Inverse seesaw model with a natural hierarchy at the TeV scale

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We propose a new kind of inverse seesaw model without any additional symmetries. Instead of the symmetries, we introduce several fermions and bosons with higher $SU(2)_L$ representations. After formulating the Higgs sector and neutrino sector, we show that the cutoff energy, which is valid for our model, is at around the TeV scale by examining behavior of $SU(2)_L$ gauge coupling. Then we show the validity of our model for testing at collider physics.

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I. INTRODUCTION

One of the important topics in particle physics beyond the standard model (SM) is to explore mechanisms generating tiny neutrino masses and their mixing. Sometimes, one relies upon a mechanism in which the tininess originates from the ultra-high-energy scale $\Lambda_H$ such as in the grand unified theory scale ($\sim 10^{15}$ GeV) or string scale ($\sim 10^{19}$ GeV), by embedding the SM gauge group into a larger one. In this case, the neutrino mass may simply be realized by running a heavy field inside a diagram for the neutrino mass; therefore, the neutrino mass is proportional to $\Lambda_{ew}/\Lambda_H$, where $\Lambda_{ew}$ is at the electroweak scale $\sim 100$ GeV. These theories are elegant in a sense that the neutrino mass is directly reflected to the high-energy scale; however, the heavy particle cannot directly be tested by any current experiments such as those at the large hadron collider (LHC). To achieve detectability at the current experiments, a theory should be closed within the TeV scale. To realize such a low-energy scenario, we apply higher multiplet fields under the $SU(2)_L$ group instead of introducing any larger gauge groups [1]. Once the neutrino mass is induced after a higher iso-spin scalar field developing the vacuum expectation value (VEV), the order of the neutrino mass is, at least, suppressed by 2 orders of magnitude compared to the origin of SM Higgs. This result follows from the bound on the $\rho$ parameter that is describe by the mass ratio between the neutral gauge boson and charged gauge boson in the SM. The higher multiplet particles there exist, the more suppression the neutrino mass receives. Thus, we realize the tiny neutrino mass with a natural hierarchy, by applying this feature.

To achieve this mechanism in a more effective manner, an inverse seesaw model [3,4] is a promising candidate to explain the miniscule neutrino masses with mild hierarchy of Majorana mass matrix for neutral Majorana fermions in a theory, and provides a lot of phenomenologies since the neutrino mass structure is more intricate than the other mechanisms such as canonical seesaw [5–8] and linear seesaw [4,9,10]. In order to realize the inverse seesaw model, heavier neutral fermions with both chiralities have to be introduced. In addition to these new fields, most of the cases, one also has to impose an additional symmetry such as the (non-)Abelian local/global one to control the texture of neutral fermion mass matrix.

In this article, we propose an inverse seesaw model without introducing any additional symmetries to the SM. Instead, we introduce several fermions (quartet and septet) and bosons (quintet and quartet) without introducing any larger gauge groups [11]. Due to such fields, the behavior of the $SU(2)_L$ gauge coupling $g_2$ blows up at the TeV scale via the renormalization group equation (RGE). It suggests that our model is tightly relevant at the low-energy scale, and the testability of our model is largely expected at various experiments such as the LHC, the ILC, and future colliders.

This article is organized as follows. In Sec. II, we review our model and formulate the lepton sector. Then we discuss phenomenologies of neutrinos. In Sec. III, we discuss extra charged particles at collider experiments. Finally, we provide the summary of our results and the conclusion.

II. MODEL SETUP AND CONSTRAINTS

In this section, we formulate our model. For the fermion sector, we introduce three families of vectorlike fermions $\psi$...
with (4,−3/2) charge under the $SU(2)_L \times U(1)_Y$ gauge symmetry, and right-handed fermions $\Sigma_R$ with (7, 0) charge under the same gauge symmetry. For the scalar sector, we add quartet and quintet scalar fields $H_4$ and $H_5$ with, respectively, 3/2 and 2 charges under the $U(1)_Y$ gauge symmetry, where SM-like Higgs field is identified as $H_2$.

Here, we denote each vacuum expectation value (VEV) of the scalar fields to be $\langle H_i \rangle \equiv v_i/\sqrt{2}$ ($i = 2, 4, 5$) that arises after the electroweak spontaneously symmetry breaking. All the field contents and their assignments are summarized in Table I, where the quark sector is exactly the same as the one of the SM and omitted. The renormalizable Yukawa Lagrangian under these symmetries is given by

$$-\mathcal{L}_Y = y_e e_L H_2 e_R^c + y_{d,a} [L_u H_4^* (\psi_L^a)^b] + f_{L,a} [\bar{\psi}^a L H_4^* \Sigma_R^b] + f_{R,a} [\bar{\psi}^a R H_4 \Sigma_R^b] + M_{a,b} \bar{\psi}^a \psi^b + M_{z,a} (\Sigma_R^a)^a \Sigma_R^b + \text{h.c.},$$

where $SU(2)_L$ index is omitted assuming it is contracted to $\gamma^5 v/\sqrt{2}$ and $m_D = y_D v_5/\sqrt{2}$.

Scalar potential and VEVs: The scalar potential in our model is given by

$$\mathcal{V} = -\mu_h^2 |H_2|^2 + M_4^2 |H_4|^2 + M_5^2 |H_5|^2 + \lambda_H |H_2|^4$$

$$+ \mu |H_4 H_2 H_5 + \text{h.c.}| + \lambda_0 [H_4^2 H_2 H_2 H_2 + \text{h.c.}]$$

$$+ \mathcal{V}_{\text{trivial}},$$

where $\mathcal{V}_{\text{trivial}}$ indicates other trivial 4-point terms and $SU(2)_L$ indices are implicitly contracted in the second line to be gauge invariant. Applying condition $\partial \mathcal{V} / \partial v_i = 0$, we obtain the VEVs as

$$v_2 \sim \sqrt{\frac{\mu_h^2}{\lambda_H}}, \quad v_4 \sim \frac{\lambda_0 v_3}{M_4^2}, \quad v_5 \sim \frac{\mu v_4 v_3}{M_5^2},$$

where we have used $v_4, v_5 \ll v_2$. Thus, $v_4$ and $v_5$ can be naturally the $O(1)$ GeV scale if $M_4$ and $M_5$ are the TeV scale.

| $L_L^a$ | $e_R^c$ | $\psi^a$ | $\Sigma_R^a$ | $H_2$ | $H_4$ | $H_5$ |
|---|---|---|---|---|---|---|
| $SU(2)_L$ | 2 | 1 | 4 | 7 | 2 | 4 | 5 |
| $U(1)_Y$ | $-\frac{1}{2}$ | -1 | -1 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 2 |

$\rho$ parameter: The VEVs of $H_4$ and $H_5$ are restricted by the $\rho$ parameter at tree level that is given by

$$\rho \approx \frac{v_4^2 + 7v_2^2 + 10v_3^2}{v_2^2 + v_4^2 + 2v_5^2},$$

where the experimental value is given by $\rho = 1.0004^{+0.0003}_{-0.0004}$ at 2$\sigma$ confidence level [20]. On the other hand, we have $v_{SM} = \sqrt{v_2^2 + 7v_4^2} \approx v_2 \approx 246$ GeV. Therefore, $v_4$ and $v_5$ are restricted via the constraint of the $\rho$ parameter. Here, we take these VEVs to be $v_4 \approx 245.9$, $v_4 \approx 1.67$, and $v_5 \approx 1.72$ GeV, which are the typical scale for the VEVs satisfying the constraint.

Exotic particles: The scalars and fermions with large $SU(2)_L$ multiplet provide exotic charged particles. Here we write components of multiplets as

$$H_4 = (\phi_4^{++}, \phi_4^{+}, \phi_4^0)^T,$$

$$H_5 = (\phi_5^{++++}, \phi_5^{++}, \phi_5^0, \phi_5^0, \phi_5^0)^T,$$

$$\psi_{L(R)} = (\psi^0, \psi^-, \psi^-, \psi^-)^T, \quad \Sigma_R = (\Sigma^{++++}, \Sigma^{+++}, \Sigma^+, \Sigma^0, \Sigma^-, \Sigma^-, \Sigma^{-})^T.$$
Note that $M_\Sigma$ and $M$ cannot be much larger than TeV scale, since $v_4$ and $v_5$ are GeV scale requiring the perturbative limit for Yukawa coupling constants. The neutrino mass matrix is diagonalized by unitary matrix $U_{\text{MNS}}$: $D_v = U^\dagger_{\text{MNS}}m_vU_{\text{MNS}}$, where $D_v = \text{diag}(m_1, m_2, m_3)$. Here we apply a convenient method to reproduce neutrino oscillation data as follows [27]:

$$m_D^* \approx U^\dagger_{\text{MNS}} \sqrt{D_v} O_{\text{mix}} \sqrt{I_N(L_N^T)^{-1}}.$$  \hfill (12)

Here $O_{\text{mix}}$ is an arbitrary 3 by 3 orthogonal matrix with complex values, $I_N$ is a diagonal matrix, and $L_N$ is a lower unit triangular [28], which can uniquely be decomposed to be $M^{-1}M^T = L_N^T I_N L_N$, since it is symmetric matrix. Note here that all the components of $m_D$ should not exceed $O(1)$ GeV, once perturbative limit of $y_D$ is taken to be 1.

Nonunitarity: Constraint of nonunitarity should always be taken into account in the case of the larger neutral mass matrix whose components are greater than three by three, since experimental neutrino oscillation results suggest nearly unitary. In case of the inverse seesaw, when nonunitarity matrix $U_{\text{MNS}}'$ is defined, one can typically parametrize it by the following form,

$$U'_{\text{MNS}} \equiv \left(1 - \frac{1}{2} FF^\dagger \right) U_{\text{MNS}},$$  \hfill (13)

where $F \equiv M^{-1}m_D^*$ is a Hermitian matrix, and $U_{\text{MNS}}'$ represents the deviation from the unitarity. Considering several experimental bounds [29], one finds the following constraints [30]:

$$|FF^\dagger| \leq \begin{bmatrix} 2.5 \times 10^{-3} & 2.4 \times 10^{-5} & 2.7 \times 10^{-3} \\ 2.4 \times 10^{-5} & 4.0 \times 10^{-4} & 1.2 \times 10^{-3} \\ 2.7 \times 10^{-3} & 1.2 \times 10^{-3} & 5.6 \times 10^{-3} \end{bmatrix}.$$  \hfill (14)

Once we conservatively take $F \approx 10^{-5}$, we find $\mu_R \approx 1$–10 GeV to satisfy the typical neutrino mass scale, which could be easy task.

Beta function of $SU(2)_L$ gauge coupling $g_2$: Here, it is worth discussing the running of gauge coupling of $g_2$ in the presence of several new multiplet fields of $SU(2)_L$ [31]. The new contribution to $g_2$ for an $SU(2)_L$ quartet fermion (boson) $\psi(H_4)$, seplet fermion $\Sigma_R$, and quintet boson $H_5$ are respectively given by

$$\Delta b^{\psi}_{g_2} = \frac{10}{3}, \quad \Delta b^{\Sigma}_{g_2} = \frac{56}{3}, \quad \Delta b^{H_4}_{g_2} = \frac{5}{3}, \quad \Delta b^{H_5}_{g_2} = \frac{10}{3}.$$  \hfill (15)

Then one finds the energy evolution of the gauge coupling $g_2$ as [17,32]

$$\frac{1}{g_2^2(\mu)} = \frac{1}{g_2^2(m_{\text{in}})} - \frac{b^{\mu}_{g_2}}{(4\pi)^2} \ln \left(\frac{\mu^2}{m_{\text{in}}^2}\right) - \theta(\mu - m_{\text{th}})$$

$$\times N_f (\Delta b^{\psi}_{g_2} + \Delta b^{\Sigma}_{g_2} + \Delta b^{H_4}_{g_2} + \Delta b^{H_5}_{g_2}) \ln \left(\frac{\mu^2}{m_{\text{th}}^2}\right).$$  \hfill (16)

where $N_f = 3$ is the number of $\psi$ and $\Sigma_R$, $\mu$ is a reference energy, $b^{\mu}_{g_2} = -19/6$, and we assume $m_{\text{in}}(= m_2) < m_{\text{th}}$ with $m_{\text{th}}$ being threshold masses of exotic fermions and bosons. The resulting flow of $g_2(\mu)$ is then given by the Fig. 2. This figure shows that $g_2$ is relevant up to the mass scale $\mu = O(10)$ TeV in the case of $m_{\text{th}} = 500$ GeV, while $g_2$ is relevant up to the mass scale $\mu = O(100)$ TeV in the case of $m_{\text{th}} = 5000$ GeV. Thus, our theory is not spoiled, as long as we work at around the scale of TeV.

III. COLLIDER PHYSICS

Here, let us discuss the collider physics of our model. We have rich phenomenology at collider experiments since there are many exotic charged particles from large $SU(2)$ multiplet scalars and fermions. As the most specific

FIG. 1. Feynman diagram to generate the masses of $\mu_L/R$.

$$m_\nu \sim \frac{v_4}{M_\Sigma} \left(\frac{v_5}{M}\right)^2 f_R^2 v_4^2.$$  \hfill (11)

FIG. 2. The running of $g_2$ in terms of a reference energy of $\mu$. 

FIG. 3. Plots for the running of $g_2$ as a function of $\mu$.
signature, we focus on the production of triply charged lepton $\Sigma^{\pm\pm}$ and its decay at the LHC. The gauge interactions associated with the triply charged lepton are obtained as

$$\Sigma_R i\gamma^\mu D_R \Sigma_L \supset \Sigma^{++} \gamma^\mu (3g_2 c_W Z_\nu + 3eA_\mu) \Sigma^{++} + \sqrt{3} g_2 \Sigma^{+++} \gamma^\mu W_\mu \Sigma^{++} + \text{H.c.},$$

where $c_W = \cos \theta_W$ with Weinberg angle $\theta_W$ and $e$ is the electromagnetic coupling: the covariant derivative for the septet can be referred to Ref. [19]. Then, we estimate cross sections for the triply charged lepton production processes using CALCHEP [33] by use of the CTEQ6 parton distribution functions (PDFs) [34], implementing relevant interactions. In Fig. 3, we show production cross section for triply charged lepton as a function of its mass; pair production $pp \rightarrow \Sigma^{++} \Sigma^{--}$ and associate productions $pp \rightarrow \Sigma^{++}(\pm \pm) \Sigma^{--}(\mp \mp)$ at the LHC 13 TeV. The cross section for pair production is the largest one and larger than 1 fb for 1 TeV mass due to large charge.

The triply charged lepton can decay via Yukawa coupling in Eq. (1) as $\Sigma^{++} \rightarrow \phi_4^{\pm} \psi_\nu^\pm$ where $Q_{\phi(\psi)}$ is the electric charge of $\phi(\psi)$ with $Q_\phi = Q_\psi = 3$, and we assume exotic scalars are lighter than exotic fermions in our discussion. In addition, $\psi_\nu^\pm$ decays as $\psi_\nu^\pm \rightarrow \phi_3^0 \ell^\pm (\nu)$ with $Q_\psi = Q_\phi = 1(0)$. There are several decay modes for exotic charged leptons due to the combination of charges in the final states which have similar size branching ratios (BRs). Here, we discuss the representative decay chain,

$$\Sigma^{++} \rightarrow \phi_4^{++} \psi^+ \rightarrow \phi_4^+ \phi_3^0 \ell^+ \rightarrow W^+ W^+ Z Z \ell^+, \quad \text{(18)}$$

![FIG. 3. The cross sections for processes $pp \rightarrow Z/\gamma \rightarrow \Sigma^{++} \Sigma^{--}$ and $pp \rightarrow W^+ \rightarrow \Sigma^{+++} \Sigma^{--}$ as a function of septet fermion mass.](image)

### Table II. Number of expected signal events for two final state from $pp \rightarrow \Sigma^{++} \Sigma^{--}$ production where integrated luminosity is taken as 300(3000) fb$^{-1}$.

| Signal | $3\ell^- 3\ell^- 8j E_T$ | $3\ell^+ 12j E_T$ |
|--------|----------------|----------------|
| $\sigma \cdot \text{BR}(M_L = 0.8 \text{ TeV})$ [fb] | 0.73 (7.3) | 7.2 (72) |
| $\sigma \cdot \text{BR}(M_L = 1.0 \text{ TeV})$ [fb] | 0.21 (2.1) | 2.0 (20) |
| $\sigma \cdot \text{BR}(M_L = 1.2 \text{ TeV})$ [fb] | 0.064 (0.64) | 0.63 (6.3) |

where $\phi_4^\pm$ and $\phi_3^0$ decay into $W^+ W^+$ and ZZ via gauge interaction [17]

$$\langle D_\mu H_4 \rangle^\dagger \langle D^\mu H_4 \rangle \supset \sqrt{\frac{3}{2}} g_2^2 W_\mu^\pm W_{\mu\pm} \phi_4^{\pm \pm} \quad \text{(19)}$$

$$\langle D_\mu H_5 \rangle^\dagger \langle D^\mu H_5 \rangle \supset \frac{1}{8 c_W^2} v_3 \phi_3^0 Z_\mu Z^\mu. \quad \text{(20)}$$

Note that the BRs for $\phi_4^{\pm \pm} \rightarrow W^\pm W^\pm$ and $\phi_3^0 \rightarrow ZZ$ are dominant when $v_4 \sim v_5 \sim 1$ GeV. When $W^\pm$ decays into leptons and ZZ decays into jets, we obtain the signal of three same-sign charged leptons with jets and missing transverse energy, which allows products of BRs; $\text{BR}(W^+ \rightarrow \ell^- \nu) \text{BR}(Z \rightarrow q\bar{q}) \sim 0.02$ with $\ell = \mu, \epsilon$. Thus, we can obtain $\sim 6(60)$ signal events containing three same-sign charged leptons for integrated luminosity of 300(3000) fb$^{-1}$ when products of the $\Sigma^{++} \Sigma^{--}$ production cross section and $\text{BR}(\phi_4^{++} \psi^\pm) \text{BR}(\psi^\pm \rightarrow \phi_3^0 \ell^\pm)$ is around 1 fb. This size of the cross section can be obtained for $M_L \sim 1$ TeV, where we show the expected number of events in Table II for several values of $M_L$ considering one or two pairs of same-sign W bosons decaying into leptons. We find that the number of events tends to be too small when both same-sign W boson pairs $W^+ W^+(W^- W^-)$ decay into leptons although the signal will be very clear. Thus, the signal of three same-sign charged leptons with jets and $E_T$ can be a good target in searching for the signature of our model. We expect a sizable discovery significance even if the number of signal events is less than 10 since the SM background is very small for three same-sign charged lepton signals.

### IV. SUMMARY AND CONCLUSIONS

We have constructed an inverse seesaw model with large $SU(2)_L$ multiplet fields in which we have formulated the neutrino mass matrix to reproduce current neutrino oscillation data, satisfying the $\rho$ parameter and nonunitarity bound. We have also checked the relevant energy scale of our theory via the RGE of $SU(2)_L$ gauge coupling $g_2$ that gives the most stringent constraint. Then, we have analyzed the collider physics focusing on triply charged lepton production at the LHC as a representative process of our model and showing a possibility of detection. We have
found the specific signal of the triply charged lepton as three same-sign charged leptons with jets and missing transverse momentum. The number of events of the signal can be at the detectable level with integrated luminosity $300(3000) \text{ fb}^{-1}$ when the triply charged lepton mass is around 1 TeV. More detailed analysis will be given elsewhere.

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