LEFT-RIGHT GAUGE SYMMETRY
AT THE TEV ENERGY SCALE

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ABSTRACT
Two first examples beyond the standard model are given which exhibit left-right symmetry \((g_L = g_R)\) and supersymmetry at a few TeV, together with gauge-coupling unification at around \(10^{16}\) GeV.

1. Introduction

What lies beyond the standard model at or below the TeV energy scale? One very well-motivated possibility is supersymmetry. In particular, the minimal supersymmetric standard model (MSSM) is being studied by very many people. Another possibility is left-right gauge symmetry, but there are a lot fewer advocates here and for good reason, as I will explain in this talk. I will also discuss how these problems may be overcome, assuming both supersymmetry and left-right gauge symmetry at the TeV energy scale.

There are two problems with the conventional left-right gauge model at the TeV energy scale with or without supersymmetry. One is the unavoidable occurrence of flavor-changing neutral currents (FCNC) at tree level. The other is the lack of gauge-coupling unification which is known to be well satisfied by the MSSM. In this talk, I will offer two new models. Both allow the gauge couplings to be unified at around \(10^{16}\) GeV. The second has the added virtue of being free of FCNC at tree level. Hence left-right gauge symmetry at a few TeV should be considered a much more attractive possibility than was previously recognized.

2. Origin of FCNC in Left-Right Models

Consider the gauge symmetry \(SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) which breaks down to the standard \(SU(3)_C \times SU(2)_L \times U(1)_Y\) at \(M_R \sim \text{few TeV}\) with particle

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content given by

$$Q \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (3, 2, 1/6), \quad Q^c \equiv \begin{pmatrix} d^c \\ u^c \end{pmatrix}_L \sim (\bar{3}, 1, 2, -1/6), \quad (1)$$

$$L \equiv \begin{pmatrix} \nu \\ l \end{pmatrix}_L \sim (1, 2, 1, -1/2), \quad L^c \equiv \begin{pmatrix} l^c \\ \nu^c \end{pmatrix}_L \sim (1, 1, 2, 1/2). \quad (2)$$

Note that each generation of quarks and leptons (i.e. \(Q + Q^c + L + L^c\)) fits naturally into a 16 representation of SO(10). In order for the quarks and leptons to obtain nonzero masses, a scalar bidoublet

$$\eta \equiv \begin{bmatrix} \eta_1^0 & \eta_2^+ \\ \overline{\eta}_1 & \overline{\eta}_2 \end{bmatrix} \sim (1, 2, 2, 0) \quad (3)$$

is required. Consider the interaction of \(\eta\) with the quarks:

$$QQ'^{\dagger} \eta = dd^c \eta_1^0 - ud^c \eta_1^- + uu^c \eta_2^0 - du^c \eta_2^+. \quad (4)$$

If there is just one \(\eta\), then the mass matrices for the \(u\) and \(d\) quarks are related by

$$M_d \langle \eta_1^0 \rangle^{-1} = M_u \langle \eta_2^0 \rangle^{-1}, \quad (5)$$

which means that there can be no mixing among generations and the ratio \(m_u/m_d\) is the same for each generation. This is certainly not realistic and two \(\eta\)'s will be required.

$$M_d = f \langle \eta_1^0 \rangle + f' \langle \eta_1^0 \rangle, \quad (6)$$

$$M_u = f \langle \eta_2^0 \rangle + f' \langle \eta_2^0 \rangle. \quad (7)$$

As a result, the diagonalizations of \(M_u\) and \(M_d\) do not also diagonalize the respective Yukawa couplings, hence FCNC are unavoidable. To suppress these contributions to processes such as \(K^0 - \bar{K}^0\) mixing, the fine tuning of couplings is required if \(M_R \sim \) few TeV. In the nonsupersymmetric case, \(\eta'\) can be simply taken to be \(\sigma \eta^* \sigma\), but that will not alleviate the FCNC problem. Similarly, if the \(f'\) terms were radiative corrections from, say, soft supersymmetry breaking, FCNC would still be present.

3. Evolution of Gauge Couplings

Consider now the evolution of the gauge couplings to one-loop order.

$$\alpha^{-1}_i(M_U) = \alpha^{-1}_i(M_R) - \frac{b_i}{2\pi} \ln \frac{M_U}{M_R}, \quad (8)$$

where \(\alpha_i \equiv g_i^2/4\pi\) and \(b_i\) are constants determined by the particle content contributing to \(\alpha_i\). Using the standard model to evolve \(\alpha_i\) from their experimentally determined
values at $M_Z$ to $M_R \sim$ few TeV and requiring that they converge to a single value at around $10^{16}$ GeV, the constraints

$$b_2 - b_3 \sim 4, \quad b_1 - b_2 \sim 14,$$

are obtained. It is easily seen that these constraints are not satisfied by the conventional left-right gauge model with or without supersymmetry. Note that $b_2 - b_3 = 4$ in the MSSM, corresponding to two SU(2)$_L$ doublets, whereas in the supersymmetric left-right model with two bidoublets (four SU(2)$_L$ doublets), $b_2 - b_3 = 5$.

4. First Example with Unification

Suppose the FCNC problem is disregarded, then the conventional left-right model with particle assignments given by Eqs. (1) and (2) can be made to have gauge-coupling unification if new particles are added at the TeV energy scale. Supersymmetry is also assumed so that $M_R$ and $M_U$ can be separated naturally. Now

$$b_S = -9 + 2(3) + n_D = -1,$$

$$b_{LR} = -6 + 2(3) + n_{22} + n_H = 3,$$

$$\frac{3}{2} b_X = 2(3) + 3n_H + n_D + 3n_E = 17,$$

and the constraints of Eq. (9) are satisfied. The gauge couplings do meet at one point as shown in Fig. 1, based on a full two-loop numerical analysis.

In this model $n_{22} = 2$ is the number of bidoublets, $n_H = 1$ is the number of an anomaly-free set of Higgs doublets needed to break the SU(2)$_R$ symmetry independent of SU(2)$_L$:

$$\Phi_L \sim (1, 2, 1, -1/2), \quad \Phi_R \sim (1, 1, 2, 1/2),$$

$$\Phi_L^e \sim (1, 2, 1, 2), \quad \Phi_R^e \sim (1, 1, 2, -1/2),$$

$n_D = 2$ is the number of exotic singlet quarks of charge $-1/3$:

$$D \sim (3, 1, 1, -1/3), \quad D^c \sim (3, 1, 1, 1/3),$$

and $n_E = 2$ is the number of exotic singlet leptons of charge $-1$:

$$E \sim (1, 1, 1, -1), \quad E^c \sim (1, 1, 1).$$

Note that $n_{22} = 2$ and $n_H = 1$ are required for fermion masses and SU(2)$_R$ breaking respectively. To obtain $b_{LR} - b_S = 4$, $n_D = 2$ is then assumed. At this stage, $(3/2)b_X - b_{LR} = 8$. To increase that to 14, $n_E = 2$ is just right. This should not be considered fine tuning because the contribution of each new set of particles comes in large chunks, 3 in the case of the $E$’s for example; so if 6 did not happen to be the desired number, it would not have been possible to achieve unification with the addition of new particles this way.
5. Left-Right Model without FCNC

Consider the $E_6$ superstring-inspired left-right model proposed some years ago. In the fundamental $27$ representation of $E_6$, there is an additional quark singlet of charge $-1/3$. An alternative to the conventional left-right assignment is then possible:

$$Q \equiv \left( \begin{array}{c} u \\ d \end{array} \right)_L \sim (3, 2, 1, 1/6), \quad d^c_L \sim (\overline{3}, 1, 1, 1/3), \quad (17)$$

$$Q^c \equiv \left( \begin{array}{c} h^c \\ u^c \end{array} \right)_L \sim (\overline{3}, 1, 2, -1/6), \quad h^c_L \sim (3, 1, 1, -1/3), \quad (18)$$

where the switch $h^c$ for $d^c$ has been made. The doublets $\Phi_{L,R}$ and the bidoublet $\eta$ are also in the $27$. Hence the following terms are allowed:

$$QQ^c\eta = dh^c\eta^0_1 - uh^c\eta^-_1 + uu^c\eta^0_2 - du^c\eta^+_2, \quad (19)$$

$$Qd^c\Phi_L = dd^c\phi^0_L - ud^c\phi^-_L, \quad (20)$$

$$hQ^c\Phi_R = hh^c\phi^0_R - hu^c\phi^+_R. \quad (21)$$

As a result,

$$\mathcal{M}_u \propto \langle \eta^0_2 \rangle, \quad \mathcal{M}_d \propto \langle \phi^0_L \rangle, \quad \mathcal{M}_h \propto \langle \phi^0_R \rangle. \quad (22)$$

Since each quark type has its own source of mass generation, FCNC are now guaranteed to be absent at tree level. This is the only example of a left-right model without FCNC.

6. Extended Definition of Lepton Number

Since the $(1,2,1,-1/2)$ component of the $27$ is now identified as the Higgs superfield $\Phi_L$, where are the leptons of this model? One lepton doublet is in fact contained in the bidoublet, *i.e.* $(\nu, l)_L$ should be identified with the spinor components of $(\eta^0_1, \eta^-_1)$, and one lepton singlet $l^c_L$ with that of $\phi^+_R$. Since

$$\Phi_L\Phi_R\eta = \phi^-_L\phi^+_R\eta^0_1 - \phi^+_L\phi^-_R\eta^-_1 + \phi^0_L\phi^0_R\eta^0_2 - \phi^-_L\phi^+_R\eta^+_2, \quad (23)$$

the lepton $l$ gets a mass from $\langle \phi^0_R \rangle$. Furthermore, from Eq. (19), it is seen that the exotic quark $h$ must have lepton number $L = 1$ and since $u^c$ and $h^c$ are linked by $SU(2)_R$, the $W^-_R$ gauge boson must also have $L = 1$. This extended definition of lepton number is consistent with all the interactions of this model and is conserved.

The production of $W_R$ in this model is very different from that of the conventional left-right model. Because of lepton-number conservation, the best scenario is to have $u + g \rightarrow h + W^+_R$, where $g$ is a gluon. The decay of $W_R$ must end up with a lepton as well as a particle with odd $R$ parity. Note also that $W_L - W_R$ mixing is strictly forbidden and $W_R$ does not contribute to $\Delta m_K$ or $\mu$ decay.

Since the absence of FCNC allows only one bidoublet, only one lepton generation is accounted for in the above. Let it be the $\tau$ lepton. The $e$ and $\mu$ generations are
then accommodated in the $\Phi_{L,R}$ components of the other two $27$’s, but they must not couple to $Qd^c$ or $hQ^c$. This can be accomplished by extending the discrete symmetry necessary for maintaining the conservation of lepton number as defined above.

7. Precision Measurements at the Z

Because of the Higgs structure of this model, there is in general some $Z-Z'$ mixing which depends on the ratio of the $W_L$ to $W_R$ masses. Let $\langle \eta^0_2 \rangle = v$, $\langle \phi^0_{L,R} \rangle = v_{L,R}$, $r = v^2/(v^2 + v^2_L)$, $x = \sin^2 \theta_W$, then

$$M^2_{W_{L,R}} = \frac{1}{2}g^2(v^2 + v^2_{L,R}),$$

(24)

and

$$M^2_Z \approx \frac{M^2_{W_L}}{1 - x} \left[ 1 - \left( r - \frac{x}{1 - x} \right)^2 \right], \quad M^2_{Z'} \approx \frac{1 - x}{1 - 2x} M^2_{W_R},$$

(25)

where $\xi = M^2_{W_L}/M^2_{W_R}$. Deviations from the standard model can now be expressed in terms of the three oblique parameters $\epsilon_{1,2,3}$ or $S, T, U$. Using the precision experimental inputs $\alpha, G_F, M_Z$, and the $Z \rightarrow e^-e^+, \mu^-\mu^+$ (but not $\tau^-\tau^+$) rates and forward-backward asymmetries, they are given by

$$\epsilon_1 = \alpha T = - \left( \frac{2 - 3x}{1 - x} - r \right) \left( r - \frac{x}{1 - x} \right) \xi,$$

(26)

$$\epsilon_2 = - \frac{\alpha U}{4x} = - \left( r - \frac{x}{1 - x} \right) \xi,$$

(27)

$$\epsilon_3 = \frac{\alpha S}{4x} = - \left( \frac{1 - 2x}{2x} \right) \left( r - \frac{x}{1 - x} \right) \xi.$$

(28)

Note that the ratio $S/T$ must be positive and of order unity here. Experimentally, $S, T, U$ are all consistent with being zero within about 1$\sigma$, but the central $S$ and $T$ values are $-0.42$ and $-0.35$ respectively. These imply that $r \sim 0.8$ and $\xi \sim 6 \times 10^{-3}$, hence the $W_R$ mass should be about 1 TeV which is exactly consistent with this model’s assumed $SU(2)_R$ breaking scale.

In this model, the $\tau$ generation transforms differently under $SU(2)_R$, hence there is a predicted difference in the $\rho$ and $\sin^2 \theta$ parameters governing $Z \rightarrow l^-l^+$ decay. Specifically,

$$\rho_{\tau} - \rho_{e,\mu} = 2 \left( r - \frac{x}{1 - x} \right) \xi \sim 6 \times 10^{-3},$$

(29)

compared with the experimental value of $0.0064 \pm 0.0048$, and

$$\sin^2 \theta_{\tau} - \sin^2 \theta_{e,\mu} = -x \left( r - \frac{x}{1 - x} \right) \xi \sim -7 \times 10^{-4},$$

(30)

compared with the experimental value of $-0.0043 \pm 0.0022$. The standard model’s prediction for either quantity is of course zero.
8. Second Example with Unification

Fig. 2 shows the two-loop evolution of gauge couplings corresponding to the following situation. Let the particle content of the proposed left-right model be restricted to only components of the $27$ and $27^*$ representations of $E_6$, then unification is achieved with

\begin{align}
  b_S &= -9 + 2(3) + n_h = 0, \\
  b_{LR} &= -6 + 2(3) + n_{22} + n_\phi = 4, \\
  (3/2)b_X &= 2(3) + n_h + 3n_\phi = 18,
\end{align}

where $n_h = 3$ and $n_{22} = 1$ are required as already discussed, and $n_\phi = 3$ is the number of extra sets of $\Phi_L + \Phi_R + \Phi^c_L + \Phi^c_R$. Note that at least one such set is needed for SU(2)$_R$ breaking and that the two constraints of Eq. (9) are simultaneously satisfied with the one choice of $n_\phi = 3$.

To complete the model, six singlets $N \sim (1,1,1,0)$ are also assumed. At the unification scale $M_U$, there are presumably six $27$’s and three $27^*$’s of $E_6$, which is then broken down to supersymmetric $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$ supplemented by a discrete $Z_4 \times Z_2$ symmetry. Of the three $27$’s and three $27^*$’s, only the combinations $\Phi_L + \Phi_R + \Phi^c_L + \Phi^c_R$ survive. Of the other three $27$’s, only two bidoublets do not survive. At $M_R \sim$ few TeV, $\Phi_R$ and $\Phi^c_R$ break SU(2)$_R \times U(1)$ down to U(1)$_Y$. Supersymmetry is also broken softly at $M_R$. The surviving model at the electroweak energy scale is the standard model with two Higgs doublets but not those of the MSSM, as already explained in my first talk at this meeting.

9. Lepton Masses

The $\tau$ gets its mass from the $\Phi_L \Phi_R \eta$ term, but there can be no such term for the $e$ and $\mu$. Hence the latter two are massless at tree level. However, the soft supersymmetry-breaking term $\Phi_L \Phi_R \tilde{\eta}$ (where $\tilde{\eta} = \sigma_2\eta^*\sigma_2$ and all three fields are scalars) is allowed, hence $m_e$ and $m_\mu$ are generated radiatively from the mass of the U(1) gauge fermion. The neutrinos obtain small seesaw masses from their couplings with the three $N_L$’s which are assumed to have large Majorana masses. The $\nu_\tau N_L$ mass comes from the $\eta\eta N_L$ term, and the $\nu_e N_L, \nu_\mu N_L$ masses come from the $\Phi_L \Phi^c_L N_L$ terms.

10. Conclusion

New physics in the framework of left-right gauge symmetry is possible at the TeV energy scale even if grand unification is required. Two examples have been given, the second of which is particularly attractive: it is free of FCNC at tree level and has negative contributions to the oblique parameters $S$ and $T$ consistent with present experimental central values.
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12. References

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