Calculation of $E_T$ from single particle spectra

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Abstract. In high energy heavy ion collisions, measurements of transverse energy ($E_T$) can constrain the initial energy density and therefore provide insight into the formation of the Quark Gluon Plasma (QGP). This is particularly interesting in regions where it is unclear if the QGP is formed. The $E_T$ in a collision can be measured either from a calorimeter or calculated from the single particle momentum spectra. We use spectra measured by the STAR collaboration to calculate the transverse energy in heavy ion collisions in Au+Au collisions at $\sqrt{s_{NN}} = 7.7 - 200$ GeV. These calculations are compared to PHENIX measurements using a calorimeter.

1. Introduction

The mean transverse energy per unit pseudorapidity $\langle dE_T/d\eta \rangle$ conveys information about how much of the initial longitudinal energy carried by the incoming nuclei is converted into energy carried transverse to the beam axis. The $\langle dE_T/d\eta \rangle$ can provide information on the energy densities using simple geometric assumptions [1]. This is particularly important for the RHIC Beam Energy Scan (BES), as it can provide insight into when conditions are conducive to the formation of the Quark Gluon Plasma.

Only the PHENIX experiment has measured $\langle dE_T/d\eta \rangle$ at all RHIC energies [2–5] using a calorimeter. The single particle spectra, $d^2N/dydp_T$, carry equivalent information, enabling a cross check between experiments. We use a method similar to the calculation of $E_T$ from spectra in [6] to calculate $E_T$ from previously published STAR spectra [7–13]. We define $E_T$ to be what would be measured in a calorimeter

$$E_T = \sum_{i=1}^{N} E^\text{cal}_i \sin(\theta_i) \quad (1)$$

where the sum over $i$ is over all $N$ particles and

$$E^\text{cal}_i = \begin{cases} 
E - mc^2 & \text{for baryons} \\
E + mc^2 & \text{for anti-baryons} \\
E & \text{for all other particles}
\end{cases} \quad (2)$$

where $m$ is the particle’s mass, $c$ is the speed of light, and $E$ is the relativistic energy,

$$E = \sqrt{(pc)^2 + (mc^2)^2}. \quad (3)$$

These calculations provide a consistency check between the experiments but will also allow the distribution of energy among different particles to be studied.
2. Method

Measurements of the single particle double differential spectra, $d^2N/dydp_T$, are available at RHIC energies for charged pions, kaons, and protons and for lambda baryons. The publications with these measurements are summarized in table 1. The $E_T$ can be calculated from these spectra by first calculating the average $E_T$ per event carried by each particle type and summing over the different particle types

$$E_T = \sum_i^M E_T^i$$

where the sum over $i$ is over each $M$ different type of particle. Since the spectra are not generally available for all particle types, the $E_T$ carried by measured particles has to be scaled up to get the total $E_T$ using

$$E_T = f_\pi(E_\pi^T + E_\pi^{-T}) + f_K(E_K^T + E_K^{-T}) + f_p(E_p^T + E_p^{-T}) + f_{\Lambda\bar{\Lambda}}(E_{\Lambda}^T + E_{\bar{\Lambda}}^T)$$

where $f_{\pi, K, p, \bar{p}, \Lambda\bar{\Lambda}}$ are constants to scale up the measured $E_T$. Each of the scale factors used in this preliminary analysis and which particles are included are listed in table 2. This is illustrated schematically in figure 1 with the particles included in each correction indicated on the figure. Each of the correction factors are discussed in greater detail below.

### Table 1. The collision energy, collaboration, particle species, reference, and maximum rapidity for midrapidity measurements of $\Lambda, \pi^\pm, K^\pm, p$ and $\bar{p}$ in Au+Au.

| $\sqrt{s_{NN}}$ (GeV) | Particle | Ref. | $y_{max}$ |
|------------------------|----------|------|----------|
| 7.7, 11.5, 19.6, 27, 39 | $\pi^\pm, K^\pm, p, \bar{p}$, $\Lambda, \bar{\Lambda}$ | 7, 8 | < 0.1, < 0.5 |
| 62.4                   | $\pi^\pm, K^\pm, p, \bar{p}$, $\Lambda, \bar{\Lambda}$ | 9, 10 | < 0.5, < 1.0 |
| 130                    | $\pi^\pm, K^\pm, p, \bar{p}$, $\Lambda, \bar{\Lambda}$ | 9, 11 | < 0.5, < 0.5 |
| 200                    | $\pi^\pm, K^\pm, p, \bar{p}$, $\Lambda, \bar{\Lambda}$ | 9, 12, 13 | < 0.5, < 1.0 |

2.1. Fits to the data

The $E_T$ carried by each particle, $dE_T^i/d\eta$, can be calculated from the double differential spectra $d^2N^i/dydp_T$ using

$$dE_T^i/d\eta = \int_0^\infty dy \frac{d^2N^i}{dydp_T} E_T^i(p_T)dp_T$$

Figure 1. Schematic diagram showing which particles are included in the correction factors. The area of the circles is approximately proportional to the $E_T$ carried by each particle.
Table 2. Correction factors

| Correction | Particles included         |
|------------|---------------------------|
| $f_\pi = 1.56 \pm 0.02$ | $\pi^\pm, \pi^0, \eta, \omega, e^\pm, \mu^\pm, \gamma$ |
| $f_K = 1.80 \pm 0.20$   | $K^\pm, K_S^0, K_L^0$     |
| $f_p = 2 - 3$, see table 4 | $p, \bar{p}, n, \bar{n}$   |
| $f_{\Lambda, \bar{\Lambda}} = 1.08 \pm 0.51$ | $\Lambda, \bar{\Lambda}, \Sigma^\pm, \Sigma^0, \Sigma^0$ |

where $\frac{d^2N}{dp_T dy}$ is the measured spectrum of particle $i$, $dy/d\eta$ is the Jacobian for the transformation from $y$ to $\eta$, and $E_T(p_T)$ is the transverse energy for a particle of type $i$ with mass $m$ and transverse momentum $p_T$. At midrapidity, $\eta \approx 0$ so $\frac{dy}{d\eta} \approx \frac{E_T}{m_T}$ and $E_T$ is as defined in (2). The momentum in each bin is estimated to be $p_T \approx p_T^c - \frac{1}{3} w$ where $p_T^c$ is the center of the bin and $w = 50$ MeV/c is the width of the bin. The data cover a finite range in momentum so a fit is used to extrapolate to both $p_T = 0$ and $p_T = \infty$.

The published spectra are fit to both a Boltzmann-Gibbs Blast Wave distribution and a variation of a modified Hagedorn distribution. The Blast Wave is given by

$$\frac{d^2N}{dp_T dy} = \frac{Np_T}{\int_0^1 r' dr' \left( \sqrt{m^2 + p_T^2} \right) \cdot I_0 \left( \frac{p_T \sinh \left( \tanh^{-1} \left( \beta_s r'' \right) \right)}{T_{\text{kin.}}} \right) \cdot K_1 \left( \frac{\sqrt{m^2 + p_T^2} \cosh \left( \tanh^{-1} \left( \beta_s r'' \right) \right)}{T_{\text{kin.}}} \right)}.$$

(7)

where $p_T$ is the transverse momentum, $y$ is the rapidity, $N$ is the normalization, $m$ is the mass of the particle, $\beta_s$ is the surface velocity, $n$ is an exponent describing the evolution of the velocity profile, and $T_{\text{kin.}}$ is the kinetic freeze out temperature. The $I_0$ and $K_1$ are modified Bessel functions. The reduced radius, $r''$, is integrated over from 0 to 1. The variation on a modified Hagedorn distribution is given by

$$\frac{d^2N}{dp_T dy} = \frac{A}{\sqrt{m^2 + p_T^2}} \left( 1 + \frac{\sqrt{m^2 + p_T^2}}{nT} \right)^{-n}.$$

(8)

where $A$, $n$, and $T$ are parameters and $m$ is the mass. The difference between these two functional forms is used as a systematic uncertainty on these extrapolations.

The $\Lambda$ and $\bar{\Lambda}$ spectra are available for fewer centrality bins than $\pi^\pm, K^\pm, p$ and $\bar{p}$. In order to allow measurements at more centralities, we interpolate $\langle dE_T/d\eta \rangle / \langle N_{\text{part}}/2 \rangle$ between centrality bins by using a polynomial between neighboring centralities.

2.2. Pion scaling factor $f_p$

There are slight deviations between the yields of different pion species, dominantly due to contributions from the decays of $\eta$ and $\omega$ mesons, both of which are short-lived and whose daughters are operationally primary particles. Decays from other short-lived resonances also contribute. The $\eta$ and $\omega$ decays which have a branching ratio of more than 1% and their branching ratios are given in table 3. We estimate the contribution from the $\eta$ and $\omega$ mesons using transverse mass scaling to estimate the shape of the spectra and matching the yields to the...
Table 3. Decays of the $\eta$ and $\omega$ mesons and their branching ratios for decays with a branching ratio greater than 1% [17].

| decay           | BR (%) |
|-----------------|--------|
| $\eta \rightarrow \gamma\gamma$ | 39.4   |
| $\eta \rightarrow \pi^0\pi^0\pi^0$ | 32.7   |
| $\eta \rightarrow \pi^+\pi^-\pi^0$ | 22.9   |
| $\eta \rightarrow \pi^+\pi^-\gamma$ | 4.2    |
| $\omega \rightarrow \pi^+\pi^-\pi^0$ | 89.2   |
| $\omega \rightarrow \pi^0\gamma$ | 8.3    |
| $\omega \rightarrow \pi^+\pi^-$ | 1.5    |

observed $\eta/\pi^0$ yield ratios [18]. This approach was also taken in [6]. We estimate $f_\pi = 1.56 \pm 0.02$. We anticipate more detailed studies of these contributions and contributions from $e^\pm$, $\mu^\pm$, and $\gamma$ before publication.

2.3. Kaon scaling factor $f_K$

At low energies, charged and neutral kaons can have different yields (and hence carry different $E_T$) due to differences in kaon production and in feeddown from short-lived resonances. We calculate the ratio of $2K^0_S/(K^+ + K^-)$ using preliminary kaon yields for Au+Au collisions at $\sqrt{s_{NN}} = 7.7, 11.5, 19.6, 27,$ and 39 GeV. The lowest value for $2K^0_S/(K^+ + K^-)$ within the uncertainties is 0.6, leading to a lower bound on $f_K$ of 1.6. At high energies, this ratio approaches 1.0, which we use as our upper bound leading to $f_K = 1.8 \pm 0.2$. We will update these estimates with the final data [8] before publication.

2.4. Proton scaling factor $f_p$

For an upper bound on the contribution from neutrons, we consider a simple picture where protons ($N_p$) and neutrons ($N_n$) at midrapidity are either generated in the collision or are primordial nucleons from the incoming nuclei. Antiprotons ($\bar{N}_p$) and antineutrons ($\bar{N}_n$) are assumed to consist entirely of generated nucleons and the number of generated particles is the same for all generated baryons and anti-baryons. In that case, the ratio of the yields can be written as

$$f_p = \frac{N_n + N_{\bar{n}} + N_p + N_{\bar{p}}}{N_p + N_{\bar{p}}} = \frac{1 + \beta + (3 - \beta)N_{\bar{p}}/N_p}{1 + N_{\bar{p}}/N_p}$$  \hspace{1cm} (9)$$

where $\beta = N/Z$ where $N$ is the number of primordial neutrons and $Z$ is the number of primordial protons. We use this to estimate the upper bound of $f_p$, listed in table [2]. At high energies, all (anti)protons and (anti)neutrons are generated in the collision, $\frac{N_n}{N_p}$ approaches one, and $f_p$ would approach 2.0. We use this combined with measurements of $\frac{N_{\bar{n}}}{N_{\bar{p}}}$ to calculate the lower bound of $f_p$. We anticipate updating [9] to take into account the different definition of $E_T$ for baryons and anti-baryons.

2.5. Lambda scaling factor $f_\Lambda$

There are two major corrections to the $\Lambda$ and $\bar{\Lambda}$ baryons, a feeddown correction to avoid double counting of the energy from proton daughters from the decay $\Lambda \rightarrow p\pi^-$ and a scale factor to take into account $\Sigma$ baryons. The dominant decays for $\Sigma$ baryons are listed in table [5]. The $\Lambda$ and the $\Sigma$ baryons have approximately the same mass and therefore are expected to carry the same amount of energy, but the $\Sigma$ baryons’ $E_T$ is divided among three different isospin partners. The
Table 4. Antiproton to proton ratios and the correction factors calculated from them for various energies.

Table 5. Weakly decaying particle decays relevant to the calculation of $E_T$ from strange baryons.

$\Sigma^0$ and its antiparticle decay to a $\Lambda$ and a $\gamma$ too rapidly for the $\Lambda$, which carries the majority of the energy, to be distinguished from primordial $\Lambda$ baryons. This means that the $E_T$ of the $\Sigma^0$ is included in the $E_T$ from the $\Lambda$. To get a lower estimate, we assume that the $\Sigma$ and $\Lambda$ baryons carry the same amount of energy, but that it is divided equally among all three $\Sigma$ baryons, leading to $f_\Lambda = 1$. For an upper limit, we use the ratio in PYTHIA and HIJING at $\sqrt{s_{NN}} = 2.76$ \[6\], $f_\Lambda = 1$. This leads to a correction factor to include the $\Sigma$ baryons of $1.585 \pm 0.085$.

Before the final publication, we will include studies at RHIC energies. The STAR (anti)proton spectra do not include corrections for feeddown from the (anti)lambda decay $\Lambda \rightarrow p\pi^-$. To avoid double counting, we estimate feeddown using the proton spectra with and without feeddown corrections reported in \[12\] for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. We calculate the fraction of $E_T$ carried by protons from $\Lambda$ decays as

$$\frac{E_{T}^{\Lambda}}{E_{T}^p} = \frac{E_{T}^{p,all} - E_{T}^{p,fd}}{E_{T}^p}$$

where $E_{T}^\Lambda$ is the $E_T$ carried by $\Lambda$ baryons, $E_{T}^{p,all}$ is the $E_T$ calculated from the proton spectra without feeddown corrections, $E_{T}^{p,fd}$ is the $E_T$ calculated from the proton spectra with feeddown corrections, and $E_{T}^{fd}$ is the amount of $E_T$ carried by protons from the decay of $\Lambda$ baryons.

We find $0.48 < \frac{E_{T}^{\Lambda}}{E_{T}^p} < 1.02$. However, this fraction is also limited by the 64% branching ratio for this decay, so $\frac{E_{T}^{\Lambda}}{E_{T}^p} < 0.68$. To be conservative, we use $\frac{E_{T}^{\Lambda}}{E_{T}^p} = 0.34 \pm 0.34$ and $f_\Lambda^{fd} = 1 - \frac{E_{T}^{\Lambda}}{E_{T}^p} = 0.66 \pm 0.34$. Since the $\Lambda$ baryons from the $\Sigma^0$ would also contribute to the feeddown correction, this factor multiplies the correction to take the $\Sigma^\pm$ into account. Combining
Figure 2. Left: $\langle dE_T/d\eta \rangle / \langle N_{\text{part}}/2 \rangle$ vs $N_{\text{part}}$ for Au+Au collisions at $\sqrt{s_{\text{NN}}} = 7.7, 11.5, 19.6, 27, 39, 62.4, 130$ and $200$ GeV. Right: Comparison of $\langle dE_T/d\eta \rangle / \langle N_{\text{part}}/2 \rangle$ at midrapidity versus $\sqrt{s_{\text{NN}}}$ in central A+A collisions at other energies [2, 20] at midrapidity. The dashed black line is a log($\sqrt{s_{\text{NN}}}$) trend which had been seen [2] to fit the RHIC and fixed-target data, while the solid red curve is a prediction from [21, 22].

this with information from the contribution of $\Sigma^+, \Sigma^-$ and $\Sigma^0$, we get an overall

$$f_{\Lambda, \bar{\Lambda}} = 1.08 \pm 0.51.$$ (11)

3. Results

Figure 2 shows $\langle dE_T/d\eta \rangle / \langle N_{\text{part}}/2 \rangle$ versus $N_{\text{part}}$ and $\sqrt{s_{\text{NN}}}$. The $N_{\text{part}}$ dependence shows similar trends to those observed by PHENIX [2–5], with a similar shape for all collision energies. The $\sqrt{s_{\text{NN}}}$ dependence looks similar to that seen in previous measurements, but there is a clear discrepancy between our calculations from STAR spectra and the published PHENIX $E_T$ measurements, particularly in the BES range.

This is unlikely to be due to the correction factors. While the contributions from neutrons and $\Sigma^+, \Sigma^-$ and $\Sigma^0$ are not constrained by data, the uncertainties on $f_p$ and $f_\Lambda$ are large and the overall contributions from baryons are small. The contributions from kaons are larger, but measurements of the $K_0^0$ constrain the reasonable range for this correction. The correction to pions is similarly constrained by extensive measurements of the $\pi^0$. The overall contribution from uncertainties in the scaling factors is indicated in figure 2 and is much less than the uncertainty on individual points reported by STAR.

There are several possible causes for this discrepancy. The PHENIX $E_T$ measurements use a Jacobian for the transformation from $y$ to $\eta$ which is calculated in simulations while these calculations use the observed momenta and particle ratios. The PHENIX measurements also use simulations to determine a single scaling factor, which may be sensitive to particle ratios, the shape of the spectra at low $p_T$ where there are few contributions to the raw measurement, and simulation of the PHENIX magnetic field. There may also be a difference in the definition of $E_T$, as we use what a calorimeter would make. STAR and PHENIX also use different centrality definitions, with PHENIX determining centrality from the multiplicity at forward rapidities well-separated from the midrapidity $E_T$ measurement while STAR determines centrality at midrapidity, in the same region as the measurements. This is not likely to cause a major discrepancy in central collisions, but may lead to some differences in peripheral collisions. There may also be issues with the spectra measurements from STAR. Further cross checks include looking at multiplicities from both measurements and calculating $E_T$ using the definition $E_T = m_T$. 

4. Conclusions
We have calculated $E_T$ from single particle spectra measured by STAR. These calculations indicate a discrepancy between the two experiments. Given the importance of global observables in establishing the formation of the QGP and how central this question is to the Beam Energy Scan program, this warrants further investigation. We will refine our calculations to the extent possible. We also recommend that STAR and PHENIX pursue studies which could provide further insight. PHENIX could measure single particle spectra and STAR could measure $E_T$ independently for BES energies.

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