A Heavy-Light Chiral Quark Model

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We present a new chiral quark model for mesons involving a heavy and a light
(anti-)quark. The model relates various combinations of a quark-meson coupling
constant and loop integrals to physical quantities. Then, some quantities may be
predicted and some used as input. The extension from other similar models is that
the present model includes the lowest order gluon condensate of the order \((300\ MeV)^4\)
determined by the mass splitting of the \(0^-\) and the \(1^-\) heavy meson states.
Within the model, we find a reasonable description of parameters such as the decay
constants \(f_B\) and \(f_D\), the Isgur-Wise function and the axial vector coupling \(g_A\) in
chiral perturbation theory for light and heavy mesons.

I. INTRODUCTION

While the short distance (SD) effects in hadronic physics are well understood within
perturbative quantum chromodynamics (pQCD), long distance (LD) effects have been hard
to pin down. Lattice gauge theory should be able to solve the problem, but the calculations
are often very difficult to perform. QCD sum rules might also give the answer in some cases.
Still, in some cases it has been fruitful to use various QCD inspired models and assumptions.
In the light quark sector, low energy quantities have been studied in terms of the (extended)
Nambu-Jona-Lasinio model (NJL)\(^1\) and the chiral quark model (\(\chiQM\))\(^2\), which is the
mean field approximation of NJL.

In the \(\chiQM\), the light quarks \((u, d, s)\) couple to the would be Goldstone octet mesons
\((K, \pi, \eta)\) in a chiral invariant way, such that all effects are in principle calculable in terms of
physical quantities and a few model dependent parameters, namely the quark condensate,
the gluon condensate and the constituent quark mass \[3, 4, 5\]. More specific, one may calculate the coupling constants of chiral Lagrangians in this way.

In this paper we will extend the ideas of the chiral quark model of the pure light sector \[2, 3, 4, 5\] to the sector involving a heavy quark (c or b) and thereby a heavy meson. Such ideas have already been presented in previous papers \[2, 4, 8, 9\]. Also in this case, one may calculate the parameters of chiral Lagrangian terms, where the description of heavy mesons are in accordance with heavy quark effective field theory (HQEFT). In the present paper we will extend the ideas of \[2, 4, 5, 9\] to include gluon (vacuum) condensates. The motivation for the inclusion of gluon condensates is that this works well in order to understand the \(\Delta I = 1/2\) rule for \(K \to 2\pi\) within the \(\chi\)QM \[3, 5\] and within generalized factorization \[10\]. Furthermore, it allows us to consider effects related to the gluonic aspect of \(\eta'\) as considered in \[11\], some aspects of \(D\)-meson decays \[12\], and also to calculate gluon condensate contributions to \(B - \bar{B}\)-mixing \[13\].

Having established our heavy - light chiral quark model (HL\(\chi\)QM), we can integrate out the light and heavy quarks and obtain chiral Lagrangians involving light and heavy mesons \[14, 15\]. Chiral perturbation theory (\(\chi\)PT) based on such Lagrangians works in the pure strong sector. In order to define the model and its parameters, we have to integrate out the quarks, and we will find some typical divergent loop integrals. We will relate all divergent loop integrals to some physical parameters, as was done in \[7\]. This means in particular that we will treat quadratic, linear, and logarithmic divergences as different. If we need to calculate a divergent integral we will do so in dimensional regularization, although various regularization procedures might be used. Thus, the regularizing prescription for divergent diagrams is to be regarded as a part of the model. Still, even if integrals are divergent, the effective UV cut-off scale is, as for \(\chi\)PT, the chiral symmetry breaking scale \(\Lambda_\chi \approx 1\) GeV, where also the matching of pQCD and HL\(\chi\)QM is performed. This is also considered a part of our model construction.

The paper is organized as follows: In the next section \[II\] we describe the HL\(\chi\)QM, and in section \[III\] we consider bosonization in the strong sector and of the weak current respectively. In section \[IV\] we discuss the relations between physical and model dependent parameters. In section \[V\] we discuss the Isgur-Wise function, and in section \[VI\] we bosonize the \(1/m_Q\) terms. Section VII contains the presentation of some necessary chiral corrections for our numerical analysis. Finally in section \[VII\] we discuss our results. Loop integrals are
listed in Appendix A. In Appendix B we list some transformation properties of the involved fields.

II. THE HEAVY - LIGHT CHIRAL QUARK MODEL (HL\chi QM)

Our starting point is the following Lagrangian containing both quark and meson fields:

$$
L = L_{HQEFT} + L_{\chi QM} + L_{int} ,
$$

where $L_{HQEFT} = \overline{Q}_v i\gamma^\mu D_\mu Q_v + \frac{1}{2m_Q} \overline{Q}_v \left( -C_M \frac{g_s}{2} \sigma \cdot G + (iD_\perp)_{\text{eff}}^2 \right) Q_v + \mathcal{O}(m_Q^{-2})$ is the Lagrangian for heavy quark effective field theory (HQEFT). The heavy quark field $Q_v$ annihilates a heavy quark with velocity $v$ and mass $m_Q$. Moreover, $D_\mu$ is the covariant derivative containing the gluon field (eventually also the photon field), and $\sigma \cdot G = \sigma^{\mu\nu} G_{\mu\nu}^a t^a$, where $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, $G_{\mu\nu}^a$ is the gluonic field tensors, and $t^a$ are the colour matrices. This chromo-magnetic term has a factor $C_M$, being one at tree level, but slightly modified by perturbative QCD. (When the covariant derivative also contains the photon field, there is also a corresponding magnetic term $\sim \sigma \cdot F$, where $F^{\mu\nu}$ is the electromagnetic tensor). Furthermore, $(iD_\perp)_{\text{eff}}^2 = C_D (iD)^2 - C_K (iv \cdot D)^2$. At tree level, $C_D = C_K = 1$. Here, $C_D$ is not modified by perturbative QCD, while $C_K$ is different from one due to perturbative QCD corrections [17].

The light quark sector is described by the chiral quark model ($\chi QM$), having a standard QCD term and a term describing interactions between quarks and (Goldstone) mesons:

$$
L_{\chi QM} = \overline{q} (i\gamma^\mu D_\mu - \mathcal{M}_q) q - m(\overline{q}_R \Sigma^\dagger \overline{q}_L + \overline{q}_L \Sigma q_R) ,
$$

where $q^T = (u, d, s)$ are the light quark fields. The left- and right-handed projections $q_L$ and $q_R$ are transforming after $SU(3)_L$ and $SU(3)_R$ respectively. $\mathcal{M}_q$ is the the current quark mass matrix, $m$ is the ($SU(3)$ - invariant) constituent quark mass for light quarks, and $\Sigma = \exp(i \sum_j \lambda^j \pi^j)$ is a 3 by 3 matrix containing the (would be) Goldstone octet $(\pi, K, \eta)$ :

$$
\xi = e^{i\Pi/f} \quad \text{where} \quad \Pi = \frac{\chi_a}{2} \phi^a(x) = \frac{1}{\sqrt{2}} \begin{bmatrix} \pi_0^0 + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi_0^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \frac{K^\ast}{\sqrt{2}} & -\frac{2}{\sqrt{6}} \eta_8 \end{bmatrix} .
$$
The $\chi$QM has a “rotated version” with flavour rotated quark fields $\chi$ given by:

$$
\chi_L = \xi^\dagger q_L \quad ; \quad \chi_R = \xi q_R \quad ; \quad \xi \cdot \xi = \Sigma .
$$

(5)

In the rotated version, the chiral interactions are rotated into the kinetic term while the interaction term proportional to $m$ in (3) become a pure (constituent) mass term $^2, ^3$:

$$
\mathcal{L}_{\chiQM} = \bar{\chi} \left[ \gamma^\mu (i D_\mu + \mathcal{V}_\mu + \gamma_5 \mathcal{A}_\mu) - m \right] \chi - \bar{\chi} \tilde{M}_q \chi ,
$$

(6)

where the vector and axial vector fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are given by:

$$
\mathcal{V}_\mu \equiv \frac{i}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \quad ; \quad \mathcal{A}_\mu \equiv - \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) ,
$$

(7)

and $\tilde{M}_q$ defines the rotated version of the current mass term:

$$
\tilde{M}_q \equiv \tilde{M}_q^V + \tilde{M}_q^A \gamma_5 ,
$$

(8)

$$
\tilde{M}_q^V \equiv \frac{1}{2} (\xi^\dagger \mathcal{M}_q \xi^\dagger + \xi \mathcal{M}_q \xi) \quad \text{and} \quad \tilde{M}_q^A \equiv - \frac{1}{2} (\xi^\dagger \mathcal{M}_q \xi^\dagger - \xi \mathcal{M}_q \xi) .
$$

(9)

Here $L$ is the left - handed projector in Dirac space, $L = (1 - \gamma_5)/2$ and $R$ is the corresponding right - handed projector, $R = (1 + \gamma_5)/2$ . The Lagrangian (3) is manifest invariant under the unbroken symmetry $SU(3)_V$. In the light - sector, the various pieces of the strong Lagrangian can be obtained by integrating out the constituent quark fields $\chi$, and these pieces can be written in terms of the fields $\mathcal{A}_\mu$, $\tilde{M}_q^V$ and $\tilde{M}_q^A$ which are manifest invariant under local $SU(3)_V$ transformations. For instance, the standard $O(p^2)$ kinetic term may (up to a constant) be written as $Tr [\mathcal{A}^\mu \mathcal{A}_\mu]$. This is easily seen by using the relations

$$
\mathcal{A}_\mu = - \frac{1}{2i} \xi (D_\mu \Sigma^\dagger) \xi = + \frac{1}{2i} \xi^\dagger (D_\mu \Sigma) \xi^\dagger ,
$$

(10)

where $D_\mu$ is a covariant derivative containing the photon field. The vector field $\mathcal{V}_\mu$ transforms as a gauge field under local $SU(3)$, and can only appear in combination with a derivative as a covariant derivative ($i\partial^\mu + \mathcal{V}_\mu$).

In the heavy - light case, the generalization of the meson - quark interactions in the pure light sector $\chi$QM is given by the following $SU(3)$ invariant Lagrangian:

$$
\mathcal{L}_{Int} = - G_H \left[ \bar{\chi}_a \bar{H}_v^a Q_v + \bar{Q}_v H_v^a \chi_a \right] + \frac{1}{2 G_3} Tr \left[ \bar{H}_v^a H_v^a \right] ,
$$

(11)

where $G_H$ and $G_3$ are coupling constants, and $H_v^a$ is the heavy meson field containing a spin zero and spin one boson:

$$
H_v^a \equiv P_+ (P_\mu^a \gamma^\mu - i P_5^a \gamma_5) ,
$$

$$
\bar{H}_v^a = \gamma^0 (H_v^a)^\dagger \gamma^0 = \left[ (P_\mu^a)^\dagger \gamma^\mu - i (P_5^a)^\dagger \gamma_5 \right] P_+ ,
$$

(12)
FIG. 1: Feynman rule for the light quark - soft gluon vertex.

where

$$P_\pm = (1 \pm \not{v})/2.$$  \hspace{1cm} (13)

are projection operators. The index \( a \) runs over the light quark flavours \( u, d, s \), and the projection operators have the property

$$P_\pm \gamma^\mu P_\pm = \pm P_\pm v^\mu P_\pm .$$  \hspace{1cm} (14)

Note that in [6, 7, 8, 9], \( G_H = 1 \) is used. However, in that case one used a renormalization factor for the heavy meson fields \( H \), which is equivalent.

The fields \( P_a(P_\mu) \) annihilates a heavy-light meson, \( 0^- (1^-) \), with velocity \( v \). The interaction term in the Lagrangian (equation (11)) can, as for the \( \chi^Q M \), be obtained from an NJL model. In the NJL model one starts with the Lagrangian for free quarks and four quark operators, thought to be generated by gluon exchange(s). Taking the heavy quark limit for heavy quarks one can obtain (after some manipulation) the interaction term (11) from the four quark operator. This has been done in [7] (as for the light sector [1]).

In our model, the hard gluons are thought to be integrated out and we are left with soft gluonic degrees of freedom. These gluons can be described using the external field technique, and their effect will be parameterized by vacuum expectation values, i.e. the gluon condensate \( \langle \alpha_s \pi G^2 \rangle \). Gluon condensates with higher dimension could also be included, but we truncate the expansion by keeping only the condensate with lowest dimension.

When calculating the soft gluon effects in terms of the gluon condensate, we follow the prescription given in [18]. The calculation is easily carried out in the Fock - Schwinger gauge. In this gauge one can expand the gluon field as:

$$A^a_\mu(k) = -\frac{i(2\pi)^4}{2} G_{\rho\mu}(0) \frac{\partial}{\partial k^\rho} \delta^{(4)}(k) + \cdots .$$  \hspace{1cm} (15)

Since each vertex in a Feynman diagram is accomplished with an integration we get the
Feynman rule given in figure 1. The gluon condensate is obtained by the replacement
\[ g_s^2 G^a_{\mu\nu} G^b_{\alpha\beta} \rightarrow \frac{4\pi^2}{(N_c^2 - 1)} \delta^{ab} \left( \frac{\alpha_s}{\pi} G^2 \right) \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}). \] (16)

We observe that soft gluons coupling to a heavy quark is suppressed by \(1/m_Q\), since to leading order the vertex is proportional to \(v_\mu v_\nu G^{\mu\nu} = 0\), \(v_\mu\) being the heavy quark velocity.

III. BOSONIZATION WITHIN THE HL\(\chi\)QM

The interaction term \(L_{\text{Int}}\) in (11) can now be used to bosonize the model, i.e. integrate out the quark fields. This can be done in the path integral formalism and the result is formally a functional determinant. This determinant can be expanded in terms of Feynman diagrams, by attaching the external fields \(H_a^\mu, \overline{H}_a^\mu, \gamma^\mu, A^\mu\) and \(\overline{M}_q^{V,A}\) of section II to quark loops. Some of the loop integrals will be divergent and have to, as for the pure light sector \([2, 3, 4, 5]\), to be related to physical parameters. The strong chiral Lagrangian has the following form \([14, 15, 19, 20, 21, 22, 23]\):

\[ L_{\text{Str}} = -(1 + \frac{\varepsilon_1}{m_Q}) \text{Tr} [\overline{H}_a (i v \cdot D) H_a] + (\Delta_Q + \frac{\delta_Q}{m_Q}) \text{Tr} [\overline{H}_a H_a] \]
\[ + (1 + \frac{\varepsilon_1}{m_Q}) \text{Tr} [\overline{H}_a H_b v_\mu V^\mu_{ba}] - (g_A - \frac{g_1}{m_Q}) \text{Tr} [\overline{H}_a H_b \gamma_\mu \gamma_5 A^\mu_{ba}] \]
\[ + 2\lambda_1 \text{Tr} [\overline{H}_a H_b (\overline{M}_q^V)_{ba}] + 2\lambda'_1 \text{Tr} [\overline{H}_a H_a] (\overline{M}_q^V)_{bb} + \ldots \]
\[ - \frac{\lambda_2}{4m_Q} \text{Tr} [\overline{H}_a \sigma^{\alpha\beta} H_a \sigma_{\alpha\beta}] + \frac{\varepsilon_2}{m_Q} \text{Tr} [\overline{H}_a \sigma^{\alpha\beta} i v \cdot D H_a \sigma_{\alpha\beta}] \]
\[ - \frac{\varepsilon_2}{m_Q} \text{Tr} [\overline{H}_a \sigma^{\alpha\beta} v_\mu V^\mu_{ba} \sigma_{\alpha\beta} H_b] + \frac{g_2}{m_Q} \text{Tr} [\overline{H}_a \gamma_\mu \gamma_5 A^\mu_{ba} H_b] + \ldots \] (17)

where the ellipses indicate other terms (of higher order, say), and \(D_\mu\) contains the photon field. The \(1/m_Q\) terms will be discarded in this section, but will be considered later in section VI. Note that the term proportional to \(\Delta_Q \equiv M_H - m_Q\) is absent in most articles considering the mesonic aspects only. The quantities \(\Delta_Q, \lambda_1, \lambda_2\) and \(\delta_Q\) are related to the masses of the heavy mesons. The trace runs over the gamma matrices (only). Note that in some conventions there is an extra factor \(M_H\) at the right-hand side in (17) (our notation is the same as used in \([3, 7, 8, 14, 15]\)).

The Feynman diagrams responsible for the kinetic term and the mass difference (self energy) term \(\sim \Delta_Q\) in the meson Lagrangian is shown in figure 2. A calculation of these
two diagrams (at zero external heavy meson momentum), leads to the identification
\[ -i G_H^2 N_c (-I_1 + m I_{3/2} + \frac{K_2}{N_c} \frac{\alpha_s}{\pi} G^2) - \frac{1}{2 G_3} = -\Delta Q , \]  
(18)

where we have added the last term of (14). Keeping the heavy meson momentum to first order (subsequently to be interpreted as the derivative of the heavy meson field), we obtain the identification for the kinetic term:
\[ -i G_H^2 N_c (I_{3/2} + 2m I_2 + \frac{K_1}{N_c} \frac{\alpha_s}{\pi} G^2) = 1 . \]  
(19)

We have denoted the divergent integrals by \( I \) and the finite integrals by \( \kappa \). They are defined in Appendix A. The quadratic -, linear - and logarithmic - divergent integrals are denoted \( I_1 \), \( I_{3/2} \), and \( I_2 \) respectively.

The relation (19) is also obtained by comparison of the loop integral for diagram in figure 3 with the vector field \( V_\mu \) attached to the light quark. This must be so because of the relevant Ward identity, but it is also realized by explicit calculation. (Note that the total covariant derivative is \( i D_\mu = i D_\mu + V_\mu \) in the quark sector and \( i D_\mu = i D_\mu - V_\mu \) in the meson sector since the meson fields transforms as an anti triplet under \( SU(3)_V \). See Appendix B for details)

From the same diagram, with the axial field \( A_\mu \) attached, we obtain the following identification for the axial vector coupling \( g_A \) :
\[ g_A \equiv i G_H^2 N_c \left( \frac{1}{3} I_{3/2} - 2 m I_2 + \frac{4}{3} \kappa_0 - \frac{K_1}{N_c} \frac{\alpha_s}{\pi} G^2 \right) , \]  
(20)

and similarly when attaching \( \tilde{M}_Y^{\lambda'} \)
\[ 2 \lambda_1 \equiv i G_H^2 N_c (I_{3/2} - 2 m I_2 + 2 \kappa_0 - \frac{K_2}{N_c} \frac{\alpha_s}{\pi} G^2) , \]  
(21)

Within the full theory (SM) at quark level, the weak current is :
\[ J_f^\alpha = \bar{q}_f \gamma^\alpha Q \]  
(22)
where $Q$ is the heavy quark field in the full theory. Within HQEFT this current will, below the renormalization scale $\mu = m_Q (= m_b, m_c)$, be modified in the following way:

$$J_\alpha^f = \chi_a \xi^\dagger_a f \Gamma^\alpha_Q + \mathcal{O}(m_Q^{-1}) ,$$  \hspace{1cm} (23)

where $\Gamma^\alpha \equiv C_\gamma(\mu) \gamma^\alpha L + C_v(\mu) v^\alpha R$. \hspace{1cm} (24)

The coefficients $C_\gamma, v(\mu)$ are determined by QCD renormalization for $\mu < m_Q$. They have been calculated to NLO and the result is the same in $\overline{MS}$ and $MS$ scheme \cite{24}. Corrections to the weak current of order $1/m_Q$ will be discussed in section \cite{VI}.

The operator in equation (23) can be bosonized by calculating the Feynman diagrams shown in figure 4:

$$J_\alpha^f (\text{Bos}) = J_\alpha^f (0) + J_\alpha^f (1) + \cdots$$ \hspace{1cm} (25)
The currents $J_\alpha^f (0)$ and $J_\alpha^f (1)$ correspond to zero and one axial field attached to the loop, the dots represents terms with more boson fields and gluon condensates. We obtain to zero order in the axial field and first order in the gluon condensate:

$$J_\alpha^f (0) = \frac{\alpha_H}{2} Tr \left[ \xi_h^1 \Gamma^\alpha H_{vh} \right] ,$$

where

$$\alpha_H \equiv -2iG_H N_c \left( -I_1 + m I_{3/2} + \frac{\kappa_2}{N_c} \left( \frac{\alpha_s}{\pi} G^2 \right) \right) .$$

Note that this expression is also related to $1/G_3$ in eq. (18). However, in the present case only $G_H$ in first power is involved.

To next order in the chiral expansion we obtain the current

$$J_\alpha^f (1) = \frac{1}{2} Tr \left[ \xi_h^1 \Gamma^\alpha H_{vh} (\alpha^{(1)}_H \gamma^\nu \gamma_5 + \alpha^{(1)}_{Hv} \gamma^\nu \gamma_5) A_\nu \right]$$

where the quantities $\alpha^{(1)}_H$ are given by

$$\alpha^{(1)}_{H\gamma} \equiv 2iG_H N_c \left( \frac{1}{3} I_{3/2} + \frac{4}{3} \kappa_0 - 2m I_2 + \frac{\kappa_3}{N_c} \left( \frac{\alpha_s}{\pi} G^2 \right) \right) \quad (29)$$

$$\alpha^{(1)}_{Hv} \equiv 2iG_H N_c \left( -\frac{2}{3} I_{3/2} - \frac{4}{3} \kappa_0 + \frac{\kappa_5}{N_c} \left( \frac{\alpha_s}{\pi} G^2 \right) \right) \quad (30)$$

We observe that these quantities, as $f_H$ in (33), contains only one power of the coupling $G_H$, in contrast to the strong sector.

The physical meaning of the $\alpha_H$’s becomes more clear by considering their contributions to the physical quantities $f_H$, $f_{H^*}$, and the semileptonic form factors $f_\pm (q^2)$. The coupling $\alpha_H$ in (26) is related to the physical decay constants $f_H$ and $f_{H^*}$, in the following way (for...
Taking the trace over the gamma matrices in (26), we obtain a relation for \( \alpha_H \) and the relations between the heavy meson decay constants \( f_H \) and \( f_H^* \) (for \( H = B, D \)):

\[
\alpha_H = \frac{f_H \sqrt{M_H}}{C_\gamma(\mu)} + \frac{f_H^* \sqrt{M_H^*}}{C_v(\mu)},
\]

where the model dictates us to put \( \mu = \Lambda \chi \). The form factors \( f_+(q^2) \) and \( f_-(q^2) \) are defined as:

\[
\langle \pi^+ (p_\pi) | \overline{u} \gamma^\alpha (1 - \gamma_5) b | H \rangle = 2 \langle \pi^+ (p_\pi) | J_\gamma^a | H \rangle = f_+(q^2) (p_H + p_\pi)^\alpha + f_-(q^2) (p_H - p_\pi)^\alpha.
\]

where \( p_H^\alpha = M_H v^\alpha \) and the index \( a \) corresponds to quark flavour \( u \) and \( q^\alpha = p_H^\mu - k_\pi^\mu \).

The diagrams in figure 5 contributes to the \( H \to \pi \) transition. \( J_\gamma^a(0) \) and \( J_\gamma^a(1) \) is responsible for the diagram to the left and the diagram to the right involve the strong Lagrangian term \( \sim g_A \) in (17). A calculation of the diagrams gives:

\[
f_+(q^2) + f_-(q^2) = \frac{-1}{\sqrt{2M_H f_\pi}} (C_\gamma + C_v - g_A C_\gamma) \alpha_H,
\]

\[
f_+(q^2) - f_-(q^2) = -C_\gamma \frac{\sqrt{M_H}}{\sqrt{2f_\pi}} \frac{g_A \alpha_H}{v \cdot k_\pi} + \alpha_H^{(1)}
\]

where we have neglected terms of first order in \( v \cdot k_\pi \) (where \( \alpha_H^{(1)} \) contributes). This means that the equations are only valid near the “no-recoil point”, where \( v \cdot k_\pi \to 0 \) and \( q^2 \to q_{\text{Max}}^2 = (M_H^2 + m_\pi^2) \). From equation (35) and (36) we see that \( (f_+(q^2) + f_-(q^2))/(f_+(q^2) - f_-(q^2)) \sim 1/M_H \), which is the well known Isgur-Wise scaling law [25]. (The 1/v \cdot k_\pi term in (36) is due to the \( H^* \) pole).
IV. CONSTRAINING THE PARAMETERS OF THE HLχQM

Within the pure light sector the quadratic and logarithmic divergent integrals are related to $f_\pi$ and the quark condensate in the following way \[3, 4, 5, 26\]:

$$f_\pi^2 = -i4m^2NcI_2 + \frac{1}{24m^2}\left(\frac{\alpha_s}{\pi}G^2\right),$$

(37)

$$\langle \bar{q}q \rangle = -4imNcI_1 - \frac{1}{12m}\left(\frac{\alpha_s}{\pi}G^2\right),$$

(38)

which are obtained by relating loop diagrams to physical quantities as for the eqs. (19) and (20). (Here the a priori divergent integrals $I_{1,2}$ have to be interpreted as the regularized ones) As the pure light sector is a part of our model, we have to keep these relations in the heavy light case studied here. In addition, in the heavy-light sector the linearly divergent integral $I_{3/2}$ will also appear. As $I_1$ and $I_2$ are related to the quark condensate and $f_\pi$ respectively, the (formally) linearly divergent integral $I_{3/2}$ is related to $\delta g_A \equiv 1 - g_A$, which is found by eliminating $I_2$ from eqs. (19) and (20) :

$$\delta g_A = -\frac{4}{3}iG_H^2Nc\left(I_{3/2} + \kappa_0\right).$$

(39)

Note that the gluon condensate drops out here. Within a primitive cut-off regularization, $I_{3/2}$ is (in the leading approximation) proportional to the cut-off in first power \[3\]. Within dimensional regularization it is finite. We will keep $I_{3/2}$ as a free parameter to be determined by the physical value of $g_A$.

Eliminating $I_{3/2}$ from the eqs. (19) and (20) and inserting the expression for $I_2$ obtained from (37) we find the following expression for $G_H$ :

$$G_H^2 = \frac{m(1 + 3g_A)}{2f_\pi^2 + \frac{m^2Nc}{4\pi} - \frac{m}{m^2}\left(\frac{\alpha_s}{\pi}G^2\right)}, \text{ where } \eta_1 \equiv \frac{\pi}{32}.\ (40)$$

Note that $G_H$ has dimension (mass)$^{-1/2}$.

In order to constrain the parameters further, we will consider the parameters $\lambda_1$, $\lambda_2$ and $\delta_Q$ related to the meson masses. The gluon condensate can be related to the chromomagnetic interaction :

$$\mu_G^2 (H) = \frac{1}{2M_H}\frac{C_M(\mu)}{H}\langle H|\bar{Q}v\frac{1}{2}\sigma \cdot GQv |H\rangle,$$

(41)

where the coefficient $C_M(\mu)$ contains the short distance effects down to the scale $\mu$ and has been calculated to next to leading order (NLO) \[27, 28\], and can be found in table \[I\] (2M_H}
is a normalization factor. The chromomagnetic operator is responsible for the splitting between the \(1^-\) and \(0^-\) state, and is known from spectroscopy,

\[
\mu_G^2(H) = 3\lambda_2 = \frac{3}{2}m_Q(M_{H^*} - M_H).
\] (42)

An explicit calculation of the matrix element in equation (41) gives

\[
\mu_G^2 = \eta_2 \frac{G_H^2}{m} \langle \frac{\alpha_s}{\pi} G^2 \rangle, \quad \text{where} \quad \eta_2 \equiv \frac{(\pi + 2)}{32}C_M(\Lambda_\chi).
\] (43)

Combining eq. (40) and eq. (43) we get the following relations:

\[
\langle \frac{\alpha_s}{\pi} G^2 \rangle = \mu_\pi^2 \frac{f_\pi^2}{\rho} \eta_2, \quad G_H^2 = \frac{2m}{f_\pi^2} \rho,
\] (44)

where the quantity \(\rho\) is of order one and given by

\[
\rho \equiv \frac{(1 + 3g_A) + \frac{\eta_3 \mu_\pi^2}{\eta_2 m^2}}{4(1 + \frac{N_c m^2}{8\pi f_\pi^2})}.
\] (45)

In the limit where only the leading logarithmic integral \(I_2\) is kept in (19), we obtain:

\[
g_A \to 1, \quad \rho \to 1, \quad G_H \to G_H^{(0)} \equiv \frac{\sqrt{2m}}{f_\pi}.
\] (46)

Note that \(g_A = 1\) is the non-relativistic value [15].

The quantity \(\delta_Q\) is found from the kinetic term as:

\[
\delta_Q = \mu_\pi^2 \equiv \frac{1}{2M_H}(H[Q_v(D_\perp)^2 Q_v]H), \quad \text{where} \quad (D_\perp)^2 \equiv D^2 - C_K(v \cdot D)^2
\] (47)

This quantity can easily be calculated in our model, and a direct calculation gives the expression:

\[
\mu_\pi^2 = iG_H^2 N_c m^2 \left\{ I_{3/2} + 2m I_2 + \frac{\kappa_1}{N_c} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right\} + \eta_3 \frac{G_H^2}{m} \langle \frac{\alpha_s}{\pi} G^2 \rangle
\]

\[
+ \frac{1}{4} C_K G_H^2 \left\{ -4imN_c I_1 - \frac{1}{12} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right\}, \quad \text{where} \quad \eta_3 = \frac{5}{48} + \frac{\pi}{64}.
\] (48)

Note that the \((v \cdot \partial)^2\) part of the kinetic term cancels the two heavy quark propagators, and the light quark condensate (38) appears naturally. Eliminating the divergent integrals, using equation (19), (38) and (43), some of the \(G_H^2\)’s can be included in physical parameters, and we obtain

\[
\delta_Q = \mu_\pi^2 = \frac{\eta_3}{\eta_2} \mu_G^2 - m^2 - \frac{1}{4} C_K \langle \bar{q}q \rangle G_H^2.
\] (49)
Note that the last term (originating from \((v \cdot D)^2 Q_v\) at quark level) gives a vanishing contribution in the free quark limit when \(G_H \to 0\). Using the expressions for \(\langle \frac{\alpha_s}{\pi} G^2 \rangle\) and \(G_H^2\) in (44), we find an expression for \(g_A\) in terms of \(m\), \(\mu^2\) and \(\langle \bar{q}q \rangle\). However, as \(\mu^2\) is not very well known, and \(C_K\) is known only to leading logarithmic approximation, we will not try to determine \(g_A\) by this relation.

From eqs (21), (37) and (43), we find

\[2\lambda_1 = \frac{1}{2} (3g_A - 1) - \frac{(9\pi - 16)\mu^2_G}{384\eta_2 m^2}.\]  

(50)

In the limit (46) we obtain \(2\lambda_1 \to 1\), as expected. The parameter \(\lambda_1\) is related to the mass difference \(M_{H_u} - M_{H_d}\). Unfortunately, this cannot be used at the present stage to constrain the parameters in our model because this quantity has large chiral corrections [23].

Using equation (19) and (38) we can write \(\alpha_H\) as:

\[\alpha_H = \frac{G_H}{2} \left( -\frac{\langle \bar{q}q \rangle}{m} - 2f^2_\pi \left( \frac{1 - \rho}{\rho} \right) + \frac{(\pi - 2)}{16m^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \right),\]

(51)

or equivalently, using the eqs. (38), (39) and (41) as

\[G_H \alpha_H = \frac{3}{2} m (1 - g_A) + \left( \frac{3 \pi + 4}{192 \eta_2} \right) \frac{\mu^2_G}{m} + G_H^2 \left( -\frac{\langle \bar{q}q \rangle}{2m} + \frac{m^2}{8\pi} \right).\]

(52)

Note that the relation (33) gives \(f_H > f_{H^*}\), which is not correct experimentally. Adding \(1/m_Q\) corrections we correctly reproduce \(f_{H^*} > f_H\) [21].

Combining (33) with (21), we obtain [12] in the leading limit (taking into account the logarithmic and quadratic divergent integrals only, and let \(C_\gamma \to 1, C_v \to 0\) and \(g_A \to 1\) as in (46) ):

\[f_H \sqrt{M_H} \to -\frac{\langle \bar{q}q \rangle}{f_\pi \sqrt{2m}},\]

(53)

which gives the scale for \(f_H\) (It is, however, numerically a factor 2 off for the \(B\)-meson).

Using the relations in equation (19), (21) and (37) we obtain for \(\alpha_{H\gamma}^{(1)}\) and \(\alpha_{Hv}^{(1)}\):

\[\alpha_{H\gamma}^{(1)} = \frac{2g_A}{G_H},\]

(54)

\[\alpha_{Hv}^{(1)} = \frac{4}{3} G_H \left( \frac{f^2_\pi}{2m} \left( \frac{1 - \rho}{\rho} \right) + \left[ mN_c \frac{\pi}{8\pi} + \frac{(\pi + 8)}{256m^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \right] \right),\]

(55)

where the latter may also be written:

\[G_H \alpha_{Hv}^{(1)} = (1 - g_A) - \frac{1}{3} G_H^2 \frac{mN_c}{4\pi} + \frac{(\pi + 8)\mu^2_G}{256\eta_2 m^2},\]

(56)
Combining equation (35) and (36), we find (up to terms of first order in $v \cdot k_\pi$):

$$f_+(q^2) = -\frac{g_A f_H \sqrt{M_H M_{H^*}}}{2 \sqrt{2} f_\pi v_\perp} - C_\gamma \frac{g_A \sqrt{M_H}}{G_H \sqrt{2} f_\pi} \left\{ f_H - g_A f_{H^*} \sqrt{\frac{M_{H^*}}{M_H}} \right\} ,$$

where we have used equation (33) and (54).

**V. THE ISGUR-WISE FUNCTION**

The Isgur-Wise function $\xi(\omega)$, relates all the form factors describing the processes $B \to D(D^*)$ in the heavy quark limit:

$$\langle D| Q_{cv}^\gamma \mu Q_{bv}| B \rangle / \sqrt{M_B M_D} = \xi(\omega) (v^{\mu} + v'^{\mu}) , \quad \omega \equiv v \cdot v' ,$$

$$\langle D^*| Q_{cv}^\gamma \mu_5 Q_{bv}| B \rangle / \sqrt{M_B M_{D^*}} = \xi(\omega) \left[ v^{\mu} \varepsilon^* \cdot v - \varepsilon^{*\mu} (1 + \omega) \right] ,$$

$$\langle D^*| Q_{cv}^\gamma \mu Q_{bv}| B \rangle / \sqrt{M_B M_{D^*}} = \xi(\omega) i \varepsilon^{\mu \lambda \sigma} \varepsilon^* \nu \varepsilon^{\nu} \varepsilon^{\lambda} \varepsilon^{\sigma} ,$$

where $Q_{cv}$ and $Q_{bv}$ are the $c$ and $b$ quark fields within HQEFT. The Isgur-Wise function (IW) can be calculated straightforward by calculating the diagrams shown in figure (6). The result is

$$\xi(\omega) = \frac{2}{1 + \omega} (1 - \rho) + \rho r(\omega) ,$$

where $\rho$ is given in (45) and $r(\omega)$ is the same function appearing in loop calculations of the anomalous dimension in HQEFT:

$$r(\omega) = \frac{1}{\sqrt{\omega^2 - 1}} \ln \left( \omega + \sqrt{\omega^2 - 1} \right)$$

We see that $\xi$ is normalized to 1 at zero recoil. Note that equation (60) is only valid in the limit of equal gluon condensate and the coupling $G_H$ in the $B$ and $D$ sector. The derivative
of the IW function at zero recoil is:

$$\left. \frac{\partial \xi}{\partial w} \right|_{w=1} = -\frac{1}{2} + \frac{1}{6} \rho$$

(62)

In the limit (46), the IW function takes the simple form:

$$\xi(\omega) = r(\omega) \quad \text{and} \quad \left. \frac{\partial \xi}{\partial w} \right|_{w=1} = -\frac{1}{3}$$

(63)

Numerically, the full result (60) is only a few percent away from (63). Adding the short distance QCD effects to (60) will slightly modify our result [27]. For the IW function describing $B \to D^*$ transition, the derivative at $\omega = 1$ gets a contribution $-0.07$ from QCD corrections such that $\xi'(0) \simeq -0.40$.

VI. $1/m_Q$ CORRECTIONS WITHIN THE HL$\chi$QM

A. Bosonization of the strong sector

The new terms of order $1/m_Q$ in (17) are a consequence of the chromomagnetic interaction (the second term in equation (2)), which break heavy quark spin symmetry, and the kinetic term (the third term in (2)). Calculating the diagrams of figure 7 gives the following identifications:

$$\varepsilon_1 = -iG_H^2 N_c \left( I_1 - mI_{3/2} + 2m^2I_2 - m\kappa_0 ight)$$

$$+ \frac{C_K}{4} \left( I_1 - m^2I_2 \right) + \frac{\kappa_5}{N_c} \frac{\alpha_s}{\pi} G^2$$

(64)

$$g_1 = -iG_H^2 N_c \left( -\frac{1}{3}I_1 + mI_{3/2} + \frac{4}{3}m^2I_2 + m\kappa_0 - \frac{2}{3}\tilde{\kappa}_0 \right)$$

$$+ \frac{C_K}{4} \left( I_1 + 3m^2I_2 \right) + \frac{\kappa_5}{N_c} \frac{\alpha_s}{\pi} (2 + C_K) \frac{\alpha_s}{\pi} G^2$$

(65)

$$g_2 = C_M(\Lambda_\chi) \left( \frac{\pi + 4}{192m^2} G_H^2 \frac{\alpha_s}{\pi} G^2 \right)$$

; \quad \varepsilon_2 = -\frac{g_2}{2} .

(66)

As the $1/m_Q$ terms break heavy quark spin symmetry, the chiral Lagrangian in (17) will split in $H(0^-)$ and $H^*(1^-)$ terms respectively. However, by allowing for a flavour and spin dependent renormalization constants we can write the Lagrangian in the compact form:

$$\mathcal{L} = -Tr \left[ \overline{H}_a(i\nu \cdot \mathcal{D}_{ba} - \Delta_Q)^t H_b^* \right] - \tilde{g}_A Tr \left[ \overline{H}_a^t H_b^* \gamma_\mu \gamma_5 A_\mu^{ba} \right] ,$$

(67)
where \( iD^\mu_{ba} = i\delta_{ba}D^\mu - \nabla^\mu_{ba} \) and we have redefined the \( H \) fields as \( H = H^r\sqrt{Z_H}, \) where \( Z_H \) and the new coupling \( \tilde{g}_A \) are now defined:

\[
Z^{-1}_H = 1 + \frac{\varepsilon_1 - 2d_M\varepsilon_2}{m_Q}, \quad \Delta^r_Q = Z_H \left( \Delta_Q + \frac{\mu^2}{m_Q} \right)
\]

\[
\tilde{g}_A = g_A \left( 1 - \frac{1}{m_Q}(\varepsilon_1 - 2d_A\varepsilon_2) \right) - \frac{1}{m_Q}(g_1 - d_Ag_2)
\]

where

\[
d_M = \begin{cases} 
3 & \text{for } 0^- \\
-1 & \text{for } 1^- 
\end{cases} \quad d_A = \begin{cases} 
1 & \text{for } H^*H \text{ coupling} \\
-1 & \text{for } H^*H^* \text{ coupling} 
\end{cases}
\]

The last term in equation (17) gives a splitting in the mass of the \( H^*(1^-) \) and \( H(0^-) \) state. This term can be absorbed in a redefinition of \( \Delta_Q \), namely \( \Delta_Q \rightarrow \Delta_Q - d_M\Delta_H/4 \), where \( \Delta_H \equiv M_{H^*} - M_{H} \) (Note that \( \mu^2 = 3m_Q\Delta_H/2 \)).

Eliminating the divergent integrals in (65) and (66) using equations (37) (38) and (39),

FIG. 7: Diagrams responsible for \( 1/m_Q \) terms in the chiral Lagrangian
we can rewrite \( \varepsilon_1, g_1 \) and \( g_2 \) as:

\[
\varepsilon_1 = -m + G_H^2 \left( \frac{\langle \bar{q}q \rangle}{4m} + f_\pi^2 + \frac{N_c m^2}{16\pi} + \frac{C_K}{16}\left( \frac{\langle \bar{q}q \rangle}{m} - f_\pi^2 \right) + \frac{\eta_5}{m^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \right)
\]

where \( \eta_5 = \frac{1}{128} (C_K + 8 - 3\pi) \) (71)

\[
g_1 = m - G_H^2 \left( \frac{\langle \bar{q}q \rangle}{12m} + \frac{f_\pi^2}{6} + \frac{N_c m^2 (3\pi + 4)}{48\pi} - \frac{C_K}{16}\left( \frac{\langle \bar{q}q \rangle}{m} + 3f_\pi^2 \right) + \frac{\eta_6}{m^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \right), \quad \text{where} \quad \eta_6 = \frac{1}{64} (C_K - 2\pi),
\]

(72)

\[
g_2 = \frac{(\pi + 4)}{2m} \mu_\pi, \quad \mu_\pi = (\pi + 2) \frac{\mu^2}{m}, \quad \mu_\pi^2 = (\pi + 2) \frac{\mu^2}{m},
\]

(73)

The masses of the particles now before SU(3) breaking terms are taken into account

\[
M(0^-) = m_Q + \Delta_Q + \frac{(\mu_\pi^2 - \mu_G^2)}{m_Q}
\]

(74)

\[
M(1^-) = m_Q + \Delta_Q + \frac{(3\mu_\pi^2 + \mu_G^2)}{3m_Q}.
\]

(75)

**B. Bosonization of the weak current**

In HQEFT the weak vector current at order \( 1/m_Q \) is [13]:

\[
J_{\nu}^\alpha = \sum_{i=1,2} C_i(\mu) J_i^\alpha + \frac{1}{2m_Q} \sum_j B_j(\mu) O_j^\alpha + \frac{1}{2m_Q} \sum_k A_k(\mu) T_k^\alpha,
\]

(76)

where the first terms are given in (23) and (24), the \( B_j \)'s and \( A_j \)'s are Wilson coefficients, and the \( O_j^\alpha \)'s are two quark operators

\[
O_1^\alpha = \bar{q}_L \gamma^\alpha D Q_v, \quad O_4^\alpha = \bar{q}_L \gamma^\alpha (-i v \cdot \vec{D}) Q_v,
\]

\[
O_2^\alpha = \bar{q}_L v^\alpha i D Q_v, \quad O_5^\alpha = \bar{q}_L v^\alpha (-i v \cdot \vec{D}) Q_v,
\]

\[
O_3^\alpha = \bar{q}_L i D^\alpha Q_v, \quad O_6^\alpha = \bar{q}_L (-i \vec{D}^\alpha) Q_v,
\]

(77)

The operators \( T_k \) are nonlocal and is a combination of the leading order currents \( J_i \) and a term of order \( 1/m_Q \) from the effective Lagrangian [2]:

\[
\frac{T_1^\alpha}{2m_Q} = i \int dy^4 T \{ J_1^\alpha(0), O_{\text{kin}}(y) \},
\]

\[
\frac{T_2^\alpha}{2m_Q} = i \int dy^4 T \{ J_2^\alpha(0), O_{\text{kin}}(y) \},
\]

\[
\frac{T_3^\alpha}{2m_Q} = i \int dy^4 T \{ J_1^\alpha(0), O_{\text{mag}}(y) \},
\]

\[
\frac{T_4^\alpha}{2m_Q} = i \int dy^4 T \{ J_2^\alpha(0), O_{\text{mag}}(y) \},
\]

(78)
where
\[ O_{\text{kin}} \equiv \frac{1}{2m_Q} Q_v (i D_\perp)_{\text{eff}}^2 Q_v , \quad \text{and} \quad O_{\text{mag}} \equiv - \frac{g_s}{4m_Q} Q_v \sigma \cdot G Q_v . \] (79)

Bosonizing the \( O^\alpha_j \) operators in (77) we obtain:
\[ \frac{1}{2} Tr \{ \tilde{\Gamma}^{\alpha \mu} H \xi^\dagger(\alpha_3^\gamma \gamma_\mu + \alpha_3^v v_\mu) \} \] (80)
where \( \tilde{\Gamma}^{\alpha \mu} \) is defined :
\[ \tilde{\Gamma}^{\alpha \mu} = B_1 \gamma^\alpha L \gamma_\mu + B_2 \gamma^\alpha R \gamma_\mu + B_3 g^{\alpha \mu} R \]
+ \[ B_4 \gamma^\alpha L v_\mu + B_5 \gamma^\alpha R v_\mu + B_6 g^{\alpha \mu} R \] (81)
The coefficients \( \alpha_3^\gamma \) and \( \alpha_3^v \) can be written after the use of (37) (38) and (39):
\[ \alpha_3^\gamma = 2G_H m_3 \left\{ \frac{f_2^2}{2m} \left( \frac{1}{\rho} - 1 \right) + \frac{\pi - 2}{64m^3} \left( \frac{\alpha_s}{\pi} G^2 \right) \right\} = \frac{m}{3} \alpha_H + \frac{G_H}{6} \langle \bar{q}q \rangle \] (82)
\[ \alpha_3^v = \alpha_3^\gamma + \frac{\langle \bar{q}q \rangle}{2} G_H = \frac{m}{3} \alpha_H + \frac{2}{3} G_H \langle \bar{q}q \rangle \] (83)
Bosonization of these non-local operators is straightforward within our model and the result is
\[ \langle 0 | (A_1 T_1^\alpha + A_2 T_2^\alpha) | H \rangle = - \frac{\mu^2_G}{G_H} Tr \{ \xi^\dagger \Gamma^\alpha H_v \} \] (84)
\[ \langle 0 | (A_3 T_3^\alpha + A_4 T_4^\alpha) | H \rangle = \frac{\mu^2_G d_M}{3G_H} Tr \{ \xi^\dagger \Gamma^\alpha H_v \} , \] (85)
where \( \Gamma^\alpha \) is given in (24).

When \( 1/m_Q \) terms are included, we find that (33) is modified in the following way:
\[ f_H \sqrt{M_H} = \alpha_H (C_\gamma + C_v)(1 - \frac{\varepsilon_1 - 6\varepsilon_2}{2m_Q}) \]
+ \[ \frac{B_\gamma \alpha_3^\gamma + B_v \alpha_3^v}{2m_Q} - (C_\gamma + C_v) \left( \mu^2_\gamma - \mu^2_G \right) \] (86)
\[ f_{H^*} \sqrt{M_{H^*}} = \alpha_H C_\gamma (1 - \frac{\varepsilon_1 + 2\varepsilon_2}{2m_Q}) \]
+ \[ \frac{B_\gamma^* \alpha_3^\gamma + B_v^* \alpha_3^v}{2m_Q} - C_\gamma \left( 3\mu^2_\gamma + \mu^2_G \right) \] (87)
where
\[ B_\gamma \equiv 2B_1 - 4B_2 - B_3 + B_4 - B_5 - B_6 , \]
\[ B_v \equiv B_1 + B_2 + B_3 + B_4 + B_5 + B_6 , \]
\[ B_\gamma^* \equiv -2B_1 + B_3 - B_4 + B_6 \quad \text{and} \quad B_v^* \equiv B_1 + B_4 . \] (88)
The ratio between the coupling constants is:

\[ \frac{f_{H^*}}{f_H} = \frac{C_{\gamma}}{C_{\gamma} + C_v} \left\{ 1 - \frac{1}{m_Q} \left( 4\varepsilon_2 + \frac{4\mu_2^2}{3\alpha_H G_H} \right) \right\} + \frac{1}{2m_Q\alpha_H} \left( (B^*_{\gamma} - B_{\gamma})\alpha_3^\gamma + (B^*_v - B_v)\alpha_3^v \right). \] (89)

VII. CHIRAL CORRECTIONS

Chiral corrections are numerically comparable with the $1/m_Q$ corrections. While the relevant mass scale of $1/m_Q$ corrections are $\Delta_H = M_{H^*} - M_H$ (cfr. eq (74)), the relevant mass scale of chiral corrections are $\delta = M_{H^*} - M_{H_u,d}$. Clearly $\Delta_H \sim \delta$ numerically. In this section we will consider just the chiral corrections to $f_H$ and $g_{A}^{H^*H\pi}$ which are necessary to include in the numerical analysis. These corrections have been calculated by many groups \[\cite{21, 30, 31}\], the result is in \(\overline{MS}\) scheme, including counter terms:

\[ f_{H_u,d}^{\chi} \sqrt{M_{H_{u,d}}} = f_{H_u,d}^{\chi*} \sqrt{M_{H^*_{u,d}}^*} = \alpha_H \left( 1 - \frac{1}{32\pi^2 f^2} \frac{11}{18} \left\{ -m^2_K (1 + g_A^2) + m^2_K (\ln m^2_K/\mu^2 + \frac{2}{11} \ln \frac{4}{3}) (1 + 3g_A^2) \right\} \right), \] (90)

\[ f_{H_s}^{\chi} \sqrt{M_{H_s}} = f_{H_s}^{\chi*} \sqrt{M_{H^*_{u,d}}} = \alpha_H \left( 1 - \frac{1}{32\pi^2 f^2} \frac{13}{9} \left\{ -m^2_K (1 + g_A^2) + m^2_K (\ln m^2_K/\mu^2 + \frac{4}{13} \ln \frac{4}{3}) (1 + 3g_A^2) \right\} 
- \frac{\omega_1}{\alpha_H \langle q\bar{q} \rangle m^2_K} \right). \] (91)

Note that this result is independent of $\Delta_Q$. We have ignored terms proportional to $m^2_\pi$ and used the mass relation $m^2_{\eta_s} = 4m^2_K/3$. The counter terms needed to make the expression above finite originates from the following terms in the weak current:

\[ J^\mu_a(M) = \frac{\omega_1}{2} Tr[\xi^\dagger_{ba} \Gamma^\mu H_{vc} \bar{\mathcal{M}}^V_{cb}] + \frac{\omega'_1}{2} Tr[\xi^\dagger_{ba} \Gamma^\mu H_{vb} \bar{\mathcal{M}}^V_{cc}], \] (92)

where $\Gamma^\mu$ is given in (24). Moreover, $\omega_1 = -4\lambda_1/G_H$, where $\lambda_1$ is given in equation (21).

We ignore $\omega'_1$ since it is subleading in $1/N_c$. (This is similar to what is found for $2L_1 - L_2$, $L_4$ and $L_6$ in the light sector \[\cite{2}\]). Therefore, in the limit where we neglect $u, d$ quark masses $f_{H_{u,d}}^{\chi}$ does not depend on counterterms. This also happens for $g_{A}^{H^*H\pi}$. The chiral corrections
to $g_A$ can be calculated with the formula listed in appendix A:

$$
g_A^\chi = g_A \left\{ 1 - g_A^2 \frac{1}{32\pi^2 f^2} \left( \frac{35}{9} m_K^2 \ln \frac{m_K^2}{\mu^2} + \frac{8}{9} m_K^2 \ln \frac{4}{3} - \frac{5}{3} m_K^2 \right) - \frac{\Delta Q^2}{3} \left( \ln \frac{4m_K^2}{3\mu^2} - 2F(m_{qs}/\Delta Q) \right) \right\}.
$$

(93)

$F(x)$ is defined in (A15). This result coincides with the one in [31] for $\Delta Q = 0$. It turns out that for $\Delta Q \sim (0 - 0.5)$ GeV, the chiral corrections vary with less than 1%. Therefore we will simply ignore $\Delta Q$.

Note that in the expressions for the chiral corrections to $f_H$ and $g_A$ considered in this section, the $1/m_Q$ corrections of the preceding section are not included. However, both chiral and $1/m_Q$ corrections are of course included in our numerical analysis in the next section.

**VIII. NUMERICAL RESULTS**

As we have seen in the previous sections, our bosonizing procedure puts restrictions, in the form of relations between the the model dependent parameters $m$, $G_H$, $\langle \alpha_s \pi G^2 \rangle$, $\langle \bar{q}q \rangle$ and the measurable parameters (quantities) such as $f_\pi$, $f_H$, $g_A$, $\mu_G^2$, and $\mu_\pi^2$. We will use a standard value of the quark condensate, $\langle \bar{q}q \rangle = -(0.240 GeV)^3$. (It is not clear if the quark and the gluon condensates defined in this paper are the same as those in QCD sume rules). In principle, we have enough relations to fix the model dependent parameters. However, some physical quantities are still relatively uncertain, which means that the values of the model dependent parameters cannot be given a very precise value.

In principle, $\Delta Q = M_H - m_Q$ is also a parameter of the model, which enters if we calculate diagrams with external momenta ($v^\mu \Delta Q$ will then be a part of an external momentum). However, the way we are bosonizing here, the external fields ($V$, $A$ and $H_v$) carry zero external momenta. Then $\Delta Q$ will not enter our loop integrals, and eq. (18) is irrelevant (so far) within our model. In the case of chiral corrections $\Delta Q$ could play a role. However, as we have seen in the previous section, $\Delta Q$ does not enter in the chiral corrections to $f_H$ and plays a very little numerical role for the corrections to $g_A$.

In the chiral corrections to $f_H$ and $g_A$, we have consequently used $\overline{MS}$ as in [3]. In pure $\chi$PT the “$(\overline{MS} + 1)$” scheme is used. We have explicitly checked that the numbers in table II and III can be reproduced in “$(\overline{MS} + 1)$” with a small change in $m$ and $g_A$ (the bare coupling constant).
The weak decay constants of heavy mesons have been calculated by many groups. Typical results from lattice calculations are $f_B = (200 \pm 30)$ MeV and $f_D = (225 \pm 30)$ MeV. QCD sum rules gives $f_B = (180 \pm 30)$ MeV and $f_D = (190 \pm 20)$ MeV \cite{33} and NRQCD gives $f_B = (147(11)^{+8}_{-12}(9)(6)$ MeV $[34]$. In order to constrain our parameters we will use the following combinations from QCD sum rules which have been evaluated rather accurately\cite{35}:

$$f_B f_{B^*} \sqrt{2} g_H^{B^*B\pi} M_B/f_\pi = (0.64 \pm 0.06) \text{GeV}^2,$$
$$f_D f_{D^*} \sqrt{2} g_H^{D^*D\pi} M_D/f_\pi = (0.51 \pm 0.05) \text{GeV}^2,$$

(94)

where $g_H^{H^*H\pi}$ (for $H = B, D$) is the chiral coupling $g_A$ with chiral and $1/m_Q$ corrections included. The left-hand side of (94) - for $H = B, D$ respectively- is a function of $m$ and (the uncorrected) $g_A$. Therefore (94) gives $g_A$ as a function of the constituent light quark mass $m$, which has been plotted in figure 8 and 9. Thus, we can then plot $f_B$, $f_D$ and other quantities as a function of the light quark mass. We may further use explicit values for $f_B$ and $f_D$ to determine a value for $m$ in the $B$- and $D$- meson sector separately. However, because we consider the Isgur-Wise function which involve both $B$- and $D$- mesons, we need a unique value of $m$, which give a reasonable value of $f_B$ and $f_D$ simultaneously. As can be seen from the plot in figure 10, this can be accomplished by taking

$$m = (220 \pm 30) \text{MeV}.$$

(95)

This is consistent with the value $m = (200 \pm 5)$ MeV used \cite{4} in the pure light sector in order to fit the $\Delta I = 1/2$ rule for $K \to 2\pi$ decays.

In table \cite{1} and table \cite{2} we have listed some of the predictions of HL$\chi$QM for decay constants and counterterms. The input parameters have been listed in table \cite{1}. It could be argued that the bare parameters, $g_A, G_H, \langle \frac{\alpha_s}{\pi} G^2 \rangle$, listed in table \cite{1} and table \cite{2} should be equal. As stated earlier in this model $m$ has to have an unique value in both the $D$- and $B$-sector. In order to fit our model to the result from QCD sumrules (equation (94)), we have to allow for a different value of $g_A, G_H, \langle \frac{\alpha_s}{\pi} G^2 \rangle$ in the $B$- and $D$-sector. From table \cite{1} and table \cite{2} we see that these parameters agrees within errorbars in the two sectors.

The inclusion of the counterterm for $f_{B_s}$ is crucial, putting $\omega_1 = 0$ would give $f_{B_s}/f_B = 1.29 \pm 0.03$ and $f_{D_s}/f_D = 1.34 \pm 0.04$ which is much higher than most lattice estimates\cite{30} $f_{B_s}/f_B \simeq f_{D_s}/f_D \simeq 1.15$ and QCD sumrules \cite{37} $f_{B_s}/f_B = 1.16 \pm 0.05$ and $f_{D_s}/f_D = $
Input parameters

| Parameter                        | Value                      |
|----------------------------------|----------------------------|
| $m$                              | $(220 \pm 30)$ MeV         |
| $f_\pi$                          | 93 MeV                     |
| $\langle \bar{q}q \rangle$       | $-(0.240 \text{ GeV})^3$  |
| $\alpha_s(\Lambda_{\chi})$      | 0.50                       |
| $\mu_{\zeta}(B)$                 | 0.36 GeV$^2$               |
| $f_B f_B^* \sqrt{2} M_B \tilde{g}_A^{B^*B \pi} / f_\pi$ | $(0.64 \pm 0.06)$ GeV$^2$ |
| $\alpha_s(m_b)$                  | 0.21                       |
| $m_b$                            | 4.8 GeV                    |
| $C_\gamma^b(\Lambda_{\chi})$    | 1.1                        |
| $C_\delta^b(\Lambda_{\chi})$    | 0.05                       |
| $C_\zeta^b(\Lambda_{\chi})$     | 0.85                       |
| $C_\kappa^b(\Lambda_{\chi})$    | 0.25                       |
| $\mu_{\zeta}(D)$                 | 0.30 GeV$^2$               |
| $f_D f_D^* \sqrt{2} M_D \tilde{g}_A^{D^*D \pi} / f_\pi$ | $(0.51 \pm 0.05)$ GeV$^2$ |
| $\alpha_s(m_c)$                  | 0.36                       |
| $m_c$                            | 1.4 GeV                    |
| $C_\gamma^c(\Lambda_{\chi})$    | 0.9                        |
| $C_\delta^c(\Lambda_{\chi})$    | 0.08                       |
| $C_\zeta^c(\Lambda_{\chi})$     | 1.15                       |
| $C_\kappa^c(\Lambda_{\chi})$    | 0.75                       |

| TABLE I: Input parameters of HL$\chi$QM in the $B$- and $D$- sector. |

1.15 $\pm$ 0.04. From table I and II, we see that we are a little low in the $B$-sector but in the $D$-sector the result agree nicely. The decay constant $f_{D_s}$ have been measured [38] $f_{D_s} = (264 \pm 15 \pm 33 \pm 2 \pm 4$ MeV and [39] $f_{D_s} = (280 \pm 19 \pm 28 \pm 34$ MeV. It is somewhat higher than our result, but we are within 1$\sigma$ of the experimental result. The ratio $f_{H^*}/f_H$ has been calculated in HQEFT sum rules with the result [40] $f_{H^*}/f_H \simeq 1.07 \pm 0.03$ and $f_{D^*}/f_D \simeq 1.37 \pm 0.04$, which also agrees perfect with our results.

The coupling $g_A^{D^*D\pi}$ has been measured [41] $g_A^{D^*D\pi} = 0.59 \pm 0.01 \pm 0.07$. Our prediction
agrees well with this result. The experimental result has also been predicted by a bag model calculation [42], \( g_{B^* B \pi} = 0.57 \) and \( g_{B^* B^\pi} = 0.57 \).

In conclusion, we have constructed a model which gives a reasonably good description of decay constants, the chiral axial coupling \( g_A \), masses and the Isgur-Wise function. We observe that the coupling \( g_A \) (leading order and corrected) is smaller in the \( B \)-sector than
for the $D$-sector. Furthermore, we find that $\mu_\pi^2 > \mu_D^2$ both in the $B$- and the $D$-sector. We have also showed that it is possible to systematically calculate the $1/m_Q$ corrections. For the decay constants in both the $B$- and $D$-sector they are of the same size as the chiral corrections as, can be seen from figure 10-13.

The model may be used to give predictions for other quantities. Especially, it will be
| Predictions of $HL\chi QM$ |
|-----------------------------|
| $G_B$ | $(7.7 \pm 0.6)$ GeV$^{-1/2}$ |
| $(\frac{G_\pi^2}{\pi})^{1/4}$ | $(0.315 \pm 0.020)$ GeV |
| $g_1$ | $(1.3 \pm 0.2)$ GeV |
| $g_2$ | $(0.39 \pm 0.05)$ GeV |
| $\varepsilon_1$ | $-(0.7 \pm 0.2)$ GeV |
| $\lambda_1$ | $1.0 \pm 0.2$ |
| $\mu_{\pi}^2$ | $(0.41 \pm 0.02)$ GeV$^2$ |
| $g_{\Lambda}$ | $(0.42 \pm 0.06)$ |
| $g_{\Lambda}^{B^*B\pi}$ | $(0.31 \pm 0.11)$ |
| $g_{\Lambda}^{B^*B^*\pi}$ | $(0.22 \pm 0.13)$ |
| $f_B$ | $(165 \pm 20)$ MeV |
| $f_{B^*}$ | $(170 \pm 25)$ MeV |
| $f_{B_s}$ | $(170 \pm 20)$ MeV |
| $f_{B_s^*}$ | $(175 \pm 25)$ MeV |
| $f_{B^*/f_B}$ | $1.06 \pm 0.03$ |
| $f_{B_s}/f_B$ | $1.07 \pm 0.02$ |

**TABLE II:** Predictions of $HL\chi QM$ in the $B$- sector, the errors in the predictions is a consequence of the error bars in the input parameters.

suitable for calculation of the $B$-parameter for $B - \bar{B}$ mixing [13].

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| Predictions of HLχ QM |
|-----------------------|
| $G_D$      | $(6.8 \pm 0.4)$ GeV$^{-1/2}$ |
| $\langle \frac{G^2}{\pi} \rangle^{1/4}$ | $(0.300 \pm 0.020)$ GeV |
| $g_1$      | $(0.8 \pm 0.1)$ GeV |
| $g_2$      | $(0.33 \pm 0.04)$ GeV |
| $\epsilon_1$ | $-(0.6 \pm 0.2)$ GeV |
| $\lambda_1$ | $0.7 \pm 0.1$ |
| $\mu_\pi^2$ | $(0.32 \pm 0.03)$ GeV$^2$ |
| $g_A$      | $0.55 \pm 0.08$ |
| $g_A^{D* \pi}$ | $0.46 \pm 0.15$ |
| $g_A^{D* \pi}$ | $(0.27 \pm 0.22)$ |
| $f_D$      | $(190 \pm 20)$ MeV |
| $f_{D^*}$  | $(220 \pm 35)$ MeV |
| $f_{D_s}$  | $(205 \pm 15)$ MeV |
| $f_{D_s^*}$ | $(235 \pm 35)$ MeV |
| $f_{D^*/f_D}$ | $1.22 \pm 0.09$ |
| $f_{D_s^*/f_D}$ | $1.16 \pm 0.03$ |

TABLE III: Predictions of HLχ QM in the $D$- sector, the errors in the predictions is a consequence of the error bars in the input parameters.

APPENDIX A: LOOP INTEGRALS

The divergent integrals entering in the bosonization of the HLχ QM are defined:

\[
I_1 = \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m^2}
\]

\[
I_{3/2} = \int \frac{d^d k}{(2\pi)^d} \frac{1}{(v \cdot k)(k^2 - m^2)}
\]

\[
I_2 = \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2)^2}
\]

(A1)

(A2)

(A3)
FIG. 12: $f_{B^*}$ as a function of $m$

FIG. 13: $f_{D^*}$ as a function of $m$
The $\kappa_i$’s are defined as:

$$
\begin{align*}
\kappa_0 &\equiv -\frac{i}{16\pi} m \\
\kappa_1 &\equiv \frac{i}{384m^3} (8 - 3\pi) \\
\kappa_2 &\equiv \frac{i}{384m^2} (3\pi - 4) \\
\kappa_3 &\equiv \frac{i}{384m^3} (8 + 3\pi) \\
\kappa_4 &\equiv \frac{i}{192m^2} (2 - 3\pi) \\
\kappa_5 &\equiv \frac{i}{96m^2}.
\end{align*}
$$

Integrals involving a heavy quark propagator and a light quark propagator can be symmetrized using the following formula:

$$
L_{m,\Delta}^{p,q} \equiv \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - m^2)^p(v \cdot k - \Delta)^q} = \frac{\Gamma(p + q)}{\Gamma(p)\Gamma(q)} \int_0^\infty d\lambda \int \frac{d^dk}{(2\pi)^d} \frac{2^q\lambda^{q-1}}{(k^2 - m^2 + 2\lambda v \cdot k)^{p+q}}
$$

(A8)

There are three interesting limits, where this integral can be written in a rather compact form: $\Delta, m \to 0$ and $\Delta = m$:

$$
\begin{align*}
L_{p,q}^{m,0} &= 2^{q-1} \frac{(-1)^{p+q}i}{(4\pi)^{d/2}} \frac{\Gamma(q/2)\Gamma(p + q/2 - d/2)}{\Gamma(p)\Gamma(q)} \left( \frac{1}{m^2} \right)^{p+q/2-d/2} \\
L_{p,q}^{0,\Delta} &= \frac{(-1)^{p+q}i}{2^{p-d}(4\pi)^{d/2}} \frac{\Gamma(d/2 - p)\Gamma(2p + q - d)}{\Gamma(p)\Gamma(q)} \left( \frac{1}{m^2} \right)^{p+q/2-d/2} \\
L_{p,q}^{m,m} &= \frac{(-1)^{p+q}i}{(4\pi)^{d/2}} \frac{\Gamma(p + q/2 + 1/2 - d/2)\Gamma(p + q/2 - d/2)}{\Gamma(p + q + 1/2 - d/2)} \left( \frac{1}{m^2} \right)^{p+q/2-d/2}
\end{align*}
$$

(A9) (A10) (A11)

where $d = 4 - 2\epsilon$ is the dimension of space. As a check equation (A9) gets the well known form in the limit $q \to 0$ ($\Gamma(q/2)/\Gamma(q) \to 2$), $L_{p,0}^{m,0} = I_p$. In the limit $p \to 0$, $L_{0,q}^{m,0} = 0$, this is because there is no mass scale entering in the integral.

In the general case for $\Delta, m \neq 0$, there is no compact form of $L_{p,q}^{m,\Delta}$, but all integrals needed in calculations can be related to the following integrals:

$$
L_{1,1}^{m,\Delta} = \frac{-i}{8\pi} \left( \frac{1}{\epsilon} - \ln(m^2) + 2 - 2F(m/\Delta) \right)
$$

(A12)
\[ \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2 - m^2)(v \cdot k - \Delta)} = A g^{\mu\nu} + B v^\mu v^\nu \]

\[ A = \frac{1}{d-1} \int \frac{d^dk}{(2\pi)^d} \frac{k^2 - (v \cdot k)^2}{(k^2 - m^2)(v \cdot k - \Delta)} \]

\[ = \frac{i\Delta}{16\pi^2} \left\{ \left( -\frac{1}{\varepsilon} + \ln(m^2) - 1 \right)(m^2 - \frac{2}{3}\Delta) - \frac{4}{3} F(m/\Delta)(\Delta^2 - m^2) - \frac{4}{3}(m^2 - \frac{5}{6}\Delta^2) \right\} \quad (A13) \]

\[ B = -A + \int \frac{d^dk}{(2\pi)^d} \frac{(v \cdot k)^2}{(k^2 - m^2)(v \cdot k - \Delta)} \]

\[ = -\frac{i\Delta}{16\pi^2} \left\{ \left( -\frac{1}{\varepsilon} + \ln(m^2) - 1 \right)(2m^2 - \frac{8}{3}\Delta) - \frac{4}{3} F(m/\Delta)(4\Delta^2 - m^2) \right. \]

\[ \left. - \frac{4}{3}(m^2 - \frac{7}{3}\Delta^2) \right\} \quad (A14) \]

where:

\[ F(x) = \begin{cases} 
-\sqrt{x^2 - 1}\tan^{-1}(\sqrt{x^2 - 1}) & x > 1 \\
\sqrt{1 - x^2}\tanh^{-1}(\sqrt{1 - x^2}) & x < 1 
\end{cases} \quad (A15) \]

In the case of \( \Delta > m \) we have ignored an analytic real part in (A12). As a check on these calculations we can use equation (A11). Equation (A12) coincides with the one obtained in [21] however equation (A14) differs by a factor \(-2/3(m^2 - 2/3\Delta^2)\) inside the parenthesis of the expressions for \( A \) and \( B \). This is presumably due to the factor \( 1/(d-1) = (1 - 2/3\varepsilon)/3 \) in \( A \).

In the case of the IW function we also need the following integral:

\[ L(p, \omega) \equiv \int \frac{d^dk}{(2\pi)^d} \frac{1}{(v \cdot k)(v' \cdot k)(k^2 - m^2)^p} \]

\[ = \frac{2i(-1)^p}{(4\pi)^{d/2}} \frac{\Gamma(p + 1 - d/2)}{\Gamma(p)(m^2)^{p+1-d/2}} r(\omega) \quad \text{where} \]

\[ r(\omega) \equiv \frac{1}{\sqrt{\omega^2 - 1}} \ln \left( \omega + \sqrt{\omega^2 - 1} \right) \quad (A16) \]

**APPENDIX B: SU(3) TRANSFORMATION PROPERTIES**

The Lagrangian of the light quark sector is

\[ \mathcal{L}_{\chiQM} = \bar{q}_L i \gamma \cdot D q_L + \bar{q}_R i \gamma \cdot D q_R - \bar{q}_L \mathcal{M}_q q_R - \bar{q}_R \mathcal{M}_q^\dagger q_L - m(\bar{q}_R \Sigma^\dagger q_L + \bar{q}_L \Sigma q_R) \quad (B1) \]

The left - and right - handed projections of the quark fields transform as

\[ q_L \to V_L q_L \quad , \quad q_R \to V_R q_R \quad (B2) \]
where $V_L \in SU(3)_L$ and $V_R \in SU(3)_R$. The octet meson field $\Sigma$ transforms as

$$\Sigma \rightarrow V_L \Sigma V_R^\dagger.$$  \hspace{1cm} (B3)

The $\xi$ field transforms more complicated as

$$\xi \rightarrow U \xi V_R^\dagger = V_L \xi U^\dagger$$ \hspace{1cm} (B4)

where $U \in SU(3)_V$. The constituent quark fields $\chi_L = \xi^\dagger q_L$ and $\chi_R = \xi q_R$, and the heavy meson field transform in a simple way under $SU(3)_V$:

$$\chi_L \rightarrow U \chi_L \ , \quad \chi_R \rightarrow U \chi_R \ , \quad H_v \rightarrow H_v U^\dagger.$$ \hspace{1cm} (B5)

The vector and axial fields transform as

$$V_\mu \rightarrow U V_\mu U^\dagger + iU \partial_\mu U^\dagger \ , \quad A_\mu \rightarrow U A_\mu U^\dagger.$$ \hspace{1cm} (B6)

Note that the weak current in (22) transforms as

$$J^\alpha_f \rightarrow J^\alpha_h \left( V_L^\dagger \right)_{hf}.$$ \hspace{1cm} (B7)
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