Secure Data Sharing With Flow Model

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Abstract

In the classical multi-party computation setting, multiple parties jointly compute a function without revealing their own input data. We consider a variant of this problem, where the input data can be shared for machine learning training purposes, but the data are also encrypted so that they cannot be recovered by other parties. We present a rotation based method using flow model, and theoretically justified its security. We demonstrate the effectiveness of our method in different scenarios, including supervised secure model training, and unsupervised generative model training. Our code is available at https://github.com/duchenzhuang/flowencrypt.

1. Introduction

Data and model are the two most important factors in machine learning. Unfortunately, sometimes no one has both of them, so multiple parties have to collaborate together to solve certain problems. For the data providers, such collaboration can be risky: once the data is sent to the other parties, they may later sell or use the data without getting permissions from the data providers. This risk is becoming larger in recent years due to the larger volume of data owned by the providers.

Multi-party computation (MPC) (Yao, 1982; 1986; Goldreich et al., 1987; Chaum et al., 1988; Ben-Or et al., 1988; Bogetoft et al., 2009) is specifically designed for such scenarios. With rigorous theoretical guarantees, it provides one way to let multiple parties jointly compute any function and also keep each party’s data private. However, this general framework comes with a price: the computational overhead of MPC is so high that currently it cannot yet support large scale computational task like neural network training.

We consider a variant of this problem, where instead of requiring the data to be completely private so that no one gets any information about it, we only require data to be partially private. That is, no one can efficiently recover the original data, but users can extract other useful information from the encrypted data. Although being different, our requirement has the flavor of differential privacy (Dwork et al., 2006), e.g., users can obtain the average salary of all employees, but cannot figure out the salary of each individual.

Let us start with the following scenario: as a data provider, we want to hire a machine learning (ML) factory to train a model for us. However, we do not trust the ML factory, and do not want it to sell the data or the trained model to the third party. Therefore, our encrypted data must be secure in the sense that no one can decrypt it. Moreover, the encrypted data can be used for machine learning training, but the trained model is only useful to the data providers with the secret encryption key, and is useless to the other parties. See Figure 1 for illustration.

To ensure security of the encryption method, we assume that the adversary is very powerful, i.e., it has unlimited computational power, and also knows exactly the underlying distribution of our input data (See Definition 3 for the formal definition). Hence, an ideal encryption method in this setting should at least satisfy three properties:

- **Security**: Any adversary with the encrypted data, the distribution of the original input data, and unlimited computational power cannot decrypt our encrypted data.

- **Integrity**: The encrypted data contain enough information for machine learning training purposes, compared with the original data.

- **Specificity**: The models trained using the encrypted data only work for the encrypted test data, and cannot be directly applied to the original test data.

Using flow-based generative models (Dinh et al., 2014; 2016; Kingma & Dhariwal, 2018), we propose an encryption method that satisfies these three properties with theoretical guarantees. Flow models are bijective functions that map the input data distribution into a Gaussian-like distribution, which we call the feature distribution. In our encryption method, we first privately sample a orthogonal matrix, and...
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**Figure 1.** Illustration of the supervised secure model training scenario. The data provider wants to encrypt its private data so that the ML factory can use it to train a model, but the model can only be used by the data provider with the secret encryption key. Moreover, the data cannot be decrypted, so the ML factory cannot sell the data or the model to the third party.

**Figure 2.** Illustration of our encryption method. We first use a flow model to map the input data into the feature space, and then apply a private orthogonal transformation to the feature, and finally use the flow model to convert the encrypted feature back into the input space. See Figure 2 for illustration. The security of the encryption is ensured because the orthogonal matrix is private and cannot be recovered, as we will prove in Section 4.

Our encryption method is a general gadget that can be used in many different scenarios, not limited to the setting in Figure 1. For example, other parties may want to train a generative model based on our private data, but we do not want to share the data with them. As another example, when multiple parties are training a machine learning model together, they may face the data leakage problem when reporting the gradient during the training (Zhu et al., 2019). Our encryption method can be applied to both scenarios for making the private data secure. We will present more details about various applications of our encryption method in Section 5, as well as experimental results in Section 6. To our best knowledge, we are the first to encrypt data using invertible generative models. We hope that our method will serve as a useful building block for data sharing and ML algorithm design in the future.

2. Related Work

Multi-party computation (MPC) (Yao, 1982; 1986; Goldreich et al., 1987; Chaum et al., 1988; Ben-Or et al., 1988; Bogetoft et al., 2009) is a very powerful framework in the sense that it will keep all the input data private. Some of the results in MPC have been applied to the training process of neural networks: Zhang et al. (2015); Li et al. (2017); Aono et al. (2017) used homomorphic encryption to encrypt the training for neural networks, and Mohassel & Zhang (2017); Riazi et al. (2018) developed new protocols for training neural networks in a 2PC setting. However, as we mentioned before, MPC-based protocols have huge computational overhead during the encryption/decryption process and people can only afford to train small networks on small datasets, so they are not yet practical in deep learning training.

There are also papers that use differential privacy (Dwork et al., 2014) to preserve the data privacy during the training. These works only care about the privacy of the data, so the learned models can be directly applied to the original data distribution. Abadi et al. (2016) added noise in the training to ensure differential privacy for the neural networks, Vaidya et al. (2013) used Naive Bayes to handle the case when there is only one data provider, and Li et al. (2018) extended that to the multiple-data-provider case.

Deep Generative models are useful tools of learning data distribution and generating new data. There are several
kinds of deep generative models, e.g., Generative Adversarial Networks (Goodfellow et al., 2014), Variational Auto-Encoders (Kingma & Welling, 2013), auto-regressive models (Oord et al., 2016), and flow-based generators (Kingma & Dhariwal, 2018). Among these, we use flow-based models because it is reversible. Flow-based generative models learn the data distribution mainly by warping Gaussian distribution and minimizing the log likelihood. To speed up the computation, people have taken several approaches, including using affine transformations (Dinh et al., 2014; 2016; Kingma & Dhariwal, 2018), combining with auto-regressive methods (Kingma et al., 2016), and normalizing flows (Rezende & Mohamed, 2015).

Recently, Huang et al. proposes InstaHide, which also aims to encrypt the private data while preserving the learnability to neural networks. They mix private labeled images with public unlabeled images and applies a random sign-flip on the pixels to produce encrypted images which look like random noise. Our method, however, is invertible and hence guaranteed to preserve all the information of the original data.

### 3. Preliminaries

We use \( D \) to denote the underlying distribution of the input data points. Assume that our private data set contains \( n \) data points \( \{s_i\}_{i=1}^{n} \sim D \), where \( s_i \in \mathbb{R}^m \). We want to construct a mapping \( Enc : \mathbb{R}^m \rightarrow \mathbb{R}^m \) for the data encryption. In this paper, we assume the data points are images. This is without loss of generality, because our approach can easily generalize to other kinds of data.

#### 3.1. Total variation distance

We use the following metric for measuring the distance between two distributions.

**Definition 1** (Total variation distance). For two distributions \( P \) and \( Q \) defined on domain \( D \), we define the total variation distance as

\[
\delta(P, Q) \equiv \sup_{A \in D} |P(A) - Q(A)| \tag{1}
\]

For the total variation distance, we later need the following lemma that effectively removes the factor \( n \) in the exponent.

**Lemma 1** (Hoeffding & Wolfowitz, 1958), Eq.(4.5)). For two distributions \( P \) and \( Q \), any integer \( n \geq 1 \), the following inequalities hold:

\[
\delta(P^n, Q^n) \leq 1 - (1 - \delta(P, Q))^n \leq n\delta(P, Q). \tag{2}
\]

#### 3.2. Flow model

Flow-based generative models (Dinh et al., 2014; 2016; Kingma & Dhariwal, 2018), or simply flow models, are one kind of generative models. They assume that the data can be converted into a feature space by a bijective function \( f \). That is, given any input \( x \in \mathbb{R}^m \), there exists a corresponding latent variable \( z \in \mathbb{R}^m \), such that \( z = f(x) \), and \( x = f^{-1}(z) \). Moreover, the latent variables follow some simple distributions, e.g., multivariate Gaussian distribution.

In other words, flow models can effectively convert the original data distribution (which does not have an analytical form) into much simpler distributions like Gaussian distribution, and this mapping is invertible. There are many good flow models, and in this paper we mainly use the Glow model (Kingma & Dhariwal, 2018). We remark that if better flow models are introduced in the future, they can also be applied to our setting to improve the empirical performance of the encryption.

#### 3.3. Encryption Algorithm

Our encryption algorithm is sketched in Algorithm 1. Overall speaking, given the private data, we first train a flow model \( f \), which learns a bijection between the input space and the feature space. After that, we map each original image to a feature, “rotate” that feature using a uniformly-sampled orthogonal matrix, and then “recover” the rotated feature into a new image, which is the encryption of the original image.

**Algorithm 1** Encryption of Private Image Data

**Input:** Original images \( \{s_i\}_{i=1}^{n} \), each of size \( m \)

- Train a flow model \( f \) using \( \{s_i\}_{i=1}^{n} \)
- Sample \( A \) uniformly from all \( m \times m \) orthogonal matrices

\[
\begin{align*}
&\text{for } i = 1 \text{ to } n \text{ do} \\
&\quad x_i = f(s_i) \\
&\quad \hat{x}_i = Ax_i \\
&\quad Enc(s_i) = f^{-1}(\hat{x}_i)
\end{align*}
\]

Although being extremely simple, Algorithm 1 has many nice properties, as we will see in Section 4 and Section 5. Based on the encryption algorithm, we can define various feature distributions.

**Definition 2** (Feature distributions). We define \( G_{\alpha} \) to be the distribution of the features \( x_i \), and \( \mathcal{H}_0 \equiv \mathcal{N}(0, I_m) \) is the standard normal distribution. Also, define \( \mathcal{H}_1 \) to be the distribution of rotated features \( \hat{x}_i \).

#### 3.4. Adversary and orthogonal matrix

Below we formally define the power of our adversary.

**Definition 3** (Strong Adversary). An adversary is called **strong** if it knows the encryption algorithm (except the private orthogonal matrix \( A \)), the encrypted data, the underlying distribution \( D \) of the original input data, and has unlimited computational power.
This kind of adversary is very strong in the sense that it not only knows the encrypted data, but also has total information about the underlying distribution of the input. Besides, it can even use the flow model \( f \) in our encryption algorithm. The goal of this adversary is to recover the orthogonal matrix \( \hat{A} \) from these information. To formally define the notion of successful recovery, we first need to define volume and ball in the space of orthogonal matrices.

**Definition 4** (Normalized volume). Let \( U \) be the set of all orthogonal matrices with size \( m \times m \), and let \( T \) be a subset of \( U \). Define the normalized volume of \( T \) be

\[
v(T) \triangleq \mathbb{P}[A \in T],
\]

where \( A \) is sampled uniformly from all \( m \times m \) orthogonal matrices.

**Definition 5** ((\( \theta, A \))-ball). Let \( U \) be the set of all orthogonal matrices with size \( m \times m \). Given an orthogonal matrix \( A \in \mathbb{R}^{m \times m} \), \( \delta > 0 \), the ball centered at \( A \) with radius \( \delta \) is defined as

\[
\mathcal{B}_\delta(A) \triangleq \{ M ||M - A||_F \leq \delta, M \in U \}.
\]

Essentially, we use Frobenius norm to define a ball centered at matrix \( A \) with normalized volume \( \theta \). Here Frobenius norm is picked for convenience, and our analysis can be applied to other metrics as well. Now we can define whether a recovery is successful.

**Definition 6** (successful recovery with tolerance \( \theta \)). A recovery of matrix \( A \) by an adversary is a successful recovery with tolerance \( \theta \) if the output \( \hat{A} \) of the adversary is within the (\( \theta, A \))-ball.

In other words, if the output \( \hat{A} \) is close to \( A \) in terms of Frobenius norm, we say the recovery is successful.

### 4. Main Theorem

In this section, we will first present the main theorem, and then give the proof for the simple case and the general case in Section 4.1 and Section 4.2, respectively.

**Theorem 1** (Main Theorem). The success probability of any strong adversary trying to recover \( A \) with tolerance \( \theta \) cannot exceed \( \delta(\mathcal{H}_0, \mathcal{H}_1) + \theta \).

Our main theorem gives an upper bound of the success probability of any strong adversary that tries to recover matrix \( A \). This upper bound has two terms, the \( \theta \) term and the total variation term, which we elaborate below.

The \( \theta \) term is completely controlled by the data provider. For example, if \( \theta = 0.5 \), that means we define the recovery to be successful if the adversary can find an orthogonal matrix that is closer to \( A \) compared with at least half of all possible orthogonal matrices. In this case, the adversary can simply do a random guess to get 0.5 success probability. Indeed, the \( \theta \) term is inevitable, because adversary can always randomly sample matrix that is in (\( \theta, A \))-ball with probability \( \theta \).

The total variation term measures the distance between the feature distribution and the Gaussian distribution. Intuitively, the Gaussian distribution is the most difficult case for the adversary because it is rotation-invariant. In other words, before and after applying the orthogonal transformation, the data look the same to the adversary, which makes the recovery of the exact \( A \) impossible. Therefore we have this term in the upper bound, indicating that if the distance to the Gaussian distribution is smaller, it will be harder to decrypt the matrix \( A \). In practice, a better flow model will make this distance smaller.

Using Lemma 1, we immediately get the following corollary, which removes the exponent \( n \) in the upper bound.

**Corollary 1.** The success probability of any adversary trying to recover \( A \) with tolerance \( \theta \) cannot exceed \( n\delta(\mathcal{H}_0, \mathcal{H}_1) + \theta \).

#### 4.1. Proof of the simple case

Let’s start the proof with a simple example where \( \delta(\mathcal{H}_0, \mathcal{H}_1) = 0 \), i.e., the distribution \( \mathcal{G}_0 \) is exactly equal to standard normal distribution. In this case, \( \delta(\mathcal{H}_0, \mathcal{H}_1^\theta) = 0 \), so our main theorem can be simplified to the following lemma.

**Lemma 2.** If \( \mathcal{G}_0 = \mathcal{H}_0 \triangleq \mathcal{N}(0,I_m) \), then the success probability of any adversary trying to recover \( A \) with tolerance \( \theta \) cannot exceed \( \theta \).

**Proof.** If \( \mathcal{G}_0 = \mathcal{N}(0,I_m) \), i.e., \( \forall i \in [n], x_i \sim \mathcal{N}(0,I_m) \), we can compute the distribution of \( \hat{x}_i \):

\[
\mathbb{E}[\hat{x}_i] = A\mathbb{E}[x_i] = A \cdot 0 = 0.
\]

\[
\text{Var}(\hat{x}_i) = A \cdot \text{Var}(x_i) A^\top = AA^\top = I_m.
\]

Note that the distribution of \( \hat{x}_i \) is still a Gaussian distribution, and it has the same expectation and variance as \( x_i \). Thus,

\[
\mathcal{H}_0 = \mathcal{H}_1.
\]

Therefore, for all orthogonal matrix \( A \), the distribution of \( \text{Enc}(s_i) \) are the same, i.e., \( A \) is independent of the distribution of the encrypted images. This means that given the encrypted images, any output from the adversary is equivalent to a random guess over all orthogonal matrices. From the definition of tolerance, we know that the success probability of any adversary trying to recover \( A \) with tolerance \( \theta \) cannot exceed \( \theta \).
This lemma tells us that if the distribution of the features is exactly standard normal distribution, i.e., the first term in the upper bound is 0, then the only possibility that the adversary can succeed is that the random guess is within the tolerance. In the next part, we will provide a formal proof for the general case, i.e., our main theorem.

4.2. Proof of the general case

In this part, we will show the proof of our main theorem, which is a generalization of the proof for Lemma 2.

We first provide the intuition of this proof: What the adversary can observe is only the encrypted data, which can be considered as some samples of the distribution of $Enc(s_i)$. From the proof of Lemma 2 we know that if $G_g = N(0, I_m)$, the adversary will output a random guess of matrix $A$. Thus, given $n$ samples of the encrypted data distribution, if the adversary cannot distinguish between $H_1$ and $H_0$, it will be impossible for him to do better than random guess in recovering matrix $A$.

Let us formalize this intuition into a full proof:

Proof of Theorem 1. Firstly, we restrict this problem in feature space: The data-to-feature map (flow model) is a bijection and known to the adversary, and the adversary has unlimited computational power. Thus, given any data or distribution in the image space, the adversary can map it to the feature space using the data-to-feature map, and vice versa. Therefore, it suffices to only consider the data and distributions in the feature space.

We will prove that if we can recover the matrix $A$ with high probability, then we can distinguish very similar distributions with high probability, which is impossible. Below we first formally define the two related problems, and then we will do a recursion between them.

Problem 1 (distinguishing distributions): Given the Gaussian distribution $H_0 = N(0, I_m)$, feature distribution $G_g$, rotated feature distribution $H_1$, and matrix $A$ uniformly sampled from the set of all orthogonal matrices. Sample $n$ i.i.d. samples $\{x_i\}_{i=1}^n \sim H_b^n$. We want to find $b$.

Problem 2 (recovering matrix $A$): Given a feature distribution $G_g$, and samples of rotated features $\{x_i\}_{i=1}^n \sim (H_1)^n$ and the we want to recover the rotation matrix $A$.

For a strong oracle $O_2$ for Problem 2, assume that its success probability is $p$. Note that this success probability is the expectation taken over the samples $\{x_i\}_{i=1}^n$, the choice of orthogonal matrix $A$, and the randomness of $O_2$ itself.

We remark that $O_2$ can also take $G_g$ and $\{x_i\}_{i=1}^n \sim (H_0)^n$ as the input because the input shape matches. However, since Problem 2 expects $\hat{x}_i$ to be sampled from $H_1$, if we use $H_0$ instead, $O_2$ may generate unexpected outputs without any guarantees on the success probability.

We now use this strong oracle $O_2$ to construct an oracle $O_1$ for Problem 1: Let

$$O_1 = \mathbb{1}_{\text{success}}[O_2(G_g, x_1, \cdots, x_n)],$$

(8)

In other words, $O_1$ first assumes $b = 1$, and then calls $O_2$ with the feature distribution and the $n$ samples, receive the output matrix $\hat{A}$ of $O_2$. $O_1$ outputs 1 if $\hat{A}$ is a successful recovery with tolerance $\theta$ to $A$, and 0 otherwise.

Define the success probability of $O_1$ be the expected probability of returning the correct bit $b$, where the expectation is taken over the randomness of bit $b$ and the oracle $O_2$.

When $b = 0$, i.e., the samples come from the Gaussian distribution, from the proof of Lemma 2 we know that no algorithm can get any information about $A$, i.e., the output of $O_2$ is independent with $A$. Since $A$ is a randomly sampled orthogonal matrix, we know that when $b = 0$, the success probability of $O_1$ (i.e., the probability of outputting 0) is $1 - \theta$.

When $b = 1$, we know that its success probability is equal to the success probability of $O_2$, which is $p$. Therefore, the overall success probability of $O_1$ is $\frac{(1 - \theta) + p}{2}$, as $b$ is uniformly sampled from $\{0, 1\}$.

However, from the Bayes risk of binary hypothesis test (Wainwright, 2019), we know that the maximum success probability for Problem 1 is $\frac{1 + \delta(H_0^n, H_1^n)}{2}$.

Thus,

$$\frac{(1 - \theta) + p}{2} \leq 1 + \delta(H_0^n, H_1^n) \frac{1}{2},$$

(9)

i.e.,

$$p \leq \delta(H_0^n, H_1^n) + \theta.$$

(10)

5. Applications

In this section we discuss various applications of our encryption method. We believe that this method can serve as an important building block for protecting privacy in machine learning.

5.1. Supervised secure model training

Our first example is the setting described in Figure 1. To solve this problem, we may directly apply our Algorithm 1 to each class of images. That is, assuming that there are $N$ classes, we train a flow model $f_i$ for the $i$-th class. In the dataset, for each pair of data points $(x, y)$, we encrypt $x$ to get $\hat{x}$ using $f_y$, and send $(\hat{x}, y)$ to the ML factory. Notice
that we cannot train a simple flow model for all classes because after the whole dataset encryption using a single model, the input-label correspondence might be lost, but independent encryption for each class does not have this problem.

This solution satisfies the three properties that we required: Security, Integrity and Specificity. The Security property holds because the ML factory cannot decrypt the orthogonal matrix for each class, as we shown in Theorem 1. Therefore, it cannot recover the original data points.

The Integrity property holds because we apply orthogonal transformation in the feature space, which has semantic meanings. After the transformation, the encrypted features follow a similar Gaussian-like distribution, so they still contain enough information for machine learning training purposes. We will further empirically demonstrate this property in our experiments.

We verify the Specificity property of our method by running simulations, as we will see in Section 6.1. That is, after being trained on the encrypted data, the model does not have good prediction accuracy on the original data distribution. In order to apply the model, one needs to first encrypt the data using the secret orthogonal matrix $A$. In other words, only the data provider can use the trained model.

### 5.2. Unsupervised generative model training

Consider the scenario that the ML factory wants to learn a generative model using the private unlabeled data from the data provider. The data provider agrees to let ML factory train the model, but does not want it to see the private dataset. In this case, we can apply our Algorithm 1 to encrypt the data, and ask the ML factory to train the generative model using the encrypted data. The intuition is, after the orthogonal transformation in the feature space, the encrypted data still contain enough semantic information for the data distribution (the Integrity property).

As we demonstrate in Section 6.2, this solution works well, and the encrypted data can indeed be used for training a very good generative model.

### 5.3. Data leakage with gradients

In this scenario, multiple agents collaborate together to train a model, and each agent has its own private data. As argued by Zhu et al. (2019), each agent faces the problem of data leakage when sharing the gradients with the other agents.

Specifically, for the synchronous distributed training, in $t$-th iteration, agent $i$ samples a minibatch $(x_{t,i}, y_{t,i})$ to compute the gradients $\nabla W_{t,i}$

$$
\nabla W_{t,i} = \frac{\partial \ell(F(x_{t,i}, W_t), y_{t,i})}{\partial W_t} 
$$

(11)

where $\ell$ is the loss function, $F$ is the neural network function, and $W_t$ is the synchronized weight for the network at the iteration $t$. Then the gradients are averaged across the $N$ agents, so the weight $W_t$ is updated as:

$$
W_{t+1} = W_t - \eta \frac{1}{N} \sum_{j=1}^{N} \nabla W_{t,j}
$$

(12)

The data leakage happens if an adversary has the information of the gradients, model architecture and weights. In this case, the input data can be recovered by minimizing the distance between the actual gradients and gradients obtained from the randomly initialized dummy input $x'$ and label $y'$ (Zhu et al., 2019), where the optimization is applied to $x'$ and $y'$. The gradient from $x'$ and $y'$ is defined as

$$
\nabla W' = \frac{\partial \ell(F(x', W), y')}{\partial W}
$$

(13)

Our encryption can help avoid data leakage under the assumption that all the agents can be trusted, and they share the common private orthogonal matrices for each class of data. In this setting, all the data points can be encrypted, and the adversary will not be able to learn the private data from the gradients without knowing the orthogonal matrices. As we will show in Section 6.3, the adversary can only recover the encrypted data using the gradients.

![Original Image](Image 327x250 to 377x300)

(a) Original Image

![Encrypted Image](Image 329x325 to 379x375)

(b) Encrypted Image

Figure 3. Visualization of the images before and after encryption. The first row contains original images and the second row contains the corresponding encrypted images.

### 6. Experiments

In this section, we present the experimental results of our method for different scenarios discussed in Section 5: supervised secure model training (Section 6.1), unsupervised generative model training (Section 6.2) and data leakage with gradients (Section 6.3). We use PyTorch (Paszke et al., 2019) for all the experiments.
Glow Model (Kingma & Dhariwal, 2018). We use Glow model in our experiments. Glow model can learn distribution on unbounded space, but the data have bounded interval range. To reduce the impact of boundary effects, we model the density as logit($\alpha + (1 - \alpha) \odot x/256$) following (Dinh et al., 2016). In our implementation, we use activation normalization as our normalization method, which is a scale and bias layer with data dependent initialization. For permutation, we use an (learned) invertible 1 × 1 convolution, where the weight matrix is initialized as a random rotation matrix. Also, we use the Affine coupling layer instead of the Additive coupling layer. We set #levels to 3, steps to 16, learning rate to 0.001 and batch size to 512.

We visualize some figures before and after the encryption in Figure 3. As we can see, original images and their corresponding encrypted images visually belong to the same class. This shows that the Glow model can indeed capture the semantic meanings of the inputs.

### 6.1. Supervised secure model training

In this subsection, we present experimental results for the two natural image datasets: CIFAR-10 (Krizhevsky et al., 2009) and LSUN (Yu et al., 2015). For every dataset, we train a separate Glow model for each individual class. Every CIFAR-10 class has 5,000 data points, while LSUN is much larger. Except 126, 227 data points for the class of outdoor church and 168, 103 data points for the class of classroom, we use randomly sampled 150,000 data points for all other classes to train Glow models. We call this case LSUN(1). We also tried another experiment where each Glow model uses only 5,000 data points for training, which is called LSUN(2).

#### Experiment Details. We train our model on two natural image datasets: CIFAR-10 (Krizhevsky et al., 2009) and LSUN (Yu et al., 2015). For every dataset, we train a separate Glow model for each individual class. Every CIFAR-10 class has 5,000 data points, while LSUN is much larger. Except 126, 227 data points for the class of outdoor church and 168, 103 data points for the class of classroom, we use randomly sampled 150,000 data points for all other classes to train Glow models. We call this case LSUN(1). We also tried another experiment where each Glow model uses only 5,000 data points for training, which is called LSUN(2). Essentially, in the first setting, the encryption method uses a more powerful Glow model trained with more data points.

All the images are resized to 32×32 and all Glow models are using the same hyper parameters as mentioned above. For LSUN, we sampled 60,000 data points for model training (50,000) and testing (10,000). As we proved in Theorem 1, our encryption method is secure. Therefore, it remains to see whether it also has the Integrity and Specificity property, which we elaborate below.

#### Integrity. In Table 1, we present the experimental results of different models on different datasets. As we can see, after being trained on the encrypted data, both ResNet and DenseNet can achieve very good accuracy on the encrypted test data, i.e., 88% for CIFAR-10, and 92% − 93% for LSUN(1). It is worth pointing out that the test accuracy for the encrypted case is not as high as the accuracy for the original case for CIFAR-10. We believe that this is because CIFAR-10 is a small dataset, and cannot be used for training good Glow models. By contrast, in LSUN(1), when we use very large dataset to train the Glow models, the accuracy is increased significantly, even more than the accuracy in the original case. This is because the original case does not have the extra information from Glow models, and only relies on the information from the 50,000 training data. As a result, in LSUN(2), when the Glow models are trained using 5,000 data points each, the accuracy in the encrypted case dramatically decreased. Hence, when well trained flow models are available, the Integrity property can be ensured in practice.

#### Specificity. In Table 1, when the models are trained using encrypted datasets, they will have good accuracy on encrypted test data, but bad accuracy on the original test data. In other words, the trained model can only be used by the data provider who knows how to encrypt data, and cannot be used by other parties.

| TrainSet          | TestSet       | Neural Network |        |        |
|-------------------|---------------|----------------|--------|--------|
|                   |               | ResNet50       | DenseNet121 |
| Original CIFAR    | Ori-CIFAR     | 95.51          | 95.54  |
|                   | Enc-CIFAR     | 44.49          | 45.83  |
| Encrypted CIFAR   | Ori-CIFAR     | 33.69          | 38.45  |
|                   | Enc-CIFAR     | 88.53          | 88.73  |
| Original LSUN     | Ori-LSUN      | 84.86          | 85.99  |
|                   | Enc-LSUN      | 69.00          | 69.77  |
| Encrypted LSUN(1) | Ori-LSUN      | 55.83          | 58.33  |
|                   | Enc-LSUN      | 92.31          | 93.21  |
| Encrypted LSUN(2) | Ori-LSUN      | 25.53          | 30.10  |
|                   | Enc-LSUN      | 77.96          | 78.69  |

| Glows-Similarity | CIFAR-10      | ImageNet       |
|------------------|---------------|----------------|
| Glow-original    | 3.49          | 3.76           |
| Glow-encrypted   | 3.67          | 3.93           |
| Glow-untrained   | 9.75          | 9.66           |

### 6.2. Unsupervised generative model training

In this subsection, we present experimental results for the unsupervised generative model training described in Section 5.2. We hope that the generative model trained on the encrypted data is as good as the one trained on the original data. For simplicity, we also use Glow as the generative...
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model for training. But in practice, one can also use the encrypted data to train GANs or VAEs. Below we show both quantitative and qualitative experimental evaluations.

![Figure 4](Image)

**Comparing the two Glows trained on original data and encrypted data.** The first row are original CIFAR-10 images. We extract their features by the glow trained on original CIFAR-10 images, and reverse the features by the glow trained on encrypted CIFAR-10 images to get the second row’s images.

**Quantitative Evaluations.** For quantitative evaluations, we use Bit per dimension (BPD) to evaluate the similarity. BPD is defined as the negative log likelihood (with base 2) divided by the number of pixels.

In this experiment, we train Glow models in both original dataset and encrypted dataset. Notice that the models are trained using data points from all classes. The results are shown in Table 2. As we can see, the BPDs of both original dataset and encrypted dataset are close, for both CIFAR-10 and ImageNet. By contrast, the BPDs for Glows without any training (i.e., at its random initialization) are much larger. In other words, from the BPD perspective, the Glow models trained on original and encrypted datasets are similar.

**Qualitative Experiments.** In this experiment, we train two Glow models, on the original CIFAR-10 and the encrypted CIFAR-10 datasets, respectively. We then first extract the feature of a given original image using the first model, and then use the second model to recover the original image. See Figure 4 for the final results.

Although they are not exactly the same (because the two Glow models are different), we can tell that they are visually similar to each other. We believe that this shows the two models contain lots of common information. Since our encryption method ensures the security of the training data, it is a good way to let others to train generative models without sharing the training data.

**6.3. Data leakage with gradients**

In this subsection, we run experiments for data leakage with gradients, as discussed in Section 5.3.

**Experimental Setup.** Our experimental settings are generally following Zhu et al. (2019). We use L-BFGS (Liu & Nocedal, 1989) as our optimizer with learning rate 1. We set batch size 1 and iteration 100 for every image. To meet the requirements that the model needs to be twice-differentiable, we replace activation ReLU to Sigmoid and remove strides. For the labels, we randomly initialize a continuous vector instead of directly optimizing the discrete vector. For the inputs, we randomly initialize a continuous vector with shape \(N \times C \times H \times W\). Here the \(C = 3\), \(N = 1\) and \(H = W = 32\). For convenience, all the experiments are using randomly initialized weights. We apply the method described in Section 5.3, i.e., all the agents will share the same secret orthogonal matrices, and use those matrices for data encryption.

**Experimental Results.** In Figure 5, We show the leaked images using gradient information. The recovery of the encrypted image is successful, but as we proved in Theorem 1, the adversary cannot further recover the original image from the encrypted image.

We also steal all the encrypted images using the gradient information for both CIFAR-10 and LSUN(1) datasets, and use those images for training. See Table 3 and Table 4 for details. We can see that the trained model has very bad accuracies on the original test data, and slightly higher test accuracy on the encrypted test data.

To sum up, leaking data from gradients can only get the encrypted data, so our encryption keeps the private data safe. Moreover, from our experimental results, the accuracy of model trained on leaked data does not work on the

![Figure 5](Image)
original test data, therefore is useless to the adversary for classification tasks.

7. Conclusion

In this paper, we proposed a data encryption algorithm for theoretically ensuring the privacy while preserving useful information from the data. Our method is a building block which can be applied in many scenarios in machine learning. We give theoretically guarantees for the security of our encrypted algorithm, and our experiments demonstrated that our methods are useful in real datasets like LSUN and CIFAR-10. For the future work, we hope to apply our method to other interesting problems.

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