Diagnostic methods of a bladed disc mode shape evaluation used for shrouded blades in steam turbines

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Abstract. This paper deals with advanced methods for the evaluation of a bladed disc behavior in terms of the wheel vibration and blade service time consumption. These methods are developed as parts of the noncontact vibration monitoring system of the steam turbine shrouded blades. The proposed methods utilize the time-frequency processing (cross spectra) and the method using least squares to analyse the data from the optical and magnetoresistive sensors, which are mounted in the stator radially above the rotor blades. Fundamentally, the blade vibrations are detected during the blade passages under the sensors and the following signal processing, which covers also the proposed methods, leads to the estimation of the blade residual service life. The prototype system implementing above mentioned techniques was installed into the last stage of the new steam turbine (LP part). The methods for bladed disc mode shape evaluation were successfully verified on the signals, which were obtained during the commission operation of the turbine.

1. Introduction
Any mechanical stress of rotating blades reduces their service life. A damage of the blades causes changes in their frequency characteristics, frequency and intensity of the excitation forces, which are measured using contact or contactless method. The contact measurement by strain gauges gives the direct information about the blades stress during a whole revolution [1]. This method is unsuitable for long-term measurements, because a sensor service life is short in a corrosive environment. Furthermore, it is very difficult and expensive to monitor all blades of the bladed disc. The contactless method BTT (blade tip-timing) is based on the measurements with the sensors that are mounted radially on the circumference of the stator above the rotor blades [2]. The times of blade tip passages under the sensors are analyzed. Changes of the frequency characteristics, which are associated with their normal wear and tear, are reflected very slowly. We can not measure these changes using conventional methods. In contrast, the immediate blade damages are manifested by the rapid changes, which enforce the immediate shutdown and repair of the turbine.

It is important to monitor the blades frequency characteristics, during the turbine runup, rundown and power changes. The measurement of vibrations gives us the information about the blade deflection and blade residual service life. Based on these informations, we monitor the state of the blades and other parts of the turbine. The long-term measurement of the blade deflection is important for the planning shutdowns and optimization of the cost of maintenance and operation of the turbine.
The blade deflections are determined by the difference between the real and expected passage times of the blades under the sensors. The deflection of the specific blade is sampled once per revolution. It can not be used the antialiasing filter in the measurement chain, because the turbine rotation frequency is in fact the blade vibration sampling frequency and furthermore, the rotation speed changes. For these reasons, the vibrations of the specific blade are measured in the limited frequency bands with lot images of higher frequencies and the low frequency resolution. An increasing of the sensor amount, and thus a broadening of the frequency bands, has little impact for the expended resources.

Instead of one blade behavior analysis, the whole bladed disc could be analyzed. The deflections are sampled by the passages of all the blades. Then, the width of the frequency band is sufficient and the aliasing manifests itself not so much. The allblade spectrum, which is spectrum involving all blades of one disc, is calculated for each sensor from deflections measured for all the blades passed under the sensor using the short-time Fourier transform. Consequently, cross spectra are calculated from the couples of the allblade spectra from different sensors.

For the estimation of the nodal diameter, a new progressive method is presented; this method is based on the analysis of data from multiple sensors using the method of least squares.

2. Blade vibration manifestation
The blades interact with each other and their behavior affects the behavior of the whole bladed disc. A mode shape of a bladed disc describes the disc behavior, because the blade vibrations are not random. There is the specific relative position of the blades, which is periodically repeated. In such cases, there are points on the disc, which are not in a motion in one time. These points lie on the straight lines, which pass through the disc centre, or on the concentric circles around the disc center. These points are known as the nodal diameter and nodal circle. The nodal diameters and circles characterize the vibrations shapes. In practice, the rotating and standing shapes (travelling or standing waves) are monitored in term of the disc vibration. The following figure 1 illustrates the selected nodal diameters including the special case - zeroth nodal diameter.

Figure 1. An example of several nodal diameters.
3. Standing wave

The bladed disc vibrates with \( ND^{th} \) nodal diameter with the frequency \( f_{osc} \). This modulation is monitored using the timing of the blades with the stationary sensor.

\[
f_{mod} = ND \cdot f_{rot} \pm f_{osc}
\]  

There are two frequencies due to the modulation.

\[
f_k = f_{osc}
\]

\[
f_d = ND \cdot f_{rot}
\]

Using two sensors \( s_1 \) and \( s_2 \), we obtain following signals

\[
s_1 = \cos(2\pi f_k t_1 + \varphi_k) \cdot \cos(2\pi f_d t_1 + \varphi_d)
\]

\[
s_2 = \cos(2\pi f_k t_1 + \varphi_k) \cdot \cos(2\pi f_d t_1 + \varphi_d)
\]

The wave with the frequency \( f_{osc} \) has the same phase shift for both signals because the sensors measure synchronously. We analyze different blades at the same time. The wave with the frequency \( f_d \) corresponds to the bladed disc shape. Different blades have different deflections at the same time. If we consider only the bladed disc shape, then the individual blade have always the same deflection. However, different blades pass under different sensors at the same time. Therefore, the vibrations on the frequency \( f_d \) have a different phase shift at the same time

\[
\varphi_d - \varphi_{d1} = \rho \cdot ND,
\]

where \( \rho \) is the angle formed by sensors. According to the goniometric equality

\[
\cos \alpha \cdot \cos \beta = \frac{1}{2} \left[ \cos(\alpha - \beta) + \cos(\alpha + \beta) \right],
\]

we monitor the signals according to equations (4) and (5) at each sensor.

\[
s_1 = \frac{1}{2} \left\{ \cos[2\pi (f_k - f_d) t_1 + \varphi_k - \varphi_{d1}] + \cos[2\pi (f_k + f_d) t_1 + \varphi_k + \varphi_{d1}] \right\}
\]

\[
s_2 = \frac{1}{2} \left\{ \cos[2\pi (f_k - f_d) t_1 + \varphi_k - \varphi_{d2}] + \cos[2\pi (f_k + f_d) t_1 + \varphi_k + \varphi_{d2}] \right\}
\]

After the discrete Fourier transform calculation, the complex frequency signals are obtained from the measured time signals. From these complex signals, we can calculate the amplitude and angle on the given frequency. Further, we focus on the determination of phase. From the equations (8) and (9), we determine the phases (\( \varphi \)) of both frequency components. For the first sensor, we apply the equation (8)

\[
\Phi(S_1(f_k - f_d)) = \varphi_k - \varphi_{d1}
\]

and for the second sensor the equation (9)

\[
\Phi(S_2(f_k - f_d)) = \varphi_k - \varphi_{d2}
\]

\[
\Phi(S_1(f_k + f_d)) = \varphi_k + \varphi_{d1}
\]

\[
\Phi(S_2(f_k + f_d)) = \varphi_k + \varphi_{d2}.
\]
It holds true for the multiply of two complex numbers \( c_1 \) and \( c_2 \), that the angle determined by the product is equal to the sum of the angles determined from the individual factors. If we will consider complex conjugate number \( c_1^* \), we will get the difference of the angles.

\[
\Phi(c_1^* \cdot c_2) = \Phi(c_2) - \Phi(c_1)
\] (14)

We obtain the cross-correlation spectrum by the multiplying of the complex conjugate Fourier image of one measured signal with the Fourier image of the other measured signal. The phase of the frequency components corresponds to the following formulas.

\[
\Phi(S_1(f_k - f_d) \cdot S_2(f_k - f_d)) = -(\varphi_k - \varphi_{d1}) + (\varphi_k - \varphi_{d2}) = -ND \cdot \rho \] (15)

\[
\Phi(S_1(f_k + f_d) \cdot S_2(f_k + f_d)) = -(\varphi_k + \varphi_{d1}) + (\varphi_k + \varphi_{d2}) = ND \cdot \rho \] (16)

If we find two integer multiples of the angle between the sensors (one positive, the other negative) in the cross-spectrum, we can identify the founded nodal diameter.

4. Travelling wave

The travelling wave is constituted by the deflections of the circumferentially blades. All blades vibrate with the same amplitude and the adjacent blades vibrate with different phase. If the total circumferentially phase delay achieves \( 2\pi \), we can see a rotating diameter, on which the blades have zero deflection at the moment. If phase delay achieves \( 4\pi \), we can see two diameters, etc. The deflection of one blade is described as follows

\[
sin(2\pi f_{osc} t + \varphi).
\] (17)

The deflection of each blade for any time is described by the expression (18), where \( b \) is the blade number, \( N \) the blade quantity and \( D \) the diameters number.

\[
sin(2\pi f_{osc} t + 2\pi \frac{b}{N} D + \varphi_{begin})
\] (18)

If we bring the disc rotation to the expression(18), we will detect the position of only one blade at one time. Assuming the uniform blades distribution around the disc circumference and constant speed, we can write

\[
sin(2\pi f_{osc} (t + b\Delta t) + 2\pi \frac{b}{N} D + \varphi_{begin}).
\] (19)

Because the initial time is constant, it is expressed as the part of the initial phase delay without loss of a generality. Correspondingly, we take the initial time as zero.

\[
sin(2\pi f_{osc} (b\Delta t) + 2\pi \frac{b}{N} D + \varphi_{begin})
\] (20)

Due to the mentioned assumptions, the time differences are replaced

\[
\Delta t = \frac{1}{N \cdot f_{rot}}
\] (21)

\[
sin(2\pi f_{osc} (\frac{b}{N} f_{rot}) + 2\pi \frac{b}{N} D + \varphi_{begin}).
\] (22)

After the modifications, we achieve

\[
sin(2\pi f_{osc} \frac{b}{N} (\frac{f_{osc}}{f_{rot}} + D) + \varphi_{begin}).
\] (23)
The sequentially coming blades, are replaced with time $T$

$$\frac{p}{N \cdot f_{rot}} = T$$  \hspace{1cm} (24)

$$\sin(2\pi T(f_{osc} + D \cdot f_{rot}) + \varphi_{\text{begin}}).$$  \hspace{1cm} (25)

In the spectrograms (whether allblade or cross), we monitor the travelling wave on the frequency

$$f_{obs} = f_{osc} + D \cdot f_{rot}. \hspace{1cm} (26)$$

The monitored wave has the form

$$\sin(2\pi T f_{obs} + \varphi_{\text{begin}}). \hspace{1cm} (27)$$

From the cross spectrogram, we find out with which diameter $D$ and on what frequency $f_{obs}$ the travelling wave seemingly vibrates. The real frequency of the blades movement is

$$f_{obs} = f_{osc} - D \cdot f_{rot}. \hspace{1cm} (28)$$

So far, we consider the travelling wave in one yet unspecified direction. The travelling waves are forward or backward, that depends on the direction, in which the wave propagates according to the disc rotation direction.

Let us take into account the rotation direction such that the blades sequentially pass under the sensor with increasing number $p$. Then, the forward wave propagates against the direction of the blades numbering. The maximum deflection passes from the blades with a higher number to the blades with a lower number $p$. This means, that the phase $\varphi$ is the higher the more blade numbers increase.

The backward wave propagates in the direction of the blades numbering. It follows, that the phase is the more downward the more blade numbers increase. The opposite wave propagation is achieved by the negative phase $\varphi$. The directions are shown in figure 2. Let’s assume that $\varphi$ is positive, we can rewrite expression (17) into forms

$$\sin(2\pi f_{osc} t + \varphi) \hspace{1cm} (29)$$

$$\sin(2\pi f_{osc} t - \varphi). \hspace{1cm} (30)$$

Expression (29) or (30) corresponds with forward or backward wave. In expression (18) the quantity of the rotating diameters $D$ was defined. Further, to the sign of $\varphi$ from the previous expression, we extend the value $D$ to the negative numbers, which maintain the validity of the expression (18) for the forward and backward wave.

5. Evaluation of nodal diameters

Based on the angle $\rho$ between the sensors, the phases are defined for each of the searched nodal diameter.

$$\pm \rho \cdot ND \hspace{1cm} (31)$$

The phases on the individual frequency components of the cross spectrum, which was discussed in the section 3, are compared with the searched phases. In the conformity with the phase tolerance, the calculated nodal diameter is assigned to the corresponding coefficient. When we process the signals from more than two sensors, we compare the coefficients for the individual frequency components of all cross spectra, which combine the signals from each sensor. The coefficient for each frequency component is assigned to the corresponding color that will be used for plotting.
The colors of the frequency components with low amplitudes are suppressed, because of the differentiation of the nodal diameters from the amplitude insignificant frequency components and noise. From these cross spectra, we obtain the cross spectrogram in figure 3, which gives us the information about the substractional and additional components (same color) of individual nodal diameters.

The figure 3 shows strong second nodal diameter 130 Hz in yellow (additional and substractional component of 230 Hz and 30 Hz) or third nodal diameter 135 Hz in red (additional and substractional component 285 Hz and 15 Hz). Less apparent is the first nodal diameter 120 Hz in light blue (additional and substractional component 170 Hz and 70 Hz). The zeroth nodal diameter in dark blue (not observed with modulation, so without additional and substractional components) is visible in the spectrogram at several frequencies.

6. Amplitude evaluation

At the first, let’s assume standing wave with amplitude $A$ vibrating on the disc. Using basic goniometric relation we can write

$$A \cdot \cos \alpha \cdot \cos \beta = A \cdot \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)].$$

The standing wave can be expressed as linear combination of two traveling waves with opposite direction of propagation. Lets define amplitudes of both traveling waves

$$A_\oplus = A_\ominus = \frac{A}{2}.$$  

These amplitudes can be calculated directly from allblade spectrum. For standing wave the following equation holds.

$$A \cdot \cos \alpha \cdot \cos \beta = A_\oplus \cdot \cos(\alpha - \beta) + A_\ominus \cdot \cos(\alpha + \beta)$$

The amplitude of standing wave is monitored on the blades in the antinodes. This is the maximum amplitude across all blades of the disc. The minimal amplitude is monitored on the blades in the nodes of the standing wave. In the ideal case, minimal amplitude is zero.

In the case of traveling wave, the amplitudes $A_\oplus$ and $A_\ominus$ are different, $A_\oplus \neq A_\ominus$. This is interpreted as the superposition of the standing and travelling wave. This behavior of the disc
can not be described by the equation (34). It is necessary to expand the equation with the description of the traveling wave. Superposition of waves travelling in opposite direction can be expressed as superposition of standing wave and single travelling wave. The travelling wave propagates in the direction with the higher amplitude of the frequency component. In following example holds $A_{\oplus} > A_{\ominus}$.

$$A_{\ominus} \cdot \cos(\alpha - \beta) + A_{\oplus} \cdot \cos(\alpha + \beta) = 2 \cdot A_{\ominus} \cdot \cos \alpha \cdot \cos \beta + (A_{\oplus} - A_{\ominus}) \cdot \cos(\alpha + \beta)$$

(35)

In the case of $A_{\oplus} > A_{\ominus}$, the amplitude of the traveling wave is equal to $A_{\oplus} - A_{\ominus}$. In the same case, the amplitude of the standing wave is equal to

$$A = 2 \cdot A_{\ominus}.$$  

(36)

In the comparison to the standing wave, the blade amplitudes are increased by the amplitude of the travelling wave. The blade amplitudes in the antinodes of the standing waves (maximum amplitude) are equal to $A_{\oplus} + A_{\ominus}$ and the blade amplitudes in the nodes of the standing wave (minimum amplitude) are equal to $|A_{\oplus} - A_{\ominus}|$.

7. Evaluation from multiple sensors

An another evaluation method of the nodal diameter provides using the method of least squares. This method, compared to the method of cross spectrum, is based on an evaluation of data from multiple sensor simultaneously. For a complex value for a given frequency in the allblade spectrum, the phase information is the superposition of the initial phase of vibration of the modulated wave, which is the same for all sensors, and phase of modulating wave, which is different for each sensor. When using the method of least squares, we use the regressive equation

$$a \cdot e^{iD\rho} = e^{i\varphi}.$$  

(37)

$D$ is a chosen diameter, which is integer, $\rho$ is an angle of sensor placement, $\varphi$ is a phase obtained from the allblade spectrum at a given frequency at a particular sensor and $a$ is a complex number. The angle of this complex number is a phase information, which includes the sum of the initial phase of vibration of the modulated wave and the initial phase of modulating wave relative to the phase marker. The absolute value of the number $a$ indicates the equivalent of the probability that the chosen diameter $D$ corresponds to the true diameter. The higher the absolute value of $a$ closer to one, the higher probability it was chosen the correct value $D$.

The sensor placement plays a major role in this method too. The placement has a great impact on the degree of credibility of the calculation. An inappropriate placement of sensors may produce falsely correct results.

8. Sensors placement

The sensors should be placed to each other at the angle, which is the integer multiple of the angle formed by the blades. In this arrangement, the blades with the corresponding angle pass under the sensor approximately at the same time. Therefore, there is no increase of the phase delay.

It is also good to choose the angle depending on the expected nodal diameter. Higher nodal diameter represents larger monitored phase. It is necessary to choose the angle between the sensors such that $ND\rho$ lies in the interval $0$ to $\pi$ rad. This prevents an overlap of the detected angles of different nodal diameters, for example the first and third nodal diameter (sensors form angle $\pi/2$ rad). It is recommended to choose the angle between the sensors on the value of prime multiples of the angle, which encloses the blades.
9. Conclusions

Any mechanical stress of blades reduces their service time. The blade vibration measurement provides the information about the long-lasting and rapid changes in the blades parameters. This allows early detection of the blade faults. With the blade tip timing method using the magnetoresistive sensors, the oscillations of the longest blades of the low-pressure part of the steam turbine 270 MW were measured. The nodal diameters were identified and the amplitudes of their oscillation were calculated. The frequencies of the nodal diameters were verified by the model, which were provided by the manufacturer of the turbine. The calculated data will be used for the determination of the blade residual service times.

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References

[1] Zielinski M, Ziller G 2005 Noncontact Blade Vibration Measurement System for Aero Engine Application American Institute of Aeronautics and Astronautics, ISABE
[2] Kadoya Y, Mase M, Kaneko Y, Umemura S, ODA T, Johnson MC 1995 Noncontact Vibrational Measurement Technology of Steam Turbine Blade JSME International Journal 38 486–493
[3] Gallego-Garrido J, Dimitriadis G, Wright JR 2007 A Class of Methods for the Analysis of Blade Tip Timing Data from Bladed Assemblies Undergoing Simultaneous Resonances-Part I: Theoretical Development International Journal of Rotating Machinery
[4] Dimitriadis G, Carrington IB, Wright JR, Cooper JE 2002 Blade-tip timing measurement of synchronous vibrations of rotating bladed assemblies Mechanical Systems and Signal Processing
[5] Ye D, Duan F, Guo H, Li Y, Wang K 2012 Turbine blade tip clearance measurement using a skewed dual-beam fiber optic sensor SPIE
[6] Mercadal M, von Flotow A, Tappert P 2000 Damage Identification By NSMS Blade Resonance Tracking in Mistuned Rotors IEEE
[7] Ivey PC, Grant KR, Lawson C 2002 Tip timing techniques for turbomachinery hcf condition monitoring The 16th Symposium on Measuring Techniques in Transonic and Supersonic Flow in Cascades and Turbomachines
[8] Heath S, Imregun M 1998 A Survey of Blade Tip-Timing Measurement Techniques for Turbomachinery Vibration ASME
[9] Kawasima T, Inuma H, Minagawa N 1994 Optical Semiconductor Blade Vibration Monitoring System for Gas Turbine Engine IEEE
[10] Allport JM, Jupp ML, Poutouvanis A, Janicki GW, Pieronczyk AI, Day AJ, Olley P, Mason B, Ebrahimi MK Turbocharger blade vibration: Measurement and validation through laser tip-timing
[11] Heath S, Imregun M 1996 An improved single-parameter tip-timing method for turbomachinery blade vibration measurements using optical laser probes Elsevier Science Ltd
[12] Hockaday BD 2011 Quantifying optical tip-timing probe error with laboratory apparatus Proceedings of ASME Turbo Expo
[13] Singh MP, Vargo JJ, Schiffer DM, Dello JD Safe diagram – a design and reliability tool for turbine blading