Modeling of Utility Tunnels as Thin-Walled Beams on an Elastic Subgrade

Norbert Jendzelovsky

STU, Faculty of Civil Engineering, Radlinskeho 11, 810 05 Bratislava, Slovakia

norbert.jendzelovsky@stuba.sk

Abstract. In the construction of municipalities and agglomerations, it is necessary to build utility tunnels and other linking underground structural systems. They enable the interconnection of individual buildings for different types of media. For the purposes of the static analysis, the tunnels’ structure can be considered to be a thin-walled beam on an elastic subgrade. The article deals with the analysis of thin-walled beam loaded by bending and torsion. Differential equations and their software solution using the transfer matrix method have been presented. In conclusion, an example of utility tunnels as thin-walled beams on an elastic subgrade has been shown. A solution of a beam using the theory of thin-walled beams has been compared with the solution of shell finite elements using the Finite Element Method.

1. Introduction

In the construction of municipalities and agglomerations, there is often a demand to place multiple distribution networks of media in a common space – utility tunnel. Utility tunnels are common on large industrial, institutional or commercial sites, where multiple large-scale services infrastructure (gas, water, power, heat, steam, compressed air, telecommunications cables, etc.) are distributed around the site to multiple buildings without impeding vehicular or pedestrian traffic above the ground. The dimensions of the utility tunnels structures can be characterized as thin-walled beams rested on an elastic subgrade. The theory presented in the paper can be used for the design of long line structures. The analyzed structure is characterized as an engineering structure - figure 1.

2. Thin walled beam on an elastic subgrade

Modeling of subsoil for the purposes of analysis of structures rested on elastic subsoil can be performed using several mathematical-physical subsoil models [1–4]. In this paper, the interaction of the thin-walled beam with the subsoil has been solved using the Winkler's subsoil model. The following equation applies to this subsoil model:

\[ p(x,y) = k \cdot w(x,y) \]  

(1)

Where \( p(x,y) \) is the contact stress in the foundation gap; the vertical deformation of the subsoil is \( w(x,y) \). Subsoil stiffness is defined by the stiffness coefficient \( k \) (N/m³).

When considering a general loading of a thin-walled beam rested on elastic subsoil we can assume that the general load can be divided into two parts. The first part - the load causing the bending of the
beam and the second part - the load causing the torsion of the beam. For the bending of the beam, the
differential equation of bending of the thin-walled beam on elastic subsoil has been derived, which is
described as follows [5]:

\[
\frac{dw(x)}{dx^4} - \frac{k \kappa}{GA} \frac{dw(x)}{dx^2} + \frac{k bw(x)}{EI} = \frac{q(x)}{EI}
\]  

Where:
- \( w(x) \) an unknown function of the vertical displacement
- \( b \) width of the beam on contact with subsoil
- \( \kappa \) cross-sectional shear coefficient
- \( E \) modulus of elasticity of the beam material
- \( G \) shear modulus of elasticity of the beam
- \( A \) cross-sectional area of the beam
- \( I \) moment of inertia of the beam

Figure 1. The utility tunnel

After obtaining the bending function \( w(x) \), for the internal forces of the beam can be formulated:
The bending moment
\[
M(x) = -EI \left( \frac{d^2w(x)}{dx^2} \right)
\]  

Transversal force
\[
V(x) = -EI \left( \frac{d^3w(x)}{dx^3} \right)
\]

If the torsional moments on the differential element of a thin-walled beam are in the equilibrium,
after modification we obtain the differential equation of the torsion of a thin-walled beam on the
elastic subsoil (5) presented in the literature [5]:
\[
\frac{d\psi(x)}{dx^4} - \frac{G I_k}{E I_\omega} \frac{d\psi(x)^2}{dx^2} + \frac{k b \psi(x)}{E I_\omega} = \frac{-m(x)}{E I_\omega}
\] (5)

Where:
- \(\psi(x)\) is an unknown function of the rotation about the longitudinal axis
- \(I_k\) is the moment of inertia in free torsion
- \(I_\omega\) is the sectional moment of inertia

For the internal forces during torsion we get:

- The bimoment:
  \[B(x) = -E I_\omega \frac{d^2\psi(x)}{dx^2}\] (6)

- The moment:
  \[M_\omega(x) = -E I_\omega \frac{d^3\psi(x)}{dx^3}\] (7)

- The free torsion moment:
  \[M_k(x) = G I_k \frac{d\psi(x)}{dx}\] (8)

The solution of equations (2) and (5) exists in a closed form. It is the sum of combinations of hyperbolic functions and trigonometric functions. The solution presented in the literature is using the method of initial parameters. A solution using the transfer matrix method has been designed for practice with computers [5]. The author of the article created the UNO program in the FORTRAN programming language. In the last chapter, the solution of an example using the UNO program has been presented.

3. The static model using the FEM

Currently, most tasks have been solved using the Finite Element Method - FEM. It is a universal method of calculation of building structures of both the static and dynamic problems. Therefore, in the second part, the contact problem of the beam on elastic subsoil using the finite elements has been solved. Many of the available software packages are based on FEM. Shell elements have been used for modeling the structure of the thin-walled beam member. It is a planar rectangular finite element that has six displacement components in the node. It is a displacement in all three directions x, y, z and three rotations about each axis. The elastic subsoil is taken into account in the additional subsoil stiffness matrix. It is added to the bending component of the stiffness matrix of the shell. These are stiffness matrices of those finite elements that are in contact with the subsoil.

4. The numerical example

The numerical example presents the solution of a thin-walled reinforced concrete beam of U-shaped cross-section. The width of the beam in contact with the subsoil is 1000 mm and its height is 500 mm; the thickness of the reinforced concrete walls is 50 mm. The length of the analyzed beam is 10 m. The reinforced concrete beam has a modulus of elasticity \(E = 21\) GPa. The cross-sectional characteristics of the beam are: area \(A = 0.10\) m\(^2\), moment of inertia \(I = 2.6x10^{-4}\) m\(^4\), sectional moment of inertia \(I_\omega = 4.6x10^{-5}\) m\(^6\). The value of the subsoil stiffness coefficient is \(k = 80\) MNm\(^{-3}\). The loading by forces (kN) and by the continuous load (kN/m) has been presented in figure 2.
Figure 2. The shape of the beam and the loads acting upon the thin-walled beam.

In the following part of the paper, the results obtained using the UNO program have been shown. The results are divided into two parts:
- the first part is bending - figure 3 shows displacement area due to the internal forces, bending moment and transversal force.
- the second part is torsion - figure 4 shows graphs of the internal forces in the thin-walled beam with torsion: the bimoment and the torsional moments.

Figure 3. Bending analysis. Deformation of the contact surface w (x) (mm); bending moment M (x) (kNm); transversal force V (x) (kN).
Figure 4. Torsion analysis: the bimoment omega B (x) (kNm²); torsional moment in the free torsion T (x) (kNm); and resulting torsional moment around the x axis Mx (x) (kNm)

The last part of the paper presents the results obtained from the FEM-based software. To compare the results graphically, figure 5 shows the deflection of the contact surface and the vertical deformation of the entire U-shaped beam. Figure 6 shows the stress sigma x on the entire beam.

Figure 5. left: overall deflection of the beam; right: deflection area of the plate on the contact with subsoil.
5. Conclusions
The performed analysis considering the beam is simpler and takes into account the perfect stiffness of the cross-section. The result is that the internal forces need to be converted to normal stresses in the cross-section. In the analysis using the shell FEM element, the deformation of the cross-section U occurs. We obtain the results directly in stresses at the individual points of the cross-section.

Acknowledgment(s)
This paper was written with the support of Slovak Grant Agency VEGA 1/0412/18, and KEGA 025STU-4/2019.

References
[1] R. Cajka, P. Mynarcik, and J. Labudkova, „Numerical solution of soil-foundation interaction and comparison of results with experimental measurements.“ International Journal of GEOMATE, Vol. 11, pp 2116-2122, 2016.
[2] N. Jendzelovsky, “Optimisation of a contact problem of an elastic beam on an elastic foundation.” Journal of Czech and Slovak Mechanical Engineering, 3 (4) pp. 183 – 187, 1994.
[3] J. Koktan, R. Cajka, and J. Brozovsky, „Finite Element Analysis of Foundation Slabs Using Numerical Integration of Boussinesq Solution.“ In Int. conference of numerical analysis and applied mathematics (ICNAAM 2017), vol. 1978, ISSN 0094-243X, 2018.
[4] K. Kotrasova, S. Harabinova, I. Hegedűšová, E. Kormaníková, and E. Panulinova, „Numerical Experiment of Fluid Structure Soil Interaction.“ In Procedia Engineering, Vol. 190, pp. 291-295, 2017.
[5] N. Jendzelovsky, “Thin walled beams of a finite length on an elastic subsoil.” Inženýrské stavby, Vol. 38 No. 7, pp. 410-416, 1990. (in Slovak).