Hall effect in charge and heat transport in a hot quark matter with $T$- and $B$-dependent quark masses

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Abstract

We investigate the transport of charge and heat in a baryon asymmetric hot QCD medium in the presence of a magnetic field. When the magnetic field strength becomes weak, some novel phenomenological aspects emerge in the transport phenomena, particularly: A) The degeneracy in left- and right-handed chiral modes of quarks gets lifted. B) Hall effect in both electrical and thermal transport develops. Both the transport coefficients assume a tensorial structure, where diagonal elements represent the longitudinal conductivities ($\sigma_{\text{Ohmic}}$ and $\kappa_0$) and off-diagonal elements represent their Hall counterparts ($\sigma_{\text{Hall}}$ and $\kappa_1$). The $B$ dependent quasiparticle parton masses serve as an input for the evaluation of charge and thermal transport properties of the medium, quantified by the electrical and Hall conductivities, and the longitudinal and transverse thermal conductivities. The Ohmic conductivity ($\sigma_{\text{Ohmic}}$) decreases whereas the Hall conductivity ($\sigma_{\text{Hall}}$) increases with magnetic field strength. There is also a rise in both the conductivities with increase in quark chemical potential. The coefficient $\kappa_0$ decreases with magnetic field whereas $\kappa_1$ increases with magnetic field similar to the trends followed by the Ohmic and Hall conductivities. The degree of baryonic asymmetry positively amplifies both $\kappa_0$ and $\kappa_1$. The value of the Knudsen number ($\Omega_0$ and $\Omega_1$) is found to be less than unity for the entire temperature range considered. Further, we find that the Lorenz number ($\frac{\kappa_0}{\sigma_{\text{Ohmic}}}$) and Hall Lorenz number ($\frac{\kappa_1}{\sigma_{\text{Hall}}}$) do not remain constant with temperature, rather increase, hence violating the Wiedemann-Franz law.

I Introduction

Quark-gluon plasma (QGP) is the deconfined phase of quarks and gluons which is believed to be existed in the early universe, about $10^{-5}$s after the cosmic Big Bang and at the core of superdense stars such as neutron stars and quark stars. The experiments are being

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carried out for the formation of QGP at European Council for Nuclear Research (CERN), Relativistic Heavy Ion Collider (RHIC), Brookhaven National Laboratory (BNL) and Large Hadron Collider (LHC). There have been successful attempts to collect the evidences for the existence of QGP [1]. It is also shown that magnetic field whose magnitude varies from \(|eB| = 0.1 \, m^2_\pi\) for SPS energy range to \(|eB| = 15 \, m^2_\pi\) for LHC, is also produced during non-central heavy ion collisions [2–4]. The strength of magnetic field is strong during the initial stage of QGP but it decays very fast with time, resulting in the weaker strength of magnetic field. The life-time of magnetic field in charged medium gets enhanced due to the medium properties [5–8]. Further, the effect of magnetic field on transport phenomena [9, 10], thermodynamical behaviour [11, 12] of quark-gluon plasma, dilepton production from QGP [13–15] has been studied.

The nonvanishing magnetic field can affect the evolution of strongly interacting matter significantly [16–23] and hence comprehensive study of transport coefficients, e.g. shear viscosity, bulk viscosity, electrical conductivity, thermal conductivity etc., became of great interest. The small ratio of shear viscosity to the entropy density (\(\eta/s\)) of QGP has lead us to use the hydrodynamical model for QGP [24–28] as \(\eta/s\) of strongly interacting plasma agrees well with the lower bound of \(\eta/s = \frac{1}{4\pi}\), where \(\hbar = 1, k_B = 1\), obtained using AdS/CFT correspondence [29]. Apart from viscosity coefficients, the study of charge and thermal transport coefficients also plays an important role in the hydrodynamical evolution of strongly interacting matter [30–32]. The topological effects induced by magnetic field can be quantified using electrical conductivity and plays crucial role in the study of chiral magnetic effect [33], which is signature of \(CP\) violation in the strong interaction. Dilepton and photon production rate are used to probe the thermalized strongly interacting matter because they hardly interact with the hadrons in region of hot and dense matter and hence carry the information about the early stage of heavy ion collisions and electrical conductivity (\(\sigma_{el}\)) can be used for phenomenological studies of heavy ion collisions [34]. Another key transport coefficient is thermal conductivity of QGP medium, which measures the transport of heat due to temperature gradient in the medium. The hydrodynamical equilibrium of the system can be determined using Knudsen number, which is the ratio of mean free path to the characteristic length of the medium. The mean free path (\(\lambda\)) is related to the thermal conductivity (\(\kappa\)) as \(\lambda = 3\kappa/(\nu C_v)\), where \(\nu\) is the relative velocity of quark and \(C_v\) is the specific heat at constant volume. Further, the relative behaviour of \(\kappa\) and \(\sigma_{el}\) can be understood in terms of Wiedemann-Franz law, which states that ratio, \(\kappa/\sigma_{el}\), of the thermal to electrical conductivity is directly proportional to the temperature, with proportionality constant to far accuracy is same for all metals. The ratio \(\kappa/(\sigma_{el}T)\) is known as Lorenz number (\(L\)), which is independent of temperature and depends on fundamental constants for all metals and follow the Wiedemann-Franz law satisfactorily [35]. However, the violation of Wiedemann-Franz law
has been observed in many systems, such as hydrodynamic electron liquid [36], high temperature superconductors [37], Luttinger liquid [38], strongly interacting QGP medium [39] and hot hadronic matter [40]. Hence, it would be interesting to see the influence of magnetic field on Lorenz number in Wiedemann-Franz law.

In the present work, we have studied the charge and thermal transport coefficients in presence of weak magnetic field at finite chemical potential. These transport coefficients can be calculated using different approaches/models, viz, NJL model [41–43], Chapmann-Enskog approximation [44–46], the correlator technique using Green-Kubo formula [47–50], effective fugacity model [51, 52], lattice simulation [53–55], relativistic Boltzmann transport equation [56]. However, we have used the kinetic theory approach by solving the relativistic Boltzmann transport equation. The calculation of transport coefficients using kinetic theory has been done [10, 56, 57] in presence of strong magnetic field (\(|q_f B| >> T^2, |q_f B| >> m_f^2\)) , where \(q_f\) and \(m_f\) is the electric charge and mass of \(f\)-th flavor. In strongly magnetized medium, the motion of charged particle is restricted to the 1 + 1-dimensional Landau level dynamics and dispersion relation in terms of Landau levels (\(n\)) can be expressed as

\[
\epsilon_{f,n} = \sqrt{p_L^2 + m_f^2 + 2n|q_f B|},
\]

where \(p_L\) is the longitudinal momentum (momentum along the direction of magnetic field). Hence, in presence of strong magnetic field, its effect are observed in the direction of original current. While in presence of weak magnetic field, where temperature is the dominant energy scale (\(T^2 >> |q_f B|\)) and effect of magnetic field enters through the cyclotron frequency (\(\omega_c\)), Hall effect becomes the important effect of magnetic field. In general, we obtain the tensorial relation between field and current which further lead to the conductivity tensor. The diagonal elements gives the conventional (longitudinal) conductivity (\(\sigma_{Ohmic}\) and \(\kappa_0\)) and off-diagonal elements gives their Hall-counterpart (\(\sigma_{Hall}\) and \(\kappa_1\)) which arises due to the transverse flow of charge and heat (perpendicular to the thermal driving force and magnetic field) in weakly magnetized medium. However, \(\sigma_{Hall}\) and \(\kappa_1\) vanishes for zero quark chemical potential due to vanishing gyration frequency of charge carriers caused by the exact cancellation of Hall current due to particles and their antiparticles. Whereas, at finite baryon chemical potential, the number of positive and negative charge carriers are not same, hence due to charge asymmetry, there is net Hall effect in electrically charged quark gluon plasma. The role of interaction among partons is incorporated using quasiparticle description of partons, where vacuum masses of partons are replaced by medium generated masses. The medium generated mass is calculated from the pole of propagator, obtained through perturbative thermal QCD in the background of weak magnetic field. In some previous work, authors have incorporated the pure thermal medium mass of quarks in the computation of transport coefficients [58, 59], whereas we have used the thermally generated mass with magnetic field correction. The dispersion relation of quasiparticle in weak magnetic field give rise to four collective mode two from left and two
from right handed mode and various properties of dispersion relation has been discussed [60, 61]. There is lifting of degeneracy in left- and right-handed chiral modes of quarks in presence of weak magnetic field which is in contrast to the case of strong magnetic field. The medium generated masses for left- and right-handed quarks has been taken into account for the estimation of transport coefficients.

The paper is organised as follows: in Sec. II, we discuss the quasiparticle model of partons and hence evaluate the medium generated mass. We will use this mass as an input to incorporate the interaction among partons to compute the transport coefficients. In Sec. III and Sec. IV, we will discuss the computation of charge and thermal transport coefficient using kinetic theory within relaxation time approximation. In Sec.V, we present and discuss the results for Ohmic and Hall conductivity as well as thermal and Hall-type thermal conductivity. Finally, we conclude our work in conclusion section.

II Quasiparticle Model

At asymptotically high temperature, a system of quarks and gluons can be treated as an ideal gas due to asymptotic freedom. The interaction among quasi quarks and quasi gluons can be incorporated through medium dependent mass of quasiparticles which can be evaluated using one-loop perturbative thermal QCD. In pure thermal medium at finite quark chemical potential ($\mu$), the thermally generated mass for quarks and gluons obtained to be as

$$m_{th}^2 = \frac{1}{8}g^2C_F\left(T^2 + \frac{\mu^2}{\pi^2}\right),$$
$$m_g^2 = \frac{1}{6}g^2T^2\left(C_A + \frac{1}{2}N_f\right),$$

respectively, where $C_F = (N_c^2 - 1)/2N_C = \frac{4}{3}$ for $N_C = 3$, $C_A(C_A = 3)$ is the group factor, $g$ is the QCD coupling constant with $g^2 = 4\pi\alpha_s$, where $\alpha_s$ is the running coupling constant, which runs with temperature as

$$\alpha_s(\Lambda^2) = \frac{1}{b_1 \ln\left(\frac{\Lambda^2}{\mu^2}\right)},$$

where $b_1 = (11N_c - 2N_f)/12\pi$ and $\Lambda_{MS} = 176$ MeV. Here, for quarks, $\Lambda = \Lambda_q = 2\pi\sqrt{T^2 + \mu^2}/\pi^2$, and for gluons, $\Lambda = \Lambda_g = 2\pi T$. As the quarks are having both thermal mass and current mass, the effective quark mass for $f$–th flavor can be written as

$$m^2 = m_f^2 + \sqrt{2}m_f m_{f,th} + m_{f,th}^2.$$
where $m_f$ and $m_{f,th}$ is the current quark mass and thermal mass for $f$–th flavor respectively. In presence of magnetic field, the effective quark mass can be generalized to

$$m^2 = m_f^2 + \sqrt{2}m_fm_{f,th,B} + m_{f,th,B}^2$$

where $m_{f,th,B}$ can be obtained from the dispersion relation of full quark propagator in magnetic field by solving Schwinger-Dyson equation. The quark propagator in the presence of background magnetic field following Schwinger formalism can be written in terms of Laguerre polynomial ($L_l(2\alpha)$) [68]

$$iS(K) = \sum_{n=0}^{\infty} \frac{-id_l(\alpha)D + d'_l(\alpha)\bar{D}}{k_L^2 + 2l|q_fB|} + \frac{i\gamma.k_\perp}{k_\perp^2},$$

where $q_f$ is the absolute charge of $f$–th flavor, $l = 0, 1, 2, \ldots$ are the Landau levels, $||$ and $\perp$ are the parallel and perpendicular components of momentum respectively with respect to direction of magnetic field, $\alpha = k_\perp^2 / |q_fB|$, $k_L^2 = m_f^2 - k_\parallel^2$ and $d_l(\alpha), d'_l(\alpha), D, \bar{D}$ are given as [69],

$$d_l(\alpha) = (-1)^l e^{-\alpha}C_l(2\alpha),$$
$$d'_l(\alpha) = \frac{\partial d_l}{\partial \alpha},$$
$$D = (m_f + \gamma.k_||) + \gamma_\perp \left( \frac{m_{f,th}^2 - k_\parallel^2}{k_\perp^2} \right),$$
$$\bar{D} = \gamma_1\gamma_2 (m_f + \gamma.k_||),$$

with $C_l(2\alpha) = L_l(2\alpha) - L_{l-1}(2\alpha)$. In weak field limit, the quark propagator can be reorganized in power series of magnetic field ($|q_fB|$) as,

$$iS(K) = \frac{i (K + m_f)}{K^2 - m_f^2} - \frac{\gamma_1\gamma_2 (\gamma.K_|| + m_f)}{(K^2 - m_f^2)^2}(|q_fB|),$$

where first term in Eq.(7) is the free fermion propagator and second term is the $O(|q_fB|)$ correction to it. Neglecting the current quark mass under the limit $(m_f^2 < |q_fB| < T^2)$ in the numerator and using the following metric tensor,

$$g^{\mu\nu} = g_||^{\mu\nu} + g_\perp^{\mu\nu};$$
$$g_||^{\mu\nu} = \text{diag}(1, 0, 0, -1); \quad g_\perp^{\mu\nu} = \text{diag}(0, -1, -1, 0);$$
$$a^{\mu} = a_||^{\mu} + a_\perp^{\mu}; \quad a_||^{\mu} = (a^0, 0, 0, a_3);$$
$$a_\perp^{\mu} = (0, a^1, a^2, 0); \quad \phi = \gamma^{\mu}a_\mu = \phi_|| + \phi_\perp;$$
$$\phi_|| = \gamma^0a_0 - \gamma^3a^3; \quad \phi_\perp = \gamma^1a^1 + \gamma^2a^2,$$
with
\[ i\gamma_1\gamma_2 k_\parallel = -\gamma_5[(K.b)\dot{u} - (K.u)\dot{b}], \] (9)

Eq. (7) can be rewritten as
\[ iS(K) = -\frac{i(K)}{K^2 - m_f^2} + \frac{i\gamma_5[(K.b)\dot{u} - (K.u)\dot{b}]}{(K^2 - m_f^2)^2}(|q_f B|), \] (10)

where \( u^\mu = (1, 0, 0, 0) \) denotes the preferred direction of heat bath which breaks the Lorentz symmetry and \( b^\mu = (0, 0, 0, 1) \) denotes the preferred direction of magnetic field which breaks the rotational symmetry. The one-loop quark self energy upto \( O(|q_f B|) \) in hot and weakly magnetized medium can be written as
\[ \Sigma(P) = g^2 C_F T \sum_n \int \frac{d^3 k}{(2\pi)^3} \frac{k}{(K^2 - m_f^2)} \gamma_\mu \frac{1}{(P - K)^2}, \] (11)

where \( T \) is the temperature of the system. The first term \( (\Sigma_{B=0}) \) is the thermal bath contribution in the absence of magnetic field whereas second one \( (\Sigma_{B\neq0}) \) is from magnetized thermal bath.

Now, the general covariant structure of quark self energy at finite temperature and magnetic field can be written as [60]
\[ \Sigma(P) = -\mathcal{A} P - B \dot{u} - C \gamma_5 \dot{b} - D \gamma_5 \dot{b}, \] (12)

where \( \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \) are the structure functions. Using Eq. (11) and (12), the general form of these structure functions are obtained as
\[ \mathcal{A}(p_0, p_\perp, p_\parallel) = \frac{1}{4} \text{Tr}(\Sigma(P)\dot{P} - (P.u)\text{Tr}(\Sigma(P)\dot{b})) - \frac{P.u}{(P.u)^2 - P^2}, \] (13)
\[ \mathcal{B}(p_0, p_\perp, p_\parallel) = \frac{-P.u\text{Tr}(\Sigma(P)\dot{P}) + P^2\text{Tr}(\Sigma(P)\dot{b})}{(P.u)^2 - P^2}, \] (14)
\[ \mathcal{C}(p_0, p_\perp, p_\parallel) = \frac{-1}{4} \text{Tr}(\gamma_5 \Sigma(P)\dot{b}), \] (15)
\[ \mathcal{D}(p_0, p_\perp, p_\parallel) = \frac{1}{4} \text{Tr}(\gamma_5 \Sigma(P)\dot{b}). \] (16)

These structure functions are found to depend upon various Lorentz scalars defined by
\[ p_0 \equiv P^\mu u_\mu = \omega, \] (17)
\[ p_3 \equiv P^\mu n_\mu = -p_z, \] (18)
\[ p_\perp \equiv [(P^\mu u_\mu)^2 - (P^\mu n_\mu)^2 - (P^\mu P_\mu)^2]^{1/2}, \] (19)

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where $\omega, p_\perp, p_z$ are termed as Lorentz invariant energy, transverse momentum and longitudinal momentum respectively. The detailed calculation of all these structure functions is shown in Appendix VI and results are quoted here,

$$\mathcal{A}(p_0, |p|) = \frac{m_{bh}^2}{|p|^2} Q_1 \left( \frac{p_0}{|p|} \right), \quad (20)$$

$$\mathcal{B}(p_0, |p|) = -\frac{m_{bh}}{|p|} \left[ \frac{p_0}{|p|} Q_1 \left( \frac{p_0}{|p|} \right) - Q_0 \left( \frac{p_0}{|p|} \right) \right], \quad (21)$$

$$\mathcal{C}(p_0, |p|) = -4g^2C_FM^2 \frac{p_z}{|p|^2} Q_1 \left( \frac{p_0}{|p|} \right), \quad (22)$$

$$\mathcal{D}(p_0, |p|) = -4g^2C_FM^2 \frac{1}{|p|} Q_0 \left( \frac{p_0}{|p|} \right), \quad (23)$$

where $Q_0$ and $Q_1$ are Legendre functions of first and second kind respectively read as

$$Q_0(x) = \frac{1}{2} \ln \left( \frac{x + 1}{x - 1} \right) \quad (24)$$

$$Q_1(x) = \frac{x}{2} \ln \left( \frac{x + 1}{x - 1} \right) - 1 = xQ_0(x) - 1, \quad (25)$$

with magnetic mass obtained as $[70]

$$M^2(T, \mu, m_f, q_f B) = \frac{|q_f B|}{16\pi^2} \left( \frac{\pi T}{2m_f} - \ln 2 + \frac{7\mu^2\zeta(3)}{8\pi^2 T^2} \right). \quad (26)$$

The general covariant structure of quark self energy Eq.(12) can be recast in terms of left handed ($P_L = (\mathbb{1} - \gamma_5)/2$) and right handed ($P_R = (\mathbb{1} + \gamma_5)/2$) chiral projection operators as

$$\Sigma(P) = -P_R \mathbb{\hat{A}} P_L - P_L \mathbb{\hat{B}} P_R, \quad (27)$$

with $\mathbb{\hat{A}}$ and $\mathbb{\hat{B}}$ defined as

$$\mathbb{\hat{A}} = \mathbb{A} \mathbb{\hat{P}} + (\mathbb{B} + \mathbb{C}) \mathbb{\hat{g}} \mp \mathbb{D} \mathbb{\hat{g}}; \quad (28)$$

$$\mathbb{\hat{B}} = \mathbb{A} \mathbb{\hat{P}} + (\mathbb{B} - \mathbb{C}) \mathbb{\hat{g}} \mp \mathbb{D} \mathbb{\hat{g}}. \quad (29)$$

The effective inverse fermion propagator using self-consistent Schwinger-Dyson equation reads as

$$S^{*-1}(P) = \mathbb{\hat{P}} - \Sigma(P). \quad (30)$$

Using (27) the inverse fermion propagator can be written as

$$S^{*-1}(P) = \mathbb{\hat{P}} + P_R \left[ \mathbb{A} \mathbb{\hat{P}} + (B + C) \mathbb{\hat{g}} + \mathbb{D} \mathbb{\hat{g}} \right] P_L + P_L \left[ \mathbb{A} \mathbb{\hat{P}} + (B - C) \mathbb{\hat{g}} - \mathbb{D} \mathbb{\hat{g}} \right] P_R, \quad (31)$$

and further using $P_{L,R}g^{\mu} = \gamma^\mu P_{R,L}$ and $P_L \mathbb{\hat{P}} P_L = P_R \mathbb{\hat{P}} P_R = P_L P_R \mathbb{\hat{P}} = 0$, we obtain

$$S^{*-1}(P) = P_R \mathbb{\hat{L}} P_L + P_L \mathbb{\hat{R}} P_R, \quad (32)$$
where \( \hat{L} \) and \( \hat{R} \) are

\[
\hat{L} = (1 + \mathcal{A}) \gamma^0 + (\mathcal{B} + \mathcal{C}) \gamma^1 + \mathcal{D} \gamma^3,
\]

\[
\hat{R} = (1 + \mathcal{A}) \gamma^0 + (\mathcal{B} - \mathcal{C}) \gamma^1 - \mathcal{D} \gamma^3.
\]

Thus, we get the effective quark propagator as

\[
S^*(P) = \frac{1}{2} \left[ P_L \frac{\hat{L}}{L^2/2} P_R + \frac{1}{2} P_R \frac{\hat{R}}{R^2/2} P_L \right],
\]

where

\[
L^2 = (1 + \mathcal{A})^2 P^2 + 2(1 + \mathcal{A})(\mathcal{B} + \mathcal{C}) p_0 - 2\mathcal{D}(1 + \mathcal{A}) p_z + (\mathcal{B} + \mathcal{C})^2 - \mathcal{D}^2,
\]

\[
R^2 = (1 + \mathcal{A})^2 P^2 + 2(1 + \mathcal{A})(\mathcal{B} - \mathcal{C}) p_0 + 2\mathcal{D}(1 + \mathcal{A}) p_z + (\mathcal{B} - \mathcal{C})^2 - \mathcal{D}^2.
\]

Further, we take the static limit \((p_0 = 0, |p| \to 0)\) of \(L^2/2\) and \(R^2/2\), after expanding the Legendre functions involved in structure functions, in power series of \(\frac{|p|}{p_0}\) considering up to \(O(g^2)\), we obtain

\[
\frac{L^2}{2} \bigg|_{p_0 = 0, |p| \to 0} = m_{th}^2 + 4g^2 C_F M^2,
\]

\[
\frac{R^2}{2} \bigg|_{p_0 = 0, |p| \to 0} = m_{th}^2 - 4g^2 C_F M^2.
\]

The degenerate left- and right-handed modes get separated out in presence of weak magnetic field and thermal mass (squared) at finite chemical potential in presence of weak magnetic field obtained as

\[
m_L^2 = m_{th}^2 + 4g^2 C_F M^2,
\]

\[
m_R^2 = m_{th}^2 - 4g^2 C_F M^2,
\]

which is opposite to the case of strong magnetic field, where left- and right-handed chiral modes have same mass \([56]\). We will incorporate both the masses for the estimation of heat and charge transport coefficients.

### III Charge Transport Coefficients

In this section, we are going to study the charge transport coefficients in the presence of weak magnetic field at finite chemical potential. The relativistic Boltzmann transport equation for relativistic particle with charge \(q\) in presence of external electromagnetic field can be written as \([62]\)

\[
p^\mu \partial_\mu f(x, p) + q F_{\mu\nu} p_\nu \frac{\partial f(x, p)}{\partial p^\mu} = C[f],
\]
where, $F^{\mu\nu}$ is the antisymmetric electromagnetic field tensor, $C[f]$ denotes the rate of change of distribution function by virtue of collisions which in relaxation time approximation can be written as

$$C[f] \approx -\frac{p^{\mu}u_{\mu}}{\tau}(f - f_0) \equiv -\frac{p^{\mu}u_{\mu}}{\tau}\delta f,$$

(43)

where $u_{\mu}$ is the fluid 4-velocity which takes the form as $u_{\mu} \equiv (1, 0)$ in rest frame of heat bath, $\tau$ is the thermal averaged relaxation time. Under relaxation time approximation, the external perturbation takes the system slightly away from equilibrium which relaxes towards the equilibrium exponentially with time scale $\tau$. $f_0$ is the equilibrium distribution function represents the equilibrium state of the system and $f$ is the out of equilibrium distribution function with $f = f_0 + \delta f$. The Eq. (42) in 3-notation can be written as

$$\frac{\partial f}{\partial t} + v \cdot \frac{\partial f}{\partial r} + q(E + v \times B) \cdot \frac{\partial f}{\partial p} = -\frac{1}{\tau}(f - f_0).$$

(44)

Considering the spatially uniformed $\frac{\partial f}{\partial r} \approx 0$ and static medium $\frac{\partial f}{\partial t} = 0$ such that there are no space-time gradient, Eq. (44) simplifies to

$$q(E + v \times B) \cdot \frac{\partial f}{\partial p} = -\frac{1}{\tau}(f - f_0).$$

(45)

Choosing electric and magnetic field as $E = E\hat{x}$ and $B = B\hat{z}$ respectively without loss of generality, we have

$$f - qB\tau \left( v_x \frac{\partial f}{\partial p_y} - v_y \frac{\partial f}{\partial p_x} \right) = f_0 - qE\tau \frac{\partial f_0}{\partial p_x}. $$

(46)

In order to solve the Eq. (46), taking the following ansatz of distribution function $f(p)$

$$f(p) = f_0 - \tau qE \frac{\partial f_0}{\partial p} - \xi \frac{\partial f_0}{\partial p},$$

(47)

with $f_0(p)$ for fermions is given by

$$f_0(p) = \frac{1}{e^{(\sqrt{p^2 + m^2} - \mu)/T} + 1},$$

(48)

which is space and time independent solution to the Boltzmann equation and $f_0$ satisfies,

$$\frac{\partial f_0}{\partial p} = v \frac{\partial f_0}{\partial \varepsilon}, \quad \frac{\partial f_0}{\partial \varepsilon} = -\beta f_0(1 - f_0),$$

(49)

where $\varepsilon = \sqrt{p^2 + m^2}$. Using the ansatz Eq. (47) in Eq. (46), we get

$$\left( f_0 - \tau qE \frac{\partial f_0}{\partial p} - \xi \frac{\partial f_0}{\partial p}, \frac{\partial f_0}{\partial p} \right) - qB\tau \left( v_x \frac{\partial f_0}{\partial p_y} - v_y \frac{\partial f_0}{\partial p_x} \right) \left( f_0 - \tau qE \frac{\partial f_0}{\partial p} - \xi \frac{\partial f_0}{\partial p} \right) = f_0 - qE\tau \frac{\partial f_0}{\partial p_x}. $$

(50)
The first term in the parenthesis in left hand side of Eq.(50) can be rewritten as,
\[
(f_0 - \tau qE \frac{\partial f_0}{\partial p} - \xi \frac{\partial f_0}{\partial p}) = f_0 + \beta \tau qEv_x f_0 + (\xi \cdot v) \beta f_0.
\] (51)

Neglecting the \( f_0^2 \) terms at high temperature and using the following second order partial derivatives,
\[
\frac{\partial^2 f_0}{\partial p_y p_x} = \frac{\beta p_x p_y f_0}{\varepsilon^2} \left( \beta + \frac{1}{\varepsilon} \right),
\] (52)
\[
\frac{\partial^2 f_0}{\partial p_y p_z} = \frac{\beta p_x p_z f_0}{\varepsilon^2} \left( \beta + \frac{1}{\varepsilon} \right),
\] (53)
\[
\frac{\partial^2 f_0}{\partial p_y^2} = -\beta \left[ \frac{f_0}{\varepsilon} - \frac{f_0 p_y^2}{\varepsilon^2} \left( \beta + \frac{1}{\varepsilon} \right) \right],
\] (54)

the second term in left hand side of Eq.(50) get reduced to
\[
qB \tau \left( v_x \frac{\partial}{\partial p_y} - v_y \frac{\partial}{\partial p_x} \right) \left( f_0 - \tau qE \frac{\partial f_0}{\partial p} - \xi \frac{\partial f_0}{\partial p} \right) = qB \tau \beta \left( \frac{\xi_y v_x}{\varepsilon} - \frac{\xi_x v_y}{\varepsilon} - \frac{v_y \tau qE}{\varepsilon} \right) f_0.
\] (55)

Combining the Eq.(51) and (55), Eq.(50) obtained as
\[
\frac{\tau qBqEv_y}{\varepsilon} - \frac{qB}{\varepsilon} (v_x \xi_y - v_y \xi_x) + \frac{1}{\tau} \left( \xi_x \frac{p_x}{\varepsilon} + \xi_y \frac{p_y}{\varepsilon} + \xi_z \frac{p_z}{\varepsilon} \right) = 0,
\] (56)

The above equation should be satisfied for any value of velocity therefore, comparing the coefficients of \( v_x, v_y \) and \( v_z \) of Eq.(56), we get \( \xi_z = 0 \). Comparing the coefficients of \( v_x \) and \( v_y \) of Eq.(56), we have
\[
\xi_z = 0,
\] (57)
\[
\omega_c \tau qE + \omega_c \xi_x + \frac{\xi_y}{\tau} = 0,
\] (58)
\[
\frac{\xi_x}{\tau} - \omega_c \xi_y = 0,
\] (59)

where \( \omega_c = \frac{qB}{\varepsilon} \) is termed as cyclotron frequency. Solving for \( \xi_x \) and \( \xi_y \), we have
\[
\xi_x = -\frac{-\omega_c^2 \tau^3 qE}{(\omega_c^2 \tau^2 + 1)}; \quad \xi_y = -\frac{-\omega_c^2 \tau^2 qE}{(\omega_c^2 \tau^2 + 1)}.
\] (60)

Using Eq.(60) in Eq.(47), the distribution function \( f(p) \) for quarks simplifies to
\[
f(p) = f_0 - \frac{qE v_x \tau}{(1 + \omega_c^2 \tau^2)} \left( \frac{\partial f_0}{\partial \varepsilon} \right) + \frac{qE v_y \omega_c \tau^2}{(1 + \omega_c^2 \tau^2)} \left( \frac{\partial f_0}{\partial \varepsilon} \right),
\] (61)

and for anti-quarks \( f \rightarrow \tilde{f}, \bar{q} \rightarrow -q, \bar{\omega}_c \rightarrow -\omega_c, \)
\[
\tilde{f}(p) = \tilde{f}_0 + \frac{qE v_x \tau}{(1 + \omega_c^2 \tau^2)} \left( \frac{\partial \tilde{f}_0}{\partial \varepsilon} \right) + \frac{qE v_y \omega_c \tau^2}{(1 + \omega_c^2 \tau^2)} \left( \frac{\partial \tilde{f}_0}{\partial \varepsilon} \right).
\] (62)
As we have taken the effect of magnetic field on an original electrical current, we have considered the two possibilities: (a) The magnetic field is at right angle to the current and its effect is observed in the direction of current. (b) The magnetic field is at right angle to the current and its effect is observed in the direction at right angles to the magnetic field and the current. Corresponding to the orientation of the magnetic field we can speak of longitudinal effects and transverse effects in the above two cases respectively. The induced current caused by external fields can be written in longitudinal and transverse direction as

\[ j^i = \sigma_{\text{Ohmic}} \delta^{ij} E_j + \sigma_{\text{Hall}} \epsilon^{ij} E_j, \]  

where \( \sigma_{\text{Ohmic}} \) and \( \sigma_{\text{Hall}} \) are the corresponding longitudinal and transverse conductivities, \( \epsilon_{ij} \) is the \( 2 \times 2 \) antisymmetric unity tensor, with \( \epsilon_{12} = -\epsilon_{21} = 1 \). Further, the induced current can be written in terms of deviation (\( \delta f \)) from \( f_0 \) as

\[ j = g \int \frac{d^3p}{(2\pi)^3} \mathbf{v} \left( q \delta f (p) + \bar{q} \delta \bar{f} (p) \right), \]  

where \( g \) is the degeneracy factor. Using Eq. (63) and (64), the Ohmic and Hall conductivity for a system of multiple charge species can be written as

\[ \sigma_{\text{Ohmic}} = \frac{1}{6\pi^2 T} \sum_f g_f q_f^2 \int \frac{dp^4}{\varepsilon_f^2} \frac{\tau_f}{(1 + \omega_c^2 \tau_f^2)} \left[ f_0^f (1 - f_0^\bar{f}) + \bar{f}_0^\bar{f} (1 - f_0^f) \right], \]  

\[ \sigma_{\text{Hall}} = \frac{1}{6\pi^2 T} \sum_f g_f q_f^2 \int \frac{dp^4}{\varepsilon_f^2} \frac{\omega_c \tau_f^2}{(1 + \omega_c^2 \tau_f^2)} \left[ f_0^f (1 - f_0^\bar{f}) - \bar{f}_0^\bar{f} (1 - f_0^f) \right], \]

where \( f \) stands for flavor and here we have used \( f = \text{up} \) (\( u \)), down (\( d \)). In the above equation \( g_f, q_f, \tau_f, f_0^f, (\bar{f}_0^\bar{f}) \) and \( \varepsilon_f \) represents the degeneracy factor, charge, relaxation time, distribution function for quark (antiquark) and single particle energy for \( f \)-th flavor respectively. As discussed in quasiparticle model, we will incorporate the quasiparticle mass which were obtained to be different for left- and right-handed chiral modes. We have analysed both the conductivities using the quasiparticle mass for left- and right-handed modes independently and added their contribution linearly. The Ohmic and Hall conductivity obtained above is defined by current which appears due to the effect of an electric and magnetic field when there is no temperature gradient or we can say isothermal Ohmic and Hall conductivity. In case of baryonic symmetry i.e. at zero quark chemical potential, the distribution function for quarks and anti-quarks become equal and hence Hall conductivity vanishes.
IV Thermal Transport Coefficients

This section is devoted to the study of thermal conductivity of hot magnetized medium. In nonrelativistic case, the heat equation is obtained by the validity of the first and second laws of thermodynamics, where the flow of heat is proportional to the temperature gradient and the proportionality factor is called the thermal conductivity.

The heat flow 4-vector is defined as the difference between energy diffusion and enthalphy diffusion [63],

\[ Q_\mu = \Delta_{\mu\alpha} T^{\alpha\beta} u_{\beta} - h \Delta_{\mu\alpha} N^\alpha, \]  

(67)

where \( \Delta_{\mu\alpha} = g_{\mu\alpha} - u_\mu u_\alpha \) is the projection operator, \( T^{\alpha\beta} \) is the energy momentum tensor, \( N^\alpha \) is the particle flow 4-vector and \( h \) is the enthalpy per particle. \( N^\alpha \) and \( T^{\alpha\beta} \) are known as the first and second moments of distribution function respectively with expressions:

\[ N^\alpha = \sum f g f \int \frac{d^3p}{(2\pi)^3} \frac{p^\alpha}{\varepsilon_f} \left[ f_f + \bar{f}_f \right], \]  

(68)

\[ T^{\alpha\beta} = \sum f g f \int \frac{d^3p}{(2\pi)^3} \frac{p^\alpha p^\beta}{\varepsilon_f} \left[ f_f + \bar{f}_f \right]. \]  

(69)

We can obtain particle number density from Eq.(68), energy density and pressure from Eq.(69) as \( n = N^\alpha u_\alpha, \varepsilon = u_\alpha T^{\alpha\beta} u_\beta \) and pressure \( P = -\Delta_{\alpha\beta} T^{\alpha\beta}/3 \). The heat flow 4-vector in rest frame of heat bath is orthogonal to fluid 4-velocity, i.e. \( Q_\mu u^\mu = 0 \). Thus, heat flow is spatial which under the action of external disturbances can be written in terms of infinitesimal changes in the distribution function as

\[ Q = \sum f g f \int \frac{d^3p}{(2\pi)^3} \frac{p}{\varepsilon_f} \left[ (\varepsilon_f - h_f) \delta f_f + (\varepsilon_f - \bar{h}_f) \delta \bar{f}_f \right]. \]  

(70)

The Navier-Stokes equation relates the heat flow with thermal potential \( (U = \mu/T) \)

\[ Q_\mu = -\kappa \frac{n T^2}{\varepsilon + P} \nabla_\mu U, \]  

(71)

where \( \kappa \) is the thermal conductivity and \( \nabla_\mu \) is the 4-gradient, \( \nabla_\mu = \partial_\mu - u_\mu u_\nu \partial^\nu \). The entropy density \( (s = s(\varepsilon, n)) \) in equilibrium state in terms of energy density, pressure and chemical potential can be written as [64],

\[ s = \left( \frac{\varepsilon + P}{T} \right) - \left( \frac{\mu}{T} \right) n. \]  

(72)

Further, the inverse of temperature \( (T^{-1}) \) and thermodynamic potential \( (U) \) can be defined as partial derivative of \( s(\varepsilon, n) \)

\[ ds = \frac{1}{T} d\varepsilon - Ud\mu. \]  

(73)
Using Eq. (72) and (73), we obtain
\[
\frac{d}{dT} \left( \frac{P}{T} \right) = -\varepsilon d \left( \frac{1}{T} \right) + ndU,
\]
\[
\frac{dP}{nT} = dU + \frac{1}{T^2} \left( \frac{P + \varepsilon}{n} \right) dT.
\] (74)

Generalizing it to the 4-gradient, the heat flow can be rewritten as
\[
Q_{\mu} = \kappa \left[ \nabla_{\mu} T - \frac{T}{\varepsilon + P} \nabla_{\mu} P \right],
\] (75)

and in local rest frame, the spatial component of heat flow can be written as
\[
Q = -\kappa \left[ \frac{T}{n \hbar} \frac{\partial P}{\partial x} \right].
\] (76)

One can thus obtain the thermal conductivity \((\kappa)\) by comparing Eq. (70) and Eq. (76). Expanding the distribution function in terms of gradients of flow velocity and temperature, the relativistic Boltzmann transport equation in relaxation time approximation can be written as
\[
p^\nu \partial_{\mu} T \left( \frac{\partial f}{\partial T} \right) + p^\nu \partial_{\mu} (p^\nu u_\nu) \left( \frac{\partial f}{\partial p^0} \right) + q \left( F^{ij} p_j \frac{\partial f}{\partial p^0} + F_{ij} p_i \frac{\partial f}{\partial p^j} \right) = -\frac{p^\nu u_{\mu}}{\tau} \delta f,
\] (77)

where \(p_0 = \varepsilon - \mu\) and for very small \(\mu\), it can be approximated as \(p_0 \approx \varepsilon\). Using the following partial derivatives
\[
\frac{\partial f_0}{\partial T} = \frac{\varepsilon}{T^2} f_0 (1 - f_0),
\] (78)
\[
\frac{\partial f_0}{\partial p^0} = -\frac{1}{T} f_0 (1 - f_0),
\] (79)
\[
\frac{\partial f_0}{\partial p^j} = -\frac{p_j}{T p_0} f_0 (1 - f_0),
\] (80)

the Eq. (77) is given as
\[
\frac{-\delta f}{\tau} = \frac{f_0 (1 - f_0)}{p^0} \left[ p^\nu \partial_{\mu} T \left( \frac{p_0}{T^2} \right) - p^\nu \partial_{\mu} (p^\nu u_\nu) \right] - \frac{q}{T} \left( F^{ij} p_j + F_{ij} p_i \right) - \frac{2 q E \cdot p}{p^0 T} \left[ \frac{\partial f}{\partial p^j} + \frac{\partial f}{\partial p^i} \right].
\]

This can be expanded further to
\[
= \frac{f_0 (1 - f_0)}{p^0} \left[ \left( p^0 \partial_0 T + p^j \partial_j T \right) \left( \frac{p_0}{T^2} \right) - \frac{(p^\nu u_\nu \partial_{\mu} p^\nu + p^\nu p^\nu \partial_{\mu} u_\nu)}{T} \right] + q (v \times B) \cdot \frac{\partial f}{\partial p},
\]

and
\[
= \frac{1}{T} f_0 (1 - f_0) \left[ \frac{1}{T} \left( p^0 \partial_0 T + p^j \partial_j T \right) + T \partial_0 \left( \frac{\mu}{T} \right) + \frac{p^j T}{p^0} \partial_j \left( \frac{\mu}{T} \right) - \frac{1}{p^0} \left( p^0 p^\nu \partial_0 u_\nu + p^j p^\nu \partial_j u_\nu \right) - \frac{q E \cdot p}{p^0 T} \right]
\]
\[
+ q (v \times B) \cdot \frac{\partial f}{\partial p},
\] (81)
where \(2F_{ij} = \epsilon_{ijk}B^k\). Now, exerting the energy-momentum conservation \((\partial_0 u^\mu = \frac{\nabla \mu}{\mathcal{h}})\) along with relativistic Gibbs-Duhem relation

\[
\partial_j \left( \frac{\mu}{T} \right) = -\frac{\hbar}{T^2} \left( \partial_j T - \frac{T}{\mathcal{h}} \partial_j P \right),
\]

we obtain Eq.(81) as

\[
\frac{-\delta f}{\tau} = \frac{1}{T} f_0 (1 - f_0) \left[ \frac{1}{T} (p^0 \partial_0 T) + \left( \frac{p^0 - \hbar}{p^0} \right) \frac{p^j}{T} \left( \partial_j T - \frac{T}{\mathcal{h}} \partial_j P \right) + T \partial_0 \left( \frac{\mu}{T} \right) - p^j \partial_\mu \left( \partial_j u^\nu \right) - 2q \frac{\mathbf{E} \cdot \mathbf{p}}{p^0} \right]
\]

\[+ q (\mathbf{v} \times \mathbf{B}) \frac{\partial f}{\partial \mathbf{p}},\]  

(82)

where \(\frac{\partial f}{\partial \mathbf{p}} = \frac{\partial}{\partial \mathbf{p}} (f_0 + \delta f)\) and \(\frac{\partial f_0}{\partial \mathbf{p}} \propto v^j\), therefore the Lorentz term vanishes for the equilibrium distribution function and we get,

\[
\frac{-\delta f}{\tau} = \frac{1}{T} f_0 (1 - f_0) \left[ \frac{p_0}{T} \partial_0 T + \left( \frac{p^0 - \hbar}{p^0} \right) \frac{p^j}{T} \left( \partial_j T - \frac{T}{\mathcal{h}} \partial_j P \right) + T \partial_0 \left( \frac{\mu}{T} \right) - \frac{p^j}{p_0} (\partial_j u^\mu) - 2q \frac{\mathbf{E} \cdot \mathbf{p}}{p^0} \right]
\]

\[+ q (\mathbf{v} \times \mathbf{B}) \frac{\partial (\delta f)}{\partial \mathbf{p}}.\]  

(83)

Now, Choosing the ansatz for non-equilibrium part of distribution function as

\[
\delta f = (\mathbf{p} \cdot \chi) \frac{\partial f_0}{\partial \varepsilon},
\]

(84)

in which \(\chi\) is related to thermal driving force and magnetic field in medium and takes the form

\[
\chi = a_1 \mathbf{c} + a_2 \mathbf{Y} + a_3 (\mathbf{Y} \times \mathbf{c}).
\]

(85)

Here, \(\mathbf{c} = \frac{\mathbf{B}}{|\mathbf{B}|}\) and \(\mathbf{Y} = \nabla T - \frac{\nabla T}{\mathcal{h}}\). Using Eq.(84) and (83), we have

\[
\frac{\mathbf{p} \cdot \chi}{\tau} = \left[ \frac{p_0}{T} \partial_0 T + \left( \frac{p^0 - \hbar}{p^0} \right) \frac{\mathbf{p}}{T} \cdot \left( \nabla T - \frac{T}{\mathcal{h}} \nabla P \right) + T \partial_0 \left( \frac{\mu}{T} \right) - \frac{p^j}{p_0} (\partial_j u^\mu) - 2q \frac{\mathbf{E} \cdot \mathbf{p}}{p_0} - q (\mathbf{v} \times \mathbf{B}) \cdot \chi \right].
\]

(86)

Since, we are concentrating on thermal transport only in presence of magnetic field and considering that all thermodynamic forces are independent and hence keeping only the thermal driving forces corresponding to thermal transport in weak magnetic field, we have

\[
\frac{p^0}{\tau} \mathbf{v} \cdot (a_1 \mathbf{c} + a_2 \mathbf{Y} + a_3 (\mathbf{Y} \times \mathbf{c})) = (p^0 - \hbar) \mathbf{v} \cdot \mathbf{Y} - q (\mathbf{v} \times \mathbf{B}) \cdot a_2 \mathbf{Y} - q (\mathbf{v} \times \mathbf{B}) \cdot a_3 (\mathbf{Y} \times \mathbf{c}),
\]

(87)

where \((\mathbf{v} \times \mathbf{B}) \cdot \mathbf{c} = 0\). Using the properties of scalar triple product, the parameters \(a_1, a_2\) and \(a_3\) can be obtained by comparing the independent terms with different tensor
structures in both sides of Eq. (87), and we have,
\begin{align}
\frac{\varepsilon}{\tau} a_1 &= a_3 q |B| (c.Y), \\
\frac{\varepsilon}{\tau} a_2 &= (\varepsilon - h) - a_3 q |B|, \\
\frac{\varepsilon}{\tau} a_3 &= a_2 q |B|.
\end{align}

(88) where \( p^0 \approx \varepsilon, |B| = B \). Employing the above equations and defining \( \omega_c = \frac{q B}{\varepsilon} \), the parameters reduced to the following forms,
\begin{align}
a_1 &= \frac{\tau^3}{\varepsilon} \frac{(\varepsilon - h)}{(1 + \omega_c^2 \tau^2)} \omega_c^2 (c.Y), \\
a_2 &= \frac{\tau}{\varepsilon} \frac{(\varepsilon - h)}{(1 + \omega_c^2 \tau^2)}, \\
a_3 &= \frac{\tau^2}{\varepsilon} \frac{(\varepsilon - h)}{(1 + \omega_c^2 \tau^2)} \omega_c.
\end{align}

(90)

Substituting \( a_1, a_2, a_3 \) in Eq. (85), we obtain the non-equilibrium correction to the distribution function in the presence of weak magnetic field from Eq. (84) as,
\begin{equation}
\delta f = \frac{\tau (\varepsilon - h)}{(1 + \omega_c^2 \tau^2)} [v.Y + \tau \omega_c v.(Y \times c) + \tau^2 \omega_c^2 (c.Y)(v.c)] \frac{\partial f_0}{\partial \varepsilon}.
\end{equation}

(92)

Similarly, \( \delta f \) can be calculated as
\begin{equation}
\delta \bar{f} = \frac{\tau (\varepsilon - \bar{h})}{(1 + \omega_c^2 \tau^2)} [v.Y - \tau \omega_c v.(Y \times c) + \tau^2 \omega_c^2 (c.Y)(v.c)] \frac{\partial \bar{f}_0}{\partial \varepsilon}.
\end{equation}

(93)

Using Eq. (92) and (93) in (70), the heat flow in weakly magnetized medium, generalizing to system of different charged particles takes the form as
\begin{equation}
Q = \sum_f g_f \tau_f \int \frac{d^3 p}{(2\pi)^3 \varepsilon_f} \left[ \frac{(\varepsilon_f - h_f)^2}{(1 + \omega_c^2 \tau_f^2)} (v.Y + \tau_f \omega_c v.(Y \times c) + \tau_f^2 \omega_c^2 (c.Y)(v.c)) \frac{\partial f_0}{\partial \varepsilon_f} + \frac{(\varepsilon_f - \bar{h}_f)^2}{(1 + \omega_c^2 \tau_f^2)} (v.Y - \tau_f \omega_c v.(Y \times c) + \tau_f^2 \omega_c^2 (c.Y)(v.c)) \frac{\partial \bar{f}_0}{\partial \varepsilon_f} \right].
\end{equation}

(94)

Simplifying the analysis by fixing the direction of \( B \) along z-axis and temperature gradient in x-y plane. Under this condition, heat flow assumes the form
\begin{equation}
Q = -\kappa_0 T Y - \kappa_1 T (Y \times c),
\end{equation}

(95) where thermal transport coefficients in weakly magnetized medium, \( \kappa_0 \) and \( \kappa_1 \), can be defined as,
\begin{equation}
\kappa_0 = \sum_f g_f \tau_f \int \frac{d^3 p}{6\pi^2 T^2} \left[ \frac{(\varepsilon_f - h_f)^2}{(1 + \omega_c^2 \tau_f^2)} f_f^0(1 - f_f^0) + \frac{(\varepsilon_f - \bar{h}_f)^2}{(1 + \omega_c^2 \tau_f^2)} \bar{f}_f(1 - \bar{f}_f) \right].
\end{equation}

(96)
and

\[ \kappa_1 = \sum_f \frac{g_f \tau_f^2}{6 \pi^2 T^2} \int \frac{dp}{\varepsilon_f} \left[ \frac{(\varepsilon_f - h_f)^2 \omega_c}{f_f^0 (1 - f_f^0)} - \frac{(\varepsilon_f - \bar{h}_f)^2 \omega_c}{(1 + \omega_c^2 \tau_f^2)} f_f^0 (1 - f_f^0) \right], \quad (97) \]

where \( f \) stands for \( f \)-th flavor. The thermal conductivity is obtained from heat current in temperature gradient on the condition that there is no electric current [65]. Similar to the discussion of charge transport coefficients, here we have longitudinal (\( \kappa_0 \)) and Hall-type thermal conductivity (thermal Hall conductivity) (\( \kappa_1 \)), where \( \kappa_1 \) vanishes for zero quark chemical potential. The thermal Hall conductivity emerges due to the transverse temperature gradient which is induced by the action of magnetic field perpendicular to longitudinal heat current. It is the thermal analog of classical Hall effect where temperature plays the role of voltage and heat flow replaces the electric current [66] and it is the Lorentz force acting on charged particles affecting the curvature of carrier’s trajectories through the magnetic field. Similar to the charge transport coefficients, we have embodied the quasiparticle masses for left- and right-handed chiral modes and taken their contribution linearly for the estimation of \( \kappa_0 \) and \( \kappa_1 \).

V Results and Discussions

In this section, we will discuss the results regarding the Ohmic and Hall conductivity, thermal and Hall-type thermal conductivity and further Knudsen number and Wiedemann-Franz law as their application.

V.A Electrical and Hall conductivity

In Fig. (1a), we have shown the variation of Ohmic conductivity with respect to temperature at different constant values of magnetic field at finite chemical potential (\( \mu = 30 \text{ MeV} \)). The increasing behaviour of \( \sigma_{\text{Ohmic}} \) with temperature could be due to the Boltzmann factor \( \exp(-\varepsilon(p)/T) \) in the distribution function. It is also shown that \( \sigma_{\text{Ohmic}} \) decreases with increase in magnetic field because with increasing magnetic field more particles are deviated from the direction of electric field, hence reduction in the Ohmic current. Fig. (1b) shows the variation of Hall conductivity with respect to temperature in which \( \sigma_{\text{Hall}} \) decreases with temperature at different constant values of magnetic field at finite chemical
potential ($\mu=30$ MeV). At zero chemical potential, number of quarks and antiquarks are same and their contribution to the Hall current is same but opposite in direction. So, the net Hall current vanishes at zero chemical potential and can be explicitly seen in Eq. (66). $\sigma_{\text{Hall}}$ increases with magnetic field and this can be attributed to the $\omega_c$ factor in the numerator of Eq. (66).

Fig. (2a) and (2b) shows the variation of Ohmic and Hall conductivity at different values of chemical potential at finite magnetic field ($eB = \frac{m^2}{\pi}$) respectively. Both the conductivities increase with increase in quark chemical potential as number density of quarks is larger than the antiquarks; hence contribution from the quarks is larger than antiquarks. With increasing quark chemical potential the Boltzmann factor $\exp(\pm \mu/T)$ increases, leading to the increasing behaviour of Ohmic and Hall conductivity. The interplay between Hall and Ohmic conductivity is shown in Fig. (3) and we found that Ohmic conductivity is dominant over Hall conductivity for the temperature range considered. We can say that, the dominance of Ohmic conductivity is more pronounced at higher temperature making the effects of Hall conductivity negligible but as the system cools down, Hall conduction...
Figure 3: Variation of ratio of \( \sigma_{\text{Hall}}/\sigma_{\text{Ohmic}} \) with temperature at different values of magnetic field.

may have non-negligible effects.

V.B Thermal conductivity

Figure 4: Variation of \( \kappa_0 \) and \( \kappa_1 \) with temperature at different fixed values of magnetic field.

Fig. (4) shows the variation of \( \kappa_0 \) and \( \kappa_1 \) with temperature at different values of magnetic field at finite chemical potential (\( \mu = 30 \) MeV). The coefficient \( \kappa_0 \) decreases with magnetic field due to the deviation of charge particle by Lorentz force. This deflected motion of particles further leads to the transverse component of thermal conductivity \( \kappa_1 \) in direction perpendicular to magnetic field and initial thermal driving force. \( \kappa_1 \) shows the non-monotonic behaviour with temperature as it first decreases say up to \( \sim 200 \) MeV and then further rise with temperature. At lower temperature regime, \( \tau^2 \omega_c \) is the dominant factor due to large relaxation time which leads to the decreasing behaviour of \( \kappa_1 \) with temperature. Whereas at high temperature, relaxation time is small and further increase
in temperature amplifies the random motion of particles and factor \((\varepsilon_f - h_f)^2\) starts to dominate leading to increasing behaviour of \(\kappa_1\) with temperature. The baryon asymmetry further amplifies both the coefficients as shown in Fig.(5). The interplay between thermal and Hall-type thermal conductivity is shown in Fig.(6) and we found the significant dominance of \(\kappa_0\) over \(\kappa_1\) for the entire temperature range at different values of magnetic field.

Figure 5: Variation of \(\kappa_0\) and \(\kappa_1\) with temperature at different fixed values of quark chemical potential.

Figure 6: Variation of ratio of \(\kappa_1/\kappa_0\) with temperature at different fixed values of magnetic field.
V.C Knudsen Number

The applicability of ideal hydrodynamic requires local thermal equilibration. The degree of thermalization in fluid produced in heavy ion collision can be characterized by dimensionless parameter which is termed as Knudsen number \((\Omega)\), which is the ratio of microscopic length scale to the macroscopic length scale \([67]\). The small value of Knudsen number implies the large number of collisions which bring the system back to local equilibrium. \(\Omega_0\) (associated to \(\kappa_0\)) and \(\Omega_1\) (associated to \(\kappa_1\)) is found to be less than unity in presence of weak magnetic field at finite chemical potential \((\mu=30 \text{ MeV})\) as shown in Fig.(7), thus ensures the system to be in thermal equilibrium.

V.D Wiedemann-Franz law

The interplay between charge and thermal transport coefficients can be understood via Wiedemann-Franz law. The temperature behaviour of Lorenz number \((\kappa_0/\sigma_{\text{Ohmic}}T)\) is plotted in Fig.(8a), where it does not remain constant rather it increases with temperature. Whereas, Lorenz number is constant for metals suggesting that metals are good conductor of heat and electricity unlike the hot QCD matter where the effect of thermal conductivity is more pronounced than Ohmic conductivity suggesting that hot QCD matter is good conductor of heat than charge. Hall Lorenz number \((\kappa_1/\sigma_{\text{Hall}}T)\) also shows same temperature behaviour along the transverse direction to thermal driving force and magnetic field as shown in Fig.(8b). From Fig.(8a) and (8b), we can infer that hot QCD matter is different from metals and also violates the Wiedemann-Franz law.
VI Conclusions

In this work, we have studied the charge and thermal transport coefficients in hot QCD matter in presence of weak magnetic field at finite chemical potential, where interactions are incorporated through effective masses using quasiparticle description. In presence of weak magnetic field, some unusual phenomenological aspects came into light in transport phenomena. We found that the left- and right-handed chiral modes get separated and become non-degenerate contrary to the strong magnetic field. Also, the transport coefficients adopts the tensorial structure whose diagonal and off-diagonal elements gives the longitudinal and transverse conductivity respectively. We have examined the transport coefficients using the effective quasiparticle mass for left- and right-handed chiral modes and taken their linear contribution. We have studied the variation of Ohmic and Hall conductivity where Ohmic decreases with magnetic field and Hall conductivity increases with magnetic field. This is due to the change in curvature of carrier’s trajectory under the action of Lorentz force. Additionally, both the conductivities increases with quark chemical potential and Hall conductivity vanishes at zero quark chemical potential due to equal and opposite contribution of quarks and anti-quarks. Analogous to Ohmic and Hall conductivity, we have studied the longitudinal and transverse thermal conductivity (thermal Hall conductivity). The thermal Hall conductivity is the manifestation of transverse temperature gradient under the action of Lorentz force. $\kappa_0$ decreases with magnetic field whereas $\kappa_1$ increases with magnetic field. Further, $\kappa_0$ shows the monotonic behaviour with temperature unlike $\kappa_1$ where it first decreases in lower temperature regime and then increases. Both the conductivities gets amplified with quark chemical potential and $\kappa_1$ vanishes at vanishing quark chemical potential. To get the further insight of the longitudinal and transverse conductivities, we have also examined the interplay of $\sigma_{\text{Hall}}/\sigma_{\text{Ohmic}}$. 

Figure 8: Variation of Lorenz number and Hall Lorenz number with temperature at different fixed values of magnetic field.
and $\kappa_1/\kappa_0$ with respect to temperature where dominance of longitudinal conductivity is very much prominent at higher temperature. In application of aforementioned conductivities, we have investigated the equilibrium property through Knudsen number ($\Omega_0$ and $\Omega_1$) where it is found to be less than unity ensuring the system to be in thermal equilibrium. Further, the relative behaviour of charge and thermal transport coefficients has been studied via Wiedemann-Franz law. Lorenz and Hall Lorenz number increases with temperature, hence violating the Wiedemann-Franz law.

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**Appendices**

**A: CALCULATION OF STRUCTURE FUNCTIONS**

Here, we will show the computation of structure functions from Eq.(20) to (23) in one-loop order for hot and weakly magnetized medium under HTL approximation. Since, trace of odd number of gamma matrices is zero, the Eq.(13) can be written as

$$A = \frac{1}{4} \left[ \text{Tr}(\Sigma^{B=0} P) - (P.u)\text{Tr}(\Sigma^{B=0} \gamma) \right] \frac{(P.u)^2 - P^2}{(P.u)^2 - P^2}, \quad \text{(A.98)}$$

where,

$$\Sigma^{B=0} = g^2 C_F T \sum_n \int \frac{d^3 k}{(2\pi)^3} \frac{k}{K^2 - m^2} \gamma_{\mu} \gamma_{\nu} \frac{1}{(P - K)^2}. \quad \text{(A.99)}$$

Using the following two traces:

$$\text{Tr} \left[ \gamma_{\mu} K \gamma_{\nu} P \right] = -8K.P, \quad \text{(A.100)}$$

$$\text{Tr} \left[ \gamma_{\mu} K \gamma_{\nu} \gamma \right] = -8K.u, \quad \text{(A.101)}$$

we obtain,

$$A(P) = \frac{1}{4|p|^2} g^2 C_F \left[ I_1(P) + I_2(P) \right], \quad \text{(A.103)}$$

where $(P.u)^2 - P^2 = |p|^2$. We will use the frequency sum to evaluate $I_1(P)$ and $I_2(P)$ with $k_0 = i\omega_n$, $p_0 = i\omega$, $E_1 = \sqrt{k^0 + m^2}$ and $E_2 = \sqrt{(p - k)^2}$. The frequency sum for
fermion-boson case is [71]

\[
T \sum_n \tilde{\Delta}_{s_1}(i\omega_n, E_1)\Delta_{s_2}(i(\omega - \omega_n), E_2) = \sum_{s_1, s_2 = \pm 1} \frac{s_1 s_2}{4E_1 E_2} \frac{1 - \tilde{f}(s_1 E_1) + f(s_2 E_2)}{i\omega - s_1 E_1 - s_2 E_2}.
\]  
(A.104)

The leading \(T^2\) behaviour will come from \(s_1 = -s_2 = 1\) with \(E_1 \approx k\) and \(E_2 = |p - k|\). Defining light-like four-vector \(\hat{K} = (-i, \hat{k})\) and \(\hat{K'} = (-i, -\hat{k})\), we have,

\[
i\omega + E_1 - E_2 \simeq i\omega + p.\hat{k} = P.\hat{K},
\]
(A.105)

\[
i\omega - E_1 + E_2 \simeq i\omega - p.\hat{k} = P.\hat{K'},
\]
(A.106)

and using the angular integration under HTL approximation,

\[
\int \frac{d\Omega}{4\pi} \frac{\hat{K}.u}{P.\hat{K}} = \frac{1}{|p|} Q_0 \left( \frac{p_0}{|p|} \right),
\]
(A.107)

we get,

\[
A(p_0, |p|) = \frac{m_f^2}{|p|^2} Q_1 \left( \frac{p_0}{|p|} \right).
\]
(A.108)

Similarly, structure function \(B\) can be evaluated as

\[
B(p_0, |p|) = -\frac{m_f^2}{|p|^2} \left[ \frac{p_0}{|p|} Q_1 \left( \frac{p_0}{|p|} \right) - Q_0 \left( \frac{p_0}{|p|} \right) \right].
\]
(A.109)

Using Eq.(11) in (15) and (16), where the contribution from \(\Sigma^{B=0}\) vanishes due to the trace of odd no. of gamma matrices and we get the non-vanishing contribution form \(\Sigma^{B\neq0}\) only and hence we get,

\[
C(p_0, |p|) = \frac{1}{4} \mathrm{Tr}(\gamma_5 \Sigma^{B\neq0} \gamma_\mu \gamma_\nu),
\]
(A.110)

\[
D(p_0, |p|) = \frac{1}{4} \mathrm{Tr}(\gamma_5 \Sigma^{B\neq0} \gamma_\mu). \tag{A.111}
\]

Using the following two traces

\[
\mathrm{Tr} [\gamma_5 \gamma_\mu \gamma_5 ((K.n)\gamma_\mu - (K.u)\gamma_\mu) \gamma^\nu \gamma_\nu] = 8(K.n),
\]
(A.112)

\[
\mathrm{Tr} [\gamma_5 \gamma_\mu \gamma_5 ((K.n)\gamma_\mu - (K.u)\gamma_\mu) \gamma^\nu] = 8(K.n),
\]
(A.113)

we obtain,

\[
C = \frac{g^2 C_F |q_f B|}{4} T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{8(K.n)}{(K^2 - m_f^2)^2(P - K)^2},
\]
(A.114)

\[
D = -\frac{g^2 C_F |q_f B|}{4} T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{8(K.u)}{(K^2 - m_f^2)^2(P - K)^2},
\]
(A.115)
which in turn requires the calculation of frequency sum [72]

\[ Y = T \sum_n \Delta^2_F(K) \Delta_B(P - K), \]  

(A.116)

\[ = \left( \frac{-\partial}{\partial m_f^2} \right) T \sum_n \Delta_F(K) \Delta_B(P - K), \]

where,

\[ T \sum_n \Delta_F(K) \Delta_B(P - K) = \sum_{s_1,s_2=\pm 1} \frac{s_1s_2}{4E_1E_2} \left( 1 - \tilde{f}(s_1E_1) + f(s_2E_2) \right). \]  

(A.117)

For \( s_1 = -s_2 = 1 \), we get,

\[ C = \frac{4g^2C_F|q_fB|}{16\pi^2} \left( \frac{\pi T}{2m_f} - \ln 2 + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} \right) \left[ \frac{-p_z|p_f}{|p_f|^2} Q_1 \left( \frac{p_0}{|p_f|} \right) \right], \]  

(A.118)

\[ D = -\frac{4g^2C_F|q_fB|}{16\pi^2} \left( \frac{\pi T}{2m_f} - \ln 2 + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} \right) \left[ \frac{1}{|p_f|} Q_0 \left( \frac{p_0}{|p_f|} \right) \right]. \]  

(A.119)

(A.120)

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