Destruction of the Kondo effect in a multi-channel Bose-Fermi Kondo model

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Abstract
We consider the SU(N) × SU(kN) generalization of the spin-isotropic Bose-Fermi Kondo model in the limit of large N. There are three fixed points corresponding to a multi-channel non-Fermi liquid phase, a local spin-liquid phase, and a Kondo-destroying quantum critical point (QCP). We show that the QCP has strong similarities with its counterpart in the single-channel model, even though the Kondo phase is very different from the latter. We also discuss the evolution of the dynamical scaling properties away from the QCP.

Key words: Bose-Fermi Kondo model, quantum phase transitions, heavy fermions

Studies of heavy fermion systems within the framework of local quantum criticality have largely been based on the self-consistent Bose-Fermi Kondo (BFK) model [1]. Here, we will consider the spin-isotropic multi-channel BFK model, whose Hamiltonian is

\[ H_{MBFK} = \left( J_K / N \right) \sum_\alpha \mathbf{S} \cdot \mathbf{s}_\alpha + \sum_{p,\sigma,\sigma'} E_p \ c_p^\dagger c_{p,\sigma} c_{p,\sigma'} \]

\[ + \left( g / \sqrt{N} \right) \sum_\sigma \mathbf{S} \cdot \Phi + \sum_p w_p \ \Phi_p^\dagger \cdot \Phi_p. \]  

(1)

We consider fixed input bath spectra: a finite \( \sum_p \delta(E_p - E_p) = N_0 \) for the conduction electrons and a sub-ohmic spectrum for bosons, \( \sum_p \delta(\omega - w_p) \sim \omega^{1-\epsilon} \), for \( 0 < \epsilon < \Lambda \). The spin and channel indices are \( \sigma = 1, \ldots, N \) and \( \alpha = 1, \ldots, M = kN \), respectively, and \( \Phi \equiv \sum_p (\Phi_p^0 + \Phi_p^1) \) contains \( N^2 - 1 \) components.

The study of the large-N limit was first reported in Ref. [2]. Our motivation is to access the physics of destruction of the Kondo effect: for the spin-isotropic case, the only alternative method capable of doing so is the perturbative (\( \epsilon \)-expansion) renormalization group (RG) [3]. The purpose of this short paper is two-fold. We will put the results of Ref. [2] in a more general context, emphasizing the similarities of the quantum critical behavior with its counterpart for the single-channel BFK model. We will also present the results on the dynamical scaling away from the QCP.

As with the multi-channel Kondo model [4], we write the local moment in the antisymmetric representation in terms of pseudo-fermions, \( S_{\alpha\alpha'} = f_{\alpha\alpha'}^0 f_{\alpha\alpha'} - \frac{1}{2} \delta_{\alpha\alpha'} \) and enforce the accompanying constraint, \( \sum_{\alpha=1}^N f_{\alpha\alpha}^0 f_{\alpha\alpha} = \frac{1}{2} N \), by a Lagrange multiplier \( i\lambda \). We then use a dynamical Hubbard-Stratonovich field \( B_\alpha(\tau) \) to represent \( \sum_\sigma f_{\alpha\sigma}^0 c_{\alpha}\sqrt{N} \). The partition function is written as \( Z = Z_0 Z_1 \), where \( Z_0 \) describes the \( c \)- and \( \Phi \)-baths alone; the corresponding free energy, \( F_0 \equiv -(1/\beta) \ln Z_0 \), is of order \( N^2 \). The effective action for \( Z_1 \) is \( S_{\text{eff}} = \int d\tau (L_1 + L_j + L_\phi) \), where

\[ L_j = \left( 1/N \right) \int d\tau' \sum_{\alpha,\alpha'} B_{\alpha} f_{\alpha}(\tau) G_0(\tau' - \tau) B_{\alpha'} f_{\alpha'}(\tau') , \]

\[ L_\phi = \left( 1/N \right) \int d\tau' \sum_{\alpha,\alpha'} f_{\alpha}^0 f_{\alpha}(\tau) \chi_0^{-1}(\tau' - \tau) f_{\alpha'}^0 f_{\alpha'}(\tau') , \]

\[ L_1 = \left( 1/J_K \right) \sum_\alpha B_{\alpha}^1(\tau) B_{\alpha}(\tau) + \sum_\alpha \int d\tau \partial_\tau f_{\alpha}(\tau) \]

\[ + \sum_\alpha i\lambda [f_{\alpha}(\tau) f_{\alpha}(\tau) - 1/2] , \]

(2)
where $G_0 = -(T_0 c_{\alpha \alpha}^{\dagger}(\tau) c_{\alpha \alpha}^{\dagger}(0))_0$ and the bosonic Weiss field $\chi_0^{\dagger} = -g^2 G_0 = -g^2 (T_0 \Phi(\tau) \Phi^{\dagger}(0))_0$. From $S_{\alpha \alpha}$, which is of order $N$, the dynamical saddle-point equations of Ref. [2] follow straightforwardly. In this dynamical large-$N$ limit, the multi-channel Kondo fixed point arises for the model without conduction electrons [5]. Does it capture a non-trivial QCP? The answer is a priori not clear: for instance, the rescaling to the appropriate powers of $1/N$ [cf. Eq. (1)] could have collapsed the transition point to one of the axes in Fig. 1. For small $\epsilon$, we have been able to show [using the RG method of Ref. [3]] that a non-trivial QCP does exist in the large-$N$ limit. The RG flow and the three fixed points are described in Fig. 1. The destruction of the (multi-channel) Kondo effect takes place as the separatrix is reached from left.

The Kondo fixed point of our model describes a multi-channel non-Fermi liquid phase, which is clearly different from the exactly-screened Fermi liquid phase of the single-channel BFK model. Nonetheless, we find that the QCPs of the two models have strong similarities. Consider the anomalous dimension, $\eta$, for the local spin susceptibility $\chi(\tau) \sim 1/|\tau|^\eta$. At the QCP of our model, we find $\eta = \epsilon$ from both the perturbative $\epsilon$-expansion RG (to infinite orders in $\epsilon$) and the saddle-point analysis (for arbitrary $\epsilon$ in the range $0 < \epsilon < 1$). This result is the same as the $\epsilon$-expansion RG result (also to infinite orders) for the QCP of the single-channel BFK model. The situation is somewhat reminiscent of the effects of spin-symmetry breaking in the single-channel BFK model. There, the bosonic fixed point with Ising anisotropy (whose local susceptibility contains a finite Curie constant [3]) is very different from its counterpart with SU(2) symmetry (whose local susceptibility is algebraic [5]). Yet, $\eta$ is the same for the QCP with either Ising anisotropy or SU(2) spin invariance [3]. The above results provide some justification to the usage of the multi-channel BFK model (which is amenable to dynamical large-$N$ approach) as a means to shed light on the quantum critical properties of the single-channel BFK model (for which no dynamical large-$N$ approach is available), even though the latter is what is physically relevant to the magnetically quantum-critical heavy fermion systems.

An important advantage of the dynamical large-$N$ saddle-point analysis lies in its ability to determine the quantum critical properties non-perturbatively in $\epsilon$. At the QCP, the dynamical spin susceptibility shows $\omega/T$ scaling [2], for arbitrary ratio of $\omega/T$ in almost the entire range $\omega, T < T^0_K$, where $T^0_K$ is the bare Kondo scale. What happens when we move away from the QCP? To address this issue, we first consider the behavior of the phases on both sides of the QCP. For the bosonic (local spin-liquid) fixed point, we show in Fig. 2(a) the dynamical spin susceptibility for a finite $g$ but zero Kondo coupling. It clearly displays an $\omega/T$ scaling. Moreover, the exponent is essentially the same as that for the QCP. The latter implies that the local spin-susceptibility should show $\omega/T$ scaling essentially everywhere on the right of the separatrix [cf. Fig. 1]; this is indeed seen in our results (not shown).

For the Kondo fixed point, we show the dynamical local spin susceptibility in Fig. 2(b). Here (as is already known [4]) an $\omega/T$ scaling also obtains; this reflects the multi-channel nature of the Kondo phase and is very different from what happens in the Kondo phase of the single-channel model. The corresponding exponent is different from that for the QCP. It follows that $\omega/T$ scaling should be violated in the region between the separatrix and the $J_K$ axis [cf. Fig. 1], as we indeed find in our saddle-point analysis (not shown).

We thank DFG (SK), NSF Grant No. DMR-0424125, and the Robert A. Welch Foundation for support.

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