Hydraulic conductivity and sorptivity at unsaturated and saturated conditions as related to water infiltration in soils

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Abstract
Sorptivity (S) has been defined in terms of the horizontal infiltration equation. At unsaturated conditions (at a very short time) S represents "maximum sorption capacity", but in saturated conditions the sorption capacity decreases with the time. Over a long time of infiltration, sorptivity was not studied as a soil water parameter that could be determined. The purpose of this study is to apply derived equations depending on the infiltration functions to predict (1) soil water sorptivity (S) at infiltration capacity (unsaturated conditions) and at basic infiltration rate (I_b) (saturated conditions), (2) the hydraulic conductivity (Saturated K_s and unsaturated K(θ)) into capillary-matrix and non-capillary macro pores of soils. Five alluvial (saline and non-saline clay) and calcareous soil profiles located in the Nile Delta were investigated for applying the assumed equations. A decrease in S value was observed with an increase in soil water content. At steady infiltration rate (I_b), S decreased from 1.04 to 0.647 cm.min^{-0.5} (i.e. S decreased by 37.79%) in average in calcareous soils and from 0.537 to 0.251 cm.min^{-0.5} (53.25%) in alluvial clay soils. The steady S_w parameter was used in prediction of the hydraulic conductivities and the basic infiltration rate I_b, whereas, S_w is a suggested term at steady infiltration rate. The calculated values of I_b were corresponding to those obtained by infiltration experiment. This confirmed the significance of steady S_w as a new functional infiltration parameter. A matching factor u was calculated as a ratio between predicted I_b and the measured saturated hydraulic conductivity, K_s. The mean values of u were 0.895, 0.685 and 0.360 for calcareous, clay and saline clay soils respectively. Unsaturated K(θ) has been discriminated into saturated macro-pore K(θ)_{mac} and matrix unsaturated K(θ)_h. The values of K(θ)_{mac} for macro pores remained higher than those for soil matrix pores (K(θ)_h) in the studied soils. The highest value of K(θ) was obvious in calcareous soil profiles, while the lowest value was existed in saline clay soil. In conclusion, the predicted values of hydraulic conductivities of soil matrix (capillary) and macro (non-capillary) pores were reasonable and existed in the normal ranges of the investigated soils, indicating that the proposed equations are applicable and can be recommended to be used in coarse and fine textured soils with large scale of different properties.

Keywords: Infiltration functions, soil pores, steady sorptivity, unsaturated hydraulic conductivity.

Introduction
Infiltration is the passage of water into the soil surface and is distinguished from percolation, which is the movement of water through the soil profile. Irrigation water is generally infiltrated into root zone during conveyance and recession of water at the soil surface (Amer, 2004). Wu (1971) and Amer (2011a) studied the infiltrated water functions into soil during surface irrigation. Infiltration of water into soil can be described quantitatively by solving the transport equation (Richards, 1931; Klute, 1952). The solutions require knowing the relationship between water content and soil water pressure (h) as well as the relation
between water content and hydraulic conductivity. The literature emphasizes the development of representative infiltration equations, e.g., those proposed by Green and Ampt (1911), Kostiakov (1932), Philip (1957), Parlange et al. (1985). The soil hydraulic properties in relation to water infiltration are essential for quantifying the rate of water flow and transport processes in the plant root zone. Water flows in the root zone occur principally through the macro non-capillary pores of soil, while the redistribution and upward flow occur in the capillary soil matrix pores (Amer, 2012). The water conductivity of soil pores is mainly controlled by pore sizes, continuity and pore size distribution in soil. On the other hand, Water flow, soil surface roughness, and infiltration rate affect the non-uniform and unsteady of flow pattern into root zone along surface irrigation (Hoogmoed and Bouma, 1980). Water inflow is expressed in a continuity equation and an equation of motion (Cahoon et al., 1995).

The purpose of this work is modeling and correlate the infiltration functions to sorptivity, hydraulic conductivity, and soil water filled pores in the root zone. In that concern, it is needed (1) to find a matching factor \((u)\) between infiltration rate and hydraulic conductivity during steady state infiltration, (2) to predict water sorptivity \((S)\) at steady state infiltration, and (3) to propose new applicable equations based on infiltration rate and soil moisture retention functions for prediction of the hydraulic conductivity \([saturated, \text{K}_s\text{ and unsaturated, K}(\theta)\]) into the rapidly (non-capillary) drainable pores (RDP) and capillary-matrix pores of soils.

**Theoretical development**

**Saturated hydraulic conductivity and sorptivity at steady infiltration rate**

The variably saturated flow process in soils is a highly nonlinear and dynamic phenomenon. The infiltration into soil is defined with the one-dimensional differential equation (Klute, 1952, Germann, 2018). The infiltration rate \((I)\) is defined as the volume of water infiltrating through a horizontal unit area of soil surface at any instant (infinitely small period of time), \([\text{LT}^{-1}]\), while the cumulative infiltration \((Z)\) is the total volume of water that has infiltrated through a unit of horizontal area of soil surface over a given period of time \(t\), measured from the beginning of infiltration, (can be expressed in depth unit, cm or mm). For many soils a plot of \(Z\) [\(L\)] as a function of time \(t\) [\(T\)] (or opportunity time, \(t_o\)) is described by the equation (Kostiakov, 1932):

\[
Z = ct^m
\]

or

\[
Z = \frac{k}{m} t_o^m
\]

where, \(c [\text{LT}^m]\) and \(m [\text{dimensionless}]\) are empirical coefficients for a given soil and a given moisture content, respectively, and \(k = c \cdot m\). Both fitting parameters \(c\) and \(m\) can be determined from a simple logarithm regression analysis over the experimental \(Z(t)\) data, as:

\[
\log Z = \log c + m \log t
\]

where, \(\log c\) is the intercept and \(m\) the slope of the linear regression, and \(Z\) is equal to \(c\) in unit of time. By differentiating the expression for \(Z\) (Eq. 2) with respect to time \(t\), the infiltration rate (instantaneous) \(I\) at the soil surface defined as:

\[
I = \frac{dZ}{dt} = cm t^{m-1} \quad \text{or} \quad I = k t_o^{m-1}
\]

where, soil infiltration rate \((I)\) is a function of time, expressed in cm/min or mm/min. Philip (1957) showed that the cumulative infiltration \(Z\), cm in soil changes with square root of time \((t^{0.5})\) depending on water sorptivity \((S)\) of the soil. The \(S [\text{LT}^{-1/2}]\) was originally defined by the Philip equation for horizontal infiltration into an initially dry soil (equation 5),

\[
Z = St^{1/2}
\]

If \(Z\) is plotted against \(t^{0.5}\) then a linear relationship is usually found for the first 1 to 3 minutes of infiltration. The slope of the linear relationship \(S\) is called the water sorptivity of the soil. Equation (5) corresponds to the first term of the semi-analytical solution of Philip (1957) to the governing differential equation for one-dimensional, unsaturated water flow, where the Philip infiltration equation was employed to calculate the cumulative infiltration \(Z\) using time series powers of \(t^{0.5}\) as follows:

\[
Z = C_1(\theta) t^{1/2} + C_2(\theta) t + C_3(\theta) t^{3/2} + C_4(\theta) t^2 + \ldots + C_m(\theta) t^{m/2}
\]

or

\[
Z = \sum_{i=1}^{m} C_i(\theta) t^{i/2}
\]
where, \( C_1(\theta), C_2(\theta), C_3(\theta), C_4(\theta), \ldots, \text{and } C_m(\theta) \) are functions of the soil water content \( \theta \), and \( t \) is time.

The first two and three terms of the Philip infiltration equation (Eq. 6) can be used to estimate the saturated hydraulic conductivity \( K_s \) (Zhang, 1997). The first two terms are applicable for relatively short times as follows:

\[
Z = St^{1/2} + uK_s t
\]  

(7)

where \( K_s \) is saturated hydraulic conductivity, and \( u \) is a constant such that 0 ≤ \( u \) ≤ 1. Philip (1969); Swartzendruber and Young (1974) suggested that a fit of Eq. 7 to the whole elapsed time range would lead to select \( u \approx 1 \).

Equations 2, 4, and 7 have the same functional form, so \( S \) can be replaced by \( C \);

\[
S = C^{0.5/m} \quad \text{or} \quad S = C^{-m} C^{1/2}
\]  

(8)

At steady-state infiltration, the infiltration rate, \( I \), becomes constant and denoted as basic (final) infiltration rate, \( I_b \). The time that must elapse before the instantaneously infiltration rate, \( I \), becomes approximately constant can be expressed in terms of the soil property, \( m \) (Amer, 2011a);

\[
t = 10 \times (1 - m) \quad \text{hr}
\]  

(9)

By differentiation of Eq.7 at unsaturated conditions, the corresponding flux equation becomes:

\[
dZ / dt = S / 2t^{0.5} + uK_s
\]  

(10)

The sorptivity \( S \) is thus:

\[
S = 2t^{0.5}(t - uK_s)
\]  

(11)

At steady state infiltration, where \( I = I_b \), the Eq. 1 and 4 can be rearranged as:

\[
Z = \frac{I_b t}{m} \quad \text{or} \quad Z = \frac{uK_s t}{m}
\]  

(12)

By combination of Eq. 7 in the form \( S = 1/t^{0.5} [Z - K_s t] \) with Eq. 12, the result is:

\[
I_{b\text{(calculated)}} = \frac{S m}{(1 - m)\sqrt{t}}
\]  

(13)

Valiantzas et al. (2009) pointed out that the variation of soil initial water content affected the value of \( S \). So, it may be of interest to define the sorptivity \( S \) at measured steady state infiltration \( I_b \) in term steady sorptivity, \( S_w \).

According to Eq.13, steady sorptivity (\( S_w \)) becomes:

\[
S_w = I_{b\text{(measured)}} \left( \frac{1 - m}{m} \right) \sqrt{t}
\]  

(14)

where measured \( I_b \) is obtained from experimental data at the time \( \sqrt{t} = [10(1-m)]^{0.5} \).

At steady state infiltration, where the gravitational potential is predominant, the basic (final) infiltration rate \( I_b \) [LT\(^{-1}\)] could be identical with the saturated hydraulic conductivity of soil as \( I_b \approx uK_s \) (Eq. 7), and then the constant \( u \) is considered as a matching factor or an adjustment between \( K_s \) and \( I_b \). By combining Eq. 7 with Eqs.10 and 13, the matching factor \( u \) (dimensionless) between \( I_b \) and \( K_s \), has been obtained as;

\[
u = \frac{mI_b \sqrt{t}}{I_{b\text{,calculated}}} \frac{I_b \sqrt{t} - mS}{m}
\]  

(15)

with respect to Eqns. 7, 12 and 13, the saturated hydraulic conductivity (\( K_s \)) can be then estimated theoretically as follows:

\[
K_{s\text{,calculated}} = \frac{Sm}{u(1 - m) \sqrt{t}}
\]  

(16)
Unsaturated hydraulic conductivity \( K(\theta) \) in transmission zone

By measuring sorptivity and using it as a scaling factor, the unsaturated hydraulic conductivity \( K(\theta) \) can be predicted fairly accurately (Moldrup et al., 1993). Application of the one-dimensional form of Darcy’s equation using the average \( K(\theta) \) and hydraulic gradient with reference to steady infiltration rate may be useful in prediction of water flow in unsaturated soils. During infiltration, the application rate of water to soil surface is often greater than \( K(\theta)_{RDP} + K(\theta)_{b} \), where \( K(\theta)_{RDP} \) is the hydraulic conductivity into the rapidly drainable pores and \( K(\theta)_{b} \) is the matrix unsaturated hydraulic conductivity (Germann and Prasuhn, 2018). At steady state infiltration and steady sorptivity \( (S = S_{e}) \), and when ponding water depth \( (h) \) on the upper surface of the soil reaches zero \( (h_{0}) \) [i.e., \( \Delta h = h_{0} - (-\psi) \), where \( \psi \) is the water potential at particular moisture content \( \theta_{i} \) that corresponds to the boundary limit of soil-water filled pore class], Amer (2011b) proposed the following model;

\[
K(\theta) = \frac{Z.S_{w}}{\Delta h.\Delta \theta.\sqrt{t}} \left[ 1 + m \right]^{-\frac{1}{2}} \left[ 2(1-m) \right]^{\frac{1}{2}} \tag{17}
\]

where \( K(\theta) \) is unsaturated hydraulic conductivity in the transmission zone of the infiltration moisture profile in soil, \( Z \) is the cumulative infiltration at the time \( \sqrt{t} = [10(1-m]^{0.5} \) which should be equal to the product of the wetting front depth \( L \) (i.e., the distance from the soil surface to the wetting front), \( \Delta h \) is the pressure head change from soil surface to the wetting front \( (h-h_{0}) \). The latter corresponds to \( \Delta \theta = \theta_{s} - \theta_{i} \) in the soil profile.

The solution of Philip's equation (Eq. 6) indicates that at small times, the advance of any \( \theta \) value proceeds as \( \sqrt{t} \) (just as in horizontal infiltration), while at larger times the downward advance of the wetting front approaches a constant rate \( (K_{0} - K_{i})/\theta(0 - \theta_{i}) \). Here \( K_{0} \) and \( K_{i} \) are the conductivities at the soil water contents of \( \theta_{0} \) (wetted surface) and \( \theta_{i} \) (initial soil wetness), respectively. For different soil pore classes, \( K(\theta) \) can be calculated by applying \( \Delta h = h_{0} - (-\psi) \) for RDP, SDP, WHP, and FCP respectively. The corresponding \( \Delta \theta \) values of water filled pore classes can be derived from soil-moisture retention curve. Thus, the equation 17 can be developed (Amer, 2011b) into:

\[
K(\theta) = \frac{C.Z.S_{w}}{\Delta h.\Delta \theta} \left[ \frac{1+m}{(1-m)^{1.5}} \right] \tag{18}
\]

The calculated values of \( K(\theta) \) (in cm/hr) by Eq. 18 represent the accumulative drainable and matrix pore classes in transmission and wetting zones. In order to calculate \( K(\theta) \) for individual class of pore size, Eq. 18 should contain saturation degree \( (a) \) as a representative for that particular pore class size:

\[
K(\theta)_{i} = \frac{C.Z.S_{w}.a}{(\Delta \theta)_{i} \Delta \theta} \left[ \frac{1+m}{(1-m)^{1.5}} \right] \tag{19}
\]

where the subscript \( i \) denotes the soil pore class, \( C \) is a numerical coefficient = 0.1581, \( \Delta \psi \) is the matric potential of the particular pore class, and \( a \) represents

\[
\frac{\Delta \theta_{RDP}}{\theta_{s}}, \frac{\Delta \theta_{SDP}}{\theta_{s}}, \frac{\Delta \theta_{WHP}}{\theta_{s}}, \text{ and } \frac{\Delta \theta_{FCP}}{\theta_{s}} \text{ for RDP, SDP, WHP, and FCP respectively.}
\]

Material and Methods

Five soil profiles; calcareous sandy loam, alluvial saline and non-saline clay, located at the Nile Delta (Egypt) were used for testing the applicability of proposed equations (Table 1). The 1st and 2nd profiles located at Nubaria and Borg El-Arab areas (northern west of the Nile Delta), and 3rd, 4th and 5th located at Shebin Elkomy, Ebshan, and El-Khamsin (middle Nile Delta) areas, respectively. Disturbed and undisturbed soil samples were taken from three successive depths of the concerned soil profiles. Soil samples were subjected to chemical and physical analyses (as given in Table 1) according to Page (1982), Sparks et al. (1996), Dane and Topp (2002). Saturated hydraulic conductivity \( (Ks) \) was measured with the constant head method as discussed in Klute (1986). Darcy’s law was applied to calculate \( Ks \);

\[
Ks = \frac{V.L}{A.t.\Delta H} \tag{20}
\]
where \( V \) is the volume of discharged water (cm\(^3\)), \( L \) is the length of the core (cm), \( A \) is the cross-sectional area of the core (cm\(^2\)), \( t \) is the discharge time (sec), and \( \Delta H \) is the hydraulic head difference across a distance \( L \) (cm). The \( \theta_{\text{soil}} \) soil water content on volume basis at suction pressure head \( h = 10 \text{ kPa} \) was determined using undisturbed samples for clay alluvial and saline soils (profiles III, IV, and V), while neutron probe and tensiometers in situ were used for calcareous soils (profiles I and II). Disturbed samples were air-dried, gently crushed, sieved through a 2 mm sieve, and used for analysis of saturation water content (\( \theta_s \)), CaCO\(_3\) salinity (\( \text{EC} \)), sodium adsorption ratio (\( \text{SAR} \)), and particle size distribution. The hydration envelopes in which water content is considered to be immobile in soil should be subtracted from FCP, can be expressed as moisture adsorption capacity (\( W_a \)) (Amer, 2009):

\[
W_a = W_m + 2W_{me}
\]

(21)

where \( W_m \) is the mono-adsorbed layer of water molecules on soil particles, and \( W_{me} \) is the external mono-adsorbed layer of water molecules. The water vapour adsorption isotherm method with applying BET theory was used to estimate \( W_m \) and \( W_{me} \).

The infiltration rate was measured using the double ring method (Ankeny, 1992; Reynolds et al., 2002) in the field for the concerned soils.

Table 1. Physical and chemical properties of the studied soils.

| Soil profile and location | Soil depth, cm | EC\(^1\), dS m\(^{-1}\) | \( \rho_b \), g cm\(^{-3}\) | CaCO\(_3\), % | Particle size distribution, Sand, % | Silt, % | Clay, % | Texture class | \( \theta_s \), m \(^{3}\) | \( K_s \), cm h\(^{-1}\) | \( W_a \), % |
|------------------------|---------------|-----------------|-----------------|--------------|-----------------|--------|--------|---------------|-----------------|-----------------|--------|
| I                      | 0-20          | 0.34            | 1.52            | 23.00        | 55.79           | 20.31  | 23.90  | SCL           | 5.12             | 3.81             | 5.90   |
| Nubaria                | 20-40         | 0.26            | 1.52            | 23.00        | 55.79           | 20.31  | 23.90  | SCL           | 5.12             | 3.81             | 5.90   |
|                       | 40-60         | 0.24            | 1.52            | 26.00        | 58.45           | 22.15  | 23.00  | SCL           | 4.89             | 3.67             | 4.98   |
| II                     | 0-20          | 0.38            | 1.46            | 36.00        | 71.33           | 17.30  | 10.37  | SL            | 4.49             | 3.32             | 5.40   |
| Borg                   | 20-40         | 0.42            | 1.48            | 38.00        | 73.32           | 15.30  | 11.38  | SL            | 4.44             | 3.10             | 5.16   |
| El-Arab                | 40-70         | 0.41            | 1.48            | 32.00        | 77.91           | 13.00  | 9.90   | LS            | 4.31             | 3.59             | 4.70   |
| III                    | 0-30          | 1.90            | 1.30            | 1.90         | 23.76           | 35.28  | 40.96  | C             | 6.57             | 2.20             | 13.46  |
| Shebin                 | 30-60         | 1.60            | 1.38            | 1.84         | 23.60           | 34.75  | 41.65  | C             | 6.65             | 1.78             | 12.17  |
| El-Kom                 | 60-90         | 2.00            | 1.35            | 0.92         | 22.29           | 32.91  | 44.80  | C             | 6.62             | 1.72             | 9.63   |
| IV                     | 0-30          | 2.30            | 1.27            | 0.84         | 21.98           | 15.37  | 62.65  | C             | 7.21             | 1.25             | 12.32  |
| Ebshan                 | 30-60         | 1.89            | 1.28            | 0.98         | 14.31           | 18.69  | 67.00  | C             | 7.68             | 1.04             | 13.70  |
| El-Khamsin             | 60-90         | 1.22            | 1.28            | 0.79         | 16.44           | 24.38  | 59.18  | C             | 7.32             | 1.13             | 13.56  |
| V                      | 0-30          | 6.00            | 1.21            | 0.67         | 8.26            | 28.50  | 63.24  | C             | 7.43             | 0.98             | 13.07  |
|                       | 30-60         | 6.64            | 1.19            | 0.82         | 7.38            | 23.62  | 69.00  | C             | 7.82             | 0.81             | 14.75  |
|                       | 60-90         | 8.12            | 1.18            | 0.56         | 9.04            | 20.46  | 70.50  | C             | 7.75             | 0.75             | 14.39  |

\(^1\)EC is electrical conductivity, \( \rho_b \) is bulk density, \( \theta_s \) is saturation water content and \( K_s \) is measured saturated hydraulic conductivity.

### Results and Discussion

**Pore class size distinctions**

Water is held in soil pores by cohesive and adhesive capillary forces. The size of pores in unsaturated soil state can be determined through the so-called hydraulic radius (\( r \)) of a section of pore space. The relation between \( r \) and capillary forces expressed as pressure head potential (\( h \)) in m is represented by the following capillary rise equation (Hillel, 1980; Amer et al., 2009):

\[
h = \frac{2\gamma \cos \alpha}{\rho_b g r}
\]

(22)

where, \( \gamma \) is surface tension between water and air (at 20°C = 0.0727 kg s\(^{-2}\)), \( r \) (in m) is equivalent cylindrical pore size (hydraulic) radius related to meniscus curvature radius (\( R \)) via equation; \( r = R \cos \alpha \), and \( \cos \alpha \) is assumed to be 1 for the wet surface, \( g \) is acceleration due to gravity (9.8 m s\(^{-2}\)), and \( \rho_b \) is density of water (998 kg m\(^{-3}\) at 20°C). As soil dries out, increasing suction occurs due to progressive empty of capillary pores. Pore size diameters were determined for the ranges of soil matrix potentials by applying Eq. (22) with respect to soil water retention curves (Table 2).

The K(\( \theta \)) of capillary pores was divided into K(\( \theta \))\(_{\text{SDP}}\), K(\( \theta \))\(_{\text{WHP}}\), and K(\( \theta \))\(_{\text{FCP}}\) within slowly drainable pores, SDP, water holding pores, WHP and fine capillary pores FCP, respectively. These categories can be combined into total draining pores (\( TDP \)) (0-330 hPa), and total water-storage pores (\( WSP \)) (>330 hPa), as well as into macro (non-capillary) pores (<100 hPa) and soil matrix (capillary) pores (>100 hPa). The pressure head corresponding with the cutoff between capillary and non-capillary pores varies widely, ranging from 1.0 hPa (Beven and Germann, 1982) to 100 hPa (Marshall, 1956). However, \( h = 100 \text{ hPa} \) is selected by Amer et al. (2009) as corresponding to the limit between capillary and non-capillary pores.
Table 2. Pore size classes as a percent of soil bulk volume (Δθ%) and ratio of total volume pores

| Soil profile and location | Soil depth (cm) | RDP | SDP | TDP | WHP | CCP | FCP | TVP | A/W |
|--------------------------|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
|                          |                | Δθ  | Δθ  | Δθ  | Δθ  | Δθ  | Δθ  | Δθ  | Δθ  |
|                          |                | %   | %   | %   | %   | %   | %   | %   | %   |
| I                        | 0-20           | 14.6| 0.369| 11.10| 0.280| 25.72| 0.649| 17.50| 0.442|
| Shebin El-Kom (Clay)     | 20-40          | 13.8| 0.385| 10.01| 0.278| 23.86| 0.663| 14.94| 0.415|
|                          | 40-60          | 12.8| 0.364| 9.42 | 0.267| 22.24| 0.631| 15.42| 0.437|
|                          | 0-30           | 1.25| 0.190| 8.10 | 0.123| 9.35 | 0.142| 31.68| 0.482|
| Shebin El-Kom (Clay)     | 30-60          | 1.33| 0.192| 16.30| 0.235| 17.63| 0.254| 28.90| 0.417|
|                          | 60-90          | 2.00| 0.302| 13.28| 0.231| 15.28| 0.231| 29.54| 0.446|

A/W is Air/Water ratio or A/W = TDP/(WHP+FCP)

The radii and volumes of the drainable and capillary pores were determined (such as in Table 2) from the soil water retention curves SWRC, ψ(θ) (Figure 1) by applying equation (22).

Figure 1. Pore size distribution (%) expressed in volumetric water content (θ%) and soil moisture suction (pF=log h) or [h(θ)] function in Shebin El-Kom soil profile.

Infiltration power functions and water sorptivity

The typical trends in cumulative infiltration, Z, cm versus time t minute and infiltration rate, I, cm/h are illustrated by empirical power functions according to Eqns.1 and 4 (Table 3 and Figure 2). The constants c and m of the equations ranged from 1.12 to 0.51 and from 0.58 to 0.38 respectively, in the investigated soils. The highest values of c and m were evident in calcareous soils (profiles I and II) and the lowest value was in alluvial saline clay soil (profile V). As the onset of wetting, the moisture gradient was the greatest, hence more rapid infiltration was obtained. The infiltration rate I (LT⁻¹) slowed gradually with time t and reached the steady state of flow (basic infiltration rate, I_b) after 4.2-4.5 h from the beginning of infiltration for calcareous I and II soil profiles, and after 5.4-6.2 h in alluvial clay IV and V soil profiles. The steady-state infiltration was occurred in Shebin El-Kom soil (III profile) after 4.7 hour. Values of steady infiltration rate can be calculated using Eq.9 or by experimental infiltration curves of Z(t) function.

Sorptivity (S) in unsaturated condition represents the highest capacity of "absorption" but the capacity decreases with increasing water content in soil due to accumulated infiltration depth. It may be of interest to propose sorptivity as a soil hydro-physical property. Thus, the term "sorptivity" (S) at unsaturated conditions [at a very short time (1 – 3 minutes)] represents "maximum sorption capacity", while at saturation conditions S represents "minimum sorption capacity". With respect to water infiltration in soil, the term wet or steady sorptivity (Sw) after a long time of infiltration (saturation case) may be suggested for application in similar way to the term "steady infiltration rate".
Table 3. Infiltration functions and hydraulic conductivities for the studied soils

| Soil profile and location | Z, cm & \( I_1 \), cm/h | \( S_w \), cm/min^{0.5} | \( S_i \), cm/min^{0.5} | \( K_0 \) (=\( I_0 \)), cm/min | \( K(\theta)_RDP \), cm/min^{1} | \( K(\theta)_h \), cm/min^{1} | (u) \( I_0 \)/\( K_i \), | \([\theta]_{0.5\cdot10^{6} \cdot P_{so}} \) |
|-------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| I Nubaria              | 0-20            | 0.97 T^{0.58}   | 0.974           | 0.633           | 0.055           | 7.11x10^{-1}    | 3.83x10^{-3}    | 0.866           | 0.0662          |
|                        | 20-40           | 33.8 T^{0.42}   | 0.661           | 0.049           | 5.45x10^{-3}    | 3.55x10^{-3}    | 0.899           | 0.0585          |
|                        | 40-60           | 36.9 T^{0.45}   | 1.108           | 1.023           | 8.23x10^{-3}    | 5.58x10^{-3}    | 0.948           | 0.0754          |
| II Borg El-Arab        | 0-20            | 1.12 T^{0.55}   | 1.108           | 0.049           | 8.23x10^{-3}    | 5.58x10^{-3}    | 0.948           | 0.0754          |
|                        | 20-40           | 36.9 T^{0.45}   | 1.108           | 0.049           | 8.23x10^{-3}    | 5.58x10^{-3}    | 0.948           | 0.0754          |
|                        | 40-70           | 36.9 T^{0.45}   | 1.108           | 0.049           | 8.23x10^{-3}    | 5.58x10^{-3}    | 0.948           | 0.0754          |
| III Shebin El-Kom      | 0-30            | 0.61 T^{0.53}   | 0.627           | 0.023           | 8.75x10^{-4}    | 7.38x10^{-4}    | 0.623           | 0.0252          |
|                        | 30-60           | 19.3 T^{0.47}   | 0.340           | 0.023           | 8.16x10^{-4}    | 6.89x10^{-4}    | 0.769           | 0.0248          |
|                        | 60-90           | 19.3 T^{0.47}   | 0.340           | 0.023           | 8.16x10^{-4}    | 6.89x10^{-4}    | 0.769           | 0.0248          |
| IV Ebshan              | 0-30            | 0.59 T^{0.46}   | 0.574           | 0.012           | 3.71x10^{-4}    | 4.59x10^{-4}    | 0.584           | 0.0221          |
|                        | 30-60           | 16.4 T^{0.54}   | 0.257           | 0.012           | 3.71x10^{-4}    | 4.59x10^{-4}    | 0.584           | 0.0221          |
|                        | 60-90           | 16.4 T^{0.54}   | 0.257           | 0.012           | 3.71x10^{-4}    | 4.59x10^{-4}    | 0.584           | 0.0221          |
| V El-Khamisn           | 0-30            | 0.51 T^{0.38}   | 0.412           | 0.005           | 1.29x10^{-4}    | 2.09x10^{-4}    | 0.306           | 0.0192          |
|                        | 30-60           | 11.6 T^{0.62}   | 0.157           | 0.005           | 1.29x10^{-4}    | 2.09x10^{-4}    | 0.306           | 0.0192          |
|                        | 60-90           | 11.6 T^{0.62}   | 0.157           | 0.005           | 1.29x10^{-4}    | 2.09x10^{-4}    | 0.306           | 0.0192          |

\([\theta]_{0.5\cdot10^{6} \cdot P_{so}} \) is drained water of macro pores, and \( K_0 \) is predicted saturated hydraulic conductivity.

Figure 2. Cumulative infiltration (Z), infiltration rate (I) and Prediction of Sorptivity (S) at square root time (T^{0.5}) for Borg El-Arab soil profile

Data of infiltration parameters and water sorptivity are given in Table 3. Hallett (2008) mentioned that sorptivity is the capacity of soil to absorb (suck up) water and is dominated by the antecedent water content of the soil. At the beginning of infiltration in an initial dry soil (unsaturation conditions), the sorptivity, S can be calculated using Z as functioned to time t and adjusted to m = 0.5 (Eq. 5). The S values were found to be ranged from 1.108 to 0.412 cm/min^{0.5} in the studied soils. The values were in the following order: Borg El-Arab>Nubaria>Shebin El-Kom>El-Bshan>El-Khamsin. Sorptivity (S) at steady-state infiltration was denoted as wet or steady sorptivity (\( S_w \)) and calculated using Equation 14. It worthy to mention that the data of the infiltration rate at steady-state infiltration \( I_0 \) which calculated via steady sorptivity (\( S_w \)) (Eq.13) were correspondent to those obtained by the experimental data. This confirm the significance of \( S_w \) in predicting the hydrological soil parameters such as \( I_0 \), \( K_0 \) and \( K(\theta)_h \). It was observed that sorptivity was decreased at steady-state infiltration by 37.79% in average in calcareous soils and by (53.25%) in average in alluvial clay soils. This means that a dry soil typically has a much greater sorptivity than a wet soil (Hallett, 2008). These results attributed to soil texture and salinity in alluvial clay soils and to abundance of CaCO_{3} fraction which has a great ability to suck up water in such calcareous soils (Ghazy, 1993).

 **Hydraulic conductivity in soil pores and matching factor**

Data presented in Table 3 show the values of hydraulic conductivity \( K(\theta) \) as calculated by the derived equations for matrix and macro pores of the investigated soils. The \( K(\theta) \) values were discriminated into saturated hydraulic conductivity \( (K_0) \), macro pore saturated \( K(\theta)_{RDP} \) and matrix unsaturated \( K(\theta)_h \) of soil (Amer, 2012; Weiler, 2017). The values of \( K(\theta)_{RDP} \) remained higher than those for lateral \( K(\theta)_h \) in I, II, III soil.
profiles, particularly, in calcareous soils. The opposite trend was observed for VI and V heavy clay soil profiles. It was evident that $K(\theta)_{RDP}$ values increased gradually with increasing sand and CaCO$_3$ fractions in soil profiles. The hydraulic conductivity $K(\theta)$ values into soil matrix were higher as much as in Borg El-Arab calcareous soil (profile II) due to the prevalence of CaCO$_3$ fraction in that calcareous soil. As expected, the values of $K(\theta)_b$ and $K(\theta)_{RDP}$ increased with increase in pore sizes, soil porosity, and water content; $\theta$ and $[\theta]_{b-10}$. On the other hand, the values are decreased by the prevailing fine clay fraction, salinity, fine and coarse capillary pores in the soil matrix of such clay soils. A matching factor $u$ was calculated as a ratio between predicted $I_b$ (Eq.13) and the measured $K_s$ (Table 2). The mean values of $u$ ranged from 0.91-0.88 in calcareous soils (I&II profiles) to 0.73-0.64 in clay soils (III&IV profiles). The $u$ values decreased to 0.36 in saline clay soil (V profile) indicating that the matching factor decreases with increasing clay fraction and salinity of soils.

**Conclusion**

Equations were proposed to estimate the hydraulic conductivity $K(\theta)$ and sorptivity (S) in soils. Five soil profiles - located in the Nile Delta - differ in their texture, salinity, and CaCO$_3$ % were used for applying the assumed equations. The equations based on the measurements of infiltration functions in particular, steady infiltration rate ($I_b$). The $K(\theta)$ was considered into macro-pore saturated $K(\theta)_{RDP}$ and matrix unsaturated $K(\theta)_b$. Using the assumed equations, the values of $K(\theta)_{RDP}$ remained higher in macro pores particularly for calcareous soils than those for soil matrix. Generally, the highest values of hydraulic conductivities [Ks, cm.h$^{-1}$, $K(\theta)_{RDP}$, and $K(\theta)_b$.cm.min$^{-1}$] were observed in calcareous soils and the lowest were existed in saline clay soil profile. The predicted values of hydraulic conductivities were reasonable and existed in the normal ranges of the investigated soils, indicating that the proposed equations are applicable and can be recommended to be used in coarse and fine textured soils with large scale of different properties. Sorptivity (S) at unsaturated conditions represents "maximum sorption capacity", while at saturation conditions S represents "minimum sorption capacity". With respect to water infiltration in soil, the term wet or steady sorptivity (S$_w$) after a long time of infiltration (saturation case) may be applied in a similar way to the term "steady infiltration rate". Water sorptivity (S) was determined for the studied soils at unsteady state (S) and at steady state (S$_w$) of infiltration. It was found that S decreased from S to S$_w$ by 37.79% in average in calcareous soils, and by (53.25%) in average in alluvial clay soils indicating that dry soils typically has a much greater sorptivity than wet soils. The steady S$_w$ parameter was used in prediction of the hydraulic conductivities and the basic (steady) infiltration rate $I_b$. The calculated values of $I_b$ were corresponding to those obtained by infiltration experiment. This confirmed the significance of steady S$_w$ as a new functional infiltration parameter. A matching factor $u$ was calculated as a ratio between predicted $I_b$ and the measured saturated Ks. The mean values of $u$ were 0.895, 0.685 and 0.360 for calcareous, clay and saline clay soils respectively.

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