Constrained Thompson Sampling for Wireless Link Optimization

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Abstract

Wireless communication systems operate in complex time-varying environments. Therefore, selecting the optimal configuration parameters in these systems is a challenging problem. For wireless links, rate selection is used to select the optimal data transmission rate that maximizes the link throughput subject to an application-defined latency constraint. We model rate selection as a stochastic multi-armed bandit (MAB) problem, where a finite set of transmission rates are modeled as independent bandit arms. For this setup, we propose Con-TS, a novel constrained version of the Thompson sampling algorithm, where the latency requirement is modeled by a linear constraint on arm selection probabilities. Since our algorithm learns a Bayesian model of the wireless link, it can be adapted to exploit prior knowledge often available in practical wireless networks. Through numerical results from simulated experiments, we demonstrate that Con-TS significantly outperforms state-of-the-art bandit algorithms proposed in the literature. Further, we compare Con-TS with the outer loop link adaptation (OLLA) scheme, which is the state-of-the-art in practical wireless networks and relies on carefully tuned offline link models. We show that Con-TS outperforms OLLA in simulations, further, it can elegantly incorporate information from the offline link models to substantially improve performance.

1. Introduction

Transmitting information over a wireless channel is a complex phenomenon. The channel capacity is determined by the signal attenuation and baseband phase rotation, which in turn depend on a variety of parameters, including the signal’s frequency, the distance between the transmitter and the receiver, and the nearby objects (scatterers) that interact with the electromagnetic waves in the intervening period between transmission and reception. Furthermore, any of the transmitter, the receiver, or the scatterers might be mobile. As a result, the response of the wireless channel is time-varying and a priori unknown (Molisch, 2010). Therefore to optimize a wireless link, the communication system needs to navigate its complex configuration parameter space, and do it quickly.

The central goal in wireless link optimization relates to maximizing the link throughput, since the transmission bandwidth is a limited and expensive resource. At the same time, ubiquitous wireless services such as telephony and video streaming need to maintain a minimum service-level latency, which is measured by the fraction of data packets transmitted successfully within a given time interval. The goals of throughput maximization and latency are inherently in conflict: aggressive data transmission rates tend to improve throughput, however, they are more likely to result in transmission failures as well. The rate selection problem deals with the selection of optimal data transmission rate for each data packet, such that the throughput is maximized under a fixed, application-specific, latency constraint. In this paper, we propose a novel rate selection approach for a finite set of candidate rates, where the transmission success probability for each candidate rate is channel-dependent and a priori unknown.

Rate selection is closely related to important problems in otherwise unrelated domains. One example relates to dynamic pricing with inventory constraints and a priori unknown demand, where a seller aims to maximize the cumulative revenue. Here, the seller is often faced with a perishable inventory or a limited selling season, in which case she must trade-off potentially higher future profits with a myopic policy of successfully selling the product. This problem of revenue optimization under inventory constraints has been extensively studied in the literature (Elmaghraby & Keskinocak, 2003). The inventory constraint problems are often interpreted as a specific instances of the so-called knapsack problem that deals with optimal allocation under budget constraints. In this paper, we show that rate selection can be interpreted as another instance of the knapsack problem with additional, application-specific, properties.
Our Contributions:

(i) We formulate the rate selection problem as a constrained optimization problem and show that it is equivalent to the well-known knapsack problem. Since the solution to this knapsack problem is NP-hard, we apply a linear programming relaxation by introducing a probabilistic selection vector over the set of available rates. Subsequently, we derive conditions under which an optimal solution for the probabilistic rate selection problem exists and comment on the nature of this solution.

(ii) We propose a novel constrained version of the Thompson sampling algorithm, Con-TS, for the rate selection problem. We argue that Con-TS converges asymptotically to the optimal rate selection policy by building upon the strong theoretical results available in the literature of non-constrained stochastic bandits.

(iii) We provide numerical results demonstrating that Con-TS significantly outperforms state-of-the-art bandit algorithms for rate selection and constrained optimization. Through further numerical evaluations, we show that Con-TS significantly outperforms the state-of-the-art rate selection algorithm employed by practical wireless systems. Finally, we demonstrate the Bayesian nature of Con-TS can be exploited to incorporate prior information commonly available in some wireless networks.

2. Related Work

Thompson sampling is a well-known Bayesian heuristic for solving sequential programming problems and was first introduced in (Thompson, 1933). Recently, (Chapelle & Li, 2011) demonstrated attractive empirical results using Thompson sampling for several common bandit problems, sparking renewed interest in the technique. Further, (Agrawal & Goyal, 2013) and (Kaufmann et al., 2012) showed analytically that Thompson sampling achieves sublinear regret in stochastic multi-armed bandit settings. In this paper, we use the ideas from (Kaufmann et al., 2012) to argue for asymptotic convergence of our proposed Con-TS algorithm.

Constrained bandit problems have recently been studied in the context of revenue maximization under a finite inventory setting, termed *bandits with knapsacks* (BwK). In (Badanidiyuru et al., 2013), a upper confidence bound (UCB)-based approach was introduced that was shown to be optimal for the stochastic BwK problem. Further in (Xia et al.), a Thompson sampling algorithms for budgeted multi-armed bandits was proposed that outperforms the UCB BwK algorithm. Subsequently in (Ferreira et al., 2018), Thompson Sampling was studied for revenue optimization for a finite inventory that contains multiple, non-identical products. In this paper, we formulate the the rate selection problem as an a knapsack problem. To the best of our knowledge, this is a novel formulation of the rate selection problem that provides new insights into its analysis. Further, unlike finite-inventory problems where the inventory costs accrue over time, we show that the latency constraint can be formulated in terms of the expected transmission success probability.

In (Chen et al., 2018), a constrained multi-play UCB approach, ConUCB, was developed to estimate a probabilistic selection vector for determining the optimal subset of links that are displayed to the user in each round. The ConUCB algorithm learns the *first-level* click-through-rate (CTR) for each link, and the *second-level* expected revenue collected after a user clicks a particular link. Subsequently, ConUCB calculates a probabilistic selection vector in each round to select the optimal subset of links, i.e., the subset of links which exceed the CTR constraint and maximize the expected cumulative revenue over a finite time horizon. The rate selection problem can be interpreted as a single-play variant of the problem studied in (Chen et al., 2018), where the second-level reward is the fixed throughput associated with a given rate. In Sec. 5, we show that our novel Bayesian algorithm for the rate selection problem, (i) empirically outperforms a suitably adapted version of the ConUCB approach, and (ii) allows us to incorporate valuable prior information available in wireless communication networks.

Rate selection is an area of active research in the context of wireless communication systems, such as WiFi networks (Lakshmanan et al., 2011) and cellular networks (Tao & Czyliwik, 2011). Stochastic MAB approaches for selecting a single optimal, throughput-maximizing rate, have recently been studied in the literature. In (Combes et al., 2014), a UCB-based, *graphical optimal rate sampling* (G-ORS) algorithm was proposed that exploits the fact that the throughput function over the rates has a unique maxima (*unimodality*). A Thompson sampling approach that also exploits the unimodality property, termed *unimodal Thompson sampling* (UTS), was proposed in (Paladino et al., 2017) and was shown to outperform G-ORS type of algorithms. However, none of the existing approaches admit latency constraints, which is a focus of our work. In Sec. 5.1, we provide empirical results that demonstrate superior performance for Con-TS compared to the UTS algorithm.

State-of-the-art wireless networks address rate selection though a combination of offline and online approaches. An offline *link-to-system* model is carefully tuned for a broad class of wireless channels and rates (Brunnerhaus et al., 2005; Saxena et al., 2018). The transmitter uses this offline model to obtain initial estimates of the transmission success probabilities for each candidate rate for the current channel conditions. The transmitter then refines these estimates through *outer loop link adaptation* (OLLA) step adjustments based on the observed transmission successes and failures.
However, OLLA suffers from two major shortcomings: (i) OLLA converges to match the latency constraint (Wu & Jindal, 2009), however, in Sec. 3.2, we show that such a rate selection strategy is not necessarily optimal. (ii) The OLLA step sizes are heuristically defined, and often lead to poor convergence and steady state performance steady-state performance of OLLA depends on the scaling factor of the step sizes (Buenestado et al., 2014). In Sec. 4.2, we describe the OLLA algorithm and in Sec. 5.2, we show that Con-TS significantly outperforms OLLA for simulated experiments.

3. Model and Objectives

3.1. Wireless Link Model

A wireless transmitter sends packetized data over discrete transmission intervals. In each transmission interval, the transmitter processes a variable number of data bits by encoding them for reliable reception and modulating them onto waveforms for transmission over the air. In each transmission interval, the choice of the rate index, \( k \in \{1, \ldots, K\} \), determines the number of data bits \( D_k \) that are prepared for transmission. The sequence of \( D_k \) data bits are mapped to a sequence of \( C_k \) coded bits by an error protecting channel code. The coded bits are then mapped onto waveforms, represented as data symbols from a symbol alphabet, and transmitted over the air. Assuming that \( \Delta f \) and \( \Delta t \) is the bandwidth and transmission duration, respectively, the (instantaneous) rate of the transmission is given by

\[
    r_k \triangleq \frac{1}{\Delta t \Delta f} D_k \quad \text{[bits / s / Hz]},
\]

i.e., the number of information bits transmitted in the transmission interval normalized by the resources utilized to do so. The specific channel code used (type and rate), and the symbol alphabet is determined by the modulation and coding scheme \( m_k \), thus allowing for adaptive control of the data rate and error protection of the transmission. Known, deterministic, pilot symbols are periodically inserted into the transmitted signal in order to allow the receiver to estimate the frequency- and time-dependent state of the wireless channel.

The wireless channel adds stochastic effects such as channel attenuation, baseband phase rotation, noise, and interference to the transmitted signal as it propagates over the air. The receiver attempts to recover the transmitted bits by first compensating for the estimated channel attenuation and phase rotation and subsequently applying inverse operations for the steps performed prior to transmission. The receiver reports back to the transmitter a positive acknowledgement, \( x_k = 1 \), in the event the reconstructed data bits are received correctly (usually validated by a cyclic redundancy check embedded in the packet), and a negative acknowledgement, \( x_k = 0 \), otherwise. Viewing \( x_k = x(r_k) \) as a family of stochastic variables, indexed by the rate \( r_k \) and with distributions induced by the random channel effects, yields the transmission success probability, \( \mu_k = Pr\{x(r_k) = 1\} \).

3.2. Optimal Rate Selection

Consider a wireless link with stationary transmission success probabilities \( \mu_1, \ldots, \mu_K \) for the finite set of available rates \( r_1, \ldots, r_K \). For the choice of rate index \( k(t) \in \{1, \ldots, K\} \) at time interval \( t \), either \( r_k(t) \) bits are delivered to the receiver with probability \( \mu_k(t) \), or zero bits are delivered with probability \( 1 - \mu_k(t) \). The transmission success event is modeled by the independent and identically distributed (i.i.d.) random variables, \( x_k(t) \sim \text{Bern}(\mu_k(t)) \). Consider the sequence of time intervals \( T = [t, \ldots, t + \Delta T - 1] \) over which the latency is constrained to be larger than \( \tau \). Our goal is to find the optimal sequence of rate indices \( k(T) = [k(t), \ldots, k(t + \Delta T - 1)] \) that maximizes the link throughput subject to the latency constraint, i.e.,

\[
    \arg\max_{k(T)} \frac{1}{\Delta T} \sum_{t \in T} r_k(t) x_k(t)
\]

\[
    \text{s.t.} \quad \frac{1}{\Delta T} \sum_{t \in T} x_k(t) \geq \tau.
\]

Since \( x_k(t) \leq 1 \), \( t \in T \), Eq. 2 is an instance of the well-known knapsack problem that is known to be NP-hard (Martello, 1990). We address this problem in the following manner: First, we reformulate Eq. 2 in terms of the number of times that each rate is selected during \( T \). Subsequently, we apply a linear programming relaxation by introducing a probabilistic rate selection vector. Finally, we derive conditions on the existence of an optimal solution.

Denote by \( n_k(T) \) and \( s_k(T) \) the number of times that rate \( r_k \) is selected and the number of observed successes respectively, during the time interval \( T \). Assuming that the transmission success events are i.i.d., we can reformulate Eq. 2 as

\[
    \arg\max_{n_1, \ldots, n_K(T)} \frac{1}{\Delta T} \sum_{k=1}^{K} r_k n_k(T) s_k(T)
\]

\[
    \text{s.t.} \quad \frac{1}{\Delta T} \sum_{k=1}^{K} n_k(T) s_k(T) \geq \tau.
\]

For sufficiently large \( T \), the we can approximate \( n_k(T) \) and \( s_k(T) \) with their corresponding probabilities. In this manner we can write the linear programming equivalent of the optimization problem stated above. Recall that the transmission success probability for rate \( r_k \) is denoted by \( \mu_k \), then by denoting the probability of selecting rate \( r_k \)
with $p_k \in [0, 1]$, we can write

$$\arg\max_{\mu_1, \ldots, \mu_K} \sum_{k=1}^K p_k r_k \mu_k$$

subject to

$$\sum_{k=1}^K p_k \mu_k \geq \tau,$$  

where $p_1, \ldots, p_K$ is the probabilistic rate selection vector and $\sum_{k=1}^K p_k \leq 1$. We are now ready to describe the condition under which feasible rate selection policies exist.

**Proposition 1.** For the set of transmission success probabilities $\mu_1, \ldots, \mu_K$ such that $\mu_k < \tau$ for all $k \in \{1, \ldots, K\}$, there does not exist any probabilistic rate selection vector $p_1, \ldots, p_K$ that satisfies Eq. 4.

**Proof.** We define the continuous linear functional $\mu = [\mu_1, \ldots, \mu_K]$ and the normed vector space $p = [p_1, \ldots, p_K]$. Then denoting the dual norms of $\mu$ and $p$ by $|\mu|$ and $|p|$ respectively, we have that $\mu^T p \leq |\mu|_\infty |p|_1$ from the property of dual norms, where $(\cdot)^T$ denotes the vector transpose. From the definition we have that $|\mu|_\infty < \tau$ and $|p|_1 = 1$. Plugging these values into the inequality above we get, $\mu^T p = \sum_{k=1}^K p_k \mu_k < \tau$. In this case, there is no feasible probabilistic rate selection vector. \hfill \Box

Next, we investigate the condition where the throughput-maximizing rate simultaneously satisfies the latency constraint.

**Proposition 2.** If there exists a rate index $k^*$ such that $k^* = \text{argmax}_k r_k \mu_k$ and $\mu_{k^*} \geq \tau$, then the optimal rate selection policies are degenerate to $p_{k^*} = 1$ and $p_k = 0 \forall k \neq k^*$.

**Proof.** Assume a rate selection vector with non-degenerate probabilities that has support for the set of rates $\{K \cup k^*\}$. For any rate $k \in K$, $r_k \mu_{k^*} \geq r_{k^*} \mu_{k^*}$ by definition. Therefore we have that $p_k r_k \mu_{k^*} \geq p_k r_{k^*} \mu_{k^*}$ and consequently, $\sum_{k=1}^K p_k r_k \mu_{k^*} \geq \sum_{k=1}^K p_k r_{k^*} \mu_{k^*}$, i.e., the expected throughput is increased by assigning the probability mass for $r_k$ to $r_{k^*}$. Further, for any valid vector $p_1, \ldots, p_K$, it holds that $\sum_{k \in \{K \cup k^*\}} p_k \mu_k \geq \tau$. Since $\mu_{k^*} \geq \tau$, $\sum_{k \in \{K \cup k^*\}} p_k \mu_{k^*} \geq \tau$, therefore, the success probability constraint holds trivially. Therefore, the optimal rate selection strategy is to always select the rate $r_{k^*}$. \hfill \Box

**Observation.** If there exists a rate index $k^*$ such that $k^* = \text{argmax}_k r_k \mu_k$ and $\mu_{k^*} \geq \tau$, then the optimal rate selection policy achieves the expected transmission success $\mu_{k^*} > \tau$.

This observation follows directly from Prop. 2, by noting that the rate $r_k$ is selected in each transmission. Consequently, the expected transmission success is the same as the success probability $\mu_{k^*}$ for the selected $r_{k^*}$. Consequently, rate selection strategies that attempt to converge to the target success probability are sub-optimal in terms of their expected throughput. In Sec. 4, we show that the state-of-the-art OLLA scheme suffers from this sub-optimality. Finally, we derive the optimal rate selection policy when throughput-maximizing rates do not satisfy the latency constraint.

**Proposition 3.** Consider a non-empty set of rate indices that strictly exceed the success probability constraint, $U := \{k | r_k > \tau\}$. If there exists at least one rate index $k^* \notin U$ such that $r_{k^*} \mu_{k^*} > \max_{k \in U} r_k \mu_k$ and $\mu_{k^*} \geq \tau$, then the optimal rate selection vector has at least one rate index with a non-zero selection probability that has success probability less than $\tau$. Further for this case, $\sum_{K=1}^K p_k \mu_k = \tau$.

**Proof.** Consider the set of arms $U' := \{k | r_k \mu_k > \max_{k \in U} r_k \mu_k, \mu_k < \tau\}$. Then for any $k_1 \in U'$ and $k_2 \in U$, and $\epsilon > 0$, there exists another constant $\delta > 0$ such that $\epsilon \mu_{k_1} + \delta \mu_{k_2} = (\epsilon + \delta) \tau$. Since $\mu_{k_1} r_{k_1} > \mu_{k_2} r_{k_2}$ by definition, we have that $\epsilon \mu_{k_1} r_{k_1} + \delta \mu_{k_2} r_{k_2} > (\epsilon + \delta) \mu_{k_2} r_{k_2}$. Setting $\epsilon + \delta = \mu_{k_1}$ we have the contribution of the rate index $k_2$ towards the success probability, we obtain $\epsilon$ and $\delta$ such that $\epsilon \mu_{k_1} + \delta \mu_{k_2} = \mu_{k_2} \tau$ and $\epsilon \mu_{k_1} r_{k_1} + \delta \mu_{k_2} r_{k_2} > \mu_{k_2} \mu_{k_2} r_{k_2}$. Repeating this process for all $k \in U$, we obtain the optimal rate selection vector that satisfies this proposition. Consequently for this scenario, the optimal rate selection policy is a probabilistic mixture of multiple rates. \hfill \Box

### 3.3. Multi-armed Bandit Formulation

The transmission success probabilities $\mu_1, \ldots, K$ for the available rates $r_1, \ldots, K$ are a priori unknown at the transmitter. At time interval $t$, the transmitter can select a single rate index $k(t)$ and observe its success or failure event $x_{k(t)}$. Therefore, the transmitter needs to trade-off between selecting those rates that have relatively large uncertainty in their success probabilities and rates that have a high likelihood of good performance. We model this exploration-exploitation tradeoff as a multi-armed bandit problem with $K$ independent arms, where $k(t)$ corresponds to the selection of rate $r_{k(t)}$ at time interval $t$.

The space of rate selection policies is defined as the sequence of rate indices $\pi := \{k^*(1), \ldots, k^*(T)\}$, $\pi \in \Pi$. We are interested in finding the optimal rate selection policy, $\pi^* \in \Pi$ that maximizes the expected link throughput for each transmission under the expected latency constraint,

$$\pi^* = \max_{\pi \in \Pi} \left\{ \mathbb{E} \left[ r_{k^*(t)} x_{k^*(t)} \right] \right\}$$

subject to

$$\mathbb{E} \left[ x_{k^*(t)} \right] \geq \tau.$$  

We define an oracle policy $\pi_0$ that has foreknowledge of the transmission success probabilities $\mu_1, \ldots, K$. We propose
the following performance metrics for any policy $\pi \in \Pi$:

(i) The expected throughput accrued until time $T$ under constraint $\tau$, $T\text{put}(T, \tau)$ (ii) The constraint violations accrued till time $T$, $\text{Vio}(T, \tau)$, and (iii) The loss in expected throughput compared to the oracle rate selection policy, $\text{Reg}(T, \tau)$:

$$T\text{put}(T, \tau) = \sum_{t=1}^{T} E[r_{k,\pi}(t) x_{k,\pi}(t)],$$  

$$\text{Vio}(T, \tau) = \sum_{t=1}^{T} \tau - E[x_{k,\pi}(t)],$$  

$$\text{Reg}(T, \tau) = \sum_{t=1}^{T} [E[r_{k,\pi}(t) x_{k,\pi}(t)] - E[r_{k,\pi}(t) x_{k,\pi}(t)]].$$  

Often, it is possible to achieve a higher throughput by sometimes selecting a higher rate that also violates the success probability constraint. Therefore, to compare with policies that do not optimize for $\tau$, we calculate the ratio of throughput to the violations, $\frac{T\text{put}(T, \tau)}{\text{Vio}(T, \tau)}$. A higher $\frac{T\text{put}(T, \tau)}{\text{Vio}(T, \tau)}$ value indicates throughput gains obtained with a relatively small number of violations, and thereby captures the throughput efficiency of the policy.

4. Algorithms

4.1. Constrained Thompson Sampling

We now present the Con-TS algorithm for the multi-armed bandit formulation above. Details of the Con-TS algorithm are described in Alg. 1. Since the reward function for Con-TS is the Bernoulli random variable $x_{k(t)}$, we choose Beta distributions to model each arm, which is conjugate to the Bernoulli distribution. Consequently in Alg. 1, the prior transmission success probabilities are modeled as a uniform distribution by setting the Beta parameters to 1 for each arm. In Sec. 5, we will also propose application-specific Beta priors that are informed by commonly available models for the wireless link. In the Con-TS algorithm, Eq. 9 solves a linear program for the sampled transmission success probability values $\tilde{\mu}_{k,t}$ from their corresponding posterior Beta distributions. This linear program is same as the one proposed in Eq. 4 for the true transmission success probabilities that are a priori unknown at the transmitter.

A complete theoretical analysis of the novel Con-TS algorithm requires careful analysis of the sampling probabilities for each arm. This analysis is further complicated by the fact that the optimal selection strategy may contain a probabilistic mixture of several arms as shown in Prop. 3. Existing results for Thompson sampling are limited to bandits with a single optimal arm for the stochastic multi-armed bandit setting (Kaufmann et al., 2012; Agrawal & Goyal, 2013).

Here we omit a full proof of the Con-TS algorithm, and instead use well-known asymptotic results to argue that the Con-TS algorithm will converge asymptotically to the optimal rate selection policy. We validate these arguments using extensive numerical experiments in Sec. 5 where Con-TS is shown to outperform state-of-the-art approaches.

We are interested in analyzing the asymptotic convergence of our proposed Con-TS algorithm. For this, we use the following lemma that was proved in (Kaufmann et al., 2012) for non-constrained version of Thompson sampling in a stochastic MAB setting:

**Lemma 4.** (Kaufmann et al., 2012) For any $\epsilon > 0$, and for any suboptimal arm $a$, the expected number of plays of the suboptimal arm, $N_{a,T}$, is bounded by

$$\mathbb{E}[N_{a,T}] \leq (1 + \epsilon) \frac{\ln(T) + \ln(T)}{d(\mu_a, \mu^*)} + C(\epsilon, \mu^*, \mu_a),$$

where $\mu^*$ is the mean reward for the optimal arm, $d$ is the Kullback-Leibler divergence and $C$ is a constant term.

Next, we argue that even for our constrained Thompson sampling approach, Con-TS, Lemma 4 provides insight into the optimal convergence of Con-TS to the oracle policy. To
show this, we note that as a consequence of this lemma the number of pulls for any sub-linear arm scales sub-linearly with $T$, while for any arm that is a part of the optimal subset of arms, the number of pulls is linear in time.

**Proposition 5.** Con-TS converges asymptotically to the oracle rate selection policy.

**Discussion:** Denote by $S^O = \{ i | p_i > 0 \}$ and $S^\pi = \{ j | y_{j,t} > 0 \}$, the set of arms with positive selection probabilities for the oracle and Con-TS rate selection policies respectively. The arms $j \in S^\pi$ are selected with a non-zero probability $y_{j,t}$, i.e., at a linear rate. Consider any arm $j' \in S^O \cup (S^\pi)^c$, where $(\cdot)^c$ denotes the complement of a set. As a corollary to Lemma 4, $j'$ is part of the optimal set and therefore sampled at a linear rate. Intuitively, this implies that any sub-optimal arm that is a returned by Eq. 9 on account of a relatively large uncertainty will be selected until it gets dropped from $S^\pi$. Additionally, any optimal arm that is not a part of $S^\pi$ will also be sampled at a linear rate and eventually become an element of $S^\pi$. As a result, $S^\pi$ asymptotically converges to $S^O$ as $t \to \infty$ and consequently the sampled probabilistic selection vector $y_{1,\ldots,K},t$ will converge to $p_{1,\ldots,K},t$.

### 4.2. Outer Loop Link Adaptation

The wireless transmitter obtains periodic, approximate channel quality index values, $q$, from the received based on measurements of the instantaneous channel state. The transmitter maps this channel quality index to approximate transmission success probabilities, $\bar{\mu}_{1,\ldots,K}$ using a parameterized, offline link-to-system (L2S) model $\mathcal{M}_\theta(q)$ : $q \mapsto \bar{\mu}_{1,\ldots,K}$. These transmission success probabilities are corrupted by modeling errors, quantization noise, and feedback delays. The transmitter compensates for these inaccuracies through a dynamic offset applied to the $\theta$ parameters of the L2S model through **outer loop link adaptation (OLLA)**. OLLA adds a fixed offset, $\Delta^u$, for every observed transmission success and subtracts another, fixed, offset, $\Delta^d$, to the model parameters for every observed transmission failure. The ratio of offset values are $\frac{\Delta^u}{\Delta^d} = \frac{\tau}{1-\tau}$, which results in the latency for OLLA converging asymptotically to the target latency constraint, $\tau$. At every transmission interval, OLLA uses the latest parameter offset, $\Delta^\text{OLLA}$, to estimate the transmission success probabilities for each rate through a mapping function $f$ over the local offline model. The OLLA algorithm steps are described in Alg. 2.

### 4.3. Non-stationary Environments

The discussion so far has assumed stationary transmission success probabilities. However the wireless channels is often time-varying on account of the movement of any of the transmitter, the receiver, and the scatterers. Consequently, the transmission success probabilities for each rate also vary with time. To deal with non-stationary environments, a common extension of bandit algorithms follows a sliding window protocol that was studied in (Garivier & Moulines, 2008). In the numerical section of this paper, Sec. 5, we extend the algorithms studied in this paper with a sliding window approach, where the historical choice of rates and corresponding observations outside a fixed-size window are discarded.

### 5. Experimental Evaluation

We conduct simulations of wireless communication networks to study the performance of algorithms discussed in this paper, and other state-of-the-art approaches. The latency constraint is chosen to be $\tau = 0.85$. For each of the throughput, violation, and reward metrics, $\text{Tput}^\tau(T)$, $\text{Vio}^\tau(T)$, and $\text{Reg}^\tau(T)$, and for a stationary environment, we calculate the expectation by averaging over 100 independent runs of each experiment over $T = 10000$ time intervals. Further, the non-stationary environments are generated by linearly interpolating between two unique pairs of transmission success probabilities over 250 time intervals, for a total of $T = 10000$ time intervals. For stationary environments, we use sliding windows spanning 100 time intervals for each algorithm, and calculate the moving averages (M.A.) of the performance metrics over every 100 time intervals.

**5.1. Bandit Algorithms**

We study Con-TS performance for optimal rate selection in WiFi networks with $K = 8$ available rates, $\{6, 9, 12, 18, 24, 36, 48, 54\}$ Mbps. We generate numerical results three sets of stationary transmission success probabilities proposed in (Combes et al., 2014) that have gradual, lossy, and steep distributions with the values, $\{95, 9, .8, .65, .45, .25, .15, .1\}$,
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Figure 1. Ratio of cumulative throughput to cumulative violation (higher is better), and cumulative violation (lower is better) for unimodal Thompson sampling (UTS), constrained UCB (Con-KL-UCB) and constrained Thompson sampling (Con-TS) algorithms.

As discussed in 3, unconstrained algorithms such as UTS can achieve a higher expected throughput, $\text{Tput}^\pi(T, \tau)$ at the cost of higher violations of the constraint, $\text{Vio}^\pi(T, \tau)$. Therefore to ensure a fair comparison, we plot the ratio $\frac{\text{Tput}^\pi(T, \tau)}{\text{Vio}^\pi(T, \tau)}$ achieved by each algorithm for the gradual, lossy, and steep distributions in Fig. 1(a), Fig. 1(b), and Fig. 1(c) respectively. A higher throughput to violation ratio indicates better performance. Further, we also plot the respective cumulative violations for each of the algorithms in Fig. 1(e), Fig. 1(f), and Fig. 1(g). For the gradual and lossy distributions, Con-TS has $3-4\times$ and $8-10\times$ better throughput-to-violation ratio than UTS and Con-KL-UCB respectively. For the steep distribution, Con-TS has a marginally better performance than UTS, this is because for the steep distribution, there exists a rate satisfying Prop. 2, i.e., the throughput-maximizing rate is also optimal with respect to the constraint. Finally, we plot the moving average of the throughput-violation ratio and the violations in Fig. 1(d) and Fig. 1(h) respectively, where Con-TS significantly outperforms UTS as well as Con-KL-UCB.

5.2 Model-guided Algorithms

Next, we study the performance of Con-TS with uniform priors, Con-TS with informed priors, termed Con-TS-Infp, and the state-of-the-art OLLA algorithm for optimal rate selection in cellular wireless networks. We simulate a 4G cellular link with 22 available rates that correspond to a subset of the transport block sizes specified in (3rd Generation Partnership Project, 2016). The offline model $M_\theta$ is gen-
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We plot the regret for three average signal-to-noise-ratios (SNR), 6, 9, and 12 dB respectively in Fig. 2(a), Fig. 2(b) and Fig. 2(c) respectively. Each SNR corresponds to a different channel quality index \( q \) and different transmission success probabilities for each rate. Further, we plot the corresponding violations in Fig. 2(e), Fig. 2(f) and Fig. 2(g) respectively. From the plots we observe that OLLA suffers from linear regret and violation, while both Con-TS and Con-TS-Infp achieve sublinear regrets and violations. This behavior of OLLA was shown analytically in Prop. 2 and discussed in Sec. 4.2. However, for small \( T \) and for relatively low SNRs, OLLA still outperforms Con-TS by exploiting prior knowledge obtained from \( M_\theta \). Con-TS-Infp overcomes the drawback of using uniform Con-TS priors, and thereby outperforms OLLA for all values of \( T \) and SNRs. Further Con-TS-Infp lower the regret by almost 50% compared to Con-TS. Finally in Fig. 2(d) and Fig. 2(h), we observe that the sliding window extension of Con-TS and Con-TS-Infp significantly outperform OLLA in terms of the moving average (M.A.) of the regret and violation metrics.

6. Conclusions and Future Work

In this work, we have developed a novel algorithm, Con-TS, for rate selection using multi-armed bandits. Further through numerical results, we have demonstrated that Con-TS outperforms the state-of-the-art bandit and model-guided algorithms in stationary as well as non-stationary wireless environments. We have provided arguments for the asymptotic convergence of Con-TS, however, further theoretical analysis of the algorithm is required to derive appropriate regret and violation bounds. Additionally in this paper, we have formulated rate selection as a knapsack problem and developed a linear programming relaxation for this problem. We believe that this novel formulation of the rate selection problem opens it up for further theoretical analysis, and allows approaches from other, unrelated, domains to be applied here. Moreover, several other important problems in wireless communications can be extended to admit practical constraints using our approach, for example in selecting the optimal beam from a set of candidate beams (Hashemi et al., 2018), and channel and rank selection problems (Combes & Proutiere, 2015).

Finally, we note that rate selection and similar problems often exhibit structure, such as the unimodality property mentioned in this paper, and correlation across the reward function. Further, rate selection in practical wireless networks can exploit valuable side information in addition to the channel quality index studied in this paper. The structural properties as well as the availability of side information encourages extensions to the algorithms studied here.
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References

3rd Generation Partnership Project. Evolved Universal Terrestrial Radio Access (E-UTRA); Physical layer procedures. Technical Report 36.213 v12.3.0, September 2016.

Agrawal, S. and Goyal, N. Further optimal regret bounds for thompson sampling. In Artificial Intelligence and Statistics, pp. 99–107, 2013.

Badanidiyuru, A., Kleinberg, R., and Slivkins, A. Bandits with knapsacks. In Foundations of Computer Science (FOCS), 2013 IEEE 54th Annual Symposium on, pp. 207–216. IEEE, 2013.

Brueninghaus, K., Astely, D., Salzer, T., Visuri, S., Alexiou, A., Karger, S., and Seraji, G.-A. Link performance models for system level simulations of broadband radio access systems. In Personal, Indoor and Mobile Radio Communications, 2005. PIMRC 2005. IEEE 16th International Symposium on, volume 4, pp. 2306–2311. IEEE, 2005.

Buendestado, V., Ruiz-Avils, J. M., Toril, M., Luna-Ramrez, S., and Mendo, A. Analysis of throughput performance statistics for benchmarking lte networks. IEEE Communications Letters, 18(9):1607–1610, Sept 2014. ISSN 1089-7798. doi: 10.1109/LCOMM.2014.2337876.

Chapelle, O. and Li, L. An empirical evaluation of thompson sampling. In Advances in neural information processing systems, pp. 2249–2257, 2011.

Chen, K., Cai, K., Huang, L., and Lui, J. Beyond the click-through rate: Web link selection with multi-level feedback. arXiv preprint arXiv:1805.01702, 2018.

Combes, R. and Proutiere, A. Dynamic rate and channel selection in cognitive radio systems. IEEE Journal on Selected Areas in Communications, 33(5):910–921, 2015.

Combes, R., Proutiere, A., Yun, D., Ok, J., and Yi, Y. Optimal rate sampling in 802.11 systems. In INFOCOM, 2014 Proceedings IEEE, pp. 2760–2767. IEEE, 2014.

Elmaghraby, W. and Keskinocak, P. Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. Management science, 49(10):1287–1309, 2003.

Ferreira, K. J., Simchi-Levi, D., and Wang, H. Online network revenue management using thompson sampling. Operations research, 2018.

Garivier, A. and Cappé, O. The kl-ucb algorithm for bounded stochastic bandits and beyond. In Proceedings of the 24th annual Conference On Learning Theory, pp. 359–376, 2011.

Garivier, A. and Moulines, E. On upper-confidence bound policies for non-stationary bandit problems. Proc. of Algorithmic Learning Theory (ALT), 2008.

Hashemi, M., Sabharwal, A., Koksal, C. E., and Shroff, N. B. Efficient beam alignment in millimeter wave systems using contextual bandits. In IEEE INFOCOM 2018-IEEE Conference on Computer Communications, pp. 2393–2401. IEEE, 2018.

Kaufmann, E., Korda, N., and Munos, R. Thompson sampling: An asymptotically optimal finite-time analysis. In International Conference on Algorithmic Learning Theory, pp. 199–213. Springer, 2012.

Kleinberg, R., Slivkins, A., and Upfal, E. Multi-armed bandits in metric spaces. In Proceedings of the fortieth annual ACM symposium on Theory of computing, pp. 681–690. ACM, 2008.

Lakshmanan, S., Sanadhya, S., and Sivakumar, R. On link rate adaptation in 802.11 n wlan. In INFOCOM, 2011 Proceedings IEEE, pp. 366–370. IEEE, 2011.

Martello, S. Knapsack problems: algorithms and computer implementations. Wiley-Interscience series in discrete mathematics and optimization, 1990.

Molisch, A. Wireless Communications. Wiley, 2010.

Paladino, S., Trovo, F., Restelli, M., and Gatti, N. Unimodal thompson sampling for graph-structured arms. In AAAI, pp. 2457–2463, 2017.

Saxena, V., Jaldén, J., Bengtsson, M., and Tullberg, H. Deep learning for frame error probability prediction in bicm-ofdm systems. In 2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pp. 6658–6662. IEEE, 2018.

Tao, T. and Czyzwick. A. Performance analysis of link adaptation in lte systems. In Smart Antennas (WSA), 2011 International ITG Workshop on, pp. 1–5. IEEE, 2011.

Thompson, W. R. On the likelihood that one unknown probability exceeds another in view of the evidence of two samples. Biometrika, 25(3/4):285–294, 1933.
Constrained Thompson Sampling for Wireless Link Optimization

Wu, P. and Jindal, N. Coding versus arq in fading channels: How reliable should the phy be? In GLOBECOM 2009 - 2009 IEEE Global Telecommunications Conference, pp. 1–6, Nov 2009. doi: 10.1109/GLOCOM.2009.5426285.

Xia, Y., Li, H., Qin, T., Yu, N., and Liu, T.-Y. Thompson sampling for budgeted multi-armed bandits.