Asymptotic Safety in the Standard Model?

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Both problems might be solved within the AS scenario.
The Higgs field is parametrized in terms of a bosonic field $\phi$ with a Lagrangian

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1-loop correction to the four-Higgs-boson coupling $\lambda \phi^4$ is represented by the diagram
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We need a non-perturbative tool to study triviality!
We observe a huge hierarchy in the standard model:

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Use effective average action $\Gamma_k$, which contains all fluctuations of the quantum fields with momenta larger than a scale $k$. 
Effective quantum field theory

- Use effective average action $\Gamma_k$, which contains all fluctuations of the quantum fields with momenta larger than a scale $k$.
- Expansion in terms of running couplings $g_{i,k}$ and all possible field operators $O_i$.

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Dependence of the effective action on the scale $k$ (or more conveniently $t = \log(k/\Lambda)$) is by definition given by the $\beta$-functions of the running couplings:

$$\partial_t \Gamma_k[\chi] = \sum_i \beta_{i,k} O_i.$$
Effective average action: $\Gamma_k[\chi] = \sum_i g_{i,k} \mathcal{O}_i$, Scale dependence: $\partial_t \Gamma_k[\chi] = \sum_i \beta_{i,k} \mathcal{O}_i$. 

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"Theory Space"
Hierarchy problem in the Asymptotic Safety scenario

- Critical exponents $\Theta_I$: Tell me how fast the effective average action changes at a FP.
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If all of them are small $\ll 1$ then the hierarchy problem is solved.
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Use exact renormalization group equations (ERGE) derived from Path-Integral representation (Wetterich '93)

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{STr} \{ [\Gamma_k^{(2)}[\Phi] + R_k]^{-1} (\partial_t R_k) \}, \quad \partial_t = k \frac{d}{dk}$$
Toy model - Chiral Yukawa system, no gauge bosons

\[ \Gamma_k = \int d^d x \left\{ i(\bar{\psi}_L^a \phi^a_L + \bar{\psi}_R^a \phi^a_R) + (\partial_\mu \phi^a)(\partial^\mu \phi^a) ight. \\
\left. + U_k (\phi^a_{\alpha} \phi^a_{\alpha}) + \bar{h}_k \bar{\psi}_R^a \phi^a_L \psi_L^a - \bar{h}_k \bar{\psi}_L^a \phi^a_{\alpha} \psi_L^a \right\} \]
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+ U_k (\phi^{a\dagger} \phi^a) + \bar{h}_k \bar{\psi}_R^a \phi^{a\dagger} \psi_L^a - \bar{h}_k \bar{\psi}_L^a \phi^{a\dagger} \psi_R^a \}

- where we define \( \rho = \phi^{a\dagger} \phi^a \).
- invariant under chiral \( U(N_L)_L \otimes U(1)_R \) transformations.
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For the phase with spontaneously broken symmetry (SSB), we expand the effective potential around its minimum: \( \kappa_k := \tilde{\rho}_{\text{min}} > 0 \),

\[ u_k = \frac{\lambda_{2,k}}{2!}(\tilde{\rho} - \kappa_k)^2 + \frac{\lambda_{3,k}}{3!}(\tilde{\rho} - \kappa_k)^3 + \ldots \]

\( \kappa, \lambda_{n_{\text{max}}}, \lambda_2 > 0. \)
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Fixed-point analysis

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Example for a leading-order truncation expanded up to $\frac{\lambda_6}{6!} \rho^6$ in the effective potential and $N_L = 10$:

$$\kappa^* = 0.0152, \quad \lambda^* = 12.13, \quad h^* = 57.41,$$
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$$\Theta_1 = 1.056, \quad \Theta_2 = -0.175, \quad \Theta_3 = -2.350$$
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The real part of the relevant direction is 1.056 and not anymore 2, so the hierarchy problem is slightly weakened.
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Choosing $v = 246\text{GeV}$ and $N_L = 10$ as an example, we find

$$m_{\text{Higgs}} = 0.81v, \quad m_{\text{top}} = 5.56v.$$
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- Next step: In a more realistic model (closer to the standard model), we have to account for gauge bosons and get rid of massless modes.
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Publication is in preparation, previous work can be found at: arXiv:0901.2459
Discussion and Outlook

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- Also gravitational effects can be included: O. Zanusso & R. Percacci and collaborators.