Numerical investigation on unstable behaviors of cellular premixed flames at low Lewis numbers based on the diffusive-thermal model and compressible Navier-Stokes equations

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Abstract
Two dimensional unsteady calculations of reactive flows were performed in large domain to investigate the unstable behaviors of cellular premixed flames at low Lewis numbers based on the diffusive-thermal (D-T) model and compressible Navier-Stokes (N-S) equations including one-step irreversible chemical reaction. The relations between the growth rate and wave number, i.e. the dispersion relations, were obtained. The growth rates obtained by the compressible N-S equations were large and the unstable ranges were wide compared with those obtained by the D-T model equations. This was because both hydrodynamic and diffusive-thermal effects were taken into account in the compressible N-S equations. A disturbance with the maximum growth rate was superimposed on a stationary planar flame to generate the cellular flame. The cellular flame formed, and the overshoot of temperature evolved. When the length of computational domain increased, the number of small cells separated from large cells of the cellular flame increased drastically. As the results, flame surface area and average burning velocity increased. The stronger unstable behaviors and the larger average burning velocities were observed especially in the numerical results based on the compressible N-S equations. In addition, the fractal dimension obtained by the compressible N-S equations was larger than that by the D-T model equations. Moreover, we confirmed that the radiative heat loss promoted the instability of premixed flames at low Lewis numbers.

Keywords : Numerical investigation, Unstable behavior, Cellular premixed flame, Low Lewis number, Large domain, Diffusive-thermal effect, Hydrodynamic effect, Fractal dimension, Radiative heat loss

1. Introduction

To solve the problems of pollutant emissions and reduce the amount of fuel, lean premixed combustion of alternative fuels such as hydrogen or methane has been paid much attention in various combustion applications such in internal combustion engines, gas turbine and industrial combustors (Brewster et al. 1999; Dunn-Rankin, 2008). Although hydrogen has possibility to use in combustion applications, lean hydrogen flames are easy to become unstable which lead us to conduct researches actively on hydrogen combustion for safe use of it. There is also an awareness on hydrogen explosion accidents in nuclear power plants or in the spent radioactive storage vessels. A kind of premixed combustion of hydrogen can be occurred when its concentration in the vessel reaches flammable limit, and there is a possibility to occur fire and gas explosion (Anderson et al. 2000). In premixed combustion, while the molecular diffusion of deficient reactant is larger than the thermal diffusivity of mixture, a planar flame front becomes wrinkle and propagates laterally, so called cellular flame formation, owing to the unbalance between molecular diffusion and thermal diffusivity. In other words, when the Lewis number which is defined as the ratio of the thermal diffusivity to the diffusion coefficient is lower than unity, e.g. lean hydrogen-air mixture, the diffusive-thermal effect has a destabilizing influence and the flame front
becomes cellular (Sivashinsky, 1977). In addition, the hydrodynamic effect caused by thermal-expansion through the flame front has a destabilizing effect on the flame front (Landau, 1944; Michelson and Sivashinsky, 1982). These two effects are significant in intrinsic instability of premixed flames (Williams, 1985). When the flame front becomes cellular, flame surface area increases, which is mainly related to the increase in burning velocity, and then premixed flames may become self-accelerated, and fire and explosion appear in severe cases, which can give loss of properties and sufferings to human (Hirano, 2002). Thus, it is important to obtain the fundamental knowledge of unstable behaviors of lean premixed flames to control the combustion system safely and effectively. The recent experimental study of Liu et al. (2012) showed that cellular flame of hydrogen-air lean premixture at the equivalence ratio of 0.2 has irregular distorted shape due to the dominant diffusive-thermal effect while the stoichiometric cellular flame has regular distorted shape due to hydrodynamic effect. Thwe Thwe Aung and Kadowaki (2013, 2015) investigated the instability induced by diffusive-thermal effect on premixed flames by the D-T model equations for various unburned gas temperatures under the adiabatic and non-adiabatic conditions. Since, constant density approximation was used in the studies of diffusive-thermal effect, hydrodynamic effect was out of consideration. To understand the mechanisms of instability induced by hydrodynamic effect which is important in all premixed flames, compressible N-S equations were adopted and numerical calculations of reactive flows were performed (Denet and Haldenwang, 1992; Kadowaki, 1997; Thwe Thwe Aung et al. 2017). Kadowaki and Goma (2000) used both the D-T model and compressible N-S equations for numerical calculations of two-dimensional and three-dimensional reactive flows, and pointed out the characteristics of cellular flame fronts at low Lewis numbers, $le = 0.5, 0.7$ and 1.0, for single wavelength in computational domain. Based on the D-T model equations, the growth rates were small and the unstable ranges were narrow compared with those based on the compressible N-S equations.

In practical combustion hazards, flame size drastically increases during the fire and gas explosion. Thus, it is indispensable to study the mechanisms of unstable flames on large scale of computational domain in numerical research. The instabilities induced by hydrodynamic and diffusive-thermal effects were studied in large scale by Kadowaki et al. (2005, 2015, 2017) based on the compressible N-S equations. Their results indicated that the scale affects strongly the unstable behaviors of cellular premixed flames. However, they have not performed for Lewis numbers less than 0.5. Long-term non-linear evolution of lean premixed hydrogen-air premixed flame fronts at low Lewis numbers, $le = 0.5, 0.7$ and 1.0, for single wavelength in computational domain. In their numerical simulations, they clarified that the flame at Lewis number equal to unity is hydrodynamically unstable but diffusively stable, and which at Lewis numbers lower than unity is unstable by both effects. In addition, fractal structure can be occurred when the flame becomes hydrodynamically unstable and the fractal dimension approximately represents the increment of flame speed. Using the Sivashinsky equation, fractal structure of spherically propagating flame in large computational domain was investigated by Altantzis et al. (2012). Their results showed that hydrodynamically unstable flame has fractal structure but diffusively unstable flame does not have. We still need the knowledge on sufficiently low Lewis numbers flames which represents lean hydrogen-air premixed flames, by adopting both the D-T model and compressible N-S equations in large domain of numerical research.

Since the effects of heat loss play an important role in combustion instability, understanding the mechanisms of non-adiabatic cellular premixed flames is worth for controlling the premixed flames in lean combustion. Heat losses in various ways cause the decrease in flame temperature and loss of usable energy. When the rate of heat loss reaches beyond the heat production rate, flame vanishes (Law, 2006; Drysdale, 2011). Ishizuka et al. (1982) studied the effects of heat loss on instability of propane-air flame fronts induced by diffusive-thermal effect experimentally. Linear analyses on the instability of non-adiabatic premixed flames were studied based on the diffusive-thermal model equations, with conductive heat loss (Joulin and Clavin, 1979) and with radiative heat loss (Kagan and Sivashinsky, 1997). Joulin and Sivashinsky (1994) investigated the effects of momentum and heat losses on the hydrodynamic instability of premixed flames propagating between two parallel plates separated by a small gap. In our previous works, the effects of radiative heat loss on low- and high-temperature premixed flames were studied in one wavelength of computational domain at Lewis number equal to and greater than 0.5 based on the D-T model and compressible N-S equations (Kadowaki and Thwe Thwe Aung, 2012; Thwe Thwe Aung and Kadowaki, 2013, 2015; Thwe Thwe Aung et al. 2017).

In this study, we performed the numerical calculations of reactive flows in large computational domain based on the D-T model and compressible N-S equations including the one-step chemical reaction and clarified the unstable behaviors of low Lewis numbers premixed flames. From these two equations, the visualizations of unstable behaviors, characteristics of cellular flames such as burning velocities and fractal dimensions were investigated. The effects of radiative heat loss on the instability were also investigated with both equations.
2. Basic equations

For both the D-T model and compressible N-S equations, we adopted the single-reactant flames, where the abundant reactant was excessive and the chemical reaction was controlled only by the deficient reactant, and used the following assumptions: The chemical reaction was a one-step irreversible exothermic reaction, and the reaction rate obeyed the Arrhenius’ law. The unburned and burned gases had the same molecular weights and the same Lewis numbers, and the ideal gas equation of state was satisfied. The specific heat and transport coefficients were independent of temperature.

Constant density approximation was used in the D-T model equations, and hydrodynamic effect was neglected. In the compressible N-S equations, the body force, Soret effect, Dufour effect, pressure gradient diffusion and bulk viscosity were all negligible, and the viscous term in the equation of energy conservation was disregarded, because its contribution was trivial in the present problem.

Using Cartesian coordinates, we took the direction tangential to the flame front as the $\gamma$-direction, with the gas velocity in the positive $\gamma$-direction. The flow variables were non-dimensionalized by the physical quantities of standard premixed flames, i.e. pressure and temperature of the unburned gas, the burning velocity and the preheat zone thickness.

The non-dimensional D-T model equations (Sivashinsky, 1977) in the Cartesian coordinates are written as follows:

Energy conservation:

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} - \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = QBY \exp\left(-\frac{E}{T}\right)$$  \hspace{1cm} (1)

Species conservation:

$$\frac{\partial Y}{\partial t} + U \frac{\partial Y}{\partial x} - \frac{1}{Le} \left(\frac{\partial^2 Y}{\partial x^2} + \frac{\partial^2 Y}{\partial y^2}\right) = -BY \exp\left(-\frac{E}{T}\right)$$  \hspace{1cm} (2)

The non-dimensional compressible N-S equations are written as follows:

Mass conservation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0$$  \hspace{1cm} (3)

Momentum conservation in $x$-direction

$$\frac{\partial}{\partial t} (\rho u) + \frac{\partial}{\partial x} \left(\rho u^2 + \frac{\rho}{\gamma M^2_a} - Pr \left(\frac{4}{3} \frac{\partial u}{\partial x} - 2 \frac{\partial v}{\partial y}\right)\right) + \frac{\partial}{\partial y} \left(\rho uv - Pr \left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right)\right) = 0$$  \hspace{1cm} (4)

Momentum conservation in $y$-direction:

$$\frac{\partial}{\partial t} (\rho v) + \frac{\partial}{\partial x} \left(\rho uv - Pr \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right) + \frac{\partial}{\partial y} \left(\rho v^2 + \frac{\rho}{\gamma M^2_a} - Pr \left(\frac{4}{3} \frac{\partial v}{\partial y} - 2 \frac{\partial u}{\partial x}\right)\right) = 0$$  \hspace{1cm} (5)

Energy conservation:

$$\frac{\partial}{\partial t} e + \frac{\partial}{\partial x} \left((e + p)u - \frac{\gamma}{\gamma - 1} \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y} \left((e + p)v + \frac{\gamma}{\gamma - 1} \frac{\partial T}{\partial y}\right) = \frac{\gamma}{\gamma - 1} QBY \exp\left(-\frac{E}{T}\right)$$  \hspace{1cm} (6)

Species conservation:

$$\frac{\partial}{\partial t} (\rho Y) + \frac{\partial}{\partial x} \left(\rho Y u - \frac{1}{Le} \frac{\partial Y}{\partial x}\right) + \frac{\partial}{\partial y} \left(\rho Y v - \frac{1}{Le} \frac{\partial Y}{\partial y}\right) = -B \rho Y \exp\left(-\frac{E}{T}\right)$$  \hspace{1cm} (7)

Equation of state:

$$p = \rho T$$  \hspace{1cm} (8)

where $\rho$ is the density, $u$ and $v$ are the velocities in the $x$- and $y$-directions, $e$ is the stored energy, $Y$ is the mass fraction of fuel, $p$ is the pressure, $T$ is the temperature, $T_u$ is the unburned-gas temperature, $\gamma$ is the ratio of two specific heats, $M_a$ is the Mach number, $Pr$ is the Prandtl number, $Le$ is the Lewis number, $Q$ is the heating value, $B$ is the frequency factor and $E$ is the activation energy. In addition, the ideal gas equation of state is satisfied. When radiative heat loss is taken into account, the term $[-R_H (T^4 - T_i^4)]$, where $R_H$ is the radiative heat loss parameter, is added to the energy conservation equation in both equations.
3. Numerical calculation procedures

We considered a standard premixed flame whose adiabatic flame temperature was 2086 K at room temperature and pressure. The value of adiabatic flame temperature was general for hydrogen-air premixed flames, \( T_{ad} = 7.0 \). Non-dimensional parameters used in both numerical calculations were \( Q = 6.0, E = 70, \gamma = 1.4, M_\infty = 0.01, Pr = 1.0 \). The non-dimensional activation energy corresponded to the activation energy of 173 kJ/mol. The frequency factor was determined under the conditions that the calculated burning velocity of an adiabatic planar flame was equal to unity at \( T_u = 1.0 \) for different Lewis numbers. The value of \( B \) at \( Le = 0.3 \) was 2948478 for the D-T model equations and 5663948 for the compressible N-S equations. We set Lewis number, \( Le \) from 0.2 to 1.0 to take account for both diffusive-thermal and hydrodynamic effects.

Both the D-T model and compressible N-S equations of two-dimensional reactive flow were solved by the explicit MacCormack scheme (MacCormack and Baldwin, 1975) with second-order accuracy both in time and space. The boundary conditions were provided by free-flow in the \( x \)-direction and by periodic in the \( y \)-direction. The length of the computational domain in the \( x \)-direction \( L_x \) was 800 times of the preheat zone thickness. In the \( y \)-direction, \( L_y \) was set to one to sixteen times of the critical wavelength of a disturbance \( \lambda_c \). A uniformly space grid used in the \( x \)-direction was 0.1 and that in the \( y \)-direction was \( \lambda_c/64 \) for single wavelength and \( \lambda_c/(64N) \) for multiple wavelength, where \( N \) was 1, 2, 4, 6, 8, 12 and 16. The computational domain was resolved by a \( 8001 \times (64+1) \) grid for the calculations of dispersion relations and a \( 8001 \times (64N + 1) \) grid for cellular flames. A time-step interval \( \Delta t \) was adopted to satisfy the Courant-Friedrichs-Lewy (CFL) condition, and the non-dimensional calculation time was \( t = 600 \) in the D-T model equations and \( t = 100 \) in the compressible N-S equations for cellular flames. The time-step interval was set to 0.0001 for all calculations, except the compressible N-S equations at \( Le = 0.3 \) in which \( \Delta t = 0.00005 \) for cellular flame calculations.

We superimposed a sinusoidal disturbance periodic in the \( y \)-direction on a planar flame and calculated the evolution of disturbed flame fronts. The displacement of flame fronts in the \( x \)-direction due to the superimposed disturbance was

\[
A_t \sin(2\pi y/\lambda)
\]

The initial amplitude \( A_t \) was set to 0.1 for calculations of the dispersion relation and to 1.0 for cellular flames. All the numerical calculations were performed by high computing server of SGI UV 300 at Nagaoka University of Technology.

4. Results and discussion

For the dispersion relation, which is the relation between the growth rate \( \omega \) and wave number \( k \), a sinusoidal disturbance with the amplitude of 0.1 is superimposed on a planar flame to investigate the intrinsic instability of premixed flames. The superimposed disturbance evolves, maintaining the sinusoidal shape, where flame front is defined as the site of maximum reaction rate. Then the amplitude of a disturbance grows exponentially with time as,

\[
A = A_t (\exp(\omega t))
\]

This type of evolution of a disturbance appears only when the amplitude is sufficiently small and it is consistent with the previous works (Thwe Thwe Aung and Kadowaki, 2013, 2015; Thwe Thwe Aung et al. 2017). When the disturbance grows to some degree, the growth rate becomes gradually lower and eventually drops to zero, which is due to the non-linearity brought about by finite amplitude.

By varying the wave number, dispersion relation is obtained for different Lewis numbers. The growth rate increases and unstable range widens when the Lewis number becomes lower. Figures 1 and 2 show the dispersion relations at \( Le = 0.2, 0.3, 0.5, 0.7 \) and 1.0, obtained by the D-T model and compressible N-S equations, respectively. The growth rates obtained by the D-T model equations are small and the unstable ranges are narrow compared with those obtained by the compressible N-S equations. This is because that hydrodynamic effect is neglected in the D-T model equations with using constant density approximation. The growth rate at the Lewis number of unity is negative by the D-T model and positive by the compressible N-S equations. This shows that the flame is diffusively stable and hydrodynamically unstable at the Lewis number of unity. These results have agreement with the results of Kadowaki and Goma (2000), and Altantzis et al. (2012). The dispersion relation yields the wave number corresponding to the maximum growth rate, i.e. the linearly most unstable wave number, and we refer to as the critical wave number \( k_c \). The critical wavelength \( \lambda_c \) is \( 2\pi / k_c \).

When we consider the radiative heat loss, the growth rate decreases and the unstable range becomes narrow owing
to the decrease of burning velocity of a planar flame. The relations between the burning velocity of a planar flame and heat loss parameter at $Le = 0.3$ are as shown in Fig.3. A standard premixed flame extinguishes when the heat loss parameter exceeds maximum $R_{hl}$ which is $1.27 \times 10^4$ by the D-T model equations and $1.23 \times 10^4$ by the compressible N-S equations. The critical wavelength increases, i.e. critical wave number decreases under non-adiabatic conditions. To understand the instability intensity, growth rate and wave number are normalized by the burning velocity of a planar flame. The normalized growth rate increases and the normalized unstable range widens. This indicates that the instability intensity becomes stronger owing to the heat loss. Figure 4 shows the dispersion relations and normalized dispersion relations of non-adiabatic premixed flames by D-T model and compressible N-S equations, at $Le = 0.3$, $R_{hl} = 4.0 \times 10^{-5}$.

![Dispersion relations at $Le = 0.2 \sim 1.0$ by the D-T model equations](image1)

![Dispersion relations at $Le = 0.2 \sim 1.0$ by the compressible N-S equations](image2)

![Burning velocity of a planar flame depending on the radiative heat loss parameter](image3)

![Dispersion relations (hollow marks) and normalized dispersion relations (solid marks) at $Le = 0.3$, $R_{hl} = 4.0 \times 10^{-5}$, by the D-T model and compressible N-S equations](image4)

By superimposing a finite disturbance, $A_i = 1.0$ with the critical wavelength on a planar flame front, we investigate the characteristics of cellular flame caused by the diffusive-thermal and hydrodynamic effects at $Le = 0.3$. The unburned gas enters from the left, and the burned gas leaves from the right side of the computational domain. In all cases, the superimposed disturbances evolve, and then cellular flames form, which is due to the diffusive-thermal effect in the D-T model equations and couple effects in the compressible N-S equations. The shape of cellular flame front drastically changes with time, and the division of several small cells from large cells and the combination of small cells to large cells appear alternatively. These behaviors become distinct when $L_f$ becomes larger. Because hydrodynamic effect is not included in the D-T model equations, the cell combination and division are not as strong as those on the cellular flame in the compressible N-S equations. Number of small cells on cellular flame in the compressible N-S equations is large.
Figures 5 and 6 show the temperature distribution of the cellular flame fronts at \( Le = 0.3 \), \( L_y = 16\lambda_c \) for different time frame by the D-T model and compressible N-S equations, respectively. In both cases, the burned-gas temperature at the convex flame front towards the unburned gas is higher than the adiabatic temperature of a planar flame. This is because of the diffusive-thermal effect through the flame fronts (Clavin, 1985). The overshoot portion of temperature which is significant in dynamic behavior of cellular flame fronts appeared by the D-T model equations is smaller than that by the compressible N-S equations. In non-adiabatic flames, we find that the unburned-gas temperature decreases towards the downstream because of the radiative heat loss. As the radiative heat loss parameter becomes larger, the unstable behavior of cellular premixed flame becomes stronger, and the overshoot of temperature becomes higher. The temperature distributions of non-adiabatic cellular flames at \( Le = 0.3 \), \( R_{ad} = 4.0 \times 10^{-5} \), \( L_y = 8\lambda_c \) are shown in Fig.7(D-T) and Fig.8(N-S).

Cellular flames have larger surface area, so that the burning velocity of a cellular flame is larger than that of a planar flame. In the present calculations, the burning velocity of a cellular flame is obtained as follows: The reaction rate is integrated throughout the computational domain, and the integrated value is divided by that of a planar flame. This value is consistent with the burning velocity of a cellular flame. Burning velocity of a cellular flame \( S_{cl} \) drastically increases when \( L_y \) becomes larger in the compressible N-S equations, from 3.933 at \( L_y = \lambda_c \) to 7.444 at \( L_y = 16\lambda_c \). This points out that the long-wavelength components of disturbances have a great influence on the dynamic behavior of cellular flames. Although \( L_y \) is increased in the D-T model equations, the difference in burning velocity is very small, 1.528 at \( L_y = \lambda_c \) to 1.558 at \( L_y = 16\lambda_c \). This means unstable behavior of cellular flames induced by pure diffusive-thermal effect is weaker than that by couple effects. The burning velocity of a cellular flame is normalized by that of a planar flame. The normalized burning velocity, \( S_{cl}/S_0 \) increases under the non-adiabatic conditions. This indicates that heat loss promotes the instability intensity of low Lewis number premixed flames. The burning velocity of adiabatic cellular flames at \( Le = 0.3 \), \( L_y = 16\lambda_c \) by the compressible N-S equations is approximately two times greater than that at \( Le = 0.5 \), \( L_y = 16\lambda_c \) obtained by Kadowaki et al. (2017). This is because that diffusive-thermal effect becomes stronger which lead to increase the instability intensity when the Lewis number becomes smaller.

Figure 9(a) shows the \( S_{cl} \) at \( Le = 0.3 \) and 0.5, \( R_{ad} = 0.0 \), and (b) shows the \( S_{cl} \) and \( S_{cl}/S_0 \) at \( Le = 0.3 \), \( R_{ad} = 4.0 \times 10^{-5} \) depending on \( L_y/ \lambda_c \) obtained by the D-T model and compressible N-S equations.

Fig. 5. Temperature distribution of adiabatic cellular flames at \( Le = 0.3 \), \( L_y = 16\lambda_c \) by the D-T model equations (\( t = 350, 450, 495, 600 \)).
Fig. 6. Temperature distribution of adiabatic cellular flames at $Le = 0.3$, $L_y = 16\lambda_c$ by the compressible N-S equations ($t = 10, 22, 39, 77$).

Fig. 7. Temperature distribution of non-adiabatic cellular flames at $Le = 0.3$, $L_y = 8\lambda_c$, $R_{hl} = 4.0 \times 10^{-5}$ by the D-T model equations ($t = 350$ and 450).
Fig. 8. Temperature distribution of non-adiabatic cellular flames at $Le = 0.3$, $L_y = 8 \lambda_c$, $R_h = 4.0 \times 10^3$ by the compressible N-S equations ($t = 22$ and 77).

Fig. 9. (a) $S_{cf}$ at $Le = 0.3$ (current) and 0.5 (Kadowaki et al., 2017), $R_h = 0.0$, and (b) $S_{cf}$ and $S_{cf}/S_u$ at $Le = 0.3$, $R_h = 4.0 \times 10^3$ by the D-T model and compressible N-S equations.

We perform fractal analysis by using binary image of the reaction rate of adiabatic cellular flames as shown in Figs 10 and 11 for the D-T model and compressible N-S equations, respectively. By adopting the box-counting method (Miyauuchi et al. 1994; Mukaiyama et al. 2013), fractal dimensions are obtained. The fractal dimension for two-dimensional flow $d$ is estimated by the relation as in equation (11), where $N_r$ is the number of circles and $r$ is the radius of circles for cellular flames.

$$d = \frac{\log(N_r)}{\log(r)}$$  \hspace{1cm} (11)

From the gradients of lines shown in Fig.12, we obtain $d$ at $Le = 0.3$, $L_y = 8 \lambda_c$, and $16 \lambda_c$ by the D-T model and compressible N-S equations. The fractal dimension $d$ is 1.047 at $8 \lambda_c$ and 1.050 at $16 \lambda_c$ by the D-T model equations. It is 1.261 at $8 \lambda_c$ and 1.289 at $16 \lambda_c$ by the compressible N-S equations. The fractal dimensions obtained by the D-T equations are much smaller than those obtained by the compressible N-S equations since dynamic behaviors of cellular flames is strong due to couple effects as shown in Fig.6. The values obtained by the compressible N-S equations at $Le = 0.3$, $L_y = 16 \lambda_c$ in this calculation is slightly larger than the value $d = 1.2$ at $Le = 0.5$, $L_y = 16 \lambda_c$ of Kadowaki et al. (2017). This is because the dynamic behavior becomes stronger and burning velocity increases when the Lewis number becomes smaller in large domain owing to hydrodynamic and diffusive-thermal effects.
Fig. 10. Binary images of the reaction rate of adiabatic cellular flames at $Le = 0.3, L_y = 16\lambda_c$ by the D-T model equations ($t = 375, 452$ and $600$).

Fig. 11. Binary images of the reaction rate of adiabatic cellular flames at $Le = 0.3, L_y = 16\lambda_c$ by the compressible N-S equations ($t = 42, 76$ and $97$).

Fig. 12. Relations between Log ($N_r$) and Log ($r$) at $Le = 0.3, L_y = 8\lambda_c, 16\lambda_c$ obtained by the D-T model equations (left) and the compressible N-S equations (right) for the calculation of $d$ under the adiabatic conditions.
Differing from the works of Shibayama and Kuwana (2014), we obtain the new results that the fractal dimensions based on the D-T model equations at $Le = 0.3$ are larger than unity. This shows that the fractal dimension becomes larger owing to the increase of strength of diffusive-thermal instability when the Lewis number is sufficiently low. In addition, larger fractal dimension is obtained when the radiative heat loss is taken into account. This is because flame instability becomes stronger owing to heat loss and the flame front becomes more wrinkle as described in Figs.7 and 8. At $R_{dl} = 4.0 \times 10^{-5}$ and $L_y = 8 \lambda_c$, $d$ is 1.067 and 1.315 by the D-T model and compressible N-S equations, respectively. To confirm the increase of fractal dimension of cellular premixed flames at sufficiently low Lewis numbers, the additional computations at $L_y = 8 \lambda_c$ based on the D-T model equations are performed for Lewis numbers 0.2 and 0.1 under the adiabatic and non-adiabatic conditions. The obtained results are shown in Fig.13 at $Le = 0.2$ (left) and at $Le = 0.1$ (right), under the adiabatic and non-adiabatic conditions ($R_{dl} = 0.0$ and $4.0 \times 10^{-5}$). The fractal dimensions for both cases are larger than unity, and $d$ of non-adiabatic flames are large compared with the adiabatic flames.

![Graphs showing Log (N) vs Log (r) for different R_{dl} values](image)

**Fig. 13.** Relations between Log ($N_i$) and Log ($r$) at $Le = 0.2$ (left) and at $Le = 0.1$ (right), $L_y = 8 \lambda_c$ obtained by the D-T model equations for the calculation of $d$ under the adiabatic and non-adiabatic conditions.

### 5. Conclusions

In this paper, we have performed the numerical calculations of reactive flows in large domain based on the D-T model and compressible N-S equations including the one-step chemical reaction. From these two equations, the visualizations of unstable behaviors, characteristics of cellular flames such as burning velocities and fractal dimensions, have been investigated.

The results show that the growth rates obtained by the D-T model equations are small and the unstable ranges are narrow compared with those obtained by the compressible N-S equations. This is because hydrodynamic effect is neglected in the D-T model equations, and the couple effects are considered in the compressible N-S equations. The growth rate at the Lewis number of unity is negative by the D-T model equations and positive by the compressible N-S equations. This indicates that the premixed flame is diffusively stable and hydrodynamically unstable at the Lewis number of unity.

The unstable behaviors of cellular flames in the compressible N-S equations are much stronger than those in the D-T model equations. This is because that both hydrodynamic and diffusive-thermal effects are taken into account in the compressible N-S equations but not in the D-T model equations. Although the cellular flames obtained by the compressible N-S equations move vigorously towards the unburned gas, those obtained by the D-T model equations move slightly. Many small cells appear on cellular flames when $L_y$ becomes larger in the compressible N-S equations, which cause the increase in flame surface area and average burning velocity. However, the number of cells is small in the D-T model equations. Thus, the average burning velocity of cellular flames from one wavelength to multiple...
wavelengths by the compressible N-S equations is large compared with that by the D-T model equations.

When the radiative heat-loss effect is taken into account, growth rate decreases and unstable range narrows as the burning velocity of a planar flame decreases. The increase in normalized growth rate and the widener in normalized unstable range indicates that the instability intensity becomes stronger due to the heat loss. The normalized burning velocity of a non-adiabatic cellular premixed flame is larger than that of an adiabatic cellular flame.

Consequently, we calculate the fractal dimension by using binary image of the reaction rate of cellular flames by box-counting method. The values of $d$ obtained by the compressible N-S equations are larger than those obtained by the D-T model equations. In addition, we confirm that the fractal dimension of diffusively unstable cellular flame is larger than unity at sufficiently low Lewis numbers. Owing to the increase of strength of instability, the fractal dimension of a non-adiabatic cellular premixed flame is larger than that of an adiabatic one.

The obtained results in this study indicate that cellular flames become more dynamic in large computational domain owing to the dominant diffusive-thermal effect together with hydrodynamic effect at sufficiently low Lewis numbers. Moreover, the radiative heat loss promotes the instability of low Lewis numbers premixed flames.

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