Imprinting Quantum Fluctuations on Hydrodynamic Initial Conditions

J.S. Morelanda,b, Z. Qiua, U. Heinz

aThe Ohio State University, 191 West Woodruff Avenue, Columbus OH 43210, USA
bDuke University, Physics Bldg., Science Dr., Box 90305, Durham, NC 27708, USA

Abstract
We have developed an algorithm to imprint quantum fluctuations onto the initial transverse energy density profile according to a given two-point covariance function. Using as an example MC-KLN initial conditions with added fluctuations satisfying the covariance function derived in [1], we find that effects from sub-nucleonic gluon field fluctuations on the eccentricity harmonics $\epsilon_n$ vary strongly with the gluonic correlation length controlled by the saturation momentum $Q_s$. Varying $Q_s$ over the range probed in Au+Au collisions at RHIC, we find gluon fluctuation induced enhancements of the eccentricity coefficients ranging from 10 to 20% in central collisions.

1. Event-by-event fluctuations

The importance of event-by-event fluctuations in the initial conditions context of heavy-ion collisions was first pointed out by Miller and Snellings who added nucleon position fluctuations to existing smooth (i.e. ensemble-averaged) Glauber initial conditions [2, 3]. These fluctuations explain the experimental observation of non-vanishing anisotropic flow in central Cu-Cu and Au-Au collisions [4] and of odd flow harmonics $v_3, v_5, ...$ [5].

Recently attention has turned to a new source of event-by-event fluctuations, namely fluctuations in the transverse distribution of color charge within the colliding nucleons. These fluctuations are evidenced by large multiplicity fluctuations observed in minimum bias $pp$ collisions and suggest that sub-nucleonic degrees of freedom may also play an important role in determining the event-by-event geometry of the initial state in nucleus-nucleus collisions [6, 7, 8].

2. From nucleonic to sub-nucleonic fluctuations

In this work, we develop a toy model for imprinting sub-nucleonic fluctuations on the transverse energy density profiles produced in relativistic heavy-ion collisions. The study is motivated by recent work of Müller and Schäfer in which they calculate the mean normalized covariance function $\text{Cov}[\epsilon(r)/\epsilon_0]$ for the transverse energy density fluctuations of gluon fields in central Au-Au collisions at 200 GeV [1]. The authors approximate the collision system by two infinite slabs of nuclear matter with fixed gluon saturation momentum $Q_s$ for which they take the value corresponding to the nuclear thickness function in the center of a central Au-Au collision.

We texture a given transverse energy density profile with additional fluctuations using a Turning Band Gaussian random field simulator (TBSIM) that includes several configurable covariance functions. The Müller-Schäfer covariance $\text{Cov}[\epsilon(r)/\epsilon_0] = (\Delta\epsilon(r)/\epsilon_0)^2$ is well described by
the Cauchy covariance included in TBSIM,
\[
\text{Cov}[r] = C(1 + (r/a)^2)^{-b}
\]  
with fit parameters \(C = 0.4679\), \(a = 0.2878\) fm and \(b = 1.7732\).

Using TBSIM we generate a large Gaussian random field (GRF) with the desired covariance on a \(4000 \times 4000\) lattice with grid spacing \(\Delta x = 0.1\) fm. The left panel in Fig. 1 shows a small slice of this GRF; in the right panel its two-point correlation function is compared to the Müller-Schäfer covariance and its Cauchy fit.

Due to the tail of the Gaussian distribution, there is a 7.2% chance that fluctuations about the mean fall into the unphysical region of negative energy density. These unphysical fluctuations can be eliminated by mapping the Gaussian random variable onto a positive definite negative binomial distribution (NBD). There is a long history of modeling fluctuations in \(pp\) collisions with negative binomial distributions (for recent examples see [6, 8]). Writing
\[
\text{NBD}(\bar{n}, k; n) = \frac{\Gamma(k + n)}{\Gamma(k) \Gamma(n + 1)} \frac{\bar{n}^k k^k}{(\bar{n} + k)^{n+k}}
\]  
where \(n\) is the sampled value, \(\bar{n}\) is its mean and \(k\) controls its variance, we identify \(n/\bar{n}\) with \(\epsilon(r)/\epsilon_0\). In the limit of large \(\bar{n}\) (we take \(\bar{n} = 100\)), the NBD becomes a continuous function \(P_{\text{NBD}}(y)\) of the reduced variable \(y = n/\bar{n}\) whose width parameter \(k\) we adjust such that its variance \((\langle n/\bar{n} - 1 \rangle) = 1/k \bar{n}\) agrees with the squared Gaussian width \(\text{Cov}[\epsilon(r)/\epsilon_0]_{r=0} = (\Delta \epsilon/\epsilon_0)^2 = (0.684)^2\). The mapping is now achieved by replacing each value of \(x = \epsilon/\epsilon_0\) from the Gaussian random field by a new value \(y = \epsilon_{\text{new}}/\epsilon_0\) such that the cumulative Gaussian distribution function at \(x\) coincides with the cumulative NBD distribution at \(n/\bar{n} = \epsilon_{\text{new}}/\epsilon_0 = y\):
\[
\int_{-\infty}^{x} P_{\text{Gauss}}(x') \, dx' = \int_{0}^{y} P_{\text{NBD}}(y') \, dy'.
\]  
The resulting negative binomial random field \(\epsilon(r)/\epsilon_0\) is positive definite and retains the two-point covariance embedded in the original GRF. A small section of this field and its two-point correlation function are shown in Fig. 2.
To imprint these energy density fluctuations onto the initial conditions for a heavy-ion collision, we take the transverse energy densities $dE/d^2r_{\perp}dy$ for events generated with the MC-KLN model [10] and multiply them with the NBD random field $\epsilon(r)/\epsilon_0$ field taken from randomly selected and appropriately sized sections of the final $4000 \times 4000$ field grid:

$$\frac{dE_{\text{fluct}}(r)}{d^2r dy} = \frac{dE_{\text{KLN}}(r)}{d^2r dy} \times \frac{\epsilon(r)}{\epsilon_0}. \quad (4)$$

3. Results and Conclusions

This texturing procedure was applied to 20,000 MC-KLN Au-Au events at $\sqrt{s} = 200$ A GeV (using the MC-KLN code with Gaussian nucleons of width $\sigma = 0.54$ fm), partitioned into equally sized bins in the number of participants of width $\Delta N_{\text{part}} = 1000$. For both the textured and untextured events we compute the harmonic eccentricity coefficients $\epsilon_n$ using the definition

$$\epsilon_n = \frac{1}{2\pi R} \int_0^{2\pi} \int_0^R r^2 e^{in\phi} \frac{dE(r,\phi)}{dr dy} dr d\phi,$$

and average them over the event ensemble. The ratios of these averages are plotted in Fig. 3 for the harmonics $n = 2, \ldots, 5$ as functions of $N_{\text{part}}$.

For the nominal correlation length $a = 0.28$ fm, shown in the left panel, we see that the gluon-field fluctuations induce only a small increase in the eccentricity harmonics $\epsilon_n$, reaching 5-10% in central collisions and falling off in more peripheral ones. We should note, however, that our fluctuation texture assumes constant (i.e. position-independent) $Q$ or $a$, with a value expected (on average) in the center of central Au-Au collisions. More realistically, $a$ should vary inversely with the position-dependent nuclear thickness function (which, on average, is largest in the fireball center):

$$a^2 \propto 1/Q^2 \propto 1/T(r). \quad (6)$$

Consequently, the Müller-Schäfer texture with constant $a = 0.28$ fm provides an approximate lower bound on the eccentricity enhancement caused by sub-nucleonic fluctuations in 200 A GeV Au-Au collisions.
To obtain an estimated upper bound on the eccentricity enhancement caused by sub-nucleonic fluctuations, we inflate the correlation length $a$ to the radius of our Gaussian nucleons, $\sigma = 0.54$ fm and repeat the texturing procedure. (This amounts to reducing $Q_s$ by about a factor 2.) As seen in the right panel of Fig. 3 with the larger correlation length the gluon fluctuations increase the eccentricities $\epsilon_n$ by larger factors, reaching now 20-25% in central collisions (and again falling off in peripheral ones). For both values of the correlation length, the sub-nucleonic fluctuation effects on $\epsilon_n$ are strongest in central collisions; in more peripheral collisions, fluctuations in the nucleon positions dominate the fluctuation effects on $\epsilon_n$.

To summarize, we generated a toy model to analyze the effects of sub-nucleonic color fluctuations on the centrality dependent eccentricity harmonics $\epsilon_n$ for MC-KLN initial conditions. While we qualitatively confirm earlier findings [7, 8] that such fluctuations tend to increase the $\epsilon_n$, only relatively small enhancements (smaller than those reported in [8]) are found for realistic values of the gluon field correlation length. Correlations over larger distances generate larger eccentricities. Due to the assumption of a position-independent saturation momentum $Q_s$ or correlation length $a$, our implementation of sub-nucleonic fluctuations is much less realistic than the one in the IP-Glasma model of Refs. [6, 7]; it has, however, the advantage of allowing us to turn the sub-nucleonic fluctuations on and off at will and thus to study their effects on $\epsilon_n$ in isolation.

Acknowledgements: This work was supported by the U.S. Department of Energy under Grants No. DE-SC0004286 and (within the framework of the JET Collaboration) DE-SC0004104, and by the Ohio Supercomputer Center.

References

[1] B. Müller and A. Schafer, Phys. Rev. D 85, 114030 (2012).
[2] M. Miller and R. Snellings, arXiv:nucl-ex/0312008.
[3] M. L. Miller, K. Reygers, S. J. Sanders and P. Steinberg, Ann. Rev. Nucl. Part. Sci. 57, 205 (2007).
[4] B. Alver et al. [PHOBOS Collaboration], Phys. Rev. Lett. 98, 242302 (2007).
[5] B. Alver and G. Roland, Phys. Rev. C 81, 054905 (2010).
[6] P. Tribedy and R. Venugopalan, Nucl. Phys. A 859, 136 (2011); Phys. Lett. B 710, 125 (2012).
[7] B. Schenke, P. Tribedy and R. Venugopalan, Phys. Rev. Lett. 108, 252301 (2012); Phys. Rev. C 86, 034908 (2012).
[8] A. Dumitru, Y. Nara, Phys. Rev. C 85, 034907 (2012).
[9] X. Emery and C. Lantuejoul, Computers and Geosciences 32, 1615 (2006).
[10] H. J. Drescher and Y. Nara, Phys. Rev. C 75, 034905 (2007).