On the applicability of Network-Oriented Modelling based on temporal-causal networks: why network models do not just model networks

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ABSTRACT

In this paper for a Network-Oriented Modelling perspective based on temporal-causal networks, it is analysed how generic and applicable it is as a general modelling approach and as a computational paradigm. It is shown that network models do not just model networks, but can be used to model many types of processes.

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1. Introduction

The notion of network itself and its use in different contexts can be traced back to the years 1920–1960 (e.g. Treur, 2016c or Treur, 2016b, Chapter 1, Section 1.4). The notion of Network-Oriented Modelling as a modelling approach (sometimes also indicated by NOM) can be found only in more recent literature, and only for specific domains. More specifically, this term is used in different forms in the context of modelling organizations and social systems (Chung, Choi, & Kim, 2003; Naudé et al., 2008), of modelling metabolic processes (Cottret & Jourdan, 2010), and of modelling electromagnetic systems (Russer & Cangellaris, 2001). The Network-Oriented Modelling approaches put forward in this literature are specific for the domains addressed, namely social systems, metabolic processes, and electromagnetic systems.

This and other Network Science literature may suggest that sometimes in real-world domains networks occur, and by some modelling process, network models are obtained that model these given networks. That might suggest a positive answer to the question in the title: network models do just model networks given in real-world domains. For example, network models can be obtained for metabolic networks, brain networks, computer networks, and social networks, all occurring (or conceived) in the real world. However, if networks occur in real-world domains, how often do they? Are networks everywhere? Or is there just a limited class of situations or processes that are conceived as networks? The scientific area of networks has developed within many disciplines, such as Biology, Neuroscience, Mathematics, Physics, Economics, Informatics or Computer Science, Artificial Intelligence, and Web Science (see, e.g. Boccalettia, Latorab, Morenod,
Chavez, & Hwanga, 2006; Giles, 2012; Valente, 2010). These developments already show how processes in quite different domains can be conceptualized as networks. Historically, the use of the concept network in different domains can be traced back roughly to the years 1930–1950, or even earlier, for studying processes such as brain processes in neuroscience by neural networks (e.g. McCulloch & Pitts, 1943; Rosenblatt, 1958), metabolic processes in Cell Biology by metabolic networks (e.g. Ouellet & Benson, 1951; Westerhoff, Groen, & Wanders, 1984), social interactions within Social Science by social networks; (e.g. Aldous & Straus, 1966; Bott, 1957; Moreno & Jennings, 1938), processes in Human Physiology (e.g. Huber, 1941; Wiener & Rosenblueth, 1946), processes in engineering in Physics (e.g. Bode, 1945; Hubbard, 1931), and processes in engineering in Chemistry (e.g. Flory, 1944; Treloar, 1943). Within such literature often graphical representations of networks are used as important means of presentation. For a historical overview of the development of social network analysis, for example, see Freeman (2004).

So, over time for more and more domains it has been shown how they can be modelled by networks: the blue area in Figure 1, with some of the examples indicated. Will this blue area eventually coincide with the pink area depicting all domains? Or is it essentially a proper subset of it?

This paper addresses this question. It is indicated how a generic, unified Network-Oriented Modelling method can be obtained that is applicable more generally. The Network-Oriented Modelling approach described here was developed initially with unification of modelling of human mental processes and social processes in mind (also see the overview in Section 5). However, it has turned out that the scope of applicability has become much wider, as is shown in the current paper. Actually, it will be indicated that in this way practically all processes in the real world can be modelled from a Network-Oriented perspective, not only those processes or situations in the real world that are generally conceived as networks. This provides a negative answer to the question in the title: network models can model all kinds of processes in the real world, not just processes generally conceived as networks.

This paper discusses how generic and applicable Network-Oriented Modelling based on temporal-causal networks is in general as a dynamic modelling approach for both continuous systems (Section 3) and discrete systems (Section 4). In Section 5, a number of actual applications of Network-Oriented Modelling are discussed, varying from mental processes to social interaction processes. First in Section 2 the Network-Oriented Modelling approach based on temporal-causal networks used in the paper is briefly introduced.

Figure 1. Domains that can be modelled by networks (blue area) within the set of all domains (pink area).
2. Network-Oriented Modelling based on temporal-causal networks

The Network-Oriented Modelling approach considered here uses temporal-causal networks as a basis (Treur, 2016a, 2016b). The temporal perspective allows to model the dynamics of the interaction processes within networks, and also of network evolution. Temporal-causal network models can be represented in two equivalent manners: by a conceptual representation or by a numerical representation. Conceptual representations can have a graphical form (as a labelled graph with states as nodes and connections as arcs, see Figure 1) or the form of a matrix (see Table 1). The following three elements define temporal-causal networks, and are part of a conceptual representation of a temporal-causal network model:

- **connection weight** $\omega_{X,Y}$
  Each connection from a state $X$ to a state $Y$ has a connection weight $\omega_{X,Y}$ representing the strength of the connection, often between 0 and 1, but sometimes also below 0 (negative impact).

- **combination function** $c_Y(...)$
  For each state $Y$ (a reference to) a combination function $c_Y(...)$ is chosen to combine the causal impacts of other states on state $Y$. This can be a standard function from a library (e.g. a scaled sum function) or an own-defined function.

- **speed factor** $\eta_Y$
  For each state $Y$ a speed factor $\eta_Y$ is used to represent how fast a state is changing upon causal impact. This is usually assumed to be in the [0, 1] interval.

These elements can be represented in a graphical conceptual representation as labels of the graph (see Figure 2) or in a conceptual matrix representation as fillings for the cells of a square connection matrix, followed by two rows for speed factors and combination functions (see Table 1).

Combination functions $c_Y(...)$ in general are similar to the functions used in a static manner in the (deterministic) Structural Causal Model perspective described, for example, in Mooij, Janzing, and Schölkopf (2011), Pearl (2000), and Wright (1921), but in the Network-Oriented Modelling approach described here they are used in a dynamic manner, as will be pointed out below. Combination functions can have different forms. The more general issue of how to combine multiple impacts or multiple sources of knowledge occurs in various forms in different areas, such as the areas addressing imperfect

Table 1. Conceptual matrix representation of a temporal-causal network model: with names for states $X$ and $Y$, and cells for connection weights $\omega_{X,Y}$ speed factors $\eta_Y$, and combination functions $c_Y(...)$.

| To from | P     | Q     | R     | S     | T     | U     | V     |
|--------|-------|-------|-------|-------|-------|-------|-------|
|        | $\omega_{P,R}$ | $\omega_{Q,R}$ | $\omega_{R,S}$ | $\omega_{R,T}$ | $\omega_{S,U}$ | $\omega_{T,U}$ | $\omega_{T,V}$ |
| $\eta_Y$ | $\eta_P$ | $\eta_Q$ | $\eta_R$ | $\eta_S$ | $\eta_T$ | $\eta_U$ | $\eta_V$ |
| $c_Y(...)$ | $c_P(...)$ | $c_Q(...)$ | $c_R(...)$ | $c_S(...)$ | $c_T(...)$ | $c_U(...)$ | $c_V(...)$ |
reasoning or reasoning with uncertainty or vagueness. For example, in a probabilistic setting, for modelling multiple causal impacts on a state often independence of these impacts is assumed, and a product rule is used for the combined effect (e.g. Dubois & Prade, 2002). In the areas addressing modelling of uncertainty also other combination rules are used, for example, in possibilistic approaches minimum- or maximum-based combination rules are used (e.g. Dubois & Prade, 2002; Dubois, Lang, & Prade, 1991; Zadeh, 1978). In another area, addressing modelling based on neural networks, yet another way of combining effects is used often. In that area, for combination of the impacts of multiple neurons on a given neuron usually a logistic sum function is used (e.g. Grossberg, 1969; Hirsch, 1989; Hopfield, 1984).

The applicability of a specific combination rule may depend much on the type of application addressed, and even on the type of states within an application. Therefore the Network-Oriented Modelling approach based on temporal-causal networks incorporates for each state, as a kind of label or parameter, a combination function indicating a way to specify how multiple causal impacts on this state are aggregated. For this aggregation a number of standard combination functions are available as options; for more details, see Treur (2016b), Chapter 2, Sections 2.6 and 2.7. These options cover, for example, scaled sum functions, logistic sum functions, product functions, and max and min functions. In addition, there is still the option to specify any other (non-standard) combination function.

The systematic generation of a numerical representation can be done in the following manner. Here for any state $Y$ and any time point $t$ the (activation) value of $Y$ at time $t$ is denoted by $Y(t)$. For any of the states $Y$ at each point in time $t$, each of the values $X_1(t), \ldots, X_k(t)$ for the states $X_1, \ldots, X_k$ connected towards $Y$ has a causal impact on the value of $Y$, due to which in principle at the next point in time $t+\Delta t$ the value of $Y$ has changed. For each of the states $X_i$ this impact on $Y$ at time $t$ is proportional both to the value $X_i(t)$ and the connection weight $\omega_{X_i,Y}$ and is defined as

$$\text{impact}_{X_i,Y}(t) = \omega_{X_i,Y} X_i(t).$$

The aggregated impact of the multiple impacts $\text{impact}_{X_1,Y}(t), \ldots, \text{impact}_{X_k,Y}(t)$ of $X_1(t), \ldots, X_k(t)$ on state $Y$ at time $t$ is modelled by a combination function $c_Y(\ldots)$ (e.g. the sum

![Figure 2. Graphical conceptual representation of a temporal-causal network model as a labelled graph: with names for states $X$ and $Y$, and labels for connection weights $\omega_{X,Y}$, speed factors $\eta_Y$, and combination functions $c_Y(\ldots)$.](image-url)
function) as

$$\text{aggimpact}_Y(t) = c_Y(\text{impact}_{X_1,Y}(t), \ldots, \text{impact}_{X_k,Y}(t)).$$

This aggregated impact is defined on the basis of the conceptual network specification in terms of the connection weights $\omega_{Xi,Y}$ and combination functions $c_Y(\ldots)$:

$$\text{aggimpact}_Y(t) = c_Y(\omega_{X_1,Y}X_1, \ldots, \omega_{X_k,Y}X_k).$$

Note that here within the combination function an ordering of the arguments (the different impacts by different connections) is used; such an ordering is usually not specified in the conceptual representation of the model. However, many often used combination functions are symmetric, in the sense that the ordering of their arguments does not matter (for example, in a sum, product, max or min function). So, in all of these cases any chosen ordering leads to the same outcome. But in some exceptional cases the order of the arguments may matter; in such cases for the combination function it has to be indicated which argument refers to which connection. Note, however, this is not a temporal order; the multiple impacts are always assumed to work simultaneously, in parallel.

So, the combination function $c_Y(V_1, \ldots, V_k)$ aggregates the multiple impacts $\text{impact}_{X_1,Y}(t), \ldots, \text{impact}_{X_k,Y}(t)$ on $Y(t)$ into one single aggregated impact value $\text{aggimpact}_Y(t)$; see Figure 3. Note that also the state $Y$ itself may be included in $X_1, \ldots, X_k$, although there are also many cases in which it will not be included. Moreover, as a special case also a combination function can be used for the case of one single impact, that is, when $k = 1$. Although in such a case it is not literally a process of combination, for convenience also the term combination function is used for a function applied to obtain the (aggregated) impact on $Y$ for this single impact case.

The aggregated impact value $\text{aggimpact}_Y(t)$ at time $t$ has an upward or downward effect on the value of state $Y$; it pushes the value of $Y$ up or down, depending on how it compares to the current value of $Y$. More specifically, this aggregated impact value $\text{aggimpact}_Y(t)$ is compared to the current value $Y(t)$ of $Y$ at $t$ by taking the difference between them (also see Figure 4): $\text{aggimpact}_Y(t) - Y(t)$. If this difference is positive (aggregated impact value $\text{aggimpact}_Y(t)$) at $t$ is higher than the current value of $Y$ at $t$), in the time step from $t$ to $t+\Delta t$ (for some small $\Delta t$) the value $Y(t)$ will increase in

![Figure 3. Aggregation of impacts on a state Y.](image-url)
the direction of the higher value \( \text{aggimpact}_Y(t) \). This increase occurs proportional to the difference, with proportion factor \( \eta_Y \): the increase is \( \eta_Y [\text{aggimpact}_Y(t) - Y(t)] \Delta t \); see Figure 4.

How fast this increase takes place depends on the speed factor \( \eta_Y \). For example, when \( \eta_Y = 0.9 \) and \( \Delta t = 0.5 \), then a fraction of 0.45 of the difference \( \text{aggimpact}_Y(t) - Y(t) \) is added to the value of \( Y(t) \). If \( \eta_Y = 1 \) holds, then the value of \( Y \) will adapt to \( \text{aggimpact}_Y(t) \) fast (big steps), and if \( \eta_Y = 0.1 \) it will be much slower (small steps). The same holds for a negative difference \( \text{aggimpact}_Y(t) - Y(t) \): in that case the value will decrease in the direction of the lower value \( \text{aggimpact}_Y(t) \). The extent to which it is increased depends on the speed factor \( \eta_Y \).

So the value \( Y(t) \) of state \( Y \) at \( t \) always moves in the direction of the aggregated impact value, and eventually may converge to this value. However, during this convergence process the value of \( \text{aggimpact}_Y(t) \) (which itself depends on other states) may change as well, which makes the process still more dynamic.

The numerical process just discussed is summarized by the following difference equation representation of the dynamical model:

\[
Y(t + \Delta t) = Y(t) + \eta_Y [\text{aggimpact}_Y(t) - Y(t)] \Delta t
\]

or in terms of the combination function \( c_Y(\ldots) \)

\[
Y(t + \Delta t) = Y(t) + \eta_Y [c_Y(\omega_{X_1,Y} X_1(t), \ldots, \omega_{X_k,Y} X_k(t)) - Y(t)] \Delta t.
\]

So, a conceptual representation of a temporal-causal network model, including the above three concepts (connection weight, combination function, and speed factor) can be transformed in a systematic and automated manner into an equivalent numerical representation of the model (Treur, 2016a, 2016b) by composing the following difference equation for each state \( Y \):

\[
Y(t + \Delta t) = Y(t) + \eta_Y [c_Y(\omega_{X_1,Y} X_1(t), \ldots, \omega_{X_k,Y} X_k(t)) - Y(t)] \Delta t,
\]

\[
\frac{dY(t)}{dt} = \eta_Y [c_Y(\omega_{X_1,Y} X_1(t), \ldots, \omega_{X_k,Y} X_k(t)) - Y(t)].
\]
Software environments in Excel, Matlab, and Python have been developed to automate the transformation from conceptual representation to numerical representation sketched above. These environments allow the modeller just to specify the conceptual representation of a temporal-causal network (in matrix format), after which automatically the numerical representation is generated by which simulation experiments can be done immediately. The Matlab and Python environments also include facilities for parameter estimation.

3. Modelling continuous dynamical systems as temporal-causal networks

In the current section it is discussed how temporal-causal networks subsume smooth continuous dynamical systems, as advocated, for example in Port and van Gelder (1995) to model human mental processes. The notion of state-determined system, adopted from Ashby (1960) was taken as the basis to describe what a dynamical system is in Port and van Gelder (1995, p. 6). That a system is state-determined means that its current state always determines a unique future behaviour. This property is reflected in modelling and simulation, as usually some rules of evolution are specified and applied that indicate how exactly a future state depends on the current state. State-determined systems can be specified in mathematical formats; see Ashby (1960, pp. 241–252) for some details. A finite set of states (or variables) $X_1, \ldots, X_n$ is assumed describing the system’s dynamics via functions $X_1(t), \ldots, X_n(t)$ of time variable $t$. In this section, it is shown how any smooth continuous dynamical system (assumed as state-determined) can be modelled by a temporal-causal network in two steps. First, it is discussed how any smooth continuous state-determined system can be described by a set of first-order differential equations, and next it is shown how any set of first-order differential equations can be modelled as a temporal-causal network.

Sets of first-order differential equations form a very general format used in computational modelling in many scientific disciplines. However, also in many scientific disciplines, processes are described and explained in terms of causal relationships. It would be helpful for understanding if these two perspectives are related in a transparent conceptual and mathematical manner. This will be discussed here. For cognitive and neurological modelling in particular, often causal relationships are used in explaining mental processes. But also in many other domains, in a wide variety of scientific disciplines causal relationships play a crucial role, and processes are often described by means of graphs with states and arrows indicating causal relationships. In this context it will be useful if it can be explained more explicitly how any state-determined system can be described or transformed into a format that more directly relates to causal relationships between states. This indeed can always be achieved in the temporal-causal network format described Section 2, in the manner shown in the following.

3.1. From state-determined systems to differential equations

From an abstract theoretical perspective the state-determined system criterion can be formalized in a numerical manner by a function $F_i(X_1, \ldots, X_n, s)$ that expresses how for each time point $t$ the future value of each state $X_i$ at time $t + s$ uniquely depends on $s$ and on the values $X_1(t), \ldots, X_n(t)$; see also Treur (2016b, Chapter 2, Section 2.9); for an alternative
treatment, see Ashby (1960, pp. 243–244). To illustrate the idea by a simple example, consider a state-determined system in one state variable \( X \) with values \( \geq 0 \) described by

\[
X(t + s) = F(X(t), s) = \sqrt{(X(t)^2 + \alpha s)}.
\]

By differentiating both sides to \( s \) and by choosing \( s = 0 \) the following is obtained:

\[
\frac{dX(t)}{dt} = \left[ \frac{\partial F(X(t), s)}{\partial s} \right]_{s=0}.
\]

The right hand side can be worked out as follows:

\[
\frac{\partial F(X(t), s)}{\partial s} = \frac{\partial \sqrt{(X(t)^2 + \alpha s)}}{\partial s} = \frac{1/2 \alpha}{\sqrt{(X(t)^2 + \alpha s)}}.
\]

So by substituting \( s = 0 \):

\[
\left[ \frac{\partial F(X(t), s)}{\partial s} \right]_{s=0} = \frac{1/2 \alpha}{\sqrt{X(t)^2}} = \frac{1/2 \alpha}{X(t)}.
\]

Thus the following differential equation for \( X \) is obtained:

\[
\frac{dX(t)}{dt} = \frac{1/2 \alpha}{X(t)}.
\]

Note that this differential equation has an analytic solution of the form

\[
X(t) = \sqrt{(X(0)^2 + \alpha t)},
\]

which indeed relates to the formula assumed at the start of the example. This illustrates how a state-determined system can be described by first-order differential equations. The more general approach is shown in Box 1.

**Box 1 Why any smooth continuous state-determined system can be represented by a set of first-order differential equations.**

Suppose any smooth continuous state-determined system is given. A sketch of why it can be described by a set of first-order differential equations is as follows. For any given time point \( t \) the future states \( x_i(t+s) \) at some future time point purely depend on \( s \) and the states \( x_i(t) \) at \( t \). This can be described by (smooth) mathematical functions \( F_i(\ldots) \):

\[
x_i(t + s) = F_i(x_1(t), \ldots, x_n(t), s) \quad \text{for} \quad s \geq 0.
\]

In the particular case of \( s = 0 \) the following holds

\[
x_i(t) = F_i(x_1(t), \ldots, x_n(t), 0).
\]

Subtracting these two expressions above and dividing by \( s \) provides:

\[
\frac{x_i(t + s) - x_i(t)}{s} = \left[ F_i(x_1(t), \ldots, x_n(t), s) - F_i(x_1(t), \ldots, x_n(t), 0) \right] / s.
\]

When the limit for \( s \) very small, approaching 0 is taken; it follows that

\[
\frac{dx_i(t)}{dt} = \left[ \frac{\partial F_i(x_1(t), \ldots, x_n(t), s)}{\partial s} \right]_{s=0}.
\]

Now define the function \( f_i(x_1, \ldots, x_n) \) by

\[
f_i(x_1, \ldots, x_n) = \left[ \frac{\partial F_i(x_1, \ldots, x_n, s)}{\partial s} \right]_{s=0}.
\]

Then the following holds

\[
\frac{dx_i(t)}{dt} = f_i(x_1(t), \ldots, x_n(t)).
\]

This shows that the given state-determined system can be described by a set of first-order differential equations.
3.2. From differential equations to temporal-causal networks

First it is shown by an example how any model described by a set of first-order differential equations can be described by a temporal-causal network. In Box 2, the general approach is discussed. Consider an arbitrary example of a model described by a set of first-order differential equations:

\[
\begin{align*}
\frac{dW(t)}{dt} &= Z(t) - Y(t)(1 - W(t)), \\
\frac{dX(t)}{dt} &= X(t)(1 - W(t)), \\
\frac{dY(t)}{dt} &= X(t) - Y(t) + Z(t), \\
\frac{dZ(t)}{dt} &= Z(t)(1 - Y(t)).
\end{align*}
\]

Box 2 Why any set of first-order differential equations can be described by a temporal-causal network.

Suppose a differential equation for any of the states \(X_i\) is given of the form:

\[\frac{dX_i(t)}{dt} = f_i(X_1(t), \ldots, X_n(t)).\]

Then this function \(f_i(X_1(t), \ldots, X_n(t))\) will depend on a subset \(D_{X_i}\) of the set of states \(\{X_1, \ldots, X_n\}\). Note that \(X_i\) may occur in \(D_{X_i}\). Usually this function \(f_i\) will be given as a formula in \(X_1, \ldots, X_n\), then this subset can be taken as the set of all states in \(\{X_1, \ldots, X_n\}\) that actually occur in this formula. For any two states \(X_j\) and \(X_i\) with \(j \neq i\) a causal connection from \(X_j\) to \(X_i\) can be defined by the criterion that \(X_j \in D_{X_i}\). Moreover, by defining the function \(h_i(X_1, \ldots, X_n)\) by

\[h_i(X_1, \ldots, X_n) = X_i + f_i(X_1, \ldots, X_n)\]

the above differential equation for \(X_i\) always can be rewritten into a differential equation of the form

\[\frac{dX_i(t)}{dt} = h_i(X_1(t), \ldots, X_n(t)) - X_i(t)\]

for some function \(h_i(X_1(t), \ldots, X_n(t))\). This form is a specific case (for \(\eta_i = 1\)) of a more general model of the form

\[\frac{dX_i(t)}{dt} = \eta_i[h_i(X_1(t), \ldots, X_n(t)) - X_i(t)],\]

where the parameter \(\eta_i\) indicates a speed factor for state \(X_i\). Note again that \(X_i\) may occur in \(h_i(X_1, \ldots, X_n)\). The obtained causal network model can be generalized further by incorporating more structure by introducing as additional parameters specific nonzero weight values \(\omega_{ij}\) for the causal connections from \(X_j\) to \(X_i\). In that case the function \(h_i(X_1, \ldots, X_n)\) can be considered a combination function \(c(X_1, \ldots, X_n)\), where for this case for the connection weights \(\omega_{ij} = 1\) holds. Then the format found above can be considered as a specific case (for \(\omega_{ij} = 1\)) of the still more general model of the form

\[\frac{dX_i(t)}{dt} = \eta_i[c(X_1(t), \ldots, X_n(t)) - X_i(t)].\]

So, having started with any arbitrary continuous, smooth state-determined system and its representation

\[\frac{dX_i(t)}{dt} = f_i(X_1(t), \ldots, X_n(t))\]

in differential equation format, finally a numerical representation of a temporal-causal network model according to Section 2.5 was obtained in the form:

\[\frac{dX_i(t)}{dt} = \eta_i[c(X_1(t), \ldots, X_n(t)) - X_i(t)]]\]

with \(c(V_1, \ldots, V_n)\) a combination function, and \(\eta_i\) and \(\omega_{ij}\) parameters for a speed factor and connection weights. The original state-determined system description is a special case of this temporal-causal network model for settings \(\eta_i = 1\) and \(\omega_{ij} = 1\) for connected states.

To determine a temporal-causal network representation for this model, the four states (or state variables) \(W, X, Y, Z\) are considered as the nodes. From each of the equations by
inspecting which states occur in the right hand sides of the differential equations it can subsequently be determined that (in addition to the effect of the state itself):

- Y and Z affect W
- W affects X
- X and Z affect Y
- Y affects Z

These causal connections can be represented in a conceptual graphical form that is shown in Figure 5. Note that the connection weights and speed factors are not mentioned as they are all assumed to be 1. The combination functions will be discussed below. Considering the numerical representation, note that, when comparing, for example, the second differential equation to the numerical representation format defined in Section 2, it can be rewritten as

\[
\frac{dX(t)}{dt} = X(t)(1 - W(t)) = [X(t)(1 - W(t)) + X(t)] - X(t).
\]

Here \([X(t)(1-W(t)) + X(t)]\) can be viewed as the result of a combination function

\[
c_X(V_1, V_2) = V_2(1 - V_1) + V_2
\]

applied to \(X(t)\) (for \(V_1\)) and \(W(t)\) (for \(V_2\)). In a similar manner the following combination functions can be identified from the differential equations (see also Table 2):

\[
c_W(V_1, V_2, V_3) = V_3 - V_2(1 - V_1) + V_1 = V_1 - V_2 + V_3 + V_1V_2,
\]

\[
c_X(V_1, V_2) = V_2(1 - V_1) + V_2 = 2V_2 - V_1V_2,
\]

\[
c_Y(V_1, V_2, V_3) = V_1 - V_2 + V_3 + V_2 = V_1 + V_3,
\]

\[
c_Z(V_1, V_2) = V_2(1 - V_1) + V_2 = 2V_2 - V_1V_2.
\]

Using these combination functions, the original differential equations transform into the following numerical representation of a temporal-causal network representation where all

![Figure 5](image)
speed factors \( \eta \) and all connection weights \( \omega \) for connected states are 1 (see also Table 3):

\[
\frac{dW(t)}{dt} = \eta_W[c_W(\omega_{Y,W}Y(t), \omega_{Z,W}Z(t), \omega_{W,W}W(t)) - W(t)],
\]

\[
\frac{dX(t)}{dt} = \eta_X[c_X(\omega_{X,X}X(t), \omega_{W,X}W(t)) - X(t)],
\]

\[
\frac{dY(t)}{dt} = \eta_Y[c_Y(\omega_{X,Y}X(t), \omega_{Y,Y}Y(t), \omega_{Z,Y}Z(t)) - Y(t)],
\]

\[
\frac{dZ(t)}{dt} = \eta_Z[c_Z(\omega_{Y,Z}Y(t), \omega_{Z,Z}Z(t)) - Z(t)].
\]

It turns out that the model described by the differential equations can be remodelled as a special case of a more general numerical temporal-causal network model representation. In Box 2, the general approach is described.

So, it has been found that any smooth continuous dynamical system can be modelled as a temporal-causal network model, by choosing suitable parameters such as connection weights, speed factors, and combination functions. In this sense this Network-Oriented Modelling approach is as generic as dynamic modelling approaches put forward, for example, in Ashby (1960), Funahashi and Nakamura (1993), Grossberg (1969), Hirsch (1989), Hopfield (1984), and Port and van Gelder (1995). This indicates that using this Network-Oriented Modelling approach does not limit the scope of applicability of the modelling in comparison to the general (smooth continuous) dynamical system approach. In Section 4, the discrete case is analysed.

### 4. Modelling discrete dynamical systems as temporal-causal networks

The numerical representations of temporal-causal network models can also be used to model any discrete and binary processes, as will be shown in this section.

### Table 2. Matrix representation for the example model based on the given differential equation representation.

| To from | \( W \) | \( X \) | \( Y \) | \( Z \) |
|---------|--------|--------|--------|--------|
| \( W \) | 1      |        |        |        |
| \( X \) | 1      | 1      |        |        |
| \( Y \) | 1      |        | 1      |        |
| \( Z \) | 1      | 1      | 1      | 1      |
| \( \eta \) | 1      | 1      | 1      | 1      |
| \( c(\ldots) \) | \( V_1 - V_2 + V_3 + V_1 V_2 \) | \( 2V_2 - V_1 \) | \( V_1 + V_3 \) | \( 2V_2 - V_1 V_2 \) |

### Table 3. Matrix representation for the general temporal-causal network model subsuming the model based on the given differential equations.

| To from | \( W \) | \( X \) | \( Y \) | \( Z \) |
|---------|--------|--------|--------|--------|
| \( W \) | \( \omega_{W,W} \) | \( \omega_{W,X} \) | \( \omega_{Y,W} \) | \( \omega_{Z,W} \) |
| \( X \) | \( \omega_{Y,X} \) | \( \omega_{X,X} \) | \( \omega_{Y,Y} \) | \( \omega_{Z,Y} \) |
| \( Y \) | \( \omega_{Z,X} \) | \( \omega_{Z,Z} \) | \( \omega_{Z,Z} \) | \( \omega_{Z,Z} \) |
| \( \eta \) | \( \eta_{W} \) | \( \eta_{Y} \) | \( \eta_{Y} \) | \( \eta_{Z} \) |
| \( c(\ldots) \) | \( c_W(V_1, V_2, V_3) \) | \( c_Y(V_1, V_2) \) | \( c_Y(V_1, V_2, V_3) \) | \( c_Z(V_1, V_2) \) |
4.1. Real-valued discrete dynamical systems

To consider discrete dynamical systems as often considered in discrete event simulation (e.g. Sarjoughian & Cellier, 2001; Uhrmacher & Schattenberg, 1998), for example, first set time step \( \Delta t = 1 \). Then the difference equation for any state \( Y \) becomes

\[
Y(t+1) = Y(t) + \eta_Y [c_Y(\omega_{X,Y} X_1(t), \ldots, \omega_{X_k,Y} X_k(t)) - Y(t)]
\]

\[
= (1 - \eta_Y) Y(t) + \eta_Y c_Y(\omega_{X,Y} X_1(t), \ldots, \omega_{X_k,Y} X_k(t)).
\]

As \( 0 \leq \eta_Y \leq 1 \) is assumed here, the new value for \( Y \) is a weighted average of the current value and the aggregated impact with \( \eta_Y \) and \((1-\eta_Y)\) as weights. Next, if the connection weights for all states \( X \) and \( Y \) with a connection from \( X \) to \( Y \) is assumed \( \omega_{X,Y} = 1 \), the following is obtained:

\[
Y(t+1) = (1 - \eta_Y) Y(t) + \eta_Y c_Y(X_1(t), \ldots, X_k(t)).
\]

Moreover, if \( \eta_Y = 1 \) for all states \( Y \) is assumed, the following is obtained:

\[
Y(t+1) = c_Y(X_1(t), \ldots, X_k(t)).
\]

This is a very general format, often used to specify iteration rules for discrete simulations. So, all such approaches are covered by temporal-causal networks.

4.2. Binary discrete dynamical systems and finite state machines

One step further is when all state values are assumed binary: 0 or 1, and all combination functions \( c_Y(\ldots) \) only generate values 0 or 1, when applied to values 0 or 1. Then the previous iteration equation

\[
Y(t+1) = c_Y(X_1(t), \ldots, X_k(t))
\]

can be taken as a general evolution or transition rule for a discrete binary dynamical system. If the overall states are defined as vectors \( X(t) = (X_1(t), \ldots, X_k(t)) \) with values 0 or 1, and for \( V = (V_1, \ldots, V_k) \) the vector combination function \( c(.) \) is defined by

\[
c(V) = (c_{X_1}(V), \ldots, c_{X_k}(V)) = (c_{X_1}(X_1(t), \ldots, X_k(t)), \ldots, c_{X_k}(X_1(t), \ldots, X_k(t))),
\]

the transitions of overall states are defined as

\[
(X_1(t+1), \ldots, X_k(t+1)) = (c_{X_1}(X_1(t), \ldots, X_k(t)), \ldots, c_{X_k}(X_1(t), \ldots, X_k(t))),
\]

or in short notation

\[
X(t+1) = c(X(t)).
\]

This is illustrated by a simple model of traffic lights at a crossing of two roads A and B, where traffic on A has priority over traffic on B. For example, if no approaching traffic is sensed on road A, then the traffic light for road B is not red, and for road A it is red. The rules describing state transitions can be described by the following transition relations:

\[
\text{traffic_on_road_A} \rightarrow \neg \text{red_light_for_road_A} \land \text{red_light_for_road_B}
\]
no traffic_on_road_A ∧ traffic_on_road_B → no red_light_for_road_B ∧
red_light_for_road_A

no traffic_on_road_A ∧ no traffic_on_road_B → no red_light_for_road_A ∧
red_light_for_road_B

These transition relations can be represented by a (vector) combination function defined by:
\[ c(v_1, v_2, v_3, v_4) = (1, v_2, 0, 1); c(0, 1, v_3, v_4) = (0, 1, 1, 0); c(0, 0, v_3, v_4) = (0, 0, 0, 1). \]
This shows how the Network-Oriented Modelling approach based on temporal-causal networks subsumes modelling by discrete binary dynamical systems.

Within theoretical analyses often variants of transition systems or finite state machines are used as universal ways to specify computational processes. In more detail and illustrated by the above traffic light example, the format for binary discrete dynamical systems described above as a special case of temporal-causal networks can be used to model transition systems or finite state machines in the format of this Network-Oriented Modelling approach. This can be done by assuming that states are described by vectors \( X \) based on a number of binary state variables \( X_i \) (with values 0 or 1) and by defining \( c(X) = X' \) if and only if within a given finite state machine or transition system there is a transition from the overall state represented by vector \( X \) to the overall state represented by vector \( X' \).

As finite state machines and transition systems are often considered to be general computational formats, this shows how very wide classes of computational processes can be covered by Network-Oriented Modelling based on temporal-causal networks.

5. Some example applications of Network-Oriented Modelling by temporal-causal networks

In Treur (2016b) in a number of chapters applications of Network-Oriented Modelling based on temporal-causal networks for the area of human mental and social processes are discussed. In Part II in Chapters 3 to 6 models are discussed that address the way in which emotions are integrated in an interactive manner in practically all mental processes. In Chapter 3, it is discussed how within Cognitive, Affective, and Social Neuroscience mechanisms have been found that indicate how emotions interact in a bidirectional manner with many other mental processes and behaviour. Based on this, an overview of neurologically inspired temporal-causal network models for the dynamics and interaction for emotions is discussed. Thus an integrative perspective is obtained that can be used to describe, for example, how emotions interact with beliefs, experiences, decision-making, and emotions of others, and also how emotions can be regulated. It is pointed out how integrated temporal-causal network models of such mental processes incorporating emotions can be obtained.

In Chapter 4, it is discussed how emotions play a role in generating dream episodes from a perspective of internal simulation. Building blocks for this internal simulation are memory elements in the form of sensory representations and their associated emotions. In the presented temporal-causal network model, under the influence of associated feeling levels and mutual competition, some sensory representation states pop up in different dream episodes. As a form of emotion regulation the activation levels of both the feelings and the sensory representation states are suppressed by control states. In Chapter 5 it is
discussed how dreaming is used to learn fear extinction. Fear extinction has been found not to involve weakening of fear associations, but instead it involves the strengthening of fear-suppressing connections that form a counterbalance against the still persisting fear associations. To this end neural mechanisms are used that strengthen these suppressing connections, as a form of learning of emotion regulation. The presented adaptive temporal-causal network model based on Hebbian learning addresses this adaptation process.

Chapter 6 addresses the role of emotions in rational decision-making. It has been found that neurological mechanisms involving emotions play an important role in rational decision-making. In this chapter an adaptive temporal-causal network model for decision-making based on predictive loops through feeling states is presented, where the feeling states function in a process of valuing of decision options. Hebbian learning is considered for different types of connections in the adaptive model. Moreover, the adaptive temporal-causal network model is analysed from the perspective of rationality. To assess the extent of rationality, measures are introduced reflecting what would be rational for a given environment’s characteristics and behaviour. It is shown how during the adaptive process this model for decision-making achieves higher levels of rationality.

Part III of Treur (2016b), consisting of Chapters 7 to 11, focuses on persons functioning in a social context. In Chapter 7, an overview is presented of a number of recent findings from Social Neuroscience on how persons can behave in a social manner. For example, shared understanding and collective power are social phenomena that serve as a form of glue between individual persons. They easily emerge and often involve both cognitive and affective aspects. As the behaviour of each person is based on complex internal mental processes involving, for example, own goals, emotions, and beliefs, it would be expected that such forms of sharedness and collectiveness are very hard to achieve. From a neurological perspective, mirror neurons and internal simulation are core concepts to explain the mechanisms underlying such social phenomena. In this chapter it is discussed how based on such neurological concepts temporal-causal network models for social processes can be obtained. It is discussed how these models indeed are an adequate basis to simulate the emergence of shared understanding and collective power in groups.

Within a social context the notion of ownership of actions is important. Chapter 8 addresses this notion. It is related to mechanisms underlying self-other distinction, where a self-ownership state is an indication for the self-relatedness of an action and an other-ownership state to an action attributed to someone else. The temporal-causal network model presented in this chapter generates prior and retrospective ownership states for an action based on principles from recent neurological theories. A prior self-ownership state is affected by prediction of the effects of a prepared action as a form of internal simulation, and exerts control by strengthening or suppressing actual execution of the action. A prior other-ownership state also plays a role in mirroring and analysis of an observed action performed by another person, without imitating the action. A retrospective self-ownership state depends on whether the sensed consequences of an executed action co-occur with the predicted consequences, and is the basis for acknowledging authorship of actions in a social context. Scenarios are shown for vetoing a prepared action due to unsatisfactory predicted effects. Moreover, it is shown how poor action effect prediction capabilities can lead to reduced retrospective ownership states, for example, in persons suffering from schizophrenia. This can explain why sometimes the own actions are attributed to others, or actions of others are attributed to oneself.
Chapter 9 addresses how in social interaction between two persons usually each person shows empathic understanding of the other person. This involves both nonverbal and verbal elements, such as bodily expressing a similar emotion and verbally expressing beliefs about the other person. Such social interaction relates to an underlying neural mechanism based on a mirror neuron system and self-other distinction. Differences in social responses of individuals can often be related to differences in functioning of certain neurological mechanisms, as can be seen, for example, in persons with a specific type of autism spectrum disorder (ASD). This chapter presents a temporal-causal network model which, depending on personal characteristics, is capable of showing different types of social response patterns based on such mechanisms, adopted from theories on the role of mirror neuron systems, emotion integration, emotion regulation, and empathy in ASD. The personal characteristics may also show variations over time. This chapter also addresses this adaptation over time. To this end it includes an adaptive temporal-causal network model capable of learning social responses, based on insights from Social Neuroscience.

Chapter 10 addresses joint decision-making. The notion of joint decision-making as considered does not only concern a choice for a common decision option, but also sharing a good feeling and mutually acknowledged empathic understanding about it. The model is based on principles from recent neurological theories on mirror neurons, internal simulation, and emotion-related valuing. Emotion-related valuing of decision options and mutual contagion of intentions and emotions between persons are used as a basis for mutual empathic understanding and convergence of decisions and their associated emotions.

In Chapter 11, it is discussed how adaptive temporal-causal network models can be used to model evolving social interactions. This perspective simplifies persons to just one state and expresses the complexity in the structure of the social interactions, modelled by a network. The states can represent, for example, a person’s emotion, a belief, an opinion, or a behaviour. Two types of dynamics are addressed: dynamics based on a fixed structure of interactions (modelled by a non-adaptive temporal-causal network model), and dynamics where the social interactions themselves change over time (modelled by an adaptive temporal-causal network model). In the case of an adaptive network model, the network connections change, for example, their weights may increase or decrease, or connections are added or removed. Different types of adaptive social network models are addressed, based on different principles: the homophily principle assuming that connections strengthen more when the persons are more similar in their state (the more you are alike, the more you like each other), and the more becomes more principle assuming that persons that already have more and stronger connections also attract more and stronger connections.

6. Discussion

The Network-Oriented Modelling approach based on temporal-causal networks, as discussed here, provides a modelling approach that enables a modeller to design high-level conceptual model representations in the form of (cyclic) labelled graphs, which can be systematically transformed in an automated manner into numerical representations that can be used to perform simulation experiments.
It sometimes is a silent assumption that a Network-Oriented Modelling approach can only work for specific application domains, where networks are more or less already given or conceived in the real world. This paper shows that this is not exactly a correct assumption. It has been shown that the applicability of the Network-Oriented Modelling approach based on temporal-causal networks is very wide. For example, it subsumes modelling approaches based on the dynamical system perspective (Port & van Gelder, 1995) not only often used to obtain cognitive models, but also to model processes in many other scientific domains. Moreover, it subsumes modelling approaches based on discrete (event) and agent simulation (Sarjoughian & Cellier, 2001; Uhrmacher & Schattenberg, 1998), including very basic computational notions such as finite state machines and transition systems. This shows that network models do not just model networks considered as given in the real world, but practically any type of process.

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No potential conflict of interest was reported by the author.

Notes on contributor

Jan Treur is an internationally well-recognized expert in multi-disciplinary research involving Network-Oriented Modelling of integrated cognitive, affective and social processes. He has been programme committee or board member of many of the main conferences and journals in these areas. His extensive list of publications covers major scientific publication media in multiple disciplines, including the top level conferences and journals. Recent involvements are on the one hand in developing a strongly multidisciplinary Bachelor and Master study program Lifestyle Informatics and Socially Aware Computing, combining subjects from Ambient and Artificial Intelligence, Biomedical Sciences, Computer Science, and Psychology, and on the other hand in developing the Network-Oriented Modeling approach based on temporal-causal networks as displayed extensively in his new book on this topic.

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