Colour Charges and Anti-Screening

E. Bagan

Grup de Física Teòrica and IFAE
Edifici Cn, Universitat Autònoma de Barcelona
E-08193 Bellaterra
Spain
E-mail: bagan@ifae.es

R. Horan, M. Lavelle and D. McMullan
School of Mathematics and Statistics
University of Plymouth
Drake Circus, Plymouth, Devon PL4 8AA
U.K.
E-mail: rhoran/mlavelle/dmcmullan@plymouth.ac.uk

If constituent quarks are to emerge from QCD they must have well defined colour and be energetically favoured. After reviewing the general properties of charges in gauge theories, a method for constructing charges is presented and applied to the infra-red structure of the theory and to the interquark potential. Both of these applications supply a physical interpretation of the structures found in the construction of charges. We will see that constituent structures arise in QCD.

Asymptotic Dynamics

The S-matrix is used to describe the scattering of in-particle states to some final out-particle states. The existence of such an S-matrix description of particle scattering is based on the overlap between the interacting and free theory in the remote past and future, i.e., in QED with the interaction Hamiltonian

\[
H_I(t) = -e \int d^3x \bar{\psi}(x)\gamma^\mu \psi(x)A_\mu(x),
\]

(1)

it is assumed that \(H_I(t) \to 0\) as \(t \to \pm \infty\). This large time limit, though, fails to be that of the free theory in QED. Instead one finds that the interaction survives:

\[
H_I(t) \to -e \int d^3x A_\mu(x)J_\mu^{as}(x),
\]

(2)

where the non-vanishing asymptotic current has the form:

\[
J_\mu^{as}(x) = \int d^3p \rho(p) \frac{p_\mu}{E_p} \delta^{(3)}(x - \frac{tp}{E_p}).
\]

(3)

\[a\text{Talk presented by D. McMullan.}\]
What this is telling us is that in such a theory with massless fields, the associated long-range nature of the interactions implies that we cannot simply “switch off the coupling”. The asymptotic dynamics for the matter is not free, and hence there is no particle description at early or late times. This is the well-known infra-red problem in QED.

The interpretation of this result that we wish to emphasize, though, is that Gauss’ law is not trivial asymptotically: the matter and gauge fields cannot be viewed as independent. In particular, the matter fields are not physical as they are not separately gauge invariant even at remote times. This raises the obvious question: if the matter fields are not physical, then what are the physical charges in this theory, and can they have a particle description?

**Construction of Charges**

Charges must be gauge invariant to be physical. The generic form for a charged field in QED is thus given by the product of the form $h^{-1}(x)\psi(x)$ where, under a gauge transformation, $\psi \rightarrow e^{ie\theta} \psi$, the dressing transforms as

$$h^{-1} \rightarrow h^{-1}e^{-ie\theta}.$$  (4)

There are, though, many ways to construct such invariant fields that are not physically acceptable. In addition, then, we impose the kinematical condition that we want charges that are, in fact, charged particles at early and late times, i.e., they should have a sharp energy-momentum at these asymptotic times. This condition is encoded in the dressing equation:

$$hu^\mu \partial_\mu (h^{-1}) = -ieu^\mu A_\mu,$$  (5)

where $u^\mu$ is the asymptotic four velocity of the particle. This equation can be motivated either from the form of the asymptotic dynamics associated with the dressed field, or from heavy charge effective theory.

Equations (4) and (5) are the fundamental equations in our approach to charges in QED and, as we will see later, QCD. For a given four velocity $u^\mu$, which we write as $\gamma(\eta + v)^\mu$ where $\eta = (1,0,0,0)$ and $v = (0,v)$, we can solve these equations to obtain the dressing

$$h^{-1} = e^{-ieK}e^{-i\chi},$$  (6)

where

$$K(x) = -\int_{\Gamma} ds(\eta + v)^\mu \frac{\partial^\nu F_{\nu\mu}}{\vec{G} \cdot \partial},$$  (7)

$$\chi(x) = \frac{\vec{G} \cdot A}{\vec{G} \cdot \partial}.$$  (8)
\[ G^\mu = (\eta + v)^\mu (\eta - v) \cdot \partial - \partial^\mu \]
while \( \Gamma \) is the past (future) trajectory of an incoming (outgoing) particle. It is immediate from this form of the solution that the dressing has structure:

- a gauge dependent part associated with \( \chi \) which makes the whole charge gauge invariant, we thus talk of this as the minimal part of the dressing;
- a gauge invariant part associated with \( K \) which we call the additional part of the dressing. It is needed to satisfy the dressing equation.

To gain a better understanding of why these different structures arise in the dressing and, indeed, to verify that the dressing equation does capture the particle structure of the asymptotic charges as we expect, we need to perform detailed perturbative tests of the construction.

In the present context, what we need to calculate are the on-shell \( n \)-point Green’s functions for the charged fields. Each charged field needs to be dressed in the form appropriate to the asymptotic velocity of its in-coming or out-going particle. The dressing for each charge can then be expanded perturbatively and the minimal and additional structure can be seen to generate two new vertices. Armed with these new Feynman rules, the on-shell Green’s functions can be calculated and we find that, at the appropriate points on the mass-shell:

- the infra-red divergences associated with the minimal part of the dressing cancel those soft divergences that arise from the perturbative expansion of the matter;
- in pair creation processes, the additional gauge invariant part of the dressing generates a phase divergence which cancels the one which arise from the matter fields.

Thus we have seen that, already at the level of Green’s functions, our construction does describe charged particles and that the structure in the dressing reflects the differing infra-red behaviour of the matter fields. In this context, we can view the minimal part of the dressing as the soft component of the charge, and the additional part as the phase.

**Charges in QCD**

In contrast to the situation in QED where the electric charge was itself gauge invariant, in an unbroken non-abelian gauge theory, such as QCD, the very definition of charge requires a restriction on the form of the gauge transformations at spatial infinity. Once this is correctly taken into account, though, the route to the construction of non-abelian charged particles proceeds in a now familiar manner:
• gauge invariance is maintained through the dressing $h^{-1}$ where now, under a non-abelian gauge transformation, $h^{-1}(x) \to h^{-1}(x)U(x)$;

• the kinematical restriction required to describe an asymptotic charged particle with four velocity $u$ is $hu \cdot \partial (h^{-1}) = gu \cdot A$.

All that needs to be done, then, is to solve these equations!

We have presented an algorithm for the perturbative construction of the soft part of the dressing to an arbitrary order in perturbation theory and we can incorporate some non-perturbative effects. We have also proven that there is a global obstruction to the construction of such dressings. This follows because it can be shown that any such dressing could be used to construct a gauge fixing which, however, we know cannot exist globally. This shows that isolated quarks are not observables and colour must be confined.

In addition to the construction of charged matter, in a non-abelian theory we also need to construct gauge invariant glue. The dressing can also be used for this and by analyzing what happens in massless QED we can see that the dressing equation for a massless charge will also lead to a vanishing asymptotic Hamiltonian. This gives us confidence that even the collinear structure associated with such charges will be correctly captured in our approach.

Rather than discuss these general results here, it is perhaps more helpful to present a concrete calculation which captures the essential new ingredients found in the non-abelian theory. This will also allow us to give a new physical interpretation to the minimal part of the dressing. Thus we shall conclude this talk by presenting a direct route to the calculation of the potential between static quarks.

**The Interquark Potential**

Up to now we have essentially been concerned with the construction of isolated charges. However, when it comes to hadrons we are faced with the question of whether these colour singlets have a constituent structure, i.e., are they constructed from individual gauge invariant constituent charges? There are various models for hadronic structure, some of which have a constituent structure and some of which do not. We wish to investigate to what extent these pictures follow from QCD.

The method we are going to follow is to construct gauge invariant constituent states and then study their energy. This will then be compared with that derived from the Wilson loop which we recall yields, at order $\alpha^2$, the
following potential between two static quarks

\[ V(r) = -\frac{g^2 C_F}{4\pi r} \left[ 1 + \frac{g^2 C_A}{4\pi} \left( 4 - \frac{1}{3} \right) \log(\mu r) \right]. \quad (9) \]

At lowest order this is a Coulombic potential and the logarithmic correction we have split here into two parts reflecting the anti-screening and screening contributions typical of a non-abelian gauge theory.

The dominant anti-screening contribution comes from longitudinal glue and the screening part from gauge invariant glue. This divide has been seen in a wide variety of studies of the \( \beta \)-function. Since our dressings factorise into two parts (a minimal part constructed out of longitudinal degrees of freedom and an additional gauge invariant term) we would expect that if a constituent picture holds then the minimal part of the dressing for the constituents should reproduce the dominant anti-screening term in the potential. We will now show this.

To calculate the potential at order \( \alpha^2 \) we will need to extend the minimal dressing to \( O(g^3) \). Having done this the potential is the separation dependent part of the expectation value of the Hamiltonian between the charged constituent states.

The minimal static dressing in QED was: \( \exp(-ie\chi) \), with \( \chi = \partial_i A_i / \nabla^2 \). In QCD this has the gauge invariant extension

\[ \exp(-ie\chi) \Rightarrow \exp(g\chi^a T^a) \equiv h^{-1} \quad (10) \]

with \( g\chi^a T^a = (g\chi^1 + g^2 \chi^2 + g^3 \chi^3 + \cdots) T^a \) where

\[ \chi^a_1 = \frac{\partial_j A^a_j}{\nabla^2}; \quad \chi^a_2 = f^{abc} \frac{\partial_j}{\nabla^2} \left( \chi^b_1 A^c_j \right) + \frac{1}{2} (\partial_j \chi^b_1) \chi^c_1; \quad \ldots \quad (11) \]

Note that for the rest of this talk we denote by \( h^{-1} \) the above minimal static dressing.

Now we take the expectation value of the Hamiltonian between such minimally dressed quark/antiquark states: \( \bar{\psi}(y) h(y) h^{-1}(y') \psi(y') |0 \rangle \). This yields for the potential

\[ - \text{tr} \int d^3 x \langle 0 | [E_i^a(x), h^{-1}(y)] [h(y) [E_i^a(x), h^{-1}(y')] |0 \rangle \quad (12) \]

At \( O(g^2) \) we obtain for \( V^{g^2}(r) \)

\[ - g^2 \text{tr} \int d^3 x \langle 0 | [E_i^a(x), \chi^d_1(y)] T^{d} T^{b} [E_i^a(x), \chi^b_1(y')] |0 \rangle. \quad (13) \]
From the equal-time commutator: $[E_i^a(x), A_j^b(y)] = i\delta^{ab}\delta_{ij}\delta(x - y)$ we thus obtain the expected Coulomb potential:

$$V^{g^2}(r) = -\frac{g^2NC_F}{4\pi r}, \quad (14)$$

At $O(g^4)$ we require $[E_i^a(x), h^{-1}(y)]h(y)$. From simple manipulations we obtain

$$g[E_i^a(x), \chi_1(y)] + g^2[E_i^a(x), \chi_2(y)] + g^3[E_i^a(x), \chi_3(y)] + O(g^4), \quad (15)$$

plus many gauge dependent terms which we can drop. This yields for the potential

$$V^{g^4}(r) = -\frac{3g^4C_FCA}{(4\pi)^3} \int d^3z \int d^3w \frac{1}{|z - w|} \times \left( \frac{1}{|z - y|} \right) \left( \frac{1}{|w - y'|} \right) \langle A^T_k(w)A^T_j(z) \rangle . \quad (16)$$

Inserting the free propagator

$$\langle A^T_k(w)A^T_j(z) \rangle = \frac{1}{2\pi^2} \frac{(z - w)_j(z - w)_k}{|z - w|^4} \quad (17)$$

and performing the integrals, we easily find

$$V^{g^4}(r) = -\frac{g^4}{(4\pi)^2} \frac{NC_FCA}{2\pi r} 4\log(\mu r). \quad (18)$$

This we recognise as the expected anti-screening contribution to the interquark potential. This calculation has shown that the non-abelian anti-screening effect can be understood as arising from the interaction of two gauge invariant constituents.

**Conclusions**

Starting from the observation that in gauge theories the asymptotic fields are not free, we have seen that Gauss’ law implies that dressings are needed to construct physical charged fields. We have presented a method of constructing dressed charges and have seen that this results in a structured dressing.

In QED we have demonstrated that the dressings remove the infra-red divergences associated with on-shell Green’s functions. We note that the different structures found in the dressings play different roles in this cancellation.
In non-abelian gauge theories such as QCD well-defined colour charges
must be dressed if colour is to be a good quantum number. The perturba-
tive construction of dressed fields has been investigated by various authors,
however, we stress that there is a global obstruction to the construction of an
isolated colour charge.

To test the physical validity of constituent structures we have studied the
potential between such gauge invariant quarks. The anti-screening contribu-
tion to the full interquark potential was recovered by considering minimally
dressed quarks. This result shows that we have determined the dominant glue
configuration and renders a constituent structure visible.

Extensions of this work to higher orders in the potential and to the usual
screening contribution are in progress. Given the global obstruction to the
construction of dressings, the above constituent picture of hadrons must break
down at some scale. The major challenge of this programme of research is to
determine this scale.

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