EFFECTIVE LAGRANGIAN FOR HEAVY AND LIGHT MESONS: SEMILEPTONIC DECAYS*

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ABSTRACT

We introduce an effective lagrangian including negative and positive parity heavy mesons containing a heavy quark, light pseudoscalars, and light vector resonances, with their allowed interactions, using heavy quark spin-flavour symmetry, chiral symmetry, and the hidden symmetry approach for light vector resonances. On the basis of such a lagrangian, by considering the allowed weak currents and by including the contributions from the nearest unitarity poles we calculate the form factors for semileptonic decays of $B$ and $D$ mesons into light pseudoscalars and light vector resonances. The available data, together with some additional assumptions, allow for a set of predictions in the different semileptonic channels, which can be compared with those following from different approaches. A discussion of non-dominant terms in our approach, which attempts at including a rather complete dynamics, will however have to wait till more abundant data become available.
1 Introduction

In this letter we shall present an analysis of semileptonic heavy meson decays into light hadrons

\[ P \to \Pi^0 \ell \bar{\nu}_\ell (1.1) \]
\[ P \to \Pi^* \ell \bar{\nu}_\ell (1.2) \]

\( P = \) heavy pseudoscalar meson, \( \Pi \) and \( \Pi^* = \) pseudoscalar and vector light mesons), based on the use of heavy quark spin flavour symmetry \([\mathbb{I}]\), chiral symmetry, and the hidden symmetry approach for light vector resonances. Specifically, our framework will make use of: (i) the heavy-light chiral lagrangian proposed in refs. \([\mathbb{II}]\) \([\mathbb{III}]\) \([\mathbb{IV}]\) \([\mathbb{V}]\), which describes the interaction of the pseudoscalar mesons belonging to the low-lying \( SU(3) \) octet and the negative parity \( J^P = 0^-, 1^- \) heavy \( Q\bar{q} \) mesons; (ii) the introduction through the hidden gauge symmetry approach of the vector meson resonances belonging to the low-lying \( SU(3) \) octet within the heavy-light chiral lagrangian \([\mathbb{VI}]\); (iii) the inclusion of low lying positive parity \( Q\bar{q} \) heavy meson states within the formalism.

We shall first summarize the well known description of the interactions of heavy mesons and light pseudoscalars in terms of an effective chiral lagrangian. In such a lagrangian we shall add a term describing the octet vector meson resonances and their interactions with the heavy mesons and the light pseudoscalars. We shall then introduce the effective lagrangian containing the low-lying positive parity heavy meson states and their interactions with the light pseudoscalars, with the negative parity heavy meson states, and the couplings of the light vector resonances of the octet to both positive and negative parity heavy meson states.

Unavoidably such an effective description, will require the introduction of a set of coupling constants. The study of the semileptonic decays (1.1) and (1.2) will be shown to yield some information on such constants. To this end one has to use all the symmetry constraints to characterize the form of the effective weak interaction of a heavy negative or positive parity meson with the light pseudoscalars and of a negative parity heavy meson with the light vector resonances. For a first numerical analysis of the leptonic decays we shall be forced to neglect higher derivative terms, which is justified only in limited portions of phase space. After having introduced such a formal setting we shall analyze the semileptonic decays (1.1) and (1.2). Their form factors will be calculated at maximum momentum transfer and at leading order in the inverse of the heavy quark mass by including the contributions of the low lying pole contributions.

2 The heavy-light chiral lagrangian

To be self-contained and to establish the notations we shall start by reviewing the description of heavy mesons and light pseudoscalars by effective field operators and of their effective chiral lagrangian. Negative parity heavy \( Q\bar{q}_a \) mesons are represented by fields described by a \( 4 \times 4 \) Dirac matrix

\[ H_a = \frac{(1 + \gamma^5)}{2} [P^*_a \gamma^\mu - P_a \gamma_5] \]  \quad (2.1)
\[ \bar{H}_a = \gamma_0 H_a^\dagger \gamma_0 \]  \quad (2.2)
Here $v$ is the heavy meson velocity, $a = 1, 2, 3$ (for $u, d$ and $s$ respectively), $P^{*\mu}_{a}$ and $P_{a}$ are annihilation operators normalized as follows

$$
\langle 0 | P_{a} | Q \bar{q}_{a}(0^-) \rangle = \sqrt{M_{H}} \tag{2.3}
$$

$$
\langle 0 | P^{*}_{a} | Q \bar{q}_{a}(1^-) \rangle = e^{i\mu} \sqrt{M_{H}} \tag{2.4}
$$

with $v^{\mu}P^{*}_{a\mu} = 0$ and $M_{H} = M_{P} = M_{P^{*}}$, the supposedly degenerate meson masses. Also $\not{\!}H = -\not{\!}H = H$, $\not{\!}H = -\not{\!}H = H$. The pseudoscalar light mesons are described by

$$
\xi = \exp \frac{iM}{f_{\pi}} \tag{2.5}
$$

where

$$
M = \begin{pmatrix}
\sqrt{\frac{1}{2}} \pi^{0} + \sqrt{\frac{1}{6}} \eta & \pi^{+} & K^{+} \\
\pi^{-} & -\sqrt{\frac{1}{2}} \pi^{0} + \sqrt{\frac{1}{6}} \eta & K^{0} \\
K^{-} & \bar{K}^{0} & -\sqrt{\frac{3}{2}} \eta
\end{pmatrix} \tag{2.6}
$$

and $f_{\pi} = 132 MeV$. Under the chiral symmetry the fields transform as follows

$$
\xi \rightarrow g_{L} \xi U^{\dagger} = U \xi g_{R}^{\dagger} \tag{2.7}
$$

$$
\Sigma \rightarrow g_{L} \Sigma g_{R}^{\dagger} \tag{2.8}
$$

$$
H \rightarrow H U^{\dagger} \tag{2.9}
$$

$$
\bar{H} \rightarrow U \bar{H} \tag{2.10}
$$

where $\Sigma = \xi^{2}$, $g_{L}$, $g_{R}$ are global $SU(3)$ transformations and $U$ is a function of $x$, of the fields and of $g_{L}$, $g_{R}$.

The lagrangian describing the fields $H$ and $\xi$ and their interactions, under the hypothesis of chiral and spin-flavour symmetry and at the lowest order in light mesons derivatives is

$$
\mathcal{L}_{0} = \frac{f_{\pi}^{2}}{8} \langle \partial^{\mu} \Sigma \partial_{\mu} \Sigma^{\dagger} > + i < H_{b}v^{\mu}D_{\mu ba} \bar{H}_{a} > + i g < H_{b} \gamma_{\mu} \gamma_{5} A_{ba} \bar{H}_{a} > \tag{2.11}
$$

where $\langle \ldots \rangle$ means the trace, and

$$
D_{\mu ba} = \delta_{ba} \partial_{\mu} + \mathcal{V}_{\mu ba} = \delta_{ba} \partial_{\mu} + \frac{1}{2} \left( \xi^{\dagger} \partial_{\mu} \xi + \xi \partial_{\mu} \xi^{\dagger} \right)_{ba} \tag{2.12}
$$

$$
A_{\mu ba} = \frac{1}{2} \left( \xi^{\dagger} \partial_{\mu} \xi - \xi \partial_{\mu} \xi^{\dagger} \right)_{ba} \tag{2.13}
$$

Besides chiral symmetry, which is obvious, since, under chiral transformations,

$$
D_{\mu} \bar{H} \rightarrow U D_{\mu} \bar{H} \tag{2.14}
$$

the lagrangian (2.11) possesses the heavy quark spin symmetry $SU(2)_{v}$, which acts as

$$
H_{a} \rightarrow \hat{S} H_{a} \tag{2.15}
$$

$$
\bar{H}_{a} \rightarrow \bar{H}_{a} \hat{S}^{\dagger} \tag{2.16}
$$

with $\hat{S} \hat{S}^{\dagger} = 1$ and $[\hat{S}, \hat{S}] = 0$, and a heavy quark flavour symmetry arising from the absence of terms containing $m_{Q}$.
Explicit symmetry breaking terms can also be introduced, by adding to $\mathcal{L}_0$ the extra piece (at the lowest order in $m_q$ and $1/m_Q$):

\[
\mathcal{L}_1 = \lambda_0 < m_q \Sigma + m_q \Sigma^\dagger > + \lambda_1 < \hat{H}_a H_b (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)_{ba} > \\
+ \lambda'_1 < \hat{H}_a H_a (m_q \Sigma + m_q \Sigma^\dagger)_{bb} > + \frac{\lambda_2}{m_Q} < \hat{H}_a \sigma_{\mu\nu} H_a \sigma^{\mu\nu} >
\]  

The last term in the previous equation induces a mass difference between the states $P$ and $P^*$ contained in the field $H$, such that

\[
M_P = M_H \quad M_{P^*} = M_H + \delta m_H
\]  

The preceding construction can be found for instance in the paper by Wise [2], and we have used the same notations.

### 3 Introduction of light vector resonances

The vector meson resonances belonging to the low-lying $SU(3)$ octet can be introduced by using the hidden gauge symmetry approach [3] (for a different approach see [4]). The new lagrangian containing these particles, to be added to $\mathcal{L}_0 + \mathcal{L}_1$, is as follows [3]:

\[
\mathcal{L}_2 = -\frac{f^2}{2} \rho < (\mathcal{V}_\mu - \rho_\mu)^2 > + \frac{1}{2g_V^2} < F_{\mu\nu} (\rho) F^{\mu\nu} (\rho) > \\
+ i\beta < H_b v^\mu (\mathcal{V}_\mu - \rho_\mu)_{ba} \hat{H}_a > \\
+ \frac{\beta^2}{2f^2 a} < \hat{H}_b H_a \hat{H}_a H_b > + i\lambda < H_b \sigma^{\mu\nu} F_{\mu\nu} (\rho)_{ba} \hat{H}_a >
\]  

where $F_{\mu\nu} (\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu]$, and $\rho_\mu$ is defined as

\[
\rho_\mu = \frac{i}{\sqrt{2}} \hat{\rho}_\mu
\]  

$\hat{\rho}$ is a hermitian $3 \times 3$ matrix analogous to (2.6) containing the light vector mesons $\rho^{0,\pm}$, $K^*$, $\omega_8$, $g_\nu$, $\beta$ and $a$ are coupling constants; by imposing the two KSRF relations [3] one obtains

\[
a = 2 \quad g_\nu \approx 5.8
\]  

We note that the quartic term in the heavy fields $H$ in (3.1) is added to obtain the simple lagrangian $\mathcal{L}_0$ in the formal limit $m_\rho \to \infty$, when the $\rho$ field decouples.

### 4 Inclusion of positive parity heavy mesons

For our subsequent analysis of the heavy mesons semileptonic decays we shall have to introduce the low-lying positive parity $Q\bar{q}_a$ heavy meson states. For $p$ waves ($l = 1$), the heavy quark effective theory predicts two distinct multiplets, one containing a $0^+$ and a $1^+$ degenerate states, the other one comprising a $1^+$ and a $2^+$ state [3], [4]. In matrix notation they are described respectively by [4, 5]

\[
S_a = \frac{1 + \gamma'}{2} [D_\mu' \gamma_5 \gamma_\mu - D_0]
\]  

3
and
\[ T^\mu_a = \frac{1 + \gamma^\rho}{2} \left[ D_2^{\mu \nu} \gamma_\nu - \frac{3}{2} \hat{D}_{1 \nu} \gamma_5 \left( g^{\mu \nu} - \frac{1}{3} \gamma^{\mu \nu} \gamma^5 \right) \right] \] (4.2)

Note that \( \gamma^S = S_R^i = S \); \( \gamma^T = T_\mu = T_\mu^\rho = \hat{T}_\mu^\rho = \hat{T}_\mu \). The two multiplets have \( s_1 = 1/2 \) and \( s_1 = 3/2 \) respectively, where \( \bar{s}_i^7 \) is the angular momentum of the light degrees of freedom which is conserved together with the heavy quark spin \( s_\bar{Q}^7 \) in the infinite quark mass limit because \( \bar{J}_l = \bar{s}_l + s_\bar{Q}^7 \). The lagrangian containing the fields \( S_a \) and \( T_\mu^a \) as well as their interactions with the Goldstone bosons and the fields \( H_a \) has been derived in ref. [12]:

\[ \mathcal{L}_3 = \mathcal{L}_{\text{kin}} + \mathcal{L}_{1\pi} + \mathcal{L}_{s} + \mathcal{L}_{d} \] (4.3)

\[ \mathcal{L}_{\text{kin}} = \frac{1}{\Lambda} < T^\mu_a (v \cdot D)_{ba} S_a > + \frac{1}{\Lambda} < T^\mu_b (v \cdot D)_{ba} \hat{T}_\mu > \]

(4.4)

\[ \mathcal{L}_{1\pi} = \frac{1}{\Lambda} < S \gamma^5 \bar{S}^\mu A^\mu_{ba} > + \frac{1}{\Lambda} < T^\mu_b \gamma^5 A^\mu_{ba} > \] (4.5)

\[ \mathcal{L}_{s} = \frac{1}{\Lambda} < T^\mu_b A^\mu_{ba} > + i \frac{1}{\Lambda} < S \gamma^5 A^\mu_{ba} > + h.c. \] (4.6)

\[ \mathcal{L}_{d} = \frac{1}{\Lambda} < T^\mu_b \gamma^5 (D^\mu A^\lambda)_{ba} > \]

(4.7)

In (4.4) \( \delta m_S = M_{D_0} - M_H = M_{D_1} - M_H \); \( \delta m_T = M_{D_2} - M_H = M_{D_3} - M_H \). Notice that a mixing term between the \( S \) and \( T_\mu \) fields is absent at the leading order. Indeed, by saturating the \( \mu \) index of \( T_\mu \) with \( v^\mu \) or \( \gamma^\mu \) gives a vanishing result, and derivative terms are forbidden by the reparametrization invariance [12].

We add here the coupling of the vector meson light resonances to the positive and negative parity states

\[ \mathcal{L}_4 = \mathcal{L}_{S_\rho} + \mathcal{L}_{T_\rho} + \mathcal{L}' \] (4.8)

\[ \mathcal{L}_{S_\rho} = \frac{1}{\Lambda} < S \gamma^\rho \bar{S}^\mu > + i \frac{1}{\Lambda} < S \gamma^\rho F_{\mu \nu} > \] (4.9)

\[ \mathcal{L}_{T_\rho} = \frac{1}{\Lambda} < T^\rho \gamma^\rho \bar{T}^\mu > + i \frac{1}{\Lambda} < T^\rho \gamma^\rho F_{\mu \nu} > \] (4.10)

\[ \mathcal{L}' = \frac{1}{\Lambda} < H \gamma^\rho F_{\mu \nu} > + i \frac{1}{\Lambda} < H \gamma^\rho F_{\mu \nu} > \] (4.11)

We shall see in the following that some information on the coupling constants \( g, \mu, \lambda \) and \( \zeta \) can be obtained by the analysis of the semileptonic decays (1.1) and (1.2).

### 5 Weak currents

At the lowest order in derivatives of the pseudoscalar couplings and in the symmetry limit, weak interactions between light pseudoscalars and a heavy meson are described by the weak current [2]:

\[ L^\mu_a = \frac{\imath \alpha}{2} < \gamma^\mu (1 - \gamma_5) H_b \xi_{ba}^\dagger > \] (5.1)
where $\alpha$ is related to the pseudoscalar heavy meson decay constant $f_H$, defined by

$$< 0|\bar{q}_a \gamma^\mu \gamma_5 Q | P_b(p) > = ip^\mu f_H \delta_{ab}$$

(5.2)

as follows:

$$\alpha = f_H \sqrt{M_H}$$

(5.3)

We can in a similar way introduce the current describing the weak interactions between pseudoscalar Goldstone bosons and the positive parity $S$ fields:

$$\hat{L}^\mu_a = \frac{i\hat{\alpha}}{2} < \gamma^\mu (1 - \gamma_5) S_b \xi^\dagger_{ba} >$$

(5.4)

and the current by which the H fields interact with the light vector mesons:

$$L^\mu_{1a} = \alpha_1 < \gamma_5 H_b (\rho^\mu - V^\mu)_{bc} \xi^\dagger_{ca} >$$

(5.5)

All these currents transform under the chiral group similarly to the quark current $q\gamma^\mu (1 - \gamma_5) Q$, i.e. as $[3_L, 1_R]$. We also observe that there is no similar coupling between the fields $T^\mu$ and $\xi$. Indeed (5.1) and (5.4) also describe the matrix element between the meson and the vacuum, and this coupling vanishes for the $1^+$ and $2^+$ states having $s_t = 3/2$. This can be proved explicitly by considering the current matrix element ($A^\mu = \bar{q}_a \gamma^\mu \gamma_5 Q$):

$$< 0|A^\mu|\tilde{D}_1 > = \tilde{f} e^\mu$$

(5.6)

Using the heavy quark spin symmetry and the methods of the first two papers in ref. [1], (5.6) turns out to be proportional to the matrix element of the vector current between the vacuum and the $2^+$ state, which vanishes.

### 6 Semileptonic decays

Let us first consider the decay (1.1). The hadronic matrix element can be written in terms of the form factors $F_0$, $F_1$ as follows

$$< \Pi(p')|V^\mu|P(p) > = [(p + p')^\mu + \frac{M^2_H - M^2_P}{q^2} q^\mu] F_1(q^2) - \frac{M^2_H - M^2_P}{q^2} q^\mu F_0(q^2)$$

(6.1)

where $q^\mu = (p - p')^\mu$, $F_0(0) = F_1(0)$ and $M_H = M_P$ (see (2.18)). The form factors $F_0$ and $F_1$ take contributions, in a dispersion relation, from the $0^+$ and $1^-$ meson states respectively.

We notice here that, by working at the leading order in $1/m_Q$, the possible parametrizations of the weak current matrix element are not all equivalent. Computed in the heavy meson effective theory, the matrix element of eq. (6.1) reads:

$$< \Pi(p')|V^\mu|P(p) > = Av^\mu + Bp'^\mu$$

(6.2)

with $A$ and $B$ both scaling as $\sqrt{M_H}$ at $q^2 = q^2_{\text{max}} = (M_H - M_H)^2$ (where the theory should provide for a better approximation). The factor $\sqrt{M_H}$ which gives rise to this scaling behaviour comes just from the wave function normalization of the $P$ operator,
and no other explicit factor $M_H$ appears in the heavy meson effective field theory. If one introduces the usual form factors $f_+$ and $f_-$ through the following decomposition:

$$<\Pi(p')|V^\mu|P(p)> = f_+(p + p')^\mu + f_-(p - p')^\mu$$  \hspace{1cm} (6.3)

one has the relations:

$$f_+ = \frac{1}{2} \left( \frac{A}{M_H} + B \right), \quad f_- = \frac{1}{2} \left( \frac{A}{M_H} - B \right)$$  \hspace{1cm} (6.4)

It would seem consistent at this point to throw away the terms proportional to $A$, obtaining

$$<\Pi(p')|V^\mu|P(p)> \simeq Bp'^\mu$$  \hspace{1cm} (6.5)

which however does not reproduce the original expression of the matrix element. This is a clear contradiction since the two terms on the right hand side of eq. (6.2) scale in the same fashion. On the other hand, by making use of the decomposition of eq. (6.1) and working at the leading order we find:

$$F_1 = \frac{B}{2}, \quad F_0 = \frac{1}{M_H} (A + BM_\Pi)$$  \hspace{1cm} (6.6)

which, inserted back in the eq. (6.1), fully reproduces the matrix element given in eq. (6.2). The previous example shows that one must be very careful in the definition of the form factors when working at the leading order in $1/m_Q$ in the heavy meson effective field theory.

Using the previous lagrangians (2.11), (4.6) and the currents (5.1), (5.4) we obtain, at the leading order in $1/m_Q$ and at $q^2 = q_{\text{max}}^2$, the following results

$$F_1(q_{\text{max}}^2) = \frac{gM_H f_H}{2f_\pi (v \cdot k - \delta m_H)}$$  \hspace{1cm} (6.7)

$$F_0(q_{\text{max}}^2) = \frac{f'' \hat{\alpha} M_\Pi}{\sqrt{M_H f_\pi (v \cdot k - \delta m_S)}} - \frac{f_H}{f_\pi}$$  \hspace{1cm} (6.8)

The r.h.s. in (6.7) and the first term in (6.8) arise from polar diagrams. Finally $k^\mu$ is the residual momentum related to the physical momenta by $k^\mu = q^\mu - M_H v^\mu$ (and $p^\mu = M_H v^\mu$).

A similar analysis can be performed for the semileptonic decay process (1.2) of a heavy pseudoscalar meson $P$ with a light vector $\Pi^*$ particle in the final state. The current matrix element is expressed as follows

$$<\Pi^*(\epsilon, p')| (V^\mu - A^\mu)|P(p)> = \frac{2V(q^2)}{M_H + M_{\Pi^*}} \epsilon^{\mu\alpha\beta} \epsilon'_{\nu\rho\sigma} p_{\rho} A_1(q^2)$$

$$+ i(M_H + M_{\Pi^*}) \left[ \epsilon'_{\mu} - \frac{\epsilon^* \cdot q}{q^2} q_{\mu} \right] A_1(q^2)$$

$$- i \frac{\epsilon^* \cdot q}{(M_H + M_{\Pi^*})} \left[ (p + p')_{\mu} - \frac{M_{\Pi^*}^2 - M_{\Pi^*}^2}{q^2} q_{\mu} \right] A_2(q^2)$$

$$+ i \epsilon^* \cdot \frac{2M_{\Pi^*}}{q^2} q_{\mu} A_0(q^2)$$  \hspace{1cm} (6.9)
where
\[ A_0(0) = \frac{M_{\Pi^*} - M_H}{2M_{\Pi^*}} A_2(0) + \frac{M_{\Pi^*} + M_H}{2M_{\Pi^*}} A_1(0) \] (6.10)

Notice that the tensor structures given in square brackets of eq. (6.9) have vanishing divergence and are constant in the limit of infinite \( M_H \). Such a decomposition satisfies the same properties discussed above for the form factors \( F_0 \) and \( F_1 \). In a dispersion relation the form factor \( V(q^2) \) takes contribution from \( 1^- \) particles, \( A_0(q^2) \) from \( 0^- \) particles and \( A_j(q^2) (j = 1, 2) \) from \( 1^+ \) states.

Using the lagrangians (3.1) and (4.11) and the currents (5.1), (5.4) and (5.5) we get at \( q^2 = q^2_{\text{max}} \) and at leading order in \( 1/m_Q \) the results
\[ V(q^2_{\text{max}}) = \frac{g_V}{\sqrt{2}} \lambda f_H \frac{M_H + M_{\Pi^*}}{v \cdot k - \delta m_H} \] (6.11)
\[ A_1(q^2_{\text{max}}) = \frac{2g_V}{\sqrt{2}} \left[ \frac{\alpha_1 \sqrt{M_H}}{M_H + M_{\Pi^*}} + \frac{\hat{\alpha} \sqrt{M_H}}{M_H + M_{\Pi^*}} \right] \] (6.12)
\[ A_2(q^2_{\text{max}}) = \frac{\mu g_V}{\sqrt{2}} \frac{\hat{\alpha}}{\sqrt{M_H}} \frac{M_H + M_{\Pi^*}}{v \cdot k - \delta m_S} \] (6.13)
\[ A_0(q^2_{\text{max}}) = \frac{g_V}{2\sqrt{2}} \frac{\beta f_H M_H}{M_{\Pi^*} (v \cdot k - \delta m')} + \frac{g_V \sqrt{M_H}}{\sqrt{2}} \frac{\alpha_1}{M_{\Pi^*}} \] (6.14)

where \( \delta m' \) arise from the chiral breaking terms of Eq. (2.17). The first term in (6.12) and the last one in (6.14) arises from the direct coupling between the heavy meson \( H \) and the \( 1^- \) light resonances of Eq. (5.5) and the other ones from polar diagrams.

### 7 Numerical analysis

The results (6.7), (6.8) and (6.11)-(6.14) are obtained in the chiral limit and for \( m_Q \to \infty \); therefore they should apply (with non-leading corrections) to the decays \( B \to \pi \ell \nu_\ell \) or \( B \to \rho \ell \nu_\ell \). Unfortunately, for those decays there are not sufficient experimental results that could be used to determine the various coupling constants appearing in the final formulae.

On the other hand, for \( D \) decays the experimental information is much more detailed and we could tentatively try to use it to fix the constants as well as to make predictions on the other decays which have not been measured yet.

In order to make contact with the experimental data, we have to know the behaviour of the form factors with \( q^2 \). Except for the direct terms in (6.8), (6.12) and (6.14) all the contributions we have collected arise from polar diagrams, which suggests a simple pole behaviour. This is also the assumption usually made in the phenomenological analysis of \( D \) semileptonic decays. Therefore we have assumed for the form factors \( F_1(q^2) \), \( V(q^2) \), \( A_1(q^2) \) and \( A_2(q^2) \) (the form factors \( F_0(q^2) \) and \( A_0(q^2) \) are not easily accessible to measurement since they appear in the width multiplied by the lepton mass) the generic formula
\[ F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m^2}} \] (7.1)
For the pole masses we use the inputs in Table I \[11\] that also agree with the masses fitted by the experimental analyses of $D$ decays \[12\].

For the $D \to \pi$ semileptonic decay one thus gets, from (6.7) and (7.1):

$$F_1(0) = -\frac{g\alpha}{2f_\pi} \sqrt{M_D} \frac{M_{D^*} + M_D - M_\pi}{M_{D^*}^2}$$

$$= -\frac{g f_D (M_{D^*} + M_D - M_\pi)}{2f_\pi M_{D^*}^2} \quad (7.2)$$

For $f_D$ we use the value suggested by lattice \[13\] QCD and by QCD sum rules analysis \[14\], $f_D = 200 MeV$.

Experimentally one has \[15\] $|F_1(0)| = 0.79 \pm 0.20$, which implies

$$|g| = 0.61 \pm 0.22 \quad (7.4)$$

This result agrees with the result that would have been obtained using as input $D \to K$ semileptonic decay \[12\]: $|g| = 0.57 \pm 0.13$ and is also in agreement with the result obtained by a recent analysis of radiative $D^*$ decays: $|g| = 0.58 \pm 0.41$ (for $m_c = 1700 MeV$) \[16\].

Let us now turn to semileptonic decays into vector mesons. The experimental inputs we can use are from $D \to K^*\ell\nu_\ell$ and are as follows:

$$V(0) = 0.95 \pm 0.20$$
$$A_1(0) = 0.48 \pm 0.05$$
$$A_2(0) = 0.27 \pm 0.11 \quad (7.5)$$

They are averages between the data from E653 \[17\] and E691 \[18\] experiments. The calculated weak couplings at $q^2 = 0$ are:

$$V(0) = \frac{g_V \lambda (M_D + M_{K^*})(M_{D^*} + M_D - M_{K^*})}{\sqrt{2} M_{D^*}} \frac{\alpha}{\sqrt{M_D}}$$

$$= \frac{g_V \lambda (M_D + M_{K^*})(M_{D^*} + M_D - M_{K^*}) f_D}{\sqrt{2} M_{D^*}} \quad (7.6)$$

$$A_1(0) = -\sqrt{2} g_V \frac{(M_{D_1} + M_D - M_{K^*})\sqrt{M_D}}{(M_D + M_{K^*})M_{B_1}^2} \times \left[ \alpha_1(M_{D_1} - M_D + M_{K^*}) - \alpha'(\frac{z}{2} - \mu M_{K^*}) \right]$$

$$A_2(0) = -\frac{g_V \mu (M_D + M_{K^*})(M_{D_1} + M_D - M_{K^*})}{\sqrt{2} M_{B_1}^2} \frac{\alpha}{\sqrt{M_D}} \quad (7.7)$$

Taking $f_D = 200 MeV$, from Eq.(7.5), (7.6) and (7.8) we obtain:

$$|\lambda| = 0.60 \pm 0.11 \quad GeV^{-1} \quad (7.9)$$

$$\hat{\alpha} \mu = -0.06 \pm 0.02 \quad GeV^{3/2} \quad (7.10)$$

By using the result $\hat{\alpha} = 0.46 \pm 0.06 \ GeV^{3/2}$ from QCD sum rules \[14\], one obtains:

$$\mu = -0.13 \pm 0.05 \quad (7.11)$$
For the $A_1$ coupling the experimental data do not allow a separate determination of $\alpha_1$ and $\zeta$. However we notice that the combination:

$$\alpha_{eff} = \alpha_1(M_{D_1} - M_D + M_{K^*}) - \hat{\alpha} \left( \frac{\zeta}{2} - \mu M_{K^*} \right) \quad (7.12)$$

is almost flavour independent and, at leading order in the $1/M_Q$ expansion is scaling invariant. From the $D \to K^*$ data given in Eq.(7.5) we find:

$$\alpha_{eff} = -0.22 \pm 0.02 \text{ GeV}^{3/2} \quad (7.13)$$

We can now give predictions for the processes which the heavy quark and chiral symmetries relate to $D \to \pi$ and $D \to K^*$. Concerning $D \to \pi$, we can use Eq.(7.3) together with the value of the constant $g$ given in Eq.(7.4) to derive $F_1(0)$ for the various decays. Taking $f_B = f_D = 200\text{MeV}$ we obtain the results given in Table II. Notice that, by using $f_B = f_D$ in Eq.(7.3), we are implicitly accounting for the large corrections to the relation $f_B/f_D = \sqrt{M_D}/\sqrt{M_B}$, which is implied by lattice QCD and QCD sum rules results\cite{18}. Had we insisted in using the leading order expression of our computation, Eq.(7.2), we would have obtained the results shown in parenthesis in Table II, by fixing from the $D \to \pi$ data the product $g\alpha$. These results agree with the previous ones in the $D$ sector, but they obviously disagree for the $B$, predicting partial widths which are smaller by almost a factor of 3.

For the decays which are related to $D \to K^*$ the situation is more complex. We have not determined all relevant couplings of the effective lagrangian, from the $D \to K^*$ data. In particular we have determined a combination of $\alpha_1$ and $\zeta$, called $\alpha_{eff}$ and given in Eq.(7.12). In the expression of $V(0)$ we shall still choose $f_D = f_B = 200\text{MeV}$, in agreement with the lattice and sum rules calculations.

This approach leads to the results given in Table III, expressed as predictions for the transverse, longitudinal and total widths $\Gamma_T$, $\Gamma_L$ and $\Gamma$. For comparison we have also displayed in parenthesis the results obtained by working strictly at the leading order in $1/m_Q$, avoiding the identifications $\alpha = f_D \sqrt{M_D}$ and fitting from the $D \to K^*$ data the combinations $\lambda \alpha$, $\alpha_{eff}$ and $\hat{\alpha} \mu$. The predictions for the form factors $A_1(0)$ and $A_2(0)$ are in this case the same and, as a consequence, the predicted values for $\Gamma_L$ coincide for all the considered decays. On the other hand $V(0)$ for the $B$ decays is smaller if computed at the leading order. This implies a transverse width $\Gamma_T$ smaller by a factor two and a total width $\Gamma$ smaller by about a factor 1.6.

The results of Table III cannot be fully compared to experiments due to the lack of data. For the decay $D^+ \to \rho^0 \ell^+\nu_\ell$ one has the upper limit\cite{15} $BR < 3.7 \cdot 10^{-3}$, which is satisfied by our result $BR(D^+ \to \rho^0 \ell^+\nu_\ell) = 2.4 \cdot 10^{-3}$.

For the decay $B^- \to \rho^0 \ell^-\bar{\nu}_\ell$ we obtain $BR = 0.44 \cdot 10^{-3}$ (resp. $0.28 \cdot 10^{-3}$ in the leading order approximation for $f_B$ and $f_D$), to be compared with the ARGUS result\cite{21}: $BR = (1.13 \pm 0.36 \pm 0.26) \cdot 10^{-3}$, which however is not confirmed by CLEO collaboration\cite{22} that finds an upper limit of about $0.3 \cdot 10^{-3}$.

\footnote{After completion of this work we received a paper by Burdman\cite{20}. There, a formal argument is provided which supports the idea that in the semileptonic transition of the kind $P \to \Pi$ the non-leading corrections are mainly reabsorbable in the $f_D$ decay constant. A straightforward extension of the argument to the transition of the kind $P \to \Pi^*$ seems problematic.}
It is curious to observe that the leading order results could have been obtained in a model independent way by assigning, in the parametrization of the matrix element, the scaling behaviour of the various form factors. For instance, for the $D \to K^*$ process we can write:

\begin{align}
V &= \frac{v}{\sqrt{M_D}} \\
(M_D + M_{K^*})A_1 &= a_1 \sqrt{M_D} \\
\frac{A_2}{(M_D + M_{K^*})} &= \frac{a_2}{\sqrt{M_D}}
\end{align}

(7.14) (7.15) (7.16)

where $v$, $a_1$ and $a_2$ are constants as $M_D$ grows. This behaviour simply follows from the definitions of $V$, $A_1$ and $A_2$, and from the fact that the matrix element $< K^* | J^\mu | D >$ scales as $\sqrt{M_D}$. The above relations are valid at $q^2 = q_{\text{max}}^2 = (M_D - M_{K^*})^2$ and they should be appropriately modified at $q^2 = 0$. To do so we assume a simple polar behaviour for the form factors. Notice that the quantities $v$, $a_1$ and $a_2$ will in general depend on $M_D$, $M_{K^*}$ and the relevant pole mass $M_{\text{Pole}}$, with the restriction that they should be constant in the large $M_D$ limit. At $q_{\text{max}}^2$ the polar behaviour provides a factor:

\begin{equation}
\frac{M_{\text{Pole}}^2}{M_{\text{Pole}}^2 - (M_D - M_{K^*})} \sim \frac{1}{2} \frac{1}{M_{\text{Pole}}(M_{\text{Pole}} - M_D + M_{K^*})} \hat{v} (7.17)
\end{equation}

This factor exhibits a certain flavour dependence, which we may account for by incorporating it in $v$, $a_1$ and $a_2$:

\begin{equation}
v = \frac{\hat{v}}{(M_{\text{Pole}} - M_D + M_{K^*})} (7.18)
\end{equation}

and similarly for $a_1$, $a_2$. We can assume that $\hat{v}$, $\hat{a}_1$ and $\hat{a}_2$ are approximately flavour independent. In this way we obtain the following expressions

\begin{align}
V(0) &= \frac{(M_D + M_{K^*})(M_{\text{Pole}} + M_D - M_{K^*})}{M_{\text{Pole}}^2 \sqrt{M_D}} \hat{v} \\
A_1(0) &= \frac{(M_{\text{Pole}} + M_D - M_{K^*})\sqrt{M_D}}{(M_D + M_{K^*})M_{\text{Pole}}^2} \hat{a}_1 \\
A_2(0) &= \frac{(M_D + M_{K^*})(M_{\text{Pole}} + M_D - M_{K^*})}{M_{\text{Pole}}^2 \sqrt{M_D}} \hat{a}_2
\end{align}

(7.19) (7.20) (7.21)

The constants $\hat{v}$, $\hat{a}_1$ and $\hat{a}_2$ are determined by the data for $D \to K^*$ given in Eq.(7.5).

A comparison with our model gives:

\begin{align}
\hat{v} &= \frac{g_V \lambda}{\sqrt{2}} \alpha \\
\hat{a}_1 &= -\sqrt{2} g_V \alpha_{\text{eff}} \\
\hat{a}_2 &= -\frac{g_V \mu}{\sqrt{2}} \hat{\alpha}
\end{align}

(7.22) (7.23) (7.24)

therefore the predictions obtained from this scaling argument coincide with those obtained at leading order from an effective lagrangian.

In Table IV we compare our results, for $f_D = f_B = 200 MeV$ with other existing calculations. The comparison is made for the ratios of the form factors at $q^2 = 0$ to the corresponding form factors for the $D$ meson, from which we have fixed our parameters.
The leptonic decays of a heavy pseudoscalar meson into a light pseudoscalar or into an octet vector resonance have been studied with our effective lagrangian by including the allowed direct coupling and the lowest contributing poles. The formalism can be reliable only at $q^2_{\text{max}}$ and to leading order in $1/m_Q$. Most of the experimental information is available only for $D$ decays. To extract information at other momentum transfers one has to assume generic pole extrapolations. In this we follow the experimental phenomenological analyses. From the present data on semileptonic decay of $D$ into pion, and by using $f_D = 200 \, \text{MeV}$, we can extract for the coupling constant $g$ appearing in the effective chiral coupling of pseudoscalars with heavy mesons a value $|g| = 0.61 \pm 0.22$ in agreement with those obtained from radiative $D^*$ decays (and also from decay into $K$). We can then try to predict the branching ratios for the related decays $D \to K, D \to \eta, D_s \to \eta, D_s \to K, B \to \pi, B_s \to K$, as shown in Table II. A similar analysis for the decays related to $D \to K^*$ through heavy quark and chiral symmetries requires additional assumptions to arrive at the predictions shown in Table III for the transverse, longitudinal, and total widths. For the $D$ decays in that table one can develop a scaling argument leading essentially to the same predictions. On the other hand the numerical estimates for $B \to$ vector resonance with the dynamical model based on the effective lagrangian differ considerably from those of such a scaling argument, as also shown in Table III. Our predictions can be compared with those of other calculations in the literature. The comparison can be made in terms of the ratios of the form factors at vanishing momentum transfer to the $D$ meson corresponding form factors used to fix the parameters. Significant differences are noticed among different models, which shows the still uncertain status of the theory. The theoretical analysis we have presented here is based on a dynamically structured approach, using an effective lagrangian including the presumably relevant degrees of freedom. The existing data would not leave much space for a more accurate treatment by including non-leading contributions. Under such a limitation the model allows for predictions for the $D \to$ pseudoscalar leptonic decays, for the $D \to$ light vector resonance and, probably with some more uncertainties, for the $B \to$ light vector resonance leptonic decays.

The present status of the subject, in particular the still insufficient experimental data, do not yet allow for a more complete theoretical approach of a precision comparable to that of low energy applications of chiral lagrangians [26]. In this sense the calculations presented here are to be considered as still exploratory. Additional experimental data would greatly help in a better determination of the parameters of the effective lagrangian proposed here and in testing for the possible necessity of non-leading corrections.
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Tables Captions

Table I Pole masses for different states. Units are GeV.

Table II Predictions for semileptonic $D$ and $B$ decays in a pseudoscalar meson. We have neglected the $\eta - \eta'$ mixing. The branching ratios and the widths for $B$ must be multiplied for $|V_{ub}/0.0045|^2$. In the first column $f_D = f_B = 200$ MeV is assumed. In parenthesis the leading order result is assumed, i.e. $f_B/f_D = \sqrt{M_D/M_B}$. We also assume $\tau_{B_s} = \tau_{B^0} = \tau_{B^+} = 1.29$ ps.

Table III Predictions for semileptonic $D$ and $B$ decays into a vector meson. Partial widths are in units of $10^{11}$ s$^{-1}$. The branching ratios and the widths for $B$ must be multiplied for $|V_{ub}/0.0045|^2$. The first column refers to the case $f_B = f_D = 200$ MeV. The results in parenthesis have been obtained in the leading order.

Table IV Comparison among our predictions and other theoretical calculations of the form factors at $q^2 = 0$. The results in parenthesis have been obtained in the leading order.