Parameter Analysis of Stepped Frequency Pulses Frequency Diverse Array Radar

Ming Tan\textsuperscript{1,a}, Chun-yang Wang\textsuperscript{2,b}, Hong-bing Li\textsuperscript{3,c}, Juan Bai\textsuperscript{4,d}, Lei Bao\textsuperscript{5,e}

\textsuperscript{1}Air and Missile Defense College, Air Force Engineering University Xi’an, Shanxi, 710051, China  86 18629679068
\textsuperscript{2}Air and Missile Defense College, Air Force Engineering University Xi’an, Shanxi, 710051, China  86 13571041693
\textsuperscript{3}Air and Missile Defense College, Air Force Engineering University Xi’an, Shanxi, 710051, China  86 13359226696
\textsuperscript{4}Air and Missile Defense College, Air Force Engineering University Xi’an, Shanxi, 710051, China  86 15249042646
\textsuperscript{5}Air and Missile Defense College, Air Force Engineering University Xi’an, Shanxi, 710051, China  86 17795834063
\textsuperscript{a}atanming_1992@163.com, \textsuperscript{b}Wcy_kgd_cn@163.com, \textsuperscript{c}84574762@qq.com, \textsuperscript{d}b_juan@163.com, \textsuperscript{e}2743681405@qq.com

ABSTRACT Owing to its range-angle-dependent transmit beampattern, the frequency diverse array (FDA) can provide the potential capability for novel radar detection, localization and interference suppression. However, the conventional FDA generates range-angle coupled beampattern, which degrades the performance of FDA in radar application. In this paper, stepped frequency pulses frequency diverse array (SFP-FDA) is investigated which can effectively remove the coupling in range-angle dimensions. Influence of parameter selections such as the frequency increment in pulse and element dimension on the proposed scheme are analyzed. The proposed method outperforms the conventional FDA, and it can be effectively utilized through reasonable parameter setting. Numerical results are given to verify the proposed scheme.

1. INTRODUCTION
Unlike phased array radar only angle-dependent, frequency diverse array (FDA) providing a range-angle-dependent beampattern by employing a tiny linearly increasing frequency offset across the array elements \[1\]-\[3\]. Due to its promising application potentials, FDA becomes a hotspot and fruitful achievements have been obtained \[4\]. The multipath characteristics of FDA are presented in \[5\]. Interference suppression by adaptive frequency offset selection is investigated in \[6\]. In \[7\], the derivation of Cramer-Rao lower bounds is made for evaluating the estimation performance in range and angle domains.

However, due to the range-angle-coupling, there might be a group of range-angle pairs hence it is difficult to distinguish the real target from numerous feedback according to the beampattern of FDA. To address this issue, several methods have been proposed recently. Through optimizing the frequency increments with genetic algorithm, a dot-shaped FDA transmit beampattern is presented in \[8\]. A non-
uniform frequency diverse array which can localize the range-dependent targets is proposed in [9]. Based on logistic map, a multi-carrier nonlinear frequency modulation system is presented in [10]. With symmetrical frequency offsets, the beampattern of FDA is discussed in [11]. To attain a required radiation performance, such as the element placements and frequency offsets, the sparse FDA technology based on artificial bee colony optimizer is presented in [12]. We proposed a novel method which called stepped frequency pulses frequency diverse array (SFP-FDA) [13], the localization performances in terms of angle-range dimensions contrast with the double-pulse FDA [14] are analyzed.

In this paper, the impact of parameter selections on SFP-FDA is investigated. By appropriate parameter selections, a narrow pencil beam can be formed which decoupled in range-angle dimensions. The transmit energy can be more concentrated in the desired area than the conventional FDA does. The frequency increment in pulse domain as a key research object is analyzed exhaustively.

The remaining sections are organized as follows. Section 2 proposes mathematical principle of conventional FDA radar. Section 3 provides the derivation of SFP-FDA and the impact analysis of parameter selections. Finally, simulations and results are presented in Section 4, and conclusions are given in Section 5.

2. CONVENTIONAL FREQUENCY DIVERSE ARRAY RADAR

Consider an \( N \) element uniform linear array, as shown in Figure 1. The signal operating frequency is \( f_0 \), the radiated frequency of the \( n \)th element is taken as

\[
f_n = f_0 + n\Delta f, \quad n = 0, 1, \ldots, N-1
\]

where \( \Delta f \) is the frequency increment between the adjacent array elements. The inter-element spacing can be represented by

\[
d = \frac{\lambda}{2} = \frac{c}{2f_0}
\]

where \( c \) and \( \lambda \) are the light velocity and the basic wavelength, respectively. The signal transmitted by the \( n \)th element can be written as

\[
x_n(t) = a_n e^{-j2\pi f_n t}
\]

where \( a_n \) is the complex weight of the \( n \)th signal.

For a far-field point target \( P(\theta, r) \), the distance between target and the \( n \)th element can be approximately given by

\[
R_n = R_0 - nd \sin \theta
\]

where \( R_0 \) is the distance between target and the reference element, note that \( R_0 = r \).

The overall signal observed by \( P(\theta, r) \) can be derived as
\[ x(t, \theta, r) = \sum_{n=0}^{N-1} a_n e^{-j2\pi f_{n}(t - \frac{R}{c})}. \]  

(5)

After approximation and arrangement, equation (5) becomes

\[ x(t, \theta, r) = e^{-j2\pi f_{n}(t - \frac{R}{c})} \sum_{n=0}^{N-1} a_n e^{-j2\pi f_{n}r/c}. \]  

(6)

Define the element weighting vector as \( \mathbf{w} = [a_0, a_1, L, a_{N-1}]^T \), where \([g]^T\) denotes the transpose operator. The steering vector of the conventional FDA is

\[ \mathbf{a}(t, \theta, r) = \left[ e^{-j2\pi f_{n}(t - \frac{R}{c})} L e^{-j2\pi f_{n}r/c} \right]^T. \]  

(7)

Then, the transmit beampattern can be expressed as

\[ B(t, \theta, r) = \left| x(t, \theta, r) \right| |w^T \mathbf{a}| \]  

(8)

where \([g]^H\) is the conjugate transpose operator. If uniform weights are applied, namely \( a_0 = a_1 = L = a_{N-1} = 1 \), equation (8) becomes

\[ B(t, \theta, r) = \frac{\sin[N\pi(f_{d}^2 \sin \theta/c - rNf_{r}^2/c)]}{\sin[2\pi(f_{d}^2 \sin \theta/c - rNf_{r}^2/c)]}. \]  

(9)

Note that when \( \Delta f_{r} = 0 \), it becomes the beampattern of phased array radar, that is

\[ B(\theta) = \frac{\sin[N\pi(f_{d}^2 \sin \theta/c)]}{\sin[\pi(f_{d}^2 \sin \theta/c)]}. \]  

(10)

By comparing (9) with (10), it implies that phased array radar is only dependent on angle and the beampattern of FDA radar is not only angle dependent, but also vary as a function of the range and time. However, it is also observed that because of its range-angle coupling, FDA have too many maximums, and it may go against focus energy on the specified location.

3. PROPOSED STEPPED FREQUENCY PULSES FREQUENCY DIVERSE ARRAY RADAR

3.1 Mathematical Principle

Figure 2 shows the radiated frequency versus time in the \( n \) th element.

In the first pulse of the SFP-FDA radar, the transmit frequency of each element is the same as that of the conventional FDA. But from pulse-to-pulse, there exists a small frequency increment \( \Delta f_{r} \), so the frequency in the \( m \) th pulse of the \( n \) th element is written as

\[ f_{n,m} = f_{n} + n\Delta f_{r} + m\Delta f_{p}, \quad m=0,1,2,3,4 \]  

(11)

where \( M \) is the number of pulses.

Define \( a_{n,m} \) as the complex weight of the \( n \) th signal in the \( m \) th pulse, the pulse width \( T_{p} \) should satisfied with \( f_{d}^2T_{p} = \text{integer} \), the \( n \) th element electric field observed at a far field target \( p(\theta, r) \) can be expressed as

\[ x_{c}(t, r) = \sum_{n=0}^{M-1} \frac{a_{n,m}}{R_{n}} \exp\left\{ -j2\pi f_{n,m} \left( t - mT_{p} - \frac{R}{c} \right) \right\}. \]  

(12)

Substituting (4) and (11) into (22) yields
\[ x_n(t,r) = \sum_{m=0}^{N-1} a_{m,n} \exp\left\{ -j 2\pi \left[ f_0 \left( t - \frac{r - nd\sin\theta}{c} \right) + \left( n f_c + m f_t \right) \left( t - \frac{r}{c} \right) - \left( f_0 + n f_c + m f_t \right) (mT) \right] \right\} \]  

(13)

The approximation \( R_m = r \) is valid in the sense of amplitude, assuming \( \Delta f_s = 1/T_p \) and \( \Delta f_c = 1/T_p \), due to the fact that the period of function \( \exp(j\theta) \) is \( 2\pi \), so the last term of equation (13) can be ignored. Assuming \( \Delta f_c = f_s \), note that \( nd\sin\theta = r \) and \( \Delta f_c = f_0 \), so the third term of equation (13) can also be negligible.

Through approximation and arrangement, (13) can be equivalently reformulated as

\[ x_n(t,\theta) = \frac{\exp(j\phi_n)}{r} \sum_{m=0}^{N-1} a_{m,n} \exp\left\{ -j 2\pi \left[ f_0 \frac{nd\sin\theta}{c} + nf_c t + m f_t \right] - n f_c \frac{r}{c} - m f_t \frac{r}{c} \right\} \]  

(14)

where \( \phi_n = -2\pi f_s (t - r/c) \). Then, the transmit beampattern of SFP-FDA can be expressed as

\[ A(t,\theta,\phi) = \sum_{n=0}^{N-1} |x_n(t,\theta,\phi)|^2 \]

\[ = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} a_{m,n} \exp\left\{ -j 2\pi \left[ n f_c t - n f_c \frac{r}{c} + f_0 \frac{nd\sin\theta}{c} \right] - n f_c \frac{r}{c} - m f_t \frac{r}{c} \right\} \]

\[ \times \exp\left\{ -j 2\pi \left( m f_t - m f_c \frac{r}{c} \right) \right\} \]  

(15)

Provided that the weights in the same order number pulses of each element are equivalent. Define \( \mathbf{a}_e(t,\theta,r) \) and \( \mathbf{a}_s(t,r) \) as the steering vector of element aspect and signal aspect, respectively, which can be written as

\[ \mathbf{a}_e(t,\theta,r) = \left[ 1 \quad \exp\left\{ -j 2\pi \left( N - 1 \right) \left( \Delta f_s - \Delta f_c \frac{r}{c} + f_0 \frac{d\sin\theta}{c} \right) \right\} \right]^T \]  

(16)

and
\[ a_{t,r} = \left[1 \ L \ \exp\left(-\frac{j2\pi(M-1)(\Delta f t - \Delta f_j \frac{r}{c})}{c}\right)\right]^T. \] (17)

Define \( w_s \) and \( w_t \) as the weighting vector in the element and signal domain, respectively. Rewritten (15) as

\[ A(t,\theta,r) = \left[ w_s^t a_{t,\theta,r} w_t^t a_{t,\theta,r} \right] \]
\[ = \left[ w_s^t \left[ a_{t,\theta,r} \otimes a_{t,\theta,r} \right] w_t^t a_{t,\theta,r} \right] \]
\[ = \left[ w_s^t a_{t,\theta,r} \right] \] (18)

where \( \otimes \) is the Kronecker product operator, and \( w \) is

\[ w = [w_{s,0} \ L \ w_{s,M-1} \ L \ w_{t,0} \ L \ w_{t,M-1}]^T. \] (19)

It is suggested that the transmit beam pattern can be designed by optimizing \( w_s \) and \( w_t \).

3.2 Parameter Selection Analysis

When using uniform weighting, which can be equivalent to \( w_{t,0} = w_{s,0} = L = w_{s,M-1} = w_{t,M-1} = 1 \), equation (18) can be rearranged to

\[ A(t,\theta,r) = \sum_{n=0}^{M-1} \exp\left(-\frac{j2\pi(n\Delta f t - n\Delta f_j \frac{r}{c} + f_n \sin \theta \frac{d}{c})}{c}\right) \times \sum_{m=0}^{M-1} \exp\left(-\frac{j2\pi(m\Delta f t - m\Delta f_j \frac{r}{c} + f_n \sin \theta \frac{d}{c})}{c}\right) \]
\[ = \left[ \sin \left( \pi \left[ \frac{M\pi \left( \Delta f t - \Delta f_j \frac{r}{c} \right) + f_n \sin \theta \frac{d}{c} - \Delta f c \right]}{c} \right) \right] \]
\[ \sin \left( \pi \left[ \frac{N\pi \left( \Delta f t - \Delta f_j \frac{r}{c} \right) + f_n \sin \theta \frac{d}{c} - \Delta f c \right]}{c} \right) \] (20)

where

\[ AF_1(t,\theta,r) = \left[ \sin \left( \pi \left[ \frac{M\pi \left( \Delta f t - \Delta f_j \frac{r}{c} \right) + f_n \sin \theta \frac{d}{c} - \Delta f c \right]}{c} \right) \right] \]
\[ \sin \left( \pi \left[ \frac{N\pi \left( \Delta f t - \Delta f_j \frac{r}{c} \right) + f_n \sin \theta \frac{d}{c} - \Delta f c \right]}{c} \right) \] (21)

and

\[ AF_2(t,r) = \left[ \sin \left( \pi \left[ \frac{M\pi \left( \Delta f t - \Delta f_j \frac{r}{c} \right) + f_n \sin \theta \frac{d}{c} - \Delta f c \right]}{c} \right) \right] \]
\[ \sin \left( \pi \left[ \frac{N\pi \left( \Delta f t - \Delta f_j \frac{r}{c} \right) + f_n \sin \theta \frac{d}{c} - \Delta f c \right]}{c} \right) \] (22)

From (20), it is observed that when we choose \( \Delta f_j = 0 \), an expression of the conventional FDA is gained, and the phased array beam pattern expression is achieved by valuing \( \Delta f_c = \Delta f = 0 \). Therefore, the conventional FDA radar and the phased array radar can be taken for two specific modalities of SFP-FDA radar.

When time \( t \) and \( \theta \) angle are fixed, the cycle of equation (21) and (22) is \( T_{t,1} = c/M_f \) and \( T_{t,2} = c/M_f \), respectively, and both are the function of \( r \). So the cycle of (20) is \( T_1 = [c/M_f, c/M_f] \), where \([c/M_f, c/M_f]\) is the least common multiple (LCM) of \( c/M_f \) and \( c/M_f \).

4. SIMULATIONS AND RESULTS

The basic parameters of these numerical results are shown in Table 1.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|

5
The additive noise is modeled as complex Gaussian zero-mean spatially and temporally white random sequences with identical variance at each antenna element.

### 4.1 Comparison with Conventional FDA

The beampatterns of the conventional FDA and the proposed scheme are presented in Figure 3 and Figure 4, respectively. From Figure 3, it is observed that the conventional FDA is range-angle-coupling, and it is difficult to localize the target in accurate position. From Figure 4, we can see that the SFP-FDA can produce decoupled beam points to the target, and it has lower sidelobe and more concentrated energy in the mainlobe area.

![Figure 3. The conventional FDA.](image3.png)

![Figure 4. SFP-FDA.](image4.png)

### 4.2 Frequency Interval Selection

By analyzing the equation (21) and (22), we can find that the parameter $\Delta f_\delta$ and $\Delta f_a$ both influence the cycle and the main lobe width in range domain, and the beampattern in angle domain is chiefly affected by $\Delta f_a$, the bigger the $\Delta f_a$, the narrower the main lobe width in angle domain. In order to facilitate the analysis, we assumed $\Delta f_\delta = \delta \Delta f_a$.

Four different $\delta$ are selected as shown in Figure 5, for a fixed $\Delta f_a = 3kHz$, $T_{\delta} = c/\Delta f_a = 100km$ is gained. The four different cycles corresponding to the different $\delta$ can be written as (a) $T_\delta = 300km$; (b) $T_\delta = 150km$; (c) $T_\delta = 75km$; (d) $T_\delta = 50km$. Owing to the equation (22) and (20), generally speaking, for a narrower and more concentrate beampattern, $\delta$ should be as big as possible. But simultaneously, the cycle will shorten, thus the number of the intersection point from (21) and (22) increased sharply, which produce many peak points that contradict with the formation of decoupled beampattern.
Notice that when $\delta > 1$, it is evitable that there are many other peak points which has different position with the desired target, consequently cannot be identified. For a detection distance no more than 100km (according to $c/\Delta f$), $\delta > 1$ can be used and another limited condition is $\delta < 2$.

![Figure 5. SFP-FDA with different $\delta$](image_url)

Figure 5. **SFP-FDA with different $\delta$:** (a) $\delta=1/3$, (b) $\delta=2/3$, (c) $\delta=4/3$, (d) $\delta=2$.

In order to get longer extent of decoupled beampattern, the parameter $\delta$ should satisfied with
\[
\delta \leq 1, \quad \frac{1}{\delta} = \text{integer}.
\] (23)

Although the main lobe width of Figure 5. (a) is wider than the other three, the result is within the acceptable limits, meanwhile, the range-angle decoupling is well realized.

### 4.3 Number of Pulses

By comparing (21) and (22), it is revealed that \( M \) to (22) is just like \( N \) to (21), which means that the larger the \( M \) is, the narrower the main lobe width will be, and the side lobe width also narrower due to the multiple of (21) and (22). The drawback is that the increase of \( M \) increases the complexity of computation. Figure 6 is the beampattern that uses parameter values \( M = 8 \), contrasting with Figure 4, which takes \( M = 16 \).

![Figure 6. SFP-FDA with \( M = 8 \).](image)

### 5. CONCLUSION

We proposed a new method for range-angle decoupled FDA transmit beamforming by stepped frequency pulses synthesizing. The parameter selections matters a lot so it is emphatically analyzed in this paper. The essence of parameter selections is to form range-angle decoupled beampattern without producing redundant peaks which can affect the performance a lot. By reasonable parameter setting, the desired beampattern can be obtained. Simulation results show the effectiveness of the proposed scheme.

### ACKNOWLEDGMENTS

This work was supported by the National Science Foundation of China (grant No.61601503).

### REFERENCES

[1] Antonik, P., Wicks, M. C., Griffiths, H. D., et al. “Frequency diverse array radars,” in *Proc. IEEE Radar Conf.*, Verona, NY, USA, pp.215-217, Apr. 2006.

[2] Wicks, M. C., and Antonik, P. “Frequency diverse array with independent modulation of frequency, amplitude, and phase,” U.S. Patent 7319427, Jan. 15, 2008.

[3] Antonik P., and Wicks, M. C. “Method and apparatus for simultaneous synthetic aperture radar and moving target indication,” U.S. Patent 20080129584A1, Jun. 5, 2008.

[4] Wang, W. Q. “Overview of frequency diverse array in radar navigation applications,” *IET Radar, Sonar Navigat.*, vol. 10, no. 6, pp. 1001-1012, 2016.

[5] Cetinepe, C., and Demir, S. “Multipath characteristics of frequency diverse arrays over a ground plane,” *IEEE Trans. Antennas Propag.*, vol. 62, no. 7, pp. 3567-3574, Jul. 2014.

[6] Shao, H. Z., Li, J. C., Chen, H., et al. “Adaptive frequency offset selection in frequency diverse array radar,” *IEEE Antennas and Wireless Propagation Letters*, vol. 13, 2014.

[7] Wang, Y. B., Wang, W. Q., and Shao, H. Z. “Frequency diverse array Cramer-Rao lower bounds for estimating direction, range and velocity,” *Int. J. Antennas Propag.*, vol. 2014, pp. 1-10, Feb. 2014.
[8] Xiong, J., Wang, W. Q., Shao, H. Z., et al. “Frequency diverse array transmit beampattern optimization with genetic algorithm,” *IEEE Antennas and Wireless Propagation Letters*, vol. 16, 2017.

[9] Wang, W. Q., So, H. C., Shao, H. Z. “Nonuniform frequency diverse array for range-angle imaging of targets,” *IEEE Sensors J.*, vol. 14, no. 8, pp. 2469-2476, Aug. 2014.

[10] Wang, Z. H., Mu, T. Song, Y. L., et al. “Beamforming of frequency diverse array radar with nonlinear frequency offset based on logistic map,” *Progress in Electromagnetics Research M*, vol. 64, pp. 55-63, 2018.

[11] Nusenu, S. Y. “Transmit/received beamforming for frequency diverse array with symmetrical frequency offsets,” *Advances in Science, Technology and Engineering Systems Journal*, vol. 2, no. 3, pp. 1-6, 2017.

[12] Yang, Y. Q., Wang, H., Wang, H. Q., et al. “Optimization of sparse frequency diverse array with time-invariant spatial-focusing beampattern,” *IEEE Antennas and Wireless Propagation Letters*, vol. 17, no. 2, pp. 351-354, 2018.

[13] Tan, M., Wang, C. Y., Li, Z. H., et al. “Stepped frequency pulse frequency diverse array radar for target localization in angle and range domains,” *International Journal of Antennas and Propagation*, 2018. DOI=http://doi.org/10.1155/2018/8962048.

[14] Wang, W. Q., and Shao, H. Z. “Range-angle localization of targets by a double-pulse frequency diverse array radar,” *IEEE Journal on Selected Topics in Signal Processing*, vol. 8, no. 1, pp. 106-144, 2014.