Investigation of the Broadband Emission of the Gamma-Ray Binary HESS J0632+057 Using an Intrabinary Shock Model

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Received 2022 April 3; revised 2022 July 27; accepted 2022 August 1; published 2022 August 29

Abstract

We investigated a wealth of X-ray and gamma-ray spectral energy distribution (SED) and multiband light-curve (LC) data of the gamma-ray binary HESS J0632+057 using a phenomenological intrabinary shock (IBS) model. Our baseline model assumes that the IBS is formed by colliding winds from a putative pulsar and its Be companion and that particles accelerated in the IBS emit broadband radiation via synchrotron (SY) and inverse Compton upscattering (ICS) processes. Adopting the latest orbital solution and system geometry, we reproduced the global X-ray and TeV LC features, two broad bumps at $\phi \sim 0.3$ and $\sim 0.7$, with the SY and ICS model components. We found that these TeV LC peaks originate from ICS emission caused by the enhanced seed photon density near periastron and superior conjunction or Doppler-beamed emission of bulk-accelerated particles in the IBS at inferior conjunction. While our IBS model successfully explained most of the observed SED and LC data, we found that phase-resolved SED data in the TeV band require an additional component associated with ICS emission from preshock particles (produced by the pulsar wind). This finding indicates a possibility of delineating the IBS emission components and determining the bulk Lorentz factors of the pulsar wind at certain orbital phases.

Unified Astronomy Thesaurus concepts: High mass x-ray binary stars (733); High energy astrophysics (739); Relativistic binary stars (1386); Gamma-ray sources (633); Radiative processes (2055)

1. Introduction

High-energy $\gamma$-ray surveys using ground-based imaging air Cerenkov telescopes (e.g., VERITAS, H.E.S.S., and MAGIC), along with X-ray telescopes, have uncovered a rare subclass of binary systems detected above $E \sim 0.1$ TeV (e.g., Aharonian et al. 2006; Fermi LAT Collaboration et al. 2012). These so-called TeV $\gamma$-ray binaries (TGBs) harbor a compact object and a massive companion (O, B, or Be star), with a wide range of orbital periods spanning 3.9 days to $\sim 50$ yr. TGBs emit orbitally modulating broadband radiation from X-ray to gamma-ray energies (e.g., Mirabel 2012). It is widely accepted that very high energy (VHE; $\gtrsim 0.1$ TeV) emission from TGBs implies that particles should be accelerated to GeV–TeV energies in the system (De Becker et al. 2017). Among $\lesssim 10$ TGBs discovered thus far, the compact object has been identified as a neutron star in only three systems: PSR B1259–63 (Johnston et al. 1992), PSR J2032+4127 (Ho et al. 2017), and LS 1+61°303 (Weng et al. 2022).

Studies of TGBs have been carried out primarily by modeling their light curves (LCs) and broadband spectra in the X-ray and gamma-ray bands. The multiwavelength spectral energy distributions (SEDs) are well characterized by two nonthermal components at low energy ($\lesssim 100$ MeV) and gamma-ray bands ($\sim$ TeV) that are mixed with thermal emission from the companion. It is thought that the low-energy nonthermal emission extending from radio to $\lesssim 100$ MeV band is produced by synchrotron (SY) radiation of energetic electrons (e.g., Tavani et al. 1994; Dubus 2013). The GeV emission may consist of the SY and the pulsar magnetospheric radiation with some contribution from inverse Compton scattering (ICS) of stellar thermal photons by low-energy electrons (e.g., with a Lorentz factor of $\sim 10^3$; Zabalza et al. 2013; Dubus et al. 2015). The VHE emission is believed to be produced by ICS of the stellar photons by high-energy electrons (e.g., Dubus 2006; Khangulyan et al. 2008; Chernyakova & Malyshev 2020). The X-ray and VHE emission, resulting from SY and ICS in the shocked region, respectively, shows a strong dependence on orbital phase due to various factors such as the intrabinary distance, anisotropic radiative processes, and relativistic Doppler boosting.

These emission mechanisms have been employed primarily in two scenarios for TGBs: a microquasar and an intrabinary shock (IBS) scenario. In the microquasar scenario, the “unknown” compact object is assumed to be a black hole with bipolar and relativistic jets. Particles are accelerated to high energies in the jets, and orbital variation of the jet viewing angle generates the orbital modulation in the emission (e.g., Bosch-Ramon 2007; Marcote et al. 2015). In the IBS scenario, the compact object is assumed to be a pulsar whose wind interacts with the companion’s outflow. The wind–wind collision produces a contact discontinuity (CD) in the IBS region where pulsar-wind particles are accelerated and emit broadband nonthermal radiation (e.g., Dubus 2006). Orbital variation of the orientation of the IBS flow with respect to the observer’s line of sight (LOS) causes the high-energy emission to modulate with the orbital period (e.g., van der Merwe et al. 2020). Given the three TGBs containing radio pulsars combined with constraints on mass function, the compact object in other TGBs is generally considered to be a neutron star (e.g., Dubus 2013).

The TeV source HESS J0632+057 (J0632 hereafter) was identified as a TGB by a detection of its $\sim 320$-day orbital modulation in the VHE band (Acciari et al. 2009). Later, X-ray and GeV modulations on the orbital period ($P_{\text{orb}}$) were detected...
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(Aliu et al. 2014; Li et al. 2017), confirming the VHE identification. An optical spectroscopic study identified the companion to be a Be star (HD 259440; Aragona et al. 2010) with an equatorial disk. The compact object has not been identified yet, but a Chandra imaging found a hint of extended emission around the source, which was interpreted as a signature of wind–wind interaction (Kargaltsev et al. 2022).

In this paper, we test the orbital solution suggested by TAH21 and determine the IBS parameters of J0632 using the \( \sim \)GeV and VHE measurements. Our phenomenological model fit to the extensive X-ray and gamma-ray data puts some constraints on magnetohydrodynamic (MHD) flows in the IBS that are useful to further MHD simulations of J0632 and other TGBs. We describe IBS structure in Section 3 and present emission model components in Section 4. We then use the model to explain the broadband data of J0632 in Section 5. We discuss implications of the modeling in Section 6 and present a summary in Section 7.

2. Orbital Solutions for J0632

The X-ray and the VHE LCs of J0632 exhibit a similar shape characterized by broad bumps at orbital phases \( \phi \approx 0.25 \) and 0.75 and a sharp spike at \( \phi = 0.35 \) (e.g., Adams et al. 2021; Tokayer et al. 2021). These features could probe the emission mechanisms and help infer the properties of particle acceleration and flow in the binary system. However, the orbital solution of J0632 has not been well determined. While optical data have provided accurate orbital solutions for other TGBs, the situation for J0632 is unclear. Orbital solutions derived from radial velocity measurements using the H\( \alpha \) line do not agree with each other. For the reference epoch of MJD 54857 used throughout this paper, the solution inferred from an optical study by Casares et al. (2012) suggests a highly eccentric orbit with an eccentricity of 0.83, periastron at \( \phi = 0.967 \), and LOS at \( \phi = 0.961 \). In contrast, Moritani et al. (2015) suggested a less eccentric orbit with eccentricity of 0.64, periastron at \( \phi = 0.663 \), and LOS at \( \phi \approx 0.17 \). Even considering that their radial velocity curves were folded on different periods, 321 days and 313 days in Casares et al. (2012) and Moritani et al. (2015), respectively, the solutions differ substantially.

X-ray LC data provide alternate orbital solutions to those obtained with optical data. Two such solutions are obtained by attributing the bumps in the X-ray LC to disk interactions (Malysh et al. 2019, Chen & Takata 2022). Most recently, Tokayer et al. (2021) derived an orbit of J0632 by modeling the most extensive X-ray LC data with an IBS model (Figure 1). This latest orbital solution seems to be well justified since the model matches the X-ray LC data well and accounts for the enhanced hydrogen column density (\( N_{\text{H}} \)) observed at some orbital phases, which Malysh et al. (2019) and Tokayer et al. (2021) attributed to the pulsar–disk interaction. These three orbits inferred from modelings of X-ray LCs folded on \( P_{\text{orb}} \approx 317 \) days are all similar to one another even though the emission models employed in those works are somewhat different. The suggested eccentricities are modest (0.4–0.5), and phases of periastron and LOS are \( \phi = 0.3–0.4 \) and 0.7–0.8, respectively. Considering that the three X-ray-inferred orbits broadly agree, this leaves three orbital solutions for J0632 that significantly disagree with one another—two from optical data and one from X-ray studies. Because it is unclear which orbital solution is correct, it will be helpful to check to see whether the suggested orbits can explain VHE data (LC and SED) using emission models, which has not been done for any of the suggested orbits.

The optical orbits of Casares et al. (2012) and Moritani et al. (2015) are incompatible with a shock emission scenario as noted by Chen & Takata (2022). Thus, they are inadequate for an IBS study of J0632. The X-ray orbits of Chen & Takata (2022) and Malysh et al. (2019) were constructed using an inclined disk model that employs a termination shock and its interaction with an inclined disk. This is essentially a one-zone model with a disk, as the shock region is assumed to be a point source (Chen & Takata 2022), whereas the IBS model of Tokayer et al. (2021, hereafter TAH21) took into account the multizone emission from a cone-shape IBS region. Because the IBS, unlike the one-zone termination shock, produces Doppler-boosted emission along the shock tail (e.g., Dubus 2013; An & Romani 2017; van der Merwe et al. 2021), the orbit of J0632 inferred by an IBS model (Figure 1; TAH21) slightly differs from that obtained by the inclined disk model. Note that TAH21 also applied a one-zone shock model to the broadband SEDs of J0632, but they did not attempt to model the VHE LC data. Given that TGBs should form an extended IBS as demonstrated by hydrodynamic simulations (e.g., Bogovalov et al. 2008; Dubus et al. 2015; Bosch-Ramon et al. 2017), it is necessary to consider a multizone shock case for modeling both the multiwavelength SED and LC data in detail. Hence, we restrict our study to an IBS scenario and the orbit of TAH21; the distinct features in the X-ray and VHE LCs allow us to determine the IBS properties more accurately.
3. Structure of the IBS

In our IBS model, a pulsar injects cold pulsar-wind (preshock; purple arrows in Figure 2) electrons and magnetic field \( B \) into the IBS (blue line in Figure 2), and the electrons are accelerated to very high energies in the shock (e.g., Tavani & Arons 1997; Dubus et al. 2015). These particle injection and acceleration schemes have been widely used in the previous models of TGBs (e.g., Sierpowska-Bartosik & Torres 2008; Dubus et al. 2015; An & Romani 2017; Chernyakova & Malyshev 2020; Xingxing et al. 2020). In this work, we adopt an analytical approach for modeling the global features of the flows, although hydrodynamics (HD) simulations showed more complexities and substructures in the particle flows (e.g., Bosch-Ramon et al. 2015). Below we describe the prescriptions, assumptions, and formulae for the IBS flows in our model.

3.1. Pulsar Wind and Stellar Outflow

The 2D shape of an IBS is determined by the pressure balance of the pulsar and the stellar winds. The pulsar wind is thought to be composed of cold and relativistic plasma. MHD simulations suggest that the pulsar wind can be anisotropic with higher flow velocities and particle densities in the equatorial plane (e.g., Tchekhovskoy et al. 2016). Massive stars emit isotropic outflows, and Be-type stars such as the companion star of J0632 may have strong equatorial outflows (decretion disk; e.g., Figure 1), which are evidenced by an infrared (IR) excess (e.g., Waters & Lamers 1984). Besides, stellar outflows can be clumpy and highly variable, inducing large variability to the IBS shape and thus to the observed emission of TGBs (e.g., Bosch-Ramon 2013).

The shape of an IBS formed by anisotropic wind interaction is difficult to determine observationally given that the pulsar’s spin axis and anisotropic geometry of the pulsar and/or stellar winds are not known. In this work, we assume an isotropic geometry for both the pulsar and the stellar winds since an IBS formed by slightly anisotropic winds does not differ much from the isotropic case (e.g., Kandel et al. 2019); the most important shock tangent angle near the tail changes only slightly, which can be accounted for by a small change in the wind momentum flux ratio and/or observer’s inclination angle in our phenomenological model. Note, however, that the strong equatorial outflow (i.e., disk) of a Be-type companion can significantly alter the IBS shape at the pulsar-crossing phases (e.g., phases 0.12 and 0.35 in Figure 1).

Under the isotropic wind assumption, our model does not properly account for the IBS–disk interaction at the disk-crossing phases. Moreover, emission produced by the interaction depends strongly on other parameters such as density and heating that are poorly known and need to be determined by simulations and observations of the interaction sites. We do not consider such interaction in this work, and hence our model does not naturally reproduce the X-ray and TeV spikes at \( \phi \approx 0.3 \). Note, however, that TAH21 arbitrarily increased \( B \) at the interaction phases to reproduce the X-ray spike. We adopt this prescription for the X-ray modeling, and thus our model phenomenologically matches the X-ray LC but not the VHE LC (see Section 5.2.1 for more details).

3.2. Shape of the IBS

The orbital variability of high-energy emission observed in TGBs is thought to be induced by a change in the emission and viewing geometry of the IBS (e.g., Romani & Sanchez 2016). In general, the IBS is assumed to have a paraboloid shape near the apex, but it is distorted significantly at large distances from the system (e.g., Bosch-Ramon et al. 2017). Because the high-energy emission in the X-ray to VHE band is mostly produced in the inner regions of the IBS (e.g., Dubus et al. 2015; Bosch-Ramon et al. 2017), the analytic formulae (Appendix A) presented in Cantó et al. (1996) for CD produced by isotropic wind–wind interaction is adequate for our modeling effort. The IBS shape is basically determined by the winds’ momentum flux ratio:

\[
\beta = \frac{\dot{E}_{\text{SD}}}{M_w v_w c},
\]

where \( \dot{E}_{\text{SD}} \) is the pulsar spin-down power, \( M_w \) is the mass-loss rate of the companion, \( v_w \) is the velocity of the companion’s
wind, and \( c \) is the speed of light. The massive companion’s wind is likely stronger than the pulsar’s (\( \beta < 1 \)); thus, it is likely that the IBS is formed around the pulsar in TGBs.

A schematic view of the vertical cross sections of a TGB system and its IBS is depicted in Figure 2. As denoted in the figure, an emission zone at a distance \( s \) (red solid line) along the IBS (blue solid) from its apex is \( r_i \) away from the star at an angle \( \theta_i \) and is \( r_p \) away from the pulsar at an angle \( \theta_p \). The blue dashed line shows the direction of the particle flow in the emission zone (polar angle \( \theta \)) and, the green line is an observer’s LOS with an inclination \( \Theta \). The asymptotic tangent angle (blue arrow; half-opening angle \( \theta_{\text{cone}} \)) of the shock is determined by \( \beta \) (Equation (A4)). These geometrical parameters for an IBS are computed using the equations given in Appendix A (see Cantó et al. 1996, for more detail).

Both \( r_i \) and \( r_p \) vary orbitally because they are proportional to the orbital separation (\( d_{\text{orb}} \)) between the pulsar and the companion (e.g., Equation (A3)): 

\[ d_{\text{orb}}(\phi) = \frac{a(1 - e^2)}{(1 + e \cos \phi_0)}, \]

where \( a \) is the semimajor axis, \( e \) is the eccentricity, and \( \phi_0 \) is the true anomaly. We assume that the emission zone extends to \( s_{\text{max}} = 3d_{\text{orb}} - 5d_{\text{orb}} \) (e.g., Dubus et al. 2015; Bosch-Ramon et al. 2017). Note that the emission-zone size (\( s_{\text{max}} \)) in our model is assumed to be a constant multiple of \( d_{\text{orb}} \), which changes with the orbital phase in eccentric orbits (Equation (2)). However, the solid angle subtended by the IBS from the pulsar (i.e., pulsar’s energy injection into IBS) remains unchanged.

### 3.3. Particles in the Pulsar-wind Zone

The preshock pulsar wind is composed of cold electrons accelerated in the pulsar-wind zone (purple arrows in Figure 2). The exact location and physical processes of the particle acceleration are not well known, and hence the energy distribution of the particles is unclear. Some physical processes may produce narrow Maxwellian-like distributions (e.g., Hoshino et al. 1992; Sironi & Spitkovsky 2011), while others may produce a broad power-law distribution (e.g., Jaroschek et al. 2008). As such, various distributions have been assumed in TGB emission models previously; a broadened delta function (e.g., Khangulyan et al. 2011), power-law (Sierpowska-Bartosik & Torres 2008), or Maxwellian distribution (Takata et al. 2017). These distributions would result in slightly different shapes for the ICS SED of the preshock particles. In this work, we assume that the preshock particles are accelerated near the light cylinder \( R_{\text{LC}} \ll r_p \) of the pulsar (e.g., Aharonian et al. 2012), flow isotropically, and follow a relativistic Maxwellian energy distribution

\[
\frac{dN_{e}}{d\gamma_e} = \frac{N_{0} \gamma_e^{2} \beta_e}{\Theta K_2(1/\Theta)} e^{-\gamma_e/\phi},
\]

where \( \beta_e \) is \( \sqrt{1 - 1/\gamma_e^2} \), \( K_2 \) is the modified Bessel function of the second kind, and \( \Theta \) is a temperature parameter that we adjust to have a Lorentz factor \( \gamma_{e,\text{peak}} \approx 10^8 \) at the peak of the distribution (e.g., Amato & Olmi 2021). The number and energy of the particles injected into the preshock by the pulsar are given by

\[
\hat{N} = \int dN_{e}^{\text{pre}} d\gamma_e
\]

and

\[
\int \gamma_e m_e c^2 dN_{e}^{\text{pre}} d\gamma_e = \eta E_{\text{SD}},
\]

where \( \eta \) is the particle conversion efficiency of the pulsar’s spin-down power (e.g., Gelfand et al. 2009; Uchiyama et al. 2009). Then, the number of particles within a radial length \( dr \) over the 4\( \pi \) solid angle in the pulsar-wind zone is given by \( \frac{dN_{e}}{dr} = \frac{N_{0}}{r^2} \). At certain orbital phases, the preshock flow along the LOS may be open if it does not cross the IBS. In this case, we stop the flow at a distance \( \approx 5d_{\text{orb}} \), where a back shock is expected to form (Dubus et al. 2015). We verified that extending the preshock flow to larger distances at these phases had no significant impact on the output emission.

### 3.4. Particles and Their Flows in IBS

The preshock particles are injected into the entire surface of the IBS and further accelerated there as in termination shocks of pulsar-wind nebulae (PWNes; Kennel & Coroniti 1984). The particles flow along the conic surface (blue in Figure 2) toward the tail of the IBS and exit the emission zone at \( s = s_{\text{max}} \). As IBS and PWN physics share some common ground, we assume that the IBS electron distribution in the flow rest frame is isotropic and follows a broken power law (e.g., as in PWN cases; Sironi & Spitkovsky 2011; Cerutti & Giacinti 2020) between a lower (\( \gamma_e,\text{min} \)) and an upper (\( \gamma_e,\text{max} \)) bound with a break at \( \gamma_b \) that is caused by particle cooling:

\[
\frac{dN_e}{d\gamma_e} = \begin{cases} 
N_{0} \gamma_e^{-p_1}, & \gamma_e,\text{min} \leq \gamma_e < \gamma_b \\
N_{0} \gamma_b^{-p_2}, & \gamma_b \leq \gamma_e \leq \gamma_e,\text{max}.
\end{cases}
\]

Here \( p_1 \) may be determined by particle acceleration modeling or more directly by X-ray observations. It is almost certain that \( p_1 \) varies over the orbit since gamma-ray binaries have shown orbital variations of their X-ray spectral index (Bosch-Ramon et al. 2005; Chernyakova et al. 2009; Fermi LAT Collaboration et al. 2012; An et al. 2015). Because the origin of the variability is still uncertain and yet it does not have a significant impact on the LCs, we assume a constant \( p_1 \) over the orbit.

In our model, the particle cooling is manifested as a spectral break of the stationary electron distribution (Equation (6)). In the case that \( B \) is uniform over the IBS, a \( p_2 - p_1 = 1 \) break is expected. However, the degree of the spectral break \( p_2 - p_1 \) may differ in IBSs because \( B \) varies with \( s \) and fresh particles are injected at every point on the IBS. The cooling timescale of the highest-energy electron is on order of \( \approx 100 \) s (e.g., \( \gamma_e,\text{max} \approx 10^8 \) and \( B \approx 1 \) G), and then a spectral break is expected at \( \gamma_b \approx 5 \times 10^6 \). For typical \( B \approx 1 \) G in IBSs of TGBs, particles at the break would emit \( \sim \)MeV photons; \( p_2 \) and \( \gamma_b \) cannot be inferred directly from data owing to the lack of sensitive observations in the MeV band. For \( \gamma_e,\text{max} \), we assume that particles are accelerated to the radiation reaction limit so that the SY photon energy emitted by the highest-energy particles is \( \sim 100 \) MeV (but see Section 5.2.1).

The flow bulk Lorentz factor \( \Gamma \) can vary along the IBS in a complex way as was seen in relativistic HD simulations (e.g.,
Bogovalov et al. 2008; Dubus et al. 2015), where a small fraction \( \xi \) (\( \ll 1 \)) of the particles in the flow is seen to be bulk accelerated to high \( \Gamma \). We follow An & Romani (2017) and TAH21, and for simplicity we assume two distinct populations of particles in the IBS, one with a small constant bulk Lorentz factor (\( \Gamma \approx 1 \); slow flow) and another with a linearly growing bulk Lorentz factor \( \Gamma(s) = 1 + \frac{s}{s_{\text{max}}} (\Gamma_{\text{max}} - 1) \) (fast flow). The flow speed (\( v_{\text{flow}} = c\sqrt{1 - 1/\Gamma^2} \)) of the “fast flow” is determined by this equation, but that of the slow flow needs to be prescribed. In an ideal model for PWNe (Kennel & Coroniti 1984), the flow speed in the post-shock region is predicted to be \( \approx c/3 \), and we use a similar value for the slow flow (but see below). Flows with different velocities are subject to instabilities and thus may not be stable. Our phenomenological description of the two flows does not account for the physical instabilities, and we assume that the two flows are physically separated. More accurate description of the flow requires relativistic HD simulations as noted above; the speed change from the “slow” to “fast” flow may be more continuous (e.g., Bogovalov et al. 2008; Dubus et al. 2015).

The number of particles per unit length along the IBS (integrated over the azimuth angle \( \phi_s \) of the shock cone) is given by the continuity condition (e.g., Cantó et al. 1996) as

\[
\frac{dN_s}{ds} = \frac{\dot{N} (1 - \cos \theta_s(s))}{2v_{\text{flow}}},
\]

where \( \dot{N} \) is the number of preshock particles injected by the pulsar given in Equation (4). The number of particles in the IBS \( N_s = \int \frac{dN_s}{ds} ds \) is controlled by particle residence time \( s_{\text{max}}/v_{\text{flow}} \). This determines the relative contribution of the preshock and the IBS particles to ICS (VHE) emission. As the pulsar’s injection into the IBS is assumed to be the same at every orbital phase, we assume that \( N_s \) in the IBS is constant over the orbit. This means that \( v_{\text{flow}} \) varies over the orbit; once this value is prescribed at a given phase, values at the other phases are determined to preserve \( N_s \). Note that this assumption does not have a large impact on matching the observational data with the phenomenological model; using a different assumption (e.g., varying \( N_s \) and constant \( v_{\text{flow}} \)) can also reproduce the measurements with slightly different \( \theta_s \) and \( \Gamma_{\text{max}} \).

We split the IBS cone, seen in Figure 2, into \( 21 \times 361 \) computational grids, which are described in greater detail in Sections 4.2, 4.3, and 5.2. We apply the same particle distribution over the IBS zones (Equation (6)), but the number of particles (Equation (8)), the fast flow \( \Gamma \) (Equation (7)), and the flow direction (Figure 2) differ in each zone, as they depend on \( s \).

### 3.5. Magnetic Field in IBS

The magnetic field in an IBS is assumed to be supplied by the pulsar and randomly oriented. For a pulsar with a surface magnetic field strength \( B_s \) and a spin period \( P_s \), \( B(s) \) in an emission zone in the IBS is estimated to be

\[
B(s) = B_s \left( \frac{2\pi R_{\text{NS}}}{cP_s} \right)^3 \left( \frac{cP_r}{2\pi r_p(s)} \right) = B_0 \left( \frac{r_0}{r_p(s)} \right). \tag{9}
\]

where \( R_{\text{NS}} \) is the radius of the neutron star and \( r_0 \) is distance to a reference point. The pulsar continuously injects particles and \( B \), which are frozen together as they stream to the IBS, potentially creating complex \( B \) structures within the IBS flow. Relativistic HD computations (e.g., Bogovalov et al. 2012; Dubus et al. 2015) indeed show very complicated structures of \( B \): nonlinear changes with \( s \), variations over the thickness of the IBS, and most importantly a \( B \) that decreases with increasing \( s \). However, the simplified binary pulsar magnetic relationship in Equation (9) is an appropriate substitute for the complex MHD results, as it captures the important inverse \( B \sim s \) relationship. Prior analyses of IBSs of pulsar binaries and TGBs have used this approach (e.g., Dubus 2013; Romani & Sanchez 2016), and we adopt Equation (9) in this work as well. In reality, deviations from the \( B \propto 1/r_p \) dependence could be present, but an investigation of such effects is beyond the scope of this paper, which attempts to account for the global IBS features and SED/LC data.

The value of \( B_0 \) can vary substantially depending on the pulsar parameters and is often assumed to be \( \approx 1 \, G \) in TGBs (e.g., Dubus 2013). In our model, \( B_0 \) at the reference point (i.e., the shock nose at the inferior conjunction of the pulsar) is prescribed, and \( B \) values at the other positions in the IBS and at different orbital phases are computed using Equation (9).

### 4. Emission in TGB Systems

The observed emission of TGB systems spans over the entire electromagnetic wavelengths from radio to TeV gamma-ray bands. The radio emission region is often observed to be elongated (e.g., > \( d_{\text{orb}} \); Ribó et al. 2008; Moldón et al. 2011) and so the radio emission is thought to be produced by particles that escaped from the IBS which we do not model. The equatorial disk of a Be companion emits in the IR to optical band, and the massive companion (blackbody) radiates primarily in the optical band. These IR and optical photons provide the preshock and IBS particles with seeds for ICS.

Below, we describe SY and ICS emissions of the preshock and IBS particles relevant to X-ray and VHE emissions in detail. IR and optical photons from the disk and the companion play an important role for the VHE emission via ICS.

#### 4.1. Stellar Emissions

Seed photons for ICS are primarily produced in the atmosphere and the disk of the companion star. The atmospheric emission is presumably isotropic blackbody radiation with absorption/emission lines and is observed as a prominent optical bump in SEDs of TGBs. This emission can be well characterized by the temperature \( T_a \) and radius \( R_a \) of the star that can be measured with spectroscopic observations. Although there are a number of absorption/emission lines in the spectrum of a star, these narrow features will be washed out in the ICS processes by electrons with featureless, nonthermal energy distributions. Hence, a blackbody spectrum, scaled to the distance to an emission zone (\( r_* \); Figure 2), is a good approximation for ICS seed photons. A blackbody spectrum of
J0632 computed with $T_\text{p} = 30,000$ K and $R_\ast = 6.6$ $R_\odot$ (Aragona et al. 2010) is presented in red in Figure 3, along with observed J0632 IR-to-optical flux densities.

Disk emission of a Be star is produced by free–free and free–bound processes in the disk (e.g., Waters & Lamers 1984; Carciofi & Bjorkman 2006) and is observed as an IR excess in the spectrum (e.g., Klement et al. 2017). While the observed disk spectra appear simple (e.g., broad IR bump; Figure 3), computing the emission spectrum is highly complicated because it is not uniform over the surface of the disk owing to changes of the disk density and thickness with distance from the star. Moreover, the disk emission is anisotropic (e.g., Carciofi & Bjorkman 2006) because of varying optical depth depending on the disk-viewing angle ($\theta_0$).

The disk emission itself is usually much weaker than the stellar atmospheric one (e.g., Figure 3; see also Klement et al. 2017), and therefore EC emission off of disk-seed photons does not contribute much to VHE emission in TGBs (e.g., van Soelen et al. 2012; see also Section 4.3). Furthermore, a detailed shape of the disk spectrum is blurred in the ICS process by the broad electron distributions. Hence, it is well justified to use an approximate continuum model for the disk emission (e.g., multitemperature blackbody), but we investigate a more complex disk emission model below.

Although a complete disk model (considering radiation transfer and radiative equilibrium) requires computations of the disk structure and its emission for arbitrary density and velocity distributions (e.g., Carciofi & Bjorkman 2006), the disk emission can be simplified for our purposes with an assumption of power laws for radial distributions of density, scale height, and temperature. In a cylindrical coordinate system for the disk, Carciofi & Bjorkman (2006) assumed $\rho(r) = \rho_0(r/R_\ast)^{3/2} \exp(-z^2/2H^2)$, $H(r) = H_0(r/R_\ast)^{3/4}$, and $T_{\text{disk}}(r) = T_\text{d}(r/R_\ast)^{-\gamma}$ for the density, scale height, and temperature, respectively, and presented analytic formulae for computation of optical depths ($\tau(r)$) and emission spectra of pole-on disks. $n_d$ and $\beta_d$ are typically $\sim 3$ and $\sim 1.5$, but they vary from source to source (Klement et al. 2017). We followed the procedure described in the Appendix of Carciofi & Bjorkman (2006) for computations of optical depths and the disk spectra. Note that these computations require Gaunt factors for which Carciofi & Bjorkman (2006) used a long-wavelength approximation. We instead used approximations from Waters & Lamers (1984), which are more accurate within our frequency range of $10^{12}$–$10^{16}$ Hz. For an inclined disk as compared to a pole-on disk, the depth of the emitting medium (disk) increases by a factor of $1/\cos \theta_\ast$, and so we assume that the optical depth of the disk varies as $1/\cos \theta_\ast$ (see van Soelen et al. 2012 for a more accurate treatment of the inclined disk case). This simplified emission model with the power-law prescriptions suffices to compute the disk spectrum in our model.

The disk parameters for J0632 are not known, and thus we adopt their typical ranges (Carciofi & Bjorkman 2006; Klement et al. 2017) to match the observed data of J0632 (Figure 3). The observed spectrum of an isothermal ($\tau_d = 0$) hydrogen disk extending to $60 R_\ast$ with a central density $\rho_0 = 10^{-17}$ g cm$^{-3}$, disk temperature $T_d = 0.7 T_\ast$, $n_d = 2.45$, and $\beta_d = 1.5$ is presented in Figure 3. The disk emission amplitude parameters (e.g., $\rho_0$) used to match the observed IR-to-optical measurements depend on the assumed disk-viewing angle ($i_\ast$) between the LOS and the surface normal vector of the disk. For the measured IR-to-optical flux densities (Figure 3), the “intrinsic” disk emission would be inferred to be stronger for a larger $i_\ast$, and then the disk will provide the preshock and IBS with a larger amount of ICS seeds. Because it was suggested that the disk of J0632 is seen nearly edge-on (e.g., Aragona et al. 2010; TAH21), we assume a large $i_\ast$ of $85^\circ$ to test a limiting case.

We verified our simple disk emission model by comparing the modeling results with those of Carciofi & Bjorkman (2006) and van Soelen et al. (2012). First, our model for the spectrum of the Be disk in J0632 (Figure 3) is similar to that in another TGB, PSR B1259–63 (van Soelen et al. 2012). Second, we found that surface brightness of the disk at high frequencies ($10^{14}$ Hz) is significantly higher in the inner region ($r \lesssim 2 R_\ast$) than in the outer regions, which is consistent with a previous result (e.g., Figure 13 of van Soelen et al. 2012). Third, emission computed by our model for a moderately inclined disk does not deviate much from that for a pole-on disk, similar to previous results (Carciofi & Bjorkman 2006; van Soelen et al. 2012). These verify that our simplified disk emission model captures the main emission features of Be disks.

4.2. Synchrotron Radiation

With particle distributions and $B$ in IBS being prescribed (Sections 3.3–3.5), an SY SED is computed using the formulae given in Appendix B (see Finke et al. 2008, for more details). We construct an IBS surface at each of 100 orbital phases and divide it into $21 \times 361$ (axial and azimuthal) emission zones. The particle distribution is computed in 300 energy ($\gamma_e$) bins, and the resulting SED is computed in 200 frequency bins. These binnings were chosen to achieve sufficient accuracy to fit the observation data while maintaining reasonable computation time. As an example, we consider a circular orbit with orbit radius $a = 3.5 \times 10^{13}$ cm, periastron at $\phi_{\text{periastron}} = 0.3$, and the inferior conjunction (IFC) of the pulsar at $\phi_{\text{IFC}} = 0.75$. For the IBS, we used $\beta = 0.07$, $p_1 = 2.3$, $p_2 = 2.6$, $\gamma_{\text{min}} = 1.5 \times 10^5$, $\gamma_0 = 5 \times 10^6$, $s_{\text{max}} = 4d_{\text{obs}}$, $\xi = 0.1$, $B_0 = 0.28$ G, $\Gamma_{\text{max}} = 5$, and $\theta_1 = 50^\circ$. Using these baseline values, we compute
parameters in the IBS flow (see Figure 2), e.g., $N_e$ (Equation (8)), $\Gamma$ and $\delta_0$ (Equations (7) and (B3)), and $B$ (Equation (9)). Then, the SY emission SED (Equation (B1)) in each zone is computed neglecting the SY emission of the "cold" preshock electrons.

Figure 4 shows an orbitally averaged SED. The computed SED matches the expected emission features at energies $10^2$, $10^3$, and $10^6$ eV, which correspond to $\gamma_{c,min}$, $\gamma_{b}$, and $\gamma_{c,max}$, respectively. Even though the fast flow is assumed to be only a small fraction ($\xi = 0.1$) of the slow flow, the fast-flow emission is highly boosted near the IFC and is noticeably strong in the phase-averaged SED. The shapes of the SY SEDs of the two flows are similar, but that of the fast flow is shifted to higher energies (blue in Figure 4) because of Doppler beaming by the bulk motion (Equation (B3)).

The SY LC of IBS flows is influenced by the orbital and the IBS parameters. Orbital variation of the "slow" flow is induced by varying $r_p$ in an eccentric orbit as $B \propto 1/r_p \propto 1/d_{orb}$ (Equations (9) and (A3)). Hence, LCs of the slow flow (dashed lines in Figure 5(a)) have a peak at periastron $\phi_{peria} = 0.3$. Because SY flux ($F_{SY}$) is proportional to $B^2(\rho_i+1)^2$ (Dermer 1995; An & Romani 2017), we anticipate the orbital variation being $\propto 1/d_{orb}^{(\rho_i+1)/2}$. For eccentricity $e$, the min-to-max ratio in the LC of the slow flow (e.g., dashed lines in Figure 5(a)) would be $\propto (1+e)(\rho_i+1)^{2/3}$ (e.g., Equation (2)).

Emission of the "fast" flow arises mostly from the tail ($s \approx s_{max}$) of the shock, where $\Gamma$ and the particle density are largest (Equations (7) and (8)). Because IBS particles flow along a cone-shape surface (blue in Figure 2) and the emission of the fast flow is highly beamed in the flow direction, the sky emission pattern of the fast flow will be ring-like in the shock tail direction (e.g., Romani & Sanchez 2016). Due to different viewing angles of the shock tail, the Doppler factor will vary with the orbital phase. Since $F_{SY} \propto B^2(\rho_i)^{2/3}$ (e.g., Dermer 1995; An & Romani 2017), the observed flux of the fast flow will be largest near the IFC, where the flow direction aligns well with the LOS (e.g., Figure 2, left). Note that $B$ ($\propto 1/d_{orb}$) also varies with $\phi$ and affects the amplitude of the variation, but this effect is small compared to the $\delta_0$ effect for the fast-flow emission. Hence, its peak occurs at the IFC.

Figure 5 shows the SY LC’s dependencies on the orbital and IBS parameters. Panel (a) shows an effect of the eccentricity ($e$). For a circular orbit ($e = 0$), a modulation of the slow-flow emission is very weak (i.e., weak beaming due to low $v_{flow}$); it was ignored in this example for clarity. The fast-flow emission modulation is large even in the $e = 0$ case. In an eccentric orbit, the phase separation between periastron and the IFC also affects the shape of the SY LC (e.g., Figure 5(b) with an assumed $e = 0.5$). The double peaks of the fast-flow emission (solid line) may have substantially different amplitudes and widths; one closer to the periastron is higher and sharper because of larger $B$ and rapid orbital motion of the pulsar.

The effects of $\theta_s$, $\beta$, $\Gamma_{max}$, and $s_{max}$ are presented in Figures 5(c)–(f), respectively. For a given IBS opening angle ($\theta_{cone}$), $\theta_s$ determines the viewing angle ($\theta_v$) and thereby $\delta_0$ (Equation (B3)) for the emission. For $\theta_s = 0$, circular orbits do not cause a modulation of the slow flow or the fast flow. As $\theta_s$ grows, the LOS becomes closer to the shock tangent (i.e., emission ring) near the IFC, and the modulation induced by orbitally varying $\theta_v$ should be observed. For a sufficiently large $\theta_s$ (i.e., $\theta_s > \pi/2 - \theta_{cone}$; Figure 2), the LOS crosses the emission ring twice near IFC, and the fast-flow emission bump in the LC splits into two peaks (red vs. purple in Figure 5(c)); the separation between them (LOS crossings of the emission ring) increases with $\theta_s$ and is $2\theta_{cone}$ for $\theta_s = 90^\circ$.

Effects of $\beta$ (Figure 5(d)) are similar to those of $\theta_s$ since $\beta$ determines $\theta_{cone}$ (Equation (A4)) and thus $\theta_v$. However, separation of the two peaks in the fast-flow LC depends more sensitively on $\beta$ than $\theta_s$. This is an obvious geometrical effect of the former ($\beta$) directly changing the emission-ring size $\theta_{cone}$. Note also that $r_p$ (see Figure 2) is determined by $\beta$ (e.g., Equation (A3)); for a smaller $\beta$, $r_p$ is smaller and thus $B$ ($\propto 1/r_p$) in the IBS is stronger, increasing the SY flux of the slow flow as well (dashed lines in Figure 5(d)). The bulk Lorentz factor of the fast flow near the shock tail (i.e., $\Gamma_{max}$) controls the width of each peak (i.e., $1/\Gamma$ beaming), as well as the emission strength of the fast flow (Figure 5(e)).

The emission strengths of the slow- and fast-flow particles change depending on the length of the IBS ($s_{max}$) even if the particle number does not change (Figure 5(f)). This is because of the decrease in $B$ at large distances from the pulsar ($r_p$); the total IBS emission is reduced with increasing $s_{max}$. This effect is more pronounced for the fast-flow emission because it mostly arises from the shock tail. Thus, the peak-amplitude ratio of the fast- to slow-flow emissions moderately decreases with increasing $s_{max}$ as is noticeable in Figure 5(f).

4.3. ICS Emission

VHE emission can be produced by ICS processes: the synchrotron self-Compton (SSC) process in the IBS and the external Compton (EC) process of the preshock and IBS particles. However, the electron density in TGBs is too low to produce significant SSC emission flux in the IBS. Hence, the SSC process is often ignored for TGBs, and we therefore only consider the EC process for VHE emission. We assume a head-on collision for the EC scattering, which is appropriate for ultrarelativistic particles ($\gamma_e \gg 1$) in the preshock and IBS, and calculate their emission SEDs using formulae given in Appendix B (see Dermer et al. 2009, for more details).

For the stellar blackbody seeds, we assume that the star is a point source. Therefore, both the incidence azimuth and polar angles $\phi_{\theta_s}$ and $\theta_{\phi_k}$ ($\phi_k \equiv \cos \theta_k$) of the seed photons (see Equation (B4)) into a scattering emission zone, as well as the distance ($r_p$) from the stellar surface to the scattering zone in the IBS or preshock, are the same over the surface of the
With the aforementioned assumptions, we compute the EC emission. In each emission zone, we compute the incidence angles and spectral energy densities ($\mu_s$) of the stellar and disk emissions, calculate the EC emission (Equation (B4)) in the zone, and integrate over the IBS surface to generate the EC SED at a phase. Figure 7 shows orbitally averaged EC SEDs constructed using the same parameters as those used for Figure 4. Note that the EC SEDs also reflect the distributions of emitting particles and seed photons; broader distributions result in a more extended EC SED. Hence, the EC SED for the disk companion (see Figure 6). This is a reasonable approximation for the companion of J0632 since $R_\star \ll d_{\text{orb}}$. In contrast, the size of the disk can be comparable to the orbit, and thus the incident azimuth and polar angles $\varphi_d$ and $\theta_d (\mu_d \equiv \cos \theta_d)$ of the disk seeds into the scattering zone vary over the disk surface (Figure 6), as does the distance from a surface element of the disk to the scattering zone ($r_d$). Additionally, the disk emission is anisotropic and varies radially (Section 4.1). These variations require computationally demanding calculations, since the EC computation is carried out in each of 21 by 361 IBS zones, 500 preshock zones, and 100 orbital phases, in addition to 300 energy ($\gamma_\nu$) bins. Hence, we simplified EC computations by assuming that the disk is a point source with $\varphi_d = \varphi_\star$, $\mu_d = \mu_\star$, and $r_d = r_\star$ over the disk surface. Note, however, that the EC scattering angles in and distance from the disk to a scattering zone vary over the IBS surface because the location and the flow direction of each zone change (Figures 2 and 6). We verified that EC spectra computed with this assumption were not significantly different from those obtained with a full integration over the disk surface at a few phases. We further note that the disk emission is weaker than the companion’s blackbody emission (see Figure 3), and therefore the contribution of the disk seeds to the EC SED is small (Figure 7).

Figure 5. SY LC’s dependencies on the IBS and orbital parameters. The baseline parameters are the same as those in Figure 4, but some of them are varied here. The IFC phase is 0.75 in all the panels (magenta vertical lines), and the periastron phase (black vertical lines) is 0.3 except for panel (b). LCs of the slow and fast flows are presented with dashed and solid lines, respectively, and different colors denote different values of the parameter varied for an investigation in each panel (red to purple from a small to large value). (a) $\epsilon$ dependence of the SY LC. Note that $B_0$ is defined to be 0.28 G for the baseline circular orbit. For eccentric orbits, we varied $B_0$ (defined at the shock nose at the IFC) by scaling with $r_{\perp}$ (e.g., Equation (9)). (b) LCs produced using $\varphi_{\text{per}} = 0.1$ (top) and $\varphi_{\text{per}} = 0.5$ (bottom) for an eccentric orbit with $\epsilon = 0.5$. (c) LCs seen by observers at different inclinations. (d-f) LCs constructed for various wind momentum flux ratios $\beta$, $r_{\text{max}}$, and $s_{\text{max}}$, respectively.

Figure 6. Geometry for ICS scattering off of the stellar and disk photons. The red circle is the stellar companion with an equatorial disk, shown as a gray disk. The pulsar is denoted by a purple circle, and the purple parabolic curve around it depicts the IBS. Both the pulsar and stellar companion are assumed to be point sources, while the orange cube represents a small volume element of the disk. The orbit of the pulsar is displayed with a gray dashed line. In this coordinate, the observer is in the $y$-direction at an inclination angle of $i_\nu$ (disk). The incident angles of the stellar and disk-seed photons into the emission zone on the IBS are shown as $(\varphi_\star, \theta_\star)$ and $(\varphi_d, \theta_d)$, respectively. Note that these angles vary over both the IBS cone and the stellar and disk surfaces. In this work, we assume that the incident angles into an emission zone of the IBS are the same over the stellar and disk surfaces, but the change of the angles into different zones of the IBS cone is taken into account. See text for more details.
photons (Figure 7, middle) is slightly broader than that for the stellar photons (Figure 7, left). Notice that any sharp features in the disk-seed spectrum (Figure 3) are not apparent in the EC SED, as they are blurred by the broad electron distributions.

The EC emission of IBS particles varies orbitally owing to changes in the seed photon density, the scattering angle $\psi$ (the angle between incoming and scattered photons), and Doppler beaming of the emission. The seed photon density varies as $\mu r_1^2 s^2$ over the orbit, and so the orbital modulation of the EC emission from the slow flow is similar to that of the SY emission (dashed curves in Figure 5). As in the SY case, the EC emission of the fast flow is strongest at the IFC, where the flow direction is well aligned with the LOS, thereby leading to the strongest Doppler beaming. However, the EC LC varies in a more complex way because of changes in the scattering geometry (e.g., scattering angle $\psi$) over the orbit, which is most favorable near superior conjunction of the pulsar (SUPC; $\psi \approx \pi$).

Dependencies of the EC LCs on the orbital and IBS parameters are similar to those of the SY LC. We use the same parameters as were used for Figure 5 and compute EC LCs. The results are displayed in Figure 8. In the EC case, the slow-flow emission exhibits strong modulation even in circular orbits owing to variation of the scattering geometry (red dashed line in Figure 8(a)). In addition, the seed photon density for ICS in eccentric orbits is highest at periastron owing to small $r_s$, and thus emission of the slow flow is further enhanced at that phase (dashed lines in Figure 8(a)). The peaks in the fast-flow EC LCs appear sharper than the corresponding peaks in the SY LCs (Figure 5). This is due to the strong dependence of EC emission on $\delta_D (3^{1/2} p_1$, Dermer 1995), with a small contribution of the $\psi$ effect. $\beta$ dependences of the strengths of the SY and the EC emissions are opposite to each other (Figure 5(d) vs. Figure 8(d)), as a smaller $\beta$ pushes the IBS closer to the pulsar (higher $B$) but farther from the companion (lower $u_\perp$). The $s_{max}$ dependence of the EC emission (Figure 8(f)) is similar to that...
Table 1
Parameters for the IBS Model in Figures 10 and 11

| Parameter                                      | Symbol | Value          |
|------------------------------------------------|--------|----------------|
| Semimajor axis                                | $a$    | $3.5 \times 10^{13}$ cm |
| Eccentricity                                   | $e$    | 0.45           |
| Inclination                                    | $\theta$ | 48°            |
| Periastron phase                               | $\phi_p$ | 0.26          |
| Pulsar inferior conjunction                    | $\phi_{p_{IC}}$ | 0.75         |
| Winds’ momentum flux ratio                     | $\beta$ | 0.045         |
| Speed of slow flow at the IFC                  | $v_{s\text{flow}}$ | $0.3c$       |
| Length of the IBS                              | $s_{\text{max}}$ | 4$d_{\text{orb}}$ |
| Max. bulk Lorentz factor                       | $\Gamma_{\text{max}}$ | 7            |
| Fraction of fast flow                          | $f_{\text{fl}}$ | 0.05          |
| Magnetic field strength of the apex ($s = 0$)  | $B_0$  | 0.25 G         |

Pulsar’s power injection

$\eta_{\text{P}} = 2 \times 10^{34}$ erg s$^{-1}$

Bulk Lorentz factor of preshock

$\gamma_{\text{break}} = 1.3 \times 10^{6}$

Min. electron Lorentz factor

$\gamma_{\text{e, min}} = 1.5 \times 10^{5}$

Max. electron Lorentz factor

$\gamma_{\text{e, max}} = \sim 10^{8}$

Lorentz factor at the break

$\gamma_b = 5 \times 10^{6}$

Low-energy spectral index

$p_1 = 2.3$

High-energy spectral index

$p_2 = 2.6$

Radius of the companion

$R_p = 6.6 R_\odot$

Temperature of the companion

$T_s = 30,000$ K

Disk size

$R_d = 60 R_s$

Disk temperature

$T_d = 0.7 T_s$

Note.

* Magnetic field strength at the apex ($s = 0$) of the IBS at the IFC.

VHE emission is absorbed by soft photons (stellar blackbody and disk emission) through the $\gamma-\gamma$ pair production process. In each of the IBS and preshock emission zones, we compute the $\gamma-\gamma$ optical depth $\tau_{\gamma\gamma}$ along the LOS using the scattering cross section given in Gould & Schréder (1967). The VHE emission in each zone (Section 4.3) is then reduced by a factor of $e^{-\tau_{\gamma\gamma}}$ appropriate for the zone. If the orbit is tight, a large companion may block part of the IBS or preshock as was seen in pulsar binaries (e.g., Corbet et al. 2022); this is not a concern for J0632.

Examples of $\gamma-\gamma$ absorption by the blackbody and disk emission (in Figure 3) for parameters appropriate for J0632 (e.g., Table 1) are presented in Figure 9. As expected, the maximum absorption occurs at S UPC for gamma-ray photons with $E \approx \left( m_e c^2 \gamma \right)^{3/2} \approx 10^{11}$ eV (Gould & Schréder 1967). Note that the emission zones are spread over the extended IBS, and thus the effect of the absorption is not very large in the spectrum integrated over the IBS even though the absorption of the emission at the shock nose (nearest to the companion) would be somewhat stronger. Secondary electrons produced by the $\gamma-\gamma$ interaction may be significant if $\tau_{\gamma\gamma}$ is large (e.g., >1; Bednarek 2013; Dubus 2013), but we do not consider them in this work since $\tau_{\gamma\gamma}$ in J0632 seems not to be very large.

5. Modeling the LCs and SEDs of J0632

5.1. Broadband SED and Multiband LCs of J0632

We compiled broadband data of J0632 from published papers. Its X-ray LCs and spectra have been measured by Swift-XRT and NuSTAR (TAH21), and a $\gamma$– keV SED (Li et al. 2017) was measured by the Fermi Large Area Telescope (LAT; Atwood et al. 2009). Note that the Fermi-LAT flux may include emission of a putative pulsar that we do not model, and thus we regard the $\gamma$– keV flux measurements as upper limits. In the VHE band, VERITAS and H.E.S.S. data presented in Adams et al. (2021) are used to construct the LCs and SEDs. For the X-ray SEDs, we plot those measured with Swift in the 0.3–10 keV band and with NuSTAR in the 3–20 keV band. Since the X-ray spectrum of J0632 is variable, we take three representative power-law models (i.e., three orbital phases) in each of the Swift and NuSTAR bands (Figure 10; Aliu et al. 2010; TAH21). For the VHE SEDs, we display six representative power-law fits reported by Adams et al. (2021). The shapes of the multiband LCs differ slightly depending on the $P_{\text{orb}}$ used to fold the data (310–320 days; Casares et al. 2012; Moritani et al. 2015; Maier et al. 2019; Adams et al. 2021). We use folded LCs (Figure 11) produced with the most recent orbital period ($P_{\text{orb}} = 317.3$ days; Adams et al. 2021). Note that the X-ray LC (Figure 11, top) was constructed with average 0.3–10 keV Swift-measured fluxes (first cycle and left ordinate; Adams et al. 2021) and average count rates (second cycle and right ordinate; TAH21); the latter was normalized to have a maximum of 1. The fluxes in the VHE LC (Figure 11, bottom; taken from Adams et al. 2021) were obtained by assuming a photon index of $\Gamma_{\text{VHE}} = 2.6$.

We base our broadband modeling of J0632 on the results of the previous X-ray study (TAH21) and use our model to explain the multiband LCs and broadband SED of J0632.

5.2. Application of the IBS Model to J0632

We apply our IBS emission model (Sections 3 and 4) to the observed SEDs and LCs of J0632. We compute the IBS emission in 21 and 361 segments along $s$ and $\phi_s$, respectively, and the preshock EC emission in 500 radial segments. Particle spectra are constructed using 800 energy bins, and each of the resulting SY and EC emissions is computed in 300 frequency
bins. The $\gamma$-$\gamma$ absorption is applied to emission in each segment separately out to $\sim 10^4 \delta_{\text{orb}}$ along the LOS, and LCs are generated by integrating the computed SEDs in the energy ranges relevant to the X-ray (0.3–10 keV) and VHE data ($>350 \text{ GeV}$). The results are displayed in Figures 10 and 11, and the model parameters are presented in Table 1.

5.2.1. Phase-averaged SED and LCs

The model reasonably describes the phase-averaged SED and the X-ray/VHE LCs of J0632 (Figures 10 and 11), except for the spike at $\phi \approx 0.35$ in the VHE LC because we do not model the VHE emission at the disk crossing, which requires deep understanding of how the pulsar and disk interact with each other. Notice that the model has a small bump at $\sim \text{TeV}$ that is produced in the preshock EC emission. It has not been commonly included in the previous basic IBS models (e.g., Dubus et al. 2015; An & Romani 2017), but $\sim \text{TeV}$ bumps seen in phase-resolved SEDs of J0632 (Figure 12) hint at a possibility of such preshock emission. By jointly modeling the X-ray and VHE data, we are able to infer the magnetic field strength $B_0$ (Equation (9)) and parameters for the particle spectrum (e.g., $\gamma_{\text{e,min}}$ and $\gamma_{\text{e,max}}$; Equation (6)) in the IBS since the spectral shapes of the IBS emission depend strongly on these parameters. The magnetic field strength $B_0$ at the shock nose at the IFC is inferred to be $B_0 \approx 0.25 \text{ G}$ by the relative flux between the SY and the EC emissions, e.g., $E_{\text{EC}} \approx \frac{\gamma_{\text{e}}}{{\gamma_{\text{e}}}}$ in the Thomson regime for EC.

For the $B_0$ value, $\gamma_{\text{e,min}}$, $\gamma_{\text{e,max}}$, and $\gamma_b$ are inferred by the observed features in the SEDs since electrons with an energy $\gamma_{\text{e}} m_{\text{e}} c^2$ emit SY photons with an energy

$$h\nu_{\text{SY}} \approx 4 \times 10^6 h \left( \frac{B}{1 \text{ G}} \right) \gamma_{\text{e}}^2 \text{ keV},$$

where $h$ is the Planck constant, and upscatter seed photons with an energy $E_{\text{seed}}$ to

$$h\nu_{\text{EC}} \approx \gamma_{\text{e}}^2 \left( \frac{E_{\text{seed}}}{1 \text{ eV}} \right) \text{ eV}.$$  

Here $\gamma_{\text{e,min}}$ is estimated to be $\approx 10^5$ so that the low-energy cutoff in the model SED is below the 0.3 keV start of the X-ray band and the low-energy EC SED is below the LAT upper limits at $<100 \text{ GeV}$ energies (Figure 10). Parameter $\gamma_{\text{e,max}}$ is computed to be $\approx 10^8$ from the radiation reaction limit as mentioned in Section 3.4. An upper bound for $\gamma_{\text{e,max}}$ can be estimated since a larger value (e.g., $\approx 4 \times 10^8$) would overpredict the $<\text{GeV}$ upper limits. Constraints on the lower bound for $\gamma_{\text{e,max}}$ are due to the Klein–Nishina (KN) effect. The observed $\sim \text{10 TeV}$ photons imply that $\gamma_{\text{e,max}} > 10^6$. It is difficult to estimate $\gamma_{\text{e}}$ without a measurement in the MeV band.

Our SY LC model for J0632 (Figure 11, top) fully matches the detected X-ray peak at $\phi \approx 0.35$ and also produces an LC that is extremely similar to that of TA921, who phenomenologically modeled the disk crossings as an enhancement of $B$ in the IBS. With this model, we ignore two complex phenomena—strong SY cooling and disk interactions—that impact the EC emission. In reality, SY cooling of particles would be boosted by the enhanced $B$. However, as our model does not include particle cooling, we are able to leverage the enhanced $B$ to fit the SY emission without altering the EC emission in the VHE band. Other processes such as additional seeds from disk heating may provide favorable conditions for EC emission (Chen et al. 2019). As noted in Section 3.1, we also ignore these complexities in our model.

The EC LC model (Figure 11, bottom) reproduces two bumps: one bump at $\phi \approx 0.25$ produced by the slow flow from high seed density and favorable ICS geometry at periastron and SUPC, and the other at $\phi \approx 0.75$ owing to the Doppler-boosted emission of the fast flow. Note that there is a small notch at $\phi = 0.75$ in the EC LC but not in the SY LC. This is due to the unfavorable ICS scattering geometry at the IFC ($\psi \approx 0$; see Section 4.3). Although the notch cannot be identified in the data shown in the bottom panel of Figure 11, a smoothed LC presented in Figure 16 of Adams et al. (2021) seems to exhibit a notched morphology.

We also display contributions of the stellar (red, green) and disk (blue, purple) EC by the IBS and by the preshock particles in the bottom panel of Figure 11. Unlike the EC emission of the IBS particles, the preshock EC exhibits only one bump at $\phi = 0.25$ (near periastron and SUPC). Hence, the preshock emission controls the relative amplitude of the EC LC at $\phi = 0.25$ and $\psi = 0.75$; using more preshock particles (e.g., large $\eta E_{\text{SD}}$; Equation (5)) and/or placing the emission zone (preshock acceleration region; Section 3.3) farther away from the pulsar would make the $\phi = 0.25$ bump more pronounced. As noted in Section 4.3, the EC off of the disk seeds is weaker than that of the stellar seeds. Although a distinction between the shapes of the stellar EC and the disk EC emissions is subtle, the ratio of the emission peaks of the slow-flow ($\phi = 0.25$) and fast-flow ($\phi = 0.75$) particles is smaller for the stellar EC (red). This is because the stellar EC at SUPC suffers from the KN effect, whereas the IR seeds of the disk alleviate that effect. At the IFC, the small scattering angle $\psi$ and Doppler deboosting push the onset of the KN suppression to much higher $\gamma_{\text{e}}$ values,
where little or no scattering electrons are available, making the KN effect observationally unimportant for both the stellar and disk seeds.

The joint modeling of the X-ray and VHE LCs allows estimations of the IBS size $s_{\text{max}}$ ($\approx 4d_{\text{orb}}$), $\Gamma_{\text{max}}$ ($\approx 7$), and $\xi$ ($\approx 0.05$) because the ratio of the peak fluxes of the slow- and fast-flow emissions generated by the SY and EC processes ($F_{\text{SY,slow,max}}^{\text{fast,max}}$ and $F_{\text{EC,slow,max}}^{\text{fast,max}}$) and the widths of the LC bumps at $\phi \approx 0.75$ depend differently on these parameters (Figures 5 and 8). Note that the ratios $F_{\text{SY,slow,max}}^{\text{fast,max}}$ and $F_{\text{EC,slow,max}}^{\text{fast,max}}$ are controlled by $\xi$, and differences between them are determined by $s_{\text{max}}$ and $\Gamma_{\text{max}}$ (panels (e) and (f) in Figures 5 and 8). The latter controls also the widths of the LC peaks at the IFC.

5.2.2. Phase-resolved VHE SEDs

As the model reproduces the observed phase-averaged SED and multiband LCs reasonably well, we check to see whether the model reproduces phase-resolved VHE SEDs (Figure 12). Because the observed SED in a phase interval was constructed by combining observations from different epochs with different exposures spread over the phase interval (Adams et al. 2021), we display model calculations at 20 phases within the interval (denoted in the panel) for comparison.

The VERITAS- and H.E.S.S.-measured phase-resolved SEDs are reproduced well by our model. Specifically, the modulation of the spectral hardness of the VHE spectra is mostly consistent with the model; the hard VHE spectra at the phase interval 0.6–0.8 (Figure 12) are explained by the enhanced contribution of the Doppler-boosted emission of the fast flow. Some noticeable discrepancies between the data and model are likely due to strong orbit-to-orbit variations of the VHE flux, as was noticed by Adams et al. (2021), and uneven phase coverage of the VHE measurements.

It is intriguing to note that the observed SEDs exhibit a bump at $\sim$TeV seen in phase intervals 0.2–0.4 and 0.8–0.2 (Figure 12). It was difficult to replicate this bump with basic IBS models that considered IBS particles only, as they produce a smooth SED. The preshock EC emission (Section 4.3 and Figure 10) can naturally reproduce the SED feature, which possibly suggests that the preshock emission is an important contributor to the VHE emission in J0632.

6. Discussion

In the previous sections, by adopting the latest orbital solution and binary system geometry suggested by a recent study (TAH21), we demonstrated that both the LC and SED data of J0632 from the X-ray to VHE band can be well explained with our IBS model. Compared to the previous investigations based on X-ray data only, we further constrained the IBS parameters by simultaneously fitting the X-ray and VHE data (Table 1). Still, we note that some of the parameters are degenerate, making error estimation difficult, and thus it is possible that the derived parameters may not represent a unique solution. In this section, we discuss several key IBS properties and observationally important parameters.

6.1. VHE Emission

We found that most of the VHE flux in J0632 arises from EC of the stellar seeds by IBS electrons and that the exact shape of the seed spectrum (e.g., features in the disk spectrum in Figure 3) does not alter the EC spectrum significantly as long as the IR-to-optical SED model matches the observed flux. For example, a phenomenological multitemperature blackbody model for the disk emission also results in a similar EC SED. Notice that the disk EC SED is slightly broader than the stellar EC SED (Section 4.3 and Figure 7). In particular, the former is less affected by the KN effect, which is severe at $h\nu_{\text{EC}} \gtrsim (m_{e}\gamma)^{2}$ eV, hence making the SED model spectrally harder if the intrinsic energy density of the disk emission is much higher. The observationally inferred $u_{\text{w}}$ of the disk depends on the assumed disk inclination $i_{d}$ with respect to the LOS. We used an extreme inclination angle of $i_{d} = 85^\circ$ for modeling the disk emission (Section 4.1); if this value were smaller, the intrinsic disk emission would be lower, thus affecting the total observed VHE emission less significantly. The “total” disk + star seed flux inferred from the optical data (Figure 3) is an important factor that determines the amplitude of the VHE emission. The inferred optical seed
density $u_\phi$ in the emission zones depends on the assumed distance to the source (here $d = 1.4\,\text{kpc}$) and orbital size quantified by the semimajor axis $a$. Hence, these parameters $d$ and $a$ have large impacts on our IBS parameter determination because one of the most important parameters, $B_0$, is related to $u_\phi$ (Section 5.2.1). Our estimations of $B_0$ and thus $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$ are sensitive to the value of $u_\phi$; see Equations (10) and (11). Therefore, accurate measurements of $d$ and $a$, as well as precise characterizations of the X-ray and VHE emission, will lead to better understanding of the particle properties and thus particle acceleration in the IBS.

6.2. Particle Flow in the IBS Region

We were able to infer $s_{\text{max}} \approx 4d_{\text{orb}}$, $\Gamma_{\text{max}} \approx 7$, and $\xi \approx 0.05$ by modeling the X-ray and VHE LCs. Since the quality of the VHE LCs is rather poor, our estimations are subject to large uncertainties. Moreover, model parameters are covariant with each other, and our simplified prescriptions such as the IBS shape and $B$ structure for the IBS flows may not be very accurate. These may add further systematic uncertainties. Although the reported parameter values (Table 1) need to be taken with some caution, we point out that the inferred parameters are in accord with the recent HD simulations supporting the general properties of IBS flows in TGBs—high-energy emission arises from the inner region of the IBS flow, and a small fraction of the particles are bulk accelerated in the flow.

6.3. A TeV Bump in the SED: EC by Preshock Particles

The most interesting emission feature in J0632 is the $\sim$TeV SED bump seen in some orbital phases, which we ascribed to EC emission of the preshock particles with $\gamma_{\text{peak}} \approx 10^6$. It has been widely believed that pulsar wind is dominated by Poynting flux near the magnetosphere, and the magnetic energy should be converted into particle energy in the pulsar-wind zone between the light cylinder and the termination shock. The location, physical processes, and energetics for such an energy conversion have not been well understood.

![Figure 12. Phase-resolved SEDs of J0632 (Adams et al. 2021) measured by VERITAS (black) and H.E.S.S. (red). Model calculations at 20 phases within the designated phase interval are presented in color: red to blue for early to late phases. The average of the computed SEDs in each interval is displayed in black.](image-url)
theoretically (e.g., Coroniti 1990; Cerutti & Giacinti 2020). But Aharonian et al. (2012) modeled pulsed VHE emission of the Crab Nebula and suggested that the Poynting flux of the pulsar wind should be converted into kinetic energy of particles in a narrow region at \( \approx 30 R_{\text{LC}} \) and that the accelerated particles have a Lorentz factor of \( \gamma_{\text{pre}}^{\text{peak}} \sim 10^9 \).

The \( \sim \text{TeV} \) bump in J0632’s SEDs can be explained by such preshock emission as we demonstrated in Section 5.2.2. Note that similar SED features have been seen in the VHE low state of the TGB PSR J2032+4127 (e.g., Abeysekara et al. 2018). The relatively sharp \( \sim \text{TeV} \) bumps imply that the energy distribution of the preshock is narrow like the Maxellian distribution we assumed (Equation (3)). Other narrow distributions such as a broadened delta function or a narrow power law with a peak at \( \gamma_{\text{pre}}^{\text{peak}} \sim 10^9 \) would predict slightly different SED shapes for the bumps but may also explain them equally well. Currently, these distributions are indiscernible because the measurement uncertainties are large. Precise characterization of the bumps with deep VHE observations may help to constrain the preshock particle distribution, possibly providing a hint to acceleration mechanisms (e.g., Hoshino et al. 1992; Jaroschek et al. 2008; Sironi & Spitkovsky 2011) in the pulsar-wind zone.

In our model, the SED amplitude, emission frequency (Figure 12), and LC shape (Figure 11, bottom) of the preshock EC depend sensitively on the conversion location and particle energy; if the conversion takes place at a larger distance from the pulsar, the model predicts weaker preshock emission and sharper LC features (particularly, a sharper peak near periastron is expected). Thus, the SED features in TGBs can provide a sensitive probe to the energy conversion process in pulsar winds. While the current measurements of the SED and LC of J0632 are somewhat constraining, more accurate measurements with deeper VHE observations (e.g., by CTA; Actis et al. 2011) in the near future will enable us to determine the fundamental parameters for energy conversion in pulsar winds.

### 6.4. A Sharp Spike in the VHE LC: A Signature of Disk–Pulsar Interaction?

Since the large spike observed at \( \phi = 0.35 \) in the VHE LC was not accounted for in our baseline IBS model, we suspect that this unique feature may be caused by a disk crossing of the pulsar. Note that the circumstellar disk material was suggested to be the origin of the higher \( N_{\text{I}} \) at the disk interaction phases (TAH21; Malyshev et al. 2019). If this is true, the disk interaction may leave some observable signatures in the multiband emission at the “interaction” phases.

H.E.S.S. Collaboration et al. (2020) reported an increase in the VHE flux near the disk-crossing phase for another TGB, PSR B1259–63, and Chen et al. (2019) attributed the enhanced VHE flux to disk heating by the IBS. Similarly, we find that the VHE spike in the LC can be reproduced if we assume that the IBS is immersed in a \( T_{\text{BB}} \approx 950 \text{ K} \) blackbody field of a heated disk. Note, however, that the existence of a \( T_{\text{BB}} \approx 950 \text{ K} \) blackbody field is not physically supported by our model since it does not include complex dynamic effects and microphysics of interaction such as disk heating. We defer further investigations to a future work.

### 6.5. Comparison with Other IBS Models: Double-bump Features in the LCs

A difference between the IBS model (TAH21) and the similar inclined disk model for J0632 (Chen & Takata 2022) is that the latter assumed one-zone shock geometry whereas the former used an extended cone-shape IBS with two particle flows. This difference has a significant impact on the LC modeling. The one-zone shock model requires a peculiar “inclined disk” geometry to account for the two bumps in the LC (Chen & Takata 2022). In contrast, the IBS cone with two particle flows can naturally reproduce two bumps, and disk crossing can add two more (bumps or peaks). Overall, the IBS model was able to accommodate the complex X-ray LC of J0632 with two bumps and one peak (TAH21).

The most distinctive feature of IBS models, as compared to one-zone inclined disk models, is a double-peak structure around IFC in the X-ray/VHE LCs of highly inclined sources (Figures 5 and 8; see also van der Merwe et al. 2020). Such double-peak features are often observed in the X-ray LCs of redback pulsar binaries and regarded as a signature of bulk-accelerated particles in the IBS (e.g., Kandel et al. 2019). Intriguingly, a hint of double-peak X-ray/VHE LCs was also found in J0632 (Figures 15 and 16 of Adams et al. 2021), suggesting that the emission at the IFC is indeed produced by IBS cone emission. While our SY model based on the parameters derived in TAH21 does not reproduce the X-ray double peaks at \( \phi = 0.75 \), the IBS model can account for the double-peak LC with a larger inclination (e.g., Figure 5(c)); in this case, the model-predicted dip at \( \phi = 0.75 \) in the EC LC (Figure 11, bottom) might be deeper. This prediction can be confirmed with more sensitive X-ray and VHE observations and can distinguish between the disk interaction case (Chen & Takata 2022) and the IBS cone emission.

### 6.6. The Origin of Short-term Variability

Broadband emission of J0632 is known to exhibit strong orbit-to-orbit variability (TAH21; Adams et al. 2021). In the IBS scenario, such short-term variability is likely due to nonuniform clumpy stellar winds that can drive varying momentum flux ratio \( \beta \). In this case, the shock opening angle \( \theta_{\text{cone}} \) (Equation (A4); see also Bogovalov et al. 2008) and distances to the emission zones from the pulsar \( r_p \) and the star \( r_s \) are also expected to vary, affecting the IBS emission. For example, a stronger stellar outflow will push the IBS closer to the pulsar, making \( B \) larger by reducing \( r_p \) and making \( u_a \) smaller by increasing \( r_s \). This will enhance the SY emission but reduce the EC emission in general as seen in panel (d) of Figures 5 and 8.

At the IFC, however, the situation is more complicated because the Doppler factor \( \delta_D \), which is also determined by \( \beta \) through a change in \( \theta_{\text{cone}} \), comes into play. Generally, stronger variability is expected at the IFC if the LOS is close to the shock tangent as it is in J0632, because the observed flux depends strongly on \( \delta_D \) (e.g., An & Romani 2017). Contemporaneous observations of multiband variability in the optical, X-ray, and VHE bands will provide a useful diagnostic to test the IBS scenario.

### 7. Summary

We showed that our phenomenological IBS model provides a good fit to the multiband SED and LC data of J0632,
suggesting that the interaction between the winds of the pulsar and companion drives the observed broadband emission. Below we summarize our results and conclusions.

1. We constructed an IBS emission model employing physical processes and emission components appropriate for TGBs, applied the model to J0632 with an orbit inferred from an X-ray LC modeling (TAH21), and found that the model and the orbit could explain not only the X-ray LC but also the VHE emission properties of the source LCs and SEDs.

2. The observed SEDs of J0632 show a bump at ∼TeV in some phase intervals. This feature is likely due to ICS emission by preshock particles, implying that the bulk Lorentz factor of the preshock is \( \gamma_{p\text{, peak}} \approx 10^4 \). Conversely, accurate characterizations of the bump, both observationally and theoretically, will help us to understand the energy conversion processes in pulsar winds.

3. Our VHE LC model for J0632 predicts a double-peak structure around IFC. This is a natural consequence of the IBS model in contrast to inclined disk models (e.g., Chen & Takata 2022).

While the model captures the main features of the broadband emission SED and LCs of J0632, some of the parameters, such as \( \gamma_{p\text{, peak}} \), are not well constrained. Further, the parameters are degenerate, and thus it was difficult to determine a unique solution set. A comparison with MHD simulations, as well as future contemporaneous observations in the optical, X-ray, and VHE bands, will allow us to determine these parameters better and to break the degeneracy. Further constraints on the IBS properties can be set by detecting a spectral break caused by particle cooling. The break is expected at ∼MeV energies, where future missions (e.g., COSI; Tomsick et al. 2019) can add valuable data in the MeV band to boost our understanding of TGBs.

It is well known that less powerful IBSs are formed in the so-called black widow and redback pulsar binaries (e.g., Romani & Sanchez 2016; Wadiasingh et al. 2017). The common signatures expected from IBSs formed in pulsar binaries are double-peak X-ray LCs (for large \( \theta_i \)) and orbital modulations in the ∼GeV band (e.g., An et al. 2018; Corbet et al. 2022). Hence, a variety of IBS models, including ours presented in this paper, have been applied to pulsar binaries in circular orbits with a very low mass companion (e.g., van der Merwe et al. 2020). However, it is yet unclear whether an (universal) IBS model can eventually account for the diverse observational properties of both the pulsar binaries and the TGBs by simply employing different geometries and energetics; e.g., the X-ray, GeV, and VHE phase variations are observed to be diverse in the sources (e.g., Fermi LAT Collaboration et al. 2012; Corbet et al. 2016; An et al. 2020). While the pulsar binary and TGB systems certainly share some common emission properties (e.g., orbital modulation), the systems seem to possess fundamentally different mechanisms (e.g., companion, orbit, energetics). A large and more comprehensive multiwavelength study of these exotic pulsar binaries and TGBs, as demonstrated for J0632 in this paper, will give deeper insights into IBS and pulsar physics.

We thank Melanie Nynka for a helpful review of the paper. We thank the anonymous referee for the careful reading of the paper and insightful comments. This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (NRF-2022R1F1A1063468).

**Appendix**

The shape of an IBS formed by interaction of isotropic winds of two stars was analytically computed using pressure balance equations (Cantó et al. 1996). For a wind’s momentum flux ratio of \( \beta \) (Equation (1)) and the geometry depicted in Figure 2, the locus of the CD in a vertical plane (to the orbital plane) containing the pulsar and the star is given by

\[
\vec{r}_p = d_{\text{orb}} \sin \theta_i \csc(\theta_p + \theta_i)
\]

(A1)

and

\[
\theta_i \cot \theta_i = 1 + \beta(\theta_p \cot \theta_p - 1)
\]

(A2)

(Equations (23) and (24) of Cantó et al. 1996). Distance to the apex of the CD (\( \theta_i = \theta_p = 0 \); shock nose) from the pulsar and the asymptotic angle of the IBS (\( \theta_{\text{cone}} \); half-opening angle of the IBS cone) are given by

\[
\vec{r}_0 = d_{\text{orb}} \frac{\sqrt{3}}{1 + \sqrt{3}}
\]

(A3)

and

\[
\tan \theta_{\text{cone}} - \theta_{\text{cone}} = \frac{\pi \beta}{1 - \beta},
\]

(A4)

respectively (Equations (27) and (28) of Cantó et al. 1996). \( \theta_{\text{cone}} \) increases monotonically with increasing \( \beta \).

**Appendix B**

**Formulae for Computation of Emission SEDs**

Suppose that particle flow has bulk motion with a Lorentz factor \( \Gamma \) with respect to an observer and that the particles in the flow rest frame move randomly (isotropically) with an energy distribution \( \frac{dN_\gamma}{d\gamma} \), where \( \gamma' \) is the Lorentz factor of the particle’s random motion in the flow rest frame (primed quantities are defined in the flow rest frame). In randomly oriented \( B \), the SY SED of the particles can be computed using the formula given in Equation (18) of Finke et al. (2008),

\[
f_{\text{SY}}(\epsilon) = \frac{\sqrt{3} \delta_4 e^3 q s^3 B}{4 \pi h d^2} \int_1^\infty d\gamma' \frac{dN_\gamma(\gamma')}{d\gamma'} R(x)
\]

(B1)
\[ R(x) = \frac{x}{2} \int_{0}^{\pi} d\theta \sin \theta \int_{s/\sin \theta}^{\infty} dt K_{S/\beta}(t) \]

with \( x = \frac{4\pi c' e'^{2} c^{3}}{3q_{e} B h \gamma^{2/7}} \),

\[ \delta_{D} = \frac{1}{\Gamma(1 - \sqrt{1 - 1/T^{2}} \cos \theta_{V})} \]
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