Light plasmon mode in the CFL phase

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Abstract. The self-energies and the spectral densities of longitudinal and transverse gluons at zero temperature in the color-flavor-locked (CFL) phase are calculated. There appears a collective excitation, a light plasmon, at energies smaller than two times the gap parameter and momenta smaller than about eight times the gap. The minimum in the dispersion relation of this mode at some nonzero value of momentum corresponds to the van Hove singularity.

In cold and dense quark matter, due to asymptotic freedom, at quark chemical potentials \( \mu \ll \lambda_{QCD} \) single-gluon exchange is the dominant interaction between quarks. Since this interaction is attractive in the color-antitriplet channel, therefore, quark matter is a color superconductor [1]. While there are, in principle, many different color-superconducting phases, corresponding to the different possibilities to form quark Cooper pairs, the ground state of color-superconducting quark matter is the so-called color-flavor-locked (CFL) phase [2].

At asymptotically large \( \mu \), the QCD coupling constant \( g \ll 1 \), thus, the gluon self-energy is dominated by the contributions from one quark and one gluon loop. The quark loop is \( \sim g^2 \mu^2 \), while the gluon loops are \( \sim g^2 T^2 \). Since the color-superconducting gap parameter is \( \phi \sim \mu \exp(-1/g) \ll \mu \) [3], and since the transition temperature to the normal conducting phase is \( T_c \sim \phi \), for temperatures where quark matter is in the color-superconducting phase, \( T < T_c \ll \mu \), the gluon loop contribution can be neglected. The full description for the 2SC phase is given in Sec. II of Ref. [8]. The full energy-momentum dependence of the one-loop gluon self-energy has also been computed, but so far only for the 2SC phase [7, 8]. Here we want to do the same calculations for the CFL phase. The detailed computation of the individual components and projections can be found in the appendix of Ref.[9].

Fig. 1 shows the imaginary part of several components of the gluon self-energy for a gluon momentum \( p = 4\phi \) as a function of the gluon energy \( p_0 \). The corresponding results for the gluon self-energy in the “hard-dense loop” (HDL) limit, \( \Pi_{HV}^{\text{HD}} \), are also shown with the dotted lines. The imaginary parts are quite similar to those of the 2SC case, cf. Fig. 1 of Ref. [8]. Nevertheless, there are subtle differences due to appearance of two kinds of gapped quark excitations, one so-called singlet excitation with a gap \( \phi_1 \), and eight so-called octet excitations with a gap \( \phi_8 \equiv \phi [2] \). In weak coupling, the singlet gap is approximately twice as large as the octet gap, \( \phi_1 \simeq 2\phi_8 \equiv 2\phi [11, 10] \). Therefore, the one-loop gluon self-energy in the CFL phase has two types of contributions, depending on whether the quarks in the loop correspond to singlet or octet excitations, cf. Eq. (23b) of Ref. [6]. For the first type, both quarks in the loop are octet excitations, and for the.
second, one is an octet and the other a singlet excitation. There is no contribution from singlet-singlet excitations.

Nonvanishing octet-octet excitations require gluon energies to be larger than $2\phi_8 \equiv 2\phi$, while octet-singlet excitations require a larger gluon energy, $p_0 \geq \phi_1 + \phi_8 \equiv 3\phi$. This introduces some additional structure in the imaginary parts at $p_0 = 3\phi$, which can be seen particularly well in Figs. 1(d) and (e). In the normal phase, the imaginary parts of the gluon self-energies vanish above $p_0 = p$. In color-superconducting phases, the imaginary parts do not vanish but fall off rapidly. This has already been noted for the 2SC phase [8], and is confirmed here by the results for the CFL phase.

The real parts of the gluon self-energy are shown in Fig. 2. When computing the real part from a dispersion integral over the imaginary part, a change of gradient in the imaginary part leads to a cusp-like structure in the real part. As one expects, for large energies $p_0 \gg \phi$ the real parts of the self-energies approach the corresponding HDL limit. Deviations from the HDL limit occur only for gluon energies $p_0 \sim \phi$.

The spectral densities are obtained from the real and imaginary parts of the gluon self-energies [8, 9]. Note that, in Fig. 3 at an energy $p_0 \simeq 0.21 m_g$, there is a delta function-like peak in the transverse spectral density. This peak corresponds to a collective excitation, the so-called “light plasmon” predicted in Ref. [13, 14]. We show the dispersion relation of this collective mode in Fig. 4(b). The mass $m_{\text{coll}} \simeq 1.35 \phi$ is roughly in agreement with the value $m_{\text{coll}} \simeq 1.362 \phi$ of Ref. [13]. As the momentum increases, the energy of the light plasmon excitation approaches $2\phi$ from below. For momenta larger than $\sim 8\phi$, the location of this excitation branch becomes numerically indistinguishable from the continuum in the spectral density above $p_0 = 2\phi$, cf. Fig. 5. Close inspection reveals that the dispersion relation of the light plasmon has a minimum at a nonzero value of $p \simeq 1.33 \phi$, indicating a van Hove singularity.

In Fig. 4 we also show the dispersion relations for the “regular” longitudinal and
transverse excitations, as well as for the Nambu-Goldstone excitation defined by the root of $P^\mu \Pi_{\mu\nu}(P) P^\nu = 0$ \cite{8,12}. For our choice of gauge, the gluon propagator is 4-transverse and this mode does not mix with the longitudinal component of the gauge field \cite{8}. Therefore, the Nambu-Goldstone mode does not appear as a peak in the longitudinal spectral density, cf. Fig. [3]. We finally note that other collective excitations have been investigated in Ref. \cite{15}.

In conclusion, we have computed the gluon self-energy in the CFL phase as a function of energy and momentum. While the imaginary parts of the gluon self-energy could be expressed analytically in terms of elliptic functions (see appendix of \cite{9}), the real parts had to be computed numerically with the help of dispersion integrals. From the real and
FIGURE 4. The dispersion relations for (a) longitudinal and (b) transverse excitations in the CFL phase. The full lines correspond to the regular longitudinal and transverse excitations. The dashed lines are for the HDL limit. The dash-dotted line in part (a) shows the dispersion relation for the Nambu-Goldstone excitation. The light plasmon dispersion relation is shown by the dash-dotted line in part (b). As in Fig. 3, the value of the gap is chosen such that $m_g = 8 \phi$.

imaginary parts we constructed the spectral densities. We confirmed the existence of a low-energy collective excitation, the so-called “light plasmon” predicted in Ref. [13].

ACKNOWLEDGMENTS

H. M. thanks the Frankfurt International Graduate School of Science for support.

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