Quantum dots as scatterers in electronic transport: interference and correlations

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Abstract

Conductance through a system consisting of a wire with side-attached quantum dots is calculated. Such geometry of the device allows to study the coexistence of quantum interference, electron correlations and their influence on conductance. We underline the differences between "classical" Fano resonance in which the resonant channel is of single-particle nature and "many-body" Fano resonance with the resonant channel formed by Kondo effect. The influence of electron-electron interactions on the Fano resonance shape is also analyzed.

Key words: A. Nanostructures, D. Electron transport, D. Kondo effects, D. Fano resonance

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1 Introduction

A model consisting of a quantum dot (QD) attached to a metallic quantum wire is conceptually analogous to the Fano model [1] consisting of a continuous spectrum and a discrete level. Such geometry is also in close relation to the behavior of magnetic impurities embedded in the host metal, where the conduction electrons are scattered resonantly on impurities while propagating through the metal. This arrangement can be opposed to the standard geometry, where a quantum dot is connected in series with leads and resonantly enhances the conductance. A model of a quantum dot side-coupled to the quantum wire is also applicable for the simplest description of a magnetic atom deposited on the metallic surface. In such a "mapping" the QD in Kondo regime mimics the adatom with nonzero magnetic moment and the wire (when assumed to be multichannel) serves as an analog of the metallic surface host. Thus, the model considered by us can provide information applicable to various nanostructures.
Fano effect, described in 1961, has been observed in many physical systems: in nuclear scattering \([2]\), scattering from donor impurities in an electron waveguide \([3]\), optical absorption \([4]\), electronic transport in heterojunctions \([5]\). Recently it is seen in new light due to the rapid progress in nanotechnology.

Apart from ‘classical’ Fano effect, where the resonant channel is provided by discrete single-particle level, a new type emerged, where the resonant channel is formed by many-body effects. This additional resonant channel is formed by the electron-electron interactions resulting in the sharp Abrikosov-Suhl (Kondo) resonance in the vicinity of Fermi surface. This kind of Fano resonance has been observed recently in scanning tunnelling microscope experiments for single atoms placed on metallic surface \([6,7]\) and also in lateral quantum dots \([8]\). There were theoretical attempts to describe these phenomena in \([9]\) and in \([10]\), respectively.

A single quantum dot modelled by Anderson Hamiltonian and side-attached to a perfect wire was investigated in resonant regime by slave boson mean field approach \([11]\), exact diagonalization \([12]\), interpolative perturbative scheme (IPS) \([13]\) and X- boson treatment \([14]\). For this arrangement we concentrate on the crossover between "classical" and "many-body" Fano resonances.

A system of two quantum dots placed in arms of Aharonov-Bohm ring has been investigated recently for non-interacting electrons \([15]\). In our model we study the influence of the strong electron correlations within the dots on transport. Repulsive Coulomb interactions in the nanoscale devices are defined by the charging energy \(\sim e^2/2C\), where \(C\) is the capacitance of the device and \(e\)-electronic charge. They cause the most celebrated phenomena as Coulomb blockade and resonant (Kondo) electron transport \([8]\).

### 2 Model and calculations

Hamiltonian of the nanodevice consisting of metallic wire and quantum dots attached parallel to it, is taken in the form:

\[
H = \sum_{k,\sigma} \epsilon_k c_{k,\sigma}^+ c_{k,\sigma} + \sum_{\gamma=1,2} \sum_{\sigma} \epsilon_\gamma d_{\gamma,\sigma}^+ d_{\gamma,\sigma} + \sum_{\gamma=1,2} U_\gamma n_{\gamma\uparrow} n_{\gamma\downarrow} + \sum_{\gamma=1,2} \sum_{\sigma, k} t_\gamma [d_{\gamma,\sigma}^+ c_{k,\sigma} + h.c.]
\]

Creation operator \(c_{k,\sigma}^+\) (\(d_{\gamma,\sigma}^+\)) describes the conduction electron in the wire with momentum \(k\), energy \(\epsilon_k\) and spin \(\sigma\) (\(\gamma\)-th QD localized electron with spin \(\sigma\)). \(n_{\gamma,\sigma}\) is the number of electrons with the spin \(\sigma\) at the localized state \(\gamma\) with the Coulomb repulsion \(U_\gamma\). The last term describes hopping between QDs and
wire. We assume spin-only degeneracy of the discrete QD’s levels and the wire
to be single-channel. The subscript $\gamma$ represents the orbital quantum number,
which follows from quantization due to spatial confinement of the 2D electron
gas within the quantum dot (see for example [16]). We underline that $\gamma$ is not
conserved in the process of hopping of electron between QD and causes an
indirect interaction between the dots.

To investigate transport through nanodevice, the wire has been connected
to external electrodes: 

$$H_{\text{con}} = \sum_{\alpha=L,R} \sum_{k,k',\sigma} t_{\alpha} [c_{k\sigma,\alpha}^+ c_{k'\sigma,\alpha} + h.c.];$$

c $c_{k\sigma,\alpha}^+$ is the creation
operator of the electron with the spin $\sigma$ and energy $\epsilon_k$ in the electrode $\alpha$.
Spectral densities of electronic states in the wire $\rho_{w,\sigma}(\omega)$ and in the electrodes
$\rho_{\text{el},\sigma}(\omega)$ have been assumed to have Lorentzian shape with a halfwidth much
larger than Kondo temperature of each QD.

The numerical parameters of the equal Coulomb repulsion $U_\gamma = 1$ meV (for
$\gamma =1, 2$) and tunneling strength $\Gamma_{\gamma,\sigma}(\omega = \epsilon_F) = 2\pi t_\gamma^2 \rho_{w,\sigma}(\omega = \epsilon_F) \equiv \Gamma_{\gamma,\sigma} = 0.28$ meV have been chosen to meet the experimental QD’s data ($\epsilon_F = 0$
defines the Fermi level in the wire). A scheme of the investigated na
nodevice is presented in the $a$-inset of Fig.(2).

Following Meir and Wingreen [23], conductance can be related to the spectral
density of the central region (in the present case - metallic quantum wire with
quantum dots attached to it) and its coupling $\Gamma_{L(R),\sigma}(\omega) = 2\pi t_{L(R)}^2 \rho_{\text{el},\sigma}(\omega)$ to
left (L) and right (R) electrode. For the symmetric coupling and in the zero
limit of drain-source voltage conductance can be written in the form:

$$G = \frac{2\pi e^2}{h} \sum_{\sigma} \int_{-\infty}^{\infty} \Gamma_{\sigma}(\epsilon) \left(-\frac{\partial f(\epsilon)}{\partial \epsilon}\right) \rho_{w,\sigma}(\epsilon) d\epsilon, \quad (2)$$

where $f(\epsilon)$ is the Fermi distribution function and $\Gamma_{\sigma}(\epsilon) = \Gamma_{L,\sigma}(\epsilon) \Gamma_{R,\sigma}(\epsilon)/(\Gamma_{L,\sigma}(\epsilon) +
\Gamma_{R,\sigma}(\epsilon))$. Spectral density of the investigated nanodevice is calculated from
the appropriate retarded Green function: $\rho_{w,\sigma}(\omega) = -(1/\pi)\text{Im} G_{\sigma}(\omega + i\delta)$. At
$T = 0$, when $-\frac{\partial f(\omega)}{\partial \epsilon} \rightarrow \delta(\epsilon)$ the formula for conductance takes a simple form

$$G = \frac{2\pi e^2}{h} \sum_{\sigma} \Gamma_{\sigma}(0) \rho_{w,\sigma}(0).$$

To calculate conductance through the considered nanodevice the Green function
of the electronic wave propagating through the wire should be calculated in presence of QDs attached to the wire. We start from the Dyson equation
(energy and spin dependence of the Green functions and selfenergies is under-
stood):

$$G = G_0 + G_0 \Sigma G,$$  \quad (3)

which can be written by iterating in the form of T-matrix:
\[ G = G_0 + G_0 T G_0, \quad T = \Sigma/(1 - \Sigma G_0). \]  

(4)

\( G_0 \) is a free conduction electron Green function. The form of selfenergy (and T-matrix) depends on the geometry of nanodevice and approximation made.

Dyson equation describing scattering of the electronic wave on both quantum dots:

\[
G = G_0 + G_0 \Sigma_1 G_0 + G_0 \Sigma_2 G_0 + G_0 \Sigma_1 G_0 \Sigma_2 G_0 + G_0 \Sigma_2 G_0 \Sigma_1 G_0 + ... 
\]

(5)

can be cast into the form Eq.(3), when \( \Sigma = \Sigma_1 + \Sigma_2 \). The selfenergy \( \Sigma_\gamma \), (\( \gamma = 1, 2 \)) arises from scattering of the conduction electron propagating through the wire on the \( \gamma \)-th quantum dot. Thus, the total selfenergy \( \Sigma \) contains infinite number of scattering events in quantum dot \( \gamma \) alone and multiple scatterings involving both quantum dots. Utilizing the second part of Eq.(4) we can express \( \Sigma_1 \) and \( \Sigma_2 \) by \( T_1 \) and \( T_2 \) and the selfenergy due to both quantum dots is:

\[
\Sigma = \sum_\gamma \Sigma_\gamma = \frac{T_1}{(1 + T_1 G_0)} + \frac{T_2}{(1 + T_2 G_0)}. 
\]

(6)

For non-interacting electrons, the expression for T-matrix due to one QD attached to the wire obtained by equation of motion (EOM) for conduction electron Green function is of the form \( T_{\gamma,\sigma}(\omega) = t^2_{\gamma} G_{QD\gamma,\sigma}(\omega) \), where the Green function of the dot \( G_{QD\gamma}(\omega) \) represents a simple single-particle level. In the interacting case, it is replaced by dressed Green function of the localized QD level, which has to be calculated in such approximation that properly describes Kondo effect. In the present work the Interpolative Perturbative Scheme (IPS) is utilized [17,18,19]. It is based on the selfconsistent second order perturbation (SOPT) in Coulomb repulsion \( U \) [20,21]. However, within the IPS the calculated selfenergy due to strong electron correlations \( \Sigma^{IPS} \) is taken in the form, that has correct limits: when \( U \to 0 \), \( \Sigma^{IPS} \to \Sigma^{SOPT} \) and when \( \Gamma \to 0 \), \( \Sigma^{IPS} \to \Sigma^{atom} \). This allows to calculate the conductance both in resonant regime and in Coulomb blockade regime. Moreover, by replacing Hartree-Fock solution for the impurity level by an effective field determined selfconsistently from the condition on particle number, the fulfillment of the Friedel-Langreth sum rule [22] is also obtained within this method. It is applicable both for \( T = 0 \) and finite temperatures.

If the expressions for \( T_1 \) and \( T_2 \)-matrices are known, then the total selfenergy Eq.(6) is calculated and finally total T-matrix by second part of Eq.(4):

\[
T = \frac{T_1(1 + T_2 G_0) + T_2(1 + T_1 G_0)}{(1 + T_1 G_0)(1 + T_2 G_0) - G_0[T_1(1 + T_2 G_0) + T_2(1 + T_1 G_0)]}. \]

(7)
The extension of the above formalism on $\gamma = N$ QDs connected to the wire in a star-like fashion is straightforward.

3 One quantum dot attached to the wire

Utilizing Eq.(4) spectral density for the propagating electron can be written in the following form:

$$\rho_{w,\sigma}(\omega) = \rho_{w0,\sigma}(\omega) \times$$

$$\{1 + ImG_{0,\sigma}(\omega)[ImT_{\sigma}(\omega)(q_\sigma^2 - 1) - 2q_\sigma ReT_{\sigma}(\omega)]\},$$

where we have denoted:

$$q_\sigma = -\frac{ReG_{0,\sigma}(\omega + i\delta)}{ImG_{0,\sigma}(\omega + i\delta)} = \frac{\Delta_\sigma}{\Gamma_\sigma},$$

which can be identified as Fano parameter. The real and imaginary parts of the free retarded conduction electron Green function are: $ReG_{0,\sigma}(\omega) = P \int d\epsilon \rho_{w0,\sigma}(\epsilon)/(\omega - \epsilon)$ and $ImG_{0,\sigma}(\omega) = -\pi \rho_{w0,\sigma}(\omega)$, and parameters $\Delta_\sigma = t_\sigma^2 ReG_{0,\sigma}$ and the $\Gamma_{\gamma,\sigma} = \pi t_\sigma^2 \rho_{w0,\sigma}(\omega)$. In the non-interacting case when the QD is represented by a single-particle level, $G_{QD\gamma,\sigma} = [\omega - \epsilon_{\gamma,\sigma} - \Delta_\sigma + i\Gamma_{\gamma,\sigma}]^{-1}$, and making substitution $\epsilon = (\omega - \epsilon_{\gamma,\sigma} - \Delta_\sigma)/\Gamma_{\gamma,\sigma}$ we get the Fano well-known formula from Eq.(8): $\rho_{w,\sigma}(\omega) = \rho_{w0,\sigma}(\omega)[(\epsilon + q_\sigma)^2/(\epsilon^2 + 1)]$.

For the interacting quantum dot, the Green function in expression for T-matrix should contain appropriate information on correlations. Within IPS it has the form:

$$G_{QD1,\sigma}^{INT}(\omega) = [\omega - \epsilon_{1,\sigma} - \Delta_\sigma - n_{1,-\sigma}U - \Sigma_{1,\sigma}^{IPS} + i\Gamma_{1,\sigma}]^{-1}.$$  

(10)

Thus, the bare quantum dot level is additionally renormalized and gets the width due to selfenergy $\Sigma^{IPS}$ by electron-electron interactions and the Kondo peak is generated by specific $\omega$-dependence of $\Sigma^{IPS}$.

Numerical calculations of the selfenergy within IPS have been performed under assumption of smooth and weakly $\omega$-dependent density of states in the wire near the Fermi level. This is a requirement for stability of numerical procedures when calculating selfenergy $\Sigma^{IPS}$. Then, the integral defining $ReG_{0,\sigma}(\omega) = P \int_{-D}^{D} d\epsilon \rho_{w0,\sigma}(\epsilon)/(\omega - \epsilon) \approx \rho_{w0,\sigma}ln\left|(D + \omega)/(D - \omega)\right| \rightarrow 0$ for halfwidth of spectral density of the wire $D \gg \omega$ implying $q_\sigma = 0$ (see Eq.(9)).
Fig. 1. Conductance through the wire with one QD side attached to it vs. QD spatial level position. Two regimes are presented: resonant regime (RR) and Coulomb blockade (CB) regime. Calculations are made for Coulomb repulsion $U = 1 \text{ meV}$. Open circles - $U = 0$ case.

Utilizing Eqs. (8) and (2), at $T = 0$ (and for $g_\sigma = 0$) the formula for conductance can be written:

$$G = \frac{2\pi e^2}{h} \sum_\sigma \Gamma_\sigma(0) \rho_{w0,\sigma}(0) \left[ 1 - \frac{\Gamma_{\text{max},\sigma}^2}{(\epsilon_\sigma)^2 + \Gamma_{\text{max},\sigma}^2} \right],$$  \hspace{1cm} (11)$$

where for non-interacting case $\epsilon_\sigma = \epsilon_{1,\sigma}$ is the bare QD level, and in presence of interactions $\epsilon_\sigma = \epsilon_{\text{eff}1,\sigma} \equiv \epsilon_{1,\sigma} + n_{1,-}\sigma U + \text{Re}\Sigma_{1}^{IP\sigma}(0)$ is the renormalized level. In this particular case, when $\text{Im}\Sigma_{1}^{IP\sigma}(\omega = 0) = 0$ as for Fermi liquid, the conductance for non-interacting and interacting cases can be expressed by the same relation. It is a consequence of the fact, that Hamiltonian Eq.(1) in this case can be formally written as for non-interacting resonant level model, with bare QD levels $\epsilon_{\gamma,\sigma}$ replaced by renormalized ones $\epsilon_{\text{eff}\gamma,\sigma}$. Conductance vs. QD level position is plotted in Fig.(1). Two types of curves are shown: in resonant regime (RR), with one broad antiresonance, and in Coulomb blockade (CB) regime with two antiresonances. In the resonant regime, for $T = 0$, the total extinction of conductance is observed for $\epsilon_\gamma < 0$ and $\epsilon_\gamma + U > 0$, i.e. when the QD level is singly occupied. In this case the destructive interference has its full strength due to Kondo resonance. For $\epsilon_\gamma > 0$ (empty level) or $\epsilon_\gamma + U < 0$ (doubly occupied level) the dot weakly disturbs transport through the wire and conductance approaches unitary limit. For this configuration Kondo temperature $T_K \sim 700 \text{ mK}$ for $\epsilon_{\gamma,\sigma} = -U/2$, as obtained from the full width of the QD density of states at half maximum. When temperature increases, the Kondo effect is gradually diminished and the scattering of the electronic waves by many-body QD’s Kondo level becomes less effective. By a decrease of QD coupling to the wire the system is driven into the Coulomb blockade regime. By this operation the Fano resonance caused by effective
Kondo peak in the vicinity of Fermi surface is depressed. Instead, two Fano resonances appear due to scattering on two single particle levels with the separation of the order of $U$. Although the Kondo temperature of the QD has been decreased by lowering hopping $t_1$, the electron correlations still exist in the system. When the temperature is increased the CB peaks get sharper due to further diminishing of strong electron correlations. For comparison, the conductance for $U = 0$ has been calculated. In this case the total suppression of conductance takes place when the bare dot level crosses Fermi level.

How Fano resonances appear conductance in Fig. (1) can be better understood in a simplified picture. At temperatures much lower than Kondo temperature, $T \ll T_K$, the Green function of interacting quantum dot can be approximated as:

$$G_{QD,\sigma}^{INT}(\omega + i\delta) \approx \frac{Z_1}{\omega - \epsilon_{1,\sigma} - \Delta_\sigma + i\Gamma_1,\sigma} + \frac{Z_U}{\omega - \epsilon_{1,\sigma} - \Delta_\sigma - U + i\Gamma_1,\sigma} + \frac{Z_K}{\omega - \epsilon_K + iT_K}$$  \hspace{1cm} (12)

The first and second terms simulate charge peaks in the spectral density and the third term represents many-body Kondo peak of the width proportional to the Kondo temperature $T_K$ and position $\epsilon_K$ in the vicinity of Fermi surface. $Z_1$, $Z_U$, and $Z_K$ are appropriate strengths of the poles: $Z_1 + Z_U + Z_K \leq 1$.

In the Kondo limit the Kondo peak can be regarded as a potential scatterer [24] and $Z_K \sim \pi T_K / \Gamma$ [25]. Utilizing Eqs.(12) and (8) the spectral density of conduction electron scattered by QD is written:

$$\rho_{w,\sigma}(\omega) = \rho_{w0,\sigma}(\omega) \left( 1 + Z_1 \frac{q_\sigma^2 - 2q_\sigma x_1 - 1}{x_1^2 + 1} + \pi \frac{q_\sigma^2 - 2q_\sigma x_K - 1}{x_K^2 + 1} \right), \hspace{1cm} (13)$$

where following substitutions were made: $x_1 = \frac{\omega - \epsilon_{1,\sigma} - \Delta_\sigma}{\Gamma_1,\sigma}$ and $x_K = \frac{\omega - \epsilon_K}{T_K}$. The second and third terms produce the shape of Fano resonance. When temperature is sufficiently low and coupling to the wire is large, electron interactions prevail in the dot. Then the third term in Eq.(13) dominates and the Fano resonance in the conductance vs. position of the QD spatial level dependence is due to many-body effects. The maximum of Kondo resonance (which is transformed into Fano resonance in conductance for present geometry) takes place for $\epsilon_1 = -U/2$, i.e. for symmetric Anderson model. The influence of the second term describing "classical" Fano effect is practically obscured by the Kondo effect. It becomes to be visible in the Coulomb Blockade regime, when Kondo temperature of the system is decreased by lowering the hybridization strength $\Gamma_\gamma$ of QD with the wire. Thus, the Kondo temperature is the energy scale which governs the transition between "classical" and "many-body" Fano resonances.

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Phase shift acquired due to scattering on the localized level can be derived when the form of T-matrix is known [24]: \( \eta(\omega) = \arg T(\omega + i\delta) \). For non-interacting electrons, at the Fermi level \( \omega = 0 \), it has the form \( \eta(\omega = 0) = \arctan(\frac{\Gamma_1}{\epsilon_1,\sigma}) \), where \( \epsilon_1,\sigma \) is the bare QD level. Thus, the phase is changed by \( \pi/2 \) when QD level \( \epsilon_1,\sigma \) crosses the Fermi level. For interacting case \( \eta(\omega = 0) \approx \arctan(\frac{T_{kk}}{\epsilon_1,\sigma}) \), because the resonant level is of many-body nature. The position of the Kondo peak is \( \epsilon_K \simeq 0 \), which gives \( \eta(\omega = 0) \approx \pi/2 \).

4 Two quantum dots side attached to the wire

The formula for the Green function of conduction electron scattered by two quantum dots was obtained from Eq.(4), for the selfenergy given by Eq.(6) and total T-matrix given by Eq.(7). The spectral density has the form of:

\[
\rho_{w,\sigma}(\omega) = \rho_{w_0,\sigma}(\omega) \times \left[ 1 - \frac{[\Gamma_{1,\sigma}(\omega - \epsilon_{2,\sigma}) + \Gamma_{2,\sigma}(\omega - \epsilon_{1,\sigma})]^2}{[(\omega - \epsilon_{1,\sigma})(\omega - \epsilon_{2,\sigma})]^2 + [\Gamma_{1,\sigma}(\omega - \epsilon_{2,\sigma}) + \Gamma_{2,\sigma}(\omega - \epsilon_{1,\sigma})]^2} \right],
\]
and conductance at \( T=0 \) can be expressed as follows:

\[
G = \frac{2\pi e^2}{h} \sum_{\sigma} \Gamma_{\sigma}(0) \rho_{w_0,\sigma}(0) \left[ 1 - \frac{[\epsilon_{1,\sigma}\Gamma_{2\max,\sigma} + \epsilon_{2,\sigma}\Gamma_{1\max,\sigma}]^2}{(\epsilon_{1,\sigma}\epsilon_{2,\sigma})^2 + [\epsilon_{1,\sigma}\Gamma_{2\max,\sigma} + \epsilon_{2,\sigma}\Gamma_{1\max,\sigma}]^2} \right].
\]

Thus, \( G = 0 \) when \( \epsilon_1 = 0 \) or \( \epsilon_2 = 0 \) is tuned to the Fermi surface in the wire. On the other hand, for \( \epsilon_{\gamma} = (-\Gamma_{\gamma}/\Gamma_{\gamma'})\epsilon_{\gamma'} \) conductance reaches unitary limit at \( T = 0 \). For such condition the total phase shift \( \eta(\omega) = -\arctan(\sum_{\gamma=1,2} \Gamma_{\gamma}(\omega - \epsilon_{\gamma}) \Gamma_{\gamma}) \) is equal to zero and the electronic wave propagates through the wire without scattering. Thus, by setting a proper configuration of the QD’s levels by gate voltages, the conductance through device can be set to unity or to zero.

When the level of one QD, say \( \epsilon_2 \), is kept fixed, while \( \epsilon_1 \) is shifted by the gate voltage, then the Fano-like resonances appear in the conductance. The QD level \( \epsilon_2 \) acts as additional background channel and the shape of Fano resonance depends on its position with respect to the Fermi level. Indeed, equation (15) can be approximated by the expression:

\[
G \simeq \frac{2\pi e^2}{h} \sum_{\sigma} \Gamma_{\sigma}(0) \rho_{w_0,\sigma}(0) \left[ 1 - \frac{\Gamma_{2\max,\sigma}^2}{\epsilon_{2,\sigma}^2 + \Gamma_{2\max,\sigma}^2} \frac{(\epsilon_{\sigma} + q_{\sigma})^2}{\epsilon_{\sigma}^2 + 1} \right],
\]
where following substitutions have been made: \( \epsilon_{\sigma} = \epsilon_{1,\sigma}/\Gamma_{\max,1,\sigma} \) and Fano parameter \( q_{\sigma} = \epsilon_{2,\sigma}/\Gamma_{\max,2,\sigma} \).
The non-conservation of the orbital quantum number while hopping of electrons between quantum dots and wire is crucial for obtaining Fano-like shaped conductance. In the opposite case no information could flow from one quantum dot to the other and the dots would act as separate entities.

To study this problem in detail we considered the Hamiltonian, Eq.(1), without Coulomb interactions which is quadratic and can be solved exactly. EOM method gives the expression for Fourier transformed retarded Green function

$$G_{\gamma,\sigma}(t-t') = \langle\langle d_{\gamma,\sigma}(t); d_{\gamma,\sigma}^+(t')\rangle\rangle$$

of $\gamma$-QD level:

$$G_{\gamma,\sigma}(\omega) = \frac{1}{\omega - \epsilon_{\gamma,\sigma} + i\Gamma_{\gamma,\sigma} + \frac{\Gamma_{\gamma,\sigma}^F}{\omega - \epsilon_{\gamma,\sigma} + i\Gamma_{\gamma,\sigma}}}.$$  (17)

Thus, apart from the broadening caused by the hopping of electrons between $\gamma$- QD and wire, the $\gamma$-level is additionally shifted and broadened due to indirect interaction with $\gamma'$-QD level.

Fourier transformed Green function $G_{\gamma\gamma',\sigma}(t-t') = \langle\langle d_{\gamma,\sigma}(t); d_{\gamma',\sigma}^+(t')\rangle\rangle$ describing virtual hopping between $\gamma$ and $\gamma'$ levels has the form:

$$G_{\gamma\gamma',\sigma}(\omega) = \frac{-i\Gamma_{\gamma\gamma',\sigma}}{(\omega - \epsilon_{\gamma,\sigma})(\omega - \epsilon_{\gamma',\sigma}) + i[\Gamma_{\gamma,\sigma}(\omega - \epsilon_{\gamma',\sigma}) + \Gamma_{\gamma',\sigma}(\omega - \epsilon_{\gamma,\sigma})]}.$$

where $\Gamma_{\gamma\gamma',\sigma} = \pi t_{\gamma}$ $t_{\gamma'}\rho_{w,\sigma}$. Utilizing Eqs.(17-18) and assuming $t_{\gamma}$ ($\gamma = 1, 2$) real, the expression for the total T-matrix due to both quantum dots can be rewritten as:

$$T_{\sigma} = t_1^2 G_{QD1,\sigma} + t_2^2 G_{QD2,\sigma} + t_1 t_2 G_{12,\sigma} + t_{21} G_{21,\sigma} = T_{1,\sigma} + T_{2,\sigma} + 2T_{12,\sigma},$$

where $T_{\gamma\gamma'} = t_{\gamma} t_{\gamma'} G_{\gamma\gamma',\sigma}$ and $T_{\gamma\gamma'} = T_{\gamma'\gamma,\sigma}$. The spectral density, Eq.(14), can now be written in a more transparent form:

$$\rho_{w,\sigma}(\omega) = \rho_{w0,\sigma}(\omega) [1 + \Gamma_{1,\sigma} Im G_{QD1}(\omega) + \Gamma_{2,\sigma} Im G_{QD2}(\omega) + 2\Gamma_{12} Im G_{12}].$$

Eq.(19) consists of terms describing scattering on each $\gamma$ quantum dot separately (but with renormalization due to indirect interaction with $\gamma'$ quantum dot), and the term of multiple scattering. When the $\gamma$- quantum dot is gradually decoupled from the system (by appropriate decrease of coupling $t_{\gamma}$, or by shifting $\epsilon_{\gamma}$ far from the Fermi level) both the $\gamma$-term and last multiple scattering term vanish and Eq.(20) coincides with Eq.(8) (with $q_0 = 0$) for a single quantum dot. When both $\gamma = 1, 2$ levels are close to the Fermi surface, the
Fig. 2. Conductance through a wire with two QDs side-attached to it vs. spatial level position $\epsilon_1$ in $QD_1$ for various values of level $\epsilon_2$ in $QD_2$. Inset $a$ - scheme of the device, inset $b$ - dependence of renormalized level $\epsilon_{eff} \sigma$ level vs. bare $\epsilon_2$ level position.

electronic wave traveling through the wire scatters resonantly in both quantum dots. Classically, this process can be seen as a multiple bouncing of the ball between two walls. This process is summed up to infinity by the Dyson equation Eq.(5). The maximum of the "bouncing effect" on the Fermi surface (for $\omega = 0$) due to the last term in Eq.(20) takes place for $\epsilon_\gamma = \frac{\Gamma_\gamma}{(\epsilon'_\gamma+\Gamma'_\gamma)^{1/2}}(-\epsilon'_\gamma)$, i.e. when the levels lie below and above Fermi surface.

When strong electron correlations within the dots are included, the explicit expression for conduction electron Green function Eq.(4) has complicated form, but it simplifies considerably at $T = 0$ (see the comment below Eq.(11)). The conductance can then be written in exactly the same form as for non-interacting case (Eq.(14, 15 and 16)), where single-particle $\epsilon_{\gamma,\sigma}$ levels have to be replaced by renormalized ones: $\epsilon_{eff\gamma,\sigma} = \epsilon_{\gamma,\sigma} + n_{\gamma,-\sigma}U + Re\Sigma^{IPS}_\gamma(\omega = 0)$ for $\gamma = 1, 2$. The discussion of Eqs.(19 and 20) also holds with the above replacement. In this special case when $Im\Sigma^{IPS} = 0$ and the bare QD levels are renormalized by static field $H_{eff} = n_{\gamma,-\sigma}U + Re\Sigma^{IPS}_\gamma(\omega = 0)$, all the dynamical many-body effects are erased. In general, apart from renormalized by interactions single-particle levels, Kondo resonances are present in the vicinity of Fermi surface which have dominant influence on conductance.

Conductance dependence on the $\epsilon_1$ QD level position for various values of level $\epsilon_2$ is plotted in Fig.(2) at $T = 0$ and in presence of correlations. The change of the Fano resonance shape can be observed as $\epsilon_2$ takes different values. A mirror reflection of the curve takes place when $\epsilon_{eff2,\sigma}$ changes sign changing $q_\sigma = \epsilon_{eff2,\sigma}/\Gamma_{2,\sigma}$ parameter. The $\epsilon_{eff2}$ dependence on bare $\epsilon_2$ is shown in the $b$-inset.
Fig. 3. Conductance through a wire with two QDs side-attached to it vs. spatial level position $\epsilon_1$ in $QD_1$ for a fixed value of level $\epsilon_2=-0.4$ meV in $QD_2$ and $U=1$ meV within the dots. Apart from $T=0$ curve, two curves are plotted for temperatures much higher than the Kondo temperature. Inset: conductance curves for interacting and non-interacting cases calculated for the same Fano parameter $q$.

To compare the conductance curves calculated with and without interactions we show two of them in the inset of Fig.(3). The curve for $U=0$ has been plotted for $\epsilon_2$ equal to corresponding $\epsilon_{eff}^2$ found selfconsistently in interacting case and the same $\Gamma_{2,\sigma}$ to have the same Fano parameter in both cases. Two substantial differences are seen between the curves: i) the curves are shifted; this is caused by renormalization of bare $\epsilon_1$ level by interactions, ii) an additional plateau emerges for $U \neq 0$ which is caused by Kondo resonant scattering when the $\epsilon_1$ level is occupied by one electron.

The $q=0$ case deserves a comment. In this case $\epsilon_{eff}^2$ coincides with Fermi level, which corresponds to the bare $\epsilon_{2,\sigma}=-U/2$ i.e. maximum Kondo resonance in $QD_2$. For such level arrangement conductance $G=0$ for any value of shifted $\epsilon_1$ because of full destructive interference caused by $QD_2$. This is a different situation comparing to classical Fano expression which for $q=0$ describes a symmetric dip. This shows a resonant nature of the background channel provided by the second quantum dot.

In Fig.(3) we show conductance curves for fixed $QD_2$ level position $\epsilon_2=-0.4$ meV at $T=0$ and at temperatures much higher than Kondo temperature. At elevated temperatures the plateau in conductance due to Kondo effect on $QD_1$ disappears, but the shape of the Fano resonance practically does not change. It leads to the conclusion that the shape of the Fano resonance is determined by the renormalized single particle level $\epsilon_{eff}^2$ and not by the many-body Kondo peak caused by resonant scattering on $QD_2$. 

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