Energy Release from Magnetospheres Deformed by Gravitational Waves

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Abstract

In this work, we consider the possibility of energy release in pulsar magnetospheres deformed by gravitational waves (GWs) from nearby sources. The strong electromagnetic fields in the magnetospheres may release non-negligible energy despite the weakness of the GW. When the background spacetime is perturbed due to the passage of a GW, the original force-free state of the inner magnetosphere will be slightly violated. The plasma-filled magnetosphere tends to evolve into new force-free states as the spacetime varies with time. During this process, a small portion of the electromagnetic energy stored in the magnetosphere will be released to the acceleration of charged particles along the magnetic field lines. When the pulsar is close enough to the GW source (e.g., $\sim 10^{-2}$ pc to the GW sources observed recently), the resulting energy loss rate is comparable with the radio luminosity of the pulsar. It is also noticed that, under very stringent conditions (for magnetars with much shorter distance to the sources), the released energy can reach the typical energy observed from fast radio bursts.

Key words: gravitational waves – pulsars: general – stars: magnetars – stars: magnetic field

1. Introduction

Magnetospheres on compact objects act as mediators through which rotational energy is extracted from the objects to accelerate charged particles and produce multiband emission. Well within the light cylinder (LC), the plasma-filled magnetosphere on a pulsar is assumed to be force-free at large scales (Goldreich & Julian 1969), except in some small gaps where the component of the electric field parallel to the magnetic field line is not sufficiently screened (Michel 1982). In the force-freeness violated gaps, charged particles will be accelerated by the parallel electric fields along magnetic field lines, which will produce copious screening charges via the pair cascade processes (Ruderman & Sutherland 1975; Arons & Scharlemann 1979; Harding & Muslimov 1998; Zhang & Harding 2000; Hibschman & Arons 2001; Mitra 2017) to keep the magnetosphere force-free.

In this work, we consider the dynamical process in the force-free magnetosphere when its background spacetime varies due to the passage of a gravitational wave (GW) from a nearby source. Though the GWs are quite weak away from the sources, the effects of GWs on magnetospheres may be compensated by the fact that the magnetic fields on neutron stars are very strong, even reaching $10^{15}$ Gauss for magnetars. On a time-varying spacetime, the force-free condition as well as Maxwell’s equations for the original magnetosphere will be slightly violated. The plasma-filled magnetosphere tends to evolve into new force-free states. During the process, part of the energy stored in the magnetosphere is triggered to be released by the GW, via various mechanisms.

Based on some simple analysis, we estimate the energy released during the process. We show that the energy is comparable to or even larger than that of the pulsar low energy emission as the pulsar is close enough to the source. This may provide an alternative detecting signal for the GW, which is in spirit similar to the method discussed in Das et al. (2018). Even when the GW sources are far away from us, we can detect them from the observation of the timing and emission variability of a pulsar located closed to the GW source.

This situation may exist in globular clusters with dense stars. In the clusters, the distance between stars is short and is typically 1 light year (even shorter near their cores). In particular, hierarchical triple-star systems can form frequently via binary–binary encounters in the globular clusters (Antonini et al. 2016). Our discussion here is mostly relevant to a hierarchical triple system in which a pulsar orbits around a close inner binary. As the binary is in the final stage of merger, GW will be radiated and the pulsar magnetosphere will be perturbed. Actually, a triple system involving a pulsar has been identified by Ransom et al. (2014; though the pulsar is in the inner binary). Interestingly, it is long realized that, in a triple system, the formation (Ivanova et al. 2010; Naoz et al. 2013, 2016) and merger (Portegies Zwart & McMillan 2000; Antonini et al. 2016; Silsbee & Tremaine 2017) of binaries can be enhanced via interactions with the outer companion.

The pulsars, especially magnetars, are usually thought to be related to the origin of the mysterious fast radio bursts (FRBs; Lorimer et al. 2007; Thornton et al. 2013; Spitler et al. 2016) in many of the models proposed so far, though they encounter various problems (Popov et al. 2018). In some of the models, FRBs arise from energy release and emission processes in pulsar magnetospheres disturbed by eternal materials or sources (Mottez & Zarka 2014; Geng & Huang 2015; Wang et al. 2016; Zhang 2017a, 2017b). Here, we examine the possibility of interpreting FRBs as energy release phenomena from pulsar magnetospheres perturbed by GWs by comparing the energy scales. The model has an attractive feature in that the pulsar system will not be contaminated, with nothing left behind, after the GW passes through the magnetosphere.

2. Electrodynamics in the Deformed Magnetosphere

The plasma-filled magnetospheres of pulsars are usually assumed to be force-free, with the inertia of plasma ignored:

$$J^\mu F_{\mu \nu} = 0,$$

where $J^\mu$ is the four-current. The spatial components of the equation reads: $\rho \dot{E} + (1/c) j \times B = 0$, i.e., the Lorentz force...
on charges vanishes. This means that the electric field must be perpendicular to the magnetic field everywhere in the force-free magnetosphere:

$$E \cdot B = 0.$$  
(2)

The charge and current densities are given by Maxwell’s equations:

$$\nabla \cdot E = 4\pi \rho_e, \quad c \nabla \times B - E = 4\pi j.$$  
(3)

The associated spacetime metric to the above equations is generally $g_{\mu\nu}$. For pulsar magnetospheres, we simply adopt flat spacetime with $g_{\mu\nu} = \eta_{\mu\nu}$, as is usually done in most simplified pulsar models.\(^4\) In the stationary and axisymmetric case the electric field induced by the rotation of the magnetic field is explicitly expressed as

$$E = -\frac{v}{c} \times B,$$  
(4)

where $v = \Omega \times r + sB$. We focus on the inner region well within the LC where the electromagnetic fields are strong. In this region, the current parallel to the magnetic field lines is negligible and the electromagnetic fields corotate with the star, i.e., the fields are purely poloidal with no toroidal component.

The energy density of the magnetosphere is electromagnetically dominated. It is

$$\varepsilon = \varepsilon_E + \varepsilon_B,$$  
(5)

where

$$\varepsilon_E = \frac{E^2}{8\pi}, \quad \varepsilon_B = \frac{B^2}{8\pi}.$$  
(6)

Now let us consider the case in which a GW passes through the magnetosphere system so that the background spacetime changes with time, with the metric becoming $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. It is easy to infer that Maxwell’s equations and the force-free equation in flat spacetime are generally not satisfied in the varying spacetime. The system is now time dependent and tries to evolve to restore new balanced force-free states. The geometry of the field lines is deformed, and the charge and current densities will redistribute in response to the variability of the spacetime. Since the magnetosphere is plasma-filled, the magnetosphere can quickly evolve into new force-free states as the spacetime varies.

As the spacetime is perturbed, the energy density of the electromagnetic system should change accordingly. The general form of the change can be given by the energy-momentum tensor and is spacetime dependent. Here, we just make an estimation of the scale by doing some simple analysis. Let us consider the magnetosphere penetrated by a GW whose propagation direction is along the $z$ coordinate and is perpendicular to the rotation axis of the magnetosphere. Under the transverse-traceless gauge, the quadrupole GW is decomposed into two polarization directions along the $x, y$ coordinates that are perpendicular to $z$. Accordingly, the poloidal electromagnetic fields can also be decomposed in the two directions. Then, in the linear order, the energy densities vary as

$$\delta\varepsilon_E = \frac{1}{8\pi}[h_{zz}(E_z^2 - E_T^2) + 2h_{zz}E_z],$$  
(7)

$$\delta\varepsilon_B = \frac{1}{8\pi}[h_{zz}(B_z^2 - B_T^2) - 2h_{zz}B_z].$$  
(8)

Here, we ignore the rotational effect because we simply take the varying energy densities as being on a similar scale for all slices. Moreover, we are more interested in pulsars with strong magnetic fields, which usually have long periods (e.g., longer than ~0.1 s), much longer than those of GWs from binary mergers (e.g., ~milliseconds for the observed GW events so far).

The axisymmetric magnetosphere can be viewed as being composed of identical slices along the toroidal direction parameterized by the toroidal angle $\phi$. On the two slices that are perpendicular to the GW propagation direction $z$ (set to be at $\phi = 0, \pi$), the component electromagnetic fields satisfy $E^2 = E_z^2 + E_T^2$ and $B^2 = B_z^2 + B_T^2$. When $E_z \sim E_T \sim E/\sqrt{2}$, $\delta\varepsilon_E \sim h\varepsilon_E$, and when $E_z \sim E$ or $E_T \sim E$, $\delta\varepsilon_E \sim \pm h\varepsilon_E$. The situation is similar for $\delta\varepsilon_B$. So we can simply take

$$\delta\varepsilon = \delta\varepsilon_E + \delta\varepsilon_B \sim h(\varepsilon_E + \varepsilon_B).$$  
(9)

On other slices at $\phi \neq 0, \pi$, the perturbed energy densities are somewhat different, but should be of similar scales. These varying energy densities are those observed by asymptotic observers. They are not necessarily dissipated or released during the GW perturbation process. In what follows, we show that part of the varying energy can indeed be transformed to the plasma in the magnetosphere.

### 3. The Energy Release Mechanisms

In this section, we investigate the release processes of the electromagnetic fields on the slices at $\phi = 0, \pi$. It is straightforward to extend the discussion to the cases on other slices.

It is known that the relative distance change induced by the GW is of the scale: $\delta L/L = h/2$. For the centered dipole magnetosphere, we can estimate that the angular change $\alpha$ of the field lines with respect to the center of the star should be $\sim \delta L/L$, i.e.,

$$\alpha \sim \frac{h}{2}.$$  
(10)

In the inner region of the magnetosphere, we can simply think that this angular change is comparable with the change of the poloidal angle $\theta$ of the magnetic field lines: $\alpha \sim \delta \theta$.

#### 3.1. The Emergence of Parallel Electric Fields

In the force-free magnetosphere, the electric and magnetic fields are originally perpendicular to each other. As the spacetime is perturbed by the GW, the relative angle between the electric and magnetic field lines is $2\alpha$ different from $\pi/2$. Thus, there emerges a non-vanishing component of the electric field along the magnetic field line, which is of the scale:

$$E_{||} = E \sin(2\alpha) \sim hE.$$  
(11)

The charges on the magnetic field lines should be accelerated by this parallel electric field. This point is easily seen in the local flat frame. In a general spacetime that is relevant here, we
can always find a local Minkowski spacetime in the neighborhood of any point. In the local frame, an observer at the point can see that the angle between the electric and magnetic fields around will be changed when the background spacetime is perturbed. Applying the laws in flat spacetime, it is easy to know that, in the local flat frame, the charges at the point should be accelerated by the electric field parallel to the magnetic field line.

The acceleration process should be efficient under particular conditions. Let us consider the ideal case in which a test electron is accelerated by this emerging parallel electric field. For \( h \sim 10^{-10} \) and \( E \sim 10^{11} \text{V m}^{-1} \), the electron can be accelerated to the relativistic speed with the Lorentz factor \( \gamma = eE_0/\delta t \frac{m_e c}{c} \sim 5.8 \) in a timescale \( \delta t \sim 10^{-3} \text{s} \). For an electron–positron pair, this is sufficient to cause them to be separated by a distance of \( \sim 100 \text{km} \).

Of course, this is not the case in a real magnetosphere because the magnetosphere is filled with plasma, with a density that is much more than the one needed to sustain the force-freeness of the magnetosphere (Ruderman & Sutherland 1975; Mitra 2017). The emerging \( E_1 \) (very small compared with \( E \)) will be quickly screened by the collection of particle pairs on the conducting magnetic field lines. During the process, its energy will be all transferred to the acceleration of the charged particles. The released energy is

\[
(L_E) = \frac{E^2}{8\pi} \sim h^2 \varepsilon_E. \tag{12}
\]

3.2. The Emergence of Toroidal Magnetic Fields

As seen from an observer corotating with the magnetic field line, its projected distance to the rotation axis varies as the GW passes. From Equation (4), this means that the electric field induced from the rotation of the magnetic field should vary accordingly under the force-free condition. In the flat spacetime, the electric field is given by Equation (4): \( E = x \Omega B \), with \( x = r \sin \theta \). When the spacetime is perturbed, the distance \( x \) changes by \( \delta x \). So the induced electric field satisfying the force-free condition is \( E' = E - \delta E = (x - \delta x) \Omega B \). With \( \delta x/x \sim h/2 \), we have

\[
\delta E \sim \frac{1}{2} hE. \tag{13}
\]

So the energy density change from the variability of the perpendicular electric field is

\[
(L_E)_\perp = \frac{1}{8\pi}(E^2 - E'^2) \sim \frac{1}{4} h(4 - h) \varepsilon_E. \tag{14}
\]

This is comparable with the variable energy density (9) of the electric field in linear order.

We now determine where the energy goes. When a magnetic field line is shifted toward the rotation axis (\( \delta x > 0 \)) at a moment, it will still keep rotating with the original angular velocity, which is larger than the angular velocity of corotation on the new “orbit.” Thus, a toroidal component \( B^\phi \) of the magnetic field appears due to the differential rotation velocities. With the emergence of \( B^\phi \), charged particles will be accelerated along the magnetic field line and a poloidal current will be excited, as given by Maxwell’s Equations (3): \( c \nabla \times B^\phi - \dot{E}^\phi = 4 \pi j^\phi \) and \( c \nabla \times B^\theta - \dot{E}^\theta = 4 \pi j^\theta \). This process proceeds until the extra magnetic field energy is exhausted and the field line corotates with the star again on the new orbit. The energy released can be measured by the difference of the energy densities (14) of the electric field induced by the magnetic field rotating on the two “orbits.”

Similarly, when the magnetic field line is shifted away from the rotation axis (\( \delta x < 0 \)), it will also keep rotating with the original velocity, which is lower than that of corotation on the new orbit. A toroidal magnetic field also appears. The poloidal current in the opposite direction is excited. But, in this process, the energy density (14) is negative, which means that the energy is compensated and extracted from the star.

3.3. The Vibrating Magnetic Field Lines

As the wave passes through the magnetosphere, the magnetic field line is rotated with a small angle \( \alpha \) by the tidal force, which can induce a transient electric field during the rotating process. On the slices at \( \phi = 0, \pi \), this transient electric field is along the toroidal direction. Its value can be estimated from Equation (4):

\[
E_T \sim -\frac{1}{c \delta t} \frac{\rho \Omega B}{r}. \tag{15}
\]

Here \( \rho \Omega / \delta t \) is the average vibrating velocity of the magnetic field line, with the time interval \( \delta t \) to be a quarter of the period of the GW.

This electric field is induced when the magnetic field line is vibrating. It disappears when the magnetic field line halts at the rotated angle \( \alpha \). So the energy of this transient electric field is completely released. The released energy is converted to the acceleration of charges along the \( B^\phi \) component mentioned in the previous subsection. The amount of the released energy divided by volume is

\[
L_B = \frac{E_T^2}{8\pi} \sim \frac{h^2 r^2}{4c^2 \delta t^2} \varepsilon_B. \tag{16}
\]

A similar process of excitation of electric field by varying magnetic field caused by star oscillation has been discussed in Rezzolla & Ahmedov (2004).

4. The Total Released Energy

From the above analysis, the energy loss contains two components. The varying electric field energy in leading order is almost all released: \( L_E = (L_E)_\| + (L_E)_\perp \sim h \varepsilon_E \). But only a small fraction of the varying magnetic field energy is released, which is of the order of \( h^2 \). The above analysis is implemented on the slices of the magnetosphere that are perpendicular to the propagation direction of the GW. For other slices away from \( \phi = 0, \pi \), we have similar energy release processes. On these slices, the GW can induce a toroidal component of the magnetic field, which can excite a current along the poloidal magnetic field lines. The fast vibrating magnetic field lines can also release some of its energy. Here, we simply take the results in the previous section as those that apply generally in all regions in the inner magnetosphere.

As usual, we assume a dipole structure of the magnetosphere within the LC. Then, the fields depending on the radial distance are:

\[
B = B_0 \left(\frac{r_0}{r}\right)^3, \quad E = E_0 \left(\frac{r_0}{r}\right)^2. \tag{17}
\]
The magnetic field at the surface of the star is $B_0 = 6.4 \times 10^{19} \sqrt{P \ G}$ (Usov & Melrose 1996; Viganò & Torres 2015; Viganò et al. 2015), where $P$ is the period of the pulsar. At the LC radius $r_{LC}$, the electric field is equal to the magnetic field $E = B$, which determines $E_0 = (r_0/r_{LC})B_0 = (r_0 \Omega/c)B_0$. The radius of the LC is $r_{LC} = cP/2\pi = 4.8 \times 10^9 P$ cm.

By integrating over the inner magnetosphere region, we can get the total released energy in the deformed magnetosphere:

$$L_E = 1.4 \times 10^{25} \left( \frac{h}{10^{-10}} \right) \left( \frac{10^7 \text{yr}}{\tau} \right) \left( \frac{r_0}{10^6 \text{cm}} \right)^5 \text{erg}, \quad (18)$$

$$L_B = 1.4 \times 10^{13} \left( \frac{h}{10^{-10}} \right) \left( \frac{10^{-3} \text{s}}{\delta t} \right)^2 \left( \frac{B_0}{10^{12} \text{G}} \right)^2 \left( \frac{r_0}{10^6 \text{cm}} \right)^5 \text{erg}, \quad (19)$$

where $\tau = P/(2P)$. The relative strain $h = 10^{-10}$ corresponds to that at a distance $d = 4.1 \times 10^{-2} \text{pc}$ to GW150914 (Abbott et al. 2016) or that at $d = 4 \times 10^{-2} \text{pc}$ to GW170817 (Abbott et al. 2017). Of course, the GW sources are not necessarily GW bursts. They may be persistent sources, like binary systems, only if the pulsar is close enough to the source.

This amount of energy is released within a timescale of a quarter of the GW period. For the above GW events, the timescale is $\delta t \sim 10^{-3} \text{s}$ (the frequency is $\nu \sim 250 \text{Hz}$). So the energy release rate is $1.4 \times 10^{28} \text{erg s}^{-1}$, which is almost the radio luminosity from a pulsar. This featured variability of radiation should be detectable and discriminable, which may provide signals indicating that a GW is passing through a pulsar magnetosphere.

We also notice that stringent conditions are needed to account for the energy observed from FRBs, whose characteristic energy is as high as $10^{39} \text{erg}$ at 1 Gpc distance (Chatterjee et al. 2017; Popov et al. 2018). For example, $L_B$ can reach the energy $10^{48} \text{erg}$ when $h \sim 10^{-4}$, $\delta t \sim 10^{-4} \text{s}$, and $B_0 \sim 5 \times 10^{15} \text{G}$. This requires a magnetar magnetosphere perturbed by a GW with the frequency $\nu = 500 \text{Hz}$ and with the strain corresponding to that at $d \sim 10^{-3} \text{A}$ to GW150914. But if FRBs are at the distance of 1 Mpc, the FRB energy is of $\sim 10^{33} \text{erg}$. The conditions for the energy are quite relieved: $h \sim 10^{-6}$, $\tau \sim 10^3 \text{yr}$ for $L_E$ and $h \sim 10^{-6}$, $\nu \sim 250 \text{Hz}$, $B_0 \sim 3 \times 10^{15} \text{G}$ for $L_B$.

Of course, here we have only done some estimation of the energy scales with some simple analysis. The details of the release process need more accurate study in more realistic situations, in particular, with the aid of numerical tools. There may exist improvement and enhancement on the estimated energy that can relieve the above strict conditions. For example, the charges should be more effectively accelerated closer to the star surface because the electromagnetic fields are stronger in inner regions. The charged particles will attain higher velocities and can catch up to those accelerated in outer regions that have lower velocities. So the charged particles accumulated in the emission region, which makes it look like there is more energy released from the magnetosphere. Moreover, we may also not ignore the effects of the GW on the polar gap. The polar gap plays crucial roles in the pulsar activities, sourcing the plasma and the energy for radiation in the magnetosphere, though it is quite small compared to the whole magnetosphere. As shown in Zanotti et al. (2012) and Lin et al. (2015), the oscillating modes of the star can drive the oscillation of the vacuum electric field in the gap, bringing prominent changes in observational features. So the perturbations of the GWs on the polar gap could also cause similar effects on the gap electric field, which is much stronger than the emergent electric fields discussed here. This may cause extra energy release, awaiting further examination.

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