Solidification Parameters Dependent on Interfacial Heat Transfer Coefficient between Aluminum Casting and Copper Mold

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The present study focused on the evaluation of the interfacial heat transfer coefficient as a function of the surface temperature of the casting material at the interface. The casting experiments of aluminum into a cylindrical copper mold were conducted. The thermal history during the experiment was used to solve the inverse heat conduction problem. The effects of coating and superheat on the interfacial heat transfer coefficient in the liquid state, during the solidification, and in the solid state were comparatively discussed. The interfacial heat transfer coefficient was categorized into three regimes according to the phase of the casting materials. In the liquid state, the interfacial heat transfer coefficient is affected by the roughness of the mold, the wettability of the casting on the mold surface, and the physical properties of the coating layer. At the initial stage of solidification, it drops to a certain value due to the abrupt surface deformation of the casting. After then, it reduces again due to contraction of the casting. In the solid state, it depends only upon the thermal conductivity and the thickness of the air gap.

KEY WORDS: interfacial heat transfer coefficient; inverse heat conduction problem; computer simulation; casting; solidification.

1. Introduction

In the casting industry, the computer simulation of solidification has played an important role in predicting the soundness of the product and optimizing the process variables. It is important to use the exact thermophysical properties of the materials, and the process variables for more precise prediction. Among them, interfacial heat transfer coefficient (IHTC) is difficult to estimate, since it depends upon many factors such as physical properties of the casting and the mold materials, coating layer, air gap formation, roughness of mold surface, superheat, and contacting pressure. The IHTC has been treated as a constant during the numerical calculation of solidification in most cases. If the IHTC is estimated more correctly, i.e., as a function of time or temperature, it would be possible to make sure the correctness of the numerical prediction of casting process.

There have been several research reports on the IHTC between the casting and the mold, concerning the effects of the air gap, superheat, the alloy composition, the mold condition, the coating, and etc. However, most of the previous studies described the IHTC as a function of time. It is difficult to utilize such data to the real situation of computer simulation. The IHTC as a function of time can be applicable only to the exact same configuration of the mold system. The IHTC expressed as a function of temperature is readily usable in computational analysis of solidification.

In the present study, the IHTC was evaluated as a function of the surface temperature of the casting material at the interface. The casting experiments of aluminum into a cylindrical copper mold were conducted. The thermal history during the experiment was used to solve the inverse heat conduction problem by Beck’s method. Effects of coating layer and superheat on the IHTC in the liquid state, during the solidification, and in the solid state were also discussed.

2. Experimental

The casting material used to estimate the IHTC during the solidification in the present study was commercially pure aluminum (99.9 wt%). The mold material was made of copper in a cylindrical form as shown schematically in Fig. 1. The mold with thickness of 24 mm was kept at room temperature prior to the casting experiments. The diameter of the die cavity was 76 mm. Top and bottom surfaces of the mold and the casting were covered with insulating bricks to maintain one-dimensional heat flow in the radial direction. In order to investigate the effect of coating layer on the mold surface, two types of coating, i.e., ceramic (Al2O3–SiO2) and carbon coating, were deposited to make 100 µm thick coating layer onto the inside surface of the mold before the experiments, and the results were compared to that without coating. Aluminum was melted in a graphite crucible with induction heating, and cast into the copper mold at 760°C ($T_M<100°C$), 810°C ($T_M<150°C$), and 860°C ($T_M<200°C$), where $T_M$ is the melting point of aluminum, to examine the effect of superheat.
The initial temperature in the casting was uniform. Then the initial temperature of the molten material was filled instantaneously in the mold, and that the temperature changes during the casting experiments: two thermocouples, marked as TC3 and TC4 in Fig. 1, were inserted into the mold, and the others, marked as TC1 and TC2 in Fig. 1, into the die cavity, respectively. The exact positions of the thermocouples used in this study were listed in Table 1. The thermocouples were connected to a personal computer via A/D converter to record the thermal history during the solidification events. The temperature profiles obtained were used in the numerical evaluation of the IHTC at the casting/mold interface.

### Table 1. Exact positions of thermocouples used in this study.

| Thermocouple | Distance from the center (mm) | Distance from the mold/casting interface (mm) |
|--------------|------------------------------|---------------------------------------------|
| TC1          | 59.5                         | 21.5                                        |
| TC2          | 40.5                         | 2.5                                         |
| TC3          | 35.0                         | 3.0                                         |
| TC4          | 0.0                          | 38.0                                        |

Four K-type thermocouples with the diameter of 0.3 mm were inserted into the mold and the die cavity to measure the temperature changes during the casting experiments: two thermocouples, marked as TC1 and TC2 in Fig. 1, were inserted into the mold, and the others, marked as TC3 and TC4 in Fig. 1, into the die cavity, respectively. The exact positions of the thermocouples are listed in Table 1. The thermocouples were connected to a personal computer via A/D converter to record the thermal history during the solidification events. The temperature profiles obtained were used in the numerical evaluation of the IHTC at the casting/mold interface.

**3. Numerical Analysis**

The heat conduction during the experiment was assumed as a one-dimensional heat conduction problem in the radial direction, since the top and the bottom of the casting were covered with insulating material. The governing equation for the heat conduction in the casting considering solidification was:

$$
\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \rho L \frac{\partial f_s}{\partial t} \quad \text{(1)}
$$

where $T$ is the temperature, $t$ is the time, $r$ is the radial coordinate, $f_s$ is the fraction of solid, and $\rho$, $c_p$ and $L$ are the density, heat capacity, and latent heat of fusion of the casting material, respectively. It was assumed that the molten material was filled instantaneously in the mold, and that the initial temperature in the casting was uniform. Then the initial condition for the equation was:

$$
T = T_p \quad \text{................................(2)}
$$

where $T_p$ is the maximum temperature in the casting during the experiment. The boundary condition for the equation at the casting/mold interface was:

$$
q = h(T_{ci} - T_{mi}) \quad \text{................................(3)}
$$

where $q$ is the heat flux at the casting/mold interface, $h$ is the IHTC, and $T_{ci}$ and $T_{mi}$ are the temperatures of the casting and the mold at the interface, respectively. Note that $h$ is a function of time or temperature of the casting material at the interface.

The heat conduction equation for the mold was:

$$
\rho_c \frac{\partial T}{\partial t} = k \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \quad \text{................................(4)}
$$

where $\rho$ and $c_p$ are the density and heat capacity of the mold material, respectively. The initial condition for the equation was:

$$
T = T_{R} \quad \text{................................(5)}
$$

where $T_{R}$ is the room temperature, since the mold had been held at the room temperature prior to the experiment. The boundary conditions for the equation at the casting/mold, and at the mold/atmosphere interfaces were:

$$
q = h(T_{ci} - T_{mi}) \quad \text{................................(6)}
$$

$$
q = h(T_{mo} - T_{a}) \quad \text{................................(7)}
$$

respectively, where $T_{mo}$ and $T_{a}$ are the temperatures of the outer surface of the mold and the atmosphere, respectively, and $h$ is the heat transfer coefficient to the atmosphere, which was considered including both of the convection and radiation heat transfers to air:

$$
h = h_{\text{air}} + \varepsilon \sigma (T_{\text{mold}} + T_{a}) (T_{\text{mold}}^2 + T_{a}^2) \quad \text{................................(8)}
$$

where $h_{\text{air}}$ is the convection heat transfer coefficient to the atmosphere, $\varepsilon$ is the emissivity, and $\sigma$ is the Stefan–Boltzmann constant, $5.67 \times 10^{-8}$ W·m$^{-2}$·K$^{-4}$. The values of $h_{\text{air}}$ and $\varepsilon$ were fixed as $15$ W·m$^{-2}$·K$^{-1}$, and $0.023$, respectively.

The above governing equations were discretized using the finite volume method. The time derivative term was approximated using fully implicit method. The resulting discretized equation systems were solved by the tri-diagonal matrix algorithm (TDMA) at each time step. To consider the solidification of the materials, the temperature recovery method was used.

In order to estimate the IHTC between the casting and the mold, it is necessary to solve the inverse heat conduction problem (IHCP). The solution algorithm by Beck was used in the present study. The Beck’s method depends upon the minimization of the following function at each time step:

$$
F(q) = \sum_{i=1}^{t} (Y_{\eta+i} - T_{\eta+i})^2 \quad \text{................................(9)}
$$

where $Y_{\eta+i}$ and $T_{\eta+i}$ are the measured and calculated temperatures at various time step, respectively, and $t$ represents
the number of future temperatures. Four future temperatures \( (I = 5) \) were used in this study to improve the stability. For the IHCP, the temperature measured at the nearest position to the casting/mold interface, \( TC2 \) in the mold, was used as the known temperature history. The other measured temperatures at \( TC1 \), \( TC3 \), and \( TC4 \), were used to verify the accuracy of the calculation. By Beck’s method, the heat flux and the surface temperatures at the interface were estimated, and then the IHTC was calculated using Eq. (3). The thermophysical properties of the casting and the mold materials used in the calculation are shown in Table 2.

### Table 2. Thermophysical properties of the casting and the mold used in the present calculation.\(^{6,15}\)

| Material          | Aluminum | Copper |
|-------------------|----------|--------|
| Density \( (kg \cdot m^{-3}) \) | 2702     | 8933   |
| Specific heat \( (J \cdot kg^{-1} \cdot K^{-1}) \) | 902      | 385    |
| Thermal conductivity \( (W \cdot m^{-1} \cdot K^{-1}) \) | 236      | 401    |
| Melting temperature \( (°C) \) | 660      | -      |
| Latent heat of fusion \( (J \cdot kg^{-1}) \) | 385000   | -      |

4. Results

The IHTCs obtained in the numerical calculation will be shown in this section. The IHTCs will be plotted against time and the temperature of the casting at the casting/mold interface to examine the effects of the pouring temperature and the coating layer on the mold surface.

#### 4.1. IHTC When No Coating Is Applied

Figure 2(a) shows the IHTC–time curves for the pouring temperatures of 760, 810, and 860°C, when no coating was applied on the mold surface. For all pouring temperatures, the IHTC curves showed their maximum values at the very initial stages. They decreased very rapidly with time at the initial stage, and then the slope of curves decreased. For the pouring temperature of 860°C, the curve had a minor peak at about 15 s, as marked as an arrow in Fig. 2(a). This peak corresponds approximately to the start of the solidification of molten aluminum. The maximum IHTCs are 1 360, 1 640, and 1 860 \( W \cdot m^{-2} \cdot K^{-1} \) for the pouring temperatures of 760, 810, and 860°C respectively: the maximum values of IHTC increased with pouring temperature. Up to about 50 s, the IHTC was higher for higher pouring temperature. After then, the curves showed an approximately similar value irrelevant to pouring temperature.

The IHTC curves are plotted against the temperature of the casting at the casting/mold interface in Fig. 2(b) to investigate the relationship between IHTC and surface temperature of the casting. For all pouring temperatures, the IHTC increased gradually with casting surface temperature up to about 610–630°C, and three curves have similar values to each other in this region. Then IHTC increased abruptly to a certain value: approximately 780 \( W \cdot m^{-2} \cdot K^{-1} \) for the pouring temperature of 760°C, 1 040 \( W \cdot m^{-2} \cdot K^{-1} \) for 810°C, and 1 380 \( W \cdot m^{-2} \cdot K^{-1} \) for 860°C, respectively. Note that this value increased with pouring temperature. The IHTC curves increased rapidly to higher values again at the melting point of aluminum, 660°C.

It is interesting to note that the line of relatively uniform values of IHCP occurred at the initial stage of solidification of the casting. The open triangle (\( ▲ \)) in Fig. 2(b) denotes the surface temperature, 660°C, of the casting when the surface temperature passed the melting point, i.e., when the casting started to solidify, since the surface is the region to freeze very first in a cylindrical system. The solid triangle (\( ▼ \)) represents the surface temperature of the casting when the center temperature passed the melting point, i.e., when the casting finished to solidify, since the center is the region to solidify very last.

#### 4.2. IHTC When Ceramic Coating Is Applied

When ceramic coating was applied to the mold surface, the IHTC–time curves for the pouring temperatures of 760, 810, and 860°C are shown in Fig. 3(a). For all pouring temperatures, the IHTC curves showed clear peaks shortly after the initial stages. They decreased very rapidly as time increased, and then the slope of curves decreased with a minor peak at about 20–35 s. Similarly to the case of no coating for the pouring temperature of 860°C, these minor peaks correspond to the solidification range of the casting. The major peak values are 1 060, 1 150, and 1 210 \( W \cdot m^{-2} \cdot K^{-1} \) for the pouring temperatures of 760, 810, and 860°C respectively: the maximum values of IHTC increased as the pouring temperature increased. Note that these maximum values are lower than those when there is no coating on the
mold surface. Similarly to the case of no coating, the IHTC was higher for higher pouring temperature, up to about 50 s. After then, the curves were approximately coincident irrelevant to the pouring temperature. The IHTCs after 50 s were reasonably similar to those when no coating applied.

The IHTC curves against temperature of the casting at the casting/mold interface are plotted in Fig. 3(b). For all pouring temperatures, the IHTC increased gradually with casting surface temperature up to about 640°C. In this region all of three curves show similar values to those when no coating applied. Then IHTC increased to a certain value and held for a while: approximately 640 W·m⁻²·K⁻¹ for the pouring temperature of 760°C, 630 W·m⁻²·K⁻¹ for 810°C, and 550 W·m⁻²·K⁻¹ for 860°C, respectively. Although the IHTCs at the initial stage of solidification decreased as the pouring temperature increased, the differences are not as big as those when no coating was applied. The IHTC curves increased rapidly to higher relatively uniform values at the melting point. The IHTC decreased slightly with increasing surface temperature. The average values of IHTC in the liquid region were 1020 W·m⁻²·K⁻¹ for the pouring temperature of 760°C, 1100 W·m⁻²·K⁻¹ for 810°C, and 1200 W·m⁻²·K⁻¹ for 860°C. Also note that the peaks of IHTC in Fig. 3(a) correspond to the values when the solidification of casting begins, i.e., when the casting surface temperature becomes the melting point.

4.3. IHTC When Carbon Coating Is Applied

The IHTC–time curves for the pouring temperatures of 760, 810, and 860°C when carbon coating was applied to the mold surface are shown in Fig. 4(a). For all pouring temperatures, the IHTC curves showed their maximum values shortly after the initial stages, although the peaks were not very obvious. They decreased with time and the slope of curves decreased gradually. The maximum IHTCs are 760, 750, and 745 W·m⁻²·K⁻¹ for the pouring temperatures of 760, 810, and 860°C, respectively: the maximum values of IHTC decreased slightly with increasing pouring temperature. These maximum values are lower than those in case of no coating or ceramic coating. For almost whole range of time, the IHTC increased with casting surface temperature up to about 650°C. In this region all of three curves show similar values to the curves in case of the other situations of coating. Then
the IHTC increased to a certain value and held for a while: approximately 480 W·m⁻²·K⁻¹ for the pouring temperature of 760°C, 520 W·m⁻²·K⁻¹ for 810°C, and 490 W·m⁻²·K⁻¹ for 860°C, respectively. It is likely that the IHTCs at the initial stage of solidification were uniform regardless of pouring temperature. The IHTC curves increased rapidly to higher uniform values at the melting point. The average values of IHTC in the liquid region were increased rapidly to higher uniform values at the melting point. The average values of IHTC in the liquid region were 755 W·m⁻²·K⁻¹ for the pouring temperature of 760°C, 745 W·m⁻²·K⁻¹ for 810°C, and 740 W·m⁻²·K⁻¹ for 860°C, respectively, which are quite identical.

5. Discussion

The IHTC calculated in previous section will be discussed in terms of coating and pouring temperature in each physical state of the casting material, i.e., in the liquid state, during the solidification, and in the solid state. A schematic description of the IHTC will be given in each states, and then the recommended values of IHTC for a numerical work data in case of the casting in a cylindrical mold will be suggested.

5.1. IHTC in the Liquid State

Figure 5 is the plot of the IHTC as a function of pouring temperature in the liquid state. The IHTCs when the mold has no coating is the maximum of the calculated values above the melting point at the casting surface. They seem to be the transient values between the liquid phase and the solidification range, considering Fig. 2(b) compared to Figs. 3(b) and 4(b). It is, therefore, difficult to define the variation tendency of IHTC as a function of pouring temperature when no coating is applied on the mold surface. For the ceramic coating, the average IHTC increased slightly with pouring temperature. For the carbon coating, there was no significant change in IHTC with pouring temperature.

It is likely that melt contact directly on the smooth mold surface in the liquid phase without air gap in an ideal situation. When no coating is applied, the IHTC approaches infinity. When coating is applied, total IHTC is affected directly by the conductivity and the thickness of the coating material. In practice, there exists some air gap due to the mold surface roughness and the imperfect wetting of the melt on the mold surface without coating. When a coating layer is introduced, interface layer composes of casting, mold, coating, and air gap. Therefore the estimation of IHTC becomes more complicated.

It is known that increasing superheat contributes the better contact at the melt/mold interface, since the viscosity and the surface tension of the melt decrease with increasing melt temperature. However Fig. 5 shows that coating must lower the IHTC considerably. In this case the thermal resistance of the coating layer should be taken into account. The IHTC is relatively insensitive to pouring temperature when the coating is applied. It is likely that the wetting of the casting/coating materials is scarcely affected by the superheat. It is noted that carbon coating is more effective to remove the effect of superheat on the IHTC than ceramic coating. The heat transfer coefficient of the coating layer is defined as:

\[ h_c = \frac{k_c}{d_c} \tag{10} \]

where \( k_c \) and \( d_c \) is the thermal conductivity and the thickness of the coating layer, respectively. For ceramic coating (\( k_c = 1.5 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}, \) \( d_c = 100 \mu\text{m} \)), \( h_c \) is 15 000 W·m⁻²·K⁻¹, and for carbon coating (\( k_c = 2.2 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}, \) \( d_c = 100 \mu\text{m} \)), \( h_c \) is 22 000 W·m⁻²·K⁻¹. These values are much higher than the IHTCs estimated in the liquid. Moreover, the IHTC when ceramic coating was applied is higher than that when carbon coating was applied. It means that there exist other factors affecting the IHTC in the liquid, such as roughness of the mold material and wettability at the mold-coating and coating-casting interfaces. It is necessary to investigate more in detail the effect of the roughness and wetting for proper estimation of the IHTC in the liquid state.

It should be noted as well that the time interval between initial pouring and the start of the solidification is so short, i.e., \(< - 15 \text{ s} \) that it is hard to estimate exact characteristic of the IHTC in the liquid state. Kim and Lee pointed out that the error arises probably due to the false initial condition of the instantaneous filling, the uniform initial temperatures, and the delay time in the response of the thermocouples.

5.2. IHTC during Solidification

When solidification of the casting started, the IHTC dropped to a certain values, holding its own value at the initial stage of solidification, as shown in Figs. 2(b), 3(b) and 4(b). The plot of the IHTC as a function of pouring temperature during the solidification of the casting materials is shown in Fig. 6. For an uncoated mold surface, the IHTC increased with pouring temperature. When a coating is applied, the IHTC keeps a uniform value regardless of pouring temperature during the solidification.

The air gap seems to be introduced between the casting and the mold due to the contraction of the casting material
during solidification again due to the density difference between the two phases, and the expansion of the mold: the direct contacts at the interface would be diminished seriously especially owing to the abrupt deformation of the casting surface. Therefore there should be a significant drop in the IHTC curves when the solidification starts. The increasing IHTC with pouring temperature when no coating is applied is also attributed to the fact the casting/mold contact improves with superheat: the direct contact between casting/mold seems to keep remained for a certain period since there must be the feeding of the molten metal from the upper part of the casting to compensate the solidification shrinkage. The coating reduced the IHTC in the solidification range, and the IHTC was insensitive to pouring temperature, since the coating layer prevent the casting material from direct contact to the mold, similarly to the case in the liquid phase. The higher IHTC in the freezing range than in the full solid state must be due to the contact between the casting and the mold at the initial stage of the solidification.

5.3. IHTC in the Solid State

In the present study, when the solidification finished, the IHTC curves dropped again to much lower values as shown in Figs. 2(b), 3(b) and 4(b), since the latent heat was no longer released. In the solid state, nine curves in this study are shown to be more or less coincident.

While the air gap forms due to contraction of the casting materials arisen from the density difference of the two phases during the solidification, the air gap tends to increase according to thermal contraction as temperature of the casting decreases in the solid state. The heat transfer coefficient variation due to the air gap becomes dominant in the heat transfer at the casting/mold surface. The direct contact between the casting and the mold seems to be almost vanished. Therefore, the casting/mold interface composes of the two layers: the air gap and the coating layer.

The IHTC may be described as the harmonic mean of the heat transfer coefficient due to the air gap, $h_g$, and that due to the coating, $h_c$:

$$\frac{1}{h} = \frac{1}{h_g} + \frac{1}{h_c} \quad \text{(11)}$$

where $h_g = k_g/d_g$, $k_g$ is the thermal conductivity and $d_g$ is the thickness of the air gap. With an assumption that the carbon was coated with a thickness of 100 μm ($d_c = 10^{-4}$ m), the heat transfer coefficient due to the air gap, $h_g$, is 132.4 W·m⁻²·K⁻¹ ($k_g = 0.331$ W·m⁻¹·K⁻¹ at 400°C), the coefficient due to the carbon coating, $h_c$, is 22 000 W·m⁻²·K⁻¹ ($k_c = 2.2$ W·m⁻¹·K⁻¹ at 400°C), respectively. The total IHTC at 400°C is 131.6 W·m⁻²·K⁻¹ which is almost identical value to $h_g$ and is of similar order of the IHTC in this study. It means that the IHTC in the solid state does not affected by sort or thickness of the coating: it only depends upon the thickness of the air gap.

5.4. Summary

The IHTC varies according to the physical state of the casting materials. In the liquid state, the IHTC is affected mainly by the direct contact of the casting and the mold. In an ideal case of perfect wetting between the casting and the mold, the IHTC goes up to infinity when no coating is applied. When a coating is applied, the IHTC is defined by the conductivity and the thickness of the coating. In practice, there exists some air gap at the interface due to the roughness of the mold surface, and the imperfect wetting between the casting and the mold materials. When solidification of the casting started, the IHTC dropped to a certain values, holding its own value at the initial stage of solidification, since the air gap is introduced between the casting and the mold due to the density difference between liquid and solid states. Therefore, the IHTC during solidification is a combination of the air gap and the direct casting/mold contact. When the solidification has finished, IHTC drops again to a much lower value, due to the solidification contraction of the casting. The IHTC in the solid phase does not affected
by sort or thickness of the coating. It only depends upon the thermal conductivity and the thickness of the air gap. This phenomenon is summarized schematically in Fig. 7, and the proposed values of IHTC during casting is shown in Table 3.

6. Conclusions

In the present study the interfacial heat transfer coefficient (IHTC) between the pure aluminum casting and a cylindrical copper mold was evaluated by solving inverse heat conduction problem. The IHTC was described as a function of surface temperature of the casting to investigate the effect of coating layer and pouring temperature. Based on the present study, the IHTC is categorized into three regimes according to the physical state of the casting materials. In the liquid state, the IHTC is affected by the roughness of the mold, the wettability of the casting on the mold surface, and the physical properties of the coating layer. At the initial stage of solidification, the IHTC is a function of the air gap thickness and the superheat in addition to the coating properties and the wettability. The IHTC in the solid state depends only upon the thermal conductivity and the thickness of the air gap.

Table 3. Proposed values of the interfacial heat transfer coefficient when pure aluminum is cast into a cylindrical copper mold.

| State                      | Interfacial Heat Transfer Coefficient (W m⁻² K⁻¹) |
|----------------------------|---------------------------------------------------|
|                            | Without coating | Ceramic coating | Carbon Coating |
| In the liquid state        | >1800           | 1107            | 746            |
| At the initial stage of solidification | 6.0T_pouring⁻3800 | 1.72T_pouring⁻291 | 495 |
| In the solid state (>400°C) | 0.90T_pouring⁻322 |                |                |

*T_pouring* pouring temperature

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