The kinematic Sunyaev Zeldovich effect and transverse cluster velocities

Edouard Audit\textsuperscript{1}, John F.L. Simmons\textsuperscript{2}

\textsuperscript{1} Laboratoire d’Astrophysique Extragalactique et de Cosmologie - CNRS URA 173 - Observatoire de Meudon - 5, Place Jules Jansen - 92 195 Meudon - France

\textsuperscript{2} Department of Physics and Astronomy, University of Glasgow Glasgow G12 8QQ - U.K

Accepted 199- Received 199- in original form 199-

\textbf{ABSTRACT}

The polarization of the CMBR scattered by galaxy clusters in the kinematic Sunyaev Zeldovich effect depends on the transverse velocity of the cluster. This polarizing effect is proportional to the transverse velocity squared, and so weaker than the change in intensity due to the radial motion in the kinematic effect. The value given by Sunyaev and Zeldovich, and which is frequently cited, underestimates the polarizing effect by a factor of ten. We show furthermore that the polarization has a strong frequency dependence. This means that the polarization should be detectable with the new generation of CMBR probes, at least for some clusters. Thus this effect offers, almost uniquely, a method of obtaining the vectorial velocity of clusters.

\textbf{Key words:} Cosmology: theory – cosmic microwave – polarization

\section{INTRODUCTION}

One of the main goals of observational cosmology in the coming years is the measurement of the CMBR and its fluctuations. Before arriving to us, the CMBR photons traverse the entire observable universe. It follows that if one can observe the effect of the interaction of these photons with matter they pass through, then one should be able to obtain crucial information about the distribution of matter in the universe. With this perspective, Sunyaev and Zeldovich studied the various scattering processes that take place between black body photons and electrons in galaxy clusters. These phenomena, which are referred to collectively as the ‘Sunyaev-Zeldovich effect’, are now being studied in great detail, and indeed are being used to probe cosmological structure formation (Birkinshaw 1998 and reference therein).

In their paper of 1980 Sunyaev and Zeldovich (Sunyaev & Zeldovich 1980) discussed the possibility of measuring the radial velocity of a cluster of galaxies from the enhancement of the CMBR temperature in the direction of the cluster. This effect is somewhat similar to the well known Sunyaev Zeldovich effect in which CMBR photons are Compton scattered by the hot gas in the cluster, a process which gives rise to a distortion in the CMBR spectrum in the direction of the cluster. For this reason the effect due to the bulk motion of the cluster has become known as the kinematic S-Z effect, and the original effect as the thermal S-Z effect. The thermal effect leads to a variation in intensity of the radiation given by

\begin{equation}
\frac{\Delta I_\nu}{I_\nu} = 2y \frac{x \exp x}{\exp x - 1} \frac{x}{2 \coth x/2} - 2,
\end{equation}

where \( y = \tau k T_e/m_e c^2 \) is the comptonization factor, \( T_e \) the temperature of the electrons in the cluster, \( x = h \nu/k T_\nu \), \( \tau \) the optical depth, and \( T_\nu \) the temperature of the CMBR. For small values of \( x \) the change in intensity is negative, whilst for large values it is positive.

The kinematic effect gives rise to a change in intensity given by

\begin{equation}
\frac{\Delta I_\nu}{I_\nu} = -\beta_r \tau \frac{x \exp x}{\exp x - 1},
\end{equation}

where \( \beta_r \) is the radial velocity in units of the speed of light. The sign of the fractional change in intensity does not change sign with frequency, and is positive if the peculiar radial motion is towards the observer. The effective temperature change in independent of frequency, and given by \( \Delta T/T = -\beta_r \tau. \)

In the same paper the question of the polarization that can be produced when the motion of the cluster has a component transverse to the line of sight is also discussed, and the degree of polarization induced in integrated light from the cluster is claimed to be of the order 0.1\( \beta_t^2 \tau \) where \( \beta_t \) is the transverse velocity of the cluster relative to the CMBR. In fact, as we
Figure 1. The intensity of the CMBR as seen by the cluster at different frequencies plotted against polar viewing angle, $\theta$. The velocity of the cluster, $\beta$, is taken to be 0.01. The small-dashed curve corresponds to 100 GHz, the long-dashed to 353 GHz and the dotted to 857 GHz (these frequencies correspond to three of the Planck-HFI instrument). The solid curve corresponds to the total (frequency integrated) intensity.

show below, the effect is exactly ten times larger, and has a strong frequency dependence. (Sunyaev and Zeldovich also argued that there would be a contribution from higher order scattering given by $1/40\beta\tau^2$, which could be of a comparable size for larger optical depths, but we shall not discuss this here as the latter effect is still more negligible than thought previously.) This polarization will be in a direction perpendicular to the plane formed by the line of sight and the velocity of the cluster. As Sunyaev and Zeldovich point out, potentially this polarizing effect of the kinematic S-Z effect can yield a great deal of information about the peculiar velocities of galaxy clusters, and indeed in theory represents one of the few ways of determining their vectorial velocity. Peculiar velocities are thought to lie in the range 500-3000 km s$^{-1}$. Cluster optical depths are badly known but appear to be in the range 0.01 to 0.05, yielding a frequency integrated degree polarization due to single scattering of the order $2 \times 10^{-7}$ to $5 \times 10^{-6}$. Such small values of polarization could be still difficult to detect.

However, the frequency dependence of this polarization is of crucial importance, firstly because this frequency dependence helps distinguish the signal from other polarizing mechanisms, and secondly because the polarization rapidly increases as a function of frequency. Although Sunyaev and Zeldovich did point out that the polarization should have a frequency dependence, they calculated only the frequency integrated polarization. In this paper we look more closely at the dependence of the polarization on frequency and find that at values of $x > 5$ the polarization will be orders of magnitude higher than the frequency integrated polarization. Furthermore this mechanism has a characteristic frequency dependence which should enable one to distinguish it from other polarizing mechanisms.

The origin of the polarization in the case of the kinematic S-Z effect is the apparent anisotropy of the CMBR radiation seen by the cluster due to its motion relative to the CMBR frame. Although the degree of anisotropy in frequency integrated light for low values of $\beta$ is not very large, the monochromatic intensity displays much greater anisotropy. Figure 1 shows the variation of incident intensity as a function of the viewing direction, described by the polar angle, $\theta$, with respect to the axis of symmetry of the radiation field (defined by the cluster velocity), at different frequencies and for $\beta = 0.01$. Now consider the case where the cluster is moving perpendicularly to the line of sight. Photons incident from the forward direction and scattered towards the observer will be scattered through 90°, and hence fully polarized perpendicular to the line of sight and velocity of the cluster. Similarly photons from the backward direction will be polarized in the same direction. Photons arriving perpendicular to this plane will be polarized in the opposite sense, and hence tend to cancel out the effect of the photons scattered from the forward and backward direction. However the intensity of the incident radiation varies with the angle of incidence, $\theta$. It is in fact the quadrupole that produces the net polarization.

We also find that the degree of polarization depends not only on the transverse velocity, but also weakly on the absolute velocity of the cluster with respect to the CMBR in a non-degenerate way. In principle this means that measurement of the degree of polarization at different frequencies provides a means of determining the vectorial velocity without reference either to the kinematic intensity S-Z effect, and independently of any knowledge of the optical depth. In practice, however, use of optical depth estimates from the thermal S-Z effect, and the radial component of the cluster velocity from the kinematic intensity S-Z effect would provide much greater precision on the vector velocity measurement.

As in the case of the Sunyaev Zeldovich paper, we shall throughout use the Thomson scattering approximation, using relativistic corrections for Doppler and aberration effects. Of course, a complete treatment should consider a distribution of hot electrons, in which case the relativistic expression for electron scattering should be used. For the points we make in this paper the simplified treatment is justified. Moreover, by formulating the problem in the cluster rest frame and including all relativistic transformations and directional effects, this treatment makes itself amenable to generalisation to the full relativistic case.
The geometric situation is represented in Figure 2. The cluster has velocity $\vec{V}$ relative to the rest frame of the CMBR, $F'$. We shall assume throughout that the CMBR has a perfect isotropic black body spectrum. The cluster will see an anisotropic background, and along the line of sight. We denote the angle of this rotation by $\chi$. The scattering plane is defined by the cluster velocity $\vec{V}$ and the line of sight.

2 ANALYTIC EXPRESSIONS FOR THE STOKES PARAMETERS

The geometric situation is represented in Figure 2. The cluster has velocity $\vec{V}$ relative to the rest frame of the CMBR, $F'$. We shall assume throughout that the CMBR has a perfect isotropic black body spectrum. The cluster will see an anisotropic background, and along the line of sight. We denote the angle of this rotation by $\chi$. The scattering plane is defined by the cluster velocity $\vec{V}$ and the line of sight.

\[ \vec{m} = \vec{k}_{out} \times \vec{k}_{in} / |\vec{k}_{out} \times \vec{k}_{in}| \]

Figure 2. Scattering geometry: Unpolarized light incident on cluster is scattered (by stationary electrons) towards the observer through an angle $\chi$. The scattering plane is defined by the cluster velocity $\vec{V}$ and the line of sight.

We take an observer basis $\vec{e}_i$, $\vec{e}_j$, $\vec{e}_k$, where $\vec{e}_k$ is equal to the direction $\vec{k}_{out}$ of the scattered photon, and $\vec{e}_j$ is perpendicular to the plane defined by $\vec{V}$ and the line of sight ($\vec{k}$). This defines the orientation of the polarimeter. To calculate the contribution of all incident photons we have to rotate from the scattering plane to the observer frame, and integrate over all solid angles, $\Omega$, and along the line of sight. We denote the angle of this rotation by $\phi$ (see figure 2). Using the phase function for Thomson scattering, and introducing for convenience the abbreviations $S_\xi$ (resp. $C_\xi$) to denote the sine (resp. cosine) of the variable $x$, we may write

\[ I_{sc}(\nu) = \frac{3\tau}{16\pi} \int I(\nu, \theta)(1 + C^2_\xi)C_{2\phi} d\Omega, \quad Q(\nu) = \frac{3\tau}{16\pi} \int I(\nu, \theta)S^2_\xi C_{2\phi} d\Omega, \quad U(\nu) = \frac{3\tau}{16\pi} \int I(\nu, \theta)S^2_\xi S_{2\phi} d\Omega. \]  

In order to obtain the frequency integrated Stokes parameters, $I(\nu, \theta)$ should be replaced by $I(\theta)$ in equations 6. The optical thickness, $\tau$, is given by an integral along the line of sight $\tau = \int \sigma_T n_e dl$, where $n_e$ is the electron density and $\sigma_T$ the Thomson cross-section. Noting that $C_\chi = C_\phi C_i - S_\phi C_i S_\phi$, $C_\phi = -(C_\theta S_i + S_\theta C_i C_i)/S_\chi$ and $S_\phi = S_i S_\theta / S_\chi$ and integrating over $\phi$, equations 6 reduce to

\[ I_{sc}(\nu) = \frac{3\tau}{8} \int_{-1}^{1} I(\nu, \theta)(1 + \mu^2 - S^2_\xi(3\mu^2 - 1)/2) d\mu, \quad Q(\nu) = \frac{3\tau}{16} S^2_\xi \int_{-1}^{1} I(\nu, \theta)(3\mu^2 - 1) d\mu, \quad U(\nu) = 0. \]
The degree of polarization is plotted against frequency in GHz. The transverse velocity of the cluster is $\beta_t = 0.01$ for the lower group (solid line) of curves and $\beta_t = 0.03$ in the upper (dashed line). In each group the lower curve corresponds to $i = 90^\circ$ and the upper to $i = 45^\circ$. The solid upper curve shows the intensity of the CMBR in arbitrary units (right vertical axis). The vertical dotted lines shows the different frequency bands of the Planck experiment and the squares display the expected sensitivity of the individual polarimetric detectors (when they exist).

The total intensity seen by $O$ is then given by

$$I_{\text{tot}}(\nu) = I + I_{sc} - I_{ab},$$

where $I = I(\nu, i)$ is the direct light, $I_{sc}$ is the light scattered toward the observer and $I_{ab} = \tau I$ is the light scattered out of the line of sight. The degree of polarization, $p$, of the net light seen by $O$ is then given by

$$p(\nu) = |Q(\nu)|/I_{\text{tot}}(\nu).$$

All the above quantities are expressed for the observer $O$. For an observer $O'$ at rest in the CMBR frame we have $p'(\nu') = p(\nu)$ and $I'(\nu') = (\nu'/\nu)^3 I(\nu)$ where $\nu' = \nu(1 + \beta C_i)^\gamma$.

The fraction change in intensity given by $\Delta I(\nu) = (I_{\text{tot}}(\nu) - I(\nu))/I(\nu)$ describe the kinematic SZ effect and, of course, coincide with the result of Sunyaev and Zeldovich (Sunyaev & Zeldovich 1980) in the case of radial motion $S_i = 0$.

We can now compute the frequency integrated degree of polarization by simply replacing $I(\nu, \theta)$ by $I(\theta)$ in equations (7). Keeping only terms up to $\beta^2$ in the expansion of $I(\theta)$ (eq. 5) and taking $I_{\text{tot}} = I$ and we obtain

$$p = S_i^2 \beta^2 \tau = \beta_i^2 \tau,$$

where we have introduced the transverse velocity $\beta_t$. This results has the same velocity and optical depth dependence as the one obtained by Sunyaev and Zeldovich, but is ten time greater. Since $p$ is Lorentz invariant this old also in the CMBR frame. Furthermore, as we shall see in the next section, the polarization has a strong frequency dependence. In the low frequency limit, the degree of polarization is independent of the frequency and is given by

$$p = S_i^2 \frac{3}{16} \tau \left(\frac{3 - \beta^2}{\beta^3}\right) \ln \left(\frac{1 + \beta}{1 - \beta}\right) - \frac{6}{\beta^2} \approx 0.1 \beta_i^2 \tau + O(\beta^4).$$

This results is similar to what was claimed by Sunyaev and Zeldovich for the frequency integrated effect.

### 3 NUMERICAL RESULTS

In this section we present the degree of polarization that should be observed for a typical galaxy cluster. All our results are given for $\tau = 0.1$ but values for other optical depths can simply be scaled linearly.

Figure 3 displays the degree of polarization against frequency for various transverse cluster speeds. We can see that $p$ slightly depends on the inclination even for constant values of $\beta_t$. As mentioned previously, the polarization reaches a constant value in the Raleigh-Jeans (low frequency) limits according to formula 11. The variation of $p$ with frequency is very strong.
The degree of polarization is plotted against $\beta$ for purely transverse motion ($i = 90^\circ$). Each curve corresponds to one of the Planck-HFI band: 857 GHz (solid line), 545 GHz (dotted line), 353 GHz (short dashed line), 217 GHz (long dashed line), 143 GHz (dot - short dashed line), 100 GHz (dot - long dashed line). The solid horizontal lines represents the degree of primordial polarization expected for a standard CDM model (Hu & White 1997) and the thick solid line the frequency integrated degree of polarization.

In the Planck higher frequency band, the expected polarization for $\beta = 0.01$ and 0.03 are of 0.01% and 0.10% respectively, although these need to be scaled according to the optical depth. Such degrees of polarization are above the detection limits foreseen by coming experiments. Figure 3 display the sensitivity of the individual polarimetric detector of the Planck experiment. Since there are between 8 and 34 polarimetric detectors per band and since a cluster could have an angular size sensibly larger than the instrument resolution, it should be possible to get the polarimetric signal by proper processing of the data. Furthermore, new experiments could be designed especially for polarimetric measurement.

Figure 4 displays the degree of polarization against $\beta$ for different frequencies. For all frequencies, as for the integrated effect, the degree of polarization has a $\beta^2$ dependence. We can see that the polarization due to Thomson scattering in the cluster is much larger than the expected primordial polarization for a wide range of parameter and should therefore be measurable.

4 CONCLUSIONS

Transverse velocities of galaxy clusters will produce polarized signal in the scattered CMBR. This polarization will be perpendicular to the cluster velocity and the light of sight. Although in frequency integrated light the level of polarization might be difficult to detect with the techniques presently available, the polarization has a strong frequency dependence, increasing towards higher frequencies. For values of transverse velocity of 1000 km s$^{-1}$, and an optical depth of 0.02, the degree of polarization at 857 GHz is around $2 \times 10^{-6}$. Since the polarization depends on the square of the transverse velocity, and linearly on the optical depth, certain clusters should show higher polarization. Other sources of polarization, in particular dust and synchrotron radiation, will contaminate the signal. However filtering techniques that take into account the characteristic frequency dependence of this polarization should allow one to extract the signal due to the kinematic effect. Measured polarization could then be used to infer transverse velocities of clusters. This question will be elaborated in future work.

REFERENCES

Birkinshaw, M., 1998, to appear in Physics Reports, astro-ph/9808050
Chandrasekhar, S., 1950, Radiative Transfer, Clarendon Press, Oxford
Hu, W. and White, M., 1997, New Astronomy, vol. 2, no 4., p323
Sunyaev R.A. and Zeldovich Y.B., 1980, MNRAS, 190, 413-420

© 1997 RAS, MNRAS 000, 000-000