Shell-Model Effective Operators for Muon Capture in $^{20}$Ne

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Abstract

It has been proposed that the discrepancy between the partially-conserved axial-current prediction and the nuclear shell-model calculations of the ratio $C_P/C_A$ in the muon-capture reactions can be solved in the case of $^{28}$Si by introducing effective transition operators. Recently there has been experimental interest in measuring the needed angular correlations also in $^{20}$Ne. Inspired by this, we have performed a shell-model analysis employing effective transition operators in the shell-model formalism for the transition $^{20}$Ne($0_{gs}^+$) + $\mu^-$ $\rightarrow$ $^{20}$F($1^+$; 1.057 MeV) + $\nu_\mu$. Comparison of the calculated capture rates with existing data supports the use of effective transition operators. Based on our calculations, as soon as the experimental anisotropy data becomes available, the limits for the ratio $C_P/C_A$ can be extracted.

Key words: Shell model; Muon capture; Effective operators

The large energy release in the ordinary (non-radiative) capture of stopped negative muons by atomic nuclei probes the hadronic current much deeper than ordinary beta decay or electron capture. In particular, the role of the induced pseudoscalar coupling $C_P$ becomes important in muon capture. Based on this, there have been many attempts in the past to extract the ratio of the induced pseudoscalar and axial-vector coupling constants, $C_P/C_A$, from measured capture rates (see e.g. [1–6]) as well as from angular correlations of the gamma emission following the capture reaction $A^ZX_N + \mu^- \rightarrow A^{X'_N+1} + \nu_\mu$ of polarized muons (see e.g. [7–9]).

The angular correlation data, available for muon capture in $^{28}$Si, has been in a key role in pointing out discrepancies in the shell-model calculations of $C_P/C_A$. In various shell-model calculations (see e.g. [7,9] and references therein) anomalously small values of this ratio ($C_P/C_A \sim 0$) have been obtained. In [9] we have proposed a method which, at least partly, lifts this discrepancy. This method is based on the use of effective transition operators.
in the shell-model formalism. Unfortunately, the anisotropy data is available only for $^{28}\text{Si}$ and thus further testing of the effective-operator method has to be done in the context of measured capture rates or future experiments on angular correlations in the capture of polarized muons. At this point it is worth pointing out that the matrix elements of muon capture can also be applied to various other problems, one of the most interesting being the search of the scalar coupling of the hadronic current [10].

A new measurement of the correlation coefficients of $\gamma$-radiation following the capture reaction in $^{28}\text{Si}$ has been reported recently [11], confirming the earlier results of Refs. [7] and [8]. So far $^{28}\text{Si}$ has been the only nucleus where this angular correlation data exists. In these experiments the parameter

$$x \equiv M_1(2)/M_1(-1)$$

(1)

can be extracted via the scaling coefficient $\alpha$ of the angular correlation between the emitted $\gamma$-radiation and the muon neutrino. This scaling coefficient is related to $x$ as [8]

$$\alpha \equiv \frac{\sqrt{2}x - x^2/2}{1 + x^2}.$$  

(2)

The quantities $M_1(-1)$ and $M_1(2)$ are linear combinations of reduced nuclear matrix elements and are given by

$$M_1(-1) = \sqrt{\frac{2}{3}} \left\{ \left( \frac{1}{3}G_P - G_A \right) [101] + G_P \frac{\sqrt{2}}{3} [121] \frac{C_A}{M} [011p] + \frac{C_V}{M} \sqrt{\frac{2}{3}} [111p] \right\},$$

(3)

$$M_1(2) = \sqrt{\frac{2}{3}} \left\{ \left( G_A - \frac{2}{3}G_P \right) [121] - G_P \frac{\sqrt{2}}{3} [101] + \frac{C_A}{M} \sqrt{2}[011p] + \frac{C_V}{M} \sqrt{\frac{2}{3}} [111p] \right\},$$

(4)

where $M$ is the nucleon mass. The definitions of the reduced nuclear matrix elements [...] can be found e.g. from [6]. The constants $G_P$ and $G_A$ are related to the weak-interaction coupling constants as

$$G_P = \left( C_P - C_A - C_V - C_M \right) \frac{E_\nu}{2M},$$

(5)

$$G_A = C_A - (C_V + C_M) \frac{E_\nu}{2M}.$$ 

(6)

Using the expressions of Eqs. (3) and (4), combined with Eq. (1), the value of $C_P/C_A$ can be extracted if the experimental value of $x$ is known. However,
calculations with different nuclear models give very different predictions for this ratio. In Ref. [8] the values $C_P/C_A = 3.4 \pm 1.0$ and $C_P/C_A = 2.0 \pm 1.6$ were extracted using the matrix elements of Refs. [2] and [12], respectively. In addition, the measurement of Ref. [7] gives the estimates $C_P/C_A = 5.3 \pm 2.0$ using the matrix elements of [2] and $C_P/C_A = 4.2 \pm 2.5$ using the matrix elements of [12].

The more realistic matrix elements, obtained from the full 1s0d shell calculation utilizing Wildenthal’s USD interaction [13], yield the value of $C_P/C_A = 0.0 \pm 3.2$ [14], far from the value $C_P/C_A \approx 7$ given by the nuclear-model independent Goldberger-Treiman relation (see e.g. [15]) obtained by using the partially-conserved axial-current hypothesis (PCAC). The estimate given by the shell-model matrix elements is very surprising, since the USD interaction is fitted to a selected set of the 1s0d-shell spectroscopic data, reproducing various spectroscopic quantities like energy spectra, Gamow-Teller decay properties and electromagnetic properties (see e.g. [16,17]) very well.

This anomaly, present in the shell-model calculations of $^{28}\text{Si}(0^+_{g.s.}) + \mu^- \rightarrow ^{28}\text{Al}(1^+_g) + \nu_\mu$, can be, at least partly, avoided by using renormalized one-body transition operators in the context of the shell model. In the work of [9] the USD and effective interactions based on the recent CD-Bonn [18] and Nijmegen [19] nucleon-nucleon (NN) interaction models, yielded the interval $0.4 \leq C_P/C_A \lesssim 2.7$, whereas with the renormalized transition operators the interval $3.4 \leq C_P/C_A \leq 5.4$ was obtained, closer to the PCAC-prediction of this ratio. This value agrees also with the recent analysis of Brudanin et al. [11]. Moreover, of special interest are the recent plans for the angular-correlation measurements following the capture reaction $^{20}\text{Ne}(0^+_{g.s.}) + \mu^- \rightarrow ^{20}\text{F}(1^+_g) + \nu_\mu$, as announced in [11]. In the present Letter we investigate this particular reaction in the shell-model framework with and without effective operators, and give predictions for the ratio $C_P/C_A$ using different sets of two-body interactions. The needed muon-capture formalism is treated in great detail in Ref. [1] and reviewed in the shell-model context e.g. in Ref. [6].

In the present shell-model calculation we have employed three different two-body interactions. In addition to the abovementioned USD interaction [13], we have derived microscopic effective interactions and operators based on the recent CD-Bonn meson-exchange NN interaction model of Machleidt et al. [18] and the Nijm-I NN interaction model of the Nijmegen group [19]. These are the same interactions which were employed by us in Ref. [9]. In order to obtain effective interactions, see Ref. [20] for more details, and operators for the muon capture studies, we use $^{16}\text{O}$ as a closed-shell nucleus and define the 1s0d shell as the shell-model space for which the effective interactions and operators are derived. Based on a $G$-matrix derived for $^{16}\text{O}$, we include all diagrams through third-order in $G$ and sum folded diagrams to infinite order employing the so-called $Q$-box approach described in e.g. Ref. [20], in order to
derive an effective two-body interaction for the 1s0d shell. In the discussions below, we will refer to these effective two-body interactions simply as CD-Bonn and Nijm-I interactions.

The effective single-particle operators are calculated along the same lines as the effective interactions. In nuclear transitions, the quantity of interest is the transition matrix element between an initial state $|\Psi_i\rangle$ and a final state $|\Psi_f\rangle$ of an operator $\mathcal{O}$ defined as

$$\mathcal{O}_{fi} = \frac{\langle \Psi_f| \mathcal{O} |\Psi_i\rangle}{\sqrt{\langle \Psi_f|\Psi_f\rangle \langle \Psi_i|\Psi_i\rangle}}.$$  \hspace{1cm} (7)

Since we perform our calculation in a reduced space, the exact wave functions $\Psi_{f,i}$ are not known, only their projections $\Phi_{f,i}$ onto the model space. We are then confronted with the problem of how to evaluate $\mathcal{O}_{fi}$ when only the model space wave functions are known. In treating this problem, it is usual to introduce an effective operator $\mathcal{O}_{eff}^{fi}$, defined by requiring

$$\mathcal{O}_{fi} = \langle \Phi_f| \mathcal{O}_{eff} |\Phi_i\rangle.$$  \hspace{1cm} (8)

Observe that $\mathcal{O}_{eff}$ is different from the original operator $\mathcal{O}$. The standard scheme is then to employ a perturbative expansion for the effective operator, see e.g. Refs. [21,22].

To obtain effective one-body transition operators for muon capture, we evaluate all effective operator diagrams through second-order in the $G$-matrix obtained with the CD-Bonn and Nijm-I interactions. Such diagrams are discussed in the reviews by Towner [21] and Ellis and Osnes [22]. Terms arising from meson-exchange currents have been neglected, similarly, also the possibility of having isobars $\Delta$ as intermediate states are omitted since the focus here is on nucleonic degrees of freedom only. Moreover, the nucleon-nucleon potentials we are employing do already contain such intermediate states. Including $\Delta$ degrees of freedom may thus lead to a possible double-counting. Intermediate-state excitations in each diagram up to $6 - 8\hbar\omega$ in oscillator energy were included in order to achieve a converged result. This is also in line with studies of effective interactions with weak tensor forces [23], such as the CD-Bonn potential employed here.

The energy spectrum of $^{20}\text{F}$, emerging from our full 1s0d-shell calculation using $^{16}\text{O}$ as closed-shell core, is shown in Fig. 1. The agreement with experiment is good. In particular, both the CD-Bonn and Nijm-I results are very close to the USD ones, and the energy of the $1^+_1$ final state of the capture reaction is reproduced almost exactly. The description of the spectrum of the double-even $^{20}\text{Ne}$ nucleus by shell-model is more trivial than the description of the spectrum
Table 1
The values of the reduced nuclear matrix elements (RNME). The recoil matrix elements $[\ldots p]$ are given in units of fm$^{-1}$.

|        | USD         | CD-Bonn  | Nijm-I   |
|--------|-------------|----------|----------|
| RNME   | bare renorm | bare renorm | bare renorm |
| [101]  | 0.0203      | 0.0218   | 0.0244   | 0.0251   | 0.0249   | 0.0256   |
| [121]  | 0.0045      | 0.0028   | 0.0039   | 0.0024   | 0.0043   | 0.0028   |
| [101−] | 0.0192      | 0.0209   | 0.0233   | 0.0241   | 0.0237   | 0.0246   |
| [121+] | 0.0055      | 0.0034   | 0.0048   | 0.0029   | 0.0052   | 0.0034   |
| [111p] | 0.0303      | 0.0231   | 0.0286   | 0.0219   | 0.0281   | 0.0220   |
| [011p] | -0.0136     | -0.0091  | -0.0165  | -0.0110  | -0.0163  | -0.0109  |

of the double-odd $^{20}\text{F}$. For this reason the agreement between the calculated and measured [24] energy spectra of $^{20}\text{Ne}$ is excellent for all interactions and thus we refrain from a detailed comparison of the $^{20}\text{Ne}$ spectra. The shell-model calculations were performed using the code OXBASH [25]. The reader should note that since the USD interaction is an effective interaction operating in the 1s0d shell only, it is not possible to calculate with this interaction the corresponding effective operators which connect to states outside the 1s0d model space. Therefore, we have employed the effective operators obtained with the CD-Bonn interaction for the USD calculation as well. Employing those from the Nijm-I interaction gives similar results.

The renormalization effects on the one-body transition matrix elements are of the order of $10–30\%$, and in almost all cases we get reduction in the absolute value. In particular, the Gamow–Teller-type single-particle matrix elements, corresponding to the matrix element $[101]$, reduce roughly by 10%. However, it should be noted, that the radial dependence in the $[101]$ matrix element differs from the radial dependence of the pure Gamow–Teller matrix element. The resulting nuclear matrix elements for the transition $^{20}\text{Ne}(0^+; \text{g.s.}) + \mu^- \rightarrow \mu^- \rightarrow ^{20}\text{F}(1^+; 1.057 \text{MeV}) + \mu$ are shown in Table 1, obtained by combining the one-body transition matrix elements with the corresponding one-body transition densities of the shell-model calculation. The corresponding capture rates obtained using the formalism of Ref. [1] are shown in Fig. 2 with the experimental value of Ref. [26]. The capture rates $W$ are calculated according to

$$W = 4P(\alpha Zm'_\mu)^3 \frac{2J_f + 1}{2J_i + 1} \left(1 - \frac{Q}{m_\mu + AM}\right) Q^2,$$

where $\alpha$ is the fine-structure constant, $m'_\mu$ is the reduced muon mass, and $Q$ is the $Q$-value of the nuclear transition. The reduced nuclear matrix elements
are included in $P$ (see Ref. [1] for further details). Instead of renormalizing
the axial vector coupling constant, the corrections are included in the effective operators. Therefore, the calculations are performed using the bare value $C_A/C_V = -1.251$.

From Fig. 2 it can be seen that the renormalization increases the capture rate for all interactions, pushing it closer to the experimental value for both the USD, CD-Bonn and Nijm-I interactions, when $C_P/C_A$ is close to the PCAC value. The USD calculation with the bare operators yield an interval $-4.9 \leq C_P/C_A \leq -3.7$, far from a reasonable expectation for the value of this ratio. The ratio $C_P/C_A$ calculated with the renormalized CD-Bonn and Nijm-I one-body operators agrees almost exactly with the PCAC prediction. For the PCAC prediction $C_P/C_A \approx 7$, the renormalized USD calculation yields a capture rate slightly below the experimental window. However, the role of the renormalization is similar to that seen with the Nijm-I and CD-Bonn interactions. As soon as the angular-correlation data on the muon capture in $^{20}$Ne are published, the predictions of Fig. 3 can be used for the extraction of the ratio $C_P/C_A$. At this point we can observe that the general trend is very similar to the $^{28}$Si case of Ref. [9]. The renormalized calculations reduce the magnitude of $x$ for a given $C_P/C_A$ ratio, and the behaviour is very similar for the USD, CD-Bonn [18] and Nijm-I [19] interactions. This supports the conclusion of Ref. [9], where the qualitative effects of the renormalization on the $x$ were found to be interaction independent.

In conclusion, our calculations support the near interaction independence of the effects of the renormalization of the one-body transition operators involved in the shell-model calculation of the muon-capture rates and the angular-correlation parameter $x$. This renormalization is introduced by replacing the bare transition operators, operating in the full Hilbert space, by effective ones, calculated with the CD-Bonn and Nijm-I interactions and now operating in the shell-model valence space. In the present work we found that these effective operators give very satisfactory results when compared to the experimental data. This is confirmed by the capture rates, where the agreement with experiment is much better with the effective operators. We have also given predictions for the ratio $x = M_1(2)/M_1(-1)$, which can be used for the determination of the ratio $C_P/C_A$ as soon as the experimental anisotropy data becomes available. If $C_P/C_A \approx 7$, as predicted by PCAC and as seen in the capture rate calculations, then $x \approx 0.35$ for all interactions employed. The results from $^{28}$Si indicate however [11,9] that $C_P/C_A \approx 5$. The latter value would yield $x \approx 0.30$ for the present reaction. With $C_P/C_A \approx 5$, the capture rates reported in Fig. 2 will not deviate much from experiment. An experimental determination of $x$ would then clarify this situation.
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Fig. 1. Calculated and experimental [24] energy spectra of $^{20}\text{F}$.

Fig. 2. Capture rates leading to the $1^+_1$ (1.057 MeV) final state in $^{20}\text{F}$.

Fig. 3. Parameter $x$ of Eq. (1) plotted as a function of the ratio $C_p/C_A$. 
