Unconventional superfluidity induced by spin-orbital coupling in a polarized two-dimensional Fermi gas

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We show the spin-orbital coupling induced by an artificial light-induced gauge field can fully restore superfluidity suppressed by population imbalance in a two-dimensional (2D) Fermi gas, leading to unconventional superfluid states either with topological Majorana fermion excitations or showing a novel mixture of triplet pairing with spin-up (down) components respectively in the $p_x \pm ip_y$ pairing channels. We self-consistently calculate the zero temperature phase diagram at the BCS side of Feshbach resonance and show that the phase transitions between different superfluid states can be revealed through measurement of the in-situ density profile of the 2D atomic cloud in a weak global trap.

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Ultradepolarized atomic gas provides an ideal platform to study superfluid states under tunable configurations. There has been great interest in looking for unconventional superfluid states beyond the conventional s-wave superfluidity observed in experiments \cite{1}. Population imbalance provides a mechanism to suppress the s-wave superfluid state and can lead to novel superfluid phase, such as the FFLO state \cite{2}. Polarized Fermi gas with tunable population imbalance has been studied extensively both experimentally and theoretically \cite{1,2}. The FFLO state, unfortunately, is typically fragile and hard to observe experimentally \cite{1,2}. Increasing population imbalance usually destroys the superfluid state instead of leading to novel superfluidity.

The spin-orbital (SO) coupling induced by an artificial gauge field emerges as a powerful control method for ultracold atomic gas \cite{3-6}. Various proposals have been made to realize SO coupling in ultracold atomic gas \cite{3,4} and remarkable experimental progress has been reported to demonstrate the artificial gauge field \cite{5}. Motivated by this progress, strong interest arises recently in studying the three-dimensional ultracold Fermi gas under artificial SO coupling \cite{6}. It has been noted that the SO coupling can enhance superfluidity for this system \cite{7}.

In this paper, we show that interesting and exotic superfluid states arise in two-dimensional (2D) Fermi gas from interplay of population imbalance and SO coupling. Without SO coupling, population imbalance above a critical value suppresses superfluidity, leading to normal phases. We show that SO coupling fully restores superfluidity in 2D, but the pairing is not in the conventional s-wave channel any more and the resultant superfluid phases have more exotic properties. Under large polarization, one of the spin components dominates with the pairing in the $p_x \pm ip_y$ channel. Such a phase supports non-abelian topological excitations by trapping a Majorana Fermion in its vortex core \cite{7}. This phase has been predicted before for a 2D Fermi gas \cite{7,8}, and the contribution of our calculation of a self-consistent phase diagram is to determine the stability region of such a phase under real physical parameters, which is important for experimental observation. Furthermore, our calculation predicts a new kind of unconventional superfluid phase under intermediate polarization, with spin-dependent pairing in the $p_x + ip_y$ channels. In this phase, the majority (minority) components pair up respectively in the $p_x + ip_y$ ($p_x - ip_y$) channels. We characterize the transition order between different superfluid states, and show through explicit calculation that these transitions can be revealed by observation of the singularity points of the density profile of a 2D atomic cloud in a weak global trap, which can be measured in experiments through the in-situ imaging \cite{9}.

In experiments, one realizes the 2D Fermi gas by applying a strong optical trapping potential (or an optical lattice) to ultracold atoms along the $z$-direction. Near the potential minimum where the atoms are located, the potential is well approximated by a harmonic trap with $V_{z} (z) = m\omega_{z}^2 z^2/2$, where $m$ denotes the atomic mass and $\omega_{z}$ is the trapping frequency. At the BCS side of the Feshbach resonance, the transverse wave function for the atoms is given by the ground state of $V_{z} (z)$, and the atomic collision can be characterized by an effective 2D interaction \cite{10}. With light induced Rashba-type of SO coupling \cite{11,12}, the Hamiltonian for the system can be described as

$$H = \sum_{k,\sigma} \xi_{k,\sigma} \hat{a}_{k,\sigma}^{\dagger} \hat{a}_{k,\sigma} + \alpha \sum_{k} \left( e^{i\varphi_{k}} \hat{a}_{k,\uparrow}^{\dagger} \hat{a}_{k,\downarrow} + h.c. \right) + \frac{U_{b}}{L^{2}} \sum_{k, k', q} \hat{a}_{k, \uparrow}^{\dagger} \hat{a}_{k+q, \downarrow} \hat{a}_{-k', q, \downarrow}^{\dagger} \hat{a}_{-k', q, \downarrow}^{\dagger} \hat{a}_{k', \uparrow}.$$  \hspace{1cm} (1)

where $a_{k,\sigma}$ and $a_{k,\sigma}^{\dagger}$ denote the fermionic field operators with in-plane wave vector $k \equiv (k_{x}, k_{y})$ and spin $\sigma = \uparrow, \downarrow$. The free particle dispersion relation is given by $\xi_{k,\sigma} \equiv \epsilon_{k} - \mu_{\sigma}$, where $\epsilon_{k} \equiv h^{2} k^{2}/(2m)$ and $\mu_{\uparrow,\downarrow} = \mu \pm \hbar$, with $\mu$ being the chemical potential and $h$ being the effective Zeeman field (equivalent to a population imbalance between spin-up and spin-down components). The second term of $H$ describes the Rashba-type SO coupling, ...
whose strength is denoted by the coefficient $\alpha$, with $k$ and $\varphi_k$ being the magnitude and azimuthal angle of the the in-plane wave vector $\mathbf{k}$. A combination of the Zeeman field and the Rashba-type SO coupling can be realized through control of a few laser beams \cite{4}. The $U_b$ term in the Hamiltonian describes the effective 2D interaction, where $L$ is the quantization length and $U_b$ is the bare coupling rate. The bare coupling $U_b$ is connected with the physical coupling $U_p$ ($U_p$ is determined by the 3D scattering length \cite{10}) through the 2D renormalization relation \cite{10}

$$U_b^{-1} = U_p^{-1} - L^{-2} \sum_k (2e_k + \hbar \omega_z)^{-1}.$$  

(2)

We determine the self-consistent phase diagram of the Hamiltonian $H$ within the mean-field framework, which is a reasonable approximation at zero temperature. Under this framework, we introduce the pairing order parameter $\Delta = U_b L^{-2} \sum_k \langle \hat{a}_{-\mathbf{k},\uparrow} \hat{a}_{\mathbf{k},\downarrow} \rangle$ to decompose the last term of $H$ into \((\Delta \sum_k \hat{a}_{\mathbf{k},\uparrow} \hat{a}_{\mathbf{k},\downarrow}^\dagger + H.c.) - |\Delta|^2 L^2 / U_b\), through the Wick theorem. The quadratic mean-field Hamiltonian can then be diagonalized into

$$H = \sum_k \left( E_{k,\uparrow} \hat{a}_{k,\uparrow}^\dagger \hat{a}_{k,\uparrow} + E_{k,\downarrow} \hat{a}_{k,\downarrow}^\dagger \hat{a}_{k,\downarrow} \right) + L^2 \Omega_0,$$

(3)

where $\hat{a}_{k,\pm}$ denote the quasi-particle modes with the excitation energies

$$E_{k,\pm}^2 = \xi_k^2 + \alpha^2 k^2 + \hbar^2 + |\Delta|^2 \pm \sqrt{(\hbar^2 + \alpha^2 k^2)\xi_k^2 + \hbar^2 |\Delta|^2},$$

(4)

and $\xi_k = e_k - \mu$. The last term in Eq. (3) denotes the zero-temperature thermodynamic potential, with $\Omega_0$ given by

$$\Omega_0 = L^{-2} \sum_k [\xi_k - (E_{k,\uparrow} + E_{k,\downarrow})/2] - |\Delta|^2 / U_b.$$

(5)

At zero temperature, the quasi-particles $\hat{a}_{k,\pm}$ are in vacuum states and we should minimize $\Omega_0$ with respect to the order parameter $\Delta$ to determine the ground state of the system.

Usually one calculates the phase diagram of the system by solving the gap equation $\partial \Omega_0 / \partial \Delta = 0$ and the number equations

$$n = n_{\uparrow} + n_{\downarrow}, \quad n_{\sigma} = - \frac{\partial \Omega_0}{\partial \mu_{\sigma}}.$$  

(6)

However, this is not a reliable approach here since the gap equation has multiple solutions corresponding to unstable and metastable states. A typical thermo-potential $\Omega_0$ is shown in Fig. 1. As one can see, it has multiple minima and maxima satisfying $\partial \Omega_0 / \partial \Delta = 0$. To find the ground state of the system, in this paper we directly calculate $\Omega_0$ for various $\Delta$ to figure out its global minimum. In our calculation, we take $\hbar \omega_z$ as the energy unit and $a_z = \sqrt{\hbar/(m \omega_z)}$ as the length unit. The units for $U_p$ and $\alpha$ are given respectively by $a_z^2 \hbar \omega_z$ and $a_z \hbar \omega_z$. With these units, all the parameters become dimensionless.

In Fig. 2 we plot the superfluid order parameter $\Delta$ (a), the number density $n$ (b), and the magnetization $S_z = (n_{\uparrow} - n_{\downarrow})/2$ (c) as functions of the effective Zeeman field $h$ under different SO coupling strengths. The chemical potential and the interaction parameter are fixed at $\mu = 0.2$ and $U_p = -5$. The inset of Fig. (a) shows the exponential decay of the superfluid order parameter with increase of the Zeeman field.

FIG. 1: The thermo-potential $\Omega_0$ as functions of the superfluid order parameter $\Delta$ under different effective fields $h$. Other parameters include the chemical potential $\mu = 0.5$, the spin-orbital coupling strength $\alpha = 0.1$, and the interaction parameter $U_p = -5$ (corresponding to a 3D $s$-wave scattering length $a_s \approx -2a_z$). Normalized with the energy unit $\hbar \omega_z$ and the length unit $a_z$, all the parameters are dimensionless in this and the following figures. The arrows indicate the global minimum of the corresponding curves.

FIG. 2: The superfluid order parameter $\Delta$ (a), the number density $n$ (b), and the magnetization $S_z = (n_{\uparrow} - n_{\downarrow})/2$ (c) as functions of the effective Zeeman field $h$ under different spin-orbital coupling strengths. The chemical potential and the interaction parameter are fixed at $\mu = 0.2$ and $U_p = -5$. The inset of Fig. (a) shows the exponential decay of the superfluid order parameter with increase of the Zeeman field.
the superfluid order parameter $\Delta$ is suppressed to zero, and we have a finite magnetization $S_z$ with $n_\uparrow > n_\downarrow > 0$. This corresponds to the normal mixture phase. Further increasing the field above $h_c \sim 0.2$, the minority component $n_\downarrow$ is reduce to zero and we have a polarized normal phase.

With a finite $\alpha$, the picture changes qualitatively. At a small $\alpha$, such as $\alpha = 0.1$ in Fig. 2, there is still a big drop of the superfluid order parameter $\Delta$ when the field $h$ is above a critical value $h_c$, but $\Delta$ does not drop to zero. The critical field $h_c$ increase with $\alpha$, which indicates that the SO coupling enhances superfluidity. The transition is still of the first order for small $\alpha$ similar to the $\alpha = 0$ case. What happens is that the thermo-potential $\Omega_{h}$ as a function of $\Delta$ has two non-zero minima with $\Delta_2 > \Delta_1 > 0$, as shown in Fig. 1. As $h$ increases, $\Delta_1$, replacing $\Delta_2$, becomes the global minimum, and the ground state undergoes a first-order transition between different types of superfluid phases when the order parameter jumps from $\Delta_2$ to $\Delta_1$. Above the critical field $h_{c2}$, the order parameter $\Delta$ eventually decreases exponentially with increase of the field $h$ as shown by the insert of Fig. 2(a), however, it does not reach exact zero within the mean-field framework. Due to the exponential decrease of $\Delta$, the order parameter becomes very small for large $h$ and it will be destroyed by thermal fluctuation even at very low temperature or likely by quantum fluctuation beyond the mean-field framework. At larger $\alpha$, the sudden drop of the order parameter $\Delta$ disappears, as indicated by the $\alpha = 0.2$ case in Fig. 2(a). If we look at the derivative of $\Delta$ as a function of $h$ at $\alpha = 0.2$, it shows a kink at the critical $h_c$, suggesting that the phase transition changes from the first order to the second order for large $\alpha$.

With nonzero SO coupling, the superfluid phase above the critical $h_c$ has novel features and is the focus of our interest in the following discussion. We concentrate on the small $\alpha$ case, as it is easier to realize this case in experiments. Below the critical $h_c$, the superfluid phase is similar to the conventional BCS state, where the singlet-pairing dominates. Above the critical $h_c$, there are two kinds of unconventional superfluid phases. From the order parameter $\Delta$, it is hard to distinguish these phases as $\Delta$ changes continuously across the phase boundary. However, from the number density and the magnetization shown in Fig. 2(b) and 2(c), one can clearly see a kink at the boundary, indicating a second order phase transition. The boundary is determined by the condition $h^2 = \mu^2 + \Delta^2$. In different superfluid phases, the excitation spectrum $E_{k,\pm}$ of the quasiparticles are always gapped, except at the critical point with $h = \sqrt{\mu^2 + \Delta^2}$, where the excitation spectrum $E_{k,\pm}$ becomes gapless. When $h > \sqrt{\mu^2 + \Delta^2}$, the phase is identified with the topological superfluid state which supports Majorana fermion excitations with exotic non-abelian fractional statistics [1, 8]. When $h_c < h < \sqrt{\mu^2 + \Delta^2}$, we have a new kind of unconventional superfluid phase whose nature will be studied below.

The overall phase diagram is shown in Fig. 3 at $\alpha = 0.1$. To understand the nature of different superfluid phases, we show in Fig. 4 the corresponding occupation number $\langle n_{k,\sigma} \rangle$ in the momentum space and the pairing functions $\langle \hat{a}_{-k,\sigma} \hat{a}_{k,\sigma'} \rangle$. In the conventional superfluid phase denoted by SF there, the pairing is dominantly in the $s$-wave channel between the spin $\uparrow$ and $\downarrow$ components with small hybridization from the triplet pairing $\langle \hat{a}_{-k,\uparrow} \hat{a}_{k,\uparrow} \rangle$ and $\langle \hat{a}_{-k,\downarrow} \hat{a}_{k,\downarrow} \rangle$ induced by the SO coupling. The momentum distribution $\langle n_{k,\sigma} \rangle$ is rounded off by the strong pairing interaction and $\langle n_{k,\uparrow} \rangle$ and $\langle n_{k,\downarrow} \rangle$ are almost identical as the magnetization is small. In the topological superfluid phase denoted by TSF there, the
Fermion excitations with non-abelian statistics [7, 8]. For phase, and it is well known that this phase has Majorana fermions, but a combination of them gives topologically trivial excitations. As a mixture of two topological superfluids, we expect the SF\(_{p±ip}\) phase to have topologically non-trivial properties, which could be activated through spin-dependent manipulation and detection of the atomic cloud [9]. This deserves further study in future.

To have experimental signature of the transition between different superfluid phases, we look at the density profile of the atomic gas in a weak global harmonic trap, which can be measured directly in experiments [10]. Assume the global trap in the \(x-y\) plane is given by \(V_\perp(x, y) = m\omega_\perp^2 (x^2 + y^2)/2 = m\omega_\perp^2 r^2/2\) with the aspect ratio \(\lambda = \omega_z/\omega_\perp \gg 1\). The density profile can be calculated through the standard local density approximation, which replaces \(\mu\) in the homogeneous case by the position-dependent \(\mu(r_z) = \mu_0 - V_\perp(x, y)\), where \(\mu_0\) denotes the chemical potential at the trap center. The typical density profiles and the corresponding superfluid order parameters are shown in Fig. 5 under two scenarios. In Fig. 5(a) and 5(b), we see a first-order transition from the conventional SF phase to the topological TSF phase, signified by a jump in the density profile for both \(\langle n_{k,↑}\rangle\) and \(\langle n_{k,↓}\rangle\). In Fig. 5(c) and 5(d), we see a second-order phase transition from the SF\(_{p±ip}\) phase to the TSF phase, indicated by a kink in the density profile of \(\langle n_{k,↓}\rangle\).

In summary, we have self-consistently calculated the phase diagram of the 2D polarized Fermi gas under SO coupling, and found two kinds of unconventional superfluid phases with either topological non-abelian excitations or spin-dependent mixing of \(p±ip\) superfluid states. The transition order between different superfluid phases is specified and the in-situ measurement of the density profile of the atomic gas provides a convenient method to reveal the phase transition and its order.

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