Application of the similarity theory for vortex chamber superchargers

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Abstract. On the basis of similarity in vortex chamber superchargers criteria of similarity are defined and their check by numerical researches is made. Vortex chamber superchargers have the big power efficiency of pumping, in comparison with classical jet pumps, at the expense of association of advantages of centrifugal and jet pumps on the basis of the vortex chamber. The concept of vortex chamber superchargers is based on a new principle of energy transfer in jet superchargers at the expense of hydrodynamic effects of rotating flows use. In consequence of pressure decrease near to an axis of the vortex chamber occurs intaking particles of a pumped over flow. Having got to the vortex chamber, particles get tangential velocity interact with a rotating flow at the expense of a momentum exchange. The received criteria take into account of three major factors of similarity: the geometrical sizes, pressure of an active stream on an input in the device, working medium density. Validity of the received criteria is confirmed by a finding of pressure-flow rate characteristics of similar operating modes of a supercharger and their further combination on one characteristic.

1. Introduction
Application of jet superchargers in adverse service conditions, and also at pumping liquids and gases with the big solid abrasive particles content is dictated by rapid wear of mechanical mobile operative parts of dynamic and displacement pumps [¹]. Jet devices, unlike classical superchargers possess low power efficiency, but high indicators of reliability and durability [²]. At work of direct-flow jet devices in hydraulic and pneumatic transport of bulk solids their use is limited to low indicators of ejection factor and high pressure values of an active stream [³]. In some way solve these problems in special service conditions [⁴, ⁵] can vortex chamber superchargers, possessing all advantages of jet technics: high reliability and durability, work possibility on any structure and concentration of phases, small overall dimensions. Besides, vortex chamber superchargers (VCS), owing to use of centrifugal force in pumping process, possess the best, in comparison with jet direct-flow pumps, characteristics at bulk solids transportation [⁶].

Works on studying of working capacity and power characteristics of vortex chamber superchargers are spent throughout last fifteen years [⁴-⁶]. But, for today, there are no the publications devoted to possibility of preliminary definition of design parameters of the projected device. Besides, there is no possibility of recalculation of characteristics of a supercharger on any standard size.
To solve this problem it is possible by use of the theory of similarity and dimension [7]. Designing of vortex chamber superchargers, as well as any another, is based on preliminary extensive researches. Among them experimental researches [4] have importance. In the dimension and similarity theory it is necessary to define conditions which should be observed in experiences with models, and then allocate the characteristic and convenient parameters defining the main effects and process modes [7].

Complicated movement character of a pumped over liquid in vortex chamber superchargers leads to that the problem of new devices designing decides along with design-theoretical engineering of a design of their flowing part by experimental researches in vitro, and also with the help of CFD-models and in natural conditions [8, 9].

Preliminary definition of design parameters of a projected supercharger, research of working processes on models and propagation of the received results on full-scale superchargers demands fulfillment of geometrical, kinematic and dynamic conditions of similarity [10].

Therefore the purpose of the given work is definition of similarity criteria of vortex chamber superchargers and their check by numerical researches.

2. Results of research

Investigations of vortex chamber superchargers similarity it was spent on the model presented in drawing 1. The VCS [6] works as follows: the basic stream with the flow rate \( Q_s \) and pressure \( p_s \) comes through the input tangential channel in the vortex mixing chamber and leaves it through the exit tangential channel. A working stream, having mixed up with the pumped over stream arriving through two input axial channels with flow rates \( Q_{in1} \) and \( Q_{in2} \), with pressure \( p_{in1} \) and \( p_{in2} \), comes in the exit tangential channel with the flow rate \( Q_e \) and pressure \( p_e \).

![Figure 1. Investigated vortex chamber supercharger.](image)

The supercharger can work in two modes [6] which are characterized by two various working processes. Realization of the first working process leads to occurrence of losses in the drainage channel (the channel in1 in drawing 1) and allows to provide relatively high pressure at the small flow rate of the pumped over medium. Realization of the second working process leads to sucking a pumped over stream through both axial channels, i.e. is fulfill undrainage variant of a supercharger work with relatively low pressure at the high flow rate on the device exit. Both working process of energy transfer in the field of centrifugal force unites, but in the first case occurs at preservation of the angular moment (tangential velocity circulation). And, in the second case energy transfer take place at a momentum exchange between interacting streams due to an eddy motion.

The concept of vortex chamber superchargers is based on a new principle of energy transfer in jet superchargers at the expense of hydrodynamic effects of rotating flows use – vacuum near a rotation axis and overpressure on periphery of the vortex chamber. In consequence of pressure decrease near to an axis of the vortex chamber occurs intaking particles of a pumped over flow. Having got to the vortex chamber, particles get tangential velocity interact with a rotating flow at the expense of a momentum exchange (Figure 2). Further under the influence of superficial (pressure forces) and mass (dominating gravity) in a potential field move on chamber periphery. Near the vortex chamber walls the particles of a pumped over liquid get in exit tangential channel of a supercharger getting kinetic energy of the overpressure in periphery chamber.
In classical jet superchargers the only method of energy transfer at the expense of an momentum exchange of interact collision flows is used that is accompanied by essential dissipative process [11]. In vortex chamber superchargers, unlike classical jet devices, the particles get the basic energy mainly in a conservative field at the expense of moving on periphery of the vortex chamber under the influence of centrifugal force. The active flow on periphery has high values of potential energy which can reach 90 % from the spent energy of a supply flow.

**Figure 2.** Justification of a pumping over process in the vortex chamber supercharger.

At physical modeling of hydromechanical processes it is accepted to distinguish geometrical, kinematic and dynamic similarity [10]. At the same time use analogous points – points of geometrically similar figures equally located in position to borders of these figures. Hydromechanical systems name geometrically similar if them analogous the sizes are proportional, and corners in analogous points are identical [7].

Geometrical similarity of superchargers is defined by a constancy of similarity linear factor:

\[
K_L = \frac{D_H}{D_M} = \frac{d_{awH}}{d_{awM}} = \frac{H_H}{H_M} = \ldots = \text{const} \tag{1}
\]

where \(D_H, d_{awH}, H_H\) and \(D_M, d_{awM}, H_M\) – diameter of the vortex chamber, diameter of the inflow channel and height of the chamber of modelling and full-scale superchargers accordingly. Geometrical similarity demands a proportion constancy of any other sizes of modeling and full-scale superchargers \(D_M / H_M = D_H / H_H = \text{const}\).

Strictly speaking, for full geometrical similarity it is necessary, that also roughness of vortex chamber surfaces also were similar, but fulfillment of this condition is seldom possible.

The kinematic similarity of superchargers says that dimensionless velocity fields in them should be equal. Besides, mechanical trajectories of liquid particles should be geometrically similar also. The conditions of the kinematic similarity are expressed in the form of ratios:

\[
K_V = \frac{V_{vH}}{V_{vM}} = \frac{V_{wH}}{V_{wM}} = \frac{V_{rH}}{V_{rM}} = \text{const} \tag{2}
\]
where $V_{rH}, V_{trH}, V_{thH}$ and $V_{rM}, V_{trM}, V_{thM}$ – the radial, tangential and axial components of points velocity of model and full-scale superchargers.

Using geometrical similarity from which follows, as $V_r \sim Q_r / d_r^2$, and also $V_c \sim Q_c / d_c^2$ similarity of flow rates in a supercharger, we gain:

$$
\left( \frac{Q_r / d_r^2}{Q_c / d_c^2} \right)_H = \left( \frac{Q_r / d_r^2}{Q_c / d_c^2} \right)_M = \text{const}, \quad \frac{Q_r}{Q_c} H = \frac{Q_r}{Q_c} M = \frac{d_r^2}{d_c^2} \frac{p_{rH}^{1/2}}{p_{cM}^{1/2}} \frac{\rho_{rH}^{1/2}}{\rho_{cM}^{1/2}}. \quad (3)
$$

As at an outflow from a supercharger in an aerosphere the total pressure is defined by formula $p_c = 0.5 \rho V_e^2$ and in the equation (3) it is necessary to use a ratio $(p_{rH} / p_{cM})^{1/2}$ as a ratio of dynamic pressures.

Dynamic similarity, except observance of geometrical and kinematic similarity conditions, demands similarity of the forces acting in matching points of a flow. Presence of dynamic similarity is a sufficient condition for existence of the kinematic and geometrical similarity. The solution of the flow equations hydrodynamically similar streams among themselves differ from each other only a scale factor. Therefore, choosing scale for quantities entering into such equations, it is possible to convert them to an aspect when factors near each member of the equation become dimensionless. The fundamentals for turbomachinery are criteria (numbers) of Reynolds, Strouhal and Euler. By flows consideration in which force of liquid weight plays the important role, necessary there is Froude number use. At achievement near- and supersonic velocity values of a flow the important role is played by a Mach number. In practice of hydro-machinery modeling very large value has Euler's similarity parameter. With reference to observed conditions and vortex chamber superchargers it can be expressed as follows:

$$
E_u_H = \left( \frac{p}{\rho V^2} \right)_H = E_u_M, \quad E_u_H = \left( \frac{p}{\rho V^2} \right)_H = \left( \frac{p \pi^2 d_e^4}{16 \rho Q_e^2} \right)_H = \left( \frac{p \pi^2 d_e^4}{16 \rho Q_e^2} \right)_M.
$$

Using (3) and (4) it is possible to express a similarity parameters for VCS:

$$
\frac{Q_{rH}}{Q_{cM}} = \frac{d_{rH}^2}{d_{cM}^2} \frac{p_{rH}^{1/2}}{p_{cM}^{1/2}} \frac{\rho_{rH}^{1/2}}{\rho_{cM}^{1/2}},
$$

$$
\frac{p_{rH}}{p_{cM}} = \frac{p_{rH}}{p_{cM}}
$$

$$
\frac{N_{rH}}{N_{cM}} = \frac{d_{rH}^2}{d_{cM}^2} \frac{p_{rH}^{1/2}}{p_{cM}^{1/2}} \frac{\rho_{rH}^{1/2}}{\rho_{cM}^{1/2}}.
$$

The wide experience of hydraulic modeling and turbomachinery modeling shows that at pump work in self-similarity region change of Reynolds number does not render noticeable influence on hydraulic efficiency [10].

For confirmation of choice correctness of vortex chamber superchargers similarity criteria in wide range of pressure, density and the geometrical sizes on the basis of the solution of equations RANS (Reynolds-Averaged Navier-Stokes) numerical research has been made for an incompressible liquid with use SST-model of turbulence with rotation-curvature correction.

Set of equations SST model looks as follows [12]:
\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \mu_{ij} \frac{\partial k}{\partial x_j} \right) + P_k - \beta \rho k \omega \tag{9}
\]

\[
\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho u_\omega)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \mu_{ij} \frac{\partial \omega}{\partial x_j} \right) - \rho \beta \omega^2 + C_{d_\omega} + \frac{\alpha P}{\mu} \tag{10}
\]

where: \( \rho \) – density; \( k \) – kinetic energy of turbulent pulsation; \( x_j \) – Cartesian coordinates; \( u_j \) – components of the mean velocity vector; \( \mu_{ij} = \mu + \mu_t \) – effective viscosity; \( \mu_t \) – turbulent viscosity; \( \mu \) – molecular viscosity; \( \omega \) – turbulence eddy frequency; \( \beta \) – production of turbulence kinetic energy; \( C_{d_\omega} \) – cross-diffusion term in SST model. Constants and the specification statement of the equations (9) - (10) can be found in [13].

Rotation-curvature correction in SST-model is realized by multiplication of production of turbulence kinetic energy in the equations (9) - (10) on function [13]:

\[
f_{\text{rotation}} = \max \left\{ \text{min} \left( f_{\text{rotation}}, 1.25 \right), 0 \right\}
\]

where

\[
f_{\text{rotation}} = (1 + c_{r1}) \frac{2 r^*}{1 + r^*} \left[ 1 - c_{r1} \tan^{-1} \left( c_{r2} \right) \right] - c_{r1}
\]

The constants \( c_{r1}, c_{r2}, c_{r3} \) are equal 1.0, 2.0 and 1.0 respectively [13, 14]. Terms \( r^* \) and \( \tilde{r} \) are calculated as follows:

\[
r^* = \frac{S}{\Omega}
\]

\[
\tilde{r} = 2\Omega_s S_a \left[ \frac{DS_{ij}}{Dt} + (\varepsilon_{mn} S_m + \varepsilon_{mn} S_m) \Omega_{mn}^{\text{rot}} \right] \frac{1}{\Omega S_a^{2/3}}
\]

Components of the mean strain tensor:

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

Components of the vorticity tensor – \( \Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) + 2 \varepsilon_{mn} \Omega_{mn}^{\text{rot}} \), where \( \varepsilon_{mn} \) – tensor of Levi-Civita.

\[
S^2 = 2 S_{ij} S_{ij}; \quad \Omega^2 = 2 \Omega_s \Omega_s; \quad D^2 = \max \left( S^2, 0.09 \Omega^2 \right)
\]

Adequacy of calculations by means of the chosen model confirmed by comparison with experimental data on integrated parameters (pressures and flow rates in all channels) and on distribution of pressure along radius of the vortex chamber on the basis of a Student’s and Fisher’s tests [8]. In [8, 14] results of verification and selection optimum on errors of mathematical model are presented.

Experimental researches of the vortex device (Figure 3) are made for a finding of the best mathematical model on errors. The error estimation of pressure distribution along radius of the vortex chamber is made at the closed outlet tangential channel of a supercharger. Thus the top end cover of
the vortex chamber was drained sixteen apertures. Experimental facility included the vortex device, blower, receiver and measuring equipment. At different stages of experimental researches of the vortex chamber superchargers characteristics standard instruments and equipment were used. Their relative error did not exceed 1 %. Pressure in channels was measured cup manometers, ambient temperature – mercury thermometers, the flow rates in a channels - flowmeters. The model of a supercharger with diameter of the vortex chamber 60 mm was used. Pressure differences on flowmeter devices it was measured by micromanometers with the maximum error in 8 Pa. The total error between computed and experimental values on pressure did not exceed 12 % in central zone of vortex chamber.

Figure 3. Experimental facility of the vortex chamber supercharger.

Calculations are made in program complex OpenFOAM (OpenCFD Ltd) [15, 16] at following boundary conditions: on a wall – no slip wall, at inflow of the supply channel was set a stagnation pressure, in outflow channels – equality to null of static pressure. The mesh made about 5 million elements for fulfillment of parameter Y+ <2.

Figure 4. Pressure-flow rate characteristics of vortex chamber superchargers: (a) variant 1 (air); (b) variant 2; (c) variant 3.

For research following parameters of a supercharger work have been chosen: 1) actuating medium - air with density 1.185 kg/m³, \( D = 0.03 \) m, a total pressure at supercharger tangential inlet \( p_{i,tot} = 500 \) Pa; 2) actuating medium – water with density 997 kg/m³, \( D = 0.05 \) m, a total pressure at supercharger tangential inlet of the \( p_{i,tot} = 50000 \) Pa; 3) actuating medium - water with density
997 kg/m³, $D = 0.05$ m, a total pressure at supercharger tangential inlet $p_{s}^{\\text{tot}} = 1.25$ MPa. Thus, wide ranges of a vortex chamber supercharger use on various actuating mediums - gas and liquid, with different levels of pressure from 0.5 kPa to 1.25 MPa are chosen.

In Figure 4 pressure-flow rate characteristics of VCS for the observed three cases are presented. For all three variants of superchargers work parameters have been selected so that conditions of geometrical, kinematic and dynamic similarity were observed.

Observance of dynamic similarity can be checked by combination of pressure-flow rate characteristics of all three observed cases on one dimensionless characteristic taking into account recalculation on the basis of the (6). I.e. for combination, for example, the first and second variants, the characteristic first needs to be converted using linear factor of similarity $K_L = D_H / D_M = 0.05/0.03 = 1.67$. Values of a supercharger pressure and flow rate are similarly computed. The result of characteristics combination is presented in Figure 5.

![Figure 5. The combined characteristics for three cases of a supercharger work.](image)

In Figure 5 characteristics of a VCS work are rated to the maximum pressure and to the maximum flow rate of the pressure-flow rate characteristic of the second variant. The points corresponding to different operating modes and transporting of different medium closely lie to the pressure curve of the second variant. It speaks about justice of the statement about similarity of operating modes of all three variants. Especially it is become apparent in the undrainage mode of a supercharger at which deviations of the majority points make no more than 10%.

Fulfillment of the kinematic similarity demands is illustrated in Figure 6.

![Figure 6. The velocity vector field in a supercharger: (a) variant 1 (air); (b) variant 3 (water).](image)
In Figure 6 it is visible that the kinematic similarity consisting in equality of dimensionless velocity field in the device, working on similar regimes is carried out.

For comparison of characteristics hydrodynamically similar pumps inter se often use, formed on the basis of mathematical expressions of similarity rules, the so-called, reduced values – dimensionless or dimensional complexes which are defined from physical parameters of hydraulic machine (flow rate, head, power and others). Equality of the last provides presence of partial dynamic working processes similarity of model and full-scale pump.

At observance of similarity all conditions, on the basis of the equations (6) - (8), equality for similar regimes of efficiency follows. However, influence of the roughness on hydraulic losses in a supercharger will lead to some difference in values of efficiency.

3. Conclusion

Similarity parameters of the vortex chamber superchargers is defined and their check by numerical way is made.

1. Criteria of geometrical, kinematic and dynamic similarity on the basis of ratios of three major factors of similarity are gained: geometrical sizes, actuating medium inlet pressure in the device, actuating medium density.

2. Validity of the gained criteria is confirmed by a finding of pressure-flow rate characteristics of three similar operating modes of a supercharger and their further combination on one characteristic. At the undrainage operating mode a deviation of the majority points make no more than 10 %.

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