Mechanical Modeling of Progressive Failure of Tieshansi Slope

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Abstract. With the rapid development of China economy, the mining environment improvement is a hot research topic. The traditional unbalanced thrust method is generalized, the partial strength reduction method (PSRM), the perfect elasto-plastic model (PEPM) and the complete process constitutive model (CPCM) are employed; the progressive failure process of a slope is simulated by the proposed generalized slice method; the safety factor of the extended slice method is closely related to deformation of a slope, the characteristics of the critical stress state movement can also revealed by the proposed generalized slice method. Taking the Tieshansi slope of Huangshi City as an example, the evolution characteristics of the physical and mechanical variables in progressive failure process of the slope are revealed by the proposed generalized unbalanced thrust method with the two sorts numerical simulations (PSRM with PEPM and CPCM); the search results show that: The proposed generalized slice methods can describe the progressive failure process of the Tieshansi slope.

Keywords. Progressive failure, partial strength reduction, slope, stability evaluation.

1. Introduction

The failure mechanism and stability analysis of a slope are traditional topics, the more than ten sorts limit equilibrium stability calculation methods are proposed, for instance: the Fellenius method [1], the simplified Bishop method [2], the Spencer method [3], the Janbu method [4], the unbalance thrust method (TCM), the Sarma method [5] and the finite element strength reduction method (SRM) [6]. The limit equilibrium method is widely used in engineering design [7]. The different limit equilibrium slice methods have different assumptions about the action point of normal force on the bottom edge of the slice and action line of thrust between the slices for the slope with a given slip surface. The stability evaluation results of different methods are not the same.

With the development of numerical analysis, the more and more calculated methods are suggested [8]. Recently, the vector sum method [9] has been applied to the evaluation of the mine slope and high dam stability. A partial strength reduction method is proposed [10-14] to simulate the progressive failure process.

The sliding surface is divided into a failure zone, a critical zone and a stable zone, the transfer law of the landslide force and deformation is analyzed, and the characteristics of the critical block (or unit) force of the slope are proposed [10-14]. Based on the failure mechanisms, deformation modes and control standards of thrust-type, pull-type and mixed landslides proposed, and the deformation-stress analysis combined with the possible failure modes of slope, the complete process constitutive model(CPCM) is proposed, the following evaluation methods are defined: comprehensive sliding-resistance, main thrust, comprehensive displacement and surplus displacement methods.
In this paper, the slice block method is extended, and the PEPM and CPCM models are used to the UTM, and the safety factor of surplus frictional method is defined. The progressive failure process is described by the comprehensive sliding-resistance method (CSRM), the main thrust method (MTM), the surplus frictional force method (SFM), the comprehensive displacement method (CDM) and the surplus displacement method (SDM).

2. Slice Method

2.1. Unbalance Thrust Method (TCM)

The limit equilibrium slice method has been used in engineering, the unbalance thrust method (TCM) is presented: Hypothesis: (1) The slice blocks are rigid and there does not exist their deformation, and are divided vertically at certain intervals; (2) The force acting on the front slice block from the back slice block is parallel to the bottom edge of the back slice block, and the force is located at the center of the front slice block; (3) The rotation of each slice block and the shear force between the two slice blocks are ignored; (4) The shear stress of the slice block bottom edge is located at a critical stress state.

Under the above assumptions, the scheme of slice blocks is shown in figure 1. The basic formula of the TCM are as follows, see equations (1-9).

The ith slice block:

Normal pressure $N_i$:

$$N_i = W_i \cos \alpha_i + P_{i-1} \sin(\alpha_{i-1} - \alpha_i) + \beta_i l_i \cos \alpha_i \cos \alpha_i + \Delta_i l_i \cos \alpha_i \sin \alpha_i$$ (1)

Normal stress $\sigma_{i,n}$:

$$\sigma_{i,n} = N_i / l_i$$ (2)

Critical frictional stress $\tau_{i,peak}$:

$$\tau_{i,peak} = c_i + \sigma_{i,n} \tan \varphi_i$$ (3)

Critical frictional force $T_{i,crit}$:

$$T_{i,crit} = c_i l_i + N_i \tan \varphi_i$$ (4)

Frictional stress after the strength reduction (SR) $\tau_{i,f}$:

$$\tau_{i,f} = (c_i + \sigma_{i,n} \tan \varphi_i) / f$$ (5)

Frictional force after the SR $T_{i,F,crit}$:

$$T_{i,F,crit} = T_{i,crit} / f$$ (6)

Sliding force $P_i^s$:

$$P_i^s = W_i \sin \alpha_i + P_{i-1} \cos(\alpha_{i-1} - \alpha_i) + \beta_i l_i \cos \alpha_i \sin \alpha_i + \Delta_i l_i \cos \alpha_i \cos \alpha_i$$ (7)

Shear stress $\tau_{i,u}$:

$$\tau_{i,u} = P_i^s / l_i$$ (8)

Surplus thrust $P_i$:

$$P_i = P_i^s - T_{i,F,crit}$$ (9)

where: $W_i, \beta_i, \Delta_i, l_i, \alpha_i, c_i, \varphi_i, \sigma_{i,n}$: the weight, the vertical uniform load, the horizontal uniform load, the length, the angle, the cohesion, the frictional angle, the normal stress of the bottom edge of the i-th slice block respectively, $f$ : the SR coefficient along the entire sliding surface in the critical stress state.
The equations (1-9) are taken the iteration to obtain the reduction coefficient $f_f$. In the above analysis, the frictional stress at the bottom edge of the slice block is located at the peak stress state, it is difficult for the shear stress of the entire bottom edge to be located at the peak stress state.

Figure 1. The scheme of slice block layout.

2.2. Partial Strength Reduction Method (PSRM)

The partial strength reduction method (PSRM) [10-14] is proposed, and its calculation steps are described as: Let the reduction coefficient is equal to 1 from the first to the m-th slice block by the above equations (1-9), while the m-th slice block is located at critical stress state. Then the calculations from the first to the m+1-th slice block, ..., from the first to the n-th slice block are carried out respectively, the reduction coefficients ($f_i$, $i \in (m,n)$) by the PSRM are obtained. The physical meaning of the safety factor is: The slice block located at the critical stress state moves forward from the m-th block to the m+1-th block, ..., to the n-th block respectively.

As the critical stress state slice block moves forward, the safety factor of a slope increases gradually. The definition of surplus safety factor ($f_{zs_i}$) is proposed: the surplus safety factor of the i-th slice block located at the critical stress state is equal to that the entire safety factor ($f_n$) minus the partial strength reduction coefficient of the i-th slice block ($f_i$), see equation (10).

$$f_{zs_i} = f_n - f_i$$  \hspace{1cm} (10)

2.3. Slice Block Method Based on PEPM

The traditional slice method is generalized based on the PSRM with PEPM, and the some basic assumptions can be changed as follows:

2.3.1. Basic Assumptions

(1) It are assumed the stress-strain relationship of the geological material of the slice block satisfies the PEPM and the slice block can produce enough self-deformation, and can be divided into vertical sections at certain intervals, and its peak shear stress can be reduced; (2) (2 and 3) are employed, and (3) The shear strains between the i-th slice block and the i+1-th slice block satisfy the vector sum relationship in the parallel and vertical bottom-edge directions in the failure zone (see figure 2).

On the basis of the above assumptions, the calculating formula of the force calculation are basically consistent with that of the TCM and the shear strain between the connected slices in failure zone has the following relationships:

$$\gamma_i^{s'} = \gamma_{i+1}^{s'} + \gamma_{i+1}^{s} \quad \text{or} \quad \gamma_{i+1}^{s'} = \gamma_i^{s'} + \gamma_i^{s}$$  \hspace{1cm} (11)

The two different physical significance of the (7) hypothesis are: the first: the i+1-th slice block goes forward, the i-th slice block provides the condition:

$$\gamma_{i+1}^{s'} = \gamma_i^{s'} \cos(\alpha_i - \alpha_{i+1})$$  \hspace{1cm} (12)
The second: the i-th slice block goes forward, the i+1-th slice block provides the condition:
\[ \gamma_i^s = \gamma_{i+1}^s \cos(\alpha_i - \alpha_{i+1}) \] (13)

When the angles between the two slice blocks are equal, the shear strains of the two slice blocks are the same. Under the above assumptions, the slice block layout is shown in figure 1. The basic formula of the PSRM of the TCM are seen the equations (1-9).

![Figure 2](image)

(a) The shear relationship of the connected slice blocks
(b) The i-th slice block goes forward, the i+1-th slice block provides the conditions
(c) The i+1-th slice block goes forward, the i-th slice block provides the conditions

Figure 2. The relationship of shear strain relation between slice blocks.

2.3.2. Perfect Elasto-Plastic Model (PEPM). The PEPM is used widely to describe the shear stress-strain of the geo-material, the linear relationship between the shear stress and strain is employed within the critical stress.

\[ \tau_i = G_i \gamma_i, \quad \text{when} \quad \gamma_i \leq \gamma_{i,\text{peak}} \] (14)
\[ \tau_i = \tau_{i,\text{peak}}, \quad \text{when} \quad \gamma_i > \gamma_{i,\text{peak}} \] (15)

where \( G_i \): shear modulus. When the critical shear stress reaches, it is difficult for the shear strain to be calculated by using the stress. There is no one-to-one correspondence between the shear stress and strain, when the stress is located at the critical stress from the equations (14-15).

2.3.3. Complete Process Constitutive Model (CPCM). The complete process shear-strain model (CPCM) must necessarily be employed to describe the progressive failure process of a slope [13]. A stress-strain equation with four parameters is a part of the CPCM. The equation can be described by taking a shear stress and strain as an example in the following form:

\[ \tau = G[1 + \varphi' \gamma / \psi] \] (16)

where \( \tau, \gamma \) are the shear stress and shear strain, respectively; \( G \) is the shear modulus dependent on the normal stress, and \( \psi, \varphi', \) and \( \psi \) are constant coefficients dependent on the normal stress. The units of \( \tau \) and \( G \) are \( \text{kPa} \).

The following conditions are needed for the rock or soil with softening mechanical behaviours.

\[ 1 + q \psi \neq 0 \quad \text{and} \quad -1 < \psi \leq 0 \]

The critical shear strain (defined as the shear strain corresponding to the peak shear stress) is satisfied in the following form:

\[ p + (1 + q \psi) \gamma_{\text{peak}} = 0 \] (17)

where \( \gamma_{\text{peak}} \) is the critical shear strain corresponding to the critical shear stress.
The Mohr-Coulomb criterion is assumed to describe the critical shear stress ($\tau_{\text{peak}}$) (note: other criteria can also describe the critical shear stress):

$$\tau_{\text{peak}} = C + \sigma_n \tan \varphi$$  \hspace{1cm} (18)

where $C$: the cohesion, $\sigma_n$: the normal stress, $\varphi$: the frictional angle. The units of $\sigma_n$ and $C$ are kPa.

The critical shear strain is assumed to be related only to normal stress, and the critical shear strain ($\gamma_{\text{peak}}$) can be described as follows:

$$\left( \frac{\gamma_{\text{peak}}}{\gamma_0^i} \right)^2 + \left( \frac{(\sigma_n - a_2)}{a_1} \right)^2 = 1$$  \hspace{1cm} (19)

where $a_1, a_2, a_3, \xi_N$ are constant coefficients which are dependent on the normal stress; the units of $a_1$ and $a_2$ are kPa and $a_3, \xi_N$ are constant coefficients without units. Finally:

$$G = G_0 + b_1 \sigma_n + b_2 \sigma_n^2$$  \hspace{1cm} (20)

where $G_0$ is the initial shear modulus when the normal stress ($\sigma_n$) is equal to zero, $b_1$ is a constant coefficient without units and $b_2 = -b_1/(2a_2)$.

The softening coefficient ($\xi$) can be presented in the following form:

$$\xi = \frac{\xi_0}{1 + \left( \frac{\xi}{\xi_0} - 1 \right) (\sigma_n/\sigma_n^*)}$$  \hspace{1cm} (21)

where $\xi_0$ is the value of $\xi$ when $\sigma_n$ is equal to zero, $\xi_c$ is the value of $\xi$ when $\sigma_n$ is equal to $\sigma_n^*$ and $\xi$ is a constant coefficient without units. The softening coefficient can be obtained by the shear stress and shear strain complete process tests with different normal stresses.

2.3.4. Displacement Determination of Sliding Surface. The stress distribution along the sliding surface can be obtained after the critical stress state slice block is gotten by using the PRSM with PEPM. The shear strain distribution is also calculated from the first to the last slice block as follows:

The slice blocks from the $m+1$-th to the $n$-th slice block are within the elastic stress state, the linear shear strain is:

$$\gamma_i = P_i / (l_i G_i), i \in (m+1, n)$$  \hspace{1cm} (22)

The shear strain of the $m$-th slice block, which is located at the critical stress state, can be presented as follows:

$$\gamma_{m, \text{peak}} = T_{m, \text{peak}} / (l_m G_m)$$  \hspace{1cm} (23)

The shear strain along the sliding surface in the failure zone can be also obtained on the basis of the hypothesis (7). The relationship between the shear strains of the bottom edge of the $i$-th and the $i+1$-th slice block can be divided into the two cases in the failure zone.

In the first case, the $i+1$-th slice block offers the conditions that the $i$-th slice block goes forward (see equation (12 or 13):

$$\gamma_i = \gamma_i^*$$  \hspace{1cm} (24)

But sometimes the strain ($\gamma_i^*$) is less than the critical strain ($\gamma_{i, \text{peak}}$), and the critical strain ($\gamma_{i, \text{peak}}$) is given ($\gamma_i = \gamma_{i, \text{peak}}$).

In the second case, the shear strain of the i-th slice block is the sum of the critical shear strain
(\(\gamma_{i,\text{peak}}\)), the shear strain produced by the reduced stress (\(\gamma'_{i,\text{peak}}\)) and the contribution shear strain (\(\gamma^*_i\)) from the \(i+1\)-th to the \(i\)-th failure slice block.

\[
\gamma_i = \gamma_{i,\text{peak}} + \gamma^*_i - \gamma'_{i,\text{peak}}
\]  

(25)

The frictional shear stress (\(\tau^*_i\)) after the strength reduction is as follows:

\[
\tau^*_i = \tau_{i,\text{peak}} / f_i
\]  

(26)

\[
\gamma'_{i,\text{peak}} = \tau^*_i / G_i
\]  

(27)

The shear strain of the critical stress state slice block:

\[
\gamma_i = \gamma_{i,\text{peak}} \cdot \gamma'_{i,\text{peak}} = \tau_{i,\text{peak}} / G_i
\]  

(28)

Under the above two assumptions, the stress-strain relationship in the post-failure zone is fixed to carry out the stability evaluation based on the displacement. On the basis of the above chapters, the solutions based on the above assumptions can be obtained for the shear strain for the CPCM too. By using the above solutions, the progressive failure stability evaluation of a slope can be carried out. The obtained stress and displacement are the current shear stress and displacement and can be compared with the in-situ monitoring stress and displacement to modify the model parameters.

2.3.5. Progressive Failure Analysis. The purpose of failure analysis is to determine the final failure state of a slope. The possible failure mode of a slope must be obtained in order to evaluate and analyze the safety factor. The failure path of a slope can be determined by using the in-situ investigation and detection, such as along the interface of rock and soil, or along the joints and fissures of rock mass. When the failure path is known, the frictional stress of slice block is gradually reduced according to the PSRM with PEPM or the CPCM, the each slice block has undergone the critical stress state until the last slice block is in a critical stress state, the stress-strain distribution along the sliding surface is obtained in the entire failure state. The frictional shear stress (\(\tau^p_i\)), driving shear stress (\(\tau^u_i\)), normal stress (\(\sigma^p_i\)) and shear strain (\(\gamma^p_i\)) are determined in the final failure state, denoted as:

\[
\tau^p_i, \tau^u_i, \sigma^p_i, \gamma^p_i, i \in (1, n)
\]

3. Stability Analysis

For a specific slope, the traditional safety factor of the whole slope can be calculated by using equations (1-9). In order to describe the characteristics of a slope at different times and evaluate its stability, the field frictional shear stress, driving shear stress, normal stress and shear strain are obtained according to the PSRM with PEPM and CPCM respectively, and recorded as \(\tau^b_i, \tau^a_i, \sigma^b_i, \gamma^b_i, i \in (1, n)\). The frictional shear stress, driving shear stress, normal stress and shear strain can be noted as \(\tau^p_i, \tau^u_i, \sigma^p_i, \gamma^p_i, i \in (1, n)\) under the failure mode for the two-dimensional engineering, the entire stability of the sliding body is evaluated, the CSRM, MTM, CDM and SDM are presented [10-14], the SFM is presented as follows:

The difference sums (\(T^{m}\) and \(T^{m}\)) between the frictional stresses in the failure state (\(\tau^p_{i,m}, i \in (m+1, n)\)) and in the current state (\(\tau^b_{i,i}, i \in (m+1, n)\)) can be obtained in the horizontal and vertical directions from the m+1-th to the n-th slice block, and their difference vector sum (\(T^m\)) can be also gotten.
\[ T^{nm} = \sum_{i=m+1}^{n} (\tau_{i}^{p,b} - \tau_{i}^{X}) \cos \alpha_{i} \]  
\[ T^{sm} = \sum_{i=m+1}^{n} (\tau_{i}^{p,b} - \tau_{i}^{X}) \sin \alpha_{i} \]  
\[ T^{m} = \sqrt{(T^{nm})^2 + (T^{sm})^2} \]  

The angle \( \alpha_{f}^{m} \) between the comprehensive vector sum and the horizontal axis can be presented as follows:

\[ \alpha_{f}^{m} = \arctan \left( \frac{T^{nm}}{T^{sm}} \right) \]  

Then the sliding force sum \( (P_{xf} \text{ and } P_{yf}) \) in the X-and Y-axis directions, and their vector sum \( (P_{f}) \) is calculated under the failure mode.

\[ P_{xf} = \sum_{i=1}^{n} \tau_{i}^{p,b} l_{i} \cos \alpha_{i} \]  
\[ P_{yf} = \sum_{i=1}^{n} \tau_{i}^{p,b} l_{i} \sin \alpha_{i} \]  
\[ P_{f} = \sqrt{(P_{xf})^2 + (P_{yf})^2} \]  

The angle \( \alpha_{pf} \) between the comprehensive vector sum \( (P_{f}) \) and the X-axis is presented as follows:

\[ \alpha_{pf} = \arctan \left( \frac{P_{xf}}{P_{yf}} \right) \]  

Safety factor of SFM in the horizontal direction:

\[ F_{SFM}^{x} = \left| \frac{T^{nm}}{P_{xf}} \right| \]  

Safety factor of SFM in the vertical direction:

\[ F_{SFM}^{y} = \left| \frac{T^{sm}}{P_{yf}} \right| \]  

Safety factor of SFM in the sliding direction:

\[ F_{SFM}^{s} = \left| \frac{T^{m} \cos(\alpha_{f}^{m} - \alpha_{pf})}{P_{f}} \right| \]  

4. Case Studies

4.1. Geological Survey

Tieshansi slope is located in the east side of Longdong iron mine entrance of the Jianlin mountain, the center point geological coordinates of the slope: East longitude: 114°53′21.83″, North latitude: 30°36′02″. The south boundary is controlled by the accumulation platform of Daye Iron Mine, the north boundary ends at the natural gully, and the back edge of the slope is located at the cement road (see figures 3-4).
The elevation of the back edge is 140m, and of the front edge is 100 m, and its area is about 3280 m², the main sliding direction of the slope is 287°, the thickness and volume of the sliding mass are 5-8 m and 1.14×10⁴ m³ respectively. The sliding surface is consisted of the highly weathered structural strata.

4.2. Characteristics of Sliding Mass, Slip Surface and Slide Bed

According to the ground investigation and borehole analysis, the slope is consisted of a rock and soil mass. The sliding mass can be divided into two layers, the residual deposit is located at the upper part, and consist of the silty clay with gravel, their colors are brown and brown yellow, and its thickness is 3.0-4.0 m, the ratio between the soil and stone is 7:3. The lower part is consisted of the strongly weathered diorite with heavy joint fissures and loose structure, its colors is grayish brown. The strongly weathered layer is about 7 m thick and the sliding body is about 6-8 m thick.

According to the analysis of the slip characteristics on the north side of the slope, the main slip surface is the interface between the weathered broken and complete diorite, its thickness is about 4-10 cm. The sliding bed is consist of the diorite, the diorite is relatively contact and with high strength.

4.3. Analysis

The calculation scheme of slice block method is formed according to the I-I’ profile (see figure 5), the sliding body’s weight is 17 KN/m³. The slice block angle and length of the bottom edge of each slice block are shown in figure 6.

![Figure 5. The geological scheme of the I-I’ profile.](image)

![Figure 6. Block division scheme of Tieshansi profile.](image)

According to the laboratory test and field experience, the model parameters of the saturated sliding surface are as follows:

- Cohesion force: $c = 19$ kPa
- Frictional angle: $\varphi = 23°$
- Shear modulus: $G = 6000$ kPa
- Shear strain: $\xi_{i,0} = -0.9999$; $\xi_{i,c} = -0.54$
- Normal stress: $\sigma_i^{n,c} = 400$ kPa
- Frictional angle: $\xi_i = 1.46$
- $a_1 = 4000$ kPa
- $a_2 = 3000$ kPa
- $a_3 = 0.0093$
- $b_1 = 60$
- $b_2 = 0$ kPa

The safety factor of Tieshansi slope under the saturated condition is 1.3493 by the traditional critical state method (TCM). The relationship between driving force ($P_i^{STCM}$), friction resistance ($T_i^{TCM}$) and surplus sliding force ($P_i^{STCM}$) and slice number (SN) of each slice block are shown in figure 7. When the safety factor is 1.3493, the maximum driving force and frictional resistance are located in the fifth and eighteenth slice blocks respectively, then they decrease. At the last slice block, the surplus sliding force is equal to zero.

The critical critical stress state slice block is located at the eleventh slice block, when the safety factor is equal to 1 according to the PSRM. As the critical stress state moves forward one by one, the safety factor of the PSRM in different critical stress state slice block becomes larger and larger. When the last slice block is in critical stress state, the safety factor of the PSRM is equal to that of the TCM.
(1.3493), and the relationship between the safety factor of the PSRM and the critical stress state slice block number (CSN: critical stress state slice block number) is shown in figure 8. The surplus safety factors ($f_{zs}^i$) of the PSRM under the different critical stress state slice block are obtained (see figure 9). The increase of the entire safety factor and the decrease of the surplus safety factor are presented in the figures 8-9 respectively with the critical stress state slice block moving forward one by one. That is to say, with the critical stress state moving forward step by step, the stability degree of the slope is getting lower and lower. The stability coefficients of the CSRM, MTM and SFM are the same for the PSRM and PSRM with PEPM.

![Figure 7](image1.png)

**Figure 7.** The force distribution of the different SN by the UTM.

![Figure 8](image2.png)

**Figure 8.** The scheme of safety factor of PSRM and critical stress state slice block.

![Figure 9](image3.png)

**Figure 9.** The scheme of surplus safety factor and CSN by PSRM.

The analysis of progressive failure by the PSRM with PEPM is carried out. The driving force ($D_i^{STCM}$), friction resistance ($T_i^{TCM}$) and surplus sliding force ($P_i^{TCM}$) under the five different critical stress state slice blocks are shown in the figures 10-14 respectively. With the progressive movement of critical stress state slice blocks, the variation law of the safety factors for the five different methods (CRSM, MTM, SFM, CDM, SDM) is shown in the figures 15-19 respectively.
Figure 10. The relationship between the force distribution and the SN under the twelfth critical stress state.

Figure 11. The relationship between the force distribution and the SN under the fifth critical stress state.

Figure 12. The relationship between the force distribution and the SN under the eighteenth critical stress state.

Figure 13. The relationship between the force distribution and the SN under the twenty-first critical stress state.

Figure 14. The relationship between the force distribution and the SN under the twenty-third critical stress state.

Figure 15. The scheme between the CRSM and the CSN of PEPM.

Figure 16. The scheme between the MTM and the CSN of PEPM.

Figure 17. The scheme between the SFM and the CSN of PEPM.
Figure 18. The scheme between the CDM and the CSN of PEPM.

Figure 19. The scheme between the SDM and the CSN of PEPM.

The maximum driving force ($P_i^{STCM}$), maximum frictional resistance ($T_i^{TCM}$) and maximum surplus sliding force ($P_i^{TCM}$) change from the seventh slice block to the seventh, eighth, fifteenth and fifteenth slice block, from the eighth slice block to the eighth, eighth and eighteenth slice block, and from the sixth slice block to the sixth, eighth, fifteenth and fifteenth slice block respectively (see the figures 10-14). When the strength reduction coefficient increases, the maximum frictional resistance (from 870 KN to 678 KN) decreases, the maximum driving force (from 1342 KN to 3233 KN) and the surplus sliding force increased (from 504 KN to 2608 KN) increase. When the critical stress state changes from the tenth to the last slice block by using the entire safety factor (CRSM, MTM, SFM, CDM, SDM) of the slope, the CRSM is changed from 1.85, 4.89, 2.54 to 0.65, 1.99 and 0.85 in X, Y-axis and comprehensive directions respectively. The MTM is changed from 1.69, 0.51, 1.40 to 0.0, 0.0 and 0.0 in X, Y-axis and comprehensive directions respectively. The SFM is changed from 0.093, 0.038, 0.085 to 0.0, 0.0 and 0.0 in X, Y-axis and comprehensive directions respectively. The CDM is changed from 4.25, 4.35, 4.27 to 1.0, 1.0 and 1.0 in X, Y-axis and comprehensive directions respectively. The SDM is changed from 0.69, 0.48, 0.64 to 0.0, 0.0 and 0.0 in X, Y-axis and comprehensive directions respectively.

Under the condition that the peak stress of the PEPM is equal to that of the CPCM, the progressive failure process is analyzed by using the CPCM, the initial critical stress state is located at the twelfth slice block. With the slope deformation increases, the critical stress state moves forward one by one along the sliding surface. When the last slice block is in the critical state, the failure of the slope occurs. In this progressive failure process, the driving force, frictional resistance and surplus sliding force corresponding to the five different critical state slice blocks are shown in figures 20-24 respectively. Additionally, with the change of displacement increase and the critical stress state slice block the variation, the five sorts safety factors are shown in figures 25-29 respectively.

Figure 20. The relationship between the force distribution and the SN under the twelfth critical stress state.

Figure 21. The relationship between the force distribution and the SN under the fifteenth critical stress state.
Figure 22. The relationship between the force distribution and the SN under the eighteenth critical stress state.

Figure 23. The relationship between the force distribution and the SN under the twenty-first critical stress state.

Figure 24. The relationship between the force distribution and the SN under the twenty-third critical stress state.

Figure 25. The scheme between the CRSM and the CSN of PCPM.

Figure 26. The scheme between the MTM and the CSN of PCPM.

Figure 27. The scheme between the SFM and the CSN of PCPM.

Figure 28. The scheme between the CDM and the CSN of PCPM.

Figure 29. The scheme between the SDM and the CSN of PCPM.
The figures 20-24 shows: the maximum driving force, frictional resistance and surplus sliding force change from the seventh slice block to the seventh, eighth, fifteenth, fifteenth slice block, from the eighth to the eighth, eighth, eighth slice block and from the sixth slice block to the sixth, eighth, fourteenth and fifteenth slice block respectively, when the critical stress state moves forward step by step. The maximum driving force (from 1342 KN to 3233 KN) and the surplus sliding force (from 504 KN to 2608 KN) increase, and the maximum frictional resistance decreases (from 870 KN to 667 KN), while the frictional resistance decreases by using CPCM with the displacement add.

The entire safety factors of the slope are evaluated by using CPCM (CRSM, MTM, SFM, CDM, SDM) during the process of the critical stress state change from the twelfth slice block to the last slice block, the CRSM changes from 1.86, 4.68, 2.43 to 0.54, 1.85, 0.72 in X, Y-axis and comprehensive directions respectively. The MTM changes from 1.32, 0.38, 1.10 to 0.0, 0.0, 0.0 in X, Y-axis and comprehensive directions respectively. The SFM changes from 0.087, 0.034, 0.079 to 0.0, 0.0, 0.0 in X, Y-axis and comprehensive directions respectively. The CDM changes from 3.29, 2.88, 3.19 to 1.0, 1.0, 1.0 in X, Y-axis and comprehensive directions respectively. The SDM changes from 0.064, 0.039, 0.058 to 0.0, 0.0, 0.0 in X, Y-axis and comprehensive directions respectively. The evolution processes of the slope are shown in figures 25-29 respectively.

From the solutions of the traditional SRM, the PSRM with PEPM and the CPCM, the safety factor obtained by the traditional slice block method is the overall stability of the slope, and the surplus safety factor obtained by the PSRM is the evolution law of the surplus degree of the overall stability of the sliding body with the critical stress state change. The MTM, SFM and SDM present the surplus degrees of the force and the displacement of the slope. The CSRM and CDM describe the evaluation law of the overall force and displacement of the slope respectively.

The traditional SRM mainly describes the force stability according to the evolution mechanism of the slope. The new methods (PSRM with PEPM and CPCM) characterized the force stability of the slope by using the CSRM, MTM and SFM. The comparative results between the traditional SRM and the new methods are as follows: the entire safety factor of Tieshansi slope obtained by SRM is 1.3493, its surplus degree can be considered to be 0.3493, the surplus stability presented by the PSRM is also 0.3493. However, the three safety factors (CSRM, MTM and SFM) for the PSRM with PEPM are the same in the X-Y-axis and comprehensive directions respectively. The four safety factors (CSRM, MTM, CDM and SDM) obtained by PSRM with PEPM are larger than that of the CPCM, but the SFM’s safety factor is less than that of the CPCM.

5. Conclusion
The progressive process from the initial cracking to the entire failure of the sliding surface is analyzed by using the PSRM with PEPM and the CPCM, the traditional slice block method is extended to make the safety factor closely related to deformation, the following conclusions are obtained:

(1) The calculation steps and physical significance of the PSRM are described in detail. The force evaluation law is described by the CSRM, MTM and SFM based on the PSRM with PEPM and the CPCM, the PSRM and CPCM can present the force characteristics during the progressive failure process.

(2) The traditional slice method hypothesis is generalized, the PEPM is introduced into the slice block method. The PSRM with PEPM and the CPCM are used to describe the whole process of progressive failure of a slope. The variation characteristics of the force and displacement during progressive failure of a slope can be well described.

(3) Based on the analysis of the deformation relationship between the two slice connected blocks in the failure zone, the shear strain formulas between the two connected blocks are proposed, and the method for determining the shear strain in the failure zone is obtained for the PEPM.

(4) The generalized slice block method for progressive failure of a slope proposed in this paper can be extended to the other methods, such as: Janbu method, the Sarma method and the simplified Bishop method.
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