Irreducible Representations of Jordanian Quantum Algebra $\mathcal{U}_h(sl(2))$ Via a Nonlinear Map

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Abstract

The generators of the Jordanian quantum algebra $\mathcal{U}_h(sl(2))$ are expressed as nonlinear invertible functions of the classical $sl(2)$ generators. This permits immediate explicit construction of the finite dimensional irreducible representations of the algebra $\mathcal{U}_h(sl(2))$. Using this construction, new finite dimensional solutions of the Yang-Baxter equation may be obtained.

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The enveloping Lie algebra $U(sl(2))$ has two distinct quantizations: The first one is called the Drinfeld-Jimbo deformation (standard $q$-deformation) \[1, 2\], whereas the second one is called the Jordanian deformation (nonstandard $h$-deformation) \[3, 4\] and may be obtained as a contraction of the Drinfeld-Jimbo ones \[5\]. Recently there is much interest in studies relating to various aspects of the $h$-deformed algebra $U_h(sl(2))$. In particular, a two parametric deformation of the dual algebra $Fun_{h,h'}(GL(2))$ was obtained in \[6\]. This author also constructed \[6\] the differential calculus in the quantum plane. Quantum de Rham complexes associated with the $h$-deformed algebra $U_h(sl(2))$ was given in \[7\]. The universal $R$-matrix of the algebra $U_h(sl(2))$ was obtained \[8, 9, 10\]. Various non-semisimple $h$-deformed algebras were constructed at contraction limits \[11, 9, 12\]. The $h$-deformation was also extended to the case of supergroups \[13\].

In \[4\] the fundamental representation of $U_h(sl(2))$ was obtained. The fundamental representation remains undeformed. The finite dimensional highest weight representations of $U_h(sl(2))$ were given in \[14\] first by a direct construction (cf. Proposition 3) and then by a factor-representation of the corresponding Verma modules. The latter was carried out by the standard singular vector construction method, the new feature being the necessity for a special construction of a homogeneous basis. In the direct construction of \[14\], however, the representation action of one of the generators was not given explicitly (except for the three-dimensional irrep). The main purpose of the present Letter is to construct a nonlinear invertible map between the generators of $U_h(sl(2))$ and the classical $sl(2)$ generators. This immediately yields the irreducible representations of $U_h(sl(2))$ in an explicit and simple manner. The known universal $R$-matrix \[8, 9, 10\] of $U_h(sl(2))$ algebra and our construction of its arbitrary irreducible representations may now be combined to generate new finite dimensional solutions of the Yang-Baxter equation.

Let $h$ be an arbitrary complex parameter. The algebra $U_h(sl(2))$ is an associative algebra over $\mathbb{C}$ generated by $H$, $T$, $T^{-1}$ and $Y$, satisfying the relations \[14\]

\[
TT^{-1} = T^{-1}T = 1, \quad [H, T] = T^2 - 1, \quad [H, T^{-1}] = T^{-2} - 1, \quad [H, Y] = -\frac{1}{2}(YT + TY + YT^{-1} + T^{-1}Y), \quad [Y, T] = -\frac{h}{2}(HT + TH), \quad [Y, T^{-1}] = \frac{h}{2}(HT^{-1} + T^{-1}H).
\]

We will not use the (non-standard) coalgebra structure \[14\] of $U_h(sl(2))$ in our work. Following \[14\], we define a new generator $X := h^{-1} \log T$. The algebra $U_h(sl(2))$ is then an associative algebra over $\mathbb{C}$ generated by $H$, $X$ and $Y$, satisfying the commutations relations \[14\]

\[
[H, X] = 2 \frac{\sinh hX}{h}, \quad [H, Y] = -Y(\cosh hX) - (\cosh hX)Y, \quad [X, Y] = H.
\]
The Casimir element of $\mathcal{U}_h(sl(2))$ is given by \[ C = \frac{1}{4h} \left( YT - YT^{-1} + TY - T^{-1}Y \right) + \frac{1}{4} H^2 + \frac{1}{16} (T^2 + T^{-2} - 2), \]
\[ = \frac{1}{2h} \left( Y (\sinh X) + (\sinh X)Y \right) + \frac{1}{4} H^2 + \frac{1}{4} (\sinh X)^2. \] (8)

The elements $Y^k H^l T^s$, $(k, l, s) \in \mathbb{N} \times \mathbb{N} \times \mathbb{Z}$ (resp. $Y^k H^l X^s$, $(k, l, s) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$) build a Poincaré-Birkhoff-Witt basis of the algebra $\mathcal{U}_h(sl(2))$ [4].

The Verma module $M$ of $\mathcal{U}_h(sl(2))$ is generated by the highest weight vector $w_0$ with the highest weight $\lambda$

\[ H.w_\lambda = \lambda w_\lambda , \quad T.w_\lambda = \varepsilon w_\lambda , \quad (\varepsilon = \pm 1), \] (9)
\[ \frac{1}{2}(T - T^{-1})w_\lambda = 0. \] (10)

Let us start with the case $\varepsilon = 1$. (The case $\varepsilon = -1$ will be included at the end.) Then the vectors $w_m := Y^m . w_0$, $m > 0$ and $w_0 := w_\lambda$ are a basis of $M$. If $\lambda = 1$, then $w_1 := w_2$ is a primitive (singular) vector in $M$ ($X.w_1 = 0$). It is the first primitive vector in $M$ below $w_\lambda$. Therefore $L^1 := M/\left( \mathcal{U}_h(sl(2)).w_1 \right)$ is a finite two ($j = \frac{1}{2}$) dimensional irreducible representation of $\mathcal{U}_h(sl(2))$ and is given by [4]

\[ H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \] (11)

Similarly, if $\lambda = 2$, then $w_1 := w_3 + h^2 w_1$ is a singular vector in $M$. The quotient space $L^1 := M/\left( \mathcal{U}_h(sl(2)).w_1 \right)$ is a finite three ($j = 1$) dimensional irreducible representation of $\mathcal{U}_h(sl(2))$ spanned by [14]

\[ H = \begin{pmatrix} 2 & 0 & -2h^2 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -h^2 \\ 0 & 1 & 0 \end{pmatrix}. \] (12)

With an eye to the future development, we present here the matrix representation, where $H$ is diagonal

\[ H = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -h^2 & 0 \\ 1 & 0 & -\frac{h^2}{2} \\ 0 & 1 & 0 \end{pmatrix}. \] (13)

For $\lambda = 3$, the first singular vector in $M$ below $w_\lambda$ reads $w_3 := w_1 + 2h^2 w_2$. Consequently, $L^3 := M/\left( \mathcal{U}_h(sl(2)).w_3 \right)$ is a finite four ($j = \frac{3}{2}$) dimensional irreducible representation of
\( \mathcal{U}_h(sl(2)) \) given by

\[
H = \begin{pmatrix}
3 & 0 & -6h^2 & 0 \\
0 & 1 & 0 & -18h^2 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -3
\end{pmatrix}, \quad X = \begin{pmatrix}
0 & 3 & 0 & -6h^2 \\
0 & 0 & 4 & 0 \\
0 & 0 & 0 & 3 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad Y = \begin{pmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & -6h^2 \\
0 & 0 & 1 & 0
\end{pmatrix}. \quad (14)
\]

When \( H \) is diagonalized, the representation (14) assumes the form

\[
H = \begin{pmatrix}
3 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -3
\end{pmatrix}, \quad X = \begin{pmatrix}
0 & 3 & 0 & 3h^2 \\
0 & 0 & 4 & 0 \\
0 & 0 & 0 & 3 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad Y = \begin{pmatrix}
0 & -\frac{3h^2}{2} & 0 & \frac{9h^4}{4} \\
1 & 0 & -3h^2 & 0 \\
0 & 1 & 0 & -\frac{2}{3h^2} \\
0 & 0 & 1 & 0
\end{pmatrix}. \quad (15)
\]

The other finite dimensional irreducible representations associated to \( \lambda = 4, 5, 6, \ldots \) may be obtained in a similar way. The singular vectors for \( \lambda \leq 3 \) were obtained analytically in [14]. For \( \lambda \leq 6 \), the singular vectors may be constructed from results derived in [14] using REDUCE. Nonetheless, the computations involved for obtaining the explicit expressions at higher dimensions become quite complicated very rapidly. However, a glance at the above representations (13) and (15), where the operator \( H \) has been diagonalized, seems to suggest a close kinship between the \( U_q(sl(2)) \) generators and their analogues for the classical \( sl(2) \) algebra. We pursue this route here.

Let \( \{ w_j^i, w_{j+1}^i, \ldots, w_m^i, \ldots, w_{j-1}^i, w_j^i \} \) be a set of basis vectors for the irreducible quotient module \( L_j := M/(\mathcal{U}_h(sl(2)).w_j^i) \) of dimension \( 2j + 1 \), where \( H \) is diagonal. We wish to point out that, with respect to the preceding examples, we introduce in the following a change of convention leading, for \( h = 0 \), to the standard \( sl(2) \) representations, where the usual hermiticity properties of the generators are satisfied. The action of the generators \( H \) and \( X \) on these basis vectors are given by

\[
H.w_m^j = 2m w_m^j, \quad (16)
\]

\[
X.w_m^j = \sum_{k=0}^{[(j-m)/2]} h^{2k} a_m^m a_{m+2k+1} w_m^{j+2k+1}, \quad (17)
\]

where \([x]\) stands for the integer part of \( x \). Using the commutation relation (5), we can easily prove that

\[
a_m^m a_{m+2k+1} = f_k a_m a_{m+1} \cdots a_{m+2k} a_{m+2k+1}, \quad k \geq 0, \quad (18)
\]

where \( a_m = \sqrt{(j-m)(j+m+1)} \) and the coefficients \( f_k, k \in \mathbb{N} \) obey the following recurrence relation

\[
f_k = \begin{cases} 
1 & k = 0 \\
\frac{1}{2k} \sum_{s=1}^{k} \frac{1}{(2s+1)!} f_{i_1} f_{i_2} \cdots f_{i_{2s+1}} & k \geq 1.
\end{cases} \quad (19)
\]
To solve (19), we introduce a generating function defined as 
\[ F(x) = \sum_{k=0}^{\infty} f_k x^{2k}. \]
The relation (19) requires \( F(x) \) to satisfy the following differential equation
\[ \frac{d(xF(x))}{dx} = \frac{\sinh(xF(x))}{x}, \quad F(0) = 1, \]
whose solution is given by
\[ F(x) = \frac{2}{x} \arctanh \left( \frac{x}{2} \right) = \sum_{k=0}^{\infty} \frac{1}{(2k+1) 2^{2k}} x^{2k}. \]
The solution for \( \{ f_k \mid k \geq 0 \} \) now reads
\[ f_k = \frac{1}{(2k+1) 2^{2k}}. \]
Using the relations (18) and (22), the action of the generator \( X \) on the basis may be written as
\[ X.w^j_m = \sum_{k=0}^{(j-m-1)/2} \frac{h^{2k}}{(2k+1) 2^{2k}} \left( \prod_{s=0}^{k-1} a_{m+s} \right) w^j_{m+2k+1} \]
and, evidently
\[ T.w^j_m = \begin{cases} 
    w^j_m + \sum_{k=1}^{j-m} \frac{h^k}{2^{k-1}} \left( \prod_{s=0}^{k-1} a_{m+s} \right) w^j_{m+k} & \text{if } m < j \\
    w^j_m & \text{if } m = j
\end{cases} \]
Now we demonstrate the previously mentioned invertible map between the generators \( U_h(sl(2)) \) algebra and the generators of the classical \( sl(2) \). Let \( J_+ \), \( J_- \) and \( J_3 \) be the classical \( sl(2) \) generators acting on the basis \( w^j_m, -j \leq m \leq j \) as
\[ J_+.w^j_m = a_m w^j_{m+1}, \quad J_-.w^j_m = a_{m-1} w^j_{m-1}, \quad J_3.w^j_m = m w^j_m. \]
Then the previous analysis indicates that the actions of the generators \( X \) and \( H \) on the vector space are equivalent to the following classical constructs
\[ X = 2 \frac{h}{h} \arctanh \left( \frac{h J_+}{2} \right), \quad H = 2 J_3 \]
and
\[ T = \frac{1 + \frac{h J_+}{2}}{1 - \frac{h J_+}{2}} = 1 + \sum_{k \geq 1} \frac{h^k}{2^{k-1}} J_+^k. \]
The inverse of the map (25) and (27) reads

\[ J_+ = \frac{2}{\hbar} \tanh \left( \frac{\hbar X}{2} \right) = \frac{2}{\hbar} \left( \frac{T - 1}{T + 1} \right), \quad J_3 = \frac{H}{2}. \]  

(28)

To obtain a similar map for the generator \( Y \), we take an ansatz

\[ Y = \varphi_h(J_+) \ J_+ \ \varphi_h(J_+) \]  

(29)

with the natural condition \( \varphi_0(x) = 1 \). The commutation relation (6) and the useful identity \([J_3, \ \phi(J_+)] = \phi'(J_+) \ J_+\) now require the function \( \varphi_h(x) \) to satisfy the following differential equation

\[ \frac{1}{\varphi_h(x)} \frac{\partial \varphi_h(x)}{\partial x} = -\frac{h^2X/4}{1 - h^2x^2/4}, \]  

(30)

whose solution is given by \( \varphi_h(x) = (1 - h^2x^2/4)^{1/2} \). The generator \( Y \) may be written finally as

\[ Y = \sqrt{1 - \frac{h^2 J_+^2}{4}} \ J_+ \sqrt{1 - \frac{h^2 J_+^2}{4}}, \]  

\[ = \sum_{k=0}^{\infty} (-1)^k \frac{h^2 k}{2^{2k}} \left( \sum_{s=0}^{k} \zeta_s \zeta_{k-s} J_+^{2s} J_- J_+^{2k-2s} \right), \]  

(32)

where, \( \zeta_0 = 1 \) and \( \zeta_k = \frac{1}{k!} \prod_{s=0}^{k-1} \left( \frac{1}{2} - s \right) \) for \( k \geq 1 \). The inverse map of (31) readily follows

\[ J_- = \cosh \left( \frac{h X}{2} \right) Y \cosh \left( \frac{h X}{2} \right), \]  

\[ = \frac{1}{4} \left( T^{1/2} + T^{-1/2} \right) Y \left( T^{1/2} + T^{-1/2} \right). \]  

(34)

The generator \( Y \) act on the basis \( w^j_m \) as

\[ Y.w^j_m = \sum_{k=0}^{[j-m+1]/2} (-1)^k \frac{h^{2k}}{2^{2k}} \left( \sum_{s=0}^{k} \zeta_s \zeta_{k-s} \psi_k s(m + 2s - 1) a_{m+2s-1} \psi_{2s}(m) \right) w^j_{m+2k-1}, \]  

(35)

where \( \psi_0(m) = 1 \) and \( \psi_s(m) = \prod_{k=0}^{2s-1} a_{m+k} \) for \( s \geq 1 \). The expressions (26), (27) and (31) constitute the realization of the Jordanian algebra \( \mathcal{U}_h(sl(2)) \) with the classical generators via a nonlinear map. This map gives the representation characterized by \( \varepsilon = 1 \). It may be directly verified that the commutation relations (5), (6) and (7) are satisfied. Expressed in terms of the classical \( sl(2) \) generators, the Casimir element (8) of \( \mathcal{U}_h(sl(2)) \) is just equal to

\[ C = \frac{1}{2} (J_+ J_- + J_- J_+) + J_3^2 \]  

(36)
and its value is equal to the classical one
\[ C.w_{m}^{j} = j(j + 1) w_{m}^{j}, \quad -j \leq m \leq j. \] (37)

Obviously the representation (16), (23), (24) and (35) \((\varepsilon = 1)\) is a \(h\)-analog to the spin \(j\) representation of \(sl(2)\).

Let us mention that there is a \(\mathbb{C}\)-algebra automorphism \(\omega\) of \(U_{h}(sl(2))\) such that
\[
\begin{align*}
\omega(T) &= T^{-1}, & \omega(T^{-1}) &= T, \\
\omega(Y) &= -Y, & \omega(H) &= H
\end{align*}
\]
and, evidently
\[ \omega(X) = -X. \] (39)

(For \(h = 0\), this automorphism reduces to the classical one \((J_{+}, J_{-}, J_{3}) \rightarrow (-J_{+}, -J_{-}, J_{3})\)). Also there is a second \(\mathbb{C}\)-algebra automorphism \(\varpi\) of \(U_{h}(sl(2))\) defined as
\[
\begin{align*}
\varpi(T) &= -T, & \varpi(T^{-1}) &= -T^{-1}, \\
\varpi(Y) &= -Y, & \varpi(H) &= -H
\end{align*}
\]
and, evidently
\[ \varpi(X) = X + \frac{i \pi}{\hbar}. \] (41)

The representation induced by the automorphism \(\varpi\) is characterized by \(X.w_{m}^{j} = \frac{i \pi}{\hbar} w_{m}^{j}\) (instead of \(X.w_{j}^{j} = 0\)). The representations characterized by \(\varepsilon = -1\) in (9) are simply obtained from those presented in this paper using the automorphism \(\varpi\). These representations have evidently no classical \((h = 0)\) limit. (See [15] for the corresponding role of \(\varepsilon\) for the \(U_{q}(sl(2))\) case.) Finally, the map associated to the irreducible representations associated to \(\varepsilon\) \((\varepsilon = \pm 1)\) is described by
\[
\begin{align*}
T &= \varepsilon \frac{1 + \frac{\hbar J_{+}}{2}}{1 - \frac{\hbar J_{+}}{2}}, \\
Y &= \varepsilon \sqrt{1 - \frac{\hbar^2 J_{+}^{2}}{4}} J_{-} \sqrt{1 - \frac{\hbar^2 J_{-}^{2}}{4}}, \\
H &= 2 \varepsilon J_{3}
\end{align*}
\]
and, where
\[
X = \frac{(1 - \varepsilon) i \pi}{2 \hbar} + \frac{2}{\hbar} \text{arctanh}(\frac{\hbar J_{+}}{2}).
\] (45)
This implies for the Casimir element (8) ($\varepsilon = \pm 1$)

$$C = \frac{1}{4h} \left( Y T - Y T^{-1} + T Y - T^{-1} Y \right) + \frac{1}{4} H^2 + \frac{1}{16} (T^2 + T^{-2} - 2),$$

$$= \frac{1}{2h} \left( Y (\sinh hX) + (\sinh hX) Y \right) + \frac{1}{4} H^2 + \frac{1}{4} (\sinh hX)^2,$$

$$= \frac{1}{2} (J_+ J_- + J_- J_+) + J_3^2.$$ (46)

The main result achieved in this Letter is the realization of the $U_h(sl(2))$ generators in terms of the undeformed $sl(2)$ generators. This facilitates immediate construction of the representation of the $U_h(sl(2))$ algebra. It is interesting to note that, via this map, the generators $(H, X, Y)$ satisfying the algebraic structure (5), (6) and (7), may also be equipped with an induced cocommutative coproduct. This way, the generators $(H, X, Y)$ are viewed as composite elements of the $U(sl(2))$ algebra. Similarly, the undeformed generators $(J_\pm, J_3)$, via the inverse map, may be viewed as elements of the $U_h(sl(2))$ algebra; and, thus, may be endowed with an induced noncocommutative coproduct [16]. An example of nonlinear mapping between $sl(2)$ and $U_q(sl(2))$ can be found in [17]. This map is also discussed in [16]. Applications of our formalism to a type of deformation of the 3-dimensional euclidean space has been presented in [18].

Using our previous construction of the arbitrary irreducible representations of the $U_h(sl(2))$ algebra via the map, we now obtain the finite dimensional solutions of the Yang-Baxter equation. To this end, we use the expression of the universal $R$-matrix of the $U_h(sl(2))$ algebra, derived in [10]:

$$R = \exp \left\{ -hX \otimes e^{hX} H \right\} \exp \left\{ h e^{hX} H \otimes X \right\}. \quad (47)$$

Our construction of the map (42)-(45) suggests that an arbitrary finite dimensional irreducible representation of the $U_h(sl(2))$ algebra may be characterized by the indices $(j, \varepsilon)$, where $(2j + 1) \in \mathbb{Z}_+$ and $\varepsilon = \pm 1$. The finite dimensional representations of the $R$-matrix now satisfy the Yang-Baxter equation

$$R_{12}^{j_1, \varepsilon_1; j_2, \varepsilon_2} R_{13}^{j_1, \varepsilon_1; j_3, \varepsilon_3} R_{23}^{j_2, \varepsilon_2; j_3, \varepsilon_3} = R_{23}^{j_2, \varepsilon_2; j_3, \varepsilon_3} R_{13}^{j_1, \varepsilon_1; j_3, \varepsilon_3} R_{12}^{j_1, \varepsilon_1; j_2, \varepsilon_2}. \quad (48)$$

The first few solutions for the classical case ($\varepsilon = 1$) read:

$$R_{12}^{j_1, \varepsilon_1; 1, \varepsilon_2} = \begin{pmatrix} 1 & 2h & 2h^2 & -2h & 2h^2 & 0 \\ 0 & 1 & 2h & 0 & 0 & 2h^2 \\ 0 & 0 & 1 & 0 & 0 & 2h \\ 0 & 0 & 0 & 1 & -2h & 2h^2 \\ 0 & 0 & 0 & 0 & 1 & -2h \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (49)$$

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\[ R^{\frac{1}{2}; \varepsilon_1 = 1; \varepsilon_2 = 1} = \begin{pmatrix}
1 & 3h & 6h^2 & 9h^3 & -3h & 3h^2 & 0 & 9h^4 \\
0 & 1 & 4h & 6h^2 & 0 & -h & 4h^2 & 0 \\
0 & 0 & 1 & 3h & 0 & 0 & h & 3h^2 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 3h \\
0 & 0 & 0 & 0 & 1 & -3h & 6h^2 & -9h^3 \\
0 & 0 & 0 & 0 & 0 & 1 & -4h & 6h^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -3h \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}. \quad (50) \]

Of special interest are the ‘non-classical’ solutions of the type

\[ R^{\frac{1}{2}; \varepsilon_1 = 1; \varepsilon_2 = -1} = \begin{pmatrix}
-1 & h & h^2 \\
0 & -1 & 0 & -h \\
0 & 0 & -1 & -h \\
0 & 0 & 0 & -1
\end{pmatrix}. \quad (51) \]

All such finite dimensional solutions of the Yang-Baxter equation may be similarly obtained from our construction of the irreducible representations of the \( U_h(sl(2)) \) algebra and its known universal \( \mathcal{R} \)-matrix [10].

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