STAR CLUSTERS, GALAXIES, AND THE FUNDAMENTAL MANIFOLD

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ABSTRACT

We explore whether global observed properties, specifically half-light radii, mean surface brightness, and integrated stellar kinematics, suffice to unambiguously differentiate galaxies from star clusters, which presumably formed differently and lack dark matter halos. We find that star clusters lie on the galaxy scaling relationship referred to as the fundamental manifold (FM), on the extension of a sequence of compact galaxies, and so conclude that there is no simple way to differentiate star clusters from ultracompact galaxies. By extending the validity of the FM over a larger range of parameter space and a wider set of objects, we demonstrate that the physics that constrains the resulting baryon and dark matter distributions in stellar systems is more general than previously appreciated. The generality of the FM implies (1) that the stellar spatial distribution and kinematics of one type of stellar system do not arise solely from a process particular to that set of systems, such as violent relaxation for elliptical galaxies, but are instead the result of an interplay of all processes responsible for the generic settling of baryons in gravitational potential wells, (2) that the physics of how baryons settle is independent of whether the system is embedded within a dark matter halo, and (3) that peculiar initial conditions at formation or stochastic events during evolution do not ultimately disturb the overall regularity of baryonic settling. We also utilize the relatively simple nature of star clusters to relate deviations from the FM to the age of the stellar population and find that stellar population models systematically and significantly overpredict the mass-to-light ratios of old, metal-rich clusters. We present an empirical calibration of stellar population mass-to-light ratios with age and color. Finally, we use the FM to estimate velocity dispersions for the low surface brightness, outer halo clusters that lack such measurements.

Key words: galaxies: clusters general – galaxies: evolution – galaxies: formation – galaxies: general – galaxies: structure

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1. INTRODUCTION

Stellar systems are conventionally divided into two broad categories, stellar clusters and galaxies, depending on whether they lie within a dark matter halo. Yet this scheme faces conceptual and practical difficulties. For example, the dynamical masses of certain “galaxies” (ultracompact dwarf galaxies or UCDs; Mieske et al. 2008) suggest little or no dark matter, as does the modeling of tidal dwarfs (Barnes & Hernquist 1992).

It is the spatial concentration of baryons relative to dark matter within the luminous region that distinguishes UCDs from other galaxies and that makes dark matter easier to measure in spirals than ellipticals. Rather than differentiating stellar systems with a binary classification—dark matter halo versus no halo—which is difficult to measure, we consider baryon versus dark matter concentration, which is easier to observe and allows for a continuum of properties. Furthermore, even for indisputable galaxies like normal ellipticals, it can be difficult to infer dark matter from kinematic measurements within the luminous radius (Cappellari et al. 2007). Our previous work on scaling relations (Zaritsky et al. 2006a, 2006b, 2008) shows that galaxies lie on a progression of star formation efficiencies and baryon concentrations within dark matter halos. The mass-to-light ratio $\Upsilon_e$ within the half-light radius $r_e$ is one gauge of the relative concentration of baryons to dark matter. The large $\Upsilon_e$ of dwarf spheroidals (dSphs) reflect the importance of dark matter within their luminous region, while the lower $\Upsilon_e$ of ellipticals arise from the dominance of the baryons there. One could hypothesize a more extreme case than that for ellipticals, where the baryons are packed so tightly that the dark matter is dynamically negligible within $r_e$. It would be difficult to differentiate such a low $\Upsilon_e$ galaxy, if it were of low mass, from a star cluster, which is devoid of dark matter. If they have any dark matter, UCDs (Mieske et al. 2008) may be examples of galaxies whose spatial and kinematic structure thus resembles that of star clusters. Are there other observables that reveal their presumably distinct formation and evolutionary paths? Remarkably, comparably low-mass systems can have low $\Upsilon_e$ (stellar clusters, UCDs) or high $\Upsilon_e$ (dSphs). What variations in dark halo properties of baryonic processes explain this wide range of baryon to dark matter concentrations?

Such questions are fundamental to understanding the nature of low-mass stellar systems, the basic characteristics of galaxies, and the ways in which baryons can settle into a dark matter halo. Here, we use a new scaling relation (the fundamental manifold, FM, of Zaritsky et al. 2006a, 2008) to explore the question above of whether star clusters can be distinguished from low-mass galaxies with similar $\Upsilon_e$ and the implications for their evolution.

In certain projections of parameter space, globular clusters are distinct from galaxies (Kormendy 1985; Burstein et al. 1997). (Then again, in certain projections, dwarf galaxies are distinct from giant galaxies.) Projections of distributions in multidimensional parameter space can be misleading. It is critical that the defined parameter space (1) has physically meaningful axes, so that one can interpret the results, (2) produces a galaxy distribution as thin as that allowed by observational errors (i.e., that has $N - 1$ dimensionality), thus ensuring that the full dimensionality of the distribution is captured, and (3) exploits the full range of galaxy properties to
maximize the discriminatory power of the observational space. In other words, we seek the minimum set of physically meaningful parameters that captures the majority of the variance in galaxy properties.

The FM, so named in reference to its antecedent, the fundamental plane (FP, Djorgovski & Davis 1987; Dressler et al. 1987), is tightly populated by galaxies of all morphological types and luminosities. By asking where star clusters lie relative to this manifold, we have a means to quantify how distinct star clusters are from galaxies. The relationship is straightforward

$$\log r_e = 2 \log V - \log I_e - \log \Upsilon_e - C,$$

(1)

where \(r_e\) is the half-light radius in kpc, \(V\) represents the internal velocity of the system and is a combination of the velocity dispersion, \(\sigma\), and the rotational velocity, \(v_r\), in km s\(^{-1}\), \(I_e\) is the mean surface brightness within \(r_e\) in solar luminosities per square parsec, \(\Upsilon_e\) is the total mass-to-light ratio of the galaxy within \(r_e\) in solar units, and \(C\) is a normalization constant. Additionally, and what leads to the finding that the systems populate a two-dimensional manifold, there is the empirical determination that \(\Upsilon_e\) is primarily a function of \(V\) and \(I_e\). The functional form of \(\Upsilon_e(V, I_e)\) is not simple, hence the broad term manifold, and while the data used to derive the fitted surface extend over many orders of magnitude along each of the relevant parameters, the function is unconstrained over much of the plausibly accessible parameter space and must be used with caution. For example, the function as described by Zaritsky et al. (2008) was unconstrained in the region of parameter space occupied by star clusters.

Equation (1) originates from the virial theorem, and when combined with the finding that \(\Upsilon_e = \Upsilon_e(V, I_e)\), it limits the region of parameter space occupied by the galaxy population. In other words, the virial theorem alone allows infinitely more possible configurations than those found to populate the FM. As such, the FM places a strong constraint on galaxy formation and evolution models. The simplicity of Equation (1) and its relation to the virial theorem satisfy our first condition above. The empirical finding that the scatter about this relation is small for the most diverse set of galaxies available, and yet captures their variety in kinematics, size, and luminosity, demonstrates that we have included the full dimensionality of the family of galaxies, at least within the observed scatter, and therefore satisfies our second and third conditions.

Before testing whether star clusters lie on the FM, it is important to understand what the relationship implies about galaxies. First, satisfying Equation (1) requires that galaxies be in virial equilibrium and that they have what we will refer to as “structural terms” that have a small degree of scatter. Hereafter, we use the term structure to encompass both the spatial distribution and kinematics of stars and dark matter. These structural terms are the calculated numerical coefficients of the \(v_2\) and \(GM/r\) terms one would obtain when evaluating the kinetic or potential energy terms in the virial theorem. For example, an imaginary galaxy of constant density out to \(r_e\) would have a different total potential energy than a galaxy that consists of the same amount of mass in a ring of radius \(r_e\), although the potential energy of either galaxy can be expressed as being proportional to \(GM/r\). It is impossible to numerically tally the terms in the virial theorem because one never has complete information regarding the phase space distributions of the stars and dark matter in a galaxy. Nevertheless, the low scatter and the lack of systematic differences among different galaxy types when applying Equation (1) indicate that the structural terms, which we have blithely incorporated into the constant term, are not detectably different among galaxies. Second, and perhaps somewhat more surprising, is that \(\Upsilon_e\) is principally determined by \(V\) and \(I_e\), and therefore only modestly related to other factors that might have had an influence on \(\Upsilon_e\), beyond whatever joint influence those factors may have had on \(V\) and \(I_e\). As such, there is a strong connection between the overall structure of a galaxy and the mass-to-light ratio within \(r_e\). It is this second part in particular, the interdependence of \(V, I_e\), and \(\Upsilon_e\), that one might suspect would be qualitatively distinct for star clusters if their formation or evolution mechanism(s) is wholly different from that of galaxies. Consider that dSphs, with their \(<10\ km\ s^{-1}\) velocity dispersions, are quite different in appearance from star clusters with similarly low velocity dispersion.

Our plan is to test whether star clusters can be incorporated easily into the FM construct. If they can be, then they can be described as a low-mass, high-concentration extension of the family of galaxies because the lack of dark matter within \(r_e\) has produced no detectable difference in their structure relative to galaxies. On the other hand, if clusters lie significantly off the FM then their structure is sufficiently different from that of galaxies to suggest differences in their formation and/or evolution. To provide a specific example, one could imagine a system like a star cluster that is instead dominated by dark matter (consider turning some fraction of the stars into dark matter). This system would have the kinematics and size typical of normal star clusters, but a very different surface brightness and mass-to-light ratio. If we then allowed the dark matter to follow a different radial density profile, all the observables would be affected and we would not necessarily expect this system to lie on the same relationship between \(\Upsilon_e, V\), and \(I_e\). The converse does not hold in the sense that systems on the FM do not necessarily have the same evolutionary history.

Although a better understanding of the origin of galaxies and star clusters is the primary goal, universal scaling relationships provide numerous ancillary benefits. For example, we are familiar with the distance determinations using the FP and Tully–Fisher (TF) relationships. Unlike those scaling relations, the FM also provides a measure of the total mass-to-light ratio within \(r_e\). Given that star clusters lack dark matter halos, we will use the FM to constrain the stellar mass-to-light ratio, \(\Upsilon_e\), as a function of cluster parameters such as main-sequence turn-off age and color. We will then compare these measures to values calculated using stellar population models as a test of those models.

In summary, by considering star clusters and UCDs, we test whether the FM formalism extends to the low-mass, stellar-dominated extremum of gravitationally bound stellar systems in Section 3. We find that star clusters do lie on the FM but with apparently greater scatter than observed among the galaxies. We then explore both observational and physical sources of that scatter. We find that there is a correlation between cluster age and deviation from the FM, in the sense expected for variations in \(\Upsilon_e\) due to the evolution of the stellar population, but in quantitative disagreement with the predictions from stellar population models. We explore this behavior and present empirical derivations of the relationships between \(\Upsilon_e,\) age, and \(B - V\) in Section 4. We find that the dominant source of scatter in the current data set is observational, arising from the uncertainties in the cluster velocity dispersions. We use the FM to make predictions for the velocity dispersions of those clusters without measured dispersions. In Section 5, we summarize our findings and discuss the implications. In the Appendix, we
revisit previous claims that are in apparent conflict with our results (Forbes et al. 2008; Tollerud et al. 2010) and provide a resolution.

2. THE DATA

We draw the measurements of structural parameters, velocity dispersions, modeled stellar population mass-to-light ratios ($\Upsilon_{\ast,max}$), ages, colors ($B - V$), and metallicities ([Fe/H]) from the compilation by McLaughlin & van der Marel (2005) of Local Group star clusters. That sample consists of 153 spatially resolved clusters from a range of nearby galaxies. Within that sample, there is a subset of 57 clusters for which they have also compiled velocity dispersion measurements. As such, it includes clusters of all ages, a wide range of metallicities, and the full range of environments found in the Local Group.

The data for stellar systems ranging from dSphs to galaxy cluster spheroids (CSphs to distinguish them from stellar clusters throughout, see Zaritsky et al. 2008) are drawn from Zaritsky et al. (2008) and for ultracompact dwarfs from Mieske et al. (2008), selecting only objects with estimated masses $> 5 \times 10^6 M_{\odot}$ to distinguish the UCDs from the stellar clusters in their sample.

3. STAR CLUSTERS AND THE MANIFOLD

To define the manifold, we use only the spheroidal galaxies and CSphs from Zaritsky et al. (2008) as our comparison sample to minimize potential problems arising from composite stellar populations and to limit ourselves to systems supported by velocity dispersion rather than rotation. In the vast majority of local galaxies, either $v_\alpha$ or $\sigma$ dominates, and therefore we have not yet truly tested whether $V^2 \equiv 0.5v^2_\alpha + \sigma^2$ is preferable to other parameterizations, such as $V^2 \equiv \max[0.5v^2_\alpha, \sigma^2]$, or whether 0.5 is the optimal coefficient for the $v_\alpha$ term (this topic will be addressed in another paper, but values of 0.3–0.5 are plausible in idealized models; Weiner et al. 2006) and such parameterizations have been used elsewhere (Burstein et al. 1997; Kassin et al. 2007; Covington et al. 2010). Here, we avoid questions regarding the incorporation of rotation supported systems in the FM formalism by only analyzing systems in which $\sigma$ dominates. The low relative rotation of Milky Way (MW) clusters is inferred from their low ellipticities (Freeman & Norris 1981). This argument is weaker for other cluster populations, such as those in the Magellanic Clouds, where larger ellipticities are sometimes found. However, even in the Clouds, the cluster ellipticities are generally less than 0.5 (Hill & Zaritsky 2006), similar to the ellipticity of one of the most extreme MW clusters, $\omega$ Cen, which has $v_\alpha/\sigma \sim 0.5$ (Reijns et al. 2006). Therefore, for the remainder of this study, we adopt $V \equiv \sigma$. Using only spheroidal galaxies further simplifies our work by removing systems with ongoing or recent star formation, for which $\Upsilon_\ast$ may have a dramatic dependence on mean stellar age.

The final relevant technical issue with the galaxy sample is the heterogeneous nature of the source samples and how one should compare it to the cluster sample. In particular, the cluster sample contains $V$-band photometry, while the galaxy sample contains a range of photometry from $V$ to $I$ bands. Equation (1) is band independent in that $I_\ast$ and $\Upsilon_\ast$ for a particular object need to be in the same band, but that band can be different for different objects. However, the manifold that we fit to describe $\Upsilon_\ast$, $\Upsilon_{\ast,\text{fit}}$, is band-specific and has been derived using the heterogeneous galaxy sample, corrected to the $I$ band. The values of $I_\ast$ used to determine $\Upsilon_{\ast,\text{fit}}$ are in solar units, so the filter dependence arises from the difference in colors between the galaxy and the Sun when converting $L_\ast$ to $I_\ast$. In effect, we are assuming solar colors for the galaxies, although any uniform color difference among all galaxies is absorbed into the calibration of $C$ in Equation (1). The remaining concerns are color differences that vary across the sample and band-related differences in $r_\ast$. If these exist, then they are inappropriately incorporated into our fit for $\Upsilon_\ast$. One clear path to progress is obtaining a single, uniform data set for the further study of the FM. However, in the interim, we proceed with the motley sample available and caution that for systems with colors much different than solar colors there will be a yet unknown color correction to $\Upsilon_{\ast,\text{fit}}$. The empirically confirmed small scatter among galaxies, even when including star-forming galaxies (Zaritsky et al. 2008), suggests that this effect will be negligible for a sample consisting only of spheroidal galaxies, but could be a factor among the star clusters, which span a wider age range.

We reevaluate $\Upsilon_{\ast,\text{fit}}$ using only those galaxies, the spheroids, from Zaritsky et al. (2008) that are $\sigma$ dominated, including CSphs. In doing so, we have also reevaluated the constant $C$ in Equation (1) using the Cappellari et al. (2007) sample, for which they have full dynamical modeling fit to two-dimensional spectral data for spheroidal galaxies, and find $C = 0.75$. In Figure 1, we show where the 57 globular

Figure 1. Edge-on view of the FM for spheroidal stellar systems, including star clusters and ultracompact dwarf galaxies (UCDs). We are extrapolating the fit from the region populated by systems with $log r_\ast > 1$ (i.e., excluding both UCDs and stellar clusters). We illustrate the statistical uncertainties in that extrapolation with dots representing projections of the surface where individual fit coefficients are altered by 1σ. Spheroidal stellar systems from Zaritsky et al. (2008) are plotted as filled black circles, clusters with tabulated $\sigma$ (McLaughlin & van der Marel 2005, and references therein) are plotted as filled red circles, and UCDs with $M > 5 \times 10^7 M_{\odot}$ from Mieske et al. (2008) are plotted as filled blue squares. The extrapolated fit, within its 1σ uncertainties, passes through the majority of the stellar clusters and UCDs, although the scatter among the clusters is greater than among the galaxies. Uncertainties are not plotted, but the dominant uncertainty in the cluster population is discussed in Section 3.2 and the uncertainties in the comparison galaxy population are comparable to the scatter (Zaritsky et al. 2008). The clusters that deviate well below the fit are the youngest clusters.

(A color version of this figure is available in the online journal.)
clusters with measured velocity dispersions and the UCDs with $M > 5 \times 10^6 M_\odot$ from Mieske et al. (2008) fall when using this fitting function, which was not derived using either the star clusters or UCDs. In addition, we show with small points the range of possible locations in this projection of the extrapolated relationship covered by random, $\leq 1\sigma$ deviations of each of the fitted coefficients. The star clusters and UCDs lie within the extrapolated version of $Y_{e,fit}$ given the fitting uncertainties. Because the extrapolated fitting function works well in the mean for stellar clusters, we infer that clusters are indeed the low-mass, limiting case of high-stellar mass-fraction systems and refit $Y_{e,fit}$ using the cluster and UCD data (Figure 2). The resulting fit is

$$Y_{e,fit} = 1.49 - 0.32 \log V - 0.83 \log I_e + 0.24 \log^2 V + 0.12 \log^2 I_e - 0.02 \log V \log I_e$$

and is used hereafter.

Projections of this function along the various axes (Figure 3) illustrate how difficult it is to see the multidimensional nature of the distribution and how certain populations of objects appear to stand out as different despite populating the same surface. Because there is no physical motivation for this particular form, the addition of new data, particularly those that probe a new part of parameter space, could require changing the functional form. Because the extrapolated fitting function works well in the mean for stellar clusters, we infer that clusters are indeed the low-mass, limiting case of high-stellar mass-fraction systems and refit $Y_{e,fit}$ using the cluster and UCD data (Figure 2). The resulting fit is $Y_{e,fit}$. This type of effect is often refereed to as a “second parameter” question in correlations, although in our case it would be a third parameter. If there is such a parameter, it has been inadvertently neglected in our formulation of the fitting function for $Y_{e,fit}$. As such, $Y_e$ as evaluated using Equation (1), $Y_{e,FM}$, should correlate with this yet unidentified parameter (as it does with $V$ and $I_e$).

In closing, we note that there are two studies (Forbes et al. 2008; Tollerud et al. 2010) that appear to conflict with our claim that stellar clusters satisfy galaxy scaling relations. We describe our interpretation of those studies in the Appendix and conclude that neither study is truly in conflict with the results presented here.

3.1. Observational Source of Apparent Scatter: Velocity Dispersions

For these low velocity dispersion systems, the uncertainties in $\sigma$ are fractionally the largest of all the input parameters. We now explore the level of scatter about the FM expected simply due to the quoted uncertainties. We use the 51 old (log(age) $> 10$ Gyr) clusters with published velocity dispersion measurements and associated uncertainties to avoid any possible effect of age on the scatter. We place each cluster exactly on the FM by adopting a value for $Y_e$ that satisfies Equation (1). We then draw a thousand cluster samples using Gaussian distributed errors on the velocity dispersion, using the corresponding value for the uncertainty in $\sigma$ for each particular cluster. In Figure 4, we plot the distribution of the simulated scatter for the thousand trials and the observed scatter for comparison. The observed scatter is somewhat larger than the typical simulated one, but we cannot exclude the possibility that the observed scatter is directly due to the $\sigma$ uncertainties with greater than 80% confidence. We conclude that the observed scatter in Figure 1 is principally due to the uncertainties in $V$, but cannot conclude that no physical present in galaxies, proportionally larger uncertainties in at least one of the observed parameters for the clusters, and the extrapolation of a fitted function beyond the parameter range over which it is constrained. Among the observed parameters, the likely source of scatter is $V$, which is more poorly measured in these low-velocity systems than in normal galaxies, and so we will begin our discussion by assessing the impact of the quoted uncertainties in $V$. However, of greater interest is the possibility that some of the scatter is driven by a parameter that has not been included in our expression for $Y_{e,fit}$. It is difficult in projection to determine whether a single surface fits all the data and, if not, which population to fit. We find that a single surface does fit all the objects.
sequences, to correct $\Upsilon$ van der Marel (2005), obtained from resolved stellar main subsequently use this fit and the cluster ages from McLaughlin & cluster a value of $\Upsilon$ because in the following three subsections.

There remain, however, at least two open questions. First, the scatter remains large for both young and old clusters. Although the observational scatter is larger than the typical simulated one, we cannot confidently exclude the possibility that it is only the result of the fractionally larger uncertainties in $\sigma_{\text{obs}}$ for these low $\sigma$ systems.

3.2. Physical Sources of Apparent Scatter: Age

There is at least one physical cause of scatter that must play a role, that we can quantify easily, and that we can model—variations in the stellar population mass-to-light ratio, $\Upsilon_\ast$. In Figure 5, we demonstrate that the observed deviations from the FM for star clusters are, as expected, different for the young and old stellar systems. We fit a linear relation in log(age) to describe the residual $(0.272 \log(\text{age}) - 10.11) + 0.044$ and subsequently use this fit and the cluster ages from McLaughlin & van der Marel (2005), obtained from resolved stellar main sequences, to correct $\Upsilon_{r,\text{fit}}$ for this systematic age deviation. Importantly, the residual is small for the older clusters, for which the calibration to galaxies is expected to be most appropriate. There remain, however, at least two open questions. First, the scatter remains large for both young and old clusters (although, at least for the old clusters, we demonstrated above that observational uncertainties can account for the scatter). Second, the mean trend only testifies to some age-related effect, not that this effect must be related to $\Upsilon_\ast$. There are three reasons that the trend is likely due to the evolving stellar population rather than due to age-dependent dynamical effects: (1) the stellar population must, to some degree, evolve and affect $\Upsilon_\ast$ because $\Upsilon_\ast$ is related to $\Upsilon_\ast$, (2) the fitting function works best (i.e., the residual is $\sim$zero) for the oldest clusters, for which the calibration for spheroidal galaxies and their similar stellar populations should be most appropriate, and (3) dynamical effects, such as those involving two-body relaxation, should trend in the opposite direction with age, becoming more, rather than less, important in the older clusters. Unfortunately, further discussion is limited by the lack of clusters with measured $\sigma$ across our entire age range, $8 < \log(\text{age[yr]}) < 10$. Although the entire McLaughlin & van der Marel (2005) cluster sample does contain clusters in this age range, we need an estimate of $\sigma$ to include them in our discussion. We next describe how we use the FM to obtain such estimates.

Because the right-hand side of Equation (1) can be written as only a function of $V$ and $I_e$ after adopting $\Upsilon_{r,\text{fit}}$, we use the tabulated values of $I_e$ and $r_e$ for the clusters to solve for $V$, which we equate with $\sigma$ (see Section 3). The drawback of this approach is that we have now assumed, on the basis of Figure 1, that both Equation (1) and $\Upsilon_{r,\text{fit}}$ are applicable to clusters. We account for the trend of $\Upsilon_{r,\text{fit}}$ with age shown in Figure 5 using the linear fit, and then we solve the equation for $V$. We test this procedure on the subset of 57 clusters with measured $\sigma$ in Figure 6. The agreement is encouraging and we proceed to estimate $\sigma$ for all 153 clusters. The agreement within the uncertainties of the calculated and observed $\sigma$ in Figure 6 demonstrates that much of the scatter among clusters on the FM must come from the uncertainties in their respective $\sigma$, as we had surmised on other grounds. In other words, we have not created a situation where we absorb errors in other quantities or intrinsic scatter into our derived values of $\sigma$. In all subsequent plots, we use the inferred $\sigma$, but highlight those clusters with measured $\sigma$ using large open circles to illustrate the degree to which any trends are driven by those clusters with unconfirmed velocity dispersion estimates.

An interesting subset of clusters are those of lower surface brightness in the outskirts of the MW. Because of their distance and naturally low $\sigma$, they tend to not have $\sigma$ measurements of high relative precision. Outside of the compilation of cluster properties by McLaughlin & van der Marel (2005) that we have used exclusively, there are measurements of $\sigma$ for Pal 5 (Dehnen et al. 2004) and Pal 14 (Jordi et al. 2009). Using these values, we find that even these clusters fall on the FM relation. As a target for future observers, we present the calculated values of $\sigma$ necessary to place similarly unusual clusters on the FM (Table 1). These low dispersions illustrate why high-precision
spectroscopy and a detailed knowledge of the binary populations (see Dehnen et al. 2004) are needed to confirm the calculated

\[ \Upsilon \]

and to resolve the question of whether such systems lie on the FM.

We now return to investigate the inferred drift in \( \Upsilon_e \) versus age by including all star clusters with or without measured \( \sigma \). We plot the total mass-to-light ratio within \( r_e \) evaluated using Equation (1), \( \Upsilon_{e,FM} \) versus age (Figure 7). For those clusters where we estimate \( \sigma \), the values of \( \Upsilon_e \) are not independent of the measured age, and so trends should be viewed with caution compared to those obtained from clusters with measured \( \sigma \). Because stellar clusters are dark matter free, \( \Upsilon_e = \Upsilon_* \). We confirm that the value of \( \Upsilon_e \) necessary to place clusters on the FM is a simple function of age and, as seen in Figure 7, that the sense of the trend (higher values of \( \Upsilon_e \) for older ages) is as expected and matches the behavior determined from only the clusters with measured \( \sigma \). Because all but the lowest four of the clusters in Figure 1 are old, the apparent scatter is not the result of age differences within the sample.

### 3.3. Physical Sources of Apparent Scatter: Structural Terms

Another potential source of scatter is variance related to the cluster structural properties. Star clusters can evolve dynamically due to their relatively short dynamical times (in comparison to the dynamical times for galaxies) and therefore should have larger variations in their structural terms, which reflect both the stellar kinematics and spatial distribution. We examine whether structural differences among the clusters have been incorrectly excluded from Equation (1), or \( \Upsilon_{e,FM} \), by looking for correlations between \( \Upsilon_{e,FM} \) and measurements of the structural state of the cluster. In a similar approach, McLaughlin (2000) found some evidence for correlations between the structural state of Galactic clusters and their location relative to the FP. Figure 8 shows the relationships between \( \Upsilon_{e,FM} \) and central potential depth, concentration, and galactocentric distance. Whatever trends there may be in the lower panels, they are clearly of much lower magnitude than that with age, which is evident as the bimodal distribution in the upper panels of the figure. We conclude that none of the measures of cluster structure that are currently available suggest a relationship between deviations from the FM and the dynamical state of the cluster beyond any directly related to age. Because dynamical state depends on more than just age, we conclude that stellar population differences, which do depend only on age, are the primary driver of the trends we observe.

#### 3.4. Physical Sources of Apparent Scatter: Metallicity

Next, we search for a dependence of \( \Upsilon_{e,FM} \) on cluster metallicity (Figure 9). There is no evident dependence of \( \Upsilon_{e,FM} \) on metallicity below (Fe/H) < −1 in the upper panel of the figure. There may be an effect for higher metallicities, although that is where age differences become important as well. In the lower panel, we apply the age correction derived from Figure 5 and find that there is little if any residual metallicity dependence. Again, the primary driver of the trends appears to be age.
3.5. Physical Sources of Apparent Scatter: Conclusions

On the basis of the correlation between scatter and age, and the agreement between the inferred sense of the effect and the qualitative expectations from stellar population models, we conclude that age is the dominant physical source of scatter. However, our data do not allow us to eliminate all possible structural differences that might play a role, nor do they statistically demonstrate the superiority of age as a driving parameter over the other parameters now available. The peculiarities of the sample, only six clusters with measured $\sigma$ at age $\sim 10^8$ yr and the remainder all at old ages, preclude application of statistical tests such as principal component analysis or partial correlation coefficients as a means to identify a dominant physical effect. However, this shortcoming can be remedied with reasonable observational effort.

4. ESTIMATING $\Upsilon_*$

4.1. Comparing $\Upsilon_{*,FM}$ to Stellar Population Models

The variation in $\Upsilon_*$, which for stellar clusters is equivalent to $\Upsilon_*$ versus age has a natural explanation in terms of stellar population evolution. In Figure 10, we compare estimates of $\Upsilon_*$ based on the FM, $\Upsilon_{*,FM}$, to those based on the model values of $\Upsilon_*$, $\Upsilon_{*,mod}$, tabulated by McLaughlin & van der Marel (2005) using their preferred Bruzual–Charlot models with a Chabrier initial mass function (IMF). The sense of the behavior of $\Upsilon_*$ with time is consistent for both $\Upsilon_{*,FM}$ and $\Upsilon_{*,mod}$ suggesting that we are indeed seeing the effects of stellar evolution on $\Upsilon_*$.

The existence of a significant difference between $\Upsilon_{*,mod}$ and $\Upsilon_{*,FM}$, particularly at old ages, is independent of the particular model or IMF, among those currently in general use. We compare $\Upsilon_{*,FM}$ to the full range of $\Upsilon_{*,mod}$ presented by McLaughlin & van der Marel (2005) in Figure 11. We find similar results after comparing to 2007 Bruzual–Charlot models, which are not included in McLaughlin & van der Marel (2005). The magnitude of the differences between $\Upsilon_{*,mod}$ and $\Upsilon_{*,FM}$ is similar to that of the differences among the stellar population models themselves. It is therefore possible that there are problems with the stellar population models at this level. Conroy et al. (2008) explore the uncertainty in stellar population models, the bulk of which comes from modeling intermediate and low-mass stars near the end of their lives where they become far more luminous but for short periods of time. Tonini et al. (2009) show that including the thermally pulsating AGB phase can increase the emission from an intermediate age galaxy by one magnitude in the K band. The level of the discrepancy suggested by our FM analysis is consistent with these potential problems in $\Upsilon_{*,mod}$. Even at the young (low $\Upsilon_*$) end of things, there are clear differences among the models, with those using the Salpeter IMF producing results more in line with those from the FM. Further discussion of the details of stellar population models is beyond the scope of this paper, but the FM provides a new constraint. The FM-derived values of $\Upsilon_*$ suggest that all currently popular stellar population model variants overpredict the stellar mass-to-light ratio of old populations (see Mieske et al. 2008, for an alternative approach that results in the same conclusion).
Figure 10. Direct comparison of two ways to evaluate \( \Upsilon_e \) as a function of age. We plot the results using the stellar mass-to-light ratio from stellar population models, \( \Upsilon^{\text{mod}} \) (small open circles), and from the FM using Equation (1) and \( \sigma_{\text{FM}} \). \( \Upsilon_{e,\text{FM}} \) (filled small circles). Large open circles highlight clusters with \( \sigma_{\text{OBS}} \). The ages for the values of \( \Upsilon_{e,\text{FM}} \) have been arbitrarily shifted by 0.2 for clarity. The two methods of estimating \( \Upsilon_e \) qualitatively track each other with age, but differ in the overall amplitude of the change from the youngest to oldest clusters.

Figure 11. Comparison of \( \Upsilon^{\text{mod}} \) and \( \Upsilon_{e,\text{FM}} \) for different stellar population models. Panels represent (a) Bruzual–Charlot models (Bruzual & Charlot 2003) with Chabrier IMF (Chabrier 2003); (b) Bruzual–Charlot models with Salpeter IMF; (c) PÉGASE models (Fioc & Rocca-Volmerange 1997) with Chabrier IMF; (d) PÉGASE models with Salpeter IMF; (e) PÉGASE models with the Chabrier globular cluster IMF (Charbrier 2003); and (f) PÉGASE models with the Kroupa et al. (1993) IMF. The sense of the deviation for the oldest ages (highest values of \( \Upsilon_e \)) is consistent among all varieties of stellar models. The stellar models consistently overpredict \( \Upsilon_e \) relative to the FM estimates for these older clusters.

Stellar population models that provide accurate stellar mass-to-light ratios are manifestly important for a variety of uses. However, star clusters on the FM provide a new way to estimate \( \Upsilon_e \) for large sets of galaxies. Currently, the FM zero point (\( C \) in Equation (1)) is calibrated using the results for \( \Upsilon_e \) from detailed dynamical studies of several tens of early-type galaxies (Cappellari et al. 2007). If we knew, from stellar models, the values of \( \Upsilon^{\text{mod}} \), and hence \( \Upsilon_{e,\text{FM}} \), for star clusters, we could use them instead to calibrate the FM. Once the FM is accurately calibrated, one can use it to solve for \( \Upsilon_e \) for any galaxy with measured \( V, r_e \), and \( I_e \).

4.2. An Independent Prediction of \( \Upsilon_e \)

We apply the FM to derive empirical relationships between parameters such as main-sequence turn-off age and color, and \( \Upsilon_e \). Figure 12 shows our crude fits to the age-\( \Upsilon_e \) (Table 2) and (\( B-V \))-\( \Upsilon_e \) (Table 3; the fit is valid for \( B-V \leq 0.75 \), for \( B-V > 0.75 \) use \( \Upsilon_e = 1.13 \)) relationships. Neither one-dimensional relationship appears satisfactory for the full range of clusters, and certain populations, such as young and metal-poor populations, are completely unconstrained. More sophisticated and well-constrained fits will be possible once more data become available for intermediate age clusters, providing an alternative to \( \Upsilon_e \) estimates that depend on stellar population models.

5. SUMMARY

We find the following:

1. On average, star clusters fall along the extrapolation of the fundamental manifold (FM) defined by spheroidal galaxies. Their larger scatter is consistent with the proportionally
larger uncertainties in their velocity dispersion measurements. Even extreme clusters such as Pal 5 and 14 fall on the relationship when precise velocity dispersions are available.

2. Individual clusters are offset from the FM in proportion to their age: older clusters fall on the manifold and younger clusters do not. Unfortunately, the sample of clusters with measured velocity dispersions does not include many clusters with log(age [yr]) < 10, so we apply the FM relationship to infer \( \Sigma \) and \( \Sigma \) simultaneously for the remainder. This procedure works faithfully for those with measured \( \sigma \).

3. Aside from the trend with age, deviations from the FM do not correlate measurably with metallicity, dynamical state, or galactocentric distance.

4. The sense of the evolution of the mass-to-light ratio with age is as predicted by stellar population synthesis models, but currently popular models fail to reproduce the estimates from the FM (see also Mieske et al. 2008). Specifically, the models overpredict the mass-to-light ratio of old, metal-rich populations. We provide empirical formulas with which to estimate the stellar mass-to-light ratios of certain cluster populations derived from the FM-estimated mass-to-light ratios.

All stellar systems that we have examined in detail so far—brightest cluster galaxies (Zaritsky et al. 2006a), various galaxy types and luminosity classes (Zaritsky et al. 2008; Mieske et al. 2008), and now star clusters—satisfy a simple, three-parameter (two-dimensional, but not planar) scaling relation. For star clusters, deviations from the scaling law are age dependent, which is qualitatively consistent with the expectation for passive stellar evolution of the population. Quantitatively, however, this description fails. We suggest, but have not definitively demonstrated, that this disagreement arises from systematic errors in the stellar population models that lead to an overestimation of the \( B - V \) mass-to-light ratio of old stellar populations. The sense and magnitude of the effect are consistent with conclusions reached by others investigating stellar evolution models in detail (Tonini et al. 2009; Maraston et al. 2006) and at the very least suggest that a factor of two uncertainty remains in modeling stellar populations. As stressed by others (see Maraston et al. 2006; Conroy et al. 2008), this uncertainty propagates in complicated ways, particularly for composite populations of unknown age distributions, in analyses of galaxy evolution.

We conclude that there are no evident structural differences, in either stellar spatial distribution or kinematics, between galaxies and star clusters beyond those of scale and those captured in the mass-to-light ratios within \( r_e \). Star clusters appear to be baryon-dominated versions of highly compact, low-mass galaxies. As such, we are not surprised by the difficulties encountered by others in distinguishing clusters from ultracompact galaxies. If there are wholesale differences between the formation process and/or evolution of star clusters and galaxies, they do not manifest themselves directly in the gross physical properties. We conclude that even if significant differences exist between the populations, due to the existence or lack of a dark matter halo, simple measurements will not reveal them. Projections of parameter space in which these populations are separate are deceptive and should not be taken to indicate that these objects are discrete families of gravitationally bound stellar systems.

Although this lack of evident distinction is a setback for attempts to differentiate the populations and probe potential differences in their formation and evolution, it is an advantage to those efforts to build a simple set of models for galaxies. Star clusters can provide an anchor for models of galaxies, empirically defining the stellar mass-to-light ratios or, alternatively, the zero point of the FM. The latter would enable us to use the FM to determine the mass-to-light ratio within \( r_e \), to a level of precision comparable to that obtained with detailed dynamical modeling or strong gravitational lensing modeling, for any galaxy with known \( V \), \( r_e \), and \( I_e \).

We return to the question of the defining characteristics of a galaxy. Among the stellar systems that we study, we have not identified a criterion that we consider a discriminating feature. Despite the popularity of the dark matter halo criterion, there exist “galaxies” that may not have a dark matter halo (tidal dwarf galaxies; Barnes & Hernquist 1992) or, at least, a dominant dark matter halo (ultracompact dwarf galaxies; Mieske et al. 2008). In fact, from state-of-the-art optical data alone, it is difficult to demonstrate that even giant ellipticals contain dark matter (Cappellari et al. 2006). An alternative, and more easily observable, classification is the relative concentration of luminous baryons to dark matter. In such a scheme, systems composed solely of baryons are not distinct from those with dark halos and whose observed kinematics are dominated by baryons. While this scheme makes it harder to connect observations with dark matter simulations, the reality is that, for systems whose baryons are extremely concentrated, we cannot determine whether a halo exists or what its properties might be.

Our primary empirical result is that the existing description of the FM (Zaritsky et al. 2008) is valid over an even larger parameter range than previously demonstrated and that now includes stellar clusters. The existence of an FM for the entire family of stellar systems implies (1) that the objects are virialized and (2) that there is limited variation in the overall structure. While the former is unsurprising, the latter implicitly places strong limits on how the systems form and how the luminous baryons settle. By extending the validity of the FM, or any scaling relation, over a larger range of parameter space and a wider set of objects, we are demonstrating that the physics that constrains the resulting baryon and dark matter distributions in stellar systems is more general than previously thought. In other words, whereas one might not have been surprised by the implication of the FP that all ellipticals have related structural properties, it is certainly a puzzle to find that spirals, dSphs, and now stellar clusters do too.

While the structure of one stellar system is not a simple scaling of another, there exists a transform function that relates the properties of stellar systems. Specifically, we have presented a fitting function for \( \Sigma \), that is valid across all stellar systems. The generality of this transform function implies that (1) the structure of one type of stellar system is not a distinct result of a process particular to that set of systems, such as violent
relaxation for elliptical galaxies, but is instead the result of an interplay of all processes responsible for the generic setting of baryons in gravitational potential wells, (2) the existence of the transform function for $\Upsilon_e$, and hence the nature of baryon settling, is independent of whether the system is embedded within a dark matter halo, and (3) differences in initial conditions or stochastic events during the lifetimes of stellar systems do not invalidate the transform function. By including stellar clusters, which we presume are devoid of dark matter, we demonstrate that the nature of the transform function for $\Upsilon_e$ depends more on the baryon physics than the source of the gravitational potential. The presence or absence of dark matter imprints no clear feature in the structure or kinematics of stellar clusters versus galaxies. By including stellar clusters, which we presume formed mostly in single, short-lived, dramatic fashion, we demonstrate that the nature of the transform function for $\Upsilon_e$ is not altered radically by differences in formation history. The extension of the FM over a larger parameter space is in reality the extension of the statement that physical processes rather than initial conditions or subsequent stochastic events dominate the final outcome in the formation of stellar systems.

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APPENDIX

Various efforts have been made to establish an FP for globular clusters (Djorgovski 1995; McLaughlin 2000; Barmby et al. 2007; Paquato & Bertin 2008). However, by necessity, this plane is not the same as that for ellipticals, whose FP requires $\Upsilon_e \propto \sigma^\alpha I^\beta$. $\Upsilon_e$ for intermediate mass systems is already comparable to $\Upsilon_e$ and cannot physically decrease further for systems with comparable surface brightnesses but lower velocity dispersions, like globular clusters (Kormendy 1985).

A recent study discussed a unification of all “spheroidal” systems from globular clusters through elliptical galaxies (Forbes et al. 2008). While prevailing generally, they found that dSphs did not fit their general formalism and that the FM (Zaritsky et al. 2008) was not valid down to globular clusters. We disagree with the latter statement and so we investigate the nature of the discrepancy.

We use their data, which they made publicly available with their paper, to remove any sample differences. Their claim regarding the FM is based on the nature of the empirical scaling relation for $\Upsilon_e$. In particular, they note that the globular clusters do not follow the “U”-shaped pattern described for $\Upsilon_e$ by Zaritsky et al. (2008). Because their data include $K$-band magnitudes, our fitting formula for $\Upsilon_e$, which is calibrated to $I$-band values of $\Upsilon_e$, is not optimal (color differences become large). Instead, we refit using their data and obtain a version of $\Upsilon_{e,\text{fit}}$ appropriate for the $K$ band. The central issue of the FM is whether the family of stellar systems can be described with a two-parameter relationship. The coefficients in the expansion of $\Upsilon_{e,\text{fit}}$, or even the functional form, are not sacrosanct, and currently have no detailed physical justification. The result of the new fit is shown in Figure 13. As Zaritsky et al. (2008) stressed, the most salient feature of this figure is not that the data straddle the 1:1 line, because this arises by construction (although it does imply that our functional form for $\Upsilon_{e,\text{fit}}$ is an acceptable one), but rather the low and uniform scatter across the full range of systems (i.e., clusters, dwarf galaxies, and giant galaxies).

In the right panel of Figure 13, we explore the crux of the argument against the FM presented by Forbes et al. (2008). In particular, they noted that star clusters, with low values of $\Upsilon_e$, do not follow the upward trend in $\Upsilon_e$ established by Local Group dSph galaxies. The difficulty with this argument is that it is based...
on a projection of the data onto the \((\sigma, \Upsilon_e)\) axes. On the right-hand plot, we show the value of \(\Upsilon_{e, \text{fit}}\) for each object in the Forbes et al. (2008) sample. By construction, because we use the values on the manifold, there is zero scatter in this distribution, and yet in this projection there appears to be significant scatter. In fact, the clusters, which populate the lower left branch, appear to be a distinct population from the dSphs, which lie directly above the clusters and extend upward along the \(\Upsilon_e\) axis. Therefore, even points distributed on a surface with zero scatter can, in projection, appear to comprise different populations of objects because the surface is not uniformly populated. As such, one cannot conclude from plots such as that shown in the left panel of the figure that there is no single scaling relationship among the populations. It is therefore critical to consider the behavior of \(\Upsilon_{e, \text{fit}}\) in terms of both \(\sigma\) and \(I_e\). We find that the FM fits the characteristics of both high- and low-surface brightness objects with low \((<10 \text{ km s}^{-1})\) velocity dispersions. In particular, in our Figure 1 the FM was derived excluding the star clusters and UCDs, and yet both populations fall within the uncertainties of the extrapolated relationship.

The apparent disagreement with Tollerud et al. (2010) stems from their aims and approach. Their aim was to identify the parameter with the greatest influence in producing the full range of spheroid properties along the scaling relationship for spheroids. They correctly focused on velocity dispersion as the dominant parameter. They proceeded to derive a one-parameter scaling relationship that they could then tie to halo parameters. Our differences are therefore ones of interpretation regarding whether it is appropriate to consider all stellar systems together or to distinguish among them. Our preference for a single descriptive formalism for all stellar systems leads us to one direction, whereas their preference for a 1:1 association of stellar systems with dark halos leads them to another.

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