Does a magnetic field modify the critical behaviour at the metal-insulator transition in 3-dimensional disordered systems?

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Abstract

The critical behaviour of 3-dimensional disordered systems with magnetic field is investigated by analyzing the spectral fluctuations of the energy spectrum. We show that in the thermodynamic limit we have two different regimes, one for the metallic side and one for the insulating side with different level statistics. The third statistics which occurs only exactly at the critical point is independent of the magnetic field. The critical behaviour which is determined by the symmetry of the system at the critical point should therefore be independent of the magnetic field.

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It is now well known that the introduction of disorder into periodic structures has dramatic effects on the properties of the system. In particular, 3 dimensional (3D) systems exhibit a metal-insulator transition (MIT) as a function of the disorder. A lot of works [1] have already been devoted to the study of this phase transition, theoretically, numerically, as well as experimentally. While all the methods seem to agree on the fact that the MIT should only be characterized by the fundamental symmetries of the system and that one can expect a second order phase transition, its description, e.g. with respect to critical disorder and critical exponents, still remains a controversial object of discussions [1].

Theoretically, the standard method applied, namely the $2+\epsilon$ expansion, for the calculation of the critical exponents turns out to be inadequate for $d = 3$ [2,3]. It seems to be a characteristic of several theories for disordered systems using perturbative methods, like the nonlinear $\sigma$-model [4] or the self-consistent theory [5], to provide quantitative results in the weak localization regime (i.e. for small disorder) but to fail in the region of the critical disorder.

Because of this problem, numerical investigations in the frame of the Anderson model of localization have played an important role in the description of the MIT. Based on extensive computations by means of the transfer matrix method (TMM) [6], it is now usually accepted that the critical exponent is $\nu \simeq 1.4$ and is independent of the distribution chosen in the Hamiltonian [7,8]. This result has been confirmed recently [9,10] using a completely different method, namely the energy level statistics method (ELSM). Meanwhile, other TMM studies have been carried out on similar models but with magnetic field, which changes the symmetry, driving the system from the orthogonal to the unitary universality class, due to the break of the time reversal invariance. Surprisingly, in spite of this change of universality class, the same value of the critical exponent has been found with and without magnetic field and that independent of the strength of the magnetic field [11,12]. In this letter, using the ELSM which will again turn out to be very suitable for such a study, we propose to explain this surprising observation with symmetry arguments shedding new light on the problem. Our conclusions are based on our numerical results for the spacing distribution.
\(P(s)\) and the \(\Delta_3\) statistics showing that there is a critical ensemble (CE) characteristic for the MIT irrespective of the magnetic field which implies that the critical exponent should be independent of the magnetic field.

In order to investigate the MIT with magnetic field we consider the usual Anderson Hamiltonian with an additional phase factor in the off-diagonal elements,

\[
H = \sum_n \epsilon_n |n\rangle \langle n| + \sum_{n \neq m} e^{i \theta_{n,m}} |n\rangle \langle m|,
\]

where the sites \(n\) are distributed regularly in 3D space, e.g. on a simple cubic lattice, with periodic boundary conditions, and \(\theta_{n,m} = -\theta_{m,n} = \theta\). Only interactions with the nearest neighbours are considered. The site energy \(\epsilon_n\) is described by a stochastic variable. In the present investigation we use a box distribution with variance \(\sigma^2 = W^2/12\). \(W\) represents the disorder and is the critical parameter. \(\theta = 0\) describes the case without magnetic field with a Hamiltonian invariant under orthogonal transformation, while \(\theta \neq 0\) corresponds to the magnetic field case which is invariant under unitary transformation. In spite of the simplicity of the model, it contains all the relevant properties necessary to describe the MIT with magnetic field. A similar Hamiltonian has been proposed by Pandey et al. \[13\] and used to study, by ELSM, the transition from the Gaussian orthogonal ensemble (GOE) to the Gaussian unitary ensemble (GUE) in a metallic ring pierced by a magnetic flux \[14\].

Based on this Hamiltonian, the MIT in presence of a magnetic field will be studied by the ELSM, i.e. via the fluctuations of the energy spectrum. This method has already given very interesting results in the case \(\theta = 0\), where the MIT corresponds to a transition from the GOE to the Poisson ensemble (PE) \[13\,16\] which reflects completely uncorrelated energy levels in the localized regime. In the thermodynamic limit one obtains two different regimes: GOE for \(W < W_c\) and PE for \(W > W_c\), which are separated by a critical ensemble (CE) \[10\,17\] occurring at the critical disorder \(W_c\). For \(\theta \neq 0\) one can expect a transition between GUE and PE, but the crucial question is what happens in the vicinity of the critical point.

Before giving the results we shortly review the ELSM. Starting from Eq.(1) the energy
spectrum was computed by means of the Lanczos algorithm (which is suited to diagonalize such very sparse secular matrices) for systems of size $M \times M \times M$ with $M = 13$ and 21, disorder $W$ ranging from 3 to 80 and phase $\theta = 0.1\pi$. The number of different realizations of the random site energies $\epsilon_n$ was chosen so that about $2 \cdot 10^5$ eigenvalues were obtained for every pair of parameters $(M, W)$ which means between 25 and 90 realizations, for which the full spectrum has been computed. For the subsequent investigations only half of the spectrum around the band center is considered so that the results are not deteriorated by the strongly localized states near the band edges. After unfolding the obtained spectrum the fluctuations can be appropriately characterized by means of the spacing distribution $P(s)$ and the Dyson-Metha statistics $\Delta_3$. $P(s)$ measures the level repulsion, it is normalized, so is its first moment because the spectrum is unfolded. $\Delta_3$ which measures the spectral rigidity is given by

$$\Delta_3(L) = \left\langle \frac{1}{L} \min_{A,B} \int_{\varepsilon'}^{\varepsilon' + L} (N(\varepsilon) - A\varepsilon - B)^2 \, d\varepsilon \right\rangle_{\varepsilon'} .$$

where $\langle \rangle_{\varepsilon'}$ means that we average over different parts of the spectrum.

Using the random matrix theory (RMT) it is possible to calculate $P(s)$ and $\Delta_3$ for the two limiting cases of the spectrum, namely the GUE and the PE. For the metallic side one obtains

$$P_{\text{GUE}}(s) \simeq \frac{32}{\pi^2} s \exp\left(-\frac{4}{\pi} s^2\right)$$

$$\Delta_3(L) = \frac{2}{L^4} \int_0^L \left( L^3 - 2L^2 r + r^3 \right) \Sigma^2_{\text{GUE}}(r) \, dr$$

where $\Sigma^2_{\text{GUE}}(r)$ is the variance of the number of levels in a spectral window of width $r$ and is given by

$$\Sigma^2_{\text{GUE}}(r) = \frac{2}{\pi^2} \left[ \ln(2\pi r) + \gamma + 1 - \cos(2\pi r) - \text{Ci}(2\pi r) + 2r \left(1 - \frac{2}{\pi} \text{Si}(2\pi r)\right) \right]$$

The formula (3) for $P_{\text{GUE}}(s)$ is exact only in the case of $2 \times 2$ matrices but remains a good approximation for the other cases. For $\Delta_3$ there is no analytical solution and the integral (4) has to be calculated numerically.
For the localized case we have

\[ P_{PE}(s) = e^{-s}, \]  

(6)

\[ \Delta_3(L) = \frac{L}{15}. \]  

(7)

In Fig.1 the results for the Dyson-Metha statistics are reported. We find, as expected, the GUE and the PE regimes for small and large disorder respectively as well as the continuous transition between them as a function of \( W \). In a previous work \[17\] it was shown for the case \( \theta = 0 \) that the \( \Delta_3 \)-curves are functions of the system size except at the critical point where they are size independent. Moreover the curves were shown to move with increasing \( M \) towards the GOE for \( W < W_c \) and towards the PE for \( W > W_c \). In Fig.2 we observe a similar behaviour for \( \theta = 0.1\pi \), accordingly this time the curves move towards the GUE and the PE. The critical value of the disorder where the curves are size independent turns out to be \( W_c \approx 16.5 \). Correspondingly, in the thermodynamic limit we expect two different regimes with the GUE and the PE, separated by the CE at the MIT. This has now to be compared to the CE obtained in the case without magnetic field. In Fig.3 one sees that the CE curves are identical and thus the symmetry should be the same, too. Moreover this was checked for \( \theta = \pi/2 \) which is the "most Hermitian" case and the same results were obtained (cp. Fig.3). Only the critical disorder, which is not a universal value, is slightly shifted upward towards \( W_c \approx 16.65 \). Such a behaviour has already been noted in a different numerical approach \[12\].

The CE curve seems to be proportional to \( L^{3/4} \) for small \( L \) and becomes linear when increasing \( L \). This would mean that the shape of the curve cannot solely be described neither by the results of Kravtzov et al. \[19\] nor by the results of Alt’shuler et al. \[16\] but rather would be in agreement with Ref. \[13\] for small \( L \) and with Ref. \[17\] for large \( L \). Moreover it has to be stressed again that the CE curve is completely independent of the magnetic field in contrast to Ref. \[19\].

The same results are derived from the study of \( P(s) \) in Fig.4. For small and large disorder we again find the GUE and the PE respectively while \( P(s) \), at the critical point,
is independent of $M$ and $\theta$. Another interesting point is the controversy \cite{10,20} about the asymptotic behaviour of $P(s)$. A careful analysis of the shape of $P(s)$ at the critical point gives

$$P(s) = As \exp(-Bs^\alpha)$$

with $\alpha = 5/4 \pm 0.05$ and $A,B$ the normalization constants which supports the results obtained in Ref. \cite{20}. Moreover one notes that the value of $\alpha$ is in very good agreement with the formula $\alpha = 1 + 1/\nu d$ obtained by Kravtzov et al. \cite{19} relating $\alpha$ to the critical exponent $\nu$ and the dimension $d$ of the system. Finally it has to be stressed that the dependence of the results in \cite{13,20} on $\beta$ ($\beta = 1$ for the GOE and 2 for the GUE) via some coefficients has not been derived from the calculation but assumed from the very beginning leaving open the question whether those results, at the critical point, depend on the magnetic field or not. Figure \ref{fig} demonstrates that this is indeed the case.

As the critical behaviour is determined by the fundamental symmetries of the system one should consider the symmetry at the critical point. From the results in Fig.\ref{fig3} and \ref{fig4} we conclude that the magnetic field has no influence on the critical behaviour. This means also that the critical exponent $\nu = 1.34 \pm 0.10$, which was obtained \cite{9} for the $\theta = 0$ case, is transferable to the case with magnetic field.

Finally we compare these results with experiments. Recent measurements performed on the persistent photoconductor Al$_{0.3}$Ga$_{0.7}$As \cite{21} and on uncompensated Si:P \cite{22} suggest, although the situation is still not completely clear, that the MIT is effectively independent of the magnetic field. About the value of the critical exponent the situation is not really clearer but it can be mentioned that in Ref. \cite{22}, considering carefully the range of the critical behaviour, a critical exponent of $\nu \approx 1.3$ has been found which would be in agreement with our results. This seems to indicate, at least in some cases, that the MIT is in fact driven purely by the disorder. Then it is not necessary to include interactions to characterize the critical behaviour although these can be important outside the critical regime. It has to be noted that some reserves have been made \cite{23} about the results obtained in Ref. \cite{22} and it
will be interesting to check carefully this problem in order to clarify the situation.

In conclusion we have shown, using the statistical properties of the energy spectrum of the Anderson model of localization, that the MIT, which is determined by the symmetry of the system at the critical point, is not influenced by the magnetic field. This means that both cases, with or without magnetic field belong to the same universality class, described by the CE, which opens new perspectives in the comprehension of the MIT. Although this universality class is defined only for the critical point it is sufficient to fix the properties of the critical behaviour. This is in agreement with previous numerical results \cite{11,12} and recent experiments \cite{21,22}. We point out that these results are valid in 3D systems and do not apply to 2D where we know that the MIT is driven by the magnetic field \cite{24}. 
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FIGURES

FIG. 1. Dyson-Metha statistics $\Delta_3(L)$ for $M = 21$. A continuous transition from GUE to Poisson statistics occurs as a function of the disorder $W$ (denoted on the right-hand side). The dotted line shows the GOE result for comparison.

FIG. 2. Dyson-Metha statistics $\Delta_3(L)$ for system sizes $M = 13$ and 21 and $W = 14, 16.5, 19$.

FIG. 3. The Dyson-Metha statistics $\Delta_3(L)$ at the critical point for different values of $\theta$.

FIG. 4. Spacing distribution $P(s)$ for $M = 21$, $W = 3, 80$ and $\theta = 0.1\pi$. The histograms are the numerical results and the full lines reflect the two expected limiting ensembles, namely the GUE for the metallic side and the PE for the insulating side. The points ◦, * represent $P(s)$ at the critical point for $M = 21$ and $M = 13$. The symbol ◦ denotes the CE for the case without magnetic field ($\theta = 0$).
