Elliptical flow – a signature for early pressure in ultrarelativistic nucleus-nucleus collisions

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Abstract

Elliptical energy flow patterns in non-central Au(11.7AGeV) on Au reactions have been studied employing the RQMD model. The strength of these azimuthal asymmetries is calculated comparing the results in two different modes of RQMD (mean field and cascade). It is found that the elliptical flow which is readily observable with current experimental detectors may help to distinguish different reasonable expansion scenarios for baryon-dense matter. The final asymmetries are very sensitive to the pressure at maximum compression, because they involve a partial cancelation between early squeeze-out and subsequent flow in the reaction plane. This cancelation can be expected to occur in a broad energy region covered by the current heavy ion fixed-target programs at BNL and at CERN.

A primary goal of current heavy-ion physics utilizing beams at ultrarelativistic energies is the creation and observation of the quark-gluon plasma (QGP), a phase in which quarks and gluons are deconfined and chiral symmetry has been restored. The extraction of flow signatures from experimental data has found considerable interest recently amid present ambiguities concerning the QGP formation. Since collective flows are driven by pressure gradients, their measurement provides a diagnostic tool to study the transient pressure in these reactions. A first-order phase transition is generically associated with the presence of a ‘softest point’ in the equation of state. The tendency of matter to expand on account of its internal pressure is reduced

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in the transition region \[1, 2\]. It has been predicted that a softening of the equation of state can be deduced from experimental measurements, e.g. by observing a minimum in the directed nucleon flow \[3, 4\] or maximum in the life time of the created hot medium \[5\] as a function of beam energy. It should be also kept in mind that there is no guidance from lattice QCD about the physics of the quark-hadron transition at nonzero baryon density which is relevant for today’s heavy ion experiments with beam energies up to 200 AGeV \[6\]. Experimental information on the pressure generated in the collision region is therefore clearly warranted.

The so-called directed transverse flow in the reaction plane and the squeeze-out, a larger flow of nucleons in the central region out-of-plane, have been studied extensively at energies around 1 AGeV \[7\], available at SIS-GSI and before at the BEVALAC. Recently, direct signals of transverse flow have been reported both at AGS and the higher CERN energies. E877 has analysed the azimuthal distributions of transverse energy and charged particle multiplicity using the technique of Fourier decomposition \[8\]. Evidence for a nonvanishing first and second moment of the azimuthal distributions has been found \[9\]. The first moment reflects the directed transverse flow and can be used to determine the reaction plane. A nonvanishing second moment signals the elliptical deformation of the flow tensor. The preliminary E877 indicate that the main flow direction is parallel to the impact parameter, not orthogonal as expected for a squeeze-out. This was predicted by Ollitrault some time ago for collisions at sufficiently high energies \[10\]. NA49 has reported preliminary data on elliptical transverse energy flow patterns for semi-peripheral Pb(158AGeV) on Pb reactions, too \[11\]. Although NA49 is not able to determine a reaction plane a clear signal above statistical fluctuations has been found by correlating the preferred directions of emission in neighboring pseudo-rapidity windows.

In this letter, I focus on elliptical flow in the central (pseudo-)rapidity region in order to extract information on the pressure in the collision zone. The highest energy and baryon densities are achieved in the central region for present experiments. Particles in the central region are presumably less affected by nonequilibrium physics. In contrast, the directed flow results mostly from a transverse momentum anticorrelation between baryons near to original projectile and target rapidity. Therefore it receives a strong pree-
Measurement of elliptical flow patterns provides a unique opportunity to study the pressure which is generated very early in the reaction. The basic idea presented here is very simple. At low beam energy (around 1AGeV), matter escapes preferentially orthogonal to the reaction plane which is spanned by the beam axis and the impact parameter. The spectator nucleons block the path of participant hadrons which try to escape from the collision zone. This is the observed squeeze-out effect. At ultrahigh energies, a larger flow of participants in the reaction plane can be expected and is confirmed by RQMD calculations for RHIC energy [12]. Since the passage time of spectator nucleons from projectile and target shrinks with the Lorentz factor gamma, produced particles do not interact with spectators. The almond-shaped geometry of the overlap region in the reaction plane clearly favors preferential emission parallel to the impact parameter. An interesting situation emerges for collision energies between the low and the ultrahigh energies, basically the whole energy region covered by present fixed target experiments at BNL-AGS and CERN-SPS (2-200AGeV). Taking collisions of equal mass nuclei moving with the speed of light, the passage time of projectile and target spectators is approximately given by $2R_A/(\gamma c)$ (numerically 5.4 fm/c at 12AGeV and 1.4 fm/c at 160AGeV for the heaviest systems). Such time scale neither covers the whole reaction time nor becomes irrelevant at these intermediate energies. As a consequence, the centrally produced matter is initially squeezed-out orthogonal to the reaction plane. After the spectator material has disappeared, the ‘confining’ spectator walls suddenly have vanished. Lateron, the geometry of the central region favors central flow parallel to the impact parameter vector. The orientation of the final azimuthal asymmetry in particle, momentum and energy flow is chiefly determined by the relative strength of the pressures during the initial passage as compared to the later expansion time. The full power of this analysis becomes apparent if the measurement of elliptical flow patterns is combined with measurements of the average flow in transverse directions, the so-called radial flow. While the elliptical flow is influenced by the difference between early and late pressures the average transverse flow reflects the time integral of these pressures. It is the main virtue of such kind of analysis to shed some light on the early pressure which is practically unknown for baryon-dense matter. The later
expansion stage characterized by baryon densities around 0-2 times ground state density is clearly more constrained from known nuclear physics and ongoing heavy ion studies at BEVALAC-SIS energies.

In the following, I am going to employ a transport model, relativistic quantum molecular dynamics (RQMD) [13], to calculate the azimuthal asymmetries in the energy deposition for Au(11.7AGeV) on Au collisions in non-central collisions. I shall discuss whether the predicted partial cancelation of in-plane and out-of-plane emission can be utilized to quantitatively distinguish different dynamical evolution scenarios for highly compressed baryon-rich matter. RQMD is constructed as a Monte-Carlo code which generates complete events under prescribed conditions (masses of the colliding nuclei, impact parameter, beam energy). RQMD is based on string and resonance excitations in the primary collisions of nucleons from target and projectile. Overlapping color strings may fuse into so-called ropes. Subsequently, the fragmentation products from rope, string and resonance decays interact with each other and the original nucleons, mostly via binary collisions. These interactions drive the system towards equilibration [14] and are responsible that collective flow develops, even in the preequilibrium stage. RQMD contains some option which allows to vary the pressure in the high-density state and to study its influence on final-state observables. Baryons acquire effective masses and thus may experience forces if they are surrounded by other baryons [15]. The effective masses are generated by introducing Lorentz-invariant quasi-potentials into the mass-shell constraints for the momenta which simulate the effect of ‘mean fields’. There are no potential-type interactions in the so-called cascade mode of RQMD. In this mode, the equilibrium pressure is simply an ideal gas of hadrons and resonances which are explicitly propagated and therefore contribute to the pressure. The resulting equation of state in the cascade mode of RQMD is very similar to the one calculated and plotted by the Bern group in Ref. [16], because the spectrum of included resonance states is nearly the same.

It should be noted that propagating strings modify the equation of state as well. This correction is small, however, in equilibrium at relevant temperatures around 150 MeV. Although it is not realized in RQMD the collision term in the equations of motion may contribute to the equilibrium pressure, in principle. For instance, repulsive trajectories are selected for colliding baryons with some probability in new versions of the ARC model [17]. In Refs. [18, 19] it was concluded that a cascade lacks some pressure in comparison to
RQMD results for ultrarelativistic nucleus-nucleus collisions have been compared to measurements by most major experimental collaborations [3], showing usually reasonable or good agreement. Various experimental data—e.g. directed and total transverse momenta [3, 20]—which have been taken in the AGS energy region around 12 AGeV seem to hint that the generated pressure in RQMD is too ‘soft’ if it is used in its cascade mode. Therefore we compare here the result obtained in the cascade mode with a calculation in which the quasi-potentials generate additional pressure due to repulsion at baryon densities larger than ground state density. Potential parameters have been selected for RQMD (version 2.3) which bring the generated transverse momenta in agreement with available data similarly as it was done in [18].

Since a first order phase transition from a resonance gas into a QGP would ‘soften’ the equation of state, its inclusion in RQMD would act in the opposite direction to repulsive mean fields. It is worthwhile to study a softening as a possible consequence of the hadron-quark transition in more detail in the RQMD framework.

In the following, the question will be addressed how sensitive the azimuthal asymmetry of energy flow in the central rapidity region is to the pressure in the early and in the late stage. The terms ‘early’ and ‘late’ are defined with respect to the passage time of target and projectile spectators. For this purpose, I have analysed the evolution of the pressure in a particular reaction Au(11.7AGeV) on Au at an impact parameter of 6 fm. The equations of motion contain a contribution from quasi-potentials. This has to be taken into account in the definition of the non-equilibrium pressure. The virial theorem has been applied to define the pressure in each of the three space directions via the equation

\[ P^i \cdot V = \left\langle \sum_M \left( p^i(M)v^i(M) + \sum_N F^i_{MN} r^i(M) \right) \right\rangle \quad i = 1, 2, 3 \quad . \tag{1} \]

\[ \mathbf{p}, \mathbf{v} \] denote the hadron’s momentum and velocity, \[ \mathbf{F}_{MN} \] the force which baryon \( N \) exerizes on \( M \). The summations over \( N \), respectively \( M \), include only hadrons inside a cylindrical volume \( V = 2\pi(xR_A)^2/\gamma \) centered at the origin (with \( x=0.3 \)). Ingoing nucleons which have not collided yet are not

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Experimental data for AA collisions at 10-15 AGeV. This result was seemingly contradicted by the findings in [17], based on ARC calculations. However, the latter calculations contain some nonideal gas pressure contributions from the repulsive trajectory prescription.
included in the evaluation of the pressure. The rhs of eq. (1) has been evaluated for approximately 400 events in the mean field mode of RQMD. An event average has been taken (indicated by the $\langle \rangle$ symbols). Analogously, local energy and baryon densities ($e$ and $\rho_B$) have been calculated. One should keep in mind that the system is initially in non-equilibrium. This means that the pressure along the beam axis is the largest at this stage. Since no nonstatistical difference between the two transverse pressure components was found, their average was taken and is dubbed ‘pressure’ in the following. The CMS time evolution of pressure, local energy and baryon density can be seen from Fig. [4]. The upper part of Fig. [4] shows the evolution in the $e$-$p$ plane, the lower part in the $e$-$\rho_B$ plane. The symbols represent the values of these quantities, with time increasing in steps of 1 fm/c each, starting 1 fm/c after the two nuclei have touched each other. It becomes apparent from this figure that the passage time is practically identical with the time of maximum compression, either of energy or baryon number. The energy and baryon density at maximum compression are approximately $1.3$ GeV/fm$^3$ and $3.5 \rho_0$, respectively. It is noteworthy that these values are close to the region for which a phase transition into a QGP is usually expected. The pressure in the expansion stage is somewhat larger than in the compression stage. This reflects the presence of a strong pre-equilibrium component which tends to soften the transverse pressure. (Remember that longitudinal momentum acts like a mass term with respect to velocity in transverse directions.) In Fig. [4] the contribution from the kinetic part of the pressure (the first term on the rhs of eq. (1)) is also shown (open symbols). Roughly, the kinetic and the potential part contribute equally at the time of maximum compression.

The azimuthal asymmetry in the energy flow can be quantified by defining the following variable:

$$E_{dir} = \sum_M E(M) \cdot \text{sgn}(\phi). \quad (2)$$

The summation over $M$ includes hadrons only within some rapidity cut around center-of-mass rapidity, set here arbitrarily to $\pm 0.7$. $\phi$ is defined as the angle of a hadron’s momentum with respect to the impact parameter vector. sgn($\phi$) is defined to be +1 in the cones with opening angle of 45° around $\phi = 0$ and 180°, -1 elsewhere. Fig. [2] displays the time evolution of $E_{dir}$ in the CMS. The time dependence of $E_{dir}$ shows the behaviour as ex-
pected from the qualitative discussion above. In the time span right before maximum compression, $E_{\text{dir}}$ acquires negative values (squeeze-out), because the pressure mostly from the repulsive potentials pushes the hadrons against the confining spectator material in the reaction plane and into the vacuum orthogonal to it. After the spectators are gone ($t > 5 \text{ fm}$) hadrons are pushed preferentially (anti-)parallel to the impact parameter vector. In course, $E_{\text{dir}}$ gets positive contributions with increasing time. Finally, the ‘in-plane flow’ effect dominates for this reaction. Therefore the major energy flow axis is parallel to $\vec{b}$. For comparison, Fig. 2 shows the evolution of the same quantity, but calculated in the RQMD cascade mode. Due to the preequilibrium effect, the effective transverse equation of state is ultrasoft. There is no visible squeeze-out present at the early times. The pressure at later times is smaller than in the mean field mode. However, the final azimuthal asymmetry expressed by its $E_{\text{dir}}$ value is approximately 60 percent larger, because the initial squeeze-out is absent. This is also visible from Fig. 3 in which the full $\phi$ dependence of the energy flow is shown, calculated with RQMD in the two different modes. I conclude that the azimuthal asymmetries are a tool of utmost importance to gain information on the pressure in the high density stage. A factor 2 effect in the early pressure is not washed out in the later evolution but shows up with practically undiluted strength in the different azimuthal asymmetries which are generated with and without mean field-type interactions.

In addition, Fig. 2 shows the evolution of the average nucleon transverse momentum (in the same rapidity window as for $E_{\text{dir}}$). Again, the results obtained in the mean field mode are compared to those of the cascade mode. The mean field effect is noticeably smaller for the inclusive single-particle observable $p_{tr}$ (10 percent). Similar observations have been made based on calculations with the transport code ART comparing the sensitivity of directed and total transverse momenta to mean fields [21]. The transverse momentum measurements favor the inclusion of repulsive potentials in RQMD. On the other side, those are most relevant at time of maximum compression. Thus they necessarily weaken the observable azimuthal asymmetry (cf. Figs. 2 and 3). This RQMD prediction is readily testable using the present experimental set-up of the E877 group. We take the qualitative agreement with preliminary E877 data which show major flow parallel to the impact parameter as
an encouraging sign. The model seems to be sensible enough to address the question of the centrally produced pressure quantitatively.

Summarizing, I have analysed calculations which were done with the RQMD model for the reaction Au(11.7AGeV) on Au at impact parameter 6 fm. Azimuthal asymmetries in the energy flow have been studied in relation to the transient pressures in the central collision zone. Employing repulsive mean fields, the elliptical flow is initially oriented orthogonal to the impact parameter and acquires lateron parallel contributions. While the total pressure integral can be constrained if not determined by other means, the azimuthal asymmetry is sensitive to the difference of the early and late pressures. I believe that this effect is present in a wide range of beam energies covered by today’s fixed target experiments. Although I have studied here only one particular reaction, the most important findings will hold true very generally. I suppose that the effect can be qualitatively reproduced by other – hydrodynamical or transport – approaches. This means in turn that the elliptical flow provides an extremely useful tool in a joint effort of experimentalists (e.g. E877 and E895 [22]) and theorists to gain quantitative information on the early pressure in the most dense – the central – collision region. Such a tool has been lacking so far, because the directed flow is concentrated in the projectile and target fragmentation regions. The partial cancelation of the azimuthal asymmetry in the flow of energy (and momentum and presumably particles) with time opens up possibilities for a rich experimental and theoretical research program. It is obvious from the presented calculations that the transition point at which the squeeze-out disappears and is replaced by an in-plane flow will strongly depend on the strength of the early pressure. Non-central asymmetric collisions and variations of centrality triggers could provide much more detailed and additional information. As it is customary in the studies of the directed flow one can impose cuts on momenta or on particle species (e.g., kaons versus protons) in order to (de-)emphasize the pressure species (e.g., kaons versus protons) in order to (de-)emphasize the pressure contribution from the early high density stage. In particular, it should be possible to considerably narrow down the pressure in matter with baryon densities of several times ground state density for which only educated guesses exist so far.

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Figure Captions:

Figure 1:
CMS time evolution of transverse pressure $p$, local energy and baryon density ($e$, respectively $\rho_B/\rho_0$) in the collision center for the reaction Au(11.7AGeV) on Au at impact parameter 6 fm. The results were obtained using the RQMD model (version 2.3) in mean field mode. The upper part shows the evolution in the $e$-$p$ plane, the lower part in the $e$-$\rho_B$ plane. The symbols represent the values of these quantities, with time increasing in steps of 1 fm/c each, starting 1 fm/c after the two nuclei have touched each other. The time direction is indicated by an arrow. The contribution from the kinetic part of the pressure alone is also displayed (open circles).

Figure 2:
CMS time evolution of the average transverse momentum of nucleons $p_{tr}$ (upper part of figure) and $E_{dir}$ (bottom). The variable $E_{dir}$ is defined in eq. (2) and is related to the direction of energy flow with respect to the impact parameter vector. Positive $E_{dir}$ values correspond to major in-plane flow and negative values to out-of-plane flow (squeeze-out). The results were obtained for the same system as in Fig. 1. Closed (open) symbols refer to the calculation in RQMD mean-field (cascade) mode. A rapidity cut ±0.7 around center-of-mass rapidity was imposed on hadrons which are included here.

Figure 3:
Differential energy flow distribution $dE/d\phi$ as a function of the angle with respect to the impact parameter vector $\phi$. The results were obtained for the same system as in Figs. 1 and 2 and the same acceptance cuts as in Fig. 2. Straight (dashed) line histogram refers to the calculation in RQMD mean-field (cascade) mode.
Figure 1:
Figure 2:
Figure 3:

Au(11.7AGeV) + Au, b=6 fm

$y_{\text{mid}} \pm 0.7$

$\frac{dE}{d\phi}$ (GeV/deg.)

$\phi$ (deg.)

RQMD 2.3