Observational constraints on neutron star crust–core coupling during glitches

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ABSTRACT

We demonstrate that observations of glitches in the Vela pulsar can be used to investigate the strength of the crust–core coupling in a neutron star and provide a powerful probe of the internal structure of neutron stars. We assume that glitch recovery is dominated by the torque exerted by the mutual friction-mediated recoupling of superfluid components of the core that were decoupled from the crust during the glitch. Then we use the observations of the recoveries from two recent glitches in the Vela pulsar to infer the fraction of the core that is coupled to the crust during the glitch. We then analyse whether crustal neutrons alone are sufficient to drive glitches in the Vela pulsar, taking into account crustal entrainment. We use two sets of neutron star equations of state (EOSs) which span crust and core consistently and cover a conservative range of the slope of the symmetry energy at saturation density $30 < L < 120$ MeV. The two sets differ in the stiffness of the high density EOS. We find that for medium to stiff EOSs, observations imply $>70$ per cent of the moment of inertia of the core is coupled to the crust during the glitch, though for softer EOSs $L \approx 30$ MeV as little as 5 per cent could be coupled. We find that only by extending the region where superfluid vortices are strongly pinned into the core by densities at least $0.016 \text{ fm}^{-3}$ above the crust–core transition density does any EOS reproduce the observed glitch activity.

Key words: dense matter – equation of state – stars: neutron – pulsars: general – pulsars: individual: Vela.

1 INTRODUCTION

Glitches – sudden increases in the rotational frequency of pulsars – offer an insight into the internal dynamics of neutron stars and potential constraints on the properties of dense matter. The quasi-periodic giant glitches exhibited by the Vela pulsar (Melatos, Peralta & Wyithe 2008) offer some of the most stringent tests of our pulsar glitch models, and our models of dense matter (Link, Epstein & Lattimer 1999). A leading class of glitch model supposes a component of the stellar interior spends most of the time decoupled from the solid crust and magnetosphere whose secular spin-down we track observationally (see Haskell & Melatos 2015 for a review of glitch models). This component, usually taken to be part of the neutron superfluid, occasionally recouples to the crust and transfers some of its angular momentum, spinning the crust up (Anderson & Itoh 1975; Alpar 1977). The superfluid component can be decoupled by the pinning of vortices to nuclei in the inner crust (Alpar 1977; Pines et al. 1980; Anderson et al. 1982; Alpar et al. 1984a), by pinning to poloidal magnetic flux tubes in the core (Sauls 1989; Ruderman, Zhu & Chen 1998; Link 2003), or by pinning to toroidal flux tubes in the core (Güercinoğlu & Alpar 2014). As a rotational lag builds between the decoupled component and the rest of the star, so the hydrodynamic Magnus force builds until it is able to overcome the pinning force and cause the vortices to move outwards collectively, transferring angular momentum to the crust. Due to the complexity of the physical glitch scenario, only recently have significant steps been made in a consistent hydrodynamical simulation of the glitch process (Andersson, Sidery & Comer 2006; Sidery, Passamonti & Andersson 2010; van Eysden & Melatos 2010; Haskell, Pizzochero & Sidery 2012; Haskell & Antonopoulou 2014).

In the most widely examined variant of the model, it is the crustal superfluid that is pinned. However, recent calculations which show that a large fraction of the superfluid neutrons are entrained by the crust via Bragg scattering off the crustal lattice, has led to a vigorous debate on the efficacy of this mechanism in the light of the observed Vela glitch activity (Andersson et al. 2012; Chamel 2013, 2012; Piekarewicz, Fattoyev & Horowitz 2014; Hooker, Newton & Li 2015; Steiner et al. 2015). From this, one can conclude that

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if the whole star spins up during the glitch, it appears that the existence of entrainment in the crust renders the crust-driven glitch scenario only marginally viable at best. If, however, a portion of the core is decoupled from the crust during the glitch, there might be sufficient angular momentum stored in the crust superfluid to drive the observed giant glitches (Hooker et al. 2015).

It is expected quite generally that the core superfluid will be coupled to the normal (non-superfluid) fluid by the mutual friction force. The main mechanism which is thought to be acting in the core is scattering of electrons off superfluid vortex cores, magnetized by entrainment of protons (Alpar, Langer & Sauls 1984b). In this case mutual friction will couple the crust and the core superfluid in less than a minute for a typical glitching pulsar with a rotation period of around 100 ms (Alpar et al. 1984a; Alpar & Sauls 1988; Andersson et al. 2006; Sidery & Alpar 2009). In the crust, however, protons are not superfluid and electron scattering is ineffective. In this case the mutual friction coupling is likely to be much weaker and due mainly to interactions with sound waves in the lattice (Jones 1990). Vortices close to the crust–core boundary, that are mostly immersed in the crust, would thus be weakly coupled, with the coupling becoming stronger at higher densities where vortices are mostly immersed in the core and electron scattering is the dominant mutual friction mechanism (Haskell et al. 2012). Comparison with the observational upper limit on the time-scale for Vela glitches <40 s (Dodson, McCulloch & Lewis 2002) suggests that it is likely a significant part of the core will not be coupled to the crust during the glitch. This uncoupled component should then recouple in the minutes after the glitch, a process that should manifest itself observationally in the recovery of the pulsar spin frequency immediately post-glitch (Haskell et al. 2012; Haskell & Antonopoulou 2014). Indeed, in the 2000 and 2004 Vela glitch, exponential fits to the recovery of the spin frequency provide tentative evidence for a short time-scale (∼1 min) component (Dodson et al. 2002; Dodson, Lewis & McCulloch 2007). Another way of probing the crust–core coupling involves analysing the power spectrum of noise in the frequency of long-period high-mass X-ray pulsars. These variations, taken to be caused by internal torques, has been shown to indicate that either the superfluid component occupies no more the 85 per cent of the star’s moment of inertia, or that the relevant crust–core coupling time-scale is less than 1 d (Boynton et al. 1984; Baykal, Alpar & Kiziloglu 1991).

Recently, much progress has been made simulating the hydrodynamic evolution of the superfluid vortices including the coupling of crust and core via mutual friction and microscopic pinning forces. Such hydrodynamic models can explain qualitatively the interglitch time-scales, Vela glitch sizes, and post-glitch rotational evolution, and have the potential to constrain the neutron star equation of state (EOS; van Eysden & Melatos 2010; Pizzochero 2011; Haskell, Pizzochero & Sidery 2012; Seveso, Pizzochero & Haskell 2012), despite remaining uncertainties in aspects of the glitch model such as the unpinning trigger mechanism and details of how the vortices subsequently unpin (Warszawski & Melatos 2008; Glampedakis & Andersson 2009; Melatos & Warszawski 2009; Warszawski, Melatos & Berloff 2012; Warszawski & Melatos 2013; Link 2014), as well as not including the effect of crustal entrainment.

In this paper, we use the qualitative picture that emerges from hydrodynamic models together with the short time-scale component of the 2000 and 2004 Vela glitches to infer the moment of inertia of that portion of the core superfluid that is coupled to the crust during the glitch via mutual friction. We then calculate the maximum moment of inertia of the crustal superfluid neutrons in the crustal regions which drive the glitch according to hydrodynamic models, including the effects of crustal entrainment, and hence infer the glitch activity for the Vela pulsar which we confront with the observed value. It is important to note that the interpretation of immediate post-glitch relaxation in the two observations as short-time-scale exponentials is tentative, with significant observational uncertainty. Our aim in this paper is to set up the framework to interpret future, more robust observations of short-time-scale components of glitch relaxations, and we use the existing two observations as a demonstration of that framework. This will provide a powerful tool for future observations to constrain the moment of inertia of the different components of the star, thus allowing us to probe the origin of pulsar glitches and the internal structure of neutron stars.

The neutron star models used are generated using two families of EOSs; one generated using the Skyrme non-relativistic nuclear model and the second using the relativistic mean field (RMF) model. Both families predict the same EOS up to saturation densities, but differ at high densities; the RMF model is stiff and able to accommodate maximum neutron star masses above 2.5 M⊙, while the Skyrme model used is softer and generally gives maximum masses around 2.0 M⊙. We generate the families of EOSs by varying the slope of the symmetry energy at saturation density, a quantity that has been shown to correlate strongly with radius and crust–core transition density. We vary it over a conservative range 30–120 MeV which encompasses constraints inferred from experiment (Tsang et al. 2012; Lattimer & Lim 2013). When L is varied, the low-density pure neutron matter (PNM) is adjusted to maintain a good fit to the results of ab initio PNM calculations (Gezerlis & Carlson 2010; Hebeler & Schwenk 2010; Gandolfi, Carlson & Reddy 2012). In doing so, we are able to explore the predictions of our glitch model over a wide range of EOS parameter space.

In Section 2, we explain how we use the observed recovery to set constraints on the fraction of the core coupled to the crust at the time of glitch. In Section 3, we apply this to our glitch model using our consistent sets of EOSs. We present our results in Section 4, and discuss them in Section 5 as well as giving our conclusions.

## 2 CRUST–CORE COUPLING VIA MUTUAL FRICTION

The Vela pulsar spins with an angular frequency of $\Omega \approx 70$ rad s$^{-1}$, and exhibits giant glitches ($\Delta \Omega_\Omega \approx 1.5 \times 10^{-4}$ rad s$^{-1}$). The time taken for a glitch to occur (the glitch rise time-scale) is constrained by observation to $\tau_\Phi \lesssim 40$ s. The secular spin-down rate due to electromagnetic torque on the Vela pulsar is $\dot{\Omega} \approx 10^{-19}$ s$^{-2}$ (Dodson et al. 2002, 2007). These are taken to be the values for the solid crust of the Vela pulsar, to which the magnetic field is anchored. The normal component of the star is coupled to the crust on short time-scales, but the superfluid neutrons are only weakly coupled, and may not react fast enough to remain coupled during a glitch. Let us take as an illustrative lower limit only 1 per cent of the total moment of inertia of the star to be coupled to the crust during the glitch. Then, by angular momentum conservation, one would expect the immediate post-glitch spin-down rate to be $\dot{\Omega} \approx 10^{-8}$ s$^{-2}$.

The key assumption in our model is that the core superfluid recouples post-glitch via mutual friction, in which superfluid vortices become magnetized by entraining protons, allowing electrons to scatter off them and hence couple to the charged components of the core and, through them, the crustal lattice. We assume that the protons and electrons in the core are rigidly rotating and coupled to the crust on short time-scales by the magnetic field (Haskell et al. 2012; van Eysden 2014).
Let us now examine how effectively mutual friction couples the crust and core on short time-scales. Mutual friction couples the two components on a time-scale (Andersson et al. 2006)

\[
\tau_{\text{mf}} \approx \frac{1}{2\Omega B},
\]

where \(B\) is the mutual friction coefficient (Andersson et al. 2006) which expresses the strength of the mutual friction force. Setting this equal to the glitch rise time-scale \(\tau_g \sim 1\) min gives us a value of \(B \sim 10^{-4}\). All regions of the core with \(B \gtrsim 10^{-4}\) will be coupled to the crust during the glitch, while regions of the core with \(B \lesssim 10^{-4}\) will begin recoupling to the crust after the glitch, resulting in a torque on the crust. A region with \(B \approx 10^{-5}\), for example, will recouple to the crust on a time-scale of several minutes after the glitch, and the resulting torque will give a contribution to the spin-down rate, due to mutual friction, of

\[
\dot{\Omega}_{\text{mf}} \approx 2\Omega B (\Delta\Omega)_{\text{eq}} \approx 10^{-7}\text{s}^{-2},
\]

where \((\Delta\Omega)_{\text{eq}}\) is the internal lag between the fluids as a consequence of the glitch, which is generally less than the change in angular frequency of the star due to the glitch \(\Delta\Omega\). \(\dot{\Omega}_{\text{mf}}\) is considerably stronger than the electromagnetic contribution, even assuming a small amount of the moment of inertia of the core is coupled during the glitch, and we can infer that core recoupling via mutual friction will give the dominant contribution to the spin-down rate for up to several hours after the glitch.

These estimates are indeed confirmed by running the code of Haskell et al. (2012) for a Vela giant glitch. In Fig. 1, we compare the spin-down rate after the glitch obtained for a standard simulation and the rate obtained by excluding the external electromagnetic torque. We can see that the two rates agree in the first few hours in which mutual friction dominates, and only after do they differ as the electromagnetic contribution takes over. Thus immediately after the glitch, the crust component of the star (including the core protons) is spinning faster than a portion of the core neutron superfluid.

If this is the case the observation of short time-scale spin recovery after a glitch represents a direct probe of the mutual friction mechanism, and therefore the vortex dynamics in the outer core. We shall now demonstrate how one can infer the moment of inertia of that part of the star coupled to the crust during the glitch, \(I_c\). It is this quantity which enters into the parameter \(G = I_d/I_c\) (where \(I_d\) is the moment of inertia of the superfluid component driving the glitch) to be compared with the pulsar’s glitch activity.

### 2.1 Extracting the coupled moment of inertia from Vela glitch observations

There are two published glitches that have an observed short-term recovery component, the Vela 2000 and 2004 glitches (Dodson et al. 2002, 2007). Unfortunately, as shown in Table 1, the amplitudes of these components, \(\Delta F_g\) vary significantly between the two by more than a factor 1000, partially due to the fact that in the Vela 2004 the short-term component was barely above the noise level. The time-scales \(\tau\) are, however, similar, and consistent with each other within the errors.

|                | 2000               | 2004               |
|----------------|--------------------|--------------------|
| \(\Delta F_g\) (Hz) | \(2.45435 \times 10^{-5}\) | \(2.2865 \times 10^{-5}\) |
| \(\Delta F_g\) (Hz) | \(2 \times 10^{-8}\) | \(5.4 \times 10^{-8}\) |
| \(\tau\) (min)     | 1.2 \pm 0.2        | 1 \pm 0.2          |

We shall refer to these two components as \(n\) and \(c\). Before the glitch, the two components will have a small equilibrium lag \((\Delta\Omega)_{\text{eq}}\), \(\Omega \approx 10^{-8}\) for the Vela pulsar, where \(\Omega\) is the secular spin-down rate dominated by the electromagnetic torque. The glitch acts to spin-up the component \(c\) by an amount \((\Delta\Omega)_c \gg (\Delta\Omega)_n\), and therefore immediately after the glitch the lag between the components will be \((\Delta\Omega)_n = (\Delta\Omega)_c\). After the time-scale over which the component \(n\) recouples, component \(c\) has decreased its frequency by an amount \((\Delta\Omega)_n\) and component \(n\) has increased its frequency by \((\Delta\Omega)_c - (\Delta\Omega)_n\); thus by angular momentum conservation

\[
\frac{I_n}{I_c} = \frac{(\Delta\Omega)_n}{(\Delta\Omega)_c - (\Delta\Omega)_n}.
\]

Taking \((\Delta\Omega)_c\) and \((\Delta\Omega)_n\) from observations, we obtain

\[
\frac{I_n}{I_c} \approx 8 \times 10^{-4} \quad \text{for the Vela 2000 glitch}
\]

\[
\frac{I_n}{I_c} \approx 2.37 \quad \text{for the Vela 2004 glitch.}
\]

Using this result and the observed time-scales, equation (3) gives us values for the mutual friction parameter above which correspond...
to regions of the core coupled at the time of glitch
\[ B = 1 \times 10^{-4} \pm 3 \times 10^{-5} \] for the Vela 2000 glitch
(7)
\[ B = 4 \times 10^{-5} \pm 1 \times 10^{-5} \] for the Vela 2004 glitch.
(8)

These numbers are still (roughly) consistent given the uncertainties that are clearly underestimated in this procedure. In the following we shall simply assume a range:
\[ 3 \times 10^{-5} < B_{\text{obs}} < 1.3 \times 10^{-4}, \]
from which we can compute the moment of inertia of the core \( I_c \) coupled via mutual friction to the crust at the time of glitch, and hence spun up by the glitch. This procedure, although limited by current observational uncertainties, allows us to derive this important quantity from observations for the first time.

We now outline how we use this result to extract \( I_c \) from neutron star models.

### 3 Modelling the Glitch

Given a particular neutron star EOS, we obtain our background neutron star model by solving the Oppenheimer–Volkoff (OV) equations, thus obtaining the density and composition profile of the star as a function of radial coordinate \( r \).

The moment of inertia of a star of radius \( R \) in the limit of small angular frequency \( \Omega \) (Hartle & Thorne 1968) is given by
\[ I_{\text{tot}} = \frac{8\pi}{3} \int_{0}^{R} r^4 e^{-\psi(r)} \frac{\hat{\omega}(r)}{\Omega} \left( \frac{e(r) + P(r)}{\sqrt{1 - 2GM(r)/r}} \right) dr, \]
(10)
where \( e(r) \) is energy density of matter in the star, \( P(r) \) is the pressure and \( M(r) \) is the mass contained in radius \( r \). \( v(r) \) is a radially dependent metric function given by
\[ v(r) = \frac{1}{2} \left( 1 - \frac{2GM}{R} \right) - G \int_{r}^{R} \left( \frac{M(x) + 4\pi x^3 P(x)}{x^2(1 - 2GM(x)/x)} \right) dx, \]
(11)
and \( \hat{\omega} \) is the frame dragging angular velocity
\[ \frac{d}{dr} \left( r^2 j(r) \frac{d\hat{\omega}(r)}{dr} \right) + 4 \frac{dj(r)}{dr} \hat{\omega}(r) = 0, \]
(12)
where
\[ j(r) = e^{-\psi(r)} - \lambda(r) \sqrt{1 - 2GM(r)/r}. \]
(13)
for \( r \leq R \).

We take the standard form for the mutual friction coefficient in the core, due to electron scattering off magnetized vortex cores (Alpar et al. 1984b; Andersson et al. 2006):
\[ B = 4 \times 10^{-4} \left( \frac{m_p - m_{\pi}^0}{m_p} \right)^2 \left( \frac{m_p}{m_{\pi}^0} \right)^{1/2} \]
\[ \times \left( \frac{0.05}{x_p} \right)^{7/6} \left( \frac{\rho}{10^{14} \text{g cm}^{-3}} \right)^{1/6}, \]
(14)
where \( x_p \) is the proton fraction, \( m_{\pi}^0 \) the microscopic effective proton mass and \( \rho \) the total density in the core. These quantities are obtained consistently from the EOS, as a function of radial coordinate \( r \) once the OV equations are solved. As detailed in Haskell et al. (2012), we average \( B \) over the \( z \)-direction to find the average mutual friction strength experienced by a vortex at cylindrical radius \( \tilde{r} = r \sin \theta \).
\[ \tilde{B}(\tilde{r}) = \frac{\int_{0}^{\pi/2} \frac{2\tilde{r}}{\sqrt{1 - 2M(\tilde{r})/\tilde{r}}} r d\theta}{\int_{0}^{\pi/2} \frac{2}{\sqrt{1 - 2M(\tilde{r})/\tilde{r}}} r d\theta}. \]
(15)

### Figure 2

Neutron star cross-section in plane of rotation axis (\( \Omega \)) depicting the geometry of the strong pinning region in inner crust (shaded area SP) and the region of the core coupled to the crust at the time of glitch via mutual friction (shaded region MF). OC and IC label the outer and inner crust, respectively. The angular locations of the boundary of the strong pinning region at the outer crust, and the outer and inner boundaries of the coupled region of the core, are given by \( \theta_{\text{outer}}, \theta_1 \) and \( \theta_2 \), respectively. Except for the softest EOSs at saturation density \( (\approx 30 \text{ MeV}) \), \( \theta_2 = 0 \).

We can now define a region of the star bounded by cylindrical radii \( \tilde{r}_1 \) and \( \tilde{r}_2 \) such that \( \tilde{B}(\tilde{r}_1 < \tilde{r} < \tilde{r}_2) > B_{\text{obs}} \). This corresponds to the region in which the superfluid neutrons are strongly coupled to the crust on the glitch rise time.

Defining
\[ r^2 \mathcal{I} = \frac{8\pi}{3} r^4 e^{-\psi(r)} \frac{\hat{\omega}(r)}{\Omega} \left( e(r) + P(r) \right) \left( \frac{1}{\sqrt{1 - 2GM(r)/r}} - 1 \right), \]
(16)
where \( e(r) \) is the energy density of the superfluid neutrons, \( P(r) \) is the pressure of the superfluid neutrons, then the moment of inertia contained in the regions of the star with cylindrical radius greater than \( \tilde{r}_1 \) and \( \tilde{r}_2 \), respectively, is given by
\[ I_{1,2} = \int_{\tilde{r}_1}^{\tilde{r}_2} \int_{\theta_{\text{outer}}}^{\pi/2} r^2 \mathcal{I} \sin \theta d\theta dr, \]
(17)
and therefore the moment of inertia of the region in which the superfluid neutrons are strongly coupled to the crust on the glitch rise time is given by
\[ I_{\text{nc}} = I_2 - I_1. \]
(18)
Here, \( R_{\text{cc}} \) is the radius of the crust–core boundary, and \( \theta_1, \theta_2 \) are the angular locations where the outer and inner boundaries of the coupled region where it meets the crust–core boundary, respectively (see Fig. 2). The total moment of inertia of that part of the star strongly coupled to the crust at glitch rise time is obtained by adding the contribution of the crustal lattice itself and the protons in the core
\[ I_c = I_{\text{nc}} + I_{\text{crust}} + I_1. \]
(19)

Hydrodynamic simulations of vortex evolution suggest that only the crustal superfluid neutrons within the strong pinning region of the crust, defined as the region within which vortices are totally immersed in the inner crust, contribute to the glitch itself. How
many contribute depends on details of the pinning force throughout the crust (Haskell et al. 2012), but here we take the entirety of the strong pinning region as an upper limit. The moment of inertia of the strong pinning region of inner crust superfluid neutrons is

\[
I^{(\theta)} = \int_{R(\theta)}^{R(0)} r^2 dr \sin \theta dh, 
\]

where \( R(\theta) \) is the distance from the core of the star to the inner boundary of the strong pinning region at an angle \( \theta \) to the rotation axis, \( R(0) = R_{\text{outer}} \) and \( R(\theta) = R_{\text{outer}} \) (see Fig. 2).

Entrainment of superfluid neutrons by the crust’s lattice mobility of the neutrons with respect to that lattice. It can be shown that this effect is encoded by introducing an effective mesoscopic neutron mass \( m'_n \) (Chamel 2005, 2012; Chamel & Carter 2006); larger values correspond to stronger coupling between the neutron superfluid and the crust, and a reduction in the fraction of superfluid neutrons able to store angular momentum for the glitch event. One can include this effect by modifying the integrand equation (16) in the inner crust:

\[
r^2 I \to r^2 I^*, 
\]

where \( m'_n(r) \) is the effective mass at radius \( r \) in the crust. We obtain \( m'_n(r) \) from the results of Chamel (Chamel 2012) by interpolating between the values calculated at specific densities to find the effective mass at arbitrary locations in the inner crust.

3.1 Nuclear matter parameters and crust and core EOSs

The glitch model requires several microscopic properties as input. These include the total pressure and energy density \( P(n_b), e(n_b) \), neutron pressures and energy densities \( P_n(n_b), e_n(n_b) \), effective proton mass in the core, proton fraction and mass density \( m'_p(n_b), x_p(n_b) \) and \( \rho(n_b) \), as a function of baryon number density \( n_b \), and the crust–core transition density and pressure \( n_{cc} \) and \( P_{cc} \). These quantities are derived from an underlying model of nuclear matter.

Experimental information about nuclear matter is predominantly extracted from nuclear systems at densities around nuclear saturation density \( n_0 = 0.16 \text{ fm}^{-3} \) and at proton fractions close to one half. As a consequence, nuclear matter models are generally characterized by their behaviour in that region of parameter space. Denoting the energy per particle of nuclear matter at saturation density by \( E(n_0, \delta) \) where \( \delta = 1 - 2x_p \) is the isospin asymmetry parameter; \( \delta = 0 \) corresponds to symmetric nuclear matter (SNM), and \( \delta = 1 \) to PNM. The nuclear symmetry energy \( S(n) \) is defined as the quadratic coefficient in the expansion of \( E(n_0, \delta) \) about \( \delta = 0 \).

Nuclear matter models can be characterized by their behaviour around nuclear saturation density \( n_0 = 0.16 \text{ fm}^{-3} \), the density region from which much of our experimental information is extracted. We can denote the energy per particle of nuclear matter around saturation density by \( E(n_0, \delta) \), where \( n_0 \) is the baryon density and \( \delta = 1 - 2x \) the isospin asymmetry, where \( x \) is the proton fraction. \( x = 0.5, \delta = 0 \) corresponds to SNM, and \( x = 0, \delta = 1 \) to PNM. By expanding \( E(n, x) \) about \( \delta = 0 \) we can define the symmetry energy \( S(n) \),

\[
E(n, \delta) = E_0(n_0) + S(n_0)\delta^2 + \cdots. 
\]

The symmetry energy is the energy cost of increasing the isospin asymmetry of matter and is a function of baryon density. Furthermore, we can expand the symmetry energy about saturation density using the density parameter \( x = \frac{n_0 - n_0}{n_0} \):

\[
S(n_0) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \cdots, 
\]

where \( J, L \) and \( K_{\text{sym}} \) are the symmetry energy, its slope and its curvature at saturation density. Over the past decade, vigorous effort has been devoted to experimentally constraining \( J \) and particularly \( L \) (Li, Chen & Ko 2008; Tsang et al. 2012). Currently the congruence of experimental results (Hebeler et al. 2013) points to the range \( 30 < L < 60 \text{ MeV} \), but stiffer (higher values of \( L \)) are not conclusively ruled out (Fattoyev & Pickarewicz 2013); we therefore examine a conservative range of \( 30 < L < 120 \text{ MeV} \) in this paper.

Crust and core EOSs and the transition density are calculated consistently using two models of the nuclear many-body interaction. We use the IUFSU parametrization of the RMF model (Fattoyev et al. 2010) and a parametrization of the non-relativistic Skyrme model, SKIUFSU, which shares the same saturation density SNM properties as IUFSU, used in previous work (Fattoyev et al. 2012, 2013; Hooker et al. 2015). Both models have isovector nuclear matter parameters obtained from a fit to state-of-the-art PNM calculations (Gezerlis & Carlson 2010; Hebeler & Schwenk 2010; Gandolfi et al. 2012), which makes them particularly suitable for describing the low-density neutron fluid in the inner crust, and both describe the bulk properties of doubly magic nuclei well (Fattoyev et al. 2012). The presence in both models of two purely isovector model parameters allows the density slope of the symmetry energy \( L \) to be systematically adjusted, while retaining the fit to PNM at low densities. The fact that they are isovector means that such adjustments leave SNM properties unchanged (Chen et al. 2009). This adjustment under the PNM constraint leads to the relation \( J = 0.16L + 23.33 \text{ MeV} \).

Note that you generally cannot separate out the individual proton and neutron pressures in these nuclear matter models – the pressure contains terms that are not separable into neutron and proton components. However, in both Skyrme and RMF models, the enthalpy density \( h = e + P \) is separable, because the non-separable terms in the pressure and energy density cancel out. It is the enthalpy that appears in the moment of inertia integrals.

Both nuclear matter models give closely identical EOSs up to saturation density for a given value of \( L \). The SKIUFSU model is softer than IUFSU at high densities and but gives maximum masses of \( M \approx 2M_\odot \) for all values of \( L \), matching the observational lower limit (Demorest et al. 2010; Antoniadis et al. 2013). The IUFSU model we have chosen to be maximally stiff at high densities by adjusting the parameter \( \zeta \) of the RMF model that controls the quartic omega-meson self-interactions (Müller & Serot 1996) and subsequently the high-density component of the EOS of SNM. It is set so that the SNM EOS is maximally stiff, resulting in maximum mass neutron stars of \( \gtrsim 2.5M_\odot \). The crustal EOS and crust–core transition densities are derived from the compressible liquid drop model for the crust (Newton, Gearheart & Li 2013) using the same nuclear matter model as the core EOS. EOS quantities obtained include the proton fraction \( x_p \) and microscopic effective proton mass \( m'_p \) as a function of density, required to calculate the mutual friction parameter \( B \) from equation (14). For a 1.4 \( M_\odot \) star, the softest EOS \( L = 30 \text{ MeV} \) produces effective proton masses that vary from 0.7 down to 0.3 from crust to core and the stiffest EOS \( L = 120 \text{ MeV} \) produces effective proton masses that vary from 0.85 down to 0.15 from crust to core.

In Fig. 3, we show the bulk properties of neutron stars for three representative values of the slope of the symmetry energy \( L = 30, 60 \) and 90 MeV for the two families of EOSs: SKIUFSU (maximum masses around 2 \( M_\odot \)) and IUFSU (maximum masses around 2.5–2.6 \( M_\odot \)). In the left-hand panel, the thickness of the crust \( \Delta R_c \) is displayed as a function of mass \( M \), while on the right the total radius of the star \( R \) is displayed as a function of mass \( M \).
4 RESULTS

Before we get into discussions of the results, we remind readers that when we refer to the portions of the core coupled and uncoupled to the crust, we are referring specifically to the superfluid core neutrons. The small proton component of the core is assumed to always be tightly coupled to the crust.

In Fig. 4, we show the averaged mutual friction profile $\overline{B}(\tilde{r})$ as a function of cylindrical radius $\tilde{r}$ throughout the core of a 1.4 $M_\odot$ star (a) and a 1.8 $M_\odot$ star (b). In each case we show the mutual friction profiles obtained with the $L = 30, 60, 90$ and 120 MeV members of the SkIUF SU EOS family. The shaded band covers the possible range of the threshold strength of mutual friction inferred from the 2000 and 2004 Vela glitches. All regions of the star with a mutual friction greater than this threshold are coupled to the crust at the time of glitch, while those regions below the threshold are uncoupled from the crust at the time of glitch.

As the slope of the symmetry energy $L$ increases (gets stiffer), the strength of the mutual friction $\overline{B}(\tilde{r})$ gets larger throughout the core of the star. A larger $L$ leads to the symmetry energy as a function density of $E_{\text{sym}}(\rho)$ that increases more rapidly, leading to a larger proton fraction at a given density, which contributes to an increasing $\overline{B}(\tilde{r})$ with density (since $\mathcal{B}$ is almost proportional to $x_p$). In addition, the microscopic effective mass $m_p^*$ generically decreases with increasing density (from 0.7–0.85 at the crust down to 0.15–0.3 in the core for a 1.4 $M_\odot$ star). Since $\mathcal{B}$ depends on $m_p - m_p^*$ and inversely on $m_p^*$, this also contributes to an increasing $\overline{B}(\tilde{r})$ with density. For the softest EOS shown, $L = 30$ MeV, the symmetry energy begins decreasing with density just above saturation density, and therefore so does the proton fraction. This behaviour dominates the behaviour of $\mathcal{B}$ at high density, which explains its non-monotonic behaviour for $L = 30$ MeV.

At the softer end of the range of $L$ we consider, the mutual friction profile changes rapidly with variations in $L$. For the softest EOS shown, $L = 30$ MeV, the non-monotonic behaviour arises because the proton fraction decreases with increasing density from the crust to the core, while the effective proton mass increases. For stiffer EOSs, both quantities increase from crust to core.

For $L = 30$ MeV the predicted mutual friction profile lies almost completely within the inferred range. The upper end of the range of $\overline{B}(\tilde{r})$ inferred from observation, $1.3 \times 10^{-4}$, implies that the core neutrons are entirely decoupled from the crust at the time of glitch, and that the superfluid neutrons in the strong pinning region in the crust have only the rest of the crust and the core protons to spin-up during the glitch event. At the lower end of the range, for a canonical 1.4 $M_\odot$ neutron star a small portion of the outer core becomes coupled to the crust. For higher masses an increasing part of the inner core also becomes decoupled.

If, on the other hand, we consider the lower end of the inferred range of $\overline{B}(\tilde{r})$, $3 \times 10^{-5}$, most of the core (within the inner $\approx 10$ km)
is coupled to the crust. For $L = 60$–120 MeV, most of the core has mutual friction significantly stronger than the inferred range, and hence a large fraction of core neutrons are coupled to the core at the time of the glitch, regardless of whether we consider the upper or lower bounds of the range.

In Fig. 5, we plot the ratio of the moment of inertia of the coupled part of the core $I_c$ (that which has a mutual friction parameter larger than the observationally inferred values) to the total moment of inertia of the star $I_{\text{tot}}$ as a function of mass for $L = 30$ MeV (a), $L = 60$ MeV (b), $L = 90$ MeV (c) and $L = 120$ MeV (d). The ranges span the upper and lower bounds inferred from observation, and correspond to taking the upper and lower bounds of the shaded regions in Fig. 4.

![Figure 5](https://example.com/figure5.png)

**Figure 5.** Fractional moment of inertia of the component of the star coupled to the crust during the glitch $I_c/I_{\text{tot}}$ as a function of stellar mass for $L = 30$ MeV (a), $L = 60$ MeV (b), $L = 90$ MeV (c) and $L = 120$ MeV (d). The ranges span the upper and lower bounds inferred from observation, and correspond to taking the upper and lower bounds of the shaded regions in Fig. 4.

Given the moment of inertia of the predicted fraction of the core $I_c$ coupled to the crust obtained as detailed above, we can calculate the ratio of the moment of inertia of the neutrons in the strong pinning region, assumed for now to be entirely within the inner crust, to the moment of inertia of the couple core fraction $I_s/I_c$. In Fig. 6, we compare this value as a function of mass to the minimum required to explain the Vela glitch activity over the past 45 yr $G = 0.016$, indicated by the horizontal dashed line in all four plots. It is important to note here that this comparison is meaningful only if the observed sizes of the glitch are a reasonable approximation to the actual change in frequency after the rise. This may not be the case if, as some simulations suggest (Haskell et al. 2012), the rise time is significantly shorter than the current upper limit of $\approx 1$ min and a larger fraction of the core may be decoupled during a glitch.

Returning to Fig. 6, the first thing to notice is that for no EOS in the SkIUFSU family within the wide range of $L$ considered does the predicted value of $I_s/I_c$ reach the required value to account for Vela glitches. Within this family of EOSs, the crustal neutrons are insufficient to explain the Vela glitches, using a core fraction consistent with the initially observed glitch recovery.

Generically, the ratio $I_s/I_c$ decreases with increasing mass as the crust becomes thinner while the radius stays relatively constant (refer to Fig. 3). At lower $L$ the predicted range for $I_s/I_c$ spans an order of magnitude owing to the wide range of possible coupled core fractions. The upper bound of the range corresponds to the lower bound of $I_s/I_c$, which in the case of $L = 30$ MeV is very small as the core is completely decoupled over the glitch rise time. Because of this, the upper bound comes closest to matching the required level from the Vela pulsar glitches. In the three other cases, between 70
10. and 100 per cent of the core is coupled, and the range predicted for $I_s/I_c$ increases with $L$ as the star’s crust increases; it does not rise enough to match the required level, though.

If one allows the region where the neutrons experience strong pinning to penetrate the core to an arbitrary amount, $I_s$ can be increased until the ratio $I_s/I_c > 0.016$. In Table 2, we give the density above the crust–core transition density to which we must extend the inner boundary of the strong pinning region in order to satisfy the Vela constraint for a 1.4 M⊙ star. Pinning could occur in the outer core, for example, on magnetic flux tubes.

We next explore the extent to which changing the high-density EOS changes the outcomes of our predictions. We now use the IUFSU family of EOSs with the parameter $\zeta = 0.00$ which leads to the stiffest EOS at supersaturation densities and a corresponding maximum mass of 2.6 M⊙. The EOS and symmetry energy is the same as the SkIUFSU up to saturation density. We show representative results in Fig. 7. In the left-hand panels, we show the predicted ranges for the moment of inertia of the coupled core fractions to the total moment of inertia $I_c/I_t$, while in the right-hand panels we show the ratio of the moment of inertia of the neutrons in the strong pinning region to the moment of inertia of the core fraction $I_s/I_c$, compared to the minimum value inferred from the Vela pulsar. We show results for a representative soft EOS $L = 30$ MeV and representative stiff EOS $L = 90$ MeV from the IUFSU $\zeta = 0.00$ family. The increased stiffness of the high-density EOS prevents the proton fraction dropping and weakening the mutual friction for $L = 30$ MeV as happens with the corresponding EOS from the SkIUFSU family. The behaviour of the coupled core fraction is thus similar to that of the stiffer EOSs including the $L = 90$ MeV shown, above $\approx 80$ per cent for all masses.

5 DISCUSSION AND CONCLUSIONS

Evidence exists for a short time-scale ($\sim 1$ min) component in the post-glitch recovery of the 2000 and 2004 Vela glitches, suggestive of the recoupling of that portion of the stellar interior that was decoupled from the crust during the glitch. Under the assumption that the recoupling is mediated by the mutual friction force which results from electron scattering off the magnetized cores of superfluid vortices, we use the observed time-scales and magnitudes to infer the moment of inertia of the region of the core coupled to the crust during the glitch.
Within the framework of hydrodynamic models of vortex dynamics, which suggest the pinning region may be confined to the region of the inner crust where the vortices are completely immersed in the crustal lattice, we calculate the ratio of the moment of inertia of the pinning region of the crust to the moment of inertia of the core coupled to the crust during the glitch which we compare with the observed Vela glitch activity. We derive the neutron star EOS from the crust to the core, including the composition, microscopic effective proton mass and crust–core transition density, consistently using the same underlying nuclear matter EOS. We choose two families of EOSs: one derived from the SkIUFSU parametrization of the phenomenological non-relativistic Skyrme model of the in-medium neutron–proton interactions and one derived from the IUFSU parametrization of the phenomenological RMF model. The Skyrme family of EOSs give maximum mass neutron stars of \( \approx 2.0 \, M_\odot \), while the RMF family give maximum mass neutron stars of \( \approx 2.6 \, M_\odot \). Each family is generated by systematically varying the slope of the symmetry energy at saturation density over a conservative range \( L = 30–120 \, \text{MeV} \), thus exploring the impact on our results of the current uncertainty in the EOS of neutron rich matter at saturation density.

The main conclusion of our analysis is that taking the strong pinning region to be confined entirely to the crust, no EOS results in large enough glitches to explain the Vela pulsar’s glitch activity for neutron stars above \( 1 \, M_\odot \). However, for the stiffest EOSs considered, neutron stars with masses slightly under \( 1 \, M_\odot \) would marginally satisfy Vela’s glitch activity, and uncertainties on glitch sizes and rise times could lead to lower fractions of the core being coupled during a glitch and make low masses consistent with the data.

We find that increasing the stiffness of the high-density EOS, has only a small effect on the results for \( 1.4 \, M_\odot \) neutron stars, generally increasing the moment of inertia coupled to the crust during the glitch slightly. This high density stiffening has two effects: increasing the radius of the star, and thus the moment of inertia of that part of the core that must be spun-up, and also increasing the crust thickness, and therefore the angular momentum reservoir which spins the star up. These two effects oppose each other, resulting in only a small effect overall.

It would thus appear that even if one allows for part of the core superfluid to decouple during a glitch it is quite generally the case that, if standard mutual friction mechanisms are at work, strong entrainment in the crust limits the amount of angular momentum available for a glitch to below the level required to explain the glitch activity of the Vela pulsar. Allowing the strong pinning region to extend by up to \( 0.05 \, \text{fm}^3 \) into the core, would allow all EOSs to produce glitches consistent with Vela’s glitch activity. Physically this could correspond to the protons in the outer core being in a type-II superconducting state and the superfluid neutron vortices being pinned to superconducting flux tubes (Sauls 1989; Ruderman et al. 1998; Link 2003; Haskell, Pizzochero & Seveso 2013), and contributing to the angular momentum reservoir available for a glitch. Pinning...
to toroidal flux lines in the outer core can also help satisfy the Vela activity condition (Güergünoğlu & Alpar 2014). Note, however, that in this case vortex–flux tubes interactions could be the leading source of mutual friction in some areas of the core (Sidery & Alpar 2009; Haskell, Glampedakis & Andersson 2014), thus invalidating our assumption that mutual friction is due to electron scattering off vortex cores.

The strength of mutual friction throughout the core increases with increasingly large symmetry energy slopes $L$. At low values of $L$ (soft EOSs), the fraction of the core coupled to the crust at the time of the glitch is quite unconstrained by the observations. At intermediate-high values of $L$ (stiffer EOSs), the fraction of core coupled to the crust is constrained by observations to be above $\approx 70$ per cent for realistic neutron star masses.

The values extracted for the coupled fractions are contingent not only on our model for the crust–core coupling mechanism, but also on the robustness of the observational data. As such, our results are tentative, as they depend on only two observations, one of which (the 2004) is particularly marginal. The fitting of exponential recovery terms can also be called into question, as on short time-scales the frequency evolution may be described by a very different functional form (Haskell et al. 2012; Link 2014). If future observations are able to resolve, or at least strongly constrain the rise time, all superfluid regions coupled to the crust on a longer time-scale than the glitch rise times will contribute to the quasi-exponential relaxation. It may thus be possible to refine our estimates and constrain the moment of inertia of different macroscopic physical regions with different coupling time-scales.

It is important to note that our conclusions also rely heavily on reliability of estimates of the frequency after a glitch. This may not be the case if the glitch rise time is significantly shorter than the current upper limit of $\approx 1$ min, e.g. closer to time-scales of a few seconds as simulations suggest (Haskell et al. 2012), and a significantly larger fraction of the core may be decoupled during a glitch. If we allow all the core neutrons to be entirely decoupled during the glitch (requiring coupling time-scales of less than 5–10 s depending on the EOS), we still do not obtain consistency with a $1.4 M_\odot$ Vela pulsar except for the stiffest EOS, $L = 120$ MeV. However, given a reasonable uncertainty of the order of 0.01 fm$^{-3}$ in the crust–core transition density (Newton et al. 2013) at a given value of $L$, our model would be consistent with Vela being a $M = 1.4 M_\odot$ neutron star if $L \gtrsim 70$ MeV and the coupling time-scale is sufficiently fast to decouple $\gtrsim 70$ per cent of the core neutrons. From our calculations, this requires the coupling time-scale to be $\approx 10$ s or less, as opposed to the minute time-scales estimated from the observed glitch recoveries. Consistency with Vela being a $M = 1.0 M_\odot$ neutron star can be achieved for $L \gtrsim 50$ MeV, provided the coupling time-scale is $\approx 30$ s or less, enough to decouple $\gtrsim 80$ per cent of the core neutrons.

The one aspect of our analysis that is not consistent with the underlying EOS is the use of equation (14) for the mutual friction coefficient; although the proton fraction and proton effective mass are derived from the underlying EOS, the form of the equation is a fit to a single microscopic calculation. Although we expect the basic functional dependence on quantities not to change much, the detailed form may depend on the EOS. One should also note that the values for the mesoscopic effective neutron masses in the crust, which encode the entrainment effect, are also not consistent with the EOS used, and could substantially alter the estimates of the angular momentum available in the crustal superfluid. Another important assumption of our model is that the protons and electrons in the core are coupled to the crust on short time-scales by the magnetic field. This assumption, however, depends strongly on the magnetic field geometry and may not hold for all regions of the core (Glampedakis & Lasky 2015).

Nevertheless, it is likely that the post-glitch response of glitches bears the imprint of the crust–core coupling dynamics, and our study illustrates one way in which such information can be extracted. It would be of considerable value to extract more robust observational constraints on the short-term components in the relaxation from Vela and similar pulsars.

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