Large-Scale Structure and Dark Matter Problem

Jaan Einasto

Tartu Observatory, EE-61602 Tõravere, Estonia

Abstract. I review the observational data most relevant for large scale structure. These data determine the system of cosmological parameters: the Hubble parameter, densities of various populations of the Universe, parameters characterizing the power spectrum of matter, including the biasing parameter of galaxies relative to matter. Recent data suggest that the overall matter/energy density is approximately equal to the critical density, and most ($0.6 - 0.7$) of the density is in the form of cosmological term or “dark (vacuum) energy”. The density of the matter is $0.3 - 0.4 \ (\text{including hot and cold dark matter and luminous matter})$, the upper limit of the density of the hot dark matter is 0.05, all in units of the critical cosmological density.

1 Introduction

Recent results from the supernova cosmology project [27], [29] and measurements of the angular spectrum of the cosmic microwave background (CMB) radiation [11], [19] have triggered a number of efforts to determine a concordant system of cosmological parameters. In this talk I shall use recent observational data to discuss values of main cosmological parameters. In addition to data on the CMB angular spectrum and supernova cosmology project I shall use data based on the large-scale distribution of galaxies and clusters of galaxies – the power spectrum, the cluster mass function etc. In this analysis I use the following assumptions: 1) the main constituents of the Universe are baryonic matter, cold dark matter (CDM) with some mixture of hot dark matter (HDM), and the dark (vacuum) energy; 2) power spectra of galaxies and CMB radiation are determined by the initial post-inflational power spectrum and by physical processes during the radiation-dominated era. These processes depend on cosmological parameters (properties of various components of the matter), and on geometrical properties of the Universe. In this analysis I try to find the possible range of cosmological parameters and to show how these are affected by the various types of data.

2 Observed quantities and functions

The Hubble parameter, $H_0 = 100 \ h \ \text{km s}^{-1} \ \text{Mpc}^{-1}$, is the observable quantity that can be estimated directly. There exist several methods to its estimation through the ladder of various distance estimators from star clusters to cepheids in nearby galaxies, through the light curves of medium-distant supernovas, and using several physical effects (gravitational lensing, SZ-effect). Summaries of
recent determinations are given by \[26\] and \[31\]). I shall use here a value \(h = 0.65 \pm 0.07\).

The baryon density, \(\Omega_b\), can be determined most accurately through observations of the deuterium, helium and lithium abundances in combination with the nucleosynthesis constrains. The best available result is \(\Omega_b h^2 = 0.019 \pm 0.002\) \[8\], \[7\].

The total density (including vacuum energy), \(\Omega_{\text{tot}} = \Omega_m + \Omega_v\), determines the position of the first Doppler peak of the angular spectrum of CMB temperature fluctuations; here \(\Omega_m\) and \(\Omega_v\) are densities of the matter and the vacuum energy, respectively. Recent observations show that the maximum of the first Doppler peak lies at \(l \approx 200\) \[11\], \[19\], \[32\]. This indicates that \(\Omega_{\text{tot}} \approx 1\). Since this is the theoretically preferred value, I assume in the following that \(\Omega_{\text{tot}} = 1\).

There exist a number of methods to estimate the total density of matter (without vacuum energy), \(\Omega_m = \Omega_b + \Omega_c + \Omega_n\), where \(\Omega_b\), \(\Omega_c\), and \(\Omega_n\) are densities of the baryonic matter, the cold dark matter (CDM), and the hot dark matter (HDM), respectively. A direct method is based on the distant supernova project, which yields (for a spatially flat universe) \(\Omega_m = 0.28 \pm 0.05\) \[27\], \[29\], \[17\]. Another method is based on X-ray data on clusters of galaxies, which gives the fraction of gas in clusters, \(f_{\text{gas}} = \Omega_b / \Omega_m\). If compared to the density of the baryonic matter one gets the estimate of the total density, \(\Omega_m = 0.31 \pm 0.05(h/0.65)^{-1/3}\) \[24\], \[20\]. A third method is based on the geometry of the Universe. Observations show the presence of a dominant scale, \(l_0 = 130 \pm 10 h^{-1}\) Mpc, in the distribution of high-density regions \[3\], \[12\], \[15\]. A similar phenomenon is observed in the distribution of Lyman-break galaxies \[6\] at high redshift, \(z \approx 3\). We can assume that this scale is primordial and co-moves with the expansion; in other words – it can be used as a standard ruler. The relation between redshift difference and linear comoving separation depends on the density parameter of the Universe; for a spatially flat Universe one gets a density estimate \(\Omega_m = 0.4 \pm 0.1\). The same method was applied for the distribution of quasars by \[30\] with the result \(\Omega_m = 0.3 \pm 0.1\). Finally, the evolution of the cluster abundance with time also depends on the density parameter (see \[3\] for a review). This method yields an estimate \(\Omega_m = 0.4 \pm 0.1\) for the matter density. The formal weighted mean of these independent estimates is \(\Omega_m = 0.32 \pm 0.03\).

Cosmological parameters enter as arguments in a number of functions which can be determined from observations. These functions include the power spectrum of galaxies, the angular spectrum of temperature fluctuations of the CMB radiation, the cluster mass and velocity distribution. I accept the power spectrum of galaxies according to a summary by Einasto et al \[13\] with the addition of the recent determination of the cluster power spectrum by Miller & Batuski \[23\]. The amplitude of the power spectrum can be expressed through the \(\sigma_8\) parameter, which describes the rms density fluctuations within a sphere of radius 8 \(h^{-1}\) Mpc. This parameter was determined for the present epoch for galaxies, \((\sigma_8)_{\text{gal}} = 0.89 \pm 0.09\) \[13\]. For the CMB angular spectrum I use recent BOOMERANG and MAXIMA I measurements \[11\], \[19\]. For the cluster mass
distribution I use determinations by Bahcall & Cen [1] and Girardi et al. [16], see Figures 2, 3.

3 Relations between cosmological parameters and observed quantities

I consider the following cosmological parameters: the Hubble parameter, \( h \); densities of the main constituents of the Universe: the baryonic matter, \( \Omega_b \); CDM, \( \Omega_c \); HDM, \( \Omega_n \); and dark energy, \( \Omega_v \) (in units of the critical cosmological density); the index of the primordial power spectrum, \( n \); the parameter \( \sigma_8 \), characterizing the amplitude of the spectrum; and the biasing parameter of the clustered matter, \( b_c \). I use the definition of the biasing parameter through the ratio of the power spectrum of all matter to that of the clustered matter, associated with galaxies,

\[
P_c(k) = b_c^2(k)P_m(k).
\]

Here \( k \) is the wavenumber in units of \( h \) Mpc\(^{-1}\). In general, the biasing parameter is a function of wavenumber \( k \). I assume that in the linear regime of the structure evolution the biasing parameter is constant. Calculations show that this assumption is correct for wavenumbers smaller than \( k \approx 0.8 \) h Mpc\(^{-1}\), or scales larger than about \( 8 \) h\(^{-1}\) Mpc [14].

The power spectra of matter and the angular spectra of CMB were calculated for a set of cosmological parameters using the CMBFAST algorithm [33]; spectra are COBE normalized. The cluster abundance and mass distribution functions were calculated using the Press-Schechter algorithm [28] for the same set of cosmological parameters.

Power spectra of matter and galaxies are related through the biasing parameter. The power spectrum is proportional to the square of the amplitude of the density contrast. The clustered population associated with galaxies does not include the matter in voids. If we subtract from the density field of all matter an approximately constant density background of void matter to get the density field of the clustered matter, then amplitudes of absolute density fluctuations remain the same, but amplitudes of the density contrast increase by a factor which is equal to the ratio of mean densities of both fields, i.e. by the fraction of matter in the clustered population, \( F_c \). We obtain [14]

\[
b_c = 1/F_c.
\]

The possible range of the bias was determined by numerical simulations. During the dynamical evolution matter flows away from low-density regions and forms filaments and clusters of galaxies. This flow depends slightly on the density parameter of the model. The fraction of matter in the clustered population was found by counting particles with local density values exceeding a certain threshold (mean density). The present epoch of simulations was expressed through the \( \sigma_8 \) parameter. This quantity was calculated by integrating the power spectrum
of matter. It is related to the observed value of \((\sigma_8)_{gal}\) by the following equation:

\[
(\sigma_8)_{gal} = b_{gal}(\sigma_8)_m. \tag{3}
\]

Fig. 1. Upper left: the fraction of matter in the clustered population associated with galaxies as a function of \(\sigma_8\) for two LCDM models (dashed curves) and the relation between \(F_{gal}\) and \((\sigma_8)_m\) (bold solid line) defined by eq. (3). Upper right: the biasing parameter needed to bring the amplitude \(\sigma_8\) of the model into agreement with the observed \(\sigma_8\) for galaxies; for LCDM and MDM models with various matter density \(\Omega_m\) and HDM density, \(\Omega_n\). Dashed box shows the range of the bias parameter allowed by numerical simulations of the evacuation of voids. Lower left: power spectra of LCDM models with various \(\Omega_m\). Lower right: angular spectra of CMB for LCDM and MDM models for various \(\Omega_m\).

We assume that \(b_{gal} = b_c\). For two LCDM models with density parameter \(\Omega_m \approx 0.4\) the growth of \(F_{gal}\) is shown in Fig. 1 [14]. Using observed \((\sigma_8)_{gal}\) in combination with relation (3) (shown in upper left panel of Fig.1 by a bold line with error corridor), and the growth of \(F_{gal}\) with epoch (dashed curves), we get for the present epoch rms density fluctuations of the matter \((\sigma_8)_m = 0.64 \pm 0.06,\)
the fraction of matter in the clustered population, $F_{gal} = 0.70 \pm 0.09$, and the biasing parameter $b_{gal} = 1.4 \pm 0.1$.

4 Analysis

The CMBFAST algorithm yields for every set of cosmological parameters the $\sigma_8$ value for matter. From observations we know this parameter for galaxies, $(\sigma_8)_{gal}$. Using eqn. (3) we can calculate the biasing parameter $b_{gal}$, needed to bring the theoretical power spectrum of matter into agreement with the observed power spectrum of galaxies. This parameter must lie in the range allowed by numerical simulations of the evolution of structure. Results of calculations for a range of $\Omega_m$ are shown in Fig. 1 (upper right), using a Hubble parameter of $h = 0.65$, a baryon density of $\Omega_b = 0.05$, and HDM densities of $\Omega_n = 0.00$, 0.05, and 0.10. The biasing parameter range shown in the Figure is larger than expected from calculations described above; this range corresponds to the maximum allowed range of the fraction of matter in the clustered population expected from analytic estimates of the speed of void evacuation.

Power spectra for LCDM models ($\Omega_n = 0; 0.2 \leq \Omega_m \leq 0.5$) are shown in lower left panel of Fig. 1. We see that with increasing $\Omega_m$ the amplitude of the power spectrum on small scales (and respective $\sigma_8$ values) increases, so that the amplitude of the matter power spectrum exceeds for high $\Omega_m$ the amplitude of the galaxy power spectrum. This leads to bias parameter values $b \leq 1$. Such values are unlikely since the presence of matter in voids always increases the amplitude of the galaxy power spectrum relative to the matter spectrum. If other constraints demand a higher matter density value, then the amplitude of the matter power spectrum can be lowered by adding some amount of HDM. However, supernova and cluster X-ray data exclude density values higher than $\Omega_m \approx 0.4$; thus the possible amount of HDM is limited. Lower right panel of Fig. 1 shows the angular spectrum of temperature anisotropies of CMB for some density parameter values. We see that a low amplitude of the first Doppler peak of the CMB spectrum prefers a higher $\Omega_m$ value: for small density values the amplitude is too high. Thus, a certain compromise is needed to satisfy all data.

The cluster mass distribution for LCDM models with $0.2 \leq \Omega_m \leq 0.3$ is shown in the left panel of Fig. 2. We see that low-density models have too low abundance of clusters over the whole range of cluster masses. The best agreement with the observed cluster abundance is achieved by an LCDM model with $\Omega_m = 0.3$, in good agreement with direct data on matter density. In this Figure we show also the effect of a bump in the power spectrum, which is seen in the observed power spectrum of galaxies and clusters [13]. Several modifications of the inflation scenario predict the formation of a break or bump in the power spectrum. The influence of the break suggested by Lesgourgues, Polarski and Starobinsky [22] was studied in [18]. Another mechanism was suggested by Chung et al [10]. To investigate this case we have used for the long wavenumber end of the bump a value $k_0 = 0.04$ h Mpc$^{-1}$, and for the amplitude parameter $a = 0.3 - 0.8$. Our results show that such bump increases only the abundance of
very massive clusters. In the right panel Fig. 2 we show the cluster abundance constraint for clusters of masses exceeding $10^{14}$ solar masses; the curves are calculated for LCDM and MDM models with $\Omega_n = 0.00, 0.05, 0.10$. We see that the cluster abundance criterion constrains the matter and HDM densities in a rather narrow range.

The power spectra of LCDM models with and without the Starobinsky break are shown in Fig. 3, upper left; these models were calculated for the parameter $\Gamma = \Omega_m h = 0.20$. In the case of the spectrum with a bump we have used MDM models as reference due to the need to decrease the amplitude of the spectrum on small scales; these spectra are shown in Fig. 3, upper right. Power spectra are compared with observed galaxy power spectrum [13] and the new cluster power spectrum by Miller & Batuski [23], reduced to the amplitude of the galaxy power spectrum. We also show the matter power spectrum based on a biasing factor $b_c = 1.3$ [14]. We see that the Starobinsky model reproduces well the matter power spectrum on small and intermediate scales, but not the new data by Miller & Batuski. The modification by Chung et al [11] with amplitude parameter $a = 0.3$ fits well all observational data. The cluster mass distribution for the Chung model is shown in lower left panel of Fig. 3, and the angular spectrum of CMB temperature fluctuations in lower right panel. In order to fit simultaneously the galaxy power spectrum and the CMB angular spectrum we have used a tilted MDM model with parameters $n = 0.90$, $\Omega_b = 0.06$, $\Omega_n = 0.05$, and $\Omega_m = 0.4$.

5 Discussion

BOOMERANG and MAXIMA I data have been used in a number of studies to determine cosmological parameters [4], [9], [11], [19], [21], [34], [35]. In addition
to CMB data various other observational data have been used. In general, the agreement between various determinations is good; however, some parameters differ. For instance, interpreted new CMB data in terms of a baryon fraction higher than expected from the nucleosynthesis constraint, \( h^2 \Omega_b = 0.03 \), and a relatively high matter density, \( h^2 \Omega_m = 0.33 \). On the other hand, velocity data suggest a relatively high amplitude of the power spectrum, \( \sigma_8 \Omega_m^{0.6} = 0.54 \), which in combination with distant supernova data yields \( \Omega_m = 0.28 \pm 0.10 \) and \( \sigma_8 = 1.17 \pm 0.2 \).

Our analysis has shown that a high value of the density of matter, \( \Omega_m > 0.4 \), and high amplitude of the matter power spectrum, \( \sigma_8 > 1 \), are difficult to explain in terms of the supernova and cluster abundance data, and the observed amplitude of the galaxy power spectrum with reasonable bias limits. This conflict can be avoided using a tilted initial power spectrum, and a MDM model with a moderate fraction of HDM, as discussed above. The best models suggested so
far have $0.3 \leq \Omega_m \leq 0.4$, $0.90 \leq n \leq 0.95$, $0.60 \leq h \leq 0.70$, $\Omega_n \leq 0.05$. Matter densities are constrained to $\geq 0.3$ by cluster abundances, and to $\leq 0.4$ by all existing matter density estimates. This upper limit of the matter density, in combination with the cluster abundance and amplitude of the power spectrum, yields an upper limit to the density of the hot dark matter. We can consider this range of cosmological parameters as compatible with all constraints. This set of cosmological parameters is surprisingly close to the set suggested by Ostriker & Steinhardt \[25\]. Now it is supported by much more accurate observational data.

A considerably lower value of matter density, $\Omega_m = 0.16$, was suggested by Bahcall et al \[2\] from the observed value of $M/L$ for galaxies and clusters of galaxies of various richness. Upper right panel of Fig. 1 shows this constraint for various fractions of matter in voids and respective bias parameter values. The reason for the deviation of this matter density determination from the rest is not clear, and we have not used it in the present analysis.

Acknowledgments

I thank M. Einasto, M. Gramann, V. Müller, E. Saar, A. Starobinsky, and E. Tago for fruitful collaboration and permission to use our joint results in this talk, Joe Silk for discussion, and H. Andernach for help in improving the style. This study was supported by the Estonian Science Foundation grant 2625.

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