Electromagnetic mode conversion: understanding waves that suddenly change their nature

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Abstract. In a magnetized plasma, such as in fusion devices or the Earth’s magnetosphere, several different kinds of waves can simultaneously exist, having very different physical properties. Under the right conditions one wave can quite suddenly convert to another type. Depending on the case, this can be either a great benefit or a problem for the use of waves to heat and control fusion plasmas. Understanding and accurately modeling such behavior is a major computational challenge.

1. Introduction

A fascinating property of magnetized plasmas is that at a given frequency, several different kinds of plasma waves can exist with very different wavelength and polarization. If the wave frequency is near the frequency at which plasma ions gyrate around the magnetic field, called the ion cyclotron frequency, \( \Omega_{ci} \) (typically 10 Mhz to 100Mhz in fusion plasmas), there are two very different electromagnetic waves, the fast magnetosonic wave and the slow ion cyclotron wave (ICW). These are similar to light waves in that their polarization is nearly perpendicular to the direction of propagation. In addition there is a wave called the ion Bernstein wave (IBW), that is similar to a sound wave in that its polarization is nearly parallel to its direction of propagation. These different types of waves, or modes, typically have very different wavelength and interact very differently with the plasma. The fusion program is developing techniques to use these different kinds of interaction to exert control on the hot magnetized plasmas in fusion devices – heating plasma electrons separately from ions, producing highly energetic populations of ions, driving electric currents in the plasma, and forcing plasma flows that in turn affect stability. This is done by injecting high power waves into the plasma, typically in the frequency range of short wave, or FM radio.

A wave launched into a non-uniform plasma can in a short distance completely change its character to another type of wave, a process called mode conversion. In order to study these effects the computer model must have very high resolution to see the small-scale structures that develop, which means that
very large computers are needed to solve for the very large number of unknowns in the equations. Also, the computers must be extremely fast in order to obtain the solutions in a reasonable time. Within our SciDAC project we have been studying mode conversion processes in tokamaks using two 2D computer codes – TORIC [1] and AORSA2D [2]. As a result of these calculations, in combination with basic analytic plasma wave theory and experiment, we have come to a new understanding of how this process can work in realistic geometries of fusion devices.

2. Basic equations of plasma waves and the mode conversion process

The starting point for the theory of hot magnetized plasmas is the kinetic equation, a form of the Boltzmann equation that describes the evolution in time of the distribution of plasma particles of species \( j \) in a six dimensional phase space of position and velocity, \( f_j(x,v,t) \).

\[
\frac{\partial f_j}{\partial t} + v \cdot \nabla f_j + \frac{q_j}{m_j} \left[ E + v \times B \right] \cdot \nabla_v f_j = C(f_j) + S_j(x,v,t), \tag{1}
\]

where \( q_j \) and \( m_j \) are respectively the charge and mass of particle species \( j \), \( E \) is the electric field and \( B \) is the magnetic field. The second term on the left describes convection in position space. The third term describes convection in velocity space due to electromagnetic forces on the charged plasma particles. The first term on the right describes redistribution of particles in velocity space due to collisions and the final term represents any particle sources. There is an equation of this sort for each plasma species including electrons and usually several different ion species. The different kinetic equations are nonlinearly coupled through the contribution of each species to the electromagnetic field and through collisions. The electric charge, \( q(x,t) \), and current, \( J(x,t) \), that act as sources for the electromagnetic field in Maxwell’s equations are obtained by taking velocity moments of \( f_j(x,v,t) \).

\[
J(x,t) = \sum_j q_j n_j(x,t) V_j(x,t) = \sum_j q_j \int d^3v f_j(x,v,t),
\]

\[
q(x,t) = \sum_j q_j n_j(x,t) \equiv \sum_j q_j \int d^3v f_j(x,v,t) \tag{2}
\]

where \( V_j \) is the mean velocity of species \( j \) and \( n_j \) is the particle density of species \( j \).

Since the RF wave period is by far the fastest timescale in the system, the fields and distribution function can be separated into a time-average, or equilibrium, part \( \langle E_0, B_0, f_{j0}(x,v) \rangle \) that is slowly varying, and a rapidly oscillating part, \( \langle E_1(x), B_1(x)e^{-i\omega t}, f_{j1}(x,v)e^{-i\omega t} \rangle \) where \( \omega \) is the frequency of the RF power source. For our applications, the time-harmonic wave fields are small compared to the equilibrium fields and we may linearize Eq (1) with respect to these amplitudes. Solving the linearized equation gives the rapidly varying part \( f_{j1}(x,v) \) in terms of the equilibrium distribution, \( f_{j0}(x,v) \), and the rapidly varying component of the electromagnetic field. This solution then allows us to relate the plasma current induced by the wave fields, \( J_1^p \), to the fields through a nonlocal, integral conductivity operator acting on the wave field.

\[
J_1^p(x) = \sigma \cdot E_i = \sum_j \int d^3x'r' K(f_{j0}^o,x,t,x',t') \cdot E_i(x',t'). \tag{3}
\]

The Maxwell’s wave equation that must be solved reduces to a generalization of the Helmholtz equation,

\[
\nabla \times \nabla \times E = J_p + J_{int} + \text{boundary conditions}. \tag{4}
\]

The source for the waves is an externally driven antenna current, \( J_{int} \), localized near the plasma edge. The interaction takes place in a bounded domain, on which are imposed appropriate boundary conditions determined by the shape of the fusion device.
In order to gain insight into these high dimensional equations, we first study the highly idealized case of plasmas with straight magnetic field lines and no spatial variation whatever. In that case the linear wave equation has plane-wave solutions of the form $E(x) \propto E_0 e^{i k \cdot x}$, where $k$ is the wave vector and $E_0$ is the electric polarization. Then Eq (4) reduces to a matrix equation for the polarization $\bar{E} \cdot E_0 = 0$. The solvability condition or dispersion relation, $D(k, \omega, B, n_j, f_j^0) = \det[\bar{E}] = 0$, determines one component of the wave vector (e.g. $k_\perp$ the component perpendicular to the magnetic field) in terms of the other components of $k$, the frequency, and the parameters of the uniform plasma. Solutions of the dispersion relation are the $k$ vectors of the plasma modes such as the fast, slow, and Bernstein modes previously mentioned. Our most basic understanding of waves in plasmas comes from analysis of such dispersion relations [3].

Although in a non-uniform plasma the plane-wave solutions are no longer valid, still if the wavelengths of the different modes are widely separated and if all wavelengths are much shorter than the scale length of the plasma variations then the waves behave essentially as a superposition of non-interacting modes. It is only when wavelengths of two or more modes become comparable that the modes interact and conversion of energy from one mode to another is possible. This tends to happen only near thin surfaces within the plasma where wavelengths are rapidly changing, called resonant surfaces. The simplest model in which this phenomenon can be studied is the idealized case in which all plasma variation is only in one coordinate, say the x coordinate, such that the fields are described by a set of coupled ordinary differential equations. Even in this simple geometry the resulting equations are extremely complicated requiring restrictive approximations for analytic solution, and they are multi-scale requiring very high resolution for numerical solution. Our basic understanding of mode conversion processes in plasmas comes from analysis of such one-dimensional models with plane waves incident on an infinite, flat mode conversion surface. [Ref 3 chapter 13]

3. Mode conversion in higher dimensions

Now, with the help of tera-scale computers and through the developments of our SciDAC project on wave-plasma interactions we are able to solve the wave equations for plasmas having 2D and 3D spatial variations, at high resolution sufficient to study mode conversion, and with far less restrictive assumptions on the physics than was ever possible before. Figure 1 shows one component of the wave electric field solution from the AORS2A2D code for fast magnetosonic waves launched from an antenna on the right (not shown) into the Alcator C-Mod tokamak at MIT. The large-scale structure on the right is the fast wave that is directly launched by the antenna. The curved blue line (seen more clearly in the blow up view at right) is the mode conversion layer.

The figure shows conversion to one type of short wavelength mode propagating to the left near the horizontal mid-line, an ion Bernstein wave. However we also see conversion to a completely different type of short wavelength mode, the slow ion cyclotron wave, above the mid-line propagating to the left and below the mid-line differential propagating back to the right. This result is a surprise. The previous expectation was that the conversion would dominantly be to the ion Bernstein wave, only propagating to the left of the conversion surface. An early, approximate calculation [4] in one dimension suggested that both types of conversion could, in principle, occur. One-dimensional analysis did give an indication of what kind of mode conversion processes might be possible, but the results are sensitive to parameters of the calculation, such as the component of $k$ parallel to the magnetic field, $k_\parallel$, which vary greatly in space in a real fusion device. The one dimensional models were not capable of giving a picture of what would happen across the whole spatially varying cross section, nor of giving quantitative measures of the electromagnetic field structures, the importance of conversion into different modes, or the amount of energy and momentum transferred to the different plasma species.
4. Experimental verification

This process has been observed experimentally on the Alcator C-Mod tokamak using an innovative new diagnostic technique, called Phase Contrast Imaging (PCI), that measures density fluctuations associated with the mode converted waves integrated along a vertical chord through the plasma [5]. The Alcator C-Mod measurements have been modeled extensively with the TORIC code incorporating a “synthetic diagnostic” that predicts the signals to be observed by the PCI instruments. Figure 2 shows the local density fluctuations calculated by TORIC for a particular C-Mod experiment having higher plasma current (higher poloidal magnetic field) than the case shown in Fig. 1. In this case the conversion is primarily to the slow ion cyclotron wave. Figure 3 shows the in-phase and quadrature components, and the modulus of the line integrated density perturbation obtained experimentally compared to the TORIC prediction. The results are in good agreement with regard to the spatial wave structure and the horizontal k spectrum.

By modifying the wave frequency and the isotopic proportions of the plasma it is possible alter the spatial location of the conversion layer and the relative strength of conversion to IBW versus ICW and the power ultimately absorbed by electrons versus ions. Many such experiments have been carried out on Alcator C-Mod and are accurately modeled using TORIC.
5. Scientific significance

These studies are an excellent example of the beneficial interaction of basic theory, computational modeling and experiment. The expectation was that fast waves would be converted to IBW propagating on the high magnetic field side of the conversion layer (left in these figures). Therefore when the new codes first began to show short waves on the high magnetic field side the results were not understood and concerns were raised about the code validity. Similarly when the newly developed PCI diagnostic indicated waves on the low field side the results were not understood at first. Going back to the three decade old 1D analytic theory suggested looking toward the ICW conversion process. Analysis of the code results from the standpoint of 1D theory verified this conjecture. Subsequent detailed comparison of the computational results with experimental measurements lead to greatly increased confidence in our understanding of both.

These results are likely to have significant practical consequences because Bernstein waves are absorbed primarily by electrons and are effective at driving current, whereas the slow ion cyclotron wave can be absorbed by ions, which would be more effective at driving plasma flow and improving the ability of the magnetic field to hold the hot plasma. Although these specific studies are directed to fusion applications, the process of mode conversion can occur when waves propagate in any non-uniform medium. It is seen in magnetized plasmas in astrophysics, such as in radio wave emission from Jupiter, in the Earth’s magnetosphere and possibly in the solar corona. Mode conversion is important in non-plasma fields such as seismology, and in the theory of Hawking particle flux due to the extremely non-uniform gravitational field near a black hole. We anticipate that physics understanding and numerical techniques developed within our project will have benefits far outside the fusion program.

References
[1] M. Brambilla, Nuclear Fusion 38, 1805 (1998)
 M. Brambilla, Plasma Phys. And Controlled Fusion 41, 1 (1999)
 J. C. Wright, P.T. Bonoli, M. Brambilla et al, Phys. Plasmas 11, 2473 (2004)
[2] E. F. Jaeger, L. A. Berry, E. D’Azevedo et al., Phys. Plasmas 9, 1873 (2002),
 E. F. Jaeger, L. A. Berry, J. R. Myra et al., Phys. Rev. Lett. 90, 195001-1 (2003)
[3] T. H. Stix, Waves in Plasmas (American Institute of Physics, New York, 1992)
[4] F. W. Perkins, Nuclear Fusion 17, 1197 (1977)
[5] S. J. Wukitch, Y. Lin, J. C. Wright et al, Phys. Plasmas 12, 056104 (2005)