Equations of motion for $N = 4$ supergravity with antisymmetric tensor from its geometric description in central charge superspace

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Abstract: We consider the geometrical formulation in central charge superspace of the $N = 4$ supergravity containing an antisymmetric tensor gauge field. The theory is on-shell, so clearly, the constraints used for the identification of the multiplet together with the superspace Bianchi identities imply equations of motion for the component fields. We deduce these equations of motion in terms of supercovariant quantities and then, we give them in terms of component fields. These equations of motion, deduced from the geometry, without supposing the existence of a Lagrangian, are found to be the same as those derived from the Lagrangian given in the component formulation of this $N = 4$ supergravity multiplet by Nicolai and Townsend.

Keywords: extended supersymmetry, supergravity, central charge superspace, equations of motion.
1. Introduction

The $N = 4$ supergravity theory containing an antisymmetric tensor was given by Nicolai and Townsend [1] already in the early eighties. The superspace formulation of the corresponding multiplet, which we call the N-T multiplet in the following, encountered a number of problems identified in [2] and overcome in [3] introducing external Chern-Simons forms for the graviphotons. Recently, a concise geometric formulation was given for this supergravity theory in central charge superspace [4].

The geometric approach adopted and described in detail in [4] was based on the superspace soldering mechanism involving gravity and 2–form geometries in central charge superspace [5]. This soldering procedure allowed to identify various gauge component fields of the one and the same multiplet in two distinct geometric structures: graviton, gravitini and graviphotons in the gravity sector and the antisymmetric tensor in the 2–form sector. Supersymmetry and central charge transformations of the component fields were deduced using the fact that in the geometric approach these transformations are identified on the same footing with general space-time coordinate transformations as superspace diffeomorphisms on the central charge superspace. Moreover, the presence of graviphoton Chern-Simons forms in the theory was interpreted as an intrinsic property of central charge superspace and a consequence of the superspace soldering mechanism.

The aim of the present paper is to emphasize that the geometric description in [4] is on-shell, that is the constraints used to identify the component fields of the N-T supergravity
multiplet imply also the equations of motion for these fields. Therefore, we begin with recalling the essential points in the identification of component fields [4] specifying the constraints we use. Then, in section 3, we derive the equations of motion directly from constraints and Bianchi identities, without any knowledge about a Lagrangian. Finally, we compare these equations of motion with those found from the component Lagrangian given in the original article by Nicolai and Townsend [1].

2. Constraints and identification of component fields

In this section we recall the essential results of [4] concerning the identification of the components of the N-T multiplet. Conventions, notations and general ideas about geometrical description of supergravity theories, central charge superspace and soldering mechanism are detailed in [4] and in references mentioned there.

Recall that in geometrical formulation of supergravity theories the basic dynamic variables are chosen to be the vielbein and the connection. Considering central charge superspace this framework provides a unified geometric identification of graviton, gravitini and graviphotons in the frame \( E^A = (E^a, E^\alpha_\Lambda, E^{\dot{\alpha}}_\Lambda, E^u) \), where \( a, \alpha, \dot{\alpha} \) denote the usual vector and Weyl spinor indices, while capital indices \( \Lambda \) count the number of supercharges and boldface indices \( u = 1 \ldots 6 \) the number of central charges:

\[
E^a = dt^m e^a_m, \quad E^\alpha_\Lambda = \frac{1}{2} dt^m \psi^\alpha_\Lambda m, \quad E^{\dot{\alpha}}_\Lambda = \frac{1}{2} dt^m \bar{\psi}^\dot{\alpha}_\Lambda m, \quad E^u = dt^m v^u_m,
\]

while the antisymmetric tensor can be identified in a superspace 2–form \( B \):

\[
B = \frac{1}{2} dt^m dt^n b_{nm}. \tag{2.2}
\]

The remaining component fields, a real scalar and 4 helicity 1/2 fields, are identified in the supersymmetry transforms of the vielbein and 2–form, that is in torsion \( (T^A = DE^A) \) and 3–form \( (H = dB) \) components. The Bianchi identities satisfied by these objects are

\[
DT^A = E^B R_B^A, \quad dH = 0, \tag{2.3}
\]

or displaying the 3–form and 4–form coefficients

\[
(D_{CB})_T : E^B E^C E^D (D_D T_{CB}^A + T_D^F T_{FB}^A - R_{DCB}^A) = 0, \tag{2.4}
\]

\[
(D_{BCA})_H : E^A E^B E^C E^D (2D_D H_{CB}^A + 3 T_{DC}^F H_{FBA}) = 0. \tag{2.5}
\]

By putting constraints on torsion and 3–form we have to solve two problems at the same time: first, we have to reduce the huge number of superfluous independent fields contained in these geometrical objects, and second, we have to make sure that the antisymmetric tensor takes part of the same multiplet as \( e^a_m, \psi^\alpha_\Lambda m, \bar{\psi}^{\dot{\alpha}}_\Lambda m, v^u_m \) (soldering mechanism).
Indeed, the biggest problem in finding a geometrical description of an off-shell supersymmetric theory is to find suitable covariant constraints which do reduce this number but do not imply equations of motion for the remaining fields. There are several approaches to this question. One of them is based on conventional constraints, which resume to suitable redefinitions of the vielbein and connection and which do not imply equations of motion [6]. However, such redefinitions leave intact torsion components with 0 canonical dimension and there is no general recipes to indicate how these torsion components have to be constrained. A simplest manner of constraining 0 dimensional torsion components together with conventional constraints give rise to the so-called natural constraints, which were analyzed in a systematic way both in ordinary extended superspace [7] and in central charge superspace [8]. Another approach is that of the geometrical constraints or integrability conditions [9], [10], which in addition to constraints on the torsion involves also constraints on the curvature, and which possess many of the integrability properties as found in the self-dual Yang-Mills system. There is no equivalence established between the two approaches, nevertheless, both imply the same second order conformal type equations of motion for $N > 4$ [11], [9].

The geometrical description of the N-T multiplet is based on a set of natural constraints in central charge superspace with structure group $SL(2, \mathbb{C}) \otimes U(4)$. The generalizations of the canonical dimension 0 “trivial constraints” [7] to central charge superspace are

$$T^C_{\gamma B} = 0, \quad T^\gamma_{\gamma B} = -2i\delta^C_B (\sigma^\alpha \epsilon)_\gamma \hat{\beta} \hat{\alpha}, \quad T^\gamma_{\hat{\alpha} B} = 0,$$

(2.6)

$$T^C_{\gamma B} L = \epsilon\gamma B T^{|CB|u}, \quad T^\gamma_{\gamma B} L = 0, \quad T^\gamma_{\hat{\alpha} B} L = \epsilon\gamma\hat{\beta} T^{|CB|u}.$$  

(2.7)

As explained in detail in the article [4], the soldering is achieved by requiring some analogous, “mirror”-constraints for the 2–form sector. Besides the -1/2 dimensional constraints $H^C_{\alpha B} = H^C_{\beta B} = H^C_{\hat{\alpha} B} = H^C_{\hat{\beta} B} = 0$, we impose

$$H^C_{\gamma B} = 0, \quad H^C_{\hat{\beta} B} = -2i\delta^C_B (\sigma^\alpha \epsilon)_\gamma \hat{\beta} L \quad H^C_{\hat{\alpha} B} = 0,$$

(2.8)

$$H^B_{\beta\alpha u} = \epsilon\beta\alpha H_u^{[BA]}, \quad H^C_{\beta B u} = 0, \quad H^C_{\hat{\alpha} B u} = \epsilon\hat{\beta}\hat{\alpha} H_u^{[BA]},$$

(2.9)

with $L$ a real superfield. The physical scalar $\phi$ of the multiplet, called also graviscalar, is identified in this superfield, parameterized as $L = e^{2\phi}$. In turn, the helicity 1/2 fields, called also gravigini fields, are identified as usual [12], [11], [2] in the 1/2–dimensional torsion component

$$\epsilon\beta\gamma T^C_{\gamma B} = 2T^{|CB|}, \quad \epsilon\beta\gamma T^C_{\hat{\alpha} B} = 2T^{|CB|}.$$  

(2.10)

The scalar, the four helicity 1/2 fields, together with the gauge–fields defined in (2.1) and (2.2) constitute the N-T on-shell N=4 supergravity multiplet. However, the 0 dimensional natural constraints listed above are not sufficient to insure that these are the only
fields transforming into each-other by supergravity transformations. The elimination of a big number of superfluous fields is achieved by assuming the constraints

\[ \mathcal{D}^{\alpha}_D T_{[CBA] \alpha} = 0, \quad \mathcal{D}^{\dot{\alpha}}_D T^{[CBA] \dot{\alpha}} = 0, \]  

(2.11)

and

\[ T_{ZB}^A = 0, \]  

(2.12)

as well as all possible compatible conventional constraints\(^1\) [7], [8].

It is worthwhile to note that even at this stage the assumptions are not sufficient to constrain the geometry to the N-T multiplet. This setup allows to give a geometrical description at least of the coupling of \( N = 4 \) supergravity with antisymmetric tensor to six copies of \( N = 4 \) Yang-Mills [13]. Nevertheless, they are strong enough to put the underlying multiplet on-shell. In order to see this, one can easily verify that the dimension 1 Bianchi identities

\[ \mathcal{D}^{\alpha}_D T_{[CBA] \alpha} = -i \delta_{DEP} G_{EF} (\mathcal{D} \mathcal{D} \mathcal{D} T_{[CBA] \alpha} - \mathcal{D} \mathcal{D} \mathcal{D} T^{[CBA] \dot{\alpha}}), \]

(2.13)

with \( G \) and \( P \) a priori some arbitrary superfields. Let us write one of the last relations as

\[ \sum_{DC} \mathcal{D}^{\delta}_D T_{[CBA] \alpha} = 0, \]  

(2.14)

take its spinorial derivative \( \mathcal{D}^{\beta}_\varepsilon \)

\[ \sum_{DC} \left( \left\{ \mathcal{D}^{\varepsilon}_\delta, \mathcal{D}^{\beta}_D \right\} T_{[CBA] \alpha} - \mathcal{D}^{\beta}_D \left( \mathcal{D}^{\varepsilon}_\delta T_{[CBA] \alpha} \right) \right) = 0, \]  

(2.15)

and observe that the antisymmetric part of this relation in the indices \( \varepsilon \) and \( \alpha \) gives rise to Dirac equation for the helicity 1/2 fields, that is \( \partial^{\alpha \dot{\beta}} T_{[CBA] \alpha} = 0 \) in the linear approach.

It turns out, that there is a simple solution of both the Bianchi identities of the torsion and 3–form, which satisfies the above mentioned constraints and reproduce the N-T multiplet. The non-zero torsion and 3–form components for this solution are listed in the appendix, we will concentrate here on its properties which are essential for the identification of the multiplet and the derivation of the equations of motion for the component fields.

Recall that the particularity of this solution is based on the identification of the scalar superfield \( \phi \) in the 0 dimensional torsion and 3–form components containing a central charge index

\[ T^{[BA]}_u = 4 e^{\phi t_{[BA]}}, \quad T_{[BA]}^u = 4 e^{\phi t_{[BA]}}, \]  

(2.16)

\[ H_u^{[BA]} = 4 e^{\phi h_u^{[BA]}}, \quad H_u^{[BA]} = 4 e^{\phi h_u^{[BA]}}, \]  

(2.17)

\(^1\)see equations (A.1) in the appendix.
with $t^{[CB]}_u$, $t^{[CB]}_v$, $h^{[BA]}_u$, $h^{[BA]}_v$ constant matrix elements satisfying the self–duality relations

$$t^{[DC]}_u = \frac{q}{2} \varepsilon^{DCBA} t^{[BA]}_u, \quad h^{[BA]}_v = \frac{q}{2} h^{[DC]}_u \varepsilon^{DCBA} \quad \text{with} \quad q = \pm 1. \quad (2.18)$$

Note, that these relations look similar to some of the properties of the 6 real, antisymmetric $4 \times 4$ matrices $\alpha^n$, $\beta^n$, ($n = 1, 2, 3$) of $SU(2) \otimes SU(2)$ [14], [15], which appear in the component formulation of $N = 4$ supergravity theories. Indeed, if we define the matrices

$$t \doteq \begin{pmatrix} t^{[DC]}_u \\ t^{[DC]}_v \end{pmatrix}, \quad h \doteq \begin{pmatrix} h^{[BA]}_u \\ h^{[BA]}_v \end{pmatrix} \quad \text{and} \quad \Sigma \doteq \begin{pmatrix} 0 & \frac{q}{2} \varepsilon^{DCBA} \\ \frac{q}{2} \varepsilon^{DCBA} & 0 \end{pmatrix}, \quad 1 \doteq \begin{pmatrix} \frac{1}{2} \delta^{[DC]}_{BA} & 0 \\ 0 & \frac{1}{2} \delta^{[BA]}_{DC} \end{pmatrix} \quad \text{satisfying} \quad \Sigma^2 = 1, \quad (2.19)$$

then the properties of the matrix elements $t^{[CB]}_u$, $t^{[CB]}_v$, $h^{[BA]}_u$, $h^{[BA]}_v$ can be written in a compact way as follows:

$$\Sigma t = t, \quad h \Sigma = h, \quad (2.21)$$

$$t h = 1 + \Sigma, \quad (h t)^v = 2 \delta^v_u. \quad (2.22)$$

These matrices serve as converters between the central charge basis (indices $u$) and the $SU(4)$ basis in the antisymmetric representation (indices $[DC]$). In particular, for the 6 vector gauge fields $v_m^u$ of the N-T multiplet there is an alternative basis, called the $SU(4)$ basis, defined by

$$\left( V_m^{[DC]} V_m^{[DC]} \right) = v_m^u \left( h^{[BA]}_u h^{[BA]}_v \right), \quad (2.23)$$

where the two components are connected by the self–duality relations\(^2\)

$$V_m^{[DC]} = \frac{q}{2} \varepsilon^{DCBA} V_m^{[BA]}. \quad (2.24)$$

Moreover, if we look at self–duality properties (2.18) as the lifting and lowering of $SU(4)$ indices with metric $\frac{q}{2} \varepsilon^{DCBA}$, then a corresponding metric in the central charge basis can be defined by

$$g_{vu} = \frac{q}{2} \varepsilon^{DCBA} h^{[DC]}_v h^{[BA]}_u, \quad g^{vu} = \frac{q}{2} \varepsilon^{DCBA} t^{[DC]}_v t^{[BA]}_u, \quad (2.25)$$

satisfying

$$g_{uv} g^{vw} = \delta^w_u. \quad (2.26)$$

These are the objects which are found to connect torsion and 3–form components containing at least one central charge index

$$H_{DCu} = T_{DC} g_{2u}, \quad T_{DC}^u = H_{DCu} g^{2u}. \quad (2.27)$$

\(^2\)Note the similarity between these self–duality relations and the reality conditions employed in the description of the $N = 4$ Yang-Mills theory [16]
insuring the soldering of the two geometries.

The four helicity 1/2 fields $T_{[CBA][\alpha]}$, $T^{[CBA]\dot{\alpha}}$ turn out to be equivalent to the fermionic partner of the graviscalar $\phi$

$$\lambda^A_{\alpha} = 2D^A_{\alpha}\phi, \quad \bar{\lambda}^{\dot{\alpha}}_A = 2D^{\dot{\alpha}}_A\phi, \quad \tag{2.28}$$

since the following duality relation holds in this $N=4$ case:

$$T_{[CBA][\alpha]} = q\varepsilon_{CBA\alpha} \quad T^{[CBA]\dot{\alpha}} = q\varepsilon^{CBA\dot{\alpha}}. \quad \tag{2.29}$$

It is the soldering mechanism between the geometry of supergravity and the geometry of the 2–form, that determines how the superfields $G$ and $P$ in the spinorial derivatives of this helicity 1/2 fields (2.13) are related to the component fields of the multiplet. In particular, we find that the superfields $G$ are related to the covariant field strength of the graviphotons $F^u_{ba}$

$$G_{(\beta\alpha)[BA]} = -2ie^{-\phi}F_{(\beta\alpha)}^{u[BA]}, \quad G^{(\dot{\beta}\dot{\alpha})[BA]} = -2ie^{-\phi}F^{(\dot{\beta}\dot{\alpha})[u_{BA}],} \quad \tag{2.30}$$

whereas the superfields $P$ contain the dual field strength of the antisymmetric tensor and the derivative of the scalar:

$$D^u_0 T^{[CBA]\dot{\alpha}} = q\varepsilon^{DCBA}P_{\delta}\dot{\alpha}, \quad \text{with} \quad P_a = 2iD_a\phi + e^{-2\phi}H^*_a - \frac{3}{4}\lambda^A\sigma_a\bar{\lambda}_A, \quad \tag{2.31}$$

$$D^{\dot{\alpha}}_0 T_{[CBA]\delta} = q\varepsilon_{DCBA}\bar{P}_{\dot{\alpha}}, \quad \text{with} \quad \bar{P}_a = 2iD_a\phi - e^{-2\phi}H^*_a + \frac{3}{4}\lambda^A\sigma_a\bar{\lambda}_A, \quad \tag{2.32}$$

where we can note that the relations

$$P_a + \bar{P}_a = 4iD_a\phi, \quad P_a - \bar{P}_a = 2e^{-2\phi}H^*_a - \frac{3}{2}\lambda^A\sigma_a\bar{\lambda}_A \quad \tag{2.33}$$

allow to separate the dual field strength of the antisymmetric tensor and the derivative of the scalar (as "real" and "imaginary" part of $P$).

Finally, let us precise that the representation of the structure group in the central charge sector is trivial, $\Phi_u^x = 0$, while the $U(4)$ part $\Phi^B_\Lambda$ of the $SL(2,\mathbb{C}) \otimes U(4)$ connection

$$\Phi^B_\Lambda = \delta^B_\Lambda\Phi^\alpha + \sigma^\alpha\Phi^B_\Lambda \quad \Phi^{\dot{\Lambda}}_\dot{\alpha} = \delta^\dot{\Lambda}_{\dot{\alpha}}\Phi^{\dot{\alpha}} - \delta^{\dot{\alpha}}_{\Lambda}\Phi_{\Lambda} \quad \tag{2.34}$$

is determined to be

$$\Phi^B_\Lambda = a^B_\Lambda + \chi^B_\Lambda, \quad \tag{2.35}$$

with $a^B_\Lambda$ pure gauge and $\chi^B_\Lambda$ a supercovariant 1–form on the superspace with components

$$\chi^B_\Lambda = \frac{1}{4}\delta^B_\Lambda \left(ie^{-2\phi}H^*_c - \frac{i}{4}\lambda^f\sigma_c\bar{\lambda}_f\right) - \frac{i}{8}(\lambda^B\sigma_c\bar{\lambda}_c), \quad \chi^{CB}_\Lambda = \frac{1}{4}\delta^B_\Lambda\chi^C_\Lambda, \quad \chi^C_\Lambda = -\frac{1}{4}\delta^B_\Lambda\chi^B_\Lambda, \quad \chi^u_\Lambda = 0. \quad \tag{2.36}$$
This situation is analogous to the case of the $16+16$ $N=1$ supergravity multiplet which is obtained from the reducible $20+20$ multiplet, described on superspace with structure group $SL(2,\mathbb{C}) \otimes U(4)$, by “breaking” the $U(1)$ symmetry [17]. By eliminating this $U(1)$ part from the $SL(2,\mathbb{C}) \otimes U(4)$ connection and putting the pure gauge part $\alpha$ to zero, one can define covariant derivatives for $SL(2,\mathbb{C})$

\[ \hat{D} u^A = Du^A - \chi_B^A \bar{u}_B \]
\[ \hat{D} u_A = Du^A + \chi_A^B \bar{u}_B \]  
(2.37) 

used in the articles [4] and [1]. Here of course $\chi_B^A$ is defined in such a way that its only non-zero components are $\chi_{\beta \alpha}^{\Lambda} = \delta_{\beta}^{\alpha} \chi^{\Lambda}_{\Lambda}$ and $\chi_{\bar{\alpha} \bar{\beta}}^{\bar{\Lambda}} = -\delta_{\bar{\alpha}}^{\bar{\beta}} \chi^{\Lambda}_{\bar{\Lambda}}$. Recall that this redefinition of the connection affects torsion and curvature components in the following way:

\[ \hat{T}_{CB}^{A} = T_{CB}^{A} - \chi_{CB}^{A} + (-)^{\bar{c}b} \chi_{BC}^{A}, \]  
(2.38) 
\[ \hat{R}_{DC}^{B \Lambda} = 0. \]  
(2.39) 

In the next section we derive the equations of motion for all the component fields of the N-T multiplet using its geometrical description presented above.

3. Equations of motion in terms of supercovariant quantities

The problem of the derivation of field equations of motion without the knowledge of a Lagrangian, using considerations on representations of the symmetry group, was considered for a long time [18], [19]. The question is particularly interesting for supersymmetric theories and there are various approaches which have been developed. Let us mention for example the procedure based on projection operators selecting irreducible representations out of superfield with arbitrary external spin [20]. About the same period Wess and Zumino suggested the use of differential geometry in superspace to reach better understanding of supersymmetric Yang-Mills and supergravity theories. The techniques used in this approach allowed to work out a new method for deriving equations of motion, namely looking to consequences of covariant constraints, which correspond to on-shell field content of a representation of the supersymmetry algebra.

In order to illustrate the method let us recall as briefly as possible the simplest example, the $N=1$ Yang-Mills theory described on superspace considering the geometry of a Lie algebra valued 1–form $A$ [16], [21]. Under a gauge transformation, parameterized by $g$, the gauge potential transforms as $A \rightarrow g^{-1}Ag - g^{-1}dg$ and its field strength $F = dA + AA$ satisfies the Bianchi identity $DF = 0$. In order to describe the off-shell multiplet one constrains the geometry by putting $F_{\alpha \beta} = F_{\alpha}^{\beta} = F^{\alpha \bar{\beta}} = 0$. Then the Bianchi identities are satisfied if and only if all the components of the field strength $F$ can be expressed in terms of two spinor superfields $W_{\alpha}$, $\bar{W}^{\dot{\alpha}}$ and their spinor derivatives:

\[ F_{\beta a} = i(\sigma_\alpha W)_{\beta}, \quad F^{\dot{\beta}}_{\alpha} = -i(\bar{\sigma}_\alpha \bar{W})^\dot{\beta}, \]  
(3.1)
and the gaugino superfields $W_\alpha$, $\bar{W}^{\dot{\alpha}}$ satisfy

$$D_\alpha \bar{W}^{\dot{\alpha}} = 0, \quad D^{\dot{\alpha}} W_\alpha = 0, \quad D^a W_\alpha = D_\alpha \bar{W}^{\dot{\alpha}}.$$ (3.4)

The components of the multiplet are thus identified as follows: the vector gauge field in the super 1–form $A \parallel = i dx^m a_m$, the gaugino component field as lowest component of the gaugino superfield $W_\alpha \parallel = -i \lambda_\alpha$, $\bar{W}^{\dot{\alpha}} = i \bar{\lambda}^{\dot{\alpha}}$, and the auxiliary field in their derivatives $D^a W_\alpha = D_\alpha \bar{W}^{\dot{\alpha}} = -2D$.

Note that the supplementary constraint

$$D^a W_\alpha = D_\alpha \bar{W}^{\dot{\alpha}} = 0$$ (3.5)

puts this multiplet on-shell. It is a superfield equation and contains all the component field equations of motion. First of all it eliminates the auxiliary field $D$ and we can derive the equations of motion for the remaining fields by successively differentiating it. We obtain the Dirac equation for the gaugino

$$D^\dot{\alpha} (D^a W_\alpha) = -2i D^{a\dot{\alpha}} W_\alpha = 0, \quad (3.6)$$

and from this we derive the relations

$$D_\beta (D^{\dot{\alpha}} W_\alpha) = -2D^{a\dot{\alpha}} F^{(\beta \alpha)} + 2i \{ W_\beta, \bar{W}^{\dot{\alpha}} \} = 0, \quad (3.7)$$

$$D^{\dot{\beta}} (D_{a\dot{\alpha}} W^{\dot{\alpha}}) = 2D_{a\dot{\alpha}} F^{(\dot{\beta} \dot{\alpha})} - 2i \{ W^{\dot{\beta}}, W_\alpha \} = 0, \quad (3.8)$$

which correspond to the well-known Bianchi identities $D_{a\dot{\beta}} F^{(\dot{\beta} \dot{\alpha})} - D^{a\dot{\beta}} F^{(\beta \alpha)} = 0$ and equations of motion $D_{a\alpha} F^{(\dot{\beta} \dot{\alpha})} + D^{a\dot{\beta}} F^{(\beta \alpha)} = 2i \{ W_\alpha, W^{\dot{\beta}} \}$ for the vector gauge field.

The case of supergravity is similar to this, the gravigino superfields $T_\parallel^{[CBA] \alpha}$, $T_\parallel^{[CBA] \dot{\alpha}}$ (or $\lambda_\alpha$, $\bar{\lambda}^{\dot{\alpha}}$ in (2.29)) play an analogous rôe to the gaugino superfields $W_\alpha$, $\bar{W}^{\dot{\alpha}}$. In order to derive the free equations of motion of component fields in a supergravity theory it is sufficient to consider only the linearized version [22], [23], [11] and the calculations are simple. Considering the full theory one obtains all the nonlinear terms which arise in equations of motion derived from a Lagrangian in component formalism.

Recall that the dimension 1 Bianchi identities in the supergravity sector imply the relations (2.13) for the spinor derivatives of the gravigino superfields. These properties can be written equivalently as

$$\sum_{DC} D^{\delta}_{D \gamma} T_{[CBA] \alpha} [CBA] \alpha = 0, \quad \sum_{DC} D^{\delta}_{D \gamma} T^{[CBA] \dot{\alpha}} = 0,$$

$$D^{\delta}_{D a \alpha} T_{[CBA] \alpha} = 0, \quad D_{D a \dot{\alpha}} T^{[CBA] \dot{\alpha}} = 0,$$

$$D_{\delta a} T_{[CBA]}^{(CBA) \alpha} - \frac{1}{4} \epsilon_{CBA} D_{\delta a} T_{[CBA]}^{(CBA) \alpha} = 0, \quad D_{(\delta a)} T_{[CBA]}^{(CBA) \dot{\alpha}} - \frac{1}{4} \epsilon_{CBA} D_{(\delta a)} T_{(CBA)}^{(CBA) \dot{\alpha}} = 0.$$ (3.10)
and they are the $N = 4$ analogues of the relations (3.3) and (3.5) satisfied by the gaugino superfield corresponding to the on-shell Yang-Mills multiplet.

Therefore, by analogy to the Yang–Mills case, the equations of motion for the gravigini, the graviphoton, the scalar and the antisymmetric tensor can be deduced from the superfield relations (2.13) by taking successive covariant spinorial derivatives.

Consider all possible spinorial derivatives of relations (2.13). They are satisfied if and only if in addition to the dimension 1 results the following relations take place:

\[
\mathcal{D}_{\gamma}^C G_{(\beta\alpha)[BA]} = \frac{1}{3} \delta_{C}^{\alpha} \left[ \frac{1}{3} \oint_{C} \mathcal{D}_{\gamma}^{E} G_{(\beta\alpha)[EF]} + i \frac{2}{3} \sum_{\beta} \epsilon_{\gamma\beta \lambda} \lambda_{\dot{\alpha}} \tilde{P}_{\alpha} \right] 
\]

\[
\mathcal{D}_{C}^{(\dot{\beta}\dot{\alpha})[BA]} = \frac{1}{3} \delta_{C}^{\dot{\alpha}} \left[ \frac{1}{3} \oint_{C} \mathcal{D}_{\beta}^{\dot{\gamma}} G_{(\dot{\beta}\dot{\alpha})[EF]} + i \frac{2}{3} \sum_{\dot{\beta}} \epsilon_{\dot{\gamma}\dot{\beta} \lambda} \lambda^{EF} P_{\dot{\alpha}} \right]
\]

Equations of motion for the helicity $1/2$ fields.

Note first, that all these relations are implied also by Bianchi identities at dim 3/2.

Secondly, note that the last equations, (3.15) and (3.16), are the Dirac equations for the helicity $1/2$ fields. The derived identity is satisfied if and only if in addition to the results obtained till dimension 3/2 the following relations take place:

\[
\mathcal{D}_{\beta\dot{\alpha}} T^{[CBA]}_\alpha = \frac{3}{2} U^{\beta\dot{\alpha}} T^{[CBA]}_\alpha 
\]

\[
\mathcal{D}^{\alpha\dot{\beta}} T^{[CBA]}_\alpha = - \frac{3}{2} U^{\alpha\dot{\beta}} T^{[CBA]}_\alpha
\]

with $U^{\beta\dot{\alpha}}_\alpha = \frac{i}{4} \left( \lambda^{\beta}_{\dot{\alpha}} \lambda^{\alpha}_{\dot{\beta}} - \frac{1}{2} \delta^{\beta}_{\dot{\alpha}} \lambda^{\alpha}_{\dot{\beta}} \right)$.

a. Consider the spinorial derivative $\mathcal{D}_{\gamma}^{\beta}$ of the Dirac equation (3.15). The derived identity is satisfied if and only if in addition to the results obtained till dimension 3/2 the following relations take place:

\[
\mathcal{D}_{\alpha\alpha} P^{\alpha\dot{\alpha}} - i \left( e^{-2\phi} H^{*}_{\alpha\dot{\alpha}} + \frac{1}{2} \lambda_{\alpha}^{\dot{\alpha}} \lambda_{\dot{\alpha}} \right) P^{\alpha\dot{\alpha}} + i q \frac{1}{2} \epsilon_{DCBA} G^{(\dot{\beta}\dot{\alpha})[DC]} G^{(\beta\alpha)[BA]} = 0
\]

\[
\sum_{\beta\dot{\alpha}} \left[ \mathcal{D}_{\beta\dot{\alpha}} P_{\alpha} \dot{\alpha} - i \left( e^{-2\phi} H^{*}_{\beta\dot{\alpha}} + \lambda_{\beta}^{\dot{\alpha}} \lambda_{\dot{\alpha}} \right) P_{\alpha} \dot{\alpha} \right] = 0.
\]
b. Consider the spinorial derivative $\mathcal{D}^\delta_D$ of (3.16). The identity is satisfied if and only if in addition to the results obtained till dimension 3/2 the following relations take place:

$$
\mathcal{D}_\alpha P^{\alpha\dot{\alpha}} + i \left( e^{-2\phi} H^*_{\alpha\dot{\alpha}} + \frac{1}{2} \lambda^\beta_{\alpha\dot{\alpha}} \bar{\lambda}^\dot{\beta}_\beta \right) \bar{P}^{\dot{\alpha}\dot{\beta}} + \frac{i q}{2} \varepsilon^{\text{DCBA}} G_{(\beta\alpha)[DC]} G^{(\beta\alpha)[BA]} = 0 \tag{3.21}
$$

$$
\sum \left[ \mathcal{D}^{\alpha\dot{\beta}} \bar{P}^{\dot{\alpha}\dot{\beta}} + i \left( e^{-2\phi} H^{*\alpha\dot{\beta}} + \lambda^\alpha_{\alpha\dot{\beta}} \bar{\lambda}^{\dot{\beta}}_\beta \right) \bar{P}^{\dot{\alpha}\dot{\beta}} \right] = 0. \tag{3.22}
$$

Equations of motion for the scalar.

Using properties (2.33) the equations of motion for the scalar can be deduced from the sum of the relations (3.19) and (3.21):

$$
2\mathcal{D}_a (\mathcal{D}^a \phi) = e^{-4\phi} H^* H^a - e^{-2\phi} H^* (\lambda^\alpha_{\alpha\dot{\beta}} \bar{\lambda}^{\dot{\beta}}_\beta) - \frac{3}{8} (\lambda^\beta_{\alpha\dot{\beta}}) (\bar{\lambda}^{\dot{\beta}}_\beta \bar{\lambda}^\beta_\beta) - \frac{1}{2} e^{-2\phi} F_{[\beta\alpha]} F^{[\beta\alpha]} \tag{3.23}
$$

This equation already shows that in the Lagrangian corresponding to these equations of motion the kinetic terms of the antisymmetric tensor and of the graviphotons are accompanied by exponentials in the scalar field.

By the way, the difference of relations (3.19) and (3.21) looks as

$$
\mathcal{D}_a H^{*a} = \frac{1}{2} e^{2\phi} (\lambda^\alpha_{\alpha\dot{\beta}} \bar{\lambda}^{\dot{\beta}}_\beta) \mathcal{D}_a^2 \phi + \frac{i}{2} F_{[\beta\alpha]} F^{[\beta\alpha]}, \tag{3.24}
$$

and it corresponds of course to the Bianchi identity satisfied by the antisymmetric tensor gauge field. The topological term $F_{[\beta\alpha]} F^{[\beta\alpha]}$ is an indication of the intrinsic presence of Chern-Simons forms in the geometry. This feature is analogous to the case of the off-shell $N = 2$ minimal supergravity multiplet containing an antisymmetric tensor [5]. It arises naturally in extended supergravity using the soldering mechanism with the geometry of a 2-form in central charge superspace.

Equations of motion for the antisymmetric tensor.

Note that relations (3.20) and (3.22) are the selfdual and respectively the anti-selfdual part of the equation of motion for the antisymmetric tensor. Putting these relations together, we obtain the equation of motion for the antisymmetric tensor:

$$
\varepsilon_{dcba} \mathcal{D}^b H^{*a} = \left[ T_{dc\lambda} \bar{\lambda}^\lambda_{\dot{\alpha}} \bar{\lambda}^{\dot{\beta}}_\beta + T_{dc\dot{\beta}} \bar{\lambda}^{\dot{\alpha}}_{\alpha} \bar{\lambda}^\dot{\beta} \right] e^{2\phi} - \frac{1}{2} H^* [\lambda^\beta_{\alpha\dot{\beta}} \bar{\lambda}^{\dot{\beta}}_\beta] 
\varepsilon_{dcba} \left[ \frac{3}{4} \mathcal{D}^b (\lambda^\alpha_{\alpha\dot{\beta}} \bar{\lambda}^{\dot{\beta}}_\beta) - (\mathcal{D}^b \phi) (\lambda^\alpha_{\alpha\dot{\beta}} \bar{\lambda}^{\dot{\beta}}_\beta) + 4 e^{-2\phi} (\mathcal{D}^b \phi) H^{*a} \right] e^{2\phi} \tag{3.25}
$$

Consider the spinorial derivative $\mathcal{D}^\delta_D$ of the Dirac equation (3.15) and the spinorial derivative $\mathcal{D}^\delta_D$ of (3.16). The identities obtained this way are satisfied if and only if in
addition to the results obtained till dimension 3/2 the following relations hold:

$$4iD_{\beta\alpha}G^{(\delta\lambda)[BA]} = q\epsilon^{BADC}(G_{(\beta\alpha)[DC]}P^{\alpha\delta} + i\bar{\lambda}_{\beta}^\alpha \bar{\lambda}_{\alpha}^\delta \bar{P})$$

$$- G^{(\delta\lambda)[BF]}\lambda^\alpha_{\beta} \lambda^\beta_{\alpha} F_{\beta\alpha} - G^{(\delta\lambda)[PA]}\lambda^\beta_{\beta} \lambda^\alpha_{\alpha} F_{\alpha\beta}$$

(3.26)

$$4iD^{\alpha\delta}G_{(\delta\alpha)[BA]} = q\epsilon_{BADC}(G^{(\beta\alpha)[DC]}\bar{P}_{\delta\alpha} + i\lambda_{\alpha}^\gamma \lambda_{\gamma}^\delta P_{\alpha\delta})$$

$$+ G_{(\delta\alpha)[BF]}\lambda^{FA}_{\alpha} \lambda^\beta_{\beta} + G_{(\delta\alpha)[PA]}\lambda^{FA}_{\alpha} \lambda^\beta_{\beta}.$$

(3.27)

**Equations of motion for the graviphotons.**

Recall that the geometric soldering mechanism between supergravity and the geometry of the 3-form implies that the fields $G_{(\beta\alpha)[BA]}$ and $G^{(\beta\alpha)[BA]}$ are related to the covariant field strength of the graviphotons, $F^a$, by (2.30). Then the previous lemma determines both the equations of motion and the Bianchi identities satisfied by the vector gauge fields of the multiplet:

$$D_\nu F^{ba\mu} = \frac{i}{2} [P_b + \bar{P}_b] F^{ba\mu} + (P_b - \bar{P}_b) F^{ba\mu}$$

$$+ \frac{i}{4} \left[ P_b (\lambda^a \sigma^{ba} \bar{\lambda}^A) t_{[BA]} u + \bar{P}_b (\lambda_B \sigma^{ba} \bar{\lambda}_A) t_{[BA]} u \right] e^\phi$$

$$- \frac{i}{4} \left[ (\lambda^\lambda \sigma^{dc} \sigma^a \bar{\lambda}_B) F_{dc} \bar{v}_\lambda^B t_{[FA]} u + (\bar{\lambda}_B \sigma^{dc} \sigma^a \lambda^B) F_{dc} \bar{v}_\lambda^B t_{[FA]} u \right],$$

(3.28)

$$D_\nu F^{ba\mu} = \frac{i}{4} \left[ P_b (\lambda^a \sigma^{ba} \bar{\lambda}^A) t_{[BA]} u - \bar{P}_b (\lambda_B \sigma^{ba} \bar{\lambda}_A) t_{[BA]} u \right] e^\phi$$

$$- \frac{i}{4} \left[ (\lambda^\lambda \sigma^{dc} \sigma^a \bar{\lambda}_B) F_{dc} \bar{v}_\lambda^B t_{[FA]} u - (\bar{\lambda}_B \sigma^{dc} \sigma^a \lambda^B) F_{dc} \bar{v}_\lambda^B t_{[FA]} u \right].$$

(3.29)

Further differentiating (3.26) and (3.27) one can obtain Bianchi identities for the gravitini and graviton, but here we would like to derive their equations of motion instead.

**Equations of motion for the gravitini.**

Unlike the equations of motion presented above, the equations of motion for the gravitini and the graviton are directly given by the superspace Bianchi identities, once the component fields are identified. For example, the Bianchi identities at dim 3/2 determine the torsion components

$$T_{(\beta\alpha)\lambda} = \frac{1}{16} \bar{P}_{\beta\alpha} \bar{\lambda}^\lambda$$

$$T^{(\delta\gamma)\beta} = \frac{1}{8} \bar{P}_{\beta} (\delta \gamma) + \frac{iq}{8} \epsilon^{DCBA} G^{(\delta\gamma)[CB]} \lambda^A_{\beta},$$

(3.30)

$$T^{(\delta\alpha)\lambda} = \frac{1}{16} P^{\alpha\beta} \lambda^\lambda$$

$$T^{(\delta\gamma)\beta} = \frac{1}{8} P_{(\delta \gamma)} \lambda^\gamma_{\beta} + \frac{iq}{8} \epsilon^{DCBA} G^{(\delta\gamma)[CB]} \lambda^\beta_{\lambda},$$

(3.31)
and these components are sufficient to give the equations of motion for the gravitini:
\[
\varepsilon^{dcba}(\tilde{\sigma}_c T_{ba}) = \frac{i}{4} \left( \tilde{\lambda}_d \sigma^d \tilde{\sigma}^c \epsilon \right)^\alpha \bar{P}_c + \frac{i}{2} (\bar{\sigma}^{ba} \sigma^d \tilde{\lambda}^e) \bar{F}_{e[\alpha]} e^{-\phi}, \tag{3.32}
\]
\[
\varepsilon^{dcba}(\sigma_c T_{ba})_\alpha = -\frac{i}{4} (\lambda^c \sigma^d \sigma^e \epsilon)_\alpha P_c - \frac{i}{2} (\bar{\sigma}^{ba} \sigma^d \lambda^e)_\alpha F_{ba}[\alpha] e^{-2\phi}. \tag{3.33}
\]

**Equations of motion for the graviton.**

In order to give the equations of motion for the graviton we need the expression of the supercovariant Ricci tensor, \( R_{db} = R_{dbca} f^a \), which is given by the superspace Bianchi identities at canonical dimension 2 (A.5). The corresponding Ricci scalar, \( R = R_{db} \eta^{db} \), is then
\[
R = -2 D^a \phi D_a \phi - \frac{1}{2} H^a H^*_a e^{-4\phi} + \frac{3}{4} e^{-2\phi} H^* H^a (\lambda^c \sigma^d \tilde{\lambda}_e) + \frac{3}{8} (\lambda^B \lambda^A) (\tilde{\lambda}_B \tilde{\lambda}_A). \tag{3.34}
\]

The knowledge of these ingredients allows us to write down the Einstein equation
\[
R_{db} - \frac{1}{2} \eta_{db} R = -2 \left[ D_d \phi D_b \phi - \frac{1}{2} \eta_{db} D^a \phi D_a \phi \right] \\
- \frac{1}{2} e^{-4\phi} \left[ H_d^* H_b^* - \frac{1}{2} \eta_{db} H^* H^a H^*_a \right] \\
- e^{-2\phi} \left[ F_{df[\alpha]} F_{bf[\alpha]} - \frac{1}{4} \eta_{db} F_{ef[\alpha]} F^{ef[\alpha]} \right] \\
- \frac{i}{8} \sum_{db} [\lambda^c \sigma_d D_b \tilde{\lambda}_e - (D_b \lambda^c) \sigma_d \tilde{\lambda}_e] \\
- \frac{1}{8} \left[ \frac{1}{4} (\lambda^c \sigma_d \tilde{\lambda}_e) (\lambda^d \sigma_b \tilde{\lambda}_a) + \eta_{db} (\lambda^B \lambda^A) (\tilde{\lambda}_B \tilde{\lambda}_A) \right] \\
- \frac{1}{8} e^{-2\phi} \left[ H^*_d (\lambda^c \sigma_b \tilde{\lambda}_e) - 3 \eta_{db} H^* H^a (\lambda^c \sigma_e \tilde{\lambda}_a) \right], \tag{3.35}
\]

where one may recognize on the right-hand-side the usual terms of the energy-momentum tensor corresponding to matter fields: scalar fields, antisymmetric tensor, photon fields and spinor fields respectively. As it will be shown by (4.12), the contribution of the gravitini is hidden in \( R_{db} \).

**4. Equations of motion in terms of component fields**

In the previous section we calculated the equations of motion for all component fields of the N-T multiplet (graviton (3.35), gravitini (3.32), (3.33), graviphotons (3.28), 1/2-spin fields (3.17), (3.18), scalar (3.23) and the antisymmetric tensor (3.25)) in terms of supercovariant objects, which have only flat (Lorentz) indices. In order to write these equations of motion in terms of component fields, one passes to curved (Einstein) indices by the standard way [24]. General formulas are easily written using the notation \( E^A = e^A = dx^m e_m^A \) [21].
4.1 Supercovariant→component toolkit

Recall that the graviton, gravitini and graviphotons are identified in the super-vielbein. Thus, their field strength can be found in their covariant counterparts using

\[ T^A = \frac{1}{2} dx^m dx^n (D_n e_m^A - D_m e_n^A) = \frac{1}{2} e^B e^C T_{CB}^A. \]  

(4.1)

For \( A = a \) one finds the relation

\[ D_n e_m^a - D_m e_n^a = i \psi_{[na]} \sigma^a \tilde{\psi}_{m]^A}, \]

(4.2)

which determinates the Lorentz connection in terms of the vierbein, its derivatives and gravitini fields. For \( A = \hat{A} \) and \( \breve{A} \), we have the expression of the covariant field strength of the gravitini

\[ T_{cbA}^\alpha = e_b^m e_c^m D_{[\psi_{m]}_{\psi_{m}\hat{A}}} - e_b^m e_c^m \frac{q}{4} e_{DCBA} \tilde{\psi}_{n} \tilde{\psi}_{m}^C \bar{\lambda}_{AB} - \frac{i}{2} e_{c}^m n (\tilde{\psi}_{n} \hat{\bar{\sigma}}_{[b]} \sigma_{d]}^a)_{\hat{A}} F_{da_{[nA]} | e^{\phi}} \]

\[ + \frac{i}{4} (\psi_{m} \sigma_{f_{[b]}^a} e_{c}^n) n [\lambda^{f \bar{\lambda}}_{A} \sigma^{f \bar{\lambda}}_{B} - \frac{1}{2} \delta^{f \bar{\lambda}}_{A} \sigma^{f \bar{\lambda}}_{B}]. \]

(4.3)

As for \( A = u \), the central charge indices, we obtain the covariant field strength of the graviphotons

\[ F_{ba}^u = e_b^m e_a^m F_{nm}^u + e_b^m e_a^m [\tilde{\psi}_{n} C \tilde{\psi}_{m}^B + i \tilde{\psi}_{[nc]} C \bar{\sigma}_{m]} \bar{\lambda}] e^{\phi} \epsilon_{[cb]} u \]

\[ + e_b^m e_a^m \psi_{nc} \psi_{mb} + i \psi_{[nc]} \bar{\sigma}_{m]} \bar{\lambda} \]  

(4.5)

with \( F_{nm}^u \) the field strength of the graviphotons \( F_{nm}^u = \partial_m v_n^u - \partial_n v_m^u \). In the \( SU(4) \) basis this becomes

\[ F_{ba}^{[BA]} = e_b^n e_a^m F_{nm}^{[BA]} + e_b^n e_a^m [\psi_{n} \psi_{m}^B + i \psi_{[n]} \bar{\sigma}_{m]} \bar{\lambda}] e^{\phi} \]

\[ + e_b^n e_a^m \frac{q}{2} e_{DCBA} [\psi_{nD} \psi_{mc} + i \psi_{[nD]} \bar{\sigma}_{m]C}] e^{\phi} ; \]

(4.6)

with the field strength \( F_{nm}^{[BA]} = F_{nm}^u u_n^{[BA]} = \partial_m V_n^{[BA]} - \partial_n V_m^{[BA]} \).

Since the antisymmetric tensor is identified in the 2–form, the development of its covariant field strength on component fields is deduced using

\[ H = \frac{1}{2} dx^m dx^n dx^k \partial_k b_{nm} = \frac{1}{3!} e^A e^B e^C H_{CB}^A. \]

(4.7)
and one finds

$$H^{\ast a} = e_i^a G^l + ie_i^a \left[ \psi_{kF} \sigma^{lk} \lambda^F - \bar{\psi}^k \sigma^{lk} \bar{\lambda}_F + \frac{1}{2} \varepsilon^{knm} \psi_{kF} \sigma_n \bar{\psi}_m \right] e^{2\phi}, \quad (4.8)$$

with

$$G^l = \frac{1}{2} \varepsilon^{knm} \left[ \partial_k b_{nm} - v_k u_{uv} F_{nm} \right] = \frac{1}{2} \varepsilon^{knm} \left[ \partial_k b_{nm} - V_{k[BA]} F_{nm} \right]. \quad (4.9)$$

Note, that the dual field strength, $\frac{1}{2} \varepsilon^{knm} \partial_k b_{nm}$, of the antisymmetric tensor appears in company with the Chern-Simons term $\frac{1}{2} \varepsilon^{knm} v_k u_{uv} F_{nm}$. We use the notation $G^l$ in order to accentuate this feature. Recall also, that one of the fundamental aims of the article [4] was to explain in detail that this phenomenon is quite general and arises as an intrinsic property of soldering in superspace with central charge coordinates.

The lowest component of the derivative of the scalar can be calculated using $D\phi \mid x = d x^m D_m \phi = e^A D_A \phi$, and it is

$$D_a \phi = e_a^m \left( D_m \phi - \frac{1}{4} \bar{\psi}^m \lambda^F - \frac{1}{4} \bar{\psi}^m \bar{\lambda}_F \right), \quad (4.10)$$

while the lowest component of the double derivative $D_a D^a \phi \mid x$, needed for the expansion of the equation of motion for the scalar (3.23), becomes

$$2 D_a D^a \phi = 2 \Box \phi + e_a^m D_m e^a \left[ 2 D_a \phi - \frac{1}{2} \psi^m \lambda^F - \frac{1}{2} \bar{\psi}^m \bar{\lambda}_F \right] - \frac{1}{2} H^{\ast a} \left( \psi^m \sigma_a \bar{\psi}^m \right) e^{-2\phi}$$

$$- \frac{1}{2} D^m (\psi^m \lambda^F + \bar{\psi}^m \bar{\lambda}_F) - \frac{1}{2} (\psi^m D^m \lambda^F + \bar{\psi}^m D^m \bar{\lambda}_F)$$

$$- \frac{3i}{32} \left( \lambda^C \sigma^n \bar{\lambda}_C \right) (\psi^m \lambda^F - \bar{\psi}^m \bar{\lambda}_F) - \frac{3}{4} (\psi^m \lambda^F)(\bar{\psi}^m \bar{\lambda}_F)$$

$$- \frac{1}{4} (\bar{\psi}^m \lambda^F)(\lambda^C \sigma^n \bar{\lambda}_C) + \frac{1}{2} (\psi^m \lambda^F)(\lambda^C \sigma^n \bar{\lambda}_C) - \frac{1}{4} (\bar{\psi}^m \bar{\lambda}_F)(\bar{\lambda}^C \sigma^n \bar{\lambda}_C)$$

$$+ F_{ba[FC]} \int \left[ \frac{1}{2} \bar{\psi}^m \sigma^{ba} \psi^m + \frac{i}{4} \bar{\psi}^m \sigma^n \sigma^{ba} \lambda^F \right] e^{-\phi}$$

$$+ F_{ba[FC]} \left[ \frac{i}{2} \psi^m \sigma^{ba} \psi^m + \frac{i}{4} \psi^m \sigma^n \sigma^{ba} \bar{\lambda}_F \right] e^{-\phi}. \quad (4.11)$$

In order to compare our results with the component expression of the scalar’s equation of motion derived from [1], we have to replace in this expression $e_a^m D_m e^a$ with

$$e_a^m D_m e^a = V^{-1} \partial_m (V g^{mn}) - ig^{mn} \psi_{[ma} \sigma^k \bar{\psi}_{k]}^c,$$

a consequence of (4.2).

Finally, using $R_{ab} \mid x = \frac{1}{2} d x^m d x^n R_{nmab} \mid x = \frac{1}{2} e^c \epsilon^c R_{CBab} \mid x$, one obtains for the lowest component of the covariant Ricci tensor $R_{db}$ the expression
\[ R_{db} = \frac{1}{2} \sum_{db} \left\{ e_d^a e_m^a R_{nmba} + \frac{1}{2} \varepsilon_{eb}^e m e f \psi_{mbd} \sigma_{d} T_{ef} D - \frac{1}{2} \varepsilon_{eb}^e m e f \psi_{mb}^D \sigma_{d} T_{ef} D \right\} + \frac{1}{4} \left( i \psi_{D}^m \sigma_{d} \sigma_{b m} \lambda^{D} - i e_d^m \delta_{b}^f \psi_{nbD} \lambda^{D} \right) P_f + \frac{1}{4} \left( \bar{\psi}_{D}^m \sigma_{d} \sigma_{b m} \bar{\lambda}^{D} - i e_d^m \delta_{b}^f \bar{\psi}_{nbD} \bar{\lambda}^{D} \right) \bar{P}_f - \frac{1}{2} e^{-\phi} F_{ef}^{\left[ pf \right]} \left( i \text{tr} \left( \sigma_{m b} \sigma_{e f} \right) \psi_{D}^m \sigma_{d} \bar{\lambda}^{F} + \frac{i}{2} e_d^m \psi_{nbD} \sigma_{e f} \sigma_{b} \bar{\lambda}^{F} \right) - \frac{1}{2} e^{-\phi} F_{ef}^{\left[ pf \right]} \left( i \text{tr} \left( \bar{\sigma}_{m b} \bar{\sigma}_{e f} \right) \bar{\psi}_{D}^m \sigma_{d} \lambda^{F} + \frac{i}{2} e_d^m \bar{\psi}_{nbD} \bar{\sigma}_{e f} \bar{\sigma}_{b} \lambda^{F} \right) + \frac{1}{2} e_d^m e^{-\phi} \left( \text{tr} \left( \bar{\sigma}_{b}^m \bar{\sigma}_{e f} \right) \psi_{mbD} \psi_{mC} F_{ef}^{\left[ pd \right]} + \text{tr} \left( \sigma_{b}^m \sigma_{e f} \right) \bar{\psi}_{mbD} \bar{\psi}_{mC} F_{ef}^{\left[ pd \right]} \right) - \frac{1}{2} e_d^m \left( \sigma_{b D}^D \sigma_{C} - 1 \right) \left[ \left( \psi_{mbD} \sigma_{b} \lambda^{B} \right) \left( \bar{\psi}_{m}^A \bar{\lambda}^{B} \right) - \left( \psi_{mbD} \lambda^{B} \right) \left( \bar{\psi}_{m}^A \sigma_{b} \bar{\lambda}^{B} \right) \right] \right\} \tag{4.12}

4.2 The equations of motion

In the last subsection we deduced the expression of all quantities appearing in the supercovariant equations of motion in terms of component fields. We are therefore ready now to replace these expressions in (3.35), (3.32), (3.33), (3.28), (3.17), (3.18), (3.23), (3.25) and give the equations of motion in terms of component fields.

It turns out that the expressions

\[ \hat{H}_t = e_t^a H_a \left| - \frac{i}{2} e^{2\phi} \psi_{tA} \lambda^{A} + \frac{i}{2} e^{2\phi} \bar{\psi}_{tA} \bar{\lambda}^{A} - \frac{3}{4} e^{2\phi} \lambda^{A} \sigma_{t} \bar{\lambda}^{A} \right. \]

\[ = G_t + \frac{i}{2} e^{2\phi} \left[ \psi_{kF} \sigma_{t} \bar{\sigma}^{k} \bar{\lambda}^{F} - \bar{\psi}_{kF} \sigma_{t} \bar{\sigma}^{k} \lambda^{F} + \varepsilon_{k m n} \psi_{kF} \sigma_{n} \bar{\psi}_{m} \right] - \frac{3}{4} e^{2\phi} \lambda^{A} \sigma_{t} \bar{\lambda}^{A} \tag{4.13} \]

and

\[ \hat{F}_{nk} = e_{nk}^{ba} F_{ba}^{z} \left| + \frac{i}{2} \varepsilon_{nkml} \left[ \psi_{mD} \bar{\psi}_{lC} - i \frac{q}{2} \varepsilon_{DCBA} \bar{\lambda}_{B} \sigma_{m} \bar{\psi}_{lA} \right] e^{\phi t_{[DC]}^{z}} \right. \]

\[ \left. - \frac{i}{2} \varepsilon_{nkml} \left[ \psi_{mD} \psi_{lC} - i \frac{q}{2} \varepsilon_{DCBA} \lambda^{B} \sigma_{m} \psi_{lA} \right] e^{\phi t_{[DC]}^{z}} \right. \]

\[ = \hat{F}_{nk} - \text{tr} \left( \sigma^{nk} \sigma^{ml} \right) \left[ \psi_{mD} \bar{\psi}_{lC} - i \frac{q}{2} \varepsilon_{DCBA} \bar{\lambda}_{B} \sigma_{m} \bar{\psi}_{lA} \right] e^{\phi t_{[DC]}^{z}} \]

\[ \left. - \text{tr} \left( \bar{\sigma}^{nk} \bar{\sigma}^{ml} \right) \left[ \psi_{mD} \psi_{lC} - i \frac{q}{2} \varepsilon_{DCBA} \lambda^{B} \sigma_{m} \psi_{lA} \right] e^{\phi t_{[DC]}^{z}} \right. \tag{4.14} \]

appear systematically, and using them, the equations take a quite simple form. Let us also denote the quantity \( \hat{F}_{nk}^{z} = e_{n}^{b} e_{k}^{a} F_{ba}^{z} \), which is called the supercovariant field strength of the graviphotons in the component approach [25], [1].
Equations of motion for the helicity 1/2 fields.

\[
\left( \sigma^m \hat{D}_m \bar{\lambda}_\lambda \right)_\beta = -ie^{-2\phi} \bar{H}_m \left[ \frac{i}{2} (\sigma^n \sigma_m \psi_{n\lambda})_\beta - \frac{3}{4} (\sigma_m \bar{\lambda}_\lambda)_\beta \right] - \frac{i}{2} (\overline{\psi}_n \sigma^n \psi_{n\lambda})_\beta
\]

\[
+ i \partial_n \phi (\sigma^n \bar{\sigma}^m \psi_{m\lambda})_\beta - e^{-\phi} \hat{F}_{kl[FA]} (\sigma^m \bar{\sigma}^n \psi_{n\lambda})_\beta - \frac{3i}{8} (\bar{\lambda}_\lambda \lambda^F)_\beta \]  

(4.15)

Equations of motion for the gravitini.

\[
\varepsilon^{lknm} (\bar{\sigma}_k \hat{D}_n \psi_{m\lambda})^{\dot{\alpha}} = -\frac{i}{4} e^{-2\phi} \bar{H}_n \left[ \varepsilon^{lknm} (\bar{\sigma}_k \psi_{m\lambda})^{\dot{\alpha}} + (\bar{\lambda}_\lambda \bar{\sigma}^l \sigma^n \varepsilon)^{\dot{\alpha}} \right] - \frac{1}{2} \partial_n \phi (\bar{\lambda}_\lambda \bar{\sigma}^l \sigma^n \varepsilon)^{\dot{\alpha}}
\]

\[
- e^{-\phi} \hat{F}_{mn[AF]} \left[ \text{tr}(\sigma^l \sigma^m \psi_k \sigma^\dot{\alpha}) + \frac{i}{2} \text{tr}(\bar{\sigma}^l \sigma^m \sigma^\dot{\alpha}) \right]
\]

\[
+ \frac{1}{8} \psi_{n\lambda} \lambda^F (\bar{\sigma}^l \bar{\sigma}^m \bar{\psi}^\dot{\alpha}) + \frac{3}{8} (\psi_{n\lambda} \sigma^l \sigma^m \bar{\psi}^\dot{\alpha}) - \frac{1}{4} (\psi_{n\lambda} \sigma^l \bar{\sigma}^m \bar{\psi}^\dot{\alpha})
\]

\[
+ \frac{q}{4} \varepsilon^{lknm} \varepsilon_{CBFA} \psi_k \gamma^C \psi_m \lambda^B (\bar{\sigma}_k \lambda^F)^{\dot{\alpha}} 
\]  

(4.16)

Equations of motion for the scalar.

\[
0 = 2V^{-1} \partial_m (V g^{mn} \partial_n \phi) + \frac{1}{2} V^{-1} \partial_m \left( V \lambda^A \sigma^n \sigma^m \psi_{n\lambda} + V \bar{\lambda}_\lambda \bar{\sigma}^n \sigma^m \bar{\psi}_n^\lambda \right)
\]

\[
- e^{-4\phi} G^m \bar{H}_m + \frac{1}{2} e^{-2\phi} F_{mn[BA]} \bar{F}^{mn[BA]} 
\]  

(4.17)

Equations of motion for the antisymmetric tensor.

\[
\partial_k \left( e^{-4\phi} V \varepsilon^{mnkl} \bar{H}_l \right) = 0 
\]  

(4.18)

Equations of motion for the graviphotons.

\[
\partial_n \left( V e^{-2\phi} \bar{F}_{nkU} \right) = \frac{1}{2} V e^{-4\phi} \varepsilon^{lnmk} \bar{H}_l \bar{F}_{mnU} 
\]  

(4.19)

Equations of motion for the graviton.

The Einstein equation in terms of component fields is also deduced in a straightforward manner from (3.35) and (4.12) with the usual Ricci tensor

\[
\mathcal{R} = \frac{1}{2} \varepsilon^{lknm} \psi_{lA} \sigma_k \hat{D}_n \bar{\psi}_{m\lambda} - \frac{1}{2} \varepsilon^{lknm} \bar{\psi}_l \bar{\sigma}_k \hat{D}_n \psi_{m\lambda} 
\]
\[-i\frac{\lambda^\Lambda}{4} \bar{\sigma}^m \bar{D}_m \bar{\lambda}_\Lambda - i\frac{\bar{\lambda}^\Lambda}{4} \sigma^m \bar{D}_m \lambda^\Lambda - 2 \partial^m \phi \partial_m \phi \]

\[-e^{-\phi} \tilde{F}_{kl}^{[nA]} \left( \text{tr}(\bar{\sigma}^{kl} \bar{\sigma}^{mn}) \psi_{mb} \psi_{nA} + \frac{i}{2} \text{tr}(\sigma^{kl} \sigma^{mn}) \psi_{mb} \sigma_n \bar{\lambda}_\Lambda \right) \]

\[-e^{-\phi} \tilde{F}_{kl}^{[nA]} \left( \text{tr}(\sigma^{kl} \sigma^{mn}) \bar{\psi}_m \bar{\psi}_n \Lambda + \frac{i}{2} \text{tr}(\bar{\sigma}^{kl} \bar{\sigma}^{mn}) \bar{\psi}_m \bar{\sigma}_n \lambda^\Lambda \right) \]

\[-\frac{1}{2} e^{-4\phi} \tilde{H}^l \left( G_l + \frac{i}{2} e^{2\phi} (\psi_{kF} \sigma_l \sigma^k \lambda^F - \bar{\psi}_k^F \bar{\sigma}_l \sigma^k \bar{\lambda}_F + 2 \varepsilon^{lkmn} \psi_{kF} \sigma_n \bar{\psi}_m^F) \right) \]

\[+\frac{1}{2} (\psi_{lA} \lambda^A)(\bar{\psi}_{lA} \bar{\lambda}_\Lambda) + \frac{i}{2} (\lambda^F \sigma^l \bar{\lambda}_F)(\psi_{lA} \lambda^A - \bar{\psi}_{lA} \bar{\lambda}_\Lambda) \]

\[-\frac{3i}{16} \varepsilon^{lmnk} (\psi_{kF} \sigma_m \bar{\psi}_n^F)(\lambda^A \sigma_k \bar{\lambda}_\Lambda) - \frac{i}{2} \varepsilon^{lmnk} (\psi_{kF} \sigma_m \bar{\psi}_n^A)(\lambda^F \sigma_k \bar{\lambda}_\Lambda) \] (4.20)

5. Conclusion

The aim of this article was to deduce the equations of motion for the components of the N-T multiplet from its geometrical description in central charge superspace, and compare these equations with those, deduced from the Lagrangian of the component formulation of the theory with the same field content [1].

We showed that the constraints on the superspace which allow to identify the components in the geometry imply equations of motion in terms of supercovariant quantities. Moreover, we succeeded in writing these equations of motion in terms of component fields in an elegant way, using the objects \( \tilde{H}_m \) and \( \tilde{F}_{mn}^u \). The equations found this way are in perfect concordance with the ones deduced from the Lagrangian of Nicolai and Townsend [1]. This result resolves all remaining doubt about the equivalence of the geometric description on central charge superspace of the N-T multiplet and the Lagrangian formulation of the theory with the same field content.

As a completion of this work one may ask oneself about an interpretation of the objects \( \tilde{H}_m \) and \( \tilde{F}_{mn}^u \), which seem to be some natural building blocks of the Lagrangian. Concerning this question let us just remark the simplicity of the relation

\[-i \chi_m^\Lambda = e^{-2\phi} \tilde{H}_m + \frac{3}{8} \lambda^\Lambda \sigma_m \bar{\lambda}_\Lambda \] (5.1)

between \( \tilde{H}_m \) and the \( U(1) \) part of the initial connection (2.36) of the central charge superspace with structure group \( SL(2, \mathbb{C}) \otimes U(4) \).

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A. Solution of the Bianchi Identities

The conventional constraints compatible with the assumptions (2.6 - 2.7) and (2.12) are the following:

\[
\begin{align*}
T^{C}_{\gamma b}a & = 0 \quad T^\gamma_{cb}a = 0 \\
T^{CB\alpha}_{\gamma \beta \Lambda} & = 0 \quad T^{\gamma \beta \Lambda}_{CB\alpha} = 0 \\
T^{C\beta \Lambda}_{\gamma \beta \Lambda} & = 0 \quad T^{\gamma \beta \Lambda}_{C\beta \Lambda} = 0 \\
T^{C\alpha}_{ca\Lambda} & = 0 \quad T^{\alpha}_{ca\Lambda} = 0 \\
T^{a}_{cb} & = 0.
\end{align*}
\] (A.1)

There is a particular solution of the Bianchi identities for the torsion and 3-form subject to the constraints (2.6 - 2.9), (2.11 - 2.12) and (A.1), which describes the N-T supergravity multiplet. Besides the constant \(T^{B\alpha}_{C\beta}\) and the supercovariant field strength of the graviphotons, \(T^{u}_{cb} = F_{cb}u\), the non-zero torsion components corresponding to this solution are then the following:

\[
\begin{align*}
T^{CBu}_{\gamma \beta} &= 4\epsilon_{\gamma \beta} t^{[CB]} u e^\phi \\
T^{CB\alpha}_{\gamma \beta \Lambda} &= q^\epsilon_{\gamma \beta} t^{CB\Lambda} \lambda_{\epsilon \Lambda} \\
T^{C\beta u}_{\gamma} &= i\epsilon_{\gamma} \sigma_{\beta \lambda} \gamma^i t_{[AC]} u \\
T^{C\alpha}_{\gamma \beta \Lambda} &= -2(\sigma_{\beta \Lambda})_{\gamma} \sigma^{\alpha} U^{AC}_{a\Lambda} \\
T^{C\Lambda}\alpha_{\beta \Lambda} &= \frac{i}{2}(\sigma_{\gamma} \sigma^{de})_{\gamma \alpha} F_{de[CA]} e^{-\phi} \\
T^{C\Lambda}\alpha_{b\alpha} &= \frac{i}{2}(\sigma_{\gamma} \sigma^{de})_{\gamma \alpha} F_{de[CA]} e^{-\phi} \\
T^{CB\alpha}_{\gamma \beta \Lambda} &= -2(\sigma_{ba})_{\gamma} \sigma^{\alpha} U^{AC}_{a\Lambda} \\
-\frac{1}{4} \text{tr}(\sigma_{ba} e_f) F_{af} &\left[ A F \right] \lambda_{\epsilon}^\Lambda \\
-\frac{1}{12} \left( \delta_\epsilon^f - \frac{i}{2} e_\epsilon^f a^f \right) P_a(\sigma_f \bar{\lambda}_\Lambda)_{\alpha} &= -\frac{1}{12} \left( \delta_\epsilon^f - \frac{i}{2} e_\epsilon^f a^f \right) P_a(\sigma_f \bar{\lambda}_\Lambda)_{\alpha}
\end{align*}
\] (A.2)

with \(U_{a \Lambda} = -\frac{i}{8}(\lambda^a \sigma_a \bar{\lambda}_\Lambda - \frac{1}{2} \delta^a_\Lambda \lambda^a \sigma_a \bar{\lambda}_\Lambda)\), \(P\) and \(\bar{P}\) are given in equations (2.31) and (2.32), while \(\Sigma_{(\gamma \beta \Lambda)}\) and \(\Sigma^{(\gamma \beta \Lambda)}\) are the gravitino "Weyl" tensors.

Furthermore, the Lorentz curvature has components

\[
\begin{align*}
R^0_{\Lambda \gamma ba} &= 2\epsilon_{\Lambda \gamma} \text{tr}(\sigma_{de} \sigma_{ba}) F_{de[DC]} e^{-\phi} \\
R^0_{\Lambda \gamma ba} &= -4\epsilon_{dcba} U^{[CD]} \text{tr}(\sigma^\epsilon) \gamma^\Lambda \\
R^0_{\Lambda \gamma ba} &= -2i(\sigma_\Lambda)_{\delta \Lambda} (\epsilon_{de \Lambda})_{\gamma} \Sigma^{(\gamma \beta \Lambda)} \\
-\frac{1}{2} \text{tr}(\sigma_{ba} e_f)(\sigma_f \bar{\lambda}_\Lambda) &\left[ A F \right] e^{-\phi} \\
+\frac{1}{4}(\sigma_\Lambda \bar{\sigma}_{de \Lambda} \lambda^D)_\delta P_e &= -\frac{1}{2} \text{tr}(\sigma_{ba} e_f)(\sigma_f \bar{\lambda}_\Lambda) \delta e^{[DA]} e^{-\phi} \\
R^0_{\Lambda \gamma ba} &= -2i(\sigma_\Lambda)_{\delta \Lambda} (\epsilon_{de \Lambda})_{\gamma} \Sigma^{(\gamma \beta \Lambda)} \\
-\frac{1}{2} \text{tr}(\sigma_{ba} e_f)(\sigma_f \bar{\lambda}_\Lambda) &\left[ A F \right] e^{-\phi} \\
+\frac{1}{4}(\sigma_\Lambda \bar{\sigma}_{de \Lambda} \lambda^D)_\delta P_e &= -\frac{1}{2} \text{tr}(\sigma_{ba} e_f)(\sigma_f \bar{\lambda}_\Lambda) \delta e^{[DA]} e^{-\phi}
\end{align*}
\] (A.3)
and

\[ R_{dcba} = (\epsilon \sigma_{dc})^{\delta \gamma} (\epsilon \sigma_{ba})^{\beta \alpha} V^{(\delta \gamma \beta \alpha)} + (\epsilon \sigma_{dc})^{\delta \gamma} (\epsilon \sigma_{ba})^{\beta \bar{\alpha}} V^{(\delta \gamma \beta \bar{\alpha})} + \frac{1}{2} (\eta_{db} R_{ca} - \eta_{da} R_{cb} + \eta_{ca} R_{db} - \eta_{cb} R_{da}) - \frac{1}{6} (\eta_{db} \eta_{ca} - \eta_{da} \eta_{cb}) R \]  

(A.4)

with the supercovariant Ricci tensor, \( R_{db} = R_{dcba} \eta^{ba} \), given by

\[
R_{db} = -2D_d \phi D_b \phi - \frac{1}{2} e^{-4\phi} H^*_d H^*_b e^{-2\phi} F_{df[a\Lambda]} F_{b}^{f[b\Lambda]} + \frac{1}{4} \eta_{db} e^{-2\phi} F_{e[f[a\Lambda]} F^{e[f[b\Lambda]} \\
+ \frac{1}{8} \sum_{db} \left\{ i(D_b \lambda^p) \sigma_d \bar{\lambda}_p - i \lambda^p \sigma_d D_b \bar{\lambda}_p + e^{-2\phi} H^*_d (\lambda^p \sigma_b \bar{\lambda}_p) \right\} \]

\[
- \frac{1}{32} (\lambda^p \sigma_d \bar{\lambda}_p) (\lambda^q \sigma_b \bar{\lambda}_q) - \frac{1}{16} \eta_{db} (\lambda^p \lambda^q) (\bar{\lambda}_p \bar{\lambda}_q) \]

(A.5)

and the corresponding Ricci scalar, \( R = R_{db} \eta^{db} \), which is then

\[
R = -2D^a \phi D_a \phi - \frac{1}{2} H^*^a H'^*_a e^{-4\phi} + \frac{3}{4} e^{-2\phi} H^*^a (\lambda^p \sigma_a \bar{\lambda}_p) + \frac{3}{8} (\lambda^B \lambda^A) (\bar{\lambda}_B \bar{\lambda}_A). 
\]  

(A.6)

The tensors \( V^{(\delta \gamma \beta \alpha)} \) and \( V^{(\delta \gamma \beta \bar{\alpha})} \) are components of the usual Weyl tensor. Like the gravitino Weyl tensors, \( \Sigma_{(\gamma \beta \alpha)\Lambda} \) and \( \Sigma^{(\gamma \beta \bar{\alpha})\Lambda} \), their lowest components do not participate in the equations of motion.

As for the 2–form sector, besides the supercovariant field strength of the antisymmetric tensor, \( H_{cba} \), the non-zero components of the 3–form \( H \), which do not have central charge indices, are

\[
H^{\gamma \beta \alpha}_{C} = -2i \delta^{\gamma}_{C} (\sigma_{\alpha} \epsilon)_{\beta} \bar{\epsilon} e^{2\phi} \quad H^{C}_{\gamma ba} = 4(\sigma_{ba} \lambda^{C})_{\gamma} e^{2\phi} \quad H^{\gamma}_{Cba} = 4(\bar{\sigma}_{ba} \bar{\lambda}_{C})_{\gamma} e^{2\phi}. 
\]  

(A.7)

The components with at least one central charge index, are related to the torsion components by

\[
H_{DCu} = T_{DC}^{\gamma} g_{zu}, \]

(A.8)

with the metric \( g_{zu} \) defined in (2.25).

References

[1] H. Nicolai and P. K. Townsend, \textit{N = 1 supersymmetry multiplets with vanishing trace anomaly : building blocks of the N < 3 supergravities}, Phys. Lett. \textbf{98B} (1981) 257–260.

[2] S. Gates, Jr., \textit{On-shell and conformal N=4 supergravity in superspace}, Nucl. Phys. \textbf{B213} (1983) 409–444.

[3] S. Gates, Jr. and J. Durachta, \textit{Gauge two-form in D=4, N=4 supergeometry with SU(4) supersymmetry}, Mod. Phys. Lett. \textbf{A4} (1989) 2007.
[4] R. Grimm, C. Herrmann, and A. Kiss, \textit{N=4 supergravity with antisymmetric tensor in central charge superspace}, Class. Quant. Grav. \textbf{18} (2001) 1027–1038. hep-th/0009201.

[5] G. Akemann, R. Grimm, M. Hasler, and C. Herrmann, \textit{N=2 central charge superspace and a minimal supergravity multiplet}, Class. Quant. Grav. \textbf{16} (1999) 1617–1623. hep-th/9812026.

[6] G. Girardi, R. Grimm, M. Müller, and J. Wess, \textit{Superspace geometry and the minimal, non minimal, and new minimal supergravity multiplets}, Z. Phys. \textbf{C26} (1984) 123–140.

[7] M. Müller, \textit{Natural Constraints for Extended Superspace}, Z. Phys. \textbf{C 31} (1986) 321–325.

[8] A. Kiss, \textit{Formulation géométrique des théories de supergravité N=4 et N=8 en superespace avec charges centrales}. PhD thesis, Université de la Méditerranée Aix-Marseille II, december, 2000. CPT-2000/P.4106.

[9] L.-L. Chau and C.-S. Lim, \textit{Geometrical constraints and equations of motion in extended supergravity}, Phys. Rev. Lett. \textbf{56} (1986) 294.

[10] L.-L. Chau, \textit{Geometrical integrability and equations of motion in physics: A unifying description}, Comments Nucl. Part. Phys. \textbf{18} (1988) 55–81. or UCD-87-38.

[11] S. Gates and R. Grimm, \textit{Consequences of conformally covariant constraints for N>4 superspace}, Phys. Lett. \textbf{133B} (1983) 192.

[12] P. Howe, \textit{Supergravity in superspace}, Nucl. Phys. \textbf{B199} (1982) 309–364.

[13] A. Chamseddine, \textit{N=4 Supergravity Coupled to N=4 Matter and Hidden Symmetries}, Nucl. Phys. \textbf{B185} (1981) 403–415.

[14] E. Cremmer, J. Scherk, and S. Ferrara, \textit{SU(4) invariant supergravity theory}, Phys. Lett. \textbf{74B} (1978) 61.

[15] D. Freedman and J. Schwartz, \textit{N=4 supergravity theory with local SU(4) \otimes SU(4) invariance}, Nucl. Phys. \textbf{B137} (1977) 225–230.

[16] M. Sohnius, \textit{Bianchi identities for supersymmetric gauge theories}, Nucl. Phys. \textbf{B136} (1978) 461–474.

[17] M. Müller, \textit{Supergravity in U(1) superspace with a two-form gauge potential}, Nucl. Phys. \textbf{B264} (1986) 292–316.

[18] B. Belinicher, \textit{Relativistic wave equations and lagrangian formalism for particles of arbitrary spin}, Theor. Math. Phys. \textbf{20} (1974) 849 (320).

[19] S. Weinberg, \textit{The quantum theory of fields}, vol. 1 : Foundations. Cambridge University Press, 1995.

[20] V. I. Ogievetsky and E. Sokatchev, \textit{Superfield equations of motion}, J. Phys. \textbf{A10} (1977) 2021–2030.

[21] P. Binétruy, G. Girardi, and R. Grimm, \textit{Supergravity Couplings: a Geometric Formulation}, Phys. Rept. \textbf{343} (2001) 255–462. hep-th/0005225.

[22] P. Howe and U. Lindström, \textit{Higher order invariants in extended supergravity}, Nucl. Phys. \textbf{B181} (1981) 487–501.
[23] W. Siegel, *On-shell $O(N)$ supergravity in superspace*, *Nucl. Phys.* **B177** (1981) 325–332.

[24] J. Wess and J. Bagger, *Supersymmetry and Supergravity*. Princeton Series in Physics. Princeton University Press, Princeton, 1983. 2nd edition 1992.

[25] E. Cremmer and J. Scherk, *Algebraic Simplifications in Supergravity Theories*, *Nucl. Phys. B127* (1977) 259.