Novel Digital Control Method with Pulse Width Modulation for Half-wave Rectified Brushless Synchronous Motors

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(Manuscript received Jan. 00, 20XX, revised May 00, 20XX)

We propose a novel digital current control method for half-wave rectified brushless synchronous motors (HWRB-SM) with a pulse width modulation (PWM) at a fixed switching frequency. For HWRB-SMs, the field magnetic flux must be excited by high-frequency stator currents. Due to the nonlinear dynamics of the diode short-circuiting the field winding, an appropriate mathematical model is required for controlling the high-frequency excitation current. In this study, we developed equivalent models for both the excitation current control and the fundamental one. The proposed method employs a synchronous PWM and resonant controllers to realize high-frequency excitation current control that is robust against the nonlinearity of the diode. The proposed method was validated using an experimental synchronous motor and a digital control system.

Keywords: variable field magnetic flux brushless synchronous motor, triangular carrier-wave comparison PWM, synchronous PWM

1. Introduction

Variable field magnetic flux brushless synchronous motors attract a lot of attention because a high-efficiency operating point is able to be optimized. This type of motors is capable of making the field magnetic flux both strong and weak depending on the operating conditions, whose feature is advantageous for such applications that have large torque variation over the wide speed range. Reference (1) proposed this kind of motor which has an external dc excitation machine, permanent magnets embedded in the rotor surface, and an excitation winding in the stator which realize the variable field magnetic flux. Although the principle and the theoretical analysis of the variable field flux were clearly revealed, an exclusive excitation machine was required.

The previous literatures (2) ~ (5) reported another type of variable magnetic flux motor whose magnetic flux was induced by an alternating magnetomotive force linking between distributed stator windings and concentrated rotor windings supplied from the stator currents. The principle of brushless self-excitation to generate the field magnetic flux to the rotor is the electromagnetic induction; the field magnetic flux is induced by the magnetomotive force of stator currents linking the field winding as shown in Fig.1. The induced field current is rectified by a diode, which generates a unidirectional field current. The synthesized air-gap flux which is the sum of the magnetic fluxes produced by the d-axis stator current and the field current becomes almost constant, then the electromagnetic torque is obtained. This motor has been called a half-wave rectified brushless synchronous motor (HWRB-SM), whose features are simple structure and no special excitation equipment. The principle of controlling HWRB-SMs and the experimental validation of the control method were revealed in the previous literatures. In addition, the previous literatures adopted a hysteresis current control which is robust against the electrical parameter variation of HWRB-SMs; an output voltage of a voltage source inverter (VSI) is determined so as to agree the reference currents with the sampled ones by comparing them. However, due to a risk of excessively high switching frequency of a VSI by the hysteresis current control, the VSI has to ensure the margins for its allowed heat dissipation and its drive circuits’ capacity.

To resolve the risk of unexpected high switching frequency, a novel current control method by a pulse width modulated (PWM) VSI with a fixed switching frequency is required. In addition, the mathematical model of HWRB-SMs for the current control with a PWM VSI operated at a fixed switching frequency is required. The references (6)-(8) pointed out that the field current is inherently

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discontinuous due to the diode short-circuiting the field winding and the field resistance; thus, the mathematical model of HWRB-SMs is hard to be derived. Reference (6) focused on the complex field circuit dynamics and proposed a block diagram expressing the field circuit adequately, which enabled dynamic numerical simulation of HWRB-SMs. However, these references lack the details of the current control method for HWRB-SMs. In addition, the references pointed out that the maximum excitation frequency is restricted by the switching frequency and a deadtime of inverters, thus the bandwidth of the current control is hard to be set to high value. Therefore, the large attenuation of the peak field current due to both the field resistance and the low excitation frequency appeared.

This paper describes a novel digital current control method with a PWM VSI operated at a fixed switching frequency for HWRB-SMs. To realize the proposed control method, the equivalent circuit of the motor that has one integrated leakage inductance has been derived. The constant switching frequency brings facilitated designs of the inverter and its peripherals. The proposed method realizes the independent stator current control of both the high-frequency excitation component and the fundamental frequency component. The proposed method has succeeded by adopting a synchronous PWM whose carrier frequency is 12 times higher than that of the excitation current. In addition, the proposed method has effectively utilized a PWM synchronous sampling, resonant controllers, and digital filters. The higher the frequency of the excitation current is, the less the unpleasant torque ripple becomes due to the small attenuation of the field current. By adopting resonant controllers, the robust excitation current control has been realized regardless of the off-period phenomenon of the diode. The proposed method has been validated by the experimental setups. Moreover, with the proposed method, the variable field current control has been confirmed experimentally.

2. Mathematical equivalent motor model for control

To control stator currents by a PWM VSI, an adequate mathematical model of HWRB-SMs is required. In the following equations, the following assumptions are made:

1. The harmonics of the spatial magnetic fluxes, stator voltages, and stator currents are neglected.
2. All parameters are time-invariant.
3. Magnetic saturation and iron loss are neglected.

The mathematical model for wound-rotor synchronous motors on the dq rotational coordinates by means of the instantaneous space vector is derived as follows:

\[ e_{di} = (p + j\omega)\Psi_{di} + r_{si}i_{di} \]  \hspace{1cm} (1)

\[ e_{dq} = p\Psi_{dq} + r_{sq}i_{dq} \]  \hspace{1cm} (2)

where \( e_{di} \) is the stator voltage vector, \( p \) represents \( (d/dt) \), \( \omega \) is the angular frequency of the coordinates, \( \Psi_{di} \) is the stator flux linkage vector, \( r_{si} \) is the stator resistance, \( i_{di} \) is the stator current vector, \( e_{dq} \) is the field voltage, \( \Psi_{dq} \) is the field magnetic flux linkage, \( r_{sq} \) is the field resistance, and \( i_{dq} \) is the field current. These equations are transformed with use of the self-inductances and the mutual inductance.

\[ e_{di} = (p + j\omega)(L_{di}i_{di} + M_{dq}i_{dq} + jL_{si}i_{si}) + r_{si}i_{di} \]  \hspace{1cm} (3)

\[ e_{dq} = p(M_{di}i_{di} + L_{dq}i_{dq}) + r_{sq}i_{dq} \]  \hspace{1cm} (4)

In order to separate the leakage inductances and to convert the field-side parameters into the stator-side ones, the following equations are derived:

\[ e_{di} = (p + j\omega)(l_{di}i_{di} + L_{sdi}i_{si}) + r_{si}i_{di} \]  \hspace{1cm} (5)

\[ e_{dq} = p\left(l_{dq}i_{dq} + L_{sdi}i_{si} + M_{dq}i_{dq}ight) + r_{sq}i_{dq} \]  \hspace{1cm} (6)

where \( l_{di} \) is the stator leakage inductance, \( L_{sdi} \) is the d-axis mutual inductance, \( L_{sdi} \) is the q-axis mutual inductance, and \( i_{si} \) is the field leakage inductance. The superscript ‘ indicates the variables converted to the stator side. Fig. 2 shows the equivalent circuit of (5) and (6).

The field leakage inductance can be integrated in the stator leakage inductance by multiplying an appropriate transform coefficient \( k \) to the rotor variables, which enables the separate control for both the torque and excitation currents.

\[ e_{di} = (p + j\omega)(l_{di}i_{di} + L_{sdi}i_{si}) + r_{si}i_{di} \]  \hspace{1cm} (7)

\[ k e_{di} = p\left(k^2\left(l_{di}i_{di} + L_{sdi}i_{si} + k^2 L_{sdi}i_{si} \right) + r_{sq}i_{dq} \right) \]  \hspace{1cm} (8)

Then, if \( k L_{sdi} \) equals \( k^2 \left( l_{di} + L_{sdi} \right) \), the following are obtained:

\[ e_{di} = (p + j\omega)\left(H_{di}i_{di} + M_{di}i_{dq} + jM_{di}i_{di} \right) + r_{si}i_{di} \]  \hspace{1cm} (9)

\[ k e_{di} = pM_{di}\left(l_{di} + \frac{L_{sdi}^2}{k} \right) + k^2 r_{sq}i_{dq} \]  \hspace{1cm} (10)

where,

\[ k = \frac{L_{sdi}}{l_{di} + L_{sdi}} \]

\[ l = (l_{di} + L_{sdi})(l_{di} + 2L_{sdi}) \]

\[ M_{di} = \frac{L_{sdi}^2}{l_{di} + L_{sdi}} \]

\[ M_{di} = \frac{L_{sdi}^2}{l_{di} + L_{sdi}} - l \]

Fig. 3 shows the equivalent circuit of (9) and (10). Then, rewrite the stator flux linkage vector as follows:

\[ \Psi_{di} = H_{di}i_{di} + M_{di}\left(l_{di} + \frac{L_{sdi}^2}{k} \right) + jM_{di}i_{di} \]  \hspace{1cm} (11)
3. Control method

3.1 Control strategy

HWRB-SMs have unique nonlinear characteristics due to the diode inserted between the field terminals. In order to avoid the adverse nonlinear effect of the diode, the frequency of the excitation current should be set to high value.

The time constant of the motor is important for the proposed current control. Table 1 shows the parameters of the prototype motor, which corresponds to the parameters of the equivalent circuit shown in Eq. 3.

The time constants of the loops shown in Fig. 3 are derived as follows: loop A: (12) shows the time constant of the d-axis circuit $\tau_{d}$, loop B: (13) shows the time constant of the field circuit $\tau_{f}$, loop C: (14) shows the time constant of the d-axis excitation circuit $\tau_{exc,d}$, and loop D: (15) shows the time constant of the q-axis circuit $\tau_{exc,q}$.

$$\tau_{d} = \frac{l}{r_d + k^2 M_{dl}} = \frac{4.294}{0.487 + 0.441} \times 10^{-3} = 4.63 \text{ ms}$$  \hspace{1cm} (12)

$$\tau_{f} = \frac{l M_{fl}}{k^2 M_{fl}} = \frac{55.31}{0.441} \times 10^{-3} = 125 \text{ ms}$$  \hspace{1cm} (13)

$$\tau_{exc,d} = \frac{l + M_{dl}}{r_d} = \frac{4.294 + 55.31}{0.487} \times 10^{-3} = 122 \text{ ms}$$  \hspace{1cm} (14)

$$\tau_{exc,q} = \frac{l + M_{ql}}{r_q} = \frac{4.294 + 9.573}{0.487} \times 10^{-3} = 28.5 \text{ ms}$$  \hspace{1cm} (15)

The equations (12) and (14) indicate that the time constant of the d-axis excitation circuit is more than 25 times longer than that of the q-axis circuit, which leads to the approximations shown in (16)-(17). The d-axis high-frequency stator current directly flows in the field windings. Therefore, in the high-frequency circuit, the inductance $M_d$ can be eliminated.

$$p M_{ia} = p M_{i,q} a$$  \hspace{1cm} (16)

$$i_a = \frac{I_a}{k}$$  \hspace{1cm} (17)

On the other hand, in the fundamental frequency circuit, the rotor circuit can be eliminated because the fundamental flux linkage of the field winding is DC. When the equivalent circuit is linear, the superposition principle is applied as shown in (18) and (19).

$$\begin{align*}
e_{d} &= (p + j \omega)\left(l_i i_d + l_a i_a + M_{d} i_q + j M_{q} i_q\right) + r_d i_d \\
&+ k\left(e_{d} + e_{q}\right) + p M_{d} i_d + k^2 r_d i_a$$  \hspace{1cm} (18)

$$\begin{align*}
e_{q} &= (p + j \omega)\left(l_i i_q + M_{d} i_d + j M_{q} i_q\right) + r_q i_q \\
&= (p + j \omega)\left(l_i i_q + M_{d} i_d + j M_{q} i_q\right) + k r_d i_a$$  \hspace{1cm} (19)

The equations at the high-frequency and the fundamental frequency are separately described as follows:

$$\begin{align*}
e_{d} &= (p + j \omega)\left(l_i i_d + M_{d} i_d + j M_{q} i_q\right) + r_d i_d \\
&= (p + j \omega)\left(l_i i_d + M_{d} i_d + j M_{q} i_q\right) + r_d i_d$$  \hspace{1cm} (20)

$$\begin{align*}
e_{q} &= (p + j \omega)\left(l_i i_q + M_{d} i_d + j M_{q} i_q\right) + k r_d i_a \\
&= (p + j \omega)\left(l_i i_q + M_{d} i_d + j M_{q} i_q\right) + k r_d i_a$$  \hspace{1cm} (21)

$$\begin{align*}
e_{d} &= (p + j \omega)\left(l_i i_d + M_{d} i_d + j M_{q} i_q\right) + r_d i_d \\
&= (p + j \omega)\left(l_i i_d + M_{d} i_d + j M_{q} i_q\right) + r_d i_d$$  \hspace{1cm} (22)

The field winding is magnetically coupled only with the d-axis stator winding. The d-axis equation is extracted as shown below:

$$e_a = p I_a - o M_q i_q + \left(r_d + k r_d\right) i_a + k e_{a}$$  \hspace{1cm} (23)

Figs.4 and 5 correspond to (22) and (20), respectively.

3.2 Effect of field current attenuation

The diode short-circuiting the field winding causes the discontinuous field current originated from brushless self-excitation.

When $p I_a$ is negative, the field current starts to flow. In other words, $e_{a_n}$ (the diode voltage) is 0 V. On the other hand, $p I_a$ is positive, the field current reduces, and finally falls in 0 A.

Here, the diode off-period is estimated by the simplified analysis as follows. Assume that $I_{a_o} = I_{a_{dc}} \cos (\omega_{a_{dc}} t)$. The attenuation of the field current can be calculated. Assuming that the diode is on, the equation (10) becomes:

$$p M_{d} \left(i_a + \frac{I_{a}}{k}\right) + k r_d i_a \frac{I_{a}}{k} = 0$$  \hspace{1cm} (24)

Substitute $i_a$ into (24), (25) is obtained:

$$p M_{d} + k^2 r_d i_a = \frac{I_{a}}{k} \cos \left(i_{a} t\right) \sin \left(i_{a} t\right)$$  \hspace{1cm} (25)

This equation is solved as (26) with the initial condition of $I_{a}=0$ at $t=0$:

$$I_{a} = \frac{L_i}{s} + \frac{M_{d}}{s} \frac{1}{1 + \frac{k r_d}{M_{d}}}$$  \hspace{1cm} (26)

The equation of the field current during the diode conducting is shown as follows:

$$I_{a} = \frac{L_i}{s} + \frac{M_{d}}{s} \frac{1}{1 + \frac{k r_d}{M_{d}}} \left\{ \exp \left(-\frac{k r_d}{M_{d}} t\right) - \cos \left(i_{a} t\right) + \frac{k r_d}{M_{d}} \sin \left(i_{a} t\right) \right\}$$  \hspace{1cm} (27)

The diode turn-off period is calculated numerically as shown in Fig.6, and the calculated values are summarized in Fig.7. In this study, the excitation frequency is chosen as 8000/12(=667 Hz), then the turn-off period is 35 μs which corresponds to 8.4 degrees of the phase angle of the excitation current frequency. Therefore, in the controller design, the turn-off period can be omitted, that is, the field terminal can be regarded as being always short-circuiting.

3.3 Synchronous PWM

A frequency of the excitation current for HWRB-SMs is desirable to be high in order to shorten the off-period in which the field current is zero by the diode.

Table 1. Equivalent parameters.

| Stator resistance | $r_s$ | 0.487 Ω | 5-axis mutual inductance | $k^2 r_{ai}$ | 0.441 Ω | 2-axis field turn ratio | $k$ | 0.4331
| Field resistance | $k^2 r_{ai}$ | 0.441 Ω | equivalent leakage inductance | $l$ | 4.294 mH | Equivalent conversion coefficient | $k$ | 0.975
| d-axis mutual inductance | $M_d$ | 55.312 mH | number of pole pairs | $n$ | 9.573 mH | 2

Fig. 4. Equivalent circuit at high-frequency.

Fig. 5. Equivalent circuit at fundamental frequency.
characteristics. In terms of the controller’s aspect, conventional current control bandwidth is limited by the interrupt process interval, the AD sampling interval, and the maximum switching frequency. When the carrier frequency and the frequency of the command current is not synchronized, the output voltage in the fundamental period is unsymmetrical. In particular, the resolution of the phase of the output voltage gets coarse, which deteriorates the accurate current control. For example, when the carrier frequency is 5 kHz and the frequency of the output current is 250 Hz, the phase of the command current shifts by 18 degree in one carrier interval, whose phase delay degrades the current control[10].

In general, the maximum current control bandwidth is limited in less than one-twentieth of the carrier interval.

A Synchronous sampling overcomes the above-mentioned limited current bandwidth by setting the carrier frequency which is a multiple of 6 times of the command current frequency. In addition, the sampling timing and the update of the modulation indices must coincide with the maximum and the minimum of the command currents. Table 2 shows the settings of the synchronous PWM. Fig.8 shows the carrier wave, the command currents and the sampled command currents. The excitation current frequency is 12 times lower than the carrier frequency. As shown in the figure, every peak value of the command current is sampled. The commands of the excitation current vector $i_{d1}$ is expressed as follows:

$$ i_{d1}^* = \frac{I_{ac} \cos (\omega_{ac} n + \frac{\pi}{12})}{12} \quad (28) $$

$$ i_{q1}^* = 0 \quad (29) $$

where $I_{ac}$ is the amplitude of the excitation current on the dq coordinates, $\omega_{ac}$ is the excitation angular frequency and $n$ is an integer variable which increases from 0 to 11. When the variable $n$ reaches to 12, then the value goes back to zero. It is important that because the incremental operation has such a problem as an accumulating error, the phase of the excitation current must be given by multiplying an integer variable to the fraction of the phase angle, which completely avoid the accumulating error. This ensures the precise synchronous PWM.

### 3.4 Torque characteristics

The electromagnetic torque of HWRB-SMs with the use of instantaneous space vector is expressed as follows:

$$\tau_e = -i_{ac} \times \mathbf{\psi}_m = -(i_d + j i_q) \times (\mathbf{\psi}_d + j \mathbf{\psi}_q) = \mathbf{\psi}_d i_q - \mathbf{\psi}_q i_d \quad (30)$$

The torque ripple caused by the excitation current was discussed in the previous literatures[2][10]. In this paper, the torque ripple only at the excitation frequency is considered. In addition, the d-axis fundamental wave current is assumed to be 0. Then, the torque ripple is expressed taking (17) into consideration:

$$\tau_{th} = -M_{i_{ac} i_q} \quad \text{........................ (31)}$$

This equation indicates that the torque ripple is mainly caused by the reluctance torque. In the actual machine, the torque ripple is weakened by the free-wheeling effect of the load inertia. By avoiding the mechanical resonant frequency, the torque ripple

| Table 2. Settings of the controller |
|-----------------------------------|
| **PWM carrier frequency** | 8 kHz |
| **Timing of sampling and interrupt** | Carrier UP |
| **Sampling frequency** | 8 kHz |
| **Timing of modulation wave** | Carrier UP |
| **Interrupt routine frequency** | 8 kHz |
| **Excitation frequency** | 8000/12 Hz (=667 Hz) |

Fig. 6. Derivation of the off-period of the diode.

Fig. 7. Diode off period derived from the equation (27).

Fig. 8. Synchronous PWM and the command currents (Top figure: carrier wave, bottom figure: the command excitation currents and the sampled values. The sampling timing is at the up edge of the carrier wave).
caused by the excitation current can be negligible for practical use \(^{31}\).

### 3.5 Control block diagram
Fig. 9 shows the proposed control diagram based on (20) and (22). In the upper side of the control block diagram, the high-frequency excitation current is controlled by the resonant controllers. The resonant controller diminishes the gain except that of the resonant frequency component. On the other hand, the notch filters pass through all the component except the resonant frequency signal. Since the sampled currents are divided into the component at the excitation frequency and that at the other frequency by the resonant controllers and the notch filters, the multiple frequency components of the currents are capable of being controlled independently. The delays of the digital control and the PWM are compensated by adding 1.5 sampling time phase shift \(^{10}\).

#### 3.6 Proportional-Integral (PI) controller in digital domain
One of a PI controller’s definition in a discrete time domain considering the output saturation is described as follows \(^{11}\):

\[
Y(z) = \frac{z^{-1}Y(z) + k_p(E(z) - z^{-1}E(z)) + k_i \frac{1 + z^{-1}}{2} E(z)}{}
\]

where \(z\) is an input, \(y\) is an output, \(k_p\) is a proportional gain, \(k_i\) is an integral gain, \(T_s\) is a discrete interval, and \(z\) is a forward shift operator. An output limiter without windup if the output deviates the limits. The block diagram is shown in Fig.10.

#### 3.7 Digital controller design at the frequency close to interrupt frequency
One of the ways to design digital controllers are to firstly design them on the continuous time domain (s-plane), then they are mapped to the discrete time domain (z-plane). Because the excitation frequency is close to the interrupt interval, the well-known mapping method cannot be enough to satisfy the required performance. With the Tustin transformation, the derivative is expressed as follows:

\[
y(z) = 2 \frac{1 - z^{-1}}{T_s(1 + z^{-1})} x(z)
\]

where \(x\) is an input, and \(y\) is an output. When the frequency of a signal is close to an interrupt interval, this approximation is not enough because the phase characteristics in the continuous time domain is different from that in the discrete time domain. Therefore, we set a correction factor \(k_T\) so that the gain at the stability margin on the z-plane is identical to the gain on the s-plane:

\[
y(z) = k_T \frac{1 - z^{-1}}{1 + z^{-1}} x(z)
\]

where \(z\) is denoted as follows:

\[z = \exp(j\omega T_s)\]

where \(\omega\) is a pre-warped angular frequency. Substituting (35) to (34) yields as follows:

\[
y(z) = k_T \frac{1 - \exp(-j\omega T_s)}{1 + \exp(-j\omega T_s)} = k_T \tan\left(\frac{\omega T_s}{2}\right)
\]

On the other hand, a derivative operator on the s-plane is as follows:

\[
\frac{y(\omega T_s)}{X(\omega T_s)} = j\omega T_s
\]

In order to agree the gains between the continuous and discrete time domains at the angular frequency of \(\omega\), the correction factor \(k_T\) is derived as follows:

\[
k_T = \tan\left(\frac{\omega T_s}{2}\right)
\]

#### 3.8 Resonant controller \(^{12}(13)\)
A resonant controller \(F(s)\) is designed by combining a positive sequence complex filter \(F(s)\) and a negative sequence complex filter \(\bar{F}(s)\):

\[
F(s) = F(s) + \bar{F}(s) = k_m \left[ \frac{\omega_0}{(s - j\omega_0) + \omega_0} - \frac{\omega_0}{(s + j\omega_0) + \omega_0} \right]
\]

where \(\omega_0\) is a gain, \(\omega_0\) is a corner angular frequency, and \(\omega\) is a resonant angular frequency. Assuming that \(\omega_0 < \omega_0\), the following equation is obtained:

\[
F(s) = \frac{2k_m \omega_0 s}{s^2 + 2\omega_0 s + \omega_0^2}
\]

Transforming this equation to the equation on the z-plane yields:

\[
F(z) = \frac{a_2 z^{-1} - a_1 z^{-2}}{b_2 - b_1 z^{-1} + b_2 z^{-2}}
\]

where:

\[
a_1 = a_2 = 2 \omega_0 \omega_1 k_T\]

\[
b_0 = k_T^2 + 2 \omega_0 k_T + \omega_0^2
\]

\[
b_1 = 2 k_T^2 - \omega_0^2
\]

\[
b_2 = k_T^2 - 2 \omega_0 k_T + \omega_0^2.
\]

The block diagram is shown in Fig.11.

#### 3.9 Phase-lead compensator
A general phase-lead compensator on the s-plane is described as follows:

\[
G_c(s) = \frac{\alpha T_s s + 1}{T_s s + 1}
\]

where \(\alpha\) must be bigger than 1. The frequency transfer function is:

\[
G_c(j\omega) = 1 + \frac{\alpha (\omega T_s)^\alpha}{1 + (\omega T_s)^\alpha}
\]

The gain becomes:

\[
\left|G_c(j\omega)\right| = \frac{\alpha}{(\omega T_s)^\alpha}
\]

The phase angle is described as follows:

\[
\angle G_c(j\omega) = \arctan\left(\frac{\alpha - 1}{\alpha}\right)
\]

In order to derive the extreme value which is the maximum phase lead, the partial differentiation of the above equation in terms of \(\omega\) is calculated. When the partial differentiation is zero, the \(\omega\) is:

\[
\omega_m = \frac{1}{T_s \sqrt{\alpha}}
\]

The maximum phase-lead angle \(\phi_m\) is derived as follows:

\[
\phi_m = \angle G_c(j\omega_m) = \arctan\left(\frac{\sqrt{\alpha - 1}}{\sqrt{\alpha}}\right)
\]
In this case, $\alpha$ is set to the following value:

$$\alpha = \frac{1 + \sin \phi}{1 - \sin \phi}$$  \hspace{1cm} (48)

In addition, the gain is derived as follows:

$$|G_i(j\alpha \omega)| = \frac{\alpha + 1}{\sqrt{\alpha^2 + 1}}$$  \hspace{1cm} (49)

The equation (42) is mapped to the $z$-plane, then the following equation is derived:

$$G_i(z) = \frac{a_0 + a_z z}{b_0 + b_z z}$$  \hspace{1cm} (50)

where,

$$b_0 = 1 + T_s K_T$$
$$b_1 = 1 - T_s K_T$$
$$a_0 = 1 + \alpha T_s K_T$$
$$a_1 = 1 - \alpha T_s K_T$$

The block diagram is shown in Fig.12.

### 3.10 Notch filter

A notch filter is originated from antiresonance and derived on the $z$-plane as follows:

$$H(z) = \frac{c_0 (1 + b_z z^{-1} + z^{-2})}{1 - a_z z^{-1} - a_z z^{-2}}$$  \hspace{1cm} (51)

where,

$$a_1 = 2 \exp(-0.5 b_0 T_s) \cos(\omega_0 T_s)$$
$$a_2 = -\exp(-b_0 T_s)$$
$$b_1 = -2\cos(\omega_0 T_s)$$
$$c_0 = \frac{1 - a_1 - a_2}{2 + b_1}$$

Here, $\omega_0$ is the notch angular frequency, and $b_0$ is the bandwidth of the notch filter. In this study, $\omega_0$ is set to the resonant angular frequency of the resonant controller. The block diagram is shown in Fig.13.

### 4. Experimental results

#### 4.1 Experimental setups

The HWRB-SM used in this study is a custom-made, whose stator and rotor structure are shown in Fig.14. In this study, the field terminals of the machine can be accessed through a slip-ring. The diode short-circuiting the field winding was connected via the slip-ring. The diode connected to the field terminal was a super-fast recovery diode (RFUH20TF6S, Rohm). An incremental type rotary encoder was used. A surge absorber (AV-13 UL, (Sankosha Inc.) for AC 200V system) was connected in parallel with the diode. The whole system diagram is illustrated in Fig.15.

The control system used in this study was the PE-Expert4 which has the DSP (TMS320C6657) and the FPGA (Kintex7 XC7K70T-1FBG484 with an external clock of 125 MHz). The FPGA creates the PWM gate signals, and controls AD conversion. The AD converter is AD7357 (Analog Devices) with the resolution of 14 bit.

A commercial inverter (MWINV-9R122C, which has the intelligent power module PM75RSD060) was used.

A general-purpose AC servo motor was used as a load machine.

The ac voltage of the inverter $v_{dc}$ is fed by a full-bridge rectifier with a 3-phase commercial ac voltage source. The magnitude of the ac voltage is adjusted so as to make the dc voltage 280 V. The parameters of the controllers are summarized in Table 3.

The offsets of the current sensors and the rotary encoder were calibrated before creating the gate signals.
4.2 Operational range of the proposed HWRB-SM control
In the proposed control method, the saturation limit of the inverter’s output voltage is critical factor to limit the operational range rather than the switching frequency. The operational limits change in accordance with the amplitude of the excitation current $I_{exc^*}$, the q-axis current, and the rotational speed. The key parameters to limit the maximum operational conditions of HWRB-SMs with the proposed method are the PWM carrier frequency (8 kHz) and the DC voltage (280 V). In addition, the allowed maximum rms currents of the stator winding and the field winding are 15 A and 4 A, respectively. The unique high-frequency excitation current control requires additional output voltage in addition to the fundamental voltage, thus the maximum output voltage determines the range of the torque-speed operational limits of HWRB-SMs. Therefore, in practice, the top speed would not be limited by the switching frequency but the maximum inverter output voltage. The operational limit where the excitation current control is possible can be increased by making the DC voltage higher than this case.

4.3 Current control at standstill
The validity of the proposed high-frequency excitation current control was examined while the fundamental frequency current control was disabled. Fig.16 shows the waveforms of the stator currents and the field current. Thanks to the synchronous PWM, the sinusoidal excitation current at the commanded frequency was successfully obtained. Fig.17 shows the waveforms of the stator currents on the dq coordinates. The actual currents tracked its commands satisfactorily. In addition, the q-axis current $i_q$ was controlled to zero. It is noticed that the amplitude of the d-axis current $i_d$ is a bit smaller than that of the command current $i_{d*}$. These results show the validity of the proposed high-frequency excitation current control.

4.4 Current control for torque control
The validity of the high-frequency excitation current control and the fundamental frequency current control were examined. In this study, the d-axis fundamental wave current command and the q-axis high-frequency current command were set to zero. Figs.18 and 19 are the cases where $I_{exc^*}$ was 4 A and $i_d^*$ was 12 A. The rotational speed was 188 min$^{-1}$ which was determined by the mechanical load. Fig.18 shows that the stator currents which were the sum of both the fundamental and the excitation currents were successfully controlled. The field current was confirmed to be induced by the high-frequency component of the stator currents. The envelop of the field current was found to fluctuate at 6 times of the fundamental frequency. It is considered that this fluctuation was caused by the space harmonics of the motor. Fig.19 shows the waveforms of the controlled currents on the dq coordinates, which indicate that the currents were successfully controlled to their commands. In particular, the q-axis current $i_q$ was independently controlled from the d-axis excitation current $i_{d*}$.

4.5 Current control for speed control
A speed controller which consists of a PI controller was examined and the validity of the proposed current control was investigated in the high-speed operation. Fig.20 is the case where the command speed was 1800 min$^{-1}$ and the load torque was 4 Nm. The error of the speed was only 1 min$^{-1}$; thus, the speed controller was validated. In addition, the current control exhibited good agreement between the actual currents and their commands irrespective of the rotational speed.

4.6 Transient performance of variable field current
The transient characteristics of changing the excitation current amplitude $I_{exc^*}$ were examined. The command speed $N^*$ was 1200 min$^{-1}$ and the load was 2 Nm. Figs.21 to 23 show the waveforms where the $I_{exc^*}$ increased from 4 A to 6 A at the ramp rate of 20 A/s. The field current tracked the command value, and the variable field flux was confirmed experimentally. The actual speed exceeded the command speed temporarily when the field magnetic flux increased because the response of the speed controller (ASR) was slow and the $i_q^*$ decreased slowly, which was due to the safety reason in the experimental room. In addition, Figs.24 to 26 show the waveforms where the $I_{exc^*}$ decreased from 6 A to 4 A at the ramp rate of 20 A/s. The field flux was confirmed to be weakened successfully by looking at the transient characteristics of the field current waveform. It is also noticed that the $i_q^*$ increased to maintain the speed because the field magnetic flux was weakened.

![Image](https://via.placeholder.com/150)

Fig. 16. Waveforms of stator field currents at standstill ($I_{exc^*} = 4$ A).

![Image](https://via.placeholder.com/150)

Fig. 17. Waveforms of d and q axes currents at standstill ($I_{exc^*} = 4$ A).

![Image](https://via.placeholder.com/150)

Fig. 18. Waveforms of stator and field currents ($I_{exc^*} = 4$ A, $i_d^* = 12$ A).

![Image](https://via.placeholder.com/150)

Fig. 19. Waveforms of dq axes currents ($I_{exc^*} = 4$ A, $i_q^* = 12$ A).

![Image](https://via.placeholder.com/150)

Fig. 20. Waveforms of dq currents and speed control results.
6. Conclusions

A novel digital current control method with a PWM for HWRB-SMs has been developed. The proposed method is not only controlling the fundamental frequency current but also realizing the high-frequency excitation current control under a constant switching frequency. The constant switching frequency brings facilitated designs of inverters for practical use. The proposed high-frequency excitation current control method overcomes the nonlinear effect of the off-state of the diode by adopting a synchronous PWM and resonant controllers. The proposed excitation current control is not affected by the magnetic saturation. The validity of the proposed current control method has been confirmed through the experimental setup. Our view for magnetic saturation of HWRB-SMs is to avoid such a deeply saturated operation in practical continuous operation. Our future tasks will be to reveal accurate torque ripple characteristics of HWRB-SMs by both developing a detailed mechanical dynamic model and developing an accurate torque measurement system. Moreover, a robust torque control method under the magnetic saturation will be addressed in our future tasks.

Acknowledgements

This work had been a sponsored research with Meidensha Corporation. We wish to thank Dr. Yuichi Yokoi for fruitful face-to-face discussion. This work has been done with the cooperation of Mr. Sota Takeishi, Mr. Kohei Kanaida, Mr. Keisuke Torigoe, Mr. Rintaro Ishihara, Mr. Taiki Ozasa, and Mr. Masaki Yamamoto, who are all the graduate students at Nagasaki University. This work was partly supported by KAKENHI 21H01315.

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