Coupling Neutrino Oscillations and Simulations of Core-Collapse Supernovae

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(Dated: October 11, 2019)

At the present time even the most sophisticated, multi-dimensional simulations of core-collapse supernovae do not (self-consistently) include neutrino flavor transformation. This physics is missing despite the importance of neutrinos in the core-collapse explosion paradigm. Because of this dependence, any flavor transformation that occurs in the region between the proto-neutron star and the shock could result in major effects upon the dynamics of the explosion. We present the first hydrodynamic core-collapse supernova simulation which simultaneously includes flavor transformation of the free-streaming neutrinos in the neutrino transport. These oscillation calculations are dynamically updated and evolve self-consistently alongside the hydrodynamics. Using a $M = 20 \, M_\odot$ progenitor, we find that while the oscillations have an effect on the neutrino emission and the heating rates, flavor transformation alone does not lead to a successful explosion of this progenitor in spherical symmetry.

Introduction Since the earliest simulations of core-collapse supernovae by Colgate and White [1], neutrinos have been recognised as important participants in the dynamics of the explosion. Accurate treatment of the neutrinos in a simulation is a difficult problem because of the huge changes in their coupling to the rest of the matter content. The most sophisticated simulations solve the classical Boltzmann equation for the neutrino distribution function [2, 3], but given its significant computational burden, many approximate neutrino transport schemes have been developed over the years. We refer the reader to O'Connor et al. [4] and Pan et al. [5] for a comparison of various 1D and 3D simulation codes which use different neutrino transport schemes.

The fundamental assumption underlying this treatment is that the neutrino flavor is conserved. This is clearly incorrect and there are multiple experiments that show neutrino flavors mix. In order to include this fundamental physics, one must treat the neutrino as a quantum particle. This necessity has been recognized for some time but little progress has been made in its direction. Firstly, a quantum treatment of neutrinos is computationally more expensive than a classical treatment. Secondly, post-processed flavor transformation calculations of supernova simulations (which had used classical treatments) indicated the flavor transformation should be suppressed during the accretion phase and should not lead to significant changes to the dynamics [6–8]. For self-consistency, however, one must consider the feedback of flavor oscillations upon the supernova dynamics.

In addition to self-consistency, there are two additional strong motivations to go beyond post-processing calculations and include flavor transformation in simulations. First, recent developments in the understanding of flavor oscillations in supernovae gives one reason to believe flavor oscillations may start much closer to the proto-neutron star than indicated in those earlier, post-processed studies. So-called ‘fast oscillations’ [9–15] may occur as a result of ‘angular crossings’ in the electron neutrino/antineutrino spectra — the electron lepton number current changes sign as a function of the angle relative to the radial direction — at radii below the shock. Second, Beyond the Standard Model physics can also lead to significant changes to the flavor evolution close to the proto-neutron star [16–23]. Whatever the origin of the flavor oscillations, if they occur well below the shock then the effect upon the dynamics would be significant and, in addition, have an impact upon the electron fraction which will affect the resulting nucleosynthesis.

In this letter we present the first core-collapse simulation which computes the flavor transformation simultaneously with the hydrodynamics in order to determine the feedback of flavor mixing upon the explosion and the neutrino emission. After describing how the coupling was achieved, we then determine the effect of the oscillations upon the dynamics of the explosion using a $20 \, M_\odot$ progenitor. We then finish by presenting our findings, the limitations of our study, and the directions for further improvement.

Including Oscillations The feedback of flavor oscillations upon the explosion dynamics was achieved by the synthesis of the neutrino oscillation code SQA, and the hydrodynamical code Agile-BOLTZTRAN. Agile-BOLTZTRAN itself is the combination of 1D Lagrangian hydrodynamics with Boltzmann neutrino transport. The hydrodynamics is calculated in general relativistic space-time on an adaptive grid to ensure high resolution where it is needed. The transport solves an implicitly finite differenced $O(\nu/c)$ Boltzmann equation for four neutrino flavors, electron (anti)neutrinos and $x$-type (anti)neutrinos representing both $\mu$ and $\tau$ flavors. A full description of the Agile-BOLTZTRAN code can be found in Liebendörfer et al. [24]. For our simulations we use 20 energy groups, 8 angle bins, and 102 radial zones.

The complete quantum description of neutrinos including flavor oscillations and coupling to matter is known as
FIG. 1. Change in $\nu_e$ (left column) and $\bar{\nu}_e$ (right column) luminosities (top) and mean energies (bottom) due to neutrino oscillations shown as percent difference relative to the case without oscillations everywhere below the shock. Luminosity contours go as (0.1, 1, 3, 10, 30, 100) FOE (where 1 FOE = $10^{51}$ erg) from thinnest line to thickest. Energy contours go as (5, 10, 15, 20, 25, 50, 100, 150, 200) MeV. Also shown are the shock radius (solid, green) and PNS radius (dot-dashed, blue). The very dark regions of red/blue show where the relative change was in excess of $\pm 2\%$.

the Quantum Kinetic Equations [25–28]. At the present time these equations are too computationally expensive to be implemented in supernova simulations. If we make the assumption that dense matter and frequent collisions strongly suppress flavor mixing, we only need to treat the flavor evolution of the free-streaming neutrinos. This is much more computationally feasible and our chosen approach. SQA is a multi-energy, free-streaming neutrino oscillation code for three neutrino and antineutrino flavors which assumes steady-state conditions. SQA uses an adaptive 6th order Runge-Kutta integrator to solve, for each energy, the Schrödinger Equation for the evolution operator $S(r, r_0)$ which relates the neutrino wavefunction at some initial point $r_0$ to the wavefunction at $r > r_0$ [29]. The flavor evolution is governed by the Hamiltonian which includes contributions from the vacuum potential, the Mikheyev-Smirnov-Wolfenstein (MSW) potential due to neutrino interactions with matter [30–32], as well as the neutrino-neutrino interaction (self-interaction) potential [33, 34]. General relativistic corrections to the Hamiltonian are included [35]. Due to its lower computational cost, we employ the single-angle approximation. We use the Particle Data Group values for the neutrino mixing angles and mass splittings [36] and consider an inverted mass hierarchy only because of the better qualitative agreement between single-angle and multi angle calculations for this ordering: see, for example [37, 38]. The SQA calculation uses 150 energy bins subdivided from the Agile-BOLTZTRAN energy grid so we maintain 0.5 MeV resolution below $\sim 50$ MeV, 1 MeV resolution up to $\sim 120$ MeV, and 8 MeV resolution up to the max BOLTZTRAN energy of 300 MeV.

The time evolution of our simulation is controlled by Agile-BOLTZTRAN. As it proceeds, SQA is periodically invoked. SQA calculates the MSW potential from the mass density and electron fraction provided by Agile-BOLTZTRAN, and constructs the neutrino spectra for each flavor from the neutrino luminosity, mean en-
energy, and mean square energy, assuming a pinched spectrum. SQA then computes the probabilities for a neutrino to change flavor across a single radial zone: that is, if $S_{i,k}$ is the evolution operator for neutrinos with energy $k$ crossing zone $i$, the electron flavor transition probability ($P_{i,k}$) is given by $P_{i,k} = 1 - |S_{i,k;ee}|^2$ where $S_{i,k;ee}$ is the ‘ee’ element of $S_{i,k}$. We do not assume flavor diagonal density matrices at the beginning of each zone: instead the ‘initial’ neutrino density matrix for the next zone $\rho_{i+1,k}$ is related to the ‘initial’ neutrino density matrix for the previous zone via $\rho_{i+1,k} = S_{i,k} \rho_{i,k} S_{i,k}^\dagger$. Only at the initial point $r_0$, taken to be 10 km above the neutrinosphere, are the density matrices at this location set to be diagonal in the flavor basis.

At the end of each SQA calculation, the transition probabilities $P_{i,k}$ and $\bar{P}_{i,k}$ are passed back to Agile-BOLTZTRAN which converts them to effective opacities for neutrino and antineutrinos, $\sigma_{i,k}$ and $\bar{\sigma}_{i,k}$, by multiplying by the speed of light $c$ and dividing by the width of the zone $r_i - r_{i-1}$, i.e.

$$\sigma_{i,k} = \frac{P_{i,k}}{r_i - r_{i-1}}, \quad \bar{\sigma}_{i,k} = \frac{\bar{P}_{i,k} c}{r_i - r_{i-1}}. \quad (1)$$

In order to conserve neutrino number, we link absorption in one flavor to emission in the other flavor resulting in the following coupled differential equations for the distribution functions $f$ of neutrino flavors $\alpha$ and $\beta$:

$$\frac{df_\alpha}{dt} = -\sigma_{i,k}(f_\beta - f_\alpha), \quad \frac{df_\beta}{dt} = \sigma_{i,k}(f_\alpha - f_\beta). \quad (2)$$

These additional flavor changing terms are added to the BOLTZTRAN transport equations for radially outward moving neutrinos only, following the same finite differencing scheme. All inward moving neutrinos are left unaltered by the oscillations. This omits the neutrino halo effect [39, 40] but, because the density in the core region is high, we do not expect the halo effect to be significant due to matter suppression [6, 41, 42]. The oscillation calculations are only executed in the post-bounce (p.b.) period after the shock has traveled beyond the neutrinosphere because only then will there be free-streaming neutrinos behind the shock where the oscillation calculations could affect the heating. We define an opacity correction term $\delta \sigma_{i,k}$ to track the effect of the changing size of the radial zones in the grid:

$$\delta \sigma_{i,k} = \sigma_{i,k} \frac{r_i - r_{i-1}^\circ}{r_i^\circ - r_{i-1}^\circ}, \quad (3)$$

where $r_i$ is the radius of zone $i$ and $r_i^\circ$ is the radius of zone $i$ the last time SQA was invoked. The oscillation opacities are updated at least every 10 ms of simulation time, but will update earlier if any $\delta \sigma_{i,k}$ or $\delta \bar{\sigma}_{i,k}$ is larger than 10% for any zone $i$ or energy $k$.

The Effects of Oscillations We consider the effects of oscillations using the 20 $M_\odot$ progenitor from Woosley & Heger [43], the same progenitor used for the comparison of 1D core-collapse supernova simulations by O’Connor et al. [44]. We performed two simulations, one including neutrino oscillations as described above and one without neutrino oscillations.

In figure 1 we show the effects of neutrino oscillations on the luminosity and mean energy of the electron (anti)neutrinos. With a few exceptions, the overall trend shows that the oscillations produce changes of a few percent, increasing the average electron (anti)neutrino energy while decreasing the electron (anti)neutrino luminosities. Starting at around 250 - 300 ms p.b. a dark red band just behind the shock shows a region with an apparent large increase in the luminosity and mean energy of the electron (anti)neutrinos. This however is due to differences in the shock position (discussed below), and not a result of the neutrino oscillations.

The changes in the luminosities and mean energies will have a direct effect upon the net neutrino heating, an important quantity in the neutrino-driven supernova mechanism. In figure 2 we show the percent change in the neutrino heating in the simulation with oscillations as compared to the simulation without oscillations. Initially the neutrino oscillations appear to have a very small effect, with no change or small ($< 1\%$) decreases in the heating After $\sim 300$ ms however, we see a larger change of generally increased heating closer to $\sim 4\%$ within the gain region.

In addition to this overall trend the other two prominent features are the bands along the shock and gain radius with $> 10\%$ relative change. The band immediately
behind the shock is not due to neutrino oscillations, but is a result of the differences in shock position between the two simulations that cause similar changes in the luminosities and mean energies (see above). Along the gain radius the large percent changes are due to the sign change in the net heating there.

The additional heating that was observed in the gain layer was not sufficient to reinvigorate the shock in our 1D simulation. Figure 3 shows the shock and proto-neutron star radii for our two simulations in the top panel and the difference in shock position between the two simulations in the bottom panel. The shock positions are initially identical. In the simulation with oscillations, however, the slightly lower heating immediately after bounce causes the shock to start falling behind around 60 ms p.b. The additional heating that begins around 300 ms p.b. is too small a change too long after bounce to make a significant change in the outcome of this 1D simulation.

**Conclusions** In this paper we have presented results from a supernova simulation with hybrid classical-quantum neutrino transport. We have found that the effects of the neutrino oscillations alter the neutrino luminosities and mean energies by a few percent which, in turn, causes changes of several percent in the neutrino heating close to the shock. The additional heating was not enough to dramatically alter the outcome of the simulation of this particular 20 $M_\odot$ progenitor. Our result agrees with the previous studies about the potential for oscillations to alter the hydrodynamics [6–8], but with greater certitude. Whether this is also true for other progenitors is a topic for future investigation.

Several improvements to the approach presented in this paper should be pursued. First, our treatment of the neutrinos as quantum particles applies only for the outgoing free-streaming neutrinos beyond the neutrinosphere. Close to the shock where we find the largest changes, this is a plausible approximation but clearly it is an approximation that needs to be relaxed. Another major assumption is the use of the single-angle approximation in order to reduce the computational burden of the flavor transformation calculation. So-called multi-angle calculations [44] increase the computational expense considerably (a single invocation would be expected to take of order $10^3 \sim 10^4$ CPU hours [45], compared to $\sim 1$ CPU hour for our single angle calculation) but nevertheless, would need to be performed eventually. Multi-angle calculations would also allow us to examine the feedback of fast-flavor oscillations should they occur. Finally, our simulation assumed spherical symmetry: multi-dimensional simulations are necessary if one wishes to examine whether additional non-linear effects found in 2D and 3D simulations might amplify the effect of oscillations. It will be interesting to see how all these future improvements might alter our results.

**Acknowledgements** We wish to thank Bronson Messer and Sherwood Richers for many useful discussions in the development of this project. This research was funded by the US Department of Education Graduate Assistance in Areas of National Need (GAANN) grant number P200A150035 and the US Department of Energy, Office of Science, Office of Nuclear Physics under Award DE-FG02-02ER41216.

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