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THE INFLUENCE OF EXPEDITED FABRICATION RATE, UNRELIABLE MACHINES, SCRAP, AND REWORK ON THE PRODUCTION RUNTIME DECISION IN A VENDOR-BUYER COORDINATED ENVIRONMENT

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Transnational manufacturing firms operate in highly competitive marketplaces. This means that they are continuously seeking ways to reduce order response and fabrication cycle times, maintain the desired product quality, manage unanticipated machine failures, and provide timely delivery to effectively minimize overall operating cost and maintain a competitive advantage over their intra-supply chains. To assist firms in achieving these operational goals, we examine a buyer-vendor coordinated system with an expedited fabrication rate, unreliable machines, scrap, and rework, with the objective of minimizing the overall operating costs. An imperfect manufacturing process is assumed, which arbitrarily produces repairable and scrap items, with the latter being reworked in each fabrication cycle. Additionally, the process is subject to a Poisson-distributed machine breakdown. The corrective action is undertaken immediately when the machine fails, and the production of unfinished/interrupted lot resumes when the process is restored. The expedited fabrication rate option is used at an extra cost to reduce the cycle length. We built a fabrication-shipment model to characterize the problem’s features explicitly. Mathematical and optimization approaches assist us in determining the optimal fabrication runtime policy. A numerical example illustrates the capability/applicability of our outcomes. Furthermore, it exposes a diverse set of information relating to the collective/individual effect of differences in the expedited rates, mean-time-to-breakdown, frequency of shipment, and rework/disposal rates of defective items on the optimal policy, utilization, total operating cost, and various cost contributors. This information can contribute to facilitating better decision-making.

Key words: production management, unreliable machine, expedited fabrication rate, rework, scrap, multi-shipment

INTRODUCTION

The present work investigates the influence of expedited fabrication rate, unreliable machines, rework, and scrap on the production runtime decision in a vendor-buyer collaborative environment. To reduce order response and fabrication cycle times, the expedited rate is an effective option at an extra cost. Gershwin [1] analyzed transfer lines with a series of machines that can take different processing rates/times to perform operations on components. A simulation approach was used to provide the research outcomes from numerical examples. Makino and Tominaga [2] examined a flexible assembly system that comprises several sub-systems. Their capacity/production rate is pre-estimated, and the standardized, flexible assembly systems are built to meet the required cycle length. The authors discussed the effect of the cycle time and the consequent fabrication rate on a flexible assembly system’s performance. Freiheit et al. [3] built several reserve-capacity models for pure serial and parallel-serial fabrication lines to estimate their productivities. The authors applied combinatorial mathematics to decide the fabrication quantities, and the system states’ occurrence probabilities. They also demonstrated how to quantify the relevant productivity improvements and to use reserve capacity in place of buffers. Singh and Sharma [4] developed a manufacturing-delivery integrated model incorporating inflation, a time-dependent demand rate, and a fabrication rate that links to the demand rate. The authors provided a numerical example to illustrate their model’s concept and purpose and conducted a sensitivity analysis of system parameters. Chiu et al. [5] explored a multiproduct inventory refilling system featuring expedited fabrication rate, rework, and multi-delivery policy. The authors built a model to portray the system’s features, derived its cost function, and simultaneously determined the optimal refilling cycle time and frequency of shipments. They demonstrated via a numerical example how their results work and crucial managerial decision-related information is accessible via their system. Other works [6-9] investigated the effects of diverse aspects of expedited fabrication rates on the production and operations planning. The imperfect fabrication process comprising the production of random nonconforming items and failures of the machine. Such instances must be carefully managed to avoid the undesirable quality and unanticipated delay
of a production schedule. They will increase the total operating cost and decrease the customer’s satisfaction. Moinzadeh and Lee [10] considered an inventory system featuring Poisson demand arrival rate, continuous-review, known resupply time, and defective items. An approximation structure with wide-ranging numerical tests was proposed to save the computation efforts on system characteristics and simplify the derivation of the optimal or near-optimal ordering policies for such a system. Hong et al. [11] analyzed a production line with multiple unreliable machines and arbitrary processing times. By using the decomposition concept and simulation techniques, the authors were able to conduct numerical experiments to explore the problem efficiently. Geren and Lo [12] studied the requirements for automating the rework process of the printed circuit board assembly (PCBA). The authors indicated that to meet the economic and crucial rework requirements, an industrial robot along with proper hardware and appropriate methods are essential. Accordingly, they developed/integrated a robotic rework cell with suggestions of the required tooling and techniques. Kyriakidis and Dimitrakos [13] examined a fabrication system with a nonstationary deteriorating installation, an intermediate buffer, and a production unit. Under the control-limit disciplines, a Markov algorithm was proposed to decide the optimal preventive maintenance policy for the system. Widyadana and Wei [14] considered rework as one of the core issues in the green supply chain for it can lower operating costs and decrease the environmental-related problem. Specifically, the authors examined the effect of rework on an economic production quantity (EPQ)-based system for deteriorating stocks. A (m, 1) fabrication scheme was explored in particular, i.e., m fabrication setups plus one rework set up in a production cycle. Through numerical illustration and sensitivity analysis, they found that the deteriorating rate’s effect on the optimal cost is not significant. Still, the impact of demand rate, fabrication setup, and serviceable holding costs on the optimal cost is significant. Nasr et al. [15] studied an EPQ model with defective items where the quality of products made in the same fabrication run is correlated. The authors explored the scheduled maintenance and fabrication policies for such a correlated binomial fabrication system. Both the impact of correlation on certain system performance measures and the maintenance and fabrication policies were investigated. They further showed that the use of their proposed approach could also analyze the interrupted geometric fabrication systems. Other works [16-19] examined the effects of diverse aspects of the imperfect manufacturing process and random failures on fabrication systems and production and operations planning.

In real vendor-buyer collaborative environments, the finished goods’ delivery policy is usually scheduled by a periodic/discontinuous multi-shipment discipline. Aderohunmu et al. [20] considered a buyer-vendor cooperation policy on cost information exchange in just-in-time manufacturing environments. The authors examined the influence of exchanging setup, ordering, stock holding, and delivering costs on the supply relationship in the long run. They found that joint optimization between vendor and buyer was not necessarily the common batch quantity. Sensitivity in various exchanges of operating parameters on the overall cost savings was analyzed. Thomas and Hackman [21] developed models with simulation approximation techniques to explore the impact of a distributor’s committed shipping strategy on a price-sensitive demand supply-chain environment, wherein a fixed-quantity fixed-frequency shipping discipline over a finite time horizon and under the normal-distribution demand was considered. As a result, the optimal ordering policy and the resale price solution were derived. Shaw et al. [22] examined the strategy of economic ordering quantity and delivery for single-vendor multi-customer supply chains featuring multi-shipment. The unequal lot sizes were considered to cope with the variation of customers’ demands. The authors used a direct search method to decide the decision variables’ optimal values, justified their proposed lot-splitting solution versus no lot-splitting approach in terms of cost savings, and provided further analyses to demonstrate their effectiveness model. Additional studies [23-27] explored the influence of different characteristics of multiple shipments on various fabrication-transportation coordinated supply chains. As few prior works focused on examining the collective influence of expedited rate, unreliable machines, scrap, rework, and multi-shipment rule on the optimal production runtime decision in a vendor-buyer collaborative business environment, this work aims to bridge the gap.

The proposed coordinated vendor-buyer system

Problem description and formulation

The proposed coordinated vendor-buyer system’s description with an expedited rate, unreliable machine, scrap, and rework is as follows. A batch fabrication plan is employed to meet a product with an annual demand λ and accelerate the manufacturing uptime at an expedited rate \( P_1 \) is used. Consequently, a higher setup \( KA \) and unit \( CA \) costs are linked to this extra percentage of manufacturing rate \( \alpha \). The relationship of speedup-rate related parameters \( P_1, KA, \) and \( CA \) and the standard-rate relevant variables are expressed as follows:

\[
P_{1k} = (1 + \alpha_k) P_1
\]

(1)

(2)

(3)

Due to various unforeseen factors, a random \( x \) portion of manufactured products is nonconforming. No shortages are permitted, so \( P_2 = d_{1x} \) \( \lambda > 0 \) is assumed (where \( d_{1x} \) is the positive element). A \( \theta_1 \) portion of nonconforming products is confirmed to be scrap (where \( 0 \leq \theta_1 \leq 1 \)) and will be disposed of at extra cost \( C_S \) per unit. Right after each cycle’s uptime completes, the rework of the other \( (1 - \theta_1) \) portion of nonconforming products starts at a rate \( P_{2a} \) and extra
unit cost $C_{RA}$. The relationship of speedup-rate related parameters $P_{2a}$ and $C_{RA}$ and the standard-rate variables are expressed as follows:

$$P_{2a}=(1+\alpha_{2})P_{2} \quad (4)$$

$$C_{RA}=(1+\alpha_{2})C_{R} \quad (5)$$

Moreover, A $\theta_{2}$ portion of reworked stocks is identified as scrap (where $0 \leq \theta_{2} \leq 1$), thus, $d_{2a}=\theta_{2}P_{2a}$ and these scrap products will also be disposed. Therefore, the overall scrap rate of nonconforming products in a cycle, $\varphi=\theta_{1}+\theta_{2}$, is subject to the Poisson distribution with $\beta$ as mean per year. An abort/resume stock controlling discipline is adopted whenever a failure happens. Under this discipline, the failed equipment is repaired immediately. The fabrication of the unfinished/interrupted lot is resumed as soon as the equipment is fixed/restore. A constant repair time of the equipment $tr$ is assumed. However, if real repair time shall exceed $tr$, then a piece of rental equipment will be in place to avoid additional fabrication delay. Because of a failure to randomly happen in manufacturing uptime $t_{2a}$, two distinct situations are studied in subsections 2.3 and 2.4. Lastly, upon completion of the regular production and rework processes, $n$ equal-size installments of the lot are transported at a fixed interval of time $t'_{3a}$ to the buyer in $t'_{3a}$.

**Situation one:** A stochastic failure happens in uptime

Figure 1 illustrates the perfect stock level in situation one (i.e., when $t_{1a}<t_{2a}$). It indicates that the stock level comes to $H_{0}$ when a stochastic failure happens; after the equipment is restored, the stock level climbs up to $H_{1}$ at the end of $t_{1a}$; and it arrives at $H$ when $t_{2a}$ ends, before the distribution time $t'_{3a}$ starts.

Figure 2 shows the safety stock level in situation one. It specifies that to meet the extra demand during $t_{r}$, the safety stocks $\lambda_{t}$ will be added to the finished lot and distributed to the buyer in $t'_{3a}$.

Figure 3 illustrates the nonconforming and scrap items' level in situation one are depicted in Figures 3 and 4. From Figure 3, one notices that the maximal nonconforming stocks reach $d_{1a}t_{1a}$ when uptime $t_{1a}$ ends; after disposal of scrap items, the level of nonconforming stocks drops to $d_{1a}t_{1a}(1-\theta_{1})$, and at the end of $t_{2a}$ it depletes to zero.

Figure 4 indicates that the level of scrap stocks comes to $(d_{2a}t_{1a})\theta_{2}$, when a stochastic failure happens; after the equipment is restored, the level scrap stocks climbs up to $(d_{2a}t_{1a})\theta_{2}$ at the end of $t_{1a}$; it reaches the maximal level $[(d_{1a}t_{1a})\theta_{1}+d_{2a}t_{2a}]$ when $t_{2a}$ ends.

---

**Figure 1:** Level of perfect stocks in the proposed coordinated vendor-buyer system with an expedited rate, unreliable machine, scrap, and rework (in blue) as compared to that of the same system without expedited rate, nor machine failure (in black)

**Figure 2:** Level of safety stocks in situation one

**Figure 3:** Level of nonconforming stocks in situation one

**Figure 4:** Level of scrap stocks in situation one
Based on problem description and Figures 1 to 4, the following equations are observed:

\[ T^*_{1A} = t_{1A} + t'_{2A} + t_{3A} \]

\[ t_{1A} = \frac{Q}{P_{1A} - d_{1A}} \]

\[ t_{2A} = \frac{Qx(1-\theta_l)}{n} \]

\[ t_{3A} = T A (t_{1A} + t_{2A} + t_{3A}) \]

\[ T A = \frac{Q(1-\psi)}{\psi} + t_{1A} \]

\[ t_{1A}' = \frac{Q(1-\psi)}{\psi} - \frac{A(x(1-\theta_l))}{P_{1A} - d_{1A}} \]

\[ H = H + (P_{1A} - d_{1A}) T A + \lambda t \]

\[ d_{1A} t_{1A}' = xQ = (x P_{1A}) t_{1A} \]

\[ \phi(xQ) = [\theta_l + (1-\psi) \theta_l](xQ) \]

Figure 5 exhibits the level of the producer’s perfect stock in distribution time in situation one. Total stocks during time \( t_{3A} \) are expressed in Eq. (17) [5].

\[ \frac{1}{H} \sum_{i=1}^{n} t_{1A}' H(t_{3A}) = \left( \frac{1}{n} \right) \frac{n(n-1)}{2} H(t_{3A}) + \frac{n}{2n} H(t_{3A}) \]

(17)

The level of the customer’s stock in situation one is displayed in Figure 6. Total stocks during the cycle time \( T A \) can be computed by Eq. (18) [5].

\[ n \left[ \frac{\bar{D} - A t_{1A}^*}{2} \right] + \frac{n(n-1)}{2} (t_{1A} + t_{2A} + T A) + \frac{1}{2} \left[ H - A t_{3A} + T A (H - A t_{3A}) \right] \]

(18)

where

\[ D = H \]

\[ I = D - (A t_{1A}^*) \]

(19)

In situation one, \( TC(t_{1A}) \) comprises the following variable and fixed expedited fabrication costs, the repair cost of failed equipment, safety stock relevant costs, rework and disposal costs, variable and fixed distribution costs, and total stock holding costs (comprising the reworked items, buyer’s items, and the finished and nonconforming items in a cycle):

\[ TC(t_{1A}) = C_s Q + K_A + M + h \left( A t_{1A} + t_{2A} + t_{3A} \right) \]

\[ + C_s x Q(1-\theta_l) + C_s x \phi Q \]

\[ + n K + C_r \left[ Q(1-\phi) + A t_{1A} + t_{2A} + t_{3A} \right] \]

\[ + \frac{h}{2} \left( \frac{H - A t_{3A} + T A (H - A t_{3A})}{n} \right) \]

\[ + h \left[ \frac{H - A t_{3A} + T A (H - A t_{3A})}{2} + \frac{H + (H - A t_{3A})}{2} - \frac{n-1}{2n} H \right] \]

To deal with the randomness of \( x \), we apply the expected values and substitute Eqs. (1) to (20) in Eq. (21) to gain \( E[TC(t_{1A})] \) as follows:

\[ E[TC(t_{1A})] = \left[ (1+\alpha_1) C_s \right] \left[ (1+\alpha_2) C_r \right] \]

\[ \cdot \left[ (1+\alpha_3) P_{1A} \right] \]

\[ \cdot \left[ (1+\alpha_4) P_{2A} \right] \]

\[ + M + C_r \left[ (1+\alpha_1) C_r \right] \]

\[ \cdot \left[ (1+\alpha_2) C_r \right] \]

\[ \cdot \left[ (1+\alpha_3) P_{1A} \right] \]

\[ \cdot \left[ (1+\alpha_4) P_{2A} \right] \]

\[ \cdot \left[ (1+\alpha_5) P_{1A} \right] \]

\[ \cdot \left[ (1+\alpha_6) P_{2A} \right] \]

\[ \cdot \left[ (1+\alpha_7) P_{1A} \right] \]

\[ \cdot \left[ (1+\alpha_8) P_{2A} \right] \]

\[ \cdot \left[ (1+\alpha_9) P_{1A} \right] \]

\[ \cdot \left[ (1+\alpha_{10}) P_{2A} \right] \]

\[ \cdot \left[ (1+\alpha_{11}) P_{1A} \right] \]

\[ \cdot \left[ (1+\alpha_{12}) P_{2A} \right] \]

\[ \cdot \left[ (1+\alpha_{13}) P_{1A} \right] \]

\[ \cdot \left[ (1+\alpha_{14}) P_{2A} \right] \]

\[ \cdot \left[ (1+\alpha_{15}) P_{1A} \right] \]

\[ \cdot \left[ (1+\alpha_{16}) P_{2A} \right] \]

\[ \cdot \left[ (1+\alpha_{17}) P_{1A} \right] \]

\[ \cdot \left[ (1+\alpha_{18}) P_{2A} \right] \]

\[ \cdot \left[ (1+\alpha_{19}) P_{1A} \right] \]

\[ \cdot \left[ (1+\alpha_{20}) P_{2A} \right] \]

(22)

\[ y_6 = \frac{1}{2} \left[ \frac{A t_{1A}}{2} \right] \]

\[ y_6 = \frac{1}{2} \left[ \frac{A t_{1A}}{2} \right] \]

(21)

Figure 5 exhibits the level of perfect stocks in situation two (i.e., when \( t_{3A} \) ends). It illustrates that the stocks climb up to \( H \) when \( t_{3A} \) finishes, and it arrives at \( H \) when \( t_{3A} \) ends, before \( t_{3A} \) begins.
Because of no failure happening in \( t_{sA} \), the safe stocks are not used during \( T_{sA} \). The maximal level of nonconforming stocks reaches \( d_{sA} t_{sA} \); when uptime \( t_{sA} \) ends; after disposal of scrap items, the level of nonconforming stocks drops to \( d_{sA} t_{sA} (1-\theta) \); and it reduces to zero when \( t_{sA} \) ends (refer to Figure 3 excepts the following parameters are utilized: \( t_{sA} \), \( t_{sA} \), and \( T_{sA} \)).

Similarly, the level of scrap stocks climbs up to \( (d_{sA} t_{sA} + d_{sA} t_{sA}) \) at the end of uptime \( t_{sA} \) and it reaches the maximal level \([d_{sA} t_{sA} + d_{sA} t_{sA}] \) at the end of rework time \( t_{sA} \) (see Figure 4 excepts the following parameters are used: \( t_{sA} \), \( t_{sA} \), and \( T_{sA} \)).

Based on problem description for situation two and Figure 5, the following equations can be observed:

\[
T_{sA} = t_{sA} + t_{sA} \tag{23}
\]

\[
t_{sA} = \frac{Q}{P_{sA}} - P_{sA} d_{sA} \tag{24}
\]

\[
t_{sA} = \frac{Q(x(1-\theta))}{P_{sA}} \tag{25}
\]

\[
t_{sA} = T_{sA} - (t_{sA} + t_{sA}) \tag{26}
\]

\[
T_{sA} = \frac{Q(1-\varphi)}{\varphi} \tag{27}
\]

\[
t_{sA} = \frac{Q}{\varphi} \left[ 1 + \varphi - \frac{\lambda}{P_{sA}} \right] \left[ 1 - E\left[ x \left( \varphi - \lambda \right) \right] \right] \tag{28}
\]

\[
H_{sA} = (P_{sA} - d_{sA}) t_{sA} \tag{29}
\]

\[
H = Q(1-\varphi) + H_{sA} + (P_{sA} - d_{sA}) t_{sA} \tag{30}
\]

Total stocks during \( t_{sA} \) [5] is expressed in Eq. (31).

\[
\frac{1}{n^2} \left( \sum_{i=1}^{n} H(t_{sA}) \right) = \frac{1}{n^2} \left( n(n-1) \right) \frac{H(t_{sA})}{n} = \frac{n-1}{2n} H(t_{sA}) \tag{31}
\]

Total stocks during the cycle time \( T_{A} \) can be calculated [5] by Eq. (32).

\[
\frac{1}{n^2} H(t_{sA}) + T_{A} (H-M_{sA}) \tag{32}
\]

In situation two, \( TC(t_{sA}) \) includes the following variable and fixed expedited production costs, safety stock holding costs, rework and disposal costs, fixed and variable distribution costs, and total holding costs (comprising reworked items, buyer’s items, and the finished and non-conforming items in a cycle):

\[
TC(t_{sA}) = C_{A} Q + K_{A} + h_{A} (t_{sA}) + C_{A} Q(1-\theta) + C_{A} \varphi Q + nK_{A} + C_{A} \left[ (1+\varphi) Q + (1+\varphi) (1-\theta) K_{A} \right] + C_{A} \left[ (1+\varphi) Q + (1+\varphi) (1-\theta) (1+\varphi) K_{A} \right] + h_{A} \left[ \frac{H_{sA} + T_{A} (H-M_{sA})}{n} \right] \tag{33}
\]

To deal with the randomness of \( x \), we apply the expected values and substitute Eqs. (15) to (16) and (23) to (32) in Eq. (33) to gain the following \( E(TC(t_{sA})) \):

\[
E\left[ TC(t_{sA}) \right] = \left[ (1+\varphi) K_{A} + (1+\varphi) \varphi C_{A}(1+\varphi) \right] \left[ (1+\varphi) P_{t_{sA}} \right] + nK_{A} + C_{A} \left[ (1+\varphi) P_{t_{sA}} \right] y_{0} + \left[ (1+\varphi) C_{A} \left[ (1+\varphi) P_{t_{sA}} \right] \right] \left[ (1-\theta) \right] \tag{34}
\]

\[
E\left[ TCU(t_{sA}) \right] = E\left[ T(t_{sA}) \right] f(t) dt = \int_{0}^{\infty} t_{sA} E\left[ T(t_{sA}) \right] f(t) dt \tag{35}
\]

where \( E[T_{sA}], E[T_{sA}], \) and \( E[T_{sA}] \) represent the following:

\[
E[T_{sA}] = \int_{0}^{\infty} t_{sA} E\left[ T(t_{sA}) \right] f(t) dt \tag{36}
\]

\[
E[T_{sA}] = Q \left[ (1-\lambda) E\left[ x \right] + \lambda t_{sA} \right] E\left[ T(t_{sA}) \right] f(t) dt \tag{37}
\]

\[
E[T_{sA}] = Q \left[ (1-\lambda) E\left[ x \right] + \lambda t_{sA} \right] E\left[ T(t_{sA}) \right] f(t) dt \tag{38}
\]

Substitute equations (22), (34), and (36) in Eq. (35), along with extra derivation efforts one gains the following \( E(TCU(t_{sA})) \) (for details, see Appendix A):
We start with the convexity test (refer to Eq. (B-3)) and obtain Eq. (40) as follows:

\[
\begin{align*}
\{z_1, z_2\} \cdot y_1(1+\alpha_1)P_1 + y_2(1+\alpha_1)P_1y_1 = 0
\end{align*}
\]

Let \(v_0, v_1, v_2\) represent the following:

\[
\begin{align*}
v_0 &= [z_1, z_2, y_1(1+\alpha_1)P_1 + y_2(1+\alpha_1)P_1y_1] \cdot y_1(1+\alpha_1)P_1 \cdot e^{-\beta t_1A} \\
v_1 &= [-z_1 + z_2, y_1(1+\alpha_1)P_1 + y_2(1+\alpha_1)P_1y_1] \cdot y_1(1+\alpha_1)P_1 = 0 \\
v_2 &= [z_1, z_2, y_1(1+\alpha_1)P_1 + y_2(1+\alpha_1)P_1y_1] \cdot y_1(1+\alpha_1)P_1 \cdot e^{-\beta t_1A} = 0
\end{align*}
\]

Applying Eq. (40) becomes as follows:

\[
\begin{align*}
0 = v_0(t_1A)^2 + v_1(t_1A) + v_2 = 0
\end{align*}
\]

To find \(t_1A^*\), we set \(e^{\beta t_1A} = 1\) and \(e^{-\beta t_1A} = 0\) initially, then compute Eq. (42) to gain the initial bounds for \(t_1A\) (i.e., \(t_{1AU}\) and \(t_{1AL}\)). The next step is to use the present \(t_{1AU}\) and \(t_{1AL}\) to calculate and obtain the updated values of \(e^{\beta t_1A}\) and \(e^{-\beta t_1A}\). Then, re-calculate Eq. (42) with the present \(e^{\beta t_1A}\) and \(e^{-\beta t_1A}\) to gain a new set of bounds \(t_{1AL}\) and \(t_{1AU}\). If \(t_{1AL} = t_{1AU}\) is true, then \(t_1A^*\) is found (i.e., \(t_1A^* = t_{1AL} = t_{1AU}\)); otherwise, repeat the iterations above until \(t_{1AL} = t_{1AU}\).

**Numerical example with discussion**

The following values of parameters are considered in the proposed vendor-buyer coordinated system with expedited rate, unreliable machine, and scrap/rework of defective items (see Table 1).

We start with the convexity test (refer to Eq. (B-3)) for \(E(TCU(t_{1A}))\) by setting \(e^{\beta t_{1A}} = 0\) and \(e^{-\beta t_{1A}} = 1\) (where \(\beta = 1.0\)). Apply Eq. (42) we obtain \(t_{1AL} = 0.3106\) and \(t_{1AU} = 0.0914\). Then, applying \(t_{1AU}\) and \(t_{1AL}\), compute \(e^{\beta t_{1A}} = 0.7330\) and \(e^{-\beta t_{1A}} = 0.9127\). Finally, apply Eq. (B-3) with the obtained values of \(e^{\beta t_{1A}}\) and \(e^{-\beta t_{1A}}\), we confirm that \(y(t_{1AU}) = 0.5388 > t_{1AU} = 0.3106 > 0\) and \(y(t_{1AL}) = 0.2931 > t_{1AL} = 0.0914 > 0\). Therefore, for \(\beta = 1\) the convexity of \(E(TCU(t_{1A}))\) is confirmed. Thus, the optimal \(t_1A^*\) exists. To demonstrate the broader applicability of our study, a wider range of \(\beta\) values were used for the convexity tests and Table C-1 (Appendix C) shows the results.

Upon verification of the convexity of \(E(TCU(t_{1A}))\), we start to find \(t_{1A}^*\). We set \(e^{\beta t_{1A}} = 0\) and \(e^{-\beta t_{1A}} = 1\), then apply Eq. (42), for \(\beta = 1.0\) we obtain \(t_{1AL} = 0.3106\) and \(t_{1AU} = 0.0914\). Then, using the current values of \(t_{1AL}\) and \(t_{1AU}\), compute \(e^{\beta t_{1A}} = 0.7330\) and \(e^{-\beta t_{1A}} = 0.9127\). Repeatedly apply Eq. (42) to update \(t_{1AU}\) and \(t_{1AL}\), and recalculate updated values of \(e^{\beta t_{1A}}\) and \(e^{-\beta t_{1A}}\) until \(t_{1AL} = t_{1AU}\). Accordingly, we obtain \(t_{1A}^* = 0.1280\) and by applying Eq. (39) \(E(TCU(t_{1A}^*)) = 13,792.94\) is obtained. Table C-2 (in Appendix C) exhibits the iterative outcomes for deriving \(t_1A^*\). The influence of changes in \(t_{1A}\) on \(E(TCU(t_{1A}))\) is illustrated in Figure 6. It specifies \(E(TCU(t_{1A}))\) increases significantly in both directions, as \(t_{1A}\) deviates from \(t_{1A}^*\).

**Figure 6: The impact of variations in \(t_{1A}\) on \(E(TCU(t_{1A}))\)**

Figure 7 depicts the influence of differences in mean-time-to-breakdown (MTDB) with different scrap rates on the optimal cost \(E(TCU(t_{1A}^*))\). It discloses that as \(1/\beta\) increases, \(E(TCU(t_{1A}^*))\) declines, and \(E(TCU(t_{1A}))\) decreases radically starting from \(1/\beta > 0.20\) (i.e., \(\beta > 5\)) and it continues to drop and reach $13,149 when \(1/\beta\) approaches infinity (i.e., when a system has no breakdown instance). Figure 7 also points out that as \(\phi\) goes up, the system cost increases slightly.

The impact of variations in the delivery’s frequency \(n\) on the system related costs is demonstrated in Figure 8. It exposes that when \(n = 1\), there is a significantly higher holding cost at the customer end; and as \(n\) increases, the system cost increases significantly. If \(n = 1\), there is a significantly higher holding cost at the customer end; and as \(n\) increases, the system cost increases significantly.

**Table 1: The values of parameters used in the example**

| Parameters  | \(C\) | \(C_{RA}\) | \(C_R\) | \(C_S\) | \(C_T\) | \(K_1\) | \(P_{1A}\) | \(P_{2A}\) | \(\alpha_1\) |
|------------|------|---------|------|------|------|------|--------|--------|--------|
| Values     | 2.5  | 2.0     | 1.25 | 1.0  | 0.1  | 0.01 | 220    | 15000  | 7500   | 0.5    |
| \(\beta\)  | \(\theta_1\) | \(\theta_2\) | \(\phi\) | \(h\) | \(h_1\) | \(K\) | \(P_1\) | \(P_2\) | \(\alpha_2\) |
| 1          | 0.3  | 0.3     | 0.51 | 0.4  | 0.4  | 200  | 10000  | 5000   | 0.1    |
| \(C_n\)    | \(n\) | \(x\)   | \(g\) | \(h_2\) | \(h_3\) | \(K_1\) | \(\lambda\) | \(M\) | \(\alpha_3\) |
| 2.0        | 3    | 20\%    | 0.018| 1.6  | 0.4  | 90   | 4000   | 2500   | 0.25   |

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Figure 7: The influence of differences in $1/\beta$ with different $\varphi$ on $E[TCU(t_{1A})]$

Figure 8: The impact of variations in $n$ on the system related costs

both the shipping cost and producer’s holding cost increase drastically (the former is due to more frequent deliveries and the latter is owing to a slow stock movement from the producer to the customer).

Figure 9: The breakup of $E[TCU(t_{1A})]$

Figure 9 exhibits the breakup of $E[TCU(t_{1A}')]$ where the cost contributors to $E[TCU(t_{1A}')]$ are exposed. For instance, 16.06% of the system cost is related to the price for expedited rate, 5.08% is the machine breakdown relevant cost, and a 5.49% cost relates to the quality assurances issues, etc.

Detailed analysis of the quality cost contributors is performed, and the results are displayed in Figure 10. It shows that the extra production cost contributes 56.79% of the quality cost for making up scrapped items.

Figure 11 depicted the combined impact of differences in mean-time-to-breakdown $1/\beta$ and the expedited-rate $\alpha_1$ on $t_{1A}$. It discloses that $t_{1A}^*$ decreases radically, as both $\alpha_1$ and $1/\beta$ goes up.

Figure 12 reveals the combined influence of changes in the expedited-rate $\alpha_1$ and overall scrap rate $\varphi$ on the optimal $E[TCU(t_{1A}')]$. It indicates that $E[TCU(t_{1A}')]$ goes up drastically as $\alpha_1$ increases, and it only goes up slightly as $\varphi$ rises. Hence, we know that $\alpha_1$ has more influence on $E[TCU(t_{1A}')]$ than that of $\varphi$.

Figure 10: A further breakup of the product quality relevant costs
Figure 11: The combined impact of differences in $\alpha_1$ and $1/\beta$ on $t_{1A^*}$

$E[TCU(t_{1A^*})]$}

Figure 12: The collective influence of variations in $\alpha_1$ and $\phi$ on $E[TCU(t_{1A^*})]$}

Figure 13: The impact of changes in $\alpha_1$ on distinct system cost contributors.

Figure 13 depicts the impact of changes in the expedited-rate $\alpha_1$ on different system cost contributors. It shows that as $\alpha_1$ increases, the expedited cost goes up radically, but other system costs change insignificantly. Besides, it reconfirms that $E[TCU(t_{1A^*})]=13,793$, at $\alpha_1=0.5$. 
The behavior of machine utilization concerning $\alpha_1$ is exhibited in Figure 14. It indicates that utilization declines from 0.4772 to 0.3188 (i.e., a 33.19% decreases), as we expedite the fabrication rate by 50% (i.e., $\alpha_1=0.5$).

Figure 15 exhibits the joint impact of differences in overall scrap rate $\phi$ and mean-time-to-breakdown $1/\beta$ on the optimal runtime $t_{1A}^*$. It exposes that $t_{1A}^*$ rises slightly as $\phi$ goes up. Conversely, $t_{1A}^*$ declines significantly as $1/\beta$ increases. Thus, we find that $1/\beta$ has more impact on $t_{1A}^*$ than that of $\phi$.

**Discussion and limitation**

This study develops the replenishment runtime model based on a case where only one or no breakdowns occur during a production cycle. Table D-1 (see Appendix D) provides the probabilities of different Poisson-distributed breakdown rates. It shows that for production equipment in “good” condition or with an average of less than one random breakdown per year, the proposed model is appropriate, as there is over 99.25% chance that only one or no breakdowns will occur (see Table D-1).

Further, for equipment in “fair” condition, or with an average of less than or equal to three breakdowns per year, our model is suitable with an over 92.94% chance of one or no breakdown occurring (see Table D-1). However, if production equipment in “worse” condition (or having over four random breakdowns per year), our proposed model’s suitability will decline to less than 80%. In this case, we suggest production researchers/managers should develop a multiple failures model for this specific condition.

**CONCLUSIONS**

Seeking to help transnational manufacturing firms achieve operating goals of reducing order response time/fabrication cycle time, keeping the desired product quality, managing the unanticipated machine failures, and providing timely delivery, a specific fabrication-shipment model is built. It captures all of the problem’s particular characteristics, such as the expedited rate, Poisson distributed breakdowns, scrap/rework of defective items, and multiple shipments policy (refer to Figures 1 to 8). Mathematical and optimization approaches and a proposed algorithm enable us to decide the optimal fabrication runtime policy (see Subsections 3 and 4, Tables C and D, and Figure 6). The applicability of our model and the expose of crucial system information are provided (see Figures 7 to 15).
This work’s contribution includes the following:

a. Developing a decision support model that assists in exploring the problem (see Subsections 2 and 3);
b. Deciding the optimal runtime of the proposed problem (refer to Subsections 3 and 4, and Figure 6); and
c. Exposing a diverse set of crucial information regarding the collective/individual impact of differences in the expedite rates, mean-time-to-breakdown, frequency of shipment, and rework/disposal rates of nonconforming items on the optimal runtime policy, utilization, total operating cost, and various cost contributors (refer to Figures 7 to 15).

This information facilitates better decision making. Future studies can extend the problem by exploring the influence of stochastic demand.

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**NOMENCLATURE**

$\lambda$ = product demand rate per year,
$P_{1A}$ = expedited manufacturing rate per year,
$T_{1A}$ = the uptime in the failure happening case – the decision variable,
$T_{2A}$ = the rework time in the failure happening case,
$T_{3A}$ = the finished stock distribution time in the failure happening case,
$T_{c}$ = the cycle time in the failure happening case,
$Q$ = the lot size,
$\beta$ = mean machine breakdown rate per year – follows Poisson distribution,
$t$ = time to a random machine failure,
$M$ = the repair cost per failure instance,
$t_{f}$ = time required to repair the failure,
$x$ = nonconforming rate – which obeys the uniform distribution,
$d_{1A}$ = fabrication rate of nonconforming items in $t_{1A}$ when $P_{1A}$ is used,
$H_{1A}$ = level of finished stock when a stochastic failure happens,
$H_{f}$ = level of finished stock when fabrication uptime completes,
$P_{2A}$ = the expedited reworking rate per year,
$d_{2A}$ = fabrication rate of scrap items during $t_{2A}$ when $P_{2A}$ is used,
$H$ = level of finished stock when the rework time completes,
$t_{NA}$ = time interval between any two deliveries in the failure happening case,
$P_{1}$ = standard annual fabrication rate,
$P_{2}$ = standard annual reworking rate,
$K_{A}$ = the setup cost when $P_{1A}$ is used,
$C_{1}$ = the unit cost if $P_{1A}$ is used,
$C_{R}$ = the reworking cost if $P_{2A}$ is used,
$K$ = the setup cost if $P_{1}$ is used,
$C$ = the unit cost when $P_{1}$ is used,
$C_{R}$ = the reworking cost when $P_{1}$ is used,
$\alpha_{1}$ = the connecting factor between $P_{1A}$ and $P_{1}$,
$\alpha_{2}$ = the connecting factor between $K_{A}$ and $K$,
$\alpha_{3}$ = the connecting factor between $C_{1}$ and $C$, and $C_{R}$ and $C_{R}$,
$t_{2A}$ = the reworking time in the failure happening case,
\( t_{3a} \) = the finished stock distribution time in the no failure happening case,
\( T_{r} \) = the cycle time in the no failure happening case,
\( t_{2a} \) = time interval between any two deliveries in the no failure happening case,
\( d_1 \) = fabrication rate of nonconforming items in \( t_{3a} \) when \( P_1 \) is used,
\( d_2 \) = fabrication rate of scrap items during \( t_\) when \( P_2 \) is used,
\( t_\) = the uptime for a system without expedited rate, nor machine failure,
\( t_{\text{r}} \) = the reworking time for a system without expedited rate, nor machine failure,
\( T \) = the distribution time for a system without expedited rate, nor machine failure,
\( h \) = unit holding cost,
\( C_s \) = unit disposal cost per scrap stock,
\( C_{\text{r}} \) = safety stock's unit cost,
\( C_{\text{f}} \) = unit distribution cost,
\( h_1 \) = reworked stock's unit holding cost,
\( h_2 \) = buyer stock's unit holding cost,
\( h_3 \) = safety stock's unit holding cost,
\( K_{\text{f}} \) = fixed distribution cost,
\( g \) = tr. fixed failure repair time,
\( I(t) \) = level of perfect stocks at time \( t \),
\( I_{\text{f}}(t) \) = level of safety stocks at time \( t \),
\( I_{\text{r}}(t) \) = level of nonconforming stocks at time \( t \),
\( \theta_1 \) = the scrap portion of the nonconforming products,
\( \theta_2 \) = the scrap portion of the reworked products,
\( \varphi \) = the overall scrap rate of the nonconforming products,
\( I_{\text{s}}(t) \) = level of scraps at time \( t \),
\( D \) = the quantity per shipment,
\( l \) = the leftover stocks after each distribution time interval,
\( l_{\text{f}}(t) \) = the buyer's stock level at time \( t \),
\( T_{\text{CU}}(t_{1a}) \) = total cost per cycle in the failure happening case,
\( E[T_{\text{CU}}(t_{1a})] \) = the expected total cost per cycle in the failure happening case,
\( E[T_{\text{CU}}(t_{2a})] \) = the expected total cost per cycle in the no failure happening case,
\( d_{\text{r}} \) = the expected cycle time in the no failure happening case,
\( E[T_{\text{CU}}(t_{3a})] \) = the expected system cost per unit time for the proposed system with or without a failure happening.

**APPENDIX – A**

Detailed derivations for Eq. (39) are given below. By substituting equations (22), (34), and (36) in Eq. (35), along with extra derivation efforts, we first gain \( E[T_{\text{CU}}(t_{1a})] \) as exhibited in Eq. (A-1):

\[
E[T_{\text{CU}}(t_{1a})] = \frac{\lambda}{Y_0 + \frac{A_0 (1-e^{-\eta t_n})}{(t_{1a})/(1+\alpha_0 P_1)}} \quad (A-1)
\]

Let \( z_p, z_{y}, z_2, z_3, z_4, z_5, z_6 \) and \( z_q \) represent the following:

\[
z_p = \frac{M}{(1+\alpha_2 P_1)} + \frac{nK}{(1+\alpha_1 P_1)}
\]

\[
z_q = \frac{M}{(1+\alpha_2 P_1)} + \frac{C_2 h_1^2 + C_1 h_2^2 + h_3 h_5^2 + h_7^2 + h_8^2}{(1+\alpha_1 P_1)} + \frac{g}{(1+\alpha_1 P_1)} \quad \beta
\]

\[
z_3 = \frac{g}{(1+\alpha_1 P_1)} \left[ (h_9^2 + h_11)^2 - 2h_2 h_3 (h_9 + h_11) + h_2^2 + h_3^2 \right]
\]

Then, Eq. (A-1) becomes the following:

\[
E[T_{\text{CU}}(t_{1a})] = \frac{\lambda}{Y_0 + \frac{A_0 (1-e^{-\eta t_n})}{(t_{1a})/(1+\alpha_0 P_1)}} \quad (39)
\]
\[
\begin{align*}
&\frac{\partial^2 E[T\{TU(t)\}]}{\partial t^2} = \frac{\partial^2}{\partial t^2}\left(\frac{dE[T\{TU(t)\}]}{dt}\right) \quad \text{(B-2)}
\end{align*}
\]

Since the first term on the right-hand side (RHS) of Eq. (B-2) is positive (i.e., if \(\gamma(t_{IA}^{}) > t_{IA}^{} > 0\) holds), then \(E[T\{TU(t)\}]\) is convex.

\[
\begin{align*}
&\frac{\partial^2 E[T\{TU(t)\}]}{\partial t^2} = \frac{\partial^2}{\partial t^2}\left(\frac{dE[T\{TU(t)\}]}{dt}\right) \quad \text{(B-2)}
\end{align*}
\]

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\end{align*}
\]

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\begin{align*}
&\frac{\partial^2 E[T\{TU(t)\}]}{\partial t^2} = \frac{\partial^2}{\partial t^2}\left(\frac{dE[T\{TU(t)\}]}{dt}\right) \quad \text{(B-2)}
\end{align*}
\]