow

Moh Vaseem Akram$^1$, Imtiyaz Ahmad Bhat$^{2,3}$,† and Anver Aziz$^{1,‡}$

$^1$Department of Physics, Jamia Millia Islamia, New Delhi, India
$^2$Center for Theoretical Physics, Jamia Millia Islamia, New Delhi, India
$^3$Department of Physics, Central University of Kashmir, Ganderbal 191201 India

Abstract

In this letter, we analyze the modification to the thermodynamics of a Schwarzschild black hole and a Kerr black hole due to gravity’s rainbow. The metric for these black holes will be made energy dependent. This will be done by using rainbow functions motivated from the hard spectra from gamma-ray bursters at cosmological distances. This modification of the metric by these rainbow functions will in turn modify the temperature and entropy of these black holes. We will also discuss how this effects the formation of virtual black holes.
It is known that the string theory can be investigated using the formalism of a two-dimensional conformal field theory. In this approach, the metric in the target space can be viewed as a matrix of coupling constants for the conformal field theory. Now it will be possible for these coupling constants to flow because of the renormalization group flow associated with the quantum field theory describing the world-sheet of a string [1, 2]. This flow would make the target space metric depend on the scale at which it is being probed. However, the scale at which the metric is probed will depend on the energy of the probe, and so it is expected that the spacetime metric will become energy dependent. A theory, where the metric depends on the energy of the probe is called gravity’s rainbow [3–6]. Such modification to gravity has also been proposed due to loop quantum gravity [7, 8], discrete spacetime [9], string field theory [9], spacetime foam [10], spin-networks [11], non-commutative geometry [12, 13], and ghost condensation [14].

In gravity’s rainbow corrections to the thermodynamics of black holes occurs at very small scales [15, 16]. This modified thermodynamics of black holes agrees with the usual thermodynamics, for large black holes. However, it differs considerably for small black holes. This difference in the behavior of thermodynamics can have important consequences for the detection of mini black holes at the LHC [17]. The modification to the thermodynamics of black branes has also been studied using gravity’s rainbow [18]. The modification to the thermodynamics in Vaidya spacetime due to gravity’s rainbow has also been investigated [19, 20]. The modification to the thermodynamics of Yang-Mills black hole from gravity’s rainbow has also been studied [21]. It has been observed that these rainbow functions also modify the black holes in higher curvature gravity in a non-trivial way [22–24]. The modification to a dilatonic black hole in gravity’s rainbow has also been studied [25].

Thus, various different black holes have been studied due to gravity’s rainbow. However, most of the discussion has been limited to a rainbow function motivated from loop quantum gravity [26]. Here, we will use an alternative rainbow function, which is motivated from cosmological data. We will use the rainbow function motivated by using the hard spectra from gamma-ray bursters at cosmological distances [27]. This has been done by observing that the fine-scale time structure and hard spectra of gamma-ray bursters emissions are very sensitive to the possible dispersion of electromagnetic waves, and can be used to estimate their modification, which in turn can be absorbed in rainbow functions. We will observe how this changes the last stage of evaporation of black holes. It has been proposed that the
virtual black holes occur at the stages of evaporation of black holes [28–31]. These virtual black holes form like virtual particles due to quantum fluctuations. However, they also form due to the last stage of evaporation of black holes. Now if the last stage of evaporation of black holes is modified then the formation of virtual black holes will also be modified. In this paper, we will analyze the formation of virtual black holes in gravity’s rainbow. We would like to point out that it has been suggested that black hole information paradox might get resolved due to gravity’s rainbow [32, 33]. Now we propose that this will happen due to the formation of virtual black holes in gravity’s rainbow.

Now we will analyze the modification of thermodynamics for Schwarzschild black hole due to gravity’s rainbow. The temperature of such a black hole that can be obtain from the metric expressed in the form

\[ ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + h_{ij}dx^idx^j \]  

(1)

It is given by[34],

\[ T_0 = \frac{1}{4\pi} \sqrt{A(r_+)B(r_+)} \]  

(2)

where \( r_+ \) is given by the largest radius at which \( B(r) = 0 \). We will analyze Schwarzschild black hole in the presence of gravity’s rainbow. We will use the rainbow function motivated using the hard spectra from gamma-ray bursters at cosmological distances [27]

\[ f(E) = e^{\frac{\alpha(E_Ep)^n}{\alpha(E_Ep)^n - 1}} \quad g(E) = 1 \]  

(3)

The temperature of a black hole in gravity’s rainbow is modified by

\[ T = T_0 \frac{g(E)}{f(E)} \]  

(4)

For Schwarzschild black hole \( r_+ = 2M \), and so \( T_0 \) for Schwarzschild black hole is given by [35].

\[ T_0 = \frac{1}{8\pi M} \]  

(5)

According to [15, 16], the uncertainty principle \( \Delta p \geq \frac{1}{\Delta x} \) can be translated to lower bound on the energy of the particle emitted in Hawking radiation and the value of the uncertainty in position can be taken to be the event horizon radius as

\[ E \geq \frac{1}{\Delta x} \approx \frac{1}{r_+} \]  

(6)
Substituting values of $E$ and $M$ in terms of area $A$ the modified temperature of the Schwarzschild black hole becomes

$$T = \frac{\alpha}{8\pi M} \left( \frac{1}{r_+ E_p} \right)^n \frac{1}{e^{\alpha \left( \frac{1}{r_+ E_p} \right)^n} - 1}$$  \hspace{1cm} (7)$$

The normal and modified temperature of Schwarzschild black hole is plotted in figure (1). As can be seen from above equation that the temperature goes to zero at some finite value of surface area. That means the black hole stops radiating and remnant forms. To calculate the entropy, we use the first law of thermodynamics,

$$dS = \frac{dM}{T}$$ \hspace{1cm} (8)$$

For Schwarschild black hole, $A = 4\pi r_+^2$ and $r_+ = 2M$. Using above equation (8), we find the expression for entropy in gravity’s rainbow,

$$S = 2^{n-4} \alpha E_p \bar{n}^{(n/2-1)} \int \frac{(\frac{1}{E_p} \sqrt{\frac{1}{A}})^{n+1}}{\sqrt{A}(e^{\alpha \left( \frac{1}{E_p} \sqrt{\frac{1}{A}} \right)^n} - 1)} dA$$ \hspace{1cm} (9)$$
The thermodynamic stability of black holes is determined by the heat capacity $C_J$ at
constant angular momentum $J$, which is given by,

$$C_J = T \left( \frac{\partial S}{\partial T} \right)_J$$ \hspace{1cm} (10)

$$C_J = \frac{A^{n+1} \left( e^{2nA^{-n/2}E_p - n\pi^{n/2}\alpha} - 1 \right)^2 E_p^{2n\pi^{n/2}}}{2^{n+1}\alpha \left( -A^{n/2} \left( e^{2nA^{-n/2}E_p - n\pi^{n/2}\alpha} - 1 \right) E_p^n (n + 1) + 2^n e^{2nA^{-n/2}E_p - n\pi^{n/2}\alpha} n\pi^{n/2}\alpha \right)}$$ \hspace{1cm} (11)

FIG. 4. Specific heat $C_J$ versus area of Schwarzschild black hole in gravity’s rainbow and normal gravity

Now we will analyze a Kerr black hole in gravity’s rainbow. We will again use the rainbow function motivated from the hard spectra from gamma-ray bursters at cosmological distances [27]. Rotating black holes in flat and (A)dS space can be cast in the form[36],

$$ds^2 = -A(r, \theta)dt^2 + \frac{1}{B(r, \theta)} dr^2 + g_{\theta\theta} d\theta^2 + g_{\phi\phi} d\phi^2 - 2g_{t\phi} dt d\phi$$ \hspace{1cm} (12)

The temperature for this metric is given by[36, 37],

$$T = \frac{1}{4\pi} \sqrt{A_x(r_+, 0)B_x(r_+, 0)}$$ \hspace{1cm} (13)

Again $r_+$ is given by $B(r_+) = 0$. The metric for Kerr black hole is given by[38]

$$ds^2 = -dt^2 + \frac{2Mr}{\Sigma} (dt - a \sin^2 \theta d\phi)^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + (r^2 + a^2) \sin^2 \theta d\phi^2$$ \hspace{1cm} (14)
where $\Sigma = r^2 + a^2 \cos^2 \theta$ and $\Delta = r^2 + a^2 - 2Mr$. The temperature for Kerr black hole is given by\cite{37, 38}

$$T_0 = \frac{1}{2\pi} \left( \frac{r_+}{a^2 + r_+^2} - \frac{1}{2r_+} \right) \quad (15)$$

which modifies in gravity’s rainbow to be

$$T = \frac{1}{2\pi} \left( \frac{r_+}{a^2 + r_+^2} - \frac{1}{2r_+} \right) \frac{\alpha \left( \frac{1}{r_+ E_p} \right)^n}{e^{\alpha \left( \frac{1}{r_+ E_p} \right)^n} - 1} \quad (16)$$

To get the modified entropy from modified temperature we use the first law of thermodynamics,

$$dS = \frac{dM}{T} - \frac{\Omega}{T} dJ - \frac{\Phi}{T} dQ \quad (17)$$

For Kerr black hole,

$$\Omega = \frac{a}{a^2 + r_+^2}, \quad J = \frac{a(a^2 + r_+^2)}{2r_+}, \quad Q = 0 \quad (18)$$

After simplification, the modified entropy turns out to be,

$$S = \frac{2E_p \pi}{\alpha} \int r_+^{n+1} \left( e^{\alpha \left( \frac{1}{r_+ E_p} \right)^n} - 1 \right) dr_+ \quad (19)$$

The thermodynamic stability of black holes is determined by the heat capacity $C_J$ at constant angular momentum $J$. Using the modified temperature and entropy in equation (10), we plot the specific heat of Kerr black hole in figure (8),

FIG. 5. Temperature of Kerr black hole in normal gravity and gravity’s rainbow versus $r_+$
FIG. 6. Comparison of Kerr black hole’s temperature in gravity’s rainbow for different values of $n = 1, 2, 3, 4$.

FIG. 7. Entropy of Kerr black hole in normal and gravity’s rainbow
It was observed that at large scales, the modification due to rainbow functions can be neglected. However, at small scales, this modification produces important changes. Due to the modification by rainbow functions, both temperature, and entropy for these black holes reduces to zero at finite radius. As the temperature reduces to zero these black holes do not radiate and will not reduce in size. Thus, they will form black black remnants. The formation of black remnants due to rainbow functions from loop quantum gravity is well known [26]. However, here we have obtained this from rainbow functions motivated from hard spectra from gamma-ray bursters at cosmological distances. Thus, these results hold for astrophysical black holes. Furthermore, at this radius, the specific heat also vanishes, and this demonstrates that the black holes do not radiate with the surroundings. These will form the last stage of evaporation of black holes. It has been proposed that the virtual black holes occur at the stages of evaporation of black holes [28–31]. Thus, these black remnants will form stable virtual black holes in to gravity’s rainbow. These virtual black holes can be used to resolve information paradox [28–31]. Now we have observed that they will be stable due to rainbow functions, and this can resolve the information paradox. As the information can reside in these stable virtual black holes at the end of the evaporation of black holes. Unlike the usual picture, where a pair of black holes form and annihilate, here a form of virtual black holes form and do not annihilate. This is similar to what happens for particles in Hawking radiation. It would be interesting to investigate the consequences of this model,
and if it can be observed using any cosmological or astrophysical observation. So, in this letter, we analyzed the effect of rainbow function on the thermodynamics of black holes. This was done for Schwarzschild black hole and Kerr black holes. We used rainbow function motivated from the hard spectra from gamma-ray bursters at cosmological distances. We obtained black remnants due to this modification, and proposed them to be stable virtual black holes.

I. ACKNOWLEDGEMENT

M. V. Akram would like to thank Prof. S. Barve for useful discussions.

[1] OJ Rosten. Fundamentals of the exact renormalization group, phys, 2012.
[2] Nicholas P Warner. Renormalization-group flows from five-dimensional supergravity. Classical and Quantum Gravity, 17(5):1287, 2000.
[3] Claudia de Rham and Scott Melville. Gravitational rainbows: Ligo and dark energy at its cutoff. Physical review letters, 121(22):221101, 2018.
[4] Ahmed Farag Ali and Mohammed M Khalil. A proposal for testing gravity’s rainbow. EPL (Europhysics Letters), 110(2):20009, 2015.
[5] Mehdi Assanioussi and Andrea Dapor. Rainbow metric from quantum gravity: Anisotropic cosmology. Physical Review D, 95(6):063513, 2017.
[6] Seyed Hossein Hendi and Mir Faizal. Black holes in gauss-bonnet gravity’s rainbow. Physical Review D, 92(4):044027, 2015.
[7] G Amelino-Camelia, John Ellis, NE Mavromatos, and Dimitri V Nanopoulos. Distance measurement and wave dispersion in a liouville-string approach to quantum gravity. International Journal of Modern Physics A, 12(03):607–623, 1997.
[8] Giovanni Amelino-Camelia. Quantum-spacetime phenomenology. Living Reviews in Relativity, 16(1):1–137, 2013.
[9] G’t Hooft. Quantization of point particles in (2+ 1)-dimensional gravity and spacetime discreteness. Classical and Quantum Gravity, 13(5):1023, 1996.
[10] V Alan Kostelecky and Stuart Samuel. Spontaneous breaking of lorentz symmetry in string theory. *Physical Review D*, 39(2):683, 1989.

[11] Rodolfo Gambini and Jorge Pullin. Nonstandard optics from quantum space-time. *Physical Review D*, 59(12):124021, 1999.

[12] Sean M Carroll, Jeffrey A Harvey, V Alan Kostelecky, Charles D Lane, and Takemi Okamoto. Noncommutative field theory and lorentz violation. *Physical Review Letters*, 87(14):141601, 2001.

[13] Mir Faizal. Noncommutative quantum gravity. *Modern Physics Letters A*, 28(10):1350034, 2013.

[14] Mir Faizal. Spontaneous breaking of lorentz symmetry by ghost condensation in perturbative quantum gravity. *Journal of Physics A: Mathematical and Theoretical*, 44(40):402001, 2011.

[15] Ahmed Farag Ali, Mir Faizal, and Mohammed M Khalil. Remnants of black rings from gravity’s rainbow. *Journal of High Energy Physics*, 2014(12):1–14, 2014.

[16] Ahmed Farag Ali, Mir Faizal, and Mohammed M Khalil. Remnant for all black objects due to gravity’s rainbow. *Nuclear Physics B*, 894:341–360, 2015.

[17] Ahmed Farag Ali, Mir Faizal, and Mohammed M Khalil. Absence of black holes at lhc due to gravity’s rainbow. *Physics Letters B*, 743:295–300, 2015.

[18] Amani Ashour, Mir Faizal, Ahmed Farag Ali, and Fayçal Hammad. Branes in gravity’s rainbow. *The European Physical Journal C*, 76(5):1–9, 2016.

[19] Yaghoub Heydarzade, Prabir Rudra, Farhad Darabi, Ahmed Farag Ali, and Mir Faizal. Vaidya spacetime in massive gravity’s rainbow. *Physics Letters B*, 774:46–53, 2017.

[20] Prabir Rudra, Mir Faizal, and Ahmed Farag Ali. Vaidya Spacetime for Galileon Gravity’s Rainbow. *Nucl. Phys. B*, 909:725–736, 2016.

[21] Houcine Aounallah, Behnam Pourhassan, Seyed Hossein Hendi, and Mir Faizal. Five-dimensional Yang–Mills black holes in massive gravity’s rainbow. *Eur. Phys. J. C*, 82(4):351, 2022.

[22] Seyed Hossein Hendi, Ali Dehghani, and Mir Faizal. Black hole thermodynamics in Lovelock gravity’s rainbow with (A)dS asymptote. *Nucl. Phys. B*, 914:117–137, 2017.

[23] Seyed Hossein Hendi, Shahram Panahiyan, Behzad Eslam Panah, Mir Faizal, and Mehrab Momennia. Critical behavior of charged black holes in Gauss-Bonnet gravity’s rainbow. *Phys. Rev. D*, 94(2):024028, 2016.
[24] Seyed Hossein Hendi and Mir Faizal. Black holes in Gauss-Bonnet gravity’s rainbow. *Phys. Rev. D*, 92(4):044027, 2015.

[25] S. H. Hendi, Mir Faizal, B. Eslam Panah, and S. Panahiyan. Charged dilatonic black holes in gravity’s rainbow. *Eur. Phys. J. C*, 76(5):296, 2016.

[26] Giovanni Amelino-Camelia. Quantum-Spacetime Phenomenology. *Living Rev. Rel.*, 16:5, 2013.

[27] Giovanni Amelino-Camelia, John Ellis, NE Mavromatos, Dimitri V Nanopoulos, and Subir Sarkar. Tests of quantum gravity from observations of γ-ray bursts. *Nature*, 393(6687):763–765, 1998.

[28] Stephen W Hawking. Virtual black holes. *Physical Review D*, 53(6):3099, 1996.

[29] Stephen William Hawking and Simon F Ross. Loss of quantum coherence through scattering off virtual black holes. *Physical Review D*, 56(10):6403, 1997.

[30] Mir Faizal. Some aspects of virtual black holes. *Journal of Experimental and Theoretical Physics*, 114(3):400–405, 2012.

[31] Yoshiaki Ohkuwa, Mir Faizal, and Yasuo Ezawa. Virtual black holes in a third quantized formalism. *Annals of Physics*, 384:105–115, 2017.

[32] Ahmed Farag Ali, Mir Faizal, Barun Majumder, and Ravi Mistry. Gravitational Collapse in Gravity’s Rainbow. *Int. J. Geom. Meth. Mod. Phys.*, 12(09):1550085, 2015.

[33] Ahmed Farag Ali, Mir Faizal, and Barun Majumder. Absence of an Effective Horizon for Black Holes in Gravity’s Rainbow. *EPL*, 109(2):20001, 2015.

[34] Marco Angheben, Mario Nadalini, Luciano Vanzo, and Sergio Zerbini. Hawking radiation as tunneling for extremal and rotating black holes. *Journal of High Energy Physics*, 2005(05):014, 2005.

[35] Ahmed Farag Ali. Black hole remnant from gravity’s rainbow. *Physical Review D*, 89(10):104040, 2014.

[36] Zheng Ze Ma. Hawking temperature of kerr–newman–ads black hole from tunneling. *Physics letters B*, 666(4):376–381, 2008.

[37] Robert M Wald. *General relativity*. University of Chicago press, 2010.

[38] Natacha Altamirano, David Kubizňák, Robert B Mann, and Zeinab Sherkatghanad. Thermodynamics of rotating black holes and black rings: phase transitions and thermodynamic volume. *Galaxies*, 2(1):89–159, 2014.