Abstract. Redactable signature allows anyone to remove parts of a signed message without invalidating the signature. The need to prove the validity of digital documents issued by governments is increasing. When governments disclose documents, they must remove private information concerning individuals. Redactable signature is useful for such a situation. However, in most redactable signature schemes, to remove parts of the signed message, we need pieces of information for each part we want to remove. If a signed message consists of \(\ell\) elements, the number of elements in an original signature is at least linear in \(\ell\). As far as we know, in some redactable signature schemes, the number of elements in an original signature is constant, regardless of the number of elements in a message to be signed. However, these constructions have drawbacks in that the use of the random oracle model or generic group model.

In this paper, we construct an efficient redactable signature to overcome these drawbacks. Our redactable signature is obtained by combining set-commitment proposed in the recent work by Fuchsbauer et al. (JoC 2019) and digital signatures.

Keywords: Redactable signature scheme · Compactness · Storing redactable signature problem · Set-commitment scheme.

1 Introduction

1.1 Background

Digital signature is an important cryptographic tool for data authentication. This allows a verifier to authenticate messages by verifying the signature. By using digital signature, we can ensure that a message has not been modified since it was signed. This property is useful for many scenarios.

However, in some scenarios, some limited modification of the signed message is desirable. For example, we consider a situation where a citizen requests a secret signed document disclosure to the government. To disclose the secret signed document, if privacy information is contained in the signed document, the government must remove this information from the signed document. In a digital signature scheme, to ensure the validity of the modified document, the signer must resign this modified document. If the original signer is not reachable anymore, or resigning the modified document produces too much overhead, it is not convenient.

Redactable signature is a useful cryptographic tool for the above situation. This scheme allows anyone to remove parts of the message from the signed message and update its signature without a signing key. We can check the validity of signed messages or submessages derived from signed messages. Note that, in the context of redactable signature, an operation that removes some parts from a signed document is called “redaction” and a message removed some parts of the signed message is called “submessage”.

The idea of redactable signature was introduced by Steinfield, Bull, and Zheng [45] as a content extraction signature. This allows generating an extracted signature on selected portions of the signed original document while hiding removed parts of portions. Johnson, Molnar, Song,
and Wagner [28] proposed a redactable signature which is similar to a content extraction signature. In addition, Miyazaki, Susaki, Iwamura, Matsumoto, Sasaki, and Yoshiura [37] proposed the digital document sanitizing problem, and proposed the redactable signature scheme called SUMI-4.

Note that, in early studies of redactable signatures, the term “sanitizing” indicates “removing”. Later, Ateniese, Chou, de Medeiros, and Tsudik [5] formalized sanitizable signature. They use the term “sanitizing” to indicate ”modifying”. That is, sanitizable signature allow some specific party to ”rewrite” some message parts. It is necessary to pay attention to distinguish whether the term “sanitizable signature” indicates redactable signature or sanitizable signature in the sense by Ateniese et al. [5].

Redactable signature has been studied for fundamental message data structures such as sets and lists. Redactable signature has been extended to more complex data structures such as trees [11,13,40,41], graphs [32], super-sets [38].

Security of redactable signature has been argued in many works. Most works consider the following two security notions in common. Unforgeability: An adversary cannot produce a signature for \( M' \) except for any redacted version of an already signed one. Privacy: Except for a signer and redactors, it is hard to derive information on redacted message parts when given a redacted message-signature pair.

Some works (e.g., [11,17,39,42]) consider the security notion called transparency which strengthens the notion of privacy. Transparency requires that it is hard to distinguish whether a signature \( \sigma \) is an original signature or redacted ones.

Camenisch, Dubovitskaya, Haralambiev, and Kohlweiss [12] proposed unlinkable redactable signature. This signature satisfies unforgeability and unlinkability which is a variant security notion of privacy. They used an unlinkable redactable signature scheme to construct an anonymous credential scheme [14]. Later, Sanders [43] also constructed an unlinkable redactable signature scheme to obtain an efficient anonymous credential scheme.

Currently, there are many studies for redactable signature and for its variants. However, due to space limitations, we only mention principal researches related to our works. See [8,15] for a more comprehensive overview of studies for redactable signature and its variants.

### 1.2 Motivation

In the use of a redactable signature, there is a problem we should consider. Let us consider the following situation. The government stores many secret signed original documents to a private cloud server. For a disclosure request of a secret signed document from a citizen, the government officer retrieves the signed document from the private cloud server, remove privacy information, and disclose subdocument of the signed document.

When the government uses the private cloud server to store original signatures, there is a problem. If the size of an original signature is too large, it takes too much time to upload the original signature to the private cloud server due to the limitation of internet communication bandwidth. Unfortunately, many redactable signature schemes have a linear signature size in the number of elements in a message. This makes it difficult to achieve quick uploading of an original signature to the private cloud server.

To overcome this problem, we requires that the size of an original signature is always constant regardless of the number of elements in a message. Moreover, it is also desirable that the government officer can remove any parts from the original document. How do we achieve these requirements? This is a natural problem for a practical use of redactable signature. Thus, we newly propose this problem as “storing redactable signature problem”.

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1. In this work, the size of the signature is measured by the bit length of signature encoded in a bit string.
To solve the “storing redactable signature problem”, we require the “compactness” for an original signature. That is, the size of an original signature is always constant regardless of the number of elements in a message.\(^2\)

Most redactable signature schemes with sets or lists seem hard to solve this problem. Since the number of elements in an original signature is at least linear in the number of elements in an original message.

As far as we know, redactable signature schemes in [4,12,43] satisfy compactness. However, these schemes have drawbacks. The redactable signature scheme by Ahn, Boneh, Camenisch, Hohenberger, Shelat, and Waters [4]\(^3\) uses the random oracle model (ROM) [7]. The redactable signature scheme by Sanders [43], its security is guaranteed in the generic group model (GGM) [44]. It is desirable to solve the “storing redactable signature problem” without the GGM or ROM.

As for the redactable signature scheme by Camenisch et al. [12], security of their redactable signature scheme relies on the \(\ell\)-RootDH assumption [12], \((\ell + 1)\)-BSDH assumption [21], and the existentially unforgeable under chosen-message attacks (EUF-CMA) security for partial randomizable structure-preserving signatures (SPS) where \(\ell\) is an upper bound for the number of elements in a message to be signed. As far as we know, two partial randomizable SPS scheme exists. One is proposed by Abe, Fuchsbauer, Groth, Haralambiev, and Ohkubo [1]. The EUF-CMA security of this scheme is proven under the \(q\)-simultaneous flexible pairing (\(q\)-SFP) assumption [1] where \(q\) is the number of signatures issued by the signer. The other is proposed by Abe, Groth, Haralambiev, and Ohkubo [2]. This scheme has an optimal signature size. That is, a signature is composed of only 3 group elements. However, the security of this scheme is proven in GGM. If we avoid using GMM and adapt the partial randomizable SPS by Abe et al. [1] to redactable signature scheme by Camenisch et al., the security of this redactable signature scheme relies on three \(q\)-type assumptions: \(\ell\)-RootDH; \((\ell + 1)\)-BSDH; and \(q\)-SFP assumptions where \(q\) is the number of original signatures issued by the signer. These assumptions are not standard. It is desirable to construct a redactable signature scheme whose security can be proven with two or less \(q\)-type assumptions.

1.3 Our Results

In this paper, we give a new redactable signature scheme with compactness from a set-commitment scheme and a digital signature scheme.

A set-commitment scheme proposed by Fuchsbauer, Hanser, and Slamanig [19] allows us to commit to a set. This supports ordinary opening and supports subsets opening. Specifically, in a set-commitment scheme, we can commit to set \(S\) and generate a commitment \(C\) and its opening information \(O\). Moreover, from \((S, C, O)\), we can generate a witness \(W\) for a subset \(S' \subseteq S\). By using \((S, C, O)\), we can verify that \(S\) is committed to \(C\). Also, by using \((S', C, W)\), we can verify that \(S'\) is a subset of \(S\) which is committed to \(C\). Fuchsbauer et al. [19] constructed a set-commitment scheme under the \(q\)-co-discrete logarithm (\(q\)-co-DL) assumption [19] and the \(q\)-co-generalized-strong-Diffie-Hellman (\(q\)-co-GSDH) assumption [19]. Moreover, they constructed attribute-based anonymous credentials by combining set-commitment and structure-preserving signatures on equivalence classes (SPS-EQ) works on the type 3 pairings.

Here, we briefly explain the idea of our redactable signature construction. The property of set-commitment is similar to the property of redactable signature for set message structures.

\(^2\) More precisely, we say that redactable signature satisfies compactness if the size of both an original signature and signature for a subdocument (redacted message) are always constant regardless of the number of elements in messages.

\(^3\) In [4], Ahn et al. proposed \(P\)-homomorphic signature schemes where \(P\) is a predicate. This scheme allows anyone to derive a signature on the object \(m'\) from a signature of \(m\) as long as \(P(m, m') = 1\) for the predicate \(P\). If we set \(P\) such that \(P(m, m') = 1\) if \(m'\) is a subdocument of \(m\) and \(P(m, m') = 0\) if \(m'\) is not a subdocument of \(m\), we can use \(P\)-homomorphic signature schemes as redactable signature schemes.
Redactable signature with sets allows us to derive a submessage $M' \subseteq M$ from the signed message $M$. The key idea is to combine the set-commitment scheme with redactable signature scheme. The signature $\sigma$ on an original set-structured message $M$ is composed of $(C, \sigma_{\text{DS}}, O)$ where $(C, O)$ is a set commitment and opening information pair computed by committing $M$, and $\sigma_{\text{DS}}$ is a digital signature on $C$. Redaction from an original message $M$ to a submessage $M'$ is can be done by deriving a witness $W$ for $M'$ by using $(M, C, O)$. The redactable signature for the message $M'$ is composed of $(C, \sigma_{\text{DS}}, W)$. See Section 5 for our construction.

Our redactable signature scheme for sets is constructed from set-commitment and a digital signature. To compare our redactable signature scheme with other redactable signature schemes, we consider the concrete instantiation for our scheme. We instantiate a redactable signature scheme by adopting the set-commitment scheme by Fuchsbeuer et al. and the structure-preserving signature by Kiltz, Pan, and Wee [30]. We explain the reason why we adopt the structure-preserving signature by Kiltz et al. Firstly, both the set-commitment scheme by Fuchsbeauer et al. and the structure-preserving signature by Kiltz et al. work on type 3 pairings. Secondly, in the set-commitment scheme proposed by Fuchsbeauer et al., the commitment $C$ belongs to $G_1$. We need a signature scheme that supports $G_1$ element signing. The structure-preserving signature by Kiltz et al. supports $G_1$ element signing. Finally, the structure-preserving signature by Kiltz et al. is efficient and its security is proven without GGM or ROM. Our instantiated redactable scheme can be proven under the $\ell$-co-DL, $\ell$-co-GSDH, and SXDH assumption [6] where $\ell$ is an upper bound for the number of elements in a message to be signed.

We summarize redactable signature scheme with compactness and major redactable signature schemes for sets or lists in Fig. 1. Our redactable signature scheme is a better solution for the “storing redactable signature problem” than other redactable signatures schemes with compactness [4,43] in that our scheme does not use the GGM or ROM. The redactable signature scheme by Camenisch et al. instantiated by the partial randomizable SPS by Abe et al. [1] relies on three $q$-type assumption. Compared with this redactable signature scheme, our scheme is milder in that our scheme can be proven with two $q$-type assumptions to two (the $l$-co-DL and $l$-co-GSDH assumptions). Moreover, in the security redactable signature by Camenisch et al., the parameter $q$ of the $q$-SFP assumption depends on the number of signatures issued by the signer. By contrast, all assumptions (the $l$-co-DL, $l$-co-GSDH, and SXDH assumptions) we need to prove the security of our scheme are independent of the number of signatures issued by the signer.

Furthermore, in the redactable signature scheme by Camenisch et al., to generate a redacted version of signature, we must prove pairing equations by using a WI-PoK proof system. For this reason, this causes somewhat large signature size. We estimate the size of redacted version of a signature of their scheme in Fig. 1 in the case of adapting the Groth-Sahai extractable WI-PoK system [22] based on the SXDH assumption. By comparing instantiations of our redactable signature scheme and that of scheme by Camenisch et al. in Fig. 1, our scheme has advantage with the concrete instantiation of their method, our scheme has shorter redacted signature size.

Our redactable signature scheme is similar to the redactable signature scheme by Camenisch et al. [12]. In their redactable signature scheme, the signature $\sigma$ on an original vector-structured (list-structured) message $M$ is composed of $(C, \sigma_{\text{PRSPS}}, O)$ where $(C, O)$ is a vector commitment and opening information pair computed by committing $M$, and $\sigma_{\text{PRSPS}}$ is partial randomizable digital signature on $C$. There is a difference in deriving the redactable signature. In their redactable signature scheme, redaction from an original message $M$ to a submessage $M'$ is proceed as follows. First, we derive a witness $W$ for $M'$, randomize $\sigma_{\text{PRSPS}}$ to $\sigma_{\text{PRSPS}}$. Then, we parse $\sigma_{\text{PRSPS}}$ as fixed elements $\sigma_{\text{PRSPS}}^{\text{Fix}}$, randomize elements $\sigma_{\text{PRSPS}}^{\text{Rand}}$ and generates a proof $\pi$ for the knowledge of $(C, W, \sigma_{\text{PRSPS}})$ by using witness-indistinguishable proof-of-knowledge (WI-PoK) system. The redactable signature for the message $M'$ is composed of $(\pi, \sigma_{\text{PRSPS}}^{\text{Rand}})$. The main difference between our scheme and scheme by Camenisch et al. is the use of witness-indistinguishable proof-of-knowledge (WI-PoK).
| Scheme | Assumption | Mstr | pp + pk size | sig size | T | U | C | R |
|--------|------------|------|--------------|----------|---|---|---|---|
| MHI | BSLS-aggregate signature | Set | | | | | | |
| §3.2 in [32] based on co-CDH [10] and ROM | | | | | | | | |
| §4.2 in [17] based on co-CDH [10] and ROM | | | | | | | | |
| §4.2 in [33] | Accumulator based on RSA and ROM | Set | | | | | | |
| §4.2 in [34] | | | | | | | | |
| §3 in [36] | GHR-signature | Set | | | | | | |
| §4 in [17] + unbounded accumulator [16] based on strong-RSA | | | | | | | | |
| §7 in [11] | HW signature [24] based on RSA | List | | | | | | |
| + unbounded accumulator [16] based on strong-RSA | | | | | | | | |
| §3.3 in [12] | SPF-1 partial randomizable SPS [1] based on q-SFP | List | | | | | | |
| §4.2 in [17] based on ℓ-RootDH and (ℓ + 1)-BSDH | | | | | | | | |
| + GS extractable WI-PoK [22] based on SXDH | | | | | | | | |
| Sanders | GGM (generic model group) | | | | | | | |
| §4.2 in [43] | (BG) + (ℓ + 1)|G| + ℓ|G| | | | |
| + set-commitment [19] based on ℓ-co-DL and ℓ-co-GSDH | | | | | | | | |
| RSP | set-commitment signature [19] based on ℓ-co-DL and ℓ-co-GSDH | | | | | | | | |
| (avoiding using ROM and GGM) | | | | | | | | |

Materials: “Mstr” indicates the message data structure supported by the corresponding scheme. The column “pp + pk size” represents the sum of the public parameters bit length and a public key bit length. The column “sig size” represents the signature bit length.

The “T”, “U”, “C” and “R” columns indicate the type of support for redaction operations. T stands for “one-time”, U stands for “unlimited multiple-time”, C stands for “compactness”, and R stands for “randomizable”. All columns of “T”, “U”, “C”, and “R” are represented by “✓”.

The “ℓ” column is the number of elements of sets or lists to be signed. The “Mstr” column indicates the structure of the message to be signed by the corresponding scheme.

In Sanders scheme, if we verify the validity of a signature, we only use O(ℓ) elements in the public key; O(ℓ^2) elements in the public key are needed to support redaction operations. Our scheme RSP_{ours} is constructed from digital signatures and set-commitments. To compare with other redactable signature schemes, we apply the structure-preserving signature scheme by Kiltz et al. [30] and the set-commitment scheme by Fuchsbauer et al. [19] to RSP_{ours}. In this instantiation, the size of a redacted signature is shorter than that of an original signature.

Fig. 1. The redaction capabilities of major redactable signature schemes with privacy security.
We briefly explain the reason why their redactable signature scheme uses WI-PoK. Their redactable signature scheme was constructed to satisfy unlinkability. Unlinkability requires that it should be hard to link back a redacted signature to its original signature. If the original signature and its redacted signature share a common (fixed) part, it is easy to link back from the redacted signature to its original signature. To hide common (fixed) parts in the redacted signature, they used a WI-PoK proof.

Although our scheme does not have transparency and unlinkability, our scheme makes sense in the following points. Non-transparent redactable signature has the drawback that an adversary can recover removed parts by collecting multiple submessages for an original signed message. That is, by comparing multiple submessages with different removed parts for the same original message, the adversary recovers removed parts of the original message. This attack can be avoided by restricting the number of redacted signatures for each original message to one. For example, we consider a situation where the government issues a subdocument only once for each signed document. In this situation, the adversary cannot obtain multiple redacted subdocuments for an original signed document, we can avoid this attack. Moreover, many redactable signatures without transparency [23,25,26,27,33,34,35,36] were constructed for real scenarios where non-transparency is desirable.

Unlinkability for a redactable signature scheme is useful to construct anonymous credential schemes. However, to solve the “storing redactable signature problem”, unlinkability security is too strong. We require that a redactable signature scheme satisfies only privacy, because it is sufficient to solve “storing redactable signature problem”. Hence, our redactable signature scheme is suitable for solving the “storing redactable signature problem” in the situation where the number of redacted signatures for each original message is restricted to one.

1.4 Related works

We present several signatures that allow editing a signed message.

- Append-only signature [29]: Kiltz, Mityagin, Panjwani, and Raghavan [29] introduced the notion of append-only signature. In this signature, we can only publicly append message blocks to a signed message and update the signature correspondingly.

- Sanitizable signature [5]: Atieh, Chou, de Medeiros, and Tsudik [5] introduced the notion of sanitizable signature. In this signature, a signer selects a sanitizer who can modify the signed message and generate a signature. The sanitizer can modify some parts of message blocks of the signed document, but he or she cannot remove message blocks. In the redactable signature, anyone can redact parts of the signed message without the secret key. However, in the sanitizable signature scheme, each sanitizer has the sanitizer’s secret key and the sanitizer designated by the signer can sanitize parts of the message using own sanitizer’s secret key.

- Protean signature [31]: Krenn, Pöhl, Samelin, and Slamanig [31] introduced the notion of protean signature. This signature allows removing and editing some parts of message blocks. They give the construction of the protean signature scheme from a sanitizable signature scheme and a redactable signature scheme in the black-box way.

1.5 Road Map

In Section 2, we introduce notations and recall digital signature. In Section 3, we review set-commitment and its security notions by Fuchsbauer et al. [19]. In Section 4, we review redactable signature and its security notions. In Section 5, we give a construction of the redactable signature scheme and its security analysis. In A, we provide a missing security proof for our scheme. In B, we review bilinear groups, the structure-preserving signature by Kiltz et al. [30], and the set-commitment construction by Fuchsbauer et al. [19].
2 Preliminaries

2.1 Notations

Let $1^\lambda$ be the security parameter. A function $f(\lambda)$ is negligible in $\lambda$ if $f(\lambda)$ tends to 0 faster than $\frac{1}{\lambda^c}$ for every constant $c > 0$. PPT stands for probabilistic polynomial time. For an integer $n$, $[n]$ denotes the set $\{1, \ldots, n\}$. For a finite set $S$, $s \in S$ denotes choosing an element $s$ from $S$ uniformly at random and $\#S$ denotes the number of elements in $S$. For a group $G$, we define $G^* := G \setminus \{1_G\}$ where $1_G$ is the identity element of the group $G$. For an algorithm $A$, $y \leftarrow A(x)$ denotes that the algorithm $A$ outputs $y$ on input $x$.

2.2 Digital Signature

**Definition 1 (Digital Signature Scheme).** A digital signature scheme $DS$ is composed of following four algorithms $(DS.\text{Setup}, DS.\text{KeyGen}, DS.\text{Sign}, DS.\text{Verify})$. $DS.\text{Setup}(1^\lambda)$ takes security parameters and generates public parameters $pp_{DS}$ which defines the message space $M_{pp_{DS}}$. $DS.\text{KeyGen}(pp_{DS})$ takes public parameters $pp_{DS}$, return a public key $pk_{DS}$ and a signing key $sk_{DS}$. $DS.\text{Sign}(pp_{DS}, sk_{DS}, m)$ takes public parameters $pp_{DS}$, a signing key $sk_{DS}$, and a message $m \in M_{pp_{DS}}$, return a signature $\sigma$. $DS.\text{Verify}(pp_{DS}, pk_{DS}, m, \sigma)$ takes public parameters $pp_{DS}$, a public key $pk_{DS}$, a message $m \in M_{pp_{DS}}$, and a signature $\sigma$, return 1 or 0.

For $DS$, we require the following correctness.

- **Correctness:** A digital signature scheme $DS$ is correct if for all $\lambda \in \mathbb{N}$, $pp_{DS} \leftarrow DS.\text{Setup}(1^\lambda)$, for all $m \in M_{pp_{DS}}$, $(pk_{DS}, sk_{DS}) \leftarrow DS.\text{KeyGen}(pp_{DS})$, $\sigma \leftarrow DS.\text{Sign}(pp_{DS}, sk_{DS}, m)$, then $DS.\text{Verify}(pp_{DS}, pk_{DS}, m, \sigma) = 1$ holds.

**Definition 2 (EUF-CMA).** Existentially unforgeable under chosen-message attacks (EUF-CMA) security for a digital signature scheme $DS$ is defined by the following unforgeability game between a challenger and an adversary $A$.

- The challenger computes $pp_{DS} \leftarrow DS.\text{Setup}(1^\lambda)$, $(pk_{DS}, sk_{DS}) \leftarrow DS.\text{KeyGen}(pp_{DS})$ initializes $Q \leftarrow \{\}$, and sends $(pp_{DS}, pk_{DS})$ to $A$.
- $A$ is given access to a signing oracle $O_{\text{Sign}}(\cdot)$. Given an input $m$, $O_{\text{Sign}}$ computes $\sigma \leftarrow DS.\text{Sign}(sk_{DS}, m)$, update $Q \leftarrow Q \cup \{m\}$ and returns $\sigma$ to $A$.
- Finally, $A$ outputs a forgery $(m^*, \sigma^*)$.

$DS$ is EUF-CMA secure if for all $\lambda \in \mathbb{N}$ and all PPT adversaries $A$, the advantage $\text{Adv}^{\text{EUF-CMA}}_{DS,A} := \Pr[DS.\text{Verify}(pp_{DS}, pk_{DS}, m^*, \sigma^*) = 1 \land m^* \notin Q]$ is negligible in $\lambda$.

3 Set-Commitment

Fuchsbauer et al. [19] proposed set-commitment which allows us to commit to a set. This scheme supports ordinary opening and subsets opening. In particular, we can commit a set $S$ and generate a commitment $C$ and its opening information $O$. Moreover, from $(S, C, O)$, we can generate a witness $W$ of a subset $S' \subseteq S$. By using $(S, C, O)$, we can verify that $S$ is committed to $C$. Also, by using $(S', C, W)$, we can verify that $S'$ is a subset of $S$ which is committed to $C$.

Now, we review the definition for set-commitment schemes.

**Definition 3 (Set-Commitment Scheme [19]).** Let $\ell$ be a polynomial in $\lambda$. A set-commitment scheme $SC$ is a tuple of algorithms $(SC.\text{Setup}, SC.\text{KGen}, SC.\text{Commit}, SC.\text{Open}, SC.\text{OSubset}, SC.\text{VSubset})$.

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4 In the syntax by Fuchsbauer et al. [19], a set commitment scheme consists of five algorithms $(SC.\text{Setup}, SC.\text{Commit}, SC.\text{Open}, SC.\text{OSubset}, SC.\text{VSubset})$. In this work, we divide $SC.\text{Setup}$ in [19] into two algorithms $SC.\text{Setup}$ and $SC.\text{KGen}$ for convenience in constructing the redactable signature scheme.
SC.Setup(1^λ) : Given a security parameter λ, return public parameters ppSC which defines the message space S_{ppSC}.

SC.KGen(ppSC, 1^λ) : Given public parameters ppSC and an upper bound ℓ for the number of elements in committed sets, return a commitment key ckSC. ckSC supports committing for a non-empty set containing at most ℓ elements.

SC.Commit(ppSC, ckSC, S) : Given public parameters ppSC, a commitment key ckSC and a non-empty set S ⊆ S_{ppSC}, return a commitment and opening information pair (C, O) or ⊥.

SC.Open(ppSC, ckSC, C, S, O) : Given public parameters ppSC, a commitment key ckSC, a commitment C, a non-empty set S ⊆ S_{ppSC}, opening information O, return 1 (Valid) or 0 (Invalid).

SC.OSubset(ppSC, ckSC, C, S, O, S') : Given public parameters ppSC, a commitment key ckSC, a commitment C, a non-empty set S ⊆ S_{ppSC}, opening information O, and a non-empty subset S' ⊆ S, return a witness W or ⊥.

SC.VSubset(ppSC, ckSC, C, S, W) : Given public parameters ppSC, a commitment key ckSC, a commitment C, and a witness W, return 1 (Valid) or 0 (Invalid).

For SC, we require the following correctness and compactness.

- Correctness: A set-commitment scheme SC is correct if for all λ ∈ N, for all ℓ(λ) > 0, ppSC ← SC.Setup(1^λ), for all non-empty set S ⊆ S_{ppSC} where #S ≤ ℓ, and for all non-empty subset S' ⊆ S, ckSC ← SC.KGen(ppSC, 1^λ), (C, O) ← SC.Commit(ppSC, ckSC, S), W ← SC.OSubset(ppSC, ckSC, C, S, O, S'), then followings holds.

\[
\text{SC.Open}(ppSC, ckSC, C, S, O) = 1 \land \text{SC.VSubset}(ppSC, ckSC, C, S', W) = 1
\]

- Compactness: A set-commitment scheme SC satisfies compactness if for all λ ∈ N, for all ℓ(λ) > 0, ppSC ← SC.Setup(1^λ), for all non-empty set S ⊆ S_{ppSC} where #S ≤ ℓ, and for all non-empty subset S' ⊆ S, ckSC ← SC.KGen(ppSC, 1^λ), (C, O) ← SC.Commit(ppSC, ckSC, S), W ← SC.O Subset(ppSC, ckSC, C, S, O, S'), the bit length of C, O, and W are independent of ℓ, #S, and #S'.

We review security notions for set-commitment.

**Definition 4 (Binding [19])**. A set-commitment scheme SC is computationally binding if for all λ ∈ N, ℓ(λ) > 0, and all PPT adversaries A, the following advantage

\[
\text{Adv}_{\text{SC}, A}^{\text{Bind}} := \Pr \left[ \begin{array}{l}
\text{SC.Open}(ppSC, ckSC, C, S, O) = 1 \\
\land \text{SC.Open}(ppSC, ckSC, C, S^*, O^*) = 1 \\
S \neq S^*
\end{array} \right] \\
\land \text{SC.KGen}(ppSC, 1^λ)
\]

is negligible in λ.

**Definition 5 (Subset-Soundness [19])**. A set-commitment scheme SC is subset-sound if for all λ ∈ N, ℓ(λ) > 0, and all PPT adversaries A, the following advantage

\[
\text{Adv}_{\text{SC}, A}^{\text{Sound}} := \Pr \left[ \begin{array}{l}
\text{SC.Open}(ppSC, ckSC, C, S, O) = 1 \\
\land \text{SC.VSubset}(ppSC, ckSC, C, S', W) = 1 \\
S' \notin S
\end{array} \right] \\
\land \text{SC.KGen}(ppSC, 1^λ)
\]

is negligible in λ.

**Definition 6 (Hiding [19])**. Hiding for a set-commitment scheme SC is defined by the following hiding game between a challenger and an adversary A.
The challenger chooses $b \in \{0, 1\}$, computes $pp_{SC} \leftarrow SC.Setup(1^\lambda)$ and $ck_{SC} \leftarrow SC.KGen(pp_{SC}, 1^\ell)$. Then, the challenger sends $(pp_{SC}, ck_{SC})$ to $A$.

$A$ sends a challenge $(S_0, S_1, \text{state})$ to the challenger.

The challenger computes $(C, O) \leftarrow SC.Commit(pp_{SC}, ck_{SC}, S_b)$. Then, the challenger sends $(C, \text{state})$ to $A$.

$A$ is given access to an open-subset oracle $O_{\text{OpenSubset}}(\cdot)$. Given an input $S$, $O_{\text{OpenSubset}}$ computes $W \leftarrow SC.OSubset(pp_{SC}, ck_{SC}, C, S, O, (S \cap S_0 \cap S_1))$ and returns $W$ to $A$.

Finally, $A$ outputs a guess $b^*$.

$SC$ is computationally hiding if for all $\lambda \in \mathbb{N}$, all $\ell(\lambda) > 0$, and all PPT adversaries $A$, the advantage $Adv_{SC,A}^{\text{Hiding}} := |\Pr[b^* = b] - \frac{1}{2}|$ is negligible in $\lambda$.

We say $SC$ is perfectly hiding if $Adv_{SC,A}^{\text{Hiding}} = 0$ holds for all $\lambda \in \mathbb{N}$, $\ell(\lambda) > 0$, and all PPT adversaries $A$.

Fuchsbauer et al. [19] gave a set-commitment scheme which satisfies correctness, compactness, binding, subset-soundness, and hiding.

### 4 Redactable Signature

We review the definition of a redactable signature scheme and its security notions. Our redactable signature scheme support sets signing. We refer to the syntax of a redactable signature by Sanders [43]. However, the syntax of the redactable signature scheme by Sanders is dedicated to redactable signature schemes for lists signing. We tailor this syntax for the redactable signature scheme with sets.

**Definition 7 (Redactable Signature Scheme).** Let $\ell$ be a polynomial in $\lambda$. A redactable signature scheme $RS$ is composed of following five algorithms $(RS.Setup, RS.KeyGen, RS.Sign, RS.Redact, RS.Verify)$.

- $RS.Setup(1^\lambda)$: Given a security parameter $\lambda$, return public parameters $pp_{RS}$ which defines the message space $M_{pp_{RS}}$.
- $RS.KeyGen(pp_{RS}, 1^\ell)$: Given public parameters $pp_{RS}$ and an upper bound $\ell$ for the number of elements in sets to be signed, return a public key $pk_{RS}$ and a signing key $sk_{RS}$. $sk_{RS}$ supports signing for a non-empty set containing at most $\ell$ elements.
- $RS.Sign(pp_{RS}, sk_{RS}, M)$: Given public parameters $pp_{RS}$, a signing key $sk_{RS}$, and non-empty set $M \subseteq M_{pp_{RS}}$, return a signature $\sigma$ on the set $M$.
- $RS.Redact(pp_{RS}, pk_{RS}, M, \sigma, M')$: Given public parameters $pp_{RS}$, a public key $pk_{RS}$, a non-empty set $M \subseteq M_{pp_{RS}}$, a signature $\sigma$, and a non-empty subset $M' \subseteq M$, return a signature $\sigma'$ on the subset $M'$ or $\bot$.
- $RS.Verify(pp_{RS}, pk_{RS}, M, \sigma)$: Given public parameters $pp_{RS}$, a public key $pk_{RS}$, a non-empty set $M \subseteq M_{pp_{RS}}$, and a signature $\sigma$, return 1 (Valid) or 0 (Invalid).

For $RS$, we require the following correctness and compactness.

- **Correctness:** A redactable signature scheme $RS$ is correct if for all $\lambda \in \mathbb{N}$, for all $\ell(\lambda) > 0$, $pp_{RS} \leftarrow RS.Setup(1^\lambda)$, for all non-empty $M \subseteq M_{pp_{RS}}$ where $\#M \leq \ell$, and for all non-empty subset $M' \subseteq M$, $(pk_{RS}, sk_{RS}) \leftarrow RS.KeyGen(pp_{RS}, 1^\ell)$, $\sigma \leftarrow RS.Sign(pp_{RS}, sk_{RS}, M)$, $\sigma' \leftarrow RS.Redact(pp_{RS}, pk_{RS}, M, \sigma, M')$, then $RS.Verify(pp_{RS}, pk_{RS}, M, \sigma) = 1$ and $RS.Verify(pp_{RS}, pk_{RS}, M, \sigma') = 1$ hold.

- **Compactness:** A redactable signature scheme $RS$ satisfies compactness if for all $\lambda \in \mathbb{N}$, for all $\ell(\lambda) > 0$, $pp_{RS} \leftarrow RS.Setup(1^\lambda)$, for all non-empty $M \subseteq M_{pp_{RS}}$ where $\#M \leq \ell$, and for all non-empty subset $M' \subseteq M$, $(pk_{RS}, sk_{RS}) \leftarrow RS.KeyGen(pp_{RS}, 1^\ell)$, $\sigma \leftarrow RS.Sign(pp_{RS}, sk_{RS}, M)$, $\sigma' \leftarrow RS.Redact(pp_{RS}, pk_{RS}, M, \sigma, M')$, the bit length of both $\sigma$ and $\sigma'$ are independent of $\ell$, $\#M$, and $\#M'$.
We review unforgeability and privacy for redactable signature. These security notions were formalized by Brzuska et al. [11] for redactable signature for tree message structures. Later, these security notions were extended to redactable signature for arbitrary data structures by Derler et al. [17].

Unforgeability requires that without a signing key $sk$, it should be infeasible to compute a valid signature $\sigma$ on $m'$ except to redact a signed message $(m, \sigma)$.

**Definition 8 (Unforgeability).** Unforgeability for a redactable signature scheme $RS$ is defined by the following unforgeability game between a challenger and an adversary $A$.

- The challenger computes $pp_{RS} \leftarrow RS.Setup(1^\lambda)$, $(pk_{RS}, sk_{RS}) \leftarrow RS.KeyGen(pp_{RS}, 1^\ell)$ initializes $Q \leftarrow \{\}$, and sends $(pp_{RS}, pk_{RS})$ to $A$.

- $A$ is given access to a signing oracle $O^{Sign}(\cdot)$. Given an input $M$, $O^{Sign}$ computes $\sigma \leftarrow RS.Sign(pp_{RS}, sk_{RS}, M)$, update $Q \leftarrow Q \cup \{M\}$ and returns $\sigma$ to $A$.

- Finally, $A$ outputs a forgery $(M^*, \sigma^*)$.

$RS$ is unforgeable if for all $\lambda \in \mathbb{N}$, $\ell(\lambda) > 0$, and all PPT adversaries $A$, the following advantage

$$Adv_{RS,A}^{Uf} := \Pr[RS.Verify(pp_{RS}, pk_{RS}, M^*, \sigma^*) = 1 \land \exists M \in Q : M^* \subseteq M]$$

is negligible in $\lambda$.

Privacy requires that except for a signer and a redactor, it is infeasible to derive information on redacted message parts when given a redacted message-signature pair.

**Definition 9 (Privacy).** Privacy for a redactable signature scheme $RS$ is defined by the following unforgeability game between a challenger and an adversary $A$.

1. The challenger computes $pp_{RS} \leftarrow RS.Setup(1^\lambda)$, $(pk_{RS}, sk_{RS}) \leftarrow RS.KeyGen(pp_{RS}, 1^\ell)$, chooses $b \leftarrow \{0, 1\}$, and sends $(pp_{RS}, pk_{RS})$ to $A$.

2. $A$ is given access to a signing oracle $O^{Sign}(\cdot)$. Given an input $M$, $O^{Sign}$ computes $\sigma \leftarrow RS.Sign(pp_{RS}, sk_{RS}, M)$ and returns $\sigma$ to $A$.

3. $A$ is also given access to a left-or-right redact oracle $O^{LoR\text{redact}}(\cdot, \cdot, \cdot)$. Given an input $(M_0, M_1, M')$, $O^{LoR\text{redact}}$ works as follows:

   1. If $M' \not\subseteq (M_0 \cap M_1)$, return $\bot$.

   2. Compute $\sigma_b \leftarrow \text{Sign}(pp_{RS}, sk_{RS}, M_b)$, $\sigma'_b \leftarrow RS.\text{Redact}(pp_{RS}, pk_{RS}, M_b, \sigma_b, M')$.

   3. Return $\sigma'_b$.

4. Finally, $A$ outputs $b^*$.

$RS$ is private if for all $\lambda \in \mathbb{N}$, $\ell(\lambda) > 0$, and all PPT adversaries $A$, the advantage $Adv_{RS,A}^{Priv} := |\Pr[b^* = b] - \frac{1}{2}|$ is negligible in $\lambda$.

5 Our Redactable Signature Scheme

In this section, we give a construction of a redactable signature scheme with compactness without the GGM or ROM. Then, we give security analysis for our redactable signature scheme.

5.1 Our Construction

Before describing our construction, we give an intuition for our construction. We can observe that a redactable signature scheme and a set-commitment scheme have similar properties. In a redactable signature scheme, we can remove parts of a signed message without invalidating the signature. That is, we can generate signatures for a subset of messages from the original signed
Here, we give a sketch of the security proof. To explain the outline of the proof, we introduce unforgeable. The construction of our redactable signature scheme is formed as RS. The digital signature scheme and a set-commitment scheme.

Algorithm RS.Setup(1^λ) :
ppRS ← SC.Setup(1^λ), ppRS ← DS.Setup(1^λ), ppRS ← (ppDS, ppSC), return ppRS.
pKRS defines message space M_{ppRS} := S_{ppSC}.

Algorithm RS.KeyGen(ppRS, 1^λ) :
ckRS ← SC.KeyGen(ppRS, 1^λ), (ckDS, skDS) ← DS.KeyGen(ppDS), ppRS ← (ppDS, ckSC), skRS ← (skDS, ckSC).
Return (pkRS, skRS).

Algorithm RS.Sign(ppRS = (skDS, ckSC), skRS, M) :
(C, O) ← SC.Commit(ppRS, ckSC, M), σC ← DS.Sign(ppDS, skDS, C).
Return σ ← (C, σC, O).

Algorithm RS.Redact(ppRS = (pkDS, ckSC), pkRS, M, σ = (C, σC, O), M') :
If DS.Verify(ppDS, pkDS, C, σC) = 0, return 1.
W ← SC.OpenSubset(ppRS, ckSC, C, M, O, M'), return σ' ← (C, σC, W).

Algorithm RS.Verify(ppRS, pkRS = (pkDS, ckSC), M, σ = (C, σC, O)) :
If DS.Verify(ppDS, pkDS, C, σC) = 0, return 0.
If (ppRS, C, M, O) is an input form of SC.Open,
If SC.Open(ppRS, C, M, O) = 1, return 1.
If (ppRS, C, M, O) is an input form of SC.VSubset,
If SC.VSubset(ppRS, ckSC, C, M, O) = 1, return 1.
Otherwise return 0.

Fig. 2. The construction of RS_{Ours}.

Clearly, the correctness of RS_{Ours} is followed by that of DS and SC. The compactness of RS_{Ours} is followed by that of SC.

5.2 Security Analysis

Theorem 1. If DS is EUF-CMA secure and SC is binding and subset-sound, then RS_{Ours} is unforgeable.

Here, we give a sketch of the security proof. To explain the outline of the proof, we introduce new notations. Let q_a be the total number of queries from an adversary to O_{Sign}, M_i be an i-th input for O_{Sign}, and σ_i = (C_i, σ_{C_i}, O_i) be an i-th output of O_{Sign}. We denote Q^E_{Sign} := \bigcup_{i=1}^{q_a} \{C_i\}.
and $Q_{\text{Sign}}^M := \bigcup_{i=1}^{\rho(M)} \{ M_i \}$. We consider three types of PPT adversaries $A_1$, $A_2$, and $A_3$ that break the unforgeability security for $\text{RS}_\text{Ours}$.

- $A_1$ generates a new commitment $C^* \notin Q_{\text{Sign}}^C$, forges a signature $\sigma_{C^*}$ for $C^*$, and outputs a valid forgery $(m^*, \sigma^* = (C^*, \sigma_{C^*}, O^*))$. That is, $A_1$ does not reuse commitments output by $O_{\text{Sign}}$. By the EUF-CMA security of DS, it is difficult for $A_1$ to forge a signature $\sigma_{C^*}$ for $C^* \notin Q_{\text{Sign}}^C$. Therefore, it is difficult for $A_1$ to output a valid forgery $(M^*, \sigma^* = (C^*, \sigma_{C^*}, O^*))$. In the security proof, we construct $B_1$ which breaking the EUF-CMA security of DS by using $A_1$.

- $A_2$ reuses a commitment $C^* \in Q_{\text{Sign}}^C$ output by $O_{\text{Sign}}$, forges opening information $O^*$ for opening a message $M^*$ against $C^*$ where there is no $M \in Q_{\text{Sign}}^M$ such that $M^* \subseteq M$, and outputs a forgery $(M^*, \sigma^* = (C^*, \sigma_{C^*}, O^*))$ where $C^* \in Q_{\text{Sign}}^C$ and $SC.\text{Open}(pp_{SC}, ck_{SC}, C^*, M^*, O^*) = 1$. Since SC is binding, it is difficult for $A_2$ to forge opening information $O^*$ for opening $M^*$ against $C^* \in Q_{\text{Sign}}^C$. In the security proof, we construct $B_2$ which breaking the binding property of SC by using $A_2$.

- $A_3$ reuses a commitment $C^* \in Q_{\text{Sign}}^C$ output by $O_{\text{Sign}}$, forges opening information $O^*$ for opening a message $M^*$ against $C^*$ where there is no $M \in Q_{\text{Sign}}^M$ such that $M^* \subseteq M$, and outputs a forgery $(M^*, \sigma^* = (C^*, \sigma_{C^*}, O^*))$ where $C^* \in Q_{\text{Sign}}^C$ and $SC.\text{VSOutput}(pp_{SC}, ck_{SC}, C^*, M^*, O^*) = 1$. Since SC is subset-sound it is difficult for $A_3$ to forge opening information $O^*$ for opening $M^*$ against $C^* \in Q_{\text{Sign}}^C$. In the security proof, we construct $B_3$ which breaking the subset-sound property of SC by using $A_3$.

Note that three types of forgers $A_1$, $A_2$, and $A_3$ cover all the possibilities of forger’s behaviors. By constructing $B_1$, $B_2$, and $B_3$, we prove the unforgeability security for $\text{RS}_\text{Ours}$.

**Theorem 2.** If SC is perfectly hiding, then $\text{RS}_\text{Ours}$ is private.

**Proof.** We consider the view of an adversary $A$ in the privacy game. To simplify the discussion, we assume that A queries $(M_0, M_1, M')$ to $O_{\text{LoRedact}}$ where $M' \notin (M_0 \cap M_1)$. Let $\sigma_b = (C_b, \sigma_{C_b}, O_b) \leftarrow \text{RS.Sign}(pp_{RS}, sk_{RS}, M)$ where $(C_b, O_b) \leftarrow \text{SC.Commit}(pp_{SC}, ck_{SC}, M_b)$, $\sigma_{C_b} \leftarrow \text{DS.Sign}(pp_{DS}, sk_{DS}, C_b)$ and $\sigma'_b = (C_b, \sigma_{C_b}, W_b) \leftarrow \text{RS.Redact}(pp_{RS}, pk_{RS}, M_b, \sigma, M')$, $W_b \leftarrow \text{SC.\text{OOutput}}(pp_{SC}, ck_{SC}, C_b, M_b, O_b, M')$ for $b \in \{0, 1\}$.

Now, we discuss distributions $\{\sigma'_0 = (C_0, \sigma_{C_0}, W_0)\}$ and $\{\sigma'_1 = (C_1, \sigma_{C_1}, W_1)\}$ output by $O_{\text{LoRedact}}$. By the perfect hiding property (in Definition 6) of SC, distributions $(C_0, W_0)$ and $(C_1, W_1)$ are identical. Hence, the distributions $\{\sigma'_0 = (C_0, \sigma_{C_0}, W_0)\}$ and $\{\sigma'_1 = (C_1, \sigma_{C_1}, W_1)\}$ are identical. From the above discussion, $\text{Adv}_{\text{Priv}}^{\text{RS}_\text{Ours},A} = 0$ holds. Therefore, we can conclude Theorem 2. \(\square\)

**6 Discussion**

We construct a redactable signature scheme with compact for sets. If redactable signature scheme for sets is used as it is, there is a problem in real scenario. For example, we consider the following submessages: $m_1 = \text{We}$, $m_2 = \text{mustn’t}$, $m_3 = \text{go}$, $m_4 = \text{must}$, $m_5 = \text{wait}$. If the adversary obtains a signature $\sigma$ on $M = \{\text{We mustn’t go. We wait.}\}$, then the adversary re-orders it to $M' = \{\text{We must go. We wait.}\}$ and generates the proper signature on it.

Re-ordering message can be easily avoided by concatenating each submessage elements with an order-ID. For instance, in above example, we change the submessages to $m_1 = \text{We}||1$, $m_2 = \text{mustn’t}||2$, $m_3 = \text{go}||3$, $m_4 = \text{We}||4$, $m_5 = \text{must}||5$, $m_6 = \text{wait}||6$. By concatenating each submessage with an order-ID, our redactable signature scheme is converted into the redactable signature scheme for lists and we can prevent re-ordering of submessages.
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A Security Proof for Theorem 1

An outline of the proof is described in Section 5.2. Now, we prove Theorem 1.

Proof. We consider the three types of adversaries $A_1$, $A_2$, and $A_3$ described as follows and evaluate the advantage $\text{Adv}_{\text{RS}_{\text{Dour}}-\text{A}_i}^{\text{DS}}$ for each $i = 1, 2, 3$.

We consider an adversary $A_1$ that generates a new commitment $C^* \notin Q_{\text{Sign}}^C$, forgives a signature $\sigma_{C^*}$ for $C^*$, and outputs a valid forgery $(M^*, \sigma^* = (C^*, \sigma_{C^*}, O^*))$. We construct $B_1$ which breaking the EUF-CMA security of DS by using $A_1$ as follows.

- **Initial setup:** Given an input $(\text{pp}_{\text{DS}}, \text{pp}_{\text{DS}})$ from the challenger of the EUF-CMA security game for DS, $B_1$ performs the following procedure.

  - $\text{pp}_{\text{SC}} \leftarrow \text{SC.Setup}(\lambda), \text{ck}_{\text{SC}} \leftarrow \text{SC.KGen}(\text{pp}_{\text{SC}}, 1^\lambda)$, $\text{pp}_{\text{RS}} \leftarrow (\text{pp}_{\text{DS}}, \text{pp}_{\text{SC}}), \text{pk}_{\text{RS}} \leftarrow (\text{pk}_{\text{DS}}, \text{ck}_{\text{SC}})$, $Q_{\text{Sign}}^C \leftarrow \{\}, Q_{\text{Sign}}^M \leftarrow \{\}$.
  - Give $(\text{pp}_{\text{RS}}, \text{pk}_{\text{RS}})$ to $A_1$ as an input.

- $\text{O}_{\text{Sign}}(M_i)$ : Given an input $M_i$, $B_1$ performs the following procedure.

  - $(C_i, O_i) \leftarrow \text{SC.Commit}((\text{pp}_{\text{DS}}, \text{ck}_{\text{SC}}, M_i)$.
  - Query the challenger for the signature on the message $C_i$ and get its signature $\sigma_{C_i}$.
  - $Q_{\text{Sign}}^C \leftarrow Q_{\text{Sign}}^C \cup \{C_i\}$, $Q_{\text{Sign}}^M \leftarrow Q_{\text{Sign}}^M \cup \{M_i\}$.
  - Return $(C_i, \sigma_{C_i}, O_i)$.

- **Output procedure:** $B_1$ receives a forgery $(M^*, \sigma^*)$ output by $A_1$. Then $B_1$ proceeds as follows.

  1. $\text{RS.Verify}(\text{pp}_{\text{RS}}, \text{pk}_{\text{RS}}, M^*, \sigma^*) = 0$, then abort.
  2. If there exists $M \in Q_{\text{Sign}}^M$, such that $M^* \subseteq M$, then abort.
  3. Parse $\sigma^*$ as $(C^*, \sigma_{C^*}, O^*)$.
  4. If $C^* \in Q_{\text{Sign}}^C$, then abort.
  5. Return $(C^*, \sigma_{C^*})$ to the challenger.

It is easy to see that $B_1$ can simulate the unforgeability game for $\text{RS}_{\text{Dour}}$. Now, we confirm that when $A_1$ successfully output a valid forgery $(M^*, \sigma^*)$, $B_1$ can forge a signature for DS. If $A_1$ successfully output a valid forgery $(M^*, \sigma^*)$, $B_1$ does not abort in Step 1, 2 and 4 of Output procedure. $\text{RS.Verify}(\text{pp}_{\text{RS}}, \text{pk}_{\text{RS}}, M^*, \sigma^*) = 1$ implies that $\text{DS.Verify}(\text{pp}_{\text{DS}}, \text{pk}_{\text{DS}}, C^*, \sigma_{C^*}) = 1$ holds. Moreover, $A_1$ outputs $C^* \notin Q_{\text{Sign}}^C$. This means that $B_1$ does not make singing query $C^*$ to the challenger. Therefore, $(C^*, \sigma_{C^*})$ is a valid forgery for DS.
Finally, we evaluate the probability that \( B_1 \) succeeds in forging a signature for DS. Let \( \text{Adv}^{\text{Uf}}_{\text{RS}_{\text{Ours}}, A_1} \) be the advantage of the unforgeability game for \( \text{RS}_{\text{Ours}} \) of \( A_1 \). The advantage of the EUF-CMA game for DS of \( B_1 \) is

\[
\text{Adv}^{\text{Uf}}_{\text{DS}, B_1} \geq \text{Adv}^{\text{Uf}}_{\text{RS}_{\text{Ours}}, A_1}.
\]  

We consider an adversary \( A_2 \) that reuses a commitment \( C^* \in Q^C_{\text{Sign}} \) output by \( O_{\text{Sign}} \), forges opening information \( O^* \) for opening a message \( M^* \) against \( C^* \) where there is no \( M \in Q^M_{\text{Sign}} \) such that \( M^* \subseteq M \), and outputs a forgery \( (M^*, \sigma^* = (C^*, \sigma_{C^*}, O^*)) \) where \( C^* \in Q^C_{\text{Sign}} \) and \( \text{SC.Open}(pp_{\text{SC}}, c_{\text{SC}}, C^*, M^*, O^*) = 1 \). We construct \( B_2 \) which breaking the binding property of SC by using \( A_2 \) as follows.

**Initial setup:** Given an input \((pp_{\text{SC}}, c_{\text{SC}})\) from the challenger of the binding security game for SC, \( B_2 \) performs the following procedure.

- **Output procedure:** \( B_2 \) receives a forgery \((M^*, \sigma^*)\) output by \( A_2 \). Then \( B_2 \) proceeds as follows.
  1. \( \text{RS.Verify}(pp_{\text{RS}}, pk_{\text{RS}}, M^*, \sigma^*) = 0 \), then abort.
  2. If there exists \( M \in Q^M_{\text{Sign}} \) such that \( M^* \subseteq M \), then abort.
  3. Parse \( \sigma^* \) as \((C^*, \sigma_{C^*}, O^*)\).
  4. If \( C^* \notin Q^C_{\text{Sign}} \), then abort.
  5. Retrieve an entry \((M', C', O')\) from \( Q^M_{\text{Sign}} \) such that \( C' = C^* \).
  6. Return \((C^*, M', O', M^*, O^*)\) to the challenger.

It is easy to see that \( B_2 \) can simulate the unforgeability game for \( \text{RS}_{\text{Ours}} \). Now, we confirm that when \( A_2 \) successfully output a valid forgery \((M^*, \sigma^*)\), \( B_2 \) can output a valid tuple \((C^*, M', O', M^*, O^*)\) for the binding game for SC. If \( A_2 \) successfully output a valid forgery \((M^*, \sigma^*)\), \( B_2 \) does not abort in Step 1, 2 and 4 of **Output procedure**. By the strategy of \( A_2 \), \( \text{SC.Open}(pp_{\text{SC}}, c_{\text{SC}}, C^*, M^*, O^*) = 1 \) holds. \( C^* \in Q^C_{\text{Sign}} \) implies that there exists an entry \((M', C', O') \in Q^M_{C,O} \) such that \( C' = C^* \). Moreover, since SC is correct, \( \text{SC.Open}(pp_{\text{SC}}, c_{\text{SC}}, C' = C^*, M', O') = 1 \) holds. Furthermore, the fact that the \( B_2 \) does not abort in Step 2 of **Output procedure** implies that \( M^* \neq M' \). Therefore, \((C^*, M', O', M^*, O^*)\) is a valid tuple for the binding game for SC.

Finally, we evaluate the probability that \( B_2 \) succeeds in outputting a valid tuple in the binding game for SC. Let \( \text{Adv}^{\text{Uf}}_{\text{RS}_{\text{Ours}}, A_2} \) be the advantage of the unforgeability game for \( \text{RS}_{\text{Ours}} \) of \( A_2 \). The advantage of the binding game for SC of \( B_2 \) is

\[
\text{Adv}^{\text{Bind}}_{\text{SC}, B_2} \geq \text{Adv}^{\text{Uf}}_{\text{RS}_{\text{Ours}}, A_2}.
\]  

We consider an adversary \( A_3 \) that reuses a commitment \( C^* \in Q^C_{\text{Sign}} \) output by \( O_{\text{Sign}} \), forges opening information \( O^* \) for opening a message \( M^* \) against \( C^* \) where there is no \( M \in Q^M_{\text{Sign}} \) such that \( M^* \subseteq M \), and outputs a forgery \((M^*, \sigma^* = (C^*, \sigma_{C^*}, O^*))\) where \( C^* \in Q^C_{\text{Sign}} \) and \( \text{SC.VSubset}(pp_{\text{SC}}, c_{\text{SC}}, C^*, M^*, O^*) = 1 \). We construct \( B_3 \) which breaking the subset-sound property of SC by using \( A_3 \) as follows.
– **Initial setup:** Given an input \((pp_{SC}, ck_{SC})\) from the challenger of the subset-soundness security game for \(SC\), \(B_3\) performs the same procedure as **Initial setup** of \(B_2\).

– \(O^{Sign}(M_i)\) : Given an input \(M_i\), \(B_3\) performs the same procedure as \(O^{Sign}(M_i)\) of \(B_2\).

– **Output procedure:** \(B_2\) receives a forgery \((M^*, \sigma^*)\) output by \(A_2\). Then \(B_2\) proceeds as follows.

1. \(RS\).Verify\((pp_{RS}, pk_{RS}, M^*, \sigma^*) = 0\), then abort.
2. If there exists \(M \in Q^M_{Sign}\) such that \(M^* \subseteq M\), then abort.
3. Parse \(\sigma^*\) as \((C^*, \sigma_{C^*}, O^*)\).
4. If \(C^* \notin Q^C_{Sign}\), then abort.
5. Retrieve an entry \((M', C', O')\) with \(Q^M_{Sign}\) such that \(C' = C^*\).
6. Return \((C^*, M', O', M^*, O^*)\) to the challenger.

It is easy to see that \(B_3\) can simulate the unforgeability game for \(RS_{Ours}\). Now, we confirm that when \(A_3\) successfully output a valid forgery \((M^*, \sigma^*)\), \(B_3\) can output a valid tuple \((C^*, M', O', M^*, O^*)\) for the subset-soundness game for \(SC\). If \(A_3\) successfully output a valid forgery \((M^*, \sigma^*)\), \(B_3\) does not abort in Step 1, 2, and 4 of **Output procedure**. By the strategy of \(A_3\), \(SC\).VSub(set)(\(pp_{SC}, ck_{SC}, C^*, M^*, O^*) = 1\) holds. When \(C^* \in Q^C_{Sign}\) holds then there exists an entry \((M', C', O')\) with \(Q^M_{Sign}\) such that \(C' = C^*\). Moreover, since \(SC\) is correct, \(SC\).Open\((pp_{SC}, ck_{SC}, C' = C^*, M', O')\) = 1 holds. Furthermore, the fact that the \(B_2\) does not abort in Step 2 of **Output procedure** implies that \(M^* \notin M'\). Therefore, \((C^*, M', O', M^*, O^*)\) is a valid tuple for the binding game for \(SC\).

Finally, we evaluate the probability that \(B_3\) succeeds in outputting a valid tuple in the subset-soundness game for \(SC\). Let \(Adv^{Uf}_{RS_{Ours}, A_3}\) be the advantage of the unforgeability game for \(RS_{Ours}\) of \(A_3\). The advantage of the subset-soundness game for \(SC\) of \(B_3\) is

\[
Adv^{Sound}_{SC,B_3} \geq Adv^{Uf}_{RS_{Ours}, A_3}. \tag{3}
\]

From inequalities (1), (2), and (3), we can conclude Theorem 1.

\[\square\]

### B Structure-Preserving Signature Scheme by Kiltz et al. [30] and Set-Commitment Scheme by Fuchsbauer et al. [19]

In this section, we review bilinear groups. Then, we review the structure-preserving signature scheme by Kiltz et al. [30] and the set-commitment scheme by Fuchsbauer et al. [19].

#### B.1 Bilinear Groups

Let \(G\) be a bilinear group generator that takes as an input a security parameter \(1^\lambda\) and outputs a description of bilinear groups \(BG := (q, G_1, G_2, G_T, e, G_1, G_2)\) where \(G_1, G_2\) are additive groups of prime order \(q\), \(G_T\) is a multiplicative group of prime order \(q\), \(e\) is an efficient computable, non-degenerating bilinear map \(e : G_1 \times G_2 \rightarrow G_T\), and \(G_1\) and \(G_2\) are generators of the group \(G_1\) and \(G_2\) respectively.

1. **Bilinear:** For all \(a, b \in \mathbf{Z}_q\), then \(e(aG_1, bG_2) = e(G_1, G_2)^{ab} = e(bG_1, aG_2)\).

2. **Non-degenerate:** \(e(G_1, G_2) \neq 1_{G_T}\). (i.e., \(e(G_1, G_2)\) is a generator of \(G_T\).)

We consider type 3 pairings where \(G_1 \neq G_2\) and there are no efficiently computable homomorphisms between \(G_1\) and \(G_2\).

We use the implicit representation of group elements by Escala, Herold, Kiltz, Ràfols, and Villar [18]. For \(s \in \{1, 2\}\) and \(a \in \mathbf{Z}_q\), we define \([a]_s := aG_s \in G_s\) as the implicit representation of
\[ a \in \mathbb{G}_a. \] For \( s = T \) and \( a \in \mathbb{Z}_q \), we define \([a]_T \equiv e(G_1, G_2)^a \in \mathbb{G}_T\) as the implicit representation of \( a \) in \( \mathbb{G}_T \). For a matrix \( A = (a_{i,j}) \in \mathbb{Z}_q^{m \times n} \) we define \([A]_1\) as

\[
[A]_1 := \begin{pmatrix}
a_{1,1}G_1 & \cdots & a_{1,n}G_1 \\
\vdots & \ddots & \vdots \\
a_{m,1}G_1 & \cdots & a_{m,n}G_1
\end{pmatrix} \in \mathbb{G}_1^{m \times n}
\]

and similarly for \([A]_2 \in \mathbb{G}_2^{m \times n}\) with a generator \( G_2 \), and \([A]_T \in \mathbb{G}_T^{m \times n}\) with a generator \( e(G_1, G_2)\).

**Definition 10 (DDH Assumption in \( \mathbb{G}_1 \)).** Let \( \mathcal{G} \) be a bilinear group generator. The decisional Diffie-Hellman (DDH) assumption holds in \( \mathbb{G}_1 \) for \( \mathcal{G} \) if for all PPT adversaries \( A \), the following advantage

\[
\text{Adv}_{\mathcal{G}, A}^{\text{DDH}} := \Pr \left[ \left| b' = b \mid \begin{array}{c}
BG = (q, G_1, G_2, G_T, e, G_1, G_2) \leftarrow \mathcal{G}(1^\lambda), \\
x, y, z \leftarrow \mathbb{Z}_q, b \leftarrow \{0, 1\}, b' \leftarrow A(BG, [x]_1, [y]_1, [bxy + (1 - b)z]_1) - \frac{1}{2}
\end{array} \right| \right.
\]

is negligible in \( \lambda \).

Note that in the case of \( b = 1 \), \( A \) receives a Diffie-Hellman tuple \(([x]_1, [y]_1, [xy]_1)\) as an input. Similarly, in the case of \( b = 0 \), \( A \) receives a random tuple \(([x]_1, [y]_1, [z]_1)\) as an input.

The dual of the above assumption is the Decisional Diffie-Hellman assumption in \( \mathbb{G}_2 \) for \( \mathcal{G} \), which is defined by changing the roles of \( \mathbb{G}_1 \) to \( \mathbb{G}_2 \) in Definition 10.

**Definition 11 (SXDH Assumption \([6]\)).** Let \( \mathcal{G} \) be a bilinear group generator outputting \( \mathbb{G}_T \equiv (q, G_1, G_2, G_T, e, G_1, G_2) \). The symmetric external Diffie-Hellman (SXDH) assumption holds for \( \mathcal{G} \) if the DDH assumption holds both \( \mathbb{G}_1 \) and \( \mathbb{G}_2 \).

**Definition 12 (q-co-DL Assumption \([19]\)).** Let \( \mathcal{G} \) be a bilinear group generator. The q-co-discrete logarithm (q-co-DL) assumption holds for \( \mathcal{G} \) if for all PPT adversaries \( A \), the following advantage

\[
\text{Adv}_{\mathcal{G}, A}^{\text{q-co-DL}} := \Pr \left[ a' = a \mid BG \leftarrow \mathcal{G}(1^\lambda), a \leftarrow \mathbb{Z}_q, a' \leftarrow A(BG, ([a^j]_1)_{j \in [q]} \right) \right]
\]

is negligible in \( \lambda \).

**Definition 13 (q-co-GSDH Assumption \([19]\)).** Let \( \mathcal{G} \) be a bilinear group generator. The q-co-generalized-strong-Diffie-Hellman (q-co-GSDH) assumption holds over \( \mathcal{G} \) if for all PPT adversaries \( A \), the following advantage

\[
\text{Adv}_{\mathcal{G}, A}^{\text{q-co-GSDH}} := \Pr \left[ \begin{array}{l}
T \in \mathbb{G}_1 \land g, h \in \mathbb{Z}_q[X] \\
\land 0 \leq \deg g < \deg h \leq q \\
\land e(T, [h(a)]_2) = e([g(a)]_1, [1]_2)
\end{array} \mid \begin{array}{c}
BG \leftarrow \mathcal{G}(1^\lambda), a \leftarrow \mathbb{Z}_q, \\
(g, h, T) \leftarrow A(BG, ([a^j]_1, [a^j]_2)_{j \in [q]})
\end{array} \right]
\]

is negligible in \( \lambda \).

**B.2 Structure-Preserving Signature Scheme by Kiltz et al.**

We review the structure-preserving signature by Kiltz et al. \([30]\). This scheme is efficient and its security is proven without GGM and supports a multi-message (vector message) signing. In this work, we only need a single-message signing scheme. Now, we describe the structure-preserving signature scheme \(\text{DS}_{\text{KPW}}\) given by Kiltz et al. in Fig. 3.

**Lemma 1 (\([30]\)).** If the SXDH assumption holds for \( \mathcal{G} \), then \( \text{SPS}_{\text{KPW}} \) is EUF-CMA secure. \(^5\)

\(^5\) Kiltz et al. \([30]\) proved that \( \text{SPS}_{\text{KPW}} \) satisfies EUF-CMA security under the \( \mathcal{D}_e \)-matrix Diffie-Hellman (\( \mathcal{D}_e \)-MDDH) assumption \([18]\). If \( k = 1 \), the \( \mathcal{D}_e \)-MDDH assumption corresponds to the SXDH assumption. In Lemma 1, we rewrite the claim of Kiltz et al. in \([30]\) as \( k = 1 \).
Lemma 6 \((\textsf{SC})\). Let \(\ell\) be an upper bound for the number of elements in committed sets. If the \(\ell\)-co-DL assumption holds for \(\mathcal{G}\), then \(\text{SC}_{\text{FHS}}\) is binding.

Lemma 5 \((\textsf{SC})\). Let \(\ell\) be an upper bound for the number of elements in committed sets. If the \(\ell\)-co-GSDH assumption holds for \(\mathcal{G}\), then \(\text{SC}_{\text{FHS}}\) is subset-sound.

Lemma 6 \((\textsf{SC})\). \(\text{SC}_{\text{FHS}}\) is perfectly hiding.
Algorithm \texttt{SC.Setup}_{\text{FHE}}(1^\lambda) :

\begin{itemize}
  \item $BG := (q, G_1, G_2, G_T, e, G_1, G_2) \leftarrow G(1^\lambda)$, return $pp_{SC} \leftarrow BG$.
  \item $pp_{SC}$ defines message space $S_{pp_{SC}} := \mathbb{Z}_q$.
\end{itemize}

Algorithm \texttt{SC.KGen}_{\text{FHE}}(pp_{SC}, 1^\ell) :

\begin{itemize}
  \item $a \leftarrow \mathbb{Z}_q$, return $ck_{SC} \leftarrow (\{[a_1^i], [a_2^i]\}_{i \in [\ell]}$).
\end{itemize}

Algorithm \texttt{SC.Commit}_{\text{FHE}}(pp_{SC}, ck_{SC} = (\{[a_1^i], [a_2^i]\}_{i \in [\ell]}), S \subseteq S_{pp_{SC}}) :

\begin{itemize}
  \item If $S \notin \mathbb{Z}_q \lor \#S = 0 \lor \ell < \#S$, return \bot.
  \item If there exist $a^i \in S$ such that $[a_1^i] = [a_2^i]$, $C \leftarrow G_1^\ast$, $O \leftarrow (1, a^i)$, return $(C, O)$.
  \item $\rho \leftarrow \mathbb{Z}_q$, $C \leftarrow [\rho \cdot f_S(a)]_1$, $O \leftarrow (0, \rho)$, return $(C, O)$.
\end{itemize}

Algorithm \texttt{SC.Open}_{\text{FHE}}(pp_{SC}, ck_{SC} = (\{[a_1^i], [a_2^i]\}_{i \in [\ell]}), C, S, O = (b, \rho)) :

\begin{itemize}
  \item If $C \notin G_1^\ast \lor \rho \notin \mathbb{Z}_q$, return 0.
  \item If $S \notin \mathbb{Z}_q \lor \#S = 0 \lor \ell < \#S$, return \bot.
  \item If $b = 1 \land [\rho]_1 = [a_1^i]$, return 1.
  \item If $b = 0 \land C = [\rho \cdot f_S(a)]_1$, return 1.
  \item Otherwise, return 0.
\end{itemize}

Algorithm \texttt{SC.OSubset}_{\text{FHE}}(pp_{SC}, ck_{SC} = (\{[a_1^i], [a_2^i]\}_{i \in [\ell]}), C, S, O = (b, \rho), S' ) :

\begin{itemize}
  \item \texttt{SC.Open}_{\text{FHE}}(pp_{SC}, ck_{SC}, C, S, O) = 0 \lor S' \notin S \lor S = \emptyset$, return \bot.
  \item If $b = 1$,
    \begin{itemize}
      \item If $\rho \in S'$, return $W \leftarrow \bot$, otherwise return $W \leftarrow f_{S'}(\rho)^{-1} \cdot C$.
    \end{itemize}
  \item If $b = 0$, return $W \leftarrow [\rho \cdot f_{S \setminus S'}(a)]_1$.
\end{itemize}

Algorithm \texttt{SC.VSubset}_{\text{FHE}}(pp_{SC}, ck_{SC} = (\{[a_1^i], [a_2^i]\}_{i \in [\ell]}), C, T, W) :

\begin{itemize}
  \item If $C \notin G_1^\ast$, return 0.
  \item If $T \notin \mathbb{Z}_q \lor \#T = 0 \lor \ell < \#T$, return 0.
  \item If there exist $\rho'$ such that $[\rho']_1 = [a_1^i]$
    \begin{itemize}
      \item If $W = \bot$, return 1, otherwise, return 0.
    \end{itemize}
  \item If $W \in G_1^\ast \land e(W, [f_T(a)]_2) = e(C, [1]_2)$, return 1.
  \item Otherwise, return 0.
\end{itemize}

\textbf{Fig. 4.} The construction of \texttt{SC}_{\text{FHE}}.
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