Research on dynamic characteristic of connecting rod with joint clearance

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Abstract. In order to study the dynamic characteristics of revolving joint with clearance, the theoretical model of the revolving joint with clearance is established by using Newton Euler method and "contact-separation" state model. The kinetic equations are solved by using the Gear algorithm in MATLAB software. The change curves of the contact force and the center trajectory of the revolving joint shaft with clearance are obtained considering different clearance, contact stiffness, and rotational speed together. The research results have laid a theoretical foundation for improving the joint forces and the accuracy by using controller.

1. Introduction

With the development of modern machinery in the direction of high speed and precision, the influence of clearance on mechanism motion has attracted more and more attention from scholars both at home and abroad. Due to the existence of clearance, the working performance of the mechanism has changed, resulting in a deviation between the actual movement of the end effector and the predetermined motion. In addition, the clearance makes a violent impact and collision between the moving pairs, which speeding up the wear between the auxiliary elements of the movement, causing the mechanical vibration and noise, and seriously deteriorating the dynamic performance of the mechanical products. Thus the operation precision, stability, reliability and service life of the machine have been reduced. The adverse effects of the kinematic pair clearance are particularly serious in high speed, heavy load and micro-mechanical systems. For example, in the field of Aeronautics and Astronautics, the unstability of the extensional mechanism, the low positioning accuracy and the failure of the antenna opening often happen, due to the nonlinear effect of clearance on the mechanism, which introduced the failure of the aerospace vehicle [1-2].

Since the 70s of last century, scholars at home and abroad have carried out related research on clearance mechanism and achieved many research results [3-5]. For dealing with the clearance problem, scholars have proposed three research methods: Newton Euler method [6], Lagrange method [7], Kaine method [8]. In addition, scholars have also proposed three types of clearance models: continuous contact model, two state model and three state model. The above research methods and models have advantages and disadvantages in dealing with dynamic problems of different clearance mechanisms.

In this paper, Newton Euler method and "contact-separation" state model are used to establish the dynamic equations of revolving joint with clearances. The contact force of the shaft and the change law of the movement track of the axis center relative to the bearing bush are analyzed, with different
clearance, different contact stiffness and different rotational speed. The research on dynamic modeling of clearance collision and dynamic simulation of clearance mechanism is developed, which provides an important reference for the design of the mechanism and is helpful to guide the practical application of the engineering.

2. Theoretical dynamic modeling of revolving joint with clearance

2.1 Brief description of the revolving joint with clearance

A three-dimensional model of a revolving joint with clearance is shown in Figure 1, and a two-dimensional simplified model with clearance joints is shown in Figure 2, where the clearance is enlarged for clarity. The length and mass of the connecting rod are \( L \), \( m \), \( S \) is the centroid of the connecting rod, and \( O_1 \) and \( O_2 \) are the centroid of the bearing bush and the shaft respectively. The distance between the center of mass of the connecting rod \( S \) and the center of mass of the shaft \( O_2 \) is \( L_s \). The radius of the bearing bush and the shaft are \( R_1 \) and \( R_2 \) respectively. The distance between the two is \( e \), and the driving torque acting on the shaft is \( T \). \( \theta \) is the angle between the central axis of the connecting rod and the counterclockwise rotation of the X-axis of the fixed coordinate system. The centroid of the bearing bush is taken as the origin of the fixed coordinate system, and the Cartesian coordinate system shown in Figure 2 is established. A clearance revolving joint is formed between the shaft and the bearing bush with clearance. A position enlarged view of clearance revolving joint is shown in Figure 3, where the parameters \( e_x \) and \( e_y \) are the projections of the axis of mass in the coordinate system \( OXY \) in the \( X \) and \( Y \) directions, and \( \phi \) is the angle between \( O_1O_2 \) and counterclockwise rotation of the \( X \)-axis.

2.2 Establishment of kinematic equation

As shown in Figure 2, the coordinate equation of the centroid of the connecting rod is:

\[
\begin{bmatrix}
x_s \\
y_s \\
\end{bmatrix} = e \begin{bmatrix}
\cos \phi \\
\sin \phi \\
\end{bmatrix} + L_s \begin{bmatrix}
\cos \theta \\
\sin \theta \\
\end{bmatrix}
\]

(1)

The acceleration formula of the link centroid can be obtained by calculating the two derivative of time on both sides of the formula (1).

\[
\begin{bmatrix}
x_{ss} \\
y_{ss} \\
\end{bmatrix} = e \begin{bmatrix}
\cos \phi \\
\sin \phi \\
\end{bmatrix} + 2e \phi \begin{bmatrix}
-sin \phi \\
cos \phi \\
\end{bmatrix} + e \phi \begin{bmatrix}
-sin \phi \\
cos \phi \\
\end{bmatrix} - e \phi^2 \begin{bmatrix}
\cos \phi \\
\sin \phi \\
\end{bmatrix} - L_s \dot{\theta} \begin{bmatrix}
2 \cos \theta \\
\sin \theta \\
\end{bmatrix} + L_s \phi \begin{bmatrix}
-\sin \theta \\
\cos \theta \\
\end{bmatrix}
\]

(2)
2.3 Establishment of dynamic equation

The model is established using Newton Euler equation \(^9\) on the assumption that the shaft of the revolving joint with clearance is regarded as rigid body and the bearing bush is regarded as a flexible body. Assuming its centroid acceleration is \(a_s\), and according to Newton's second law \(F_s = ma_s\), the following equation are given:

\[
\begin{align*}
-F_{Os} &= m\ddot{x}_s \\
-F_{Oy} - mg &= m\ddot{y}_s
\end{align*}
\]

According to the Euler equation \(\tau = J\alpha + \omega \times (\omega \times J\omega)\), the following two equations are obtained:

\[
\begin{align*}
F_{o_x}R_n \sin \phi - F_{o_y}R_n \cos \phi &= \tau_{o_x} \\
-mgL_n \cos \theta + T &= J\ddot{\theta}
\end{align*}
\]

The shaft collided with the bearing bush during the movement, and assume the normal deformation produced by the contact collision is \(\delta\), then there is the following relationship between \(\delta\) and \(e\) and the ideal clearance \(c\):

\[
\delta = e - c
\]

In the above formula: \(c = R_n - R_s\);

\[
\begin{align*}
&\delta < 0 \quad \text{Free movement} \\
&\delta = 0 \quad \text{Start to contact or start to separate} \\
&\delta > 0 \quad \text{Contact deformation}
\end{align*}
\]

In order to solve the contact force problem in the collision hinge model, the nonlinear spring damping model proposed by Lankari-Nikravesh \(^{10}\) is used to describe the contact-separation process between the motion pairs. The normal contact force \(F_n\) is expressed as:

\[
F_n = u(\delta)\left(K\delta^n + C_\delta \dot{\delta}\right)
\]

Where \(K\) represents the Stiffness coefficient, \(C_\delta\) represents the Normal damping coefficient, and \(m\) represents the Nonlinear spring contact force coefficient.

The expression of the tangential impact force is:

\[
F_t = u(\delta)(-fF_n \text{sign}(\nu) - C_t \nu)
\]

Where \(C_t\) represents the Tangential damping coefficient, \(f\) represents the Sliding friction coefficient, and \(\nu\) represents the Motion tangential relative speed.

The \(u(\delta)\) in equations (7) and (8) is the step function.

\[
u(\delta) = \begin{cases} 0 & \delta < 0 \\ 1 & \delta \geq 0 \end{cases}
\]

As shown in figure 4, project \(F_n\) and \(F_t\) toward the \(X\) and \(Y\) axes to get the expressions of \(F_{o_x}\) and \(F_{o_y}\).
At the point of contact between the shaft and the bearing bush, the normal and tangential relative velocity expressions of the shaft relative to the bearing bush are represented as:

\[
\begin{align*}
\nu_n &= -\dot{e}_x \sin \phi + \dot{e}_y \cos \phi + \dot{\theta} R_z \\
\nu_t &= \dot{e}_x \cos \phi + \dot{e}_y \sin \phi
\end{align*}
\]  
(11)

According to equations (1) to (11), the dynamic expressions of the clearance-containing hinges are obtained.

\[
\ddot{x} = M^{-1}Q
\]  
(12)

Where \(\ddot{x}\) represents the Generalized acceleration array, \(M\) represents the Generalized mass matrix, and \(Q\) represents the Generalized force array.

\[
\ddot{x} = \begin{bmatrix}
\ddot{e} \\
\ddot{\phi} \\
\ddot{\theta}
\end{bmatrix} \quad M = \begin{bmatrix} 0 & 0 & J_{o2} \\ \cos \phi & -e \sin \phi & -L_s \sin \theta \\ \sin \phi & e \cos \phi & L_s \cos \theta \end{bmatrix} \quad Q = \begin{bmatrix} F_{o2} R_z \sin \phi - F_{o2} R_z \cos \phi - mgL_s \cos \theta + T \\ -\frac{F_{o2}}{m} \ddot{x} + 2 \dot{e} \dot{\phi} \sin \phi + e \dot{\phi}^2 \cos \phi + L_s \dot{\theta}^2 \cos \theta \\ -\frac{F_{o2}}{m} - g - 2 \dot{e} \dot{\phi} \cos \phi + e \dot{\phi}^2 \sin \phi + L_s \dot{\theta}^2 \sin \theta \end{bmatrix}
\]  
(13)

According to the formula (12), the function expression of the time of variable parameters \(e, \phi, \theta\) is calculated by numerical calculation, in which the coordinate \((e, \phi)\) is the trajectory of the center of the pin axis. The expression of the contact force \(F_n\) of the axis can be obtained by taking the function expression of \(e, \phi, \theta\) about time into equation (7).

3 Analysis of dynamic characteristics of revolving joint with clearance

The Gear algorithm of MATLAB is used to solve the equation (12), and the variation of the contact force \(F_n\) of the shaft with time and the change curve of the center track \(\dot{e} - e\) are obtained. The specific parameters of the revolving joint with clearance are shown in Table 1.

| Connecting rod length \(L\) | 0.2 [m] |
| Connecting rod centroid \(L_s\) | 0.15 [m] |
| Connecting rod quality \(m\) | 5 [kg] |
| Connecting rod inertia \(J_{o2}\) | 0.24 [kg*m^2] |
| Axial radius \(R_z\) | 0.005 [m] |
| Gravitational acceleration \(g\) | 9.8 [N/kg] |

3.1 Contact force and central motion locus of shaft at different clearances

Under the condition of constant contact stiffness and speed, the contact force and central motion trajectory of the shaft under different clearance conditions are analyzed. The given simulation parameters are as follows: \(K = 1 \times 10^7 \text{ N/m}\); \(C_s = 175 \text{ N*s/m}\); \(f = 0.05\); \(C_i = 0\); \(\dot{\theta} = 2\pi \text{ rad/s}\); \(t = 4s\)

In the initial state, the connecting rod is stationary, and the change diagram of the contact force of the shaft with different clearance is obtained by MATLAB, as shown in figure. 5.
a) A change map of $F_s$ when the clearance is $0.1\text{mm}$

b) A change map of $F_s$ when the clearance is $0.2\text{mm}$

**Figure 5.** $F_s$ change diagram at different clearance

Figure 5 shows that with the increase of clearance between axle and bearing bush, the magnitude of contact force increases. The above analysis shows that the clearance is very destructive to the mechanism at the beginning stage of the movement. The greater the clearance, the greater the damage. When the connecting rod is stable, the clearance has little effect on the force of the shaft.

As shown in figure 6, with the increase of the range of the center of the axis, the larger the clearance is, the deeper the shaft is in the axle bush, the greater the contact force is. When the connecting rod moves steadily, the center of the shaft moves periodically on both sides of the gravitational direction under different clearance conditions.

### 3.2 Contact force and center motion trajectory of pin shaft with different contact stiffness

Under the condition of constant clearance and speed, the contact force and central motion trajectory of the shaft with different contact stiffness are analyzed. The given simulation parameters are as follows: $c=0.2\text{mm}$; $C_m=175\text{N}\cdot\text{s/m}$; $f=0.05$; $C_s=0$; $\dot{\theta}=2\pi\text{rad/s}$; $t=4\text{s}$.

Figure 7 shows that the change of the contact force of the shaft changes abrupt at the beginning of the movement. It can be seen from the above that the instantaneous contact force increases with the increase of contact stiffness at the beginning of the movement stage. With the movement of the moving arm, the contact force of the contact shaft has no obvious difference when the connecting rod is running smoothly.

a) The change diagram of $F_s$ under the condition of $K=1\times10^7\text{N/m}$

b) The change diagram of $F_s$ under the condition of $K=5\times10^7\text{N/m}$

**Figure 7.** Change diagram of $F_s$ at different contact stiffness

Figure 8 shows that with the increase of the contact stiffness between the shaft and the bearing bush, the range of motion of the shaft center in the horizontal direction increases, but the range of motion in the direction of gravity decreases. The reason is that the greater the contact stiffness when the shaft is subjected to the same force, the smaller the deformation of the bearing bush. When the bearing bush is deformed, the shaft will have a large slip along the inner wall of the bearing bush.
3.3 Contact force and center motion trajectory of pin at different speeds

Under the condition of constant clearance and contact stiffness, the contact force and central motion trajectory of the shaft at different speeds are analyzed. The given simulation parameters are as follows:

\[ K=1\times10^7\ N/m; \quad C_i = 175N\cdot s/m; \quad f = 0.05; \quad C_c = 0; \quad e = 0.2mm; \quad t = 4s \]

![Contact force and center motion trajectory of pin at different speeds](image)

Figure 9, F change diagram at different rotational speeds

Figure 9 shows that with the increase of the rotational speed of the connecting rod, the force applied to the shaft also increases, and the larger the rotational speed, the larger the fluctuation of \( F_c \).

In addition, the greater the rotational speed, the slower the attenuation of the contact force of the shaft, that is, the greater the rotational speed, the longer the contact time of the shaft reaches a stable value. As shown in Figure 10, it can be seen that the maximum penetration depth of the shaft with the revolving joint with clearance relative to the bearing bush increases with the increase of the rotational speed. After the shaft is stable, its motion trajectory exhibits a periodic variation. In the gravitational field, the central trajectory of the shaft moves downward as a whole due to the effect of gravity.

4. Summary

In this paper, the dynamic model of the revolving joint with clearance with "contact-separation" state is established based on Newton Euler method. The dynamic equation is solved, and the general change law is obtained, which lays a foundation for the future analysis of dynamics and control of hinges with clearances. The specific laws are as follows:

1. The clearance has a strong destructive effect on the mechanism in the early stage, and the greater the clearance, the greater the damage. When the connecting rod moves steadily, the clearance has little influence on the stress of the shaft.

2. The greater the contact stiffness between the shaft and the bearing bush, the greater the force of the shaft, but the smaller the deformation of the bearing bush. When the connecting rod moves steadily, the contact stiffness between the shaft and the bearing bush has little effect on the stress of the shaft.

3. The greater the rotational speed, the greater the contact force of the shaft, the longer the time it experienced when the contact force reaches the stable value, the higher the connecting rod speed, the deeper the corresponding puncture depth.

5. References

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