$B$ anomalies in the nonminimal universal extra dimension model

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Abstract

We investigate the $B$ anomalies in the framework of the nonminimal universal extra dimension models. Newly measured polarization parameters in $B \rightarrow D^{(*)}\tau\nu$, $P_T(D^{(*)})$ and $F_L(D^*)$ as well as the ratios $R(D^{(*)})$ are considered altogether. The Kaluza-Klein modes of the $W$-boson and charged scalar contributes as the new physics effects. We find that the model parameters fit the global data very well with the minimum $\chi^2$/d.o.f. near unity, rendering $B_c \rightarrow \tau\nu$ branching ratios to be a few percents. The best-fit values of $R(D)$ and $R(D^*)$ are still far from ($\gtrsim 2\sigma$) the standard model predictions.
I. INTRODUCTION

The standard model (SM) of particle physics has been up to now very successful to explain many phenomena in our universe. The last missing piece of the SM, the Higgs particle was finally discovered in 2012. But there must be some new physics (NP) beyond the SM. Flavor physics is a good testing ground for the NP. Recently, some anomalies are reported in $b \rightarrow c$ semileptonic decays. The fraction of the branching ratios

$$R(D^{(*)}) \equiv \frac{\text{Br}(B \rightarrow D^{(*)}\tau \nu)}{\text{Br}(B \rightarrow D^{(*)}\ell \nu)},$$  \hspace{1cm} (1)

reveals an excess over the SM predictions \cite{1},

$$R(D)_{\text{SM}} = 0.299 \pm 0.003,$$

$$R(D^*)_{\text{SM}} = 0.258 \pm 0.005.$$  \hspace{1cm} (2)

Experiments including BABAR, Belle, and LHCb have reported somewhat larger values of $R(D^{(*)})$ than those of Eq. (2) by about $2 \sim 3\sigma$ \cite{2,11}. Recently the Belle collaboration announced new results \cite{9}

$$R(D)_{\text{Belle1904}} = 0.307 \pm 0.037 \pm 0.016,$$

$$R(D^*)_{\text{Belle1904}} = 0.283 \pm 0.018 \pm 0.014,$$  \hspace{1cm} (3)

which are rather closer to Eq. (2) than the previous data and consistent with the SM within $1.2\sigma$. Combined results for all data by the heavy flavor averaging group (HFLAV) collaboration \cite{12}

$$R(D)_{\text{HFLAV}} = 0.340 \pm 0.027 \pm 0.013,$$

$$R(D^*)_{\text{HFLAV}} = 0.295 \pm 0.011 \pm 0.008,$$  \hspace{1cm} (4)

give a discrepancy between the SM predictions and experimental data at $3.08\sigma$ level. The BABAR measurements \cite{2,3} exclude at the 99.8% confidence level the type-II two-Higgs-doublet model (2HDM) where a charged Higgs boson contributes to $R(D^{(*)})$, while the Belle measurements \cite{11} are compatible with the type-II 2HDM. It was shown that an anomalous $\tau$ coupling to the charged Higgs in the 2HDM can explain the data very well \cite{13}. In extra dimension models the overlapping between the wave functions of $\tau$ and the neutral scalar could be weak to make $\tau$ screened from the scalar vacuum, resulting in an enhancement of
couplings to charged Higgs. There are many other NP scenarios to explain the $R(D^{(*)})$ anomaly.

On top of the ratio $R(D^{(*)})$ the Belle collaboration measured the relevant polarizations in $B \rightarrow D^{(*)}\tau\nu$ decays. One can consider observable parameters associated with $D^*$ as well as $\tau$. The $\tau$-polarization asymmetry is defined as

$$P_{\tau}(D^{(*)}) \equiv \frac{\Gamma_{\tau}^{D^{(*)}}(+) - \Gamma_{\tau}^{D^{(*)}}(-)}{\Gamma_{\tau}^{D^{(*)}}(+) + \Gamma_{\tau}^{D^{(*)}}(-)},$$

where $\Gamma_{\tau}^{D^{(*)}}(\pm)$ is the decay width for $(\pm)\tau$ helicity. The SM predictions are [13] [15]

$$P_{\tau}(D)_{\text{SM}} = 0.325 \pm 0.009, \quad P_{\tau}(D^*)_{\text{SM}} = -0.497 \pm 0.013.$$ (6)

The experimental result is [7] [8]

$$P_{\tau}(D^*) = -0.38 \pm 0.51^{+0.21}_{-0.16}.$$ (7)

The longitudinal $D^*$ polarization is

$$F_L(D^*) \equiv \frac{\Gamma(B \rightarrow D^*_L\tau\nu)}{\Gamma(B \rightarrow D^*\tau\nu)},$$

where the Belle’s measurement is [16]

$$F_L(D^*) = 0.60 \pm 0.08 \pm 0.035,$$ (9)

while the SM value is estimated to be [17]

$$F_L(D^*)_{\text{SM}} = 0.46 \pm 0.04.$$ (10)

The polarization parameters could provide more information about the Lorentz structure of possible NP.

In this paper we consider the nonminimal universal extra dimension (nmUED) model [18] [23] to fit the global data on $R(D^{(*)})$ and polarization parameters. In the universal extra dimension (UED) models there is an extra spacelike dimension with a flat metric compactified on an $S^1/Z_2$ orbifold, where the SM particles could reside. Each SM particles is accompanied by infinite towers of Kaluza-Klein (KK) states. There are two branes at the endpoints of the orbifold. The reflection symmetry of the bulk space provides with the KK-parity conservation. As discussed in [24], in the minimal version of the UED (MUED) there
are no new couplings at the tree level relevant to \( R(D^{(*)}) \). The radiative corrections include bulk corrections and boundary localized ones. In the MUED models the latter is adjusted to cancel the cutoff dependent corrections. The nmUED models allow the boundary localized terms to be free parameters. In this analysis we include the boundary localized kinetic terms (BLKTs) with strength parameters. The NP effects enter through the possible interactions between a pair of zero-mode fermion and even KK-modes of charged gauge boson or scalar, associated with the BLKTs \([24, 25]\). These kinds of interactions are not allowed in the MUED because of the KK-wave function orthogonality.

The paper is organized as follows. In the next section the nmUED model is introduced. Section III provides the various observables in numerical forms. The results and discussions are given in Sec. IV, and we conclude in Sec. V.

II. NMUED MODEL

We assume that there is one flat extra dimension \((y)\) compactified on an \(S^1/Z_2\) orbifold with radius \(R\). Two branes are located at the endpoints \(y = 0\) and \(y = \pi R\) where both boundary terms are equal. The 5D action for fermions \(f\) is \([24]\)

\[
S_f = \sum_{f=q,\ell} \int d^4x \int_0^{\pi R} dy \left\{ i\bar{\Psi}_L^f \Gamma^M \mathcal{D}_M \Psi_L^f + r_f [\delta(y) + \delta(y - \pi R)] i\bar{\Psi}_L^f \gamma^\mu \mathcal{D}_\mu P_L \Psi_L^f \\
+ i\bar{\Psi}_R^f \Gamma^M \mathcal{D}_M \Psi_R^f + r_f [\delta(y) + \delta(y - \pi R)] i\bar{\Psi}_R^f \gamma^\mu \mathcal{D}_\mu P_R \Psi_R^f \right\},
\]

where \(\Psi_L^f(x,y)\) are the 5D four component Dirac spinors for fermions \(f = q, \ell\). In terms of two component spinors,

\[
\Psi_L^f(x,y) = \begin{pmatrix} \psi_{L,R}^f(x,y) \\ \chi_{L,R}^f(x,y) \end{pmatrix} = \sum_n \begin{pmatrix} \psi_{L,R}^{f(n)}(x) F_{L,R}^{f(n)}(y) \\ \chi_{L,R}^{f(n)}(x) G_{L,R}^{f(n)}(y) \end{pmatrix},
\]

where \(F_{L,R}^{f(n)}(y)\) and \(G_{L,R}^{f(n)}(y)\) are the \(n\)-th KK-wave functions. In Eq. (11) \(r_f\) is the strength of the boundary localized terms. They are related to the mass of the \(n\)th KK-excitation \(m_{f(n)}\) by the transcendental equation

\[
\frac{r_f m_{f(n)}}{2} = \begin{cases} -\tan\left(\frac{m_{f(n)} \pi R}{2}\right) & \text{for even } n \\ \cot\left(\frac{m_{f(n)} \pi R}{2}\right) & \text{for odd } n \end{cases}.
\]

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As for the gauge boson sector, the 5D action is

$$S_{\text{gauge}} = -\frac{1}{4} \int d^4x \int_0^{\pi R} dy \left\{ W_{iMN}^i W^{iMN} + r_V [\delta(y) + \delta(y - \pi R)] W_{\mu\nu}^i W^{i\mu\nu} + B_{MN} B^{MN} + r_V [\delta(y) + \delta(y - \pi R)] B_{\mu\nu} B^{\mu\nu} \right\} ,$$

(14)

where $W_{iMN}^i, B_{MN}$ are the 5D gauge field strength tensors. The $n$th KK-mass of the gauge boson is

$$M_{W(n)} = \sqrt{M_W^2 + m_{V(n)}^2} ,$$

(15)

where $m_{V(n)}$ satisfies the same transcendental equation as Eq. (13). For the 5D scalar field $\Phi(x, y)$, the action is

$$S_{\Phi} = \int d^4x \int_0^{\pi R} dy \left\{ (D_M \Phi)^\dagger (D_M \Phi) + r_{\phi} [\delta(y) + \delta(y - \pi R)] (D_\mu \Phi)^\dagger (D_\mu \Phi) \right\} .$$

(16)

We choose $r_{\phi} = r_V$ for proper gauge fixing [26], and consequently the mass of the KK-scalar is $m_{\phi(n)} = m_{V(n)}$. The Yukawa interaction is described by

$$S_Y = -\sum_f \int d^4x \int_0^{\pi R} dy \left\{ \lambda_5 \bar{\Psi_f^L} \tilde{\Phi} \Psi_f^R + r_Y [\delta(y) + \delta(y - \pi R)] \lambda_5 \bar{\psi_f^L} \tilde{\Phi} \chi_f^R + \text{H.c.} \right\} ,$$

(17)

where $\lambda_5$ is the 5D Yukawa coupling and $r_Y$ is the boundary strength.

In nmUED, new KK particles contribute to $B$ decays. As mentioned in Sec. I even KK-modes of $W$-boson as well as charged Higgs couple to a pair of zero-mode fermions, which provide new vector and scalar interactions respectively. The effects are encoded in the overlap integrals

$$I_{n}^{fg} = \sqrt{\pi R \left( 1 + \frac{r_V}{\pi R} \right)} \int_0^{\pi R} dy \left\{ 1 + r_f [\delta(y) + \delta(y - \pi R)] \right\} a^n F^{(0)}_L F^{(0)}_L ,$$

$$I_{n}^{fY} = \sqrt{\pi R \left( 1 + \frac{r_V}{\pi R} \right)} \int_0^{\pi R} dy \left\{ 1 + r_Y [\delta(y) + \delta(y - \pi R)] \right\} f^n F^{(0)}_L G^{(0)}_R ,$$

where $a^n$ and $h^n$ are $n$th KK-mode of the $W$-boson and scalar, respectively. For $r_{\phi} = r_V$, $a^n = h^n$, and further if $r_f = r_Y$ then [24]

$$I_{n}^{fg} = I_{n}^{fY} \equiv I_{n}^{f} = \frac{\sqrt{2}(\hat{r}_f - \hat{r}_V)\sqrt{1 + \hat{r}_V}}{(1 + \hat{r}_f)\sqrt{1 + r_V^2 m_{V(n)}^2}/4 + \hat{r}_V} ,$$

(19)

where $\hat{r} \equiv r/(\pi R)$. Actually, $I_{n}^{f}$ is the interaction term between a pair of zero-mode fermion $f$ and $n$th KK-modes of $W$-boson or scalar, which encodes the NP effects on observables.
III. OBSERVABLES

Now the effective Hamiltonian for $b \rightarrow c \ell \nu$ is

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \sum_{\ell=\mu,\tau} \left\{ (1 + C^\ell_V) \mathcal{O}^\ell_V + C^\ell_S \mathcal{O}^\ell_S \right\},$$

(20)

where the operators $\mathcal{O}^\ell_{V,S}$ are defined by

$$\mathcal{O}^\ell_V = (\bar{c}_L \gamma^\mu b_L) (\bar{\ell}_L \gamma_\mu \nu LL),$$

(21)

$$\mathcal{O}^\ell_S = (\bar{c}_L b_R) (\bar{\ell}_R \nu LL).$$

(22)

The NP effects are encapsulated in the Wilson coefficients $C^\ell_{V,S}$ given as [24]

$$C^\ell_V = \sum_{n \geq 2} \left[ \frac{M^2_W}{M^2_{W(n)}} \right] I^q_n I^\ell_n,$$

(23)

$$C^\ell_S = \sum_{n \geq 2} \left[ \frac{m_b m_\ell}{M^2_{W(n)}} \right] \left[ \frac{M^2_W}{M^2_{W(0)}} \right] \left\{ \cos \left( \frac{1}{2} \tan^{-1} \left[ \frac{m_\ell}{m_{f(0)}} \right] \right) - \sin \left( \frac{1}{2} \tan^{-1} \left[ \frac{m_\ell}{m_{f(0)}} \right] \right) \right\} I^q_n I^\ell_n.$$

(24)

From $\mathcal{H}_{\text{eff}}$ one can calculate the transition amplitudes and decay rates for $B \rightarrow D^{(*)}$ decays, and construct various observable parameters. We only concentrate on the numerical results for the observables in our analysis. Numerically the observables for $B \rightarrow D^{(*)} \ell \nu_\ell$ decays are

(at $\mu = m_b$ scale) [27]

$$R(D) = 2R_{\text{SM}}(D) \frac{(1 + C^\tau_V)^2 + 1.54 (1 + C^\tau_V) C^\tau_S + 1.09 (C^\tau_S)^2}{1 + (1 + C^\mu_V)^2 + 1.54 (1 + C^\mu_V) C^\mu_S + 1.09 (C^\mu_S)^2},$$

(25)

$$R(D^*) = 2R_{\text{SM}}(D^*) \frac{(1 + C^\tau_V)^2 + 0.13 (1 + C^\tau_V) C^\tau_S + 0.05 (C^\tau_S)^2}{1 + (1 + C^\mu_V)^2 + 0.13 (1 + C^\mu_V) C^\mu_S + 0.05 (C^\mu_S)^2},$$

(26)

$$P_\tau(D) = \frac{0.32 (1 + C^\tau_V)^2 + 1.54 (1 + C^\tau_V) C^\tau_S + 1.09 (C^\tau_S)^2}{(1 + C^\tau_V)^2 + 1.54 (1 + C^\tau_V) C^\tau_S + 1.09 (C^\tau_S)^2},$$

(27)

$$P_\tau(D^*) = -\frac{0.49 (1 + C^\tau_V)^2 + 0.13 (1 + C^\tau_V) C^\tau_S + 0.05 (C^\tau_S)^2}{(1 + C^\tau_V)^2 + 0.13 (1 + C^\tau_V) C^\tau_S + 0.05 (C^\tau_S)^2},$$

(28)

$$F_L(D^*) = \frac{0.46 (1 + C^\tau_V)^2 + 0.13 (1 + C^\tau_V) C^\tau_S + 0.05 (C^\tau_S)^2}{(1 + C^\tau_V)^2 + 0.13 (1 + C^\tau_V) C^\tau_S + 0.05 (C^\tau_S)^2},$$

(29)

$$\text{Br}(B_c \rightarrow \tau \nu) = 0.02 \left( \frac{f_{B_c}}{0.43 \text{ GeV}} \right) \left[ 1 + 2 C^\tau_V + 4.3 C^\tau_S \right]^2.$$  

(30)

The results are obtained from the numerical values of the relevant form factors of $B \rightarrow D$ [28] and $B \rightarrow D^*$ transitions [11, 29].
TABLE I. Experimental data for $R(D)$, $R(D^{(*)})$, and $P_{\tau}(D^{*})$. The uncertainties are ±(statistical)±(systematic). For the third uncertainty of LHCb(1711), see [11] for details. For BABAR, Belle(2015), and Belle(2019) results, the correlations between $R(D)$ and $R(D^{(*)})$ are $-0.31$, $-0.50$ and $-0.51$ respectively [12].

The branching ratio of $B_c \to \tau \nu$, $\text{Br}(B_c \to \tau \nu)$ is estimated to be $< 10\%$. Since $\text{Br}(B_c \to \tau \nu) \sim (1 + C_V + 4.3C_S)^2$, the requirement of $\text{Br}(B_c \to \tau \nu) < 10\%$ would impose strong constraints on the relevant Wilson coefficients. The experimental data for various observables used in this analysis are listed in Table I.

IV. RESULTS

We implement the global $\chi^2$ fit for the observables in Table I. We first define the $\chi^2$ as

$$\chi^2 \equiv \sum_{i,j} \left[ O_{i}^{\text{exp}} - O_{i}^{\text{th}} \right] C_{ij}^{-1} \left[ O_{j}^{\text{exp}} - O_{j}^{\text{th}} \right],$$

where $O_{i}^{\text{exp}}$ are the experimental data while $O_{i}^{\text{th}}$ are the theoretical predictions of Eqs.(25)-(30), and $C_{ij}$ are the correlation matrix elements.

There are two major constraints. One is from the oblique parameters of the electroweak precision test (EWPT) [30,33]. In the nmUED model, the Fermi constant is modified by the
tree level contributions of even $n$th KK-modes of $W$-bosons to the four-fermion interactions. This kind of correction is absent in the MUED scenario. The Fermi constant in mmUED is now written as

$$G_F = G_F^0 + \delta G_F .$$

(32)

Here $G_F^0$ is the Fermi constant in the SM and $\delta G_F$ is the correction from the new contributions of $W^\pm$ KK-modes. Explicitly [24],

$$G_F^0 = \frac{g^2}{4\sqrt{2}M_W^2} , \quad \delta G_F = \sum_{n \geq 2} \frac{g^2 (I_n^f)^2}{4\sqrt{2}m_{W(n)}^2},$$

(33)

where $g$ is the gauge coupling constant. Note that $\delta G_F \sim (I_n^f)^2$ because the Fermi constant is derived from the muon lifetime. We only consider the 2nd KK contributions for simplicity.

Now the Fermi constant is related to the Peskin-Tacheuchi parameters as [30]

$$S_{mmUED} = 0 , \quad T_{mmUED} = -\frac{1}{\alpha} \frac{\delta G_F}{G_F} , \quad U_{mmUED} = \frac{4\sin^2 \theta_W \delta G_F}{\alpha G_F},$$

(34)

where we neglect possible loop effects which are subdominant compared to the tree-level contributions to $\delta G_F$. We use the data [34]

$$S = 0.05 \pm 0.11 , \quad T = 0.09 \pm 0.13 , \quad U = 0.01 \pm 0.11 ,$$

(35)

where the correlation coefficients are

$$\rho_{ST} = 0.90 , \quad \rho_{TU} = -0.83 , \quad \rho_{US} = -0.59 .$$

(36)

Following the methods of [33], we impose the $S, T, U$ constraints by requiring $\chi^2_{STU} < 6.18$ at 2$\sigma$ where $\chi^2_{STU}$ is defined by the covariant matrix relevant for the $S, T, U$ parameters, similarly to Eq. (31).

The other major constraint comes from the LHC dilepton resonance searches. At the LHC the second KK gauge boson $A^{(2)}$ can be produced via the KK number violating interactions, subsequently decaying into the SM particles. Recent results from ATLAS dilepton resonance searches at the 13 TeV with 13.3 fb$^{-1}$ provide a stringent constraint on the mmUED parameters [35]. We reflect the results of [35] on the strength of the BLKT in the gauge sector to constrain our analysis to the region $0 \leq r_V/R \leq 0.5$. The best-fit values for the minimum $\chi^2$ are listed in Table II.

In Fig. 1, we plot the allowed regions of the mmUED parameters at the 2$\sigma$ level. We scanned over the range $0 \leq 1/R \leq 3$ TeV. A noticeable feature is that the allowed range of
\begin{equation*}
R(D) \quad R(D^*) \quad P_\tau(D) \quad P_\tau(D^*) \quad F_L(D^*) \quad \text{Br}(B_c \to \tau\nu) \quad \chi^2/\text{d.o.f.}
\end{equation*}

\begin{tabular}{ccccccc}
0.343 & 0.296 & 0.320 & -0.490 & 0.460 & $2.75 \times 10^{-2}$ & 1.25 \\
\end{tabular}

\textbf{TABLE II.} Best-fit values.

\begin{figure}
\centering
\begin{subfigure}{.49\textwidth}
\includegraphics[width=\linewidth]{fig1a.png}
\caption{(a)}
\end{subfigure}
\begin{subfigure}{.49\textwidth}
\includegraphics[width=\linewidth]{fig1b.png}
\caption{(b)}
\end{subfigure}
\begin{subfigure}{.49\textwidth}
\includegraphics[width=\linewidth]{fig1c.png}
\caption{(c)}
\end{subfigure}
\begin{subfigure}{.49\textwidth}
\includegraphics[width=\linewidth]{fig1d.png}
\caption{(d)}
\end{subfigure}
\caption{Allowed regions of model parameters of nmUED at the 2\sigma level.}
\end{figure}

$r_q/R$ is rather narrow with negative values, contrary to that of $r_l/R$ as shown in Fig. 1 (a). Figure 2 shows the 2nd KK masses $m_{W(2)}$ and $m_{\tau(2)}$. Allowed values of various observables at 2\sigma are given in Fig. 3. As can be seen in Fig. 3 (a), $R(D)$ is still far away from the SM predictions beyond 2\sigma level while $R(D^*)$ values have small overlaps at the edge of the SM-allowed range within 2\sigma. But the best-fit values of $R(D^{(*)})$ in Table I are still beyond the SM by more than 2\sigma. Other polarization observables $P_\tau(D^{(*)})$ and $F_L(D^*)$ are consistent with the SM. Figure 3 (b) shows that the branching ratio $\text{Br}(B_c \to \tau\nu)$ lies safely within a
FIG. 2. Mass scales of nmUED at 2σ.

FIG. 3. Allowed values for various observables at 2σ. In (a) the horizontal lines are the SM predictions at 2σ for $R(D)$ (blue) and $R(D^*)$ (cyan). Other polarization parameters $P_\tau(D^{(*)})$ and $F_L(D^*)$ are consistent with the SM values at 2σ. In (b) the branching ratio of $\text{Br}(B_c \to \tau\nu)$ vs $R(D)$ is plotted.

few percents.

Contributions of the Wilson coefficients to observables at 2σ are depicted in Fig. 4. We find that the pattern for $C_V^\mu$ is very similar to that of $C_S^\mu$. Note that the Wilson coefficients are

$$ C_{V,S} \sim I_{\mu}^n I_{\mu}^\ell , $$(37)
while the EW precision parameters are

\[ T_{\text{nmUED}}(U_{\text{nmUED}}) \sim \delta G_F \sim (I_n^\ell)^2. \]  

(38)

In case of \( r_q = r_\ell \) the overlap integrals become \( I_n^q = I_n^\ell \) and \( C_{V,S} \sim (I_n^\ell)^2 \), which are directly affected by the oblique parameters of Eq. (38). According to Eq. (35) EWPT prefers small \((I_n^\ell)^2\). It means that for \( r_q = r_\ell \) EWPT requires smaller \( C_{V,S} \), which results in smaller \( R(D^{(*)}) \) and does not fit data so well. In other words, we find that \( R(D^{(*)}) \) anomalies require \( r_q \neq r_\ell \) in nmUED. The situation is depicted in Fig. 5 where \( R(D) \) vs \( \chi^2/\text{d.o.f.} \) are compared for \( r_\ell/R = r_q/R \) and \( r_\ell/R \neq r_q/R \) cases.

In Fig. 6 we compare the cases of \( r_\ell/R = r_q/R \) and \( r_\ell/R \neq r_q/R \). Figure 6(a) shows that the allowed regions of \( r_q/R \) are quite different from each other. The effect of \( r_\ell/R \neq r_q/R \) appears dramatically on \( I_2^q \), as shown in Figs. 6(b)-(d). As mentioned above, this is due to the constraints on the oblique parameters. If \( r_q/R = r_\ell/R \), then \( I_2^q = I_2^\ell \) and it should be kept small to satisfy the EWPT (Fig. 6(b)). In case of \( r_q/R \neq r_\ell/R \), \( I_2^q \) can be very large compared to \( I_2^\ell \) (Fig. 6(c)). As a result, \( R(D) \) is allowed to have large values to fit the data (Fig. 6(d)).

In our analysis \( C_{V} = C_{V}^\mu \), and we checked the influence of nonzero \( C_{V,S}^\mu \). Figure 7 shows some of the results. Figure 7(a) depicts \( 1/R \) vs \( r_q/R \) while (b) does \( R(D) \) vs \( C_{V}^\mu \). We have similar figure for \( R(D^*) \) to Fig. 7(b). Whether \( C_{V,S}^\mu = 0 \) or not does not affect the observables including the polarizations so much, but the allowed range of \( r_q/R \) or \( C_{V}^\mu \) could be slightly different. The effect of \( C_{S}^\mu \) is negligible because its values are very small compared
FIG. 5. $R(D)$ vs $\chi^2$/d.o.f. for $r_{\ell}/R = r_{q}/R$ and $r_{\ell}/R \neq r_{q}/R$. 

to $C'_{5 \gamma}$. Note that $C'_{5 \gamma}$ is suppressed by $\sim m_{\mu}/m_{\tau}$ with respect to $C'_{5 \gamma}$.

V. CONCLUSIONS

In conclusion, we investigated the $B \rightarrow D(\ast)$ anomalies in the nmUED model. In the model, $n$th KK-modes of $W$-boson and scalar couple to a pair of zero-mode fermions to result in nonzero NP Wilson coefficients. We found that the nmUED model successfully fits the current data including $D^*$ polarizations, at the sacrifice of $r_{\ell}/R = r_{q}/R$. The EWPT plays a significant role in the model. We also found that the branching ratio $\text{Br}(B_c \rightarrow \tau \nu)$ stays at a few percents, well below 10%. In our analysis $R(D)$ values have no overlap with the SM predictions at the $2\sigma$ level while $R(D^*)$ touches the SM-allowed region. Future measurements of more observables would check further the validity of the nmUED model.

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FIG. 6. Comparisons of various parameter spaces for $r_q/R = r_{\ell}/R$ (red) and $r_q \neq r_{\ell}/R$ (blue) at 2$\sigma$.

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FIG. 7. Comparisons of parameter spaces for $C_{V,S}^\mu = 0$ (red) and $C_{V,S}^\mu \neq 0$ (blue) at $2\sigma$.

[12] Average of $R(D)$ and $R(D^*)$ for Spring 2019, Heavy Flavor Averaging Group, https://hflav-eos.web.cern.ch/hflav-eos/semi/spring19/html/RDsDsstar/RDRDs.html

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