Do observations prove that cosmological neutrinos are thermally distributed?

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It is usually assumed that relic neutrinos possess a Fermi-Dirac distribution, acquired during thermal equilibrium in the Early Universe. However, various mechanisms could introduce strong distortions in this distribution. We perform a Bayesian likelihood analysis including the first moments of the three active neutrino distributions as free parameters, and show that current cosmological observations of light element abundances, Cosmic Microwave Background (CMB) anisotropies and Large Scale Structures (LSS) are compatible with very large deviations from the standard picture. We also calculate the bounds on non-thermal distortions which can be expected from future observations, and stress that CMB and LSS data alone will not be sensitive enough in order to distinguish between non-thermal distortions in the neutrino sector and extra relativistic degrees of freedom. This degeneracy could be removed by additional constraints from primordial nucleosynthesis or independent neutrino mass scale measurements.

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I. INTRODUCTION

The large amount of observations of the Cosmic Microwave Background (CMB) which have been accumulated through forty years, since its discovery till the most recent detailed CMB map provided by the Wilkinson Microwave Anisotropy Probe (WMAP) Collaboration 1, shows that relic photons are distributed according to a black body distribution to an incredibly high accuracy. This fact is indeed one of the main cornerstones of the hot Big Bang model, strongly supporting the idea that our Universe expanded starting from high density and temperature conditions.

It is well known that in this framework we expect the Universe to be also filled by a large amount of neutrinos, with present densities of the order of $10^2$ cm$^{-3}$ per flavor. These relic neutrinos decoupled from the electromagnetic plasma quite early in time, when weak interaction rates became slower than the Hubble rate for (photon) temperatures in the range $2 \div 4$ MeV, just before Big Bang Nucleosynthesis (BBN) took place. Unfortunately the fact that this Cosmic Neutrino Background (CνB) has today a very small kinetic energy, of the order of $10^{-4}$ eV, and that neutrinos only interact via weak interactions, prevents us from any possible direct detection of this background on the Earth (see e.g. 2, 3 for recent reviews). Nevertheless, there are several indirect ways to constrain the CνB by looking at cosmological observables which are influenced either by the fact that neutrinos contribute to the Universe expansion rate at all stages, or also via their interactions with the electromagnetic plasma and baryons before their decoupling. In this respect the most sensitive probe is represented by the values of the light nuclide abundances produced during BBN. Actually, the final yields of Deuterium, $^7$Li and in particular of $^4$He strongly depend on the number of neutrino species as well as on their distribution in phase space at about 1 MeV when the neutron to proton density ratio freezes. In fact, since neutrinos were in chemical equilibrium with the electromagnetic plasma till this epoch we know by equilibrium thermodynamics that they were distributed according to a Fermi-Dirac function, yet BBN can constrain exotic features like the value of their chemical potential 4, 5, 6, 7. Furthermore, a detailed analysis carried out by several authors 8, 9 shows that neutrinos benefit to a small extent of the entropy released by electron-positron pairs at their annihilation stage, and that this phenomenon reheat neutrinos in a non-thermal way, introducing a small distortion of the order of few percent which grows with the neutrino comoving momentum.

Summarizing, neutrinos are likely to emerge from the
BBN epoch as decoupled species with an equilibrium distribution, a smaller temperature with respect to photons by the approximate factor \((4/11)^{1/3}\), possibly a small chemical potential to temperature ratio \(\xi \lesssim 0.1\) and finally, tiny momentum dependent non-thermal features.

Once decoupled, neutrinos affect all key cosmological observables which are governed by later stages of the evolution of the Universe only via their coupling with gravity. A well known example is their contribution to the total relativistic energy density, which affects the value of the matter-radiation equality point, which in turn influences the CMB anisotropy spectrum, in particular around the scale of the first acoustic peak. Similarly, their number density and their masses are key parameters in the small scale suppression of the power spectrum of Large Scale Structures (LSS), smaller than \(l_{nr}\), the horizon when neutrinos become non-relativistic.

\[
l_{nr} \sim 38.5 \left( \frac{1 \text{ eV}}{m_{\nu}} \right)^{1/2} \omega_m^{-1/2} \text{ Mpc} \ ,
\]

where \(\omega_m = \Omega_m h^2\), with \(\Omega_m\) the fraction of the critical density due to matter and \(h\) the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). Several authors have investigated in details these issues assuming a standard Fermi-Dirac distribution for neutrinos, e.g. \(6\, 11\, 12\, 13\, 14\, 15\, 16\). Indications for primordial anisotropies in the neutrino distribution function have been pointed out in \(17\). In this paper we consider the possibility that the relic neutrino distribution in phase space may be sensibly different and address the following point: how present (and future) cosmological observations can prove that indeed neutrinos are thermally distributed? The implications of a relic neutrino spectrum significantly different from a Fermi-Dirac distribution have been also considered by several authors.

As we already said, till temperatures of the order of MeV neutrinos are in chemical equilibrium with the electromagnetic plasma, so that they keep an equilibrium distribution. After weak interaction decoupling and in absence of any other interaction process, the collisionless kinetic equations in a Lemaitre-Friedman-Robertson-Walker universe state that, assuming for simplicity instantaneous decoupling, the neutrino and antineutrino distribution in phase-space is a Fermi-Dirac function. Therefore, in terms of the comoving momentum \(y = k a\), with \(a\) the scale factor and neutrino temperature \(T_{\nu}\), one has

\[
df_\alpha(k,T_\nu) = \frac{1}{\pi^2} T_\nu^3 y^2 f_{\alpha}^{ih}(y) dy = \frac{1}{\pi^2} T_\nu^3 y^2 \frac{1}{e^{y}+1} dy
\]

for each mass eigenstate. We assume zero chemical potential in the following. Regardless of the specific case at hand, we can specify the neutrino distribution \(df_\alpha(y)\) by the set of moments \(Q^{(n)}_\alpha\)

\[
Q^{(n)}_\alpha = \frac{1}{\pi^2} \left( \frac{4}{11} \right)^{(3+n)/3} T^{3+n} \int y^{2+n} f_\alpha(y) dy \ ,
\]

where \(T\) is the photon temperature, and we have defined the \(Q^{(n)}_\alpha\) in terms of the standard value of the neutrino temperature in the instantaneous decoupling limit \(T_{\nu} = (4/11)^{1/3} T\). We assume in the following that neutrino distribution decays at large comoving momentum as \(\exp(-y)\), so that the distribution admits moments of all orders. Actually, this is not a particularly severe constraint, since we expect that at very high \(y\) the shape of the distribution is ruled by the behaviour imprinted by neutrino decoupling as hot relics at the MeV scale.

In order to answer the question raised above, we consider in detail (Section \(18\)) a simple model where the out of equilibrium decay of a light scalar produces non-thermal features in the neutrino spectra, although our results are quite general and can be also applied to different scenarios. We describe the current bounds on this

\[^{1}\] We point out that the bound on \(\xi\) is shared by all neutrino species due to flavor oscillations \(1\), and that a much looser bound is obtained if we allow for extra relativistic degrees of freedom contributing to the Hubble rate during the BBN epoch (see e.g. \(3\)).

II. THE PHASE-SPACE DISTRIBUTION OF RELIC NEUTRINOS

As we already said, till temperatures of the order of MeV neutrinos are in chemical equilibrium with the electromagnetic plasma, so that they keep an equilibrium distribution. After weak interaction decoupling and in absence of any other interaction process, the collisionless kinetic equations in a Lemaitre-Friedman-Robertson-Walker universe state that, assuming for simplicity instantaneous decoupling, the neutrino and antineutrino distribution in phase-space is a Fermi-Dirac function. Therefore, in terms of the comoving momentum \(y = k a\), with \(a\) the scale factor and neutrino temperature \(T_{\nu}\), one has

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If we denote by \(P_m(y)\), \(m\) being the degree of \(P_m(y)\),

\[
P_m(y) = \sum_{k=0}^{m} c^{(m)}_k y^k \ ,
\]

the set of polynomials orthonormal with respect to the measure \(y^2/(\exp(y) + 1)\)

\[
\int_0^\infty dy \frac{y^2}{e^y+1} P_n(y) P_m(y) = \delta_{nm} \ ,
\]

it is easy to write the neutrino distribution in terms of its moments

\[
df_\alpha(y) = \frac{y^2}{e^y+1} \sum_{m=0}^{\infty} F_{\alpha,m} P_m(y) dy \ ,
\]
where we have defined
\[ F_{\alpha,m} = \sum_{k=0}^{m} c^{(m)}_{k} Q^{(k)}_{\alpha} T^{-k}_\nu, \]  
(7)
i.e. a linear combination of moments up to order \( m \) with coefficients \( c^{(m)}_{k} \).

For a Fermi-Dirac distribution all moments can be expressed in terms of the number density \( Q^{(0)}_{\alpha} \) or, equivalently, as functions of the only independent parameter \( T_\nu \). The first two moments, namely the neutrino energy density defined during the relativistic regime and their number density, enter all present analyses of cosmological observables. More precisely, it is customary to define the effective number of neutrinos
\[ N_{\text{eff}} = \frac{120}{\pi^2} \left( \frac{11}{4} \right)^{4/3} T^{-4} \sum_{\alpha} Q^{(1)}_{\alpha}, \]  
(8)
where \( N_{\text{eff}} = 3 \) for a thermal distribution, while it gives \( N_{\text{eff}} = 3.04 \) when including the small effect of neutrino heating during the \( e^+ - e^- \) annihilation phase. Similarly, \( Q^{(0)}_{\alpha} \) are related to the present neutrino energy density normalized to the critical density today
\[ \omega_\nu = \Omega_\nu h^2 = 0.058 \frac{m_0}{11 \text{ eV}} \frac{1}{4} T^{-3} Q^{(0)}_{\alpha}, \]  
(9)
where \( m_0 \) is the sum of the neutrino masses and we have assumed for simplicity that the three neutrinos share the same distribution. Flavor neutrino oscillations are in fact very effective after BBN, since for temperatures significantly lower than MeV the background effects of the medium are negligible, assuming the neutrino mixing parameters favoured by present experimental results, see[4]. Therefore, the initial distortions on the flavor neutrino spectra are fastly redistributed according to oscillations in vacuum. Averaging over all oscillatory terms, it is easy to find the final energy spectra of the three neutrino mass states. Here we take the approximation of equal (non-thermal) distributions for all neutrino mass states. In light of present sensitivity of data to neutrino distribution, to be discussed in the following, this approximation is quite reasonable.

In the standard (thermal) case we recover the usual result
\[ \omega_\nu = \frac{m_0}{94.1[93.2] \text{ eV}}, \]  
(10)
where the value between brackets takes into account the entropy release to neutrinos from \( e^+ - e^- \) annihilation. Actually, there are more parameters which may affect the LSS power spectrum, such as the individual neutrino masses, but their effect is negligible for a fixed \( m_0 \).

What about higher moments? In principle for an arbitrary distribution function, with the only requirement of an exponential decay for large comoving momentum, all moments may be relevant in the way this distribution is reconstructed by using the expansion. In the following we will consider only the first two moments \( \omega_\nu \) and \( N_{\text{eff}} \) as free and independent parameters, to be constrained using cosmological data. In fact, we start observing that in the limit of a thermal distribution, the value of \( df_\alpha(y) \) is uniquely determined by \( Q^{(0)}_{\alpha} \), see Eq. 4. Adopting therefore a conservative approach one may think that if no extremely strong deviations from equilibrium are present, the expansion can be truncated for some \( m \) without losing relevant pieces of information. The only way to decide how many parameters should be included in the analysis is by studying the sensitivity of the available observational data to the distortions of the neutrino distribution. As we will see, using all available data from CMB, LSS and possibly BBN, it is already very hard to get quite strong constraints on the first two moments, so that it appears that presently it is not worth comparing more complicated models with observations and introduce further parameters \( Q^{(n)}_{\alpha} \). In fact more moments means more parameter degeneracies, so that the main conclusion of the analysis would remain unchanged. Of course, in case future observations would reach a higher sensitivity on neutrino distribution, it would be desirable to include higher order moments, such as the skewness or the kurtosis, related to \( Q^{(2)}_{\alpha} \) and \( Q^{(3)}_{\alpha} \), respectively, as free parameters in the analysis.

We now turn to the main point: how the neutrino distributions may acquire non-thermal shapes after the weak interaction freeze-out. A possible source of such non standard contribution may arise from the decays of unstable massive particles provided

i) decays take place out of equilibrium, i.e. for temperatures smaller than the decaying particle mass \( M \). In fact, if decays take place at equilibrium for \( T \sim M \), the resulting neutrino distribution would be still thermal, possibly with a different ratio of neutrino-photon temperatures if the entropy is released in different amounts to neutrinos and photons. Therefore, in this case the effect consists at most in an overall constant factor in neutrino distribution, changing the energy density in relativistic species.

ii) occur after weak interaction freeze-out.

The simplest scenario which we will be considering in details in the following is the two body decay of a light \((M \leq 1 \text{ MeV})\) neutral scalar particle \( \Phi \) whose dynamics is dictated by the following lagrangian density
\[ L = \frac{1}{2} \partial \mu \Phi \partial^\mu \Phi - \frac{1}{2} M^2 \Phi^2 - \frac{\lambda}{\sqrt{3}} \Phi \sum_i \bar{\nu}_i \nu_i + L_{\text{int}}, \]  
(11)
with \( i \) the flavor index and we have assumed a flavor independent coupling \( \lambda \) to neutrinos. With \( L_{\text{int}} \) we denote the interaction terms of \( \Phi \) with all other fields except neutrinos and responsible for the thermalization of the \( \Phi \) quanta at high temperatures. The \( \Phi \) particles are assumed to decay in out of equilibrium conditions before
the photon last scattering epoch, so that the produced neutrino burst directly influences the CMB anisotropy spectrum, as well as the late LSS formation. Different scenarios can be envisaged, as for example the case of unstable neutrinos $\nu_l$ decaying into a (pseudo) scalar particle $\varphi$ and a lighter neutrino $\nu_l$,

$$\nu_l \rightarrow \varphi \nu_l \ , \quad \text{(12)}$$

or three-body decays. It is worth stressing that the aim of this paper is to show how combining different cosmological observables we can constrain the non-thermal contribution to the neutrino background. Despite of the fact that we will be mainly dealing for definiteness with the out of equilibrium $\Phi$ decay case, all our results are quite general and can be applied to different scenarios as well. Using Eq. (11) we can easily compute the decay rate

$$\Gamma(\Phi \rightarrow \nu \bar{\nu}) = \frac{\lambda^2 M}{8\pi} \ . \quad \text{(13)}$$

If decays take place out of equilibrium for temperatures smaller than the $\Phi$ mass it follows that the coupling $\lambda$ should be very small. In the radiation dominated regime

$$\lambda \sim 10^{-10} \frac{T_D/\text{MeV}}{\sqrt{M/\text{MeV}}} \ , \quad \text{(14)}$$

where $T_D$ is the temperature of the thermal part of the neutrino background at decay. The coupling $\lambda$ also produces scattering processes like $\Phi\nu \leftrightarrow \Phi\nu$ and crossed reactions, which however receive contribution at order $\lambda^4$. It is easy to see that for the small values implied by Eq. (14) these processes were never in equilibrium before $T_D$, so they cannot thermalize the $\Phi$ particles with neutrinos after decoupling of weak interactions. When the $\Phi$ particles decay, the neutrino distribution gets an additional contribution, which in the narrow width limit and in the instantaneous decay approximation at $T_D$ corresponds to a peaked pulse at $y_* = M/(2T_D)$ so that

$$y^2 f(y)dy = \frac{y^2}{e^{y^2} + 1}dy + \pi^2 \frac{A}{\sqrt{2\pi}\sigma^2} \exp \left[-\frac{(y - y_*)^2}{2\sigma^2}\right] \ , \quad \text{(15)}$$

where $\sigma \ll y_*$. Relaxing this assumption would be one of the many ways to generalize our study. We show an example of this non-thermal neutrino spectrum in Figure 1 where a second peak is clearly seen around $y_*$. With this distribution, and accounting for tiny neutrino heating effects from $e^+ - e^-$ annihilations, the lower moments expressed in terms of the parameters of Eqs. (8) and (9) read

$$\omega_\nu = \frac{m_0}{93.2 \text{eV}} \left(1 + 0.99 \frac{2\pi^2}{3\zeta(3)} A\right) \ , \quad \text{(16)}$$

$$N_{\text{eff}} = 3.04 \left(1 + 0.99 \frac{120}{7\pi^2 A y_*}\right) \ . \quad \text{(17)}$$

The value of the constant $A$, which measures the number of neutrinos and antineutrinos produced at decay can be expressed in terms of the total number of $\Phi$ particles before decay in a comoving volume

$$A = \frac{2}{3} \frac{n_\Phi}{T^\nu} \ , \quad \text{(18)}$$

with $n_\Phi$ the $\Phi$ number density. Since these particles are decoupled from the thermal bath since at least the BBN epoch, we see that the right hand side of Eq. (18) can be evaluated at this early epoch, so that the largest value of $A$ can be bound by BBN as a function of the $\Phi$ mass and number density. We will discuss this issue in the next Section along with the constraints on $A$ and $y_*$ arising from present data on BBN, CMB and LSS. Later we will show in Section 4 how future CMB and LSS data could improve these constraints. Our results, which when expressed in terms of the parameters $A$ and $y_*$ get a direct interpretation in the $\Phi$ particle decay model described so far, nevertheless can be looked upon as quite general and give the expected sensitivity that future data will likely have in pointing out non-thermal features, if any. This sensitivity, as unfortunately it is quite common the case, will be crucially depending on the specific cosmological model which is assumed as a reference one, namely on the number of parameters which enter the data fitting procedure. For example, we will stress that if we enlarge this set of parameters allowing for extra relativistic degrees of freedom in addition to ordinary photons and neutrinos, a parameter degeneracy would make it very difficult to distinguish between non-thermal corrections to the ordinary active neutrino background and the presence of extra exotic light particles.
III. CURRENT BOUNDS FOR A SCENARIO WITH THREE NON- THERMAL NEUTRINOS

A. Constraints from BBN

The amount of light nuclei produced during BBN is rather sensitive to the value of the Hubble parameter $H$ during that epoch, as well as to its time dependence. In particular the $^4$He mass fraction $Y_p$ strongly depends on the freeze-out temperature of weak processes which keep neutrons and protons in chemical equilibrium. Changing the value of $H$ affects the neutron to proton number density ratio at the onset of the BBN, which is the key parameter entering the final value of $Y_p$ and more weakly the Deuterium abundance. For a fixed baryon density parameter, the energy and number density of $\Phi$ particles can be written as

$$\rho_\Phi = \frac{1}{2\pi^2} T_\Phi^4 \int x^2 \left( \frac{x^2 + M^2}{T_\Phi^2} \right)^{1/2} \frac{1}{e^x - 1} \, dx \quad , (19)$$

$$n_\Phi = \frac{1}{2\pi^2} T_\Phi^3 \int x^2 \frac{1}{e^x - 1} \, dx = \frac{\zeta(3)}{\pi^2} T_\Phi^3 \quad , (20)$$

where we have assumed, as from our discussion in the previous Section, that the $\Phi$’s decoupled as hot relics well before the onset of BBN. We mention that the effect of massive particles with mass of the order of 1–10 MeV which are instead coupled to neutrons or photons during BBN has been studied in details in [30]. The two Eqs. (19) and (20) can be combined getting

$$\rho_\Phi = T_\nu^4 \frac{1}{2\zeta(3)} \left( \frac{n_\Phi}{T_\nu^3} \right) \times$$

$$\int x^2 \left[ \left( \frac{\pi^2 n_\Phi}{\zeta(3) T_\nu^3} \right)^{2/3} \frac{x^2 + M^2}{T_\nu^2} \right]^{1/2} \frac{1}{e^x - 1} \, dx \quad ,$$

so that the upper bound on $\rho_\Phi$ from light nuclei abundances translates into an upper limit to $n_\Phi/T_\nu^3$ and so to $A$, see Eq. (18), as a function of $M$. To get this bound we use a standard likelihood procedure, as discussed e.g. in [7], and the BBN numerical code described in details in [28]. We use the WMAP prior on $\omega_b$ mentioned before and the experimental result of [31] for D/H

$$D/H = (2.78^{+0.44}_{-0.38}) \times 10^{-5} \quad . (22)$$

We note that, allowing for non-thermal features in neutrino distributions may in principle change the preferred value of $\omega_b$ and the corresponding uncertainty from a CMB data analysis. However, we will show in the following that the effect on the baryon density parameter is completely negligible and a likelihood analysis provides the same value of a standard ΛCDM model, as considered in [13]. It is therefore consistent to adopt this result in our BBN study.

The $^4$He mass fraction $Y_p$ obtained by extrapolating to zero the metallicity measurements performed in dwarf irregular and blue compact galaxies is still controversial and possibly affected by systematics. There are two different determinations [32, 33] which are only compatible by invoking the large systematic uncertainty quoted in [32]

$$Y_p = 0.238 \pm (0.002)_{\text{stat}} \pm (0.005)_{\text{sys}} \quad , (23)$$

$$Y_p = 0.2421 \pm (0.0021)_{\text{stat}} \quad . (24)$$

In the present analysis we consider a conservative estimate for the experimental $^4$He abundance obtained by using the results of [32]

$$Y_p^{\text{exp}} = 0.245 \pm 0.007 \quad . (25)$$

Finally, we do not use the $^7$Li abundance, since the experimental estimates of this nuclide may be affected by a large depletion mechanism with respect to the primordial value, see e.g. [28, 29].

Our results are summarized in Figure 2 where we show the $1\sigma$ and $2\sigma$ bounds on $n_\Phi/T_\nu^3$ obtained using the $^4$He mass fraction and D/H, assuming that the $\Phi$’s decay after BBN. As expected the most stringent constraint is due to $Y_p$, since this parameter is very sensitive to any change of the value of the Hubble parameter. For $\Phi$ masses as large as 0.1 MeV, the scalars are relativistic during the neutron to proton density freeze-out, so we get an almost mass independent bound from $Y_p$ of the order of

![Figure 2](image-url)
\[ n_\phi / T_\nu^3 \leq 0.17 \text{ (or } A \leq 0.1 \text{ at } 2\sigma). \] This corresponds to a limit on the non-thermal component of the neutrino number density of the order 54\% of the thermal neutrino contribution. On the other hand, for larger masses the upper bound on \( n_\phi / T_\nu^3 \) decreases approximately as \( 1/M \) for \( M \) larger than few MeV, see Eq. (21). For this mass range, the BBN bound from the \( D \) abundance actually becomes slightly more stringent than that from \(^4\text{He}.\) Note that the BBN bounds still allow a large contribution of the decay products to the relativistic neutrino energy density \( N_{\text{eff}}. \) This value depends quite crucially on the ratio \( M/T_D, \) which measures how far from equilibrium conditions the \( \Phi \) decays take place. For values of \( M \) smaller than 0.1 MeV we get

\[ N_{\text{eff}} \leq 3.04 \left(1 + 0.1 \frac{M}{T_D}\right), \quad (26) \]

where \( T_D \leq 0.01 \text{ MeV, since we assumed that the } \Phi \text{ decays take place after BBN. If } T_D \sim 0.1M \text{ or smaller the non-thermal component can give a large or even dominant contribution to } N_{\text{eff}}. \) For higher values \( M \geq 1 \text{ MeV}, \) our simple scenario becomes unlikely because it requires the \( \Phi \) decays to take place after BBN, and therefore one needs a very large ratio \( M/T_D \geq 10^2. \) However, in this case the contribution to \( N_{\text{eff}} \) can still be very large, despite of the fact that the \( \Phi \) number density rapidly decreases with increasing values of \( M. \) In fact, the upper bound on \( N_{\text{eff}} \) scales as \( 1/T_D \) in this regime, see Eq. (17).

### B. Adding constraints from cosmological perturbations (CMB and LSS)

We use the public code\(^2\) CAMB \(^34\) in order to compute the theoretical prediction for the \( C_\ell \) coefficients of the temperature and polarization power spectra of CMB, as well as the matter spectra \( P(k). \) We explicitly implement into the code the expression of the non-thermal phase-space distribution function, in the same fashion as massive neutrino chemical potentials were added to cmbfast \(^35\) in \(^37\): We focus on three models: the minimal \( \Lambda \text{CDM} \) model including three massive neutrinos (with equal mass); our extended model with three non-thermal massive neutrinos (\( \Lambda \text{CDM+NT} \)) and finally, for comparison a model with three ordinary massive thermal neutrinos plus extra relativistic degrees of freedom (\( \Lambda \text{CDM+R} \)), already analyzed in \(^12\) \(^38\) (using a grid-based method and a smaller dataset). We compute the Bayesian likelihood of each cosmological parameter with a Monte Carlo Markov Chain method, using the public code Cosmomc \(^39\) with option MPI on the MajoRana cluster at Naples University. The likelihood of each model is found using three groups of data sets, as implemented in the public version of Cosmomc: (i) CMB data: WMAP \(^13\), VSA \(^40\), CBI \(^41\), ACBAR \(^42\); (ii) LSS data: 32 correlated points from 2dFGRS (up to \( k_{\text{max}} = 0.1h \text{ Mpc}^{-1} \) \(^43\) \(^44\), 14 uncorrelated points from SDSS \(^45\) (up to the same scale); and (iii) SNIa data from Riess et al. \(^46\). For SDSS, we compare directly the data with the real space lineal matter power spectrum multiplied by a scale-independent bias with flat prior, which we marginalize out. For 2dF, we include a prior on the bias: the linear matter power spectrum (at redshift \( z = 0 \)) is rescaled by a redshift-space correction at \( z = 0.17 \) and a bias factor \( b \)

\[ P(k)_{\text{red}, sp} = b^2 \left(1 + \frac{2}{3} \beta_{\text{eff}} + \frac{1}{5} \beta_{\text{eff}}^2\right) P(k)_{\text{red}, sp} \quad (27) \]

with \( \beta_{\text{eff}} = 0.85 \beta \) and \( b = \Omega_0^{0.6} / \beta \) (see \(^17\) and references therein). For the redshift-space distortion factor \( \beta \) we adopt the prior \( \beta = 0.43 \pm 0.08 \) \(^43\). We do not include any data from Lyman-\( \alpha \) forests\(^3\). Finally, we impose the gaussian prior \( h = 0.72 \pm 0.08 \) (1\(\sigma \)) from the HST Key Project \(^40\), and for the non-thermal model we add the BBN prior described in the previous Section: \( \Delta < 0.1 \) at the 2-\(\sigma \) level (the prior is implemented as a half-gaussian centered at \( A = 0 \) and with a standard deviation of 0.05). As we already said, a remarkable feature of the \( \Lambda \text{CDM+NT} \) model is that BBN does not impose an upper limit on \( N_{\text{eff}} \) at the CMB epoch, so the results shown for comparison in the \( \Lambda \text{CDM+R} \) case will not include an \( N_{\text{eff}} \) prior, and will not always be in agreement with predictions from standard BBN. For the \( \Lambda \text{CDM} \) model, the basis of cosmological parameters used by the Markov Chain algorithm is as follows: \(^1\) the overall normalization parameter \( \ln[10^{10} R_{\text{rad}}] \) where \( R_{\text{rad}} \) is the curvature perturbation in the radiation era; \(^2\) the primordial spectrum tilt \( n_s; \) \(^3\) the baryon density \( \omega_b = \Omega_b h^2; \) \(^4\) the (cold+hot) dark matter density \( \omega_{\text{dm}} = \Omega_{\text{dm}} h^2; \) \(^5\) the ratio of the sound horizon to the angular diameter distance multiplied by 100; \(^6\) the optical depth to reionization \( \tau; \) \(^7\) the 2dF redshift-space distortion factor \( \beta; \) \(^8\) the total mass \( m_\nu \) of the three neutrinos. In addition, the \( \Lambda \text{CDM+R} \) model includes: \(^9\) the effective neutrino number \( N_{\text{eff}} = 3.04 + \Delta N_{\text{rel}} \). Finally, the \( \Lambda \text{CDM+NT} \) model includes: \(^9\) the statistical moment \( Q_r \) of the three non-thermal neutrinos, or equivalently, their relativistic energy density in units of that of a standard massless neutrino \( N_{\text{eff}} \geq 3.04, \) see Eq.

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\(^2\) CAMB Code homepage [http://camb.info/]

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\(^3\) The non-thermal model includes massive neutrinos which are individually more energetic and with a higher average momentum than their thermal counterpart. In presence of such a fluid, the non-linear small-scale evolution might differ significantly from that of ordinary \( \Lambda \text{CDM} \) models. Therefore, we believe that data from Lyman-\( \alpha \) forests are inappropriate for a conservative analysis. We are aware that the bias constraint does rely on observations in the mildly non-linear range \( 0.2 < k < 0.4 \text{ h Mpc}^{-1} \), and we assume that on these scales the non-thermal neutrinos do not generate unusually large non-linear corrections which would invalidate our bias prior.
FIG. 3: Marginalized likelihood for the parameters of each model: ΛCDM+NT (black, solid), ΛCDM+R (red, dashed), or ΛCDM (blue, dotted). The independent basis parameters are included in the first ten plots only. The last two plots show the related parameters ($A$, $y_*$) for the non-thermal model.
The only bound being instead on the scalar particle number density at the BBN epoch.

FIG. 4: Two-dimensional likelihoods illustrating the main degeneracy affecting the ΛCDM+NT model (for the ΛCDM+R model the plots are extremely similar). This degeneracy correlates $N_{\text{eff}}$ mainly with $\omega_{\text{dm}}$ (top left) and $m_0$ (middle left), and to a smaller extent with $\theta$ (top right) and $n_s$ (middle right). We also show how $N_{\text{eff}}$ is correlated with two related parameters: the age of the Universe (bottom left) and the Hubble parameter (bottom right). The solid curves refer to the 1-σ and 2-σ contours of the marginalized likelihood. For comparison, the average sample likelihood is shown in color levels.

strained, and large values for it are still allowed. In the $\Phi$ decay scenario this implies that these decays can take place in highly out of equilibrium conditions, $T_D \ll M$, the only bound being instead on the scalar particle number density at the BBN epoch.

IV. FUTURE PROSPECTS: CAN WE REMOVE THE DEGENERACIES?

A. Error forecast assuming no extra relativistic degrees of freedom

We will now compute the error expected from future experiments on the parameters of the ΛCDM+NT model. Of course, the results that will be derived here are model-dependent, since we assume that the cosmological evolution can be described with a given set of cosmological parameters. The issue of generalization to larger classes of models will be considered in the next Section. As usually done in this kind of analysis, we first assume that future experiments with known sensitivity. Inverting the Fisher matrix

$$F_{ij} = -\frac{\partial^2 \ln {\mathcal L}}{\partial x_i \partial x_j} (\bar{x}_i) \ ,$$

which represents the curvature of the likelihood distribution $\mathcal{L}$ near its maximum for a given set of future experiments with known sensitivity. Inverting the Fisher matrix, we obtain the 1-σ error on each parameter, assuming that all other parameters are unknown

$$\Delta x_i = \left( F^{-1} \right)_{ii}^{1/2} .$$

The present analysis follows the lines of that in [50], and we refer the reader to this work for details on the expression of the likelihood distribution, for the description of future experimental sensitivities and for references of seminal papers on the subject. An analysis for massless neutrinos and aimed to study the sensitivity on $N_{\text{eff}}$ of WMAP and Planck experiments has been also discussed in [51]. When computing numerically the derivative of the likelihood with respect to each cosmological parameter, we choose the smallest possible step size above the threshold set by the precision of cmbfast, and we check that the final result is almost independent of the exact step value. Our first “conservative” set of experiments consists in the Planck satellite ⁵ (without lensing information [52]) combined with the completed SDSS redshift

|                              | ΛCDM | ΛCDM+R | ΛCDM+NT |
|------------------------------|------|--------|---------|
| $\chi_{\text{min}}$         | 1688.2 | 1688.0 | 1688.0 |
| $\ln[10^{15}{\mathcal R}_{\text{min}}]$ | 3.2±0.1 | 3.2±0.1 | 3.2±0.1 |
| $n_s$                        | 0.97±0.02 | 0.99±0.03 | 1.00±0.03 |
| $\omega_b$                   | 0.0235±0.0010 | 0.0231±0.0010 | 0.0233±0.0011 |
| $\omega_{\text{dm}}$        | 0.121±0.005 | 0.17±0.03 | 0.17±0.03 |
| $\theta$                     | 1.043±0.005 | 1.033±0.006 | 1.033±0.006 |
| $\tau$                       | 0.13±0.05 | 0.13±0.06 | 0.15±0.07 |
| $\beta$                      | 0.46±0.04 | 0.48±0.04 | 0.48±0.04 |
| $m_0$ (eV)                   | 0.3±0.2 | 0.8±0.5 | 0.7±0.4 |
| $N_{\text{eff}}$             | 3.04 | 6±2 | 6±2 |
| $q$                          | 1 | 1 | 1.25±0.13 |
| $h$                          | 0.67±0.02 | 0.76±0.06 | 0.76±0.05 |
| Age (Gyr)                    | 13.8±0.2 | 12.1±0.9 | 12.1±0.8 |
| $\Omega_{\Lambda}$          | 0.68±0.03 | 0.67±0.03 | 0.67±0.03 |
| $z_{re}$                     | 14±4 | 16±5 | 18±6 |
| $\sigma_s$                  | 0.76±0.06 | 0.77±0.07 | 0.77±0.07 |

| TABLE I: Minimum value of the effective $\chi^2$ (defined as $\chi^2$, where $\mathcal{L}$ is the likelihood function) and 1σ confidence limits for the parameters of the three models under consideration. The first ten lines correspond to our basis of independent parameters, while the last five refer to related parameters. |
The results are shown in Table I for a fiducial model comfortably allowed by current bounds, with $N_{\text{eff}} = 4$, $q = 1.1$ and $m_0 = 0.5$ eV, corresponding to non-thermal correction parameters $A = 0.018$ and $y_e = 10.5$. For this model, the neutrinos enter the non-relativistic regime well after decoupling, and only contribute to 0.5% of the critical density today. As a consequence, the two parameters $q$ and $m_0$ are measurable only from the free-streaming effect in the matter power spectrum, while all other parameters have a clear signature in the CMB anisotropies. More precisely, a combination of these two parameters (close to $q \times m_0 \propto \omega_0$ at first approximation) controls the amplitude of the free-streaming suppression on small scales, while the orthogonal combination controls the details of the transition on intermediate scales. Since LSS experiments do not have enough resolution for constraining this last combination, we expect a large degeneracy between $q$ and $m_0$, which is confirmed by the diagonalization of the Fisher matrix. The direction $\Delta q = (\Delta m_0/0.5$ eV) appears to be poorly constrained. As a consequence, the errors quoted in Table I for $m_0$ are larger (typically by a factor two) than those expected for thermal neutrinos, and $q$ has the biggest relative error. Notice that we are not applying in this case any prior on $q$, as was instead the case in the previous Section, where this parameter was related to the $\Phi$ number density at the BBN epoch in the $\Phi$ to neutrino decay scenario. Therefore, results in Table I are independent of any particular model that produces a distortion on the relic neutrino spectrum. The remarkable result of this Section is that in spite of this degeneracy, future experiments can provide some insight on non-thermal features and still measure the neutrino mass with good precision even when the thermal assumption is relaxed. The error for all parameters but $(q, m_0)$ are close to the usual expectations based on thermal neutrinos, see [51] for comparison. We also find a sensitivity on $^4\text{He}$ from CMB of the same order of magnitude found in [52] for Planck experiment. A small distortion of the first moment described by $q \sim 1.16$ would be sufficient for a 2-$\sigma$ detection with CMBPOL+LSST. Notice that large distortions (close to the current upper bounds) would lead to smaller errors than those quoted in Table I due to their additional direct effect on the CMB.

As a final remark we note that in the CMBPOL+LSST scenario the uncertainty on the $^4\text{He}$ mass fraction would be reduced by a factor two with respect to present sensitivity from direct measurements of this nuclide via spectroscopic methods. In the framework of the scalar decay scenario considered in this paper it is worth seeing whether this result from CMB may further constrain the value of $q$ and/or $N_{\text{eff}}$. The result of course depends on the fiducial model which is considered, namely what is the value of $^4\text{He}$ which future measurements will suggest to be the preferred one. We consider as an example a fiducial model predicting $Y_p = 0.245$, in order to compare with constraints presently available, see Eq. $[28]$, with an error of 0.004 (see Table II, “ambitious” set of data). Even in this optimistic scenario, comparing this determination with the results of the nucleosynthesis theoretical code of [28] we find the bound $q < 0.16$, a factor two larger than what is already obtained by CMB/LSS data (0.08, see again Table II). Finally, we recall that in the particle decay scenario considered in the paper we cannot bound the value of $N_{\text{eff}}$ at CMB using BBN, since we are assuming that decays take place after BBN and the value of $N_{\text{eff}}$ can be quite large depending on the $M/T_D$ ratio. Much stronger constraints come in fact from CMB/LSS data alone, order 0.1 and 0.02 in the conservative and ambitious scenarios respectively, see Table II.

### B. Degeneracy between non-thermal corrections and extra relativistic degrees of freedom

The bounds derived in the previous Sections depend on the assumption that the cosmological observations can be described within the framework of the $\Lambda$CDM+NT model. In the future, we would be in a position to make

| name | $\ln[10^{10}R_{\text{fid}}]$ | $n_s$ | $\omega_0$ | $\omega_m$ | $\Omega_{\Lambda}$ | $\tau$ | $Y_{\text{He}}$ | $b^2$ | $m_0$ (eV) | $N_{\text{eff}}$ | $q$ |
|------|-----------------|------|--------|--------|---------|------|-------|-----|---------|-------------|-----|
| fiducial values | 3.1 | 0.96 | 0.023 | 0.14 | 0.70 | 0.11 | 0.24 | 1.0 | 0.5 | 4.0 | 1.1 |
| $1\sigma$ error | 0.01 | 0.009 | 0.0003 | 0.004 | 0.02 | 0.005 | 0.02 | 0.08 | 0.3 | 0.1 | 0.7 (conservative) |
| | 0.006 | 0.003 | 0.0001 | 0.0007 | 0.001 | 0.003 | 0.004 | 0.03 | 0.02 | 0.08 (ambitious) |

Table II: Expected errors on the parameters of the $\Lambda$CDM+NT model. The first line shows the fiducial values, i.e. the parameter values assumed to represent the best fit to the data. The next lines give the associated $1\sigma$ errors for the “conservative” and “ambitious” sets of future experiments as described in the text.

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6 http://www.sdss.org
7 http://universe.nasa.gov/program/inflation.html
8 http://www.lsst.org
such an analysis if we had some independent motivations for the introduction of non-thermal distortions in the neutrino background. Otherwise, even if the best-fit $\Lambda CDM+NT$ model turned out to be much more likely than the best-fit $\Lambda CDM$, many alternative models would compete with each other.

The $\Lambda CDM+NT$ model has a natural “competitor”, which is the previously discussed $\Lambda CDM+R$ model, with three standard massive neutrinos and extra relativistic degrees of freedom. Indeed, in the two models the amplitude and scale of the free-streaming effect can be tuned independently, which is not usually the case (and which cannot be mimicked by the introduction of tilt running, spatial curvature, tensor modes, isocurvature modes, etc.)

The results of Section 3.3 suggest that at least within current experimental sensitivities, the two models are roughly equivalent. However, many theoretical arguments suggest that the intrinsic differences between the two cases can lead to distinct observable signatures. For instance, the non-thermal corrections boost the average neutrino momentum, so at the level of the equations the free-streaming effect is never strictly identical in the two cases.

We have investigated in more details the issue of a possible (at least approximate) one-to-one correspondence between $\Lambda CDM+NT$ and $\Lambda CDM+R$ models. We studied many examples and found a systematic way to find the “twin $\Lambda CDM+R$ model” of a given $\Lambda CDM+NT$ model:

1. for any $\Lambda CDM+NT$ model with a small total mass $m_0 < 0.9$ eV, the conclusion is straightforward: the $\Lambda CDM+R$ model sharing the same values of $N_{\text{eff}}$ and $\omega_\nu$ has identical CMB anisotropy spectra, and a matter power spectrum differing by less than one percent on all scales. Note that the “twin $\Lambda CDM+R$ model” has a different total neutrino mass than the original $\Lambda CDM+NT$ one, given by the simple relation $m_0^{(R)} = q m_0^{(NT)}$.

2. for any $\Lambda CDM+NT$ model with larger neutrino mass – such that the neutrinos are not ultra-relativistic at decoupling – the situation is a bit more complicated: the “twin $\Lambda CDM+R$ model” does not share that same values of $N_{\text{eff}}$ and $\omega_\nu$, because the time of equality is not strictly the same for the two models. Therefore, it is necessary to increase slightly $N_{\text{eff}}$ and $\Omega_\Lambda$ in the $\Lambda CDM+R$ model in order to obtain the same position and amplitude of the first peak in the CMB temperature spectrum. Finally, a simultaneous increase of $N_{\text{eff}}$, $\omega_m$ and $n_s$ makes it possible to match the matter power spectra without changing the time of equality and the shape of the CMB temperature spectrum.

In both cases the differences are below the percent level for the matter power spectrum, even less for the CMB anisotropy spectra. This is too small for allowing any detection with the next generation of experiments\textsuperscript{9}. Of course, this conclusion applies to a pure CMB and LSS analysis where all parameters are assumed to be unknown. Strong evidence for the $\Lambda CDM+NT$ could still arise from external data sets: for instance, in case of contradiction (assuming standard neutrinos) between future CMB and BBN predictions for the value of $N_{\text{eff}}$; or between the neutrino mass scale measured by cosmology and by beta decay experiments.

V. CONCLUSIONS

In this paper we have studied the possibility that the phase space distribution of cosmological relic neutrinos may be non-thermal, and how present observations can constrain any departure from a standard Fermi-Dirac function. The latter is expected to hold to a very good accuracy (percent level) in the standard scenario, where neutrinos last interact at the freeze out of weak interactions for temperatures of the order of MeV, and then simply free stream keeping an equilibrium distribution. However, if neutrino evolution is influenced at some later stage by some unknown and exotic phenomena, we expect that their distribution could be modified. For example, if some decoupled particle eventually decay into neutrinos in out of equilibrium conditions and after weak interaction freezing, the neutrino distribution can have large departures from a simple Fermi-Dirac function. We have investigated this model in details, for the case of light ($M < 1$ MeV) decoupled scalar particles decaying in neutrino/antineutrino pairs.

Neutrinos both influence the CMB spectrum and the clustering of structures on large scales. Their distribution, in fact, enters the value of the matter-radiation equality point, as well as for massive neutrinos the matter power spectrum suppression at scales smaller than the free streaming scale. The neutrino distribution can be always fully characterized by the set of its moments $Q_n^{(m)}$ which in principle may all affect the cosmological evolution of perturbations. However, present data are only sensitive to the first two distribution moments. Actually, the total neutrino mass $m_0$ and the moment $Q_1^{(1)}$ (corresponding to $N_{\text{eff}}$) are well constrained by the CMB and LSS data, while $Q_0^{(1)}$ (corresponding to the ratio $\omega_\nu/m_0$) is not. This fact, along with the existence of a degeneracy among $\omega_m$, $N_{\text{eff}}$ and $m_0$, implies that it is presently impossible to establish whether neutrinos are thermally distributed or not, even in the most conservative scenario of no extra relativistic particles at CMB epoch apart from photons and neutrinos. In particular,

\textsuperscript{9} This statement is easy to prove quantitatively by repeating the Fisher matrix analysis of the previous section, including an extra parameter describing the “linear deformation” of one model into another. This parameter results to be far from detectable.
in the framework of the Φ decay scenario described in Section II, the most stringent constraint on \( Q^{(0)} \) comes from BBN only, \( A \lesssim 0.1 \), rather than from CMB and LSS. The non-thermal contribution to the total neutrino energy density can then be very large or even dominant, since it is ruled by the parameter \( y_\nu = M/(2T_D) \) which is very poorly constrained and can take very large values, \( y_\nu \sim 10 \), even for \( A \) close to the BBN upper bound.

We have also studied how these results may likely change when new data on CMB and LSS will be available. Adopting a Fisher matrix analysis we find that combining the sensitivity of the Planck experiment with the completed SDSS redshift survey, the uncertainty on the neutrino first moment \( N_{\text{eff}} \) would be reduced by almost an order of magnitude, but still the neutrino number density will be affected by a large error, see for example the results of Table IV. The sensitivity to non-thermal features in the neutrino distribution highly improves only by considering a survey with the properties of a shear survey like the LSST project, and also the sensitivity of a CMB experiment like CMBPOL. Even in this case, however, it may be quite involved to test the hypothesis of a non thermal neutrino background since, as we stressed in Section IV, a purely thermal neutrino background with extra relativistic degrees of freedom would provide an almost degenerate scenario, with the exception of predicting a different energy density at the time of primordial nucleosynthesis, and a different value for the neutrino mass scale. In the future, evidence for non-thermal
distorsions could possibly result from an apparent contradiction between the values of \( N_{\text{eff}} \) and/or \( m_0 \) probed by cosmological perturbations (assuming standard neutrinos) and those indicated respectively by primordial nucleosynthesis and particle physics experiments. In particular, an independent source of information on \( m_0 \) would greatly help. Direct neutrino mass measurements like the Tritium beta decay experiment KATRIN [54] or experiments on neutrinoless double beta decay [55], which are expected to reach a sensitivity on the neutrino mass scale of the order of \( 0.1-0.3 \) eV, combined with cosmological observations could reduce this degeneracy.

Despite of the fact that by Ockham’s razor principle it is presently worth assuming a purely thermal distribution for relic neutrinos, nevertheless we think that it is remarkable that present knowledge of the C\( \nu \)B features still allows for quite non standard and exotic physics which affected the evolution of the Universe at late stages. As frequently happened in the past, neutrinos can play a role in constraining such possibilities.

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