3.5 keV X-ray line and R-Parity Conserving Supersymmetry

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Abstract

We present some R-parity conserving supersymmetric models which can accommodate the 3.5 keV X-ray line reported in recent spectral studies of the Perseus galaxy cluster and the Andromeda galaxy. Within the Minimal Supersymmetric Standard Model (MSSM) framework, the dark matter (DM) gravitino (or the axino) with mass of around 7 keV decays into a massless neutralino (bino) and a photon with lifetime $\sim 10^{28}$ sec. The massless bino contributes to the effective number of neutrino species $N_{\text{eff}}$ and future data will test this prediction. In the context of NMSSM, we first consider scenarios where the bino is massless and the singlino mass is around 7 keV. We also consider quasi-degenerate bino-singlino scenarios where the mass scale of DM particles are $O(\text{GeV})$ or larger. In such a scenario we require the mass gap to generate the 3.5 keV line. We comment on the possibility of a 7 keV singlino decaying via R parity violating couplings while all other neutralinos are heavy.

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1 Introduction

Two recent independent studies \cite{1, 2} based on X-ray observation data show a photon emission line at 3.5 keV energy in the spectra from Perseus galaxy cluster and the Andromeda galaxy. This observation can be interpreted as a possible signal of dark matter (DM) decay with the emission of a 3.5keV photon, with the DM mass \( m_{\text{DM}} \) and lifetime \( \tau_{\text{DM}} \) given by,

\[
m_{\text{DM}} \simeq 7 \text{ keV} \\
\tau_{\text{DM}} \simeq 2 \times 10^{27} - 10^{28} \text{ sec}.
\]  

A variety of explanations of this line have already been proposed \cite{3, 4, 5, 6}. However, there exist just a few supersymmetric scenarios which contain such a light neutral particle. For instance, it could be an axino \cite{4}, gravitino \cite{5} or neutralino (bino) \cite{6}. These particles are able to produce the observed X-ray line \cite{1, 2} by decaying through R-parity violating processes \cite{6} to a photon and neutrino, for example.

In this paper we present some simple scenarios which can accommodate the 3.5 keV X-ray line in the context of R-parity conserving supersymmetry (SUSY). They include the minimal supersymmetric standard model (MSSM) and Next-to-MSSM (NMSSM). It is interesting to note that in the MSSM, the lightest neutralino can be massless \cite{7, 8, 9} while satisfying the current experimental constraints. In order to realize this scenario \cite{7}, we assume that the soft supersymmetry breaking (SSB) MSSM gaugino masses are arbitrary, and we impose the requirement that the neutralino mass matrix at the weak scale has zero determinant. This can be achieved by suitable choice of parameters, while having very small \( \lesssim 1 \text{ eV} \) or even zero mass bino, with the charginos (and the next to lightest neutralino \( \tilde{\chi}_1^0 \)) heavier then 420 GeV to satisfy the mass bounds on the chargino from LHC \cite{10}. In our scenarios where the ‘near massless’ bino is accompanied by a 7 keV gravitino, axino, or singlino which behave as warm DM, arising in different models around keV scale giving rise to warm DM. The 7 keV DM particle decays to a bino and a photon with an appropriate long lifetime to explain the observed X-ray line. The warm dark matter scenario which is under investigation for a long time \cite{11}, proposes solution to the missing satelite problem of the local group of galaxies \cite{12}. The massless bino contributes to the effective number of neutrino species, \( N_{\text{eff}} \), which is expected to be strongly constrained in the near future. We also consider an almost degenerate bino-singlino scenario in the NMSSM framework, such that the mass scale of cold DM particles are \( \text{O}(\text{GeV}) \) or larger.

We can retain gauge coupling unification in the presence of non-universal gaugino masses at \( M_{\text{GUT}} \), which are realized via non-singlet \( F \)-terms compatible with the underlying grand unified theory (GUT) \cite{13}. Nonuniversal gauginos can also be generated from an \( F \)-term which is a linear combination of two distinct fields of different dimensions \cite{14}. It is also possible to have non-universal gaugino masses \cite{15} in the
SO(10) GUT with unified Higgs sector [16], or utilize two distinct sources for supersymmetry breaking [17]. In general, in the gauge mediated supersymmetry breaking (GMSB) scenario, all gaugino masses can be independent of each other [18]. With so many distinct possibilities available for realizing non-universal gaugino masses while keeping universal sfermion mass \((m_0)\) at \(M_{\text{GUT}}\), we employ non-universal masses for the MSSM gauginos in our study without further justification.

One of the motivations for non-universal gauginos can be related to the interplay between the 125 GeV Higgs boson and the explanation of the apparent muon g-2 anomaly. A universal SSB mass term for sfermions \((m_0)\) is needed to suppress flavor-changing neutral current processes [19]. On the other hand, in order to accommodate the 125 GeV [20, 21] light CP even Higgs boson mass and to resolve the discrepancy between the SM and the measurement of the anomalous magnetic moment of the muon [22] in the framework of universal sfermion SSB masses, we need to have non-universal gaugino masses at \(M_{\text{GUT}}\) [23].

The outline of our paper is as follows. In section 2, we discuss the 3.5 keV line in the context of MSSM scenarios. In section 3, we discuss possible NMSSM scenarios, followed with our conclusion in Section 4. In the Appendix we present technical details regarding two massless neutralinos in the NMSSM and provide a few representative solutions of interest.

2 MSSM

In this section, we outline several scenarios that can accommodate a 3.5 keV X-ray line in the MSSM. Let us start by examining how it might be possible to obtain a massless neutralino in the framework of the MSSM. The neutralino mass matrix in the gauge eigenbasis \(\Psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)^T\) has the form [19]

\[
M_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 & 0 & -M_Z s_w c_\beta & M_Z c_w s_\beta \\
0 & M_2 & M_Z c_w c_\beta & -M_Z c_w s_\beta \\
-M_Z s_w c_\beta & M_Z c_w c_\beta & 0 & -\mu \\
M_Z s_w s_\beta & -M_Z c_w s_\beta & -\mu & 0
\end{pmatrix}.
\]  

(2)

Here \(M_1, M_2\) are the supersymmetric gaugino mass parameters for the \(U(1)\) and \(SU(2)\) sector respectively, while \(\mu\) is the bilinear Higgs mixing parameter. \(M_Z\) denotes the \(Z\) gauge-boson mass and \(s_w \equiv \sin \theta_w, \ c_w \equiv \cos \theta_w\), where \(\theta_w\) is the weak mixing angle. \(s_\beta \equiv \sin \beta, \ c_\beta \equiv \cos \beta,\) while \(\tan \beta\) is the ratio of the vacuum expectation values (VEVs) of the MSSM Higgs doublets.

To realize a massless neutralino [7, 9], the following relation must be satisfied:

\[
M_1 = \frac{M_2 M_Z^2 \sin(2\beta) s_w^2}{\mu M_2 - M_Z^2 \sin(2\beta) c_w^2} \approx \frac{2 M_Z^2 s_w^2}{\mu \tan \beta}.
\]  

(3)
Implementing the chargino mass bound \((|\mu|, M_2) > 420 \text{ GeV}\) in Eq. (3) leads to \(M_1 \ll (M_2, |\mu|)\). In the Appendix we give one example of an MSSM scenario with very small LSP neutralino (mostly bino) mass. Such a bino is consistent with current experimental data from LEP, structure formation etc [9]. The LHC provides constraints on the next to lightest neutralino, chargino, and slepton masses when the lightest neutralino is almost massless.

The relation in Eq. (3) has been obtained at tree level, but radiative corrections do not significantly modify it. Notwithstanding radiative corrections, since \(M_1, M_2\) and \(\mu\) are free parameters, there is no problem to ensure that the determinant in Eq. (2) is zero. Thus, it is possible to have an essentially massless neutralino in the framework of the MSSM, and an example is presented in the Appendix.

The existence of a near massless bino, however, would contribute to \(\Delta N_{\text{eff}} \equiv N_{\text{eff}} - N_{\text{eff,SM}} = 1\). The reason for this is that the essentially massless bino decouples from the thermal background around the same time as the neutrinos. The present observational bound on \(\Delta N_{\text{eff}}\) from Planck + WMAP9 + ACT + SPT + BAO + HST at 2\(\sigma\) is \(\Delta N_{\text{eff}} = 0.48^{+0.48}_{-0.45}\) [24]. The value of \(N_{\text{eff}}\) depends on Hubble constant where there is a discrepancy between Planck and HST [25]. A reconciliation can occur using larger \(\Delta N_{\text{eff}}\) [26]. The new BICEP2 data [27] also requires a larger \(\Delta N_{\text{eff}} (=0.81 \pm 0.25)\) in order to reconcile with the Planck data [28]. Future data hopefully will settle this issue.

### 2.1 Gravitino dark matter and massless bino

One way to accommodate a 3.5 keV X-ray line via a massless neutralino comes from the gauge mediated SUSY breaking (GMSB) scenario. As a consequence of the flavor blind gauge interactions responsible for generating the SSB terms [29], this scenario provides a compelling resolution of the SUSY flavor problem. In both the minimal [29] and general [18] GMSB versions, the gravitino, which is the spin \(3/2\) superpartner of the graviton, acquires mass through spontaneous breaking of local supersymmetry. The gravitino mass can be \(\sim 1 \text{ eV} - 100 \text{ TeV}\). Additionally, in the general GMSB scenario, the SSB mass terms for the MSSM gauginos are arbitrary. In particular, it is possible to have a massless neutralino (essentially a bino) in this framework. With all other sparticles being much heavier, the gravitino dominantly decays to the neutralino (bino) and photon \((\tilde{G} \to \tilde{\chi}^0_1 + \gamma)\).

The relevant diagram for this decay is shown in Figure 1, and the decay rate is given by [30]

\[
\Gamma(\tilde{G} \to \tilde{\chi}^0_1 \gamma) = \frac{\cos^3 \theta_W m_{\tilde{G}}^3}{8\pi M_P^2}.
\]  

Using Eq. (4) and assuming the gravitino mass to be 7 keV, the gravitino lifetime is estimated to be \(3 \times 10^{29}\) sec, which is approximately a factor of 10 more than
what we need. This estimate assumes, of course, that in 4D the reduced Planck mass is $\mathcal{M}_P = 2.4 \times 10^{18}$ GeV. However, physics around the Planck scale $\mathcal{M}_P$ is largely unknown because one is in a domain where gravity becomes strong. It has been noted in ref. [31] that the fundamental mass scale ($\mathcal{M}_ \Lambda$) can be reduced to $\mathcal{M}_P/\sqrt{N}$ in the presence of a nonzero number of degrees of freedom ($N$). In this case, in the framework of the MSSM, $\mathcal{M}_ \Lambda$ is a factor 10 or so smaller than $\mathcal{M}_P$, which yields the desired lifetime for the gravitino according to Eq. (4).

One important issue for gravitino dark matter is the reproduction of the correct dark matter relic density. The initial thermal abundance is diluted because of a late reheat temperature ($T_R$) arising from heavy field/moduli decay. The relic density ($\Omega_{\tilde{G}}h^2$) of gravitinos which arise from the scattering of gluinos, squarks etc. is given by [32, 33],

$$\Omega_{\tilde{G}}h^2 \approx 0.27 \left( \frac{100 \text{ GeV}}{m_{\tilde{G}}} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left( \frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^2 \left( \frac{2.4 \times 10^{18} \text{ GeV}}{\mathcal{M}_\Lambda} \right)^2. \quad (5)$$

To realize $\Omega_{\tilde{G}}h^2 \approx 0.1$ with $m_{\tilde{g}} \gtrsim 1.4$ TeV and $\mathcal{M}_\Lambda \approx 10^{17}$ GeV, we require $T_R \lesssim 10^4$ GeV.

2.2 Axino dark matter and massless bino

A very compelling way of solving the strong CP problem is via the Peccei-Quinn (PQ) mechanism [34], which yields a light pseudo-scalar field (axion $a$) associated with the spontaneously broken global $U(1)$ symmetry. An inevitable prediction from a combination of PQ mechanism and low scale supersymmetry is the existence of the supersymmetric partners of the axion, the axino ($\tilde{a}$) and saxion $s$ [36]. The axion superfield $A$ can be expressed as,

$$A = \frac{1}{\sqrt{2}}(s + ia) + \sqrt{2} \tilde{a} \theta + F_A \theta \theta, \quad (6)$$

where $F_A$ denotes the auxiliary field and $\theta$ is a Grassmann coordinate. In general, the axino mass is very model dependent [35] and can lie anywhere from eV to multi-
It was shown that a stable axino with keV mass is a viable warm dark matter candidate \([37, 38]\). The 3.5 keV X-ray line can be explained by a decaying axino dark matter. For this purpose, the authors in \([4]\) introduce R-parity violating couplings, with strength \(\sim 10^{-1} - 10^{-3}\) in order to accommodate desired axino life time.

In this paper, we propose an alternative way to explain the X-ray line using 7 keV axino dark matter. As mentioned above, within the MSSM framework, it is possible to have a massless neutralino in the spectrum which is consistent with all experimental constraints. We know that the axino couples to the gauginos and gauge bosons via the anomaly induced term. In particular, we are interested in the interaction of the axino to the bino \((\tilde{B})\) and the hypercharge vector boson \((B)\). This interaction takes the form \([39]\),

\[
\frac{i \alpha_Y C_Y}{16 \pi f_a} \gamma_5 [\gamma^\mu, \gamma^\nu] \tilde{B}_{\mu\nu}.
\]

Here \(\alpha_Y = Y^2/4\pi\) is the hypercharge gauge coupling constant and \(C_Y\) is a model dependent coupling associated with the \(U(1)_Y\) gauge anomaly interaction. The axion decay constant is denote by \(f_a\). The axino decays to a neutralino (bino) and photon without requiring R-parity violating interaction. The relevant diagram for this decay is shown in Figure 2, and the decay rate is given by \([38]\),

\[
\Gamma(\tilde{a} \to \chi_1^0 \gamma) = \frac{\alpha_{em}^2 C_{a\chi\gamma}^2 m_{\tilde{a}}^3}{128 \pi^3 f_a^2},
\]

where \(m_{\tilde{a}}\) is axino mass, \(C_{a\chi\gamma}^2 = (C_y/\cos \theta_W)Z_{11}\), and \(Z_{11}\) denotes the bino part of the lightest neutralino.

The axino lifetime can be expressed as:

\[
\tau(\tilde{a} \to \chi_1^0 \gamma) = 1.3 \times 10^{23} \text{sec} \left(\frac{f_a}{10^{12} \text{GeV}}\right)^2 \left(\frac{7.1 \text{keV}}{m_{\tilde{a}}}\right)^{3}
\]

\[\tag{9}\]
From Eq. (9) we see that we need to have $f_a \approx 10^{14} \text{GeV}$ is required. On the other hand, in order not to overproduce axion dark matter, we need to have $f_a \lesssim 10^{12} \text{GeV}$ is preferred. One resolution of this is to invoke a small initial axion mis-alignment angle $\theta \approx 0.1 - 0.01$ [40], which yields the required axion dark matter abundance while allowing $f_a \approx 10^{14} \text{GeV}$. An alternative solution [41] is to add additional massive fields whose late decay can inject substantial entropy into the universe at times after axion oscillations begin, but before BBN starts.

It is, furthermore, possible to have an axion-like particle (and associated axino) [42] in the low scale spectrum, which may be obtained from string theory. Axino-like particles can decay into a bino and photon. In this case the bound on $f_a$ can be more flexible and also the coefficient $C_y$ can be suitably adjusted to be $O(10^{-2})$ or so, since it is not tied to the solution of the strong CP problem.

## 3 NMSSM

As shown in the previous section, in the MSSM it is possible to have a massless bino, while keeping all other neutralinos heavier than 400 GeV. In the NMSSM, the neutralinos have a singlino component from the gauge singlet chiral superfield $S$ (with even $Z_2$ matter parity) added to the MSSM with new terms in the superpotential:

$$W \supset \mu H_u H_d + \lambda H_u H_d S - \frac{1}{3} \kappa S^3,$$

(10)

$H_u$ and $H_d$ are the standard MSSM Higgs doublets and $\kappa$ and $\lambda$ are dimensionless couplings. Once the $S$ field acquires a VEV $\langle S \rangle$, we obtain an effective $\mu$-term for MSSM Higgs fields, $\mu_{\text{eff}} = \mu + \lambda \langle S \rangle$. The neutralino mass matrix in the gauge eigenstate basis $\Psi^0 = (\tilde{B}, \tilde{W}_0, \tilde{H}_0^d, \tilde{H}_0^u, s)^T$ has the following form:

$$\mathcal{M}_N = \begin{pmatrix}
M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W & 0 \\
0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W & 0 \\
-m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu_{\text{eff}} & -\lambda v s_\beta \\
m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu_{\text{eff}} & 0 & -\lambda v c_\beta \\
0 & 0 & -\lambda v s_\beta & -\lambda v c_\beta & 2\kappa \langle S \rangle 
\end{pmatrix}.$$  

(11)

It was shown in [8] that a massless neutralino requires that

$$\kappa = \frac{\lambda}{2} \left( \frac{\lambda v}{\mu} \right)^2 \frac{0.6 m_Z^2 M_2 - 0.5 \mu M_2^2 \sin 2\beta}{-\mu M_1 M_2}.$$  

This solution is obtained for the case when $(\mu_{\text{eff}}, M_1, M_2) > M_Z$ and the singlino is the lightest neutralino. We can, however, easily make the lightest neutralino to be mostly bino and the next to lightest neutralino essentially the singlino. The technical
details for obtaining two massless neutralinos in the framework of NMSSM are given in appendix A.

In order to explain the 3.5 keV X-ray line, we propose that one of the neutralinos, which is mostly bino, is almost a massless ($\lesssim 1$ eV) particle and does not, therefore, contribute to the warm or cold dark matter relic abundance. The second neutralino, in this scenario, is mostly singlino with a mass of 7 keV and gives rise to the correct dark matter relic abundance [43]. The annihilation of thermal NMSSM Higgs produce singlinos, and it was shown that the correct relic abundance requires the singlino mass to be a few keV. Thus,

$$\Omega_{\tilde{\chi}^0_1} h^2 \approx \frac{4(1.2)^2}{\pi^5} \left( \frac{(\kappa/3 + \lambda^2) v^2 \sin 2\beta}{M_s M_{\tilde{\chi}}} \right)^2 \frac{g(T_{\gamma})}{g(T_R)} \left( \frac{T_R T_{\gamma}^3}{k T v^4 \sin^2 2\beta} \right)^2 \frac{M_{\tilde{\chi}}^3 M_{pl}}{\rho_c}. \quad (12)$$

Here $M_s$ is the mass of the scalar singlet, $g(T_R) = 228.75$, $g(T_{\gamma}) = 2$, $T_R \sim 10^3 - 10^5$ GeV, $k_T = (4\pi^3 g(T)/45)^{1/2}$ and $T_{\gamma}$ is the present CMB temperature. Choosing $\kappa = 3 \times 10^{-2}$, $\lambda = 10^{-10}$, $M_1 = 0.23$ GeV and $M_2 = -\mu = -550$ GeV (shown in point 1 of Table 1 in the Appendix), we can have the masses for the lightest neutralino (mostly bino) and the next to lightest neutralino (mostly singlino) to be essentially massless and 7 keV respectively. This scenario satisfies the dark matter relic abundance constraint.

The singlino can radiatively decay to a bino and photon with a long lifetime, which allows us to obtain the 3.5 keV X-ray line. The relevant diagram [44] for this decay is shown in Figure 3 and the decay rate is given by

$$\Gamma(\tilde{\chi}^0_2 \rightarrow \tilde{\chi}^0_1 \gamma) \approx \frac{\lambda^2 \alpha_{em}^2 m_{\tilde{\chi}}^3}{8\pi^3 M_H^2}. \quad (13)$$

Here we assume that the charginos ($m_{\tilde{\chi}^\pm_1}$) and charged Higgs ($m_{H^\pm}$) have approximately the same mass.

The $\tilde{\chi}^0_2$ lifetime can be written as:

$$\tau(\tilde{\chi}^0_2 \rightarrow \tilde{\chi}^0_1 \gamma) \approx 2 \times 10^{27} \text{sec} \left( \frac{M_H}{10^5 \text{GeV}} \right)^2 \left( \frac{10^{-10}}{\lambda} \right)^2. \quad (14)$$

In the NMSSM, an alternative explanation for the 3.5 keV emission line requires one to have two quasi-degenerate neutralinos (bino and singlino), with mass difference arranged to be $\sim 3.5$ keV. We present one such example in the Appendix. We require the next to lightest supersymmetric particle (NLSP), which is a mixture of singlino and bino, to be long-lived on cosmological time scales. The decay of this NLSP to the LSP, which again may be a bino-singlino mixture, can explain the 3.5 keV emission line.

The relevant Feynman diagrams for the NLSP neutralino decay are given in Figure 3. The decay width is given by [45].
Figure 3: Decay of NLSP neutralino to the LSP neutralino with the associated emission of a photon.

\[ \Gamma \sim \frac{\alpha_{em}^2 \lambda^2 (\Delta m_\chi)^3}{64\pi^4} \frac{m_\chi^2}{m_{H^\pm}^2}, \]  

where \( \alpha_{em} \) is the electromagnetic coupling constant, \( m_\chi \) is the quasi-degenerate mass of the two lightest neutralinos, \( \Delta m_\chi \) is their mass splitting, and \( m_{H^\pm} \) is the mass of the charged Higgs.

Assuming \( m_{\chi_0^2} \approx m_{\chi_0^1} \approx 1 \text{ GeV} \) and \( \Delta m_\chi \approx 3.5 \text{ keV} \), and as an example we consider \( \lambda \approx 10^{-8} \) and \( m_{H^\pm} = 500 \text{ GeV} \) in order to have \( \tau(\chi_0^2 \rightarrow \chi_1^0 \gamma) \approx (10^{27} - 10^{28}) \) sec. The dark matter in this case is cold compared to the previous scenarios.

The singlino/bino dark matter can be produced non-thermally from the decay of some heavy field/moduli (\( \phi \)) with a reheat temperature \( T_R \gtrsim 2 \text{ MeV} \) in order to avoid problems with big bang nucleosynthesis. As shown in [46], if the abundance of DM production (combination of dilution factor due to decay and branching ratio into DM particles) is small enough to satisfy the DM content, the annihilation cross-section of dark matter becomes irrelevant.

The DM abundance is given as \( n_{DM}/s = \text{min}[ (n_{DM}/s)_{obs} (3 \times 10^{26}/<\sigma v>_f ) (T_f/T_R), Y_\phi BR_{DM}] \), where \( (n_{DM}/s)_{obs} \approx 5 \times 10^{-10} (1 \text{ GeV}/m_{DM}) \), \( T_R \) is the reheat temperature, \( Y_\phi = 3T_R/4m_\phi \approx 1/\pi \sqrt{cm_\phi/M_P} \), and \( BR_{DM} \) denotes the branching ratio for \( \phi \) decay into singlino/bino. The singlino DM does not reach thermal equilibrium after production from the decay of the heavy field since the decoupling temperature is much larger than the reheat temperature \( T_R \).

It is also interesting to note that the singlino can be the lightest sparticle, and it can then decay via some R-parity violating couplings. We present an example in the Appendix. A slight change in the parameter values corresponding to the existence of massless neutralinos will make the neutralino mass around keV. A keV scale singlino
LSP can decay at loop level in the presence of R-parity violating couplings. Here we consider only the lepton number violation operators:

\[ \mathcal{L}_R = \lambda_i L_i H_u S + \lambda'_{ijk} L_i L_j E_k^c + \lambda'_{ijk} Q_i L_j d_k + \mu_i H_u L_i. \]  

(16)

The neutralino-neutrino mass matrix in the gauge eigenstate basis \( \Psi_0 \equiv (\tilde{B}_0, \tilde{W}_3^0, \tilde{h}_d^0, \tilde{h}_u^0, \tilde{s}, \nu_i) \) is given by

\[ \mathcal{M}_\tilde{\chi}^0 = \begin{pmatrix} M_N & \xi_R^T \\ \xi_R & \mathcal{M}_{3\times3}^\nu \end{pmatrix}, \]  

(17)

where

\[ \xi_R = \begin{pmatrix} \frac{-g' v_1}{\sqrt{2}} & \frac{g_1}{\sqrt{2}} & 0 & \mu_1 + \lambda_1 \langle s \rangle & \lambda_1 v_u \\ \frac{-g' v_2}{\sqrt{2}} & \frac{g_2}{\sqrt{2}} & 0 & \mu_2 + \lambda_2 \langle s \rangle & \lambda_2 v_u \\ \frac{-g' v_3}{\sqrt{2}} & \frac{g_3}{\sqrt{2}} & 0 & \mu_3 + \lambda_3 \langle s \rangle & \lambda_3 v_u \end{pmatrix}, \]  

(18)

and \( \mathcal{M}_{3\times3}^\nu \) is the 3 \times 3 light neutrino majorana mass matrix.

One of the dominant diagrams for the decay \( \tilde{\chi}_1^0 \to \nu + \gamma \) is given in Figure 4, and the corresponding decay rate is given by

\[ \Gamma(\tilde{\chi}_1^0 \to \nu \gamma) \sim \alpha_{em} \frac{(\lambda \lambda_1)^2}{32\pi^3} \frac{\tilde{\chi}_1^3}{M_H^2}. \]  

(19)

Here we assume, for simplicity, that the charged Higgs and charginos have similar masses \( M_H \equiv (m_{\chi^+_1} \approx m_{H^+}) \). The singlino lifetime can be expressed as

\[ \tau(\tilde{\chi}_1^0 \to \nu \gamma) \approx 2 \times 10^{27} \text{sec} \left( \frac{M_H}{10^5 \text{GeV}} \right)^2 \left( \frac{10^{-11}}{\lambda_1^2} \right)^2, \]  

(20)

and if we assume \( \lambda_1 \approx \lambda \approx 3 \times 10^{-6} \), the desired singlino life time is obtained. The LSP singlino, as mentioned above, can provide the correct DM abundance.
Conclusion

In summary, we have presented several scenarios that can accommodate the 3.5 keV X-ray line in the context of R-parity conserving SUSY. In the MSSM, the LSP neutralino can be massless and the gravitino or axino dark matter of mass around 7 keV can decay into the LSP neutralino and a photon with lifetime $\sim 10^{28}$ sec. To realize this scenario, we assume that the soft SUSY breaking MSSM gaugino masses are non-universal and they satisfy the requirement that the determinant of the neutralino mass matrix vanishes at the weak scale. This can always be achieved with a suitable choice of parameters, while keeping the charginos (and second lightest neutralino $\tilde{\chi}_2^0$) heavier than 420 GeV to avoid the LHC constraint. A keV mass dark matter is of considerable interest since it can provide potential solutions to the missing satellites problems of the Local Group of Galaxies. The massless bino, however, contributes to $N_{\text{eff}}$ and future data should seriously test this scenario. In the context of NMSSM, we consider scenarios where the bino is massless and the dark matter singlino mass is around 7 keV. Within the NMSSM, we also consider quasi-degenerate bino-singlino scenarios where the DM mass scale is $O(\text{GeV})$ or larger. We require, in this scenario, a small mass gap to generate the 3.5 keV X-ray line. In passing, we also consider scenarios where the singlino is the lightest SUSY particle, and it decays via R parity violating couplings which give rise to the 3.5 keV X-ray line.

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Appendix

A Two massless neutralinos in the NMSSM

The neutralino mass matrix is given in Eq. (11) and we seek a solution with two massless neutralinos. Assuming that $\gamma$ is an eigenvalue of $\mathcal{M}_N$, we can write the characteristic equation in the form

$$|\mathcal{M}_N - \gamma I_5| = \gamma^5 + A\gamma^4 + B\gamma^3 + C\gamma^2 + D\gamma + E = 0,$$

(21)

where $I_5$ is the $5 \times 5$ identity matrix, and $A, B, C, D, E$, of course, depend on the entries in $\mathcal{M}_N$. It is known that $A, B, C, D$ and $E$ are invariants (under similarity transformations).
transformations) of the matrix and, in particular, $E$ is the determinant of $\mathcal{M}_N$. We can express the coefficients in Eq. (21) in terms of the mass eigenstates:

$$E = m_1^2 m_2^2 m_3^2 m_4^2 m_5^2; \quad D = \sum_{i \neq j \neq k \neq l}^n m_i^2 m_j^2 m_k^2 m_l^2; \quad C = \sum_{i \neq j}^n m_i^2 m_j^2 m_k^2; \quad B = \sum_{i \neq j}^n m_i^2 m_j^2; \quad A = \sum_{i=1}^5 m_i^2. \quad (22)$$

A necessary and sufficient condition for any one eigenvalue to be zero is for the determinant of the matrix to be zero (i.e. $E = 0$). The quintic characteristic equation then reduces to a quadratic one. Proceeding in this fashion, if we now also set $D = 0$, we will ensure that two eigenvalues of the mass matrix are zero. It is then possible to adjust the parameters to get the desired small mass eigenvalues.

While the general expression for the determinant and the coefficient of $\gamma$ in the characteristic equation (variously known as the fourth invariant) is rather complicated, the conditions to obtain two massless neutralinos simplifies in the limit of large $\tan \beta$. Setting $s_\beta \to 1$ and $c_\beta \to 0$ in the neutralino mass matrix, we obtain the following conditions for two massless neutralinos,

$$D = -M_1 M_2 (\lambda^2 v^2 + \mu^2) - 2 \kappa x \mu^2 (M_1 + M_2) + 2 \kappa x m_Z^2 (M_1 c_W^2 + M_2 s_W^2) + m_1^2 \lambda^2 = 0$$

$$E = 2 M_1 M_2 \kappa x \mu^2 - m_Z^2 \lambda^2 (M_1 c_W^2 + M_2 s_W^2) = 0 \quad (23)$$

There can, however, be issues while using this approximation because of the large differences in orders of magnitudes of the various terms. In practice it is much simpler to numerically fine-tune the parameters in the exact expressions to obtain two zero eigenvalues. We are essentially interested in a quasi-degenerate (\lesssim 1 \text{ GeV}) bino-singlino mixture. With $\lambda$ small, there is very little mixing between the singlino and the higgsinos, particularly for $\mu \gtrsim 100$ GeV (which is needed as previously explained). Furthermore, if we choose $M_1, 2 \kappa x \sim 1$ GeV and $M_2 \gtrsim 400$ GeV, we should naively expect to get the required neutralino masses.

In Table 1 we display three representative solutions that correspond to the three scenarios for obtaining the 3.5 keV X-ray line within the NMSSM framework. Point 1 corresponds to a massless bino with a 7 keV singlino. Point 2 shows the quasi-degenerate scenario involving the bino and singlino, with a mass of 1 GeV and a mass splitting of 3.5 keV. Point 3 describes the scenario in which the singlino is \sim 7 keV and all other neutralinos are heavy.

As far as the MSSM case is concerned, things are even simpler. For example, one could take, $\tan \beta = 30, M_2 = \mu = 550$ GeV, $M_1 = 0.23$ GeV where, $M_1$ is chosen to obtain a massless bino. The masses of the three heavier neutralinos are 499 GeV, 555 GeV and 606 GeV.
|                  | Point 1 | Point 2 | Point 3 |
|------------------|---------|---------|---------|
| $M_2$ (GeV)      | 550     | 500     | 550     |
| $\mu$ (GeV)     | 550     | 500     | 550     |
| $x$ (GeV)        | 0.0001  | 1       | $7 \times 10^{-6}$ |
| $\tan \beta$    | 30      | 30      | 50      |
| $M_1$            | 0.234   | 1.267   | 550     |
| $\kappa$        | $3.5 \times 10^{-2}$ | 0.5 | 0.5 |
| $\lambda$       | $10^{-10}$ | $10^{-9}$ | $10^{-5}$ |
| $m_{\tilde{\chi}^0_1}$ (GeV) | $6.69 \times 10^{-13}$ | 1 | $7 \times 10^{-6}$ |
| $\tilde{\chi}^0_1$ composition | $\simeq 100\% \tilde{B}$ | $99\% \tilde{B}$ | $\simeq 100\% \tilde{S}$ |
| $m_{\tilde{\chi}^0_2}$ (GeV) | $7 \times 10^{-6}$ | 1 | 1.08 |
| $\tilde{\chi}^0_2$ composition | $\simeq 100\% \tilde{S}$ | $99\% \tilde{S}$ | mixture |
| $m_{\tilde{\chi}^0_3}$ (GeV) | 498     | 445     | 550     |
| $m_{\tilde{\chi}^0_4}$ (GeV) | 554     | 505     | 554     |
| $m_{\tilde{\chi}^0_5}$ (GeV) | 605     | 560     | 617     |

Table 1: Three representative solutions.

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