Utilizing Explainable AI for Quantization and Pruning of Deep Neural Networks

Muhammad Sabih, Frank Hannig and Jürgen Teich
Hardware/Software Co-Design, Department of Computer Science
Friedrich-Alexander University Erlangen-Nürnberg (FAU)
Cauerstr. 11, 91058 Erlangen, Germany
{muhammad.sabih, frank.hannig, juergen.teich}@fau.de

Abstract

For many applications, utilizing Deep Neural Networks (DNNs) requires their implementation on a target architecture in an optimized manner concerning energy consumption, memory requirement, throughput, etc. DNN compression is used to reduce the memory footprint and complexity of a DNN before its deployment on hardware. Recent efforts to understand and explain AI (Artificial Intelligence) methods have led to a new research area, termed as explainable AI. Explainable AI methods allow us to better understand the inner working of DNNs, such as the importance of different neurons and features. The concepts from explainable AI provide an opportunity to improve DNN compression methods such as quantization and pruning in several ways that have not been sufficiently explored so far. In this paper, we utilize explainable AI methods: mainly DeepLIFT method. We use DeepLIFT for (1) pruning of DNNs; this includes structured and unstructured pruning of Convolutional Neural Network (CNN) filters pruning as well as pruning weights of fully connected layers, (2) non-uniform quantization of DNN weights using clustering algorithm; this is also referred to as Weight Sharing, and (3) integer-based mixed-precision quantization; this is where each layer of a DNN may use a different number of integer bits. We use typical image classification datasets with common deep learning image classification models for evaluation. In all these three cases, we demonstrate significant improvements as well as new insights and opportunities from the use of explainable AI in DNN compression.

1 Introduction

DNNs have achieved rapid success in many image processing applications, including image classification [1], [2], image segmentation [3], [4], object detection [5], etc. The key ingredients in this success of DNNs have been the usage of deeper networks and a large amount of training data. However, as the network gets deeper, the model complexity also increases rapidly. The training of DNNs can be carried out on high-performance clusters with Graphics Processing Unit (GPU) acceleration; however, for implementing these networks on hardware, the complexity of the DNNs needs to be reduced. This includes decreasing the memory requirements, energy consumption, latency or throughput of the implementation of DNNs on the hardware. Pruning and quantization are two of the main techniques that are utilized for compression and complexity-reduction of DNNs. Our work is motivated by the increasing requirement of taking the DNNs to the hardware as well as the progress in understanding and explaining AI. DNN pruning and quantization have been a popular topic for research recently, and a number of works have been published in this area.

Few of the challenges in this area of research are: lack of standard metrics, datasets and benchmarks and the lack of explainable algorithms that can achieve state-of-the-art or close to the state-of-the-art...
for a broad range of problems: we typically have one approach performing better for one application
and another approach performing better for another application. With this research work, we aim to not
only use explainable AI for better performing pruning and quantization algorithms but also to address
a broad range of cases with a unified approach: this includes structured and unstructured pruning,
CNN filter pruning and neuron pruning, mixed-precision integer-based quantization and weight
sharing quantization. Additionally, our proposed approach can optimize for Multiply-Accumulate
Operations (MACs), Number of Parameters (NPs), or both.

1.1 Contributions

Our novel contributions in this paper can be summarized as follows:

1. Utilizing explainable AI, in particular, DeepLIFT, for obtaining the importances of neurons
   for the pruning and quantization of DNNs.
2. Proposing a method to calculate sensitivities of layers for quantization and pruning.
3. Using (1) and (2) with iterative pruning algorithm for structured and unstructured pruning
   can optimize MACs, NPs, or both.
4. Using (1) with a weighted MSE criteria to obtain better clustering of weights.
5. Using (1) and (2) and (4) to obtain bit-widths for mixed-precision integer quantization.
6. Obtaining state-of-the-art or competitive results for these methods.

2 Explainable AI

Obtaining the importances of layers, neurons and filters is a common challenge in quantization and
pruning of DNNs. This can be addressed using explainable AI. Among the different types of explain-
able AI algorithms, we are most interested in the saliency methods. They explain an algorithm’s
decision by assigning values that reflect the importance of input components in their contribution to
that decision [6]. These methods such as Class Activation Map (CAM) [7], Layer-wise Relevance
Propagation (LRP) [8], Deep Learning Important Features (DeepLIFT) [9], Conductance [10], etc.,
are types of saliency methods that can be used to obtain the importance measures.

2.1 DeepLIFT

In [9], the authors describe the algorithm as a method for decomposing the output prediction of a
neural network on a specific input by back-propagating the contributions of all neurons in the network
to every feature of the input. DeepLIFT compares the activation of each neuron to its reference
activation and assigns contribution scores according to the difference. The choice of reference
activation is important for the algorithm’s outcome, and it often requires domain-specific knowledge.
In order to specify a reference activation, we must understand the intuition behind the DeepLIFT
algorithm. It compares the effect of the features to a baseline of what the model would predict when
it can not see the features. Therefore, a good reference activation for MNIST [11] is an all-black
image. For more complex datasets such as CIFAR10, CIFAR100, and Large Scale Visual Recognition
Challenge 2012 (ILSVRC2012), different possibilities for reference activations are mean-image of
the training set, mean-image of the training set with random noise, all zero image, noisy versions of
input images, etc.

Mathematically, the DeepLIFT algorithm is formulated by the authors as follows. Let $t$ represent some
target output neuron of interest and let $x_1, x_2, \ldots, x_n$ represent some neurons in some intermediate
layer or set of layers that are necessary and sufficient to compute $t$. Let $t^0$ represent the reference
activation of $t$. The quantity $\Delta t$ is defined as the difference-from-reference, that is $\Delta t = t - t^0$.
DeepLIFT assigns contribution scores $C_{\Delta x_i, \Delta t}$ to $\Delta x_i$ s.t.:

$$\sum_{i=1}^{n} C_{\Delta x_i, \Delta t} = \Delta t \quad (1)$$

$C_{\Delta x_i, \Delta t}$ can be thought of as the amount of difference-from-reference int $t$ that is attributed to the
difference-from-reference of $x_i$. $\Delta t$ is the DeepLIFT score, which can also be represented as follows.
\[ DL(t, x) = \sum_{i=1}^{n} C_{\Delta x_i \Delta t} \]  

Our choice for using the DeepLIFT is because it solves the problem of saturation with the gradient methods such as LRP. Conductance is a closely related algorithm that gives similar results. However, DeepLIFT is a more computationally efficient method. The drawback of DeepLIFT and Conductance is that they need a baseline or a reference input, against which the hidden representation of the DNN is compared. To obtain this reference or baseline, one needs domain-specific knowledge. In their work, the authors suggested using three types of references: (1) a blank image with all zero pixels, (2) an image that is the mean of the training images in the datasets, and (3) a blurred version of the input image.

3 DNN Pruning

Pruning refers to removing the undesired parameters of a DNN that have little influence on the output of the neural network [12], [13]. This leads to fewer MACs operations and fewer NPs.

Pruning can be structured or unstructured. In structured pruning, the filters and weights are removed by removing all their input and output connections, and this means that no extra compilation effort or hardware optimization is required to get the gain on hardware in compression or the inference efficiency. In unstructured pruning, the unimportant filters or weights are set to zero, a compiler utilizes these zeros to get a gain in compression or efficiency in inference. Utilizing the unstructured pruning at the hardware has additional cost in terms of compilation effort or computational effort in order to exploit the irregular sparsity. Typically unstructured pruning provides more pruning ratios.

Pruning can be neuron pruning, filter pruning, weight pruning, and layer pruning. In neuron pruning, individual neurons are removed, i.e., all the incoming and outgoing connections to a neuron are also removed [14]. In filter pruning, CNN filters are removed [15]. In layer pruning, some of the layers can also be pruned [16]. Weight pruning is used synonymously for unstructured pruning, where the redundant weights are set to zero.

The two fundamental objectives for pruning the model are (1) reducing the memory by lowering the NPs and reducing the latency and energy consumption of the hardware by reducing MACs. These two objectives are often conflicting in a DNN because the MACs are concentrated in the lower layers of a typical image classification network and the number of parameters are concentrated in the higher layers of a typical image classification network.

The key challenge in the pruning of DNN is identifying criteria by which the importance of parameters can be measured. This importance can be of two types: one is the importance of parameters based on their contribution or effect on the output of the network: this is typically measured by the degradation in accuracy of the DNN output after pruning that parameter, the other type of importance is the reduction in memory or number of multiplications that is obtained after removing the parameters. The effect that a DNN parameter has on MACs or memory is typically constant for each layer. However, the effect that a parameter has on the output of the network varies considerably within the layer.

3.1 Related work and State-of-the-art

Some of the examples of commonly used criteria for pruning are \( \ell^1 \) norm and \( \ell^2 \) norm of the weights. The closest work to ours for CNN filter pruning is [8], where authors used explainable AI method: LRP, to calculate the importance of features. In their work, authors provide useful insights to utilize pruning to focus on some of the output classes. LRP is a gradient-based method and suffers from similar limitations against saturation of activations, as shown by Shrikumar et al. [9]. In [17], a second-order Taylor expansion based on the Hessian matrix of the loss function was used to select parameters for pruning, however computational cost of calculating the exact Hessian is prohibitive and approximations lead to incorrect importance measures.

The state-of-the-art for DNNs is challenging to identify. There are various reasons for this such as: (1) this is a relatively new research area and there are a lack of standardized benchmarks, (2) there is a difference in implementation of networks on different deep learning platforms, (3) the choice of the un-pruned reference network has different effects on the results, (4) the training recipes and
training budgets considerably vary, (5) different types of metrics and combinations of networks and datasets are used by different authors, etc. In the work titled “What is the State of Neural Network Pruning?” [18], the authors survey 81 papers and summarize the state-of-the-art results related to pruning of DNNs and also suggested standard practices for benchmarking DNN pruning.

3.2 DeepLIFT-based local pruning

We propose a DeepLIFT-based local pruning approach that prunes CNN filters or neurons, locally in a layer according to the importances calculated from DeepLIFT method. In local pruning, we prune a layer or a group of layers with the same specific amount and optionally fine-tune it. This type of pruning is useful, if the goal is to reduce the memory requirement or MACs of a layer or a group of layers to a certain amount, for example, if on a Field Programmable Gate Arrays (FPGA), a certain layer cannot fit on the memory or if the latency of a particular layer is too much, then pruning of unimportant parameters is needed. Local pruning without fine-tuning or retraining is also useful to evaluate the performance of the ranking criteria for pruning because it gives a measure of how well the ranking criteria identifies the redundant features. With training or fine-tuning, the performance of a pruning method can be attributed to a good training or fine-tuning recipe rather than the pruning criteria. We compare the results of our local pruning approach by comparing it with the pruning based on $\ell_1$ norm of the weights, in order to have an initial assessment of the performance of our proposed method. The results are shown in Figure 1.

Another interesting application of DeepLIFT-based pruning criteria is to prune such that the accuracy of a subset of image classes is given more preference than the others. For our work, we do not use this degree of freedom.

We describe our algorithm as follows in Algorithm 1.

**Algorithm 1:** DeepLIFT-based local pruning

```
Input layers_to_prune, prune_amount, trained_model
Output pruned_model

1: for each_layer in layers_to_prune do
2:   calculate the $\ell_1$ norm of DeepLIFT importances.
3:   sort the importance.
4:   prune the lowest weights specified by prune_amount.
5: end for
6: return pruned_model
```

3.3 DeepLIFT-based layer sensitivity analysis

The importance measures obtained from DeepLIFT explain how much one neuron or filter effects the output of the layer, these importances do not tell the global importance.

This importance can be obtained by scaling the local importance measures with the sensitivities of each layer. In order to obtain these sensitivities, we propose a DeepLIFT-based sensitivity analysis of the DNN. DNNs can be modeled as a Markov chain [20], such that the layered structure of a network generates a successive Markov chain of intermediate representations, which together form (approximate) sufficient statistic. We are interested in measuring how much distortion in a layer (whether by pruning or quantization) results in how much degradation of the network’s output. We use the interpretation of a DNN as a Markov chain to say that each layer in a DNN takes the representation from the previous layer and adds to the clustering ability of the DNN such that in an intermediate representation, the two images belonging to the same class are closer to each other and farther away from the images of the different classes (for example, a cat image and a dog image). This means we can measure the pairwise distances from the important parts of the intermediate representation of image classes to determine how good is the separability of the image classes at a layer. So we obtain the sensitivities for each layer by introducing distortion into that layer by quantization or pruning and then measuring the separability of classes in terms of difference between the distribution of same-class distances and the distribution of different-class distances. In Figure 2, the separability
of first layer and last layer of a ResNet-20 model on CIFAR10 is shown in terms of distributions of same class distances and different class distances.

3.4 DeepLIFT-based global pruning

Often, the goal of DNN pruning is not just to prune a specific layer by a specified number, but the goal is to globally prune the DNN the filters of a CNN and neurons of fully-connected layer, in order to minimize the NPs (memory), MACs (latency) or both. To obtain the global importance, a typical approach is to rank the filters or neurons according to criteria, prune a fraction of these filters, tune the DNN and repeat the process until a certain requirement is met. We take a slightly different approach, we first calculate the number of filters or neurons that are need to be pruned in each layer, this number is calculated from the sensitivities of the layers in addition to an objective such as the number of MACs or NPs in the layer, after this, the importance measures are used within the layers to prune the least important filters or neurons. One can use this algorithm to prune the DNN to optimize for MACs, NPs, or both. Our algorithm is described in Algorithm 2.

3.4.1 Evaluation

We choose the Taylor approximation of the Hessian approach [17] and the LRP [8] approach as our comparison methods. One datasets and model pair used in these two papers is AlexNet [1] with Oxford Flowers Dataset [21]. This is a 102 category-dataset with around 2000 images for training and around 8000 for testing. The resolution of images roughly matches that of ImageNet [22], so this
Figure 2: In the figure, we show how the distributions of same-class distances and different-class distances vary from layer to layer. The figure on the left shows how distributions of pairwise distance at the last convolutional layer look like after introducing distortion in the first layer of ResNet20. The figure on the right shows the same for the second last convolutional layer of ResNet20.

Algorithm 2: DeepLIFT-based global pruning

```
1 Input trained_model.
2 Output pruned_model.
3 while error < maximum error and iterations < maximum iterations do
4   get layer sensitivities
5   if optimization criteria == MACs then
6     get load of layers by MACs.
7   else if optimization criteria == NPs then
8     get load of layers by NPs.
9   end if
10  multiply sensitivities of layers with their load to get pruning number per layer.
11  for each_layer in model do
12    calculate ℓ1 norm of DeepLIFT importances for the layer.
13    sort the importances.
14    prune the number of filters or neurons for this layer.
15  end for
16 end while
17 return pruned_model.
```

is a good lighter alternative to ImageNet for benchmarking. In both the LRP-based approach and the Taylor approximation approach, the goal is to prune the CNN filters for optimal MACs and NPs. In our case, we do not restrict ourselves to CNN filters alone. However, our goal is also to prune AlexNet for optimal MACs and NPs. We start with AlexNet pre-trained on ImageNet and perform transfer learning to obtain an initial accuracy of 81.9% on the testing dataset. We use our DeepLIFT global pruning algorithm by using one pruning round for optimizing MACs and one pruning round for optimizing NPs. In each round, 10 to 15 percent of MACs or NPs need to be reduced. Between each pruning round, we fine-tune or retrain the AlexNet weights by training for 30 epochs with learning rate varying between 0.01 and 0.001 and using SGD [23] optimization. For DeepLIFT, we use blurred images corresponding to the input image as a reference. These blurred images are obtained from Gaussian blurring, and the DeepLIFT importances are obtained from 512 training images. The results of our experiment are shown in Figure 3.

In terms of MACs, our approach can keep up with the Taylor approach in terms of accuracy up to the MAC reduction of around 70%; these are 237 Million MACs (MMACs) or 0.47 GFLOPs in absolute terms and roughly 3x less than the original MACs. The accuracy of the model at this point on the test dataset is 0.70. This is despite the fact, that in terms of NPs, our model is orders of magnitude more compressed, because in the Taylor approach and the LRP approach, only CNN filter maps are considered for pruning. However, we argue that in terms of the end goal, we are mostly interested in the overall reduction in MACs and NPs, rather than specifically pruning either the CNNs or the fully connected layers. Our final results are shown in Table 1.
Figure 3: This figure shows the results from DeepLIFT global pruning method. The figure on the left compares the reduction in the number of parameters with the degradation in accuracy as the pruning algorithm iterates. The iterations start from right. The figure on the right compares the number of MACs with the degradation in accuracy. One pruning iteration reduces MACs, and the other reduces NPs. Both objectives, i.e., MAC reduction and NP reduction, are supporting each other in the start because the reduction in NPs of an over-parameterized network helps reduce MACs without loss in accuracy. However, later on, these two objectives conflict with each other.

Table 1: Our most pruned model contains 30x less parameters (0.30%) and 3x less GFLOPs (0.30%) than the un-pruned model.

| AlexNet on Flowers 102 | Accuracy | Reference Accuracy | NPs   | MACs  |
|------------------------|----------|--------------------|-------|-------|
|                        | 0.698    | 0.819              | 2.03 million | 0.47 GLOPs |

3.5 Unstructured Pruning

For the unstructured pruning, we use the same approach as in global pruning, except that in the case of unstructured pruning, we can prune the individual weights in the CNN filter’s kernel. The DeepLIFT importance measures can only be calculated at the granularity of a CNN channel, so we multiply the DeepLIFT importance measures with the $\ell_1$ norm of the weights, with this modification, we can use the DeepLIFT-based global pruning algorithm for unstructured pruning as well.

For evaluating our unstructured pruning approach, we refer to the shrink-bench [18] for the state-of-the-art. We choose ResNet56 on CIFAR10 as our competitor, where the state-of-the-art compression scores are reported by Carreira-Perpinan 2018 (CP2018) [19]. We iteratively prune the model in 10 iterations, and training with SGD and varying learning rates between 0.1 and 0.001 are employed, each pruning iteration trains for 40 epochs. Our pruning method gives an error of 0.066 on the testing set of CIFAR10 dataset, with 83% of the weights pruned. Our scores are well above the various baselines for CIFAR10 and Resnet56, reported in shrink-bench [18]. In CP2018, the reported score for ResNet56 CIFAR10 is an error of 0.0692, with 85% of the weights pruned. A lack of a common reference model and non-availability of a full trade-off curve makes our comparison difficult. Our trade-off curve is shown in fig. [1].

4 DeepLIFT-based quantization for weight sharing

Quantization for DNNs refers to approximating a neural network using reduced bit precision. This was first used in [24] and [25]. DNNs can be quantized using a clustering algorithm, whereby, all the weights that fall into a cluster share the same value. For example, 32-bit weight values can be quantized to a 2-bit value using four clusters. This is an example of non-uniform quantization (with non-uniformly separated quantization points), and the idea is also known as weight sharing [26]. Han et al. [27] used weight sharing quantization along with pruning and Huffman coding to reduce the DNN size by 35 to 50 times. We propose DeepLIFT Weighted Mean Squared Error (DWMSE)
Figure 4: This figure compares Hessian Weighted MSE (HWMSE) criteria with DeepLIFT Weighted MSE (DLWMSE) and MSE. The figure on the left shows results after batch-norm calibration or fine-tuning. The figure on the right shows results without fine-tuning. The accuracies are calculated on the testing set of CIFAR 10. Our approach gives better accuracy, especially when no fine-tuning is employed.

We compare our results with [29]; in this work, the authors used the weights from the Taylor approximation of Hessian to calculate the weighting factor of k-means. As the authors use ResNet20 with CIFAR10 for evaluation, we use the same dataset and model pair. The plain k-means quantization with Mean Square Error (MSE) criteria is used as an additional baseline. We first quantize the weights without recalibrating the batch norm and with recalibrating the batch norm. We do not retrain our weights. In [29], the authors also presented their results without fine-tuning and with fine-tuning. Their fine-tuning might also mean additional training. They additionally utilize pruning to compress the model further. However, we only compare our results with their quantization. These results are shown in Figure 4.

5 DeepLIFT based mixed-precision integer quantization

The DNN can also be quantized using integer-based uniform quantization. This speeds up inference on different kinds of hardware and also reduces the memory requirement.

We propose DeepLIFT based search, whereby, instead of reducing the bit-precision one by one, we collectively reduce the bit-precisions in all layers, according to the sensitivities of the layers. We call this coarse search. At some point when the accuracy goes below a certain threshold, we can stop these coarse iterations and start the iterations, whereby, the bit-precision of each layer can be changed, in the order of their sensitivities. In [30], the Hessian is used to sort the layers according to their sensitivities, and then a greedy search is performed to obtain the optimum bit-widths. Their results along with ours for ResNet20 CIFAR10 are briefly summarized as: We obtain an accuracy of 91.2% on CIFAR10 dataset with the 79 Cumulative Bits (CB) for weights and 79 CB for activations, whereas in [30], an accuracy of 92.22% was obtained with 81 CB for weights and 88 CB for activations. For our experiment, was used five coarse iterations and a training step in between utilizing SGD with a learning rate varying from 0.01 to 0.001 for 30 epochs. We did not optimize specially for MACs or NPs.

6 Hardware and software setup

For all experiments, we used NVIDIA TITAN RTX GPU with 24 GB of memory and Intel's i7-9700 CPU. For software, we used Python [31] programming language for all experiments, PyTorch [32] for deep learning, NumPy [33] for array manipulation, Matplotlib [34] for visualization, scikit-learn [35] for machine learning.
for k-means and pairwise distances, Captum \cite{36} for DeepLIFT and explainable AI, PyTorchCV \cite{37} for DNN implementations, NEMO \cite{38} for mixed-precision quantization-aware training, ptflops \cite{39} for profiling MACs and NPs.

7 Conclusion

In conclusion, we utilize the DeepLIFT method for DNN quantization and pruning. DeepLIFT is part of the algorithms in the explainable AI area, and it is considered to have an advantage over other methods for similar tasks. This advantage is also reflected in DNN compression and inversely, we can utilize DNN compression to assess the explainable AI methods. We additionally propose our novel algorithms utilizing DeepLIFT for a wide range of tasks in DNN pruning and quantization like structured pruning, unstructured pruning, CNN filter pruning, weights pruning, mixed precision integer quantization and weight sharing quantization, for optimizing memory as well as latency. Besides achieving state-of-the-art or competitive results on most problems, this work also provides a unified approach for a diverse set of compression problems, which can be very useful for its application for real-world problems.

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