Shear viscosity in $SU(2)$ lattice gluodynamics

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Abstract. The calculation of the gluon plasma viscosity has been performed by lattice simulations in the $SU(2)$ gluodynamics at $T/T_c = 1.2$ on the supercomputers. The evaluation is based on the Kubo formula that relates viscosity to the spectral function of the correlation functions of the energy-momentum tensor. For the extraction of the spectral function from the Euclidean correlator the linear method is applied.

1. Introduction
One of the most important results obtained at RHIC experiment on heavy ion collisions is the measurement of the elliptic flow of final particles[1, 2]. Obtained value can be explained within the hydrodynamical approximation if one assumes that quark-gluon plasma formed during heavy ion collisions is almost a superfluid liquid[3, 4, 5]. Numerical simulations of a relativistic liquid lead to the following bound on the ratio of the viscosity to the entropy density of the system[6]: $\eta/s \leq 0.4$, which is the smallest ratio for all systems known to date. Thereby one of the most important problems in the modern theoretical physics is a first-principle calculation of the viscosity of quark-gluon plasma. One of the most powerful approaches to this problem is the numerical simulation of quark-gluon plasma. It is worth mentioning, that there were the works, in which the viscosity of quark-gluon plasma was studied on the lattice[7, 8, 9], but due to the complicated calculations, the results obtained are not exhaustive.

In the present paper the calculation of the viscosity of the $SU(2)$ gluodynamics is performed by means of lattice modeling. Instead of the $SU(3)$ QCD we consider the $SU(2)$ model as both theories are rather similar physically and at the same time numerical calculations in the $SU(2)$ gluodynamics are much simpler than in the $SU(3)$ theory. This leads to the significant advances in the study of transport coefficients of quark-gluon plasma.

2. A brief description of the method
Kubo formulas [10] relating transport coefficients to the correlators of energy-momentum tensor are used in this calculation. In particular, for the viscosity the Kubo formula reads:

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho_{12,12}(\omega, q = 0)}{\omega},$$

(1)
where $\rho_{12,12}$ is a spectral function, which is determined as an imaginary part of the two-point retarded Green function: $\rho_{12,12}(\omega, q) = \frac{1}{\pi} \text{Im}(T_{12,12})_{\text{ret}}(\omega, q)$. The Green function of energy-momentum tensor measured in lattice calculations is defined as follows:

$$
C_{12,12}(x_0, p) = \beta^5 \int d^3x e^{ipx} \langle T_{12}(0) T_{12}(x_0, x) \rangle_{\text{ret}},
$$

where $\beta = 1/T$ is an inverse temperature of the system. Function (2) is an analytical continuation of the retarded Green function: $p_0 \rightarrow i\omega$ and is related to the spectral function in the following way:

$$
C_{12,12}(x_0, p) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, p) \frac{\cosh(\frac{1}{2} \beta - x_0)}{\sinh(\frac{\omega}{2})} d\omega
$$

In order to determine the value of viscosity one needs to measure on the lattice the correlator $C_{12,12}$ (1) as a function of $x_0$ for $p = 0$, and then invert (3) to find the spectral function $\rho_{12,12}(\omega, q = 0)$. Low-frequency behaviour of spectral function $\rho_{12,12}$ determines the value of the viscosity.

For the inversion of (3) we use the linear method proposed in [9], which is based on the assumption that the spectral function is rather smooth and can be decomposed into certain basis functions.

3. Measurement of the viscosity

![Figure 1: Correlation function $C_{12,12}(x_0)$ of the energy-momentum tensor. The points show the data measured on the lattice. The red line corresponds to the spectral function extracted from this data.](image1)

![Figure 2: Spectral density $\rho_{12,12}(\omega)$ divided by $\sinh(\frac{\omega}{2T})$. The filled region corresponds to statistical uncertainties of our data.](image2)

The measurement of energy-momentum tensor was performed in lattice gluodynamics with the $SU(2)$ gauge group and the Wilson action:

$$
S_g = \beta \sum_{x, \mu<\nu} \left( 1 - \frac{1}{N_c} \text{tr} U_{\mu,\nu}(x) \right).
$$

Energy-momentum tensor in gluodynamics has the form:

$$
T_{\mu\nu} = F_{\mu\alpha}^a F_{\nu\alpha}^a - \frac{1}{4} \delta_{\mu\nu} F_{\rho\sigma}^a F_{\rho\sigma}^a
$$
For the chromo-electromagnetic tensor the clover-shaped discretization is used:

$$F_{\mu\nu}(x) = \frac{1}{4}(V_{\mu,\nu}(x) + V_{\nu,-\mu}(x) + V_{-\mu,-\nu}(x) + V_{-\nu,\mu}(x))$$

$$V_{\mu,\nu}(x) = \frac{1}{2}(U_{\mu,\nu}(x) - U_{\nu,\mu}(x)).$$

The two-level algorithm is used for measuring correlator (1). It significantly improves the accuracy of the calculations [12]. The simulations are performed on the lattice $8 \times 32^3$, $\beta = 2.643$, which corresponds to the temperature $T/T_c \approx 1.2$. The results of the evaluation of correlator (2) are shown in Fig. 1.

For the extraction of spectral function the linear method described in [9] is used. The following function that describes the behaviour of the spectral function at large frequencies (which is determined by the asymptotical freedom) is used as an initial approximation:

$$m(\omega) = \frac{A\omega^4}{\tanh^2 \frac{\omega}{T} \tanh^2 \frac{\omega}{T}}$$

In Fig. 2 the obtained data for the spectral function is shown. Statistical errors are determined in accordance with the method described in [9].

The results for the measuring of the viscosity are presented as the ratio $\eta/s$, where $s$ is an entropy density. The latter is determined using the relation: $s = \frac{\epsilon + p}{T}$ and the method for the evaluation of $\epsilon + p$, described in [13]. Applying this method we obtain the following value for this ratio:

$$\frac{\eta}{s} = 0.111 \pm 0.032$$

An error in this result corresponds to the statistical uncertainty of our data. Value (8) is close to the so-called KSS bound $\eta/s \geq 1/4\pi \approx 0.08$, obtained by means of $AdS/CFT$-correspondence [14]. Moreover, result (8) satisfies the bound obtained from the description of RHIC experiments $\eta/s < 0.4$ [6].

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