Thermodynamic analysis of geothermal heat pump during the cold season

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Abstract. The paper is analysing the performances (COP, power and, heating heat rate function of time) for a ground-coupled heat pump that is used to heat a space during winter, for a period of 180 days. The analysis purpose is to evaluate the time based changes in values of COP and, energy transfers of a geothermal heat pump, considering a scenario for the variation of the ambient temperature in time and an analytical solution for the time dependence of the soil one. The temperatures and the energy transfer rates were determined on the basis of the irreversible entropy balance equation.

1. Introduction

The ground-coupled heat pump based energy systems are using geothermal underground energy for space heating and cooling. Because of large heating efficiency and environmental protection, the interest in these systems is increasing today. There is a wide variety of these systems to be used in office buildings, houses and historical buildings. The study[1], regarding the office building heating systems using ground coupled heat pumps with different borehole diameters, showed that the highest savings to investment ratio, during a period of thirty years, is 4.80. The paper [2]presents an open loop system that uses for cooling a water lake heat capacity. For heating most heat pump systems use closed loop. A general review[3], presenting the most common heat pumps based energy systems that are used today, is comparing their performances and operational parameters but for quasi-steady state operation. The thermodynamic analysis of ground-coupled heat pump based energy systems emphasizes the important influence of underground soil temperature profile and the geometry of the heat exchangers extracting the geothermal energy, e.g. either the superficial temperature profile of soil influenced by the natural convection on the surface [4], or that after a depth of 15m, the initial underground temperature is almost constant.

This paper perform a thermodynamic analysis of a closed loop heat pump based heating system coupling two external heat “reservoirs” with different variable features. The heat “sink” is the heated space that asks a heating rate function of the environmental temperature and the heat “source” is the soil with a variable temperature in time. The heat pump cycle is an irreversible one, and the overall entropy balance equation allowed linking these two heat “reservoirs” by choosing a scenarios for the heating rate during the winter and an analytical solution in time for the geothermal heat rate. The heat pump based heating system is presented in figure 1.
2. Restrictive conditions

- The heated space temperature $T_{HS} = 295.15K = \text{const.}$ while outside ambient temperature $T_a$ is varying from $253.15K$ to $288.15K$ (from $–20°C$ to $+15°C$).
- For the ambient temperature was adopted a scenario, respectively we considered a continuous heating during the winter (180 days) with a time dependence, see figure 2

\[
T_a = 288.15 - 35 \cdot \sin \left( \frac{\pi \cdot t}{4320} \right)
\]  

(1)

where $t$ is the time in hours.

- Constant initial soil temperature $T_{Si} = 283.15 \text{ K}$. For the soil temperature variation in time was imposed an analytical dependence, see figure 3:
\[ T_S = 283.15 \cdot \exp\left(-0.0025 \cdot t^{0.375}\right) \]  

where \( t \) is the time in hours.

\[ \frac{\dot{Q}_{\text{lost}}(T_a)}{\dot{Q}_{\text{lost}}(T_{aN})} = \frac{h_{HS} \cdot A_{HS} \cdot (T_{HS} - T_a)}{h_{HS} \cdot A_{HS} \cdot (T_{HS} - T_{aN})} = \frac{\dot{Q}_K(T_a)}{\dot{Q}_K(T_{aN})} = \frac{\dot{Q}_K(T_a - T_{N})}{\dot{Q}_K(T_{aN} - T_{N})}. \]  

**Figure 3.** \( T_S = T_S(t) \).

- Heat carriers were assumed to have constant heat capacities, \( \dot{C}_k = 500 \text{ W/K} = \text{const.} \) and \( \dot{C}_e = 1000 \text{ W/K} = \text{const.} \). The heating system uses the air as heat carrier flowing in a loop between the condenser and the heated space.
- Heat pump heat exchangers (E, K and soil borehole) were presumed to have constant effectiveness (\( \varepsilon_E = 0.75 = \text{const.} \), \( \varepsilon_K = 0.75 = \text{const.} \), and \( \varepsilon_S = 0.95 = \text{const.} \)).

3. **Mathematical algorithm**

3.1. **The heated space and the condenser**

The heated space needs to be maintained at constant temperature, \( T_{HS} = 295.15 \text{ K} \), while outside ambient temperature \( T_a \) is varying from 253.15 K to 288.15 K. Thus the heating rate supplied by K equalizes the lost heat rate, see figure 4.

\[ \dot{Q}_{\text{lost}} = h_{HS} \cdot A_{HS} \cdot (T_{HS} - T_a) = \dot{Q}_K = \dot{C}_K \cdot (T_{okK} - T_{IK}) = \varepsilon_K \cdot \dot{C}_K \cdot (T_{K} - T_{IK}) \]  

**Figure 4.** Thermal energy balance of the heated space.
When the ambient temperature fluctuates between +15°C and –20°C, the air input/output temperatures are: \( T_{ik} = T_{ikN} = T_{HS} = 295.15 \text{K} = \text{const.} \) and \( T_{ok} = T_{ok}(T_a) \) function of the heating system design temperatures. We imposed \( T_{okN} = 313.15 \text{K} \) at \( T_{aN} = 253.15 \text{K} \).

The dependences \( \dot{Q}_K = \dot{Q}_K(T_a) \) and, \( T_K = T_K(T_a) \) and, \( T_{ok} = T_{ok}(T_a) \) are obtained on the basis of the heat balance equations1, 3 and 4, see figure 5:

\[
\Rightarrow T_{ok}(T_a) = T_{ik} + \frac{T_{HS} - T_{aN}}{T_{HS} - T_{aN}} \cdot (T_{okN} - T_{ikN}) = 421.64286 - \frac{3}{7} \cdot T_a, \quad [\text{K}] \tag{5}
\]

\[
T_K = T_{ik} + \frac{T_{ok}(T_a) - T_{ik}}{\epsilon_K}, \quad [\text{K}] \tag{6}
\]

\[
\dot{Q}_K = \dot{C}_K \cdot \left[ T_{ok}(T_a) - T_{ik} \right] = \epsilon_K \cdot \dot{C}_K \cdot \left[ T_K(T_a) - T_{ik} \right], \quad [\text{W}] \tag{7}
\]

![Figure 5.](image)

**Figure 5.** The dependences (a) \( T_{ok} = T_{ok}(T_a) \), (b) \( T_K = T_K(T_a) \), (c) \( \dot{Q}_K = \dot{Q}_K(T_a) \) for 253.15 K ≤ \( T_a \) ≤ 288.15 K.

### 3.2. The soil and the evaporator

The heat balance equations related to the thermal connection evaporator – soil are:

\[
\dot{Q}_E = \dot{C}_E \cdot (T_{IE} - T_{OE}) = \epsilon_E \cdot \dot{C}_E \cdot (T_{IE} - T_E), \quad [\text{W}] \tag{8}
\]

\[
\dot{Q}_S = \epsilon_S \cdot \dot{C}_S \cdot (T_{IS} - T_{OE}) = \dot{Q}_E, \quad [\text{W}] \tag{9}
\]

From equations 2, 8, and 9 we derived:

\[
T_{IE} = \frac{\epsilon_E \cdot \epsilon_S \cdot T_E - \epsilon_E \cdot T_E - \epsilon_S \cdot T_S}{\epsilon_E \cdot \epsilon_S - \epsilon_E - \epsilon_S}, \quad [\text{K}] \tag{10}
\]

\[
T_{OE} = \frac{\epsilon_E \cdot \epsilon_S \cdot T_S - \epsilon_E \cdot T_E - \epsilon_S \cdot T_S}{\epsilon_E \cdot \epsilon_S - \epsilon_E - \epsilon_S}, \quad [\text{K}] \tag{11}
\]

\[
\dot{Q}_E = \frac{\dot{C}_E \cdot \epsilon_E \cdot \epsilon_S \cdot (T_E - T_S)}{\epsilon_E \cdot \epsilon_S - \epsilon_E - \epsilon_S}, \quad [\text{W}] \tag{12}
\]
3.3. Heat pump irreversibility

The heat pump irreversibility (Irr) considered that the isentropic efficiency of the compressor is 0.95, and pressures drops in evaporator and condenser, on the suction and discharge and liquid lines are of 0.1K.

Figure 6[5] present a basic irreversible reverse cycle, consisting of processes: 1 – 2r irreversible adiabatic compression, 2r – 3r irreversible cooling, 3r – 4r irreversible adiabatic process (expansion or throttling), 4r – 1 irreversible heating. The internal irreversibility is also “emphasized” by the entropy variations of the working fluid during the heat exchanges with external heat “reservoirs”. Hence, the entropy variations are:

- $\Delta s_{4-1} = s_{4t} - s_{4r}$ is the entropy variation along the heating process 4r-1t (4-1)

$$\Delta s_{2-3} = s_{2r} - s_{3r} = \Delta s_{4-1} + \Delta s_{4r}^{1-2} + \Delta s_{4r}^{2-3} + \Delta s_{4r}^{3-4} + \Delta s_{4r}^{4-1} = \Delta s_{4-1} \cdot N_{irr}$$

is the entropy variation along the cooling process 2r-3t (2-3).

![Figure 6. Scheme of an irreversible reverse cycle, temperature – entropy diagram.](image)

The irreversible entropy internally generated by flow with friction is stored in the entropy variation during the cooling process, and accordingly we can identify it in the first law efficiency (COP):

$$COP = \frac{\dot{Q}_{2-3}}{\dot{W}} = \frac{\dot{Q}_{2-3}}{\dot{Q}_{2-3} - \dot{Q}_{4-1}} = \frac{\dot{m} \cdot T_{mq}^{2r-3t} \cdot \Delta s_{2-3}}{\dot{m} \cdot T_{mq}^{2r-3t} \cdot -\dot{m} \cdot T_{mq}^{4r-1t} \cdot \Delta s_{4-1}} = \frac{T_{mq}^{2r-3t}}{T_{mq}^{4r-1t} \cdot N_{irr} - 1} = \frac{T_K}{T_E} \cdot N_{irr} \cdot T_{mq}^{2r-3t}$$

where:

- $T_{mq}$ is the mean thermodynamic temperature during the heating and cooling processes, and $T_K$ and $T_E$ are the condensing and evaporating ones in K and E.

- $N_{irr} = \left(1 + \frac{\sum \Delta s_{irr}^{int}}{\Delta s_{4-1}}\right)$ and $Irr = N_{irr} \frac{T_{mq}^{2r-3t}}{T_K} \cdot \frac{T_E}{T_{mq}^{4r-1t}}$, Irr are the numbers of internal irreversibility related to the chosen reference temperatures of processes 2r-3t and 4r-1t.

For an ideal reverse cycle, both numbers of irreversibility are equal to one, i.e. Carnot. Thus the
entropy balance equation becomes \[6\]:

\[
\frac{\dot{Q}_E}{T_E} \text{ Irr} = \frac{(COP + 1) T_E^K}{COP \ T_K} = COPR \frac{T_E}{T_K}
\] (15)

and, by using CoolPack we get for R717 (ammonia):

\[
COPR = \frac{1}{COP} \left( 1.5672 - 2.1225 \frac{T_K}{T_E} + 1.5656 \left( \frac{T_K}{T_E} \right)^2 \right)
\] (17)

or simplified:

\[
COPR = -0.60229 + 1.56833 \frac{T_K}{T_E}, \text{ (errors: } +1.1\%, -1.8\%).
\] (18)

Solving the entropy balance equation it is obtained the reference temperature \(T_E(t)\). This issue allows to evaluate \(T_K\), \(T_{Ei}\) and \(\dot{Q}_E\) from equations (14), (15) and (16).

3.4. Power balance and, first law efficiency

\[
\dot{W} = \dot{Q}_K - \dot{Q}_E
\] (19)

\[
COP = \frac{\dot{Q}_K}{\dot{W}}.
\] (20)

4. Numerical results

This simulation was developed for a period of 6 months, around 180 days. Effectiveness of heat exchangers, condenser and, evaporator and, soil heat exchanger were assumed to be \(\varepsilon_E = 0.75 = \text{const.}, \varepsilon_K = 0.75 = \text{const.}, \text{ and } \varepsilon_S = 0.95 = \text{const.}\). Initial soil temperature for this simulation is \(10^\circ\text{C}, T_{Si} = 283.15\text{K}\).

Underground heat exchanger is considered to be a borehole 50m depth and radius of 0.2m. Overall heat transfer surface for extracting heat by convection is \(A_S = 62.8\text{m}^2\).

Solving the problem starts by using equations from evaporator temperature section, simulating different time steps.

Results according to time for soil temperature \(T_S\), ambient temperature \(T_a\), output temperature from condenser \(T_{OK}\), temperature inside condenser \(T_K\), temperature input to evaporator \(T_{EI}\), temperature output from evaporator \(T_{EO}\) and temperature inside evaporator are presented in table 1.

The graph in figure 7 indicates the temperatures according to time dependences.

Table 2 presents the heat rate inside condenser, evaporator and the power of the heat pump.

Heat energy rates function of time, are presented in figure 8. Their dependences with ambient temperature/time show that heat rates in condenser, evaporator and, the power have their peaks during the lowest ambient temperature.

In figure 9 is shown the COP function of time.
Table 1. Temperatures in relation to time.

| Time [h] | $T_S$ [K] | $T_a$ [K] | $T_{ak}$ [K] | $T_K$ [K] | $T_{IE}$ [K] | $T_{oe}$ [K] | $T_E$ [K] |
|----------|-----------|-----------|--------------|-----------|-------------|-------------|----------|
| 1        | 282.44    | 288.12    | 298.16       | 299.16    | 282.36      | 280.98      | 280.51   |
| 50       | 280.09    | 286.87    | 298.69       | 299.87    | 280.01      | 278.40      | 277.86   |
| 100      | 279.19    | 285.60    | 299.23       | 300.60    | 279.10      | 277.25      | 276.64   |
| 200      | 278.03    | 283.07    | 300.32       | 302.04    | 277.91      | 275.61      | 274.84   |
| 300      | 277.20    | 280.57    | 301.39       | 303.47    | 277.05      | 274.31      | 273.40   |
| 400      | 276.53    | 278.11    | 302.45       | 304.88    | 276.36      | 273.20      | 272.15   |
| 500      | 275.96    | 275.70    | 303.48       | 306.26    | 275.77      | 272.20      | 271.01   |
| 1000     | 273.86    | 264.88    | 308.12       | 312.44    | 273.58      | 268.31      | 266.55   |
| 1500     | 272.37    | 257.10    | 311.45       | 316.89    | 272.03      | 265.65      | 263.53   |
| 1750     | 271.74    | 254.69    | 312.48       | 318.26    | 271.39      | 264.69      | 262.46   |
| 2160     | 270.82    | 253.15    | 313.15       | 319.15    | 270.46      | 263.59      | 261.30   |
| 2320     | 270.49    | 253.38    | 313.04       | 319.01    | 270.13      | 263.31      | 261.04   |
| 2820     | 269.56    | 257.10    | 311.45       | 316.89    | 269.23      | 262.95      | 260.86   |
| 3220     | 268.72    | 264.88    | 308.12       | 312.44    | 268.45      | 263.33      | 261.62   |
| 3820     | 267.96    | 275.70    | 303.48       | 306.26    | 267.78      | 264.36      | 263.22   |
| 3920     | 267.82    | 278.11    | 302.45       | 304.88    | 267.66      | 264.63      | 263.63   |
| 4020     | 267.68    | 280.57    | 301.39       | 303.47    | 267.54      | 264.93      | 264.06   |
| 4120     | 267.54    | 283.07    | 300.32       | 302.04    | 267.42      | 265.24      | 264.51   |
| 4220     | 267.40    | 285.60    | 299.23       | 300.60    | 267.31      | 265.57      | 264.99   |
| 4270     | 267.33    | 286.87    | 298.69       | 299.87    | 267.25      | 265.74      | 265.23   |
| 4320     | 267.27    | 288.15    | 298.15       | 299.15    | 267.20      | 265.91      | 265.48   |

Figure 7. Operational temperatures($T_S$, $T_a$, $T_{ak}$, $T_K$, $T_{IE}$, $T_{oe}$, $T_E$) function of time.
Table 2. Heat energy rates functions of time.

| Time [h] | $\dot{Q}_K$ [W] | $\dot{Q}_E$ [W] | $\dot{W}$ [W] | COP |
|----------|----------------|----------------|----------------|-----|
| 1        | 1505.45        | 1388.39        | 117.05         | 12.86 |
| 50       | 1772.64        | 1611.37        | 161.27         | 10.99 |
| 100      | 2044.93        | 1843.18        | 201.75         | 10.13 |
| 200      | 2586.98        | 2298.51        | 288.47         | 8.96  |
| 300      | 3123.29        | 2739.16        | 384.12         | 8.13  |
| 400      | 3651.02        | 3162.88        | 488.14         | 7.47  |
| 500      | 4167.38        | 3567.95        | 599.42         | 6.95  |
| 1000     | 6485.96        | 5271.48        | 1214.48        | 5.34  |
| 1500     | 8152.58        | 6375.40        | 1777.17        | 4.58  |
| 1750     | 8669.08        | 6691.37        | 1977.71        | 4.38  |
| 2160     | 9000           | 6868.32        | 2131.67        | 4.22  |
| 2320     | 8949.28        | 6823.24        | 2126.04        | 4.21  |
| 2820     | 8152.58        | 6275.03        | 1877.55        | 4.34  |
| 3320     | 6485.96        | 5125.65        | 1360.31        | 4.76  |
| 3820     | 4167.38        | 3423.89        | 743.48         | 5.60  |
| 3920     | 3651.02        | 3026.01        | 625            | 5.84  |
| 4020     | 3123.29        | 2611.89        | 511.40         | 6.10  |
| 4120     | 2586.98        | 2183.14        | 403.84         | 6.40  |
| 4220     | 2044.93        | 1741.59        | 303.33         | 6.74  |
| 4270     | 1772.64        | 1516.65        | 255.99         | 6.92  |
| 4320     | 1500           | 1289.28        | 210.71         | 7.12  |

Figure 8. Heat rate in evaporator, condenser and power in time.
5. Conclusions
The paper deals with entropy balance equation in order to evaluate the time dependent performances of heating system, during the winter season, involving a geothermal heat pump. The thermodynamic assessment considered restrictive conditions describing imposed time based evolution of heat pump external heat “reservoirs”. The entropy balance equation of the irreversible heat pump allowed complete linking of all variable in time parameters, temperatures, energy transfer rates and COP.

The mathematical model of this assessment might be applied to other real time based restrictive conditions.

The geothermal heat pump based heating systems seem to be very competitive to cogeneration based ones. Thus, for this assessment, the mean value of the COP is around 5.46. If we evaluate the first law efficiency related to the fossil based heat energy operating the engine delivering the power for the heat pump, we obtain:

\[
FLE_{HP,mean} = COP_{mean} \times \eta_E = 5.46 \times (0.35 \text{ to } 0.45) = (1.911 \text{ to } 2.457).
\]

Here \(\eta_E\) is the first law efficiency of the engine delivering the power to the heat pump.

**Nomenclature**
- \(T\) – temperature [K]
- \(h\) – convection heat coefficient [W/(m²K)]
- \(Q\) – heat rate [W]
- \(W\) – power consumed [W]
- \(A\) – surface [m²]
- \(a\) – heat exchanger efficiency [-]
- \(m\) – mass flow [kg/h]
- \(C\) – heat capacity of heat carrier [W/K]
- \(s\) – entropy [J/K]
- \(Irr\) – irreversibility
- \(COP\) – coefficient of performance

**Subscripts:**
- \(a\) – ambient
- \(E\) – evaporator
- \(K\) – condenser
- \(o\) – output
- \(i\) – input
- \(N\) – nominal
- \(hc\) – heat carrier
- \(S\) – soil
- \(HS\) – heated space

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