Electronic Aharonov-Bohm Effect Induced by Quantum Vibrations

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Mechanical displacements of a nanoelectromechanical system (NEMS) shift the electron trajectories and hence perturb phase coherent charge transport through the device. We show theoretically that in the presence of a magnetic field such quantum-coherent displacements may give rise to an Aharonov-Bohm-type of effect. In particular, we demonstrate that quantum vibrations of a suspended carbon nanotube result in a positive nanotube magnetoresistance, which decreases slowly with the increase of temperature. This effect may enable one to detect quantum displacement fluctuations of a nanomechanical device.

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Following the ubiquitous trend of downsizing devices, micromechanical systems (MEMS) are today evolving into nanoelectromechanical systems (NEMS) rapidly approaching the limits set by the laws of quantum mechanics. The ultimate potential for nanomechanical devices is governed by the ability to detect the NEMS motional response to various external stimuli. In the quantum regime of operation optical and other sensing methods used in MEMS are not practical and one has turned instead to various mesoscopic sensing devices. Much work, including fundamental research, still needs to be done. For example, even though one has recently been able to detect flexural vibrations of a SIN beam resonator with the amazing sensitivity of $10^{-13}$ m using a radio-frequency single-electron transistor, this is still about 6 times the quantum limit set by the amplitude of the zero-point oscillations of the beam.

In this Letter we propose a different approach to sensing ultrasmall quantum vibrations of a beam — we have a suspended carbon nanotube in mind — and show that coherent tube vibrations can induce an effectively multi-connected electron path through the tube. Through an Aharonov-Bohm-type effect this in turn gives rise to a negative magneto-conductance that can be detected. We propose that this is but one example of how employing quantum coherence in both the electronic and mechanical degrees of freedom may lead to new functionality and novel applications.

Fundamental research on NEMS in the quantum coherent regime can profit from analogies with mesoscopic phenomena in confined structures, where the conductance may depend on quantum interference between electron waves. It is, e.g., well known that the conductance of a mesoscopic sample changes if a single impurity is displaced a small distance and that varying a magnetic field leaves a sample-specific “magnetic fingerprint” conductance pattern. We will show that a similar effect can be achieved by the mechanical displacement corresponding to quantum vibrations of a suspended carbon nanotube in the presence of a magnetic field. To this end, consider first the structure shown in Fig. 1(b). Here a beam of electrons can pass through two openings in an otherwise opaque screen. Interference between the quantum amplitudes for going through one or the other of the two holes will determine the probability for electrons to hit the detector. Obviously no such interference effect is expected with only one hole, as in Fig. 1(a). Even if the position of the hole in the screen were to move, no interference would occur if the trajectory of the hole is classically well defined. In every instant, only one classical trajectory would be relevant. This situation changes qualitatively if the hole motion is in the quantum regime, where its position cannot be precisely defined and “quantum alternative” paths for the electrons through the screen appear.

Figures 1(c) and (d) show two examples of such a “quantum geometry”, giving rise to electron paths through the hole that are multiply connected. Such an intimate, coherent nanoelectromechanical coupling between electronic and mechanical degrees of freedom leads to characteristic quantum interference among the electron paths and in particular to an Aharonov-Bohm-type of effect in the presence of an external magnetic field.

Due to their low mass and unique mechanical and electronic properties, single-wall carbon nanotubes (SWNTs) offer perhaps the best possibility for studying nanoelectromechanical phenomena. Figure 2 shows a sketch of the system we have in mind to achieve coherent coupling between quantum electronic transport and quantum flexural vibrations of a nanotube: a freely-hanging SWNT, doubly clamped to two metallic leads and subject to a transverse magnetic field, $H$. This system is described by the Hamiltonian,

$$\hat{H} = \hat{H}_{\text{leads}} + \hat{H}_{\text{el}} + \hat{H}_{\text{mech}} + \hat{H}_{\text{tunn}},$$

where the first term,

$$\hat{H}_{\text{leads}} = \sum_k \varepsilon_{\ell,k} \hat{a}_{\ell,k}^\dagger \hat{a}_{\ell,k} + \sum_k \varepsilon_{r,k} \hat{a}_{r,k}^\dagger \hat{a}_{r,k},$$
models electrons in states \( k \) in the left (\( \ell \)) and right (\( r \)) leads and \( \hat{a}^{\dagger}_{\ell/r,k} \) [\( \hat{a}_{\ell/r,k} \)] is the corresponding creation [annihilation] operator. The second term,

\[
\hat{H}_{el} = \int d^2 r \left\{ -\frac{\hbar^2}{2m} \hat{\psi}^{\dagger}(\vec{r}) \left( \frac{\partial^2}{\partial \vec{r}^2} - \frac{i e}{\hbar} \vec{A}(\vec{r}) \right) \hat{\psi}(\vec{r}) \right. \\
+ U(y - \hat{u}(x), z) \hat{\psi}^{\dagger}(\vec{r}) \hat{\psi}(\vec{r}) \left\}, \quad (3)
\]
describes the SWNT electrons, confined in the transverse direction by a potential \( U(y, z) \) that depends on the deflection \( u(x) \) of the tube (in the \( y \)-direction). The operator \( \hat{\psi}^{\dagger}(\vec{r}) [\hat{\psi}(\vec{r})] \) creates [annihilates] an electron at \( \vec{r} = (x, y, z) \); \( \{ \hat{\psi}^{\dagger}(\vec{r}), \hat{\psi}(\vec{r}) \} = \delta(\vec{r} - \vec{r}') \) and \( \vec{A}(\vec{r}) = (-Hy, 0, 0) \).

The bending of the tube is modelled by the third term in the Hamiltonian (1) as

\[
\hat{H}_{\text{mech}} = \int_{-L/2}^{L/2} dx \left\{ \frac{1}{2\rho} \dot{\pi}^2(x) + \frac{EI}{2} \left( \frac{\partial^2 \hat{u}(x)}{\partial x^2} \right)^2 \right\}. \quad (4)
\]

Here \( \dot{\pi}(x) \) is the momentum density operator conjugate with the deflection field operator \( \hat{u}(x) \), i.e. \( [\hat{u}(x), \hat{\pi}(x')] = i\hbar\delta(x - x') \), \( \rho \) is the linear mass density of the SWNT, \( I \) is its area moment of inertia and \( E \) is the Young’s modulus.

The tube is doubly clamped, which gives the boundary conditions \( u(x) = 0 \) and \( u'(x) = 0 \) for \( |x| \geq L/2 \).
By analogy one may think of the elementary excitations created by \( \hat{p}^\dagger(x) \) in the transformed Hamiltonian as polarons. It is important that due to the quantum vibrations of the nanotube the wavefunction of this polaronic state is extended in the direction perpendicular to the the tube axis. These “swinging states” serve as intermediate states for electrons tunneling through the vibrating SWNT. The quantum phase acquired by the tunneling electrons depends on magnetic field and below we will show that a nanomechanically induced magneto-conductance may follow.

So far our approach has been quite general, but here we will make some approximations that allow us to find an analytical solution. First, we consider the weak coupling limit, where the spacing \( \Delta E \approx v_F \hbar L_0 \) of quantized energies associated with the longitudinal motion of electrons along the SWNT of total length \( L_0 \) is much larger than the level broadening caused by the coupling to the continuous spectrum of energy levels in the leads (\( v_F \) is the Fermi velocity). In this case only a few longitudinal states with energy \( \varepsilon_n \) are relevant. Further simplifications arise if one restricts the mechanical SWNT dynamics to the fundamental bending mode, which gives the most important contribution, and expresses the corresponding deflection operator as \( \hat{u}(x) = Y_0 u_0(x/L)(\hat{b}^\dagger + \hat{b})/\sqrt{2} \), where \( u_0(\xi) \) is the normalized profile of the fundamental bending mode, \( Y_0 = (\hbar^2 L^2/\beta_0 \rho E) \) is the amplitude of zero-point fluctuations in the fundamental mode, \( \beta_0 \) is the dimensionless eigenvalue associated with this mode and \( \hat{b}^\dagger [\hat{b}] \) is a boson operator that creates [annihilates] one vibration quantum.

Next we consider a non-resonant case where no electron energy level in the SWNT is close enough to the chemical potential \( \mu \) in the leads to be involved in even inelastic electron tunneling via a real state on the tube. This requires the criteria \( |\varepsilon_n + \varepsilon_1 - \mu| \gg \nu|T_{ir}|^2, \hbar \omega, eV \) to be fulfilled, where \( \nu \) is the electron density of states in the leads, \( \omega = (\beta_0 I E/\rho)^{1/2} / L^2 \) is the eigenfrequency of the fundamental mode and \( V \) is the applied bias voltage. Under such conditions the coupling between electronic states in the left and right leads occurs via virtual states on the tube and may be described by an overlap integral. As a result one comes to a tunneling Hamiltonian,

\[
\hat{H}_{\mathrm{eff}} = \sum_{k,\sigma = \uparrow, \downarrow} \varepsilon_{\sigma, k} \hat{a}^\dagger_{\sigma, k} \hat{a}_{\sigma, k} + \hbar \omega \hat{b}^\dagger \hat{b} + \sum_{k, k', \sigma = \uparrow, \downarrow} \langle \hat{T}_{\mathrm{eff}}(k, k') \rangle \hat{a}^\dagger_{\sigma, k'} \hat{a}_{\sigma, k} + \h.c.,
\]

that describes non-resonant charge transfer processes through the suspended SWNT. In this equation \( \phi = 4\pi Y_0 \hbar L H / \Phi_0 \) is a dimensionless magnetic flux, \( \Phi_0 = hc/e \) is the flux quantum, \( g = (\pi / \sqrt{2}) \int d\xi u_0(\xi) \) is a geometric factor of order unity, and \( \langle \hat{T}_{\mathrm{eff}}(k, k') \rangle = \sum_n \hat{T}_{n, \ell/k}(k)\hat{T}_{n, \ell/k}^\dagger(k')/(\mu - \varepsilon_1 - \varepsilon_n) \) is the effective overlap integral [\( \hat{T}_{n, \ell/k}(k) = \int dx T_{\ell/k}(x, k)|\psi_n(x)|^2 \)]. Below we will use Eq. \( \text{6} \) to analyze the magneto-conductance of the system shown in Fig. 2. In doing so we assume that the overlap integral \( T_{n, \ell/k}(k, k') \) does not depend on the momenta \( k \) and \( k' \).

It is clear from the effective Hamiltonian \( \text{6} \) that in the presence of a magnetic field electronic transport through the SWNT may be accompanied by inelastic processes. Although in principle these may drive the vibration modes out of thermal equilibrium, we will assume here that the coupling to the thermal bath is strong enough for this not to happen. Under this condition the current \( I \) through the SWNT is to leading order in the tunneling coupling given by the expression

\[
I = G_0 \sum_{n=0}^{\infty} \sum_{l=-n}^{n} P(n)|\langle e^{i\phi(\hat{b}^\dagger + \hat{b})}| n + l \rangle|^2 \times
\int \! dc \left[ f_i(c)(1 - f_i(c - \hbar \omega)) - f_i(c)(1 - f_i(c - \hbar \omega)) \right].
\]

Here \( G_0 = (2e/h)\nu|T_{\mathrm{eff}}|^2 \) is the zero-field conductance, \( P(n) = (1 - e^{-\beta_0 \hbar \omega})e^{-\beta_0 \hbar \omega} \), where \( \beta \equiv 1/\kappa B T \), is the probability that the fundamental mode is in state \( |n \rangle \) with energy \( n\hbar \omega \), and \( f_i(c) \equiv (1 + e^{(c - \mu)/\beta_0 \hbar \omega})^{-1} \) are Fermi distribution functions in the leads. The restrictions on the tunneling channels imposed by the Pauli principle are crucial; it easy to see from Eq. \( \text{6} \) that if all electron states in the right lead were empty it would follow from the completeness of the set of vibronic states that there is no effect of the magnetic field.

Finally, after integrating over the electron energy \( \varepsilon \) in Eq. \( \text{6} \) and using the completeness of the set of vibronic states, \( \sum_{n}|\langle e^{i\phi(\hat{b}^\dagger + \hat{b})}| n \rangle|^2 = 1 \), one arrives at the final result for the linear conductance \( G \),

\[
\frac{G}{G_0} = \sum_{n=0}^{\infty} \sum_{l=1}^{\infty} \frac{P(n)|\langle e^{i\phi(\hat{b}^\dagger + \hat{b})}| n + l \rangle|^2 \left. 2\beta_0 \hbar \omega \right| e^{\beta_0 \hbar \omega} - 1}. \]

The first term in this expression gives the contribution to the conductance from the elastic tunneling processes. The second term is due to the inelastic processes, which contribute at finite temperature.

Equation \( \text{7} \) has to be evaluated numerically, but the asymptotic forms in the limits of high and low temperature,

\[
\frac{G}{G_0} = \begin{cases} 
1 - \frac{\hbar \omega}{k_B T} \left( \frac{2eY_0 L H}{\Phi_0} \right)^2, & \frac{\hbar \omega}{k_B T} \ll 1, \phi \gtrsim 1 \\
\left( 1 - \frac{2eY_0 L H}{\Phi_0} \right)^2, & \frac{\hbar \omega}{k_B T} \gg 1.
\end{cases}
\]

follow readily from the identities \( \langle 0|e^{i\phi(\hat{b}^\dagger + \hat{b})}|0 \rangle = e^{\phi^2/2} \) and \( \sum_{n=0}^{\infty} (n^2 - n)|\langle n|e^{i\phi(\hat{b}^\dagger + \hat{b})}| n \rangle|^2 = \phi^2 \), respectively.

It is instructive to express the probability amplitude \( A_n = \langle n|e^{i\phi(\hat{b}^\dagger + \hat{b})}| n \rangle \) for elastic transitions involving the
As more and more inelastic channels are switched on with the strong nonlinearity of the magneto-conductance. As we see Eq. (10). A complete analysis has to account for the elastic tunneling amplitude corresponds to quasiclassical trajectories passing through points where the probability to “find” the SWNT is maximal, while electronic trajectories passing through nodes of the SWNT wave functions do not contribute. This can be interpreted as an effective multi-connectivity of the system. We note, however, that this interpretation is strictly valid only in the limit of vanishing temperature and bias voltage.

Figure 3 shows that quantum oscillations of a suspended SWNT result in a positive magnetoresistance. The most striking feature of this electronic conduction effect is its temperature dependence. It comes from the dynamics of the entire nanotube, rather than from the electron dynamics. Consequently, the characteristic temperature scale of the resulting magnetoresistance — which decays as $T^{-1}$ for $T \gg h\omega$, where $\omega$ is the “swinging” frequency of the SWNT — is anomalously small, see Eq. (10). A complete analysis has to account for the strong nonlinearity of the magneto-conductance. As more and more inelastic channels are switched on with increasing bias voltage $V$, the influence of the magnetic field is suppressed in the same way as by temperature. As a result the change in current due to a magnetic field $H$ does not exceed $|G - G_0| h \omega / e$ as $eV$ becomes larger than the vibration quantum $h\omega$. Hence, in order to easily detect the magneto-conductance effect discussed here, one needs to be in the ballistic transport regime, where $G \approx G_0 = 2e^2/h \sim 10^{-4}$ mho. For a 1 $\mu$m long SWNT at $T = 30$ mK and $H \sim 20 - 40$ T we find from Eq. (10) a relative conductance change $|G - G_0|/G_0$ of about 1-3%, which corresponds to a maximal magneto-current $I(H) - I(0)$ of 0.1-0.3 pA. Its characteristic dependence on temperature and transverse magnetic field makes this effect readily distinguishable from the recently observed SWNT magneto-conductance in a longitudinal field.

In conclusion we have shown that quantum vibrations of a current-carrying suspended single-wall carbon nanotube (SWNT) subject to a transverse magnetic field couple electronic and vibronic states — forming what we call “swinging states” — in such a way that a negative magneto-conductance can follow. This effect may be used to detect nanomechanical SWNT vibrations in the quantum regime. Other effects due to electron-phonon coupling can be expected to occur in mesoscopic rings and in superconducting Josephson junctions containing nanomechanical elements. A possible consequence of the coupling to the macroscopic phase of a superconductor is the prospect of using SQUIDs to detect nanomechanical displacements in the quantum limit.

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