State prediction based control schemes for nonlinear systems with input delay and external disturbance

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Abstract

In this paper, a control scheme based on state prediction is proposed for a class of nonlinear systems subject to input delay and external disturbance. Firstly, as for constant input delay, an improved control scheme that combining state prediction with dynamic surface control (DSC) technique is proposed, which can compensate for the input delay completely. Secondly, by considering the constant input delay and external disturbance, an effective controller based on state prediction, DSC technique and disturbance observer is presented to tackle the disturbed input-delayed nonlinear systems. Thirdly, as for time-varying input delay and external disturbance, a control scheme that includes state prediction, DSC technique, input saturation and sliding mode disturbance observer is further proposed to tackle the tracking control for the addressed nonlinear systems. Besides, some theorems are given to ensure that all signals in the closed-loop systems can be semi-globally ultimately uniformly bounded (UUB) under the proposed control schemes. Finally, by utilising the dynamic model of quadrotor unmanned aerial vehicle (UAV), some simulations are presented to illustrate the effectiveness of our proposed results.

1 | INTRODUCTION

Input delay in control system always causes many adverse effects such as performance deterioration or even instability. It occurs between the control input and the plant that arises from the communication latencies, computation time, and sensor measurement [1]–[5]. In order to deal with input delay of linear systems in frequency domain, a Smith predictor was initially presented [1], in which the main idea was that for a single-input and single-output (SISO) system with known dynamic characteristic of input delay, a predictor was designed to compensate for the negative effects. For multi-input and multi-output (MIMO) linear time-invariant systems with input delay, a receding horizon method was proposed for the stabilisation [2], and the controller was composed of two parts: a linear feedback controller and an integral compensator. In [3], a model reduction technique was proposed, and it could transform the linear input-delayed systems into the linear delay-free ones, in which the controllability, stabilisation and optimisation were analysed for the reduced systems. Due to the truth that a chain of integrators cannot be stabilised by using a proportional and memory controller, a multiple delay controller was proposed [4]. Furthermore, for unknown input delay in a chain of integrators, an adaptive output feedback controller was designed [5]. In [6], a new predictive control scheme was proposed for the linear-invariant systems subject to input delay and unknown disturbance. Nevertheless, the majority of practical systems always exhibit the nonlinear dynamic behaviours and have inherent model uncertainties including the parametric uncertainties, and external disturbances, such as quadrotor UAVs [7]. Hence, it will be of great significance to study the control problems on the MIMO nonlinear systems under the input delay.

For various control issues of nonlinear systems with input delay, some results have been reported [8]–[14]. In [8], an adaptive quantised control method was proposed to tackle a class of uncertain nonlinear systems with state constraints and time-varying delays. Based on Laplace transformation and Taylor formula, a Padé approximation was used to transform time-delayed system into the delay-free one [13]. However, the approximation error could not be neglected when input delay was large. In [15], the stabilisation for linear and nonlinear
systems with state-dependent input delay was introduced, in which the region of attraction in the state space was estimated. For discrete-time systems with input delay, ref. [17] studied the design of feedback controller and a backstepping procedure was involved. In [18]–[20], the stabilisation of delayed input-affine dynamics was discussed, in which the delay was set in the sampled-data-based context and the delay was compensated via a discrete-time predictor. In [21], [22], a compensating method for long actuator input delay was proposed for forward complete and strict-feedback nonlinear systems such that the addressed system could be stabilised in an asymptotic stable way. Since the controllers were designed based on state prediction and controllability and stabilisation could be ensured. Yet, in [21], [22], the difference between the current state and its prediction was not fully studied, and it would degrade the tracking performance especially for the time-varying reference signals, which will be considered in this work. Meanwhile, since external disturbances widely exist in many control systems, how to eliminate its negative effects has become a heated topic in recent years [31], [32]. In [28], a generalised disturbance observer capable of removing higher-order disturbances in the time series expansion was proposed. In [29], [30], a nonlinear disturbance observer-based approach was presented to enhance the anti-disturbance ability and performance robustness against uncertain aerodynamic coefficients. Furthermore, a generalised disturbance observer was presented for estimating a broad range of disturbances including fast-varying ones [31].

Then in our work, in order to tackle the external disturbance, an improved control scheme combined state prediction with disturbance observer will be proposed for a class of MIMO nonlinear systems, in which the compounded disturbance induced by input delay will be tackled. The main contributions of this paper are listed as follows:

1. For the nonlinear system with constant input delay, an improved controller combining state prediction with the DSC technique is proposed to compensate for the input delay completely;
2. Considering the external disturbance and constant input delay, a control scheme based on state prediction, DSC technique, and disturbance observer is proposed for the disturbed input-delayed nonlinear systems;
3. As for time-varying input delay and external disturbance, an effective control scheme including state prediction, DSC technique, input saturation and sliding mode disturbance observer is presented for the control of the addressed MIMO nonlinear systems.

Notations
Throughout this paper, the term $\mathbb{R}^{*n}$ denotes $n$-dimensional Euclidean space, $\mathbb{R}^{n \times m}$ represents the set of all $n \times m$ real matrices, $I$ denotes an identity matrix of appropriate dimension. Besides, for any vector $\mathbf{x}(t) \in \mathbb{R}^{n}$, the norm of $\mathbf{x}(t)$ is defined as $\|\mathbf{x}(t)\| = \sqrt{x_1^2(t) + \cdots + x_n^2(t)}$ and $\text{sign}(\mathbf{x}(t)) = [\text{sign}(x_1(t)), ..., \text{sign}(x_n(t))]^T$.

2 | PROBLEM FORMULATIONS AND ASSUMPTIONS
Consider the following MIMO nonlinear systems described by

$$
\begin{align*}
\dot{x}(t) &= f(x(t)) + g(x(t))u(t - \tau(t)) + d(t), \\
x(0) &= x_0, \\
u(t) &= u_0(t), \ t \in [-\tau_{\text{max}}, 0],
\end{align*}
$$

where $\tau(t) \in \mathbb{R}^+$ is the input delay with $\tau_{\text{max}}$ being the maximum value of $\tau(t)$, $\mathbf{x}(t) \in \mathbb{R}^n$ is the system state with an initial state $x_0 \in \mathbb{R}^n$, $f(\mathbf{x}(t)) \in \mathbb{R}^n$ is a nonlinear function vector and the nonlinear item $g(\mathbf{x}(t)) \in \mathbb{R}^{n \times a}$ denotes the nonsingular gain matrix, $d(t) \in \mathbb{R}^{n}$ means the external disturbance, and $u(t) \in \mathbb{R}^{n}$ denotes the control input with $u_0(t)$ is known.

This paper mainly aims to realise that the system in Equation (1) can exponentially track a given reference signal $w(t)$ and the tracking errors can be limited in a tolerable bound. In the next, in order to better clarify the control schemes proposed in this paper, some assumptions are listed below.

Assumption 1. The nonsingular matrix $g(\mathbf{x}(t))$ in Equation (1) is norm-bounded and satisfies $\|g(\mathbf{x}(t))\| \leq a$, where $a$ is a known positive constant. Besides, the $g(\mathbf{x}(t))$ is invertible, which means that there must exist a $g^{-1}(\mathbf{x}(t))$ such that $g(\mathbf{x}(t))g^{-1}(\mathbf{x}(t)) = I$.

Assumption 2. The disturbance $d(t)$ in Equation (1) is bounded and slowly variable. Thus, there exists two known positive constants $\lambda_i$ ($i = 1, 2$) such that $\|d(t)\| \leq \lambda_1$ and $\|\dot{d}(t)\| \leq \lambda_2$.

Assumption 3. The reference signal $w(t)$ is continuous and smooth that satisfies $\|w(t)\| \leq \eta_1$ and $\|\dot{w}(t)\| \leq \eta_2$, where $\eta_i$ ($i = 1, 2$) are known positive constants.

Assumption 4. The nonlinear system is globally controllable in the absence of input delay. Meanwhile, as for given $d(t)$ satisfying Assumption 2 and initial conditions $x(0)$, $w(t), t \in [-\tau_{\text{max}}, 0]$, the implicit equality

$$
\begin{align*}
\dot{x}(0) + \int_{0}^{t} H(s)ds = x(t), \ \forall \ t \in [0, \tau_{\text{max}}]
\end{align*}
$$

is always satisfied, where $H(t) = f(x(s)) + g(x(s))w(s - \tau(s)) + d(s)$.

Remark 1. In Assumption 4, it is obvious that implicit Equation (2) holds if $t = 0$. If $t > 0$, according to the continuous property of the system, the Equation (2) is solvable. Hence, the Assumption 4 is reasonable and denotes $\tau_{\text{max}}$ as the maximum solution of (2). Besides, different from the forward complete system in [22] which can be globally stabilised with an appropriate controller, the more general input-delayed nonlinear system
presented in Equation (1) could be collapsed before the control signals’ arrival. Hence, the Assumption 4 is necessary if the input-delayed system is controllable.

3 | MAIN RESULTS

3.1 | Control scheme for nonlinear system with constant input delay

In this subsection, we consider the MIMO nonlinear system with constant input delay. Then as for \( d(t) = 0 \) and \( \tau(t) = \tau_0 \leq \tau_{\text{max}} \), the system (1) is reduced to

\[
\begin{align*}
\dot{x}(t) &= f(x(t)) + g(x(t))u(t - \tau_0), \\
x(0) &= x_0, \\
u(t) &= u_0(t), & t \in [-\tau_0, 0].
\end{align*}
\]

(3)

Based on the Assumption 4, there exist the initial states \( x(0) \) and \( u(t), t \in [-\tau_0, 0] \) are in the set \( \Pi_0 := \{(x(0), u(t)), t \in [-\tau_0, 0] : x(0) + \int_0^t H_1(s)ds = x(t), t \leq \tau_0 \} \), which can ensure the controllability of (3).

As illustrated in [22], an input delay compensation method was proposed to ensure the desired stability. However, in [22], since the new predicted state instead of current one was utilised to track the reference signals, the difference between these two signals was existent and the effects of input delay was not literally eliminated especially when the reference signal was time-varying. Then in order to solve this problem, an improved control scheme based on state prediction and DSC technique will be proposed in our work.

Firstly, a state prediction \( \hat{x}(t + \tau_0) \) based on Equation (3) is given as

\[
\hat{x}(t + \tau_0) = x(t) + \int_t^{t+\tau_0} H_1(s)ds,
\]

(4)

where \( H_1(s) = f(\hat{x}(s)) + g(\hat{x}(s))u(s - \tau_0) \) with an initial condition

\[
\hat{x}(t) = x(0) + \int_0^t H_1(s)ds
\]

(5)

defined for \( t \in [0, \tau_0] \). Based on Assumption 4, it is known that the Equation (5) is solvable and \( \hat{x}(t), t \in [0, \tau_0] \) is bounded. As for \( \hat{x}(t + \tau_0) \), it is given by the implicit Equation (4) that can be approximated by many various strategies. A simplest discretisation method can be referred in [22], which will not be discussed here.

By differentiating \( \hat{x}(t + \tau_0) \) in Equation (4) with respect to \( t \), it yields

\[
\dot{\hat{x}}(t + \tau_0) = f(\hat{x}(t + \tau_0)) + g(\hat{x}(t + \tau_0))u(t),
\]

(6)

which means that the input-delayed system (3) has been transformed into a delay-free one. Based on the fact that the initial condition in Equation (5) is bounded, it can be concluded that the system (6) is controllable. Besides, the difference \( \delta_1(t) \) between Equations (3) and (6) is defined as

\[
\delta_1(t) = x(t) - \hat{x}(t + \tau_0).
\]

(7)

In order to avoid differentiating \( \delta_1(t) \), a DSC technique is introduced to let \( \delta_1(t) \) pass through a first-order filter with time constant \( Y_1 \in \mathbb{R}^{\delta \times \delta} > 0 \), and it has

\[
Y_1 \dot{\delta}_1(t) + \delta_1(t) = \delta_1(t), \quad \delta_1(0) = \delta_1(0).
\]

(8)

By defining the filtering error as \( \zeta_1(t) = \ddot{\delta}_1(t) - \delta_1(t) \) and invoking Equation (8), it yields \( \ddot{\delta}_1(t) = -Y_1^{-1}\zeta_1(t) \). Furthermore, by invoking Equations (4) and (7), the derivative of \( \zeta_1(t) \) can be obtained as

\[
\dot{\zeta}_1(t) = -Y_1^{-1}\zeta_1(t) - f(\hat{x}(t + \tau_0)) - g(\hat{x}(t + \tau_0))u(t)
\]

(9)

\[
= -Y_1^{-1}\zeta_1(t) + M_1(\cdot),
\]

where \( M_1(\cdot) = M_1(\hat{x}(t), u(\cdot)) \) for \( s \in [t - \tau_0, t] \) is a continuous function. It is found that all the components of \( M_1(\cdot) \) are in the compact set \( \Pi_0 \). Hence, by considering the continuous property, the function \( M_1(\cdot) \) has a maximum value as \( b_1 \) for the given initial condition in the compact set \( \Pi_0 \).

For the reference signal \( w(t) \) given in Assumption 3, the tracking error of Equation (3) is defined as \( \epsilon_1(t) = \hat{x}(t + \tau_0) + \delta_1(t) - w(t) \) and the derivative of \( \epsilon_1(t) \) is derived as

\[
\dot{\epsilon}_1(t) = f(\hat{x}(t + \tau_0)) + g(\hat{x}(t + \tau_0))u(t) - Y_1^{-1}\zeta_1(t)
\]

(10)

\[
- \dot{w}(t).
\]

Then the controller \( u(t) \) is designed as follows:

\[
u(t) = g^{-1}(\hat{x}(t + \tau_0)) \left[ -f(\hat{x}(t + \tau_0)) + Y_1^{-1}\zeta_1(t) + \dot{w}(t) - P_1\epsilon_1(t) \right],
\]

(11)

where \( P_1 \in \mathbb{R}^{\delta \times \delta} > 0 \). In what follows, a theorem is given to summarise the proposed control scheme.

**Theorem 1.** Considering the system (3), the predictive system (6), the first-order filter Equation (8) and the controller Equation (11) under Assumptions 1-4, all signals in the closed-loop nonlinear system are semi-globally UUB if \( p_1 > 0 \) where

\[
p_1 = \min\{\lambda_{\min}(P_1), \lambda_{\min}(Y_1^{-1} - 0.5I)\}.
\]

(12)
Proof. In order to prove the effectiveness of our proposed control scheme, a Lyapunov function is chosen as

$$V_1(t) = \frac{1}{2} \xi_1^T(t)\xi_1(t) + \frac{1}{2} \xi_1^T(t)\xi_1(t).$$

(13)

By invoking Equations (9)–(11), and the condition in Equation (12), the derivative of $V_1(t)$ with respect to $t$ is obtained as

$$\dot{V}_1(t) = -\xi_1^T(t)P_1\xi_1(t) + \xi_1^T(t)[-Y_1^{-1}\xi_1(t) + M_1(t)]$$

$$\leq -\xi_1^T(t)P_1\xi_1(t) - \xi_1^T(t)Y_1^{-1} - 0.5I\xi_1^T(t)$$

$$+ 0.5b_1^2$$

$$\leq -p_1V_1(t) + c_1,$$

(14)

where $p_1 > 0$ is defined in Equation (12) and $c_1 = 0.5b_1^2$. From Equations (13) and (14), we can further yield

$$V_1(t) \leq e^{-p_1t}V_1(0) + \frac{c_1}{p_1}(1 - e^{-p_1t})$$

(15)

and

$$\|\xi_1(t)\| \leq \sqrt{2e^{-p_1t}V_1(0) + \frac{2c_1}{p_1}(1 - e^{-p_1t})},$$

$$\|\xi_1(t)\| \leq \sqrt{2e^{-p_1t}V_1(0) + \frac{2c_1}{p_1}(1 - e^{-p_1t})}.$$

(16)

It is obvious that all signals in the closed-loop system are semi-globally UUB, and the ultimate bound of $\xi_1(t), \xi_1(t)$ is $\sqrt{\frac{2c_1}{p_1}}$. By invoking $\xi_1(t) = x(t) - \hat{x}(t + \tau_0)$ and $\xi_1(t) = \delta_1(t) - \tilde{\delta}_1(t)$, it can further yield

$$\xi_1(t) = \frac{\hat{x}(t + \tau_0) + \tilde{\delta}_1(t) - \hat{x}(t)}{\xi_1(t)}$$

(17)

Therefore, it can be concluded that $x(t) - \hat{x}(t)$ is semi-globally UUB. Meanwhile, by choosing a group of appropriate parameters, the ultimate bound can be small enough which the control performance is satisfied. It completes the proof.

\[ \square \]

3.2 Control scheme for disturbed nonlinear system with constant input delay

In this subsection, we begin to study the disturbed nonlinear system with constant input delay. Then as for Equation (1), if $\tau(t) = \tau_0 \leq \tau_{\text{max}}$ and $d(t) \neq 0$, we have

$$\begin{cases} 
\dot{x}(t) = f(x(t)) + g(x(t))u(t - \tau_0) + d(t), \\
x(0) = x_0, \\
u(t) = \mu_0(t), \quad t \in [-\tau_0, 0].
\end{cases}$$

(18)

According to Assumption 4, for $d(t)$ satisfying Assumption 2, there exist the initial states $x(0)$ and $u(t), t \in [-\tau_0, 0]$ are in the set $\Pi_{d} : = \{(\dot{x}(0), u(\tau)), \dot{t} \in [-\tau_0, 0] : x(0) + \int_{0}^{t}H(t)ds = \dot{x}(t), t \leq \tau_0\}$ to ensure that the system (18) is controllable.

In what follows, a control scheme based on state prediction, DSC technique and disturbance observer will be proposed to study the tracking control for the system (18). Firstly, a state prediction $\hat{x}(t + \tau_0)$ based on Equation (18) is given as

$$\hat{x}(t + \tau_0) = x(t) + \int_{0}^{t+\tau_0}H_2(\dot{x})d\tau$$

(19)

where $H_2(\dot{x}) = f(\dot{x}(\sigma)) + g(\dot{x}(\sigma))u(\tau - \tau_0) + \dot{d}(t - \tau_0)$ and $\dot{d}(t - \tau_0)$ is the estimated value of $d(t - \tau_0)$ with an initial condition

$$\hat{x}(t) = x(0) + \int_{0}^{t}H_2(\dot{x})d\tau$$

(20)

defined for $t \in [0, \tau_0]$. From the definition of $\hat{x}(t + \tau_0)$ in Equation (19), it has $\dot{\hat{x}}(t) = \dot{x}(t)$. By differentiating $\hat{x}(t + \tau_0)$ in Equation (19) with respect to time $t$, it yields

$$\begin{cases} 
\dot{\hat{x}}(t + \tau_0) = f(\hat{x}(t + \tau_0)) + g(\hat{x}(t + \tau_0))u(t) + d(t) \\
\dot{\hat{x}}(t + \tau_0) = \dot{x}(t) - \dot{d}(t - \tau_0).
\end{cases}$$

(21)

For $d(t)$ existing in Equation (21) and invoking Assumption 2, a disturbance observer is designed as

$$\dot{\tilde{d}}(t) = \Gamma[\hat{x}(t + \tau_0) - \eta(t)]$$

(22)

with

$$\eta(t) = f(\hat{x}(t + \tau_0)) + g(\hat{x}(t + \tau_0))u(t) + 2\tilde{d}(t)$$

$$- \ddot{d}(t - \tau_0),$$

(23)

where $\Gamma \in \mathbb{R}^{\infty} > 0.5I$ is a parameter to be designed and $\dot{\tilde{d}}(t)$ is the estimation of $\dot{d}(t)$. The estimated error is denoted as $\ddot{d}(t) = \ddot{d}(t) - \ddot{d}(t)$ and by invoking Equations (21) and (22), it yields

$$\ddot{d}(t) = \ddot{d}(t) - \Gamma\ddot{d}(t).$$

(24)
In order to prove the stability of disturbance observer error, a Lyapunov function is chosen as

\[
V_d(t) = \tilde{a}^T(t) \tilde{a}(t)
\]  
(25)

and the derivative of it with respect to time \( t \) can be further obtained as

\[
\dot{V}_d(t) = \tilde{a}^T(t)[\dot{\tilde{a}}(t) - \Gamma \tilde{a}(t)] \\
\leq - \tilde{a}^T(t)(\Gamma - 0.5I)\tilde{a}(t) + 0.5\|\tilde{a}(t)\|^2 \\
\leq - p_d V_d(t) + c_d,
\]  
(26)

where \( p_d = \lambda_{\min}(2\Gamma - I) > 0 \) and \( c_d = 0.5\lambda_2^2 \). Furthermore, by invoking Equations (25) and (26), it has

\[
\|\tilde{a}(t)\| \leq \sqrt{2 e^{-p_d t} V_d(0) + \frac{2c_d}{p_d} (1 - e^{-p_d t})}
\]  
(27)

and the estimated error of disturbance observer is UUB. By invoking the Assumption 2, it yields that \( \tilde{a}(t) \) belongs to a compact set denoted as \( \Pi_d \).

Besides, the difference \( \delta_2(t) \) between Equations (18) and (21) is defined as

\[
\delta_2(t) = \tilde{x}(t) - \hat{x}(t + \tau_0).
\]  
(28)

In order to avoid differentiating \( \delta_2(t) \), a DSC technique is introduced. Here, a first-order filter is utilised as

\[
Y_2 \dot{\delta}_2(t) + \delta_2(t) = \delta_2(t), \quad \delta_2(0) = \delta_2(0),
\]  
(29)

where \( Y_2 \in \mathbb{R}^{\infty \times 2} > 0 \). Defining the filtering error as \( \xi_2(t) = \delta_2(t) - \delta_2(t) \) and invoking Equation (29), it has \( \dot{\xi}_2(t) = -Y_2^{-1} \xi_2(t) \). Furthermore, by invoking Equation (28), the derivative of \( \xi_2(t) \) can be obtained as

\[
\dot{\xi}_2(t) = -Y_2^{-1} \xi_2(t) - [f(\tilde{x}(t)) + g(\tilde{x}(t))u(t - \tau_0) \\
- f(\hat{x}(t + \tau_0)) - g(\hat{x}(t + \tau_0))u(t) \\
- \dot{\hat{a}}(t) + \hat{a}(t - \tau_0)] \\
= -Y_2^{-1} \xi_2(t) + M_2(\cdot),
\]  
(30)

where \( M_2(\cdot) = M_2(\tilde{x}(t), u(t), \hat{a}(t)) \) for \( s \in [t - \tau_0, t] \) is a continuous function. Due to the fact that \( \tilde{x}(t) \) and \( u(t) \), \( s \in [t - \tau_0, t] \) are in a compact set \( \Pi_0 \) and \( \hat{a}(t) \) is limited in a compact set \( \Pi_2 \).

According to the continuous property of the controlled system, we have that \( \Pi_0 \times \Pi_2 \) is also a compact set. Hence, the function \( M_2(\cdot) \) has the maximum value \( b_2 \) for the given initial condition in the compact set \( \Pi_0 \times \Pi_2 \).

For a reference signal \( w(t) \) defined in the Assumption 3, the tracking error for the system (18) is presented as

\[
e_2(t) = \tilde{x}(t + \tau_0) + \delta_2(t) - w(t).
\]  
(31)

Differentiating \( e_2(t) \) and invoking (21), it yields

\[
e_2(t) = f(\tilde{x}(t + \tau_0)) + g(\tilde{x}(t + \tau_0))u(t) + \dot{d}(t) + \dot{\hat{a}}(t) \\
- \dot{\hat{a}}(t - \tau_0) - Y_2^{-1} \xi_2(t) - \dot{w}(t),
\]  
(32)

then the controller \( u(t) \) is designed as follows:

\[
u(t) = g^{-1}(\hat{x}(t + \tau_0)) \left[ -f(\hat{x}(t + \tau_0)) + \dot{d}(t - \tau_0) \\
-2\dot{\hat{a}}(t) + Y_2^{-1} \xi_2(t) + \dot{w}(t) - P_2 e_2(t) \right],
\]  
(33)

where \( P_2 \in \mathbb{R}^{\infty \times 2} > 0 \). In what follows, a theorem is presented to verify the proposed control scheme.

**Theorem 2.** Considering the input-delayed nonlinear system (18) under external disturbance satisfying Assumptions 1–4, then all signals of the resulted closed-loop nonlinear system are semi-globally UUB under the predictive system (21), the disturbance observer (22), (23), the first-order filter (29) and the controller (31) if \( P_2 > 0 \) holds, where

\[
p_2 = \min \left\{ \lambda_{\min}(P_2 - 0.5I), \lambda_{\min}(\Gamma - I), \lambda_{\min}(Y_2^{-1} - 0.5I) \right\}.
\]  
(34)

**Proof.** In order to prove this theorem, a Lyapunov function is chosen as

\[
V_2(t) = \frac{1}{2} e_2^T(t)e_2(t) + \frac{1}{2} \tilde{a}^T(t) \tilde{a}(t) + \frac{1}{2} \xi_2^T(t) \xi_2(t).
\]  
(35)

By invoking Equations (24), (30), (32), (33), and the condition in Equation (34), the derivative of \( V_2(t) \) with respect to time \( t \) is

\[
\dot{V}_2(t) = e_2^T(t) [\dot{\tilde{a}}(t) - P_2 \xi_2(t)] + \tilde{a}^T(t) [\dot{\tilde{a}}(t) - \Gamma \dot{\tilde{a}}(t)] \\
+ \xi_2^T(t) [Y_2^{-1} \xi_2(t) + M_2(\cdot)] \\
\leq -c_2 e_2^2(t) + \frac{1}{2} \xi_2^T(t) \left[ -y_2^{-1} \xi_2(t) + M_2(\cdot) \right] \\
\leq -p_2 V_1(t) + c_2,
\]  
(36)

where \( p_2 > 0 \) is defined in Equation (34) and \( c_2 = 0.5(\delta_2^2 + \lambda_2^2) \).

From Equations (35) and (36), it yields

\[
V_2(t) \leq e^{-p_2 t} V_2(0) + \frac{c_2}{p_2} (1 - e^{-p_2 t}),
\]  
(37)
and

\[
\begin{align*}
\|e_2(t)\| & \leq \sqrt{2e^{-\beta \tau} V_2(0) + \frac{2\alpha}{\beta^2} (1 - e^{-\beta \tau})}, \\
\|\tilde{z}(t)\| & \leq \sqrt{2e^{-\beta \tau} V_2(0) + \frac{2\alpha}{\beta^2} (1 - e^{-\beta \tau})}, \\
\|\xi_2(t)\| & \leq \sqrt{2e^{-\beta \tau} V_2(0) + \frac{2\alpha}{\beta^2} (1 - e^{-\beta \tau})}.
\end{align*}
\]

(38)

It is obvious that the signals of the resulted closed-loop system are semi-globally UUB, and the ultimate bounds of \(e_2(t)\), \(\xi_2(t)\) are \(\sqrt{\frac{2\alpha}{\beta^2}}\). By invoking \(\delta_2(t) = x(t) - \tilde{x}(t + \tau_0)\) and \(\xi_2(t) = \tilde{\delta}_2(t) - \delta_2(t)\), it can further yield

\[
e_2(t) = \dot{x}(t + \tau_0) + \tilde{\delta}_2(t) - \delta_2(t).
\]

(39)

Therefore, it can be concluded that \(x(t) - \tilde{x}(t)\) is semi-globally UUB. Meanwhile, by choosing a group of appropriate parameters, the ultimate bound can be small enough which the control performance is satisfied. It completes the proof.

\[\square\]

3.3 Control scheme for disturbed nonlinear system with time-varying input delay

In practice, the input delay \(\tau(t)\) in Equation (1) is always time-varying in a certain interval. Hence, in this subsection, \(\tau(t)\) is assumed as \(\tau(t) = \tau_0 + \varepsilon(t)\) with \(\varepsilon(t) \leq \varepsilon^*\). where \(\varepsilon^*\) is a small positive constant and \(d(t) \neq 0\) satisfying Assumption 2. According to Assumption 4, for \(d(t)\) satisfying Assumption 2, there exist the initial states \(x(0)\) and \(u(t)\) in \([−\tau_{\text{max}}, 0]\) are in the set \(\Pi_0 := \{x(0), u(t)\}, t \in [−\tau_{\text{max}}, 0]\) to ensure that the system (1) is controllable. Furthermore, a saturation function is introduced and it yields

\[
\begin{align*}
\dot{x}(t) & = f(x(t)) + g(x(t))u(v(t - \tau(t))) + d(t), \\
x(0) & = x_0, \\
u(t) & = u_0(t), \quad t \in [−\tau_{\text{max}}, 0]
\end{align*}
\]

(40)

with

\[
u_i(v(t)) = \begin{cases} 
\tilde{n}, & \text{if } v_i(t) \geq \tilde{n}, \\
v_i(t), & \text{if } -\tilde{n} < v_i(t) < \tilde{n}, \\
-\tilde{n}, & \text{if } v_i(t) \leq -\tilde{n},
\end{cases}
\]

(41)

where \(i = 1, 2, \ldots, n, \tilde{n} \in \mathbb{R}^+\) is the known bound of saturation function, and \(v(t) \in \mathbb{R}^n\) is the ideal controller. Besides, there exists \(\omega \geq 0\) such that \(\|\Delta u(t)\| = \|v(t) - u(v(t))\| \leq \omega\).

Based on (40), a state prediction \(\hat{x}(t + \tau_0)\) is given as

\[
\hat{x}(t + \tau_0) = x(t) + \int_0^{t + \tau_0} H_1(s)ds,
\]

(42)

where \(H_1(s) = f(\hat{x}(s)) + g(\hat{x}(s))u(s - \tau_0) + \dot{D}(s - \tau_0)\) with an initial condition

\[
\hat{x}(t) = x(0) + \int_0^t H_1(s)ds
\]

(43)

defined for \(t \in [0, \tau_0]\). By differentiating \(\hat{x}(t + \tau_0)\) in Equation (42) with respect to time \(t\), it yields

\[
\dot{\hat{x}}(t + \tau_0) = f(\hat{x}(t + \tau_0)) + g(\hat{x}(t + \tau_0))u(v(t)) + D(t)
\]

\[+ \dot{D}(t - D(t - \tau_0)),
\]

(44)

where \(D(t) = d(t) + g(x(t))u(v(t - \tau(t))) - u(v(t - \tau_0))\) \((t) \in \mathbb{R}^n\) is regarded as a compounded disturbance. From the Assumptions 1 and 2, it is known that \(\|d(t)\|\) and \(\|g(x(t))\|\) are bounded. Besides, from the saturated controller \(u(v(t))\) in Equation (41), it can be checked that \(\|u(v(t - \tau(t))) - u(v(t - \tau_0))\|\) is also bounded. Therefore, the compounded disturbance \(D(t)\) is bounded and satisfies \(\|D(t)\| \leq \lambda_1 + 2\tilde{n}\).

In order to obtain the estimation of \(D(t)\) in Equation (44), a sliding mode disturbance observer modified from [7] is given as

\[
\dot{\hat{D}}(t) = -K\sigma(t) - \Lambda \text{sign}(\sigma(t)),
\]

(45)

where \(\sigma(t) = \chi(t) - \hat{x}(t + \tau_0)\) and

\[
\dot{\chi}(t) = -K\sigma(t) - \Lambda \text{sign}(\sigma(t)) + g(\hat{x}(t + \tau_0))u(v(t))
\]

\[+ f(\hat{x}(t + \tau_0)) + \dot{D}(t) - \dot{D}(t - \tau_0).
\]

(46)

Here \(K \in \mathbb{R}^{n \times n} > 0\) and \(\Lambda \in \mathbb{R}^{n \times n}\) satisfies \(\|\Lambda\| \geq \lambda_1 + 2\tilde{n}\).

In order to prove the effectiveness of the sliding mode disturbance observer, a Lyapunov function is chosen as

\[
V_D(t) = \frac{1}{2} \sigma^T(t)\sigma(t).
\]

(47)

By invoking Equations (44), (46) and the condition \(\|\Lambda\| \geq \|\dot{D}(t)\|\), the derivative of Equation (47) with respect to time \(t\) is

\[
\dot{V}_D(t) = -K\sigma(t) - \Lambda \text{sign}(\sigma(t)) - D(t)
\]

\[\leq -\sigma^T(t)K\sigma(t),
\]

(48)
and it can be further obtained as
\[
\begin{align*}
V_D(t) & \leq e^{-2\lambda_{\min}(K)} V_D(0), \\
\| \sigma(t) \| & \leq \sqrt{2e^{-2\lambda_{\min}(K)} V_D(0)}.
\end{align*}
\] (49)

From Equation (49), it is obvious that $\sigma(t)$ tends to zero in an exponential rate. Defining the estimate error as $\hat{D}(t) = D(t) - \hat{D}(t)$ and invoking Equation (45), it yields
\[
\begin{align*}
\hat{D}(t) & = D(t) - [-K\sigma(t) - \Delta \text{sign}(\sigma(t))] \\
& = -\hat{\sigma}(t).
\end{align*}
\] (50)

Integrating $\hat{D}(t)$ in Equation (50) from 0 to $t$, it yields
\[
\int_0^t \hat{D}(t)dt = -\sigma(t) + \sigma(0).
\] (51)

From Equation (51), it is found that $\int_0^t \hat{D}(t)dt$ tends to zero in an exponential rate. Furthermore, we can conclude that $\hat{D}(t) \leq 0$. Hence, the disturbance observer in Equations (44) and (45) can exponentially estimate the compound disturbance $D(t)$. By invoking the bounded $D(t)$, it can conclude that $\hat{D}(t)$ belongs to a compact set denoted as $\Pi_D$.

From the definition of $\hat{x}(t + \tau_0)$ in Equation (42), it has $\hat{x}(t) = x(t)$. Besides, the difference $\delta_3(t)$ between Equations (40) and (44) is defined as
\[
\delta_3(t) = x(t) - \hat{x}(t + \tau_0).
\] (52)

In order to avoiding differentiating $\delta_3(t)$, a DSC technique is introduced and a first-order filter is given as
\[
Y_3\hat{\delta}_3(t) + \delta_3(t) = \delta_3(t), \quad \delta_3(0) = \delta_3(0).
\] (53)

where $Y_3 \in \mathbb{R}^{\infty \times \infty} > 0$. Defining the filtering error $\xi_3(t) = \delta_3(t) - \delta_3(t)$ and invoking Equation (53), it has $\hat{\delta}_3(t) = -Y_3^{-1}\xi_3(t)$. Furthermore, by utilising Equation (52), the derivative of $\xi_3(t)$ can be obtained as
\[
\begin{align*}
\dot{\xi}_3(t) &= -Y_3^{-1}\xi_3(t) - [\dot{x}(t) - \hat{x}(t + \tau_0)] \\
& = -Y_3^{-1}\xi_3(t) - [g(u(t))) + d(t) + f(x(t) - f(\hat{x}(t + \tau_0))] \\
& + g(\hat{x}(t + \tau_0))] u(t - \tau(t))) + d(t) + f(x(t) - f(\hat{x}(t + \tau_0))] u(t - \tau(t)))
\end{align*}
\] (54)

and $\delta_3(t)$, $\hat{D}(t)$ are also in a compact set, it can be concluded that $M_3(\cdot)$ also belongs to a compact set. According to the continuous property of the system, the function $M_3(\cdot)$ has a maximum value $\beta_3 \geq 0$ for a given initial condition in the compact set.

Based on the saturated controller $u(v(t))$ in Equation (44), an auxiliary system is introduced as
\[
\dot{\xi}_3(t) = -Q_3\xi(t) + g(\hat{x}(t + \tau_0)) \Delta u(t),
\] (55)

where $Q_3 \in \mathbb{R}^{\infty \times \infty} > 0$, and $\Delta u(t) = u(t) - \dot{v}(t)$ denotes the difference between the actual control input $u(t)$ and the ideal one $v(t)$.

For the reference signal $v(t)$ in the Assumption 3, the tracking error is defined as $e_3(t) = \hat{x}(t + \tau_0) - \hat{x}_3(t) = -v(t) - \xi_3(t)$. By invoking Equations (44), (53), (54), and differentiating $e_3(t)$ with respect to time $t$, it yields
\[
\dot{e}_3(t) = \hat{\xi}_3(t + \tau_0) + \hat{\xi}_3(t) - \dot{v}(t) - \dot{\xi}_3(t)
\] (56)

The controller $v(t)$ is designed as follows:
\[
v(t) = g^{-1}(\hat{x}(t + \tau_0)) [f(\hat{x}(t + \tau_0))] + D(t) - \tau_0)
\] (57)

where $\omega_3 \in \mathbb{R}^{\infty \times \infty} > 0$.

In what follows, a theorem is given to illustrate the effectiveness of the proposed control scheme.

**Theorem 3.** Considering the disturbed nonlinear system (40) satisfying Assumptions 1–4 and input delay $\tau(t)$, it is in $\mathbb{R}^n$ with $\|v(t)\| \leq \varepsilon$, then all signals of the closed-loop nonlinear system are semi-globally UUB under the state predictive system (44), the $\Sigma$MDO (43), (46), the first-order filter (53), the auxiliary system (55) and the controller (57) if $\|\Delta\| \geq \lambda_1 + 2a\pi$ and $p_3 > 0$ where
\[
\begin{align*}
p_3 &= \min \left\{ \lambda_{\min}(P_3 - 0.5I), \lambda_{\min}(K), \lambda_{\min}(L_3 - 0.5I), \lambda_{\min}(Q - 0.5a^2I) \right\}.
\end{align*}
\] (58)

**Proof.** In order to verify the proposed control scheme, a Lyapunov function is chosen as
\[
\begin{align*}
V_3(t) &= \frac{1}{2}\xi_3(t)^T \xi_3(t) + \frac{1}{2} \sigma^T(t) \sigma(t) + \frac{1}{2} \xi(t)^T \xi(t) \\
& + \frac{1}{2} \xi_3(t)^T \xi_3(t).
\end{align*}
\] (59)

By invoking Equations (54), (55), (56), (57) and (48), the derivative of $V_3(t)$ with respect to time $t$ is
\[
\begin{align*}
\dot{V}_3(t) & \leq \xi_3(t)^T [D(t) - P_3 \xi_3(t)] - \sigma^T(t) K \sigma(t) \\
& + \frac{1}{2} \xi_3(t)^T - Y_3^{-1} \xi_3(t) + M_3(\cdot)]
\end{align*}
\]
the imprecise state prediction yields \( \delta \) of quadrotor UAV are presented below. Due to the existence of communication latencies, computation time, and sensor measurement transmission, the dynamic model of the quadrotor UAV is given as [32]

\[
\begin{align*}
\dot{\Theta}(t) &= f_1(\Theta(t))\Omega(t), \\
\dot{\Omega}(t) &= f_2(\Omega(t)) + g(\Omega(t))u(t - \tau(t)) + d(t),
\end{align*}
\]

where \( \Theta(t) = [\zeta(t) \phi(t) \theta(t) \psi(t)]^T \) with \( \zeta(t) \) denoting the altitude, \( \phi(t) \) being the roll angle, \( \theta(t) \) being the pitch angle, and \( \psi(t) \) being the yaw angle of the UAV. Meanwhile, \( \Omega(t) = [w(t) p(t) q(t) r(t)]^T \) with \( w(t) \) denoting the velocity in altitude channel, \( p(t) \) denoting the roll angular velocity, \( q(t) \) meaning the pitch angular velocity, and \( r(t) \) being the yaw angular velocity of quadrotor UAV. Here, \( d(t) = [d_1(t) d_2(t) d_3(t) d_4(t)]^T \) denotes the external disturbance in each channel. Besides,

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & s_4s_2 & s_4c_2 \\
0 & 0 & s_4 & c_4 \\
0 & 0 & s_4c_2 & c_4c_2
\end{pmatrix}
\]

and \( f_2(\Omega(t)) =

\[
\begin{pmatrix}
\frac{1}{m} & 0 & 0 & 0 \\
0 & \frac{1}{J_z} & 0 & 0 \\
0 & 0 & \frac{1}{J_y} & 0 \\
0 & 0 & 0 & \frac{1}{J_x}
\end{pmatrix}
\]

with \( m = 1.2 \text{ kg}, g = -9.8 \text{ m/s}^2, J_x = J_y = 6.23 \times 10^{-3} \text{ kg} \cdot \text{m}^2, \text{ and } J_z = 1.12 \times 10^{-2} \text{ kg} \cdot \text{m}^2. \)

Then, as for the tracking control of quadrotor UAV, the initial state of quadrotor UAV is chosen as \( \zeta(0) = 0 \text{m}, \phi(0) = 0^\circ, \theta(0) = 0^\circ, \psi(0) = 0^\circ, \text{ and the initial controller } u(0) = 0 \text{ with } t \leq 0. \text{ The reference signals are chosen as } \zeta_r = 0.2t, \phi_r = 3 \sin(t)^\circ, \theta_r = 0^\circ, \text{ and } \psi_r = 5^\circ. \)

### 4.1 Tracking control of quadrotor UAV with constant input delay

In this subsection, the tracking control of quadrotor UAV with constant input delay is considered with setting \( \tau_0 = 0.1r. \) Then based on Theorem 1, the parameters in the controller (11) are chosen as \( P = \text{diag}[5, 5, 5, 5] \) and \( Y_1^{-1} = \text{diag}[5, 5, 5, 5]. \)

The simulation results are showed in Figures 1 and 2. From Figure 1, it is noticed that the quadrotor UAV falls in the attitude channel at the beginning time due to the existence of input delay. Then, the quadrotor UAV begins to rise after 0.3 s and it can exactly follow the desired signal after 2 s. Similarly, it is found that the Euler angles do not change at the beginning 0.1 s due to the existence of input delay. Yet, after 0.1 s, the Euler angles begin to track the desired signals quickly and they can

It is obvious that all signals of the resulted closed-loop system can be semi-globally UUB with the ultimate bound as \( \sqrt{\frac{2\zeta}{\rho_3}}. \)

Meanwhile, by choosing a group of appropriate parameters, the ultimate bound can be small enough which the control performance is satisfied. It completes the proof.

Remark 2. In this subsection, \( \tilde{x}(t + \tau_0) \) is not a precise state prediction as for \( \hat{x}(t + \tau(t)) \). In fact, it can be regarded as a transition variable that transforms the time-delayed system into a delay-free one. From the definition of \( \tilde{c}_3(t) = \tilde{x}(t + \tau(t)) + \tilde{\delta}_3(t) - w(t) - \tilde{x}(t), \) it can be obtained that \( \dot{x}(t + \tau(t)) + \tilde{\delta}_3(t) - w(t) \) is semi-globally UUB when the controlled system is semi-globally UUB. By invoking the definition of \( \tilde{\delta}_3(t) \) and the fact that the error of first-order filter is semi-globally UUB, it can be concluded that \( x(t) - w(t) \) is semi-globally UUB. Therefore, the imprecise state prediction \( \hat{x}(t + \tau_0) \) at time \( t + \tau(t) \) does not influence the covergences of tracking error between \( x(t) \) and \( w(t) \).

### 4. NUMERICAL EXAMPLES

In order to illustrate the effectiveness of our proposed control schemes, some simulations on the altitude and attitude model of quadrotor UAV are presented below. Due to the existence of
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FIGURE 1 The altitude $z(t)$ and Euler angles $\phi(t), \theta(t), \psi(t)$

FIGURE 2 Total thrust $u_1$ and control torques $u_2, u_3, u_4$

FIGURE 3 The altitude $z(t)$ and Euler angles $\phi(t), \theta(t), \psi(t)$

FIGURE 4 Total thrust $u_1$ and control torques $u_2, u_3, u_4$

4.2 Tracking control of quadrotor UAV with disturbance and constant input delay

In this subsection, the tracking control of quadrotor UAV with constant input delay and external disturbance is considered. Here, the input delay $\tau_0$ is selected as 0.1 s and the external disturbances are chosen as $d_i(t) = 3\cos(t), i = 1, 2, 3, 4$. Then based on Theorem 2, the parameters in the controller (33) are chosen as $P_2 = \text{diag}\{3, 3, 3, 3\}, Y_{21}^{-1} = \text{diag}\{3, 3, 3, 3\}$, and parameter in disturbance observer (22) is chosen as $\Gamma = \text{diag}\{4, 4, 4, 4\}$.

Furthermore, the simulation results are showed in Figures 3 and 4. From Figure 3, it is noticed that the quadrotor UAV falls in altitude channel at the beginning time due to the existence of input delay and disturbance. Then, the quadrotor begins to follow around the desired signal after 2 s. Similarly, the Euler angles change slowly at the beginning 0.1 s due to the existence of external disturbance. Yet, after 0.1 s, these angles begin to track the desired signals due to the arrival of control signal. Besides, the controller $u_1 - u_4$ are given in Figure 4. Therefore, from Figures 3 and 4, it can further prove the effectiveness of Theorem 2 and the closed-loop system is semi-globally UUB.
input delay and external disturbance. Then, it follows around the desired signal after 2 s. Similarly, the Euler angles are variable at the beginning time due to the existence of disturbance. Then, the Euler angles begin to track the desired signal after 2 s. Besides, the controller $u_1 = u_4$ are given in Figure 6. Therefore, from Figures 5 and 6, it can further prove the validity of Theorem 3 and the closed-loop system is semi-globally UUB.

5 | CONCLUSIONS

In this paper, some control schemes based on state prediction have been proposed to investigate the disturbed MIMO nonlinear systems with input delay. Firstly, for the nonlinear systems with constant input delay, an improved controller has been obtained to eliminate the tracking error between predicted state and original state. Secondly, for the disturbed nonlinear systems with constant input delay, an effective control scheme combining state prediction with disturbance observer has been proposed. Thirdly, as for the disturbed nonlinear systems with time-varying input delay, an elegant control scheme combining state prediction, input saturation with sliding mode disturbance observer has been presented. Finally, by using the dynamic model of the quadrotor UAV, some simulations have been exploited to illustrate the effectiveness of our proposed control schemes. In the future works, some meaningful issues including the input-delayed nonlinear system with uncertain dynamics and nonlinear system with unknown constant input delay will be further investigated.

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