Relation between the lepton and quark mixings: $U_{PMNS} \approx V_{CKM}^\dagger U_X$, where $U_X$ is the BM or TBM mixing matrices, implies the quark-lepton (Grand) unification and existence of hidden sector with certain flavor symmetries. The latter couples to the visible sector via the neutrino portal and is responsible for $U_X$, as well as for smallness of neutrino mass. GUT ensures appearance of $\sim V_{CKM}$ in the lepton mixing. General features of this scenario (inverse or double seesaw, screening of the Dirac structures, basis fixing symmetry) are described and two realizations are presented. The high energy realization is based on $SO(10)$ GUT with the hidden sector at the Planck scale. The low energy realization includes the 100 TeV scale $L - R$ symmetry and the hidden sector at the keV - MeV scale.

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1 Introduction

It seems, another phase in developments of the field related to the discovery of large lepton mixing and wide exploration of the non-abelian discrete flavor symmetries is nearly over. No simple and convincing model has been proposed which could explain small quark mixing and large lepton mixing in the same framework. Several different symmetries with \textit{ad hoc} charge prescriptions, non-renormalizable interactions, complicated flavons content, \textit{etc.} are some generic features of the models. The bottom line is that in understanding of neutrino mass and mixing we are not far from the very beginning, that is, from experimental results.

In this connection the guidelines could be that

1. After all, the Grand Unification is the best proposed physics beyond the Standard Model (SM). It provides unification of forces: explanation of why the strong interactions are strong, the weak interactions are weak, and the EM interactions are as they are. GUT unifies quarks and leptons and gives explanation of the SM symmetry charges. SO(10) perfectly embeds all known fermions including RH neutrinos in a single 16-plet.

The simplest versions of GUT’s predicted

\[ m_q \sim m_l \sim m^\nu_D \quad \text{or} \quad m_l \sim m_d, \quad m^\nu_D \sim m_u. \]  

(1)

Beauty of the seesaw mechanism is that it allows to reconcile relations (1), \textit{i.e.} “normal” values of the Dirac Yukawa couplings of neutrinos, and smallness of neutrino mass with only one assumption – existence of large Majorana masses of the RH neutrinos, \( M_R \). Furthermore, \( M_R \sim M_{GUT} \). In many models the “hybrid” seesaw is employed which uses two assumptions (and in this sense less attractive): large Majorana masses of the RH neutrinos and smallness of the Dirac Yukawa couplings (Dirac masses).

2. Existing GUT picture is not complete (hierarchy problem, proton decay, \textit{etc.}), something important is still missing, but adding these “extra” may produce small perturbation of the main picture for visible sector. E.g. hidden sector interacting via different portals may exist, which is also needed for explanation of dark matter, inflation, \textit{etc.}

3. Testability, especially in forthcoming and planned experiments, is not the problem of Nature. It is our problem. Simplicity, minimality, symmetry still have great value.

2 Scenario: \( \nu - \text{mixing from the hidden sector} \)

Starting point is that the data are in a good agreement with the relation \cite{[1][2][3][4]}:

\[ U_{PMNS} = U_l^\dagger U_X, \]  

(2)
where $U_l \approx V_{CKM}$ is the quark mixing matrix and $U_X \approx U_{BM}$ or $U_{TBM}$ are the BM- or TBM- mixing matrices. The diagonal matrix of phases $\Gamma_\alpha$ can be attached to $U_X$.

The equality (2) leads to prediction $\sin^2 \theta_{13} = 0.5 \sin^2 \theta_C$, and in general, for $U_X = \Gamma_\alpha U_{\theta_{23}} (\theta_{12}^X) U_{\theta_{12}} (\theta_{12}^X)$, one can obtain relation between the observables [3]:

$$\sin^2 \theta_{13} = \sin^2 \theta_{23} \sin^2 \theta_C [1 - O(\sin^2 \theta_C)].$$

(3)

The present experimental status of this relation is summarized in Fig. 7. The relation can be modified due to RGE running if it is fixed at high (GUT) scale. Further $\sim 20\%$ correction to $\sin^2 \theta_{13}$ can be due to deviation of $\theta_{12}^l$ from $\theta_C$: $\theta_C \rightarrow \theta_{12}^l$ in (3). This brings the prediction to the best fit point.

Figure 1: Relation between the 1-3 and 2-3 leptonic mixings according to Eq. (3) for $\theta_{12}^l = \theta_C$. Two lines show the band of predictions obtained by varying the phases in $\Gamma_\alpha$. The $1\sigma$, $2\sigma$ and $3\sigma$ allowed regions are taken from the global fit [5].

The form of equality (2) implies that two different contributions from two different sectors of theory with different symmetries are involved in generation of the lepton mixing:

$V_{CKM}$ follows from common sector for quarks and leptons, which gives Eq. (1). This requires the $q - l$ unification, GUT. The CKM physics is characterized by hierarchy of masses and mixings as well as relations between masses and mixing which can be achieved with, e.g., Froggatt-Nielsen mechanism.

$U_X$ originates from new sector related to neutrinos via what we call now the neutrino “portal”. It is responsible for large (maximal) neutrino mixing and smallness of neutrino mass. It can have special symmetry which leads to the BM or TBM mixing.

General setup of this scenario (Fig. 2) is the following.

- Visible sector contains particles of the Standard Model: $l, \nu_L$, as well as $\nu_R$ with mass matrices $m_l, m_H^\nu$. It can be embedded into the $L - R$ symmetry model and then GUT. This sector produces $V_{CKM}$. 

2
Neutrino mixing via the neutrino portal. Shown is the general setup. The lepton mixing has two different sources.

- **Neutrino portal:** $\nu_R$ and singlet fermions $S$ have Dirac mass terms which form the matrix $M_D$.

- **Hidden sector:** Apart from $S$ it contains flavons - scalar fields with non-zero flavor charges which couple with $S$. Flavons develop non-zero VEV’s, break the flavor symmetry, and generate non-diagonal mass matrix $M_S$ which is the origin of (diagonalized by) $U_X$.

Concerning scale of the hidden sector, $M_S$, there are two extreme possibilities: high scale, $M_S \sim M_{Pl}$, or low scale, $M_S \sim \mu = (\text{keV} - \text{MeV})$.

In this scenario all the interactions are renormalizable and it is easy to realize flavor symmetries.\footnote{Notice that more economic version without singlets and with usual seesaw type-I also has structure which does not exist in the quark sector and so can be responsible for the difference of mixing. This, however, does not allow to disentangle the CKM and $X-$ sectors and therefore implement symmetries.}

In general, there can be many singlets in the hidden sector but some of them decouple or their effects can be reduced to the effect of three singlets. For three $S$ which couple to neutrinos the mass matrix is

$$
\mathcal{M} = \begin{pmatrix} 0 & m_D^T & m_D' \\
 m_D & 0 & M_D \\
 m_D' & M_D^T & M_S \end{pmatrix}.
$$

Block diagonalization of $\mathcal{M}$ gives the mass matrix of light neutrinos

$$
m_\nu = m_D^T M_D^{-1} M_S M_D^{-1} m_D - (m_D^T M_D^{-1} m_D' + m_D'^T M_D^{-1} m_D). \quad (5)
$$
We consider the situations when the double or inverse seesaw \footnote{\[6\]} (the first term in \eqref{eq:5}) dominates, while the linear seesaw (the second term) is suppressed. If \(m_D = A M_D\), where \(A\) is a constant, the first term of Eq. \eqref{eq:5} gives
\[
m_\nu = A^2 M_S,
\]
In this case structure of \(m_\nu\) is determined by \(M_S\), and it does not depend on structure the Dirac mass matrices (what was called in \cite{7} the Dirac screening). The screening allows to disentangle the sectors, and at the same time - transfer the flavor information from the hidden sector to the visible one.

In general, the information about mixing in the hidden sector should be communicated to the visible one. For this, the simplest possibility is to introduce symmetry which fixes bases in all three sectors, and the simplest basis fixing symmetry is \(G_{\text{basis}} = Z_2 \times Z_2\) \cite{8}. Indeed, the \(G_{\text{basis}}\) transformations of the fermionic multiplets and singlets \((-,-), (+,-), (-,+))\) allow to distinguish three generations. If the Higgs multiplets of visible sector are singlets of \(G_{\text{basis}}\), then \(m_D \sim M_D = \text{diagonal}\).

To ensure the proportionality of the diagonal elements \(m_D^{\text{diag}} \sim M_D^{\text{diag}}\), required by complete screening, one needs to introduce additional (e.g., permutation) symmetry or rely on further unification of \(S\) and other fermions. Flavons \(F\) are charged with respect to \(G_{\text{basis}}\) and spontaneously break \(G_{\text{basis}}\), which leads to non-diagonal \(M_S\), and consequently, to mixing \(U_X\).

\(G_{\text{basis}}\) is a part of the intrinsic symmetry \((Z_2)^3\) of Majorana mass matrix which is always present, \(i.e., G_{\text{basis}}\) is given “for free” \cite{9}.

\(M_S\) diagonalized by \(U_X\) has another unbroken symmetry \((Z_2 \times Z_2)_H\). Thus, \(U_X\) connects bases determined by \((Z_2 \times Z_2)_V\) and \((Z_2 \times Z_2)_H\). To fix \(U_X\) one can assume embedding of \((Z_2 \times Z_2)_V\) and \((Z_2 \times Z_2)_H\) into a finite discrete group (residual symmetry approach): Using the symmetry group condition \cite{10} one finds that embedding of two Klein groups leads to general expression for elements of mixing matrix \cite{9}, \cite{11}
\[
|(U_X)_{ij}|^2 = \cos^2 \pi \frac{n_{ij}}{p_{ij}}, \quad p, n - \text{integer}.
\]
This expression and the unitarity condition
\[
\sum_i \cos^2 \pi \frac{n_{ij}}{p_{ij}} = 1,
\]
(and similar equalities hold for the sum over \(j\)) allow to reconstruct the matrix \(U_X\) up to discrete number of possibilities. Taking into account that elements of \(U_X\) are in general complex, 5 matrices have been found \cite{11}. Among them are \(U_{q/p}, U_{BM}\):
\[
U_{q/p} = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \frac{2\pi}{p} & \sin \frac{2\pi}{p} \\
0 & -\sin \frac{2\pi}{p} & \cos \frac{2\pi}{p}
\end{pmatrix}, \quad U_{BM} = \begin{pmatrix}
1/\sqrt{2} & 1/\sqrt{2} & 0 \\
-1/2 & 1/2 & -1/\sqrt{2} \\
-1/2 & 1/2 & 1/\sqrt{2}
\end{pmatrix}.
\]
as well as the golden ratio matrix $U_{GR}$ which could be important for phenomenology. In the case of $U_{BM}$ the covering symmetry is $S_4$. $U_{TBM}$ can not be obtained. One could consider more complicated basis fixing symmetry which has different embeddings and covering groups.

3 High energy GUT-Planck realization

In this realization $M_S \sim M_{Pl}$, we assume SO(10) GUT, and the portal scale $M_D \sim M_{GUT}$ \cite{11}. The linear seesaw contribution is “automatically” suppressed and neutrino masses are generated via the double seesaw. The RH neutrinos get mass via see-saw $M_R \approx M_D^T M_S^{-1} M_D$.

![Figure 3: Scheme of generation of mixing in the high scale scenario.](image)

The scheme with $G_f = G_{hidden} = S_4$ is shown in Fig. 3. $G_f = S_4$ is explicitly broken down to $G_{basis} = (Z_2 \times Z_2)_V$ in the visible and portal sectors, and spontaneously - down to $(Z_2 \times Z_2)_H$ in the hidden sector. The explicit breaking gives very small corrections to the PMNS-mixing.

With respect to $S_4$ the fermionic 16-plets and singlets $S$ transform as $\sim 3$, flavon form triplet $\phi \sim 3'$ and doublet $\xi \sim 2$, and other fields are $S_4$ singlets. The VEV alignment

$$\langle \phi \rangle^T \sim (0, 0, 1), \quad \langle \xi \rangle^T \sim (0, 1)$$

gives $M_S = M_{BM}$, and consequently, $U_X = U_{BM}$.

$U_l$ and $V_{CKM}$ follow from the down components of the fermion EW doublets. The required equality

$$U_l \approx U_d = V_{CKM}$$

should be reconciled with difference of masses at the GUT scale:

$$m_\mu \sim 3m_s, \quad m_d \gg m_e.$$
Here, general idea is that the mass matrices have two different contributions:

\[ M_d = M_d^{(10)} + M_C, \quad M_l = M_d^{(10)} - 3M_C, \]  

but only one contribution dominates in generation of a given 2 fermion mixing 1-2 and 1-3. In (8) \( M_d^{(10)} \) is the contribution from 10\(_H\). It is diagonal and strongly hierarchical, as up quark masses \( M_d^{(10)} = v_d/v_u M_u^{(10)} \), and therefore dominates in generation of mass and mixing of the third generation states.

\( M_C = \mathcal{O}(m_s) \) is sub-dominant for the third generation but is less hierarchical and therefore dominant for the 2nd and 1st generations. The matrix \( M_C \) is off-diagonal, it breaks \( G_{\text{basis}} = G_{\text{visible}} \) and produces the CKM mixing. The required form is

\[ M_C \approx \begin{pmatrix} d_1 & f & f' \\ f & d & d' \\ f' & d' & d \end{pmatrix}, \quad \frac{f}{d} = \sin \theta_C, \quad d_1 \ll d \sim d', \quad f \sim f'. \]  

Interestingly, \( M_C \propto M_{BM} \), and therefore both could originate from the same Planck scale physics. The largest elements of \( M_C \) are of the order \( d, d' \sim 0.1 v_{EW} M_{GUT}/M_{Pl} \). \( M_C \) can be generated by additional 126-plet of Higgses with Planck scale mass or by effective 126: \( M_{Pl}^{-1}16_F16_F16_H16_H' \) with \( \langle 16_H16_H' \rangle \sim v_{EW} M_{GUT} \) [11].

Fit of the observed masses and mixing of quarks with (8) and (9) gives the angles in \( U_l \): \( \theta_{12}^l \sim \theta_C, \theta_{13}^l \sim (4 - 5)\^\circ, \theta_{23}^l \approx 1\^\circ \). Using uncertainties in the quark masses and phases involved we have

\[ \theta_{12}^l/\theta_C = 0.87 - 1.35. \]

According to \( U_{PMNS} = U^\dagger_{BM} \), the parameters of the PMNS matrix equal

\[ s_{13}^2 = \frac{s_{12}^2}{2}, \quad s_{12}^2 = \frac{1}{2} - \frac{\sqrt{2} c_{12} s_{12} \cos \phi_l}{2 - s_{12}^2}, \quad s_{23}^2 = \frac{c_{12}^2}{2 - s_{12}^2} \approx \frac{1}{2} \left( 1 - \frac{1}{2} s_{12}^2 \right), \]

\[ \sin \delta_{CP} = - \sin \phi_l \left( 1 + s_{12}^2 \cos^2 \phi_l \right) + \mathcal{O}(s_{12}^4), \]  

where \( s_l \equiv \sin \theta_{12}^l \), and \( \phi_l \) is the phase of the 1-2 element: \( (U_l)_{12} = s_l e^{i\phi_l} \).

Excluding \( s_l \) and \( \phi_l \) from Eqs. (10) and (11) we find relations between observables:

\[ s_{12}^2 \approx \frac{1}{2} + \frac{s_{13} \cos \delta_{CP}}{c_{13}^2}, \quad s_{13} = \frac{3}{\sqrt{2}} \sin \theta_C \frac{|m_s - m_d e^{i\phi_d}|}{|m_\mu + m_\tau e^{i\phi_\tau}|}, \]

where \( \phi_d = \phi_d(\delta_{CP}), \phi_\tau = \phi_\tau(\delta_{CP}) \) are known functions of \( \delta_{CP} \) [11], see Fig. 4. From the figure we obtain \( \delta_{CP} = (0.80 - 1.16)\pi \) which touches the 1\(\sigma\) region from the global fit: \( (1.17 - 1.53)\pi \). Notice that \( \cos \delta_{CP} \approx -1 \) is a generic prediction for the BM mixing case [12]. RGE can change this result, so that \( \delta_{CP} = -0.5\pi \) becomes possible.

Tests and problems: in this realization one expects that (i) flavons, new fermions and new Higgses are at the GUT Planck scale; (ii) nothing should be observed at
Figure 4: Dependence of 12 (left) and 13 (right) mixing angles on the CP phase. Blue points correspond to values of the charged fermion masses randomly generated within 1σ allowed regions. From [11].

LHC which is responsible for neutrino mass generation; (iii) proton decays; (iv) new elements of theory related to the CKM physics may show up; (v) the RH neutrinos have very strong hierarchy of masses; leptogenesis with second RH neutrino is possible [13]; (vi) other particles from the hidden (Dark) sector can be found such as sterile neutrinos, DM particles, etc.

4 Low scale realization with the L-R symmetry

The low scale scheme with $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$ symmetry and one singlet $S$ per generation [14] is shown in Fig. 5 left. Here $P$ is the parity. The $B-L$ charges of the fields ($L_L, L_R, \chi_L, \chi_R, S$) equal $(-1, -1, 1, 1, 0)$. For Dirac mass matrices of Eq.(1) small neutrino mass can be obtained using the inverse seesaw (ISS) mechanism [4] with $M_S = \mu \sim 10$ keV [14].

In general, in the low scale case, the linear seesaw dominates. Due to $P$ symmetry the Yukawa couplings of two Higgs doublets are the same (in the lowest order), and consequently $m_D^L \propto M_D$. Therefore the linear seesaw contribution reduces to

$$m^{LSS}_\nu = \frac{\langle \chi_L \rangle}{\langle \chi_R \rangle} (m_D^T + m_D). \quad (12)$$

It has wrong (too strong) mass hierarchy and therefore should be suppressed, which requires $\langle \chi_L \rangle / \langle \chi_R \rangle < 10^{-12}$. For this the interactions $h\chi_L\chi_R\phi$, which leads to VEV of $\chi_L$, should have small coupling $h < 40$ keV. Even if $h = 0$ and therefore $\langle \chi_L \rangle = 0$ at tree level due to certain symmetry, the interaction term is generated at 1 loop (Fig. 6 left). The corresponding induced VEV equals $\langle \chi_L \rangle \sim 1/16\pi^2 v_L (u_L \mu / v_R^2)$, which satisfies the bound. Here $v_R \equiv \langle \chi_R \rangle$ and $v_L \equiv \langle \Phi \rangle$ is the bi-doublet VEV.

The neutrino mass determines via the ISS the $L - R$ symmetry breaking scale.
For $m_D^\nu \sim m_{top}$ it equals

\[ v_R = \langle \chi_R \rangle = 3.5 \cdot 10^5 \text{ GeV} \left( \frac{\mu}{0.1 \text{ MeV}} \right)^{1/2} \left( \frac{0.05 \text{eV}}{m_{\nu 3}} \right)^{1/2}. \]

Figure 5: Generation of masses and mixing in the low scale scenarios. Left panel: scheme with one singlet $S$ per generation. Right panel: scheme with two singlets, $S_L$ and $S_R$, per generation.

Figure 6: Left panel: one loop diagram that generated $\chi^\dagger_L \Phi \chi_R$ interactions. Right panel: the leading radiative correction to the Majorana mass $\mu$. From [14].

As in the high scale scheme, at $m_D \propto M_D$ the Dirac structures are screened, and

\[ m_\nu = \xi^2 \mu, \quad \xi = \frac{m_D}{M_D} = \frac{v_L}{v_R}. \]

Symmetries in the $S$-sector can lead to special form of $\mu$, and consequently, to special mixing from the hidden sector. The symmetry is broken (explicitly) in the portal, by $M_D$. But in spite of the fact that $\mu \ll M_D$, corrections due to symmetry breaking (see Fig. 6) are small:

\[ \Delta \mu = \frac{1}{16 \pi^2} h Y_L Y_R Y \sim 10 \text{ eV}, \]
while $\mu \gg 10$ eV for $h \sim 0.1$ MeV.

The components $N_i$ and $S_i$ form pairs of the pseudo-Dirac leptons with masses and mass splittings

$$|M_i| \approx M_{Di}(1 + \xi^2)^{1/2}, \quad \Delta M_i = m_{ii}. $$

Their production and decay proceed, mainly, via mixing in the light flavor neutrinos:

$$\nu_f = U_{PMNS}\nu - \frac{1}{\sqrt{2}}\xi U_{l}^\dagger(N^- - N^+),$$

where $N^-$ and $N^+$ are the mass eigenstates. Thus, mixing of the heavy lepton in the flavor state $\nu_\alpha$ equals

$$|U_{\alpha i}|^2 = \frac{1}{2} \left(\frac{m_{Di}}{M_{Di}}\right)^2 |U_{\alpha i}|^2.$$

The dependencies (13) together with experimental bounds on mixing parameters of $N_i$ [14] are shown in Fig. 7. From this figure for $m_u \approx 2$ MeV we obtain the lower bound $M_1 > 2$ GeV, so that $\xi = m_u/M_1 < 10^{-3}$. Consequently, $M_2 > 600$ GeV, and $M_3 > 2.5 \cdot 10^5$ GeV. SHiP [15] can further improve the bound on $M_1$ or discover $N_1$. Presently, there is no direct experimental bounds on $M_2$ and $M_3$. In future, 100 TeV collider may be sensitive to them.

Figure 7: Mixing of the heavy leptons in the neutrino flavor states $\nu_e$ (left) and $\nu_\mu$ (right) as functions of their masses. Solid black lines show predictions. Colored regions and lines show bounds from the existing and future experiments. From [14].

Interesting variation of this scenario is a scheme with two singlets: left and right per generation (see Fig. 5 right). It is invariant under global $U(1)_L$ with charge prescription $(L_L, L_R, S_L, S_R) = (1, 1, -1, -1)$. This symmetry is broken in the hidden sector by the $\mu$-terms.
Mass matrix of neutral leptons in the basis \((\nu_L, N_L, S_L, S^c_R)\) reads

\[
\mathcal{M} = \begin{pmatrix}
0 & m_D & m'_D & 0 \\
m_D & 0 & 0 & M_D \\
m'_D & 0 & \mu & \mu_{LR} \\
0 & M_D & \mu^T_{LR} & \mu
\end{pmatrix}.
\] (14)

Pairs of the pseudo-Dirac heavy leptons formed by \(N\) and \(S^c_R\) have similar phenomenology as before. After decoupling of these heavy states the mass matrix in the basis \((\nu_L, S_L)\) becomes

\[
\begin{pmatrix}
\mu s^2_{\xi} \\
c_{\xi} m'_D - s_{\xi} \mu_{LR} \\
\end{pmatrix}.
\] (15)

In contrast to the high scale scheme now light Majorana leptons with masses \((10 - 100)\) keV exist which nearly coincide with \(S_{Li}\). They mix very weakly with usual active neutrinos:

\[
\sin \theta_s \approx -\frac{\xi_{\mu LR}}{\mu}.
\]

If \(\mu_{LR}/\mu < 10^{-2}\), \(S_L\) with mass \(\sim 10\) keV can be the Dark matter candidate [14].

5 Conclusion

1. If not accidental, the relation between the lepton and quark mixings \(U_{PMNS} = V_{\text{CKM}} U_X\), where \(U_X \sim U_{BM}\) or \(U_{TBM}\) implies Grand Unification and existence of the hidden sector which has certain symmetry and interacts with the visible sector via the neutrino portal.

2. The hidden sector with non-abelian flavor symmetries generates large neutrino mixing of special type and is responsible for smallness of neutrino mass.

3. The key elements of this scenario are (i) existence of two sectors with different symmetries; (ii) the basis fixing symmetry which communicates flavor information from the hidden sector to the visible one.

4. The high scale realization of such a scenario is the \(SO(10)\) GUT with hidden sector at the Planck scale. Neutrino masses are generated by the double seesaw. The residual symmetry approach can lead to the BM mixing for \(U_X\).

5. Similar scenario can be realized at low energies in the scheme with \(L - R\) symmetry and the inverse seesaw. The scale of \(L - R\) symmetry breaking is about 300 TeV. The pseudo-Dirac heavy leptons can be searched at existing and future accelerator experiments. In version with two singlets per generation the kev mass scale leptons exist which can be candidates for Dark Matter particles.
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