Origin of the anomalous Hall Effect in overdoped n-type cuprates: current vertex corrections due to antiferromagnetic fluctuations

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The anomalous magnetotransport properties in electron doped (n-type) cuprates were investigated using Hall measurements at THz frequencies. The complex Hall angle was measured in overdoped Pr$_{2-x}$Ce$_x$CuO$_4$ samples ($x=0.17$ and 0.18) as a continuous function of temperature above $T_c$ at excitation energies 5.24 and 10.5 meV. The results, extrapolated to low temperatures, show that inelastic scattering introduces electron-like contributions to the Hall response. First principle calculations of the Hall angle that include current vertex corrections (CVC) induced by electron interactions mediated by magnetic fluctuations in the Hall conductivity reproduce the temperature, frequency, and doping dependence of the experimental data. These results show that CVC effects are the source of the anomalous Hall transport properties in overdoped n-type cuprates.

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The anomalous properties of the cuprates above the superconducting transition temperature have presented many puzzling challenges to the paradigms of condensed matter physics. In particular the unusual magnetotransport has often been cited as evidence that the cuprates are not Fermi liquids [1, 2]. Despite the large simple convex hole-like Fermi surfaces observed by angular resolved photoemission (ARPES), both optimally doped n- and p-type cuprates exhibit anomalous and strongly temperature dependent Hall coefficients. In overdoped n-type cuprates, $R_H$ has zero crossings that suggest a mixed electron and hole response. Consensus on an explanation for the apparent non-Fermi liquid behavior of the cuprates has not been achieved despite much theoretical and experimental effort [1, 2].

One proposed explanation involves current vertex corrections (CVC) to the standard relaxation time approximation (RTA) for the Hall conductivity [2]. In this Fermi-liquid scenario, inelastic electron interactions mediated by antiferromagnetic (AF) fluctuations are the operative mechanism. In an alternative explanation thermally induced magnetic fluctuations at finite temperatures reconstruct the FS dynamically leading to fluctuating electron and hole-like Fermi surface segments over an area determined by the AF correlation length and a time scale associated with the AF correlation time [3]. In both of these scenarios, Fermi liquid behavior is recovered at $T=0$ where inelastic scattering and thermal fluctuations vanish.

In this letter we directly address these issues by extending magnetotransport measurements into the frequency domain in the electron doped cuprates. We report temperature and doping dependent Hall data on overdoped Pr$_{2-x}$Ce$_x$CuO$_4$ (PCCO) at THz frequencies $\ll 10$ meV. At finite frequencies near zero temperatures where inelastic scattering is still operative but magnetic fluctuations are weak, we observe a strong suppression of the Hall response which corresponds to electronlike contributions coming into $\sigma_{xy}$, even at $T=0$. Our experimental observations together with direct comparisons between our magnetotransport data and first-principle calculations of the frequency dependent Hall angle which include CVCs offer compelling evidence in support of the Fermi liquid interpretation.

Thin film Pr$_{2-x}$Ce$_x$CuO$_4$ c-axis oriented samples were grown via pulsed laser deposition onto 100 µm thick LaSrGaO$_4$ (001) substrates. The two samples reported here have a chemical doping of $x=0.17$ and 0.18 with thicknesses 40 and 125 nm and $T_c$’s of 13 and 9 K, respectively. Resistivity and dc Hall measurements were similar to previously reported data [4, 5]. However, the low temperature dc Hall coefficient $R_H$ measured on these new films is consistent with the simple large hole-like Fermi surface centered at $(\pi,\pi)$ measured by ARPES [5]. Luttinger’s thereom is satisfied since the carrier number density associated with the FS volume measured by ARPES is consistent with the stoichiometric doping level [4, 5, 6]. As temperature is raised, the dc Hall coefficient rapidly decreases and becomes negative at a temperature which increases with $x$, and $R_H$ eventually returns to positive values around room temperature.

Below a doping of $\sim16\%$ where PCCO undergoes a quantum phase transformation to an antiferromagnetic state, the low temperature dc Hall coefficient deviates sharply from the observed overdoped behavior [5]. At and below the critical doping, ARPES data show the
fractionalization of the Fermi surface into Fermi arcs [7, 10] consistent with recent quantum oscillations experiments [11]. In this letter we address the overdoped paramagnetic phase in which the large holelike Fermi surface remains intact.

The Faraday rotation and circular dichroism were measured (expressed as the complex Faraday angle, $\theta_F$) at discrete frequencies as a continuous function of temperature. The output of a far-infrared molecular vapor laser was polarization modulated with a rotating quartz quarter-wave plate and subsequently transmitted through the c-axis oriented sample at normal incidence in an applied magnetic field up to 8 T. The detector signal was harmonically analyzed to directly extract the complex Faraday angle, a technique that is detailed elsewhere [12].

In the thin film limit, the complex Hall angle is related to the Faraday angle via $\theta_H = (1 + \frac{\gamma}{2\sigma_{xx}d})\theta_F$, where $\sigma_{xx}$ is the longitudinal conductivity, $n$ is the index of refraction of the substrate, $Z_0$ is the impedance of free space, and $d$ is the thickness of the film [12]. FTIR-spectroscopic transmission measurements were performed in the spectral range from 2 to 13 meV at a set of discrete temperatures ranging from 5 to 300 K, and the complex conductivity $\sigma_{xx}$ was extracted by fitting to a Drude form.

We have measured the real and imaginary parts of the Hall angle at 10.5 and 5.24 meV. The Hall signals were found to be linear in field over the measured magnetic field range of ±8 T. The complex Hall angle as a function of temperature is plotted in Fig. 1. As in the case of the dc Hall effect in overdoped PCCO [4], we observe a complex temperature dependence with zero crossings that depend on doping.

The observed Hall angle differs in important ways compared to the dc behavior (see Fig. 1). Most importantly, $\text{Im}(\theta_H)$ becomes nonzero. $\text{Im}(\theta_H)$ is negative at high temperature crossing zero at a temperature that increases with doping and frequency. The low temperature $\text{Im}(\theta_H)$ values are positive as expected for a holelike Fermi surface as observed by ARPES. However the negative values at high temperatures is inconsistent with the holelike Fermi surface within Drude-like models. The differences between the $\text{Re}(\theta_H)$ and the dc Hall angle increase with frequency and doping. The peak in $\text{Re}(\theta_H)$ is expected in the simple Drude model for $\omega \sim \gamma$.

Motivated by the simple Fermi liquid-like behavior of the low temperature dc Hall coefficient, we begin our discussion by comparing the infrared (IR) Hall response...
with a simple Drude model where the complex Hall angle in the weak field approximation is given by:

$$
\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}} = \frac{\omega_H}{\gamma_H - i\omega}
$$

where $\sigma_{xy}$ is the Hall conductivity, $\sigma_{xx}$ is the longitudinal conductivity, $\omega_H = qB/m_H$ is the Hall frequency, $m_H$ is the effective Hall mass, $\omega$ is the radiation frequency, $\gamma_H$ is the Hall scattering rate, $B$ is the applied magnetic field, and $q$ is the effective charge of the quasi-particle. While Eq. (1) can not be expected to accurately represent either the dc or IR Hall data of Fig. 1, at finite temperature because of the apparent combined electron- and hole-like response, it may be expected to correctly describe the low temperature results.

To accomplish this it is necessary to extrapolate the Hall data to $T=0$. Generally in conducting condensed matter systems when the energy scale associated with temperature becomes less than the frequency, the temperature dependent conductivity rolls over to a flatter response. The exponent which may describe the low temperature finite frequency power law response is expected to be larger than that of the dc response. Therefore, we use as a guide the reported dc Hall coefficient, Hall angle, and longitudinal resistivity data for 17% and 18% PCCO which were fit between 0 and 50 K and demonstrate temperature power laws with exponents ranging from 0.8 to 1.5. The zero temperature extrapolations of similar power law fits to the IR Hall angle data between 0 and 50 K with exponents ranging from 0.8 to 2 are shown in Fig. 1. Even with these generous overestimates of uncertainties associated with the extrapolations, the IR Hall angle is clearly much smaller than the dc Hall angle (2.6 mrad/T and 3.4 mrad/T for the 17% and 18% doped samples, respectively). This suppression of the finite frequency Hall response is more quantitatively characterized by comparing the low temperature Hall frequency $\omega_H$ in the Drude representation to that expected from ARPES measurements. As noted previously, the low temperature dc Hall coefficient above the quantum critical doping is correctly given by the Fermi surface properties measured by ARPES. All the magnetotransport response functions including $\omega_H$ can be derived directly from ARPES data within the RTA in which $\sigma_{xx}$ and $\sigma_{xy}$ are both expressed as integrals around the Fermi surface involving the Fermi velocities and scattering rates. ARPES measurements determine the size and shape of the Fermi surface, Fermi velocities, and momentum distribution widths. For optimal doping, the Hall frequency derived from the large holelike Fermi surface observed by ARPES yields $\omega_H^0 = 0.052$ meV/T, equivalent to an effective Hall mass $m_H = 2.2 m_e$.

The observed reduction in the Hall angle with frequency (and/or temperature) corresponds to a large suppression in $\sigma_{xy}$ caused by the presence of electronlike contributions. The electronlike contributions at $T=0$ and finite frequency cannot be due to antiferromagnetic (AF) fluctuations since thermally excited AF fluctuations vanish at zero temperature. Instead these results suggest a breakdown of the RTA except when both temperature and frequency are zero where inelastic scattering vanishes. While the experiments implicate inelastic scattering as the underlying cause of the anomalous Hall effect in overdoped PCCO, the mechanism is not obvious from the experimental findings. However, current vertex corrections to the conductivity in the presence of antiferromagnetic fluctuations can lead to currents that are not necessarily parallel to the quasiparticle velocity which can produce negative contributions to the Hall conductivity. In these nearly antiferromagnetic metals, the Hall conductivity is strongly modified by the CVCs which represents electron-electron (Umklapp) scattering processes associated with the magnetic Brillouin zone. Including the CVCs has successfully reproduced the strong temperature dependence of dc Hall coefficient in both electron- and hole-doped cuprates. These effects also modify the frequency dependent response since in-

| doping, x (%) | $\omega_H$ (meV) | $\omega_H^0$ (meV) |
|--------------|-----------------|-----------------|
| 17           | 5.21            | 0.20±0.13       |
| 17           | 10.5            | 0.04±0.02       |
| 18           | 10.5            | 0.16±0.04       |

TABLE I: Utilizing the extrapolated zero temperature complex Hall angle values in Fig. 2, the finite frequency zero temperature Hall frequency $\omega_H$ may be extracted normalized to the Hall frequency predicted by ARPES measurements and Boltzmann transport theory, $\omega_H^0 = 0.052$ meV/T.
elastic scattering is frequency dependent and occurs even at $T \geq 0$.

To examine the role of CVCs on the IR Hall response, we have calculated the frequency and temperature dependent conductivity for overdoped PCCO within the fluctuation-exchange (FLEX) conserving approximation. The starting point is the Hubbard model with model parameters for the tight binding hopping amplitudes representing the band structure $(t, t', t'')$ and the Hubbard $U$ [2]. In the FLEX approximation the effective interaction potential is proportional to the derived dynamical spin susceptibility which has the following functional form: $\chi(q, \omega) \propto \frac{1}{1 + \xi^2(q - Q)^2 - i\omega/\omega_D}^{-1}$, where the AF correlation length $\xi$ is much larger than the lattice spacing in PCCO at low temperatures [10]. With this interaction, an electron with $k$ on the Fermi surface is scattered to $k + (-)Q$ by absorbing (or emitting) a low-energy AF magnon fluctuation. This scattering process in nearly AF metals not only shortens the quasiparticle lifetime (i.e., hot-spot), but also changes the direction of electron motion. In Fermi liquid theory, the former and the latter effects are described by the self-energy and the CVC, respectively. How the electron scattering due to magnetic fluctuations leads to strong CVC effects is illustrated in Fig. 3. An excited electron at $k$, after a quasiparticle lifetime $1/2\gamma_{AF}$, is scattered to $k + Q$ due to AF fluctuations $Q \approx (\pm \pi, \pm \pi)$. Therefore, $J_k \propto v_k + v_{k+Q}$ in the hydrodynamic regime $\omega \ll \gamma_{AF}$.

Figure 2 shows the numerical results for the ac Hall angle obtained by the FLEX+CVC approximation. The nearest neighbor hopping integral is taken as $t = -0.36$ eV and the quasiparticle damping rate due to elastic scattering was set at $\gamma_{imp} = 3.5$ meV corresponding to $\rho_{imp} \sim 5 \mu \Omega \text{cm}$. In the absence of CVC the complex frequency dependent $\theta_H$ is positive due to the large hole-like Fermi surface corresponding to the ARPES data on electron overdoped cuprates. In Fig. 2 in which CVCs are included, both $\Re(\theta_H)$ and especially $\Im(\theta_H)$ take on increasingly negative values as temperature is lowered from room temperature and the AF fluctuations become stronger. Below 100 K when the damping rate due to AF fluctuations $\gamma_{AF}$ becomes smaller than the elastic scattering rate, CVCs become less important. However, since $\gamma_{AF}$ increases with $\omega$, CVC effects will be more important for larger $\omega$. As $\omega$ and $T$ approach zero, the inelastic scattering reduces and both $\Re(\theta_H)$ and $\Im(\theta_H)$ become positive. In the collisionless regime where $\omega \gg \gamma_{AF}$ the electron-electron scattering becomes less important and $J_k \sim v_k$.

Also, as can be seen in Fig. 2 as $T$ approaches zero, $\Re(\theta_H)$ at 5 and 10 meV is much smaller than the zero frequency value as observed directly. In addition, the figure shows the calculated results for two different doping levels which demonstrate the reduction of the CVC effects as the system is doped further from the AF phase boundary.

Although the temperature, frequency, and doping dependence of $\theta_H$ is qualitatively well described by the FLEX+CVC theory, the overall amplitude of the experimental data in Fig. 2 is smaller by a factor of 2 to 3 compared with the theoretical results in Fig. 2. This discrepancy may result from the fact that the FLEX approximation, as a low energy theory, can provide an accurate description of the magnetic correlation effects but can not give a good account of the Mott correlations in the Hubbard model. In particular, the theory fails to account for the high frequency optical transitions for $\omega \gtrsim U$. This implies, by the optical conductivity sum rules, that the theory overestimates the weight of the low energy Drude-like response. The suppression of the Drude-like spectral weight in the cuprates has been discussed by Millis et al. [14, 17]. Experiments suggest that the Drude oscillator strength associated with $\sigma_{xx}$ is more strongly suppressed than that associated with $\sigma_{xy}$ [14, 19]. Therefore, since $\theta_H = \sigma_{yy}/\sigma_{xx}$, there is an overall suppression of the Hall angle Drude-like oscillator strength [20] that is not captured by the FLEX theory. Further study of these Mott correlation effects on $\sigma_{xy}$ are underway [21].

In conclusion, the low temperature Hall response of PCCO is observed to be strongly suppressed at finite frequencies. The full complex temperature, frequency, and doping dependencies of $\theta_H$ in overdoped PCCO are found to be naturally reproduced in the FLEX+CVC approximation for this strongly interacting nearly antiferromagnetic electron system. These results show that current vertex corrections contribute significantly to the magnetotransport properties of PCCO and, by extension, suggest that CVC effects may also be the underlying mechanism causing the anomalous Hall transport in the hole doped cuprate systems [2].

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