Effect of the design parameters of the spindle knot on natural frequencies

A F Denisenko
Samara State Technical University, Samara, Russia

Abstract. The results of the analysis of the influence of various design parameters on the dynamics of the spindle knot of the machine-tool are presented. As a dynamic characteristic, the natural frequencies were chosen. When choosing a dynamic model, the features of spindle knots were taken into account, which do not allow to take full advantage of the rich experience accumulated for rotary systems. The first feature is that modern spindles are massive shafts with areas that are insignificantly different in diameter. This makes it possible to neglect gyroscopic forces when considering the spindle dynamics. The second feature is the asymmetric location of the center of mass relative to the supports. Another feature of the spindle knots is the need to provide high rigidity of the spindle unit in the processing area, whereupon the radial stiffness of the front support is usually several times higher than the rigidity of the back support. Based on a single-mass dynamic model with two degrees of freedom, which performs translational-angular oscillations, the influence of design parameters on the values of natural frequencies was analyzed. As the design parameters, the mass and the moment of inertia of the spindle, the rigidity of the supports, the inter-bearing distance, the position of the center of mass (mass distribution over the front and back supports) were chosen. It was found that the anisotropy of the elastic properties of the spindle supports creates a band of natural frequencies. This complicates the monitoring task, since it requires taking into account the orientation of the hodographs of rigidity and a clear fixation of the direction of control of the oscillations of the spindle. In order to increase the spindle's natural frequencies, an increase in the stiffnesses of the front and back supports, an increase in the distance determining the position of the back support relative to the center of mass, a reduction in the mass of the spindle and the moment of inertia can be envisaged during the design. It is noted that the fastening of the workpiece in the chuck of the machine-tool practically does not affect the values of the natural frequencies. The results of the analysis allow at the design stage to select rational design solutions for the projected machine, create a system for ensuring the predicted dynamic characteristics and organize monitoring of the technical condition of the equipment by a dynamic criterion.

1. Introduction

The consideration of the dynamic criterion in the design of machine tools is an integral part of the design process. The absence of results of the analysis of the influence of various design parameters on the dynamics of the machine does not allow creating a system with the provision of predictable dynamic characteristics and makes it difficult to monitor the technical condition of the equipment by the dynamic criterion [1, 2]. The most important dynamic characteristic of any system is the natural frequencies. Their knowledge at the design stage allows you to choose the rational design solutions of the projected machine-tool.

These comments particularly concern the design of the spindle knot, as the most important unit in terms of ensuring the performance characteristics of the machine-tool.
When evaluating dynamic processes in a spindle knot, one can use the rich experience accumulated for rotor systems [3, 4].

However, the development of dynamic models should take into account a number of structural features of the knot that have not been reflected in existing models [6].

The first feature is that modern spindles are massive shafts with areas that are insignificantly different in diameter, which allows neglecting gyroscopic forces when considering the spindle dynamics.

The second feature is the asymmetric location of the center of mass relative to the supports.

Another feature of the spindle knots is the need to provide high rigidity of the spindle unit in the machining area, whereupon the radial stiffness of the front support is usually several times higher than the stiffness of the back.

The use of CAE-systems can significantly shorten the design time of the equipment. However, this is possible only if there is a mathematical model that adequately describes the projected design.

Taking into account the complexity of the dynamic system of the spindle knot of modern machine-tools and the variety of factors that directly influence the formation of the frequencies of the natural vibrations of the unit, it becomes necessary to determine the most significant system parameters that must be taken into account in the modeling process [7].

2. Dynamic model

As shown by theoretical and experimental studies [8, 9], the characteristics of the oscillatory processes at the natural frequencies of the spindle knots depend on the inertial, elastic and structural parameters of both the spindles themselves and their supports.

Despite the constant improvement of the design of spindle knots of modern machine tools, the number of variable parameters during design practically does not change. This allows you to compile a list of these parameters and outline the ways in which they affect the final result.

These parameters include: the mass and moment of inertia of the spindle; rigidity of supports; inter-bearing distance; position of the center of mass (mass distribution over the front and back supports).

We will analyze the influence of design parameters on natural frequencies of spindle oscillations on the example of a spindle knot of a lathe mod. 16B16T1 produced by the CJSC "Srednevolzhskiy Machine Tools" (Samara, Russia) (figure 1).

![Figure 1. The design of the spindle knot of the lathe mod. 16B16T1](image)

Since the lower natural frequencies are of particular importance in dynamic analysis, the design scheme of a rigid shaft that performs translational-angular vibrations on elastic supports in the form of a single-mass system with two degrees of freedom can be chosen as a dynamic model (figure 2).
Figure 2. The translational-angular dynamic model

For the spindle construction shown in figure 1, mass \( m_0 \) is determined by the mass of the spindle body, coupling half, drive gear and cartridge (figure 3). The mass \( m_0 \) also includes masses of bearings, calculated from the inside diameter and the diameter of the centers of the balls.

Figure 3. The design scheme of the spindle unit of the lathe mod. 16B16T1

For the construction shown in figure 1 \( m_0 = 58.855 \) kg; \( I = 2.818 \) kg·m²; \( a = 0.349 \) m; \( b = 0.016 \) m.

When assessing the radial stiffness of the supports of the spindle knot on rolling bearings, several parameters affecting the elastic characteristics of the supports should be taken into account [10, 11]: type, material, number and size of the rolling elements; angle of contact; preload value; variable number of contact areas that accept load; displacement of contact zones relative to the fixed ring; pressure redistribution in contact areas, etc.

It is not possible to take these parameters into account fully, and therefore the existing dependencies make it possible to calculate some averaged values and have a theoretical-empirical character in which the bearing features are taken into account by a number of coefficients [12].

The equations for the natural oscillations of the mass \( m_0 \) (figure 2) with translational movements along the coordinate \( x_0 \) and rotation \( \phi \) around the center of mass are written in the form of a system of differential equations (without taking into account dissipative characteristics)

\[
[M]{\ddot{x}} + [C]{x} = 0, \quad (1)
\]

where \( M, C \) is the inertial and elastic matrix, respectively; \( \{x\} \) – matrix-column of coordinates.

For the translational-angular dynamic model under consideration (figure 2), these matrices have the form

\[
M = \begin{bmatrix} m_0 & 0 \\ 0 & I \end{bmatrix}; \quad C = \begin{bmatrix} c_b + c_f & c_b a - c_f b \\ c_b a - c_f b & c_b a^2 + c_f b^2 \end{bmatrix}; \quad \{x\} = \begin{bmatrix} x_0 \\ \phi \end{bmatrix}, \quad (2)
\]

where \( c_b, c_f \) – rigidity of the back and front support, respectively.

The solution of equation (1) allows us to determine the natural frequencies, Hz:
\[ f_{1,2} = \frac{1}{2\pi} \left[ k_{c00} + m_0 c_{11} \pm \left( f^2 c_{00}^2 + 4m_0 k_{c1}^2 - 2m_0 k_{c00} c_{11} + m_0^2 c_{11}^2 \right)^{1/2} \right]^{1/2}, \]  

(3)

where \(c_{00} = c_b + c_f\); \(c_{11} = c_b a^2 + c_f b^2\); \(c_1 = c_b a - c_f b\).

3. Influence of Anisotropy of Elastic Properties Support of the Spindle Knot

In [9, 13, 14], the presence of anisotropy of the elastic properties of the supports of the spindle knot was confirmed experimentally. In this case, the range of the elastic properties of the support, depending on the direction of action of the load, is \(\Delta s_{\max} / \min\Delta s = 1.5 \ldots 2.5\), where \(\Delta s\) is the elastic deformation of the support.

The simplest variant of anisotropy is the case when the hodograph of rigidity of supports has the shape of an oval. We denote the ovality of the travel-time hodographs of the front support \(W\) and the back support \(V\):

\[
\frac{c_{f\max}}{c_{f\min}} = \frac{c_{b\max}}{c_{b\min}} \quad (4)
\]

and consider the case when \(W = V = 2\).

Assuming that the values of the averaged stiffnesses of the supports \(c_f\) and \(c_b\) are related to the limit values by expressions

\[
c_f = \frac{c_{f\max} + c_{f\min}}{2}; \quad c_b = \frac{c_{b\max} + c_{b\min}}{2}, \quad (5)
\]

we have

\[
c_{f\max} = \frac{2W c_f}{1 + W}; \quad c_{f\min} = \frac{2c_f}{1 + W}; \quad (6)
\]

\[
c_{b\max} = \frac{2V c_b}{1 + V}; \quad c_{b\min} = \frac{2c_b}{1 + V}. \quad (7)
\]

In addition, taking into account the random nature of the mutual orientation of the hodographs of the stiffness of the front and back supports, it should be borne in mind that the angle between the directions \(c_{f\max}\) and \(c_{b\max}\) can vary from 0 to 90°.

Averaged stiffness of the front and back supports of the lathe spindle mod. 16B16T1, calculated from the dependences given in [12], was \(c_f = 872 \cdot 10^6\) N/m; \(c_b = 375 \cdot 10^6\) N/m.

Taking into account the anisotropy of the stiffness of the supports, the natural oscillation frequency will have some deviations with respect to the values obtained with the average stiffness values (table 1).

In accordance with the data given in the table 1, due to the anisotropy of the stiffness of the supports, the natural frequencies can vary between \(f_1 = 689.4 \ldots 975.0\) Hz and \(f_2 = 397.0 \ldots 561.4\) Hz.
Table 1. The natural frequencies of the system when the stiffness of the spindle supports are varied due to anisotropy, Hz

| $c_b$, N/m | $c_f$, N/m | $c_{f_{min}}=581 \cdot 10^6$ | $c_f=872 \cdot 10^6$ | $c_{f_{max}}=1162 \cdot 10^6$ |
|-----------|-----------|---------------------|---------------------|---------------------|
|           | $f_1$     | $f_2$   | $f_1$     | $f_2$   | $f_1$     | $f_2$   |
| $c_{b_{min}}=250 \cdot 10^6$ | 689     | 397     | 746     | 449     | 810     | 478     |
| $c_b=375 \cdot 10^6$        | 807     | 415     | 844     | 486     | 889     | 533     |
| $c_{b_{max}}=500 \cdot 10^6$ | 914     | 424     | 942     | 503     | 975     | 561     |

4. Influence of Design Parameters of the Spindle Knot

In figure 4 and figure 5 show the results of modeling the change in natural frequencies depending on the average stiffness of the supports $c_f$ and $c_b$. As can be seen from the plotted graphs, the natural frequency $f_1$ is practically linearly dependent on the stiffnesses of the supports $c_f$ and $c_b$, increasing with their growth. The natural frequency $f_2$ also increases with the stiffness of the supports $c_f$ and $c_b$, but this dependence is non-linear. Moreover, the nonlinearity is more pronounced for small values of the rigidity of the supports.

Figure 4. The effect of changing the stiffnesses of the front $c_f$ and back $c_b$ supports to the natural frequency $f_1$
An increase in the distance $a$, which determines the position of the back support relative to the center of mass, leads to an increase in frequencies $f_1$ and $f_2$ (figure 6). However, the nature of the influence is different: for small values of $a$, its change is more significant for $f_2$, and for large values it is $f_1$.

Making a spindle with a more massive back part of the spindle results in an insignificant decrease of both frequencies $f_1$ and $f_2$ (figure 7).
Figure 7. The effect of the change in the mass of the back part of the spindle (additional mass $M$ at a distance of 0.5 m from the center of mass of the initial design scheme)

Fixing the workpiece in the spindle holder does not change the frequency $f_1$ and very slightly reduces the frequency $f_2$ (figure 8).

Figure 8. The effect of the change in the mass of the front part of the spindle due to the fastening in the chuck of the workpiece with a diameter of $d$ and a length of 170 mm (free removal of the workpiece from the chuck 100 mm)

The picture of the influence of the mass of the spindle $m_0$ and the moment of inertia $I$ is qualitatively qualitative: as the project parameters increase, the natural frequencies $f_1$ and $f_2$ decrease (figure 9, 10).
5. Conclusions
The anisotropy of the elastic properties of the spindle supports creates a band of natural frequencies. This complicates the monitoring task, since it requires taking into account the orientation of the hodographs of rigidity and a clear fixation of the direction of control of the oscillations of the spindle.

To increase the natural frequencies of the spindle, an increase in the stiffnesses of the front and back supports, an increase in the distance $a$ determining the position of the rear support relative to the center of mass, a reduction in the mass of the spindle $m_0$ and the moment of inertia $I$ can be envisaged during the design.

The fastening of the workpiece in the chuck of the machine practically does not affect the values of the natural frequencies.
6. References

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