Airfoil aerodynamics optimization under uncertain operating conditions

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Abstract: An aerodynamic robust optimization of a 2D airfoil is presented. Robust optimization aims to find a design which is less sensitive to small changes in uncertain quantities. Aerodynamic performance of airfoil is studied under variation of Mach number and angle of attack as operational uncertainties. The optimal aerodynamic shape of airfoil is computed by maximizing lift and drag coefficient ratio. Airfoil aerodynamic performance is evaluated using XFOIL solver. NACA 0012 is adopted as a basic airfoil design and optimized for robustness using Taguchi method. Signal-to-Noise ratio (S/N) of Taguchi method for larger the better is applied as robustness index. Nine of eleven variables of PARSEC airfoil parameters with 3 levels for each factors are selected as design variables and distributed using L27 fractional orthogonal array as the design of experiment. The optimization process does not only increase the average performance of lift-to-drag ratio but also decrease the standard deviation. The optimization strategy significantly improves the aerodynamic performance of the basic NACA 0012 from 21.76 to 36.79 or improves 69% of the robustness index.

1. Introduction

Airfoil is a cross sectional shape of an aircraft wing or a wind turbine blade. Improving the aerodynamic shape of an airfoil could increase wing or wind turbine blade performance. Conventionally aerodynamic performance of an airfoil design is optimized in a certain operating condition such as angle of attack, Mach number, and Reynold number [1, 2]. Practically, those operating conditions may fluctuate during flight. Without considering fluctuation in those operating conditions, the aerodynamic performance of wing design, for an example, may be highly sensitive to small changes of operating conditions, which leads to performance loss. In this work, aerodynamic performance of an airfoil shape is optimized by considering angle of attack and Mach number as the operational uncertainties. The optimization method by considering uncertain conditions is known as robust optimization in which the optimum design is less sensitive to small changes in uncertain quantities [3-5]. Airfoil NACA 0012 is selected as aerodynamic shape optimization problem. NACA 0012 is a symmetric airfoil, commonly used as cross-sectional airfoil of helicopter, un-manned vehicles, and wind turbine blades [6], [7]. PARSEC parameter method is adopted to configure the airfoil shape. Aerodynamic performance, lift-to-drag ratio, for each airfoil design is evaluated using XFOIL under different Mach number and angle of attack as uncertain operating conditions. Taguchi method [8] is conducted for robust optimality of airfoil NACA 0012.
2. Airfoil parameterization and optimization

Airfoil parameterization to describe airfoil sections are known for many applications, like the NACA 4 and 5 digit airfoils and other standard sections such as PARSEC and CST [9, 10]. NACA uses 4 or 5 digits number to describe an airfoil shape, as shown in Figure 1 for NACA 0012 as an example.

Beside airfoil NACA parameterization, PARSEC airfoil parameter is also commonly found in literature [7, 9, 11]. PARSEC uses eleven parameters to describe an airfoil shape as shown in Figure 2 and Table 1.

![Figure 1. Airfoil NACA 0012](image1)

![Figure 2. PARSEC airfoil parameterization](image2)

| Parameters               | Symbol  | Parameters               | Symbol  |
|--------------------------|---------|--------------------------|---------|
| Leading edge radius      | \( R_{LE} \) | Lower crest curvature    | \( Z_{XXLO} \) |
| Upper crest absissa      | \( X_{UP} \) | Trailing edge ordinate   | \( Z_{TE} \) |
| Upper crest ordinate     | \( Z_{UP} \) | Trailing edge thickness  | \( \Delta Z_{TE} \) |
| Upper crest curvature    | \( Z_{XXUP} \) | Trailing edge direction  | \( \alpha_{TE} \) |
| Lower crest absissa      | \( X_{LO} \) | Trailing edge wedge angle| \( \beta_{TE} \) |
| Lower crest ordinate     | \( Z_{LO} \) || |

The PARSEC parameter of NACA 0012 is adopted from [6] and presented in
Table 2. PARSEC parameter of NACA 0012

| PARSEC Parameters                  | Value     |
|-----------------------------------|-----------|
| Leading edge radius, $R_{LE}$     | 0.01550   |
| Upper crest absissa, $X_{UP}$     | 0.29663   |
| Upper crest ordinate, $Z_{UP}$    | 0.06002   |
| Upper crest curvature, $Z_{XXUP}$ | -0.45150  |
| Lower crest absissa, $X_{LO}$     | 0.29663   |
| Lower crest ordinate, $Z_{LO}$    | -0.06002  |
| Lower crest curvature, $Z_{XXLO}$ | 0.45150   |
| Trailing edge ordinate, $Z_{TE}$  | 0.00000   |
| Trailing edge thickness, $\Delta Z_{TE}$ | 0.00000 |
| Trailing edge direction, $\alpha_{TE}$ | 0.00000 |
| Trailing edge wedge angle, $\beta_{TE}$ | 0.00000 |

The aerodynamic performance of an airfoil is indicated by lift and drag force acting on it. Lift and drag force generated by air flow depend on the angle of attack between the direction of the motion of the airfoil through the air and the chord line of the airfoil (Figure 3). Lift force generally depend on pressure distribution along the airfoil shape, while drag force depends on both pressure and friction distribution.

![Figure 3. Lift force and drag force on an airfoil](image)

There are many numerical studies about lift and drag coefficients of airfoils under different conditions. Airfoil improvement is commonly approached as single objective or multi-objective optimization under a certain operational condition. Single objective optimization of an airfoil by minimizing the drag force (indicated by drag coefficient, $C_d$) [12], maximizing the lift force (indicated by lift coefficient, $C_l$) [11, 13] or simultaneously maximizing lift-to-drag ratio ($C_l/C_d$) [14]. Some works used multi-objective approach to improve an airfoil performance by maximizing $C_l$ as the first objective and minimizing $C_d$ as the second objective [2, 15]. In this work, aerodynamic performance of airfoil is optimized by maximizing $C_l/C_d$.

3. Robust Optimization using Taguchi Method

Various optimization methods have been developed to handle uncertain quantities i.e., reliability-based method and robust design method. Robust design such as Taguchi method [8] considers the influence of life-cycle uncertainties and applies parameter design to search for the optimum. Taguchi
method is adapted from fractional factorial experiments featuring signal-to-noise ratio (S/N), orthogonal arrays (OA), and analysis of means (ANOM). The optimal combination of parameter levels with maximum S/N is identified from the effect analysis to improve the average performance and at the mean time to reduce the performance deviation. Some successful works had been reported in the literature [16-18].

Depending on the performance requirement, there are three types of S/N adapted from Taguchi’s method which are defined as follows:

2.1. Smaller-the-better problems:

\[
S / N_{STB} = -10 \cdot \log_{10} \left( \frac{1}{n} \sum_{i=1}^{n} y_i^2 \right)
\]  

(1)

2.2. Larger-the-better problems:

\[
S / N_{LTB} = -10 \cdot \log_{10} \left( \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2} \right)
\]  

(2)

2.3. Nominal-the-best problems:

\[
S / N_{NTB} = -10 \cdot \log_{10} \left( \frac{1}{n} \sum_{i=1}^{n} (y_i - m)^2 \right)
\]

where \(y_i\) are the output responses, \(n\) is the number of samples in the outer array, and \(m\) is the design target value.

Nine variables of eleven PARSEC airfoil parameters are considered as the design variables. At this work, the trailing edge thickness (\(\Delta Z_{TE}\)) and the trailing edge direction (\(\alpha_{TE}\)) was omitted from the analysis and fixed at zero. Herein a minimal orthogonal array of \(L_{27}\) design of experiment is selected for design variables to distribute samples in inner array, as shown in Table 3. Lower and upper bound of PARSEC variables are ±20% fitted of nominal NACA 0012 and selected as lower and upper level of orthogonal array. The parameter values of the basic NACA 0012 are assumed to be the second levels to explore the design space centered at the initial design.

A full factorial design of two 2-level variables is selected for the uncertain quantities or noise factors. Different Mach number and angle of attach are applied to each airfoil design to estimate fluctuation of aerodynamic performance. The robust design of airfoil obtained from Taguchi method is compared to basic NACA 0012 for aerodynamic performance.
Table 3. L27 fractional orthogonal array for nine 3-levels design variables

| Design | RL_E | XL_P | ZUP | ZZXUP | XLO | ZLO | ZZXLO | ZTE | Beta (°) |
|--------|------|------|-----|-------|-----|-----|-------|-----|----------|
| 1.     | 0.0100 | 0.2500 | 0.0500 | -0.5000 | 0.2500 | -0.0500 | 0.4000 | -0.0100 | 0.0000 |
| 2.     | 0.0100 | 0.2500 | 0.0500 | -0.5000 | 0.3000 | -0.0600 | 0.4500 | 0.0000 | 2.5000 |
| 3.     | 0.0100 | 0.2500 | 0.0500 | -0.5000 | 0.3500 | -0.0700 | 0.5000 | 0.0100 | 5.0000 |
| 4.     | 0.0100 | 0.3000 | 0.0600 | -0.4500 | 0.2500 | -0.0500 | 0.4000 | 0.0000 | 5.0000 |
| 5.     | 0.0100 | 0.3000 | 0.0600 | -0.4500 | 0.3000 | -0.0600 | 0.4500 | 0.0100 | 0.0000 |
| 6.     | 0.0100 | 0.3000 | 0.0600 | -0.4500 | 0.3500 | -0.0700 | 0.5000 | -0.0100 | 2.5000 |
| 7.     | 0.0100 | 0.3500 | 0.0700 | -0.4000 | 0.2500 | -0.0500 | 0.4000 | 0.0100 | 2.5000 |
| 8.     | 0.0100 | 0.3500 | 0.0700 | -0.4000 | 0.3500 | -0.0600 | 0.4500 | -0.0100 | 5.0000 |
| 9.     | 0.0100 | 0.3500 | 0.0700 | -0.4000 | 0.3500 | -0.0700 | 0.5000 | 0.0000 | 0.0000 |
| 10.    | 0.0130 | 0.2500 | 0.0600 | -0.4000 | 0.2500 | -0.0600 | 0.5000 | -0.0100 | 0.0000 |
| 11.    | 0.0130 | 0.2500 | 0.0600 | -0.4000 | 0.3000 | -0.0700 | 0.4000 | 0.0000 | 2.5000 |
| 12.    | 0.0130 | 0.2500 | 0.0600 | -0.4000 | 0.3500 | -0.0500 | 0.4500 | 0.0100 | 5.0000 |
| 13.    | 0.0130 | 0.3000 | 0.0700 | -0.5000 | 0.2500 | -0.0600 | 0.5000 | 0.0000 | 5.0000 |
| 14.    | 0.0130 | 0.3000 | 0.0700 | -0.5000 | 0.3000 | -0.0700 | 0.4000 | 0.0100 | 0.0000 |
| 15.    | 0.0130 | 0.3000 | 0.0700 | -0.5000 | 0.3500 | -0.0500 | 0.4500 | -0.0100 | 2.5000 |
| 16.    | 0.0130 | 0.3500 | 0.0500 | -0.4500 | 0.2500 | -0.0600 | 0.5000 | 0.0100 | 2.5000 |
| 17.    | 0.0130 | 0.3500 | 0.0500 | -0.4500 | 0.3000 | -0.0700 | 0.4000 | -0.0100 | 5.0000 |
| 18.    | 0.0130 | 0.3500 | 0.0500 | -0.4500 | 0.3500 | -0.0500 | 0.4500 | 0.0000 | 0.0000 |
| 19.    | 0.0160 | 0.2500 | 0.0700 | -0.4500 | 0.2500 | -0.0700 | 0.4500 | -0.0100 | 0.0000 |
| 20.    | 0.0160 | 0.2500 | 0.0700 | -0.4500 | 0.3000 | -0.0500 | 0.5000 | 0.0000 | 2.5000 |
| 21.    | 0.0160 | 0.2500 | 0.0700 | -0.4500 | 0.3500 | -0.0600 | 0.4000 | 0.0100 | 5.0000 |
| 22.    | 0.0160 | 0.3000 | 0.0500 | -0.4000 | 0.2500 | -0.0700 | 0.4500 | 0.0000 | 5.0000 |
| 23.    | 0.0160 | 0.3000 | 0.0500 | -0.4000 | 0.3000 | -0.0500 | 0.5000 | 0.0100 | 0.0000 |
| 24.    | 0.0160 | 0.3000 | 0.0500 | -0.4000 | 0.3500 | -0.0600 | 0.4000 | -0.0100 | 2.5000 |
| 25.    | 0.0160 | 0.3500 | 0.0600 | -0.5000 | 0.2500 | -0.0700 | 0.4500 | 0.0100 | 2.5000 |
| 26.    | 0.0160 | 0.3500 | 0.0600 | -0.5000 | 0.3000 | -0.0500 | 0.5000 | -0.0100 | 5.0000 |
| 27.    | 0.0160 | 0.3500 | 0.0600 | -0.5000 | 0.3500 | -0.0600 | 0.4000 | 0.0000 | 0.0000 |

4. Optimization results

Analysis of mean shows effect plot for S/N ratio of Taguchi Method, as shown in Figure 4. Red dots are the optimum levels of PARSEC parameter. As shown in the figure, trailing edge ordinate position (ZTE) is the most significant effect to S/N ratio, while leading edge radius (RL_E), upper crest abissa (XL_P) and lower crest curvature (ZZXLO) give smaller effect to S/N ratio. Verification result of the estimated optimum is presented in Table 4. It is shown that optimized airfoil does not only improve aerodynamics performance but also decrease fluctuation under different operating conditions. Robust optimization improves the S/N ratio from 21.76 of base design NACA 0012 airfoil to 36.79 of optimized airfoil.

Figure 4. Effect plot for the S/N of Cl/Cd
Table 4. Comparison results of airfoil optimization

| \( \text{R}_{\text{LE}} \) | \( \text{X}_{\text{UP}} \) | \( \text{Z}_{\text{UP}} \) | \( \text{Z}_{\text{OXUP}} \) | \( \text{X}_{\text{LO}} \) | \( \text{Z}_{\text{LO}} \) | \( \text{Z}_{\text{XXLO}} \) | \( \text{Z}_{\text{TE}} \) | \( \beta_{\text{TE}} \) (\(^\circ\)) | \( \bar{\text{C}}_{\text{L}}/\overline{\text{C}}_{\text{D}} \) | \( s \) | S/N |
|----------------|----------|----------|-------------|----------|----------|-------------|----------|---------------|----------|------|-----|
| 0.0155         | 0.2966   | 0.0600   | -0.4515     | 0.2966   | -0.0600  | 0.4515      | 0.0000   | 0.0000        | Baseline | 29.25| 36.62| 21.76|
| 0.0130         | 0.3500   | 0.0700   | -0.4000     | 0.2500   | -0.0500  | 0.4000      | -0.0100 | 5.0000        | Optimized| 78.26| 23.99| 36.79|

Figure 5. Airfoil shape comparison

Figure 5 shows airfoil shape comparison of basic NACA 0012 and optimized shape, while Figure 6 shows pressure distribution comparison along the airfoil surface. The pressure distributions are plotted under the same operating conditions which are \( M = 0.3 \), \( \alpha = 3^\circ \), and \( \text{Re} = 3.0 \times 10^6 \). As shown in the figure, the pressure on lower surface reaches the lowest value relatively close to the leading edge. This means that pressure gradient extend only no more than 5% of the cord length. Also note that the pressure distribution along upper surface starts with stagnation condition. \( C_p = 1 \) occurs at leading edge, \( X/C = 0 \), for both airfoil design. Figure 6 also shows difference between the upper and lower surface pressure. It is negative along the entire cord, indicating that all segments of the cord are contributing on the lift force. On the lower optimized surface, the pressure distribution is almost flat. Pressure difference of the optimized airfoil shape is larger than pressure distribution of basic NACA 0012, indicating also better in lift coefficient compared with basic NACA 0012.

Figure 6. Pressure distribution (\( M = 0.5 \); \( \alpha = 3 \), and \( \text{Re} = 3.0 \times 10^6 \))
5. Conclusions

The aerodynamic airfoil optimization under the uncertainty of free stream Mach number or angle of attack has been successfully applied to optimize the Cl/Cd of a NACA 0012 airfoil using Taguchi method. In both cases of Mach number and angle of attack uncertainty, the mean and variance of Cl/Cd coefficient of robust airfoils are significantly improved as compared to the basic NACA 0012.

The optimized airfoil improves for both average performance and standard deviation, or increases 69% in S/N ratio.

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