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An algorithm for roundness evaluation of crankshaft main journal

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Abstract: The roundness of crankshaft main journal is a key parameter which affects the performance of automobile engine. When the crankshaft comprehensive measuring machine is used to measure roundness error, the result of roundness is deviated because the workpiece is eccentric installed. In this paper, an technique of eccentric error separation is proposed to separate and correct the eccentric error existing in the original measurement data. On this basis, the least square evaluation algorithm based on polar coordinate is used to evaluate the roundness error of crankshaft main journal. The result shows that this method can effectively eliminate the installation eccentricity introduced in the measurement results and has high accuracy.

1. Introduction

Crankshaft is the key part of engine, whose quality directly determines and restricts the power and economy of automobile. In the structure of crankshaft, the shape error of spindle journal and pin journal directly affects the running stability of reciprocating engine. Therefore, the accurate inspection of the shape error of crankshaft spindle journal, especially the roundness error is particularly important. At present, the measurement methods [1-9] of roundness error mainly include the contact measurement method and the non-contact measurement method. Among them, the contact relative measurement method has high precision, while the measurement range is limited, it can’t be used to measure the large axis workpiece; the non-contact measurement method has high efficiency, but there are imaging aiming error and reading error in the measurement process, the data processing process is complex, and the measurement accuracy is poor. In this paper, according to the definition of roundness error and the characteristics of the measuring machine, an eccentricity error separation technique is proposed to separate and correct the eccentricity error existing in the original measuring data, the separation principle of eccentricity error is deduced theoretically and the results were analyzed between before and after the separation of the eccentricity.
2. The measuring principle of roundness error of main journal

The roundness error of main journal refers to the variation of the profile, which reflects the character of a section that is perpendicular to the measured main journal axis, relative to the ideal circle, and it is an important index of crankshaft accuracy and assembly quality.

The crankshaft comprehensive measuring machine is composed of a rotating shaft which drives the crankshaft to rotate and two orthogonal linear shafts. The structure of the crankshaft comprehensive measuring machine is shown in Fig.1. When measuring the roundness of the spindle journal, the plate measuring probe contacts with a certain point on the outline of the measured spindle journal, and the spindle motor rotates to drive the crankshaft to rotate, thus forming the theoretical track of the surface of the measured section of the workpiece—$F' = (r', z', \theta')$. But because of the existence of workpiece errors, there is a deviation between the actual trajectory—$F = (r, z, \theta)$ and the theoretical trajectory, where we can use the deviation to calculate the roundness error of the measured section. But there is the installation eccentricity, and the random error because of the installation eccentricity is introduced into the original data so as to effect the result of the measured section, so it has practical significance of engineering to separate the installation eccentricity.

![Fig. 1 The structure diagram of crankshaft comprehensive measuring machine](image)

3. The algorithm of roundness error for spindle journal

Roundness error evaluation algorithms have always been the focus of foreign scholars [10-16]. Among all those method, Iterative method, Simplex method and Genetic algorithm are often used for optimization algorithm of roundness error, while there may be difficult and complicated to determine the center and radius. According to the definition of roundness error and the characteristics of measuring machine, a roundness error evaluation algorithm for polar coordinate measuring data based on eccentric error separation technology is proposed in this paper.
In actual measurement, the axis of the measuring machine is usually not coincided with the axis of the measured crankshaft. As shown in Fig. 2, the symbol of \( o' \) is the rotating center of the measuring machine and \( xo'y \) is the measuring coordinate system of the measuring machine. When measuring, the crankshaft rotates with the rotating platform and the rotating angle of the rotating axis is detected by the circular grating. The non-circular closed curve in Fig. 2 is the actual profile which determined by the measuring point on the surface of the crankshaft journal, and \( p_i(r_i, \theta_i) \) is the \( i \)th sampling point \((i = 1, 2, 3 \cdots n)\), while \( n \) represents the number of sampling points and \( o \) is the center of the least square circle whose coordinates and the radius respectively are \((a, b)\) and \( R \). \( r_i \) is the measuring polar diameter corresponding to the \( p_i \) point. \( e = o o' \) is the eccentricity of the center of rotation to the least square center, \( \varepsilon_i \) is the radial distance from \( p_i \) point to least squares circle, \( \phi \) is the initial phase angle and \( \theta_i \) is the angle between \( o'p_i \) and the axis of \( o'x \). The geometric relationship obtained in Fig.2 is as follow:

\[
r_i = e \cos(\theta_i - \phi) + \sqrt{(R + \varepsilon_i)^2 - e^2 \sin^2(\theta_i - \phi)} \quad (1)
\]

Where, because of \( e \ll R \) and \( |\sin^2(\theta_i - \phi)| \leq 1 \), Formula (1) can be simplified to (2):

\[
\varepsilon_i = r_i - R - e \cos(\theta_i - \phi) \quad (2)
\]

According to the principle of the least square, the formula—\( \sum_{i=1}^{n} \varepsilon_i^2 \) must be minimization for the purpose of minimizing the sum of squares of distances between the actual profile and the least square circle, that is:

\[
\sum_{i=1}^{n}[r_i - R - e \cos(\theta_i - \phi)]^2 = \text{min}F(e, \phi, \) \quad (3)
\]

The partial differential of parameters \( R, e \) and \( \phi \) in Equation (3) is obtained respectively as:

\[
\begin{align*}
\frac{\partial F}{\partial R} &= \sum_{i=1}^{n} \varepsilon_i^2 / \partial e = 0 \\
\frac{\partial F}{\partial e} &= \sum_{i=1}^{n} \varepsilon_i^2 / \partial e = 0 \\
\frac{\partial F}{\partial \phi} &= \sum_{i=1}^{n} \varepsilon_i^2 / \partial \phi = 0
\end{align*}
\]

And solving the Formula (4) can get the center coordinates and radius as:

\[
\begin{align*}
R &= \left( \frac{1}{n} \right) \sum_{i=1}^{n} r_i \\
a &= \left( \frac{2}{n} \right) \sum_{i=1}^{n} r_i \cos \theta_i \\
b &= \left( \frac{2}{n} \right) \sum_{i=1}^{n} r_i \sin \theta_i
\end{align*}
\]

According to the geometric relationship in Fig.2, we can get the equation as:

\[
e \cos(\theta_i - \phi) = a \cos \theta_i + b \sin \theta_i \quad (6)
\]

And then simplify the Formula (2) to Formula (7) as:
\[
\epsilon_i = r_i - R - a \cos \theta_i - b \sin \theta_i 
\]  
\( i = 1, 2, \ldots, n \)  

In project, the parameter of \( \Delta r_i \) is easy to get, so after making \( r_i = r_0 + \Delta r_i \) and \( \Delta R = R - r_0 \) can we get the Formula (8).

\[
\epsilon_i = \Delta r_i - \Delta R - a \cos \theta_i - b \sin \theta_i 
\]  

Where,

\[
\Delta R = \frac{1}{n} \sum_{i=1}^{n} \Delta r_i 
\]

\[
a = \frac{2}{n} \sum_{i=1}^{n} \Delta r_i \cos \theta_i 
\]

\[
b = \frac{2}{n} \sum_{i=1}^{n} \Delta r_i \sin \theta_i 
\]  

When measuring the roundness of the crankshaft spindle journal, both the values of \( \Delta r_i \) and the angle of \( \theta_i \) corresponding to a certain point \( p_i \) on the actual profile of the measured section can be obtained. Then introduce the values of \( \Delta r_i \) and the angle of \( \theta_i \) into the Formula (9), and the distance of the sampling data to the least square circle—\( \epsilon_i \) can be obtained. So the roundness error of the measured section can be calculated by the relation as:

\[
E = \epsilon_{max} - \epsilon_{min} 
\]  

Where, the symbols of \( \epsilon_{max} \) and \( \epsilon_{min} \) relatively represent the maximum and the minimum distance of the actual profile to the least square circle.

The simulation results with MATALAB show that the algorithm used in this paper is basically consistent with the expected results when the eccentricity is less than 0.01 mm, and the evaluation accuracy is higher and the evaluation results with this algorithm can reflect the actual roundness error. But when the eccentricity is larger, the algorithm error is larger.

4. The separation and correction principle of Installation eccentricity

4.1 The separation principle of eccentricity

According to the traditional processing method, the roundness error signal is a periodic signal, which can be expanded into Fourier trigonometric series. If the revolving center of the workpiece coincides with the geometric center of the workpiece, the Fourier series of the error function has only the second harmonic, but not the first harmonic, so it is easy to prove the roundness error as:

\[
\epsilon = \sum_{i=2}^{N} a_i \cos [i(\theta - \phi) - \beta_i] 
\]  

Where, \( a_i \) and \( \beta_i \) are the amplitude and initial phase of the \( i \)th harmonic component in the Fourier series respectively. Then bring (11) into (1) and express \( h(\theta) \) as a difference of the distance of the point—\( p(\theta) \) to \( R \):

\[
h(\theta) = p(\theta) - R 
\]

\[
= e \cdot \cos(\theta - \phi) + (R - \epsilon)\sqrt{1 - K^2 \sin^2(\theta - \phi)} - R 
\]  

\[
= e \cos(\theta - \phi) + \sqrt{1 - K^2 \sin^2(\theta - \phi)} \cdot \sum_{i=2}^{N} a_i \cos [i(\theta - \phi) - \beta_i] + R \sqrt{1 - K^2 \sin^2(\theta - \phi)} - R 
\]

Where, the equation of \( K^2 \ll 1 \) is satisfied because of \( K = \frac{e}{R+\epsilon} \), so make the Formula (12) into power series and take the first two terms as below:

\[
\sqrt{1 - K^2 \sin^2(\theta - \phi)} = 1 - \frac{K^2}{2} \left[ 1 - \cos 2(\theta - \phi) \right] 
\]  

\[
\frac{1}{2} K^2 \left[ 1 - \cos 2(\theta - \phi) \right] \]
And introduce (13) into (12) as:

\[
    h(\theta) = ecos(\theta - \phi) - \frac{1}{4} RK^2 [1 - \cos 2(\theta - \phi)] + \sum_{i=2}^{N} a_i \cos [i(\theta - \phi) - \beta_i] \\
    - \frac{1}{4} K^2 [1 - \cos 2(\theta - \phi)] \cdot \sum_{i=2}^{N} a_i \cos [i(\theta - \phi) - \beta_i]
\]  

(14)

Formula (14) is the original function of the roundness error measured by the probe when the revolving center of the workpiece does not coincide with the geometric center. The first two terms are caused by eccentricity—e, but the second term compared to the first are negligible. The third is caused by the true roundness error, and the fourth is caused by the interaction of eccentricity and the true roundness error. So the essence of eccentric separation is to try to find and eliminate the first in Formula (14).

After analyzing the Formula (14), only the first term is the same as the sampling frequency, and the rest is the high-order harmonic of the sampling frequency. According to the principle of co-frequency detection of the related technology, the cross-correlation function between the complex periodic function and a certain cosine signal only retains the same component as the cosine signal frequency in the complex signal. Therefore, it is easy to separate the first item by correlation analysis.

So, a standard cosine function is set up and the initial phase is zero, and the frequency is equal to the sampling frequency as:

\[
    S(\theta) = 2 \cos(\theta)
\]

(15)

The cross correlation function—\(G_{hs}(v)\) of \(h(\theta)\) and \(S(\theta)\) is:

\[
    G_{hs}(v) = \frac{1}{2\pi} \int_{0}^{2\pi} h(\theta) \cdot S(\theta + v) d\theta \\
    = \frac{1}{2\pi} \int_{0}^{2\pi} ecos(\theta - \phi) \cdot \cos(\theta + v) d\theta + \frac{1}{2\pi} \int_{0}^{2\pi} \left\{ 1 - \frac{1}{4} K^2 [1 - \cos 2(\theta - \phi)] \right\} \\
    \sum_{i=2}^{N} a_i \cos [i(\theta - \phi) - \beta_i] \cdot 2 \cos(\theta + v) d\theta
\]

(16)

According to the principle that the same frequency is correlated and the different frequencies are not correlated, the integrals of the last two terms of equation (16) are equal to 0, so:

\[
    G_{hs}(v) = \frac{1}{\pi} \int_{0}^{2\pi} ecos(\theta - \phi) \cdot \cos(\theta + v) d\theta = ecos(v + \phi)
\]

(17)

It can be seen that the amplitude of correlation function is the magnitude of eccentricity—e, and the initial phase is the initial angle of eccentricity, so the eccentricity can be separated from the original data of roundness error by correlation function method.

4.2 The correction principle of eccentricity

An eccentric compensation function is constructed by calculating the estimated values \(C\) and \(D\) of amplitude \(A\) and phase \(B\) from the correlation function of equation(17):

\[
    \Delta h(\theta) = -\hat{e} \cos(\theta - \bar{\phi})
\]

(18)

The error function after compensating eccentricity is constructed as:

\[
    y(\theta) = h(\theta) + \Delta h(\theta)
\]

(19)

In the measurement of roundness error, discrete digital signal processing is often used. According to
the principle of Fig.2, roundness error function \( h(\theta) \) is measured at equal angle intervals. Two sequences \( \{h(n)\} \) and \( \{S(n)\} \) are obtained by making discretization of \( h(\theta) \) and \( S(\theta) \) and the correlation function sequence \( \{G_{hs}(m)\} \) are obtained by fast discrete correlation algorithm, whose magnitude is the size of the eccentricity and whose serial number of \( m_p \) corresponding to the first peak indicates the direction of eccentricity, that is:

\[
\begin{align*}
\hat{\theta} &= \max \{G_{hs}(m)\} \\
\hat{\epsilon} &= \Delta \theta \cdot m_p = \frac{360^\circ}{M} \cdot m_p
\end{align*}
\]

(20)

And the eccentricity compensation sequence is:

\[
\Delta h(n) = -\hat{\epsilon} \cos \left(\frac{360^\circ}{M} \cdot n - \hat{\theta}\right), \quad n = 0,1,2,\cdots,M - 1
\]

(21)

Where, the symbol of \( M \) represents the number of sampling points. So the roundness error sequence after compensating eccentricity is:

\[
y(n) = h(n) + \Delta h(n), \quad n = 0,1,2,\cdots,M - 1
\]

(22)

5 The simulation about the algorithm of roundness error

In order to verify the correctness and feasibility of the principle of error separation, algorithm simulation is carried out in MATLAB. The standard diameter of the spindle journal is 50mm, and 150 sampling points are obtained by polar coordinate method.

Then, on the premise of ensuring the same cross-section, the workpiece is eccentrically installed at 0.05 mm and 150 sampling points are measured by polar coordinate method with the same equipment. Subset data are shown in Table 1.

| No. | Angle (degree) | radius (mm) | No. | Angle (degree) | radius (mm) |
|-----|---------------|-------------|-----|---------------|-------------|
| 1   | 0.0000        | 50.0431     | 11  | 24.1611       | 50.0530     |
| 2   | 2.4161        | 50.0483     | 12  | 26.5722       | 50.0500     |
| 3   | 4.8322        | 50.0496     | 13  | 28.9933       | 50.0487     |
| 4   | 7.2483        | 50.0493     | 14  | 31.4049       | 50.0473     |
| 5   | 9.6644        | 50.0490     | 15  | 33.8255       | 50.0502     |
| 6   | 12.0805       | 50.0500     | 16  | 36.2416       | 50.0476     |
| 7   | 14.4966       | 50.0522     | 17  | 38.6577       | 50.0451     |
| 8   | 16.9128       | 50.0444     | 18  | 41.0738       | 50.0449     |
| 9   | 19.3289       | 50.0475     | 19  | 43.4899       | 50.0443     |
| 10  | 21.7450       | 50.0445     | 20  | 45.9060       | 50.0472     |

The original data in Table 1 are processed by the least square method and the minimum area method respectively, and the error curve is shown in Figure 3. Then, according to the principle of the error separation technique mentioned above, the cross-correlation function is obtained by calculating the sample data in Table 1 and \( S(\theta) = 2 \cdot \cos(\theta) \) with the correlation discrete Fourier algorithm and that is:

\[
G_{hs}(v) = 0.05 \cos \left(\frac{\pi \cdot v}{6}\right)
\]

(23)

Therefore, the compensation sequence can be constructed according to the formula(23) as:
\[ \Delta h = -0.05 \cos \left( \frac{\pi}{7.5} - \frac{\pi}{6} \right) \]  

(24)

The simulation results after compensation are shown in Table 2.

![Simulation curve with eccentricity of 0.05mm](image)

**Fig. 3** the simulation curve with eccentricity of 0.05mm

| Method   | Eccentricity(mm) | Result  |
|----------|------------------|---------|
| Unrevised| LSC 0.05         | 0.0106  |
|          | LZC 0.05         | 0.0098  |
| Revised  | LSC 0           | 0.0102  |
|          | LZC 0           | 0.0097  |

**Table 2** The simulation results with eccentricity

The simulation results in Table 2 show that the evaluation results of roundness errors for crankshaft spindle journal based on the combination of the eccentric separation algorithm and the least square algorithm proposed in this paper are basically consistent with the expected results, and the evaluation accuracy is high. The evaluation results by this algorithm can reflect the actual roundness errors very well. However, the eccentricity of the crankshaft measuring machine is usually very small, so the new algorithm fully meets the actual requirements.

6 Conclusion

In this paper, the influence of eccentricity on the measurement of roundness error for crankshaft spindle journal is analyzed. According to the characteristics of the measurement and the theory of error separation, an algorithm based both on eccentricity separation and correction for the evaluation of roundness error is proposed. Theoretical analysis and simulation results show that the algorithm has more advantages in data processing method, and effectively avoids the influence which is introduced by the installation eccentricity on roundness evaluation results at a certain extent, thus this method improves the accuracy of the roundness evaluation of spindle journal, and the effect of installation eccentricity on roundness error evaluation of spindle journal is well solved.

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