Consistent relativistic mean field models constrained by GW170817

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We have obtained the Love number and corresponding tidal deformabilities (Λ) associated with the relativistic mean-field parametrizations shown to be consistent (CRMF) with the nuclear matter, pure neutron matter, symmetry energy and its derivatives [Dutra et al., Phys. Rev. C 90, 055203 (2014)]. Our results show that CRMF models present very good agreement with the recent data from binary neutron star merger event GW170817. They also confirm the strong correlation between Λ1.4 and the radius of canonical stars (R1.4). When a recently GW170817 constraint on Λ1.4 and the corresponding radius R1.4 is used, the majority of the models tested are shown to satisfy it.

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I. INTRODUCTION

The discovery of the first binary pulsar PSR1913+16 by Russel Hulse and Joseph Taylor in 1974 [1] with its very stable and precise pulse period, and the observation in 1978 that its orbit period was declining with time [2], opened a clear possibility for the detection of gravitational waves (GW). A probable explanation for the change in the period was the loss of energy by the binary system in the form of GW. The detection of these waves was expected since then until in 2015 the first signal was clearly seen (GW150914) and shown to be produced by two colliding black holes [3]. Finally, in 2017, LIGO and Virgo made the first detection of GW170817 produced by colliding neutron stars [4] and the event was observed also as light in the optical, UV, IR, X-ray and γ-ray emissions [5], what was then called a multi-messenger observation.

When one of the neutron stars in a binary system gets close to its companion just before merging, a mass quadrupole develops as a response to the tidal field induced by the companion. This is known as tidal deformability [6, 7] and can be used to constrain neutron star macroscopic properties [8, 9], which in turn, are obtained from appropriate equations of state (EOS). A nice and simple review on the basic ingredients necessary to construct an EOS is given in [10].

If one searches the literature for an equation of state, hundreds of models are found. Not too long ago, 263 relativistic mean-field (RMF) parametrizations were analyzed in [11] and confronted with different sets of constraints, all of them related to symmetric nuclear matter, pure neutron matter, symmetry energy and its derivatives. The different sets differ from one another in the choice of validity ranges of certain quantities and in the level of restriction. Only a small number of parametrizations of these models (35) were shown to satisfy adequately the chosen constraints. This fact reinforces the idea that the proliferation of models and the production of new parameter sets with a limited range of application should not be encouraged.

In [11], the relativistic models were divided into 7 families, namely, linear finite range models (Walecka-type models [12], type 1), nonlinear σ models (Boguta-Bodmer models [13], type 2), nonlinear σ and ω models with a self-quartic interaction in the ω field (type 3), nonlinear σ and ω terms and cross terms involving these fields (type 4), density dependent models [14] with couplings adjusted to nuclear properties (type 5), nonlinear point coupling models [15] (type 6) and models with δ mesons (type 7). Thirty of the approved models are of type 4, two are of type 5, one of type 6, and two of type 7 being both density dependent.

Later on, 34 RMF models that were shown to satisfy several nuclear matter constraints in [11], namely, the same previous 35 models excluding the point-coupling one as discussed in [16], were confronted with astrophysical constraints [16]. The more important of these constraints are the neutron stars with maximum mass in the range of 1.93 ≤ M/M⊙ ≤ 2.05 [17, 18], but the direct Urca process and the sound velocity also give hints on the star cooling mechanism and its internal matter distribution. From the 34 analyzed models with nucleonic matter included, only 15 can sustain massive stars and none if hyperons are included, a result that accounts for the famous hyperon puzzle. Once hyperons are included in the calculations, the situation becomes more complicated because the EOS must be soft at sub-saturation densities and hard at higher densities to predict massive stars, but hyperons soften the EOS. A possibility that reconciles the measurements of massive stars with canonical stars with small radii present in the same family (another recently imposed constraint), is either the inclusion of strange mesons or of a new degree of freedom (not necessarily known) in the calculations [19].

The measurements and analyses of data from this specific gravitational wave established limits both on the dimensionless tidal deformability of the binary system Λ and on the tidal deformability of the canonical star Λ1.4 as being ≤ 800 for the low spin priors upper boundary [4] and contributed to the exclusion of very stiff EOS that would give rise to values larger than 800. A lower limit
was estimated as $\tilde{\Lambda} > 400$ [20]. Recently, LIGO and Virgo collaboration updated the $\Lambda_{1,4}$ values to be constrained to the range of $70 \leq \Lambda_{1,4} \leq 580$ [21]. Moreover, the chirp mass, which relates the masses of both NS in the binary system was observed to be $M = 1.188 M_\odot$ [4]. The above mentioned boundaries combined with the chirp mass can be used to calculate the bounds on the tidal deformability of the individual neutron stars in the binary system [22].

Since the detection of GW170817, several studies were dedicated to look for correlations and sensitivity of important nuclear bulk properties, i.e., the symmetry energy, its slope, compressibility and values of the tidal deformability for the canonical $1.4 M_\odot$ and other slightly less and slightly more massive stars. In [23] the authors analyzed 4 Skyrme-type models and 1 obtained from a density functional theory; in [22] 18 relativistic and 24 nonrelativistic models were analyzed; in [24] many Skyrme type models were investigated and in [25] 67 RMF models were considered. In the 3 last works, different correlations were found between, for instance, $\Lambda$ and $R$, $\Lambda_{1,4}$ and $R_{1,4}$, $\Lambda_{1,4}$ and $M_{max}$ or $\Lambda_{1,4}$ and $\Lambda$.

In [21], a parametrized EOS was built at high-densities and one Skyrme EOS at low densities and was confronted with GW170817 tidal deformability information to obtain NS radii. The suggested values for the two neutron stars in the binary system lie in the range $R_1 = 10.8^{+2.9}_{-1.7}$ km and $R_2 = 10.7^{+2.1}_{-1.5}$ km. If a further restriction is imposed to account for EOS that support massive stars, both radii are constrained to the range of $11.9 \pm 1.4$ km. All models analyzed in [16] with a maximum mass of $(1.97 \pm 0.04) M_\odot$ or larger bear radii within the proposed range. In [25], the authors established the upper limit on the canonical stars radii as $R_{1,4} \leq 12.9$ km. In [16], only 7 models are excluded by this constraint.

Also, in a recent paper [26], the authors show that an infinite number of combinations of EOS with large slopes and small compressibilities or small slopes and large compressibilities can lead to the same $\Lambda_{1,4}$ and $R_{1,4}$, pointing to the need of more observables so that the density dependence of the symmetry energy be completely determined.

In all above mentioned papers, the models used to test constraints and to look for correlations were randomly chosen. However, in the present work we follow a more direct line of work by choosing models we have already tested previously based on exactly the same constraints, avoiding models either with flamboyant degrees of freedom or that have been forcefully corrected with extra mixed meson interactions and return to the more conventional 34 RMF parametrizations that were shown to satisfy the nuclear matter constraints in [11] to confront them with tidal deformabilities inferred from GW170817.

II. RESULTS AND DISCUSSION

In a binary neutron star system, the tidal deformability is the measurement of the perturbation generated by the quadrupole moment in one star as a response to the external field created by its companion. From the mathematical point of view, the dimensionless tidal deformability, in terms of the Love number $k_2$, is given by

$$\Lambda = \frac{2k_2}{3C_5},$$

where $C = m/R$ is the compactness of the neutron star of mass $m$. The Love number $k_2$ is calculated by the following expression,

$$k_2 = \frac{8C^5}{5}(1 - 2C)^2[2 + C(y_R - 1) - y_R] \times$$

$$\times \left\{2C[6 - 3y_R + 3C(5y_R - 8)] + 4C^2[13 - 11y_R + C(3y_R - 2) + 2C^2(1 + y_R)] + 3(1 - 2C^2)[2 - y_R + 2C(y_R - 1)]\ln(1 - 2C)\right\}^{-1},$$

where $y_R = y(r)$ is found from the solution of

$$r \frac{dy(r)}{dr} + y(r)^2 + y(r)F(r) + r^2 Q(r) = 0,$$

with

$$F(r) = \frac{r - 4\pi \rho^3 [\epsilon(r) - p(r)]}{r - 2M(r)},$$

and

$$Q(r) = \frac{4\pi r}{r - 2M(r)} \left[5\epsilon(r) + 9p(r) + \frac{\epsilon(r) + p(r)}{\partial\rho(r)/\partial\epsilon(r)} - \frac{6}{4\pi \rho^4} \right]$$

$$- 4 \left[\frac{M(r) + 4\pi \rho^3 p(r)}{r^2 (1 - 2M(r)/r)}\right]^2.$$

The set of Eqs. (1)-(5) can also be found in Refs. [27–31], for instance, in which earlier new developments were performed by using a large number of EOS’s used to calculate $k_2$ and $\Lambda$.

Actually, Eq. (3) must be solved together with the well known TOV equations [32], in which $\epsilon$ and $p$ are the energy density and pressure, respectively, given as input. In our case, these quantities are given by the CRMF parametrizations with protons, neutrons, electrons and muons with the charge neutrality and $\beta$-equilibrium conditions together with the Baym-Pethick-Sutherland (BPS) equation of state [33] in the low density regime, namely, $0.1581 \times 10^{-10}$ fm$^{-3} \leq \rho \leq 0.008907$ fm$^{-3}$. The initial condition for Eq. (3) is $y(0) = 2$ (related to the Love number order) and $M(r)$ is the neutron star mass enclosed within the radius $r$. At the surface of the star, in which $r = R$, one has $M(R) = m$. For detailed discussions on such calculations, we address the reader to Refs. [28–31, 34], for instance.
The compactness of one recently measured isolated neutron star [35] is equal to $0.105 \pm 0.002$. Notice that the GW170817 constrains NS in a binary system and hence, more measurements are necessary before this value is used as a constraint. Nevertheless, in Table I, we show the compactness for the cases of $m = m_{\text{max}} (C_{\text{max}})$, $m = 1.4 M_\odot (C_{1,4})$, and 3 more cases obtained from the limits of possible masses in the binary system. The mass-radius diagrams used to calculate these $C$ values for the CRMF parametrizations are found in Ref. [16].

TABLE I. Compactness, in units of $M_\odot$/km, related to the maximum neutron star mass $(C_{\text{max}})$, canonical one $(C_{1,4})$, and for $m_1 = 1.37$, 1.48 and 1.60 solar masses, with their respective values of $m_2$ for the CRMF models analyzed.

| Model       | $C_{\text{max}}$ | $C_{1,4}$ | $C_{\text{DD-ME}}$ | $C_{\text{DD}}$ | $C_{\text{DDH}}$ | $C_{\text{DD-HD}}$ |
|-------------|------------------|----------|---------------------|-----------------|-----------------|-------------------|
| BKA29       | 0.170            | 0.105    | 0.105               | 0.115           | 0.095           | 0.123             |
| BKA22       | 0.170            | 0.105    | 0.105               | 0.115           | 0.095           | 0.123             |
| BKA24       | 0.169            | 0.104    | 0.102               | 0.111           | 0.093           | 0.121             |
| BSR8        | 0.171            | 0.108    | 0.105               | 0.115           | 0.097           | 0.125             |
| BSR9        | 0.170            | 0.106    | 0.105               | 0.115           | 0.097           | 0.125             |
| BSR10       | 0.170            | 0.107    | 0.104               | 0.113           | 0.096           | 0.123             |
| BSR11       | 0.169            | 0.105    | 0.095               | 0.112           | 0.094           | 0.123             |
| BSR12       | 0.170            | 0.106    | 0.103               | 0.112           | 0.094           | 0.122             |
| BSR13       | 0.169            | 0.109    | 0.109               | 0.119           | 0.099           | 0.132             |
| BSR14       | 0.159            | 0.111    | 0.109               | 0.119           | 0.099           | 0.132             |
| BSR15       | 0.159            | 0.111    | 0.108               | 0.119           | 0.098           | 0.131             |
| BSR16       | 0.158            | 0.110    | 0.107               | 0.118           | 0.097           | 0.130             |
| BSR17       | 0.158            | 0.109    | 0.106               | 0.117           | 0.096           | 0.130             |
| BSR20       | 0.157            | 0.107    | 0.104               | 0.115           | 0.095           | 0.128             |
| FSU-IH      | 0.158            | 0.111    | 0.108               | 0.119           | 0.098           | 0.132             |
| FSU-IV      | 0.160            | 0.114    | 0.111               | 0.122           | 0.101           | 0.135             |
| FSUGold     | 0.159            | 0.112    | 0.109               | 0.121           | 0.100           | 0.134             |
| FSUGold4    | 0.160            | 0.114    | 0.111               | 0.122           | 0.101           | 0.135             |
| FSUZG03     | 0.170            | 0.108    | 0.105               | 0.115           | 0.097           | 0.125             |
| FSUG206     | 0.159            | 0.112    | 0.108               | 0.119           | 0.099           | 0.132             |
| BSR19       | 0.158            | 0.109    | 0.106               | 0.117           | 0.096           | 0.130             |
| BSR20       | 0.157            | 0.107    | 0.104               | 0.115           | 0.095           | 0.128             |
| Z271s2      | 0.155            | 0.111    | 0.108               | 0.120           | 0.098           | 0.136             |
| Z271s3      | 0.153            | 0.114    | 0.110               | 0.122           | 0.100           | 0.139             |
| Z271s4      | 0.153            | 0.115    | 0.112               | 0.124           | 0.101           | 0.141             |
| Z271s5      | 0.153            | 0.116    | 0.113               | 0.125           | 0.103           | 0.142             |
| Z271s6      | 0.154            | 0.117    | 0.114               | 0.126           | 0.104           | 0.143             |
| Z271s7      | 0.149            | 0.115    | 0.111               | 0.124           | 0.100           | 0.147             |
| Z271s8      | 0.149            | 0.115    | 0.112               | 0.125           | 0.101           | 0.149             |
| Z271s9      | 0.150            | 0.116    | 0.113               | 0.126           | 0.102           | 0.150             |
| DD-F        | 0.191            | 0.117    | 0.114               | 0.125           | 0.105           | 0.137             |
| TW99        | 0.196            | 0.114    | 0.111               | 0.121           | 0.102           | 0.132             |
| DD-H        | 0.192            | 0.111    | 0.109               | 0.126           | 0.100           | 0.127             |
| DD-ME5      | 0.194            | 0.118    | 0.115               | 0.145           | 0.105           | 0.137             |

Models belonging to the same families present very similar compactness both for the maximum mass star and for the canonical one. In the low limit mass case, both stars of the binary system present practically the same mass, $m_1 \approx m_2 \approx 1.37 M_\odot$. This is reason they present the same compactness. However, this is no longer true in the other cases, when one of the star is always more compact than its companion.

In Fig. 1 we display the Love number $k_2$ for the CRMF parametrizations, obtained through the definition given in Eq. (2) with $y_2$ calculated from the solution of Eq. (3) coupled to the TOV equations. The pattern exhibited by $k_2$ is similar to that found in calculations involving other relativistic hadronic model, as one can verify in Ref. [34], for instance. It is important to point out that $k_2$ is very sensitive to the description of the crust of the star. Once $k_2$ is calculated, it is possible to analyze the dimensionless tidal deformability by using the definition presented in Eq. (1).

In Fig. 2 we display a diagram of the dimensionless tidal deformabilities of each star in the binary system. $\Lambda_1$ is associated to the neutron star with mass $m_1$ which corresponds to the integration of every EOS in the range $1.37 \leq m/M_\odot \leq 1.60$ obtained from GW170817. On the other hand, the mass $m_2$ of the companion star is determined by solving the chirp mass $M = (m_1 m_2)^{3/5}$, whose value is $1.188 M_\odot$, as determined in Ref. [4]. One can see that all the investigated models lie in between the confidence lines, what corroborates the fact that the previously constrained models to satisfy nuclear bulk properties are reliable to investigate neutron stars in binary systems, although many of them do not describe massive neutron stars accurately.
stars, as explained in the introduction of this letter.

As already shown in Refs. [22, 24, 25], we have also found a strong correlation between the tidal deformability of the canonical star and its radius both in linear and log scale (not shown), namely, \( \Lambda_{1,4} \equiv 2.65 \times 10^{-5} R_{1,4}^{5.58} \), as can be seen in Fig. 3. Since the second Love number \( k_2 \) depends on \( R \), for a given EOS, through a nontrivial differential equation coupled to the TOV one, \( \Lambda_{1,4} \) as a function of \( R_{1,4} \) is not simply given by \( \Lambda_{1,4} \propto R_5^{4} \) as Eq. (1) suggests.

When we analyze the constraint for \( \Lambda_{1,4} \) in the range \( 70 \leq \Lambda_{1,4} \leq 580 \), as proposed in [21], with corresponding \( R_{1,4} \) values, depicted in Fig. 3 by the shaded square, we observe that 24 parametrizations (out of 34) are in accordance with this proposition. They are: BSR15, BSR16, BSR17, BSR18, BSR19, BSR20, FSU-III, FSU-IV, FSUGold, FSUGold4, FSUGZ06, G2*, IU-FSU, Z271s2, Z271s3, Z271s4, Z271s5, Z271s6, Z271v4, Z271v5, Z271v6, DD-F, TW99, and DD-MEδ. It is interesting to notice that not all models capable of describing massive stars in the range \( 1.93 \leq M/M_\odot \leq 2.05 \) [17, 18] discussed in [16] lie inside the box with values obtained from GW170817, and not all the 24 models inside the gray area can describe massive stars. Only 5 models satisfy both constraints, namely, G2*, IU-FSU, DD-F, TW99, and DD-MEδ.

Although the current range for \( \Lambda_{1,4} \) is not very restrictive, we remind the reader that its values were still more imprecise, as one can verify in Ref. [4] in which the range was computed as \( \Lambda_{1,4} \leq 800 \). Furthermore, there is a huge number of hadronic parameterizations coming from relativistic and non-relativistic models, around 500 if we take into account only those from RMF and Skyrme models. Thus, it is important to find which particular set of parameterizations among these huge number is able to describe simultaneously different nuclear environments. In that sense, a constraint coming from the analysis of the recent GW170817, even being not so restrictive (\( 70 \leq \Lambda_{1,4} \leq 580 \)) can be useful for this purpose.

III. FINAL REMARKS

In the present work, we have revisited 34 relativistic mean-field parametrizations shown to be consistent (CRMF) with the nuclear matter, pure neutron matter, symmetry energy and its derivatives in [11] and used them to compute the Love number and corresponding tidal deformabilities. We have checked that all analyzed models lie in between the confidence lines in the plot \( \Lambda_2 \) versus \( \Lambda_1 \). They also confirm previously obtained correlation between the tidal deformability and the radius of canonical stars. Once we use the GW170817 constraints on the tidal deformabilities to identify the corresponding neutron star radii range, as proposed in [21], 24 parametrizations are shown to satisfy them. As far as the compactness, an important ingredient in the calculation of the Love numbers, is investigated, we have seen that, generally, one of the star is always more compact than its companion, except in the low limit mass case \( m_1 = 1.37 \), when both stars in the binary system present the same compactness.

It is also worth pointing out that only 5 parametrizations of the CRMF models, namely, G2*, IU-FSU, DD-F, TW99, and DD-MEδ, can simultaneously describe massive stars in the range \( 1.93 \leq M/M_\odot \leq 2.05 \) [17, 18], as shown in [16], and constraints from GW170817.

To further constrain the existing EOS or confirm the results obtained so far, we look forward to the next detections of gravitational waves.

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[1] R. A. Hulse and J. H. Taylor, Astrophys. J. 191 L59 (1974); 195 L51 (1975).

[2] J. H. Taylor, L. A. Fowler and P. M. McCulloch, Nature 277 437 (1979).
[3] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 061102 (2016); 116, 241103 (2016); Phys. Rev. X 6, 041015 (2016).

[4] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 119, 161101 (2017).

[5] P. S. Cowperthwaite et al. (Ligo Scientific Collaboration and Virgo Collaboration), Astrophys. Jour. Lett. 848: L12 (2017).

[6] T. Damour, M. Soffel and C.-M. Xu, Phys. Rev. D 45, 1017 (1992).

[7] E. E. Flanagan and T. Hinderer, Phys. Rev. D 77, 021502 (2008).

[8] F. J. Fattoyev, J. Piekarewicz and C. J. Horowitz, Phys. Rev. Lett. 120, 172702 (2018).

[9] Tuhin Malik et al., Phys. Rev. C 98, 035804 (2018).

[10] J. Piekarewicz, Proceedings of the XIV International Workshop on Hadron Physics, Florianópolis, 18-23 March 2018 pages 12-37, arXiv:1805.04780.

[11] M. Dutra, O. Lourenço, S. S. Avancini, B. V. Carlson, A. Delfino, D. P. Menezes, C. Providência, S. Typel, and J. R. Stone, Phys. Rev. C 90, 055203 (2014).

[12] J. D. Walecka, Ann. Phys. 83, 491 (1974).

[13] J. Boguta and A. R. Bodmer, Nucl. Phys. A 292, 413 (1977).

[14] S. Typel and H. H. Wolter, Nucl. Phys. A 656, 331 (1999).

[15] B. A. Nikolaus, T. Hoch, and D. G. Madland, Phys. Rev. C 46, 1757 (1992); O. Lourenco, M. Dutra, A. Delfino, and R. L. P. G. Amaral, Int. J. Mod. Phys. E, 16, 3037 (2007).

[16] Mariana Dutra, Odilon Lourenço and Débora Peres Menezes, Phys. Rev. C 93, 025806 (2016); 94, 049901(E) (2016).

[17] P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts, and J. W. T. Hessels, Nature 467, 1081 (2010).

[18] J. Antoniadis, P. C. C. Freire, N. Wex et al., Science 340, 448 (2013).

[19] L. Lopes and D. Menezes, Journal of Cosmology and Astroparticle Physics 5, 038 (2018).

[20] D. Radice, A. Perego, F. Zappa, and S. Bernuzzi, The Astrophysical Journal Letters 852, L29 (2018).

[21] B. P. Abbott et al. (The LIGO Scientific Collaboration and the Virgo Collaboration), Phys. Rev. Lett. 121, 161101 (2018).

[22] Tuhin Malik, N. Alam, M. Fortin, C. Providência, B. K. Agrawal, T. K. Jha, Bharat Kumar and S. K. Patra, Phys. Rev. C 98, 035804 (2018).

[23] Young-Min Kim, Yeunhwan Lim, Kyujin Kwak, Chang Ho Hyun and Chang-Hwan Lee, arXiv 1805.00219, to appear in Phys. Rev. C.

[24] M.B. Tsang, C.Y. Tsang, P. Danielewicz, and W.G. Lynch, arXiv: 1811.04888.

[25] Rana Nandi, Prasanta Char and Subrata Pal, arXiv: 1809.07108.

[26] Nai-Bo Zhang and Bao-An Li, J. Phys. G: Nuclear and Particle Physics 46, 014002 (2019).

[27] S. Postnikov, M. Prakash, and J. M. Lattimer, Phys. Rev. D 82, 024016 (2010).

[28] T. Hinderer, Astrophys. J. 677, 1216 (2008).

[29] T. Hinderer, B. D. Lackey, R. N. Lang, and J. S. Read, Phys. Rev. D 81, 123016 (2010).

[30] T. Binnington and E. Poisson, Phys. Rev. D 80, 084018 (2009).

[31] T. Damour and A. Nagar, Phys. Rev. D 81, 084016 (2010).

[32] R. C. Tolman, Phys. Rev. 55, 364 (1939); J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. 55, 374 (1939).

[33] N. K. Glendenning, Compact Stars, 2nd ed. (Springer, New York, 2000); G. Baym, C. Pethick, and P. Sutherland, Astrophys. J. 170, 299 (1971).

[34] Bharat Kumar, S. K. Biswal, and S. K. Patra, Phys. Rev. C 95, 015801, (2017)

[35] V. Hambaryan, V. Suleimanov, F. Haberl, A. D. Schwope, R. Neuhauser, M. Hohle, and K. Werner, Astronomy & Astrophysics 601, A108 (2017).