The Two-Point Angular Correlation Function and BATSE Sky Exposure

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Abstract. The Two-Point Angular Correlation Function is a standard analysis tool used to study angular anisotropies. Since BATSE’s sky exposure (the angular sampling of gamma-ray bursts) is anisotropic, the TPACF should at some point identify anisotropies in BATSE burst catalogs due to sky exposure. The effects of BATSE sky exposure are thus explored here for BATSE 3B and 4B catalogs. Sky-exposure effects are found to be small.

INTRODUCTION

The Two-Point Angular Correlation Function (TPACF), denoted as \( w(\theta) \), is the frequency distribution of angular separations \( \theta \) between celestial objects in the interval \( (\theta, \theta + \delta\theta) \). The TPACF is normalized to zero when integrated over all separation angles. A positive value of \( w(\theta) \) indicates that objects are often found with these separations, zero indicates no correlation, and a negative value indicates anti-correlation. The TPACF was developed for studying galaxy clustering [10], but has since been applied to gamma-ray burst angular distributions [6]. It has been used extensively in this field, primarily for studying repetition and clustering [4,7,8]. The technique has been found to be superior in repetition testing to nearest neighbor analysis [1], although it has not been compared directly to detailed multipole analysis [11]. The TPACF has also been used in burst distance scale studies as an M31 test [3].

Angular sky exposure is important to the modeling of burst angular distributions. The gamma-ray burst angular distribution detected by BATSE is highly isotropic [9]. However, BATSE’s sky exposure is slightly anisotropic, and has been specified via a sky exposure map [2,5]. This anisotropy is due primarily to earth blockage. It causes a small systematic deviation from isotropy that could show up in the gamma-ray burst angular distribution.

In this study, we identify the form of the TPACF taken by BATSE sky exposure in the case of a large (infinite) number of detected bursts. We also search the BATSE 3B and 4B catalogs to determine to what extent this effect is present.
ANALYSIS METHOD

Two methods have been used to calculate the TPACF while including sky exposure effects: (a) Monte Carlo simulations of random burst catalogs are performed using sky exposure as a mask, and (b) summation of isotropic sampled points with the declination-dependent sky exposure are performed.

The first technique requires simulation of gamma-ray burst catalogs, using an assumed isotropic gamma-ray burst angular distribution. The sky exposure function is used as a mask to selectively decide whether or not a simulated burst was detected. Using the detected bursts, the number of distinct burst pairs with different angular separations \((\theta, \theta + \delta \theta)\) is identified, and is used to calculate the TPACF via:

\[
1 + w(\theta) = \frac{2n_p \Omega}{N(N - 1) \langle \delta \Omega \rangle}
\]

where \(n_p\) is the expected number of pairs with separations \(\theta\) to \(\theta + \delta \theta\), and \(\langle \delta \Omega \rangle\) is the mean value of the solid angle subtended by the annulus \(\theta\) to \(\theta + \delta \theta\). This form of the TPACF \(w(\theta)\) is valid when it is estimated from a discrete set of sky objects [10], that is, via a limited number of detected bursts. Statistical variations between Monte Carlo catalogs are removed by numerous Monte Carlo runs using the same number of detected bursts.

The second technique calculates \(w(\theta)\) for a large number of isotropically-sampled points using a weighted sky exposure. Equal sampling is obtained by using a technique for pixelizing the celestial sphere in a smooth and regular way [12]. The total number of observed bursts \(N\) can be described in terms of the sky exposure \(e_i\) at each sampled sky location \(i\) as

\[
N = A \sum_{i=1}^{N} e_i
\]

where \(A\) is a normalization constant. Notice that when all sky regions are completely-sampled \((e_i = 1\) everywhere), then \(A = 1\). Otherwise, \(A > 1\). For each burst detected, \(\frac{1}{e_i}\) bursts should have been detected. Similarly, for each evenly-sampled point on the celestial sphere, bursts should be observed only a fraction of the time \(e_i\). The weighted exposure \(y_i\) is

\[
y_i = \frac{Ne_i}{\sum_{i=1}^{N} e_i} = Ae_i,
\]

and

\[
\sum_{i=1}^{N} y_i = N
\]

Equation (1) is used to calculate the TPACF \(w(\theta)\) for discrete points, with the number of pairs now given as
\[ n_p(\theta) = \sum_{i=1}^{N} \sum_{j=i+1}^{N} y_i y_j = A^2 \sum_{i=1}^{N} e_i \sum_{j=i+1}^{N} e_j \]  

(5)

This technique produces an exact solution in the limit \( N \to \infty \).

A total of 6252 sampling points is used for the equal sampling technique, with the separation between any two neighboring points being \( 1.45^\circ \). This separation is smaller than the \( 1.5^\circ \) BATSE systematic burst localization error and has been chosen because localization uncertainties make it difficult to obtain meaningful data from smaller angular scales.

Some systematic errors in the TPACF are still found using the equal sampling technique. These are due to the discrete placement of pixel separations into \( w(\theta) \) angular bins. Exact corrections can be obtained for the values in each angular bin, since for ideal exposure \( e_i = 1 \) the number of expected pairs per bin can be found from direct integration.

**RESULTS AND DISCUSSION**

Figure 1 demonstrates application of the equal sampling technique to the BATSE 3B and 4B catalogs. Differences between the 3B and 4B exposures are insignificant. The Monte Carlo technique produces similar results when averaged over a large number of Monte Carlo runs, although computational constraints limited us to ten runs per model.

The effects of sky exposure in the TPACF are found to be very small, although they produce a distinct TPACF functional form. Figure 1 exhibits a depletion of burst pairs for \( 60^\circ \leq \theta \leq 120^\circ \), with corresponding pair excesses for \( \theta < 60^\circ \) and \( \theta > 120^\circ \). The pair excesses disappear in the limits of small \( (\theta \to 0^\circ) \) and large \( (\theta \to 180^\circ) \) separation angles. There is a slight enhancement in the peak at \( \theta \approx 30^\circ \) relative to that at \( \theta \approx 90^\circ \).

Enhanced polar to equatorial coverage produces a lower angular density band around the celestial equator along with two higher angular-density polar regions. Bursts form an excess number of pairs on angular scales \( \theta \approx 30^\circ \) and \( \theta \approx 150^\circ \) since both polar and equatorial bursts sample a larger number of their pair partners from the enhanced polar regions on these angular scales. In contrast, the total number of burst pairs near \( \theta \approx 90^\circ \) is depleted because polar region bursts have relatively few neighbors on angular scales of \( \theta \approx 90^\circ \) while equatorial bursts produce some \( \theta \approx 90^\circ \) pairs from polar bursts along with others from the lower-density equatorial region. On small angular scales \( (\theta \approx 0^\circ) \) bursts draw their companions from the surrounding region with a similar angular density; thus the excess of polar burst pairs is balanced overall by the depletion of equatorial burst pairs. The same reasoning applies to large angular scales \( (\theta \approx 180^\circ) \), since bursts also draw their companions from a region of similar density on this angular scale. The enhanced peak at \( \theta \approx 30^\circ \) (relative to that at \( \theta \approx 90^\circ \)) is probably due to better northern hemisphere exposure.
TABLE 1. Effects of sky exposure on the BATSE 3B and 4B catalogs. The reduced $\chi^2$ values shown are obtained by comparing the TPACF ($\nu = 89$ degrees of freedom) for the specified catalog to models accounting for exposure and those not accounting for it (e.g. $w(\theta) = 0$ everywhere). The calculations are performed using both Monte Carlo analysis and the equal sampling technique.

| Technique   | 3B no exposure | 3B exposure | 4B no exposure | 4B exposure |
|-------------|----------------|-------------|----------------|-------------|
| Monte Carlo | 1.12           | 1.03        | 1.47           | 1.30        |
| equal sampling | 0.94        | 1.01        | 1.12           | 1.08        |

When properly normalized, the distribution of distinct pairs found in all angular annuli ($\theta, \theta + \delta \theta$) becomes the TPACF. The number of distinct pairs in each annulus can be used to identify the importance of sky exposure effects on the BATSE catalogs, since the number of bursts is needed to obtain the measurement error. A $\chi^2$ value can be found by comparing the BATSE distinct pair distribution with the number expected (a) in the absence of sky exposure, and (b) in the presence of sky exposure.

The results of this analysis are obtained using the two methods described previously, and are demonstrated in Table 1. The reduced $\chi^2$ values are given for the BATSE 3B and 4B catalogs both when sky exposure effects are not taken into account (e.g. an isotropic distribution) and when these effects are included. Different data binnings have also been used in order to determine their effect on $\chi^2_\nu$. The results do not depend strongly on data binning, whether based on the sizes or on angular offsets of the bins.

No significant improvement in $\chi^2$ is obtained when sky exposure is taken into account, as indicated by $\chi^2_\nu \approx 1$ for all models. This result is not terribly surprising since statistical errors due to burst counting statistics produce errors that are typically 10 to 100 times as large as the maximum TPACF amplitude of the sky exposure (for data binned on $2^\circ$ intervals).

FIGURE 1. Two-Point Angular Correlation Function indicating the extent of BATSE 3B (dotted line) and 4B (solid line) sky exposure using the equal sampling method.
However, other anisotropy tests appear to be capable of detecting sky exposure. For example, dipole and quadrupole moments of exposure are marginally detectable [9]. There are several possible explanations for the apparent inadequacy of the TPACF to statistically detect exposure effects:

- Anisotropies are more statistically-significant when described using the coordinate system for which the anisotropy is largest. Generalized coordinates (such as $\theta$) lose some of this significance by not taking advantage of this a priori knowledge.

- By using only the two-point angular correlation function, rather than all n-point orthogonal functions, we are not making use of all available information. Just as the dipole moment can identify some anisotropies to which the quadrupole moment is insensitive, it may be that the TPACF is less sensitive to sky exposure effects than the higher-order correlation functions. Limiting the analysis to just one component of all correlation functions may also limit the ability of the test to identify BATSE sky exposure in the gamma-ray burst angular distribution.

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