A New Derivation of the Twist-3 Gluon Fragmentation Contribution to Polarized Hyperon Production

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Abstract

A novel method of formulating the twist-3 gluon fragmentation function contribution to hyperon polarization in the proton-proton collision is presented. The method employs a covariant gauge and takes full advantage of the Ward-Takahashi identities before performing the collinear expansion. It provides a robust way of constructing the general cross section formula, and also a clear understanding for the absence of the ghost-like terms in the twist-3 cross section in the leading order with respect to the QCD coupling constant.
1 Introduction

In our recent paper\footnote{To get a gauge- and frame-independent twist-3 cross section, the $q\bar{q}g$-type FF contribution shown in Fig. 2 of \cite{1} needs to be added to \cite{2}. Since the calculation of the contribution is straightforward, it is not considered in this paper. Readers should refer to \cite{1} for that contribution.}, we presented a formalism for calculating the twist-3 gluon fragmentation function (FF) contribution to the polarized hyperon production in the proton-proton collision:

$$p(p) + p(p') \to \Lambda^\uparrow (P_h, S_\perp) + X,$$

where $p$, $p'$ and $P_h$ are the momenta of the particles and $S_\perp$ is the transverse spin vector of final $\Lambda^\uparrow$. This contribution is diagrammatically shown in Fig. 1 and the corresponding twist-3 cross section can be calculated from the following formula: \cite{1}

$$E_h \frac{d\sigma(p, p', P_h; S_\perp)}{d^3 P_h} = \frac{1}{16\pi^2 S_E} \sum_{i,j=q,\bar{q},g} \int_0^1 \frac{dx}{x} f_i(x) \int_0^1 \frac{dx'}{x'} f_j(x')$$

$$\times \left[ \Omega^\mu_\alpha \Omega^\nu_\beta \int_0^1 dz \Tr \left[ \hat{\Gamma}^{\alpha\beta} z S_{\mu\nu}(P_h/z) - i \Omega^\mu_\alpha \Omega^\nu_\beta \Omega_{\gamma} \int_0^1 dz \Tr \left[ \hat{\Gamma}_{\gamma}^{\alpha\beta\gamma}(z) \frac{\partial S_{\mu\nu}(k)}{\partial k^\gamma} \right]_{c.l.} \right] \right]$$

$$+ \Re \left[ i \Omega^\mu_\alpha \Omega^\nu_\beta \Omega_{\gamma} \int_0^1 dz \int_z^\infty dz' \left( \frac{1}{1/z - 1/z'} \right) \Tr \left[ \hat{\Gamma}_{\gamma}^{\alpha\beta\gamma}(z') S_{Fabc}(z) \left( \frac{1}{z'} \frac{1}{z} \right) \right] \right]$$

where $f_i(x)$ ($i = q, \bar{q}, g$) is the twist-2 unpolarized quark, antiquark, and gluon distributions in the unpolarized proton with the parton’s momentum fraction $x$, $S_E = (p + p')^2$ is the center-of-mass energy squared, and $\hat{\Tr}$ indicates the sum over all spinor or Lorentz indices depending on the channels. The correlation functions $\hat{\Gamma}^{\alpha\beta}(z)$, $\hat{\Gamma}^{\alpha\beta\gamma}(z)$ and $\hat{\Gamma}_{\gamma}^{\alpha\beta\gamma}(z)$, respectively, define intrinsic, kinematical and dynamical twist-3 gluon FFs. (For the precise definition, see section 2.) $S_{\mu\nu}(k)$ and $S_{L\mu\nu\lambda}(z', z)$ are the partonic hard parts which are, at the beginning, convoluted with the Fourier transform of the hadronic matrix elements $\sim \langle 0|A_\nu^a(0)|hX\rangle \langle hX|A_\mu^a(\xi)|0 \rangle$ and $\sim \langle 0|A_\nu^a(0)|hX\rangle \langle hX|A_\mu^a(\xi)gA_\rho^c(\eta)|0 \rangle$, respectively. The symbol $\Omega^\mu_\alpha$ is defined as $\Omega^\mu_\alpha = g^\mu_\alpha - P_h^\mu w_\alpha$ with another lightlike vector $w$ satisfying $P_h \cdot w = 1$, and $|_{c.l.}$ implies the collinear limit, $k \to P_h/z$. In \cite{1}, the formula \cite{2} was applied to the process \cite{1} and the cross section was calculated in the leading order (LO) with respect to the QCD coupling constant. This completed the LO twist-3 cross section for \cite{1} together with the known results for the contribution from the twist-3 distribution function \cite{2,4} and the twist-3 quark fragmentation function \cite{5}. Since the formula \cite{2} is a very general one, it can be easily adopted for other processes such as $e^+e^- \to \Lambda^\uparrow X$ \cite{6} and $ep \to e\Lambda^\uparrow X$ \cite{7,8}, etc.

To derive the general formula \cite{2}, we applied in \cite{1} the collinear expansion to the hard parts $S_{\mu\nu}(k)$ and $S_{L\mu\nu\lambda}(k, k')$. Using the Ward-Takahashi identities for the partonic hard parts, we could eventually rewrite the twist-3 cross sections in terms of the low derivatives of the hard parts and the gauge invariant correlation functions of the gluon’s field strengths as in \cite{2}. Actual calculation, however, is extremely complicated and lengthy, and is not easy to see how the correlation functions of the gauge field $A_\mu^a$ is converted into those of the field strength $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$. Furthermore, vanishing of the ghost-like terms appearing in the Ward-Takahashi identities is essential to reach \cite{2}, which was not clearly shown in \cite{1}. Therefore an easier way of deriving \cite{2} is very useful.
Figure 1: Generic diagrams representing the twist-3 gluon FF contribution to $pp \to \Lambda^\uparrow X$ for the quark and antiquark distributions in the initial unpolarized protons. The gluon distribution functions in the initial protons also contribute. The diagrams (a), (b) and (c), respectively, correspond to $W_g^{(a)}$, $W_g^{(b)}$ and $W_g^{(c)}$ in (10).

In this paper, we present a much more robust and concise way of deriving (2). In this method, we use Ward-Takahashi identities from the outset to convert gauge fields into a part of the field strengths, which results in substantial saving in actual calculation. This procedure was once adopted for deriving the twist-3 three-gluon distribution contribution to the single spin asymmetry in $e p^\uparrow \to e DX$, where three-gluon distribution contributes as an only source for the asymmetry and appears as a "pole contribution" [9]. For the present case of the twist-3 gluon FF for (1), three types of FFs contribute as a "nonpole contribution", and hence the situation is much more complicated. Furthermore, our present method provides a clear proof for the absence of the ghost-like terms which appear in the Ward-Takahashi identities. This is crucial to guarantee the gauge invariance of the twist-3 cross section.

The remainder of the paper is organized as follows: In section 2, a brief summary of the twist-3 gluon FFs which appear in (2) is given. In section 3 and Appendix, we present a novel derivation of (2) and prove the absence of the ghost-like terms in the LO twist-3 cross section. Section 4 is devoted to a brief summary.

2 Gluon Fragmentation Functions

Here we summarize the twist-3 gluon FFs in our notation which appear in (2). The twist-3 intrinsic gluon FFs are defined as

$$\hat{\Gamma}^{\alpha\beta}(z) = \frac{1}{N^2 - 1} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda(z)} \langle [\infty w, 0] | F^{w\alpha}(0) \rangle a h(P_h, S_h) X \langle h(P_h, S_h) X | (F^{w\alpha}(\lambda w) | \infty w) \rangle a | 0 \rangle$$

$$= -g^{\alpha\beta}_\perp \hat{G}(z) - iM_h \epsilon^{P_h w\alpha \beta}(S_h \cdot w) \Delta \hat{G}(z) - iM_h \epsilon^{P_h w S_\perp \alpha w \beta} \Delta \hat{G}_3(z) + M_h \epsilon^{P_h w S_\perp \alpha w \beta} \Delta \hat{G}_3(z),$$

(3)

See also [6,10,11] for earlier references and more details about the gluon FFs.
where \( N = 3 \) is the number of colors for a quark, \(|h(P_h, S_h)\rangle\) is the spin-1/2 hyperon state with the four momentum \( P_h \) \((P_h^2 = M_h^2)\) and the spin vector \( S_h \) \((S_h^2 = -M_h^2)\), and \(|\lambda w, \infty w\rangle\) is the gauge link in the adjoint representation connecting \( \lambda w \) and \( \infty w \). For the transversely polarized baryon, we use the spin vector \( S_\perp \) normalized as \( S_\perp^2 = -1 \). In the twist-3 accuracy \( P_h \) can be regarded as lightlike. For a baryon with large momentum, \( P_h \simeq (|\vec{P}_h|, \vec{P}_h) \), another lightlike vector \( w \) is defined as \( w = 1/(2|\vec{P}_h|^2)(|\vec{P}_h|, -\vec{P}_h) \) which satisfies \( P_h \cdot w = 1 \). \( G(z) \) and \( \Delta \hat{G}(z) \) are twist-2, and \( \Delta \hat{G}_{3T}(z) \) and \( \Delta \hat{G}_{3T}(z) \) are twist-3. We also note \( \Delta \hat{G}_{3T}(z) \) is naively \( T \)-odd, contributing to hyperon polarization. Each function in (3) has a support on \( 0 < z < 1 \).

The kinematical FFs are defined from the transverse derivative of the correlation functions of the field strengths:

\[
\tilde{F}_{\hat{\partial} \gamma}^\alpha(z) = \frac{1}{N^2 - 1} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{z}{\lambda}} \langle 0 | [\infty w, 0] F^{\omega\beta}(0) | h(P_h, S_\perp) X \rangle \langle h(P_h, S_\perp) X | [\infty w, \infty w] a | 0 \rangle \tilde{\partial}^\gamma
\]

\[
= -i \frac{M_h}{2} \alpha \beta \gamma F_{h o w} S_\perp \gamma \hat{G}_T^{(1)}(z) + \frac{M_h}{2} \epsilon [P_h o w \beta S_\perp \gamma \hat{G}_T^{(1)}(z)]
- i \frac{M_h}{8} \left( \epsilon P_h o w \alpha g_\perp + \epsilon P_h o w \alpha S_\perp \gamma \right) \Delta \hat{G}_T^{(1)}(z),
\]

where each function is defined to be real and has a support on \( 0 < z < 1 \).

The dynamical FFs are defined from the lightcone correlation functions of three field strengths:

\[
\tilde{\Gamma}_{F a b c}^\alpha(\frac{1}{z_1}, \frac{1}{z_2})
\]

\[
= \sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{\lambda}} e^{-i\mu(\frac{1}{\lambda} - \frac{1}{\mu})} \langle 0 | F_{h o \gamma}^{\omega\beta}(0) | h(P_h, S_\perp) X \rangle \langle h(P_h, S_\perp) X | F_{a o \gamma}^{\omega\beta}(\lambda w) g F_e^{\omega\gamma}(\mu w) | 0 \rangle
\]

\[
= i \frac{f_{a b c}}{N} \tilde{\Gamma}_{F A}^\alpha(\frac{1}{z_1}, \frac{1}{z_2}) + d_{a b c} \frac{N}{N^2 - 4} \tilde{\Gamma}_{F S}^\alpha(\frac{1}{z_1}, \frac{1}{z_2}),
\]

where the gauge link operators are suppressed for simplicity, and \( f_{a b c} \) and \( d_{a b c} \) are the antisymmetric and symmetric structure constants of color \( \text{SU}(N) \). The dynamical FFs can be defined as the decomposition of the two correlation functions in (4) as

\[
\tilde{\Gamma}_{F A}^\alpha(\frac{1}{z_1}, \frac{1}{z_2})
\]

\[
= -i \frac{f_{a b c}}{N^2 - 1} \sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{\lambda}} e^{-i\mu(\frac{1}{\lambda} - \frac{1}{\mu})} \langle 0 | F_{h o \gamma}^{\omega\beta}(0) | h(P_h, S_\perp) X \rangle \langle h(P_h, S_\perp) X | F_{a o \gamma}^{\omega\beta}(\lambda w) g F_e^{\omega\gamma}(\mu w) | 0 \rangle
\]

\[
= -M_h \left( \tilde{N}_1 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) g_\perp^{\alpha \beta} P_h o S_\perp \beta + \tilde{N}_2 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) g_\perp^{\beta \gamma} P_h o S_\perp \alpha \right) - \tilde{N}_2 \left( \frac{1}{z_2} - \frac{1}{z_1}, \frac{1}{z_2} \right) g_\perp^{\alpha \beta} P_h o S_\perp \gamma \right),
\]
\[ \hat{\Gamma}_{FS}(\frac{1}{z_1}, \frac{1}{z_2}) = \frac{d_{abc}}{N^2 - 1} \sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\lambda \frac{1}{z_1} - i\mu \frac{1}{z_2}} \langle 0| F^\mu_{ab}(0)| h(P_h, S_\perp)X \rangle \langle h(P_h, S_\perp)X| F^\mu_{ab}(\lambda) g F^\gamma_{\rho^c}(\mu) | 0 \rangle \]

\[ = -M_h \left( \hat{O}_1 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) \right) g^{\alpha\gamma}_1 \epsilon_{\mu^n S_\perp} + \hat{O}_2 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) g^{\beta\gamma}_1 \epsilon_{\mu^n S_\perp} + \hat{O}_2 \left( \frac{1}{z_2} - \frac{1}{z_1}, \frac{1}{z_2} \right) g^{\alpha\beta}_1 \epsilon_{\mu^n S_\perp} \right). \]

Correlation functions (6) and (7), respectively, define two independent set of the complex functions \( \{ \hat{N}_1, \hat{N}_2 \} \) and \( \{ \hat{O}_1, \hat{O}_2 \} \) due to the exchange symmetry of the field strengths. Functions \( \hat{N}_1 \) and \( \hat{O}_1 \) satisfy the relations

\[ \hat{N}_1 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) = -\hat{N}_1 \left( \frac{1}{z_2}, \frac{1}{z_1}, \frac{1}{z_2} \right), \]

\[ \hat{O}_1 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) = \hat{O}_1 \left( \frac{1}{z_2} - \frac{1}{z_1}, \frac{1}{z_2} \right). \]

The real parts of these four FFs are \( T \)-even and the imaginary parts are \( T \)-odd, the latter being the sources of SSAs. \( \hat{N}_{1,2} \left( \frac{1}{z_1}, \frac{1}{z_2} \right) \) and \( \hat{O}_{1,2} \left( \frac{1}{z_1}, \frac{1}{z_2} \right) \) have a support on \( \frac{1}{z_2} > 1 \) and \( \frac{1}{z_2} > \frac{1}{z_1} > 0 \).

### 3 Twist-3 gluon fragmentation contribution to \( pp \to \Lambda^\uparrow X \)

In this section we present a robust way to derive the basic formula (2). The twist-3 gluon FF contribution to (1) can be written as

\[ E_h \frac{d\sigma(p, p', P_h; S_\perp)}{d^3 P_h} = \frac{1}{16\pi^2 S_E} \sum_{i,j=q,g,q'} \int \frac{dx}{x} f_i(x) \int \frac{dx'}{x'} f_j(x') W_g(xp, x'p', P_h/z, S_\perp), \]

where \( W_g \) represents the partonic hard scattering followed by the fragmentation of a gluon into the final \( \Lambda^\uparrow \). Figure 4 shows the generic structure of the LO diagrams for this contribution. Corresponding to Figs. 1 (a), (b), (c), \( W_g \) consists of three parts:

\[ W_g(xp, x'p', P_h; S_\perp) \equiv W_g^{(a)} + W_g^{(b)} + W_g^{(c)} \]

\[ = \int \frac{d^4 k}{(2\pi)^4} \left[ S_{ab}^{\mu\nu}(k) \hat{\Gamma}_{ab}^{\mu\nu}(k) \right] \]

\[ + \frac{1}{2} \int \frac{d^4 k'}{(2\pi)^4} \int \frac{d^4 k''}{(2\pi)^4} \left[ S_{abc}^{\mu\nu\lambda}(k, k') \hat{\Gamma}_{abc}^{\mu\nu\lambda}(k, k') + S_{Rab}^{\mu\nu\lambda}(k, k') \hat{\Gamma}_{Rabc}^{\mu\nu\lambda}(k, k') \right], \]

where \( \hat{\Gamma}_{ab}^{\mu\nu}(k), \hat{\Gamma}_{abc}^{\mu\nu\lambda}(k, k') \) and \( \hat{\Gamma}_{Rab}^{\mu\nu\lambda}(k, k') \) are the hadronic matrix elements of the gauge (gluon) fields with \( k \) and \( k' \) the four momenta of the gluons fragmenting into the final \( \Lambda \), and \( S_{ab}^{\mu\nu}(k), \)}
where shellness and the nonphysical polarization of the gluon lines entering the fragmentation matrix \( \Gamma \) indices \( a, b, c \) part of the field strength \( F \) use the Ward-Takahashi identities for the hard parts to convert some of the gluon field and procedure to extract the twist-3 effect is the collinear expansion of the hard parts with respect to Hadronic matrix elements are defined as

\[
\hat{\Gamma}_{ab}(k) = \sum_X \int d^4 \xi e^{-i k \xi} \langle 0 | A_\mu^a(0) | hX \rangle \langle hX | A_\mu^b(\xi) | 0 \rangle,
\]

(11)

\[
\hat{\Gamma}_{L \mu \nu \lambda}(k, k') = \sum_X \int d^4 \xi \int d^4 \eta e^{-i k \xi} e^{-i (k' - k) \eta} \langle 0 | A_\mu^a(0) | hX \rangle \langle hX | A_\mu^b(\xi) g A_\lambda^c(\eta) | 0 \rangle,
\]

(12)

\[
\hat{\Gamma}_{R \mu \nu \lambda}(k, k') = \sum_X \int d^4 \xi \int d^4 \eta e^{-i k \xi} e^{-i (k' - k) \eta} \langle 0 | A_\mu^a(0) g A_\lambda^c(\eta) | hX \rangle \langle hX | A_\mu^b(\xi) | 0 \rangle,
\]

(13)

where the gauge coupling \( g \) associated with the attachment of the extra gluon line to the hard part is included in \( \hat{\Gamma}_{L \mu \nu \lambda}(k, k') \) and \( \hat{\Gamma}_{R \mu \nu \lambda}(k, k') \). Therefore the hard parts \( S_{ab}^{\mu \nu}(k), S_{ab}^{\mu \nu}(k, k') \) and \( S_{R \mu \nu \lambda}(k, k') \) are of \( O(g^4) \) in the LO calculation. From hermiticity, one has \( \hat{\Gamma}_{L \mu \nu \lambda}^*(k, k') = \hat{\Gamma}_{R \mu \nu \lambda}(k', k) \) and \( S_{R \mu \nu \lambda}^*(k, k') = S_{R \mu \nu \lambda}(k, k') \), which guarantees the reality of \( W_\mu \). A standard procedure to extract the twist-3 effect is the collinear expansion of the hard parts with respect to \( k \) and \( k' \) around \( P_h \). We followed the method in [1] to get (2).

Here we present an alternative method which leads to (2) more easily. In this method we fully around (1)

\[
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\]

\[
\text{Ward-Takahashi identities for the hard part read}
\]

\[
k^\mu S_{ab}^{\mu \nu}(k) = k^\mu S_{ab}^{\mu \nu}(k) = 0,
\]

(14)

\[
(k' - k)^\lambda S_{R \mu \nu \lambda}(k, k') = \frac{-i f^{abc}}{N^2 - 1} S_{\mu \nu}(k') + G_{\mu \nu}^{abc}(k, k'),
\]

(15)

\[
k^\mu S_{L \mu \nu \lambda}(k, k') = \frac{i f^{abc}}{N^2 - 1} S_{\lambda \nu}(k') + G_{\lambda \nu}^{abc}(k - k', k'),
\]

(16)

\[
k^\mu S_{L \mu \nu \lambda}(k, k') = 0,
\]

(17)

\[
(k' - k)^\lambda S_{R \mu \nu \lambda}(k, k') = \frac{i f^{abc}}{N^2 - 1} S_{\mu \nu}(k) - \left( G_{\mu \nu}^{bac}(k', k) \right)^*,
\]

(18)

\[
k^\mu S_{R \mu \nu \lambda}(k, k') = \frac{i f^{abc}}{N^2 - 1} S_{\lambda \nu}(k) + \left( G_{\lambda \nu}^{cab}(k - k', k) \right)^*,
\]

(19)

\[
k^\mu S_{R \mu \nu \lambda}(k, k') = 0,
\]

(20)

where \( S_{\mu \nu}(k) \equiv S_{\mu \nu}^{ab}(k) \delta_{ab} \). The \( G \)-terms are the ghost-like terms which appear due to the off-shellness and the nonphysical polarization of the gluon lines entering the fragmentation matrix.
where \( F^{abc} \) contribution to \( \vec{pp} \) elements. Actual forms of those ghost-like terms for \( pp \to \Lambda^* X \) were given in Appendix A of [1] in the LO with respect to the QCD coupling. They are proportional to \( f^{abc} \) and satisfy the relation

\[
k^\mu G^{abc}_{\mu
u}(k, k') = k'^\nu G^{abc}_{\mu
u}(k, k') = 0. \tag{21}
\]

We will see that use of the relations \([14]-[21]\) from the outset brings enormous saving in the actual calculation and clearer understanding on the absence of the ghost-like terms in the LO twist-3 cross section

We first consider \( W_g^{(a)} \). The integration momentum \( k \) can be decomposed as

\[
k^\mu = (k \cdot w) P^\mu_h + \Omega^\mu_\nu k^\nu, \tag{22}
\]

where \( \Omega^\mu_\nu \equiv g^\mu_\nu - P^\mu_h w_\nu \). Inserting \( (22) \) into \( (14) \), one gets

\[
S_{F_h \nu}^{ab}(k) = -\frac{1}{k \cdot w} \Omega^\mu_\kappa k^\kappa S_{\mu\nu}^{ab}(k), \tag{23}
\]

\[
S_{\mu F_h}^{ab}(k) = \frac{1}{k \cdot w} \Omega^\nu_\tau k^\tau S_{\mu\nu}^{ab}(k). \tag{24}
\]

Then we can write

\[
S_{\mu\nu}^{ab}(k) \Gamma_{ab}^{\mu\nu}(k) = S_{\mu\nu}^{ab}(k) g^{\mu_\alpha} g^{\nu_\beta} \Gamma_{ab}^{\alpha\beta} (k) = S_{\mu\nu}^{ab}(k) (P_{\mu_h}^\nu w_\alpha + \Omega^\mu_\alpha)(P_{h}^\nu w_\beta + \Omega^\nu_\beta) \Gamma_{ab}^{\alpha\beta}(k)
\]

\[
= \frac{1}{(k \cdot w)^2} S_{\mu\nu}^{ab}(k) \Omega^\kappa_\mu \Omega^\tau_\nu (-k^\kappa w_\alpha + k \cdot w g^{\kappa}_\alpha)(-k^\tau w_\beta + k \cdot w g^{\tau}_\beta) \Gamma_{ab}^{\alpha\beta}(k), \tag{25}
\]

where we have used \( (23) \) and \( (24) \) in the last equality. Noting that one can write

\[
(-k^\kappa w_\alpha + k \cdot w g^{\kappa}_\alpha)(-k^\tau w_\beta + k \cdot w g^{\tau}_\beta) \Gamma_{ab}^{\alpha\beta}(k)
\]

\[
= \sum_X \int d^4 \xi e^{-i k^\nu (0) F^{(0)}_{\kappa\nu}(0) X} \langle hX | F^{(0) \kappa\nu}_a(\xi) | 0 \rangle \equiv \bar{\Gamma} F_{F_{ab}}(k), \tag{26}
\]

where \( F_a^{(0) \kappa\nu} \equiv \partial^\kappa A^\nu_a - \partial^\nu A^\kappa_a \) is the \( O(A) \) piece of the gluon’s field strength, one obtains

\[
\int \frac{d^4 k}{(2\pi)^4} \left[ S_{\mu\nu}^{ab}(k) \bar{\Gamma}_{ab}^{\mu\nu}(k) \right] = \int \frac{d^4 k}{(2\pi)^4} \left[ \frac{1}{(k \cdot w)^2} S_{\mu\nu}^{ab}(k) \Omega^\kappa_\mu \Omega^\tau_\nu \bar{\Gamma} F_{F_{ab}}(k) \right]. \tag{27}
\]

To extract the twist-3 contribution, we apply the collinear expansion to the RHS of \( (27) \). Writing \( k \cdot w = 1/z \), the contribution up to twist-3 from \( (27) \) can be obtained as

\[
W_g^{(a)} = \int \frac{d^4 k}{(2\pi)^4} \left[ S_{\mu\nu}^{ab}(k) \bar{\Gamma}_{ab}^{\mu\nu}(k) \right] \text{twist-3}
\]

\[
= \int \frac{d}{z} \left[ \frac{1}{z} \right] z^2 \Omega^\mu_\kappa \Omega^\nu_\tau S_{\mu\nu}(z) \bar{\Gamma}_F^{\kappa\tau}(z) + \Omega^\lambda_\rho \frac{\partial S_{\mu\nu}(k)}{\partial k^\lambda} \bar{\Gamma}_F^{\kappa\tau}(z) \right|_{k=P_h/z} \bar{\Gamma}_F^{\kappa\tau}(z), \tag{28}
\]

\[\text{Absence of the ghost term contribution to the twist-3 cross sections was discussed for the 3-gluon distribution contribution to } \bar{p}p^+ \to DX \text{ and twist-3 quark FF contribution to } ep^+ \to e\pi X.\]
where $S_{\mu\nu}(z) \equiv S_{\mu\nu}^a(P_h/z)\delta^{ab}$ and

$$
\tilde{\Gamma}^{\sigma\tau}_{\mu\nu}(z) = \frac{1}{N^2 - 1} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0|e^{(0)}_{\tau}w(0)|hX\rangle \langle hX|e^{(0)}_{\mu\nu}\lambda w(\lambda w)|0\rangle,
$$

(29)

$$
\tilde{\Gamma}^{\sigma\tau}_{\partial F}(z) = \frac{1}{N^2 - 1} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0|e^{(0)}_{\tau}w(0)|hX\rangle \langle hX|(-i)\partial^\rho e^{(0)}_{\mu\nu}\lambda w(\lambda w)|0\rangle.
$$

(30)

Equations (29) and (30) are, respectively, identified as the $O(g)$-parts of (3) and (4), and (28) represents the lowest order contribution to the first and second terms in (2).

Next we proceed to analyze $W^{(b)}_g$. Using (22) in (16), we have

$$(k \cdot w)S_{L,\mu\nu}\lambda(k,k') + \Omega^\mu_{\rho} k^\rho S_{L,\mu\nu}\lambda(k,k') = \frac{i f_{abc}}{N^2 - 1} S_{\lambda\nu}(k') + G_{\lambda\nu}(k' - k,k'),
$$

(31)

from which we obtain

$$
S_{L,\mu\nu}(k,k') = \frac{1}{k \cdot w} \left(-\Omega_{\rho}^\mu k^\rho S_{L,\mu\nu}\lambda(k,k') + \frac{i f_{abc}}{N^2 - 1} S_{\lambda\nu}(k') + G_{\lambda\nu}(k' - k,k')\right).
$$

(32)

Likewise, from (22) and (17), we have

$$
S_{L,\mu\nu}(k,k') = \frac{-1}{k' \cdot w} \Omega_{\rho}^\nu k'^\rho S_{L,\mu\nu}\lambda(k,k').
$$

(33)

As in (25), (32) and (33) can be used to rewrite integrand of $W^{(b)}_g$ as

$$
S_{L,\mu\nu}\lambda(k,k') \tilde{\Gamma}^{\sigma\tau}_{L,abc}(k,k') = S_{L,\mu\nu}\lambda(k,k') g_{\mu}^\nu g_{\sigma}^\lambda \tilde{\Gamma}^{\sigma\tau}_{L,abc}(k,k')
$$

$$
= S_{L,\mu\nu}\lambda(k,k') \left(P_{h}^\mu w_{\kappa} + \Omega_{\kappa}^\mu\right) \left(P_{h}^\nu w_{\tau} + \Omega_{\tau}^\nu\right) \left(P_{h}^\sigma w_{\sigma} + \Omega_{\sigma}^\lambda\right) \tilde{\Gamma}^{\sigma\tau}_{L,abc}(k,k')
$$

$$
= \frac{1}{k \cdot w} \frac{1}{k' \cdot w} S_{L,\mu\nu}\lambda(k,k') \Omega_{\alpha}^\mu \Omega_{\beta}^\nu \left(-k^\alpha w_{\kappa} + k \cdot wg_{\kappa}\right) \left(-k^\beta w_{\tau} + k' \cdot wg_{\tau}\right)
$$

$$
\times \left(P_{h}^\lambda w_{\sigma} + \Omega_{\sigma}^\lambda\right) \tilde{\Gamma}^{\sigma\tau}_{L,abc}(k,k')
$$

$$
+ \frac{1}{k \cdot w} \left(\frac{i f_{\alpha\beta\gamma}}{N^2 - 1} S_{\sigma\tau}(k') + G_{\sigma\tau}(k' - k,k')\right) \tilde{\Gamma}^{\sigma\tau}_{L,abc}(k,k').
$$

(34)

Similarly to (26), we have

$$
(-k^\alpha w_{\kappa} + k \cdot wg_{\kappa}) \left(-k^\beta w_{\tau} + k' \cdot wg_{\tau}\right) \tilde{\Gamma}^{\sigma\tau}_{L,abc}(k,k')
$$

$$
= \sum_X \int d^4\xi \int d^4\eta e^{-ik\xi e^{-i(k' - k)\eta}} \langle 0|e^{(0)}_{\beta\tau}(0)|hX\rangle \langle hX|e^{(0)}_{\alpha\nu}(\xi)gA_{\nu}^\sigma(\eta)|0\rangle
$$

$$
\equiv \tilde{\Gamma}^{\alpha\beta\sigma}_{L,\partial A,abc}(k,k').
$$

(35)
Using this equation, (34) can be rewritten as

\[
S_{\lambda \mu \nu \lambda}^{abc}(k, k') \hat{\Gamma}_{L \mu \nu \lambda}^{abc}(k, k') =
\]
\[
= S_{\lambda \mu \nu \lambda}^{abc}(k, k') \frac{1}{k \cdot w} \frac{1}{k' \cdot w} \Omega_\alpha^\mu \Omega_\beta^\nu \left( P^\lambda_h \hat{\Gamma}_{L \mu \nu \lambda}^{abc}(k, k') + \Omega_\alpha^\lambda \hat{\Gamma}_{L \mu \nu \lambda}^{abc}(k, k') \right) + \frac{1}{k \cdot w} \left( i f_{abc} \right) \frac{1}{N^2 - 1} S_{\sigma \tau}^{abc}(k') + G_{\sigma \tau}^{abc}(k') \hat{\Gamma}_{L \mu \nu \lambda}^{abc}(k, k').
\]

(36)

Since (36) is integrated over \( k \) and \( k' \) in (10), one can change the integration variable as \( k \rightarrow k' - k \) in the last term of (36) containing the ghost-like term. Furthermore, because of \( \hat{\Gamma}_{L \mu \nu \lambda}^{abc}(k') = \hat{\Gamma}_{L \alpha \beta \gamma}^{abc}(k') \), one can change this term as

\[
\frac{1}{k' \cdot w} G_{\sigma \tau}^{abc}(k' - k, k') = \frac{1}{k' \cdot w} \hat{\Gamma}_{L \mu \nu \lambda}^{abc}(k, k').
\]

(37)

Using this form for the last term in (36) and applying the collinear expansion to the first term in (36) up to twist-3, one obtains

\[
\left[ S_{\lambda \mu \nu \lambda}^{abc}(k, k') \hat{\Gamma}_{L \mu \nu \lambda}^{abc}(k, k') \right]_{\text{twist-3}} =
\]
\[
= z' \Omega_\alpha^\mu \Omega_\beta^\nu \left\{ S_{\lambda \mu \nu \lambda}^{abc} \left( 1, 1 \right) + \Omega_\gamma^\lambda \frac{\partial S_{\lambda \mu \nu \lambda}^{abc}(k, k')}{\partial k^\lambda} \right\}_{\text{c.l.}} + \Omega_\gamma^\lambda k' \frac{\partial S_{\lambda \mu \nu \lambda}^{abc}(k, k')}{\partial k^\lambda} \left\{ \hat{\Gamma}_{L \mu \nu \lambda}^{abc}(k, k') \right\}_{\text{c.l.}}
\]
\[
+ z' \Omega_\alpha^\mu \Omega_\beta^\nu \Omega_\sigma^\lambda \Omega_\tau^\gamma G_{\lambda \mu \nu \lambda}^{abc} \left( 1, 1 \right) \hat{\Gamma}_{L \sigma \tau \gamma}^{abc}(k, k')
\]
\[
+ z' \frac{i f_{abc}}{N^2 - 1} S_{\sigma \tau}^{abc}(k') \hat{\Gamma}_{L \mu \nu \lambda}^{abc}(k, k') + \frac{1}{1/z' - 1/z} G_{\sigma \tau}^{abc}(k, k') \hat{\Gamma}_{L \mu \nu \lambda}^{abc}(k, k'),
\]

(38)

where we have set \( k \cdot w = \frac{1}{z} \) and \( k' \cdot w = \frac{1}{z'} \). The first term of (38) can be further rewritten by the Ward-Takahashi identity (15). Collinear limit of (15) gives

\[
S_{\lambda \mu \nu \lambda}^{abc} \left( 1, 1 \right) = \frac{1}{1/z' - 1/z} \left\{ -i f_{abc} \frac{1}{N^2 - 1} S_{\mu \nu}^{abc} \left( 1 \right) + G_{\mu \nu}^{abc} \left( 1 \right) \right\},
\]

(39)

Similarly from the collinear limit of the first derivatives of (15) with respect to \( k \) and \( k' \), one
obtains
\[
\frac{\partial S_{\mu\nu\lambda}^{abc}(k, k')}{\partial k^\lambda}\bigg|_{c.l.} = \frac{1}{1/z' - 1/z} \left\{ S_{\mu\nu\lambda}^{abc} \left( \frac{1}{z'}, \frac{1}{z} \right) + \frac{\partial G_{\mu\nu}^{abc}(k, k')}{\partial k^\lambda} \bigg|_{c.l.} \right\},
\]
(40) and
\[
\frac{\partial S_{\mu\nu\lambda}^{abc}(k, k')}{\partial k^\lambda}\bigg|_{c.l.} = \frac{1}{1/z' - 1/z} \left\{ -S_{\mu\nu\lambda}^{abc} \left( \frac{1}{z}, \frac{1}{z'} \right) + \frac{\partial G_{\mu\nu}^{abc}(k, k')}{\partial k^\lambda} \bigg|_{c.l.} - i f_{abc} \frac{\partial S_{\mu\nu}(k')}{\partial k^\lambda} \bigg|_{c.l.} \right\}.
\]
(41)

In the last term in the RHS of (38), using the relation (21) and following a similar procedure as (34) and (35), one can rewrite
\[
G_{\mu\nu}^{abc}(k, k') \tilde{\Gamma}_{Labc}(k, k') = z z' G_{\mu\nu}^{abc}(k, k') \Omega_\mu^\alpha \Omega_\nu^\beta \tilde{\Gamma}^{\alpha\beta\omega}_{LFAabc}(k, k').
\]
(42)

Using this form, one sees the collinear expansion of the last term in (38) yields the identical terms as the ghost-like terms in (39), (40) and (41). This way one obtains the ghost-like terms (i.e., terms containing \(G_{\mu\nu}^{abc}\)) in (38) as
\[
\left[ S_{\mu\nu\lambda}^{abc}(k, k') \tilde{\Gamma}_{Labc}(k, k') \right]^{\text{ghost}} = 2 \Omega_\mu^\alpha \Omega_\nu^\beta \frac{z z'}{1/z' - 1/z} \left\{ G_{\mu\nu}^{abc} \left( \frac{1}{z}, \frac{1}{z'} \right) + \Omega_\lambda^\gamma \left( k^\gamma \frac{\partial G_{\mu\nu}^{abc}(k, k')}{\partial k^\lambda} \bigg|_{c.l.} \right) \right\} \tilde{\Gamma}^{\alpha\beta\omega}_{LFAabc}(k, k').
\]
(43)

Using the actual forms of the ghost-like terms given in Appendix A of [1], we will show in Appendix that (43) does not contribute to the LO twist-3 cross section. Hence we will discard (43) below.

Remaining terms in (38) can be written as
\[
\left[ S_{\mu\nu\lambda}^{abc}(k, k') \tilde{\Gamma}_{Labc}(k, k') \right]^{\text{twist-3}} = \Omega_\mu^\alpha \Omega_\nu^\beta \Omega_\lambda^\gamma \frac{1}{1/z + i \epsilon 1/z' - 1/z + i \epsilon} \left( -i f_{abc} \right) S_{\mu\nu}(z') \tilde{\Gamma}^{\alpha\beta\omega}_{LFAabc}(k, k')
\]
\[
+ \Omega_\mu^\alpha \Omega_\nu^\beta \Omega_\lambda^\gamma \frac{1}{1/z + i \epsilon 1/z' - 1/z + i \epsilon} \left( \frac{z'}{1/z'} \right) S_{\mu\nu}^{abc} \left( \frac{1}{z'}, \frac{1}{z} \right) \tilde{\Gamma}^{\alpha\beta\gamma}_{LFAabc}(k, k'
\]
\[
\times \left\{ (k - k') \tilde{\Gamma}^{\alpha\beta\omega}_{LFAabc}(k, k') + \left( \frac{1}{z'} - \frac{1}{z} \right) \tilde{\Gamma}^{\alpha\beta\gamma}_{LFAabc}(k, k') \right\}
\]
\[
+ \Omega_\mu^\alpha \Omega_\nu^\beta \Omega_\lambda^\gamma \frac{1}{1/z + i \epsilon 1/z' - 1/z + i \epsilon} \left( -i f_{abc} \right) k^\gamma \frac{\partial S_{\mu\nu}(k')}{\partial k^\lambda} \bigg|_{c.l.} \tilde{\Gamma}^{\alpha\beta\omega}_{LFAabc}(k, k')
\]
\[
+ \Omega_\mu^\alpha \Omega_\nu^\beta \frac{z'^2}{1/z + i \epsilon} \left\{ S_{\mu\nu}(z') + \Omega_\gamma^\delta \frac{\partial S_{\mu\nu}(k')}{\partial k^\lambda} \bigg|_{c.l.} \right\}
\]
\[
\times \left( -k^\alpha w_\sigma + k' \cdot w_\sigma^\alpha \right) \left( -k'^\beta w_\tau + k' \cdot w_\tau^\beta \right) \tilde{\Gamma}^{\sigma\tau\gamma}_{Labc}(k, k'),
\]
(44)
where the third term in (38) was rewritten as the last term by using (44) and subsequent collinear expansion of $S_{\mu\nu}(k')$. We have introduced $ie$ in the denominators, which gives rise to the future pointing gauge links. Integration of (44) over $k$ and $k'$ proceeds as follows. We first write for the last term of (44)

\[
(-k^\alpha w_\sigma + k' \cdot wg^\alpha_\sigma)(-k'^\beta w_\tau + k' \cdot wg^\beta_\tau)\hat{\Gamma}_{\alpha\beta}^{\mu\nu}(k, k')
\]

\[
= \int d^4\xi \int d^4\eta e^{-ik\xi} e^{-i(k' - k)\eta} \sum_X \langle 0| F_b^{(0)} w^\beta(0)|hX\rangle \times \langle hX| g(\partial^w A^w_\alpha(\xi)) A^\alpha_\beta(\eta) - g(\partial^w A^w_\alpha(\xi')) A^w_\beta(\eta) + g A^w_\alpha(\xi) F^{(0)} w^\alpha(\eta) + g F^{(0)} w^\alpha(\xi) A^w_\beta(\eta)|0\rangle
\]

\[
\equiv \hat{\Gamma}_{\alpha\beta}^{\mu\nu}(k, k').
\]

Then the contribution from the first term in \{ \} of the last term of (44) reads

\[
\int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \Omega^\mu_{\alpha\beta} \Omega^\nu_{\alpha\beta} \frac{z'^2}{1/z + i N^2 - 1} i f_{abc} S_{\mu\nu}(z') \hat{\Gamma}_{\alpha\beta}^{\mu\nu}(k, k')
\]

\[
= \int d \left( \frac{1}{z} \right) \int d \left( \frac{1}{z'} \right) \Omega^\mu_{\alpha\beta} \Omega^\nu_{\alpha\beta} \frac{z'^2}{1/z + i N^2 - 1} i f_{abc} \int d\lambda \int d\mu \frac{1}{2\pi} e^{-i\lambda/z} e^{-i\mu(1/z' - 1/z)}
\]

\[
\times \sum_X \langle 0| F_b^{(0)} w^\beta(0)|hX\rangle \langle hX| g A^w_\alpha(\lambda w) A^\alpha_\beta(\mu w) - g A^w_\alpha(\lambda w) A^w_\beta(\mu w)|0\rangle
\]

\[
+ \int d \left( \frac{1}{z} \right) \int d \left( \frac{1}{z'} \right) \Omega^\mu_{\alpha\beta} \Omega^\nu_{\alpha\beta} \frac{z'^2}{1/z + i N^2 - 1} i f_{abc} \int d\lambda \int d\mu \frac{1}{2\pi} e^{-i\lambda/z} e^{-i\mu(1/z' - 1/z)}
\]

\[
\times \sum_X \langle 0| F_b^{(0)} w^\beta(0)|hX\rangle \langle hX| g A^w_\alpha(\lambda w) F^{(0)} w^\alpha(\mu w) + g F^{(0)} w^\alpha(\lambda w) A^w_\beta(\mu w)|0\rangle,
\]

where, in the first term, we have performed integration by parts for $\lambda$-integration, which kills the factor $\frac{1}{1/z + i\epsilon}$. Integration over $1/z$ of this equation can be done immediately. The second term in \{ \} of the last term of (44) can be integrated parallelly. Following this procedure, $W^{(b)}_g$ is obtained by the integral of (44) as

\[
W^{(b)}_g = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \left[ S_{\gamma\delta\lambda}^{\mu\nu}(k, k') \hat{\Gamma}_{\mu\nu\lambda}^{\gamma\delta}(k, k') \right]^{\text{twist-3}}
\]

\[
= -\Omega^\mu_{\alpha\beta} \Omega^\nu_{\alpha\beta} \int d \left( \frac{1}{z} \right) f_{abc} z'^2 S_{\mu\nu}(z')
\]

\[
\times \frac{1}{N^2 - 1} \sum_X \int d\lambda \frac{1}{2\pi} e^{-i\lambda/z'} \langle 0| F_b^{(0)} w^\beta(0)|hX\rangle \langle hX| g A^w_\alpha(\lambda w) A^\alpha_\beta(\lambda w)|0\rangle
\]

\[
+ \Omega^\mu_{\alpha\beta} \Omega^\nu_{\alpha\beta} \int d \left( \frac{1}{z} \right) f_{abc} z'^2 S_{\mu\nu}(z')
\]

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where

\[ \hat{\Gamma}_{\mu}^{\alpha\beta\gamma} \left( \frac{1}{z}, \frac{1}{z'} \right) = \sum \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\lambda/z} e^{-i\mu(1/z' - 1/z)} \langle 0 | F_b^{(0) w\beta}(0) | hX \rangle \langle hX | F_a^{(0) w\alpha}(\lambda w) gF_c^{(0) w\gamma}(\mu w) | 0 \rangle. \]  

(48)

The first four terms in (47) come from the combination of the first, third and the last terms in (44), while the last term in (47) is from the second term of (44). \( \hat{\Gamma}_{\mu}^{\alpha\beta\gamma} \left( \frac{1}{z}, \frac{1}{z'} \right) \) can be identified as the lowest order part of the dynamical FF \( \hat{\Gamma}_{ab} \).

Calculation of \( W^{(c)}_g \) can be performed in the same way. The result reads

\[ W^{(c)}_g = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \left[ S_{\mu\nu\lambda}(k, k') \tilde{F}_{\mu\nu\lambda}^{\mu\nu\lambda}(k, k') \right]_{\text{twist} - 3} \]

\[ = \Omega_{\alpha}^{\mu} \Omega_{\beta}^{\nu} \int d \left( \frac{1}{z} \right) f^{abc} z^2 S_{\mu\nu}(z) \]

\[ \times \frac{1}{N^2 - 1} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | A_b^{w}(0) gA_c^{\beta}(0) | hX \rangle \langle hX | F_a^{(0) w\alpha}(\lambda w) | 0 \rangle \]

\[ + \Omega_{\alpha}^{\mu} \Omega_{\beta}^{\nu} \sum_X \int d \left( \frac{1}{z} \right) f^{abc} z^2 S_{\mu\nu}(z) \]

\[ \times \frac{1}{N^2 - 1} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | \int_0^\infty d\mu A_c^{(\mu w)}(\mu w) F_b^{(0) w\beta}(0) | hX \rangle \langle hX | gF_c^{(0) w\alpha}(\lambda w) | 0 \rangle \]

\[ \times \frac{1}{N^2 - 1} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | \int_0^\infty d\mu A_c^{(\mu w)}(\mu w) F_b^{(0) w\beta}(0) | hX \rangle \langle hX | gF_c^{(0) w\alpha}(\lambda w) | 0 \rangle \]
\[-\Omega^\mu \Sigma^\nu \Sigma^\lambda \sum \int d \left( \frac{1}{z} \right) z^2 i f^{abc} \frac{\partial S_{\mu \nu}}{\partial k} \Bigr|_{\text{e.l.}} \times \frac{1}{N^2 - 1} \sum \int \frac{d \lambda}{2\pi} e^{-i \lambda/z} \langle 0 | A^w_b(0) g A^\beta_c(0) | hX \rangle \langle hX | \partial^\gamma F^{(0)w\alpha}(\lambda w) | 0 \rangle \]

\[-\Omega^\mu \Sigma^\nu \Sigma^\lambda \int d \left( \frac{1}{z} \right) z^2 i f^{abc} \frac{\partial S_{\mu \nu}}{\partial k} \Bigr|_{\text{e.l.}} \times \frac{1}{N^2 - 1} \sum \int \frac{d \lambda}{2\pi} e^{-i \lambda/z} \langle 0 | d \mu g A^w_c(\mu w) F^{(0)w\beta}_b(0) | hX \rangle \langle hX | \partial^\gamma F^{(0)w\alpha}(\lambda w) | 0 \rangle \]

\[-\frac{i}{2} \Omega^\mu \Omega^\nu \Omega^\lambda \int \frac{\infty}{1} d \left( \frac{1}{z} \right) \int \frac{1/z'}{1/z' - 1/z} \sum \int \frac{d \mu}{2\pi} \frac{e^{-i \mu/z} e^{-i(1/z'-1/z)}}{d \lambda} e^{-i \mu/z} \langle 0 | F^{(0)w\beta}_b(0) g F^{(0)w\alpha}(\mu w) | hX \rangle \langle hX | F^{(0)w\alpha}(\lambda w) | 0 \rangle. \]

where

\[\hat{\Gamma}_{RF^{abc}}^{\alpha\beta\gamma} \left( \frac{1}{z}, \frac{1}{z'} \right) = \sum \int \frac{d \lambda}{2\pi} \int \frac{d \mu}{2\pi} e^{-i \lambda/z} e^{-i \mu/z'} \langle 0 | F^{(0)w\beta}_b(0) g F^{(0)w\alpha}(\mu w) | hX \rangle \langle hX | F^{(0)w\alpha}(\lambda w) | 0 \rangle. \]

We can now compare the sum of \( W_g^{(b)} \) [47] and \( W_g^{(c)} \) [49] with [2]. We first remind that the gluon’s field strength is \( F^{\mu\nu}_a = \partial^\mu A^\nu - \partial^\nu A^\mu + g f^{abc} A^\alpha_c A^\beta_b = F^{(0)\mu\nu}_a + g f^{abc} A^\alpha_c A^\beta_b \). The first terms of [47] and [49] are the \( O(g) \)-contribution to [2] from the \( O(A^2) \)-term of the field strength in the intrinsic FF [1]. The second terms of [47] and [49] are the \( O(g) \)-contribution to [2] from the gauge links in the intrinsic FF. The third terms of [47] and [49] are the \( O(g) \)-contribution to [2] from the \( O(A^3) \)-term of the field strength in the kinematical FF [4]. The fourth terms of [47] and [49] are the \( O(g) \)-contribution to [2] from the gauge links in the kinematical FF. To identify the fifth terms of [47] and [49], we note the following relations:

\[\hat{\Gamma}_{RF^{abc}}^{\alpha\beta\gamma} \left( \frac{1}{z}, \frac{1}{z'} \right) = \hat{\Gamma}_{LF^{bac}}^{\beta\gamma \alpha} \left( \frac{1}{z'}, \frac{1}{z} \right)^*, \]

\[S^{abc}_{RF\alpha\beta\gamma} \left( \frac{1}{z}, \frac{1}{z'} \right) = S^{bac}_{LF\beta\alpha\gamma} \left( \frac{1}{z'}, \frac{1}{z} \right)^*. \]

From these relations, the sum of the last terms in [47] and [49] are the \( O(g) \) contribution to [2] from the dynamical FF [5]. This way, the basic formula [2] has been proved in the leading order with respect to the QCD coupling constant.

The method used here for the twist-3 gluon FF contribution to \( pp \to \Lambda^\uparrow X \) can also be applied to the twist-3 gluon distribution function contribution to the double-spin asymmetry in \( \bar{p}p^\uparrow \to DX \), which occurs as a nonpole contribution [12]. Our method provides a clearer understanding for the absence of the ghost-like terms in the corresponding LO twist-3 cross section.
Summary

In this paper we presented a new derivation of the basic formula (2) for the twist-3 gluon FF contribution to \( pp \to \Lambda^+ X \). Our method uses the Ward-Takahashi identities for the partonic hard parts from the outset before performing the collinear expansion. This method provides a robust shortcut to convert the correlation functions of the gauge (gluon) fields into the gauge invariant correlation functions for the gluon’s field strengths. Furthermore it provides a clear understanding that the ghost-like terms appearing in the Ward-Takahashi identities do not contribute to the LO twist-3 cross section. Since this method is quite general, it will become a useful tool to extend the formula in the next-to-leading order calculation.

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Appendix: Absence of the ghost-like contribution at LO twist-3

Here we show that the ghost-like term (43) does not contribute to the twist-3 cross section. Actual forms of \( G_{\mu
u}^{abc}(k,k') \) are given in (A7) for the \( q\bar{q} \to gg \) channel, (A9) for the \( qg \to gq \) channel, and (A10)-(A14) for the \( gg \to gg \) channel in [1]. In all channels they take the structure

\[
G_{\mu
u}^{abc}(k,k') = (k^2 g_{\mu\rho} - k_\mu k_\rho) f_{L\rho\lambda}^{abc} \tilde{G}_\lambda^{\mu\nu}(k,k'),
\]

where \( \tilde{G}_\nu^{\rho}(k,k') \) is some function of \( k \) and \( k' \) (and \( xp \) and \( x'p' \)). Inserting this form into (43), one obtains

\[
\left[ S_{L\mu\nu\lambda}^{abc}(k,k') \tilde{G}_\nu^{\mu\lambda}(k,k') \right]_{\text{ghost}}
\]

\[
= 2\Omega_\alpha^\mu \Omega_\beta^\nu \frac{f_{L\rho\gamma}^{abc}(k,k')}{1/z' - 1/z} \left\{ -k_\mu \bar{G}_{\rho\nu}(k,k') \left( \frac{1}{z}, \frac{1}{z'} \right) + 2P_h \cdot k \bar{G}_{\mu\nu}(k,k') \left( \frac{1}{z}, \frac{1}{z'} \right) - \Omega_\gamma^\lambda k_\gamma \left( \frac{1}{z}, \frac{1}{z'} \right) \right. \\
- \frac{P_{h\mu}}{z} \left( \Omega_\gamma^\lambda k^\gamma \left( \frac{1}{z}, \frac{1}{z'} \right) \right) \left. \right\} \Gamma_{\alpha\beta\omega}^{L\mu\nu\lambda}(k,k').
\]

From this form one sees that all terms in \{\cdots\} except for the first one contribute only at twist-4: \( \Omega_\alpha^\mu P_{h\mu} \) extracts \( \alpha = "c" \) component from \( \Gamma_{L\alpha\beta\omega}^{\mu\nu\lambda}(k,k') \) which is subleading, and \( \Omega_\gamma^\lambda k_\gamma \) and \( P_h \cdot k \), respectively, causes additional one- and two- power suppressions. For the first term which
contributes at twist-3, we obtain from (35)
\[
\Omega^\mu_\alpha k^\mu \Gamma^\alpha_{\beta w} L_{FA_{abc}}(k, k') = \sum_X \int d^4 \xi \int d^4 \eta e^{-ik \xi} e^{-i(k' - k) \eta} \langle 0 | F_{b}^{(0) \beta w}(0) | hX \rangle \langle hX | (-i) \frac{\partial}{\partial \xi^\alpha} F_{a}^{(0) \alpha w}(\xi) g A_c^w(\eta) | 0 \rangle.
\]
(55)

Here we note that the QCD equation of motion \( D_\alpha F^{\alpha w} + g \bar{\psi} t^a \psi \psi = 0 \) implies \( \partial_\alpha F_{a}^{(0) \alpha w}(\xi) \) is of \( O(g) \), and hence the first term in \( \{ \cdots \} \) of (54) becomes \( O(g^6) \). Consistent treatment of this term requires the inclusion of all \( O(g^6) \) diagrams, which is beyond the scope of this work. We thus conclude that it does not contribute to the LO cross section. This proves that (43) does not contribute to the LO twist-3 cross section.

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