Geometric calculation of planetary rotor hydraulic machines

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Abstract. Hydraulic and pneumatic volume displacement machines are commonly the most important and integral elements of modern mechanical systems. One of the known types of such machines is planetary rotor hydraulic machines (PRHM) with floating satellites in contact with central gearwheels. The numbers of the central gearwheels «waves» M and N may be different or the same. They are characterized by the absence of valves and loaded kinematic sliding pairs. The main task of design and calculation of PRHM is geometrical calculation of their gear elements. Such a task has to be solved in the case of a PRHM with two non-circular centre gearwheels and a PRHM with round and non-circular centre gearwheels. The developed method of geometric synthesis of PRHM allows to obtain profiles of PRHM gear elements.

1. Introduction
In various technical systems, volume hydraulic machines are widely used: pumps and motors. Such machines include planetary rotary hydraulic machines (PRHM). The principal advantages of the PRHM are the great payload volume of working spaces, the absence of loaded sliding pairs, and the insensitivity to gear.
In the mode of hydraulic motor, they exceed all currently produced hydraulic machines by their mass-power indicators. In pump they are characterized by record capacity at medium pressure up to 20 MPa [1, 2]. For a number of reasons, the PRHM remains poorly understood, especially the geometric calculation of gear elements.

2. Structure of planetary rotor hydraulic machines
A typical PRHM (figure 1) comprises [3] a non-circular central gearwheel 1 with external teeth (solar) and a non-circular central gearwheel 2 with internal teeth (epicyclic), as well as floating satellites 3. The number of «waves» of the center gearwheel with internal teeth N and external teeth M may be different or the same. For the scheme shown in figure 1, M = 2, N = 4 (or 2 × 4).
In another PRHM (figure 2), known from the patent [4], both central gearwheels are circular, that is have «wave» number M = 1, N = 1 (or 1 × 1).
In figure 3 shows the diagram of the PRHM [5], characterized by two non-circular central gearwheels. With this, the number of «waves» M and N is two. Both central gearwheels 1 and 2 have the same number of teeth. Positioning of satellites between central gearwheels having the same number of «waves» (M = N) and, accordingly, the same number of teeth (Z1 = Z2) becomes possible at extreme geometric parameters of gearing. Since the numbers of the teeth of the Z1 and Z2 center gearwheels are the same, the angular velocity of the center gearwheel 1 is twice the portable angular velocity of the satellite system. At that one moving of the central gearwheel 1 corresponds to one cycle of configuration change of the satellite pinion system 3.
Figure 1. Planetary Rotor Hydraulic Machines (PRHM): 1 – solar gearwheel; 2 – epicyclic gearwheel; 3 – satellites.

Figure 2. Planetary Rotor Hydraulic Machines (PRHM) with round central gearwheels.

Figure 3. Planetary Rotor Hydraulic Machines (PRHM) with non-circular central gearwheels (2 × 2).
As a result of element moving, volumes of working cavities. The maximum advantages of this scheme are realized by using several sections installed in series.

[6; 7] are also known as PRHM in which one of the gearwheels is circular (has a «wave» number equal to 1), and the other has a non-circular (has a «wave» number greater than 1). Diagrams of such PRHM are shown in figure 4.

From a technological point of view, such schemes can be constructed using 2-D technologies (electroerosion machining, laser and waterjet cutting) and 3-D (3D printers). Such technologies allow effective surface treatment of any complexity.

Existing and proposed schemes of planetary rotary hydraulic machines can be used in pumps for oil and fuel oil pumping, in pumps for water (drilling and fire), in pumps-dispensers for various liquids, in pumps and engines of hydraulic drives, in pneumatic motors, in vacuum pumps of low vacuum.

3. Theoretical bases of geometric calculation of the PRHM

It should be noted that the properties and structural features of the PRHM mechanisms have been little studied. In design and production of PRHM the main stage is geometric calculation of their gear elements. Until recently, there were no engineering techniques for geometric calculation of the PRHM. Existing calculation methods [3,8,9,10] have complex mathematical apparatus for practical application.

The geometric calculation of the PRHM is reduced to three cases: 1. Both central gearwheels are non-circular (N > 1, M > 1). 2. Both center gearwheels are round (N = M = 1). 3. Circular and non-circular central gearwheels (M = 1, N > 1 or N = 1, M > 1).

For the first case, [11,12] the method of geometric synthesis of PRHM is developed using software complexes Kompas and Mathcad. It includes: selecting a pair of satellite center paths in coordinate systems associated with each of the center gearwheels; obtaining the centroid of the solar and epicyclic gearwheels as an equidistant to centre paths; «involves» the corresponding centroids, that is, obtaining gear profiles based on these centroids. In proceeding [11,12], the center paths of the satellite are changed by the simplest «cosine» law:

\[
\begin{align*}
  r_1 &= r_0 \cdot \left(1 + k \cdot \cos(M \cdot \varphi_1)\right); \\
  r_2 &= r_0 \cdot \left(1 + k \cdot \cos(N \cdot \varphi_1)\right),
\end{align*}
\]

Figure 4. Planetary Rotor Hydraulic Machines (PRHM) diagrams with round and non-round central gearwheels 2 × 4.
where \( r_1 \) and \( r_2 \) is the radius-vectors of paths; \( \varphi_1 \) and \( \varphi_2 \) is the current angles in the polar coordinates connected to the respective elements; \( k \) is the coefficient of «non-roundness» of paths; \( r_0 \) is the radius of the calculated circle (in which both paths degenerate at \( k = 0 \)).

In the second case \((N = M = 1 \text{ (figure 2)})\), when the centroids of the central gearwheels of the PRHM are circles, the trajectory of the central point of the satellite is also a circle. The geometric synthesis method is the same as for conventional planetary gears.

For the third case \((M = 1, N > 1 \text{ or } N = 1, M > 1)\) the geometric synthesis method is given in [13], the centre paths of the satellite in polar coordinates connected to the fixed non-circular wheel 2 in contrast to the case of two non-circular central gearwheels change according to the law:

\[
r_2 = r_0 \cdot \left( k \cdot \cos(N \cdot \varphi_2) + (1 - k^2 \cdot \sin^2(N \cdot \varphi_2))^{1/2} \right). \tag{3}
\]

4. Obtaining profiles of central gearwheel rings by envelope teeth of satellite moving along trajectory

For all PRHM diagrams, the geometric synthesis of central gearwheels can be performed in two ways [11, 12] is approximate and theoretically accurate. It consists of finding tooth profiles. In the case of the approximate method, the profiles of the ring gears are obtained by «involves» the corresponding equidistants of the non-circular gearwheels. The teeth are transferred to non-circular equidistants of the synthesized gearwheel from the calculated round-link mechanism.

In order to obtain tooth profiles, it is theoretically necessary to have an analytical or graphical description of the desired profiles. These profiles are typically envelopes to the corresponding curve population. The problem of obtaining a population of curves is characteristic of geometric synthesis of elements of higher pairs. Traditionally, this problem can be solved analytically using gearing theory. The system of equations obtained using the matrix method of coordinate transformation, to which the so-called gearing equation is added, is compiled. The system is solved by numerical methods. The result is an array of profile point coordinates. However, this approach requires a high qualification of the calculator, accompanied by a high labour intensity of the calculation process and the need to analytically set all sections of the profile enveloped by the object.

Theoretically, precisely the profiles of the central gearwheel teeth are obtained as envelopes to the satellite in its motion relative to the corresponding central gearwheel (figure 5). The satellite rotation angle \( \varphi \) relative to the stationary coordinate systems associated with the solar and epicyclic gearwheels is calculated analytically:

\[
\varphi_c = (1 - Z_1/Z_c) \cdot \int_0^\varphi \left( (1 + k \cdot \cos(M \cdot \varphi))^2 + (M \cdot k \cdot \sin(M \cdot \varphi))^2 \right)^{1/2}; \tag{4}
\]

\[
\varphi_c = (1 + Z_2/Z_c) \cdot \int_0^\varphi \left( (1 + k \cdot \cos(N \cdot \varphi))^2 + (N \cdot k \cdot \sin(N \cdot \varphi))^2 \right)^{1/2}, \tag{5}
\]

where \( \varphi_c \) is the rotation angle of the satellite relative to the given non-circular central gearwheel.

**Figure 5.** Obtaining of PRHM tooth profile by envelope.
In the case of round and non-round centre gearwheels, [13] the equation derived from equation (3) by angle $\varphi_2$ is required to calculate the coordinates of the elements. This equation will be:

$$\frac{dr}{d\varphi_2} = -N \cdot k \cdot r_0 \cdot \sin(N \cdot \varphi_2) \cdot \left(1 + k \cdot \cos(N \cdot \varphi_2) / \left(1 - k^2 \cdot \sin^2(N \cdot \varphi_2)\right)^{1/2}\right).$$  

(6)

The geometric synthesis [14] of curved element profiles has been developed, based on the use of the KOMPAS-3D graphics package and Mathcad engineering mathematical software. The choice of these programs is due to the fact that graphical other programs of the design systems (for example, in SolidWorks (Solidworks) and T-FLEX) have relatively little distribution in the design environment. Some initial profile of the "tool" is selected (for example, the satellite profile of the planetary rotor hydraulic machine) and set graphically. Based on the data array obtained in Mathcad in the software package KOMPAC-3D a population of PRG tooth profile curves is built (figure 6 and figure 7).

![Figure 6. Geometric synthesis of PRHM solar gearwheel.](image1)

![Figure 7. Geometric synthesis of PRHM epicyclic gearwheel.](image2)

Thus, the problem of finding the profile of the PRHM gear ring with the same and different number of «waves» M and N is solved using software complexes Kompas and Mathcad.

5. Conclusions

The method of geometric synthesis of PRHM allows to obtain profiles of PRHM gear rings of different structural schemes in the form necessary for their manufacture using 2-D technologies using standard packages of computer programs. In practice, the method can be used in designing planetary rotary hydraulic machines (pumps and hydraulic and pneumatic motors) by designers of machine-building industry enterprises.
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