Application of FPGA technologies for modeling hydrophysical processes in reservoirs with complex bottom topography

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Abstract. The article is devoted to the development and research of methods for accelerating calculations and processing input data for mathematical modeling of hydrodynamic processes in reservoirs with complex bottom geometry. The mathematical model of hydrodynamics considers: microturbulent diffusion; the influence of salinity and temperature. To increase the speed of calculations, it is proposed to transfer part of the computational load to reconfigurable computing systems (RVS). The paper also proposes an algorithm for adding input data by analyzing the existing database of environmental data, which was successfully verified analytically. The practical significance of the work is that a hardware implementation of the IP core was proposed, which allows solving systems of linear algebraic equations that arise when sampling the model of hydrodynamics of the Caspian Sea. It is experimentally established that the resulting implementation is characterized by low power consumption and is capable of processing large-dimensional matrices. The prospects of using the proposed algorithm are determined.

1. Introduction

To predict the situation of waters with complex bottom geometry, mathematical models are constructed, considering the unique features of the researched water object – climatic factors and hydrodynamic regimes. Among the papers of Russian scientists devoted to the research and forecast of aquatic ecosystems can be identified the papers of Marchuk G.I., Matishov G.G., Sukhinov A.I., etc. [1-8] The development of models, software and information systems for monitoring and forecasting the situation of water ecosystems is performed by leading foreign research centers and organizations such as Sweden’s Meteorological and Hydrological Institute, Center for Water Research, National Oceanic and Atmospheric Administration, Centre for Ecology and Hydrology [9-14].

Many approaches are used to model spatially inhomogeneous three-dimensional Hydrophysics processes in reservoirs with complex bottom topography. The most acceptable of them is the modeling of hydrodynamic processes of a reservoir as a single body of water. It is known that this approach increases the computational complexity of algorithms and increases the time spent.

Another problem is the lack of sufficient input data. For example, a three-dimensional digital map was developed for the Caspian Sea, including 2.5 billion nodal points, while existing databases on
hydrometeorological characteristics have coarser measurement grids and do not allow modeling with the required accuracy (figure 1).

![Figure 1. Three-dimensional map of the Caspian Sea](image)

Problems that arise can be solved by using reconfigurable computing systems. A high degree of parallelization allows you to process databases at high speed and analyze multiple indicators in near-real-time mode.

2. Statement of the problem the hydrodynamics of the Caspian Sea

One of the computationally labor-intensive tasks is the model of hydrodynamics of the Caspian Sea:

\[
\begin{align*}
    u_t' &+ uu_x' + vu_y' + wu_z' = -\frac{1}{\rho} P_x' + \left(\mu u_x'\right)_x' + \left(\mu u_y'\right)_y' + \left(\mu u_z'\right)_z' + 2\Omega (v \sin \vartheta - w \cos \vartheta), \\
    v_t' &+ uv_x' + vv_y' + wv_z' = -\frac{1}{\rho} P_y' + \left(\mu v_x'\right)_x' + \left(\mu v_y'\right)_y' + \left(\mu v_z'\right)_z' + 2\Omega \mu \sin \vartheta, \\
    w_t' &+ uw_x' + vv_y' + ww_z' = -\frac{1}{\rho} P_z' + \left(\mu w_x'\right)_x' + \left(\mu w_y'\right)_y' + \left(\mu w_z'\right)_z' + 2\Omega \mu \cos \vartheta + g, \\
    \rho_t' &+ (\rho u)'_x + (\rho v)'_y + (\rho w)'_z = 0, \\
    T_t' &+ uT_x' + vT_y' + wT_z' = (\mu T_x')_x' + (\mu T_y')_y' + (\mu T_z')_z' + f_T, \\
    S_t' &+ uS_x' + vS_y' + wS_z' = (\mu S_x')_x' + (\mu S_y')_y' + (\mu S_z')_z' + f_S,
\end{align*}
\]

where \( t \) is a time; \( \mathbf{V} = \{u, v, w\} \) is the velocity vector of the water flow; \( P \) is full hydrodynamic pressure; \( T, S \) are water temperature and salinity; \( \rho \) is density of water; \( \mu, \nu \) are horizontal and vertical diffusion coefficients, respectively [15]; \( \Omega = \Omega (\cos \vartheta j + \sin \vartheta k) \) is angular velocity of the Earth's rotation; \( \vartheta \) is the latitude of the place; \( g \) is acceleration of gravity; \( f_T, f_S \) are sources of heat and salt (located on the border of the region). In this case, two components are conditionally distinguished from the total hydrodynamic pressure: the pressure function of the liquid column and the hydrodynamic part [16]

\[
P(x, y, z, t) = p(x, y, z, t) + \rho_0 g z,
\]
where \( p \) is the hydrostatic pressure of the unperturbed fluid, \( \rho_0 \) is fresh water density under normal conditions (at 20°C); the water density is determined by the formula:
\[
\rho = \tilde{\rho} + \rho_0,
\]
where \( \rho_0 \) is fresh water density under normal conditions, \( \tilde{\rho} \) determined by the equation recommended by UNESCO:
\[
\tilde{\rho} = \tilde{\rho}_w + (8.24493 \cdot 10^{-1} - 4.0899 \cdot 10^{-3} T + 7.6438 \cdot 10^{-5} T^2 - 8.2467 \cdot 10^{-7} T^3 + 5.3875 \cdot 10^{-9} T^4) S + (-5.72466 \cdot 10^{-3} + 1.0227 \cdot 10^{-4} T - 1.6546 \cdot 10^{-6} T^2) S^{3/2} + 4.8314 \cdot 10^{-4} S^2,
\]
where \( \tilde{\rho}_w \) is the density of fresh water is specified by the polynomial:
\[
\tilde{\rho}_w = 999.842594 + 6.793952 \cdot 10^2 T - 9.09529 \cdot 10^3 T^2 + 1.001685 \cdot 10^4 T^3 - 1.120083 \cdot 10^6 T^4 + 6.536332 \cdot 10^9 T^5.
\]

Add the boundary conditions to the system (1) – (6).
At the bottom:
\[
\rho, \mu(V_n) = -\tau, V_n = 0, P_n = 0, T_n = 0, S_n = 0, f_T = 0, f_S = 0.
\]
On the side surface:
\[
(V_n) = 0, V_n = 0, P_n = 0, T_n = 0, S_n = 0, f_T = 0, f_S = 0.
\]
On the upper border:
\[
\rho, \mu(V_n) = -\tau, w = -\frac{\rho' g}{\rho}, P_n = 0, T_n = 0, S_n = 0, f_T = k_r(T - T), f_S = \frac{\omega}{h_t - h_w} - S.
\]
Here \( \omega \) is the rate of evaporation of liquid; \( V_n, V_i \) are normal and tangential components of the water flow velocity vector; \( \rho \) is density of the water environment; \( \rho_v \) is the density of the suspended impurities; \( T_a \) is atmospheric temperature; \( k_r \) is heat transfer coefficient between the atmosphere and the water environment; \( h_t \) is depth step; \( h_w = \omega \tau \) is the thickness of the liquid layer that evaporates over time \( \tau \); \( \tau = \{\tau, \tau, \tau, \} \) is the tangential stress vector, which is calculated using the formulas: for a free surface \( \tau = \rho_a CD_l |w| w \), where \( w \) is wind speed vector relative to water, \( \rho_a \) is atmospheric density, \( CD_l \) is a dimensionless coefficient of surface resistance that depends on wind speed and is in the range of 0.0016–0.0032 [17]; for the bottom \( \tau = \rho b CD_b |v| v \), where \( CD_b = g k_x^2 / h^{3/2} \), at the same time \( k_x \) is the group roughness coefficient in the manning formula (varies in the range 0.025-0.2), \( h = H + \eta \) is water depth, \( H \) is the depth of the undisturbed surface, \( \eta \) is surface height relative to the geoid.

3. Methods of accelerating the calculations using FPGA technology
Modern technologies can be used to solve the problems that arise when solving the described problem. Methods related to parallelization of algorithms for solving tasks at the software and hardware levels are developed to increase the speed of calculations. Among the high-speed equipment used for solving model problems of hydrodynamics, it is necessary to single out supercomputers built on the basis of programmable logic integrated circuits (FPGAs), the main advantage of which is a high degree of parallelization. The time of processes occurring in the water area of a reservoir is comparable to the time of modeling these processes on conventional computers. In the process of discretization of a model problem of the form (1) – (6), a system of linear algebraic equations of large dimension arises, the solution of which requires large computational costs.

To optimize calculations, we developed a library element-the IP-core for organizing the computational process using the algorithm of an alternately triangular method for solving grid equations that arise during the discretization of model hydrophysics problems for the self-adjoint case [18].

The IP core was implemented using the following algorithm.
Algorithm 1. Solving a system of grid equations

1. Arrays are Formed for the input values of the SLA.
2. Is filling the arrays with data from files.
3. Constants are used for the calculations.
4. A library element is Connected that implements the square root calculation operation.
5. Arrays of initial values for variables are Formed.
6. Enter the condition of the calculations when you change the time signal on the rising edge.
7. Start of the cycle of comparing the maximum discrepancy and the maximum error value.
8. Calculation of the vector of residuals and its norms on the input values.
9. Calculation of the amendment.
10. The iteration counter is being built up.
11. End of the cycle of comparing the maximum discrepancy with the maximum error value.
12. Creating an array of SLOUGH solutions and writing it to a file.

The implementation of this algorithm on the xc7vx485tffg1157-1 FPGA of the seventh series used in the “Taigeta” RVS (figure 2) uses a small part of the available resources (table 1) with the power consumption shown in table 2.

![Figure 2. “Taigeta” computing block.](image)

Table 1. Resources involved in implementation

| Resource | Utilization | Available | Utilization % |
|----------|-------------|-----------|---------------|
| LUT      | 3721        | 303600    | 1.23          |
| FF       | 127         | 607200    | 0.02          |
| DSP      | 27          | 2800      | 0.96          |
| IO       | 33          | 600       | 5.50          |
| BUFG     | 1           | 32        | 3.12          |
Table 2. Power consumption

|                           |       |
|---------------------------|-------|
| Total On-Chip Power       | 0.258 W |
| Junction Temperature      | 25.4 °C |
| Thermal Margin            | 59.6 °C (41.1 W) |
| Power supplied to off-chip devices | 0 W |
| Confidence level          | Medium |

Methods based on interpolation or the use of neural networks to assimilate satellite sensing data can be used to address the problem of data scarcity. Currently, existing databases are not being updated as actively and regularly, and satellite data does not provide all the necessary input data for predictive modeling.

Within the framework of this research, an algorithm for expanding input data databases for predictive modeling of hydrophysical processes has been developed. The main idea of this method is to find unknown characteristics based on retrospective data correlated with SPZ data.

Algorithm 2. Forecast of ecosystem development based on the observation model

1. Gathering information from public databases (DB).
2. Acquisition and processing of neural network is not less than two consecutive data packets of the SPZ.
3. Determine the gradients of characteristics obtained during data processing in the previous step for each node point.
4. Search for the most similar to the received retrospective data.
5. The definition of missing for the simulation of the characteristics of brute force.
6. Predictive modeling of hydrodynamic processes of a reservoir.
7. Determining the error of modeling the main calculation functions.
8. If the modeling error is more than 15%, we recalculate the gradients of the obtained characteristics considering the data received during the execution of points 4)-7) of this algorithm, otherwise we update the database.

In [19] it is proved that points 4) and 5) are solved most effectively using FPGA technologies. The solution of the problem from point 6) of this algorithm for a given prediction time is described in [20].

4. Conclusion

An algorithm for implementing the alternately triangular SLOUGH solution method for the self-adjoint case is proposed. It is used for solving the problem of distribution of the density of sea water in the Caspian Sea. The application of this algorithm will allow modeling the processes of hydrodynamics and biological kinetics occurring in reservoirs with complex bathymetry in conditions of lack of input data with minimal time costs and error values. Presents data on the results of the modeling involved in the implementation of resource and power consumption.

In the future, it is planned to develop an IP core that implements a modified alternating-triangular method for solving large-dimensional slows for the non-self-adjoint case and its application in solving problems of Hydrophysics and biological kinetics in a mode close to real-time.

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