Does one observe chiral symmetry restoration in baryon spectrum?

Thomas D. Cohen\textsuperscript{a} and Leonid Ya. Glozman\textsuperscript{b}

\textsuperscript{a} Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA
\textsuperscript{b} Institute for Theoretical Physics, University of Graz, Universitätsplatz 5, A-8010 Graz, Austria

Abstract

It has recently been suggested that the parity doublet structure seen in the spectrum of highly excited baryons may be due to effective chiral symmetry restoration for these states. We review the recent developments in this field. We demonstrate with a simple quantum-mechanical example that it is a very natural property of quantum systems that a symmetry breaking effect which is important for the low-lying spectrum of the system, can become unimportant for the highly-lying states; the highly lying states reveal a multiplet structure of nearly degenerate states. Using the well established concepts of quark-hadron duality, asymptotic freedom in QCD and validity of the operator product expansion in QCD we show that the spectral densities obtained with the local currents that are connected to each other via chiral transformations, very high in the spectrum must coincide. Hence effects of spontaneous breaking of chiral symmetry in QCD vacuum that are crucially important for the low-lying spectra, become irrelevant for the highly-lying states. Then to the extent that identifiable hadronic resonances still exist in the continuum spectrum at high excitations this implies that the highly excited hadrons must fall into multiplets associated with the representations of the chiral group. We demonstrate that this is indeed the case for meson spectra in the large $N_c$ limit. All possible parity-chiral multiplets are classified for baryons and it is demonstrated that the existing data on highly excited $N$ and $\Delta$ states at masses of 2 GeV and higher is consistent with approximate chiral symmetry restoration. However new experimental studies are needed to achieve any definitive conclusions.
1 Introduction

QCD is a very strange theory. On the one hand, it is formulated in terms of quarks and gluons; on the other hand, these particles are never observed experimentally. Due to confinement, a fundamental but poorly understood property of the theory, only color neutral (color singlet) particles are possible as asymptotic states. Thus only hadrons will hit our detectors but not quarks or gluons.

Another interesting property of QCD is that its Lagrangian has an almost perfect $SU(2)_L \times SU(2)_R$ chiral symmetry, which is broken only by the very small up and down quark masses. However this symmetry is not directly observed in the world - it is hidden, i.e. spontaneously broken.

One of the most intriguing fields of study in physics, both experimentally and theoretically, is the exploration of regimes where the color degree of freedom is not trapped into a small volume of hadrons, and where chiral symmetry is restored. These regimes are expected to be achieved as phases of bulk matter at high temperature or/and at high density. Experimentally it is believed these conditions can be reached in heavy ion collisions.

As has been argued very recently [1, 2], however, it is possible that a regime exists where the chiral symmetry is (almost) restored but hadrons as entities still exist. To see this regime one needs to only study very highly excited hadrons. Such a task is experimentally feasible with present facilities and with a careful analysis of older data. Evidence of effective chiral restoration might then be seen in the spectroscopic patterns of the highly excited hadrons. It is amusing to note, that data which hint the onset of this regime have existed for many years, but have not attracted much attention.

The aim of this paper is to review the recent developments in this field. As will be seen very little can be calculated directly. However we can reach some definitive conclusions on the asymptotic symmetry properties of spectral functions using only very general theoretical grounds such as the well-established idea of quark-hadron duality, the property of asymptotic freedom and the apparatus of operator product expansion (OPE) in QCD. In particular one can show that the effects of spontaneous symmetry breaking must smoothly switch off once we go up in the spectrum. This then implies that the spectrum of highly excited states should reveal the chiral symmetry of QCD. This will be reflected in multiplet structures for the highly excited states. There is a caveat which must be made at this point. On theoretical grounds we have no \textit{a priori} way to establish whether or not hadron states remain as identifiable resonances when one studies the spectrum high enough for chiral symmetry to be effectively restored. We conjecture, however, that it is possible, and then ask whether the experimental data support this conjecture. Moreover we have
strong theoretical evidence that the conjecture is not impossible: in large $N_c$ QCD the meson spectrum can be shown to behave in precisely this way.

The idea that the fundamental strong interaction theory should possess an approximate $SU(2)_L \times SU(2)_R$ (or $SU(3)_L \times SU(3)_R$) chiral symmetry dates back to the 1960s [3, 4, 5]. One of the most important insights from this was that this symmetry must be spontaneously broken in the vacuum (i.e. realized in the Nambu-Goldstone mode). The most important early arguments were: (i) the absence of parity doublets in the hadron spectrum (if the chiral symmetry were realized in the Wigner-Weil mode—i.e. if the vacuum were trivial—then the hadron spectrum would have to reveal the multiplets of the chiral group which are manifest as parity doublets); (ii) the exceptionally low mass of pions, which are taken to be pseudo-Goldstone bosons associated with the spontaneously broken axial symmetry. Substantial phenomenological work in this field occurred during the 1960s. However, the microscopic foundations of the symmetry were not well understood. When QCD appeared in the 1970s, one of the reasons for its rapid acceptance was that it very naturally explained all of the successes of chiral current algebra.

In parallel with the development of the current algebra, the physics of excited hadron states also attracted significant attention. One important result for the present context is that the famous linear-like behavior of Regge trajectories required the parity doubling of baryonic states, due to the so called generalized McDowell symmetry [6]. And indeed, the “Regge physicists” have observed that high in the baryon spectrum there appear more and more parity doublets. On the other hand, such doublets were clearly absent low in the spectrum, a fact which could not be understood from the Regge physics perspective.

Given these facts, it is difficult to understand why the simple idea that chiral symmetry is effectively restored for states high in the spectrum was not explored long ago. From the present perspective there is no contradiction between the linear-like behavior of the highly-lying baryon Regge trajectories and the absence of parity doublets low in the spectrum. We know, that the well established spontaneous breaking of chiral symmetry prevents the low-lying states from doubling. This, together with the McDowell symmetry suggests then that there should be no systematic linear parallel Regge trajectories for positive and negative parity states low in the baryon spectrum. Of course, this is precisely what is seen [7].

To the best of our knowledge the first speculations that the parity doublets seen in the highly lying baryon states probably reflect the chiral symmetry restoration have appeared only recently [3]. This possibility was taken seriously in ref. [1]. It was argued in the latter work that it is quite natural to expect chiral symmetry restoration high in the spectrum because in this case the typical momenta of quarks should be high, and once it is high enough and approach the chiral symmetry...
restoration scale the dynamical (constituent) mass of quarks should drop off and as a consequence the chiral symmetry should be restored. In other words, at high momenta the valence quarks in hadrons should decouple from the quark condensates. This smooth chiral symmetry restoration is seen by the presence of approximate parity doublets high in the spectrum. This perspective, while interesting in its own right, suggests strong limitations on constituent quark models. Thus it becomes evident that the constituent quark model is not applicable high in the spectrum.

That chiral symmetry must indeed be restored high in the spectrum was shown in ref. [2]. This can be seen directly from quark-hadron duality, asymptotic freedom property of QCD and the OPE. It was shown that even if the chiral symmetry is strongly broken in the vacuum, and hence the low-lying states do not manifest chiral symmetry, one should expect effective chiral symmetry restoration in the spectral density for highly lying states in the spectrum. This then suggests that the highly excited hadrons should fall into multiplets of nearly degenerate states which are associated with representations of the chiral group. All such possible multiplets have been classified and it was demonstrated that the existing data on highly lying baryon resonances are appear to be compatible with this. New experimental studies are needed, however, in order to make any definite statements whether we see approximate chiral symmetry restoration at baryon masses of 2 GeV and higher. If it does take a place, then the spectrum of highly excited baryons should consist exclusively of approximate parity doublets.

These ideas have immediately been extended by Beane [9] for highly excited vector and axial vector mesons, which also should be degenerate once the chiral symmetry is restored. Beane has proved that it is impossible in the large $N_c$ limit to satisfy the chiral symmetry of QCD Lagrangian and the quark-hadron duality if the highly excited vector and axial vector mesons do not form parity doublets. Yet, there appear to be no experimentally observable parity doublets in this meson case, in contrast to baryons.

We should stress that this smooth chiral symmetry restoration should not be confused with a phase transition. If one defines a phase transition in the usual way as an abrupt transition of the vacuum from one phase to the other, then the phase transition implies symmetry restoration through the whole spectrum of the system, not only for the highly lying states. On the contrary, the property which we discuss, is a particular case of a different, but quite general, physical property. Namely, if one studies a system (in our case the QCD vacuum) with a high frequency (or short distance) probe that is sensitive to distances that are much smaller than the length associated with the symmetry breaking in the system, then the response of the system to this probe is essentially the same as if there were no symmetry breaking in the system. For example, if one probes a metal in the superconducting phase with photons $\hbar \omega \gg \Delta$, then the superconducting coherence in the ground state (which
is analogous with the QCD vacuum for our case) become unimportant and response of the superconductor is the same as of normal metal. There is a smooth transition from the regime $\hbar \omega \sim 2\Delta$—where the effects of the superconducting (phase coherent) structure of the metal are crucially important, to the regime $\hbar \omega \gg \Delta$—where they are not.

This short review consists of a number of sections. In the second one we give a short overview of chiral symmetry in QCD, which may be omitted by any reader who is familiar with the subject. In the third section we give a simple pedagogical quantum mechanical example. We show that it is a very natural property of quantum systems that a symmetry breaking effect which is important for the low-lying spectrum of the system, can become unimportant for the highly-lying states; the high lying states reveal a multiplet structure of nearly degenerate states. In the next section we discuss a tool, the spectral density, which can be used to study systems with continuous spectra. In that section, we show why one should expect the smooth chiral symmetry restoration high in the spectrum. The fifth section is devoted to somewhat delicate question about to what extent one can use the language of highly excited resonance states once we are in the continuum. In the sixth section we classify all possible parity-chiral multiplets that should be recovered once we approach the regime of chiral symmetry restoration. The experimental data for both low-lying baryon states and the highly lying states are analyzed in the seventh section, where we show that the existing experimental pattern of $N$ and $\Delta$ excitations is such that it can be interpreted that one achieves the regime of approximate chiral symmetry restoration at baryon masses of 2 GeV. Nevertheless, we stress that the new experimental studies are called for to make this conjecture a more definite statement. Finally we present a general outlook in a concluding section.

2 Short overview of chiral symmetry in QCD

In the chiral limit the quarks are massless. In reality the masses of $u$ and $d$ quarks are quite small compared to the typical hadronic scale of 1 GeV; to a good approximation they can be neglected. In this limit the right and left components of quark fields

$$
\psi_R = \frac{1}{2} (1 + \gamma_5) \psi, \quad \psi_L = \frac{1}{2} (1 - \gamma_5) \psi
$$

are decoupled. This is because the quark-gluon interaction is vectorial, $\bar{\psi} \gamma^\mu \psi A_\mu$, which does not mix the right- and left-handed components of quark fields. On the other hand in the chiral limit the quark-gluon interaction is insensitive to the specific flavor of quarks. For example one can substitute, the $u$ and $d$ quarks by properly normalized orthogonal linear combinations of $u$ and $d$ quarks (i.e. one can perform a rotation in the flavor space) and nothing will change. Since the left- and right-
handed components are completely decoupled, one can perform two independent flavor rotations of the left- and right-handed components:

\[ \psi_R \rightarrow \exp \left( \frac{i \theta^R R}{2} \right) \psi_R; \quad \psi_L \rightarrow \exp \left( i \frac{\theta^L L}{2} \tau^a \right) \psi_L, \]

(2)

where \( \tau^a \) are the isospin Pauli matrices and the angles \( \theta^R \) and \( \theta^L \) parameterize rotations of the right- and left-handed components. These rotations leave the QCD Lagrangian invariant. The symmetry group of these transformations,

\[ SU(2)_L \times SU(2)_R, \]

(3)

is called chiral symmetry.

Now generally if the Hamiltonian of a system is invariant under some transformation group \( G \), then one can expect that one can find states which are simultaneously eigenstates of the Hamiltonian and of the Casimir operators of the group , \( C_i \). Now, if the ground state of the theory, the vacuum, is invariant under the same group, i.e. if for all \( U \in G \)

\[ U |0 \rangle = |0 \rangle, \]

(4)

then eigenstates of this Hamiltonian corresponding to excitations above the vacuum can be grouped into degenerate multiplets corresponding to the particular representations of \( G \). This mode of symmetry is usually referred to as the Wigner-Weyl mode. Conversely, if (4) does not hold, the excitations do not generally form degenerate multiplets in this case. This situation is called spontaneous symmetry breaking.

If chiral symmetry were realized in the Wigner-Weyl mode, then the excitations would be grouped into representations of the chiral group. The representations of the chiral group are discussed in detail in one of the following sections. The important feature is that the every representation except the trivial one (which by quantum numbers cannot include baryons) necessarily implies parity doubling. In other words, for every baryon with the given quantum numbers and parity, there must exist another baryon with the same quantum numbers but opposite parity and which must have the same mass. In the case of mesons the chiral representations combine, e.g. the vector mesons with the axial vector mesons, which should be degenerate. This feature is definitely not observed for the low-lying states in hadron spectra. This means that eq. (4) does not apply; the chiral symmetry of QCD Lagrangian is spontaneously (dynamically) broken in the vacuum, i.e. it is hidden. Such a mode of symmetry realization is referred to as the Nambu-Goldstone one.

The independent left and right rotations (2) can be represented equivalently with independent isospin and axial rotations.
\[ \psi \rightarrow \exp \left( i \frac{\theta^a V}{2} \right) \psi; \quad \psi \rightarrow \exp \left( i \frac{\gamma_5 \theta^a A}{2} \right) \psi. \]  

In the Wigner-Weyl mode, the invariance under these transformations implies three conserved vector and three conserved axial vector currents. The existence of approximately degenerate isospin multiplets in hadron spectra suggests that the vacuum is invariant under the isospin transformation. Indeed, from the theoretical side the Vafa-Witten theorem guarantees that in the local gauge theories the vector part of chiral symmetry cannot be spontaneously broken. The axial transformation mixes states with opposite parity. The fact that all states do not have parity doublets implies that the vacuum is not invariant under the axial transformations. In other words the almost perfect chiral symmetry of the QCD Lagrangian is dynamically broken down by the vacuum to the vectorial (isospin) subgroup

\[ SU(2)_L \times SU(2)_R \rightarrow SU(2)_I. \]  

The noninvariance of the vacuum with respect to the three axial transformations requires existence of three massless Goldstone bosons, which should be pseudoscalars and form an isospin triplet. These are identified with pions. The nonzero mass of pions is entirely due to the explicit chiral symmetry breaking by the small masses of \( u \) and \( d \) quarks. These small masses can be accounted for as a perturbation. As a result the squares of the pion masses are proportional to the \( u \) and \( d \) quark masses

\[ m^2_{\pi^{+,-}} = -\frac{1}{f^2_\pi} \left( m_u + m_d \right) (\bar{u}u + \bar{d}d) + O(m_{u,d}^2), \]

\[ m^2_{\pi^0} = -\frac{1}{f^2_\pi} \left( m_u \bar{u}u + m_d \bar{d}d \right) + O(m_{u,d}^2). \]

That the vacuum is not invariant under the axial transformation is directly seen from the nonzero values of the quark condensates, which are an order parameter for spontaneous chiral symmetry breaking. These condensates are the vacuum expectation values of the \( \bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L \) operator and at the renormalization scale of 1 GeV they approximately are

\[ \langle \bar{u}u \rangle \simeq \langle \bar{d}d \rangle \simeq -(240 \pm 10 MeV)^3. \]

The values above are deduced from phenomenological considerations. Lattice gauge calculations also confirm the nonzero and rather large values for quark condensates. However, the quark condensates above are not the only order parameters for chiral symmetry breaking. There exist chiral condensates of higher dimension (vacuum expectation values of more complicated combinations of \( \bar{\psi} \) and \( \psi \) that are not invariant under the axial transformations). Their numerical values are difficult to extract from phenomenological data, however, and they are still unknown.
To summarize this section. There exists overwhelming evidence that the nearly perfect chiral symmetry of the QCD Lagrangian is spontaneously broken in the QCD vacuum. Physically this is because the vacuum state in QCD is highly nontrivial which can be seen by the condensation in the vacuum state of the chiral pairs. These condensates break the symmetry of the vacuum with respect to the axial transformations and as a consequence, there is no parity doubling in the low-lying spectrum. However, as we shall show, the role of the chiral symmetry breaking quark condensates becomes progressively less important once we go up in the spectrum, i.e. the chiral symmetry is effectively restored, which should be evidenced by the systematical appearance of the approximate parity doublets in the highly lying spectrum. This is the subject of the following sections.

3 A simple pedagogical example

One key theoretical idea underlying this review is that effect of spontaneous chiral symmetry breaking on the spectrum becomes progressive less important as one studies states higher in the spectrum. As will be discussed later, this can be seen directly from QCD. It is none-the-less instructive to consider a simple quantum mechanical system with some underlying symmetry that is broken and accordingly is not readily apparent in the low lying spectrum but which is effectively restored to good approximation for high lying states. In this section we will discuss such a system in the context of single particle quantum mechanics. This example will illustrate how this general phenomenon can come about. The example we consider is a two-dimensional harmonic oscillator (with an underlying $U(2)$ symmetry) with an added strong symmetry breaking term. We choose the harmonic oscillator only for simplicity; the property that will be discussed below is quite general one and can be seen in systems with other type of symmetry.

Before discussing the example in any detail we wish to stress that the example is not in perfect analogy to the problem of interest in a number of ways. In the first place, the symmetry breaking in our pedagogical example is explicit while in the case of chiral symmetry breaking in QCD the principal effect of symmetry breaking on the spectrum is due to spontaneous symmetry breaking. Secondly, in our pedagogical example the spectrum is discrete while in the QCD case the spectrum is continuous. Thus, one of the essential question in the QCD case—“Are the states still resonant when they are high enough in the spectrum for symmetry breaking to effective turn off?”—simply does not arise in this section. However, the example will make clear one essential thing: it is quite possible to have a system in which a symmetry breaking effect destroys the effect of a symmetry for the low lying spectrum (as seen by the lack of a multiplet structure) while to very good approximation the symmetry is manifest high in the spectrum.
The unperturbed system we consider is a two dimensional harmonic oscillator. We can always choose our units of time such that the vibration frequency is unity and our units on distance so that the spring constant is unity. The Hamiltonian for such a system is

\[ H_{\text{HO}} = \frac{1}{2} \left( p_x^2 + p_y^2 + x^2 + y^2 \right). \]  

(10)

This Hamiltonian can be rewritten in terms of creation and annihilation operators

\[ a_x = \frac{1}{\sqrt{2}}(x + ip_x), \quad a_x^+ = \frac{1}{\sqrt{2}}(x - ip_x); \]  

(11)

\[ a_y = \frac{1}{\sqrt{2}}(y + ip_y), \quad a_y^+ = \frac{1}{\sqrt{2}}(y - ip_y) \]  

(12)

with

\[ H_{\text{HO}} = a_x^+ a_x + a_y^+ a_y + 1. \]  

(13)

The Hamiltonian above is a quadratic form in both \( x \) and \( p \) and as such is invariant under \( U(2) \) (or equivalently \( SU(2) \times U(1) \)) transformations:

\[ \begin{pmatrix} x + ip_x \\ y + ip_y \end{pmatrix} \rightarrow \begin{pmatrix} x' + ip'_x \\ y' + ip'_y \end{pmatrix} = U \begin{pmatrix} x + ip_x \\ y + ip_y \end{pmatrix} \quad \text{with} \quad U \in SU(2) \times U(1). \]  

(14)

This symmetry has profound consequences on the spectrum of the system. The energy levels of this unperturbed system are trivially found and are given by

\[ E_{N,m} = (N + 1); \quad m = N, N-2, N-4, \ldots, -(N-2), -N, \]  

(15)

where \( N \) is the principle quantum number (which is the sum of the harmonic excitation quanta, \( N = N_x + N_y \)), and \( m \) is the (two dimensional) angular momentum. The interesting point is that as a consequence of the symmetry, the levels are \((N + 1)\) -fold degenerate. In contrast, without the additional \( SU(2) \) symmetry inherent to the harmonic oscillator, the two-dimensional rotations imply the \( U(1) \) symmetry which requires that levels be two-fold degenerate for all \( m \neq 0 \).

Now suppose we add to the Hamiltonian a \( SU(2) \) symmetry breaking interaction (but which is still \( U(1) \) invariant) of the form

\[ V_{\text{SB}} = A \theta(r - R), \]  

(16)

where \( A \) and \( R \) are parameters and \( \theta \) is the step function. Clearly, \( V_{\text{SB}} \) is not invariant under the transformation of eq. (14). Thus the \( SU(2) \) symmetry is explicitly broken by this additional interaction, that acts only within a circle of radius \( R \). As a result one would expect that the eigenenergies will not reflect the degeneracy structure of seen in eq. (13). Of course, if either \( A \) or \( R \) is sufficiently small one expects
to find nearly degenerate multiplets ([15]) everywhere in the spectrum. However, if the coefficients are sufficiently large one would expect to find no obvious remnants of the multiplets low in the spectrum. Indeed, we have solved numerically for the eigenstates for the case of \( A = 4 \) and \( R = 1 \) (in these dimensionless units) and one does not see an approximate multiplet structure in the low lying spectrum as can be seen in Fig. 1. This is not so surprising, the parameters were chosen to be big enough to wipe out the “would be” degeneracy structure.

What is interesting for the present context is the high-lying spectrum. In Fig. 1 we have also plotted the energies between 70 and 74 for a few of the lower \( m \)’s (again for the case of \( A = 4, R = 1 \)). A multiplet structure is quite evident—to very good approximation the states of different \( m \)’s form degenerate multiplets and, although we have not shown this in the figure these multiplets extend in \( m \) up to \( m = N \). Thus the symmetry breaking interaction of eq (16) plays a dominant role in the spectroscopy for small energies and becomes insignificant at higher energies. At higher energies, the spectroscopy reveals the \( SU(2) \) symmetry of the two-dimensional harmonic oscillator.

How does this happen? After all, the symmetry breaking term in this case is explicitly there in the Hamiltonian so how is it that it is not seen high in the spectrum? In fact, it is seen high in the spectrum. The highly-lying levels shown in Fig. 1 are not degenerate, they are merely almost degenerate. The key point is that the effect of the symmetry breaking term is very small high in the spectrum and vanishes asymptotically high. Our central thesis is that something very similar happens for the case of spontaneous chiral symmetry breaking in QCD.

In the pedagogic case considered above, it is quite clear why the symmetry breaking term becomes unimportant high in the spectrum. For very high lying states the energy of the state is much larger than the symmetry breaking interaction over the entire range where the symmetry breaking interaction acts and the wavefunction

Figure 1: The low-lying (left panel) and highly-lying (right panel) spectra of two-dimensional harmonic oscillator with the \( SU(2) \)-breaking term.
amplitude is very small within the circle where the perturbation acts. Thus, the wave function starting at the origin and over entire space is, to very good approximation is simply solution to the Schrödinger equation with the Hamiltonian \((10)\). The effect of the symmetry breaking term then acts as a perturbation

\[
\Delta E_{N,m} = \langle N, m | V_{SB} | N, m \rangle.
\]

Clearly as \(E \to \infty\) the effect of the symmetry breaking term on the wave function and energies vanishes.

One thing about this simple problem is worth noting. While the effect of symmetry breaking becomes increasingly unimportant as one goes higher up in the spectrum, there is nothing discontinuous about the changes in the spectrum; the spectrum smoothly changes from one regime to another. While there is a transition from the symmetry breaking regime low in the spectrum to the explicit symmetry regime asymptotically high in the spectrum, it should not be confused with a phase transition in the thermodynamic sense for a number of reasons. In the first place we are describing single states and not an intensive quantity. Secondly, the transition is smooth and to the extent that it has a thermodynamic analog it would correspond to a cross-over and not to a phase transition. We expect that in QCD as in our toy model there will be a gradual transition in the spectrum from the low energy regime where spontaneous chiral-symmetry-breaking effects are central to the physics to the high energy regime where they are very small.

To summarize this section, we have shown an explicit example in two dimensional quantum mechanics where a system with an underlying symmetry is subjected to a symmetry breaking effect (in this case an explicitly symmetry breaking interaction) which destroys the multiplet structure in the spectrum associated with the symmetry for low lying states but for which the high lying spectrum retains the multiplet structure with nearly degenerate members of the multiplet. Our central argument is that something analogous happens in QCD: a symmetry breaking effect (in this case due to spontaneous symmetry breaking) affects the low-lying part of the spectrum and ruins the “would be” multiplet structure. High in the spectrum however this symmetry breaking effect plays a very small role and to good approximation the spectrum reflects the underlying chiral symmetry. As mentioned above, the analogy between the two cases is not perfect—the QCD case depends on spontaneous symmetry breaking while the simple example is based on explicit symmetry breaking. Moreover, in the QCD the states of interest are in the continuum while in the toy problem they are discrete.

The distinction between a continuous versus a discrete spectrum however raises critical issues. Accordingly, in the following section we will discuss a tool, the spectral density, which enables us to ask sensible questions about a continuous spectrum.
4 Spectral Densities, Correlation Functions and All That

As noted above, the fact that all high lying states in the QCD spectrum are in the continuum complicates any discussion about possible multiplets of hadrons high in the spectrum. The difficulty is that there are states at any energy and thus some means must be found for differentiating between the states at different masses. To proceed, one must find some probe that samples these states and one can ask how strongly the probe couples to states at various masses. This strength can be parameterized in a spectral density. Before discussing in any mathematical detail how this is done a few general comments are in order. The spectral density does not simply tell us about the states, it also tells about the probe, thus it may seem to be ill-suited to providing fundamental information about the spectrum. However, there are cases where information about the spectrum more generally can be learned. For example, if the system has discrete bound states then the spectral density represents a set of $\delta$ functions at masses corresponding to the bound states. While the strength multiplying the $\delta$ functions are properties of the probe as well as the states, the masses in the arguments of the $\delta$ function are independent of the probe and only tell you about the states in the spectrum. Unfortunately, the case of interest to us here is in the continuum so in principle one always contaminates the information about the underlying spectrum with the information about the probe. However, if the spectrum is strongly resonant and the resonances are well separated then there will be a narrow region with large spectral strength—i.e. a large bump. Clearly the height of the bump will depend on the details of the probe however the position of the bump will be large insensitive. There is some sensitivity of the position of the spectral bump to the probe as there is always some ambiguity in separating the resonance contribution from the background but for narrow resonances this ambiguity is small. Thus, provided the spectrum of interest is in a region of well-defined resonances we can use any convenient probe and use the spectral density to determine the resonance position.

The easiest context to deal with a spectral density is the two-point correlation function. Consider a “local current”, $J(x)$ which we can construct entirely out of quark and gluon fields. We will choose $J(x)$ to be gauge invariant and to carry the quantum numbers we wish to study. A useful listing of a number of such currents for various spin-flavor quantum numbers along with their transformation properties under chiral transformations is given in ref. [21]. We can use $J$ to probe the vacuum—i.e. we act with $J$ on the vacuum and create the state with the quantum numbers of $J$, let this state propagate and act again with $J$ to bring the system back to the vacuum. In the processes we learn about the propagation of all possible states with the quantum numbers of $J$.

Sometimes such a current is directly accessible experimentally. For example, if
one chooses the usual electromagnetic current, one is able to study the response of the vacuum to this vector probe, which is represented by the excitations of vector mesons at low \( s \) and by jet production at the very high \( s \). These vector mesons which smoothly transform into jets once the momentum transfer increases, are directly observable in the process \( e^+e^- \rightarrow \text{hadrons} \). The total cross-section of this process is directly connected to the imaginary part of the 2-point correlator of the current in the time-like (Minkowski) domain. The experimental study of this process, in particular in the regime \( s \rightarrow \infty \), was historically one of the most important arguments for acceptance of QCD. Indeed, because of asymptotic freedom, QCD predicts that in the regime \( s \rightarrow \infty \) the correlator is adequately described by the free quark loop diagram (i.e. the photon creates from the vacuum the quark-antiquark pair, which freely propagates to the point where they are annihilated by the photon). The spectral density is given by the imaginary part of this diagram in the time-like region and is measurable as a total cross-section. This process is described in standard texts on QCD \[12\].

If one uses the weak axial vector current, which is experimentally accessible in decays of \( \tau \)-lepton, then one studies the response of the vacuum to the axial-vector probe; this is reflected in the excitations of axial vector mesons, etc. For our purposes it is not necessary that the spectral function associated with the current to be experimentally accessible. For example, there is no experimental probe of a local current that creates three quarks from the vacuum. The two-point correlators with these currents give information about the baryon spectrum. Such currents can be constructed theoretically \[19\], however, and they represent a tool to study baryons in QCD sum rules and lattice gauge calculations \[14, 15\].

Consider for simplicity the two-point correlation function for a Lorentz scalar (or pseudoscalar) \( J \) defined as

\[
\Pi_J(q^2) = i \int d^4xe^{-iq \cdot x} \langle 0 | T [J(x)J(0)] | 0 \rangle
\]  

(18)

where \( |0\rangle \) is the vacuum state and \( T \) represents a time-ordered product. This correlation function can be written in standard Källen-Lehmann form \[22\]

\[
\Pi_J(q^2) = - \int ds \frac{\rho_J(s)}{q^2 - s + i\epsilon}
\]  

(19)

The spectral density \( \rho_J(s) \) is defined as

\[
\rho_J(s) \equiv \frac{1}{\pi} \text{Im} (\Pi_J(s))
\]  

(20)

and has the physical interpretation of being proportional to the probability density that the current \( J \) when acting on the vacuum creates a state of a mass of \( \sqrt{s} \). Analogous expressions for nonscalars are slightly more complicated and will not be
written down here but they can all be expressed using the same general dispersive structure seen in eq. (20).

The key point from the present perspective is that the spectral density for two currents which are related to each other by chiral rotations become essentially equal high in the spectrum:

$$\lim_{s \to \infty} \left[ \rho_J(s) - \rho_{J'}(s) \right] \to 0$$

(21)

for $J' = UJU^\dagger$ where $U \in SU(2)_L \times SU(2)_R$ is a chiral rotation.

Equation (21) is easily understood from considering asymptotic properties of the correlation function at the large space-like momenta and then using the dispersion relation of eq. (19) to relate this to the spectral density. The tool for calculating the correlator at large asymptotic $Q^2 = -q^2$ is the operator product expansion (OPE). The operator product $\int d^4x e^{-iq \cdot x} T[J(x)J(0)]$ in this regime can be written as the following operator series:

$$\int d^4x e^{-iq \cdot x} T[J(x)J(0)] = \sum_k C_k(Q^2, \alpha_s) O_k,$$

(22)

where the Wilson coefficients, $C_k(Q^2, \alpha_s)$ are calculable in perturbation theory, the $O_k$ are local gauge invariant operators constructed from the quark and gluon fields and $Q^2 = -q^2 \gg \Lambda_{QCD}$. Examples of these $O_J$ operators include identity operator $1$, $\bar{q}q$, $F_{\mu \nu} F^{\mu \nu}$, etc. Thus the correlator can be expressed as

$$\Pi_J(Q^2) = \sum_k C_k(Q^2, \alpha_s) \langle 0|O_k|0 \rangle,$$

(23)

where the vacuum expectation values of the $O_k$ are referred to as “condensates” [17]. In such an analysis all of nonperturbative effects including symmetry breaking effects resides in the condensates. The only effect that chiral symmetry breaking can have on the correlator is through the nonzero value of condensates associated with operators which are chirally active (i.e. which transform nontrivially under chiral transformations). To these belong $\langle \bar{q}q \rangle$ and higher dimensional condensates that are not invariant under axial transformation.

Simple dimensional analysis indicates that

$$\frac{C_m}{C_n} \sim Q^{\text{dim}(O_n) - \text{dim}(O_m)},$$

(24)

so that at large $Q^2$ the terms associated with high dimensional operators are suppressed by powers of $1/Q^2$. This structure in which the higher order condensates are increasingly suppressed by higher powers of $1/Q^2$ is essential to the usefulness of the OPE. At large $Q^2$ only a small number of condensates need be retained to
get an accurate description of the correlator. At asymptotically high $Q^2$, the correlator is well described by a single term—the perturbative term which multiplies the identity operator 1. The essential thing to note from this OPE analysis is that the perturbative contribution knows nothing about chiral symmetry breaking as it contains no chirally nontrivial condensates. In other words, though the chiral symmetry is broken in the vacuum and all chiral noninvariant condensates are not zero, their influence on the correlator at asymptotically high $Q^2$ vanishes. This is in contrast to the situation of low values of $Q^2$, where the role of chiral condensates is crucial. Accordingly it is clear that if we consider two operators $J$ and $J'$ related to each other through chiral rotations then, it must be true that for large $Q^2$ the two correlators become equal up to power law corrections.

$$\Pi_J(Q^2) - \Pi_{J'}(Q^2) \sim \frac{1}{Q^n} n > 0 .$$ (25)

The preceding relation merely shows that at large space-like momentum transfers the two correlation functions are identical, whereas the spectral functions describe the time-like region. However, the dispersion relation of eq. (19) provides a connection between the space-like and time-like regions. In essence one understands the fact that correlators for $J$ and $J'$ differ at low $Q^2$ and agree at large $Q^2$ up to power law corrections in the following way—the spectral densities agree (up to small corrections) at large $s$ and disagree only at small $s$. Such a structure guarantees the result of eq. (22). Thus one expects eqn. (21) to hold. Strictly speaking at large $Q^2$ one does not require $\rho_J(s)$ to equal $\rho_{J'}(s)$ on a point by point basis since at large space-like $Q^2$ one cannot resolve the fine structure at large $s$. One does require however that coarse-grained integrals of the two must be essentially equal if integrated over moderate ranges in $s$.

Nevertheless, there are some limitations and ambiguities in the procedure of analytical continuation from the deep Euclidean domain to the Minkowski one [20]. Such a continuation were unambiguous if only the function $\Pi_J(Q^2)$ had been known exactly on some finite interval. In practice, however, one always truncates the expansion (23). In addition, there could be also intrinsically nonperturbative contributions to the coefficients functions $C_k$, e.g. a direct contribution from the small size instantons (these nonperturbative contributions are however suppressed and do not contribute in the limit $Q^2 \to \infty$). More importantly, the OPE by itself does not determine the function $\Pi_J(Q^2)$ everywhere. This is because the OPE is an expansion that picks only the light-cone singularities of the correlator. Thus some possible singularities that are far from the light cone intervals are not properly reflected in OPE. These singularities could be quite important, however, in Minkowski domain. So the question arises, what is a solid basis, nevertheless, for arguing that the symmetries seen at large space like $Q^2$ will be reflected as symmetries in the spectral function. In part, this reflects theoretical prejudice—it is hard to imagine a situation in which it were to fail grossly. After all, as the space-like $Q^2$ goes to $\infty$ one “sees”
more and more of the spectral function at large $s$ as seen from the K"allen-Lehmann representation of eq. (15). Providing the correlator does not vanish in the $Q^2 \to \infty$ limit, the large $Q^2$ correlator will be totally dominated by the large $s$ spectral function and one thus expects them to have the same symmetry behavior, justifying eq. (21). Moreover, there is an empirical basis for the validity of this type of argument. It is a well established fact that, e.g. the deep inelastic processes (which are sensitive to Euclidean kinematics) and the process $e^+e^- \to \text{hadrons}$ (that happens in Minkowski domain) at $|q^2| \to \infty$ are both described by the same free quark loop diagram which represents the first term in OPE [18]. This free quark loop diagram is obviously insensitive to spontaneous chiral symmetry breaking.

Thus, one sees that at large $s$ the spectral density for two operators associated with each other via chiral rotations must become the same. Thus, the spectra at sufficiently large $s$ cannot manifest any effects of chiral symmetry breaking and the spectral densities for any chirally active currents must reflect a chiral multiplet structure: the spectral strength for one channel (corresponding to one current) must be very close to the spectral strength for all channels related to via chiral transformations. We can refer to the phenomenon of the spectral densities becoming close as “effective chiral restoration”. We should note as we did in previous sections, this effective restoration is smooth and is not associated with a phase transition in the thermodynamic sense. Rather it indicates that the effects of spontaneous chiral symmetry breaking on the spectrum are becoming progressively more irrelevant as one goes high in the spectrum so the symmetry is effectively restored.

As noted in the beginning of this section the spectral density tells us about both the “probe” (in this case the current) and the spectrum. It only gives relatively unambiguous information about the spectrum independent of the probe if the spectrum is strongly resonant. Thus the question we must address is whether the spectrum remains resonant when one is high enough in the spectrum. This subject will be discussed in the next section.

5 Are There Hadronic Resonance Way Up There?

The crux of the argument as outlined above is that from general considerations of the OPE in QCD, one deduces that the spectral densities at large $s$ for currents related by chiral transformations become identical. This need not imply that hadrons form multiplets, however, since it may happen that by the time one is high enough in the spectrum for the spectral densities to be essentially equal, one is beyond the region of identifiable hadronic resonances. Thus the conjecture that high in the spectrum of hadronic resonances there are multiplets associated with effective chiral restoration, comes down to the conjecture that the effects of chiral symmetry break-
ing on the spectrum turn off with increasing $s$ more or equally rapidly than certain other nonperturbative effects—namely those effects responsible for the formation of hadronic resonances.

As a matter of principle it is therefore useful to demonstrate that it is at least possible that the effects of chiral symmetry breaking can die off in a region in which resonance are still well defined. There is a very elegant argument due to Beane that demonstrates this. The essence of this argument is that the phenomenon occurs for the problem of meson spectroscopy for QCD in the large $N_c$ limit. As noted by ‘t Hooft in his seminal paper [23], mesons become narrow in the large $N_c$ limit: three meson couplings scale as $N_c^{-1/2}$ implying mesonic widths which scale as $N_c^{-1}$ as $N_c \to \infty$, the widths go to zero and all mesons become stable. Thus, in the large $N_c$ limit, the dispersion integral of eq. (19) becomes a discrete sum:

$$\Pi_J(q^2) = -\sum_l \frac{|a_l|^2}{q^2 - m_l^2 + i\epsilon}$$  \hspace{1cm} (26)$$

where $l$ labels the meson and $a_l$ is the amplitude for the current acting on the vacuum to make the meson. It has long been known that the interplay between the perturbative results and the fact that mesons become narrow imposes strong constraints on the high part of the meson spectrum.

For example as noted by Witten [24], one immediately sees that as $N_c \to \infty$ there must be an infinite number of narrow mesons. The argument goes as follows. If there were only a finite number of terms in eq. (26) then at large space-like $q^2$ the sum would fall off like $1/q^2$. However, perturbation theory is valid in this region and perturbatively the correlation functions grow with increasing space-like $q^2$. These are incompatible and thus the hypothesis that only a finite number of mesons with finite mass contribute must be false. Beane’s argument is a variant of this: even if we are sufficiently high in the spectrum so that chiral symmetry breaking effects are unimportant, if $N_c$ is large enough eq. (23) remains valid. One key point in this is the fact the chiral symmetry breaking condensates do no grow with $N_c$ while the widths of the mesonic resonance decrease with $N_c$. Thus, for the case of mesons in large $N_c$, the spectra indeed do form chiral multiplets of narrow resonant states if one goes sufficiently high in the spectrum [3]. This demonstrates by an explicit construction that it is possible for a system to be sufficiently high in the spectrum so that chiral symmetry breaking effects become negligible while still being in a regime where the hadronic states are narrow. This large $N_c$ argument cannot be extended readily to the baryon spectrum since baryons do not become narrow in the large $N_c$ limit.

In fact, there is a small subtlety with the argument even in the mesonic case. The form of the spectral density in eq. (26) is strictly valid for $N_c = \infty$. For finite $N_c$ the mesons are narrow but, never-the-less are of finite width. The widths of these states go as $N_c^{-1}$ since the coupling constants for three meson couplings go as
\( N_c^{-1/2} \) and the widths are proportional to the coupling constants squared. However the proportionality constants depend on the available phase space for the decay and as one goes up in the spectrum the available phase for decay increases both since there are an increasing number of open decay channels and because each channel has larger phase space. Thus the widths shrink with increasing \( N_c \) but grow with increasing mass. Accordingly the behavior of the spectral density in the combined large \( N_c \) and large \( s \) limits depends on which limit one takes first. For any mass and sufficiently large \( N_c \) the mesons are narrow while for any \( N_c \) for sufficiently large mass the mesons are wide. This noncommutativity of limits might potentially be an issue when one formulates a large \( N_c \) argument along the line of the one given by Beane. Since the states of interest are high in the spectrum, how do we know that they are not so high as to be too wide to be isolated? The key point is that the chiral symmetry breaking effects do not grow with \( N_c \) so we expect that high in the spectrum (where chiral symmetry breaking effects become insignificant) they are independent of \( N_c \). In contrast the value of \( s \) where the mesons become so broad as to strongly overlap with other resonance of the same quantum numbers increases with increasing \( N_c \). Thus one can always find an \( N_c \) big enough so that the mesons are still narrow but chiral symmetry is effectively restored.

Beane’s argument demonstrates that it is possible to be in a regime where hadrons are well-defined resonances while at the same time the effects of chiral symmetry breaking have become insignificant. The question, however, is does this happen in the real world of \( N_c = 3 \)? Here we really do not have any reliable theoretical tools of calculations. One might hope that eventually lattice QCD calculations may be able to shed light on this question. However all presently tractable formulations of lattice QCD are in Euclidean space and one must extrapolate this information into the time like region to learn about the spectrum. This is straightforward for the lightest state with given quantum numbers but it is increasingly difficult as one goes up in the spectrum to separate out contributions of higher states. Picking out resonances high in the spectrum is an intrinsically difficult task for lattice QCD. Thus for the foreseeable future we are unlikely to answer our question from ab initio calculations in QCD. Instead we will rely on an analysis of the experimental data to see if there is evidence for this phenomenon. This will be discussed in the following sections.

6 Classification of the parity-chiral multiplets

As discussed above, we know on very general grounds that the effects of chiral symmetry breaking high in the hadronic spectrum become small. The key issue which needs to be addressed is whether or not more-or-less well-defined hadronic resonances exist in this regime. Our conjecture is that for the baryon spectrum they do.
Now if this is the case then high lying baryon states will fall into multiplets of chiral group. To see if the experimental data supports such a conjecture one must first determine what chiral multiplets are expected and then see if the observed states fall into these multiplets.

A simple way to generate the multiplets for baryon states in the chirally restored regime is to use a model in which the high-lying baryon is constructed out of three quark fields. Such a scheme is simple, however it makes model-dependent assumptions. In particular, the construction of a baryon out of three quark fields is common in constituent quark models. However, the constituent quarks (which are massive - in contrast to current quarks - and do not belong to any irreducible representation of the chiral group) do not transform under the chiral group in the same manner as current quarks. Here, we assume three quark states and quarks which transform chirally in the manner of massless current quarks. For pedagogical purposes we will first outline the classification scheme based on this model and only after that will describe a model-independent one.

Assume that baryon properties are determined by the properties of the three valence quarks only. If chiral symmetry is effectively unbroken, then the right and left components of valence quark fields are decoupled

\[ q = \frac{1 - \gamma_5}{2} q + \frac{1 + \gamma_5}{2} q \equiv q_l + q_r \]  

and can be independently rotated in the flavor (isospin) space. The irreducible representations of \( SU(2)_L \times SU(2)_R \) may be labeled as \((I_L, I_R)\) where \(I_L\) and \(I_R\) represent the isospin of the left- and right handed \(SU(2)\) groups. The left component of the quark field is isodoublet with respect to the left rotations, while it is not affected by right rotations, i.e. it is singlet (scalar) with respect to the right rotations. Hence, the left component transforms as \((\frac{1}{2}, 0)\) irreducible representation, while the right component transforms as \((0, \frac{1}{2})\) irreducible representation of the chiral group. Consequently, according to (27), the one-quark field transforms as a direct sum of two irreducible representations

\[ q \sim \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right). \]

We will refer such a representation as fundamental.

Since a baryon within this model picture consists of three quarks only, the possible representations for baryon in the chirally restored phase can be obtained as a direct product of three fundamental representations (28). Using the standard isospin coupling rules separately for the left and right quark components, one easily obtains a decomposition of this direct product
\[
\left[\left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right)\right]^3 = \left[\left(\frac{3}{2},0\right) \oplus \left(0,\frac{3}{2}\right)\right]
\]

\[+ 3 \left[\left(\frac{1}{2},\frac{1}{2}\right) \oplus \left(\frac{1}{2},1\right)\right] + 3 \left[\left(0,\frac{1}{2}\right) \oplus \left(1,\frac{1}{2}\right)\right] + 2 \left[\left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right)\right].\]  

(29)

The last two representations in the expansion above are identical group-theoretically, so they can be combined with the common multiplicity factor 5. Thus, according to the simple-minded model above, baryons in the chirally restored regime will belong to one of the following representations

\[
\left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right); \quad \left(\frac{3}{2},0\right) \oplus \left(0,\frac{3}{2}\right); \quad \left(\frac{1}{2},\frac{1}{2}\right) \oplus \left(\frac{1}{2},1\right).
\]

(30)

As will be discussed below, the fact that parity is unbroken by strong interactions restricts the baryon state to a sum of two irreducible chiral representations.

The result (30) is essentially the correct one. However, since it was obtained with strong model assumptions on the baryon structure and since these assumptions are questionable, it is important to develop a classification based only on the fundamental symmetries and with no assumptions about baryon structure \[2\]. This will be done in what follows.

Effective chiral symmetry restoration implies that the physical states must fill out representations of the chiral group and transform into each other under chiral transformations. At the same time all these physical states must be eigenstates of parity, since the strong interaction does not break parity.\[3\] Parity transforms “left” into “right” and vice versa. Hence, the states that fill out a general irreducible representation \((I_a, I_b)\) cannot be at the same time eigenstates of the parity, because under parity transformation those states transform into the states that belong to a different irreducible representation,

\[
P|\langle I_a, I_b\rangle\rangle = |\langle I_b, I_a\rangle\rangle.
\]

(31)

Irreducible chiral representations are invariant under parity transformations only for the case \(I_a = I_b\). However, the states in the representation \((I_a, I_a)\) only have integral isospin in the range \(I = 0, 1, \ldots, 2I_a\) and thus cannot be baryons in two flavor QCD. (Recall that with two flavors baryons must have a half integral isospin). Thus multiplets must correspond to reducible representation of the chiral group. We have to construct the minimal possible representations of the chiral group for half-integral isospin that are compatible with definite parity for the states. This task is simple

\[1\] In QCD there is a fundamental parameter \(\Theta\), and the parity is conserved if \(\Theta = 0\). The present phenomenological data limits the possible value of this parameter to at most to a very small number. For purposes of strong interaction physics we can safely set it to 0.
because there is an automorphism of the group \( SU(2)_L \times SU(2)_R \) with respect to a mapping of the left and right subgroups. This mapping is an interchange of the left and right chiral charges, \( Q^i_L \leftrightarrow Q^i_R \), where \( i \) refers to isospin projection. Under this operation the vector charge (isospin) \( Q^i = Q^i_L + Q^i_R \) is not affected, while the axial charge, \( Q^i_5 = Q^i_R - Q^i_L \), changes its sign. Since such an operation is the parity transformation, the minimal possible representation of the chiral group that is invariant under parity operation (i.e. under parity transformation every state in the given representation transforms into a state within the same representation) must contain two distinct irreducible representations of \( SU(2)_L \times SU(2)_R \) that transform into each other under parity operation

\[
(I_a, I_b) \oplus (I_b, I_a). \tag{32}
\]

We refer such a representation as a \textit{parity-chiral} multiplet. When chiral symmetry is (almost) restored in spectral density and if the identifiable baryon resonances still exist, then the baryons high in the spectrum fall into such multiplets. While this representation is a reducible one with respect to the chiral group \( SU(2)_L \times SU(2)_R \), it is an irreducible one with respect to the wider symmetry group

\[
SU(2)_L \times SU(2)_R \times C_i, \tag{33}
\]

where the group \( C_i \) consists of two elements: identity and inversion in 3-dim space. This symmetry group is the symmetry of the QCD Lagrangian (neglecting quark masses), however only its subgroup \( SU(2)_I \times C_i \) survives in the broken symmetry mode. The dimension of the representation \((32)\) is

\[
dim_{(I_a, I_b) \oplus (I_b, I_a)} = 2(2I_a + 1)(2I_b + 1). \tag{34}
\]

The isospin group \( SU(2)_I \) is a subgroup of the chiral group, and the isospin symmetry survives in the broken chiral symmetry mode. Hence the isospin is a good quantum number that can be used to classify states in both (approximately) restored and broken chiral symmetry regimes. In the explicit chiral symmetry regime the isospin of the state can be obtained from the left and right isospins according to a standard angular momentum rules. The given representation of the parity-chiral group contains states with all possible isospins

\[
I = |I_b - I_a|, |I_b - I_a| + 1, \ldots, I_b + I_a. \tag{35}
\]

The states of definite parity in the chirally restored regime are constructed from the states with definite chirality and isospin, that we denote \( |I_{(I_a, I_b)}\rangle \). The states of positive and negative parity are

\[
2^{-1/2} \left( |I_{(I_a, I_b)}\rangle + P|I_{(I_a, I_b)}\rangle \right) = 2^{-1/2} \left( |I_{(I_a, I_b)}\rangle + |I_{(I_b, I_a)}\rangle \right) \tag{36}
\]

\footnote{In the literature language is sometimes used in a sloppy way and the representation \((32)\) is referred to erroneously as an irreducible representation of the chiral group.}
and

\[ 2^{-1/2} \left( |I_{I_a}I_{I_b}\rangle - P|I_{I_a}I_{I_b}\rangle \right) = 2^{-1/2} \left( |I_{I_a}I_{I_b}\rangle - |I_{I_b}I_{I_a}\rangle \right), \quad (37) \]

respectively.

Empirically, there are no known baryon resonances within the two light flavor sector which have an isospin greater than 3/2. Thus we have a constraint from the data that if chiral symmetry is effectively restored for very highly excited baryons, the only possible representations for the observed baryons have \( I_a + I_b \leq 3/2 \), i.e. the only possible representations are \((1/2, 0) \oplus (0, 1/2), \ (1/2, 1) \oplus (1, 1/2) \) and \((3/2, 0) \oplus (0, 3/2)\). Since chiral symmetry and parity do not constrain the possible spins of the states these multiplets can correspond to states of any fixed spin. Note this empirical constraint reduces the allowable representations to precisely those seen in the simple valence quark model discussed above. It is worth noting that the constraint on the allowable representations stemming from the lack of known resonances with isospin larger than 3/2 is somewhat weak. It is conceivable that there do exist baryon resonances with \( I > 3/2 \) that simply have not yet been observed. If such states do exist, they greatly expand the possible multiplets. In the analysis that follows we assume that these states either do not exist as well-defined resonances or do so at an energy above the present data (where picking any resonance out of the data is hard). This constrains the allowable multiplets to the ones enumerated above. However, if there were an unidentified resonance with \( I > 3/2 \) with an energy comparable to the known resonances, it is possible that our assignments of states to multiplets would need to be altered.

The \((1/2, 0) \oplus (0, 1/2)\) multiplets contain only isospin 1/2 states and hence correspond to parity doublets of nucleon states (of any fixed spin). Similarly, \((3/2, 0) \oplus (0, 3/2)\) multiplets contain only isospin 3/2 states and hence correspond to parity doublets of \( \Delta \) states (of any fixed spin). However, \((1/2, 1) \oplus (1, 1/2)\) multiplets contain both isospin 1/2 and isospin 3/2 states and hence correspond to multiplets containing both nucleon and \( \Delta \) states of both parities and any fixed spin.

Summarizing, the phenomenological consequence of the effective restoration of chiral symmetry high in \( N \) and \( \Delta \) spectra is that the baryon states will fill out the irreducible representations of the parity-chiral group \((33)\). If \((1/2, 0) \oplus (0, 1/2)\) and \((3/2, 0) \oplus (0, 3/2)\) multiplets were realized in nature, then the spectra of highly excited nucleons and deltas would consist of parity doublets. However, the energy of the parity doublet with given spin in the nucleon spectrum \( a-priori \) would not be degenerate with the doublet with the same spin in the delta spectrum; these

---

3 If one distinguishes nucleon states with different electric charge, i.e. different isospin projection, then this “doublet” is actually a quartet.

4 Again, keeping in mind different charge states of delta resonance it is actually an octet.

5 This representation is a 12-plet once we distinguish between different charge states.
doublets would belong to different representations of eq. (33), i.e. to distinct multiplets and their energies are not related. On the other hand, if $(1/2, 1) \oplus (1, 1/2)$ were realized, then the highly lying states in $N$ and $\Delta$ spectrum would have a $N$ parity doublet and a $\Delta$ parity doublet with the same spin and which are degenerate in mass. In either of cases the highly lying spectrum must systematically consist of parity doublets.

We stress that this classification is the most general one and does not rely on any model assumption about the structure of baryons. The only assumption beyond those of effective symmetry restoration and the lack of parity breaking is that the states fall into representations with $I \leq 3/2$. This last constraint is empirical in nature.

7 Review of the experimental data

Before reviewing the experimental situation in detail, a few words of caution should be given. We rely on the Particle Data Group’s compilation of the resonances and we use the masses of these resonances in attempting to assess whether states are nearly degenerate. It is worth noting at the outset however, that strictly speaking, the resonance masses reported by the PDG are not experimental quantities. The actual experimental quantities are various scattering observables such as differential cross-sections. The resonance parameters can only be extracted from these observables via some type of modeling. For example an amplitude written as the sum of a resonant contribution plus a background term of some prescribed form. Clearly, there is some model dependence in the extraction of the parameters so technically the extracted masses are not purely experimental quantities. However, for strong resonances the model dependence is weak.

In Fig. 2 we show all the well established states in $N$ and $\Delta$ spectra below 2 GeV. Up to approximately 1.8 GeV the spectrum is well explored experimentally. However, this is not the case higher in the spectrum and it would not be surprising if future experiments or reanalysis of old data yield the new resonant states.

What is immediately evident from the low-lying spectrum is that positive and negative parity states with the same spin are not nearly degenerate. Even more, there is no one-to-one mapping of positive and negative parity states of the same spin with masses below 1.8 GeV. This means that one cannot describe the low-lying spectrum as consisting of sets of chiral partners.\[6\]
Figure 2: The low-lying $N$ and $\Delta$ experimental spectra. The shadowed boxes represent experimental uncertainties for baryon masses.
The absence of systematic parity doublets low in the spectrum is one of the most direct pieces of evidence that chiral symmetry in QCD is spontaneously broken. However, as follows from the discussion in previous sections, there are good reasons to expect that chiral symmetry breaking effect become progressively less important higher in the spectrum. As a phenomenological manifestation of this smooth chiral symmetry restoration one should expect an appearance of systematic parity-chiral multiplets high in the spectrum.

The question of relevance is whether the observed baryon highly lying resonances fall into these representations. This is not trivial to determine for a number of reasons. The first is that the theoretical arguments discussed in the previous section suggest that the effective chiral restoration is only approximate both because of finite quark mass effects and due to residual effects of spontaneous symmetry breaking which we have argued are expected to turn off gradually. Moreover, we have no tools to estimate in an a priori fashion the expected size of these symmetry-breaking effects high in the baryon spectrum. Thus some judgment is need to assert that two levels are “nearly degenerate”. A second complication stems from the fact that this high in the spectrum there are many levels close together and one cannot rule out the possibility that two states are near each other in energy by accident. Moreover, as noted above the extraction of the “experimental data” in [26] is somewhat model dependent. In addition high in the spectrum some resonant states may well have been missed and some states may have masses which have been misestimated.

Keeping all these in mind, we note however, that the known empirical spectra of the highly lying $N$ and $\Delta$ baryon resonances suggest remarkable regularity. Below we show all the known $N$ and $\Delta$ resonances in the region 2 GeV and higher and include not only the well established baryons (“****” and “***” states according to the PDG classification [26]), but also “**” states that are defined by PDG as states where “evidence of existence is only fair”. In some cases we will fill in the vacancies in the classification below by the “*” states, that are defined as “evidence actual world (i.e. into the physical states in the Nambu-Goldstone mode), which is not observed. There is a fundamental reason why such attempts cannot be grounded in QCD where hadrons are composite objects. It could be correct if there were a continuous smooth transition for all hadron states from the Nambu-Goldstone mode to the Wigner-Weyl one. This is in conflict, however, with the Coleman-Witten theorem [25] which states that in the large $N_c$ limit in the confining regime (i.e. in the regime where all hadrons as color-singlet entities still exist) the chiral symmetry must be spontaneously broken to the vectorial subgroup. Hence at least in the large $N_c$ limit in QCD it is not possible to provide at the same time the existence of all hadrons and the explicit (Wigner-Weyl) mode of chiral symmetry. This then implies that at least in the large $N_c$ limit in QCD there is no continuous smooth transition for all hadron states from the spontaneously broken to the explicit mode of chiral symmetry. In other words, it is impossible to define the baryon fields that are classified into chiral multiplets and that one-to-one map into physical baryons. This is also consistent with the standard wisdom that transition from the Nambu-Goldstone mode to the Wigner-Weyl one is a phase transition that is a-priory discontinuous.
of existence is poor”. We mark both the 1-star and 2-star states in the classification below.

\[ J = \frac{1}{2} : N^+(2100) (\ast), N^-(2090) (\ast), \Delta^+(1910), \Delta^-(1900)(\ast\ast); \]

\[ J = \frac{3}{2} : N^+(1900)(\ast\ast), N^-(2080)(\ast\ast), \Delta^+(1920), \Delta^-(1940) (\ast); \]

\[ J = \frac{5}{2} : N^+(2000)(\ast\ast), N^-(2200)(\ast\ast), \Delta^+(1905), \Delta^-(1930); \]

\[ J = \frac{7}{2} : N^+(1990)(\ast\ast), N^-(2190), \Delta^+(1950), \Delta^-(2200) (\ast); \]

\[ J = \frac{9}{2} : N^+(2220), N^-(2250), \Delta^+(2300)(\ast\ast), \Delta^-(2400)(\ast\ast); \]

\[ J = \frac{11}{2} : ?, N^-(2600), \Delta^+(2420), ?; \]

\[ J = \frac{13}{2} : N^+(2700)(\ast\ast), ?, ?, \Delta^-(2750)(\ast\ast); \]

\[ J = \frac{15}{2} : ?, ?, ?, \Delta^+(2950)(\ast\ast), ? . \]

The data above suggest that the parity doublets in \( N \) and \( \Delta \) spectra are approximately degenerate; the typical splitting in the multiplets are \( \sim 200 \text{ MeV} \) or less, which is within the decay width of those states. Of course, as noted above,”nearly degenerate” is not a truly well-defined idea. In judging how close to degenerate these states really are one should keep in mind that the extracted resonance masses have uncertainties which are typically of the order of 100 MeV.

We stress that the 1-star-states by no means should be taken very seriously. The uncertainty interval for the masses of 1-star and 2-star should be taken essentially bigger, than for the well established states. This is because the masses of these states were extracted from rather old phase shift analysis and the results of different groups often do not coincide with each other.

Though one cannot rule out the possibility that the approximate mass degeneracy between the \( N \) and \( \Delta \) doublets is accidental (which would presumably mean
that the baryons are organized according to \((1/2, 0) \oplus (0, 1/2)\) for \(N\) and \((3/2, 0) \oplus (0, 3/2)\) for \(\Delta\) parity-chiral doublets), we believe that this fact supports the idea (ii) that the highly excited states fall into approximately degenerate multiplets \((1/2, 1) \oplus (1, 1/2)\).

It can also be possible that in the narrow energy interval more than one parity doublet in the nucleon and delta spectra is found for a given spin. This would then mean that different doublets would belong to different parity-chiral multiplets.

While a discovery of states that are marked by (?) would support the idea of effective chiral symmetry restoration, a definitive discovery of states that are beyond the systematics of parity doubling, would certainly be strong evidence against it. The nucleon states listed above exhaust all states \((\ast\ast\ast\ast\ast\ast, \ast\ast\ast\ast, \ast\ast\ast, \ast\ast\ast)\) in this part of the spectrum included by the PDG. However, there are some additional candidates (not established states) in the \(\Delta\) spectrum. In the \(J = 5/2\) channel there are two other candidate states \(\Delta^+(2000)(\ast\ast)\) and \(\Delta^-(2350)(\ast)\); there is another candidate for \(J = 7/2\) positive parity state - \(\Delta^+(2390)(\ast)\) as well as for \(J = 1/2\) negative parity state \(\Delta^-(2150)(\ast)\). Certainly a better exploration of the highly lying baryons is needed. This task is just for the facilities like in JLAB, BNL, SAPHIR, SPRING-8 and similar.

Recent experimental data from SAPHIR (Bonn) \cite{27} indicate two additional states in the nucleon spectrum:

\[
J = \frac{1}{2} : N^+(1986 \pm 26^{+10}_{-30}), N^-(1897 \pm 50^{+30}_{-2}.
\]

What is interesting, the states again appear as approximate parity doublets. It is not clear at the moment whether or not these new states should be actually identified with those states above mentioned in PDG.

8 Conclusion and outlook

In this short review we have shown that the chiral symmetry of QCD must be restored smoothly as one goes up in the hadron spectra. The arguments are based very general properties such as quark-hadron duality, asymptotic freedom in QCD and the validity of the operator product expansion in QCD. Using these we have demonstrated that asymptotically high in the spectrum, the spectral densities obtained with local currents that transform into each other under chiral transformations, must coincide. This is in marked contrast to the low-lying part of the spectra where these spectral densities are very different due to the spontaneous breaking of chiral symmetry of the vacuum. Physically this is quite easy to understand. The chiral symmetry breaking of the vacuum is simply not important high in the spectrum,
while it is crucial for the low-lying states.

To the extent that the identifiable hadronic resonances still exist in the continuum spectrum at high excitations the effective chiral symmetry restoration in the spectral densities implies that the highly excited hadrons should fall into multiplets associated with the representations of the chiral group. In other words, the spectrum of highly excited $N$ and $\Delta$ states should consist of the approximate parity doublets or higher multiplets. There are two possibilities: (i) the parity doublet in the nucleon spectrum is not degenerate with the doublet of the same spin in the delta spectrum. This would imply that these doublets belong to different chiral multiplets – to $(1/2,0) \oplus (0,1/2)$ for $N$ and to $(3/2,0) \oplus (0,3/2)$ for $\Delta$; (ii) the parity doublets of the same spin in nucleon and delta spectra are degenerate. Then it would support the possibility that both parity doublets belong to the same multiplet – $(1/2,1) \oplus (1,1/2)$. The small splitting in these doublets is due to the explicit chiral symmetry breaking by small masses of $u$ and $d$ quarks as well as due to the remaining small effects of spontaneous symmetry breaking. The latter effects vanish completely only asymptotically high, where the identifiable hadrons probably do not exist in the real world.

We have shown that the existing old data on highly excited $N$ and $\Delta$ baryons in the region of 2 GeV and higher do support this picture. However new experimental studies are necessary to reach definitive conclusion on whether nature realizes approximate chiral symmetry restoration in this region. Such studies could be performed with the existing facilities.

One question which arises is why do we see the chiral symmetry restoration in baryon spectrum but no such evidences in meson spectrum? Indeed, the approximate chiral symmetry restoration would require that e.g. the highly excited vector and axial vector mesons also form approximate parity doublets. If one looks at the PDG tables, then one finds the vector mesons at the following masses $\rho(770)$, $\rho(1450)$, $\rho(1700)$, $\rho(2150)$. There are, however, only two axial vector meson states: $a_1(1260)$ and $a_1(1640)$. As expected from spontaneous chiral symmetry breaking, there is no parity doubling low in the spectrum. (Compare the mass of $\rho(770)$ and $a_1(1260)$). However, there is no hint of parity doubling high in the spectrum at masses of 2 GeV, as one sees in the baryon spectrum. One possible reason is trivial however – with the present experimental possibilities available to date one might expect to have trouble seeing such a doubling in meson spectrum. In the case of baryons, the nucleon targets exist and one can perform direct experiments on high excitation of nucleons by different projectiles such as pions, protons, electrons or photons. The multipole analysis needed to extract resonances in various channels is relatively straightforward. On the other hand, the meson targets do not exist; except in a few cases the only way to extract the meson spectrum is from indirect experiments. For example, the vector mesons are obtained directly from the $e^+e^-$. 

28
annihilation process, which is well explored and the $\rho$ mesons are indeed known up to 2 GeV region. The extraction of higher vector mesons is more difficult due to the opening of the hidden charm production channel. The axial vector mesons are obtained directly from the weak decay of the $\tau$-lepton. The mass of $\tau$ is 1777 MeV, which guarantees that we could not observe the possible axial vector mesons in the 2 GeV region. Other types of experiments are necessary.

References

[1] L. Ya. Glozman, Phys. Lett. B475, 329 (2000).

[2] T.D. Cohen and L. Ya. Glozman, hep-ph/0102206, Phys. Rev. D65, 016006 (2002).

[3] M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).

[4] S. L. Adler and R. F. Dashen, Current Algebras And Applications to Particle Physics, W. A. Benjamin, Inc; New York, 1968.

[5] H. Pagels, Phys. Rep. 16, 219 (1975).

[6] P. D. B. Collins, An Introduction to Regge Theory and High Energy Physics, Cambridge University Press, Cambridge, 1977.

[7] L. Ya. Glozman, hep-ph/0105225.

[8] L. Ya. Glozman and D.O. Riska, Phys. Rep., 268, 263 (1996).

[9] S. R. Beane, hep-ph/0106022, Phys. Rev. D64, 116010 (2001).

[10] C. Vafa and E. Witten, Nucl. Phys. B234, 173 (1984).

[11] M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968).

[12] T.P. Cheng and L.F. Li, Gauge Theory of Elementary Particle Physics, Clarendon Press, Oxford, 1984.

[13] B. L. Ioffe, Nucl. Phys. B188, 317 (1981); E:B191, 591.

[14] P. Colangelo and A. Khodjamirian, ”QCD Sum Rules, a Modern Perspective”, in: At the Frontier of Particle Physics/ Handbook of QCD, ed. M. Shifman, World Scientific, Singapore, 2001.

[15] I. Montvy and G. Münster, “Quantum Fields on the Lattice”, Cambridge University Press, Cambridge, 1994.
[16] K. G. Wilson, Phys. Rev. 179, 1499 (1969).

[17] M. A. Shifman, A. I. Vainstein, and V. I. Zakharov, Nucl. Phys. B147, 385 (1979).

[18] H. Georgi and H. D. Politzer, Phys. Rev. D9, 416 (1974).

[19] B. L. Ioffe, Nucl. Phys. B188, 317 (1981); E:B191, 591.

[20] M. Shifman, ”Quark-Hadron Duality”, in: At the Frontier of Particle Physics/Handbook of QCD, ed. M. Shifman, World Scientific, Singapore, 2001.

[21] T. D. Cohen and X. Ji, Phys. Rev. D55, 6870 (1997).

[22] G. Källen, Hev. Phys. Acta 25,417 (1952); H. Lehmann, Nuovo Cimento 11 342 (1954). This topic is discussed in standard field theory texts. See for example S. Weinberg, The Quantum Theory of Fields, Cambridge University Press, Cambridge, 1995.

[23] G. ’t Hoft, Nucl. Phys. B72, 461 (1974).

[24] E. Witten, Nucl. Phys. B156, 269 (1979).

[25] S. Coleman and E. Witten, Phys. Rev. Lett. 45, 100 (1980).

[26] Particle Data Group, Europ. Phys. Journ. C15,1 (2000).

[27] R. Plötzke et al, Phys. Lett. B444 555 (1998).