Single- and double-scattering production of four muons in ultraperipheral \( {\text{PbPb}} \) collisions at the Large Hadron Collider

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Abstract

We discuss production of two \( \mu^+\mu^- \) pairs in ultraperipheral ultrarelativistic heavy ion collisions at the LHC. We take into account electromagnetic (two-photon) double-scattering production and for a first time direct \( \gamma\gamma \) production of four muons in one scattering. We study the unexplored process \( \gamma\gamma \rightarrow \mu^+\mu^-\mu^+\mu^- \). We present predictions for total and differential cross sections. Measurable nuclear cross sections are obtained and corresponding differential distributions and counting rates are presented.

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I. INTRODUCTION

Ultraperipheral heavy ion scattering is a special category of nuclear collisions [1–5]. This field got a new impulse with the start of the LHC. Several final states are possible in ultraperipheral heavy ion collisions (UPC). Charged particles are final states that are relatively easy to measure. The present experiments on UPC concentrated rather on two-body final states. Typical examples are $e^+e^−$ [6], $\mu^+\mu^−$ [7], $\pi^+\pi^−$ [8]. $pp$ scattering is another interesting possibility [9]. Recently the ATLAS collaboration has demonstrated that also $\gamma\gamma$ final states are available [10] leading to a first experimental verification of elastic $\gamma\gamma \to \gamma\gamma$ scattering (for a theoretical work see e.g. [11]).

In principle, also four-body final states can be measured. For example the STAR collaboration obtained some results for $\pi^+\pi^−\pi^+\pi^−$ [12]. Here both resonant and nonresonant contributions can be present [13]. Here the double-scattering (two subsequent $\gamma\gamma$ interactions) in UPC may play an important role.

Recently we made a first estimation of the production of four electrons in UPC of two lead ions [6]. Only the double-scattering contribution was considered there. Here we do similar calculation but for four muon production. However, in addition to the double-scattering mechanism we include here for a first time also the contribution of the single-scattering process with underlying $\gamma\gamma \to \mu^+\mu^−\mu^+\mu^−$ elementary process. We wish to explore the competition of the two mechanisms. The single-scattering mechanism in nuclear collisions was not discussed so far in the literature. Therefore we start from considering the elementary $\gamma\gamma \to \mu^+\mu^−\mu^+\mu^−$ subprocess, also not discussed in the literature.

II. SOME THEORETICAL ASPECTS OF MUON PRODUCTION IN HEAVY ION UPC

![Diagrams for (a) double-scattering and (b) direct (single-scattering) production of two $\mu^+\mu^−$ pairs in ultrarelativistic UPC of heavy ions.](image)

The differential cross section for double-scattering production of four muons can be expressed through a probability density to produce a first (superscript $I$) and second

\footnote{Charge conservation is understood. We will regularly refer to “electrons” and “muons” where we mean neutral combinations of electron-positron pairs and muon-antimuon pairs.}
In Eq. (2.2) \( \mu^+ \mu^- \) pair

\[
\frac{d\sigma_{AA\rightarrow AA\mu^+\mu^-}}{dy_{\mu^+}dy_{\mu^-}dp_{t,\mu}} = \frac{1}{2} \int d^2b \times \frac{dP^I_{AA\gamma\gamma\gamma AA\mu^+\mu^-}}{dy_{\mu^+}dy_{\mu^-}dp_{t,\mu}} \times \frac{dP^I_{AA\gamma\gamma\gamma AA\mu^+\mu^-}}{dy_{\mu^+}dy_{\mu^-}dp_{t,\mu}}. (2.1)
\]

The additional factor 1/2 comes from identity of the two pairs. The probability for each of pair is the same, but the correlations between muons of the same sign are different than correlations for an opposite-sign pair. Above \( b \) is an impact parameter i.e. the distance between colliding nuclei in the transverse direction, \( y_{\mu} \) is the rapidity of a muon and \( p_{t,\mu} \) is transverse momentum of a muon. Here we use the fact that both muons in a given scattering (both first and second scattering) have the same transverse momenta. The formula for the differential cross section (Eq. (2.1)) allows to control (calculate) the whole kinematics of the outgoing particles (e.g. scattering angle of each of muons and invariant mass of two/four muons). The probability for the production of the two muon pairs reads:

\[
P_{AA\gamma\gamma\gamma AA\mu^+\mu^-}(b; y_{\mu^+}, y_{\mu^-}, p_{t,\mu}) = \int \frac{d\sigma_{\gamma\gamma\rightarrow \mu^+\mu^-}(W_{\gamma\gamma})}{dz} N(\omega_1, b_1) N(\omega_2, b_2) S_{abs}^2(b) \times \frac{W_{\gamma\gamma}}{2} dW_{\gamma\gamma} dY_{\mu^+\mu^-} dz d\overrightarrow{b}_x d\overrightarrow{b}_y. (2.2)
\]

The cross section for the production of a single muon pair in heavy-ion collisions

\[
\sigma_{AA\rightarrow AA\mu^+\mu^-} = \int d^2b \times \frac{dP_{AA\gamma\gamma\gamma AA\mu^+\mu^-}}{dy_{\mu^+}dy_{\mu^-}dp_{t,\mu}} \times dy_{\mu^+}dy_{\mu^-}dp_{t,\mu}. (2.3)
\]

In Eq. (2.2) \( W_{\gamma\gamma} = \sqrt{4\omega_1\omega_2} \) is the energy available in the \( \gamma\gamma \) system and \( \omega_i \) is energy of the photon which is emitted from the first or second nucleus. \( Y_{\mu^+\mu^-} = \frac{1}{2}(y_{\mu^+} + y_{\mu^-}) \) is the rapidity of the outgoing muon pair. The quantities \( \overrightarrow{b}_x, \overrightarrow{b}_y \) are the components of the \( \overrightarrow{b} = (b_1 + b_2)/2 \) vector where \( b_1 \) and \( b_2 \) indicate the point (transverse distance from the first and second nucleus) in which the photons collide with each other and particles (muons) are produced. A diagram illustrating the quantities in the impact parameter space can be found for example in Ref. [7]. In our calculations we use the so-called realistic form factor which is a Fourier transform of the charge distribution in the nucleus. A detailed study of this form factor and its role was presented e.g. in Ref. [7]. In the same paper an expression for the photon flux \( N(\omega_i, b_i) \) and its dependence on the charge form factor of the “emitting” nucleus was given explicitly.

In the case of direct four-muon production we use the following formula:

\[
\sigma_{4\mu} = \int \sigma_{\gamma\gamma\rightarrow 4\mu}(W_{\gamma\gamma}) N(\omega_1, b_1) N(\omega_2, b_2) S_{abs}^2(b) d^2b d\omega_1 d\omega_2 d\overrightarrow{b}_x d\overrightarrow{b}_y. (2.4)
\]

By appropriate binning one can obtain the distribution in \( M_{4\mu} = W_{\gamma\gamma} \).
A. Dimuon production in Pb+Pb UPC

Before going to the production of two (neutral) dimuon pairs we wish to check for the first time our predictions for the production of one muon pair with the preliminary data of the ATLAS collaboration [14]. In our calculations we imposed cuts on muon rapidities $-2.4 < y < 2.4$ and on muon transverse momenta $p_{t,i} > 4$ GeV. In Fig. 2 we show distributions in the rapidity of the pair $Y_{\mu\mu}$ (left panel) for different windows of dimuon invariant mass specified in the figure and in dimuon invariant mass (right panel) for two windows of $Y_{\mu\mu}$ specified in the figure. We get slightly larger cross sections than observed experimentally. There can be a few reasons. Perhaps not all experimental conditions were included by us. In our present simplified treatment we assume $S^2_{abs} = \theta (b - |R_1 + R_2|)$ which is a rather crude approximation for collisions when peripheries of nuclei interact with each other. We think this is, however, a sufficiently precise approximation for our first estimation of four-muon production in UPC. A slightly better agreement was obtained in [6] for the reaction PbPb→PbPbe$^+\!\!e^-\!$ measured by the ALICE collaboration [15].

III. $\gamma\gamma \rightarrow \mu^+\mu^-\mu^+\mu^-$ ELEMENTARY CROSS SECTION

In our approach, the cross section for the elementary $\gamma\gamma \rightarrow \mu^+\mu^-$ process is one of the basic ingredients for the nuclear double-scattering mechanism. The elementary process with on-shell photons could not yet be studied experimentally. It was demonstrated recently that in nuclear collisions the subprocess $\gamma\gamma \rightarrow \mu^+\mu^-$ can be observed [14]. The total cross section for $\gamma\gamma \rightarrow \mu^+\mu^-$ is well known [1] and can be obtained based on standard Feynman diagram technique. In our calculations of one pair production as well
as the double parton scattering we use rather differential cross section as a function of $z = \cos \theta$, where $\theta$ is muon scattering angle. The corresponding formula is also well known:

$$\frac{d\sigma}{dz} = \frac{(4\pi\alpha_{em})^2 v^2 (1 - v^2)(1 - z^2) - v^4 z^4}{8\pi W^2 (1 - v^2 z^2)^2},$$

where $v = \sqrt{1 - \frac{4m_\mu^2}{W^2}}$.

The calculations for $\gamma\gamma \rightarrow \mu^+\mu^-$ were performed with the help of KATIE [16]. It is an event generator that is specially designed to deal with initial states that have an explicit transverse momentum dependence, but can also deal with on-shell initial states, like the ones we consider here. Furthermore, KATIE is a parton-level generator for hadron scattering, but requires only a few adjustments to deal with photon scattering. All necessary tools to sample the phase space and to calculate the scattering amplitude are available in the library AVHLIB [17], which KATIE employs. Amplitudes for processes with several final-state particles are calculated numerically via recursive methods keeping the computational complexity under control. Large algebraic expressions are avoided, and the numerical approach suffices for the generation of event files, which can then be used to obtain the distributions of the desired variables.

Fig. 3 presents the elementary cross section for the production of $\mu^+\mu^-$ (dotted lines) and $\mu^+\mu^-\mu^+\mu^-$ (dashed lines) as a function of $\gamma\gamma$ collision energy for different cuts on transverse momenta of all muons in the final state.

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Fig. 3 presents the elementary cross section for the production of $\mu^+\mu^-$ (dotted lines) and $\mu^+\mu^-\mu^+\mu^-$ (dashed lines). Here we consider the single-scattering mechanism only. This means that muons are produced directly from $\gamma\gamma$ fusion. We show the total cross section as a function of energy for four different ranges of transverse momentum ($p_{t,\mu}$ in full range, $p_{t,\mu}$ larger than 0.2, 0.5 and 1 GeV). We see that the value of $p_{t,\mu}^{\text{min}}$ has an influence mainly on small $W_{\gamma\gamma}$ for $\gamma\gamma \rightarrow \mu^+\mu^-$. For four-muon production, the larger value of $p_{t,\mu}^{\text{min}}$ the smaller the cross section over the whole range of energy. Simultaneously, it
is important to note that the cross section for single muon pair production is at least one order of magnitude larger at $W = 20$ GeV, and that it is about four orders of magnitude larger than the cross section for four-muon production for smaller values of $W$ over the whole range of $p_t$. In contrast to two-muon production, the cross section for four-muon pair production increases with larger $W$.

![Graph](image)

**FIG. 4.** Differential cross section as a function of transverse momentum of muon (left panel) and muon rapidity (right panel) for different values of the $\gamma\gamma$ collision energy specified in the figure.

Direct production of two $\mu^+\mu^-$ pairs is a new mechanism which was never considered before. It seems interesting to examine this reaction more closely. We would like to study kinematic characteristics of one muon or one pair of muons out of four muons in the final state. The left panel of Fig. 4 shows the differential cross section as function of the transverse momentum of one randomly selected muon. The three lines present results for three different values of the $\gamma\gamma$ collision energy ($W = 5, 10$ and $20$ GeV). Obviously, the larger energy the larger the total cross section. The right panel of the same figure presents the differential cross section as a function of the muon rapidity. In general, muons in the four-body final state are produced in the forward/backward directions rather than in the mid-rapidity range.

Experimentally, some times, it is more useful to make distributions of harder and softer muons separately for positive and negative ones separately. By harder we mean the muon with $p_t = \max\{p_{1t}, p_{3t}\}$ (or $p_t = \max\{p_{2t}, p_{4t}\}$) and by softer we mean the muon with $p_t = \min\{p_{1t}, p_{3t}\}$ (or $p_t = \min\{p_{2t}, p_{4t}\}$). In Fig. 5 we show an example for the $\gamma\gamma$ collision energy $W = 10$ GeV.

![Graph](image)

**FIG. 5.** Differential cross section as function of the rapidity difference for different combinations of muons. The left panel depicts the distribution in the difference of the rapidities of opposite-sign muons and the right panel is for the same-sign muons. We see the largest probability for the case when muons of the opposite-sign fly both at small and large rapidity distances while muons of the same-sign fly preferably at large rapidity distances.
IV. PREDICTIONS FOR FOUR MUONS PRODUCTION IN UPC OF HEAVY IONS

At present the process $\gamma\gamma \to \mu^+\mu^-\mu^+\mu^-$ cannot be studied experimentally. This process can be, however, studied in collisions of heavy ions. In the present calculation we assume $\sqrt{s_{NN}} = 5.02$ TeV.

In Fig. 7 we show the (rather academic) distribution of the impact parameter for lead-
lead collisions. Of course such a distribution cannot be measured experimentally. The solid line on top corresponds to double-scattering mechanism. No cuts on rapidities nor on muon transverse momenta are included here. The contribution of single-scattering is shown by the dashed lines below, for different cuts on the muon transverse momenta specified in the figure. The single-scattering cross sections are smaller than the double-scattering contribution. This confirms our naive expectations in [6].

![Graph showing differential cross section](image)

**FIG. 7.** The differential cross section for \( \text{PbPb} \rightarrow \text{PbPb} \mu^+ \mu^- \mu^+ \mu^- \) as function of the impact parameter. The top solid line corresponds to double-scattering contributions while the dashed lines below are for single scattering. Each of the dashed lines corresponds to a distinct cut on transverse momenta of each muon.

In the left panel of Fig. 8 we show rapidity distributions of muons from the double-scattering mechanism (dashed line at the bottom) as well as for the single \( \mu^+ \mu^- \) pair production (top dashed line). Clearly the cross section for the two-pair production is more than three orders of magnitude smaller than that for the single-pair production. In the right panel we show corresponding differential distributions in several differences of rapidities of the produced muons. The muons from the same scattering are strongly correlated. The corresponding distribution peaks at \( y_{\text{diff}} = 0 \). There is no similar correlation for muons produced in different scattering (the same-sign muons). It is not easy to show similar distributions for single-scattering with the four-body final state. This can be understood by inspecting the organization of our nuclear calculations, where first a simple energy dependent grid of the cross section for \( \gamma \gamma \rightarrow 4\mu \) is prepared with the help of the Monte Carlo code KA\textsc{tie} which is then used for calculating the nuclear cross section (see Eq. 2.4). Preparing more detailed multi-dimensional distributions that could be used in nuclear calculations clearly goes beyond the scope of the present paper.

In Fig. 9 we show differential distributions in the invariant mass of the four-muon system (left panel) as well as distributions in the transverse momentum of individual muons (right panel). The distributions in the left panel are done for different cuts on the transverse momenta of each of the muons. There is a strong dependence on the cuts. However, the cross sections for double-scattering are considerably larger than those for...
single scattering. We conclude that four-muon events measured in a future will originate mostly from the double-scattering mechanism, which is easier to handle in the case of nuclear calculations. In the right panel we show the transverse momentum distributions of a muon for four-muon events (bottom curve) and for two-muon events (top curve). The two distributions have the same shape in transverse momentum. The distribution in transverse momentum is very steep which explains the strong dependence on the transverse momentum cut as demonstrated e.g. in the left panel.

Finally we wish to show some two-dimensional distributions. We start with distributions in rapidities of two of the four muons in the final state, see Fig. 10. The left panel is for the opposite-sign muons from the same scattering, while the right panel is for the same-sign muons, originating evidently from different scatterings. There is also a similar distribution for opposite sign muons originating from different scatterings. In a realistic experiment one cannot exactly determine from which scatterings the two randomly chosen muons are. However, muons coming from the same scattering have practically the same transverse momenta. This (transverse momentum balance) can be used to construct experimental distributions similar to those presented here. Real experiments have, however, a limited range of muon rapidities. In principle, ATLAS and CMS could try to construct such two-dimensional distributions. However, we do not expect too large statistics (see Table I below), so one-dimensional distributions in $y_{\text{diff}}$ may be the only practical option.

In Fig. 11 we show two-dimensional distributions in the invariant masses for the first and second scattering. On the experimental side one would need to select opposite-sign muons in each pair and use the transverse momentum balance check to identify the “first” and “second” scattering. We expect that there will be cases when such an identification may not be possible due to finite transverse momentum resolution. The actual efficiency of such an identification of the two scatterings requires dedicated Monte
FIG. 9. Differential cross section for \( \text{PbPb} \rightarrow \text{PbPb} \mu^+ \mu^- \mu^+ \mu^- \) as function of the invariant mass of four muons (left panel) and transverse momentum of each of the muons (right panel).

FIG. 10. Two-dimensional cross section as function of the rapidities of muons for the double-scattering mechanism. The left panel is for \( d\sigma/dy_{\mu^+}dy_{\mu^-} \) and the right panel is for \( d\sigma/dy_{\mu^+}dy_{\mu^+} = d\sigma/dy_{\mu^-}dy_{\mu^-} \). No extra cuts on rapidities or transverse momenta were imposed here.

Carlo studies. This goes beyond the scope of the present paper.

Finally, in Table I we show predicted numbers of counts for different transverse momentum and rapidity cuts specified in the table. In this calculation we have assumed an integrated luminosity \( L_{\text{int}} = 1 \, \text{nb}^{-1} \).

Rather small numbers of counts are predicted for the selected luminosity \( L = 1 \, \text{nb}^{-1} \). For the cuts relevant for the present ATLAS measurement of dimuons, \( p_{t,\text{cut}} = 4 \, \text{GeV} \).
FIG. 11. Two-dimensional cross section as function of the invariant masses of the first and second pair of muons in the double-scattering mechanism. No extra cuts on rapidities or transverse momenta were imposed here.

TABLE I. The cross section in nb for selected cuts and number of counts for selected integrated luminosity $L_{int} = 1$ nb$^{-1}$.

| experimental cuts | cross section | number of counts |
|-------------------|---------------|------------------|
| $-2.5 < y_i < 2.5 , p_t > 0.5$ GeV | 815 | 815 |
| $-2.5 < y_i < 2.5 , p_t > 1.0$ GeV | 53 | 53 |
| $-0.9 < y_i < 0.9 , p_t > 0.5$ GeV | 31 | 31 |
| $-0.9 < y_i < 0.9 , p_t > 1.0$ GeV | 2 | 2 |
| $-2.4 < y_i < 2.4 , p_t > 4.0$ GeV | $1.9 \times 10^{-3}$ | $\ll 1$ |

it would be very difficult to observe any event with four muons. For CMS and ALICE the situation seems to be better with $p_t,\text{cut} = 1$ GeV. A real measurement would be a first experimental confirmation of double-scattering effects in UPC.

V. CONCLUSIONS

In the present paper we have discussed production of two pairs of muons (four muons) in ultraperipheral ultrarelativistic heavy ion collisions at $\sqrt{s_{NN}} = 5.02$ TeV. We have included both the double-scattering mechanism discussed before for production of two electron pairs and the single-scattering mechanism for the first time.

We have presented several distributions for the elementary process $\gamma \gamma \rightarrow \mu^+ \mu^- \mu^+ \mu^-$. The calculation have been done using the automated code KAT1E adapted for the double-photon induced processes. The cross section for different cuts on muon transverse momenta has been calculated as a function of the (sub)collision energy.

The elementary cross sections have been calculated on a grid, and this grid was used next for calculating nuclear $PbPb \rightarrow PbPb4\mu$ cross sections. The flux of photons has
been calculated using a realistic nuclear charge form factor, being a Fourier transform of the realistic charge distribution. Several differential single-particle distributions and correlation distributions have been calculated and presented. For a first time, we have shown explicitly that the cross section for the single-scattering mechanism is considerably smaller than the cross section for the double-scattering mechanism. This shows that the double-scattering mechanism is sufficient for detailed studies and planning experiments.

We have shown a few one- (rapidity, transverse momentum, invariant masses) and some two-dimensional distributions that could be measured at the LHC. The cross sections for four-muon production strongly depend on the cuts on muon transverse momenta. We have discussed how to study some correlation observables from the same $\gamma\gamma$ scattering by using the transverse momentum balance of opposite-sign muons.

Finally, we have presented counting rates for experimental situations, i.e. including cuts on transverse momenta and rapidities. Measurable cross sections and counting rates of the order of hundreds of events have been predicted. We hope that some LHC collaborations (CMS and ALICE) will be able to start such studies in the near future while for the present ATLAS cuts the situation seems rather difficult.

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