COSMOLOGICAL IMPLICATIONS OF CONFORMAL FIELD THEORY

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Requiring all massless elementary fields to have conformal scaling symmetry removes a conflict between gravitational theory and the quantum theory of elementary particles and fields. Extending this postulate to the scalar field of the Higgs model, dynamical breaking of both gauge and conformal symmetries determines parameters for the interacting fields. In uniform isotropic geometry a modified Friedmann cosmic evolution equation is derived with nonvanishing cosmological constant. Parameters determined by numerical solution are consistent with empirical data for redshifts \( z \leq z^* = 1090 \), including luminosity distances for observed type Ia supernovae and peak structure ratios in the cosmic microwave background (CMB). The theory does not require dark matter.

Keywords: Conformal theory; modified Friedmann equation; dark energy

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1. Introduction

Massless fields of the standard model\(^{1,2}\) have definite conformal character for local Weyl scaling\(^3,4\) such that \( g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x)e^{2\alpha(x)} \). If an action integral is conformally invariant, the implied energy-momentum 4-tensor is traceless. The Einstein tensor is not. Compatibility can be imposed if Einstein-Hilbert gravitational field action is replaced by a uniquely determined action integral \( I_g \) constructed using the conformal Weyl tensor.\(^3\) This preserves phenomenology on the distance scale of the solar system, while providing an alternative explanation of excessive rotational velocities in galaxies, without invoking dark matter.\(^3\)

Higgs symmetry-breaking invokes a spacetime constant scalar field \( \Phi \). Conformal symmetry requires Lagrangian density \( \mathcal{L}_\Phi \) to contain a term proportional to Ricci curvature scalar \( R = g_{\mu\nu}R^{\mu\nu} \), where Ricci tensor \( R^{\mu\nu} \) is the symmetric contraction \( R^\mu_{\lambda\nu} \) of the gravitational Riemann tensor. Thus conformal \( \Phi \) is a cosmological entity.\(^3\) Its energy-momentum tensor contains a residual cosmological constant.\(^3\) An earlier study\(^5\) showed that a conformal scalar field can modify Einstein-Hilbert gravitation, but did not consider the conformally invariant Weyl tensor.
These results suggest a unifying postulate, that all massless elementary fields have conformal symmetry. The cosmological consequences of universal conformal symmetry are explored here. It will be shown that identifying the Higgs scalar field as the source of a cosmological constant (aka dark energy) produces an internally consistent theory, in agreement with all relevant cosmological data.

Variational formalism of classical field theory \(^6\) is easily extended to the context of general relativity. \(^3\) Generic Lagrangian density \( \mathcal{L} \) defines action integral

\[
I = \int d^4x \sqrt{-g} \mathcal{L},
\]

where \( g \) is the determinant of metric tensor \( g_{\mu\nu} \). The metric functional derivative \( \frac{\delta I}{\delta g_{\mu\nu}} \) is

\[
X_{\mu\nu} = x_{\mu\nu} + \frac{1}{2} \xi g_{\mu\nu},
\]

if \( \delta L = x_{\mu\nu} \delta g_{\mu\nu} \). This defines symmetric energy-momentum tensor \( \Theta_{\mu\nu} = -2 X_{\mu\nu} \). Evaluated for solutions of the conformal field equations, trace \( g_{\mu\nu} \Theta_{\mu\nu} = 0 \).

Conformally invariant action integral \( I_\Phi \) is defined for scalar field \( \Phi \) by Lagrangian density

\[
L_\Phi = \left( \partial_\mu \Phi \right)^\dagger \partial^\mu \Phi - \frac{1}{6} R \Phi^\dagger \Phi - \lambda \left( \Phi^\dagger \Phi \right)^2,
\]

where \( R \) is the Ricci scalar. \(^3\) The Higgs mechanism \(^2\) postulates incremental Lagrangian density \( \Delta L_\Phi = w^2 \Phi^\dagger \Phi - \lambda \left( \Phi^\dagger \Phi \right)^2 \). This augments \( L_\Phi \) by a term \( w^2 \Phi^\dagger \Phi \) which breaks conformal symmetry. In conformal theory, this term must be produced dynamically.

Mannheim\(^3\)[arXiv:astro-ph/0505266] has recently reviewed conformal gravitational theory. The Einstein-Hilbert Lagrangian density is replaced by a uniquely determined quadratic form \(^3\)[Section 8.7]

\[
L_g = -\alpha g^{\lambda\mu\kappa\nu} C_{\lambda\mu\kappa\nu}^\lambda,
\]

where \( C_{\lambda\mu\kappa\nu}^\lambda \) is the conformally invariant Weyl tensor. This tensor is the traceless component of Riemann tensor \( R_{\mu\kappa\nu}^\lambda \), obtained by removing a linear combination of contracted terms depending on the Ricci tensor and scalar \(^3\)[Eq.(180)]. The Weyl tensor vanishes identically in uniform, isotropic Robertson-Walker (RW) geometry. Vanishing of the metric functional derivative of action integral \( I_g \) \(^3\)[Eq.(185)], for the RW metric given below, can be verified by direct evaluation.

Thus the conformal gravitational action integral replaces the standard Einstein-Hilbert action integral, but in the uniform model of cosmology its functional derivative drops out completely from the gravitational field equations\(^3\)[Section 10.1]. The observed Hubble expansion requires an alternative gravitational mechanism. This is supplied by a postulated conformal scalar field. A nonvanishing conformal scalar field determines gravitational field equations that differ from Einstein-Hilbert theory. The Newton-Einstein gravitational constant is not relevant. As shown by Mannheim\(^3\)[Eq.(224)], the gravitational constant determined by the scalar field is inherently negative, appropriate to Hubble expansion of the early universe.

The argument here differs from Mannheim by noting that the Lagrangian terms proportional to \( \Phi^\dagger \Phi \) in Higgs and conformal theory have opposite algebraic signs. A consistent theory must include both. The consequences of this are examined here, leading in particular to a modified Friedmann cosmic evolution equation that differs from the standard form used in all previous work. An important consequence is that the spatial curvature parameter implied by the modified Friedmann equation is now consistent with current cosmological data, removing a severe problem in fitting type Ia supernovae redshift data using the standard equation. \(^3\) The anomalous
imaginary-mass term in the Higgs scalar field Lagrangian becomes a cosmological constant (dark energy) in the modified Friedmann equation. This term dominates the current epoch.

It should be noted that fitting conformal gravitation to galactic rotation data[3][Section 9.3] implies a universal nonclassical linear gravitational potential $V = \gamma_0 c^2 r/2$. Coefficient $\gamma_0$, independent of galactic luminous mass, must be attributed to the background Hubble flow[3] On converting the local Schwarzschild metric to conformal RW form, this produces a curvature parameter $k = -\frac{1}{4} \gamma_0^2$ which is small and negative, consistent with other current empirical data. This supports the present argument for modifying the standard Friedman equation in RW geometry.

For redshift $z(t)$, the modified Friedmann equation determines scale parameter $a(t) = 1/(1 + z(t))$ and Hubble function $H(t) = \frac{\dot{a}}{a}(t)$. Acceleration parameter $\ddot{a}/a^2$ is always positive and occurs explicitly in the modified dimensionless sum rule. In the current epoch, dark energy and acceleration terms are of comparable magnitude, the curvature term is small, and other terms are negligible. As $t \to \infty$, $H(t)$ descends asymptotically to a finite value determined by the cosmological constant, while the acceleration parameter goes to zero.

The present analysis derives Ricci scalar $R$ as a time-dependent function. This indicates that other nominally constant parameters deduced here are time-dependent on a cosmological scale (ten billion years). If these parameters were strictly constants, fitted here to data for $z \leq z_* = 1090$, the Hubble function would increase from zero at some initial minimum $a_0$ ($z_0 > z_*$) to a maximum value $\bar{H}$ at $a < a_*$, defining an inflationary epoch. Comoving radius $1/aH$ would decrease monotonically from infinity, which eliminates any inherent horizon problem[7] Details of this early epoch require accurate time-dependent parameters, not currently available. An initial big-bang singularity, or initial $a_0$ small enough that the corresponding temperature would support nucleosynthesis[7] cannot be ruled out.

2. Scalar and tensor field equations

The scalar field equation is $\partial_{\mu} \partial^{\mu} \Phi = ( -\frac{1}{6} R + w^2 - 2\lambda \Phi^4 \Phi) \Phi$. Generalizing the Higgs construction, for constant $R$ the scalar field equation has a global solution[5] such that $\Phi^4 \Phi = \phi_0^2 = (w^2 - \frac{1}{6} R)/2\lambda$, if this ratio is positive. $\phi_0^2$ determines gauge boson masses. Empirical parameters imply $\frac{1}{6} R > w^2$, so that $\lambda < 0$. Although this differs from the standard electroweak model, which assumes positive $w^2$ and $\lambda$, conformal theory determines a stable scalar field solution with a finite energy-momentum tensor so long as $\phi_0^2 > 0[5]$.

If Ricci $R$ were neglected, for positive parameter $\lambda$ fluctuations about scalar field $\Phi^4 \Phi = \phi_0^2$ would satisfy a Klein-Gordon equation with mass parameter $m_H = \sqrt{2w^2}$, defining a Higgs boson[2]. The empirical value of $w$ deduced here would imply $m_H \simeq 10^{-33} eV$. However, the implied value of $\lambda$ is negative, inconsistent with a Klein-Gordon equation. This does not define a Higgs mass. This issue is discussed
in more detail elsewhere.\footnote{\textsuperscript{9}}

Given matter/radiation action integral $I_m$, the gravitational field equation in terms of functional derivatives is $X_g^\mu \phi^\mu + X_m^\mu = 0$. Gravitational energy-momentum exactly cancels that of matter and radiation. Conformal $X_g$ vanishes identically in uniform, isotropic geometry.\footnote{\textsuperscript{3}} RW gravitation is determined by $X_\Phi$, due to Ricci scalar $R$ in $\mathcal{L}_\Phi$. Finite $X_m$ determines finite $X_\Phi$. For the quantized scalar field, finite energy density precludes destabilization of the vacuum.\footnote{\textsuperscript{9}} The resulting gravitational field equation is $X_g^\mu = \frac{1}{2} \Theta_m^\mu$, where $X_g^\mu = \frac{1}{2} R^{\mu \nu} \phi^\mu + \frac{1}{2} \mathcal{L}_g g^{\mu \nu}$, evaluated for $\phi^\mu = (w^2 - \frac{1}{2} R)/2 \lambda$.

For $\mathcal{L}_\Phi = \frac{1}{2} \phi^2 (w^2 - \frac{1}{2} R - \lambda \phi^2) = \frac{1}{2} \phi^2 (3w^2 - \frac{1}{2} R)$, the gravitational functional derivative is $X_g^\mu = \frac{1}{6} \phi^2 (R^{\mu \nu} - \frac{1}{4} R g^{\mu \nu} + \frac{3}{2} w^2 g^{\mu \nu})$. The first two terms here replace the Einstein tensor of standard theory by a traceless modified tensor. This removes an obvious inconsistency in the context of universal conformal symmetry. Defining $\bar{\kappa} = -3/\lambda$ and $\bar{\Lambda} = \frac{3}{2} w^2$, the gravitational field equation is $R^{\mu \nu} - \frac{1}{4} R g^{\mu \nu} + \Lambda g^{\mu \nu} = -\bar{\kappa} \Theta_m^\mu$. The effective cosmological constant $\bar{\Lambda}$ here is identified with the Higgs scalar field parameter $w^2$.

3. The modified Friedmann equation

For Robertson-Walker metric $ds^2 = dt^2 - a(t)^2 (\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2)$, Ricci tensor $R^{\mu \nu}$ depends on $a(t)$ through two independent functions, $\xi(t) = \frac{\dot{a}}{a}$ and $\xi_1(t) = \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{2a^2}$, such that $R^{00} = 3 \xi_0$ and scalar $R = 6(\xi_0 + \xi_1)$.

In a consistent conformal theory, vanishing trace eliminates one of the two independent Friedmann equations of standard theory. Energy density $\rho = \Theta_m^{00}$ implies modified Friedmann equation $-\frac{3}{2} (R^{00} - \frac{1}{4} R) = \xi_1(t) - \xi_0(t) = \frac{\dot{a}}{a} + \frac{\ddot{a}}{2a} - \frac{\dot{a}^2}{2a^2} = \frac{3}{2} (\bar{\kappa} \rho + \bar{\Lambda})$, which determines RW scale parameter $a(t)$ and Hubble function $H(t)$.\footnote{\textsuperscript{8}} For consistency with electroweak theory $\bar{\Lambda} = \frac{3}{2} w^2 > 0$. For positive energy density $\rho$, $\bar{\kappa} \rho$ is negative, as shown by Mannheim.\footnote{\textsuperscript{11}}

At present time $t_0$, $a(t_0) = 1$. $H(t_0) = 1$ in Hubble units $H_0 = 100 h$ km/s/Mpc, with $h = 0.705\textsuperscript{11}$ and $h, c = 1$. Scaled energy densities $\rho_m a^3$ and $\rho_r a^4$, for matter and radiation respectively, are constant. In the absence of dark matter, $\rho_m \simeq \rho_r$, the baryon density.

It is convenient to define constant parameters $\alpha = \frac{\dot{a}}{a} = w^2 > 0$, $k \simeq 0$, $\beta = -\frac{k}{6} \rho_m a^3 > 0$, and if $\rho_m \rightarrow \rho_r$, $\gamma = 3 \beta / 4 R_b(t_0) > 0$, where $\frac{3}{2} R_b(t) = \frac{h}{H_0} a(t)$ is the ratio of baryon to radiation energy densities. Empirical value $R_b(t_0) = 688.6$ is assumed here. The equation to be integrated is $\frac{\dot{a}^2}{a^2} - \frac{\dot{a}}{a} = -\frac{d}{dt} \frac{a}{a} = \dot{a} = \alpha - \frac{k}{a^2} - \frac{\beta}{a} - \frac{\gamma}{a^3}$.

Dividing by $H^2(t)$ implies dimensionless sum rule $\Omega_m(t) + \Omega_r(t) + \Omega_\Lambda(t) + \Omega_k(t) + \Omega_q(t) = 1$, where $\Omega_m(t) = \frac{\dot{a}^2}{3 H^2(t)} < 0$, $\Omega_r(t) = \frac{\dot{a}^2}{3 H^2(t)} < 0$, $\Omega_\Lambda(t) = \frac{\bar{\Lambda}}{H^2(t)} > 0$, $\Omega_k(t) = \frac{k}{a^2 (H(t))^2}$, and $\Omega_q(t) = \frac{\dot{a} a}{H(t)} = -q(t)$. Acceleration parameter $\Omega_q(t)$ appears explicitly in the sum rule.

Because parameters $\alpha, \beta, \gamma$ are all necessarily positive, $\dot{a}$ must vanish for some $a(t) = \bar{a}$, defining a maximum value $\frac{a}{\bar{a}} = H(\bar{t}) = \bar{H}$. Assuming constant parameters,
$H(t) = 0$ for a finite minimum $a_0$, defining time $t = 0$. There are no mathematical singularities. $a(t)$ increases monotonically from $a_0$ to $a(\bar{t}) = \bar{a}$, where $\dot{a}$ vanishes and Hubble function $H(t) = \frac{\dot{a}}{a}$ rises to its maximum value $\bar{H}$. Acceleration parameter $\Omega_q = 1 - \frac{\bar{H}}{H^2} = 1 + \frac{\dot{a}^2}{a^2} + \frac{k a^2}{a^2} - \frac{6 \dot{a}^2}{a^2} \rightarrow \infty$ at $a_0$. It decreases monotonically to unity at $\bar{t}$, and asymptotically to zero, while $\Omega_\Lambda \rightarrow 1$.

In standard theory an initial inflationary epoch is postulated, in which the co-moving Hubble radius $\frac{1}{aH}$ decreases as time increases, for positive $\Omega_q$. This condition is satisfied by the modified Friedmann equation. For $H(t)$ in units of $H_0$, \[
\frac{\dot{a}}{\sqrt{H}} = -\frac{\dot{a}}{a} \frac{dH}{dt} = -H(1 + \frac{\dot{a}^2}{H^2}) = -H(1 + \frac{1}{H^2} - \frac{6 \dot{a}^2}{H^2}) = -H \frac{6 \ddot{a}}{H^2} = -H \Omega_q \leq 0.
\]

Scaled Hubble radius $\frac{1}{aH}$ decreases monotonically to present value unity.

4. Numerical solution in the current epoch

The modified Friedmann equation has been solved for vanishing $k$, $\beta$, and $\gamma$. Parameter $\alpha$ is adjusted to match a fit to observed magnitudes of type Ia supernovae, using scaled luminosity distance $H_0d_L/c$ computed as a function of redshift $z$. $\Omega_q$ is determined by the modified equation.

| $z$ | $\Omega_\Lambda$ | $\Omega_q$ | $H_0d_L/c$(calc) | $H_0d_L/c$(\cite{10}) |
|-----|------------------|------------|------------------|------------------|
| 0.000 | 0.732 | 0.268 | 0.000 | 0.000 |
| 0.063 | 0.672 | 0.328 | 0.066 | 0.066 |
| 0.133 | 0.619 | 0.381 | 0.145 | 0.145 |
| 0.211 | 0.571 | 0.429 | 0.240 | 0.241 |
| 0.298 | 0.530 | 0.470 | 0.355 | 0.357 |
| 0.395 | 0.492 | 0.508 | 0.494 | 0.497 |
| 0.503 | 0.459 | 0.541 | 0.663 | 0.666 |
| 0.623 | 0.428 | 0.572 | 0.868 | 0.871 |
| 0.758 | 0.401 | 0.599 | 1.118 | 1.121 |
| 0.909 | 0.376 | 0.624 | 1.424 | 1.426 |
| 1.079 | 0.353 | 0.647 | 1.799 | 1.799 |

Results for $z \leq 1$ agree to graphical accuracy with the $\Lambda$CDM model. Parameter $\alpha = \Omega_\Lambda(t_0) = 0.732$ for $\Omega_k(t) = 0$ is consistent with current empirical values $\Omega_\Lambda = 0.726 \pm 0.015, \Omega_k = -0.005 \pm 0.013$. Any significant discrepancy would invalidate the present theory. Mannheim fits type Ia supernovae luminosities setting $\Omega_m = 0$ and using the standard Friedmann equation, which requires $\Omega_k = 1 - \Omega_\Lambda$. The implied $\Omega_k$ is much larger than empirical limits $\pm \Omega_k \simeq 0.01$. This is corrected by modified sum rule $\Omega_k = 1 - \Omega_\Lambda - \Omega_q \simeq 0$. Here $d_L(z) = (1+z)d_a(z)$ for geodesic
distance $d_\chi$ such that $r_\chi = \chi(z) = \int_{t_z}^{t_0} cdt/a(t)$, for $z(t_z) = z$. Parameters $\Omega_m, \Omega_r$, which scale as $a^{-3}, a^{-4}$ respectively, can apparently be neglected for $z \leq 1$.

5. Recombination epoch

In standard theory $aH|_{a=0} \rightarrow (\Omega_m H_0^2)^{1/2} (1+z)^{3/2}$ is used to define dimensionless shift ratio \[ R(z) = (\Omega_m H_0^2)^{1/2} (1+z) d_A(z) = aH(1+z)^{3/2} d_A(z), \] for angular diameter distance $d_A = d_L/(1+z)^2$. Not restricted to a particular limiting form, $R(z) = \frac{H_0 d_A(z)}{c/(1+z)^{3/2}} \frac{H(z)}{H_0}$. The empirical value for recombination epoch $z_*=1090$, at $t = t_*$, is $R(z_*) = 1.710 \pm 0.019$.11

A second dimensionless ratio is acoustic scale $\ell_A(z) = (1+z)^{-\alpha} \frac{H_0 d_A(z)}{a(z)}$.12 CMB data determine $\ell_A(z_*) = 302.10 \pm 0.86$.11 Comoving sound horizon $d_s$, in Hubble length units $c/H_0$, is determined by $r_s(z_*) = \int_{t_*}^{t_0} \frac{cdt}{a(t)\sqrt{H(t)+H_0}}$.

Fitting the modified Friedmann equation to $H_0 d_L(z)/c$ for $z \leq 1$,10 3 to shift parameter $R(z_*)$, and to acoustic scale ratio $\ell_A(z_*)$ determines model parameters $\alpha = 0.7171, k = -0.01249, \beta = 0.3650 \times 10^{-5}$. The fourth parameter is $\gamma = 0.3976 \times 10^{-8}$, if fixed at $\gamma = 3\beta/4R_0(t_0)$, which neglects dark matter. There is no significant inconsistency with current empirical data, given the demonstration by Mannheim3 that type Ia supernovae redshifts can be fitted neglecting $\Omega_m = -\beta/a^3 H^2$ in the current epoch ($z \leq 1$). Clearly parameter $\Omega_m$ must be reconsidered in the context of conformal theory.

Parameters determined by the modified Friedmann equation fit model-independent data from the recombination epoch $z_* = 1090$ to the present $z = 0$. Relevant computed parameters are

| $z$  | 1090 | 100  | 10   | 1    | 0  |
|-----|------|------|------|------|----|
| $H$ | 92.4 | 11.50| 2.44 | 1.43 | 1.00|
| $\Omega_q$ | 0.474 | 0.063 | 0.625 | 0.622 | 0.271 |
| $\Omega_\Lambda$ | 0.000 | 0.005 | 0.121 | 0.353 | 0.717 |
| $\Omega_k$ | 1.740 | 0.963 | 0.255 | 0.025 | 0.012 |
| $\Omega_m$ | -0.555 | -0.028 | -0.001 | 0.000 | 0.000 |
| $\Omega_r$ | -0.659 | -0.003 | -0.000 | -0.000 | -0.000 |
| $\frac{H_0}{c} d_H$ | 15.54 | 10.26 | 4.71 | 1.41 | 1.00 |
| $\frac{H_0}{c} d_\chi$ | 614.1 | 47.78 | 5.68 | 0.81 | 0.00 |

$H(t)$ has its maximum value $\ddot{H} = 100.9$ for $\ddot{z} = 1371$, where $\Omega_q = 1$. As redshift $z$ increases, dark energy term $\Omega_\Lambda$ becomes negligible. For $z > \ddot{z}$, $H(t)$ decreases to zero, while $a(t) \rightarrow a_0 = 0.508 \times 10^{-3}$ ($z_0 = 1967$) and $\Omega_q \rightarrow \infty$.

For negative $k$, as implied here, the geodesic Hubble radius becomes $d_H = \sinh(\sqrt{-k}/aH)/\sqrt{-k}$, tabulated above as $H_0 d_H/c$. Similarly, geodesic distance to given redshift $z$ is $d_\chi = \sinh(\sqrt{-k} \chi(z))/\sqrt{-k}$, where the comoving distance is
\[ \chi(z) = \int_{t_z}^{t_0} \frac{c dt}{a(t)} . \]

Meaningful cosmological structure requires the wavelength of a periodic perturbation to be smaller than the Hubble horizon, \( \kappa, \kappa d_H > 1 \). For angular structure, multipole index \( \ell \simeq \kappa d_H \). The relative scale criterion is \( \ell > \frac{d_{H}}{d_{\chi}} \). Using parameter values listed above, conformal theory implies \( \ell(z_*) > 39.52 \), consistent with observed CMB anisotropies.

Defining criterion \( \zeta = \frac{1}{6} R - w^2 \), the dimensionless sum rule determines \( \zeta = \xi_0 + \xi_1 - w^2 = H(t)^2(2\Omega_q + \Omega_m + \Omega_r) \). For \( a \to 0 \), when both \( a \) and \( k \) can be neglected, the sum rule implies \( \zeta = H(t)^2(\Omega_q + 1) \). For large \( a \), \( \zeta = H(t)^2(2\Omega_q) \). \( \zeta > 0 \) in both limits, regardless of numerical values, since \( \Omega_q > 0 \). Using the present empirical parameters, \( \zeta \) is positive for all \( z \). For nonzero \( \phi_0^2 \), this implies that empirical scalar field parameter \( \lambda \) is negative.

To test consistency, parameter \( H_0d_V(z)/c \), where \( d_V(z) = [d_{\chi}^2(z)cz/H(z)]^{1/3} \), was computed for \( z = 0.35 \). For \( \alpha = 0.732 \) and vanishing \( k, \beta, \gamma \), computed value 0.3087 agrees with empirical 0.322±0.015. Using all four adjusted parameters, the computed value changes only to 0.3083.

6. Conclusions and speculations

Conformal theory provides a straightforward explanation of dark energy: it appears in the energy-momentum tensor of the scalar field required by the Higgs mechanism to produce the masses of gauge bosons. It should be possible to compute the implied cosmological constant as the self-interaction of the Higgs scalar field. The required transition amplitude, which creates an induced gauge field, depends on the extremely small cosmological time derivative of the dressed scalar field. The time constant is of the order \( 10^{10} \) years. Solving the coupled field equations for \( g_{\mu \nu}, \Phi, \) and induced \( U(1) \) gauge field \( B_\mu \), using this computed time derivative, gives \( w \simeq 2.651hH_0 = 3.984 \times 10^{-33} eV \). This approximate calculation agrees in order of magnitude with parameter value \( w = 1.273 \times 10^{-33} eV \) implied by present value \( \Omega_A(t_0) = \alpha = 0.7171 \). Details will be published separately. These numbers justify the conclusion that conformal theory explains both the existence and magnitude of dark energy.

The modified Friedmann equation predicts a stationary value of the Hubble function at redshift somewhat greater than \( z_* \), preceded by an inflationary epoch. A time-dependent theory of the relevant field parameters is needed in order to compare with current big-bang theory.

Dark matter is not required for \( z \leq 1 \) supernovae redshifts, for anomalous galactic rotation, or for the present empirical parametrization of the modified Friedmann equation. Interpretation of the parameter \( \Omega_m \) requires substantial revision if the modified Friedmann equation is correct.

Models of gravitational lensing should consider that geodesic deflection is due to the difference between nonuniform galactic matter and the nonvanishing averaged background that determines the RW metric. This subtracts the isotropically dis-
persed repulsive field considered here from the attractive field due to the observed galaxy. Subtracting this isotropic repulsive field would have the same effect as an additive attraction attributed to a dark-matter halo.

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