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Evidence of Two-Source King Plot Nonlinearity in Spectroscopic Search for New Boson

Joonseok Hur,$^{1,*}$ Diana P. L. Aude Craik,$^{1,*}$ Ian Counts,$^{1,*}$ Eugene Knyazev,$^{1}$ Luke Caldwell,$^{2}$ Calvin Leung,$^{1}$ Swadha Pandey,$^{1}$ Julian C. Berengut,$^{3}$ Amy Geddes,$^{3}$ Witold Nazarewicz,$^{4}$ Paul-Gerhard Reinhard,$^{5}$ Akio Kawasaki,$^{5}$ Honggi Jeon,$^{7}$ Wonho Jhe,$^{7}$ and Vladan Vuletić$^{1}$

$^1$Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
$^2$JILA, NIST and University of Colorado, Boulder, Colorado 80309, USA
$^3$School of Physics, University of New South Wales, Sydney, New South Wales 2052, Australia
$^4$Facility for Rare Isotope Beams and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA
$^5$Institut für Theoretische Physik, Universität Erlangen, Erlangen, Germany.
$^6$National Metrology Institute of Japan (NMIJ), National Institute of Advanced Industrial Science and Technology (AIST), 1-1-1 Umezono, Tsukuba, Ibaraki 305-8563, Japan
$^7$Department of Physics and Astronomy, Seoul National University, Seoul 151-747, Korea

Optical precision spectroscopy of isotope shifts can be used to test for new forces beyond the Standard Model, and to determine basic properties of atomic nuclei. We measure isotope shifts on the highly forbidden $^2S_{1/2} \rightarrow ^2F_{7/2}$ octupole transition of trapped $^{168,170,172,174,176}$Yb$^+$ ions. When combined with previous measurements in Yb$^+$ and very recent measurements in Yb, the data reveal a King plot nonlinearity of up to 240σ. The trends exhibited by experimental data are explained by nuclear density functional theory calculations with the Fayans functional. We also find, with 4.3σ confidence, that there is a second distinct source of nonlinearity, and discuss its possible origin.

Despite ample evidence for the existence of dark matter$^{[1-4]}$ and concerted experimental searches for candidate particles$^{[5-8]}$, its origin and composition remain unknown. Isotope-shift (IS) spectroscopy has been recently proposed as a tabletop method to search for dark matter candidates in the intermediate mass range $\lesssim 100\text{MeV}/c^2$$^{[9,10]}$. In particular, IS spectroscopy can be used to search for a hypothetical new boson, $\phi$, that mediates interactions between quarks and leptons. An observable consequence is an additional isotope shift that arises from the effective interaction between neutrons and electrons. Such a shift could be detected as a deviation from linearity in a King plot$^{[11]}$ that compares the normalized isotope shifts for two different transitions. If at least three isotope shifts in each transition are measured, a deviation from linearity can be detected. The nonlinearity can also be caused by higher-order nuclear effects$^{[12,13]}$.

In our previous work, we reported evidence, at the 3σ level, for a nonlinearity in a King plot that compared two optical quadrupole transitions ($^2S_{1/2} \rightarrow ^2D_{3/2}, ^2D_{5/2}$) in a trapped Yb$^+$ ion$^{[19]}$. The measurement was performed for five even isotopes, one more than required, and we also proposed a new method to assign the nonlinearity to different possible physical origins based on the observed nonlinearity pattern. At the reported measurement accuracy of $\sim 300\text{Hz}$ on two relatively similar electronic excited states, the source of the nonlinearity could not be discriminated, and was consistent both with a new boson and with Standard-Model (SM) nuclear shifts. IS spectroscopy in Ca$^+$, which has lighter nuclei and therefore lower sensitivity to both new physics and nuclear effects than Yb$^+$$^{[10]}$, showed no King nonlinearity at the 20Hz level$^{[20]}$. At the time of completion of the present work, large King nonlinearities were also reported when comparing transitions in neutral Yb$^{[21,22]}$ with the quadrupole transitions in Yb$^+$.

In this Letter, we report IS laser spectroscopy for the highly forbidden octupole transition $^2S_{1/2} \rightarrow ^2F_{7/2}$ in Yb$^+$. The electron configuration in the F state is very different from the previously measured D states$^{[19]}$, which increased the size of the observed King nonlinearity 20-fold (see Fig. 1). At a measurement resolution of $\sim 500\text{Hz}$, we observe a King plot nonlinearity with 41 standard deviations $\sigma$. Including the recent data for neutral Yb$^{[21,22]}$, into our analysis, the significance of the nonlinearity rises to $240\sigma$, and analyzing the patterns$^{[19]}$ we show that the measurements can be consistently explained by microscopic calculations carried out within nuclear density functional theory (DFT), which provides agreement with ground-state properties of complex deformed Yb isotopes$^{[16,23]}$. Combining all measured transitions in Yb$^+$ and Yb, we further find evidence, at the 4.3σ level, of a second, smaller source of nonlinearity, and discuss implications for limits on a new boson. Finally, we also extract nuclear data that can be used to further fine-tune nuclear energy density functionals.

Our IS measurements are performed on individual cold trapped $^4$Yb$^+$ ions with zero nuclear spin ($A \in \{168, 170, 172, 174, 176\}$). To make an IS measurement on the octupole transition $^2S_{1/2} \rightarrow ^2F_{7/2}$ near 467 nm...
TABLE I. Isotope shifts $\nu_{A',\alpha} = \nu_{A,\alpha} - \nu_{A',\alpha}$ measured for the $\gamma : 2^1S_{1/2} \rightarrow 2^3F_{7/2}$ (this work) and $\alpha : 2^1S_{1/2} \rightarrow 2^3D_{5/2}$ transitions for pairs $(A, A')$ of stable Yb$^+$ even isotopes. Inverse-mass differences $\mu_{A'}} = 1/m_A - 1/m_{A'}$ calculated from $^{24-27}$ with the Yb ionization energy set to 6.254 eV are also listed. Numbers in parentheses indicate 1σ statistical uncertainties.

| (A, A') | $\nu_{A',\alpha}$ [MHz] | $\nu_{A,\alpha}$ [MHz] | $\mu_{A'}}$ [10^{-6} a.u.] |
|---------|-----------------|-----------------|-----------------|
| (108,170) | -4.483.160 30(50) | 2.179.098 93(21) | 70.13.6195(36) |
| (170,172) | -4.149.190 38(45) | 2.044.854 78(34) | 68.506.89049(63) |
| (172,174) | -3.132.321 60(50) | 1.583.068 42(36) | 66.958.65195(64) |
| (174,176) | -2.976.391 60(48) | 1.509.055 29(28) | 65.474.07821(65) |
| (168,172) | -8.587.352 00(47) | 3.627.922 95(50) |
| (170,174) | -7.281.511 88(45) | 3.627.922 95(50) |
| (172,176) | -6.108.712 93(44) | |

TABLE II. Absolute frequencies of the $\gamma : 2^1S_{1/2} \rightarrow 2^3F_{7/2}$ transition extracted from our IS measurements and the absolute frequency measurement in Ref. $^{29,30}$. A second isotope is then loaded into the trap and its center frequency $\nu_{n}$ is measured for $\nu_{n}$ | A, A' |
|-----------------|-----------------|-----------------|-----------------|
| Isotope | Absolute frequency [THz] | Ref. | |
| 168 | 642.108 197 799 37(37) | [this work] | |
| 170 | 642.112 635 960 21(32) | [this work] | |
| 172 | 642.116 785 150 879 5(24) | $^{29,30}$ | |
| 174 | 642.119 917 472 25(33) | | |
| 176 | 642.122 893 863 83(36) | [this work] | |

that we label $\gamma$, we first load a single ion of one isotope $A$ into the trap, Doppler cool it to $\sim 500$ μK, and measure the excitation probability when scanning the frequency of our probe laser, a frequency-doubled Ti:Sapphire laser which is locked to an ultrahigh-thermal-expansion (ULE) cavity with linewidth $\kappa_c/(2\pi) = 30$ kHz. We measure two transitions between Zeeman sublevels that are symmetrically detuned from the zero-field transition $\nu_{n}$, and determine the center frequency $\nu_{A}$ as the mean (see Supplemental Material (SMat) $^{28}$). A second isotope $A'$ is then loaded into the trap and its center frequency $\nu_{A'}$ is measured. We alternate several times between the two isotopes, achieving an accuracy of $\sim 500$ Hz in our measurement of the IS $\nu_{A,A'} = \nu_{A'} - \nu_{A}$, limited mainly by the long-term stability of the ULE cavity. Our measured ISs $\nu_{A,A'}$ are given in Table I. Table II lists the absolute transition frequencies derived from our measured IS in combination with the absolute transition frequency for $^{172}$Yb$^+$ $^{29,30}$. To a very good approximation, the IS can be factored into an electronic component, which is transition dependent (labeled by a greek letter subscript) but does not depend on the isotope, and a nuclear contribution, which depends on the isotopes (labeled by $A,A'$) but not on the electronic transition $^{9,11,14,19}$:

$$\nu_{A,A'} = F_{\gamma} \delta(r)^{A,A'} + K_{\gamma} \mu^{A,A'} + G_{\gamma}^{(4)} \delta(r)^{A,A'} + G_{\gamma}^{(2)} \delta(r^{2})^{A,A'} + \nu_{n} D_{\gamma} a^{A,A'} + \cdots \quad (1)$$

Here $\delta(r^{n})^{A,A'} \equiv (r^{n})^{A} - (r^{n})^{A'}$ is the difference in the $n$-th nuclear charge moment between isotopes $A$ and $A'$, $\mu^{A,A'} = 1/m_A - 1/m_{A'}$ is the inverse-mass difference, and $[\delta(r^{2})^{A,A'}] = (\delta(r^{2})^{A})^{2} - (\delta(r^{2})^{A'})^{2}$, with $A''$ denoting a reference isotope (we use $A'' = 172$). The quantity $\nu_{n} = -(1)^{n+1} y_{n}/(4\pi \hbar c)$ is the product of the coupling constants of the new boson to the neutron $y_{n}$ and electron $y_{e}$, resulting in a Yukawa-like potential given by $\nu_{n}(r) = h\nu_{n} \exp(\gamma_{r}/r)$ for a boson with spin $s$, mass $m_{b}$, and reduced Compton wavelength $\lambda_{c} = h/(m_{b}c)$ $^{9,11,14,19}$. $\nu_{n}^{AA'} = A - A'$ is the neutron-number difference between the two isotopes. The coefficients $F_{\gamma}$, $K_{\gamma}$, $G_{\gamma}^{(4)}$, $G_{\gamma}^{(2)}$, and $D_{\gamma}$ are transition-dependent quantities that quantify the field shift, the mass shift, the fourth-moment shift, the quadratic field shift (QFS), and the sensitivity to the new boson, respectively.

To eliminate the large field shift $F$ (associated with the size change of the nucleus $\delta(r)^{2}$, of order $\sim 4$ GHz), and mass shift $K$ (of order $\sim 0.2$ GHz) contributions, one can use a second set of isotope shifts measured on a different reference transition $\tau$ to generate a King plot $^{11}$. In its frequency-normalized version $^{19}$, the relationship studied can be written as

$$\nu_{\tau}^{AA'} = f_{\gamma} + K_{\gamma} \nu_{\tau}^{AA'} + \nu_{n} D_{\tau} \tau^{AA'} \quad (2)$$

where the notation $\tau^{AA'} \equiv x^{AA'}/\nu_{\tau}^{AA'}$ indicates frequency-normalized terms. We define $z_{\gamma,\tau} \equiv Z_{\gamma}/Z_{\tau}$ as the ratio of coefficients for transitions $\gamma$ and $\tau$, and $Z_{\gamma,\tau} \equiv Z_{\gamma}(1 - f_{\gamma}/z_{\gamma,\tau})$ for $Z \in \{F,K,G_{\gamma}^{(2)},G_{\gamma}^{(4)},D\}$. The first two terms in Eq. 2 represent the linear relation between $\tau_{\gamma}$ and $\gamma$ in the King plot, while the remaining terms possibly violate the linearity.

Figure 1 shows a frequency-normalized King plot using the previously measured transition $\alpha : 2^1S_{1/2} \rightarrow 2^3D_{5/2}$ near 411 nm $^{19}$ as the reference transition $\tau$. The residuals from the linear fit reveal a nonlinearity at the 10⁻⁵ level, corresponding to 41σ. The nonlinearity is 20 times larger than the nonlinearity we observed previously $^{19}$ comparing the two quadrupole transitions, $\alpha$ and $\beta : 2^1S_{1/2} \rightarrow 2^3D_{3/2}$, that have a more similar electronic structure. The recent measurements in neutral Yb $^{21,22}$, when combined with our $\alpha$ or $\beta$ transition data, confirm a nonlinearity of a similar size (see also Fig. 2).

Having unambiguously established a King nonlinearity, we can gain information about the sources of nonlinearity by analyzing the deviation patterns $^{19}$. With four isotope-shift data points, we can rewrite Eq. 2 in terms of four-dimensional vectors as follows:
\[ \mathbf{\nu}_\gamma = f_{\gamma} \mathbf{1} + K_{\gamma} \mathbf{\mu} + (\lambda_+ \mathbf{A}_+ + \lambda_- \mathbf{A}_-) \]  

where the vector space inhabited by the vectors \( z = (z_1, z_2, z_3, z_4) \) with \( z_k \equiv z^{A,A+2} \) \( (A = 166 + 2k \) for \( k = 1, 2, 3, 4, \) \( z \in \{ \mathbf{\mu}, \mathbf{\nu}_\gamma \} \) is spanned by the basis \( (\mathbf{1}, \mathbf{\mu}, \mathbf{A}_+ \mathbf{+}, \mathbf{A}_-) \).

The first two vectors, \( \mathbf{1} \equiv (1, 1, 1, 1) \) and \( \mathbf{\mu} \), define a plane of King linearity (i.e. the component of \( \mathbf{\nu}_\gamma \) in this plane does not give rise to King nonlinearities), while the unit vectors \( \mathbf{A}_+ \) and \( \mathbf{A}_- \), defined as \( \mathbf{A}_+ \propto (\mathbf{p}_4 - \mathbf{p}_2, \mathbf{p}_1 - \mathbf{p}_4, \mathbf{p}_4 - \mathbf{p}_1, \mathbf{p}_2 - \mathbf{p}_3) \) and \( \mathbf{A}_- \propto (\mathbf{p}_4 - \mathbf{p}_2, \mathbf{p}_1 - \mathbf{p}_4, \mathbf{p}_2 - \mathbf{p}_4, \mathbf{p}_3 - \mathbf{p}_1) \), span the out-of-plane space of vectors that produce a King nonlinearity (see SMat). Any vector with nonzero residuals from the linear King plot fit hence has components in the space spanned by \( (\mathbf{A}_+, \mathbf{A}_-) \), and can be expressed as in terms of its scalar components \( \lambda_+ \) and \( \lambda_- \) along \( \mathbf{A}_+ \) and \( \mathbf{A}_- \), respectively. \( (\mathbf{A}_+, \mathbf{A}_-) \) correspond approximately to the zigzag +++−− and curved +−−++ patterns of residuals introduced in Ref. [19].) Both SM and new-boson effects produce nonlinearities with a defined \( \lambda_+/\lambda_- \) ratio, given by the associated nuclear factors \( x^{AA} \), and are characterized by lines along definite directions in the \( \lambda_+ \)-plane (see Fig. 2).

Figure 2 displays the measured nonlinearity in the \( \lambda_+ \) plane for the \( \gamma \) transition as well as for the previously measured \( \alpha \) and \( \beta \) transitions in Yb\(^+\) [19], and the recently measured \( \epsilon \) \( 1S_0 \rightarrow 1D_2 \) transition in Yb [22]. For the reference transition \( \tau \) in Eq. (2), we choose in Fig. 2 the transition \( 1S_0 \rightarrow 3P_0 \) in Yb that has been very recently measured with the highest frequency accuracy [21]. All measured transitions \( \alpha, \beta, \gamma, \epsilon, \delta \) are consistent with each other in that they lie nearly along the same direction in the \( \lambda_+ \) plane, indicating that the nonlinearity originates from a common dominant source for all transitions. This direction corresponds neither to a new boson \( \alpha^{AA} \) nor to the QFS \( \delta^{(r^2,r^4)} \).

To interpret the IS measurements, we performed quantified nuclear calculations of \( (r^2) \) and \( (r^4) \) using nuclear density functional theory (DFT) with realistic energy density functionals (EDFs). The nuclear charge radial moments were obtained directly from calculated charge densities as discussed in Refs. [10] [23]. To explore a possible span of predictions, we consider four different
EDFs: Skyrme functionals SV-min and UNEDF1, extended Skyrme functional RD-min, and the Fayans functional $F_y(\Delta r)$. The calculated $\delta(r^4)$ are multiplied by $G_{\alpha \alpha}^{(4)}$ from atomic structure calculations to predict the nonlinearity for $G_{\alpha \alpha}^{(4)} \delta(r^4)$. For details on the calculations, see Refs. \[32, \[43\], and SMat.

The predicted values of $\langle r^2 \rangle$ and $\langle r^4 \rangle$ are impacted by several effects \[16, \[23\], \[34\], \[35\], including: the surface thickness of nuclear density that shows a pronounced particle-number dependence due to shell effects; the relativistic corrections that contain contributions from the intrinsic nucleon form factors; and nuclear deformation and pairing effects, which also give rise to the fragmentation \[23\] of the single-particle spin-orbit strength that affects spin-orbit contributions to charge moments. Our DFT calculations take all these effects into account. In this respect, a King plot nonlinearity may be rooted in several nuclear structure effects impacting $\langle r^2 \rangle$ and $\langle r^4 \rangle$, not just one as discussed in Ref. \[13\]. As shown in the large inset to Fig. 2, our DFT results agree well with the observed direction in the $\lambda_{\pm}$ plane (see SMat for details).

We can also directly compare the calculated changes in the nuclear size $\delta(r^2)$ to the measured values. In order to be sensitive to the electronic factor $F$ in Eq. (1), which can currently only be calculated with a typical uncertainty of $\lesssim 30\%$, we plot in Fig. 3 the ratios $\delta(r^2)_{A,A+2}/\delta(r^2)_{A-2,A}$ that can be determined from the experimental data with much higher accuracy. The nuclear calculations agree with the IS data to within $20\%$. The ratios obtained from nuclear theory show monotonically increasing trends for the three EDFs SV-min, RD-min, and UNEDF1. Only $F_y(\Delta r)$ produces a trend that is consistent with data. This is yet another demonstration that the Fayans functional is better adapted to local nonmonotonic trends in charge radius data, see also Refs. \[13\], \[34\]. We note that $F_y(\Delta r)$ also provides a better description of nuclear quadrupole deformations as compared to other EDFs, see SMat for details. This demonstrates that high-precision data on nuclear radii deliver important information for discrimination and further development of nuclear models.

Our data also provide strong tests for electronic-structure calculations, as shown in Fig. 3(c,d): The field (mass) shift coefficient $F_\tau (K_\tau)$ on one transition $\tau$ determines the coefficients on all other transitions $\kappa$ via the experimentally determined value of $F_{\kappa \tau} (K_{\kappa \tau})$ (see SMat for details).

While all transitions $\alpha, \beta, \gamma, \epsilon$ lie near a line through the origin in Fig. 2, there is a deviation from that line for all four transitions (plus the reference transition $\delta$) with $4.3\sigma$ significance. (In contrast, the generalized King plot proposed in previous studies \[13\], \[40\] provides a test only for three transitions, giving significance less than $4\sigma$ for any choices of three transitions, see SMat). This second nonlinearity is too large to be explained by the QFS, which is expected to be the next largest source of nonlinearity within the SM (see SMat). In Fig. 4, we show the strength of the coupling constant $y_\alpha y_\alpha$ for a new boson vs boson mass under the assumption that the new boson is the sole source of the second nonlinearity. Different combinations of measured transitions give similar values or bounds for the coupling strength $y_\alpha y_\alpha$; that is near or slightly exceeds the best other laboratory bounds given by the combination of $g-2$ measurements on the electron and neutron scattering experiments \[10\], \[47\], \[54\].

In the future, it should be possible to reduce the experimental uncertainties by up to four orders of magnitude to sub-Hz levels, as has been demonstrated with simultaneously trapped Sr$^+$ ions \[55\]. In combination with improved electronic and nuclear calculations, it should then be possible to determine unambiguously if some part of the observed nonlinearity cannot be explained by physics.
FIG. 4. Product of coupling constants $y_ey_n$ of a new boson with spin $s$ versus boson mass $m_\phi$, derived from generalized-King-plot analyses [14, 46] of groups of three transitions $(\alpha, \gamma, \delta)$ (blue), $(\gamma, \delta, \epsilon)$ (red), and $(\beta, \gamma, \delta)$ (green), assuming that the observed second nonlinearity is dominated by a new boson. Dashed lines indicate the lower bounds of $y_ey_n$’s excluded magnitude. Solid lines and shaded areas are center values and confidence intervals for configuration-interaction calculations using AMBIT. [we show the $95\%$ confidence interval (see SMat) that arises from the statistical uncertainty in the measured ISs. The systematic uncertainty due to the atomic structure calculations is larger; the dash-dotted line shows the center value of $y_ey_n$ for the $(\alpha, \gamma, \delta)$ transition combination using GRASP2018 calculation results, for comparison.] The yellow line indicates the bound derived from electron $g_e$ – 2 measurements [17][32] in combination with with neutron scattering measurements [31][54] from Ref. [10].

within the SM. Besides better measurements on (more) transitions, it may also become possible to perform further measurements on unstable isotopes, which would allow the direct extraction (and elimination) of additional nuclear effects.

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* These authors contributed equally to this work. (vuletic@mit.edu)

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