Note on the pseudo-Nambu-Goldstone Boson of Meta-stable SUSY Violation

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Abstract: Many models of meta-stable supersymmetry (SUSY) breaking lead to a very light scalar pseudo-Nambu Goldstone boson (PNGB), $\mathcal{P}$, associated with spontaneous breakdown of a baryon number like symmetry in the hidden sector. Current particle physics data provide no useful constraints on the existence of $\mathcal{P}$. For example, the predicted decay rates for $K \to \pi + \mathcal{P}$, $b \to s + \mathcal{P}$ and and $\Upsilon \to \gamma + \mathcal{P}$ are many orders of magnitude below the present experimental bounds. We also consider astrophysical implications of the PNGB and find a significant constraint from its effect on the evolution of red giants. This constraint either rules out models with a hidden sector gauge group larger than SU(4), or requires a new intermediate scale, of order at most $10^{10}$ GeV, at which the hidden sector baryon number is explicitly broken.
1. Meta-stable SUSY breaking and PNGBs

The idea that, within the quantum field theory approximation, supersymmetry (SUSY) is broken in a meta-stable vacuum, has a very long history (for a comprehensive set of references, see e.g., [1] and [2]). It is however only rather recently, that this idea has been pursued with vigor. To a large extent, this is due to the ground-breaking paper of Intriligator, Seiberg and Shih [1], which demonstrated that a large class of vector-like SUSY gauge theories (based on classical gauge groups) possess meta-stable SUSY violating vacua. These authors also argued that field-theoretic meta-stability was more or less required by a combination of theoretical and phenomenological arguments [2].

One of the striking features of these models is that the meta-stable vacuum breaks the analog of baryon number in the hidden sector (we will call this meta-baryon number, after the meta-stable state in the hidden sector), giving rise to a Nambu-Goldstone boson. General models of this type give rise to other pseudo-Nambu-Goldstone bosons, many of which would be ruled out by experiment if they were light. This can be avoided by a variety of methods, but it is interesting that the models based on SUSY QCD with $N_F = N_C$ have only the universal penta-baryon pseudo-Nambu-Goldstone boson. For $N_F = N_C = 5$, that particle was called the penton in [3]. In this paper, we shall be more general and consider the model-independent properties of the universal pseudo-Nambu-Goldstone boson (henceforth designated by PNGB), and determine constraints on models that are imposed by current experimental bounds on light CP-odd scalars.
Figure 1: Diagrams giving rise to a dimension-five coupling of the PNGB, $\mathcal{P}$. The PNGB couples to colored hidden sector quark and squark fields, which generates an effective operator (indicated by the darkened circle) in which $\mathcal{P}$ is derivatively-coupled to gluons. The gluons couple to hadronic flavor-neutral vector currents of the Standard Model (through its couplings to the quarks $q$).

In [3], one of the present authors argued that the dominant coupling of the PNGB to Standard Model fields arises from its coupling to hidden sector quarks and squarks that transform non-trivially with respect to the ordinary color SU(3) gauge group of the Standard Model. This is generic in models of direct gauge mediation [4], in which an SU(3)×SU(2)×U(1) subgroup of the global flavor symmetry group of the hidden sector superfields is gauged and identified as the Standard Model gauge group. At low-energies, an effective operator arises in which the PNGB is derivatively-coupled to gluons, as shown in Fig. 1. This diagram yields a local interaction of the PNGB (henceforth denoted by $\mathcal{P}$) to neutral flavor hadronic currents of Standard Model, denoted below by $F^\mu$,

$$\mathcal{L}_{\text{int}} \sim \alpha_3^3 (\Lambda_{\text{ISS}}) \frac{\Delta m_q^2}{\Lambda_{\text{ISS}}^{2}} \frac{F^\mu}{\Lambda_{\text{ISS}}} \partial_\mu \mathcal{P}. \quad (1.1)$$

For example, $F^\mu$ can be any one of the neutral flavor currents, $I_3$, strangeness, charm, truth or beauty and $\Delta m_q$ is an appropriate quark mass difference. Integrating by parts, we see that the dominant coupling comes from the breaking of flavor symmetries by the charged current weak interactions. The claim in [3] was that the PNGB would be produced primarily in association with a conventional flavor changing weak decay.

A brief explanation is in order as to why the effective coupling of the PNGB involves three gluons,\(^1\) to leading order in the QCD coupling, $\alpha_3$. In typical models of the hidden sector, the theory of the hidden sector quarks and squarks is vector-like (hence the meta-baryon number symmetry is non-anomalous) and separately conserves the discrete symmetries C, P and T. The meta-baryon current, $J_{\text{MB}}^\mu$ is a vector current.

\(^1\)This result corrects the previous estimates of the coefficient of the PNGB interactions with neutral flavor currents of the Standard Model given in [3].
which can create a PNGB from the vacuum,

$$\langle 0 | J_{MB}^\mu (0) | \mathcal{P} \rangle = f_\mathcal{P} q^\mu ,$$  

(1.2)

where $q^\mu$ is the PNGB four-momentum and $f_\mathcal{P}$ is the PNGB decay constant. It follows from eq. (1.2) that $C(\mathcal{P}) = -1$ and $P(\mathcal{P}) = +1$. That is, $\mathcal{P}$ is a CP-odd, C-odd scalar. Color and C conservation then imply that the minimum number of gluons that the PNGB can couple to is three. Remarkably, the form of the P and C-conserving invariant amplitude for the coupling of a CP-odd, C-odd scalar to photons was first obtained by Dolgov and Ponomarev in 1967 [5], and subsequently rewritten as an effective Lagrangian by Dolgov [6]. Generalizing the latter result to gluons, the relevant C and P-invariant effective Lagrangian with the least number of derivatives is unique:

$$L_{\text{eff}} = \frac{g_3^3}{\Lambda_{\text{ISS}}^6} d_{abc} (D_\rho G_{a\beta})^a (D^\beta G_{\sigma\tau})^b (D^\sigma D^\tau G_{c})^c \mathcal{P} ,$$  

(1.3)

where $(D_\rho G_{a\beta})^a \equiv D_\rho^{ab} G_{b\alpha}^\beta$, etc. In eq. (1.3), $G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{abc} A_\mu^b A_\nu^c$ is the gluon field strength tensor, $A_\mu^a$ is the gluon field, $D_\mu^{ab} \equiv \delta^{ab} \partial_\mu + g f_{abc} A_\mu^c$ is the covariant derivative acting on an adjoint field, and $d_{abc} \equiv 2 \text{Tr}(\{T_a, T_b\} T_c)$ is the totally symmetric tensor of color SU(3).

In Fig. 1, the dominant contribution to the graph arises when the internal gauge boson lines are significantly off-shell and carry momenta of order $\Lambda_{\text{ISS}}$. As a result, the $\Lambda_{\text{ISS}}$ dependence in eq. (1.1) is simply a consequence of dimensional analysis. In contrast, in cases where the internal gauge bosons carry low momenta (which for gluons would lead to strong coupling), one should perform an operator product expansion first in the gauge theory coupled only to the hidden sector and then consider the effect of those pure gauge operators in the Standard Model. The leading gluonic operator is given in eq. (1.3), and the resulting effective coupling is highly suppressed for momenta small compared to $\Lambda_{\text{ISS}}$. Similarly, the decay of the PNGB into three photons is governed by eq. (1.3) with $d_{abc} = 1$, $g_3$ replaced by $e$, $D_\mu$ replaced by $\partial_\mu$, and $G_{\mu\nu}^a$ replaced by the electromagnetic field strength $F_{\mu\nu}$. Due to the $\Lambda_{\text{ISS}}^{-6}$ suppression in eq. (1.3), the decay rate of the PNGB into three photons is many orders of magnitude smaller than the inverse lifetime of the universe, and poses no significant cosmological constraint.

In this paper we will examine more closely the properties of the PNGB in order to determine whether the experimental non-observation of a light CP-odd particle can impose significant constraints on model building. In Section 2, we will estimate the mass of the PNGB. To the extent that the global flavor symmetries of the hidden-sector Lagrangian are exact, the PNGB would be an exactly massless Nambu-Goldstone boson. However, in realistic models, we expect a small violation of these flavor symmetries.
arising at energy scales much higher than $\Lambda_{\text{ISS}}$. Taking such explicit breakings into account implies that the Nambu-Goldstone boson is actually a \textit{pseudo}-Nambu-Goldstone boson with a small mass proportional to the relevant symmetry-breaking parameter.

In Sections 3 and 4, we will sharpen the estimates of the coefficients of the effective interactions of the PNGB with Standard Model particles. We will clarify the energy scales involved in determining the relevant effective operators. We also note that there is an important new process which is made possible by this interaction: the direct conversion of the hadronic PNGBs (namely, the pions and kaons) into each other, with the emission of a PNGB. Bounds on such processes have been investigated in connection with the idea of “flavor Goldstone bosons” \cite{7}. The experimental constraints on $K \to \pi \mathcal{P}$ are the strongest. Nevertheless, we demonstrate that the predicted decay rate for $K \to \pi \mathcal{P}$ is significantly below the present experimental bounds.

When the gauge bosons of Fig. 1 are SU(2)$\times$U(1) gauge bosons, then such operators generate effective flavor-diagonal and flavor non-diagonal Yukawa couplings, whose magnitudes can potentially be constrained by experimental data. For example, the former provides a mechanism for the decay $Y \to \gamma \mathcal{P}$, whereas the latter provides the dominant contribution to $K \to \pi \mathcal{P}$ and $b \to s \mathcal{P}$. In all cases, the predicted decay rates are many orders of magnitude below the corresponding experimental bounds. In contrast, astrophysical consequences of the PNGB investigated in Section 5 imply that its effective Yukawa coupling to electrons is larger than that allowed by bounds on stellar cooling. Thus, one must arrange that its mass be large enough, such that it cannot be produced in stars. We present a preliminary analysis of the model building constraints imposed by this bound.

Although our analysis was motivated by the PNGB of models of meta-stable SUSY breaking, it is actually much more general. It applies to any hidden sector theory whose characteristic energy scale is in the TeV to multi-TeV range (the scale enters only into the explicit numerical estimates), which produces a light CP-odd, C-odd spin zero PNGB. In this connection, it is worth noting that the famous Vafa-Witten result \cite{8}, that vector symmetries are not spontaneously broken in QCD-like gauge theories does \textit{not} apply to supersymmetric theories. The positivity of the Dirac determinant, crucial to the Vafa-Witten argument, is violated by the Yukawa couplings in the presence of scalar backgrounds. Thus the existence of CP-odd, C-odd PNGBs in SUSY theories might be much more general than a particular class of models. Our results can be viewed as the first step in a general analysis of the properties of such particles.

Finally, we note that another paper on hidden sector Nambu-Goldstone bosons \cite{9} has appeared recently. These authors make different assumptions about the leading coupling of the PNGB to the Standard Model, from those that are natural in models of meta-stable SUSY breaking. There is little overlap between our analysis and theirs.
2. The PNGB mass

Meta-baryon number is an anomaly free global accidental symmetry of all models based on the ISS mechanism of meta-stable SUSY breaking, and is spontaneously broken in the meta-stable vacuum. In this approximation, the PNGB is an exact Nambu-Goldstone boson. However, we expect that there are irrelevant corrections to the effective Lagrangian that explicitly break this symmetry. In a model with SU($N_C$) color, the lowest dimension gauge invariant operator carrying meta-baryon number, has dimension $N_C$. This is a chiral operator, and contributes terms of dimension $\geq N_C + 1$ to the Lagrangian. For example, in the Pentagon model, $N_C = 5$ but there is an R symmetry, under which the penta-baryon operators have R charge 0, so the lowest dimension allowed operator is dimension 7. Note that for $N_C = 2, 3$, meta-baryon number violation is relevant or marginal and there is no PNGB in the spectrum.\footnote{Actually, this depends on how the Standard Model fits into the flavor group SU($N_F$). There may be no components of the meta-baryon that are Standard Model singlets.} In this paper, we will assume $N_C \geq 4$.

We will also assume that the high energy scale associated with these operators is the unification scale, $M_U = 2 \times 10^{16}$ GeV. The reader can easily modify our results to replace this by the reduced Planck scale, $m_P = 2 \times 10^{18}$ GeV, or the apparent neutrino see-saw scale $M_{SS} \sim 5 \times 10^{14}$ GeV (which is a dangerous scale for ordinary dimension 6 baryon violating operators). The lowest gauge-invariant operator (involving fields of the electric theory) that violates meta-baryon number, which contributes to the hidden sector superpotential, is given by

$$\delta W \sim \frac{1}{\Lambda_U^{N_c - 3}} Q^{N_c},$$

where $Q$ is a hidden sector quark superfield. If the above operator is disallowed (due, say, to discrete symmetries preserved at the scale $M_U$), then one can introduce an extra singlet field $S$ and choose,

$$\delta W \sim \frac{1}{\Lambda_U^{N_c + P - 3}} Q^{N_c} S^P,$$

for some suitably chosen $P$. In either case, the PNGB, $P$, acquires a non-trivial potential due to the explicit breaking,

$$V = \Lambda_{ISS}^4 \left( \frac{\Lambda_{ISS}}{M_U} \right)^{N_c + P - 3} U(P/\Lambda_{ISS}),$$

where $U(x) = U_0 + cx^2 + \cdots$, for some constant $c \sim \mathcal{O}(1)$.\footnote{Actually, this depends on how the Standard Model fits into the flavor group SU($N_F$). There may be no components of the meta-baryon that are Standard Model singlets.}
The PNGB mass is then

\[ m_P \sim \Lambda_{\text{ISS}} \left( \frac{\Lambda_{\text{ISS}}}{M_U} \right)^{\frac{N_C + P - 3}{2}}, \]

The choice of \( M_{\text{ISS}} \) and \( M_U \) is highly model-dependent. In the framework of gauge-mediated SUSY-breaking models, one expects \( \Lambda_{\text{ISS}} \) to be in the TeV to multi-TeV range. In this work we choose:

\[ \Lambda_{\text{ISS}} \sim 2 \text{ TeV}, \]

which is an optimistic choice (most probably this scale is significantly larger). If we allow a possible range of unification masses, \( 5 \times 10^{14} \text{ GeV} \lesssim M_U \lesssim 2 \times 10^{18} \text{ GeV} \), as indicated above and consider three possible values of \( N_C + P = 4, 5, 6 \), then we find possible PNGB masses lying in the range:

\[ 6 \times 10^{-11} \text{ eV} \lesssim m_P \lesssim 4 \text{ MeV}, \]

which is a huge dynamic range of possible masses.

In [10], a version of the Pentagon model in which the PNGB accounted for both baryogenesis and dark matter requires a meta-baryon number breaking scale of order \( 10^8—10^{10} \text{ GeV} \) in place of \( M_U \). In this latter case, the corresponding PNGB mass is given by \( m_P \sim 100 \text{ MeV—100 keV} \).

3. PNGB interactions via QCD interactions

The dominant coupling of PNGBs to the Standard Model comes, at lowest order in Standard Model couplings, through the diagrams of Fig. 1. We can split this diagram, and QCD corrections to it, into pieces where the gluon lines carry momentum of order \( \Lambda_{\text{ISS}} \) and contributions from lower scales. The form of the lower scale contributions involves an operator expansion in gauge invariant pure glue operators, which is then inserted into low energy QCD. All higher order QCD corrections must be included in such contributions.

The \( P \) field is a Nambu-Goldstone boson, so the resulting effective action is a sum of contributions of the form

\[ g_3^3(\Lambda_{\text{ISS}})\partial_{\mu}P \frac{V^\mu}{\Lambda_{\text{ISS}}^{D-2}}, \]

where \( V^\mu \) is a gauge invariant, pure glue operator of dimension \( D \). In QCD, if we assume the hidden sector conserves \( P \) and \( C \), then \( D \geq 8 \) for such operators. There are also terms higher order in the \( P \) field, or with higher derivatives, that are even more suppressed.
The high momentum part of the one-loop (QCD) diagram, gives interactions of the form
\[ \alpha_3^3(\Lambda_{\text{ISS}}) \partial_\mu P \frac{F^\mu}{\Lambda_{\text{ISS}}}, \]
where \( F^\mu \) is some neutral hadronic vector current.\(^3\) The third power of the QCD coupling is a consequence of C-invariance which requires at least three gluons in Fig. 1. It is convenient to decompose \( F^\mu \) into a sum of the ordinary baryon number current, \( J_B^\mu \equiv \sum_i \bar{Q}_i \gamma_\mu Q_i \) and
\[ F^\mu \sim Q_\gamma T^a Q, \quad \text{where } \text{Tr } T^a = 0, \]

Consider first \( F^\mu \) given by eq. (3.2). The dominant momentum passing through Fig. 1 is of \( \mathcal{O}(\Lambda_{\text{ISS}}) \). Thus, we can treat the quark propagators in the mass insertion approximation. An even number of mass insertions is required. The triangle with no mass insertions vanishes, since \( \text{Tr } T^a = 0 \). Thus, the first non-vanishing result derives from the diagram with two mass insertions and yields a result proportional to \( \text{Tr}(M^2 T^a) \). The latter vanishes for degenerate quark masses; hence the result given in eq. (3.1) must be suppressed further by a factor of
\[ \frac{\Delta m_q^2}{\Lambda_{\text{ISS}}^2}, \]
where \( \Delta m_q^2 \) is an appropriate quark squared-mass difference. Thus, we confirm the estimate given in eq. (1.1). Note that in the low momentum part of the diagram, flavor violation only costs powers of \( \Delta m_q/\Lambda_{\text{QCD}} \). Nevertheless, the high dimension required by the pure glue operators makes these contributions smaller than the ones we have just estimated.

To proceed further with our analysis, we will use the fact that the couplings we have calculated are all small, and will be used only in first order perturbation theory. Thus, we can integrate by parts, and use the equations of motion of the flavor currents in the unperturbed Lagrangian to get a non-derivative coupling between the PNGB and Standard Model fermions,
\[ \mathcal{L}_{\text{int}} \sim \alpha_3^2(\Lambda_{\text{ISS}}) \frac{\Delta m_Q^2}{\Lambda_{\text{ISS}}^2} P \partial_\mu F^\mu. \]

The correct all-orders procedure is to perform a field redefinition, which has the form of a flavor gauge transformation, to eliminate the derivative coupling in favor of an

\(^3\)In order to conserve C, the hadronic current \( F^\mu \) must be a vector current and not a pseudo-vector current.
exponential coupling of the PNGB to flavor changing operators. This would supplement the couplings obtained by our method by multi-PNGB operators, which are of higher order.

Using this prescription, we see that the coupling of the PNGB to the baryon number current, which does not have a quark mass difference suppression, vanishes for low momentum PNGBs (since the baryon number current is conserved). In principle it could give rise to PNGB emission in the scattering of weak bosons, but apart from that it would correspond to emission only from virtual weak boson lines in high order weak interactions.

In contrast, the non-singlet flavor current divergences are predominantly due to first order charged current weak interactions. As an example, consider the strangeness current
\[ J^\mu_S = \bar{s}\gamma^\mu s. \]
Due to the effective \( \Delta S = \pm 1 \) four-Fermi weak interaction. Inserting this result into eq. (3.3) yields the effective interaction Lagrangian
\[ \mathcal{L}_{\text{int}} = \alpha_3^\Lambda_{\text{ISS}} \frac{m_s^2 G_F \sin \theta_c}{\Lambda_{\text{ISS}}^3} P (J^\mu_0 - J^\mu_1 + \text{h.c.}), \]

where \( m_s \) is the strange quark mass and \( J^\mu_S \) and \( J^\mu_{\pm} \) indicate the part of the hadronic weak current that changes strangeness by \( \pm 1 \) unit. To compute the contribution of Fig. 1 to the decay rate for \( K^{\pm} \rightarrow \pi^{\pm}P \), we use \( \langle 0 | \bar{u} \gamma_{\nu} (1 - \gamma_5) d | \pi^- \rangle = i f_{\pi} q_{\pi}^e \) and similarly for \( K \). This corresponds to replacing the two currents by \( J^\mu_0 - f_{\pi} \partial_{\mu}\pi^- \) and \( J^\mu_1 \rightarrow f_K \partial_{\mu} K^+ \). The invariant amplitude, evaluated in the kaon rest frame, is then
\[ \mathcal{M}(K^{\pm} \rightarrow \pi^{\pm}P) = \frac{G_F \alpha_3^\Lambda_{\text{ISS}}}{\sqrt{2}} m_s^2 \sin \theta_c f_{\pi} f_K \frac{m_K^2}{2}. \]

where we have used \( E_\pi \simeq m_K/2 \). A rough estimate of the partial width for this decay then gives
\[ \Gamma(K^{\pm} \rightarrow \pi^{\pm}P) \sim 10^{-48} \text{ GeV} \left( \frac{2 \text{ TeV}}{\Lambda_{\text{ISS}}} \right)^6. \]

We will show in Section 4 that there are other contributions to the decay rate for \( K^{\pm} \rightarrow \pi^{\pm}P \) that are significantly larger than the one computed above. These arise from effective flavor-changing Yukawa couplings that are generated by purely weak interaction effects.
4. PNGB interactions via the electroweak interactions

In Section 3, we discussed the PNGB interactions that arise due to strong QCD interactions that couple the Standard Model quark sector and the hidden sector quarks and squarks. However, diagrams such as those exhibited in Fig. 1 do not yield flavor diagonal couplings of the PNGB to the Standard Model fields in the limit of low-momentum PNGBs. As previously noted, after integration by parts, the divergence of the corresponding flavor-diagonal quark current vanishes.

However, flavor-diagonal couplings of the PNGB to quarks and leptons can be generated directly by diagrams involving electroweak gauge bosons. Two typical diagrams are shown in Fig. 2. In computing the flavor-conserving couplings, one can neglect the deviation of the diagonal CKM matrix elements from unity. Moreover, as $g_2 > g_1$, we will keep only the leading contribution that is proportional to $\alpha_3^2$ [and neglect, e.g., the contribution of the hypercharge gauge boson ($B$) exchange graph in Fig. 2(b)].

![Diagrams of electroweak contributions to the coupling of the PNGB to Standard Model fermion currents.](image)

**Figure 2:** Typical electroweak contributions to the coupling of the PNGB to Standard Model fermion currents, where $F_L \equiv (u_L, d_L)$ in the case of a left-handed quark doublet, $F_L \equiv (\nu_L, e_L)$ in the case of a left-handed lepton doublet, and $f_R$ is a right-handed quark or charged lepton singlet (with generation indices suppressed in all cases). In (a), couplings to both the singlet and triplet currents can appear.
As in our analysis of the graphs of Fig. 1, the dominant momentum in the loops are of \( \mathcal{O}(\Lambda_{\text{ISS}}) \). Hence, the resulting PNGB interaction is governed by an operator of the form:

\[
\frac{\alpha^3(\Lambda_{\text{ISS}})}{\Lambda_{\text{ISS}}} f \gamma^\mu (1 - \gamma_5) f \partial_\mu P .
\]

Integrating by parts and using the free field equations then yields the desired Yukawa coupling:

\[
\mathcal{L}_{Pf} \sim \frac{\alpha^3(\Lambda_{\text{ISS}}) m_f}{\Lambda_{\text{ISS}}} i f \gamma_5 f P . \tag{4.1}
\]

In contrast to the flavor-changing couplings that are suppressed by \( 1/\Lambda_{\text{ISS}}^3 \), the flavor-conserving couplings of \( P \) scale as one inverse power of \( \Lambda_{\text{ISS}} \) (since no mass insertions on the fermion lines are required).

As an application to the above result, we compute the decay rate for \( \Upsilon \to \gamma P \). In general, if \( V \) is a \( ^3S_1 \) quarkonium bound state of \( \bar{Q}Q \) (for a heavy quark \( Q = c \) or \( b \)), and if the \( P \bar{Q}Q \) coupling is given by \( \lambda_Q m_Q i\bar{Q} \gamma_5 Q P \), then a tree-level computation yields [11]:

\[
\frac{\Gamma(V \to \gamma P)}{\Gamma(V \to e^+e^-)} = \frac{\lambda^2 Q m^2 V}{8\pi \alpha} .
\]

According to eq. (4.1), \( \lambda_Q \sim \frac{\alpha^3}{\Lambda_{\text{ISS}}} \). Hence, using \( \text{BR}(\Upsilon \to e^+e^-) = 2\% \) [12] and \( \alpha_2 \sim 0.03 \), we find:

\[
\text{BR}(\Upsilon \to \gamma P) \sim 2 \times 10^{-15} \left( \frac{2 \text{ TeV}}{\Lambda_{\text{ISS}}} \right)^2 ,
\]

which is many orders below the experimental bound. The corresponding branching ratio for \( \psi \to \gamma P \) is about a factor of three smaller, which is again much too small to be experimentally ruled out.

Next, we examine the non-diagonal (flavor-changing) Yukawa coupling generated by the electroweak interactions. As an example, we compute the effective \( dsP \) Yukawa interaction, which arises from diagrams such as the one depicted in Fig. 3. Inserting the factors of the CKM matrix \( V \) at the two charged \( W \) vertices of the triangle, we note that if the up-type quark masses were degenerate one would produce a factor of \( V^\dagger V = 1 \), resulting in a diagonal coupling. Thus, in the case of a non-diagonal coupling, we have a GIM suppression [13]. Treating the quark masses in the mass-insertion approximation, one needs two mass insertions to obtain a flavor-changing

\[\text{In a C-conserving theory, } P \text{ is a } J^{PC} = 0^{+} - \text{ scalar. However, the Yukawa couplings of } P \text{ to Standard Model fermions generated by electroweak physics necessarily introduces C and P violation. Consequently } P \text{ behaves as a linear combination of } J^{PC} = 0^{+} \oplus 0^{-} \text{ (we neglect small CP-violating effects). As a result, } P \text{ can now couple diagonally to a fermion-antifermion pair via the pseudoscalar } \gamma_5 \text{ coupling.} \]
coupling. This yields a suppression factor of $\Delta m_{q}^{2}/\Lambda_{\text{ISS}}^{2}$. The contribution from the top quark in the loop dominates, and the resulting effective operator for the $ds\mathcal{P}$ interaction is given by:

$$\frac{\alpha^{2}}{\Lambda_{\text{ISS}}^{3}}m_{t}^{2}V_{ts}^{\ast}V_{td}^{\ast}\gamma_{\mu}(1 - \gamma_{5})s \partial_{\mu}\mathcal{P} + \text{h.c.}$$  \hspace{1cm} (4.2)$$

In order to compute the decay rate for $K \to \pi\mathcal{P}$, we employ the techniques of the low-energy chiral Lagrangian. Following [14], we identify

$$\bar{d}\gamma_{\mu}(1 - \gamma_{5})s + \text{h.c.} \longleftrightarrow -2f_{K}\partial^{\mu}K_{2}^{*} - i \left[ K^{-} \gamma_{\mu} \pi^{-} - K^{+} \gamma_{\mu} \pi^{-} + K_{1}^{0} \gamma_{\mu} \pi^{0} \right] + \cdots,$$  \hspace{1cm} (4.3)$$

where we have isolated those terms that are linear and quadratic in the pion and kaon fields. In the notation above, $K_{1}^{0} \equiv (K^{0} - \bar{K}^{0})/\sqrt{2} \simeq K_{S}^{0}$ is CP-even and $K_{2}^{0} \equiv (K^{0} + \bar{K}^{0})/\sqrt{2} \simeq K_{L}^{0}$ is CP-odd. The matrix element for $K^{\pm} \to \pi^{\pm}\mathcal{P}$ is thus proportional to $(p_{K} + p_{\pi}) \cdot p_{\mathcal{P}} = m_{K}^{2} - m_{\pi}^{2} \simeq m_{K}^{2}$, and thus we find

$$\Gamma(K^{\pm} \to \pi^{\pm}\mathcal{P}) \simeq \frac{\alpha^{2}m_{t}^{4}|V_{td}V_{ts}^{\ast}|^{2}m_{K}^{3}}{16\pi\Lambda_{\text{ISS}}^{3}} \sim 2.5 \times 10^{-30} \text{ GeV} \left(\frac{2 \text{ TeV}}{\Lambda_{\text{ISS}}}\right)^{3},$$  \hspace{1cm} (4.4)$$

where we have used $\alpha^{2} \sim 10^{-3}$ and $|V_{td}V_{ts}^{\ast}| \sim 3 \times 10^{-4}$ [12].

Remarkably, the QCD contribution to the decay rate for $K^{\pm} \to \pi^{\pm}\mathcal{P}$, obtained in Section 3, is negligible compared with the result of eq. (4.4). The $K^{\pm}$ lifetime is of order $10^{-8}$ sec., corresponding to a width of about $5 \times 10^{-16}$ GeV. Thus, the branching ratio for the rare kaon decay into pion plus PNGB is roughly

$$\text{BR}(K^{\pm} \to \pi^{\pm}\mathcal{P}) \sim 5 \times 10^{-15} \left(\frac{2 \text{ TeV}}{\Lambda_{\text{ISS}}}\right)^{6}.$$  \hspace{1cm} (4.5)$$

Likewise, we can estimate the rate for $K_{L}^{0} \to \pi^{0}\mathcal{P}$. Using eq. (4.3), we see that this process is absent in the limit of CP conservation. Including the CP-violating effects,
we can write $K_1^0 \simeq K_3^0 - \epsilon K_0^0$, where $|\epsilon| \sim 2 \times 10^{-3}$. Thus, $\Gamma(K_L^0 \to \pi^0 P)$ is suppressed by an additional a factor of $|\epsilon|^2$. Noting that the $K_L^0$ lifetime is of order $5 \times 10^{-8}$ sec., we estimate

$$BR(K_L^0 \to \pi^0 P) \sim 10^{-19} \left( \frac{2 \text{ TeV}}{\Lambda_{\text{ISS}}} \right)^6.$$  

The strongest experimental bounds on such decays [15] are for approximately massless PNGBs, and give a branching ratio below $7 \times 10^{-11}$. For MeV scale masses, the bounds of [16] are about three orders of magnitude weaker. Our predicted branching ratios are significantly below these experimental bounds. However, future experiments anticipate the possibility of detecting branching ratios for $K \to \pi + \text{ invisible}$ as small as $10^{-14}$ [17], which is approaching the estimate given in eq. (4.5).

For the related process $B^\pm \to K^\pm P$, the chiral Lagrangian technique is no longer appropriate. In this case, it is more useful to consider the inclusive partonic decay process $b \to s P$. In analogy with eq. (4.2), the effective operator for the $sb P$ interaction is also dominated by the top quark loop and is given by:

$$\frac{\alpha_3^2}{\Lambda_{\text{ISS}}^3} m_t^2 V_{ts}^* V_{tb} \bar{s} \gamma^\mu (1 - \gamma_5) b \partial_\mu P .$$

Integrating by parts and using the field equations yield the desired Yukawa coupling:

$$\mathcal{L}_{sb P} \sim \frac{\alpha_3^2 m_t^2 V_{ts}^* V_{tb} m_b}{\Lambda_{\text{ISS}}^3} (i \bar{s} \gamma_5 b P + \text{h.c.}).$$

We now compute the decay width for $b \to s P$. The effective Yukawa coupling is

$$\lambda \sim \frac{\alpha_3^2 m_t^2 V_{ts}^* V_{tb} m_b}{\Lambda_{\text{ISS}}^3} \sim 2 \times 10^{-11} \left( \frac{2 \text{ TeV}}{\Lambda_{\text{ISS}}} \right)^3 ,$$

where we have used $|V_{ts} V_{tb}^*| \sim 0.04$ and $m_b = 4.2$ GeV [12]. Thus,

$$\Gamma(b \to s P) \simeq \frac{\lambda^2 m_b}{16\pi} \sim 3 \times 10^{-23} \text{ GeV} \left( \frac{2 \text{ TeV}}{\Lambda_{\text{ISS}}} \right)^6 .$$

The $b$ lifetime is roughly $1.6 \times 10^{-12}$ sec., corresponding to a width of about $3 \times 10^{-12}$ GeV. Thus, the inclusive branching ratio for $B$ meson decay into strange mesons plus a PNGB is roughly

$$BR(b \to s P) \sim 10^{-11} \left( \frac{2 \text{ TeV}}{\Lambda_{\text{ISS}}} \right)^6 .$$

Unfortunately, such a small branching ratio is out of the reach of the next generation of super $B$ factories [18].

-12-
5. Cosmological and astrophysical effects

We have been unable to find any cosmological constraints on the PNGB. For example, if the PNGBs were the dark matter (e.g., as in [10]), the extra contributions to proton/neutron conversion due to the interactions

\[ n \rightarrow p + e^- + \bar{\nu}_e + \mathcal{P}, \]

and especially

\[ \mathcal{P} + p \rightarrow n + e^+ + \nu_e, \]

could in principle alter the results of nucleosynthesis. However, PNGB interactions decouple at a higher temperature than neutrinos and are found to be negligible.

Because the neutrino mass is nonzero, it too will have a Yukawa coupling to \( \mathcal{P} \) given by

\[ \lambda_{\nu\nu\mathcal{P}} \sim \frac{\alpha_3^3 m_\nu}{\Lambda_{\text{ISS}}} \sim 10^{-19} \left( \frac{2 \text{ TeV}}{\Lambda_{\text{ISS}}} \right). \]

This leads to a new energy loss mechanism for supernovae. Neutrinos trapped in the hot plasma can bremsstrahlung the very weakly interacting PNGBs, which transport energy out of the star. However, the coupling \( \lambda_{\nu\nu\mathcal{P}} \) is too small for this to be a significant effect.

The coupling of the PNGB to electrons in red giants is a more significant constraint, because of the marvelous limits on energy loss processes in these stars. The electron Yukawa coupling, \( \lambda_{ee\mathcal{P}} \), can be estimating from eq. (4.1). Defining the “PNGB fine-structure constant” by \( \alpha_\mathcal{P} \equiv \lambda_{ee\mathcal{P}}^2 / 4\pi \), it follows that:

\[ \alpha_\mathcal{P} \sim 10^{-23} \left( \frac{2 \text{ TeV}}{\Lambda_{\text{ISS}}} \right). \]

The actual observational bound of Raffelt and Weiss [19] for the coupling of a light pseudoscalar (e.g., the axion) to electrons is \( \alpha_a < 0.5 \times 10^{-26} \), assuming that the boson is light enough to be produced in the star (by Compton scattering or by bremsstrahlung), and assuming that it subsequently escapes. This constraint would rule out all PNGB models with \( m_\mathcal{P} \lesssim 10^4 - 10^5 \text{ eV} \), if \( \Lambda_{\text{ISS}} \lesssim 4000 \text{ TeV}. \)

Thus, models with \( N_C > 4 \) (and in particular the Pentagon model) are ruled out if the scale of meta-baryon number violation is \( M_U \). The model of [10] is safe, as the scale of penta-baryon number violation (in the intermediate range of \( 10^8 - 10^{10} \text{ GeV} \))

\[ ^5 \text{Although we have optimistically assumed that } \Lambda_{\text{ISS}} \sim 2 \text{ TeV in this paper, in realistic models of direct gauge mediation [4], } \Lambda_{\text{ISS}} \text{ is no larger than a few hundred TeV, well below the value needed to escape the Raffelt and Weiss constraint.} \]
is considerably lower than $M_U$, in which case the PNGB is too massive to be produced appreciably in the star. The biggest unmet challenge for this latter model is to find an explanation for this intermediate scale, with enough symmetry to ensure that it does not contribute to dimension 6 operators that violate ordinary baryon number. The most likely candidate for such a symmetry is a discrete remnant of baryon number\footnote{Note that this discrete symmetry does not prevent the generation of the baryon asymmetry. Indeed, in the model of \cite{[10]} electroweak baryon number violation, in combination with an asymmetry in penta-baryon number, generates the cosmological baryon asymmetry through spontaneous baryogenesis \cite{[20]}} that is sufficient to prevent proton decay (and prevent or suppress neutron-anti-neutron conversion). Electroweak instantons conserve baryon number modulo 6 (in units where the baryon number of the proton is 1).

6. Conclusions

We have investigated constraints on the PNGB of hidden sector baryon number in SUSY breaking models based on the meta-stable states of ISS. We found no current laboratory constraints, but constraints from the cooling of red giants require the PNGB mass to be such that it cannot be produced in these stars. This puts strong restrictions on the structure of the hidden sector gauge group, if the irrelevant operators that break meta-baryon number explicitly, are scaled by the unification scale. Models in which this scale is lower have to incorporate some mechanism for suppressing dimension 6 operators that violate ordinary baryon number.

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