Hubble Meets Planck: A Cosmic Peek at Quantum Foam

Y. Jack Ng

Institute of Field Physics, Department of Physics and Astronomy,
University of North Carolina, Chapel Hill, NC 27599-3255, USA
E-mail: yjng@physics.unc.edu

If spacetime undergoes quantum fluctuations, an electromagnetic wavefront will acquire uncertainties in direction as well as phase as it propagates through spacetime. These uncertainties can show up in interferometric observations of distant quasars as a decreased fringe visibility. The Very Large Telescope and Keck interferometers may be on the verge of probing spacetime fluctuations which, we also argue, have repercussions for cosmology, requiring the existence of dark energy/matter, the critical cosmic energy density, and accelerating cosmic expansion in the present era.

Keywords: detection of quantum foam, holography, critical energy density, dark energy/matter

1. Quantum Fluctuations of Spacetime

Conceivably spacetime, like everything else, is subject to quantum fluctuations. As a result, spacetime is “foamy” at small scales,\(^1\) giving rise to a microscopic structure of spacetime known as quantum foam, also known as spacetime foam, and entailing an intrinsic limitation \(\delta l\) to the accuracy with which one can measure a distance \(l\). In principle, \(\delta l\) can depend on both \(l\) and the Planck length \(l_P = \sqrt{\hbar G/c^3}\), the intrinsic scale in quantum gravity, and hence can be written as \(\delta l \gtrsim l^{1-\alpha} l_P^\alpha\), with \(\alpha \sim 1\) parametrizing the various spacetime foam models. (For related effects of quantum fluctuations of spacetime geometry, see Ref. 2.) In what follows, we will advocate the so-called holographic model corresponding to \(\alpha = 2/3\), but we will also consider the (random walk) model with \(\alpha = 1/2\) for comparison. The holographic model has been derived by various arguments, including the Wigner-Salecker gedanken experiment to measure a distance\(^3\) and the holographic principle.\(^4,5\) (See my contribution to the Proceedings of MG10.\(^6\)) Here in the two subsections to follow, we use instead (1) an approach based on quantum computation, and (2) an argument over the maximum number of particles that can be put inside a region of space respectively.

1.1. Quantum Computation

This method\(^7,8\) hinges on the fact that quantum fluctuations of spacetime manifest themselves in the form of uncertainties in the geometry of spacetime. Hence the structure of spacetime foam can be inferred from the accuracy with which we can measure that geometry. Let us consider a spherical volume of radius \(l\) over the amount of time \(T = 2l/c\) it takes light to cross the volume. One way to map out the geometry of this spacetime region is to fill the space with clocks, exchanging signals with other clocks and measuring the signals’ times of arrival. This process of mapping the geometry is a sort of computation; hence the total number of operations (the ticking of the clocks and the measurement of signals etc) is bounded by the...
Margolus-Levitin theorem in quantum computation, which stipulates that the rate of operations for any computer cannot exceed the amount of energy $E$ that is available for computation divided by $\pi \hbar / 2$. A total mass $M$ of clocks then yields, via the Margolus-Levitin theorem, the bound on the total number of operations given by $(2Mc^2 / \pi \hbar) \times 2l/c$. But to prevent black hole formation, $M$ must be less than $lc^2 / 2G$. Together, these two limits imply that the total number of operations that can occur in a spatial volume of radius $l$ for a time period $2l/c$ is no greater than $\sim (l/l_P)^2$. (Here and henceforth we neglect multiplicative constants of order unity, and set $c = 1 = \hbar$.) To maximize spatial resolution, each clock must tick only once during the entire time period. And if we regard the operations partitioning the spacetime volume into “cells”, then on the average each cell occupies a spatial volume no less than $\sim l^3/(l^2/l_P^2) = l/l_P^2$, yielding an average separation between neighboring cells no less than $l^{1/3}l_P^{2/3}$. This spatial separation is interpreted as the average minimum uncertainty in the measurement of a distance $l$, that is, $\delta l \gtrsim l^{1/3}l_P^{2/3}$.

Parenthetically we can now understand why this quantum foam model has come to be known as the holographic model. Since, on the average, each cell occupies a spatial volume of $l/l_P^2$, a spatial region of size $l$ can contain no more than $l^3/(l/l_P^2) = (l/l_P)^2$ cells. Thus this model corresponds to the case of maximum number of bits of information $l^2/l_P^2$ in a spatial region of size $l$, that is allowed by the holographic principle, according to which, the maximum amount of information stored in a region of space scales as the area of its two-dimensional surface, like a hologram.

It will prove to be useful to compare the holographic model in the mapping of the geometry of spacetime with the one that corresponds to spreading the spacetime cells uniformly in both space and time. For the latter case, each cell has the size of $(l^2/l_P^2)^{1/4} = l^{1/2}l_P^{1/2}$ both spatially and temporally, i.e., each clock ticks once in the time it takes to communicate with a neighboring clock. Since the dependence on $l^{1/2}$ is the hallmark of a random-walk fluctuation, this quantum foam model corresponding to $\delta l \gtrsim (l/l_P)^{1/2}$ is called the random-walk model. Compared to the holographic model, the random-walk model predicts a coarser spatial resolution, i.e., a larger distance fluctuation, in the mapping of spacetime geometry. It also yields a smaller bound on the information content in a spatial region, viz., $(l/l_P)^2/(l/l_P)^{1/2} = (l^2/l_P^2)^{3/4} = (l/l_P)^{3/2}$.

1.2. Maximum Number of Particles in a Region of Space

This method involves an estimate of the maximum number of particles that can be put inside a spherical region of radius $l$. Since matter can embody the maximum information when it is converted to energetic and effectively massless particles, let us consider massless particles. According to Heisenberg’s uncertainty principle, the minimum energy of each particle is no less than $\sim l^{-1}$. To prevent the region from collapsing into a black hole, the total energy is bounded by $\sim l/G$. Thus the total number of particles must be less than $(l/l_P)^2$, and hence the average interparticle distance is no less than $\sim l^{1/3}l_P^{2/3}$. Now, the more particles there are (i.e., the
shorter the interparticle distance), the more information can be contained in the region, and accordingly the more accurate the geometry of the region can be mapped out. Therefore the spatial separation we have just found can be interpreted as the average minimum uncertainty in the measurement of a distance \( l \); i.e., \( \delta l \gtrsim l^{1/3} l_P^{2/3} \).

Two remarks are in order. First, this minimum \( \delta l \) just found corresponds to the case of maximum energy density \( \rho \sim (ll_P)^{-2} \) for the region not to collapse into a black hole, i.e., the holographic model, in contrast to the random-walk model and other models, requires, for its consistency, the critical energy density which, in the cosmological setting, is \( (H/l_P)^2 \) with \( H \) being the Hubble parameter. Secondly, the numerical factor in \( \delta l \), according to the four different methods alluded to above, can be shown to be between 1 and 2, i.e., \( \delta l \gtrsim l^{1/3} l_P^{2/3} \) to \( 2l^{1/3} l_P^{2/3} \).

2. Probing Quantum Foam with Extragalactic Sources

The Planck length \( l_P \sim 10^{-33} \) cm is so short that we need an astronomical (even cosmological) distance \( l \) for its fluctuation \( \delta l \) to be detectable. Let us consider light (with wavelength \( \lambda \)) from distant quasars or bright active galactic nuclei.\(^{12,13}\) Due to the quantum fluctuations of spacetime, the wavefront, while planar, is itself “foamy”, having random fluctuations in phase\(^{13}\) \( \Delta \phi \sim 2\pi \delta l/\lambda \) as well as the direction of the wave vector\(^{14}\) given by \( \Delta \phi/2\pi \). In effect, spacetime foam creates a “seeing disk” whose angular diameter is \( \sim \Delta \phi/2\pi \). For an interferometer with baseline length \( D \), this means that dispersion will be seen as a spread in the angular size of a distant point source, causing a reduction in the fringe visibility when \( \Delta \phi/2\pi \sim \lambda/D \). For a quasar of 1 Gpc away, at infrared wavelength, the holographic model predicts a phase fluctuation \( \Delta \phi \sim 2\pi \times 10^{-9} \) radians. On the other hand, an infrared interferometer (like the Very Large Telescope Interferometer) with \( D \sim 100 \) meters has \( \lambda/D \sim 5 \times 10^{-9} \). Thus, in principle, this method will allow the use of interferometry fringe patterns to test the holographic model! Furthermore, these tests can be carried out without guaranteed time using archived high resolution, deep imaging data on quasars, and possibly, supernovae from existing and upcoming telescopes.

The key issue here is the sensitivity of the interferometer. The lack of observed fringes may simply be due to the lack of sufficient flux (or even just effects originated from the turbulence of the Earth’s atmosphere) rather than the possibility that the instrument has resolved a spacetime foam generated halo. But, given sufficient sensitivity, the VLTI, for example, with its maximum baseline, presumably has sufficient resolution to detect spacetime foam halos for low redshift quasars, and in principle, it can be even more effective for the higher redshift quasars. Note that the test is simply a question of the detection or non-detection of fringes. It is not a question of mapping the structure of the predicted halo.

---

\(^{a}\)Using \( k = 2\pi/\lambda \), one finds that, over one wavelength, the wave vector fluctuates by \( \delta k = 2\pi \delta \lambda/\lambda^2 = k \delta \lambda/\lambda \). Due to space isotropy of quantum fluctuations, the transverse and longitudinal components of the wave vector fluctuate by comparable amounts. Thus, over distance \( l \), the direction of the wave vector fluctuates by \( \Delta k_T/k = \Sigma \delta \lambda/\lambda \sim \delta l/\lambda \).
3. From Quantum Foam to Cosmology

In the meantime, we can use existing archived data on quasars or active galactic nuclei from the Hubble Space Telescope to test the quantum foam models. Consider the case of PKS1413+135, an AGN for which the redshift is $z = 0.2467$. With $l \approx 1.2 \text{ Gpc}$ and $\lambda = 1.6 \mu\text{m}$, we find $\Delta \phi \sim 10^{-9} \times 2\pi$ for the random-walk model and the holographic model of spacetime foam respectively. With $D = 2.4 \text{ m}$ for HST, we expect to detect halos if $\Delta \phi \sim 10^{-6} \times 2\pi$. Thus, the HST image only fails to test the holographic model by 3 orders of magnitude.

However, the absence of a quantum foam induced halo structure in the HST image of PKS1413+135 rules out convincingly the random-walk model. (In fact, the scaling relation discussed above indicates that all spacetime foam models with $\alpha \lesssim 0.6$ are ruled out by this HST observation.) This result has profound implications for cosmology. To wit, from the (observed) cosmic critical density in the present era, a prediction of the holographic-foam-inspired cosmology, we deduce that $\rho \sim H_0^2 / G \sim (R_H l_P)^{-2}$, where $H_0$ and $R_H$ are the present Hubble parameter and Hubble radius of the observable universe respectively. Treating the whole universe as a computer, one can apply the Margolus-Levitin theorem to conclude that the universe computes at a rate $\nu$ up to $\rho R_H^3 \sim R_H l_P^2$ for a total of $(R_H l_P)^2$ operations during its lifetime so far. If all the information of this huge computer is stored in ordinary matter, then we can apply standard methods of statistical mechanics to find that the total number $I$ of bits is $(R_H l_P^2)^{3/4} = (R_H l_P)^{3/2}$. It follows that each bit flips once in the amount of time given by $I/\nu \sim (R_H l_P)^{1/2}$. On the other hand, the average separation of neighboring bits is $(R_H l_P)^{1/3} \sim (R_H l_P)^{1/2}$. Hence, the time to communicate with neighboring bits is equal to the time for each bit to flip once. It follows that the accuracy to which ordinary matter maps out the geometry of spacetime corresponds exactly to the case of events spread out uniformly in space and time discussed above for the case of the random-walk model of quantum foam. Succinctly, ordinary matter only contains an amount of information dense enough to map out spacetime at a level consistent with the random-walk model. Observationally ruling out the random-walk model suggests that there must be other kinds of matter/energy with which the universe can map out its spacetime geometry to a finer spatial accuracy than is possible with the use of ordinary matter. This line of reasoning then strongly hints at the existence of dark energy/matter independent of the evidence from recent cosmological (supernovae, cosmic microwave background, gravitational lensing, galaxy configuration and clusters) observations.

Moreover, the fact that our universe is observed to be at or very close to its critical energy density $\rho \sim (H/l_P)^2 \sim (R_H l_P)^{-2}$ must be taken as solid albeit indirect evidence in favor of the holographic model because, as aforementioned, this model is the only model that requires the energy density to be critical. The holographic model also predicts a huge number of degrees of freedom for the universe in the present era, with the cosmic entropy given by $\sim I \sim R_H^3 l_P^2 \sim (R_H l_P)^2$. Hence the average energy carried by each bit is $\rho R_H^3 I / I \sim R_H^{-1}$. Such long-wavelength
bits or “particles” carry negligible kinetic energy. Since pressure (energy density) is given by kinetic energy minus (plus) potential energy, a negligible kinetic energy means that the pressure of the unconventional energy is roughly equal to minus its energy density, leading to accelerating cosmic expansion as has been observed. This scenario is very similar to that for quintessence.

How about the early universe? Here a cautionary remark is in order. Recall that the holographic model has been derived for a static and flat spacetime. Its application to the universe of the present era may be valid, but to extend the discussion to the early universe may need a judicious generalization of some of the concepts involved. However, there is cause for optimism: for example, one of the main features of the holographic model, viz. the critical energy density, is actually the hallmark of the inflationary universe paradigm. Further study is warranted.

Acknowledgments

This work was supported in part by the US Department of Energy and the Bahnson Fund of the University of North Carolina.

References

1. J.A. Wheeler, in Relativity, Groups and Topology, eds. B.S. DeWitt and C.M. DeWitt (Gordon & Breach, New York, 1963), p. 315. Also see S.W. Hawking et al., Nucl. Phys. 170, 283 (1980); A. Ashtekar et al., Phys. Rev. Lett. 69, 237 (1992); J. Ellis et al., Phys. Lett. B 293, 37 (1992).
2. L. H. Ford, Phys. Rev. D51, 1692 (1995); B. L. Hu and E. Vergagner, Living Rev. Rel. 7, 3 (2004).
3. H. Salecker and E.P. Wigner, Phys. Rev. 109, 571 (1958); Y.J. Ng and H. van Dam, Mod. Phys. Lett. A9, 335 (1994): A10, 2801 (1995). Also see F. Karolyhazy, Nuovo Cimento A42, 390 (1966).
4. Y. J. Ng, Phys. Rev. Lett. 86, 2946 (2001), and (erratum) 88, 139902-1 (2002).
5. Y. J. Ng, Int. J. Mod. Phys. D11, 1585 (2002).
6. Y. J. Ng, in Proc. of the Tenth Marcel Grossman Meeting on General Relativity, eds. M. Novello et al. (World Scientific, Singapore, 2005), p. 2150.
7. S. Lloyd and Y.J. Ng, Sci. Am. 291, # 5, 52 (2004).
8. V. Giovannetti, S. Lloyd and L. Maccone, Science 306, 1330 (2004).
9. N. Margolus and L. B. Levitin, Physica D120, 188 (1998).
10. G. t Hooft, in Salamfestschrift, eds. A. Ali et al. (World Scientific, Singapore, 1993), p. 284; L. Susskind, J. Math. Phys. (N.Y.) 36, 6377 (1995). Also see J.D. Bekenstein, Phys. Rev. D7, 2333 (1973); S. Hawking, Comm. Math. Phys. 43, 199 (1975).
11. G. Amelino-Camelia, Mod. Phys. Lett. A9, 3415 (1994); Nature 398, 216 (1999).
12. R. Lieu and L. W. Hillman, Astrophys. J. 585, L77 (2003); R. Ragazzoni, M. Turatto, and W. Gaessler, Astrophys. J. 587, L1 (2003).
13. Y. J. Ng, W. Christiansen, and H. van Dam, Astrophys. J. 591, L87 (2003).
14. W. Christiansen, Y. J. Ng, and H. van Dam, Phys. Rev. Lett. 96, 051301 (2006).
15. E. S. Perlman, et al., 2002, Astro. J. 124, 2401 (2002).
16. M. Arzano, T. W. Kephart, and Y. J. Ng, arXiv:gr-qc/0605117.
17. S. Lloyd, Phys. Rev. Lett. 88, 237901-1 (2002).