Rubber tyre waste is a huge burden on environmental system. In the present study, performance of gravity retaining wall system is assessed, both in deterministic and probabilistic framework, by utilizing rubber waste mix in the backfill. In the probabilistic approach, performance of a geotechnical system is measured in terms of an index popularly called “reliability index $\beta$”, a measure of probability of success of system. Concept of response surface methodology (RSM) has been applied to establish an approximate functional relationship between input geotechnical parameters, and output response, i.e. factor of safety through numerical analysis and then reliability index $\beta$ is evaluated using FORM approach. Present study demonstrates the two aspects, i.e., by considering the case of a gravity retaining wall, i.e., reliability of computed value of factor of safety and role of mixing tyre chips in improving the performance of gravity wall system.
1. INTRODUCTION

Scraped tyres are the major source of concern for society and its safe disposal presents a major challenge to people working in the field of waste management and waste utilization. Majority of scraped tyres either go to landfills or brick kilns, especially in developing countries. Utilization in later case is more dangerous and poses serious threat to environment. Several researchers in the past have demonstrated that these waste tyres can effectively be utilized in the civil engineering construction works and excellent reviews are presented (Sofi, 2018; Eldin & Senouci, 1992). Pertaining to earth retaining structure, type of backfill soil in the retaining walls is a major factor that decides the performance under static as well as dynamic loading conditions. Usually, cohesionless backfill materials such as clean gravels are used, but nowadays different lightweight fills materials such as geofoam, shredded tyre chips, plastic bottles are being used as instead of conventional material. Such lightweight materials are not only advantageous in terms of reducing earth pressures as well as lateral displacements of the retaining walls but also are economical. Reddy and Krishna (2015) researched that sand–tyre chips (STC) mixtures were used as a backfill and it was found that wall deformations and earth pressures for the model retaining wall were reduced. STC mixtures can be effectively used as a backfill which reduces the displacements and earth pressures by about 50–60 %. Lee and Roh (2007) stated that not only in static pressures, soil – tyre mix as lightweight backfill materials can also be safely used to minimize the dynamic earth pressure on retaining structure. Also, in comparison with traditional granular backfill, tyre-derived aggregates geosynthetically reinforced wall showed better performance in both static and dynamic loading conditions (Xiao et al., 2012). Sand–Tyre Chips (STC) mixtures in different proportions were used as lightweight backfill materials behind the cantilever retaining wall and performance and stability characteristic were analyzed numerically with the finite element software. It was found that total displacement, lateral and vertical displacements, lateral pressures, maximum bending moments and shear forces value were decreased and overall stability of the retaining wall is improved after using STC mixtures rather than sand alone (Djadouni et al., 2019). Hence, it can be advocated that rubber tyre waste can be effectively utilized in the backfill of the earth retaining system. Such utilization will dramatically reduce the burden of disposal of waste tyres and will ensure safety against environment protection. After utilizing rubber tyre in the backfill, it is again important to assess the stability of the structure.

In the geotechnical system, conventional factor of safety approach is used to ensure the stability against overturning, sliding and bearing failure. Analysis and design of geotechnical system requires input parameters that is always uncertain and that uncertainty is contributed due to inherent variability due to nature, testing errors and model transformation uncertainty (Phoon & Kulhawy, 1999a, 1999b). A single factor of safety value assigned to a geotechnical system is based on the assumption that all sources of uncertainties are inherently taken care of and choice of appropriate value of factor of safety is purely subjective in nature. Owning to this issue, in recent times, analysis of different geotechnical issues also involves probabilistic theory to bring rationality and justification while making decision (Duncan, 2000; Fenton & Griffiths, 2007). Probabilistic approach gives a better insight into the extent of uncertainty and its implications on the performance study of a geotechnical system. Although mathematically demanding, but with time and experience it is proved that probabilistic approach when used in conjunction with conventional factor of safety approach gives better understanding of problem in hand. Methods of reliability analysis include different approach like first order reliability method (FORM), Monte Carlo Simulations (MCS), Point Estimate
method (PEM) are available in the standard literature. The complete objectives of the present study are to visualize performance measures in terms of factor of safety and reliability based approach and present a discussion that may prove to be useful for geotechnical experts little reluctant in adapting the probabilistic approach. For the numerical analysis, commercially available finite element code PLAXIS 2D is utilized. Objectives of the present study are further highlighted as below:

1. To first perform the numerical analysis of gravity retaining wall and compare the results of distribution of back pressure with conventional Rankine theory for numerical model and analysis verifications purposes.
2. To estimate the factor of safety of retaining walls for a given set of input parameters using strength reduction technique option available as in build in the numerical tool.
3. Performing reliability analysis of the gravity retaining wall using combination of response surface methodology, numerical analysis and first order reliability method by considering extent of uncertainty in the input parameters.

Reliability based performance assessment of gravity retaining wall for backfilled material mixed with tyre chips by taking input properties of STC mix from published literature.

2. RELIABILITY BASED APPROACH
2.1. Quantification of Uncertainty

Uncertainties in the input parameters are quantified using statistical descriptors like mean, variance, covariance and auto-covariance. A detailed discussion is available in published literature (Fenton & Griffiths, 2007). In the present study, input parameters are considered as uncorrelated normally distributed random variables as this is to simplify the analysis using conventional statistics. Coefficient of variation (COV) which is nothing but the ratio of standard deviation to mean is commonly used as, being dimensionless, it given a better picture of extent of variation in the parameters. In the absence of sufficient test data, published literature are referred for choosing the appropriate value of COV for the geotechnical parameters (Duncan, 2000). For the probabilistic analysis, input parameters are treated as random variables and mathematical description of these random variables are in terms of probability distribution function. Conventionally, in geotechnical engineering, input parameters are assumed as Normally distributed. The advantage of assuming normal distribution is that parameters of distribution are directly defined by its mean (μ) and variance (σ²).

2.2. Response Surface Methodology (RSM)

Response surface method (RSM) is well utilized in optimization techniques for design of experiments and establishing an approximate explicit functional relationship between input variables and output response, when it is not available. That functional relationship is must for conducting the reliability analysis, for simplification purpose otherwise approach like Monte Carlos simulations become computationally demanding and time consuming (Babu & Srivastava, 2008). Regression modelling techniques, Statistical experimental design and optimization methods are the techniques or methods in which RSM is useful. Such approaches are very well utilized by different researches; for reliability analysis of shallow foundation in the surrounding area of the existing buried pipe (Malhotra & Srivastava, 2020); Uses of response surface methods on the reliability analysis of laterally loaded piles (Tandjiria et al., 2000); Usage of numerical analysis as well as RSM in Reliability-Based stability analysis of Rock slopes (Dadashzadeh et al., 2017) and many more studies have been done using RSM.

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The approach involves fitting a linear and nonlinear model by choosing set of design points. A 2n factorial design is used for the same, where n is the number of input parameters. For example, if there are two input parameters, then 4 design points (coordinates) will be selected as they are lying on the corner of a square. The size of square depends on combination of values of ($\mu \pm \sigma m$); where $\mu$ is the mean and $\sigma$ is standard deviation of input parameter; $m$ is a numerical parameter taken from 1 to 3. Similarly, for 3 input parameters, there will be 8 such combinations of design points. For each design point, the corresponding output, i.e., factor of safety (FS) is the measure of performance of geotechnical system. Definitely, uncertainty in the input parameters brings uncertainty in the estimation of factor of safety.

A functional relationship between input parameters and output response is must to perform the reliability analysis. For each combination of input parameter, the corresponding output, i.e., factor of safety is obtained using the numerical analysis and an explicit functional relationship is established. In the present study, an approximate linear regression model is fitted using least square error approach, the same is available as an inbuilt option in data analysis tool pack of Microsoft excel. The approach is not so useful when input parameters are too large as this lead to large number of combinations. In such scenario, it is always suggested to perform sensitivity analysis of input parameters to identify and filter those parameters that actually influence the output. The fitted model is checked for its adequacy by using statistical technique or getting information on $R^2$ or adjusted $R^2$ values. Once a functional relationship is established, next step is performing the reliability analysis.

2.3. Reliability Index ($\beta$)

Reliability is defined as probability of success of a system in a given environment of loading conditions over a period of time. For the reliability analysis, performance function is defined as $g() = R - S$; where $R$ is resistance and $S$ is the load. In terms of factor of safety, $R$ is FS and $S$ is 1.0, i.e., a system will fail if $g()$ is less than 1.0. For normally distributed uncorrelated $R$ and $S$, the reliability index ($\beta$) is evaluated as Equation [1] below:

$$\beta = \frac{R - S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

(1)

For more complicated cases, one may refer standard literature. USACE made specific recommendations on the choice of reliability index and associated expected performance level of the geotechnical system. It is suggested that for above average, good and excellent performance level, reliability index value should be minimum, 3, 4 and 5, respectively.

3. PERFORMANCE EVALUATION OF GRAVITY RETAINING WALL

3.1. Numerical Simulation Procedure

Step by step procedure for the numerical analysis involves drawing a geometric model with defined scale and unit; assigning material properties; generating finite element mesh; establishing initial stress conditions and then proceeding for the calculations for plastic analysis and factor of safety using $\phi$-c reduction technique. Figure 1 shows the geometric model for the gravity retaining wall. It is to be noted that the boundaries are well beyond the location of the retaining wall. It is purposefully done to avoid the boundary effect. Also, in situ soil and backfill materials are assumed to be same. It is noted that numerical code applies the standard boundary conditions where horizontal fixity is given to vertical boundaries and fixities in both the directions are given to bottom boundary. Top boundary has no fixity as it
the case for natural ground surface. Properties for in situ soil are taken for sand with the assumption that it follows Mohr-Coulomb failure hypothesis with non-associated flow rule. The input geotechnical parameters are $c$ (cohesion), $\phi$ (angle of internal friction), $\gamma$ (unit weight), $E$ (Elastic modulus) and $\nu$ (Poisson’s ratio) and numerical values are assumed as 0.01 (to avoid numerical instability), $41^\circ$, 18 kN/m$^3$, 5 MPa and 0.35, respectively.

Next step is the finite element discretization of the geometric model using 15 noded triangular element. It is noted that fine mesh is generated to get more accurate results with less time consumption. Interface element is used to model frictional component between soil backfill and back of the wall. Standard values for rigidity and fixities are selected as the same was not available either through experimentation or in literature. Also, it was not of much relevance when comparative studies were performed and results of the performance analysis were relatively studied. After that in situ stress condition calculated due to gravity. The numerical tool estimates the values using Terzhagi’s effective stress concept. The effect of GWT was ignored as it is known that retaining walls are never designed for such conditions and this is ensured by providing proper drainage conditions and also through provision of weep holes.

Figure 2 shows the deformation pattern obtained for the gravity retaining wall. It is noted that there is no boundary effect and wall is deforming in active state. Total displacement is 13.56 mm which is sufficient to get the active conditions. There is no differential movement in the body of the retaining wall, which ensures rigid body movement and compatible to the conditions of a gravity retaining wall that is rigid in nature.

Figure 3 shows the comparison of the results of the numerical analysis (blue dots) and rankine theory (orange line with dots). It is noted that the variation of earth pressure behind the retaining wall estimated through numerical analysis very well compares with the well-established Rankine’s theory. This confirms the correctness of numerical model, and calculation procedure. This fulfills the requirements of objective one of the present study. Factor of safety for the case considered was evaluated as 2.04 which is much above the minimum accepted value of 1.5 against sliding but definitely not acceptable for overturning conditions. Hence, it is suggested to use the Tyre chips for improving the factor of safety and bringing it to more than 3.0 so that the wall is ensured to be safe against sliding as well as overturning. Considering the in situ properties for STC as indicated in Table 1, numerical analysis was performed to estimate the factor of safety, numerically. Table 2 summarizes the results of the analysis for all the 5 cases considered.
**Figure 2.** Deformation pattern for the gravity retaining wall under active condition.

**Figure 3.** Comparison of earth pressure behind retaining wall from numerical analysis and Rankine’s earth pressure theory.
Table 1. Properties of STC assuming Mohr-Coulomb material behavior (Adapted from Ahmed 1993).

| Chip mix ratio by % weight | Cohesion (c) in kPa | Angle of internal friction (φ) | Unit weight in kN/m³ | Elastic modulus (kPa) | Poisson’s ratio |
|---------------------------|---------------------|-------------------------------|----------------------|----------------------|----------------|
| 16.5                      | 14                  | 38°                           | 16.0                 | 45145                | 0.306          |
| 29.2                      | 31                  | 35°                           | 15.0                 | 42155                | 0.309          |
| 39.4                      | 43                  | 33°                           | 14.0                 | 39513                | 0.312          |
| 50.0                      | 22                  | 27°                           | 11.5                 | 36037                | 0.314          |
| 66.5                      | 22                  | 17°                           | 8.5                  | 29980                | 0.316          |

Table 2. Factor of safety evaluation of all cases of tyre chip mix.

| Cases          | Chip mix ratio by % weight | Factor of safety | Remarks (FS>3.0) |
|----------------|-----------------------------|------------------|------------------|
| Case 0         | Without Tyre mix            | 2.0356           | Not acceptable   |
| Case 1         | 16.5                        | 1.9858           | Not acceptable   |
| Case 2         | 29.2                        | 3.1319           | Acceptable       |
| Case 3         | 39.4                        | 3.5705           | Acceptable       |
| Case 4         | 50.0                        | 2.3877           | Not acceptable   |
| Case 5         | 66.5                        | 1.9153           | Not Acceptable   |

It can be noted that adding 16.5% tyre chip does not make much difference in terms of improvement in the factor of safety. Highest factor of safety is achieved in Case 3 when 39.4% of tyre chips are introduced. Although, both case 2 and 3 fulfils the requirement of safety through conventional approach as the values are higher than minimum required 3.0. The factor of safety is improved due to reduction in the earth pressure on the wall due to lighter backfill mix. Again for case 4 and 5, the factor of safety values is reduced due to reduction in the shear strength parameters of mix in spite of appreciable amount of reduction in the wall pressure may have been achieved due to lighter backfill materials. This fulfils the study of second objective.

3.2. RSM and Reliability Index Evaluation

For establishing a functional relationship between input parameters and output response, i.e., Factor of safety of retaining wall, Case 2 and 3 were considered as they were giving acceptable amount of factor of safety from conventional approach. It is noted that there are 3 input geotechnical parameters and while using 2^n factorial design, 8 combinations of input parameters will be required. For the present study, value of m is taken as 1.65, to ensure 95% coverage and 5% acceptable error. The coefficient of variation in the input parameters c, φ and γ were assumed as 20, 15 and 10%, respectively.

Table 3 provides information on 8 such combinations of input parameters and corresponding output obtained for each set of design points for case 2 analyses. Using least square error approach, a linear functional relationship between input and output is established as Equation [2]:

\[
FS = 0.293 + 0.0435c + 0.0674\phi - 0.0533\gamma
\]  

(2)

As it is a simple linear functional relationship between input and output, the mean and variance in FS will be estimated using conventional statistical approach and the values are obtained as 3.18 and 0.453, respectively. The reliability index value using Equation [1] is obtained as 4.81. Similar exercise was done for other cases and results of the reliability analysis are provided in Table 4.
Table 3. Design points for 8 combinations of input parameters and estimated FOS.

| Sr. No | $c$ | $\phi$ | $\gamma$ | $c$ | $\phi$ | $\gamma$ | FOS  |
|--------|-----|-------|-------|-----|-------|-------|-----|
| 1      | +   | +     | +     | 41.51 | 43.66 | 17.35 | 4.0537 |
| 2      | +   | +     | -     | 41.51 | 43.66 | 12.43 | 4.4088 |
| 3      | +   | -     | +     | 41.51 | 26.34 | 17.35 | 2.8738 |
| 4      | +   | -     | -     | 41.51 | 26.34 | 12.43 | 3.1981 |
| 5      | -   | +     | +     | 20.91 | 43.66 | 17.35 | 3.1920 |
| 6      | -   | +     | -     | 20.91 | 43.66 | 12.43 | 3.4218 |
| 7      | -   | -     | +     | 20.91 | 26.34 | 17.35 | 2.0783 |
| 8      | -   | -     | -     | 20.91 | 26.34 | 12.43 | 2.2565 |

Table 4. Reliability index evaluation of all cases of tyre chip mix.

| Cases           | Chip mix ratio by % weight | Reliability index ($\beta$) | Remarks ($\beta$>5.0) |
|-----------------|-----------------------------|----------------------------|-----------------------|
| Case 0          | Without Tyre mix            | 4.81                       | Not acceptable        |
| Case 1          | 16.5                        | 3.76                       | Not acceptable        |
| Case 2          | 29.2                        | 4.81                       | Not Acceptable        |
| Case 3          | 39.4                        | 5.26                       | Acceptable            |
| Case 4          | 50.0                        | 4.38                       | Not acceptable        |
| Case 5          | 66.5                        | 3.57                       | Not Acceptable        |

For the present study, if it is assumed that acceptable level of performance should fall in excellent category, then in a given environment of uncertainty in input parameters, case 2 will not qualify and the only case 3 satisfied the expected performance level. Definitely, in all situations, retaining wall will perform above average. Hence, it can be noted that conventional factor of safety approach when used in conjunction with reliability based approach, a better insight and rationality in decision making is achieved. One may get clear demarcation of case 2 and case 3, where both were satisfying the requirement of factor of safety from conventional approach, but the one failed to ensure the expected performance level when a due consideration was given to extent of uncertainty involved in input parameters.

4. DISCUSSION ON RESULTS

The study first advocates the use of rubber tyre waste in the backfill of the retaining structure. Such utilization will not only help reducing the burden of disposal but also it will reduce pressure on the backfill which will in turn reduce the section of the wall and result in cost saving. Further, considering the case of gravity retaining wall, it is demonstrated, through numerical analysis, to assess the stability in terms of factor of safety using strength reduction technique available as an inbuilt option in the numerical tool. Considering different cases, it was found that factor of safety reaches the satisfactory level for two scenarios but considering uncertainty factor the performance of wall is found two be Satisfactory only in one scenario. Hence, probabilistic approach in conjunction with deterministic approach is essential in reliability based decision making. Response surface method (RSM) is useful in establishing an explicit approximate functional relationship between input parameters and output response to perform the reliability analysis for simple cases. Number of simulations required for establishing such functional relationship is quite less and it saves time, effort and cost.
paper demonstrates a simple approach for performing the reliability analysis and it becomes useful in initial stages of preliminary analysis. For more complex situations with consideration of correlation among input parameters along with spatial variability modeling requires complex mathematical modeling and numerical analysis procedure which is also time consuming and requires efforts.

5. CONCLUSION

- Reliability based approach is much useful when used in conjunction with conventional factor of safety approach. It treats uncertainty in mathematical framework and brings rationality in decision making.
- Response surface method when used in conjunction with numerical analysis and FORM provides a better alternative to mathematically demanding approach.
- The approach is applicable for preliminary analysis with simplified conditions. For more complicated cases, there is no replacement of simulations techniques with. Although, it may require high end computers for facilitating the computational efforts, yet time consuming for getting basic preliminary information.

6. AUTHORS’ NOTE

The authors declare that there is no conflict of interest regarding the publication of this article. The authors confirmed that the paper was free of plagiarism.

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