Iterative adaptive approach STAP algorithm based on arc
antenna array

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Abstract. The radar with arc antenna array has the advantages of all-orientation space scanning, simultaneous estimation of pitch-azimuth angle and good antenna pattern. However, due to the special geometry of the antenna, the echoes from different range cells are not independently and identically distributed (IID) samples, resulting in performance degradation in space-time adaptive processing technology. In this paper, the model of airborne radar with arc antenna array is established, and the clutter distribution is analyzed. The conclusion is that the nonlinear variation of spatial angular frequency with array element number results in the non-uniform clutter distribution in different range cells. Therefore, the traditional space-time adaptive processing is not fit for the target detection of arc array radar. Hence, the arc array-based iterative adaptive approach (IAA) algorithm is studied. The IAA algorithm is utilized to estimate the space-time spectrum, which can distinguish clutter and targets. Simulation experiments verify the effectiveness of the IAA algorithm in arc array radar.

1. Introduction
More and more countries have been paid more attention to the research and development of airborne radar. One of the important reasons is that space-time adaptive (STAP) technology can be used to improve the performance of clutter and interference suppression. The traditional STAP methods are based on uniform linear array which has a single viewing angle. Compared with it, the arc array radar can scan the omni directional range around the platform[1], which can estimate the pitching angle and azimuth angle of signals simultaneously, and have good beam bearing. The airborne radar is always used to detect targets by STAP methods. The traditional statistical STAP algorithms are based on a side-looking uniform linear array radar, and the clutter in different range cells satisfies the condition of independently and identically distribution (IID)[2]. Since there are enough IID samples used to estimate the clutter covariance matrix, the performance of STAP is optimal. However, due to the particularity of the arc array radar structure, the clutter distribution is non-uniform[3-4], namely, the clutter in the training and detection samples doesn’t satisfy the IID condition. Hence, most of the STAP methods based on linear array radar cannot be used in arc array radar directly. There are some non-uniform STAP technologies can be utilized in arc array radar. Local processing technology reduces the number of training samples and the degree of clutter spectrum broadening, thereby reducing the degree of clutter non-uniformity[5-7]. Clutter compensation technology makes the clutter
spectrum of each range cell coincide at a certain point through clutter translation, which reduces the clutter inhomogeneity by improving the clutter similarity. Derivative based updating (DBU) technology can reduce the number of training samples of STAP[8-9].

In this paper, the arc array-based iterative adaptive approach (IAA) algorithm is proposed. First, the space-time two-dimensional signal model is established according to the antenna structure of the arc array, the space-time steering vector of the arc array is calculated, and the clutter distribution and clutter statistical information of the arc array radar are analyzed. The IAA method is introduced into the arc array STAP to achieve the space-time spectrum of clutter and targets[10]. Finally, the feasibility and effectiveness are verified through simulation.

2. Clutter characteristics of arc array radar

Helicopter-borne arc array radar is shown in figure 1. The aircraft is flying with an average speed of $v$ along the positive action of the X-axis. Assuming that the airborne radar antenna is an arc array with $p$ elements evenly distributed on the circumference, whose radius is $R$. The coordinates shown in figure 2 are established.

\[ \theta_n = \frac{2\pi(n-1)}{P}, \quad n = 1, 2, \ldots, P. \]

At any working moment, $N$ adjacent array elements on the circumference are selected for beam scanning (e.g., array element 1~$N$). At the next working moment, array element selection is realized through phased array antenna technology. The first array element is removed to next, from which the $N$ array elements are selected to work. Hence, the azimuth $360^\circ$ scan is realized. The transmitting signal of radar is pulse sequence with length $K$ shown by

\[ S(t) = Au(t) \exp(j\omega_t t) \] (1)

\[ u(t) = \sum_{k=1}^{K} u_r [t - (k - 1)T_r] \] (2)

where, $u_r(t)$ is the complex envelope of a single matrix pulse, $A$ is the amplitude value of the transmitted signal, $T_r$ is pulse repetition interval, $f_t$ is pulse repetition frequency, $\omega_0 = 2\pi f_0$ is the angular frequency and $f_0$ is the carrier frequency of the signal. For the target in direction $(\theta, \phi)$, the echo signal of the $n$th element is

\[ \tilde{S}_n(t) = A_r u(t-t_n) \exp[j2\pi(f_0 + f_d)(t-t_n)] \] (3)

where, $A_r$ is the amplitude of the received signal, $f_d = 2v \cos \theta \cos \phi / \lambda$ is the Doppler frequency of the target, $t_n$ is the time delay of the echo signal relative to the $n$th element, and $\lambda$ is the radar working wavelength. Assuming that the number of radar transmitted pulses in a coherent processing interval is
$k$, given a certain range cell, the echo data of $K$ pulses of all $N$ elements is a $NK+1$ dimensional vector, which is expressed as

$$X = \int_0^s \rho(\varphi)S_i(f_d) \otimes S_s(\varphi)d\varphi + N$$

(4)

where, $\rho(\varphi)$ is the clutter scattering coefficient. Since the time domain steering vector has nothing to do with the geometry of the array, it has the same form as the conventional array. The $K \times 1$ dimension time domain steering vector is shown by

$$S_t = \left[1, \exp\left(j \frac{4\pi v}{\lambda f_r} \cos(\varphi)\cos(\theta)\right), \ldots, \exp\left(j(K-1) \frac{4\pi v}{\lambda f_r} \cos(\varphi)\cos(\theta)\right)\right]$$

(5)

Because of the particularity of the arc array structure, the center of the circle is taken as the reference of the array. The geometric relationship of the echo signal is shown in figure 2. The phase delay of the $n^{th}$ element can be expressed as:

$$W_s(n) = \frac{2\pi}{\lambda} R \cos(\varphi_s - \varphi) \cos(\theta)$$

(6)

The $N \times 1$ dimension space domain steering vector is given by

$$S_s(n) = \exp[j \cdot W_s(n)], n = 1 \ldots N$$

(7)

Then the space time steering vector is gotten as

$$S = S_T \otimes S_s$$

(8)

where $\otimes$ is the Kroncker product. If the target exists, the signal has the same structure as clutter.

Assuming that the aircraft platform flies in a straight line with a horizontal uniform velocity $v$, which is shown in figure 3, the Doppler frequency of the clutter scattering unit in the direction $(\varphi, \theta)$ is

$$f_d = \frac{2v}{\lambda} \cos \theta \cos \varphi = \frac{2v}{\lambda} \cos \psi$$

(9)

where, $\psi$ is the spatial cone angle. It satisfies the relation with the pitching angle $\theta$ and azimuth angle $\varphi$ shown by

$$\cos \theta \cos \varphi = \cos \psi$$

(10)
When the viewing angle is non-side-looking, the normalized Doppler frequency of the clutter is

\[ \bar{f}_d = 2f_d / f_r = 4v \cos \theta \cos (\varphi - \beta) / \lambda f_r \] (11)

where, \( \beta \) is offset angle. When the viewing angle is side-looking (\( \beta = 0^\circ \)), the relationship between ratio of cone angle and normalized Doppler frequency is

\[ \rho = \frac{\cos \psi}{f_d} = \frac{\lambda f_r}{4v} \] (12)

It can be seen that the clutter spectrum is linear distribution in the \([\cos \psi, 2f_d / f_r]\) plane. When the viewing angle is non-side-looking (\( \beta \neq 0^\circ \)), \( \rho \) is

\[ \rho = \frac{\cos \psi}{f_d} = \frac{\lambda f_r}{4v \cos \theta \cos (\varphi - \beta)} \] (13)

The clutter spectrum is non-linear distribution in the \([\cos \psi, 2f_d / f_r]\) plane. This is similar to uniform linear array[11].

3. Iterative adaptive approach

The IAA algorithm can directly form a high-resolution angle-Doppler image of the data from the cell under test, which includes clutter, moving targets and interference[10]. Therefore, the moving target can be directly positioned and indicated according to the image. The IAA algorithm is non-parametric spectrum estimation method, which can obtain high-resolution and low sidelobe spectral estimation by only a few or even one snapshot. The basic idea of IAA algorithm is to solve the following optimization problems of weighted least squares.

\[ \min \left\| X - \gamma_{k,i} S \right\|_{Q_{k,i}}^2 \] (14)

Where, \( \gamma = [\gamma_{1,1}, \gamma_{1,2}, ..., \gamma_{N_d, N_s}] \) represents the complex amplitude of the clutter components in the angle-Doppler domain. \( k = 1,...,N_d, i = 1,...,N_s, N_d = \rho_d K, N_s = \rho_s N \). \( \rho_d \) and \( \rho_s \) represent the resolution scale of Doppler frequency and spatial frequency, respectively, which can control the magnitude of quantization error.

In formula (14),

\[ \left\| X \right\|_{Q_{k,i}}^2 = X^H Q_{k,i} X \] (15)

where, \( Q_{k,i}^{-1} \) represents the covariance matrix of clutter (no target) or clutter-target (target presence) plus noise (all signal components except the spatial frequency and Doppler frequency are \((f_d,k, f_s,i)\) signal components), specifically written as

\[ Q_{k,i} = R - a_{k,i} S (f_d,k, f_s,i) S^H (f_d,k, f_s,i) \] (16)

where, \( a_{k,i} = |\gamma_{k,i}|^2 \) represents the signal component power at point \((f_d,k, f_s,i)\), and \( R \) represents the space-time covariance matrix obtained from the discretization hypothesis model, shown by

\[ R = \sum_{k=1}^{N_d} \sum_{i=1}^{N_s} a_{k,i} S (f_d,k, f_s,i) S^H (f_d,k, f_s,i) \] (17)

By solving the optimization problem (14), parameter \( a_{k,i} \) can be estimated as[10]

\[ \hat{a}_{k,i}^{\text{IAA}} = |\gamma_{k,i}|^2 = \frac{S^H (f_d,k, f_s,i) R^{-1} X}{S^H (f_d,k, f_s,i) R^{-1} S (f_d,k, f_s,i)} \] (18)
Unlike the traditional STAP methods, the IAA-STAP algorithm obtains the space-time power spectrum of the cell under test, and the moving target is detected by the space-time power spectrum through the median filter directly. The specific steps are shown as follows.

Step 1: The background clutter level is determined by the space-time power spectrum of the adjacent range cell. Assuming that the Doppler frequency and spatial frequency of the interested target are $f_{d,t}$ and $f_{t,t}$ respectively, the median of the space-time power spectrum of $L$ neighboring range cells at $f_{d,t}$ and $f_{t,t}$ is taken and denoted as $\eta_m(f_{d,t}, f_{t,t})$.

Step 2: Design the median filter according to the following formula.

$$10 \log_{10} a_{k,t} - 10 \log_{10} \eta_m(f_{d,t}, f_{s,t}) > \xi$$

Or

$$10 \log_{10} a_{k,t} - 10 \log_{10} \eta_m(f_{d,t}, f_{s,t}) < \xi$$

where, $a_{k,t}$ is the value of the space-time power spectrum of the cell under test at $f_{d,t}$ and $f_{s,t}$, and $\xi$ is the detection threshold of the median filter.

4. Simulation results and performance analysis
The simulation parameters are listed in Table 1.

| Parameter                        | Value  |
|----------------------------------|--------|
| Platform velocity                | 300(\text{ms}^{-1}) |
| Platform height                  | 3000m  |
| Radar wavelength                 | 0.2m   |
| Pulse repetition frequency       | 6000Hz |
| Total elements number            | 900    |
| Elements number                  | 9      |
| Pulses number                    | 10     |
| Inter-element spacing            | 0.1m   |
| Spacer ring width                | 30m    |
| Clutter-to-noise ratio           | 60dB   |

Figure 4. Space-time power spectrum estimation.
Figure 4 shows the simulation results. The space-time power spectrum estimation of IAA-STAP based on linear arrays is shown in (a), and space-time power spectrum estimation of IAA-STAP based on arc arrays is shown in (b).

It can be seen from figure 4 that the IAA method can be well combined with the arc array structure, and the clutter is distributed on the diagonal clutter ridge. The two target points in the figure can already be distinguished.

5. Conclusion
This paper introduces the geometric structure of the arc array radar and analyzes the characteristics of the clutter based on the arc array. Because the clutter distribution depends on the range cells, the traditional STAP method is unable to detect target in the cell under test. Hence, the iteration adaptive approach is utilized to detect moving targets effectively with the arc array radar. Combining the advantages of the iterative adaptive approach and reducing the requirement of STAP for the number of IID samples, the high-resolution angle-Doppler image can be formed directly on the data of cell under test, which includes clutter, moving targets and interference. The accuracy of clutter distribution is guaranteed and the possibility of moving target detection based on arc antenna array is increased.

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