Revealing the pulsational properties of the V777 Her star KUV 05134+2605 by its long-term monitoring

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ABSTRACT

Context. KUV 05134+2605 is one of the 21 pulsating DB white dwarfs (V777 Her or DBV variables) known so far. The detailed investigation of the short-period and low-amplitude pulsations of these relatively faint targets requires considerable observational efforts from the ground, long-term single-site or multisite observations. The observed amplitudes of excited modes undergo short-term variations in many cases, which makes the determination of pulsation modes difficult.

Aims. We aim to determine the pulsation frequencies of KUV 05134+2605, find regularities between the frequency and period components, and perform its asteroseismic investigation for the first time.

Methods. We re-analysed the data already published, and collected new measurements. We compared the frequency content of the different datasets from the different epochs and performed various tests to check the reliability of the frequency determinations. The mean period spacings were investigated with linear fits to the observed periods, Kolmogorov-Smirnov and Inverse Variance significance tests, and Fourier analysis of different period sets, including a Monte Carlo test simulating the effect of alias ambiguities. We employed fully evolutionary DB white dwarf models for the asteroseismic investigations.

Results. We identified 22 frequencies between 1280 and 2530 µHz. These form 12 groups, which suggests at least 12 possible frequencies for the asteroseismic investigations. Thanks to the extended observations, KUV 05134+2605 joined the group of rich white dwarf pulsators. We identified one triplet and at least one doublet with a ≈ 9 µHz frequency separation, from which we derived a stellar rotation period of 0.6 d. We determined the mean period spacings of ≈ 31 and 18 s for the modes we propose as dipole and quadrupole, respectively. We found an excellent agreement between the stellar mass derived from the ℓ = 1 period spacing and the period-to-period fits, all providing Ms = 0.84 – 0.85 Ms solutions. Our study suggests that KUV 05134+2605 is the most massive amongst the known V777 Her stars.

Key words. Techniques: photometric – Stars: individual: KUV 05134+2605 – Stars: interiors – Stars: oscillations – white dwarfs

1. Introduction

The discovery of the first pulsating DB-type white dwarf (GD 358 or V777 Her) was not serendipitous: it was the result of a dedicated search for unstable modes in these stars, driven by the partial ionization of the atmospheric helium (Winget et al. 1982).

The V777 Her stars are similar to their ‘cousins’, the ZZ Ceti (or DAV) stars, in their pulsational properties. Both show light variations due to non-radial g-mode pulsations with periods between 100 – 1400 s. However, the V777 Her stars have larger effective temperatures (22 000–29 000 K) than the DAVs (10 900–12 300 K). Another significant difference is the composition of their atmospheres: hydrogen dominates the DAV, while helium constitute the DBV atmospheres. We note that the classification of the recently discovered cool (Teff ≈ 8000–9000 K), extremely low-mass pulsating DA white dwarfs as ZZ Ceti stars was proposed by the modelling results of Van Grootel et al. (2013). However, except their similar driving mechanism and surface composition, they are completely different, e.g. they likely have binary origin, their cores are dominated by helium, and they pulsate with longer periods associated to g-modes than the ZZ Ceti stars (Hermes et al. 2013). About 80% of the white dwarfs belong to the DA spectral class (e.g. Kleinman et al. 2013) and most of the known white dwarf pulsators are ZZ Ceti variables. With the discovery of pulsations in KIC 8626021, the number of known V777 Her stars has reached only 21 (Østensen et al. 2011).

The regularities between the observed frequencies and periods offer great potential for the asteroseismic investigations of pulsating white dwarfs. Theoretically, rotational splitting of dipole and quadrupole modes into three or five equally spaced components allow the determination of the stellar rotation period, but the internal differential rotation (Kawaler et al. 1999) and the presence of the magnetic field (Jones et al. 1989) can complicate this picture. Finding these rotationally split frequencies also can be complicated: we need a long enough dataset to resolve the components, the overlapping splitting structures of the closely spaced modes can make the finding of the regularities difficult, and the frequency separations can be close to the daily alias of ground-based measurements. Furthermore, observational results indicate that in many cases not all of the com...
ponents reach an observable amplitude at a given time, for unknown reasons (Winget & Kepler 2008).

The other theoretically predicted regularity is the quasi equally spaced formation of the consecutive radial overtone pulsation periods having the same spherical harmonic index ($\ell$). The inner chemical stratification breaks the uniform spacings, while the mean period spacing is an indicator of the total stellar mass.

Typical feature of the white dwarf pulsation is that the light curves are not sinusoidal, and we can detect combination ($nf \pm m f$), harmonic ($nf$), or even subharmonic ($n/2f$) peaks in the corresponding Fourier transforms. These features reveal the importance of the non-linear effects on the observed light variations.

For further details on the general properties of white dwarf pulsators, we refer to the reviews of Winget & Kepler (2008), Fontaine & Brassard (2008) and Althaus et al. (2010).

The pulsation frequencies of V777 Her stars are reported to undergo short-term amplitude and phase variations (see e.g. Handler et al. 2013 and Handler et al. 2002). The most famed example is the so-called sforzando effect observed in GD 358. This star showed a high-amplitude sinusoidal light variation for a short period of time, instead of the non-linear variability observed just a day before (Kepler et al. 2003, Provencal et al. 2009). To find an explanation for the short-term changes in the pulsational behaviour of a star is challenging. As convection plays an important role in the driving of pulsations, the pulsation/convection interactions could explain some of the temporal variations (Montgomery et al. 2010). In the case of the pre-white dwarf GW Vir pulsator PG 0122+200, Vauclair et al. (2011) found that the temporal variations of pulsation modes may be the result of resonant coupling induced by rotation within triplets. In some cases, observed amplitude variations are the result of the insufficient frequency resolution of the datasets, and do not reflect real changes in the frequencies’ energy content.

We assume that the structure of a star, which determines which pulsation frequencies can be excited, does not change on such short time scales as the number of detectable frequencies and their amplitudes in many white dwarfs. As a consequence of amplitude variations, one dataset reveals only a subset of the possible frequencies, but with observations at different epochs, we can add more and more frequencies to the list of observed ones, essential for asteroseismic investigations (see e.g. the pioneer case of G29-38 by Kleinman et al. 1998). Here, observations of KUV 05134+2605 at different epochs provide a more complete sampling of the pulsation frequencies, which will then lead to a more detailed asteroseismic analysis.

Asteroseismology can provide information on the masses of the envelope layers, the core structure and the stellar mass. These are crucial for building more realistic models and improving our knowledge of stellar evolution. Since more than 95% of the stars will end their life as white dwarfs, and the information we gather on pulsating members is representative of the non-pulsating ones – as most, if not all stars pulsate when they cross the instability strip –, asteroseismic studies of DAVs and DBVs provide important constraints on our models of stellar evolution and galactic history.

In the case of KUV 05134+2605, previous observations suggested a rich amplitude spectrum showing considerable amplitude variations from season to season (Handler et al. 2003). This made the star a promising target for further monitoring. Our main goal was to find the excited normal modes with data obtained in different seasons. In this paper, we discuss our findings and the results on the asteroseismic investigations based on the set of pulsation modes.

| Run No. | UT date | Start time (BJD-2 450 000) | Points | Length (h) |
|---------|---------|---------------------------|--------|-----------|
| 01      | 2007 Nov 29 | 4434.434                | 308    | 3.33      |
| 02      | 2007 Dec 01  | 4435.582                | 120    | 1.08      |
| 03      | 2007 Dec 04  | 4439.379                | 521    | 4.72      |
| 04      | 2007 Dec 05  | 4440.341                | 765    | 7.97      |
| 05      | 2008 Jan 23  | 4489.196                | 506    | 4.58      |
| 06      | 2008 Feb 09   | 4506.266                | 375    | 3.43      |
| 07      | 2008 Feb 11   | 4508.260                | 497    | 4.55      |
| 08      | 2008 Feb 12   | 4509.244                | 590    | 5.57      |
| 09      | 2008 Feb 18   | 4515.276                | 347    | 3.12      |
| 10      | 2008 Feb 19   | 4516.238                | 463    | 4.20      |
| 11      | 2008 Feb 20   | 4517.227                | 544    | 5.11      |
| 12      | 2011 Feb 03   | 5596.235                | 233    | 2.12      |
| 13      | 2011 Feb 06   | 5599.298                | 179    | 1.71      |
| 14      | 2011 Feb 08   | 5601.229                | 354    | 3.19      |
| Total:  |          |                         | 4936+766 | 47.66+7.02 |

2. Observations and data reduction

The first detection of the light variations of KUV 05134+2605 ($V = 16.3$ mag, $\delta m_{3000} = 0.516^{+0.28}_{-0.28}$, $\delta m_{3000} = +26^{+08}_{-38}$) was announced by Grauer et al. (1989). Later, several multi- and single-site observations followed the discovery run. Data were collected in 1992, 2000 and 2001, respectively, including the Whole Earth Telescope (WET, Nather et al. 1990) run in 2000 November (XCov20), when KUV 05134+2605 was one of the secondary targets. In this paper, we re-analyse these datasets. The log of observations and the reduction process applied in these cases are summarized in Handler et al. (2003).

To complement the earlier measurements, we collected data on KUV 05134+2605 on 11 nights in the 2007/2008 observing season with the 1-m Ritchey-Chrétien-Coudé telescope at Piszkéstető mountain station of Konkoly Observatory. We continued the observations in 2011 for 3 nights. The measurements were made with a Princeton Instruments VersArray:1300B back-illuminated CCD camera in white light and with 30 s integration times. Table 1 shows the journal of the Konkoly observations. We obtained $\approx 48$ and 7 hours of data in 2007/2008 and 2011, respectively.

We followed the standard reduction procedure of the raw data frames: we applied bias, dark and flat corrections, and performed aperture photometry of the variable and comparison stars, using standard IRAF routines. We converted the JD times to Barycentric Julian Dates in Barycentric Dynamical Time (BJD$_{\text{TDB}}$) using the applet of Eastman et al. (2010). After the aperture photometry, we checked whether the light curves of the possible comparison stars are free from any instrumental effects or variability. Finally, we selected three stars and used their average brightness as a comparison for the differential photometry. Fig. 1 shows the variable and the selected comparisons in the CCD field. We corrected for the atmospheric extinction and for the instrumental trends by applying low-order polynomial fits to the resulting light curves. This affected the low-frequency re-

1 IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.

2 http://astroutils.astronomy.ohio-state.edu/time/utc2bjd.html
We performed Fourier analysis of the datasets using the program MuFaAn (Multi-Frequency Analyser, Kolláth 1990; Csubry & Kolláth 2004). This tool can handle gapped and unequally spaced data, has Fast Fourier and Discrete Fourier Transform abilities, and is also capable of performing linear and non-linear fits of Fourier components. The results were also checked form abilities, and is also capable of performing linear and non-

### Table 2. Comparison of the different observing runs on KUV 05134+2605.

| Run          | Length (d) | $f_R$ ($\mu$Hz) | Cov. (%) | Noise (mmag) | Points |
|--------------|------------|-----------------|----------|--------------|--------|
| 1988         | 5.99       | 1.9             | 7.5      | 1.2          | 1257   |
| 1992         | 1.16       | 9.9             | 34.0     | 1.0          | 3415   |
| 2000 Oct     | 5.11       | 2.3             | 8.2      | 0.8          | 3163   |
| WET2000      | 13.07      | 0.9             | 11.1     | 0.4          | 9516   |
| 2001         | 2.09       | 5.5             | 14.8     | 1.4          | 1147   |
| Konkoly2007  | 83.01      | 0.1             | 2.4      | 0.3          | 4936   |
| Konkoly2011  | 5.13       | 2.3             | 5.7      | 1.1          | 766    |

...in the Fourier Transforms (FTs) below 350 $\mu$Hz. Therefore we did not investigate this frequency domain. Fig. 2 presents the light curves from the 2007/2008 and 2011 seasons, respectively. The Rayleigh frequency resolutions (1/ΔT) of the whole light curves for these two epochs are 0.1 and 2.3 $\mu$Hz.

The most important parameters of the individual datasets used in this study can be found in Table 2. This way, we can compare the different runs quantitatively, which can also be useful in the interpretation of their frequency content.

### 3. Frequency content of the datasets

We performed Fourier analysis of the datasets using the program MuFaAn (Multi-Frequency Analyser, Kolláth 1990; Csubry & Kolláth 2004). This tool can handle gapped and unequally spaced data, has Fast Fourier and Discrete Fourier Transform abilities, and is also capable of performing linear and non-linear fits of Fourier components. The results were also checked with PyRoot04 (Lenz & Breger 2005) and the photometry modules of FAMIAS (Zima 2008).

As Table 2 shows, the long time-base of the data obtained in the 2007/2008 season (hereafter ‘Konkoly2007 data’) and the WET data from 2000 (hereafter ‘WET2000 data’) provide the best Rayleigh frequency resolution and have the lowest noise levels. We review the results of the Fourier analyses of these two datasets first.
the frequencies and amplitudes of the 9 and 13 peaks exceeding the 4(A) significance level, respectively.

In the WET2000 dataset, we detect two closely spaced frequencies as dominant ones (at 1901.9 and 1892.3 µHz) with a 9.6 µHz frequency spacing (see the first two panels on the left of Fig. 3). The main pulsation frequency region is between ≈ 1200 and 2000 µHz.

The 1893.8 µHz frequency is close to a higher-amplitude one (δf = 1.5 µHz, beat period: 7.5 d). We considered and investigated in Sect. 3.3 the possibility that this is not a real pulsation frequency, but represents an example for an artificial peak which emerges as a result of temporal amplitude variations.

Similarly to the WET2000 data, two closely spaced frequencies dominate the Konkoly2007 dataset, but at 1623.7 and 1614.5 µHz. The dominant frequencies of the WET2000 data are below the detection limit. It is also important to note that an additional frequency can be detected at 1633.0 µHz in the Konkoly2007 data, which completes the dominant doublet into an almost exactly equidistant triplet with spacings of 9.2 and 9.3 µHz (and corresponding beat periods of 1.3 and 1.2 d), respectively. This is the only complete triplet structure detected in a single dataset.

In the Konkoly2007 data, besides the triplet, we find a doublet around 1450 µHz, but with a different spacing: only 6.9 µHz. In the 1700−1900 µHz region, the number of peaks is rather high and the identification of the real pulsation peaks is difficult. We consider the three-frequency solution, in which one of the frequencies, at 1831.0 µHz, can be detected also in a Konkoly2007 data subset (see Sect. 3.3). The other two frequencies at 1767.1 and 1800.5 µHz are the same within the uncertainties as the 1762.9 and 1798.6 µHz ones detected in the 2001 and WET2000 data, respectively.

The 2275.1 µHz frequency is also considerably close to a larger amplitude one at 2277.6 µHz, similarly to the case of the 1893.8 µHz frequency of the WET2000 data.

The 3902.1 µHz frequency in the Konkoly2007 dataset is a combination (f1 + f2) of the two large-amplitude ones at 1623.7 and 2277.6 µHz. We did not detect other combination frequencies neither in the WET2000 nor in the Konkoly2007 dataset.

In conclusion, we find a common frequency of the WET2000 and Konkoly2007 datasets at 1800 µHz. The frequencies at 1633.0 µHz (2007) and 1644.2 (2000) are also common, if we consider that these are most likely 1 d⁻¹ (11.574 µHz) aliases.
Table 3. Results of the frequency analyses of the datasets from different observing seasons. The largest amplitude frequency of a given dataset is marked with an asterisk. The abbreviation ‘K2011’ refers to the new observations obtained at Konkoly Observatory in 2011. The numbers in parentheses are the uncertainties of the frequencies and amplitudes, calculated by Monte Carlo simulations (see Sect. 3.3.1).

| Dataset | Frequency (µHz) | Amplitude (mmag) | Frequency (µHz) | Amplitude (mmag) | Frequency (µHz) | Amplitude (mmag) | Frequency (µHz) | Amplitude (mmag) |
|---------|----------------|------------------|----------------|------------------|----------------|------------------|----------------|------------------|
| WET2000 | 1390.6(2.8)    | 3.2(0.3)         | 1449.7(3.3)    | 3.3(0.3)         | 1531.4(1.7)    | 3.6(0.3)         | 1699.9(2.3)    | 4.2(0.3)         |
| Konkoly2007 | 3.2(0.3) | 1456.6(4.1)    | 4.4(0.4)        | 1633.0(4.5)    | 3.9(0.3)        | 1681.7(1.6)    | 3.8(0.3)        | 1767.1(5.7)    |
| Other measurements, 1988 – 2011 | | | | | | | | |
| 1988    | 1287.6(3.1)    | 10.4(0.9)        | 1414.7(0.1)    | 26.6(1.0)       | 1514.7(0.1)    | 26.6(1.0)       | 1549.7(4.9)    | 12.5(0.8)       |
| 1992    | 1465.4(5.1)    | 12.5(1.7)        | 1762.9(4.0)    | 18.8(1.4)       | 1509.2(5.3)    | 13.7(0.7)       | 2023.7(6.8)    | 8.4(1.3)        |

The peaks found at 1681.7 and 1767.1 µHz (2007) are also slightly different from the ones detected at 1699.9 and 1781.7 µHz (2000). Different scenarios can explain the emergence of these frequency pairs (alias ambiguities, rotational splitting, the mixing of dipole and quadrupole modes). We discuss them in Sect. 3.4.

3.2. Analysis of the short datasets obtained between 1988 and 2011

Table 3 summarizes the results of the Fourier analysis of the short light curves, too. The largest number of frequencies are detected in the 1988 and 1992 data (5-5 frequencies). In 2000, 2001, and 2011, two, one, and two frequencies exceeded the 4(σ) significance level, respectively. The dominant frequency, around 1901 µHz, did not change from 2000 October to 2000 November-December (WET2000), while it was different in all the other observing seasons.

1988: The 1287.6 and the 1504.5 µHz frequencies are candidates for new pulsation modes. The 1414.7 µHz frequency is close to the 2d⁻¹ alias of the 1390.6 µHz peak of the WET2000 dataset, but it could still be an independent frequency. The two frequencies above 2800 µHz can be interpreted as combination terms, considering the alias ambiguities.

1992: In the 1992 dataset, we detect a doublet at 1465.4 and 1474.5 µHz with 9.1 µHz separation. We note that this separation is slightly lower than the Rayleigh frequency resolution of the dataset (9.9 µHz). We checked the reliability of this doublet in the course of the frequency determination tests, see Sects. 3.3 and 3.4. The 1612.9 µHz frequency corresponds to the 1614.5 µHz one from 2007. The 2504.7 µHz frequency might be a -2 d⁻¹ alias of the 2530.7 µHz one detected also in 2007, or might represent a new pulsation frequency.

2000: The two significant frequencies obtained in 2000 October correspond to the two largest amplitude ones detected in the WET2000 dataset.

2001: The only significant frequency of the 2001 data at 1762.9 µHz is the same within the uncertainties as the 1767.1 µHz one observed in 2007, so this confirms the choice of this latter during the pre-whitening process.

2011: The 2023.7 µHz peak is a promising candidate for a new pulsation mode. The frequency at 1472 µHz was found in the 1992 data.

3.3. Frequency determination tests

We performed several different tests based both on observational and synthetic data to investigate how reliable are the frequency solutions presented in Table 3. These tests also help to calcu-
late more realistic uncertainties for the frequency values than the simple formal uncertainties derived from the light curve fits.

**Analysis of data subsets:** The Konkoly2007 dataset provides the longest time base, but it has also the worst temporal coverage. We selected four data subsets for independent analysis: subset 1 (runs No. 3 and 4 in Table 1), subset 2 (runs No. 1 – 4), subset 3 (runs No. 6 – 8) and subset 4 (runs No. 9 – 11). Comparing to the result of the Fourier analysis of the whole dataset (Table 3), we found five frequencies being significant also in the subset data. Four of them (at 1456, 1624, 1633 and 1831 µHz) could be detected only once, but we found the 2277 µHz frequency in every subset.

**Analysis of synthetic datasets:** We performed this test separately for every dataset: the 1988, 1992, 2000 Oct, WET2000, 2001, Konkoly2007, Konkoly2011, and also the subset 1 – 4 data. In this step, we generated at least 50 synthetic light curves with the same timings as the original data and with Gaussian random noise added, using the frequencies, amplitudes and phases of the corresponding light curve solution presented in Table 3. The applied noise level was calculated according to the residual scatter of the given light curve solution. We carried out independent and automated frequency analysis of each synthetic dataset with the program SpecTrum (Reeseg 2007). This way we could eliminate the subjective personal factor e.g. in the selection of the starting frequency. We checked finally in what percentage the program found the input frequencies instead of their aliases. The success rates we obtained for the different frequencies are between 34 and 100 perc cent, but above 70 perc cent in most cases.

**Analysis of synthetic datasets with amplitude variations:** The presence of temporal variations in pulsating white dwarfs are well-known, and comparing the FTs of different epochs’ datasets, amplitude variations of KUV 05134+2605 are obvious. However, the Fourier analysis method we apply for the whitening assumes constant amplitudes, frequencies and phases for the pulsation. Any deviations from this assumption may result in the emergence of additional peaks in the Fourier spectrum.

We divided the Konkoly2007 dataset into five subsets: subset A, B, C, D and E (runs No. 1–2, 3–4, 5, 6–8 and 9-11), and generated synthetic light curves for the times of these subsets. We fixed the Konkoly2007 frequencies and phases according to the ones determined by the whole dataset, but let the amplitudes vary from subset to subset for every frequency. We omitted only the combination term at 3902.1 µHz and the 2275.1 µHz frequency from the test. This latter may be an artefact resulting from temporal variations in the pulsation, and this way we checked, whether amplitude changes can really cause such closely spaced peaks. We chose the amplitudes applied to represent both slight and major variations from subset to subset, based on the observed values. Constructing this way a new, synthetic Konkoly2007 dataset with variable amplitudes, we then created 50 of them adding Gaussian random noise as in the case of the fixed-amplitude tests, and analysed them with SpecTrum. The program found the original frequencies with 22–98 per cent success rates, and 6 out of the 11 frequencies reached or exceeded the 70 per cent rate.

The other finding is that amplitude variations can cause indeed the emergence of closely spaced, and low amplitude peaks in the Fourier transforms. For example, the doublet at 2277.6 and 2278.5 µHz showing 3.9 and 2.8 mmag amplitudes and 0.9 µHz frequency separation in one of the test datasets. We note that similarly closely spaced peaks were detected, e.g., in the case of the ZZ Ceti stars KUV 02464+3239 (Bognár et al. 2009) and EC14012-1446 (Provenčal et al. 2012). Both stars show variable amplitudes on a time-scale of weeks.

**3.3.1. Conclusions**

The main goal of these tests was to investigate the robustness of the light curve solutions presented in Table 3. Based on the results, we constructed the list of frequencies considered real (see Table 4 applying the following rules: (1) in the case of the Konkoly2007 frequencies, we considered real a frequency if it was detected at least 75 per cent success rate at least in 2 test cases (considering the analyses of data subsets and the whole dataset with fixed or variable amplitudes). (2) In the case of the other datasets, we considered real a frequency if was found at least 75 per cent success rate in the corresponding test data. (3) If two frequencies found to be the same within the uncertainties, we chose the one having the lower uncertainty. (4) Finally, we do not consider real the 1893.8 and 2275.1 µHz frequencies, as based on the test with variable amplitude frequencies, we cannot exclude the possibility that these closely spaced peaks are artefacts from such variations.

However, these simple rules do not help to choose between the 1644.2 and 1633.0 µHz 1 d⁻¹ alias frequencies. We included the 1633.0 µHz one in the list of Table 3, but with the additional note that its +1 d⁻¹ alias solution is also acceptable.

The uncertainties presented both in Tables 3 and 4 are calculated by the analyses of the test light curves. In calculations with Monte Carlo simulations, we determine the standard deviations of the input frequencies derived from a large number of test datasets. In this case, we calculated the uncertainties including the cases when the program found not the original frequency, but one of its alias peaks. This way we took into account the effect of alias ambiguities in the uncertainties.

**3.4. Investigation of the set of frequencies**

In total, 22 frequencies were considered as the most probable set of frequencies after the tests described in Sects. 3.3 and 3.3 Table 4 lists these and also summarizes the frequency spacings. Considering these spacing values, we can form groups of frequencies, in which the separation of the consecutive members are less than 26 µHz (~ 2 × 1 d⁻¹, taking the uncertainties of frequencies into account). These groups are also indicated in Table 4. The frequency separation was chosen to be ~ 2 × 1 d⁻¹ in order to ensure that even if one or both frequencies suffer from 1 d⁻¹ alias ambiguities, they remain in the same group.

The first conspicuous thing is the large number of excited modes. The grouping of frequencies suggests that at least 12 normal modes can be detected by the ensemble treatment of the different datasets. This means that KUV 05134+2605 has become one of the ‘rich’ white dwarf pulsators known. We know only a few of them (Bischoff-Kim 2009, Bischoff-Kim & Metcalfe 2012), and more modes give stronger constraints for the asteroseismic investigations. Note that when the modes are asymptotic, adding one or two new frequencies to the list of equidistant modes brings limited new information. This is not the case of KUV 05134+2605 (see Table 4). Consequently, KUV 05134+2605 has a particular importance among the white dwarf variables, especially the V777 Her-type ones.

We have already mentioned in Sects. 3.3.1 and 3.3.2 the doublets and the triplet showing ~ 9 µHz frequency spacings. The 1992 dataset’s doublet failed in the frequency determination test, as we could get back both peaks in the same dataset only in 26 per cent of the test cases, i.e., with a low success rate. However, this is not surprising because their separation is close to the dataset’s Rayleigh frequency resolution. The 1892 ~ 1901 µHz doublet found by the WET2000 data seems to be a robust finding, as


Table 4. Frequencies and periods of the accepted pulsation frequencies of KUV 05134+2605. The third column shows the frequency differences of closely spaced peaks. The last two columns summarize the results of the preliminary mode identification based on rotational multiplets and period spacing investigations (see Sect. 3.4.2). The frequency uncertainties are the same as in Table 3.

| Frequency ($\mu$Hz) | Period (s) | $\delta f$ | $\ell$ | $m$ |
|---------------------|------------|-----------|-------|-----|
| 1287.6±3.1          | 76.6±1.9   | 1         |       |     |
| 1390.6±2.8          | 719.1±1.4  | 24.1      | 1     |     |
| 1414.7±0.1          | 706.8±0.1  | 21        |       |     |
| 1456.6±4.1          | 686.6±1.9  | 15.9      | 1.2   |     |
| 1472.5±1.2          | 679.1±0.6  | 1.2       |       |     |
| 1504.5±0.1          | 664.7±0.1  | 2         |       |     |
| 1531.4±1.7          | 653.0±0.7  | 18.3      | 1     | -1  |
| 1549.7±4.9          | 645.3±2.0  | 1         | +1    |     |
| 1614.5±1.6          | 619.4±0.6  | 9.2       | 1     | -1  |
| 1623.7±4.8          | 615.9±1.8  | 9.3       | 1     | 0   |
| 1633.0±4.5          | 612.4±1.7  | 1         | +1    |     |
| 1681.7±1.6          | 594.6±0.6  | 18.2      | 1     | -1  |
| 1699.9±2.3          | 588.3±0.8  | 1         | +1    |     |
| 1762.9±4.0          | 567.2±1.3  | 18.8      | 2     |     |
| 1781.7±2.3          | 561.2±0.7  | 16.9      | 1     | -1  |
| 1798.6±2.3          | 556.0±0.7  | 1         | +1    |     |
| 1831.1±5.7          | 546.1±1.7  | 2         |       |     |
| 1892.3±0.1          | 528.5±0.1  | 9.6       | 1     | -1.0|
| 1901.9±0.1          | 525.8±0.1  | 1         | 0+1   |     |
| 2277.6±1.6          | 439.1±0.3  | 2.1       |       |     |
| 2504.7±6.6          | 399.2±1.1  | 26.0      | 1     |     |
| 2530.7±4.4          | 395.1±0.7  | 2.1       |       |     |

Notes. Periods marked with plus symbol (+) are reported previously by Handler et al. (2003) as dominant signals at different epochs. (+) $1\text{d}^{-1}$ solution (1644.2 $\mu$Hz) is also possible by the WET2000 dataset.

3.4.1. Stellar rotation

Because of the better visibility of dipole ($\ell=1$) modes over quadrupole ($\ell=2$) ones (see e.g. Castanheira & Kepler [2008] and references therein), we expect that most of the detected frequencies are $\ell=1$. We see that there is a recurring $\approx9\mu$Hz spacing in our datasets, and the most robust finding is the $\delta f=9.6\mu$Hz doublet in the WET2000 data. Supposing that the frequency components are high-overtone ($k \gg 1$) $\ell=1, m=-1,0$ or $m=0,1$ modes, the rotation period of the star can be calculated by the asymptotic relation: $\delta f_{k,\ell,m}=\Omega m(1-C_{k,\ell})\Omega$, where the coefficient $C_{k,\ell} \approx 1/(\ell+1)$ and $\Omega$ is the rotation frequency. Thus, the 9.6 $\mu$Hz splitting value implies a 0.6 d rotation period for KUV 05134+2605.

According to theory (Winget et al. [1991]), the ratio of the rotational splitting for $\ell=1$ and $\ell=2$ $g$-modes is $\delta f_{1,\ell}/\delta f_{2,\ell}$ = 0.60 in the asymptotic limit, as $\delta f \sim 1/(1/\ell+1)$. We find two closely spaced frequencies having $\delta f \approx 16.9\mu$Hz in the WET2000 dataset. Assuming that these are components of rotationally split $\ell=2$ modes, the corresponding splitting ratio is $(9.6 \pm 0.1)/(16.9 \pm 3.3) = 0.57 \pm 0.11$. This is the same within the errors as the theoretical 0.60 value. However, these two frequencies still can be $m=-1,+1, \ell=1$ components, too. Note that the 15.9 $\mu$Hz separation of the 1456 and 1472 $\mu$Hz frequencies also could be interpreted as rotational splitting of $\ell=2$ modes (9.6/15.9 = 0.60).

3.4.2. Period spacings

For $g$-modes with high radial order $k$ (long periods), the separation of consecutive periods ($\Delta k = 1$) becomes nearly constant at a value given by the asymptotic theory of non-radial stellar pulsations. Specifically, the asymptotic period spacing (Tassoul et al. [1990]) is given by:

$$\Delta \Pi_{k}^2 = \Pi_0 / \sqrt{\ell + 1},$$  \hspace{1cm} (1)

where

$$\Pi_0 = 2\pi^2 \left[ \int_{r_1}^{r_2} \frac{N}{r} dr \right]^{-1},$$  \hspace{1cm} (2)

$N$ being the Brunt-Väisälä frequency. In principle, one can compare the asymptotic period spacing computed from a grid of models with different masses and effective temperatures with the mean period spacing exhibited by the star, and then infer the value of the stellar mass. This method has been applied in numerous studies of pulsating PG 1159 stars (see, for instance, Córsico et al. [2007, 2008, 2009] and references therein). For the method to be valid, the periods exhibited by the pulsating star must be associated with high order $g$-modes, that is, the star must be within the asymptotic regime of pulsations. In addition, the star should be chemically homogeneous for Eq. (2) to be strictly valid. However, the interior of DB white dwarf stars are supposed to be chemically stratified and characterized by strong chemical gradients built up during the progenitor star life. So, the direct application of the asymptotic period spacing to infer the effective temperature and stellar mass of KUV 05134+2605

the test resulted 100 per cent success rate for the determination of both components. Considering the triplet at 1623 $\mu$Hz in the Konkoly2007 dataset, we could detect all three components in 74 per cent of the fixed-amplitude cases. We also find two of the components in the subset 3 data, with 82 per cent doublet-finding rate. The recurring emergence of the $\approx9\mu$Hz spacing in different datasets (epochs) with different window functions strongly suggests that this is not the result of the 1 $\text{d}^{-1}$ alias problem, but these frequency components have pulsational origin.

It is also conspicuous that frequencies were found not only with $\approx9\mu$Hz spacings, but also with its double. However, we must be cautious with the interpretation of these spacings; as the doublet components originate from different datasets, there is a risk that we see outcome of the frequency determination uncertainties and the 1 $\text{d}^{-1}$ alias problem. If these frequencies are real, a possible explanation is that the frequencies showing $\approx18\mu$Hz spacings are $m \pm 1$ peaks of the same triplet.

Table 4 also shows doublets with $\approx16$ and $26\mu$Hz spacings. We discuss them in Sects. 3.4.1 and 3.4.2.
is questionable. A better way to compare our models to the period spacing of the observed pulsation spectrum is to calculate the average of the computed period spacings, using:

$$\overline{\Delta \Pi(M_*, T_{\text{eff}})} = \frac{1}{(n-1)} \sum_{k} \Delta \Pi_k,$$

(3)

where $\Delta \Pi_k$ is the ‘forward’ period spacing defined as $\Delta \Pi_k = \Pi_{k+1} - \Pi_k$, and $n$ is the number of computed periods laying in the range of the observed periods. The theoretical period spacing of the models as computed through Eqs. (1) and (3) share the same general trends (that is, the same dependence on $M_*$ and $T_{\text{eff}}$), although $\overline{\Delta \Pi}$ is usually somewhat higher than $\overline{\Delta \Pi}$. As a preliminary mode identification, we attempted to determine the $\ell$ values of the observed frequencies using not only the effect of rotational splitting, but the period spacing values derived. The period values of the frequencies are given in Table 4.

Assuming that the frequencies showing $\delta f \approx 9 \mu\text{Hz}$ spacings or its double are $\ell = 1$, we created an initial set of the (supposed) $\ell = 1$ modes: $527.2, 591.4, 615.9,$ and $649.1$ s. These are the averages of the triplet and doublet peaks observed. This way we can avoid any assumption on the actual $m$ values. The period differences in this case are $64.3$ ($2 \times 32.2$), $24.5$ and $33.2$ s, respectively. We did not use the average of the peaks at 560 s, but investigated the three frequencies separately.

The effective temperature and surface gravity parameters of KUV 05134+2605 are $T_{\text{eff}} = 24700 \pm 1300 \text{K}$ and $\log g = 8.21 \pm 0.06 (M_* = 0.73 \pm 0.04 M_\odot)$, from model atmosphere fits to a high signal-to-noise optical spectrum of the star. Bergeron et al. (2011) also found that KUV 05134+2605 belongs to the DBA spectral class, which means that it shows not only helium, but hydrogen lines.

Theoretical calculations show that period spacings of $\ell = 1$ modes of 0.7 and $0.8 M_*$ model stars can vary between $\approx 25 - 40$ s, depending on mode trapping (Bradley et al. 1993). It is also known that the mean period spacing decreases with increasing stellar mass. Considering Fig. 8 of Bradley et al. (1993), the minima in the period spacing diagrams of 0.7 and $0.8 M_*$ model stars are $\approx 30$ and $25$ s for the $\ell = 1$ modes, respectively. Trapped modes have lower kinetic energies and minima in period spacing diagrams correspond to minima in kinetic energies (Bradley & Winget 1991). Assuming that there is a missing overtone in our period list, the period spacing values are $\approx 30$ s, except one case with the lower value of 25 s. These are acceptable values of period spacings for this relatively high-mass variable.

Fu et al. (2013) demonstrated in the case of the ZZ Ceti star HS 0507+0434B, that the mean period spacing value can also be determined by the linear least-squares fit of the $m = 0$ modes plotted in the $\Delta k$ – period plane. Assuming that most of the observed frequencies correspond to $\ell = 1$ modes, we fitted almost all of the periods on the top left panel of Fig. 4, excluding only the periods at 664 and 546 s; because of the high frequency density in these regions, it seems improbable that they are all $\ell = 1$. This solution reflects the expectation that the period differences should be higher and lower than 25 s for the $\ell = 1$ and 2 modes, respectively. The formula used for the fits is $P_k = P_0 + \Delta P \times \Delta k$.
In the case of the supposed \( \ell = 1 \) modes, we selected the 615.9 s mode as \( k_0 \).

The top panels of Fig. 4 show the corresponding \( \ell = 1 \) and 2 fits and their residuals. Filled circles denote the four average periods of the presumed \( \ell = 1 \) modes by using the Kolmogorov-Smirnov (K-S) significance tests. The average \( \ell = 1 \) period spacing determined this way is \( \Delta \Pi_{1s} = 31.2 \pm 0.4 \) s. We present a possible fit to the two \( \ell = 2 \) periods (\( \Delta \Pi_{2s} = 16.9 \) s) for comparison with the next fit’s result.

As we see some outliers in the \( \ell = 1 \) fit, we assumed that their previously attributed \( \ell \) values are not correct, and in the next step, we moved them to the \( \ell = 2 \) group in an attempt to optimize our fit. The bottom panels of Fig. 4 show the results of this solution. The corresponding \( \Delta \Pi \) values for the \( \ell = 1 \) and 2 modes are 31.4 \( \pm 0.3 \) and 18.0 \( \pm 0.3 \) s, respectively. The results show that with the change of the \( \ell \) values of some periods, the average period spacings change little. Note that this method alone cannot be used for mode identification, but strongly suggests that even if there are frequencies in our list which are not correctly determined because of the alias ambiguities, the frequencies are not randomly distributed, and especially in the case of the presumed \( \ell = 1 \) ones, points to a definite mean spacing value.

According to theoretical calculations (e.g. Reed et al. 2011), the asymptotic ratio of the period spacings of \( \ell = 1 \) and 2 modes is \( \Delta \Pi_{2s}/\Delta \Pi_{1s} = \sqrt{3} = 1.73 \). For the second fits presented above, this ratio is \((31.4 \pm 0.3)/(18.0 \pm 0.3) = 1.74 \pm 0.03 \). That is, the result agrees with the theoretical prediction within the uncertainty.

In the case of the frequency pairs, we can conclude that (1) the 719 and 706 s periods, if both are real pulsation frequencies and not aliases, cannot have the same \( \ell \), and the 706 s one may be the \( \ell = 2 \). (2) The 686 – 679 s peaks either can be rotationally split or 1 d\(^{-1} \) alias \( \ell = 1 \) or 2 frequencies. (3) According to the period fits, the 561 and 556 s periods may be rotationally split \( \ell = 1 \) frequencies \((m = -1, +1)\) and not \( \ell = 2 \) ones. In this case, the 567 s period is an \( \ell = 2 \), and its 18.8 \( \mu \)Hz frequency separation with the 561 s peak is a coincidence, and have nothing to do with the supposed 9 \( \mu \)Hz rotational splitting. (4) The 399 s member of the 395 – 399 s pair fits better into the series of \( \ell = 1 \) modes. The last two columns of Table 4 show the presumed \( \ell \) and \( m \) values determined by the supposed rotational multiplets and the period spacing investigations.

3.4.3. Tests on period spacings

All of our efforts aim to know more about the stellar structure, specially the mass of the observed object. As we mentioned above, the mean period spacing is an indicator of the stellar mass, and applying linear fits to the observed periods, we determined 31.4 s value for the presumed \( \ell = 1 \) modes. Considering the 1 d\(^{-1} \) alias ambiguities, the reliability of this value must be investigated. We performed several tests to check our solution.

We searched for a characteristic period spacing between the periods listed in Table 4 by using the Kolmogorov-Smirnov (K-S) (Kawaler 1988) and the Inverse Variance (I-V) (O’Donoghue 1994) significance tests. In the K-S test, the quantity \( Q \) is defined as the probability that the observed periods are randomly distributed. Thus, any characteristic period spacings present in the period spectrum should appear as a minimum in \( Q \). In the I-V test, a maximum of the inverse variance will indicate the presence of a constant period spacing. Taking all the periods of Table 4 thus without any assumptions based on frequency or period spacings, both methods strongly point to the existence of a characteristic period spacing of about 31 s (see Fig. 5). The minimum in the K-S test is at 31.0 s, while the maximum in the I-V plot is at 31.4 s, respectively. These values are in very good agreement with the mean period spacings found by linear fits for \( \ell = 1 \) modes.

Another well-known method to find a characteristic spacing value is the Fourier analysis of the periods themselves (see e.g. Handler et al. 1997). At first, we performed the Fourier analysis of the periods listed in Table 4 and found a clear maximum at 31.4 s. This also supports the presence of the spacing derived by the linear fits, and the K-S and I-V tests. In the next step, we modified the period list used: we chose randomly 8 frequencies and (also randomly) added or subtracted 11.574 \( \mu \)Hz (1 d\(^{-1} \)) to these values. That is, we changed almost 40 per cent of the periods. Afterwards, we generated a new period list with these modified frequencies, simulating the effect of the 1 d\(^{-1} \) aliasing, present in all datasets. We generated 20 such aliased period lists and performed their Fourier analyses. Applying this Monte Carlo test, we still find a common maximum around 31 s in each simulation. The average of the maxima for the 20 different cases is 31.4 \( \pm 0.2 \) s.

Finally, we created a period list containing the periods presented in Handler et al. (2003) and marked in Table 4 which were selected as the dominant and well-separated peaks from the different epochs’ datasets. We complemented this with five periods from the Konkoly2007 dataset (686.6, 615.9, 594.6, 439.1 and 395.1 s), and also performed their Fourier analysis. This list contains 14 periods only, but we still obtain a maximum at 31.3 s in the AI = 25 – 40 s region. Figure 6 shows the FT of the original period set of Table 4 five of the FTs of the aliased period lists, and the FT of the 14-period list.

These tests demonstrate that even if the dataset suffers from alias problems, the results from different methods remain consistent and thus the mean period spacing at 31 s is a robust finding. Considering all of the mean period spacings derived by the linear fits, the K-S, I-V and Monte Carlo (Fourier) tests, we consider 31.4 \( \pm 0.3 \) s as the best estimation for the mean spacing of \( \ell = 1 \) modes.
4. Stellar models

4.1. New spectroscopic mass of KUV 05134+2605

We employed the DB white dwarf evolutionary tracks of Córsico et al. (2012) to infer the stellar mass of KUV 05134+2605 from the spectroscopic determination of $T_{\text{eff}} = 24 700 \pm 1300$ K and $\log g = 8.21 \pm 0.06$ by Bergeron et al. (2011). These tracks cover a wide range of effective temperatures ($35 000 \geq T_{\text{eff}} \geq 18 000$ K) and masses ($0.51 \leq M/\odot \leq 0.870$). The sequences of DB white dwarf models have been obtained with a complete treatment of the evolutionary history of progenitor stars (see Althaus et al. 2009 for details or Córsico et al. 2012 for a brief discussion). We varied the stellar mass and the effective temperature parameters in our model calculations, while the helium content, the chemical structure at the core, and the thickness of the chemical transition regions were fixed by the evolutionary history of progenitor objects.

In Fig. 7, we plot the evolutionary tracks along with the location of all the DBV stars known to date. We infer a new value of the spectroscopic mass for this star on the basis of this set of evolutionary models. This is relevant because these same set of DB white dwarf models will be utilized in the next sections to derive the stellar mass from the period spacing and the individual pulsation periods of KUV 05134+2605. We found $M_* = 0.72 \pm 0.04 M_\odot$, in agreement with the value quoted in Sect. 3.4.2 ($M_* = 0.73 \pm 0.04 M_\odot$).

4.2. Clues of the stellar mass from the period spacing

The observed mean period spacing of the dipole modes of KUV 05134+2605, as derived and checked in Sect. 3.4.3, is $\Delta \Pi_{e,1} = 31.4 \pm 0.3$ s. This period spacing is certainly shorter than the values measured in other DBV stars, that is, EC20058-5234 ($\Delta \Pi = 36.5$ s; Bischoff-Kim & Ostensen 2011), GD 358 ($\Delta \Pi = 38.8$ s; Provencal et al. 2009), and WD J1929+4447 ($\Delta \Pi = 35.9$ s; Bischoff-Kim & Ostensen 2011). This suggests that KUV 05134+2605 could be more massive than other pulsating stars within the V777 Her class.

In Fig. 8, we show the run of the average of the computed period spacings [Eq. (3)] with $\ell = 1$, in terms of the effective temperature for all of our DB white-dwarf evolutionary sequences. The $g$-mode pulsation periods were computed with the LP-PUL adiabatic pulsation code described in Córsico & Althaus (2006). The curves displayed in the plot are somewhat jagged. This is because the average of the computed period spacings is evaluated for a fixed period interval, and not for a fixed $k$-interval. As the star evolves towards lower effective temperatures, the periods generally increase with time. At a given $T_{\text{eff}}$, there are $n$ computed periods laying in the chosen period interval. Later, when the model has cooled enough, it is possible that the accumulated period drift nearly matches the period separation between adjacent modes ($|\Delta k| = 1$). In these circumstances, the number of periods laying in the chosen (fixed) period interval is $n \pm 1$, and $\Delta \Pi$ exhibits a small jump. In order to smooth the curves of $\Delta \Pi$, in constructing Fig. 8, we have considered pulsation periods in a wider range of periods ($100 \leq \Pi \leq 1200$ s) than that observed in KUV 05134+2605 ($390 \leq \Pi \leq 780$ s). The location of KUV 05134+2605 is indicated by a red asterisk symbol according to the obtained solution for $\Delta \Pi_{e,1}$ (Sect. 3.4.3).

We can have an estimate of the stellar mass free from any possible uncertainty in the spectroscopic analysis. If we look...
for the possible range of masses for the star to be within the DBV instability strip\((T_{\text{eff}} = 24\,700 \pm 1300\, K)\) for the observed \(\Delta T_{1} = 31.4 \pm 0.3\) solution of mean period spacing. By simple linear interpolation, we found that the mass of the star according to its period spacing is \(M_* = 0.85 \pm 0.05\, M_\odot\).

Fig. 8. The average of the computed period spacings for \(\ell = 1\) corresponding to our DB white dwarf sequences with different stellar masses. The location of KUV 05134+2605 is shown with a red asterisk symbol\((T_{\text{eff}} = 24\,700 \pm 1300\, K)\) for the observed \(\Delta T_{1} = 31.4 \pm 0.3\) solution of mean period spacing. By simple linear interpolation, we found that the mass of the star according to its period spacing is \(M_* = 0.85 \pm 0.05\, M_\odot\).

\(\Delta T_1\) is a necessary condition in order that the comparison between the two values to be consistent.

We infer a mass for the star of \(0.870 \pm 0.008\). This conclusion relies only on the measured mean period spacing, and on the average of the computed period spacing, a global quantity that is not affected by the precise shape of the chemical transition regions of the models. It is particularly worth noting that this result is independent of the mass of the He-rich envelope. Indeed, Tassoul et al. (1990) have shown that the asymptotic period spacing of DBV white dwarfs is particularly insensitive to the mass of the He-rich mantle, changing less than 1 s for a change of 10 orders of magnitude in the mass of the He-rich mantle (see their Fig. 42). Also, this conclusion is independent of the effective temperature of the star. So, the result that the mass of KUV 05134+2605 is higher than the average mass of DB white dwarfs (see Kepler et al. 2007; Bergeron et al. 2011; Kleinman et al. 2013) is a robust finding.

If we adopt the spectroscopic effective temperature of KUV 05134+2605\((T_{\text{eff}} = 24\,700 \pm 1300\, K)\) as a constraint, we infer a mass for the star of \(M_* = 0.85 \pm 0.05\, M_\odot\). This value is larger than the spectroscopic mass\((-0.72 \pm 0.73\, M_\odot)\). Note that both estimates of the stellar mass of KUV 05134+2605 are based on the same DB white dwarf evolutionary models, which is a necessary condition in order that the comparison between the two values to be consistent.

4.3. Constraints from the individual observed periods

The usual way to infer the stellar mass, the effective temperature and also details of the internal structure of pulsating white dwarfs is investigating their individual pulsation periods. In this approach, we seek a pulsation DB white dwarf model that best matches the pulsation periods of KUV 05134+2605. The goodness of the match between the theoretical pulsation periods \(\Pi_k\) and the observed individual periods \(\Pi_{\text{obs},i}\) is measured by means of a quality function defined as:

\[
\chi^2(M_*, T_{\text{eff}}) = \frac{1}{N} \sum_{i=1}^{N} \min[(\Pi_{\text{obs},i} - \Pi_k)^2],
\]

where \(N\) is the number of observed periods. The DB white dwarf model that shows the lowest value of \(\chi^2\) is adopted as the ‘best-fit model’. Our asteroseismological approach combines (i) a significant exploration of the parameter space, and (ii) a detailed and updated input physics, in particular, regarding the internal structure, that is a crucial aspect for correctly disentangling the information encoded in the pulsation patterns of variable white dwarfs (see Corsico et al. 2007a,b, 2008, 2009a, 2012; Romero et al. 2012, 2013).

We evaluate the function \(\chi^2(M_*, T_{\text{eff}})\) for stellar masses of \(0.515, 0.530, 0.542, 0.565, 0.584, 0.609, 0.664, 0.741,\) and \(0.870\, M_\odot\). For the effective temperature, we employed a finer grid \(\Delta T_{\text{eff}} = 10 \pm 30\, K\). The quality of our period fits is assessed by means of the average of the absolute period differences, \(\bar{\delta} = \left(\frac{\sum_{i=1}^{N} |\delta_i|}{N}\right)\), where \(\delta_i = \Pi_{\text{obs},i} - \Pi_k\), and by the root-mean-square residual, \(\sigma = \left(\frac{\sum_{i=1}^{N} |\delta_i|^2}{N}\right)^{1/2}\).

Given the unusually large (for DBV standards) number of pulsation periods observed in KUV 05134+2605, we have to consider several cases regarding the set of periods to be employed in our asteroseismological fits.

4.3.1. Case 0

Here, we consider that all the identified periods shown in Table 4 are associated with either \(\ell = 1\) or \(\ell = 2\), i.e., no mixture of \(\ell\). Given the presence of very small separations between some periods in the pulsation spectrum of KUV 05134+2605, then the modes should be all \(\ell = 2\). However, it is improbable that a pulsating white dwarf shows only quadrupole modes (Winget et al. 1994; Costa et al. 2008), and we discard it.

4.3.2. Case 1

In this case, we assume that the periods exhibited by KUV 05134+2605 correspond to a mix of dipole and quadrupole modes. Here, we perform a period fit in which the value of \(\ell\) for the theoretical periods is not fixed at the outset but instead is obtained as an output of our period fit procedure, with the allowed values of \(\ell = 1\) and \(\ell = 2\). We assume as real the existence of five rotational multiplets of frequencies and we select the period of the \(m = 0\) component (if present) to perform our period fit, and the period corresponding to the average of the frequencies supposed to be the \(m = -1\) and \(m = +1\) components if the \(m = 0\) component is absent. We also assume that the component \(m = 0\) in the supposed triplet at periods in the interval 525.8 – 528.5 s is the period at 525.8 s. Specifically, we adopt a set of 16 periods.

We investigated the quantity \(\left(\chi^2\right)^{-1}\) in terms of the effective temperature for different stellar masses, and found one strong maximum for a model with \(M_* = 0.87\, M_\odot\) and \(T_{\text{eff}} = 25\,980\, K\). Such a pronounced maximum in the inverse of \(\chi^2\) implies a good agreement between the theoretical and observed periods. The effective temperature of this model is somewhat higher than that of the DB white dwarf model that best matches the pulsation periods of KUV 05134+2605.
but still in agreement with) the spectroscopic effective temperature of KUV 05134+2605 ($T_{\text{eff}} = 24,700 \pm 1300 \, \text{K}$). Another maximum, albeit much less pronounced, is encountered for a model with the same stellar mass and lower effective temperature ($T_{\text{eff}} \sim 24,000 \, \text{K}$). However, because the agreement between the observed and theoretical periods for these models are much poorer, we adopt the model with $T_{\text{eff}} = 25,980 \, \text{K}$ as the best-fit asteroseismological model. In order to have an indicator of the quality of the period fit, we computed the Bayes Information Criterion (BIC; Koen & Lane 2000):

$$\text{BIC} = N_p \left( \log \frac{N}{N_p} + \log \sigma^2 \right),$$

(5)

where $N_p$ is the number of free parameters, and $N$ is the number of observed periods. The smaller the value of BIC, the better the quality of the fit. In our case, $N_p = 2$ (stellar mass and effective temperature), $N = 16$, and $\sigma = 2.8 \, \text{s}$. We obtained BIC = 1.06, which means that our fit is relatively good.

An important caveat should be kept in mind: from the period-fit procedure, most of the periods are associated with $\ell = 2$ modes. This is in contradiction with the well-known property of periods that cannot be considered as components of triplets (395.1, 399.2) s is fitted by a single period at 398.1 s. Here, we assume that the periods close to each other (the ‘pairs’ in the third column of this Table, and the results shown in Table 4 and the lower panels of Fig. 4. Case 3

Here, we assume the periods close to each other (the ‘pairs’ of periods) that cannot be considered as components of triplets with a frequency separation of $\sim 9 \, \mu\text{Hz}$ are actually a single period, and then we consider the period resulting from the average of the frequencies present. That is, we replace the pair (395.1, 399.2) s by a single period at 397.2 s, the pair (679.1, 682.7) s by a period at 689.5 s, and the pair (1555.2, 1557.2) s by a period at 1559.3 s. The reduced list contains 13 periods. In this case, the best-fit model has $M_* = 0.87 \, M_{\odot}$ and $T_{\text{eff}} = 25,890 \, \text{K}$ ($\chi^2 = 0.13$), thus the solution found in Case 1 still persists. However, in this case, other solutions [$T_{\text{eff}} \sim 27,100 \, \text{K}$, $\chi^2 = 0.12$, and $T_{\text{eff}} \sim 24,150 \, \text{K}$, $\chi^2 = 0.11$] with the same stellar mass are equally valid from a statistical point of view. The number of $\ell = 2$ modes still exceeds the number of $\ell = 1$ modes in the fit.

4.3.4. Case 3

At variance with the previous cases, here we assume the $\ell$-identification shown in Table 4 to be correct and then we constrain the $\ell = 1$ observed modes to be fitted by $\ell = 1$ theoretical modes, and the $\ell = 2$ observed modes to be fitted by $\ell = 2$ theoretical modes. Note that, according to Table 4, there are several possible assumptions for the mode identification. Specifically, the period at 439.1 s can be associated to $\ell = 1$ or $\ell = 2$; the $m = 0$ component in the (525.8, 528.5) s doublet can be either 525.8 s or 528.5 s; and the period at 679.1 s may be associated to $\ell = 1$ and the period at 686.6 s to $\ell = 2$ or vice versa. On the other hand, the period at 395.1 s must be an $\ell = 2$ mode if the period at 399.2 s is an $\ell = 1$ mode. Similarly, the 706.8 s period must be associated to $\ell = 2$ if the 719.1 s period is and $\ell = 1$ mode. We have taken into account all the possible combinations in our period fits. No matter what assumptions we adopt for the periods at 439.1, 525.8, 528.5, 679.1, and 686.6 s, we obtain a clear and recurrent single solution for a model with $M_* = 0.87 \, M_{\odot}$ and $T_{\text{eff}} = 24,060 \, \text{K}$. The set of observed periods and their assigned $\ell$ values (9 modes with $\ell = 1$ and 7 modes with $\ell = 2$) are listed in the first two columns of Table 5. The theoretical periods of this fit are shown in the third column of this Table, and the results of the period fit are displayed in Fig. 5. The quality of the fit is described by a BIC index of 1.17.

4.4. Zooming in on the best-fit model region

To search for possible closer solutions in the region of the parameter space where the best-fit model from period-to-period fits ($M_* \sim 0.87 \, M_{\odot}$) was found, we have extended our exploration by generating a set of additional DB white dwarf evolutionary

| $\Pi_{\text{obs}}$ | $\ell$ | $\Pi_{\text{Case 3}}$ | $k_{\text{C}}$ | $\Pi_{\text{zoom}}$ | $k_{\text{zoom}}$ |
|---|---|---|---|---|---|
| 395.1 | 2 | 399.9 | 19 | 401.3 | 19 |
| 399.2 | 1 | 398.8 | 10 | 400.7 | 10 |
| 439.1 | 2 | 439.2 | 21 | 440.7 | 21 |
| 525.8 | 1 | 522.6 | 14 | 524.7 | 14 |
| 546.1 | 2 | 549.6 | 27 | 548.2 | 27 |
| 558.6 | 1 | 555.2 | 15 | 557.2 | 15 |
| 567.2 | 2 | 567.7 | 28 | 568.7 | 28 |
| 591.4 | 1 | 590.3 | 16 | 591.1 | 16 |
| 615.9 | 1 | 620.2 | 17 | 618.0 | 17 |
| 649.1 | 1 | 644.0 | 18 | 649.9 | 18 |
| 664.7 | 2 | 668.2 | 33 | 666.4 | 33 |
| 679.1 | 1 | 682.7 | 19 | 682.1 | 19 |
| 686.6 | 2 | 689.5 | 34 | 685.6 | 34 |
| 706.8 | 2 | 703.6 | 35 | 699.7 | 35 |
| 719.1 | 1 | 717.4 | 20 | 716.9 | 20 |
| 776.6 | 1 | 772.4 | 22 | 774.8 | 22 |

Fig. 9. The inverse of the quality function of the period fit in terms of the effective temperature for Case 3. The vertical grey strip indicates the spectroscopic $T_{\text{eff}}$ and its uncertainties ($T_{\text{eff}} = 24,700 \pm 1300 \, \text{K}$).
sequences with masses between 0.74 $M_\odot$ and 0.87 $M_\odot$. These sequences were constructed by scaling the stellar mass from the 0.87 $M_\odot$ sequence at very high effective temperatures. It is worth noting that the internal chemical profiles (of utmost importance regarding the pulsation properties of white dwarfs) of these new sequences are consistently assessed through linear interpolation between the chemical profiles of the sequences with 0.74 $M_\odot$ and 0.87 $M_\odot$.

Specifically, we have computed $\ell = 1, 2$ g-mode periods of these additional sequences with $M_\star = 0.76, 0.78, 0.80, 0.805, 0.82, 0.84, 0.85, 0.86 M_\odot$ and we have carried out period fits to these models. We will focus the discussion on Case 3. The main results of the period fits are displayed in Fig. 10. The best solution found is a model with a stellar mass of 0.84 $M_\odot$ and an effective temperature of 25050 K, characterized by a BIC index of 1.06, a bit lower than for Case 3.

This period fit corresponds to the case in which the period at 679.1 s is an $\ell = 1$ mode, and the 686.6 s period is an $\ell = 2$ mode. If not, we obtain a second period of 682.8 s, corresponding to the total content of He of the envelope of the model.

We have also searched for asteroseismological solutions taking other combinations of these assumptions into account. Interestingly, the assignment of the harmonic degree to the periods at 679.1 s and 686.6 s proved to be crucial for our results. Specifically, we obtain the solution $(M_\star/M_\odot, T_{\text{eff}}) = (0.84, 25 050 \text{ K})$ whenever we suppose that the period at 679.1 s is an $\ell = 1$, and the 686.6 s period is an $\ell = 2$ mode. If not, we obtain a second seismological solution with $M_\star = 0.82 M_\odot$ and $T_{\text{eff}} = 25 700 \text{ K}$. The quality of the period fit, however, is worse (BIC $\approx 1.19$).

We also performed fits replacing the pair (679.1, 686.6)s by a single period at 682.8 s. Assuming that this (average) period is $\ell = 1$, the best-fit solution is $M_\star = 0.85 M_\odot$ and $T_{\text{eff}} = 24 690 \text{ K}$. The preferred model is $M_\star = 0.84 M_\odot$ and $T_{\text{eff}} = 25 050 \text{ K}$, if the 682.8 s period is $\ell = 2$. Finally, we adopted the model with $(M_\star/M_\odot, T_{\text{eff}}) = (0.84, 25 050 \text{ K})$, when the 679.1 s period is an $\ell = 1$ and the 686.6 s period is an $\ell = 2$, as our asteroseismological solution for KUV 05134+2605. This solution is characterized by the lowest BIC index, and has an effective temperature that is in excellent agreement with the value predicted by spectroscopy. The set of theoretical periods of this fit is listed in Table 5 (fifth column).

4.4.1. The best-fit model

The main features of our best-fit model considering the additional evolutionary sequences are summarized in Table 5 where we also include the parameters from spectroscopy (Bergeron et al. 2011). In the Table, the quantity $M_{\text{He}}$ corresponds to the total content of He of the envelope of the model.

We briefly describe the main properties of our best-fit DB model for KUV 05134+2605. In Fig. 11 we depict the internal chemical structure of such model (upper panel), where the abundance by mass of the main constituents ($^4\text{He}$, $^{12}\text{C}$, and $^{16}\text{O}$) is shown in terms of the outer mass fraction $-\log(1 - M_{\text{He}}/M_\star)$. The chemical structure of our models consists of a C/O core – resulting from the He burning of the previous evolution – shaped by processes of extra mixing like overshooting. The core is surrounded by a mantle rich in He, C, and O which is the remnant of the regions altered by the nucleosynthesis during the thermally pulsing asymptotic giant branch. Above this shell, there is a pure He mantle with a mass $M_{\text{He}}/M_\star \sim 1.4 \times 10^{-3}$, constructed by the action of gravitational settling that causes He to float to the surface and heavier species to sink.

The lower panel of Fig. 11 displays the run of the two critical frequencies of non-radial stellar pulsations, that is, the Brunt-Väisälä frequency and the Lamb frequency ($L_\ell$) for $\ell = 1$. The precise shape of the Brunt-Väisälä frequency largely determines the properties of the $g$-mode period spectrum of the model. In particular, each chemical gradient in the model contributes locally to the value of $N$. The most notable feature is the very peaked structure at the C/O chemical transition $-\log(1 - M_{\text{He}}/M_\star) \sim 0.5$. On the other hand, the He/C/O transition region at $-\log(1 - M_{\text{He}}/M_\star) \sim 2 - 5$ is very smooth and does not affect the pulsation spectrum much.

4.4.2. Periods from a DB model

Bergeron et al. (2011) reported that KUV 05134+2605 belongs to the DBa spectral class. Therefore, we performed exploratory computations of the period spectrum of a DB white dwarf model with pure helium atmosphere (‘DB model’) and a similar model that has small impurities of hydrogen (‘DBa model’). In the DBa model, the hydrogen is mixed with helium all across the helium envelope (like in the DBa models of Córtesco et al. 2009b, sect. 4). Both models have $M_\star = 0.74 M_\odot$ and $T_{\text{eff}} \sim 24 600 \text{ K}$. In particular, we have analysed the case in which $\log(N_{\text{He}}/N_{\text{H}}) \sim -3.8$ (the value corresponding to KUV 05134+2605 according to Bergeron et al. 2011).

We have found that the relative differences in the pulsation periods of these two models are in average lower than 0.2%. This means an absolute difference lower than 0.8 s for periods around...
sational properties and thus studying their interiors are still
2011; Hermes et al. 2014). This means that exploring their pul-
space (Greiss et al. 2014; Hermes et al. 2011; Østensen et al.
one V777 Her and three ZZ Ceti stars were monitored from
photometric space missions completely revolutionized thevari-
The almost continuous, high-precision light curves provided by
long monitoring) are needed to determine enough (at least half
pulsators. It can help to detect new modes, as observing the star
short time scales seem to be common among the white dwarf
modes. Moreover, our study suggests that this star is the most
massive one amongst the known DBVs.

5. Summary and conclusions
The almost continuous, high-precision light curves provided by
photometric space missions completely revolutionized the vari-
able star studies in many fields. However, up to know only one
V777 Her and three ZZ Ceti stars were monitored from
space (Greiss et al. 2014; Hermes et al. 2011; Østensen et al.
This means that exploring their pulsational properties and thus studying their interiors are still
mainly based on ground-based observations. Even thought their pulsation periods are short, in many cases extended observ-
ations (e.g. multisite campaigns and/or at least one season-
long monitoring) are needed to determine enough (at least half
dozen) normal modes for asteroseismic investigations (see e.g.
the cases of KUV 02464+3239, Bogdár et al. 2009, and GD 154,
Paparó et al. 2013). Otherwise, the low number of detected modes can result in models with fairly different physical param-
eters that can fit the observed periods similarly well. This shows
the particular importance of the rich pulsators, in which at least
a dozen normal modes can be fitted. Amplitude variations on
short time scales seem to be common among the white dwarf
pulsators. It can help to detect new modes, as observing the star
in different epochs, we can find new frequencies and comple-
ment the list of known excited modes.
In the case of KUV 05134+2605, we used photometric datasets obtained in seven different epochs between 1988 and
2011. We re-analysed the data published by Handler et al.
2003, and also collected data at the Piszkéstető mountain sta-
tion of Konkoly Observatory in two different observing seasons. Comparing the frequency content of the different datasets and
considering the results of the frequency determination tests, we
arrived at 22 pulsation frequencies between 1280 and 2530µHz.
The grouping of frequencies suggests at least 12 possible modes
for the asteroseismic model fits. With this, KUV 05134+2605
joined the – not populous – group of rich white dwarf pulsators.
We also showed that considering a $\approx 9\mu$Hz frequency separa-
one triplet and at least one doublet can be determined. Assum-
ing that the frequencies showing the $\approx 9\mu$Hz separation are
$\ell = 1$ modes, we estimated the stellar rotation period as 0.6d.
We determined a 31 s mean period spacing using different meth-
ods, as linear fits to the observed periods, Kolmogorov-Smirnov
and Inverse Variance tests, and Fourier analysis of different pe-
riod lists. We also simulated the effect of alias ambiguities on the
mean period spacing determination. All of the results were con-
sistent and point to the 31 s characteristic spacing for the dipole
modes.
The extended list of frequencies provided the opportunity for
asteroseismic investigations of KUV 05134+2605 for the first
time. In the case of period-to-period fits, the effective tempera-
ture of the selected model is $T_{\text{eff}} = 25,050 K$, which is the same
as the spectroscopic one ($T_{\text{eff}} = 24,700 \pm 1300 K$, Bergeron et al.
2011) within the uncertainties. This model’s helium layer mass
is $1.2 \times 10^{-3} M_\odot$.
We found an excellent agreement between the stellar mass
derived from the period spacing data and the period-to-period fits, all providing $M_*=0.84 - 0.85 M_\odot$ solutions. Note that the mass inferred from spectroscopy is 17 – 18% lower than these values. One should keep in mind, however, that around the V777
Her instability strip, the helium lines are at maximum strength
and therefore there is an ambiguity about a hot and a cool solu-
tion, just like the ambiguity of the high and low mass from the
asteroseismological side. Also, there are still uncertainties in the
synthetic line profile calculations. Similarly, we make assump-
tions on the physics of stars in the asteroseismic modelling. The
discussion of which method is better to estimate the stellar mass
and why, is out of the scope of this paper.
Therefore, we consider the agreement between the results of
three methods to be satisfactory. All three methods result a mass for KUV 05134+2605 higher than the average mass of DB white
dwarfs. Moreover, our study suggests that this star is the most
massive one amongst the known DBVs.

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Fig. 11. The internal chemical structure (upper panel), and the squared
Brunt-Väisälä and Lamb frequencies for $\ell = 1$ (lower panel) corre-
sponding to our best-fit DB white dwarf model with a stellar mass
$M_* = 0.84 M_\odot$, an effective temperature $T_{\text{eff}} = 25,050 K$, and a He en-
velope mass of $M_{\text{He}}/M_* \approx 1.5 \times 10^{-1}$.

400 s, and below 1.6 s for periods around 800 s. That is, the
presence of small abundances of hydrogen in the almost pure helium
atmospheres of DB white dwarfs does not significantly alter the
pulsation periods. As a result, pure helium envelope models like
the ones we employed in this study are suitable for asteroseismic
modelling of DBA white dwarfs like KUV 05134+2605.

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