Entropy optimized flow of Jeffrey fluid with radiation effect over a stretched surface

Tasawar Hayat¹,², Zobia Kainat¹, Sohail A Khan¹ and Ahmed Alsaedi³

Abstract
The theme of this paper is to scrutinize hydromagnetic flow of Jeffrey fluid subject to stretched curved sheet. Heat expression is developed through dissipation, magnetic force and radiation. Entropy generation is also studied. First order isothermal reaction is examined. Nonlinear ordinary differential systems are found through adequate transformation. Here we have used the ND-based numerical solution method to develop numerical results. Impact of sundry variables on temperature, fluid flow, concentration and entropy rate are discussed. Performance of skin friction and heat transport rate via flow parameters are graphically studied. An increase in curvature variables lead to improve velocity and thermal field. Higher approximation of radiation enhances temperature. An intensification in drag force is seen versus Deborah number. Larger approximation of Brinkman number boosts up entropy analysis.

Keywords
Jeffrey fluid, joule heating, thermal radiation, dissipation and entropy analysis

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Introduction
Flow due to stretched surface is significant in fiber spinning, glass fiber, rubber sheet, nuclear reactor, fission reactions, glass blowing, paper production, hybrid-powered engines, annealing of copper wires, cooling of large metallic plates, micro-manufacturing and many others. Thermal and solutal transportation phenomena subject to stretched surface with suction and injection effects are investigated by Gupta and Gupta.¹ Khan et al.² reported the hydrothermal effect of nanomaterials toward a stretching surface with thermal transport rate. Thermal transfer analysis in magnetized viscoelastic fluid with variable heat source/sink over a stretched sheet is demonstrated by Abel and Nandeppanavar.³ Lok et al.⁴ analyzed the non-orthogonal flow subject to stretchable medium. Bao and Yang⁵ analyzed the bifurcation of the wrap flow in a generalized manner. Melting and activation energy analyses for viscoelastic bioconvective nanofluids toward a stretchable surface with random and thermophoretic motion is illustrated by Khan and Alzahrani.⁶ Thermal transfer analysis for time-dependent viscous fluid flow subject to stretching permeable cylinder is inspected by Si et al.⁷ Mustafa et al.⁸ examined a mathematical model for two-dimensional steady and incompressible nanomaterials flow. Some studies regarding stretched medium are presented in Refs.⁹–²⁴

¹Department of Mathematics, Quaid-I-Azam University, Islamabad, Pakistan
²Pakistan Academy of Science, G-5/2, Islamabad, Pakistan
³Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia

Corresponding author:
Sohail A Khan, Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan.
Email: sakhan@math.qau.edu.pk

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Non-Newtonian liquids are to be more efficient than Newtonian liquids. Non-Newtonian liquids are those fluid which hold Newton law viscosity in nonlinear way. Exotic lubricants, paints, oils, clay coating, cosmetics and shampoos are examples of non-Newtonian liquids. Because of their applications in industry, engineering, physiology, and biosciences, non-Newtonian fluids have gained considerable attention. Hayat et al.\textsuperscript{25} investigated the MHD Jeffrey liquid flow in the presence of a radially growing surface. Thermal and solutal transport analyses for Jeffrey liquid due to a stretchable sheet is explained by Shehzad et al.\textsuperscript{26} Abbasi et al.\textsuperscript{27} analyzed the thermal influence in hydromagnetic Jeffrey nanoliquid flow with random and thermophoretic motion. Hydromagnetic peristaltic flow of Jeffrey liquid in a cylinder-shaped tube was investigated by Tripathi et al.\textsuperscript{28} Maninaga Kumar and Kavitha\textsuperscript{29} studied Jeffrey liquid flow between two fixed and revolving disk with suction two parallel circular disks. Farooq et al.\textsuperscript{30} explored the features of Jeffrey liquid flow with isothermal chemical reactions. Mohd Zin et al.\textsuperscript{31} investigated heat conduction augmentation for time-dependent Jeffrey nanomaterials flow with Lorentz force. Hayat et al.\textsuperscript{32} analyzed the hydrothermalf Jeffrey liquid flow subject to a variable stretching sheet. Few investigations regarding non-Newtonian are presented in Refs.\textsuperscript{33–37}

The Objective of this research is to explore entropy rate in magnetohydromagnetic Jeffrey liquid flow over a curved stretchable surface. Heat equation is discussed in presence of dissipation, Lorentz force and radiation. Furthermore, binary reaction is discussed. Here our prime concentration is on heat and entropy analysis. Ordinary differential equations are obtained through suitable parameters. Here we used ND-solve based numerical method to develop computational results. Variation of thermal field, entropy rate, concentration and fluid flow against emerging parameters are examined. Significance performance of drag force and Nusselt number graphically analyzed. Figure 1 sketch to show the residual error. Individual residual errors with CPU time are mentioned in Table 1.

![Figure 1. Total residual error.](image)

**Statement**

Here an incompressible magnetohydromagnetic flow of Jeffrey fluid toward a bended surface is considered. Dissipation, Lorentz force and radiation impacts are deliberated in heat equation. Isothermal chemical reaction is considered. Entropy communication is developed through thermodynamics second law. Consider stretching velocity is \( U_w = (as) \). Magnetic force of strength \((B_0)\) is applied.

Governing equations satisfy

\[
\begin{align*}
R \frac{\partial u}{\partial s} + \frac{\partial}{\partial r} (r \left( r + R \right) u) &= 0, \\
\rho \left( \frac{\partial u}{\partial s} + \frac{u}{r} \frac{\partial u}{\partial r} \right) &= -\frac{\partial p}{\partial r} - \frac{\tau_{ss}}{r} + \frac{R}{r} \frac{\partial}{\partial r} \left( r \tau_{rr} \right), \\
\rho \left( \frac{\partial u}{\partial s} + \frac{u}{r} \frac{\partial u}{\partial r} \right) &= -\frac{R}{r} \frac{\partial p}{\partial r} - \frac{R}{r} \frac{\partial}{\partial r} \left( r \partial u \right), \\
\frac{\partial T}{\partial s} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + S : L &= \frac{16 \sigma T^3}{3(\rho_c)T_c} \frac{\partial^2 T}{\partial r^2} + \frac{\sigma}{(\rho_c) T_c} B_0^2 \mu^2 \frac{\partial u}{\partial r}, \\
\frac{\partial C}{\partial s} + \frac{\partial C}{\partial s} \frac{u}{r} \frac{\partial C}{\partial r} &= D_B \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) - k_f (C - C_\infty),
\end{align*}
\]

with

| \( m \) | \( v_m \) | \( v^2_m \) | \( v^3_m \) | CPU time (sec) |
|---|---|---|---|---|
| 2 | 0.0000321452 | 0.0000231405 | 0.0003245 | 0.503205 |
| 6 | 0.000012451 | 0.000103214 | 0.000213541 | 12.2789 |
| 10 | 0.0000014563 | 0.0000542145 | 0.000124573 | 149.511 |
| 14 | 5.8656 \times 10^{-7} | 0.0000321451 | 0.000124573 | 15.198 |
| 16 | 3.4021 \times 10^{-8} | 0.0000124532 | 0.000045213 | 256.601 |
\[
\begin{align*}
\nu = & \text{as, } v = 0, \quad C = C_u, \quad T = T_u, \quad \text{at } r = 0 \\
u \to 0, \quad \frac{\partial u}{\partial r} \to 0, \quad C \to C_\infty, \quad T \to T_\infty, \quad \text{as } r \to \infty \\
\end{align*}
\]
(6)

Here \( S : L \) is defined as
\[
S : L = \frac{u}{1 + \lambda_1}
\]
\[
\left( \frac{d\nu}{d\nu} \right)^2 - \frac{u}{r + R} \frac{d\nu}{d\nu} + \left( \frac{u}{r + R} \right)^2 + \lambda_2 \left( \frac{u}{r + R} \frac{d\nu}{d\nu} + \frac{u v}{(r + R)^2} \frac{d\nu}{d\nu} \right) - \lambda_1 \left( \frac{u}{r + R} \frac{d\nu}{d\nu} - \frac{u v}{(r + R)^2} \frac{d\nu}{d\nu} \right)^2 - \frac{u}{r + R} \frac{d\nu}{d\nu} \right)
\]
(7)

Here \((u, v)\) indicate the velocity component, \((r, s)\) curvilinear coordinates, \(\sigma\) the electrical conductivity, \(\rho\) the density, \(R\) the radius, \((\rho c_p)\) heat capacity of fluid, \(\sigma^*\) the Stefan-Boltzman constant, \(T\) temperature,

\[\begin{align*}
2P = & \frac{f'' + f'^2}{\eta + K} - M f' + \frac{1}{1 + \lambda_1} \left( \eta + \frac{1}{K} f' + f'' - \frac{1}{K} f'^2 \right) \\
+ & \frac{1}{1 + \lambda_1} \left( \beta \left( \frac{1}{(r + K)^2} f' - \frac{1}{(r + K)^2} f'' \right) \right)
\end{align*}\]
(15)

\[\begin{align*}
\frac{1}{R} \left( \theta'' + \frac{1}{K} \theta' \right) + \frac{K}{(r + K)^2} f'' + M E C f'^2 + \frac{E}{1 + \lambda_1} \left( \frac{1}{K} f' - \frac{1}{(r + K)^2} f'' \right) \\
+ \frac{K}{(r + K)^2} f'' + \frac{K}{(r + K)^2} f'^2 \right) = \frac{1}{(r + K)^2} f'' + \frac{K}{(r + K)^2} f'^2 \right) = 0
\end{align*}\]
(16)

\[\begin{align*}
f'(0) = 1, \quad f(0) = 0, \quad f'(\infty) = 0, \quad f''(\infty) = 0 \\
\phi(0) = 1, \quad \phi(\infty) = 0, \quad \theta(0) = 1, \quad \theta(\infty) = 0
\end{align*}\]
(19)

In which \( M = \frac{-(\rho c_p)}{a_0} \) denotes the magnetic variable, \( K = \sqrt{\frac{\beta}{\rho}} \), is curvature variable, \( R d \left( \frac{16 \sigma T^2}{3 M_0} \right) \) the radiation variable, \( \beta = (a \lambda_2) \) Deborah number, \( P_r = \frac{a}{c_p} \) the Prandtl number, \( E_C = \frac{a c_p}{c_p - T_0} \) the Eckert

\[
\begin{align*}
\begin{align*}
R \frac{\partial u}{\partial s} + \frac{\partial}{\partial r} \left[ (r + R) v \right] = 0, \\
\rho u \frac{\partial u}{\partial r} + \frac{u v}{r + R} = - \frac{R}{r + R} \frac{\partial}{\partial s} \left( \frac{\partial u}{\partial s} \right) - \frac{1}{r + R} \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} \right) + \frac{u}{1 + \lambda_1} \left( \frac{\partial u}{\partial r} + \frac{R}{r + R} \frac{\partial}{\partial s} \right)
\end{align*}
\end{align*}
\]
(8)

(9)

(10)

(11)
number, \( k_1 \left(= \frac{k^2}{\nu} \right) \) the reaction parameter and 
\( Sc \left(= \frac{D_\phi}{\nu} \right) \) the Schmidt number.

By avoiding the pressure from equations (13) and 
(14) we get

\[
f''' + (\eta + K) f'' + 2(\eta + K) f' + 3(\eta + K)^2 f'' + (\eta + K)^3 f'''
\]
\[
+ \beta K(2(\eta + K)^2 f'' + (\eta + K)^3 f'' - (\eta + K)^3 f''')
\]
\[
+ 3(\eta + K)^2 f'' - (\eta + K)^3 f'''
\]
\[
- 3(\eta + K)^2 f'' - (1 + \lambda_1) K((\eta + K)^2 f''
\]
\[
- 3(\eta + K)^2 f'' - 3f''')
\]
\[
- (\eta + K)^2 f'' - (\eta + K)^2 f'' - (\eta + K)^2 f''
\]
\[
+ (\eta + K)^2 f'' + M((\eta + K)^2 f'' = 0,
\]
\[
(20)
\]

**Engineering quantities**

The velocity and temperature gradient are defined as

\[
C_p = \frac{\tau_{xy}}{\mu_0}, \quad Nu = \frac{sq_w}{k(T_w - T_0)},
\]
\[
(21)
\]

with \( \tau_{xy} \) (shear stress) and \( q_w \) (heat flux) are expressed as

\[
\tau_{xy} = \frac{\mu}{1 - \lambda_1} \left( (1 + \lambda_2 \left(v_r^2 + \frac{R_w}{\mu} \frac{\dot{\alpha}}{\pi} \right)) \left[ w - \frac{w}{\mu} + \frac{R}{\mu \pi} \frac{\dot{\alpha}}{\pi} \right] \right)
\]
\[
qu_w = -k \left(1 + \frac{16r_c^2}{4\pi^2} \frac{\dot{\alpha}}{\pi} \right) \left[ w \right] + 0,
\]
\[
(22)
\]

One can found

\[
C_p Re^{1/2} = \frac{2}{1 - \lambda_1} \left( f''(0) - 2f'(0) + \frac{2\beta}{\lambda_1} \right)
\]
\[
\left( f'(0)^2 f''(0) - f''(0) \right)
\]
\[
\left( f'(0)^2 f''(0) - f'(0)^2 \right)
\]
\[
\left( f'(0)^2 f''(0) - f'(0)^2 \right)
\]
\[
\frac{Nu}{Re^{1/2}} = - (1 + Rd) \theta'(0),
\]
\[
(23)
\]

**Entropy generation**

It is expressed as

\[
E_G = \frac{1}{T^2} \left[ \left( \frac{\dot{\alpha}}{\pi} \right)^2 + \frac{16r_c^2}{4\pi^2} \frac{\dot{\alpha}}{\pi} \left( \frac{\dot{\alpha}}{\pi} \right) \right] + \frac{\mu}{\nu} S : L + \frac{\alpha T^2}{\pi^2}
\]
\[
+ \frac{R_D}{\pi^2} \left( \frac{\dot{\alpha}}{\pi} \right)^2 + \frac{R_D}{\pi^2} \left( \frac{\dot{\alpha}}{\pi} \right)^2
\]
\[
\left( \frac{\dot{\alpha}}{\pi} \right)^2
\]
\[
(25)
\]

We have

\[
N_G = \alpha_1 (1 + Rd) \theta^2 + MB + L \theta^2 \phi^2 + L \theta^2 \phi^2 + L \theta^2 \phi^2
\]
\[
+ \frac{R_D}{\pi} \left( f''(0) - 2f'(0) + \frac{2\beta}{\lambda_1} \right)
\]
\[
\frac{16r_c^2}{4\pi^2} \frac{\dot{\alpha}}{\pi} \left( f''(0) - 2f'(0) + \frac{2\beta}{\lambda_1} \right)
\]
\[
+ \frac{16r_c^2}{4\pi^2} \frac{\dot{\alpha}}{\pi} \left( f''(0) - 2f'(0) + \frac{2\beta}{\lambda_1} \right)
\]
\[
+ \frac{16r_c^2}{4\pi^2} \frac{\dot{\alpha}}{\pi} \left( f''(0) - 2f'(0) + \frac{2\beta}{\lambda_1} \right)
\]
\[
(26)
\]

In above expression \( N_G \left(= \frac{E_G}{\dot{c}T^2} \right) \) denotes the
entropy rate, \( R_1 \) the gas constant, \( \alpha_1 \left(= \frac{E_G}{\dot{c}T^2} \right) \) the
thermal ratio variable, \( L \left(= \frac{E_G}{\dot{c}T^2} \right) \) the diffusion
parameter, \( Br \left(= \frac{\mu_0^2}{\nu T^2} \right) \) the Brinkman number and
\( \alpha_2 \left(= \frac{C_f - C_c}{C_f} \right) \) the solutal ratio variable.

**Graphical results**

Effect of sundry variable on temperature, fluid flow, entropy rate and concentration are examined. Thermal transport rate and friction force are addressed.

**Velocity**

Impact of fluid flow versus curvature parameter is portrayed in Figure 2. An increment in velocity profile is observed for increasing the curvature parameter \( K \). Physical performance of fluid flow versus Deborah number is drafted in Figure 3. An amplification in \( \beta \) leads to diminish \( f''(\eta) \).

\[
K \text{ via } f'(\eta)\ldots
\]
\[
\beta \text{ via } f'(\eta)\ldots
\]

**Temperature**

Influence of \( (K) \) on temperature is designed in Figure 4. An augmentation in \( \theta(\eta) \) is observed for curvature variable \( (K) \). Impact of radiation on \( \theta(\eta) \) is represented in Figure 5. Clearly temperature improves for thermal radiation variable. Figure 6 highlighted the characteristic of \( (Pr) \) on thermal field. It is observed that larger
(Pr) declines the temperature. Larger estimation of magnetic ($M$) parameter rises temperature (see Figure 7).

Concentration

Impact of concentration versus curvature parameter is shown in Figure 8. An increment in concentration is noted through increasing the curvature parameter ($K$).

Figure 9 highlighted the characteristic of concentration via Schmidt number. Clearly concentration boosts up versus larger Schmidt variable.

Entropy generation

Figure 10 show the effect of $N_G$ against temperature difference parameter ($\alpha_1$). A progress in entropy rate is seen for temperature difference parameter. Figure 11 shows the increasing effect of entropy ($N_G$) rate against higher Brinkman ($Br$) number. Figure 12 highlighted the behavior of relaxation time variable ($\lambda_1$) on entropy rate. Here entropy ($N_G$) rate declines for ($\lambda_1$). Impact of ($N_G$) versus magnetic ($M$) parameter is interpreted in Figure 13. Higher approximation of ($M$) increases the entropy ($N_G$) generation.

Quantities of interest

Figures 14 to 17 highlighted the characteristics of influential variables on drag force ($C_{f}$) and Nusselt number ($Nu$).
Velocity gradient. Influence of Deborah number ($\beta$) and relaxation time ($\lambda_1$) variables on velocity gradient is illustrated in Figures 14 and 15. Clearly drag force increased via Deborah number ($\beta$) and relaxation time ($\lambda_1$) variables.

Nusselt number. Effect of thermal transport rate against ($K$) and ($M$) is portrayed in Figures 16 and 17. An inverse result for thermal transport rate is seen through ($M$) and ($K$).
Concluding remarks

Main findings are given below.

- Velocity distribution is boosted for curvature ($K$) variable, while an opposite effect is seen for Deborah ($\beta$) number.
- An intensification in $(\theta(\eta))$ is seen for $(M)$ and $(Rd)$.

- Reduction in temperature is noted for Prandtl number.
- Concentration decreases versus Schmidt number, while opposite result seen for curvature parameter.
- An amplification in entropy rate is noted through thermal ratio variable and Brinkman number.
- An amplification in entropy is noticed for magnetic variable.
- An intensification in drag force is observed for relaxation time and Deborah number.
- Heat transfer rate boosts up versus curvature variable, while reverse trend holds for magnetic variable.

Declaration of conflicting interests

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ORCID iD

Sohail A Khan https://orcid.org/0000-0001-8240-6044

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