Metamagnetic phase transition in ferromagnetic superconductor URhGe

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Ferromagnetic superconductor URhGe has orthorhombic structure and possesses spontaneous magnetisation along c-axis. Magnetic field directed along b-axis suppresses ferromagnetism in c-direction and leads to the metamagnetic transition to polarised paramagnetic state in b-direction. The theory of these phenomena based on the specific magnetic anisotropy of this material in (b,c) plane is given. Line of the first order metamagnetic transition ends at a critical point. The Van der Waals - type description of behaviour of physical properties near this point is developed. The triplet superconducting state destroyed by orbital effect is recreated in vicinity of the transition. It shown that the reentrance of superconductivity is caused by the sharp increase of magnetic susceptibility in b direction near the metamagnetic transition.

I. INTRODUCTION

Investigations of uranium superconducting ferromagnets UGe2, URhGe and UCoGe continue attract attention mostly due to the quite unusual nature of its superconducting states created by the magnetic fluctuations (see the recent experimental [1] and theoretical [2] reviews and references therein). They have orthorhombic crystal structure and the anisotropic magnetic properties. The spontaneous magnetisation is directed along a axis in UGe2 and along c-axis in URhGe and UCoGe. The ferromagnetic state in the two last materials is suppressed by the external magnetic field $H_y$ directed along $b$ crystallographic direction. In URhGe at field $H_y = H_{cr} \approx 12$ T the second order phase transition to ferromagnetic state is transformed to the transition of the first order [3]. The superconducting state suppressed [4] in much smaller fields $H_y \approx 2$ T is reappeared in vicinity of the first order transition in field interval (9,13)T. The phenomenological theory of this phenomenon has been developed in Ref.5 (see also [2]). According to this theory the state arising in fields above the suppression of spontaneous magnetisation in c-direction is the paramagnetic state.

There was established, however, [3][6][7] that in fields above $H_{cr}$ the magnetisation along b direction looks like it has field independent ”spontaneous” component

$$M_y = M_{y0} + \chi_y H_y.$$  \hspace{1cm} (1)

This state is called polarised paramagnetic state. The formation of this state is related with so called metamagnetic transition observed in several heavy-fermion compounds (see the paper [8] and the more recent publication [9] and references therein). To take into account the formation of polarised paramagnetic state one must introduce definite modifications in the treatment performed in [5]. Here I present the corresponding derivation.

The paper is organized as follows. In the Chapter II after the brief reminder of results of the paper [3] the description of the metamagnetic transition is presented. It is based on the specific phenomenon of magnetic anisotropy in URhGe obtained with a local spin-density approximation calculations by Alexander Shick [10]. After the general consideration of the metamagnetic transition the modifications introduced by the uniaxial stress are considered. Then the Van der Waals - type theory of phenomena near the metamagnetic critical point is developed and some physical properties are discussed.

The phenomenon of the reentrant superconducting state is the subject of Chapter III. It is shown that the recreation of superconductivity is caused by the sharp increase in the magnetic susceptibility [7] in b direction near the metamagnetic transition. The Conclusion contains the summary of the results.

II. METAMAGNETIC TRANSITION IN URhGe

As in the previous publications (ex [2]) I shall use $x,y,z$ as the coordinates pinned to the corresponding crystallographic directions $a,b,c$. The Landau free energy of an orthorhombic ferromagnet in magnetic field $\mathbf{H}(r) = H_y \hat{y}$ is

$$F = \alpha_z M_z^2 + \beta_z M_z^4 + \delta_z M_z^6 + \alpha_y M_y^2 + \beta_y M_y^4 + \delta_y M_y^6 + \beta_{yz} M_z^2 M_y^2 - H_y M_y.$$  \hspace{1cm} (2)

Here

$$\alpha_z = \alpha_{z0}(T - T_c^0), \hspace{0.5cm} \alpha_y > 0,$$  \hspace{1cm} (3)

and I bear in mind the terms of the sixth order in powers of $M_z, M_y$ with the coefficients $\delta_z > 0, \delta_y > 0$ and also the fact that in the absence of a field in x-direction the magnetisation along hard x-direction $M_x = 0$.
A. Transition ferro-para

Let us remind first the treatment developed in Ref.5 undertaken in the assumption $\beta_y > 0$. Then in constant magnetic field $\mathbf{H} = H_y \hat{y}$ the equilibrium magnetisation projection along the $y$ direction

$$M_y \approx \frac{H_y}{2(\alpha_y + \beta_y z M_z^2)}$$

(4)
is obtained by minimisation of free energy \[2\] in respect of $M_y$ neglecting the higher order terms. Substituting this expression back to \[2\] we obtain

$$F = \alpha_z M_z^2 + \beta_z M_z^4 + \delta_z M_z^6 - \frac{1}{4} \frac{H_y^2}{\alpha_y + \beta_y z M_z^2},$$

(5)

that gives after expansion of the denominator in the last term,

$$F = -\frac{H_y^2}{4 \alpha_y} + \tilde{\alpha}_z M_z^2 + \tilde{\beta}_z M_z^4 + \tilde{\delta}_z M_z^6 + \ldots,$$

(6)

where

$$\tilde{\alpha}_z = \alpha_{z0}(T - T_{c0}) + \frac{\beta_y \gamma H_y^2}{4 \alpha_y},$$

(7)

$$\tilde{\beta}_z = \beta_z - \frac{\beta_y \gamma H_y^2}{2 \alpha_y},$$

(8)

$$\tilde{\delta}_z = \delta_z + \frac{\beta_y \gamma H_y^2}{4 \alpha_y}.$$  

(9)

Thus, in a magnetic field perpendicular to the direction of spontaneous magnetization the Curie temperature decreases as

$$T_c = T_c(H_y) = T_{c0} - \frac{\beta_y \gamma H_y^2}{4 \alpha_y \alpha_{z0}}.$$  

(10)

The coefficient $\tilde{\beta}_z$ also decreases with $H_y$ and reaches zero at

$$H_y = H^* = \frac{2 \alpha_y^{3/2} \beta_z^{1/2}}{\beta_y \gamma}.$$  

(11)

At this field under fulfilment the condition,

$$\frac{\alpha_{z0} \beta_y \gamma}{\alpha_y \beta_z} > 1$$

(12)

the Curie temperature [10] is still positive and the phase transition from the ferromagnetic to the paramagnetic state becomes the transition of the first order (Fig 1a). The point $(H^*, T_c(H^*))$ on the line paramagneto-ferromagnet phase transition is a tricritical point. The qualitative field dependencies of the normalised Curie temperature $t_c(H_y) = \frac{T_c(H_y)}{T_{c0}}$ and $b(H_y) = \frac{\tilde{\beta}_z}{\tilde{\delta}_z}$ are plotted in Fig 1a.

On the line of the first order phase transition from the ferromagnet to the paramagnet state the $M_z$ component of magnetisation drops from $M_z^\ast$ to zero [2]. The $M_y$ component jumps from $M_y \approx \frac{H_y}{2(\alpha_y + \beta_y z M_z^2)}$ to $M_y \approx \frac{H_y}{2 \alpha_y}$. Then at fields $H_y > H^*$

$$M_y \approx \frac{H_y}{2 \alpha_y}$$

(13)

proportional to the external field. This contradicts experimental observations [3] [6] [7] which demonstrate the presence of a "spontaneous" part of magnetization in the field above the transition in accordance with eq. (1).

B. Transition ferro-polarised para

The part of free energy depending from $M_y$

$$F_y = \alpha_y M_y^2 + \beta_y M_y^4 + \delta_y M_y^6 + \beta_y z M_z^2 M_y^2 - H_y M_y,$$

(14)
is valid also far from the transition to the ferromagnetic state in the temperature region where $M_z$ is not small. The important fact obtained with the local spin-density approximation calculations [10] is that the coefficient $\beta_y < 0$.

In frame of isotropic Fermi liquid model the negativity of the fourth order term in the expansion of the free energy in power of magnetic moment is usually ascribed to the peculiar behaviour of the electron density of states (see the review [11] and references therein). In the orthorhombic URhGe this specific property of energy anisotropy in $(b, c)$ plane is hardly related with a naive itinerant electron picture. In the paper [12] there was evaluated the magnetic dipole moment $\langle T_z \rangle$ "related to the anisotropy of the local magnetic field produced by the spin when the valence cloud is distorted either by spin-orbit and/or crystal field interaction. The high value of the $\langle T_z \rangle/\langle S_z \rangle$ ratio found in URhGe could be expected since this compound presents a high magnetocrystalline anisotropy, so it should not be too close to the itinerant limit, where $\langle T_z \rangle$ is expected to be strongly suppressed."

The $M_y$ component of magnetisation is determined by the equation

$$2 \tilde{\alpha}_y M_y + 4 \beta_y M_y^3 + 9 \delta_y M_y + H_y = 0,$$

(15)

where

$$\tilde{\alpha}_y = \alpha_y + \beta_y z M_z^2.$$  

(16)

Taking into account the third order term we obtain

$$M_y \approx \frac{H_y}{2 \tilde{\alpha}_y} - \frac{\beta_y H_y^3}{2 \tilde{\alpha}_4^4}.$$  

(17)

The coefficient $\beta_y < 0$ and we see that the increase of magnetisation occurs faster than it was according to Eq. (1).

The shape of $M_y(H_y)$ depends from the temperature and pressure dependence of coefficients $\alpha_y, \beta_y, \delta_y$. At
temperature decrease the field dependence of $M_y$ can transfer from the monotonous growth taking place at $\beta_y^2 < \frac{5}{3} \tilde{\alpha_y} \delta_y$ to the S-shape dependence. This transformation occurs at some temperature $T_{cr}$ such that in the dependence $H_y(M_y)$ appears an inflection point. It is determined by the equations

$$ \frac{\partial H_y}{\partial M_y} = 0, \quad \frac{\partial^2 H_y}{\partial M_y^2} = 0 \quad (18) $$

having common solution

$$ M_{cr}^2 = -\frac{\beta_y}{5\delta_y} \quad (19) $$

at $\beta_y^2 = \frac{5}{3} \tilde{\alpha_y} \delta_y$. The corresponding critical field is

$$ H_{cr} = H_y(M_{cr}) = \frac{16}{5\sqrt{3}} \frac{\tilde{\alpha}_y^{3/2}}{|\beta_y|^{1/2}} \quad (20) $$

At $T < T_{cr}$ the inequality

$$ \beta_y^2 > \frac{5}{3} \tilde{\alpha_y} \delta_y \quad (21) $$

is realised and the equation $\frac{\partial H_y}{\partial M_y} = 0$ acquires two real solutions, hence, the field dependence of $M_y$ acquires the S-shape plotted at Fig.1b. Equilibrium transition from the lower to the upper part of the curve $M_y(H_y)$ corresponds to a vertical line connecting the points $M_1$ and $M_2$ defined by the Maxwell rule $\int_{y_1}^{y_2} M(H) dH = 0$. The integration is performed along the curve $M_y(H_y)$. The $M_y$ component of magnetisation jumps from $M_1$ to $M_2$ (see Fig1b). At the same time the Curie temperature drops from

$$ T_{c}(H_{cr}) = T_{c0} - \frac{\beta_y M_{cr}^2}{\alpha_{z0}} \quad (22) $$

to zero or even to negative value and the ferromagnetic order along $z$-direction disappears. Here we assume that the Curie temperature given by Eq. $22$ exceeds the critical temperature $T_{cr}$.

The described jump-like transition is realised in the cylindrical specimen in the magnetic field parallel to the cylinder axis. In specimens of the arbitrary shape with demagnetisation factor $n$ the transition occurs in some field interval where the specimen is filled by the domains with different magnetisation.

Thus, at $T < T_{cr}$ and $H_y = H_{cr}$ we have the phase transition of the first order from the ferromagnetic state with spontaneous magnetisation along $z$-direction to the polarised paramagnetic state with induced magnetisation along $y$-direction.

When the critical field $H_{cr}$ is smaller than the critical field of transition ferro-para $H^*$, the ferro-para transition, discussed in the previous section does not occurs.

At $T < T_{cr}$ in fields $H_y$ exceeding $H_{cr}$ the field dependence of $M_y$ component of magnetisation behaves in accordance with Eq.(1) corresponding to the experimental observations.

C. Uniaxial stress effects

It is known that a hydrostatic pressure applied to URhGe crystals stimulate ferromagnetism and at the same time suppresses the superconducting state \[10\] and the reentrant superconducting state \[13\] as well. The later is also shifted to a bit higher field interval. On the contrary, the uniaxial stress along $b$-direction suppresses the ferromagnetism decreasing the Curie temperature and stimulates the superconducting state so strongly that it leads to the coalescence of the superconducting and reentrant superconducting regions in the $(H_y, T)$ phase diagram \[15\]. The phenomenological description of these phenomena was undertaken in the paper \[16\]. There was shown that both coefficients $\alpha_z$ and $\alpha_y$ in the Landau free energy Eq.(2) acquire the linear uniaxial pressure dependence

$$ \alpha_z(P_y) = \alpha_{z0}(T - T_{c0}) + A_z P_y, \quad (23) $$

$$ \alpha_y(P_y) = \alpha_y - |A_y|P_y \quad (24) $$

corresponding to the moderate uniaxial pressure suppression of the Curie temperature

$$ T_{c}(P_y) = T_{c0} - \frac{A_z P_y}{\alpha_{z0}}, \quad (25) $$

reported in \[15\] in the absence of an external field. However, under the external field along $y$-direction the drop of the Curie temperature Eq. \[16\] is accelerated

$$ T_{c}(H_y, P_y)) \approx T_{c0} - \frac{A_z P_y}{\alpha_{z0}} - \frac{\beta_y H_y^2}{4(\alpha_y(P_y))^2 \alpha_{z0}} \quad (26) $$

in correspondence with the observed behaviour. Moreover, the uniaxial stress causes strong decrease of the critical field Eq. \[20\]

$$ H_{cr} = H_y(M_{cr}) = \frac{16}{5\sqrt{3}} \frac{(\tilde{\alpha_y}(P_y))^{3/2}}{|\beta_y|^{1/2}} \quad (27) $$

D. Van der Waals-type theory near the critical point

The critical end point temperature for the first order transition in URhGe is $T_{cr} = 4$ K and the critical field is $H_{cr} = 12T$. Let us expand the function $H_y(M_y)$ at temperature slightly deviating from critical temperature $T = T_{cr} + t$ and the magnetisation near its critical value $M_y = M_{cr} + m$. We have

$$ h = H_y - H_{cr} = bt + \left[ \frac{\partial H_y}{\partial M_y} \right]_{t=0} + 2at \right] m $$

$$ + \frac{1}{2} \left[ \frac{\partial^2 H_y}{\partial M_y^2} \right]_{t=0} m^2 + \frac{1}{6} \left[ \frac{\partial^3 H_y}{\partial M_y^3} \right]_{t=0} m^3, \quad (28) $$

Here, we neglected by the temperature dependence of the second and the third order terms. Taking into account
that \( \frac{\partial H_y}{\partial M_y} |_{t=0} = \frac{\partial^2 H_y}{\partial M_y^2} |_{t=0} = 0 \) we obtain

\[
h = bt + 2atm + 4Bm^3, \tag{29}
\]

which obviously corresponds to the expansion of pressure

\[ p = P - P_{cr} \]

in powers of density \( \eta = n - n_{cr} \) near the Van der Waals critical point \(17\).

At \( t < 0 \) according to the Maxwell rule the magnetisation

densities of two phases in equilibrium with each other are:

\[
m_2 = -m_1 = \sqrt{-at \over 2B}. \tag{30}
\]

The line of phase equilibrium between the two phases below and above the transition is given by the equation \( h = bt \), \( t < 0 \). Hence, according to the Clausius-Clapeyron relation

\[
b = {dh \over dt} = \frac{s_2 - s_1}{m_1 - m_2} = \frac{T_{cr}q}{m_2 - m_1}, \tag{31}
\]

where \( q \) is the transition latent heat. Near the critical point the coefficient \( b \) is positive and finite, \( q \propto -1 \). At \( T \to 0 \) the latent heat tends to zero, and, according to Eq. (31) \( b \to 0 \). In whole temperature interval \( (0, T_{cr}) \) the line of the phase equilibrium is almost vertical.

### 1. Specific heat

The specific heat at fixed external field (see \(17\)) is

\[
C_h \propto T \left( \frac{\partial h}{\partial T} \right)_m \frac{(\partial h}{\partial m} \right) \tag{32}
\]

Then, using Eq. (29) we obtain

\[
C_h \propto \frac{b^2 T}{2at + 12Bm^2}. \tag{33}
\]

Thus, the contribution to heat capacity according to the equation of state (33) near the critical point grows so long \( m^2 \) decreases till to \( m_1^2 \) and then begins to fall when \( m^2 \) increases starting from \( m_2^2 \) (see Fig. 1.c). This is the contribution to the specific heat of the whole system and cannot be directly attributed to the specific heat of itinerant electrons proportional to the electron effective mass.

The low temperature behaviour of the URhGe specific heat in magnetic field has not been established by a direct measurement but was derived \(6\) by the application of the Maxwell relation \( \left( \frac{\partial S}{\partial H_y} \right)_T = \left( \frac{\partial M_y}{\partial T} \right) \) from the temperature dependence of the magnetisation \( M_y(T, H_y) \) in the fixed field. The changes of the ratio \( C(T)/T \) have been ascribed to the electron effective mass dependence from magnetic field \(6\) \(15\). This was done in the assumption that URhGe is a weak itinerant ferromagnet, in other words, all the low temperature degrees of freedom in this material belong to the itinerant electron subsystem. As we already mentioned above, the strong magnetic anisotropy of this material \(12\) points on the importance of the magnetic degrees of freedom localised on the uranium ions and related with crystal field levels. See also the papers \(2\) \(19\) \(20\).

### 2. Resistivity

The magnetic field dependence of effective mass was also found \(15\) \(21\) by the application of the Kadowaki-Woods relation \( A(H_y) \propto (m^*)^2 \) where coefficient \( A \) is a pre-factor in the low-temperature dependence of resistivity \( \rho = \rho_0 + AT^2 \).

The \( A(H_y) \) behaviour is determined by the processes of inelastic electron-electron scattering which in the multi-band metals interfere with scattering on impurities (see \(22\) \(26\)) and on magnetic excitations with field dependent spectrum. The non-spherical shape of the Fermi surface sheets and the screening of ee-coulomb interaction can introduce deviations from \( T^2 \) resistivity dependence. So, the physical meaning of the coefficient \( A(H_y) \) behaviour is not so transparent and its relationship with the electron effective mass is questionable.

One can also note, that the temperature fit of the experimental data was done in very narrow temperature interval and the \( T^2 \) temperature dependence claimed in \(21\) seems somewhat unreliable. Compare with the results reported in \(13\) \(27\).

### 3. Correlation function

The correlation function of fluctuations of the magnetisation density \( m \) near the critical point at \( t < 0 \) behaves similar to the specific heat \(17\)

\[
\varphi(k) = \frac{T}{2(at + 6Bm^2 + \gamma_{ij}k_i k_j)}. \tag{34}
\]

This is in correspondence with a marked increase of the NMR relaxation rate \(1/T_2\) with field \( H_y \) increasing toward \( 12 T \) reported in \(28\) \(29\).

### III. REENTRANT SUPERCONDUCTIVITY

The superconducting state in URhGe is completely suppressed by the magnetic field \( H_{c2}(T = 0) \approx 2 T \) in \( y \)-direction due to the orbital depairing effect. Then superconductivity recovers in the field interval \( 9 - 13 T \) around the critical field \( H_{c2} \approx 12 T \) of the transition of the first order from the ferromagnetic state with spontaneous magnetization along \( z \)-direction to the state with induced magnetization along \( y \)-direction. Evidently such type behaviour is possible if the magnetic field somehow stimulates the pairing interaction surmounting the orbital depairing effect.
In numerous publications starting from the paper by A. Miyake et al. [18] the treatment of this phenomenon was related with the assumption of an enhancement of electron effective mass \( m^* = m(1 + \lambda) \) leading to the enhancement of pairing interaction and consequently of the temperature of transition to superconducting state according to the Mc-Millan-like formula [30]

\[
T_{sc} \approx \epsilon \exp\left( -\frac{1 + \lambda}{\lambda} \right)
\]

derived in the paper [31] for the superconducting state with \( p \)-pairing in an itinerant isotropic ferromagnetic metal. Similar to the liquid He-3 in this model there are two independent phase transition to the superconducting state in the subsystems with spin-up and spin-down electrons. The constant \( \lambda \) determined by the Hubbard four-fermion interaction [31] increases as we approach but not too much close to ferromagnetic instability. In frame of this model the question of why the growth of the magnetic field \( H_s \) approaches the ferromagnetic transition remains unanswered.

The following development of this type approach has been undertaken by A. Chubukov and co-authors [33]. The reentrant superconductivity and mass enhancement have been associated with the Lifshitz transition [34] which occurs in one of the bands in a finite magnetic field stimulating the splitting of spin-up and spin-down bands. There was established modest enhancement of the transition critical temperature in the field about 10 T. Thus, the model can claim to the qualitative explanation of the superconducting state reentrance. However, it should be noted that the measured [34] quasiparticle mass in the corresponding band does not increase but decreases and remains finite, implying that the Fermi velocity vanishes due to the collapse of the Fermi wave vector. The cross-section of the Fermi surface of this band corresponds to 7% of the Brillouin zone area. Thus, the reentrance of superconductivity is hardly could be associated with the observed Lifshitz transition.

The models [31,33] describe the physics of pure itinerant electron subsystem. Such a treatment is approved in application to the \(^3\)He Fermi-liquid. The measurements by x-ray magnetic circular dichroism [12] point to the local nature of the URhGe ferromagnetism. Namely, the comparison of the total uranium moment \( \mu_{tot} \) to the total magnetisation \( M_{tot} \) at different magnitude and direction of magnetic field indicates that the uranium ions dominate the magnetism of URhGe. The same is true also in the parent compound UCoGe [35]. So, the magnetic susceptibility \( \chi_{ij}(q, \omega) \) is mostly determined by the localised moments subsystem. Hence, an approach based on the exchange interaction between conduction electrons and magnetic moments localised on uranium atoms seems more appropriate.

Using the standard functional-integral representation of the partition function of the system (see fi [36]), we obtain the following term in the fermionic action describing an effective two-particle interaction between electrons:

\[
S_{int} = -\frac{1}{2} \int dx dx' S(x) D_{ij}(x - x') S(x'),
\]

where \( S(r) = \psi^\dagger(r) \sigma_{\alpha\beta} \psi_{\beta}(r) \) is the operator of the electron spin density, \( x = (r, \tau) \) is a shorthand notation for the coordinates in real space and the Matsubara time, \( \int dx(\ldots) = \int dt \int_0^\beta d\tau(\ldots), \) \( I \) is the coupling constants of electrons with spin fluctuations, \( D_{ij}(x - x') \) is the spin-fluctuation propagator expressed in terms of the dynamical spin susceptibility \( \chi_{ij}(q, \omega) \).

Making use the interaction [36] one can calculate the electron self energy and find the dependence of the electron effective mass from magnetic field as well the temperature of transition to the superconducting state with triplet pairing. In application to UCoGe in magnetic field parallel to direction of spontaneous magnetisation this program has been accomplished in the paper [37]. In the simplified case of a single-band (say spin-up) equal-spin pairing superconducting state the critical temperature without including the orbital effect of the field is

\[
T_{sc} = \epsilon \exp\left( -\frac{1 + \lambda}{\langle N_0(k)\chi_{zz}(H)\rangle} \right),
\]

where, as in the McMillan formula, \( 1 + \lambda \) corresponds to the effective mass renormalisation, whereas the pairing amplitude expressed through the odd in momentum part of static susceptibility

\[
\chi_{zz}^u = \frac{1}{2} \left[ \chi_{zz}(k - k') - \chi_{zz}(k + k') \right],
\]

which is the main source of the critical temperature dependence from magnetic field. Here,

\[
\chi_{zz}(k) = \frac{1}{\chi^2 + 2\gamma_{ij} k_i k_j},
\]

and \( \chi_z = \chi_z(H_z) \) is the \( z \)-component of susceptibility in the finite field \( H_z \). Its magnitude at \( H_z \to 0 \) we will denote \( \chi_{z0} \). The angular brackets denote averaging over the Fermi surface and \( N_0(k) \) is the angular dependent density of electronic states on the Fermi surface,

\[
\langle N_0(k)\chi_{zz}(H_z) \rangle \approx \frac{2\langle N_0(k)k_z^2\rangle^2 k_z^2 \chi_z}{(2\chi_z)^{-1} + 4\xi_{zz}^2 k_F^2}.
\]

The denominator in the exponent of Eq. (37) can be expressed through its value at \( H_z \to 0 \)

\[
\frac{\langle N_0(k)\chi_{zz}(H_z) \rangle}{\langle N_0(k)\chi_{zz}(H_z \to 0) \rangle} = \frac{\chi_z}{\chi_{z0}} \frac{1 + 2(\xi_m k_F)^2}{\chi_{z0} \frac{\xi_m}{\chi_{z0}} + 2(\xi_m k_F)^2}.
\]

Here the product \( \gamma_{zz} k_F^2 \chi_{z0} = (\xi_m k_F)^2 \) is expressed through the magnetic coherence length \( \xi_m \) which near the zero temperature is of the order several interatomic distances.
In assumption $(\xi_m k_F)^2 \gg 1$ one can rewrite the Eq.(40) as
\[
\langle N_0(k)\chi_{zz}^u(H_y) \rangle \approx \frac{\chi_z(H_y)}{\chi_{yz}} \langle N_0(k)\chi_{zz}^u(H_z \to 0) \rangle. \quad (41)
\]
This very rough estimation presents the qualitative dependence of exponent in equation (37) from magnetic field. The longitudinal susceptibility drops with the augmentation of magnetic field parallel to the spontaneous magnetisation leading to the suppression of the temperature of transition to the superconducting state. The same mechanism works in the opposite sense in the field perpendicular to spontaneous magnetisation.

In field perpendicular to the spontaneous magnetisation the similar approach applied to the simplified single band model in weak coupling approximation yields (see Eq.(169) in the review [2]) the critical temperature
\[
T_{sc} \approx \frac{1}{\epsilon} \exp \left( -\frac{1}{\chi_{zz}^u(H_y) \cos^2 \varphi + \chi_{yy}^u(H_y) \sin^2 \varphi \varphi^2} \right), \quad (42)
\]
where $\tan \varphi = H_y/h$ and $h$ is the exchange field acting on the electron spins. This is the critical temperature of transition to the superconducting state without including the orbital effect.

The orbital effect suppresses the superconducting state and near the upper critical field at zero temperature
\[
H_{c2y}(T = 0) = H_y = cT_{sc}^2 \quad (43)
\]
the actual critical temperature is
\[
T_{sc}^{orb} = a\sqrt{H_y - H_y}, \quad (44)
\]
where $a\sqrt{c}$ is the numerical constant of the order of unity. This is the usual square root BCS dependence of the critical temperature from magnetic field in low temperature - high field region such that $T_{sc}^{orb}(H_y = H_y) = 0$. However, in the present case the magnitude $H_y$ itself is a function of the external field $H_y$. Let us look on its behaviour.

Similar to Eq.(41) we get
\[
\langle N_0(k)\chi_{zz}^u(H_y) \rangle \cos^2 \varphi + \langle N_0(k)\chi_{yy}^u(H_y) \rangle \sin^2 \varphi \approx \frac{\chi_z(H_y)}{\chi_{yz}} \langle N_0(k)\chi_{zz}^u(H_y \to 0) \rangle \cos^2 \varphi
\]
\[
+ \frac{\chi_y(H_y)}{\chi_{yy}} \langle N_0(k)\chi_{yy}^u(H_y \to 0) \rangle \sin^2 \varphi \]. \quad (45)
\]
Here $\chi_z(H_y)$ and $\chi_y(H_y)$ are the z and y components of susceptibility in finite field $H_y$ and $\chi_{yz}$ and $\chi_{yy}$ are the corresponding susceptibilities at $H_y \to 0$. Unlike to the Eq.(41) the field dependence of the Eq.(45) is not so visible. One can note, however, the different field dependence of two summands in the Eq.(45).

(i) The susceptibility along z direction $\chi_z(H_y)$ increases with magnetic field $H_y$ following to the decreasing of the Curie temperature according to Eq.(10). The growth of susceptibility along z direction at the approaching the field $H_y$ to $H_{sc}$ is confirmed by the field dependence of the NMR scattering rate $1/T$ reported in [28, 29]. At the same time, the increase of $\chi_y(H_y)$ is limited by the decrease of $\cos^2 \varphi$. We do not know how fast it is because the magnitude of the exchange field is not known.

(ii) As the field approaches to $H_{cr}$ the low temperature susceptibility $\chi_y(H_y)$ has a high delta-function-like peak [41] with magnitude more than 10 times greater than it is at $H_y \to 0$ (see Fig2a). The factor $\sin^2 \varphi$ is also increased. This indicates that in URhGe the more important is the second term connected with the metamagnetic transition.

Thus, in vicinity of metamagnetic transition one can expect the increase of the critical temperature estimated without including the orbital effect according to Eq.(12). The radicand in the equation (44) after being negative in some field interval acquires the positive value as the field approaches to $H_{cr}$. The critical temperature Eq.(44) reaches maximum in vicinity of metamagnetic transition.

Similar arguments in favour of stimulation superconductivity near the metamagnetic transition in field parallel to b axis can be applied to the discovered recently other superconducting compound UTe$_2$ [38, 41] isostructural with URhGe. However, in view of many particular properties of this material we leave this subject for future studies.

In the parent compound UCoGe the metamagnetic transition is absent (at least at $H_y < 40T$) [38]. Hence, in this material the unusual temperature dependence of the upper critical field parallel to b axis is probably mostly determined by the first term in the Eq.(45).

Near $H_y = H_{cr}$ at temperatures $T < T_{cr}$ the NMR spectrum is composed of two components indicating that the transition is of the first order accompanied by the phase separation [28]. Thus, in almost whole interval near $H_{cr}$ the superconductivity is developed in mixture of ferromagnetic state with polarisation along z direction and the field polarised state with polarisation along y-direction.

**IV. CONCLUSION**

We have demonstrated that in the orthorhombic ferromagnet URhGe the ferromagnetic ordering along c-axis suppressed in process of increase of magnetization in the perpendicular b-direction induced by the external magnetic field. This process is accelerated by the tendency to the metamagnetic transition which occurs at $H_y = H_{cr} = 12.6$ T. The transition of the first order is accompanied by the suppression of the ferromagnetic state with polarization along c-axis and the arising of magnetic state polarized along b-axis. The line of first order phase transition is finished at the critical end point with
The uniaxial stress along b-axis causing moderate suppression of the Curie temperature in the absence of magnetic field accelerates the Curie temperature drop in finite magnetic field $H_p$ and quite effectively decreases the critical field of metamagnetic transition. As result, the superconducting state recovers itself in much smaller field and can even merged with superconducting state in the small fields region.

The superconducting state suppressed in field $H_p \approx 2T$ is recovered in fields interval $(9 - 13) \, T$ in the vicinity of the critical field. This phenomenon is related to the strong increase of the pairing interaction caused mostly by the strong augmentation of the magnetic susceptibility along b-direction in vicinity of the metamagnetic transition.

[1] D.Aoki, K.Ishida and J.Flouquet, J. Phys. Soc. Jpn. 88, 022001 (2019).
[2] V.P. Mineev, Usp. Fiz. Nauk 187, 129 (2017) [Phys.-Usp. 60, 1 (1958)].
[3] Levy F, Sheikin I, Grenier B, Huxley A D Science 309 1343 (2005).
[4] F. Hardy, A.D. Huxley, Phys. Rev. Lett. 94, 247006 (2005).
[5] V.P. Mineev, Phys. Rev. B 91 045107 (2015).
[6] A. Gourgout, A. Pourret, G. Knebel, D. Aoki, G. Seyfarth, and J. Flouquet, Phys. Rev. B 104, 224408 (2016).
[7] T.D. Matsuda, H. Sugawara, H. Sato, H. Ohkuni, R. Settai, Y. Onuki, E. Yamamoto, H. Haga, A.V. Andreev, V. Sechovsky, L. Havela, H. Ikeda, K. Miyake, Journ. Magn. Magn. Mat. 177-181 (2005).
[8] D.Aoki, T.Combier, V. Taufour, T.D. Matsuda, G.Knebel, H.Kotegawa, and J.Flouquet, J.Phys. Soc. Jpn. 80, 094711 (2011).
[9] A.B. Shick, Phys.Rev. B 65, 180509(R) (2002).
[10] R.Z. Levitin, A.S. Markosyan, Usp. Fiz. Nauk 155, 623 (1988) [Phys.-Usp. 31, 730 (1988)].
[11] F. Hardy, J. Huxley, J. Flouquet, B. Sulce, G. Knebel, D. Braithwaite, D. Aoki, M. Uhlarz, C. Pfeiffer, Physica B 359-361, 1111 (2005).
[12] M.Taupin, J.P. Sanchez, J.-P. Brison, D. Aoki, G. Lapertot, M.Nardone, A.Zitouni, D. Braithwaite, E. Yamamoto, Y. Onuki, K.M. Koh, and E.E. Haller, J. Phys. Soc. Jpn. 84, 054710 (2015).
[13] Y. Tokunaga, D. Aoki, H. Mayaffre, S. Krämer, M.-H. Julien, C. Berthier, M. Horvatić, H. Sakai, S. Kambe, and S. Araki, Phys. Rev. Lett. 114, 216401 (2015).
[14] W.L. McMillan, Phys.Rev. 167, 331 (1968).
[15] D. Fay and J. Appel: Phys. Rev. B 22 (1980) 3173.
[16] V.P. Mineev, Annals of Physics (NY), to be published (2020).
[17] W. F. Brinkman and S. Engelsberg, Phys. Rev. 169, 417 (1968).
[18] Yu.Sherkunov, A.V. Chubukov, and J.J. Betouras, Phys.Rev.Lett. 121, 097001 (2018).
[19] E.A. Yelland, J.M. Barraclough, W. Wang, K. V. Kamenev, and A.D. Huxley, Nat. Phys. 7, 890 (2011).
[20] M. Taupin, J.P. Sanchez, J.-P. Brison, D. Aoki, G. Lapertot, F. Wilhelm, and A. Rogalev, Phys.Rev. 92, 035124 (2015).
[21] N. Karchev, Phys.Rev.B 67, 054416 (2003).
[22] W. Knafo, T.D. Matsuda, D. Aoki, F. Hardy, G.W. Scheerer, G. Ballon, M. Nardone, A. Zitouni, C. Meingast and J.Flouquet, Phys. Rev. B 86, 184416 (2012).
[23] S. Ran, I-Lin Liu, YunSuk Eo, D.J. Campbell, P.M. Neves, W.T. Fuhrman, S.R. Saha, C. Eckberg, H. Kim, D. Graf, F. Balakirev, J. Singleton, J. Paglione and N. Butch, Nature Physics 15, 1250 (2019).
[24] G. Knebel, W. Knafo, A. Pourret, Qun Niu, M. Valisaka, D. Braithwaite, G. Lapertot, M. Nardone, A. Zitouni, S. Mishra, I. Sheikin, G. Seyfarth, J.-P. Brison, D. Aoki, J. Flouquet, J. Phys. Soc. Jpn. 88, 063707 (2019).
[25] Q. Niu, G. Knebel, D. Braithwaite, D. Aoki, G. Lapertot, M. Valisaka, G. Seyfarth, W. Knafo, T. Helm, J.-P. Brison, J. Flouquet, and A. Pourret, arXiv:2003.08986 [cond-mat] (2020).
FIG. 1: (Color online) a) Schematic behaviour of the normalised Curie temperature $t_c(H_y) = \frac{T_c(H_y)}{T_{c0}}$ and coefficient $b(H_y) = \frac{\hat{\beta}_z}{\tilde{\beta}_z}$. b) Schematic dependence $M_y(H_y)$ at $T < T_{cr}$ and $H_{cr} < H^*$. c) Schematic behaviour $C_h/T$. 
FIG. 2: (Color online) a) Schematic field dependence of $y$ component susceptibility $\chi_y(H_y)$ at temperature $T \to 0$ according to [7]. b) Schematic field dependence of superconducting critical temperature $T_{sc}^{orb}$. 