Relaying Swarms of Low-Mass Interstellar Probes

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Abstract

Low-mass probes propelled by directed energy from earth are an early option for exploration of nearby star systems. A challenging aspect of such technology is returning scientific observational data to earth. We compare two configurations for achieving this. A direct configuration utilizes optical transmission from the probe to a terrestrial receiver employing a large photon collector. In a relay configuration, probes spaced at uniform intervals act as regenerative repeaters for the scientific data, which eventually arrives at a terrestrial receiver from the most recently launched probe. A number of advantages and disadvantages of the relay configuration are discussed. A numerical comparison approximates equal probe mass in the two cases by using the same optical transmit power and equivalent total transmit plus receive aperture area. When the total downlink data rate is equal, the relay configuration benefits from a smaller terrestrial receive collector, but also requires very frequent launches to achieve higher data rates due to the limitations on relay probe receive aperture area. The direct configuration can achieve higher data rates without such frequent launches by increasing terrestrial collector area. A single-point failure problem in the relay configuration can be addressed by introducing relay-bypass modes, but only at the expense of further increases in launch rate or reductions in data volume, as well as a considerable increase in design and operational complexity. Taking into account launch and collector area costs, the direct configuration is found to achieve lower overall cost by a wide margin over a range of cost parameter values and data rates.

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1. Introduction

Previous studies of communication of scientific data from low-mass interstellar probes have assumed direct probe-to-earth transmission [1, 2, 3]. An alternative that invariably arises in discussions of the data downlink system configuration is the idea of opportunistically tasking more recently launched probes as relays servicing the downlink communication needs of probes launched earlier. Particularly attractive is the opportunity to devote otherwise-unused electrical power resources and communication capabilities to the relaying function while probes traveling to the target star, during which time the opportunities for scientific observations are more limited.

1.1. History

The basic idea behind this relaying of data has a long and storied history in the digital transmission and storage of data. The periodic regeneration of digitally-represented data is a powerful technique used widely in terrestrial communication systems as well as in storage systems. In such systems the data is inevitably copied to create new replicas or to replace an aging copy. Each such copy is called a new generation, which is the origin of the term regeneration. With multiple generations of a medium like audio or video that is communicated or stored in analog (continuous in time or amplitude) form, successive generations inevitably deteriorate due to the accumulation of noise and distortion. On the other hand, multiple generations of a digital representation of information are almost totally free of these accumulating impairments. There may be some impairment due to occasional errors in the recovery of a digital representation, but these can usually be rendered insignificant with little penalty.

In communications, the idea is to periodically position regenerative repeaters, which partitions the end-to-end communication into links. Repeaters encompass receiver/transmitter combinations that recover the original digital data almost error-free and retransmit it free of any noise and distortion introduced on the previous link. This simple idea largely prevents the accumulation of noise and distortion effects over long distances. Periodic regeneration is the primary reason that local vs long-distance voice and video calls are today largely indistinguishable in quality. Regeneration is the original (and probably still the single most important) motivation for migration from analog to digital transmission [4], as well as for the long-term storage of various media in digital rather than analog form [5].

Just as regenerative repeaters are applicable to terrestrial communications, they have been used in space communication as well, where the term relay is applied to a single regenerative repeater node. A common configuration uses an orbital satellite as a relay for a surface vehicle or other orbital satellites in the vicinity of Earth [5], the Moon [6], and Mars [7]. This has the benefit (relative to direct-to-earth transmission) of reducing the surface vehicle’s resources (aperture area, electrical power, etc) devoted to communication.

1.2. Application to low-mass probes

We quantitatively explore the merits of the relay idea as applied to low-mass interstellar probes by considering and comparing two alternative configurations for a swarm of probes performing a flyby of a target star. These configurations are:

Direct configuration. The probes operate completely independently of one another. Each probe serves as a scientific data collection platform during encounter with the target star, as well as incorporates a dedicated and separate communication downlink for that scientific data which transmits directly

In fiber optics systems, due to technological opportunities like low-loss and low-dispersion fibers, regenerative repeaters are often replaced by simpler optical amplifiers, which harks back to the practice in earlier analog systems.
post-encounter to a terrestrial receiver. Such a receiver will typically receive data from multiple post-encounter probes simultaneously, and thus multiple downlinks operate concurrently. This configuration was analyzed in [1].

**Relay configuration.** The probes operate independently in their scientific observations, but work cooperatively to return the resulting scientific data to earth. There is a single downlink originating from a post-encounter probe with scientific data to share with earth. The downlink then passes through regenerative repeaters carried by all more recently launched probes. In each repeater, the scientific data embodied in the downlink is recovered at the highest available fidelity before being re-encoded and re-transmitted to another probe closer to earth.

These configurations are illustrated in Fig. 1. In the direct configuration, multiple post-encounter probes are involved in downlink communication, but they have no other function during downlink operation. In particular, their scientific observations have been completed during a previous encounter with the target star, so that each probe carries both scientific instrumentation and communication capability. Since the probes operate independently they can be heterogeneous (in launch interval, mass/velocity, data rate, scientific mission etc).

In the relay configuration the probes are launched at fixed intervals, and are homogeneous in mass/velocity so that they cruise with unchanged inter-probe distance. Their scientific instrumentation and missions may still be differentiated, but there is no opportunity to adjust probe mass to instrumentation and data volume needs as proposed in [1].

Also in the relay configuration each probe deploys a communication receiver (as well as transmitter) and performs the regeneration function. From a communication perspective, the probes perform four distinct functions during their lifetime. There is a distinction between a relay probe and a post-encounter probe, as the relay probe is required to receive downlink data from a more distant probe, regenerate that data, and then retransmit it to a probe closer to earth. There are three distinct operational phases for relay probes:

- The most recently launched relay probe (called the near relay probe) transmits directly to a terrestrial receiver, similarly to the direct configuration but benefiting from a much shorter propagation distance. As a result of that shorter distance, the area of the terrestrial receive collector can be considerably smaller.
  - Following the launch of another probe but still pre-encounter, the relay probe has a single function, which is to serve as a regenerative repeater in support of the downlink.
  - During its subsequent encounter with the target star, the relay probe performs its scientific observations and concurrently performs the relay function (with its regenerative repeater in operation).

1.3. **Comparison**

The goal here is to understand the performance limitations and cost implications of the two configurations, and to compare them. This comparison is divided into two categories: technical and economic. To approximate a fixed probe mass in all cases, the transmit power and total aperture areas are held fixed. From there the technical and economic comparisons diverge:

- In the technical comparison the goal is to appreciate the difference in terrestrial aperture...
area due to the much greater propagation distance in the direct configuration. To accomplish this, the launch interval and total scientific data volume conveyed to earth by each probe are forced to be the same, and the terrestrial collector areas are compared. This does not account for any difference in total data latency.

- In the economic comparison, it is important to allow the launch frequency and interval to be different for the direct and relay configurations. This is because the direct configuration derives economic benefit from reducing the number of probes launched and compensating for this with a larger data volume per probe, and this is not an option in the relay configuration. For a fair comparison, the total cumulative downlink data rate is held fixed between the two configurations, and the costs are compared. Only costs that differ strongly between the two configurations, namely the cost of launch and the cost of collector area, are considered.

1.4. Summary of outcomes

The advantage of the relay configuration is a significantly smaller receive terrestrial collector area, not much larger than the receive aperture on a relay probe (see §4). The price paid for this advantage is a small data volume per probe, constrained by the receive aperture size on each relay probe. The principle technical advantage of the direct configuration is the possibility of much larger data volumes per probe, because the receive terrestrial collector area is not constrained by probe mass considerations and can therefore be much larger. In the direct configuration, the data volume per probe can be freely matched to science requirements by adjusting the terrestrial collector area, but in the relay configuration the data volume is constrained by the probe parameters (and hence probe mass considerations).

The relay configuration is much more complex from a design and operational perspective. In addition it is quite vulnerable to launch and probe failures, with individual missing or failed probes compromising the operation of the entire system (see §5). To an extent this can be overcome by building in robustness to failure, but only at a significant cost in terms of reduced data volumes per probe as well as some significant additional complications in the design and operation. In the direct configuration each probe operates independently, and launch failures affect a single probe. However, the system remains vulnerable to generalized shutdowns when there are failures or problems with the terrestrial receiver, which is shared across all probes currently in their post-encounter phase.

Ultimately cost considerations will be important in choice of a configuration (see §6 and §7). There is a tradeoff between larger launch costs (relay configuration) and the capital cost of a larger receive terrestrial collector (direct configuration). For the relay configuration, achieving higher data volumes cumulative across all probes requires an investment in a shorter launch interval, with the attendant recurrent energy costs of launches. The data volume per probe is severely limited, which may be in conflict with scientific requirements. Launches must be frequent, regular, and uninterrupted for the entire life of the system, which precludes commensal uses of the launch infrastructure. On the other hand, replication of the receiver for parallel uses is less expensive due to the small collector aperture size.

Achieving higher data volumes for the direct configuration requires a one-time capital investment in a significantly larger terrestrial receive collector, which is compensated by less frequent launches. The launch interval and probe mass can be freely varied according to scientific objectives or advancing technology. Commensal uses of the launch infrastructure and cost sharing across different missions objectives (such as outer planet and a multitude of nearby stars) is feasible due to a flexible launch schedule. An estimate of total costs and unit costs finds that overall the direct configuration is more cost-effective by a wide margin across a range of cost parameter values and downlink data rates.

Overall, taking into account both technical and cost considerations, the direct configuration appears to have overwhelming advantages.

2. Technical comparison

From a technical perspective, the relay configuration has advantages and disadvantages.

2.1. Advantages

Near terrestrial receiver. Since a terrestrial receiver on earth receives downlink data from the most recently launched probe, the short
propagation distance from that probe implies that this receiver’s collector area is significantly smaller. In fact the terrestrial receiver is comparable in its performance metrics to a relay probe receiver, with the added complication that it has to deal with atmospheric impairments and with the additional propagation distance that accounts for the launch.

**Outage mitigation.** The relay probe transmit function does not need to account for atmospheric outages (sunlight and weather). Only the near probe has to communicate through the atmosphere, and thus suffer the additional redundancy overhead to counter outages due to weather, sunlight scattering, etc. This larger overhead can be compensated by a modest increase in the area of the terrestrial collector.

**Wavelength flexibility.** With the unique exception of the near relay probe, the transmit wavelength can be chosen freely without regard to atmospheric transparency. This could result in smaller transmit/receive apertures, and operation at UV wavelengths would in addition significantly reduce the background radiation originating from the target star.

**Simple multiplexing.** The complexities of multiplexing signals from multiple probes and separating them at the receiver are avoided. While the single downlink is time-shared between all the probes, in the simplest case assumed here only one probe at a time makes use of the downlink. Even if more than one probe originates downlink data concurrently, the time-division multiplexing this implies is easy to implement.

**Limited coverage.** Each relay probe and the relay near-link terrestrial receiver require limited coverage because they transmit to a single probe or terrestrial receiver, or receive from a single probe. This reduced coverage increases aperture sensitivity, and may also simplify tracking.

**Limited transmission time.** In the relay configuration, each post-encounter probe transmits all the scientific data it has accumulated over a limited time (equal to the inter-launch interval), and subsequently shuts itself down permanently. Thus, the lifetime of the electrical power source is advantageously fixed and short.

**Fixed power and distance.** The propagation of all signals between relay and post-encounter probes is the same. Only the near-link probe need compensate for variations in propagation distance, and that is as simple as designing for the worst-case. However, in practice improving the system reliability by allowing for relay probe bypass (see §5) negates this advantage.

### 2.2. Disadvantages

**Multiple functionality.** Although the communications functionality is distinctive between post-encounter and relay probes, each and every probe serves all roles (scientific and communications) at different times during the mission and thus has to embody all the requisite capabilities with all the implications of that to mass, electrical power, attitude control, etc.

**Multi-probe data.** For the direct configuration, there are parallel downlinks operating concurrently. In the relay configuration, there is a single downlink that is used sequentially by each probe post-encounter to convey its scientific data back to earth. All else equal, the data rate on that single downlink is significantly higher, with all the implications to processing and electrical power requirements.

**Concurrent observation and regeneration.** In the direct configuration the scientific observations and downlink communications need not be concurrent, and this moderates the electrical power requirements. In the relay configuration a probe performing scientific observations must concurrently serve as a regenerative repeater for the downlink serving a post-encounter probe. Thus the electrical power requirements are correspondingly higher, although this may possibly be offset by photovoltaic power generation available in the vicinity of the target star.

**Smaller data volume** We will find that the direct configuration can achieve considerably
greater data volumes per probe. The reason for this is that the receive terrestrial collector is unconstrained in size, whereas in the relay configuration the receive aperture on each relay probe is highly constrained by probe mass limitations. Within one relay-to-relay link, the small apertures for both transmit and receive functions are not overcome by the shorter propagation distance unless the inter-launch interval is very short, in which case the launch energy costs become very large.

Coupling of launch interval and data volumes. In the direct configuration, the number of probes and data volume per probe can be chosen independently based on maximization of scientific return. In the relay configuration these two parameters are closely coupled and dependent due to the relationship between data volume and launch interval.

Speed uniformity. All probes must travel at the same velocity to maintain a consistent maximum propagation distance. Thus any opportunity to trade larger probe mass for lower speed, greater data latency, and higher data volume as in the direct configuration [1], is abandoned.

Parallax and pointing complications. Since probe launches occur at different times in the earth’s orbital cycle, and are aimed in slightly different directions to track the proper motion of the target star, the trajectories of probes are not co-linear. The pointing of each relay probe to a probe nearer to earth (rather than earth itself) will be complicated by parallax effects, and further there is no apparent reference for this pointing function. In the direct configuration, the sun provides a convenient reference for pointing back to earth.

Simultaneous attitude adjustment. In the direct configuration, attitude adjustment for scientific observations and for data communications can be separated, since the probe need not do both functions simultaneously. For the relay configuration, a probe performing attitude adjustments for its own scientific observations has to simultaneously maintain pointing accuracy in both its data transmission and reception functions. This presumably implies independence of aperture pointing and probe attitude.

Continuous launches. Probes must be launched at regular intervals, without breaks for maintenance, weather events, etc. Any interruption in the launch schedule cuts off the scientific data downlinks from all previously launched probes unless there is a probe bypass capability (see §5) which substantially reduces data volumes.

Inter-probe interference. Earlier-launched probes transmitting to their respective relays will be a source of unwanted interference with relay links closer to earth. With considerable complication this interference would be largely eliminated by using different wavelengths on the different links.

Regeneration processing. Although all-optical regeneration of signals has been studied for fiber systems [8], in a photon-starvation mode with high photon efficiency neither optical amplification nor optical regeneration is an option. Thus, each relay probe is a full regenerative repeater which performs optical-to-electrical conversion and implements the complete receive and transmit stacks (modulation decoding, error-correction decoding, error-correction coding, modulation coding). The processing required for error-correction decoding in particular is quite significant. This processing is replicated in every probe, and may consume an electrical power that is significant in comparison to the optical transmit power.

Error accumulation. In the relay configuration bit errors in recovery of scientific data will accumulate through multiple regeneration steps. Thus, the permissible error rate objective in each relay link will have to be correspondingly tighter, resulting in slightly higher transmit powers.

Timing recovery. High photon efficiency using pulse-position modulation (PPM) with photon starvation presents a difficult challenge in deriving the timing of the PPM slots. This would presumably have to be performed in real-time (as opposed to post-processing following completion of the mission as in the direct case). It is likely this would only be feasible at higher received power levels and poorer photon efficiencies, reducing data rate. It would also consume considerable processing power.
Cryogenic. Cryogenic (low) temperatures are desired for the receive photonics and optics. This will likely be accomplished with passive cooling in general as the probes will run cold during the cruise phase, but may be a challenge in the vicinity of the target star. The direct configuration requires no receiver and thus no cryogenic temperatures.

Numerous single points of failure. The failure of a single probe results in partial or complete mission failure for all earlier-launched probes. This can be offset by adding the complication of bypassing probes, although this comes at the expense of lower data volumes.

Complexity. A relay approach will impose considerable design and operational complexity. In contrast to planetary missions there is no opportunity to upgrade software, and thus extensive operational testing will be required before the first launch. Aspects of the operation such as attitude adjustment would probably be impossible to test in advance, raising the risk of malfunction.

Many abandoned probes. At the end of the system life, a full cohort of relay probes will be precluded from communicating data back to earth for lack of subsequent relay capability.

3. Basis of comparisons

The quantitative results in §4 and §6 focus on a comparison between the terrestrial collector size and system cost for the direct and relay configurations. For both configurations, the quantitative models for data rate and data volume are adopted from II. The assumed parameters are listed in Tbl.1 for three cases: Direct, and in the case of a relay configuration, a relay-to-relay and near-relay-to-earth downlink.

The chosen parameters are notable in several respects:

Propagation distance. The launch interval \( T_\Delta \) is a major consideration in the relay configuration, since it governs the inter-relay distance and hence the downlink data rate. The propagation distance is greater than the target star distance \( D_0 \) for the direct configuration, and is generally far smaller in the relay configuration.

| Parameter                                | Direct | Relay |
|-------------------------------------------|--------|-------|
| Propagation distance \( D_0 \) to \( D_1 \) |        |       |
| \( T_\Delta \)                             |        |       |
| \( D_1 + T_\Delta \)                       |        |       |
| Average transmit power \( P_T^0 \)         |        |       |
| Transmit aperture location Probe           |        | Probe |
| Transmit aperture effective area \( A_T^0 \) |        | \( A_T^0/2 \) |
| Receive collector location Terrestrial     |        | Probe |
| Receive aperture or collector total area \( A_d \) | \( A_n/2 \) | \( A_n \) |

Equal probe resources. In the interest of a comparison emphasizing the distinctions between the two configurations, parameters of the probe are fixed to approximate equivalent mass probe requirements. In particular, the transmit power \( P_0 \) and the total probe aperture area (for transmission and reception) \( A_0 \) are kept equal.

Terrestrial collector area. The receive terrestrial collector in the near relay case has a larger area \( A_n > A_n/2 \) than the other relay probes due to a propagation distance that is larger by launch distance \( D_l \). Near relay probe relay downlink operation is assumed to begin following the completion of launch, and hence the propagation distance is larger by \( D_l \).

Downlink data rate. To approximate a comparable scientific return for the two configurations, the total cumulative downlink data rate across all probes is assumed to be the same in the direct and relay configurations.

3.1. Comparison shortcomings

Our comparison of the two configurations neglects several factors that will differ:

Electrical power. The electrical power is consumed in functions other than communications, including attitude control and data processing. In particular, in the relay configuration each probe has to have adequate electrical power for concurrent scientific observation and communications, including attitude control, but not in the direct configuration. The data rates are also considerably higher in the relay configuration, and this will consume additional processing electrical power.
Processing overhead. We expect the processing overhead to be considerably higher for the relay probes because of their higher data rate (all else equal) and because of the significant processing associated with regeneration (principally receiver error-correction decoding).

Error rate objectives. Due to the accumulation of errors through multiple regenerative repeater stages, the error rate objective has to be tightened to account for an increase in regeneration stages. Since the models used here are based on theoretical limits on communication with reliable recovery of scientific data, this issue is not directly relevant. However, in practice tighter error rate objectives will increase the transmission power requirement slightly.

One-at-a-time downlink capture. It would be feasible for more than one post-encounter probes to access and share the single relay downlink concurrently. However, as pictured in Fig.1 it is assumed that a single probe at a time accesses the downlink. For a fixed cumulative downlink data rate, this simplification does not affect the total data volume \( V_0 \) downloaded from each probe, but it does adversely impact a probe’s ability to do interleaving to counter atmospheric outages on the near-link.

These factors all serve to make the relay configuration less attractive than suggested by the numerical results to follow.

4. Technical comparison

Our basis for a quantitative comparison between the direct and relay configurations is to set the total cumulative data rate \( b \) (with units of bits/second) equal for the two configurations, as well as the probe parameters. There are two primary parameters that will differ:

- The launch interval \( T_\Delta \) can be varied. This controls the number of probes per unit time which conduct scientific observations and return the resulting scientific data. The number of probes and the data volume per probe \( V_0 \) determine the total data rate \( b \). For a given \( b \), as \( T_\Delta \) is adjusted the granularity of the partitioning of that data between probes (as measured by \( V_0 \)) changes.

- For the relay configuration, \( b \) and \( V_0 \) are determined by the probe parameters and by \( T_\Delta \), which determines the inter-relay propagation distance. For the direct configuration, for any assumed \( T_\Delta \) the collector area \( A_d \) has to be adjusted to match \( b \) and \( T_\Delta \).

For a fixed \( b \), the primary difference between the two configurations is the \( T_\Delta \) as well as the area of the terrestrial receive collector.

For purposes of a numerical example, we adopt the specific values listed in Tbl.2 for the probe parameters defined in Tbl.1. As a basis of comparison there are at least two approaches. In the technical comparison of this section not only the data rate \( b \) but also the launch interval \( T_\Delta \) are constrained to be the same for the two configurations. The motivation here is to isolate the effect on the collector area metric by fixing all other parameters and making them identical. Naturally we will find that this collector area is larger for the direct configuration because its signal is propagating a greater distance. This makes it appear that the relay configuration is advantageous. Subsequently in the cost comparison, the constraint that \( T_\Delta \) is the same is removed (see §6). In that case, we find that the direct configuration becomes strongly advantageous from an economic (as opposed to technical) perspective.

The collector area for the direct and relay configurations are plotted in Fig.2 and compared to the area of the receive aperture on a relay probe. The collector area for the direct configuration is considerably larger, and that difference grows with \( b \). As \( b \) increases a shorter launch interval \( T_\Delta \) is required, and the resulting number of probes in the

| Parameter                              | Value   |
|----------------------------------------|---------|
| Distance \( D_0 \)                     | 4.24 ly |
| Direct rate of propagation distances \( D_1 − D_0 \) | 0.424 ly |
| Wavelength \( \lambda_0 \)             | 400 nm  |
| Average transmit power \( P_0 \)       | 100 mW  |
| Photon efficiency BPP                  | 10.9 bits/ph |
| Aperture area \( A_0 \)                | 100 cm² |
| Atmospheric outage probability         | 0.58    |

3 Countering atmospheric outages could still be performed in later regenerative repeaters, but this would require one probe (and hence every probe) to have sufficient storage for buffering a substantial quantity of data.
pre-encounter phase of their trajectories increases as shown in Fig.3. At higher data rates the launch rate becomes impractical for a single launcher, and the energy cost of those launches will be large.

Also as $b$ increases the data volume per probe $V_0$ increases as shown in Fig.4. Thus, the larger $b$ is accommodated by simultaneously increasing the number of probes and the data volume $V_0$ for each probe. This coupling in $V_0$ and $b$ is a disadvantage of the relay mode, due to the inflexible partitioning of observations and resulting data volume among probes.

This comparison in which the launch interval $T_{\Delta}$ is constrained to be equal for the two configurations favors the relay configuration due to its smaller terrestrial collector area. However, the direct configuration offers the freedom to increase $T_{\Delta}$ and compensate for the fewer probes with a larger data volume $V_0$ per probe (achieved by increasing the terrestrial collector area). Due to the substantial cost of each launch, this is strongly advantageous from a cost standpoint (see §6).

5. System reliability

In the straightforward relay system pictured in Fig.1 even a single missed launch or a single failed probe will render useless and forever abandoned all probes further from earth than the point of missing or failed probe. This is clearly an unacceptable level of system reliability. Consider for example a relay system in which there are $J_i$ relay probes at any given time. Whenever one or more of these probes fails or is not successfully launched, then the down-
A mandatory adjustment to data rate for the missing or failed link requires knowledge of the failure at the transmitter so that it can adjust its transmit energy per PPM pulse (likely by increasing pulse duration) to compensate for the greater propagation distance and still achieve the expected number of photon detections per PPM slot at the receiver.

A probe has no means of knowing about failures or missing probes closer to earth based on its own observations, although it can detect failed or missing probes farther from earth due to the absence of received signal.

As a result, probe bypassing adds an additional requirement that probes closer to earth inform probes farther from earth of intermediate missing or failed probes. This required level of coordination between probes could be obtained by a telemetry transmission from the operational probe closer to earth, which can observe the failure directly, requesting that transmission be redirected to it. Thus, each probe requires a limited transmission and receiving capability for control telemetry communication in the direction away from earth, adding two additional apertures and associated transmitter circuitry and electrical power consumption to the probe mass budget. In the following we neglect the adverse mass implications of this enhanced capability.

More generally, if $J_r$ failed or missing probes can be bypassed, increasing the transmission distance by a factor of $(J_r + 1)^2$ to account for the greater transmission distance.

### 5.3. Probability of system failure

The expectation of improved system reliability with an increase in the number of bypassed probes $J_r$ is confirmed by a calculation of the probability of that event.

Suppose we have a system configuration in which $J_r$ contiguous probe failures can be accommodated with the bypass of $J_r$ probes in the relay system. The system as a whole then fails (meaning that the supported data volume $V_0$ per probe goes to zero for multiple probes) if $J_r+1$ or more contiguous probes fail. In that case, the propagation dis-
The primary conclusion of Fig[6] and Fig[7] is that the probability of system failure can be rendered much smaller than the probability of probe failure. In other words, a reliable system can be constructed of unreliable probes by building in tolerance for missing or failed probes. As the reliability of individual probes and their launches is improved, the value of $J_r$ can be reduced.

6. Economic considerations

The preceding analysis has focused on the technical considerations in a comparison between the direct and relay configurations. Following from these technical considerations are major implications to the economic structure of the low-mass probe project. There are three principle assets in the system: The ground infrastructure for launch and data reception, the relay probes, and the scientific return in terms of scientific observational data returned reliably to earth.

6.1. Fixed and recurring costs and returns

The relative costs and returns are strongly affected by the choice of direct or relay configuration:

Directed-energy launch. The up-front fixed cost of the terrestrial launcher infrastructure is dominated by the peak energy delivered to a probe’s sail, and related to that the velocity of the probe and data latency [3]. This fixed cost is not expected to be materially affected by the choice of the direct vs relay configuration.

Pre-encounter probes. The relay configuration will generally require significantly more frequent launches if a similar scientific data volume per probe is to be maintained. The energy cost of each launch is significant, resulting in a significantly larger recurring cost overall. The recurring cost per unit of data volume is higher, and strongly dependent on the launch interval $T_\Delta$.

Receive terrestrial collector. For the direct configuration the fixed cost of the collector will be roughly proportional to its area, and hence to the data volume returned from each probe. For the relay configuration the terrestrial collector area is essentially invariant to data volume since it is constrained by the probe mass.
**Science return.** The direct configuration can generally return a larger data volume from each probe, and hence the recurring cost per unit of scientific data will be lower. On the other hand the relay configuration probe swarm will generally fly a larger number of probes by the target, increasing the total scientific return in proportion to the number of probes (assuming that the scientific data can be more fragmented across probes without reducing its value).

In summary, the direct configuration will generally have a significantly larger fixed cost and significantly lower recurring cost than the relay configuration. These costs are compared numerically in \( \textsection 7 \).

### 6.2. Risk

The relay configuration definitely increases the risk involved in infrastructure investments due to the greater complexity and greater number of failure modalities (see \( \textsection 4 \)). The performance metrics are generally rendered less favorable by technical measures taken to reduce or limit these risks.

### 6.3. Commensal usage

In addition to fixed and recurring costs related to exploration of a single target, there are longer-term issues related to the commensal utilization of the fixed-investment ground infrastructure. These differ considerably for the two configurations.

**Launcher.** The directed-energy launcher in the relay configuration will be generally preoccupied with keeping a regular repetitive launch cycle for probes to a single target. Thus, the opportunity for commensal usage to serve a second target will be severely limited. The direct configuration offers more opportunity for commensal uses, or example launching probes to different outer planet or exoplanet explorations, since the launch schedule has more flexibility.

**Receiver.** Due to the need for continuous operation in downlink reception from probes performing flybys of a single target, as well as the need to maintain pointing at the probe trajectories to that target, generally each receiver collector will be dedicated to a single target regardless of whether a direct or relay configuration is adopted. Thus generally we expect that a separate receive infrastructure, with its substantial fixed costs, will be dedicated to each separate target. In this case the relay configuration has an advantage in its considerably smaller collector area, which makes it more economically attractive to invest in multiple receivers for multiple targets.

In summary, the direct configuration is more favorable in its support for commensal missions with respect to the launcher, while the relay configuration is more favorable with respect to the terrestrial receiver.

### 6.4. Long-term reuse

A second even longer-term issue is the non-commensal reuse of investments made in the launcher and terrestrial receiver for new science missions in the future. This opportunity favors a solution with higher fixed costs (since these costs are amortized over more usage) and lower recurring costs (so that each usage is less expensive). Thus it favors the direct configuration.

### 7. Numerical cost comparisons

A major difference between the direct and relay configurations is the cost structure. To achieve comparable scientific return (measured here by data rate), the relay configuration requires more frequent launches with less data volume returned per probe, while the direct configuration is able to reduce the frequency of launches but increases the area of the receive collector. "The only criterion which can establish the relative merits of these two approaches is the relative cost. In particular for the direct configuration this can answer the quandry "as a way to increase the scientific data return, which option is more cost effective, increasing the launch frequency or increasing the collector area?"

Our cost model incorporates the cost of launches and collector area. These are incremental costs which strongly differ between the direct and relay configurations. The cost model does not include other costs (such as real estate and the capital cost of the launch beamer infrastructure) that are less materially affected by which configuration is chosen. Thus the total budgetary expenditure required will be considerably higher than the numerical values listed here. We do expect that the system design and testing effort will be considerably higher
Table 3: Relevant cost metrics

| Metric   | Units       | Interpretation                                      |
|----------|-------------|-----------------------------------------------------|
| Data rate\(b\) | bits/sec    | Average rate that scientific data is recovered, cumulative over all probes |
| Total cost\(d\) | dollars/year | Average rate of expenditure on probe launches and the downlink receiver amortized over time |
| Unit cost\(e = d/b\) | dollars/bit | Average expenditure for each scientific data bit returned from the probes |

for the relay configuration due to its greater complexity, but the costs of this are difficult to quantify and thus are not modeled.

The overall conclusion is that the direct configuration is strongly advantageous from an overall cost perspective, even as other cost disadvantages for the relay configuration are neglected.

7.1. Cost metrics

For any particular probe and ground parameters the cost metrics of interest are listed in Tbl\(3\). For both configurations we assume that probes are launched at a regular interval \(T\Delta\), where \(T\Delta\) will generally differ numerically between the two configurations. Total cost \(d\) is interpreted as the annual budget outlay for launches and the amortized cost of terrestrial collector area. Unit cost \(e\), the ratio of \(d\) and \(b\), is a measure of the cost of each unit of science return (one bit). When \(e\) is larger, each unit of scientific data costs more to acquire. Scientists will prefer the configuration that achieves a smaller \(e\) because it maximizes the science return associated with whatever budget is available.

Both \(b\) and \(d\) are cumulative over all probes, rather than on a per-probe basis, and each can be manipulated by choosing the launch interval \(T\Delta\). With regard to the terrestrial collector area \(A\) there is a big distinction. For the relay configuration, \(A\) is predetermined by the probe receive aperture area \(A_0/2\) (adjusted to account for outages), and thus cannot be used to manipulate \(b\) and \(d\). The direct configuration is more complicated in that \(b\) can be manipulated by the unconstrained choice of terrestrial collector area \(A\) as well as \(T\Delta\), and the tradeoff between \(T\Delta\) and \(A\) strongly affects \(d\). This ambiguity is resolved by consistently choosing the cost-optimized combination \(\{T\Delta, A\}\) that minimizes \(d\) for a given \(b\) (see Appendix C.1).

Could we base cost metrics on individual probes rather than averaging across all probes? Although simpler, this would not be a meaningful comparison. While the launch energy cost is per-probe, the fixed capital cost of the receiver has to be amortized across the various probes over the receiver’s lifetime. That amortization depends strongly on the number of probes launched during that lifetime and hence the launch interval \(T\Delta\). In other words, if probes are launched with greater frequency, then the amortized cost of the receiver is lower per launch than if probes are launched at lower frequency.

To avoid any unnecessary complexities introduced by end effects (startup of operations and end of system life) we assume that \(b\) and \(d\) are “steady state” averages that overlap neither the beginning nor end of operations.

For both configurations, increasing \(b\) (by decreasing \(T\Delta\) or increasing \(A\)) will result in a larger \(d\). In other words, there is inevitably a larger cost associated with acquiring more scientific data. Our comparison methodology chooses the same \(b\) for both configurations, determines how \(d\) differs between them, and repeats this for different values of \(b\).

7.2. Cost model

We utilize a simple model for cost \(d\),

\[
d = \frac{C_e}{T\Delta} + \frac{k_a A}{T_L} \tag{2a}
\]

\[
= C_e \left( \frac{1}{T\Delta} + \rho A \right) \text{ where } \rho = \frac{k_a}{T_L C_e}. \tag{2b}
\]

Version Eq.(2a) includes three cost parameters \(\{C_e, k_a, T_L\}\) listed in Tbl\(4\) and makes two assumptions regarding receiver cost. First, the dominant cost contribution to the receiver that differs between the two configurations is terrestrial aperture area \(A\), and Eq.(2) assumes that the total collector area cost \(k_a A\) is proportional to the area \(A\) with proportionality cost factor \(k_a\). The justification is that for fixed building blocks (optical elements, filters, detectors, etc), the required number of such building blocks will be proportional to \(A\). Second, Eq.(2a) assumes straight-line depreciation over receiver lifetime \(T_L\); that is, the capital cost is amortized equally over all probes launched during that lifetime.

In version Eq.(2b) the number of cost metrics is reduced to just two, \(\{C_e, \rho\}\). The relative collector-launch cost metric \(\rho\) displays clearly the relative importance of launch frequency \(T\Delta^{-1}\) and collector area \(A\) in contributing to cost \(d\).
### Table 4: Cost metrics

| Units | Interpretation |
|-------|-----------------|
| $C_a$ | dollars         |
| $k_a$ | dollars/m²      |
| $T_L$ | years⁻¹          |
| $\rho$ | years            |

The incremental cost of each probe launch (primarily energy and the cost of the probe itself) per unit area.

Cost of a terrestrial receiver collector of the probe itself.

Operational lifetime of the receiver.

Relative measure of the launch and collector costs.

#### 7.3. Rate model

The determination of cumulative scientific rate $b$ differs between the two configurations. For the relay configuration, $b$ equals $R_d$, which is the total data rate supported between two relay probes during downlink operation. For the direct configuration, $b$ has to be determined in two steps. It equals $b = V_0/T_d$, where $V_0$ is the total volume that reaches the terrestrial receiver from each individual probe. This volume is determined from the initial data rate $R_0$ and the duration of the downlink transmission (see Eq. (A.2)). Because $b$ is an average over all probes, the fact that individual probe downlink transmissions actually time-overlap one another is irrelevant to the determination of $b$.

#### 7.4. Numerical cost results

In the following we calculate and plot and compare total cost $d$ and unit cost $e$ vs average data rate $b$ for the direct and relay configurations. Although there are many design and technology elements that will establish the cost parameters, a generous range of possible values is listed for each individual parameter in Tbl. 5. Specific cost comparisons are based on specific chosen values for cost parameters. These fall within the range and are consistent with values used in a system study of the sail and propulsion system for StarShot 3.

#### 7.4.1. Is relay or direct more cost effective?

The bottom line in the cost comparison is "which is most cost effective, the direct or the relay configuration?" for equivalent $b$. This question can be answered simply in terms of $\rho$ as defined in Eq. (20). For any set of probe and trajectory parameters as in Tbl. 2, there is a threshold value $\rho_0$. For equal $b_d = b_r$, if $\rho < \rho_0$ then $d_d < d_r$ and $e_d < e_r$, or in other words the direct configuration is more cost effective than the relay configuration (see Appendix C.2). Conversely, whenever $\rho > \rho_0$ the relay configuration is most cost effective. This is expected because larger $\rho$ increases the relative cost of the collector area $A$.

For the probe and trajectory parameters in Tbl. 2, the threshold is $\rho_0 = 2.96$ m⁻² yr⁻¹. The actual value $\rho$, which is listed in Tbl. 5, is about five orders of magnitude smaller, putting the cost parameters chosen in Tbl. 5 well into the regime wherein the direct configuration is more cost effective. The collector area would have to be very expensive for the relay configuration to be cost effective; specifically $k_a > 5.3 \times 10^8$ $$/m²$$ (more than half a billion dollars per square meter) when $T_L = 30$ yr.

The three cost parameters {Cₐ, kₐ, Tₙ} contribute to $\rho$, and the boundary for which $\rho = \rho_0$ is plotted in Fig. 8 for three different receiver lifetimes. These contours separate the parameters into two regions, one region above (within which the relay configuration is more cost effective) and the other region below (within which the direct configuration is more cost effective). Also illustrated by a shaded box is the range of cost parameters listed in Tbl. 5. The direct configuration is more cost effective (by a large margin) over this entire range.

#### 7.4.2. Total cost comparison

In the following cost comparisons between the direct and relay configurations, the chosen values
for cost parameters listed in Tbl 5 are adopted. The value of \( \rho \) establishes in advance that these chosen cost parameters strongly favor the direct configuration.

The total cost \( d \) is shown in Fig. 9 over a range of data rates \( b \). Note that both \( d \) and \( b \) are cumulative over all probes currently operating downlinks. As expected, \( d \) increases with \( b \); that is, obtaining more scientific data requires a larger budget.

### 7.4.3 Unit cost comparison

The unit cost \( e \) is shown in Fig. 10 This value is the same whether determined on a per-probe basis or cumulative over all probes. The value of \( e \) decreasing with \( b \) is indicative of economies of scale. That is, it is more cost effective to build one large system rather than replicating multiple smaller systems.

**Table 5: Numerical cost parameters**

| Parameter range | Chosen value |
|-----------------|--------------|
| \$100K ≤ \( C_a \) ≤ \$10M | \$6M |
| \$100K m\(^{-2}\) ≤ \( k_a \) ≤ \$100K m\(^{-2}\) | \$10K m\(^{-2}\) |
| 10 yr ≤ \( T_L \) ≤ 100 yr | 30 yr |
| \( \rho \) | \( 5.5 \times 10^{-3} \) m\(^{-2}\) yr\(^{-1}\) |

Figure 9: A log-log plot of the total launch and collector area cost \( d \) in dollars/yr vs the total scientific data rate \( b \) in bits/sec. Many other budgetary expenditures that do not materially affect the relay vs direct configuration comparison are neglected. The probe and trajectory parameters are those of Tbl 3 and in the relay configuration it is assumed that \( J_c = 3 \). Both \( d \) and \( b \) are aggregated over all probes concurrently contributing to the downlink (the data volume per probe is shown later in Fig. 12). Thus \( d \) is the total annual budget required to fund the launches plus the amortized cost of the terrestrial collector area. The relay configuration is consistently a factor of 239 more expensive. The annual budget ranges from \$335K to \$10.6B for the direct configuration, and from \$77M to \$2.4T in the relay configuration.

Figure 10: For the same conditions as Fig. 9, a log-log plot of the unit cost \( e \) in dollars/bit vs the scientific data rate \( b \) in bits/sec. The unit cost ranges from \$336M per bit to \$11K per bit for the direct configuration, and from \$78B per bit to \$2.4M per bit for the relay configuration.

Figure 11: For the same conditions as Fig. 9, a log-log plot of the frequency of probe launches in probes/yr vs the scientific data rate \( b \) in bits/sec. Also identified are the probe launch frequencies for a launch interval equal to an hour, day, week, month, and year. The frequency ranges from 0.03 to 885 launches/yr in the direct configuration, and from 13 to 409K in the relay configuration.

#### 7.4.4 Probe launch frequency

Focusing on the total cost and data rate across all probes concurrently operating downlinks obscures significant differences between the direct and relay configurations in launch interval, number of probes, and data volume per probe. In particular, the frequency of probe launches \( T^{-1} \) is considerably higher in the relay configuration as shown in Fig. 11. This is because the unit cost \( e \) of the direct configuration can be reduced by launching probes less often and returning a larger volume of scientific data \( V_0 \) per probe.
Figure 12: For the same conditions as Fig. 9, a log-log plot of the per-probe data volume $V_0$ returned to a terrestrial receiver in bits vs the total data rate $b$ in bits/sec cumulative over all probes. This data volume is about three orders of magnitude larger in the direct configuration, compensating for the lower launch frequency shown in Fig. 11. The relay configuration $V_0$ ranges from 2.4 kb to 77 Mb, and the direct configuration from 1.1 Mb to 36 Gb.

7.4.5. Data volume per probe

The data volume returned from each probe is shown in Fig. 12. While the number of probes making scientific observations (for equivalent $b$) is much larger in the relay configuration, the volume of scientific data returned per probe is about three orders of magnitude smaller. This has considerable implications to the science mission, since the volume of scientific data acquired by each probe is much smaller. In other words, for equivalent total data rate $b$ the scientific observations need to be considerably more fragmented among probes.

It is likely that the data volume of a direct configuration probe as assumed in Fig. 12 will also be a mismatch to scientific needs. Fortuitously the ground rules of this comparison (periodic probe launches with launch frequency chosen for cost optimization) can be readily violated in the direct (but not relay) configuration. For the direct configuration there is no constraint that launches occur at regular intervals, nor is homogeneity in the mass/velocity of the various probes required. Thus there is the intrinsic flexibility to accommodate heterogenous scientific needs among the various probes in a swarm.

8. Conclusions

Although a relay configuration has some identifiable advantages, overall the results of this study are consistently favorable to the direct configuration. Its greatest advantages are its relative simplicity and reliability and the flexibility to achieve greater data volume per probe through the unconstrained choice of a larger terrestrial collector area. The direct configuration is also superior on measures of annual cost (annual budgetary outlays) and unit cost (cost per unit of scientific data) over a wide range of cost parameter values.

This study suffers several limitations. By neglecting many adverse technical factors outlined in §2.2, it presents a decidedly optimistic bound on the performance of the relay configuration. These issues would have to be further and more accurately quantified before serious consideration of the relay configuration.

This study has assumed a homogeneous-probe system in which all probes are functionally equivalent, both collecting and communicating scientific data back to earth, and are launched at fixed intervals. It would be useful to study alternatives in which probes are functionally specialized, for example with some probes devoted to scientific observations and communications and others exclusively to communications relay. Launch schedule alternatives in which probes are launched at different velocities and the launch schedule is manipulated such that probes arrive at the target in spatial groupings could also be considered.

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Appendix A. Technical model

Appendix A.1. End-to-end metric

The numerical results in §3 and §5.2 make use of the end-to-end power-area² metric \[ V \]

\[ P_A^T A_e^T A_e^R = \alpha_0 \cdot D^2 R, \quad \alpha_0 = \frac{hc \lambda_0}{\eta \cdot \text{BPP}}. \quad (A.1) \]

This relates the product of transmitted average power \( P_A^T \), transmit aperture effective area \( A_e^T \), and receive collector total effective area \( A_e^R \) to the propagation distance \( D \) and rate \( R \) at which scientific data is recovered reliably at the receiver. The value of \( \alpha_0 \) is held fixed across all comparisons. We have made the simplification \( A_e^R = N^S A_e^S \) where \( N^S \) is the number of apertures that comprise the total collector, and \( A_e^S \) is the effective area of an aperture. It is notable that \( A_e^R \) is not an effective area in the sense of antenna theory, because the receive collector is not a single-mode diffraction-limited aperture, but rather is a total of the effective areas \( A_e^S \) of \( N^S \) such apertures.

Eq. \( (A.1) \) is an invariant relation in the sense that all consistent sets of parameters must satisfy it. It is simple to apply because it is not dependent on the background radiation sources and model, and the resulting signal-to-background ratio \( \text{SBR} \) at the receive aperture. However, it will \( \text{not} \) apply to any arbitrary set of parameters because they may be inconsistent with one another. The principle situation where Eq. \( (A.1) \) is invalid occurs when the signal-to-background ratio \( \text{SBR} \) at the receiver is too small to support the assumed photon efficiency \( \text{BPP} \) for a particular background radiation model. In this event, it will be necessary to increase \( P_A^T \) sufficiently to bring \( \text{SBR} \) into line with \( \text{BPP} \) before Eq. \( (A.1) \) becomes valid. In spite of this shortcoming, Eq. \( (A.1) \) is suitable for upper bounding the data rate \( R \) that can be achieved for any set of parameters, which is the goal here.

The terrestrial collector areas are determined from Eq. \( (A.1) \), substituting values from Tab. 1 and setting equal data rates \( b_e = b_d = b \) and equal launch intervals \( T_\Delta \) in the two cases,

\[ A_n = \frac{2\alpha_0 b \left( \frac{1}{2} A_0 \sqrt{\frac{P_0}{m_0}} + D_1 \right)^2}{A_0 P_0 (1 - P_0) \alpha_0 b D_0 D_1}, \quad \alpha_0 = \frac{hc \lambda_0}{\eta \cdot \text{BPP}}. \quad (A.2) \]

Under these assumptions the data volume \( V_0 \) per probe is the same in the two cases,

\[ V_0 = \frac{A_0}{2(J_r + 1) a_0} \cdot \sqrt{\frac{b P_0}{\alpha}}. \]

Likewise the number of probes in transit is the same in the two cases,

\[ D_0 = \frac{2D_0 (J_r + 1)}{A_0} \cdot \sqrt{\frac{\alpha_0 b}{P_0}}. \]

The data volumes per probe are calculated differently because the different downlinks operate concurrently in the direct configuration but not the relay configuration. For the relay configuration, the downlink operation duration equals the launch interval \( T_\Delta \), and thus \( V_0 = T_\Delta \cdot R_r \), where \( R_r \) is the data rate between relay probes. For the direct configuration, the data volume is \[1\]

\[ V_0 = \frac{R_0 D_0}{a_0} \left( 1 - \frac{D_0}{D_1} \right). \quad (A.2) \]

where \( R_0 \) is the initial data rate following target-star encounter at \( D = D_0 \). Eq. \( (A.2) \) takes into account the declining value of \( R \) with distance \( D \).

Appendix B. Bernoulli trial statistics

In the language of statistics, a missing or non-operational probe is called a failure and a present and fully operational probe is called a success. Assume that failures are statistically independent and uniformly distributed and occur with probability \( p \), and define \( q = 1 - p \). A sequence of \( n \) failures & successes and labeled with index \( 1 \leq i \leq n \) is termed a Bernoulli trials sequence of length \( n \). A run of \( k \) failures is defined as \( k \) consecutive failures that are preceded and followed by successes. Note that there can be multiple such runs, as long as they are interspersed with one or more successes. Failures in positions \([1, k]\) followed by a success, as well as in positions \([n - k + 1, n]\) preceded by a success, also qualify as runs.

Define \( L \) as the length of the longest run of failures in a Bernoulli trials sequence with total length \( n \). \( L \) is a random variable with cumulative distribution function given by

\[ \text{Pr} \{ L \leq k - 1 \} = \sum_{m=0}^{\frac{n-k}{2}} (-1)^m p^m q^{m-1} \left( \binom{n-m-1}{m} + q \binom{n-m}{m} \right) \]

\[ 1 \]
for \( k \geq 1 \). [9]

In the relay configuration, all runs of length \( k \leq J_r \) can be tolerated without relay system failure, where \( J_r \) is defined in [18]. Relay system failure results whenever \( L > J_r \), and hence the probability of system failure is

\[
\Pr \{ \text{system failure} \} = \Pr \{ L > J_r \} = 1 - \Pr \{ L \leq J_r \}
\]

for \( J_r \geq 0 \).

### Appendix C. System cost results

The cost model is given in Eq. (2). This can be combined with Eq. (A.1) and Eq. (A.2) to yield formulas for cost metrics \( b, d, \) and \( e \) as a function of probe and probe trajectory parameters. Expressed in terms of rates \( b_d \) and \( b_t \), these are

\[
\frac{d_d}{C_e} = \frac{1}{T_\Delta} + \frac{\alpha_0 b_d D_0 D_1 \rho T_\Delta u_0}{A_0 P_0 (1 - P_o) (D_1 - D_0)} \tag{C.1a}
\]

\[
d_e = \frac{2 \alpha_0 b_d D_t^2 \rho}{A_0 P_0 (1 - P_o)} + \frac{2 (J_r + 1) u_0 \sqrt{\alpha_0 b_t P_0}}{P_0 (1 - P_o)} + \frac{D_r \rho u}{2 (1 - P_o)} + \frac{2 D_T \rho u \sqrt{\alpha_0 b_t P_0}}{P_0 (1 - P_o)} \tag{C.1b}
\]

### Appendix C.1. Direct configuration cost optimization

While \( d_e \) in Eq. (C.1b) is not dependent on \( T_\Delta \) (because this parameter was manipulated to achieve rate \( b_t \)), \( d_d \) in Eq. (C.1a) is dependent on \( T_\Delta \) (because \( A_0 \) rather than \( T_\Delta \) was manipulated to achieve \( b_d \)). There is therefore an opportunity to minimize \( d_d \) by choosing the cost-optimum value of \( T_\Delta \). A restatement is that the two performance metrics \( \{b_d, d_d\} \) depend on the two parameters \( \{T_\Delta, A_0\} \), and there is thus an opportunity to choose the combination that both achieves rate \( b_d \) and at the same time minimizes \( d_d \). The resulting values of \( T_\Delta \) and \( d_d \) are

\[
T_\Delta^2 = \frac{A_0 P_0 (1 - P_o) (D_1 - D_0)}{\alpha_0 b_d D_0 D_1 \rho u_0} \tag{C.2a}
\]

\[
\frac{d_d}{C_e} = 2 \sqrt{\frac{\alpha_0 b_d D_t^2 \rho}{A_0 P_0 (1 - P_o) (D_1 - D_0)}}. \tag{C.2b}
\]

The conclusion is that \( d_d \) in Eq. (C.2b) is proportional to \( \sqrt{b_d} \), while \( d_e \) in Eq. (C.1b) has a more complicated dependence on \( b_d \).

### Appendix C.2. When the relay configuration is cost effective

Although the expression for \( d_e \) in Eq. (C.1b) is complex, it simplifies considerably when \( A = 0 \). Since \( A \) is very small in the relay configuration in any case (it is constrained by the probe receive aperture, and hence the probe mass), arbitrarily setting \( A = 0 \) should not affect \( d_e \) materially. Thus to find the region where the relay configuration has lower cost, it is a good approximation to use the modified criterion

\[
d_e | A = 0 < d_d . \tag{C.3}
\]

This criterion slightly expands the region wherein the relay configuration is more cost effective, since it deliberately ignores the cost associated with \( A \). Thus, the “relay is cost effective” region in Fig. 8 is actually slightly smaller than shown. The critical value of \( \rho \) where equality is achieved in Eq. (C.3) is

\[
\rho_0 = \frac{(J_r + 1)^2 (1 - P_o) u_0 (D_1 - D_0)}{A_0 D_0 D_1} . \tag{C.4}
\]

Thus, the relay option becomes more cost-effective when probe aperture area \( A_0 \) is larger or probe by-pass parameter \( J_r \) is smaller.

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