Abstract—With the prevalence of accessible depth sensors, dynamic human body skeletons have attracted much attention as a robust modality for action recognition. Previous methods model skeletons based on RNN or CNN, which has limited expressive power for irregular joints. In this paper, we represent skeletons naturally on graphs and propose a generalized graph convolutional neural networks (GGCN) for skeleton-based action recognition, aiming to capture space-time variation via spectral graph theory. In particular, we construct a generalized graph over consecutive frames, where each joint is not only connected to its neighboring joints in the same frame strongly or weakly, but also linked with relevant joints in the previous and subsequent frames. The generalized graphs are then fed into GGCN along with the coordinate matrix of the skeleton sequence for feature learning, where we deploy high-order and fast Chebyshev approximation of spectral graph convolution in the network. Experiments show that we achieve the state-of-the-art performance on the widely used NTU RGB+D, UT-Kinect and SYSU 3D datasets.

I. INTRODUCTION

Action recognition is an active research direction in computer vision, with widespread applications in video surveillance, human computer interaction, robot vision, autonomous driving and so on. Among the multiple modalities [1], [2], [3], [4], [5] that are able to recognize human action, such as appearance, depth and body skeletons [6], [7], the skeleton-based sequences are springing up in recent years, due to the prevalence of affordable depth sensors (e.g., Kinect) and effective pose estimation algorithms [8]. Skeletons convey compact 3D position information of the major body joints, which are robust to variations of viewpoints, body scales and motion speeds [9]. Hence, skeleton-based action recognition has attracted more and more attention [10], [11], [12], [13], [14], [15], [16].

Different from modalities defined on regular grids such as images or videos, dynamic human skeletons are non-Euclidean geometric data, which consists of a series of human joint coordinates. This poses challenges in capturing both the intra-frame features and temporal dependencies. Recent methods learn these features via deep models like recurrent neural networks (RNN) [6], [7], [17], [18], [19], [20], [21], [22], [23] and convolutional neural networks (CNN) [21], [24], [25], [26], [27]. Nevertheless, the topology in skeletons is not fully exploited in the grid-shaped representation of RNN and CNN.
restricted by small partitions, graphs in [33] only model joints bridged by a bone, while there is no explicit temporal graph in [34]. Further, theoretical analysis of the graph construction has not been provided.

In order to further improve the graph construction of skeleton data for stronger expressive power, we propose Generalized Graph Convolutional Networks, where generalized spatial-temporal graphs are constructed over consecutive frames to model both spatial correlations and causal temporal dependencies, providing an alternate view of the action sequence. This is based on our modeling of the variation of skeletons via spectral graph theory [35], where the Laplacian matrix of the generalized spatial-temporal graph extracts the variation of the coordinates of joints via Chebyshev approximation [31] of graph convolution. The captured variation is then leveraged to learn action features for final classification. As graph construction is crucial to the variation modeling, we carefully design the generalized graph according to the correlation among joints. Each joint in the target frame is not only connected to its neighboring joints in the same frame, but also linked with relevant joints in the other two frames, which enables learning the variation of the position of joints both spatially and temporally. Also, we model both strong and weak correlations among joints in space and time with graph edges of different weights. Strong correlations reflect physical connections or strong relationship among non-physical joints, while weak correlations represent potential relationship among joints that are not physically connected. This strengthens learning actions which are accomplished by joints that are not bridged by bones, such as “drink water” with the interaction between one hand and the head. Based on the constructed generalized graphs, we then deploy spectral graph convolutions with fast Chebyshev approximation for feature learning, which leads to final classification scores.

In summary, our contributions include the following aspects:

- We theoretically model the variation in a skeleton sequence on graphs based on spectral graph theory, which leads to the proposed generalized graph representation for capturing the variation.
- We apply GCNN to the input skeleton sequences and the constructed graph. When integrated with the generalized graph, the network is able to capture the variation of joint coordinates, thus leading to effective action feature learning.
- We achieve the state-of-the-art performance on the widely used NTU RGB+D, UT-Kinect and SYSU 3D datasets, and validate the effectiveness of the proposed generalized graph construction.

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1This matrix is an algebraic representation of the topology/connectivity of the corresponding graph in spectral graph theory, which will be defined in Sec. III
order to get the eigenvector matrix. Hence, it is improved by [31] through fast localized convolutions, where the Chebyshev expansion is deployed to approximate GFT. Besides, Susnjara et al. introduce the Lanczos method for approximation [39]. Spectral GCNN has shown its efficiency in various tasks such as segmentation and classification [30], [40].

**Nodal-domain methods.** Many techniques are introduced to implement graph convolution directly on each node and its neighbors, in the nodal domain. Gori et al. introduce recurrent neural networks that operate on graphs in [41]. Duvenaud et al. propose a convolution-like propagation to accumulate local features [29]. Bruna et al. deploy the multiscale clustering of graphs in convolution to implement multiscale representation [28]. Furthermore, Niepert et al. define convolution on a sequence of nodes and perform normalization afterwards [42]. Wang et al. propose edge convolution on graphs by incorporating local neighborhood information, which is applied to point cloud segmentation and classification [43]. Nodal-domain methods provide strong localized filters, but it also means it might be difficult to learn the global structure.

### III. PRELIMINARIES

We consider an undirected graph \( G = \{V, E, A\} \) composed of a vertex set \( V \) of cardinality \(|V| = n\), an edge set \( E \) connecting vertices, and a weighted adjacency matrix \( A \). \( A \) is a real symmetric \( n \times n \) matrix, where \( a_{i,j} \) is the weight assigned to the edge \((i, j)\) connecting vertices \(i\) and \(j\). We assume non-negative weights, i.e., \( a_{i,j} \geq 0 \).

The Laplacian matrix, defined from the adjacency matrix, can be used to uncover many useful properties of a graph. Among different variants of Laplacian matrices, the combinatorial graph Laplacian used in [44], [45] is defined as

\[
L = D - A,
\]

where \( D \) is the degree matrix—a diagonal matrix where \( d_{i,i} = \sum_{j=1}^{n} a_{i,j} \). \( L \) will be leveraged in the proposed variation modeling in Sec. IV-B. The symmetric normalized Laplacian is defined as \( \mathcal{L} = D^{-\frac{1}{2}}LD^{-\frac{1}{2}} \).

Graph signal refers to data that resides on the vertices of a graph, such as social, transportation, sensor, and neuronal networks. In our context, we treat each joint in a skeleton sequence as a vertex in a graph, and construct a spatial-temporal graph. Then we define the corresponding graph signal as the coordinates of each joint.

### IV. GENERALIZED GRAPH CONVOLUTIONAL NETWORKS

We first overview the architecture of the proposed GGCN. Then we dive into our method starting from modeling of the variation in a skeleton sequence, which leads to the generalized graph construction. Based on this, we discuss generalized graph convolution and feature learning.

#### A. GGCN architecture

As illustrated in Fig. 2, the input is a skeleton-based action sequence organized as a \( P \times T_0 \times N_0 \times 3 \) tensor, where \( P \) is the number of frames, \( T_0 \) is the number of frames, \( N_0 \) is the number of joints in each frame, and 3 means the dimension of \( x, y, z \) coordinates. In order to exploit the spatial-temporal dependencies, we firstly concatenate the input sequence in the unit of 3 consecutive frames, e.g., the \( \{1, 2, 3\} \)th frames are concatenated into the first spatial-temporal frame, and the \( \{2, 3, 4\} \)th frames into the second one, etc. Thus, the sequence length is changed to \( T_1 \), and the number of joints in each frame is \( N_1 \) after frame concatenation, where \( T_1 = T_0 - 2 \) and \( N_1 = N_0 \times 3 \). We then construct a generalized graph on each spatial-temporal frame, which will be elaborated in Sec. IV-C, and compute the corresponding symmetric normalized graph Laplacian for describing the connectivities among joints. Secondly, we feed a feature matrix containing the coordinates of skeleton joints in the concatenated sequence and the graph Laplacian into the designed GGCN layer and standard 2D convolution layers for feature extraction. Average pooling is then employed for feature aggregation. Finally, the global feature matrix will go through a fully connected layer followed by a Softmax activation function to output the classification score for \( C \) classes. Also, batch normalization is used for all layers before the ReLU activation function.

#### B. The Modeling of Skeleton’s Variation

The fundamental of skeleton-based action recognition is to capture the variation of joints so as to learn motion features for
classification. We propose to model the variation in skeletons via the graph Laplacian defined in Eq. 1.

As discussed in [46], the Laplacian matrix \( \mathbf{L} \) is essentially a high-pass operator which captures the variation in the underlying signal. For any signal \( \mathbf{x} \in \mathbb{R}^n \), it satisfies

\[
(\mathbf{Lx})(i) = \sum_{j \in \mathcal{N}_i} a_{i,j}(x_i - x_j),
\]

(2)

where \( \mathbf{L} \in \mathbb{R}^{n \times n} \), and \( (\mathbf{Lx})(i) \) denotes the \( i \)-th component of \( \mathbf{Lx} \). \( \mathcal{N}_i \) is the set of vertices connected to \( i \). This presents that when operating \( \mathbf{L} \) on \( \mathbf{x} \), for each vertex, it computes the signal difference among the vertex and its neighboring vertices. In other words, \( \mathbf{Lx} \) captures the variation in \( \mathbf{x} \). We thus represent the coordinates of joints in a skeleton sequence as signal defined on a graph, and model its variation via Eq. 2.

As will be discussed in Sec. IV-D, a Chebyshev polynomial of \( \mathbf{Lx} \) approximates graph convolution in GCNN, thus elegantly enabling learning the variation in a skeleton sequence.

Furthermore, from Eq. 2 we see that the difference among neighboring vertices is weighted by edge weights \( a_{i,j} \) in the underlying graph. If the signal difference between vertices \( i \) and \( j \) is large but the corresponding weight \( a_{i,j} \) is rather small or even 0, then this variation will be neglected. On the other hand, if the signal difference is relatively small but \( a_{i,j} \) is large, then this variation will be magnified. Hence, the choice of edge weights, i.e., graph construction, plays a crucial role in the variation modeling, which is also the key contribution of our method. We elaborate on the proposed generalized graph construction below.

C. Generalized Graph Construction

The generalized graph construction includes spatial connectivity and temporal connectivity.

Spatial connectivity. For each frame, we model the human body via a connected graph, based on two types of connectivities in particular: strong connections \( \mathcal{E}_s \) and weak connections \( \mathcal{E}_w \) for describing different correlations. Strong connections aim to capture strong correlations with large weights to emphasize the variation, including physical connectivity and some physical disconnection among joints, while weak connections are used to represent potential correlations among joints that are not physically connected. As shown in Fig. 1, whereas the “head” joint and “hand” joint are not bridged by a bone, we build a weak connectivity between them because of the latent relationship during some actions (e.g., “drink water”). In particular, different weights are assigned to strong and weak edges, i.e., edge weights within a frame are set as

\[
a_{i,j} = \begin{cases} w_1, & (i, j) \in \mathcal{E}_s \\ w_2, & (i, j) \in \mathcal{E}_w \\ 0, & \text{otherwise}, \end{cases}
\]

(3)

where \( w_1 > w_2 \). We empirically set \( w_1 = 5 \) and \( w_2 = 1 \) in our experiments. Note that, while physical connections are intrinsically fixed, it is our freedom to define strong physical disconnections and weak connections. The choice is dependent on the content of skeleton sequences and prior knowledge, which will be discussed in detail in Sec. V-C.

Temporal connectivity. Unlike previous works where each joint is disconnected in the temporal domain or only connected to its corresponding joints in the adjacent frames, we further connect each joint in frame \( \mathbf{x}_t \) to the neighborhood of its correspondence in the previous frame \( \mathbf{x}_{t-1} \) and subsequent frame \( \mathbf{x}_{t+1} \), which is referred to as potential edge, as shown in Fig. 3. This is to capture the latent variation between one joint in frame \( \mathbf{x}_t \) and its neighboring joints in the adjacent frames. The receptive field in the temporal domain is thus enlarged by exploiting more neighboring joints, which contributes to learning temporal variation. Taking the action “typing on a keyboard” as an example, the left thumb may have little motion in a short period. However, the left index finger moves relative to the left thumb both spatially and over time, which can be captured by the proposed potential edge. Hence, the final generalized adjacency matrix of consecutive frames \( \{\mathbf{x}_{t-1}, \mathbf{x}_t, \mathbf{x}_{t+1}\} \) is defined as

\[
\mathbf{A}_g = \begin{bmatrix}
\mathbf{A}_{t-1,t-1} & \mathbf{A}_{t-1,t} & \mathbf{O} \\
\mathbf{A}_{t,t-1} & \mathbf{A}_{t,t} & \mathbf{A}_{t,t+1} \\
\mathbf{O} & \mathbf{A}_{t+1,t} & \mathbf{A}_{t+1,t+1}
\end{bmatrix},
\]

(4)

where \( \mathbf{O} \in \mathbb{R}^{n \times n} \) is a zero matrix, \( \mathbf{A}_{i,i} \in \mathbb{R}^{n \times n} \) is the weighted adjacency matrix of frame \( i \) for representing the intra-frame connectivity, while \( \mathbf{A}_{i,j} \in \mathbb{R}^{n \times n} \) is the adjacency matrix between frame \( i \) and \( j \) for description of the inter-frame connectivity. Based on \( \mathbf{A}_g \in \mathbb{R}^{3n \times 3n} \), we compute the graph Laplacian \( \mathbf{L} = \mathbf{D} - \mathbf{A}_g \).

D. Generalized Graph Convolution

Following the definition of graph convolution in [31], we adopt the approximation of spectral convolution by Chebyshev
polynomials for efficient implementation:
\[
g_\theta \ast \mathbf{x} \approx \sum_{k=0}^{K-1} \theta_k T_k(\mathcal{L}) \mathbf{x}, \tag{5}
\]
where \( \mathcal{L} = \mathbf{D}^{-\frac{1}{2}} \mathbf{L} \mathbf{D}^{-\frac{1}{2}} \) is the symmetric normalized graph Laplacian as defined in Sec. III, which is employed because the domain of Chebyshev polynomials lies in \([-1, 1]\). \( \theta_k \) denotes the \( k \)-th Chebyshev coefficient and \( g_\theta \) denotes a convolution kernel. \( T_k(\mathcal{L}) \) is the Chebyshev polynomial of order \( k \). It is recurrently calculated by \( T_k(\mathcal{L}) = 2C T_{k-1}(\mathcal{L}) - T_{k-2}(\mathcal{L}) \), where \( T_0(\mathcal{L}) = 1, T_1(\mathcal{L}) = \mathcal{L} \). Hence, the 1-st order Chebyshev polynomial computes \( \mathcal{L} \mathbf{x} \), which exactly corresponds to Eq. 2, thus capturing the variation in the skeleton data. When \( k > 1 \), \( \mathcal{L}^k \) essentially describes \( k \)-hop connectivity, thus incorporating more neighbors and leading to convolution over a larger receptive field.

E. Feature Learning

Having designed the generalized graph convolution, we define the transfer function as follows:
\[
\mathbf{y} = \text{ReLU}(\sum_{k=0}^{K-1} T_k(\mathcal{L}) \mathbf{x} \mathbf{W}_k + \mathbf{b}), \tag{6}
\]
where \( \mathbf{W}_k \in \mathbb{R}^{F_1 \times F_2} \) is a matrix of weight parameters \( \theta'_k \) as in Eq. 5, which will be learnt from the network, and \( F_1, F_2 \) are the dimensions of generated features in two connected layers respectively. \( \mathbf{b} \in \mathbb{R}^{n \times F_2} \) is the bias, while ReLU is an activation function.

After the graph convolution layer, we employ standard 2D convolution to the output \( \mathbf{y} \), followed by feature aggregation via average pooling. Thereafter, a fully-connected layer and a Softmax activation function are adopted to generate the output classification scores. We adopt the categorical cross-entropy loss to train the network. The implementation details of our model will be discussed in Sec. V-B.

V. Experiments

We evaluate our proposed GGCN on four widely used datasets and compare with state-of-the-art skeleton-based action recognition methods. Experimental details and results are discussed below.

A. Datasets and Evaluation Metrics

NTU RGB+D Dataset [17]: This dataset was captured from 40 human subjects by 3 Microsoft Kinect v2 cameras. It consists of 56880 action sequences with 60 classes. Actions 1-49 were performed by one actor, and actions 50-60 were performed by the other two actors. Each body skeleton was recorded with 25 joints. The benchmark evaluations include Cross-Subject (CS) and Cross-View (CV). In the CS evaluation, 40320 samples from 20 subjects were used for training, and the other samples for testing. In the CV evaluation, samples captured from camera 2 and 3 were used for training, while samples from camera 1 were employed for testing.

Florence 3D Dataset [47]: This dataset contains 215 action sequences of 10 actors with 9 classes. Each body skeleton was collected from Kinect, and recorded with 15 joints. We follow the standard experimental settings to perform leave-one-actor-out validation protocol: we use all the sequences from 9 out of 10 actors for training and the remaining one for testing, and repeat this procedure for all the actors. The resulting 10 classification accuracy values are averaged to get the final accuracy.

UT-Kinect Dataset [10]: This dataset was captured using a single stationary Kinect. It consists of 200 sequences with 10 classes, and each skeleton includes 20 joints. The dataset was recorded by three channels: RGB, depth, and skeleton joint locations, whereas we only use the 3D skeleton joint coordinates. We also adopt the leave-one-actor-out validation protocol to evaluate our model on this dataset.

SYSU 3D Dataset [48]: On this dataset, 40 subjects were asked to perform 12 different activities. Therefore, there are totally 480 action videos on this dataset. For each video, the corresponding RGB frames, depth sequences and skeleton information were captured by a Kinect. We use the sequences performed by 20 subjects for training, and the remaining 20 subjects for testing. We employ the 30-fold cross-subject validation and report the mean accuracy on the dataset.

B. Implementation Details

Our proposed model was implemented with the PyTorch frame work. The number of actors \( P \) is set to be 2, 1, 1, 1 for NTU RGB+D, Florence 3D, UT-Kinect, and SYSU 3D dataset respectively. The construction of the generalized graph Laplacian for each dataset will be discussed in detail in Sec. V-C.

Basic Model: Prior to the graph convolution layer, we set a Batch Normalization layer for the batched input data in order to be less careful about data initialization and speed up the training process [49]. In the graph convolution layer, we set the Chebyshev order \( K \) to be 4, and the dimension of the weight matrix \( \mathbf{W}_k \) in Eq. 6 to be \( 3n \times 3n \) (i.e., the same as the generalized Laplacian matrix \( \mathcal{L} \)). In terms of the standard 2D convolution layer, we set the stride to be 1 and the kernel size to be 9. Each convolution layer follows a Batch Normalization layer. We choose ReLU as the activation function after each convolution layer, and assign the dropout rate 0.5.

Deep Stacking: The above convolutional model can be easily extended into a deep architecture. Taking the above model as one basic layer, we stack it into a multi-layer network architecture, in which the output at the previous layer is used as the input of the next layer. With the increase of layers, the receptive field of convolutional kernels become larger, thus enabling abstracting more global information.

Next, we employ three average pooling layers to pool the \( P, N, \) and \( T \) dimension respectively, followed by a fully connected layer and a Softmax activation function to output the final classification score. The number of neurons depends

2https://pytorch.org
on the output channel of the last convolution layer of the network. We apply Adam [50] optimizer to train the whole model with the initial learning rate 0.1, and decrease it on the 10th epoch. Note that we did not perform any normalization on the skeleton coordinates during data preprocessing.

C. Data Preprocessing

NTU RGB+D Dataset: Due to some missing skeletons in this dataset, we only use the cleaned data3 for action recognition [51]. In order to enhance the robustness of model training, we split the sequences into several segments of equal size in a way similar to [33]. Specifically, we split the whole sequence into 32 segments, and pick the {1, 2, 3, 4}th frame respectively from each segment to generate a large amount of training data.

During the graph construction, according to the characteristics of the actions, we build strong edges across physically connected joints, as well as potentially correlated joint pairs including both hands, both finger tips, both thumbs, both wrists, and both elbows. Both hands and head are linked with weak edges.

Florence 3D Dataset: Since the sequences in this dataset contain few frames, we design two ways to generate the training set: sampling and interpolation. For longer sequences (i.e., the length of the sequence is greater than 32), we randomly choose 32 frames; for the other sequences, we calculate the mean of two adjacent frames and insert it into the sequence as a new frame, eventually forming a sequence of 32 frames. For all the sequences, we repeat this operation 3 times to generate the training set.

Regarding the graph construction, we build strong edges for physical connections, as well as both hands and neck, and both hands and head. The weak edges are set to be an empty set. Note that we change the weak edges between both hands and head in the NTU RGB+D dataset to strong edges because they are more closely related in this dataset.

UT-Kinect Dataset: We also adopt sampling and interpolation methods to generate the training set. Here, we set the length of each training sequence to be 64, and repeat the process twice. During the graph construction, we treat physical connections, as well as the connections between both hands and head, and both wrists and head as strong edges, while setting weak edges to be an empty set.

SYSU 3D Dataset: Similar to the NTU RGB+D dataset, we split each sequence into 32 segments, and pick the {1, 2, 3, 4, 5}th frame from each segment to generate the training set. However, this dataset does not provide vertex labels, hence we only adopt the adjacency matrix of physical connections provided by the author as the graph within each frame.

Note that, the temporal graph construction is the same on the above datasets, where each joint is not only connected to its corresponding joints in the previous and subsequent frames, but also linked with the neighbors of the corresponding joints.

3https://github.com/InwoongLee/TS-LSTM

D. Results on NTU RGB+D Dataset

As reported in Tab. I, our model achieves accuracy of 87.5% in CS and 94.3% in CV respectively. Also, as will be discussed in the ablation study, the proposed intra-connections improve the performance by 0.7% in CS and 1.4% in CV over the baseline method (GGCN+Bone), while the proposed temporal connectivities lead to 3.2% gain in CS and 3.1% gain in CV, validating the effectiveness of our method.

Comparison with the State-of-the-arts: We present the comparison with the state-of-the-art methods in Tab. I. We see that our method outperforms all the other state-of-the-art methods. Specifically, compared with the latest state-of-the-art method SR-TSL [52], our model leads to 2.7% gain in CS and 1.9% gain in CV respectively, which demonstrates the superiority of our method.

Ablation Study: In order to validate the advantages of the proposed generalized graph construction in our method, we evaluate various graph construction methods progressively and design the following incomplete models. Model 1 is GGCN + Bone, in which only joints connected with a bone are linked with graph edges. This kind of graph construction is commonly used in existing graph-based skeleton recognition [32], [33], [34], and thus is the baseline. Model 2 is GGCN + Bone + Intra-connection (non-physical), where connectivities are further added to joints that are not physically connected within each frame, including strong and weak edges for capturing latent dependencies. This kind of connectivities are previously exploited in [34]. Model 3 is our complete model with extra temporal connections included. We observe that Model 1 already achieves competitive performance with the state-of-the-art methods, which shows the effectiveness of the proposed GGCN. With additional intra-connectivities, Model 2 improves the accuracy by 0.7% in CS and 1.4% in CV over Model 1, validating the benefits of non-physical connections. Further, when the temporal connections are exploited, the complete model achieves 2.5% gain in CS and 1.7% gain in CV over Model 2. We thus conclude that both
the proposed non-physical intra-connectivities and the explicit temporal connections make contributions to skeleton-based action recognition, in which the temporal connectivities are more crucial.

Analysis of the Training Process: Moreover, Fig. 4 shows the training process of our model on the NTU RGB+D dataset in CS validation. The horizontal axis is the index of the training epoch, while the vertical axis refers to the mean training loss. We observe that the mean training loss decreases rapidly in the first 10 epochs due to the large learning rate. We update the learning rate at the 10\textsuperscript{th} epoch, after which the mean training loss decreases slowly. Until the 20\textsuperscript{th} epoch, the mean training loss basically converges, validating the effectiveness of our model.

E. Results on SYSU 3D Dataset

We compare our method with the state-of-the-art skeleton-based action recognition methods on SYSU 3D Dataset, which are presented in Tab. II. Our proposed method outperforms all the other state-of-the-art methods on this dataset, achieving accuracy improvement of 1.0% over the previous best method DPRL \cite{34}.

Note that, as vertex labels are not provided by this dataset, we can only build strong physical connections from the given adjacency matrix within each frame while abandoning weak edges. Hence, we provide ablation study with Model 1 (GGCN + Bone) in Tab. II. We see that our complete model achieves 2.7% improvement over the baseline method with additional intra-connectivities. Further, when the temporal connectivities are built, the complete model achieves 1.1% improvement over Model 2, which demonstrates the advantages of the proposed generalized graph construction.

G. Results on Florence 3D Dataset

We present the performance comparison with the state-of-the-art methods on the Florence 3D dataset in Tab. IV. Our methods. Note that the performance difference among all the methods is rather small in general. The reason is that this dataset includes several very similar actions, which are difficult to distinguish without RGB or depth data.

Also, we perform the same ablation study as in Sec. V-D, as reported in Tab. III. We observe that Model 2 improves the accuracy by 0.5% over Model 1 with additional intra-connectivities. Further, when the temporal connectivities are built, the complete model achieves 1.1% improvement over Model 2, which demonstrates the advantages of the proposed generalized graph construction.

TABLE I

| Methods | CS | CV | Year |
|---------|----|----|------|
| Dynamic Skeletons \cite{48} | 60.2 | 65.2 | 2015 |
| Part-aware LSTM \cite{17} | 62.9 | 70.3 | 2016 |
| Geometric Features \cite{20} | 70.3 | 82.4 | 2017 |
| LSTM-CNN \cite{21} | 82.9 | 91.0 | 2017 |
| Two-Stream CNN \cite{24} | 83.2 | 89.3 | 2017 |
| ST-LSTM (Tree) + Trust Gate \cite{23} | 69.2 | 77.7 | 2018 |
| Deep STGC \textsuperscript{K} \cite{33} | 74.9 | 86.3 | 2018 |
| ST-GCN \cite{32} | 81.5 | 88.3 | 2018 |
| DPRL \cite{34} | 83.5 | 89.8 | 2018 |
| SR-TSL \cite{52} | 84.8 | 92.4 | 2018 |
| GGCN + Bone | 84.3 | 91.2 |
| Complete GGCN model | 87.5 | 94.3 |

TABLE II

| Methods | Accuracy | Year |
|---------|----------|------|
| Dynamic Skeletons \cite{48} | 75.5 | 2015 |
| LAFF (SKL) \cite{53} | 54.2 | 2016 |
| ST-LSTM (Tree) \cite{23} | 73.4 | 2018 |
| ST-LSTM (Tree) + Trust Gate \cite{23} | 76.5 | 2018 |
| DPRL \cite{34} | 76.9 | 2018 |
| GGCN + Bone | 75.2 |
| Complete GGCN model | 77.9 |

TABLE III

| Methods | Accuracy | Year |
|---------|----------|------|
| Lie Group \cite{14} | 97.1 | 2014 |
| LARP+mfPCA \cite{54} | 94.9 | 2015 |
| SPGK \cite{13} | 97.4 | 2016 |
| ST-NBNN \cite{16} | 98.0 | 2017 |
| Bi-LSTM \cite{22} | 96.9 | 2018 |
| ST-LSTM(Tree) + Trust Gate \cite{23} | 97.0 | 2018 |
| DPRL \cite{34} | 98.5 |
| GGCN + Bone | 96.9 |
| Complete GGCN model | 97.4 |
method achieves classification accuracy of 98.5%, outperforming all the other state-of-the-art methods significantly except Deep STGC$_K$ [33]. The reason is that Deep STGC$_K$ benefits from the design philosophy of autoregressive moving average model, which is tailored for time sequences. Due to the few joints in each frame and few frames in the sequence, our model is difficult to capture subtle variation from few joints. Thus we misclassify “drink from a bottle” and “answer phone”, “read watch” and “clap”, which is difficult to distinguish even with human vision.

Moreover, Tab. IV reports the results of ablation study. We achieve 0.1% improvement from non-physical intra-connections compared with GGCN + Bone, and another 2.8% improvement from temporal connections compared with GGCN + Bone + Intra-connection. This validates the effectiveness of the proposed graph construction, in which the temporal connectivities are vital.

VI. CONCLUSION

We propose a generalized graph convolutional network (GGCN) for skeleton-based action recognition, aiming to fully exploit both spatial and temporal dependencies among human joints via generalized graph construction. The proposed generalized graph not only captures intrinsic physical connections, but also models strong and weak non-physical connectivities over consecutive frames so as to represent latent correlations for better recognition. We then employ spectral graph convolution with high-order Chebyshev approximation for feature extraction. Extensive experiments demonstrate the superiority of our method.

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TABLE IV

| Methods                  | Accuracy | Year |
|--------------------------|----------|------|
| Lie Group [14]           | 90.9     | 2014 |
| LARP+mfPCA [54]          | 89.7     | 2015 |
| Rolling Rotations [55]   | 91.4     | 2016 |
| SPGR [13]                | 91.6     | 2016 |
| Transition Forests [56]  | 94.2     | 2017 |
| MIMTL [57]               | 95.3     | 2017 |
| Bi-LSTM [22]             | 93.0     | 2018 |
| Deep STGC$_K$ [33]       | 99.1     | 2018 |
| GGCN + Bone              | 95.5     |      |
| GGCN + Bone + Intra-connection | 95.6 |      |
| Complete GGCN model      | 98.4     |      |
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