Complete account of randomness in the EPR-Bohm-Bell experiment

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Abstract

We show that paradoxical consequences of violations of Bell’s inequality are induced by the use of an unsuitable probabilistic description for the EPR-Bohm-Bell experiment. The conventional description (due to Bell) is based on a combination of statistical data collected for different settings of polarization beam splitters (PBSs). In fact, such data consists of some conditional probabilities which only partially define a probability space. Ignoring this conditioning leads to apparent contradictions in the classical probabilistic model (due to Kolmogorov). We show how to make a completely consistent probabilistic model by taking into account the probabilities of selecting the settings of the PBSs. Our model matches both the experimental data and is consistent with classical probability theory.
Keywords: EPR-Bohm-Bell experiment, violation of Bell’s inequality, complete account of randomness, incorporation of conditional probabilities in Kolmogorov model.

1 Introduction

We remind that Bell’s type inequalities [1]–[4] are purely probabilistic statements which a priori have no direct relation with QM. They could be easily derived if one starts with a Kolmogorov probability space

\[ P = (\Omega, \mathcal{F}, \mathbf{P}) \]

and a few random variables, say \( A^{(i)}(\omega), B^{(j)}(\omega) \). Here \( \Omega \) is a set of chance parameters \( \omega \), \( \mathcal{F} \) is a collection of subsets of \( \Omega \) (so called \( \sigma \)-algebra), and \( \mathbf{P} \) is a probability measure.

The problem arises when one puts (by Bell’s recommendation) statistical data collected in a few experiments into those of these inequalities which could be experimentally verified. They are violated! (see e.g. [5]–[7]) Physicists typically point out to such mystical things as non-locality or (and) ”death of realism” to explain why experimental data does not match Bell’s type inequalities, [1]–[4].

However, we could not ignore a possible purely probabilistic source of violation of Bell’s type inequalities. We recall that the use of a single probability space for statistical data collected with respect to a few different experimental contexts is not a custom of probability theory. It is clear that if one goes against the rules of the well established mathematical formalism (as well as experimental statistical methodology), then paradoxical conclusions might be expected.

We recall that Andrei Nikolaevich Kolmogorov (the founder of modern probability theory) emphasized that each experiment is described by its own probability space, see [8], see also Gnedenko [9]. Thus the use of a single probability space in the derivation of Bell’s type inequalities, see [1]–[4] for details, was not justified. As was pointed out in [10]–[20] operating with a few probability measures (corresponding to different experiments) blocks the derivation. As was recently pointed out in [16], in probability theory

\[ \text{1 There is the evident matching between views of Kolmogorov and Bohr. The latter pointed out that experimental arrangement should be taken into account.} \]
such a problem – a possibility to realize a system of random variables on a
given probability space – were studied and finally solved by Soviet probabilist
Vorobjev in the 1960s [21].

If one wants to use classical probability theory to describe the EPR-Bohm-
Bell experiment, it should be done properly. To be sure that paradoxical con-
clusions of violation of Bell’s inequality are not artifacts of the misuse of the
mathematical formalism, we should demand Weirstrassian rigorousness of
any probabilistic description. The aim of this paper is to provide an alterna-
tive probabilistic model for the EPR-Bohm-Bell experiment. If one wants to
apply the classical probabilistic model, a single Kolmogorov probability space,
then random experiments for different settings of PBS should be unified in
a single random experiment in an intelligent way. We shall describe such an
experiment and we shall show that quantum (experimental) statistical data
is harmonically combined with the classical probabilistic description. In this
paper we shall consider the Clauser-Horne-Shimony-Holt (CHSH) inequality,
although results obtained here apply to any Bell-type inequality.

2 CHSH inequality

We recall the rigorous mathematical formulation of the CHSH inequality:

Theorem. Let $A^{(i)}(\omega)$ and $B^{(i)}(\omega)$, $i = 1, 2$, be random variables tak-
ing values in $[-1, 1]$ and defined on a single probability space $\mathcal{P}$. Then the
following inequality holds:

$$| < A^{(1)}, B^{(1)} > + < A^{(1)}, B^{(2)} > + < A^{(2)}, B^{(1)} > - < A^{(2)}, B^{(2)} > | \leq 2.$$  (1)

The classical correlation is defined as it is in classical probability theory:

$$< A^{(i)}, B^{(j)} > = \int_{\Omega} A^{(i)}(\omega)B^{(j)}(\omega)d\mathbf{P}(\omega).$$

J. Bell proposed the following methodology. To verify an inequality of this
type, one should put statistical data collected for four pairs of PBSs settings:

$$\theta_{11} = (\theta_1, \theta'_1), \theta_{12} = (\theta_1, \theta'_2), \theta_{21} = (\theta_2, \theta'_1), \theta_{22} = (\theta_2, \theta'_2),$$

2 The EPR-Bohm experiment was about precise correlations. Bell completed it by
combining statistical data collected for different experimental settings. Our point is that
this combining is responsible for paradoxical results. Therefore we speak about “EPR-
Bohm-Bell experiment”.

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into it. Here $\theta = \theta_1, \theta_2$ and $\theta' = \theta'_1, \theta'_2$ are selections of angles for orientations of respective PBSs.

Following Bell, the selection of the angle $\theta_i$ determines the random variable

$$A^{(i)}(\omega) \equiv a_{\theta_i}(\omega).$$

There are two detectors coupled to the PBS with the $\theta$-orientation: "up-spin" (or "up-polarization") detector and "down-spin" (or "down-polarization") detector. A click of the up-detector assigns to the random variable $a_{\theta}(\omega)$ the value +1 and a click of the down-detector assigns to it the value -1. However, since a lot of photons disappear without any click, it is also permitted for random variables to take the value zero in the case of no detection. Therefore in Bell’s framework it is sufficient to consider $a_{\theta}(\omega)$ taking values $-1, 0, +1$.

In the same way selection of the angle $\theta'$ determines

$$B^{(i)}(\omega) \equiv b_{\theta'_i}(\omega),$$

where $b_{\theta'_i}(\omega)$ takes values $-1, 0, +1$.

It seems that Bell’s random model is not proper for the EPR-Bohm-Bell experiment. Bell’s description does not take into account probabilities of choosing pairs of angles (orientations of PBSs) $\theta_{11}, \ldots, \theta_{22}$. Thus his model provides only incomplete probabilistic description. This allows to include probabilities of choosing experimental settings $P(\theta_{ij})$ into the model; this way completing it.

In the next section we shall provide such a complete probabilistic description of the EPR-Bohm-Bell experiment. We point out that random variables of our model (which will be put into the CHSH inequality) does not coincide with Bellian variables.

### 3 Proper random experiment

a). There is a source of entangled photons.

b). There are four PBSs and corresponding pairs of detectors. PBSs are labelled as $i = 1, 2$ and $j = 1, 2$.

c). Directly after source there is a distribution device which opens at each instance of time, $t = 0, \tau, 2\tau, \ldots$ ways to only two (of four) optical

\[\text{ways to only two (of four) optical}\]

\[\text{It is just the form of labelling which is convenient to form pairs (i, j).}\]
fibers going to the corresponding two PBSs. For simplicity, we suppose that each pair \((i, j) : (1, 1), (1, 2), (2, 1), (2, 2)\) can be opened with equal probability:

\[ P(i, j) = 1/4. \]

We now define "proper random variables". To simplify considerations, we consider the ideal experiment with 100% detectors efficiency. Thus in Bell’s framework random variables \(a_\theta(\omega)\) and \(b_\theta'(\omega)\) should take only values \(\pm 1\). The zero-value will play a totally different role in our model.

1) \(A^{(i)}(\omega) = \pm 1, i = 1, 2\) if the corresponding (up or down) detector is coupled to \(i\)th PBS fires;
2) \(A^{(i)}(\omega) = 0\) if the \(i\)-th channel is blocked. In the same way we define random variables \(B^{(j)}(\omega)\) corresponding to PBSs \(j = 1, 2\).

Of course, the correlations of these random variables satisfy CHSH inequality.

Thus if such an experiment were performed and if CHSH inequality were violated, we should seriously think about e.g. quantum non-locality or death of realism.

It would be really interesting to perform such a "proper random experiment" for photon polarizations.

However, to see that CHSH inequality for \(<A^{(i)}, B^{(j)}>\) correlations does not contradict to experimental data, we could use statistical data which has been collected for experiments with fixed pairs \(\theta_{ij} = (\theta_i, \theta'_j)\) of orientations of PBS. We only need to express correlations of Bell’s variables \(<a_\theta, b_{\theta'}\) via correlations \(<A^{(i)}, B^{(j)}>\).

### 4 Frequency analysis

Suppose that our version of EPR-Bohm-Bell experiment was repeated \(M = 4N\) times and each pair \((i,j)\) of optical fibers was opened only \(N\) times.

The random variables took values

\[ A^{(i)} = A^{(i)}_1, \ldots, A^{(i)}_M, i = 1, 2, B^{(j)} = B^{(j)}_1, \ldots, B^{(j)}_M, j = 1, 2. \]
Then by the law of large numbers:

\[ < A^{(i)}, B^{(j)} > = \lim_{M \to \infty} \frac{1}{M} \sum_{k=1}^{M} A^{(i)}_k B^{(j)}_k. \]

We remark that, for each pair of gates \((i, j)\), only \(N\) pairs \((A^{(i)}_k, B^{(j)}_k)\) have both components non zero. Thus

\[ < A^{(i)}, B^{(j)} > = \lim_{N \to \infty} \frac{1}{4N} \sum_{l=1}^{N} A^{(i)}_{k_l} B^{(j)}_{k_l}, \]

where summation is with respect to only pairs of values with both nonzero components.

Thus the quantities \(< A^{(i)}, B^{(j)} >\) are not estimates for the \(< a_{\theta_i}, b_{\theta_j} >\) obtained in physical experiments. The right estimates are given by

\[ \frac{1}{N} \sum_{l=1}^{N} A^{(i)}_{k_l} B^{(j)}_{k_l}. \]

Hence the CHSH inequality for random variables \(A^{(i)}, B^{(j)}\) induces the following inequality for "traditional Bellian random variables":

\[ | < a_{\theta_1}, b_{\theta'_1} > + < a_{\theta_1}, b_{\theta'_2} > + < a_{\theta_2}, b_{\theta'_1} > - < a_{\theta_2}, b_{\theta'_2} > | \leq 8. \quad (2) \]

It is not violated for known experimental data for entangled photons. Moreover, this inequality provides a trivial constraint on correlations: each correlation of Bellian variables is majorated by 1, hence, their linear combination with \(\pm\)-signs is always bounded above by 4.

5 "Proper probability space"

We now construct a proper Kolmogorov probability space for the EPR-Bohm-Bell experiment. This is a general construction for combining of probabilities produced by a few incompatible experiments. We have probabilities

\[ \text{We assume that different trials are independent. Thus the law of large numbers is applicable} \]
$p_{ij}(\epsilon, \epsilon'), \epsilon, \epsilon' = \pm 1$, to get $a_{\theta_i} = \epsilon, b_{\theta_j'} = \epsilon'$ in the experiment with the fixed pair of orientations $(\theta_i, \theta_j')$. From QM we know that

$$p_{ij}(\epsilon, \epsilon) = \frac{1}{2} \cos^2 \frac{\theta_i - \theta_j'}{2}, p_{ij}(\epsilon, -\epsilon) = \frac{1}{2} \sin^2 \frac{\theta_i - \theta_j'}{2}. \quad (3)$$

However, this special form of probabilities is not important for us. Our construction of unifying Kolmogorov probability space works well for any collection of probabilities $p_{ij} : \sum_{\epsilon, \epsilon'} p_{ij}(\epsilon, \epsilon') = 1$. We remark that $p_{ij}(\epsilon, \epsilon')$ determine automatically marginal probabilities:

$$p_i(\epsilon) = \sum_{\epsilon'} p_{ij}(\epsilon, \epsilon'),$$

$$p_j(\epsilon') = \sum_{\epsilon} p_{ij}(\epsilon, \epsilon').$$

In the EPR-Bohm-Bell experiment they are equal to $1/2$. Let us now consider the set $\{-1, 0, +1\}^4$: the set of all vectors

$$\omega = (\omega_1, \omega_2, \omega_3, \omega_4), \omega_l = \pm 1, 0.$$ 

It contains $3^4$ points. Now we consider the following subset $\Omega$ of this set:

$$\omega = (\epsilon_1, 0, \epsilon_1', 0), (\epsilon_1, 0, 0, \epsilon_2'), (0, \epsilon_2, \epsilon_1', 0), (0, \epsilon_2, 0, \epsilon_2').$$

It contains 16 points. We define the following probability measure on $\Omega$:

$$P(\epsilon_1, 0, \epsilon_1', 0) = \frac{1}{4} p_{11}(\epsilon_1, \epsilon_1'), P(\epsilon_1, 0, 0, \epsilon_2') = \frac{1}{4} p_{12}(\epsilon_1, \epsilon_2')$$

$$P(0, \epsilon_2, \epsilon_1', 0) = \frac{1}{4} p_{21}(\epsilon_2, \epsilon_1'), P(0, \epsilon_2, 0, \epsilon_2') = \frac{1}{4} p_{22}(\epsilon_2, \epsilon_2').$$

We remark that we really have

$$\sum_{\epsilon, \epsilon_1'} P(\epsilon_1, 0, \epsilon_1', 0) + \sum_{\epsilon_1, \epsilon_2'} P(\epsilon_1, 0, 0, \epsilon_2') + \sum_{\epsilon_2, \epsilon_1'} P(0, \epsilon_2, \epsilon_1', 0) + \sum_{\epsilon_2, \epsilon_2'} P(0, \epsilon_2, 0, \epsilon_2') =$$

$$\frac{1}{4} \left[ \sum_{\epsilon, \epsilon_1'} p_{11}(\epsilon_1, \epsilon_1') + \sum_{\epsilon_1, \epsilon_2'} p_{12}(\epsilon_1, \epsilon_2') + \sum_{\epsilon_2, \epsilon_1'} p_{21}(\epsilon_2, \epsilon_1') + \sum_{\epsilon_2, \epsilon_2'} p_{22}(\epsilon_2, \epsilon_2') \right] = 1.$$
We now define random variables \( A^{(i)}(\omega), B^{(j)}(\omega) \):

\[
A^{(1)}(\epsilon_1, 0, \epsilon_1', 0) = A^{(1)}(\epsilon_1, 0, 0, \epsilon_2') = \epsilon_1, A^{(2)}(0, \epsilon_2, \epsilon_1', 0) = A^{(2)}(0, \epsilon_2, 0, \epsilon_2') = \epsilon_2;
\]
\[
B^{(1)}(\epsilon_1, 0, \epsilon_1', 0) = B^{(1)}(0, \epsilon_2, \epsilon_1', 0) = \epsilon_1', B^{(2)}(\epsilon_1, 0, 0, \epsilon_2') = B^{(2)}(0, \epsilon_2, 0, \epsilon_2') = \epsilon_2'.
\]

We find two dimensional probabilities

\[
P(\omega \in \Omega : A^{(1)}(\omega) = \epsilon_1, B^{(1)}(\omega) = \epsilon_1') = P(\epsilon_1, 0, \epsilon_1', 0) = \frac{1}{4}p_{11}(\epsilon_1, \epsilon_1'), \ldots,
\]
\[
P(\omega \in \Omega : A^{(2)}(\omega) = \epsilon_2, B^{(2)}(\omega) = \epsilon_2') = \frac{1}{4}p_{22}(\epsilon_2, \epsilon_2').
\]

We also consider the random variable which is responsible for selection of pairs of gates:

\[
\eta(0, \epsilon_2, 0, \epsilon_2') = 22, \eta(0, \epsilon_2, \epsilon_1', 0) = 21, \eta(\epsilon_1, 0, 0, \epsilon_2') = 12, \eta(\epsilon_1, 0, \epsilon_1', 0) = 11.
\]

It is uniformly distributed (by our assumption on equal frequency to open each of pair of channels):

In probability theory we have the notion as conditional expectation of a random variable (under the condition that some event occurred).

Let \((\Omega, \mathcal{F}, P)\) be an arbitrary probability space and let \(\Omega_0 \subset \Omega, \Omega_0 \in \mathcal{F}, P(\Omega_0) \neq 0\). We also consider an arbitrary random variable \(\xi : \Omega \to R\). Then

\[
E(\xi|\Omega_0) = \int_{\Omega} \xi(\omega) dP_{\Omega_0}(\omega),
\]

where the conditional probability is defined by the Bayes’ formula:

\[
P_{\Omega_0}(U) \equiv P(U|\Omega_0) = P(U \cap \Omega_0) / P(\Omega_0).
\]

Let us come back to our unifying probability space. Take \(\Omega_0 \equiv \Omega_{ij} = \{\omega \in \Omega : \eta(\omega) = ij\}\). We have \(P(\Omega_{ij}) = 1/4\). Thus

\[
E(A^{(i)}B^{(j)}|\eta = ij) = \int_{\Omega} A^{(i)}(\omega) B^{(j)}(\omega) dP_{\Omega_{ij}}(\omega) = 4 \int_{\Omega_{ij}} A^{(i)}(\omega) B^{(j)}(\omega) dP(\omega)
\]
\[
= 4 \int_{\Omega} A^{(i)}(\omega) B^{(j)}(\omega) dP(\omega) = 4 < A^{(i)}, B^{(j)} > = < a_{\theta_i}, b_{\theta_j} >.
\]

Thus QM-correlations for fixed choice of settings of PBSs can be represented as conditional expectations:

\[
< a_{\theta_i}, b_{\theta_j} > = E(A^{(i)}B^{(j)}|\eta = ij).
\]
Remark. (Jaynes critique of derivation of Bell’s inequality) Jaynes [22] criticized derivation of Bell’s inequality which was based on Bell-Clauser-Horne-Shimony (CHSH) locality condition (factorization condition). Jaynes emphasized that Bell did a mistake in operation with conditional probabilities, because he used the objective interpretation of probability, instead of the subjective one. Opposite to Jaynes, we do not appeal to subjective probability. Moreover, our aim is not critique of some special types of derivations of Bell’s type inequality. We point out that Bell’s description of the random experiment for measurement of polarization (or spin) projections for a few incompatible pairs of setting was incomplete. By completing this description we obtain a classical probabilistic model which matches the experimental data.

6 Two-valued random variables

We showed in the last section how to give a complete probabilistic description of an EPR-Bohm-Bell experiment with random variables \( A^{(1)}, A^{(2)}, B^{(1)}, B^{(2)}, \) and \( \eta \). In that description the \( A^{(i)}, B^{(j)} \) took three values: \( \pm 1 \) and 0. In this section we show that it is also possible to do this when the \( A^{(i)}, B^{(j)} \) take only the values \( \pm 1 \).

By way of illustration, let us take the standard idealized EPR-Bohm-Bell experiment described in the beginning of the previous section with fixed orientations \( \theta_1 = \pi/4, \theta_2 = 0, \theta'_1 = \pi/8, \theta'_2 = 3\pi/8 \). The probabilities of the experimental outcome \( a_{\theta_i} = \epsilon, b_{\theta'_j} = \epsilon' \) are given by [3] and yield the expected values

\[
< a_{\theta_1}, b_{\theta'_1} >= < a_{\theta_1}, b_{\theta'_2} >= < a_{\theta_2}, b_{\theta'_1} >= \frac{1}{\sqrt{2}}, < a_{\theta_2}, b_{\theta'_2} >= -\frac{1}{\sqrt{2}} \tag{5}
\]

Therefore we have

\[
< a_{\theta_1}, b_{\theta'_1} > + < a_{\theta_1}, b_{\theta'_2} > + < a_{\theta_2}, b_{\theta'_1} > - < a_{\theta_2}, b_{\theta'_2} > = 2\sqrt{2}, \tag{6}
\]

obtaining the Tsirelson bound [23] on the maximum quantum ”violation” of the CHSH inequality.

We construct a Kolmogorov probability space \( P = (\Omega, F, P) \) with sixteen outcomes and five random variables: \( A^{(1)}, A^{(2)}, B^{(1)}, B^{(2)}, \eta \). The first four random variables take values \( \pm 1 \) and \( \eta \) takes values from 11, 12, 21, 22.
The first eight outcomes each occur with equal probability $x$:

|   | $A^{(1)}(\omega)$ | $A^{(2)}(\omega)$ | $B^{(1)}(\omega)$ | $B^{(2)}(\omega)$ | $\eta(\omega)$ |
|---|------------------|------------------|------------------|------------------|------------------|
| 1 | 1 | 1 | 1 | 11 |
| -1 | -1 | -1 | -1 | 11 |
| 1 | 1 | 1 | 1 | 12 |
| -1 | -1 | -1 | -1 | 12 |
| 1 | 1 | 1 | 1 | 21 |
| -1 | -1 | -1 | -1 | 21 |
| 1 | 1 | 1 | -1 | 22 |
| -1 | -1 | -1 | -1 | 22 |

The remaining eight outcomes each occur with equal probability $y$:

|   | $A^{(1)}(\omega)$ | $A^{(2)}(\omega)$ | $B^{(1)}(\omega)$ | $B^{(2)}(\omega)$ | $\eta(\omega)$ |
|---|------------------|------------------|------------------|------------------|------------------|
| -1 | -1 | 1 | 1 | 11 |
| 1 | 1 | -1 | -1 | 11 |
| -1 | -1 | 1 | 1 | 12 |
| 1 | 1 | -1 | -1 | 12 |
| -1 | -1 | 1 | 1 | 21 |
| 1 | 1 | -1 | -1 | 21 |
| -1 | -1 | 1 | -1 | 22 |
| 1 | 1 | -1 | -1 | 22 |

The probabilities $x$ and $y$ must be non-negative and $8x + 8y = 1$. One may verify that for $i = 1, 2$ and $\epsilon = \pm 1$:

$$P(\omega \in \Omega : A^{(i)}(\omega) = \epsilon) = \frac{1}{2}.$$  

Furthermore we can check that for $i, j = 1, 2$ and $\epsilon = \pm 1$:

$$P(\omega \in \Omega : A^{(i)}(\omega) = \epsilon | \eta(\omega) = i1 \text{ or } i2) = P(\omega \in \Omega : A^{(i)}(\omega) = \epsilon | \eta(\omega) = ij) = \frac{1}{2}$$

and so the non-signalling condition holds. A similar set of equations hold for the random variables $B^{(j)}$. We see that

$$< A^{(i)} > = < B^{(j)} > = 0,$$

$$< A^{(1)}, B^{(1)} >= < A^{(2)}, B^{(1)} >= 8x - 8y.$$
and
\[ <A^{(1)}, B^{(2)}>= <A^{(2)}, B^{(2)}>= 4x - 4y. \]
The left hand side of inequality (1) becomes \(|16x - 16y|\), and so (unsurprisingly) (1) holds since \(0 \leq x, y \leq 1/8\).

A further calculation shows that
\[ E(A^{(i)}B^{(j)}|\eta = ij) = 8x - 8y, \quad ij \neq 22 \quad (7) \]
and
\[ E(A^{(2)}B^{(2)}|\eta = 22) = 8y - 8x. \quad (8) \]

It suffices to set
\[ x = \frac{\sqrt{2} + 1}{16\sqrt{2}}, \quad y = \frac{\sqrt{2} - 1}{16\sqrt{2}} \]
in (7) and (8) to see that equation (1) is indeed satisfied for the expected values given in (5). Again we conclude that there is a probabilistic model consistent with the experimental outcomes given by (5).

Even more striking, perhaps, is the case when \(x = 1, y = 0\). From (7) and (8) we have that
\[
E(A^{(1)}B^{(1)}|\eta = 11) + E(A^{(1)}B^{(2)}|\eta = 12) +
E(A^{(2)}B^{(1)}|\eta = 21) - E(A^{(2)}B^{(2)}|\eta = 22) = 4
\]
and so the left hand side obtains its maximum mathematical value for any distribution of \(\pm 1\) valued random variables. Since this is larger than Tsirelson’s bound of \(2\sqrt{2}\) these outcomes are not obtainable in QM. The above construction gives a perfectly satisfactory probability space consistent with these conditional expectations that satisfies the non-signalling condition.

**Remark.** The probability space constructed in this section gives values to random variables corresponding to values that are not measured in the EPR-Bohm-Bell experiment. For example, in the probability space \(\omega = (1, 1, 1, -1, 22)\) asserts that \(A^{(1)}(\omega) = B^{(1)}(\omega) = 1\) and \(\eta(\omega) = 22\). In an EPR-Bohm-Bell experiment when the PBS’s are in their second position there are no readings for \(a_{\theta_1}(\omega)\) and \(b_{\theta_1}(\omega)\), and QM gives no predictions about their value. We do not assert that ”in reality” for this outcome \(a_{\theta_1}(\omega) = b_{\theta_1}(\omega) = 1\). After all, as pointed out, there may be many consistent ways to assign values to \(A^{(1)}(\omega)\) and \(B^{(1)}(\omega)\). One interpretation of Bell’s
theorem is that there does not, however, exist any such probability space consistent with (5) for which for all i=1,2 and j=1,2:

\[ E(A^{(i)}B^{(j)}|\eta = ij) = E(A^{(i)}B^{(j)}). \]  

We merely assert that probability spaces exist that are consistent with all the available experimental data. Calculations made within the probability space yielding formulae for which all the parameters can be measured may be tested experimentally.

7 Macroscopic realization of the experiment

The experimental setting which we described in this paper, ”proper EPR-Bohm-Bell experiment”, can be realized in various situations outside quantum physics, e.g. in ”classical engineering.”

An example for an experiment with the outcomes described above is the following. A device is equipped with four sensors, \( A_1, A_2, B_1, B_2 \). Both \( A \)-sensors operate on a common power supply, as do the \( B \)-sensors. A measurement of any of the sensors needs the full capacity of its power supply. Thus only one of the \( A \)-sensors can be active at any time and the same for the \( B \)-sensors. Inactive sensors return a default value 0. The device randomly switches between \( A_1 \) and \( A_2 \) respectively \( B_1 \) and \( B_2 \). When the device is polled exactly one \( A \) and one \( B \) sensor return a non-default readings.

One might examine similar experimental settings outside quantum physics. However, it seems that it would not surprise anybody from engineering. It is not a custom to combine the data from sensors which could not operate simultaneously.

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