Mathematical Model and Analysis Method of Fowfield Separation

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In the field of fluid mechanics, the more difficult but important problems are transition and separation. Hitherto, the separation and transition problems cannot be described accurately and mathematically, which leads to plenty of errors in design and problem prediction of fluid machine engineering. Because of the nonlinear uncertainty obvious state of separation and transition, it is difficult to accurately analyze the general law of separation and transition by using experimental methods. Because at the moment of fluid separation and transition, physical field is a state field. It will bring new ideas and methods to the mathematical prediction of fluid separation and transition. In this paper, after axiomatic treatment of fluid mechanics, the concept of excited state is given by generating fluctuation velocity, it is revealed that fluid separation and transition are special forms of excited state, and the state conditions of fluid separation and transition are clarified. Through the mathematical analysis of the famous Navier-Stokes equations, a general excited state theorem suitable for flowfield are obtained. Finally, the conditions of separation and transition are given, and the general laws of transition and separation are established. It lays a foundation for the future researches on the mechanism of fluid turbulence and solving engineering problems.

I. INTRODUCTION

The earliest study of separation and transition phenomenon began in 1883. Renault, a British scientist, discovered two different fluid forms by using the experiment of circular pipe flow: one is laminar flow, which is characterized by orderly movement and mutual immiscibility; the other is turbulent, which is characterized by irregular and mixed fluid particles, with tortuous and chaotic traces. The dimensionless parameters are defined, i.e. Reynolds number and Sherwood number[1, 2]. Reynolds number is mainly used to distinguish these two forms. Sherwood number is the ratio of flow mass transfer and diffusion mass transfer. The current research show that Sherwood number is related to the spatial distribution of velocity. However, up to now, it is impossible to give an exact definition of turbulence.

In theoretical research, based on the fluctuation velocity method in the time domain of Reynolds equation, analyzing the physical phenomena described by transition separation, summarising the fluid stability theory of transition separation, the basic equations of transition and separation are established. Thus, a semi-empirical mathematical model[3] that can predict the turning point of separation based on experimental data is established. Magdy Saeed Hussin[4] discretized compressible Navier-Stokes equations with finite volume method and solved them with semi-implicit pressure linking algorithm on unstructured grids. Compared with the classical method, the numerical dissipation within the program is reduced and the accuracy of the solution is improved. Maryam Azarpira[5] developed a 3D model to study the flow field in hollow vortices, which can well predict the characteristics of vortices. Qian Weiqi[6] proposed a K-SST two-equation turbulence model considering the influence of intermittent function, and proved through experiments that this model has good ability to predict transition position.

By describing the physical significance of the mathematical model of vorticity stability and exp the law of turbulence occurrence and expounding development as well as the stress response of small disturbances, the turbulence which has not been strictly defined in the world is given a qualitative explanation, which is expected to provide a new methodology for turbulence researches. In February 2018, Jakob Kuhnen[7] in Nature Physics in a paper published found that the turbulent strength quickly reduces by using the rotor in the circular tube to enhance the turbulence level and making severe turbulence flow through the downstream rotor. It is found that the turbulent strength quickly reduces, a laminar flow. The experiments showed the interaction between the eddy and at last completely dissipation phenomenon, but the study did not quantitatively describe separation and the transition phenomenon: In June 2019, Johnstone Shaun P. [8] published an article in Science: observing the non-equilibrium distribution of vorticity through experiments, and obtaining the characteristics of inverse energy cascade driven by evaporating heating of vorticity, which provides a novel method for studying critical points of...
II. GENERAL MATHEMATICAL MODEL OF FLOWFIELD

In the field of fluid mechanics, Navier-Stokes equations are important equations to describe flowfield. Assuming that the fluid flow space is \( \Omega \subseteq \mathbb{R}^3 \), and the velocity vector field is in the differential manifold, which is constructed by \( M = (\Omega, \mathcal{T}) \), whose cotangent vector field represents its velocity field \( u_i \in \mathcal{T}_M \). The \( m \)-th submanifold in \( M \) is \( M_m \). Since the bounded space \( \Omega \) is homeomorphic with the smooth submanifold \( M_m \) with edges, we can find a mapping \( \varphi_m \) satisfying the following conditions:

\[
\varphi_m : x_j \mapsto u_i, x_j \in \Omega, u_i \in \mathcal{T}_{M_m}
\]

In this way, the complex flow can be represented by the embedding of different submanifolds and different states can be considered as different submanifolds, but in this paper, in order to simplify the difficulty of mathematical symbol expression and reveal the physical meaning clearly, the mathematical laws of submanifolds are not discussed, they will be discussed in the follow-up studies. Therefore, for any velocity fields, the space-time evolution of the physical field should satisfy the Navier-Stokes equations:

**Lemma 1 (The Navier-Stokes equations).** For \( \Omega \subseteq \mathbb{R}^3 \), \( u_i = \varphi_m (x_j) \in \mathcal{T}_M \) and \( x_j \in \Omega \), so \( u_i \) satisfies continuity equation

\[
\frac{\partial \rho}{\partial t} + \rho u_i \overset{\cdot}{u}_i = 0;
\]

and momentum equation for viscous medium

\[
\rho \frac{\partial u_i}{\partial t} + \rho u_j u_i \overset{\cdot}{u}_j = \sigma_{ij} \overset{\cdot}{u}_j + \rho f_i
\]

where \( \rho \) is the density of fluid; \( \rho f_i \) is the volumetric force of the fluid; \( \sigma_{ij} = -\rho \delta_{ij} + \tau_{ij} \) is the stress state tensor of the fluid, which is expressed in following expression for Newton fluid in isomorphism.

\[
\sigma_{ij} = -p\delta_{ij} + \psi u_k \overset{\cdot}{u}_k \delta_{ij} + \mu (u_{j,i} + u_{i,j})
\]

In the construction of Reynolds equation, the concept of fluctuation velocity is used to reveal the randomness and uncertainty of turbulence. Similarly, the concept of fluctuation velocity is introduced here to describe the mechanism of separation and transition phenomenologically.

**Definition 1 (Fluctuation velocity).** Let \( \Omega \subseteq \mathbb{R}^3 \) is flowfield space, and the velocity vector field is in the differential manifold, which is constructed by \( M = (\Omega, \mathcal{T}) \), whose cotangent vector field represents its velocity field \( u_i \in \mathcal{T}_M \). The velocity field can be expressed as

\[
u_i = u'_i + \bar{u}_i
\]

where \( u'_i \in \mathcal{T}_M \) is the fluctuation velocity field and \( \bar{u}_i \in \mathcal{T}_M \) is the average velocity field.

It is worth noting that fluctuation velocity may bring nonlinearity and bifurcation. Since we define submanifolds, through different submanifolds embed, here still satisfy the mapping conditions. But this is an important problem, which will be our next topic.
If the above relationship is combined with the Navier-Stokes equations, the Reynolds average equation can be obtained. Turbulence in fluid dynamics is a chaotic state of motion, which is difficult to explain with the current research situation, but it is also a very common phenomenon, causing many problems in practical engineering applications.

It is noted that the fluctuation velocity defined here is not necessarily irregular and disordered. Apparently, this definition is only a simple superposition principle of vector field, but we can try to define transition and separation by it.

The keys of transition are random and irregular, so the fluctuation velocity induced by different coordinates should be defined differently. The transition position is a key position, which makes the laminar flow turbulent and makes the fluid flow from stable to unstable. From the perspective of phenomenological physics, transition means the generation of irregular and random flow mechanism. According to the second law of thermodynamics, the system spontaneously moves towards chaos. Therefore, it is very important to define the first point or region, which irregular change occurs. The first point which makes the fluid chaotic must have the same velocity as other points, but the fluctuation velocity is different at the same time.

**Definition 2 (Transition)**. Let \( U (x_h, \epsilon) \) is the neighbourhood at point \( x_h \in \Omega \), the laminar fluid with velocity \( u_i \in \mathcal{I}_M \) flows through the neighborhood, and it excited the fluctuation velocity \( u'_{ji} \in \mathcal{I}_M \) by kinds of disturbance, assuming \( \forall x_k \in U (x_h, \epsilon) \), if \( \exists x_s \in U (x_h, \epsilon), x_s \neq x_k \) has

\[
 u_i (x_k) = u_i (x_s), \quad u'_{ji} (x_k) \neq u'_{ji} (x_s), \quad u'_{ji} (x_h) = 0 \quad (6)
\]

there is transition in the flowfield at point \( x_h \), at this time, point \( x_h \) is called transition point, and the set of transition points is called transition position.

Separation is another key physical feature in fluid mechanics, which represents the huge loss of flow energy. It is one of the problems that can not be ignored in the process of fluid machinery design. As for separation, it is not important that the fluctuation velocity is regular or not. In phenomenological physics, whether the fluctuation velocity in the neighborhood is opposite to the original velocity field is more important. The reason of separation is backflow, that is, a reverse velocity equal to or greater than the original velocity is produced, so we define separation as follows

**Definition 3 (Separate)**. Let \( U (x_h, \epsilon) \) is the neighborhood at point \( x_h \in \Omega \), the fluid with velocity \( u_i \in \mathcal{I}_M \) flows through the neighborhood, and it excited the fluctuation velocity \( u'_{ji} \in \mathcal{I}_M \) by disturbance, if \( \exists x_k \in U (x_h, \epsilon) \) has

\[
 u_i (x_k) u'_{ji} (x_k) \leq 0, \quad |u_i (x_k)| \leq |u'_{ji} (x_k)| \quad (7)
\]

\[
 u'_{ji} (x_h) = 0
\]

there is separation in the flowfield at point \( x_h \), at this time, point \( x_h \) is called separation point, and the set of transition points is called separation position.

Now we find that the emergence of fluctuation velocity is one of the key problems to solve the transition and separation. The fluctuation velocity is a physical quantity that can not be described directly, so we first study the characteristics of the fluctuation velocity and related physical quantities at the transition and separation positions.

### III. Mathematical and Physical Conditions of Generating Fluctuation Velocity

In order to accurately describe the flowfield characteristics from the mathematical and physical point of view, it is assumed that a new fluctuation velocity field is generated in the flowfield, which exists in the manifold \( u'_{ji} \in \mathcal{I}_M \).

Before the fluctuation velocity is generated, fluctuation velocity should be zero. But it will not be equal to zero at the next moment, which means that the derivative of fluctuation velocity to time is not equal to zero at this moment. Therefore, we can draw the following conclusions:

**Theorem 1 (Conditions of Generating Fluctuation Velocity)**. Let position \( x_k \in \Omega \) has a neighborhood \( U (x_k, \epsilon) \), there is \( x_j \in \{ x_i | x_i \in U (x_k, \epsilon), ||x_k - x_i|| < \epsilon, \forall \epsilon > 0 \} \), conditions of generating fluctuation velocity can be expressed as:

\[
 u'_{ji} = 0, \quad (8)
\]

for first order derivative process equation, when the spatial derivative of the fluctuating velocity at the separation position is not zero, is expressed as:

\[
 
\frac{du'_{ji}}{dt} = \frac{\partial u'_{ji}}{\partial t} + u_k w'_{ji,k} \neq 0 \quad (9)
\]

when \( \epsilon \rightarrow 0 \), \( x_k \) is coordinates of key points, such as transition and separation points.

The formula (8) and formula (9) can be regarded as the necessary condition of separation and transition. When the particle is at the key position, although the fluctuation velocity has not been occurred, there is fluctuation acceleration. As time goes by, the fluctuation acceleration will induce the appearance of fluctuation velocity at the next moment, which may make the fluid begin separate or transite.

It is worth mentioning that when the conclusion is opposite to that of conditions (8) and (9) appears, the transition and separation will end, and then the flowfield state will quickly enter into another equilibrium state. Whether it is transition or separation, it is the result of
fluid resisting external disturbance. Therefore, for the whole system, transition and separation are means to make the system tend to equilibrium.

Although we have given very important conditions for the occurrence and even the end of the fluctuation velocity, as a random and uncertain physical quantity, it is difficult to evaluate its development law. Therefore, we introduce the superposition principle of quantum mechanics to obtain the general dynamic law of fluctuating velocity in fluid mechanics.

**IV. SPACE-TIME EVOLUTION LAW OF GENERATING FLUCTUATING VELOCITY IN FLUIDFLOW**

Here, we introduce the concept of superposition state in quantum mechanics. We assume that any uncertain state in an event exists in different parallel spaces. These parallel spaces will eventually collapse into a state for some reasons (according to the current research progress in the field of physics, this reason can be regarded as subjective in the field of quantum mechanics, but it is generally regarded as objective in the field of classical mechanics). By using the idea of superposition state, the reason of separation and transition can be obtained. Firstly, the space-time evolution law of fluctuating velocity is studied.

In order to simply describe states, considering the generation of fluctuation velocity is relatively difficult to judge, we assume that once the fluctuation velocity is generated in flowfield, the state is called excited state. Finally, we can judge the direction and get the final dynamic law.

The mathematical relationship between excited state and unexcited state is an important physical quantity. In order to clearly describe the process of excited flowfield, we set up a new physical quantity. Here, we consider the generation of fluctuation velocity as the excited flowfield state.

In order to simplify the operation, we define a new physical quantity in physics, it can be used to evaluate the measurement tensor of the ratio of fluctuation velocity to average velocity.

**Definition 4 (Velocity strain tensor).** Let \( \Omega \subseteq \mathbb{R}^3 \), if second-order tensor \( \lambda_i^j \in \mathcal{T}_M \) satisfies the following relationship

\[
\bar{u}_i^j = \bar{u}_j \lambda_i^j, \quad \bar{u}_i^j, \bar{u}_j \in \mathcal{T}_M
\]  

then tensor \( \lambda_i^j \) is called by velocity strain tensor.

According to the sensitivity principle of the initial boundary value problem of nonlinear dynamics, considering the external physical factors, the fluid separation phenomenon will also occur. So the reasons of fluid separation or transition can be divided into two factors: active factor and passive factor. Between them, the active factors are all kinds of influence factors caused by the initial state of the fluid, while the passive factors are all kinds of influence and interference caused by the geometry and physical conditions of the fluid. So far, it is not specifically analyzed, which factor causes the separation and transition phenomena of flowfield, but has been discussed the basic situation of the two states of flowfield from the perspective of phenomenological physics.

Under the unexcited state, due to various reasons, in the flowfield, there is no fluctuation velocity. Therefore, the fluctuation velocity of fluid \( u_i^j \) in formula [5] can be regarded as 0, so formula [5] can be simplified as:

\[
u_i = \bar{u}_i
\]

By introducing formula [11] into the Navier-Stokes equations describing fluid motion, the unexcited state equations can be obtained as follows. For continuity equation

\[
\frac{\partial \rho}{\partial t} + (\rho \bar{u}_i)^{\cdot i} = 0
\]

and for momentum equation under the action of viscosity, the temporal and spatial variation of the flowfield can be described as follows:

\[
\rho \frac{\partial \bar{u}_i}{\partial t} + \rho \bar{u}_k \bar{u}_i^{\cdot k} = \sigma_i^{\cdot k} + \rho f_i
\]

However, it can be seen from the subscript of formula [12] that the frames of the two fields of the index coincide. In order to make the expression of the formula more clear and avoid problems in later calculation and simplification, formula [12] and formula [13] can be established the following theorem.

**Theorem 2 (Unexcited state).** Let \( \Omega \subseteq \mathbb{R}^3 \), \( \bar{u}_j \in \mathcal{T}_M \), for unexcited state \( u_j \) satisfies continuity equation

\[
\frac{\partial \rho}{\partial t} + \rho \bar{u}_j \bar{u}_i^{\cdot j} = 0
\]

and momentum equation for viscous medium

\[
\rho \frac{\partial \bar{u}_i \delta_i^j}{\partial t} + \rho \bar{u}_j \bar{u}_i^{\cdot k} \sigma_i^{\cdot k} + \rho f_i = 0
\]

where \( \rho \) is the density of fluid; \( \rho f_i \) is the volumetric force of the fluid; \( \sigma_i^{\cdot k} \) is the general stress state tensor of the fluid, external force field \( g_i^{(1)} = \sigma_i^{(1)\cdot k} + \rho f_i \).

Under the excited state, when the fluid has the necessary conditions for separation or transition, due to the generation of fluctuation velocity, the velocity of fluid particles should not only take into account the average velocity of the fluid, but also its fluctuation velocity. Therefore, in order to calculate the instantaneous velocity of the fluid particle at this time more simply, the pulsation velocity is replaced by the average velocity by formula [10] and substituted into formula [5] to obtain the simplified expression for the velocity of the fluid particle, establishing the concept of excited ratio tensor.
Definition 5 (Excited ratio tensor). For $\Omega \subseteq \mathbb{R}^3$, $\xi_i^j$ and $\lambda_i^j \in \mathcal{F}_M$, if they satisfy the following relationship
\[ \xi_i^j = \delta_i^j + \lambda_i^j \] (16)
then tensor $\xi_i^j$ is excited ratio tensor, which can be used to evaluate the measurement tensor for describing the ratio of average velocity to instantaneous velocity.

Therefore, we can draw the following conclusion
\[ u_i = \bar{u}_j \delta_i^j + \bar{u}_j \lambda_i^j = \bar{u}_j (\delta_i^j + \lambda_i^j) = \bar{u}_j \xi_i^j \] (17)

At the same time, the condition of flowfield excited state changes accordingly. According to the above analysis and formula [8] and formula [9], when the fluctuation velocity field is 0 and the fluctuation acceleration is not 0, this moment is the moment when the particle is the beginning point of the excited state position. Therefore, the following theory is obtained:

**Theorem 3 (Conditions of Excited State).** In the beginning point of excited state position, the velocity strain tensor $\lambda_i^j \in \mathcal{F}_M$ satisfies continuity equation
\[ \lambda_i^j = 0 \] (18)
when the spatial derivative of the fluctuation velocity at the excited state position is not zero, the time derivative of the velocity strain tensor is expressed as:
\[ \frac{d\lambda_i^j}{dt} = \frac{\partial \lambda_i^j}{\partial t} + u_k \lambda_i^j \Rightarrow 0 \] (19)
where $u_k \in \mathcal{F}_M$ is the velocity field of the flowfield.

Therefore, in order to make a better comparison with the phenomena without excited flowfield, formula [17] and formula [19] are combined with Navier-Stokes equations, and the following equations can be obtained:

**Theorem 4 (Excited State).** Let $\Omega \subseteq \mathbb{R}^3$, $\bar{u}_j \in \mathcal{F}_M$, for excited state $\bar{u}_j$ satisfies continuity equation
\[ \frac{\partial \rho}{\partial t} + \rho \bar{u}_j \xi_i^j + \rho \bar{p}u_j \xi_i^j = 0 \] (20)
and momentum equation for viscous medium
\[ \rho \frac{\partial (u_j \xi_i^j)}{\partial t} + \rho u_j \bar{u}_k \xi_i^{1j2} + \rho \bar{p}u_j \bar{u}_k \xi_i^{1j2} = g_i \] (21)
where $\xi_i^j \in \mathcal{F}_M$ is the excited ratio tensor; $\rho$ is the density of fluid; $\rho f_i$ is the volumetric force of the fluid; $\sigma_{ik}^{(1)}$ is the stress state tensor of the fluid under excitation effect, here adjustment tensor coefficient $\epsilon_{ik}^{1j2} = \xi_i^j \xi_i^j$, external force field $g_i^{(2)} = \sigma_{ik}^{(2)} \delta_i^j + \rho f_i$.

In order to better compare the two assumed flowfield states, the following relationship are needed and easy to get.
\[ \xi_i^{1j2} = \xi_i^j \xi_i^j - \delta_i^j \delta_i^j = \lambda_i^j \lambda_i^j + \lambda_i^j \delta_i^j + \delta_i^j \lambda_i^j \] (22)

The flowfield is excited by the physical properties of the fluid and changes in external conditions, included in formula [14] [15] [20] and [21]. Therefore, if the excited field is subtracted from the assumed unexcited state field, the equation causing the excited state can be obtained:

**Theorem 5 (Excited Equation).** Let $\Omega \subseteq \mathbb{R}^3$, $\bar{u}_j \in \mathcal{F}_M$, for excited state $\bar{u}_j$ satisfies continuity equation
\[ \rho \frac{\partial u_j}{\partial t} + \rho \bar{u}_j \lambda_i^j + \rho \bar{p}u_j \lambda_i^j = 0 \] (23)
and momentum equation for viscous medium
\[ \rho \frac{\partial (u_j \lambda_i^j)}{\partial t} + \rho \bar{p}u_j \frac{\partial (\lambda_i^j)}{\partial t} + \rho \bar{u}_j \lambda_i^j \lambda_i^j \lambda_i^j = g_i \] (24)
where $\lambda_i^j \in \mathcal{F}_M$ is the velocity strain tensor, $\rho$ is the density of fluid; $\epsilon_{ik}^{1j2}$ is the adjustment tensor coefficient, external force field $g_i = \sigma_{ik}^{(2)} \delta_i^j - \sigma_{ik}^{(1)} \delta_i^j$, here $\sigma_{ik}^{(1)}$ and $\sigma_{ik}^{(2)}$ is the stress state tensor of the fluid under the excited and unexcited.

At the same time, equation [14] and equation [19] with the excitation condition are substituted into equations [20] and [24] and the following equations can be obtained:

**Theorem 6 (Excited Equation).** For $\Omega \subseteq \mathbb{R}^3$, $\bar{u}_i = \varphi_m(x_k)$, $x_k \in \Omega$, if the flowfield is into excited state under the action of physical properties, $\bar{u}_i$ satisfies continuity equation
\[ \lambda_i^{1i} = 0, \quad \lambda_i^j \in \mathcal{F}_M \] (25)
and momentum equation for viscous medium
\[ \rho \bar{u}_j \lambda_i^j \lambda_i^j = g_i \] (26)
where $\lambda_i^j \in \mathcal{F}_M$ is the velocity strain tensor, $\rho$ is the density of fluid; $\epsilon_{ik}^{1j2}$ is the adjustment tensor coefficient, external force field $g_i = \sigma_{ik}^{(2)} \delta_i^j - \sigma_{ik}^{(1)} \delta_i^j$,

here $\sigma_{ik}^{(1)}$ and $\sigma_{ik}^{(2)}$ is the stress state tensor of the fluid under the excited and unexcited.

At this point, the mathematical significance of formula [25] can be described according to Gauss theorem: when fluid separation is about to take place, the separation position is taken as the origin and an open sphere with radius $\varepsilon > 0$ is established. Once again, according to formula [25] and using Gauss’s theorem, the following conclusions can be drawn:
\[ \lambda_i^{1i} = \lim_{\Delta \tau \to 0} \frac{1}{\Delta \tau} \int_{\sigma} \lambda_i^j n^i d\sigma = 0 \] (27)
In the study of fluid mechanics, the velocity of fluid is one of the important parameters to describe the fluid motion, and there are many application methods to measure the velocity of any particles in the fluid, such as pitot tube, hot-wire anemometer and so on. According to the derivation of the above equations, it can be seen that the generation of separation flow and transition phenomena have important relationship with the flow velocity of the fluid. Due to the difference of velocity field, many factors will lead to changes in speed, such as the shape of flow, the flow of the variable form, the friction factor of fluid, heat and other factors, for example surface roughness, so based on the research of the speed, most of the factors are taken into account, to improve the accuracy and completeness of the mathematical model.

As there are various small vortices produced in the turbulence of the separation flow, these vortices can be regarded as the results of the change in the direction and magnitude of the pulsation velocity caused by the pulsation acceleration at the separation position before that, is the vortices with uncertain magnitude and direction of the velocity are generated. According to formula 27, which can be seen that the velocity strain tensor is in the open sphere with radius of adjacent domain at the separation position, from which the streamline formed by the fluid particles to the original velocity is a closed curve. In 1972, lugt vortex proposed this phenomenon is defined as many fluid particles around a common center of rotation, but was unable to predict the orientation of the center of rotation and movement, so confirmed when, namely, the divergence is zero, the particle in the position of the separation position, which validate the correctness of the mathematical model. In summary, the streamline formed by the stress on the original velocity field is a closed curve, and the divergence of the velocity strain tensor is zero, indicating that the fluid particle is passive.

V. STRESS STATE ANALYSIS OF EXCITED EQUATIONS

In fluid mechanics, the stress state at any point in the fluid is uniquely determined by the stress vectors on the three orthogonal planes of action at that point, and each stress vector can again be represented by three components. Therefore, this study assumes that the fluid in the flowfield is isotropic, the stress tensor and the deformation rate tensor are linear, and their homogeneous function relationship can be expressed as formula 23.

For the unexcited state, the stress tensor of the fluid particle can be expressed as the following equation:

\[ \sigma_{ik}^{(1)} = -p^{(1)} \delta_{ik} + \psi \mu \bar{u}_{ik} + \mu \bar{u}_{ik} \]

(28)

Thus, above formula is calculated and its gradient is as follows:

\[ \sigma_{ik}^{(1)\bullet} = -p^{(1)} \delta_{ik} + \psi \mu \bar{u}_{ik} + \mu \bar{u}_{ik} \]

(29)

According to the consistency of the flowfield frame, the index of variable subscript can be transformed, so formula 29 can be expressed as follows:

\[ \sigma_{ik}^{(1)\bullet k} = -p^{(1)} \delta_{ik} + \psi \mu \bar{u}_{ik} + \mu \bar{u}_{ik} \]

(30)

For the excited state, the stress tensor of the fluid particle can be expressed as the following equation:

\[ \sigma_{ik}^{(2)\bullet k} = -p^{(2)} \delta_{ik} + \psi \mu \bar{u}_{ik} + \mu \bar{u}_{ik} \]

(31)

Combining formula 16 and formula 31 we can get its gradient as:

\[ \sigma_{ik}^{(2)\bullet k} = -p^{(2)} \delta_{ik} + \psi \mu \bar{u}_{ik} + \mu \bar{u}_{ik} \]

(32)

In order to better and more obviously see how the stress of the fluid particle changes when the fluid is excited, formula 30 and formula 32 are used to subtract, and the change of the stress of the particle at the moment of two states' transformation can be obtained, as shown in the following:

\[ \sigma_{ik}^{(2)\bullet k} - \sigma_{ik}^{(1)\bullet k} = p^{(1)} - p^{(2)} + \psi \mu \bar{u}_{ik} + \mu \bar{u}_{ik} \]

Combining with the coordinate transformation law, we can develop the following theorem

Theorem 7 (Excited stress state equation). Let \( \Omega \subseteq \mathbb{R}^3 \), \( g_i \in \mathcal{F}_M \), if the flowfield is into excited state under the action of physical properties, when the stress state is

\[ g_i = P_i + \psi \mu \lambda_i \]

(33)

where \( \lambda_i \in \mathcal{F}_M \) is the velocity strain tensor, \( \mu \) is the dynamic viscosity coefficient of fluid, \( \theta = u_{ij}^0 \) is expansion volume for velocity and \( g_i = \sigma_{ik}^{(2)\bullet k} - \sigma_{ik}^{(1)\bullet k} \), here \( \sigma_{ik}^{(1)} \) and \( \sigma_{ik}^{(2)} \) is the stress state tensor of the fluid under excitation and nonexcitation, \( P_i \) is the pressure gradient difference \( P_i = p^{(1)} - p^{(2)} \).

Thus, the final Newtonian fluid excitation law is obtained by synthesizing formula 29 and formula 33.

Theorem 8 (Excitation Law). Let \( \Omega \subseteq \mathbb{R}^3 \), \( g_i \in \mathcal{F}_M \), in order to reach the excited state, velocity strain tensor \( \lambda_i \in \mathcal{F}_M \) is necessary to meet the requirements: for continuity condition

\[ \lambda_i^0(x_k) = 0, \quad \lambda_i^{1,j}(x_k) = 0 \]

(34)
and for momentum equation with viscous medium

\[
\rho u_j \frac{\partial \lambda^j_i (x_k)}{\partial t} = P_i + \psi \mu \theta \lambda^j_{k,i} (x_k) + \mu u_{j,k} \lambda^{i,k}_{j,i} (x_k)
\]  

(35)

where \( \rho \) is the density of fluid, \( \mu \) is the dynamic viscosity coefficient of fluid, \( \theta = u^*_{k,i} \) is expansion volume for velocity, \( x_k \in \Omega \) is the beginning point of excited state position.

VI. DEGENERATE FORM OF EXCITATION LAW

The degenerate form of the excitation equation means that the velocity strain tensor in the excitation law is degenerated into the pulse velocity, so as to solve the problem of difficult calculation of complex equations.

In accordance with the above analysis methods, let a very short time \( \Delta t \), the fluid had previously advanced a small distance, such as in 3D space \( \Delta x \), \( \Delta y \), \( \Delta z \) are developed in space. The velocity strain tensor is developed from 0 to \( \lambda^j_i \). By using the difference principle, the formula is expanded and we can get degenerate form of excitation law for unsteady flow of viscous compressible fluid.

**Theorem 9** (Degenerate form of excitation law). Let \( \Omega \subseteq \mathbb{R}^3 \), \( u_j \in \mathcal{M} \), if the system is exited, the fluctuation velocity \( u^\prime_j \in \mathcal{M} \) satisfies equation

\[
\frac{d u^\prime_j}{d t} = \frac{1}{1 - \beta - \gamma} P_i
\]

(36)

where \( P_i \) is the pressure gradient difference; \( C^j_k \) is velocity gradient; \( \beta \) is dimensionless coefficient of viscous-inertial force ratio; \( \gamma \) is dimensionless coefficient of dilatation-inertial force ratio, if \( |u^i_j u^k_j| \neq 0 \), \( |u^i_j u^l_j| \neq 0 \), and then these dimensionless coefficients have the following forms

\[
\beta = \frac{\mu C^j_k}{\rho u_j u^k_j}; \quad \gamma = \frac{\mu \theta}{\rho u_j u^j}
\]

\( \rho u_j u^k_j \) is inertial stress, which represents the inertia force formed by the product of spatial acceleration and density caused by the spatial distribution of velocity. \( \mu C^j_k \) is the viscous stress, \( \mu \) is dilatation stress.

**Proof.** For \( \Omega \subseteq \mathbb{R}^3 \), \( u_j \in \mathcal{M} \), if there is the excited flowfield under the action of physical properties, velocity strain tensor \( \lambda^j_i \in \mathcal{M} \) satisfies continuity equation

\[
\lambda^j_i (x_k) = 0, \quad \lambda^{i,j}_{j,i} (x_k) = 0
\]

(37)

and momentum equation for viscous medium

\[
\rho u_j \frac{\partial \lambda^j_i (x_k)}{\partial t} = P_i + \psi \mu \theta \lambda^j_{k,i} (x_k) + \mu u_{j,k} \lambda^{i,k}_{j,i} (x_k)
\]

(38)

At this time, according to the equation as time goes on, the fluid will gradually be excited under the effect of pressure derivative differences. In a short period of time \( \Delta t \), the velocity strain rate tensor changes from 0 to \( \lambda^j_i \), so we can get the following relationship, we can obtain

\[
\frac{\rho u^\prime_j}{\Delta t} = P_i + \mu \frac{\Delta u^j_i}{\Delta x_k \Delta x^k_j} u_j + \psi \mu \theta \lambda \frac{\Delta x^i_j}{\Delta x^j_i}
\]

where \( \lambda = \lambda^k_k \) is the sum of diagonal elements of velocity strain matrix.

After a simple mathematical arrangement, the above formula becomes

\[
\left( \rho - \mu \frac{\Delta u^j_i}{\Delta x_k} \right) \frac{\Delta t}{\Delta x_k} = P_i \Delta t + \psi \mu \theta \frac{\Delta t}{\Delta x^j_i} \lambda
\]

when the change of time infinitely approaches zero, \( \Delta t \rightarrow 0 \), if \( |u^i_j u^k_j| \neq 0 \), \( |u^i_j u^l_j| \neq 0 \), the change of each physical quantity in the above formula is as follows:

\[
\Delta u_j \rightarrow \frac{d u^j_j}{d x_k} = C^j_k, \quad \Delta x_k \rightarrow \frac{d x_k}{d t} = u_k, \quad \Delta x^i_j \rightarrow \frac{d x^i_j}{d t} = u^i
\]

The above formula is divided by the vector \( u_i \). The above formula becomes

\[
\left( \rho - \mu \frac{C^j_k}{u_k u_j} \right) \lambda = \frac{P_i}{u_i} \Delta t + \psi \mu \theta \frac{\Delta t}{u_j u^j} \lambda
\]

After a simple mathematical transformation, multiply both sides by \( u_i \) and divide by \( \Delta t \), we can obtain

\[
\rho \left( 1 - \frac{\mu C^j_k}{u_k u_j} - \psi \frac{\mu \theta}{\rho u_j u^j} \right) \frac{d u^\prime_j}{d t} = P_i
\]

since the pulse velocity increases from 0 during separation, it has the following form

\[
\frac{u^\prime_j}{\Delta t} = \frac{\Delta u^\prime_j}{\Delta t} \rightarrow \frac{d u^\prime_j}{d t}
\]

We come to the final conclusion

\[
\rho \frac{d u^\prime_j}{d t} = \frac{1}{1 - \beta - \gamma} P_i, \quad P_i n^j > 0
\]

where \( P_i \) is the pressure gradient difference; \( C^j_k \) is velocity gradient; \( \beta \) is dimensionless coefficient of viscous-inertial force ratio; \( \gamma \) is dimensionless coefficient of dilatation-inertial force ratio, they have the following forms

\[
\beta = \frac{\mu C^j_k}{\rho u_j u^k_j}; \quad \gamma = \frac{\mu \theta}{\rho u_j u^j}
\]

Then, this theorem is proved. □

Since Reynolds number is defined as the ratio of inertial force to viscous force, \( \beta \) can be regarded as the reciprocal value of “Reynolds number at each point” in the flowfield. So the statistical method from the micro world to the macro world is defined as follows
**Definition 6 (Statistical method).** If mapping \( \phi : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R} \), satisfies
\[
DV_\infty = \phi \left( \frac{u_j u_k}{u_j^2} \right)
\]
where \( V_\infty \) is the velocity of incoming stream at infinity, \( D \) is characteristic diameter, and \( \phi \) is evaluated fluid excited and returns to the original state again, such as the separation of pressure because the pulse velocity is too small, state, it is difficult to maintain more time under the action of pressure gradient, the stable excited states is more correlated to the velocity divergence of the flowfield.

\[
Re = \phi \left( \frac{1}{\beta} \right)
\]

Such mapping is a new statistical method, which is similar to the potential theory of infinite field integral.

This is a new statistical method from micro to macro. It can be widely used to connect micro tensor space and macro scalar space. This will be described in detail in future research.

The other new dimensionless number \( \gamma \) is closely related to the velocity divergence of the flowfield.

In engineering, due to the randomness and uncertainty of pressure gradient, the stable excited states is more concerned. Otherwise, even if the system goes into excited state, it is difficult to maintain more time under the action of pressure because the pulse velocity is too small, and returns to the original state again, such as the separation bubble. The coefficient of evaluating fluid excited state ability is \( 1 - \beta - \gamma \), which can effectively evaluate the magnitude of viscous fluid affected by pressure gradient. We can see that the pressure gradient changes at any moment due to the effect of viscosity and expansion, so there is fluctuation velocity, which makes the fluid enter the excited state near this point, and then if the pressure gradient projection is positive along outer normal vector, the flowfield will be excited. Therefore, we need to further study the deep physical meaning of pressure gradient.

**VII. PHYSICAL MEANING OF PRESSURE GRADIENT DIFFERENCE**

According to the derivation of the above equations, the root cause of excitation obtained is determined by external and internal causes, among which the external reason is caused by the change of pressure difference around the excitation position, and the internal reason is caused by the viscous term and the diffusion term of velocity.

Internal cause, viscous term and diffusion term of velocity. In the Navier-Stokes equations, the pressure term and the viscous term (diffusion term) describe the effect of pressure and viscous force on the fluid respectively.

\[
Re = \frac{U_0 L}{\nu} = \frac{u \mu u^2}{L^2} \approx \frac{\| u \nabla u \|}{\| \mu \nabla^2 u \|}
\]

As can be seen from the definition of Reynolds number (see formula (39), when the Reynolds number is high, it means that in the Navier-Stokes equations, relative to the viscous term, convection term is dominant. And different from the viscous term of the linear term, the convection term is a nonlinear term with certain uncertainty and is very sensitive to the slight change of parameters, which will increase all kinds of influence of the slight disturbance. And the higher the Reynolds number is, the more easily the flow becomes unstable. Therefore, when the Reynolds number of fluid flow is relatively high, it is more likely to lead to the occurrence of turbulence, that is, the viscous term and the diffusion term of velocity both have a certain influence and relationship with fluid excitation and transition.

As for external causes, the changes of pressure difference around the separation position. The studies of pressure can reflect the changes of other flow parameters. According to formula (39), it can be seen that there is a non-negligible relationship between the stress changes of fluid particles and the pressure differences.

However, for practical application and experimental measurement, the pressure parameters are usually not directly measured. Direct numerical simulation method, experimental method and large eddy simulation method are generally used, but the power spectrum semi-empirical model is mostly used for the balanced turbulence boundary of the plate, and the performance of the experimental instrument is too high and the experiment time is too long, so it is not widely used. In engineering applications, in the design of aircraft and ships, pressure difference is generated due to the change of pressure inside the fluid, leading to a sharp change in fluid velocity, that is, the velocity gradient becomes larger, so there is a greater friction force inside the fluid, which also means that a larger strength vortex will be generated.

Therefore, pressure is an important research direction both from the perspective of mathematics or physics and from the perspective of practical experimental application. Here we make more detailed analysis and explanation of the concept of pressure gradient difference and excitation law.

Let’s go back to the idea of superposition. If there are \( n \) possibilities non-interlaced states in an uncertain event, the difference and ratio of the state characteristic equation can be used to describe the cause of the induced state. The difference describes the additional effect. The ratio describes the rate of change of the additional effect.

As mentioned before, when the system goes from one equilibrium state to another, the existence of difference indicates that a new equilibrium form needs to be added in the process of breaking the equilibrium, and the pressure gradient difference is such a form.

\[
P_i = p_{i,1}^{(1)} - p_{i,1}^{(2)} = -\Delta p_{i,1}
\]

where \( \Delta p_{i,1} \) is the variation of pressure gradient. For external reasons, in order to achieve the equilibrium after
excitation, we must supplement the equilibrium form of equation \[ \text{38} \] on the basis of the original equilibrium. This change is caused by the temporal or spatial distribution of pressure gradient.

Divergence is a common operator to describe the convergence degree of vector field spatial distribution, the Laplace operator distribution of pressure represents the ability of external influence on the system. It needs to be reminded again, in Minkowski space, the Laplace operator is the d'Alembert operator. Let \( \zeta \in \mathcal{F}_M \), \( M = (\Omega, \mathcal{S}) \), \( \Omega \subseteq \mathbb{R}^3 \) evaluate the spatial variation rate of pressure gradient difference, and \( \Delta x_i \in \Omega \) represents the scale of spatial change, and then according to the equation \[ \text{40} \] we can obtain

\[
\zeta = \lim_{\Delta x_i \rightarrow 0} \frac{\Delta p_{s,i}}{\Delta x_i} = \mathcal{L} p
\]

where \( \mathcal{L} \) is Laplace operator.

In addition, the wall condition is another important factor to excite the flowfield. Because there is a fact in nature that the flowfield cannot pass through the solid. Therefore, for excited the flowfield the direction of the fluctuation acceleration only along the normal vector outside the wall. According to the equation \[ \text{36} \] the direction of pressure gradient difference can only follow the direction of outer normal vector on the wall.

**Lemma 2** (Wall condition). Let the fluid \( \Omega_f \subseteq \mathbb{R}^3 \) flow around the rigid body \( \Omega_s \subseteq \mathbb{R}^3 \), then the fluctuation velocity \( u'_i \in \mathcal{F}_M \), pressure gradient difference \( p'_i \in \mathcal{F}_M \), \( M = (\Omega_f \cup \Omega_s, \mathcal{S}) \) near the key point should satisfy the following relation.

\[
u'_i n^i > 0, \quad p'_i n^i > 0 \quad (42)
\]

where \( n^i \) is the outer normal vector of the solid boundary.

So far, we have established the excited state theory of flowfield completely by mathematical analysis. Now we need to describe and analyze the specific problems according to the definition of separation and transition.

**VIII. THEOREMS OF TRANSITION AND SEPARATION**

Transition and separation are two special cases in the excited state, one is the generation of irregular fluctuation velocity, the other is the generation of reflux near the boundary layer. Therefore, we will use the excitation law to find out the laws of transition and separation.

**Theorem 10** (Transition). Let \( U(x_h, \epsilon) \) is the neighbourhood at point \( x_h \in \Omega \), the laminar fluid with velocity \( u_i \in \mathcal{F}_M \) flows through the neighborhood, and it excited the transition by kinds of disturbance, if \( \forall x_h \in U(x_h, \epsilon) \) has

\[
\mathcal{L} p(x_h) = 0, \quad \mathcal{L} p(x_k) \neq 0, \quad \Delta p_{s,i}, n^i < 0 \quad (43)
\]

where \( \mathcal{L} \) is Laplace operator, then all points \( x_j \), satisfying the above equation, constitute set \( K \), the set \( K \subseteq \Omega \) is called the transition position.

**Proof.** Let \( U(x_h, \epsilon) \) is the neighborhood at point \( x_h \in \Omega \), the laminar fluid with velocity \( u_i \in \mathcal{F}_M \) flows through the neighborhood, and it excited the fluctuation velocity \( u_j' \in \mathcal{F}_M \) by kinds of disturbance. By definition \( \forall x_k \in U(x_h, \epsilon) \), then \( \exists x_s \in U(x_h, \epsilon) \)

\[
u'_j (x_k) \neq u'_j (x_s), \quad u'_j (x_h) = 0
\]

so, it means that no matter how \( x_k \) changes, \( u'_j \) is not equal. In this case, field \( u'_j \) can not be regarded as an isolated system, and there are sources and sinks everywhere in the field, so we can get the divergence field of fluctuation velocity is not 0, except point \( x_h \).

\[
u'_j (x_k) \neq 0, \quad \nu'_j (x_h) = 0, \quad \forall x_k \in U(x_h, \epsilon)
\]

As we can see from the previous discussion, \( u'_j \) is developed from 0 in a very short time, so it can be obtained

\[
\left( \frac{du'_j}{dt} \right) (x_k) \neq 0, \quad \left( \frac{du'_j}{dt} \right) (x_h) = 0, \quad \forall x_k \in U(x_h, \epsilon)
\]

So we combine the excitation equation \[ \text{44} \] and \( u_i (x_s) = u_i (x_k) \) we can get

\[
P_{s,j} (x_k) \neq 0, \quad P_{s,j} (x_h) = 0, \quad \forall x_k \in U(x_h, \epsilon)
\]

Combine with equation \[ \text{40} \] we can obtain

\[
\mathcal{L} p(x_h) = 0, \quad \mathcal{L} p(x_k) \neq 0, \quad \forall x_k \in U(x_h, \epsilon)
\]

In this way, combined with the wall condition, we can prove the necessity of the theorem. On the contrary, we can prove its sufficiency. \( \square \)

**Theorem 11** (Separation). Let scalar pressure field \( p \in \mathcal{F}_M \), \( M = (\Omega, \mathcal{S}) \), and velocity vector field \( u_i \in \mathcal{F}_M \). If the separation is excited, there exists a set of solutions \( x_j \) satisfying the following time domain integral equations

\[
u_i (x_j) = \int \frac{1}{1 - \beta - \gamma - \Delta p_{s,i}(x_j) \rho} dt, \quad \Delta p_{s,i}, n^i < 0
\]

then all points \( \forall x_j \in K \), satisfying the above equation, constitute set \( K \), the set \( K \subseteq \Omega \) is called the separation position.

**Proof.** By definition, let \( U(x_h, \epsilon) \) is the neighborhood at point \( x_h \in \Omega \), the fluid with velocity \( u_i \in \mathcal{F}_M \) flows through the neighborhood, and it excited the fluctuation velocity \( u_j' \in \mathcal{F}_M \) by disturbance, if \( \forall x_k \in U(x_h, \epsilon) \) has

\[
u_i (x_k) = -u'_j (x_k), \quad u'_j (x_h) = 0
\]

According to the excitation equation \[ \text{44} \] we can obtain

\[
\rho \frac{du_i}{dt} = -\frac{1}{1 - \beta - \gamma} (-\Delta p_{s,i})
\]
After simple mathematical arrangement, take the time integral on both sides, we can get the final result

\[ u_i(x_j) = \int_T \frac{1}{1 - \beta - \gamma} \Delta p_{ij}(x_j) \rho \, dt, \forall x_j \in K \]

In this way, combined with the wall condition, we prove the necessity of the theorem. On the contrary, we can prove its sufficiency.

In summary, we have established the basic theory of separation and transition in general, but it is still not conducive to engineering application, especially in the active or passive flow control technology which is more common in the aerospace field in recent years. According to the excitation law, we define a new physical quantity by divergence of fluctuating acceleration field, it can be called the intensity of flow excitation.

**Definition 7 (Intensity of flow excitation).** Let \( \Omega \subseteq \mathbb{R}^3 \), and pressure field \( p \in \mathcal{T}_M \), if the system is exited, there is scalar physical quantity \( \Pi \in \mathcal{T}_M \) satisfies

\[ \Pi = \frac{1}{\beta + \gamma - 1} \mathcal{L} p \quad (44) \]

where \( \mathcal{L} \) is Laplace operator; \( \rho \) is the density of fluid; \( \beta \) is dimensionless coefficient of viscous-inertial force ratio; \( \gamma \) is dimensionless coefficient of dilatation-inertial force ratio. The physical quantity \( \Pi \) is called flow excitation intensity.

Therefore, the goal of active or passive flow control is to eliminate the excitation intensity at the begining point of the excited state. In addition, the turbulent flow can be transformed into laminar flow by using this physical quantity. As long as the excitation intensity of each point in the flow field is clear, and try to offset the excitation intensity.

**IX. EXPERIMENTAL VERIFICATION OF THEORIES AND DISCUSSION ON CORRELATION ANALYSIS METHODS**

The experiment is the final verification of all the physical theory results. Therefore, we will use several simple one-dimensional flow experiments to reveal the applicability of the separation and transition theory. Because it is an one-dimensional flow, the sum of all physical quantities is related to one degree of freedom. The relationship is introduced into the above equation \( (43) \) we can get the transition is only related to the second derivative of pressure. The solution satisfying the following equations is the transition position, take infinitesimal \( \epsilon \), if \( x \) satisfy the following relationship

\[ \frac{d^2 p}{dx^2}(x) = 0, \quad \frac{d^2 p}{dx^2}(x - \epsilon) n_x < 0 \]

then \( x \) is the transition position, along the chord of airfoil. \( n_x \) is the component of the outer normal vector of the upper surface of the airfoil on the \( x \) axis.

For separation, a simple difference method is used to discretize the equation, if \( x \) satisfy the following relationship.

\[ u \frac{d^2 u}{dx^2}(x) = \frac{1}{\rho (1 - \beta - \gamma)} \frac{d^2 p}{dx^2}(x), \quad \frac{d^2 p}{dx^2}(x - \epsilon) n_x < 0 \]

then \( x \) is the separation position, along the chord of airfoil. \( n_x \) is the component of the outer normal vector of the upper surface of the airfoil on the \( x \) axis.

In the experiment, NACA2412 with inflow velocity of 30m/s was used. In order to analyze transition and separation better, let its angle of attack is \( 12^\circ \) and \( -2^\circ \). The installation diagram of the experimental device is shown in Fig. 1.

**FIG. 1. Installation and fixation of airfoil NACA2412**

The figures 2 and 3 show the comparison between the upper surface oil flow test results and the theoretical prediction results. It is obvious from the figure that the separation or transition caused by the installation wall is not a straight line, but a parabola. This is because viscosity reduces the conditions of transition and separation. This is also the error caused by calculating only one dimension. Attention to the importance of the wall condition. Point A in Fig. 3 does not meet the wall condition, so the transition will not occur.

Except the obvious errors due to the wall effect, from the comparison chart that the theoretical prediction is in good agreement with the experimental results, and the prediction error is less than 1\%. In addition, this method can promote the improvement of active or passive flow control technology.
X. CONCLUSIONS

In the process of completing this study, we found that the principle of quantum superposition state as a methodology can be widely applied to the theoretical research of physics or mechanics. If there are $n$ possibilities non-interlaced states in an uncertain event, the difference and ratio of the state characteristic equations can be used to describe the cause of the induced state. The difference describes the additional effect. The ratio describes the rate of change of the additional effect.

In addition, we explained the state conditions of fluid separation and transition. Through the mathematical analysis of Navier-Stokes equations, a general equation applicable to compressible fluid is obtained. By setting up a variable, the root cause of the separation and transition phenomena are divided into excited state problems due to external and internal reasons. The external reason is mainly the pressure gradient change around the key position, which shows that there is a non-negligible relationship between the stress changes of flowfield and the pressure difference; the internal reason is the viscosity and diffusion terms of velocity. The pressure and viscosity terms in the Navier-Stokes equations describe the effects of pressure and viscous forces on the fluid, respectively. Through setting variables, a specific calculation equation that can describe flowfield separation and transition is given, and the position of the separation and transition can be calculated more accurately and simply, laying a foundation for the research of turbulence mechanism and solving theoretical problems.

Mathematically, the solution of the Navier-Stokes equation can be regarded as the sum of the average velocity and the fluctuation velocity. Therefore, it can be proved that the Navier-Stokes equations have a smooth solutions because the fluctuation velocity can be excited in a smooth manifold. However, it is still a problem to be proved whether the solutions of Navier-Stokes equations are in smooth domain.

In engineering, the determination of separation and transition position directly or indirectly affects the accuracy of test results, the feasibility of design results and the applicability of test standards, which is a very important problem. The theory of transition and separation theorems here can promote the solved problem obviously. And it can promote the development and improvement of active or passive flow control technology.

In the future, the study of dimensionless parameters $\beta$ and $\gamma$ can effectively help the development of nonlinear terms in fluid mechanics, especially find a statistical method to connect $\beta$ with Reynolds number $Re$, and connect $\gamma$ with Mach number $Ma$. This will be a new statistical method from the micro world to the macro world.

This is a new research direction. On the one hand, this direction will lead to the development of nonlinear unsteady flow such as separation, transition and shock wave, on the other hand, it also reveals the development and discussion of many related dimensionless coefficients, which is an important link between Euler method and Lagrangian method, and has positive significance in fluid mechanics. Axiomatic fluid mechanics system research
will make fluid mechanics research more rigorous, logical and quantitative. Hope to get more support and help from scholars.

XI. AUTHORS’ CONTRIBUTIONS

P.Y. conceived of the presented research idea, proposed and obtained mathematical model and analysis method; J.X. designed the experiment to verify mathematical model and analysis method; Other authors proof-read the mathematical model and constructed the experimental setup and performed the experiments.

ACKNOWLEDGMENTS

Thanks to the University of Electronic Science and Technology of China and China Aerodynamics Research and Development Center for providing excellent conditions for project. At same time, thanks for the financial support of Sichuan Province Expert Service Center (No. M162019LXHGKJJD18) and the Fundamental Research Funds for the Central Universities (No. A03019023801181) in the course of the project development.