1. Introduction

First data reported by the STAR Collaboration at RHIC [1] has a significant meaning that the observed large magnitude of elliptic flow for charged hadrons is consistent with hydrodynamic predictions [2]. This suggests that large pressure possibly in the partonic phase is built at the early stage ($\tau \sim 0.6$ fm/$c$) in Au+Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV. This situation at RHIC is in contrast to that at lower energies such as AGS or SPS where hydrodynamics always overpredicts the data. Hadronic transport models are very good to describe experimental data at lower energies, while they fail to reproduce such large values of elliptic flow parameter at RHIC (see, e.g., Ref. [3]). So the importance of hydrodynamics is rising in heavy ion physics. In this review, I discuss first how well hydrodynamics can fit the data in Sec. 2 and next how hydrodynamics is related with other non-flow phenomena such as jet quenching and thermal photon emission in Sec. 3. In Sec. 4, I argue several attempts to incorporate initial conditions from other models. Finally, I briefly mention about viscosity in Sec. 5.

2. Results from the hydrodynamic approach at RHIC

After the first STAR data were published [1], other groups at RHIC have also obtained the data concerning with flow phenomena [4]. To understand these experimental data, hydrodynamic analyses are also performed extensively [5]. Here I pick up several results on elliptic flow and emphasize on the momentum regions where hydrodynamics gives a good description. Elliptic flow is very sensitive to the degree of secondary interactions [6]. The indicator is the second harmonic coefficient of azimuthal distributions [7]

$$v_2(p_T, y) = \frac{\int d\phi \cos(2\phi) \frac{dN}{dy dp_T}}{\int d\phi \frac{dN}{dy dp_T}} = \langle \cos(2\phi) \rangle.$$

(1)
(For recent progress of higher harmonics, see Refs. [8, 9].) In ideal hydrodynamics, the mean free path of particles is assumed to be zero. So the hydrodynamic prediction of $v_2$ should be maximum among transport theories. The scaled elliptic flow, which is defined as $v_2$ divided by initial spatial eccentricity $\varepsilon = \langle y^2 - x^2 \rangle / \langle x^2 + y^2 \rangle$, becomes almost constant around 0.2 from hydrodynamic simulations [10]. Interestingly, the experimental data reach this hydrodynamic limit for the first time in central and semi-central collisions at RHIC energies [11]. The data of scaled $v_2$ at various collision energies monotonically increase with multiplicity per unit transverse area $(1/S)dN/dy$ and, eventually, comes to the hydrodynamic limit. The key quantity to achieve thermalisation is the number of particles in a unit volume. This can be seen also in the centrality dependence of $v_2$. Deviation of the hydrodynamic result from experimental data starts from $N_{ch}/N_{max} \sim 0.5$ which corresponds to an impact parameter $b \sim 5$ fm [2]. The $p_T$ dependence of $v_2$ contains rich physics. In low $p_T$ region, $v_2$ increases with $p_T$ almost linearly. On the other hand, the data points of $v_2$ saturate at high $p_T$ [12] and deviate from hydrodynamic predictions. In the intermediate $p_T$ regions, the interplay between soft physics and hard physics is very important in understanding $v_2$ for identified hadrons. This will be discussed in the next section. Hydrodynamic predictions give good agreements with experimental data of $v_2$ including mass dependences in low $p_T$ region [13]. In low $p_T$ region at midrapidity, hydrodynamics reproduces experimental data very well. While, in forward/backward rapidity regions where multiplicity becomes small, hydrodynamics overpredicts largely [14] the experimental data observed by the PHOBOS Collaboration [15]. This suggests thermalisation is achieved only near midrapidity. The regions where hydrodynamics works well for charged hadrons are summarised as follows: $|b| \lesssim 5$ fm, $p_T \lesssim 1.5$ GeV and $|\eta| \lesssim 2$ [5]. The scaled $v_2$ suggests that thermalisation is only partially achieved in small $(1/S)dN/dy$. Heinz tried to discuss a possible mechanism of deviation between hydrodynamic calculations and experimental data in forward/backward rapidity regions or in peripheral collisions by introducing “thermalisation coefficient” [16].

3. Information inside fluids

Due to strong interaction among secondary particles, hadrons can be emitted only from freezeout hypersurface. So hadron spectra reflect the information simply about accumulation of the space-time evolution. Unlike the blast-wave model fitting, the space-time dependences of thermodynamic variables are obtained in hydrodynamic simulations. These informations are significantly helpful to understand what happens inside the reaction regions. Here I discuss two phenomena, i.e., jet quenching and electromagnetic radiations, which are directly related with information inside bulk matter.
Hydrodynamic models

3.1. Jet quenching

Minijets produced in initial semihard collisions have to traverse the reaction region where bulk matter evolves. During traveling, these minijets lose their energies through interactions with the medium. So high $p_T$ hadrons originated from fragmentation of minijets contain information about the medium [17]. Gyulassy et al. first employed results from (2+1)-dimensional hydrodynamic simulations in a jet quenching analysis [18]. The first order term of an energy loss formula in the opacity expansion becomes [19]

$$\Delta E = C \int_{\tau_0}^{\infty} d\tau \rho(\tau, \vec{x}(\tau)) (\tau - \tau_0) \ln \left( \frac{2E_0}{\mu^2 L} \right).$$

Here kinematics of emitted gluons are neglected. A dimensionless parameter $C$ includes strong running coupling constant and colour Casimir factors. One needs the space-time evolution of parton density $\rho$ in quantitative analysis for the energy loss of a parton. Note that the parton density appeared in Eq. (2) does not need to be thermalised because it simply comes from the inverse mean free path of an energetic parton. Nevertheless, the parton density in Eq. (2) should be a thermalised one at RHIC energies from the analyses of $v_2$ discussed in the previous section. In Ref. [20], it is assumed that the parton density in an energy loss formula is a solution of hydrodynamic equations in full 3D space which is compatible with low $p_T$ data such as $dN/dy$ and $p_T$ spectra [21] (the hydro+jet model). Systematic studies based on the hydro+jet model with Eq. (2) are performed for $p_T$ spectra [22], back-to-back correlation functions [23], pseudorapidity dependence of nuclear modification factors [24], and $v_2$ [22, 24] in high $p_T$ regions. It is not obvious where hydrodynamic $p_T$ spectrum eventually turns into pQCD power law spectrum. Moreover, the transition point can depend on particle species. Interplay between radial flow and jet quenching is discussed in Ref. [22]. It is found that the transition point increases with hadron mass due to mass dependent effects of radial flow. The resultant $v_2$ divided by the number of constituent quarks is represented in Fig. 1 (left). This looks very similar to the results from recombination/coalescence models [25]. The nuclear modification factor $R_{CP}$ for $\phi$ mesons is found to be smaller than for protons even though $m_\phi > m_p$ [26]. This may be the meson-baryon effect [25] rather than the particle mass effect resulting from radial flow. But there is a possibility that $\phi$ mesons do not participate in the hydrodynamic flow. This is suggested by a hybrid model in which hadronic afterburner is described by RQMD [27]. Theoretically, one needs to study hadronic afterburner by combining a hadronic cascade model with the hydro+jet approach. Experimentally, it is important to observe $v_2$ for $\phi$ mesons in very low $p_T$ region where hydrodynamics is expected to work. These will reveal the production mechanism in the intermediate $p_T$ region at RHIC.

3.2. Electromagnetic radiation

Photons and dileptons emitted from thermalised matter are signals of the QGP in heavy ion physics proposed many years ago [28]. Here I focus on the discussion of thermal
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Photon emissions. As emphasised by Moore [29], the yield of thermal photons is much more sensitive to temperature profile and space-time volume than the production rate (the invariant yield per unit space-time volume). In a hydrodynamic approach, the invariant yield of thermal photon is evaluated by accumulation of the production rate over all fluid elements above the kinetic freezeout temperature

$$E \frac{dN_{\gamma}}{d^3p} = \sum_i \Delta V_i \vec{E} \frac{dR_{\gamma}}{d^3\vec{p}}(T(x_i), \mu(x_i), \vec{E} = p_{\mu} u^{\mu}(x_i))$$

(3)

Here $\vec{E}$ and $\vec{p}$ are, respectively, energy and momentum of an emitted photon measured in a local rest frame. Information from hydrodynamic simulations is taken through temperature $T$, chemical potential $\mu$ (or fugacity $\lambda$), four fluid velocity $u^{\mu}$ and four dimensional volume $\Delta V_i$ for each fluid element at $x_i$. A novel calculation of thermal photon yield is based on the hydrodynamics with rate equations for quarks and gluons [30, 31]. At collider energies, the QGP at the very early stage is supposed to be the “gluon plasma (GP)” since multiparticle production is dominated by small $x$ gluons in the wave function of colliding nuclei at ultrarelativistic energies. Initial gluons achieve a thermalised state, but it may not be a chemically equilibrium one yet. Through the process $gg \rightarrow q\bar{q}$, the system of the GP goes towards to the QGP. Due to the smaller degree of freedom, the initial temperature in the GP is larger than in the QGP when one assumes a fixed initial energy density. The photon yield would be enhanced due to higher initial temperature. On the other hand, small values of fugacity $\lambda$ for quarks and gluons make the photon yield per unit space-time volume suppressed. The competition between these two effects is studied in terms of a hydrodynamic model with rate equations. It is found that unexpected cancellation happens between initial temperature effect and fugacity effect in the final spectrum [31]. I note that it is worth studying thermal photon emissions.
yields from chemically non-equilibrium hadronic phase in order to conclude real excess of thermal photon yield from the QGP phase. Related with chemical non-equilibrium properties, it is interesting to see whether hydrodynamic results for elliptic flow will be changed at RHIC due to the insufficient chemical equilibrium.

4. Improvement of initial conditions in hydrodynamic models

All the hydrodynamic results mentioned in Sec. 2 are based on ideal hydrodynamics with parametrised initial conditions which lead to the reproduction of the multiplicity and $p_T$ spectra for hadrons in low $p_T$ region. Towards description of heavy ion collisions from initial colliding nuclei to final spectra in a unified way, it is much better to employ effective theories/models which are relevant in the very early stage of collisions. There are some attempts to incorporate results from event generators, e.g., HIJING [32], URASiMA [33], VNI [34] or NeXus [35], into initial conditions of hydrodynamic simulations. In other approaches, effective theories, e.g., string ropes/flux tubes [36], the final state saturation [37] or the initial state saturation [38], are employed. Here I pick up two approaches which are, to my mind, important for the RHIC physics.

4.1. Fluctuation of initial conditions

In conventional hydrodynamic simulations for non-central collisions, one parametrises initial transverse profile by using the number density of participants or binary collisions at a fixed impact parameter. This gives us a smooth profile for thermodynamic variables. However, if one picks up one event, the energy density distribution in the transverse plane has an extremely bumpy structure shown in Fig. 1 (right). In SPheRIO [35], initial conditions for hydrodynamic simulations are taken from an event generator NeXus [39] as an event-by-event basis. The final spectra in this approach are calculated like conventional event generators: they perform hydrodynamic simulations, calculate particle spectra at each event, accumulate many events, and average particle spectra over simulated events. It is worth noting that the numerical cost of the hydrodynamic code SPheRIO is very cheap and that they are able to simulate many events even in full 3D space. It is found that the multiplicity from initial energy densities with fluctuations are always smaller than without fluctuations. One can easily show [40] the following relation between initial and final total entropy in a simple EoS case:

$$\langle S \rangle_{\text{final}} \sim \langle S \rangle_{\text{initial}} \left[ 1 - \alpha \frac{\langle \Delta E^2 \rangle}{\langle E \rangle^2} \right]$$

where $\langle \cdot \cdot \cdot \rangle$ means the event average and $\alpha$ is a positive constant. One should keep this fact in mind in detailed analyses based on hydrodynamic models. Needless to say, a systematic study of the effect of fluctuation on $v_2$ would be interesting since elliptic flow is sensitive to the initial geometry.

4.2. Initial conditions from the colour glass condensate

Jet quenching and large elliptic flow are two important findings measured at RHIC. The common key for these two phenomena is the dense partonic medium. What is a possible
origin of this dense matter in Au+Au collisions at RHIC? This can be traced to the initial parton density inside the colliding nuclei. It is well known that small $x$ gluons are dominant for multi-particle production. In the ultra-relativistic limit, these gluons in colliding nuclei could form the Colour Glass Condensate (CGC) [41, 42]. So initial conditions in relativistic heavy ion collisions can be described by melting the CGC [43]. Hirano and Nara employed a nuclear wave function discussed in Ref. [44] and calculate gluon number density at each transverse point through $k_T$ factorised formula. Regarding this gluon number density as a thermalised one, they evaluate energy density at each space point at initial time $\tau_0$. By throwing it into hydrodynamic simulations as an initial condition, they obtain pseudorapidity distribution in the usual hydrodynamic manner [38] and compare these results with the PHOBOS data [45]. The CGC initial conditions are found to work very well for describing the energy, centrality, and (pseudo)rapidity dependences of charged hadrons [38].

5. Viscosity

Finally, I briefly mention about the viscosity. There is no a priori reason why ideal hydrodynamics works so well at RHIC as discussed in Sec. 2 (For a perspective from a strong coupling gauge theory, see Refs. [46, 47]). Viscous corrections to momentum distribution functions are considered in Refs. [48, 49]. The first order correction to the momentum distribution function becomes $\delta f \propto \frac{\Gamma_s}{\tau} f_0 (1 + f_0) p^\mu p^\nu X_{\mu\nu}$, where $\Gamma_s = \frac{4}{3} \frac{\eta}{(e + p)}$ is the sound attenuation length, $f_0$ is the Bose distribution function, and $X_{\mu\nu}$ is the tensor part of the thermodynamic force. Relation between the inverse Reynolds number [50] and the attenuation length becomes $R^{-1} \approx \Gamma_s / \tau$ in the Bjorken flow case [51]. $R^{-1}$ is found to be very small from the blast-wave fitting with the above correction term [49]. This suggests that the hadronic fluid in heavy ion collisions is nearly perfect one. It is known that naive relativistic extension of Navier-Stokes equations breaks down due to infinite signal velocity since the equations are parabolic ones (see, e.g., Ref. [52]). One can introduce relaxation terms to avoid this problem [53]. The existence of relaxation terms are essential since they change the type of equations to hyperbolic ones in which signal velocity remains finite. More systematically, one takes account of the second order viscous terms [54] from the prescription of extended thermodynamics (see, e.g., Ref. [55]). Dynamical studies of viscous fluids are mandatory for comprehensive understanding of the QGP.

6. Summary and Discussion

I reviewed the hydrodynamic results at RHIC and discussed some related topics such as jet quenching and electromagnetic radiations from a hydrodynamic point of view. I also discussed recent attempts for initial conditions of hydrodynamic simulations and for viscous effects. In addition to the photon emission discussed in Sec. 3 dileptons are also interesting to study chiral properties of hadrons by using hydrodynamics. In the
low invariant mass regions, dilepton spectrum is the best tool to see the spectral change of hadrons in hot/dense medium (see, e.g., Ref. [56]). On the other hand, in the high invariant mass region, it is interesting to see whether $J/\psi$ really melts in the QGP phase [57, 58, 59]. Hydrodynamics provides the temperature profile and can be also useful for quantitative analyses of these phenomena. Some open questions are as follows: Usually, the initial time in hydrodynamic simulations is chosen as around $\tau_0 \sim 0.6$-1.0 fm/c.

How do we get such an early thermalisation time? Although $gg \rightarrow ggg$ process seems to play an important role in thermalisation, the resultant thermalisation time is still a few fm/c [60]. Note that a typical life time of the QGP phase from hydrodynamic simulations is around 3 fm/c in central collisions at RHIC. In this review, I did not go into details about the HBT puzzle [61]. The important point is to find a solution of the HBT puzzle which is compatible with other observables such as $p_T$ spectra and $v_2$.

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