Surface Flows of Soft Monopole Modes of $^{40}$Mg

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Abstract. In order to explore the nature of collective modes in weakly bound nuclei, we have investigated surface flow patterns of isoscalar monopole modes in a shape-coexisting nucleus $^{40}$Mg. The calculations were done in a fully self-consistent continuum finite-amplitude-method quasiparticle-random-phase-approximation (FAM-QRPA) in a large deformed spatial mesh. Our results demonstrated that the transition current densities at surfaces are nontrivial and dependent on deformations. The toroidal mode is favorable in the oblate shape, demonstrating the geometric deformation effects on nuclear surface flows. The surface flow patterns become more complicated as excitation energies increase.

1. Introduction
The low-energy collective modes associated with neutron skin or halo are of great interests in the area of radioactive nuclear beams. Nuclei close to drip lines are weakly bound nuclei and can have significant extended nuclear surfaces. The dilute surfaces are loosely interacting with the cores, and nuclei become a quantum liquid-gases coexisted system. The spatial extended surface allows low energy surface-core relative motions which is novel in contrast to the giant resonances. The resulting soft or pygmy modes are related to a variety of physics aspects [1] such as the equation of state, symmetry energy, size of neutron skin/halo, the incompressibility. The low energy resonances can also enhance the neutron capture rates in astrophysical r-process.

The strengths of the low-energy modes have been extensively studied theoretically and experimentally. However, the detailed nature of the low-energy modes is still of great interests. In the framework of QRPA, the transition current density can be a useful tool to directly understand the collective modes. In the superfluid quantum systems, the current flows are irrotational and can have various self-organized patterns. In the finite nuclei, the superfluid flows can also dependent on the excitation energies, deformations, the multipole operators and the weakly binding effects. For the pygmy dipole modes, the superfluid flow can be compressional
Figure 1. (Color online) Transition strength functions of isoscalar monopole resonances in the shape-coexisting nucleus $^{40}$Mg as a function of excitation energies $\omega$. (a) prolate case; (b) oblate case.

Figure 2. (Color online) Neutron transition current densities $\vec{j}(r, z, \phi = 0)$ of isoscalar monopole soft modes in $^{40}$Mg. The color scales denote the logarithm of the current density and the arrows denote the flow directions. (a) $\omega=1$ MeV of prolate shape; (b) $\omega=4$ MeV of prolate shape; (c) $\omega=3$ MeV of oblate shape.

or toroidal modes [2, 3, 4]. In this work, we are interested in the soft monopole modes. The monopole modes are also called breathing modes. The dilute surfaces can have significant soft monopole modes since the incompressibility is very small [5]. It is interesting to explore the superfluid flows of the soft monopole modes.

The ideal tool is the fully self-consistent continuum QRPA [6]. However, the QRPA calculations are very expensive in deformed cases with continuum configurations [7, 8, 9]. Recently, FAM-QRPA has been proposed [10] and it is very efficient compared to the conventional QRPA method [11, 12, 13, 14]. The FAM-QRPA solves the non-linear QRPA equations iteratively and avoids the computations of huge QRPA matrix elements. We recently implement the FAM-QRPA approach in the large deformed spatial mesh with high precision so that the details of the multipole collective modes can be revealed [15].
2. Calculations of transition strengths

In this work, we studied the monopole modes of a weakly-bound deformed $^{40}$Mg, which has N=28 magic neutron number but has a well-established prolate-oblate shape coexistence [16, 17]. The shape coexistence provides a good opportunity to study the deformation dependent surface flows. Recently there are strong experimental interests in the spectroscopy of $^{40}$Mg as studied in RIKEN.

Firstly, the Hartree-Fock-Bogoliubov equation is solved in the axial-deformed coordinatespaces of 27.6 fm, using the B-spline techniques [18]. The precise HFB solutions are essential for describing deformed halo structures and continuum discretizations. In our HFB calculations, the ground state of $^{40}$Mg is prolate with $\beta_2=0.4$, and its energy is 1.9 MeV lower than the prolate shape with $\beta_2=-0.31$. Then obtained HFB wavefunctions and quasiparticle energies are inputs for the solutions of the non-linear FAM-QRPA equations which is implemented in a Gauss-Legendre lattice. Our goal is to iteratively solve the FAM-QRPA non-linear equation [10],

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\begin{align*}
(E_\mu + E_\nu - \omega) X_{\mu\nu}(\omega) + \delta H_{\mu\nu}^{20}(\omega) &= -F_{\mu\nu}^{20}, \\
(E_\mu + E_\nu + \omega) Y_{\mu\nu}(\omega) + \delta H_{\mu\nu}^{02}(\omega) &= -F_{\mu\nu}^{02},
\end{align*}
\]

where $X_{\mu\nu}(\omega)$ and $Y_{\mu\nu}(\omega)$ are the FAM amplitudes; $\delta H_{\mu\nu}^{20}$, $\delta H_{\mu\nu}^{02}$ are the induced oscillations in Hamiltonian; $F$ is the external time-dependent field. Then the transition strength can be obtained with $X$ and $Y$. The expressions of $H^{20}$ and $H^{02}$ in the HFB approach can be found in Ref. [10], which are not needed in the ground state calculations. Note that the time-odd terms [19] of the Skyrme functional are very important in QRPA calculations and have been included. The non-linear equation is solved iteratively using the modified Broyden method. In the solution of the FAM-QRPA equation, a smooth factor of 0.5 MeV has been adopted. The FAM-QRPA solver is implemented with hybrid OpenMP+MPI parallel scheme. The calculations adopt the Skyrme force, new-fitting extended SLy4 force [20] and the density dependent pairing interactions as described in Refs. [15, 21]. The parameters of the extended SLy4 force are given in Ref. [20].

The monopole transition strengths of the prolate and oblate energy minima of $^{40}$Mg are displayed in Fig. 1. We see that there are remarkable soft monopole modes in $^{40}$Mg. In the prolate case, the peak of a soft monopole mode is at 4 MeV. The soft monopole mode is very
broad and is close to the threshold. There is another possible peak at 1 MeV. In the oblate case, the peak of a soft monopole mode appears at 3 MeV. In contrast to the prolate case, the soft monopole mode is narrow and evident. The features of the strengths of monopole modes can be understood as the influences of deformation effects, which has been demonstrated in isovector dipole modes [15]. We like to point out that the monopole strength of the prolate shape is almost close to zero energy, which is special and very different from other multipole modes of $^{40}$Mg and it is interesting to probe it by experimental cross sections near zero degrees.

3. Transition Current Densities
The transition current densities $\vec{j}(r, z, \phi = 0)$ of monopole modes of $^{40}$Mg have been studied to reveal the surface flow patterns. Fig.2 displays the neutron current flows of the soft monopole modes at $E=1$ and 4 MeV of the prolate shape and at $E=3$ MeV of the oblate shape, respectively. In Fig.2(a), the in-phase compressional flow pattern is very simple at $E=1$ MeV. In Fig.2(b), at $E=4$ MeV of the prolate shape, the compressional flow pattern has two boundary lines at 10 fm and 15 fm respectively. This is similar to the flow patterns of dipole modes around 4 MeV [15]. In Fig.2(c), with $E=3$ MeV of the oblate shape, the compressional flow forms a distinct circulation, or a toroidal mode. In the oblate $K=1$ dipole mode, there is also an ambiguous flow circulation. Thus we say the toroidal mode is more favorable in the oblate shape in contrast to the prolate shape, demonstrating the geometric deformation effects on nuclear surface flows.

Fig.3 displays the neutron current flows of the giant monopole modes at $E=15$ MeV of the prolate shape and at $E=18$ MeV of the oblate shape. The flows are compressional but are more complicated compared to the flows of soft modes in Fig.2. This is similar to the flows of giant dipole modes. Both the soft and giant monopole modes have an in-phase pattern of transition densities. It is difficult to identify the geometric deformation effects in flows of giant monopole modes. In both soft and giant resonances, the flows of protons involve a small region with a distance less than 12 fm. Thus the novel surface flow patterns we demonstrated in Fig.2 and Fig.3 are unique phenomena in weakly bound nuclei with largely-extended neutron surfaces.

4. Summary
We have developed the deformed continuum FAM-QRPA solver for multipole collective excitations in a large spatial mesh. This enable us to reveal the detailed nature of soft collective excitations. In the previous work, we studied the surface flows of the pygmy dipole modes. In this work, we studied the surface flows of the soft monopole modes in the shape coexisting weakly-bound $^{40}$Mg. We see the monopole modes are compressional and the flow patterns are dependent on the geometric deformations and excitation energies. The lowest-energy mode has the simplest flow patterns. The oblate shape is favorable for the toroidal mode. The flow patterns certainly become complex as excitation energies increase.

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