Gravitational Corrections to $\Phi^4$ Theory with Spontaneously Broken Symmetry

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We consider a complex scalar $\Phi^4$ theory with spontaneously broken global $U(1)$ symmetry, minimally coupling to perturbatively quantized Einstein gravity which is treated as an effective theory at the energy well below the Planck scale. Both the lowest order pure real scalar correction and the gravitational correction to the renormalization of the Higgs sector in this model have been investigated. Our results show that the gravitational correction renders the renormalization of the Higgs sector in this model inconsistent while the pure real scalar correction to it leads to a compatible renormalization.

PACS numbers: 04.60.-m, 11.10.Gh, 11.30.Qc

I. INTRODUCTION

It is well known that Einstein’s general relativity quantized on a fixed background is not a renormalizable quantum field theory since the mass dimension of its coupling constant $\kappa = \sqrt{32\pi G_N}$ is negative [1,2]. Furthermore, the coupling of the Einstein-Hilbert theory to any type of matter fields leads to nonrenormalizable theories as well [2–4]. Provided that our interest is restricted to the physics at the energy well below the Planck scale $M_{\text{Planck}} = G_N^{-1/2} \approx 10^{19}$GeV, perturbatively quantized gravity can be treated as an effective field theory, which has been established by Donoghue [5]. It can provide the results that should coincide with the results predicted by the underlying fundamental quantum theory of gravitation.

In their initiative paper [6], within the framework of the effective quantum theory of gravity, Robinson and Wilczek presented a calculation that claimed the behavior of running gauge coupling constants in Abelian and non-Abelian gauge theories could be altered by quantum gravitational correction which would render all gauge theories asymptotically free. However, doubts have been cast on their conclusion by some authors and the result has been studied from different approaches. After careful reconsideration of the calculation, Pietrykowski [7] first showed that Robinson and Wilczek’s result was not gauge condition independent, and that the gravitational correction to the $\beta$ function at one-loop order was absent in the harmonic gauge. Using a gauge condition independent background field method [8], along with dimensional regularization (DR) [9], Toms [10] showed that it did not lead to nonvanishing gravitational contributions to the running of gauge coupling constants. This result has been confirmed by a traditional Feynman diagram approach calculation using standard Feynman rules in Ref.[11], where if a momentum space cutoff was used the quadratic divergences could be made to cancel, leaving a result that was consistent with DR. Subsequent works of Toms et al. have investigated the cases with the cosmological constant [12, 13]. Further, various approaches were used to discuss the applications to the gravitational corrections to a series of theories [14–30].

In Ref.[22], the authors concluded that outside of some special cases, such as the ordinary $\varphi^4$ interaction, the gravitational contribution to the running coupling constant of other theories, such as Yukawa and gauge theories, is not a useful and universal idea in the perturbative regime. So special attention should be paid to the gravitational corrections to the $\varphi^4$ interaction. Moreover, Rodigast and Schuster [16] considered gravitational corrections to the ordinary $\varphi^4$ interaction and related the real scalar field to the Higgs boson (hereafter Higgs field is denoted by $H$). However, since there is not only $H^4$ interaction but also $H^3$ interaction in the standard model (SM) because of the spontaneous breaking of electroweak $SU(2)_L \times U(1)_Y$ symmetry, a relatively physical SM-like case for the Higgs sector is a model in which the three real scalars interaction as well as the four real scalars interaction should be included.

Motivated by this, following the approach of Ref.[16], we investigate $\Phi^4$ theory with spontaneously broken global $U(1)$ symmetry, minimally coupling to perturbatively quantized gravity which is treated as an effective theory. We study

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the lowest order pure real scalar correction and gravitational correction to the renormalization of the Higgs sector in this model. It has been found that the gravitational correction renders the renormalization of this model inconsistent while the pure real scalar correction to it leads to a consistent renormalization.

This paper is organized as follows. First, we provide the framework of calculating the quantum corrections to our model in Sec. II. Then in Sec. III the pure real scalar correction and the gravitational correction to this model are studied. In Sec. IV the comparison with scalar quantum electrodynamics (SQED) with spontaneously broken global U(1) symmetry and discussion are presented. Finally, we give our conclusion in Sec. V.

II. FRAMEWORK OF CALCULATION

Before investigating our model, we first sketch the approach of Ref.[16]. The full original Lagrangian takes the following form:

\[ \mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_{gf} + \mathcal{L}_{ghost} + \mathcal{L}_{gs}. \]  

In Eq.(1), the Einstein-Hilbert Lagrangian \( \mathcal{L}_{EH} \) reads

\[ \mathcal{L}_{EH} = \frac{2}{\kappa^2} \sqrt{-g} R, \]

where \( g \) is the determinant of the metric \( g_{\mu\nu} \) and \( R = g^{\mu\nu} R_{\mu\nu} \) is the Ricci scalar defined by

\[ R_{\mu\nu} = \partial_{\mu} \Gamma^\rho_{\nu\sigma} - \partial_{\nu} \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma}, \]

\[ \Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} (\partial_{\mu} g_{\nu\sigma} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu}). \]

In order to quantize gravity, the metric \( g_{\mu\nu} \) should be perturbed about the flat Minkowski background \( \eta_{\mu\nu} \), then

\[ g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \]

with the symmetric tensor field \( h_{\mu\nu} \) being the graviton, the spacetime fluctuations. Note that hereafter indices are raised and lowered with the background metric \( \eta_{\mu\nu} \). The inverse metric becomes

\[ g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\alpha} h^{\nu}_\alpha + \mathcal{O}(\kappa^3), \]

and to \( \mathcal{O}(\kappa^2) \) the expansion of the measure in terms of \( h_{\mu\nu} \) is given by

\[ \sqrt{-g} = 1 + \frac{1}{2} \kappa \eta^{\mu\nu} h_{\mu\nu} - \frac{1}{2} h_{\alpha\beta} \kappa^2 P^{\alpha\beta\mu\nu} h_{\mu\nu}, \]

with

\[ P^{\alpha\beta\mu\nu} = \frac{1}{2} \left( \eta^{\alpha\mu} \eta^{\beta\nu} + \eta^{\alpha\nu} \eta^{\beta\mu} - \eta^{\alpha\beta} \eta^{\mu\nu} \right). \]

General coordinate invariance implies that \( \mathcal{L}_{EH} \) is invariant under the infinitesimal transformation

\[ \delta h_{\mu\nu} = h_{\sigma\mu} \partial_{\nu} \xi^\sigma + h_{\sigma\nu} \partial_{\mu} \xi^\sigma + \xi^\sigma \partial_{\sigma} h_{\mu\nu} + \frac{1}{\kappa} (\partial_{\mu} \xi_{\nu} + \partial_{\nu} \xi_{\mu}). \]

The Faddeev-Popov procedure [31] is used to fix this gauge freedom by employing the harmonic (de Donder) gauge fixing condition

\[ G_\mu = \partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_{\mu} h^\nu_\nu, \]

and this leads to the gauge fixing term Lagrangian \( \mathcal{L}_{gf} \) as well as the corresponding ghost term Lagrangian \( \mathcal{L}_{ghost} \),

\[ \mathcal{L}_{gf} = G_\mu G^\mu, \]

\[ \mathcal{L}_{ghost} = -c^\mu \left( \frac{\delta G_\mu}{\delta \xi^\sigma} \right) c^\sigma. \]
FIG. 1: Graviton propagator

Then in the harmonic gauge the graviton propagator shown in Fig. 1 takes the form

\[ D^{\alpha\beta,\mu\nu}(k) = \frac{i}{k^2} P^{\alpha\beta\mu\nu}. \]  

The graviton-scalar Lagrangian is

\[ \mathcal{L}_{gs} = \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4 \right]. \]  

Expanding Eq. (11) in orders of \( \kappa \) leads to an infinite series of interactions involving arbitrary numbers of gravitons, e. g., as shown in Fig. 2, two scalars can couple to any number of gravitons.

FIG. 2: Two scalars can couple to any number of gravitons. (a)-(d) are of order \( \mathcal{O}(\kappa^0) \), \( \mathcal{O}(\kappa^1) \), \( \mathcal{O}(\kappa^2) \) and \( \mathcal{O}(\kappa^3) \), respectively. Hereafter the solid line represents Higgs field \( \phi \) and the double wavy line denotes graviton \( h_{\mu\nu} \).

In the calculation, the gravitational ghosts are irrelevant since only one-loop diagrams with no external gravitons are concerned. The remaining thing to do is to calculate the renormalizations of the scalar field and coupling constant using the standard Feynman rules approach in the minimal subtraction scheme and the DR scheme with spacetime dimension \( D = 4 - 2\epsilon \), where the one-loop divergences manifest themselves as poles at \( D = 4 \), in which the \( Z \) factors contain solely the divergent pole terms proportional to \( \frac{1}{\epsilon} \). One subtle issue is that because the squared-momentum-dependent terms appear in the \( \mathcal{O}(\kappa^2) \) correction to the four-point function of the \( \phi^4 \) interaction there should be one possible higher-derivative counterterm of the form

\[ \mathcal{L}_{hdc} \sim \sqrt{-g} \ g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) \phi^2; \]  

Different from that in Ref. [16], we consider a complex scalar \( \Phi^4 \) theory with spontaneously broken global U(1) symmetry. The graviton-scalar Lagrangian for it is taken to be

\[ \mathcal{L}_{gs} = \sqrt{-g} \left[ g^{\mu\nu} (\partial_\mu \Phi)^*(\partial_\nu \Phi) - V(\Phi) \right], \]  

with the potential taking the form

\[ V(\Phi) = \frac{\lambda}{2} (\Phi^* \Phi - v^2)^2, \]  

where

\[ v = \sqrt{\frac{2\mu^2}{\lambda}}. \]  

Evidently, this Lagrangian is invariant under the global U(1) transformation

\[ \Phi \rightarrow \Phi' = e^{-i\theta} \Phi; \]  

where \( \theta \) is independent of spacetime coordinate \( x \).

It is straightforward to obtain the vacuum expect value (VEV) of \( \Phi \) at tree-level,

\[ \langle 0 | \Phi | 0 \rangle = \pm \frac{v}{\sqrt{2}}. \]
We can express $\Phi$ in terms of the real fields $\Phi_1$ and $\Phi_2$

$$\Phi = \frac{\Phi_1 + i\Phi_2}{\sqrt{2}},$$  \hspace{1cm} (18)

and then choose the vacuum,

$$<0|\Phi_1|0> = v,  \hspace{1cm} <0|\Phi_2|0> = 0.$$  \hspace{1cm} (19)

By paralleling the procedure carried out in Ref.[33], after the spontaneous breaking of global U(1) symmetry, in the Lagrangian, a tadpole constant $\delta_t$ should appear that is zero in the lowest order and must be adjusted in such a way that the VEV of the Higgs field remains zero order by order in perturbation theory, then

$$\Phi_1 = \phi + v(1 + \delta_t),$$  \hspace{1cm} (20)

$$\Phi_2 = \rho.$$  \hspace{1cm} (20)

To $O(\delta)$, Eq.(13) can be reexpressed in terms of the Higgs field $\phi$ and Goldstone field $\rho$,

$$\mathcal{L}_{gs} = \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{1}{2} g^{\mu\nu} (\partial_\mu \rho)(\partial_\nu \rho) \right] - \frac{\lambda}{8} \phi^4 - \frac{\lambda}{2} v^2 \phi^2 - \frac{\lambda}{8} \rho^4 - \frac{\lambda}{2} v \phi^3 - \frac{\lambda}{4} \phi^2 \rho^2$$

$$- \lambda v^2 \phi \delta_t - \lambda v \phi \rho \delta_t - \frac{\lambda}{2} v \phi \rho^2 \delta_t - \frac{\lambda}{2} v^2 \rho^2 \delta_t.]$$

(21)

Obviously, there is no cosmological term in this model, so the expansion of the metric around a flat Minkowski background, as done in Eq.(3), is valid. In order to investigate the contribution of the gravitational correction to the renormalization of the Higgs sector in $\Phi^4$ theory with spontaneously broken global U(1) symmetry, we replace the quantities $\phi$, $\rho$, $\lambda$, and $v$ by the corresponding bare quantities $\phi_0$, $\rho_0$, $\lambda_0$, and $v_0$, respectively, but leave the graviton field $h_{\mu\nu}$ and the gravitational coupling $\kappa$ unchanged as done in Ref.[16]. This treatment is reasonable since their contributions of gravitational corrections to the renormalization of the quantities $h_{\mu\nu}$ and $\kappa$ are higher order and therefore can be neglected.

It is convenient to introduce the following relations

$$\phi_0 = \sqrt{Z_{\phi}} \phi,$$

$$\rho_0 = \sqrt{Z_{\rho}} \rho,$$

$$\lambda_0 = Z_{\lambda} \lambda Z_{\lambda},$$

$$v_0 = \sqrt{Z_v} v Z_v,$$

(22)

with

$$Z_{\phi} = 1 + \delta_{\phi},$$

$$Z_{\rho} = 1 + \delta_{\rho},$$

$$Z_{\lambda} = 1 + \delta_{\lambda},$$

$$Z_v = 1 + \delta_v.$$  \hspace{1cm} (23)

The $Z$ factors (or equivalently $\delta$ factors) and $\delta_t$ are used to absorb all the ultraviolet divergences arising from the pure real scalar correction and gravitational correction in the model. Since we are interested only in one-loop diagrams with no external gravitons and the lowest order gravitational corrections to the renormalizations of $\phi$, $\lambda$, and $v$, other terms with both $\delta$’s and $h_{\mu\nu}$ or without $\phi$, $\rho$ in Eq.(21) are omitted, hence to $O(\delta)$ in terms of renormalized quantities, the part of Eq.(21) relevant for our considerations reads explicitly

$$\mathcal{L}_{gs} = \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + \frac{1}{2} g^{\mu\nu} (\partial_\mu \rho)(\partial_\nu \rho) - \frac{\lambda}{8} \phi^4 - \frac{\lambda}{2} v^2 \phi^2 - \frac{\lambda}{8} \rho^4 - \frac{\lambda}{2} v \phi^3 - \frac{\lambda}{4} \phi^2 \rho^2 - \frac{\lambda}{2} v \phi \rho^2 \right]$$
\[-\lambda v^3 \phi \delta t + \frac{1}{2} (\partial_\mu \phi)^2 \delta \phi - \frac{\lambda}{2} v^2 \phi^2 (\delta \lambda + 2 \delta v + 3 \delta t) \]
\[-\lambda \phi^3 (\delta \lambda + \delta v + \delta t) - \frac{\lambda}{8} \phi^4 \delta \lambda + \frac{1}{2} (\partial_\mu \rho)^2 \delta \rho \]
\[-\lambda \phi^2 \rho^2 \delta t - \frac{\lambda}{4} \phi^2 \rho^2 (\delta \lambda - \delta \phi + \delta \rho) \]
\[-\lambda \phi \rho^2 (\delta \lambda + \delta v - \delta \phi + \delta \rho + \delta t) \]
\[-\lambda \phi^4 \delta \lambda = \frac{1}{8} \phi^4 (\delta \lambda - 2 \delta \phi + 2 \delta \rho) + \cdots \]
\[(24)\]

where the ellipsis represents the terms with both \( \delta \)'s and \( h_{\mu \nu} \), the constant terms without \( \phi, \rho \) or \( h_{\mu \nu} \), and the higher order terms involving scalars as well (e.g., higher-derivative counterterms).

From Eq. (24), the counterterms for one-point, two-point, three-point, and four-point functions of Higgs field \( \phi \) listed in Fig. (3) are

\[
i \delta \Gamma^{(1)} = -i \lambda v^3 \delta t, \]
\[
i \delta \Gamma^{(2)} = i [p^2 \delta \phi - \lambda v^2 (\delta \lambda + 2 \delta v + 3 \delta t)], \]
\[
i \delta \Gamma^{(3)} = -3i \lambda v (\delta \lambda + \delta v + \delta t), \]
\[
i \delta \Gamma^{(4)} = -3i \lambda \delta \lambda. \]
\[(25)\]

\[
(a) \quad (b) \quad (c) \quad (d)
\]

**FIG. 3**: Counterterms for the one-point, two-point, three-point, and four-point functions of \( \phi \).

It should be pointed out that these four counterterms are valid for both pure real scalar correction and gravitational correction. And in our model there is not only \( \phi^4 \) but also \( \phi^3 \) coupling to any number of gravitons, which is one of the main differences from Ref. [16].

**III. PURE REAL SCALAR CORRECTION VS GRAVITATIONAL CORRECTION**

Before considering the gravitational correction part, we should investigate the pure real scalar case, namely, the \( \mathcal{O}(\kappa^0) \) part, for comparison. Equation (24) reduces to the following pure real scalar case:

\[
\mathcal{L}_{ps} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} (\partial_\mu \rho)^2 - \frac{\lambda}{4} \phi^2 \rho^2 - \frac{\lambda}{2} \phi^2 \delta \lambda - \frac{\lambda}{8} \rho^4 \\
- \frac{\lambda}{2} \phi^3 (\delta \lambda + \delta v + \delta t) - \frac{\lambda}{2} \phi \rho^2 (\delta \lambda + \delta v + \delta \rho + \delta t) \\
- \frac{\lambda}{8} \phi^4 \delta \lambda + \frac{1}{2} (\partial_\mu \rho)^2 \delta \rho - \frac{\lambda}{4} \phi^2 \rho^2 (\delta \lambda - \delta \phi + \delta \rho) \\
- \frac{\lambda}{2} \phi^2 \rho^2 \delta t - \frac{\lambda}{2} \phi \rho^2 (\delta \lambda + \delta v - \delta \phi + \delta \rho + \delta t) \\
- \frac{\lambda}{8} \rho^4 (\delta \lambda - 2 \delta \phi + 2 \delta \rho) + \cdots \]
\[(26)\]

It is interesting to note that there is no odd number of Goldstone field(s) in every term of the pure scalar Lagrangian above, namely, the Goldstone field must appear in pairs. Taking into account that groups O(2) and U(1) are...
locally isomorphic while $O(1)$ and $Z_2$ are exactly the same, according to Goldstone theorem \[34\], after spontaneous symmetry breaking, the original $U(1)$ [or equivalently $O(2)$] symmetry is broken, but a residual $Z_2$ [or equivalently $O(1)$] symmetry is still respected by the Goldstone field. As a result, terms with not only even but also odd numbers of Higgs fields can survive but terms with odd numbers of Goldstone field(s) are forbidden.

The one-point, two-point, three-point and four-point functions corresponding to the diagrams listed in Fig. (4), Figs. (5)-(7) are in turn,

\[
\begin{align*}
\langle 1 \rangle_s &= \frac{3i\lambda^2v^2}{2(4\pi)^2\epsilon}, \\
\langle 2 \rangle_s &= \frac{13i\lambda^2v^2}{2(4\pi)^2\epsilon}, \\
\langle 3 \rangle_s &= \frac{10i\lambda^2v}{2(4\pi)^2\epsilon}, \\
\langle 4 \rangle_s &= \frac{10i\lambda^2}{2(4\pi)^2\epsilon}.
\end{align*}
\]

(27)

FIG. 4: $O(\lambda)$ corrections to the one-point function of $\phi$. The diagrams (a) and (b) are both of weight $\frac{1}{2}$, which leads to a factor $\frac{1}{2}$ to them. Hereafter the dashed line represents Goldstone field $\rho$.

FIG. 5: $O(\lambda)$ corrections to the two-point function of $\phi$. (a)-(d) are all of weight $\frac{1}{2}$, which leads to a factor $\frac{1}{2}$.

FIG. 6: $O(\lambda)$ correction to the three-point functions of $\phi$. (a) and (b) are both of weight $\frac{1}{2}$ and 3 permutations, which leads to a factor $\frac{1}{2}$.

From one-point, two-point and three-point functions, it is easy to get

\[
\begin{align*}
\delta_t &= \frac{3\lambda}{2(4\pi)^2\epsilon}, \\
\delta_\phi &= 0, \\
\delta_\lambda &= \frac{10\lambda}{2(4\pi)^2\epsilon}, \\
\delta_v &= -\frac{3\lambda}{2(4\pi)^2\epsilon}.
\end{align*}
\]

(28)
And exactly the same results are easy to reach from one-point, two-point, and four-point functions. To put it differently, the pure real scalar correction to the renormalization of the Higgs sector is consistent. The result is natural but nontrivial because both the coupling constants and the interactions of two-point, three-point and four-point pure real scalar interactions after the spontaneous breaking of global U(1) symmetry obey some strong constrains which are dictated by the original symmetry. Even though the original global U(1) symmetry is no longer apparent, it is still respected in such a special way. However, if the coupling constants of two-point, three-point, and four-point real scalar interactions are corrected by an alien field which does not carry any information of the original symmetry, one cannot expect it to render the correction consistent.

Using the approach presented in Sec. II, we consider the lowest order gravitational correction to the renormalization of the Higgs sector of $\Phi^4$ theory with spontaneously broken global U(1) symmetry. Below, only the lowest order $O(\kappa^2)$ gravitational correction are listed, and the $O(\kappa^0)$ terms listed in Eqs.(28) are omitted. Since we are interested only in the lowest order gravitational corrections to the renormalization of the Higgs sector, the one-loop one particle irreducible (1PI) Feynman diagrams relevant for our concern must satisfy three conditions. The first is all the external line particles must be Higgs field. The second condition is there must exist and only exist a graviton in the internal lines. And the last one is that the Feynman diagrams must be divergent.

Because of the absence of the interaction vertex as shown in Fig. (8a) in this model, there is no such one-loop tadpole diagram corrected by the graviton as listed in Fig. (8b), which is the only diagram of $O(\kappa^2)$ that contributes to the one-point function of the Higgs field. Hence there is no contribution from the $O(\kappa^2)$ one-loop tadpole diagram in the calculation, and then it can be obtained that,

$$\delta_{\ell} = 0.$$ 

The diagrams for two-point, three-point and four-point functions are listed in Figs. 9[11], respectively, where the weight factor and permutations of external legs have been taken into account.

As discussed above, there is no odd number of Goldstone field(s) in every term of Lagrangian. Considering in the lowest order gravitational correction there must exist and only exist a graviton in the internal lines, the divergent one-loop 1PI Feynman diagram with one graviton and one Goldstone field as internal lines and Higgs fields as external lines is absent.

The corresponding two-point, three-point and four-point functions are in turn,

$$i\Gamma^{(2)}_g = \frac{i\kappa^2 \lambda v^2 p^2}{(4\pi)^2 \epsilon} - \frac{i\kappa^2 \lambda^2 v^4}{(4\pi)^2 \epsilon},$$

$$i\Gamma^{(3)}_g = \frac{i\lambda \kappa^2}{(4\pi)^2 \epsilon} [-3\lambda v^2 + \frac{1}{2}(p_1^2 + p_2^2 + p_3^2)],$$

$$i\Gamma^{(4)}_g = \frac{i\lambda \kappa^2}{(4\pi)^2 \epsilon} [-13\lambda v^2 + \frac{1}{2}(p_1^2 + p_2^2 + p_3^2 + p_4^2)].$$

The squared-momentum-dependent terms in the $O(\kappa^2)$ gravitational corrections to the three-point and four-point functions of the Higgs field do not contribute to the corresponding three-point and four-point counterterms and are the contributions to the higher-derivative counterterms [32], respectively. The only possible higher-derivative
counterterms for three-point and four-point functions can be derived from the additional U(1) symmetry preserved term,

\[ \mathcal{L}_{h\text{det}} \sim \sqrt{-g} \, g^{\mu
u} \, (\partial_{\mu}\Phi_0)^*(\partial_{\nu}\Phi_0)\Phi_0^*\Phi_0. \]  

(31)

After the spontaneous breaking of global U(1) symmetry, using Eqs. (4), (5), (22) and (23) and neglecting the higher order terms and terms with gravitons, it is straightforward to get the high derivative counterterms for Higgs field \( \phi \),

\[ \mathcal{L}_{h\text{det}} \sim \phi^2(\partial_\mu\phi)^2 + 2v\phi(\partial_\mu\phi)^2 + v^2(\partial_\mu\phi)^2. \]  

(32)

The three terms in (32) are related to higher-derivative counterterms for four-point, three-point and two-point functions, respectively. Hence part of the correction in the two-point function is removed by the contribution of the third term in (32). These three terms correspond to the following contributions to the four-point, three-point, and two-point functions and should be added to them, respectively:

\[
\begin{align*}
\phi^2(\partial_\mu\phi)^2 & \sim \frac{i\lambda\kappa^2}{2(4\pi)^2\epsilon} \left( p_1^2 + p_2^2 + p_3^2 + p_4^2 \right) = i\delta\Gamma^{(4)}_g, \\
2v\phi(\partial_\mu\phi)^2 & \sim -\frac{i\lambda v\kappa^2}{2(4\pi)^2\epsilon} \left( p_1^2 + p_2^2 + p_3^2 \right) = i\delta\Gamma^{(3)}_g, \\
v^2(\partial_\mu\phi)^2 & \sim \frac{-3i\kappa^2\lambda v^2}{(4\pi)^2\epsilon} p^2 = i\delta\Gamma^{(2)}_g.
\end{align*}
\]  

(33)

A key point is that after the spontaneous breaking of global U(1) symmetry the squared-momentum-dependent terms in the \( \mathcal{O}(\kappa^2) \) corrections to the three-point and four-point functions are simultaneously removed by the higher-derivative counterterms derived from (31) for four-point and three-point functions, respectively.

Taking into account (32), from two-point and three-point functions and Eq. (29), it is straightforward to get

\[
\begin{align*}
\delta_\phi & = \frac{\kappa^2\lambda v^2}{2(4\pi)^2\epsilon}, \\
\delta_\lambda & = \frac{-5\kappa^2\lambda v^2}{(4\pi)^2\epsilon}, \\
\delta_v & = \frac{2\kappa^2\lambda v^2}{(4\pi)^2\epsilon}.
\end{align*}
\]  

(34)
while from two-point and four-point functions and Eq. (29), one can obtain

\[ \delta \phi = \frac{\kappa^2 \lambda v^2}{2(4\pi)^2 \epsilon} \]
\[ \delta \lambda = -\frac{13\kappa^2 \lambda v^2}{(4\pi)^2 \epsilon} \]
\[ \delta v = \frac{6\kappa^2 \lambda v^2}{(4\pi)^2 \epsilon} \]

It is easy to find that the results for \( \delta \lambda \) and \( \delta v \) obtained from two-point and three-point functions of the Higgs field [see Eqs. (34)] and that from two-point and four-point functions [see Eq. (35)] contradict each other, which indicates that the lowest order gravitational correction to the renormalizations of the Higgs sector in this model is inconsistent.

If we reduce the complex scalar field \( \Phi \) to a real scalar field (namely, \( \Phi^* = \Phi \)), the Lagrangian has a \( Z_2 \) symmetry, \( \Phi \rightarrow -\Phi \), then we arrive at a real scalar \( \Phi^4 \) theory with spontaneously broken \( Z_2 \) symmetry. There is no Goldstone field after the spontaneous breaking of the discrete \( Z_2 \) symmetry. Therefore, compared with the case of the \( \Phi^4 \) theory with spontaneously broken global \( U(1) \) symmetry, all of the Feynman diagrams with Goldstone field \( \rho \) in the pure real scalar correction part should be removed here. It is easy to check that the renormalization of the Higgs field \( \phi \) self-correction is also consistent, and the result reads

\[ \delta t = \frac{3 \lambda}{2(4\pi)^2 \epsilon} \]
\[ \delta \phi = 0 \]
\[ \delta \lambda = \frac{9 \lambda}{2(4\pi)^2 \epsilon} \]
\[ \delta v = -\frac{3 \lambda}{2(4\pi)^2 \epsilon} \]

For gravitational correction to \( \Phi^4 \) theory with spontaneously broken \( Z_2 \) symmetry, the interactions between Higgs field \( \phi \) and graviton \( h_{\mu\nu} \), hence the related Feynman diagrams, and the high derivative counterterms, do not change compared with the case of \( \Phi^4 \) theory with spontaneously broken \( U(1) \) symmetry. On the other hand, as pointed out in Sec. III, the Goldstone field does not contribute to the gravitational correction to the Higgs sector in the case of \( \Phi^4 \) theory with spontaneously broken \( U(1) \) symmetry. Therefore even though the Goldstone field is absent in the case of \( \Phi^4 \) theory with spontaneously broken \( Z_2 \) symmetry, there is no difference between these two cases for \( \mathcal{O}(\kappa^2) \) gravitational correction to the Higgs sector. The contradiction of the gravitational correction to Higgs field still holds.

### IV. COMPARISON AND DISCUSSION

In order to reveal the reason of the inconsistence of the gravitational correction to the Higgs sector, we make some comparison with SQED with spontaneously broken global \( U(1) \) symmetry, whose Lagrangian takes the form

\[ \mathcal{L}_{SQED} = T(\Phi, A_\mu) - V(\Phi), \]

where the potential \( V(\Phi) \) is given by Eq. (14), and the kinetic term is

\[ T(\Phi, A_\mu) = (D_\mu \Phi)^*(D^\mu \Phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \]

with \( D_\mu = \partial_\mu + i e A_\mu \). The renormalizability of this model with local \( U(1) \) transformation had been studied in Ref. [35]. Here we are interested in the behavior of this model under the global \( U(1) \) transformation Eq. (16). Using Eqs. (17), (18), (19) and (20), and setting \( \delta t = 0 \) in Eq. (20), one arrives at

\[ T(\Phi, A_\mu) = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} (\partial_\mu \rho)^2 + e^2 v \phi A_\mu A^\mu + \frac{1}{4} e^2 \phi A_\mu A^\mu + \frac{1}{2} e^2 \rho A_\mu A^\mu - e \rho A^\mu \partial_\mu \phi \]
\[ + e \phi A^\mu \partial_\mu \rho - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} e^2 v^2 A_\mu A^\mu + e v A^\mu \partial_\mu \rho. \]

(39)
In this case, the covariant derivative in the kinetic term \((D_\mu \Phi)^*(D^\mu \Phi)\) consists the gauge field \(A_\mu\), since the complex scalar field \(\Phi\) carries charge. This directly leads to the appearance the mixing between \(A_\mu\) and \(\rho\) (see the last term in Eq. (39)) after the spontaneous breaking of global \(U(1)\) symmetry, which is proved to play a crucial role in the renormalization of this model. Because of such mixing term, the gauge field will be massive after eating the Goldstone boson and the mass of the gauge field, which is related to the VEV, is just the reflection of the information from the spontaneous breaking of global \(U(1)\) symmetry. The reason that the massive gauge field can give a consistent correction to the Higgs field \(\phi\) is that the massive gauge field contains not only the original massless gauge field but also the Goldstone field which is a part of the original complex scalar field \(\Phi\).

In fact, considering the coupling constants and interactions of three-point and four-point Higgs field self-interactions are not independent after the spontaneous breaking of global \(U(1)\) symmetry, one cannot expect that an ordinary alien gauge field could give a consistent correction to such model. But only if the parameters in the theory, e.g., mass of gauge boson \(m_A = ev\), is connected with the VEV of the original complex scalar field by the mixing term between \(A_\mu\) and \(\rho\), and the interactions among the gauge field, the Higgs field, and the Goldstone field are derived from the spontaneous breaking of global \(U(1)\) symmetry, the expectation for a consistent correction is reasonable.

But something changes in the case of the gravitational correction to the Higgs sector in our model. When the complex scalar field \(\Phi\) couples to gravity, the connection in the covariant derivative of the kinetic term of the field, e.g., \(D_\mu \Phi\), vanishes, resulting in \(D_\mu \Phi \rightarrow \partial_\mu \Phi\). This will directly lead to the absence of the two-point mixing terms between the graviton and the Goldstone field. On the other hand, considering the symmetry is spontaneously broken in the internal charged field space, which is different from the external spacetime, one will find this is natural that the Goldstone field can mix with gauge field \(A_\mu\) but not graviton \(h_{\mu\nu}\). Therefore the information of the spontaneous breaking of symmetry puts no influence on the graviton, and the Goldstone field, which appears after the spontaneous breaking of symmetry, no longer contributes to the gravitational correction to the Higgs sector. Actually this inconsistent result is not surprising when we take into account what we considered is the correction of an alien field, which carries no information of the original symmetry, to the Higgs field after the spontaneous breaking of symmetry.

It is also interesting to compare a graviton in this model with a gluon in the quantum chromodynamics. Since the Higgs field is a color singlet, the coupling between the gluon field and the Goldstone field is forbidden. Therefore the two-point mixing term between the gluon and the Goldstone field is also absent. But unlike the fact that the correction of the gluon to the Higgs sector is meaningless, gravity can couple to Higgs field and contribute to the correction of the Higgs field due to the fact that gravity is a reflection of the feature of spacetime and could couple to any kind of energy.

Hence in the SM, every field that contributes to the correction to the Higgs sector will be influenced by the spontaneous breaking of symmetry, specifically speaking, the masses of \(W^\pm\) and \(Z^0\) are related to the VEV of the original complex scalar field, while on the other hand, the gluon and photon, whose couplings with the Higgs field are absent, will not contribute to such a correction. For this reason, the consistence of the gravitational correction to the SM is questionable considering that the gravity, even not influenced by the spontaneous breaking of symmetry, will still contribute to the correction to the Higgs sector in the SM.

V. CONCLUSION

In summary, in this paper we considered a model in which perturbatively quantized Einstein gravity couples to the \(\Phi^4\) theory with spontaneously broken global \(U(1)\) symmetry and calculated the lowest order pure real scalar correction and gravitational correction to the renormalizations of the Higgs sector. It is found that there is a contradiction in the gravitational correction to the renormalization of the Higgs sector while the pure real scalar correction to it leads to a compatible renormalization of such model.

Based on the analysis above, the consistence for the gravitational correction to the renormalization of the Higgs sector in the SM, where the electroweak \(SU(2)_L \times U(1)_Y\) symmetry is spontaneously broken, is open to doubt. It is expected that this contradiction will still hold there, which is the subject of future study.

Acknowledgments

We are grateful to Professor Mu-Lin Yan and Professor Dao-Neng Gao for helpful discussions. Hao-Ran Chang would like to express his special thanks to Professor Jun Yan for favorable correspondence. This work was supported
by the National Natural Science Foundation of China under Grants No.11075149, No.10975128, and No.11074234.

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