Threshold resummation for the prompt-photon cross section revisited

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Abstract

We study the resummation of large logarithmic perturbative corrections to the partonic cross sections relevant for the process $pp \to \gamma X$ at high transverse momentum of the photon. These corrections arise near the threshold for the partonic reaction and are associated with soft-gluon emission. We especially focus on the resummation effects for the contribution to the cross section where the photon is produced in jet fragmentation. Previous calculations in perturbation theory at fixed-order have established that this contribution is a subdominant part of the cross section. We find, however, that it is subject to much larger resummation effects than the direct (non-fragmentation) piece and therefore appears to be a significant contribution in the fixed-target regime, not much suppressed with respect to the direct part. Inclusion of threshold resummation for the fragmentation piece leads to some improvement in comparisons between theoretical calculations and experimental data.
Introduction. Prompt-photon production at high transverse momentum \[ pp, p\bar{p}, pN \rightarrow \gamma X \], has been a classic tool for constraining the nucleon’s gluon density, because at leading order a photon can be produced in the Compton reaction \[ qg \rightarrow \gamma q \]. The “point-like” coupling of the photon to the quark provides a potentially clean electromagnetic probe of QCD hard scattering. However, a pattern of disagreement between theoretical next-to-leading order (NLO) predictions \[ 2, 3 \] and experimental data \[ 4, 5, 6 \] for prompt photon production has been observed in recent years, not globally curable by “fine-tuning” the gluon density \[ 7, 8, 9 \]. The most serious problems relate to the fixed-target regime, where NLO theory shows a dramatic shortfall when compared to some of the data sets \[ 5, 6 \]. We note that the mutual consistency of the data sets has been questioned \[ 9 \]. Nevertheless, for the related single-inclusive neutral-pion production, \[ pp \rightarrow \pi^0 X \], comparisons between NLO calculations and data from mostly the same experiments have also shown a systematic disagreement \[ 10, 11 \].

In a recent paper \[ 12 \], we have shown that a drastic improvement of the theoretical description of single-inclusive pion production in the fixed-target regime is found when certain large perturbative contributions to the partonic hard-scattering cross sections are taken into account to all orders in perturbation theory. These terms, known as threshold logarithms, arise near partonic threshold, when the initial partons have just enough energy to produce a high-transverse momentum parton (which subsequently fragments into the observed hadron) and a massless recoiling jet. In this case, the phase space available for gluon radiation vanishes, resulting in large logarithmic corrections to the partonic cross section. For the cross section integrated over all rapidities, the most important (“leading”) logarithms are of the form \[ \alpha_s^k \ln^{2k} (1 - \hat{x}_T^2) \] at the \( k \)th order in perturbation theory, where \( \alpha_s \) is the strong coupling and \( \hat{x}_T \equiv 2p_T/\sqrt{\hat{s}} \), with \( p_T \) the parton transverse momentum and \( \sqrt{\hat{s}} \) the partonic center-of-mass (cms) energy. Sufficiently close to threshold, NLO, which captures only the term for \( k = 1 \), will not be adequate anymore; instead, all logarithmic terms will become relevant and thus need to be taken into account. This is achieved by threshold resummation \[ 13, 14, 15 \].

The improvement of the comparison between data and theory due to threshold resummation in pion production has motivated us to revisit prompt-photon production. Here, too, large logarithmic corrections arise near partonic threshold. There is an extensive earlier literature \[ 16, 17, 18, 19, 20, 21 \] on QCD resummations for the “direct” partonic processes \[ qg \rightarrow \gamma c \] and \[ q\bar{q} \rightarrow \gamma g \]. The corresponding phenomenological studies for threshold resummation \[ 18, 19, 20 \] have found only a relatively small enhancement of the theoretical prediction by threshold resummation, not generally sufficient to provide satisfactory agreement with the fixed-target prompt-photon data. In the present paper, we will extend the previous studies of threshold resummation effects in prompt-photon production by including also the resummation for the “fragmentation” component in the cross section, to which we turn now.

As is well-known \[ 22 \] (for discussion and references, see also Ref. \[ 1 \]), high-\( p_T \) photons are not only produced by the “direct” contributions from the partonic hard processes \( ab \rightarrow \gamma c \), but also in jet fragmentation, when a parton \( f \) emerging from the hard-scattering process fragments into a photon plus a number of hadrons. The need for introducing a fragmentation contribution is physically motivated from the fact that a fragmentation process may produce, for example, a \( \rho \) meson that converts into a photon, leading to the same signal. In addition, at higher orders, the perturbative direct component contains divergencies from configurations where a final-state quark becomes collinear to the photon. These long-distance contributions naturally introduce the need for non-perturbative fragmentation functions \( D_{f \rightarrow \gamma} \) into which they can be absorbed. The
fragmentation component is of the same perturbative order as the direct one, $O(\alpha_{em}\alpha_s)$, since the underlying lowest-order (LO) partonic processes are the $O(\alpha_s^2)$ QCD scatterings $ab \to fc$, and the fragmentation functions $D_{f \to \gamma}$ are of order $\alpha_{em}/\alpha_s$ in QCD. There is some knowledge about the photon fragmentation functions from the LEP experiments [23]. Theoretical model predictions [24, 25, 26] for the photon fragmentation functions are compatible with these data. Using these sets of $D_{f \to \gamma}$, one can then estimate the fragmentation contribution to the prompt photon cross section.

NLO calculations in the fixed-target regime show [17] that fragmentation photons contribute about $10 - 30\%$ to the prompt-photon photon cross section. Here, the precise value depends both on the photon transverse momentum, but also on the type of hadron beams used. Generally, because of the additional fragmentation function and because of the different underlying hard-scattering processes, the fragmentation component is suppressed and also expected to fall off more rapidly in $p_T$ than the direct one. On the other hand, in $pp$ or $pN$ (as opposed to $p\bar{p}$) collisions, the direct channels $qq \to \gamma q$ and $q\bar{q} \to \gamma g$ always involve either a sea quark or gluon distribution in the initial state, which both decrease rapidly towards larger momentum fractions, leading to a rapid decrease of the cross section at high $p_T$. In contrast, the fragmentation piece has contributions from $qq$ scattering [18], involving two valence densities. As a result, for $pp$ or $pN$ collisions, the fragmentation component may continue to be sizable relative to the direct part out to quite high transverse momenta.

Despite the fact that according to the NLO calculation the fragmentation contribution is only a subdominant part of the cross section, in the light of the results of Ref. [12] it deserves a closer investigation. There, as we mentioned above, very large enhancements were found for $pp \to \pi^0 X$ in the fixed-target regime. In the theoretical calculation, the only difference between $pp \to \pi^0 X$ and the fragmentation component to $pp \to \gamma X$ is the use of different fragmentation functions. One therefore expects that also for the fragmentation component to prompt photon production there could be a large increase from resummation. Since it is known from the previous studies [18, 19, 20] that the direct part receives only moderate resummation effects, it is likely that the relative importance of the fragmentation contribution in the fixed-target regime is actually much larger than previously estimated on the basis of the NLO calculations. The precise details will of course depend on the photon fragmentation functions. The $D_{f \to \gamma}$ are much more peaked at large momentum fractions $z$ than pion fragmentation functions, due to the perturbative (“point-like”) piece in the evolution [24, 25, 26]. On the other hand, the gluon fragmentation function will be relatively much less important than in the pion case, meaning that some partonic channels with large resummation effects, such as $gg \to gg$, are less important. In the present paper, we present a brief phenomenological study of the resummation effects for the fragmentation part of the prompt photon cross section, and their implications for the comparison with the fixed-target data. Irrespective of how well theory and fixed-target data sets agree after the resummation of the fragmentation part is included, the latter is an important ingredient of the theoretical calculation of the cross section.

**Resummed cross section.** The cross section for $H_1H_2 \to \gamma X$ may be written as

$$\frac{p_T^3 d\sigma(x_T)}{dp_T} = \sum_{a,b,f} \int_0^1 dx_1 f_{a/H_1}(x_1, \mu^2) \int_0^1 dx_2 f_{b/H_2}(x_2, \mu^2) \int_0^1 dz z^2 D_{f \to \gamma}(z, \mu^2) \int_0 x_T dx_T \delta \left(\frac{x_T}{z\sqrt{x_1x_2}}\right) \int_\eta^\eta d\eta \hat{d}\hat{x}_T \frac{1}{2} \frac{d\sigma_{ab \to fX}(\hat{x}_T^2, \hat{\eta}, \mu)}{d\hat{x}_T^2 d\hat{\eta}}.$$  \hspace{1cm} (1)

We have integrated over all pseudorapidities $\eta$ of the produced photon. $\hat{\eta}$ is the pseu
At momentum level, with \( \eta_+ = -\eta_- = \ln \left( 1 + \sqrt{1 - \hat{x}_T^2} / \hat{x}_T \right) \). The sum in Eq. \((1)\) runs over all partonic subprocesses \( ab \to fX \), with partonic cross sections \( d\hat{\sigma}_{ab\to fX} \), parton distribution functions \( f_{a/H_1} \) and \( f_{b/H_2} \), and parton-to-photon fragmentation functions \( D^{f\to\gamma} \). The direct contributions are included and are obtained by setting \( f = \gamma \) and \( D^{f\to\gamma} = \delta(1 - z) \). \( \mu \) denotes the factorization/renormalization scales, which we have chosen to be equal for simplicity.

The partonic cross sections are computed in QCD perturbation theory. Their expansions begin at \( \mathcal{O}(\alpha_s\alpha_{em}) \) for the direct part, and at \( \mathcal{O}(\alpha_s^2) \) for the fragmentation part. Defining

\[
\Sigma_{ab\to fX}(\hat{x}_T^2, \mu) \equiv \int_{\eta_-}^{\eta_+} d\hat{\eta} \frac{\hat{x}_T^4}{2} \, \frac{d\hat{\sigma}_{ab\to fX}(\hat{x}_T^2, \hat{\eta}, \mu)}{d\hat{x}_T^2 d\hat{\eta}},
\]

one finds at NLO the structure \([2, 3]\)

\[
\Sigma_{ab\to fX}(\hat{x}_T^2, \mu) = \Sigma_{ab\to fX}^{(\text{Born})}(\hat{x}_T^2) \left[ 1 + \alpha_s(\mu) \left\{ A \ln^2(1 - \hat{x}_T^2) + B \ln(1 - \hat{x}_T^2) + C + \ldots \right\} \right],
\]

where \( \Sigma_{ab\to fX}^{(\text{Born})} \) is the Born cross section for the process \( ab \to fX \), and \( A, B, C \) are coefficients that depend on the partonic process. Finally, the ellipses denote terms that vanish at \( \hat{x}_T = 1 \).

The logarithmic terms are the leading and next-to-leading logarithms (LL, NLL) at this order. At higher orders, the logarithmic contributions are enhanced by terms proportional to \( \alpha_s^k \ln^m(1 - \hat{x}_T^2) \), with \( m \leq 2k \), at the \( k \)th order of \( \Sigma_{ab\to fX} \). As we discussed earlier, these logarithmic terms are due to soft-gluon radiation and may be resummed to all orders in \( \alpha_s \). The resummation discussed in this work deals with the “towers” for \( m = 2k, 2k - 1, 2k - 2 \).

As follows from Eq. \((1)\), since the observed \( x_T = 2p_T / \sqrt{S} \) is fixed, \( \hat{x}_T \) assumes particularly large values when the partonic momentum fractions approach the lower ends of their ranges. Since the parton distributions rise steeply towards small argument, this generally increases the relevance of the threshold regime, and the soft-gluon effects are relevant even for situations where the transverse momentum of the final state hadrons. This effect, valid in general in hadronic collisions, is even enhanced in the fragmentation contribution since only a fraction \( z \) of the available energy is actually used to produce the final-state photon.

The resummation of the soft-gluon contributions is carried out in Mellin-\( N \) moment space, where the convolutions in Eq. \((1)\) between parton distributions, fragmentation functions, and subprocess cross sections factorize into ordinary products. We take Mellin moments in the scaling variable \( x_T^2 \) as

\[
\sigma(N) \equiv \int_0^1 dx_T^2 \, (x_T^2)^{N-1} \, \frac{p_T^2 \, d\sigma(x_T)}{dp_T}.
\]

In \( N \)-space Eq. \((1)\) becomes

\[
\sigma(N) = \sum_{a,b,f} f_{a/H_1}(N + 1, \mu^2) \, f_{b/H_2}(N + 1, \mu^2) \, D^{f\to\gamma}(2N + 3, \mu^2) \, \Sigma_{ab\to fX}(N),
\]

with the usual Mellin moments of the parton distribution functions and fragmentation functions. As before, for the direct contributions, one has \( D^{f\to\gamma} = \delta(1 - z) \) and therefore \( D^{f\to\gamma}(2N + 3, \mu^2) = 1 \). In addition,

\[
\Sigma_{ab\to fX}(N) \equiv \int_0^1 d\hat{x}_T^2 \, (\hat{x}_T^2)^{N-1} \, \Sigma_{ab\to fX}(\hat{x}_T^2).
\]
Here, the threshold limit \( \hat{x}_T^2 \to 1 \) corresponds to \( N \to \infty \), and the leading soft-gluon corrections arise as terms \( \propto \alpha_s N \ln N \).

In Mellin-moment space, threshold resummation results in exponentiation of the soft-gluon corrections. In case of a single-inclusive cross section, the structure of the resummed result reads for a given partonic channel \[12, 17, 27, 28\]

\[
\Sigma_{ab \to cd}^{(\text{res})}(N - 1) = C_{ab \to cd} \Delta^a_N \Delta^b_N \Delta^c_N \Delta^d_N \sum_I G_I^{ab \to cd} \Delta_I^{(\text{int}) ab \to cd} \Sigma_{ab \to cd}^{(\text{Born})}(N - 1) .
\] (7)

Each of the functions \( \Delta^a_N, \Delta^b_N, \Delta^c_N, \Delta^d_N \) is an exponential. The \( \Delta^a_N, \Delta^b_N, \Delta^c_N \) represent the effects of soft-gluon radiation collinear to initial partons \( a, b \) or the “observed” final-state parton \( c \). The function \( \Delta^d_N \) embodies collinear, soft or hard, emission by the non-observed parton \( d \). Large-angle soft-gluon emission is accounted for by the factors \( \Delta_I^{(\text{int}) ab \to cd} \), which depend on the color configuration \( I \) of the participating partons. The sum runs over all possible color configurations \( I \), with \( G_I^{ab \to cd} \) representing a weight for each color configuration, such that \( \sum_I G_I^{ab \to cd} = 1 \). Finally, the coefficient \( C_{ab \to cd} \) contains \( N \)-independent hard contributions arising from one-loop virtual corrections.

The explicit NLL expressions for all the factors in Eq. (7) may be found in Refs. [12, 17]. The factors \( \Delta^a_N, \Delta^b_N, \Delta^d_N \) contain the leading logarithms and are universal in the sense that they only depend on the color charge of the parton they represent. Eq. (7) applies to the direct as well as to the fragmentation component. In the former, the “observed” parton is the photon, and thus \( \Delta^c_N = 1 \). Also, in this case there is only one color structure of the hard scattering, so that the sum in Eq. (7) contains only one term. In contrast, several color channels contribute to each of the \( 2 \to 2 \) QCD subprocesses relevant for the fragmentation part. As a result, there are color interferences and correlations in large-angle soft-gluon emission at NLL, and the resummed cross section for each subprocess becomes a sum of exponentials, rather than a single one. The complete expressions for the \( \Delta_I^{(\text{int}) ab \to cd} \), \( G_I^{ab \to cd} \) and \( C_{ab \to cd} \) are also given in Ref. [17] for the direct case, and in [12] for the fragmentation part.

In the resummed exponent, the large logarithms in \( N \) occur only as single logarithms, of the form \( \alpha_s \ln^{k+1}(N) \) for the leading terms. Subleading terms are down by one or more powers of \( \ln(N) \). Soft-gluon effects are partly already contained in the (\( \overline{\text{MS}} \)-defined) parton distribution functions and fragmentation functions. As a result, it turns out that they enhance the cross section \[13, 17\]. We also note that the factors \( \Delta_N \) depend on the factorization scale in such a way that they will compensate the scale dependence (evolution) of the parton distribution and fragmentation functions. One therefore expects a decrease in scale dependence of the predicted cross section \[15, 29, 30\].

We finally note that from the large Mellin-\( N \) point of view the fragmentation component is at first sight suppressed by \( 1/N \) \[18\] since the photon fragmentation functions always involve a “quark-to-photon” splitting function \( P_{\gamma q} \) which in moment space is \( \propto 1/N \). However, as was pointed out in \[18\], this suppression may be compensated in particular for \( pp \) or \( pN \) collisions by the fact that the fragmentation component involves quark-quark scattering, whereas the direct piece proceeds through quark-antiquark or quark-gluon scattering (see above). At large \( N \), the quark channels with their valence component dominate. In any case, the resummed corrections for the fragmentation component constitute by themselves a well-defined set of higher-order corrections.
which has much phenomenological relevance as we will see below. That said, we emphasize that a more detailed analysis of $1/N$-suppressed contributions also in the direct part would be desirable for future work.

**Phenomenological results.** We will now present some phenomenological results for the prompt photon cross section, taking into account the resummation for both the direct and the fragmentation parts. This is not meant to be an exhaustive study of the available data for direct-photon production; rather we should like to investigate the overall size and relevance of the resummation effects and in particular the question in how far they change the relative importance of direct and fragmentation contributions. We therefore select only a few representative data sets to compare to: the E706 data for prompt-photon production in $pBe$ scattering \cite{E706} at $\sqrt{S} = 31.5$ GeV, the $pp$ data from UA6 \cite{UA6} ($\sqrt{S} = 24.3$ GeV), and the data from R806 \cite{R806} taken in $pp$ collisions at the ISR at $\sqrt{S} = 63$ GeV.

In order to obtain a resummed cross section in $x_T$ space, one needs an inverse Mellin transform. As previous studies \cite{12,18,20} we will use the “Minimal Prescription” developed in Ref. \cite{29}, for which one chooses a Mellin contour in complex-$N$ space that lies to the left of the poles at $\lambda = 1/2$ and $\lambda = 1$ in the resummed Mellin integrand, where $\lambda = \alpha_s(\mu^2)b_0\ln(N)$ with $b_0 = (33 - 2N_f)/12\pi$, but to the right of all other poles.

When performing the resummation, one of course wants to make full use of the available fixed-order cross section, which in our case is NLO ($O(\alpha_{em}\alpha_s^2)$). Therefore, a matching to this cross section is appropriate, which may be achieved by expanding the resummed cross section to NLO, subtracting the expanded result from the resummed one, and adding the “exact” NLO cross section \cite{2,3}:

$$\frac{p_T^3}{dp_T} d\sigma^{(match)}(x_T) = \sum_{a,b,f} \int_C \frac{dN}{2\pi i} (x_T^2)^{-N} f_{a/h_1}(N + 1, \mu^2) f_{b/h_2}(N + 1, \mu^2) D^{f\rightarrow\gamma}(2N + 3, \mu^2) \times \left[ \left. \frac{\Sigma^{(res)}_{ab\rightarrow fd}(N) - \Sigma^{(res)}_{ab\rightarrow fd}(N)}{\Sigma^{(res)}_{ab\rightarrow fd}(N)} \right|_{NLO} \right] + \frac{p_T^3}{dp_T} d\sigma^{(NLO)}(x_T),$$

where $\Sigma^{(res)}_{ab\rightarrow cd}(N)$ is the resummed cross section for the partonic channel $ab \rightarrow cd$ as given in Eq. \cite{7}. In this way, NLO is taken into account in full, and the soft-gluon contributions beyond NLO are resummed to NLL. Any double-counting of perturbative orders is avoided. Note that, as before, this cross section is the sum of both direct and fragmentation contributions.

As we have discussed earlier, we perform the resummation for the fully rapidity-integrated cross section. In experiment always only a certain limited range of rapidity is covered. In order to be able to compare to data, we therefore approximate the cross section in the experimentally accessible rapidity region by \cite{12,18}:

$$\frac{p_T^3}{dp_T} d\sigma^{(match)}(\eta \text{ in exp. range}) = \frac{d\sigma^{(match)}(\text{all } \eta)}{d\sigma^{(NLO)}(\text{all } \eta)} \frac{p_T^3}{dp_T} d\sigma^{(NLO)}(\eta \text{ in exp. range}).$$

In other words, we rescale the matched resummed result by the ratio of NLO cross sections integrated over the experimentally relevant rapidity region or over all $\eta$, respectively.

Our choice for the parton distribution functions will be the CTEQ6 set \cite{32}. For the photon fragmentation functions we use those of \cite{25}. We note that other sets have been proposed \cite{24,26} for the latter.
We start by comparing the relative importance of the photon fragmentation contribution at NLO and after NLL resummation of the threshold logarithms. Figure 1 shows the corresponding ratios

\[
\frac{\text{direct}}{\text{direct + fragmentation}}, \quad \frac{\text{fragmentation}}{\text{direct + fragmentation}}
\]

as functions of the photon transverse momentum \( p_T \), for \( \sqrt{S} = 31.5 \) GeV, corresponding to a typical fixed-target energy. Here we have considered \( pp \) collisions, and we have chosen the factorization/renormalization scales as \( \mu = p_T \). One can see that the NLO fragmentation component contributes about 40% of the cross section at the lowest \( p_T \) shown and then rapidly decreases, becoming lower than 10% at \( p_T \approx 11 \) GeV. As we anticipated earlier, threshold resummation affects the fragmentation component much more strongly than the direct part. After resummation, the fragmentation contribution is relatively much more important, as shown in Fig. 1 yielding almost 60% of the cross section at smaller \( p_T \) and still more than 20% at \( p_T = 11 \) GeV.

Similar conclusions are reached when one analyzes the additional enhancement that NLL resummation gives over NLO. In Fig. 2 we show the “K-factors”

\[
K \equiv \frac{d\sigma^{(\text{match})}}{d\sigma^{(\text{NLO})}}
\] (10)

for the case where only the direct contribution is resummed (and the fragmentation one taken into account at NLO), and for the case when both contributions, direct and fragmentation, are
Figure 2: “K-factors” as defined in Eq. (10) for the case where only the direct component is resummed, and for the case where NLL resummation is applied to both the direct and the fragmentation contributions. Parameters are as for Fig. 1. The insert shows the individual “K-factors” for the direct and the fragmentation pieces.

From Fig. 2 we may conclude that NLL resummation of the fragmentation component leads to a significant enhancement of the theoretical prediction and will have some relevance for comparisons of data and theory. Such comparisons are shown in Figs. 3-5. In Fig. 3 we show the data for $pBe \rightarrow \gamma X$ from the E706 experiment [5], along with our theoretical calculations at NLO and for the NLL resummed case. The energy is $\sqrt{S} = 31.5$ GeV, as used for the previous figures, and the data cover $|\eta| \leq 0.75$. We give results for three different choices of scales, $\mu = \zeta p_T$, where $\zeta = 1/2, 1, 2$. It is first of all evident from the figure that the NLO result falls far short of the data. As we shall see below, this shortfall is particularly pronounced for the E706 data. Furthermore, there is a very large scale dependence at NLO. When the NLL resummation is taken resummed. We have chosen the same energy and other parameters as in the previous figure. In agreement with earlier studies [18, 19, 20], resummation of the direct contribution alone is fairly unimportant at lower $p_T$, yielding a “K-factor” close to unity. In contrast to this, taking into account the NLL resummation of the fragmentation component as well leads to a much bigger “K-factor”, roughly a 50% enhancement over NLO at the lower $p_T$, and even a factor 2.5–3 at the highest $p_T$ considered. The insert in the figure shows the individual “K-factors” for the direct and the fragmentation components. The one for the fragmentation piece is very large, albeit not as large as what was found for the case of $\pi^0$ production in our previous study [12]. This finding is explained by the fact that gluonic channels receive much larger resummation effects than quark ones, but that the such channels are relatively suppressed in the photon production case since the gluon-to-photon fragmentation function is much smaller than the gluon-to-pion one.
Figure 3: Comparison of NLO and NLL resummed calculations of the cross section for $pBe \rightarrow \gamma X$ to data from E706 [5], at $\sqrt{S} = 31.5$ GeV and $|\eta| \leq 0.75$, for three different choices of the renormalization/factorization scale $\mu$. The scale dependence is drastically reduced. This observation was already made in the previous phenomenological studies of the resummed prompt-photon cross section [18, 19, 20], in which however only resummation for the direct component was implemented. As can also be seen from Fig. 3, at the lower $p_T$ the full resummed result is roughly at the upper end of the “band” generated by the scale uncertainty at NLO, whereas at the higher $p_T$ it is considerably higher. Overall, as we saw in Fig. 2, there is a further significant enhancement over previous NLL resummed results [18, 19, 20]. This additional enhancement leads to a moderate improvement of the comparison between theory and the E706 data. Clearly, even with NLL resummation of the fragmentation component the calculated cross section remains far below the E706 data, except for $p_T \gtrsim 8$ GeV.

Figure 4 shows similar comparisons with the data for $pp \rightarrow \gamma X$ from UA6 [6] at $\sqrt{S} = 24.3$ GeV. Here, the resummed calculation, which again shows a very small scale dependence, is in very good agreement with the data. As before, resummation of the fragmentation component leads to a non-negligible enhancement of the cross section, pushing the theoretical NLL results to or slightly beyond the upper end of the NLO scale uncertainty band. Finally, in Fig. 5 we show R806 results for $pp \rightarrow \gamma X$ from the ISR at $\sqrt{S} = 63$ GeV. Similar features as before are observed. Note that we are further away from threshold here, due to the higher energy of the ISR. It is likely that the NLL resummation is not completely accurate here, but that terms subleading in $N$ could have some relevance. We reserve the closer investigation of this issue to a future study.

Conclusions and outlook. We have studied the NLL all-order resummation of threshold loga-
Figure 4: Same as Fig. 3, but comparing to data from UA6 [6] for $pp \rightarrow \gamma X$ at $\sqrt{S} = 24.3$ GeV with $-0.2 \leq \eta \leq 1$.

arithms in the partonic cross sections relevant for high-$p_T$ prompt-photon production. The novel feature of our study is that we have also taken into account the NLL resummation of the photon fragmentation component. Here we were motivated by the rather large enhancements that we had found in a previous study of threshold resummation for the process $pp \rightarrow \pi^0 X$. The theoretical description for this process is the same as that for the fragmentation component to the prompt photon cross section; the only difference arises in the use of pion vs. photon fragmentation functions.

We have found that indeed the fragmentation component is subject to much larger resummation effects than the direct one. This implies that probably a substantially larger fraction of observed photons than previously estimated are produced in jet fragmentation. On the other hand, we also found that the enhancement of the fragmentation component due to the threshold logarithms is smaller than the enhancement previously observed for $\pi^0$ production, mostly as a result of the smallness of the photon-to-gluon fragmentation function as compared to the gluon-to-pion one. We note, however, that fairly little is known about the function $D^{g\rightarrow\gamma}$. It is probably not ruled out that this function is much bigger than expected in the set [25] of photon fragmentation functions that we have used, in which case resummation effects would become yet more substantial.

The fully resummed prompt-photon cross section shows a much reduced scale dependence. We find that the comparison of the NLL resummed cross section with experimental data shows varied success, with the theoretical calculations still lying much lower than the E706 data, but consistent with the UA6 and R806 $pp$ data. In the light of this, further studies and more detailed com-
comparisons are desirable. We note that generally any residual shortfall of the resummed theoretical results would likely need to be attributed to non-perturbative contributions that are suppressed by inverse powers of the photon transverse momentum. These could for example be related to small “intrinsic” parton transverse momenta \[33\]. Resummed perturbation theory itself may provide information on the structure of power corrections, through contributions to the resummed expressions in which the running coupling constant is probed at very small momentum scales. A recent study \[34\] addressed this issue in the case of the prompt-photon cross section at large \(x_T\) and estimated power corrections to be not very sizable.

Our study improves the theoretical description and thus is a step towards a better understanding of the prompt-photon cross section in the fixed-target regime.

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