Symplectic Fermi Liquid and its realization in cold atomic systems

A. Ramires, arXiv:1705.04080

Aline Ramires
Institute for Theoretical Studies - ETH - Zürich

SPS Meeting - Geneva - August 24th 2017
Outline

- Why Enlarged Symmetries?
  SU(N) and SP(N)

- Why Symplectic Symmetry?
  Time-reversal symmetry

- SU(N) and SP(N) in cold atoms

- SP(N) FL Theory

- Final Remarks
  Experimental realization
  Further connections to theory
Why Enlarged Symmetries?

In the context of strongly correlated systems, Large-N treatments are useful:

1/N is an artificial small parameter
Why Enlarged Symmetries?

In the context of strongly correlated systems, Large-N treatments are useful:

\[ \frac{1}{N} \text{ is an artificial small parameter} \]

Quantum Chromodynamics

\[ SU(3) \Rightarrow SU(N) \]

Barions
(N-body singlets)

G. t'Hooft, Nucl Phys B 71, 461 (1973)
E. Witten, Nucl Phys B 160, 57 (1979)
Why Enlarged Symmetries?

In the context of strongly correlated systems, Large-N treatments are useful: $1/N$ is an artificial small parameter.

Quantum Chromodynamics

$$SU(3) \Rightarrow SU(N)$$

Condensed Matter

$$SU(2) \Rightarrow SU(N)$$

Distinct Large-N approaches capture different kinds of correlations and are appropriate to describe different physical phenomena.

G. t'Hooft, Nucl Phys B 71, 461 (1973)
E. Witten, Nucl Phys B 160, 57 (1979)
Why SP(N) Symmetry?

Consistency requirement:
Enlarged symmetry and time-reversal symmetry (TRS):

\[ [\theta, U] = 0 \]

Why SP(N) Symmetry?

So the most appropriate generalisation to treat condensed matter systems is to SP(N), not SU(N) symmetry.

\[ SU(2) \Rightarrow SP(N) \]
Cold Atomic Systems

Spin-orbit coupling + Hyperfine coupling = Large Hyperfine Spin
Ex.: $^{40}\text{K}$ (f=9/2), $^{87}\text{Sr}$ (f=9/2), $^{41,43}\text{Ca}$ (f=7/2)

\[ H = H_0 + H_I, \]

The non-interacting part:

\[ H_0 = \int \sum_{\alpha=\pm f} \Psi_\alpha^\dagger(r) \left( -\frac{1}{2m} \nabla^2 + V(r) \right) \Psi_\alpha(r), \]
Cold Atomic Systems

Spin-orbit coupling + Hyperfine coupling = Large Hyperfine Spin

Ex.: $^{40}$K ($f=9/2$), $^{87}$Sr ($f=9/2$), $^{41,43}$Ca ($f=7/2$)

\[ H = H_0 + H_I, \]

The non-interacting part:

\[ H_0 = \int \sum_{\alpha=-f}^{f} \Psi_\alpha(r)^\dagger \left( -\frac{1}{2m} \nabla^2 + V(r) \right) \Psi_\alpha(r), \]

At ultra-low temperatures and in the low density limit, we can model interacting atoms with contact interactions.

\[ H_I = \frac{1}{2} \int \sum_{\alpha,\beta,\mu,\nu=-f}^{f} \Psi_\beta(r)^\dagger \Psi_\alpha(r)^\dagger \Gamma_{\alpha\beta;\mu\nu} \Psi_\mu(r) \Psi_\nu(r), \]

\[ \Gamma_{\alpha\beta;\mu\nu} = \sum_{F=0}^{2f} g_F \sum_{M=-F}^{F} \langle f\alpha, f\beta | FM \rangle \langle FM | f\mu, f\nu \rangle. \]

f: Hyperfine Spin (Total angular momentum of the atom)
F: Total angular momentum of the PAIR of atoms which are scattering

* Only the even-F channels contribute to scattering

T. D. Lee, K. Huang and C. N. Yang, Phys. Rev. 106, 1135 (1957)
Cold Atoms and enlarged symmetries

\[ \Gamma_{\alpha\beta;\mu\nu} = \sum_{F=0}^{2f} g_F \sum_{M=-F}^{F} \langle f_{\alpha}, f_{\beta} | F M \rangle \langle F M | f_{\mu}, f_{\nu} \rangle. \]

**SU(N) Symmetry**

- All N colours are equivalent

  ![Colorful diagram](image1)

- Scattering is colour-independent

  ![Reciprocal scattering](image2)

- Number of particles in each colour is preserved

  Realization: Alkaline-Earth atoms

\[ L = 0, \; S = 0 \Rightarrow g_F = g \]

M. A. Cazalilla and A. M. Rey, Rep. Prog. Phys. 77, 124401 (2014)
Cold Atoms and enlarged symmetries

\[\Gamma_{\alpha \beta; \mu \nu} = \sum_{F=0}^{2f} g_F \sum_{M=-F}^{F} \langle f \alpha, f \beta | FM \rangle \langle FM | f \mu, f \nu \rangle.\]

**SU(N) Symmetry**
- All N colours are equivalent
- Scattering is colour-independent
- Number of particles in each colour is preserved
  
  Realization: Alkaline-Earth atoms
  
  \[L = 0, \ S = 0 \Rightarrow g_F = g\]

**SP(N) Symmetry**
- Colours come in pairs: (↑, ↓)
- Scattering can allow for colour transmutation in the F=0 channel
- Colour magnetisation is preserved
  
  \[m_\alpha = n_\alpha - n_{-\alpha}\]

  Realization under the condition
  
  \[g_0 \neq g_F > 0 = g\]

*SP(4) naturally realized for f=3/2

M. A. Cazalilla and A. M. Rey, Rep. Prog. Phys. 77, 124401 (2014)
SP(N) Fermi Liquid Theory

As for the usual Fermi Liquid theory we can write:

$$\delta \epsilon_{\alpha\beta}(k) = \sum_{k'} \sum_{\mu, \nu} f_{\alpha\mu, \beta\nu}(k, k') \delta n_{\nu\mu}(k'),$$

Now we parametrize the interaction function in terms of three FL parameters:

$$f_{\alpha\mu, \beta\nu}(k, k') = f_s(k, k') \delta_{\alpha\beta} \delta_{\mu\nu} + f_a(k, k') \sum_A \Gamma^A_{\alpha\beta} \Gamma^A_{\mu\nu} + f_\epsilon(k, k') \epsilon_{\alpha\mu} \epsilon_{\beta\nu}$$

Symmetric  Anti-symmetric  Symplectic
SP(N) Fermi Liquid Theory

As for the usual Fermi Liquid theory we can write:

\[ \delta \epsilon_{\alpha \beta}(k) = \sum_{k'} \sum_{\mu, \nu} f_{\alpha \mu, \beta \nu}(k, k') \delta n_{\nu \mu}(k'), \]

Now we parametric the interaction function in terms of three FL parameters:

\[ f_{\alpha \mu, \beta \nu}(k, k') = f_s(k, k') \delta_{\alpha \beta} \delta_{\mu \nu} + f_a(k, k') \sum_A \Gamma^A_{\alpha \beta} \Gamma^A_{\mu \nu} + f_\epsilon(k, k') \epsilon_{\alpha \mu} \epsilon_{\beta \nu} \]

from which we are able to extract the renormalisation of measurable quantities:

**Effective Mass**

\[ \frac{m^*}{m} = 1 + N F_s(\theta) \cos \theta, \]

**Inverse Compressibility**

\[ \frac{u'^2}{u^2} = \frac{1 + N F_s(\theta)}{1 + N F_s(\theta) \cos \theta}. \]

**Generalized Susceptibility**

\[ \chi_G = \frac{2 \mu_B^2 \rho^*(E_f)}{1 + F_a(\theta)} \]

where:

\[ F_s(\theta) = \rho^*(E_f) \left(f_s(\theta) + \frac{1}{N} f_\epsilon(\theta)\right), \quad F_a(\theta) = \rho^*(E_f) (f_a(\theta) - f_\epsilon(\theta)) \]

A. Ramires, arXiv:1705.04080
SP(N) Fermi Liquid Behaviour

Given the interaction term with SP(N) symmetry one can determine the explicit form of the FL parameters and the renormalization of the physical quantities:

\[ H_{I}^{SP(N)} = \frac{g}{2} \sum'_{\{k\}} \sum_{\alpha,\beta, \alpha \neq \beta} \Psi_{k_1,\alpha}^{\dagger} \Psi_{k_2,\beta}^{\dagger} \Psi_{k_3,\beta} \Psi_{k_4,\alpha} + \frac{\Delta g}{2N} \sum'_{\{k\}} \sum_{\alpha,\beta} (-1)^{\alpha+\beta} \Psi_{k_1,\alpha}^{\dagger} \Psi_{k_2,\alpha}^{\dagger} \Psi_{k_3,\beta} \Psi_{k_4,\beta}, \]

where: \( \Delta g = g_0 - g \)
SP(N) Fermi Liquid Behaviour

Given the interaction term with SP(N) symmetry one can determine the explicit form of the FL parameters and the renormalization of the physical quantities:

\[
H_{I}^{SP(N)} = \frac{g}{2} \sum' \sum_{\{k\}}^{\alpha,\beta} \Psi_{k_1,\alpha}^{\dagger} \Psi_{k_2,\beta}^{\dagger} \Psi_{k_3,\beta} \Psi_{k_4,\alpha} + \frac{\Delta g}{2N} \sum' \sum_{\{k\}}^{\alpha,\beta} (-1)^{\alpha+\beta} \Psi_{k_1,\alpha}^{\dagger} \Psi_{k_2,-\alpha}^{\dagger} \Psi_{k_3,\beta} \Psi_{k_4,-\beta},
\]

where: \( \Delta g = g_0 - g \)

Effective Mass

\[ \frac{m^*}{m} \]

\[ \begin{align*}
\bar{g} &< \Delta \bar{g} \\
\bar{g} &< \Delta \bar{g} \\
\bar{g} &\geq \Delta \bar{g}
\end{align*} \]

\( \bar{g} = 0.005 \) and \( \Delta \bar{g} = 0.5 \)

\( \bar{g} = 0.1 \) and \( \Delta \bar{g} = 0.5 \)

\( \bar{g} = 0.1 \) and \( \Delta \bar{g} = 0.1 \)

S.-K. Yip et al., Phys. Rev. A 89, 043610 (2014)
A. Ramires, arXiv:1705.04080
SP(N) Fermi Liquid Behaviour

Given the interaction term with SP(N) symmetry one can determine the explicit form of the FL parameters and the renormalization of the physical quantities:

\[ H_{I}^{SP(N)} = \frac{g}{2} \sum_{\{k\}} \sum_{\alpha,\beta,\alpha \neq \beta} \Psi_{k_{1},\alpha}^{\dagger} \Psi_{k_{2},\beta}^{\dagger} \Psi_{k_{3},\beta} \Psi_{k_{4},\alpha} + \frac{\Delta g}{2N} \sum_{\{k\}} \sum_{\alpha,\beta} (-1)^{\alpha+\beta} \Psi_{k_{1},\alpha}^{\dagger} \Psi_{k_{2},-\alpha}^{\dagger} \Psi_{k_{3},\beta} \Psi_{k_{4},-\beta}, \]

where: \( \Delta g = g_{0} - g \)

Effective Mass

Wilson Ratio

This is in contrast with the results for SU(N), since for \( N > 2 \) the second order corrections always take the system away from a magnetic instability.

S.-K. Yip et al., Phys. Rev. A 89, 043610 (2014)
A. Ramires, arXiv:1705.04080
Cold Atoms and enlarged symmetries

1) Not strong dipole-dipole interaction
Cold Atoms and enlarged symmetries

1) Not strong dipole-dipole interaction

2) Stable Elements
Cold Atoms and enlarged symmetries

1) Not strong dipole-dipole interaction
2) Stable Elements
3) Fermionic Isotopes with $f > 1/2$
Cold Atoms and enlarged symmetries

1) Not strong dipole-dipole interaction
2) Stable Elements
3) Fermionic Isotopes with $f > 1/2$

SP(6) Realizes $SU(N)$

SP(8) Realizes $SU(N)$

SP(10)

Dipolar character $m\mu_m^2$

T. Maier, PhD Thesis (2015)
Final Remarks

- Enlarged symmetries can be realised in cold atomic systems;

- Symplectic symmetry requires a smart experimental setup; Candidates $^{40}\text{K}$ ($f=9/2$), $^{10}\text{B}$ ($f=5/2$), $^{73}\text{Ge}$ ($f=7/2$)

- The behaviour of a SP($N$)-FL is characterised and can be accessed experimentally (measuring density profiles or fluctuations);

- Order is more easily realised in SP($N$) systems (enhancement of the Wilson Ratio for $\Delta g<0$);

- Realisation of concepts previously taken only as a theoretical tool:
  - different Large-$N$ schemes (SU($N$) and SP($N$))
  - physics in higher dimensions (accidental isomorphism: Spin (5) $\approx$ SP(4))

A. Ramires, arXiv:1705.04080