Probing the lightest new gauge boson $B_H$ in the littlest Higgs model via the processes $\gamma\gamma \rightarrow f \bar{f} B_H$ at the ILC

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Abstract

The neutral gauge boson $B_H$ with the mass of hundreds GeV, is the lightest particle predicted by the littlest Higgs(LH) model, and such particle should be the first signal of the LH model at the planed ILC if it exists indeed. In this paper, we study some processes of the $B_H$ production associated with the fermion pair at the ILC, i.e., $\gamma\gamma \rightarrow f \bar{f} B_H$. The studies show that the most promising processes to detect $B_H$ among $\gamma\gamma \rightarrow f \bar{f} B_H$ are $\gamma\gamma \rightarrow l'^+ l'^- B_H (l' = e, \mu)$, and they can produce the sufficient signals in most parameter space preferred by the electroweak precision data at the ILC. On the other hand, the signal produced via the certain $B_H$ decay modes is typical and such signal can be easily identified from the SM background. Therefore, $B_H$, the lightest gauge boson in the LH model would be detectable at the photon collider realized at the ILC.

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I. INTRODUCTION

The little Higgs models[1, 2, 3, 4, 5] were recently proposed to solve the hierarchy problem in the standard model(SM) by protecting the Higgs mass from quadratical divergences at one-loop order, and thus can be regarded as one of the important candidates of the new physics beyond the SM. The key feature of this kind of models is that the Higgs boson is a pseudo-Goldstone boson of a global symmetry breaking at a scale $\Lambda \sim 10^{10}$TeV, so that the Higgs boson mass is naturally light. The light Higgs boson mass is protected from the one-loop quadratic divergences by introducing a few new particles with the same statistics as the corresponding SM particles.

Among various little Higgs models, the littlest Higgs(LH) model[5] is a simplest and phenomenologically viable one to realize the little Higgs idea. In this model, new charged heavy vector bosons $W^\pm_H$, neutral heavy vector bosons $Z_H, B_H$, a heavy vector-like quark $T$ and charged or neutral heavy Higgs scalars are present which just cancel the quadratic divergences by the SM gauge boson loops, the top quark loop and Higgs self-interaction, respectively. These new particles might produce characteristic signatures at present and future high energy collider experiments[6, 7, 8]. In the literatures[7, 8], the phenomenologies of the LH model at the LHC have been studied, showing that the LHC has the potential to detect these particles. In the LH model, however, we find that the globe symmetry structure $SU(5)/SO(5)$ allows a substantially light $B_H$, light enough to be produced on shell at a 500 GeV linear collider. $B_H$, as the lightest new particle in the LH model, would play an important role in the phenomenological studies of the LH model. Such gauge boson can be probed indirectly through its contributions to some processes[9, 10]. On the other hand, such particle would be the first signal of the LH model at high energy experiments, and the direct detection of it can provide a robust evidence of the model.

Although the LHC has the considerable potential to detect $B_H[7, 8]$, the detailed study of its properties needs the precision measurement at the future high energy and luminosity linear collider, and such work will be performed at the planned International Linear Collider(ILC), with the center of mass(c.m) energy $\sqrt{s} = 300$ GeV-1.5 TeV and the integrated luminosity 500 $fb^{-1}$ within the first four year running[11]. The gauge boson $B_H$ would be light enough to be produced at the first running of the ILC. So people also pay much attention to study the $B_H$ production mechanism at the ILC. Some $B_H$ production processes via $e^+e^-$ collision at the ILC have been done[12]. A unique feature of the ILC is that it can be transformed to $\gamma\gamma$ or $e\gamma$ collisions with the photon beams generated by using the Compton backscattering of the initial electron and laser beams. In this case, the energy and luminosity of the photon beams would be the same order of magnitude of the original electron beams, and the set of final states at a photon collider is much richer than that in the $e^+e^-$ mode. So the realization of the photon collider will open a wider window to probe $B_H$. Some $B_H$ production processes at the photon collider have also been studied[13]. In this paper, we study other interesting $B_H$ production processes at the photon collider, i.e., $\gamma\gamma \rightarrow f\bar{f}B_H$. Here $f$ represent all fermions in the SM. Our results show that the productions of $B_H$ associated with $e^+e^-$ or $\mu^+\mu^-$ can also provide an ideal way to probe $B_H$ with clean background.

This paper is organized as follows. In Sec. II, we first present the key idea of the LH model and summarize some couplings in the LH model related to our study, then we give our calculations of the cross sections for the processes $\gamma\gamma \rightarrow f\bar{f}B_H$. The numerical results and conclusions will be shown in Sec. III.
II. THE KEY IDEA OF THE LH MODEL AND THE CROSS SECTIONS OF THE PROCESSES $\gamma\gamma \rightarrow f\bar{f}B_H$

The LH model is the most economical one among various little Higgs models. It is based on a non-linear sigma model and consists of a global SU(5) symmetry which is broken down to SO(5) by a vacuum condensate $f \sim \frac{m}{4\pi} \sim$ TeV. Such breaking scenario results in 14 Goldstone bosons. Four of them are eaten by the broken gauge generators and will become the longitudinal modes of four new massive gauge bosons, leaving 10 states that transform under the SM gauge group as a complex Higgs doublet and a complex scalar triplet. A subgroup $[SU(2) \times U(1)]^2$ of the global $SU(5)$ which is gauged with gauge couplings $g_1, g_2, g'_1, g'_2$, respectively, is spontaneously broken down to its diagonal $SU(2)_L \times U(1)_Y$ subgroup. Such diagonal group is identified as the SM electroweak gauge group and the mass eigenstates of the gauge bosons after the symmetry breaking are

$$W = sW_1 + cW_2, \quad W' = -cW_1 + sW_2, \quad (1)$$
$$B = s'B_1 + c'B_2, \quad B' = -c'B_1 + s'B_2.$$  

Where the $W, B$ are the massless gauge bosons associated with the generators of the electroweak gauge group $SU(2)_L \times U(1)_Y$. The $W'$ and $B'$ are the massive gauge bosons associated with the four broken generators of $[SU(2) \times U(1)]^2$. Using the mixing parameters $c(s = \sqrt{1 - c^2})$ and $c'(s' = \sqrt{1 - c'^2})$, one can represent the SM gauge coupling constants as $g = g_1s = g_2c$ and $g' = g_1's' = g_2'c'$.

After electroweak symmetry breaking, the final observed mass eigenstates are obtained via mixing between the heavy($W', B'$) and light ($W, B$) gauge bosons. They include the light SM-like bosons $W^\pm, Z_L$ and $A_L$ observed at experiments, and new heavy bosons $W_H^\pm, Z_H$ and $B_H$ that could be observed in future experiments. The masses of neutral gauge bosons are given to $O(v^2/f^2)$ by $[7, 9]$

$$M^2_{A_L} = 0, \quad (2)$$
$$M^2_{B_H} = (M^2_Z)^2 s_W^2 \left\{ \frac{f^2}{5s^2c^2v^2} - 1 + \frac{v^2}{2f^2} \left[ \frac{5(c^2 - s^2)}{2s^2} \right] + \frac{gH_{c'}}{cc's's'} \right\},$$
$$M^2_{Z_L} = (M^2_Z)^2 \left\{ 1 - \frac{v^2}{f^2} \left[ \frac{1}{6} + \frac{1}{4}(c^2 - s^2)^2 + \frac{5}{4}(c^2 - s^2)^2 \right] + \frac{v^2}{v_H^2} \right\},$$
$$M^2_{Z_H} = (M^2_W)^2 \left\{ \frac{f^2}{s^2c^2v^2} - 1 + \frac{v^2}{2f^2} \left[ \frac{(c^2 - s^2)^2}{2c^2} + \frac{gH_{c'}}{g} \cdot \frac{c^2s^2}{cc's's'} \right] \right\}.$$  

Where, $\chi_H = \frac{5}{2} gg'H_{c's's'c^2}/g_{c's's'c^2}, s_W(c_W)$ represents the sine(cosine) of the weak mixing angle, $v=246$ GeV is the electroweak scale and $v'$ is the vev of the scalar $SU(2)_L$ triplet.

The effective non-linear lagrangian invariant under the local gauge group $[SU(2) \times U(1)]^2$ can be written as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_\Sigma + \mathcal{L}_Y - V_{CW}(\Sigma). \quad (3)$$

$\mathcal{L}_G$ consists of the pure gauge terms; $\mathcal{L}_F$ is the fermion kinetic terms, $\mathcal{L}_\Sigma$ consists of the $\sigma$-model terms of the LH model, $\mathcal{L}_Y$ is the Yukawa couplings of fermions and pseudo-Goldstone bosons, and $V_{CW}(\Sigma)$ is the Coleman-Weinberg potential generated radiatively.
from $\mathcal{L}_V$ and $\mathcal{L}_\Sigma$. The couplings related to our work are the couplings of the neutral gauge bosons with fermion pair which are included in $\mathcal{L}_F$. Such fermion kinetic terms take the generic form

$$\mathcal{L}_F = \sum_f \bar{\psi}_f i \gamma_\mu D_\mu \psi_f,$$

with

$$D_\mu = \partial_\mu - i \sum_{j=1}^2 (g_j W_{j\mu} + g'_j B_{j\mu}).$$

The couplings of the neutral gauge bosons $V_i$ with fermion pair can be written in form of $V_i^{V;ff} = i g_i^{V;ff} \bar{f} \gamma_\mu D_\mu f$, and the explicit expressions of the couplings related with $B_H$ are \[ \begin{align*}
B_{V_i} l_i^+ l_i^- &= g'_V \left( 2 y_e - \frac{9}{5} + \frac{3}{5} c'^2 \right), \\
B_{V_i} \bar{u}_u &= g'_V \left( 2 y_u + \frac{17}{15} - \frac{5}{6} c'^2 \right), \\
B_{V_i} d_i &= g'_V \left( 2 y_d + \frac{11}{15} + \frac{1}{6} c'^2 \right), \\
B_{V_i} \bar{t} &= g'_V \left( 2 y_t + \frac{17}{15} - \frac{5}{6} c'^2 - \frac{1}{5} x_L \right), \\
B_{V_i} \bar{u} &= g'_V \left( 2 y_u + \frac{17}{15} - \frac{5}{6} c'^2 - \frac{1}{5} x_L \right).
\end{align*} \]

$l_i, u_i, d_i$ denote $(e, \mu, \tau), (u, c), (d, s, b)$, respectively. We also define $x_L = \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}$, and $x_L$ is the mixing angle parameter between the SM top quark $t$ and the vector-like quark $T$, in which $\lambda_1$ and $\lambda_2$ are the Yukawa coupling parameters. As we can see above, the gauge invariance alone can not unambiguously fix all the $U(1)$ hypercharge values, and the two parameters $y_e$ and $y_u$ are undetermined. If one requires that the $U(1)$ hypercharge assignments be anomaly free, they can be fixed as $y_e = \frac{3}{5}, y_u = -\frac{2}{5}$. In the following calculation, we take $y_e = \frac{3}{5}, y_u = -\frac{2}{5}$ as an example. In this case, we can see that an apparently special point at $c' = \sqrt{2/5}$ exists where all the couplings of the gauge boson $B_H$ to light fermion pair vanish. An exception is the coupling of $B_H$ to the top quark pair, and an additional term is attributed to such coupling.

In the LH model, the custodial $SU(2)$ global symmetry is explicitly broken, which can generate large contributions to the electroweak observables. In the early study, global fit to the experimental data puts rather severe constraints on the $f > 4$ TeV at 95% C.L. However, their analyses are based on a simple assumption that the SM fermions are charged only under $U(1)_1$. If the SM fermions are charged under $U(1)_1 \times U(1)_2$, the bounds become relaxed. The substantial parameter space allows $f = 1 \sim 2$ TeV, with $c = 0 \sim 0.5, c' = 0.62 \sim 0.73$. Due to the existence of $B_H f f$ couplings, $B_H$ can be produced associated with fermion pair via $\gamma \gamma$ collision. The Feynman diagrams for the processes are shown in Fig.1, in which the cross diagrams with the interchange of the two incoming photons are not shown.
The amplitudes for the processes are given by

\[ M_f^a = iG(p_3 + p_5, m_f)G(p_1 - p_4, m_f) \cdot \]

\[ \bar{\nu}_f(p_3)\psi(p_5)g^{\gamma^*f}\bar{f}(p_3 + p_5 + m_f)\psi(p_2)g^{\gamma f}\bar{f}(p_1 - p_4 + m_f)\psi(p_4)g^{\gamma f}\bar{f}(p_4), \]

\[ M_f^b = iG(p_3 - p_2, m_f)G(p_1 - p_4, m_f) \cdot \]

\[ \bar{\nu}_f(p_3)\psi(p_5)g^{\gamma^*f}\bar{f}(p_3 - p_2 + m_f)\psi(p_5)g^{\gamma f}\bar{f}(p_1 - p_4 + m_f)\psi(p_4)g^{\gamma f}\bar{f}(p_4), \]

\[ M_f^c = iG(p_3 - p_2, m_f)G(p_4 + p_5, m_f) \cdot \]

\[ \bar{\nu}_f(p_3)\psi(p_2)g^{\gamma^*f}\bar{f}(p_3 - p_2 + m_f)\psi(p_1)g^{\gamma f}\bar{f}(p_1)(-p_4 - p_5 + m_f)\psi(p_5)g^{\gamma f}\bar{f}(p_4). \]

The amplitudes of the diagrams with the interchange of two incoming photons can be directly obtained by interchanging \( p_1, p_2 \) in above amplitudes. Where \( \hat{G}(p, m) = \frac{1}{p^2 - m^2} \) is the propagator of the particle.

With the above amplitudes, we can directly obtain the production cross sections \( \hat{\sigma}(\hat{s}) \) for the subprocesses \( \gamma \gamma \rightarrow f \bar{f}B_H \) and the total cross sections at the \( e^+e^- \) linear collider can be obtained by folding \( \hat{\sigma}(\hat{s}) \) with the photon distribution function \( F(x) \) which is given in Ref. [16],

\[ \sigma_{\text{tot}}(s) = \int_{x_{\text{min}}}^{x_{\text{max}}} dx_1 \int_{x_{\text{min}}x_{\text{max}}/x_1}^{x_{\text{max}}} dx_2 F(x_1)F(x_2)\hat{\sigma}(\hat{s}), \]

where \( s \) is the c.m. energy squared for \( e^+e^- \). The subprocesses occur effectively at \( \hat{s} = x_1x_2s \), and \( x_i \) are the fractions of the electron energies carried by the photons. The explicit form of the photon distribution function \( F(x) \) is

\[ F(x) = \frac{1}{D(\xi)} \left[ 1 - x + \frac{1}{1 - x} - \frac{4x}{\xi(1 - x)} + \frac{4x^2}{\xi^2(1 - x)^2} \right], \]

with

\[ D(\xi) = \left( 1 - 4 \frac{\xi}{\xi^2} - \frac{8}{\xi^2} \right) \ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2}, \]

and

\[ \xi = \frac{4E_{\text{coll}}}{m_e^2}. \]
\(E_0\) and \(\omega_0\) are the incident electron and laser light energies, and \(x = \omega/E_0\). The energy \(\omega\) of the scattered photon depends on its angle \(\theta\) with respect to the incident electron beam and is given by

\[
\omega = \frac{E_0(\xi)}{1 + (\frac{\theta}{\omega_0})^2}.
\] (12)

Therefore, at \(\theta = 0\), \(\omega = E_0\xi/(1 + \xi) = \omega_{\text{max}}\) is the maximum energy of the backscattered photon, and \(x_{\text{max}} = \frac{\omega_{\text{max}}}{E_0} = \frac{\xi}{1 + \xi}\).

To avoid unwanted \(e^+e^-\) pair production from the collision between the incident and back-scattered photons, we should not choose too large \(\omega_0\). The threshold for \(e^+e^-\) pair creation is \(\omega_{\text{max}}\omega_0 > m_e^2\), so we require \(\omega_{\text{max}}\omega_0 \leq m_e^2\). Solving \(\omega_{\text{max}}\omega_0 = m_e^2\), we find

\[
\xi = 2(1 + \sqrt{2}) = 4.8.
\] (13)

For the choice \(\xi = 4.8\), we obtain \(x_{\text{max}} = 0.83\) and \(D(\xi_{\text{max}}) = 1.8\). The minimum value for \(x\) is determined by the production threshold

\[
x_{\text{min}} = \frac{\hat{s}_{\text{min}}}{\omega_{\text{max}} s}, \quad \hat{s}_{\text{min}} = (2M_f + M_{BH})^2.
\] (14)

Here we have assumed that both photon beams and electron beams are unpolarized. We also assume that, the number of the backscattered photons produced per electron is one.

### III. Numerical Results and Conclusions

In our numerical calculations, we take the SM input parameters as: \(m_e = 0\), \(m_\mu = 0.106\) GeV, \(m_\tau = 1.77\) GeV, \(m_u = 0.002\) GeV, \(m_d = 0.005\) GeV, \(m_c = 1.25\) GeV, \(m_s = 0.1\) GeV, \(m_t = 174.2\) GeV, \(m_b = 4.7\) GeV, \(M_Z = 91.187\) GeV, \(v = 246\) GeV, \(s_W^2 = 0.23\) [17]. Another SM parameter, the electromagnetic fine structure constant \(\alpha_e\), can be fixed at a certain energy scale by calculating from the simple QED one-loop evolution formula with the boundary value \(\alpha = 1/137.04\) [18]. There are still four free parameters \((f, c, c', \sqrt{s})\) involved in the cross sections, except for the production mode \(t\bar{t}B_H\) with an extra parameter \(x_L\). Because the mixing parameter \(c\) only has a little effect on the \(B_H\) mass, the cross sections are insensitive to \(c\) and we fix \(c = 0.3\) as an example. The influences of parameters \(c', f, \sqrt{s}, x_L\) on the cross sections are shown in Figs.2-4, and there we also take into account the constraints of electroweak precision data on the parameters, i.e., \(f = 1 \sim 2\) TeV is allowed for the mixing parameters \(c, c'\) in the ranges of \(0 \sim 0.5, 0.62 \sim 0.73\), respectively.

As we know, \(c'\) has strong influence on the couplings \(B_H f\bar{f}\), and also effects the \(B_H\) mass. So the cross sections should be very sensitive to \(c'\). The curves shown in Fig.2 are the cross sections as a function of \(c'\). We can see that the cross sections sharply drop to zero when \(c'\) approaches \(\sqrt{\frac{2}{5}}\) except for the \(t\bar{t}B_H\) production. This is because the couplings \(B_H f\bar{f}\), except for \(B_H t\bar{t}\), are proportional to \(c'^2 - \frac{2}{5}\) (with \(y_e = \frac{3}{5}, y_u = -\frac{2}{5}\) for the anomaly free case) and these couplings become decoupled when \(c' = \sqrt{\frac{2}{5}}\). While, for \(c' > \sqrt{\frac{2}{5}}\), the
FIG. 2: The production cross sections of the processes $\gamma\gamma \to f\bar{f}B_H$ as a function of the mixing parameter $c'$, with $f = 2$ TeV, $c = 0.3$, and $\sqrt{s} = 1.5$ TeV. Fig.2(a) for the associated lepton pair productions, Fig.2(b) for the associated light up-type quark pair productions, Fig.2(c) for the associated down-type quark pair productions, Fig.2(d) for the associated heavy top pair production with $x_L = 0.2, 0.5, 0.8$, respectively.

cross sections increase sharply with $c'$ increasing. The typical orders of magnitude of the cross sections are: $O(1)$ fb for $l^+l^-B_H$ productions, $O(10^{-1})$ fb for $u\bar{u}(c\bar{c})B_H$ productions, $O(10^{-3})$ fb for $d\bar{d}(s\bar{s},b\bar{b})B_H$ productions, $O(10^{-2})$ fb for $t\bar{t}B_H$ production. The significant differences of the cross sections among the associated lepton pair productions, associated up-type quark pair productions, and associated down-type quark pair productions are mainly caused by the different coupling strengths of $f\bar{f}B_H$. The couplings of $B_H$ to lepton pair are the strongest among $f\bar{f}B_H$, and the couplings of $B_H$ to up-type quark pair are much stronger than those of $B_H$ to down-type quark pair. The large mass of the top quark sharply depresses the phase space of the $t\bar{t}B_H$ production which makes the cross section of the $t\bar{t}B_H$ production is much smaller than those of $u\bar{u}(c\bar{c})B_H$ productions.

To see the effect of the scale parameter $f$ on the cross sections, we plot the cross sections as a function of $f$ in Fig.3. From the expression of the $B_H$ mass, we can see that the large value of $f$ can make the $B_H$ mass increase sharply, meanwhile depresses the phase space significantly. So the cross sections decrease with the $f$ increasing for the given $c'$. We also show the plots of the cross sections as a function of c.m. energy $\sqrt{s}$.
FIG. 3: The production cross sections of the processes $\gamma\gamma \rightarrow f\bar{f}B_H$ as a function of scale parameter $f$, with $c' = 0.68$, $c = 0.3$, and $\sqrt{s} = 1.5$ TeV. Fig.3(a) for the associated lepton pair productions, Fig.3(b) for the associated light up-type quark pair productions, Fig.3(c) for the associated down-type quark pair productions, Fig.3(d) for the associated heavy top pair production with $x_L = 0.2, 0.5, 0.8$, respectively.

in Fig.4. The cross sections significantly increase with $\sqrt{s}$ increasing at first and then are insensitive to $\sqrt{s}$. Therefore, the ILC, with c.m. energy $\sqrt{s} = 300 - 1500$ GeV, can provide an ideal collision energy to probe $B_H$ via $\gamma\gamma \rightarrow f\bar{f}B_H$.

The integrated luminosity of the ILC can reach 500 fb$^{-1}$ within the first four year running. With the cross sections at the order of $10^{-3} - 10^3$ fb in most case, there are enough $B_H$ events can be produced via the production modes $l^+l^-B_H$. The cross sections of other processes are too small to probe $B_H$. So we only focus on discussing the production modes $l^+l^-B_H$. The potential to detect a particle is not only depended on the event number of the signal produced, but also depended on the branching ratio of certain decay mode to search the particle and the efficiency to reconstruct the signal. On the other hand, the possibility of detection of a particle also depends critically on the width of the associated resonance, and wide resonance can be difficult to detect. The decay modes of $B_H$ have been studied in reference[7]. For $B_H$, the parameter spaces where the large decay width would occur are beyond current search limits in any case. So if $B_H$ would be produced it can be detected via the measurement of the peak in the
invariant mass distribution of its decaying particles. The main decay modes of $B_H$ are $e^+e^- + \mu^+\mu^- + \tau^+\tau^-$, $u\bar{u} + c\bar{c}, d\bar{d} + s\bar{s}, W^+W^-,ZH$. The most interesting decay modes of $B_H$ should be $l'^+l'^-$ ($l' = e, \mu$). This is because the leptons $l'$ can be identified easily and the number of $l'^+l'^-$ background events with such a high invariant mass is very small. So, a search for a peak in the invariant mass distribution of $l'^+l'^-$ is sensitive to the presence of $B_H$. Based on the discussion above, we know that the most interesting final signals would be $l'^+l'^-l'^+l'^-$ with one $l'^+l'^-$ being reconstructed to $B_H$. These signals that we are interested in, are not free from the SM backgrounds. In the SM, with $Z \rightarrow l'^+l'^-$, the same sufficient final states can also be produced via the processes $\gamma\gamma \rightarrow l'^+l'^-Z (\sim \text{pb at TeV scale})$, $\gamma\gamma \rightarrow ZZ (\sim 10^2 \text{ fb at TeV scale})$. However, it should be very easy to distinguish $B_H$ from $Z$ when we look at the $l'^+l'^-$ invariant mass distributions, because there might exist significantly different $l'^+l'^-$ invariant mass distributions between $B_H$ and $Z$. Therefore, the measurement of the $l'^+l'^-$ invariant mass distributions can greatly depress the background, and the production modes $\gamma\gamma \rightarrow l'^+l'^-B_H$ with $B_H$ decaying to $l'^+l'^-$ would open an ideal window to detect $B_H$ with clean background. As we know, the
decay branching ratios of $l'^+l'^-$ approach zero when $c'$ is near $\sqrt{\frac{2}{5}}$. In this case, one could not search $B_H$ via its leptonic decay modes, and the bosonic decay modes $W^+W^-$, $ZH$ would play an important role in searching $B_H$. The decay branching ratios of these bosonic decay modes are significant with $c'$ near $\sqrt{\frac{2}{5}}$, and one can assume enough $W^+W^-$ and $ZH$ signals to be produced with high luminosity. For the decay mode $B_H \rightarrow W^+W^-$, the main SM background should arises from $\gamma\gamma \rightarrow W^+W^−Z$ ($\sim$pb at TeV scale[20]) with Z decaying to $l'^+l'^-$. But the existence of a narrow peak in the $W^+W^-$ invariant mass distribution for the signal can help us to distinguish $B_H$ from the huge background, so the typical signal can also be obtained via the decay mode $W^+W^-$. Another interesting decay mode of $B_H$ is $ZH$ which involves the off-diagonal coupling $HZB_H$ and the experimental precision measurement of such off-diagonal coupling is more easier than that of diagonal coupling. So, the decay mode $ZH$ would provide a better way to verify the crucial feature of quadratic divergence cancellation in Higgs mass. Furthermore, such signal would provide crucial evidence that an observed new gauge boson is of the type predicted in the little Higgs models. For $B_H \rightarrow ZH$, the main interesting final states for the production mode $l'^+l'^-B_H$ should be $l'^+l'^-l'^+l'^-b\bar{b}$. Two b jets reconstruct to the Higgs mass and $l'^+l'^-$ pair reconstructs to the Z mass. The main SM production processes via the $\gamma\gamma$ collision can not produce the same final states[20], so the SM backgrounds for such signal are also very clean.

In summary, the new gauge bosons in the LH model are crucial ingredients for the model. Among these gauge bosons, the $U(1)$ gauge boson $B_H$ with the mass in the range of hundreds GeV is the lightest one and might provide an early signal of the LH model at the ILC. With the realization of photon-photon collision at the ILC, the new gauge boson $B_H$ can be produced via the processes $\gamma\gamma \rightarrow ffB_H$ which are studied in this paper. We find that there are the following features for these processes: (i) The cross sections of these processes are sensitive to the parameters $f,c'$. (ii) Due to the large couplings, the cross sections of the processes $\gamma\gamma \rightarrow l^+l^-B_H(l = e, \mu, \tau)$ are within $10^{-1} - 10^{0}$ fb in most parameter spaces allowed by the electroweak precision data which are the largest among those of $\gamma\gamma \rightarrow ffB_H$. With the high luminosity at the ILC, the sufficient events can be produced to detect $B_H$ via $\gamma\gamma \rightarrow l^+l^-B_H$. Specially, the processes $\gamma\gamma \rightarrow l'^+l'^-B_H(l' = e, \mu)$ could provide a good chance to detect $B_H$ because the leptons $l'$ can be easily identified. (iii) For the other processes except $\gamma\gamma \rightarrow l^+l^-B_H$, the cross sections are too small to detect gauge boson $B_H$. (iv) In most case, the most interesting decay modes of $B_H$ should be $l'^+l'^-$, and a search for a peak in the $l'^+l'^-$ invariant mass distributions are sensitive to the presence of $B_H$ with clean background. When $c'$ is near $\sqrt{\frac{2}{5}}$, the decay modes $W^+W^-$, $ZH$ would complement the search for $B_H$. Therefore, the photon collider realized at the ILC can provide more opportunities to probe $B_H$ and test LH model.
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