Auxiliary particle filter-model predictive control of the vacuum arc remelting process

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Abstract. Solidification control is required for the suppression of segregation defects in vacuum arc remelting of superalloys. In recent years, process controllers for the VAR process have been proposed based on linear models, which are known to be inaccurate in highly-dynamic conditions, e.g. start-up, hot-top and melt rate perturbations. A novel controller is proposed using auxiliary particle filter-model predictive control based on a nonlinear stochastic model. The auxiliary particle filter approximates the probability of the state, which is fed to a model predictive controller that returns an optimal control signal. For simplicity, the estimation and control problems are solved using Sequential Monte Carlo (SMC) methods. The validity of this approach is verified for a 430 mm (17 in) diameter Alloy 718 electrode melted into a 510 mm (20 in) diameter ingot. Simulation shows a more accurate and smoother performance than the one obtained with an earlier version of the controller.

1. Introduction
Vacuum Arc Remelting (VAR) is a melting process used in the production of nickel-based superalloys and various titanium alloys. In this process, a continuous arc is used to melt a metal alloy electrode in a water-cooled copper crucible. Molten metal falls to the bottom of the crucible and solidifies into an ingot. By performing this process in a vacuum, some oxides and other volatile compounds are removed from the material improving its properties [12]. Although ingots obtained with VAR have elevated chemical and mechanical homogeneity, they are prone to segregation defects, such as freckles (solute channels) and white spots.

The study of the mechanisms that describe the formation and growth of solidification defects in remelting processes is still an active area of research, but it is commonly believed that they can be prevented by maintaining appropriate solidification conditions throughout the melt. For example, solute-lean white spots are believed to be caused by material falling into the liquid pool atop the ingot and resulting in solidification defect precursors [7]. For that reason, deep liquid pools - which result in longer local solidification times (LST) - are used to dissolve fall-in material. On the other hand, solute-rich freckles are believed to be caused by convective instabilities and buoyancy-driven flows [5]. It has been reported that the tendency for freckles decreases when melting with shallower liquid pools because LST decreases. Prevention of both freckles and white spots can be thought of as a balancing act that requires having a liquid pool that is deep enough to prevent white spots while being shallow enough to prevent freckles.
In VAR process control, not only the shape of the solidification front is important for prevention of defects. For example, tree rings - another kind of solidification defect - are found when thermal instabilities occur in the liquid pool [17]. Thus, for accurate prevention of solidification defects in remelted ingots, process controllers must drive the liquid pool profile to desired values (to prevent white spots and freckles) while attenuating process perturbations (to prevent tree rings).

In usual industrial practice, ingot solidification is not monitored nor controlled in the melt. Most process controllers attempt to attain a desired melt rate hoping that it will result in controlled solidification, but this is valid only at steady-state conditions. The first attempt at solidification control was developed by Beaman et al., who designed a model-based controller for pool depth based on melting current [2]. The pool depth controller was designed for quasi-steady conditions - which occur during most of the melt - and worked in cooperation with a separate controller that corrected ram speed to keep a desired electrode gap. The controller was tested experimentally showing close agreement between references and manually-measured pool depths [16]. Although successful in its primary goal, the authors reported that inaccuracies in the ingot solidification model resulted in oscillatory behavior when the reference depth was modified.

This article presents an improved version of the pool depth controller where:

- A nonlinear model is used for improved performance when moving away from the linearization point, e.g. melt rate perturbations, start-up, hot-top, etc.
- A model predictive controller (MPC), an advanced method of process control widely used in chemical applications, is adopted to optimize the predicted system response in the near future.
- The electrode gap dynamics are coupled to the solidifying ingot in a unified system, instead of using two separate controllers as in the previous version of the pool depth controller.
- Both the estimation and control problems were expressed in the form of Sequential Monte Carlo (SMC) problems.

The rest of the paper is organized as follows. Section 2 presents the stochastic model used for estimation and control. In Section 3, an auxiliary particle filter – an SMC method - is used for state estimation. Section 4 describes the MPC problem and presents a solution using SMC methods. The model predictive controller is coupled to the auxiliary particle filter to form a novel method for nonlinear estimation and control of liquid pool depth for the VAR process. The effectiveness of the method is verified via simulation for a melt of Alloy 718 from an electrode of 430 mm (17 in) in diameter to a 510 mm (20 in) ingot in Section 5. Finally, Section 6 contains the concluding remarks.

2. Stochastic model

A nonlinear stochastic model is proposed for nonlinear estimation and control of the VAR process. It is a variation of the model proposed by Beaman et al. [2] that, although simpler, shows improved accuracy for ingot solidification. The nonlinear ingot solidification model (initially obtained from a numerical approximation of the heat equation) was substituted by a linear model. This is justified by observations where liquid pool depth showed exponential convergence to a new equilibrium when process inputs were perturbed, suggesting a first-order response.

The state at time $n$ is given by $x_n = [\Delta_n \ G_n \ X_{ram} \ M_e \ \mu_n \ S_c \ S_m \ phe \ i] \superscript{T}$; where $\Delta$ is the electrode thermal boundary layer, $G$ is the electrode gap, $X_{ram}$ is the ram position, $M_e$ is the electrode mass, $\mu$ is the melting efficiency$^2$, $S_c$ and $S_m$ are the pool depths at the center of the ingot and mid-radius, $phe$ is the helium pressure in the crucible and $i$ is the current. The process

$^1$ The superscript $T$ denotes the transpose of a matrix.

$^2$ Defined as the instantaneous fraction of supplied power that is used in melting the electrode $\mu = \frac{P_{melt}}{P_{total}}$. 

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is controlled with the control signal \( u_n = [i_{e_n} \ V_{ram_n}]^T \), where \( i_{e_n} \) is the commanded current and \( V_{ram_n} \) is the commanded ram speed. The model is defined as:

\[
\Delta_{n+1} = \Delta_n + \left[ \frac{\alpha \ C_{\Delta \Delta}}{\Delta_n} - \frac{C_{\Delta \rho}}{\Delta_n} \mu_n (V_c + R_i i_n) \right] \Delta t + G_{11} d_i n
\]

\[
G_{n+1} = G_n - \left[ \frac{\alpha \ C_{\Delta \Delta}}{\Delta_n} - \frac{C_{\Delta \rho}}{\Delta_n} \mu_n (V_c + R_i i_n) \right] \Delta t + G_{21} d_i n - dV_{ram_n}
\]

\[
X_{ram_{n+1}} = X_{ram_n} + V_{ram_n} \Delta t + dV_{ram_n}
\]

\[
M_{e_{n+1}} = M_{e_n} + \left[ \rho C_p \ C_{\Delta \Delta} \Delta_n - \frac{\rho C_p}{\Delta_n} \mu_n (V_c + R_i i_n) \right] \Delta t + G_{41} d_i n
\]

\[
\mu_{n+1} = \mu_n + d\mu_n
\]

\[
S_{C_{e_{n+1}}} = S_{C_{e_n}} + [-A_{C} (S_{C_{e_n}} - S_{C_0}) + B_{MC} (\Delta_n - \Delta_0) + B_{MC} (i_n - i_0 + di_n)
\]

\[
\quad + B_{MC} (\mu_n - \mu_0) + B_{MC} (p_{he_n} - p_{he_0}) \Delta t]
\]

\[
S_{M_{e_{n+1}}} = S_{M_{e_n}} + [-A_{MC} (S_{M_{e_n}} - S_{M_{e_0}}) + B_{MC} (\Delta_n - \Delta_0) + B_{MC} (i_n - i_0 + di_n)
\]

\[
\quad + B_{MC} (\mu_n - \mu_0) + B_{MC} (p_{he_n} - p_{he_0}) \Delta t]
\]

\[
i_{n+1} = i_n + (i_n - i_{e_n}) e^{-\Delta t / \tau} + di_n
\]

where 

\[
G_{11} = -(V_c + 2R_i i_n) \mu_0 C_{\Delta \rho} / \Delta_i h_m, \quad G_{21} = (V_c + 2R_i i_n) \rho \mu_0 C_{\rho} / \Delta_i h_m,\]

\[
G_{41} = -(V_c + 2R_i i_n) \rho \mu_0 C_{\rho} / \Delta_i h_m^3, \text{ and variables } A_{C_{or \ M}}, B_{MC_{or \ M}}, B_{MC_{or \ M}}, B_{MC_{or \ M}} \text{ and } B_{MC_{or \ M}} \text{ are obtained from a linear model of ingot solidification}^4.
\]

The proposed model is stochastic due to the presence of the random inputs \( d_i n \), \( dV_{ram_n} \), \( d\mu_n \) and \( dp_{he} \) which model uncertainty in melting current (caused by unknown current partition and distribution in the arc), ram speed (because of friction and error in the translation mechanism), melting efficiency (due to inhomogeneous electrodes and end effects) and helium pressure in the cooling system, respectively. These random inputs are modeled as 

\[
di_n \sim N(0, \sigma_i^2 \Delta t^2), \quad dV_{ram_n} \sim N(0, \sigma_{V_{ram}}^2 \Delta t^3), \quad d\mu_n \sim N(0, \sigma_{\mu}^2 \Delta t) \text{ and } dp_{he} \sim N(0, \sigma_{p_{he}}^2 \Delta t)^5.
\]

The stochastic model can be summarized in two equations: 1) the conditional probability of the current state given its previous value, \( p(x_n | x_{n-1}) \), defined by equations (1) to (9); and 2) the

\[3\] These coefficients come from the electrode melting model proposed in Ref. [3]; \( V_c \) is the cathode voltage fall; \( \alpha \) is the area ratio given by \( \alpha = A_e / A_y \), where \( A_e \) and \( A_y \) are the cross sectional areas of the electrode and ingot respectively; and \( R_i \) is the electric resistance in the electrode; and the coefficients \( C_{\Delta \rho}, C_{sp}, C_{\Delta \rho} \), and \( C_{sp} \) depend on thermo-physical properties.

\[4\] In this application, they were obtained from simulations of Basic Axisymmetric Remelting (BAR) using system identification based on first order systems [4]. The subscript 0 denotes nominal value in steady-state operation.

\[5\] \( x \sim N(\mu, \sigma^2) \) denotes a random variable \( x \) sampled from a normal distribution of mean \( \mu \) and variance \( \sigma^2 \).
probability of measurements given the current state, \( p(y_n | x_n) \). The second equation is required because, just like in most industrial processes, the state is not observed directly but through an observation sequence \( y_n \), described by:

\[
p(y_n | x_n) \sim N \left( \begin{bmatrix} G_n & X_{\text{ram}_n} & i_n & M_{c_i} & S_{c_i} & S_{M_{c_i}} & p_{\text{he}_n} \end{bmatrix}^T, R \right),
\]

\[
R = \text{diag} \left( \sigma_g^2, \sigma_x^2, \sigma_{i_n}^2, \sigma_{M_{c_i}}^2, \sigma_{S_{c_i}}^2, \sigma_{S_{M_{c_i}}}^2, \sigma_{p_{\text{he}_n}}^2 \right).
\]

The stochastic model was developed for two purposes: 1) estimation, the determination of the probability of the current state based on all the available observations up to this instant; and 2) control, the computation of the optimal control signal that optimizes the performance in the near future.

### 3. Auxiliary particle filter

State estimation determines the probability of the state \( x_n \) by updating the conditional probability \( p(x_{n-1} | y_{1:n-1}) \) when a new set of measurements \( y_n \) becomes available. The update is performed using Bayes’ rule and it involves stochastic integrals. Unfortunately, these calculations have no analytical solutions in nonlinear non-Gaussian scenarios, such as the one presented in this article. In these cases it is common to construct approximate solutions using particle filters, a family of Monte Carlo methods. These methods use discrete approximations of probability density functions allowing calculations to be performed in the form of finite sums [6].

The proposed VAR stochastic model has more process uncertainty than measurement uncertainty. Studies have shown that the auxiliary particle filter (APF) [6,7] outperforms other particle methods in statistical efficiency in this application [8]. The superior performance was obtained by incorporating knowledge of the next-step measurements in the proposal mechanism, preventing particles from moving to unlikely regions of the state-space.

![Figure 1](image1.png)

Figure 1. Estimates for electrode gap \( G \), efficiency \( \mu \), and pool depth \( S_{C\text{or}M} \) in a VAR melt of Alloy 718. Maximum likelihood estimates are plotted in blue and 95% confidence intervals in red.

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6. \( A = \text{diag}(x_1, \ldots, x_n) \) denotes an \( n \times n \) diagonal matrix where \( A_{ii} = x_i \) for \( i = 1, \ldots, n \).

7. Process is uncertain due to disturbances that may occur in the process that make it difficult to accurately predict how much metal is melted at each sampled time. These uncertain predictions are then corrected with precise measurements of ram position and electrode weight, and noisy electrode gap measurements (inferred from drip-short frequency or voltage).

8. Incorporation of the measurement reduces the variability of the weights \( \{w_n^j\}_{i=1}^N \). And since the variance of the estimates is proportional to the variance of the weights, less variable weights result in more precise predictions.
As in other Monte Carlo methods, precision of the estimates is dependent on the number of samples (particles) used in the approximation. The desired precision for the estimates was defined as 95% confidence intervals of 0.1 cm for electrode gap and 0.04 cm for liquid pool depth. It was observed experimentally that at least 10,000 particles were required to meet the desired precision. As an example, Figure 1 shows state estimates for the APF running with 65,536 particles.

4. Model predictive control

The linear pool depth controller proposed by Beaman et al. was based on a Linear-Quadratic-Gaussian (LQG) controller, a classical solution in the optimal control literature. In this article, a model predictive controller is proposed as a better alternative. The basic concept in MPC is to use a dynamic model to solve an open-loop optimal control problem at every time step for a prediction horizon of length $P$. At a fixed time $n$, the cost function is defined as

$$\min J(x_n, \bar{u}_{n:n+P})$$

which penalizes variations in control signals and deviations from desired references. In this expression, $\bar{u}_{n:n+P}$ is the tentative control input, $\bar{x}_{n:n+P}$ the predicted states, $\bar{y}_{n:n+P}$ the predicted observations and $s_j$ are the process set points (desired trajectory) [13].

Monte Carlo methods, previously described as estimators, have also been explored as tools to solve the MPC problem. The solution described in this article is based on Stahl and Hauth’s PF-MPC (Particle filter-Model predictive control), where the MPC optimization problem is expressed in the form of an estimation problem and two particle filters are used in the same application: one for estimation and one for control [14]. Because estimation is performed in the usual way, only the control problem is discussed in this section.

In this approach, the proposed control sequence $\bar{u}_j$ and the desired reference $s_j$ are treated as random variables. The closer the prediction and reference are, the more optimal the proposed control sequence will be. Predictions are made following $\bar{x}_j = f(\bar{x}_{j-1}, \bar{u}_{j-1})$ for the state and $\bar{y}_j = g(\bar{x}_j)$ for the observations. In these predictions, every entry of the control sequence has a normal distribution around the previous value of the sequence, i.e. $\bar{u}_j \sim N(\bar{u}_{j-1}, Q^{-1})$. The likelihood of each control sequence is evaluated with a normal distribution that penalizes the error between the predicted output and the reference, following $p(s_j \mid \bar{y}_j) = N(\bar{y}_j, R^{-1})$. In this approach, the solution of the optimization problem is found at the mode of the conditional distribution $p(\bar{u}_{n:n+P} \mid s_{n+1:n+P})$.

The method is illustrated in Figure 2, where measurements $y_n$ are taken from the plant and fed to the first particle filter, which acts as an estimator. The estimator takes the previous state estimate, in the form of samples $x_{n-1}$ and their respective weights $w_{n-1}$, inputs $u_{n-1}$ and measurements $y_n$, and

9Electrode gap and liquid pool depth are the most significant control variables in liquid pool depth control. Error bounds were defined based on usual practice in other liquid pool depth control experiments.

10$\|x\|_M^2$ denotes the 2-norm of a vector $x$ evaluated with a weighing matrix $M$.

11A bar as a superscript means that the variable is a prediction of the future performance of the system based on the model.

12The functions $f$ and $g$ are deterministic versions of the two conditional probabilities used in the stochastic model. Function $f$ returns the most likely new state given its previous value and the control inputs, and function $g$ returns the most likely observations for a given state.
returns updated samples \( x'_{\text{n}} \) and weights \( w'_{\text{n}} \). The samples are fed to the second particle filter, which evaluates control sequences and their likelihood in resulting in the desired reference. In this stage, the samples and weights from the particle filter are used to predict states \( \tilde{x}'_{\text{j}} \) and observations \( \tilde{y}'_{\text{j}} \), for a proposed control sequence \( \tilde{u}'_{\text{j}} \). A control sequence is proposed for each sample from the particle filter, and a weight \( \tilde{w}'_{\text{j}} \) is assigned to each control sequence dependent on how close the predictions are to the reference \( x_{\text{j}} \). The control sequences and their weights are propagated to a point estimator that returns the optimal signal \( u'_{\text{n}} \), which is sent back to the plant, closing the loop.

![Figure 2. Schematic of a PF-MPC controller.](image)

The original method, as proposed by Stahl and Hauth, was modified for this application to use an auxiliary particle filter (hence, APF-MPC) in the estimation stage while still having a basic particle filter in the control one. The approaches are interchangeable as they both return discrete approximations of the same probability density function.

5. Numerical study

The performance of the proposed APF-MPC controller was tested via simulation for a VAR melt of Alloy 718, from an electrode that was 430 mm (17 in) in diameter to a 510 mm (20 in) ingot.

13 The thermo-physical properties used for Alloy 718 are: thermal diffusivities at 300 K and 1623 K (\( \alpha_{\text{M}} \)) equal to 2.4×10^{-5} cm²/s and 6.0×10^{-5} cm²/s, while enthalpies at 1623 K and 1673 K (\( \Delta H \)) are equal to 5.4×10^{3} J/cm³ and 8.1×10^{3} J/cm³, respectively. The electrical parameters are: electric resistance (\( R \)) of 4.37×10^{-6} Ω and cathode voltage fall (\( V_{\text{C}} \)) of 21.2 V. The nominal operating conditions are: centerline and mid-radius pool depths (\( S_{\text{CL}} \) and \( S_{\text{ML}} \)) of 15.7 cm and 13.2 cm, current (\( I_{\text{o}} \)) of 6000 A, melt rate (\( m_{\text{o}} \)) of 60 g/s, helium pressure (\( \rho_{\text{He}} \)) of 3.0 Torr and melting efficiency (\( \mu_{\text{o}} \)) of 0.44. Other model parameters include: \( C_{\text{sp}} \) equal to 40, \( C_{\text{op}} \) is 6.7, \( C_{\text{hp}} \) is 13, \( A_{\text{C}} \) is 1.9×10^{-3} m³/s, \( A_{\text{M}} \) is 1.4×10^{-3} m³/s, \( B_{\text{sc}} \) is 2.6×10^{-3} m³/s, \( B_{\text{sm}} \) is -1.3×10^{-4} m³/s, \( B_{\text{sc}} \) is 6.6×10^{-6} m³/s-A, \( B_{\text{sc}} \) is 3.2×10^{-6} m³/s-A, \( B_{\text{pc}} \) is 6.9×10^{-5} m³/s, \( B_{\text{pm}} \) is 2.7×10^{-5} m³/s, \( B_{\text{pc}} \) is -8.1×10^{-5} m³/s-Torr, \( B_{\text{pc}} \) is -6.5×10^{-5} m³/s-Torr, \( G_{\text{ij}} \) is -5.6×10^{-6} cm²/A, \( G_{\text{ij}} \) is 5.4×10^{-6} cm²/A, \( G_{\text{ij}} \) is -2.2×10^{-3} g/A, \( \Delta t \) is 0.01 s and \( \tau \) is 1 s.

14 Standard deviations of process uncertainties are defined as: 20 A for current (\( \sigma_{\text{i}} \)), 5.0×10^{-3} cm/s for ram speed (\( \sigma_{\text{ram}} \)), 0.01 for melting efficiency (\( \sigma_{\text{e}} \)), and 0.001 for helium pressure (\( \sigma_{\text{p}} \)). Meanwhile, standard deviations of measurement uncertainties are defined as: 0.2 cm for electrode gap (\( \sigma_{\text{e}} \)), 0.005 cm for ram position (\( \sigma_{\text{e}} \)), 15 A for current (\( \sigma_{\text{e}} \)), 200 g for electrode mass (\( \sigma_{\text{m}} \)), 1.0 cm for both centerline and mid-radius pool depths (\( \sigma_{\text{d}} \) and \( \sigma_{\text{c}} \)), and 0.01 Torr for helium pressure (\( \sigma_{\text{p}} \)).

15 In the optimization step, the length of the prediction horizon (\( P \)) was set to 15. The optimization weights were set as: 10^6/\( \alpha^2 \) for current variations (\( Q_{\text{i}} \)), 10^4/\( V_{\text{ram}}^2 \) for ram speed variations (\( Q_{\text{v}} \)), 50^2/\( G^2 \) for electrode gap and 50^2/\( S_{\text{c,0}}^2 \) for pool depth.
this simulation, a constant reference was defined for electrode gap while a time-varying one is used for liquid pool depth at the center of the ingot. Initially, the reference starts at nominal conditions and is driven 5% deeper in a 20 minutes ramp, where it stays for 60 minutes. Then, it is ramped down to 95% of the nominal value in 20 minutes, where it stays for 60 minutes more. Finally, it is brought back to nominal conditions in a 20 minutes ramp, where it stays until the end of the simulation.

The closed-loop system was simulated for 10,000 particles, and nominal helium pressure and melting efficiency, resulting in Figure 3. Precise control is achieved for both electrode gap and liquid pool depth. The prediction horizon chosen for this simulation was 1.5 minutes, but longer prediction horizons (close to 5 minutes) show a smoother behavior, preventing oscillations in input signals at the expense of increasing the computational cost of the algorithm.

One of the main advantages of the proposed APF-MPC controller as compared to the LQG controller is that future variations in pool depth reference are taken into account when computing optimal control signals, resulting in a superior response as the commanded current no longer shows jumps. The controller proposed by Beaman et al. was able to drive the ingot to desired pool depths at the expense of having big current perturbations, which resulted in tree rings [16]. This problem seems to have been solved. Additionally, this controller is expected to result in improved performance in highly-nonlinear operation, e.g. start-up and hot-top.

Figure 3. Process inputs and outputs in the closed-loop stochastic simulation of the VAR system using APF-MPC.

Although promising, the APF-MPC controller is too slow to be implemented in closed-loop operation using a personal computer. The controller, when simulated with MATLAB on an Intel Core i7 CPU 860 at 2.8 GHz x 8 processor with Windows 7, was 14 times slower than the sampling time of the process. The inability to perform in real time prevents its implementation in process control applications. However, the method can be accelerated if implemented on appropriate computer architectures. Both estimation and control problems were formulated as SMC problems, which are parallel by nature. Studies suggest that the implementation of these algorithms in parallel computer architectures accelerates their performance by several orders of magnitude. Graphics processing units (GPUs), common in the gaming industry, allow massive parallelization by distributing independent tasks over different computing units, speeding up the total execution time [9].
The application of GPU architectures to accelerate estimation in VAR has been reported by Lopez et al. [8]. A similar study was performed for the APF-MPC controller using NVIDIA GTX TITAN. The simulation took 5.8 ms for 16,384 particles, 9.6 ms for 32,768 particles, 14 ms for 65,536 particles, 23 ms for 131,072 particles, 40 ms for 262,144 particles, 75 ms for 524,288 particles and 144 ms for 1,048,576 particles. The obtained results suggest that when implemented on GPUs, the APF-MPC controller is much faster than typical sampling times for VAR processes (around 5 seconds for large Nickel ingots).

6. Conclusions
In this paper it was shown that model predictive control, widely used in chemical applications, should be explored as an alternative to the LQG approach followed in the pool depth controller developed by Beaman et al. The most substantial advantage of the proposed approach is that nonlinearities – important to describe the thermal dynamics of the melting electrode – are included in the dynamic model, ensuring a more accurate behavior during highly-dynamic conditions, e.g. melt rate perturbations, start-up, hot-top, etc. Also, in the proposed approach a centralized controller monitors both electrode gap and liquid pool depth control, instead of having two separate controllers that ignored the interaction between the two sub-systems. And finally, this approach takes into account the desired near future when choosing an optimal input signal, resulting in smoother current variations. The major drawback is that melt recipes will have to be pre-defined and not changed during operation.

SMC methods, commonly used as estimators, have been proposed to solve the MPC problem in a semi-stochastic form following the formulation of Stahl and Hauth. The computational cost of the algorithm would be prohibitive if implemented on personal computers. However, simulation time is well under typical sampling times when parallel computer architectures (such as GPUs) are used.

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