Abstract. After a brief review of the quark-based model for nuclear matter, and some pion properties in medium presented in our previous works, we report new results for the pion valence wave function as well as the valence distribution amplitude in medium, which are presented in our recent article. We find that both the in-medium pion valence distribution and the in-medium pion valence wave function, are substantially modified at normal nuclear matter density, due to the reduction in the pion decay constant.

Keywords In-medium pion properties · Form Factors · Distribution Amplitude

1 Introduction

One of the most exciting and challenging topics in hadronic physics is to study the modifications of hadron properties in nuclear medium, and to study how such modifications give impact on observables in medium. Since hadrons are composed of quarks, antiquarks and gluons, one can naturally expect that hadron internal structure would change when they are immersed in nuclear medium [1; 2; 3; 4; 5]. This question becomes particularly interesting when it comes to that of pion. To be able to answer it, one first needs, simpler, effective quark-antiquark models of pion, which are successful in describing its properties in vacuum. Among such models, light-front constituent quark model has been very successful in describing the electromagnetic form factors, electromagnetic radii and decay constants of pion and kaon [6; 7; 8; 9; 10; 11]. Recent advances in experiments, indeed suggest to make it possible to access to the pion properties in a nuclear medium [12; 13; 14].

Among the all hadrons, pion is the lightest, and it is believed as a Nambu-Goldstone boson, which is realized in nature emerged by the spontaneous breaking of chiral symmetry. This Nambu-Goldstone boson, pion, plays very important and special roles in hadronic and nuclear physics. However, because of its special properties, particularly the unusually light mass, it is not easy to describe the pion properties in medium as well as in vacuum based on naive quark models, even though such models can be successful in describing the other hadrons.

Recently, we studied the properties of pion in nuclear medium [12; 13; 14], namely, the electromagnetic form factor, charge radius and weak decay constant, distribution amplitude (DA) by using a

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light-front constituent quark model. There, the in-medium input was calculated by the quark-meson coupling (QMC) model \cite{3 15}. The main purpose of this article is, to report our new results for the in-medium pion DA studied in Ref. \cite{14}, supplemented by the other pion properties already presented in Refs. \cite{12 13}.

Below, we first briefly review the QMC model, the quark-based model of nuclear matter, to study the pion properties in medium. The QMC model was invented by Guichon \cite{15} to describe the nuclear matter based on the quark degrees of freedom. The self-consistent exchange of the scalar-isoscalar $\sigma$ and vector-isoscalar $\omega$ mean fields coupled directly to the relativistic confined quarks, are the key and novelty for the new saturation mechanism of nuclear matter. The model was extended to finite nuclei \cite{16}, and has successfully been applied for various nuclear and hadronic phenomena \cite{3}.

The effective Lagrangian density for a uniform, spin-saturated, and isospin-symmetric nuclear system (symmetric nuclear matter) at the hadronic level is given by \cite{3 15 13},

$$\mathcal{L} = \bar{\psi} i\gamma \cdot \partial - m_N^*(\sigma) - g_\sigma \omega^\mu \gamma_\mu \psi + \mathcal{L}_{\text{meson}}, \quad (1)$$

where $\psi$, $\sigma$ and $\omega$ are respectively the nucleon, Lorentz-scalar-isoscalar $\sigma$, and Lorentz-vector-isoscalar $\omega$ field operators with,

$$m_N^*(\sigma) \equiv m_N - g_\sigma(\sigma)\sigma. \quad (2)$$

Note that, in symmetric nuclear matter isospin-dependent $\rho$-meson mean filed is zero, and thus we have omitted it. Then the relevant free meson Lagrangian density is given by,

$$\mathcal{L}_{\text{meson}} = \frac{1}{2} (\partial_\mu \hat{\sigma} \hat{\partial}^\mu \hat{\sigma} - m_\sigma^2 \hat{\sigma}^2) - \frac{1}{2} \partial_\mu \hat{\omega}_\nu (\hat{\partial}^\nu \hat{\omega}^\mu - \hat{\partial}^\mu \hat{\omega}^\nu) + \frac{1}{2} m_\pi^2 \hat{\omega}^\mu \hat{\omega}_\mu. \quad (3)$$

Hereafter, we consider in the rest frame of symmetric nuclear matter. Then, within Hartree mean-field approximation, the nuclear (baryon) and scalar densities are respectively given by,

$$\rho = \frac{4}{(2\pi)^3} \int dk \theta(k_F - |k|) = \frac{2k_F^2}{3\pi^2}, \quad \rho_s = \frac{4}{(2\pi)^3} \int dk \theta(k_F - |k|) \frac{m_N^*(\sigma)}{\sqrt{m_N^*(\sigma) + k^2}}, \quad (4)$$

here, $m_N^*(\sigma)$ is the value (constant) of effective nucleon mass at given density (see also Eq. (2)). In the standard QMC model \cite{3 15 16} the MIT bag model is used, and the Dirac equations for the light quarks inside a nucleon (bag) composing nuclear matter, are given by,

$$\begin{bmatrix}
    i\gamma \cdot \partial_x - (m_q - V_{\sigma}^q) + \gamma_0 \left( V_{\rho}^q + \frac{1}{2} V_{\omega}^q \right) \\
    i\gamma \cdot \partial_x - (m_q - V_{\sigma}^q) + \gamma_0 \left( V_{\omega}^q - \frac{1}{2} V_{\rho}^q \right)
\end{bmatrix}
\begin{bmatrix}
    \psi_u(x) \\
    \psi_d(x)
\end{bmatrix} = 0, \quad (5)
$$

$$\begin{bmatrix}
    i\gamma \cdot \partial_x - (m_q - V_{\sigma}^q) + \gamma_0 \left( V_{\omega}^q - \frac{1}{2} V_{\rho}^q \right) \\
    i\gamma \cdot \partial_x - (m_q - V_{\sigma}^q) + \gamma_0 \left( V_{\rho}^q + \frac{1}{2} V_{\omega}^q \right)
\end{bmatrix}
\begin{bmatrix}
    \psi_d(x) \\
    \psi_u(x)
\end{bmatrix} = 0. \quad (6)$$

As usual, Coulomb interaction is neglected, and SU(2) symmetry is assumed, $m_{u,d} = m_{d,u} \equiv m_{q,d}$. The corresponding effective (constituent) quark masses are defined by, $m_{u,a}^*, m_{d,a}^*, m_{q,d}^*, m_{q,u}^* \equiv m_{q,d} - V_{\sigma}^q$, to be explained later.

As mentioned already, in symmetric nuclear matter within Hartree approximation, the $\rho$-meson mean field is zero, $V_\rho = 0$, in Eq. (3), and we ignore it. The constant mean-field potentials are defined as, $V_{\sigma}^q \equiv g_\sigma^q \sigma = g_{\sigma}^q < \sigma >$, and, $V_{\omega}^q \equiv g_{\omega}^q \omega = g_{\omega}^q < \omega >$, with $g_{\sigma}^q$, and $g_{\omega}^q$, are the corresponding quark-meson coupling constants, where the quantities with the brackets stand for the expectation values in symmetric nuclear matter \cite{3}.

The same meson mean fields $\sigma$ and $\omega$ for the quarks in Eqs. (5) and (6), satisfy self-consistently the following equations at the nucleon level (see Eq. (3)):

$$\sigma = \frac{g_\sigma}{m_{\sigma}^*} C_N(\sigma) \frac{4}{(2\pi)^3} \int dk \theta(k_F - |k|) \frac{m_N^*(\sigma)}{\sqrt{m_N^*(\sigma) + k^2}} = \frac{g_\sigma}{m_{\sigma}^*} C_N(\sigma) \rho_s, \quad (7)$$

$$\omega = \frac{g_\omega}{m_{\omega}^*} \frac{1}{g_{\omega}(\sigma = \sigma_0)} \frac{1}{\rho_s} \left[ \frac{\partial m_N^*(\sigma)}{\partial \rho_s} \right] = \frac{g_\omega}{m_{\omega}^*} C_N(\sigma) \rho_s, \quad (8)$$

where $C_N(\sigma) \equiv \frac{1}{g_{\omega}(\sigma = \sigma_0)} \left[ \frac{\partial m_N^*(\sigma)}{\partial \rho_s} \right]$ is the constant value of the scalar density ratio \cite{3 15 16}. Because of the underlying quark structure of the nucleon used to calculate $M_N^*(\sigma)$ in nuclear medium (see Eq. (7)), $C_N(\sigma)$ gets nonlinear $\sigma$-dependence, whereas the usual point-like nucleon-based model yields
Fig. 1: Negative of the binding energy per nucleon for symmetric nuclear matter, $(E^{\text{tot}}/A - m_N)$, v.s. $\rho$ ($\rho_0 = 0.15$ fm$^{-3}$) with the vacuum quark mass $m_q = m_{\bar{q}} = 220$ MeV ($q = u, d$), calculated by the QMC model (left panel), and effective constituent quark mass $m^*_q$ ($q = \bar{q} = u, \bar{d}, d$) (right panel). (Both figures are taken from Ref. [12]). The incompressibility $K$ obtained is $K = 320.9$ MeV.

Once the self-consistency equation for the $\sigma$, Eq. (7), has been solved, one can evaluate the total energy per nucleon:

$$E^{\text{tot}}/A = \frac{4}{(2\pi)^3} \rho \int d\mathbf{k} \frac{\theta(k_F - |\mathbf{k}|)}{\sqrt{m_N^2(\sigma) + \mathbf{k}^2 + m_{\bar{q}}^2 + g_{\omega}^2 \rho}}.$$

We then determine the coupling constants, $g_\sigma$ and $g_\omega$, so as to fit the binding energy of 15.7 MeV at the saturation density $\rho_0 = 0.15$ fm$^{-3}$ ($k_F^0 = 1.305$ fm$^{-1}$) for symmetric nuclear matter.

In Refs. [11; 12], the quark mass in vacuum was used $m_q, \bar{q} = 220$ MeV to study the pion properties in symmetric nuclear matter. With this value the model can reproduce the electromagnetic form factor and the decay constant well in vacuum [8]. So, we use the same value in this study. Then, all the nuclear matter saturation properties are generated by using this light-quark mass value. In other words, the different values of $m_q$ in vacuum generate the corresponding different nuclear matter properties, except for the saturation point of the symmetric nuclear matter, $\rho = \rho_0$ (normal nuclear matter density, 0.15 fm$^{-3}$) with the empirically extracted binding energy of 15.7 MeV. This saturation point condition is generally used to constrain the models of nuclear matter. Thus, we have obtained the necessary properties of the light-flavor constituent quarks in symmetric nuclear matter with $m_q = 220$ MeV, namely, the density dependence of the effective mass (scalar potential) and vector potential. The same in-medium constituent quark properties which reproduce the nuclear saturation properties will be used as input to study the pion properties in symmetric nuclear matter.

In Figs. 1 we show negative of the binding energy per nucleon $(E^{\text{tot}}/A - m_N)$ (left panel), and effective constituent light-quark mass, $m^*_q$ (right panel), in symmetric nuclear matter [12].

2 Pion properties in medium

The light-front constituent quark model used here [8], although simple, is quite successful in describing the properties of pion in vacuum.

As examples, we show in Fig. 2 pion charge form factor (left panel) and root mean-square pion charge radius in symmetric nuclear matter calculated in Ref. [12].
Here, we note that the pion mass up to normal nuclear matter density is expected to be modified only slightly, where the modification $\delta m_\pi$ at nuclear density $\rho = 0.17 \text{ fm}^{-3}$, averaged over the pion isospin states, is estimated as $\delta m_\pi \approx +3 \text{ MeV}$ [18, 19, 20, 21]. Therefore, we approximate the effective pion mass in nuclear medium to be the same as in vacuum, $m_\pi^\ast = m_\pi$, up to $\rho = \rho_0 = 0.15 \text{ fm}^{-3}$, the maximum nuclear matter density treated. Furthermore, since the present light-front constituent quark model is rather simple, the model cannot discuss the chiral limit of vanishing (effective) light-quark masses.

The effective interaction Lagrangian density for the quarks and pion in medium is given by,

$$\mathcal{L}_{\text{eff}} = -ig^\ast (\bar{q}\gamma^5 \tau q \cdot \phi) A^\ast,$$

where the coupling constant, $g^\ast = m_\pi^\ast / f_\pi^\ast$, is obtained by the "in-medium Goldberger-Treiman relation" at the quark level, with $m_\pi^\ast$ and $f_\pi^\ast$ being respectively the effective constituent quark mass and pion decay constant in medium, $\phi$ the pion field [8, 10], and $A^\ast$ is the $\pi$-$\bar{q}$-$q$ vertex function in medium. Hereafter, the in-medium quantities are indicated with the asterisk, $\ast$.

The pion valence wave function used in this study is symmetric under the exchange of quark and antiquark momenta. This $\pi$-$\bar{q}$-$q$ vertex function, $A(k, P)$ in vacuum is the same as that used for studying the properties of pion [8] and kaon [10]. However, for the in-medium $A^\ast$, the arguments of the function are replaced by those of the in-medium [12]:

$$A^\ast(k + V, P) = \frac{C^\ast}{((k + V)^2 - m_R^2 + i\epsilon)} + \frac{C^\ast}{((P - k - V)^2 - m_R^2 + i\epsilon)},$$

where $V^\mu = \delta_0^\mu V^0$ is the vector potential felt by the light quarks in the pion immersed in medium, and can be eliminated by the variable change in the $k$-integration, $k^\mu + \delta_0^\mu V^0 \rightarrow k^\mu$. The normalization factor associated with $C^\ast$ is modified by the medium effects. The regulator mass $m_R$ represents soft effects at short range of about the 1 GeV scale, and $m_R$ may also be influenced by in-medium effects. However, we employ $m_R^\ast = m_R$ in Eq. (11), since there exists no established way of estimating this effect on the regulator mass. This can avoid introducing extra source of uncertainty.

The Bethe-Salpeter amplitude in medium, $\Psi_\pi^\ast(k + V, P)$ with the vertex function in medium $A^\ast$ is given by,

$$\Psi_\pi^\ast(k + V, P) = \frac{\bar{k} + \bar{V} + m_\pi^\ast}{(k + V)^2 - m_\pi^2 + i\epsilon} \gamma^5 A^\ast(k + V, P) \frac{k + V - P + m_\pi^\ast}{(k + V - P)^2 - m_\pi^2 + i\epsilon}.$$  

By eliminating the instantaneous terms, or eliminating the terms with the matrix $\gamma^5$ in the numerators and $k^\ast$ and $(P^\ast - k^\ast)$ in the denominators with the light-front convention $a^\pm \equiv a^0 \pm a^3$, and integrating over the light-front energy $k^\pm$, we obtain the in-medium pion valence wave function $\Phi_\pi^\ast$,

$$\Phi_\pi^\ast(k^+, k_\perp; P^+, P_\perp) = \frac{P^+}{m_\pi^2 - M_0^2} \left[ (1 - x)(m_\pi^2 - M_1^2(m_\pi^2, m_R^2)) + \frac{N_1^*}{x(m_\pi^2 - M_2^2(m_R^2, m_\pi^2))} \right],$$

**Fig. 2** Pion charme form factor (left panel), and root mean-square charge radius in symmetric nuclear matter (right panel). Both are taken from Ref. [12].
where, \( N^* = C^* \left( m_q^*/f_\pi^* \right) (N_c)^{1/2} \) is the normalization factor with the number of colors \( N_c \) \cite{8 12},

\[ x = k^+/P^+ \text{ with } 0 \leq x \leq 1, \quad M^2 \left( m_\pi^2, m_\rho^2 \right) \equiv \frac{k^+ + m_\rho^2}{1 + \frac{k^+ + m_\rho^2}{P^+}} \text{, the square of the mass } \]

\( M_0^2 = M^2 \left( m_\pi^2, m_\rho^2 \right) \), and \( m_B \) is the regulator mass with the value \( m_B = m_R = 600 \text{ MeV} \) \cite{8 12}. Note that the model used in Refs. \cite{7 9} does not have the second term in Eq. (13). This means that the pion transverse momentum probability density in medium, and the time component is shown to be model independent and directly associated with the in-medium “quark condensate” value with that extracted experimentally \cite{18}.

The result is shown in Fig. 4 (left panel) as a function of nuclear density \( (\rho/\rho_0) \). Note that, \( f_\pi^* \) above is calculated with the plus-component of the light front axial-vector current (light-front time component). Thus the \( f_\pi^* \) cannot be separated into the usual sense of the time and space components, where the corresponding two components of \( f_\pi^* \) decouple with the presence of background matter (in nuclear medium), and the time component is shown to be model independent and directly associated with the observables, and also with the Gell-Mann-Oakes-Renner (GMOR) relation \cite{22}. As nuclear density increases, the in-medium pion decay constant \( f_\pi^* \) decreases, although the amount of the reduction may be larger than that calculated by the NJL model \cite{20 21}. However, in the treatment of the NJL model \cite{20 21}, the decoupling of the time and space components of \( f_\pi^* \) is not clear with the presence of background matter, and may not directly be compared with our result, as well as the empirical value extracted from the pionic-atom experiment \cite{18}.

In the following, we further discuss the in-medium “quark condensate” and the GMOR-like relation, just for an illustration. As we mentioned already, the present model is a light-front constituent quark model with the constituent quark mass of \( m_q = 220 \text{ MeV} \) in vacuum, and thus we cannot discuss the chiral limit (model limitation). Keeping this point in mind, however, to get some idea, we simply write down the GMOR-like relation in vacuum and in medium, so that we try to compare the “quark condensate” value with that extracted experimentally \cite{18}:

\[ m_\pi^2 f_\pi^* = -2m_q \langle \pi q \rangle, \]

\[ m_\pi^2 f_\pi^* = -2m_q \langle \pi q \rangle^* . \]

Then, the ratio of the in-medium to vacuum quark condensates in the present approach may be estimated as,

\[ \frac{\langle \pi q \rangle^*}{\langle \pi q \rangle} = \frac{m_q m_\pi^2 f_\pi^*}{m_q^2 m_\pi^2 f_\pi^*} \approx \frac{m_q}{m_q^2} \frac{f_\pi^*}{f_\pi^*} . \]

At normal nuclear matter density, \( \rho_0 (0.15 \text{ fm}^{-3}) \), the ratio gives \( \approx 0.52 \) \cite{12} (and also one can calculate by using the values listed in Table 1 to be given next). This implies a larger reduction in “quark condensate” compared to the value \( 0.67 \pm 0.06 \) extracted in Ref. \cite{18} at a density \( 0.17 \text{ fm}^{-3} \) (their value for the normal nuclear matter density). This feature may also be understood from the larger reduction in \( (f_\pi^*/f_\pi^*)^2 \) in our approach compared with that obtained in Ref. \cite{18} (see also Ref. \cite{12} for more details). We repeat that the discussions above are not rigorous, but just for giving some intuition, since the vacuum structure in the light-front approach is usually considered as “trivial”, and it is very
difficult to study the quark condensate (or spontaneously broken chiral symmetry) in vacuum as well as in medium. This is a very interesting issue for the future elaboration within the light-front approach.

Some properties of the pion in symmetric nuclear matter obtained in Ref. [12], are summarized in Table 1. The results listed in Table 1 are summarized as follows. [See also Fig. 1 (right panel) for \( m_0^\pi \), Fig. 2 (right panel) for \( f_\pi^* \) to the reduced \( f_\pi^* \) and Fig. 4 (left panel) for \( f_\pi^* \) to the reduced \( f_\pi^* \).] As the nuclear density increases, \( m_0^\pi \) and \( f_\pi^* \) decrease, while \( < r_\pi^* >^2 > 1/2 \) and the probability of valence component in the pion, \( \eta^* \), increase. This can be understood as follows. Since the pion decay constant in nuclear medium is modified, the pion valence wave function in nuclear medium is also modified via this normalization.

Next, we discuss the in-medium pion valence distribution amplitude. Pion DA provides information on the nonperturbative regime of the bound state nature of pion due to the quark and antiquark at higher momentum transfer. The pion valence wave function in vacuum is normalized by \( 2 \sqrt{6} \) (aside from the factor \( \sqrt{2} \) difference):

\[
\int_0^1 dx \int \frac{d^2 k_\perp}{16 \pi^3} \phi_\pi(x, k_\perp) = \frac{f_\pi}{2 \sqrt{6}}.
\]

This is an important constraint on the normalization of the \( q\bar{q} \) wave function [24; 25], associated with a probability of finding a pure \( q\bar{q} \) state in the pion state. According to this normalization, the in-medium pion valence wave function is normalized by replacing \( f_\pi \to f_\pi^* \) in the above. Since the pion decay constant in nuclear medium is modified, the pion valence wave function in nuclear medium is also modified via this normalization.

In order to examine more in detail as to how the change in \( f_\pi^* \) impacts on the in-medium pion valence wave function, we show in Fig. 8 the pion valence wave functions in vacuum (left panel) and \( \rho = \rho_0 \) (right panel). One can notice that the in-medium pion valence wave function in momentum space has a sharper peak and localized in narrower regions both in \( x \) and \( k_\perp \) than those in vacuum. Of course, the total volume, the quantity integrated over \( x \) and \( k_\perp \), is reduced in medium, corresponding to the reduced \( f_\pi^* \). This fact is reflected in the wave function in coordinate space, that it becomes spread wider, and generally its height is reduced.

The corresponding pion valence DA in medium, denoted by \( \phi_{DA}^\pi(x) \) (not normalized to unity), is calculated as

\[
\phi_{DA}^\pi(x) = \int \frac{d^2 k_\perp}{16 \pi^3} \phi_\pi^*(x, k_\perp).
\]

Note that, Eq. 21 holds also for the other pseudoscalar mesons \( M_{ps} \) such as kaon and D-meson, by replacing \( \phi_\pi^*(x, k_\perp) \to \phi_{M_{ps}}^*(x, k_\perp) \) in the above.

We show in Fig. 4 (right panel) the obtained pion DAs, \( \phi_{DA}^\pi(x) \), in vacuum (\( \rho/\rho_0 = 0 \)) and in medium for several nuclear densities. The significant reduction of the in-medium pion valence DA is obvious, that reflects the reduction of \( f_\pi^* \) (left panel).

Next, we study pion valence DA normalized to unity, or normalized pion valence DAs in vacuum and in medium. By this, we can study the change in shape due to the medium effects. We show in Fig. 5 (left panel) the normalized pion valence DAs, \( \phi^*(x) \), both in vacuum (\( \rho/\rho_0 = 0 \)) and in medium. The in-medium change in shape is moderate when the nuclear matter densities are small, but it becomes evident when the density becomes \( \rho_0 \).

### Table 1: Properties of pion in medium, taken from Ref. [12], with \( \rho_0 = 0.15 \text{ fm}^{-3} \).

| \( \rho/\rho_0 \) | \( m_0^\pi \) [MeV] | \( f_\pi^* \) [MeV] | \( < r_\pi^* >^2 > 1/2 \) [fm] | \( \eta^* \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.00            | 220             | 93.1            | 0.73            | 0.782           |
| 0.25            | 179.9           | 80.6            | 0.84            | 0.812           |
| 0.50            | 143.2           | 68.0            | 1.00            | 0.843           |
| 0.75            | 109.8           | 55.1            | 1.26            | 0.878           |
| 1.00            | 79.5            | 40.2            | 1.96            | 0.930           |
Fig. 3 Pion valence wave functions in vacuum ($\rho = 0$) [left panel] and in medium ($\rho = \rho_0$) [right panel] v.s. $x$ and $k_\perp = |k_\perp|$, where $P^+ = m_\pi = m_\pi^*$ and $P_\perp = |P_\perp| = 0$. The wave functions are given in the units, $10^{-8} \times (\text{GeV})^{-1}$. Notice that the differences in the vertical axis scales for the left and right panels. (Taken from Ref. [9].)

Fig. 4 Pion decay constant calculated in symmetric nuclear matter (left panel) taken from Ref. [12], and pion valence distribution amplitudes (right panel), taken from Ref. [14].

Furthermore, it may be useful to define effective pion valence DA using the valence probability in vacuum $\eta$ and in medium $\eta^*$. (See Eq. (15) and table 1.) The pion states in vacuum, $|\pi>$, and in medium, $|\pi^*>$, can respectively be written as,

$$|\pi> = \sqrt{\eta}|qq> + a|qqq> + b|qqg> + \cdots,$$

$$|\pi^*> = \sqrt{\eta^*}|qq^*> + c|qqq^*> + d|qqg^*> + \cdots,$$

where $a$, $b$, $c$ and $d$ are constants, and $g$ denotes a gluon, and $\cdots$ stands for the higher Fock components in the pion states. The quantity $\eta^*$ in table 1 indicates that the valence $qq$ component in the pion state increases in medium as nuclear density increases. The effective pion valence DAs, $\sqrt{\eta^*} \phi(x)^*$, in vacuum ($\rho/\rho_0 = 0$) and in medium, are shown in Fig. 5. They may respectively correspond to the first terms of Eqs. (22) and (23).

Since $\eta^*/\eta$ is enhanced in medium, effective pion valence DA in medium is also enhanced, on the top of the corresponding medium-(shape)modified normalized pion valence DA. The obvious enhancement of the effective pion valence DA in medium can be seen around $x = 0.5$. This quantity may be useful when one studies some reactions in medium (in a nucleus) involving a pion, based on a constituent quark picture of pion.
3 Summary

We have studied the impact of in-medium effects on the pion valence distribution amplitudes using a light-front constituent quark model, combined with the in-medium input calculated by the quark-meson coupling model. The in-medium constituent light-quark properties inside the pion are consistently constrained by the saturation properties of symmetric nuclear matter.

The in-medium pion mass is assumed to be the same as that in vacuum, based on the extracted information from the pionic-atom experiment, and some theoretical studies. This information extracted is valid up to around the normal nuclear matter density. Thus, the results obtained in this study, combined with the light-front constituent quark model, are valid up to around the normal nuclear matter density, but cannot discuss reliably the chiral limit, the vanishing limit of the (effective constituent) light-quark masses. We need to rely on more sophisticated models of pion to be able to discuss the chiral limit in medium, as well as in vacuum.

Due to the reduction in the pion decay constant in medium, the pion distribution amplitude in medium normalized with the pion decay constant, is substantially reduced at relatively higher nuclear densities. Because the valence component probability in medium increases as nuclear density increases, we have defined an effective pion distribution amplitude normalized to the square root of the valence probability in the pion state. This may give some information for the effectiveness of the valence quark picture of pion in nuclear medium. Within the present light-front constituent quark model approach, the effectiveness of the valence quark picture of the pion in medium, becomes more enhanced as nuclear density increases, due to the increase of the valence component in the pion state.

Although the present study is based on a simple, light-front constituent quark model, this is a first step to understand the impact of the medium effects on the internal structure of the pion immersed in nuclear medium.

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References

1. Brown, G.E., Rho, M.: Scaling effective Lagrangians in a dense medium. Phys. Rev. Lett. 66, 2720 (1991).
2. Hatsuda, H., Lee, S.H.: QCD sum rules for vector mesons in the nuclear medium. Phys. Rev. C 46, no. 1, R34 (1992).
3. Saito, K., Tsushima, K., Thomas, A.W.: Nucleon and hadron structure changes in the nuclear medium and impact on observables. Prog. Part. Nucl. Phys. 58 (2007) 1.
4. Hayano, R.S., Hatsuda, T.: Hadron properties in the nuclear medium. Rev. Mod. Phys. 82, 2949 (2010).
5. For a review, Brooks, W.K., Strauch, S., Tsushima, K.: Medium Modifications of Hadron Properties and Partonic Processes. J. Phys. Conf. Ser. 299, 012011 (2011).
6. de Melo, J.P.B.C., Frederico, T., Tomio, L., Dorokhov, A.E.: Quark - anti-quark correlation in the pion. Nucl. Phys. A 623 (1997) 456.
7. de Melo, J.P.B.C., Frederico, T.: Pion electromagnetic current in the light cone formalism. Phys. Rev. C 59 (1999) 2278.
8. de Melo, J.P.B.C., Frederico, T., Pace, E., Salmè, G.: Pair term in the electromagnetic current within the front form dynamics: Spin-0 case. Nucl. Phys. A 707 (2002) 399.
9. da Silva, E.O., de Melo, J.P.B.C., El-Bennich, B., Filho, V.S.: Pion and kaon elastic form factors in a refined light-front model. Phys. Rev. C 86 (2012) 035202.
10. Yabusaki, G.H.S., Ahmed, I., Paracha, M.A., de Melo, J.P.B.C., Bennichi, B.: Pseudoscalar mesons with symmetric bound state vertex functions on the light front Phys. Rev. D92(2015) 034017.
11. de Melo, J.P.B.C., Ahmed, I., Tsushima, K.: Parton Distribution in Pseudoscalar Mesons with a Light-Front Constituent Quark Model. AIP Conf. Proc. 1735, 080012 (2016).
12. de Melo, J.P.B.C., Tsushima, K., El-Bennich, B., Rojas, E., Frederico, T.: Pion structure in the nuclear medium. Phys. Rev. C 90 (2014) 035201.
13. de Melo, J.P.B.C., Tsushima, K., Frederico, T.: Pion in the Medium with a Light-Front Model. AIP Conf. Proc. 1735, 080006 (2016).
14. de Melo, J.P.B.C., Tsushima, K., Ahmed, I: In-medium Pion Valence Distributions in a Light-Front Model. Phys. Lett. B 766, 125 (2017).
15. Guichon, P.A.M.: A Possible Quark Mechanism for the Saturation of Nuclear Matter. Phys. Lett. B 200, 235 (1988).
16. Guichon, P.A.M., Saito, K., Rodionov, E.N., Thomas, A.W.: The Role of nucleon structure in finite nuclei. Nucl. Phys. A 601 (1996) 349; Saito, K., Tsushima, K., Thomas, A.W.: Selfconsistent description of finite nuclei based on a relativistic quark model. Nucl. Phys. A 609, 339 (1996); Variation of hadron masses in finite nuclei. Phys. Rev. C 55, 2637 (1997); Tushima, K., Saito, K., Haidenbauer, J., Thomas, A.W.: The Quark - meson coupling model for Lambda, Sigma and Xi hypernuclei. Nucl. Phys. A 630, 691 (1998); Guichon, P.A.M., Thomas, A.W., Tsushima, K.: Binding of hypernuclei in the latest quark-meson coupling model. Nucl. Phys. A 814 (2008) 66.
17. Serot, B.D., Walecka, J.D.: The Relativistic Nuclear Many Body Problem. Adv. Nucl. Phys. 16, 1 (1986).
18. Kienle, R., Yanuzaki, T.: Pions in nuclei, a probe of chiral symmetry restoration. Prog. Part. Nucl. Phys. 52, 85 (2004).
19. Meissner, U.G., Oller, J.A., Wirzba, A.: In-medium chiral perturbation theory beyond the mean field approximation. Annals Phys. 297, 27 (2002).
20. Weise, W., Vogl, U.: The Nambu and Jona Lasinio model: Its implications for hadrons and nuclei. Prog. Part. Nucl. Phys. 27, 195 (1991).
21. Lutz, M., Klimt, S., Weise, W.: Meson properties at finite temperature and baryon density. Nucl. Phys. A542, 521 (1992).
22. Kirchbach, M., Wirzba, A.: In-medium chiral perturbation theory and pion weak decay in the presence of background matter. Nucl. Phys. A 616, 648 (1997).
23. Van Royen, R., Weisskopf, V.F.: Hadron Decay Processes and the Quark Model. Nuovo Cim. A 50, 617 (1967) Erratum: Nuovo Cim. A 51, 583 (1967).
24. Lepage, G.P., Brodsky, S.J., Huang, T., Mackenzie, P.B.: Hadronic Wave Functions In QCD. CLNS-82-522, FERMILAB-CONF-81-114-T.
25. Brodsky, S.J., Lepage, G.P.: Exclusive Processes in Quantum Chromodynamics. Adv. Ser. Direct. High Energy Phys. 5, 93 (1989).