A potential instructional theory for meaning of minus sign

N F Fuadiah¹,²*, D Suryadi² and T Turmudi²

¹ Departement of Mathematics Education, Universitas PGRI Palembang, Jl. Jendral Ahmad Yani, Palembang 30251, Indonesia
² Departement of Mathematics Education, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No. 229, Bandung 40154, Indonesia

* fahrizafuadiah@student.upi.edu

Abstract. This study presents the results of teaching experiment of 7th-grade students about the meaning of the minus sign as the initial knowledge toward the concept of negative integers and its operations. A hypothetical learning trajectory (HLT) was designed on the basis of findings in a preliminary study showing that most students did not understand the meaning of the minus sign that resulted in students' ability in operations involving negative integers. Implementation and revision of HLT were carried out by involving 32 seventh grade students and a math teacher by taking into account the stages of the didactical situation. Furthermore, group learning can encourage students to identify the meaning of minus sign encountered in a context. All data in the form of learning video recording, interview, and students worksheet are analyzed qualitatively based on the perspective of the theory of didactical situation to get the instruction design according to the condition and requirement of the students. The learning practices indicate that the students can differentiate the meaning of the minus sign according to the context of the given problem.

1. Introduction

Some research results show that most students construct negative numbers with the context of debt [1,2]. A student interviewed by the researcher stated that the operation \(-4 - 3 = -7\) is true for the reason "to have debt then debt 3, the debts being 7. The same student constructs the problem \(-4 - (-3) = -1\) with the reason "negative met negative equals positive; \(-4 + (-3) = -4 + 3\). After exploring, the student equates the meaning minus "\(-\)" on \((-3)\), whereas the minus sign in the first part is the subtraction operation, while the minus sign in the second part is the symbol of the number negative. In this case, the student answers correctly but the reason is wrong (pseudo true). Furthermore, there is an incorrect process in constructing the concept of integer operation, which equates the meaning between the number symbol and the number operation on the "\(-\)" [2].

When the negative number material was introduced, there is a conceptual and procedural change involving knowledge that is a number less than zero [3]. Their perception is that numbers are real so there can be no "less than nothing" numbers [4]. About one-quarter of high school students interviewed were correct in identifying \(-4\) as larger than \(-4\) [5]. Furthermore, they did not realize that they had enough information to determine which one was larger from \(-x \text{ or } x\). It appears that students are still experiencing obstacles in the meaning of the minus sign. The error of meaning of this symbol can of course affect the next mathematical problem of algebra [6]. Therefore students must be equipped with an understanding on minus sign in math learning before they understand the count operation that involves this minus sign.
because in this count operation there will be many minus marks in the subtraction operation and the symbol of the number.

For the world of science, mathematics has a role as a symbolic language that allows the realization of accurate and precise communication. As a symbolic language, every symbol in mathematics has its own meaning so that students who study mathematics need to know the language of this symbol to avoid miscommunication and ambiguity of meaning. The realization of various symbols may differ in different contexts as well (such as variables or parameters) and evolve from an intuition to those differences [7].

The minus sign "-" is used in three ways: the common binary, unary, and symmetric function [5,8,9]. The minus sign serves as a binary operator in the two inputs used to generate one output. In addition, subtraction, multiplication, and division are examples of binary operators. The minus sign is used as a unary operator, in that it involves only one input and one output. When one thinks of \((-4)\) as "opposite of negative 4", then the first minus sign serves as a unary operator, i.e. the opposite of. However, in the case of \(-7 + 5 = -2\), some people may see the minus sign at -2 as the unary operator, not as part of the number but as opposed to 2. The minus sign in -2 can be used flexibly with both minus sign meaning. Although students may initially face difficulties because three meanings are used for the same symbol; has the same symbol but represents several different concepts.

The application of the theory of didactic situational through the design of didactical situations created by teachers in classroom learning activities is expected to develop students' potential, which they can build on their own knowledge to be achieved through a series of abstraction processes [10]. Actions and feedback through the strategy will enable the formation of new knowledge. According to that the authors focus on the problem of "how is the instructional theory that has good potential to embed the meaning of minus sign to students?" This design is expected to support learning integer operations in junior secondary school.

2. Method
This study is part of a series of teaching experiments on Didactical Design Research in learning negative integers. This research begins by designing HLT to build the meaning of minus sign to the students. HLT is made up of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning process a prediction of how the students' thinking and understanding will evolve in the context of the learning activities [11]. This HLT design has been through the validation and revision phase. This design was implemented to 32 seventh grade students and a math teacher at one of Junior Secondary Schools in Palembang, Indonesia. The conformity of HLT with its actual state in the learning process becomes the basis in examining how the construction of the meaning of minus sign is constructed according to the characteristics of the students.

Data is taken through recording all learning activities and student responses in audio visual, field notes and student work. Interviews were also conducted on several students to explore information about their understanding of the material they had learned. All data were then analyzed qualitatively descriptive as reference material in the evaluation of instructional design. The mathematical practice analysis will provide for the feedback of the emerging instructional theory on integers [12].

3. Results and discussion
There were 4 phases in the HLT that we designed at the beginning of the lesson. The next stage is integrated in the HLT integer arithmetic operation. In this paper, we use HLT specifically designed to introduce students to the meaning of minus sign. Figure 1 represents HLT for the meaning of the minus sign in the class practices.
Figure 1. HLT for meaning of minus sign.

3.1 Activities 1: Identify the use of minus signs in various contexts
The didactical situation is built on independent tasks assigned to students in groups without any intervention from the teacher in the sense that the teacher does not lead to a particular answer but only gives direction or explanation when there are questions that students are not familiar with [13]. This activity is intended as a stage of action situations in accordance with the students' thinking stage of assigning tasks to the students by playing; the teacher gives some questions in the question card that must be answered by the students. Each group should be able to answer the question in just 10 minutes. This situation is an action situation that encourages students to a meaningful activity through a process of adaptation to a new knowledge. Each group has its own strategy and each member of the group provides parcels in solving the problem.

3.2. Activities 2: Provide examples of contexts related to the minus sign and explain its function
This activity is the opposite of the first stage aiming for students to represent the minus sign in the appropriate context. Research shows that all these contexts have their strengths, and the students demonstrate a better negative understanding of negative numbers [12]. The mental object formed from this context is the beginning of the abstraction process as a result of the relationship between mental action [15]. At the first students will be have difficulty in this task. Something like this happens when explaining the minus sign as a unary function that is as 'opposite of'. Explicitly, the teacher mentions the context of the floor of a building. Students are familiar with the context of the opposite, such as the opposite of the right is left, the opposite of the top is bottom, and the opposite of the positive is negative or otherwise, but not yet accustomed to interpret the 'opposite of' with the minus sign.

3.3. Activities 3: The challenge from the teacher
This learning activity is done through teacher action showing revelation containing minus sign and student giving answer with argument. Teacher shown some cards that contain arithmetic form and students mentioned the function of the minus sign contained in the card. The most challenging question of students is how to represent - - 5. To explain this, didactic interventions still need to be given by teachers in anticipation of student responses. Didactic interventions are a fairly effective way of fostering children's understanding to facilitate the process of adaptation and acculturation [14]. The interaction between teacher and students aims to construct piece of knowledge as a situation of formulation [13]. Such a process can lead to a situation of validation, the teacher starts with what is known and ends with mathematical knowledge through the construction process [16]. This stage can be used as a means of communicating explicitly for students as well as a means of learning to build logical thinking based on evidence [13]. Based on an analysis of student work, 85% of students can represent a
minus sign to a context appropriately. Students can interpret the 83 meters underwater context as -83, 12°C below freezing point as -12 and can declare the opposite of -9 is 9.

3.4. Activity 4: Problem solving

The last stage of this learning hope is the moment when the student can use the concepts he or she has known to a new knowledge without the help of the context he has learned. This stage is called institutionalization [13]. The process of this situation gives the value of the truth of the science learned in the classroom, usually related to concepts, symbols and possible knowledge to be used at times and for different purposes [17]. The institutionalization process encourages teachers to ensure that students have these skills so that knowledge becomes a permanent part of the student [18]. This is illustrated by the process that students pass through in transforming their knowledge, i.e. the concept - (-3) into new knowledge: +3, through empowerment by the teacher who gives them the value of truth and makes it possible to use the new knowledge gained to solve the next problem, - (-3) = 6 + 3. Based on our preliminary studies, 172 of the 174 students in grade 8 and 9 failed to resolve this problem. The problem BS5.4 is a problem that requires answers with concepts and procedures are quite complex compared with other problem solving. Table 1 show that students can interpret the minus sign better after HLT implementation.

Problem Code: BS5.4

Miranda lives in Lanny Jaya, Papua. The following table shows the temperature change in Lanny Jaya for 2 hours:

| Time       | Temperature Changes |
|------------|---------------------|
| 03.00 – 04.00 | +3°C                |
| 04.00 – 05.00 | -8°C                |

Table 1. Students answers on BS5.4.

| Variety of Answers | Code | Clarification                                                                 | %   |
|--------------------|------|-----------------------------------------------------------------------------|-----|
| -9 + 4 - 3 = -8°C  | PS4.1| Using the meaning of minus sign as a opposite. Subjects used counting operations and appropriate strategies. In this case the students have an abstraction ability. | 65,2|
| Temperature changes = end temperature - initial temperature  | PS4.2| The students understands the context of the visible problem and knows the concepts used to solve the problem of using formal formulas of difference with the correct strategy and proper counting procedures by compiling the equations of the given problem | 13,1|
| -4 = -9 – initial temperature |                     |                                                                             |     |
| End temperature = -9 + 4 = -5 |                     |                                                                             |     |
| +3 = -5 – initial temperature |                     |                                                                             |     |
| Initial temperature = -5 – 3 = -8 |                     |                                                                             |     |
| Draw a vertical number line then specify dots 3 and -4 on a vertical number line. Operation count starts from -9 then moves 4 steps up followed by 3 numbers down. The result is -8. | PS4.3| Visualize the context on a number line and use appropriate counting operations in representing the context. Because the calculation is done 'backwards' then operation is done using the meaning of the opposite. | 4,3 |
| -9 + 3 = -6 | PS4.4| Using the counting operation but not in the context of the question | 17,4|

Miranda noted that the temperature at 5:00 is 9°C below freezing. What is the temperature in Lanny Jaya at 03.00?
Based on Table 1, 65% of students can even state the problem in proper counting operations (PS4.1) as in PS4.2 (Figure 2.a) and PS4.3 (Figure 2.b). These findings show that students understand that the meaning of the minus sign changes during the troubleshooting process of a binary operator (subtraction) to unary operator (the opposite of), as well as very good in the abstract context of the problem. Overall only 17% of students are unable to solve the problem thoroughly and correctly (PS4.4).

The minus sign as the unary operator, 'opposite of', is an important concept in supporting the understanding of variables, algebraic expressions, and the symbolic representation of a definition [5]. The meaning of the minus sign may change during the troubleshooting process that requires a change in the meaning of minus sign from the binary operator (subtraction) to the unary operator (opposite of). Based on design implementation, HLT gives good contribution in constructing students' understanding on the meaning of minus sign. The instructions contained in the HLT can support student achievement of the symbol sense minus sign through activity in the group. Classroom learning has been shown to improve student creativity through argumentation and discussion [19,20]. Group discussions can explore students' creative thinking when responding or responding to an idea both within their own group and within the classroom.

4. Conclusion
Besides the meaning of the minus sign as the operator in the subtraction operation, representing the negative number through the context of the opposite direction allows students to recognize the minus sign function as "opposite of" as well as its function as a symbol of the negative number. Negative numbers as the first abstract concept accepted by students as well as other symbols that have different meanings with minus marks on subtraction, encourage teachers to begin preparing HLTs that support students understanding the meaning of minus signs with instructions that are easily understood by students remembering students at this stage still in the arithmetic thinking stage. Activity by identifying different forms of use of minus signs may help the student in using the minus sign according to the context. This concept is empirically proven to assist students in performing a counting operation involving negative numbers.

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