Chiral dynamics and s-wave exotic hadrons

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Abstract. Based on chiral dynamics, existence of exotic hadrons is discussed in the SU(3) symmetric limit. The low energy s-wave interaction of the Nambu-Goldstone boson with a hadron is known to be determined model-independently, which has been used to generate some hadron resonances in nonexotic channels. We show that this interaction in any exotic channels is not strong enough to generate a bound state.

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Experimentally, more than hundred of hadrons have been discovered so far, whose properties are summarized by the Particle Data Group (PDG) [1]. Most of the hadrons can be, in principle, described in terms of \( \bar{q}q \) or \( q\bar{q}q \), and the only state with exotic flavor quantum numbers in PDG is the \( S = +1 \) baryon \( \Theta^+ \) [2]. Thus, exotic hadrons are indeed “exotic” from an experimental point of view, while there is no clear theoretical explanation of the non-observation (or non-existence) of the exotic hadrons. This is certainly a non-trivial issue to be explained theoretically, irrespective to the existence of the \( \Theta^+ \).

Here we report our recent work on this issue [3,4], in which we have studied the existence of the exotic hadrons in s-wave scattering of a hadron and the Nambu-Goldstone (NG) boson. We utilize the theoretical framework based on the SU(3) limit. In 60’s, the framework was used to describe hadron resonances such as \( \Lambda(1405) \) with the effective vector meson exchange interaction. Recently, the interaction has been founded by chiral symmetry [7], leading to a successful description of the hadron resonances in chiral unitary approaches [11,12,13].

In recent applications of the chiral unitary approach, it was shown that some resonances obtained in the coupled channel dynamics with SU(3) breaking became bound states of a single channel in the flavor SU(3) limit [11,14,15,16,17,18]. Therefore, we expect that the origin of the physical resonances may be clarified by studying the bound states in the SU(3) limit. We focus on the s-wave scatterings, since the low energy interaction of the NG boson with any hadrons in s-wave are uniquely determined by chiral symmetry. In this framework, we examine the possibility to generate the exotic hadron as a bound state of the NG boson and a hadron.

The low energy s-wave interaction of the NG boson \((Ad)\) with a target hadron \((T)\) is model-independently given by

\[
\alpha \begin{array}{c} \text{Ad} \\ \text{T} \end{array} = -\frac{\omega}{2f_T^2} C_{\alpha,T},
\]

Fig. 1. (a) : Notation of the representations \( \alpha \), \( \text{Ad} \) and \( T \) for the WT term. (b) : The bound state pole diagram after unitarization of the amplitude.

with the decay constant \((f)\) and energy \((\omega)\) of the NG boson. The group theoretical factor \( C_{\alpha,T} \) is determined by specifying the flavor representations of the target \( T \) and the scattering system \( \alpha \in T \otimes \text{Ad} \) (see Fig. 1):

\[
C_{\alpha,T} = -\langle 2F_T \cdot F_{\text{Ad}} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3,
\]

where \( C_2(R) \) is the quadratic Casimir of SU(3) for the representation \( R \), and we use \( C_2(\text{Ad}) = 3 \) for the adjoint representation of the NG boson.

For the target hadron with an arbitrary SU(3) representation \( T = [p,q] \), possible representations \( \alpha \) for the scattering channels are obtained as

\[
ap, q \otimes [1,1] = [p+1,q+1]
\]
\[
\oplus [p+2,q-1] \oplus [p-1,q+2] \oplus [p,q] \oplus [p,q]
\]
\[
\oplus [p+1,q-2] \oplus [p-2,q+1] \oplus [p-1,q-1],
\]

where the labels of representations \( [a,b] \) should satisfy \( a,b \geq 0 \), and one of the two \([p,q]\) representations on the right hand side should satisfy \( p \geq 1 \) and the other \( q \geq 1 \). Using Eq. (2), we evaluate the coupling strengths \( C_{\alpha,T} \) for
Table 1. Properties of the WT interaction in the channel $\alpha$ of the NG boson scattering on the target hadron with the $T = [p, q]$ representation. The coupling strengths of the WT term is denoted as $C_{\alpha,T}$. $\Delta E$ is the differences of the exoticness $E$ between the channel $\alpha$ and the target hadron $T$, and $C_{\alpha,T}(N_c)$ denotes the coupling strengths for arbitrary $N_c$.

| $\alpha$ | $C_{\alpha,T}$ | $\Delta E$ | $C_{\alpha,T}(N_c)$ |
|-------|----------------|----------|-----------------
| $p + 1, q + 1$ | $-p - q$ | 1 or 0 | $\frac{5 - N_c}{2} - p - q$ |
| $p + 2, q - 1$ | $1 - p$ | 1 or 0 | $1 - p$ |
| $p - 1, q + 2$ | $1 - q$ | 1 or 0 | $\frac{5 - N_c}{2} - q$ |
| $[p, q]$ | 3 | 0 | 3 |
| $p + 1, q - 2$ | $3 + q$ | 0 or $-1$ | $\frac{5 - N_c}{2} + q$ |
| $p - 2, q + 1$ | $3 + p$ | 0 or $-1$ | $\frac{3 + p}{2}$ |
| $p - 1, q - 1$ | $4 + p + q$ | 0 or $-1$ | $\frac{5 + N_c}{2} + p + q$ |



Fig. 2. Critical coupling strength $C_{\text{crit}}$ for $f = 93$ MeV and $m = 368$ MeV (Solid line). The dashed line denotes the universal attractive coupling strength in exotic channels $C_{\text{exotic}} = 1$.

the possible representations of the channel $\alpha$ as shown in Table 1.

In order to specify the exotic channels, we define the exoticness quantum number $E$ [21,22,23,24] as the number of valence quark-antiquark pairs to construct the given flavor multiplet $[p, q]$ with the baryon number $B$ carried by the $u$, $d$, and $s$ quarks. For $B > 0$ the exoticness $E$ is given by

$$E = \epsilon \theta(\epsilon) + \nu \theta(\nu),$$

where we define $\epsilon \equiv (p + 2q)/3 - B$, $\nu \equiv (p - q)/3 - B$ and $\theta(x)$ is the step function. For each $\alpha$, we evaluate the difference of the exoticness $\Delta E$ between the target $T$ and $\alpha$, as shown in Table 1. Taking $p, q \geq 0$ into account, we find that most of exotic channels are repulsive ($C_{\alpha,T} < 0$) and that the attractive interaction ($C_{\alpha,T} > 0$) is realized with the universal strength

$$C_{\text{exotic}} = 1.$$ (3)

By looking at the construction in detail, we can further specify the attractive channels as $\alpha = [p - 1, 2]$ for $T = [p, 0]$ and $p \geq 3B$ [34].

Next we study the scattering problem with the WT interaction $V_{\alpha}$ of Eq. 11 in order to examine whether the attractive interaction [3] can accommodate an exotic bound state. We utilize the N/D method [9] to construct the scattering amplitude which satisfies the elastic unitarity condition. The scattering amplitude of the NG boson and the target hadron in the channel $\alpha$ is given by

$$t_{\alpha}(\sqrt{s}) = \frac{1}{1 - V_{\alpha}(\sqrt{s})G(\sqrt{s})}V_{\alpha}(\sqrt{s}),$$

as a function of the center-of-mass energy $\sqrt{s}$. Here $G(\sqrt{s})$ is given by the once-subtracted dispersion integral

$$G(\sqrt{s}) = -\tilde{a}(s_0) - \frac{1}{2\pi} \int_{s_0}^{\infty} ds' \left( \frac{\rho(s')}{s' - s} - \frac{\rho(s)}{s' - s_0} \right),$$

with $\rho(s) = 2M_T \sqrt{(s - s^+)(s - s^-)}/(8\pi s)$ and $s^\pm = (m \pm M_T)^2$.

The subtraction constant $\tilde{a}(s_0)$ should be in principle determined so as to reproduce some experimental observables. Here we would like to discuss a prescription which can be applicable to the cases without experimental information. For this purpose, we adopt the prescription given in Refs. [24,10]

$$G(M_T) = 0,$$ (4)

which is equivalent to $t_{\alpha}(\sqrt{s}) = V_{\alpha}(\sqrt{s})$ at $\sqrt{s} = M_T$. This allows us to match the amplitude with that of the $u$-channel resummation at this energy. In addition, we show that this prescription provides a “natural size” of the subtraction parameter [4], with which the experimental observables in the $S = -1$ meson-baryon channel are well reproduced [9].

Since the WT interaction is the leading order term of the chiral perturbation theory, the condition (4) means that the full amplitude becomes that of the chiral perturbation theory at $\sqrt{s} = M_T$. It is worth noting that $t_{\alpha}(\sqrt{s}) = V_{\alpha}(\sqrt{s})$ can be achieved only in the region where $G(\sqrt{s})$ is real, since the subtraction constant $\tilde{a}(s_0)$ is a real number. Therefore, in order to match the full amplitude to the chiral perturbation theory, we need

$$G(\sqrt{s}) = 0 \quad \text{within} \quad M_T - m \leq \sqrt{s} \leq M_T + m.$$ (5)

If this condition is not satisfied, the unitarized amplitude $t_{\alpha}(\sqrt{s})$ does not coincide with the tree level amplitude obtained in chiral perturbation theory.

With the renormalization condition (4), we derive the smallest attractive coupling strength $C_{\text{crit}}$ to produce a bound state of the NG boson with the target hadron as

$$C_{\text{crit}} = \frac{2f^2}{m \left[ -G(M_T + m) \right]}.$$ (5)

Using $f = 93$ MeV and $m = 368$ MeV, we plot $C_{\text{crit}}$ in Fig. 2 as a function of $M_T$ together with the universal attractive strength $C_{\text{exotic}} = 1$ found in Eq. 3. It is seen that $C_{\text{exotic}}$ is not strong enough to generate a bound state in this region. We also examine the dependence of the $C_{\text{crit}}$ on $f$ and $m$ and find that $C_{\text{crit}}$ becomes smaller, either for the larger $m$ or the smaller $f$. However, $C_{\text{crit}}$ cannot be smaller than $C_{\text{exotic}} = 1$ in physically reasonable parameter region.

For a baryon target, we can evaluate the WT interaction in large-$N_c$ limit using the analytic form of the coupling strength (4). With the standard extension of the
The interaction is smaller than the critical value to generate a bound state $C_{\text{exotic}} < C_{\text{crit}}$. Combining these two, we have shown that the exotic hadrons are not generated in s-wave scattering of the NG boson with a target hadron in the SU(3) symmetric limit.

It should be, however, noted that the present analysis is based on the simplest framework with the leading order interaction in the SU(3) limit. In practice, the effect of the flavor SU(3) breaking and higher order terms in the chiral Lagrangian should be taken into account. In addition, we do not exclude the existence of the exotic state with different origins, such as genuine quark states or the rotational excitation of the chiral solitons. Nonetheless, the conclusion drawn here should contribute to partly explain the difficulty to observe of the exotic hadrons in Nature.

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References

1. Particle Data Group, W. M. Yao et al., J. Phys. G33, 1 (2006).
2. T. Nakano et al., (LEPS Collaboration), Phys. Rev. Lett. 91, 012002 (2003).
3. T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. Lett. 97, 192002 (2006).
4. T. Hyodo, D. Jido, and A. Hosaka, hep-ph/0611004.
5. R. H. Dalitz and S. F. Tuan, Ann. Phys. (N.Y.) 10, 307 (1960).
6. J. H. W. Wyld, Phys. Rev. 155, 1649 (1967).
7. N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. A594, 325 (1995).
8. E. Oset and A. Ramos, Nucl. Phys. A635, 99 (1998).
9. J. A. Oller and U. G. Meissner, Phys. Lett. B500, 263 (2001).
10. M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002).
11. D. Jido, J. A. Oller, E. Oset, A. Ramos, and U. G. Meissner, Nucl. Phys. A725, 181 (2003).
12. T. Hyodo, A. Hosaka, E. Oset, A. Ramos, and M. J. Vicente Vacas, Phys. Rev. C 68, 065203 (2003).
13. V. K. Magas, E. Oset, and A. Ramos, Phys. Rev. Lett. 95, 052301 (2005).
14. C. Garcia-Recio, M. F. M. Lutz, and J. Nieves, Phys. Lett. B582, 49 (2004).
15. E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B585, 243 (2004).
16. S. Sarkar, E. Oset, and M. J. Vicente Vacas, Nucl. Phys. A750, 294 (2005).
17. M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A730, 110 (2004).
18. E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B582, 39 (2004).
19. S. Weinberg, Phys. Rev. Lett. 17, 616 (1966).
20. Y. Tomozawa, Nuovo Cimento 46A, 707 (1966).
21. V. Kopeliovich, Phys. Lett. B259, 234 (1991).
22. D. Diakonov and V. Petrov, Phys. Rev. D 69, 056002 (2004).
23. V. Kopeliovich, hep-ph/0310071.
24. E. Jenkins and A. V. Manohar, Phys. Rev. Lett. 93, 022001 (2004).
25. K. Igi and K.-i. Hikasa, Phys. Rev. D59, 034005 (1999).
26. R. F. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D49, 4713 (1994).
27. T. Hyodo, S. I. Nam, D. Jido, and A. Hosaka, Prog. Theor. Phys. 112, 73 (2004).
28. B. Borasoy, R. Nissler, and W. Weise, Phys. Rev. Lett. 94, 213401 (2005).
29. B. Borasoy, R. Nissler, and W. Weise, Eur. Phys. J. A25, 79 (2005).
30. J. A. Oller, J. Prades, and M. Verbeni, Phys. Rev. Lett. 95, 172502 (2005).
31. J. A. Oller, Eur. Phys. J. A28, 63 (2006).