Pendulum tuned mass damper: optimization and performance assessment in structures with elastoplastic behavior

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ABSTRACT

Different types of tuned mass dampers (TMD) have been applied to reduce wind and seismic induces vibrations in buildings. We analyze a pendulum tuned mass damper (PTMD) to reduce vibrations of structures that exhibit elastoplastic behavior subjected to ground motion excitation. Using a simple dynamic model of the primary structure with and without the PTMD and a random process description of the ground acceleration, the performance improvement of the structure is assessed using statistical linearization. The Lyapunov equation is used to estimate the mean-square response in the stationary condition of the random process and optimize PTMD parameters. The optimum values of the PTMD frequency and damping ratio are defined as PTMD design values for a specific maximum seismic intensity design criterion. The results show that: (1) The values of the PTMD effectiveness criterion and the optimal design values of the frequency ratio are higher when the damping ratio of the primary structure decreases. (2) The performance of the optimized PTMD is higher when the structure exhibits a linear hysteresis loop (low seismic intensity). (3) The optimized PTMD controls the development of structural plasticity reducing vulnerability. (4) There is a strong dependence of the optimum PTMD parameters on the dynamic soil properties of the building foundation. (5) The PTMD performance improves as its mass increases. The optimum frequency ratio decreases, and the damping ratio increases as the mass of the pendulum increases. The PTMD designed and optimized with the proposed methodology reduces vibrations, controls the development of plasticity, and protects the primary structure, particularly in low and medium-intensity earthquakes.

1. Introduction

The TMD is a practical and efficient method for structural response reduction to ground acceleration input [1]. This technology is an alternative for building structural design in seismic regions [2]. The TMD is a mechanical system that consists of a mass, a spring that provides stiffness, and a viscous damper. The mass is attached to the structure through the spring and damper [3]. When the element that provides stiffness to the TMD is a pendulum, the device is called PTMD [4]. The PTMD absorbs a large portion of the energy produced by external forces, minimizes the vibration amplitude, and reduces the probability of damage to structural elements [5]. The PTMD is very simple and responds quickly to structure motion [6]. One of PTMD's most exciting features is that its design is straightforward. The pendulum's length controls its natural frequency, and its design can integrate the viscous dampers without difficulty [7]. A noteworthy PTMD advantage is that the pendulum can oscillate in all directions providing energy dissipation for loads applied in different directions [8]. However, the PTMD's performance in reducing vibration depends on its optimum design parameters' values: the pendulum's mass, length, and damping, amount other. Under seismic loads, PTMD performance depends on the soil dynamic properties that define dominant frequency content of ground acceleration and the main structure dynamic properties (frequency and damping). It also depends on the ductility demand level of the main structure, mainly due to the intensity of seismic excitation.

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In practice, PTMDs are mainly used to reduce wind-induced vibration of tall and slender structures, wind turbine towers, and high-rise steel towers [9, 10]. Moreover, PTMD has become a popular device in controlling and reducing structural vibration due to wind loads. In practical applications, PTMD has shown to be effective in low and medium-intensity earthquake excitation. Mainly where the fundamental vibration mode of the main structure controls most of the response of the building [6]. An example of a building with a PTMD conceived to reduce wind-induced vibration is the Taipei 101 Tower in the capital of Taiwan. This building has 101 floors, with a total height of 508 m. At its inauguration, the PTMD was the biggest ever built, with a large solid steel sphere weighing 660 metric tons. The PTMD attached to the Taipei 101 Tower reduces wind-induced vibration levels and has also shown the capability to alleviate seismic induced vibrations [8]. The difference between the structure response to strong intensity earthquakes and to wind load is the amount of energy dissipation in main structural elements. The building's motion during an earthquake induces plastic behavior in some structural members, and the structure vibrates for a few seconds. During a wind load, the building oscillates during several minutes in the elastic range [11].

PTMD performance deteriorates when the structure shows elastoplastic behavior. When a structure enters the nonlinear range, there is a loss of effective stiffness, which generates a loss of tuning between the PTMD frequency and the structure's primary frequency. A large amount of energy dissipation is provided by yielding structural elements, which imply a marginal contribution of the energy dissipation of the TMD to the building response. Under stronger dynamic loading induced by earthquakes, larger changes can occur in the effective stiffness of the structure because of inelastic effects potentially coupled with damage. These changes cause an increase in the structural period and, consequently, a much more significant detuning effect [12]. Sung-Sik et al. [13] showed that TMD's performance, whose design parameters were optimized for an elastic structure, considerably deteriorated when the structural response's hysteretic portion increased. Sgobba and Marano [14] and Duque et al. [15] reported that the TMD performance decreases when the structural hysteretic response increases, even when the TMD design parameters are optimized for a structure with elastoplastic behavior. While the achieved reductions are not significant, they are not negligible in structures subjected to medium and high seismic intensities.

An alternative to the TMD limitations attached to an elastoplastic structure is the semi-active TMD (S-TMD) or the active TMD (A-TMD), whose control method adapts the TMD to the structure with variables parameters. S-TMD and A-TMD are an alternative to passive TMD, especially if demand reductions are to be achieved in structures that enter the plastic regime during their response. A control system optimizes (tune) the TMD to get the best performance and adapt it to the dynamic structure regime. In this regard, Sung and Nagarajaiah [16] found that the S-TMD can effectively attenuate the seismic responses and outperform the optimal passive TMD. Also, these authors reported that the S-TMD remains tuned with the primary structure.

In contrast, the optimal passive TMD becomes off-tuned when damage occurs. Lourenco [17] described the design, construction, implementation, and performance of a prototype adaptive PTMD. The experimental studies' results demonstrate the importance of optimizing the PTMD frequency and damping ratio to reduce structural vibrations. Finding the PTMD's optimum design parameters is not a trivial task. Gerges and Vickery [18] reported in design charts the optimum design parameters and the corresponding efficiency of the PTMD under both wind and earthquake dynamic loads considering an elastic response of the main structure. Oliveira et al. [4] found a general dimensionless optimal parameter for a PTMD considering structure elastic response; they concluded that the dimensionless parameters could be employed to design a pendulum to control any tall building subjected to dynamic loads, with different mass and damping ratios. Hassani and Aminafshar [19] study the numerical optimization of PTMD attached to a tall building that shows elastic behavior and under horizontal earthquake excitation. Deramaeker and Soltani [10] extended Den Hartog's equal peak methods to the PTMD and observed an excellent agreement between the PTMD performance tuned by these analytical formulae and the numerical results obtained by the Oliveira et al. [4] optimization process. Colherinhas et al. [20] assumed a structure elastic behavior. Cloherinhas et al. modeled a tower with a PTMD using Finite Element and ANSYS to find the relation between the mass, length, stiffness, and damping coefficient of the pendulum as a function of the high vibration amplitudes at the top of the tower. Amrutha and Amritha [21] assessed the seismic response reduction by PTMD on regular high-rise RC buildings using SAP2000 V19 software. They concluded that installing PTMD on the structure 10–25% top story displacement reduction was observed.

Few authors have considered a PTMD optimized considering the elastoplastic behavior of the main structure and its foundation soil dynamic properties. For the classical TMD, this job has been done by Sgobba and Marano [14] and Duque et al. [15]. Also, Jia and Jianwen [22] investigated the performance degradation of TMDs arising from ignoring soil-structure interaction effects. They showed that a well-tuned damper performs better than an off-tuned one by up to 25%, although an off-tuned one may reduce the structure responses by up to 30%. Similarly, Salvi et al. [23] investigated an optimum TMD's effectiveness in reducing the linear structural response to strong-motion earthquakes by embedding soil-structure interaction within the dynamic and TMD optimization model.

Regarding the classical TMD and structure with an elastoplastic behavior, Sgobba and Marano [14] studied the optimum design of TMD for structures with nonlinear behavior. These authors use the de Bouc-Wen model to describe the nonlinear behavior of the main structure. The Kanai-Tajimi stochastic seismic model describes the earthquake ground acceleration. Sgobba y Marano confirmed that the TMD reduces the amount of the hysteretic dissipated energy, which directly measures damage in the structure. So, it is beneficial to protect buildings that develop a nonlinear behavior under severe dynamic loadings. Woo et al. [13] assessed the seismic response control of elastic and inelastic structures by using passive and semi-active TMDs. They performed a numerical analysis for a structure with hysteretic described by the Bouc-Wen model. The results indicated that the passive TMD’s performance, whose design parameters were optimized for an elastic structure, considerably deteriorated when the hysteretic portion of the structural responses increased. The semiactive TMD showed about 15–40% more response reduction than the TMD. Duque et al. [15] found that if the TMD is optimized considering the seismic intensity and a structure with elastoplastic behavior, the TMD reduces the structure's displacements in seismic events. While the achieved reductions are not significant, they are not negligible in structures subject to high seismic intensities. Although qualitatively, we can expect similar results and performance for a PTMD. These results cannot be quantitatively extrapolated to a PTMD. More exploratory research on the PTMD performance is needed considering the pendulum optimum design parameters' values, the soil's dynamic properties, the primary structure dynamic properties, and the primary structure behavior due to seismic intensity and design parameters.

The purpose of this study is to analyze the PTMD performance when its design parameters are optimized considering: (1) the main structure dynamic properties (frequency and damping), (2) the structure exhibits elastoplastic behavior depending on the intensity (PGA) of the seismic excitation, (3) the main structure is on soils with different dynamic properties (soft, medium, and firm soil), and (4) the soil is on firm soil. The equations of motion of the system were solved using Monte Carlo simulation and the stochastic pseudo-linear equivalent system (SPLIES) to achieve our objective. The numerical optimization scheme applies optimization methods to the SPLIES parameters in an iterative scheme. This scheme allows finding the optimal design parameters of the PTMD (frequency and damping ratio) given the dynamic parameters of the main structure and the soil. The validation of the stochastic pseudo linear equivalent system was performed by comparing the solution using Monte Carlo simulation and the theoretical solution.
Carlo and the solution using the stationary regime of the stochastic pseudo linear equivalent system. Finally, case study results are presented for seismic excitation records measured during Pedernales Ecuador 2016 earthquake.

2. Methodology

Seismic excitation is represented through a stationary random process of filtered white noise (Kanai-Tajimi filter, KTF) and the main structure’s elastoplastic behavior represented by the Bouc Wen model (BWM). A SDOF model modeled the primary structure.

2.1. Kanai-Tajimi filter

The KTF is a model frequently used to represent a seismic acceleration and achieve artificial accelerograms. Kanai-Tajimi’s model considers the earthquake represented by a spectrum of filtered white Gaussian noise [24]. Kanai [25] and Tajimi [26] showed that a second-order linear oscillator is suitable to filter white noise and obtain a spectrum that match frequency content of registered accelerograms (Figure 1). Therefore, the filter parameters are related to the soil characteristics and, consequently, to different frequency contents of the ground acceleration signal (Figure 1).

The acceleration process is characterized by its power spectral density (PSD), named excitation power spectral density (EPSD). The white noise PSD obtained with the KTF considering the values of different EPSD.

The parameters of the KTF are associated with soil dynamics characteristics. However, their values depend on the distance to the epicenter, earthquake magnitude, and soil rigidity, among other factors. In the context of our research, the parameters of the KTF reported in the literature are listed in Table 1. Unless otherwise specified, in our study, we consider the white noise PSD obtained with the KTF considering the values of and reported by Sues et al. [24].

The parameters and are associated with soil dynamics characteristics. However, their values depend on the distance to the epicenter, earthquake magnitude, and soil rigidity, among other factors. In the context of our research, the parameters and do not provide information on the soil-structure interaction and only serve to generate three different EPSD.

Eq. (1) models the KTF, where , , and represent the relative acceleration, velocity, and displacement of the filter, respectively; and represents the absolute bedrock acceleration, which is modeled as white noise with a constant PSD .

\[ \ddot{x}_f(t) + 2\xi f \dot{x}_f + \omega_0^2 x_f = -W \]  
(1)

The absolute acceleration of the building foundation \( \ddot{x}_g \) is expressed in Eq. (2).

\[ \ddot{x}_g(t) = \ddot{W}(t) + \ddot{x}_f(t) = -2\xi f \dot{x}_f(t) - \omega_0^2 x_f(t) \]  
(2)

2.2. Bouc-Wen model for structures with elastoplastic behavior

Bouc [30] and Wen [31] proposed and generalized the hysteresis BWM. Eq. (5) expresses the BWM for an SDOF exhibiting elastoplastic behavior, where \( m \) represents the mass of the system (primary structure), \( c_\alpha \) and \( c_\beta \) the viscous damping coefficient (primary structure), \( F(x_f, x, t) \) the restoring force, \( f(t) \) the applied external force, and \( x_f, \dot{x}_f, \ddot{x}_f \) represents the base displacement, speed, and acceleration, respectively.

\[ m_0 \ddot{x}_f + c_\alpha \dot{x}_f + F(x_f, \dot{x}_f, t) = f(t) \]  
(5)

The restoring force \( F(x_f, \dot{x}_f, t) \) results from the sum of an elastic part \( F^e(t) \) and a hysteretic component \( F^h(t) \), thus:

\[ F(x_f, \dot{x}_f, t) = F^e(t) + F^h(t) \]  
(6)

Where: \( \alpha \) is the pre-yielding, \( k_i \) post-yielding; \( x_y \) yielding displacement. The differential equation of the state variable \( z(t) \) that represents the hysteresis loop is:

\[ z(t) = A \dot{x}_f(t) - \beta |\dot{x}_f(t)|z(t)^{n-1}z(t) - \gamma x_f(t) z(t)^n \]  
(7)

In Eq. (7) A, \( \beta, \gamma, n, \alpha \) are the dimensionless parameters used to set up the BWM. They play the role of governing and controlling the scale and general shape of the hysteresis loop. Constant A determines the amplitude of the hysteresis loop at \( z = 0 \). Constants \( \beta \) and \( \gamma \) control the level of hysteresis and the level of energy dissipation per cycle, respectively. The value of \( n \) provides the linearity of the system. If \( n = 1 \) the system is linear and if \( n = 0 \) the system is entirely nonlinear. When \( \beta = 0 \) there is no hysteresis, if \( \beta = 0 \) and \( \gamma > 0 \) softening occurs, hardening occurs when \( \beta = 0 \) and \( \gamma < 0 \) [32, 33, 34, 35]. In this study, the

\[ \sigma_y^2 = \frac{1}{4} \frac{\omega_n^2 (1 + 4\xi^2)}{\xi f} S_0 \]  
(3)

Considering that the peak ground acceleration (PGA) is of the order of three times the standard deviation \( \sigma_y \) of the soil acceleration \( x_g(t) \), therefore \( \text{PGA} = 3\sigma_y \) [29] and:

\[ S_0 = \frac{4}{9} \frac{\xi f (\text{PGA})^2}{\omega_n^4 (1 + 4\xi^2)} \]  
(4)
Table 1. Soil dynamic parameters.

| Source   | Soil profile description | Frequency, \( \omega_f \) (rad/s) | Damping, \( \xi_f \) |
|----------|--------------------------|-------------------------------|------------------|
| [27]     | Medium                   | 20                            | 0.5              |
| [28]     | Firm                     | 20                            | 0.65             |
| [24]     | Soft                     | 4.5                           | 0.1              |
|          | Medium                   | 16.5                          | 0.8              |
|          | Firm                     | 16.9                          | 0.94             |

values assumed for the BWM hysteretic model were \( A = n = 1, \beta = \gamma = 0.5 \), and \( a = 0.5 \) since with them, the model captures the response of structures and structural members exposed to earthquakes [36].

2.3. Stochastic linearization of the hysteretic Bouc-Wen model

The statistical linearization method replaces Eq. (7) by the equivalent linear form given in the Eq. (8) [37, 38, 39] so that the mean square error is minimized.

\[
\dot{z} = S_{eq} \ddot{x}_C + C_{eq} \dot{x}_C + K_{eq} z
\]

(8)

The calculate \( S_{eq}, C_{eq} y K_{eq} \) we denote \( h(x, \dot{x}, z) \) the function that defines the hysteretic displacement \( z(t) \) in Eq. (7). Then, we proceed to minimize the mean-square error, \( \dot{z}^2 \), where the error is

\[
\dot{z}^2 = \left( h(x, \dot{x}, z) - \left( S_{eq} x + C_{eq} \dot{x} + K_{eq} z \right) \right)^2
\]

(9)

The parameters that define the minimum value of the mean-square error are:

\[
S_{eq} = E \left[ \frac{\partial h}{\partial x} \right] ; \quad C_{eq} = E \left[ \frac{\partial h}{\partial \dot{x}} \right] ; \quad K_{eq} = E \left[ \frac{\partial h}{\partial z} \right]
\]

(10)

If the joint probability distribution of the state vector \( Q^T = [x, \dot{x}, z] \) is Gaussian with zero mean and \( \beta = \gamma, A = n = 1 \), the values of the coefficients of the equivalent linear system are [37, 38]:

\[
C_{eq} = 1 - \sqrt{2} \beta \left( \frac{\gamma_{x,z}}{\sigma_{x,z}} \right) ; \quad K_{eq} = -\sqrt{2} \beta \left( \frac{\gamma_{x,x}}{\sigma_{x,x}} \right) ; \quad S_{eq} = 0
\]

(11)

Where \( \sigma_{x,x} \) represents the standard deviation of the hysteretic displacement, \( \sigma_{x,z} \) standard deviation of the deformation rate, and \( \gamma_{x,x}, \gamma_{x,z} \) the covariance of the structure’s hysteretic displacement variable and deformation rate.

2.4. Dynamic equations of the primary structure with elastoplastic behavior

The primary structure model is a SDOF model with mass \( m_s \), stiffness \( k_s \), damping \( c_s \), and hysteretic behavior defined by the state variable \( z \) as shown in Figure 2.

It considers the set of forces (proportional to the relative movement) acting on the structure.

\[
x_i = x_m - x_g \Rightarrow x_m = x_i + x_g
\]

(12)

\[
m_s \ddot{x}_m = -ak_s (x_m - x_g) - c_s (x_m - x_g) - k_s (1 - \alpha) z
\]

(13)

\[
\ddot{x}_s = -\frac{k_s}{m_s} x_s - \frac{c_s}{m_s} \dot{x}_s - \frac{k_s}{m_s} (1 - \alpha) z - \dot{x}_g
\]

(14)

Replacing Eq. (2) and the parameters: (i) Stiffness: \( k_i = m_i \omega_i^2 \). (ii) Damping ratio: \( \xi_i = c_i / (2 \sqrt{m_i k_i}) \) in the Eq. (14):

\[
\ddot{x}_i = -a \omega_i^2 x_i + \omega_i^2 \dot{x}_i - \omega_i^2 (1 - \alpha) z - 2 \omega_i \alpha \dot{x}_i + 2 \xi_i \alpha \dot{x}_i
\]

(15)

Thus, the dynamic equations of the soil-structure system are shown in Eq. (16).

\[
x_i = x_s
\]

\[
x_i = \dot{x}_s
\]

\[
\ddot{x}_s = K_{eq} z + C_{eq} \dot{x}_s
\]

(16)

\[
\ddot{x}_i = -a \omega_i^2 x_i + \omega_i^2 \dot{x}_i - \omega_i^2 (1 - \alpha) z - 2 \omega_i \alpha \dot{x}_i + 2 \xi_i \alpha \dot{x}_i
\]

Therefore, the state-space formulation of the soil-structure system’s linearized model is given in Eqs. (17) and (18).

\[
X = AX + BW
\]

(17)

\[
X = [\dot{x}_s \ \dot{x}_f \ \ddot{x}_s \ \ddot{x}_f]^T
\]

\[
X = [x_s \ \dot{x}_s \ \ddot{x}_s \ \ddot{x}_f]^T
\]

\[
B = [0 \ 0 \ 0 \ 1]^T
\]

\[
W = 2\pi S_0 \ S_0 = \begin{bmatrix} 4 \pi^2 & \xi_f (\text{PGA})^2 \\ 0 & 1 + 4 \xi_f^2 \end{bmatrix}\]

(18)

\[
A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ -\omega_i^2 & -\omega_i^2 (1 - \alpha) & -2 \omega_i \alpha & 2 \xi_i \alpha \end{bmatrix}
\]

Figure 2. Graphic representation of the soil and the primary structure without PTMD.
2.5. Dynamic equations of the combined system

The combined system consists of the passive PTMD model attached to the structure (Figure 3a). The passive PTMD device consists of a pendulum with a viscous damper $c_d$. The pendulum is a solid sphere of mass $m_p$ connected by a cable of length $l$ to the structure. We assume that the pendulum rotational inertia and the cable mass are negligible, and the angle $\theta$ is small. The pendulum oscillation frequency is $\omega_p = g/l$, where $g$ represents the acceleration of gravity. The primary structure model is an SDOF model with mass $m_s$, stiffness $k_s$, and damping $c_s$.

Considering the set of forces acting on the structure and the PTMD with viscous damping, the force on the mass $m_s$ and the damping $c_s$ is proportional to the relative movement.

$$x_t = x_m - x_g \Rightarrow x_m = x_s + x_g$$ (19)

The dynamic equilibrium equation of $m_s$ (Figure 3b) projected on the horizontal direction leads to Eq. (20).

$$-m_p x_m \cos \theta - m_s \dot{g} \theta - m_s g \sin \theta - c_s \dot{\theta} \cos \theta = 0$$ (20)

When rotations $\theta$ are small $\cos \theta \approx 1$ and $\sin \theta \approx \theta$, and considering the Eq. (19), we can rewrite the Eq. (20) into Eq. (21).

$$x_t = -l \ddot{\theta} - g \theta - \frac{c_d}{m_d} \dot{\theta} - \ddot{x_g}$$ (21)

The following non-dimensional parameters should be defined: (1) Ratio between the mass of the PTMD and the structure’s mass $\mu = m_p/m_s$. (2) The ratio between the frequency of the PTMD and the frequency of the structure’s main mode of vibration $\omega = \omega_p/\omega_s$. (3) Stiffness: $k_t = m_p \omega_p^2$. (4) Damping ratio: $\xi = c_d/(2 \sqrt{k_t m_p})$. Replacing $c_d/m_d = 2 \xi \omega_p$ and Eq. (2) in the Eq. (21):

$$\ddot{x}_t = -l \ddot{\theta} - g \theta - 2 \xi \omega_p \dot{\theta} \dot{x}_t + 2 \xi \omega_p \dot{\theta}$$ (22)

Considering the forces acting on the mass of the primary structure, we have:

$$m_s \ddot{x}_m = -m_p x_m - m_p \dot{\theta} \dot{x}_m - c_s (x_m - x_g) - c_s (\dot{x}_m - \dot{x}_g) - k_s (\dot{x}_m - \dot{x}_g) - \mu (1 - \alpha) \ddot{x}_g$$ (23)

Replacing Eqs. (19) and (21), as well as the terms $c_d/m_d = 2 \xi \omega_p$; $k_t/m_s = \omega_s^2$; $\mu = m_d/m_s$ in Eq. (23):

$$\ddot{\theta} = -\frac{2(1 + \mu) \xi \omega_p}{l} \theta - \frac{\alpha \omega_s^2}{l} \dot{x}_s + \frac{2 \xi \omega_p \dot{x}_s}{l} + \frac{2 \xi \omega_p (1 - \alpha) \dot{x}_g}{l}$$ (24)

Replacing dimensionless terms $g/l = \omega_s^2$ and $\omega_p^2/l = \omega_s^2 \omega_p^2/g$: $\omega_s/l = \omega_p \omega_s^2/g$ in Eq. (24):

$$\ddot{\theta} = -2(1 + \mu) \xi \omega_p \dot{\theta} - (1 + \mu) \omega_s^2 \dot{\theta} + \frac{\alpha \omega_s^2}{l} \dot{x}_s + \frac{2 \xi \omega_p \dot{x}_s}{l} + \frac{(1 - \alpha) \omega_s^2}{l} \dot{x}_g$$ (25)

Replacing the Eq. (25) and the relationship $l = g/\omega_s^2$ in Eq. (22):

$$\ddot{x}_t = \frac{2 \mu \omega_s^2}{\omega_t} \dot{\theta} + \alpha \omega_s^2 \dot{x}_s - 2 \xi \omega_p \dot{x}_s - \omega_s^2 (1 - \alpha) \ddot{x}_g + \omega_s^2 \ddot{x}_s + 2 \xi \omega_p \dot{x}_g$$ (26)

Therefore, the state-space formulation of the linearized model of the combined primary structure and PTMD is given in Eqs. (27), (28), and (29).

$$X = AX + BW$$ (27)

$$X = \begin{bmatrix} \dot{\theta} & \dot{x}_s & \ddot{x}_s & \dot{\theta} & \dot{x}_g \end{bmatrix}^T$$

$$W = 2\pi S_0; S_0 = \begin{bmatrix} 4 & \xi/(PGA)^2 \\ \frac{\pi}{2} & 1 + 4 \xi^2 \end{bmatrix}$$ (28)

$$\ddot{x}_s = -\dot{\theta} - g \theta - \frac{c_t}{m_t} \ddot{\theta} - \ddot{x}_g$$ (21)

2.6. PTMD design parameters optimization and validation

The PTMD design parameters optimization is performed for a stationary random ground acceleration process. The optimization problem is handled via a numerical search algorithm. The algorithm finds the parameters of the SPLES and computes expected performance with the linearized model. The search for optimum PTMD design parameters is performed by solving the dynamic equation for a given mean maximum ground acceleration and finds the optimum design parameter in a steady-state condition. Model validation is performed by comparing the time solution (Monte Carlo simulation) and the SPLES response.

2.6.1. Monte Carlo simulation

The Monte Carlo simulation is a method for estimating the exact response statistics of randomly excited nonlinear systems within any desired confidence level. This approach is applicable for the estimation of the stationary and non-stationary response statistics. The stochastic differential equation governing the system’s motion is interpreted as an
The effectiveness of PTMD in reducing vibration is defined by the covariance matrix $P_{xx}$ and the intensity of white noise. Eq. (33) is solved for $\Delta$ and represents the intensity of white noise. Therefore, we obtain a matrix $OF_{gi}$, which contains the performance indicator OF values for the frequency and damping combinations assumed for the PTMD. Figure 5 illustrates the optimization procedure for various PGA. The maximum effectiveness $OF_{max}$ is given by $f_{opt}, \xi_{opt}$ values.

A closed look at Figure 4b shows the optimum frequency ratio trend $f_{opt}$ when we keep the damping ratio constant and equal to $\xi_{opt}$ (Figure 5a). Figure 5a shows a peak around $f_{opt}$, while Figure 5b shows a highly asymmetric peak around $\xi_{opt}$. The symmetric peak in Figure 5a suggests that the effectiveness of the PTMD is more sensitive to the frequency ratio $f$ values when the damping ratio is equal to $\xi_{opt}$ (Figure 5a). The asymmetric peak around $\xi_{opt}$ suggests that the PTMD effectiveness is very sensitive to the damping ratio before reaching the optimum value $\xi_{opt}$. In contrast, the PTMD effectiveness is robust to changes in the damping ratio when the damping ratio value is bigger than the $\xi_{opt}$ (Figure 5b).

We develop a model validation to assess equivalent linear modal accuracy for estimating the nonlinear model stochastic response. Model validation involves comparing the time domain simulation (Monte Carlo simulation) and the SPLS response. Therefore, we consider a SDOF system that represents a structure described with the following parameters: $\alpha_i = 10 \text{ rad/s}, \xi_i = 0.02, A = 1, \beta = \gamma = 0.25, a = 0.5, n = 1$. The soil profile is described as a firm soil with $\alpha = 16.9 \text{ rad/s}$ and $\xi = 0.94$ (Table 1). The mass ratio between the PTMD and primary structure is considered equal to 0.1 ($\mu = 0.1$). The PTMD design parameters are $f_{opt} = 0.853$ and $\xi_{opt} = 0.153$. We have considered two levels of seismic intensity PGA = 0.01 g (Figure 6a) and PGA = 0.6 g (Figure 6b), and one thousand observations (samples) at each time. Figure 6a and b show the structural response in terms of its standard deviation of the displacement of the protected structure and $\sigma_{z}$, which are obtained from the covariance matrix $P_{xx}$.

OF = $1 - \frac{\sigma_{z}}{\sigma_{0}}$ (33)

The PTMD design parameters are optimized by numerically solving Eq. (33) for several frequency factor values $f = \omega_i/\omega_0$ and several damping coefficient values $\xi_i$. Therefore, we obtain a matrix $OF_{gi}$, which contains the performance indicator OF values for the frequency and damping combinations assumed for the PTMD. Figure 5 illustrates the optimization procedure for various PGA. The maximum effectiveness $OF_{max}$ is given by $f_{opt}, \xi_{opt}$ values.

A closed look at Figure 4b shows the optimum frequency ratio trend $f_{opt}$ when we keep the damping ratio constant and equal to $\xi_{opt}$ (Figure 5a). Figure 5a shows a peak around $f_{opt}$, while Figure 5b shows a highly asymmetric peak around $\xi_{opt}$. The symmetric peak in Figure 5a suggests that the effectiveness of the PTMD is more sensitive to the frequency ratio $f$ values when the damping ratio is equal to $\xi_{opt}$ (Figure 5a). The asymmetric peak around $\xi_{opt}$ suggests that the PTMD effectiveness is very sensitive to the damping ratio before reaching the optimum value $\xi_{opt}$. In contrast, the PTMD effectiveness is robust to changes in the damping ratio when the damping ratio value is bigger than the $\xi_{opt}$ (Figure 5b).

We develop a model validation to assess equivalent linear modal accuracy for estimating the nonlinear model stochastic response. Model validation involves comparing the time domain simulation (Monte Carlo simulation) and the SPLS response. Therefore, we consider a SDOF system that represents a structure described with the following parameters: $\alpha_i = 10 \text{ rad/s}, \xi_i = 0.02, A = 1, \beta = \gamma = 0.25, a = 0.5, n = 1$. The soil profile is described as a firm soil with $\alpha = 16.9 \text{ rad/s}$ and $\xi = 0.94$ (Table 1). The mass ratio between the PTMD and primary structure is considered equal to 0.1 ($\mu = 0.1$). The PTMD design parameters are $f_{opt} = 0.853$ and $\xi_{opt} = 0.153$. We have considered two levels of seismic intensity PGA = 0.01 g (Figure 6a) and PGA = 0.6 g (Figure 6b), and one thousand observations (samples) at each time. Figure 6a and b show the structural response in terms of its standard deviation of the displacement of the protected structure and $\sigma_{z}$, which are obtained from the covariance matrix $P_{xx}$.

The effectiveness of PTMD in reducing vibration is defined in terms of the performance index $OF$ defined in Eq. (33). Where $\sigma_{z}$ is the standard deviation of the displacement of the protected structure and $\sigma_{0}$ is the standard deviation of the displacement of the unprotected structure [15].

\[
\text{OF} = 1 - \frac{\sigma_{z}}{\sigma_{0}}
\]
Figure 4. Optimization algorithm. (a) The workflow of the optimization process. (b) An illustration of the objective function characteristic response surface and the optimal $OF, f, \xi_d$ values.

Figure 5. (a) An illustration of how the PTMD effectiveness changes if we keep constants the damping ratio and change de frequency ratio. (b) An illustration of how the PTMD effectiveness changes if we keep constants the frequency ratio and change the damping ratio.

Figure 6. Comparison of the standard deviation of the primary system base displacement obtained by Monte Carlo simulation and the SPLES and 1000 samples at each instant. (a) System response to PGA = 0.01 g. (b) System response to PGA = 0.6 g. (c) Structural response $\sigma_x$ calculated by the Monte Carlo method and solving the SPLES while searching the PTMD optimum design parameters.
linear system. Thus, we use the equivalent linear system response in the steady-state to find the PTMD optimum design parameters and the mean structural response for different seismic intensities.

2.7. PTMD seismic performance: a case study

The primary structure response with an optimized PTMD and without PTMD are compared upon been subjected to the Pedernales earthquake. Thus, the PTMD effectiveness was verified using the Pedernales earthquake’s horizontal acceleration registered by the APED station at Pedernales, Ecuador. The APED station is localized at Latitude 0.068, Longitude -80.057, Altitude 15 m.a.s.l., Epícenter distance 36 km, PGA EW 1380.49 cm/s², PGA NS 812.70 cm/s², and PGA Z 727.38 cm/s². Event: 0001. Date UTM: 2016 4 16. Registration time 18:58 (local time). Component EW. Sampling frequency 100 Hz. Units cm/s² [40].

3. Results

3.1. Optimum PTMD design parameters

The PTMD design parameters depend mainly on the level of incursion into the elastoplastic behavior by the primary structure (Figure 7). The mean value of the standard deviation of the primary structure base displacement $\langle \sigma_{x_s} \rangle$ is related to the level of elastoplastic incursion. Therefore, the PTMD is highly effective when its design parameters have been optimized to show the best performance in the “elastic” region – $\sigma_{x_s}$ mean values close to zero (Figure 7a). In comparison, the PTMD effectiveness design value decreases when the PTMD design parameters are optimized considering its performance in the region with high elastoplastic behavior – high $\sigma_{x_s}$ mean values (Figure 7a). The structure with the lowest “damping ratio” naturally removes more vibratory energy and reduces oscillation amplitude. Unlike a structure that remains in the elastic range, a structure that has a substantial incursion into the plastic range dissipates much energy due to the plastic behavior of its structural elements so that the energy dissipated by the PTMD is marginal.

Furthermore, the results show that the removal of vibratory energy is enhanced with the PTMD. The PTMD effectiveness decreases as the primary structure damping ratio increases. However, these results suggest that a low structure damping ratio makes the PTMD more effective when the structure exhibits elastoplastic behavior. Therefore, the primary structure ductility and softening development affect the PTMD performance, mainly when the structure exhibits a high level of incursion into the elastoplastic behavior.

Figure 7b shows how the design value of the PTMD frequency ratio decreased when the considered degree of incursion into the elastoplastic behavior increases. However, the primary structure damping ratio slightly affects the design frequency ratio. An increment of the primary structure damping ratio decreases the optimum design frequency ratio. The PTMD optimum design damping ratio looks less sensitive to the primary structure damping ratio (Figure 7c). Similarly, the optimum design PTMD damping ratio increases and changes according to the seismic intensity design criterion. However, the optimum design PTMD damping ratio does not change with the soil’s dynamic properties used in the simulation. The PTMD design parameters for structures with damping ratios of 0.02 and 0.05 show that they are not affected by the dynamic parameters of the three soil conditions considered (reported by Sues et al. [24]).

3.2. PTMD’s effectiveness design criterion

The PTMD exhibits high effectivity (above 45%), reducing primary structure base displacement when the maximum seismic design criterion is low. However, the PTMD effectiveness values decrease (between 10% and 20%) when the maximum seismic design criterion increases (Figure 8b). The high effectiveness design criterion values still significant, especially at high values of the standard deviation of the structure displacement (high PGA values) (Figure 8b). Even when the standard

Figure 7. Optimum PTMD design parameters: effectiveness OF (a), frequency ratio (b), and damping ratio (c) versus the standard deviation of the primary structure displacement (seismic intensity criterion).
Figure 8. (a) Primary structure hysteresis cycle without PTMD to different design seismic intensities criterion. (b) PTMD's effectiveness in reducing the primary structure base displacement. (c) Primary structure hysteresis cycle with PTMD to different design seismic intensities. The primary structure parameters were $\omega_s = 10$ rad/s, and $\xi_s = 0.02$. The PTMD mass ratio design value was $\mu = 0.1$. The soil's dynamic parameters were $\omega_f = 16.9$ rad/s and $\xi_f = 0.95$ (Firm soil).

Figure 9. Optimum PTMD design parameters: effectiveness OF (a), frequency ratio (b), and damping ratio (c) versus the standard deviation of the primary structure displacement (seismic intensity criterion). For these calculations, we used the dynamic soil properties proposed by Greco and Marano [27] and Marano and Greco [28] (Table 1). The structure frequency is $\omega_s = 10$ rad/s, and the PTMD design mass ratio is $\mu = 0.1$. 
Figure 10. Optimum PTMD design parameters: frequency ratio OF (a), damping ratio (b), and effectiveness (c) versus the design mass ratio \( \mu \) (pendulum mass). (d) The optimum PTMD effectiveness design parameter values vs. the standard deviation of the structure’s displacement and the design mass ratio \( \mu \) (pendulum mass). The structure frequency is \( \omega_s = 10 \text{ rad/s} \). The soil parameters are \( \omega_f = 16.9 \text{ rad/s} \) and \( \xi_f = 0.95 \) (Firm soil).

Figure 11. (a) The displacement history of the primary structure: without and with the optimized PTMD. (b) Primary structure hysteresis loops without and with the optimized PTMD. (c) A short segment magnification of the primary structure time response.

Structure:
Steel structure \([\omega_s = 10 \text{ rad/s} ; \xi_s = 0.02]\)

Soil dynamic parameters:
Firm soil \([\omega_f = 16.9 \text{ rad/s} ; \xi_f = 0.95]\)

Bouc Wen model:
\( A = 1 ; \alpha = 1 ; \beta = \gamma = 0.25 ; n = 1 \)

PTMD design parameters:
\( f_{opt} = 0.64 \); \( \xi_{opt} = 0.168 \); Mass ratio \( \mu = 0.1 \); OF \( \text{opt} = 0.13 \)
\( \text{PGA} = 0.5 \text{ g} \)

without PTMD \quad \text{with PTMD}
deviation of the structure displacement is high, these relatively high design effectiveness values are of great relevance to reduce structural damage in structural elements.

Figure 8a shows the elastoplastic behavior of the primary structure response without the PTMD when the maximum seismic design criterion changes from low to high seismic intensity. Similarly, Figure 8c shows the primary structure response with the PTMD. Figure 8a and c display elastoplastic behavior characteristic hysteresis loop. The area enclosed by each loop is a measure of the energy dissipated due to plasticity in structural members. When the seismic intensity criterion value is close to zero, the hysteresis loops are slim, and the energy dissipated is minimum. This hysteresis loop occurs when the primary structure is loaded within its elastic range.

The high PTMD effectiveness in this elastic range justifies its frequent application to reduce vibration in the highly elastic primary structure. However, when the primary structure is loaded at high levels of inelastic behavior, the dissipated energy becomes more apparent. The hysteresis loops are significantly larger. Moreover, the loops display the characteristic pointed shape shown in Figures 8a and c (nonlinear hysteresis loops) [39]. However, the center of symmetry of the hysteresis loops does not remain centered at the origin of the coordinate axis.

We expected a reduction in the hysteresis loops area of the primary structure with the optimized PTMD (Figure 8c). However, their hysteresis loops show a shifting of the center of the loop. This shifting made it harder to observe a reduction in the loop’s areas. The presence of this area suggests that the PTMD controls the development of structural plasticity and protects the primary structure’s safety (see Figure 8c PGA = 0.05). We will study the optimized PTMD attached to an elastoplastic primary structure in a different section.

Furthermore, Figure 8c suggests a larger shift of the center of the hysteresis loop of the structure with PTMD. This shift is associated with a more significant base displacement. This shift appears to be induced by the PTMD attached to an elastoplastic primary structure. These larger base displacements could have a detrimental effect on the primary structure. However, the average standard deviation of the base displacement is reduced regarding the structure without the PTMD.

3.3. Soil’s dynamic properties effects

The effect of soil’s dynamic properties on PTMD design parameters values was studied using the soil’s dynamic properties reported by Greco and Marano [27] and Marano and Greco [28] (Table 1). Figure 9 shows the PTMD design parameters values as a function of the mean values of the standard deviation of the primary structure base displacement. Figure 9 confirms the results shown in Figure 7 for the firm and medium soil. In contrast, the PTMD design effectiveness value is around 8% when the primary structure is above the soft soil (Reference [28] in Table 1). The substantial design effectiveness reduction may be due to the frequency that characterizes the soil’s dynamic properties (4.5 rad/s). This frequency value is lower than the primary structure’s frequency of 10 rad/s the PTMD’s optimization frequency of about 3.5 rad/s. However, the soil’s damping ratio is 0.1, the lowest of all soils considered in this study. It has the highest damping capacity and removes oscillatory energy from the excitation. Therefore, the PTMD device may behave more like a dissipater than an absorber. This result suggests an interaction between the soil’s dynamic properties and the optimization parameters of the PTMD device, which should be studied further.

The design effectiveness values (Figure 9a) display a behavior opposite to other soil considered in this study. The design effectiveness value is lower in the region where the linear hysteresis loop is apparent. However, the effectiveness increases and reaches the value of 15% when the seismic intensity is between 0.3 and 0.4 g. This value remains constant and equal to 15% for higher seismic intensity where the nonlinear hysteresis loop is evident. These results suggest that PTMD performance turns up to be independent of the primary structure plasticity development and softening. The optimum design frequency ratio (Figure 9b) remains constant (around 0.35) and seismic intensity independent. Therefore, the PTMD optimum design frequency is 3.5 rad/s, and this value is around 78% of the soil frequency (4.5 rad/s). The PTMD design frequency value is around 35% of the primary structure frequency (10 rad/s). These results suggest that the PTMD remains tuned with the soil; it is independent of the primary structure of elastoplastic behavior. The PTMD optimum design damping ratio (Figure 9c) shows similar behavior to the optimum design frequency ratio (Figure 9b). The optimum design damping ratio values remain constant and equal to 0.06, well below the values obtained with the other soils studied. We recall that the soil damping ratio is 0.1.

These results show a strong interaction between the soil’s dynamic properties and the optimization parameters of the PTMD device, which should be studied further. Salvi et al. [41] reported on an optimum tuned mass damper under seismic soil-structure interaction. They concluded that the soil-structure interaction effects require a dedicated TMD tuning, specifically in the case of a soft soil-foundation system. However, these results open the way for further studies and engineering the PTMD, one tuned with the structure and the other tuned with the soft soil.

3.4. Optimum design mass ratio

The results in Figure 10 suggest that the PTMD design effectiveness improves when the mass ratio (pendulum’s mass) increases. Figure 10a shows that the optimum frequency ratio decreases when the design mass ratio increases. The optimum design frequency ratio decreases as the primary structure damping ratio increases. However, Figure 10b shows that the optimum damping ratio increase when the optimum mass ratio increases. The results in Figure 10b suggest an increment of the optimum damping ratio value when the primary structure damping ratio increases, mainly to the high value of the design mass ratio. Figure 10c shows how the design effectiveness depends on the design mass ratio (PGA = 0.8). The result shows that as the design mass ratio µ increases, the effectiveness of the optimized PTMD increases asymptotically. Thus, as the mass of the pendulum increases, the optimized PTMD exhibits better performance. However, the maximum value of the design effectiveness depends on the primary structure damping ratio value. High design effectiveness values are reaches when with a lower primary structure damping ratio (Figure 10c).

The differences between successive design effectiveness values decrease as the pendulum mass increases. Very little effectiveness is gained when the pendulum mass is greater than 10% of the structure’s mass (µ = 0.1). Similar results were reported by Hassanian and Aminafshar [19]. They reported on the optimization of PTMD in a tall building under horizontal earthquake excitation. They concluded that the mass ratio’s desirable range is between 0.04 to 0.1. Figure 10d validates the improvement of the PTMD design effectiveness when increases the mass ratio in a wide range of seismic intensities. One remarkable fact is that the design effectiveness to high seismic intensities also increases, which is relevant to avoid primary structure collapse and save a life.

3.5. PTMD performance: case of study

We chose the recorded ground motions of the Pedernales earthquake to verify the control effect of the PTMD optimized following the methodology described in this study. Therefore, the model was evaluated using time-domain simulation. The displacement history of the primary structure was calculated: without PTMD and with the optimized PTMD.
It should be kept in mind that the simulations result from substantial simplification. They shed light on understanding the PTMD performance under several dynamic conditions. It is hoped that the results will serve to guide further studies on the subject.

Declarations

Author contribution statement

Víctor J. García: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.
Edwin P. Duque: Performed the experiments; Contributed reagents, materials, analysis tools or data.
José Antonio Inaudi & Carmen O. Márquez: Analyzed and interpreted the data; Wrote the paper.
Josselyn D. Mera: Performed the experiments; Analyzed and interpreted the data.
Anita C. Ríos: Contributed reagents, materials, analysis tools or data.

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Data will be made available on request.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

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