Towards Optimal Path Allocation for Unreliable Reconfigurable Intelligent Surfaces

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Abstract—Terahertz (THz) communications and reconfigurable intelligent surfaces (RISs) have been recently proposed to enable various powerful indoor applications, such as wireless virtual reality (VR). For an efficient servicing of VR users, an efficient THz path allocation solution becomes a necessity. Assuming the RIS component is the most critical one in enabling the service, we investigate the impact of RIS hardware failure on path allocation performance. To this end, we study a THz network that employs THz operated RISs acting as base stations, serving VR users. We propose a Semi-Markov decision Process (SMDP)-based path allocation model to ensure the reliability of THz connection, while maximizing the total long-term expected system reward, considering the system gains, costs of link utilization, and the penalty of RIS failure. The SMDP-based model of the RIS system is formulated by defining the state space, action space, reward model, and transition probability distribution. We propose an optimal iterative algorithm for path allocation that decides the next action at each system state. The results show the average reward and VR service blocking probability under different scenarios and with various VR service arrivals and RIS failure rates, as first step towards feasible VR services over unreliable THz RIS.

Index Terms—SMDP, path allocation, virtual reality, reconfigurable intelligent surfaces, reliability.

I. INTRODUCTION

Virtual Reality (VR) applications are envisioned as one of the key technologies that will advance the human to machine interactions where a user is present and acts in a virtual world [1]. In order to satisfy the high demand of data rates that VR applications require, the THz frequency band is currently one of the most promising communications technology to provide a high quality VR experience. As THz communication is characterized by directional and sensitive to attenuation beams, a new technology called Reconfigurable Intelligent Surfaces (RIS) is used to enhance the performance of THz wireless communications [2]. In an indoor setting, the usage of RIS in THz communications is rather critical, since by controlling the reflective properties of the underlying channels, we can mitigate various THz transmission impairments [3].

It is known that RIS elements, also called unit cells or meta-atoms, are vulnerable to failures, which can cause deterioration of the antenna radiation pattern, and in severe cases, the RIS failures can affect the functioning of RIS meta-surface elements [4]. In addition, THz links between users and RISs can be blocked, due to various obstacles. In order to design reliable RIS based networks in practical environments, it is becoming essential to develop path allocation schemes that are aware of RIS meta-surface failures, as well as link failure between RISs and users. The path allocation problem in RIS networks is indeed one of the key problems to solve for any efficient RIS design, where several sub-problems can be investigated, including channel allocation, power allocation, RIS-to-user assignment, and phase shifts design. Few works studied resource allocation problems, but no work to date considered the problem of path allocation under faulty RIS scenarios. To the best of our knowledge, path allocation of VR over THz RIS, considering the unreliability of RIS devices has not yet been investigated.

In this paper, we consider path allocation in the VR applications over RIS network with controlled access of VR users request by Semi-Markov decision Process (SMDP). We introduce an optimal path allocation scheme to ensure the reliability and maximize the system reward in a set of RISs, whereby a RIS device consists of blocks of meta-surfaces which are vulnerable to failures. We formulate the problem of RIS unreliability-aware path allocation as an SMDP model considering multiple RIS devices in an indoor environment used to allocate THz meta-surfaces blocks to VR users. Following the definition of SMDP [5], the decision is taken at the event occurrence, while an iterative algorithm proposed maximizes the total long-term reward of the VR over RIS system, considering the factors of user’s income of the accepted services, the costs for occupying paths, and penalty of RIS and meta-surface failures. We make realistic assumptions on the reference THz network model, where a set of backup RIS devices are used to ensure the reliability of the system. When a RIS or a meta-surface fails, other blocks of meta-surfaces in the same RIS or in the corresponding backup RIS are allocated to transfer the existing services, which allows the RIS network to operate in a reliable fashion. Numerical results show the average reward and blocking probability with various service arrival and RIS failure rates, and under different network configurations. We show that our scheme improves the QoS of THz communication with regard to the arrival rate of service requests and RIS meta-surfaces availability. The results show that increasing the number of RISs and meta-surfaces does not always improve the performance in terms of long-term reward and service blocking for service arrival rates below a certain threshold. Hence, our model can help dimensioning RIS THz networks in terms of costs and performance based on the expected service rate.

The rest of this paper is organized as follows. Section II
discusses the related work. Section III presents the SMDP based path allocation model in indoor THz network, with unreliable RIS. Section IV describes the proposed optimal iterative-based solution. Section V evaluates the performance. We conclude the paper in Section VI.

II. RELATED WORK

VR communication has been recently investigated in several seminal works [6], [7], focusing on studying the VR Quality of Service (QoS). In [6], a VR model was studied that detect the tracking and delay components of VR QoS. In [7] the downlink of VR requests over RISs network, operating over the terahertz (THz) frequency bands, was considered, and a solution was proposed to the problem of associating RISs to virtual reality users.

Due to the importance of hardware failures, the baseband complex received signal from a RIS element has been modeled in [8], considering faulty reflecting elements of RIS. Thus, in order to design reliable RIS based networks in practical environments, it is essential to develop resource allocation schemes aware of RIS array failures. The optimal number of reflecting elements in a RIS has been studied theoretically in [9] to maximize the transmission rate in a point-to-point link. Motivated by this work, which concluded that new, sophisticated resource allocation schemes are needed, we use a model of a VR over THz RIS network under the similar failure assumptions to solving path allocation problem. Recent works studied the resource allocation problem in THz RIS networks specifically. Paper [10] studied different types of active RISs (active and passive), considering their hardware architectures, operation modes, and applications in communications, and also highlighted the need of developing efficient resource allocation solutions, which is still an open challenge. In [7], a virtual reality network was considered and a solution was proposed to the problem of associating RISs to virtual reality users operating over the terahertz frequency bands. In particular, the paper formulates a risk-based framework based on the entropic value-at-risk and optimize the transmission rate and reliability. A single RIS device was considered and the formulated problem aims at achieving higher order statistics of the queue length, in order to guaranteeing continuous reliability. We adopt a similar VR over RIS model as in [7], however our focus is not on optimizing the transmission rate by adjusting the queue length. Instead, we assume the existence of multiple RISs that can provide the same set of VR users, as well as the possibility to allocate multiple paths for the same VR user request, while considering the failure possibility of RISs and meta-surfaces.

It should be noted that path allocation problem has been well investigated in other contexts such as in vehicular networks. In [11], an SMDP-based path allocation scheme was proposed in vehicular ad-hoc networks to maximize the overall system rewards, while servicing user requests through roadside units. The paper concluded that due to the high mobility of users, an SMDP policy for the path allocation problem can improve the QoS. Motivated by this work, we could see the VR over RIS network as a cognitive-enabled vehicular ad hoc networks, where VR users can be mapped to vehicle users, RISs could replace the roadside units (RSUs), and the allocation of meta-surface blocks can be seen as path allocation, which is our approach in this paper. SMDP-model was also proposed in [12] to solve path allocation problem in order to maximizing throughput in the context of vehicular networks. The related work is primarily based on linear programming which is known to be an NP hard problem, and that it works for off-line processing, whereas our proposed model works for online channel assignment to users using an iterative solution, that assumes the knowledge about some system statistics, such as the RIS failure, service arrival, and service departure. Moreover, our approach assumes that the system statistics are known, and that the RIS configuration such as phase shift and power control are managed by other algorithms independently. Thus, our approach is focused on path allocation only, as a function of VR user’s THz channel request behavior. We also do not consider the dependency between RIS failures, which is a limitation, as is the intrinsic limitation of SMDP to scale to larger systems.

III. SMDP BASED CHANNEL ALLOCATION OF VR OVER THz RIS

In this section, we model a RIS-based wireless reference THz network in an indoor area, provide the channel model, analyze the channel states and actions, transmission probabilities, as well as rewards analysis.

A. The reference network and assumptions

The reference THz network is illustrated in Fig. 1. The THz network consists of a set of RISs, small base station (SBS) operating over THz frequency, RIS controller, and VR users distributed in an indoor environment. RISs are placed in an indoor environment where they can be reached by users in certain areas based on the distance, direction and the existence of obstacles. The users are assumed to be mobile and can have different locations in the indoor environment. The VR user’s end-devices can send several service requests through the RIS network. Each mobile wireless VR user is serviced by a main RIS, using a certain allocated blocks of meta-surfaces. We assume that the block of meta-surfaces can be non-functional due to hardware problems, e.g., due to dirty surfaces or failure. Thus, the RIS controller allows to re-allocate service request to another available meta-surfaces of the same RIS, or meta-surfaces of a backup RIS. The SBS is responsible for receiving requests from VR users and service them via RIS components. The uplink of VR service requests is adopting an ultra reliable low latency communications (URLLLC) scheme, similar to [7].

In this study, we consider the downlink of the RIS-based network only, where each RIS is covering a part of the indoor environment and can provide THz communication channel to VR users. The end-devices of VR users can send several service requests through the RIS network. Each RIS can reflect the beams received from a main THz SBS, which generally uses a MIMO system. A RIS contains a
very large number of passive reflective elements called meta-surfaces, capable of reconfiguring properties of electromagnetic (EM) waves impinging upon them. We assume that all RISs are connected to the RIS Controller responsible for channel tuning by changing the phase of the reflected signal [13]. We assume that a RIS contains several blocks of meta-surfaces, configured to create a path from the SBS to a user. These blocks can be allocated based on the system availability. In order to ensure the reliability of the system, we pre-reserve a backup RIS to each operating RIS. We assume that services can be transferred from the operating RIS to the backup RIS when operating RIS or a single block of meta-surfaces or a THz connection to a RIS fails. Figure 1 illustrates three failure scenarios. First, when a block of meta-surfaces fails, we assume the failure of the block will affect at most one user, while all other RIS blocks remain unaffected. To insure the reliability, we first check whether the main operating RIS has available meta-surfaces for re-allocation, otherwise connection from the main RIS is transferred to an available backup RIS. When an entire RIS fails, all connections will be affected and transferred to a backup RIS. The same recovery strategy is applied, is a THz connection between RIS and VR users fails, as shown in Fig. 1.

Without loss of generality, we assume that the RIS network can include heterogeneous RIS devices, of various sizes and containing different number of blocks of meta-surfaces, and thus servicing different number of user through multiple paths. When a VR service requested by VR user is detected, the RIS controller accepts or rejects it based on the availability of paths, i.e the RIS resources. In this paper, we assume that the THz link between the meta-surface $n$ of the RIS $r$ and the user $u$ at time slot $t$ is available and $0$ otherwise. The corresponding random channel gain is defined as in [7] as:

$$h_{r,n,u,t} = \begin{cases} \left( \frac{c}{4\pi f d_{r,u,t}} \right)^2 e^{-k(f)d_{r,u,t}}, & \text{if } P(L_{r,n,u,t} = 1), \\ 0, & \text{if } P(L_{r,n,u,t} = 0). \end{cases}$$

where $c$ is the speed of light in the vacuum, $d_{r,u,t}$ is the distance between RIS $r$ and the VR user at time slot $t$, $f$ is the operating frequency, $k(f)$ is the overall molecular absorption coefficients of the medium at THz band, available from HITRAN database [14].

The transmission rate between RIS $r$ and the VR user $u$ at time slot $t$ is given by:

$$c_{r,u,t} = W \cdot \log_2 \left( 1 + \frac{P_{RIS} h_{r,n,u,t} \sum_{n=1}^{N} |e^{j(\phi_{r,n,u,t} - \psi_{r,n,u,t})}|^2 L_{r,n,u,t}}{N(d_{r,u,t},p,f)} \right)$$

where $N$ is the number of meta-surface blocks in a RIS, $W$ is the total bandwidth, $P_{RIS}$ is the transmit power related to each RIS, $\psi_{r,n,u,t}$ is the phase shift of meta-surface element $n$ of RIS $r$ with respect to user $u$ at time slot $t$, $\phi_{r,n,u,t}$ is the phase shift of the channel between meta-surface element $n$ of RIS $r$ with respect to user $u$ at time slot $t$, $N(d_{r,u,t},p,f) = N_0 + \sum_{r=1}^{R} P_{RIS} A_0 e^{-K(f) d_{r,u,t}}(1 - e^{-K(f) d_{r,u,t}})$, $N_0 = \frac{W \lambda^2}{4\pi^2} k_B T_0$, $T_0$ is the temperature (Kelvin), $k_B$ is the Boltzmann constant, $A_0 = \frac{c^2}{16\pi^2 f^2}$ [15].

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**Figure 1:** The reference indoor THz RIS network.
The obtained transmission rate can be used to calculate the service rate of a specific VR service $s$, given by:

$$C_{r,u,t,s} = \frac{c_{r,u} \tau_0}{O_s}$$  \hspace{1cm} (3)

where $\tau_0$ is the duration between two discrete time slots, $O_s$ is the size of the object related to the service $s$, such as the size of a VR image.

C. Problem Formulation

We consider a RIS-based THz network including a set of RISs, i.e., a set of operating RISs $R = \{r_1, ..., r_i, ..., r_R\}$ and each RIS with index $i$ is assigned to a different RIS called a backup RIS, the set of backup RISs is $B = \{b_1, ..., b_1, ..., b_R\}$, managed by the RIS controller. We study the channel allocation problem regarding different RIS components, where VR users are requesting service connections. The RIS $r_i$ and $b_i$ contain $N(i)$ meta-surface blocks that can be allocated to create THz paths. A service request can occupy $k \in \{1, ..., K\}$ block based on the resource availability, where $K$ represents the maximum number of blocks a RIS can allocate to a single service. We model the arrival and service processes of VR service requests from a user as a Poisson process with mean rates $\lambda_s$ and $\mu_s$, respectively, where $\mu_s$ can be concluded from eq. (3) for different services.

D. System States

The system state $s$ represents the number of VR service requests with different number of allocated block $k$ in a RIS $r_i$, the availability of the RIS and the next event that can happen in the system:

$$S = \{s | s = (\Delta, X, e)\},$$  \hspace{1cm} (4)

where a set $\Delta = \delta^1, ..., \delta^r, ..., \delta^R$ collects all subsets $\delta^r$, which describe a state $s$ and indicate a number of VR services requested $\delta^r_i = \{\delta_{\delta_1}(r_i), ..., \delta_{\delta_2}(r_i), ..., \delta_{\delta_K}(r_i)\}$. The variable $\delta_{\delta_k}(r_i)$ denotes the number of VR services allocated with $k$ blocks in the main RIS $r_i$. A set $X = \{X^r_1, ..., X^r_n\}$ describes the availability of RIS devices and indicates the number of available meta-surface blocks: $X^r = N(i)$ if all the elements of the RIS $r_i$ are available, $X^r_j = N(i) - j$ if $j$ elements are failed and $X^r = 0$ if all elements are failed or the RIS hardware fails. The variable $e$ describes an event that occurs in the VR over THz RIS network, such as $e = \{Ar, D^r_1, F^m, Re^m\}$, where a set $Ar$ denotes the arrival of any VR service request, a subset $D^r = \{D^r_1, ..., D^r_i, ..., D^r_n\}$, and a subset $D^r_k = \{D^r_1, ..., D^r_{i-k}, ..., D^r_{R-1}\}$ collects the set of departure events of a VR service from a main RIS $r_i$, and $D^r_k$ defines the departure of a VR service allocated in $k$ blocks from a RIS $r_i$, a set $F^m = \{F^m_1, ..., F^m_i, ..., F^m_R\}$ and a variable $F^m_k$ define the failure process of a meta-surface block in the RIS $r_i$, a set $Re^m = \{Re^m_1, ..., Re^m_i, ..., Re^m_R\}$ and a variable $Re^m_k$ define the return of a path/meta-surface block in the RIS $r_i$ to working state after a failure.

E. Actions

The system controller has several possibilities of actions a to take when it receives a service request, whether to accept or reject it. The action space $A(s)$ is described as follows:

$$A(s) = \begin{cases} \{0, (i, k)\}, & e = Ar, k \in \{0, 1, 2, ..., K\}, i \in \{1, ..., R\} \\ (-1), & e = \{D^r_1, Re^m_i\} \\ (-2, T), & e = F^m_i \end{cases}$$  \hspace{1cm} (5)

where $a(s) = (i, k), \forall k \in \{1, ..., K\} \forall i \in \{1, ..., R\}$ when a VR service request is accepted and $k$ blocks are allocated in RIS $r_i$, $a(s) = 0$ denotes the action of rejecting a service request. When a service completes and departs the system, a RIS node or meta-surface block returns into the system after a failure, no action is required, and the controller needs only to update the system state, we represent the action as $a(s) = -1$. When a RIS fails, all the allocated blocks are transferred to the backup RIS, and when a meta-surface block fails, a corresponding block will be allocated in the same RIS if resources are available or in a backup RIS otherwise, we denote the information update and the transfer action as $a(s) = (-2, T)$, where $T$ is the transfer vector from current RIS state to re-allocation state.

F. Transition Probabilities

We assume that the time period between two continuous decision epoches follows an exponential distribution and denoted as $\tau(s,a)$, given the current state $s$ and action $a$. Thus the mean rate of events for a specific state $s$ and action $a$ denoted as $\gamma(s,a)$, is the sum of the rates of all events in the RIS system, which is expressed as follows:

$$\forall i \in \{1, ..., R\}, \forall k \in \{1, ..., K\} : \tau(s,a) = \gamma(s,a)^{-1} = \begin{cases} \lambda_s + \Lambda^m + \Theta^m + \Theta + k\mu_s, & e = Ar, a = (i, k), \\ \lambda_s + \Lambda^m + \Theta^m + \Theta, & e = Ar, a = 0, \\ \lambda_s + (\Lambda^m - \lambda^m) + (\Theta^m + \mu^m) + \Theta, & e = Re^m_i, a = -1, \\ \lambda_s + \Lambda^m + \Theta^m + \Theta - k\mu_s, & e = D^r_k, a = -1, \\ \lambda_s + (\Lambda^m + \lambda^m) + (\Theta^m - \mu^m) + \Theta, & e = F^m_i, a = (-2, T), \end{cases}$$  \hspace{1cm} (6)

where $\lambda_s$ is the arrival rate of VR service requests, $\Lambda^m = \lambda^m \sum_{i=1}^{R} (N(i) - X^r)$ is the arrival rate of non available RIS blocks, and $\Theta^m = \mu^m \sum_{i=1}^{R} N(i)$ is the failure rate of
available RIS blocks.

When an arriving service request is rejected, or a RIS or a RIS block returns to system, the total number of blocks allocated in RISs is \( \sum_{i=1}^{R} \sum_{k=1}^{K} \delta_k(r_i) \), so the departure rate of a VR service in RIS system is \( \Theta = \sum_{i=1}^{R} \sum_{k=1}^{K} k \delta_k(r_i) \mu_s \). When a service request is accepted and \( k \) blocks are allocated, one more service is added to the system, thus the departure rate becomes \( \Theta + k \mu_s \). When a departure of a VR service from a RIS \( ri \) allocated in \( k \) blocks occurs, the departure rate becomes \( \Theta - k \mu_s \). When a RIS block fails, the arrival rate and failure rate of the RIS block in the system are adjusted such that the failed block can return in the future and cannot fail again while it is already allocated. All services admitted in the case of a failed RIS block are transferred to a backup RIS block, so it should be accounted in the departure rate, which ensures the system reliability, and thus the number of existing services remains equal to \( \Theta \). The transition probability in our Markov decision model from state \( s \) to state \( s' \) when an action \( a \) is selected is denoted as \( p(s'|s, a) \), which can be determined under different events.

- **State** \( s = (\Delta, X, Ar) \), and \( a = 0 \).

\[
p(s'|s, a) = \begin{cases} 
\frac{\lambda_s}{\tau(s,a)} & s' = (\Delta, X, Ar) \\
\frac{k \delta_k(r_i) \mu_s}{\tau(s,a)} & s' = (\Delta, X, D_k^r) \\
\frac{\lambda_m}{\tau(s,a)} & s' = (\Delta, X, Re_i^m) \\
\frac{\mu_m}{\tau(s,a)} & s' = (\Delta, X, F_i^m) 
\end{cases} 
\]  

where \( \delta(s) = \sum_{i=1}^{R} X_{ri} \) denotes the number of RIS blocks that are available at the state \( s \), and \( \Delta = \delta_{ri} \ldots, \delta_{ri} + I_k, \ldots, \delta_{ri} \), \( I_k \) denotes a vector having \( K \) elements, with k-th element equal to 1 and 0 for the others.

- **State** \( s = (\Delta, X, D_k^r) \), \( a = -1 \).

\[
p(s'|s, a) = \begin{cases} 
\frac{\lambda_s}{\tau(s,a)} & s' = (\Delta, X, Ar) \\
k \delta_k(r_i) \mu_s - 1 & s' = (\Delta, X, D_k^r) \\
\frac{\lambda_m}{\tau(s,a)} & s' = (\Delta, X, Re_i^m) \\
\frac{\mu_m}{\tau(s,a)} & s' = (\Delta, X, F_i^m) 
\end{cases} 
\]  

where \( \Delta = \delta_{ri} \ldots, \delta_{ri} - I_k, \ldots, \delta_{ri} \).

- **State** \( s = (\Delta, X, Re_i^m) \), \( a = -1 \), where \( X_{ri} = N(i) - j \), and \( j \) is the number of failed RIS blocks.

\[
p(s'|s, a) = \begin{cases} 
\lambda_s \left( k \delta_k(r_i) \mu_s \right) & s' = (\Delta, X, Ar) \\
\lambda_m \left( k \delta_k(r_i) \mu_s \right) & s' = (\Delta, X, D_k^r) \\
\mu_m \left( k \delta_k(r_i) \mu_s \right) & s' = (\Delta, X, F_i^m) 
\end{cases} 
\]  

where \( \tilde{X}_{ri} = N(i) - j + 1 \)

- **State** \( s = (\Delta, X, F_i^m) \), \( a = (-2, T) \), where \( X_{ri} = N(i) - j \geq 1 \). The vector \( T \) transfers one or more allocated RIS blocks from \( ri \) to the available RIS.

\[
p(s'|s, a) = \begin{cases} 
\lambda_s \left( k \delta_k(r_i) \mu_s \right) & s' = (\Delta, X, Ar) \\
\lambda_m \left( k \delta_k(r_i) \mu_s \right) & s' = (\Delta, X, D_k^r) \\
\mu_m \left( k \delta_k(r_i) \mu_s \right) & s' = (\Delta, X, F_i^m) 
\end{cases} 
\]

where \( \tilde{X}_{ri} = N(i) - j - 1 \), \( \tilde{\Delta} = \Delta - T \), \( T \) is determined using Algorithm 1.

**Algorithm 1 Finding Service Transfer Vector**

1. **Input:** \( s = (\Delta, X, F_i^m) \)
2. **Initialization:** \( T = T^1, \ldots, T^R, T^i = \{T_i^1, \ldots, T_i^K\} \)
3. \( T_k^i = 0, \forall i \in \{1, \ldots, R\}, \forall k \in \{1, \ldots, K\} \)
4. **if** \( \tilde{X}_{ri} = 0 \) **then** \( k = 1 \)
5. **while** \( \delta_k(r_i) = 0 \) **do**
6. \( k = k + 1 \)
7. **end while**
8. \( T_k^i = -1, T_k^b = 1 \)
9. **end if**
10. **Return** \( T \)

**G. Rewards**

Given the system state \( s \) and the corresponding action \( a \), the system reward of the VR over THz RIS system is denoted by

\[
r(s,a) = w(s,a) - g(s,a) 
\]

where \( w(s,a) \) is the net lump sum incomes of VR users at the state \( s \) when action \( a \) is taken and an event \( e \) occurs, and \( g(s,a) \) is the expected system costs.

\[
w(s,a) = \begin{cases} 
R_k & e = Ar, a = (i,k) \\
0 & e = Ar_k, a = 0 \\
0 & e = Re_i^m, a = -1 \\
0 & e = D_i^k a = -1 \\
-\varepsilon \sum_{k=1}^{K} k T_i^k & e = F_i^m a = (-2, T) 
\end{cases} 
\]

where the variable \( R_k = Q - \frac{Z}{t} \) denotes the reward of the RIS system for accepting of the requested service and
allocating \( k \) blocks. \( Q \) denotes the income reward from VR user satisfaction, \( \xi \) denotes the transmission cost of occupying \( k \) blocks. The constant \( \varepsilon \) denotes the penalty of a meta-surface block failure. When a meta-surface block fails, a number of services are transferred to the backup RIS, and thus we consider a penalty proportional to the number of re-allocated blocks as \( -\varepsilon \sum_{k=1}^{K} kT_k \). In our work, we don’t penalize rejected services, neither reward accomplished services or returned RISs or returned blocks.

The expected system cost \( g(s, a) \) is defined as:
\[
g(s, a) = c(s, a) \cdot \tau(s, a) \quad (14)
\]
where \( \tau(s, a) \) is the expected service time defined by eq. (6) from the state \( s \) to the next state in case that action \( a \) is chosen and \( c(s, a) \) is the service holding cost rate when the RIS system is in state \( s \) in case that action \( a \) is selected. Furthermore, \( c(s, a) \) can be described by the number of occupied blocks in the RIS system, as follows:
\[
c(s, a) = \sum_{i=1}^{R} \sum_{k=1}^{K} c \cdot k \cdot \delta_{i}^{k} \quad (15)
\]
where \( c \) represents the utilization cost of a block unit.

IV. SMDP-BASED CHANNEL ALLOCATION MODEL

In this section, we develop an SMDP-based path allocation model to study the performance of a RIS system considering the unreliability of RIS devices. We aim to take optimal decisions at every decision epochs; arrival of new service request, departure of a service, failure of a RIS, failure of a meta-surface, return of a failed meta-surface block, where our goal is to maximize the long-term expected system rewards. The expected discounted reward is given based on the model in [5] as follows:
\[
r(s, a) = w(s, a) - c(s, a) \cdot E_{\pi}^{s} \{ \int_{0}^{\infty} e^{-\alpha t} dt \}
\]
\[
= w(s, a) - c(s, a) \cdot E_{\pi}^{s} \{ \frac{1 - e^{-\alpha \tau}}{\alpha} \} \quad (16)
\]
where \( \alpha \) is a continuous-time discount factor.

Using the defined transition probabilities eq. (7), (8), (9), (10), (11), we can obtain the maximum long-term discounted reward using a discounted reward model defined in [5] as
\[
\nu(s) = \max_{a \in A(s)} \left\{ r(s, a) + \lambda \sum_{s' \in S} p(s'|s, a)\nu(s') \right\} \quad (17)
\]
where \( \lambda = \tau(s, a)/(\alpha + \tau(s, a)) \). In the SMDP model, the value of \( \nu(s) \) in a strategy \( \psi \) is computed based on the value \( \nu(s') \) obtained in the strategy \( \psi - 1 \), and as an initial value, the discounted reward can be set to zero for all states to initialize the computation, which will converge afterwards to the optimal solution.

To simplify the computation of the reward, let \( \rho \) be a finite constant, where \( \rho = \lambda_{s} + \lambda^{m} \prod_{i=1}^{R} N(i) + \mu^{m} \prod_{i=1}^{R} N(i) + \mu_{s} \sum_{i=1}^{R} N(i) < \infty \). We define \( p(s'|s, a) \), \( r(s) \), and \( \tau(s, a) \) as the uniformed transition probability, long-term reward, and reward function, respectively, and given by:
\[
\tau(s, a) = r(s, a) \frac{\tau(s, a) + \alpha}{\rho + \alpha}, \quad \lambda = \frac{\rho}{\rho + \alpha} \quad (18)
\]
\[
p(s'|s, a) = \begin{cases} 1 - \frac{(1 - p(s'|s, a))\tau(s, a)}{p(s'|s, a)\tau(s, a)} & s' = s \ 
\frac{p(s'|s, a)}{\rho} & s' \neq s \end{cases} \quad (19)
\]
After uniformization, the optimal reward is given by:
\[
\nu(s) = \max_{a \in A(s)} \left\{ \tau(s, a) + \lambda \sum_{s' \in S} p(s'|s, a)\nu(s') \right\} \quad (20)
\]

In order to solve our SMDP-based path Allocation (PA) model, we consider an iterative algorithm described as follows:

Algorithm 2 Iterative SMDP-PA Algorithm

1: **Step 1** (Initialization): \( \pi^{0}(s) = 0 \), for all \( s \in S \). Set the value of \( \epsilon > 0 \), and iteration \( t = 0 \).
2: **Step 2**: Using eq. 20, compute the discounted reward for each state \( s \):
\[
\pi^{t+1}(s) = \max_{a \in A(s)} \left\{ \tau(s, a) + \lambda \sum_{s' \in S} p(s'|s, a)\pi^{t}(s') \right\}
\]
3: **Step 3**: if \( \|\pi^{t+1} - \pi^{t}\| > \epsilon \) then \( t \leftarrow t + 1 \), go to **Step 2**
5: else go to **Step 4**
6: **Step 4**: Compute the optimal policy for all \( s \in S \)
\[
d^{*}_{\pi}(s) \in \arg \max_{a \in A(s)} \left\{ \nu(s, a) + \lambda \sum_{s' \in S} p(s'|s, a)\nu(s') \right\}
\]

After obtaining the optimal policy from Algorithm 2, the steady states probabilities are computed using the following system of equations:
\[
\pi(P - J) = 0, \quad \sum_{s \in S} \pi(s) = 1 \quad (21)
\]
where \( \pi(s) \) represents the steady state probability at state \( s \), \( P \) is the transition probabilities matrix, considering the optimal policy \( d^{*}_{\pi} \), and \( J \) denotes the all-ones matrix.

V. NUMERICAL RESULTS

In this section, we validate and evaluate the proposed SMDP-based unreliability-aware path allocation for VR over THz RIS system using a Python simulator to implement the model and the algorithms proposed. The parameters used for simulation are summarized as follows: \( R = 1 - 3 \), \( K = 1 - 2 \), \( c = 1 \), \( \epsilon = 100 \), \( Q = 150 \), \( Z = 100 \), \( \alpha = 0.1 \), \( \lambda_{s} = 1 - 10 \), \( \mu_{s} = 5 \). To investigate the performance of the RIS system under different system settings, the simulation results present a function of the service arrival rate, service departure rate,
RIS blocks failure rate, RIS blocks return rate. We define 4 scenarios with following settings: scenario 1) the RIS network is operating using a two RISs: a main RIS with a single backup RIS \((R = 1)\) containing 5 meta-surfaces \((N(1) = 5)\), and allowing users to allocate only one block for each VR service \((K = 1)\); scenario 2) the RIS network is operating using two main RIS with two backup RISs \((R = 2)\), similar to scenario 1, \(K = 1\); scenario 3) the RIS network is operating with three main RISs and three backup RISs \((R = 3)\), the system allows the allocation of two blocks at maximum for a single VR user request \((K = 2)\), where each RIS contains 5 blocks \((N(r_i) = 5, \forall i \in [1, 2, 3])\); and scenario 4) the RIS network is operating with three main RISs and three backup RISs \((R = 3)\), the system allows the allocation of two blocks at maximum for a single VR user request \((K = 2)\), whereby the amount of blocks in RISs was set to \(N(r_1) = 4, N(r_2) = 3, N(r_3) = 2\). Similar to [11], the discount factor \(\alpha\) is 0.1.

Fig. 2.a) shows the acceptance probability as a function of the RIS block failure rate under different scenarios defined above. Here, we set \(\lambda_s = 1\). As expected, the acceptance probability of a service decreases with increasing failure rate of RIS blocks. The decrease of the acceptance probability can be explained with the increasing request blocking due to the capacity of the RIS system reduced by failed RIS blocks. The highest acceptance probability of around 68% could be reached in scenario 3 with 3 RISs and low RIS block failure rate. This can be explained by the fact that the high RIS blocks/path availability with 4 RISs and 4 backup RISs, where hardware failure is managed by the backup RISs. The RIS system configured based on the scenario 1 and 2 show the highest acceptance probability for the higher RIS blocks failure rate starting from \(\mu^m = 0.1\) and decreases from 45 % to 30%. The RIS system in scenario 4 shows the lowest service acceptance probability up to 37 %, as the RIS system can not always satisfy the amount of VR user requests with the limited amount of RISs and additional RIS failures.

Next we investigate the average system reward as a function of different arrival rates of service requests and different RIS block failure rates. Fig. 3 illustrates that the highest and the lowest average reward could be provided in scenario 1, and in scenarios 4 and 3 with \(\lambda_s > 9\), respectively. In general, the average reward decreases with increasing arrival rate of service requests, which is a result of the limited capacity of RIS system. Additionally, in scenarios 3 and 4, the service requests can be blocked, when available RIS do not provide enough meta-surfaces/channels for the requested service. When the service arrival rate increases, the overall capacity needed to provide all service requests is higher than the RIS capacity. As a result, the system rejects any new incoming service requests, which decreases the average reward value. The average reward is a good metric for future work to compare an optimal path allocation policy with heuristic or machine learning based allocation algorithms.

Fig. 2.b) shows an average reward as a function of the RIS block failure rate. In our settings we highly penalize the RIS block failure, which explains the drastic decrease of the average reward, since the probability of accepting services becomes lower, and where we increased the number of RIS. In Fig. 2.b) the average reward follows a convex decreasing function in terms of RIS block failure rates, with service arrival rate \(\lambda_s\) equal to 1. The figure shows that with low failure rate the reward is the same under all scenarios, and then it becomes the worst with scenarios that have more RIS. The minimum reward for scenario 1 is 0, which means that the single RIS has failed and no service is accepted, while for scenario 3, the system can have failed RIS and still accepts service which will have a high probability to be lost in the case of failure, causing a negative reward value.

Finally we illustrate in Fig. 4-2.c) the blocking probabilities with different arrival rates of services and with different RIS failure rates under different scenarios. We vary the service arrival rate and \(\lambda_s = 1\) when varying the RIS block failure rate. The blocking probability is very low with low arrival rates, less than 0.001. It increases sharply after reaching the value \(\lambda_s = 5\), in scenario 1, due to the lack of blocks to allocate of a single RIS system and reaches the value 0.31 when \(\lambda_s = 10\). For high service arrival rate, the more we increase the number of RISs the more stable our system gets. However, for low and medium arrival rates, scenario 1 with less RISs performs better than scenario 4 with 3 RISs, thus increasing the number of RISs and meta-surface blocks does not always improve the performance in terms of service blocking and long-term reward. In Fig. 2.c), we fix the service arrival rate to 1 and we vary the RIS blocks failure rate, we remark that the blocking probability is equal to 0 all values under \(\mu^m = 0.1\) which shows the stability of our channel allocation algorithm. For \(\mu^m = 0.5\), the blocking increases then to reach a maximum of 0.14, 0.03 and 0.01 for scenarios 1, 2 and 3, respectively. We see that increasing the number of RISs and meta-surface blocks highly improves the performance of RIS network in terms of service blocking.

VI. Conclusion

We considered path allocation in the virtual reality (VR) applications over RIS network with controlled access of VR users request by Semi-Markov decision Process (SMDP). We introduced an optimal path allocation scheme to ensure the reliability and maximize the system reward in a set of RISs, in which a RIS device and its meta-surface elements are vulnerable to failures. We formulated the problem as an SMDP model considering multiple RIS devices in an indoor environment used to provide services to VR users. Numerical results showed the average reward and blocking probability with various service arrival and RIS block failure rates, and under different network configurations. The system computed corresponding policies to each service arrival rate to ensure the reliability and maximize long-term rewards. We showed that the proposed scheme was generally applicable, dynamic and provided an efficient solution to the path allocation problem. We showed that increasing the number of RISs and meta-surface blocks does not always improve the performance in terms of service blocking and long-term reward, for low and
medium service arrival rate. Such finding allows us to better dimensioning RIS networks based on the expected service rate, in order to improve costs and performance at the same time. In our future work, a large-scale solution based on reinforcement learning can be proposed to solve value iteration algorithm, which is known by its exponential growth.

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