\( \Omega_m \) – Different Ways to Determine the Matter Density of the Universe

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Abstract.
A summary of various measurements of the mean matter density in the universe, \( \Omega_m \), is presented. Results from very different kinds of methods using various astronomical objects – from supernovae to large-scale structure – are shown. There is a remarkable preference for \( \Omega_m \) values around 0.3, but there are also some measurements that favour a higher or a smaller value.

1. Introduction

\( \Omega_m \) – the mean matter density in the universe – is one of the key parameters for cosmological models. It is usually expressed as a fraction of the critical density

\[
\Omega_m = \frac{\rho_m}{\rho_{\text{crit}}}
\]

with \( \rho_{\text{crit}} = 3H_0^2/(8\pi G) = 1.88h^2 \times 10^{-29} \text{g/cm}^3 \). Several years ago the philosophically appealing value of \( \Omega_m = 1 \) was favoured by most scientists. But this value led more and more to contradictions with various other measurements, which require a lower value.

Recently, many different methods using different kinds of astronomical objects have been developed to determine \( \Omega_m \). In this article measurements of the matter density with very different methods and their results are summarised. It is impossible to present a complete compilation of all results on this topic due to the limited space. Therefore only a selection of methods and results are presented, whereby I concentrate on recent determinations of \( \Omega_m \).

This article is organised as follows. In Sect. 2 the combined results from distant supernovae and measurements of the cosmic microwave background radiation are given. Sect. 3 lists \( \Omega_m \) values determined with the gravitational lensing effect. In Sects. 4 and 5 the evolution and the baryon fraction of galaxy clusters, respectively, are used to determine the matter density. The results from mass-to-light ratios are listed in Sect. 6. \( \Omega_m \) determinations from cosmic flows, and from correlation functions and power spectra are given in Sect. 7 and 8, respectively.
Sect. 9 summarises all the results obtained by the different methods. Throughout this article a Hubble constant of $H_0 = 65 \text{ km/s/Mpc}$ is used.

2. Supernovae and Cosmic Microwave Background

Currently, the most discussed results for $\Omega_m$ are derived from a combination of supernova and Cosmic Microwave Background measurements.

Recent measurements of the Cosmic Microwave Background Radiation (CMBR) determined the small-angle anisotropies of this radiation over a significant part of the sky. The angular power spectrum of these measurements yields values for the total density $\Omega_{tot}$ around unity. Two balloon experiments find the following results: Boomerang (de Bernardis et al. 2000)

$$0.88 < \Omega_{tot} < 1.12$$

and MAXIMA (Hanany et al. 2000, Balbi et al. 2000)

$$\Omega_{tot} = 1.0^{+0.15}_{-0.30},$$

Distant Type Ia supernovae can be used as standard candles and hence they can be used to determine cosmological parameters. With the assumption of the total density $\Omega_{tot} = \Omega_m + \Omega_\Lambda = 1$ – as suggested by the CMBR measurements – quite stringent constraints can be set on the matter density. Two independent groups measured supernovae for this purpose and found very similar results:

$$\Omega_m = 0.28^{+0.09}_{-0.08}$$ Perlmutter et al. (1999)

and

$$\Omega_m = 0.32 \pm 0.1$$ Riess et al. (1998).

The main concerns about the interpretation of these data are the evolution of supernovae Ia and dimming by dust.

3. Gravitational lensing

There are several ways to determine $\Omega_m$ by the gravitational lensing effect. A very interesting method is the weak gravitational lensing by of large-scale structure – the cosmic shear. Four independent groups discovered the effect recently (Bacon et al. 2000; Kaiser et al. 2000;
van Waerbeke et al. 2000; Wittman et al. 2000). The first values for $\Omega_m$ were given by van Waerbeke et al. (2001)

$$0.2 < \Omega_m < 0.5$$

for an open universe

and

$$\Omega_m < 0.4$$

for a flat universe.

Wilson et al. (2001) determined mass-to-light ratios from the gravitational shear of many faint galaxies implying a very low value for

$$\Omega_m = 0.10 \pm 0.02.$$

Another method is arc statistics, i.e. the number of giant gravitational arcs caused by lensing of foreground objects. X-ray selected clusters are ideal objects for this purpose. Bartelmann et al. (1998) and Kaufmann & Straumann (2000) applied this method to the cluster sample of the EINSTEIN Medium Sensitivity Survey (EMSS) (Gioia & Luppino 1994; Luppino et al. 1999). Kaufmann & Straumann (2000) derive with semi-analytic methods a mean matter density between

$$0.2 < \Omega_m < 0.5.$$

Bartelmann et al. (1998) find from numerical simulations also a low value for $\Omega_m$. In principle also the arcs found in radio surveys can determine $\Omega_m$ (Helbig 2000). The CLASS/JVAS surveys already give some results, but up to now only relatively weak constraints can be set on $\Omega_m$.

A new method was suggested by Golse et al. (2001). They show that strong lensing in galaxy clusters with several image systems can constrain cosmological parameters without any further assumptions. When high resolution images and the redshifts of the gravitational arcs are available a single galaxy cluster with 3 multiple image systems can determine $\Omega_m$ with an uncertainty of about $\pm 0.3$.

4. Cluster evolution

For $\Omega_m = 1$ a strong evolution is predicted in the number space density of rich clusters, because in this cosmological model the growth of structure continues to the present day. In a low-$\Omega$ universe, on the other hand, relatively little change in the cluster number is expected since a redshift of 1. Therefore much work has been done to test whether there is evolution or not in the cluster number density.
Table I. Various projects evaluating the cluster X-ray luminosity function which find a deficit of high-redshift clusters. The acronym of the project (Column 2) and the significance of deficit of distant luminous clusters (Column 3) are listed. The study by Gioia & Luppino (1994) is based on the EINSTEIN Medium Sensitivity Survey. All the others are based on ROSAT data.

| Study                        | Acronym | Significance |
|------------------------------|---------|--------------|
| Gioia & Luppino (1994)       |         | 3σ           |
| Nichol et al. (1999)         | SHARC   | 1.7σ         |
| Vikhlinin et al. (2000)      |         | 3.5σ         |
| Rosati et al. (2000)         | RDCS    | 3σ           |
| Gioia et al. (2001)          | NEP     | 5σ           |

4.1. SINGLE CLUSTERS

The existence of a single distant, massive cluster – MS1054-03 at a redshift of $z=0.83$ with a mass $M \approx 10^{15} M_\odot$ – is by itself a strong indication for a low $\Omega_m$ universe (Donahue et al. 1998). This cluster and two more clusters were used in an analysis by Bahcall & Fan (1998), in which they also found a low value

$$\Omega_m = 0.2^{+0.3}_{-0.1}.$$  

4.2. X-RAY LUMINOSITY FUNCTION

To put this type of analysis on a broader statistical basis the evolution of the cluster mass function, i.e. the evolution of the number of clusters of different masses, would be the ideal quantity to measure. But as it is not easy to determine the mass for a large number of clusters, the cluster luminosity function and the cluster temperature function have generally been studied instead. This is possible because both quantities – the X-ray luminosity and the temperature – correlate quite well with the cluster mass.

The luminosity function of X-ray selected clusters has been measured by several groups. In many measurements a deficit of distant luminous clusters (see Table 1) was found. These results point towards a high $\Omega_m$ universe, but the current results have still a large uncertainty and therefore they cannot exclude an $\Omega_m = 0.3$. There is one analysis that yields a different result although it is based on the same data from ROSAT: Jones et al. (2000) found no deficit of distant clusters. Hence
they concluded that there is “no evolution” in the cluster luminosity function, which is an indication for a low $\Omega_m$ universe.

4.3. Temperature Function

Several groups have investigated the temperature function of galaxy clusters and found discordant results. Evidence for “no evolution” was found by Eke et al. (1998). They determined a matter density of

$$\Omega_m = 0.45 \pm 0.25.$$

Henry (2000) also did not find any evolution and concluded

$$\Omega_m = 0.49 \pm 0.12 \quad \text{for an open universe}$$

$$\Omega_m = 0.44 \pm 0.12 \quad \text{for a flat universe}.$$  

Two other groups found evidence that there is evolution in the temperature function. Viana & Liddle (1999) derived

$$\Omega_m = 0.75 \pm 0.3$$

and Blanchard et al. (2000) found

$$\Omega_m = 0.92^{+0.26}_{-0.22} \quad \text{for an open universe}$$

$$\Omega_m = 0.87^{+0.35}_{-0.25} \quad \text{for a flat universe}$$

Maybe the sample selection must be done more carefully in order to find agreement. It also might be that the temperature function is only a weak test in the redshift range used here as it was suggested by Colafrancesco et al. (1997).

4.4. Mass Function

The evolution of the mass function has been measured directly by Carlberg et al. (1997a) with the CNOC (Canadian Network for Observational Cosmology) sample. The clusters in this sample were selected from the EMSS. The masses were obtained from optical measurements of the galaxy velocity dispersion. Carlberg et al. (1997a) find a low value for $\Omega_m$:

$$\Omega_m = 0.2 \pm 0.1.$$
4.5. X-ray luminosity – Temperature Relation

The evolution of the X-ray luminosity – temperature relation is another test for the mean matter density, because it evolves differently in different cosmological models. Several authors concluded that there is no significant detectable evolution in the relation: Mushotzky & Scharf (1997) for a sample out to redshift $z \lesssim 0.4$, Donahue et al. (1998) and Della Ceca et al. (2000) out to $z \lesssim 0.8$, Schindler (1999) out to $z \lesssim 1.0$, Fabian et al. (2001) out to $z \lesssim 1.8$. From a detailed comparison of the ROSAT Deep Cluster Survey (Rosati et al. 1995) and the EMSS Sample Borgani et al. (1999) derived

$$\Omega_m = 0.4^{+0.3}_{-0.2}$$

for an open universe

and

$$\Omega_m \lesssim 0.6$$

for a flat universe.

5. Cluster baryon fraction

With the assumption that the matter is accumulated indiscriminately in the potential wells of clusters the baryon fraction in galaxy clusters is a measure for the baryon fraction of the universe as a whole. The advantage of measuring the baryon fraction in clusters is that both the baryon mass and the total cluster mass can be determined reliably (Schindler 1996). For the analysis only the gas density and the gas temperature are required which can both be inferred from X-ray observations. Several groups determined gas mass fractions from X-ray observations in samples of nearby and distant clusters, e.g.

- Mohr et al. (1999): $f_{gas} = 0.14$
- Ettori & Fabian (1999): $f_{gas} = 0.11$
- Arnaud & Evrard (1999): $f_{gas} = 0.12$
- Schindler (1999): $f_{gas} = 0.12$

All these determinations depend on the radius where the mass fraction is determined, because the gas mass fraction increases slightly with radius. In the above mentioned analyses the mass was determined within a radius $r_{500}$ from the cluster centre. This radius encompasses a volume that has a density of 500 $\times$ the critical density of the universe $\rho_{crit}$.
Out to this radius the X-ray profile necessary for the analysis could be measured reliably.

To determine $\Omega_m$, the gas mass fraction $f_{\text{gas}}$ must be compared to the baryon density in the universe $\Omega_B \lesssim 0.05$ determined from primordial nucleosynthesis (see e.g. Burles & Tytler 1998a,b). The ratio of the baryon density and the gas mass fraction yields an upper limit for the matter density $\Omega_m$:

$$\Omega_m < \frac{\Omega_B}{f_{\text{gas}}} = 0.3 - 0.4$$

The baryon fraction can also be determined in a different way: measurements of the Sunyaev-Zel’dovich effect – inverse-Compton scattering of the Cosmic Microwave Background photons by the hot intra-cluster gas shifts the CMBR spectrum to slightly higher energies. As this effect is proportional to the gas density, the density profile can be determined directly. Only an additional measurement of the gas temperature is necessary from X-rays. The gas mass fraction found

- Grego et al. (2001): $f_{\text{gas}} = 0.13$

is very similar to the X-ray results. Hence they derive also a similar upper limit for the matter density

$$\Omega_m < 0.4.$$ 

In these analyses only the mass in the intra-cluster gas was taken into account. Baryons in the galaxies were neglected. If they were to be included, the baryon fraction would increase slightly and hence ever more stringent constraints on $\Omega_m$ could be placed.

6. Mass-to-light ratio

The matter density in the universe $\Omega_m$ is defined as the ratio of the mean matter density $\rho_m$ and the critical density $\rho_{\text{crit}}$

$$\Omega_m = \frac{\rho_m}{\rho_{\text{crit}}} = \frac{M}{L} \frac{j}{\rho_{\text{crit}}}.$$ 

$\rho_m$ can also be expressed as the mass - to -(optical)light ratio times the field luminosity density $j$. The assumption here is that mass-to-light ratios in galaxy clusters are representative for the whole universe. This is probably a good assumption because clusters draw mass and galaxy content from regions of about 40 Mpc in size.
Table II. $\Omega_m$ - values derived by several groups. Column (2) lists the catalogues used: MARK III and SFI are catalogues of galaxies, Abell is a catalogue of galaxy clusters.

| Willick & Strauss (1998) | MARK III | $\Omega_m \approx 0.3$ |
|-------------------------|----------|----------------------|
| Susperregi (2001)       | MARK III | $\Omega_m \approx 0.3$ |
| Zaroubi et al. (1997)   | MARK III | $\Omega_m = 0.5 \pm 0.1$ |
| Freudling et al. (1999) | SFI      | $\Omega_m = 0.5 \pm 0.1$ |
| Bridle et al. (2001)    | SFI      | $0.25 < \Omega_m < 0.89$ |
| Sigad et al. (1998)     | MARK III | $\Omega_m \approx 1$ |
| Branchini et al. (2000) | Abell    | $\Omega_m \approx 1$ |

Carlberg et al. (1997b) inferred mass-to-light ratios from the CNOC sample. They could also measure directly with their data the value for the field luminosity density. The resulting matter density is

$$\Omega_m = 0.19 \pm 0.06.$$  

From a comparison of cosmological hydrodynamic simulations by Cen & Ostriker (1999), and observations, Bahcall et al. (2000) also determined mass-to-light ratios and concluded that the matter density is

$$\Omega_m = 0.16 \pm 0.05.$$  

7. Cosmic flows

Measurements of peculiar velocities of galaxies and clusters on large scales can be used to determine the large-scale potential and hence the mass content of the universe. In linear perturbation theory there is a linear relation between the peculiar velocity and the gravity field. The only uncertainty is the proportionality factor $\beta = \frac{\Omega_0}{b}$ – the biasing, which reflects that the visible matter does not exactly trace the total matter. Unfortunately, this factor is up to now not very well defined (see e.g. Strauss 1999).

Many groups determined $\Omega_m$ from these cosmic flows. Some of the results are summarised in Table 2 (see also Fig. 1). Although for the various analyses the same catalogues were used very different results were obtained. The reason for the discrepancies is probably the uncertainty in the biasing parameter.
8. Correlation functions and power spectra

As was shown by Mo et al. (1996) the cluster correlation function can be used to determine $\Omega_m$. Different cluster samples have been used: optically selected clusters (Croft et al. 1997: APM) and X-ray selected samples (Moscardini et al. 2000; Collins et al. 2000; Schuecker et al. 2001: ROSAT). All analyses favour a low $\Omega_m$, but no ranges for $\Omega_m$ are given so far because the constraints are not very stringent yet.

The power spectrum of the Ly$\alpha$ forest was used by Croft et al. (1999). The authors find a matter density of

$$\Omega_m \approx 0.4.$$  

Weinberg et al. (1999) combined galaxy clusters and measurements of the Ly$\alpha$ forest. They adopted a shape parameter of the power spectrum $\Gamma = 0.2$ which is favoured by a number of studies of large-scale galaxy clustering. Their results for the matter density are

$$\Omega_m = 0.46^{+0.12}_{-0.10}$$ for an open universe

and

$$\Omega_m = 0.34^{+0.13}_{-0.09}$$ for a flat universe.

9. Summary on $\Omega_m$

An overview of results for $\Omega_m$ obtained with the different methods is shown in Fig. 1. Unfortunately, it is impossible to plot all the results in one diagram because of the large number of publications on this topic. Therefore only a selection of results is shown. The diagram is simplified in the sense that for some methods assumptions had to be made which cannot be shown in such a simple figure. Some authors do not state error ranges, therefore for some data points errors had to be assumed.

Some publications distinguish between open and flat models. In these cases the flat models are shown as full lines and the open models as dashed lines. There is a systematic shift between the results for these two models. Therefore the region constrained e.g. by galaxy clusters is not a vertical bar in $\Omega_m - \Omega_\Lambda$ diagram but a bar slightly tilted towards the line defining $\Omega_m + \Omega_\Lambda = 1$.

As shown in Fig. 1 most values cluster around $\Omega_m = 0.3$. There is a remarkable agreement between determinations from completely different methods using various astronomical objects from supernovae
Figure 1. Summary of $\Omega_m$ values as derived by the different methods (selection only). The methods are listed on the left hand side, the references on the right hand side. If the authors distinguished between open and flat models, the flat models are shown as full lines and the open models as dashed lines. Most methods are in agreement with an $\Omega_m \approx 0.3$, but there are also some results that favour other values.
to the mass distribution on large scales. But there are also measurements that favour higher or lower values. It is not that a particular method yields systematically lower or higher values, but it seems rather a scatter which depends on certain assumptions made by the authors or simply on large uncertainties in the measurements. Therefore new and future observational facilities, e.g. CHANDRA, XMM and PLANCK, will ensure that the coming years remain exciting for cosmology.

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