Students’ mathematical proficiency in solving calculus problems after Maple implementation

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Abstract. This research aimed to analyze and explore the capability of mathematical proficiency second semester math education students of Faculty of Teacher Training and Education (FKIP) Khairun University academic year 2017/2018. In this study mathematical problems are presented in Calculus, and given the opportunity for students to understand the problems provided both from the theoretical aspects related to indicators of mathematical proficiency namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition, which further uses Interactive Maple Tutor, Maple Assistant, and Show Solution commands. Students in this study can take some steps themselves, and ask Maple to take the next step. Research data were collected and analysed with a descriptive-qualitative approach of student work for each indicator of mathematical proficiency with research subject in total 128 students. The results showed that the ability of the mathematical proficiency of students of mathematics education in the medium category.

1. Introduction

NCTM [1] indicates the importance of using multi representation as one of the standards for learning concepts and mathematical relationships. Furthermore, Godarzi, Q.S [2] states that in addition to multi-representation, visualization also plays an important role in understanding the concepts of multivariate calculus. Therefore, the role of visualization is very significant in the learning process of calculus. This is supported also by the statement of Bishop [3] that the interactive computer, especially when making a visual representation of dynamic, can support improved visualization capabilities of students to a certain extent. Integration of technology into the calculus learning process is highly prioritized because calculus is not only part of mathematics, but calculus is also a core curriculum in the fields of engineering, science, and some social sciences. Utilization of this technology provides space for researchers to find alternatives and innovative teaching-learning approaches, as well as provide opportunities for software developers to help students who have difficulties in learning conventional calculus [4]. If this is related to the improvement of mathematical skills of prospective teachers, it is necessary for the role of educators in preparing the instructional materials, especially Calculus materials that always integrated the Maple application and teaching methods in accordance with the constructivist approach, because Computer Algebra System (CAS), one of which is Maple.
has the advantages of visualization, symbolic conversion, graphical, and algebraic representation will greatly help students build their knowledge actively and constructively. This is also supported by the concept put forward by the pioneers of the "calculus reform movement" known as the Harvard Calculus Consortium defending the need for conceptual understanding of procedural understanding; for this reason, they suggest that the context of learning must be enhanced by multiple representations and must be supported by technology [5, 6]. In addition to mastering the mathematical content and pedagogical skills, graduate mathematics teacher candidates must also be equipped with skills mastery Computer Algebra System (CAS), one of which is a Maple application. Figure 1 presents the results of the work of prospective math teacher for one of the given questions.

\[
\int_{0}^{5} f(x) \, dx \quad \text{using an integral definition, where} \\
\quad f(x) = \begin{cases} 
    x + 2 & \text{jika} \quad 0 \leq x < 2 \\
    6 - x & \text{jika} \quad 2 \leq x \leq 5 
\end{cases}
\]

![Figure 1. Examples of student difficulties when completing a lecture problem](image)

Student work as shown in Figure 1 when discussing lecture questions in Calculus shows that the person concerned is able to draw a lot of functions correctly, but in solving integrals with predetermined boundaries using integral definitions, it seems that the student has not been able to express boundaries graph that has been made so there is an error in resolving the problem. Errors in solving problems concerning Differential and Integral Calculus show that students cannot repeat concepts that have been learned, choose and use the right procedures in solving derivative and integral problems, of course, by using integral definitions, the difficulty of describing graphs of functions with many equations and not yet skilled in using the problem solving algorithm of the problem given. If traced the inability of students as mentioned above, it is very closely related to the development of mathematical proficiency is not a student in the learning process Calculus. Mathematical proficiency
according to Kilpatrick [7] consist of: (1) conceptual understanding; (2) procedural fluency; (3) strategic competence; (4) adaptive reasoning; and (5) productive disposition. These five strands of mathematical proficiency are intertwined and interdependent, how students acquire the mathematical proficiency of how lecturers develop their abilities, because students are educated to achieve that goal. Further, according to Widjajanti [8], mathematical skill is not a "congenital" skill of the students, but a combination of students' knowledge, skills, abilities and beliefs with teacher support, curriculum and learning environment dependable. This is in line with what was stated by Bartell et al. [9] illustrates that mathematical proficiency can improve mathematical knowledge and skills. Groves [10] views that mathematical proficiency is related to the ability of understanding, computing, application, reasoning, and engaging. For that it is necessary to improve the quality of lectures, such as using the Maple application on the Integral Calculus concept in Calculus courses that will help students of mathematics teacher candidate to improve mathematical proficiency. Therefore, to facilitate the improvement of students' mathematical proficiency, the following will be introduced at a glance at the utilization of Maple on the concept of integral calculus:

Table 1. Examples of materials to be taught by using Maple

| Some Concepts | Sample Course Material | Explanation |
|---------------|------------------------|-------------|
| Area of the area under the curve | ![Calculus 1 - Approximate Integration](image) | Discusses the area under the curve based on outside and inside polygons, which are further developed for definite definitions. |
Interpret the Riemann sum as the approximating volume of the flat circular cylinders (or disks).
Technique of Integration

Besides the material as in Table 1, many researchers have used software Maple in math education, for example: (1) Animation of Essential Calculus Concepts in Maple [11]; (2) The Effects of Maple Integrated Strategy on Engineering Technology Students’ Understanding of Integral Calculus [12]; (3) A Mathematics E-book Application by Maple Animations [13]; (4) Interactive Mathematics for Engineers with Maple Usage [14], etc. Further according to Ralitsa Vasileva-Ivanova [14] some mathematical problems could not be solved using Maple Tutor or Maple Assistant. In that case, the students have to use the correct Maple commands. Therefore, they need to know the commands and their syntax. Tonchev, J. [15] describe the Maple command in solving mathematical problems as presented in Table 2.

Table 2. List of Maple commands

| Syntax | Application |
|--------|-------------|
| > factor(a, K); | Computes the factorization of a multivariate polynomial with integer, rational, (complex) numeric, or algebraic number coefficients. |
| a – expression | |
| K - field extension over which to factor | |
| > int(expression, x=a..b, options); | The command to find the integral of a function with variable x at intervals [a, b]. |
| > diff(f, [x1$n$]); | Derivatives of nth order, where n is not specified as a number, can be constructed as in diff(f(x),[x$n$]) and are interpreted as integer order derivatives, that is, computed assuming n is an integer. |
| > solve(equations, variables); | Solves one or more equations or inequalities for their unknowns. |
| > fsolve( equations, variables, complex ); | Numerically computes the zeroes of one or more |
Equations, expressions or procedures.

> pivot(A, i, j,);

The `pivot(a, i, j)` function pivots about the non-zero entry `a[i, j]`. Multiples of the `i`th row are added to every other row in `a`, with the result that all of the entries in the `j`th column of `a` are zero except for the `(i, j)`th element.

> dsolve(ODE);

Solve ordinary differential equation solutions.

> dsolve({ODE, ICs}, y(x), options);

Solving odes or a system of them with given initial conditions (boundary value problems).

> limit(f, x=a);

This function attempts to compute the limiting value of `f` as `x` approaches `a`.

> extrema(expr, {constraints});

The function can be used to find extreme values of a multivariate expression with zero or more constraints.

2. Method

This research uses a descriptive qualitative approach, and the subject of research is the second semester student of Faculty of Teacher Training and Education (FKIP) Universitas Khairun academic year 2017/2018. Further subjects involved in this study are given six target test item descriptions of the courses Calculus. To analyze and track their mathematical proficiency, four students were chosen based on three categories of ability (low, medium and high) to analyze their work. The four students are presented in Table 3.

| No. | Respondent Code | Category |
|-----|-----------------|----------|
| 1.  | R5              | low      |
| 2.  | R10             | medium   |
| 3.  | R17             | medium   |
| 4.  | R21             | high     |

3. Results and Discussion

3.1 Utilization of Maple towards final test of mathematical proficiency

The final test used in this study is expected to trace the desired mathematical proficiency in the study. Problem number (1), calculate the following integral:

\[ \int |x - 2| \, dx \]

This problem can reveal the ability of students' mathematical understanding in relating the concepts and related methods in an appropriate way, including linking the definition of the absolute value of the known integrand making it easier to calculate the integral. For students that are expected to define

\[ |x - 2| = \begin{cases} -(x - 2), & x < 2 \\ (x - 2), & x \geq 2 \end{cases} \]

based on this definition, the student can answer as shown in Figure 2.
\[
\int |x - 2| \, dx = \begin{cases} 
-\frac{x^2}{2} + 2x + C, & x < 2 \\
\frac{x^2}{2} - 2x + C, & x \geq 2
\end{cases}
\]

or

\[
\int |x - 2| \, dx = \begin{cases} 
-(x - 2) \, dx, & x < 2 \\
(x - 2) \, dx, & x \geq 2
\end{cases}
\]

\[
= \begin{cases} 
-\frac{x^2}{2} + 2x + C, & x < 2 \\
0, & x = 2 \\
\frac{x^2}{2} - 2x + C, & x > 2
\end{cases}
\]

Figure 2. Output Maple for problem-1

Problem number (2), complete the following integral,

\[
\int \sin^{-4} x \cos^3 x \, dx
\]

The above question reveals the students' mathematical understanding of the ability to associate the use of trigonometric similarities of the type of trigonometric integrals known. Utilizing trigonometric similarity namely \( \cos^2 x + \sin^2 x = 1 \), then by issuing the exact factor \( \cos x \) so that the calculation can be solved. solve the problem (2), the student will usually use, the nature of the trigonometric rank (\( m \) or \( n \) odd), from this problem \( n \) odd show:

\[
\int \sin^{-4} x \cos^3 x \, dx = \int (\sin^{-4} x) (\cos^2 x) (\cos x) \, dx
\]

\[
= \int (\sin^{-4} x) (1 - \sin^2 x) (\cos x) \, dx = \int (\sin^{-4} x - \sin^{-2} x) (\cos x) \, dx
\]

\[
= \int (\sin^{-4} x - \sin^{-2} x) (\cos x) \, dx = \int (\sin^{-4} x - \sin^{-2} x) \, d(\sin x)
\]

\[
= \left[ \frac{(\sin x)^{-3}}{-3} - \frac{(\sin x)^{-1}}{-1} \right] + C
\]

\[
= \left[ -\frac{1}{3 (\sin x)^3} + \frac{1}{\sin(x)} \right] + C = cosec \ (x) - \frac{1}{3} cosec^3 \ (x) +
\]
Problem number (3), calculate the following integral:

\[
\int 4x \sin^4(2x^2 + 1) \cos^3(2x^2 + 1) \, dx
\]

This problem may reveal procedural fluency, ability, confident using the procedure of integrand known flexible, accurate and efficient in analogy \( u = 2x^2 + 1 \) and determine \( du = 4x \, dx \) precisely and calculate the integrals correctly. In solving this problem, students should be able to use the namely:

\[ u = 2x^2 + 1 \implies du = 4x \, dx \text{, so that} \]

\[
\int 4x \sin^4(2x^2 + 1) \cos^3(2x^2 + 1) \, dx
\]

\[
= \int \sin^4 u \cos^3 u \, du = \int \sin^4 u \cos^2 u \cos u \, du
\]

\[
= \int \sin^4 u (1 - \sin^2 u) \, d(\sin u)
\]

\[
= \int \sin^4 u \, d(\sin u) - \int \sin^6 u \, d(\sin u) = \frac{\sin^5 u}{5} - \frac{\sin^7 u}{7} + C
\]

by replacing \( u \); obtained:

\[
\int 4x \sin^4(2x^2 + 1) \cos^3(2x^2 + 1) \, dx
\]

\[
= \frac{\sin^5 (2x^2+1)}{5} - \frac{\sin^7 (2x^2+1)}{7} + C.
\]
Problem number (4), calculate the following integral

\[ \int \frac{x^2}{\sqrt{16 - x^2}} \, dx \]

This problem can reveal the ability of procedural fluency, confident of using procedures flexibly from known, accurate and efficient integrity of choice: \( x = 4 \sin \theta \), karena \( x = 4 \sin \theta \Rightarrow dx = 4 \cos \theta \, d\theta \); so the student will finish it as follows:

\[ \int \frac{x^2}{\sqrt{16 - x^2}} \, dx = \int \frac{(4 \sin \theta)^2}{\sqrt{16 - (4 \sin \theta)^2}} \cdot 4 \cos \theta \, d\theta = \int \frac{(4 \sin \theta)^2}{\sqrt{16(1 - \sin^2 \theta)}} \cdot 4 \cos \theta \, d\theta = \int \frac{(4 \sin \theta)^2}{\sqrt{16 \cos^2 \theta}} \cdot 4 \cos \theta \, d\theta = 16 \int \frac{1 - \cos 2\theta}{2} \, d\theta = 8 \int \theta - 8 \cos 2\theta \, d\theta = 8\theta - 4 \sin 2\theta + C \]

\[ x = 4 \sin \theta \text{, maka } \theta = \sin^{-1} \left( \frac{x}{4} \right); \sin 2\theta = 2 \sin \theta \cos \theta = 2 \left( \frac{x}{4} \right) \frac{\sqrt{16 - x^2}}{4} = \frac{x \sqrt{16 - x^2}}{8} \]

\[ \int \frac{x^2}{\sqrt{16 - x^2}} \, dx = 8 \theta - 4 \sin 2\theta + C = 8 \sin^{-1} \left( \frac{x}{4} \right) - 4 \left( \frac{x \sqrt{16 - x^2}}{8} \right) + C = 8 \sin^{-1} \left( \frac{x}{4} \right) - \frac{x \sqrt{16 - x^2}}{2} + C \]
Figure 5. Output Maple for problem 4

Problem number (5), sketch the following function \( f(x) = 3x^2 + 2 \); \([-2, 1]\], and calculate \( \int_{-2}^{1} (3x^2 + 2) \, dx \) by definition!

This problem can reveal the ability of procedural smoothness, which is able to present graphically, select the limits of the specified intervals, and use the appropriate method in calculating the area of the curve in question.
Problem number (6), consider the area of $R$ (as shown below), calculate the volume of the object formed when the $R$ region is rotated as far as $360^0$ around the $x$ axis!

**Figure 6.** Output Maple for problem-5
3.2 Analysis of student outcomes on mathematical proficiency ability

Here are some examples of student answers that have not been perfect in the aspect of students' mathematical skills. Mistakes that often arise in answering questions and other types of mistake from the results of their answers.

Problem number (1), calculate the following integral:

$$\int |x - 2| dx$$

$$\int |x-2| \, dx$$

$$|x - 2| = \begin{cases} (x - 2) & \text{if } x - 2 \geq 0 \text{ atau } (x - 2) < 0 \\ -(x - 2) & \text{if } x - 2 < 2 \text{ atau } x < 2 \end{cases}$$

Untuk

$$|x - 2| = \begin{cases} x - 2 & \text{if } x - 2 \geq 0 \text{ atau } (x - 2) < 0 \\ -x + 2 & \text{if } x - 2 < 2 \text{ atau } x < 2 \end{cases}$$

$$\int |x-2| \, dx = \int x - 2 \, dx + \int -x + 2 \, dx = \frac{1}{2} x^2 - 2x + (\frac{1}{2} x^2 + 2x) + C = 0$$

Figure 7. Output Maple for problem-6

Figure 8. Example of student work (R5) for question number-1
The result of student work (R$_5$) in Figure 8 is one example of an incomplete and inappropriate answer. It appears that the results of work on the second line indicate that the student has utilized the absolute value definition $|x - 2| = \begin{cases} -(x - 2), & x < 2 \\ (x - 2), & x \geq 2 \end{cases}$ appropriately, it shows that students have successfully linked concepts and methods appropriately. But in the next step that is on the seventh row $\int |x - 2| \, dx = \int x - 2 \, dx + \int -x + 2 \, dx = \cdots$, students misconstrue the concepts of resolving the integral of the absolute value in question and the improper formation of mathematical symbols, so this becomes illogical in the final settlement. It seems that students have difficulty in attributing the definition of the absolute value of the known integrand so that there is an error in calculating the integral.

Problem number (2), complete the following integral, $\int \sin^{-4} x \cos^3 x \, dx$

$$\int \sin^{-4} x \cos^3 x \, dx = \int (\sin^2 x)^{-2} \cos^3 x \, dx = \int \left(\frac{1 - \cos^2 x}{2}\right)^{-2} \cos^2 x \cos x \, dx$$

$$= \int \left(\frac{1 - \cos^2 x}{2}\right)^{-2} (1 - \sin^2 x) \cos x \, dx$$

$$= \int \left(\frac{1 - \cos^2 x}{2}\right)^{-2} (1 - \sin^2 x) \, d (\sin x)$$

$$= \int \left(\frac{1}{1 - \cos^2 x}\right)^{-2} (1 - \sin^2 x) \, d (\sin x)$$

$$= \frac{1}{3} \int \frac{1 - \sin^2 x}{\frac{1}{4}(1 - \cos^2 x)^2} \, d (\sin x)$$

$$= \frac{4}{3} \int \frac{1 - \sin^2 x}{(1 - 2 \cos^2 x + \cos^4 x)} \, d (\sin x)$$

$$= \frac{4}{3} \ln \left|\frac{1}{1 - 2 \cos^2 x + \cos^4 x}\right| + C$$

**Figure 9.** Example of student work (R$_{10}$) for question number-2

Problem number (2) above relates to students’ mathematical understanding ability to relate the usage of trigonometric similarity of trigonometric integral types known. Utilizing trigonometric similarities namely $\cos^2 x + \sin^2 x = 1$, further by issuing the appropriate factor $\cos x$ so that the calculation can be completed. Appears on the second line for the student sorting “$\cos^3 x = \cos^2 x \cdot \cos x$”, whereas for “$\sin^{-4} x = (\sin^2 x)^{-2}$”. Students should just sit for $\cos^3 x$, so it can exploit the properties of the integrity ranks of the properties $\int \sin^m x \cos^n x \, dx$ (m or n odd). A mistake made in sorting and selecting utilize trigonometric similarities namely $\cos^2 x + \sin^2 x = 1$ This resulted in the final completion of the work of students. The work of students shows that the ability to link the concepts and methods related in the right way still needs to be done comprehensively through the practice of solving the varieties and the form of the variability problem especially on the material trigonometric integrals.
Problem number (4), calculate the following integral: $\int \frac{x^2}{\sqrt{16-x^2}} \, dx$

$$\int \frac{x^2}{\sqrt{16-x^2}} \, dx$$

Misal: $\theta = \sqrt{16-x^2} = \sqrt{4^2 - x^2}$

$\theta = 4 - x$ (Bentuk I)

$x = 4 \sin \theta$

$dx = 4 \cos \theta \, d\theta$

Substitusi nilai $x = 4 \sin \theta$ ke persamaan $\int \frac{x^2}{\sqrt{16-x^2}} \, dx$

$$\int \frac{x^2}{\sqrt{16-x^2}} \, dx = \int \frac{(4 \sin \theta)^2}{\sqrt{16 - (4 \sin \theta)^2}} \, 4 \cos \theta \, d\theta = \int \frac{16 \sin^2 \theta}{\sqrt{16 - (4 \sin \theta)^2}} \, 4 \cos \theta \, d\theta$$

$$= \int \frac{16 \sin^2 \theta}{\sqrt{16 - (4 \sin \theta)^2}} \, 4 \cos \theta \, d\theta$$

$$= 64 \int \frac{\sin^2 \theta \cos \theta}{\sqrt{16 \cos^2 \theta}} \, d\theta = 64 \int \frac{\sin^2 \theta \cos \theta}{4 \cos \theta} \, d\theta$$

$$= 24 \int \sin^2 \theta \, d\theta$$

$$= 24 \cos^3 \theta + C$$

**Figure 10.** Example of student work (R17) for question number-4

This can reveal the knowledge of the procedure, when and how to use the procedure appropriately, the knowledge of how to estimate the outcome of the procedure, and to use the procedure flexibly, accurately and efficiently. The work of students in the second row, third, and fourth, which is preceded by the instance “$\theta = \sqrt{16-x^2}$...” indicates that this step seems unfamiliar with the procedure and uses the procedure appropriately. But in the next step for the fifth row up to the twelfth, with the instance “$x = 4 \sin \theta$...” until it substitutes into the integral to be obtained “$\int \frac{x^2}{\sqrt{16-x^2}} \, dx = \int \frac{(4 \sin \theta)^2}{\sqrt{16-(4 \sin \theta)^2}} \, 4 \cos \theta \, d\theta$”, continued from “$\int \frac{x^2}{\sqrt{16-x^2}} \, dx = \cdots = 64 \int \frac{\sin^2 \theta \cos \theta}{4 \cos \theta} \, d\theta$” shows that the knowledge of the procedure, estimating the results of the procedure, and skillfully using the procedure flexibly, accurately and efficiently put forward by the students is correct in solving this problem until the twelfth step. However, in the next step, students make mistakes of mathematical concepts that ultimately affect the outcome of solving this problem. To avoid any misconceptions by students, it should be in the fifth step when the instance “$x = 4 \sin \theta$”, the student must represent the instance.

Problem number (5), sketch the following function $f(x) = 3x^2 + 2$; $[-2, 1]$ , and calculate $\int_{-2}^{1}(3x^2 + 2) \, dx$ by definition!
This problem still reveals the ability of procedural smoothness, which is able to present graphically, select the limits of the specified intervals, and use the appropriate method in calculating the area of the curve in question. It appears from the work of the students sat on the second row up to twelve, the students begin by deciding “\( \Delta x = \frac{3}{n} \) \( \ldots \), \( x_n = 1 \) “, and in working on this matter, the student appropriately determines “\( x_i = -2 + \frac{3i}{n} \)”. The next step students are able to represent in the form of graphical sketches precisely and followed by formulating the integral definition of the intended problem, but in the completion of the fifteenth row to the twenty-third row, it appears that the student does not precisely write the limit symbol, it is impressed that this step is not related to the previous step. For the second row, the students write back the limit symbol so that the process of calculating the limit in question is correct. It appears that the error of estimating the results of flexible, accurate, and efficient selection of procedures is still frequently performed by students.

4. Conclusion
Maple-assisted learning is feasible to be implemented as an alternative learning model in Calculus to improve the ability of mathematical proficiency in students with early skills (high, medium, and low). Aspects of mathematical proficiency, ability of mathematics education students in presenting concepts...
for various forms of mathematical representation, linking concepts and related methods in appropriate ways, and estimating the results of flexible, accurate, and efficient selection of procedures indicated in the medium category. This is because students do not practice to solve problems with high-level thinking qualifications. Maple is very helpful to students of mathematics education to understand the solution algorithm of mathematical problems or problems that are designed based on indicators of aspects of mathematical proficiency. Maple provides opportunities for interactive use associated with the use of the Tutor, Assistant or Show Solution commands. It is expected to extend the research topics to mathematical problems and increase mathematical competence, not only on calculus, but can be used in differential equations, linear algebra, and algebraic structures using Maple.

Acknowledgment
Thanks addressed to the Rector Prof. Dr. Husen Alting, SH, MH, who has programmed research grants for lecturers and was organized in the DIPA FKIP of Khairun University.

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