Nonequilibrium Phenomena in Liquid Crystals

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The briefest glance through the literature on nonequilibrium phenomena shows that complex fluids, particularly liquid crystals, are often favored for experimental investigations. This might seem surprising in that complex fluids, as befits their name, are difficult materials: experiments require subtle tricks to prepare reproducible samples; theoretical descriptions lead to notoriously messy equations. Given the prejudice of physicists towards simple, well-controlled systems, and given the success that studies of simple fluids have enjoyed, why use complex fluids to study nonequilibrium phenomena? In this paper, I shall offer two answers, one practical and obvious, the other more fundamental and subtle.

The obvious, practical reason is that the dynamics of complex fluids display a variety of interesting “effects” that have been – and will continue to be – exploited for gain. Indeed, the use of liquid crystals in flat-screen displays is perhaps the best-known and most widely exploited of such special effects.

The very first observations of liquid crystals [26,21] noted that although clearly fluid, they were uniaxially birefringent, a property that had been associated only with solids. Even more interestingly, the optical axis could be aligned along an imposed electric or magnetic field; soon after, Mauguin [22] and Grandjean [17] discovered that suitably prepared solid surfaces would also align the optical axes of nematics. In the 1930s, Freedericksz and Tsvetkov [15] and Zöcher [33] put these two effects together in an experiment illustrated in Fig. 1. In the Freedericksz experiment, the surface and external field tend to align the molecules in the nematic phase along different directions. For small electric field, the orientation imposed by the surfaces wins out; but above some threshold $E^*$, the molecules align along the imposed
electric field. The transition is really a supercritical bifurcation from one uniform state to a second, stationary state. This is perhaps the simplest of bifurcations, and one can immediately expect to see all of the universal behavior associated with such transitions. (For example, the maximum deflection of molecules grows as the square root of the distance above threshold, as suggested in Fig. 3a.)

In 1971, Schadt and Helfrich [27] modified the Freedericksz experiment slightly by rotating one plate 90° with respect to the other – thereby twisting the molecules in the sample – and by adding crossed polarizers. In this configuration, as shown in Fig. 4, the transmitted light-intensity curve follows that of the molecular distortion. The configuration was the basis for the first commercially successful liquid crystal display and is still extensively used for small displays where a limited amount of information is to be shown.

In 1982, it was found that if the twist angle is increased past 270°, the bifurcation becomes subcritical [32,28]. (See Fig. 3.) Although the resultant hysteresis causes difficulties for display switching, the limiting case of a 270° twist angle is useful. As Fig. 3c shows, the transmission curves switches more abruptly for such “tricritical” bifurcations than for supercritical bifurcations. (An elementary analysis shows that the intensity now rises as the distance from threshold to the 1/4 power [1].) This “supertwist” display is the dominant one used for the large flat-screen displays found in notebook computers.

I have outlined the history of liquid-crystal displays in some detail because – at least in hindsight – simple ideas from nonequilibrium science are relevant. A good display requires a sharp transition from the “off” state to the “on” state. Thus, it makes sense to use a supercritical transition, as opposed to a design in which the intensity is an analytic function of the control parameter. Changing the bifurcation from supercritical to sub- or tricritical further speeds the switching.

Simple ideas from nonequilibrium science can thus be combined with the special properties of complex fluids (birefringence, electric-field alignment of the optical axis) to create useful devices. The large markets for such devices – well over $3 billion per year for liquid-crystal displays [24] – certainly justifies continued research into understanding and further
cataloguing of analogous special effects. Other special effects I could have cited include drag reduction in turbulent flows by adding small amounts of polymer [13], which has been used to make fire hoses shoot farther and submarines move faster; the giant swelling transition in gels [25], which promises robotic “fingers” that can grasp delicate parts without damage; and electrorheological fluids [18], which are being tested in active automobile suspensions. In this conference, K. Amundson [2], R. Larson [20], and H. R. Brand [7] have discussed other interesting polymer effects. I could go on, but I hope the point is clear.

In addition to the “bestiary” of special effects, there is a second, more fundamental reason to study nonequilibrium phenomena in complex fluids. Nonequilibrium science can loosely be characterized as the systematic exploration of systems as some “stress” is increased. And, simply put, complex fluids are easier to drive out of equilibrium than simple ones.

To understand this remark, consider what I shall call – with no disapproval implied – the “conventional” view of the progression of nonequilibrium phenomena. This view, largely shaped by work in fluid dynamics, is sketched in Fig. 4: unstressed or lightly stressed systems are in a simple “lamellar” state. As the stress is increased, the system undergoes a sequence of bifurcations that results in a time-dependent, chaotic state with limited temporal but full spatial coherence. As one further increases the stress, a second series of transitions – less well understood – progressively destroys the spatial coherence of the system and results in a fully turbulent flow. Well-studied examples that illustrate this progression include Rayleigh-Bénard convection and Taylor vortex flow [11], where “stress” is measured by the Rayleigh and the Reynolds numbers, respectively.

At first glance, the behavior in complex fluids would seem to parallel that of simple fluids. For example, when the Freedericksz experiment is performed on a nematic that tries to align perpendicularly to the applied field, convective motion is observed. (See, for example, W. Zimmerman’s contribution to these proceedings [34].) I want to suggest, though, that there is an important difference between the behavior of complex and simple fluids when driven out of equilibrium: In simple fluids, for reasonable driving stresses, the fluid is always in local – but not global – thermodynamic equilibrium. For simple fluids, this observation has a
number of consequences. If, during an experiment on simple fluids (e.g., Rayleigh-Bénard convection using water), you were to sample the fluid used, you would find its material properties to be the same as in equilibrium. Moreover, at the end of the day, when you switched off the experiment, the fluid would settle down to its equilibrium state. Water that has been churned about at Reynolds numbers of $10^5$ cannot be distinguished from water that has spent all day sitting at rest in a glass. Such observations – trivial as they may be – stand in contrast to the case of complex fluids where, I shall argue, modest driving forces can push a system out of equilibrium on length and time scales comparable to the microscopic scales that characterize the structure of the fluid.

Rather than discuss fluid dynamics, I want to focus on a phenomenon that is equally rich and about which I have personal experience: solidification. As is well-known, freezing fronts are often unstable to shape undulations. (See Fig. 5.) This instability was first analyzed in detail by Mullins and Sekerka [23] and is relevant whenever front growth is controlled by diffusive processes (typically, these are either the diffusion of latent heat or chemical impurities away from the interface). If one freezes more rapidly, however, one finds another regime, the kinetics-limited regime, where front behavior is controlled by local ordering processes at the interface itself. As we shall see, the velocity separating the diffusion- from the kinetics-limited regimes, $v_0$, sets the scale for nonequilibrium phenomena. Fronts moving with $v \ll v_0$ are nearly in equilibrium, while fronts moving with $v \gtrsim v_0$ are strongly out of equilibrium. I shall call the former regime one of slow solidification and the latter regime one of rapid solidification.

To understand why $v_0$ sets the scale for nonequilibrium “stress” in solidification, we need to recall two facts: On the one hand, fronts have a finite thickness $\ell$. This means that an interface moving at velocity $v$ will take a time $t_p = \ell/v$ to pass a given observation point. On the other hand, a front may be viewed as an “ordering wave” that propagates through the fluid. As the front passes through an observation point, fluid molecules that were formerly in a disordered state now have to order. The ordering takes time – call it $t_0$. If the ordering time $t_0 \ll t_p$, then we have slow solidification, since the front has ample time to order. If
$t_0 \gtrsim t_p$, then the front will have already passed through the observation point before the ordering is complete, and one may expect new phenomena to be observed. Equating the two time scales gives the velocity $v_0 \sim \ell/t_0$ described above.

The characteristic solidification speed of a front, $v_0$, is the ratio of a microscopic length, $\ell$, to a microscopic ordering time, $t_0$. For simple fluids, this scale velocity turns out to be roughly the sound speed, and one can imagine that concocting a controlled experiment on fronts moving a kilometer a second is not easy! It turns out, though, that in a complex fluid, $v_0$ can be dramatically reduced, so that controlled experiments become feasible. This is then the second reason that complex fluids are useful in the exploration of nonequilibrium phenomena.

To understand where this reduction of $v_0$ comes from, let us first consider a **simple fluid** – nice examples include the noble elements, such as krypton and xenon – where the molecules (or atoms) are small and spherical and where interactions are short-ranged and isotropic. For such fluids, the interface width is roughly equal to an atomic diameter, so $\ell \approx 10^{-8}$ cm. Since all atoms are identical and spherical, the ordering time is set by the time it takes to remove energy (heat) from the interface. This is given by the heat diffusion time, so that $t_0 \sim \ell^2/D_h \approx 10^{-16}$ cm$/10^{-3}$ cm$^2$/sec $\approx 10^{-13}$ sec. This gives $v_0 \sim D_h/\ell \sim 10^{-3}/10^{-8} \sim 10^{+5}$ cm/sec. (10$^3$ m/sec), which is roughly the velocity of sound in a simple fluid.

Next, consider a **simple alloy**, made of a mixture of two simple fluids. The fundamental length scale is still about an angstrom ($\ell \approx 10^{-8}$ cm), but now the solid phase is formed with an additional constraint: not only must energy be removed from the interface, but also the $A$ and $B$ molecules must be arranged in a precise pattern in the solid phase. In addition, the relative concentration of $B$ and $A$ molecules will differ in the two phases. Thus, freezing an alloy requires rearranging atoms, so that the time scale is set by mass diffusion and not by heat diffusion. Since the mass diffusivity $D \approx 10^{-5}$ cm$^2$/sec is a hundred times smaller than the heat diffusivity, we expect $t_0 \approx 10^{-11}$ sec. and $v_0 \approx 10^{-5}/10^{-8} \approx 10^3$ cm/sec (10 m/sec). Indeed, rapid solidification experiments on metallic alloys do show interesting phenomena.
when fronts move faster than about 10 m/sec. When fronts move faster than about 10 m/sec.

Notice that the microscopic time scale $t_0$ determining $v_0$ is set by the slower of the two relaxational processes (heat and mass diffusion). This is a general feature of complex fluids: the slowest relaxational process sets the microscopic ordering time scale. Notice, too, that although the length and time scales both increase as we go from a simple pure fluid to a simple alloy, the ratio $v_0$ decreases. This, too, is general.

Next, we consider **thermotropic liquid crystals**, which are pure materials made up of rigid, anisotropic molecules. In most cases, the molecules are rod-shaped, but disk-shaped molecules also form liquid-crystal phases [9]. The small dimension measures 5 Å across typically and the large dimension about 30 Å. Motions on the scale of the *large* dimension—the slowest process—set the length scale and the diffusion time scale. Although we once again have a pure fluid, the transition from an isotropic to a nematic state requires orientation alignment, so that one must consider rotational diffusivities in addition to heat diffusion. Using $\ell \approx 10^{-7}$ cm and $D \approx 10^{-7}$ cm$^2$/sec, we obtain $t_0 \approx 10^{-7}$ sec and $v_0 \approx 1$ cm/sec.

**Lyotropic liquid crystal phases** [10] are formed when large amounts of amphiphilic molecules are forced into an aqueous or oily solvent. (Amphiphilic molecules have two parts, one hydrophilic, the other hydrophobic. Examples include soaps and phospholipids, the constituents of biomembranes. See Fig. 3) A large variety of phases—lamellar, cubic, hexagonal, and others—can be observed for different temperatures and amphiphile concentrations. Here, the repeat distances are larger ($\ell \approx 50\) Å). Diffusivities vary greatly, ranging from $10^{-7}$ to $10^{-10}$ cm$^2$/sec. The small values occur because phase transitions can require topology changes in the amphiphile sheets that are the building blocks of the different configurations. Using $D \approx 10^{-8}$ cm$^2$/sec, we expect $t_0 \approx 10^{-6}$ sec and $v_0 \approx 1$ mm/sec.

My final example is the ordering of **diblock copolymers**, which consist of a chain of $A$ monomers joined covalently to a chain of $B$ monomers [4]. (See Fig. 4) At high temperatures, the $A$ and $B$ chains are miscible and form a disordered solution. Below a critical temperature, the $A$ and $B$ chains phase separate. In contrast to a polymer blend,
the phase separation must remain local, since the $A$ and $B$ chains remain joined together. Depending on the relative lengths of $A$ and $B$ chains, the microscopic ordering will vary. The phases that are formed have structures similar to those found in lyotropics. In contrast to liquid crystals, polymers – diblock or normal – are highly flexible molecules. There are new relaxational processes corresponding to the intricate meshing and disentangling of long polymer strands. To estimate this time scale theoretically, we use de Gennes’s “reptation model,” in which the polymer molecule is assumed to be confined to a tube enclosing the molecule [12]. This gives $t_0 \sim \tau N^3$, where $\tau \approx 10^{-11}$ cm is the time scale of the monomer (assumed to be a simple molecule of size $a \sim 10$ Å). Alternatively, $t_0$ may be estimated experimentally from rheological measurements. (I thank Karl Amundson for pointing this out.) The appropriate length scale is the radius of gyration of the molecule, which in a random-walk model is simply $\ell \sim aN^{1/2}$. The diffusion constant then is $D \sim \ell^2/t_0 \sim (a^2/\tau)N^{-2}$ and the scale velocity for front growth is $\ell/t_0 \sim (a/\tau)N^{-5/2}$. Clearly, for large enough $N$, the velocity scale can be as small as one wishes. To get reasonable values, one might want to look at short molecules. For $N = 150$, we estimate $\ell \approx 120$ Å, $t_0 \approx 3 \times 10^{-5}$ sec, $D \approx 4 \times 10^{-8}$ cm$^2$/sec, and $v \approx 300 \mu$/sec.

The scales for the five examples discussed above are collected in Table I, where it is immediately clear that increasing the complexity of the fluid dramatically reduces the velocity scale for rapid solidification. (For lyotropics, we selected a middle value, $D \approx 10^{-8}$ cm$^2$/sec, and for the diblocks, we chose $N = 150$.) Notice that liquid crystals – both thermotropic and lyotropic – have convenient values of $v_0$. Simple fluids and alloys have $v_0$’s so high that fronts cannot be followed during an experiment. Polymers, by contrast, have scales that are painfully slow, except perhaps for very short-chained molecules.

In my own work, I have studied the solidification of thermotropic liquid crystals with Patrick Oswald, Adam Simon, and Albert Libchaber [14]. Our directional solidification apparatus allowed a maximum speed of about 300 µm/sec. This is still somewhat slower than the scale speed of $v_0 \approx 1$ cm/sec, but already interesting phenomena were observed. In particular, we observed that in addition to a velocity threshold above which a flat interface
destabilized, there was a second threshold above which the flat interface reappeared. In fact, the original study of a flat interface had predicted that for large freezing velocities and for large thermal gradients, the front would restabilize. The front restabilization velocity is indirectly linked to $v_0$ and occurs at about 300 $\mu$m/sec for the nematic-isotropic interface of a thermotropic liquid crystal lightly doped with ordinary impurities (i.e., impurities that are themselves simple molecules). A typical stability curve is shown in Fig. 8. These observations were significant in that the restabilization velocity of simple alloys is on the order of meters/sec. We were thus able to explore the entire bifurcation diagram, while previous experiments had probed just a small piece of it. We tested the linear stability analysis in the restabilization regime and also found a number of interesting secondary instabilities in the interior of the bifurcation diagram (parity breaking, traveling waves, breathing modes, etc.). [29,14]

One answer, then, to the question “why use liquid crystals and other complex fluids to study nonequilibrium phenomena” is that they can facilitate the study of instabilities that were already known in the context of simpler fluids. A second answer is that they allow access to the locally nonequilibrium regime. What can one expect to see here? In contrast to the usual nonlinear regime, much less is known, and I can only suggest what is to be learned. If we consider the case of solidification, we see that if we were to freeze a liquid instantaneously, the disorder of the fluid would be quenched in and produce a glassy state. One possibility, then, is that in the kinetics-limited regime, the ordered state will be progressively disrupted as the velocity is increased. The defect density in the ordered phase would then be a smoothly increasing function of the freezing velocity [30].

Another – and to my mind, more interesting – possibility is that the route from the ordered state of near-equilibrium freezing to the glassy state of extremely rapid solidification will be marked by a series of transitions analogous to the phase transitions of equilibrium physics or the bifurcations of weakly nonlinear dynamics. With my colleagues Laurette Tuckerman and Hartmut Löwen, I have studied a simple theoretical model of solidification that displays such behavior [351]. As illustrated in Fig. 9, we have proposed that a rapidly
moving front can split into two separately moving fronts, one dividing the disordered phase (phase 0) from a new metastable phase (phase 1), the second dividing this metastable phase from the ordered, thermodynamically stable phase (phase 2). A necessary condition for the front to split is that the velocity of the leading edge $v_{10}$ exceed that of the trailing edge $v_{21}$. If this condition is met and if reasonable initial conditions favor splitting, then an ever-widening region of phase 1 will be created. Our description of this transition turns out to be mathematically equivalent to surface melting and wetting transitions, so that one may view the appearance of phase 1 as being equivalent to the condensation of a liquid at a solid-vapor interface. Because the mathematics are the same, one expects to observe a pretransitional thickening of the 20 interface (logarithmic or power-law divergence, depending on the nature of the interactions). In addition, one can show that the transition can be continuous, hysteretic, or finite amplitude. An important difference from, say, surface melting, is that the transition need not occur in the vicinity of a special point on the equilibrium phase diagram (for example, the triple point), but can occur even if phase 1 is metastable at all temperatures. We require only that its free energy not greatly exceed that of the stable phase 2 and that it should somehow “resemble” the ordered phase. (For example, one phase might have an FCC lattice, the other a BCC or simple cubic.)

Referring to the list of complex fluids in Table I, one might expect that lyotropic liquid crystals would be good candidates to search for such behavior. Not only is the scale velocity $v_0$ modest, but also lyotropics display a large variety of phases with weak first-order transitions separating them. Such experiments are currently being started in Lyon under Patrick Oswald and at Simon Fraser University, with Nancy Tamblyn and Anand Yethiraj. So far, these transitions have yet to be observed, but the experiments are still preliminary.

In the meantime, poor man’s versions of the splitting transition have been observed in thermotropic liquid crystals. The transition is not between two thermodynamically distinct phases but between two configurations of the nematic phase. In Fig. 10, I show a side view of the meniscus of the nematic-isotropic (NI) interface discussed above. The glass plates are treated to align surface molecules perpendicular to the plates (homeotropic orientation).
There is another, globally incompatible condition at the NI interface itself. The resulting frustration forces a singularity in the nematic phase. (See Fig. 10a.) Topologically, the defect can either be next to the interface or be deep in the nematic phase. (See Fig. 10b.) In the latter situation, the twisted region has a higher elastic energy than the homeotropic region. The defect line will then move back towards the NI interface at a velocity $v_{\text{defect}}$ set by the nematic’s viscosity and elastic constants. However, if the isotropic phase is moving faster than the defect line, the defect cannot catch up and we have the splitting transition described above. In this case, the isotropic is phase 0, the homeotropic phase 2, and the new (planar) orientation of the nematic is the metastable phase 1. If the freezing velocity $v$ is low, we expect to see a homeotropic-isotropic interface (20 interface). For $v > v_{\text{defect}}$, we would expect to see the defect line peel back, creating a widening region of phase 1.

In fact, something slightly different happens. (Fig. 11.) The defect line detaches only when $v$ substantially exceeds $v_{\text{defect}}$ and then only when the interface passes through a dust particle. The interface detaches locally, and a planar region spreads out, creating a triangular shape that is a record of the space-time history of the new domain. Note that in Fig. 11 there are simultaneously 20 and 10 interfaces present. This means that the splitting transition here is hysteretic. Finally, while physicists tend to be intrigued by the triangular shape of the domain, metallurgists are distinctly unimpressed: in the rapid casting of metal alloys, they see these shapes all the time.

Summing up, Fig. 12 shows what the complete spectrum of behavior of a front might be as the driving force is systematically increased. In the near-equilibrium regime, the front is unstable to undulations whose size decreases with velocity. Above, $v_0$, one can expect to see front splitting and, eventually, disordering of the low-temperature phase. For lack of time, my discussion of rapid solidification has been incomplete, and I regret not talking about oscillatory instabilities [19] and solute trapping [3,33]. Moreover, my focus on solidification was purely for personal convenience; someone else could have easily rephrased this talk in terms of the Taylor-Couette experiment, where interesting features – including metastable phase formation – have been observed for complex fluids undergoing shear.
I began my discussion by saying that there were two reasons for using liquid crystals and other complex fluids to study nonequilibrium phenomena. The first was that there are a number of special effects that have great practical application, and I reviewed the history of liquid-crystal displays by way of illustration. The second point was the alteration of microscopic length, time, and velocity scales to values that are convenient experimentally. In the end, these two reasons happily do not separate as neatly as that. The metastable states that can result from strongly nonequilibrium processes are themselves new materials, and they may have useful properties. Indeed, metallurgists during the past 30 years have created thousands of new alloys through rapid solidification, and some of these are widely manufactured. A very old example is martensitic steel, which is significantly harder than the equilibrium austenite steel that is formed at slower cooling rates. Thus, although the more fundamentally minded scientist may wish to focus on strongly nonequilibrium phenomena, the result may be a better understanding of how to make new materials.

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FIGURES

FIG. 1. The Freedericksz transition. Glass plates are treated to align molecules in the nematic phase horizontally. (a) For small fields, the molecules lie flat throughout the sample. (b) Above a critical field strength $E^*$, the molecules tilt to align themselves along the electric field. The maximum distortion is at the midplane of the sample; the boundaries still force the molecules to lie flat.

FIG. 2. The twisted nematic display. The configuration is similar to that of the Freedericksz transition, but the bottom plate is rotated 90°, imposing a twist through the sample. Crossed polarizers are added to top and bottom. (Left.) With no field applied, the plane of polarization follows the nematic molecules adiabatically and light is transmitted through the display. The changing length of molecules represents rotation out of the plane of the illustration. (Right.) With a field applied, the polarization is no longer rotated and the analyzer blocks all light transmission.

FIG. 3. Transmitted light intensity curves for liquid-crystal displays. (a) Ordinary twisted nematic cell. The bifurcation is supercritical; $I(E)$ is continuous and rises as $(E - E^*)^{1/2}$. (b) Twist-angle exceeds 270°. The bifurcation is subcritical, and there is hysteresis in the switching. (c) “Supertwist display,” with a twist angle of 270°. The bifurcation is tricritical, and the intensity increases above threshold as $(E - E^*)^{1/4}$.

FIG. 4. “Conventional” view of nonlinear phenomena as the driving “stress” is increased.

FIG. 5. A moving nematic-isotropic interface goes unstable as the velocity is increased. The interface is on average horizontal, with the isotropic phase on top and the nematic phase on bottom. For $v < v^* = 2.5 \, \mu m/sec$, the front is flat. For $v > v^*$, the front is wavy. The bifurcation is supercritical.

FIG. 6. Sketch of an amphiphilic molecules. When mixed in high concentration with water, molecules such as these order in lyotropic liquid crystalline phases.
FIG. 7. Sketch of a diblock copolymer. These are the polymer equivalent of amphiphilic molecules and form phases of similar structure to lyotropics.

FIG. 8. Linear stability of a flat interface under different combinations of front velocity and the temperature gradient normal to the interface (after Flesselles et al.)

FIG. 9. Schematic illustration of a splitting transition. In (a) and (b), we plot spatial profiles of a non-conserved order parameter that distinguishes the two phases. For example, in a solid-liquid transition, \( q \) could be the amplitude of one of the Fourier amplitudes of a reciprocal lattice vector. It is non-zero in the solid but zero in an isotropic fluid. For low velocities, the front between phases 2 and 0 propagates normally. For high velocities, the 20 front splits into a 21 and 10 fronts. The 10 front moves faster than the 21 front, leaving a widening region of the new metastable phase 1. The dependence of the free energy \( f \) on the order parameter \( q \) is sketched at right.

FIG. 10. Side view of the nematic-isotropic meniscus spanning the gap between two plates. (a) Conflicting boundary conditions at the sidewalls and at the NI interface imply frustration in the nematic, leading to a defect (disclination line, denoted by the large black dot) in the nematic phase (here denoted by “H” for homeotropic orientation). (b) The defect may detach from the interface, creating a region of planar-oriented nematic (denoted “P”).

FIG. 11. Front splitting in a moving nematic-isotropic interface. The isotropic phase (phase 0) is on top. The homeotropically oriented nematic (phase 2) is the on the bottom. A region of metastable planar nematic (phase 1) is present inside the bright triangle. The simultaneous presence of 20 and 10 interfaces indicates that the splitting transition here is hysteretic.

FIG. 12. A “different” view of front behavior.
TABLES

TABLE I. Microscopic scales of simple and complex fluids

|                          | length scale $\ell$ (cm) | diffusion constant $D$ (cm$^2$/sec) | time scale $t_0 = \ell^2/D$ (sec) | velocity scale $v_0 = \ell/t_0$ (cm/sec) |
|--------------------------|--------------------------|--------------------------------------|-----------------------------------|---------------------------------------|
| simple fluid             | $10^{-8}$                | $10^{-3}$                            | $10^{-13}$                        | $10^5$                                |
| binary alloy             | $10^{-8}$                | $10^{-5}$                            | $10^{-11}$                        | $10^3$                                |
| thermotropic liq. cryst. | $10^{-7}$                | $10^{-7}$                            | $10^{-7}$                         | $1$                                   |
| lyotropic liq. cryst.    | $10^{-7}$                | $10^{-8}$                            | $10^{-6}$                         | $10^{-1}$                             |
| diblock copolymer        | $10^{-6}$                | $10^{-8}$                            | $10^{-5}$                         | $10^{-2}$                             |