Theoretical description of the ferromagnetic $\pi$-junctions near the critical temperature

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Abstract

The theory of ferromagnetic $\pi$-junction near the critical temperature is presented. It is demonstrated that in the dirty limit the modified Usadel equation adequately describes the proximity effect in ferromagnets. To provide the description of an experimentally relevant situation, oscillations of the Josephson critical current are calculated as a function of ferromagnetic layer thickness for different transparencies of the superconductor-ferromagnet interfaces.

1. INTRODUCTION

The strong exchange field acting on the electrons in a ferromagnet provokes the damped oscillatory behavior of the superconducting order parameter. This effect is at the origin of the $\pi$-junction realization in superconductor-ferromagnet-superconductor (S/F/S) systems [1,2]. The study of the superconducting $\pi$-state sheds more light on the coexistence of superconductivity and magnetism in general and may be also important for superconducting electronics (see for example [3]).

Recently such $\pi$-junctions have been successfully fabricated and experimentally studied in [4–6]. While the existing theoretical approach [2] provides a good qualitative description
of the observed damped oscillations of the critical current as a function of ferromagnetic layer thickness [5,6], a more detailed description is needed to take into account the finite S/F boundary transparency. Indeed, in [5] Nb/Al/Al$_2$O$_3$/PdNi/Nb junctions have been studied. In such junctions, the presence of the Al$_2$O$_3$ barrier can be modeled by a very low transparency of one of the S/F interfaces.

In this work we develop a theory of ferromagnetic junctions with different transparencies of the interfaces providing a general description of such junctions near the critical temperature.

II. GENERAL FORMALISM

We will concentrate on the studies of the properties of an S/F/S junction with a thin F-layer of thickness $d$ and large superconducting electrodes, see Fig. 1. The very convenient set of equations describing an inhomogeneous superconductivity has been elaborated by Eilenberger [7]. These are transport-like equations for the energy-integrated Green’s functions $f$ and $g$, assuming that relevant length scales are much larger than atomic length scales. For our geometry all quantities depend only on one coordinate $x$, and in the presence of the ferromagnetic exchange field $h(x)$ acting on the electron spins in the F-layer, the Eilenberger equations take the form (see for example [8])

\[
\left(\omega + ih(x) + \frac{1}{2}\tau G(x, \omega)\right) f(x, \theta, \omega) + \frac{1}{2} v_F \cos \theta \frac{\partial f(x, \theta, \omega)}{\partial x} = \left(\Delta(x) + \frac{1}{2}\tau F(x, \omega)\right) g(x, \theta, \omega),
\]

\[
G(x, \omega) = \int \frac{d\Omega}{4\pi} g(x, \theta, \omega), \quad F(x, \omega) = \int \frac{d\Omega}{4\pi} f(x, \theta, \omega),
\]

\[
f(x, \theta, \omega) f^+(x, \theta, \omega) + g^2(x, \theta, \omega) = 1,
\]

where $\omega = 2\pi T (n + 1/2)$ are the Matsubara frequencies and $\theta$ is the angle between the $x$ axis and the direction of the Fermi velocity $v_F$, and $\tau$ is the elastic scattering time. In the following, we consider the behavior of S/F/S systems close to the critical transition temperature $T_c$, so we may linearize equations (1) by putting $g = \text{sign} (\omega)$.

Usually the electron scattering mean free path in S/F/S systems is rather small. In such
a dirty limit the angular dependence of the Green’s functions is weak, and the Eilenberger equations can be replaced by much simpler Usadel equations [9]. In fact, the conditions of the applicability of Usadel equations are $T_c \tau \ll 1$ and $h \tau \ll 1$. The second condition, is much more restrictive due to a large value of the exchange field ($h \gg T_c$). Therefore the attempts to retain in the Usadel equations the first correction in the parameter $h \tau$ have been made in [11–13]. They have resulted in the renormalization of the diffusion constant in F-layer $D_f \rightarrow D_f(1 - 2ih\tau)$, where $D_f = v_F^2 \tau / 3$. The critical analysis of this renormalization in [14] has revealed its inaccuracy, but has not provided the right correction. To resolve this controversy we perform below a careful derivation of the Usadel equation for an F-layer retaining the linear correction over parameter $h \tau$.

In the ferromagnet the Cooper pairing is absent, so $\Delta = 0$, and considering the case of the strong exchange field $\omega \ll h$, we may write the linearized Eilenberger equation for a function $f$ as

$$\left(ih + \frac{1}{2\tau} \text{sign}(\omega)\right)f + \frac{1}{2} v_F \cos \theta \frac{\partial f}{\partial x} = \frac{1}{2\tau} \text{sign}(\omega) F,$$

(2)

where $v_F$ and $\tau$ refer to the Fermi velocity and the scattering time in the ferromagnet. Let us recall that $F$ is an anomalous Green function averaged over the angles. Then we may present $f$ as $f = F(x) + f_1(x, \theta)$, where $f_1 \ll F$. So for $\omega > 0$

$$ihf + \frac{f_1}{2\tau} + \frac{1}{2} v_F \cos \theta \frac{\partial f}{\partial x} = 0.$$  

(3)

The averaging of this equation over the angles gives

$$ihF + \frac{1}{2} v_F \left\langle \cos \theta \frac{\partial f_1}{\partial x} \right\rangle = 0.$$  

(4)

Multiplying (3) by $\cos \theta$, averaging and taking the derivative over $x$ we obtain

$$\left\langle \cos \theta \frac{\partial f_1}{\partial x} \right\rangle = -\frac{\tau v_F}{(2ih\tau + 1)} \left(\left\langle \cos^2 \theta \frac{\partial^2 f_1}{\partial x^2} \right\rangle + \frac{1}{3} \frac{\partial^2 F}{\partial x^2}\right).$$  

(5)

Then combining (3) and (4) we have

$$ihF - \frac{\tau v^2}{2 (2ih\tau + 1)} \left(\left\langle \cos^2 \theta \frac{\partial^2 f_1}{\partial x^2} \right\rangle + \frac{1}{3} \frac{\partial^2 F}{\partial x^2}\right) = 0.$$  

(6)
Neglecting in this equation the $f_i$ term, we obtain the usual Usadel equation with the renormalized diffusion constant $D_f \to D_f(1 - 2i\hbar\tau)$. However the omitted term gives a correction of the same order as that coming from the diffusion constant renormalization [14]. It effectively means that it is needed to keep higher order terms in the expansion of $f$ over $\cos \theta$ (Legendre polynomials). Multiplying (3) consecutively by $\cos^2 \theta$ and $\cos^3 \theta$ and taking derivative over $x$, we obtain after some algebra the following exact equation

$$ihF - \frac{\tau v_F^2}{2(2i\hbar\tau + 1)} \left\{ \frac{-1}{(2i\hbar\tau + 1)} \left[ \frac{2i\hbar\tau}{3} \frac{\partial^2 F}{\partial x^2} - \frac{v_F^2 \tau^2}{(2i\hbar\tau + 1)} \left( \frac{\cos^4 \theta \frac{\partial^4 f_1}{\partial x^4}}{5 \frac{\partial^4 x^4}} \right) \right] + \frac{1}{\partial x^2} \right\} = 0. \tag{7}$$

Here we may already safely neglect the $f_1$ term, as it gives a contribution $\sim (\hbar\tau)^2$. In the result, in the linear approximation over $\hbar\tau$, the Usadel equation is written as

$$ihF - \frac{\tau v_F^2}{6(4i\hbar\tau + 1)} \frac{\partial^2 F}{\partial x^2} - \frac{\tau^3 v_F^4 \frac{\partial^4 F}{\partial x^4}}{10} = 0. \tag{8}$$

Taking in mind that in zero approximation over $\hbar\tau$, the function $F$ satisfies the equation

$$ihF - \frac{\tau v_F^2}{6} \frac{\partial^2 F}{\partial x^2} = 0, \tag{9}$$

we may finally rewrite (8) in the following form applicable for positive and negative $\omega$

$$isign(\omega) \hbar F - \frac{D_f [1 - i(2/5)\hbar\tau sign(\omega)]}{2} \frac{\partial^2 F}{\partial x^2} = 0. \tag{10}$$

So we conclude that the first correction in the parameter $\hbar\tau$ to the Usadel equation leads to the somewhat different renormalization of the diffusion constant $D_f \to D_f(1 - i(2/5)\hbar\tau sign(\omega))$, comparing to what has been suggested before [11–13] ($D_f \to D_f [1 - i2\hbar\tau sign(\omega)]$). This result has been also independently obtained by Tagirov [15].

In the superconducting layer, the linearized Usadel equation is

$$|\omega| F(x, \omega) - \frac{D_s}{2} \frac{\partial^2 F(x, \omega)}{\partial x^2} = \Delta(x), \tag{11}$$

$$\Delta(x) = |\lambda| \pi T \sum_\omega F(x, \omega), \tag{12}$$
where \( \lambda \) is the BCS coupling constant and \( D_s \) is the diffusion coefficient in S layer.

The equations (10) and (11) are completed by the general boundaries conditions at the S/F interfaces [16]

\[
F_s(d/2) = F_f(d/2) + \xi_f \gamma_B \frac{\partial F_f}{\partial x}_{d/2},
\]

\[
F_s(-d/2) = F_f(-d/2) - \xi_f \gamma_B \frac{\partial F_f}{\partial x}_{-d/2},
\]

\[
\frac{\partial F_s}{\partial x}_{\pm d/2} = \frac{\sigma_n}{\sigma_s} \frac{\partial F_f}{\partial x}_{\pm d/2},
\]

where the notation \( F_s(F_f) \) is used for the anomalous Green function in a superconductor (ferromagnet) and \( \sigma_n (\sigma_s) \) is the conductivity of the F-layer (S-layer above \( T_c \)). \( \xi_f = \sqrt{\frac{D_f}{\tau}} \) is the characteristic length of superconducting correlations decaying in F-layer, and \( \xi_s = \sqrt{\frac{D_s}{2T_c}} \) is the superconducting coherence length of the S-layer, the parameter \( \gamma_B = \frac{R_B \sigma_f}{\xi_f} \), where \( R_B \) is the S/F boundary resistance per unit area. Here we introduce two different parameter \( \gamma_B_1 \) and \( \gamma_B_2 \), for the left and right S/F interfaces, assuming that their boundary resistances may be different. Note that the parameter \( \gamma_B \) is directly related to the transparency of the S/F interface \( T = \frac{1}{1+\gamma_B} \) [17]. The limit \( T = 0 (\gamma_B = \infty) \) corresponds to a vanishingly small boundary transparency, and the limit \( T = 1 (\gamma_B = 0) \) corresponds to a perfectly transparent interface.

In principle, the equations (10) and (11) with boundary conditions (13) give the complete description of the S/F/S junction near the transition temperature.

III. CALCULATION OF THE JOSEPHSON CRITICAL CURRENT

Let us start with a general solution of the Usadel equation in the ferromagnet for \( \omega > 0 \)

\[
F_f(\omega > 0, x) = A_\omega \sinh(kx) + B_\omega \cosh(kx),
\]

where \( k = [(1 - 0.2h\tau) + i(1 + 0.2h\tau)] \sqrt{\frac{D_f}{\hbar}} \). Similarly for \( \omega < 0 \)

\[
F_f(\omega < 0, x) = C_\omega \sinh(k^*x) + D_\omega \cosh(k^*x)
\]
We see that the complex wave-vector $k$ describes the oscillating exponential damping of the anomalous Green function inside the ferromagnet. In the case of very small scattering time ($\tau \to 0$), the imaginary and real parts of the wave-vector $k$ coincide, and so the characteristic lengths for oscillating and damping are exactly the same. The finite scattering length decreases the oscillating period and increases the damping length. This is consistent with the fact that in a pure limit the exponential damping is replaced by a much slower power damping, while the oscillations have a much shorter period comparing to the dirty limit [1,13,18].

To obtain the analytical description of the critical current in S/F/S junctions, it is useful to introduce the functions $F^+$ and $F^-$ [2]

$$F^+(\omega > 0, x) = F(\omega > 0, x) + F(\omega < 0, x), \quad (16)$$

$$F^-(\omega > 0, x) = F(\omega > 0, x) - F(\omega < 0, x). \quad (17)$$

In a superconductor, the equation for $F_s^+$ is the same as (11) with $\Delta \to 2\Delta$, while the equation for $F_s^-$ is much simpler

$$|\omega| F_s^-(x, \omega) - \frac{D_s}{2} \frac{\partial^2 F_s^-(x, \omega)}{\partial x^2} = 0, \quad (18)$$

and the self consistency equation reads as

$$\Delta = |\lambda| \pi T \sum_{\omega > 0} F_s^+(x, \omega). \quad (19)$$

The boundary conditions (13) will be the same for $F^+$ and $F^-$ functions too. Using the solution of (18) for $F_s^-(x, \omega)$, it may be demonstrated (see also [2]) that in the case of a rather large resistivity of the F layer ($\frac{\sigma_n \xi_s}{\sigma_f \xi_f} \ll 1$) or in the case low transparency of the interfaces, when the condition $\frac{\sigma_n \xi_s \xi_f}{\sigma_f \xi_f} \ll 1$ is satisfied, the boundary conditions for $F_f^-$ functions are essentially simplified

$$F_f^-(d/2) + \xi_f \gamma_B \left( \frac{\partial F_f^-}{\partial x} \right)_{d/2} = 0, \quad (20)$$

$$F_f^-(d/2) - \xi_f \gamma_B \left( \frac{\partial F_f^-}{\partial x} \right)_{-d/2} = 0. \quad (21)$$
Further on, introducing the functions

\[ \mathcal{F}^+ = |\lambda| \pi T \sum_{\omega > 0} F^+ (\omega > 0, x), \]  
\[ \mathcal{F}^- = |\lambda| \pi T \sum_{\omega > 0} F^- (\omega > 0, x), \]  

we obtain the following system of equations for the order parameter and its derivatives on the left and right sides of the junction

\[ \mathcal{F}_f^- (d/2) + \xi f \gamma B_2 \left( \frac{\partial \mathcal{F}_f^-}{\partial x} \right)_{d/2} = 0, \]  
\[ \mathcal{F}_f^- (-d/2) - \xi f \gamma B_1 \left( \frac{\partial \mathcal{F}_f^-}{\partial x} \right)_{-d/2} = 0, \]  

\[ \Delta (x = d/2) = \Delta^+ = \mathcal{F}_f^+ (d/2) + \xi f \gamma B_2 \left( \frac{\partial \mathcal{F}_f^+}{\partial x} \right)_{d/2}, \]  
\[ \Delta (x = -d/2) = \Delta^- = \mathcal{F}_f^+ (-d/2) - \xi f \gamma B_1 \left( \frac{\partial \mathcal{F}_f^+}{\partial x} \right)_{-d/2}, \]

\[ \left( \frac{\partial \Delta^+}{\partial x} \right) = \sigma_n \left( \frac{\partial \mathcal{F}_f^+}{\partial x} \right)_{d/2}, \]  
\[ \left( \frac{\partial \Delta^-}{\partial x} \right) = \sigma_n \left( \frac{\partial \mathcal{F}_f^+}{\partial x} \right)_{-d/2}. \]

This system of six equations, after the elimination of four coefficients of the type \( \left( |\lambda| \pi T \sum_{\omega > 0} A_\omega \right) \) may be reduced to a standard linear form of \( \left( \frac{\partial \Delta^+}{\partial x} \right) \), \( \left( \frac{\partial \Delta^-}{\partial x} \right) \), \( \Delta^+ \) and \( \Delta^- \)

\[ \Delta^+ = M_{11} \Delta^- + M_{12} \left( \frac{\partial \Delta^-}{\partial x} \right), \]  
\[ \left( \frac{\partial \Delta^+}{\partial x} \right) = M_{21} \Delta^- + M_{22} \left( \frac{\partial \Delta^-}{\partial x} \right), \]

which was used by De Gennes in his general description of the Josephson junction close to \( T_c [19] \). Following the approach [19], the critical current of the SFS junction is given directly by the expression

\[ I_c = \frac{-ie\pi N_s(0)D_s}{4TcM_{12}} \left( \Delta^* - \Delta^+ - c.c \right), \]  

where \( N_s(0) \) is the electron density of states in S electrodes when they are in the normal state.
The calculation of the coefficient $M_{12}$ is straightforward but rather cumbersome for a general case. The corresponding formula is to intricate to be presented here. So, to discuss the essential physics, we consider below only the most interesting limits.

In the case of highly transparent interfaces, it is possible to obtain an analytical expression of the critical current taking into account the first order correction over $\hbar\tau$ in the Usadel equation (10)

$$I_c = \frac{\pi \Delta^2 \sigma_n}{8eT_c \xi_f} \left| \frac{k}{\sinh \left( \frac{k d}{2} \right) \cosh \left( \frac{k d}{2} \right)} + c.c. \right|$$

where the complex wave vector $k = [(1 - 0.2\hbar\tau) + i(1 + 0.2\hbar\tau)] \sqrt{\frac{F}{D_f}}$. The dependences $I_c(d)$ for different parameter $\hbar\tau$ are presented in Fig. 2.

When the critical current goes through zero, the transition from 0 to $\pi$-shift Josephson contact takes place. As expected, with the increase of the scattering time the period of oscillations becomes somewhat shorter, while its amplitude increases. This is consistent with a power low decrease of the critical current with F-layer thickness in the clean limit [1].

Further we concentrate on the influence of the S/F boundary transparency on the $I_c$ oscillations, and then we neglect the $\hbar\tau$ correction in the diffusion coefficient. Starting with the case of completely transparent interfaces $\gamma_{B1,2} \to 0$, we retrieve the corresponding expression previously obtained in [2]

$$I_c = \frac{\sigma_n \pi \Delta^2}{\xi_f eT_c} \left| \frac{\sin(x) \cosh(x) + \cos(x) \sinh(x)}{\cos(2x) - \cosh(2x)} \right|, \quad (34)$$

where $x = d/\xi_f$. In the case of low transparency of both interfaces $\gamma_{B1,2} \gg 1$, the critical current is

$$I_c = \frac{1}{\gamma_{B1} \gamma_{B2} \xi_f eT_c} \left| \frac{\cos(x) \sinh(x) - \sin(x) \cosh(x)}{\cos(2x) - \cosh(2x)} \right|. \quad (35)$$

If one interface (the left S/F interface) is transparent, $\gamma_{B1} \to 0$, the critical current is

$$I_c = \frac{\sigma_n \pi \Delta^2}{\xi_f eT_c} \left| \frac{\cosh(x) \sin(x) + \cos(x) \sinh(x) + 2\gamma_{B2} \cos(x) \cosh(x)}{(1 - 2\gamma_{B2}^2) \cos(2x) - (1 + 2\gamma_{B2}^2) \cosh(2x) - 2\gamma_{B2} \sin(2x) + \sinh(2x)} \right|. \quad (36)$$
If one interface (the first S/F interface) has a low transparency, $\gamma_{B1} \gg 1$, the critical current is

$$I_c = \frac{\sigma_n \pi \Delta^2}{\gamma_{B1} \xi_f e T_c} \left| \frac{\cos(x) \cosh(x) - \gamma_{B2} \left[ \cosh(x) \sin(x) - \cos(x) \sinh(x) \right]}{(1 - 2\gamma_{B2}^2) \cos(2x) + (1 + 2\gamma_{B2}^2) \cosh(2x) - 2\gamma_{B2} \left[ \sin(2x) - \sinh(2x) \right]} \right|$$

(37)

The evolution of the $I_c(d)$ dependence with the decrease of the transparency of the second interface is presented in Fig. 3. We observe that with the increase of $\gamma_{B2}$ the amplitude of oscillations decreases, as well as the thickness of the ferromagnetic layer corresponding to the first zero of the critical current.

In the experiment [5], the presence of Al$_2$O$_3$ barrier at one S/F boundary can be modeled by a low transparency interface ($\gamma_{B1} \gg 1$) while the other boundary is quite transparent $\gamma_{B2} \to 0$. The resistance $R_n$ per unit area of the S/F/S junction in this experiment is dominated by the resistance of the tunnel barrier and thus $R_n \sim \frac{\gamma_{B1} \xi_f}{\sigma_n}$. So we may expect that the experimental situation [5] must be described by the expression (37) with $\gamma_{B2} = 0$ i.e.

$$I_c R_n = \frac{\pi \Delta^2}{e T_c} \left| \frac{\cos(x) \cosh(x)}{\cos(2x) + \cosh(2x)} \right| .$$

(38)

Note that this expression is quite different from that used in [5] to fit the experimental data. Unfortunately in [5] there is no information which could shed light not only on the reasons of the discrepancy between our formula but even at the origin of the used theoretical expression. The experimental results on the $I_c(d)$ oscillations presented in [5] have been obtained at low temperature. Nevertheless we think that the overall shape of the $I_c(d)$ curve is not very sensitive to the temperature and we compare the experimental data [5] with our theoretical expression (38) in Fig. 4. We use as the fitting parameter $\xi_f \sim 30 \, \text{Å}$, while the experiment [20] provided a value of $\xi_f$ around 35 Å in our notation. The value of the other fitting parameter $\frac{e \Delta^2}{\xi_f e T_c}$ is 110 $\mu$V; this parameter is quite difficult to estimate because of the uncertainty on the value of $\Delta$ at the S/F interface in the geometry of experiment [5]. Also following the analysis of the authors [5] presented in their previous publication [20],
we have taken into account that the actual ferromagnetic thickness of PdNi layer is reduced by 15 Å due to some interdiffusion at the S/F interface. The obtained description of the experimental data [5] is quite satisfactory.

In conclusion, we have presented the general theoretical description of the ferromagnetic π-junctions near superconducting transition temperature and proposed a simple way to take into account the finite elastic scattering time in Usadel equation. The obtained analytical expressions may be useful for the analysis of different experimental realizations of such junctions. Our analysis may be also easily generalized to the situation when the superconducting electrodes are fabricated from different materials.

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REFERENCES

[1] A. I. Buzdin, L. N. Bulaevskii, and S. V. Panyukov, Pis'ma Zh. Eksp. Teor. Fiz. 35, 147 (1982) [JETP Lett. 35, 178 (1982)].

[2] A. I. Buzdin and M. Y. Kuprianov, Pis'ma Zh. Eksp. Teor. Fiz. 53, 308 (1991) [JETP Lett. 53, 321 (1991)].

[3] L. B. Ioffe, V. B. Geshkenbein, M. V. Feigelman, A. L. Fauchere, and G. Blatter, Nature (London) 398, 679 (1999).

[4] V. V. Ryazanov, V. A. Oboznov, A. Yu. Rusanov, A. V. Veretennikov, A. A. Golubov, and J. Aarts, Phys. Rev. Lett. 86, 2427 (2001).

[5] T. Kontos, M. Aprili, J. Lesueur, F. Genet, B. Stephanidis, and R. Boursier, Phys. Rev. Lett. 89, 137007 (2002).

[6] Y. Blum, A. Tsukernik, M. Karpovski, and A. Palevski, Phys. Rev. Lett. 89, 187004 (2002).

[7] G. Eilenberger, Z. Phys. 214, 195 (1968).

[8] L. N. Bulaevskii, A. I. Buzdin, M. L. Kubic, and S. V. Panyukov, Adv. Phys. 34, 175 (1985).

[9] L. Usadel, Phys. Rev. Lett. 95, 507 (1970).

[10] L. R. Tagirov, Physica C 307, 145 (1998).

[11] Y. N. Proshin and M. G. Khusainov, Zh. Eksp. Theor. Fiz. 113, 1708 (1998) [Sov. Phys. JETP 86, 930 (1998)]; 116, 1887 (1999) [Sov. Phys. JETP 89, 1021 (1999)].

[12] Y. A. Izyumov, Y. N. Proshin, M. G. Khusainov, Phys. Uspekhi. 45, 109 (2002).

[13] I. Baladié and A. Buzdin, Phys. Rev. B 64, 224514 (2001).

[14] Ya. V. Fominov, N. M. Chetkatchev, and A. A. Golubov, Phys. Rev. B 66, 14507
(2002).

[15] L. Tagirov, private communication.

[16] M. Y. Kuprianov and V. F. Lukichev, Zh. Eksp. Theor. Fiz. 94, 139 (1988) [Sov. Phys. JETP 67, 1163 (1988)].

[17] J. Aarts, J. M. E. Geers, E. Brück, A. A. Golubov, and R. Coehoorn, Phys. Rev. B 56, 2779 (1997).

[18] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. B 65, 134505 (2002)

[19] P. G. De Gennes, *Superconductivity of Metals and Alloys*, W. A. Benjamin, New York (1966).

[20] T. Kontos, M. Aprili, J. Lesueur, and X. Grison, Phys. Rev. Lett. 86, 304 (2001).

**Figure captions**

**FIG. 1** Geometry of the considered S/F/S system. The thickness of the ferromagnetic layer is \(d\). The transparency of the left S/F interface is characterized by the coefficient \(\gamma_{B1}\) and the transparency of the right F/S interface is characterized by \(\gamma_{B2}\).

**FIG. 2** Critical current of the S/F/S junction as a function of the thickness of the ferromagnetic layer normalized by \(\xi_f\) for different scattering time. Both S/F interfaces are completely transparent. The parameter \(h\tau\) is equal to 0, 0.25 and 0.5.

**FIG. 3** Critical current of the S/F/S junction as a function of the thickness of the ferromagnetic layer normalized by \(\xi_f\). The first S/F interface has a low transparency \((\gamma_{B1} \gg 1)\). The parameter \(\gamma_{B2}\) characterizing the transparency of the second interface is chosen as 0.2, 1.5, 3.0, and 10.

**FIG. 4** The experimental points correspond to the measurement of the critical current, done by Kontos et al [5], vs the PdNi layer thickness. The theoretical curve is the best fit obtained by using formula (38). The fitting parameters are \(\xi_f \sim 30\) Å and \(\frac{\Delta^2}{eT_c} \sim 110\) \(\mu\)V.
$I_c \frac{\xi_f}{\sigma_n} \frac{\pi \Delta^2}{eT_c}$

Figure 2

- $h \tau = 0.5$
- $h \tau = 0.25$
- $h \tau = 0$
$I_c \gamma_{B1} \xi_f \frac{\pi \Delta^2}{\sigma_n} \frac{\pi \Delta^2}{eT_c}$

- $\gamma_{B2} = 0.2$
- $\gamma_{B2} = 1.5$
- $\gamma_{B2} = 3.0$
- $\gamma_{B2} = 10$

$\gamma_{B1} \gg 1$

**Figure 3**

$d_f/\xi_f$
