Economical Aspects of the Vehicle Scheduling Optimization

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The paper deals with the vehicle scheduling problem related to regional public transport. Linear programming methods are used to solve the problem. A mathematical model is created including the constraints and the objective function minimizing costs and the number of vehicles. A minimum costs and a number of vehicles are forced at the same time by special economical input data analysis and an allocation of costs. Determining of the costs coefficients is done by three methods, which differs primarily by how much of the total costs they take into account. The decomposition of the set of lines into disjoint subsets can be used instead of the “direct” optimization. The decomposition has proven to be a suitable alternative in solving large optimization problems. The problem was applied to optimize vehicle scheduling in the region, which is situated in the north-east of the Czech Republic. There is used Xpress-IVE software, which solve the problem by simplex algorithm and branch and bound method. Research results show that there are large reserves in the organization of public transport. The implementation of the new vehicle scheduling would bring significant costs reductions in amount of at least 10% for the optimal solution and in amount of about 10% for the decomposition solution. The number of drivers could be decreased and the total time of the vehicles being outside the garage could be also reduced by at least 10%.

Keywords: public transport, optimization, vehicle scheduling, linear mathematical modeling, transport economy, decomposition of input data

Introduction

Grant demands of the public transport have increased substantially in the Czech and Slovak Republic in the last 20 years. It is an interest of the state to reduce subsidy for the public transport. The main goal for carriers to the future should be preparation and intensive searching for the savings in their management. Significant savings have to be achieved simply by changing an organization of work. It is important to use linear programming methods (LP) (Daněk & Teichmann, 2005; Černý & Kluvánek, 1990) for solving the optimization problem of the vehicle scheduling (Baita, Pesenti, Ukovich, & Favaretto, 2000). What objectives can be achieved using these methods? Which opportunities are opening for carriers by solving the problem (see Figure 1)?

Basic Reasoning

The main goal in the task of the optimizing the vehicle scheduling is proposing such a sequence of every single transport link (i.e., train, airplane, bus connection etc.) operations for each vehicle, in order to achieve
the minimum or the maximum value of an optimization criteria. It seems most appropriate to choose as an optimization criterion one of two: the total costs of operation and the number of vehicles (Surovec, 2000). However, the minimum would be necessary to search all at once. Using a multi-criterion optimization function has many disadvantages. That’s why it is appropriate to use the following solution. The transport link operation costs are chosen as an optimization criterion. The depreciation (i.e., the cost of the vehicle) for one planning period includes costs of the vehicle crossing from the garage to the first stop of the first circuit which operates the vehicle that day. If the vehicle throughout the planning period does not serve any link, it is not necessary to put the vehicle into the fleet carrier. Then the objective function is not burdened by a depreciation expense. It is ensured that the given set of transport links will be operated at the lowest costs. It is also expected that the number of deployed vehicles will be minimal.

**Figure 1.** SWOT analysis of solving vehicle scheduling problem with linear programming methods.

**The Mathematical Model**

First, it is necessary to define what is about to decide in the vehicle scheduling problem (Palúch, 1993):

- \( z_{ijk} \) bivalent variable modeling the decision of whether the vehicle \( i \in I \) may pass between the end stop of the transport links \( j \in J \) and \( k \in K \).

The variable \( z_{ijk} \) is introduced only in those cases for which the introduction is necessary:

- \( z_{ijk} \in \{0, 1\} \) for \( i \in I, j \in J \) and \( k \in K \).

The definition of the sets can be elaborate as follows:

- \( I \): the set of vehicles, which can be deployed on the operation of the transport link;
- \( J \): the set of the transport links \( j \) after which, you can change the default stop of transport link \( k \) to the time of departure of the link \( k \);
- \( K \): the set of the transport links \( k \) to which it is possible to drive after finishing of operating the link \( j \) in the time of departure of the link \( k \).
• $S$: the set of the garages.

The objective function minimizing the costs takes the form of equation (1):

$$
\min f(z) = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K \cup S} a_{ijk} \cdot z_{ijk}
$$

(1)

• $a_{ijk}$: total costs of the vehicle operation of the transport link $j$ plus the costs of the crossing between the links $j$ and $k$ plus the costs of vehicle’s waiting between the operation of the link $j$ and $k$.

It is necessary to specify all the requirements on the mathematical model:

• deploying the required type of the vehicle on the link (1a);
• avoid the “return of the vehicle at the time” (1b);
• the correct sequence of the technological activities (ensured by the constraint 2);
• the operating each transport link (3);
• every vehicle exits the garages only once during the planning period (4);
• limiting the length of the shifts and the hours of operation of the vehicle (5).

The simplicity of the model is ensured by the introduction of the variable $z_{ijk}$ only in cases when the declaration is necessary. This declaration also meets some of the above requirements: (1a) and (1b). The “return at the time” means a situation where the vehicle after finishing operating link in a given time shall be deployed on operation of the other link with the start time earlier.

The correct sequence of technological operations (vehicle crossing from the garage to stop of the link, operating link, vehicle crossing between the final stop of links, crossing between the final stop of the vehicle and garages) can ensure equation (2):

$$
\sum_{k \in K \cup S} z_{ijk} = \sum_{k \in J \cup S} z_{ikj} \quad \text{for } \forall i \in I
$$

(2)

The operating of each link guarantees (3):

$$
\sum_{i \in I} \sum_{k \in K} z_{ijk} = 1 \quad \text{for } \forall j \in J
$$

(3)

The constraint (4) ensures that each vehicle exits from the garages at most once:

$$
\sum_{k \in K} z_{ijk} \leq 1 \quad \text{for } \forall i \in I, \ j \in S
$$

(4)

It is necessary to define a variable $t_{ijk}$. It represents the duration time of the transport link $j$ plus the transit time from the end of the link $j$ to the starting point of link $k$ plus waiting to start operating link $k$.

The constraint (5) ensures that the time of the operation of the vehicle $i$ will not exceed $T_{i}$. If it is assumed that the driver is over the whole operation “assigned” to the same vehicle, the constraint analogous limits the length of the driver’s shifts to $T_{i}$.

$$
\sum_{j \in J} \sum_{k \in K} t_{ijk} \cdot z_{ijk} \leq T_{i} \quad \text{for } \forall i \in I
$$

(5)

The constraints (5), (6) define the domain of variables:

$$
z_{ijk} \in \{0, 1\} \quad \text{for } \forall i, j, k
$$

(6)

$$
t_{ijk} \in Z_{0}^{+} \quad \text{for } \forall i, j, k
$$

(7)

**Decomposition of the Set of Transport Links**

The number of the transport links is a major factor affecting the computational time and the time of
processing the input data. The decomposition set of links to \( n \) disjoint subsets can be also used to achieve optimum solution. The components of the set of links are given by the time interval. The link is inserted into the subset of links based on time of departure from the default stop. The problem is solved separately for each set of components. Separated links sequences operated by the vehicle are suitably connected to achieve the minimum total costs. This procedure does not guarantee an optimal solution, but it can be assumed that the obtained solution is not far from the optimum.

The Economical Input Data Analysis

It is necessary to accurately determine the value of all inputs appearing in the mathematical model in order to achieve valid results with good predictive value. It is difficult to obtain relevant information about the management of private carriers. That is the reason why it is appropriate to calculate the costs of driving and parking of vehicles in several ways, based on different and independent data (Surovec, 2000). The results are compared in Table 2. Among other the following procedures may be used:

1. The determination of the relative costs coefficients;
2. The determination of the costs coefficients of the costing formula (Byrtusová, 2009; Guzej, 2009);
3. The determination of the costs coefficients according to the Ostrava Transport Company (DPO).

![The cost structure of the DPO in 2008](image)

**Table 1**

| The type of driving the vehicle                      | Percentage of the total costs (%) |
|-----------------------------------------------------|----------------------------------|
| Driving and crossing the vehicle by driver          | 80.61                            |
| Parking of the vehicle with driver                  | 47.91                            |
| Parking of the vehicle without driver               | 5.64                             |
| Vehicle ownership (depreciation + repair, service)  | 19.39                            |

Finding the minimum costs and the minimum number of vehicles is needed at the same time. It is done the following way. The depreciation (i.e., the cost of the vehicle) for one planning period is included into costs of the vehicle crossing from the garage to the first stop of the first circuit which operates the vehicle that day.

Procedure No. I allocate the costs into several groups (see Figure 2 and Table 1).

Procedure No. II is based on the calculation formula for the road transport. This formula divides the cost items in dependent and independent. To the first group belong:

- direct materials (fuel and oil, tires and other direct material);
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- direct wages (drivers wage, health and social insurance).

To the second group belong:
- depreciation;
- the other direct costs (repairs and maintenance, insurance, road tax, fees, other direct costs). Results of the calculations are listed in Table 2.

Procedure No. III is based on the information, which carriers are willing to tell. It is costs in $CZK \cdot km^{-1}$ and average speed of vehicles in $km \cdot h^{-1}$. According to the DPO the costs are:
- bus (standard construction) 1,184 $CZK \cdot h^{-1}$;
- “long” bus 1,408 $CZK \cdot h^{-1}$.

Results done by three mentioned methods are listed in Table 2. The methods of calculation differ primarily by how much of the total costs they take into account (see comments).

Table 2

| The computational method                                      | The costs of the driving vehicle | The costs of the parking vehicle | Comments                                                                 |
|---------------------------------------------------------------|---------------------------------|---------------------------------|--------------------------------------------------------------------------|
| The determination of the relative costs coefficients          | 1.0000 [−]                       | 0.6755 [−]                      | The calculation takes into account the 100% of the costs of the carrier.  |
| The determination of the costs coefficients of the costing formula | 614.75 $CZK \cdot h^{-1}$ (8 years depreciation) | 251.59 $CZK \cdot h^{-1}$ (8 years depr.) | The calculation takes into account of the costs referred in the costing formula. |
|                                                               | 666.12 $CZK \cdot h^{-1}$ (5 years depreciation) | 302.96 $CZK \cdot h^{-1}$ (5 years depr.) |                                                                  |
| The determination of the costs coefficients according to the DPO | 1184/1408 $CZK \cdot h^{-1}$ (standard/long bus) | -                               | The calculation takes into account all of the costs arising when the vehicle is in motion. |

It is necessary to use only the values referred in the costing formula with a depreciation of vehicles five years, because the optimization of the vehicle scheduling can only affects the operating costs. The costs are therefore considered 666 $CZK \cdot h^{-1}$ for a vehicle in motion and at the 303 $CZK \cdot h^{-1}$ for the parking vehicle with a driver.

Application Into the Practice

There are used Xpress-IVE software (Dash Optimization, 2008, 2009), which solve the problem by simplex algorithm and branch and bound method. Results are listed in Table 3.

The proposed attitude to the problem was applied to optimize vehicle scheduling in the region, which is defined in the south by the towns of Ostrava, Hlučín, Dolní Benešov, Kravaře and in the north by the border between the Czech Republic and Poland. This is the link, No 70, 72, 281, 282 and 283. All of them are finished at the stop called Přívoz, Muglinovská in the urban district of Ostrava-Přívoz (IDOS - Vlaky + Autobusy - Vyhledání spojení, 2012).

The optimization was performed in a relatively small group of links. The implementation of the new vehicle scheduling would bring significant costs reductions in amount of 16.4% for the optimal solution and in amount of 14.3% for the decomposition solution. The number of drivers has decreased from 14 to 13 because the total time of the vehicles being outside the garage has also decreased by about 13%.

If the minor timetable changes are accepted, further substantial savings could be achieved by reducing the number of vehicles and the number of drivers.
Table 3

The Comparison of the Current State With Decomposition and With Optimal Solution

|                                | The current state | The optimum solution | The index of the original/optimal solution | The solution obtained by the decomposition | The index of the original/decomposition |
|--------------------------------|-------------------|----------------------|-------------------------------------------|-------------------------------------------|-----------------------------------------|
| The total calculated costs [CZK] | 65,526            | 62,937               | 0.960                                     | 63,268                                    | 0.966                                   |
| The costs (without the operating links and the depreciation) [CZK] | 15,739            | 13,150               | 0.836                                     | 13,481                                    | 0.857                                   |
| The total time of the vehicles being outside the garage [hours] | 131               | 113                  | 0.863                                     | 114                                       | 0.870                                   |
| The number of vehicles [-]     | 8                 | 8                    | 1.000                                     | 8                                         | 1.000                                   |
| The number of drivers [-]      | 14                | 13                   | 0.929                                     | 13                                        | 0.929                                   |
| The number of crossings between the links [-] | 0                 | 5                    | -                                        | 6                                         | -                                       |

Conclusions

The decomposition of the set of links has proven to be the suitable alternative in solving large optimizing problems. It affects the value of the objective function by only 0.6%. It is believed that using linear programming techniques with powerful computers is the way to the rational organization of public passenger transport, which will leads to its improvement and subsequent revival.

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