Current induced Spin Torque in a nanomagnet

X. Waintal
Service de Physique de l’État Condensé, CEA Saclay, 91191 Gif-sur-Yvette cedex, France

O. Parcollet
Service de Physique Théorique, CEA Saclay, 91191 Gif-sur-Yvette cedex, France
(Dated: February 8, 2020)

In a nanomagnet very small polarized currents can lead to magnetic reversal. Treating on the same footing the transport and magnetic properties of a nanomagnet connected to magnetic leads via tunneling barriers, we derive a closed equation for the time evolution of the magnetization. The interplay between Coulomb blockade phenomena and magnetism gives some additional structure to the current induced spin torque. In addition to the possibility of stabilizing uniform spin precession states, we find that the system is highly hysteretic: up to three different magnetic states can be simultaneously stable in one region of the parameter space (magnetic field and bias voltage).

PACS numbers:}

Nanomagnets (typically 3 – 4 nm thick grains of magnetic materials with a total spin \(S_0 \sim 1000\)) are model systems for the study of the interplay between transport and magnetism. Their magnetization can be considered as a large macrospin without spatial variations \([1]\) (the magnetic exchange length is of the order of 7 nm in cobalt so that degenerate magnons with non-zero momentum can be ignored). Moreover, their mean level spacing is of the order of 1 meV, bigger than temperatures accessible experimentally so that the discreteness of the grain spectrum can be resolved \([2]\). A first set of experiments used single electron tunneling spectroscopy (connecting the grain to electrodes with tunnel junctions) to probe the grain’s transport properties \([2, 3]\). The differential conductance-voltage characteristic of such a device displays peaks corresponding to excitations of the grain from which the grain’s spectrum can be measured \([2, 3]\). In a second set of experiments the magnetization of the grain is measured directly using a microSQUID technique \([1, 4]\). Reversal of magnetic moments as small as 1000 \(\mu_B\) have been observed providing information on the static (switching field as a function of the external field direction that allowed the reconstruction of the anisotropy tensor) and dynamical (switching times) magnetic properties of the system.

Magnetic reversal by injection of a spin-polarized current was predicted in 1996 by Slonczewski \([5]\) and observed later in magnetic multilayers \([6]\). A very large interest for these systems has emerged since, due in particular to their large potential for practical applications (including current driven Magnetic Random Access Memories and Radio Frequency components) motivating works on magnetic tunnel junctions \([4, 11, 11]\). The spin dynamics in those systems is usually modeled with the Landau-Lifshitz-Gilbert (LLG) equation to which a spin-torque term is added \([7]\).

In this letter, we study the spin dynamics of a nanomagnet connected to magnetic leads through tunneling contacts (see Fig. 1, in the sequential tunneling regime. We show that the Coulomb blockade \([12]\) strongly affect the spin dynamics in this system. As a consequence, the standard LLG equation has to be replaced by Eq. \([8]\) which is the main result of this letter. In the following, we first outline the derivation of this equation, then compare it to the standard LLG equation and study its phase diagram as a function of the bias voltage \(V_{bias}\) and magnetic field \(H\). We find that highly hysteretic current-voltage characteristics and metastable magnetic precessional states could be observed in those systems.

![FIG. 1: Nanomagnet connected through tunneling barriers to two (magnetic or not) leads with chemical potential \(\mu_i\) and magnetization directions \(\vec{m}_i\) (in arbitrary directions). The grain magnetization direction is \(\vec{m} (||\vec{m}|| = 1)\) and its easy axis \(z\). The magnetic field \(H\) is along \(z\).](image-url)
alyzed in 14, 15 and reads:

\[ H_g = \sum_{\alpha\sigma} \epsilon_\alpha d_{\alpha\sigma}^\dagger d_{\alpha\sigma} + E(N) - J \hat{S}_z \hat{S}_z - \kappa \frac{e}{S_0} \hat{S}_z^2 - \hbar \gamma H \hat{S}_z \]

where \( d_{\alpha\sigma}^\dagger \) creates an electron on a one-body level \( \alpha \) with energy \( \epsilon_\alpha \), \( N \) is the number of electrons in the grain, \( E(N) = E_C(N - N_g)^2 \), with \( E_C \) the charging energy and \( N_g \) an offset (controlled by a gate voltage), \( \gamma \) is the gyromagnetic ratio, \( H \) the magnetic field. The exchange energy (\( \propto J \)) and uniaxial anisotropy (\( \propto \kappa \)) depend on the total spin \( \hat{S} = \frac{1}{2} \sum_{\alpha,\sigma_1,\sigma_2} d_{\alpha\sigma_1}^\dagger \sigma_{\sigma_1\sigma_2} d_{\alpha\sigma_2} \) (\( \sigma \) are Pauli matrices). Upon increasing \( J \) the grain undergoes a Stoner instability and acquires a macroscopic spin \( S_0 \) (equal to half of the number of singly occupied one-body states). Its low energy excitations \( |A\rangle \) are entirely characterized by the set of occupation numbers \( n_\alpha = 0, 1, 2 \) of the single energy levels together with the spin \( S_z \) along the easy axis 14, 15. Intrinsic relaxation processes (like spin-orbit or phonon-magnon scattering) are described phenomenologically via a weak coupling \( \Gamma_B \) of the form \( S^+ \phi \)

to a bosonic bath \( \phi \) of spectral density \( \rho_B(E) \).

The grain is then coupled via tunneling barriers to magnetic leads and we note \( \Gamma_a^\beta \), the tunneling rate of an electron with spin \( \sigma \) (\( \uparrow \) or \( \downarrow \) along the \( z \)-axis) coming from lead \( a \) (Left or Right). For simplicity, we restrict ourselves to tunneling events occurring on only one one-body state \( \alpha_0 \), i.e. to one step in the current-voltage Coulomb staircase \( (k_B T \ll \Delta, \text{where} \Delta \text{is the mean level spacing}) \). For larger systems where more levels come into play, spin accumulation effects have to be taken into account 16. This state can be a majority level \( (i.e. \text{unoccupied}, n_{\alpha_0} = 0) \) or a minority level \( (n_{\alpha_0} = 1) \). The different tunneling events (or interactions with the bosonic bath) induce a random walk in the Hilbert space of the isolated grain, its inner state \( |A\rangle \) changing at each event. This dynamics is described by the Master equation for the probability \( P_A(t) \) for the system to be in state \( |A\rangle \) at time \( t \). The first part is a direct generalization 14 of the standard theory of sequential tunneling 12, while the second part is due to the coupling to the bosonic bath:

\[
\partial_t P_A = \sum_{A',a,\sigma} \left\{ \Gamma^\sigma_a \langle A'|d_{\alpha_0\sigma}|A\rangle \right\}^2 \left[ n(\Delta E - \mu_a) P_{A'} - \left( 1 - n(\Delta E - \mu_a) \right) P_A \right] + \Gamma^\sigma_a \langle A'|d_{\alpha_0\sigma}|A\rangle \left[ -n(-\Delta E - \mu_a) P_A \right. \\
+ \left. \left( 1 - n(-\Delta E - \mu_a) \right) P_{A'} \right] \right\} + \Gamma_B \sum_{A',\epsilon = \pm} \epsilon \langle A'|S_x - i \epsilon S_y|A\rangle \left[ \rho_B(\epsilon \Delta E) n_B(\Delta E) P_{A'} + \rho_B(\epsilon \Delta E) n_B(-\Delta E) P_A \right]
\]

(2)

where \( \Delta E = E_A - E_{A'} \) is the energy difference between state \( |A\rangle \) and \( |A'\rangle \), \( n(E) = 1/(1 + e^{E/kT}) \) \((n_B(E) = 1/(e^{E/kT} - 1)) \) is the Fermi (Bose) function, \( T \) the temperature and \( \mu_a \) the chemical potential of lead \( a \). For Eq. (2) to be valid, the different tunneling events (and interactions with the bosonic bath) have to be incoherent, which requires \( \Gamma, \Gamma_B \ll kT/\hbar \). In addition, Eq. (2) does not include the dynamics of the magnetization in the \( xy \) plane (off-diagonal terms of the density matrix).

This is valid when internal precession is fast enough to average out contributions transverse to the easy axis. The time needed for \( S_z \) to change significantly is of order \( S_0/\Gamma \) (one tunneling event can only change \( S_z \) by half a unit) so that this condition implies \( \Gamma \ll \kappa S_0/\hbar \). In this paper, we restrict ourselves to that case. The effective tunneling rates \( \Gamma^\sigma_a \) along the easy axis \( z \) can be related to the tunneling rates for electrons polarized along the magnetization of the leads \( \Gamma^\sigma_a = \Gamma_a^\beta (U, D \text{Up,Down}) \) through \( \Gamma^U_a = \Gamma^\sigma_a \cos^2(\theta_a/2) + \Gamma^\sigma_a \sin^2(\theta_a/2) \) and \( \Gamma^D_a = \Gamma^\sigma_a \cos^2(\theta_a/2) + \Gamma^\sigma_a \sin^2(\theta_a/2) \) where \( \theta_a \) is the angle between the \( z \)-axis and the magnetization of lead \( a \). The \( \Gamma^\beta_a \) are given by the Fermi golden rule. A complete derivation of Eq. (2) from the microscopic model will be provided elsewhere 17. Since \( S_0 \) is macroscopic, we proceed with expanding Eq. (2) with respect to \( 1/S_0 \), introducing the continuous variable \( m_z = S_z/S_0 \). To zeroth order in \( 1/S_0 \) the magnetization does not change and the charge equilibrium (between \( N_0 \) and \( N_0 + 1 \) electrons in the grain) takes place. Denoting by \( P_+(m_z) \) (resp. \( P_-(m_z) \)) the probability for the grain to contain \( N_0 + 1 \) (resp. \( N_0 \)) electrons, it reads:

\[
\partial_t P_+ = \sum_{a,\sigma} \frac{\Gamma^\sigma_a}{2} \left\{ \left( 1 + \eta_{\sigma} m_z \right) \left[ n(\Delta E^\sigma - \mu_a) P_- - \left( 1 - n(\Delta E^\sigma - \mu_a) \right) P_+ \right] \right\}
\]

where \( \Delta E^\sigma(m_z) = \epsilon - \kappa \eta_{\sigma} m_z - \hbar \gamma H \sigma/2 \) is the energy to add an electron in the nanomagnet with spin \( \sigma \). The offset energy \( \epsilon \) includes the charging energy, a possible capacitive coupling to an additional gate and the energy of the level \( \alpha_0 \). \( \eta = 1 \) (resp. -1) when \( \alpha_0 \) is a majority (resp. minority) level. On short time scales \( \propto \hbar/\Gamma \), the charge equilibrate and \( P_+/P_- \) reaches its stationary value. The latter is inserted into the first order part of the \( 1/S_0 \) expansion to obtain an equation.
for $P(m_z) = P_+(m_z) + P_-(m_z)$ of the form \( \partial_t P(m_z) = \partial_{m_z} [-R(m_z) P(m_z) + \frac{1}{2S_0} \partial_{m_z} \{ D(m_z) P(m_z) \}] \). This is a Fokker-Planck equation in the Itô form. The associated Langevin equation for \( m_z \) reads \( \partial_t m_z = R(m_z) + \sqrt{2D(m_z)/S_0} \xi(t) \), where \( \xi(t) \) is a white noise. The noise part is small at large \( S_0 \) (the complete expression of \( D(m_z) \) will be given in \( \text{[12]} \)) so that in this letter, we focus on the deterministic part \( R(m_z) \). Defining the asymmetry parameter $q = (\sum_a \Gamma_a^+ - \Gamma_a^-) / (\sum_a \Gamma_a^+ + \Gamma_a^-)$, the total tunneling rate $\Gamma = \sum_a \Gamma_a^+ + \Gamma_a^-$ and the functions \( f_a(m_z) = \sum_{\alpha} \Gamma_a^0 n(\Delta E^{\alpha} - \mu_{\alpha}) \), we obtain the equation governing the spin dynamics of the nanomagnet:

\[
\frac{dm_z}{dt} = (1 - m_z^2) \left[ \alpha_B \left( \gamma H + \frac{2\kappa}{\hbar} m_z \right) + \frac{(1 - q)f_1(m_z) - (1 + q)f_2(m_z)}{4S_0(1 + q\mu_{m_z})} \right] \tag{3}
\]

where $\alpha_B \propto \Gamma_B$ is the Gilbert damping associated to the bosonic bath. Equation \( \text{[3]} \) is the main result of this letter.

Let us now compare Eq. \( \text{[3]} \) with the LLG equation that has been widely used in multilayer systems,

\[
\frac{\partial \vec{m}}{\partial t} = \vec{m} \times \left[ \frac{1}{\hbar S_0} \frac{\partial E}{\partial \vec{m}} + \alpha \frac{\partial \vec{m}}{\partial t} + \frac{IP}{\epsilon S_0} \vec{z} \times \vec{m} \right] \tag{4}
\]

It consists of (i) a conservative term (the magnetization precesses around trajectories of constant energy $E(\vec{m}) = -\kappa S_0 m_z^2 - \hbar \gamma S_0 H m_z$); (ii) a phenomenological Gilbert damping term $\alpha$ that allows the system to relax to its equilibrium; (iii) a current induced spin torque term \( \text{[4]} \), written here assuming a magnetic left lead (with $\vec{m}_L \parallel \vec{z}$) and a non-magnetic right lead. The torque is asymmetric and modulated by $0 \leq \bar{q} < 1$ ($\bar{q} = 0$ however in a tunneling junction \( \text{[4]} \)). Using the cylindrical symmetry of Eq. \( \text{[4]} \), we obtain:

\[
\frac{dm_z}{dt} = \frac{(1 - m_z^2)}{1 + \alpha^2} \left[ \alpha \left( \gamma H + \frac{2\kappa}{\hbar} m_z \right) + \frac{PI/(eS_0)}{1 + \bar{q}m_z} \right] \tag{5}
\]

Even though Eq. \( \text{[5]} \) looks similar to Eq. \( \text{[4]} \), its third term is more complex leading to richer physics: \( i) \) conducting channels for up/down electrons can open or close when $m_z$ varies: there is a magnetic blockade induced by the anisotropy, and the torque term varies abruptly with the bias voltage as the different channels open; \( ii) \) this term is non-zero even at zero current; it is more than the current-induced spin-torque effect, and contains in particular relaxations processes due to the leads \( \text{[21]} \). We emphasize that while the usual treatment of spin torque physics is done in two separate steps (magnetization dynamics on one hand and transport properties on the other hand), here we treat on the same footing the transport and magnetic degrees of freedom. Some steps in this direction were taken previously by one of us \( \text{[14]} \) for grains close to equilibrium.

Let us now investigate the possibility of voltage-driven spin reversal. In the framework of the LLG equation, a current $I$ with a polarization $P$ flowing through a macrospin $S_0$ reverses the magnetization with a rate $PI/(eS_0)$ (2$S_0$ polarized spins +1/2 are needed to fully reverse a large spin $S_0$ from $S_z = -S_0$ to $S_z = +S_0$ provided the system “absorbs” all the injected polarized spins). This torque term $PI/eS_0$ competes with the relaxation rate $\alpha \gamma / \hbar$ (as opposed to $\kappa$ alone in magnetic-field-driven reversals) so that the reversal occurs for $\Theta_{\text{LLG}} = PIh/(eS_0 \alpha \gamma) \sim 1$. For example, for the nanopillars studied in \( \text{[8]} \), $I = \alpha j (A$ cross section of the pillar and $j$ current density) while $S_0 = M_s Ad/\hbar \gamma$ ($M_s$ saturation magnetization and $d$ thickness of the magnetic layer) leads to a critical value of the current density $j_c = eM_s \alpha \gamma \gamma / (\hbar^2 \gamma)$ which gives the observed value $j_c \sim 10^7 A.cm^{-2}$ for typical parameters. In Eq. \( \text{[8]} \), the general condition for voltage-driven spin reversal (as a function $V_{\text{bias}}$ and $H$) is $\Theta > 2(1 - q)$ where $\Theta = A \left[ (1 - q) \Gamma_L^\uparrow - (1 + q) \Gamma_L^\downarrow \right] / (8\kappa \alpha_B S_0)$ (corresponding to the value of the torque when all channels are open \( \text{[17]} \)).

The complete phase diagram of Eq. \( \text{[8]} \) as a function of $V_{\text{bias}}$ and $H$ is found by studying its stable fixed points (i.e. the values of $m_z$ satisfying $R(m_z) = 0$ with $\partial_{m_z} R(m_z) < 0$). We restrict the following discussion to low temperature, $\gamma = 1$ and to the case $FR = 0$ (Coulomb blockade situation, $\epsilon$ is the largest energy). Two identical magnetic leads with anti-parallel magnetizations is the most favorable case for voltage-driven switching: not only up electrons have a higher probability to enter the grain than the down electrons, but in addition, once in, they have a lower probability to get out so that the imbalance between up and down current is enhanced. In this case however, the asymmetry parameter $q \neq 0$ vanishes. The situation gets more interesting when $q \neq 0$ which is most easily achieved by taking one lead to be magnetic while the other is not. The torque can be strong enough to destabilize the $m_z = -1$ configuration but since its magnitude decreases with increasing $m_z$, a stable fixed point $-1 < m_z^* < +1$ can appear. These fixed points are called (uniform) spin precession states (SP) in the following. A typical phase diagram is given in Fig. \( \text{[2]} \) (where the different regions are labeled by the list of the stable fixed points). In addition to the regions that can exist in the LLG equation, (SP), $(SP, +1)$ and $-1, (SP, +1)$, a region $(-1, SP, +1)$ where three states are stable can also exist. In this region, the up spin channel is blocked at $m_z = -1$ so that the torque term does not destabilize the $(-1)$ phase. For $m_z$ slightly higher, this channels opens up, and stabilizes a spin precession fixed point. The condition for observing a $(SP, +1)$ and $(-1, SP, +1)$ region is $\Theta > (1 - q)^2/q$: the condition for observing a $(SP)$ and $(-1, SP)$ region is $\Theta > (1 - q^2)/q$ \( \text{[17]} \). This phase diagram is highly hysteretic as can be inferred from the
are of a few (mainly due to surface anisotropy). Typical currents \(0.1 \text{nA} \) are estimated total damping constant of the order of 0.1 due to spin relaxation through the conducting electrons \([20, 23]\). Also, 20 nm particles probably have modes of relaxations that disappears for smaller sizes. Hence, we estimate \(\alpha_B \approx 0.001 - 0.005 \) (0.005 being the bulk value for Cobalt). Putting everything together, we get \(\Theta \sim 1\). This is of the order of the critical value for voltage-driven spin reversal but is probably too small to excite spin precession states. Better candidates are magnetic alloys that would allow to decrease the anisotropy (permalloy) and/or the total magnetization (weak ferromagnets like NiPd should be good candidates \([24]\)) and increase the parameter \(\Theta\) by one or two orders of magnitude. Magnetic semiconductors (like GaMnAs) could also be material of choice to study the effects described in this letter.

We thank K. Mallick, F. Portier and P. Roche for useful comments.

1. M. Jamet et al., Phys. Rev. Lett. 86 4676 (2001).
2. S. Guéron et al., Phys. Rev. Lett. 83 4148 (1999).
3. M. M. Deshmukh et al., Phys. Rev. Lett 87 226801 (2001).
4. S. Kleff et al., Phys. Rev. B 64 220401(R) (2001).
5. S. Kleff and J. von Delft, Phys. Rev. B 65 214421(R) (2002).
6. C. Thirion, W. Wernsdorfer and D. Mailly, Nature Mat. 2 524 (2003).
7. J.C. Slonczewski, J. Magn. Magn. Mater. 159 L1 (1996).
8. J. A. Katine et al., Phys. Rev. Lett 84 3149 (2000).
9. J.C. Slonczewski, Phys. Rev. B 71 024411 (2005).
10. G.D. Fuchs et al., Appl. Phys. Lett. 85 1205 (2004).
[11] Y.M. Huai et al., Appl. Phys. Lett. 84 3118 (2004).
[12] C. W. J. Beenakker, Phys. Rev. B 44 1646 (1991).
[13] M. Braun, J. König and J. Martinek, Phys. Rev. B 70 195345 (2004).
[14] X. Waintal and P.W. Brouwer, Phys. Rev. Lett 91 247201 (2003).
[15] C. M. Canali and A. H. MacDonald, Phys. Rev. Lett. 85 5623 (2000).
[16] J. Inoue and A. Brataas, Phys. Rev. B 70 140406 (2004).
[17] O. Parcollet and X. Waintal, to be published.
[18] "Handbook of stochastic methods" C.W. Gardiner, Springer (1983).
[19] X. Waintal, E.B. Myers, P.W. Brouwer and D.C. Ralph, Phys. Rev. B 62 12317 (2000).
[20] Y. Tserkovnyak, A. Brataas and G. E. W. Bauer, Phys. Rev. Lett 88 117601 (2002).
[21] M. Jamet et al., Phys. Rev. B 69 024401 (2004).
[22] M. Respaud et al., Phys. Rev. B 57 2925 (1998).
[23] C. H. Back et al., Science 285 864 (1999).
[24] T. Kontos et al., Phys. Rev. Lett 89 137007 (2002).