Possible Molecular States of $D_s^*\bar{D}_s^*$ System and $Y(4140)$

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The interpretation of $Y(4140)$ as a $D_s^*\bar{D}_s^*$ molecule is studied dynamically in the one boson exchange approach, where $\sigma$, $\eta$ and $\phi$ exchange are included. Ten allowed $D_s^*\bar{D}_s^*$ states with low spin parity are considered, we find that the $J^{PC} = 0^{++}$, $1^{++}$, $0^{-+}$, $2^{++}$ and $1^{-+}$ $D_s^*\bar{D}_s^*$ configurations are most tightly bound. We suggest the most favorable quantum numbers are $J^{PC} = 0^{++}$ for $Y(4140)$ as a $D_s^*\bar{D}_s^*$ molecule, however, $J^{PC} = 0^{-+}$ and $2^{++}$ can not be excluded. We propose to search for the $1^{-+}$ and $1^{--}$ partners in the $J/\psi\phi$ and $J/\psi\eta$ final states, which is an important test of the molecular hypothesis of $Y(4140)$ and the reasonability of our model. The $0^{++} B_s^*B_s^*$ molecule is deeply bound, experimental search in the $Y(1S)\phi$ channel at Tevatron and LHC is suggested.

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I. INTRODUCTION

Recently the CDF Collaboration reported a narrow structure $Y(4140)$ in the decay $B^+ \to K^+ Y(4140)$ followed by $Y(4140) \to J/\psi\phi$ with a statistical significance of $3.8 \sigma$. The mass and the width of this structure are fitted to be $4143.0 \pm 2.9$ (stat) $\pm 1.2$ (syst) MeV and $11.7^{+3.3}_{-5.0}$ (stat) $\pm 3.7$ (syst) MeV respectively [1,2]. Since the quantum numbers of both $J/\psi$ and $\phi$ are $I^G(J^{PC}) = 0^-(1^{--})$, $Y(4140)$ is an isospin singlet with positive $C$ parity.

$Y(4140)$ is very similar to the charmonium like state $Y(3940)$, which is observed in $B \to KY(3940)$ $\to KJ/\psi\omega$ [3,4]. It was argued by the CDF Collaboration that $Y(4140)$ can not be a conventional charmonium state, because a $c\bar{c}$ charmonium state with mass about 4143 MeV would dominantly decay into open charm pairs, and the branch ratio into the double OZI forbidden modes $J/\psi\phi$ or $J/\psi\omega$ is negligible. Although the decay $Y(4140) \to J/\psi\phi$ is unusual, the decay $\chi_{b1,2}(2P) \to \omega Y(1S)$ has been observed [5]. Therefore $Y(4140)$ as a conventional $c\bar{c}$ state can not be ruled out completely due to the scarcity of current experiment data. Comparing with the theoretical predictions for the charmonium spectrum [6,7], we suggest that $Y(4140)$ would most likely to be the $2^1D_2$ state with $I^G(J^{PC}) = 0^+(2^{--})$, if it is a $c\bar{c}$ charmonium state. It would dominantly decay into $DD^*$, $D^*D^*$ and $D_sD_s^*$ in this scenario. A possible explanation for the unusual decay mode $Y(4140) \to J/\psi\phi$ is that the rescattering through $D_sD_s^*/D_s^*D_s$ or $D_s^*D_s^*$ may be responsible, i.e. $Y(4140) \to D_sD_s^*(D_s^*D_s$ or $D_s^*D_s^*) \to J/\psi\phi$. Mixing between the charmonium state and the $D_s^*\bar{D}_s^*$ molecule may also contribute to this extraordinary decay.

There are already some theoretical discussions about this structure. The authors in Ref. [8] suggested that $Y(4140)$ is a molecular partner of $Y(3940)$, and that it is a $J^{PC} = 0^{++}$ or $2^{++}$ $D_s^*\bar{D}_s^*$ molecule. In Ref. [9], the author argued that $Y(4140)$ is a $D_s^*\bar{D}_s^*$ molecule or a exotic $1^{--}$ charmonium hybrid. The interpretation of $Y(4140)$ as a $0^{++}$ $D_s^*\bar{D}_s^*$ molecule has been studied by QCD sum rules [10,11], however, different conclusions were reached. The decay of $Y(4140)$ as a hadronic molecule or conventional charmonium $\chi_{c0,1}$ were discussed as well [12,13].

In the past years, some new states near threshold have been observed, and they are usually suggested to be hadronic molecules [14,15,16,17,18,19,20]. Being different from familiar hadronic molecule candidates, $Y(4140)$ is about 81.6 MeV below the $D_s^*\bar{D}_s^*$ threshold, its binding energy is not small if it is identified as a $D_s^*\bar{D}_s^*$ molecule. There are various theoretical methods to study the hadronic molecule dynamically, such as the QCD sum rule, the QCD effective field theory [20,21], the potential models with pairwise interactions between quarks [16,17] and so on. Tornqvist’s original work on hadronic molecule from one pion exchange is especially attractive [22,23]. Guided by the binding of deuterons, he performed a systematic study of possible deuteronlike two mesons bound states with long distance one pion exchange. Being different from the above mentioned theoretical methods, he took into account the contribution of the tensor force. Since the tensor force turns out to be very important in the deuteron binding, one expects that it would make a significant contribution to the binding of the general hadronic molecule. At short distance, the interaction should be induced by the interactions among the quarks in principle. However, a detailed and reliable modeling of the short range interaction is not an easy matter, and various phenomenological models have been proposed [24,25], although one pion exchange is expected to be dominant for the hadronic molecule. Inspired by the nucleon-nucleon interactions, we further extended the one pion exchange model to represent the short distance contributions by the heavier mesons $\eta$, $\sigma$, $\rho$ and $\omega$ exchanges in Ref. [26]. From these studies, we learn that one pion exchange really dominates the physics of hadronic molecule, and the tensor forces indeed plays a critical role in
providing the binding, which results in the S-D wave mixing. Since in general the molecular state is weakly bound, the separation between the two hadrons in the molecule should be large. Consequently the dominance of one pion exchange can be easily understood. A remarkable success of the model is its prediction for the existence of the famous X(3872) long ago [18, 23]. Based on one pion exchange, Frank Close et al. recently suggested that the charmed mesons molecular state is a possible solution to the enigmatic states observed above 4 GeV in $e^+e^-$ annihilation [27]. Therefore we think that the one boson exchange model can give a reasonable and reliable description for the hadronic molecule. In this work, we shall study the possible molecular states of the $D_s^*D_s^*$ system and discuss the most favorable quantum numbers of $Y(4140)$ as a $D_s^*D_s^*$ molecule in the one boson exchange model. Ten allowed $D_s^*D_s^*$ states with total angular momentum smaller than 3 will be discussed.

The paper is organized as follows. In section II, the effective potentials of the ten $D_s^*D_s^*$ molecular states are given. The numerical results and the possible $D_s^*D_s^*$ molecules are presented in section III. Section IV discusses the most favorable quantum numbers of $Y(4140)$ and the implications, if it is interpreted as a $D_s^*D_s^*$ molecule. Finally we give our conclusions and some discussions in section V.

II. THE EFFECTIVE POTENTIALS FOR THE $D_s^*D_s^*$ STATES WITH LOW SPIN PARITY

In the one boson exchange model [22, 23, 26], the effective potentials between two hadrons are obtained by summing the interactions between light quarks or antiquarks via one boson exchange, and one boson exchange only between up quarks or down quarks has been considered. We note that the present one boson exchange is different from the well-known Goldstone boson exchange constituent quark model [28], where one boson exchange between quarks inside a hadron is used to describe the light baryon spectrum. Since strange quark is involved in the present problem, we should extend our consideration to the one boson exchange between strange quarks. Because $D_s^*$ is an isospin singlet, the effective potential between $D_s^*$ and $D_s^*$ is induced by $\eta$, $\sigma$ and $\phi$ exchanges, whereas the $\pi$, $\rho$ and $\omega$ exchanges don’t contribute. Taking into account the overall sign difference $(-1)^G$ between the quark-quark interactions and the quark-antiquark interactions, where $G$ is the $G$-parity of the exchanged meson, we obtain the effective potential

$$V(r) = V_\eta(r) + V_\sigma(r) - V_\phi(r) \equiv V_C(r) + V_S(r)\sigma_i \cdot \sigma_j + V_T(r)S_{ij}(\vec{r}) + V_{L_S}(r)L \cdot S_{ij}$$

where $V_\eta(r)$, $V_\sigma(r)$ and $V_\phi(r)$ respectively denote the effective potentials from $\eta$, $\sigma$ and $\phi$ exchange between two quarks, and their expressions are given explicitly in Eq.(9)-Eq.(11) of Ref. [26]. Noting that the $\phi$ exchange distinguishes the $D_s^*D_s^*$ from the exotic $D_s^*D_s^*$ systems. The subscript $i$ and $j$ are the indexes of the strange quark and strange antiquark. $S_{ij}(\vec{r}) = 3(\sigma_i \cdot \vec{r})(\sigma_j \cdot \vec{r}) - \sigma_i \cdot \sigma_j$ is the well-known tensor operator, $S_{ij} = \frac{1}{2}(\sigma_i + \sigma_j)$ is the total spin of light quarks, and $L$ is the relative angular momentum operator. Comparing with the general results in Ref. [26], the isospin dependent interactions vanish here. We would like to note that the spin operator $\sigma_i$ or $\sigma_j$ only acts...
on the strange quark or antiquark here. The potentials \( V_C(r) \), \( V_S(r) \), \( V_T(r) \) and \( V_{LS}(r) \) are given as follows:

\[
\begin{align*}
V_C(r) & = -\frac{g_{\text{ass}}^2}{4\pi} m_\sigma H_0(\Lambda, m_\sigma, r) - \frac{g_{\text{ass}}^2}{4\pi} \frac{m^2}{8m_s^2} H_1(\Lambda, m_\sigma, r) - \frac{g_{\text{ass}}^2}{4\pi} m_\phi H_0(\Lambda, m_\phi, r) + \frac{g_{\text{ass}}^2 + 4g_{\phi ss}f_{\phi ss}}{4\pi} \frac{m^2}{8m_s^2} H_1(\Lambda, m_\phi, r) \\
V_S(r) & = -\frac{g_{\text{ass}}^2}{4\pi} \frac{m^3}{12m_s^2} H_3(\Lambda, m_\sigma, r) + \frac{(g_{\phi ss} + f_{\phi ss})^2}{6m_s^2} H_1(\Lambda, m_\phi, r) \\
V_T(r) & = \frac{g_{\text{ass}}^2}{4\pi} \frac{m^3}{12m_s^2} H_3(\Lambda, m_\sigma, r) + \frac{g_{\phi ss}^2 + 2g_{\phi ss}f_{\phi ss}}{4\pi} \frac{m^3}{12m_s^2} H_3(\Lambda, m_\phi, r) \\
V_{LS}(r) & = -\frac{g_{\text{ass}}^2}{4\pi} \frac{m^3}{2m_s^2} H_2(\Lambda, m_\sigma, r) + \frac{3g_{\text{ass}}^2 + 4g_{\phi ss}f_{\phi ss}}{4\pi} \frac{m^3}{2m_s^2} H_2(\Lambda, m_\phi, r)
\end{align*}
\]

where \( m_s \) is the mass of the constituent strange quark, and we take \( m_s = 0.55 \text{ GeV} \) in this work. \( m_\sigma, m_\sigma \) and \( m_\phi \) are the masses of \( \eta, \sigma \) and \( \phi \) mesons respectively. \( g_{\text{ass}}(M = \eta, \sigma \text{ and } \phi) \) and \( f_{\phi ss} \) denote the effective coupling constants between the exchanged meson and the strange quarks. In the above formulae, we have introduced form factor at each interaction vertex, and the form factor in momentum space is taken as

\[
F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}
\]

where \( \Lambda \) is the so-called regularization parameter, \( m \) and \( q \) are the mass and the four momentum of the exchanged boson respectively. The form factor suppresses the contribution of high momentum, i.e. small distance. The presence of such a form factor is dictated by the extended structure of the hadrons. The parameter \( \Lambda \), which governs the range of suppression, can be directly related to the hadron size which is approximately proportional to \( 1/\Lambda \). However, since the question of hadron size is still very much open, the value of \( \Lambda \) is poorly known phenomenologically, and it is dependent on the models and applications. In the nucleon-nucleon interactions, the \( \Lambda \) in the range of 0.8-1.5 GeV has been used to fit the data. The extended structure of hadrons also has the following obvious consequence: because the mass of the exchanged meson determines the range of the corresponding contribution to the \( D_s^*D_s^* \) interactions, one should restrict oneself to meson exchange with the exchanged meson mass below a certain value, typically on the order of the regularization parameter \( \Lambda \). The \( \eta, \sigma \) and \( \phi \) exchange are considered in the present work, therefore the value of \( \Lambda \) should be at least larger than the \( \phi \) meson mass. The functions \( H_0(\Lambda, m, r), H_1(\Lambda, m, r), H_2(\Lambda, m, r) \) and \( H_3(\Lambda, m, r) \) involved in Eq. (2) are given by

\[
\begin{align*}
H_0(\Lambda, m, r) & = \frac{1}{mr} (e^{-mr} - e^{-\Lambda r}) - \frac{\Lambda^2 - m^2}{2mA} e^{-\Lambda r} \\
H_1(\Lambda, m, r) & = \frac{1}{mr} (e^{-mr} - e^{-\Lambda r}) + \frac{\Lambda(\Lambda^2 - m^2)}{2m^3} e^{-\Lambda r} \\
H_2(\Lambda, m, r) & = (1 + \frac{1}{mr}) \frac{1}{m^2} e^{-mr} - (1 + \frac{1}{m^2}) \frac{\Lambda}{m^2} e^{-\Lambda r} - \frac{\Lambda^2 - m^2}{2m^2} (1 + \frac{3}{m^2} \frac{\Lambda}{m^2} \frac{e^{-\Lambda r}}{mr}) \\
H_3(\Lambda, m, r) & = (1 + \frac{3}{mr} + \frac{3}{m^2} \frac{e^{-mr}}{m^2}) \frac{\Lambda^2}{m^2} e^{-\Lambda r} - \frac{\Lambda^2 - m^2}{2m^2} (1 + \frac{3}{m^2} \frac{\Lambda}{m^2} \frac{e^{-\Lambda r}}{mr})
\end{align*}
\]

| States | \( J^P \) |
|--------|----------|
| \( \eta^0 \) | \( ^1S_0 \) |
| \( \eta^+ \) | \( ^3S_1 \) |
| \( \eta^- \) | \( ^3S_1 \) |
| \( ^1P_1 \) | \( ^3P_1 \) |
| \( ^3P_1 \) | \( ^3P_1 \) |
| \( ^1F_1 \) | \( ^3F_1 \) |
| \( ^3F_1 \) | \( ^3F_1 \) |
| \( ^1D_2 \) | \( ^5D_2 \) |
| \( ^3D_2 \) | \( ^5D_2 \) |
| \( ^1F_2 \) | \( ^5F_2 \) |
| \( ^3F_2 \) | \( ^5F_2 \) |

**TABLE I:** The allowed states of the \( D_s^*D_s^* \) system with low spin parity, where we cut off the total angular momentum to \( J = 2 \).
The four components of the interaction potentials $V_C(r)$, $V_S(r)$, $V_T(r)$ and $V_{LS}(r)$ are displayed in Fig. [1]. We see that the central potential $V_C(r)$ is negative, whereas the remaining three components are positive. These potentials are the cancellation results of different contributions, especially for the tensor potential $V_T(r)$, whose amplitude is smaller than that of the other three potentials. For the $D^*\bar{D}^*$ system, the spatial parity is determined by $P = (-1)^L$ and the $C$-parity is $C = (-1)^{L+S}$, where $L$ is the relative angular momentum between $D^*_1$ and $D^*_3$, and $S$ is the total spin of the system. We cut off the total angular momentum to $J = 2$, the allowed states with low spin parity are listed in Table [II]. The matrix elements of operators $\sigma_i \cdot \sigma_j$, $S_{ij}(\hat{r})$ and $L \cdot S_{ij}$ can be calculated straightforwardly with the help of angular momentum algebra, and the results are given analytically in the appendix of Ref. [26]. Consequently we can write out the one boson exchange potentials for the allowed states in matrix form as follows

$$V_{0++}(r) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_C(r) + \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_S(r) + \begin{pmatrix} 0 & 0 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\sqrt{2} & -\frac{\sqrt{6}}{2} \\ -\frac{\sqrt{6}}{2} & \sqrt{2} & 0 \end{pmatrix} V_T(r) + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -4 \end{pmatrix} V_{LS}(r) \tag{5}$$

$$V_{1+-}(r) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_C(r) + \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_S(r) + \begin{pmatrix} 0 & \sqrt{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\sqrt{2} & 0 \\ -\frac{\sqrt{6}}{2} & \sqrt{2} & \frac{\sqrt{6}}{2} \end{pmatrix} V_T(r) + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -4 \end{pmatrix} V_{LS}(r) \tag{6}$$

$$V_{0-+}(r) = V_C(r) - V_S(r) - 2V_T(r) - V_{LS}(r) \tag{7}$$

$$V_{1++}(r) = V_C(r) + V_S(r) - V_T(r) - \frac{5}{2} V_{LS}(r) \tag{8}$$

$$V_{1-+}(r) = V_C(r) - V_S(r) + V_T(r) - \frac{1}{2} V_{LS}(r) \tag{9}$$

$$V_{2+-}(r) = V_C(r) - V_S(r) + V_T(r) - \frac{1}{2} V_{LS}(r) \tag{10}$$

$$V_{1--}(r) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_C(r) + \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_S(r) + \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{6}}{2} \\ \frac{\sqrt{3}}{2} & -\sqrt{2} & \frac{\sqrt{6}}{2} \\ -\frac{\sqrt{6}}{2} & \sqrt{2} & 0 \end{pmatrix} V_T(r) + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{pmatrix} V_{LS}(r) \tag{11}$$

$$V_{2++}(r) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_C(r) + \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_S(r) + \begin{pmatrix} 0 & -\frac{\sqrt{6}}{2} & \frac{\sqrt{6}}{2} \\ -\frac{\sqrt{6}}{2} & 0 & 0 \\ \frac{\sqrt{6}}{2} & 0 & -\frac{\sqrt{6}}{2} \end{pmatrix} V_T(r) + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} V_{LS}(r) \tag{12}$$

$$V_{2-+}(r) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} V_C(r) + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} V_S(r) + \begin{pmatrix} 0 & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} V_T(r) + \begin{pmatrix} 0 & 0 \\ 0 & -4 \end{pmatrix} V_{LS}(r) \tag{13}$$

$$V_{2--}(r) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} V_C(r) + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} V_S(r) + \begin{pmatrix} 0 & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} V_T(r) + \begin{pmatrix} 0 & 0 \\ 0 & -4 \end{pmatrix} V_{LS}(r) \tag{14}$$

In the following, we will perform the same analysis as for the deuteron and the possible heavy flavor molecules in Ref. [26]. One can then determine for which quantum numbers the one boson exchange potential is attractive and strong enough so that $D^*_1\bar{D}^*_3$ bound states are expected.
III. NUMERICAL RESULTS AND POSSIBLE $D_s^*D_s^*$ MOLECULES

The input parameters in our model are the involved meson masses, the effective coupling constants $g_{\text{M}ss}(\mathcal{M} = \eta, \sigma$ and $\phi)$ and $f_{\phi ss}$, and the regularization parameter $\Lambda$. The meson masses are taken from the compilation of the Particle Data Group [23]: $\eta = 547.853$ MeV, $\eta_\sigma = 600$ MeV, $\phi = 1019.456$ MeV and $m_{D^*_s} = 2112.3$ MeV. In the SU(3) flavor symmetry limit, the coupling constants $g_{\text{M}qq}(\mathcal{M} = \eta, \sigma$ and $\phi)$ and $f_{\phi qq}$ between the exchanged mesons and strange quarks are related to coupling constants $g_{\text{M}qq}(\mathcal{M} = \eta, \sigma$ and $\phi)$ and $f_{\phi qq}$ between the exchanged mesons and up/down quarks via the following relations

$$g_{\text{ps}ss} = -2g_{\text{ps}qq}, \quad g_{\text{ss}ss} = g_{\sigma qq}, \quad g_{\text{oss}ss} = \sqrt{2}g_{\omega qq}, \quad f_{\phi ss} = \sqrt{2}f_{\omega qq}$$

The coupling constants $g_{\text{M}qq}(\mathcal{M} = \eta, \sigma$ and $\omega)$ and $f_{\omega qq}$ can be estimated from the phenomenologically known $\eta NN, \sigma NN$ and $\omega NN$ coupling constants. Riska and Brown have explicitly demonstrated that the nucleon resonance transition couplings to $\pi, \rho$ and $\omega$ can be derived in the single quark operator approximation [30], which are in good agreement with the experimental data. Adopting the same method, we can straightforwardly derive the following relations between the meson-quark couplings and the meson-nucleon couplings [26, 30],

$$g_{\text{ps}qq} = \frac{m_q}{m_N}g_{\eta NN}, \quad g_{\text{ss}qq} = \frac{1}{3}g_{\sigma NN}, \quad g_{\text{oss}qq} = \frac{1}{3}g_{\omega NN}, \quad f_{\omega qq} = \frac{m_q}{m_N}f_{\omega NN} - \left(\frac{1}{3} - \frac{m_q}{m_N}\right)g_{\omega NN}$$

where $m_N$ is the nucleon mass. In this work, the effective couplings between the exchanged bosons and the nucleons are taken from from the well-known Bonn model [31]: $g_{\eta NN}^{\sqrt{2}/4\pi} = 3.0, g_{\omega NN}^{\sqrt{2}/4\pi} = 7.78, g_{\sigma NN}^{\sqrt{2}/4\pi} = 20.0$ and $f_{\omega NN}^{\sqrt{2}/4\pi} = 0$. In nature and in QCD, the flavor SU(3) symmetry is broken by non-equal masses of the up and down quarks and the strange quark or the electromagnetic effects. It is commonly believed that the error of SU(3) predictions is approximately 20% – 30%. Consequently the uncertainty of the coupling constants $g_{\text{ps}ss}, g_{\sigma ss}, g_{\omega ss}$ and $f_{\phi ss}$ is about 20% – 30% as well. As a demonstration for the consequence induced by the uncertainties of the effective couplings, all the coupling constants would be reduced by 20 percent later, and the corresponding predictions are analyzed seriously. Taking into account the centrifugal barrier and solving the coupled channel Schrödinger equations numerically, we can obtain the predictions for the binding energy and the static properties, which are listed in the tables of the Appendix. We notice that these predictions are rather sensitive to the regularization parameter $\Lambda$ and the effective couplings, this is common to the one boson exchange model [8, 26, 32]. We also find that the binding energy increases with $\Lambda$, the reason is that increasing $\Lambda$ increases the strength of the potential at short distance.

For the $0^{++} D_s^*D_s^*$ state, the system can be in $^1S_0$ or $^5D_0$ configuration, this is very similar to the deuteron, which can be in $^3S_1$ or $^3D_1$. The S wave state mixes with the D wave state under the tensor force, as is shown explicitly in Eq. [4]. The energy of the system would be lowered substantially due to the freedom of flipping from the $^1S_0$ configuration to the $^5D_0$ configuration and back. For $\Lambda$ in the range of 1350 MeV-1600 MeV, we can find a bound state with the binding energy $\varepsilon = 6.46 - 168.73$ MeV. The D wave probability increases with the binding energy $\varepsilon$, and it is about 12.73% for $\varepsilon = 97.73$ MeV, the importance of the tensor force is obvious. If all the coupling constants are reduced by 20%, we need increase $\Lambda$ by about 200 MeV in order to obtain similar binding energy. However, the value of $\Lambda$ is still in the reasonable range. Since the molecular state is widely extended, the decay into light mesons via annihilation is generally suppressed by the form factor. The leading source of decay is dissociation, to a good approximation the dissociation will proceed via the free space decay of the constituent mesons. Consequently the $0^{++} D_s^*D_s^*$ molecule mainly decays into $D_s^* D_s^- \gamma\gamma$, and $D_s^* D_s^- \gamma\pi^0$, and the mode $D_s^* D_s^- \pi^0\pi^0$ is forbidden by the phase space.

For the axial vector $1^{-+}$ state, there are two channels $^3S_1$ and $^3D_1$. The coupling between the S wave and D wave has the same strength as the $0^{++}$ state, while the S wave spin-spin interaction potential $V_S(r)$ is weaker than the corresponding one of the $0^{++}$ state. Therefore the predictions for the binding energy and the static properties have similar pattern with the $0^{++}$ sector, and the binding energy of the $1^{-+}$ state is somewhat smaller than that of the latter for the same $\Lambda$ value. We note that the unnatural spin parity forbids its decay into $D_s\bar{D}_s$, while the decay mode $D_s^*\bar{D}_s^*$ is allowed.

The $0^{--}$ state involves only one channel $^3P_0$. In contrast with the $0^{++}$ and $1^{-+}$ cases, the tensor interaction potential $-2V_T(r)$ is attractive as a first order effect instead of a second order effect. The contributions of both spin-spin interaction and spin-orbit interaction are attractive as well, since $V_S(r)$ and $V_{LS}(r)$ are positive as shown in Fig. [4] and their potential Eq. [7] for this pseudoscalar is displayed in Fig. [2]. With $\Lambda = 1600$ MeV, we see that the potential is strong enough so that the P wave centrifugal barrier can be partly compensated, there then remains a weak attractive interaction in the intermediate range. Therefore bound state solution can be found for reasonable values of $\Lambda$, as can be seen from Table [LV]. For $\Lambda = 1500 - 1600$ MeV, we find the binding energy $\varepsilon = 1.40 - 114.81$ MeV. The binding energy is more sensitive to $\Lambda$ than the $0^{++}$ and $1^{-+}$ two coupled channels cases.
FIG. 2: The potentials for the single channel states with $\Lambda = 1600$ MeV. (a), (b) and (c) correspond to $0^{-+}$, $1^{++}$ and $1^{-+}$ states respectively. The solid line represents the potential from one boson exchange, and the dashed line denotes the effective potential after taking into account the centrifugal barrier.

The results for $1^{++}$ state is similar to the single channel $0^{-+}$ case. Because the D wave centrifugal barrier is higher than the P wave centrifugal barrier, the total effective potential is less attractive than the $0^{-+}$ state, this point can be seen clearly in Fig. 2. If the coupling constants are reduced by 20\%, bound state solution can be found only for $\Lambda$ larger than 1920 MeV. $1^{++}$ $D_s^*\bar{D}_s^*$ is harder to be bound than the $0^{-+}$ state due to the repulsive D wave centrifugal barrier.

We then come to the CP exotic $1^{-+}$ and $2^{+-}$ states, only one channel is involved in both sectors. As has been shown in Eq. (9) and Eq. (10), the potentials from one boson exchange are exactly the same, and they are less attractive than the potentials of the $0^{-+}$ and $1^{+-}$ states. For the $1^{-+}$ configuration, bound state solution appears only for $\Lambda$ as large as 2000 MeV, and we can find a $2^{+-}$ bound state only if the regularization parameter $\Lambda$ is larger than 3160 MeV. If we reduce all the coupling constants by 20\%, larger value of $\Lambda$ is required to find bound state solutions. Because the value of $\Lambda$ is so large that it is far beyond the range of 0.8 to 1.5 GeV favored by the nucleon-nucleon interactions, we tend to conclude that the CP exotic $1^{-+}$ and $2^{+-}$ $D_s^*\bar{D}_s^*$ states can not be bound by the one boson exchange potential. This conclusion is consistent with the fact that no such CP exotic states have been observed so far.

For the $1^{-+}$ states, there are three configurations $^1P_1$, $^5P_1$ and $^5F_1$. In spite of the P wave centrifugal barrier, bound state solutions can be found for reasonable value of $\Lambda$, the reason is the large attractive contributions from the tensor interaction and the spin-orbit interaction. From the numerical results in Table VII, we see that the binding energy is rather sensitive to $\Lambda$, and $^5P_1$ is the dominant component. This is because that the 22 element of the potential in Eq. (11) is more attractive than the 11 element, and the $^5F_1$ component is strongly suppressed by the large F wave centrifugal barrier. The $1^{-+}$ $D_s^*\bar{D}_s^*$ state is very interesting, it can be produced via the $e^+e^-$ annihilation or with the help of the initial state radiation (ISR) technique at $B$ factory. The existence of such a state can be confirmed or rejected, if more detailed $e^+e^-$ annihilation data in the range of 4100-4200 MeV become available. We strongly urge the Babar and Belle collaboration to search for this state, so that our prediction can be checked. In addition to the dominant decay modes $D_{s}^{*}\bar{D}_{s}\gamma\gamma$ and $D_{s}^{*}\bar{D}_{s}\gamma n\bar{n}$, the $1^{-+}$ $D_s^*\bar{D}_s^*$ molecule can also decay into $D_sD_s$ and $D_s\bar{D}_s$ which are other important decay modes.

For the $2^{+-}$ states, there are four channels $^1D_2$, $^5S_2$, $^5D_2$ and $^5G_2$. We note that both the tensor interaction and the spin-orbit interaction vanish in the $^5S_2$ configuration, and the spin-spin interaction is repulsive. However, bound state solution can be found for appropriate value of $\Lambda$ albeit its value is somewhat larger than the corresponding ones in the $0^{++}$, $1^{+-}$, $0^{-+}$ and $1^{-+}$ cases. This is because that the mixing of $^5S_2$ with $^1D_2$, $^5D_2$ and $^5G_2$ under the tensor force increases the binding of the system through higher order iterative processes. From the numerical results in Table IX, we see that the $^5S_2$ component is dominant, $^5D_2$ probability is larger than $^1D_2$, and $^5G_2$ has the smallest probability.

Finally two states $2^{-+}$ and $2^{--}$ remain. For both states, one has a two coupled channels (P wave and F wave configurations). Because of the P wave and F wave centrifugal barrier, bound state appears only for $\Lambda$ as large as 2080 MeV and 1890 MeV respectively. Therefore both the $2^{-+}$ and $2^{--}$ $D_s^*\bar{D}_s^*$ states may not be bound by the one boson exchange potential.

In short summary, ten allowed $D_s^*\bar{D}_s^*$ states with low spin parity have been studied. We find that $J^{PC} = 0^{++}, 1^{+-}, 0^{-+}, 2^{++}$ and $1^{-+}$ states are more tightly bound. We expect that the $0^{++}$ and $1^{+-}$ $D_s^*\bar{D}_s^*$ molecular states very likely exist, where the system can be in S wave or D wave and the two configurations mix with each other under the tensor interaction. Due to the remarkably strong contributions from the spin-spin interaction, the tensor interaction and the spin-orbit interaction, the $0^{+-} D_s^*\bar{D}_s^*$ state likely exist as well, and the $^5P_1$ component is dominant. One expects that the one boson exchange potential could
support the $2^{++} D_s^* \bar{D}^*_s$ bound state because of the presence of the S wave configuration and mixing with the other three higher partial waves. However, the CP exotic $1^-_+$ and $2^{--}$ states, $2^{--}$ and $2^{--}$ states should not exist.

IV. THE INTERPRETATION OF Y(4140) AS A $D_s^* \bar{D}^*_s$ MOLECULE AND ITS IMPLICATION

Since the one boson exchange potential does not support $1^-, 2^{--}$, $2^{++}$ and $2^{--} D_s^* \bar{D}^*_s$ molecular states, we concentrate on $0^{++}, 1^{++}, 0^{--}, 1^{--}$ and $2^{++}$ states in the following. Because the $C^-$ parity of Y(4140) is positive, $J^{PC} = 1^{++}$ and $1^{--}$ states are not possible. From the numerical results in Table II Table III, it is obvious that the $0^{++} D_s^* \bar{D}^*_s$ is most tightly bound by the one boson exchange potential. Consequently the most favorable quantum number of Y(4140) is $J^{PC} = 0^{++}$, however, $J^{PC} = 0^{-}$ or $2^{++}$ can not be excluded at present. It is crucial to perform a partial wave analysis in future, if the spatial parity turns out to be positive, $J^{PC} = 0^{++}$ and $2^{++}$ are favored, otherwise it may be a $0^{--}$ state.

If Y(4140) is confirmed to be a $0^{++} D_s^* \bar{D}^*_s$ molecule with masses about 4143 MeV by future experiments, $0^{--}$ and $2^{++} D_s^* \bar{D}^*_s$ molecules should be observed as well, whose masses are in the range of 4190 MeV to 4210 MeV. For the parameters that allow Y(4140) to emerge as a $0^{++} D_s^* \bar{D}^*_s$ molecule, one expects that the $1^{--}$ and $1^{--} D_s^* \bar{D}^*_s$ states may exist as well. We note that the $C^-$ parity of both states is negative. The $1^{--}$ state is particularly interesting, its mass is about 4120-4150 MeV, and it can be produced directly in the $e^+e^-$ annihilation or via the initial state radiation at B factory. Both states can decay into $D_s^* \bar{D}^*_s$, whereas the spin parity of the first state forbids its decay into $D_s \bar{D}^*_s$. Since Y(4140) is observed in the $J/\psi \phi$ channel, the $1^{--}$ and $1^{--}$ states should be searched for in the $J/\psi \eta$ and $J/\psi \eta'$ final states. We suggest the CDF, Babar and Belle collaboration to search for this state, which would be a critical check to the $D_s^* \bar{D}^*_s$ molecule interpretation of Y(4140).

If Y(4140) is identified to be a $D_s^* \bar{D}^*_s$ molecule, it is interesting to investigate whether a $D^* \bar{D}^*$ molecular state exists and what its most favorable quantum numbers are. In Ref. [3], the authors identified Y(3940) as the $D^* \bar{D}^*$ molecular partner of Y(4140), then its binding energy is about 77.5 MeV (Belle Collaboration) or 105.9 MeV (Babar Collaboration). However, in one pion exchange model Törnqvist demonstrated that the $D^* \bar{D}^*$ molecule should be near the threshold of 4020 MeV with $J^{PC} = 0^{++}$, $0^{--}$, $1^{--}$ or $2^{++}$. It is necessary to reanalysis the $D^* \bar{D}^*$ system dynamically. In Ref. [33], the contributions from $\pi, \eta, \sigma, \rho$ and $\omega$ exchanges are included, the binding energy and other static properties are found to be sensitive to the regularization parameter $\Lambda$. However, we qualitatively confirmed that the $J^{PC} = 2^{++}$, $1^{--}$, $0^{++}$ and $0^{--} D^* \bar{D}^*$ states are more deeply bound. Consequently its huge binding energy implies the existence of more $D^* \bar{D}^*$ molecules with different quantum numbers, if Y(3940) is identified as a $D^* \bar{D}^*$ bound state. However, no such candidates have been reported experimentally so far, therefore we tend to conclude that Y(3940) as a $D^* \bar{D}^*$ molecule is not favored. Further experimental and theoretical efforts are needed to understand the structure and the properties of Y(3940). With the same argument as that in the introduction section, Y(3940) as a canonical $c\bar{c}(2F)$ charmonium can not be completely excluded. The charmonium assignment of Y(3940) can be tested by searching for $DD$ and $DD^*/DD^*$ final states and by studying their angular distributions [5,4].

For the $B_s^* B_s^*$ system, the repulsive kinetic energy is greatly reduced due to larger mass of $B_s^*$ meson, therefore the $B_s^* B_s^*$ state should be more deeply bound than $D_s^* \bar{D}^*_s$. We expect that at least $0^{--}$ $B_s^* B_s^*$ molecular state should exist with larger binding energy. Obviously this state could be searched for in the $Y(1S)\phi$ channel. Because of its large mass, the most promising places to produce this state conspicuously are the large hadron colliders such as Tevatron and LHC.

V. CONCLUSIONS AND DISCUSSIONS

In this work, we have dynamically studied the possible $D_s^* \bar{D}^*_s$ molecular states and the interpretation of Y(4140) as a $D_s^* \bar{D}^*_s$ molecule in the one boson exchange model, where $\sigma, \eta$ and $\phi$ exchanges are taken into account. Ten allowed states with low spin parity have been considered, we would like to stress that only S wave configuration is usually considered in the familiar phenomenological models such as the one boson exchange model in heavy quark effective theory [21,22] and the potential model with pairwise interactions [16,19]. We find that the binding energy and static properties are sensitive to the regularization parameter $\Lambda$ and the effective coupling constants. Since the regularization parameter $\Lambda$ is poorly known so far, we are not able to precisely predict the binding energies of the possible $D_s^* \bar{D}^*_s$ molecular states bound by one boson exchange potential. However, we can reliably predict which ones of the ten allowed states are much easier to be bound, and the predictions are rather stable even if the uncertainty of the coupling constants is considered, as is obvious from the numerical results listed in the manuscript. Further research on X(3872), which is a promising $DD^*/DD^*$ molecule, would severely constrain the parameters of the one boson exchange model, especially the regularization parameter $\Lambda$, so that the predictions presented in the work could become more precise.
We quantitatively confirm that the $0^{++}$ $D_s^* D_s^*$ state is most easily to be bound. Our numerical results imply that the $J^{PC} = 0^{++}, 1^{+-}, 0^{-+}, 2^{++}$ and $1^{--}$ configurations are rather more strongly bound so that the corresponding $D_s^* D_s^*$ molecules may exist, whereas the CP exotic $1^{++}$ and $2^{++}$, $2^{++}$ and $1^{--}$ $D_s^* D_s^*$ states are not be bound by the one boson exchange potential. We note that the possible existence of a number of bound state is not a specific prediction of our model. Finally we would like to stress that we still can not completely rule out $Y(4140)$ as a conventional $c\bar{c}$ charmonium at present, in spite of its peculiar decay mode $J/\psi\phi$. From the theoretical predictions for the charmonium spectrum, $Y(4140)$ is most likely to be the $2^1D_2$ state with $I^G(J^{PC}) = 0^+(2^{++})$, if it is a $c\bar{c}$ charmonium state. It unusual large branch ratio into $J/\psi\phi$ may be explained by the rescattering mechanism or the mixing between charmonium and molecule. Compared with other charmonium like states, the experimental information for $Y(4140)$ is scarce, and further experiment data are critically needed.

For $Y(4140)$ as a $D_s^* D_s^*$ molecule, we suggest that its most favorable quantum numbers are $J^{PC} = 0^{++}$, although $J^{PC} = 0^{-+}$ and $2^{++}$ can not be ruled out by the present experimental data. It mainly decays into $D_s^* D_s^* \gamma\gamma$ and $D_s^* D_s^* \pi^0\pi^0$ via almost free decay of $D_s^*$ and $D_s^*$, and the decay mode $D_s^* D_s^* \gamma\pi^0$ is forbidden by phase space constraints. The search for the four body decays $Y(4140) \rightarrow D_s^* D_s^* \gamma\gamma$ and $Y(4140) \rightarrow D_s^* D_s^* \gamma\pi^0$ is crucial to test the hadronic molecule hypothesis of $Y(4140)$. If $Y(4140)$ is confirmed to be a $0^{++}$ $D_s^* D_s^*$ molecule by future theoretical and experimental efforts, the $0^{++}$ and $2^{++}$ partners should exist with mass in the range of 4190-4210 MeV. We argue that the $1^{++}$ and $1^{--}$ $D_s^* D_s^*$ states with negative $C$–parity should be observed as well. The $1^{--}$ states can be produced largely in the $e^+e^-$ annihilation or with the help the initial state radiation at B factory, and detailed $e^+e^-$ annihilation data near 4100-4200 MeV are important to confirm or refute the existence of such state. Both the $1^{++}$ and $1^{--}$ $D_s^* D_s^*$ states can be searched for in the $J/\psi\eta$ and $J/\psi\eta'$ final states. We strongly urge the CDF, Babar and Belle Collaborations to search for these two negative $C$–parity states, which would be another important test to the molecular hypothesis of $Y(4140)$ and the reliability of our one boson exchange model.

If we identify $Y(3940)$ as a $D^* D^*$ molecule, its large binding energy requires the existence of more $D^* D^*$ bound states, which has not been observed so far. Therefore the interpretation of $Y(3940)$ as a $D^* D^*$ molecule may not be favored in our opinion. With the present experimental data, the charmonium assignment for $Y(3940)$ can not be ruled out. We suggest that the $0^{++}$ $B^* B^*$ molecular state should exist, and it is bound more tightly than $D_s^* D_s^*$. We should search for this state at Tevatron or LHC in the $Y(1S)\phi$ channel.

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APPENDIX A: NUMERICAL RESULTS FOR THE TEN ALLOWED $D_s^* \bar{D}_s^*$ STATES

| $\Lambda$(MeV) | $\varepsilon$(MeV) | $r_{\text{rms}}$(fm) | $P_S : P_D$(%) |
|----------------|-------------------|-------------------|----------------|
| 1350           | 6.46              | 1.49              | 99.48 : 0.52   |
| 1400           | 16.09             | 1.03              | 98.83 : 1.17   |
| 1450           | 32.11             | 0.79              | 97.51 : 2.49   |
| 1500           | 57.33             | 0.63              | 94.56 : 5.44   |
| 1550           | 97.73             | 0.52              | 87.27 : 12.73  |
| 1600           | 168.73            | 0.44              | 69.44 : 30.56  |

and couplings are reduced by 20 percents

| $\Lambda$(MeV) | $\varepsilon$(MeV) | $r_{\text{rms}}$(fm) | $P_S : P_D$(%) |
|----------------|-------------------|-------------------|----------------|
| 1500           | 4.20              | 1.75              | 99.51 : 0.49   |
| 1550           | 10.28             | 1.20              | 98.94 : 1.06   |
| 1600           | 20.14             | 0.91              | 97.88 : 2.13   |
| 1650           | 35.21             | 0.73              | 95.77 : 4.22   |
| 1700           | 58.25             | 0.60              | 91.35 : 8.65   |
| 1750           | 95.26             | 0.50              | 81.68 : 18.32  |

TABLE II: The predictions for the static properties of the $J^{PC} = 0^{++}$ $D_s^* \bar{D}_s^*$ hadronic molecule, where $\varepsilon$ denotes the binding energy, $r_{\text{rms}}$ is the root of mean square radius, $P_S$ and $P_D$ represent the S state and D state probabilities respectively.

| $\Lambda$(MeV) | $\varepsilon$(MeV) | $r_{\text{rms}}$(fm) | $P_S : P_D$(%) |
|----------------|-------------------|-------------------|----------------|
| 1350           | 6.13              | 1.53              | 99.65 : 0.35   |
| 1400           | 10.34             | 1.12              | 99.36 : 0.64   |
| 1450           | 13.74             | 0.89              | 98.95 : 1.05   |
| 1500           | 16.74             | 0.75              | 98.35 : 1.65   |
| 1550           | 55.74             | 0.65              | 97.48 : 2.52   |
| 1600           | 78.42             | 0.58              | 96.16 : 3.84   |
| 1650           | 106.87            | 0.52              | 94.15 : 5.85   |

and couplings are reduced by 20 percents

| $\Lambda$(MeV) | $\varepsilon$(MeV) | $r_{\text{rms}}$(fm) | $P_S : P_D$(%) |
|----------------|-------------------|-------------------|----------------|
| 1600           | 10.55             | 1.20              | 99.29 : 0.71   |
| 1650           | 17.09             | 0.98              | 98.91 : 1.09   |
| 1700           | 25.48             | 0.84              | 98.91 : 1.61   |
| 1750           | 35.97             | 0.74              | 97.67 : 2.33   |
| 1800           | 48.92             | 0.66              | 96.67 : 3.33   |
| 1850           | 64.84             | 0.59              | 95.27 : 4.73   |
| 1900           | 84.49             | 0.54              | 93.29 : 6.71   |
| 1950           | 109.01            | 0.49              | 90.48 : 9.52   |

TABLE III: The predictions about the binding energy, the root of mean square radius($r_{\text{rms}}$) and the probabilities of the different components for the $1^{+-} D_s^* \bar{D}_s^*$ molecule.
| Λ(MeV) | ɛ(MeV) | r_{rms}(fm) |
|--------|--------|-------------|
| 1500   | 1.40   | 1.52        |
| 1520   | 14.99  | 0.86        |
| 1540   | 33.36  | 0.70        |
| 1560   | 56.13  | 0.62        |
| 1580   | 83.26  | 0.56        |
| 1600   | 114.81 | 0.51        |

All couplings are reduced by 20 percents.

| Λ(MeV) | M(MeV) | r_{rms}(fm) |
|--------|--------|-------------|
| 1680   | 4.21   | 0.62        |
| 1690   | 28.16  | 0.54        |
| 1700   | 54.83  | 0.50        |
| 1710   | 84.04  | 0.47        |
| 1720   | 115.71 | 0.45        |

TABLE IV: The predictions for the binding energy and the rms of the $0^{-+} D_s^*\bar{D}_s^*$ molecule.

| Λ(MeV) | ɛ(MeV) | r_{rms}(fm) |
|--------|--------|-------------|
| 1920   | 12.37  | 0.54        |
| 1930   | 33.19  | 0.49        |
| 1940   | 55.70  | 0.46        |
| 1950   | 79.81  | 0.44        |
| 1960   | 105.50 | 0.43        |

TABLE V: The predictions for the binding energy and the rms of the $1^{++} D_s^*\bar{D}_s^*$ molecule.
| Λ(MeV) | ε(MeV) | r_{rms}(fm) |
|--------|--------|-------------|
| 2000   | 10.72  | 0.77        |
| 2020   | 23.60  | 0.63        |
| 2040   | 38.60  | 0.56        |
| 2060   | 55.61  | 0.51        |
| 2080   | 74.60  | 0.47        |
| 2100   | 95.54  | 0.44        |
| 2120   | 118.44 | 0.42        |

All couplings are reduced by 20 percents

| Λ(MeV) | ε(MeV) | r_{rms}(fm) |
|--------|--------|-------------|
| 2480   | 9.94   | 0.72        |
| 2500   | 20.67  | 0.60        |
| 2520   | 32.81  | 0.53        |
| 2540   | 46.25  | 0.49        |
| 2560   | 60.96  | 0.45        |
| 2580   | 76.92  | 0.42        |
| 2600   | 94.11  | 0.40        |
| 2620   | 112.54 | 0.38        |

Table VI: The predictions for the binding energy and the rms of the $1^{-+}$ $D^*_s\bar{D}^*_s$ molecule.

| Λ(MeV) | ε(MeV) | r_{rms}(fm) |
|--------|--------|-------------|
| 3160   | 6.48   | 0.37        |
| 3170   | 22.41  | 0.34        |
| 3180   | 39.00  | 0.33        |
| 3190   | 56.20  | 0.32        |
| 3200   | 74.00  | 0.31        |
| 3210   | 92.40  | 0.30        |
| 3220   | 111.38 | 0.29        |

All couplings are reduced by half

| Λ(MeV) | ε(MeV) | r_{rms}(fm) |
|--------|--------|-------------|
| 4420   | 11.51  | 0.28        |
| 4430   | 28.97  | 0.26        |
| 4440   | 46.92  | 0.25        |
| 4450   | 65.36  | 0.25        |
| 4460   | 84.27  | 0.24        |
| 4470   | 103.63 | 0.23        |

Table VII: The predictions for the binding energy and the rms of the $2^{+-}$ $D^*_s\bar{D}^*_s$ molecule.
| \( \Lambda (\text{MeV}) \) | \( \varepsilon (\text{MeV}) \) | \( r_{\text{rms}} (\text{fm}) \) | \( P_{D0} : P_{S} : P_{D2} : P_{G} (\%) \) |
|-----------------|-----------------|-----------------|-----------------|
| 1470            | 5.38            | 1.10            | 3.40:96.50:0.10  |
| 1480            | 13.13           | 0.89            | 3.18:96.71:0.11  |
| 1490            | 22.23           | 0.78            | 3.01:96.87:0.13  |
| 1500            | 32.59           | 0.71            | 2.86:97.00:0.14  |
| 1510            | 44.18           | 0.66            | 2.73:97.12:0.16  |
| 1520            | 56.97           | 0.62            | 2.61:97.22:0.17  |
| 1530            | 70.95           | 0.59            | 2.51:97.30:0.19  |
| 1540            | 86.12           | 0.56            | 2.42:97.38:0.20  |
| 1550            | 102.49          | 0.53            | 2.34:97.44:0.22  |

| \( \Lambda (\text{MeV}) \) | \( \varepsilon (\text{MeV}) \) | \( r_{\text{rms}} (\text{fm}) \) | \( P_{D0} : P_{S} : P_{D2} : P_{G} (\%) \) |
|-----------------|-----------------|-----------------|-----------------|
| 1650            | 11.28           | 0.88            | 2.46:97.45:0.09  |
| 1660            | 18.49           | 0.78            | 2.36:97.53:0.11  |
| 1670            | 26.53           | 0.71            | 2.29:97.60:0.12  |
| 1680            | 35.36           | 0.67            | 2.22:97.66:0.13  |
| 1690            | 44.96           | 0.63            | 2.15:97.71:0.14  |
| 1700            | 55.33           | 0.60            | 2.10:97.75:0.15  |
| 1710            | 66.45           | 0.57            | 2.04:97.79:0.16  |
| 1720            | 78.32           | 0.55            | 2.00:97.83:0.18  |
| 1730            | 90.95           | 0.53            | 1.95:97.86:0.19  |
| 1740            | 104.33          | 0.51            | 1.91:97.89:0.20  |

all couplings are reduced by 20 percents

| \( \Lambda (\text{MeV}) \) | \( \varepsilon (\text{MeV}) \) | \( r_{\text{rms}} (\text{fm}) \) | \( P_{D0} : P_{S} : P_{D2} : P_{G} (\%) \) |
|-----------------|-----------------|-----------------|-----------------|
| 1650            | 8.09            | 1.36            | 0.11:98.92:0.07:0.00 |
| 1700            | 11.69           | 1.18            | 0.15:98.44:1.00:0.00 |
| 1750            | 16.09           | 1.05            | 0.21:97.81:1.98:0.00 |
| 1800            | 21.41           | 0.94            | 0.27:96.97:2.75:0.01 |
| 1850            | 27.91           | 0.86            | 0.35:95.86:3.79:0.01 |
| 1900            | 35.91           | 0.78            | 0.44:94.36:5.17:0.02 |
| 1950            | 45.91           | 0.72            | 0.56:92.36:7.04:0.04 |
| 2000            | 58.62           | 0.66            | 0.70:89.64:9.56:0.09 |
| 2050            | 75.18           | 0.60            | 0.89:85.95:12.95:0.22 |
| 2100            | 97.42           | 0.55            | 1.13:80.82:17.48:0.57 |
| 2150            | 129.26          | 0.50            | 1.49:73.08:23.51:1.93 |

**TABLE VIII:** The predictions about the binding energy, the root of mean square radius and the probabilities of the different components for the \( ^1D_s^*\bar{D}_s^* \) molecule, where \( P_{D0} \) and \( P_{D2} \) denote the \( ^1P_1 \) state and \( ^5P_1 \) state probabilities respectively.

| \( \Lambda (\text{MeV}) \) | \( \varepsilon (\text{MeV}) \) | \( r_{\text{rms}} (\text{fm}) \) | \( P_{D0} : P_{S} : P_{D2} : P_{G} (\%) \) |
|-----------------|-----------------|-----------------|-----------------|
| 1350            | 6.81            | 1.47            | 0.07:99.48:0.36:0.00 |
| 1400            | 12.03           | 1.18            | 0.10:99.14:0.75:0.00 |
| 1450            | 18.59           | 1.00            | 0.15:98.67:1.18:0.00 |
| 1500            | 26.55           | 0.88            | 0.21:98.01:1.78:0.00 |
| 1550            | 36.10           | 0.80            | 0.29:97.07:2.63:0.00 |
| 1600            | 47.59           | 0.73            | 0.40:95.74:3.85:0.01 |
| 1650            | 61.65           | 0.67            | 0.53:93.85:5.60:0.02 |
| 1700            | 79.28           | 0.62            | 0.70:91.12:8.13:0.05 |
| 1750            | 102.10          | 0.57            | 0.91:87.18:11.77:0.14 |

all couplings are reduced by 20 percents

| \( \Lambda (\text{MeV}) \) | \( \varepsilon (\text{MeV}) \) | \( r_{\text{rms}} (\text{fm}) \) | \( P_{D0} : P_{S} : P_{D2} : P_{G} (\%) \) |
|-----------------|-----------------|-----------------|-----------------|
| 1650            | 8.09            | 1.36            | 0.11:98.92:0.09:0.00 |
| 1700            | 11.69           | 1.18            | 0.15:98.44:1.00:0.00 |
| 1750            | 16.09           | 1.05            | 0.21:97.81:1.98:0.00 |
| 1800            | 21.41           | 0.94            | 0.27:96.97:2.75:0.01 |
| 1850            | 27.91           | 0.86            | 0.35:95.86:3.79:0.01 |
| 1900            | 35.91           | 0.78            | 0.44:94.36:5.17:0.02 |
| 1950            | 45.91           | 0.72            | 0.56:92.36:7.04:0.04 |
| 2000            | 58.62           | 0.66            | 0.70:89.64:9.56:0.09 |
| 2050            | 75.18           | 0.60            | 0.89:85.95:12.95:0.22 |
| 2100            | 97.42           | 0.55            | 1.13:80.82:17.48:0.57 |
| 2150            | 129.26          | 0.50            | 1.49:73.08:23.51:1.93 |

**TABLE IX:** The predictions about the binding energy, the root of mean square radius and the probabilities of the different components for the \( ^2D_s^*\bar{D}_s^* \) molecule.
| Λ(MeV) | ε(MeV) | r_{rms}(fm) | P_F : P_P(%) |
|--------|--------|-------------|--------------|
| 2080   | 21.35  | 0.54        | 32.02:67.98  |
| 2090   | 50.76  | 0.46        | 25.30:74.70  |
| 2100   | 84.10  | 0.42        | 21.08:78.92  |
| 2110   | 120.80 | 0.39        | 18.09:81.91  |

All couplings are reduced by 20 percents

| Λ(MeV) | ε(MeV) | r_{rms}(fm) | P_F : P_P(%) |
|--------|--------|-------------|--------------|
| 2510   | 18.99  | 0.47        | 24.99:75.01  |
| 2520   | 47.32  | 0.40        | 20.26:79.74  |
| 2530   | 78.24  | 0.37        | 17.38:82.62  |
| 2540   | 111.36 | 0.35        | 15.34:84.66  |

TABLE X: The predictions about the binding energy, the root of mean square radius and the probabilities of the different components for the $2^{-+}$ $D_s^*\bar{D}_s^*$ molecule.

| Λ(MeV) | ε(MeV) | r_{rms}(fm) | P_F : P_P(%) |
|--------|--------|-------------|--------------|
| 1890   | 1.59   | 1.12        | 65.79:34.21  |
| 1900   | 21.17  | 0.61        | 51.31:48.69  |
| 1910   | 46.97  | 0.51        | 41.55:58.46  |
| 1920   | 78.03  | 0.46        | 34.08:65.92  |
| 1930   | 113.83 | 0.42        | 28.31:71.69  |

All couplings are reduced by 20 percents

| Λ(MeV) | ε(MeV) | r_{rms}(fm) | P_F : P_P(%) |
|--------|--------|-------------|--------------|
| 2230   | 5.92   | 0.71        | 49.37:50.63  |
| 2240   | 28.13  | 0.50        | 38.73:61.27  |
| 2250   | 54.53  | 0.44        | 31.95:68.05  |
| 2260   | 84.34  | 0.40        | 26.99:73.01  |
| 2270   | 117.14 | 0.38        | 23.20:76.80  |

TABLE XI: The predictions about the binding energy, the root of mean square radius and the probabilities of the different components for the $2^{--}$ $D_s^*\bar{D}_s^*$ molecule.