Applying the Data Envelopment Analysis method for evaluating the efficiency of the complex system operations in fuel and energy companies

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Abstract. This work proposes the Data Envelopment Analysis method (DEA) as a tool for evaluating the efficiency of the complex systems operations on the example of fuel and energy companies. It is also presented a comparative analysis of different methods for evaluating the efficiency of the complex systems operations. The output-oriented DEA model is used in the research. The task with one input and two outputs is solved. In order to test the method, a complex system was chosen – the heat supply system for the heat and power plants on the left bank of Krasnoyarsk. The calculations were made using four heat and power plants in Krasnoyarsk.

1. Introduction
Operation efficiency is one of the defining characteristics of the production system. The concept reflects the effectiveness that the system under study has during the conversion of certain inputs to outputs.

Operation of complex objects implies, in particular, the decision-making on of their efficiency controlling. Efficiency controlling includes evaluating the current level of an object’s effectiveness, predicting its future level, and making recommendations for efficiency improving. These recommendations may relate to determining the target values of the object’s indicators, as well as ways to achieve the recommended values. The problem of studying the systems effectiveness in all areas of human activity has received increased attention in recent years [1,2,3,4]. Different approaches are used to conduct such research, depending on the subject area. However, a number of methods that are used to evaluate the efficiency of objects in fairly broad classes of systems have also been developed.

To determine the efficiency of the complex systems operation, it is necessary to have special tools. In our opinion, the Data Envelope Analysis (DEA) method can serve as one of the components of such tools [5].

The DEA method was proposed in 1978 by American scientists A. Charnes, W. W. Cooper, and E. Rhodes [2,5]. They based their research on the ideas outlined in the article by M. J. Farrell published in 1957 [6,7].
Over the past 25 years, it has been widely used to evaluate the performance of complex objects in various fields. However, the method was not used to evaluate the efficiency of heat and power plants.

2. Efficiency evaluation methods

Let’s consider the organization effectiveness from the point of view of correlating the results of the organization with the costs that are required to achieve these results [6,8].

There are many different methods for evaluating the operational efficiency of the complex systems, which differ from each other in the type of indicators obtained, the approach used, etc.

Let’s single out some of the most commonly used methods and conduct their comparative analysis.

The diagram of methods for evaluating the efficiency of the complex systems is presented in figure 1 [4,6,7,9].

![Diagram of methods for evaluating the efficiency of the complex systems](image)

**Figure 1.** Scheme of methods for evaluating the efficiency of the complex systems operation.

Further, table 1 containing the information about the comparative analysis of the above-mentioned methods for evaluating efficiency by the advantage criteria is presented [2,4,7,9,10].

**Table 1.** Comparative analysis of the efficiency evaluation methods based on the advantage criteria.

| Method                        | Method advantages                                                                 |
|-------------------------------|-----------------------------------------------------------------------------------|
| Simple regression analysis    |  - simplicity of computational algorithms;                                        |
|                               |  - visibility of results (for a linear model);                                    |
|                               |  - the form of the relationship between values is nonparametric;                  |
|                               |  - is applicable to various areas of human life.                                  |
| SFA                           |  - takes into account random errors;                                              |
|                               |  - evaluates the “true” boundary, not the average of all objects;                |
|                               |  - technically separates the value of random error and efficiency.               |
| Index method                  |  - relative ease of use;                                                          |
|                               |  - allows to evaluate long-term productivity trends.                              |
| DEA                           |  - allows to calculate one aggregated indicator for each of the objects, using input and output indicators; |
- takes into account the presence of many inputs and outputs, that is, it has the ability to process them simultaneously, and each of them can have its own units of measurement;
- allows to represent the efficiency of an object as a numeric value;
- there is no need to assign weight coefficients to input and output parameters when solving the optimization problem;
- does not require defining a specific type of production function;
- generates a Pareto-optimal set of points corresponding to effective objects;
- allows to determine what needs to be changed at the input/output of an inefficient object to output to the efficiency boundary;
- allows to take environmental factors into account;
- is aimed at identifying best practices;
- allows to take wishes regarding the significance of any input / output variables into account.

Further, table 2 containing the information about the comparative analysis of the above-mentioned methods for evaluating efficiency by the disadvantage criteria is presented [2,4,7,9,10,11].

**Table 2.** Comparative analysis of the efficiency evaluation methods based on the disadvantage criteria.

| Method            | Method disadvantage                                                                                                                                 |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|
| Simple regression analysis | - the choice of the dependency type has the subjective character (fitting the model to the original data);                                      |
|                   | - the original data must be converted;                                                                                                                                 |
|                   | - the absence of important independent variables in the model leads to an inadequate representation of the simulated object;                        |
|                   | - outliers have a negative impact on the regression coefficients.                                                                                           |
| SFA               | - random variable of inefficiency is not defined a priori;                                                                                                   |
|                   | - the method is designed to produce a single product (output);                                                                                                 |
|                   | - the need to choose the functional form of the production functional;                                                                                         |
|                   | - outliers can cause estimates to shift.                                                                                                                        |
| Index method      | - does not take random errors into account;                                                                                                                    |
|                   | - implicitly in the growth assessment the full technical efficiency is implied, constant returns to scale, behavioral objectives such as cost minimization. |
| DEA               | - DEA does not take random errors into account in the source data, and has sensitivity of the results to outliers;                                                   |
|                   | - the efficiency boundary is based on the effective objects of the system, therefore, after a certain period of time, it can move. This is due to the fact that the boundary is not based on the “average” value, as in the case of non-boundary methods of evaluating effectiveness; |
|                   | - the solution of the problem may be too resource-intensive with a large sample size.                                                                           |

Based on table 1 and table 2, we can conclude that the DEA is the most convenient and effective tool for measuring the object’s operation efficiency.

Further, let’s look at the Data Envelope method in more detail [4,11]. The DEA is based on a nonparametric methodology, because no form of production function is defined. The DEA refers to boundary methods. This is due to the fact that the method is based on constructing an efficiency boundary and analyzing objects located relative to this boundary [12,13].

Let’s consider the main idea of the DEA method. Let there be data for K input parameters and M output parameters for each of the N objects (the term “object” can mean regions, industries, enterprises,
educational institutions, etc.). For the \(i\)\(^{th}\) object, it is represented by the vector columns \(x_i\) and \(y_i\). Then the matrix \(X\) of dimension \(K*N\) represents the matrix of input parameters for all \(N\) objects, and the matrix \(Y\) of dimension \(M*N\) represents the matrix of output parameters for all \(N\) objects. One can come to the task of mathematical programming and, using the duality theory, formulate it in this form [2,4,10]:

\[
\begin{align*}
\min_{\theta, \lambda} & \ (\theta), \\
-y_1 + Y\lambda & \geq 0, \\
\theta x_1 - X\lambda & \geq 0, \\
\lambda & \geq 0
\end{align*}
\]  

(1)

where \(\theta\) is a scalar, and \(\lambda\) is a vector of dimension constants \(N*1\). The value \(\theta\) obtained when solving the task will be a measure of the \(i\)\(^{th}\) object efficiency. In this case, the efficiency cannot exceed 1 (one). It is important to remember that the same task is solved \(N\) times, i.e. for each object. Thus, the objects for which the value of the efficiency indicator was equal to one are located on the border of efficiency. As a result, a piecewise linear efficiency boundary can be formed. Points corresponding to objects that have the efficiency score less than one can be projected onto the efficiency boundary in such a way that each of these points is equal to a linear combination \((X\lambda, Y\lambda)\). Some elements of the vector \(\lambda\) have non-zero values.

These elements correspond to the objects that are the reference for the object being evaluated. The linear combination of reference objects forms a hypothetical object located at the efficiency boundary. A hypothetical object would be effective if it actually existed. But since it doesn’t exist, the values of its variables are the target for a real – inefficient – object. As a result, targets can be set for objects with \(\theta < 1\), which are to proportionally reduce their input factors by a value of \(\theta\) while maintaining the output values at the same level. The closer a point corresponding to a given object is to the efficiency boundary, the higher its efficiency measure is [2].

The above-mentioned model is called a model oriented to input and receiving the presence of constant scale effect. In order to take the possibility of a variable scale effect into account, it is necessary to add a limit on the amount of weight coefficients \(\lambda\) to this model [3,7]:

\[
\Sigma \lambda = 1
\]  

(2)

The consequence of introducing this restriction is the formation of a convex linear combination of reference objects.

3. Results and discussion

In previous publications, the process of constructing an effective performance evaluation model with the help of the decision support system (DSS) using the DEA method has already been considered in sufficient detail. The concept of the complex system and its features have also been considered. The efficiency indicators of four heat and power plants on the right bank of Krasnoyarsk have been calculated [14].

In this research, a complex system – the heat supply system for the heat and power plants on the left bank of Krasnoyarsk was selected for the method testing.

The Data Envelopment Analysis Program (DEAP) of the University of New England in Australia by Professor T. Coelli was used for calculations [10,15].

On the basis of the project [14] “The heat supply scheme for Krasnoyarsk until 2033” four boiler heating systems were selected on the left bank of the city.

The output-oriented DEA model will be used in the research. The task with one input and two outputs will be described in the article.

The initial data are presented in table 3. This table includes information about the initial data of heat and power plant indicators – Decision Making Unit (DMU). Input indicators are given in the table as INPUT (\(x\)). Output indicators are shown in the table as OUTPUT (\(y\)). The column in the table “Emission.
mass (deviation from the threshold value)” contains the deviation of the output indicator from the threshold value. For the threshold, we take a value equal to 22,000.0 thousand tons per year, focusing on the maximum value for this indicator, in this case for the heat and power plant D.

Table 3. Initial data – list of DMU, input, outputs, deviation indicator.

| № | Name of DMU                  | INPUT (x)                                      | OUTPUT (y)                                      |
|---|-------------------------------|-----------------------------------------------|-----------------------------------------------|
|   |                               | Available heat power, Gcal / hour             | Heat energy output to the heating system, thousand Gcal per year | Emission mass, thousand tons per year | Emission mass (deviation from the threshold value) |
| 1 | Heat and power plant A        | 631.50                                        | 1914.00                                        | 12655.9817                         | 9344.018                           |
| 2 | Heat and power plant B        | 120.00                                        | 157.55                                        | 1755.0000                         | 20245.0000                        |
| 3 | Heat and power plant C        | 57.60                                         | 98.95                                         | 1839.6038                         | 20160.4000                        |
| 4 | Heat and power plant D        | 1464.00                                       | 3531.00                                       | 21165.6953                        | 834.3047                          |

We solve the task set in the research: the output volume is increased without increasing the input. The results of the research calculations are presented in table 4. This table reflects the values of efficiency indicators for all considered heat and power plants.

Table 4. Efficiency indicators according to the DEA method.

| №  | Division (DMU) Inputs and outputs | Efficiency indicator | Difference between values | Difference in % |
|----|----------------------------------|----------------------|---------------------------|----------------|
| 1  | Heat and power plant A           | 1                    | 0                         | 0%             |
|    | Available equipment capacity     | 631.5                | 631.5                     | 0              | 0%             |
|    | Heat energy output to the heat supply system | 1914.00          | 1914.00                   | 0              | 0%             |
|    | Emission mass                    | 9344.018             | 9344.018                  | 0              | 0%             |
| 2  | Heat and power plant B           | 0.972                | 0                         | 0%             |
|    | Available equipment capacity     | 120                  | 120                       | 0              | 0%             |
|    | Heat energy output to the heat supply system | 157.55            | 165.982                   | 8,432          | 5.35%           |
|    | Emission mass                    | 20245.0000           | 20830,757                 | 585.757        | 2.9%            |
| 3  | Heat and power plant C           | 1                    | 0                         | 0%             |
|    | Available equipment capacity     | 57.6                 | 57.6                      | 0              | 0%             |
|    | Heat energy output to the heat supply system | 98,95           | 98,95                     | 0              | 0%             |
|    | Emission mass                    | 20160,4              | 20160,4                   | 0              | 0%             |
| 4  | Heat and power plant D           | 1                    | 0                         | 0%             |
|    | Available equipment capacity     | 1464.00              | 1464.00                   | 0              | 0%             |
|    | Heat energy output to the heat supply system | 3531.00         | 3531.00                   | 0              | 0%             |
|    | Emission mass                    | 834,3047             | 834,3047                  | 0              | 0%             |

Thus, table 4 shows that the heat and power plants A, B, D of the heat supply system of Krasnoyarsk operate as efficiently as possible, since their efficiency indicator is the maximum possible value and is
equal to 1 (one). The heat and power plant C has the lowest operating efficiency indicator, which is 0.972, relative to the other objects under consideration, and therefore is the least efficient object of the heat supply system of the left bank of Krasnoyarsk. But in general, it can be noted that all the considered objects of the heat supply system of the left bank of Krasnoyarsk operate quite effectively, since their threshold values of efficiency do not deviate from the permissible ones.

4. Conclusion
This research considers the DEA method as an analysis tool for evaluating the efficiency of the complex systems operation on the example of fuel and energy enterprises. The article presents a comparative analysis of methods for evaluating the complex systems efficiency. The description of the specificity of the DEA method use is given. Based on the comparative analysis, conclusions about the effectiveness of using the DEA method are made, since it allows to represent the efficiency of an object as a numerical value by calculating one aggregated indicator for each of the objects, using input and output indicators.

The output-oriented DEA model is used in research. The task with one input and two outputs is solved.

In order to test the method, a complex system was chosen – the heat supply system for the heat and power plants on the left bank of Krasnoyarsk. The calculations were made on the example of four heat and power plants in Krasnoyarsk.

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