Fine Structure Constant, Domain Walls and Generalized Uncertainty Principle in the Universe

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Abstract

In this paper we study the corrections to the fine structure constant from the generalized uncertainty principle in the spacetime of a domain wall. We also calculate the corrections to the standard formula to the energy of the electron in the hydrogen atom to the ground state, in the case of spacetime of a domain wall and generalized uncertainty principle. The results generalize the cases known in literature.
1. Introduction

In the last years there has been an interest in cosmology with a space-time variation of the constants of nature. In the 1920 in order to explain the relativistic splits of the atomic spectral lines, Arnold Sommerfeld introduced the fine structure constant

\[ \alpha_0 = \frac{e^2}{4 \pi \epsilon_0 \hbar c} \]  

where \( c \) is the speed of light in vacuum, \( \hbar = h/2\pi \) is the reduced Planck constant, \( e \) is the electron charge magnitude and \( \epsilon_0 \) is the permittivity of free space, all quantities measured in the laboratories on Earth. The numerical value of the constant is \( \alpha_0 \approx 1/137.035999710 \) that can be determined without any reference to a specific system of the units and \( \alpha \) gives the strength of the electromagnetic interaction. In the recent years possible variations of the fine structure constant have been observed, these observations suggest that about \( 10^{10} \) years ago \( \alpha \) was smaller than today. On the other hand time variation of fundamental constants has been an intriguing field of theoretical research since the propose by Dirac in 1937 \[1\] where in the large numbers hypothesis he conjectured that the fundamental constants are functions of the epoch. The physical motivation to search a time or a space dependence of fundamental constants originates because the effort to unify the fundamental constant imply variations of the coupling constants \[3\]. Let us introduce \( \alpha(z) \) that is the value that might be dependent on the time. The variations of \( \alpha \) can be measured by the so called "time shift density parameter"

\[ \frac{\Delta \alpha}{\alpha} \equiv \frac{\alpha(z) - \alpha_0}{\alpha_0} \]  

with \( \alpha_0 \) value of \( \alpha \) today.

From an experimental point of view there are two ways to test the validity of the "constant" hypothesis of \( \alpha \): local and astronomical methods. The former connected with local geophysical data, the natural reactor \( 1.8 \times 10^9 \) years ago \( (z \approx 0.16) \) in Oklo \[4\], these data give \[5\] \( |\dot{\alpha}/\alpha| = (0.4 \pm 0.5) \times 10^{-17} \text{yr}^{-1} \) (or \( |\Delta \alpha/\alpha| \leq 2 \times 10^{-8} \) that is one of the most stringent constrain on the variation of \( \alpha \) over cosmological time scales. The latter methods consider deep-space astronomical observations, they mainly consider the analysis of spectra from high red-shift quasar absorption system. Evidence of time variation of \( \alpha \) derive from these data \[6\]. It is important to say that these data, coming from the Keck telescope in the Northern hemisphere, give for a range of the red-shift \( 0.2 < z < 4.2 \) \[7\]: \( \Delta \alpha/\alpha = (-0.543 \pm 0.116) \times 10^{-5} \). If we assume a linear increase of \( \alpha \) with time, we have a drift rate \( \frac{d \ln \alpha}{dt} = (6.40 \pm 1.35) \times 10^{-16} \) per year. In any case \( \Delta \alpha/\alpha \) may be more complex \[5\] and a linear extrapolation may not be valid when we consider a cosmic time scale. However, independent analysis of the same phenomena with VLT telescope, in Chile, does not find any variation of \( \alpha \) \[9\], in fact we find \( \Delta \alpha/\alpha = (-0.06 \pm 0.06) \times 10^{-5} \). There is an intensive debate in literature about possible reasons for disagreement, for example a possible reason may be that the Keck telescope is in the Northern hemisphere and VLT telescope is in the Southern hemisphere. Recently \[10\] a re-analysis of Ref. \[9\] varying \( \alpha \) by means of the multiple heavy element transition on the Southern hemisphere has been reported, obtaining \( \Delta \alpha/\alpha = (-0.64 \pm 0.36) \times 10^{-5} \). On the other hand this search may connected in astronomical observations for variations in the fundamental constants in quasar absorption spectra and in laboratory \[11\].

The experimental physics has reached very high precision therefore in order to search corrections very fine to our theories in the description of the nature it is necessary to introduce logical systems more and more sophisticated. In this context, to search corrections to the fine structure constant, it is only possible if we study very complex fields of the knowledge. The conceptual utilization of the GUP may be usefull in order to calculated the corrections to the fine structure.
constant. The paper follows this line in which we want to built a bridge between corrections to the alpha and GUP. On the other hand if we consider a cosmological ambit these corrections may have important consequences if we also consider a topological defect has a domain wall on large scale in the universe. For these reasons it is important to employ gup and $\alpha$ evolution.

The search for a quantum theory of the gravitation is one of the most intriguing problems in physics. The generalized uncertainty principle is a consequence of incorporating a minimal length from a theory of quantum gravity. When we consider a quantum gravity theory we need a fundamental distance scale of order the Planck length $l_p$. These reasonings induce the possibility to have corrections to the Heinsenberg principle in order to have a more general uncertainty principle (GUP). Thus the Heinsenberg principle

$$\Delta x \Delta p \gtrsim h$$

has to be replaced by

$$\Delta x \Delta p \gtrsim h + \beta l_p^2 \frac{\Delta p^2}{\hbar}$$

here $\Delta x$ and $\Delta p$ are the position and momentum uncertainty for a quantum particle, $\beta$ is a positive dimensionless coefficient that may depend on the position $x$ and momentum $p$, usually assumed to be of order one and $l_p = (G\hbar/c^3)^{1/2} \sim 1.66 \times 10^{-33}$ cm is the Planck length. It is important to stress that $l_p^2 \hbar$ may be replaced with the Newtonian constant $G$, therefore the second term in (1.4) is a consequence of gravity. The physical reason consider that the quantum mechanics limits the accuracy of the position and momentum of the particle by the well known rule $\Delta x \geq \hbar/\Delta p$, moreover if we consider general relativity the energy cannot be localized in a region smaller than the one defined by its gravitational radius, $\Delta x \geq l_p^2 \Delta p$. If we combine the results, there is a minimum observable length $\Delta x \geq \max(1/\Delta p; l_p^2 \Delta p) \geq l_p$. This final result is the Generalized Uncertainty Principle, that can be summarized as eq.(1.4).

Generally speaking the GUP is obtained when the Heinsenberg uncertainty principle is considered combining both quantum theory and gravity and it may be obtained from different fields and frameworks as strings [12], black holes [13] and gravitation [14], where the gravitational interaction between the photon and the particle modifies the Heinsenberg principle, adding an additional term in eq. (1.4) proportional to the square of the Planck length $l_p^2$. From a physical point of view very interesting consequences can be found in [15].

The initial stages of the primordial Universe according to the standard model of the particles physics, are often described as the era of the phase transition. In the recent years the cosmological consequence of primordial phase transitions has been the subject of many studies in the early Universe. When we have a cosmological phase transition, topological defects necessarily can be formed [16] [17]: they are domain walls, cosmic strings or monopoles. These phenomena are expected to be produced at a phase transition in various area of physics, for example also in condensed matter physics several examples have been observed, while up today in particle physics, astrophysics and cosmology it is not the case; on the other hand they could have very important cosmological consequences. Generally people study cosmic strings because they present interesting properties and there are not any bad cosmological consequences, instead domain walls scenarios have attracted less attention since there is the so called Zeldovich bound [19], in which in a linear scaling regime would dominate the energy density of the Universe violating the observed isotropy and homogeneity. A domain wall network was proposed to explain dark matter and dark energy [20].

The connection between topological defects and variation of the fundamental constants is an intriguing field of work. The corrections to the fine structure constant has been calculated in
the spacetime of a cosmic strings [21]. In a recent paper [22] it has been studied the correlation of time variation of the fine structure constant in the space-time of a domain wall and in particular it has been shown that the gravitational field generated by a domain wall acts as a medium with spacetime dependent permittivity $\epsilon$. In this way the fine structure constant will depend on a time-dependent function at a fixed point. A further step has been obtained with the calculation of the corrections to the fine structure constant in the spacetime of a cosmic string from the generalized uncertainty principle [23]. In this paper we study the corrections to the fine structure constant in the spacetime of a cosmic domain wall taking into account the generalized uncertainty principle, are calculated. In other terms we generalize our previous study [22]. The paper is organized as follows: in Section 2 we summarize our previous results obtained considering the time variation of the fine structure constant in the space-time of a cosmic domain wall, in Section 3 we generalized the results taking into account the generalize uncertainty principle, in Section 4 we calculate, as application, the correction to the energy ground state of the hydrogen atom, the results are summarized in the concluding Section 5.

2. $\alpha$ in the spacetime of a domain wall

As it is well known a domain wall is a topologically stable kink produced when a vacuum manifold of a spontaneously broken gauge theory is disconnected [17]. A very important concept regards the surface energy density $\sigma$ of a domain wall because it determines the dynamics and gravitational properties, but unfortunately $\sigma$ is very large and this implies that cosmic domain walls would have an enormous impact on the homogeneity of the Universe. It is possible to have constraint on the wall tension $\sigma$ from the isotropy of the cosmic microwave background, in fact if a few walls stretch across the present horizon we have an anisotropy fluctuation temperature of CMB $\frac{\Delta T}{T} \sim 2\pi G\sigma H_0^{-1}$ with G Newton’s constant and $H_0$ Hubble constant. The anisotropy $\frac{\Delta T}{T} \leq 3 \times 10^{-5}$ arises from WMAP therefore it is not possible to have topologically stable cosmic walls with $\sigma \geq 1 MeV^3$.

A cosmic domain wall in the Universe modifies the electromagnetic properties of the free space and in particular if we consider the gravitational field generated by a wall, it acts as a medium with space and time dependent permittivity. Therefore eq. (1.1) implies that the fine structure constant at fixed point will be a time-dependent function. In this Section we follow the way of [22].

Let us consider the line element associated to the spacetime of a thin wall [18]

$$ds^2 = e^{-4\pi G\sigma|x|} (e^2 dt^2 - dx^2) - e^{4\pi G\sigma(ct-|x|)} (dy^2 + dz^2),$$

(2.1)

in which we have considered a model with infinitely static domain walls in the $zy$-plane. Generally speaking in a curved spacetime the electromagnetic field tensor $F_{\mu\nu}$ has electric and magnetic fields respectively defined as

$$E_i = F_{0i}; \quad B^i = -\frac{1}{2\sqrt{\gamma}} \epsilon^{ijk} F_{jk},$$

(2.2)

with $\gamma = \det || \gamma_{ij} ||$ determinant of the spatial metric and $\epsilon^{ijk}$ Levi-Civita symbol. If we consider a charged particle $q$, the charge density at rest in $x = x_0$ is

$$\rho = \frac{q}{\sqrt{\gamma}} \delta(x - x_0).$$

(2.3)

We write the divergence and curl operators in curved spacetime as

$$\text{div } v = \frac{\partial_i (\sqrt{\gamma} v^i)}{\sqrt{\gamma}}$$

(2.4)
and

$$(\text{curl } \mathbf{v})^i = \frac{\epsilon^{ijk} (\partial_j v_k - \partial_k v_j)}{2\sqrt{\gamma}}$$

respectively, therefore Maxwell’s equation in three dimensions are

$$\text{div } \mathbf{B} = 0 \quad \text{curl } \mathbf{E} = -\frac{1}{\sqrt{\gamma}} \frac{\partial (\sqrt{\gamma} \mathbf{B})}{\partial t},$$

$$\text{div } \mathbf{D} = 4\pi \rho \quad \text{curl } \mathbf{H} = \frac{1}{\sqrt{\gamma}} \frac{\partial (\sqrt{\gamma} \mathbf{D})}{\partial t}.$$  

where

$$\mathbf{D} = \frac{\mathbf{E}}{\sqrt{g_{00}}} \quad \mathbf{H} = \sqrt{g_{00}} \mathbf{B}.$$  

If we indicate with $\nabla$ the three-dimensional nabla operator in Euclidean space, we can rewrite the first equation of eq. (2.7) as

$$\nabla \cdot (\epsilon \mathbf{E}) = 4\pi q \delta(x - x_0),$$

where $\epsilon = \sqrt{\gamma}/\sqrt{g_{00}}$. The solution of Poisson equation, eq.(2.9), is $\epsilon \mathbf{E} = q/4\pi \epsilon r^3$ that gives for the electric field the expression

$$\mathbf{E} = \frac{q}{4\pi \epsilon r^3} \mathbf{r}.$$  

It is interesting to note that if we consider the metric (2.1), a domain wall produce a gravitational field that acts as a medium with a permittivity $\epsilon$ that has the expression

$$\epsilon = \epsilon_0 e^{4\pi G \sigma (ct - |x|)}.$$  

Therefore a cosmic domain wall in the Universe modifies the electromagnetic properties of the free space and taking into account eq.(2.11), we can say that in the free space the constant $\alpha$ is given by eq.(1.1) and in the spacetime of a domain wall is

$$\alpha = \frac{e^2}{4\pi \epsilon h c},$$

that is to say the fine structure constant in the spacetime of a domain wall is spacetime dependent.

3. $\alpha$ in the spacetime of a domain wall from the generalized uncertainty principle

Now we calculate the corrections to the fine structure constant in the spacetime of a domain wall taking into account the generalized uncertainty principle. If we take into account the gravitational interactions, the Heisenberg principle must be revised with the generalized uncertainty principle, that is to say $\Delta x \Delta p \geq h$ becomes $\Delta x \Delta p \geq h + \beta l_{Pl}^2 (\Delta p/h)^2$, this suggests to introduce a kind of ”effective” Planck constant, $h_{eff}$, due to the generalized uncertainty principle, defined as

$$h_{eff} = h \left[ 1 + \beta l_{Pl}^2 \left( \frac{\Delta p}{h} \right)^2 \right]$$

(3.1)
in order to write $\Delta x \Delta p \geq \hbar_{\text{eff}}$. Therefore the constant will be

$$\alpha_{\text{eff}} = \frac{e^2}{4 \pi \epsilon \hbar_{\text{eff}}} \quad (3.2)$$

with $\epsilon$ given by (2.11). In this way the GUP is be able to introduce "itself" in the expression and change the structure of $\alpha$.

In order to obtain $\alpha_{\text{eff}}$ let us consider eq. (1.4) that we solve as a second order equation for the momentum uncertainty in terms of the distance uncertainty, we have

$$\frac{\Delta p}{\hbar} = \frac{\Delta x}{2\beta l_{\text{pl}}} \left[ 1 - \sqrt{1 - \frac{4\beta l_{\text{pl}}^2}{(\Delta x)^2}} \right] \quad (3.3)$$

(we do not consider the sign + in the parenthesis because non physical, in fact if we impose correct classical limit $l_{\text{pl}} \to 0$ we only have minus sign).

We obtain $\Delta x$ considering Bohr’s radius in the spacetime of a domain wall. In absence of a domain wall a Bohr’s atom has the radius ($n=1$) $r_0 = 4\pi\epsilon_0 h^2/m_e^2$, with $m$ mass of the electron, but in presence of a domain wall and the GUP, it becomes

$$\tilde{r}_0 = \frac{4 \pi \epsilon h^2}{m e^2} \equiv \Delta x. \quad (3.4)$$

In other terms Bohr’s radius in a spacetime of a domain wall, $\tilde{r}_0$, is connected with $r_0$ classical Bohr’s radius by the relation

$$\tilde{r}_0 = r_0 e^{4\pi G\sigma (ct - |x|)}. \quad (3.5)$$

Now introducing (3.3) in (3.1) we obtain $\hbar_{\text{eff}}$ as a function of $\Delta x$. This $\hbar_{\text{eff}}$ introduced in (3.2), finally gives the fine structure constant in the spacetime of a domain wall with the generalized uncertainty principle:

$$\alpha_{\text{eff}} = \frac{e^2}{4 \pi \epsilon \hbar \frac{r_0^2}{4 l_{\text{pl}}}} \left[ 1 + \frac{(\Delta x)^2}{4 \beta l_{\text{pl}}^2} \left( 1 - \sqrt{1 - \frac{4\beta l_{\text{pl}}^2}{(\Delta x)^2}} \right)^2 \right]^{-1} \quad (3.6)$$

We discuss eq. (3.6) starting from the case without the spacetime of a domain wall, in other terms $\alpha$ with the generalized uncertainty principle. There are several studies [24] that consider non-commutativity spacetime and quantum gravitational effects in the calculation of the fine structure constant with $\Delta x$ given by (3.4). If we only consider the GUP effect on the fine structure constant we have

$$\alpha_{\text{gup}} \simeq \alpha_0 \left[ 1 - 3.6 \times 10^{-50} \right], \quad (3.7)$$

but in presence of the cosmic domain wall it is possible to render explicit the expression of $\alpha$

$$\alpha_{\text{eff}} = \alpha_0 e^{-4\pi G\sigma (ct - |x|)} \left[ 1 + \frac{r_0^2}{4 l_{\text{pl}}^2} e^{8\pi G\sigma (ct - |x|)} \left( 1 - \sqrt{1 - \frac{4\beta l_{\text{pl}}^2}{r_0^2}} e^{-8\pi G\sigma (ct - |x|)} \right)^2 \right]^{-1} \quad (3.8)$$

4. Corrections to the energy groung state of hydrogen atom

It is interesting to calculate the corrections to the energy ground state $E_0$ of the hydrogen atom in presence of a domain wall and considering the GUP. Classically the hydrogen atom decays
and it is just the Heisenberg Uncertainty Principle that assures the stability. The energy of the electron in the hydrogen atom is

$$E_{\text{dw}}^{\text{gup}} \sim \frac{p^2}{2m} - \frac{e^2}{4\pi \epsilon \bar{r}_0},$$

(4.1)

the GUP gives

$$\Delta p \geq \frac{\hbar}{\Delta x} + \frac{l_{\text{pl}}^2 (\Delta p)^2}{\Delta x \hbar}.$$  

(4.2)

Now, let us iterate eq.(4.2), neglecting the terms $O(l_{\text{pl}}^2)$ and squaring both members we have

$$p^2 \geq (\Delta p)^2 \geq \frac{\hbar^2}{(\Delta x)^2} + 2 \frac{\hbar^2 l_{\text{pl}}^2}{m \bar{r}_0^4}.$$  

(4.3)

Therefore eq. (4.1) for the energy becomes

$$E_{\text{dw}}^{\text{gup}} = \frac{\hbar^2}{2m \bar{r}_0^2} - \frac{e^2}{4\pi \epsilon \bar{r}_0} + \frac{\hbar^2 l_{\text{pl}}^2}{m \bar{r}_0^4}.  

(4.4)$$

From a physical point of view, eq. (4.4) is very interesting. If we ”switch off” the domain wall contribution, the first two terms on the second member, are the energy of the ground state of the electron in the hydrogen atom, $E_0 = -m e^2/8\pi^2 \epsilon_0^2 \hbar^2 = 13.6 \text{ eV}$. The third term is the correction to the ground state energy due to the generalized uncertainty principle, that is to say:

$$\Delta E_{\text{gup}} = \frac{m^3 l_{\text{pl}}^2 e^8}{(4\pi \epsilon_0)^4 \hbar^6} \sim 10^{-48} \text{ eV}.  

(4.5)$$

This corrective term, due to the GUP, is very little to be experimentally tested actually. If now we ”switch on” the domain wall contribution, we have

$$E = -\frac{m e^4}{8\pi^2 \epsilon_0^2 \epsilon^2} e^{-8\pi G \sigma (ct - |x|)} + \frac{m^3 e^8 l_{\text{pl}}^2}{(4\pi \epsilon)^4 \hbar^6} e^{-16\pi G \sigma (ct - |x|)}.  

(4.6)$$

In other terms, when we consider the domain wall, the classical and the GUP contribution to the energy are exponentially modulated, therefore an integrate effect, starting from the early Universe, may be relevant into the amplification to the correction to the energy of the electron in a hydrogen atom from an experimental point of view.

5. Conclusion

In conclusion, if we consider that the gravitational interactions may modify the Heisenberg principle with the so called generalized uncertainty principle and if we also consider that the fine structure constant may be different in different epochs, it is possible to study the right expression of the fine structure constant in the spacetime of a domain wall, taking into account the generalized uncertainty principle. In this paper we have examined the effects of these two contributions on $\alpha$. We have found the most general expression given by (3.8). The modification of $\alpha$ involves two aspects, the domain wall’s contribution influences the value of $\epsilon_0$ that becomes $\epsilon$ given by (2.11), while the GUP’s contribution acts in order to modify the Planck constant $\hbar$ into $\hbar_{\text{eff}}$ given by (3.1). $\alpha$ is very near at $\alpha_0$ as we can see in (3.7), this means that the GUP does not change the numerical value in an appreciable way. The domain wall’s contribution consists into exponentially modulate the $\alpha_0$ value and from a numerical point of view if we set $ct - |x| = H_0^{-1}$, we does not change the value of $\alpha$. On the other hand it is possible to think as a kind of ”integrate effect” in the spacetime, in this way it is possible to have a different evolution
of $\alpha$ in the spacetime. These arguments are also very interesting because recently a sample of 153 measurements from the ESO Very Large Telescope indicate that $\alpha$ appears on average to be larger than in the past [25]. Moreover manifestations of a spatial variation in $\alpha$ must be independently confirmed by means terrestrial measurements as laboratory, meteorite data and nuclear reactor [26] and by means a new test connected by big bang nucleosynthesis [27]. For completeness we have also studied the corrections to the energy of the hydrogen atom if we add both the actions: gap and domain wall. Also in this case the corrections are still too small for the actual experiments. Future investigations are in progress by the author.

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