Space(-Time) Emergence as Symmetry Breaking Effect

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Abstract

The microscopic origin of space(-time) geometry is explained on the basis of an emergence process associated with the condensation of infinite number of microscopic quanta responsible for symmetry breakdown, which implements the basic essence of “Quantum-Classical Correspondence” and of the forcing method in physical and mathematical contexts, respectively. From this viewpoint, the space(-time) dependence of physical quantities arises from the “logical extension” to change “constant objects” into “variable objects” by tagging the order parameters associated with the condensation onto “constant objects”; the logical direction here from a value $y$ to a domain variable $x$ (to materialize the basic mechanism behind the Gel’fand isomorphism) is just opposite to that common in the usual definition of a function $f : x \mapsto f(x)$ from its domain variable $x$ to a value $y = f(x)$.

1 Outline of the Problem

Before going into the main context, a comment would be necessary on the parenthesis in “Space(-Time) Emergence” in the title: in sharp contrast to the case of space, the emergence of time axis seems now doubtful. To justify this suspicion, we need re-examine its consistency with the space-time picture essential in special and general theories of relativity, which is not undertaken yet here. This is the reason for the expression “Space(-Time)”.

1.1 “Theory of Everything” vs. Duheme-Quine thesis

In search for a new theory to incorporate the old and standard one as a special case, one usually attempts trial-and-error searches in a heuristic way which seems to be unavoidable. How and to which extent can this be made systematic by the method for solving “inverse problem”? In this context, we note the existence of an obstruction to this possibility in such a form as “Duheme-Quine thesis”. This is just a No-Go theorem telling the

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impossibility to determine uniquely a theory from phenomenological data so as to reproduce the latter, because of unavoidable finiteness in number of measurable quantities and of their limited accuracy:

non-unique choices of starting “Micro”:

predictions: not 1-to-1 finite data with limited accuracy

inference: not onto at “Macro” level

Owing to inevitable errors in the measured data, the agreements between theoretical predictions and experimental data justify the former only as one of the possible candidates to explain the latter:

\[ \text{Theory 1} \not\xrightarrow{\sim} \text{Theory 2} \rightarrow \text{Experimental data + errors} . \]

Fortunately, our bi-directional method of “Micro-Macro duality”[1] can resolve this universal dilemma in harmony with the necessary and sufficient levels of accuracy determined by the inevitable restrictions on focused aspects and degrees of accuracy inherent in a certain pre-chosen context. Within such a context, a theoretical explanation can be unificed by “Micro-Macro duality” as a context-dependent “matching condition”[2] between the phenomena (“Macro”) to be described and the theory to describe (“Micro”); “Macro” in this mathematical formulation plays the roles of a standard reference frame characterized by its “universality” (in the mathematical and categorical sense). This naturally leads us to the idea of “matching condition” between inductive & deductive aspects for judging the correctness, which demarcates and characterizes the target domain of discourse.

1.2 “Geometrization principle” vs. Physical emergence of space-time

In contrast to the above resolution of Duheme-Quine thesis by Micro-Macro duality:

\[ \text{Micro} \xleftarrow{\text{deduction}} \xrightarrow{\text{induction}} \text{Macro}, \]

the “standard” approach regards the “rigorous” deductions from Micro to Macro as the only possible scientific paths to be followed. In this context, the starting point of Micro Theory consists simply of ad hoc postulates which cannot be justified within a theory itself, to be justified experimentally up to certain limited accuracy. With this point neglected, however, theoretical hypotheses on Micro quantum systems are always absolutized, in combination with the basic principle of “geometrization of physics” prevailing in modern physics. However, we need to ask what its foundation is:
it turns out to be based upon the successes of methods of modern geometry in mathematics and in its physical applications, such as general relativity, gauge theories, etc., which are essentially **of macroscopic nature**!! Almost all the basic principles governing modern geometries (differential, complex-analytic, algebraic, etc.) have been extracted from and applied to **classical and macroscopic levels** based on commutative algebras (of observables), and its **quantum versions** have only started to be sought for, without reaching mature stages yet!

In spite of strong emphasis there on the rigorous derivations of Macro-(scopically observable predictions) from Micro, neither the origin of **space-time as Macro**, nor **"Macro principle of geometrization"** seem to be well founded!? These pitfalls at both the Micro and Macro ends seem to be the two fatal defects of the fashionable trends in modern physics hidden in its blind spots. Therefore, we need explain the **microscopic origins of macroscopic structures of space-time geometry itself**: This is just the problem to search the physical origin and emergent processes of spacetime structure in microscopic physics, to be pursued in the following.

### 2 Universality inherent in Macro-levels

For this latter purpose also, the methods based on “Micro-Macro duality” will turn out to be quite effective, as shown below. In this context, what plays the most crucial roles can be found in the construction of a **Micro-Macro composite system** consisting of Micro and Macro levels based upon the **duality** between the two directions, deduction and induction:

i) deduction [Micro $\implies$ Macro] = a **bundle** structure

\[ A^i \hookrightarrow \mathcal{F} \xrightarrow{p} \mathcal{F}/A \simeq \text{Gal} (\mathcal{F}/A) \]

(formulated as an exact sequence, $\text{Im } i = \ker p$, of “triples” equipped with a tri-linear multiplication, e.g., \( A \times A \times A \ni (A, B, C) \mapsto \{A, B, C\} \in A \), depending linearly on $A$ and $C$ and anti-linearly on $B$) and

ii) [Micro $\iff$ Macro] induction = the corresponding **connections** defined as its **splittings**, \( \mathcal{A}^m \overset{h}{\hookleftarrow} \mathcal{F}/A \), characterized by one of the mutually equivalent three conditions,

\[ m \circ i = 1_A, p \circ h = 1_{\mathcal{F}/A}, i \circ m + h \circ p = 1_{\mathcal{F}}. \]

Example: The first law of thermodynamics describes a dilation $\Delta E = Q + W$ of **heat** $Q$ and of **work** $W$ into a closed dynamical system with conserved energy $\Delta E$. Here, the heat $Q$ symbolically represents the **uncontrollable** component in Macro-manifestation of invisible Micro-motions as **holonomy** in the thermal classifying space (consisting of the basic thermodynamic order parameters) and the work $W$ Macro-aspects of thermal
system directly controllable at Macro-level, both of which are unified via dilation into Micro-Macro composite system as a closed dynamical system with conserved energy. This relation can be expressed concisely in terms of the exact sequence in such a form as [Micro fibre: \( Q \to [\Delta E = Q + W: \text{Micro-Macro composite system}] \to [W: \text{Macro “base space”}] \)], where \( \text{Im } i \subset \ker p \) means that heat \( \in \text{Im } i \) cannot be transformed into controllable work as its projection by \( p \) equals 0, and, conversely, \( \ker p \subset \text{Im } i \) means that any energy \( \in \ker p \) unchangeable to work should be regarded as heat \( \in \text{Im } i \). Thus, the bundle structure + exactness can be seen to carry relevant physical or operational meanings.

Other interesting examples can also be found, for instance, in the theories of Maxwell and of Einstein in such forms as:

\[
\begin{array}{ccc}
\text{Micro} & \text{Micro-Macro} & \text{Macro} \\
J_\mu & F_{\mu \nu} & A_\mu \\
\uparrow & \rightleftharpoons & \uparrow \\
\Downarrow \text{Maxwell Eqn} & \Downarrow \text{Electromagmetoc Forces} & \Downarrow \text{Gravitational Force}
\end{array}
\]

and

\[
\begin{array}{ccc}
\text{Micro} & \text{Micro-Macro} & \text{Macro} \\
T_{\mu \nu} & R_{\mu \nu} & \Gamma^\lambda_{\mu \nu} \\
\uparrow & \rightleftharpoons & \uparrow \\
\Downarrow \text{Einstein Eqn} & \Downarrow & \\
\Downarrow \text{Gravitational Force} & &
\end{array}
\]

In many cases the above exact sequence takes such a form that \( \mathcal{A} \) and \( \mathcal{F} \) are \((C^\ast\text{-})\)algebras, and the triple \( \mathcal{F}/\mathcal{A} \) can be viewed as \( \mathcal{A}\text{-module } \mathcal{F} \) controlled by Galois group \( G = Gal(\mathcal{F}/\mathcal{A}) \) defined by such a subgroup of automorphisms \( Aut(\mathcal{F}) \) of \( \mathcal{F} \) as consisting of elements \( \in G \) fixing \( \mathcal{A} = \mathcal{F}_G \) pointwise.

In this case, \( Gal(\mathcal{F}/\mathcal{A}) = \hat{G} \) can be regarded as the totality of irreducible unitary representations of \( G \) (if such is meaningful) or the tensor category consisting of unitary representations of \( G \) and the map \( p: \mathcal{F} \to \hat{G} \) extracts the \( G \)-representation contents of each element in \( \mathcal{F} \). If we equip \( \mathcal{F} \) with an \( \mathcal{A} \)-valued inner product, \( \mathcal{F} \times \mathcal{F} \ni (F_1, F_2) \mapsto \langle F_1 | F_2 \rangle \in \mathcal{A} \), it becomes a Hilbert module with a right action of \( \mathcal{A} \), and a splitting, \( \mathcal{A} \rightleftarrows \mathcal{F} \rightleftarrows \mathcal{F}/\mathcal{A} = \hat{G} \), can be specified by the conditional expectation value \( \mathcal{A} \rightleftarrows \mathcal{F} \) arising from an \( \mathcal{A} \)-valued inner product of \( \mathcal{F} \) by \( m(F) = \langle 1 | F \rangle \) if \( 1 \in \mathcal{F} \) (or considering an
approximate unit of $\mathcal{F}$). Then, $\mathcal{F}$ can be recovered from $\mathcal{A}$ and $\hat{\mathcal{G}}$ as a Galois extension by a crossed product: $\mathcal{F} = \mathcal{A} \rtimes \hat{\mathcal{G}}$ which gives a typical example of **dilation** from Macro to Micro.

In this way, the duality between bundle structure $\mathcal{A} \overset{i}{\rightarrow} \mathcal{F} \overset{p}{\rightarrow} \mathcal{F}/\mathcal{A} \simeq Gal(\mathcal{F}/\mathcal{A})$ and its connection $\mathcal{A} \overset{h}{\twoheadleftarrow} \mathcal{F}/\mathcal{A}$ can be seen to condense the essence of **Fourier-Galois duality**, especially because the functors $Gal$ and $G \mapsto \hat{G}$ assign, respectively, a group $Gal(\mathcal{F}/\mathcal{A})$ to the $\mathcal{A}$-module $\mathcal{F}/\mathcal{A}$ and the representation contents $\hat{G}$ of $G$ to a group $G$.

Extending this Fourier-Galois theoretical machinery to a symmetry breaking situation of a $G$-dynamical system $\mathcal{F} \triangleright G$ with a fixed-point subalgebra $\mathcal{A} = \mathcal{F}^G$, we see below that a process of space(-time) emergence can be formulated as a kind of **symmetry breaking** in terms of the notions of an **augmented algebra** and of an associated **sector bundle** [2].

### 3 “Sector bundle” associated with broken symmetry

Breakdown of a symmetry of $\mathcal{F}$ in a state $\omega \in E_{\mathcal{F}}$ with a group $G$ into a subgroup $H \subset G$ of a remaining symmetry is characterized [2] by the non-invariance of the “central extension” of $\omega$ on the centre $\mathfrak{Z}_{\pi\omega}(\mathcal{F}) := \pi_{\omega}(\mathcal{F})' \cap \pi_{\omega}(\mathcal{F})'$ under the corresponding $G$-action on $\mathfrak{Z}_{\pi\omega}(\mathcal{F})$. In this situation, the role of algebra $\mathcal{A} = \mathcal{F}^G$ of observables is known in algebraic QFT to be replaced by the Haag-dual extension $\mathcal{A}^d$ owing to the breakdown $\mathcal{A} \nsubseteq \mathcal{A}^d$ of Haag duality, where the Haag-dual net $\mathcal{A}^d$ is defined with respect to a vacuum representation $\pi_0$ by $\mathcal{A}^d(\mathcal{O}) := [\pi_0^{-1}]((\pi_0(\mathcal{A}(\mathcal{O}')))')$ (for $\forall \mathcal{O}$: double cones in Minkowski spacetime), so that the sector structure is determined by the factor spectrum $\mathcal{A}^d = Spec(\mathfrak{Z}(\mathcal{A}^d)) = \hat{H}$: the group dual of a compact Lie group $H$ consisting of its irreducible unitary representations: $\mathcal{A} \mapsto \mathcal{A}^d = \mathcal{F}^{\hat{H}}$ ($\mathcal{F} = \mathcal{A}^d \rtimes \hat{H}$).

A general and desirable definition of the group $G$ of broken symmetry is not known yet in terms of the above data coming from the Haag dual net $\mathcal{A}^d$, but such a definition as $G := Gal(\mathcal{F}/\mathcal{A})$ with the field algebra $\mathcal{F} = \mathcal{A}^d \rtimes \hat{H} \supseteq \mathcal{A}^d = \mathcal{F}^{\hat{H}}$ and the group of unbroken symmetry: $\hat{H} = Gal(\mathcal{F}/\mathcal{A}^d) \subset G$, is sufficient for our purposes when the obtained $G$ is a finite-dimensional Lie group.

With $\tilde{\mathcal{F}} := \mathcal{A}^d \times \hat{G} = \mathcal{F} \rtimes (\hat{H}\setminus\hat{G})$ called an **augmented algebra** [2], we have a **split** bundle exact sequence $\mathcal{A}^d \overset{\tilde{m}}{\hookrightarrow} \tilde{\mathcal{F}} \twoheadrightarrow \tilde{\mathcal{F}}/\mathcal{A}^d \simeq \hat{G}$. In this situation, the **minimality** of $G$ and $\tilde{\mathcal{F}}$ is guaranteed by the $G$-**central ergodicity**, i.e., $G$-ergodicity of the centre $\mathfrak{Z}_{\tilde{\pi}}(\tilde{\mathcal{F}})$ in the representation $\tilde{\pi}$ given by the GNS representation of $\omega_0 \circ \tilde{m}$ induced from the vacuum state.
ω₀ of \( A^d \) [2], and we have the following commutativity diagram:

\[
\begin{array}{ccc}
\mathcal{F}^\mathcal{H} & = & \tilde{\mathcal{F}}^\mathcal{G} : \text{unbroken alg.} \\
\downarrow 1:1 & & \downarrow 1:1 \\
\mathcal{F} & \searrow & \tilde{\mathcal{F}}^\mathcal{H} : \text{extended observables} \\
\downarrow \text{onto} & \downarrow \text{onto} & \downarrow \text{onto} \\
\tilde{\mathcal{F}} : \text{augmented alg.} & \searrow & \tilde{\mathcal{F}}^\mathcal{H} : \text{extended observables} \\
\downarrow \text{onto} & \downarrow \text{onto} & \downarrow \text{onto} \\
\hat{H} & \leftarrow & \hat{G}
\end{array}
\]

whose dual version describes the sector structure:

\[
\begin{array}{ccc}
\tilde{\mathcal{F}}^\mathcal{G} & = & \tilde{\mathcal{F}}^\mathcal{H} \simeq \hat{H} : \text{unbroken sectors} \\
\downarrow \text{onto} & \downarrow \text{onto} & \downarrow 1:1 \\
\tilde{\mathcal{F}} & \leftarrow & \tilde{\mathcal{F}}^\mathcal{H} \simeq \hat{G} \times \hat{H} : \text{sector bdle} \\
\downarrow \text{onto} & \downarrow \text{onto} & \downarrow 1:1 \\
\hat{H} & \leftarrow & \hat{G} : \text{broken} \\
\end{array}
\]

where \( \tilde{\mathcal{F}} = \text{Spec}(\mathfrak{Z}(\mathcal{F})) \) denotes the factor spectrum of \( \mathcal{F} \), etc.

**Remark:** The physical essence of the extension \( A \rightarrow A^d \) from the original observable algebra \( A \) to its Haag-dual net algebra \( A^d = \mathcal{F}^\mathcal{H} \) can now be interpreted as an “extension of coefficient algebra \( A \)” by (the dual of) \( G/H \) to parametrize the degenerate vacua: \( A^d = \mathcal{F}^\mathcal{H} = \tilde{\mathcal{F}}^\mathcal{G} = [(\mathcal{F} \times (\hat{H} \backslash \hat{G}))^G = \mathcal{F}^G \times (\hat{H} \backslash \hat{G}) = A \times (\hat{H} \backslash \hat{G})]. \) In this extension, a part \( G/H \) of originally invisible \( G \) becomes visible through the emergence of degenerate vacua parametrized by \( G/H \) due to the condensation of order parameters \( \in G/H \) associated with \( S(\text{ponteous symmetry}) \) of \( G \) to \( H \). As a result, observables \( A \in A \) acquire \( G/H \)-dependence: \( \tilde{A} = (G/H \ni \hat{g} \mapsto \tilde{A}(\hat{g}) \in A) \in A \times (\hat{H} \backslash \hat{G}) \), which should just be interpreted as an example case of **logical extension** transforming a “constant object” \( A \in A \) into a “variable object” \( \tilde{A} \in A \times (\hat{H} \backslash \hat{G}) \) having **functional dependence** on the universal classifying space \( G/H \) for (multi-valued) semantics, as is familiar in the non-standard and Boolean-valued analyses. By replacing \( G/H \) with the space(-time), the above consideration can be utilized as a prototype for the origin of the functional dependence of physical quantities on space(-time) coordinates, due to the physical emergence of space(-time) from microscopic physical world.
Along this line, we prescribe the similar logical extension procedure on the observable algebra $A^d = \mathcal{F}^H$ adding $G/H$-dependence:

$$A^d \times (\widehat{H \backslash G}) = \mathcal{F}^H \times (\widehat{H \backslash G}) = (\mathcal{F} \times (\widehat{H \backslash G}))^H = \tilde{\mathcal{F}}^H.$$ 

Then, the whole sector structure of $\tilde{\mathcal{F}}^H = (\mathcal{F}^H \times (\widehat{H \backslash G}))$ can be identified with its factor spectrum $\tilde{\mathcal{F}}^H = G \times \hat{H}$; this is seen to constitute a bundle structure, $\hat{H} \hookrightarrow \tilde{\mathcal{F}}^H = G \times \hat{H} \twoheadrightarrow G/H$, called a sector bundle consisting of the classifying space $G/H$ of degenerate vacua, each fibre over which describes the sector structure $\hat{H}$ corresponding to the unbroken remaining symmetry $H$ (or, more precisely, the conjugated group $gHg^{-1}$ for the vacuum parametrized by $\dot{g} = gH \in G/H$).

Namely, the sector bundle, $\hat{H} \hookrightarrow \tilde{\mathcal{F}}^H = G \times \hat{H} \twoheadrightarrow G/H$, can be understood as the connection = splitting of the dual, $\tilde{\mathcal{F}}^H = \hat{H} \hookrightarrow \tilde{\mathcal{F}}^H = G \times \hat{H} \hookrightarrow G/H$, of the bundle exact sequence of observable triples, $\mathcal{F}^H \hookrightarrow \tilde{\mathcal{F}}^H = \mathcal{F}^H \times (\widehat{H \backslash G}) \to (\widehat{H \backslash G})^H$.

### 4 Emergence of space(-time) as symmetry breaking

We can now apply the above scenario to the situation with the group $G$ describing both the external (= space-time) and the internal symmetries. For simplicity, the latter component described by a subgroup $H$ of $G$ is assumed to be unbroken, and hence, the broken symmetry described by $G/H$ represents the space-time symmetry. It would be convenient to take $H$ as a normal subgroup of $G$, though not essential. To be precise, $G/H$ may contain such non-commutative components as spatial rotations (and Lorentz boosts) acting on space(-time), we simply neglect this aspect to identify $G/H$ as the space(-time) itself. Then, by identifying $G/H$ with a space(-time) domain $R$, we can notice a remarkable parallelism between the commutative diagram in the previous section:

$$\mathcal{F}^H = \tilde{\mathcal{F}}^G$$

$$\xymatrix{ & \mathcal{F}^H \ar@{^{(}->}[d] \ar@{^{(}->}[l]_{\hat{H}} & \times_{\mathcal{G}/\mathcal{H}} \tilde{\mathcal{F}}^G \ar@{^{(}->}[d] \ar@{^{(}->}[l]_{\tilde{H}} & \times_{\mathcal{G}/\mathcal{H}} \tilde{\mathcal{F}}^G \ar@{^{(}->}[d] \ar@{^{(}->}[l]_{\tilde{\hat{H}}} & \mathcal{F}^H \ar@{^{(}->}[d] \ar@{^{(}->}[l]_{\hat{H}} & \times_{\mathcal{G}/\mathcal{H}} \tilde{\mathcal{F}}^G \ar@{^{(}->}[d] \ar@{^{(}->}[l]_{\tilde{H}} & \times_{\mathcal{G}/\mathcal{H}} \tilde{\mathcal{F}}^G \ar@{^{(}->}[d] \ar@{^{(}->}[l]_{\tilde{\hat{H}}} & \mathcal{F}^H \ar@{^{(}->}[d] \ar@{^{(}->}[l]_{\hat{H}} & \times_{\mathcal{G}/\mathcal{H}} \tilde{\mathcal{F}}^G \ar@{^{(}->}[d] \ar@{^{(}->}[l]_{\tilde{H}} & \times_{\mathcal{G}/\mathcal{H}} \tilde{\mathcal{F}}^G \ar@{^{(}->}[d] \ar@{^{(}->}[l]_{\tilde{\hat{H}}}}$$
and the diagram controlling Doplicher-Roberts reconstruction of the local net $\mathcal{R} \rightarrow \mathcal{F}(\mathcal{R})$ from $\mathcal{R} \rightarrow \mathcal{A}(\mathcal{R})$: 

\[
\begin{array}{c}
\mathcal{O}_d \\
\downarrow \\
\mathcal{R} \\
\downarrow \\
\hat{\mathcal{R}}
\end{array}
\begin{array}{c}
\mathcal{O}_\mu = \mathcal{O}_d^G \\
\downarrow \\
\mathcal{A}(\mathcal{R}) \\
\downarrow \\
\mathcal{F}(\mathcal{R}) \\
\downarrow \\
\hat{\mathcal{Q}}
\end{array}
\]

where $\mathcal{O}_d := C^*(\{\psi_i, \psi_j^*\})$ is a Cuntz algebra consisting of $d$-isometries $\psi_i$ ($i = 1, 2, \cdots, d$): $\psi_i^* \psi_j = \delta_{ij} 1$, $\sum_{i=1}^d \psi_i \psi_i^* = 1$.

The crucial ingredients for this scenario are as follows:

1) The essence of transitions from invisible Micro with dynamical motions into visible Macro equipped with universal indices can be found physically and typically in the processes of condensation (to form condensed states), whose mathematical expression is nothing but the so-called “B-construction” (or “bar-construction”, “basic construction”, etc., with variety of names) and/or “heat kernel method” to extract topological and/or homotopical invariants forming a classifying space and playing the universal roles in classifying objects in question. Such a classical object as $G/H$ to classify degenerate vacua plays universal roles in the sector structure of SSB in such a form as the base space of the sector bundle, $\hat{H} \hookrightarrow \hat{\mathcal{H}} \rightarrow G/H$.

2) The notions of sectors or pure phases and mixed phases have been introduced to clarify the mutual relations between quantum Micro and classical Macro [2]. For this purpose, we need classify representations of the algebra of physical variables on the basis of quasi-equivalence which is just the unitary equivalence up to multiplicities. The minimal units in this classification are factor states or factor representations whose centres are trivial, and they are called sectors mathematically or pure phases physically. If a state or its GNS representation is not a factor because of its non-trivial centre, it is called a mixed phase which can be canonically decomposed into sectors or pure phases.

3) In the context of measurement processes, the above Micro $\Rightarrow$ Macro transitions are taking place in the amplification process to magnify the microscopic changes of quantum states at the contact points between the systems and the microscopic ends (= probe systems) of measuring apparatus into the macroscopic motions of measuring pointers [1, 3]. In these papers, the process of this sort is shown to be formulated as a Lévy process with (or, ideally without) small deviations from it. What is most important
in this process is the transitions from a mixed phase as a virtual probabilistic mixture of many different sectors or phases into their spatial configurations in the “real space”, each subdomain (= pointer position) of which is occupied by a single sector or phase. In the context of measurements, this phase separation is allowed to take place chronologically as is indicated by the Born statistics rule, whereas it can occur spatially or synchronically in some such thermal contexts as non-equilibrium states with certain degrees of stability.

4) The above problem of phase separation from the physical viewpoint can be viewed logically as the localized selections of the single truth value (in the sense of standard two-valued logic) from a multi-valued logic (in the context of Boolean-valued analysis of probability space). This process can be controlled by a logical method called “forcing” \[7\] (famous for P.Cohen’s use of it in the context of proving independence of continuum hypothesis from the ZF axioms of set theory), resulting in a topos of sheaves on the classifying space \(G/H = \mathcal{R}\) (consisting of degenerate vacua) whose core member is given by a sheaf \(\Gamma(G \times \hat{H})\) of sections of the sector bundle

\[
\hat{H} \hookrightarrow \hat{\mathcal{F}}^H = G \times \hat{H} \rightarrow G/H = \mathcal{R}
\]

describing the sector structure of the algebra \(\hat{\mathcal{F}}^H\) of extended observables in terms of its factor spectrum \(\hat{\mathcal{F}}^H\). This is to be put in parallelism with the sheaf \(\mathcal{R} \mapsto E_{\mathcal{A}(\mathcal{R})}\) of local states in DHR sector theory, which means that the notion of local states \(E_{\mathcal{A}(\mathcal{R})}\) of a local algebra \(\mathcal{A}(\mathcal{R})\) in a spacetime domain \(\mathcal{R}\) should be understood to correspond to a choice of a family of degenerate vacua in \(G/H = \mathcal{R}\) arising from SSB, namely, to the context of considering states of extended observables \(\hat{\mathcal{F}}^H\) in reference to each member of degenerate vacua belonging to \(G/H\). This parallelism clearly shows the existence of quantum fluctuations inside of each space(-time) point \(x \in G/H = \mathcal{R}\) to which extent space(-time) points are highly non-trivial physical objects!!

5) The differences in the degrees of stability mentioned in 3) may be related in a meaningful way to the corresponding differences in changeability between some items to be put one place to another and the certain stable behaviours of the “names” attached to specific places. To systematize such degrees of stability may be quite relevant for satisfactory understanding of the various stabilized domains appearing in different levels in nature and also of hierarchical structures of biological organisms, from the viewpoint of Grothendieck’s topoi and sites.

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