We consider spontaneous CP violation in the Next to Minimal Supersymmetric Standard Model (NMSSM), without the usual $Z_3$ discrete symmetry. CP violation can occur at tree level, raising a potential conflict with the experimental bounds on the electric dipole moments of the electron and neutron. One escape from this is to demand that the CP violating angles are small, but we find that this entails an unacceptably light neutral Higgs.

1 Electroweak Baryogenesis

Two requirements of Electroweak Baryogenesis underlie this work. The generation of baryon asymmetry from an initially symmetric state requires stronger CP violation than is provided by the CKM matrix in the Standard Model, and the Higgs sector is a possible source of this. Once baryon number has been created, whether in the electroweak transition or earlier, there arises the problem of preventing its wipeout by sphaleron transitions below the electroweak transition, and this needs a strongly first order phase transition, which in the Standard model entails an unwanted light Higgs. The Next to Minimal Supersymmetric Standard Model (NMSSM) has some advantages over the Minimal model (MSSM) in both these areas. As well as producing and preserving baryon number, models have to avoid dangerous by-products, in particular a large electric dipole moments arising from the CP violating phases. We find that a light neutral Higgs also accompanies weak spontaneous CP violation.

Sphaleron transitions produce an irreconcilable conflict between electroweak baryogenesis and the Standard Model. The MSSM can avoid this, at the expense of a light stop and a neutral Higgs just above current experimental reach. If these do not materialise, the NMSSM will deserve increased consideration, as it has an extensive parameter space, much of which is secure from baryon washout by sphalerons.

CP violation (CPV) can arise explicitly, via complex couplings in the Lagrangian, or spontaneously when the minima of a real potential occur at com-

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plex vevs. The MSSM can incorporate explicit CPV by including complex phases in the soft SUSY breaking terms involving the squarks and gauginos. It has however no spontaneous CP violation (SCPV) at tree level, and generation by radiative corrections entails an unacceptably light Higgs. In contrast, in the NMSSM CPV can occur spontaneously even at tree level if there is no additional discrete Z$_3$ symmetry. The phases in all these CPV models give rise to potentially large electric dipole moments. The NMSSM with Z$_3$ has the attractive feature of permitting SCPV at finite temperature where it can play a role in baryogenesis, but not at zero temperature, thus avoiding problems with the electric dipole moments. However, this model with the additional constraint of universal supersymmetry breaking terms at the GUT scale has had difficulty fitting the observational constraints on the Higgs spectrum.

In order to disentangle the CP and Higgs spectrum properties we consider an unconstrained NMSSM, without Z$_3$ and with general soft SUSY breaking terms.

2 NMSSM

Our model is based on the superpotential

$$W = \lambda N H_1 H_2 - \frac{k}{3} N^3 - r N + \mu H_1 H_2 + W_{\text{Fermion}}$$

where $H_1$ and $H_2$ are the doublets of the MSSM and $N$ is a singlet. We do not impose the common restriction $\mu = r = 0$, which adds a discrete Z$_3$ symmetry. Nor do we require the soft SUSY-breaking terms to evolve perturbatively from a universal high energy form. This allows two additional Z$_3$-violating terms as well as more freedom in the coupling constants.

At the electroweak scale the effective potential is

$$V_0 = \frac{1}{2} \lambda_1 (H_1^\dagger H_1)^2 + \frac{1}{2} \lambda_2 (H_2^\dagger H_2)^2$$

$$+ (\lambda_3 + \lambda_4) (H_1^\dagger H_1) (H_2^\dagger H_2) - \lambda_4 \left| H_1^\dagger H_2 \right|^2$$

$$+ (\lambda_5 H_1^\dagger H_1 + \lambda_6 H_2^\dagger H_2) N^* N + (\lambda_7 H_1 H_2 N^* N + h.c.)$$

$$+ \lambda_8 (N^* N)^2 + \lambda_\mu (N + h.c.) (H_1^\dagger H_1 + H_2^\dagger H_2)$$

$$+ m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 N^* N - m_4 (H_1 H_2 N + h.c.)$$

$$- \frac{1}{3} m_5 (N^3 + h.c.) + \frac{1}{2} m_6^2 (H_1 H_2 + h.c.) + m_7^2 (N^2 + h.c.)$$

$^6$Spontaneous breaking of a discrete symmetry raises a cosmological domain wall problem.
where the quartic couplings $\lambda_i, i = 1 \ldots 8$ at the electroweak scale are related via renormalization group equations to the gauge couplings and the $\lambda, k$ of the superpotential, assuming that the electroweak scale $M_{\text{Weak}} < M_S$, the supersymmetry scale, taken to be 1 TeV. $m_i, i = 1 \ldots 7$, are taken as arbitrary parameters. We consider real coupling constants, so that the tree level potential is CP conserving, but admit complex vevs for the neutral fields, $\langle H^0_i \rangle = v_i e^{i\theta_i} (i = 1, 2), \langle N \rangle = v_3 e^{i\theta_3}$, giving

$$V_0 = \frac{1}{2} \left( \lambda_1 v_1^4 + \lambda_2 v_2^4 \right) + (\lambda_3 + \lambda_4) v_1^2 v_2^2 + (\lambda_5 v_1^2 + \lambda_6 v_2^2) v_3^2$$

$$+ 2\lambda_7 v_1 v_2 v_3^2 \cos(\theta_1 + \theta_2 - 2\theta_3) + \lambda_8 v_3^4 + 2\lambda_9 (v_1^2 + v_2^2) v_3 \cos(\theta_3)$$

$$+ m_1^2 v_1^2 + m_2^2 v_2^2 + m_3^2 v_3^2 - 2\mu v_1 v_2 v_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$- \frac{2}{3} m_5 v_3^3 \cos(3\theta_3) + 2m_6^2 v_1 v_2 \cos(\theta_1 + \theta_2) + 2m_7^2 v_3^2 \cos(2\theta_3)$$

where, without loss of generality, $\theta_2 = 0$. We trade some $m_i$ for chosen vevs $v_0 = \sqrt{v_1^2 + v_2^2} = 174$ GeV, $\tan \beta \equiv v_2/v_1$, $R \equiv v_3/v_0$, $\theta_1, \theta_3$, as well as the tree level charged Higgs mass $M_{H^+}$, leaving $m_5$ and $\mu$ free.

Sets of parameters are chosen which satisfy the conditions for a stationary value of the potential, and then numerical searches are performed to ensure that this is the global minimum.

3 Spontaneous CP violation and the Higgs spectrum

The NMSSM with $Z_3$ does not allow SCPV. If SUSY is broken by radiative corrections to the quartic potential SCPV is possible but is accompanied by a light scalar. As noticed by Pomarol, inclusion of general soft breaking terms, namely those in $m_6^2$ and $m_7^2$ above, does allow SCPV. Large neutron and electron dipole moments can be suppressed by three mechanisms, alone or in combination: (i) large squark and gaugino masses (few TeV); (ii) small phases, $O(0.01)$; or (iii) cancellations between the graphs contributing. This last possibility is more generic in the constrained MSSM than was hitherto realised.

We find that consideration of the Higgs spectrum disfavours the small phase option in the SCPV case.

The scalar mass matrix gives rise to 1 charged and 5 neutral particles. An acceptable mass spectrum can readily be obtained. For example, the parameters in Table with $\lambda = k = 0.5$, $M_S = 1$ TeV give neutral masses 89 to 318 GeV. This example corresponds to large angles in the vevs, and indeed such a spectrum arises for a wide range of parameters in the potential. Fig. shows the upper bound on the lightest neutral when we require SCPV and allow
large angles $\theta \approx 1$ radian. $\theta_1$ was fixed at $\theta_1 = 1$ and $\theta_3$ increased in steps from 0.5 $\theta_1$ to 2.0 $\theta_1$. 100,000 sets of other parameters in the potential were randomly chosen, in the ranges $2 < \tan \beta < 3$, and, in GeV, $10 \leq v_3 \leq 500$, $-500 \leq \mu \leq +500$, $0 \leq m_5 \leq 500$, $200 \leq M_{H^+} \leq 800$. When both angles are large the lightest neutral can have a mass $m_h > 80$ GeV. The mass of the lightest Higgs is always $< 122$ GeV which is not far above current experimental reach at LEP, but it can contain a significant admixture of the singlet $N$ field, which reduces its experimental visibility.

Similar scans with small CP violating phases exhibit a neutral light scalar. Phases $\theta \approx 0.1$ radians give a mass $m_h < 30$ GeV, and, as shown in Fig. 2, $\theta_1 = 0.01$, and $0.05 < \theta_3 < 0.02$ give $m_h < 3$ GeV. Thus we can probably exclude possibility (ii) above.

4 Discussion
The result that weak spontaneous CP breaking implies a light Higgs is quite general, and may be understood by an argument similar to that used by Georgi.
and Pais\cite{Pais} in proving a theorem on the conditions under which radiative corrections can trigger SCPV. If CP is weakly broken, the potential has two nearby minima at

$$\epsilon_1 = (v_1, v_2, v_3, v_1\theta_1, v_2\theta_2, v_3\theta_3),$$  \hspace{1cm} (4)

$$\epsilon_2 = (v_1, v_2, v_3, -v_1\theta_1, -v_2\theta_2, -v_3\theta_3).$$  \hspace{1cm} (5)

Performing a Taylor expansion

$$\epsilon_2 - \epsilon_1 \approx \frac{\partial^2 V}{\partial \phi_j \partial \phi_i} \bigg|_{\epsilon_1} \approx \frac{\partial V}{\partial \phi_i} \bigg|_{\epsilon_2} - \frac{\partial V}{\partial \phi_i} \bigg|_{\epsilon_1} = 0 - 0,$$  \hspace{1cm} (6)

so the mass squared matrix must be singular. To zeroth order there is a zero mass particle, with eigenvector along the direction in the 6-dimensional neutral Higgs space joining the two CP violating minima. If $\theta_i \neq 0$, the neutral matrix does not decouple into sectors with CP = +1 and -1, but it does so approximately as the off diagonal blocks of the matrix are proportional to the small angles $\theta$. This light particle is thus in the nearly CP odd sector. Depending on the parameters in the potential this particle can be a varying admixture of singlet and doublet fields, so it may be difficult to detect.

We conclude that SCPV with small phases is disfavoured. It can occur with large phases, but then heavy squarks or cancellations are required to suppress electric dipole moments.

References

1. M. Carena, M. Quiros and C.E.M. Wagner, Phys. Lett. B 380, 81 (1996).
2. D. Delepine, J.M. Gerard, R. Gonzalez Filipe and J. Weyers, Phys. Lett. B 386, 183 (1996).
3. D. Comelli, M. Pietroni and A. Riotto, Phys. Rev. D 50, 7703 (1994).
4. A.T. Davies, C.D. Froggatt, G. Jenkins and R.G. Moorhouse, Phys. Lett. B 372, 88 (1996).
5. N. Maekawa, Phys. Lett. B 282, 392 (1992).
6. A. Pomarol, Phys. Lett. B 287, 331 (1992).
7. A. Pomarol, Phys. Rev. D 47, 273 (1993).
8. A.T. Davies, C.D. Froggatt and A. Usai, Proc. of the International Europhysics Conference on High Energy Physics, Jerusalem (1997), Eds. D. Lellouch, G. Mikenberg and E. Rabinovici, Springer-Verlag (1998), p391.
9. N. Haba, M. Matsuda and M. Tanimoto, Phys. Rev. D 54, 6928 (1996).
10. J. Gunion and H.E. Haber, Nucl. Phys. B 272, 1 (1986).
11. J. C. Romao, Phys. Lett. B 173, 309 (1986).
12. K.S. Babu and S.M. Barr, *Phys. Rev.* D **49**, R2156 (1994).
13. T. Ibrahim and P. Nath *Phys. Rev.* D **57**, 478 (1998).
14. H. Georgi and A. Pais, *Phys. Rev.* D **10**, 1246 (1974).