We investigate the emergence of single spin asymmetries (SSA) in hard processes using transverse momentum dependent (TMD) distribution and fragmentation functions. Specifically, the description of SSA involves time reversal-odd functions. Process-dependence (non-universality) in measurements of SSA can be attributed to the non-trivial gauge link structure in the TMD correlator. Finding the appropriate gauge links, however, also enables us to characterize the non-universality [1, 2].

1 Introduction

In recent years many theoretical and experimental studies aimed for an understanding of the mechanisms that lead to single spin asymmetries (SSA) in hard hadronic scattering processes. In collinear approximation (integrating over all transverse momenta) all leading twist distribution (and fragmentation functions) only depend on the longitudinal momentum fraction $x$ (or $z$) and involve double spin asymmetries, i.e. polarized quarks are only found in polarized hadrons (and vice versa). Single spin asymmetries (SSA) involve twist-three collinear quark-gluon matrix elements. In the specific limit of a zero-momentum gluon, referred to as gluonic pole matrix elements such as the Qiu-Sterman matrix elements [3], the effects can appear at leading order. Also in model calculations the effects of these soft gluon interactions between the target remnant and the hard part parton have been demonstrated, giving rise to specific effects for initial or final state interactions [4].

Going beyond the collinear approximation and including the effects of intrinsic transverse momenta of partons provides another mechanism to generate leading order SSA, which can be traced back to correlations between the intrinsic transverse motion and spin of partons and/or hadron. The effects are described by transverse momentum dependent (TMD) distribution functions, containing both T-even and T-odd parts and depending on longitudinal momentum fraction $x$ and the transverse momentum $p_T$ as appearing in the Sudakov decomposition $p = x P + p_T$ (or $p = (1/z) P + p_T$ for fragmentation). The TMD correlators include Wilson lines, which besides ensuring gauge-invariance are in the case of distribution functions the sole cause of T-odd contributions. Upon $p_T$-integration one finds after weighing with $p_T$ the so-called transverse moments of the TMD distribution functions, which can be separated into T-even and T-odd parts that are universal and of which the T-odd part can be identified with the gluonic pole matrix elements.

2 Transverse momentum dependent (TMD) correlators

The TMD distribution functions are projections of the TMD quark correlator defined on the light-front (LF: $\xi \cdot n = 0$)

$$\Phi_{ij}^{[C]}(x,p_T;n) = \int \frac{d(\xi\cdot P)d^2\xi_{T}}{(2\pi)^3} e^{ip_{\xi} \cdot P} \langle P,S| \bar{\psi}_j(0) U_{[0,\xi]}^{(C)} \psi_i(\xi) |P,S\rangle \bigg|_{\text{LF}}.$$  (1)
The Wilson line or gauge link $U^{[C]}_{[\eta;\xi]} = \mathcal{P}\exp\left[-ig \int_{\eta}^{\xi} ds A^a(s) t^a\right]$ is a path-ordered exponential along the integration path $C$ with endpoints at $\eta$ and $\xi$, ensuring gauge-invariance. In the TMD correlator \(^{[1]}\) the integration path $C$ in the gauge link is process-dependent.

In the diagrammatic approach the Wilson lines arise by resumming all collinear gluons exchanged between the soft and the hard partonic parts of the hadronic process. The integration path $C$ is fixed by the (color-flow structure of) the hard partonic process. The TMD correlator \(^{[1]}\) is fixed by the resummation of all final-state interactions leads to the future pointing Wilson line $U_t^{[-]}$, and Drell-Yan scattering where the initial-state interactions lead to the past pointing Wilson line $U_l^{[-]}$. These links connect the parton fields in the correlator, running along the light-like direction $n$, conjugate to $P$ (satisfying $P \cdot n = 1$ and $n^2 = 0$) and closing in the transverse direction at lightcone infinity \(^{[1]}\). For gluons the correlators including links are given by

\[
\Gamma^{[C,C']}_{\alpha\beta}(x, p_T; n) = \int \frac{d(\xi \cdot P) d^2 \xi}{(2\pi)^3} e^{i P \cdot \xi} \, Tr \left( F_{\alpha}^{\eta}(0) \, U^{[n,C]}_{[\eta;\xi]} \, F_{\beta}^{\eta}(\xi) U^{[n,C']}_{[\xi;\eta]} \right) |P|_{LF}, \tag{2}
\]

with the simplest possibilities also shown in Fig. \(^{[1]}\).

### 3 Observables

Considering intrinsic transverse momenta is useful as it is possible to access them in experiments. The collinear fractions ($x$ or $z$) in the Sudakov expansion of the parton momenta can be related to kinematical ratios of hard momenta (e.g., $x \approx x_B = Q^2/2P \cdot q$ and $z \approx z_h = P_h \cdot P / P \cdot q$ in semi-inclusive deep inelastic scattering) up to $O(1/Q^2)$ corrections. Therefore the quantity $q_T = q + x_B P - P_h / z_h \approx k_T - p_T$ can be measured in semi-inclusive deep inelastic scattering (SIDIS), $\gamma^* (q) + N(P) \rightarrow h(P_h) + X$. It is zero at leading order ($O(Q)$ in the hard scale), but relates to the intrinsic transverse momenta at $O(M)$. The vector $q_T$ is the transverse momentum of $q$ in a frame in which $P$ and $P_h$ are chosen parallel or (experimentally more useful) related to the transverse momentum of $P_h$, $q_T = -P_{h \perp} / z_h$ in a frame in which $q$ and $P$ are chosen parallel. With $Q_T^2 = -q_T^2$, one needs TMD functions when $Q_T \sim O(M)$ and one needs a collinear description involving a subprocess with one more parton radiated off when $Q_T \sim O(Q)$. Matching of these approaches was considered in Ref. \(^{[7]}\). Not only in electroweak processes like SIDIS or the Drell-Yan process transverse momenta can be accessed, but one can also consider inclusive hadron-hadron scattering. The experimental signature in this case is the non-collinearity of the produced particles/jets in the plane perpendicular to the colliding beam particles, outlined in detail in Ref. \(^{[8]}\).

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Accessing intrinsic transverse momenta in most cases requires a study of azimuthal dependence in high energy processes. Although the effects are in principle not suppressed by powers of the hard scale in comparison with the leading collinear treatment, it requires measuring hadronic scale quantities (transverse momenta) in a high momentum environment. In applications to explain SSA time reversal (T) invariance plays an important role:

1. The theory of QCD is T-invariant. This allows to distinguish quantities and observables according to their T-behavior. Collinear correlators $\Phi(x)$ and $\Gamma(x)$, obtained after integration over transverse momenta, are T-even. For the TMD correlators, however, the T-operation interchanges $\Phi^+(x,p_T) \leftrightarrow \Phi^-(x,p_T)$ (and similar relations for gluon TMD correlators), allowing T-even and T-odd combinations.

2. For fragmentation functions the appearance of a hadronic out-state in the definition, prohibits the use of T-symmetry as a constraint and one has always both T-even and T-odd parts in the correlator, although one can separate the correlators into two classes containing T-even or T-odd operator combinations in analogy with the case of distributions, referred to as naive T-even or T-odd.

3. In a scattering process, in which T-symmetry can be used as a constraint, SSA would be forbidden. In fact the only real example of this is DIS (omitting electromagnetic interaction effects). For hadron-hadron scattering, e.g. the Drell-Yan process, one has a two-hadron initial state and only the assumption of a factorized description would imply absence of SSA. We now know that this assumption is not valid, even not at leading order! Similarly for processes with identified hadrons in the final state T-invariance does not give constraints.

4. At leading order in $\alpha_s$, however, it is possible to connect SSA (being T-odd observables) to the T-odd soft parts, since the hard process will be T-even at this leading order. Collins and Sivers effects as explanation for SSA are the best known examples.

4 TMD treatment

As already referred to in section 2 the gauge links in the correlators are the result of resumming leading matrix elements with collinear gluons. The presence of links, differing for each partonic sub-diagram and its color-flow, results in the following expression for a hard cross section at measured $q_T$ (involving in general complex diagram-dependent gauge-link paths),

$$\frac{d\sigma}{d^2q_T} \sim \sum_{D,abc...} \Phi_a^{[C_1(D)]}(x_1,p_{1T}) \Phi_b^{[C_2(D)]}(x_2,p_{2T}) \hat{\sigma}_{ab\rightarrow c...} \Delta_c^{[C_1(D)]}(z_1,k_{1T}) + ...$$

where the sum $D$ runs over diagrams distinguishing also the color flow and $abc...$ is the summation over quark and antiquark flavors and gluons. Dirac and Lorentz indices, traces are suppressed. The ellipsis at the end indicate contributions of other hard processes.

The results for cross sections after integration over the transverse momenta $q_T$ involve the path-independent integrated correlators $\Phi(x)$ rather than the path-dependent TMD correlators $\Phi^{[C(D)]}(x,p_T)$. Thus, from Eq. 3 one gets the well-known result

$$\sigma \sim \sum_{abc...} \Phi_a(x_1) \Phi_b(x_2) \hat{\sigma}_{ab\rightarrow c...} \Delta_c(z_1) + ...$$

where $\hat{\sigma}_{ab\rightarrow c...} = \sum_D \hat{\sigma}_{ab\rightarrow c...}^{[D]}$ is the partonic cross section.

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Constructing a weighted cross section (azimuthal asymmetry) by including a weight $q_T^\alpha$ in the $q_T$-integration leads to the *transverse moments*:

$$\Phi_\alpha^{[C]}(x) = \int d^2 p_T \, p_T^\alpha \Phi^{[C]}(x, p_T) = \tilde{\Phi}_\alpha(x) + C_G^{[U]} \pi \Phi_G(x, x).$$

These moments still contain a path dependence, so Eq. 3 cannot be simplified immediately but as shown the path dependence is contained in a (gluonic pole) factor $C_G$, which can easily be calculated. The first term, $\tilde{\Phi}_\alpha(x)$, is a collinear correlator containing matrix elements with T-even operators, while $\Phi_G(x, x - x_1)$ is a collinear correlator with a structure of a quark-gluon-quark correlator involving the gluon field $F_{\mu\alpha}$. In Eq. 5 one needs the zero-momentum ($x_1 = 0$) limit for the gluon momentum. This matrix element is known as the gluonic pole matrix element. The operators involved are T-odd. Both collinear correlators on the RHS in Eq. 5 are link-independent. Using this decomposition one can write down a parton-model like expansion for the single-weighted cross section $\langle q^\alpha T \sigma \rangle$ in which $\tilde{\Phi}_\alpha(x)$ is multiplied with the partonic cross section, while $\pi \Phi_G(x, x)$ is multiplied with the gluonic pole cross section, $\tilde{\sigma}_{[ab\ldots]} = \sum_D C_G^{[U]} C_D^{[U]} \pi \Phi_G(x, x)$, which just like the normal partonic cross sections also constitutes a different gauge-invariant combination of the squared amplitudes [9]. For more complex weightings or trying to stay at the unintegrated level, one has to make additional assumptions outlined in Ref. [2]. In this paper also the split-up of TMD functions in

$$\Phi^{[U]}(x, p_T) = \frac{1}{2} \left( \Phi^{[even]}(x, p_T) + G_G^{[U]} \Phi^{[odd]}(x, p_T) \right) + \delta \Phi^{[U]}(x, p_T),$$

with $\Phi^{[even/odd]} = \frac{1}{2}(\Phi^+ \pm \Phi^-)$ is discussed, with $\delta \Phi^{[U]}(x, p_T)$ satisfying $\delta \Phi^{[U]}(x, p_T) = \delta \tilde{\Phi}_\alpha^{[U]}(x) = 0$.

The approach to understand T-odd observables like single spin asymmetries via the TMD correlators and the non-trivial gauge link structure unifies a number of approaches to understand such observables, in particular the collinear approach and the inclusion of soft gluon interactions. Although the treatment of fragmentation correlators also separates into parts with T-even and T-odd operator structure, gluonic pole contributions (T-odd parts) in the case of fragmentation might vanish. Indications come from the soft-gluon approach [10] and a recent spectral analysis in a spectator model approach [11].

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