Cores and cusps in the dwarf spheroidals

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ABSTRACT

We consider the problem of determining the structure of the dark halo of nearby dwarf spheroidal galaxies (dSphs) from the spherical Jeans equations. Whether the dark haloes are cusped or cored at the centre is an important strategic problem in modern astronomy. The observational data comprise the line-of-sight velocity dispersion of a luminous tracer population. We show that when such data are analysed to find the dark matter density with the spherical Poisson and Jeans equations, then the generic solution is a dark halo density that is cusped like an isothermal \((\rho_D \propto r^{-2})\). Although milder cusps (like the Navarro–Frenk–White \(\rho_D \propto r^{-1}\)) and even cores are possible, they are not generic. Such solutions exist only if the anisotropy parameter \(\beta\) and the logarithmic slope of the stellar density \(\gamma_\ell\) satisfy the constraint \(\gamma_\ell = 2\beta\) at the centre or if the radial velocity dispersion falls to zero at the centre. So, for example, a dSph with an exponential light profile can exist in a Navarro–Frenk–White halo and have a flat velocity dispersion, but anisotropy in general drives the dark halo solution to an isothermal cusp. The identified cusp or core is therefore a consequence of the assumptions (particularly of spherical symmetry and isotropy), and not the data.

Key words: galaxies: kinematics and dynamics – galaxies: structure.

1 INTRODUCTION

Hot stellar systems are held up by the stellar velocity dispersion and have little or no rotation. In fact, many such stellar systems – giant elliptical galaxies and dwarf spheroidals – have a velocity dispersion profile that is constant to a good approximation. The case of the dwarf spheroidals (dSphs) has received particular attention in recent years. Kleyna et al. (2001, 2002) showed that the velocity dispersion profile of Draco is flatish throughout the bulk of the galaxy, although later work by Wilkinson et al. (2004) found evidence for kinematically cold populations in the very outermost parts. Walker et al. (2007) presented stellar velocity dispersion profiles for seven Milky Way dSphs and found almost all to be constant to a good approximation right in to the very centre, although the profile of Sextans seems to dip somewhat. Koch et al. (2007a,b) studied Leo I and Leo II, and also found essentially flatish profiles.

The hope has been that gathering line-of-sight velocities of bright giant stars in the Milky Way dSphs may provide evidence of the structure of the halo in these extremely dark matter dominated galaxies. A question of great interest is whether the dark haloes of the dSphs are cusped, as predicted by numerical simulations in cold dark matter cosmogonies, or cored – but results so far have been inconclusive. For example, Koch et al. (2007b) found that the velocity dispersion data on Leo II are consistent with halo dark matter densities that are both cored and cusped. Wilkinson et al. (2006) and Gilmore et al. (2007) presented an analysis of the velocity dispersion data for six dSphs based on the Jeans equations and argued that cored density profiles were favoured, partly because this also explains the persistence of kinematically cold substructure in the Ursa Minor dSph (Kleyna et al. 2003) and the maintenance of globular clusters in the Fornax dSph (Goerdt et al. 2006).

Here, we are less concerned with modelling observational data on any given dSph than with understanding the generic qualities of the light and dark matter profiles. We consider the general problem of a tracer stellar population with a known line-of-sight velocity dispersion residing in a spherical dark matter halo of an unknown density law. Given the data, Section 2 considers what can be legitimately inferred concerning the properties of the dark halo. Motivated by the flatness of the observed dispersion profiles, Section 3 considers constant velocity dispersion solutions of the Jeans equations. We show that the generic solution gives a halo density law with an isothermal cusp \((\rho_D \propto r^{-2})\). Although other solutions are possible – in particular with cores or with the milder cusps preferred by cosmologists \((\rho_D \propto r^{-1})\) – they are not generic. Section 4 gives some
examples, which show why Jeans modelling in the isotropic case has yielded results consistent with both cores and cusps. In our examples, however, any anisotropy drives the dark halo solution to the isothermal cusp. Finally, in Section 5, we discard the assumption of constancy of the velocity dispersion. Solely within the framework of the spherical symmetric Poisson and Jeans equations, we show that solutions of these equations almost always possess isothermal cusps, unless some very special conditions are satisfied either by the radial velocity dispersion or by the anisotropy and the logarithmic gradient of the light profile.

2 THE JEANS DEGENERACY

The observables are the surface brightness and the line-of-sight velocity dispersion of a stellar population. Given a mass-to-light ratio ($\Upsilon$), the surface mass density of the stellar populations ($\Sigma_s$) can be deduced from the surface brightness. If the system is spherically symmetric, $\Sigma_s(R)$ is then related to the three-dimensional density associated with the luminous material $\rho_s(r)$ via an Abel transform. Here, $R$ is the projected distance, whilst $r$ is the three-dimensional distance measured from the centre of the halo. The inverse transform provides us with the unique $\rho_s$ (Binney & Tremaine 1987):

$$\rho_s(r) = -\frac{1}{\pi} \int_0^\infty \frac{d\Sigma_s}{dR} \frac{dR}{\sqrt{R^2 - r^2}}. \quad (1)$$

However, even if one assumes spherical symmetry, the behaviour of the line-of-sight velocity dispersion does not produce a unique solution for the radial dependence of the radial and tangential velocity dispersions. The ‘luminosity weighted’ (assuming a constant $\Upsilon$ for the stellar population) line-of-sight velocity dispersion $\sigma_{los}^2$ is given by the integral:

$$\sigma_{los}(R)^2 = 2 \int_0^\infty \left( 1 - \frac{\beta}{\sigma^2} \right) \frac{\rho_s \sigma^2_r}{\sqrt{R^2 - r^2}} dr. \quad (2)$$

where $\beta = 1 - (\sigma^2_\Phi/\sigma^2_r)$ is the anisotropy parameter for a spherical system, and $\sigma_r$ and $\sigma_\Phi$ are the radial and (one-dimensional) tangential velocity dispersions. It has long been known (see e.g. Merrifield & Kent 1990; Dejonghe & Merritt 1992) that the line-of-sight velocity second moment is degenerate, in that there exist many sets of solutions $-\sigma^2_\Phi(r)$ and $\beta(r)$ that reproduce the observables.

For example, suppose that we have $\rho_s(r)$ from equation (1). Then, for any given behaviour of $\sigma^2_\Phi(r)$, the anisotropy parameter $\beta(r)$ can be found1 to reproduce the observables as such:

$$\beta(r) = \frac{1}{\pi \rho_s \sigma^2_r} \int_0^\infty \frac{dR}{\sqrt{R^2 - r^2}} \frac{d}{dR} \left[ \Sigma_s(\sigma^2_r - \sigma^2_{los}) \right]$$

$$= 1 + \frac{1}{\rho_s \sigma^2_r} \int_0^\infty \frac{d\tilde{r}}{\sqrt{\tilde{r}^2 - r^2}} \frac{d}{d\tilde{r}} \left[ \tilde{\Sigma}_s(\tilde{\sigma}^2_r - \tilde{\sigma}^2_{los}) \right]$$

$$+ \frac{r^2}{\rho_s \sigma^2_r} \int_0^\infty \frac{dR}{\sqrt{R^2 - r^2}} \frac{d}{dR} \left[ \Sigma_s(\sigma^2_r) \right], \quad (3)$$

where

$$\tilde{\sigma}^2_r(R) = \frac{2}{\Sigma_s(R)} \int_0^\infty \frac{\rho_s \sigma^2_r dr}{\sqrt{\tilde{r}^2 - r^2}}$$

is the luminosity-weighted mean projected radial velocity dispersion. [We have not found equation (3) for the anisotropy parameter

1 The first part of equation (3) is easily verified after re-arranging equation (2) and applying an inverse Abel transform. The derivation of the second part requires switching the order of a double integral and performing explicit integrations on it. in the existing literature.] Not all solutions are necessarily physical since $-\infty \leq \beta \leq 1$, which has to be checked a posteriori. However, subject to this condition, for any given arbitrary $\sigma^2_\Phi(r)$, an anisotropy parameter $\beta(r)$ can be found to reproduce any observable $\sigma^2_{los}$.

Once we have $\rho_s(r)$, guessed (any) $\sigma^2_\Phi(r)$ and found the anisotropy parameter $\beta(r)$ to reproduce the observables, then the spherically symmetric Jeans equation

$$\frac{d}{dr} \left( r^2 \beta \rho_s \sigma^2_r \right) + 2r \beta \rho_s \sigma^2_r = \frac{-4\pi G \rho_s}{r^2} \int_0^r \rho_s(\tilde{r}) \tilde{r}^2 d\tilde{r}. \quad (4)$$

Here, $\rho_s$ is the total density such that $\rho_s = \Upsilon \rho_c + \rho_D$, where $\rho_c$ and $\rho_D$ are the stellar and dark matter densities, respectively. With the self-consistency assumption, if there is no dark matter ($\rho_D = 0$) and the mass-to-light ratio $\Upsilon$ is additionally constant, then the choice of $\sigma^2_\Phi$ is uniquely determined by the coupled Poisson and Jeans equations (see e.g. Binney & Mamon 1982; Tonry 1983; Dejonghe & Merritt 1992).

However, if $\Upsilon$ is varying or $\rho_D(0) \neq 0$, there is no unique choice of $\sigma^2_\Phi$. That is, any $\sigma^2_\Phi$ is allowed subject only to the constraints that $-\infty \leq \beta(r) \leq 1$ and $\rho_c \geq 0$. Consequently, without further observational constraints or simplifying assumptions, determining a model to reproduce the observed $\Sigma_s(R)$ and $\sigma^2_{los}(R)$ is completely indeterminate in the spherical case.

3 JEANS MODELING WITH CONSTANT VELOCITY DISPERSSIONS

3.1 Isotropy

The simplest assumption to make is that of isotropy ($\beta = 0$). Then, $\sigma^2_\Phi(r)$ is recovered uniquely from $\sigma^2_{los}(R)$ by an inverse Abel Transform. The Jeans equation reduces basically to hydrostatic equilibrium with the ‘pressure’ being equal to $P = \rho_\sigma \sigma^2$, that is,

$$\nabla \cdot (\rho_\sigma \sigma^2) = -\rho_\sigma \nabla \psi. \quad (5)$$

where $\sigma$ is the one-dimensional velocity dispersion of the tracer population and $\psi$ is the gravitational potential.

If we further assume that $\sigma^2_{los}(R)$ is a constant $\sigma^2_0$, as suggested by most of the observations, then the unique solution is $\sigma^2_\Phi(r) = \sigma^2_0$. Under this assumption, the central properties of the halo are severely restricted, as we now show. Since $\nabla (\rho_\sigma \sigma^2) = \sigma^2_0 \nabla \rho_\sigma$, the Jeans equation indicates that $\nabla \rho_\sigma$ and $\nabla \psi$ are (anti)parallel everywhere. This further implies that the surfaces of constant $\rho_\sigma$ and $\psi$ coincide, and thus $\rho_\sigma$ can be considered as a function of $\psi$. Consequently, $\nabla \rho_\sigma = (d\rho_\sigma/d\psi) \nabla \psi$ and equation (5) reduces to

$$\sigma^2_0 \frac{d\rho_\sigma}{d\psi} + \rho_\sigma = 0. \quad (6)$$

By solving this differential equation, we find that

$$\rho_\sigma = \rho_0 \exp \left( -\frac{\psi}{\sigma^2_0} \right); \quad \psi = \psi_0 - \sigma^2_0 \ln \rho_\sigma, \quad (7)$$

where $\psi_0 = \sigma^2_0 \ln \rho_0$ is an integration constant. Combined with the Poisson equation under the assumption that the potential is generated by the dark matter halo of a density $\rho_D$ (i.e. $\rho_D \gg \rho_c$ and so $\rho_\sigma \approx \rho_D$), we obtain

$$\rho_D = \frac{\nabla^2 \psi}{4\pi G} = -\frac{\sigma^2_0}{4\pi G} \nabla^2 \ln \rho_\sigma. \quad (8)$$

This is an interesting equation, as the dark matter density, which we wish to know, depends on the Laplacian of the luminosity density.
The combined integro-differential equations (1) and (8) now relate the dark matter density to the observables, $\sigma_0$ and $\Sigma_i(R)$.

The implications of equations (7) and (8) on the behaviour of tracer populations in dark matter haloes are of considerable interest, even before we apply to any specific example. First, equation (7) indicates that the potential is finite for any finite luminosity density. Consequently, we find that any cored luminosity density implies that the central potential well cannot be infinitely deep and so the dark matter halo density profile is also cored, or diverges strictly slower than the singular isothermal sphere ($r^{-2}$) if it is cusped. Similar argument also leads us to the conclusion that any cusped luminosity density would only be supported by a cusped halo diverging at least as fast as a singular isothermal sphere. In fact, this latter conclusion can be sharpened with further analysis. Under the assumption of spherical symmetry, equation (8) may be written to be

$$\gamma_t + r \frac{d\gamma_t}{dr} = \frac{4\pi G}{\sigma_0^2} \rho_D r^2,$$

where $\gamma_t = -\left(\frac{d\ln \rho_D}{dr}\right)$ is the logarithmic slope of the density of the tracers. By taking the limit to the centre ($r \to 0$), we find that

$$\lim_{r \to 0} \rho_D r^2 = \frac{\sigma_0^2}{4\pi G} \gamma_t(0),$$

where $\gamma_{t,0}$ is the limiting value of $\gamma_t$ towards $r \to 0$, i.e. the logarithmic cusp slope of the luminous tracer density. This indicates that if the luminosity density is cusped with $\gamma_{t,0} > 0$, the halo density must be cusped as $\rho_D \sim r^{-2}$ like a singular isothermal sphere. On the other hand, any cored $\rho_t$ indicates that $\gamma_{t,0} = \lim_{r \to 0} (\rho_D r^2) = 0$ and thus the halo density may not diverge as fast as or faster than the cusp of the singular isothermal sphere. Finally, $\rho_t$ with a central hole implies that $\gamma_{t,0} < 0$ and is therefore unphysical as it will lead to $\rho_D < 0$ (cf. An & Evans 2006).

3.2 Anisotropy

In reality, the velocity dispersion tensor of a ‘collisionless’ stellar system is not necessarily isotropic. However, provided the gravitational potential is still spherically symmetric, the Jeans equation reduces to

$$\frac{1}{I} \frac{d}{dr} \left( I \rho_t \sigma^2 \right) = -\rho_t \frac{d\psi}{dr},$$

where $I = \exp \int (2\beta/r)dr$ is the integrating factor (e.g. $I = \exp(2\beta_0)$ if $\beta$ is constant).

Inspired by the discussion in the preceding section, we now consider the case of constant radial velocity dispersions with an arbitrary functional form of $\beta$. Note that it is possible, from equation (3), to find $\beta(r)$ that is consistent with the observed $\Sigma_i(R)$ and $\sigma^2_{\varphi}(R)$ once we ascribe a particular behaviour to $\sigma^2_t(r)$. Hence, we can, in principle, find such a model that produces a constant line-of-sight velocity dispersion (or any other desired form), which is indicated by the observations using equation (3).

If $\sigma^2_t = \sigma^2_0$ is a constant, equation (11) becomes

$$\frac{d\psi}{dr} = -\frac{\sigma^2_0}{dr} \ln(I \rho_t),$$

and consequently, we find that

$$\psi = \psi_0 - \sigma^2_0 \ln(I \rho_t) = \psi_0 - \sigma^2_0 \left( \ln \rho_t + \int \frac{2\beta}{r} dr \right),$$

$$\rho_D = \frac{\nabla^2 \psi}{4\pi G} = -\frac{\sigma^2_0}{4\pi G r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} \ln(I \rho_t) \right).$$

Here, the last equation can be recast similarly to equation (9) so that

$$\gamma_t - 2\beta + r \left( \frac{d\gamma_t}{dr} - 2 \frac{d\beta}{dr} \right) = \frac{4\pi G \rho_D r^2}. $$

This is basically the same as equation (9), except $\gamma_t$ is replaced by $\gamma_t - 2\beta$. We note that the result does not require the assumption that $\beta$ is a constant. The implication of this result is quite similar to that of equation (9). At the limit towards the centre ($r \to 0$), we find that $\gamma_{t,0} > 2\beta_0$ implies that $\rho_D \sim r^{-2}$ whilst we infer that $\lim_{r \to 0} (\rho_D r^2) = 0$ if $\gamma_{t,0} = 2\beta_0$. Here, $\beta_0$ is the limiting value of anisotropy parameter at the centre. As per An & Evans (2006), $\gamma_{t,0} < 2\beta_0$ is unphysical (although $\rho_{t,0}$ does not self-consistently generate $\psi$, the potential well depth is finite at the centre so that their result holds) since it indicates negative halo density.

In summary, therefore, given a tracer population with a constant radial velocity dispersion, the generic solution of the Jeans equation for the dark matter is cusped like a singular isothermal sphere ($\rho_D \propto r^{-2}$). Milder cusps (like $\rho_D \propto r^{-\gamma}$) and cores are possible, but they are not generic. Such solutions only exist if the anisotropy parameter $\beta$ and the logarithmic slope of the stellar density $\gamma_t$ satisfy $\gamma_{t,0} = 2\beta_0$.

4 EXAMPLES

4.1 A plummer light profile

Plummer’s law is commonly used to model the light of dSphs (e.g. Lake 1990; Wilkinson et al. 2002). Assuming a constant mass-to-light ratio $\Upsilon$, then the surface density is

$$\Sigma_i(R) = \frac{\Sigma_0}{(1 + R^2/r_0^2)^{3/2}}.$$  

Here, $r_0$ is the radius of the cylinder that encloses half the light, whilst the total luminosity is $L = \pi r_0^2 \Sigma_0 / \Upsilon$. It is straightforward to establish via equation (1) that the stellar density is

$$\rho_t(r) = \frac{3\Sigma_0}{4\pi r_0^2 (1 + r^2/r_0^2)^{3/2}}.$$  

Now, using equation (8), the dark matter density must be

$$\rho_D(r) = \frac{5\Sigma_0^2}{4\pi G r_0^2} \left( 3 + r^2/r_0^2 \right)^{3/2},$$  

which is a cored isothermal sphere (see Evans 1993). However, as the model is isotropic ($\beta = 0$) and the stellar density is cored at the centre ($\gamma_{t,0} = 0$), this dark halo solution corresponds to the special case $\gamma_{t,0} = 2\beta_0$.

Now suppose the assumption of isotropy is dropped. Using equation (13), we find that the dark halo density acquires an additional term that behaves near the centre as

$$\rho_D(r) \simeq \frac{\beta_0 \sigma^2_0}{2\pi G r^2},$$

from which we deduce that $\beta_0 \leq 0$ so that the model is tangentially anisotropic as $r \to 0$. Note that this has changed the behaviour of the density at the origin – the halo law has now an isothermal cusp. This is in accord with the general result that provided $\gamma_{t,0} > 2\beta_0$, the cusp is isothermal.

4.2 Exponential light profiles

Another set of profiles often used to model dSph light distributions is based on the exponential law (Sérsic 1968; Faber & Lin 1983;
Bessel profile. The half-light radius is \( \sim \frac{\Sigma_1}{\rho} \).

Frenk & White (1996). In fact, a three-dimensional density law \( \rho \propto r^{-\beta} \) is log-divergent at the centre. The dark matter density inferred from equation (8) is then

\[
\rho_0 = \frac{\sigma_0^2}{4\pi G r_0^2},
\]

which is the \( r^{-1} \) cusp beloved of cosmologists (e.g. Navarro, Frenk & White 1996). In fact, a three-dimensional density law \( \rho_0 = \rho_0 e^{-v/r_0} \) leads to an infinite dark matter cusp of form \( \rho_0 \propto r^{-2-\omega} \).

Suppose instead the surface brightness profile is modelled with an exponential law

\[
\Sigma_\ell(R) = \Sigma_0 \exp\left(-\frac{R}{R_\sigma}\right),
\]

where \( \Sigma_0 = 2\pi R_0 \rho_0 \) and \( K_\ell(x) \) is the modified Bessel function of the second kind and of the order of \( \nu \). We refer to this as the Bessel profile. The half-light radius is \( \sim 2.027 r_\sigma \), whilst the total luminosity is \( L = 4\pi r_\sigma^2 \Sigma_0 / \Upsilon \). The dark matter density inferred from equation (8) is then

\[
\rho_\ell(r) = \rho_0 \exp\left(-\frac{r}{r_\sigma}\right) \frac{R}{R_\sigma} K_\ell\left(\frac{R}{R_\sigma}\right),
\]

(20)

where \( \Sigma_\ell = 2\pi R_\ell \rho_\ell \) and \( K_\ell(x) \) is the modified Bessel function of the second kind of order \( \nu \), and the logarithmic slope of the stellar density \( \gamma \).

The luminosity density is then (see e.g. Kent 1992)

\[
\rho_\ell(R) = \frac{\Sigma_0}{\pi R_\sigma} K_\ell\left(\frac{R}{R_\sigma}\right),
\]

(22)

which is logarithmically divergent at the centre. The dark matter density is now

\[
\rho_\ell(r) = \frac{\sigma_\ell^2}{4\pi G r^2} \left[ \left( \frac{K_1}{K_0} \right)^2 - 1 \right] \frac{r}{R_\sigma} + \frac{K_1}{K_0},
\]

(24)

where \( K_\ell = K_\ell(r/R_\sigma) \) and \( n = 0, 1 \). It is singular as \( r \to 0 \),

\[
\rho_\ell \sim \frac{\alpha^2}{4\pi G r^2 \ln(r)}.
\]

which exhibits strictly slower divergence than a singular isothermal sphere. In other words, once the luminosity density has been assumed to be of exponential form (either in projection or three dimensions), then the inferred dark matter density is cusped, but the cusp is always weaker than the isothermal cusp \( \rho_0 \propto r^{-2} \).

It is easy to see that equation (9) still applies since \( \gamma_\ell \to 0 \) as \( r \to 0 \). Again, the isotropic models are somewhat unusual – the introduction of anisotropy drives the dark halo solution towards an isothermal cusp. The fact that the terms in the density and the anisotropy decouple in equation (13) means that the assumption of any central anisotropy gives the same additional contribution to the halo density (equation 18) as in the previous example.

5 THE GENERAL CASE

Let us now gain insight into the general case by discarding the assumption that the radial velocity dispersion is constant. We derive the extension of our result to the general spherically symmetric case. By rewriting the spherical steady-state Jeans equation (4), we obtain (under the assumption that \( \rho_\ell = \rho_0 \))

\[
4\pi G \int_0^r \rho_0(r) \rho_0(r) \frac{d\ell}{d\nu} = r \sigma_\ell^2 \left( \gamma_\ell - 2\beta - \frac{d\ln \sigma_\ell^2}{d \ln r}\right),
\]

(26)

Now note that if the dark matter density has an isothermal or steeper cusp, then the left-hand side of equation (26) tends to a non-zero value as \( r \to 0 \) if the density diverges more slowly than \( r^{-2} \), then the left-hand side vanishes as \( r \to 0 \). Then, for the right-hand side also to vanish as \( r \to 0 \), one of the following three conditions must hold at the same limit

(i) \( \frac{d\ln \sigma_\ell^2}{d \ln r} \to -1 \),

(ii) \( \frac{d\ln \sigma_\ell^2}{d \ln r} \to \gamma_\ell - 2\beta \),

(iii) \( \sigma_\ell^2 \to 0 \).

Case (i) implies that \( \sigma_\ell^2 \sim r^{-1} \). Excluding the possibility that there is a black hole at the centre, then the central potential must be finite (as \( \rho_0 \to 0 \)) and the divergent velocity dispersion cannot be supported. Case (ii) implies that \( \sigma_\ell^2 \sim r^{\nu - 2\beta} \). If \( \gamma_\ell > 2\beta \), then \( \sigma_\ell^2 \to 0 \) and so this may be subsumed into Case (iii). If \( \gamma_\ell < 2\beta \), then \( \sigma_\ell^2 \) diverges and so is again unphysical. This leaves only \( \gamma_\ell = 2\beta \) as an independent possibility.

Consequently, we have established that if the dark matter density is cored or falls off with a cusp less severe than \( r^{-2} \), then either \( \gamma_\ell = 2\beta \) or \( \sigma_\ell^2 = 0 \) at the centre. The discarding of the assumption of constancy of the radial velocity dispersion permits one additional possibility, namely that \( \sigma_\ell^2 \to 0 \) as \( r \to 0 \).

In summary, therefore, we have a surprisingly strong and general result. Given a tracer population in the spherical Jeans equation, the generic solution for the dark matter is cusped like a singular isothermal sphere (\( \rho_0 \propto r^{-2} \)). Milder cusps and cores are possible, but they are not generic. Such solutions exist either if the anisotropy parameter \( \beta \) and the logarithmic slope of the stellar density \( \gamma_\ell \) satisfy \( \gamma_\ell = 0 \) or if the central radial velocity dispersion \( \sigma_\ell^2 \) vanishes.

Such a strong result may seem astonishing. It is worth remarking that this result is essentially due to the coordinate singularity at the origin. A real dSph is not likely to be exactly spherically symmetric at the centre. So, it may be possible for a dSph to have a finite velocity dispersion towards the centre by mild symmetry breaking. Outside of the very central region, a dSph can often be well approximated by the idealized spherically symmetric fiction. Our theorem therefore really lays bare the dangers of inferring the central density profile from the almost universally made assumption of spherical symmetry.

6 SUMMARY

We have studied the problem of deducing the dark halo density from the surface brightness and velocity dispersion profiles of a tracer population. This has immediate application to the dSphs of the Milky Way.

If the stellar population generates the gravity field, then the Jeans and Poisson equations give a unique solution for the density, the mass-to-light ratio and the anisotropy of the spherical model, consistent with the observed surface brightness and the line-of-sight velocity dispersion (Binney & Mamon 1982; Tonry 1983). Of course, this does not apply to the case of dSphs, in which the density of the stellar population is dominated by the dark halo density. Now, the problem suffers from the well-known mass-anisotropy degeneracy.
The line-of-sight velocity dispersion profiles of the Milky Way dSphs appear to be usually flat (see e.g. Kleyna et al. 2001, 2002; Koch et al. 2007a; Walker et al. 2007), which suggests the simple assumption that the velocity dispersion tensor is isotropic and has a constant value. Then, any inference as to the central behaviour of the dark matter potential is controlled by the assumption as to the light profile of the tracer population. If the light profile is cored, then a dark matter halo density that is itself cored is deduced from Jeans modelling. If the light profile is cusped like an exponential law (or its variants), then a dark halo density that has a milder cusp than isothermal (such as the Navarro–Frenk–White (1996) cusp of \( \rho \propto r^{-1} \)) is deduced. This provides the explanation as to why previous investigators (Wilkinson et al. 2006; Koch et al. 2007b) have concluded that the data are consistent with both cusps and cores.

However, velocity anisotropy in the stellar population has a dramatic effect on the dark halo density recovered from Jeans modelling. If a tracer population has a constant radial velocity dispersion, then the generic solution for the dark halo is always cusped like a singular isothermal sphere (\( \rho_D \propto r^{-2} \)). Milder dark matter cusps (like \( \rho_D \propto r^{-1} \)) and cores are possible, but they are not generic. They can occur only when the condition \( \gamma_\ell = 2\beta \) is fulfilled at the centre, where \( \beta \) is the anisotropy parameter and \( \gamma_\ell \) is the logarithmic slope of the stellar density. Note that many of the commonly used dSph models (such as Plummer or exponential profiles with isotropic velocities) correspond to the special case \( \gamma_\ell = 2\beta \) and so any conclusions inferred as to the dark halo law may not be beyond reproach.

Finally, even if the assumption as to the constancy of the radial velocity dispersion is discarded, then almost the same theorem holds true. If \( \gamma_\ell = 2\beta \) at the centre or if \( \sigma_r^2 \) falls to zero at the centre, then dark matter cores and milder cusps than isothermal are possible. The generic solution, however, remains the isothermal dark matter cusp, at least within the framework of the spherical symmetric Jeans and Poisson equations.

Here, our examples and analysis have shown how Jeans solutions may be telling modellers more about their assumptions rather than the theoretical implications of the data. Of course, there is, in principle, more information in the discrete velocities than in the velocity dispersion profile and the Jeans equations. The best response to the degeneracy of the problem is to seek further observational constraints (perhaps higher moments from the line profile, e.g. Merrifield & Kent 1990) or additional insights on the behaviour of the anisotropy (maybe from simulations or a detailed analysis on the underlying physics).

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