Moduli stabilization in (string) model building:
gauge fluxes and loops

A. P. Braun\(^1\), A. Hebecker\(^2\), and M. Trapletti\(^2\,^3\)

\(^1\) Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16 und 19, D-69120 Heidelberg, Germany.
\(^2\) Laboratoire de Physique Theorique, Bat. 210, Université de Paris-Sud, F-91405 Orsay, France.
\(^3\) Centre de Physique Théorique, École Polytechnique, F-91128 Palaiseau, France.

Abstract. We discuss the moduli stabilization arising in the presence of gauge fluxes, R-symmetry twists and non-perturbative effects in the context of 6-dimensional supergravity models. We show how the presence of D-terms, due to the gauge fluxes, is compatible with gaugino condensation, and that the two effects, combined with the R-symmetry twist, do stabilize all the Kähler moduli present in the model, in the spirit of KKLT. We also calculate the flux-induced one-loop correction to the scalar potential coming from charged hypermultiplets, and find that it does not destabilize the minimum.

PACS. 04.65.+e Supergravity – 11.10.Kk Field theories in dimensions other than four – 11.25.Mj Compactification and four-dimensional models

1 Introduction

One of the perceived problems of the KKLT construction \([1]\), is the presence of D3-branes (‘anti-D3-branes’), which break SUSY explicitly and do not have a supergravity description.\(^3\)

Following \([4]\), one could avoid such a problem by trading the D3-branes for the introduction of two-form flux on the worldvolume of D7-branes, that has a supergravity description in terms of a SUSY-breaking D-term potential (see e.g. \([5–8]\)). Unfortunately, there are two fundamental problems with this proposal: one related to the intimate connection between F- and D-terms, the other to the gauge invariance of the superpotential \([2, 9–11]\). Namely, the D-terms originate from the gauging of an isometry of the scalar manifold of the supergravity model. In KKLT, such an isometry should act on the single Kähler modulus \(T\) by shifting its imaginary part. This clashes with the fact that the superpotential \(W = W_0 + A e^T\) is not invariant under such a shift. This clash can obviously be avoided if light fields other than \(T\) are present \([8, 9, 12, 13]\), but in this case a reanalysis of the whole stabilization/uplifting proposal is needed.

In our following investigation we approach the problem from a 6d supergravity perspective \([14–16]\), in the presence of 2-form-flux.

In Sect. 2 we introduce a \(T^2/Z_2\) model in which two moduli superfields \(S\) and \(T\) encode the dilaton and the compactification volume. We calculate the scalar potential arising in the presence of 2-form-flux in two ways: by integrating the \(F_{56}^2\) term over the compact space and by finding the D-term that arises from the gauge transformation of \(T\). Since the superfield \(S\), which governs all gauge-kinetic functions, does not transform, no gauge invariance problem arises in the presence of gaugino condensation (see also \([8]\)).

In Sect. 3 we introduce Scherk-Schwarz SUSY breaking as a source for a constant superpotential \(W_0\). We study the compatibility of such an option with a \(T^2/Z_n\) orbifold compactification, finding that only the \(n = 2\) case is actually viable.

In Sect. 4 we calculate the one-loop correction to the scalar potential. This is done by the explicit computation of the correction that arises if hypermultiplets charged under the fluxed U(1) are present. Since the constituents of the charged hypermultiplet feel the flux directly, we expect this to be the dominant contribution to the corrections.

In Sect. 5 we discuss options for moduli stabilization using the various ingredients analysed above. Working on a \(T^2/Z_2\) orbifold and ignoring, for simplicity, the shape modulus of the torus, one still has to deal with the stabilization of the superfields \(S\) and \(T\) simultaneously. At fixed \(T\), the modulus \(S\) is stabilized à la KKLT by the interplay of \(W_0\) and gaugino condensate. The depth of the resulting SUSY AdS vacuum depends

\(^3\) It has, however, been argued that a phenomenologically motivated description in terms of non-linearly realized supersymmetry is sufficient for most practical purposes \([2]\). Indeed, when modelling the D3-brane effect by F-term breaking, the phenomenology turns out to be independent of the detailed dynamics of this SUSY breaking sector (unless extra fields violate the underlying sequestering assumption) \([3]\).
on $T$, driving $\Re T$ to small values. This is balanced by the $T$ dependence of the flux-induced $D$-term, leading to a stable non-SUSY AdS vacuum. Thus, while the 2-form flux does not provide the desired uplift, it plays an essential role in the simultaneous stabilization of two moduli. Finally, we show that the loop-corrections do not spoil the stabilization.

Acknowledgements: The work of MT is supported by the European Community through the contract 041273.

2 A six-dimensional model: gauge fluxes as a source for $D$-term potential

We start from the following bosonic action for supergravity coupled to gauge theory in six dimensions [14, 17] (for details see [18] and references therein)

$$\mathcal{L} = -\frac{\sqrt{-g}}{2} \left[ R_{MN} + (\partial \phi)^2 + \frac{e^{2\phi}}{12} H^2 + \frac{e^\phi}{2} F^2 \right],$$

(1)

where $H \equiv dB + F \wedge A$. This action is invariant under the gauge transformations

$$\delta A = d\Lambda, \quad \delta B = -A F + dC.$$  

(2)

We consider a compactification on $\mathbb{M}^4 \times T^2$, with

$$g_{\mu \nu} = \begin{pmatrix} r^2 g_{\mu \nu} & 0 \\ 0 & g_{2M} \end{pmatrix}, \quad g_{2M} = \frac{1}{\tau_2} \left( \begin{array}{cc} 1 & \tau_1 \\ \tau_1 & |\tau_1|^2 \end{array} \right),$$

(3)

where $\mu, \nu = 0,3, m, n = 5,6$, and $\tilde{r} \equiv \tau_2 + \mathrm{i} \tau_1$. The domain of $x_5$ and $x_6$ is taken to be a square of unit length, so that $\int \sqrt{g_{2M}} dx^5 dx^6 = 1$.

We introduce a constant background for the field strength $F_{mn} = f_{mn}$, $f$ being a quantized number. We split the gauge potential $A$ into a fluctuation term $\tilde{A}$ and a background term $A$, such that $(F) = d(A)$. The background $\langle A \rangle$ cannot be globally defined in the internal space, thus, also $\tilde{B}$ is not globally defined. Rather, a new field $\tilde{B} = B - \langle A \rangle \wedge A$ is globally defined [19] and has a standard Kaluza-Klein expansion.

The next step is to pass to the 4d theory arising from the compactification on a supersymmetric $T^2/Z_2$ orbifold. We achieve this by disregarding all 4d vector multiplets arising from 6d gravity, as well as the Wilson line degrees of freedom associated with the 6d gauge theory (we neglect the possibility of localized matter). What remains is 4d supergravity, the $A_4$ vector multiplet, and three chiral multiplets. The latter contain the degrees of freedom $r, \phi, \tau$ and two scalars related to the 2-form $B$. The lowest components of the three modulus superfields are [15, 20]

$$S \equiv \frac{1}{2}(s + ic), \quad T \equiv \frac{1}{2}(t + ib), \quad \tau \equiv \frac{1}{2}(\tau_2 + \mathrm{i} \tau_1).$$

(4)

where we have used the definitions $t \equiv e^{-\phi} r^2, s \equiv e^\phi r^2$ and $b_{mn} = B_{mn}, \epsilon_{\mu \rho \sigma \nu \kappa \lambda} \partial^\sigma c \equiv r^4 e^{2\phi} (dB)_{\mu \rho \sigma \nu \kappa \lambda}$. The Kähler potential, which can be inferred from the kinetic terms for the scalars, is

$$K = -\log(T + \mathcal{T}) - \log(S + \mathcal{S}) - \log(r + \mathcal{r}).$$

(5)

Similarly, the gauge-kinetic function is found to be $h(S) = 2S$.

The 4d model is invariant under the gauge transformations inherited from those described in Eq. (2). In particular, the gauge transformations of $B$ follow from its definition together with Eq. (2). Considering 4d gauge transformations $A = A(x')$ and restricting to the zero-mode level only, we find $\delta B_{56} = -2A(F_{56})$, i.e. $\delta b = -2fA$. This implies that the only nonvanishing component of the Killing vector is $X^2 = -if$. Thus, the resulting $D$-term, $D = iK_T X^2$, leads to the $D$-term potential

$$V_D = f^2/2s t^2.$$  

(6)

The same potential also follows directly from the 6d gauge-kinetic term, evaluated in the flux background. This represents a nontrivial check of the fact that the flux is described by the gauging of an isometry from the 4d perspective. (See [21] for a similar computation in heterotic string theory.) Note in particular that the gauge transformation acts only on $T$, while the gauge kinetic function depends only on $S$. Hence, no clash between gaugino condensation and $D$-term potential arises. A related situation occurring in the presence of both flux and gaugino condensation on the same D7-brane-stack has been discussed in [8].

3 Scherk-Schwarz twists as a source for $W_0$

The 6d supergravity theory studied in Sect. 2 possesses an $SU(2)_R$ R-symmetry, thus we can compactify it on $T^2$ imposing non-trivial field-identifications. Given a generic $SU(2)_R$ doublet $\Phi(x^\mu, x^5, x^6)$ (e.g. the gaugino) we require

$$\Phi(x^\mu, x^5, x^6) = T_5 \Phi(x^\mu, x^5 + 1, x^6), \quad (7)$$

$$\Phi(x^\mu, x^5, x^6) = T_0 \Phi(x^\mu, x^5 + 1, x^6).$$

(8)

where the matrices $T_i$ embed the translations $t_i$ along the torus coordinate $x^i$ in the R-symmetry group. Since $t_5 t_6 = t_6 t_5$, we also require $T_5 T_6 = T_6 T_5$. In case one (or both) of the matrices are non-trivial, we obtain a Scherk-Schwarz (SS) dimensional reduction [22].

For an orbifold compactification of the 6d theory, the rotation operator $r \in SO(2)$ is also embedded in the R-symmetry group via a matrix $R$. A non-trivial embedding is crucial to avoid a hard SUSY breaking, indeed, in case $R = 1$, the net action of the orbifold on any 4d spinor would result in a non-trivial phase, projecting it out of the spectrum. Having such a non-trivial embedding, extra consistency conditions must be fulfilled.

In the case of a $Z_2$ orbifold, $r^2 = 1, rt_i = t_i^{-1} r$, and we have to impose these conditions also on the corresponding transformations of the spinors. Non-trivial solutions to these conditions exist [23], as can be easily demonstrated explicitly: The transformation associated with $r$ is $R = S(r) R$, where $S(r)$ is the phase rotation of the two 4d Weyl spinors coming from a 4 of SO(1,5). In the $Z_2$ case, we have $S(r) = i \mathbb{I}$. Choosing
\( R = \text{diag}(-i, i) \), we find \( \tilde{R} = \text{diag}(1, -1) \). This matrix satisfies the required consistency relations with \( T_\alpha = \exp(i \alpha_0 \sigma_2) \). In case only one of the \( T_\alpha \)'s is non trivial, e.g. \( \alpha_0 = 0 \) and \( \alpha_5 = \alpha \), we can shrink the \( x^6 \) direction, obtaining a 5d effective field theory compactified on \( S^1/Z_2 \). In this case it is well known that the continuous SS parameter \( \alpha \) can be described by a tunable constant superpotential \( W \) mimicking a one-dimensional harmonic oscillator with unit ant derivative \( [\frac{1}{2}] \). Algebraically, this is equivalent to a one-dimensional harmonic oscillator with unit momentum operators having to be interchanged but the \( \text{Kähler} \) potential if we assume that the gauge symmetries of the states of opposite chirality have the same degeneracy, because we are considering the same Laplace operator to which merely a constant is added, and thus we find precisely the same eigenfunctions.

Thus the monopole number equals the degeneracy of the state with vanishing mass. It is clear that the ground state of the fermions of opposite chirality has the same degeneracy, because we are considering the same Laplace operator to which merely a constant is added, and thus we find precisely the same eigenfunctions. By the same argument we conclude that the bosonic ground state is \( N \)-fold degenerate.

With this particle spectrum we directly compute the one-loop effective potential from a four-dimensional perspective (see [18] for details), finding

\[
V_C = \frac{7}{4} \frac{|N|^3}{(st)^2} \zeta_R(-2) \approx -0.053 \frac{|f|^3}{(2\pi)^3 (st)^2} \quad (14)
\]

Here we have used the quantization condition for the flux, Eq. (13).

The computation is analogous, albeit technically more involved, in the \( T^2/Z_2 \) case. The result is:

\[
V_C^\pm = 7 \frac{N^2}{(st)^2} \zeta'(-2) J_N^\pm \approx -0.053 \frac{f^2}{(2\pi)^2 (st)^2} J_N^\pm, \quad (15)
\]

where we have defined

\[
J_N^\pm = |N| \pm 4. \quad (16)
\]

The two signs in \( V_C^\pm \) stem from the different internal parity that may be assigned to the fermions on the massless level.

This correction can be understood as a correction to the Kähler potential. We found a non-zero Casimir energy because SUSY is broken, which in turn is a result of the flux. The flux was shown to generate a \( D \)-term potential back to a correction to the \( \text{Kähler} \) potential, Eq. (13).

The monopole number equals the degeneracy of the state with vanishing mass. It is clear that the ground state of the fermions of opposite chirality has the same degeneracy, because we are considering the same Laplace operator to which merely a constant is added, and thus we find precisely the same eigenfunctions.

Thus the monopole number equals the degeneracy of the state with vanishing mass. It is clear that the ground state of the fermions of opposite chirality has the same degeneracy, because we are considering the same Laplace operator to which merely a constant is added, and thus we find precisely the same eigenfunctions.

Thus the monopole number equals the degeneracy of the state with vanishing mass. It is clear that the ground state of the fermions of opposite chirality has the same degeneracy, because we are considering the same Laplace operator to which merely a constant is added, and thus we find precisely the same eigenfunctions.
so that we can conclude
\[ \Delta K = -\frac{1}{(2\pi)^2} \frac{7}{4} \zeta(-2) \log(T + \mathcal{T}) f^\pm_N. \] (18)

5 Moduli stabilization

In this section we study the stabilization of our model. We start by considering the effect of a gaugino condensate, a constant superpotential term and the D-term due to the gauge flux. We neglect for a moment the contribution due to the loop corrections. We have

\[ K = -\log(T + \mathcal{T}) - \log(S + \overline{S}) - \log(\tau + \overline{\tau}), \] (19)
\[ W = \mu^3 \exp(-aS) + W_0, \] (20)

where we assume for simplicity that \( a \) and \( \mu \) are real and positive, and \( W_0 \) is real and negative. The complete scalar potential, including the D-term is then

\[ V = \frac{\tilde{V}(s)}{t(\tau + \overline{\tau})} + \frac{f^2}{2st^2}, \] (21)

with

\[ \tilde{V}(s) = a\mu^6(\alpha s + 2)e^{-\alpha s} + 2W_0\mu^3a \cos\left(\frac{\alpha s}{2}\right) e^{-\frac{\alpha s}{2}}. \] (22)

This potential stabilizes both \( s \) and \( t \) at a negative value of \( V \), as is shown in the following.

Consider first the ‘axionic’ partner of \( s \), denoted by \( c \). As \( W_0 \) is taken to be negative, while \( a \) and \( \mu^3 \) are positive, \( a \) is always stabilized at a value where the cosine is unity. Thus we assume \( a = 0 \) in the following. Since the shift symmetry acting on the modulus \( b \) (the ‘axionic’ partner of \( t \)) is gauged, \( b \) is absorbed in the massive vector boson. Further effects have to be taken into account to stabilize the complex structure modulus \( \tau \), for which we assume \( 2\tau = 1 \) from now on.

To get some intuition for the stabilization of \( s \) and \( t \), it is advantageous to first set \( f = 0 \) and \( t = 1 \). Then the remaining modulus \( s \) enters the potential in exactly the same fashion as in the KKLMT model. At the minimum of the potential, \( s \) has to solve \( D_s W = 0 \), so that we find \( W_0 + \mu^3 e^{-\frac{\alpha}{2}}(1 + a s) = 0 \). This is equivalent to minimizing \( \tilde{V}(s) \). For small \( W_0 \) we find the approximate solution

\[ a_0 \sim 2 \ln(-\mu^3/W_0). \] (23)

This equation shows that \( a_0 \) can be made parametrically large by tuning \( W_0 \) to have small negative values.

The approximate value at which \( t \) is stabilized can be found by setting \( s = s_0 \). This is reasonable as the extra \( 1/s \) contribution coming from the \( D \)-term potential will not alter the value of \( s \) at the minimum significantly. The resulting potential for \( t \) is then

\[ V(t) = \frac{f^2}{2s_0^2 t^2} + \frac{\tilde{V}(s_0)}{t}, \] (24)

which is minimized by \( t_0 = -f^2/s_0 \tilde{V}(s_0) \).

The perturbative corrections of Sect. 3 do not alter the stabilization qualitatively. As a contribution to the effective action, they can simply be added to the scalar potential, and slightly drive the minimum to larger values of \( s \) and \( t \).

References

1. S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D 68 (2003) 046005 [arXiv:hep-th/0301240].
2. K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. B 718 (2005) 113 [arXiv:hep-th/0503216].
3. F. Brummer, A. Hebecker and M. Trapletti, Nucl. Phys. B 755 (2006) 186 [arXiv:hep-th/0605232].
4. C. P. Burgess, R. Kallosh and F. Quevedo, JHEP 0310 (2003) 056 [arXiv:hep-th/0309187].
5. S. Kachru and J. McGreevy, Phys. Rev. D 61 (2000) 026001 [arXiv:hep-th/9908135].
6. H. Jockers and J. Louis, Nucl. Phys. B 718, 203 (2005) [arXiv:hep-th/0502059].
7. G. Villadoro and F. Zwirner, JHEP 0603 (2006) 087 [arXiv:hep-th/0602120].
8. M. Haack, D. Kreff, D. Lüst, A. Van Proeyen and M. Zagermann, JHEP 0701 (2007) 078 [arXiv:hep-th/0609211].
9. E. Dudas and S. K. Vempati, Nucl. Phys. B 727, 139 (2005) [arXiv:hep-th/0506172].
10. G. Villadoro and F. Zwirner, Phys. Rev. Lett. 95, 231602 (2005) [arXiv:hep-th/0508167].
11. S. P. de Alwis, [arXiv:hep-th/0602182].
12. A. Achucarro, B. de Carlos, J. A. Casas and L. Doplicher, JHEP 0606 (2006) 014 [arXiv:hep-th/0601190].
13. E. Dudas and Y. Mambrini, JHEP 0607 (2006) 077 [arXiv:hep-th/0607077].
14. A. K. Kashani-Poor and A. Tomasiello, Nucl. Phys. B 728 (2005) 135 [arXiv:hep-th/0505208].
15. A. Salam and E. Sezgin, Phys. Lett. B 147 (1984) 47.
16. Y. Aghababaie, C. P. Burgess, S. L. Parameswaran and F. Quevedo, JHEP 0303 (2003) 032 [arXiv:hep-th/0212091].
17. G. W. Gibbons, R. Guven and C. N. Pope, Phys. Lett. B 595 (2004) 498 [arXiv:hep-th/0307238].
18. H. Nishino and E. Sezgin, Phys. Lett. B 144 (1984) 187 and Nucl. Phys. B 278 (1986) 353.
19. A. P. Braun, A. Hebecker and M. Trapletti, JHEP 0702 (2007) 015 [arXiv:hep-th/0611102].
20. N. Kaloper and R. C. Myers, JHEP 9905 (1999) 010 [arXiv:hep-th/9904045].
21. A. Falkowski, H. M. Lee and C. Lüdeling, JHEP 0510 (2005) 090 [arXiv:hep-th/0504091].
22. G. Villadoro, PhD Thesis, University of Rome “La Sapienza” (2006), available at http://padis.uniroma1.it/search.py.
23. J. Scherk and J. H. Schwarz, Nucl. Phys. B 153 (1979) 61.
24. H. M. Lee, JHEP 0506 (2005) 044 [arXiv:hep-th/0502093].
25. E. Dudas and C. Grojean, Nucl. Phys. B 507 (1997) 553 [arXiv:hep-th/9704177].
26. C. Bachas, [arXiv:hep-th/9503030].