A comment on Bell's Theorem Logical Consistency

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Abstract

Lambare and Franco correctly claim that Bell’s deterministic model and inequalities may be derived using only local causality, perfect correlations and measurement independence, without talking about joint probabilities. However, measurement independence, as we explain, should not be called “no-conspiracy” or “freedom of choice”. Measurement independence should be called noncontextuality, because it allows implementing random variables, describing incompatible random experiments, on a unique probability space, on which they are jointly distributed. Using a precise terminology proposed by Dzhafarov and Kujala in Contextuality-by-Default approach, such implementation defines a probabilistic coupling, which we explain in this paper. The authors’ frequentists proof fails, if this probabilistic coupling and joint probabilities do not exist. We construct also a probabilistic coupling for their counterexample to prove, that there is no contradiction with Fine’s Theorem. Nobody questions Bell’s Theorem logical consistency and nobody claims that Fine disproved Bell’s Theorem. Various metaphysical assumptions, such as local realism, classicality or counterfactual definiteness may motivate a choice of a probabilistic model. However, once a model is chosen, its meaning and its implications may only be discussed rigorously in a probabilistic framework. Bell inequalities are violated in various Bell Tests; for us it proves that hidden variables depend on settings confirming contextual character of quantum observables and an active role played by measuring instruments. Bell was a realist, thus he thought that he had to choose between nonlocality and super-determinism. From two bad choices he chose nonlocality. Today he would probably choose contextuality.

Keywords: Bell inequality, quantum nonlocality, freedom of choice, measurement independence, contextuality, local causality, local realism
1 Introduction

Quantum mechanics (QM) provides probabilistic predictions and the main question debated since nearly 100 years is: are these probabilities irreducible or do they emerge from some more detailed description of physical reality and experiments used to probe it. Einstein strongly believed that QM should emerge from more detailed description of individual physical systems [1, 2]

Bell was a realist and also believed that physical objects possess definite properties [3, 4]. In 1964, he proposed probabilistic local realistic hidden variable model (LRHVM) trying to reproduce quantum predictions for an ideal EPRB experiment [3]. Pairwise expectations deduced using LRHVM have to satisfy Clauser-Horne-Shimony–Holt inequalities (CHSH) [5] which for some experimental settings are violated by quantum predictions and by experimental data in Bell Tests. The violation of inequalities is a source of unfounded speculations about the nonlocality of Nature, free will and superdeterminism.

In their paper in Foundation of Physics [6], Lambare and Franco make some statements, which we want to rectify. They correctly claim, that there is no need for a counterfactual reasoning in the context of Bell inequalities and explain, that LRHVM, which they call LHV model, may be derived using local causality (LC), perfect correlations and measurement independence (MI). However they do not realize, that the conjunction of these three assumptions implies, the existence of a counterfactual joint probability distribution (JP) of 4 random variables, which is used to deduce pairwise-correlations of 4 only pairwise measurable random variables. The existence of such JP is used implicitly in their frequentist proof of inequalities. A counterfactual JP may also be constructed, in their counterexample, by which they claimed to prove the fallacy of bizarre claims related to joint probabilities.

We explain that the claims in [7-11] related to joint probabilities are correct and not bizarre. Nobody believes that Fine [12, 13] has disproved Bell’s Theorem. Bell’s Theorem is a mathematical theorem which says: if LRHVM is used to describe EPRB, then some pairwise cyclic expectations obey Bell inequalities, which for some settings are violated by quantum predictions.

In LRHVM it is assumed that hidden variables do not depend on experimental settings. This assumption is now called MI and understood as a statistical independence of setting and hidden variables: p(λ, x, y) = p(λ) p(x, y), where λ are hidden variables and (x, y) are setting variables. It is believed that MI, called also free choice or no conspiracy, is a direct consequence of experimenters’ free will (FW). For the majority of scientists experimenters’ freedom of choice is a prerequisite of science and its violation would imply superdeterminism. As we explained in [14-19] a violation of MI does not restrict FW. The misunderstanding is based on a questionable use of Bayes Theorem and on incorrect causal interpretation of marginal probability distributions [16-19]. If hidden variables depend on setting variables, then p(λ|x, y) ≠ p(λ). It means also that p(x,y|λ) ≠ p(x, y), but it does not mean that λ may causally influence setting variables (x, y) which in QM are simply labels of different settings. A stochastic dependence of random variables does not imply a causal dependence. In statistics correlation does not mean causation.
If hidden variables depend on settings, it does not imply superdeterminism and experimenters’ freedom of choice is not compromised: thus it is misleading to talk about the violation of measurement independence or violation of free choice. In our opinion, MI should be called noncontextuality, because it allows implementing random variables, describing different random experiments, on a unique probability space, on which they are jointly distributed. Such implementation defines a probabilistic coupling, which we explain in detail in this paper hoping to cut short unfounded criticism and speculations. CHSH inequalities are simply noncontextuality inequalities for a 4-cyclic scenario [20].

There are two locally causal probabilistic hidden variable models. One is LRHVM and the second is a stochastic hidden variable model (SHVM) [21]. Contrary to what several authors believe, LRHVM is not a special case of SHVM in which we have a family of stochastically independent random experiments labelled by λ, a detailed discussion of two models and corresponding different experimental protocols may be found in [14].

In LRHVM, outcomes (clicks on detectors coded ±1) are locally predetermined by variables describing correlated photonic signals, produced by a source. Local predetermination of outcomes of all experiments, by some ontic properties of the signals, is called usually: local realism, classicality or counterfactual definiteness (CFD. Local realism automatically implies MI and the existence of JP. Bell inequalities are violated in various Bell Tests, what only proves that LRHVM provides an incorrect and oversimplified description of these experiments. Peres correctly concluded that unperformed experiments have no results [22].

Various metaphysical assumptions may motivate a choice of a probabilistic model. However, once a model is chosen, its meaning and its implications may only be rigorously discussed in a probabilistic framework. Hidden variables are values of some random variables describing details of an experimental protocol consistent with a given probabilistic model [14].

The ideal EPRB and perfect correlations do not exist [11, 23]. The random variables describing the data in Bell Tests are inconsistently connected and should be analyzed using Contextuality-by-Default approach (CbD) [18, 24-31]. In CbD proposed and studied by Dzhafarov and Kujala all empirical scenarios are described by systems of random variables representing measurements of properties q in contexts c. Properties of experimental scenarios and possible hidden variable models are studied without evoking any metaphysical assumptions. Free choice is equivalent to context-independent mapping and experimenters’ free will assumption is completely redundant [29-31]. In this paper we are not using CbD approach and its notation. We define and explain only specific probabilistic couplings in Bell scenario and in the counterexample of Lambare and Franco.

Bell inequalities are violated. For us, it means only, that hidden variables depend on settings confirming contextual character of quantum observables and an active role played by measuring instruments. Moreover, if hidden variables describing measuring instruments are correctly incorporated in a probabilistic model the experimental data and an apparent violation of non-signalling may be explained in a locally causal way [11, 14-16, 18, 19].
The paper is organized as follows.

In Sect.2 we explain how LRHVM defines a probabilistic coupling of random variables describing outcomes of an ideal EPRB experiment.

In Sect.3 we discuss an experimental protocol consistent with LRHVM and the properties of finite samples generated using this protocol.

In Sect.4 we point out, that data spreadsheets obtained for 4 different settings cannot be reordered to prove CHSH. To explain experimental data in Bell Tests, hidden variables have depend on settings what can be done by incorporating, into a probabilistic model, variables describing measuring instruments. We also explain on a concrete example, why setting dependence of hidden variables (the violation of MI) has nothing to do with super-determinism and/or conspiracy. Contrary to what Bell and the authors claimed one may not average over these instrument variables [11, 14-16].

In Sect.5 we analyze in detail authors’ counterexample and we reject their criticism of the arguments given in [7-11]. Contrary to what they claim, JP does not exist for the incompatible random variables describing their 4 incompatible experiments; nevertheless the correlations in their experiments may be explained using a particular probabilistic coupling and a corresponding JP which we construct.

Sect.6 contains few final conclusions.

2. LRHVM and probabilistic coupling

We discuss LRHVM and its implications using a rigorous probabilistic framework, what avoids misunderstanding.

The experimental protocol of an ideal EPRB is the following [23]:

1. A beam (ensemble) E of entangled pairs of particles is created by a source. One particle is sent to Alice and its twin partner to Bob in distant laboratories, who chose independently experimental settings (x, y) of their polarization beam splitters (PBS). In general (x, y) are labels and not necessarily values of random variables.

2. Particles pass by corresponding beam splitters (PBS) and produce clicks on detectors, which are coded by two random variables $A_x$ and $B_y$ taking values ±1.

It does not matter in QM how settings (x, y) are chosen. Experiments performed using different settings are incompatible and they are described, by specific, setting dependent, probability distributions. In particular pairwise expectations for a setting (x, y) are given by:

$$E(A_xB_y) = Tr \rho \hat{A}_x \hat{B}_y$$

(1)

where $\rho$ is a density matrix describing the ensemble E prepared by a source, $\hat{A}_x$ and $\hat{B}_y$. 
are operators representing spin projection measurements made by Alice and Bob respectively.

As Cetto et al. pointed out [32], the equation (1) can be rewritten as:

\[ E(A_x B_y) = \sum_{a,b} ab p_{a,b}(a,b) \]  

where \( a=\pm 1 \) and \( b=\pm 1 \) are experimental outcomes being eigenvalues of the operators \( \hat{A}_x \) and \( \hat{B}_y \). The quantum probabilistic models (1-2) explicitly depend on settings.

For a singlet state \( \rho \) and for identical settings \((x, x)\), QM predicts \( p(A_x = 1) = 1/2 \), \( p(B_x = 1) = 1/2 \) and \( p(A_x = 1, B_x = -1) = 1 \). It is mind boggling, if one believes that quantum probabilities are irreducible, because randomly created outcomes may not be perfectly correlated [11]. The only rational explanation is that experimental outcomes are predetermined by correlated properties of particles prepared at the source. The apparent randomness and a statistical scatter of outcomes are then due, like in classical physics, to the lack of knowledge of the statistical ensemble \( E \).

Let us cite Bell [33]: *For me, it is so reasonable to assume that the photons in those experiments carry with them programs, which have been correlated in advance, telling them how to behave. This is so rational that I think that when Einstein saw that, and the others refused to see it, he was the rational man.* This is the correct reasoning underlying LRHVM [3] in which clicks on detectors coded \( \pm 1 \) are predetermined:

\[ E(A'_x B'_y) = \sum_{\lambda \in \Lambda} A_1(\lambda) B_2(\lambda) p(\lambda) \]  

Please note that, contrary to Bell, we replaced in (3) \( A_x \) by \( A'_x \), and \( B_y \) by \( B'_y \). We did it, because, there is no JP of \((A_x , B_y, A'_x, B'_y)\), but there exists a JP of \((A'_x , B'_y, A'_x', B'_y')\). Namely for 4 experimental settings \((x, y) = (1,1), (1, 2), (2, 1) \) or \((2,2)\) we have:

\[ E(A'_1 B'_1 A'_2 B'_2) = \sum_{\lambda \in \Lambda} A_1(\lambda) B_2(\lambda) A_2(\lambda) B_2(\lambda) p(\lambda) . \]  

Moreover, there exists a mapping \( M : \Lambda \Rightarrow \Omega = \{ \omega = (a_1, b_1, a_2, b_2) \} \), where \( a_i = A'_i(\lambda) = \pm 1 \) and \( b_j = B'_j(\lambda) = \pm 1 \) thus:

\[ E(A'_1 B'_1 A'_2 B'_2) = \sum_{\omega \in \Omega} a_1 b_1 a_2 b_2 p(a_1, b_1, a_2, b_2) \]  

and instead of (3) we may use:

\[ E(A'_i B'_j) = \sum_{\omega \in \Omega} a_i b_j p_{A'_i B'_j}(a_i, b_j) \]  

where \( \Omega_{ij} = \{ (a_i, b_j) \} \) and \( p_{A'_i B'_j}(a_i, b_j) \) is a standard marginal distribution obtained from \( p(a_1, b_1, a_2, b_2) = p(\omega) = \sum_{\lambda \in M^{-1}(\omega)} p(\lambda) . \) Please note, that sample space \( \Omega \) contains exactly 16
elements and each sample space \( \Omega_{ij} \) only 4 elements. Using (5-6) one easily obtains CHSH inequalities [4]:

\[
| E(A'_1, B'_1) + E(A'_1, B'_2) + E(A'_2, B'_1) - E(A'_2, B'_2) | \leq 2
\]  

(7)

As Fine demonstrated [12, 13], the inequalities (7) are necessary and sufficient conditions for the existence of JP defined above.

In EPRB we have only pair-wise measurable random variables \((A_i, B_j)\) and their JP does not exist and Bell never claimed the opposite. Nevertheless Bell postulated that:

\[
E(A_i B_j) = E(A'_i B'_j) = \sum_{\lambda \in \Lambda} A_i(\lambda) B_j(\lambda) p(\lambda)
\]

(8)

and derived BI inequalities for \( E(A_i, B_j) \), without noticing that his prove implicitly relies on the existence of a counterfactual JP. He demonstrated that, for some experimental settings, the inequalities were violated by quantum predictions (1-2), but in 1964 he still hoped that experimental data might agree with his model.

In CbD [24-31], the equation (8) defines a non-contextual coupling of only pairwise jointly measurable observables:

\[
P(A_i = a) = P(A'_i = a); P(B_j = b) = P(B'_j = b); E(A_i B_j) = \langle A_i B_j \rangle = \langle A'_i B'_j \rangle
\]

(9)

which in general does not exist.

Ideal EPRB experiments, with perfectly correlated clicks on distant detectors, do not exist [11, 23]. Nevertheless, a significant violation of (7) was reported in several Bell Tests, thus the data in these experiments can neither be described using LRHVM nor by SHVM.

For a mathematician, the violation of (7) means only, that a non-contextual probabilistic coupling (9) does not exist and that CHSH inequalities are simply noncontextuality inequalities for a 4-cyclic scenario [20].

3 Experimental protocols and finite samples

Probabilistic models describe a scatter of observed outcomes without entering into details how these data were produced. However, there is an intimate relation between probabilistic models and experimental protocols [14, 34]. The model (3) describes a three-step random experiment.

1. A marble is drawn from an urn (or a box) \( E \). Properties of marbles in \( E \) are described by \( \lambda \) being values of some random variable \( L \) distributed according to a probability distribution \( p(\lambda) \) on a unique probability space \( \Lambda \).

2. Experimenters, choose at random one among 4 available incompatible settings \((i, j)\) of their instruments, which output two numbers \( A'_i = a = A_i(\lambda) \) and \( B'_j = b = B_j(\lambda) \).
3. The marble is returned to the box and another marble is drawn from the box.

Since $A'_i = A_i(L)$ and $B'_j = B_j(L)$, there exists JP of these random variables. It is obvious that the random variable $L$ and its probability distribution do not depend on how the settings $(i, j)$ are chosen in the step 2 of the experimental protocol. As in QM, $(i, j)$ are only labels of 4 incompatible experimental settings and experimenters’ freedom of choice is never compromised.

This explains why LRHVM is often called Bertlmann’s socks model. Pairs of photons, atoms or electrons are described, as they were pairs of socks. Similarly in so called stochastic hidden variable model (SHVM), they are described as pairs of dice. More detailed discussion of these probabilistic models and their intimate relation with experimental protocols may be found in [14].

In LRHVM, each experiment $(i, j)$ is described as a fair sampling from $\Lambda$ followed by a deterministic assignment of outcomes $(A'_i(\lambda), B'_j(\lambda))$. If we limit ourselves to 4 settings, then as we saw in (5, 6), instead of $\Lambda$, we may use a finite sample space containing only 16 elements: $\Omega = \{(a_1, b_1, a_2, b_2)\}$, where $a_1 = \pm 1$ and $b_2 = \pm 1$. Every finite random sample of size $4N$, drawn from $\Omega$, may be displayed in a $4N \times 4$ spreadsheet [35], such that each line obeys strictly the inequality (7). For an experiment performed in a setting $(i, j)$, only the entries $(a_i, b_j)$ are outputted and displayed in a specific $N \times 2$ spreadsheet, which is a random simple sample drawn from the corresponding columns of the $4N \times 4$ spreadsheet. If such four different $N \times 4$ spreadsheets are used to estimate expectations of $E(A'_i, B'_j)$ the inequalities (7) are violated approximately 50% of time [15,35-37] but not as significantly as predicted by QM and reported in Bell Tests. We see that in each trial of this thought experiment outcomes are predetermined and measuring instruments passively register corresponding predetermined values.

4. Bell Tests, instrument variables and contextuality.

In real experiment a $4N \times 4$ spreadsheet does not exist and four $N \times 4$ spreadsheets are not simple random samples drawn from the columns of $4N \times 4$ spreadsheet [11,15]. They cannot be reordered to satisfy (7) and the only constraint, on estimated $E(A_i, B_j)$, without additional assumptions, is: $S \leq 4$.

Moreover in Bell Tests, there are no perfect correlations. Some data violate no-signalling, thus they are also inconsistent with quantum predictions (1, 2) for an ideal EPRB. Using CbD terminology [24-31], the data used to estimate pairwise expectations are described by inconsistently connected random variables, thus they should be analyzed using CbD approach [18,19].

Therefore, it is clear that LRHVM is an oversimplified probabilistic model unable to describe the experimental data from Bell Tests. As Nieuwenhuizen [38-40] concluded, it suffers from contextuality loophole, because it does not incorporate correctly hidden variables describing measuring devices, as they are perceived by incoming photonic signals.
Lambare and Franco claim, that Bell recognized this point and showed how to deal with it. He averaged over instrument variables and derived CHSH [41]. Bell did not realize that, after such averaging, one obtains a hidden variable model describing a different random experiment with an experimental protocol impossible to implement [11, 14-16]. If instrument variables are correctly incorporated, then instead of (8) we obtain a contextual local causal probabilistic model which is consistent with the experimental protocol used in Bell Tests:

\[
E(A_j B_j) = \sum_{\lambda \in \Lambda_j} A_i(\lambda_1, \lambda_4) B_j(\lambda_2, \lambda_5) p_j(\lambda)
\]

where \( \lambda = (\lambda_1, \lambda_2, \lambda_4, \lambda_5) \), \( \Lambda_j \cap \Lambda_{i,j} = \emptyset \) and

\[
p_j(\lambda) = p(\lambda | i, j) = p_i(\lambda_i) p_j(\lambda_j) p(\lambda_i, \lambda_j)
\]  

Using (11) and Bayes Theorem we obtain:

\[
p(\lambda, i, j) = p_i(\lambda_i) p_j(\lambda_j) p(\lambda_i, \lambda_j) p(i, j) = p(\lambda) \rightarrow p(i, j | \lambda) = 1
\]

It means only that, if an invisible events \( \{\lambda\} \) happened, thus the settings \( (i, j) \) were used [16-19]. It has nothing to do with conspiracy and experimenters’ freedom of choice is not compromised. Therefore, the assumption \( p(\lambda | x, y) = p(\lambda) \) should be called noncontextuality and not MI.

In the model (10) hidden variables depend on settings, thus different random experiments may not be described using a non-contextual probabilistic coupling (9). There is no JP distribution (4), which may be used to prove CHSH (7). In order to explain faithfully experimental data in Bell Tests: \( A_i(\lambda_1, \lambda_4) = \pm 1 \) or \( 0 \) and \( B_j(\lambda_2, \lambda_5) = \pm 1 \) or \( 0 \) where \( 0 \) denotes the absence of a click [11, 16, 18, 19].

It is clear, that the frequentist proof of CHSH, given in [6], fails, if \( p(\lambda | x, y) \neq p(\lambda) \). Lambare and Franco realize this, but they believe, as many do, that the violation of MI, would mean conspiracy or superdeterminism, thus they dismiss such solution.

5 Fine’s Theorem, joint probabilities and rejection of criticism

Fine demonstrated, that CHSH are necessary and sufficient conditions for the existence of JP of 4 only pairwise measurable random variables [12,13]. Nobody claims that Fine has disproved Bell’s Theorem.

Lambare and Franco claim to show the fallacy of arguments given in [7-11]. We reject this claim by analyzing in detail their counterexample (Eqs.35-41 in [6]).

JP of \( n \)-random variables may only exist, if in each trial n-results are outputted [34,37]. Therefore, JP neither exists in Bell scenario nor in their counterexample. However, the correlations in their experiment and in the experiment with metal balls discussed in [11] may be
derived using a counterfactual JP and an appropriate probabilistic coupling. This is possible because the outcomes of these incompatible experiments are predetermined.

In their counterexample we have: 7 random variables: L taking values \( \lambda \in \Lambda = \{1, 2, 3, 4, 5, 6\} \), \( X \) taking values \( x = \{1, -1\} \), \( Y \) taking values \( y = \{1, -1\} \), \( A_x \) and \( B_y \). L describes an experiment in which hidden variables are sampled from \( \Lambda \) by rolling a dice, \( X \) and \( Y \) are random variables describing flipping fair coins in order to determine experimental settings \( (x, y) \), \( A_x = A(x, L) \) and \( B_y = B(y, L) \) are random variables describing predetermined outcomes. Namely:

\[
A_x(\lambda) = A(x, \lambda) = x^3, \quad B_y(\lambda) = B(y, \lambda) = y^{i+1}
\]  

(13)

We have 4 incompatible experiments, labelled by \( (x, y) \), and only 2 outcomes are outputted in each trial, thus JP of 4 random variables \( (A_1, A_{-1}, B_1, B_{-1}) \) does not exist. It is easy to evaluate 4 expectations [6] entering the inequality (7):

\[
E(A,B) = 1, \quad E(A_{-1},B) = E(A,B_{-1}) = 0, \quad E(A_{-1},B_{-1}) = -1
\]  

(14)

Expectations (14) do not violate the inequality (7). The authors incorrectly conclude: ``according to Fine’s theorem A, a joint probability \( P(A_1; A_{-1}; B_1; B_{-1}) \) exists, although the experiments are incompatible``.

The random variables \( A_1, A_{-1}, B_1 \) and \( B_{-1} \) are not jointly distributed. Nevertheless, there exist 4 random jointly distributed variables \( A_1', A_{-1}', B_1' \) and \( B_{-1}' \), which define a non-contextual coupling \( E(A_x) = E(A_x'), E(B_y) = E(B_y'), E(A_x B_y) = E(A_x' B_y') \).

Instead of (4) using (13) we have now:

\[
E(A_1 A_{-1} B_1 B_{-1}) = \sum_{\lambda \in \Lambda} A_1(\lambda) A_{-1}(\lambda) B_1(\lambda) B_{-1}(\lambda) p(\lambda) = \sum_{\lambda = 1}^6 1^3(-1)^11^{i+1}(-1)^1 = -1
\]  

(15)

The random variables \( A_1', A_{-1}', B_1' \) and \( B_{-1}' \) define a mapping \( M : \Lambda \Rightarrow \Omega = \{(1,1,-1,1), (1,1,1,-1)\} \) and their joint probability distribution is:

\[
p_1 = p(1,1,-1,1) = \frac{1}{2}, \quad p_2 = p(1,1,1,-1) = \frac{1}{2}
\]  

(16)

Using (16) we immediately obtain:

\[
E(A_1' A_{-1}' B_1' B_{-1}') = 1 \times (-1) \times 1 \times 1 \times p_1 + 1 \times 1 \times (-1) \times p_2 = -1
\]  

(17)

\[
E(A_1' B_1') = 1, \quad E(A_{-1}' B_{-1}') = E(A_1' B_{-1}') = 0 \quad \text{and} \quad E(A_{-1}' B_{-1}') = -1.
\]

We use only (16) and we do not need to mention hidden variables.
Jointly distributed $A', A''$, $B'$ and $B''$ describe outcomes of a different random experiment in which in each trial one obtains one of two quadruplets with probability $\frac{1}{2}$. For example, after receiving the same $\lambda$, both Alice and Bob flip two fair coins each, and output their outcomes calculated using (13). After $N$ trials Bob sends his $Nx2$ spreadsheet to Alice, who displays her and his results (strictly preserving the order) in a new $Nx4$ spreadsheet. Only these data are described by JP of 4 random variables and now various pair-wise correlations between them may be estimated. This is the main problem in real Bell Tests, because there is no unambiguous ordering between distant clicks produced by entangled photonic signals [18, 23].

It is difficult to understand, why such arguments are not understood and are still a minority stance. Already in 1984, we wrote [42]:” To describe random events in any particular experiment we do not need to abandon the Kolmogorov axioms of probability theory. However, the measured probabilities in the different experiments may not be determined by conditionalization from a unique probability space. The last assumption was used in all the proofs of Bell inequalities.”

6. Conclusions

The claims related to joint probabilities [8-11] are not bizarre or fallacious. The criticism of these claims in [6] is unfounded. We explained that measurement independence (MI) should be called noncontextuality, because it allows implementing random variables, describing incompatible random experiments, on a unique probability space, on which they are jointly distributed. Using CbD terminology such implementation defines a probabilistic coupling and MI is equivalent to context-independent mapping. In EPRB and in Bell Tests JP of random variables describing the outcomes of 4 incompatible experiments does not exist. In LRHVM, which defines a particular probabilistic coupling, such JP does exist.

We agree with the authors, that the counterfactual reasoning does not underlie Bell Theorem. However in LRHVM clicks on detectors (coded ±1) are locally predetermined by variables describing correlated photonic signals, Local predetermination of outcomes of experiments, by some ontic properties of signals, is called usually: local realism, classicality or counterfactual definiteness (CFD). Since different authors attach a different meaning to the notion of realism, thus CFD understood as local predetermination of outcomes is less ambiguous.

Such assumption was proven incorrect, but it was not stupid. Reinhold Bertlmann remembers, what his friend John said to him: ”I’m a realist…I think that in actual daily practice all scientists are realists, they believe that the world is really there, that it is not a creation of their mind. They feel that there are things there to be discovered, not a world to be invented but a world to be discovered. So I think that realism is a natural position for a scientist and in this debate about the meaning of quantum mechanics I do not know any good arguments against realism.”[43].

Local realism understood as CFD, automatically implies MI and the existence of JP. Bell Theorem and its implications are now well understood but nobody questions Bell’s Theorem
logical consistency. Bell inequalities are violated in various Bell Tests, what only proves that LRHVM and SHVM provide an incorrect and an oversimplified description of these experiments. Several authors arrived several years ago and often independently, to such correct conclusion e.g.[7-19, 22-23, 34, 36-40, 44-67], where more references may be found.

The violation of Bell inequalities neither proves completeness of QM nor impossibility of a local and causal description of experimental data. It only proves, that hidden variables have to depend on settings confirming contextual character of quantum observables and an active role played by measuring instruments.

It is high time to stop speculating about nonlocality, freedom of choice, retro-causality etc. Bell was a realist, thus he thought that he had to choose between nonlocality and superdeterminism. From two bad choices he chose nonlocality. Today probably he would choose contextuality.

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