Creating Bell states and decoherence effects in quantum dots system

X. X. Yi, G. R. Jin, D. L. Zhou

Institute of Theoretical Physics, Academia Sinica, P.O.Box 2735, Beijing 100080, China

We show how to improve the efficiency for preparing Bell states in coupled two quantum dots system. A measurement to the state of driven quantum laser field leads to wave function collapse. This results in highly efficiency preparation of Bell states. The effect of decoherence on the efficiency of generating Bell states is also discussed in this paper. The results show that the decoherence does not affect the relative weight of $|00\rangle$ and $|11\rangle$ in the output state, but the efficiency of finding Bell states.

PACS number(s): 03.67.-a, 71.10.Li, 71.35.-y

The EPR paradox was proposed in the earliest days of quantum mechanics[1]. Since then quantum entanglement became a interesting subject, which lies at the heart of quantum mechanics[1]. Since then quantum entanglement and the photon creation (annihilation) operator $a$ stands for the interdot interaction, and $\xi(t)$ denotes the laser pulse times the strength of the coupling of the laser field to the excitons. This Hamiltonian governs the time evolution of the excitons in $N$ identical quantum dots system with interdot transfer. The $J_i$ operator obey the usual angular momentum commutation relations $[J_z, J_\pm] = \pm J_\pm$, $[J_+, J_-] = 2J_z$. We consider a laser field $\xi(t) = aAe^{-i\omega t} + a^\dagger A^*e^{i\omega t}$ with electron-photon coupling and the electric field strength $A$, the frequency $\omega$, and the photon creation (annihilation) operator $a^\dagger$ ($a$).

For a coupling two quantum dots system, we introduce notions $|0\rangle = |J = 1, M = -1\rangle$, $|1\rangle = |J = 1, M = 0\rangle$, $|2\rangle = |J = 1, M = 1\rangle$ to describe the vacuum state, the single exciton state and the biexciton state, respectively. In terms of $|0\rangle$, $|1\rangle$ and $|2\rangle$, the Hamiltonian (1) can be re-expressed as

$$H = eJ_z + W(J^2 - J_z^2) + \xi(t)J_+ + \xi^*(t)J_-,$$ (1)

where

$$J_+ = \sum_{i=1}^N c_i^h h_i^\dagger, \quad J_- = (J_+)^\dagger,$$

$$J_z = \frac{1}{2} \sum_{i=1}^N \{c_i^h c_i - h_i h_i^\dagger\},$$ (2)

$c_i^h$ ($h_i^\dagger$) is the electron (hole) creation operator in the ith quantum dot, $e$ is the QD band gap, $W$ stands for the interdot interaction, and $\xi(t)$ denotes the laser pulse times the strength of the coupling of the laser field to the excitons. This Hamiltonian governs the time evolution of the excitons in $N$ identical quantum dots system with interdot transfer.

The EPR paradox was proposed in the earliest days of quantum mechanics[1]. Since then quantum entanglement and the photon creation (annihilation) operator $a^\dagger$ ($a$).

For a coupling two quantum dots system, we introduce notions $|0\rangle = |J = 1, M = -1\rangle$, $|1\rangle = |J = 1, M = 0\rangle$, $|2\rangle = |J = 1, M = 1\rangle$ to describe the vacuum state, the single exciton state and the biexciton state, respectively. In terms of $|0\rangle$, $|1\rangle$ and $|2\rangle$, the Hamiltonian (1) can be re-expressed as

$$H = \sum_{i=0}^2 E_i |i\rangle\langle i| + \sqrt{2}Aa|1\rangle\langle 0| + \sqrt{2}A^*a^\dagger|2\rangle + H.c,$$ (3)

where $E_0 = W - e + 0.5\omega^2$, $E_1 = 2W - 0.5\omega^2$, $E_2 = W + e + 0.5\omega$. We now show that this Hamiltonian leads to the generation of Bell states from suitably initialized states. In an invariant subspace spanned by $|1,n\rangle = |1\rangle \otimes |n\rangle$, $|0,n+1\rangle = |0\rangle \otimes |n+1\rangle$ and $|2,n+1\rangle = |2\rangle \otimes |n+1\rangle$. The Hamiltonian takes the following form

$$H_e = \begin{pmatrix}
E_1 & \sqrt{2(n+1)}A^* & \sqrt{2(n+1)}A \\
\sqrt{2(n+1)}A & E_0 & 0 \\
\sqrt{2(n+1)}A^* & 0 & E_2
\end{pmatrix},$$ (4)
where \( |n\rangle \) is Fock state of the laser field. For explicitness, we study here the case of \( E_0 \sim E_2 \), and choose \( A = A^* \). Defining \( \Omega_1 = \sqrt{2(n+1)}A \), and \( \Omega^2 = 8\Omega_1^2 + (E_1 - E_0)^2 \), the Hamiltonian can be rewritten as

\[
H_e = E_0 + \Omega \left( \begin{array}{ccc}
\cos \theta & \sqrt{\frac{\theta}{2}} \sin \theta & \sqrt{\frac{\theta}{2}} \sin \theta \\
\sqrt{\frac{\theta}{2}} \sin \theta & 0 & 0 \\
\sqrt{\frac{\theta}{2}} \sin \theta & 0 & 0
\end{array} \right),
\]

with \( \tanh \theta = \frac{2\sqrt{2}\Omega_1}{E_1 - E_0} \). Then, the eigenvalues and the corresponding eigenstates are

\[
E_d(n) = E_0,
\]
\[
E_{\pm}(n) = \Omega \frac{\cos \theta \pm 1}{2} + E_0,
\]

and

\[
|E_d\rangle = \frac{\sqrt{2}}{2} (|0\rangle - |2\rangle) \otimes |n + 1\rangle
\equiv B_{d,0}|0, n + 1\rangle + B_{d,2}|2, n + 1\rangle,
\]
\[
|E_{+}\rangle = \frac{\sqrt{2}}{2} \sin \frac{\theta}{2}(|0\rangle + |2\rangle) \otimes |n + 1\rangle + \cos \frac{\theta}{2}|1, n\rangle
\equiv (B_{+,0}|0, n + 1\rangle + B_{+,2}|2, n + 1\rangle) \otimes |1, n\rangle,
\]
\[
|E_{-}\rangle = -\frac{\sqrt{2}}{2} \cos \frac{\theta}{2}(|0\rangle + |2\rangle) \otimes |n + 1\rangle + \sin \frac{\theta}{2}|1, n\rangle
\equiv (B_{-,0}|0, n + 1\rangle + B_{-,2}|2, n + 1\rangle) \otimes |1, n\rangle,
\]

respectively. The above solution shows that the system of the exciton with \( J = 1 \), and \( \Delta = 0 \) has a dark state \( |E_d\rangle \), which decouples with the state \(|1\rangle \). In the process of adiabatic evolution the probability of transition from \(|E_d\rangle\) to \(|1, n + 1\rangle\) is zero when the system is initially in the dark state \(|E_d\rangle\).

Let us consider an initial state \(|0, n + 1\rangle\), i.e., the excitons are in their vacuum state, while the laser field is in a Fock state \(|n + 1\rangle\) initially. At time \( t \), the wave function of system that consists of the excitons and the laser field is

\[
|\psi(t)\rangle = \frac{1}{\sqrt{2}} |\sin \frac{\theta}{2} e^{-iE_{+}(n)t}(|0\rangle + |2\rangle)\rangle + \frac{1}{\sqrt{2}} |\cos \frac{\theta}{2}(|0\rangle + |2\rangle)e^{-iE_{-}(n)t}\rangle
+ \frac{1}{\sqrt{2}} |e^{-iE_{d}(n)t}(|0\rangle - |2\rangle)\rangle \otimes |n + 1\rangle
+ \frac{\sqrt{\frac{\theta}{2}}}{2} |\sin \frac{\theta}{2} e^{-iE_{+}(n)t}\rangle|1, n\rangle
+ \frac{\sqrt{\frac{\theta}{2}}}{2} |\cos \frac{\theta}{2} e^{-iE_{-}(n)t}\rangle|1, n\rangle.
\]

The probability of finding one exciton in the two coupled semiconductor quantum dots is

\[
P_1(t) = \sum_n P(n + 1) \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2} \sin^2 \frac{\Omega t}{2},
\]

where we choose \(|\alpha\rangle = \sum_n P(n)|n\rangle\) and \(|0\rangle\) as the initial states of the laser field and the excitons, respectively. The probability \(P_1(t)\) versus time \( t \) is illustrated in figure 1. As seen from figure 1, there are evidences of collapses and revivals in the coupled quantum dots system. The collapse time obviously depend on the strength of the driven laser field. The stronger the laser field, the shorter the collapse time (from fig.1,a to fig.1,b). Despite the fact that there is no experimental study on the collapses and revivals in this system. The observation of excitonic Rabi oscillations in semiconductor quantum well[25] lead us to believe that we are not too far away from the experimental observation of the existence of these effects. Now we take a measure to the laser field, if we observe that the laser field is in its initial state \(|n + 1\rangle\), then the excitons are in state(unnormalized)

\[
|\phi(t)\rangle = \frac{1}{\sqrt{2}} |\sin \frac{\theta}{2} e^{-iE_{+}(n)t}(|0\rangle + |2\rangle)\rangle + \frac{1}{\sqrt{2}} |\cos \frac{\theta}{2}(|0\rangle + |2\rangle)e^{-iE_{-}(n)t}\rangle
+ \frac{1}{\sqrt{2}} |e^{-iE_{d}(n)t}(|0\rangle - |2\rangle)\rangle
\]

certainly, this is a superposition of Bell state

\[
\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),\text{ and }\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle).
\]

Figure 2 shows the ratio of \(P_1(t)\) to \(P_r(t)\) versus time \( t \) for the initial condition \(|\psi(0)\rangle = |0, n + 1\rangle\). Where \(P_1(t)\) is the probability for finding the Bell state \(\frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)\) in the systems of two coupled QDs. For example, if the length of the laser field satisfies \(\cos E_{+}(n)T/\cos E_{-}(n)T = ctanh^{2}\theta/2\), the output state is exactly in Bell state \(\frac{1}{\sqrt{2}}(|0\rangle - |2\rangle)\). Figure 2-b is the same as figure 2-a, but with different initial photon number \(n\), while energy \(W\) is kept fixed in figure 2. The results indicate that the time \(T\) is increased with decreasing the photon number \(n\) (from figure 2-a to figure 2-b).

It is well known that the decoherence is the most enemy to prepare entangled state and keep a state entangled in the quantum information processing. In what follows, we analyze the reliability of the preparation of Bell states when decoherence is taken into account, the decoherence of excitons in semiconductor quantum dots mainly results from the electron-acoustic phonon interactions, this process is governed by the Hamiltonian[24]

\[
H = H + \sum_k \omega_k b_k^\dagger b_k + \sum_k g_k J_z (b_k^\dagger + b_k),
\]

where \(H\) is given by eq.(2), and \(b_k^\dagger (b_k)\) stands for the creation (annihilation) operator of the acoustic phonon with
wavevector $k$. By the general procedure, we can deduce a master equation for the reduced density operator under Markov approximation

$$i \frac{\partial}{\partial t} \rho = [H, \rho] - i \Gamma [J_z, [J_z, \rho]],$$  \hspace{1cm} (12)

where $\Gamma$ is the decoherence rate which depends on the mode distribution of phonons as well as the cut-off frequency. Furthermore, we assume that $\Gamma / A \ll 1$, i.e., the decoherence rate $\Gamma$ is much smaller than the coupling between the excitons and the laser field, this assumption allows us to solve the master equation by small-loss expansions[26]. To begin with, we write the density operator that is a solution to the master equation in powers of $\Gamma$

$$\rho(t, \Gamma) = \rho(t, 0) + \rho_1 \Gamma + \frac{1}{2} \rho_2 \Gamma^2 + ..., \hspace{1cm} (13)$$

where $\rho_1 = \frac{\partial^2 \rho}{\partial \Gamma^2}|_{\Gamma=0}$, $\rho_2 = \frac{\partial^3 \rho}{\partial \Gamma^3}|_{\Gamma=0}$. Substituting this expansions into the master equation, we obtain the following set of equations

$$i \dot{\rho}(t, 0) = [H, \rho(t, 0)], \hspace{1cm} (14)$$
$$i \dot{\rho}_1 = [H, \rho_1] - i [J_z, [J_z, \rho(t, 0)]], \hspace{1cm} (15)$$
$$i \dot{\rho}_2 = [H, \rho_2] - i [J_z, [J_z, \rho_1]], \hspace{1cm} (16)$$

In general, we can solve eq.(14) exactly for an given initial condition, which gives the zeroth order solution $\rho(t, 0)$. Substituting the zeroth order equation into eq.(15), $\rho_1$ can be calculated. Following this procedure, successive terms of the expansion could be worked out, though the calculation is complicated. The average value of a given operator, say $B$, could be calculated through

$$\langle B \rangle = Tr(\rho B) = Tr(\rho(t, 0)B) + \Gamma Tr(\rho_1 B) + ..., \hspace{1cm} (17)$$

For an initial state $|0, n + 1 \rangle$, it is obvious from eq.(14) that

$$\rho(0, t) = |\psi(t) \rangle \langle \psi(t)|, \hspace{1cm} (18)$$

where $|\psi(t) \rangle$ is given by eq.(8). A straightforward calculation shows that

$$\rho_1^{a,b} = \langle E_a | \rho_1 | E_b \rangle, a, b = d, +, -$$
$$= - \int_0^t d\tau f_{a,b} e^{i(E_a - E_b)(t - \tau)}, \hspace{1cm} (19)$$

where

$$f_{a,b} = 4 B_{a,0} B_{b,0} \rho^{0,2}(t, 0) + 4 B_{a,2} B_{b,2} \rho^{2,0}(t, 0),$$
$$\rho^{0,2}(t, 0) = |0, n + 1 \rangle \langle 0, n + 1|, \hspace{1cm} (20)$$

Equations(19)and (13) together give

$$\rho^{a,b}(t, \Gamma) = \rho^{a,b}(t, 0) - \Gamma \int_0^t d\tau f_{a,b} e^{i(E_a - E_b)(t - \tau)}, \hspace{1cm} (21)$$

it is easy to show that the output state after measuring the laser field state $|n + 1 \rangle$ is

$$\rho_e = Tr_f (|n + 1 \rangle \langle n + 1| \rho(t, \Gamma))$$
$$= |\psi(t) \rangle \langle \psi(t)| - \Gamma \sum_{a,b=+,-,d} \rho_1^{a,b}(t)(B_{a,0}|0\rangle + B_{a,2}|2\rangle)$$
$$\cdot (\langle 0|B_{b,0} + \langle 2|B_{b,2}), \hspace{1cm} (22)$$

The first term is just the output state (10) in the situation without decoherence, while the last are represent the effect of decoherence on the generating Bell states. In the case of exact resonance, $f_{a,b}$ is time independent, hence $\rho_1^{a,b}$ for $a = b$ is proportional to $t$. For $a \neq b$,

$$\rho_1^{a,b} = \frac{i f_{a,b}}{E_a - E_b} (1 - e^{-i(E_a - E_b)t}).$$

Noticing the second term in eq.(21) is also a superposition of Bell states, we come to a conclusion that the decoherence does not influence the relative weight of $|00\rangle$ and $|11\rangle$, but diminish the efficiency of finding Bell state. For instance, the probability of finding Bell state $\frac{\sqrt{2}}{2}(|00\rangle - |11\rangle)$ is $P_- = \frac{1}{2} - \Gamma f_{a,d} t$, which decrease proportionally with time.

In summary, we generalized the proposal proposed in [24] to the case of quantum laser field, the advantages of this generalized proposal are that we can select the output state through measuring the laser field state. This provide with us a method to generate Bell states with highly efficiency. Decoherence caused by acoustic phonon scattering diminish the efficiency of preparing Bell states. Up to the first order of $\Gamma$, the decrease in probability of finding Bell state $\frac{\sqrt{2}}{2}(|00\rangle - |2\rangle)$ is proportional to time $t$. Generally speaking, the dependence of the decrease on time is complicated, although we only consider the first order on $\Gamma$.

**ACKNOWLEDGEMENT:**
This work is supported by the Chinese postdoctoral Fund, and NNSF of China.

[1] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47(1935)777.
[2] C. A. Kocher, E. D. Commins, Phys. Rev. Lett. 18(1967)575.
[3] J. S. Bell, Speakable and unspeakable in quantum mechanics (Cambridge University Press, 1987).
[4] C. H. Bennett, G. Brassard, A. K. Ekert, Sci. Am. 267(1992)50.
[5] C. H. Bennett, etal. Phys. Rev. Lett. 70(1993)1395.
[6] D. Bouwmeester etal., Nature 390(1997)575.
[7] Y. H. Shih, C. O. Alley, Phys. Rev. Lett. 61(1987)2921.
[8] E. Hagley, X. Maitre, G. Nogues, etal., Phys. Rev. Lett. 70(1997)1.
[9] M. G. Moore, P. Meystre, E-print quant-ph/0004083.
[10] D. J. Wineland, J. J. Bolinger, W. M. Itano, etal. Phys. Rev. 46(1992) R6797.
[11] M. Kitagawa, M. Ueda, Phys. Rev. A 47(1993)5138.
[12] A. Kuzmich, N. P. Bigelow, L. Mandel, Europhys. Lett. 42(1998)481.
[13] C. A. Sackett, Phys. Rev. A 59(1999)4202.
[14] M. G. Moore, P. Meystre, quant-ph/0004083. H. Pu, P. Meystre, quant-ph/0007012. M. G. Moore, P. Meystre, Phys. Rev. A 59(1999)R1754.
[15] L. M. Duan A. Sorensen, J. I. Cirac, P. Zoller, quant-ph/0007048. A. Sorensen, L. M. Duan, J. I. Cirac, P. Zoller, quant-ph/0006114.
[16] U. V. Poulsen, K. Molmer, cond-mat/0006030.
[17] E. S. Polzik, Phys. Rev. A 59(1999)4202.
[18] M. D. Lukin, S. F. Yelin, M. Fleischhauer, Phys. Rev. Lett. 84(2000)4232.

[19] J. Hald, J. L. Sorensen, C. Schori, E.S. Polzik, Phys. Rev. Lett. 83(1999) 1319.
[20] A. Ekert, R. Jozsa, Rev. Mod. Phys. 68(1996) 733.
[21] D. Loss, D. P. DiVincenzo, Phys. Rev. A 57(1998)120.
[22] A. Imamoglu, etal., Phys. Rev. Lett. 83(1999) 4204.
[23] G. Chen, N. H. Bonadeo, D. G. Steel, D. Gannon, D. S. Datzer, D. Park, L. J. Sham, Science, 289(2000)1906.
[24] L. Quiroga, N. F. Johnson, Phys. Rev. Lett. 83(1999)2270; J. H. Reina, L. Quiroga, N. F. Johnson, Phys. Rev. A 62(2000)012305; quant-ph/0009033.
[25] A. Schulzgen, R. Binder, M. E. Donovan, etal. Phys. Rev. Lett. 82(1999)2346.
[26] X. X. Yi, C. Li, J. C. Su, Phys. Rev. A 62(2000)013819.

Figure captions

Fig. 1: Population of the one exciton state in two coupled QDs, as a function of time. The parameter chosen are $a: \alpha = 5, \omega = 10^{15}, W = 0.1\omega, A = 0.4W, b$: are the same as those in a, but with $A = 0.8W$.

Fig. 2: The ratio of the population in Bell state $\sqrt{\frac{\omega}{2}}(|00\rangle + |11\rangle)$ to that in $\sqrt{\frac{\omega}{2}}(|00\rangle - |11\rangle)$. We choose $a: n = 10, b: n = 5$. The other parameters are the same as those in figure 1.
Figure 1

(a) Time $t$ (in units of $0.1 \omega^{-1}$)

(b) Probability $P_1(t)$
Ratio $P_+ (t)/P_- (t)$

Time $t$ (in units of $\omega^{-1}$)

Figure 2