POLARIZED STRUCTURE FUNCTIONS:
A STATUS REPORT

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Abstract

We review the present status of polarized structure functions measured in deep-inelastic scattering. We discuss the $x$ and $Q^2$ dependence of the structure function $g_1$, and how it can be used to test perturbative QCD at next-to-leading order and beyond. We summarize the current knowledge of polarized parton distributions, in particular the determination of the first moment of the quark and gluon distributions and the axial charge of the nucleon. We critically examine what future experiments could teach us on the polarized structure of the nucleon.

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We review the present status of polarized structure functions measured in deep-inelastic scattering. We discuss the \(x\) and \(Q^2\) dependence of the structure function \(g_1\), and how it can be used to test perturbative QCD at next-to-leading order and beyond. We summarize the current knowledge of polarized parton distributions, in particular the determination of the first moment of the quark and gluon distributions and the axial charge of the nucleon. We critically examine what future experiments could teach us on the polarized structure of the nucleon.

1. From structure functions to polarized partons.

Structure functions measured in deep-inelastic scattering provide the cleanest access to the structure of the nucleon, as probed in hard processes, and understood in perturbative QCD. In the polarized case only very recently have theory and experiment progressed to the point that it is possible to extract from experiment physically meaningful information. Indeed, it is now possible to measure the nucleon matrix elements of quark and gluon operators, and compare experimental results on parton distributions with the \(x\) and \(Q^2\) dependence predicted by perturbative QCD at next-to-leading order (NLO).

Here we will chart the current status of this knowledge, by summarizing how recent data test perturbative QCD, reviewing what we have learnt on the polarized parton structure of the nucleon, and discussing what future experiments could teach us. In Sect. 2 we will review recent developments related to the study of the Bjorken sum rule, which singles out the hard structure of perturbative QCD. In Sect. 3 we will discuss the behavior of the structure function \(g_1(x, Q^2)\) in next-to-leading order (NLO) in the \((x, Q^2)\) plane, with special regard to the small-\(x\) region. In Sect. 4 we will review the current knowledge of polarized parton distributions, and in particular their first moments, which are related to the parton interpretation of the nucleon spin. In Sect. 5 we will finally compare the information which could be in principle obtained on polarized partons with current and future experimental prospects.*

* For a more detailed introduction to the aspects of the theory of the polarized structure function \(g_1(x, Q^2)\) relevant to this paper see e.g. ref. 1, whose notation and conventions we will follow; a general discussion of the phenomenology of \(g_1\) is e.g. in ref. 2. For a comprehensive introduction to the problems related to the determination of the first moment of \(g_1\) (the “spin crisis”) see ref. 3.
2. The Bjorken sum rule.

The nonsinglet first moment of the structure function $g_1$ has the peculiar feature of being related to the matrix element of a conserved current through a nontrivial coefficient function:

$$\Gamma_{1}^{\text{NS}}(Q^2) \equiv \int_{0}^{1} dx \, g_{1}^{\text{NS}}(x, Q^2) = \frac{1}{2} C_{\text{NS}}[\alpha_s(Q^2)] a_{\text{NS}}.$$  \hspace{0.5cm} (2.1)

This implies that its measurement probes the hard part of the total polarized inclusive nonsinglet $\gamma^* p$ cross section, which is proportional to the coefficient function $C_{\text{NS}}(Q^2)$, up to a scale-independent coefficient, the nucleon matrix element of the nonsinglet axial current $a_{\text{NS}}$, which incorporates all the soft, target-dependent physics. Hence, determining $C_{\text{NS}}(Q^2)$ at various scales probes the perturbative computation of this cross-section, which is under theoretical control and indeed has been accomplished up to order $\alpha_s^3$.

In the particular case of the isotriplet combination the current matrix element can in turn be related, using isospin algebra, to that which governs nucleon $\beta$ decay, and thus expressed in terms of the corresponding decay constant $g_A$:

$$a^{t=1} = \frac{1}{6} \langle p, s | j_5^{\mu} \lambda_5 | p, s \rangle \frac{s^\mu}{M} = \frac{1}{6} g_A$$ \hspace{0.5cm} (2.2)

where $M$ and $s^\mu$ are the nucleon mass and spin and the axial current is $j_5^\mu = \bar{\psi} \gamma^\mu \gamma_5 \lambda_5 \psi$, with $\lambda_5$ being an isospin matrix. This gives an absolute prediction for the isotriplet first moment (Bjorken sum rule): testing the normalization of isospin symmetry in this channel, and measuring its scale dependence tests perturbative QCD, in particular the size and running of the strong coupling $\alpha_s(Q^2)$.

The predicted scale dependence of the sum rule (with $\alpha_s(M_Z) = 0.119 \pm 0.006$) is shown in Fig. 1, which demonstrates its sensitivity both to the value of the strong coupling, and to higher order corrections. Notice the jump at the charm threshold, which is due to the explicit $n_f$ dependence of the coefficient function at order $\alpha_s^2$ and beyond (the discontinuity would be smoothened in a more refined treatment of thresholds). Recently, both the the SMC$^5$ and the E142-E143 collaboration$^6$ have released preliminary determinations of the isotriplet first moment, respectively obtained at 10 GeV$^2$ combining proton and deuteron data, and at 3 GeV$^2$ also including $^3$He data; the results are manifestly consistent with the QCD prediction. Apparent from Fig. 1 is also

\footnote{Strictly speaking, $a_{\text{NS}}$ is only scale independent far from heavy quark thresholds, otherwise it acquires a scale dependence due to the appearance of a new heavy quark contribution as the threshold is passed.}
Fig. 1: Scale dependence of the Bjorken sum rule up to order $\alpha_s$ (dotted), $\alpha_s^3$ (solid) and with higher twist corrections (dashed). The three sets of curves correspond to $\alpha_s(M_Z) = 0.119 \pm 0.006$. The data points are from ref. 5 (square), 6 (diamond) and the global average of ref. 7 (circle) (slightly offset from $Q^2=3$ GeV$^2$ to improve readability).

the much greater sensitivity of the sum rule to higher order corrections and the value of $\alpha_s$ when the scale is low. It follows that a measurement of equal accuracy leads a priori to a more stringent test of the sum rule (or, equivalently, to a better determination of $\alpha_s$) if performed at a low scale, but it is then affected by a larger theoretical uncertainty related to lack of knowledge of higher order corrections.

In fact, as we will discuss in Sect. 3 and 4, the experimental values of the first moment are themselves obtained by evolving to a common scale data taken at several values of $Q^2$, and then extrapolating to $x = 0$ and $x = 1$. The evolution is done\textsuperscript{5,6} by means of an approximate procedure (which at best approximates NLO evolution), involving a systematic error which is neglected, but is likely to be large at low scales: it follows that the apparent greater precision of the low $Q^2$ data point in fig. 1 is in part due to the neglect of this systematics.\textsuperscript{9} If, disregarding these complications, the published values of the first moment are combined (at a common scale), one may arrive at a determination of the sum rule which taken at face value is very precise, and would determine $\alpha_s$ to competitive accuracy.\textsuperscript{7} (see Fig. 1). This can only be correct if the aforementioned systematic error cancels out in the isotriplet combination; which is presumably true to a certain extent, but has not been quantitatively established. An example of unavoidable uncertainty is that related to the treatment of the charm threshold, which, as shown in fig. 1, is at least as large as the $O(\alpha_s^4)$ correction to $C_{NS}(Q^2)$.

The perturbative computation of $C_{NS}(Q^2)$ is in fact extended to high enough order that one may wonder whether the asymptotic nature of perturbation theory in QCD may already manifest itself. Specifically, the asymptotic series for $C_{NS}(Q^2)$ can be summed by the Borel method but only up to an ambiguity of order $\frac{1}{Q^2}$; one can then try to estimate the size of this ambiguity, either
by approximate computation of the high-order behavior of the series,\(^\text{10}\) or on the basis of the terms which are already known (extrapolated with the method of Padé approximants).\(^\text{11}\) This ambiguity must cancel against an equal and opposite ambiguity in the contribution \(e_{\text{HT}}\) to the sum rule from higher twist operators (such as \(\bar{\psi}\gamma^\mu\gamma_5\psi F_{\mu\nu}\)): 

\[
\Gamma^{I=1} = \frac{1}{6} \left[ C_{\text{NS}}(Q^2)g_A + \frac{C_{\text{HT}}}{Q^2} + O(1/Q^4) \right].
\]

The estimates of ref.s \(^{10,11}\) lead to a value of the ambiguity which is of the same size as the contributions of such operators computed with QCD sum rule methods, which give \(C_{\text{HT}} \approx -0.01\) GeV\(^2\).\(^\text{12}\) This suggests that even though the sum rule estimates are of the right order of magnitude, their actual size is entirely uncertain. The logarithmic corrections to the scale dependence of the twist four, spin one operators which contribute to the first moment of \(g_1\) have also been computed recently.\(^\text{13}\)

### 3. Parton distributions at next-to-leading order

The moments of \(g_1(N, Q^2) = \int_0^1 dx x^{N-1} g_1(x, Q^2)\) generically depend on scale both because of the \(\alpha_s\) dependence of coefficient functions, and because of the scale dependence of the matrix elements of quark and gluon operators to which they are related by

\[
g_1(N, Q^2) = \frac{\langle e^2 \rangle}{2} [C_{\text{NS}}(N, \alpha_s)\Delta q_{\text{NS}}(N, Q^2) + C_S(N, \alpha_s)\Delta \Sigma(N, Q^2) + 2n_f C_g(N, \alpha_s)\Delta g(N, Q^2)],
\]

where \(\Delta q_{\text{NS}} = \sum_{i=1}^{n_f} \left( \frac{\langle e_i^2 \rangle}{\langle e_i \rangle} - 1 \right) (\Delta q_i + \Delta \bar{q}_i)\) and \(\Delta \Sigma = \sum_{i=1}^{n_f} (\Delta q_i + \Delta \bar{q}_i)\) (with \(\langle e^2 \rangle = \frac{1}{n_f} \sum_{i=1}^{n_f} e_i^2\)) are respectively the nonsinglet and singlet combinations of moments of the quark distributions. The scale dependence of the polarized quark and gluon distributions is in turn governed by the evolution equations

\[
\begin{align*}
\frac{d}{dt} \Delta q_{\text{NS}} &= \alpha_s(t) \frac{\gamma^N}{\gamma_{\bar{q}q}} \Delta q_{\text{NS}} \\
\frac{d}{dt} \left( \frac{\Delta \Sigma}{\Delta g} \right) &= \alpha_s(t) \left( \frac{\gamma^N}{\gamma_{\bar{q}q}} \right)^2 \left( \frac{\Delta \Sigma}{\Delta g} \right).
\end{align*}
\]

The physically relevant moments of quark and gluon distributions can thus be extracted from the measurement of the moments of \(g_1\) at a pair of different scales. However, the moments of \(g_1\) cannot be determined directly because the structure function can only be measured in a finite range of \(x\) (\(x \to 0\) corresponds to the energy of the \(\gamma^*\)-nucleon collision going to infinity); furthermore, present-day experiments only determine \(g_1\) along a curve \(Q(x)\) (rather than at a fixed scale) with \(Q\) increasing with \(x\). This implies that
the moments of $g_1$ can only be extracted indirectly, through an analysis of its $x$ and $Q^2$ dependence: all the available experimental information is then summarized in a set of polarized parton distributions at a reference scale, from which $g_1$ at all $x$ and $Q^2$ are found by solving the evolution equations and taking (and inverting) moments. The potential accuracy of such an analysis has substantially improved recently since the complete set of anomalous dimensions $\gamma^N_{ij}$ has now been determined up to NLO.\textsuperscript{14}

Given the need to extrapolate the data to $x = 0$ in order to compute moments, it is particularly interesting to study the implications of perturbative evolution for the small $x$ behavior of $g_1$, which follows from the behavior of the anomalous dimensions $\gamma^N_{ij}$ eq. (3.2) around their rightmost singularity in $N$ space. This is located at $N = 0$ and is on general grounds of the form\textsuperscript{15} $\alpha_s \gamma_{N \rightarrow 0} \sim (\alpha_s/N)^{2n+1}$ at $n$-th perturbative order. Already at LO this implies that any starting parton distributions leads to a growth of $|g_1(x, Q^2)|$ at small $x$ and large $Q^2$;\textsuperscript{16} in fact in this limit the ratio of $\Delta g$ and $\Delta \Sigma$ is fixed and negative, so that $g_1 < 0$ asymptotically at small $x$ and large $Q^2$ for any reasonable input parton distribution.\textsuperscript{9} The generic form of the rise at $n$-th perturbative order is then

$$\Delta f \sim \frac{1}{(\xi \zeta)^{1/4}} \epsilon^{2\gamma_f \sqrt{\xi \zeta}} \left( 1 + \sum_{i=1}^{n} \epsilon_f^i \left( \sqrt{\frac{\xi}{\zeta}} \right)^{2i+1} \alpha_s^i \right), \quad (3.3)$$

where $\xi \equiv \ln \frac{Q_0^2}{Q^2}$, $\zeta \equiv \ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}$, and $\Delta f$ denotes either the nonsinglet quark distribution, or the two linear combinations of the gluon and singlet quark distributions which correspond to eigenvectors of the small $N$ anomalous dimension matrix. Notice that this rise contradicts the Regge expectation\textsuperscript{17} that $g_1$ should at most be constant at small $x$.

The coefficients $\epsilon_f$ in eq. (3.3) at NLO can be extracted\textsuperscript{18} from the full two-loop anomalous dimension;\textsuperscript{14} beyond NLO they have been obtained at fixed coupling both in the nonsinglet\textsuperscript{19,20} and singlet\textsuperscript{21} case on the basis of the analysis of the leading double logarithmic singularities. These results (unlike their unpolarized counterparts) are not supported by suitable factorization theorems, even though they do agree with the NLO result\textsuperscript{18}. If correct, they lead to the prediction that the series in eq. (3.3) exponentiates, thus leading to a power-like rise of $g_1 \sim x^{-\lambda}$. The value of $\lambda$ cannot be computed precisely since it depends on $\alpha_s$, which is assumed to be constant in these computations; taking $\alpha_s \sim 0.2$ (which corresponds to a scale of roughly 10 GeV$^2$) would lead to $\lambda \approx 0.5$ in the nonsinglet and $\lambda \approx 1$ in the singlet case, implying that the first moment of $g_1$ would then diverge. This casts some doubts on the applicability of the analysis, at least in its naive form. However, it suggests that a strong rise of $g_1$, which might be of considerable phenomenological relevance, should be generated by perturbative QCD. The all-order analysis confirms the LO result
that this rise will preserve the sign of the nonsinglet, but lead to a negative singlet contribution (which asymptotically dominates \( g_1 \) at small \( x \)) even if \( g_1 \) is positive at the starting scale.

4. Phenomenology of \( g_1 \).

The scaling violations displayed by current data for \( g_1^p \) are shown in Fig. 2: what might appear as a systematic discrepancy of the two data sets\(^{22,23} \) is in fact a consequence of the fact that in each \( x \) bin the SMC data are taken at a higher value of \( Q^2 \) than the E143 data. These scaling violations are partly due to scale dependence of the asymmetry \( A_1 = g_1/F_1 \), i.e. they would be significantly smaller if the asymmetry were scale independent. Thus, even though the individual experiments, because of their limited kinematic coverage, cannot measure\(^{26,5} \) the scale dependence of the asymmetry directly, the data do show evidence for this scale dependence when the two data sets are combined.

4.1 Parton distributions

The observation of sizable scaling violations, together with the availability of proton\(^{22,23} \) and deuteron\(^{24,25} \) data which allow the disentangling of singlet and nonsinglet contributions, make a full NLO determination of polarized parton distributions possible.\(^{18,28,29} \) A detailed study of the shape of polarized parton distributions\(^{29} \) shows (see Fig. 3) that while it is possible to separate out valence (defined as \( \Delta q - \Delta \bar{q} \)) and sea components (assumed to be SU(3) flavor symmetric) the constraints on the gluon distribution are only relatively loose at large \( x \) (\( x \approx 0.1 \)) where scaling violations driven by it are very small: in particular, the data disfavor but cannot rule out a change of sign of \( \Delta g \) at
large $x$. There is, however, remarkable agreement between the shape of $\Delta g$ obtained by different groups under rather different assumptions.\(^{30}\)

The small $x$ behavior of the parton distributions is particularly interesting. The proton data (Fig. 4a) display a marked rise at small $x$ (and low $Q^2$, where the small $x$ data are taken): since $g_1^p$ at small $x$ is positive, this appears to be due to a rise of the nonsinglet contribution, because a rising singlet would produce substantial scaling violations which turn it into a rapid drop. Indeed, the data for the deuteron, which is almost pure singlet (the octet contribution to it is quite small) do not display such a rise (Fig. 4b). This then implies an effective behavior $\Delta g_{NS} \sim \frac{1}{\sqrt{x}}$ in the measured region, in contradiction to Regge expectations, and reminescent of the predictions\(^{19,20}\) based on the summation of double logs discussed in Sect. 2.\(^*\) Much more precise data at smaller values of $x$ will be required to elucidate this issue.

The first precision data on the neutron structure function $g_1^n$ (from scattering on a $^3$He target, see sect. 5 below) have recently been released in preliminary form.\(^{31}\) Comparison with these data of the predicted $g_1^n$, based on parton distributions derived from fits to previous data shows excellent agreement (Fig. 4c), demonstrating that our current understanding of the triplet/singlet separation and of the small $x$ behavior of the nonsinglet are reasonably good.

\(^*\) Interestingly, these predictions also imply that the small $x$ behavior of the polarized and unpolarized nonsinglet should be similar; this also appears to agree with the data.
4.2 First moments

The NLO analysis can in particular be used to give a precision determination of the first moment of $g_1$, which is of primary theoretical interest because of its relation to the singlet axial current:

$$\Gamma_S(Q^2) \equiv \int_0^1 dx \, g_S^S(x, Q^2) = \frac{\langle e^2 \rangle}{2} C_5 [\alpha_s(Q^2)] a_0(Q^2), \quad (4.1)$$

where $a_0$ is the singlet axial charge $a_0(Q^2) = \langle p, s | j_0^5 | p, s \rangle$. Unlike its non-singlet counterpart eq. (2.2), $a_0(Q^2)$ depends on scale (see Fig. 5) because the classical conservation of the singlet axial current is spoiled by the axial anomaly in the quantized theory. Its nucleon matrix element measures a non-trivial combination [eq. (3.1)] of the first moments of the quark and gluon
The singlet axial charge $a_0$ extracted from the values of the first moment $\Gamma_1$ published by the experimental collaborations, and evolved to a common scale. The data in the last column are all preliminary.

| Ref. | Targ | $a_0(5\text{ GeV}^2)$ | Ref. | Targ | $a_0(5\text{ GeV}^2)$ |
|------|------|------------------------|------|------|------------------------|
| 22   | $p$  | $0.27 \pm 0.15$        | 6    | $p$  | $0.24 \pm 0.09$        |
| 23   | $p$  | $0.31 \pm 0.11$        | 5    | $d$  | $0.22 \pm 0.09$        |
| 24   | $d$  | $0.16 \pm 0.11$        | 6    | $^{3}\text{He}$ | $0.37 \pm 0.12$        |
| 25   | $d$  | $0.28 \pm 0.06$        | 35   | $^{3}\text{He}$ | $0.41 \pm 0.22$        |

The singlet axial charge $a_0$ extracted from the values of the first moment $\Gamma_1$ published by the experimental collaborations, and evolved to a common scale. The data in the last column are all preliminary.

distributions (the total quark and gluon spin), which at LO simply reduces to $a_0(Q^2) = \Delta \Sigma(1,Q^2)$. The first moment is usually extracted from the data by assuming $Q^2$ independence of the asymmetry $A_1 = g_1/F_1$ to evolve all data to a common scale, and extrapolating the data to $x = 0$ on the basis of the Regge expectation. The axial charge can then be obtained directly by using SU(3) symmetry to separate off the nonsinglet component, and the perturbative expression of $C_S(\alpha_s)$ (which is known up to order $\alpha_s^2$). The results thus found from the published (or preliminary) values of the first moment, evolved to a common scale of 5 GeV$^2$, are collected in the table.*

Neither of these evolution and extrapolation procedures is consistent with perturbative QCD, as discussed in Sect. 2: the information contained in the data can instead be consistently extracted from a global NLO fit. In such case, the primary quantities which enter the analysis are the first moment of the quark and gluon distribution, from which the first moment of $g_1$ is then extracted using eq. (3.1), and the singlet charge using eq. (4.1). The main result of such an analysis is that the naive procedure systematically underestimates the value of $a_0$, and substantially underestimates the theoretical uncertainty on it. This is partly due to the fact that the small $x$ behavior is overconstrained by the Regge ansatz, and partly to the fact that corrections due to perturbative evolution (which are only known up to NLO) are potentially large. In the NLO analysis the small $x$ behavior is fitted to the data and constrained by assuming a reasonably smooth form of the starting parton distributions, while higher order corrections to evolution are estimated by varying the renormalization and factorization scale. Using the data of Refs. 22–25 leads to the value

$$a_0(5\text{ GeV}^2) = 0.15^{+0.16}_{-0.11}$$

of the axial charge, to be compared to those of the table.

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* We used the $O(\alpha_s^3)$ expression of the nonsinglet and the $O(\alpha_s^2)$ expression of the singlet coefficient function with $\alpha_s(M_z) = 0.119 \pm 0.006$, and the values of the triplet and octet charges from Ref. 34.
Experimental results for the singlet axial charge can be compared to expectations based on the quark model. In particular, the Zweig rule suggests that the strange quark component of the nucleon should be small: this then leads to the prediction that the singlet and the octet axial charge should be approximately equal to each other (Ellis-Jaffe sum rule\textsuperscript{36}). Because the octet axial charge is scale independent to all orders, while the singlet charge, starting at NLO, is not, this prediction can only be exactly satisfied at one particular scale. However, the scale dependence of $a_0$ is rather slight even at relatively small scales (Fig. 5b), so that the Ellis-Jaffe prediction can be considered to be an approximate one, which the result eq. (4.2) would then strongly contradict (the octet charge is\textsuperscript{34} $a_8 = 0.579 \pm 0.025$). However, if one were to boldly extrapolate the perturbative scale dependence of $a_0$ outside the perturbative region proper (Fig. 5a), then the sum rule could be satisfied by assuming it to be valid at a “hadronic” low scale of order of several hundreds MeV.

Now, quark model predictions should presumably apply to partonic observables, i.e. in this case to the first moments of the polarized quark and gluon distributions. However, the definition of the first moment of the quark distribution is entirely scheme dependent. This follows from the fact that $\alpha_s \Delta g(1, Q^2)$ is scale independent at LO\textsuperscript{37} if one performs a scheme change $\Delta \Sigma'(1, Q^2) = [1 + \alpha_s \Delta g(1, Q^2)] \Delta \Sigma(1, Q^2)$ then $\Delta \Sigma'(1, Q^2) - \Delta \Sigma(1, Q^2)$ is asymptotically constant, so even asymptotically the ambiguity on $\Delta \Sigma$ is of the same order as its size.* It is then possible in particular to choose a scheme where $\Delta \Sigma(1)$ is scale independent. In this scheme one can meaningfully compare the singlet quark distributions with the Ellis-Jaffe prediction since then

\* Notice that this is not the case for $\Delta g(1, Q^2)$ itself: if $\Delta g'(1, Q^2) = [1 + \alpha_s \Delta g(1, Q^2)] \Delta g(1, Q^2)$ then even though $\Delta g'(1, Q^2) - \Delta g(1, Q^2)$ is asymptotically constant $\Delta g$ itself diverges (as $\alpha_s^{-1}$) in the same limit, so the relative ambiguity vanishes asymptotically.

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*Fig. 5: Scale dependence of the axial charge $a_0(Q^2)$ eq. (4.2) computed at order $\alpha^2$ at low (a) and intermediate (b) scales. The three curves correspond to $\alpha_s(M_Z) = 0.119 \pm 0.006$. The dot-dashed line is the value of the octet charge $a_8$. 

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the first moments of all polarized quark distributions (and linear combinations thereof) are separately scale independent.\textsuperscript{37}

As the NLO analysis actually derives the axial charge from the polarized quark and gluon distributions, and specifically their first moments, these are also determined in the process. Using a scheme\textsuperscript{18,38} where $\Delta \Sigma(1)$ is scale independent leads to\textsuperscript{18}

\[
\Delta \Sigma(1) = 0.5 \pm 0.1, \quad \Delta g(1, 5 \text{ GeV}^2) = 2.6 \pm 1.4. \tag{4.3}
\]

An analysis\textsuperscript{29} with a different scheme choice for the quark leads essentially to the same result for $\Delta g(1, Q^2)$. This value of $\Delta \Sigma(1)$ agrees with the Ellis-Jaffe prediction; the size of $\Delta g(1, Q^2)$ may appear to be large, but is in fact natural recalling that it scales as $1/\alpha_s$: if one were to use NLO evolution down to very low scales the value eq. (4.3) would correspond to vanishing of $\Delta g(1)$ around 700 MeV.

5. What can we learn?

The present generation of polarized experiments has brought our understanding of polarized parton distributions from the parton model to the level where it makes sense to use QCD at NLO. The data have shown evidence for scaling violations (Fig. 2), have given us a first understanding of the shape of parton distributions (Fig. 3, 4), and have led to a determination of the singlet axial charge and the first moment of the gluon distribution [eq.s (4.2),(4.3)] which, given the uncertainties involved, should be considered surprisingly precise.

While the determination of the first moment of the gluon distribution is dominated by the statistics and will thus improve somewhat as more abundant data in the currently explored kinematic region become available (specifically from SMC\textsuperscript{5} and E155\textsuperscript{6}), the determination of the singlet axial charge is already dominated by the systematics related to higher order corrections, as well as by the uncertainty in the small $x$ behavior. Only the availability of data with a more complete coverage of the $(x, Q^2)$ plane (such as those from the proposed GSI collider\textsuperscript{39} or from HERA with a polarized proton beam\textsuperscript{40}) would allow a substantial improvement here, as well as a determination of higher moments and in general of the $x$ dependence of parton distributions. The study of scaling violations at small $x$ in particular should turn out to be very fruitful: it could confirm the violation of the Regge-based expectation for the small $x$ shape of $g_1$ suggested by current data, provide evidence for the non-Regge behavior eq. (3.3) predicted by perturbative QCD (and seen\textsuperscript{41} in unpolarized singlet structure functions at HERA), show the dramatic change of sign of $g_1^p$ (see Fig. 4a) predicted by the QCD evolution equations as the
scale is raised (thereby leading to a very precise determination of the polarized gluon distribution\(^{42}\)), and possibly show evidence of the double logarithmic effects which characterize the polarized small \(x\) behavior.\(^{20,21}\) On the theoretical side, there is room for improvement from a better understanding of double logs at small \(x\), consistent with the running of \(\alpha_s\), and from a more detailed understanding of higher twist effects, both kinematical\(^{43}\) (such as related to target mass effects) and dynamical\(^{12}\) which have been so far studied systematically only at the level of first moments.

The theory and phenomenology which we have discussed so far are based on fully inclusive measurements of longitudinal polarization with lepton beams and nucleon (proton and deuteron, actually) targets. These allow (at leading twist) only the determination of the combinations of polarized parton distributions which contribute to \(g_1\) eq. (3.1). Firstly, this only measures the \(C\)-even combination \(q + \bar{q}\) of polarized quark distributions: the “valence” \(q - \bar{q}\) component can only be disentangled\(^{28,29}\) by making specific assumptions on the symmetry of the sea. Furthermore, whereas comparing proton and deuteron data allows the separation of the isotriplet component, a further flavor decomposition can only be done at the level of first moments by using SU(3) symmetry and data from hyperon \(\beta\) decays, while for the full \(x\) dependence it again requires\(^{28,29,18}\) assumptions on the symmetry of the sea.

For all moments beyond the first the singlet quark and gluon contributions can then separated by the observation of scaling violations. The case of the first moment is special because, whereas the gluon is again determined by scaling violations, the quark is completely ambiguous (see Sect. 3), and can thus only be determined within a specific scheme choice. It is important to notice that this is an inevitable consequence of the scaling properties of \(\Delta g(1, Q^2)\), which allow an asymptotically nonvanishing redefinition of the singlet quark, and it does not depend on the specific process. Thus, a determination of \(\Delta \Sigma(1, Q^2)\) from any hard process will be subject to the same ambiguity, and there is no piece of data which could eliminate it, even in principle. Notice however that the singlet valence component, which decouples from the gluon, is not subject to this ambiguity.

A simple way of widening the source of available information is to consider nuclear targets. In particular, \(^3\)He targets provide a direct measurement of the neutron structure function, to the extent that only the neutron carries the nuclear spin. Corrections due to higher spin components in the nuclear wave function\(^{44}\) have been studied and seem under theoretical control (Fermi motion effects appear\(^{45}\) to be negligible). On the contrary, there are only preliminary studies\(^{46}\) (in the nonsinglet sector only) of shadowing effects, which suggest that they may be quite large (up to 20% around \(x = 0.1\) where \(g_1\) is sizable): a systematic investigation of these effects is required if data from the \(^3\)He experiments (in particular the statistically very precise E154\(^{31}\) data of Fig. 4c)
are to be seriously taken as a precise determination of the neutron polarized structure function.

More ambitious programs (perhaps more theoretically than experimentally so) involve going beyond fully inclusive measurements. Tagging hadrons in the final state of a DIS event leads to information on the struck current, provided one can separate out the fragmentation of the current itself (as described by fragmentation functions) from the fragmentation of the target (parametrized by fracture functions). In principle, measurements in the current fragmentation region makes then possible a separation into individual flavor components, and also into valence and sea contributions: however, beyond LO there is no simple way of associating simple observables (such as asymmetry ratios) to individual matrix elements (such as the isoscalar valence component), other than by making assumptions, for instance about the symmetry of the sea, which is what one would want to test in the first place. However, these data can be used in a global NLO analysis (the general framework of which can be already set up) where they may significantly constrain the final result. Such experiments are currently underway at HERMES, while an ambitious program has been proposed at CERN. On the other hand, measurements of target fragmentation may provide nonperturbative information, and specifically allow to test whether the smallness of the singlet axial charge discussed in sect. 4.2 is a peculiar property of the nucleon, or rather a target-independent property of the QCD vacuum.

A separate class of exclusive or semi-inclusive experiments can be used to measure the polarized gluon distributions in processes where it contributes at leading order (such as heavy quark production by photon-gluon fusion). It is then possible to evade the tradeoff of inclusive DIS, where the gluon is measured by scaling violations, which are larger at low scales where however scheme ambiguities are also large; even though there are then other theoretical difficulties characteristic of non-inclusive processes (such as scale ambiguities). The cleanest measurement here is possibly heavy quark photoproduction, where since only one large scale (the quark mass) is present the cross section can be reliably computed in perturbation, so that a study of the $p_T$ dependence of the production asymmetry can lead to a sensitive determination of the gluon distribution. A NLO computation of this process, which is still lacking, would be highly desirable.

As more precise data become available, it will also be possible to take the ambitious step of going beyond longitudinal polarization. So far, the structure function $g_2$ has been considered a higher twist background to the measurement

* An alternative method to disentangle valence and sea components for individual flavors is to use charged-current events. Even though this would be feasible at HERA with a polarized proton beam, it does not appear to be competitive.
of \(g_1\), and recent determinations\(^{56}\) of it have essentially achieved the aim of reducing this background. A study of the \(x\) and \(Q^2\) dependence of \(g_2\) as might be possible in a dedicated experiment would open Pandora’s box of twist-3 operators and their mixing,\(^{57}\) and lead beyond the safe context of parton distributions defined by twist two operators.

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