Estimating Origin-Destination Matrix of Bogor City Using Gravity Model

I Eko Wilicaksono¹, F Bukhari¹, and A Aman¹

¹Department of Mathematics, Bogor Agriculture University, Jl. Meranti, Kampus IPB Dramaga Bogor 16680, Indonesia

Abstract. Origin-Destination (O-D) Matrix describes people movement in a certain area. An O-D matrix is necessary for planning a good public transportation system. However, the exact values of O-D matrix are difficult to measure. There are several ways to estimate O-D matrix such as gravity model, gravity opportunity model, etc. In this study, gravity model was used to estimate the O-D matrix in Bogor city. The following assumptions were used to estimate the O-D matrix: (i) forces between two different zones are related to some existing parameters such as population, social-economic condition, etc. (ii) the people movements are influenced by accessibility from origin to destination, and the accessibility affected by distance, time, and/or cost.

1. Introduction
Nowadays, the world is growing fast, including Bogor city. In 2013, Bogor city has more than 1 million citizen who lived, but they still encounter the same transportation problem. The main transportation problem is about making a good transportation system in Bogor. The transportation system including the comfort infrastructure for people, the effective and efficient route for mass rapid transit, the number of bus or another transportation infrastructure and its frequency in every route. The most important thing is the transportation system should have to fulfill the demand of transportation.

In order to fulfill the demand of transportation, the demand of transportation should be known[11]. The demand of transportation can be presented by origin-destination matrix[7]. The origin-destination matrix is a matrix which is each cell represent the number of trip from origin (row) to the destination (column). There are several ways to estimate the origin-destination matrix. In conventional way, we can count the people trip from their origin to the destination by conducting a survey. But, conducting a survey would spend a lot of resources[8].

The scientist have conducted research about estimating the origin-destination matrix[3,9]. They used synthetic model to estimate the origin-destination matrix. This study used synthetic model, the gravity model for transportation, to estimate the origin-destination matrix.

2. Gravity Model
This study used gravity model to estimate the O-D matrix. The idea of gravity method originally came from Newton’s gravitational law. Casey was the first who suggested such an approach to synthesise shopping trips and catchment areas between towns in region[9]. In its simplest formulation to the model has the following functional form:
\[ T_{ij} = \frac{\alpha P_i P_j}{d_{ij}^2} \]  

(1)

where \( P_i \) and \( P_j \) were the following populations of the origin and destination, \( d_{ij}^2 \) was the distance between origin and destination, and \( \alpha \) was a proportional factor. These gravity model was not ready to use for transportation purpose. So, there were some improvements to be applied including the use of total trip ends instead of population and several parameter to calibrate. After the improvement have been applied to equation (1), the gravity model for transportation can be used for transportation purpose[6].

The gravity model for transportation uses some existing parameter such as population and social-economic condition. The formulation of gravity model for transportation is shown below.

\[ T_{ij} = O_i D_j A_i B_j f(c_{ij}) \]  

(2)

Here, \( T_{ij} \) is a people movement from zone \( i \) to zone \( j \), \( O_i \) the number of people from zone \( i \), \( D_j \) the number of fieldwork in zone \( j \), \( f(c_{ij}) \) the deterrence function, \( A_i \) and \( B_j \) the balancing factor which are defined as follow:

\[ A_i = \left( \sum_j B_j D_j f(c_{ij}) \right)^{-1} \]  

(3)

\[ B_j = \left( \sum_i A_i O_i f(c_{ij}) \right)^{-1} \]  

(4)

In equation (1), we use negative exponential function for the deterrence function[2],

\[ f(c_{ij}) = e^{-\beta c_{ij}} \]  

(5)

The deterrence function in equation (1) is affected by the value of \( \beta \). \( \beta \) means the average cost trip in the research area. If the value of \( \beta \) is increasing, the average cost trip is decreasing. There are some calibration techniques to get the value of \( \beta \), such as Hyman method. This study used Hyman method to calibrate the value of \( \beta \).

3. Hyman Method

The Hyman method is one of the techniques to calibrate the parameter in deterrence function[4]. Williams was compared several method to calibrate the parameter, and the Hyman method was robust and effective than other methods[10]. We would describe the Hyman method by the following paragraph.

Define \( T(\beta) \) the O-D matrix of the current estimate of \( \beta \). The matrix also defined by the equation:

\[ \sum_{i,j} T_{ij}(\beta) = T(\beta) \]  

(6)

The method based on the requirement of \( c(\beta) = c^* \), where:

\[ c(\beta) = (T(\beta))^{-1} \sum_{ij} T_{ij}(\beta) c_{ij} \]  

(7)

\[ c^* = \left( \sum_j N_j \right)^{-1} \sum_{ij} N_j c_{ij} \]  

(8)
$c^*$ is the mean cost from observation and $N_{ij}$ is the number of observed trip from each origin destination pair. The Hyman method can be described as follows:

1. Start the first iteration by making $m = 0$ and an initial estimate of $eta_0 = (c^*)^{-1}$.

2. Using the value of $\beta_0$ calculate a trip matrix using the standard gravity model. Obtain the mean modelled trip cost $c_0$ and estimate a better value for $\beta$ as follows:

   $$\beta_m = \beta_0 (c^*)^{-1}$$

3. Make $m = m+1$. Using the latest value for $\beta$ (i.e. $\beta_{m-1}$) calculate a trip matrix using standard gravity model and obtain the new mean modelled trip cost $c_{m-1}$ and compare it with $c^*$. If they are sufficiently close, stop and accept $\beta_{m-1}$ as the best estimate for this parameter; otherwise go to step 4.

4. Obtain the better estimate of $\beta$ as:

   $$\beta_{m+1} = \frac{(c^* - c_{m-1}) \beta_m - (c^* - c_m) \beta_{m-1}}{c_m - c_{m-1}}$$

5. Repeat steps 3 and 4 as necessary, i.e. until the last mean modelled cost $c_{m-1}$ is sufficiently close to the observed value of $c^*$.

4. Numerical Result

This study was applied to Bogor city. In 2013, Bogor city had more than 1 million population including 403,628 workers. An assumption was applied to this study which was the worker trip used to estimate the origin-destination matrix of Bogor city. Bogor city assumed to be a closed system, so that the worker trip had the origin and destination within Bogor city.

From the observation, we got the trip matrix of the worker’s trip as shown in Table 1 below. The distance between subdistrict was shown in Table 2. There were assumed no distance between the same subdistrict. We use distance for deterrence function.

**Table 1.** Observed worker’s trip matrix

|       | Central | West | South | East | North | Tn. Sareal |
|-------|---------|------|-------|------|-------|------------|
| Central | 24680   | 6250 | 1068  | 8380 | 511   | 2759       |
| West   | 50950   | 12903| 2204  | 17299| 1054  | 5697       |
| South  | 42105   | 10663| 1822  | 14296| 871   | 4708       |
| East   | 22624   | 5730 | 979   | 7681 | 468   | 2530       |
| North  | 41674   | 10554| 1803  | 14150| 862   | 4660       |
| Tn. Sareal | 46193 | 11699| 1998  | 15684| 956   | 5165       |

**Table 2.** Distances between subdistrict (km)

|       | Central | West | South | East | North | Tn. Sareal |
|-------|---------|------|-------|------|-------|------------|
| Central | 0       | 1.047| 2.852 | 3.442| 1.833 | 2.486      |
| West   | 1.047   | 0    | 2.988 | 4.005| 2.914 | 3.138      |
| South  | 2.852   | 2.988| 0     | 1.645| 3.818 | 5.194      |
| East   | 3.442   | 4.005| 1.645 | 0    | 3.54  | 5.274      |
| North  | 1.833   | 2.914| 3.818 | 3.54 | 0     | 1.913      |
| Tn. Sareal | 2.486 | 3.138| 5.194 | 5.274| 1.913 | 0          |
Numerical simulation using Fortran[5] yield the value of $\beta = 1.1679 \times 10^{-7}$ and the calculated origin-destination matrix of Bogor city shown in table 3. In figure 1 below, we served the calculated origin-destination matrix in a map of Bogor city to facilitate the government for determine the best policy to solve any problem that occurred in transportation in Bogor city.

**Table 3.** Calculated origin-destination matrix in Bogor city

|        | Central | West | South | East | North | Tn. Sareal |
|--------|---------|------|-------|------|-------|------------|
| Central| 24680   | 6250 | 1068  | 8379 | 511   | 2759       |
| West   | 50950   | 12904| 2204  | 17299| 1054  | 5697       |
| South  | 42105   | 10663| 1822  | 14296| 871   | 4708       |
| East   | 22623   | 5729 | 979   | 7682 | 468   | 2530       |
| North  | 41674   | 10554| 1803  | 14150| 862   | 4660       |
| Tn. Sareal | 46193 | 11698| 1998  | 15683| 956   | 5165       |

**Figure 1.** Worker’s trip pattern in Bogor city.

5. Conclusion
This study estimated the worker’s trip in the morning. The worker’s trip would presented as origin-destination matrix. The origin-destination matrix can be estimated by gravity model for transportation. In gravity model for transportation, there was a parameter which was need to calibrate. The parameter was in the deterrence function. There were some calibration techniques that have been used in different study, one of them was Hyman method. This study used Hyman method as the calibration method.

In the numerical simulation, the parameter which was calibrated by Hyman method, has been evaluated. This study got the value of $\beta = 1.1679 \times 10^{-7}$, and the origin-destination matrix of Bogor city as shown in table 3 above. For the further study, the value of $\beta$ can be used for estimating the origin-destination matrix of Bogor city for coming years.
References

[1] Casey H J 1955 Traffic Quarterly 9 23-35
[2] Evans A W 1971 Transpn. Res. 5 15-38.
[3] Fathoni M 2005 Estimasi matriks asal dan tujuan perjalanan penumpang angkutan umum trans jawa – sumatera melalui lintasan merak – bakauheni (Yogyakarta: Gadjah Mada University).
[4] Hyman G M 1969 The calibration of trip distribution models (London: Environment and Planning A) pp 105-112.
[5] Lemmon D R and Schafer J L 2005 Developing Statistical Software in Fortran 95 (New York: Springer).
[6] Ortúzar J de D and Willumsen L G 2011 Modelling Transport 4th Edition (New Delhi: John Wiley & Sons).
[7] Peterson A 2007 The Origin-Destination Matrix Estimation Problem – Analysis and Computation (Norrköping: University of Linköping).
[8] Roziqin C 2012 J. Tek. Sipil. 12 28-34.
[9] Tamin O Z 2008 Perencanaan dan Pemodelan Transportasi (Bandung: Penerbit ITB).
[10] Williams I 1976 Transpn. Res. 10 91-104.
[11] Willumsen L G 1978 Estimation of an O-D Matrix From Traffic Counts – A Review (Leeds: Institute of Transport Studies).