The Same Physics Underlying SGRs, AXPs and Radio Pulsars

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Abstract Unexpected sign, significant magnitude and variable frequency second derivative exists not only in singular radio pulsars but also in Soft Gamma repeaters (SGRs) and Anomalous X-ray pulsars (AXPs). This paper shows that these phenomena are related, and can be interpreted by a simple unified model, long-term orbital effect. Thus many of previous “isolated” pulsars may be binary pulsars, i.e., orbital period $P_b \approx (47, 72)$ min for AXP 1E 2259+586, and $P_b \approx (20, 34)$ min for PSR J1614–5047, $P_b \approx (3, 6.4)$ min for SGR 1900+14, and $P_b \approx (1.5, 5.8)$ min for SGR 1806–20. In this model, the frequency first derivative of these pulsars is still dominated by magnetic dipole radiation. Therefore, it is not contradictory to the magnetar interpretation of SGRs and AXPs. In other words, the model of this paper provides new interpretation to higher order of derivatives of pulse frequency (second and third...) instead of the first.

Key words: pulsars: binary pulsars — timing noise: individual (1E 2259+586, SGR 1806–20, SGR 1900+14, PSR J1341–6220, PSR J1614–5047)

1 INTRODUCTION

Pulsars are powered by rotational kinetic energy and lose energy by accelerating particle winds and by emitting electromagnetic radiation at their rotational frequency, $\nu$. The slowdown is usually described by

$$\dot{\nu} = -\kappa \nu^n,$$

(1)

where $\kappa$ is a positive constant which determined by the moment of inertia and the magnetic dipole moment of the pulsar and $n$ is the braking index. By Equation (1) we have,

$$\dot{\nu}/\nu = n\dot{\nu}/\nu,$$

(2)

$n = 3$ for constant spin-dipole angle and dipole moment. Distortion of the magnetic field lines in the radial direction from that of a pure dipole, pulsar wind, and time-variable effective magnetic moment results in $1 \leq n \leq 3$ (Manchester et al. 1985; Blandford & Romani 1988). However the frequency first and second derivatives of observed pulsars show that most pulsars differ from $n = 3$ substantially (Hobbs et al. 2004),

$$|\dot{\nu}_{\text{obs}}/\nu_{\text{obs}}| \gg |\ddot{\nu}/\nu| = 3|\dot{\nu}/\nu|. $$

The main characteristics of the frequency second derivative are: (1) the magnitude of it depends on the length of the data span; (2) the sign of it can be both positive and negative; (3) the magnitude of $\dot{\nu}_{\text{obs}}$ can be orders of magnitude larger than that expected by magnetic dipole radiation. These effects are usually attributed to the long-term timing noise.

Gong (2005a) introduced long-term orbital effect to explain the puzzles of radio quiet neutron star 1E 1207.4–5209, in which the discrepancy between the measured $\dot{\nu}_{\text{obs}}$ and the magnetic dipole radiation induced $\dot{\nu}$ is attributed to the long-term orbital effect of an ultra-compact binary system. This model actually provides a mechanism that can interpret the three characteristics of timing noise.

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Further more, Gong & Bignami (2006) analyzes why the orbital period of 1E1207.4–5209 can escape different tests, like modulation of flux density, Doppler shift of pulse frequency, and Roemer delay, and how to test the binary nature by XMM-Newton and Chandra data in an alternative way.

This paper extends the model in two aspects: firstly to different type of pulsars, SGRs, AXPs and radio pulsars; secondly to higher order of frequency derivatives, which can explain puzzles in braking index and timing noise. The orbital period of 5 different pulsars are estimated which can also be tested by the method proposed by Gong & Bignami (2006).

2 THE ULTRA-COMPACT BINARY MODEL

As analyzed by Gong & Bignami (2006), an ultra-compact neutron star (NS) binary with very short orbital period, corresponds to small projected semi-major axis, and hence short Roemer time delay, the time for the pulsed light to travel across the projection of the orbit into observer’s line of sight. In X-ray pulsar, like 1E1207.4–5209, the corresponding Roemer time delay may smaller than the time resolution of the instrument XMM-Newton, which observes it.

Moreover the small projected semi-major axis corresponds to small amplitude and the separation of the sideband (induced by orbital motion) in the Fourier response of the fundamental spin harmonic corresponding to the signal pulse phase. The side bands due to orbital motion are very difficult to resolve from the noise. Therefore, it is the short orbital period that prevent some ultra-compact binary from detecting.

Although a direct test of orbital effect is very difficult, but the long-term effect of an ultra-compact binary can still affect the timing behavior of the pulsar, which may result strange phenomena, such as on 1E 1207.4–5209 (Gong 2005a).

The Roemer time delay from the instantaneous position of the pulsar is

$$\frac{z}{c} = \frac{r \sin i}{c} \sin(\omega + f),$$  \hspace{1cm} (3)

where $c$ is speed of light, $r$ is the distance between the focus and the pulsar, $f$ is the true anomaly, $\omega$ is the angular distance of periastron from the node, $i$ is orbital inclination. The orbital motion induced change of pulse frequency is given, $\Delta \nu$,

$$\frac{\Delta \nu}{\nu} = \frac{v \cdot n_p}{c} = K [\cos(\omega + f) + e \cos \omega],$$  \hspace{1cm} (4)

where $K \equiv \frac{2\pi a_p \sin i}{P_b (1 - e^2)^{1/2}}$ is the semi-amplitude, $e$, $P_b$, $a_p$ are eccentricity, orbital period, and pulsar semi-major axis respectively.

What if a pulsar is in a binary system, however the effect of Roemer time delay and Doppler shift, as given in Equation (3) and Equation (4), respectively, has not been measured? In such circumstance the observational effect is neither as a true isolated pulsar, no as an usual binary pulsar (which has measured Roemer delay or Doppler shift directly).

When a pulsar has a companion, the time received by the observer (Baryon centric time) is

$$t_b = t_p + \frac{z}{c},$$  \hspace{1cm} (5)

where $t_p$ is the proper time of the pulsar, and $z/c$ is dependent of Kepler equation,

$$E - e \sin E = \bar{M} = \bar{n} t,$$  \hspace{1cm} (6)

where $\bar{M}$, $E$ and $\bar{n}$ are mean anomaly, eccentric anomaly and mean angular velocity, respectively. Notice that $t$ is the time of periastron passage, which is uniform.

Obviously for a true isolated pulsar, we have $z/c = 0$ in Equation (5), thus $t_b = t_p$, which means both $t_b$ and $t_p$ are uniform. But for a binary pulsar system, $t_b$ is no longer uniform, whereas $t_p$ is still uniform.

Therefore, the proper time of the pulsar, $t_p$, can be used to replace the uniform time, $t$ of Equation (6), then we have $\bar{M} = \bar{n} t_p$.

If $\Delta \nu$ of Equation (4) is averaged over one orbit period by the measured time, $t_b$, then it gives (Gong 2005a)

$$\langle \Delta \nu \rangle = \frac{xK \nu}{P_b} \pi \left(1 - \frac{e^2}{4}\right) + O(e^4),$$  \hspace{1cm} (7)
where \( x \) is the projected semi-major axis, \( x \equiv a_\text{p} \sin i/c \).

In practical observation, an observer may average \( \Delta \nu \) from 0 to \( T \) (\( T \gg P_b \)) through \( t_b \), the time received by observer, without knowing the orbital period, \( P_b \), at all. Thus the observer tells that the averaged \( \Delta \nu \) is (Gong 2005a)

\[
\langle \Delta \nu \rangle \equiv \beta = \frac{x K \nu}{P_b} \pi (1 - \frac{a_\text{p}^2}{4}) + o(\frac{P_b}{T}) .
\]  

(8)

The second term at the right hand side of Equation (8) is negligible due to \( T \gg P_b \). By Equation (8), if a pulsar is in a binary system, then \( \Delta \nu \) (for convenience bracket, \( \langle \cdot \rangle \) is not used hereafter) measured by the observer is actually contaminated by orbital effect, \( \beta \). Comparatively, for a truly isolated pulsar, there is no orbital effect, and thus \( \beta = 0 \). And for a pulsar that has already been recognized as in a binary system, the effect of \( \beta \) has been absorbed by binary parameters, such as, \( P_b, c, \) and \( \dot{\omega}_{GR} \), the well known advance of periastron given by General Relativity.

The contaminated \( \Delta \nu \) can lead to unexpected effect on the derivative, and derivatives of pulse frequency. Differentiating \( \Delta \nu \) of Equation (8) gives (Gong 2005a)

\[
\dot{\nu}_L = \beta \frac{\dot{a}}{a}(1 - \xi) ,
\]  

(9)

where \( a \) is the semi major axis of the orbit, and \( \xi \equiv \frac{(1 - e^2) \sin^2 \beta}{2(1 + e^2)(1 - e^2/4)} \). By the expression of \( \dot{a}/a \) (Gong 2005b), it is related to the advance of precession of periastron, \( \dot{\omega}_{GR} \) by

\[
\frac{\dot{a}/a}{\dot{\omega}_{GR}} = \frac{M_1 M_2}{3 M^2} \left[ 1 + \frac{e^2}{(1 - e^2/4)} \right] + \frac{e^2}{1 + e^2},
\]  

(10)

where \( M_1 \) and \( M_2 \) are the mass of the pulsar, companion, respectively and \( M \) is the total mass of a binary system. \( P_x, P_y, Q_x, Q_y \) are sine and cosine functions of \( \omega \) (the angle distance of periastron from the node) and \( \Omega \) (the longitude of the ascending node) (Gong 2005b). Notice that \( \dot{a}/a \) is a long periodic term, whereas, \( \dot{\omega}_{GR} \) is a secular term.

Considering Equation (9), the observational \( \dot{\nu}_{\text{obs}} \) is given

\[
\dot{\nu}_{\text{obs}} = \dot{\nu} + \dot{\nu}_L ,
\]  

(11)

where \( \dot{\nu} \) is the intrinsic one, which caused by magnetic dipole radiation and \( \dot{\nu}_L \) is caused by orbital effect.

3 APPLICATION TO SGRS

SGRs and AXPs are believed to be magnetars, the spinning down and magnitude of timing noise of which are as large as 100 times than that of radio pulsars. The derivatives of SGR 1900+14 satisfies following relation,

\[
\frac{\dot{\nu}_{\text{obs}}}{\dot{\nu}_{\text{obs}}} \sim \frac{\nu^{(3)}_{\text{obs}}}{\nu^{(3)}_{\text{obs}}} \sim \frac{\nu^{(4)}_{\text{obs}}}{\nu^{(3)}_{\text{obs}}} .
\]  

(12)

The derivatives of SGR 1806–20 satisfies following relation,

\[
\frac{\dot{\nu}_{\text{obs}}}{\dot{\nu}_{\text{obs}}} \gg \frac{\nu^{(3)}_{\text{obs}}}{\nu^{(3)}_{\text{obs}}} \sim \frac{\nu^{(4)}_{\text{obs}}}{\nu^{(3)}_{\text{obs}}} .
\]  

(13)

The magnetic dipole radiation predicts:

\[
\dot{\nu}/\dot{\nu} = 3\dot{\nu}/\nu , \quad \nu^{(3)}/\nu = 5\dot{\nu}/\nu , \quad \nu^{(4)}/\nu^{(3)} = 7\dot{\nu}/\nu .
\]  

(14)

Thus it seems that the relation, Equation (12), can be explained by magnetic dipole radiation. However the magnitude of \( \dot{\nu}/\dot{\nu} \) is much smaller than observational ratio given by Equation (12). Therefore, the relation Equation (12) cannot be explained by magnetic dipole radiation, neither can Equation (13).
Is the long-term orbital effect induced contamination of pulse frequency responsible for the phenomena in Equation (12) and Equation (13)? Through Equation (11) following relation can be obtained,

$$\frac{\dot{\nu}_{\text{obs}}}{\nu_{\text{obs}}} = \frac{3\dot{\nu}}{\nu} + \frac{\dot{\nu}_{\text{L}}}{\nu_{\text{obs}}} + \dot{\omega}_1 \frac{\dot{\nu}_{\text{L}}}{\nu_{\text{obs}}},$$

(15)

where $\dot{\omega}_1 \equiv \frac{\dot{\nu}_{\text{L}}}{\nu_{\text{L}}}$. The first term at the right hand side of Equation (15) corresponds to the magnetic dipole radiation and the second one corresponds to the orbital effect which can change sign and be much larger in magnitude than that of the first term. This explains why $|n| \gg 3$ is inevitable when the second term is ignored.

Similarly by Equation (11) we can have,

$$\frac{\dot{\nu}_{\text{obs}}^{(3)}}{\nu_{\text{obs}}} = \frac{5\dot{\nu}}{\nu} + \frac{\dot{\nu}_{\text{L}}}{\nu_{\text{obs}}} + \frac{\dot{\omega}_2 \dot{\nu}_{\text{L}}}{\nu_{\text{obs}}},$$

(16)

where $\dot{\omega}_2 \equiv \frac{\nu_{\text{L}}^{(3)}}{\dot{\nu}_{\text{L}}}$. Again the left hand side of Equation (16) can be explained by the second term at the right hand side which is dominant. $\dot{\omega}_1$ and $\dot{\omega}_2$ are generally in order of magnitude, $\dot{\omega}$ or $\dot{\Omega}$. However since they are all long periodic terms which is dependent on $\omega$ and $\Omega$, at certain time the trigonometric function of $\omega$ and $\Omega$ may cause a relative large or small values in the derivative or derivatives of pulse frequency, and hence results the discrepancy between Equation (12) and Equation (13).

The magnetic dipole radiation induced frequency first derivative is always negative; whereas, the long-term orbital effect can cause both negative and positive $\dot{\nu}_{\text{L}}$. The fact that most pulsars have negative frequency derivative indicates that $|\dot{\nu}| > |\nu_{\text{L}}|$. In other words, for most pulsars $\nu_{\text{obs}}$ is dominated by $\dot{\nu}$, thus $\dot{\nu}_{\text{obs}}$ and $\dot{\nu}$ have the same sign. Therefore we can assume: $\dot{\nu} = \sigma \nu_{\text{obs}}$, and in turn $\dot{\nu}_{\text{L}} = (1 - \sigma) \nu_{\text{obs}}$, where $\sigma > 0.5$. For convenience define $\alpha = 1 - \sigma$, notice that $\alpha$ can both be positive and negative. Equation (15) can be rewritten,

$${\dot{\nu}_{\text{obs}} = \frac{3(1 - \alpha)^2 \dot{\nu}_{\text{obs}}}{\nu} + \dot{\omega}_1 \alpha},$$

(17)

Actually $\alpha$ can be obtained from observation, from which $\dot{\omega}_1$ of Equation (15) can be obtained. Whereas, the discrepancy between $\dot{\omega}_1$ and $\dot{\omega}_{\text{GR}}$ can be expressed as,

$$\dot{\omega}_{\text{GR}} = \gamma \dot{\omega}_1,$$

(18)

where $\gamma$ is an arbitrary value, from which the orbital period can be written,

$$P_b = \frac{2\pi}{\frac{3(GM)^{2/3}}{c^2(1 - e^2)^{5/3}}},$$

(19)

where $G$ is the gravitational constant. Having $P_b$, the semi-major axis of orbit, $a$ can be obtained. Finally putting $a$, $P_b$, as well as estimated $M_2$ ($M_1 = 1.4M_{\odot}$) into following equation (which is given by rewritten Equation (9)), $\rho$ can be obtained,

$$\alpha \dot{\nu}_{\text{obs}} = \dot{\nu}_{\text{L}} = \frac{GM\nu}{2\pi c^2 a} \rho \left(1 - \xi\right),$$

(20)

where $\rho \equiv \pi \sin^2 i (M_2/M)^2 (1 - e^2/4) / \sqrt{1 - e^2}$. Then one can adjust the companion mass, $M_2$ and parameter $\gamma$, to check whether $|\sin i| \leq 1$ is satisfied or not in the expression of $\rho$. Moreover putting $M_2$, $\sin i$ and $P_b$ into Equation (8) one can obtain $\Delta \nu / \nu$ induced by the orbital effect, and further check whether it can be consistent with $\Delta \nu / \nu$ given by observation or not. In other words, one can adjust two parameters, $\gamma$ and $M_2$ to satisfy the three constraints, $\alpha$, $\sin i$ and $\Delta \nu / \nu$.

By the observation of SGR 1900+14, $P_{\text{obs}} = 8.2(6) \times 10^{-11}\text{s}^{-1}$ in May 31-Jun 9, 1998; and $P_{\text{obs}} = 5.93(3) \times 10^{-11}\text{s}^{-1}$ in Aug 28-Oct 8, 1999 (Woods et al. 1999), $\nu_{\text{obs}}$ varies significantly ($\nu_{\text{obs}} = 1/P_{\text{obs}}$). By $\alpha = 0.32$, and through Equation (17) and Equation (19), we can obtain $P_b \approx 6.4\text{~min}$. By assuming $M_1 = 1.4M_{\odot}$, $M_2 = 1.4M_{\odot}$ and $\gamma = 10$, we have $\sin i = 0.2$ and $x = 0.04$ through the definition of $\rho$. Another solution is $P_b \approx 3.6\text{~min}$, $M_1 = 1.4M_{\odot}$, $M_2 = 0.5M_{\odot}$ and $\gamma = 20$ as shown in Table 1.
Table 1 Calculated Rotational Parameters and Estimated Orbital Parameters of Possible Binary Pulsars

| Pulsars          | $n$  | $\Delta \nu$ | $\nu_\ell / \nu_{\text{obs}}$ | refs | $P_b$(min) | $m_2$(M$_\odot$) | sin $i$ | $x$ | $\gamma$ | $c$ |
|------------------|------|---------------|-------------------------------|------|-------------|------------------|---------|-----|----------|-----|
| 1E 2259+586      | 3302 | 0.42          | 0.10                          | 1    | 72          | 0.1              | 0.6     | 0.06| 10       | 0.2 |
| SGR 1806–20      | –826 | 5.5           | 0.32                          |      | 5.8         | 0.5              | 0.4     | 0.03| 1        | 0.2 |
| SGR 1900+14      | –3921| 4.5           | 0.30                          | 3    | 6.4         | 1.4              | 0.2     | 0.04| 1        | 0.2 |
| PSR J1341–6220   | –1   | 0.032         | 0.0012                        | 4    | 3.6         | 0.5              | 0.2     | 0.01| 20       | 0.1 |
| PSR J1614–5047   | –81  | 0.12          | 0.010                         | 4    | 20          | 0.1              | 0.2     | 0.01| 10       | 0.1 |

$^a$ and $^b$ correspond to observational data of 1999 and 2000 respectively. Notice that the orbital parameters of SGR 1806–20 are estimated through $\dot{\omega}_2$ of Equation (16), instead of $\dot{\omega}_1$ of Equation (15) as the rest of pulsars in this table. $\dot{\omega}_2$ is very close to $\dot{\omega}_{\text{GR}}$, which corresponds to $\gamma \approx 1$. The test of the binary nature of these pulsars can be performed by setting $\nu_{\text{obs}} = \nu_{\text{b}}$ and $\nu_{\text{obs}} = \nu_{\text{e}}$ in other words, $\nu_{\text{obs}}$ and $\nu_{\text{b}}$ satisfies the expectation of magnetic dipole radiation, and then use orbital parameters to fit the quasi-sinusoidal residual. Orbital period, $P_b$, predicted in this table is obtained in the case that the mass of the pulsar is $M_1 = 1.4M_\odot$. The error of the estimated $P_b$ mainly comes from the assumption $\nu_{\text{GR}} = \nu_{\text{b}}$, typically the error in $\gamma$ is less than 10, thus by Equation (19), the error of $P_b$ is $P_b \pm \sigma_1$ where $\sigma_1 \equiv 3P_b$ and $\sigma_2 \equiv 3P_b/2$. $n$, $\Delta \nu$ and $\nu_\ell / \nu_{\text{obs}}$ are obtained by references: 1 (Kaspi et al. 2003); 2 (Wood et al. 2000); 3 (Wood et al. 2002); 4 (Wang et al. 2000).

Nevertheless $|\alpha| < 0.5$ guarantees that the magnitude of the frequency first derivative caused by the long-term orbital effect is smaller than that of the intrinsic spin down. Therefore the long-term orbital effect model is not contradictory to the assumption that SGRs are magnetars (Duncan & Thompson 1992), in the sense that $\dot{\nu}$ is still dominated by the magnetic dipole radiation. The model is not contradictory to other possibilities either (Marsden et al. 1999; Mosquera 2004).

On the other hand, the second order frequency derivatives caused by the long-term orbital effect is much larger than that of the magnetic dipole radiation induced one for SGRs, in other words, $\ddot{\nu}_{\text{obs}}$ is dominated by $\ddot{\nu}_L$. And since $\ddot{\nu}_L$ can change sign at time scale $\sim 2\pi/\dot{\omega}_{\text{GR}}$, it is expected that $\ddot{\nu}_{\text{obs}}$ (also $\nu^{(3)}_{\text{obs}}, \nu^{(4)}_{\text{obs}}$) will change sign at time scale $\sim 2\pi/\dot{\omega}_{\text{GR}}$. This can be tested by observation.

4 APPLICATION TO AXPS AND RADIO PULSARS

The variation of $\ddot{\nu}_{\text{obs}}$ and $\dot{\nu}_{\text{obs}}$ between 1990 January and 1998 December of 11 pulsars is measured using ATNF Parkes radio telescope (Wang et al. 2000). The signs of $\ddot{\nu}_{\text{obs}}$ of PSR J1614–5047 and PSR 1341–6220 change for two times, which have been attributed to glitch.

The change on $\dot{\nu}_{\text{obs}}$ of PSR J1614–5047 is about 1%, thus $\alpha = 0.01$, and similarly through Equations (15)–(19), the orbital period of PSR J1614–5047 can be estimated, $P_b \approx 34$ min, which corresponds to $M_2 = 0.7M_\odot$, $\sin i = 0.2$ and $x = 0.05$ s as shown in Table 1.

The measured time scale of change sign on $\dot{\nu}_{\text{obs}}$ of PSR J1614–5047 is $\sim 3.2$ yr, which is consistent $2\pi/\dot{\omega}_{\text{GR}} \sim 3$ yr corresponding to $P_b \approx 34$ min.

By Equations (8) and (9), $\Delta \nu$ and $\dot{\nu}_L$ contains orbital elements, $a$, $e$, and $i$, which are all long-periodic under the Spin-Orbit coupling model. Thus $\Delta \nu$ and $\dot{\nu}_L$ are trigonometric functions of $\omega$ and $\Omega$ (Gong 2005a). In such case $\omega$ and $\Omega$ are non-uniform which may induce abrupt change in $\Delta \nu$ and $\dot{\nu}_L$, and therefore mimic glitches in pulsars. The absence of glitches in binary pulsars may be due to that the additional times delay caused by long-term orbital effect can be absorbed by the uncertainties in parameters such as $P_1$, $\omega_{\text{GR}}$, and $\dot{\omega}$. Whereas, for isolated pulsars, the only possible absorption of long-term orbital effect is by rotational parameters, $\nu_{\text{obs}}$ and $\nu_{\text{b}}$, etc. This explains why glitches always happens in young isolated pulsars. As shown in Table 1, these pulsars usually have short orbital period, i.e., from a few minute to 10$^4$ min, which can make significant timing noise, but the small semi-major axis, $x \sim 10^{-2}$ to $10^{-3}$, as shown in Table 1, prevents them from being observed as binary pulsars directly.

The timing noise parameter is defined as $\Delta(t) = \log(|\dot{\nu}|^2/6\nu)$, and $\Delta_8$ means $\Delta(t = 10^8)$. By Equation (20), the second derivatives of $\nu_L$ is given,

$$\ddot{\nu}_L \approx \frac{GM\nu}{2\pi^2a^3} \rho \dot{\omega}_1,$$  (21)
Equation (21) indicates that $\dot{\nu}_L \propto \rho$, and by the definition of $\rho$, we have $\dot{\nu}_L \propto \sin^2 i, M_2^2, P_{\nu}^{-2/3}$. By the estimated orbital parameters of Table 1, one can found out why PSR J1314–6220 has the minimum timing noise, $\Delta_8 \approx 0.0012$, and SGR 1806–20 has the maximum, $\Delta_8 = 5.5$ among the five pulsars.

5 DISCUSSION

Different timing behaviors shown in different kinds of pulsars may caused by the same physics. The long-term orbital effect provides possible explanations to following phenomena:

1. Magnitude and time scale on the variation of pulse frequency, $\Delta\nu$.
2. Magnitude, sign and variation of frequency second derivative, $\ddot{\nu}_{\text{obs}}$.
3. The relationship of ratios of derivatives of pulse frequency like $\dot{\nu}_{\text{obs}}/\nu_{\text{obs}}, \nu_{\text{obs}}^{(3)}/\dot{\nu}_{\text{obs}}$.
4. Why unexpected sign of $\ddot{\nu}_{\text{obs}}$ appears much more often than that of $\dot{\nu}_{\text{obs}}$.
5. Why timing noise parameters, $\Delta_8$, of SGRs are much larger than that of radio pulsars.

The new model leads to two predictions. The first one is that $\ddot{\nu}_{\text{obs}}, \nu_{\text{obs}}^{(3)}$ and $\nu_{\text{obs}}^{(4)}$ of SGRs, AXPs and radio pulsars should change sign at time scale $\sim 2\pi/\dot{\omega}_{\text{GR}}$. The test of this prediction may tell us whether timing noise is caused by the long term orbital effect or not.

The second one is the orbital period of different pulsars listed in Table 1. The two SGRs may be ultra-compact binary with orbital period of a few minutes, this is a natural extension from the binaries with orbital period of $10^1$ min. If confirmed the population of sources for gravitational wave detectors, like LIGO and LISA, may increase considerably.

A simple method of searching binary motion is proposed by Gong & Bignami (2006), which uses the large Doppler shift of an ultra-compact binary and the length of time span ($\tau \gg P_b$) simultaneously. One can split the time span evenly into a number of segments, and each segment corresponds to a time scale of $\sim P_b/2$. Then fold the odd segments, which may correspond to one shift (say blue); and fold separately the even segments, which may correspond to the other shift (red).

Having the two groups of folding, one can adjust the length of each segment, $\tau_i$, which corresponds to the orbital period of the binary, and the initial time, $t_0$, of the segments which corresponds to the initial orbital phase of the binary, to see if $\dot{\nu} > 0$ in the folding corresponding to blue shift, and $\dot{\nu} < 0$ in the folding corresponding to red shift can be obtained.

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