Universal properties of three-dimensional magnetohydrodynamic turbulence: do Alfvén waves matter?

Abhik Basu$^{1,2}$ and Jayanta K Bhattacharjee$^3$

$^1$Max-Planck-Institut für Physik Komplexer Systeme, Nöthnitzer Strasse 38, D-01187, Dresden, Germany
$^2$Poornaprajna Institute of Scientific Research, Bangalore, India
$^3$Department of Theoretical Physics, Indian Association for the Cultivation of Sciences, Jadavpur, Calcutta 700032, India
E-mail: abhik@mpipks-dresden.mpg.de and tpjkb@mahendra.iacs.res.in

Received 7 March 2005
Accepted 25 June 2005
Published 8 July 2005

Online at stacks.iop.org/JSTAT/2005/P07002
doi:10.1088/1742-5468/2005/07/P07002

Abstract. We analyse the effects of the propagating Alfvén waves, arising due to non-zero mean magnetic fields, on the nonequilibrium steady states of three-dimensional (3D) homogeneous magnetohydrodynamic (MHD) turbulence. In particular, the effects of Alfvén waves on the universal properties of 3DMHD turbulence are studied in a one-loop self-consistent mode-coupling approach. We calculate the kinetic- and magnetic-energy spectra. We find that even in the presence of a mean magnetic field the energy spectra are Kolmogorov-like, i.e., scale as $k^{-5/3}$ in the inertial range where $k$ is a Fourier wavevector belonging to the inertial range. We also elucidate the multiscaling of the structure functions in a log-normal model by evaluating the relevant intermittency exponents, and our results suggest that the multiscaling deviations from the simple Kolmogorov scaling of the structure functions decrease with increasing strength of the mean magnetic field. Our results compare favourably with many existing numerical and observational results.

Keywords: renormalization group, driven diffusive systems (theory), magnetohydrodynamics
Universal properties of three-dimensional magnetohydrodynamic turbulence: do Alfvén waves matter?

Contents

1. Introduction 2
2. Model equations 4
3. Correlation and structure functions in MHD 5
4. Symmetries of the equations of motion 6
5. Energy spectra for incompressible fluids 8
6. Kolmogorov’s constants 12
7. Possibilities of variable multifractality 13
8. Conclusion 14
   Acknowledgments 15
References 15

1. Introduction

The effects of propagating waves on the statistical properties of systems out of equilibrium remain an important topic of discussion. In the context of a coupled spin model in one dimension ($d = 1$) [1] it has been shown that the presence of such waves leads to weak dynamic scaling in that model. In contrast, in a coupled Burgers-like model in $d = 1$ propagating waves do not affect the scaling properties of the correlation functions at all [2]. So far such issues have been considered only within very simplified one-dimensional nonequilibrium models [2, 3]. Magnetohydrodynamic (3DMHD) turbulence, which is a hydrodynamic description of the coupled evolution of the velocity fields $\mathbf{u}$ and the magnetic fields $\mathbf{b}$ in a quasi-neutral plasma, stands as a very good candidate for a natural system with propagating waves in three dimensions as most of its natural realizations have propagating Alfvén waves arising due to the presence of mean magnetic fields. Examples of such physical situations include solar wind, neutral plasma in fusion confinement devices etc. The presence of the propagating Alfvén modes, in addition to the usual dissipative modes due to the fluid and magnetic viscosities, makes MHD turbulence a good natural example to study the interplay between the propagating and the dissipative modes in a system and their combined effects on the scaling properties of the correlation and structure functions, which are important issues from the point of view of nonequilibrium statistical mechanics.

The scaling of magnetic- ($E_b(k)$) and kinetic- ($E_u(k)$) energy spectra in the inertial range (i.e., wavevector $k$ lies in the region $L^{-1} \ll k \ll \eta_D^{-1}$, $L$ and $\eta_D$ being the integral scale given by the system size and the dissipation scale, respectively) in 3DMHD in the presence of Alfvén waves originating due to a mean magnetic field $\mathbf{B}_o$ remains controversial to date. Numerical simulations, due to the lack of sufficient resolution, failed to conclusively distinguish between the Kolmogorov and Kraichnan predictions (see below). In this communication, within one-loop self-consistent mode-coupling (SCMC)
approximations, we obtain the following results for homogeneous but anisotropic (due to
the mean magnetic field) 3DMHD.

- The bare Alfvén wave speed, proportional to $B_0$, renormalizes to acquire a singular
  $k$-dependence to become $\sim B_0 k^{-1/3}$ where $k$ is a Fourier wavevector belonging to the
  inertial (scaling) range. From this result we are able to conclude that even in the
  presence of a mean magnetic field the kinetic- and the magnetic-energy spectra scale
  as $k^{-5/3}$ in the inertial range, identical with the situation without any mean magnetic
  field.
- The dimensionless Kolmogorov constants for the kinetic- and magnetic-energy spectra
  depend on a dimensionless parameter $\beta$ which we identify as the ratio of the
  renormalized Alfvén wave speed and the renormalized viscosities (see below).
- The intermittency exponents of the Elsässer fields $z^\pm = u \pm b$ which approximately
  and qualitatively characterize the multiscaling properties of the structure functions
  in a log-normal model depend on the parameter $\beta$ and decrease with increasing
  $\beta$.

Thus our results show that Alfvén waves in 3DMHD do not affect the scaling
properties of the two-point correlation functions. However, the multiscaling properties
of the structure functions are shown to be affected by the mean magnetic fields.

The famous Kolmogorov arguments [4] for fluid turbulence can be easily extended to
3DMHD turbulence. In MHD, in the unit where mass density $\rho = 1$, the kinetic energy
dissipation rate per unit mass ($\epsilon_K$) and the magnetic energy dissipation rate per unit
mass ($\epsilon_M$) have same physical dimensions. Thus, as in fluid turbulence, by claiming that
the structure functions of the velocity and magnetic field differences in the inertial range
must be constructed out of the mean energy dissipation rate $\epsilon$ per unit mass and the local
length scale $r$ (belonging to the inertial range), one obtains for the $n$th order structure function [5,6]

$$ S_n^a(r) = \langle [a_i(x + r) - a_i(x)]^n \rangle \sim (\epsilon r)^{n/3}, \quad \eta_D \ll r \ll L \tag{1} $$

where $\epsilon = \epsilon_K$ or $\epsilon_M$ and $a = u, b$ for the velocity and the magnetic fields, $\eta_D$ is the (small)
dissipation scale and $L$ is the (large) system size. This yields, for the energy spectra in
the inertial range (as a function of wavevector $k$),

$$ E_a(k) \sim k^{-5/3}, \quad a = u, b. \tag{2} $$

This is known as the K41 theory in the relevant literature. However, in the presence of a mean magnetic field $B_0$, the Alfvén waves are generated (see below) with the propagation speed $\sim B_0$. Thus in such a case in addition to the usual Kolmogorov timescale $\sim k^{2/3}$ [7, 8]
there exists another timescale constructed from the mean magnetic field $B_0$, known as the
Kraichnan timescale $\sim (B_0 k)^{-1}$ [9]. Kraichnan argued that this timescale would determine
the energy cascade process and hence would enter the expression of the structure function
yielding $E_a(k) \sim r^{-3/2}, a = u, b$. There has not been any satisfactory resolution of this
issue to date; due to the particularly difficult vectorial nature of the 3DMHD equations
(see below), it is rather difficult to achieve high Reynolds number in direct numerical
solutions (DNSs) of the 3DMHD equations. Numerical solutions of an MHD shell-model
in the presence of a small mean-magnetic-field-like term did not find any dependence of
the multiscaling of the structure functions on the mean magnetic field [6]. Analytically, it
has been shown within the context of a 1D coupled Burgers-like model [2] that the energy
spectra are independent of a mean magnetic field and in the case of 1DMHD turbulence it exhibits the Kolmogorov scaling for the energy spectra. Similar conclusions followed from an analogous 1D model [3]. Various phenomenological approaches, including weak turbulence theories and three-wave interaction models, yield, in general, mean magnetic field dependences of the energy spectra [10]–[13] when the mean magnetic field is strong. However, these theories, despite their predictions, are either not directly derivable from the underlying 3DMHD equations of motion or involve additional assumptions on the flow fields. Analyses starting from the 3DMHD equations in this regard are still lacking. Most observational results on astrophysical systems seem to favour the K41 results [14]. Simulations of incompressible MHD [15] find results close to the K41 result. Simulations of compressible MHD [16], even though in some cases they find energy spectra closer to K41, in general yield a less clear picture. Recent numerical results of Müller et al [17] suggest that in the presence of finite magnetic helicity structure functions parallel and perpendicular to the mean magnetic fields are affected differently by the mean magnetic field. In this paper, we address some of these issues by starting from the 3DMHD equations without making any further assumptions on the velocity and the magnetic fields, except for the validity of the perturbative approaches. In particular, we show by applying one-loop mode coupling methods to the 3DMHD equations with a mean magnetic field and in the absence of any magnetic helicity that the one-dimensional energy spectra in 3DMHD turbulence are independent of the mean magnetic field $B_0$ and scale as $k^{-5/3}$ in the inertial range where $k$ is a Fourier wavevector belonging to the inertial range. We then proceed to calculate the Kolmogorov constants for the kinetic- and the magnetic-energy spectra and show that they depend on $B_0$. Lastly, we calculate the intermittency exponents for the velocity and the magnetic fields and find that they decrease with increasing $B_0$. We do not distinguish between the longitudinal and the transverse structure functions. Even though for analytical convenience we assume a weak mean magnetic field, we are able to obtain new and interesting results concerning scaling and multiscaling in 3DMHD in the presence of a mean magnetic field of small magnitude. In this respect our results can be considered as complementary to some of the existing results [10,11,13]. The rest of the paper is organized as follows: in section 2 we discuss the 3DMHD equations for incompressible fluids. In section 5 we show that for incompressible fluids the bare Alfvén wave speed $\sim B_0$ renormalizes to pick a correction $k^{-1/3}$ in the inertial range. This, as we argue, implies that the energy spectra scale as $k^{-5/3}$ in the inertial range even in the presence of a mean magnetic field. In section 6 we calculate the Kolmogorov constants for the kinetic and the magnetic energy spectra. We introduce a parameter $\beta$ which is the dimensionless ratio of the renormalized mean magnetic field and renormalized viscosities and show that the Kolmogorov constants depend on $\beta$. In section 7 we elucidate the multiscaling properties of the structure functions by calculating the intermittency exponents and show that they decrease with increasing $\beta$, i.e., with increasing $B_0$. This suggests that a mean magnetic field tends to reduce multiscaling corrections to the K41 results for the structure functions. In section 8 we summarize our results.

2. Model equations

We begin by writing down the 3DMHD equations for the velocity fields $u$ and the magnetic fields $b$: the velocity field $u$ is governed by the Navier–Stokes equation modified by the
inclusion of the Lorentz force \[18\]
\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{\nabla p}{\rho} + \frac{(\nabla \times \mathbf{b}) \times \mathbf{b}}{\rho} + \nu \nabla^2 \mathbf{u} + \mathbf{f},
\]
and the dynamics of the magnetic field \( \mathbf{b} \) is governed by the induction equation \[18\] constructed out of the Ohm law for a moving frame and the Ampere law:
\[
\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{b}) + \eta \nabla^2 \mathbf{b} + \mathbf{g}.
\]
Here, \( \rho \) is the mass density, \( p \) is the pressure, \( \nu \) is the fluid viscosity and \( \eta \) is the magnetic viscosity (inversely proportional to the electrical conductivity of the fluid medium concerned). Functions \( \mathbf{f} \) and \( \mathbf{g} \) are external forces needed to maintain a statistical steady state. In the present approach these are taken to be stochastic forces. We assume them to be zero mean and Gaussian distributed with specified variances (see below). In addition to equations (3) and (4) we also have \( \nabla \cdot \mathbf{b} = 0 \) (Maxwell’s equation) and, for incompressible fluids, \( \nabla \cdot \mathbf{u} = 0 \).

If the magnetic fields \( \mathbf{b}(\mathbf{x}, t) \) are such that \( \langle \mathbf{b} \rangle = \mathbf{B}_0 \) (a constant vector) then replacing \( \mathbf{b} \) by \( \mathbf{b} + \mathbf{B}_0 \) where now \( \langle \mathbf{b} \rangle = \mathbf{0} \), in equations (3) and (4) one obtains additional linear terms proportional to wavevector \( \mathbf{k} \) leading to wave-like excitations, known as Alfvén waves. The resulting equations are
\[
\frac{\partial \mathbf{u}}{\partial t} + \lambda_1 (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{\nabla p}{\rho} + \lambda_2 \frac{(\nabla \times \mathbf{b}) \times \mathbf{b}}{\rho} + \frac{(\nabla \times \mathbf{b}) \times \mathbf{B}_0}{\rho} + \nu \nabla^2 \mathbf{u} + \mathbf{f},
\]
and
\[
\frac{\partial \mathbf{b}}{\partial t} = \lambda_3 \nabla \times (\mathbf{u} \times \mathbf{b}) + \nabla \times (\mathbf{u} \times \mathbf{B}_0) + \eta \nabla^2 \mathbf{b} + \mathbf{g}
\]
with \( \langle \mathbf{b}(\mathbf{x}, t) \rangle = \mathbf{0} \). The parameters \( \lambda_1, \lambda_2, \lambda_3 \) are kept for book keeping purposes and can be set to unity (see section 4). Note that on dropping the nonlinear and the dissipative terms from equations (5) and (6) the resulting linear coupled partial differential equations admit wave-like solutions with dispersion relation linear in wavevector \( k \). These are known as the Alfvén waves \[19\] in the literature which propagate with speed proportional to \( B_0 \).

In the following sections we would calculate the kinetic and the magnetic energy spectra, the Kolmogorov constants and the intermittency exponents in the presence of the Alfvén waves, i.e., for \( B_0 \neq 0 \).

3. Correlation and structure functions in MHD

In the statistical steady state the time dependent correlation functions of \( \mathbf{u} \) and \( \mathbf{b} \) exhibit scaling which is characterized by the roughness exponents \( \chi_u \) and \( \chi_b \), respectively, and the dynamic exponent \( z \). In terms of the scaling exponents \( \chi_u, \chi_b \) and \( z \) the velocity and the magnetic field correlators have the form (as a function of wavevector \( \mathbf{k} \) and frequency \( \omega \))
\[
C_{ij}^u(k, \omega) = \langle u_i(k, \omega) u_j(-k, -\omega) \rangle = D^u P_{ij} k^{-d-2\chi_u-z} f_u(k^2/\omega),
\]
\[
C_{ij}^b(k, \omega) = \langle b_i(k, \omega) b_j(-k, -\omega) \rangle = D^b P_{ij} k^{-d-2\chi_b-z} f_b(k^2/\omega),
\]
where \( f_u \) and \( f_b \) are scaling functions, \( P_{ij} \) is the transverse projection operator: \( P_{ij} = \delta_{ij} - k_i k_j / k^2 \) to account for the incompressibility of the fields. The kinetic and magnetic
energies are simply related to the correlation functions: they are just the equal time velocity and magnetic field auto-correlation functions multiplied by appropriate phase factors. We now set out to find whether the Alfvén waves are relevant perturbations on the system. It should be noted that the correlators (7) are chosen as they would be in the fully isotropic case. In the presence of a mean magnetic field $B_\circ$ there would however be additional anisotropic terms in the expression for the correlators above. In the small $B_\circ$ limit the lowest order corrections to the expressions (7) are $O(B_\circ)^2$. Neglecting these correction terms does not affect our scaling analyses below, and in any case we are interested to see whether anisotropic Alfvén waves are relevant perturbations in the large scale, long time limit on the isotropic 3DMHD as characterized by expressions (7). Therefore, it suffices for us to work with the expressions (7) for our mode-coupling analyses below.

A complete characterization of the nonequilibrium steady state (NESS) of MHD, however, requires information about not just the scaling of the energy spectra, but also of the $n$th order equal time structure functions of the velocity and the magnetic fields in the inertial range. These are defined by

$$S^a_n(r) \equiv \langle |a_i(x+r) - a_i(x)| r_i^n \rangle, \quad a = u, b, \quad \eta_D \ll r \ll L,$$

which scale as $r^{\zeta^n_a}$ where $r$ belongs to the inertial range ($\eta_D \ll r \ll L$). According to the Kolmogorov theory (K41) the multiscaling exponents $\zeta^n_a = n/3$; i.e., they are linear in $n$. Subsequent numerical and observational results for homogeneous and isotropic 3DMHD, i.e., without any mean magnetic fields [20, 21], suggested deviations from the K41 results which are similar to those of pure fluid turbulence [22]. Far fewer results for the multiscaling of the structure functions are available when there are mean magnetic fields. Recent numerical simulations of Müller et al [17] suggested that structure functions parallel and perpendicular to the mean magnetic fields are differently affected by it when there is a finite magnetic helicity. Below we investigate the issue of multiscaling in the presence of a mean magnetic field $B_\circ$ within the context of a log-normal model by evaluating the relevant intermittency exponents which are found to depend explicitly on $B_\circ$. In our analyses we ignore the distinction between the structure functions parallel and perpendicular to the direction of the mean magnetic field; nevertheless, as we discuss below, our results are significantly new.

4. Symmetries of the equations of motion

We begin by re-expressing 3DMHD equations (3) and (4) by writing the magnetic fields $b$ as a sum of a space-time dependent part and a constant vector: $b(x, t) \rightarrow b(x, t) + \bar{B}_\circ$. In terms of these fields and parameters, the 3DMHD equations become

$$\frac{\partial u}{\partial t} + \lambda_1(u \cdot \nabla)u = -\frac{\nabla p}{\rho} + \lambda_2 \left( \nabla \times b \right) \times b + \left( \nabla \times b \right) \times \bar{B}_\circ \frac{\rho}{\rho} + \nu \nabla^2 u + f,$$

and

$$\frac{\partial b}{\partial t} = \lambda_3 \nabla \times (u \times b) + \nabla \times (u \times \bar{B}_\circ) + \eta \nabla^2 b + g.$$  

It should be noted that in equations (9) and (10) $\bar{B}_\circ$ is not the mean magnetic field in general; it would be so only if $\langle b(x, t) \rangle$ is zero in equations (9) and (10). Note that our
above way of splitting the magnetic fields fields does not change the actual mean magnetic field in the system: it is still given by $\tilde{B}_0 + \langle b \rangle = B_0$. The equations of motion (9) and (10) are invariant under the following continuous transformations [8, 23].

- The Galilean transformation (TI): $u(x, t) \rightarrow u(x + u_0(t), t) + u_0$, $(\partial/\partial t) - u_0 \cdot \nabla$, and $b \rightarrow b$ [2, 8, 24] with $\lambda_1 = \lambda_3 = 1$ in equations (5) and (6). This implies nonrenormalization of $\lambda_1$ [2, 24, 25].

- The transformation (TII) $\tilde{B}_0 \rightarrow B_0 + \lambda_2 \delta$, $b(x, t) \rightarrow b(x, t) - \delta$, $u \rightarrow u$. This allows one to work with the *effective* magnetic fields defined by $\sqrt{\lambda_2} b$ such that the coefficient of the Lorentz force vertex constructed out of the effective magnetic fields does not renormalize. This, therefore, ensures $\lambda_2$ can be set to unity [8, 23] by treating all magnetic fields as *effective fields*. Here the shift $\delta$ is a vector. A transformation similar to TII above exists in a problem of passive scalar turbulence [26].

The transformation TII essentially signifies the freedom to split the total magnetic fields as a sum of a constant part and a space time dependent part. It should be noted that the transformation TII keeps the mean magnetic field unchanged. In fact, nonrenormalization of $\lambda_2$ can be shown in a simpler way: let us assume that under mode eliminations and rescaling $\lambda_2 \rightarrow \alpha \lambda_2$. This scale factor of $\alpha$ can now be absorbed by redefining the units of the magnetic fields by $\sqrt{\lambda_2} b \rightarrow b$. Therefore, $\lambda_2$ can be set to unity if all magnetic fields are considered as *effective* or rescaled magnetic fields. Since the induction equation (4) is linear in the magnetic fields $b(x, t)$, such rescaling leaves every conclusion unchanged. Of course, the external force $g$ is also scaled by a factor $\sqrt{\alpha}$ which does not affect our analysis here as the assignment of canonical dimensions to various fields and parameters is done after absorbing $\lambda_2$ in the definition of $b$. More specifically, under the rescaling $x \rightarrow lx$, $t \rightarrow lt$, $u_i \rightarrow l^{z_\chi} u_i$, $b_i \rightarrow l^{z_\chi} b_i$ the bare parameters scale as $\lambda_{1,3}(l) \rightarrow l^{x_\chi + z_\chi - 1} \lambda_{1,3}$, $\lambda_2(l) \rightarrow l^{x_\chi - x_\lambda + z_\chi - 1} \lambda_2$ and $B_0(l) \rightarrow l^{x_\chi - x_\lambda + z_\chi - 1} B_0$. Since there are no fluctuation corrections to the nonlinearities $\lambda_1$, $\lambda_2$, and $\lambda_3$, which are the consequences of the invariances under the transformations TI and TII, they can be kept invariant under rescaling of space and time as mentioned above leading to $\chi_\lambda = \chi_\beta = \chi$ and $\chi + z = 1$. Thus, under naive rescaling the bare parameter $B_0 \sim B_0 l^{z - 1}$. Furthermore, the invariance under the transformation TII and the resulting Ward identity ensure different ways of implementing one-loop RG by having different values for $\langle b \rangle$ and $B_0$ in equations (9) and (10) subject to the same bare mean magnetic field $B_0 = \tilde{B}_0 + \langle b \rangle$. Since any renormalization scheme must respect a freedom of choice as represented by the transformation TII, respective terms must scale the same way under the rescaling of space and time [23]. Hence, due to fluctuation corrections $B_0(l) \sim l^{z}$. Furthermore, since both $\tilde{B}_0$ and $b$ have the same scaling dimensions given by the exponent $\chi$, the physical mean magnetic field in the system $B_0 = \tilde{B}_0 + \langle b(x, t) \rangle$, if it receives fluctuation corrections under mode eliminations, must scale as $B_0(l) \sim l^{z}$, or if there are no fluctuation corrections to it due to some special symmetries (as in the one-dimensional Burgers-like model for MHD in [2]) will be irrelevant (in an RG sense) as it will flow to zero as $l^{z - 1}$ (since $z$ for fully developed turbulence is less than unity). This clearly suggests the possibility of a renormalization of the Alfvén wave speed, akin to the renormalization of the sound speed in compressible fluid turbulence [27]. As we will see below, for 3DMHD there are infra-red singular fluctuation corrections to the bare mean magnetic field leading it to renormalize in a way consistent with our predictions from the Ward identities above. It should be
noted that our analysis above is independent of the strength of the bare value of the mean magnetic field. Although in the discussion above we have rescaled space isotropically (i.e., \(x, y\) and \(z\) coordinates are rescaled the same way) and analyse the scale dependence of the resulting effective parameters and the fields, it should be noted that such a rescaling does not imply that the effective parameters have isotropic structures.

5. Energy spectra for incompressible fluids

We begin by writing down the 3DMHD equations in the incompressible limit (i.e., \(\nabla \cdot \mathbf{u} = 0\)). We write down the equations of motion in \(k\)-space in terms of the Elsässer variables \(\mathbf{z}^\pm = \mathbf{u} \pm \mathbf{b}\). The equations are (we take the mean magnetic field \(\mathbf{B}_0\) to be along the \(\hat{z}\) direction)

\[
\frac{\partial z_i^+(k, t)}{\partial t} - iB_0k_zz_i^+ + iP_{lp}k_s \sum z_s^+(q, t)z_p^+(k - q, t) + \eta_+^o k^2z_i^+ + \eta_+^o k^2z_i^- = \theta_i^+(k, t),
\]

\[
\frac{\partial z_i^-(k, t)}{\partial t} + iB_0k_zz_i^- + iP_{lp}k_s \sum z_s^-(q, t)z_p^-(k - q, t) + \eta_-^o k^2z_i^- + \eta_-^o k^2z_i^+ = \theta_i^-(k, t).
\]

The stochastic forces \(\theta_i^\pm\) are linear combinations of \(f_l, g_l\), the tensor \(P_{lp}\) is the transverse projection operator which appears due to the divergence-free conditions on \(\mathbf{u}\) and \(\mathbf{b}\), and \(\eta_{\pm}^o = \nu \pm \eta\).

In the absence of any cross-correlations between the velocity and the magnetic fields, the simplest choice for the noise variances, consistent with the divergence-free conditions on the velocity and the magnetic fields, are

\[
\langle \theta_i^+(k, t)\theta_m^+(\mathbf{-k}, t) \rangle = 2P_{lm}D_1k^{-y}\delta(t),
\]

\[
\langle \theta_i^+(k, t)\theta_m^-(\mathbf{-k}, t) \rangle = 2P_{lm}D_2k^{-y}\delta(t).
\]

In equations (12) we choose \(y > 0\). In particular, the choice of \(y = d\) in \(d\) space dimensions ensures that the energy flux in the inertial range is a constant (up to a weak logarithmic dependence on wavevector \(k\)) which forms the basis of the K41 theory [4], and as a result the parameters \(D_1\) and \(D_2\) pick up dimensions of energy dissipation rate per unit mass. Hence, we will use \(y = d\) in our calculations below. In experimental realizations of MHD, external forces act on the large scales. In analytical approaches, such forces are replaced by stochastic forces with variances given by equations (12) for calculational convenience. It should however be noted that in numerical solutions of the Navier–Stokes equation driven by stochastic forces with variances similar to equations (12) structure functions of the velocity fields are shown to exhibit multiscaling similar to the experimental results [28]. Our preliminary results from the numerical solutions of the isotropic 3DMHD equations (i.e., no mean magnetic fields) yield multiscaling similar to those obtained from the 3DMHD equations driven by large-scale forces [29]. Although no such numerical results exist for the case with a mean magnetic field, it will presumably be true for such a situation also. In a stochastically driven Langevin description, correlation functions are proportional to the noise correlations and hence it is necessary to force the induction equation (6) stochastically, as is common in the literature (see, e.g., [2,8] and references therein). For such noises in the absence of any mean magnetic field (i.e., for the isotropic case) it has been shown by using a one-loop mode coupling theory that the scaling exponents have values \(\chi = 1/3, z = 2/3\) [8].

doi:10.1088/1742-5468/2005/07/P07002
Universal properties of three-dimensional magnetohydrodynamic turbulence: do Alfvén waves matter?

The equations of motion (11) are coupled even at the linear level leading to a bare propagator matrix for the equations (11) of the form (a ‘0’ refers to bare quantities)

\[ G^{-1}_0 = \begin{pmatrix} -i\omega - iB_0 k_z + \eta^0_{\perp} k^2 & \eta^0_{\parallel} k^2 \\ \eta^0_{\parallel} k^2 & -i\omega + iB_0 k_z + \eta^0_{\perp} k^2 \end{pmatrix}. \]

We use a one-loop mode coupling theory which is conveniently formulated in terms of the self-energy matrix \( \Sigma \) and the correlation functions given by equations (7). The self-energy matrix \( \Sigma \) is defined by \( G^{-1} = G^{-1}_0 - \Sigma \) where \( G \) is the renormalized propagator matrix. In terms of the scaling exponents \( \chi \) and \( z \), and the renormalized parameters the self-energy matrix \( \Sigma \) is given by

\[ \Sigma = \begin{pmatrix} -i\omega - iB(k) k_z + \eta_{\perp} k^2 & \eta_{\parallel} k^2 \\ \eta_{\parallel} k^2 & -i\omega + iB(k) k_z + \eta_{\perp} k^2 \end{pmatrix} \]

where \( B(k) \) is the renormalized or the effective Alfvén wave speed. If there are diagrammatic corrections to the imaginary parts of \( \Sigma(k,\omega) \) at frequency \( \omega = 0 \) which are singular in the infra-red limit, then \( B(k) \) is different from \( B_0 \), the bare Alfvén wave speed, else they are the same.

There are two one-loop diagrams which contribute to the fluctuation corrections to \( \Sigma_{11}(k,\omega = 0) = -iB(k) k_z + \eta_{\perp} k^2 \). These are shown in figure 1. These have both real and imaginary parts at frequency \( \omega = 0 \). Thus there are fluctuation corrections to the bare Alfvén wave speed which are singular in the small wavenumber limit. We assume, in the spirit of mode-coupling methods, \( B(k) = Bk^{-\chi} \) (we assume an isotropic scale dependence for \( B(k) \) for simplicity which suffices our purpose of finding the scale dependence of the effective Alfvén wave speed). Clearly, if \( -s + 1 < z \) the small wavenumber limit of the problem is dominated by underdamped waves; if \( -s + 1 > z \) the Alfvén waves are damped out in the small wavenumber limit. In contrast, if \( -s + 1 = z \) then in the small wavenumber limit both the propagating and the dissipative modes are present. Our analyses in section 4 clearly suggest that if there are fluctuation corrections to \( B_0 \) then the effective scale dependent Alfvén wave speed \( B(l) \sim k^{-\chi} \) yielding \( s = \chi \). Furthermore, since \( \chi + z = 1 \) we have \( -s + 1 = z \) leading to the co-existence of the underdamped Alfvén waves and the dissipative modes in the large scale, long time limit. Note that this situation allows us to define a dimensionless parameter \( \beta \equiv B/\eta_{\perp} \) where \( B \) and \( \eta_{\perp} \) are the renormalized Alfvén speed and the viscosity respectively.

In the SCMC approach vertex corrections are neglected. Lack of vertex renormalizations in the zero wavevector limit in 3DMHD allows SCMC to yield exact relations between the scaling exponents \( \chi \) and \( z \) as in the noisy Burgers/Kardar–Parisi–Zhang equation [25]. In this model the nonlinearities and the noise variances do not renormalize, thus leading to \( z = 2/3, \chi = 1/3 \), satisfying the exponent identity \( \chi + z = 1 \). In the present case, due to the singular nature (in the small wavevector limit) of the bare noise correlations (12), they do not pick up any further singular corrections to their scaling; however, the amplitudes are modified (this is similar to the results in [8]). We denote the renormalized amplitudes by \( D \) and \( \tilde{D} \), respectively. Furthermore, for the 3DMHD equations (5) and (6) there are infra-red singular fluctuation corrections to \( B_0 \) (see the one-loop diagrams in figure (1)) leading to \( B_0(l) \sim l^\chi \) consistent with our arguments above. The SCMC approach involves consistency in the scaling and the amplitudes of the mode coupling equations. It should be noted that the exponent values \( z = 2/3, \chi = 1/3 \) satisfy the mode coupling integral equations regardless of the strength of the
Universal properties of three-dimensional magnetohydrodynamic turbulence: do Alfvén waves matter?

![Diagram](image)

Figure 1. One-loop diagrams contributing to renormalization of $G^+_0$. A single line refers to a propagator and a line with a filled circle refers to a correlator.

Mean magnetic field. For amplitude consistency the one-loop integrals are required to be evaluated. Due to their complicated structure, we evaluated them by assuming that the strength of the mean magnetic field is small. Therefore, only our amplitude relations and not the scaling exponents are affected by the approximation of small mean magnetic fields.

Demanding amplitude consistency in the mode coupling equations we obtain,

$$
\eta_+ = \frac{D}{\eta_+^2} \left[ 1 - \frac{2}{d} + \frac{1}{d(d+2)} \right] + \frac{D}{d(d+2)\eta_+^2} - \frac{\beta^2 D}{\eta_+^2 d} \left[ 1 - \frac{2}{d} + \frac{1}{d(d+2)} - \frac{\beta^2 D}{\eta_+^2 d(d+2)(d+4)} \right],
$$

$$
\beta \equiv \frac{B}{\eta_+^3}, \quad B = B_0 \frac{D}{d(d+2)\eta_+^2},
$$

$$
\frac{1 - \Gamma}{1 + \Gamma} = \frac{(1 + \Gamma^2)(1 - (2/d) + (2/d(d+2))) - 0.5\Gamma(1 - (2/d)) + F_1(\beta)}{(1 + \Gamma^2)(1 - (2/d) + (2/d(d+2))) + 0.5\Gamma(1 - (2/d)) + F_2(\beta)},
$$

where $F_1(\beta) \equiv 2\beta^2(1 + 3\Gamma^2)(-1/d + (1/d(d+2)) - (1/d(d+2)(d+4)))$, $F_2(\beta) \equiv 2\beta^2(3 + \Gamma^2)(-1/d + (1/d(d+2)) - (1/d(d+2)(d+4)))$ are the $\beta$-dependent parts of the amplitude-ratio. While calculating the amplitude consistency relations we worked in the limit of small $\beta \equiv B/\eta_+$ and the renormalized magnetic Prandtl number $(\eta/\nu = (\eta_+ - \eta_-)/(\eta_+ + \eta_-))$ close to unity. Away from these limits the amplitudes of the underlying one-loop integrals
become much more complicated functions of \( \beta \) and also of the renormalized magnetic Prandtl number, but the qualitative picture remains unchanged. The main physical picture that emerges from the expressions (13) is that in the absence of a bare mean magnetic field \( B_0 \) the effective Alfven wave speed is also zero, which is a restatement of the fact that if the original theory is isotropic, so will be the renormalized theory. Moreover the renormalized Alfven speed increases with increasing \( B_0 \). With these results we are now in a position to calculate the energy spectra in the inertial range. We use the form of the effective (scale dependent) viscosity and the Alfven wave speed to obtain the equal time correlation function of \( z_i^\pm \) in the long wavelength limit. We define (in 3D)

\[
C_{ij}^{++}(k, \omega) = \langle z_i^+(k, \omega) z_j^+(−k, −\omega) \rangle = \frac{2Dk^{-3}P_{ij}(k)}{(\omega − Bk^{-1/3}k_z)^2 + \eta_+ k^{4/3}},
\]

\[
C_{ij}^{−−}(k, \omega) = \langle z_i^−(k, \omega) z_j^−(−k, −\omega) \rangle = \frac{2Dk^{-3}P_{ij}(k)}{(\omega + Bk^{-1/3}k_z)^2 + \eta_+ k^{4/3}},
\]

\[
C_{ij}^{+-}(k, \omega) = \langle z_i^+(k, \omega) z_j^−(−k, −\omega) \rangle = \frac{2\tilde{D}k^{-4/3}P_{ij}(k)}{−i(\omega − Bk^{-1/3}k_z) + \eta_+k^{2/3}[i(\omega + Bk^{-1/3}k_z) + \eta_+k^{2/3}]}. \tag{14}
\]

As discussed before, we have omitted anisotropic corrections of \( O(B_0)^2 \) or of \( O(\beta)^2 \) to the correlation functions as we are trying to find out the relevance (in an RG sense) of Alfven waves on the scaling of the isotropic correlation functions in the absence of any mean magnetic fields. Therefore, the equal time correlation functions have the following form (in \( d = 3 \)):

\[
C_{ij}^{++}(k, t = 0) = C_{ij}^{−−}(k, t = 0) = Dk^{-3−2/3}P_{ij}(k). \tag{15}
\]

Therefore, the equal time autocorrelation functions of \( z^\pm \) are independent of any mean magnetic field. This holds true regardless of the scaling of the noise variances. The equal time cross correlation function \( C_{ij}^{+-}(k, t = 0) \) requires more careful considerations. On integrating \( C_{ij}^{+-}(k, \omega) \) over all frequencies \( \omega \) one obtains (in 3D)

\[
C_{ij}^{+-}(k, t = 0) = \frac{\tilde{D}k^{-3}P_{ij}}{iB^{-1/3}k_z + \eta_+ k^{2/3}}. \tag{16}
\]

a form which is valid in the inertial range. It is clear from the expression (16) that both the real and the imaginary parts of \( C_{ij}^{+-}(k, t = 0) \) scale as \( k^{-3−2/3} \) in the inertial range. Thus the one-dimensional kinetic- and magnetic-energy spectra (which are simply related to the correlators defined in equations (14)) scale as \( k^{-5/3} \) in the inertial range. The emerging physical picture is as follows: we find from the expression for \( C_{ij}^{+-}(k, \omega) \) above that this, as a function of frequency \( \omega \), has maxima at \( \omega = ±Bk^{-1/3}k_z \) and the width at half-maxima \( \sim k^{2/3} \). In contrast, the auto-correlations of \( z^+, z^- \) are maximum at \( \omega = 0 \) and their widths scale as \( \sim k^{2/3} \). Thus in the long wavelength limit the width and the location of the maxima of \( C_{ij}^{+-}(k, \omega) \) scale in the same way leading to the presence of the underdamped Alfven waves in the hydrodynamic limit. Therefore, it immediately follows that the kinetic- and the magnetic-energy spectra, being linear combinations of the correlators discussed above times appropriate phase factors, scale as \( k^{-5/3} \) even in the presence of a non-zero mean magnetic field. It should be noted that we have used a small \( \beta \) approximation to
arrive at our results for the self-consistent amplitude relations. For a finite $\beta$ one would require to work with a fully anisotropic form of the correlation functions and obtain self-consistent relations for scaling and anisotropic amplitudes; this task, which is analytically challenging, remains to be done in the future. However, based on our calculations above, the exponent identity $\chi + z = 1$ and the Ward identity suggesting that the renormalized Alfvén wave speed should scale as $k^{-1/3}$ with wavevector $k$ being in the inertial range, we argue that even for a finite $\beta$, i.e., a finite mean magnetic field, the energy spectra will scale as $k^{-5/3}$ in the inertial range, a result supported by much observational evidence [14]. The self-consistent amplitude equations will then be anisotropic, reflecting the presence of underdamped Alfvén waves in the inertial range. The full correlation matrix will be anisotropic in the hydrodynamic limit; its eigenvalues have different amplitudes, but all of them scale the same way. Our confidence in our result that the scaling of the correlation functions along directions parallel and perpendicular to the direction of the mean magnetic field is same is derived from the fact that our exponent values $z = 2/3$, $\chi = 1/3$ satisfy the one-loop integral equations regardless of the strength of the mean magnetic field and are consistent with the Ward identity discussed above.

6. Kolmogorov’s constants

According to the Kolmogorov hypothesis for fluid turbulence [4], in the inertial range energy spectrum $E(k) = K_0 \epsilon^{2/3} k^{-5/3}$, where $K_0$, a universal constant, is the Kolmogorov constant and $\epsilon$ is the energy dissipation rate per unit mass. Various calculations, based on different techniques by different groups [7], [30]–[32], show that $K_0 \sim 1.5$ in three dimensions. Having noted that the energy spectra, even in the presence of a mean magnetic field, scale as $k^{-5/3}$ extensions of Kolmogorov’s hypothesis for 3DMHD allows one to define Kolmogorov’s constants for the Elsässer fields: $E_{\pm}(k) = K_0^\pm \epsilon_{\pm}^{2/3} k^{-5/3}$. Since $z^\pm = u \pm b$, $(z_i^+(k, \omega)z_j^-(\mathbf{k}, -\mathbf{\omega})) = (z_i^-(k, \omega)z_j^+(\mathbf{\mathbf{k}}, -\mathbf{\omega}))$ and $\epsilon_+ = \epsilon_- = \epsilon_{\text{MHD}}$ in the absence of any cross-correlations between the velocity and the magnetic fields, we have $K_0^+ = K_0^- = K_{\text{MHD}}$. The noise strength $D$ and the rate of energy dissipation per unit mass is connected by the Novikov theorem [8,33]: $\epsilon = 2D(S/\pi^3)$. Noting that the energy spectra $E_{\pm}(k)$ of $z^\pm$ in the inertial range is given by

$$E_{\pm}(k) = Dk^{-3}/\eta_+k^{2/3};$$

where $D$ is the effective or renormalized noise strength, we identify

$$K_{\text{MHD}} = 1.6 \left[ 1 + 0.7 (3\Gamma^2 - 6\Gamma - \beta^2/105) \right]^{2/3}. \quad (18)$$

The notable feature of the expression (18) is that the constant $K_{\text{MHD}}$ depends on the dimensionless parameter $\beta$ which we introduced before. For $\Gamma = 0$, i.e., for no magnetic fields, we find $K_{\text{MHD}} = K_0 = 1.6$ for pure fluid turbulence which is well within the accepted range of values [31]. Before closing this section we would like to point out that the presence of multiscaling raises questions about Kolmogorov’s constant $K_{\text{MHD}}$ being universal: a small but finite intermittency correction (i.e., multiscaling) over the simple K41 scaling implies the presence of an arbitrary scale which may spoil the universality of $K_{\text{MHD}}$. We refrain however from getting into this question and adopt a point of view that regardless of whether or not $K_{\text{MHD}}$ remains universal due to multiscaling, the numerical value of this constant is likely to be affected by the presence of Alfvén waves in the system, which is reflected by expression (18).
7. Possibilities of variable multifractality

Experiments and numerical simulations [20, 34] find nonlinear multiscaling corrections to the K41 prediction of $\zeta_p = p/3$ for the structure functions in the inertial range. To date, no controlled perturbative calculation for $\zeta_p$ is available. To account for multiscaling in fluid turbulence, however, Obukhov [35] and Kolmogorov [36] assumed a log-normal distribution for dissipation $\epsilon$ to arrive at

$$S_p(r) = \langle |\Delta u|^p \rangle = C_p \bar{\epsilon}^{\eta/p} r^{\eta/p} \left( \frac{L}{r} \right)^{(5/2)p(\beta - 3)} ,$$

where $\bar{\epsilon}$ is the mean value of $\epsilon$ and (a bound for $\epsilon$ in fluid turbulence has been discussed in [37]):

$$\langle \epsilon(x + r)\epsilon(x) \rangle \propto \langle (\Delta u)^6 / r^2 \rangle \sim (L/r)^{\delta_6}.$$  \hspace{1cm} (20)

For small $\delta$, $\delta \simeq 9\bar{\delta}$. A standard calculation on the randomly stirred model yields intermittency exponent $\delta = 0.2$ [32] where $\delta = 9\bar{\delta}$, whereas the best possible estimate from experiments is $0.23$ [32]. This model, despite having well known limitations and difficulties [38], serves as a qualitative illustration of multiscaling. As in fluid turbulence, in MHD the dissipations $\epsilon_m$ of the Els"asser variables $z^\pm$ fluctuate in space and time, and as a result one may define two intermittency exponents $\delta_\pm$ for them. In the present problem \(\delta_+ = \delta_- = \delta_{\text{MHD}}\). Below we calculate the exponent $\delta_{\text{MHD}}$ in a one-loop expression which will give us an estimate of the Alfvén wave speed-dependent deviation of the scaling of the structure functions from their simple-scaling values as predicted by the K41 theory. We closely follow [32]. We work with the self-consistent forms for the self-energies and correlation functions given above along with the consistency relations for the amplitude ratios $\Gamma$ and $\beta$. Following [32], we find the dissipation correlation functions in 3D to be

$$\langle \epsilon(x + r)\epsilon(x) \rangle \simeq 12.4 \epsilon_{\text{MHD}}^2 K_{\text{MHD}}^2 \alpha^2 K_{\text{MHD}}^2 \ln \frac{L}{r} ,$$

with $\alpha$ being defined by the relation $\nu_+ = \alpha \epsilon_{\text{MHD}}^{1/3}$. From the self-consistent amplitude-relations (13) we find

$$\alpha = 0.4 \left[ 1 + 0.7 \left( 3\Gamma^2 - 6\Gamma - \beta^2 (29/105) \right) \right]^{\nu_+ / 3} \left[ 1 - 0.5\Gamma - 4\beta^2 (0.7 - 0.03\Gamma) \right]^{\nu_- / 3} .$$

Thus, $\alpha$, which is a universal coefficient in ordinary fluid turbulence, varies with the parameter $\beta$, or with the Alfvén wave speed in MHD. Substituting the values of $K_0$ and $\alpha$ we find

$$\delta_+ = \delta_- = \delta_{\text{MHD}} = 0.2 \left[ 1 + 0.7 \left( 3\Gamma^2 - 6\Gamma - \beta^2 (29/105) \right) \right]^{\nu_+ / 3} .$$

Thus we find a decrease in the value of $\delta$ with an increase in $\beta$, i.e., with increasing mean magnetic field. Despite the limited applicability of log-normal models in characterising multiscaling, we can conclude, from our expression (23) for the intermittency exponent in MHD in the presence of a mean magnetic field, that the intermittency corrections to the simple K41 scaling are likely to be affected (reduced in our calculations) in the presence of Alfvén waves. Further calculations and/or numerical simulations are needed to find the exact extent of the dependence of multiscaling on the mean magnetic field and the possibilities of anisotropic multiscaling for the structure functions parallel and
perpendicular to the mean magnetic field as demonstrated recently in [17]. It should be noted that our conclusions on the multiscaling properties of the structure functions depend on a log-normal model for 3DMHD. Such a description, unfortunately, is unable to distinguish between the structure functions parallel and perpendicular to the direction of the mean magnetic field in the system. Moreover, the intermittency exponents above (expressions (23)) are evaluated in the lowest order in mean magnetic field. Therefore, even though from our results we are not able to make firm comments on the possibility of parallel and perpendicular structure functions exhibiting different multiscaling, the real importance of our results lies in their elucidation of the multiscaling properties depending on the mean magnetic field.

8. Conclusion

In this paper we have considered the effects of the Alfvén waves on the statistical properties of the correlation functions of the velocity and the magnetic fields or the Elsässer fields. We considered the case when the velocity fields are incompressible. In a one-loop approximation we find that the effective or the renormalized Alfvén wave speed scales as $k^{-1/3}$ where the wavevector $k$ is in the inertial range. This immediately yields that the energy spectra, even in the presence of a mean magnetic field, scale as $k^{-5/3}$ in the inertial range. We identify a dimensionless parameter $\beta$ which is the ratio of the effective Alfvén wave speed and the renormalized viscosity. We obtain self-consistent relations between the amplitude ratio $\Gamma$ of the correlation functions and $\beta$. These relations allow us to calculate the dimensionless Kolmogorov constant and we show that it depends explicitly on $\beta$ or on the mean magnetic field. Finally, we calculate the intermittency exponent which in a log-normal model gives a qualitative account of the multiscaling in terms of the deviation from the K41 scaling for the structure functions. We would like to emphasize that although the one-dimensional Burgers-like model of MHD of [2] and the 3DMHD equations yield energy spectra independent of the Alfvén waves the long wavelength physical pictures are different. In the former case, due to the nonrenormalization of the bare Alfvén wave speed, Alfvén waves are overdamped in the hydrodynamic limit; the dominant process in that limit is viscous dissipation. In contrast, in 3DMHD, the Alfvén wave speed picks up a singular correction in the hydrodynamic limit leading to the K41 scaling of the energy spectra and the presence of underdamped Alfvén waves in the hydrodynamic limit. Despite similar mathematical structures these crucial differences arise principally because the one-dimensional model decouples completely when written in terms of the Elsässer variables, allowing comovement with the waves of each of them separately. As a result correlators of each of them are independent of the Alfvén waves and the bare Alfvén wave speed remains unrenormalized, leading to overdamped Alfvén waves in the hydrodynamic limit. However, the 3DMHD equations do not decouple when written in terms of the Elsässer variables and hence oppositely propagating waves cannot be made to vanish by comoving. Despite the limitations of the one-loop methods [7] and the small $\beta$ approximation to facilitate easier analytical manipulations for the amplitude relations, we obtain results which are significantly new and open the intriguing possibilities of MHD multiscaling universality classes being parametrized by the mean magnetic field. Some of the quantitative details will change if one retains terms which are higher order in $\beta$; however, we believe that the qualitative picture will essentially remain the same. As MHD
turbulence forms a natural example of a driven nonequilibrium system with Alfvén waves, our results are the first of its kind for a natural system. In our log-normal model approach, we did not distinguish between the longitudinal and the transverse structure functions; our results cannot be compared directly with those of [17], where the longitudinal and the transverse structure functions are found to scale differently, with multiscaling exponents which depend on the magnitude of the mean magnetic field in the presence of a finite magnetic helicity. In contrast, our results apply to the multiscaling of the usual structure functions in the absence of any magnetic helicity which are combinations of the transverse and the longitudinal structure functions as considered in [17] which would also then exhibit a mean magnetic field dependent scaling. In particular we find that the usual structure functions multiscale less in the presence of an increasingly strong mean magnetic field which is in qualitative agreement with those of [17]. Although here we have restricted ourselves to the study of Alfvén waves as relevant perturbations on the amplitude and the scaling of the isotropic correlation functions in the limit of small $\beta$, our results indicate the possibility of the multiscaling exponents varying with the amplitude of the mean magnetic field. Our analyses, when extended for finite values of $\beta$ and for full anisotropic structure of the correlation functions, are likely to provide further understanding of and resolve some of the discrepancies between the various phenomenological scenarios available for situations with large mean magnetic fields [10,11,13]. We leave this task for the future.

From a broader point of view our results demonstrate the critical role of wave-like excitations in determining the statistical properties of 3DMHD. The presence of propagating waves is not confined to MHD only; they are a generic feature in many other naturally occurring soft-matter systems where such waves can be present in a viscous environment, e.g., active polar gels in cytoskeletal dynamics [39] and in the dynamics of self-propelled particles [40] etc. We believe our results will lead to similar theoretical and experimental studies in relevant nonequilibrium soft-matter systems.

Acknowledgments

This work was started when one of the authors (AB) was an Alexander von Humboldt Fellow at the Hahn–Meitner Institut, Berlin. AB wishes to thank the AvH Stiftung, Germany, for partial financial support.

References

[1] Das D, Basu A, Barma M and Ramaswamy S, 2001 Phys. Rev. E 64 021402
[2] Basu A, Bhattacharjee J K and Ramaswamy S, 1999 Eur. Phys. J. B 9 425
[3] Verma M K, 1999 Phys. Plasmas 6 1455
[4] Kolmogorov A N, 1941 C. R. Acad. Sci. USSR 30 301
[5] Goldreich P and Sridhar S, 1995 Astrophys. J. 438 763
[6] Basu A, 2000 PhD Thesis Indian Institute of Science, Bangalore
[7] Yakhot V and Orzag S A, 1986 Phys. Rev. Lett. 57 1722
[8] Basu A, 2004 Europhys. Lett. 65 505
[9] Kraichnan R H, 1965 Phys. Fluids 8 1385
[10] Galtier S et al., 2000 Preprint astro-ph/0008148
[11] Ng C S and Bhattacharjee A, 1996 Astrophys. J. 465 845
[12] Shebalin J V et al., 1983 J. Plasma Phys. 29 525
[13] Montgomery D and Turner L, 1981 Phys. Fluids 24 825
[14] Rickett B J and Lyne A G, 1990 Mon. Not. R. Astron. Soc. 244 68
   Spangler S R and Gewinn C R, 1990 Astrophys. J. 353 L29

doi:10.1088/1742-5468/2005/07/P07002
Universal properties of three-dimensional magnetohydrodynamic turbulence: do Alfvén waves matter?

Gupta Y, Rickett B J and Coles W A, 1993 Astrophys. J. 403 183
[15] Cho J and Vishniac E T, 2000 Astrophys. J. 539 273
Cho J, Lazarian A and Vishniac E T, 2002 Astrophys. J. 564 291
[16] Cho J and Lazarian A, 2003 Mon. Not. R. Astron. Soc. 345 325
[17] Müller W-C, Biskamp D and Grappin R, 2003 Phys. Rev. E 67 066302
[18] Jackson J D, 1975 Classical Electrodynamics 2nd edn (New Delhi: Wiley Eastern)
Raichaudhuri A, 1998 The Physics of Fluids and Plasmas (Cambridge: Cambridge University Press)
[19] Montgomery D, 1989 Lecture Notes on Turbulence ed J R Herring and J C McWilliam
(Shingapore: World Scientific)
[20] Basu A, Sain A, Dhar S K and Pandit R, 1998 Phys. Rev. Lett. 81 2687
[21] Biskamp D and Müller W, 2000 Phys. Plasmas 7 4889
[22] Sreenivasan K R and Antonia R A, 1997 Ann. Rev. Fluid Mech. 29 435
Dhar S K, Sain A, Pande A and Pandit R, 1997 Pramana—J. Phys. 48 325
[23] Basu A, unpublished
[24] Forster D et al, 1977 Phys. Rev. A 16 732
Bhattacharjee J K, 1998 J. Phys. A: Math. Gen. 31 L93
[25] Frey E and Tauber U, 1994 Phys. Rev. A 50 1024
[26] Antonov N V, 2000 Physica D 144 370
[27] Taroseisky I S, Yakhot V, Kida S and Orszag S A, 1990 Phys. Rev. Lett. 65 171
Bhattacharjee J K, 1993 Mod. Phys. Lett. 7 881
[28] Sain A, Manu and Pandit R, 1998 Phys. Rev. Lett. 81 4377
[29] Basu A and Pandit R, 2005 work in progress
[30] Kraichnan R H, 1971 J. Fluid Mech. 47 525
[31] Leslie C D, 1973 Developments in the Theory of Turbulence (Oxford: Clarendon)
[32] Das A and Bhattacharjee J K, 1994 Europhys. Lett. 25 527
[33] Woyczynski W A, 1998 Burgers-KPZ Turbulence–Göttingen Lectures (Berlin: Springer)
McComb W D, 1990 The Physics of Fluid Turbulence (Oxford: Clarendon)
[34] Cho J, Lazarian A and Vishniac E T, 2002 Astrophys. J. 564 291
Biskamp D and Müller W, 2000 Phys. Plasmas 7 4889
[35] Obukhov A M, 1962 J. Fluid Mech. 13 77
[36] Kolmogorov A M, 1962 J. Fluid Mech. 13 82
[37] Doering C R and Foias C, 2002 J. Fluid Mech. 467 289
[38] Frisch U, 1995 Turbulence: The Legacy of A.N. Kolmogorov (Cambridge: Cambridge University Press)
[39] Kruse K et al, 2004 Preprint physics/0406058
[40] Simha R A and Ramaswamy S, 2002 Phys. Rev. Lett. 89 058101

doi:10.1088/1742-5468/2005/07/P07002 16