Six Fermion Production at LC

Alessandro Ballestrero

I.N.F.N., Sezione di Torino, Italy

and

Dipartimento di Fisica Teorica, Università di Torino, Italy

v. Giuria 1, 10125 Torino, Italy.

We review some features and results of the calculations performed with the program SIXPHACT for six fermion final states at Linear Collider.

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e-mail: ballestrero@to.infn.it
1. Introduction

An important part of LEP2 physics is concerned with 4-fermion final states. From their studies\cite{1}, it has become evident that on shell computations are often not sufficient for processes like for instance $WW$ production. If on one hand production $\times$ decay approximation makes it possible to compute various electroweak and strong corrections, on the other one is neglecting important issues such as irreducible backgrounds, finite width effects, spin correlations. The full set of diagrams for a given process is required for a complete description at parton level and to analyze distributions and experimental cuts. The two approaches can be regarded at present as complementary, while corrections to the full set of diagrams start to be evaluated.

With the advent of future $e^+e^-$ colliders, important physical processes, such as $t\bar{t}$, $WWZ$ and Higgs production, will be concerned with 6-fermion final states. It is immediate for instance to realize that a final state like that of $e^+e^- \rightarrow \mu\bar{\nu}d\bar{b}b$ can be the result of the above mentioned production processes:

$$e^+e^- \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow \mu\bar{\nu}d\bar{b}b$$
$$e^+e^- \rightarrow W^+W^-Z \rightarrow \mu\bar{\nu}d\bar{b}b$$
$$e^+e^- \rightarrow hZ \rightarrow W^+W^-Z \rightarrow \mu\bar{\nu}d\bar{b}b$$

and of their irreducible backgrounds corresponding to all non resonant diagrams. The same final state gets in reality many different contributions which can be regarded as signal or background depending on the physical process we are interested in. Present helicity amplitude techniques allow to compute the tree-level full set of diagrams for such final states in reasonable time.

Six fermion (6f) final states can be divided in charged, neutral and mixed current processes:

**CC** : 6f can form 2 W’s and a Z but not 3 Z’s (e.g. $\mu\bar{\nu}d\bar{e}^+e^-$)

**NC** : 6f can form 3 Z’s but not 2 W’s and a Z (e.g. $u\bar{u}\mu^+\mu^- e^+e^-$)

**Mixed** : 6f can form both 3 Z’s and 2 W’s and a Z (e.g. $u\bar{d}d\bar{e}^+e^-$) The program SIXPHACT\cite{2} can compute all CC final states. It has been used to perform phenomenological studies of $t\bar{t}$, $WWZ$ and intermediate higgs physics. CC states are particularly interesting as they often allow to exclude NC and QCD backgrounds. Specific processes have also been computed and analyzed by other groups. In particular the reaction $e^+e^- \rightarrow \mu\bar{\nu}d\bar{b}b$ has been studied in connection with $t\bar{t}$ production\cite{3} while $e^+e^- \rightarrow \mu^+\mu^- \bar{\nu}d\bar{q}$ ($l = e, \tau$) and $e^+e^- \rightarrow e^+e^- \nu\bar{\nu}q\bar{q}$ ($q \neq b$) have been examined for their relevance to intermediate mass higgs\cite{4}.
2. SIXPHACT. Method of calculation

All diagrams for six fermion production can be subdivided into five different topologies, corresponding to diagrams with quartic gauge coupling, two triple gauge couplings, one triple gauge coupling, two fermion lines with two boson insertions, one fermion line with three boson insertions. In spite of the fact that there are only few different topologies, the number of diagrams in CC processes is of the order of 200 when no exchanges among identical particles are possible and it can raise to more than 1000 as it happens for $e^{-}\bar{\nu}ud\bar{e}^+e^-$ (1254). If we consider final states with one isolated lepton and four quarks, $\mu\bar{\nu}db\bar{b}$ has 232 diagrams and $e\bar{\nu}ud\bar{u}$ 840.

The program for the amplitudes has been written with the help of PHACT \[5\], a set of routines which implement the helicity method of ref.\[6\]. With it, one can easily perform fast modular massive and massless amplitude computations. The key ingredients are so-called $\tau$ matrices which assume a trivial expression for fermion propagators. The strategy used has been that of computing subdiagrams of increasing complexity, sum them together when needed and compose the sums to form the final results. In such a way repeated computations are avoided and the number of matrix multiplications is optimized.

Taking as an example $e^+e^- \rightarrow \mu\bar{\nu}db\bar{b}$, we have first evaluated the subdiagrams corresponding to a boson in four fermions or to the emission from the upper part of a fermion line of four outgoing or two outgoing and two incoming fermions. If we indicate these subdiagrams with the symbols:

\[ W_{ud\bar{b}b} \quad \gamma_{\mu\bar{\nu}} \quad u\bar{d} \quad u\bar{d} \quad u\bar{d} \]

the computation of the full set of diagrams for $e^+e^- \rightarrow \mu\bar{\nu}db\bar{b}$ is then reduced to those in fig. 1.

SIXPHACT can account for initial state radiation (ISR), beamstrahlung (BMS) with a link to CIRCE\[6\], naive QCD corrections (exact in production $\times$ decay no cuts limit), exact $b$ and $t$ fermion masses. All results given in the following have been computed in this way.

In order to account for the different peaking structures, SIXPHACT has several mappings for the 15-dimensional phase space. It moreover automatically performs all requested distributions and any cut can be implemented. Apart from other specific cuts, we will require in the following jet(quark) energy $> 3$ GeV, lepton energy $> 1$ GeV, jet-jet invariant mass $> 10$ GeV, lepton-beam angle $> 10^\circ$, jet-beam angle $> 5^\circ$, lepton-jet angle $> 5^\circ$. 
Fig. 1. Diagrams for the process $e^+e^- \rightarrow \mu\bar{\nu}udd$.

CPU times depend of course on the particular final processes, on the cuts, and the options required. As an indication, a process like $e^+e^- \rightarrow \mu^+\mu^-e\bar{\nu}_eud$ with ISR and BMS takes about 20 minutes for 3 per mille, 3 hours for per mille and one day for .2 per mille accuracy on a DEC ALPHA station.
3. Applications

Full six fermion calculations have been used to perform phenomenological studies of $t\bar{t}$ production in the continuum, $W W Z$ and intermediate mass higgs physics at Linear Collider.

In this section we will limit ourselves to the last two issues, where for various final channels $t\bar{t}$ production is to be considered as a huge background to be suppressed with appropriate cuts. For brevity, we will not attempt a complete discussion of the results but just illustrate some examples.

3.1. $WWZ$

Triple gauge boson ($WWZ$) production processes are important as they allow a direct investigation of triple and quartic gauge boson couplings. Accurate analyses of the sensitivities to genuine quartic coupling have already been performed using only on shell calculation, where the three boson final state was considered. The cross sections are not very large: at $\sqrt{s} = 500$ GeV $WWZ$ production is of the order of 39 fb, which corresponds to about 2000 events in all six fermion final states for an integrated luminosity of $50 \, fb^{-1}$. The announced possibility of having a final luminosity about ten times higher is therefore extremely important.

| process | WWZ NWA (fb) | WWZ signal (fb) | complete (fb) |
|---------|--------------|----------------|--------------|
| $\mu\nu d\bar{c}c$ | 0.13836(2) | 0.13464(2) | 0.16218(9) |
| $e\nu d\bar{c}c$ | 0.17780(3) | 0.17303(3) | 0.1803(1) |
| $\mu\nu d\bar{s}s$ | 0.12815(2) | 0.12469(2) | 0.1512(1) |
| $e\nu d\bar{s}s$ | 0.16468(3) | 0.16025(3) | 0.16733(9) |

Table 1. Cross section for the processes $e^+e^- \rightarrow l\bar{\nu}l + 4$ light quarks ($l = \mu, e$) at $\sqrt{s} = 500$ GeV

We have considered final states with an isolated electron or muon and four final quarks: $e^+e^- \rightarrow \mu(e)\nu d\bar{q}q$. The isolated leptons give a signature that in the processes two $W$'s have been produced, and are therefore useful in reducing the background. We have also summed over all possible flavours for $q = u, d, c, s, b$. With no $b$ tagging one has the full contribution from $q = b$ which will be dominated by $t\bar{t}$ production events. With a
realistic $b$ tagging and appropriate cuts such background can however be well under control\[\text{[2]}\]. In table 1 we report the cross sections computed with the whole set of diagrams, with only those diagrams corresponding to $WWZ$ production $\times$ decay (signal) and the production $\times$ decay approximation itself (Narrow Width Approximation). The difference between full calculation and on shell results is remarkable and shows the relevance of so called irreducible background.

Fig. 2. Cross section for the process $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}s\bar{s}$ at $\sqrt{s} = 500$ GeV (lower) and $\sqrt{s} = 800$ GeV (upper) as a function of $M_{cut}$. Quarks are required to form two pairs whose invariant masses $m_i (i = 1, 2)$ satisfy the conditions $|M_V - m_i| < M_{cut}$, $V = W, Z$. The dot lines represent the cross section for $WWZ$ on shell, the dashed ones the contribution of resonant $WWZ$ diagrams only, the continuous the complete cross section. The markers indicate the points effectively computed.

In fig. 2 one uses cuts to force two of the invariant masses formed by two quarks to be in the vicinity of the $W$ and $Z$ mass. Even with such a requirement, there is an evident difference between full calculation and resonant $WWZ$ diagrams. The effect grows with the energy. We have found that it is less important for $e^+e^- \rightarrow \mu\bar{\nu}_\mu u\bar{d}s\bar{s}$, but in order to better isolate genuine $WWZ$ production, cuts on the invariant mass formed by the lepton and the missing momentum have to be studied.
Fig. 3. *Reconstructed* mass distributions. Quarks are required to form two pairs whose invariant masses $m_i$ ($i = 1, 2$) satisfy the conditions $|M_V - m_i| < 20$ GeV, $V = W, Z$. The continuous line represents the total background. The others correspond to the total cross sections for (from left to right) $m_h = 150, 170, 200, 250$ GeV.

### 3.2. Intermediate mass Higgs

If the mass of the Higgs is greater than about 140 GeV, it will mainly decay to two $W$'s. In such a case, as the most important production channels are $hZ$ production (up to 500 GeV) and $WW$ fusion (which dominates at higher energies), the higgs events will effectively result in 6 fermion final states.

We have examined in detail processes with one isolated lepton like $l \nu_l + 4q's$ or $l \nu_l + l' \bar{l'} + 2q's$ and two leptons of different flavour and missing energy like $l \nu_l + l' \nu_{l'} + 2q's$. In the first two cases, which represent respectively about 31% and 4.4% of $hZ$ signal, one can study the distribution of the reconstructed mass, i.e. the invariant mass formed by the isolated lepton, the reconstructed neutrino momentum and the two quarks reconstructing the $W$. To the neutrino is attributed all missing 3-momentum and its energy is taken to be equal to its modulus. As far as $l \nu_l + l' \bar{l'} + 2q's$ is concerned, we have verified that requiring that the invariant mass of the 2 $q's$ and of $l' \bar{l'}$ be within 20 GeV from $m_W$ and $m_Z$ respectively, the irreducible backgrounds become completely harmless. An analogous conclusion can be drawn from the upper part of fig. 3 for the case of 4 light quarks and one isolated lepton.
Missing mass distributions. $s\bar{s}$ are required to have an invariant mass $M$ such that $|M_Z - M| < 20$ GeV. The continuous line represents the total background. The others correspond to the total cross sections for (from left to right) $m_h = 150$, 170, 200, 250 GeV. In the lower plot a 5 GeV gaussian error in missing mass determination is assumed.

One would tend to exclude the $b$ quarks in such processes to get rid of the huge $t\bar{t}$ irreducible background. In practice however $b$ flavour cannot be completely excluded with a realistic $b$ tagging and one would like to keep also the $b$ signal. The lower part of fig. 3 shows that the cuts can still keep the background under control, at least for not too high higgs masses, even if we sum also over $q = b$ events.

In the case of $l \nu_l + l' \nu_{l'} + 2 q'$s (about 5.2% of $hZ$ signal) one cannot determine the reconstructed mass due to the presence of the two neutrinos, but the missing mass can be used instead. This is the invariant mass of the four-momentum recoiling against the particles decaying from the $Z$ (2 $q'$s in our case). Fig. 4 shows that indeed one can very well see the signal also in such a case, but the distribution around the higgs mass becomes asymmetric. In the more realistic case in which a 5 GeV gaussian error in the determination of the missing mass is assumed (lower part of fig. 4) one can see that the asymmetry implies a consistent shift of the maximum with respect to the higgs mass.
4. Conclusions

Modern helicity techniques allow complete computation of many body final states, which are of primary interest for future colliders. We have described some features of $e^+e^- \rightarrow 6$ fermion calculations relevant to $t\bar{t}$, WWZ, intermediate higgs physics. The relevance of such an approach has been illustrated with some examples of phenomenological results.

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