A Hybrid Calculation Method of Tectonomagnetic Effect Using BEM and the Surface Integral Representation of the Piezomagnetic Potential —Two Dimensional Case Study—

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In order to evaluate changes in the geomagnetic field due to various tectonic models, a new method using the surface integrals over magnetized two dimensional bodies is presented. This method is based on combination of the boundary element method (BEM) with the surface integral formula. BEM allows calculations for arbitrarily shaped magnetoelastic bodies. The technique using surface integrals saves CPU time compared with a code using volume integral formula. In particular, BEM method can deal with complicated tectonomagnetic problems including actual surface topography.

In order to test the new method, we make comparative calculations for what is called Yukutake model whose solution has been well investigated. The result for Yukutake model obtained using the present method is essentially equivalent to those obtained using numerical volume integrals.

The method is also applied to tectonomagnetic problems in which topographic effect is taken into account. Initial calculations of topographic effect indicate model corrections are about 1nT or so.

1. Introduction

It has been often reported that locally anomalous changes in the geomagnetic field were observed in association with tectonic activities such as earthquake occurrences or volcanic eruptions. Since the proton magnetometer has been practically used in 1960's, geomagnetic measurements become more accurate. The proton magnetometer is sufficiently stable for long term operation and does not require any skillfulness in measurement.

It is, however, no easy matter to detect the anomalous geomagnetic changes with a sufficient reliability. Except for the tectonomagnetic changes observed in active volcanic areas, large changes more than 10 nT have not been observed since 1960's (Rikitake, 1968). The problem is mainly due to difficulty in discriminating small changes of the geomagnetic field in association with tectonic activity against large geomagnetic variations originating from external sources and/or those from artificial noises particularly around urban areas. In order to eliminate non-regional changes included in observed values, we usually calculate simple differences in the total intensity between an observation site and a reference station. Also we often adopt a particular technique of the weighted mean difference to obtain higher accuracy in geomagnetic noise reductions (e.g. Rikitake, 1966; Mori and Yoshino, 1970).

Locally anomalous changes in the geomagnetic field in association with crustal activities can result from the following possible mechanisms: changes in rock magnetization due to stress changes, changes in electrical conductivity beneath the earth's ground, demagnetization in rocks due to thermal sources, movement of magnetized rocks, the electrokinetic phenomena due to ground water flow and so on. In this study, only the stress-magnetization effect is considered. Changes in the rock magnetization caused by applied stresses are called the piezomagnetic ef-
fect, which is regarded as a probable cause of the seismomagnetic changes and a part of the volcanomagnetic changes. This is the inverse effect of magnetostriction, i.e. a phenomenon where a magnetic substance is strained under an applied magnetic field.

The piezomagnetic experiments of rocks were first carried out on the magnetic susceptibility by Kalashinikov and Kapitsa (1952), and on the thermal remanent magnetization (TRM) of volcanic rocks by Ohnaka and Kinoshita (1968). Both the experiments were carried out under uniaxial compression. Nagata (1970), and Stacey and Johnston (1972) also made theoretical investigations for the piezomagnetic effect based on the experimental results. The experimental and theoretical studies show that the component of magnetization and susceptibility of rocks parallel to the direction of compression decreases, whereas the component perpendicular to the applied compression increases. Though some irreversible changes in magnetization may occur, a large portion of the changes in magnetization due to stress are reversible. In this study we are concerned only with the reversible piezomagnetic effect, as has been assumed in the previous theoretical studies (e.g. Sasai, 1991).

From experimental results of uniaxial compression, the reversible change in rock magnetization, $\Delta J$, due to mechanical stress is expressed as follows:

$$\Delta J// = \beta \sigma J_0//$$  (1a)
$$\Delta J\perp = -\frac{1}{2} \beta \sigma J_0\perp$$  (1b)

where the superscripts $// \quad \perp$ represent components of magnetization parallel and perpendicular to the direction of an applied stress, respectively, while $J_0// \quad J_0\perp$ indicate the initial value of magnetization without any stress. We follow the conventional sign of stress $\sigma$ in the elasticity theory: i.e. if compression is applied to rock samples, $\sigma$ is negative. $\beta$ is called the stress sensitivity or the stress-magnetization coefficient. The linear relations between changes in magnetization and mechanical stress as shown in (1a) and (1b) are valid under condition in which applied stresses are relatively small (less than about 10 MPa = $10^8$ dyn/cm² = 100 bar). The stress-magnetization relationships expressed by Eqs. (1) are basically used in this study.

Piezomagnetic modeling relates distribution of stress changes in the earth's crust to observed geomagnetic changes. We cannot determine a unique distribution of stress changes only from observed geomagnetic data. However, forward modeling, in which a distribution of stress changes is assumed beforehand, provides us with some indication of stress changes under the ground.

An analytical approach to calculation of the piezomagnetic effects in tectonomagnetism has been reported by Sasai (1991) who computed piezomagnetic effects for several configurations of magnetized bodies and several distributions of stress changes. Numerical approaches are also useful to calculate the piezomagnetic changes, because they would be able to include not only complicated distribution of stress changes, and non-uniform distribution of rock magnetization in the earth's crust (e.g. Zlotnicki and Cornet, 1986; Oshiman et al., 1991), but also actual topographies. In this study, a numerical method using BEM (e.g. Brebbia et al., 1984) is pursued for complex problems of which analytical solutions are not available.

2. A New Technique for Evaluating Tectonomagnetic Changes

An incremental magnetic potential $W_k(r)$ due to the magnetization change $\Delta M_k$, induced by stress changes in an element $dV$ is represented using volume integral as follows:

$$W_k(r) = \iiint_V \Delta M_k \cdot \nabla \left( \frac{1}{\rho} \right) dV$$  (2)

where $r$ is a position vector of the observation point and $\rho$ is a distance between each element of magnetization and an observation point. $\Delta M_k$ is originated from the $k$-th component of the
initial value of rock magnetization. In place of $\Delta J^\parallel$ and $\Delta J^\perp$ in Eqs.(1), the components of $\Delta M_k$ are used in the following calculations.

In this study, we will use an equation expressed by surface integral over magnetized bodies instead of Eq. (2) in order to obtain the incremental magnetic potential. According to Sasai’s (1983) expression (also Sasai, 1991), an incremental magnetic potential is expressed by

$$W_k(r) = \int_S \left\{ -C_k \frac{\partial u_k(r')}{\partial n'} + \Delta M_k \cdot n' \right\} \frac{1}{\rho} + C_k u_k(r') \frac{\partial}{\partial n'} \left( \frac{1}{\rho} \right) dS \quad (3a)$$

where $u_k$ is assigned to the displacement of $k$-th component. $n'$ is the outward normal vector on the surface of the magnetized bodies. Primes on characters represent quantities on the boundary surface. A distance between a point on the boundary surface and an observation site is denoted by $\rho$. $\Delta M_{kl}$ denotes the $l$-th component of vector $\Delta M_k$ given by

$$\Delta M_{kl} = A_k \delta_{kl} \text{div} u(r') + B_k \left( \frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right). \quad (3b)$$

For later simplification we introduce the constants such as

$$P_2 = \frac{3}{4} \beta, \quad P_1 = -\frac{2}{3} P_2 = -\frac{1}{2} \beta \quad (3c)$$

$$A_k = \{(3\lambda + 2\mu)P_1 + 2\lambda P_2\} J_k = 2\mu \left( \frac{1 + \nu}{1 - 2\nu} P_1 + \frac{2\nu}{1 - 2\nu} P_2 \right) J_k \quad (3d)$$

$$B_k = 2\mu P_2 J_k \quad (3e)$$

$$C_k = \frac{-3\lambda + 2\mu}{\lambda + \mu} P_1 J_k = -2(1 + \nu)\mu P_1 J_k \quad (3f)$$

where $\beta$ is stress sensitivity, $\lambda$ and $\mu$ are elastic constants, $\nu$ is Poisson ratio and $J_k$ is $k$-th components of the initial magnetization. The outline of deducing Eq. (3a) is shown in Sasai (1983) or Sasai (1991).

In computing the right hand side of Eq. (3a), only surface quantities, that is, displacements and their strain components are necessary, whereas quantities at internal and surface points are required for Eq. (2). Therefore, using Eq. (3a) is advantageous because it saves much calculation time than the volume integral method. In particular Eq. (3a) may have a merit in application to the three-dimensional cases.

Combining Eq. (3a) with the boundary element method (BEM), we propose a technique for evaluating the piezomagnetic effect. In BEM, only either displacement components or tractions on the surface boundary nodes are initially required. Calculating BEM gives us both displacement components and tractions on boundary nodes without any of internal quantities of magneto-elastic bodies.

In this study we deal with two dimensional problems within the $x$-$z$ plane. First we calculate displacement components $u_x$, $u_z$ and their partial derivatives $\partial u_x/\partial x$, $\partial u_z/\partial x$, $\partial u_x/\partial z$, $\partial u_z/\partial z$ using the two dimensional BEM. Secondly, substituting the values at the boundary surface obtained by BEM into Eq. (3a) and operating partial differentiations on Eq. (3a), we can calculate the geomagnetic field induced by stress changes. $\partial u_x/\partial n'$ and $\partial u_z/\partial n'$ in Eq. (3a) can be expressed by $\partial u_x/\partial x$, $\partial u_z/\partial x$, $\partial u_x/\partial z$, $\partial u_z/\partial z$.

If the north component of the piezomagnetic field, $\Delta X$ and the vertical component, $\Delta Z$ at any point are thus given, changes of the geomagnetic total intensity, $\Delta F$ is obtained by

$$\Delta F = \Delta X \cos I + \Delta Z \sin I \quad (4)$$
where $I$ is the inclination of the geomagnetic field at an observation point and the declination is assumed to be negligibly small.

The flow chart of computing piezomagnetic changes in this study is briefly shown in Fig. 1.

3. Application to Calculation of Tectonomagnetic Changes of Yukutake Model

3.1 Construction of a model

We first test the new hybrid method by applying it to a well-examined model called Yukutake model (Yukutake and Tachinaka, 1967). Yukutake model assumes an infinitely long cylinder embedded in a semi-infinite elastic medium. The cylinder is subjected to an internal hydrostatic pressure from the inside. Figure 2 shows a rough picture of Yukutake model with the coordinate system. The $x$-axis is oriented north. The $z$-axis is vertically downward and the $y$-axis is east. In Yukutake model, a two dimensional structure which is uniform in the $y$-direction is assumed. Expected piezomagnetic changes are calculated at various points on the ground surface above the cylinder when an amount of hydrostatic pressure inside the cylinder is changed in the model.

Since Yukutake model is regarded as the two dimensional case of Mogi model frequently investigated in the volcanology (Mogi, 1958), the model also attempts to replica to magma chambers or sources of high temperature vapor in the earth's crust. Accordingly, the rock magnetization inside the cylinder is assumed to be zero because of higher temperature than the Curie point.

Yukutake and Tachinaka (1967) adopted an analytical solution for elastic displacements obtained using the technique of mirror images. They used the stress-magnetization relationships given by Eqs. (1a) and (1b) to calculate magnetization changes at the two dimensional grid points beneath the ground. Substituting the changes in magnetization into Eq. (2), they carried out numerical integrations with respect to $x$ and $z$ to get the piezomagnetic changes. Integrations with respect to $y$ had been done analytically. Oshimame (1990) corrected the piezomagnetic effect for this same model and revised the Yukutake and Tachinaka (1967) solution. In both the studies, they displayed distribution of stress components, stress induced magnetization and geomagnetic changes for the models in which the inclinations of the initial magnetization are assumed to be
0°, 50° and 90° respectively, being assumed that the declination of the initial magnetization is 0°. According to their results, geomagnetic changes are at most a few nanoteslas above the cylinder. Let us trace their results using present method in keeping the same parameters as those adopted Yukutake and Tachinaka (1967) and Oshiman (1990).

First we construct the discretized boundary surface for Yukutake model to which BEM is applied. The schematic boundary surface is shown in Fig. 3. This figure lies in the x-z plane as illustrated in Fig. 2 and the elastic region in BEM calculation is taken as $-100 \text{ km} \leq x \leq +100 \text{ km}, \ 0 \text{ km} \leq z \leq 100 \text{ km}$. The computation is carried out over an area broad enough to get a reasonable distribution of displacement fields using BEM. The boundary conditions in BEM calculation are such that 1) the displacements at $z = 100 \text{ km}$ are assumed to be zero in the $x$-direction, 2) the displacements at $x = \pm 100 \text{ km}$ are zero in the $x$-direction, 3) the upper
The free surface is stress free and the embedded cylinder is subject to an internal hydrostatic pressure $P_0$ of $10^8$ dyn/cm² in cgs unit (10 MPa in SI units). This value of $P_0$ is close to the critical pressure where fractures in rocks take place. The total number of nodal points in BEM calculation are about 800. The nodal points on the free surface are at 10 km intervals for the range of $40 \leq |x| \leq 100$ km, 250 m for $20 \leq |x| \leq 40$ km, and 100 m for $0 \leq |x| \leq 20$ km. Both the nodal intervals at far boundaries such as $|x| = 100$ km and $z = 100$ km are 10 km. The boundary surface of the cylinder is divided into 120 elements by 120 nodes. The length of each element on the embedded cylinder is about 260 m. Model parameters are set up as follows. The center axis of the cylinder lies at a depth $f = 10$ km and the radius of the cylinder $a = 5$ km. The depth of the Curie point isotherm is 20 km. $\lambda$ and $\mu$, Lamé's elastic constants, are assumed to be $1.0 \times 10^{12}$ dyn/cm² ($1.0 \times 10^{11}$ Pa). The depth and the radius of the cylinder, the internal pressure and Lamé's elastic constants adopted above are all the same as those adopted by Yukutake and Tachinaka (1967) and Oshiman (1990).

Before an actual calculation of Eq. (3a), analytical integration is first conducted with respect to $y$ from negative infinity to positive infinity for the purpose of applying Eq. (3a) to the two

![Fig. 4](image_url)

Fig. 4. The distribution of the changes in the magnetization components due to stress in Yukutake model. Unit of numerals on contour lines is $10^{-5}$ emu/cc. The inclinations of the initial magnetization are (a) $I = 0^\circ$ (b) $I = 50^\circ$ (c) $I = 90^\circ$ and the declination is $D = 0^\circ$. 
dimensional problems. Partial differentiations of the potential Eq. (3a) with respect to \( x \) and \( z \) are necessary in order to obtain the expression for the magnetic field. Thus we can directly calculate the piezomagnetic changes by determining the surface integrations over the two dimensional boundary surface only using boundary values of displacement and strain calculated with the aid of BEM. The details of the partial differentiations and their integrations on Eq. (3a) are shown in Appendix A.

When the surface integrations on each element are carried out to get the piezomagnetic changes, linear interpolations are adopted inside each element to obtain values of the components of displacement and their partial derivatives. We employ the Romberg integral of trapezoidal formulae for the numerical integrations.

We calculate the distributions of the stress induced magnetization in the magneto-elastic bodies when the inclinations are \( I = 0^\circ, 50^\circ \) and \( 90^\circ \) respectively, in Yukutake model. \( J_0 \), the initial value of magnetization, is designated as \( 3.0 \times 10^{-3} \) emu/cc (3.0 A/m), and \( \beta \), the stress sensitivity as \( 1.0 \times 10^{-10} \) cm\(^2\)/dyn (1.0 \times 10^{-9} \) m\(^2\)/N, 1.0 \times 10^{-4} \) bar\(^{-1}\). The results are shown in Figs. 4(a), 4(b) and 4(c). Wedgelike arrows indicate the directions for the change of magnetizations and the contour lines show the changes of their magnitudes.

The \( x \)- and \( z \)-components and the total intensity of the piezomagnetic field in the cases of

![Graphs showing piezomagnetic field changes](image)

**Fig. 5.** The piezomagnetic field for Yukutake model obtained by the hybrid method, where the inclinations (a) \( I = 0^\circ \) (b) \( I = 50^\circ \) (c) \( I = 90^\circ \) and the declination is \( D = 0^\circ \). The direction of positive \( x \) is north and that of positive \( z \) is vertically downward. The observation points are located at 10 m above the ground surface.
the inclinations of $I = 0^\circ$, $50^\circ$ and $90^\circ$ are shown in Figs. 5(a), 5(b) and 5(c) respectively. The observation height is assumed to be 10 m from the earth’s surface for the model calculation. These results almost equal to both the magnitudes and the spatial patterns of changes in the geomagnetic field which were calculated by Oshiman (1990). Thus this new method for calculating the piezomagnetic effects appears valid and expectable in application to more complex models.

3.2 Size dependency of surface elements

In this subsection, we examine element size dependence on the surface integral of magnetic changes.

We used components of displacements and their partial derivatives obtained by BEM in the above calculations. Eq. (3a) also can be evaluated using an analytical solution of an elastic model if it exists. In order to clear the size dependence of the surface elements on Eq. 3(a), we use the analytical elastic solution shown in Yukutake and Tachinaka (1967). The analytical displacements and their derivatives for Yukutake model are shown in Appendix B.

We examine changes in the computed piezomagnetic field by reducing the size of elements on the earth’s surface from 3 km to 10 m for $-20 \text{ km} \leq x \leq 20 \text{ km}$, since the surface elements around the areas immediately above the embedded cylinder contribute sensitively to the piezomagnetic changes. We define a “computational error” by a difference between a calculated change for a given element size and the change for the smallest length of 10 m. For examples, we show computational errors of the $z$-component just above the center of cylinder ($x = 0$) as a function of the element size when the inclination $I = 90^\circ$ and those of the $x$-component when $I = 0^\circ$ in Fig. 6. It can be concluded from Fig. 6 that the element size should be 500 m or smaller than it in order to attain the accuracy of 0.1 nT. We, therefore, use the surface element size of 500 m in the following calculations.

![Graph showing computational errors as a function of element size](image)

**Fig. 6.** The dependency of “computational error” on the length of boundary elements. Triangles and circles indicate the differences between each computed field component for a given element size and the “true” value obtained by the finest size of the element (10 m).

3.3 Parameter dependence of the model

In order to understand the physical characteristics of Yukutake model, we calculate the piezomagnetic effects by changing various parameters such as the observation height, the radius of the cylinder, the depth of the cylinder and the Curie point isotherm. Such evaluations will help us in applying the piezomagnetic model to observed tectonomagnetic changes from which the subterranean stress condition is inferred.

First we examine the height effect of observation points on the piezomagnetic changes. The model parameters and the physical quantities are the same as those of Section 3.1 except for
the height of observation. The $z$-component of the piezomagnetic changes are calculated when $I = 90^\circ$ and the $x$-component when $I = 0^\circ$ above the center of the cylinder. The results are shown in Fig. 7. Even if the height of observation varies from 1 m to 100 m the computed magnetic field is almost unchanged. We thus recognize that Oshiman’s (1990) results calculated at points 10 m above the free surface are applicable to changes observed at a height of a few meters.

Secondly, we calculate the magnetic field for various radii and depths of the embedded cylinder keeping the observation height at 10 m from the ground surface above the cylinder. Figure 8(a) shows the piezomagnetic changes as a function of radius of the cylinder which is

![Figure 7](image)

**Fig. 7.** Computed magnetic field components along the vertical axis above the pressure source cylinder.

![Figure 8](image)

**Fig. 8.** (a) Computed magnetic field components for varying radius, $a$, of the cylinder. (b) Computed magnetic field components for varying depth, $f$, of the cylinder.
embedded at a fixed depth of 10 km. On the other hand, the piezomagnetic changes as a function of depth of the cylinder with a fixed radius of 5 km are shown in Fig. 8(b). Comparing Fig. 8(a) with Fig. 8(b), we can see that enlargement of the radius produces almost similar effect when the cylinder depth decreases but the radius is fixed.

Thirdly, the relationship between the magnetic field and the depth of the Curie point isotherm is examined as shown in Fig. 9(a). Both curves of the z- and the x-component in Fig. 9(a) show similar variation but are offset. Both the components first decrease then turn to increase after reaching minimum values at depth of a few kilometers. After they reach maximum values at depth around 12 km, then they approach certain values asymptotically. The maximum values occur when the Curie point isotherm passes a depth deeper than half of the embedded cylinder. Two inflection points on the curves in Fig. 9(a) occur when the Curie point isotherm is at 5 km and 15 km depth, i.e. the top and bottom of the cylinder.

There is a discrepancy between the results of Oshiman (1990) and this study. The x-component of magnetic field in Fig. 9(a) approaches zero nT around the Curie depth of 12 km.
However, according to Oshiman (1990), the $x$-component is below zero for all depths of the Curie point isotherm.

The difference between our result and Oshiman's (1990) might arise from the reason that the stress component $t_{yy}$ was neglected in Oshiman's (1990) calculation. Since we adopt plane strain in our model, it is assumed to be free from strain in the direction of $y$-axis: the strain components $e_{yy}$, $e_{x}y$ and $e_{xy}$ do not exist but apparently the stress component $t_{yy}$ does exist, which produces piezomagnetization arising from the $x$-component of the initial magnetization, $J_x$, and the $z$-component, $J_z$. In the present calculation $t_{yy}$ is implicitly taken into consideration, because the surface integral representation in Eq. (3a) was derived from substituting the equation (Cauchy-Navier equation) of elastic equilibrium into the modified equation of the dipole law of force (Sasai, 1983; 1991).

For comparison, the effect of the Curie point isotherm using the volume integrals such as Oshiman (1990) calculated but neglected $t_{yy}$ is shown in Fig. 9(b). In calculation, the magnitude of the initial magnetization is only different from Oshiman's (1990) calculation: the initial magnetization used in this study is $3.0 \times 10^{-3}$ emu/cc (3.0 A/m), while the initial magnetization used in Oshiman (1990) is $5.0 \times 10^{-3}$ emu/cc (5.0 A/m). The result of the numerical volume integrals taking $t_{yy}$ into consideration is exactly the same as the result of the surface integrals shown in Fig. 9(a). In these volume integrals, the size of elements within the range of about 200 m from the observation point is $2 \times 2$ m and that far from the observation point is $200 \times 200$ m.

As discussed above, the discrepancy between Oshiman's (1990) result and the present study finally proved to be due to the disregard of $t_{yy}$ component. The discrepancy makes serious difference in the signs of changes even if the magnitude is very small, when the Curie point isotherm is around the depth of the cylinder. We should keep this discrepancy in mind, because most of the important models with two dimensional configurations appear in the in-plane problems which is always subject to this kind of troubles.

4. An Application to a Model Having Surface Topography

We can also treat a more complicated version of Yukutake model, in which a horizontally embedded cylinder inflates below an irregular surface topography.

Let us take up the case of Unzendake volcano in Kyusyu district, Japan, as an example. Unzendake volcano became active recently and detailed tectonomagnetic investigation has been done by Tanaka (1995). We here don't intend to interpret the observed magnetic changes as those due to piezomagnetic effect, but simply examine the topographic effect for piezomagnetic changes.

We estimate the topographic effect for a Yukutake model having the topography of Unzendake volcano as shown in Fig. 10(a). The topography is approximated with a 20 m element length on the surface of the ground. According to Tanaka (1995), we assume that the center of an infinite cylinder with a radius, $a = 84$ m, is embedded at the depth, $f = 250$ m, beneath the top of the mountain and that the inside of the cylinder is demagnetized in association with the volcanic activity of Unzendake volcano. Tanaka (1995) determined the location of the demagnetized body from the changes in the geomagnetic total force observed from October 1991 to January 1992 around the summit. The maximum change was larger than 5 nT per 100 days.

In order to evaluate the piezomagnetic effect around Unzendake volcano, let us adopt the parameters in the model as $P_0 = 10^8$ dyn/cm$^2$ (10 MPa) for hydrostatic pressure, $\mu = 1.0 \times 10^{12}$ dyn/cm$^2$ (1.0 $\times 10^{11}$ Pa) for rigidity and $\nu = 0.25$ for Poisson's ratio. The elastic region in BEM calculation is taken as $-20$ km $\leq x \leq +20$ km, $0$ km $\leq z \leq +20$ km. The components of displacement due to $P_0$ which are calculated from BEM are shown in Fig. 10(b). Also, $u_x$ derivatives and $u_z$ derivatives with respect to $x$ and $z$, respectively are shown in Figs. 10(c) and 10(d).
Fig. 10. (a) The topography of the mountain and the pressure source cylinder used in the present calculation. (b) The displacement components obtained by BEM for a model including the topography of the mountain. (c) The derivatives of $x$-components of displacement with respect to $x$ and $z$ for a model including the topography of the mountain. (d) The derivatives of $z$-components of displacement with respect to $x$ and $z$ for a model including the topography of the mountain.
We calculate the piezomagnetic changes assuming $2.9 \times 10^{-3}$ emu/cc (2.9 A/m) for the initial magnetization (Nakatsuka, 1994) and $1.0 \times 10^{-10}$ cm$^2$/dyn ($1.0 \times 10^{-9}$ Pa$^{-1}$) for the stress sensitivity. The inclination is assumed to be 50°, the declination to be 0° and the depth of the Curie point isotherm to be 7 km (Okubo et al., 1985). Observation points are at a height of 10 m from the ground surface.

Figure 11(a) shows the influence of the surface topography. For comparison, the piezomagnetic change without the topography of the mountain is shown in Fig. 11(b). These changes, which don't exceed 1 nT in Figs. 11(a) and 11(b), are far smaller than those observed by Tanaka (1995) at points close to the summit. In this case, it is clear that the piezomagnetic effect is not significant in comparison with the demagnetized effect.

However, we can notice in Figs. 11(a) and 11(b) that the local irregularities above the pressure source have almost the same amplitude as the flat-surface model. In addition, a kind of the enhancement effects of the piezomagnetic signals in the magnetic field is recognized. It is evident from Figs. 10(a), 10(b), 10(c) and 10(d) that this topographic effect is due to local stress concentration on the jagged topography. This new kind of the enhancement effects should be distinguished from the effect of the inhomogeneous magnetization (Oshiman, 1990) and also from the effect which emerges when we observe the magnetic field within the magnetized bodies (Sasai, 1983; 1991).

The topographic effects should be investigated more by the present method in the future.
5. Summary

We have investigated whether BEM can be combined with Sasai’s (1983) technique that uses surface integral to calculate tectonomagnetic effects. BEM allows calculation of arbitrarily shaped magnetized bodies and Sasai’s surface integral technique saves the calculating time. The new method was tested by comparison with solutions for Yukutake model obtained by Yukutake and Tachinaka (1967) and Oshiman (1990).

Investigation of the topographic effect for Yukutake model showed that the irregular piezomagnetic changes due to the topographic effect caused by the local stress concentration are comparable to the overall changes of the flat-surface model. This stress concentration causes a new kind of the enhancement effect of the tectonomagnetism.

It is difficult to uniquely determine a distribution of magnetized bodies and the stress condition under the ground from observed changes in the geomagnetic fields. However, it is feasible and necessary to build up standard models for various tectonomagnetic phenomena related to the crustal activities and to examine characteristics of them. Such a study is helpful to infer the stress state within the crust by magnetic observations. It is concluded that the generalized method based on the surface integral and BEM gives us a powerful tool for testing the piezomagnetic models.

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APPENDIX A: Piezomagnetic Field Derived from Sasai’s Formula

According to Sasai’s (1983) formula, the piezomagnetic potential outside the magnetized bodies is expressed by surface integral.

\[ W_k(r) = \iint_S \left\{ -C_k \frac{\partial u_k(r')}{\partial n'} + \Delta M_k \cdot n' \right\} \frac{1}{\rho} + C_k u_k(r') \frac{\partial}{\partial n'} \left( \frac{1}{\rho} \right) dS. \quad (A.1) \]

In the two dimensional problem, Eq. (A.1) is integrated from negative to positive infinity with respect to \( y \). Incremental magnetic field components \( \Delta X \) and \( \Delta Z \) are obtained by partial differentiation with respect to \( x \) and \( z \). In this case, the integrand is smooth enough that integral operator and differential operator can be interchanged. Here differentiation is first done on Eq. (A.1).

Eq. (A.1) is rewritten by using variable \( x \) and \( z \) as follows:

\[ W_k(r) = \iint_S \left\{ -C_k \frac{\partial u_k(r')}{\partial n'} + \Delta M_k \cdot n' \right\} \frac{1}{\rho} \left\{ (x-x')^2 + y^2 + (z-z')^2 \right\}^{-\frac{3}{2}} dS \]

\[ + \iint_S C_k u_k(r') (x-x') \left\{ (x-x')^2 + y^2 + (z-z')^2 \right\}^{-\frac{3}{2}} x'_n dS \]

\[ + \iint_S C_k u_k(r') (z-z') \left\{ (x-x')^2 + y^2 + (z-z')^2 \right\}^{-\frac{3}{2}} z'_n dS \quad (A.2) \]

where \( x'_n = \partial x'/\partial n' \), \( z'_n = \partial z'/\partial n' \). The partial derivatives of Eq. (A.2) with respect to \( x \) and \( z \) are given by Eqs. (A.3) and (A.4), respectively.

\[ \frac{\partial W_k(r)}{\partial x} = - \iint_S \left\{ -C_k \frac{\partial u_k(r')}{\partial n'} + \Delta M_k \cdot n' \right\} (x-x') \left\{ (x-x')^2 + y^2 + (z-z')^2 \right\}^{-\frac{3}{2}} dS \]

\[ - \iint_S C_k u_k(r') (x-x') \left\{ (x-x')^2 + y^2 + (z-z')^2 \right\}^{-\frac{3}{2}} x'_n dS \]

\[ - \iint_S C_k u_k(r')(z-z') \left\{ (x-x')^2 + y^2 + (z-z')^2 \right\}^{-\frac{3}{2}} z'_n dS \]
Integration of Eqs. (A.3) and (A.4) with respect to \( y \) gives Eqs. (A.5) and (A.6), respectively.

\[
\int_{-\infty}^{+\infty} \frac{\partial W_k(r)}{\partial x} dy' = - \int_S \left\{ -C_k \frac{\partial u_k(r')}{\partial n'} + \Delta M_k \cdot n' \right\} \frac{(x - x')}{(x - x')^2 + (z - z')^2} \cdot \frac{y'^2}{(x - x')^2 + y'^2 + (z - z')^2} \bigg|_{y'=-\infty}^{y'=+\infty} dx' dz' \\
+ \int_S C_k u_k(r') \frac{1}{(x - x')^2 + (z - z')^2} \cdot \frac{y'^2}{(x - x')^2 + y'^2 + (z - z')^2} \bigg|_{y'=-\infty}^{y'=+\infty} x_n dx'dz' \\
- \int_S C_k u_k(r') \frac{(x - x') \{(x - x')x_n + (z - z')z_n\}}{(x - x')^2 + (z - z')^2} \cdot \frac{y' \{3(x - x')^2 + 2y'^2 + 3(z - z')^2\}}{\{(x - x')^2 + y'^2 + (z - z')^2\}^{\frac{3}{2}}} \bigg|_{y'=-\infty}^{y'=+\infty} dx'dz' \\
= - \int_S \left\{ -C_k \frac{\partial u_k(r')}{\partial n'} + \Delta M_k \cdot n' \right\} \\
\cdot \frac{2(x - x') \{(x - x')^2 + (z - z')^2\}^{\frac{1}{2}} - 1}{\{(x - x')^2 + (z - z')^2\}^{\frac{3}{2}} dx'dz' \\
+ \int_S 2C_k u_k(r') \{(x - x')^2 + (z - z')^2\}^{\frac{1}{2}} - 1 \frac{1}{\{(x - x')^2 + (z - z')^2\}^{\frac{3}{2}}} x_n dx'dz' \\
- \int_S 4C_k u_k(r') \{(x - x')^2 + (z - z')^2\}^{\frac{1}{2}} dx'dz' \\
= - \int_S \left\{ -C_k \frac{\partial u_k(r')}{\partial n'} + \Delta M_k \cdot n' \right\} \frac{2(x - x') (\rho - 1)}{\rho^3} dx'dz' \\
+ \int_S C_k u_k(r') \left\{ \frac{2(\rho - 1)}{\rho^3} - \frac{4(x - x')^2}{\rho^4} \right\} x_n dx'dz'.
\[\int_{-\infty}^{+\infty} \frac{\partial W_k(r)}{\partial z} dy' = -\int_S \left\{ -C_k \frac{\partial u_k(r')}{\partial n'} + \Delta M_k \cdot n' \right\} \frac{(z - z')}{(x - x')^2 + (z - z')^2} \]
\[-4(x - x')(z - z') \rho^4 x'_n \right\} dx'dz', \quad (A.5)\]

\[+ \int_S C_k u_k(r') \frac{1}{(x - x')^2 + (z - z')^2} \]
\[\left\{ \frac{y'^2}{(x - x')^2 + y'^2 + (z - z')^2} \right\}^{\frac{1}{2}} dy' \]
\[\int_{-\infty}^{+\infty} \left\{ (x - x')x'_n + (z - z')z'_n \right\} \]
\[\left\{ (x - x')^2 + (z - z')^2 \right\}^{\frac{3}{2}} dy' \]
\[-\int_S C_k u_k(r') \frac{(z - z')}{(x - x')^2 + (z - z')^2} \]
\[\left\{ y' \frac{3(x - x')^2 + 2y'^2 + 3(z - z')^2}{(x - x')^2 + y'^2 + (z - z')^2} \right\}^{\frac{3}{2}} dy' \]
\[\int_{-\infty}^{+\infty} \left\{ (x - x')^2 + (z - z')^2 \right\}^{\frac{3}{2}} dy' \]
\[-\int_S \left\{ -C_k \frac{\partial u_k(r')}{\partial n'} + \Delta M_k \cdot n' \right\} \]
\[\frac{2(z - z')}{(x - x')^2 + (z - z')^2} \left\{ (x - x')^2 + (z - z')^2 \right\}^{\frac{3}{2}} dx'dz' \]
\[+ \int_S 2C_k u_k(r') \left\{ (x - x')^2 + (z - z')^2 \right\}^{\frac{1}{2}} - 1 \]
\[\left\{ (x - x')^2 + (z - z')^2 \right\}^{\frac{3}{2}} z'_n dx'dz' \]
\[-\int_S 4C_k u_k(r')(z - z') \left\{ (x - x')x'_n + (z - z')z'_n \right\} \]
\[\left\{ (x - x')^2 + (z - z')^2 \right\}^{\frac{3}{2}} dx'dz' \]
\[= -\int_S \left\{ -C_k \frac{\partial u_k(r')}{\partial n'} + \Delta M_k \cdot n' \right\} \frac{2(z - z')(\rho - 1)}{\rho^3} dx'dz' \]
\[+ \int_S C_k u_k(r') \left\{ -\frac{4(x - x')(z - z')}{\rho^4} \right\} x'_n \]
\[\left\{ 2(\rho - 1) - 4(z - z')^2 \right\} z'_n \right\} dx'dz' \quad (A.6)\]

\[\rho = \{ (x - x')^2 + (z - z')^2 \}^{\frac{1}{2}}.\]

**APPENDIX B: Displacement Field Components of Yukutake Model and Their Partial Derivatives**

\[u_x = \frac{a^2 P_0}{2\mu} \left\{ \frac{x}{x^2 + (z - f)^2} + \frac{x}{x^2 + (z + f)^2} \right\} \]
\[-\frac{a^2 P_0}{2\mu} \frac{x(3z^2 + 2fz - f^2 - x^2)}{x^2 + (z + f)^2} \quad (B.1)\]
A Hybrid Calculation Method of Tectonomagnetic Effect

\[ u_z = \frac{a^2 P_0}{2\mu} \left\{ \frac{z-f}{x^2+(z-f)^2} + \frac{z+f}{x^2+(z+f)^2} \right\} \]
\[ - \frac{a^2 P_0}{2\mu} \left\{ \frac{5 z^3 + 13 f z^2 + 11 f^2 z + x^2 z + 3 f^3 + 3 f z^2}{x^2+(z+f)^2} \right\}, \]  
(B.2)

\[ \frac{\partial u_z}{\partial x} = \frac{a^2 P_0}{2\mu} \left\{ \frac{-x^2 + (z-f)^2}{x^2+(z-f)^2} + \frac{-x^2 + (z+f)^2}{x^2+(z+f)^2} \right\} \]
\[ - \frac{a^2 P_0}{2\mu} \left\{ \frac{3 z^4 + 8 f z^3 + 6 f^2 z^2 - 12 x^2 z^2 - 12 f x^2 z - f^4 + x^4}{x^2+(z+f)^2} \right\}, \]  
(B.3)

\[ \frac{\partial u_z}{\partial z} = \frac{a^2 P_0}{2\mu} \left\{ \frac{-2x(z-f)}{x^2+(z-f)^2} + \frac{-2x(z+f)}{x^2+(z+f)^2} \right\} \]
\[ - \frac{a^2 P_0}{2\mu} \left\{ \frac{2x (-3 z^3 - 3 f z^2 + 3 f^2 z + 5 x^2 z + 3 f^3 + 3 f z^2)}{x^2+(z+f)^2} \right\}, \]  
(B.4)

\[ \frac{\partial u_z}{\partial x} = \frac{a^2 P_0}{2\mu} \left\{ \frac{-2x(z-f)}{x^2+(z-f)^2} + \frac{-2x(z+f)}{x^2+(z+f)^2} \right\} \]
\[ - \frac{a^2 P_0}{2\mu} \left\{ \frac{2x (-9 z^3 - 21 f z^2 - 15 f^2 z + x^2 z + 3 f^3 - 6 f^2 - 3 f x^2)}{x^2+(z+f)^2} \right\}, \]  
(B.5)

\[ \frac{\partial u_z}{\partial z} = \frac{a^2 P_0}{2\mu} \left\{ \frac{x^2 - (z-f)^2}{x^2+(z-f)^2} + \frac{x^2 - (z+f)^2}{x^2+(z+f)^2} \right\} \]
\[ - \frac{a^2 P_0}{2\mu} \left\{ \frac{(-5 z^4 - 16 f z^3 - 18 f^2 z^2 + 12 x^2 z^2 - 8 f^3 z + 12 x^2 f z - f^4 + x^4)}{x^2+(z+f)^2} \right\}. \]  
(B.6)

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