Moduli stabilization with non-geometric fluxes
—
comments on tadpole contributions
and de-Sitter vacua

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Abstract
We study moduli stabilization for type IIB orientifold compactifications on Calabi-Yau three-folds with (non-)geometric fluxes. For this setting it is possible to stabilize all closed-string moduli classically without the need for non-perturbative contributions, and examples of stable de-Sitter constructions can be found in the literature which violate a prominent swampland conjecture.
In this paper we derive general properties of non-geometric flux-compactifications, we argue that the contribution of fluxes to the tadpole cancellation conditions should be similar to D-branes (and not anti-D-branes), and we exclude supersymmetric Minkowski vacua for certain cases. We also reassess known stable de-Sitter constructions with (non-)geometric $H$-, $F$-, $Q$- and $R$-fluxes and argue that these are not consistent in string theory.
Constructing stable de-Sitter vacua in string theory is notoriously difficult. An overview of existing de-Sitter constructions and discussions of their validity can be found in the reviews [1] and [2], but to the best of our knowledge no generally-accepted, fully-consistent and explicitly worked-out example of a stable de-Sitter vacuum in string theory is known. This observation may have led the authors of [3–5] to conjecture that de-Sitter vacua cannot be realized in string theory, or any consistent theory of quantum gravity. As a potential counter-example to this conjecture non-geometric flux-compactifications are often mentioned, which are the topic of this work.

Non-geometric backgrounds are configurations which cannot be described in terms of Riemannian geometry. Such spaces are inconsistent for point particles but are well-defined for strings, and for these backgrounds the transition functions between local charts are required to include T-duality transformations [6]. For a review of this topic we refer to [7]. The standard example for a non-geometric background is obtained by applying successive T-duality transformations to a three-
torus with $H$-flux, leading to a twisted torus with geometric $F$-flux \cite{8,9}, a T-fold with non-geometric $Q$-flux \cite{6} and a non-associative space with a non-geometric $R$-flux \cite{10}. When compactifying string theory in the presence of fluxes, the $H$-flux generates the familiar Gukov-Vafa-Witten superpotential \cite{11} in the lower-dimensional theory. Applying then T-duality transformations leads to contributions of the geometric $F$- and of the non-geometric $Q$- and $R$-fluxes \cite{10,12,13}, but an uplift of these four-dimensional theories to ten-dimension is often not known. However, we note that certain compactifications with geometric and non-geometric fluxes fit into the framework of $SU(3) \times SU(3)$-structure compactifications which have been studied in \cite{14–17}.

Coming back to de-Sitter vacua, for compactifications including positive-tension objects such as D-branes the Maldacena-Nunez no-go theorem excludes the existence of de-Sitter vacua \cite{18}. However, this no-go theorem does not hold in the presence of orientifold planes or for non-geometric backgrounds, and the purpose of the present work is to investigate the latter case. We focus on type IIB string-theory compactifications on Calabi-Yau orientifolds with $O3$-/O7-planes and (non-)geometric $H$, $F$, $Q$- and $R$-fluxes which, in the context of de-Sitter vacua, have been considered before for instance in \cite{19–29}. Unfortunately, we were neither able to show that stable de-Sitter vacua from non-geometric fluxes cannot exist — nor able to construct a consistent de-Sitter model for this setting. Nevertheless, we made progress in understanding non-geometric flux vacua and, in particular, we argue that all stable de-Sitter constructions with $H$, $F$, $Q$- and $R$-fluxes known to us are inconsistent in string theory. Our findings are reported in the present paper, which is organized as follows:

- In section 2, we review type IIB Calabi-Yau orientifold compactifications with $O3$-/O7-planes and non-geometric $H$, $F$, $Q$- and $R$-fluxes. This section contains no new results, but it clarifies for instance how non-geometric fluxes should be quantized and it may serve as a modern introduction to the topic.

- In section 3, we discuss how (non-)geometric fluxes contribute to the tadpole cancellation conditions. By requiring the rank of the four-dimensional gauge group to be bounded (in agreement with \cite{30}), we present arguments that fluxes should contribute in the same way as supersymmetric D-branes and not as anti-D-branes. This condition excludes a known de-Sitter construction with non-geometric fluxes.

- In section 4, we study the scalar potential induced by non-geometric fluxes. We derive a simple expression for the scalar potential at the minimum, and we derive the necessary conditions $h_1^{1,1} \leq h_+^{2,1}$ and $h_-^{2,1} = 0$ on the Hodge numbers for stabilization of all closed-string moduli.
In section 5 we specialize to the situation $h^{1,1} = h^{2,1}$. We rewrite the scalar potential and eliminate the contribution of the Ramond-Ramond three-form flux in favor of tadpole charges, which is a form suitable for computer-aided scans for vacua. We furthermore show that supersymmetric Minkowski vacua do not exist and we discuss necessary conditions for stable de-Sitter minima.

In section 6 we review known de-Sitter constructions with non-geometric fluxes. For the stable models which in addition to the Ramond-Ramond three-form flux only involve $H_-, F_-, Q_-$ and $R$-fluxes we conclude – using the constraint from section 3 – that none of them are consistent in string theory.

In section 7 we summarize our findings.

## 2 Type IIB compactifications with fluxes

We start by introducing our notation and conventions for compactifications of type IIB string theory on Calabi-Yau orientifolds with fluxes. This section contains no new results, and for the reader familiar with the topic it is safe to only skim over the formulas and move to the next section.

### 2.1 Type IIB orientifolds with O3-/O7-planes

In this subsection we summarize some properties of type IIB orientifold compactifications with O3- and O7-planes which are relevant for our subsequent discussion.

**Cohomology**

We consider type IIB string theory on $\mathbb{R}^{3,1} \times X$ subject to an orientifold projection of the form $(-1)^{F_L} \Omega_P \sigma$. $F_L$ denotes the left-moving fermion number and $\Omega_P$ is the world-sheet parity operator, and their action on the type IIB field content can be found for instance in [31]. The compact space $X$ is chosen to be a Calabi-Yau three-fold, and we impose a holomorphic involution $\sigma$ on $X$ such that its Kähler and holomorphic three-form satisfy $\sigma^* J = +J$ and $\sigma^* \Omega = -\Omega$. The fixed loci of $\sigma$ in $\mathbb{R}^{3,1} \times X$ correspond to O3- and O7-planes.

Since $\sigma$ is an involution, the cohomology groups of $X$ split into even and odd eigenspaces as $H^{p,q}(X) = H^{p,q}_+(X) \oplus H^{p,q}_-(X)$ with dimensions $h^{p,q} = h^{p,q}_+ + h^{p,q}_-$. For the even cohomology groups of $X$ we introduce bases in the following way

\begin{align}
\omega_0 & \in H^{3,3}_+(X), \\
\omega_a & \in H^{1,1}_+(X), \\
\sigma^a & \in H^{2,2}_+(X), \\
\sigma^\alpha & \in H^{2,2}_-(X), \\
\sigma^0 & \in H^{0,0}_+(X), \\
\omega_a & \in H^{1,1}(X), \\
\sigma^a & \in H^{2,2}(X), \\
\sigma^\alpha & \in H^{2,2}(X), \\
\sigma^0 & \in H^{0,0}(X), \\
\end{align}

(2.1)
| fields                        | multiplicity | symbol   |
|------------------------------|--------------|----------|
| four-dimensional metric      | 1            | g        |
| $U(1)$ vector fields         | $h^{2,1}_{\pm}$ | $U^\lambda$ |
| complex-structure moduli     | $h^{2,1}_-$  | $z^i$    |
| axio-dilaton                 | 1            | $\tau$  |
| Kähler moduli                | $h^{1,1}_{\pm}$ | $T_a$ |
| axionic moduli               | $h^{1,1}_-$  | $G^\alpha$ |

Table 1: Bosonic massless field content corresponding to the closed-string sector after compactifying type IIB string theory on Calabi-Yau orientifolds with O3-/O7-planes.

which can be chosen such that the only non-vanishing pairings satisfy the relations

\[ \int_X \omega_0 \wedge \sigma^0 = 1, \int_X \omega_a \wedge \sigma^b = 1 \text{ and } \int_X \omega_\alpha \wedge \sigma^\beta = \delta_\alpha^\beta. \]

For the third cohomology group we introduce symplectic bases as

\[ \{ \alpha_I, \beta_J \} \in H^3(X), \quad I = 0, \ldots, h^{2,1}_-, \]

\[ \{ \alpha_\lambda, \beta_\lambda \} \in H^3_+(X), \quad \lambda = 1, \ldots, h^{2,1}_+ \]  \hspace{1cm} (2.2)

with the only non-vanishing pairings \( \int_X \alpha_I \wedge \beta_J = \delta_I^J \) and \( \int_X \alpha_\kappa \wedge \beta_\lambda = \delta_\kappa^\lambda \).

**Moduli**

The effective theory obtained after compactification can be described in terms of four-dimensional \( \mathcal{N} = 1 \) supergravity. The bosonic field content originating from the closed-string sector is summarized in Table 1 and for details we refer to [32]. The complex-structure moduli \( z^i \) with \( i = 1, \ldots, h^{2,1}_- \) are encoded in the holomorphic three-form \( \Omega \), while the axio-dilaton, Kähler moduli and axionic moduli are contained in the multiform [33]

\[ \Phi = e^B C + i \text{ Re} \left( e^{-\phi} \lambda \left[ e^{B-iJ} \right] \right). \]  \hspace{1cm} (2.3)

Here, \( C \) denotes the sum over all Ramond-Ramond potentials \( C = \sum_p C_p \), \( B \) denotes the Kalb-Ramond field and we introduced an operator \( \lambda \) acting on a \( p \)-form \( A_{(p)} \) as \( \lambda [A_{(p)}] = (-1)^\left[ \frac{p}{2} \right] A_{(p)} \) with \([\ldots]\) denoting the integral part. Expanding \( \Phi \) in the bases (2.1) defines the moduli in the following way

\[ \Phi = \tau + G^\alpha \omega_\alpha + T_a \sigma^a, \]  \hspace{1cm} (2.4)

and the precise form of \( \tau, G^\alpha \) and \( T_a \) can be found for instance in [32] or by working out the definition of \( \Phi \). For later convenience we group the Kähler-sector moduli
together as

\[ T^A = (\tau, G^\alpha, T_a), \quad T^A = c^A + i\tau^A, \]  

(2.5)

where \( c^A \) and \( \tau^A \) denote the real and imaginary parts of \( T^A \) and where we introduced a collective index \( A = 0, \ldots, h^{1,1} \).

**Kähler potential**

The moduli-space geometry is captured by a Kähler potential \( K \), which splits into the Kähler sector containing the axio-dilaton, Kähler moduli and axionic moduli and the complex-structure sector as follows

\[ K = K_K + K_{cs}, \]

\[ K_K = -\log \left[ -i (\tau - \overline{\tau}) \right] - 2 \log \left[ \mathcal{V} + \xi \right], \]

\[ K_{cs} = -\log \left[ +i \int_X \Omega \wedge \overline{\Omega} \right]. \]

(2.6)

For the Kähler sector we note that the Einstein-frame volume of the Calabi-Yau three-fold is denoted by \( \mathcal{V} \) which depends implicitly on \( \tau, G^\alpha \) and \( T_a \), and we included \( \alpha' \)-corrections encoded in \( \xi = -\frac{\xi(3\chi(X))(\tau - \overline{\tau})^{3/2}}{2(2\pi)^3(2i)^{3/2}} \) [33]. In the following we will employ the notation

\[ K_A = \partial_{T^A} K, \quad G_{\overline{A}B} = \partial_{\overline{T}^A} \partial_{T^B} K, \]

\[ G^A_{\overline{B}} = \frac{1}{4} \delta^A_B, \quad G^{AB} \equiv (G^{-1})^{AB}, \]

(2.7)

and from (2.6) we can determine the relations

\[ K^A = -(T - \overline{T})^A, \quad K_A G^{A\overline{B}} K_{\overline{B}} = 4. \]

(2.8)

We emphasize that our analysis below depends on (2.8), which we have checked explicitly for the \( \alpha' \)-corrected Kähler potential \( K_K \) shown in (2.6). However, we do not know whether (2.8) also holds for the full quantum expression.

In the complex-structure sector \( \Omega \) denotes again the holomorphic three-form of \( X \), and the Kähler covariant derivatives \( D_i = \partial_i + \partial_i K \) acting on \( \Omega \) provide a basis \( \chi_i = D_i \Omega \) for \( H^2(X) \). In addition to (2.2), an alternative basis for \( H^3(X) \) is therefore given by \( \{ \Omega, \chi_i, \overline{\chi}_i, \overline{\Omega} \} \) where the bar denotes complex conjugation. A general three-form can then be expanded as

\[ A = a^0 \Omega + a^i \chi_i + \overline{a}^i \overline{\chi}_i + a^0 \overline{\Omega}, \]

(2.9)

and, with the Hodge-star operator acting as \( *\Omega = -i \Omega \) and \( *\chi_i = +i \chi_i \), for two three-forms \( A \) and \( B \) we obtain the relation

\[ a^\overline{\Omega} + a^i G^{ij}_\overline{\delta} b = \frac{e^{K_{cs}}}{2} \int_X A \wedge (B - i B). \]

(2.10)
2.2 Fluxes

To generate a potential for the moduli in the effective theory we introduce fluxes, and to preserve four-dimensional Poincaré invariance they are chosen to extend only along the compact space. The type of fluxes we consider in the Ramond-Ramond (R-R) and Neveu-Schwarz-Neveu-Schwarz (NS-NS) sectors are

\[
\begin{align*}
\text{R-R sector:} & \quad F_3, \\
\text{NS-NS sector:} & \quad H, F, Q, R.
\end{align*}
\] (2.11)

(Non-)geometric fluxes

In the NS-NS sector \(H\) denotes the familiar \(H\)-flux, and its T-dual completions are the geometric \(F\)-flux and the non-geometric \(Q\)- and \(R\)-fluxes \([8,9,6,10]\). A detailed discussion of non-geometric fluxes can be found for instance in \([7]\), but here we only note that they can be interpreted as operators acting on the cohomology as

\[
\begin{align*}
H \wedge & : \text{p-form} \to (p + 3)\text{-form}, \\
F & : \text{p-form} \to (p + 1)\text{-form}, \\
Q & : \text{p-form} \to (p - 1)\text{-form}, \\
R & : \text{p-form} \to (p - 3)\text{-form}.
\end{align*}
\] (2.12)

These operators can be conveniently summarized using a generalized derivative of the form \([10]\)

\[
D = d + H \wedge + F + Q \cdot + R,
\] (2.13)

where \(d\) is the ordinary exterior derivative. We furthermore note that \(H, Q\) and \(F_3\) are odd under the action of the combined world-sheet parity and left-moving fermion number \(\Omega_p(-1)^{F_L}\) whereas \(F\) and \(R\) are even \([10,35]\). When acting with \(D\) on the cohomology bases shown in (2.1) we obtain

\[
\begin{align*}
D(\sigma^0, \omega, \sigma^a) & \equiv (H, F_a, Q^a) \equiv \Xi_A \in H^3_-(\mathcal{X}), \\
D(\omega^0, \sigma^0, \omega_a) & \equiv (R, Q^a, F_a) \equiv \Theta^A \in H^3_+\mathcal{X},
\end{align*}
\] (2.14)

where we employ the same collective index \(A\) as in \([2.5]\). Note also that the three-forms \(F_3, \Xi_A\) and \(\Theta^A\) can be expanded in the integral bases \([2.2]\), for which the expansion coefficients are quantized due to the familiar flux-quantization condition. This includes in particular the quantization of non-geometric fluxes.
Bianchi identities and tadpole cancellation condition

In our setting we assume the absence of localized NS-NS sources such as NS5-branes, Kaluza-Klein monopoles or non-geometric branes, and hence the Bianchi identities for the NS-NS fluxes can be expressed as $D^2 = 0 \ [10, 15]$. Using the three-forms defined in (2.14), these conditions can be written as

$$
\int_X \Xi_A \wedge \Xi_B = 0, \quad \int_X \Theta^A \wedge \Theta^B = 0, \quad \Xi_A \otimes \Theta^A = 0. \quad (2.15)
$$

Turning to the R-R sector, performing an orientifold projection typically gives rise to orientifold planes and requires the introduction of D-branes. Both are charged under the R-R potentials and therefore the Bianchi identities for the R-R field strengths are in general non-trivial. Their integrated versions – or equivalently their formulation in cohomology – are known as the tadpole cancellation conditions and read (see for instance [7] for more details of this formulation)

$$
DF = \sum_{Dp+Dp'} Q_{Dp} + \sum_{Op} Q_{Op}, \quad (2.16)
$$

where the combined R-R field strength is given by $F = DC$, $Q_{Dp}$ and $Q_{Op}$ are multi-forms encoding the D-brane and O-plane charges, and the sums are over all D-branes, their orientifold images and O-planes present in the background. We discuss these conditions in more detail in section 3 below. In our situation the only non-trivial R-R flux is $F_3$ and we obtain for the left-hand side in (2.16)

$$
DF = N^0_0 \omega_0 + N^\alpha_0 \sigma^\alpha + N^a_0 \omega_a, \quad (2.17)
$$

where

$$
N_A = \int_X \Xi_A \wedge F_3 = \left( (H \wedge F_3)^0, \ (F \circ F_3)_{\alpha}, \ (Q \bullet F_3)^a \right) \quad (2.18)
$$

are the contributions of the $H$-, $F$- and $Q$-fluxes to the D3-, D5- and D7-brane tadpoles, respectively. We will refer to the $N_A$ also as tadpole charges, and we note that since the $\Xi_A$ and $F_3$ are quantized the $N_A$ take integer values.

2.3 Scalar potential

The fluxes introduced above generate a scalar potential $V = V_F + V_D$ in the effective four-dimensional $\mathcal{N} = 1$ supergravity theory. The F-term potential $V_F$ is expressed in terms of a superpotential $W$ and the Kähler potential (2.6) as

$$
V_F = e^K \left[ D_1 W G^{\mathcal{I}} D_2 \overline{W} + D_A W G^{A\overline{B}} D_{\overline{B}} \overline{W} - 3|W|^2 \right], \quad (2.19)
$$
where $D_i W$ and $D_A W$ are the Kähler-covariant derivatives and $G^\mathcal{J}$ and $G^{\mathcal{A}\mathcal{B}}$ denote the inverse Kähler metrics in the complex-structure and Kähler sector, respectively. The fluxes generate a generalization of the Gukov-Vafa-Witten superpotential of the form $W = \int_X \Omega \wedge (F_3 - \mathcal{D} \Phi)$ which, using (2.14) and (2.4), can be expressed as

$$W = \int_X \Omega \wedge G, \quad G = F_3 - \Xi^A T^A. \quad (2.20)$$

The D-term potential in the effective theory takes the form $V_D = [\text{Re} f]^{-1}|\kappa\lambda|D_\kappa D_\lambda$, where $f_{\kappa\lambda}$ is the gauge-kinetic function for the $U(1)$ vector fields shown in table 1. We emphasize that $f_{\kappa\lambda}$ only depends on the complex-structure moduli, and we note that the D-term potential is generated by the fluxes $\Theta^A$ [36,37,35]. It can be written as

$$V_D = 2 \int_X (\tilde{\Theta}^A K_A) \wedge *(\tilde{\Theta}^{\mathcal{B}} K^{\mathcal{B}}), \quad (2.21)$$

where, due to the self-duality of the R-R four-form $C_4$ (see [32] for details), only the restricted three-forms $\tilde{\Theta}^A = -\Theta^A \chi^\beta$ appear.

### 3 Tadpole charges

In equation (2.18) we have defined tadpole charges $N_A$, which play an important role for moduli stabilization. They connect the open- and closed-string sectors to each other and give rise to strong restrictions (see for instance [38–46] for recent discussions of this question). In this section we present arguments that the $N_A$ should contribute to the tadpole cancellation conditions (2.16) in the same way as D-branes and not as anti-D-branes. This implies

$$N_A \leq 0. \quad (3.1)$$

#### D-branes and orientifold planes

Let us start by extending our review of type IIB orientifold compactifications from section 2.1. When performing an orientifold projection of the form $(-1)^{F_L} \Omega_P \sigma$ the fixed loci of $\sigma$ correspond to orientifold planes which fill-out four-dimensional space-time $\mathbb{R}^{3,1}$ and wrap cycles $\Gamma_{Op}$ in the compact space $\mathcal{X}$. In our case these are O3- and O7-planes which are point-like and are wrapping four-cycles in $\mathcal{X}$, respectively. When orientifold planes are present then D-branes can be introduced, which in our case are D3- and D7-branes filling-out four-dimensional space-time $\mathbb{R}^{3,1}$ and wrapping cycles $\Gamma_{Dp}$ in $\mathcal{X}$. Additionally, D-branes can have a non-trivial
The gauge flux $\mathcal{F}_{Dp} = F_{Dp} + B \in H^2(\Gamma_{Dp})$ on their world-volume, where $F_{Dp}$ denotes the quantized open-string gauge flux and $B$ is understood as pulled-back to the D-brane. We do not consider discrete torsion for the $B$-field.

The data characterizing D-branes and O-planes can be encoded in the charges $Q_{Dp}$ and $Q_{Op}$ which already appeared in (2.16). These are multi-forms defined as

$$Q_{Dp} = \lambda \left[ [\Gamma_{Dp}] \wedge \text{tr} (e^{F_{Dp}}) \wedge \sqrt{\hat{A}(\mathcal{R}_T)} \right],$$

$$Q_{Op} = Q_p \lambda \left[ [\Gamma_{Op}] \wedge \sqrt{\mathcal{L}(\mathcal{R}_T/4)} \sqrt{\mathcal{L}(\mathcal{R}_N/4)} \right],$$

where $[\Gamma_{Dp}]$ is the Poincaré dual of the cycle wrapped by the D-brane, $F_{Dp}$ denotes the open-string gauge-flux and the trace is over the fundamental representation. The expressions $\mathcal{R}_T$ and $\mathcal{R}_N$ stand for the restrictions of the curvature two-form $\mathcal{R}$ to the tangent and normal bundle of $\Gamma_{Dp}$, and we used the $\mathcal{A}$-genus and the Hirzebruch polynomial $\mathcal{L}$. The charge $Q_p$ of the orientifold planes is given by $Q_p = -2^{p-4}$, and the operator $\lambda$ was introduced below equation (2.3).

**Calibrations**

In order for D-branes to preserve $\mathcal{N} = 1$ supersymmetry, calibration conditions have to be satisfied. In our situation they are of the form [47] (see for instance [48] for a review)

$$\text{Vol}(\Gamma_{Dp}) = e^{i\theta_{Dp}} \int_{\Gamma_{Dp}} e^{-\phi} \lambda \left[ \text{tr} (e^{F_{Dp}}) \wedge e^{-iJ} \right],$$

where the volume is computed using the DBI action and $J$ denotes the Kähler form of $\mathcal{X}$. The operator $\lambda$ was introduced below equation (2.3), and $\theta_{Dp}$ is an up to now undetermined phase which selects a particular $\mathcal{N} = 1$ combination of $\mathcal{N} = 2$ supercharges. Turning to orientifold planes, since the involution $\sigma$ is holomorphic, the cycles $\Gamma_{Op}$ are holomorphic and satisfy the calibration conditions

$$\text{Vol}(\Gamma_{Op}) = e^{i\theta_{Op}} \int_{\Gamma_{Op}} e^{-\phi} \lambda \left[ e^{-iJ} \right], \quad \theta_{Op} = 0.$$

Note that since a gauge flux $\mathcal{F}_{Dp}$ is odd under the orientifold projection $\sigma$ it vanishes when restricted to the fixed-point set of $\sigma$. We can therefore trivially extend (3.4) by $\text{tr} (e^{F_{Dp}})$ and compare with (3.3). In order for all D-branes and O-planes to preserve the same $\mathcal{N} = 1$ supersymmetry, we then have to require

$$\theta_{Dp} = \theta_{Op} = 0.$$

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1. It would be desirable to include curvature terms in the calibration condition, but we are not aware of any work addressing this question.
Tadpole cancellation conditions

The tadpole cancellation conditions have been stated schematically already in (2.16), but here we want to make them more precise. For our setting with O3-/O7-planes and D3-/D7-branes the tadpole cancellation conditions can be written in the following way (see for instance [49] for details of the derivation)

\[ N^a \omega_a = \sum_{D7_i} N_{D7_i} \left( [\Gamma_{D7_i}] + [\Gamma'_{D7_i}] \right) - 8 \sum_{O7_j} [\Gamma_{O7_j}], \]

\[ N_a \sigma^a = - \sum_{D7_i} \left( \text{tr} \left[ F_{D7_i} \right] \wedge [\Gamma_{D7_i}] + \text{tr} \left[ F'_{D7_i} \right] \wedge [\Gamma'_{D7_i}] \right), \]

\[ \frac{N^0}{2} = N_{D3} - \frac{N_{O3}}{4} - \sum_{D7_i} \left( \frac{1}{2} \int_{\Gamma_{D7_i}} \left[ F^2_{D7_i} \right] + N_{D7_i} \frac{\chi(\Gamma_{D7_i})}{24} \right) - \sum_{O7_j} \frac{\chi(\Gamma_{O7_j})}{12}, \]

where \( N_{D7_i} \) denotes the number of D7-branes in a stack labelled by \( i \) and \( N_{D3} \) is the total number of D3-branes. Both of these numbers are counted without the orientifold images which are denoted by a prime. Furthermore, \( N_{O3} \) is the total number of O3-planes, and \( \chi(\Gamma) \) denotes the Euler number of the cycle \( \Gamma \).

Gauge groups

Let us also recall that D-branes give rise to gauge theories on their world-volume. In type IIB the corresponding gauge groups are either \( U(N_{Dp}) \), \( SO(2N_{Dp}) \) or \( Sp(2N_{Dp}) \), where \( N_{Dp} \) denotes the number of \( Dp \)-branes on top of each other. Since we can have several such stacks of D-branes wrapping cycles in the compact space, the four-dimensional gauge group is of the schematic form

\[ G = \prod_i U(N_{Dp_i}) \times \prod_j SO(2N_{Dp_j}) \times \prod_k Sp(2N_{Dp_k}) \times U(1)^{l_+^2}, \]

where the last factor corresponds to the closed-string \( U(1) \) gauge fields \( U^\lambda \) mentioned in table I.

Now, in [30] conjectures have been presented which imply that the rank of the total gauge group in a consistent quantum-gravity theory should be bounded from above. This requirement is supported for instance by extensive experience in string-theory model building, and progress in establishing such a bound for different settings has been made for instance in [52–54]. And indeed, in the absence of fluxes we see from (3.6) that the rank of the gauge group \( G \) is bounded by the

\[ \text{In case the D7-branes have double intersection points the corresponding Euler number has to be corrected by the number of pinch points [50,51]. This subtlety will however not be relevant here.} \]
orientifold data. More concretely, for a fixed compactification space and orientifold projection, for supersymmetric D-brane configurations and in the absence of fluxes, the rank of \(G\) is bounded by the number of O-planes, by the Euler numbers of four-cycles in \(X\), and by \(h^{2,1}_+\). Our main requirement is then that also when introducing fluxes, the rank of the gauge group \(G\) remains bounded from above.

**Contribution of closed-string fluxes**

We now want to argue that the closed-string fluxes encoded in the tadpole charges \(N_A\) should contribute in the same way as D-branes to the tadpole cancellation conditions (3.6) and not as anti-D-branes or orientifold planes. Loosely speaking, our main requirement is that in string theory one should not be able to generate arbitrarily-large gauge groups.

- Let us start by considering the D3-brane tadpole condition and assume that we found a configuration of branes and fluxes which solves (3.6c), say for \(N^0 = 0\). A new solution can be obtained by changing the fluxes and number of D3-branes as

  \[ N^0 \rightarrow N^0 + 2\alpha, \quad N_{D3} \rightarrow N_{D3} + \alpha, \quad \alpha \in \mathbb{Z}, \quad (3.8) \]

Since for supersymmetric D-branes \(N_{D3}\) is non-negative (cf. the calibration condition (3.3)), the integer \(\alpha\) is bounded from below. Requiring furthermore the rank of the gauge group to be bounded implies that \(\alpha\) also has to be bounded from above. These restrictions then translate into bounds on \(N^0\). It is not clear to us how to derive an upper bound on \(N^0\) from first principles, but since the tadpole cancellation conditions contain only topological data in the form of integers, the only natural bound is

  \[ N^0 \leq 0. \]

This means that the combination of fluxes contained in \(N^0\) contributes like a D-brane to the tadpole conditions and not like an anti-D-brane or orientifold plane. Note also that (3.9) is of course realized in standard flux compactifications.

- A similar reasoning can be applied to the D7-brane tadpole (3.6a). For simplicity we consider a setting with \(h^{1,1}_- = 0\) and \(\chi(\Gamma_{D7}) = 0\), which is satisfied for instance in toroidal-orbifold compactifications. Let us then again assume a configuration of D7-branes has been found which solves (3.6a). As a physical process this is known as brane-flux transmutation \[55\], however, in general (3.8) will lead to a physically-different configuration solving the tadpole condition (3.6c).
for $N^a = 0$. A new solution can be obtained by replacing $N^a \rightarrow N^a + 2\alpha^a$, where $\alpha^a \in \mathbb{Z}$, and introducing/removing $\alpha^a$ D7-branes without gauge flux wrapping a cycle dual to $\omega_b$. The $\alpha^a$ are bounded from below by the requirement that $N_{D7} \geq 0$, and from above by requiring the rank of the gauge group to be bounded. It is again not clear to us how to derive an upper bound on the $N^a$, but the most natural choice is

$$N^a \leq 0.$$ (3.10)

Let us emphasize that this restriction can be derived from (3.9) via T-duality. From (3.10) we then see that the flux combinations $N^a$ contribute like D-branes to the tadpole conditions and not like anti-D-branes or orientifold planes. Finally, for more general situations with $h^{1,1} \neq 0$ or $\chi(\Gamma_{D7}) \neq 0$ also the D3- and D5-brane tadpole might have to be adjusted, but the main conclusion doesn’t change.

- For the D5-brane tadpole (3.6b) we have to slightly extend our setting by including D5-branes. In this case the corresponding tadpole condition has been determined for instance in [49], and following similar arguments as above we can infer that

$$N_\alpha \leq 0.$$ (3.11)

To summarize, we have argued that in order for the total rank of the gauge group (shown in (3.7)) to be bounded – in agreement with the swampland conjectures made in [30] – the tadpole charges $N_A$ have to be bounded from above. We then identified

$$N_A \leq 0$$ (3.12)

as the most natural choice for such a bound, which is consistent with known constructions as well as with T-duality. This implies that fluxes should contribute in the same way as D-branes to the tadpole cancellation conditions.

**Remarks**

We close this section with the following comments and remarks concerning the tadpole charges $N_A$:

- Let us recall the general form of the tadpole cancellation conditions (2.16), multiply both sides with $e^{-\phi} \lambda [e^{B+iJ}]$ and integrate over $\mathcal{X}$. Using the definition of the moduli shown in (2.4) and the calibration conditions (3.3) and
\( (3.4) \), we obtain
\[
\frac{N_A \tau^A}{2} = e^{-\phi} N_{D3} + \sum_{D_7} \left[ \text{Vol}(\Gamma_{D_7}) - e^{-\phi} N_{D_7} \frac{\chi(\Gamma_{D_7})}{24} \right] - e^{-\phi} N_{O3} \frac{4}{4} - \sum_{O_7} \left[ 4 \text{Vol}(\Gamma_{O_7}) + e^{-\phi} \frac{\chi(\Gamma_{O_7})}{12} \right].
\] (3.13)

Our requirement of having the closed-string fluxes contributing in the same way as D-branes to the tadpole conditions (the first line on the right-hand side) then implies
\[
N_A \tau^A \leq 0.
\] (3.14)

- In the absence of non-geometric fluxes, the presence of O7-planes requires the introduction of D7-branes via the tadpole cancellation condition. However, with non-vanishing Q-flux we can obtain configurations without D-branes. In particular, in the absence of D-branes the tadpole conditions (3.6) read
\[
N^a \omega_a = -8 \sum_{O_7} |\Gamma_{O_7}|, \quad N_\alpha = 0, \quad N^0 = -\frac{N_{O3}}{2} - \sum_{O_7} \frac{\chi(\Gamma_{O_7})}{6},
\] (3.15)
which satisfy the requirement \( N_A \leq 0 \). Note that especially with respect to stabilizing all moduli, this is a desirable situation since no open-string moduli (typically coming with the introduction of D-branes) need to be stabilized.

- Our requirement \( N_A \leq 0 \) for the tadpole charges is restrictive but has non-trivial solutions. Indeed, in [38] we studied supersymmetric AdS vacua for the \( T^6/Z_2 \times Z_2 \) type IIB orientifold and found solutions with \( N_A \leq 0 \) and all moduli stabilized (see equation (5.14) in [38] and note that our conventions are related as \( N_{\text{here}} = -Q_{\text{there}} \)). Similarly, also some of the AdS vacua found in [56] satisfy the requirement \( N_A \leq 0 \).

4 Moduli stabilization

In this section we discuss some general features of moduli stabilization with non-geometric fluxes. This question has been studied before, for instance in [10,13,57, 19,20,23,24,27] for toroidal orbifold compactifications and in [16,58,62] for more general settings. Our results in this section are the following: in subsection 4.1 we determine for Calabi-Yau orientifold compactifications the form of the scalar potential at an extremum as
\[
V \bigg|_{\text{ext}} = -\frac{e^{K_k}}{2} \left[ \int_X (\text{Re } \mathcal{G} \wedge \star \text{Re } \mathcal{G}) + N_A \tau^A \right]_{\text{ext}},
\] (4.1)
where $G$ denotes the three-form flux defined in (2.20), $\tau^A = \text{Im} T^A$ and $N_A$ denotes the tadpole charges. This surprisingly simple form will be employed later on. In subsection 4.2.2 we show that in order to stabilize all moduli by non-geometric fluxes, the Hodge numbers have to satisfy

$$h^{1,1}_- \leq h^{2,1}_-, \quad h^{2,1}_+ = 0.$$ (4.2)

4.1 Non-geometric flux vacua

In this section we discuss necessary conditions for (non-)geometric flux vacua for the setting introduced in section 2. We first determine an expression for extrema of the scalar potential and derive a necessary relation for stable minima.

Rewriting the F-term potential

The scalar potential in $\mathcal{N} = 1$ supergravity is given by $V = V_F + V_D$, where the D-term potential has been given in (2.21). The F-term potential (2.19) for the superpotential (2.20) can be worked out using the relation (2.10) as

$$V_F = e^K \left[ \frac{e^{-K_{ca}}}{2} \int_X G \wedge (\ast \bar{G} + i \bar{G}) + e^{-2K_{ca}} \Xi^\Omega \Xi^\omega \Xi^\varphi \Xi^\psi \right],$$ (4.3)

where the three-form flux reads $G = F_3 - T^A \Xi^A$ and where $\Xi^\Omega$ and $\Xi^\varphi$ denote the $\Omega$ and $\varphi$ components of $\Xi^A$ and $\Xi^\psi$ as in (2.9). These components together with the matrix $\omega^{AB}$ are given by

$$\Xi^\Omega = i e^{K_{ca}} \int_X \Omega \wedge \Xi^A, \quad \omega^{AB} = G^{AB} - K^A K^B,$$ (4.4)

where the former only depends on the complex-structure moduli and the latter only depends on the imaginary parts $\tau^A$ of the Kähler-sector moduli. (In equation (4.21) we show the form of $\omega^{AB}$ for a particular example.) In order to make (4.3) more symmetric we can use $\ast^2 = -1$ when acting on three-forms on an Euclidean six-dimensional space and rewrite the scalar potential as

$$V_F = e^K \left[ \frac{e^{-K_{ca}}}{4} \int_X (G + i \ast G) \wedge (\ast G - i \ast \bar{G}) + e^{-2K_{ca}} \Xi^\Omega \Xi^\omega \Xi^\varphi \Xi^\psi \right].$$ (4.5)

When ignoring $\alpha'$-corrections to the Kähler potential and only considering $H$-flux, the second term in (4.5) vanishes. This corresponds to the situation studied by Giddings, Kachru and Polchinski in [63].
Some identities

In order to proceed, we determine some identities for derivatives of the Kähler potential and the Kähler metric in the Kähler-moduli sector. We first note that the expression $K_A$ and $K^A$ are purely imaginary and hence $K_A = -K^A$ and $K^A = -K_A$. Using then the relations shown in equation (2.8), we compute

$$K^C \partial_C K^A = K^C (-\delta^A_C) = -K^A,$$

$$K^C \partial_C K^\overline{A} = K^C (+\delta^\overline{A}_C) = -K^\overline{A},$$

and in a similar way we determine for the first derivatives of the Kähler potential

$$K^C \partial_C K_A = -K^C G_{C\overline{A}} = +K_A,$$

$$K^C \partial_C K_{\overline{A}} = +K^C G_{C\overline{A}} = +K_{\overline{A}}.$$  (4.7)

Furthermore, for the Kähler metric $G_{AB}$ and for the matrix $\omega^{AB}$ defined in (4.4) we compute

$$K^C \partial_C G_{AB} = K^C \partial_A G_{CB} = \partial_A (K^C G_{CB}) - (\partial_A K^C) G_{CB} = +2G_{AB},$$

$$K^C \partial_C \omega^{AB} = -G^{A\overline{M}} (K^C \partial_C G_{\overline{M}\overline{N}}) G^{\overline{N}B} + 2K^A K^\overline{B} = -2 \omega^{AB}.$$  (4.8)

A necessary condition for extrema

In order to obtain extrema of the combined potential $V = V_F + V_D$ we have to solve $\partial_i V = 0$ and $\partial_A V = 0$, where $i$ and $A$ label the complex-structure and combined Kähler moduli. These are in general complicated equations which are typically solved numerically, however, let us consider the necessary condition

$$0 = K^A \partial_A V.$$  (4.9)

With the help of the relations (4.6), (4.7) and (4.8) we find that the D-term potential satisfies $K^A \partial_A V_D = 2V_D$, and for the full potential we obtain

$$0 = K^A \partial_A V = 2V + \frac{e^{K_K}}{2} \int_X (G + \overline{G}) \wedge (\ast G + i \overline{G}).$$  (4.10)

Note that the last term in (4.10) contains an imaginary contribution which, since the scalar potential $V$ is real, implies

$$0 = \int_X (G + \overline{G}) \wedge \ast (G - \overline{G}).$$  (4.11)

We now use this relation together with the Bianchi identity (2.15) and the definition of $N_A$ and $\tau^A$ given in (2.18) and (2.5) to evaluate the scalar potential at the extremum as

$$V \bigg|_{\text{extr}} = -\frac{e^{K_K}}{2} \left[ \int_X (\text{Re} G \wedge \ast \text{Re} G) + N_A \tau^A \right]_{\text{extr}}.$$  (4.12)
A necessary condition for stability

Next, we derive a necessary condition for stability of the extremum. The combined mass matrix for the Kähler-sector moduli $T^A$ and complex-structure moduli $z^i$ takes the following general form

$$M^2 = \begin{bmatrix}
  m^2_{AB} & m^2_{AB} & m^2_{A} & m^2_{A} \\
  m^2_{AB} & m^2_{AB} & m^2_{A} & m^2_{A} \\
  m^2_{i} & m^2_{i} & m^2_{i} & m^2_{i} \\
  m^2_{i} & m^2_{i} & m^2_{i} & m^2_{i}
\end{bmatrix},$$

(4.13)

where $m^2_{AB} = \partial_A \partial_B V|_{\text{extr}}$ and similarly for the other entries. For our purpose we do not need to determine the complete mass matrix, but we are only interested in the expression

$$K^A m^2_{AB} K^B = K^A K^B \partial_A \partial_B V|_{\text{extr}}.$$

(4.14)

Using again the relations (4.6), (4.7) and (4.8) we compute

$$K^A K^B \partial_A \partial_B V = K^A \partial_A V + K^B \partial_B (K^A \partial_A V)$$

$$= K^A \partial_A V + 2K^B \partial_B V + \frac{e^{K_K}}{2} \int_X \left[ G \wedge *G + 5G \wedge *G + 2 \overline{G} \wedge * \overline{G} + 3i G \wedge \overline{G} \right].$$

(4.15)

and, employing (4.10) and (4.11), we evaluate (4.14) at the extremum as

$$K^A m^2_{AB} K^B = \left[ -6V + \frac{e^{K_K}}{6} \int_X G \wedge *G \right]|_{\text{extr}}.$$

(4.16)

For a stable minimum of the potential the mass matrix (4.13) has to be positive definite, which implies that the left-hand side of (4.16) has to be positive. This allows us to determine the following necessary condition

$$V|_{\text{min}} < \left. \frac{e^{K_K}}{6} \int_X G \wedge *G \right|_{\text{min}}.$$

(4.17)

Comments on de-Sitter vacua

Let us briefly return to our discussion of the tadpole charges in section 3, where we argued that $N_A \tau^A \leq 0$. From the form of the potential in the minimum (4.12), we see that a necessary condition for a de-Sitter minimum is

$$0 > \int_X (\text{Re} G \wedge * \text{Re} G) + N_A \tau^A.$$

(4.18)
Since the Hodge star is positive definite the first term on the right-hand side is always non-negative, and hence $N_A \tau^A$ has to be negative for a de-Sitter solution. This shows that our requirement (3.14) of having fluxes contributing like D-branes to the tadpole cancellation conditions does not exclude de-Sitter vacua. Furthermore, together with the stability condition (4.17) we find the following constraint at the minimum

$$0 < - \int_X (\text{Re} G \wedge \ast \text{Re} G) - N_A \tau^A < \frac{1}{3} \int_X (\text{Re} G \wedge \ast \text{Re} G + \text{Im} G \wedge \ast \text{Im} G).$$ (4.19)

These inequalities restrict stable de-Sitter minima, but they do not exclude them. We come back to this relation below.

**Remark**

To provide some intuition for the matrix $\omega^{AB}$ defined in (4.4), let us consider the type IIB $\mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with O3-/O7-planes (where we ignore the twisted sectors). This compactification space is characterized by $h^{1,1} = 3$ and $h^{1,1}_- = h^{2,1}_+ = 0$, and the Kähler potential for the combined Kähler-sector moduli is given by

$$K = - \sum_{A=0}^{3} \log \left[ -i (T^A - \overline{T}^A) \right].$$ (4.20)

With $\tau^A = \text{Im} T^A$ the matrix $\omega^{AB}$ in (4.4) then takes the form

$$\omega^{AB} = -4 \begin{bmatrix} 0 & \tau^A \tau^B \\ \tau^A \tau^B & 0 \end{bmatrix}.$$ (4.21)

### 4.2 Constraints on the Hodge numbers

Let us now require that all complex-structure moduli $z^i$ and combined Kähler moduli $T^A$ are stabilized classically by the fluxes introduced in section 2.2. In order to achieve that, Bianchi identities imply constraints on the Hodge numbers which we derive in this section.

**Notation**

To derive constraints on the Hodge numbers for stabilizing all closed-string moduli, it is useful to introduce some more notation. First, for ease of presentation, we define the two numbers

$$d_1 = 2(h^{2,1}_- + 1), \quad d_2 = h^{1,1} + 1.$$ (4.22)
Next, for the symplectic basis \( \{ \alpha_I, \beta^I \} \in H^3(\mathcal{X}) \) shown in (2.2) we define two \( d_1 \times d_1 \) matrices as follows (we suppress indices most of the time)

\[
\mathcal{M} = \int_X \left( \alpha \wedge \beta \right), \quad \mathcal{M}^T = +\mathcal{M},
\]

\[
\eta = \int_X \left( \alpha \wedge \beta \right) = \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}, \quad \eta^T = -\eta,
\]

where \( \mathcal{M} \) is positive definite. We also expand the three-forms \( F_3 \) and \( \Xi_A \) introduced in (2.11) and (2.14) into \( \{ \alpha_I, \beta^I \} \) as

\[
F_3 = (\alpha, -\beta) \hat{F}_3, \quad \hat{F}_3 = \begin{pmatrix} F_{31} \\ F_3^I \end{pmatrix},
\]

\[
\Xi_A = (\alpha, -\beta) \hat{\Xi}_A, \quad \hat{\Xi}_A = \begin{pmatrix} \Xi_I^A \\ \Xi^A_I \end{pmatrix},
\]

and to avoid confusion let us state that vector \( \hat{F}_3 \) has \( d_1 \) integer components and that \( \hat{\Xi} \) is a \( d_1 \times d_2 \) dimensional matrix with integer components.

**Stabilizing the \( c^A \)**

We now turn to the stabilization of the real parts \( c^A \) of the Kähler-sector moduli \( T^A \). The Kähler potential \( K_K \) shown in (2.6), its derivatives \( K_A = \partial_A K \) as well as the Kähler metric \( G_{AB} \) do not depend on the \( c^A \), and in the F-term potential (4.3) we can isolate the \( c^A \) contributions as follows

\[
V_F = e^{K_K} \frac{1}{2} \int_X \left( F_3 - \Xi_A c^A \right) \wedge \star \left( F_3 - \Xi_B c^B \right) + \ldots
\]

\[
= e^{K_K} \frac{1}{2} \left( \hat{F}_3^T - c^T \hat{\Xi} \right) \mathcal{M} \left( \hat{F}_3 - \Xi c \right) + \ldots,
\]

where we used the first Bianchi identity in (2.15) and employed the matrix notation introduced in (4.23) and (4.24). By similar arguments we see that the D-term potential (2.21) does not depend on \( c^A \). The extremum of the combined scalar potential for the \( c^A \) is then determined as

\[
0 = \partial_{c^A} V \implies 0 = \Xi^T \mathcal{M} \hat{F}_3 - (\Xi^T \mathcal{M} \Xi) c,
\]

and in order to stabilize all of the \( c^A \) the \( d_2 \times d_2 \) matrix \( \Xi^T \mathcal{M} \Xi \) has to have maximal rank equal to \( d_2 \). Since \( \mathcal{M} \) is a \( d_1 \times d_1 \)-dimensional matrix, we therefore obtain the restriction

\[
d_2 \leq d_1.
\]
Bianchi identities I

We can however make (4.27) more precise by taking into account the Bianchi identities for the fluxes. To do so, we first perform a singular value decomposition of the flux matrix $\hat{\Xi}$ as

$$\hat{\Xi} = U \Sigma V^T,$$

where $U$ is a $d_1 \times d_1$ dimensional orthogonal matrix, $V$ is a $d_2 \times d_2$ dimensional orthogonal matrix and $\sigma$ is a $d_2 \times d_2$-dimensional diagonal matrix with the singular values of $\hat{\Xi}$ on the diagonal. Note that since we require $(\hat{\Xi}^T M \hat{\Xi})$ to have maximal rank, $\sigma$ has to have maximal rank and therefore is invertible. Turning then to the first Bianchi identity in (2.15), we see that using matrix notation we can write

$$0 = \hat{\Xi}^T \eta \hat{\Xi} \implies 0 = V (\sigma, 0) U^T \eta U \sigma V^T,$$

$$\implies U^T \eta U = \begin{pmatrix} 0 & B \\ -B^T & C \end{pmatrix},$$

where the upper-left block of $U^T \eta U$ has dimensions $d_2 \times d_2$ and $B$ has dimensions $d_2 \times (d_1 - d_2)$. Now, the determinant of the left-hand side in (4.30) is equal to one, and hence also the right-hand side has to have a non-vanishing determinant (equal to one). This is however only possible if $(d_1 - d_2) \geq d_2$, that is $d_2 \leq d_1/2$, which means

$$h^{1,1} \leq h^{2,1}.$$  \hspace{1cm} (4.31)

Hence, in order to stabilize all closed-string moduli by fluxes and satisfy the Bianchi identities, a necessary requirement is given by (4.31).

Bianchi identities II

Let us also consider the third Bianchi identity in (2.15). Expanding the three-form $\Theta^A$ in the basis $\{\alpha, \beta^\lambda\} \in H^3_+ (X)$ similarly as in (4.24) gives rise to a matrix $\Theta$ of dimensions $2h^{2,1}_+ \times (h^{1,1} + 1)$. Using matrix notation we then have

$$0 = \hat{\Xi} \Theta^T \implies 0 = U \Sigma V^T \Theta^T \implies 0 = \Theta^T,$$

where we used that $\hat{\Xi}$ has maximal rank and hence the diagonal matrix $\sigma$ in $\Sigma$ is invertible. We can therefore conclude that when stabilizing all closed-string moduli by fluxes, Bianchi identities imply that $\Theta^A = 0$ and therefore the D-term potential
vanishes. Without loss of generality, for the purpose of moduli stabilization we can therefore restrict ourselves to
\[ h_{2,1}^{-} = 0. \]  
(4.33)

5 The minimal case: \( h_{2,1}^{-} = h_{1,1} \)

We now consider the situation \( h_{2,1}^{-} = h_{1,1} \) and \( h_{2,1}^{+} = 0 \), which is the minimal case for stabilizing all closed-string moduli by fluxes. In this section we discuss the following questions:

- In section 5.1 we rewrite the scalar potential and replace the R-R three-form flux \( F_3 \) by the tadpole charges \( N_A \). This formulation is suitable for performing computer-based scans for flux vacua.
- In section 5.2 we show that supersymmetric Minkowski vacua with all moduli stabilized by fluxes are always singular and should be excluded, and
- in section 5.3 we discuss moduli stabilization of the radial Kähler-sector modulus and determine restrictions for de-Sitter vacua.

5.1 Manifest tadpole charges

We start by showing how, after the moduli \( c^A = \text{Re} \, T^A \) have been stabilized, the R-R three-form flux \( F_3 \) can be replaced by the tadpole charges \( N_A \). Such a rewriting is useful for eliminating the dependence on \( F_3 \) in computer-based searches for flux vacua.

Singular value decomposition

Let us recall and expand our discussion from section 4.2. In equation (4.28) we have performed a singular value decomposition of the flux matrix \( \hat{\Xi} \) as
\[ \hat{\Xi} = U \Sigma V^T, \quad \Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & 0 \end{pmatrix}, \]  
(5.1)
where \( \sigma \) is an invertible diagonal matrix. Here we specialize to the situation \( h_{1,1} = h_{2,1}^{-} \), and for ease of presentation we use \( d = h_{1,1} + 1 = h_{2,1}^{-} + 1 \). The orthogonal \( 2d \times 2d \) matrix \( U \) can be expressed in terms of two \( 2d \times d \) blocks \( u_1 \) and \( u_2 \), and we introduce the notation
\[ U = (u_1, u_2), \quad U^T \hat{F}_3 = \begin{pmatrix} f^1 \\ f^2 \end{pmatrix}, \quad U^T M U = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \]  
(5.2)
where the $2d \times 2d$ matrix $\mathcal{M}$ was defined in (4.23) and the $2d$ vector $\hat{F}_3$ was defined in (4.24). Note that since $\mathcal{M}$ is positive definite, also $\mathcal{M}_{11}$ is positive definite. Furthermore, we have argued before that $\hat{\Xi}$ has to satisfy the Bianchi identity $0 = \hat{\Xi}^T \eta \hat{\Xi}$, which puts restrictions on $u_1$ and $u_2$. In particular, in the present situation we find that

$$U^T \eta U = \begin{pmatrix} 0 & b \\ -b^T & 0 \end{pmatrix}, \quad \det b = \pm 1,$$

(5.3)

where $b = u_1^T \eta u_2$ is an invertible $d \times d$ matrix, and the lower-right block vanishes by requiring $(U^T \eta U)^2 = 1$. With a bit of algebra we can then compute

$$\int_X \operatorname{Re} G \wedge \ast \operatorname{Re} G = F^T M_{11} F + N^T m^{-1} N,$$

$$\int_X \operatorname{Im} G \wedge \ast \operatorname{Im} G = \tau^T m \tau,$$

(5.4)

where $N_A$ denotes again the vector of tadpole charges, where $\tau^A = \Im T^A$ are the imaginary parts of the combined Kähler-sector moduli, and where we defined the $d \times d$ matrix and $d$-vector

$$m = \Xi^T \mathcal{M} \Xi, \quad F = f^1 + M_{11}^{-1} M_{12} f^2 - \sigma V^T c.$$

(5.5)

Stabilizing the $c^A$

Next, we turn to the scalar F-term potential shown in (4.3). As mentioned already in section 4.2, the moduli $c^A = \Re T^A$ only appear through $\Re G$, and using (5.4) we obtain

$$V_F = \frac{e^{K_K}}{2} \int_X \Re G \wedge \ast \Re G + \ldots = \frac{e^{K_K}}{2} F^T M_{11} F + \ldots.$$

(5.6)

Since by assumption the $d \times d$ matrix $M_{11}$ has maximal rank $d$, minimizing (5.6) with respect to $c^A$ fixes all $c^A$ in terms of the flux matrix $\hat{\Xi}$ and complex-structure moduli as

$$F = 0 \quad \implies \quad c = V^{-1} \sigma \left( f^1 + M_{11}^{-1} M_{12} f^2 \right).$$

(5.7)

However, for our purpose the precise values of the stabilized $c^A$ are not important. We furthermore note that with $\phi$ denoting all real moduli except the $c^A$, the mass matrix takes the form

$$M^2 = \begin{bmatrix} e^{K_K} m & 0 \\ 0 & M_{\phi \phi}^2 \end{bmatrix}, \quad m = \Xi^T \mathcal{M} \Xi,$$

(5.8)

where $m$ is positive definite and $M_{\phi \phi}^2$ denotes the mass matrix for all moduli $\phi$. Hence, the stabilization of the $c^A$ can be separated from the problem of stabilizing the remaining moduli.
Scalar potential

We finally turn to the full form of the F-term potential (4.3) after the $c^A$ moduli have been stabilized. We denote this potential by $\tilde{V}_F = V_F|_{\text{min}}$. Separating the three-form flux $G$ into real and imaginary components, using the first Bianchi identity in (2.15) and employing equation (5.4), this potential can be written as

$$\tilde{V}_F = e^{K_K} \left[ \frac{1}{2} N^T m^{-1} N + N^T \tau + \frac{1}{2} \tau^T m \tau + e^{-K_{cs}} \Xi^A \omega A^B \Xi_0 \right].$$  \hspace{1cm} (5.9)

From here we see that indeed, after the $c^A$ have been stabilized, we can express the F-term potential using the tadpole charges $N_A$ instead of the R-R three-form flux $F_3$. However, of course one has to check whether for a given $N$ and flux matrix $\hat{\Xi}$ there exists a corresponding $\hat{F}_3$ with integer components such that $N = \hat{\Xi}^T \eta \hat{F}_3$.

Coming then back to the expression for the scalar potential at the minimum derived in (4.12), using (5.4) together with (5.7) we find

$$V|_{\text{extr}} = -\frac{e^{K_K}}{2} \left[ N^T m^{-1} N + N^T \tau \right]_{\text{extr}},$$  \hspace{1cm} (5.10)

where we recall that the matrix $m = \Xi^T M \Xi$ was defined in (5.5). We discuss this expression in more detail below.

5.2 No supersymmetric Minkowski vacua

We now want to show that in the case $h_{2,1} = h_{1,1}$, supersymmetric Minkowski vacua require the complex-structure moduli to be stabilized at the boundary of moduli space. Such solutions should therefore be excluded. Note that for type IIA compactifications with non-geometric fluxes a similar result has been derived in [16].

Analysis

To show the above statement, we recall the general form of the F-term potential (2.19) and note that for supersymmetric Minkowski vacua we have to require

$$0 = W, \quad 0 = F_A = \partial_A W, \quad 0 = F_i = \partial_i W,$$  \hspace{1cm} (5.11)

where $A = 0, \ldots, h_{1,1}$ labels the combined Kähler-sector moduli and $i = 1, \ldots, h_{2,1}$ labels the complex-structure moduli. The superpotential $W$ was defined in (2.20), for which the second condition in (5.11) reads

$$0 = \int \Xi_A \wedge \Omega.$$  \hspace{1cm} (5.12)
Let us now expand the holomorphic three-form $\Omega$ into the symplectic basis (2.2) and define

\[ \Omega = \begin{pmatrix} \alpha & -\beta \end{pmatrix} \begin{pmatrix} X \\ F \end{pmatrix}, \quad U^T \begin{pmatrix} X \\ F \end{pmatrix} = \begin{pmatrix} x^1 \\ x^2 \end{pmatrix}, \] (5.13)

where $X$ and $F$ denote the periods of the Calabi-Yau three-fold, the $2d \times 2d$ matrix $U$ appeared in the singular value decomposition (5.1) and where we suppressed indices. Using this notation together with the expansion of $\Xi_A$ given in (4.24), the singular value decomposition of $\hat{\Xi}$ shown in (5.1) and the form of $U^T \eta U^T$ given in (5.3) allows us to write (5.12) as

\[ 0 = \int_X \Xi \wedge \Omega = V(\sigma, 0) U^T \eta U U^T \begin{pmatrix} X \\ F \end{pmatrix} = V \sigma b x^2 \quad \implies \quad x^2 = 0, \] (5.14)

where we used that the $d \times d$ matrices $V$, $\sigma$ and $b$ are invertible. We can now use this solution to evaluate

\[ \int_X \Omega \wedge \bar{\Omega} = (X, F) \eta \begin{pmatrix} \bar{X} \\ \bar{F} \end{pmatrix} = (x^{1T}, x^{2T}) \begin{pmatrix} 0 & b \\ -b^T & 0 \end{pmatrix} \begin{pmatrix} \tau^1 \\ \tau^2 \end{pmatrix} x^2 = 0. \] (5.15)

This implies that the Kähler potential $K_{cs}$ shown in (2.6) is singular, and hence the complex-structure moduli are stabilized at the boundary of their moduli space. Note that since we did not specify the form of the periods this result holds for the quantum-corrected periods, and we have to exclude such situations.

**Remark**

We close this subsection with the following remark. For $h_1^{1,1} < h_2^{2,1}$ the argument presented above is modified and supersymmetric Minkowski vacua stabilizing all moduli cannot be excluded. The condition (5.12) is equal to $\Xi^\eta_A = 0$, which when inserted in (4.5) gives the condition

\[ \text{supersymmetric Minkowski vacuum:} \quad \star G = iG. \] (5.16)

This is the well-known requirement of $G$ being imaginary self-dual for the case of only $H$-flux present [63].

### 5.3 Stabilizing the radial Kähler-sector modulus

In this section we study moduli stabilization and stability conditions for the radial Kähler-sector modulus, and discuss conditions for obtaining de-Sitter minima.
Moduli stabilization

To do so, we first split the combined Kähler-sector modulus $\tau^A$ and the tadpole-charge vector $N_A$ into radial and angular parts, that is, using the Euclidean norm $\delta \equiv \delta_{AB}$ and suppressing indices we define

$$\tau = r e_+, \quad \text{where} \quad \tau^T \delta \tau = r^2, \quad e_+ = 1,$$
$$N = n e_N, \quad \text{where} \quad N^T \delta^{-1} N = n^2, \quad e_N \delta^{-1} e_N = 1.$$  \hspace{1cm} (5.17)

Our convention is $r \geq 0$ and $n \geq 0$. Next, we note that the Kähler potential $K_K$ and the matrix $\omega^{AB}$ appearing in the scalar potential (5.9) depend on the radial modulus $r$ as $e^{K_K} \sim r^{-4}$ and $\omega \sim r^2$. We can then express the scalar potential (5.9) as

$$\tilde{V}_F = \frac{1}{r^4} \left[ a + b r + \frac{c}{2} r^2 \right], \hspace{1cm} (5.18)$$

where the coefficients $a, b, c$ are independent of $r$. Using $e^{\tilde{K}_K} = r^4 e^{K_K}$ and $\tilde{\omega}^{AB} = r^{-2} \omega^{AB}$, they are given by

$$a = e^{\tilde{K}_K} n^2 e_N^T m^{-1} e_N, \quad b = e^{\tilde{K}_K} n e_N^T e_+, \quad c = e^{\tilde{K}_K} \left( e_+^T m e_+ + 2 e^{-K_{\phi}} \Xi \tilde{\omega} \Xi^0 \right).$$  \hspace{1cm} (5.19)

where the matrix $m$ was defined in (5.5), and since $m$ is positive definite we have $a \geq 0$. We can then minimize the potential (5.18) with respect to $r$ by solving $\partial_r \tilde{V}_F = 0$. Imposing in addition the necessary condition $\partial^2_r \tilde{V}_F > 0$ for the minimum to be stable, we obtain the following necessary conditions for stable minima:

| Type | $a$ | $b$ | $c$ |
|------|-----|-----|-----|
| AdS minimum | $a > 0$ | $b \leq 0$ | $c \leq \frac{b^2}{a}$ |
| Minkowski minimum | $a > 0$ | $b > 0$ | $c < 0$ |
| dS minimum | $a > 0$ | $b < 0$ | $\frac{b^2}{a} < c < \frac{9}{8} \frac{b^2}{a}$ |

Note however, that our requirement (3.14) excludes AdS minima with $b > 0$.

Stable de-Sitter minima

Let us now focus on stable de-Sitter minima. As we can see from (5.20), after $r$ has been stabilized the remaining moduli have to be fixed in a rather restricted region. This is in agreement with the observation made in [23, 24] that stable
de-Sitter vacua are located in a narrow band in moduli space. For the stabilized radial modulus $r$ the constraints on $c$ shown in (5.20) translate into

$$dS \text{ minimum: } \frac{a}{|b|} < r_{\text{min}} < \frac{4}{3} \frac{a}{|b|} \quad \text{with} \quad \frac{a}{|b|} = n - \frac{e_N^T m^{-1} e_N}{|e_N^T e_\tau|}, \quad (5.21)$$

and from here we see that a large value for $r_{\text{min}}$ – necessary for a large-volume and small-coupling limit – can be achieved in three different ways:

1. The first possibility to make $a/|b|$ large is to require $n \gg 1$. This means that the tadpole charges have to be large, requiring a large number of O3- and O7-planes to be present. Such configurations exist – but these usually have a complicated topology and hence a large number of moduli have to be stabilized. Problems related to the latter issue have been discussed recently for instance in [43–45].

2. A second possibility for obtaining a large $r_{\text{min}}$ is to stabilize the complex-structure moduli such that $e_N^T m^{-1} e_N \gg 1$, where $m$ was defined in (5.5). Since $e_N$ is normalized to one, such a condition is possible only near a boundary of complex-structure moduli space.

3. A third way to make $a/|b|$ large is to have $|e_N^T e_\tau| \ll 1$. Demanding the moduli to be stabilized in a large-volume and weak-coupling regime implies that all components of $\tau^A$ have to be large (excluding the $h_{1,1}$ moduli). This means that approximately we have $e_\tau \sim (1, 1, \ldots, 1)/\sqrt{h_{1,1}}$. Since we require all components of the tadpole charges to be non-positive, to achieve $|e_N^T e_\tau| \ll 1$ we therefore may look for compactifications with $e_N = (0, \ldots, 0, -1, 0, \ldots)$ and $h_{1,1} \gg 1$.

To summarize, for stable de-Sitter vacua the moduli have to be stabilized in a narrow region of moduli space and one typically has to consider non-generic situations. This explains why it is difficult to find de-Sitter solutions, however, from our analysis here we do not see any reason why they cannot exist.

6 Comments on de-Sitter vacua

A number of approaches for constructing de-Sitter vacua using non-geometric fluxes have appeared in the literature [19–29]. We briefly review them in this section. For the stable constructions which in addition to the R-R three-form flux $F_3$ only use (non-)geometric $H$-, $F$-, $Q$- and $R$-fluxes to stabilize all closed-string moduli, we argue that none of them satisfy all the consistency conditions discussed above.
The de Carlos-Guarino-Moreno model

The authors of [19, 20] have constructed a family of stable de-Sitter minima for type IIB string theory compactified on the isotropic $T^{6}/Z_{2} \times Z_{2}$ orientifold with O3-/O7-planes in the presence of geometric and non-geometric fluxes. Let us recall a particular representative of this family using our conventions:

- The isotropic torus can be seen as a compactification manifold with topology characterized by $h_{1}^{1,1} = h_{2}^{2,1} = 1$ and $h_{3}^{1,1} = h_{4}^{2,1} = 0$. Up to an irrelevant additive constant, the corresponding Kähler potential in the large-volume, small-coupling and large-complex-structure limit is given by

$$K = - \log[-i(T^{0} - \overline{T}^{0})] - 3 \log[-i(T^{1} - \overline{T}^{1})] - 3 \log[-i(z^{1} - \overline{z}^{1})]. \quad (6.1)$$

- The R-R three-form flux, the NS-NS $H$- and $Q$-flux and the tadpole charges can be expressed using the matrix notation introduced in (4.24). In particular, we have

$$\hat{F}_{3} = \begin{pmatrix} -47 \\ -44 \\ 44 \\ 51 \end{pmatrix}, \quad \hat{\Xi} = \begin{pmatrix} 44 & 0 \\ 1 & -1 \\ -1 & -3 \\ 132 & 0 \end{pmatrix}, \quad N = \begin{pmatrix} -7748 \\ 192 \end{pmatrix}, \quad (6.2)$$

where the first column in $\hat{\Xi}$ corresponds to the $H$-flux and the second column to the non-geometric $Q$-flux. Note also that the fluxes contained in $\hat{\Xi}$ satisfy the Bianchi identity (2.15) and that $\hat{\Xi}$ is of maximal rank equal to 2.

- Using this data for instance in (5.9) and minimizing the resulting potential, the authors of [19, 20] find a de-Sitter minimum with $V|_{\text{min}} = 1.43 \cdot 10^{-3}$ at

$$\tau_{0}^{\text{min}} = 2.008, \quad \tau_{1}^{1|\text{min}} = 48.684, \quad \bar{\tau}_{1}^{1|\text{min}} = -1.034 + 1.144i. \quad (6.3)$$

(As explained above, the $e^{A}$ dependence can be decoupled.) This minimum is stable, and the eigenvalues of the corresponding canonically-normalized mass matrix are

$$\text{eigenvalues of } M_{\text{can}}^{2} : \quad \{ 4.25, 1.75, 1.46, 2.29 \cdot 10^{-3} \}. \quad (6.4)$$

This example illustrates that the supergravity equations following from non-geometric flux compactifications can have stable de-Sitter solutions. However, we believe that this model is not consistent in string theory: in section 3 we argued that in order to avoid arbitrary-large gauge groups the tadpole charges $N_{A}$ should be non-positive, and from (6.2) we see that the present model violates this requirement. (This observation extends to the whole family of models.) We therefore do not consider it to be a consistent string-theory construction.
Other de-Sitter constructions with non-geometric fluxes I

Other papers known to us which construct de-Sitter vacua – using in addition to the R-R three-form flux $F_3$ only (non-)geometric $H$-, $F$-, $Q$- and $R$-fluxes – are the following:

- From a supergravity point of view the gauge algebras originating from non-geometric fluxes haven been studied in [21,22]. However, no stable de-Sitter models have been found.

- In [23] a search for de-Sitter vacua has been performed for the $T^6/Z_2 \times Z_2$ orientifold, but no fully stable de-Sitter extrema without any tachyonic directions have been found.

- In [24] the authors found three stable de-Sitter minima with the help on an evolutionary algorithm. However, one can check that these models (summarized in appendix A of [24]) do not satisfy the Bianchi identities (2.15). We therefore do not consider them to be consistent string-theory models.

- In [25] as well as in [26] the authors considered compactifications of type IIB string theory on the $T^6/Z_2 \times Z_2$ orientifold with O3-/O7-planes and (non-)geometric fluxes. With the help of a genetic algorithm a number of stable de-Sitter minima are obtained for this setting, however, we were not able to reproduce these results.

- In [29] the authors consider non-geometric flux compactifications of type IIB string theory on the isotropic $T^6/Z_2 \times Z_2$ orientifold. Using machine-learning techniques the authors obtain de-Sitter extrema which, however, have at least one tachyonic direction.

We conclude that none of the non-geometric de-Sitter models reviewed here are fully-stable and consistent in string theory. This observation is in agreement with the swampland de-Sitter conjecture made in [3].

Other de-Sitter constructions with non-geometric fluxes II

In addition to the $H$-, $F$-, $Q$- and $R$-fluxes discussed in this paper, one can consider the non-geometric $P$-flux. This flux is the the S-dual completion of the $Q$-flux [64], which generates quadratic couplings among the Kähler-sector moduli in the superpotential.

- In [26,28] the authors include non-geometric $P$-flux for the construction of de-Sitter vacua. We have not considered such fluxes in our work and therefore do not comment on these examples.

\footnote{We thank C. Damian and O. Loaiza-Brito for correspondence on this question.}
7 Summary, conclusions, outlook

In this paper we have studied compactifications of type IIB string theory on Calabi-Yau orientifolds with O3-/O7-planes and (non-)geometric $H$, $F$, $Q$- and $R$-fluxes. For this setting it is possible to stabilize all closed-string moduli classically without the need for non-perturbative contributions, and examples of stable de-Sitter constructions can be found in the literature.

Summary

Our motivation for this work was to investigate whether non-geometric flux compactifications allow for stable de-Sitter vacua. Although we were neither able to show that de-Sitter vacua cannot be obtained from non-geometric fluxes nor able to construct a consistent de-Sitter solution, we made progress in understanding non-geometric flux compactifications. More concretely,

- based on the requirement that the total rank of the gauge group should be bounded from above (cf. [30]), in section 3 we argued that the contributions of closed-string fluxes to the tadpole cancellation conditions should be similar to D-branes and not anti-D-branes. In our conventions this means $N_A \leq 0$.

- In section 4 we derived a simple expression for the scalar potential at the minimum (cf. equation (4.12)). We furthermore showed that in order to stabilize all closed-string moduli classically by $H$, $F$, $Q$-, $R$- and $F_3$-fluxes, the topology of the compactification manifold has to be restricted as $h^{1,1} \leq h^{2,1}$, and we can choose $h^{2,1}_+ = 0$ without loss of generality.

- When specializing to the case $h^{1,1} = h^{2,1}_-$, we were able to eliminate the dependence on the R-R three-form flux $F_3$ in favor of the tadpole charges $N_A$ in the scalar potential (cf. equation (5.9)). This form promises to be very useful for performing computer-based searches for flux vacua. We also showed that for $h^{1,1} = h^{2,1}_-$ it is not possible to obtain supersymmetric Minkowski vacua, and we derived constraints for stable de-Sitter vacua. In particular, for the latter we concluded that moduli have to be stabilized in a rather restricted region of moduli space.

- In section 6 we reviewed known de-Sitter constructions based on non-geometric flux compactifications. For the fully-stable constructions with only $H$, $F$, $Q$- and $R$-fluxes we concluded that these are not consistent in string theory.
Conclusions

The swampland conjecture made in [3] proposes that stable de-Sitter vacua cannot exist in any consistent theory of quantum gravity. Non-geometric flux compactifications provide potential counter-examples to this conjecture, however, in this paper we have argued that all examples known to us (which only involve $H$-, $F_3$-, $Q$-, $R$- and $F_3$-fluxes) are not consistent in string theory. On the other hand, we were not able to show in general that stable de-Sitter vacua cannot be obtained from non-geometric flux-compactifications. In fact, the scalar potential we derived shows a close resemblance to ordinary Calabi-Yau compactifications with the familiar $H$- and $F_3$-flux. Whether non-geometric stable de-Sitter vacua exist in string theory remains an open question.

Outlook

The results obtained in this paper provide a starting point for further research on non-geometric flux-compactifications:

- For the case $h^{1,1}_+ = h^{2,1}_-$ we derived an expression for the scalar potential where the R-R three-form flux $F_3$ has been replaced by the tadpole charges $N_A$. Using our requirement from section 3 for a given compactification manifold there are finitely-many choices for $N_A$ which in turn simplifies computer-based searches for flux vacua.

- The complex-structure moduli sector is subject to a $Sp(2h^{2,1}_+ + 2, \mathbb{Z})$ duality acting on the third cohomology. This implies that the NS-NS $H$-, $F_3$-, $Q$- and $R$-fluxes contained in the flux matrix (4.24) are unique up to $Sp(2h^{2,1}_+ + 2, \mathbb{Z})$ transformations. As a preliminary study we considered $h^{1,1}_+ = h^{2,1}_- = 1$ and flux quanta with values $-3, \ldots, +3$ appearing in $\hat{\Xi}$. In this case there are 318 784 combinations for which $\hat{\Xi}$ has maximal rank, which by $Sp(2h^{2,1}_+ + 2, \mathbb{Z})$ transformations are related to 70 inequivalent choices. We therefore have a reduction of the size of the flux landscape as

$$318\,784 \text{ flux choices} \xrightarrow{Sp(4,\mathbb{Z})} 70 \text{ inequivalent flux choices}.$$  

Note however that $Sp(2h^{2,1}_+ + 2, \mathbb{Z})$ also acts on the periods of the holomorphic three-form in a non-trivial way and typically does not preserve for instance a large-volume limit. It would be interesting to extend this observation to more general NS-NS flux configurations.

We are planning to come back to these questions in the future.
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