Experimental Study of Positionally Disordered Josephson
Junctions Arrays

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Abstract

We experimentally studied the effect of positional disorder on a Josephson junction array with $f = n, \frac{1}{2} + n, \text{or } \frac{3}{2} + n$ flux quanta per unit cell for integral $n$. This system provides an experimental realization of a two-dimensional $XY$ model with random phase shifts. Contrary to many earlier numerical and analytical investigations, our results suggest that low-temperature superconductivity is never destroyed by positional disorder. As the disorder strength increased, the Kosterlitz-Thouless (KT) type order in the $f = 0$ and $\frac{1}{2}$ systems changed to a non-KT type order with a long-range phase coherence, which persisted even in the maximal disorder limit. A possible finite-temperature glass transition is discussed.

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The \( XY \) model with quenched random phase shifts, also known as the random-gauge \( XY \) model, has been intensively studied with a focus on the maximal disorder limit as a model for a vortex glass phase \[1\] of high-temperature superconductors in a magnetic field. Despite a great deal of effort for the last two decades, the model in two dimensions (2D) remains not well understood. Conflicting evidences, mostly numerical or analytical ones, forbid a consensus about the possibility of a finite-temperature glass transition in 2D. The purpose of this work is to see experimentally how the low-temperature ordered phase of the 2D random-gauge \( XY \) system evolves with the disorder strength increased.

The random-gauge \( XY \) model is described by the Hamiltonian

\[
H = -J \sum_{ij} \cos(\theta_i - \theta_j - A_{ij}),
\]

where \( \theta_i \) is the phase of the order parameter on site \( i \) and the quenched random phase shifts \( A_{ij} \)'s are uncorrelated and distributed uniformly in \([A^0_{ij} - r\pi, A^0_{ij} + r\pi]\) with \( 0 \leq r \leq 1 \). The case with maximal disorder corresponds to the disorder strength \( r = 1 \). The effect of random phase shifts in 2D was investigated primarily with respect to the Kosterlitz-Thouless (KT) transition for \( f = 0 \), where frustration \( f = (1/2\pi) \sum A^0_{ij} \). The directed sum of the \( A^0_{ij} \) is around a unit cell. A prevailing conclusion in many numerical and analytical studies \[2\] for \( f = 0 \) is that there exists a KT transition at finite temperatures when \( r < r_c \approx 0.37 \) and the case with maximal disorder, termed the gauge glass, has no ordered phase at any finite temperature. The lack of ordering at finite temperatures has been proven \[3\] rigorously in the limit of maximum disorder. Nevertheless, numerical evidences against this conclusion have been also provided by several different research groups \[4, 5\]. The counter evidences point to a finite-temperature superconducting glass order in the gauge glass limit. It has been shown \[6\] numerically that a breaking of ergodicity due to the large energy barrier against vortex motion may open up the possibility of a finite-temperature glass transition which was denied by the analytical equilibrium studies \[3\]. The confused situation is much the same for \( f = 1/2 \), where the \( XY \) system without disorder has the Ising-like structural order as well as the KT-like phase coherence at low temperatures \[7\]. Along with arguments \[8\] for the persistence of both the Ising-like order and the KT-like order for at least weak disorder strength, it has been also suggested \[9\] that the critical disorder for \( f = 1/2 \) is zero, or that any amount of disorder, no matter how weak it is, destroys the Ising-like order and the phase coherence even at zero temperature.
The disagreement among the analytical and numerical results may be settled by experimental evidences for the random-gauge effect. The random-gauge XY model in 2D can be most closely realized by a Josephson junction array (JJA) with positional disorder in a uniform magnetic field [10]. For a JJA, \( J \) is the Josephson coupling energy per junction and given by \( J = \frac{\hbar i_c}{2e} \) with \( i_c \) the single-junction critical current. \( \theta_i \) is the phase of the superconducting island at site \( i \) and \( A_{ij} = \frac{2\pi}{\phi_0} \int_i^j A \cdot dl \) is the integral of the vector potential across a junction with \( \phi_0 \) being the flux quantum. The frustration \( f \) is equal to the mean number of flux quanta penetrating each cell. Random phase shifts can be introduced by randomly displacing the centers of superconducting islands while keeping the junctions on their regular positions in a periodic lattice. For a square array with frustration \( f \) and with the random displacements in \( x \) and \( y \) directions distributed uniformly in the range \( [-\Delta, \Delta] \) with \( \Delta \) in units of the lattice constant, the corresponding disorder strength is given by the relation \( r \approx 2.2f\Delta \) [10, 11]. Since the XY Hamiltonian is invariant under the transformation of \( f \) to \( f + n \) for an integer \( n \), the disorder strength of a JJA for a specific \( f \) can be controlled by changing either \( n \) or \( \Delta \). In this paper, we present an experimental investigation of a positionally disordered JJA with \( f = n, \frac{1}{2}n, \) or \( \frac{4}{5}n \) for various \( n \)'s. Transport measurements revealed that the three different systems superconduct at sufficiently low temperatures for any disorder strength \( r \) and, despite the essential differences among them at weak disorder, belong to the same universality class in the limit of maximal disorder, proposing the possible occurrence of a finite-temperature glass transition. For \( f = 0 \) and \( \frac{1}{2} \), the nature of the superconducting transition was found to change from a KT type to a non-KT type with \( r \) increased.

Experiments were performed on a square array of 200\( \times \)800 Nb/Cu/Nb Josephson junctions with a controlled amount positional disorder, a section of which is shown in Fig. 1(a). Nb islands were disposed on a 0.3-\( \mu \)m-thick Cu film periodically with a lattice constant of 13.7 \( \mu \)m, a junction width of 4 \( \mu \)m, and a junction separation of 1.4 \( \mu \)m. The random displacements of the centers of the crosses in \( x \) and \( y \) directions were uniformly distributed in the range \( [-\Delta, \Delta] \) with \( \Delta = 0.20 \) in units of the lattice constant. For an ideal JJA with zero junction width, this amount of displacement corresponds to the disorder strength \( r \approx 0.44f \). However, the actual disorder strength for our sample with a finite junction width must be weaker than designed, due to the asymmetric diamagnetic responses of the distorted superconducting islands which inhomogeneously changes the effective area of each.
cell to reduce the amount of positional disorder. If the relocation of the effective center of the Nb island is limited within the middle square of the cross, the smallest possible value of the effective displacement $\Delta_{\text{eff}}$ for $\Delta = 0.20$ is $\sim 0.05$. The actual amount of $\Delta_{\text{eff}}$ of the sample will be determined in the stage of comparing the experimental results with the numerically proposed phase diagram. In the absence of a magnetic field, $i_c$ and $J (= \hbar i_c/2e)$ can be obtained by extrapolating the low-temperature $i_c$ vs $T$ data, obtained from the $I$ vs $dV/dI$ curves, by using de Gennes formula for proximity-coupled junctions in the dirty limit. In the presence of a magnetic field, $i_c$ depends on the magnetic flux $\phi$ through the junction area as $i_c(B) = i_c(0) \sin(\pi\phi/\phi_0)/(\pi\phi/\phi_0)$. Since it is not possible to directly measure $i_c(B)$ of a positionally disordered array and $i_c(B)/i_c(0) (= \sin(\pi\phi/\phi_0)/(\pi\phi/\phi_0))$ depends mainly on the geometry of the junction, $i_c(B)$ of the sample was determined by combining $i_c(0)$ of the sample and $i_c(B)/i_c(0) (= \sin(\pi\phi/\phi_0)/(\pi\phi/\phi_0))$ of another array with the same junction dimensions and properties but no positional disorder. The resistance ($R$) and the current-voltage ($IV$) characteristics were measured by employing the phase-sensitive voltage-signal-detection method using a lock-in voltmeter and a square-wave current at 23 Hz. The magnetic field or the frustration $f$ was adjusted from the $f$ vs $R$ curve of the sample exhibiting distinct resistance minima at integral $f$'s. More details of the experiments are described in Ref. 

Figure 1(b) shows the resistive transition of the sample for three different sets of frustrations $f = n, \frac{1}{2} + n,$ and $\frac{2}{5} + n$. Although the transition broadens appreciably with increasing $n$, it is evident that for all the frustrations, the transition to a zero-resistance state takes a place at a finite temperature. Figure 2 illustrates the $f$ dependence of the superconducting transition temperature $T_c$ determined at $R = 5 \times 10^{-6}$ $\Omega$. Note that $T_c$ is plotted in units of $J/k_B$ in order to scale out the temperature and magnetic-field dependences of the critical current $i_c(T, B)$ and to make a direct comparison of the $T_c$ with the numerical results possible. The $T_c$'s determined from the resistance measurements agree within error bars with those independently determined from the analysis of the $IV$ characteristics (to be commented on in detail below). We first consider the $f = 0$ case. As $n$ (or the disorder strength) increases, $T_c$ decreases at first from $0.84J/k_B$ and then stays at $\sim 0.35J/k_B$ for $f > f_c \sim 3$. The phase line never ends at $T = 0$, contrary to many earlier investigations. A similar phase boundary with $f_c \sim 5$ was observed for the sample with $\Delta = 0.15$. The nature of the superconducting transition could be identified from the development of
the IV characteristics with temperature. Measurements of the IV characteristics were carried out for six different frustrations \( f = 0, 2, 3, 4, 6, \) and 8. Shown in Fig. 3 are those for \( f = 2 \) and 6. The IV curves were obtained by averaging 15-240 measurements for each current. For \( f = 2 \), the IV curves below the transition display a power-law behavior (a straight line in a logI vs logV plot or a flat line in a logI vs \( d(\log V)/d(\log I) \) plot) at low currents, indicative of the KT-type order with a quasi-long-range phase coherence and vortex-antivortex pairs as dominant excitations. The fast drop of the slope \( d(\log V)/d(\log I) \) of the IV curves with temperature above the transition is also characteristic of the KT transition. The IV curves for \( f = 6 \), on the other hand, have an exponential form (a convex upward curve in a log-log plot) below the transition, indicating a non-KT type order with a long-range phase coherence. As shown at the bottom of Fig. 3, the IV curves for \( f = 6 \) can be fitted to the Fisher-Fisher-Huse scaling form \([1]\) for a non-KT type superconducting transition in 2D, 

\[
TV/I|T-T_c|^{\nu} = \varepsilon_\pm (I/T|T-T_c|^{\nu}) ,
\]

where \( \varepsilon \) is the dynamic exponent and \( \nu \) is the correlation length exponent \([15]\). The critical exponents from the scaling analysis are \( \nu = 2.0 \pm 0.3 \) and \( z = 1.8 \pm 0.3 \) for \( f > 3 \), insensitive to \( f \). Crossing over from the KT-like behavior to the non-KT-like one arose at \( f \sim f_c (\sim 3) \), the frustration above which \( T_c \) became approximately independent of \( f \). The evolution of the IV characteristics with \( f \) discloses that the nature of the superconducting transition for \( f = 0 \) changes from a KT type at weak disorder to a non-KT type at strong disorder. These experimental findings except the values of \( T_c \) and \( \nu \) at strong disorder are in a good agreement with the numerical results reported in Ref. \([5]\). Comparing the phase boundary for \( f = 0 \) in Fig. 2 with the numerically proposed phase diagram in Ref. \([5]\), we find that \( f_c \sim 3 \) for our sample corresponds to the critical disorder strength \( r_c \approx 0.37 \) and thus \( f \sim 8 \) to \( r = 1 \). This implies a finite-temperature superconducting transition even in the gauge glass limit of \( r = 1 \). The correspondence of \( f_c \sim 3 \) with \( r_c \approx 0.37 \) also gives \( \sim 0.06 \) for \( \Delta_{\text{eff}} \) of the sample.

The effect of positional disorder on the \( f = \frac{1}{2} \) system which has the KT-like phase coherence and the Ising-like structural order in the absence of disorder were found quite similar to the observed for \( f = 0 \). As shown in Fig. 2 with increasing \( n \), \( T_c \) of the \( f = \frac{1}{2} \) system drops fast from \( 0.45J/k_B \) (the \( T_c \) in the absence of disorder) to \( \sim 0.25J/k_B \) and then increases slowly until it reaches \( \sim 0.3J/k_B \). The IV characteristics for \( f = \frac{1}{2}, 1\frac{1}{2}, 3\frac{1}{2}, 5\frac{1}{2}, \) and \( 7\frac{1}{2} \), some of which are shown in figures 4 and 5(b), indicate that similarly to the \( f = 0 \) case, the nature of the superconducting transition changes from a KT type at weak disorder...
to a non-KT type at strong disorder. The critical exponents for the non-KT type transitions are $\nu = 2.0 \pm 0.3$ and $z = 1.8 \pm 0.3$. The major difference from the $f = 0$ case is that the crossing over appears at lower $f_c \gtrsim 1.5$.

Dissimilarly from the $f = 0$ and $\frac{1}{2}$ systems, the $f = \frac{2}{5}$ system without disorder has a superconducting vortex-solid phase with a long-range phase coherence at low temperatures and experiences a melting transition driven by domain wall excitations [16]. The IV characteristics for $f = \frac{2}{5}, \frac{1}{5}, 3\frac{2}{5}, 5\frac{2}{5},$ and $7\frac{2}{5}$, some of which are shown in figures 4 and 5(b), reveal that for $f = \frac{2}{5}$, the introduction of positional disorder does not alter the essential features of the low-temperature ordered state and phase transition. The phase diagram of Fig. 2 displays $T_c$ for $f = \frac{2}{5}$ as a monotonously increasing function of the disorder strength. The scaling analyses of the IV data present 1.8-2.0 for $\nu$ and 1-1.8 for $z$, dependent on the disorder strength.

The experimental results for the three different sets of frustrations demonstrate that as the disorder strength (or the amount of the random phase shifts) increases, a KT type order of a 2D XY system weakens and is eventually replaced by a non-KT type order with a long-range phase coherence, which never weakens even at the maximum disorder strength. A remaining important question to be answered through this work is whether the finite-temperature superconducting phase in the maximal disorder limit is a glass in nature. Figure 6(a) exhibits the variation of the sample resistance with $f$ at $T = 4.20$ K. The resistance oscillation fades away as $f$ approaches the maximal disorder limit ($f \sim 8$). Three sets of IV curves for $f = 8, 7\frac{1}{2},$ and $7\frac{2}{5}$ plotted in a single figure [Fig. 5(b)] also shows that the IV characteristics near maximal disorder are independent of frustration, excluding the slightly $f$-dependent $T_c$. Consequently, the three sets of IV data collapse onto the same curves with $I$ and $V$ scaled by the Fisher-Fisher-Huse scaling form with the same $\nu (= 2.0)$ and $z (= 1.85)$. This suggests that the 2D XY systems with the maximal disorder strength belong to the same universality class, regardless of the amount of frustration imposed on the system, or that the natures of the superconducting state and phase transition of a 2D XY system become independent of $f$ in the gauge glass limit, despite the essential differences among the systems with different $f$’s at weak disorder. Although the data do not provide a full account for the nature of the low-temperature superconducting state, these features of the data seem to propose the possible occurrence of a finite-temperature glass transition in the gauge glass limit.
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FIG. 1: (a) Photograph of the positionally disordered array with $\Delta = 0.20$. (b) Resistive transition of the disordered array for $f = n, \frac{1}{2} + n$, and $\frac{2}{5} + n$.

FIG. 2: Superconducting transition temperature $T_c$ in units of $J/k_B$ for three different sets of frustrations $f = n$ (filled circles), $\frac{1}{2} + n$ (open squares), and $\frac{2}{5} + n$ (filled triangles), determined from the resistance measurements.

FIG. 3: $I$ vs $V$ curves in a log-log scale for $f = 2$ at $T = 4.200$ to 5.200 K and for $f = 6$ at $T = 3.800$ to 4.900 K; the slope $d(\log V)/d(\log I)$ of the $I$ vs $V$ curves as a function of $I$; and scaling plots of the $I$ vs $V$ curves at 12-18 different temperatures. The dashed lines in the $I$ vs $V$ plots are drawn to show where the phase transition occurs. The insets of the scaling plots show the values of $T_c$, $\nu$, and $z$ used to scale the data.

FIG. 4: Development of $I$ vs $V$ curves with temperature for $f = \frac{1}{2}, \frac{3}{2}, \frac{2}{5}$, and $\frac{3}{5}$. The dashed lines are drawn to show where the phase transition occurs.

FIG. 5: (a) Frustration dependence of the resistance of the sample with an excitation current of 30 $\mu$A at $T = 4.20$ K. (b) $I$ vs $V$ curves for three different frustrations $f = 8, \frac{7}{2},$ and $\frac{7}{5}$ in a single plot.
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