Reheating constraints to modulus mass for single field inflationary models

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Abstract. We consider string and supergravity motivated scenarios in which moduli fields dominate the energy density of the Universe in a post-inflationary epoch. For the case of a single light modulus we show that considering the evolution of a specific scale from the time of its Hubble crossing during inflation to the present time, one can obtain a relation among the lightest modulus mass, reheating parameters ($T_{\text{reh}}$, $w_{\text{reh}}$ and $N_{\text{reh}}$) and the inflationary observables. Using this relation, the value of the modulus mass and the reheating parameters can be constrained by the CMB data. Next, the analysis is extended to include features in the inflaton potential as a source of CMB low multipole anomalies, which further constrains the mass of the modulus to be substantially higher than without such a constraint.

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1 Introduction

The data from the recently concluded Cosmic Microwave Background (CMB) experiments [1–14] are in perfect agreement with the scale invariant spectrum [15–19] as predicted by the theory of inflation [20–26], and this puts forth slow-roll inflation as the leading candidate for the early Universe cosmology. The slow-roll conditions of inflation are ultraviolet sensitive, and one should embed inflationary models in quantum theory of gravity. String theoretical models of inflation take care of these ultraviolet issues. However, in string or supergravity models [27–29], there are moduli fields which play a central role. These are generic scalar fields which are massless in the basic construction, but acquire masses much lighter than the string scale through subleading corrections. At the end of the inflationary phase, the inflaton field oscillates and brings the Universe to thermal equilibrium; this phase is generically referred to as reheating [30–36]. The presence of moduli whose masses are lighter than the value of the Hubble parameter after the Universe returns to a thermal equilibrium post inflation are important to subsequent cosmology, while the heavier ones can be considered to have been inflated away. The light moduli with almost flat potentials are displaced from their minima during inflation, but subsequently begin to oscillate and manifest as light particles. In this paper we consider the case of a single modulus field as an example, and consider it in conjunction with a duly parametrised reheating scenario.

The possible physics underlying the reheating process remains open to many scenarios, and so far there are no direct observational data available. However, one can parametrize the reheating phase as, the temperature of reheating ($T_{\text{reh}}$), the duration of reheating ($N_{\text{reh}}$), and a somewhat coarse but useful equation of state parameter during reheating ($w_{\text{reh}}$). Scale-invariant perturbations are considered to have arisen during the extensive slow-roll phase of inflation, during which the effective equation of state parameter is $w = -1$. The end of this phase should be marked by moving closer to positive values of $w$ and therefore accompanied by the equation of state parameter $w$ attaining at least $w > -\frac{1}{3}$. Hence it is convenient to assume that $\bar{w}_{\text{reh}} = -\frac{1}{3}$ at the onset of the reheating. After the reheating era, $w$ should grow to $\frac{1}{3}$ as appropriate for radiation dominated Universe. Therefore, in various scenarios $\bar{w}_{\text{reh}}$ has been considered to be $-\frac{1}{3} \leq \bar{w}_{\text{reh}} \leq 1$. As for $T_{\text{reh}}$, the lower and upper bounds are $10^{-2}$ GeV in order to satisfy the big-bang nucleosynthesis (BBN) temperature [37] and $10^{16}$
GeV to accord with the scale of inflation. The relation between the reheating parameters \((T_{\text{reh}}, N_{\text{reh}} \text{ and } w_{\text{reh}})\) and the inflationary observables can be obtained by considering the evolution of the observable cosmological scales from the time of Hubble crossing during inflation to the present time [38–42]. Although the reheating parameters seem to be hopelessly far away from being observationally determined, the natural range for \(w_{\text{reh}}\) being limited, provides substantial constraints on the rest of the reheating scenario.

After reheating, the Universe becomes radiation dominated. The expansion of the Universe redshifts the energy density associated with it, and the Hubble parameter \((H)\) value decreases. When \(H\) becomes comparable with the mass of the light moduli, the moduli fields start oscillating around the minimum of their respective potentials [43–45]. The energy associated with the moduli dilutes like matter which is at a rate slower than the radiation. Hence, very quickly the energy density of the Universe becomes modulus dominated. Ultimately, the moduli decay, and as a consequence, the Universe should reheat for the second time. Due to the prompt decay of the moduli, we can consider the second reheating phase to be short enough that it may be treated as instantaneous. In this paper we consider the case of single light modulus field of mass \(m_\chi\), and while the duration of modulus domination is finite, the second reheating is instantaneous.

According to the standard cosmology, the early Universe passed through the following epochs: inflation, reheating, radiation domination and matter domination. Now, if the Universe becomes modulus dominated after the radiation dominated epoch and reheats the Universe for the second time, then the Universe has gone through the epochs: inflation, reheating (inflaton decay), radiation dominated, modulus dominated, reheating (modulus decay) and matter dominated eras. In this paper, we relate the reheating and inflationary parameters to the lightest modulus mass, \((m_\chi)\), by considering the second reheating phase (modulus decay) is instantaneous, and obtain tight constraints on the modulus mass as well as on the reheating parameters.

As another input, we consider the fact that at lower multipoles, specifically around \(\ell = 22\) and 40, the Planck data points lie outside the cosmic variance associated with the power law primordial spectrum. If not of a completely accidental origin, this raises the possibility of non-trivial inflationary dynamics, in turn providing important phenomenological inputs about the inflationary model. Model independent approaches to reconstruct the primordial power spectrum from the CMB anisotropies have been reported in [46–54]. The consideration of a burst of oscillations in the primordial power spectrum leads to a good fit to the CMB angular power spectrum, including the anomalies [55–57]. In order to generate these oscillations in the primordial power spectrum, one has to consider a short period of deviation from slow-roll inflation [58, 59]. A possible approach to such a deviation is to introduce a step in the inflaton potential [11, 60–66]. A step with suitable height and width at a particular location of the inflationary potential has resulted in a better fit to the CMB data near the multipole \(\ell = 22\). In [67] a possible origin for an unusual phase at the onset of inflation that can produce such a deviation has been considered in the context of SO(10) grand unification.

It can be shown that the generic relation between late time observables and reheating phase in a single field inflation can be strengthened by demanding successful explanation of the low multipole anomalies. The link is the specific position, \(\phi_k/M_{\text{Pl}}\), of the inflaton in the course of its slow roll, at which it encounters the step in the potential. The location of such a step in the inflaton potential was obtained in Ref. [55]. Then it can be shown [42] that such a step makes the constraints on the reheating parameters more stringent for different
inflationary models. In this paper we apply this method in conjunction with a single late time light modulus.

This article is organized as follows: Sec 2 deals with the slow-roll inflation and late time modulus dominated cosmology. In this section we derive the relation of $m_\chi$ with $T_{\text{reh}}$, $N_{\text{reh}}$, $w_{\text{reh}}$ and the inflationary parameters ($V_{\text{end}}$ and $\Delta N_k$). The expression for $m_\chi$ is derived as a function of the scalar spectral index $n_s$ for different single field inflationary models in Sec 3. Sec. 4 studies the additional constraint on the modulus mass $m_\chi$ due to the addition of a step in the potential along the lines of [42]. Conversely in Sec. 5, using a previously obtained bound on the modulus mass [68], it is shown that quantitative features of the primordial reheating phase are modified. Finally, Sec 6 contains the conclusions.

We work with $\hbar = c = 1$ units and the following values are used. $M_{\text{Pl}} = \sqrt{\frac{1}{8\pi G}} = 2.435 \times 10^{18}\text{GeV}$ is the reduced Planck mass and the redshift of last scattering surface is $z_{\text{ls}} = 1100$. The $z_{\text{eq(MR)}} = 3402$ is the redshift of matter radiation equality and the present value of the Hubble parameter $H_0 = 100h \text{ km s}^{-1}\text{Mpc}^{-1}$ with $h = 0.6736$ [1].

2 Reheating parameters: extension to modulus dominated case

We begin with recapitulating the essentials of the formalism. The inflaton $\phi$ is considering to be governed by a potential $V(\phi)$ undergoing slow-roll evolution with parameters $\epsilon$ and $\eta$, resulting in scalar curvature power spectrum $P_\zeta(k)$ and tensor power spectrum $P_h(k)$ as a function of the Fourier transform variable $k$ of the argument of the spatial correlation functions, with corresponding indices $n_s - 1$ and $n_T$. The details of the definitions and notation are standard [69], and can be found also in the references [19, 26, 70]. We shall use $A_s$ and $A_T$, the amplitude of scalar and tensor power spectra at the pivot scale $k_*$ as used by Planck collaboration, $\frac{k_*}{a_0} = 0.05\text{Mpc}^{-1}$. For $k = k_*$, these amplitudes are given in terms of $H_*$ as

$$ A_T = P_h(k_*) = \frac{2H_*^2}{\pi^2 M_{\text{Pl}}^2}, \quad A_s = P_\zeta(k_*) = \frac{H_*^2}{8\pi^2 M_{\text{Pl}}^2 \epsilon_*}. \quad (2.1) $$

In terms of the slow-roll parameters $\epsilon$ and $\eta$, the tensor to scalar ratio $r$, the scalar spectral index $n_s$ and the tensor spectral index $n_T$ satisfy the relations

$$ r = 16\epsilon, \quad n_s = 1 - 6\epsilon + 2\eta, \quad n_T = -2\epsilon. \quad (2.2) $$

The total number of e-foldings, $N_T$, is defined as the logarithm of the ratio of the scale factor at the final time $t_e$ to it’s value at initial time $t_i$ of the era of inflation.

$$ N_T \equiv \ln \left(\frac{a(t_e)}{a(t_i)}\right) = \int_{t_i}^{t_e} H dt = \int_{\phi_i}^{\phi_{\text{end}}} \frac{H}{\dot{\phi}} d\phi = \frac{1}{M_{\text{Pl}}} \int_{\phi_{\text{end}}}^{\phi_i} \frac{1}{\sqrt{2\epsilon}} d\phi. \quad (2.3) $$

Where $\phi_i$ and $\phi_{\text{end}}$ are the initial and final values of the inflaton field $\phi$ and $\epsilon$ is the slow-roll parameter defined as $\epsilon = -\dot{H}/H^2 = \dot{\phi}^2/(2H^2 M_{\text{Pl}}^2)$. Likewise, given a mode $k$, the number of e-foldings between the time when it crosses the Hubble horizon and the end of inflation is given by

$$ \Delta N_k = \int_{\phi_k}^{\phi_{\text{end}}} \frac{H}{\dot{\phi}} d\phi = \frac{1}{M_{\text{Pl}}} \int_{\phi_{\text{end}}}^{\phi_k} \frac{1}{\sqrt{2\epsilon}} d\phi, \quad (2.4) $$
where $\phi_k$ is the value of the inflaton field at the time of Hubble crossing of the scale $k$. For the slow-roll approximation i.e., $V(\phi) \gg \dot{\phi}^2$ and $\ddot{\phi} \ll 3H \dot{\phi}$ the Eq. (2.4) becomes

$$\Delta N_k \approx \frac{1}{M_{Pl}^2} \int_{\phi_{end}}^{\phi_k} \frac{V}{V'} d\phi.$$  

(2.5)

In the inflationary model of cosmology, at the end of the inflation, the inflaton field decays and reheat the Universe. The energy stored in the inflaton gets converted to radiation. The Hubble parameter ($H$) and the energy density of the Universe decrease with the expansion of the Universe. When the Hubble parameters value becomes equivalent to the mass of a modulus, the modulus field start oscillating in its potential minimum [43–45, 68, 71]. The energy density of the modulus field redshifts like matter (which is slower than the redshift rate of radiation); hence, the energy density of the Universe becomes modulus dominated. After this, the modulus decays and reheat the Universe for the second time. During inflation, The equation of motion of a scalar field $\chi$ is given by

$$\dddot{\chi} + (3H + \Gamma_\chi) \ddot{\chi} + \frac{\partial V}{\partial \chi} = 0.$$  

(2.6)

Where $\Gamma_\chi$ is the decay width of the scalar field $\chi$ and $H$ is the Hubble parameter. If the value of the Hubble parameter is greater than the mass of the scalar, $m_{\chi}$, then the field will freeze at its initial displacement $\chi_{in}$. This initial displacement is of the order of $M_{Pl}$. The quantum fluctuation [72] of the field during inflation or the dependence of the modulus potential on the vacuum expectation value [68, 73–76] of the inflaton are the possible reasons for this initial displacement of the scalar field. The modulus starts oscillating around its minimum, and then the energy density of modulus (matter) and radiation becomes equal, and is given by [68]

$$\rho_{eq} = m_{\chi}^2 \chi_{in}^2 \left( \frac{\chi_{in}^2}{6M_{Pl}^2} \right)^3.$$  

(2.7)

The energy density of the modulus then dominates, and the modulus decays at energy density

$$\rho_{\text{decay}} \sim M_{Pl}^2 \Gamma_{\chi}^2.$$  

(2.8)

The lifetime of the modulus $\tau_{\text{mod}}$ is expressed as

$$\tau_{\text{mod}} \approx \frac{1}{\Gamma_\chi} \approx \frac{16\pi M_{Pl}^2}{m_{\chi}^3}.$$  

(2.9)

Using Eqs. (2.8) and (2.9) the reheat temperature can be written in terms of the modulus mass as

$$T_{\text{reh}} \sim m_{\chi}^{3/2} M_{Pl}^{-1/2}.$$  

(2.10)

The lower bound of the reheat temperature is around a few MeV (the BBN temperature). Hence, using Eq. (2.10) one obtains the bound on the modulus mass (known as cosmological moduli problem bound) as [43–45]

$$m_{\chi} \geq 30\text{TeV}.$$  

(2.11)
2.1 Connecting to observable scales

We now adapt our method of [42] to show that the mass of the lightest modulus field can be related to cosmological observational parameters. This can be done by considering the evolution of a cosmological scale from the time of Hubble crossing during inflation to present time. While the discussion parallels our earlier work, the specific expressions differ and for completeness we include all the reasoning.

A physical scale today \( \frac{k}{a_0} \) can be related to it’s value at the time of Hubble crossing during inflation, \( \frac{k}{a_k} \), as

\[
\frac{k}{a_k} = H_k = \frac{k}{a_0} \frac{a_{eq(MR)}}{a_{decay}} \frac{a_{eq(mod)}}{a_{reh}} \frac{a_{end}}{a_k},
\]

(2.12)

where \( a_k, a_{end}, a_{reh}, a_{eq(mod)}, a_{decay}, a_{eq(MR)} \) and \( a_0 \) represents the value of the scale factor at the time of Hubble crossing, end of inflation, end of reheating (decay of inflaton), modulus radiation equality, end of modulus decay, matter radiation equality and at the present time respectively. Throughout this paper, the subscripts “reh” and “decay” represent the end of reheating due to inflaton decay and modulus decay respectively. The Eq. (2.12) can be written as

\[
H_k = \frac{k}{a_0} \left( 1 + Z_{eq(MR)} \right) \left( \frac{\rho_{decay}}{\rho_{eq(MR)}} \right)^{1/4} e^{N_{mod} - N_{rad} e^{N_{reh} e^{\Delta N_k}}}. 
\]

(2.13)

Here \( Z_{eq(MR)} \) is the redshift of matter radiation equality, and \( \Delta N_k \) indicates the number of e-folds remaining after the scale \( k \) has crossed the Hubble radius during inflation. The quantity \( N_{mod} \) and \( N_{rad} \) represent the number of e-foldings in modulus dominated era and radiation dominated era (after inflaton decay). The energy density at the end of modulus decay and at the time of matter radiation equality are represented by \( \rho_{decay} \) and \( \rho_{eq(MR)} \), respectively. \( N_{reh} \) is the number of e-folds during the period of reheating, in which an epoch of preheating [34, 77–81] is followed by the thermalization process. Subsequent evolution of the Universe governed by an energy density

\[
\rho_{reh} = \frac{\pi^2}{30} g_{reh} T_{reh}^4,
\]

(2.14)

where \( g_{reh} \) is the effective number of relativistic species, and \( T_{reh} \) is the temperature at the end of inflaton decay. The Eq. (2.13) can be re written as

\[
H_k = \frac{k}{a_0} \left( 1 + Z_{eq(MR)} \right) \left( \frac{\rho_{decay}}{\rho_{eq(MR)}} \right)^{1/4} e^{N_{mod} - N_{reh} e^{\Delta N_k}},
\]

(2.15)

where \( \rho_{reh} \) and \( \rho_{eq(mod)} \) are energy density at the end of inflaton decay and at the time of modulus radiation equality respectively. We can further parametrize the reheating phase (decay of inflaton) by considering that during that time the Universe was dominated by a fluid [26, 82] of pressure \( P \) and energy density \( \rho \), with an equation of state \( w_{reh} = \frac{P}{\rho} \). Imposing the continuity equation, we have

\[
\dot{\rho} + 3H(\rho + P) = 0, \quad (2.16)
\]

\[
\dot{\rho} + 3H\rho(1 + w_{reh}) = 0. \quad (2.17)
\]
In view of this equation, we have

$$\rho_{\text{reh}} = \rho_{\text{end}} e^{-3N_{\text{reh}}(1 + \bar{w}_{\text{reh}})}; \quad (2.18)$$

where

$$\bar{w}_{\text{reh}} = \langle w \rangle = \frac{1}{N_{\text{reh}}} \int_{N_e}^{N_e} w_{\text{reh}}(N)dN \quad (2.19)$$

Here $\bar{w}_{\text{reh}}$ is the average equation of state parameter during reheating [41]. From Eq. (2.18) we have

$$e^{N_{\text{reh}}} = \left( \frac{\rho_{\text{reh}}}{\rho_{\text{end}}} \right)^{-\frac{1}{3(1 + w_{\text{reh}})}}, \quad (2.20)$$

and the Eq. (2.13) can now be rewritten as

$$H_k = \frac{k}{a_0} a_{\text{eq(mod)}}^{1/4} \rho_{\text{decay}}^{-1/4} \rho_{\text{eq(MR)}} e^{N_{\text{mod}}} = \frac{3\bar{w}_{\text{reh}}^{-1}}{12(1 + \bar{w}_{\text{reh}})} \rho_{\text{reh}}^{-1/4} \rho_{\text{eq(mod)}}^{-1/4} \rho_{\text{end}}^{-1} \rho_{\text{reh}}^{1/4} \rho_{\text{end}}^{1/4} e^\Delta N_k. \quad (2.21)$$

During the modulus dominated era, the energy density of the Universe scales as $\rho \sim a^{-3}$, and we can write

$$e^{-N_{\text{mod}}} = \left( \frac{\rho_{\text{decay}}}{\rho_{\text{eq(mod)}}} \right)^{1/3 + \alpha}. \quad (2.22)$$

During the epoch that the moduli dominate the energy density, $\alpha \approx 0$, which is what we assume in the following. Taking natural logarithm on both sides of Eq. (2.22), one obtains

$$-N_{\text{mod}} = \frac{1}{3} \ln \left( \frac{\rho_{\text{decay}}}{\rho_{\text{eq(mod)}}} \right). \quad (2.23)$$

From Eq. (2.23) we can write the energy density at the time of modulus radiation equality as

$$\ln \rho_{\text{eq(mod)}} = \ln \rho_{\text{decay}} + 3N_{\text{mod}}. \quad (2.24)$$

Using Eq. (2.24) in Eq. (2.20) one obtains

$$\ln H_k = \ln \left( \frac{k}{a_0} \right) + \ln (1 + Z_{\text{eq(MR)}}) - \ln \rho_{\text{eq(MR)}}^{1/4} + \frac{1}{4} N_{\text{mod}} + \frac{3\bar{w}_{\text{reh}}^{-1}}{12(1 + \bar{w}_{\text{reh}})} \ln \rho_{\text{reh}}$$

$$+ \frac{1}{3(1 + \bar{w}_{\text{reh}})} \ln \rho_{\text{end}} + \Delta N_k. \quad (2.25)$$

From Eq. (2.25) we can write

$$-\frac{1}{4} N_{\text{mod}} = \ln \left( \frac{k}{a_0} \right) + \ln (1 + Z_{\text{eq(MR)}}) - \ln \rho_{\text{eq(MR)}}^{1/4} + \frac{3\bar{w}_{\text{reh}}^{-1}}{12(1 + \bar{w}_{\text{reh}})} \ln \rho_{\text{reh}}$$

$$+ \frac{1}{3(1 + \bar{w}_{\text{reh}})} \ln \rho_{\text{end}} + \Delta N_k - \ln H_k. \quad (2.26)$$

Now, the $N_{\text{mod}}$ can be expressed in terms of modulus mass, $m_\chi$, and lifetime of the modulus by computing the evolution of the scale factor from the time of modulus radiation equality $t_{\text{eq}}$.
to time of modulus decay \( t_{\text{decay}} \). The scale factor at any time between the modulus radiation equality and modulus decay can be written as

\[
a(t) = a_{\text{eq(mod)}} \left( \frac{3}{2} H_{\text{eq(mod)}} \left( t - t_{\text{eq(mod)}} \right) + 1 \right)^{\frac{2}{3}},
\]

which gives the number of e-folds during modulus dominated era as

\[
N_{\text{mod}} = \int H \, dt = \int_{t_{\text{eq(mod)}}}^{t_{\text{decay}}} H_{\text{eq(mod)}} \left( \frac{3}{2} H_{\text{eq(mod)}} \left( t - t_{\text{eq(mod)}} \right) + 1 \right)^{-1} \, dt
\]

\[
= \frac{2}{3} \ln \left( \frac{3}{2} H_{\text{eq(mod)}} \left( t - t_{\text{eq(mod)}} \right) + 1 \right)
\]

(2.28)

If we consider that the lifetime of modulus \( \tau_{\text{mod}} \) is the time elapsed between \( t_{\text{end}} \) and \( t_{\text{decay}} \) then one can write

\[
H_{\text{eq(mod)}} \left( t_{\text{decay}} - t_{\text{eq(mod)}} \right) = H_{\text{eq(mod)}} \left( t_{\text{decay}} - t_{\text{end}} \right) - H_{\text{eq(mod)}} \left( t_{\text{eq(mod)}} - t_{\text{reh}} \right) - H_{\text{eq(mod)}} \left( t_{\text{reh}} - t_{\text{end}} \right)
\]

\[
= H_{\text{eq(mod)}} \tau_{\text{mod}} - \frac{H_{\text{eq(mod)}}}{2H_{\text{reh}}} \left( \frac{a_{\text{reh}}}{a_{\text{end}}} \right)^{\frac{2}{3}(1+\bar{w}_{\text{reh}})} - \frac{2}{3(1+\bar{w}_{\text{reh}})} H_{\text{end}} \left( \frac{a_{\text{reh}}}{a_{\text{end}}} \right)^{\frac{3}{2} \left(1+\bar{w}_{\text{reh}}\right)} e^{-2N_{\text{rad}}}
\]

(2.29)

Substituting Eq. (2.29) in Eq. (2.28) we obtain

\[
N_{\text{mod}} = \frac{2}{3} \ln \left( \frac{3}{2} H_{\text{eq(mod)}} \tau_{\text{mod}} - \frac{3}{4} - \frac{1}{1+\bar{w}_{\text{reh}}} e^{-2N_{\text{rad}}} + 1 \right)
\]

\[
\approx \frac{2}{3} \ln \frac{3}{2} + \frac{2}{3} \ln \left( H_{\text{eq(mod)}} \tau_{\text{mod}} \right)
\]

(2.30)

Now, employing Eq. (2.7) we can compute \( H_{\text{eq(mod)}} \), and substituting it in Eq. (2.30) we obtain

\[
N_{\text{mod}} \approx -\frac{2}{3} \ln 3 - \frac{5}{3} \ln 2 + \frac{2}{3} \ln m_{\chi} \tau + \frac{8}{3} \ln Y
\]

(2.31)

where \( m_{\chi} \) is the modulus mass, and the initial displacement of the modulus field \( \chi \) is defined as \( \chi_{\text{in}} = Y M_{\text{Pl}} \). Now, substituting Eq. (2.31) in Eq. (2.26) we get

\[
\frac{2}{3} \ln 3 + \frac{5}{3} \ln 2 - \frac{1}{6} \ln (m_{\chi} \tau) - \frac{2}{3} \ln Y = \ln \left( \frac{k}{a_0} \right) + \ln \left( 1 + Z_{\text{eq(MR)}} \right) - \ln \rho_{\text{eq(MR)}}^{1/4}
\]

\[
+ \frac{3\bar{w}_{\text{reh}} - 1}{12(1 + \bar{w}_{\text{reh}})} \ln \rho_{\text{end}} - \frac{1}{3(1 + \bar{w}_{\text{reh}})} \ln \rho_{\text{end}} + \Delta N_k - \ln H_k.
\]

(2.32)

To make a contact with the slow-roll inflation, we begin by the definition of the slow-roll parameter, \( \epsilon \), as

\[
\epsilon = \frac{\dot{H}}{H^2} = \frac{3\dot{\phi}^2}{2\dot{\phi}^2 + V(\phi)}.
\]

(2.33)
From Eq. (2.33), the kinetic energy of the inflaton field can be expressed in terms of the slow-roll parameter \( \epsilon \) as

\[
\frac{1}{2} \dot{\phi}^2 = \frac{\epsilon V(\phi)}{3 - \epsilon}.
\]  

(2.34)

Now, we can write the energy density of the Universe and the Hubble parameter during inflation as a function of the slow-roll parameter \( \epsilon \) as follows

\[
\rho(\phi) = \frac{1}{2} \dot{\phi}^2 + V(\phi) = \frac{3V(\phi)}{3 - \epsilon},
\]  

(2.35)

\[
H^2 = \frac{\rho}{3M_{Pl}^2} = \frac{1}{M_{Pl}^2} \left( \frac{V(\phi)}{3 - \epsilon} \right).
\]  

(2.36)

At the end of inflation, the slow-roll parameter becomes of the order of unity, \( \epsilon \sim 1 \). Hence, the energy density of the Universe at the end of inflation is

\[
\rho_{\text{end}} = \frac{3}{2} V_{\text{end}},
\]  

with \( V_{\text{end}} \) being the potential at the end of inflation. Therefore, employing Eq. (2.35) in Eq. (2.32), the modulus mass can be expressed in terms of the inflationary and reheating parameters as given below

\[
m_\chi \approx 4\sqrt{\pi} M_{Pl} \exp \left\{ \left( - \frac{2}{3}\ln 3 + \frac{2}{3}\ln 2 \right) - \ln \left( \frac{k}{a_0} \right) - \ln (1 + Z_{\text{eq}}(MR)) + \ln \rho_{\text{eq}}^{1/4}(MR) \right\}
\]

\[
- \frac{3\bar{\omega}_{\text{reh}} - 1}{12(1 + \bar{\omega}_{\text{reh}})} \ln \left( \frac{e^2}{3 \pi g_{\text{reh}} T_{\text{reh}}^4} \right) - \frac{1}{3(1 + \bar{\omega}_{\text{reh}})} \ln \left( \frac{3}{2} V_{\text{end}} \right) - \Delta N_k + \ln H_k \}
\]  

(2.37)

Eq. (2.37) is the key relationship we shall use for relating the moduli mass, late time observables and reheating parameters for different inflationary models. We consider \( Y = 1/10 \) as per Refs. [68, 71], and \( g_{\text{reh}} \sim 100 \) [40] for our calculations.

3 Inflationary models and constraints on the lightest modulus mass

Quadratic large field model:

The quadratic large field model [26, 40, 70, 83] of inflation is described by the potential \( V(\phi) = \frac{1}{2} m^2 \phi^2 \). Now, consider the mode \( k_* \) corresponding to the pivot scale introduced above, Eq. (2.1), which crosses the Hubble radius \( H_* \) during inflation when the field \( \phi \) has attained the value \( \phi_* \). The number of e-folds remaining after the pivot scale \( k_* \) crosses the Hubble radius is

\[
\Delta N_* \approx \frac{1}{M_{Pl}^2} \int_{\phi_{\text{end}}}^{\phi_*} \frac{V}{V'} d\phi = \frac{1}{4} \left[ \left( \frac{\phi_*}{M_{Pl}} \right)^2 - 2 \right],
\]  

(3.1)

where we have used the condition defining the end of inflation, \( \epsilon = 1 \) which gives \( \frac{\dot{\phi}_{\text{end}}}{M_{Pl}} = 2 \). Using \( \epsilon = 2M_{Pl}^2/\phi^2 \) as arises in this model, the spectral index \( n_s \), Eq. (2.2), can be written as

\[
n_s = 1 - 8 \left( \frac{M_{Pl}}{\phi_*} \right)^2.
\]  

(3.2)
And thus $\Delta N_s$ as a function of the scalar spectral index $n_s$ and is given by

$$\Delta N_s = \left( \frac{2}{1 - n_s} - \frac{1}{2} \right), \quad (3.3)$$

Further, in this model one obtains the relation

$$H_* = \pi M_{Pl} \sqrt{2A_s(1 - n_s)} \quad (3.4)$$

where $n_s$ although strictly $k$ dependent has been replaced by its almost constant value. This, along with the relation of $H$ and field $\phi$ in this model, and the criterion for the end of inflation as used in (3.1), gives the value of $V$ at the end of the inflation, $V_{end}$, as a function of $A_s$ and $n_s$,

$$V_{end} = \frac{1}{2} m^2 \phi^2_{end} \approx \frac{3}{2} \pi^2 A_s M_{Pl}^4 (1 - n_s)^2. \quad (3.5)$$

Substituting Eqs. (3.2), (3.4) and (3.5) in Eq. (2.37) the modulus mass can be expressed as a function of $n_s$ as

$$m_\chi \approx 4 \sqrt{\pi} M_{Pl} \exp\left\{ -\left( \frac{2}{3} \ln 3 + \frac{2}{3} \ln 2 - \frac{2}{3} \ln Y - \ln \left( \frac{k_s}{a_0} \right) - \ln (1 + Z_{eq(MR)}) + \ln \rho_{eq(MR)}^{1/4} \right) 
- \frac{3(\bar{w}_{reh})^{-1}}{12(1 + \bar{w}_{reh})} \ln \left( \frac{2}{3} \pi^2 T_{reh}^4 \right) - \frac{1}{3(1 + \bar{w}_{reh})} \ln \left( \frac{3}{2} \pi^2 A_s M_{Pl}^4 (1 - n_s)^2 \right) 
- \left( \frac{2}{1 - n_s} - \frac{1}{2} \right) + \ln \left( \pi M_{Pl} \sqrt{2A_s(1 - n_s)} \right) \right\} \quad (3.6)$$

The variation of the modulus mass, $m_\chi$, as a function of the scalar spectral index $n_s$ for different values of $\bar{w}_{reh}$ and reheating temperature $T_{reh}$ are shown in figure 1. Planck’s central value of $A_s = 2.1 \times 10^{-9}$ and $z_{eq(MR)} = 3402$ are used, and the parameter $\rho_{eq(MR)}$ is computed to be $10^{-9}$ GeV [1] to obtain the figure 1. For reheating temperature $T_{reh} < 10^{10}$ GeV, within Planck’s 1σ bounds on $n_s$, curves with $\bar{w}_{reh} < 0$ require $m_\chi > M_{Pl}$. However, for $T_{reh} \geq 10^{10}$ GeV and $\bar{w}_{reh} > -\frac{1}{6}$, we obtain $m_\chi < M_{Pl}$ within Planck’s 1σ bounds on $n_s$. From figure 1 we see that with larger values of the reheating temperature, $T_{reh}$, all curves come closer, and at a temperature around $T_{reh} \sim 10^{15}$ GeV all curves converge which corresponds to an instantaneous reheating (see Ref. [42]).
Quartic hilltop potential:

In this model, inflation occurs at very small value of the field and at the top of the flat potential. The potential for this kind of inflation is described by [22, 26, 84].

\[
V(\phi) = V_0 \left[ 1 - \left( \frac{\phi}{\mu} \right)^2 \right].
\]  

(3.7)
The field value at the end of inflation is calculated by setting $\epsilon = 1$ and $\phi_{\text{end}} < \mu$ which leads to the following equation
\[
\left( \frac{\phi_{\text{end}}}{\mu} \right)^p + \frac{p}{\sqrt{2}} \frac{M_{\text{Pl}}}{\mu} \left( \frac{\phi_{\text{end}}}{\mu} \right)^{p-1} = 1. \tag{3.8}
\]

As per Ref. [42, 55], we have considered $p = 4$ and $\mu = 15M_{\text{Pl}}$ and obtained $\frac{\phi_{\text{end}}}{M_{\text{Pl}}} = 14.34$. For this quartic hilltop model we obtain
\[
\Delta N_s = 6.328 \times 10^3 \left[ \frac{(M_{\text{Pl}}/\phi_s)^2}{2} - \left( \frac{1}{14.34} \right)^2 \right] + \frac{1}{8} \left[ \left( \frac{\phi_s}{M_{\text{Pl}}} \right)^2 - (14.34)^2 \right]. \tag{3.9}
\]

We can write the field value at the time of horizon crossing of the pivot scale as a function of $n_s$ as given below
\[
n_s = 1 - 6\epsilon_s + 2\eta_s \]
\[
= 1 - 3M_{\text{Pl}}^2 \left( - \frac{4\phi_s^3}{(15M_{\text{Pl}})^2} - \phi_s^4 \right) - \frac{24\phi_s^2}{(15M_{\text{Pl}})^4} - \phi_s^4 M_{\text{Pl}}^2 \tag{3.10}
\]

The $H_s$ and $V_{\text{end}}$ can be expressed as a function of $A_s$ and $n_s$ as
\[
H_s = 8\pi M_{\text{Pl}} \left( \frac{\chi^3(n_s)}{15^4 - \chi^4(n_s)} \right) \sqrt{A_s} = 8\pi M_{\text{Pl}} \beta(n_s) \sqrt{A_s}, \tag{3.11}
\]
\[
V_{\text{end}} = \gamma A_s M_{\text{Pl}}^2 \frac{\beta^3(n_s) (3 - 8\beta^2(n_s))}{\chi^3(n_s)}, \tag{3.12}
\]

where, $\chi(n_s) = \frac{\phi_s}{M_{\text{Pl}}}(n_s)$ is the solution of equation Eq. (3.10). Here we define $\beta(n_s) = \frac{\chi^3(n_s)}{15^4 - \chi^4(n_s)}$ and $\gamma = 5.28 \times 10^6$. Using the above expressions one can write out $m_\chi$ as a function of $A_s$ and $n_s$ for this quartic hilltop potential, and is given below
\[
m_\chi \approx 4\sqrt{\pi} M_{\text{Pl}} \text{Exp} \left\{ - \left( \frac{2}{3} \ln 3 + \frac{2}{3} \ln 2 - \frac{2}{3} \ln Y - \ln \left( \frac{\epsilon_s}{\chi_s} \right) - \ln (1 + Z_{\text{eq(MR)}}) \right) + \ln \rho_{\text{eq}}^{1/4}
- \frac{3\rho_{\text{reh}}^{-1}}{12(1+w_{\text{reh}})} \ln \left( \frac{\pi^2}{30} g_{\text{reh}} T_{\text{reh}}^4 \right) - \frac{1}{3(1+w_{\text{reh}})} \ln \left( \frac{\gamma A_s M_{\text{Pl}}^2 \beta(n_s) (3 - 8\beta^2(n_s))}{\chi^3(n_s)} \right)
- \Delta N_s(n_s) + \ln \left( 8\pi M_{\text{Pl}} \beta(n_s) \sqrt{A_s} \right) \right\}. \tag{3.13}
\]

Using the above expression, Eq. (3.13), we plot the variation of $m_\chi$ with $n_s$ for six different values of $\tilde{\omega}_{\text{reh}}$ and $T_{\text{reh}}$ in figure 2. Within Planck’s 1σ bounds on $n_s$, for reheating temperature $T_{\text{reh}} < 10^5$ GeV, curves with $\tilde{\omega}_{\text{reh}} < \frac{1}{6}$ predict $m_\chi < M_{\text{Pl}}$. However, for higher reheating temperature, $T_{\text{reh}} > 10^5$ GeV, one obtains $m_\chi < M_{\text{Pl}}$ for all possible $\tilde{\omega}_{\text{reh}}$. Within Planck’s 2σ bounds on $n_s$, only curves with $\tilde{\omega} < -\frac{1}{6}$ and reheating temperature $T_{\text{reh}} \leq 10^5$ GeV gives $m_\chi > M_{\text{Pl}}$. 

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Figure 2: Plots of allowed modulus mass values $m_\chi$ as a function of $n_s$ for the quartic hilltop potential. All curves and shaded regions are as for figure 1.

Starobinsky model:

The potential for the Starobinsky model can be written as [85, 86]

$$V(\phi) = \Lambda^4 \left( 1 - e^{-\sqrt{2/3} \phi/M_{Pl}} \right)^2,$$

(3.14)

Where $\Lambda$ is the energy scale. Similar to the large field and hilltop model, the $\Delta N_s$, $H_s$ and $V_{\text{end}}$ can be expressed as a function of $n_s$, and are given below (see Ref. [42] for details...
\[
\Delta N_* = \frac{3}{4} \left[ \frac{8}{3(1 - n_s)} - \left(1 + \frac{2}{\sqrt{3}}\right) \ln \left(\frac{8}{(1 - n_s)(3 + 2\sqrt{3})}\right) \right], \quad (3.15)
\]

\[
H_* \approx \pi M_{Pl}(1 - n_s) \sqrt{\frac{3}{2} A_s} \quad (3.16)
\]

and

\[
V_{\text{end}} \approx \frac{9}{2} \pi^2 A_s M_{Pl}^4 (1 - n_s)^2 \frac{1}{\left(1 + \frac{\sqrt{3}}{2}\right)^2}. \quad (3.17)
\]

Using the above expressions one can write \(m_\chi\) as a function of \(n_s\) and \(A_s\) for the Starobinsky model, and is given by

\[
m_\chi \approx 4\sqrt{\pi} M_{Pl} \text{Exp} \left\{ -\left(\frac{2}{3} \ln 3 + \frac{5}{3} \ln 2 - \frac{2}{3} \ln Y - \ln \left(\frac{k_{\text{eq}}}{a_0}\right) - \ln(1 + Z_{\text{eq(MR)}}) + \ln \rho_{\text{eq}}^{1/4} \right.ight.

\[
- \frac{3\tilde{w}_{\text{reh}}-1}{12(1+\tilde{w}_{\text{reh}})} \ln \left(\frac{\pi^2 g_{\text{reh}} T_{\text{reh}}^4}{3(1+\tilde{w}_{\text{reh}})}\right) - \frac{1}{3(1+\tilde{w}_{\text{reh}})} \ln \left(\frac{27}{4} \pi^2 A_s M_{Pl}^2 (1 - n_s)^2 \frac{1}{(1+\frac{\sqrt{3}}{2})^2}\right)
\]

\[
- \frac{3}{4} \left(\frac{8}{3(1 - n_s)} - \left(1 + \frac{2}{\sqrt{3}}\right) - \ln \left(\frac{8}{(1 - n_s)(3 + 2\sqrt{3})}\right) \right) + \ln \left(\pi M_{Pl} \sqrt{2A_s(1 - n_s)}\right) \right\}. \quad (3.18)
\]

For the Starobinsky model, using Eq. (3.18), the relation between \(m_\chi\) and \(n_s\) for different values of \(\tilde{w}_{\text{reh}}\) and \(T_{\text{reh}}\) are shown in figure 3. For this model when the reheating temperature is less than \(10^5\) GeV, curves with \(\tilde{w}_{\text{reh}} < 0\), within Planck’s 1σ bounds on \(n_s\) requires the modulus mass to lie in the super Planckian regime. However, for higher values of the reheating temperature all curves move toward the centre and predict the modulus mass in the sub-Planckian range.
by considering a feature in the inflaton potential the reheating parameters $\bar{w}_{\text{reh}}$ and $N_{\text{reh}}$ can be constrained for the large field, quartic hilltop and Starobinsky models. Using these constraints on the reheating parameters (the upper bounds on $\bar{w}_{\text{reh}}, T_{\text{reh}}$), we plot the modulus mass as a function of the scalar spectral index $n_s$ for different single field inflationary models in Fig. 4. For Planck’s central value of $n_s = 0.965$, we obtain $m_\chi \approx 1.64 \times 10^{16}, 3.17 \times 10^{18}$ and $2.18 \times 10^{18}$ GeV respectively for the quadratic large field, quartic hilltop and Starobinsky model. 

Figure 3: Plots of allowed modulus mass values $m_\chi$ as a function of $n_s$ for the Starobinsky model. All curves and shaded regions are as for figure 1.

4 Features in the inflaton potential and constraints on modulus mass

In Ref. [42] we showed that the successful explanation of the CMB low multipole anomalies by considering a feature in the inflaton potential the reheating parameters $T_{\text{reh}}, \bar{w}_{\text{reh}}$ and $N_{\text{reh}}$ can be constrained for the large field, quartic hilltop and Starobinsky models. Using these constraints on the reheating parameters (the upper bounds on $\bar{w}_{\text{reh}}, T_{\text{reh}}$), we plot the modulus mass as a function of the scalar spectral index $n_s$ for different single field inflationary models in Fig. 4. For Planck’s central value of $n_s = 0.965$, we obtain $m_\chi \approx 1.64 \times 10^{16}, 3.17 \times 10^{18}$ and $2.18 \times 10^{18}$ GeV respectively for the quadratic large field, quartic hilltop and Starobinsky model.
model. The $1\sigma$ lower limit of $n_s$ gives $m_\chi \approx 4.74 \times 10^{13}$GeV, $2.84 \times 10^{15}$GeV and $5.94 \times 10^{15}$ GeV for the quadratic large field, quartic hilltop and Starobinsky model respectively. The $1\sigma$ upper limit of $n_s$ gives $m_\chi > M_{Pl}$ which rules out late time modulus cosmology.

Figure 4: Plots of allowed modulus mass $m_\chi$ as a function of $n_s$ for different inflationary models by considering the constraints of Ref. [42] obtained for a successful explanation of the CMB low multipole anomalies by considering a step in the inflaton potential. The solid red, dotted green and dashed blue curves represent the quadratic large field, quartic hilltop, and Starobinsky model respectively. The dark gray and light gray shaded regions correspond to the $1\sigma$ and $2\sigma$ bounds respectively on $n_s$ from Planck 2018 data (TT, TE, EE + lowE + lensing) [1].

5 Reheating and the modulus mass

In Sec. 4, we obtained constraints on the modulus mass by using the bounds of the reheating parameters ($T_{reh}$ and $\bar{w}_{reh}$). Now, we are going to do the reverse, i.e., we will use the modulus mass as an input parameter to constrain the primordial reheating phase. For this, we use the older cosmological moduli bound $m_\chi \geq 30$TeV [43–45], and the bound obtained by Ref. [68] which is $m_\chi \geq 10^6$ TeV. To implement this, we have to invert Eq. (2.37), then, one can write out $T_{reh}$ as a function of $m_\chi$, $A_s$, $n_s$ and $\bar{w}_{reh}$. Using the bounds of $m_\chi$ as mentioned above, we plot $T_{reh}$ as a function of $n_s$ for different $\bar{w}_{reh}$ value for different single field inflationary models in Fig. 5. The left and right panel of Fig. 5 represent $m_\chi \approx 30$ TeV and $m_\chi \approx 10^6$ TeV respectively.

From Fig. 5a, 5c and 5e we see that for $m_\chi \approx 30$ TeV none of the three models predicts reheating temperature in the reasonable range within Planck’s $1\sigma$ bounds on $n_s$ from Planck 2018 data. On the other hand, from Fig. 5b, 5d and 5f for $m_\chi \approx 10^6$ TeV, only the Starobinsky model predicts the reheating temperature within the reasonable range. For the lower bound of $m_\chi$ as obtained in [68], the Starobinsky model predicts $T_{reh} \leq 10^5$ GeV for $\bar{w}_{reh} \leq 0$ within Planck’s $1\sigma$ bounds on $n_s$. However, for $0 \leq \bar{w}_{reh} \leq \frac{1}{3}$ any reheating temperature in the reasonable range is allowed within Planck’s $1\sigma$ bounds on $n_s$. 
Figure 5: Plots of $T_{reh}$ as a function of $n_s$ for different inflationary models. (a) and (b) are for the quadratic large field model with the modulus mass 30 TeV and $10^6$ TeV respectively. Similarly, (c) and (d) are for the quartic Hilltop potential and (e) and (f) are for the Starobinsky model with $m_\chi = 30$ TeV and $10^6$ TeV respectively in each case. All curves and shaded regions are as for Figure 1.
6 Discussion and conclusions

We have obtained a relation among the reheating parameters, the modulus mass \( m_\chi \) and the inflationary observables for single field inflationary models. This is done by tracing the evolution of specific observable scales in CMB from the time of their Hubble crossing during inflation to the present time. It is possible to obtain a relation between \( m_\chi \) and the scalar spectral index \( n_s \) which can be further constrained by the requirement that \( \bar{w}_{\text{reh}} \) and \( T_{\text{reh}} \) lie within physically reasonable range. This is shown in Figs. 1, 2, and 3. For all the single field inflationary models considered here, we found that for \( \bar{w}_{\text{reh}} < \frac{1}{3} \), the reheating temperature \( T_{\text{reh}} \) should be greater than \( 10^5 \) GeV to obtain the modulus mass in the sub-Planckian regime, \( m_\chi < M_{\text{Pl}} \).

Further, we considered modeling the CMB low multipole anomalies through feature in the inflaton potential. This gives a further handle on the reheating parameters. The simultaneous demand of explaining the CMB anomalies and disappearance of the heavy moduli to accord with observed cosmology requires that \( 10^{13} \) GeV \( \lesssim m_\chi \lesssim 10^{15} \) GeV depending on the model.

Finally we considered the recently improved bound on the lightest modulus mass, Ref. [68] taking account of precision CMB data, where it is estimated that \( m_\chi \gtrsim 10^6 \) TeV. This is expected to replace the older bound \([43–45]\) 30TeV. Figs. 5 contain plots of \( T_{\text{reh}} \) vs \( n_s \) implementing the constraint of Eq. (2.37). It is found that the lower bound \( m_\chi \approx 30 \) TeV is incompatible with physically reasonable \( T_{\text{reh}} \) and \( \bar{w}_{\text{reh}} \) for all of the single field inflationary scenarios considered. In the case of the newer limit \( m_\chi \approx 10^6 \) TeV as input, it is found that only the Starobinsky model predicts the reheating parameters in the physically acceptable range, and within Planck’s 1\( \sigma \) and 2\( \sigma \) bounds on \( n_s \).

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