Abstract

Programming in Prolog is hard for programmers that are used to procedural coding. In this manual the method of drawing search trees is introduced with the aim to get a better understanding of how Prolog works. After giving a first example of a Prolog database, query and search tree, the art of drawing search trees is systematically introduced giving guidelines for queries with variables, conjunction, disjunction, and negation. Further examples are provided by giving the complete search trees that are shown in Learn Prolog Now!
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1 Introduction

Prolog, a *declarative* programming language, requires a different mindset for programmers who are used to code in *procedural* programming languages like C, Python, or Java. It is astonishingly hard to achieve this different attitude to coding. One approach that sheds light on the way Prolog executes programs is to apply the brutal method of drawing complete search trees. It is well worth the effort: mastering Prolog will enrich the life of a programmer considerably, because it adds a completely new (and beautiful) perspective on solving computer science problems.

Prolog’s backbone is based on a depth-first search strategy that operates on facts and rules declared in a database (Clocksin and Mellish, 1987; Bratko, 2001; Blackburn et al., 2006). This sounds harmless, but it is not always straightforward to predict how a Prolog program behaves in response to a certain query (sometimes also called *question* or *goal*) given to it. The best way to find out how Prolog reacts to a query, given a certain knowledge base, is to draw a search tree. In the standard logic programming course books there is too little space devoted to drawing search trees.

Clocksin and Mellish (1987), one of the first complete introductions to programming in Prolog, look at the way Prolog solves queries by providing a *flow of satisfaction*, which is based on the tracing model that Prolog offers for debugging programs. Bratko (2001) probably was the first to introduce search trees, but he calls them *execution traces*. The search trees in this manual are inspired by those of Bratko.

Even though drawing search trees requires some effort, students of logic programming will get a much better understanding of how their Prolog programs work by doing so, instead of peeking at the output of overrated tools such as the trace facility. This manual is therefore meant to serve as an aid in the classroom, both for students and teachers of logic programming. Trees can be drawn in a free fashion by paper and pencil, with the aid of sophisticated drawing software, or just by a simple straightforward standard text characters. The latter method will be used in this manual.

This manual is not an introduction to Prolog, and assumes the reader has some basic experience with logic programming, and is familiar with the concepts clause, fact, rule, predicate, backtracking, and unification. We will first show the basic idea of drawing search trees in Section 2 where we give an example for a simple query, a Prolog database, and the corresponding search tree. This will give the reader a first feeling of what search trees are and what their potential is. In Section 3 we show the search tree expansion rules, for simple queries (with and without variables), conjunctive queries, queries with disjunction, and queries for some built-in predicates In Section 4 we show how to draw search trees for queries with negation and the cut. Finally, in Section 5 we give our version of all search trees that occur in Blackburn et al. (2006).
2 Drawing Search Trees

2.1 The general idea

The programming language Prolog, one of the oldest (and one of the oddest) programming languages in the world, has its roots in (classical) logic. It therefore divides everything in true or false. Like classical logic, Prolog’s ingredients are based on conjunction, disjunction, implication, and negation, although the use of these operators is restricted and Prolog’s negation is based on the closed world assumption (see Section 4).

Prolog works by answering queries with respect to a knowledge base. Queries are like yes/no-questions in natural language: they are answered by Prolog as yes (following the information in the knowledge base), or no (the knowledge base does not support the query). A Prolog database comprises clauses that are either rules or facts. Rules introduce new queries, facts lead to positive answers, and if a piece of information cannot be deduced, it will be considered false. Together, the facts and rules define one or more predicates in the knowledge base. Queries can also contain variables (recall that variables are denoted by uppercase letters in Prolog). A query with variables for which Prolog returns a positive answer, will also instantiate the variable with values that make the query true. Such queries are the natural language equivalent of wh-questions (questions starting with who, what, where, and when).

A search tree shows the way Prolog finds an answer to a query. It shows all the steps of reasoning to come to an answer, so you may also call it a proof tree, using more logical terminology. In this manual, we use the following strategy for drawing search trees:

1. Write down the query in a box on the top of the page—the box acts as a sort of to-do list;
2. Draw the branches based on the number of clauses defined for the predicate that the query addresses;
3. Proceed with the branches in a left-to-right fashion. If a branch corresponds to a fact, go to Step 4. If a branch corresponds to a rule, go to Step 5.
4. If the query does not unify with the fact draw a cross (x, denoting a dead branch); otherwise (the query unifies with the fact) draw an empty box ( ), signalling that there is nothing left to do;
5. If the query does not unify with the head of the rule draw a cross; otherwise draw a new box with the body of the rule as new query. Go to Step 2.

Of course, search trees can be drawn in many different ways. It doesn’t matter which style you’re using, as long as you’re consistent (for instance, instead of the cross you can also write fail or no, and instead of the empty box you can also write yes or success). You can draw them with pencil and paper, or use a handy graphical visualisation tool.

In this manual we follow, by and large, the graphical conventions introduced in Learn Prolog Now! by Blackburn et al. (2006). It is a very simple way to draw trees in a normal text format. However, note that not all search trees in Learn Prolog Now! are complete (see Section 5).

Let’s have look at a first example.

2.2 A first example

The example we consider is taken from Learn Prolog Now! (Blackburn et al., 2006, p.38), and describes a situation with rather complicated love relationships between three people and a (rather naive) definition of jealousy, as shown in Listing 1.
Suppose we are interested in the answers to the query `jealous(X,Y)`. Following Step 1 of the recipe given above, we start our first search tree by putting the query in an empty box (we use square brackets to visualise boxes):

```
[jealous(X,Y)]
```

This query addresses the predicate `jealous/2`. There is only one clause defining this predicate in the database. Hence, we only draw one branch:

```
[jealous(X,Y)]
```

Because the clause in the database is a rule, we do two things: first we check whether the head of the rule unifies with the query. It does, because we have for both arguments variables in the query, and variables always unify. What happens next is that the body of the rule in line 4 of Listing 1 will form the new query, taking into account the variable bindings that occurred while matching the query with the head of the rule. So this gives us:

```
[jealous(X,Y)]
```

```
| X=A
| Y=B
| [loves(A,C), loves(B,C)]
```

We now proceed at looking at this new query. It is a compound query, and it contains two sub-queries combined with a conjunction. In this case we first look at the left conjunct: `loves(A,C)`. The `loves/2` predicate is defined by two clauses in the database as Listing 1 shows in line 1–2. So we will draw two new branches—one for each clause:

```
[jealous(X,Y)]
```

```
| X=A
| Y=B
| [loves(A,C), loves(B,C)]
```

```
/  \
|    |
/  \
```

Note, at this point, that the order of the clauses in the database—from top to bottom—corresponds to a left-to-right order in the search tree that we draw. This is just a conventional habit that we adopt (in a culture where one would write from right to left, we probably would have picked a different order). But this order is important. As Prolog searches answers to queries by examining clauses from top to bottom, the order of answers (in case there is more than one answer) should be reflected in the search tree. Hence, the order of the answers in a search tree should be read from left to right. And this explains why we proceed with the first branch from the left, and do not look at the second, open, branch (because recall, Prolog’s search strategy is depth-first!). The left-most branch targets the first clause of `loves/2`, shown in line 1 of Listing 1. This is a fact. So the answer is either true or false. It is true if the fact matches with the query, which it does, because once again, the query contains only variables: variable `A` unifies with atom `vincent`, and variable `C` unifies with atom `mia`.
Does this mean we have our first answer? No! We were dealing with a compound, conjunctive, query. We did the first part of this compound query, but still have to solve the remaining sub-query. So our new query becomes $\text{loves}(B, \text{mia})$ and the search tree is expanded as follows:

```
[jealous(X,Y)]
  |  1
  |  2
  |  3
  |  4
  |  5
  |  6
  |  7
  |  8
  |  9
  |  10
  |  11
  |  12
  |  13
  |  14

  [loves(A,C),loves(B,C)]
    /  6
   /   7
  /    8
 [loves(B,\text{mia})]
```

Once more, we deal with the predicate $\text{loves}/2$, and there are two possibilities because it is defined by two clauses, so again we draw two new branches:

```
[jealous(X,Y)]
  |  1
  |  2
  |  3
  |  4
  |  5
  |  6
  |  7
  |  8
  |  9
  |  10
  |  11
  |  12
  |  13
  |  14

  [loves(A,C),loves(B,C)]
    /  6
   /   7
  /    8
 [loves(B,\text{mia})]
```

We proceed by examining the left-most branch. The corresponding fact in the database matches with the query, and variable $B$ unifies with atom $\text{vincent}$. Because a fact does not introduce new queries, we have our first answer, and we signify this with an empty box in the search tree, showing that we satisfied all queries: there is nothing more left to do on our to-do list (instead of an empty box in line 15 you could also label the node with “yes” or “true”):

```
[jealous(X,Y)]
  |  1
  |  2
  |  3
  |  4
  |  5
  |  6
  |  7
  |  8
  |  9
  |  10
  |  11
  |  12
  |  13
  |  14

  [loves(A,C),loves(B,C)]
    /  6
   /   7
  /    8
 [loves(B,\text{mia})]

  /  11
 /   12
/    13
\  14
[]
```

The answer that Prolog will give to the original query is: $\text{true}$, with the variable $X$ unified with the atom $\text{vincent}$ (because $X$ unifies with $A$), and variable $Y$ unified with atom $\text{vincent}$ (because $Y$ unifies with $B$). (So the answer to our original query, “Who is jealous of whom?”, is “Vincent is jealous of himself”. This isn’t perhaps the answer you would expect, but as we have remarked before, the definition if jealousy in Listing 1 is a rather naive one and incomplete.)

So far so good—we got our first search tree, but it is not complete yet. There are still open branches that might give us alternative (positive) answers. To make the search tree exhaustive,
we perform, like Prolog does if you ask more for solutions, backtracking. We go up in the tree, and find the first open branch. We continue to expand this branch. This will give us the following search tree and a second solution for the query:

```
[jealous(X,Y)]
  |   
  | X=A
  | Y=B
  |
[loves(A,C),loves(B,C)]
  |
  |
  |
  |
A=vincent
C=mia
|
|
|
|
|
[loves(B,mia)]
  |
  |
B=vincent
|
|
[
[]
]

Is this search tree exhaustive? No, it isn’t: there is yet another open branch. When we turn to that, we arrive at the following search tree:

```
[jealous(X,Y)]
  |   
  | X=A
  | Y=B
  |
[loves(A,C),loves(B,C)]
  |
  |
  |
  |
A=vincent
C=mia
|
|
|
|
|
[loves(B,mia)]
  |
  |
B=vincent
|
|
[
[]

The current query that we work on the search tree, loves(B,mia), is a query we have already seen before, and is already part of the larger search tree. So we know how it will expand, as the queries in search trees are independent from each other. This means the complete, exhaustive search tree can be graphically realised as follows:

```
[jealous(X,Y)]
  |   
  | X=A
  | Y=B
  |
[loves(A,C),loves(B,C)]
  |
  |
  |
  |
A=vincent
C=mia
|
|
|
|
|
[loves(B,mia)]
  |
  |
B=vincent
|
|
[
[]
]

Note that variable bindings, when they appear, are decorated along the branches of the search tree. Each Prolog answer to a query corresponds to a path from the top query to an empty box. All the variable bindings along this path are the instantiations used in answering the query. For
this particular query we get four different answers: Vincent is jealous of Vincent, Vincent is jealous of Marsellus, Marsellus is jealous of Vincent, and Marsellus is jealous of Marsellus.

2.3 Next steps

In the previous example we got acquainted with a first search tree with a couple of exciting developments. But there are a lot of other things that can happen when constructing search trees. Not all paths may lead to an answer: they die, instead, leading to dead branches. There are compound queries as we have seen, but sometimes the queries are disjunctive instead of conjunctive, and there are cases where we have to rename the variables. The next section gives a schematic overview of all cases that may appear (excluding negation and the cut, to which we define a separate section).

3 The Search Tree Expansion Rules

3.1 A simple query without variables

First we place the query in a box or list. Next we check how many clauses are defined in the Prolog database for the predicate matching the query. Then we draw a branch for each clause that is defined by the matching predicate. For instance, say you get a query p(a), and the predicate p/1 happens to be defined by two clauses in the knowledge base. Then we can draw a tree with two branches, and we concentrate on the left-most branch. There are three possibilities: (i) there is no fact p(a) in the database: in this case we draw a cross to indicate that this possibility is “dead”; (ii) there is a fact p(a) in the database: we draw an empty box to indicate we found an answer; (iii) there is a rule whose head matches the query, e.g. p(a):- Q., in the database: draw a new box for the query Q. All three options are shown here:

| NO FACT: p(a). | FACT: p(a). | RULE: p(X):- Q. |
|----------------|-------------|----------------|
| []             | [p(a)]      | [p(a)]        |
| / \            | / \         | / \           |
| / \            | / \         | / \           |
| x ...          | [] ...      | [Q] ...       |

Figure 1: Search tree expansion rules for queries without variables.

Let’s have a look at a concrete example where we can apply the rules shown in Figure 1. Consider the following knowledge base about a small animal kingdom with various mammals, reptiles, and birds (Listing 2).

```
1  mammal(fox).
2  mammal(whale).
3  mammal(bat).
4  bird(pelican).
5  bird(swan).
6  reptile(snake).
7  animal(X):- mammal(X).
8  animal(X):- reptile(X).
9  animal(X):- bird(X).
```

Listing 2: Prolog database for a small animal kingdom.
This knowledge base contains the definition for four predicates using six facts and three rules. Suppose we want to know whether a bat is an animal, according to this database. The corresponding search tree for \texttt{animal(bat)} is drawn as follows:

And indeed, a bat is an animal, because a bat is a mammal, and every mammal is an animal. The search tree shows this by the branch with the empty box in line 9.

What about queries that contain variables? How do the expansion rules look for such queries? Very similar, but we need to keep track of the variable instantiations. Let’s have a look.

### 3.2 Simple queries with variables

Here we consider search trees for queries with variables. As with any query, we place it in a box. Then we check how many clauses are defined in the Prolog database for the predicate matching the query. We draw a branch for each clause that is defined by the matching predicate. But now, because there are variables in the query, each branch also needs to be annotated in case variable bindings occur. Like before, we need to concentrate on the left-most branch first. There are three possibilities: (i) there is a fact in the database that matches the query: in this case we draw an empty box and decorate the branch with variable bindings; (ii) there is a rule whose head matches the query, for instance \texttt{p(Y):- q(Y) .}, and there no variables in the rule with the same name as in the query: add the body of the rule as a new query to the tree; (iii) there is a rule whose head matches the query that contains variables with the same name as in the query (e.g. \texttt{p(X):- q(X) .}: in this case you need to rename the variables of the rule to avoid confusion. All three options are shown in Figure 2.

![Search tree expansion rules for queries with variables.](image)

Consider again the knowledge base about the animal kingdom (Listing 2). The search tree for \texttt{animal(Animals)} (what is an animal?) is drawn as follows.

![Search tree for animal(Animals).](image)

9
3.3 More than one query: conjunctive queries

In case there is a conjunctive query (two or more queries separated by a comma), we first consider the left-most query. The remaining queries are carried over if the first one succeeds.

\[
\begin{array}{c}
\text{FACT: } p(a) \\
[p(X),Q] \\
\text{RULE: } p(A) :- q(A). \\
[p(X),Q] \\
[x=A] \\
[q(A),Q] \\
\end{array}
\]

Figure 3: Search tree expansion rules for conjunctive queries.

3.4 A disjunctive query

The semi-colon denotes a disjunctive query. Here it is important to look at the scope of the disjuncts (it is good practice to use brackets to disambiguate scope ambiguities). Disjunction always creates two new branches, one for each disjunct. Nothing happens to the variables, and further queries in the box are duplicated (Figure 4).

\[
\begin{array}{c}
[p(X);q(X)] \\
[p(X)] \\
[q(X)] \\
\end{array}
\]

Figure 4: Search tree expansion rules for disjunctive queries.

3.5 Lists and variable renaming

Sometimes variables need to be renamed when drawing a search tree. This is required when there is a variable occurring in a query that is also used in the rule. We illustrate this procedure of renaming with the help of the standard definition for \texttt{member}:

\[
\begin{align*}
1 & \text{ member}(X,[X|L]). \\
2 & \text{ member}(X,[\_|L]):- \text{ member}(X,L).
\end{align*}
\]

Listing 3: Definition of \texttt{member}

Say we follow the example in Learn Prolog Now! and have a query \texttt{member(X,a,b,c)} (Blackburn et al., 2006, p.244). The query happens to contain the variable \texttt{X}. This variable is also used by the rule in the definition of \texttt{member/2}. To avoid confusion we need to rename the variables in the rule each time we use it. It is safe to do this: the scope of Prolog variables is restricted to the rule they occur in. So the rule above can be replaced by \texttt{member(Y,[\_|L]):- member(Y,L)}, for instance. You can pick any (unused so far in the search tree) variable name that you like. In the tree below we have chosen \texttt{X'}, \texttt{X''}, and so on, but we could have also chosen \texttt{X1} and \texttt{X2} or other variables.
The result of this query are three solutions: $X=a$, $X=b$ (because $X=X'$ and $X'=b$), and $X=c$ (because $X=X'$, $X'=X''$ and $X''=c$).

4 Search Trees with Negation and the Cut

4.1 The built-in control predicates

There are a couple of built-in predicates that affect the way search trees are constructed. One important one is the cut. In order to get good understanding of the cut, we first need to look at the logical predicates. The predicate true/0 always succeeds; the predicates fail/0 (or the equivalent false/0) always fail. Hence, their corresponding search trees are as follows:

| fail | fail,Q | false | false,Q | true | true,Q |
|------|--------|-------|---------|------|--------|
|      |        |       |         |      |        |
| x    | x      | x     |         | x    | []     |

Figure 5: Search tree expansion rules for the logical predicates.

4.2 The cut

Without doubts, one of the most interesting (and one of the most trickiest) tools of Prolog is the cut predicate. It is a predicate that always succeeds. But it has an extremely useful side-effect: it cuts open branches from the search tree, thereby giving direct control on memory management. Cuts are used to make Prolog programs efficient. This often goes at the cost of making Prolog programs less declarative and readable. The fact or the matter is that cuts are needed in practical applications of logic programming.

The cut is not an easy concept to understand. Some newcomers to Prolog confuse cut with a halt or a stop. But it has little to do with that, because it always succeeds. What cut does is pruning open branches (choice points) in the search tree. But, and here is where it gets tricky, not necessarily all open branches in the search tree. The effect of the cut predicate is to discard all choice points (open branches) created since entering the predicate in which the cut appears. In other words, commit to the clause (rule) in which the cut appears and discard choice points that have been created by goals to the left of the cut in the current clause. So the general schemata look like:

But these rules don’t show anything about cutting branches. So let’s have a look at an example using cut.
### 4.3 An example using cut

Consider the following Prolog database:

```prolog
1 a(X):- b(X).
2 a(X):- c(X).
3 b(X):- d(X), !.
4 b(X):- e(X).
5 c(1).
6 c(2).
7 d(3).
8 d(4).
9 e(5).
```

Listing 4: Prolog database to illustrate the cut

This Prolog database defines four predicates: `a/1`, `b/1`, `c/1`, and `d/1`, using three rules and four facts. It doesn’t have any practical usage, except from explaining how the cut works. Now consider the query `a(A)`. As there are two rules for `a/1`, we can draw two branches:

![Search tree expansion rules for the cut.](image)

The first branch corresponds to a rule, introducing a new query:

![Search tree expansion rules for the cut.](image)

This query can be satisfied by two clauses, so we create two new branches, and concentrate on the first one. This is a query with a cut:

![Search tree expansion rules for the cut.](image)

As this is a compound, conjunctive query (the second part just happens to be a cut), we first look at the left conjunct, the query `d(X)`. There are two possibilities to satisfy this query, so we draw two branches. The first branch succeeds, and the variable `X` is unified with atom `3`. And now we have to deal with the cut:
This cut, as all cuts do, succeeds. But it cuts down all open branches that were created since it was introduced. There are three open branches in the tree above. The first open branch was introduced before the cut came in to play, so it is not affected (just check the Prolog database to verify this). What about the second branch? This branch was triggered by the predicate `b/1` that also introduced the cut. As a consequence, this branch is pruned away from the search tree, as is the third open branch. This gives the following tree, with the two branches removed (signalled by `X`):

At this point, backtracking will jump to the one open branch that is left, corresponding to the second possibility for satisfying the query `a(A)`. The final search tree, then, looks like this:

In sum, there are three solutions for this query. Without the cut in the program there would have been five answers.
4.4 Negation as failure

In Prolog, a negated query \( Q \) is written as \( \text{not}(Q) \) or \( \\neg Q \). A negated query succeeds if an attempt to falsify it fails (Clocksin and Mellish, 1987). This is called negation as failure (Bratko, 2001). Prolog’s negation makes use of the cut and the built-in predicate \( \text{fail}/0 \). When used, it creates two new branches in the tree. In the first branch the query in the scope is placed followed by the cut and \( \text{fail}/0 \). The second branch succeeds (but it will be cut from the tree in case the query of the first branch succeeds). This is how the search tree expansion rule for negation looks like:

\[
\begin{array}{c}
[\neg p(X)] \\
\downarrow \\
[p(X),!,\text{fail}] \\
\end{array}
\quad
\begin{array}{c}
[\nach p(X),Q] \\
\downarrow \\
[p(X),!,\text{fail}] \\
\end{array}
\]

Figure 7: Search tree expansion rules for negation.

The expansion rules in Figure 7 also show why variables in the scope of a Prolog negation are never bound. This is because Prolog negation introduces two branches, and variables on different branches are independent of each other. So even if the query \( Q \) in in Figure 7 would contain an occurrence of the variable \( X \), it would be effected by any bindings that were made while trying to proof the query \( p(X) \).

Now let’s look at an example. We have three children (Ann, Ben and Cai) with their ages stored in a Prolog database. And we have defined a predicate that computes who is the youngest child (Listing 5).

```
1 age(ann, 5).
2 age(ben, 4).
3 age(cai, 6).

4 youngest(X):− age(X,Y), \n+ (age(_,Z), Z < Y).
```

Listing 5: A Prolog database and definition for youngest/1.

Here, in line 5 of Listing 5 “the youngest” is defined as a child with a certain age \( Y \), such that there is no child that has an age that is smaller than \( Y \). Well, who is the youngest? We can find out by drawing the search tree for \( \text{youngest}(\text{Who}) \). There is only one rule for \( \text{youngest}/1 \), so we draw just one branch with a complex query that is formed by the body of the rule:

\[
\begin{array}{c}
[\text{youngest}(\text{Who})] \\
| \\
| \text{Who}=X \\
| \\
[\text{age}(X,Y), \n+ (\text{age}(_,Z), Z < Y)]
\end{array}
\]

This is a compound query, with the first conjunct a simply query with variables, and the second conjunct a negated query of another compound query. We look at the left conjunct first. There are three facts for the predicate \( \text{age}/2 \) in the database (line 1–3 in Listing 5), so we draw three branches. We look at the first branch first: perhaps Ann with the age of 5 is the youngest.
Now we deal with the negation, using the search tree expansion rule for negation, introducing the cut-fail combination:

Once again, for age/3 we have three possibilities, so we draw three branches. The first branch fails (because Ann is not younger than Ann), the second initially succeeds (Ben is younger than Ann), but then the cut removes the two open branches:

No, Ann is not the youngest. The cut was activated and removed two of the open branches. The fail/0 predicates ensures that we look at the two other possibilities. Perhaps Ben or Cai is (we don’t redraw the part of the search tree for reasons of limited space):
Yes, we have a positive answer. The cut is not activated, and the top query succeeds with variable Who unified with atom ben. There is still one open branch, but backtracking will not lead to more solutions here.
5 Search Trees in Learn Prolog Now!

Some of the search trees that you find in Learn Prolog Now! (Blackburn et al., 2006, 2007) are incomplete, or are not drawn in the way shown in this manual. We will present them here in the order they appear in the book.

5.1 Proof Search, Chapter 2 from Learn Prolog Now!

In this chapter of Learn Prolog Now, proof search is illustrated with the following knowledge base (Blackburn et al., 2006, p.34):

```
1 f(a).
2 f(b).
3 g(a).
4 g(b).
5 h(b).
6 k(X):- f(X), g(X), h(X).
```

Listing 6: Prolog database to illustrate proof search.

The query posed is k(Y). There is only one answer. The way Prolog works this out is illustrated by the following search tree:

```
[k(Y)]
  |   |
  X=Y | |
  |   | [f(Y), g(Y), h(Y)]
  /   /    |
Y=a /   Y=b |
  |   |
  [g(a), h(a)] [g(b), h(b)]
  /    / |
 /    /  |
[h(a)] x x [h(b)]
 /    |    |
/     |
x  []
```

Even though in Learn Prolog Now! the tree is shown with variable renaming, this is not required for this example. Another difference with Learn Prolog Now! are the two branches that we draw for the query g(a). Here it is correct to draw two branches following our rules for drawing search trees, as the predicate g/1 is defined by two clauses. Having said that, it is also clear that the second branch fails immediately, and in cases like these we could consider some flexibility and be less strict in showing all search possibilities in the tree, leaving out obvious pre-mature dead branches.
5.2 Recursion, Chapter 3 from Learn Prolog Now!

The search tree for the query \texttt{descent(anne,donna)} in Learn Prolog Now! is incomplete \citep[Blackburn et al., 2006, p.54]{Blackburn2006}. The search tree requires variable renaming for the variable \(Z\), and we do so by introducing \(Z_1, Z_2\), and so on. The complete search tree is enormous, but we managed to get it on one page:

\[
\begin{array}{c}
\text{[descend(anne,donna)]} \\
\text{[child(anne,donna)]} & \text{[child(anne,Z), descend(Z,donna)]} \\
\text{[descend(bridget,donna)]} \\
\text{[child(bridget,donna)]} & \text{[child(bridget,Z1), descend(Z1,donna)]} \\
\text{[descend(caroline,donna)]} \\
\text{[child(caroline,donna)]} & \text{[child(caroline,Z2), descend(Z2,donna)]} \\
\text{[descend(emily,donna)]} \\
\text{[child(emily,donna)]} & \text{[child(emily,Z4), descend(Z4,donna)]} \\
\end{array}
\]

Note that there is only one successful branch, shown in line 27 with the empty box.

5.3 Recursion, Chapter 3 from Learn Prolog Now!

Example 4 of Chapter 3 in Learn Prolog Now! is about the recursive predicate \texttt{add/3} \citep[Blackburn et al., 2006, p.57]{Blackburn2006}, which we give here again in Listing 7:

```
1 add(0,Y,Y).
2 add(succ((X),Y,succ(Z)):- add(X,Y,Z).
```

Listing 7: Prolog definition for \texttt{add/3}.

The search tree given two pages later is incomplete \citep[Blackburn et al., 2006, p.59]{Blackburn2006}. Here is the complete search tree:
The query succeeds, and the variable \( R \) unifies with \( \text{succ(succ(succ(succ(succ(0))))))} \).

5.4 More Lists, the predicate append/3

In Chapter 6 of *Learn Prolog Now!*, the idea behind the standard Prolog predicate `append/3` is explained (Blackburn et al., 2006, p.106). The recursive definition provided contains two clauses (a fact and a rule):

1. `append([], L, L).`
2. `append([H | T], L2, [H | L3]) :- append(T, L2, L3).`

Listing 8: Definition of append/3.

This is done with the help of a search tree for the query `append([a,b,c],[1,2,3],X)` a couple of pages later. The search tree is incomplete: it only shows the successful branches (Blackburn et al., 2006, p.108). The complete search tree is realised as follows:

There is one closed branch, so Prolog will answer in the affirmative way and unify the variable \( X \) with the list \( [a,b,c,1,2,3] \).
5.5 Search tree for the last query of Exercise 2.2 from Learn Prolog Now!

In Exercise 2.2 we are asked to draw the search tree for `magic(Hermione)`. The answer given is correct (Blackburn et al., 2006, p.239) but it consists of unnecessary variable renaming. A simplified search tree is:

```
[\texttt{magic(Hermione)}]
  / \  /
Hermione=X / Hermione=X  \ Hermione=X
  / \  |  \  |
[\texttt{house\_elf(X)}] [\texttt{wizard(X)}] [\texttt{witch(X)}]
  |  /  |  |
X=dobby / x /  |  \ X=\texttt{rita\_skeeter}
  /  |  |  |
[] X=hermione / []  \
  |  |
  \ X=\texttt{\textquotesingle McConagall\textquotesingle} 
[] []
```

5.6 Complete answers to Exercise 4.7 from Learn Prolog Now!

The answers to Exercise 4.7, drawing three search trees for the predicate `member/3`, are all incomplete (Blackburn et al., 2006, p.244). Here are the complete search trees:

```
[\texttt{member(a,[c,b,a,y])}]
  /  \  /  \
  x /  x  /  x
  /  |  |  |
[\texttt{member(a,[b,a,y])}]
  |  /  |  |
  \ x  |  |  |
  \  |  |  |
[\texttt{member(a,[a,y])}]
  |  /  |  |  |
  \ x  |  |  |  |
  \  |  |  |  |
[\texttt{member(a,[])}]
  |  /  |  |  |
  \ x  |  |  |  |
  \  |  |  |  |

[\texttt{member(x,[a,b,c])}]
  /  \  /  \
  x /  x  /  x
  /  |  |  |
[\texttt{member(x,[b,c])}]
  |  /  |  |
  \ x  |  |  |
  \  |  |  |
[\texttt{member(x,[c])}]
  |  /  |  |  |
  \ x  |  |  |  |
  \  |  |  |  |
[\texttt{member(x,[])}]
  |  /  |  |  |
  \ x  |  |  |  |
  \  |  |  |  |
```

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References

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Bratko, I. (2001). Prolog Programming for Artificial Intelligence. Addison-Wesley.

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