Transverse Energy production
in Ultrarelativistic Heavy Ion Collisions *

Bin Zhang$^1$, Yang Pang$^{1,2}$, and Miklos Gyulassy$^1$

$^1$Physics Department, Columbia University, New York, NY 10027, USA
$^2$Brookhaven National Laboratory, Upton, New York, 11973

Abstract

Kinetic theory is used to check the applicability of parton cascade in 1 dimensional expansion. Using the information provided by 3 dimensional parton cascade, we model the transverse expansion by an effective area. With this model, kinetic theory is able to give prediction of the time development of transverse energy which is in good agreement with the parton cascade results.

*This work was supported by the U.S. Department of Energy under contract No. DE-FG02-93ER40764, DE-FG-02-92-ER40699, and DE-AC02-76CH00016
1 Introduction

Cascade method has long been used to study nucleus-nucleus collisions [1]. Recently, the OSCAR standard [2] has been passed to provide some objective scientific criteria for cascade simulations. One goal of the open standard working group is to develop and perform a set of standardized tests to ensure the applicability of the cascade method. One test we proposed is to compare the cascade prediction for time dependence of transverse energy with the scaling kinetic theory prediction. It’s shown that ZPC gives result which is in good agreement with the kinetic theory prediction when the isolated 2 body collision criteria is satisfied. The we go from 1 dimensional expansion to 3 dimensional expansion. The cascade gives us the transverse expansion information. With a model of the transverse expansion, we see the kinetic theory gives consistent result with the cascade prediction.

2 Boost Invariant Relativistic Transport

Parton transport equation:

\[(p \cdot \partial_x)f(x, p) = (p \cdot u)(C(x, p) + S(x, p))\]

can be simplified by changing variables to: \(\tau, \xi = \eta - y,\) and \(p_T\) [3].

We put the partons in on a hyperbola of constant \(\tau_0\) with Boltzmann distribution, i.e., the source term is:

\[S(\tau, \xi, p_T) = C\delta(\tau - \tau_0)e^x(-\frac{p_T}{T_0}\cosh\xi).\]

Under relaxation time approximation, the solution of the above transport equation is:

\[f(\tau, \xi, p_T) = \Theta(\tau - \tau_0)e^{x(-\int_{\tau_0}^{\tau} d\tau' \frac{d\tau'}{\tau_c(\tau')}} C e^{x(-\frac{p_T}{T_0}\cosh\xi(\tau_0))}
+ \int_{\tau_0}^{\tau} d\tau' e^{x(-\int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_c(\tau')}}} f_{eq}(\tau', \xi'(\tau'), p_T)\]

The proper energy density

\[\epsilon(\tau) = \int d\xi d^2p_T(p_T\cosh\xi)^2f(\tau, \xi, p_T)\]
and the transverse energy per unit rapidity

\[ e_T(\tau) = \frac{dE_T}{dy} = \tau A \int d\eta d^2p_T p_T^2 \cosh \xi f(\tau, \xi, p_T) \]

can be calculated from the above phase space distribution. We get:

\[ \epsilon(\tau) = \Theta(\tau - \tau_0)e^{\exp(- \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau c(\tau')})} \frac{\tau_0}{\tau} h(\frac{\tau_0}{\tau}) \epsilon(\tau_0) + \int_{\tau_0}^{\tau} d\tau' \frac{\exp(- \int_{\tau_0}^{\tau} \frac{d\tau''}{\tau c(\tau'')})}{\tau c(\tau')} \tau' h(\frac{\tau_0}{\tau}) \epsilon(\tau'), \]

in which:

\[ h(x) = \frac{1}{2}(|x| + \frac{\text{Arccos} \sqrt{1-x^2}}{\sqrt{1-x^2}}), \]

and

\[ e_T(\tau) = \Theta(\tau - \tau_0)e^{\exp(- \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau c(\tau')})} e_T(\tau_0) + \int_{\tau_0}^{\tau} d\tau' \frac{\exp(- \int_{\tau_0}^{\tau} \frac{d\tau''}{\tau c(\tau'')})}{\tau c(\tau')} \tau' e(\tau') \]

The relaxation time

\[ \tau_c = \frac{1}{n\sigma_\theta}, \]

in which

\[ \sigma_\theta = \int \sin^2 \theta \frac{d\sigma}{d\Omega} d\Omega \]

is the transport cross section.

We take the differential cross section to be:

\[ \frac{d\sigma}{dt} = \frac{9\pi \alpha^2}{2} \left( \frac{\mu^2}{\hat{s}} + 1 \right) / (\hat{t} - \mu^2)^2, \]

such that the total cross section is

\[ \sigma = \frac{9\pi \alpha^2}{2\mu^2} \]

Notice many authors have studied the energy density equation \[ \text{[3] [4]}, \] and a variable similar to transverse energy (proper time times the energy density)
is introduced for calculational convenience. But experiments measure only transverse energy, and the transverse energy is not simply a product of proper time, transverse area and the energy density. It can not be simply calculated from one integral equation, but has to be calculated from a set of couple integral equations including the energy density evolution equation.

The parton cascade gives reasonably good result for the time development of the transverse energy per unit rapidity as seen in Fig. 1.

3 Effect of transverse expansion on the transverse energy production

It is difficult to solve kinetic equation semianalytically with the transverse expansion. But we can get the transverse expansion information from the cascade and using a model for the transverse expansion to make the semianalytical calculation possible.

A model for freezeout is to assume that the $e_T$ freezes out outside the rarefaction front, i.e.,

$$\frac{d e_{T_{\text{out}}} (\tau)}{d \tau} = - \frac{e_{T_{\text{in}}} A(\tau)}{d \tau}$$

The contribution to $e_T$ from inside the rarefaction front is just the rescaled (by the effective area inside the rarefaction front) contribution derived before:

$$e_{T_{\text{in}}} (\tau) = \frac{A(\tau)}{A(\tau_0)} \Theta (\tau - \tau_0) \exp (- \int_{\tau_0}^{\tau} \frac{d \tau'}{\tau_c (\tau')} e_T (\tau_0))$$

$$+ \frac{\pi}{4} A(\tau) \int_{\tau_0}^{\tau} d \tau' \frac{\exp (- \int_{\tau_0}^{\tau} \frac{d \tau''}{\tau_c (\tau'')})}{\tau_c (\tau')} \tau' \epsilon (\tau').$$

The cascade gives the weight function for the transverse area:

$$w(r, \tau) = \begin{cases} 1 & r < R - v_c \tau \\ \left( \frac{R + v_c \tau - r}{2 v_c \tau} \right)^4 & R - v_c \tau < r < R + v_c \tau \\ 0 & r > R + v_c \tau \end{cases}$$

This weight function is almost independent of interaction. The effective transverse area can be calculated from:

$$A(\tau) = \int_{0}^{\infty} w(r, \tau) dr$$
The model results (Fig. 2) are similar to the cascade prediction. Both of them give more transverse energy production than the 1-d expansion.

4 Conclusion

For the 1 dimensional expansion, ZPC's prediction for the time development of the transverse energy production is consistent with the kinetic theory prediction. With the model motivated by the cascade, we can perform the semianalytic calculation of the kinetic theory and give out consistent result with the cascade's.

References

[1] For example, Y. Pang, GCP, [http://rhic.phys.columbia.edu/rhic/gcp](http://rhic.phys.columbia.edu/rhic/gcp); Klaus Werner, VENUS, [http://www-subatech.in2p3.fr/Sciences/Theorie/venus/venus.html](http://www-subatech.in2p3.fr/Sciences/Theorie/venus/venus.html); Klaus Kinder Geiger, [http://rhic.phys.columbia.edu/rhic/vni](http://rhic.phys.columbia.edu/rhic/vni); Bin Zhang, ZPC, [http://nt1.phys.columbia.edu/people/bzhang/ZPC/zpc.html](http://nt1.phys.columbia.edu/people/bzhang/ZPC/zpc.html).

[2] [http://rhic.phys.columbia.edu/oscar/](http://rhic.phys.columbia.edu/oscar/).

[3] K. Kajantie, and T. Matsui, *Phys. Lett. B* 164 (1985) 373.

[4] G. Baym, *Phys. Lett. B* 138 (1984) 18; S. Gavin, *Nucl. Phys. B* 1991 561.
Figure 1: 20 event averaged 1-d expansion. Initially (at $\tau = 0.1 \, fm$), $T_0 = 500 \, MeV$, $\eta \in [-5, 5]$, $\frac{dN}{d\eta} = 400$. Screen mass $\mu = 3 \, fm^{-1}$, strong interaction coupling constant $\alpha_S = 0.47$. Interaction length $0.3 \, fm$. Initial mean free path $0.1 \, fm$, smaller than the interaction length. This is the reason that initially the cascade gives a slight increase. The comparison starts at $\tau = 0.2 \, fm$ when the mean free path is close to the interaction length and good agreement is achieved.
Figure 2: Comparison of 3 dimensional expansion cascade and kinetic theory results. The initial condition is similar to 1-d case except that the transverse positions of particles are within a disc of radius $5 \text{ fm}$. The dash-dotted and the long dashed curve are corresponding 1-d kinetic theory and ideal hydro predictions for comparison.