Correlator expansion approach to stationary states of weakly coupled cavity arrays

Elena del Valle and Michael J Hartmann

Physik Department, Technische Universität München, James-Franck-Straße, D-85748 Garching, Germany
E-mail: elena.delvalle.reboul@gmail.com

Received 12 April 2013, in final form 7 June 2013
Published 7 November 2013
Online at stacks.iop.org/JPhysB/46/224023

Abstract
We introduce a method for calculating the stationary state of a translation invariant array of weakly coupled cavities in the presence of dissipation and coherent as well as incoherent drives. Instead of computing the full density matrix, our method directly calculates the correlation functions which are relevant for obtaining all local quantities of interest. It considers an expansion of the correlation functions and their equations of motion in powers of the photon tunnelling rate between adjacent cavities, leading to an exact second-order solution for any number of cavities. Our method provides a controllable approximation for weak tunnelling rates applicable to the strongly correlated regime that is dominated by nonlinearities in the cavities and thus of high interest.

(Some figures may appear in colour only in the online journal)

1. Introduction

The study of light–matter interactions that are enhanced by confining light fields in electromagnetic cavities has been a thriving discipline of quantum optics throughout the past decades. Particularly with the advent of realizations of the so-called strong coupling regime where the strength of the interaction between a photon and a quantum emitter exceeds the decay mechanisms for photons and emitters, experimental investigations of the coherent interactions between single emitters and individual photons became possible [1].

In recent years, a new direction of cavity quantum electrodynamics (cavity-QED) has developed, in which multiple cavities that are coupled via the exchange of photons are considered. Such setups are particularly intriguing if the cavities are connected forming an array and the strong coupling regime is achieved in each cavity of the array. These devices would then give rise to quantum many-body systems of strongly interacting photons and polaritons [2–4]. As an alternative to a cavity array, one may also consider optical fibres that couple to nearby atoms [5, 6] or even clouds of Rydberg atoms that are optically thick in free space [7]. Both these systems avoid the need to build mutually resonant cavities, which is possible [8] but can be rather challenging in the optical range. For microwave photons, it is however perfectly feasible to build large arrays of mutually resonant cavities on one chip in an architecture known as circuit-QED [9, 10].

For strongly interacting polaritons and photons in coupled arrays of micro-cavities and optical fibers, possibilities of observing equilibrium phenomena, such as a Mott insulator [2–4] or a Tonks–Girardeau gas [5, 6], have mostly been addressed so far and the development has been summarized in the reviews [11, 12, 9]. In every experiment that involves light–matter interactions, some photons will however inevitably be lost from the structure due to imperfect light confinement or emitter relaxation. To compensate for such losses, coupled cavity arrays are thus most naturally studied in a regime where a coherent or incoherent input continuously replaces the dissipated excitations, so that thermal states are not an appropriate description of the system. This mode of operation eventually gives rise to a driven dissipative regime, where the dynamical balance of loading and loss processes leads to the emergence of stationary states. Yet the properties of stationary states of driven dissipative systems are only explored to a much lesser degree than the properties of thermal equilibrium states.
For coupled cavities, small arrays have been considered exactly [13] and mean field approaches for larger arrays have been employed [14]. Moreover, numerical studies found signatures for crystallization [15] and photon solids were predicted for arrays with cross-Kerr interactions [16].

As driven dissipative quantum many-body systems have to date only barely been explored, there is a need for technical tools for their efficient description. Here we introduce a perturbative technique for the calculation of the physical properties of stationary states of driven dissipative cavity arrays. In order to focus on the supposedly simpler cases and obtain a better understanding of their main physical properties, our approach assumes a large, translation invariant cavity array where the quantum states of all cavities are identical. Of course, imperfections in fabrication procedures may induce some disorder in the array. For implementations of identical cavities is very small [17]. Moreover, studying identical cavities is the first step in the analysis, whereas small inhomogeneities in the strength can be applied. Within these limits, we allow the interaction strengths \( J \), \( \Delta \), \( \Omega \) to describe the full \( N \)-cavity system, i.e., \( \mathcal{O} \) includes all the sought observables as well as operators which couple to them through the equations of motion, \( \{ a_j^{†m} a_j^n, a_k^{†r} a_k^s \} \).

To simplify notation, we express this general correlator as \( \{m, n, \alpha, \beta \} \). Naturally, in the case of an anharmonic mode or in the presence of any anharmonicity, one must choose an appropriate truncation in the number of excitations, i.e. \( n, m \leq n_{\text{max}} \), that also truncates the number of correlations in this set. For a given driving intensity, the truncation must be high enough to yield converged, accurate results.

From the master equation (2) one can obtain the set of coupled equations for the full set of operators \( \mathcal{O} \), which we write in the matrix form [18]

\[
\hat{\rho}_t = \hat{M} \hat{\rho}_t + \hat{I},
\]

(4) with all correlators forming the vector \( \hat{\nu} \), i.e. \( \hat{\nu}^T = (\langle a_1 \rangle, \langle a_1^+ \rangle, \ldots, \langle a_1 a_2 \rangle, \langle a_1^+ a_2^+ \rangle, \ldots) \), where the exponent \( T \) denotes the transpose. The coefficient matrix \( \hat{M} \) and vector \( \hat{I} \) are derived from the master equation in a systematic way [19, 21], as we explicitly show in the appendix. The solution of equation (4) is completely equivalent to computing the correlators as in equation (3), from the density matrix obtained by solving equation (2). The exact solution in the steady state (if it is unique and exists) simply reads

\[
\hat{\nu} = -\hat{M}^{-1} \hat{I}.
\]

(5)

We can considerably reduce the number of operators to a minimal set by making use of the translational symmetry in the 1D chain (circle). That is, all correlators that are left or right circular rotations of the elements in \( \{m, n, \alpha, \beta \} \) are redundant because they are exactly the same. There is,
therefore, a maximum of \(2N\) representations of the same correlator (less if some of the pairs are \([0,0]\) or mutually equal). We can choose an arbitrary rule to systematically keep only one representative of such a set of redundant correlators, for instance, we choose the ones where \((N = 4)\)

(i) The nonzero sets are always to the left and as cluttered together as possible, such as \([4, 2], [3, 0], [0, 0], [0, 0]\). This is naturally achieved if the set of operators is built starting from the element \([\eta_{\max}, \eta_{\max}]; [0, 0], [0, 0], [0, 0]\).

(ii) The choice of circulation in the array is made so that the largest sum of indices is most to the left: \(\mu + \nu \geq \alpha + \beta\), such as \([3, 3], [0, 2], [1, 1], [1, 2]\) would become \([3, 3], [1, 2], [1, 1], [0, 2]\] (inverting the circulation).

(iii) If there are two pairs with an equal sum, reduced system of equations is now given by correlator (less if some of the pairs are equal).

\[\sum_{\beta} \{a_{\mu}^\dagger a_{\nu}^\dagger a_{\alpha}^\dagger a_{\beta}^\dagger \} = \cdots = \sum_{\beta} \{a_{\mu}^\dagger a_{\nu}^\dagger a_{\alpha}^\dagger a_{\beta}^\dagger \} = \{m, \mu, \nu, \alpha, \beta\}; [0, 0], [0, 0], [0, 0]\].

We denote by \(b\) the photon annihilation operator for the second cavity, and apply the rules to extract the minimal set of operators \((m + n \geq \mu + \nu, \nu \geq \mu\) in the case of the degeneracy of the sum).

For example, for a truncation in each cavity system with \(n_{\max} = 2\) photons, we thus have a dimension of 8 for \(v_a\) and 36 for \(v_b\). A third subset, \(v_c\), includes correlators with three cavities, \(\{a_{\mu}^\dagger a_{\nu}^\dagger a_{\alpha}^\dagger a_{\beta}^\dagger \} = \{a_{\mu}^\dagger a_{\nu}^\dagger a_{\alpha}^\dagger a_{\beta}^\dagger \} = \cdots = \{a_{\mu}^\dagger a_{\nu}^\dagger a_{\alpha}^\dagger a_{\beta}^\dagger \} = \{m, \mu, \nu, \alpha, \beta\}; [0, 0], [0, 0], [0, 0]\].

Similar to \(b\), we denote by \(c\) the photon annihilation operator in the third cavity, where we have applied the circular rules to obtain the minimal set of operators. Finally, specifically to \(N = 4\), we need to consider \(v_d\), which includes correlators with four cavities \(\{a_{\mu}^\dagger a_{\nu}^\dagger a_{\alpha}^\dagger a_{\beta}^\dagger \} = \cdots = \{a_{\mu}^\dagger a_{\nu}^\dagger a_{\alpha}^\dagger a_{\beta}^\dagger \} = \{m, \mu, \nu, \alpha, \beta\}; [0, 0], [0, 0], [0, 0]\].

We can rewrite equation (4) in terms of these subsets of correlators, each with a different dimension, as five coupled matrix equations,

\[\partial_t v_a = (M_a + i\mathcal{J}_S) v_a + I_a + iJR_{ab} v_b, \quad (9)\]

\[\partial_t v_b = (M_b + i\mathcal{J}_S) v_b + (B_{bc} + iJR_{bc}) v_a + iJR_{cd} v_c, \quad (10)\]

\[\partial_t v_c = (M_c + i\mathcal{J}_S) v_c + (B_{cd} + iJR_{cd}) v_b + iJR_{de} v_d + (B_{de} + iJR_{de}) v_c, \quad (11)\]

\[\partial_t v_d = (M_d + i\mathcal{J}_S) v_d + (B_{de} + iJR_{de}) v_c, \quad (12)\]

\[\partial_t v_e = (M_e + i\mathcal{J}_S) v_e + (B_{ea} + iJR_{ea}) v_a + iJR_{ec} v_c, \quad (13)\]

Here, we have separated the effect of the hopping \(J\) into the self-renormalization matrices \(S\) and the linking matrices \(R\) that only contain integer numbers. The vector \(I\) and matrices \(B\) contain only the driving parameters \(\Omega\) and \(P\) (coherent or incoherent). Other internal parameters such as \(\Delta, \gamma\) and \(U\) enter in the matrices \(M\).

One can solve these equations recurrently in the steady state, from bottom to top. This may be useful for a small number of cavities where the expressions are simple. For instance, for \(N = 2\), the system reduces to \(v_a\) and \(v_b\) with equations (9) and (10), and solutions

\[v_a = -(M_a + i\mathcal{J}_S) v_a + iJR_{ab} F_{ba}^{-1} I_a, \quad (14)\]

\[v_b = F_{ba} v_a, \quad (15)\]

where \(F_{ba} = -(M_b + i\mathcal{J}_S)^{-1} (B_{bc} + iJR_{bc}) v_a\). Similarly, for \(N = 3\), we have

\[v_a = -(M_a + i\mathcal{J}_S) v_a + iJR_{ab} F_{ba}^{-1} I_a, \quad (16)\]

\[v_b = F_{ba} v_a \quad \text{and} \quad v_c = F_{cb} v_b, \quad (17)\]

where \(F_{ba} = -(M_b + i\mathcal{J}_S)^{-1} (B_{bc} + iJR_{bc}) v_a\) and \(F_{cb} = -(M_c + i\mathcal{J}_S)^{-1} (B_{cb} + iJR_{cb})\). This recursive procedure
is possible in principle for any $N$, although, in general not very practical, given that the exact solution can also be obtained by simply inverting one matrix, $M$, as in equation (6).

Anyhow, obtaining the exact solution becomes exceedingly cumbersome for a large number of cavities, $N \gg 1$. In the following, we therefore concentrate on finding an approximate solution for the weak coupling regime, where $J \ll \gamma$ and there is no manifestation of the array’s normal modes in the dynamics [20]. For this purpose, we divide all equations by $\gamma$, meaning that all parameters become dimensionless, in the form $J/\gamma$, $\Omega/\gamma$, $U/\gamma$, $\alpha/\gamma$, $P/\gamma$ as well as time, $t^\prime = \gamma t$.

3. Approximated solution to second order in $J/\gamma$

In order to find an approximate semi-analytical expression for the steady state of a cavity, $v_a$, we expand both the correlators in $v$ and the set of equations (9)-(13) in powers of $J/\gamma$ up to second order. More precisely, second order for $v_a$ requires for consistency the following lower orders in the other subsets:

$$v_a = v_a^{(0)} + (J/\gamma)v_a^{(1)} + (J/\gamma)^2 v_a^{(2)} + \cdots, \quad (18)$$

$$v_b = v_b^{(0)} + (J/\gamma)v_b^{(1)} + \cdots, \quad (19)$$

$$v_c = v_c^{(0)} + \cdots, \quad (20)$$

$$v_d = v_d^{(0)} + \cdots, \quad (21)$$

$$v_e = v_e^{(0)} + \cdots. \quad (22)$$

The expanded equations read in these terms

$$\partial_t v_a = 0 = [M_a v_a^{(0)} + h_a] + (J/\gamma)[M_a v_a^{(1)} + iS_a v_a^{(0)} + iR_{ab} v_b^{(0)}] + (J/\gamma)^2 [M_a v_a^{(2)} + iS_a v_a^{(1)} + iR_{ab} v_b^{(1)}] + \cdots. \quad (23)$$

$$\partial_t v_b = 0 = [M_b v_b^{(0)} + h_b v_b^{(0)}] + (J/\gamma)[M_b v_b^{(1)} + iS_b v_b^{(0)} + B_{ba} v_a^{(1)} + iR_{ba} v_a^{(0)} + iR_{bc} v_c^{(0)}] + \cdots. \quad (24)$$

We truncate the equations at this point, since the solutions obtained by setting each square bracket to zero,

$$v_a^{(0)} = -M_a^{-1} h_a, \quad v_b^{(0)} = -M_b^{-1} B_{ba} v_a^{(0)}, \quad (25)$$

$$v_a^{(1)} = -M_a^{-1} [iS_a v_a^{(0)} + iR_{ab} v_b^{(0)}], \quad (26)$$

$$v_b^{(1)} = -M_b^{-1} [iR_{ba} v_a^{(0)} + B_{ba} v_a^{(1)} + iS_b v_b^{(0)} + iR_{bc} v_c^{(0)}], \quad (27)$$

$$v_a^{(2)} = -M_a^{-1} [iS_a v_a^{(1)} + iR_{ab} v_b^{(1)}], \quad (28)$$

ultimately depend on $v_c^{(0)}$ only. Obtaining $v_c^{(0)}$ from the equations would in turn require the knowledge of $v_e^{(0)}$, but this is not needed given that the zero order is simply the uncoupled limit, that is, products of the solutions for $N = 1$ as in equation (8). For instance, the uncorrelated solution for $v_b^{(0)}$, corresponding to $(\langle a^m a^* b^m b^* \rangle^{(0)}) = (\langle a^m a^* d^m \rangle \langle a^m a^* \rangle)$, can be directly obtained through the product of twice $v_a^{(0)}$, as $v_b^{(0)} = v_a^{(0)} X_b v_a^{(0)}$, where $X_b$ is the corresponding mixing matrix

obtained by inspection. This is completely equivalent to the linear algebra solution of equation (25). The same applies for $v_c^{(0)}$ but with two mixing matrices, $v_c^{(0)} = v_a^{(0)} X_c v_a^{(0)} X_c X_c v_a^{(0)}$.

These solutions are valid for $N \geq 4$, since adding more cavities to the array does not produce any structural qualitative change to second order in $J/\gamma$. This is independent of the boundary conditions, having a linear or circular array configuration. It is formally the same for $N = 2$ and $3$ (setting $v_c^{(0)} = 0$ for $N = 2$), but differs quantitatively to first and second orders, respectively, due to different coefficients in the equations.

Note that quantities related to the cooperativity or interaction between neighbouring cavities, such as $(ab)^7$ or $(a^3 a^* b^2 b^3)$, are included in $v_b$ and, therefore, obtained to first order in $J/\gamma$ through this method.

4. Comparison between the exact and approximated results

Since the approximated solutions are single valued, this method cannot reproduce regimes where several steady states are compatible for the individual system (corresponding to different steady states of the ensemble) or any other instability regions like lasing. Its perturbative nature allows it only to describe regimes where the coupling is smaller than the effective decoherence or driving (weak coupling regime).

We illustrate the interest of this method by comparing in figure 1 the exact solution for $N = 4$ (in solid black line) and the DMRG solution for $N = 21$ (black circles) with the approximated ones, valid for $N \geq 4$. Here the DMRG results have been obtained with the method explained in [15] for a chain of cavities with open boundary conditions where the quoted values have been found for cavity number 11 in the centre of the chain. The agreement of the results thus shows that boundary effects are negligible in this example.

We have chosen two quantities of interest, the mean cavity population, $n_a = \langle a^\dagger a \rangle$, and its second-order coherence function at zero delay, $g^{(2)}(0) = \langle a^\dagger a^\dagger a a \rangle / n_a^2$. Figures 1(a) and (b) show that both are well approximated by the second-order solution (in dashed red) as long as $J < \Omega, \gamma$. Lower order approximations (in blue and green) deviate from the exact solution at even lower $J/\gamma$. In figures 1(c) and (d), we fix $J = 0.3 \gamma < \Omega = 0.5 \gamma$ and scan the system resonances by tuning the laser frequency. In this case, we observe that the second-order approximation remains very close to the exact solution for all frequencies, while the first and zeroth orders deviate from it, notably, close to the various cavity resonances at 0, $U/2$, $U$ (marked with vertical lines). The first-order approximation breaks down near the one-photon resonance at $0$ as evidenced by the negative value of $g^{(2)}(0)$ in the zoomed inset of figure 1(d).

Due to the form of the dissipation terms in equation (2) we cannot illustrate the method in the absence of a coherent drive, only under the action of an incoherent pumping. An incoherent pump bringing the system into a steady state is equivalent to letting each cavity interact with a thermal bath, where $P = n_T \gamma_0$ and $\gamma = (1 + n_T) \gamma_0$. Here, $\gamma_0$ is the decay rate at zero temperature and $n_T$ is the occupation number of
Figure 1. Mean cavity population, $n_a$, and second-order coherence function, $g^{(2)}(0)$. The exact solution for $N = 4$ cavities (solid black line) and the DMRG solution for $N = 21$ (black circles) are compared to second-order (dashed red line), first-order (dashed blue line) and zero-order (dotted green line) approximations, valid for $N \geq 4$. In (a) and (b), we fix $\Omega = 0.7\gamma$ and $\Delta = 0$ and vary the photon tunnelling rate $J/\gamma$. In (c) and (d), we fix $\Omega = 0.5\gamma$ and $J = 0.3\gamma$ and vary the laser frequency $\omega_L$. In the inset of (d), we have magnified the region around 0. Other parameters: $U = 6\gamma$, $P = 0$, $n_{\text{max}} = 2$.

Figure 2. (a) Mean cavity population, $n_a$, and (b) second-order coherence function, $g^{(2)}(0)$, as a function of the thermal bath occupation. The exact solution for $N = 4$ cavities (solid black) is compared to second-order (dashed red), first-order (dashed blue) and zero-order (dotted green) approximations, valid for $N \geq 4$. Parameters are chosen to maximize the cavity population in figure 1(c): $U = 6\gamma_0$, $\Omega = 0.5\gamma_0$, $J = 0.3\gamma_0$, $\omega_L = \omega_0 + 0.25\gamma_0$ and $n_{\text{max}} = 2$.

antibunched and each cavity behaves like a single-photon emitter, first- and zero-order approximations differ strongly from the exact solution, especially when computing $n_a$, the bath at temperature $T$, which are identical for all cavities in our considerations. Hence for the case of purely dissipative dynamics with $H_N = 0$ in equation (2), the steady state $\rho_\text{th}$ is a product of thermal states at temperature $T$ for each cavity, that is, $\rho_\text{th} = Z^{-1} \sum_n e^{-\omega_n/\hbar \beta} / (2\pi \hbar \beta)$, where $\hbar \beta$ is the Boltzmann constant, $n$ is the total number of excitations in the system and $Z$ is the partition sum. For $\Omega = 0$, the Hamiltonian in equation (1) conserves the number of excitations in the system and thus $[H_N, \rho_\text{th}] = 0$, so that $\rho_\text{th}$ is the steady state even in the presence of unitary dynamics generated by $H_N$, independent of the value of $J$. Of course, the rotating wave approximation that has been applied to derive the Hamiltonian (1) is only valid for $U, J \ll \omega_0$. In this regime, a thermal bath as described by the dissipation terms in equation (2) is ‘blind’ to the energy scales $U$ and $J$ and all cavities will eventually be in thermal equilibrium with their bath, $n_a = n_T$ and $g^{(2)}(0) = 2$, regardless of the hopping $J$ and therefore other neighbouring cavities. The dynamics and spectrum of emission (out of the scope of the present study) do, however, depend on the microscopic properties of the cavities. For example, larger $J$ and $\gamma_0$ would accelerate the thermalization of the cavity array.

If, on the other hand, a coherent and an incoherent drive are both present, the steady state becomes nontrivial. Moreover, our approach is suited for exploring this experimentally relevant scenario that describes coherently driven cavities in the presence of thermal background radiation. Figure 2 shows $n_a$ and $g^{(2)}(0)$ as a function of the thermal bath occupation number $n_T$, which increases with increasing temperature, for the case of maximum cavity population in figure 1(c). Both the exact and the approximated solutions converge at high temperatures to the thermalized steady state ($n_a = n_T$ and $g^{(2)}(0) = 2$). At low temperatures, where the emission is
while, again, the second-order approximation follows it quite smoothly.

It is interesting to note that the steady state cavity spectrum of emission could also be obtained from this approach without recurring to the quantum regression theorem and, therefore, to deriving any time dynamics. The alternative to such complications is to look into the steady state occupation, as a function of its natural frequency, of a mode that is very weakly coupled to the cavity array and which plays the role of the detector. We showed with co-workers the equivalence and zero everywhere else. The vector $\tilde{I}$ is constructed from the elements that provide an independent term for the equations, that is, $\tilde{R}_{m,n,\mu,\nu}$, while the matrix $\tilde{M}$ corresponds to all other elements.

5. Conclusions

We have presented a method to solve for the steady state of coupled identical cavities in a circular 1D array with a translational symmetry to second order in the photon tunnelling rate, $J/\gamma$. This method can be generalized to any set of identical weakly coupled systems, being in a 1D, 2D or 3D arrangement.

We consider any type of driving of the cavities (coherent or incoherent), dissipation and nonlinearities. We first derive the equations of motion for a minimal set of relevant correlators, $v_n$, and then perform a power expansion of both the equations and the solutions to obtain semi-analytical expressions for $v_n \approx v_n^{(0)} + (J/\gamma) v_n^{(1)} + (J/\gamma)^2 v_n^{(2)}$. The approximated solution is invariant for $N \geq 4$ cavities due to the nearest neighbour nature of the coupling. We have finally illustrated the performance of our method with an example of four weakly coupled cavities under a coherent drive and temperature.

Acknowledgments

We thank Peter Degenfeld-Schonburg for fruitful discussions. EdV acknowledges support from the Alexander von Humboldt foundation and MJH from the German Research Foundation (DFG) via the Emmy Noether project HA 5593/1-1 and the CRC 631.

Appendix. Equations for the correlators

In this appendix we provide the matrix $\tilde{M}$ and vector $\tilde{I}$ appearing in equation (4), which are the starting point of the described procedure. Let us consider operators for only two adjacent cavities, $(a_i^{m\alpha} a_i^{n\beta})$, as the general case of an array is an straightforward generalization. Then, we can obtain their equations of motion from the full master equation as

$$\begin{align*}
\hat{\rho} & = P_{mn}, \\
\tilde{R}_{m,n,\mu,\nu} & = P_{\mu,\nu}.
\end{align*}$$

(A.3)

$$\begin{align*}
\tilde{R}_{m,n,\mu,\nu} & = iU(m-n), \\
\tilde{R}_{m,n,\mu,\nu} & = iU(\mu - v), \\
\tilde{R}_{m,n,\mu,\nu} & = i\Omega m, \\
\tilde{R}_{m,n,\mu,\nu} & = i\Omega \mu, \\
\tilde{R}_{m,n,\mu,\nu} & = -i\Omega n, \\
\tilde{R}_{m,n,\mu,\nu} & = -i\Omega \nu, \\
\tilde{R}_{m,n,\mu,\nu} & = iJm, \\
\tilde{R}_{m,n,\mu,\nu} & = iJ\mu, \\
\tilde{R}_{m,n,\mu,\nu} & = -iJn, \\
\tilde{R}_{m,n,\mu,\nu} & = -iJ\nu.
\end{align*}$$

(A.4)

and zero everywhere else. The vector $\tilde{I}$ is constructed from the elements that provide an independent term for the equations, that is, $\tilde{R}_{m,n,\mu,\nu}$, while the matrix $\tilde{M}$ corresponds to all other elements.

References

[1] Raimond J M, Brune R and Haroche S 2001 Rev. Mod. Phys. 73 656
[2] Hartmann M J, Brandão F G S L and Plenio M B 2006 Nature Phys. 2 849
[3] Angelakis D G, Santos M F and Bose S 2007 Phys. Rev. A 76 031805
[4] Greenestree A D, Tahan C, Cole J H and Hollenberg L C L 2006 Nature Phys. 2 856
[5] Chang D E, Gritsev V, Morigi G, Vuletic V, Lukin M D andDemler E A 2008 Nature Phys. 4 884
[6] Kiffner M and Hartmann M J 2010 Phys. Rev. A 81 021806
[7] Peyronel T, Firstenberg O, Liang Q-Y, Hofferberth S, Gorshkov A V, Pohl T, Lukin M D and Vuletic V 2012 Nature 488 57
[8] Ritter S, Nölleke C, Hahn C, Reiserer A, Neuzner A, Uphoff M, Mucke M, Figueroa E, Bochmann J and Rempe G 2012 Nature 484 195
[9] Houck A A, Tureci H E and Koch J 2012 Nature Phys. 8 292
[10] Lucero E et al 2012 Nature Phys. 8 719
[11] Hartmann M J, Brandão F G S L and Plenio M B 2008 Laser Photon. Rev. 2 527
[12] Tomadin A and Fazio R 2010 J. Opt. Soc. Am. B 27 A130
[13] Carusotto I, Gerace D, Tureci H, De Liberato S, Ciuti C and Imamoglu A 2009 Phys. Rev. Lett. 103 033601
[14] Nissen F, Schmidt S, Biondi M, Blatter G, Türeci H E and Keeling J 2012 Phys. Rev. Lett. 108 236303
[15] Hartmann M J 2010 Phys. Rev. Lett. 104 113601
[16] Jin J, Rossini D, Fazio R, Leib M and Hartmann M J 2013 Phys. Rev. Lett. 110 163605
[17] Underwood D L, Shanks W E, Koch J and Houck A A 2009 Phys. Rev. B 79 235326
[18] del Valle E, Laussy F P and Tejedor C 2009 Phys. Rev. Lett. 103 083601
[19] del Valle E, Laussy F P and Tejedor C 2008 Phys. Rev. Lett. 101 083601
[20] del Valle E, Gonzalez-Tudela A, Laussy F P, Tejedor C and Hartmann M J 2012 Phys. Rev. Lett. 109 183601