Research Article

Research on Optimization of Customized Bus Routes Based on Uncertainty Theory

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Received 24 November 2020; Revised 28 January 2021; Accepted 17 March 2021; Published 9 April 2021

Abstract

In the optimization process of the routes of customized buses, there are numerous uncertainties in the route planning and setting. In this study, the uncertainty theory is introduced into the optimization problem of a customized bus route, and an uncertain customized bus route optimization model is established, which aims at the minimizing the total mileage of vehicle operation. An improved genetic algorithm is used to solve the model, whose feasibility is verified by a case study. The results show that the optimization model based on the uncertainty theory can yield a reasonable customized bus route optimization scheme, and the total mileage reduced from 35.6 kilometers to 32.2 kilometers. This research provides the theoretical support for the optimization of customized bus routes.

1. Introduction

Public transport priority is an important approach to alleviate traffic congestion. Customized public transport is a public transport service mode between regular public transport and taxis, which takes the form of multiperson transportation. Its objective is to customize a public transport service for people located in similar areas with similar travel times and similar travel needs [1]. Scientific and reasonable driving routes are crucial for customized bus operation. Customized reasonable and appropriate bus routes can not only save the travel costs and travel times of the passengers but also improve the economic benefits of customized bus companies, reduce carbon emissions, and protect the environment.

With the promotion of customized public transport in various cities, there is an increasing number of studies on the subject. To ensure the punctuality of customized public transport, some scholars set a time window to reach each stop and built the relevant model as constraints. Wang [2] adopted numerous customized bus routes as objects of study and considered the limitations of the passenger travel starting point and stop time window. Accordingly, the optimal vehicle scheduling scheme was formulated, and the optimal solution of the model was obtained by combining a greedy algorithm with a genetic algorithm. Li [3] established a mixed-load custom bus routing model that satisfies the requirements of the time window and used the Cplex software to solve the model. Carol Tong [4] established an optimization model that satisfies the time window and passenger needs, to optimize vehicle paths and allocate vehicles. Finally, an example analysis verified the effectiveness of the algorithm and the sensitivity under different actual operating conditions. Guo [5] built a mixed-integer programming model for custom bus route optimization problems with time window constraints. Branch-cut, genetic, and tabu search algorithms were used to solve the problem and were compared separately. The above papers all set the time window for the customized bus to arrive at each station and use it as a constraint in the establishment of the model, so as to regulate that the customized bus can arrive at the station more on time and improve the satisfaction of passengers on the customized bus.

In addition, Guo [6] established a mixed-integer programming model to describe the problem of customized bus routing and proposed the location and route of a bus stop.
Similar to the studies of Rongge Guo, there are numerous others on the design and optimization of customized bus routes. Huang [7] divided the operation of customized buses into dynamic and static stages and used two-stage optimization models to optimize the design of customized bus routes. Wang [8] proposed a solution method for real-time customized bus route optimization problems under a random user demand and employed a two-stage method to solve the model. Yan [9] proposed a comprehensive planning method to plan customized bus routes. This model can meet the needs of passengers, minimize the number of stops, and maximize the profit by the walking distance, and was empirically verified. Han [10] introduced a customized bus network planning method to balance the interests of operators, society, and passengers and discussed the influence of the customized bus model, fixed operating cost, and minimized the design of customized bus routes. Some optimization goals are to reduce the travel time of customized buses, some are to save bus company operating costs, and some are to optimize for comprehensive goals.

The relevant models and algorithms commonly used for customized public transport include bi-level programming models and genetic algorithm. For example, Lei [11] constructed a customized dynamic network dispatching model for public transportation on the Internet. The objectives of this model are to maximize the demand service rate and minimize the cost. Moreover, its constraints are factors such as maximum passenger capacity and passenger time threshold. He [12] established a bus line design model that simultaneously considers the minimum cost of responsive customized bus operators and the requirements of passenger travel reliability and comfort and used genetic algorithm to solve the model. Xue [13] considered the uncertain factor of number of passengers at a bus stop in reality, introduced the uncertainty theory to construct an uncertain bi-level programming model of bus line allocation, and solved it using MATLAB. Other scholars [14–18] analyzed customized buses from the aspects of passenger flow prediction and influencing factors. However, in real life, the time for a vehicle to arrive at each stop is not fixed, and few studies regard it as an uncertain variable. Most studies assume that a vehicle travels on a road at a predetermined speed and therefore arrives at its stop at a fixed point in time. However, this is different from the actual scenario. Simultaneously, most articles on the optimization of customized buses regard the needs of passengers as known conditions. However, after a period of actual operation, some customized buses, especially companies and schools, may no longer collect passenger demand every day or every week. Thus, passenger demand should be analyzed as an uncertain event, which is rarely mentioned in existing research.

To quantitatively analyze the abovementioned uncertain factors, Liu [19] proposed the uncertainty theory in 2007 and improved it in the following years. Accordingly, the author ensured the uncertainty theory satisfies normative axioms, duality axioms, axiomatized mathematical systems with secondary axioms, and product axioms. Subsequently, numerous scholars have conducted in-depth research on the uncertainty theory and applied it to logic, transportation, logistics, finance, and other fields. In transportation, the uncertainty theory is extensively used in logistics distribution and vehicle scheduling. For example, Huang [20] described the delivery time of a third-party logistics supplier as an uncertain variable, subsequently converted the model into an equivalent deterministic model, and designed several improved genetic algorithms to solve it. Hua [21] used the time of logistics project development activities as an uncertain variable to establish a model, designed an intelligent algorithm based on simulated annealing and conducted a logistics project as an example to illustrate the effectiveness of the model and algorithm. Zhang [22] used the quantities of available resources as uncertain variables and the minimum total cost as the objective. Accordingly, a project scheduling model with uncertain resource availability was established and subsequently solved by genetic algorithm. Jiao [23] considered stop demand and vehicle travel time as two uncertain factors and proposed a new uncertain planning model for vehicle scheduling. It can be seen from the related literature the uncertainty theory is applied in logistics distribution and location selection, vehicle scheduling, and transportation; however, it is rarely applied for customized bus optimization.

Uncertainty theory is a method of studying uncertain events in real life. It can help us build customized bus route planning models that are more in line with actual conditions. In the existing research, few studies consider both vehicle operating times and passenger demands as uncertain variables simultaneously. By the analysis of previous survey data, it is found that the arrival of vehicles and the passenger demand conform to an uncertain distribution [13]. Therefore, we consider applying the uncertainty theory to the optimization of customized bus routes and provide new concepts for the same. The structure of this paper is organized as follows: Section 2 introduces the uncertainty theory. Section 3 establishes the customized bus route optimization model that uses vehicle arrival time and passenger demand as uncertain variables. Section 4 introduces the model solution algorithms, and Section 5 presents a case study. The Section 6 summarizes the research conclusion.

2. Uncertainty Theory

The uncertainty theory was proposed and established by Liu Baoding, and numerous researchers have subsequently studied it. At present, the uncertainty theory is a branch of axiomatic mathematics. The uncertainty theory is essentially a measurement theory with four axioms: normative, duality, subadditivity, and product measurement [19].

Let $\Gamma$ be a nonempty set, and $L$ is the $\sigma$– algebra of $\Gamma$. Each element $\Lambda \in L$ is called as an event, and an aggregate function $M: L \rightarrow [0, 1]$ is called as an uncertain measure, whose properties are as follows.

Property 1: for an uncertain measure $M$, for any event $\Lambda_1 \subset \Lambda_2$, $M[\Lambda_1] \leq M[\Lambda_2]$, where $M$ is a monotonically increasing function.

Property 2: the uncertain measure, $M$, of the empty set, $\emptyset$, is zero, i.e., $M[\emptyset] = 0$. 

$\Lambda \subset \Lambda_{1}$, $M[\Lambda] = \int_{\Lambda} df(x)$.
Definition 1. For an uncertain variable $\xi$, its uncertain distribution $\Phi$ is defined as $\Phi(x) = M(\xi \leq x)$, where $x$ is any real number.

Definition 2. For an uncertain variable $\xi$, if its uncertain distribution $\Phi$ has an inverse function $\Phi^{-1}(a)$, which exists and is unique for any $a \in (0, 1)$, then $\Phi^{-1}$ is called as the inverse uncertain distribution of $\xi$.

Commonly used uncertain distributions include linear and normal uncertain distributions, which are introduced in Section 3.1.

3. Customized Bus Route Optimization Model

3.1. Problem Description. In actual operation, due to factors such as the operating speed of the vehicle, the road conditions during operation, and the difference in actual conditions, the operating time of the vehicle between the two stations is an uncertain variable. This is typically ignored when optimizing a route, so that the actual arrival time at each stop does not meet the specified time, and the punctuality of the customized buses is reduced, which leads to passenger dissatisfaction with the route optimization. In addition, due to the influence of practical factors, there will be a certain error between the number of people getting on and off at each stop during the actual operation of these customized buses and the number of passengers initially collected. Specifically, the number of passengers on and off each stop is uncertain. Based on the information provided by the passengers, the condition of uncertain passenger demand can be added to the optimization objective, to obtain better optimization results. When optimizing this type of a problem, the minimum operating cost of the public transport enterprise is generally chosen as the optimization goal, and the operating cost includes fuel consumption, salary of the driver, and vehicle wear and tear. This study directly adopts the shortest total mileage of the customized public transport vehicles as the optimization objective because a shorter mileage will reduce the operating costs, and the two of them are consistent. A shorter mileage can also shorten the travel time of passengers and improve their travel satisfaction. At the same time, shortening the mileage of vehicles will also reduce environmental pollution. Therefore, this article chooses the shortest vehicle mileage as the optimization goal.

Therefore, the examination of the problems in this study can be summarized as the optimization of the direction of a customized bus route considering the uncertain running time of the vehicle and the uncertain demand of the passengers. Moreover, the minimum total mileage of all the vehicles of the customized bus route is considered as the optimization objective.

3.2. Model Construction. First, define the symbols mentioned below. The variables and their definitions involved in the model are shown in Table 1.

Suppose there are $K$ customized buses in a certain area, and each bus can transport $Q$ passengers. Let there be a total of $N$ customized bus stops in the area. All the customized buses have different routes and pass through different stops. Suppose the $n$-th stop passed by the $k$-th car is $x_{kn}$, then the driving path of the $k$-th bus is $s_{x_{kn}} \longrightarrow x_{kn} \longrightarrow x_{k1} \longrightarrow \ldots \longrightarrow x_{k3}$. Specifically, this bus first reaches the first stop, then passes through other stops of the bus, and finally reaches the terminal stop. At the same time, we set that one stop can only be served by one customized bus, and each customized bus will be sent out from the parking lot as the starting point. When designing the route, we must include all stops.

Owing to the randomness of the actual traffic conditions, it is difficult for buses to arrive at stops accurately based on the time requirements of their passengers. Therefore, there is a certain time range fluctuation in the allowed bus pick-up times of the passengers, which is called as the time window in the paper. Let the actual time that the $k$-th bus arrives at stop $x_{k1}$ be $f_{x_{k1}}$, and the time window for the passengers to allow the buses to arrive at a stop be $[a_{x_{k1}}, b_{x_{k1}}]$. For example, let the expected departure time of the passengers be 7:10 in the morning, and the bus be allowed to arrive early or late by 5 min. Therefore, the window is $[7:05, 7:15]$, and the actual arrival time of the vehicle may be 7:13.

3.2.1. Vehicle Arrival Time. Because each stop has a time window restriction on the arrival of the customized buses, the time for each customized bus to reach the different stops needs to be discussed. Suppose the time that the $k$-th bus departs from a parking lot is $t_k$ and the running time from the parking lot to the first stop is $T_{0x_{k1}}$, then the time that the $k$-th bus arrives at the stop, $x_{k1}$, is

$$f_{x_{k1}} = t_k + T_{0x_{k1}}.$$  

(1)

In an actual operation, some customized buses may arrive at a stop outside the time window; therefore, we stipulate the following:

(1) The customized buses arrive at a stop earlier than the time specified in the time window and must wait for the earliest time allowed to the passengers before they depart

(2) The customized buses arrive at a stop within the time specified in the time window, or arrive at a stop later than the time specified in the time window, and can pick up and drop off passengers directly from the stop

Thus, the time that the $k$-th bus arrives at stop $x_{kn}$ is

$$f_{x_{kn}} = \max \left\{ f_{x_{k1}}, a_{x_{kn}} \right\} + T_{x_{kn}},$$  

(2)

Therefore, for the $k$-th vehicle, because its running time $T_{x_{kn}}$ between stops $x_{kn}$ and $x_{kn}$ is an uncertain variable,
time $f_{x_k}$ it takes to arrive at stop $x_k$ is also an uncertain variable, and the corresponding inverse uncertain distribution is

$$
\Psi^{-1}_{x_k} = t_k + \Phi^{-1}_{x_k}.
$$

The inverse uncertainty distribution corresponding to time $f_{x_k}$ of arrival at stop $x_k$ is

$$
\Psi^{-1}_{x_k} = \max \left\{ \Psi^{-1}_{x_k,1}, \Phi^{-1}_{x_k,2} \right\}.
$$

The times for each customized bus to reach different stops can be calculated according to the above formula.

### 3.2.2. Passenger Demand

In order to ensure the comfort of passengers, we must ensure that each passenger has a seat. Because of the uncertainty of the passengers getting on and off at each stop, we restrict the total number of passengers on the bus. When the $k$-th bus is at stop $x_k$, the difference between the number of passengers getting on the bus and that getting off is $h_{x_k}$. If the number of passengers in the bus when departing from stop $x_k$ is $q_{x_k}$, then

$$
q_{x_k} = h_{x_k}.
$$

The corresponding inverse uncertain distribution is

$$
\Psi^{-1}_{x_k} = \phi^{-1}_{x_k}.
$$

Similarly, the number of passengers in the bus when departing from stop $x_k$ is

$$
q_{x_k} = q_{x_{k-1}} + h_{x_k}.
$$

The corresponding inverse uncertain distribution is

$$
\Psi^{-1}_{x_k} = \Psi^{-1}_{x_{k-1}} + \phi^{-1}_{x_k}.
$$

The number of passengers in each customized bus can be calculated according to the above formula.

### 3.2.3. Model Building

Let the mileage between stops $x_{k-1}$ and $x_k$ be $l_{x_{k-1}, x_k}$, and the total mileage of the $k$-th bus be

$$
l_k = l_{x_k} + \sum_{j=1}^{n-1} l_{x_j, x_{j+1}}.
$$

Thus, the total mileage of all the vehicles is $L = \sum_{k=1}^{K} l_k$, and the objective of the optimization model is that $L$ is minimum.

To ensure the punctuality of the customized bus operation, we expect that the customer of each stop receives the service within the time window determined by the customer with a reliability of $\alpha$. Specifically, the customized bus arrives at stop $x_k$ within time window $[a_{x_k}, b_{x_k}]$ with a reliability of $\alpha$, which can be expressed as

$$
M \left\{ a_{x_k} \leq f_{x_k} \leq b_{x_k} \right\} \geq \alpha.
$$

Similarly, to ensure the comfort of the customized bus, we expect the reliability of the passengers within the rated passenger number of the vehicle to be $\beta$. Specifically, when the customized bus, $x_k$, departs from the stop, $q_{x_k}$, the number of passengers on the vehicle is less than the rated passenger number, $Q$, and this can be expressed as

$$
M \left\{ q_{x_k} \leq Q \right\} \geq \beta.
$$

In addition, each stop is only served by one customized bus, i.e., the $n$-th stop passed by the $k$-th vehicle is not the same as the $m$-th stop passed by the $j$-th vehicle, which can be expressed as

$$
x_k \neq x_j.$$

**Table 1: Definition of each symbol in model.**

| Symbol   | Definition                                                                 |
|----------|---------------------------------------------------------------------------|
| $K$      | Number of customized buses                                                |
| $Q$      | Rated passenger capacity of customized bus                                |
| $N$      | Total number of bus stops in area                                         |
| $x_k$    | $n$-th stop passed by $k$-th bus                                         |
| $f_{x_k}$| Time when $k$-th vehicle arrives at stop $x_k$                            |
| $[a_{x_k}, b_{x_k}]$ | Time window for passengers to allow bus to arrive at stop $x_k$        |
| $t_k$    | Time when the $k$-th bus left the parking lot                           |
| $T_{x_{k-1}, x_k}$ | The running time of the $k$-th bus between station $x_{k-1}$ and station $x_k$ |
| $\Psi^{-1}_{x_k}$ | The inverse uncertainty distribution corresponding to $f_{x_k}$       |
| $\Phi^{-1}_{x_k}$ | The inverse uncertainty distribution corresponding to $T_{x_{k-1}, x_k}$|
| $q_{x_k}$ | Customize number of passengers on bus when bus departs from stop $x_k$   |
| $\psi^{-1}_{x_k}$ | The inverse uncertainty distribution corresponding to $q_{x_k}$   |
| $\phi^{-1}_{x_k}$ | The inverse uncertainty distribution corresponding to $h_{x_k}$     |
| $l_{x_{k-1}, x_k}$ | Mileage between stops $x_{k-1}$ and $x_k$                           |
| $l_k$    | Total mileage of $k$-th bus                                             |
| $L$      | Total mileage of all vehicles                                            |

**Symbol Definition**

- Total mileage of all vehicles, $L = \sum_{k=1}^{K} l_k$
- Time when the inverse uncertainty distribution corresponding to $f_{x_k}$
- Time window for passengers to allow bus to arrive at stop $x_k$
- Time when the $k$-th bus left the parking lot
- The running time of the $k$-th bus between station $x_{k-1}$ and station $x_k$
- The inverse uncertainty distribution corresponding to $f_{x_k}$
- The inverse uncertainty distribution corresponding to $T_{x_{k-1}, x_k}$
- Customize number of passengers on bus when bus departs from stop $x_k$
- The inverse uncertainty distribution corresponding to $q_{x_k}$
- The inverse uncertainty distribution corresponding to $h_{x_k}$
- Mileage between stops $x_{k-1}$ and $x_k$
- Total mileage of the $k$-th bus, $l_k = l_{x_k} + \sum_{j=1}^{n-1} l_{x_j, x_{j+1}}$
Subsequently, the model expression is as follows.

\[ \min L, \]
\[ b \cdot x_n \leq \psi^{-1}_{x_n} \leq a \cdot x_n, \]
\[ \psi^{-1}_{x_n} \geq Q, \]
\[ 1 \leq n \leq N, \]
\[ x_i \neq x_j. \]  

4. Model Solution

4.1. Uncertain Distribution

In order to facilitate subsequent calculations [23], we convert the constraints expressed by the uncertain measures in the above model into the form of inverse uncertain distribution:

\[ \min L, \]
\[ \begin{align*}
M \{ a \cdot x_n \leq f \cdot x_n \leq b \cdot x_n \} & \geq \alpha, \\
M \{ \beta \cdot x_n \leq Q \} & \geq \beta, \\
1 \leq n \leq N, \\
x_i \neq x_j.
\end{align*} \]  

(13)

Among them, \( e \) and \( \sigma \) are constants and \( \sigma > 0 \), and the above distribution is referred to as \( N(e, \sigma) \). The inverse uncertain distribution of the normal uncertain distribution, \( N(e, \sigma) \), is

\[ \Phi^{-1}(\alpha') = e + \frac{\alpha'}{\sqrt{\pi}} \ln \left( \frac{\alpha'}{1 - \alpha'} \right). \]  

(18)

By giving the expected confidence, \( \alpha \) and \( \beta \), and transforming the uncertain distribution into the inverse uncertain distribution, the distribution of the uncertain variables under this confidence can be obtained.

4.2. Simulated Annealing Genetic Algorithm

At present, the solution method of this type of problem model is mainly a heuristic algorithm, and a genetic algorithm is a type of heuristic algorithm. It is a random search method that evolves from the genetic mechanism of the survival of the fittest in the biological world. It has a high calculation speed and high global optimization ability. Because genetic algorithms are prone to premature maturity, resulting in failure to obtain the optimal solutions, and considering that simulated annealing algorithms have strong local search capabilities, genetic and simulated annealing algorithms are combined. Moreover, the solution mechanism of simulated annealing algorithms is adopted, accept the difference with a certain probability, and improve the genetic algorithm. In this study, a customized bus route is used as a chromosome in the genetic algorithm. The specific steps and processes are shown in Figure 1.

Step 1: the basic data of the customized bus route optimization are input, and the control parameters are initialized.

Step 2: \( N \) chromosomes are randomly generated to form the initial population, the initialization of the population is completed, and the steps for generating the initial solution are as follows:

(i) The stop with the earliest departure time is selected among all the stops as the starting point of the first vehicle

(ii) The next stop is randomly selected from the remaining stops, and it is determined whether the time window limit is met.

(iii) If the time window limit is satisfied, determination of whether the passenger demand limit is met is continued.

(iv) If the time window restriction is not met, the site is not being regarded as the next site, and Step Ⓜ is returned.

(v) If the time window and passenger demand restrictions are met, the current stop is taken as the next stop. Steps Ⓜ-⓿ and the search for subsequent stops is continued.

(vi) If the time window limit is met but the passenger demand limit is not met, that is, the sum of the passengers on the bus and the passengers who are about to board the bus exceeds the approved passenger capacity of the customized bus. Then,
Figure 1: Flow chart of improved genetic algorithm.
Table 2: Time windows and requirements of passengers at different stops.

| Bus stop | Time window limit | Uncertain distribution of demand | Bus stop | Time window limit | Uncertain distribution of demand |
|----------|-------------------|----------------------------------|----------|-------------------|----------------------------------|
| 1        | [8:10, 8:20]      | L (6, 5)                         | 6        | [8:30, 8:40]      | L (3, 9)                         |
| 2        | [8:05, 8:15]      | L (1, 7)                         | 7        | [8:25, 8:35]      | L (5, 8)                         |
| 3        | [8:20, 8:30]      | L (7, 6)                         | 8        | [8:35, 8:45]      | L (3, 6)                         |
| 4        | [8:15, 8:25]      | L (2, 8)                         | 9        | [8:40, 8:50]      | L (4, 9)                         |
| 5        | [8:20, 8:30]      | L (4, 7)                         | 10       | [8:35, 8:45]      | L (1, 9)                         |

Table 3: Distances between stops (km).

| Stop     | Starting point | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | Destination |
|----------|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|--------------|
| Starting point | 0           | 2.2 | 2.1 | 1.9 | 3.1 | 3.4 | 5.7 | 5.4 | 3.6 | 5.2 | 6.1 | 8.5          |
| 1        | 2.2           | 0   | 2.6 | 3.1 | 1.9 | 2.2 | 2.7 | 2.5 | 1.8 | 2.1 | 7.6 |             |
| 2        | 2.1           | 2.6 | 0   | 3.3 | 2.6 | 1.7 | 2.4 | 2.7 | 2.2 | 3.1 | 1.7 | 6.4          |
| 3        | 1.9           | 3.1 | 3.3 | 0   | 2.6 | 3.3 | 3.7 | 2.5 | 1.6 | 3.4 | 2.8 | 5.7          |
| 4        | 3.1           | 1.9 | 2.6 | 2.6 | 0   | 2.1 | 1.5 | 2.4 | 0.7 | 3.1 | 2.9 | 4.9          |
| 5        | 3.4           | 2.2 | 1.7 | 3.3 | 2.1 | 0   | 2.5 | 2.8 | 2.2 | 1.2 | 3.4 | 4.8          |
| 6        | 5.7           | 2.7 | 2.4 | 3.7 | 1.5 | 2.5 | 0   | 3   | 2   | 1.9 | 2.2 | 5.2          |
| 7        | 5.4           | 3   | 2.7 | 2.5 | 2.4 | 2.8 | 3   | 0   | 1.7 | 2.2 | 2.4 | 5.1          |
| 8        | 3.6           | 2.5 | 2.2 | 1.6 | 0.7 | 2.2 | 2   | 1.7 | 0   | 2.6 | 2.1 | 4.6          |
| 9        | 5.2           | 1.8 | 3.1 | 3.4 | 3.1 | 1.2 | 1.9 | 2.2 | 2.6 | 0   | 2.8 | 3.2          |
| 10       | 6.1           | 2.1 | 1.7 | 2.8 | 2.9 | 3.4 | 2.2 | 2.4 | 2.1 | 2.8 | 0   | 2.8          |
| Destination | 8.5          | 7.6 | 6.4 | 5.7 | 4.9 | 4.8 | 5.2 | 5.1 | 4.6 | 3.2 | 2.8 | 0            |

Figure 2: Schematic of stop distances.

Table 4: Comparison of optimization schemes obtained by different algorithms.

| Comparison index | Conventional genetic algorithm | Improved genetic algorithm |
|------------------|--------------------------------|---------------------------|
| Optimization     | Customized bus 1: 0 → 1 → 3 → 6 → destination | Customized bus 1: 0 → 1 → 4 → 7 → destination |
|                  | Customized bus 2: 0 → 2 → 5 → 7 → 10 → destination | Customized bus 2: 0 → 2 → 5 → 6 → 10 → destination |
|                  | Customized bus 3: 0 → 4 → 8 → 9 → destination | Customized bus 3: 0 → 3 → 8 → 9 → destination |
| Total operating mileage (km) | 35.6 | 32.2 |
the stop will not be regarded as the next stop, and
the destination is directly reached.
(vii) After the route of a customized bus is generated,
the stop with the earliest departure time is selected
from the remaining stops as the starting point of
the second bus, and Steps ②–⑥ are repeated to
generate other customized bus routes.

Step 3: each chromosome in the i-th generation pop-
ulation is decoded, and the fitness value of each
chromosome is calculated by the simulated annealing
stretching method.

Step 4: a roulette is used to select the new populations.

Step 5: the crossover operation is performed on the
chromosomes according to crossover probability
Pc.

Step 6: the mutation operation is performed on the
chromosomes according to mutation probability
Pm.

Step 7: the simulated annealing process is conducted on
the generated chromosomes, and the replication
strategy is used based on the Metropolis criterion to
generate the next generation of population. First, the
optimal retention strategy is implemented, and then a
new individual j is randomly generated in the neigh-
borhood of chromosome i, and chromosomes i and j
compete in the next generation.

Step 8: if the convergence condition is met and the
predetermined evolution algebra, M, is reached, then
Step 9 is entered; otherwise, the selection operation is
continued to the iteration.

Step 9: the calculation is stopped, and the individuals in
the population that minimize the objective function are
output as the final calculation result.

5. Case Study

To test the practicality of the model, we conduct a simple
case analysis. The case scenario is set as the operating
scene of a shuttle bus of a company at work. The pas-
sengers take customized buses from different stops to the
same company, i.e., each customized bus starts from the
same starting point, passes through different routes, and
finally reaches the same destination. Assume that the bus
company has three customized buses and there are 12 bus
stops in the area including the starting stop and the
terminal stop. Each customized bus has a rated capacity
of 30 people. The passenger demand at the different stops
satisfies different uncertain distributions. We set the
scene as a company shuttle bus of a company in Nan-
chang, measured the distance between bus stops around
the company, and used this as the data for the calculation
example. The corresponding time window restrictions
and the uncertain distribution of the passenger demand
are listed in Table 2.

The reliability of the passengers of the bus stops in the
designated time window is 0.9. The reliability of each cus-
tomized bus during the operation does not exceed its
maximum rated capacity of 0.95. For simplicity, we assume
that the time that each bus arrives at the first stop is the
earliest time specified by the time window limit for the
passengers. We also assume that the distance between two
bus stops \(x_{kn} \) and \(x_{kn} \) is \(l_{x_{kn} x_{kn}} \), where \(k = 1, 2, 3 \) and \(n = 1, 2, 3, 4 \). Furthermore, the transit time of the customized bus
between the two stops follows a normal uncertain distri-
bution, \(T_{x_{kn} x_{kn}} \sim N(2l_{x_{kn} x_{kn}}, 1) \), where \(k = 1, 2, 3 \) and \(n = 1, 2, 3, 4 \). The distances between two stops are listed in Table 3.
A schematic of the distances between the stops is shown in
Figure 2.

We set the population size as 100, crossover probability
\(Pc = 0.25 \), mutation probability \(Pm = 0.1 \), and the maximum
number of iterations = 500. The solutions using the con-
tventional genetic algorithm and the improved genetic
algorithm are listed in Table 4. The bus driving schematic
diagram is shown in Figure 3, since the calculation content
of this example is relatively small, the calculation time of the
two algorithms is very fast, so only the calculation results are
compared.

By setting the vehicle arrival time and the passenger
demand as the uncertain factors, optimizing the customized
bus route can be closer to reality and can make the designed
route more user-friendly, and this optimization method has
been proved to be feasible by calculation. Simultaneously,
solving the calculation example with an improved genetic
algorithm can reduce the total operating mileage by 9.55%
comparing to the conventional genetic algorithm, reducing
the mileage will result in a reduction in fuel costs, depre-
ciation fees, etc., resulting in a reduction in the operating
costs of the bus company.

6. Conclusions

This study considers the uncertain factors encountered in
the operation of customized buses, analyzes the vehicle
running time and passenger demand as uncertain variables,
and introduces the uncertainty theory. Moreover, it estab-
lishes the uncertainty theory with the objective of mini-
mizing the total mileage of the running vehicles. The
customized bus route optimization model and the feasibility
of the model are demonstrated by calculation examples. The
established model comprehensively considers the two fac-
tors of vehicle punctuality and passenger capacity and can
output a scientific customized bus line design plan, which
provides theoretical support to bus operators in the design of
bus lines. Nanchang customized public transportation is in
the development stage. Correlating the research concepts
presented in this paper with the actual scenario in Nanchang
can provide new approaches for the formulation and future
development of Nanchang customized public transportation
routes, which requires further research.

In reality, there are numerous different restrictions or
objectives to be achieved in the customized bus route op-
timization problem, such as vehicle configuration and route
design for multiple models, and customized bus route op-
timization considering other uncertain factors. In addition,
further optimization of the algorithm is also an important
research direction in the future.

Data Availability

The data used to support the findings of this study are
available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was sponsored by the project of Jiangxi
Provincial Department of Education (Grant nos. 2020H0053
and GJ[190331]), the National Natural Science Foundation of
China (Grant nos. 71961006 and 71971005), and the Post-
doctoral Research Foundation of Southeast University
(Grant no. 1121000301).

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