Correlations between 21-cm radiation and the cosmic microwave background from active sources

Aaron Berndsen, Levon Pogosian, and Mark Wyman
1Department of Physics, Simon Fraser University, Burnaby, BC V5A 1S6, Canada
2Department of Physics & Astronomy, University of British Columbia, Vancouver, BC V6T 1Z1, Canada
3Perimeter Institute for Theoretical Physics, Waterloo, ON L8H 2A4, Canada

Accepted 2010 May 4. Received 2010 April 27; in original form 2010 March 16

ABSTRACT
Neutral hydrogen is ubiquitous, absorbing and emitting 21-cm radiation throughout much of the Universe’s history. Active sources of perturbations, such as cosmic strings, would generate simultaneous perturbations in the distribution of neutral hydrogen and in the cosmic microwave background (CMB) radiation from recombination. Moving strings would create wakes leading to 21-cm brightness fluctuations, while also perturbing CMB light via the Gott–Kaiser–Stebbins effect. This would lead to spatial correlations between the 21-cm and CMB anisotropies. Passive sources, like inflationary perturbations, predict no cross-correlations prior to the onset of reionization. Thus, observation of any cross-correlation between CMB and 21-cm radiation from dark ages would constitute evidence for new physics. We calculate the cosmic string-induced correlations between CMB and 21-cm radiation and evaluate their observability.

Key words: cosmic background radiation – cosmology: observations – dark ages, reionization, first stars – large scale structure of Universe.

1 INTRODUCTION
Neutral hydrogen emits and absorbs radiation at 21 cm in its rest frame throughout the history of the Universe. The brightness of the 21-cm line observed today from various redshifts is determined by the spatial distribution and peculiar velocity of the neutral hydrogen that emitted it. The radiation from any particular redshift has small angular variations, similar to those in the cosmic microwave background (CMB) radiation. There is also variation along the line of sight, with a correlation depth of ~10 Mpc (Eisenstein et al. 2005; Furlanetto, Oh & Briggs 2006). In the absence of physics that violates free streaming, one does not expect a correlation among slices in the line-of-sight direction that are separated by more than this correlation length. Since observations can measure the redshift of 21-cm emission precisely, we should be able to extract information about this line-of-sight variation from 21-cm observations.

It is this third dimension of information encoded in 21-cm radiation that can make it a powerful new tool to learn about fundamental physics. One path forward is to look for physics that violates free streaming and generates correlations among distinct redshift shells (Furlanetto et al. 2006). Several sources of such a signal have been identified, though they are limited to the epoch during and after reionization (Alvarez et al. 2006; Adshead & Furlanetto 2008; Slosar, Cooray & Silk 2007; Sarkar, Datta & Bharadwaj 2009). Though passive perturbations, such as those seeded by inflation, cannot generate such correlations, active sources of perturbation could. Active sources of perturbation include cosmic strings and other topological defects, such as textures (Turok 1989), as well as the field gradients that are left behind after global phase transitions (Jones-Smith, Krauss & Mathur 2008). Of these, cosmic strings are the most thoroughly studied and have perhaps the richest phenomenology (Myers & Wyman 2009), so they will be our focus.

Moving strings generate density wakes in the material through which they pass (Vilenkin 1981; Zeldovich 1980), including neutral hydrogen. As a result, the brightness of the 21-cm emission, which depends on the density and the velocity of the emitters, will be directly perturbed in the string’s wake (Khatri & Wandelt 2008). The same moving strings also perturb the background CMB light, monotonically shifting the spectra and creating line discontinuities in the temperature screen of the sky – a phenomenon known as the Gott–Kaiser–Stebbins (GKS; Kaiser & Stebbins 1984; Gott 1985) effect. GKS is a special case of the Integrated Sachs–Wolfe (ISW) effect (Sachs & Wolfe 1967) caused by time-varying gravitational potentials sourced by moving strings. Thus, there will be some correlation between string-sourced 21-cm brightness fluctuations at any redshift and a part of the CMB temperature anisotropy. Strings will also induce some cross-correlation in 21-cm fluctuations coming from different redshifts. However, as we discuss in Section 2, we expect this effect to be relatively small.
Correlations between 21-cm and CMB radiation

21-CM/CMB CORRELATION INDUCED BY COSMIC STRINGS

Cosmic strings are relativistically moving linear topological defects that are remnants of the high-energy early Universe. As they move, strings affect matter by kicking it into overdense wakes and perturb light by shifting its spectrum via the GKS effect. Strings form in the early Universe and remain active through all epochs, spreading their effects throughout the Universe’s history.

The effect of cosmic string networks on the CMB power spectrum has been well studied. Strings affect the CMB in both the ways described above: their wakes generate a single peak at an angular scale determined by the strings’ correlation length ($\lesssim c/H$) and strings in the foreground of the CMB generate an interlacing network of linear discontinuities through the GKS effect. The fact that these signatures have not been observed provide a bound on the density and tension of cosmic strings (Landriau & Shellard 2004; Pogosian et al. 2004; Wyman et al. 2005; Battye et al. 2008; Bevis et al. 2008).

These effects are also present during the dark ages, when the neutral hydrogen is releasing 21-cm radiation. Moving strings produce wakes and thus seed hydrogen density contrasts that grow and generate observable variations in the brightness temperature. Interestingly, though, the GKS effect is much less observable for 21-cm radiation. Strings shift the spectrum of light that passes them, so their effect on passing light is only observable when there is a noticeable change in the known background source, such as a shift in a spectral peak or discontinuity in a smoothly varying brightness. However, in the presence of a flat spectrum of background light, the string is invisible, since a lateral shift of a horizontal spectrum results in no detectable change. In the dark ages, strings will be swimming in a background of continuously renewed radio waves, with a very weak gradient in brightness and spectrum. Thus, the strings’ GKS effect on 21-cm radiation will be considerably less observable than the same effect applied to CMB photons.

To calculate these string-sourced effects quantitatively, we must give a formal characterization of the string perturbations. The most important difference between string-sourced perturbations and those from inflation, besides the active/passive distinction, is that strings source vector-mode perturbations. For strings, vector modes are as large as scalar modes, and they also generate much of the novel phenomenology of string networks (Pogosian et al. 2006; Pogosian & Wyman 2008). Vector modes are usually neglected since they decay in systems described by General Relativity and ordinary matter. Strings, however, actively source vector perturbations faster than they can decay away. We will reproduce the most relevant equations for vector-mode perturbations in Appendix A.

Calculating CMB and 21-cm observables also requires a string model and a code for tracing the history of light streaming to us now from these early times. To this end, we have incorporated a model for strings (Albrecht et al. 1997, 1999; Pogosian & Vachaspati 1999; Pogosian et al. 2006; Pogosian 2009; see Section 2.1.1 for a description) into the 21-cm extension of CAMB (Lewis et al. 2000; Lewis & Challinor 2007); this code will be publicly released (Berndsen & Pogosian, in preparation). Our major addition to the underlying CAMB code is the inclusion of vector-mode fluctuations. Details about how string-sourced vector modes enter the Boltzmann equation can be found in Albrecht et al. (1997, 1999), Hu & White (1997), Pogosian & Vachaspati (1999), Pogosian et al. (2006) and Lewis & Challinor (2007), and will be more comprehensively explained in Berndsen & Pogosian (in preparation).

Now we can ask what physics will control the shape and amplitude of string-sourced-two-point functions in the 21-cm fluctuations and in the cross-correlation. The two physical effects controlling the shape of these spectra are (1) the intrinsic brightness of the 21-cm photons and (2) the correlation length of the string network. The brightness temperature sets the amplitude of the spectrum. It is, in turn, determined by the difference between the 21-cm spin temperature and the CMB temperature at that epoch, which are mismatched over a long period during the dark ages (Furlanetto et al. 2006). It is worth noting that the brightness temperature is negative throughout most of the dark ages. That is, the neutral hydrogen is absorbing.

1 We thank Mark Halpern for a discussion of this effect.
not emitting, radiation. This shows up as an observable because we can infer what the flux of long wavelength CMB photons should be and hence detect the dimming due to neutral hydrogen. Since we are looking for small variations in the brightness temperature, a large intrinsic brightness makes detections easier. The anisotropy of 21-cm brightness is chiefly caused by variations in the density and the peculiar velocity of the neutral hydrogen, both of which are sourced by cosmic strings. These effects and others are more comprehensively discussed in Lewis et al. (2000) and Lewis & Challinor (2007).

The correlation length of the string network sets the location of the peak in the \( \ell \) space, the angular scale. Since this correlation length is some fraction of the Hubble scale, the spectrum from dark ages strings peaks for multipoles \( \ell < 100 \). This peak in the two-point function corresponds to the size of the wakes generated by the string network at that epoch. Hence, the peak signal migrates to lower-\( \ell \) as the redshift decreases since the network is scaling with the horizon, and the horizon is growing.

Strings generate a cross-correlation between matter overdensity and the CMB with similar power throughout all epochs. However, the brightness temperature of the neutral hydrogen peaks at a redshift of \( z \approx 50 \). The signal is a product of these two trends: a flat cross-correlation power multiplied by a redshift-dependent brightness temperature. The brightness temperature evolves via competing gas physics. At early times, neutral hydrogen collision times are short enough to couple the spin and gas temperatures, resulting in a net deficit of 21-cm photons against the CMB and, hence, a growing, negative brightness temperature (Seager, Sasselov & Scott 1999). At later times, the gas becomes diffuse and the spin temperature is driven towards the CMB temperature, resulting in a weaker, then vanishing, brightness temperature. The net result is a peak in brightness temperature at a redshift \( z \approx 50 \) (cf. Lewis & Challinor 2007 for a detailed discussion). It is not until after reionization that the spin temperature exceeds the CMB temperature and the 21-cm signal can be seen in emission.

Both trends can be seen in Fig. 1, a plot of the CMB–21-cm cross-correlation \( C_{\ell}^{XY} \) from a string network, which we have generated with our code.

### 2.1 The string model

The string code we used is based on the segment model (Albrecht et al. 1997, 1999; Pogosian & Vachaspati 1999; Pogosian et al. 2006). In this model, the string network is represented by a collection of uncorrelated straight string segments moving with uncorrelated, random velocities. There are two fundamental length-scales in such a model: \( \xi \), the length of a string segment, which represents the typical length of roughly straight segments in a full network, and \( \xi' \), the typical length between two string segments, which sets the number density of strings in a given volume \( N_s \propto 1/\xi^2 \). The model also tracks the root-mean-square (rms) velocity of the string segments. These parameters are deduced by assuming network scaling – i.e. the string energy density tracks the dominant background energy density in the radiation and matter eras. Numerical simulations of networks have confirmed that these assumptions accurately describe network physics. The positions and orientations of the segments are drawn from uniform distributions in space and on a 2-sphere, respectively. The model’s parameters have been calibrated to produce source correlation functions in agreement with those in Vincent et al. (1997). Because of the approximations built into the string segments, the model can only produce two-point correlation functions. It cannot be used for making simulated string maps or other more sophisticated observables, though it could, with some modification, generate three-point functions (Gangui, Pogosian & Winitzki 2001).

On the cosmological scales we consider, finer details of the string evolution do not play a major role. It is the large-scale properties – such as the scaling distance, the equation of state (wiggliness) and the rms velocity – that determine the shape of the string-induced spectra. Further details of the computational methods necessary to compute the 21-cm signal will be included in a subsequent publication (Berndsen & Pogosian, in preparation).

### 2.2 Cross-correlations

Let \( \delta T_x(x, \hat{n}) \) denote the anisotropy in the 21-cm brightness temperature emitted at redshift \( z \) and observed at a spatial position \( x \) coming from the propagation direction \( \hat{n} \). Similarly, the CMB brightness temperature is characterized by \( \delta T_{\text{CMB}}(x', \hat{n}') \). For computational purposes, these quantities are expanded into normal modes (Hu & White 1997):

\[
\delta T^X(x, \hat{n}) = \int \frac{d^3k}{(2\pi)^3} \sum_{\ell} \sum_{m=-2}^{2} \delta T^X_{\ell m}(k) G^m_{\ell},
\]

where \( k \) is the Fourier conjugate to \( x \), \( X \in \{z, \text{CMB}\} \) and \( \delta T^X_{\ell m} \) are the associated multipole moments in the basis

\[
G^m_{\ell} = (-i)^f \sqrt{\frac{4\pi}{2\ell + 1}} Y^m_{\ell} \exp (i k \cdot x).
\]

In our analysis, we use the angular correlation function between quantities \( X \) and \( Y \) as measured by an observer at \( x = 0 \), defined as

\[
C_{XY}(\theta) = \langle \delta T^X(\hat{n}) \delta T^\text{CMB}(\hat{n}') \rangle_{\hat{n}' = \cos \theta} = \frac{1}{4\pi} \sum_{\ell = 0}^{\infty} (2\ell + 1) C^X_{\ell} P_\ell (\cos \Theta),
\]

where \( P_\ell \) are the Legendre polynomials and \( C^X_\ell \) is the anisotropy spectrum calculated from the associated multipoles.
\( \delta T^X(k) \) (Seljak & Zaldarriaga 1996; Lewis et al. 2000; Lewis & Challinor 2007) as

\[
C^X_\ell = (4\pi)^2 \int d^3k \sum_{m=-2}^2 \delta T^X_{im} \delta T^Y_{em}.
\]

(4)

In the above notation, \( m = 0 \) corresponds to scalar modes, \( m = \pm 1 \) labels vector modes of right-handed and left-handed vorticity and \( m = \pm 2 \) denotes the tensor modes. See Appendix A for an expanded discussion of our calculational method.

### 3 SOURCES OF NOISE

Having formed the cross-correlation spectrum (equation 4), a density distribution of S/N can be formed in redshift and \( \ell \) space as

\[
\left( \frac{S}{N} \right)^2 (z, \ell, \Delta z) = \frac{f_{\text{sky}}(2\ell + 1) (C^X_\ell)^2}{C^T_\ell C^z_\ell + (C^z_\ell)^2},
\]

(5)

where \( \Delta z \) is the width of the redshift shell set by the chosen frequency interval and \( f_{\text{sky}} \approx 0.8 \) is the fraction of observable sky. The total S/N is then

\[
\left( \frac{S}{N} \right)^2_{\text{tot}} = \sum_i \sum_{\Delta z_i} \left( \frac{S}{N} \right)^2 (z_i, \ell, \Delta z_i).
\]

(6)

where the sum is over the harmonic multipoles \( \ell \) and the (independent) observed redshift slices \( \Delta z_i \), determined by the observational set-up. Each multipole can receive contributions from the inflationary background \( C^X_\ell \), the active sources \( C^T_\ell \) and astrophysical and telescope noise \( C^z_\ell \), so we can write

\[
C^X_\ell [\text{total}] = C^X_\ell [\text{sky}] + C^X_\ell [\text{tot}] + C^X_\ell [\text{noise}].
\]

(7)

Strings are the only source of the cross-correlation, so the numerator in equation (5) receives only one contribution. The denominator, however, receives a large contribution from inflationary perturbations that are at least 10 times the size of the string power spectrum. There is also a large contribution from the detector noise which is dominated by the Galactic emission at the low frequencies we need to probe the high-redshift 21-cm brightness temperature. The detector noise is uncorrelated between the CMB and 21-cm frequencies and is given by

\[
C^N_\ell = \frac{(2\pi)^3 T^2_\text{sys}(v)}{\Delta v f_{\text{obs}} f^2_{\text{cover}} f^2_{\text{max}}(v)}. \tag{8}
\]

where \( T_\text{sys} \) is dominated by the sky temperature and, for regions away from the Galactic plane, can be approximated as (Furlanetto et al. 2006)

\[
T_\text{sys}(v) = 150 \text{K} \left( \frac{v}{180 \text{ MHz}} \right)^{-2.6}. \tag{9}
\]

In the previous expressions \( \Delta v \) is the width of a frequency bin, which is set by the experiment, \( f_{\text{obs}} \) is the observing time, \( f_{\text{cover}} \equiv N_{\text{dish}} A_{\text{obs}} / A_{\text{total}} \) is the fraction of the total area enclosed by the experiment and characterizes the total collecting area and \( f_{\text{max}}(v) = 2\pi D_{\text{array}} v / c \), where \( D_{\text{array}} \) is the diameter of the array and \( c \) is the speed of light.

Foregrounds include galactic and extragalactic point sources, as well as the galactic and extragalactic free-free emission, all of which are expected to induce correlations between different frequencies (Zaldarriaga, Furlanetto & Hernquist 2004; Santos, Cooray & Knox 2005). In this study, however, astrophysical sources will have a negligible effect since we are only including 21-cm cross-correlations with CMB photons, and these signals quickly decorrelate with increasing frequency separation (Furlanetto et al. 2006). Furthermore, these signals dominate on very small scales while string-induced correlations reside on large scales, \( \ell \lesssim 100 \).

We fold the observational parameters into a single observational ambition parameter, \( x \), defined as

\[
x = \text{obs} f^2_{\text{cover}} D^2. \tag{10}
\]

with units of \( [x] = [\text{yr}] \times [\text{km}]^2 \). There is always a practical trade-off between making \( f_{\text{cover}} \) large and making \( D \) large. For our signal, though, it is clear what we should choose. The main reason for going to large \( D \) is to resolve very small angular scales. However, string-sourced cross-correlations reside only on large angular scales. Hence, we can make due with a relatively small baseline \( D \), which should let us make \( f_{\text{cover}} \) nearly unity; i.e. a ‘moderately’ sized telescope or array with nearly a complete covering area would be ideal for searching for strings.

### 4 RESULTS

We have calculated cross-correlation power spectra from redshifts throughout the dark ages. In producing our results, we have fixed the observing bandwidth to \( \Delta v = 0.5 \text{ MHz} \); however, varying this does not change our result. Although increasing the bandwidth diminishes the autocorrelator signal (noise ‘\( N \)’ \( \propto 1/\Delta v \)) and thus increases the S/N in each redshift bin, one observes fewer independent redshift slices as a result (signal ‘\( S_{\text{tot}} \)’ \( \propto 1/\Delta v \)) so the overall S/N remains unchanged to first order. We have numerically verified this for bandwidths in the range 0.01 MHz < \( \Delta v < 2 \text{ MHz} \).

We have excluded the epoch of reionization (\( z < 20 \)) from our S/N results. There are several reasons for this. The main reason is that reionization physics is still too uncertain for reliable calculations to be done, and the details of reionization, once begun, will dominate the 21-cm physics. Moreover, if cosmic strings are present during reionization, they will likely play a role in speeding up the process of reionization through their early generation of non-linear overdensities (Rees 1986; Pogosian & Vilenkin 2004). Reionization itself will also create cross-correlations between the CMB and 21-cm radiation (Alvarez et al. 2006; Adshead & Furlanetto 2008; Slosar et al. 2007; Sarkar et al. 2009). Though it is possible that the string and reionization signals could be disentangled if they are both present, we restrict ourselves in this study to redshifts where strings are the only possible source of cross-correlation.

At redshifts \( z \lesssim 7 \), reionization is complete and the surviving neutral hydrogen only exists in islands that are presumably correlated with large-scale structure. In principle, we can continue calculating the string-sourced cross-correlation between the matter distribution (traced via 21-cm or any other observable light) and the CMB. However, there will be competing sources of cross-correlation. For instance, the rescattering of CMB photons off ionized particles will induce a correlation of CMB with the large-scale plasma distribution (Gianantonio & Crittenden 2007). Eventually, at lower redshifts, there will be a correlation between large-scale structure and the ISW distortion of the CMB caused time-varying potentials when dark energy becomes dynamically important. None the less, we have made an approximate calculation of what a hypothetical radio survey at these redshifts might see in the presence of strings, extending an analytical distribution of large-scale structure tracers used in Hu & Scranton (2004) to the range 0 < \( z < 7 \). The resulting theoretical S/N for the cross-correlation was too small. This is not surprising since string wakes that are sourced at such late times have not had time to grow. By contrast, the potential wells that attract
the ionized particles which then rescatter CMB have been growing through gravitational instability for some time since entering the horizon. Hence, we do not include the $z \leq 7$ range into our S/N results.

Let us turn to our results, shown in Fig. 2, which begin with analysis of a hypothetical Universe where inflationary perturbations are not present and cosmic strings are the only source of both CMB and 21-cm anisotropies. We plot the cross-correlation S/N distribution for this case in Fig. 2(a). When only strings are present and noise is disregarded, as we do here, the overall normalization, the string tension $G\mu$, drops out of the S/N calculation. In this imaginary Universe, the signal is so strong that an observation at each redshift would generate a detection $\left[ (S/N)(\ell, z, \Delta z) > 1 \right]$.

In the real Universe, the inflationary perturbations overwhelm those from strings, as we see in Fig. 2(b), even without including the detector noise. No single measurement now has significant S/N ($\gg 1$). Instead, we must sum over many independent redshift bins in order to accumulate a significant signal, giving a statistical detection. The case presented in this figure represents the best-case scenario: no noise and a maximal, 10 per cent contribution to primordial anisotropy from strings. The signal is similar in shape to, though lower in magnitude than, the idealized case of a system seeded only by strings. The signal is suppressed more significantly on small angular scales, since the inflationary autocorrelations monotonically increase for $\ell$, while the string-induced signal peaks at $\ell \approx 60$. Although any particular observation in the $z-\ell$ plane will result in a contribution $(S/N)^2 \approx (10^{-3})$, the statistical benefit of observing roughly 50 multipoles ($2 < \ell < 50$) in roughly 50 redshift bins (observations with a 1-MHz bandwidth in the range $20 < z < 120$) gives rise to a total $(S/N)^2 \approx \sqrt{(50)^2(10^{-3})} \approx (1)$. Summing our simulated data over all $\ell$ and over $70 \leq z \leq 20$ reveals a $(S/N) \approx 3$. However, this does not include experimental noise, which corresponds to a survey-strategy parameter of $x = \infty$.

Fig. 2(c) shows the remaining signal once the sky temperature is incorporated. These data were generated assuming an all-sky survey from a single dish of diameter 1 km observed for 10 yr ($x = 10$); note that sky temperature completely overwhelms the signal for redshift $z > 50$. The estimated S/N remaining in our analysis volume is $(S/N) \approx 0.28$.

### 5 Observational Outlook

Finally, let us look at Fig. 3, which summarizes the detectability of our signal for various string tensions, $G\mu$, as a function of our observational parameter, $x$. It is clear that the noise induced by the large-sky temperatures severely limits the possibility of observing a string network through cross-correlation. However, the calculation reported here does not exhaust the possible locations of the cross-correlation signal from strings. For instance, our estimate is restricted to the domain $z > 20$ that we are certain will not be affected by reionization. Our present ignorance about the physics of reionization means that we cannot trust our calculation into the $z < 20$ range. This is a major impediment, though: lower $z$ is the redshift range least affected by noise, since noise goes as an inverse power law with frequency. Hence, we can hope to boost our signal significantly once the physics of reionization are better understood.

We have also only looked at the CMB–21-cm cross-correlations in this study. Although they are likely very small, we are currently analysing contributions from the $O(10^5)$ 21-cm–21-cm cross-correlations. These will also give some boost to our signal.

---

**Figure 2.** Distribution of $(S/N)^2$ from cross-correlation studies between CMB and 21-cm photons in the presence of a string network. The observing bandwidth has been fixed to $\Delta v = 0.5$ MHz, which determines the redshift bin width $|\Delta z| = (1 + z)^2 \Delta v/v_0$ being the rest frequency of the 21-cm radiation. The signal along horizontal slices is modulated by the brightness temperature, which nulls around the epoch of first light as the H$_\text{I}$ regions transit from absorption to emission. The signal along vertical slices is set by the correlation length of the network, a fraction of the Hubble scale. (a) An idealized situation where structure is seeded only by strings, in the absence of foreground noise. Contours of 0.5 and 1.0 are shown. (b) and (c): $G\mu = 4 \times 10^{-5}$, representing a system primarily seeded by inflation, but with a 10 per cent contribution from strings. (b) includes no sky noise and has contours representing 0.001 and 0.002. (c) includes noise and assumes 10-yr observation from a square kilometre array ($D = 1$ km, $x = 10$); recall that $x \equiv \ell_{\text{rad}}^2/\Omega^2$. Contours of 1, 2, 3 and $4 \times 10^{-5}$ are shown. These plots primarily serve to indicate where the signal resides in the observing volume.
6 DISCUSSION

By incorporating the segment model of strings into the CAMB-sources Einstein–Boltzmann code, we have calculated the cross-correlation between the 21-cm brightness temperature and the CMB. Our chief result is plotted in Fig. 3 which represents the S/N available in CMB–21 cm correlation studies given an all-sky survey, a string tension $G\mu$ and our observing strategy, which we characterize by a single integration time and telescope properties parameter, $x$. Unfortunately, noise from the sky temperature overwhelms the signal we are calculating for observationally allowed cosmic string tensions.

However, there is room for more work. We have thus far only included the signal from the CMB–21 cm correlations for $z > 20$, where we do not need to know the physics of reionization. Since the signal should extend until the epoch of reionization $z \approx 10$, we may eventually be able to include $O(10^3)$ more redshift bins and $O(10^5)$ data points in the $z$–$t$ plane, once we can accurately model reionization. Another contribution to our signal that was neglected in this study can be found from a cross-correlation study between different redshift bins, and this signal will be addressed in future work (Berndsen & Pogosian, in preparation). It is possible that we will find a detectable signal when these extra sources of data are included in the analysis.

Though our calculations have not predicted a detectable signal, we reiterate that any detection of a cross-correlation between the 21-cm radiation from $z > 20$ and the CMB would be a clear sign of new physics. Active sources, such as cosmic strings, are a promising candidate for the sort of new physics that could generate this correlation. However, the fact that inflation very likely provides the dominant source of perturbations in the Universe makes detectability of any signal difficult: the inflationary perturbations typically much larger than any alternate sources of perturbation, at least prior to recombination where measurements of the CMB limit any non-inflationary perturbations. One way around this would be to contrive some active source that was suppressed prior to recombination which subsequently sourced a larger fraction of perturbations in the dark ages. On the other hand, a non-detection of such a cross-correlation would constitute a strong constraint on active sources with such unusual properties.

ACKNOWLEDGMENTS

AB is supported by the National Sciences and Engineering Research Council of Canada (NSERC) National Postdoctoral Fellowship. LP is supported by an NSERC Discovery grant. The work of MW at the Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research & Innovation.

REFERENCES

Adshedd P., Furlanetto S., 2008, MNRAS, 384, 291
Albrecht A., Battye R. A., Robinson J., 1997, Phys. Rev. Lett., 79, 4736
Albrecht A., Battye R. A., Robinson J., 1999, Phys. Rev. D, 59, 023508
Alvarez M. A., Komatsu E., Dore O., Shapiro P. R., 2006, ApJ, 647, 840
Battye R. A., Garbrecht B., Moss A., Stoica H., 2008, J. Cosmol. Astropart. Phys., 0801, 020
Bevis N., Hindmarsh M., Kunz M., Ureštilla J., 2008, Phys. Rev. Lett., 100, 021301
Eisenstein D. J. et al., 2005, ApJ, 633, 560
Furlanetto S., Oh S. P., Briggs F., 2006, Phys. Rep., 433, 181
Gangui A., Pogosian L., Winitzki S., 2001, Phys. Rev. D, 64, 043001
Ghannantionio T., Crittenden R., 2007, MNRAS, 381, 819
Gott J. R. III, 1985, ApJ, 288, 422
Hu W., Scrandon R., 2004, Phys. Rev. D, 70, 123002
Hu W., White M. J., 1997, Phys. Rev. D, 56, 596
Jones-Smith K., Krauss L. M., Mathur H., 2008, Phys. Rev. Lett., 100, 131102
Kaiser N., Stebbins A., 1984, Nat, 310, 391
Khatri R., Wandelt B. D., 2008, Phys. Rev. Lett., 100, 091302
Landriau M., Shellard E. P. S., 2004, Phys. Rev. D, 69, 023003
Lewis A., Challinor A., 2007, Phys. Rev. D, 76, 083005
Lewis A., Challinor A., Lasenby A., 2000, ApJ, 538, 473
Myers R. C., Wyman M., 2009, in Eridmejer J., ed., String Cosmology. Wiley, Hoboken, p. 121
Pogosian L., 2009, http://www.sfu.ca/~levon/cmbact.html
Pogosian L., Vachaspati T., 1999, Phys. Rev. D, 60, 083504
Pogosian L., Vilenkin A., 2004, Phys. Rev. D, 70, 063523
Pogosian L., Wyman M., 2008, Phys. Rev. D, 77, 083509
Pogosian L., Wyman M., Wasserman L., 2004, J. Cosmol. Astropart. Phys., 09, 008
Pogosian L., Wasserman L., Wyman M., 2006, preprint (astro-ph/0604141)
Rees J. M., 1986, MNRAS, 222, 27
Sachs R. K., Wolfe A. M., 1967, ApJ, 147, 73
Santos M. G., Cooray A., Knox L., 2005, ApJ, 625, 575
Sarkar T. G., Datta K. K., Bharadwaj S., 2009, J. Cosmol. Astropart. Phys., 0908, 019
Seager S., Sasselov D. D., Scott D., 1999, ApJ, 523, L1
Seljak U., Zaldarriage M., 1998, ApJ, 496, 437
Slosar A., Cooray A., Silk J., 2007, MNRAS, 377, 168
Turok N., 1989, Phys. Rev. Lett., 63, 2625
Vilenkin A., 1981, Phys. Rev. Lett., 46, 1167
Wilson G. R., Hindmarsh M., Sakellariadou M., 1997, Phys. Rev. D, 55, 573
Wyman M., Pogosian L., Wasserman L., 2005, Phys. Rev. D, 72, 023513
Zaldarriaga M., Furlanetto S. R., Hernquist L., 2004, ApJ, 608, 622
Zeldovich Y. B., 1980, MNRAS, 192, 663

APPENDIX A: 21-CM VECTOR MODES

In the absence of active sources vector perturbations quickly decay and, for that reason, are frequently ignored. In particular, they are not included in the 21-cm extension to CAMB (Lewis et al. 2000; Lewis & Challinor 2007). Cosmic strings, however, actively source vector modes and provide a significant contribution to CMB and 21-cm anisotropies. A large effort was spent incorporating their
effects into our numerical code (Berndsen & Pogosian, in preparation), and we elaborate on some of the details here.

Applying the harmonic decomposition described in Hu & White (1997) and equation (2) to (fluctuations in) the photon distribution function $f$ provided in Lewis et al. (2000) and Lewis & Challinor (2007) gives scalar $m = 0$, vector $m = \pm 1$ and tensor $m = \pm 2$ contributions to the expanded field:

$$\delta f(\eta, x, \epsilon, \hat{n}) = \int \frac{d^3k}{(2\pi)^3} \sum_{l,m} F_l^m(\eta, \epsilon, k) \delta G_l^m. \quad \text{(A1)}$$

The photon fluid is characterized at some conformal time $\eta$ and position $x$ as photons coming from some direction $\hat{n}$ with an energy $\epsilon = a(\eta)E_{\text{21}}$. The quantity $\epsilon$ represents the Doppler-shifted energy of a 21-cm photon, $E_{\text{21}}$, stretched by the scalefactor $a(\eta)$. $k$ is the Fourier conjugate to the spatial coordinate $x$; we denote $k = |k|$. Expressions for the harmonic multipoles $F_l^m$ are formally obtained by integrating the Boltzmann equation for 21-cm photons along the line of sight (Seljak & Zaldarriaga 1996). The scalar contribution $F_l^0$ has been reported in Lewis et al. (2000) and Lewis & Challinor (2007), while we report the vector modes in equation (A2):

$$F_l^{(1)}(\eta, \epsilon, k) \simeq -e^{-\tau_c} \bar{f}(\epsilon) \frac{r_l}{H_{\epsilon}} \left( j_l(k \chi_c) \frac{k \chi_c}{k \chi_c} - j_l(k \chi_c) \right)$$

$$+ \int_{\eta}^{\eta_{\text{obs}}} d\eta' \tau_c e^{\tau_c} \bar{f}_{\text{in}}(\epsilon) v_{(1)}^l \frac{j_l(k \chi_c)}{k \chi_c}$$

$$- e^{-\tau_c} \left\{ \bar{f}(\epsilon) \left[ v_{(1)}^l + \frac{T_v}{T_v - T_p} (v_{(1)}^l - v_{(1)}^l) \right] \right.$$

$$+ \frac{r_l}{h} \left[ (r_c - 1) v_{(1)}^l + j_{\text{in}}(\epsilon) v_{(1)}^l \right] \frac{j_l(k \chi_c)}{k \chi_c}$$

$$- e^{-\tau_c} \bar{f}(\epsilon) \frac{T_v}{T_v - T_p} \sum_{l' = 2}^{\infty} \Theta_{l'} P_{l'} \left( \frac{-i}{k} \frac{d}{d\chi_c} \right) j_{l'}(k \chi_c)$$

$$+ \int_{\eta}^{\eta_{\text{obs}}} d\eta' \tau_c e^{\tau_c} \bar{f}_{\text{in}}(\epsilon) \frac{\sqrt{3} F_2}{4}$$

$$\times \frac{j_l(k \chi_c)}{k \chi_c} \left. \right). \quad \text{(A2)}$$

This expression holds for $l \geq 1$, where a prime denotes a derivative with respect to the argument and a bar denotes a background value. As well, $\tau_c$ is the Thomson scattering optical depth, $\tau_c$ is the optical depth to 21-cm radiation, $v^{(1)}$ is the vector component to the baryon velocity, $u^{(1)}$ is the velocity (dipole) of the photon distribution, $\bar{f}$ is the background photon distribution function, $T_v$ is the photon temperature, $T_p$ is the spin-flip temperature of the neutral hydrogen and determines how much the hydrogen is emitting, $r_c = \tau_c \left( 1 - e^{-\tau_c} \right)$, $H_\epsilon$ is the conformal Hubble parameter, $\chi_c = \eta_b - \eta_c$ is the conformal distance along the line of sight and $j_l(k \chi_c)$ is the spherical Bessel function. Quantities subscripted with $\epsilon$ are evaluated at the conformal time $\eta_c$. The $\Theta_{l'}$ are the angular moments of the Fourier expansion of the CMB temperature anisotropy. All quantities are discussed at length in Lewis et al. (2000) and Lewis & Challinor (2007).

The 21-cm power spectra referenced in the main text were given in terms of the brightness temperature $T_b$. This relates to the photon distribution function $f$ through the relation

$$T_b = \frac{E_{\text{obs}} h_p}{k_B} f(\eta, x, \epsilon, \hat{n}). \quad \text{(A3)}$$

where $E_{\text{obs}}$ is the observed frequency of the 21-cm emission, $h_p$ is the Planck constant and $k_B$ is the Boltzmann constant. Thus, anisotropies in the 21-cm brightness temperature emitted at a redshift $z$ are given as

$$\delta T^i(x, \hat{n}) = \frac{E_{\text{obs}} h_p^2}{2k_B} \delta f(\eta, x, \epsilon, \hat{n}). \quad \text{(A4)}$$

This paper has been typeset from a \TeX\/LaTeX\ file prepared by the author.