Robust Controller for Permanent Magnet Linear Motor Based on Interval Matrix Theory

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Abstract. Parameter perturbation, end effect and external interference of AC permanent magnet linear motor affect the accuracy and performance of servo system. In this paper, the PMLSM servo system controller designed by the interval matrix theory is used to solve this problem. The uncertainty of servo system is described by interval matrix. Compared with the precise mathematical model, the controller designed by this theory has strong robustness. The simulation and experiment of the system show that the strategy has excellent dynamic and static performance, it has strong parameter robustness.

1. Introduction
Compared with the traditional rotary motor servo system, Permanent magnet linear synchronous motor (PMLSM) servo system has long service life, fast response speed and high precision. There is no transmission component between the permanent magnet synchronous motor and the load, so the external interference and load change will directly affect the dynamic performance of the system. In addition, the control accuracy and stability of the servo system are also affected by the end effect of permanent magnet linear synchronous motor. In the modern high performance direct drive linear servo system, the traditional PID controller cannot meet the system requirements [1-3].
In this paper, the PMLSM servo system is affected by load change, external disturbance, parameter perturbation and end effect. The servo system controller is designed by using interval matrix. The system designed by this method has the advantages of rapidity and simple implementation. The system is robust to the changes of electrical parameters, parameter uncertainty, mathematical model uncertainty and external disturbance. In this way, the control precision of direct drive AC linear servo system is improved. The static and dynamic performance is improved.

2. Mathematical Model of AC PMLSM Servo System
PMLSM is a thrust device which can convert alternating current energy into linear motion directly. The inner current loop of linear servo system adopts stator flux oriented vector control technology. The current vector of PMLSM is orthogonal to the stator magnetic field, and makes \( i_d = 0 \). In this way, the model of the servo system in coordinate system \( d - q \) is [2, 4].
\begin{align}
\begin{cases}
   u_d = -\omega_e L_q i_q \\
   u_q = R_s i_q + L_q \frac{di_q}{dt} + \omega_e \psi_f
\end{cases} 
\end{align} 
(1)

\[ F_e = K_f i_q = \frac{\pi}{\tau} \Phi f i_q = m \frac{dv}{dt} + \mu_B v + F_L \] 
(2)

\[ L = \int_0^t v dt \] 
(3)

The thrust coefficient is expressed in \( K_f \); the viscous friction coefficient is expressed in \( \mu_B \); the mass of the mover is expressed in \( m \); the linear velocity is expressed in \( v \); the electromagnetic thrust is expressed in \( F_e \); the effective flux of the permanent magnet is expressed in \( \Phi_f \); the pole distance is expressed in \( \tau \); the load resistance is expressed in \( F_L \); the mechanical displacement is expressed in \( L \).

Considering the fluctuation of system parameters and external disturbance, the system model is established as follows:

\[ F_e = (m + \Delta m) \frac{dv}{dt} + (\mu_B + \Delta \mu_B) v + F_L + F_d \] 
(4)

In the equation, the offset of parameter \( m \) is expressed by \( \Delta m \); the offset of parameter \( \mu_B \) is expressed by \( \Delta \mu_B \); the equivalent resistance caused by the end effect of PMLSM is expressed as \( F_d \).

Define the generalized perturbation as

\[ f = \Delta m \frac{dv}{dt} + \Delta \mu_B v + F_L + F_d \] 
(5)

The following figure shows the AC linear servo system, in which the controller can be designed as PI regulator, sliding mode variable structure regulator or Robust Regulator designed in this paper.

Figure 1. Diagram of AC PMLSM servo system.

Specifies that the state variable: \( x_1(t) \) is the position of the PMLSM, \( x_2(t) \) is the speed of the PMLSM, \( x_3(t) \) is the acceleration of the PMLSM, assume \( df/dt = \delta(x_1, x_2, x_3) \).

Available from Equation (1) (2) (3) (4) (5)

\[
\begin{bmatrix}
   \dot{x}_1 \\
   \dot{x}_2 \\
   \dot{x}_3
\end{bmatrix}
= 
\begin{bmatrix}
   0 & 1 & 0 \\
   0 & 0 & 1 \\
   0 & \frac{K_f^2 + \mu_B R_q}{mL_q} & -\mu_B \frac{R_q}{mL_q}
\end{bmatrix}
\begin{bmatrix}
   x_1 \\
   x_2 \\
   x_3
\end{bmatrix}
+ 
\begin{bmatrix}
   0 \\
   0 \\
   \frac{K_f}{mL_q}
\end{bmatrix}
\frac{d}{dt} \begin{bmatrix}
   x_1 \\
   x_2 \\
   x_3
\end{bmatrix}
+ 
\begin{bmatrix}
   0 \\
   0 \\
   -\frac{1}{m} \delta(x_1, x_2, x_3)
\end{bmatrix}
\] 
(6)
When $\delta(x_1, x_2, x_3) = 0$ (without considering the generalized disturbance), the controller of system (6) can be designed by linear quadratic regulator (LQR).

For a given real symmetric matrix $R, Q$. There is a feedback gain vector $K = -R^{-1}B^TP$, where $P$ satisfies the Riccati equation $A^TP + PA - PBR^{-1}B^TP + Q = 0$. So the controller is:

$$U = [u_1, u_2]^T = -KX$$

However, in fact, disturbance is inevitable, so LQR method cannot be used to design controller when considering generalized disturbance.

3. Robust Controller Design of Servo System Based on Interval Matrix

When considering generalized disturbance ($\delta(x_1, x_2, x_3) \neq 0$), System (6) can be described as

$$\dot{X} = (A + \Delta A)X + BU, \text{ Where } \Delta A \text{ is the parameter undetermined interval matrix describing the system uncertainty.}$$

According to the literature [5], the solution of system (6) control law is as follows:

$$M^TP + PM - PBR^{-1}B^TP + Q = 0 \quad (7)$$

In the equation, the $R, Q$ represent real symmetric matrix, the $P$ represent the solution of Riccati algebraic equation (7), and $PBR^{-1}B^TP \geq 0$ holds. Where $M = [(A + A^T)/2] + E_{\Delta A}$ ($E_{\Delta A}$ belongs to matrix set $\Psi = \{(\Delta A_1 + \Delta A^T)/2, (\Delta A_2 + \Delta A^T)/2, ..., (\Delta A_d + \Delta A^T)/2\}$), $d$ is the vertices of the $\Delta A$. Then $U = -R^{-1}B^TX$ makes the closed-loop asymptotic stability of the system.

Prove as follows:

Let $M = M - BR^{-1}B^TP$, then:

$$P\left(\frac{M + M^T}{2}\right) + \left(\frac{M + M^T}{2}\right)P + Q$$

$$= P\left(\frac{M + M^T}{2} - \frac{BR^{-1}B^TP + PBR^{-1}B^T}{2}\right) + \left(\frac{M + M^T}{2} - \frac{BR^{-1}B^TP + PBR^{-1}B^T}{2}\right)P + Q$$

$$= PM + M^TP - PBR^{-1}B^TP - \frac{PBR^{-1}B^TP + PBR^{-1}B^T}{2} + Q$$

$$\leq PM + M^TP - PBR^{-1}B^TP + Q = 0$$

So $\frac{M + M^T}{2}$ is stable,

Re $\lambda\left(\frac{M + M^T}{2}\right) < 0$

Set $M_0 = A + \Delta A$, $i = 1, 2, \cdots, n$, and $\partial_i = \lambda_i\left(\frac{M + M^T}{2}\right)$, $\eta_i = \lambda_i\left(\frac{M_0 + M_0^T}{2}\right)$,

$$\varepsilon_{\max} = \max \{ \lambda_i(\frac{\Delta A + \Delta A^T}{2}) \} \quad \varepsilon_{\min} = \min \{ \lambda_i(\frac{\Delta A + \Delta A^T}{2}) \}$$

$$\therefore \quad M = M_0 + (E_{\Delta A} - \Delta A) \therefore \quad \frac{M + M^T}{2} = M_0 + M_0^T + (E_{\Delta A} - \Delta A)$$

According to the literature [6]: $\eta_i + \varepsilon_{\max} \leq \partial_i \leq \eta_i + \varepsilon_{\min}$

$\therefore \eta_i < 0$, And every element of matrix $(E_{\Delta A} - \Delta A)$ is nonnegative, then $0 \leq \varepsilon_{\min} \leq \varepsilon_{\max}$,

$\therefore \eta_i < 0$, and then Re $\lambda\left(\frac{M + M^T}{2}\right) < 0$, so $\dot{X} = (A + \Delta A)X + BU$ stability. q. e. d..

In reference [7], the minimum upper bound of interval matrix is defined, and the solution of the minimum upper bound is given. If there are N uncertain parameters in $\Delta A$, then $\Delta A$ has $2^N$ vertices.
The matrix $E_{\Delta t}$ is used to deal with the uncertainty $(\Delta A_t + \Delta A_t^T)^2/2$. This not only improves the possibility of the system having a solution, but also reduces the size of the matrix $P$.

Consider the uncertainty of the system (6), $\Delta(x_i, x_j, x_k) \neq 0$.

Assume $\delta(x_i, x_j, x_k) = \Delta(x_i, x_j) - \lambda x_j$, $|\Delta(x_i, x_j)| \leq \beta_1|x_i| + \beta_2|x_j|

In the formula, $\lambda \in [-\alpha, +\alpha]$, $\beta_1, \beta_2 \in [-\beta, +\beta]$.

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & \frac{-K_f^2 + \mu_B R_q}{m L_q} \\
0 & \frac{\mu_B}{m} & \frac{R_q}{L_q}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & \beta_1 - \lambda & \beta_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
\frac{K_f}{m L_q}
\end{bmatrix}
u
$$

According to the theorem:

$$
E_{\Delta t} = \begin{bmatrix}
0 & 0 & \frac{\beta}{2} \\
0 & 0 & -\frac{\alpha}{2} \\
\frac{\beta}{2} & -\frac{\alpha}{2} & \beta
\end{bmatrix},
M = [(A + A^T)/2] + E_{\Delta t} = \begin{bmatrix}
0 & 1 & \frac{1}{2} & \frac{\beta}{2} \\
1 & 0 & \frac{1}{2} & \frac{1 - \alpha}{2} \frac{K_f^2 + \mu_B R_q}{2m L_q} \\
\frac{\beta}{2} & \frac{1 - \alpha}{2} & \frac{1}{2} & \frac{-K_f^2 + \mu_B R_q}{2m L_q} \\
\frac{\beta}{2} & \frac{1 - \alpha}{2} & \frac{K_f^2 + \mu_B R_q}{2m L_q} & \frac{\mu_B}{m} - \frac{R_q}{L_q}
\end{bmatrix}
$$

For the selected matrix $R, Q$, the solution $P_0$ of equation (7) is obtained. According to the above conclusion, it can be deduced that the control law of system (6) is $U = -R^{-1}B'P_0X$.

4. Simulation Studies

For the PMLSM servo system shown in Figure 1, the robust controller is compared with the traditional PI controller by using MATLAB. The simulation parameters are: $T = 0.5ms$; $m = 11kg$; $\mu_B = 8N \cdot s/m$; $F_{eN} = 100N$; $K_f = 28.5N/A$; $L_q = 41.4mH$; $R_q = 2\Omega$; $v_N = 1.0m/s$.

Selected weighting matrix $Q = \text{diag}(600, 100, 800)$; $R = \begin{bmatrix} 1000 \end{bmatrix}$; $\alpha = 0.2$; $\beta = 2.2$.

$$
M = [(A + A^T)/2] + E_{\Delta t} = \begin{bmatrix}
0 & 0.5 & 1.1 \\
0.5 & 0 & -1.16 \\
1.1 & -1.16 & -46.8
\end{bmatrix}; B = \begin{bmatrix}
0 \\
0 \\
62.6
\end{bmatrix}
$$

In simulation, the given setting is $2m$, that is $L' = 2m$. It can be seen from equation (4) that the generalized disturbance includes load, friction, and equivalent resistance due to the variation of system parameters (mass and viscous friction coefficient), etc. Therefore, the generalized disturbance can be simulated by adding different load resistance $F_L$.

Under $t = 0.4s$ sudden load $F_L = 50N$, Figure 2 (a) (b) shows the speed response of traditional PI controller and robust controller, respectively. Fig. 2 shows that the speed drop and overshoot are very large in PI controller, while the speed is hardly affected in robust controller.
Figure 2. Speed response curve of the system (constant disturbance). The simulation curve of the speed response when the time varying load \( F_L = 30 \sin(25t)N \) is imposed is shown in Fig. 3. The results show that the PI controller cannot suppress the time-varying resistance disturbance, and the system speed hardness and control performance are poor. When the robust controller is used, the time-varying disturbance can be effectively suppressed. It shows that the robust controller is robust to parameter perturbation and disturbance.

Figure 3. Speed response curve of the system (Time-varying load resistance). The simulation of parameter perturbation assumes that the mass \( M \) of the mover changes, and here it simulates the situation when \( M \) becomes 1.5 times. The simulation curve with parameter perturbation is shown in Fig. 4. The velocity response curve before and after parameter perturbation is expressed by 1 and 2. Fig. 4 (a) (b) shows the speed response curves of PI control and robust control with parameter perturbation.

Figure 4. Speed response curve of the system (Parameter perturbation). From Fig. 4, when the servo system has parameter perturbation, the speed response of the traditional PI controller has overshoot, which is obviously unable to adapt. The speed response curves of the robust controller are basically consistent before and after the parameter perturbation.
5. Conclusion
The robust controller of AC permanent magnet linear synchronous motor servo system is designed by interval matrix method, and compared with traditional PI controller. Simulation results verify the effectiveness of the proposed control strategy. Compared with traditional PI control, it has superior static and dynamic performance, not only has strong parameter robustness, but also can eliminate the thrust ripple caused by the end effect. The design method is easy to implement.

Acknowledgments
Liaoning Provincial Department of education Basic Research Project (project number: L2017LFW009) funded the research work of this project.

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