Spatial measurement of axial and radial momentum fluxes of a plasma expanding in a magnetic nozzle

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Abstract
Spatial profiles of axial and radial momentum fluxes of a plasma expanding in a magnetic nozzle is revealed by using a momentum vector measurement instrument located downstream of a radiofrequency plasma source, where the radial and axial forces exerted to the detector plate facing the source side are independently, directly, and simultaneously obtained. It is shown that a conical structure having high electron temperature and plasma density is significantly responsible for the axial momentum flux, which corresponds to the thrust in an electric propulsion device. The radially outward momentum flux is detected at the outer region of the conical structure, where the electron pressure gradient is formed; implying that the ions are radially accelerated by an electric field. The increase in the radially integrated axial momentum flux along the axis is demonstrated, where the gain of the axial momentum flux occurs at the radially peripheral region of the plasma expanding along the magnetic nozzle.

1. Introduction
Momentum flux of plasmas is one of fundamental physical quantities affecting the plasma dynamics; its gain, loss, and conversion processes have been important topics associated with plasma acceleration and deceleration in space [1, 2], astrophysical objects [3], and terrestrial plasmas in laboratories [4]. Especially, the processes spontaneously occurring in electric and magnetic fields have attracted much attention, e.g., the solar plasma acceleration and coronal mass ejection from the Sun [5], the formation of the accretion disk and the jet [6], particle accelerations in magnetosphere [7], and confinement of the fusion plasmas [8]. Since the momentum flux is balanced with external forces exerted to the plasmas according to the momentum equation, the identification of the momentum flux is crucial to understand the interaction between the plasmas and the electromagnetic fields, the resultant plasma structural formations, and the acceleration and deceleration processes.

One of the fascinating applications of the momentum flux of the plasmas is a thruster for space propulsion, where the axial force exerted to the thruster, i.e., the thrust, corresponds to the axial momentum flux exhausted from the system, as derived from the momentum conservation law [9], where the momentum flux is given by the sum of the static pressure and the dynamic momentum flux. In typical low-pressure gaseous plasmas, in which the electric power for the plasma production is transferred to the electrons, the ion temperature is much lower than the electron temperature \( T_e \) [10]. Assuming the negligible ion temperature (i.e., the ion static pressure) and electron inertia (i.e., the electron dynamic momentum), the axial momentum flux \( \tau \) delivered by the plasma is given by

\[
\tau = p_e + m_i n_p u_e^2, \tag{1}
\]

where \( p_e \), \( m_i \), \( n_p \), \( u_e \) are the electron pressure (\( p_e = n_p k_B T_e \) with the Boltzmann constant \( k_B \)), the ion mass, the plasma density, and the axial ion velocity, respectively. According to equation (1), the momentum flux can be principally obtained from the spatial profile of \( n_p, T_e, u_e \), while the measurements of the absolute
values of the density, the temperature, and the velocity require great effort. For example, a significant error likely occurs in the density estimation even in the well-known Langmuir probe due to the effects of the sheath expansion [11, 12] and magnetic fields [13]. Furthermore, both the electron and ion energy distributions in the low-pressure plasmas are often non-Maxwellian due to their kinetic behaviors [14–17]. In addition to the precise measurements of the physical quantities, the spatial profiles have to be measured to estimate the thrust from the spatial integration of the quantities. The thrust, i.e., the momentum exhausted from the system per unit time, has been measured in the field of the electric propulsion by mounting a thruster to a thrust balance [18–20] or locate a target having larger diameter than the plasma column [21–24], while these techniques cannot provide the spatial information of the momentum flux, i.e., it is difficult to understand how and where the momentum flux increases or decreases. In experiments on a variable specific impulse magnetoplasma rocket, a radial measurement of the momentum flux has been used to assess the thruster performance and the total thrust has been obtained by integrating the data in the cross section [25].

Fundamental investigations on the thrust imparted by a radiofrequency (rf) plasma thruster containing a magnetic nozzle, which is a new type of the thruster and often called a helicon thruster, have been carried out over the last decade in analytical, numerical, and experimental studies [26–28]. The studies have involved many aspect of physics ranging from particle acceleration [29], magnetohydrodynamic phenomena [30, 31], electron thermodynamics [32–35], and new application to space debris removal [36]. A number of laboratory experiments have shown inherent structural formations in the configuration of the magnetic nozzle rf plasma thruster, e.g., the formation of a current-free double layer and the subsequent ion acceleration [15, 37], a conical high density and temperature plasmas [38–42], generation of a low-divergence ion beam [43, 44]. Some models have shown that the thrust can be given by the pressure force in the source, the loss of the axial momentum flux to the radial wall, and the Lorentz force arising from the azimuthal internal plasma current and the radial magnetic fields [45–47]. These forces are exerted to the source back plate, the radial source wall, and the magnetic fields produced by the solenoid, respectively; the force components have been individually measured by attaching one of them to the thrust balance [48–50], which has provided the discoveries of the axial force exerted to the radial wall and of the thrust gain by the magnetic nozzle. The measurement using the thrust balance has yielded the force integrated over the surface area of the source or over the volume in the magnetic nozzle, while a recently developed momentum vector measurement instrument (MVMI) has been used to identify the spatial profiles of the axial and radial momentum fluxes lost to the radial wall [51, 52]. The spatial measurement in the source has provided the insight that the external magnetic field can inhibit the momentum and energy losses to the radial wall.
In the present paper, detailed two-dimensional profiles of the axial and radial momentum fluxes in the magnetic nozzle downstream of the rf plasma source are measured by using the MVMI mounted on the radially and axially movable stage immersed in vacuum, where the detector surface of the MVMI faces the source tube. The results on the axial momentum flux demonstrate that the high electron temperature or density conic is responsible for the thrust imparted by the magnetic nozzle rf plasma thruster. Furthermore, it is shown that the thrust gain occurs near the surface of the plasma column expanding along the magnetic nozzle, where the radial density gradient exists. The radial force exerted to the detector plate facing the source, which corresponds to the radial momentum flux delivered by the ions, implies the radial acceleration of the ions at the region of the radial density gradient; indicating the radial electrostatic ion acceleration in the magnetic nozzle region.

2. Experimental setup

Schematic diagram of the experimental setup is shown in figure 1(a) and the experiments are performed with a rf plasma source having a 10 cm-outer-diameter and 9.5 cm-inner-diameter Pyrex glass source tube. The source is attached via an O-ring seal to a 60 cm-diameter and 140 cm-long vacuum chamber evacuated by a turbomolecular pumping system to a base pressure of about $10^{-4}$ Pa, where the axial location of the open source exit is defined as $z = 0$. The upstream side of the source tube is terminated at $z = -20$ cm by an insulator vacuum flange having a gas injection port. Argon gas is introduced from the gas injection port and the gas flow rate is maintained at 40 sccm, giving the chamber pressure of about 150 mPa. The solenoid is located at $z = -1$ cm and a dc electric current of 20 A is supplied to the solenoid; the calculated magnetic field on axis is shown in figure 1(b). The field strength has the peak value of about 380 G at the solenoid center and decreases to about 30 G at $z = 30$ cm. A double-turn rf loop antenna is wound around the source tube at $z = -10$ cm and powered by a 13.56 MHz rf generator via an impedance matching box, where the rf power is chosen as 1 kW in the present experiment. Two variable capacitors in the matching box is tuned so as to make the rf power reflection undetectable, providing the plasma density of about $3 \times 10^{17}$ m$^{-3}$ near the open source exit.

To identify the local flux of the axial and radial momentums in the magnetic nozzle, the previously reported momentum vector measurement instrument (MVMI) is mounted on an axially and radially movable motor stage installed inside the vacuum chamber as shown in figure 1(a). The detailed structure of the MVMI is shown in figure 2 and can be found in references [51, 52]. Very briefly, the MVMI has a 2 cm-square insulator detector plate facing the source side and attached to an arm mounted on a rotational pivot. A radial ($F_r$) component of the force exerted to the surface of the detector plate acts as a torque to the arm and induces the rotational displacement as shown by the solid arrow in figure 2. This structure is further mounted on an axially movable pendulum stage suspended by two flexible metallic plates from the top metallic part. An axial ($F_z$) component of the force exerted to the surface of the detector plate is transmitted to the axially movable pendulum via the arm and the pivot, while it does not induce the rotational displacement since the torque to the arm is zero. Hence the axial component of the force induces only the axial displacement of the pendulum structure as shown by the dashed arrow in figure 2. Therefore the whole structure enable the radial ($F_r$) and axial ($F_z$) forces exerted to the detector surface to induce the rotational and axial displacements, respectively. The rotational and axial displacements measured by light
Figure 3. Two-dimensional (r–z) profiles of the measured (a) electron temperature $T_e$ and (b) ion saturation current $I_{is}$ of the Langmuir probe, together with the magnetic field lines, where the representative magnetic field lines are labeled as circles 1–5.

emitting diode displacement sensors can provide the absolute values of the radial and axial forces by multiplying the calibration coefficients as reported earlier [52].

When the detector surface faces the axial direction, the radial and axial momentum fluxes, i.e., the force densities $f_r$ and $f_z$, to the detector plate are generally given by

$$f_r = m_i n_s u_{isz}, \quad (2)$$

$$f_z = m_i n_s u_{isz} + n_e k_B T_e, \quad (3)$$

where $m_i$, $n_s$, $u_{isz}$, $u_{isz}$, $u_{isz}$ are the ion mass, the ion density at the sheath edge, the axial ion velocity at the sheath edge, the radial ion velocity at the detector surface, and the axial ion velocity at the detector surface, respectively. The second term in equation (3) is the electron pressure given by the electron density $n_e$ at the detector surface, the Boltzmann constant $k_B$, and the electron temperature $T_e$. Since both the momentum and the particle flux should be conserved between the front of the sheath and the detector surface, even if the sheath accelerates the ions, equation (3) can be rewritten as

$$f_z = m_i n_p u_z^2 + n_p k_B T_e, \quad (4)$$

where $n_p$ and $u_z$ are the plasma density and the axial ion velocity in the plasma, respectively. It should be noted that $f_z$ in equation (4) corresponds to the momentum flux $\tau$ given by equation (1). As the sheath at the axially facing detector plate has only the axial electric field, $f_z$ in equation (2) mirrors only the flux of the radial ion momentum, i.e.,

$$f_r = m_i n_p u_z u_r, \quad (5)$$

where $u_r$ is the radial ion velocity in the plasma. Therefore, the presence of $f_r$ to the axially facing detector plate implies that the ions impinging the detector surface have a finite radial velocity.

To discuss the spatial profiles of the momentum fluxes and the physical quantities of the plasma, a 3 mm-diameter radially facing Langmuir probe is alternatively mounted on the stage. The electron temperature is estimated from the current–voltage characteristic of the Langmuir probe with assuming the Maxwellian energy distribution and the ion saturation current is also measured to discuss the brief plasma
structure in the magnetic nozzle. It should be mentioned that the zero-order value of the ion saturation current \( I_{is} \) of another Langmuir probe located near the thruster exit is measured for both the cases with and without the MVMI in the chamber. No visible change in the dc component of the ion saturation current is detected by the presence of the MVMI. Therefore, it can be deduced that the disturbance of the plasma by the MVMI is negligible for the zero-order quantities in the present experiment.

3. Experimental results

Figure 3 shows the two-dimensional \((r–z)\) profiles of (a) the electron temperature \( T_e \) and (b) the ion saturation current \( I_{is} \) of the Langmuir probe, together with the magnetic field lines (white solid lines), where the measurement is not performed in the black region in the figures and the representative magnetic field lines intersecting the open source exit are labeled as 1–5 from the center to the periphery. Figure 3(a) shows the electron temperature of about 3 eV on axis, while the high electron temperature of about 6 eV is detected along the magnetic field line labeled as 3. Simultaneously the large ion saturation current is detected near the magnetic field line 3 as seen in figure 3(b). It should be noted that the ion saturation current is proportional to both the plasma density and the square root of the electron temperature; the ion saturation current profile does not directly mirror the density profile in the non-uniform electron temperature case. However, the very similar structure including the high electron temperature and high density conic have been often observed in some experiments [38–42]. Some of the studies have interpreted that the high electron temperature is due to the electron heating near the rf antenna and transport along the magnetic field lines; the results in figure 3 well reproduce the previous studies, implying the conic of the high electron pressure.

The measured two-dimensional profiles of the axial \( f_z \) and radial \( f_r \) force densities exerted to the MVMI detector plate are shown by the contour color in figures 4(a) and (b), respectively. Along the magnetic field line labeled as 3, the large axial force density \( f_z \) is detected as seen in figure 4(a). The location of the large axial force density is very close to that of the conical structure of \( I_{is} \) shown by figure 3(b). As the thrust corresponds to the axial momentum flux integrated over the cross section of the plasma flow, it can...
be deduced that the high electron pressure (temperature or density) conic significantly contributes to the thrust generation in the helicon thruster configuration. The measured profile of $f_r$ also shows the large radial force density at the peripheral region along the magnetic nozzle as seen in figure 4(b); the maximum location is found to exist between the magnetic field lines labeled by 3 and 4, i.e., the location of the maximum radial force density exists at the radially outer region of the maximum axial force density. As the radial force density exerted to the detector plate is given by the product of the radial ion momentum ($m_i u_r$) and the axial ion flux ($n_p u_z = n_s u_{zs}$), the conical structure of $f_r$ implies the presence of the radially accelerated ions.

To compare the measured profiles of the force densities and the ion saturation current in detail, the representative radial profiles of the data are extracted from figures 3(b) and 4. The radial profiles of $f_r$, $f_z$, and $I_{is}$ at $z = 14$ cm are plotted by open squares in figure 5(a), by filled squares in figure 5(b), and by open circles in figure 5(c), respectively. The open squares in figure 5(c) is the first derivative $-dI_{is}/dr$ of the radial profile of $I_{is}$, which very roughly sketches the radial electron pressure gradient force ($-dp_e/dr$). The solid lines in figure 5(c) are added as visual guides. It can be seen that the location of the radially outward force ($f_r > 0$) corresponds to that showing the positive value of $-dI_{is}/dr$ when comparing figures 5(a) and (c). It is considered that the ions are radially accelerated by the radial electric field induced by the radial electron pressure gradient; indicating that the radial motion of the ions are significantly affected by the electrostatic field rather than the magnetic nozzle. The radial plasma transport is often induced by the presence of low-frequency instabilities, e.g., a drift wave [53], a Simon–Hoh instability [54], a lower-hybrid wave [55], and so on. The low-frequency instabilities have not been investigated yet in the present configuration and the effect of the instabilities on the detected radial momentum flux remains further challenge. Unlike the profile of the radial force density $f_r$, both the axial force density $f_z$ and the ion saturation current $I_{is}$ has their maximum values at the same radial location of $r = 5$ cm as seen in figures 5(b) and (c), as already discussed with the two-dimensional profiles.

Figure 5. (a) Radial profile of the measured radial force density $f_r$. (b) Radial profile of the measured axial force density $f_z$, together with the fitted curve (solid line) superimposing two Gaussian deconvolutions (dashed lines). (c) Radial profile of the ion saturation current $I_{is}$ (open circles) and the first derivative $-dI_{is}/dr$ (open squares), together with the solid lines added as visual guide. The data are measured at $z = 14$ cm.
Figure 6. (a) Two-dimensional profile of the fitted axial force density $f_z$, together with the magnetic field lines. (b) The axial force $T_z$ integrated over the cross section of the plasma as a function of the axial position $z$, where the typical error bar is estimated from the several measurements of the radial profile.

Figure 7. Two-dimensional profile of the thrust gain $G$ given by $G = \frac{\partial f_z}{\partial z}$, together with the magnetic field lines.

The momentum flux integrated over the cross section and the axial differentiation of the momentum flux ($G = \frac{\partial \tau}{\partial z}$) corresponds to the thrust and the local gain of the thrust, respectively [45, 46, 56]. To take the momentum flux at the radially outer region beyond the measurement into account and to minimize the numerical noise in the analysis, especially in the differentiation of the discrete data, the radial profile of $f_z$ are fitted by two Gaussian deconvolutions. Representatively, the solid line in figure 5(b) shows the fitted curve by superimposing the two Gaussian curves (dashed lines). This procedure is carried out at each axial position of the radial measurement and the two-dimensional profile of the fitted data of $f_z$ is shown in figure 6(a), duplicating the data in figure 4(a). The radially integrated axial force, i.e., the thrust $T_z$, is plotted by open squares as a function of the axial position $z$ in figure 6(b), together with the typical error bar. The data demonstrates that the thrust increases along the axis, which is consistent with the
Figure 8. Sketch of the directions of the pressure gradient ($\nabla p$), the azimuthal diamagnetic current ($j_\theta$), and the resultant Lorentz force ($j_\theta \times B_r$), where the radially outward and inward pressure gradients induce the axially upward and downward forces exerted to the plasma, respectively.

prediction by the previous models, e.g. [45], and the individual measurement of the thrust components exerted to the magnetic fields [48].

The local thrust gain $G$ is obtained from the partial differentiation of the axial force density $f_z$ corresponding to the axial momentum flux $\tau$, i.e., $G = \partial \tau / \partial z = \partial f_z / \partial z$. Figure 7 shows the two-dimensional profile of the thrust gain $G$ obtained from the data in figure 6(a). Between the magnetic field lines labeled by 3 and 4, the positive thrust gain can be seen, while the negative thrust gain is shown in the central region of the plasma. The thrust gain profile can be interpreted as a result of the Lorentz force ($j_\theta \times B_r$) due to the radial magnetic field ($B_r$) and the azimuthal diamagnetic current ($j_\theta$), where the radially outward ($dp_e/dr > 0$) and inward ($dp_e/dr < 0$) pressure gradients induce the azimuthal currents imparting axially upward and downward forces to the plasma, respectively. The directions of the pressure gradient, the azimuthal diamagnetic current, and the Lorentz force for the radially outward and inward pressure gradients are sketched in figure 8; the direction of the Lorentz force is consistent with the observed thrust gain $G$ in figure 7. Hence the present results demonstrate the thrust gain at the peripheral region of the plasma along the magnetic nozzle, where the radially inward pressure gradient exists. The results are indeed consistent with the previously measured internal plasma current [57].

4. Conclusion

The local fluxes of the radial and axial momentums of the plasma are measured by the momentum vector measurement instrument (MVMI) installed downstream of the magnetic nozzle rf plasma source. The results on the axial momentum flux show that the high density or temperature conical structure significantly contributes to the thrust generation. It is demonstrated that the thrust gain occurring at the peripheral region of the plasma increases the thrust along the axis in the magnetic nozzle. On the other hand, the measurement of the radial force exerted to the axially facing detector presents that the ions are accelerated in the radially outward direction at the pressure gradient region, where the pressure gradient is considered to induce the electrostatic field. Hence, the behavior of the ions in the magnetic nozzle seems to be dominated by the electrostatic field rather than the magnetic field.

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