Summary A detailed analysis of the existence of high energy secondary diffractive dips and structures in the extrapolations of the fits to the data is given. The existence of these dips and \textit{a fortiori} their position is found to be rather model-dependent: present in all eikonalized models including Pomeron, Odderon and secondary Reggeons they disappear when an additional large-\(t\) term is added (as sometimes advocated).

1. Layout of the paper

Few years ago, the suggestion was made\cite{1} that, at increasingly high energies, secondary (diffractive) dips and structures should develop at intermediate |\(t\)| values in both \(pp\) and \(p\bar{p}\) angular distributions. In particular, in\cite{1} it was suggested that such effects should be well visible at LHC while only extremely precise data could perhaps show up at RHIC energies. As a matter of fact, predictions of secondary structures have appeared many times in the past\cite{2}. The large spectrum of predictions in the position of these secondary dips shows that things are actually more complicated than anticipated long ago\cite{3}. It is not enough that a given scheme inherently generates oscillations (like the Bessel function of an impact parameter representation); interference effects are very important in determining their position. The model dependence of these predictions, however, is not so important; it is the prediction itself of the existence of secondary structures which matters. This was received with great interest and plans are under the way to investigate this point experimentally\cite{4}. In\cite{1} the prediction was based on the extrapolation to higher energies of high quality fits to all existing data and proposed \"...with the greatest reservation...\" since it was the result of dealing with the data within fairly sophisticated "Born amplitudes" devised to provide the best possible fit. The question which was left open to further investigation was how credible this extrapolation could be and how much it could be generalized. For this reason, a careful analysis of different models has now been carried out using various refinements. In particular, we have undertaken an extensive analysis of several models where the input (or Born amplitude) is variously eikonalized as a sort of unitarization\cite{5}.

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1 E-mail: desgrolard@ipnl.in2p3.fr
2 E-mail: giffon@ipnl.in2p3.fr
3 E-mail: martynov@bitp.kiev.ua
4 E-mail: predazzi@to.infn.it
As a result of these investigations, in the present paper we reconsider the entire question by:
(i) using more general schemes of eikonalization \([5, 6, 7]\); (ii) choosing various kinds of input amplitudes (in particular, for their simplicity, monopoles \([8]\) and dipoles \([9, 10]\), see below); (iii) occasionally superimposed to a \("large-|t|\) term" attributed \([11]\) to Odderon exchange and behaving like \(t^{-4}\) \([11]\) at height \(|t|\).

Here, we will not report all the results obtained so far but we will rather give a general summary moving quickly to the conclusions (a more complete analysis and discussion will be reported soon \([7]\)).

At high (LHC) energies, if we strictly respect the unitarity constraints \([1]\) and confining ourselves to just one case (chosen as an example to be a dipole \([9, 10]\) for both Pomeron and Odderon Born amplitudes plus secondary Reggeons) our conclusions are:

(a) secondary diffractive structures are invariably predicted after eikonalization of the Born amplitude (so long as a large-\(|t|\) term is not included).

(b) The position of the dips is quite model dependent. Loosely speaking, the first secondary dip at \(\sqrt{s} = 14\) TeV is predicted around \(|t_2| \sim 1.5 - 3\) GeV\(^2\).

(c) When a large-\(|t|\) term is altogether omitted, the differential cross section is dominated by the eikonalized Odderon term as \(|t|\) increases (this term being quite negligible at small \(|t|\) values).

(d) All secondary structures tend to disappear if a large-\(|t|\) term is surimposed to the previous amplitude. Possibly, this amounts to double counting but this point is far from clear.

At lower (RHIC) energies, the situation appears even more problematic because we find, at best, some hint of new structures which manifest themselves as breaks of the slopes.

### 2. The input Born

We focus on the (dimensionless) crossing-even and -odd amplitudes \(a_{\pm}(s, t)\) of the \(pp\) and \(\bar{p}p\) reactions

\[
A_{pp, \text{Born}}^{\pm}(s, t) = a_+(s, t) \pm a_-(s, t),
\]

for which we have data on

i) the total cross-sections

\[
\sigma_t = \frac{4\pi}{s} \Re A(s, t = 0),
\]

ii) the differential cross-sections

\[
\frac{d\sigma}{dt} = \frac{\pi}{s^2} |A(s, t)|^2,
\]

iii) the ratio of the real to the imaginary forward amplitudes

\[
\rho = \frac{\Re A(s, t = 0)}{\Im A(s, t = 0)}.
\]

The crossing even part in the Born amplitude is a Pomeron (to which an \(f-\)Reggeon is added) while the crossing odd part is an Odderon (plus an \(\omega-\)Reggeon)\(^5\).

\[
a_+(s, t) = a_{\mathbb{P}}(s, t) + a_f(s, t), \quad a_-(s, t) = a_{\mathbb{O}}(s, t) + a_\omega(s, t).
\]

\(^5\)The data at the present highest available energies are, notoriously, still far from asymptopia (and this appears quite clearly from our analysis \([\text{1}]\)). It might even be that it will never be reached in actual experiments. In this sense it will be necessary to check at each step that unitarity is not violated; as an example, it is not sufficient to simply state that \(\text{Froissart’s bound is not violated.}\)
For simplicity the two Reggeons that have been retained have been taken in the standard form (with fixed parameters [1] for economy)

\[ a_R(s, t) = a_R s^{\alpha_R(t)} , \quad \alpha_R(t) = \alpha_R(0) + \alpha_R' t , \quad (R = f \text{ and } \omega) , \]

where \( a_f (a_\omega) \) is real (imaginary).

For the Pomeron \( a_{\Pi} (s, t) \) and the Odderon \( a_O (s, t) \), in this paper we take just a dipole \( D(s, t) \) (a double pole in the complex angular momentum) of the form [2]

\[ D(s, t) = a s^{\alpha(t)} \left[ e^{b(\alpha(t) - 1)}(b + \ell n \tilde{s}) + d\ell n \tilde{s} \right] . \tag{7} \]

As usual

\[ \tilde{s} = \frac{s}{s_0} e^{-i \frac{\pi}{2}} , \quad (s_0 = 1 \text{ GeV}^2) \]

(to respect \( s - u \) crossing) and \( \alpha(t) \) is the Regge trajectory taken of the linear form [3]

\[ \alpha(t) = 1 + \delta + \alpha' t . \tag{9} \]

The case where the input is a monopole (i.e. a simple pole in the angular momentum plane) will be considered in [6] (the difference between a monopole and a dipole is essentially that the amplitude for the second grows with an additional power of \( \ell n s \)). It is quite difficult to discriminate between these forms on general grounds as well as phenomenologically since a reasonable or good agreement with the data is obtained with both and the reason behind this is, presumably, that the data are not yet asymptotic as already mentioned.

Some authors maintain that a perturbative large-\(|t|\) term behaving like \(|t|^{-4}\) (and complying with perturbative QCD requirements according to [11]) is to be added to the Odderon [8]. We believe that, when the Born amplitude is eikonalized, the rescattering corrections implied by eikonalization should be the end of story especially for trajectories rising slower than linearly. Adding another term could lead to double counting. In [7] however, we will investigate for completeness the rôle of incorporating in the Odderon an additional large \(|t|\) term.

### 3. Eikonalization procedure

A positive \( \delta \) value (for either \( \Pi \) or \( O \) or both) in (9) signals a supercritical situation necessitating some kind of regularization (like eikonalization) to avoid conflicts with unitarity. Several eikonalization procedures can be found in the litterature (see [5, 6, 7] and references therein). They all amount to taking rescattering (therefore, hopefully, unitarity) corrections into account. If \( a_\pm (s, t) \) are our Born amplitudes (5), the crossing even and crossing odd input amplitudes in the impact parameter or \( b\)- representation are proportional to the eikonal function \( \chi_\pm (s, b) \)

\[ h_\pm \equiv h_\pm (s, b) = \frac{1}{2} \chi_\pm (s, b) = \frac{1}{2s} \int_0^\infty dq \ J_0(bq) \ a_\pm (s, -q^2) , \quad (q^2 = -t) . \tag{10} \]

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6In (7) and (9), a suffix \( \Pi \) or \( O \) is understood according to wether we are referring to the Pomeron or to the Odderon. With our choices, \( a_{\Pi} \) is real and \( a_O \) is imaginary.

7Linear trajectories are an oversimplification that, strictly, violates analyticity. In addition, at large-\(|t|\) this may be dangerous in practice. We ignore this complication.

8We should, however, not forget that at large-\(|t|\), the ratio \(|t|/s\) is really rather small so that we are in a domain closer to the usual Regge kinematics than to that of perturbative QCD.
In a generalized scheme that accounts for rescattering corrections, one can prove (see [7] for details) that the impact parameter amplitude takes the two parameter form

\[ H_{pp}(s, b) = h_+ \pm h_- + \left( \frac{h_+ \sqrt{\lambda_+} \pm h_- \sqrt{\lambda_-}}{h_+ \lambda_+ \pm h_- \lambda_-} \right)^2 \left( \frac{e^{2i(h_+ \lambda_+ \pm h_- \lambda_-)} - 1}{2i} - (h_+ \lambda_+ \pm h_- \lambda_-) \right). \]  

(11)

This corresponds to the so-called generalized eikonalization [5]. When \( \lambda_+ = \lambda_- \), (11) reduces to the quasi-eikonalization [6]. This reduces further to the special (and traditional) eikonal form

\[ H_{pp}(s, b) = e^{2i(h_+ \pm h_-)} - \frac{1}{2i}, \]  

(12)

when \( \lambda_+ = \lambda_- = 1 \).

Once the eikonal amplitude \( H_{pp}(s, b) \) is known in the \( b \)-representation, the inverse Fourier-Bessel transform leads, finally, to the eikonalized amplitude

\[ A_{pp,Eik}(s, t) = 2s \int_0^\infty \, db \, b \, J_0(b \sqrt{-t}) \, H_{pp}(s, b). \]  

(13)

The consequences of the unitarity constraint \( |H(s, b)| \leq 1 \) and \( \Im H(s, b) \geq 0 \)

\[ \alpha'_P \geq \alpha'_O, \delta_P \geq \delta_O \]  

(14)

and other limitations are discussed in great details elsewhere [12]. The role of unitarity and its constraints on (11) will be discussed at length in [7]). Here, however, they are carefully taken into account.

4. (Some) results and conclusions.

As already stated, in this paper we consider only the dipole case without a large-\(|t|\) term added to the Odderon and we compare the full case of (11) (\( \lambda_+ \neq \lambda_- \)) with the one for which \( \lambda_+ = \lambda_- \). A good reproduction of the data is obtained where the most relevant parameters are \( \lambda_+ = 0.5; \lambda_- = 1.32; \delta_P = 0.058; \delta_O \approx 0 \) for the first case and \( \lambda = 0.5; \delta_P = 0.068; \delta_O = 0 \) for the second one.

The extrapolation to 500 GeV and 14 TeV, (the energies to be reached in the near future [4]), are shown in Fig.1. We find

(a) for the first fit (\( \lambda_+ \neq \lambda_- \)) \( \chi^2_{dof} = 11.1 \);

(b) for the second one (\( \lambda_+ = \lambda_- \)) \( \chi^2_{dof} = 14.1 \).

Several comments are in order:

Secondary structures are present in both cases (\( \lambda_+ \neq \lambda_- \) and \( \lambda_+ = \lambda_- \)) and a second dip clearly appears as the energy increases. However, from an extensive trial of all possibilities (not reported here), we find that its location moves considerably from one case to the other. If by fault of better argument, we had to choose the best fit, the high energy extrapolation

9 This procedure, roughly speaking, mimicks a situation whereby the particle-Pomeron-particle and the particle-Odderon-particle amplitude vertices (\( g_+ \) and \( g_- \)) are rescaled by a priori different positive constants (\( \sqrt{\lambda_+} \) and \( \sqrt{\lambda_-} \)).

10 We do not expect this standard eikonalization process to give satisfactory results with the simple Born models of Sect.2; this is why a generalization is considered.
(LHC) would favor $|t_2| \simeq 2.7 \text{ GeV}^2$ for the case where $\lambda_+ \neq \lambda_-$ and $|t_2| \simeq 1.7 \text{ GeV}^2$ for the case where $\lambda_+ = \lambda_-$. Nothing really conclusive we feel may be said at RHIC energy where, at best, a break in the slope can be seen.

The addition of a large-$|t|$ term as advocated in [11] leads in both cases to fits qualitatively good and comparable to those which may be obtained at the Born level [1]. As already stated, in both cases, if a large-$|t|$ term is added, the secondary structures disappear from our predictions. We will discuss this in [7] but we really do not know why this happens; the best we can offer is that adding a large-$|t|$ term to the eikonalized Odderon tautamounts to double counting and destroys the simplicity of the approach (see also comments at the closing of Section 2).

We conclude that a careful reanalysis based on full eikonalization confirms our early prediction about the existence of secondary diffractive structures in the case of a crude dipole model unless a large-$|t|$ term is added.

Searching for the origin of secondary structures we find that this is a cooperative effect due to the eikonalized Pomeron and Odderon. If one removes all components one by one, when we are left with a dipole Pomeron alone, structures are present but the fit to the large-$|t|$ data becomes disastrous. The large-$|t|$ domain is dominated by the eikonalized Odderon and this alone shows also structures (but the fit is *a fortiori* disastrous). Keeping both eikonalized Pomeron and Odderon (no secondary Reggeons) reproduces roughly the data and shows secondary structures.

Based on a wide exploration of simple models, we feel that we can commit ourselves to saying that the second minimum at the LHC may be expected anywhere between

$$1.5 \leq |t_2| \text{ (GeV}^2) \leq 3.$$  

(15)

Only the experiment can answer the question of the existence and location of secondary dips and discriminate among models.

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Fig. 1 Extrapolations to RHIC and LHC energies of the angular distributions for two fits using the dipole model (dipole Pomeron, dipole Odderon and Reggeons) respecting the unitarity constraints: (a) dash lines: quasi-eikonalization procedure (QE). (b) solid lines: generalized eikonalization procedure (GE).