Metal-insulator transition in 2D: 
Anderson localization by temperature-dependent disorder?

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Abstract

A generalization of the single-parameter scaling theory of localization is proposed for the case when the random potential depends on temperature. The scaling equation describing the behavior of the resistance is derived. It is shown that the competition between the metallic-like temperature dependence of the Drude resistivity and localization leads to a maximum (minimum) at higher (lower) temperatures. An illustration of a metal-insulator transition in the model of charged traps whose concentration depends on temperature is presented.

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Until recently, the one-parameter scaling theory of localization (STL) [1], supplemented by the perturbative treatment of interactions between electrons, had enjoyed an overwhelming agreement with the experiment. This agreement strengthened a common belief that the STL’s description of weak and strong localization regimes, as well as of crossover between the two, is qualitatively correct even in the presence of interactions. Recent observations of an unexpected metallic behavior in 2D electron- and hole-gas systems, starting from the pioneering work by Kravchenko et al. [2], have raised concerns about the general validity of the STL. It has also been suggested that the metallic-like temperature dependence of the resistance $\rho$ is a signature of a novel and perhaps non-Fermi-liquid state, which either results from the interplay of strong Coulomb interactions and disorder or is a non-trivial superconducting state [3]. We believe though that a number of experimental results indicates strongly that the metallic state is quite conventional and manifestly Fermi-liquid–like, and that the $\rho(T)$-dependence is not determined by the interactions. These results include:

i) well-pronounced Shubnikov-de Haas oscillations in the same systems that demonstrate a strong $\rho(T)$-dependence at higher temperatures [4];

ii) negative magnetoresistance of the weak-localization (WL) type at low temperatures [5];

iii) almost absent $T$-dependence of the low-field Hall resistance $\rho_{xy}$ (as compared to that of $\rho_{xx}$) [6]. (If the correction to $\rho$ were due to electron-electron interactions in a disordered system, the following relation should have been satisfied: $\delta\rho_{xy}/\rho_{xy} = 2\delta\rho_{xx}/\rho_{xx}$ [7]);

iv) an insulating upturn of the resistivity at low temperatures and high densities [8];

v) persistence of the metallic-like $\rho(T)$-dependence up to densities of about 30 times larger than the density at the transition and, correspondingly, down to resistivities of about 100 times smaller than the resistivity at the transition [9]. In this region, quantum interference of interacting electrons is already amenable to the perturbative treatment [10] and is known to result only in small (of the order of few %) corrections to the resistance, whereas the observed variation of $\rho$ still constitutes $200-300\%$. This suggests that the $\rho(T)$-dependence in the metallic phase is not due to quantum effects of interference and interactions.

With this in mind, we are going to generalize the STL for the case when the disorder po-
tential has two components: a $T$-independent and $T$-dependent one. The latter component may result from, e.g., charged defects (traps) whose concentration changes with $T$ or from the $T$-dependence of the screening radius. Correspondingly, the Drude (classical) resistance of such a system is

$$\rho_d(T) = \rho_1 + \rho_0(T),$$

where $\rho_1$ is the $T$-independent residual resistance, whereas $\rho_0$, resulting from scattering at the $T$-dependent component of disorder, is metallic-like ($d\rho_0/dT > 0$). The goal of this work is: given that $\rho_d(T)$ is of the form (1), to account for localization effects and to describe the crossover from the metallic to the insulating phase.

Let us choose $T$ to be low enough so that the phase-breaking length $L_\varphi$ is much larger than the system size $L$. Once $T$, and thus the random potential is fixed, we perform the conventional scaling of $\rho$ with $L$, which leads to an RG equation:

$$\frac{\partial \ln \rho(L, T)/\partial \ln L}{\partial \ln T} = -\beta(\rho),$$

where $\beta(\rho)$ is the same scaling function which appears in the STL. The only difference between Eq. (2) and the STL is that now $\rho$ depends on both $L$ and $T$, the latter entering the equation as a parameter, via the $T$-dependence of disorder. Integrating Eq. (2), we get

$$\ln L/\ell = -\int_{\rho_d}^{\rho} d\rho'/\rho'\beta(\rho'),$$

where $\rho_d \equiv \rho(L = \ell(T), T)$ and $\ell(T)$ is the ($T$-dependent) elastic mean free path. The $L$-dependence of $\rho$, given (implicitly) by Eq. (3), crosses over to the $T$-dependence when $L$ becomes comparable to $L_\varphi$. To account for this crossover, we substitute $\ln L/\ell \to \ln L_\varphi/\ell$ in Eq. (3) and differentiate both side of this equation with respect to $T$, taking into account the $T$-dependences of both $\rho_d$ and $L_\varphi$. When doing this, we assume that $\rho_d$ depends on $T$ only via $\ell(T)$, i.e., $\rho_d \propto \ell(T)^{-1}$. The resulting equation reads:

$$\frac{1}{\beta(\rho)} \frac{d \ln \rho}{d \ln T} = - \frac{d \ln L_\varphi}{d \ln T} + \left[ \frac{1}{\beta(\rho_d)} - 1 \right] \frac{d \ln \rho_d}{d \ln T}. $$

(4)
The phase breaking time $\tau_\phi$ usually diverges as a negative power of $T$ at $T \to 0$. In simplest cases, e.g., when dephasing is due to phonons, $\tau_\phi$ does not depend on disorder. In other cases, $\tau_\phi$ may depend on disorder itself. We will consider a general case, when $\tau_\phi \propto T^{-p} \rho^{1-2\gamma}$, $p$ and $\gamma$ being some constants. Note that $\tau_\phi$ depends on the observable, rather than Drude, resistance. $L_\phi$ is determined by $\tau_\phi$ and by the observable diffusion constant of electrons $D = \propto \rho^{-1}$:

$$L_\phi = \sqrt{D \tau_\phi} \propto \sqrt{\tau_\phi / \rho} \propto \rho^{-\gamma} T^{-p/2}.$$  \hspace{1cm} (5)

With the dependences of $L_\phi$ on $T$ and $\rho$ taken into account, the RG-equation reduces to

$$\left[ \frac{1}{\beta(\rho)} - \gamma \right] \frac{d \ln \rho}{d \ln T} = \frac{p}{2} + \left[ \frac{1}{\beta(\rho_d)} - 1 \right] \frac{d \ln \rho_d}{d \ln T}.$$  \hspace{1cm} (6)

Eq. (6) is supplemented by the boundary condition $\rho(T_0) = \rho_{d0}(T_0)$, where $T_0$ is a cut-off temperature at which localization effects become negligible, i.e., $L_\phi(T_0) = \ell(T_0)$. Eq. (6) is the main technical result of this paper which allows one to find the observable $\rho(T)$-dependence, arising both from the phase-breaking processes [the first term on the RHS] and the $T$-dependence of the classical resistance $\rho_d$ [the second term]. The $T$-dependence of the random potential implies that, in fact, disorder is dynamic. For our approach to be valid, the time scale of disorder variation $\tau_{\text{dis}}$ should be larger than $\tau_\phi$ but smaller than the measurement time. In what follows, we will focus on the 2D case, for which the $\beta$-function can be approximated with a reasonable accuracy by $\beta(\rho) = -\ln(1 + a\rho)$, where $a = 2/\pi$ and $\rho$ is a resistance per square measured in units of $\hbar/e^2$.

A number of conclusions about the behavior of $\rho$ can be extracted just from the general form of Eq. (6): i) a metallic (insulating)-like $T$-dependence of $\rho_d$ impedes (facilitates) Anderson localization; ii) the system remains metallic, i.e., $d \ln \rho / d \ln T > 0$ at $T \to 0$, provided that $\rho_d$ vanishes with $T$ rapidly enough, namely, $\rho_d < 1 / \ln 1/T$; iii) if $\rho_d(T = 0) = \rho_{d0} \neq 0$, then the system will eventually become an insulator at low enough $T$. This happens, however, only at exponentially low temperatures [$\propto \exp(-\rho_{d0}^{-1})$].

When both $\rho$ and $\rho_d$ are small, Eq. (6) gives $\rho^{-1}(T) - \rho_d^{-1}d(T) = -a \ln [L_\phi(T)/\ell(T)]$, which is simply a WL result.
It is instructive to analyze the case of a power-law, metallic-like $T$-dependence of $\rho_d$:

$$\rho_d(T) = \rho_0(T) = \bar{\rho} \left(\frac{T}{T_0}\right)^q, \quad q > 0.$$  \hfill (7)

In this case, Eq. (6) takes the form

$$\left[\frac{1}{\beta(\rho)} - \gamma\right] \frac{d\ln \rho}{d\ln T} = \frac{p}{2} + q \left(\frac{1}{\beta(\rho_d)} - 1\right).$$  \hfill (8)

As $T \to 0$, $\rho_d \to 0$ and $\beta(\rho_d) \to 0$. The second term on the RHS of Eq. (8) diverges and a metallic-like dependence (7) prevails. At higher temperatures, however, when $\rho_d$ is still large enough, the localization may be effective. If $p$ and $\bar{\rho}$ are large enough, $\rho$ first increases with decreasing $T$, goes through a maximum and then decreases, approaching the classical $T$-dependence (7) for $T \to 0$ [cf. Fig. 1, top solid curve]. The maximum in $\rho$ exists if

i) $p > 2q$ and ii) $\bar{\rho} > \rho_c = \frac{1}{a} \left[\exp\left(\frac{1}{(p/2q)} - 1\right) - 1\right].$  \hfill (9)

As $\bar{\rho}$ approaches $\rho_c$ from above, the maximum becomes more shallow (Fig. 1, medium solid curve) and it disappears for $\bar{\rho} < \rho_c$ (Fig. 1, bottom solid curve).

Even if the resistance is metallic-like at higher temperatures, the $T$-independent part of disorder leads eventually to localization. Therefore, $\rho$ should have a minimum at low temperatures. If the residual resistance $\rho_1 \ll 1$, this minimum occurs already in the WL regime. The temperature of the minimum is to be determined from the following equation

$$T_{\min} = \frac{pa}{2} \left(\frac{\rho_1^2}{\rho_0 / dT}\right)_{T=T_{\min}}.$$  \hfill (10)

In all experiments on the metal-insulator transition (MIT) in 2D, both $\rho(T)$ and $d\rho(T)/dT$ decrease as the carrier density increases, i.e., as the system becomes more metallic. Eq. (10) tells then that the minimum is shifting to higher temperatures as one is getting deeper into the metallic phase. This behavior is illustrated in Fig. 2 for $\rho_0(T)$ given by Eq. (7).

We now turn to a particular model for $\rho_d(T)$ \cite{10}, which describes scattering of 2D electrons at charged traps located in the oxide layer of thickness $d$ and dielectric constant $\epsilon_{ox}$. The energy level of a trap is shifted by the electrostatic field between the gate and
the 2DEG as well as by that between the trap and its image charge in the 2DEG. The competition between these two forces leads to a sharp peak in the spatial distribution of charged traps. The resistance is obtained by integrating the scattering cross-section due to a single trap over the spatial distribution of the traps. In what follows, we assume that $T \ll E_F$, and that the chemical potential $\mu$ of the 2D gas is determined entirely by the gate voltage $V_g$, and is therefore $T$-independent. The $\rho_d(T)$-dependence results then only from the thermal population of charged traps. Let $\mu$ coincide with the trap level at $V_g = V_g^c$. We call this point “critical”, although what appears to be a critical point of the metal-insulator transition may be shifted from $V_g^c$ due to effects of localization. The distance from the “critical” point can be characterized by the parameter

$$\delta = \frac{V_g - V_g^c}{eV_g^c} \sqrt{\frac{e^2}{2\epsilon_{ox}d}} \propto \frac{n - n_c}{n_c},$$

(11)

where $n$ is the electron concentration. $\rho_d$ in this model can be written as

$$\rho_d = \rho_1 + \bar{\rho}(T/T_0)^q \times \begin{cases} (1 + |\delta|/cT)^q, & \text{for } \delta \leq 0; \\ e^{-\delta/T}, & \text{for } \delta > 0, \end{cases}$$

(12)

where $q = 1/2$, $c \approx 0.62$, and $\bar{\rho}$ depends on the microscopic details of the model (trap concentration, etc.). $\rho_1$ describes the $T$-independent part of disorder, arising from, e.g., residual impurities and interface roughness. When $\mu$ is above the trap level ($\delta > 0$, “metallic” regime) the number of charged traps $N_+$, and thus $\rho_d$ decreases with $T$ exponentially. When the chemical potential is below the trap level ($\delta < 0$, “insulating” regime), $N_+$ and $\rho_d$ saturate for $T \ll |\delta|$. In the “critical” region ($|\delta| \leq T$), $\rho_d \propto T^q$.

An illustration of an MIT-like behavior, corresponding to $\rho_d$ given by Eq. (12), is presented in Fig. 3 for $\bar{\rho} = 2.0$ and $p = 3$. For this choice of parameters, localization prevails over the metallic-like temperature dependence over a range of “metallic” densities ($\delta > 0$) and $\rho(T)$ exhibit a maximum at $T = 0.1 \ldots 0.2T_0$ (Fig. 4). As $\delta$ becomes negative and increases in its absolute value (which corresponds to increasing $\rho_d$), the maximum turns over into a monotonic, insulating-like $T$-dependence of $\rho$. 

6
The $\rho(T)$ behavior at relatively high temperatures $T = (0.1 - 1)T_0$ leaves one with an impression of a true MIT. As $\rho_d(0) \neq 0$, however, curves which look “metallic” at higher $T$, exhibit localization behavior at lower $T$. Fig. 5 shows the behavior of “metallic” curves in a wide temperature range. As was explained earlier, the temperature of the minimum is higher for more “metallic” curves. In the experiment, the high-density, low-resistance curves show an upturn at low temperatures, whereas no such upturn is observed for lower-density, higher-resistance curves. Fig. 5 suggests a simple, if not trivial, explanation of this effect: if the lowest temperature attained experimentally is still higher than $T_{\text{min}}$, the curve may be interpreted as “metallic”, whereas in fact it is “insulating”.

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REFERENCES

[1] E. A. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979).

[2] a) S. V. Kravchenko et al., Phys. Rev. B 50, 8039 (1994).

[3] C. Castellani, C. Di Castro, and P. A. Lee, Phys. Rev. B 57, R9381 (1998); Q. Si and C. M. Varma, Phys. Rev. Lett. 81, p. 4951 (1998); S. Chakravarty, S. Kivelson, C. Nayak, and K. Völkner, cond-mat/9805383; Song He and X. C. Xie, Phys. Rev. Lett. 80, 3324 (1998); P. Phillips et al., Nature 395, 253 (1998); D. Belitz and T. R. Kirkpatrick, Phys. Rev. B 58, 8214 (1998); F.-C. Zhang and T. M. Rice, cond-mat/9708050.

[4] V. M. Pudalov et al. (to be published).

[5] V. M. Pudalov et al., JETP Lett. 65, 932 (1997); G. Brunthaler et al. (to be published).

[6] V. M. Pudalov et al. JETP Lett., 70, 48 (1999); S. V. Kravchenko et al. (unpublished).

[7] B. L. Altshuler and A. G. Aronov, in Electron-electron interactions in disordered systems, edited by A. L. Efros and M. Pollak (Elsevier, Amsterdam, 1985).

[8] V. M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, JETP Lett. 68, 534 (1998), Phys. Rev. B 50, R2154 (1999).

[9] V. M. Pudalov et al., Phys. Rev. B 50, R2154 (1999) and to be published.

[10] B. L. Alsthuler and D. L. Maslov, Phys. Rev. Lett. 82, 145 (1999).

[11] A. Gold and V. T. Dolgopolov, Phys. Rev. B 33, 1076 (1986); S. Das Sarma, ibid. 33, 5401 (1986).

[12] M. Batsch, I. Kh. Zharekeshev, and B. Kramer, Sol. State Commun. 95, 353 (1995); I. Kh. Zharekeshev, private communication.
FIGURES

FIG. 1. \( \rho(T) \) (solid) corresponding to the Drude resistance \( \rho_d(T) \) [Eq. (7)] (dashed) for \( \bar{\rho} = 1, 2, 3 \). \( p = 3, q = 1/2, \gamma = 0.5 \). For this value of \( p/q \), the critical value of \( \bar{\rho} \) for the existence of the maximum \( \rho_c \approx 1 \). Inset: characteristic resistance \( \rho_c \) as a function of \( p/q \).

FIG. 2. Variation of the resistivity minimum with parameter \( \bar{\rho} \) in Eq. (7). \( \rho_1 = 0.1 \). From top to bottom: \( \bar{\rho} = 0.5, 1, 2 \).

FIG. 3. Illustration of an MIT in the model of Ref. [10]. \( p = 3, q = 1/2, \gamma = 0.5, \bar{\rho} = 2.0, \rho_1 = 0.01 \). From top to bottom:

FIG. 4. Evolution of the maximum in \( \rho(T) \) for the same parameters as in Fig. 3.

FIG. 5. Evolution of the WL minimum in \( \rho \) for the same parameters as in Fig. 3. From left to right: \( \delta/T_0 = 0.1 \ldots 0.8 \) with increment 0.1.
\[ \rho_c \]

\[ \rho = 1 \]

\[ \rho = 2 \]

\[ \rho = 3 \]

\[ T/T_0 \]
Fig. 2

\( \frac{\rho}{h/e^2} \)

\[ \frac{T}{T_0} \]
Fig. 3

\[ \delta = 0 \]

\[ \rho (h/e^2) \]

\[ T/T_0 \]
Fig. 4

\[ \frac{\delta}{T_0} = \begin{array}{cccccc}
0.00 & 0.02 & 0.04 & 0.06 & 0.08 & 0.10 \\
0.12 & 0.14 & 0.16 & 0.18 & 0.20 & 0.22 \\
0.24 & 0.26 & 0.28 & 0.30 & 0.32 & 0.34 \\
\end{array} \]

\[ \rho \left( \frac{h}{e^2} \right) \]

\[ \frac{T}{T_0} \]
