Predictions via large $\theta_{13}$ from cascades

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Abstract

We investigate a relation among neutrino observables, three mixing angles and two mass squared differences, from a cascade texture of neutrino mass matrix. We show an allowed region of the correlation by use of current data of neutrino oscillation experiments. The relation predicts sharp correlations among neutrino mixing angles as $0.315 \lesssim \sin^2 \theta_{12} \lesssim 0.332$ and $0.480 \lesssim \sin^2 \theta_{23} \lesssim 0.500$ with a large $\theta_{13}$ ($0.03 < \sin^2 2\theta_{13} < 0.28$). These magnitudes are modified $0.310 \lesssim \sin^2 \theta_{12} \lesssim 0.330$ and $0.540 \lesssim \sin^2 \theta_{23} \lesssim 0.560$ when the charged lepton mass matrix also has the cascade form.

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1 Introduction

Current neutrino oscillation experiments suggest an existence of two large mixing angles among three generations in lepton sector \([1]\). It is well known that the two large mixing angles is suitably approximated by so-called tri-bimaximal mixing (TBM) \([2, 3]\),

\[
V_{\text{TBM}} = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix},
\]

(1)

which induces mixing angle,

\[
\sin^2 \theta_{12} = \frac{1}{3}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin \theta_{13} = 0.
\]

(2)

Such a characteristic form of mixing matrix strongly motivates a study of flavor structure in the lepton sector. Actually, there are a large number of models which try to realize the TBM based on a flavor symmetry, neutrino mass textures, and so on.

In a study of suitably realization of the TBM, one peculiar relation among the neutrino observables, three mixing angles and two mass squared differences, has been proposed in \([4]\), that is

\[
\frac{1}{9} \left( \sin^2 \theta_{23} - \frac{1}{2} \right) - \frac{r}{4} \left( \sin^2 \theta_{12} - \frac{1}{3} \right) - \frac{\sqrt{2} r}{27} \sin \theta_{13} = 0,
\]

(3)

where \(r \equiv \sqrt{\Delta m^2_{21}/\Delta m^2_{31}}\), and \(\theta_{ij}\) \((i, j = 1, 2, 3)\), \(\Delta m^2_{21}\) and \(\Delta m^2_{31}\) are the leptonic mixing angles and two mass squared differences of neutrinos, respectively. Notice that the exact TBM satisfies this relation independently of the mass squared differences. Moreover, this relation also shows correlations among deviations from the TBM. In fact, this attractive relation is derived from so-called cascade texture \([4]\) with hierarchical neutrino masses.

A typical cascade texture is represented by

\[
M_{\text{cas}} = \begin{pmatrix}
\delta & \delta & \delta \\
\delta & \lambda & \lambda \\
\delta & \lambda & 1
\end{pmatrix} v \quad \text{with} \quad |\delta| \ll |\lambda| \ll 1,
\]

(4)

where \(v\) denotes an overall mass scale. In ref. \([4]\), it has been pointed out that the TBM can be realized at a leading order in type-I seesaw mechanism \([5]-[7]\) if the neutrino Dirac mass matrix is taken as the cascade form\(^1\). Realizations of such a cascade texture have

\(^1\)There are some kinds of textures, which can lead to two large leptonic mixing angles with vanishing or non-vanishing \(\theta_{13}\). For instance, the Fritzsch-type \([8]\) lepton mass matrices, which is classified to two-zero textures, can predict non-vanishing \(\theta_{13}\) \([9, 10]\). One of interesting features of cascade texture is that it can also lead to two large leptonic mixing angles with either vanishing or non-vanishing \(\theta_{13}\) even though the texture is hierarchical structure as we will show below. Such a hierarchical mass structure might be realized by the Froggatt-Nielsen mechanism \([11]\).
also been discussed in terms of flavor symmetries and extra-dimensions [4, 12]. We call the model which induces the cascade texture “cascade model”. We here comment on a slightly modified cascade texture, called hybrid cascade texture, which is given by

\[ M_{\text{hyb}} = \begin{pmatrix} \epsilon & \delta' & \delta' \\ \delta' & \lambda' & \lambda' \\ \delta' & \lambda' & 1 \end{pmatrix} v', \quad \text{with} \quad |\epsilon| \ll |\delta'| \ll |\lambda'| \ll 1. \]  

(5)

This can naturally fit a quark sector, masses and mixing angles. Thus, there have been some researches, where the (hybrid) cascade textures can really reproduce the suitable masses and mixing angles of the SM fermions at a low energy regime in the frameworks of SUSY SU(5) [12] and SUSY SO(10) [13] GUTs. However, it should be noticed that the TBM in the lepton sector is hardly realized by any seesaw mechanism [4, 12, 13].

In this letter, we investigate the relation (3) predicted from the original cascade model, and examine verifiability of cascade in the lepton sector. The latest global analyses of three-flavor neutrino parameters [1] give

\[ \sin^2 \theta_{12} = 0.316 \pm 0.016, \quad \sin^2 \theta_{23} = 0.51 \pm 0.06, \quad \sin^2 \theta_{13} = 0.017^{+0.007}_{-0.009}, \]  

(6)

at 1σ level for normal neutrino mass hierarchy (NH). In addition there are recent observations of \( \nu_\mu \rightarrow \nu_e \) oscillation by T2K experiment [14], which suggested

\[ 0.03 < \sin^2 2\theta_{13} < 0.28, \]  

(7)

at 90% C.L. for NH with \( \delta_{CP} = 0 \) [15]. This experimental result of (a non-vanishing or) large \( \sin \theta_{13} \) motivates studies of deviation from the TBM to search a true physics which determine the lepton flavor structure, and screens a large number of neutrino (lepton) flavor models.\(^2\)

The letter is organized as follows: In section 2, we will give a brief review of the cascade model and investigate predictions from the model as focusing on the recent data of neutrino oscillation experiments. The section 3 is devoted to a summary.

2 Cascade model and probing a relation among neutrino observables

In this section, we present a brief review of the cascade texture and investigate predictions from it as focusing on recent data of neutrino oscillation experiments.

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\(^2\)See e.g. [16] for early and general discussions of deviations from TBM (in particular, see e.g. [17, 18] for the recent discussions of a large \( \theta_{13} \)), [19] for general discussions of deviations from TBM and quark-lepton complementarity [20, 21], and [22, 30] for discussions of deviations from TBM including the latest T2K results.
2.1 Cascade neutrino texture

At first, we give a brief review of the cascade neutrino texture \cite{4}. In the cascade neutrino model, the neutrino Dirac mass matrix takes the following cascade form:

\[
M_{\nu D} = \begin{pmatrix}
\delta & \delta & \delta \\
\delta & \lambda & -\lambda \\
\delta & -\lambda & 1
\end{pmatrix} v \quad \text{with} \quad |\delta| \ll |\lambda| \ll 1.
\] (8)

This mass matrix can lead to experimentally favored (nearly TBM) mixing angles with NH in the context of type-I seesaw mechanism. Such types of mass texture have often been seen in the lepton mass models, e.g. with the vacuum alignments and non-Abelian flavor symmetry (see, for example, refs. [7] in \cite{4}). Mass eigenvalues of light neutrinos, \(m_i\), in the model are given by

\[
m_1 = \frac{v^2}{6M_3},
\] (9)

\[
m_2 = \left(\frac{1}{3M_3} + \frac{3\delta^2}{M_1}\right)v^2,
\] (10)

\[
m_3 = \left(\frac{1}{2M_3} + \frac{2\lambda^2}{M_2}\right)v^2,
\] (11)

in the diagonal basis of right-handed neutrino mass matrix, \(M_R = \text{Diag}[M_1, M_2, M_3]\). The cascade neutrino model leads to the NH in order to realize the tri-bimaximal mixing at the leading order \cite{4}. Thus, we perturbatively computed to give eqs. (9)-(11) around \(m_1/m_{2,3}\) and \(\delta/\lambda\), which are small quantities to be consistent with experimental values. In the same perturbation, the mixing angle are evaluated as

\[
sin^2 \theta_{12} = \left| \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} \frac{m_1}{m_2} \right|^2,
\] (12)

\[
sin^2 \theta_{23} = \left| -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2} m_3(m_3 - m_2)} + \frac{\delta}{3\sqrt{2} \lambda m_3} \frac{m_2}{m_3 - m_2} \right|^2,
\] (13)

\[
sin^2 \theta_{13} = \left| \frac{\delta}{\sqrt{2} \lambda} \frac{m_3 - 2m_2/3}{m_3 - m_2} + \frac{\sqrt{2} m_1 m_2}{m_3(m_3 - m_2)} \right|^2,
\] (14)

in the diagonal basis of the charged lepton mass matrix. Notice again that this cascade neutrino model leads to the TBM at leading order. In other words, the corrections of next-leading order shift the mixing angles form the exact TBM. We will focus on this point in the following subsections.

It can be seen that there are four combinations of independent model parameters, \(m_i\) and \(\delta/\lambda\), while the five observables exist (three mixing angles and two mass squared differences can be expressed by \(m_i\) and \(\delta/\lambda\)). Therefore, one parameter independent relation among neutrino observables must exist, that is just (3), with real parameters in the model.
Figure 1: Correlations among mixing angles in the cascade model of neutrino mass matrix with $\delta_{\text{CP}} = 0$.

### 2.2 Probing a relation among neutrino observables

Now let us investigate a predicted relation (3) from the above cascade model to examine the verifiability of the model through the data of neutrino oscillation experiments.

We give numerical plots in Fig. 1. These figures show predicted regions from a relation among neutrino observables in the cascade model. This numerical simulation is based on random plots including a mild hierarchical cascade, $0 < \delta < \lambda < 0.1$. Therefore, this simulation could scan all classes of cascade model, that is, from a mild hierarchy up to a rapid one. From this complete scan, about 1600 viable models have been chosen among 10000 models. Too mild hierarchical models are automatically screened by experimental data. The Fig. 1 (a), (b) and (c) are drawn in $(\sin^2 \theta_{13}, \sin^2 \theta_{12})$, $(\sin^2 \theta_{13}, \sin^2 \theta_{23})$ and $(\sin^2 \theta_{12}, \sin^2 \theta_{23})$ planes, respectively. The upper and lower flat (yellow) shaded regions in all figures indicate regions out of $3\sigma$ level for the vertical axes. The horizontal regions in all figures are shown within the $3\sigma$ levels. The (red) lighter solid and dashed lines correspond to the best fit and $1\sigma$ lines for all mixing angles. The (black) darker solid lines
in the Fig. (a) and (b) are the lower bound ($\sin^2 2\theta_{13} = 0.03$) at 90% C.L. for NH with $\delta_{\text{CP}} = 0$ as reported by the latest T2K experiment (the upper bound, $\sin^2 2\theta_{13} = 0.28$ is out of the figure). The (green) darker regions show where the (3) is satisfied with each value of three mixing angles in $3\sigma$ ranges and the best fit values of two mass squared differences. The (blue) plots are the predicted points from the cascade model. Note that since predicted mixing angles from the cascade model are strongly correlated each other as shown in (12)-(14), the (blue) plots are on the partial regions of the (green) darker ones, which are covered by all $3\sigma$ data without correlations of the mixing angles.

We can see that there are relatively strong correlations among each mixing angle compared with other neutrino flavor models. This is one of advantages of the cascade neutrino model to check the model. In particular, we can predict $0.315 \lesssim \sin^2 \theta_{12} \lesssim 0.332$ and $0.480 \lesssim \sin^2 \theta_{23} \lesssim 0.500$ with a relatively large $\theta_{13}$, e.g. $\sin^2 \theta_{13} \sim 0.008$, which corresponds to the lower bound from T2K. The above computation has been done with the vanishing Dirac CP phase, $\delta_{\text{CP}}$. When $\delta_{\text{CP}} \neq 0$, the above correlations slightly weaken but the predictions of mixing angles do not change drastically as $0.320 \lesssim \sin^2 \theta_{12} \lesssim 0.333$ and $0.480 \lesssim \sin^2 \theta_{23} \lesssim 0.510$ for $0.008 \lesssim |\sin \theta_{13}|^2$ which are shown in Fig. 2. Note that only scatter plots are shown in Fig. 2 with $\delta_{\text{CP}} \neq 0$ case, since the relation (3) is established only for real parameters. Anyhow, we emphasize that the cascade model is surely predictive, and thus it would be checked by the T2K and other future neutrino experiments such as the Double Chooz [31] (whose future sensitivity is $\sin^2 \theta_{13} = 0.07$ at 2011~year), RENO [32] ($\sin^2 \theta_{13} = 0.03$ at 2011~year) and Daya-Bay [33] ($\sin^2 \theta_{13} = 0.01$ at 2012~year) collaborations [34] thanks to the above strong correlations among mixing angles.

2.3 Cascade charged lepton mass texture

It might be more natural that the charged lepton mass matrix also takes the cascade form in the sense that the neutrino Dirac mass matrix of the cascade form is obtained from an $U(1)$ flavor symmetry and/or other dynamics [4]. Thus, here we research the case when the charged lepton mass matrix also has the cascade form,

$$M_e = \begin{pmatrix} \delta_e & \delta_e & \delta_e \\ \delta_e & \lambda_e & \lambda_e \\ \delta_e & \lambda_e & 1 \end{pmatrix} v \quad \text{with} \quad |\delta_e| \ll |\lambda_e| \ll 1. \quad (15)$$

Note that the magnitudes of cascade parameters, $\delta_e$ and $\lambda_e$, should be evaluated from the experimentally observed values of charged lepton masses, $m_e$, $m_\mu$ and $m_\tau$, which are given by

$$|\lambda_e| \simeq \frac{m_\nu}{m_\tau} \simeq 6 \times 10^{-2}, \quad (16)$$
$$|\delta_e| \simeq \frac{m_e}{m_\tau} \simeq 3 \times 10^{-4}. \quad (17)$$
Figure 2: Correlations among mixing angles in the cascade model of neutrino mass matrix with $\delta_{\text{CP}} \neq 0$.

It can be found from (15)-(17) that the contributions to the mixing angles from the charged lepton sector are small. Therefore, the total lepton mixing angles can be estimated at the first order perturbation as

$$\sin^2 \theta_{12} = \left| \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} \frac{m_1}{m_2} - \frac{1}{\sqrt{3}} \frac{m_e}{m_\mu} \right|^2, \quad (18)$$

$$\sin^2 \theta_{23} = \left| -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \frac{m_1 (m_3 - m_2)}{m_3 (m_3 - m_2)} + \frac{\delta}{3 \sqrt{2} \lambda} \frac{m_2}{m_3 - m_2} - \frac{1}{\sqrt{2}} \frac{m_\mu}{m_\tau} \right|^2, \quad (19)$$

$$\sin^2 \theta_{13} = \left| \frac{\delta}{\sqrt{2} \lambda} \frac{m_3 - 2m_2/3}{m_3 - m_2} + \frac{\sqrt{2} m_1 m_2}{m_3 (m_3 - m_2)} + \frac{1}{\sqrt{2}} \frac{m_e}{m_\mu} \right|^2. \quad (20)$$

One can see that the solar neutrino mixing is little affected, on the other hand, as for the atmospheric neutrino mixing, the charged lepton effect often dominates. And the magnitude of the contribution to the reactor neutrino mixing is of negligible order. Since the hierarchy in the charged lepton mass matrix can be expressed by the observables as (16) and (17), the strong correlation among neutrino observables still holds but (3) is slightly
modified as
\[
\frac{1}{9} \left( \sin^2 \theta_{23} - \frac{1}{2} - \frac{m_\mu}{m_\tau} \right) - \frac{r}{4} \left( \sin^2 \theta_{12} - \frac{1}{3} \right) - \frac{\sqrt{2}r}{27} \sin \theta_{13} = 0,
\] (21)
by including the charged lepton contributions in the first order approximation.

We show numerical plots with the relation (21) in Fig. 3. A fundamental setup of the numerical simulation is the same as one in the previous subsection. The cascade parameters in the charged lepton mass matrix are determined by the experimentally observed charged lepton masses. About 1600 possible models have been also chosen among 10000 complete sets of model. This means that the contributions from the charged lepton mass matrix of the cascade form do not drastically change the results from models with the diagonal charged lepton mass matrix in the previous subsection. It can be seen that the prediction of the value of \( \sin^2 \theta_{12} \) with a large \( \theta_{13} \) becomes slightly smaller compared to the case of diagonal charged lepton mass matrix, as \( 0.310 \lesssim \sin^2 \theta_{12} \lesssim 0.330 \). On the other hand, \( \sin^2 \theta_{23} \) becomes larger as \( 0.540 \lesssim \sin^2 \theta_{23} \lesssim 0.560 \). In the case of \( \delta_{\text{CP}} \neq 0 \), the
correlations slightly weaken and mixing angles are predicted as $0.320 \lesssim \sin^2 \theta_{12} \lesssim 0.345$ and $0.530 \lesssim \sin^2 \theta_{23} \lesssim 0.580$ for $0.008 \lesssim |\sin \theta_{13}|^2$, which are shown in Fig 4. Therefore, we conclude that all leptonic Dirac mass textures of the cascade model, also predict explicit deviations from the exact TBM, and the deviations are strongly correlated with each other. These would be also checked in the future neutrino oscillation experiments with higher sensitivities.

3 Summary

We have studied a relation among neutrino observables, which are three mixing angles and two mass squared differences. This relation is predicted from a cascade texture with hierarchical neutrino masses. The neutrino cascade model is favored by the current neutrino oscillation experiments and is supported by theoretical studies of new physics such as the realizations from flavor symmetry, extra-dimensional theory, and embedding the model
into GUTs. Since recent data of the neutrino oscillation experiments including the latest T2K result might suggest the deviations from the exact TBM, we have motivated for the precise investigations of the relation. The relation gives strong correlations among each deviation of leptonic mixing angle from the TBM.

We have numerically shown predicted regions of the relation and scatter plots from a cascade model by use of recent data of neutrino oscillation experiments in two cases. One is the model with the diagonal charged lepton mass matrix, and the other is the case of cascade form of charged lepton mass matrix also. In both cases, we can see that predictions of the cascade model and deviations from the TBM are strongly correlated among three lepton mixing angles. This is a strong advantage of the cascade model for the verifiability of the model compared with other neutrino flavor models. We have predicted $0.315 \lesssim \sin^2 \theta_{12} \lesssim 0.332$, $0.480 \lesssim \sin^2 \theta_{23} \lesssim 0.500$ in the case of the diagonal charged lepton mass matrix, and $0.310 \lesssim \sin^2 \theta_{12} \lesssim 0.330$, $0.540 \lesssim \sin^2 \theta_{23} \lesssim 0.560$ in the case of the cascade charged lepton mass matrix. Hence, we conclude that the cascade model has predicted deviations of all mixing angles from the exact TBM with a relatively large $\theta_{13}$. These would be also checked in the future neutrino oscillation experiments with higher sensitivities.

At the end of this letter, we comment on recent MINOS result, where $\theta_{13}$ can be still consistent with zero \cite{MINOS}. Figures \cite{figs} suggest the correlations among $\theta_{ij}$ even if $\theta_{13} = 0$, where the cascade predictions are slightly modified as $0.315 \lesssim \sin^2 \theta_{12} \lesssim 0.335$. Anyhow, the cascade model predicts the strong correlation in wide range of $\theta_{13}$ (as $\sin^2 2\theta_{13} < 0.28$).

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