Spin-orbit coupling and perpendicular Zeeman field for fermionic cold atoms: 
Observation of the intrinsic anomalous Hall effect

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We propose a scheme for generating Rashba spin-orbit coupling and perpendicular Zeeman field simultaneously for cold fermionic atoms in a harmonic trap through the coupling between atoms and laser fields. The realization of Rashba spin-orbit coupling and perpendicular Zeeman field provides opportunities for exploring many topological phenomena using cold fermionic atoms. We focus on the intrinsic anomalous Hall effect and show that it may be observed through the response of atomic density to a rotation of the harmonic trap.

PACS numbers: 03.75.Ss, 72.10.-d, 03.65.Vf, 71.70.Ej

Two important ingredients for manipulating electron spin dynamics and designing spin devices in spintronics [1] are spin-orbit coupling and Zeeman field. For instance, Rashba spin-orbit coupling (RSOC) (assume in the x-y plane), together with a perpendicular Zeeman field (PZF) (along e_y), yield a transverse (along e_x) topological Hall current with an applied electric field (along e_z). Such an intrinsic current was proposed to be one feasible explanation of the experimentally observed anomalous Hall effect (AHE) and spin Hall effect (SHE) in ferromagnetic semiconductors [2, 3]. However, the scattering of electrons from impurities and defects in the solid, leading to extrinsic AHE and SHE, makes the experimental observation of intrinsic AHE and SHE very difficult [4].

Ultra-cold atomic gases experience an environment essentially free from impurities and defects, and therefore provide an ideal platform to emulate many condensed matter models or even observe new phenomena. One important recent effort along this line is in study of atomtronics that, in analogy to electronics and spintronics, aims to realize devices and circuits using cold atoms [5]. One natural and important question in atomtronics is how to generate effective spin-orbit coupling and Zeeman fields. Great progress has been made recently on the generation of RSOC by considering the coupling between cold atoms and laser fields [6-9], which leads to a series of important applications [10-13].

However, the direction of the generated Zeeman field in these previous schemes [6, 8-13] is in the spin-orbit coupling plane. Such in-plane Zeeman fields cannot open a band gap between different energy branches in the energy spectrum. The band gap, together with RSOC, is the physical origin of many topological phenomena. For instance, the band gap is necessary for the observation of the intrinsic AHE [2]. It is also the key ingredient of the recently broadly discussed schemes on the creation of a chiral psf + ipsf-wave superfluid/superconductor from an s-wave superfluid/superconductor [14, 15] for the observation of non-Abelian statistics and topological quantum computation [16]. In contrast, a band gap can be opened in the presence of RSOC and PZF, but a scheme for generating them simultaneously for cold atoms is still absent.

In this paper, we propose a scheme to create RSOC and PZF simultaneously for a fermionic atomic gas in a harmonic trap. The realization of RSOC and PZF may open opportunities for the observation of many topological phenomena in cold atoms because of the non-zero Berry phase induced by RSOC and PZF. Here we focus on one of them: observation of the intrinsic AHE. In solid state systems, the AHE has been observed in transport experiments for electrons (i.e., measuring charge currents or voltages). Such transport experiments are not suitable for cold atoms in a harmonic trap. We find that the time-of-flight of cold atoms in the presence of RSOC and PZF can only yield a small asymmetry of the atomic density, therefore it may not be suitable for observing the intrinsic AHE. Instead, we consider the response of atom density to an external rotation of the trap, which corresponds to an effective magnetic field for atoms. Because the intrinsic AHE is not a quantized effect, the St" reda formula [17] that was proposed for studying quantum Hall effects in cold atoms [18] does not apply. We find that the atomic density response to the rotation contains not only contributions from the anomalous Hall conductivity, but also a new term from the orbit magnetic moments of atoms that is absent in previous literature [19]. Both contributions originate from the topological properties of RSOC and PZF.

Consider ultra-cold Fermi atoms with a tripod electronic level scheme (Fig. 1b). States |1⟩, |2⟩, |3⟩ are three hyperfine ground states, and state |4⟩ is an excited state. The fermi gas is confined in a quasi-two dimensional (x-y-plane) harmonic trap. Along the z direction, the atomic dynamics is “frozen” by a deep optical lattice, leading to a multiple layered system. The ground states |1⟩, |2⟩, |3⟩ are coupled with state |4⟩ by three lasers with corresponding Rabi frequencies \( \Omega_{a1} \), \( \Omega_{a2} \), and \( \Omega_{a3} \). In the interaction representation, the single-particle Hamiltonian is

\[
H = \frac{p^2}{2m} + V_{ext} + H_I - \mu, \tag{1}
\]

where \( H_I = \hbar \Delta |4⟩⟨4| - \hbar (\Omega_{a1} |4⟩⟨1| + \Omega_{a2} |4⟩⟨2| + \Omega_{a3} |4⟩⟨3| + H.c.) \) describes the laser-atom interaction, where \( \Delta \) is the detuning to state |4⟩. \( V_{ext} \) is the external potential that includes the harmonic trap as well as

\[
V_{ext} = \frac{1}{2} m \omega_z^2 r^2 + \frac{1}{2} m \omega_x^2 z^2 + \frac{1}{2} m \omega_y^2 y^2.
\]
The configuration of the laser beams. All lasers are uniform plane waves.

The effective low-energy Hamiltonian \( H_{\text{eff}} \) is given by

\[
H_{\text{eff}} = \gamma k^2 - \mu - \alpha (k_x \sigma_y - k_y \sigma_x) + V (r),
\]

where the third term is the RSOC, the fourth term is the PZF. All eight lasers used for the generation of RSOC and PZF are uniform plane waves, therefore they do not lead to spatial periodic modulation of the atomic density. In addition, these lasers propagate only along three different directions (the same as other tripod schemes [11, 13]), therefore our scheme should be feasible in experiments.

Under the local density approximation with the local chemical potential \( \mu (r) = \mu - m \omega_t^2 r^2 / 2 \), the Hamiltonian \( \mathcal{H}_1 \) has two eigenenergies \( \varepsilon_{+} \) and \( \varepsilon_{-} \), \( \varepsilon_{+} = \gamma k^2 - \mu \pm \varepsilon_0 \). There is an energy gap \( \varepsilon_{\gamma} = 2 \hbar_0 \), opening between two spin orbit bands at \( k = 0 \) (Fig. 2a). The intrinsic AHE is nonzero only when the chemical potential \( \mu (r) \) lies inside the gap.

The dynamics of cold fermi atoms are described by the semiclassical equations of motion

\[
f = \partial \varepsilon_k / \partial k - \mathbf{k} \times \mathbf{\Gamma}_z, \quad \dot{\mathbf{k}} = \mathbf{F} / \hbar,
\]
about the intrinsic AHE. Consider a rotation of the harmonic trap along the z axis, the Hamiltonian can be written as

$$H = \hbar^2 q^2/2m + \omega^0 (q_x \sigma_y - q_y \sigma_x) + h_0 \sigma_z + m (\omega^2 - \omega^2)^2/2 - \mu$$

in the rotation frame, where \(\hbar\) is the Planck constant, \(m\) is the mass of the atom, \(\omega^0\) is the mechanical frequency of the trap, and \(\omega\) is the rotation frequency of the trap. The density of atoms is

$$n_\rho = (2\pi)^{-2} \int d^2 \mathbf{q} [1 + m\omega \Gamma_z/\hbar] f(\mathbf{q}, \mathbf{r}, \omega),$$

where \(m\omega \Gamma_z/\hbar\) is a correction to the well-known constant density of states \(1/(2\pi)^2\) in the presence of nonzero Berry curvature fields and the rotation. The expression for the rotation frequency \(\omega\) is

$$\omega = \omega_0 + \mu^2 \omega^2/2 - \mu$$

for fixed \(\mu\) and \(T\). At \(T = 0\) and \(\omega = 0\), Eq. 9 reduces to

$$\frac{dn}{d\omega} = \frac{m}{\hbar^2} \int d^2 \mathbf{q} \left[ \frac{m}{\hbar} \Gamma_z f + \left( 1 + \frac{m\omega \Gamma_z}{\hbar} \right) \frac{df}{d\mu} M_z \right]$$

for fixed \(\mu\) and \(T\). At \(T = 0\) and \(\omega = 0\), Eq. 9 reduces to

$$\frac{dn}{d\omega} = \frac{m}{\hbar} \int d^2 \mathbf{q} \left[ \frac{m}{\hbar^2} \Gamma_z f + \frac{M_z}{4\pi} \frac{1}{d\varepsilon_q/dq^2} \right]_{\varepsilon = \mu}.$$
the Fermi wavevector \( q_F (r) \) is obtained from \( \varepsilon q_F = \mu (r) \). In the parameter region \( \alpha^2 / \gamma \gg h_0 \), \( q_F^2 \approx \alpha^2 / \gamma^2 \) and \( \sigma_{xy} \approx m/4 \pi \hbar \). The second term \( \sigma_{xy} \), originating from the non-zero orbital magnetic moment \( M_z \), is an additional contribution to \( \partial n / \partial \varpi \) that was missing in the previous literature \cite{ref19} for electron systems. Eq. \cite{ref10} is a generalization of Str"eda formula for the anomalous Hall effects. By varying parameters and measuring the density response, we can extract information not only about the anomalous Hall conductivity, but also the magnetic moment that is generally hard to measure in solid state systems. In the parameter region \( \alpha^2 / \gamma \gg h_0 \), \( \sigma_{xy} \approx \gamma m n_0 / 8 \pi \hbar \alpha^2 \ll m / 4 \pi \hbar \approx \sigma_{xy} \). Therefore \( \sigma_{xy} \) dominates in Eq. \cite{ref9} in this region, and the density response \( \partial n / \partial \varpi \) yields a rough measurement for the anomalous Hall conductivity.

We numerically calculate the density \( n \) and density response \( \partial n / \partial \varpi \) as functions of the parameters \( (\mu, \varpi, T) \), and plot them in Figs. \ref{fig3} and \ref{fig4} for \( T = 200 \) nK and 2 nK. The presence of the harmonic trap changes the chemical potential at \( r \) by \( -m (\omega_t^2 - \varpi^2) r^2 / 2 \). In a realistic experiment, the effective trapping frequency \( \sqrt{\omega_t^2 - \varpi^2} \) in the presence of rotation may be slightly different from the initial trapping frequency \( \omega_t \) without rotation to keep the same temperature of the system \cite{ref22}. This can be overcome by comparing the densities at different spatial points \( r, r' \) such that \( \mu = -\frac{1}{2} m (\omega_t^2 - \varpi^2) r'^2 = \mu - \frac{1}{2} m \omega_{t0}^2 r^2 \) to keep the same local chemical potential \cite{ref22}. With this method, we can measure the density response to the rotation with the fixed temperature and chemical potential. In addition, the rotation of the system requires an asymmetric harmonic trap \cite{ref21}, which does not affect our results because it only change the spatial potential dependence \( \alpha^2 / \gamma^2 \). The density variation at a medium temperature \( T = 200 \) nK is at the same order as that at a low temperature \( T = 2 \) nK. Note that the multiple layer structure \( \omega_t = 2 \pi R / 8 \text{KHz}, \omega_t = 2 \pi R / 425 \text{Hz}, \omega_{t0} = 2 \pi \times 50 \text{Hz}, \omega_{max} = 0.05 \omega_R = 2 \pi \times 425 \text{Hz} \), \( \omega_{max} = 2 \pi / 425 \text{Hz} \), \( \omega = \omega_t + \omega_{t0} / (\omega_t + \omega_{t0}) \approx 2 \pi / 3 \text{Hz} \). At \( \omega = 0.1 \omega_R / \omega_{t0} = 4.2 \mu m \), the chemical potential \( \mu \) changes by 0.01 \( \mu R \). From Figs. \ref{fig3} and \ref{fig4} we see a maximum density change at the order of \( 3 \times 10^7 \) cm\(^{-2} \) can be observed with a rotation frequency \( \omega_{max} \), which corresponds to about 10% of the total density and can be observed in a realistic experiment. The density variation at a medium temperature \( T = 200 \) nK is at the same order as that at a low temperature \( T = 2 \) nK.

In summary, we propose a scheme to create RSOC and PZF simultaneously for cold atomic gases. We show that, by measuring the atomic density response to a rotation of the trap, the intrinsic AHE can be observed for cold fermionic atoms in a harmonic trap. We emphasize that the creation of RSOC and PZF brings new opportunities for studying many topological phenomena, such as chiral \( \nu \)-wave superfluids, anomalous and spin Hall insulators, etc.

Acknowledgments: We thank Di Xiao and Qian Niu for helpful discussion. This work is supported by the ARO (W911NF-09-1-0248) and DARPA-YFA (N66001-10-1-4025).

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