The periodic signals application for the estimation of the unstable object parameters

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Abstract. The parameters active identification of an unstable object like the inverted pendulum with the regulator is considered when the periodic signal in the form of a meander is given on the system input. The sinusoidal signal is usually used as a periodic signal. It is simpler to give a sign-variable signal (meander) in systems in some cases. Two special nomograms are constructed for the object parameters determination. The first nomogram is used for determination of coefficient \( k \). The second nomogram is used for determination of coefficient \( b \). Also, in the article a detailed algorithm of determination of parameters \( k \) and \( b \) is given.

The received results show that the system is steady for fixed values of parameters.

1. Introduction

The identification problem (parameters estimation) can be carried out with two methods: by the active identification method and by the passive identification method. In this work, the use of the active identification method is offered for the object parameters estimation. It is proposed to estimate the parameters of the object included in the control system by means of a periodic signal. Usually the sinusoidal signal is used as a periodic signal. In technical systems, in some cases, it is simpler to give a signal in the meander form (the sign-variable signal with values \( \pm \text{const} \)). It is convenient to form a signal in the meander form by means of a harmonious signal, for example, using bipolar relay \( \text{sign}(\sin(\omega t)) \). The identification offered a technique, which is illustrated by the example of the second order unstable object with two parameters and it is represented by the inverted pendulum with an artificially introduced integrator, covered by a negative feedback \([1, 2]\). The control contour includes the second-order regulator calculated by a polynomial method. The considered object is unstable therefore the regulator is necessary. Usually the parameters identification procedure is unknown for an unstable object. Many methods are offered for the solution of the parameters determination problem \([3, 4]\). The optimal control problems for the dynamic system identification are reflected, for example, in works \([5, 6]\). We will note that the Kalman filter is used extensively for state vector estimation \([7, 8]\). The steady-state is considered for the dynamic objects parameters estimation in works \([9, 10, 11]\). In this article, two special nomograms are used for the object parameters determination.

2. Control object

The “inverted pendulum on the cart” model in the linear approach can be presented in the following form \([1, 10]\):
where \( u \) – the input signal on the platform, \( \theta \) – the core deviation corner from the balance position. Equation (1) can be transformed to the following form:

\[
s^2 \theta(s) - b \theta(s) = k_\theta u(s),
\]

where \( b = \alpha g / L \). Below, parameters \( k_\theta \) and \( b \) are defined as

\[
b = \frac{M + m}{M} \cdot \frac{g}{l} \quad k_\theta = \frac{1}{MI}.
\]

The object transfer function can be presented in the following form:

\[
W_{ob} = -\frac{k_\theta}{s^2 - b}.
\]

The object can be described as follows:

\[
W_{ob} = -\frac{k_\theta}{s^3 - bs}.
\]

The regulator calculation is given further. The regulator is calculated by a polynomial method of synthesis:

\[
W_{R}(s) = \frac{x(s)}{y(s)} = \frac{-0.44s^2 - 1.4075s - 1}{0.0003s^2 + 0.008s + 0.0815}.
\]

The system transfer function is defined on the basis of equations (4) – (5):

\[
W_{sys} = \frac{(-0.44s^2 - 1.4075s - 1) \cdot (-k_\theta)}{(-0.44s^2 - 1.4075s - 1)(-k_\theta) + (0.0003s^2 + 0.008s + 0.0815)(s^3 - bs)}.
\]

We will use object parameters values \( k = 1 \), \( b = 5 \) as the basic values. Further, the basic values of the object parameters are substituted in equation (6):

\[
W_{sys} = \frac{0.44s^2 + 1.4075s + 1}{0.0003s^5 + 0.008s^4 + 0.08s^3 + 0.4s^2 + s + 1}.
\]

The system transition process at the basic values of the object parameters is shown in Figure 1.

As is seen from Figure 1, the transition process time in the system at the object parameters basic values is \( t_{mn} = 2.5 \) sec. Therefore, the output signal amplitude should be measured upon the transition process termination.
3. The Nyquist pseudo–godograf for the object parameters variation

In this section, the active identification task of the object parameters is considered. The diagram shown in Figure 2 is used for pseudo–godograf construction.

Let us suppose that the object parameters may be changed relative to basic values in limits \( b \in [4; 6] \) and \( k \in [0.8; 1.4] \). The amplitude and the phase dependence of an output signal from a meander frequency is investigated. Further we will call it Nyquist pseudo–godograf. Let us build Nyquist pseudo–godografs for the object parameters values \( k = 1, k = 1.2 \), and \( b = 5 \) (Figure 3).

4. The nomograms construction

For the nomograms construction at the system input (Figure 2), the signal in the meander form with the unit amplitude and frequency \( \omega_1 \) is given. The signal amplitude is measured in the steady state at the system exit (Figure 4) and the dependence (nomogram) of the gain coefficient on the amplitude is formed. The signal like a meander with the unit amplitude and frequency \( \omega_2 \) is also given at the system input for the second nomogram construction. The output signal in time \( t = 5.85 \text{ sec} \) is measured at the system exit (Figure 5). Let us use the value of this output signal together with the gain coefficient, defined earlier, and build the second nomogram. Values of frequencies \( \omega_1 \) and \( \omega_2 \) and the measurement moments are chosen as a result of Nyquist pseudo–godograf analysis. In need of the measurements errors reduction, it is necessary to perform several measurements, multiplied by the period.
5. The example of the object parameters estimation

We will show the procedure of object parameters determination at values $k = 0.9$, $b = 5$ in the assumption that these values are unknown to us. The signal in the form of a meander with frequency $\omega = 10$ rad/sec and the unit amplitude are given at the system input (Figure 2). The output signal amplitude is measured at the system exit: in this case, $A = 0.85$, which allows one to determine the object gain coefficient by the nomogram of Figure 6 (it is shown by an arrow).

Figure 4. The transition processes in the system for an input signal like a meander with frequency $\omega = 10$ rad/sec

Figure 5. The transition processes in the system for an input signal like a meander with frequency $\omega = 2.5$ rad/sec

Figure 6. Amplitude dependence $A$ on gain coefficient $k$ for the frequency of $\omega = 10$ rad/sec
For the second parameter definition, we use the nomogram in Figure 7. The signal in the form of a meander with frequency $\omega = 2.5 \text{ rad/sec}$ is given at the system input. At the system exit, we measure the amplitude which is equal to $A = 3.36$ at $t = 2.5s$. Coefficient $b = 5.75$ is determined by values $A = 3.36$ and $k = 0.9$ (Figure 7). Thus, the unstable object parameters representing the inverted pendulum included in the control system are determined.

![Figure 7](image)

Figure 7. Amplitude dependence $A$ on coefficient $k$ and coefficient $b$ for the frequency of $\omega = 2.5 \text{ rad/sec}$

6. Conclusion

In the work, the active identification method consisting in the fact that the signal like a meander with fixed frequency $\omega_1$ given at the system input is investigated. The output signal amplitude is measured at the system exit. The value of this amplitude allows defining the object gain coefficient (the nomogram represented in Figure 6 is used). The signal like a meander with frequency $\omega_2$ is given at the system input for the object second parameter determination. The output signal value in fixed timepoint $t_2$ is measured at the system exit. The value of this output signal together with the earlier defined gain coefficient allows one to determine the object second parameter. Further, the offered technique is proposed to develop by the example of the object of a high order.

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