Gravitational waves from neutron stars described by modern EOS

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Abstract. The frequencies and damping times of neutron star (and quark star) oscillations have been computed using the most recent equations of state available in the literature. We find that some of the empirical relations that connect the frequencies and damping times of the modes to the mass and radius of the star, and that were previously derived in the literature need to be modified.

Asteroseismology, that is, the study of stellar properties through the analysis of the proper oscillation frequencies, is a very useful tool. For instance, it has been successfully applied to study the internal composition of the Sun.

The oscillation modes of compact stars, like neutron stars (NS) or quark stars, give a clear signature in the spectrum of the gravitational waves that these stars may emit in several astrophysical processes. It should be stressed that, according to General Relativity the modes of compact stars are not normal modes, because they are damped by gravitational wave (GW) emission; for this reason they are called “quasi-normal modes” (QNM), with characteristic frequencies and damping times, that carry information on the structure of the star and on the behaviour of nuclear matter in the interior. The study of QNM is called gravitational wave asteroseismology [1].

Some years ago, Andersson and Kokkotas computed the frequencies and damping times of the most relevant oscillation modes [1] (that is, the modes that most likely would be excited by a perturbing event) of a non rotating NS for a number of equations of state (EOS) available at that time. They fitted the data with appropriate functions of the radius and the mass of the star, showing how these empirical relations could be used to put constraints on these parameters if the frequency of one or more modes could be identified in a detected gravitational signal. Knowing the mass and the radius, we would gain information on the behaviour of matter in a NS core, at density that cannot be reproduced in a laboratory.

In recent years, a number of new EOS have been proposed to describe matter at supranuclear densities, some of them allowing for the formation of a core of strange baryons and/or deconfined quarks. In ref. [2], that we summarize here, we have verified whether, in the light of the recent developments, the empirical relations derived in [1] are still appropriate or need to be updated.

We have considered a variety of EOS. For any of them we have obtained the equilibrium configurations for assigned values of the mass, and solved the equations of stellar perturbations computing the frequencies and damping times of the QNM. Then, we have...
fitted our data with suitable functions of M and R to see whether the fits agree with those of [1]. We have extended the results of [1] in two respects: we have considered more recent EOS, and we have studied a larger set of QNM.

A NS is believed to be composed mainly by three different layers of different composition: an outer crust, composed by heavy nuclei and free electrons; an inner crust, composed by heavy nuclei, free electrons and neutrons; a core, composed by leptons, nucleons, and, in some models, also hyperons or quarks. There is an overall agreement on the EOS describing the crust [3, 4], while the composition of the core is poorly known, due to the present limited understanding of hadronic interactions. We have modeled the crust as in [3, 4], and used various models for the matter in the core, which we summarize in Table 1.

TABLE 1. EOS included in our study

| Model   | Description                                                   |
|---------|---------------------------------------------------------------|
| APR2    | Akmal, Pandharipande, Ravenhall [5, 6]                        |
| APRB200, APRB120 | APR2 [5, 6] + quark inner core [7], [8]               |
| BBS1    | Baldo, Burgio, Schultze without hyperons [9]                 |
| BBS2    | Baldo, Burgio, Schultze with hyperons [9]                    |
| G240    | Glendenning, mean field approximation [10]                   |

In addition to the above models we have considered the possibility that a star entirely made of quarks (strange star) may form. The models denoted SS1 and SS2 correspond to a quark star, described by the MIT bag model [7], with or without a crust.

In order to find the frequencies and damping times of the quasi-normal modes, we have solved the equations describing non radial perturbations of a non rotating star in general relativity [11, 12]. In [2] we have considered several oscillation modes. Here we only report the results on the fundamental mode (f-mode), which gives the major contribution to GW emission.

The data we derived can be fitted by the following expressions:

\[
\nu_f = a + b \sqrt{\frac{M}{R^3}}, \quad \tau_f = \frac{R^4}{cM^3} \left[ c + d \frac{M}{R} \right]^{-1}
\]

with \( a = 0.79 \pm 0.09 \) kHz, \( b = 33 \pm 2 \) km kHz, \( c = [8.7 \pm 0.2] \cdot 10^{-2} \) and \( d = -0.271 \pm 0.009 \). Frequencies are expressed in kHz, masses and radii in km, damping times in s. The data for the f-mode and the fits are shown in Fig. 1. In the left panel we plot \( \nu_f \) versus \( \sqrt{\frac{M}{R^3}} \), for all considered stellar models. Our fit (1) is plotted as a thick solid line, and the fit computed by Andersson and Kokkotas in [1], which is based on the EOSs considered in that paper, is plotted as a dashed line labelled as ‘AK-fit’. In the right panel we plot the damping time \( \tau_f \) versus the compactness \( M/R \), our fit and the corresponding AK-fit. We can see that our new fit for \( \nu_f \) is systematically lower than the AK fit by about 100 Hz. Conversely, our fit for the damping time is very similar to that found in [1]. The empirical relations derived above can be used, as described in [1, 13], to determine the mass and the radius of the star from the knowledge of the frequency and damping time of the modes.

By comparing our results with the sensitivity curves of existing gravitational detectors, we have shown in [2] that it is unlikely that the first generation of interferometric antennas will detect the GW emitted by an oscillating neutron star. However, new detectors are under investigation that should be much more sensitive at frequencies above 1-2
FIGURE 1. The frequency of the fundamental mode is plotted in the left panel as a function of the square root of the average density for the different EOS considered in this paper. The new fit is systematically lower (about 100 Hz) than that previously derived in the literature. The damping time of the fundamental mode is plotted in the right panel as a function of the compactness $M/R$.

kHz and that would be more appropriate to detect these signals. If the frequencies of the modes will be identified in a detected signal, the simultaneous knowledge of the mass of the emitting star will be crucial to understand its internal composition.

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