Electroweak-scale mirror fermions, $\mu \to e\gamma$ and $\tau \to \mu\gamma$

P.Q. Hung

Dept. of Physics, University of Virginia,
382 McCormick Road, P. O. Box 400714,
Charlottesville, Virginia 22904-4714, USA

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The Lepton Flavour Violating (LFV) processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are estimated in a model of electroweak-scale right-handed neutrinos. The present bounds on the branching ratios, $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$, $B(\tau \to \mu\gamma) < 6.8 \times 10^{-8}$ (BaBar) and $< 4.5 \times 10^{-8}$ (Belle), puts strong constraints on the parameters of the model. This constraint links low energy rare decay processes to high-energy phenomena (e.g. decay lengths of the mirror charged leptons which are important in the search for the telltale like-sign dilepton events present in the model of electroweak-scale right-handed neutrinos). The model can be tested at future colliders (LHC, ILC) and at MEG and/or B factories.

I. INTRODUCTION

The nature and origin of neutrino masses are undoubtedly among the most important questions in particle physics, experimentally and theoretically. Neutrino oscillation experiments and astrophysical arguments indicate that neutrinos that interact with normal matter at tree level are very light compared with all known charged particles, typically with masses less than an electronvolt. There exists plausible models of light neutrino masses which could, in principle, be tested experimentally. One of such models is the famous see-saw mechanism [1] where a lepton-number conserving $(\Delta L = 0)$ Dirac neutrino mass term, $m_D \bar{\nu}_L \nu_R$, is combined with a lepton-number violating $(\Delta L = 2)$ Majorana mass term, $M_\nu \bar{\nu}_L \sigma_2 \nu_R$ where it is usually assumed that $M_\nu \gg m_D$, to yield a “tiny” mass eigenvalue $-m_D^2/M_\nu$ and a “very large” one $M_\nu$. Since in a generic framework, the only “knowledge” that one has at the present time is the smallness of the ratio $-m_D^2/M_\nu$, the question of how small $m_D$ is and how large $M_\nu$ could be is rather model-dependent. The most popular scenario is one in which $m_D$ is related to the electroweak symmetry breaking scale while $M_\nu$ is in general related to some Grand Unified scale. It is in particular the relationship of the Majorana scale $M_\nu$ to some new physics that is of great interest since its probe would reveal not only the see-saw mechanism but also what type of new physics one might be dealing with.

A very interesting connection between $M_\nu$ and the scale above which parity is restored was made by [2] in which the Standard Model (SM) is extended to $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. In this model, the finiteness, albeit small, of the light neutrino mass $m_\nu$ is related to the finiteness of the $SU(2)_R$ gauge boson masses $M_{W_R} \gg m_\nu$ where $M_{W_R}$, $m_\nu$ vanishing in the limit $M_{W_R} \to \infty$. The V-A nature of the weak interactions of the SM fermions is recovered in this limit. It is extremely intriguing that parity restoration is linked in the left-right (LR) symmetric model to the non-vanishing value of the neutrino mass. The strength of V+A interactions vanishes in the limit of zero neutrino masses.

Can parity restoration be accomplished within the SM gauge sector? This issue was addressed in [3, 4] where the gauge group is simply the SM $SU(2) \otimes U(1)$. (Notice that the subscripts $L$ and $R$ are deliberately omitted for reasons to be given below.) However, one now has, for every SM left-handed doublet such as e.g. $(\nu_e, e)_L$, a heavy mirror right-handed doublet e.g. $(\nu_e^M, e^M)_R$. Similarly, for every SM singlet such as e.g. $e_R$, one has a heavy left-handed mirror singlet $e^M_L$. (The content for the quarks and their mirror counterparts are listed in [3, 4].) One word of caution is in order here. What we mean by “mirror” is simply the aforementioned assignments for fermions and nothing else, e.g. there is no mirror gauge group etc...

There are two remarks we would like to make concerning these types of mirror fermions. First, experimental constraints on mirror quarks and leptons masses are not very well studied since they depend on the specifics of decay modes. However, as discussed in [3, 4], one can safely conclude that the right-handed neutrinos have to be more massive than $M_Z/2$ because of constraints from the width of the Z boson. The lower bound on the charged mirror lepton masses can be taken to be around $\sim 100$ GeV (long-lived heavy leptons) if not lower. The bounds on the mirror quarks are murkier but one can assume that they might be heavier than $\sim 200$ GeV (from bounds on long-lived quarks) [5]. The second remark has to do with electroweak precision parameter constraints. As mentioned in [3, 4], the positive contribution of mirror fermions to the $S$ parameter can be compensated by a negative contribution from the Higgs triplet sector present in the model [6]. In [5], it was shown that, depending on the mass splitting inside the triplet Higgs scalar(s), its contribution to $S$ can be negative and relatively large in magnitude, and can offset the positive contributions coming from the mirror fermions. Furthermore, it is well-known that Majorana fermions can also have a negative contribution to $S$ [6]. Notice that it is straightforward to satisfy the $\rho$-parameter constraint [6].
The above remarks and other phenomenological issues concerning mirror fermions will be dealt with elsewhere.

Since \( \nu_R \)'s are now members of \( SU(2) \) doublets, it was shown in [3] that the Majorana masses \( M_R \) coming from the term \( M_R \nu_R^T \sigma_2 \nu_R \) is now related to the electroweak breaking scale \( \Lambda_{EW} \sim 246 \text{GeV} \), rendering the detection of a low-scale \( \nu_R \) at colliders and observing lepton-number violating processes a real possibility. In this scenario, one can directly test the see-saw mechanism at collider energies as well as the Majorana nature of \( \nu_R \)'s.

The Dirac mass term now comes from the mixing between the SM left-handed doublet and the mirror right-handed doublet through the coupling with a singlet scalar field which develops a non-vanishing vacuum-expectation value (VEV) \( v_S \), giving \( m_D = g_{SI} v_S \), where \( g_{SI} \) is the Yukawa coupling. The light neutrinos becomes massless in the limit \( g_{SI} \to 0 \), i.e. when the SM particles and their mirror counterparts decouple. It is this mixing which will prove important for the LFV processes considered in this paper.

What is parity violation? Basically, this is equivalent to the question of why the weak interactions of SM particles are of the V-A type. This rather old question never really disappears and occasionally has new twists to it, especially in light of the upcoming explorations by the LHC and hopefully also by the ILC. For the SM, this is built in because of the absence of right-handed neutrinos. In the LR model, this is due to \( M_{W_R} > M_{W_L} \). Parity is restored for \( E \gg M_{W_R} \). For \( E \gg M_{W_R} > M_{W_L} \), the V+A interactions have equal strength to that of the V-A interactions and parity is restored.

In the model of electroweak-scale right-handed neutrinos, the question of parity violation takes on a slightly different meaning as compared with the LR model. Here, \( SU(2) \otimes U(1) \) as a gauge theory is actually a vector-like model in the sense that fermions of both chiralities are present. In consequence, “parity restoration” refers to the existence of the mirror fermions in our model. Furthermore, the SM fermions have V-A weak interactions at tree-level but, because of the existence of a mixing between SM and mirror fermions, receive V+A interactions through one-loop diagrams which are of course suppressed with respect to the tree-level V-A interactions. As with the LR model, these (radiative) V+A contributions vanishes in the limit \( g_{SI} \to 0 \) which also implies a vanishing Dirac mass \( m_D \) and hence a vanishing light neutrino mass. Here, one expects deviations of the SM couplings of the SM quarks and leptons due to the mixing between SM fermions and their mirror counterparts. This includes corrections to the electroweak processes involving electrons and neutrinos as well as lepton flavor violating processes such as \( \mu \to e \gamma \) and \( \tau \to \mu \gamma \). It is the latter processes that we will concentrate on in this paper. Details of the former processes will be presented elsewhere.

We will see below that there is a deep connection between the mixing parameters involved in LFV processes and the Dirac neutrino mass matrix which participates in the see-saw mechanism. Since these same parameters participate in the determination of the decay length of the mirror charged leptons, \( l_R^M \), it will be shown below that were these LFV processes to be detected the decay length would be microscopic. If the charged mirror leptons were to be found with “macroscopic” decay lengths, LFV processes would be practically unobservable in our model. One word of reminder is in order here on why it is important to gain a good understanding on the size of the decay lengths of the mirror leptons. Because the model presented in [3] deals with right-handed neutrinos with electroweak-scale masses, it is possible to probe the Majorana nature of \( \nu_R \)'s and the see-saw mechanism at present and future colliders (TEvatron, LHC, ILC). The telltale signatures would be SM like-sign dileptons which are produced e.g. in processes such as

\[
q + \bar{q} \to Z \to \nu_R + \nu_R. \tag{1}
\]

Since \( \nu_R \)'s are Majorana particles, they can have transitions such as \( \nu_R \to l_R^M \pm + W^\mp \). A heavier \( \nu_R \) can decay into a lighter \( l_R^M \) and one can have

\[
\nu_R + \nu_R \to l_R^M \pm + l_R^M \mp + W^\pm + W^\mp
\to l_L^+ + l_L^- + W^\pm + W^\mp + \phi_S + \phi_S, \tag{2}
\]

where \( l_R^{M, \mp} \to l_L^+ + \phi_S \) and where \( \phi_S \), the singlet scalar field in the model, would constitute the missing energy.

## II. BRIEF REVIEW OF THE ELECTROWEAK SCALE RIGHT-HANDED NEUTRINO MODEL

In our model, the dominant contribution to the process \( l_i \to l_j \gamma \) comes from the diagrams shown in Fig. 1. The Feynman rules for the diagrams can be extracted from the Lagrangians given below. Let us first write down the Lagrangian of our model and enumerate the particle content and various symmetries.

The gauge group is \( SU(2) \otimes U(1) \).

Below is a list of the fermion content. Because of the problem at hand, namely the processes \( \mu \to e \gamma \) and \( \tau \to \mu \gamma \), we shall concentrate mainly on the leptons in this paper.

- (2, \( Y/2 = -1/2 \)) (Lepton doublets):

\[
l_L = \left( \begin{array}{c} \nu_L \\ e_L \\
\end{array} \right)_i, \tag{3}
\]

\[
l_R^M = \left( \begin{array}{c} \nu_R^M \\ e_R \\
\end{array} \right)_i, \tag{4}
\]

where \( i \) stands for the family number. In [3], \( \nu_R^M \) is identified with the right-handed neutrino, a fact which gives rise to an electroweak-scale right-handed neutrino.
\( 1, Y/2 = -1 \) (Lepton singlets):

\[ e_i R; e^M_i L. \]  

Next, we write down the Lagrangian involving the mirror leptons and their interactions with the SM leptons. In so doing, we will also include for clarity the term that gives the Majorana mass to the right-handed neutrinos although it is not needed for the present discussion. Also for completeness we will write down the Lagrangian for the SM leptons in order to make a comparison. We have the following interactions.

- Charged current interactions:

\[ \mathcal{L}^{CC} = \mathcal{L}_{SM}^{CC} + \mathcal{L}_M^{CC}, \]  

where

\[ \mathcal{L}_{SM}^{CC} = -\left( g \frac{g}{2\sqrt{2}} \right) \sum_i \bar{\psi}_i^{SM} \gamma^\mu (1 - \gamma_5)[T^+ W^+_{\mu}] + T^- W^-_{\mu} ] \psi_i^{SM}, \]  

\[ \mathcal{L}_M^{CC} = -\left( g \frac{g}{2\sqrt{2}} \right) \sum_i \bar{\psi}_i^M \gamma^\mu (1 + \gamma_5)[T^+ W^+_{\mu}] + T^- W^-_{\mu} ] \psi_i^M, \]

and

\[ \psi_{SM} = \begin{pmatrix} \nu \\ e \end{pmatrix} \]

\[ \psi^M = \begin{pmatrix} \nu^M \\ e^M \end{pmatrix}. \]

Above the subscripts \( SM \) and \( M \) refer to SM and mirror particles respectively. Notice that Eq. (7) has \((1 - \gamma_5)\) as opposed to \((1 + \gamma_5)\) of Eq. (8).

- Neutral current interactions:

\[ \mathcal{L}^{NC} = \mathcal{L}_{SM}^{NC} + \mathcal{L}_M^{NC}, \]

where

\[ \mathcal{L}_{SM}^{NC} = -\left( g \frac{g}{4 \cos \theta_W} \right) Z_\mu \left\{ \sum_i \bar{\psi}_i \gamma^\mu (1 - \gamma_5) \nu_i + \sum_i \bar{e}_i \gamma^\mu \left[ (-1 + 4 \sin^2 \theta_W) + \gamma_5 \right] e_i \right\}, \]

\[ \mathcal{L}_M^{NC} = -\left( g \frac{g}{4 \cos \theta_W} \right) Z_\mu \left\{ \sum_i \bar{\psi}_i^M \gamma^\mu (1 + \gamma_5) \nu_i^M + \sum_i \bar{e}_i^M \gamma^\mu \left[ (-1 + 4 \sin^2 \theta_W) - \gamma_5 \right] e_i^M \right\}. \]

- Electromagnetic interactions:

\[ \mathcal{L}^{EM} = e \sum_i (\bar{e}_i \gamma^\mu e_i + \bar{e}_i^M \gamma^\mu e_i^M) A_\mu. \]

- Yukawa interactions which contribute to the neutrino Dirac mass term:

This is the most important part which contributes to the aforementioned LFV rare processes. The gauge-invariant Yukawa Lagrangian can be written as follows:

\[ \mathcal{L}_S = -\bar{\nu}^0_L g_{Sl} \nu_R^{0,M} \phi_S + H.c., \]

\[ = -\bar{\nu}^0_L g_{Sl} \nu_R^{0,M} + \bar{e}^0_L g_{Sl} e_R^{0,M} \phi_S + H.c., \]  

where \( \phi_S \) is the singlet scalar field whose vacuum expectation value (VEV) gives rise to the neutrino Dirac mass term [3]. This is combined with the Majorana mass term for the right-handed neutrinos in a see-saw mechanism as shown in [3]. In (15), \( \nu^0_L, \nu^0_R, e^0_L, e^0_R \) denote column vectors with \( n \) components for \( n \) families and \( g_{Sl} \) denotes an \( n \times n \) coupling matrix. From hereon, we will take for definiteness \( n = 3 \).

- Yukawa interactions involving \( SU(2)_L \) singlets, \( e_R \) and \( e^M_i \):

\[ \mathcal{L}_S' = -\bar{e}_R^0 g_{Sl} e_L^{0,M} \phi_S + H.c., \]

where \( g_{Sl}' \) is a \( 3 \times 3 \) matrix for \( 3 \) families.

- Yukawa interactions giving rise to the Majorana mass for the right-handed neutrinos:

\[ \mathcal{L}_M = l_R^{M,T} \sigma_2 \gamma_2 g_M \tilde{\chi} l_R^M, \]

where

\[ \tilde{\chi} = \frac{1}{\sqrt{2}} \tilde{\chi}_l = \begin{pmatrix} \frac{1}{\sqrt{2}} \chi^0 & \chi^{++} \end{pmatrix}, \]

and \( g_M \) is a \( 3 \times 3 \) matrix for \( 3 \) families. Notice that Eq. (17) is written down for completeness in this manuscript. As shown in [3], this term gives rise to an electroweak-scale Majorana mass for the right-handed neutrinos when the \( SU(2)_L \)-triplet Higgs field develops a VEV, \( \langle \chi^0 \rangle = v_M \), of the order \( O(\Lambda_{EW}) \). (The subtleties associated with such a large triplet VEV are discussed in [3].) As of now, there is no direct evidence for Higgs triplets, and for that matter, also for Higgs doublet(s). However, one of the characteristics of this model is the existence of particles such as doubly-charged Higgs bosons. Present limits of around 100 GeV are rather model-dependent. A study of the Higgs sector of our model is under investigation [11].
The above mixings between the SM fermions and their mirror counterparts also give rise to mass mixings in the charged lepton sector (as well as in the quark sector). A brief review of the points made in §3 goes as follows. Take, for example, one family of fermions. When \( \phi_S \) develops a VEV \( v_S \), one obtains a Dirac mass for the neutrino and a mass mixing between the SM and mirror charged leptons as follows (assuming \( g_{SI} = g_{SI} \))

\[
m^{D}_\nu = g_{SI} v_S ,
\]

for the Dirac neutrino and

\[
M_l = \left( \begin{array}{cc} m_l & m^{D}_\nu \\ m^{D}_\nu & m_{lM} \end{array} \right) ,
\]

for the charged SM and mirror leptons. In Eq. (20), \( m_l \) is a generic notation for the mass of a SM charged lepton obtained from the coupling to the SM Higgs doublet. Similarly, \( m_{lM} \) is a generic notation for the mirror charged lepton mass obtained in the same manner. The off-diagonal element in Eq. (20) comes from the cross coupling to the singlet scalar field through Eq. (15). The diagonalization of (20) gives the following eigenvalues for the charged (SM and mirror) lepton masses

\[
\tilde{m}_l = m_l - \frac{(m^{D}_\nu)^2}{m_{lM} - m_l},
\]

\[
\tilde{m}_{lM} = m_{lM} + \frac{(m^{D}_\nu)^2}{m_{lM} - m_l}.
\]

As shown in §3, the Majorana mass of the right-handed neutrino \( M_R = g_M v_M \), where \( v_M \sim O(\Lambda_{EW}) \). The seesaw mechanism then gives a light neutrino a mass \( (m^{D}_\nu)^2/M_R \) of \( O(1 \text{ eV}) \) and a heavy neutrino of mass \( M_Z^2/2 < M_R < O(\Lambda_{EW}) \). This makes the right-handed neutrinos detectable by future collider experiments. This also implies that \( m^{D}_\nu \sim 10^7 \text{ eV} \). Since the mirror lepton mass is of the order of the electroweak scale \( \Lambda_{EW} \), it follows that the second terms of the right-hand side of (21a) and (21b) are tiny compared with the first terms and can be ignored. Hence, for all practical purposes, the masses of the charged fermions are those obtained in a SM way i.e. through the Yukawa couplings with the SM Higgs doublet(s). As a result, the mass eigenstates for the charged (SM and mirror) leptons are principally obtained in this way. This will be the (rather good) approximation that we will use below in the discussion of LFV rare processes involving charged leptons.

In what follows, we will make use of the remarks made above concerning mass eigenstates for the charged leptons. To express (15) in terms of mass eigenstates, let us define

\[
e^0 = U^\dagger_l e_L ; e^{0,M}_R = U^{-1}_R e^M_R .
\]

Using (22), one can now rewrite the charged lepton part of (15) in terms of the mass eigenstates as follows

\[
\mathcal{L}_{S,\text{charged}} = - (\tilde{e} L U^L e^M_R) \phi_S + H.c. ,
\]

where

\[
U^L = U^L_{SI} g_{SI} U^{M}_R .
\]

From §3, one can deduce from (15) the Dirac mass matrix of the neutrino sector when \( \langle \phi_S \rangle = v_S \) as follows

\[
m^{D}_\nu = v_S g_{SI} .
\]

In terms of (26), the matrix \( U^L \) can now be written as

\[
U^L = U^L_{SI} \left( \frac{m^{D}_\nu}{v_S} \right) U^{M}_R .
\]

One can see that the matrix \( U^L \) which mixes different families of SM charged leptons with those of the mirror leptons now involves the Dirac neutrino mass matrix. LFV processes to be discussed in this paper will in consequence indirectly probe the Dirac part of the neutrino mass matrix. In fact, one can invert Eq. (26) to obtain

\[
\frac{m^{D}_\nu}{v_S} = U^L U^L U^{M}_R .
\]

As discussed in §3, let us recall that the see-saw mechanism yields the following mass matrix for the light neutrino sector

\[
m_{\nu,light} = -m^{D}_\nu T M_R^{-1} m^{D}_\nu ,
\]

while the mass matrix for the heavy right-handed sector is simply \( M_R \). Once again, let us notice that \( U^L \to 0 \) in the limit \( m_{\nu,light}(m^{D}_\nu) \to 0 \), and there will be no mixing between SM and mirror fermions.

For the \( SU(2)_L \) singlets, one has

\[
e^0 = U^L_{SI} e_L ; e^{0,M}_R = U^{M}_L e^M_R .
\]

is rewritten as

\[
\mathcal{L}^\prime_S = - (\tilde{e} R e^M_L) \phi_S + H.c. ,
\]

where

\[
U^R = U^L_{SI} g_{SI} U^{M}_L .
\]

Although it is not necessary to do so, one can further simplify the problem by assuming that \( g_{SI} = g_{SI} \) in which case we obtain

\[
U^R = U^L_{SI} \left( \frac{m^{D}_\nu}{v_S} \right) U^{M}_L .
\]

Similarly to (27), one can invert (32) to obtain

\[
\frac{m^{D}_\nu}{v_S} = U^L_{SI} U^R U^{M}_L .
\]

From the above discussion, one cannot fail but notice the deep connection between the Dirac part of the neutrino mass matrix \( m^{D}_\nu \) and the matrices which are involved in LFV processes in our model, namely \( U^L \) and \( U^R \).
The decay rate and the branching ratio are respectively

\[ \sigma^{[12]} \] side the loop. But this is entirely negligible as noticed by
diagrams with a W and a light neutrino propagating in-
be read off Eqs. (8, 13, 14, 23, 30). Notice that there is
listed above. The Feynman rules needed for Fig. 1 can
computed in our model by using the interaction Lagrangians
the photon line attached to the external legs need not be
one that we concentrate on. (Two other diagrams with
B\( \mu \rightarrow e \gamma \) and \( \tau \rightarrow \mu \gamma \) can now be com-
puted in our model by using the interaction Lagrangians
listed above. The Feynman rules needed for Fig. 1 can
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be computed in our model by using the interaction Lagrangians
the photon line attached to the external legs need not be
one that we concentrate on. (Two other diagrams with

\[ B(\mu \rightarrow e \gamma) = \frac{\Gamma(\mu \rightarrow e \gamma)}{\Gamma(\mu \rightarrow e \nu_e \nu_\mu)} = \frac{12\pi^2}{m_\mu^2 G_F^2} \left( |c_L^{(\mu)}|^2 + |c_R^{(\mu)}|^2 + |c_L^{(\mu)} - c_R^{(\mu)}|^2 \right). \]  

\[ B(\tau \rightarrow \mu \gamma) = \frac{\Gamma(\tau \rightarrow \mu \gamma)}{\Gamma(\tau \rightarrow \mu \nu_\mu \nu_\tau)} = \frac{12\pi^2}{m_\tau^2 G_F^2} \left( |c_L^{(\tau)}|^2 + |c_R^{(\tau)}|^2 + |c_L^{(\tau)} - c_R^{(\tau)}|^2 \right). \]  

\[ \Gamma(l_i \rightarrow l_j \gamma) = \frac{m_i^3}{16\pi} \left| |c_L^{(l_i)}|^2 + |c_R^{(l_i)}|^2 + |c_L^{(l_i)} - c_R^{(l_i)}|^2 \right. \]  

\[ \left. + c_R^{(l_i)} \left( \frac{1}{2} + \frac{\gamma \mu}{2} \right) \right| \]  

\[ \gamma \mu \text{ is the polarization of the photon. Also, as discussed in [3, m_\mu \ll m_l], Notice in [36] and [37] that whereas B(\mu \rightarrow e \nu_e \nu_\mu) \sim 100\%, one has B(\tau \rightarrow \mu \nu_\mu \nu_\tau) \sim 17.4\%. \]  

A few special cases are worth noticing.

- \( g_{Sl} = g_{Sl}', U_L' = U_L \) and \( U_R' = U_R' \):

The last two above equalities simply imply that one
assumes that the mass matrices for the SM charged
leptons and those of the mirror leptons are such that the “left” and “right” diagonalization matrices are identical. With these assumptions, one obtains

\[ U_L = U_R = U_E. \]  

With these assumptions, one obtains

\[ c_L^{(\mu)} = c_R^{(\mu)} = \frac{1}{64\pi^2} \sum_i \frac{U_{Ei}^{*} U_{Ei}^{\mu}}{m_i}, \]  

\[ c_L^{(\tau)} = c_R^{(\tau)} = \frac{1}{64\pi^2} \sum_i \frac{U_{Ei}^{*} U_{Ei}^{\tau}}{m_i}. \]  

It is also convenient to rewrite the above expres-
sions in terms of a mass \( m_E \) (more like an average of
the three mirror lepton masses) as follows

\[ c_L^{(\mu)} = c_R^{(\mu)} = \frac{1}{64\pi^2 m_E} \sum_i \frac{m_i}{m_i} (U_{Ei}^{*} U_{Ei}^{\mu}), \]  

\[ c_L^{(\tau)} = c_R^{(\tau)} = \frac{1}{64\pi^2 m_E} \sum_i \frac{m_i}{m_i} (U_{Ei}^{*} U_{Ei}^{\tau}). \]  

What are the implications of the above assumption?
• Degenerate charged mirror leptons: $m_i = m_E$.

In this case one is left with the factors $\sum_i U_{\mu i}^* U_{\mu i}^E$ and $\sum_i U_{\tau i}^* U_{\tau i}^E$ which would vanish identically if $g_{\ell \ell}$ (or equivalently $m_\nu^2/v_S$) were proportional to the unit matrix because in this case the matrix $U^E$ would be unitary. One would then have $c_{\ell E}^{(\mu)} = c_{\ell E}^{(\tau)} = 0$ and $c_{\ell E}^{(\mu)} = c_{\ell E}^{(\tau)} = 0$, implying $B(\mu \rightarrow e \gamma) = 0$ and $B(\tau \rightarrow \mu \gamma) = 0$. However $U^E$ would no longer be unitary if $m_\nu^2/v_S$ were not proportional to the unit matrix, a fact which implies a non-vanishing value for the aforementioned branching ratios. Once again, one notices the implication of the form of neutrino mass matrices on LFV processes.

• Non-degenerate charged mirror leptons: $m_1 = m_E$, $m_2 = m_E + \delta m_2$, and $m_3 = m_E + \delta m_3$ with the assumption $|\delta m_{2,3}| \ll m_E$.

The non-degenerate case implies that the above branching ratios can be non-vanishing even if $m_\nu^2/v_S$ were proportional to the unit matrix.

IV. CONSTRAINTS ON THE MODEL FROM $B(\mu \rightarrow e \gamma)_{\text{exp}}$ AND $B(\tau \rightarrow \mu \gamma)_{\text{exp}}$

The experimental constraints we will use here are those from BaBar [14] and Belle [15] for $B(\tau \rightarrow \mu \gamma)_{\text{exp}}$ and from PDG [4] for $B(\mu \rightarrow e \gamma)_{\text{exp}}$. They are $B(\tau \rightarrow \mu \gamma)_{\text{exp}} < 6.8 \times 10^{-8}$ for BaBar [14] and $B(\tau \rightarrow \mu \gamma)_{\text{exp}} < 4.5 \times 10^{-8}$ for Belle [15]; $B(\mu \rightarrow e \gamma)_{\text{exp}} < 1.2 \times 10^{-11}$ [6]. Also we will be using $B(\tau \rightarrow \mu \nu \tau) \sim 17.4\%$.

Although the most general discussion would involve the non-degenerate case with $U^L \neq U^R$, it is probably more illuminating to first investigate the non-degenerate scenario with $U^L = U^R$. For this, we will use Eq. (44). We obtain for $B(\mu \rightarrow e \gamma)$

$$B(\mu \rightarrow e \gamma) = \left( \frac{3}{256 \pi^2} \right) \frac{1}{m_\mu^2 G^2 F m_E^2} \times \left| \sum_i \left( \frac{m_E}{m_i} \right) |U_{\mu i}^E U_{e i}^E|^2 \right|.$$  \hspace{1cm} (45)

and for $B(\tau \rightarrow \mu \gamma)$

$$B(\tau \rightarrow \mu \gamma) = \left( \frac{3}{256 \pi^2} \right) \frac{1}{m_\tau^2 G^2 F m_E^2} \times 0.174 \times \left| \sum_i \left( \frac{m_E}{m_i} \right) |U_{\tau i}^E U_{\mu i}^E|^2 \right|.$$  \hspace{1cm} (46)

It is also useful to relate one branching ratio to another as follows

$$B(\mu \rightarrow e \gamma) = \left| \sum_i \left( \frac{m_E}{m_i} \right) |U_{\mu i}^E U_{e i}^E|^2 \right| \times \frac{B(\tau \rightarrow \mu \gamma)}{0.174} \times \left( \frac{m_\tau}{m_\mu} \right)^2.$$  \hspace{1cm} (47)

We will illustrate the previous results with two examples: $m_E = 100$ GeV and $m_E = 200$ GeV.

• $m_E = 100$ GeV:

$$\left| \sum_i \left( \frac{m_E}{m_i} \right) |U_{\mu i}^E U_{e i}^E|^2 \right| < 1.25 \times 10^{-15},$$  \hspace{1cm} (48)

$$\left| \sum_i \left( \frac{m_E}{m_i} \right) |U_{\tau i}^E U_{\mu i}^E|^2 \right| < \left\{ \begin{array}{c} 7.1 \\ 4.7 \end{array} \right\} \times 10^{-12},$$  \hspace{1cm} (49)

where the first and second numbers on the right-hand-side of Eq. (49) refer to BaBar and Belle respectively.

• $m_E = 200$ GeV:

$$\left| \sum_i \left( \frac{m_E}{m_i} \right) |U_{\mu i}^E U_{e i}^E|^2 \right| < 5.0 \times 10^{-15},$$  \hspace{1cm} (50)

$$\left| \sum_i \left( \frac{m_E}{m_i} \right) |U_{\tau i}^E U_{\mu i}^E|^2 \right| < \left\{ \begin{array}{c} 28.4 \\ 18.8 \end{array} \right\} \times 10^{-12}.$$  \hspace{1cm} (51)

As we have mentioned in the beginning of the manuscript, the matrix elements of $U^E$ determine the lifetime and hence the decay length of the mirror charged leptons. Also, as we have seen above, some of these elements are constrained by the LFV processes $\mu \rightarrow e \gamma$ and $\tau \rightarrow \mu \gamma$. Let us make some rough assumptions on the above bounds in order to gain some insights into what one might expect.

Let us take the case $m_E = 100 GeV$ for definiteness and let us assume:

$$|\delta m_i| \ll m_E.$$

Let us also assume the following hierarchy (Case I) with $i = 1, 2, 3$:

$$U_{i e}^E \sim \lambda^3; \ U_{i \mu}^E \sim \lambda^2; \ U_{i \tau}^E \sim \lambda.$$  \hspace{1cm} (53)

With the above assumptions, one can now rewrite the bounds [19] [48] (taking e.g. the Belle value) as

$$\lambda^6 < 5.2 \times 10^{-13},$$  \hspace{1cm} (54)

$$\lambda^{10} < 1.4 \times 10^{-16}.$$  \hspace{1cm} (55)
where (54) refers to the bound from $\tau \to \mu \gamma$ while (55) comes from the bound on $\mu \to e \gamma$. (54) gives $\lambda < 0.009$. This gives $\lambda^{10} < 10^{-21}$ which satisfies (55). Suppose that the process $\tau \to \mu \gamma$ can be experimentally probed with a branching ratio not too far below the current bound. What does it say about the detectability of $\mu \to e \gamma$? This can be easily estimated by looking at the relation

$$B(\mu \to e \gamma) \approx 1.6 \times 10^{3} \lambda^{4} B(\tau \to \mu \gamma). \quad (56)$$

With $\lambda < 0.009$, one would have $B(\mu \to e \gamma) \sim 10^{-8} B(\tau \to \mu \gamma) < 4 \times 10^{-16}$ which makes the process $\mu \to e \gamma$ unobservable in the near future, about two orders of magnitude below the sensitivity of the MEG proposal\cite{16}.

A more interesting hierarchy (Case II) is as follows: (i = 1, 2, 3)

$$U_{i e}^{E} \sim \lambda^{3}; \ U_{i \mu}^{E} \sim \lambda; \ U_{i \tau}^{E} \sim \lambda^{2}. \quad (57)$$

We now obtain the following bounds for (49) (48)

$$\lambda^{6} < 5.2 \times 10^{-13}, \quad (58)$$

$$\lambda^{8} < 1.4 \times 10^{-16}. \quad (59)$$

Note that (58) is identical to (54) since one is simply switching the role of $\mu$ and $\tau$, but now the exponent on the left-hand-side of (59) is lower 8 instead of 10. Again (58) gives $\lambda < 0.009$. One now has $\lambda^{8} < 4.3 \times 10^{-17}$, a factor of three below (59) which is clearly satisfied. There is in addition a huge advantage: If $\tau \to \mu \gamma$ is discovered e.g. slightly below the present bound then one might expect to discover $\mu \to e \gamma$ with a rate of about a factor of three below its present experimental limit! In fact, the relationship (47) is now

$$B(\mu \to e \gamma) \approx 1.6 \times 10^{3} \lambda^{2} B(\tau \to \mu \gamma). \quad (60)$$

With $\lambda < 0.009$, one now has $B(\mu \to e \gamma) \sim 10^{-4} B(\tau \to \mu \gamma) < 5 \times 10^{-12}$. The observability of one process implies that of the other. Note that the estimated bound on $B(\mu \to e \gamma)$ is well within the range of the MEG proposal\cite{16}.

Last but not least, we could also consider the “inverted hierarchy” scenario (Case III):

$$U_{i e}^{E} \sim \lambda; \ U_{i \mu}^{E} \sim \lambda^{2}; \ U_{i \tau}^{E} \sim \lambda^{3}. \quad (61)$$

This case results in for (49) (48)

$$\lambda^{10} < 5.2 \times 10^{-13}, \quad (62)$$

$$\lambda^{6} < 1.4 \times 10^{-16}. \quad (63)$$

This case is the mirror of Case I. Using (62), one obtains $\lambda < 0.06$. For $\lambda$ close to this upper limit, (63) cannot be satisfied. If we satisfy (63) with $\lambda < 0.002$, $B(\tau \to \mu \gamma)$

| TABLE I: Bounds on branching ratios for three mixing scenarios (25, 27, 30) |
|------------------|-----------------|-----------------|
| $\lambda \leq B(\tau \to \mu \gamma) \leq B(\mu \to e \gamma)$ |
| Case I: (53) 0.009 4.5 $\times 10^{-8}$ 4 $\times 10^{-16}$ |
| Case II: (54) 0.009 4.5 $\times 10^{-8}$ 5 $\times 10^{-12}$ |
| Case III: (55) 0.002 1.2 $\times 10^{-25}$ 1.2 $\times 10^{-11}$ |

which is proportional to $\lambda^{10} < 10^{-27}$ will be hopelessly small.

The above constraints (Cases I, II, and III) are listed for convenience in Table I. Notice that the upper bounds on $\lambda$ listed in Table I are those which satisfy the experimental constraints from both $B(\tau \to \mu \gamma)$ and $B(\mu \to e \gamma)$.

Notice that in all of the cases listed above the bound on $B(\tau \to e \gamma) < 1.1 \times 10^{-7}$ which is weaker than the other two is trivially satisfied. That is the primary reason for using the constraints from the LFV processes $\mu \to e \gamma$ and $\tau \to \mu \gamma$.

In summary, Case II appears to be the most interesting one in that both LFV branching ratios could in principle be observed. This scenario has also another interesting phenomenological consequence: The primary decay mode of a given mirror charged lepton is into a muon instead of a tau as in the first scenario or an electron as in the third scenario. What would a typical decay length be?

In the simple scenario described above, the primary decay mode of the mirror charged leptons is $\mu + \phi_{S}$ (assuming e.g. that $\nu_{R}^{M}$ is approximately degenerate with its charged counterpart) with a coupling of the order $< 9 \times 10^{-3}$. For example, the decay rate of $e_{3}^{M}$ is approximately $\Gamma(e_{3}^{M} \to \mu + \phi_{S}) \sim m_{E} \lambda^{2}/(32 \pi)$ and a decay length $l = 1/\Gamma(e_{3}^{M} \to \mu + \phi_{S})$. With the bound on $\lambda < 9 \times 10^{-3}$, one estimates the decay length to be $l > 2445$ fm, which is microscopic. A macroscopic decay length of the order of a few centimeters would imply a much smaller $\lambda$ rendering the LFV processes discussed here practically unobservable.

V. CONCLUSION

The electroweak-scale right-handed neutrino model\cite{3} has a number of phenomenological implications which could be explored experimentally in the near future. As mentioned in\cite{3}, the see-saw mechanism could be directly tested at colliders by searching for like-sign dilepton events coming from the production and decays of a pair of right-handed neutrinos into a pair of like-sign lighter charged mirror leptons. The subsequent decays of those charged mirror leptons into SM leptons, $e_{R}^{M} \to e_{L} + \phi_{S}$ ($e_{R}^{M}$ and $e_{L}$ are generic notations), provide the desired signals. How far from the beam pipe these decays occur will depend on the strength of the Yukawa interactions written down in Eqs. (25) (30). These Yukawa interac-
tions are found to be proportional to the Dirac neutrino mass matrix which enters the see-saw mechanism and vanish in the limit where the light neutrino mass goes to zero. It turns out that these interactions also generate at one-loop level LFV processes such as the ones discussed here, namely $\mu \rightarrow e \gamma$ and $\tau \rightarrow \mu \gamma$. (The subject of $\mu - e$ conversion will be treated elsewhere.) These LFV processes put some interesting constraints on the model. In one example, it is found that if one process is observed (which by itself is already astonishing), the other will not be far from the current bound. Furthermore, within the framework of this model, a “macroscopic” decay length (a cm or so) of the mirror charged lepton, if discovered, implies that these LFV processes would be practically unobservable. Conversely, if any of these LFV processes is observed, the decay length would be tiny. This would require an extremely careful analysis to distinguish these like-sign dilepton events from possible backgrounds.

In summary, our model contains several phenomena which can be tested at future experimental facilities: 1) LFV processes at MEG and/or B factories; 2) Electroweak-scale right-handed neutrinos at future colliders (LHC, ILC).

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