Two-color surface lattice solitons

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We consider propagation of light at the interface of semi-infinite waveguide arrays. The existence of two-color surface lattice solitons is studied. By using a discrete model, the existence of two-color twisted surface solitons with different properties is revealed.

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Surface solitons are usually studied for cubic or saturable nonlinear media. However, Siviloglou et al. [6] reported on the first observation of discrete quadratic surface solitons in periodically poled lithium niobate waveguide arrays. By operating on either side of the phase-matching condition and using the cascading nonlinearity, they observed both in-phase and staggered discrete surface solitons. They also employed a discrete model with decoupled waveguides at the second harmonics to model some of the effects observed experimentally.

The purpose of this Letter is two-fold. First, we extend substantially the earlier theoretical analysis performed by Siviloglou et al. [6] and study, for the first time to our knowledge, two-color quadratic surface solitons in a continuum model with a truncated periodic potential. We analyze the effect of the mismatch on the existence, stability, and generation of surface solitons located at the edge of a semi-infinite waveguide array in nonlinear quadratic media. Second, we reveal the existence of novel classes of surface solitons which are stable in a large domain of their existence.

We study the properties of surface solitons generated at the edge of a semi-infinite photonic lattice in nonlinear quadratic media, namely two-color surface lattice solitons. We analyze the impact of phase mismatch on existence and stability of surface modes, and find novel classes of two-color twisted surface solitons which are stable in a large domain of their existence.

Fig. 1. (Color online) Profiles of two-color odd surface solitons with (a) b1 = 1.5 and (b) b1 = 2. Profiles of two-color twisted surface solitons with (c) b1 = 1.75 and (d) b1 = 2.2. Black and red curves show the profiles of FF and SH fields, respectively. Lattice depth p = 1, and phase mismatch β = 0. In the white regions R(x) < 1, while in the gray regions R(x) ≥ 1.

semi-infinite lattice imprinted in quadratic nonlinear media, which involves the interaction between fundamental frequency (FF) and second-harmonic (SH) waves. Light propagation is described by the following coupled nonlinear equations [12,13]

\[ i \frac{\partial q_1}{\partial z} = \frac{d_1}{2} \frac{\partial^2 q_1}{\partial x^2} - q_1^* q_2 \exp(-i \beta z) - p R(x) q_1 \]

\[ \frac{\partial q_2}{\partial z} = \frac{d_2}{2} \frac{\partial^2 q_2}{\partial x^2} - q_2^* \exp(i \beta z) - 2p R(x) q_2. \] (1)

where \( q_1 \) and \( q_2 \) represent the normalized complex amplitudes of the FF and SH fields, \( x \) and \( z \) stand for the normalized transverse and longitudinal coordinates, respectively, \( \beta \) is the phase mismatch, and \( d_1 = -1, d_2 = -0.5 \); \( p \) is the lattice depth; the function \( R(x) = 0 \) at \( x < 0 \) and \( R(x) = 1 - \cos(Kx) \) at \( x ≥ 0 \) describes the profile...
face solitons can be found in the form $q_\delta V_K$ including the power propagation constant $\beta$ of a truncated periodic lattice with modulation $K$ where both FF and SH fields reach their maxima. Due to the lattice truncation, the solitons have asymmetric profiles at lower power [Fig. 1(a)]. There exists a lower cutoff ($b_{c0}$) of the propagation constant for the existence of odd solitons. The power of odd solitons is a nonmonotonic function of the propagation constant, and there is a narrow region close to the cutoff where the power dependence changes its slope, $dP/db_1 < 0$ [Fig. 2(a)]. We find that the cutoff $b_{c0}$ is a monotonically increasing function of lattice depth $p$, and it is a decreasing function of the phase mismatch [Fig. 2(b)]. It should be noted that the critical power for the existence of two-color surface lattice solitons depends on the phase mismatch, it reaches the minimum value for the exact phase matching ($\beta = 0$). Linear stability analysis reveals that these odd surface solitons are stable almost in the whole domain of their existence except a very narrow region near the cutoff $b_{c0}$ where the exponential instability develops. Direct numerical simulations of the model confirm the results of the linear stability analysis.

In addition to the simplest solitons described above, we find various families of higher-order surface lattice solitons, which can be viewed as the combination of several in-phase and out-of-phase odd solitons. Here we only focus on the case where the FF field features the out of phase combinations because the modes with in-phase combinations are unstable. Being reminiscent to its discrete counterparts such as twisted localized modes, here we term the mode with out-of-phase combination as twisted surface lattice soliton. Typical profiles of twisted modes residing at the edge of a semi-infinite lattice are shown in Figs. 1(c,d). Similar to the properties of odd solitons, the power of twisted surface solitons is a nonmonotonic function of the propagation constant, and there exists a narrow region close to the cutoff where $dP/db_1 < 0$ [Fig. 2(c)]. However, one should note that the cutoff $b_{c0}$ is a nonmonotonic function of the lattice depth $p$, as shown in Fig. 2(d). Likewise, the critical power for the existence of twisted solitons reaches the minimum value at $\beta = 0$.

Importantly, we find that the twisted surface lattice solitons become completely stable when the power exceeds a certain threshold value [Fig. 3(a)], while the instability domain expands with the growth of the absolute value of phase mismatch $\beta$. Figure 3(b) shows the real
part of the perturbation growth rate versus the propagation constant, where the left part corresponds to the exponential instability, while in the right part solitons suffer from oscillatory instabilities. Results from linear stability analysis are confirmed by the direct numerical simulation of the model \( \Box \).

We also study the properties of two-color surface modes which are shifted from the lattice edge, similar to the analysis performed earlier for the discrete cubic model [3]. Figures 4(a,b) show some illustrative examples of odd and twisted states residing in the second channel of the semi-infinite waveguides from the interface. Such modes require much lower critical power comparing with surface modes residing at the interface [Fig. 4(c,d)]. Linear stability analysis reveals that two-color surface solitons in the second channel are stable when their power exceeds a certain threshold.

To address the issue of excitation of two-color surface solitons, we perform a comprehensive study of the soliton generation by two Gaussian beams \( q_1, q_2 = a_1, a_2 \exp(-x^2) \). As shown in Figs. 5(a-d), for lower powers the beams just experience repulsion from the surface and diffraction [(a)]. For high enough powers, surface solitons can be excited either from only FF field [(b)] or both FF and SH fields [(c)] with a proper phase matching. Two-color surface solitons can be formed for other conditions (\( \beta = -3 \)), but for higher input powers [(d)].

In conclusion, we have analyzed the existence, stability, and generation of two-color quadratic surface solitons in a continuum model with a truncated periodic potential, and also revealed the existence of novel classes of stable parametrically coupled surface states.

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