Evaluation of color image interpolation based on incompressible Navier Stokes technique

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ABSTRACT
Color image interpolation encompasses reconstructing parts of a video or an image based on information from the neighbor. Technique involves the restoration of noised photos and animation or image denoising. The Navier-Stokes (NS) technique has been widely investigated as an essential research by image restoration. These NS equations contribute spectacular results for producing an animation as they augment reality. They can boost real-time video games to be more sensible than ever. In this paper, we present the Incompressible NS approach (INS) for color image interpolating. The method per se is based on fluid flow concept to circulate directed lines from the peripheral into the area to be interpolated. The image intensity represents stream function in a computational flow of 2D fluid dynamics. The algorithm is implemented to carry on lines regarding gradient vectors at the edge of the interpolating region. It uses the improvement of powerful numerical analysis. It is also proven as an innovative idea for easing problems in image analytics as well as computer vision.

Keywords:
Computer vision
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1. INTRODUCTION
The research on dynamic flows [1] plays a noticeable role in a broad range of computer applications. The automobile industries, game companies, and space studies are regularly in need for an animation model of the 3D graphics to simulate video game and image restoration that can experience the adversity of reality. As a human, the investigation of image processing can bring substantial enhancements to existing computer graphic systems. For example, the reconstruction and denoising of underlying damaged images introduced by [2] use the recursive shrinking threshold algorithm, then the image is reconstructed by the gradient vector method. This approach is attentive for X-ray luminescence tomography industries as it identifies the dissemination of nanophosphors in object images. Above all, we spend most of the time delimited by thermo dynamic flows which agitate human activities regularly. Over the past few years, computational fluid dynamics (CFD) model [3] arises from the dynamic flows concept to examine the performance of streams. It reconstructs mathematically the progression of unsteady or steady streams from the originality. The experimental fluid dynamics (EFD) model [4] detects 3D annotations of each element of the line direction.

Digital interpolation algorithm [5] has extended presentations in photo reconstruction, image restoration, and object zooming. It brings up the use of algorithm to discard degraded fragments of object images. In computer vision [6], we kick off with a visual image or an array of them. They are image systems such as biomedical images, satellite images or artistic paintings. Most digital devices are equipped with computer vision and image processing algorithms [7]. A basic maneuver in image processing includes the
filtering of image’s intensity. It estimates the 1st order derivative filters for animation or border detection, and the 2nd order or higher for characteristics collection. For animation control and shape detection [8], the approximation of gradient descends [9] of the intensity is critical. It can result that the calculation of both magnitude of gradient vectors and its orientation is invalid. Then, option must be selected based on the orthogonality of gradient vectors. Therefore, it is confirmed that the direction of the gradient descends is more vital than the magnitude. The filtering technique is based on finite values using convolution optimization [10]-[12] regarding the presence of noise in the image or numerical model assumption.

The fundamental impression for most of the interpolating methods is to perform a refined dissemination of the information in the area encompassing the interpolating region and inpainting curvatures in an accurate manner. K. V. Madhavi et al. [13], authors propose to reconstruct the missing information using correlation between the missing block and its surroundings. In an associated article [14], the idea is magnified to guarantee that the image segmentation is applied for a combination of curvatures in order to restore the image. O. O. Akanni, et al. [15], a relation between the NS equations and the isophote (level line) direction of the image is presented and authors propose to simulate a propagation of carbonate matrix to seal the interpolating domain. Another connected article is introduced in [16]. Authors minimize the divergence to develop optical flow functions.

The article in [17] motivates a sophisticated technique to denoise based on total variation (TV) approach and discretization. The inpainting algorithm in [18] is restricted to forming straight level-lines, easily discontinued from the hole edge, and is evolved for a small hole. Note that it is to cultivate a model that allows inpainting of isophotes through large gaps, in which linking with straight connections is unattractive for modest images although direct lines provide obviously useful results for small holes. To find a continuation and smooth interpolation of level lines, the technique based on high-order partial differential equations (PDE) is presented by M. C. Leseduarte and R. Quintanilla [19]. An estimation approach to the fragmental derivation [20] using finite integration with the quadratic interpolation polynomial in PDE is also given in [21]. W. Zhou et al. [22] maintain the concept of executing a smooth continuation of the arrival angle of the isophotes. On the other hand, reflection waveform inversion [23] has been suggested to form the velocity model by confining the sensitivity of full waveform regarding two ways communication.

2. INCOMPRESSIBLE NS (INS) METHOD

The INS approach [24] to find out the image interpolation problem is a way to impress how color image can be restored physically. Authors inpaint extended boundaries from the edge of $\Omega$, link these stretched edges, and then fill-in the region correspondingly. The concept has been scientifically verified to fabricate acceptable result, or if negative satisfied solution. The article demonstrates an algorithm intended to resolve the smoothing gradient of the color image convolution in the lines direction. The derivative design is a discontinuous estimation of the PDE [19] as follows.

$$I_c = \nabla^\perp \cdot \nabla \Delta f$$  \hspace{1cm} (1)

$I$ denotes the image convolution, $\Delta$ is the Laplacian operation ($\partial_x^2 + \partial_y^2$), and $\nabla^\perp$ represents the vertical gradient ($-\partial_y, \partial_x$). Image’s anisotropic diffusion can be calculated by

$$I_c = \nabla \cdot (g(|\nabla|)\nabla) + \nabla^\perp \cdot \nabla \Delta f.$$  \hspace{1cm} (2)

The objective is to determine a steady state result from either (1) or (2), but befitting the circumstance in the intra-region that lines are in the trend of vertical gradient ($-\partial_y, \partial_x$). It has to be analogous to curvatures of smoothing image using the convolution ($I$), and when $\nu = 0$ we have the solution for the interpolation as shown in (3) and (4).

$$\nabla^\perp \cdot \nabla \Delta f = 0$$  \hspace{1cm} (3)

Incompressible Newtonian flow follows the NS equations,

$$\nu_c + \nu \cdot \nabla \nu = \nu \Delta \nu - \nabla p, \nabla \cdot \nu = 0,$$  \hspace{1cm} (4)

Where $\nu$ denotes the velocity vector field, $\nu$ represents the anisotropic diffusion, and $p$ is a scalar pressure. In two dimensional flows, the free velocity field introduces a stream function $\Psi$, where $\nabla \cdot \Psi = \nu$. Let the smoothness, $\omega = \nabla \times \nu$, then the smoothness-stream function can be calculated by eliminating $p$ in (4), taking the curl of (1), and applying some facts about the geometry in two dimension:

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\[ \omega_c + v \cdot \nabla \omega = \nu \Delta \omega. \]  \hspace{1cm} (5)

In case of absence of the anisotropic diffusion, \( v \approx 0 \), the smoothness-stream function shown in (5) implies the steady state solution as follows.

\[ v \cdot \nabla \omega = \nabla^2 \Psi \cdot \nabla \Psi \approx 0 \]  \hspace{1cm} (6)

In image processing, the counterpart to the smoothness-stream function can be formulated as listed below.

\[ \frac{\partial w}{\partial c} + v \cdot \nabla w = \nabla \cdot (g(|\nabla w|) \nabla w). \]  \hspace{1cm} (7)

Where \( w = \Delta I \), the smoothness of image convolution, and \( v = \nabla^2 I \), the velocity field of image convolution. The \( g \) explains a boundary-preserving diffusion and deserves \( g(0) = 1, \lim_{n \to 0} g(n) = 0 \), and is always declining uniformly.

The image convolution, \( I \), can be figured out by resolving the Poisson’s formulation

\[ \Delta I = w, I_{\frac{\partial}{\partial n}} = I_0. \]  \hspace{1cm} (8)

In case of small \( v \), the aforementioned flow takes lengthy time to reach the steady state; therefore there is pseudo-steady state that accounts for the Poisson problem, so called a relaxing equation [25] shown as follows.

\[ I_c - \alpha (w + \Delta I) = 0, \alpha > 0, I_{\frac{\partial}{\partial n}} = I_0. \]  \hspace{1cm} (9)

3. EXPERIMENTAL RESULTS AND ANALYSIS

We illustrate using deteriorated images to check the performance of the INS approach. In each image presented below, the INS equations are executed in the interpolating region. Let us begin with calculating the smoothness \( \omega \) from the individual image \( I \), using information from the outside to verify the edge of the smoothness. We can obtain the smoothness-stream function through an advance Euler timing, with alternative centers in space domain for the anisotropic diffusion (which helps improve edges) and the relocation. After each step we calculate the image convolution \( I_c \) by solving the Poisson formulation iteratively. The algorithm shown in Figure 1 is implemented in number of lines of Matlab code. The results shown below are achieved in seconds of CPU time of a regular Window-based PC. We work with color images and execute interpolation on two gears separately (a color image and a mask), and the results in the end. In the experiment we consider a deteriorated image as shown in figure left and the recovered result upon INS interpolation is listed in figure right. Note that the result is improved regarding denosing, edges are illuminated (artifacts disappear) and restored. However, these are examples for the interpolation, not a complete recovered metric. The image interpolation is illustrated in Figure 2. Next examples use a different masking approach to investigate whether or not the mask will affect the quality of end results of an image. We have taken an inverse mask and \( r \), radius of the interpolation process into our account. Note how the mask and variable \( r \) can recover the original image. These results are illustrated in Figure 3.

| Proposed Algorithm |
|--------------------|
| **Require:** Defected image \( I \) |
| **Ensure:** \([I]_{xy} = \text{Interpolating region (} x, y \text{)} \) of the image \( I \) |
| for \( I = 1 \) to \( x \) do |
| for \( J = 1 \) to \( y \) do |
| Compute the smoothness \( \omega \); |
| Verify the edge using Euler timing; |
| Allocate different centroids in space domain for the anisotropic diffusion; |
| Compute the convolution \( I_c \) upon Poisson function; |
| end for |
| end for |

Figure 1. Proposed algorithm
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Figure 2. Deteriorated images (left) and their associated recovered images (right)

Figure 3. Deteriorated images (left) and their recovered images using inverse mask with $r=0.3$ (middle), and $r=3$ (right)
4. CONCLUSION

The work evaluates the application of INS approach for color image recovery. The used technique in this article leverages an image convolution and transforms a stable input from deteriorated image to a recovered output. Comparisons are also conducted for an inverse masking scheme with the INS algorithm. Image recovery is executed by varying radius parameter into an account during the process. The experiment presented in this article confirms that the INS will not applicable to the specific damaged images. The different levels of noise degrade an original image that cannot always be perfectly recovered although it is anticipated that a state-of-the-art method like INS would complete comparatively well. Rather, this method does not provide the optimized solution for some image recovery experiments. Future work includes the investigation how to determine the degree of noise for limiting deterioration and the implementation of a practical image recovery algorithm.

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