GAMMA-RAY EMISSION FROM CRUSHED CLOUDS IN SUPERNOVA REMNANTS

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ABSTRACT

It is shown that the radio and gamma-ray emission observed from newly-found “GeV-bright” supernova remnants (SNRs) can be explained by a model, in which a shocked cloud and shock-accelerated cosmic rays (CRs) frozen in it are simultaneously compressed by the supernova blastwave as a result of formation of a radiative cloud shock. Simple reacceleration of pre-existing CRs is generally sufficient to power the observed gamma-ray emission through the decays of $\pi^0$-mesons produced in hadronic interactions between high-energy protons (nuclei) and gas in the compressed-cloud layer. This model provides a natural account of the observed synchrotron radiation in SNRs W51C, W44 and IC 443 with flat radio spectral index, which can be ascribed to a combination of secondary and reaccelerated electrons and positrons.

Subject headings: cosmic rays — acceleration of particles — ISM: individual objects (W51C, W44, IC 443) — radiation mechanisms: nonthermal

1. INTRODUCTION

Luminous extended GeV $\gamma$-ray emission associated with middle-aged supernova remnants (SNRs) has recently been unveiled by the Large Area Telescope (LAT) on the Fermi Gamma-ray Space Telescope. Specifically, SNRs W51C, W44, IC 443, and W28 are spatially resolved with the Fermi LAT (Abdo et al. 2009, 2010a, b, c). The four SNRs are interacting with molecular clouds, as evidenced e.g., by 1720 MHz OH maser emission. The radio and $\gamma$-ray emission from these SNRs share similar characteristics. The synchrotron radio emission has a large flux of 160–310 Jy at 1 GHz with flat spectral index of $\alpha \approx 0.26$–0.40. The GeV $\gamma$-ray spectrum commonly exhibits a spectral break at around 1–10 GeV, and the luminosity ranges $L_{\gamma} \approx (0.8$–9$) \times 10^{35}$ erg s$^{-1}$ in the 1–100 GeV band. Other cloud-interacting SNRs associated with the LAT sources, such as CTB 37A, also emit $\gamma$-rays at a luminosity of $L_{\gamma} \sim 10^{35}$ erg s$^{-1}$ (Castro & Slane 2010).

A prototypical example of the GeV-bright SNRs is SNR W44. The radio continuum map of W44 exhibits filamentary and sheet-like structures of synchrotron radiation well correlated with the shocked H$_2$ emission (Reach et al. 2005; Castelletti et al. 2007). According to Reach et al. (2005), the bulk of the synchrotron radiation can be ascribed to a fast molecular shock of a velocity $v_s \sim 100$ km s$^{-1}$ advancing through a molecular cloud of a preshock density $n_0 \sim 200$ cm$^{-3}$. By passage of the blastwave of the supernova remnant, the shocked molecular cloud forms a thin sheet due to radiative cooling. The radio filaments are thought to come from the compressed zone behind the shock front.

The synchrotron radio emission arising from such a “crushed cloud” was modeled by Blandford & Cowie (1982). It was shown that reacceleration of pre-existing CR electrons at a cloud shock and subsequent adiabatic compression results in enhanced synchrotron radiation, capable of explaining the radio intensity from evolved SNRs. Bykov et al. (2000) discussed a similar scenario, in which direct electron acceleration from the thermal pool is also invoked.

The $\pi^0$-decay $\gamma$-ray emission should be enhanced in the crushed clouds in the same manner as the synchrotron radiation. In this Letter, we demonstrate that the newly-found $\gamma$-ray emission from middle-aged SNRs can be readily understood within the crushed cloud scenario, in which the $\gamma$-ray emission comes from shocked clouds overrun by SNR blastwaves. It should be emphasized that our model is essentially different from the scenarios adopted in the recent papers (e.g., Fujita et al. 2010; Torres et al. 2010), where molecular clouds in the vicinity of SNRs are assumed to be illuminated by runaway CRs (Aharonian & Atoyan 1996; Gabici et al. 2009). While the GeV and TeV $\gamma$-rays outside the southern boundary of SNR W28 (Aharonian et al. 2008; Giuliani et al. 2010; Abdo et al. 2010c) may be explained by such runaway CRs, we argue here that the luminous GeV $\gamma$-ray emission in the directions of cloud-interacting SNRs emerges from the radiatively-compressed clouds.

2. THE MODEL

2.1. Cloud Shock Structure

Let us consider a strong shock driven into a molecular cloud by the high pressure behind a supernova blastwave. Using the number density of hydrogen nucleus in the preshock cloud, $n_0$, the preshock magnetic field is described by a dimensionless parameter $b$:

$$B_0 = b \sqrt{(n_0 / \text{cm}^{-3})} \mu\text{G}.$$  \hspace{1cm} (1)

Zeeman measurements of self-gravitating molecular clouds show that $b$ is roughly constant from one cloud to another with $b \sim 1$ (Crutcher 1999). We are concerned primarily with a fast ($v_s \gtrsim 50$ km s$^{-1}$) J-type shock (Draine 1980), in which ambipolar diffusion and radiative cooling are unimportant in a shock dissipation layer. The steady-state postshock structure of a fast molecular shock is described in Hollenbach & McKee (1989), which is referred to as HM89 hereafter.

The initial temperature immediately behind the shock front is $T_s \approx 3.2 \times 10^5 v_{\text{sh}}^2 / x_1 \text{K}$, where $v_{\text{sh}} \equiv v_s / (100$ km s$^{-1})$, and $x_1$ is the number of particles per hydrogen nucleus (HM89). The density rises from a preshock density $n_0$ by a factor of $x_{\text{sh}} = 4$,...
in the strong shock limit. For \( \nu > 1 \), ionizing radiation produced in the immediate postshock layer is strong enough to fully predissociate and preionize the upstream cloud.

As the gas radiatively cools downstream, the temperature decreases and the density increases. The compression of the cooled gas, \( n_m \), is determined as

\[
n_m \approx 94 n_0 v_s n_0^{-1},
\]

which is obtained by equating \( B_0^2/8\pi \) with the shock ram pressure \( n_0 u_0^2/2 \), where \( B_0 = \sqrt{2/3} \) (the compressed field pressure, and \( u_0 \) is the mass per hydrogen nucleus (HM89). We have assumed that the preshock magnetic field is randomly directed.) The \( [\text{O} \text{I}] \) line (632\( \mu \text{m} \)) surface brightness of a face-on J-shock is proportional to the particle flux into the shock, \( n_0 v_s \). The peak surface brightness of the \( [\text{O} \text{I}] \) line is \( 1 \times 10^{-3} \) erg cm\(^{-2}\) s\(^{-1}\) sr\(^{-1}\) in SNR W44 (Reach & Rho 1996), and a factor of 2 smaller in IC 443 (Rho et al. 2001), indicating the particle flux of order \( n_0 v_s \sim 10^6 \) cm\(^{-2}\) s\(^{-1}\) (HM89) and therefore \( n_m \sim 9 \times 10^{-1} \) cm\(^{-3}\).

A certain column density, \( N_{\text{cool}} \), has to be transmitted by a shock for gas to cool down to \( 10^8 \) K and become radiative. This occurs in a column density of \( N_{\text{cool}} \approx 10^{17} \) \( v_s^2 \) cm\(^{-2}\) for \( \nu > 0.6 \) (McKee et al. 1987). Recombination and photoionization by the ultraviolet radiation produced upstream are balanced in \( N_{\text{cool}} < N < N_{\text{ion}} \sim 10^{19} \) cm\(^{-2}\). Beyond \( N_{\text{ion}} \), the ionizing photons are absorbed and molecular chemistry commences. By setting a typical elapsed time since shocked to be \( t_s = t/2 \), where \( t = 10^7 t_4 \) yr is the age of the remnant, and using \( n_0 \equiv n_0/(100 \text{ cm}^3) \), the column density of the compressed cloud is written as \( N_s \equiv n_0 v_s t_s \sim 1.5 \times 10^{20} n_0 v_s t_s \) cm\(^{-2}\).

The bulk of synchrotron radio waves and \( \gamma \)-rays should be emitted in the compressed gas with a constant density \( n_m \) (within a factor of 2) and magnetic field \( B_m \). Note that we idealize the shock as one-dimensional and ignore any effects, such as lateral compression, caused by the secondary shocks. The secondary shocks could affect the leakage of high-energy particles from the compressed cloud, playing an indirect role in the \( \gamma \)-ray production. Also, the ultraviolet radiation produced in the secondary shocks may change the ionization level of the precursor of the main cloud shock.

2.2. Shock Acceleration and Adiabatic Compression

Following Blandford & Cowie (1982), we consider a conservative case in which only pre-existing CRs are accelerated by the process of diffusive shock acceleration at a cloud shock. Suprathermal particles may be injected to the acceleration process at the shock front, despite slow acceleration and fast Coulomb losses. However, we shall show below that the shock acceleration of pre-existing CRs alone appears to suffice as the origin of the observed \( \gamma \)-ray emission from the cloud-interacting SNRs, and therefore consider the reacceleration case only.

Let \( n_{\text{acc}}(p) dp \) be the CR number density in \( p \sim p+dp \) transmitted by a shock, and \( n_{\text{GCR}}(p) \) be the pre-existing ambient CR density. According to the theory of diffusive shock acceleration (Blandford & Eichler 1987), for \( p < p_{\text{br}/\text{max}} \) (see below),

\[
n_{\text{acc}}(p) = (\alpha + 2) p^{-\alpha} \int_0^p dp' n_{\text{GCR}}(p') p'^{(\alpha - 1)},
\]

where \( \alpha = (r_{\text{sh}} + 2)/(r_{\text{sh}} - 1) \) and \( r_{\text{sh}} \) is the shock compression ratio, which is assumed to be \( r_{\text{sh}} = 4 \).

Assuming that the density of the Galactic CRs in the molecular cloud is same as that in the general interstellar medium, we adopt the Galactic CR proton spectrum of the form:

\[
n_{\text{GCR}}(p) = 4\pi J_p p^{(\beta + 2)/3} p_0^{-2},
\]

where \( p_0 = p/(\text{GeV}/c) \), \( J_p = 1.9 \text{ cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{GeV}^{-1} \), \( \beta \) is the proton velocity in units of \( c \). A low-energy cutoff at a kinetic energy of \( 50 \) MeV is applied. This spectrum lies in between Shikaze et al. (2007) and Strong et al. (2004) at 100 MeV. For the CR electron+positron spectrum, we use:

\[
n_{\text{GCR}}(e) = 4\pi J_e p_0^2 \left( 1 + \frac{p}{p_0} \right)^{-0.55},
\]

extending down to a low-energy cutoff of \( 20 \) MeV at which point ionization losses in the Galaxy should make the spectrum flat. The normalization factor is \( J_e = 2 \times 10^{-2} \text{ cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{GeV}^{-1} \).

The preshock upstream gas is only partially ionized for \( \nu > 1 \). Recently, Malkov et al. (2010) have proposed that strong ion-neutral collisions accompanying Alfvén wave evanescence lead to steepening of the spectrum of accelerated particles; the slope of the particle momentum distribution becomes steeper by one power above \( p_{\text{br}} = 2 B_0 V_s/\mu_n \), where \( V_s \) is the Alfvén velocity and \( \nu \approx 9 \times 10^9 n_0 T_{\text{sh}}^{-2/3} \) is the ion-neutral collision frequency. Here \( n_0 \) denotes the density of neutrals in units of \( \text{cm}^{-3} \) and \( T_s \) is the precursor temperature in units of \( 10^8 \) K. Interestingly, the parameters of Reach et al. (2005) estimated for the radio filaments of W44, namely \( \nu \approx 1 \) and \( n_0 \approx 2 \), together with \( B_0 = 30 \mu \text{G} \) and \( T_s = 1 \) predict \( p_{\text{br}} = 10 \text{ GeV}/c \) using the precursor ionization fraction calculated by HM89. This agrees with the break value measured by the Fermi LAT (Abdo et al. 2010a). We introduce spectral steepening to \( n_{\text{acc}}(p) \), by multiplying a factor of \( p_{\text{br}}/p \) above \( p_{\text{br}} \).

Even if the preshock gas is fully ionized, spectral steepening due to a finite acceleration time would be unavoidable already in the Fermi bandpass. The timescale of diffusive shock acceleration can be written as \( t_{\text{acc}} \sim (10/3) \tau \nu r_s v_s^2 \), where \( r_s = c p/e B_0 \) is the gyroradius and \( \tau \sim 1 \) is the gyrofactor; \( \eta \sim 1 \) has been obtained in young SNRs like RX J1713.7-3946 (Uchiyama et al. 2007). Equating \( t_{\text{acc}} \) with \( t_s \), the maximum attainable energy is obtained as \( p_{\text{max}} \sim 50 (\eta/10)^{-1/2} B_{-5} T_{24} \text{ GeV} \), where \( B_{-5} = B_0/10^{-5} \) G. A factor of \( \exp[-p/(p_{\text{max}})] \) is multiplied with \( n_{\text{acc}}(p) \) to introduce the maximum attainable energy.

The high-energy particles accelerated at the shock experience further heating due to adiabatic compression, as the gas density increases until the pressure is magnetically supported. Each particle gains energy as \( p \sim s^{1/3} \), where \( s \equiv (n_m/n_0)/r_0 \), and the density increases by a factor of \( s \) (see Blandford & Cowie 1982). Therefore, the number density of accelerated and compressed CRs at the point where the density becomes \( n_m \) is

\[
n_{\text{acc}}(p) = s^{2/3} n_{\text{acc}}(s^{1/3} p).
\]

2.3. Evolution in the Compressed Region

In the compressed region of the cloud with a constant density \( n_m \), high-energy particles suffer from energy losses such as Coulomb/ionization losses. Also, the production of
and for secondary gas. The particle spectrum for accelerated electrons, are assumed to be effectively trapped within the compressed volume of the preshock cloud is then \( fV \) following the prescription given by Kamae et al. (2006). Let \( N_p(p,t)dp \) represent the number of protons in \( p \sim p+dp \) integrated over the emission volume at time \( t \). We employ a usual kinetic equation to obtain the proton spectrum \( N_p(p,t) \):

\[
\frac{\partial N_p(p,t)}{\partial t} = \frac{\partial}{\partial p} \left[ b(p)N_p(p,t) \right] + Q_p(p),
\]

where \( Q_p \) is the proton injection rate, and \( b(p) = -\dot{p} \) represents the proton energy losses. Charged particles with \( p < p_{\text{max}} \) are assumed to be effectively trapped within the compressed gas. The particle spectrum for accelerated electrons, \( \tilde{N}_e(p,t) \), and for secondary \( e^\pm \) resulting from hadronic interactions, \( N_{\text{sec}}(p,t) \), are computed similarly.

The injection rates of primaries are related to \( n_{\text{ad}}(p) \):

\[
Q(p) = n_0 f V \frac{\dot{V}}{n_{\text{ad}}},
\]

where \( V = \frac{4\pi R^3}{3} \) is the SNR volume with a radius \( R \), and \( f \) is the preshock filling factor of the compressed cloud. The volume of the preshock cloud is then \( fV \). For simplicity, we assume that \( Q(p) \) for primaries is time-independent; \( v_s \) is assumed to be constant and \( p_{\text{max}} \) is evaluated simply at \( t = t_c \). The injection rate of secondary \( e^\pm \), \( Q_{\text{sec}}(p_e,t) \), is determined by \( N_p(P_p,t) \) following the prescription given by Kamae et al. (2006).

### 2.4. SNR Dynamics

It may be desirable to relate the cloud shock velocity with the physical parameters that describe the blastwave of the remnant. We consider a remnant in the Sedov stage:\(^4\)

\[
R_{12.5} = \left( \frac{E_{51}}{n_{a0}} \right)^{1/5} \left( \frac{t_c}{4} \right)^{1/5},
\]

\(^4\) This implies \( t < t_0 \), where \( t_0 \) is an age for transition to the radiative phase of SNR evolution (Blondin et al. 1998): \( t_0 \sim 3 \times 10^4 E_{51}^{4/17} n_{a0}^{-1/17} \) yr.

where \( R_{12.5} = R/(12.5 \text{ pc}) \), \( E_{51} \) denotes the kinetic energy released by the supernova in units of \( 10^{51} \text{ erg} \), and \( n_{a0} = n_a/(\text{cm}^{-3}) \) represents the ambient (intercloud) density. The blastwave velocity is \( v_b = 0.4R/t \). When a molecular cloud is struck by the blastwave, a strong shock is driven into the cloud with a shock velocity of

\[
v_s = k \left( \frac{n_0}{n_{a0}} \right)^{1/2} v_b,
\]

\[
\cong 65 n_{a0}^{-1/2} E_{51}^{-1/2} R_{12.5}^{-3/2} \text{ km s}^{-1},
\]

where \( k \approx 1.3 \) is adopted. Note that \( k \) depends weakly on \( v_s/v_b \) (McKee & Cowie 1975), ranging \( k = 1-1.5 \) in the circumstances of our interest. To drive a fast molecular shock of \( v_s \gtrsim 0.5 \) into a molecular cloud, a condition of

\[
n_{a0} \lesssim k^2 E_{51} R_{12.5}^3
\]

must be satisfied.

**TABLE 1**

| Parameters | Values |
|------------|--------|
| Assumed SNR Dynamics | |
| Distance: \( D \) | 2.9 kpc\(^a\) |
| Radius: \( R \) | 12.5 pc\(^a\) |
| Age: \( t \) | \((9 \approx 15\text{')}\) |
| Explosion energy: \( E_{51} \) | 100000 yr\(^a\) |
| Preshock Cloud Parameters | |
| Density: \( n_0 \) | 200 cm\(^{-3}\)\(^a\) |
| Filling factor: \( f \) | 0.18 |
| Magnetic field: \( B_0 \) | 25 \( \mu \text{G} \) |
| Dependent Parameters | |
| Cloud shock velocity: \( v_s \) | 100 km s\(^{-1}\)\(^a\) |
| Break momentum (\( f_b = 2 \)): \( p_b \) | 7 GeV/c |
| Maximum momentum (\( f > 10 \)): \( p_{\text{max}} \) | 122 GeV/c |

\(^a\) Taken from Reach et al. (2005).
3. Results

Let us calculate the nonthermal radiation arising from the radiatively compressed clouds. The gas density and magnetic field strength are constant over the emission volume in which high-energy particles are distributed with the volume-integrated spectrum of \( N(p, t) \). Leptonic components include synchrotron radiation, inverse-Compton scattering, and relativistic bremsstrahlung; the inverse-Compton emission is negligible as compared to the bremsstrahlung component (see, e.g., Abdo et al. 2010a). Inelastic collisions between CR protons/nuclei and gas nuclei lead to \( \pi^0 \)-decay \( \gamma \)-ray emission, which constitutes the main component in the Fermi-LAT band under the assumption that only pre-existing CRs can be accelerated at the cloud shock.

We do not consider the densest part of the interacting molecular cloud. For example, detections of OH(1720 MHz) maser emission (Frail et al. 1996) indicate the presence of slow \( C \)-type shocks in dense molecular clumps, say \( n_\pi \sim 0.3 \) and \( n_{\rm H_2} \sim 10 \), in addition to the fast \( J \)-type shock described in §2. However, the \( \gamma \)-ray emission from the slow non-dissociative shocks is not expected to be strong because of inefficient shock acceleration and weaker gas compression (see also Bykov et al. 2000). Indeed, the contribution from such dense molecular clumps to the total radio intensity appears small in SNR W44 (Reach et al. 2005). The largest preshock density we should consider is given by Eq. (11). Also, we ignore the blastwave region, since the radiative compression is essential in our model.

3.1. Gamma-ray Luminosity

We show here that the \( \gamma \)-ray luminosity anticipated within this scenario agrees well with the observed luminosity of \( \sim 10^{35} \text{ erg s}^{-1} \). In Figure 1, the \( \gamma \)-ray spectra of SNRs W51C, W44, and IC 443 measured with the Fermi-LAT are shown in the so-called \( \nu L_\nu \) form in units of \( 10^{35} \text{ erg s}^{-1} \). The LAT spectral points are taken from Abdo et al. (2009, 2010a,b) and converted into the \( \nu L_\nu \) form using the distances of 6 kpc (W51C), 2.9 kpc (W44), and 1.5 kpc (IC 443).

To demonstrate the expected level of the \( \gamma \)-ray luminosity, we present the spectra of \( \pi^0 \)-decay \( \gamma \)-rays with varying \( R \), \( n_0 \), and \( E_{51} \). The following parameters are fixed: \( b \equiv 2 \), \( f = 0.2 \), \( n_{\rm H_2} = 1 \). Also, \( p_{\mu} \) is set by adopting \( n = 10 \) and \( p_{\pi} \) is set by \( T_\pi = 2 \) and by the ionization fraction calculated based on HM89, here and hereafter. The \( \gamma \)-ray spectra simply scale as \( \propto f \) and depend very weakly on \( n_\pi \). The black lines in Fig. 1 show the results obtained for \( n_{\rm H_2} = 0.3 \) (solid curve) and \( n_{\rm H_2} = 3 \) (dashed curve) in the case of \( R = 10 \text{ pc} \) and \( E_{51} = 1 \). The \( \gamma \)-ray luminosity around 1 GeV varies only within a factor of \( \sim 2 \) between \( n_{\rm H_2} = 0.3 \) and \( n_{\rm H_2} = 3 \), while that at 100 GeV changes more than an order of magnitude. On the other hand, the blue lines in Fig. 1 show the spectra calculated for \( R = 5 \text{ pc} \) (solid curve) and \( R = 15 \text{ pc} \) (dashed curve) in the case of \( n_{\rm H_2} = 1 \) and \( E_{51} = 1 \). The \( \gamma \)-ray luminosity differs by a factor of \( \sim 3 \). We note that the shocked cloud mass amounts to \( \sim 10^4 M_\odot \) in the case of \( R = 15 \text{ pc} \). Finally, to explore the most luminous scenario, we adopt \( E_{51} = 5 \) together with \( R = 30 \text{ pc} \) and \( n_{\rm H_2} = 1 \) (red curve). The \( \gamma \)-ray luminosity reaches \( \sim 10^{36} \text{ erg s}^{-1} \) in good agreement with the observations of SNR W51C, which is indeed the most luminous SNRs in gamma-rays. Our model generally predicts \( L_\gamma \sim 10^{35} (f/0.2) E_{51}^{2/3} \text{ erg s}^{-1} \) to the first order, which led us to conclude that the Fermi-detected \( \gamma \)-rays are quite likely due to the decays of \( \pi^0 \)-mesons produced by the pre-existing CRs accelerated and subsequently compressed in the shocked cloud.

3.2. Flat Radio Spectra

The GeV-bright SNRs W51C, W44, and IC 443 are also radio-bright objects. As shell-type SNRs, their radio spectra are remarkably flat with spectral index of \( \alpha \sim 0.26 \) (W51C; Moon & Koo 1994), \( \alpha \simeq 0.37 \) (W44: Castelletti et al. 2007), and \( \alpha \approx 0.36 \) (IC 443; Erickson & Mahoney 1985), with a typical uncertainty of 0.02, being inconsistent with \( \alpha = 0.5 \) that is expected by shock-acceleration theory. Our model naturally explains the flat radio spectrum. Let us demonstrate by presenting the radio and \( \gamma \)-ray modeling of SNR W44 how the radio and \( \gamma \)-ray spectra can be simultaneously reproduced.

Radio (Castelletti et al. 2007) and \( \gamma \)-ray (Abdo et al. 2010a) spectra of SNR W44 are shown in Figure 2. One half of the total synchrotron flux measured for W44 is assumed to originate in a fast molecular shock, which is roughly consistent with Table 2 of Reach et al. (2005). The rest is attributed to the blastwave region. Table 2 of Reach et al. (2005) was chosen as an initial set of model parameters: \( R_{1.2} = 1, \alpha = 1, E_{51} = 5 \), and \( n_{\rm H_2} = 2 \). We then attempted to reproduce the non-thermal radiation spectra by varying \( f \) and \( B_{\mu} \) and found that \( f = 0.18 \) and \( B_{\mu} = 25 \mu G \) provide a good fit to the data (see Table 1 and Fig. 2). The radio measurements can be reconciled with this model, in which the synchrotron radiation is largely contributed by secondary electrons and positrons. The flat radio spectrum is generically expected in our model.

4. Discussion

The radiatively-compressed cloud provides a simple explanation for the radio and \( \gamma \)-ray data. Interestingly, the observed steepening in the \( \gamma \)-ray spectra is successfully reproduced by \( p_{\mu} \) in the case of SNR W44. However, there may be other explanations for the steepening. For example, high-energy particles may be prone to escape from the compressed magnetized cloud. Also, the spectral break may be due to the fact that the crushed clouds have a range of \( n_0 \). A superposition of \( \gamma \)-ray spectra characterized by different \( p_{\mu} \) (as a result of different \( n_0 \)) could look like a break. We are primarily interested in understanding the \( \gamma \)-ray luminosity rather than the spectral shape in this paper, and therefore we did not explore this issue.

The simple re-acceleration of pre-existing CRs and subsequent compression alone would not fully explain the \( \gamma \)-rays associated with cloud-interacting SNRs. For example, the GeV-TeV \( \gamma \)-ray emission found outside the radio boundary of SNR W28 (Aharonyan et al. 2008; Giuliani et al. 2010; Abdo et al. 2010c) may represent the molecular cloud illuminated by runaway CRs (Aharonyan & Atoyan 1996; Gabici et al. 2009). Also, we assumed pre-existing CRs in the cloud to have the same spectra as the galactic CRs in the vicinity of the solar system. However, the ambient CRs in the pre-shock cloud may deviate from the galactic pool due to the runaway CRs that have escaped from SNR shocks at earlier epochs. If this is the case, modeling of the \( \gamma \)-ray spectrum, at TeV energies in particular, should take into account modified pre-existing CRs in the pre-shock cloud.

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