Magnetic Monopole Content of Hot Instantons
R. C. Brower$^a$, D. Chen$^b$, J. Negele$^b$, K. Orginos$^c$ and C-I Tan$^d$

$^a$Physics Department, Boston University, 590 Commonwealth Ave, Boston, MA 02215, USA
$^b$Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
$^c$Physics Department, University of Arizona, Tucson, AZ 85721, USA
$^d$Physics Department, Brown University, Providence, RI 02912, USA

We study the Abelian projection of an instanton in $R^3 \times S^1$ as a function of temperature ($T$) and non-trivial holonomic twist ($\omega$) of the Polyakov loop at infinity. These parameters interpolate between the circular monopole loop solution at $T = 0$ and the static ’t Hooft-Polyakov monopole/anti-monopole pair at high temperature.

1. INTRODUCTION

Although many qualitative features of QCD are well described by a vacuum state dominated by an instanton “liquid”, confinement appears to be an exception [1]. Instead, magnetic monopoles are thought to be the crucial ingredient. This raises the question of how magnetic degrees of freedom can be incorporated into (or reconciled with) an instanton “liquid”. A recent step in this direction was taken by Brower, Orginos and Tan (BOT)[2] who studied in detail the magnetic content of a single isolated instanton, defining magnetic currents via the Maximally Abelian (MA) projection. They found a marginally stable direction for the formation of a monopole loop. Now with the more general caloron solution of T. Kraan and P. van Baal [3], and K. Lee and C. Lu [4], this analysis can be extended to an isolated SU(2) instanton at finite temperature ($T$) with a non-trivial holonomy ($\omega$) for the Polyakov loop.

The resultant picture that emerges is appealing (see Fig. 1). For $\omega = 0$, the small monopole loop at the core of a cold instanton grows in size as one increases the temperature and is transformed into a single static ’t Hooft-Polyakov monopole at infinite temperature, as noted earlier by Rossi [5].

The MA projection provides a fully gauge and Lorentz invariant definition of monopole currents by introducing an auxiliary adjoint Higgs field, $\vec{\phi}(x)$, fixed at the classical minimum,

\[
G = \frac{1}{2} \int ([D_\mu(A)\vec{\phi}]^2 + \lambda(\vec{\phi} \cdot \vec{\phi} - 1)^2) dx ,
\]

(1)

in a fixed background gauge field, $A_\mu$. This yields the Abelian projected field strength,

\[
f_{\mu\nu} = n \cdot F_{\mu\nu} - n \cdot D_\mu n \times D_\nu n ,
\]

(2)

*Talk presented by R.C. Brower. Supported by the US DOE. Computations were done at the Theoretical Physics Computing Facility at Brown University.
with its U(1) monopole current,
\[ k_\mu = \frac{1}{4\pi} \partial_\nu \tilde{f}_{\nu\mu} = \frac{1}{8\pi} \epsilon_{\mu \nu \rho \sigma} \partial_\nu n \cdot \partial_\rho n \times \partial_\sigma n, \] (3)
where \( n(x) = \tilde{\phi}(x)/|\tilde{\phi}(x)| \). It is conventional to identify the MA gauge by the rotation \( \Omega(x) \) in the coset \( SU(2)/U(1) \) that aligns \( n \) along the 3 axis.
We have extended the conventional Abelian projection (\( \lambda = \infty \)) to a continuous family including the analytically more tractable BPS limit (\( \lambda = 0 \)), where the difficult problem of minimizing the MA functional \( G \) reduces to an eigenvector problem for the Higgs field, \( D_\mu (A)^2 \tilde{\phi}_E = E \tilde{\phi}_E \).

2. COLD MONOPOLE LOOP

We begin at the origin of Fig. 1, where there is a single isolated instanton at zero temperature. A trivial, but essential, observation is that the singular gauge instanton in the ’t Hooft ansatz,
\[ A_\mu^n = \tilde{\eta}_{\mu \nu} \partial_\nu \log(1 + \rho^2/|x|^2), \] (4)
is also the MA projection which minimizes the Higgs action \( G \). In the BPS limit, this is equivalent to having a zero eigenvalue solution,
\[ \phi_0(x) = \frac{x^2}{x^2 + \rho^2} [0, 0, 1], \]
for the Higgs field aligned with the 3-axis. Consequently there is no magnetic content to the MA projection. This would be the entire story except that there is another zero eigenvalue that implies a flat direction for the formation of an infinitesimal monopole loop.

The geometry of this loop is interesting. The gauge singularity at the origin is caused by a rotation, \( g(x) = x_\mu \tau_\mu/|x| \) and \( \tau_\mu = (1, i \hat{r}) \). Locally it is advantageous to “unwind” this singularity to a distance \( R \) further reducing the MA functional \( G \). This almost wins, creating a monopole loop of radius \( R \) which only slightly increases \( G \), \( \delta G/G \simeq R^4 \log(R) \). Almost any local disturbance, due to a nearby instanton for example, will stabilize the loop \([3]\). The second zero mode vector in the BPS limit is easily constructed using conformal invariance.
\[ \tilde{\phi}_1(x) = \frac{1}{x^2(x^2 + \rho^2)} [\sin \beta \cos \alpha, \sin \beta \sin \alpha, \cos \beta], \]
where \( \alpha = \varphi - \psi \), \( \cos \beta = (u^2 - v^2)/(u^2 + v^2) \), with \( x_1 + ix_2 = u e^{i \varphi}, x_3 + ix_0 = v e^{i \psi} \).

The superposition of the two zero modes produces a loop. Finally note that the topological charge is related to the magnetic charge, through a surface term on the boundary of the loop, \( \Sigma \),
\[ Q \rightarrow \int_\Sigma \frac{Tr[\Omega_R \delta \Omega_R^\dagger \wedge \Omega_R \delta \Omega_R^\dagger \wedge \Omega_R \delta \Omega_R^\dagger]}{24 \pi^2}, \]
where \( \Omega_R \) is the singular gauge transformation providing a Hopf fibration for the infinitesimal “non-contractible” loop.

3. HOT BPS MONOPOLES

At low temperature with \( \omega = 0 \) the MA projection is similar. The periodic instanton in the ’t Hooft ansatz,
\[ A_\mu^n = \tilde{\eta}_{\mu \nu} \partial_\nu \log(1 + \pi T \rho^2 \sinh(2\pi T r)) \]
is again equivalent to the MA projection. “Unwinding” the periodic copies of the singularities at \( x = 0 \) is now accomplished by \( g = X_\mu \tau_\mu/|X| \), \( X_\mu = [\tan(\pi Tr), -\tanh(\pi Tr)] \), leaving a monopole loop. However surprisingly at infinite temperature, or equivalently \( \rho = \infty \) as noted by Rossi, the instanton is gauge equivalent to the static ’t Hooft-Polyakov monopole solution. With \( n(x) = \hat{r} \), this is the correct MA projection (or unitary gauge). For the case of the BPS limit, the solution is simply \( \tilde{\phi}(x) \equiv \tilde{A}_\mu(x) \), as one might expect. Consequently the MA projection correctly identifies the standard static monopole.

4. CALORON T-\( \omega \) PLANE

For \( \omega \neq 0 \), we encounter the full complexity of the new caloron solution \([3]\).
\[ A_\mu = r^3 \tilde{\eta}_{\mu \nu} \partial_\nu \log(\phi(x) + (\tau^+ \tilde{\eta}_{\nu \rho} \partial_\nu \chi(x) + c.c.)\psi(x)), \]
in the singular gauge. In the limit of \( T \rightarrow \infty \), we have verified that MA projection now gives a pair of ’t Hooft-Polyakov BPS monopole/anti-monopole separated by distance \( D = \pi \rho^2 T \) as expected. However, now the singular gauge caloron no longer satisfies the MA projection and it is difficult to find the MA projection analytically.
Thus we have minimized $G$ numerical in the interior of the $T-\omega$ phase plane of Fig. 1 by placing the functional on a grid. On symmetry grounds, one can prove the existence of cylindrical solutions with $\alpha = \varphi + \hat{\alpha}(u,z,t)$, $\beta(u,z,t)$. The Dirac sheet is located by a jump in $\beta(t,x)$ by $\pi$. In Fig. 2, we give the profile for $\beta(u,z,t)$ in the $z$-$t$ plane slicing through the instanton centered at $u=0$.

To explore further the transition from the monopole loop to a pair of monopole lines, we plot in Fig. 3 the area of the minimal spanning Dirac sheet. At $\rho T \simeq 0.56$, there is a clear transition separating the two regimes.

Based on the absence of a loop for a single isolated instanton \[2\], we anticipate that the formula for the size of the loop (or separation of the lines) must involve a new length scale, $L$. This scale represents the distance to nearby perturbations such as the anti-instanton presented in Ref. \[3\]. For the single caloron plotted here, the new scale is $L = \beta = 1/T$. This suggests a simple scaling form: $R \sim \rho(\rho T)^{\gamma}$ with $\gamma > 0$. Indeed in Fig. 3 for $(\rho T)^2 < 0.32$, we do see a positive curvature for the area, $A/\beta^2 \sim (RT)^2 \sim (\rho T)^{2+2\gamma}$, i.e., $\gamma > 0$, consistent with our expectation. On the other hand, at high temperatures, the monopole/anti-monopole trajectories are known \[3\] to be separated asymptotically by $D = \pi \rho(\rho T)^{\gamma}$ with $\gamma = 1$, which is also confirmed by a linear fit to $A = D \beta \simeq \pi \rho^2$ for $(\rho T)^2 > 0.3$.

Finally it is interesting to note that the kinematical “transition” seen in Fig. 3 is near to the Yang-Mills deconfinement temperature for a typical instanton size of $\frac{1}{3}$ fermi. However, a serious analysis of deconfinement dynamics and its possible relations to the monopole content of the caloron is left to future investigations.

REFERENCES

1. Talks by J. W. Negele and D. Chen at this conference.
2. R. Brower, K. Orginos and C-I Tan, Phys. Rev. D55 (1997) 6313.
3. T. Kraan and P. van Baal, “Periodic Instantons with non-trivial Holonomy”, hep-th/9805168.
4. K. Lee and C. Lu, Phys.Rev. D58 (1998) 025011.
5. P. Rossi, Nucl. Phys. B149 (1979) 170.