Residential Area Modelling Using Cellular Automata with Estimated Water Resources
- A Case Study in Darkhan, Mongolia -

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Abstract: Development and economic growth of cities in Northern Mongolia seriously depend on its national urban planning policy and also the accompanying social infrastructure. In addition, the city expansion might be restricted by water resources, as the area has only 300-400 mm of annual precipitation. In this research, despite of its inherent difficulty thus, development of Darkhan city in Northern Mongolia is simulated using a Cellular Automata Model for better decision-making in planning. The model is modified to accommodate to the locality of the city growth, in which ger that is a typical temporal residence of nomads is introduced as a part of the city. Population growth is assumed to be expressed by a logistic function with coefficients derived originally from an actual statistical record and fluctuated according to dynamically-changing water resources deficiency. The urban growth is regulated by these population dynamics as well as a potential of respective land for development that is given by the distances from a station, roads and a cluster of residence. Calibration is successfully executed and the fractal dimensions of the real urban area and simulated one become 1.56 and 1.60, respectively. The simulated results with several scenarios during 2008-2030 indicate that Darkhan might develop with fluctuation and attain to a certain magnitude, only if water demand increases gradually. Otherwise, Darkhan city, especially ger area might shrink due to water resources deficiency.

Keywords: Cellular Automata, Urban Model, Water Resources, System Dynamics, Northern Mongolia, Ger

1 Introduction
Darkhan is the second largest city in Mongolia, located in northern part of Mongolia, 220 km far from the capital city Ulaanbaatar (Figure 1). Darkhan had been developed with extensive economic assistance from Soviet Union since early 1960’s and it is now known as an industrial region famous for steel production. Its transportation condition is superior to other Mongolian cities except the capital city, as it is connected to Russia and China through Trans-Mongolian railway. While its climate is categorized as Monsoon-influenced extremely cold subarctic climate (Dwd) and the amount of precipitation is as little as approximately 300-400 mm, Kharaa River flows along the city and the city has considerable water resources in the groundwater. Currently almost all the municipal water is pumped up from battery of 18 wells and distributed through pipelines to the city. Besides, several factories have their own wells, but the amount of yield is not clear. As the city develops and water demand increases, the groundwater level seems to decline as shown in Figure 2.

The city is expected to develop more as an industrial center of Mongolia due to its convenient location and therefore its urban planning policy might determine the expansion of the city considerably. There are, however, unexpected factors, i.e., Ger, a typical temporal residence of nomads moves autonomously being independent of the planning and also groundwater resources inexactly understood in this region. Excessive groundwater-drawing might induce the shortage of municipal and industrial water and cause the stagnation of the city development. Thus, simulation and forecasting of the urban expansion of Darkhan using scenarios are of importance to assist the decision-making for its sustainable development.

2 Modelling
Cellular automata (CA) are models of which cells change their states according to simple repetitive rules with their
contiguous and adjacent cells, which often express unexpected complexity. CA were initially developed for fluid dynamics by Ulam, S. M. and J. von Neumann in 1950’s and Wolfram (2002) showed its applicability to natural complex phenomena. Tobler (1979) applied the concept of the cellular space to geographic modelling.

Since, CA is frequently utilized in various fields with complexity such as physics, biology, and urban expansion. First appearance of the application of CA to urban expansion was Coulcels in 1985, which considered a hypothetical but fairly complex problem of individual decision and large-scale urban change. It was mainly explanation of the CA theory, but even somewhat prescient to mention the concept of agent-based model. Realistic application may begin from Batty and Xie (1994) and Batty (1997). They proposed versatile and flexible algorithm, so various urban expansion models based on the algorithm are developed so far (e.g. Santé et al., 2010; Uesugi, 2009). Many of them are modified to include particular relaxation rules such as “distant-decay effect”, “zoning”, “constraints”, etc., which enables to reproduce more complex phenomena, while initial CA model adopts a simple transition rule in which the state of cell depends on the previous state of neighboring cells and itself. At present, versatile models like SLEUTH model, iCity, and CLEU-S are widely utilized to simulate urban dynamics and a number of models are still under development.

In this paper, CA model based on Batty and Xie (1994) is relaxed to accommodate a northern Mongolian city, Darhan, in semi-arid steppes. As one of several relaxations the model introduces, each cell has four states, i.e., 1) binary index of urbanity and seven types of land-use, 2) potential that shows how much the cell affords to develop up to the capacity, 3) development which indicates the degree of development of the cell, often similar to population, and 4) geographical condition (gradient). Transition between each land-use occurs by repetition of birth and death of agents that mean residents. These birth and death determine the population, the development of the city and approximate water demand in this area. These three factors are simultaneously cast into a simple system dynamics model and interact each other based on several hypothetical scenarios.

2.1 Land use
The land-use of the domain is categorized into seven patterns, i.e., residential area (apartment, ger), industrial area, commercial area, road, station, others (unused) and unavailable area. Simulation occurs on a regular lattice of which size is 100 m × 100 m in this research and the model does not have a concept of vector data, which means roads and stations cannot be expressed by cells properly. Then, the categories of roads and stations can exist simultaneously with other categories in a cell. Residential area consists of apartment and ger of which existence is quite unique characteristics of this region. A ger is originally a round tent covered with animal’s skins or felt for nomads in Mongolia and adjacent regions. Nomads generally immigrate for better grassland with their gers and livestock. In these years, however, quite a large number of nomads assemble to suburb of cities and often settle down in such areas unexpectedly within the regional and city planning. There are poor infrastructures for water supply, electricity, sewage and so on in the ger area. Therefore, though the ger area is certainly a residential area, it can be thought as a kind of temporal resident compound with relatively low population density. In addition, it is often the case that they move to another suburb or apartments in the city without hesitation, when the ger area becomes too crowded. Considering this property, ger area is discriminated from the apartment and its mobility is included in the transition rule. Compared with ger area, apartment has higher capacity for residents and higher consumption rate of municipal water. The location of apartment is much more stable than that of ger. The location and magnitude of commercial area, road and station cannot be determined autonomously by the model. They are given exogenously by the local government or scenarios for prediction.

2.2 Potential
Potential is defined as an indicator for prosperity and convenience of the cell and around the cell, and it also shows how much the cell affords to develop up to the capacity. It is utilized to decide the allocation of birth, transition and decay of the cells. In this research the potential depends on the ratio of urbanity in the extended Neumann neighborhood (7 cells × 7 cells, Ω2), distance from the station, that from the road, slope condition and room for new agents as given by Eq. (1).

$$PT = RU \cdot \left\{ AR + AS + (CP - DL) \right\} \cdot SL \quad (1)$$

where $PT$ is potential, $RU$ is a ratio of urban cells that consist of gers and apartments in the neighborhood, $AR$ is an index of distance from roads, $AS$ is an index of distance from the station, $DL$ is an index of prosperity or conceptual number of agents, $CP$ is a capacity for agents (constant for ger and equals to $DL$ for apartment) and $SL$ is an index of slope (classified slope into 5 levels, 1.0 for flat, 0.0 for more than 1/5), respectively. Figure 3 shows the cumulative number of gers against the distance from roads and that from the station, respectively, in this region, which apparently indicates logarithmic distributions. Hence, $AR$ and $AS$ are expressed by Eqs. (2) and (3), respectively.

$$AR = CR_1 \ln (dr/100) + CR_2$$

$$AS = CS_1 \ln (ds/100) + CS_2$$
where \( d_r \) is a distance from nearest road in (m), and \( d_s \) is a distance from the station in (m), \( CR_1, CR_2, CS_1, \) and \( CS_2 \) are coefficients to accommodate to the statistical data, respectively.

### 2.3 Birth and decay

This CA model proceeds by birth and decay of the cell, similar to Conway’s famous life game. The birth of this model is, however, controlled by externality, while the life game determines it intrinsically. It is due to immigrants from outside of the domain (\( \Omega_1: \) objective domain), which is frequently seen in every city and moreover the mobility of nomads is very high. Judging from a statistical data we assume that the population of this city \((PP)\) obeys to a logistic function expressed as;

\[
d_{PP} = rPP\left(1 - \frac{PP}{k}\right)
\]  

where \( r \) is the intrinsic growth rate and \( k \) is the carrying capacity, respectively. These coefficients are sought by Monte Carlo method for the calibration period and they are assumed to be valid throughout our simulation. Herein, additional assumption that the growth is restricted by water deficiency is introduced and then the growth rate \((\lambda)\) is modified as;

\[
\lambda' = \frac{PP_{\text{ini}}}{PP} - \left(1 - \frac{WR_0}{WR_0}\right) = \lambda - \delta
\]  

where \( \lambda' \) is a modified growth rate, \( WR_0 \) and \( WR_t \) are amount of water resources of initial and time \( t \), respectively. Subscript \( t \) and \( t+1 \) means time steps (year).

As is often the case in CA, some ratio of cells decays or dies randomly in the time marching. This model has a system almost similar to other CAs, but cells with lower potential and with more water deficiency are easy to die in this process as;

if \( n(t) = 1, PT < PT_{\text{av}}, \) and rand\((\lambda' + PT) < 1 - \sigma + w_{df},\)

then \( n(t+1) = 0 \)

where \( n(t) \) is a binary index of urbainity for a cell at time \( t, \) \( \text{rand}(x) \) is a random number between 0 and \( x, \) \( \sigma \) is a survival rate defined between 0 and 1, and \( w_{df} \) is an index of water deficiency given by a scenario, respectively. Subscript \( av \) means average over the domain. Then, the decrease of population \((DA; \text{non-dimensional value})\) is computed by;

\[
DC = \sum [DL(t) - DL(t+1)], \text{ for } n(t) - n(t+1) > 0
\]  

\[
NG_{\text{new}} = \beta(\lambda' - 1)NG_{\text{old}}
\]  

where \( NG_{\text{old}} \) are number of gers in the domain, and \( \beta \) is a fraction coefficient that indicates the ratio of inflowing immigrants in the population growth. Then, each parent cell picks up a candidate unused cells (child cells) in their neighborhood according to the following probability \((P_j)\);

\[
P_j = \sum_{i, t \in \Omega_1} PT_j DS_{ij}^{\text{av}}
\]  

where subscript \( i, j \) indicates the location of the candidate cell, \( DS \) is distance from the parent cell, and \( \alpha \) is an empirical coefficient so as to make probability half on the edge of neighborhood \((\Omega_2)\).

Subsequently, the rest of the parent cells contribute to develop the prosperity index of the existing gers. When a ger cell is selected similarly according to Eq. (9), the index of prosperity increases by one. In addition, when the prosperity of a cell attains to the capacity, agents begin to move to a newly built apartment and then the prosperity of the cell will be mitigated. The location of the new apartment is selected randomly at an unused cell adjacent to the existing apartment \((3 \text{ cells} \times 3 \text{ cells}, \Omega_2)\). The probability for this case is almost same to Eq. (9) in which \( \Omega_1 \) is used in place of \( \Omega_2 \).

Finally, the population \((\text{non-dimensional value})\) in the model is computed by the sum of \( DL \) of all the cells.

### 2.4 Transition rules and development

Transition of cells is an essential part of CA. Originally CA has simple rules, which is a superiority of the model, but the relaxations are inevitably introduced to simulate realistic urban expansions and such introductions become the originality of the models.

As time proceeds, \((\lambda' - 1)NG_{\text{old}} \) number of ger cells are selected randomly as parent cells and divided by a certain rate \((\beta)\) into two types, i.e., parent cell to produce a new ger cell (child cell) and that to enhance the prosperity of an existing ger cell. The number of parent cells to produce child cells \((= \text{the number of child cells}, NG_{\text{new}})\) is given by;

Where \( NG_{\text{old}} \) are number of gers in the domain, and \( \beta \) is a fraction coefficient that indicates the ratio of inflowing immigrants in the population growth. Then, each parent cell picks up a candidate unused cells (child cells) in their neighborhood according to the following probability \((P_j)\);

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Finally, the population \((\text{non-dimensional value})\) in the model is computed by the sum of \( DL \) of all the cells.
The growth of the city depends on the water resources as seen in Eq. (5), and vice versa. The amount of water resources is expressed by a groundwater level at battery of wells and it is initially set as 200 cm above the impermeable layer. The consumption of municipal water ($WD_{m}$) is also expressed by the index in unit of length and computed using $PT$. The consumption for a ger cell is set as $1.0\ PT$, and that for apartment is $2.0\ PT$. The consumption for a factory cell ($WD_{f}$) is given by;

$$WD_{f} = 10.0 \times \sum_{i} DL_{f} \quad \text{(11)}$$

where the subscript t and 0 indicates time. The water consumption ($WD$) is given by the sum of $WD_{f}$ and $WD_{m}$. Contrarily, groundwater is recharged and gains or recovers its water level of which amount is derived from the calibration.

In the model, a simple SD model with a single loop is complementarily introduced as in Figure 4. Consequently, the stock of water resources makes the growth of the city either increase or decrease.

### 3 Simulation

#### 3.1 Calibration

In the simulation for 1998-2009 the actual population data is applied to calibrate the model. The domain is a center of Darkhan of 12 km north to south and 10 km east to west. It is divided into 120 by 100 cells of which side lengths are 100 m. The coefficients used in the simulation is shown in Table 1.

Figure 5(a) indicates the land-use of Darkhan in 1998 that is the initial stage of the simulation. Figures 5(b) and 5(c) show the observed and simulated land-use maps in 2009, respectively. As seen, they do not differ much to show the calibration successfully executed qualitatively.

In addition to confirmation of the calibration quantitatively, the computed and actual properties of the urbanity are compared with fractal dimensions ($FD$) by Box-Count method (Eq. (12)).

$$FD = \lim_{\varepsilon \rightarrow 0} \frac{\ln NB_{\varepsilon}}{\ln(1/\varepsilon)} \quad \text{(12)}$$

where $\varepsilon$ is a side length of the box, and $NB_{\varepsilon}$ is a number of box requires to cover the urban cells. The calibration is basically carried out by trial and error and $FD$ for actual Darkhan and simulated one finally become 1.5488 and 1.5586, respectively (Figure 6). $FD$ indicates only rough feature of the city, but we guess their similarity is appropriate enough by these parameters.

#### 3.2 Scenario and results

In the simulation urban growth is assumed to depend only on the coefficients (intrinsic growth rate and carrying capacity) of the logistic equation for population and water deficiency, though urban expansion actually includes miscellaneous factors unknown a priori. The coefficients are basically computed from the statistical data of Darkhan, but it might change abruptly due to the local economic growth and the government policy, e.g. a new factory is invited or some enterprise goes bankrupt. Herein, the coefficients are also considered variables suitable for each scenario. Then, the following three scenarios are considered:

### Table 1: Coefficients and parameters in the calibrated run

| Coefficient/ parameter          | Symbol | Value (unit) | Note                                           |
|---------------------------------|--------|--------------|------------------------------------------------|
| Survival rate                   | $\sigma$ | 0.15         | estimated from fractal dimension               |
| Intrinsic growth rate           | $r$    | 0.1431(-)    | sought from calibration data                   |
| Carrying capacity               | $k$    | 75,618(-)    | sought from calibration data                   |
| Capacity for ger                | $CP_{\text{for ger}}$ | 5(-)         | arbitrarily decided                            |
| Initial prosperity of ger       | $DL_{\text{for ger}}$ | 3(-)         | arbitrarily decided                            |
| Initial prosperity of apartment  | $DL_{\text{for amparment}}$ | 5(-)         | arbitrarily decided                            |
| Rate of gers in transition      | $\beta$ | 0.4(-)       | estimated from fractal dimension               |
| Damping coefficient for neighborhood ($\Omega_2$) | $\alpha_2$ | -2.71        | empirically decided                            |
| Damping coefficient for adjacent area ($\Omega_3$) | $\alpha_3$ | -2.71       | empirically decided                            |
| Coefficient of logarithmic distribution for road  | $CR_1$ | 2.83         | sought from calibration data                   |
| Coefficient of logarithmic distribution for road  | $CR_2$ | 0.527+ln100  | sought from calibration data                   |
| Coefficient of logarithmic distribution for station | $CS_1$ | 3.23         | sought from calibration data                   |
| Coefficient of logarithmic distribution for station | $CS_2$ | -0.673+ln100 | sought from calibration data                   |
| Increase of groundwater level without withdrawal | $ws$   | 6(cm)        | sought from calibration data                   |

![Figure 4: Conceptual diagram of the model supported by SD procedure](image-url)
3.2.1 Scenario 0: Continuing current condition
All the parameters and coefficients sought by calibration are used to simulate the urban expansion for coming 41 years (2010-2050). Simulated results are illustrated in Figure 7. Figure 8 shows the simulated and observed groundwater levels and the number of gers and apartments.

They indicate that ger settlement is gradually increasing and apartment area is growing rapidly between 2010 and 2030. Then expansion of the apartment area is getting slow and almost converges during two decades after 2030 due to relative deficiency of groundwater. In this scenario water deficiency, however, is not critical to maintain the city. In contrast to the apartment area, the expansion of ger area that consumes less water continues to increase gradually throughout 2010-2050. In addition, sudden drop of the groundwater level in 2009 is due to a newly built factory given in the scenario a priori.

Here it should be noted that Ger area expansion implies neither population growth nor economic growth of Darkhan.
city exactly. It is due to several reasons listed below:
1) In comparison with apartment area, ger area has much less population density.
2) Installation of ger settlement does not require much investment, furthermore the market price of apartment is higher than the income of people living in the ger area.
3) Though there are a number of households using small wells in their own properties, the water quality of the well and their demand/consumption are unclear.
4) Massive migration from countryside to suburb area has started since 2000s in Mongolia.

If the development of the city continues after 2050 similarly, the expansion of the ger area might induces water deficiency problems. Or even if the water is kept sufficient enough to maintain the city, its quality of life would be worse instead of the growth in the number of gers.

3.2.2 Scenario 1: Gradual decay with water deficiency
The supply of water to the city depends on the increase of groundwater level without withdrawal (ws). In this scenario water supply is assumed to be restricted by one-third due to some factor, e.g. heavy consumption by factories. Then, the number of the employee might increase, and so the carrying capacity is assumed to 100,000. In addition, we assume the development of the city depends more seriously on the amount of water resources than Scenario 0. Then, we let the index of water deficiency (wdef) 0.1, so as to make the effective survival rate (σ) low, when the amount of water resources reduces by half. Moreover, when the amount of water resources reduces by one-third, wdef increases to 0.3. The intrinsic growth rate and the carrying capacity of the logistic equation are not unchanged from the value in calibration.

There are two concentrated zones of ger, located in northern and southern sides of the city as seen in Figure 9. The maps show that northern ger settlement area is declined year by year, while the southern area of ger settlement expands gradually. Compared with the expansion of the ger settlement area in scenario 0, the area in this scenario became much smaller. It means the residents of the ger area quickly react against the water deficiency (Figure 10). The resistance to the water deficiency of the residents in the ger area is actually not known well, but the results might show the possibility of the decay of the ger area with severe water deficiency.

Although apartment area is growing between 2010-2030, it decreases and becomes sparse after the period. In the model the mobility of the residents in the ger area is considered high, while the residents in the apartment do not move easily after their settlement. This assumption might give the slow response to water deficiency of the apartment area. It demonstrates that the decay of the city and accordingly reduction of economic condition of the city might occur due to severe water deficiency.

3.2.3 Scenario 2: Development of new water resources
When the amount of water resources reduces by half, new wells are assumed to be installed to supply sufficient municipal water in Scenario 2. Then ws increased twice as that of the Scenario 0. It indicates that water deficiency can be avoided by new well installation, even if extensive factory building such as mineral exploitation factory in 2009 happens. In such a case available water resource increases, so that the growths of both the ger and apartment areas are kept high. Then, the carrying capacity is assumed 300,000. Figure 11 shows that ger settlement area is remarkably increasing, much more extensively than other scenarios.

The behavior of the ger areas seems quite similar to that of the Scenario 0 (Figure 12), though their magnitudes are slightly different. On the other hand, the apartment area shows different tendency, namely, the apartment of Scenario 2 keeps expanding, while that of Scenario 0 shows its peak. They are due to the difference of the response to the water deficiency between ger and apartment defined in our model.

4 Conclusion
Simulation results may change to any degree according to the given parameters, scenarios and also inherent randomness of the model. However, they must show one of the possible results in the development of Darkhan city and can alert the sign of the decay. At least in our scenarios, water resources are one of critical factors to maintain the development of the city, especially for ger area.

The model includes some simple SD model regarding with water resources and the survival rate, which must effect
significantly to the development of the city. However, the remarkable fluctuations of the water resources, ger, and apartments are not seen, indicating that the response of the city to the SD model is not sensitive, compared with other factors.

In the calibration of the model, simulated results are consistent with the observed data to a certain degree, proved by the fractal dimensions. Some scenarios are successfully carried out for the prediction of the city. However, the brushing up of the model is still required, because of its lacks of some information and some subsystems to express the transition of the city, such as well withdrawal in the ger area and factories, the relationship between employment and population and so on.

In the coming years, Darkhan city is expected to develop as an industrial center of Mongolia, according to the planning and decisions of the Mongolian government. Therefore, the model suggests that water management is essential for its sustainable development.

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