Population of dipole states via isoscalar probes: the splitting of pygmy dipole resonance in $^{124}$Sn

E. G. Lanza,$^{1,2}$ A. Vitturi,$^{3,4}$ E. Litvinova,$^{5,6}$ and D. Savran$^{7,8}$

$^1$INFN Sezione di Catania, Catania, Italy
$^2$Dipartimento di Fisica e Astronomia, Università di Catania, Italy
$^3$INFN Sezione di Padova, Padova, Italy
$^4$Dipartimento di Fisica e Astronomia, Università di Padova, Italy
$^5$Department of Physics, Western Michigan University, Kalamazoo, MI 49008-5252, USA
$^6$National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824-1321, USA
$^7$ExtreMe Matter Institute EMMI and Research Division, GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany
$^8$Frankfurt Institute for Advanced Studies, Frankfurt am Main, Germany

Inelastic $\alpha$-scattering excitation cross sections are calculated for electric dipole excitations in $^{124}$Sn based on the transition densities obtained from the relativistic quasiparticle time-blocking approximation (RQTBA) in the framework of a semiclassical model. The calculation provides the missing link to directly compare the results from the microscopic RQTBA calculations to recent experimental data measured via the $(\alpha,\alpha')\gamma$ reaction, which show a structural splitting of the low-lying $E1$ strength often denoted as pygmy dipole resonance (PDR). The experimentally observed splitting is reproduced by the cross section calculations, which allows to draw conclusion on the structure of the PDR.

The study of nuclei with neutron excess have been pursued in the last years with focus to the properties of collective states. Special attention has been devoted to the electric dipole strength at low excitation energy, the so-called Pygmy Dipole Resonance (PDR), both in theory and as experimentally. While the PDR represents an interesting new nuclear phenomenon itself, the connection of the (low-lying) $E1$ strength to the neutron skin of atomic nuclei as well as to isovector parameters in the equation of state of nuclear matter as well as its possible importance for reaction rates in astrophysical scenarios has further increased the interest in the PDR region. Different microscopic models predict the presence of the low-lying PDR states below the well-known (isoscalar) electric giant dipole resonance (IVGDR), see and references therein. Most calculations show similar results for the transition densities of the low-lying $E1$ strength, which is often used to “define” the PDR states: The neutron and proton transition densities are in phase inside the nucleus and at the surface only the neutron part considerably contributes. The investigation of the PDR by means of isovector (IV) as well as isoscalar (IS) probes provides information on this structure of the involved transition densities, which show a strong mixing of isospin character. While for isovector probes (i.e. photons) data exist for various nuclei obtained mainly in nuclear resonance fluorescence (NRF) and Coulomb excitation (for a recent overview see ), only recently data on the PDR using an isoscalar probe became available . Applying the $(\alpha,\alpha')\gamma$ coincidence method in combination with high-resolution $\gamma$-ray spectroscopy allowed to study the PDR for several semi magic stable nuclei in $\alpha$-scattering experiments. The comparison to results from NRF revealed a surprising splitting of the PDR strength below the particle threshold into two groups: the lower lying group of states is excited by both isoscalar and isovector probes while the states at higher energy are excited by photons only. A first qualitative comparison to calculations within the relativistic quasiparticle time-blocking approximation (RQTBA) as well as quasi-particle phonon model (QPM) has shown good agreement to the experimental observation . While the low-lying part of the $E1$ strength shows the described pattern of transition densities, which lead to an enhancement in the isoscalar $E1$ response, the higher lying states are of transition character towards the GDR and, thus, are suppressed in the isoscalar channel. Meanwhile, similar IS/IV energy behaviour of the low-lying $E1$ strength has been reported experimentally in inelastic scattering of $^{17}$O off $^{208}$Pb at low bombarding energies as well as in other microscopic model calculations for different nuclei, see e.g. . However, the comparison to the experimental data lacks on the isoscalar part, since the calculated IS $B(E1)$ strength cannot be directly compared to the measured $\alpha$-scattering cross sections. The comparison thus remained on the “qualitative level”. In this manuscript we present calculation of $\alpha$-scattering cross sections, based to the microscopic transition densities obtained in the RQTBA, within the framework of a semiclassical model to overcome this drawback in the comparison of experiment and calculation.

The relation between the inelastic cross section and the reduced transition probabilities $B(E1)$ is clear for the Coulomb excitation or NRF (they are proportional) while it is not so evident in the relation between the isoscalar response and the inelastic excitation cross section due to an isoscalar probe. In this case it is better to cal-
ulate explicitly the inelastic cross section due to the nuclear interaction in order to establish its connection with the isoscalar transition probability. Recently, calculation along this line have been performed [22, 28] within a semiclassical model. Here we present the result for the system $\alpha + ^{124}\text{Sn}$ at $E_\alpha = 136$ MeV which have been experimentally studied in refs. [19, 21].

The calculation of the inelastic cross sections are performed within a semiclassical model. The basic assumption is that the colliding nuclei move on a classical trajectory determined by the real part of the optical potential, while the internal degrees of freedom are described quantum mechanically. This model is known to hold for heavy-ion grazing collisions. In our specific case we consider the alpha particle projectile scattered off by $^{124}\text{Sn}$ at $E_\alpha = 136$ MeV and analyze the excitation process of the target. Under these conditions, we can write the Hamiltonian as $H_T = H_T^0 + W(t)$ where $H_T^0$ is the internal hamiltonian of the target and the external field $W$ describes the excitation of $T$ by the mean field of the projectile nucleus, whose matrix elements depend on time through the relative coordinate $R(t)$. By solving the Schrödinger equation in the space spanned by the eigenstates of the internal hamiltonian $|\Phi_\alpha >$ one can calculate, non perturbatively, the final population for each of the $|\Phi_\alpha >$ states. Then the time dependent state, $|\Psi(t) >$, of the target nucleus can be expressed as

$$|\Psi(t) > = \sum_\alpha A_\alpha(t)e^{-iE_\alpha t}|\Phi_\alpha >$$

where the ground state is also included in the sum as the term $\alpha = 0$. The Schrödinger equation can be cast into a set of linear differential equations for the amplitudes $A_\alpha(t)$,

$$A_\alpha(t) = -i\sum_{\alpha'} e^{i(E_\alpha - E_{\alpha'})t} <\Phi_\alpha |W(t)|\Phi_{\alpha'} > A_{\alpha'}(t). \quad (2)$$

Their solutions are then used to construct the probability of exciting the internal state $|\Phi_\alpha >$ as

$$P_\alpha(b) = |A_\alpha(t = +\infty)|^2 \quad (3)$$

for each impact parameter $b$. Finally, the total cross section is obtained

$$\sigma_\alpha = 2\pi \int_0^{+\infty} P_\alpha(b)T(b)bdb. \quad (4)$$

by integrating $P_\alpha$ over the whole range of the impact parameters. The integral is modulated by the transmission coefficient $T(b)$ which takes into account processes not explicitly included in the model space and that take flux away from the elastic channel. A standard practice is to construct it by integrating the imaginary part of the optical potential associated to the studied reaction. When the imaginary part is not available from the experimental data we use the simple assumption of taking it as half of the real part.

The internal structure of the nucleus $^{124}\text{Sn}$ is provided by the relativistic quasiparticle time blocking approximation developed in refs. [24, 31] and based on the covariant density functional with NL3 parametrization [31]. This approach is a self-consistent extension of the relativistic quasiparticle random phase approximation [32] accounting for the quasiparticle-vibration approximation. The details of the calculations for $^{124}\text{Sn}$ are given in ref. [23]. The RQTBA calculation scheme has been approved in various applications and justified by the full self-consistency and renormalization technique. The phonon space is truncated by the angular momentum of the phonons at $J^p = 6^+$ and by their frequencies at 15 MeV. The two-isoscalar space is sufficiently large to provide decoupling of the translational spurious mode from the physical ones. Coupling to vibrations within the RQTBA does not affect this mode due to the subtraction of the static contribution of the particle-vibration coupling (PVC) amplitude. This subtraction also removes double counting of the PVC effects from the residual interaction, guarantees the stability of the solutions for the response function and provides fast convergence of the renormalized PVC amplitude [33]. Here we perform a so-called bunching procedure which is illustrated in Fig. 1 (a) for the isoscalar dipole strength distribution. The strength obtained with the smearing parameter $\Delta = 20$ keV is transformed into the reduced transition probabilities via the relation: $B^{\nu} = \pi\Delta S(\nu, \Delta)$, where $S(\nu, \Delta)$ is the value of the strength at the $\nu$-th maximum. The bunched states obtained in this way accumulate the transition probabilities of all the states in $\approx 40$ keV bins. The distributions of the probabilities obtained by this procedure are displayed in Fig. 1 (b,c) for the isoscalar and electromagnetic transitions, respectively. The transition densities of these bunched states are determined as described in Ref. [24] and will be used in the subsequent cross section calculations.

The real part of the optical potential, which together with the Coulomb interaction determines the classical trajectory, is constructed with the double folding procedure [35], as well as the nuclear formfactors by double folding the RQTBA transition densities. In both cases the nucleon nucleon interaction has been chosen to be the M3Y-Reid type [36]. Since the excitation is produced by a probe which is essentially isoscalar and with $N=Z$, only the isoscalar part of the nucleon nucleon interaction and the isoscalar densities and transition densities contribute to the potential and to the formfactors. More details are given in ref. [23].

For the given conditions the important part of the inelastic excitation at $E_\alpha = 136$ MeV is due to the nuclear interaction. Nevertheless, the small contribution due to the Coulomb interaction is important because of interference effects. A comparison of pure nuclear and coulomb cross section as well as total cross section including the interference for the RQTBA bunched states is given in Fig. 3. The contribution of the Coulomb interaction, panel (b), is small as compared to the cross section gen-
FIG. 1: (Color online) Bunching procedure for the strength distribution calculated within RQTBA (a) and the obtained isoscalar (b) and electromagnetic (c) reduced transition probabilities of the bunched states.

FIG. 2: (Color online) Inelastic cross sections as function of the excitation energy for the systems $\alpha + {^{124}\text{Sn}}$ at $E_{\alpha} = 136$ MeV. The calculations are performed when only the nuclear (panel (a)) and Coulomb (panel (b)) interaction is switched on. The results when both interactions are working are in panel (c).

The gross features of the strength distributions (see Fig. 1) is retrieved in the cross section calculation both for the isoscalar and isovector cases. Indeed, while we know that this is mathematically true for the Coulomb case, it is not clear that the same is actually correct for the nuclear interaction. One can verify it by performing a calculation by putting the energies of the states to zero. This eliminates the contributions due to the dynamic of the reaction, such as the Q-value effect, and, at least for the Coulomb case should produce a cross section which is proportional to the $B_{em}(EL)$ values.

Indeed, in panel (b) of Fig. 2 this is shown by plotting for each state the ratio between the Coulomb inelastic cross section and the $B_{em}(EL)$ value, calculated by putting to zero the energy of the corresponding state. As it is expected, the ratio is constant for all the states. Conversely, in the case of nuclear excitation the ratios do not have a constant value, as presented in Fig. 2 (a). This corroborate the fact that for the nuclear excitation the individual cross section depends on the characteristics of the transition densities and one has to make calculations of the sort presented here in order do gain useful information on the excitation process.

So far the presented cross sections are implicitly integrated over the full solid angle. However, the experimental data of refs. are taken at the angle range from about $1.5^\circ$ to $5.5^\circ$ corresponding to about $1.53^\circ$ to $5.94^\circ$ in the c.m. system. From the deflection function...
shown in Fig. 4, one can deduce the ranges of impact parameters whose corresponding trajectories will end up to the experimental $\alpha$ scattering angle range. As given in Fig. 4, only the impact parameters between 8.2 fm to 9.0 fm and between 13.3 fm to 52.0 fm give contributions within the experimental angular range. Therefore, in order to be consistent with the experiment, the calculations have been repeated taking into account only the above given impact parameters ranges. The results are shown in Fig. 5. Apart from the scale and a small variation, the general features of the three considered cross sections distributions are maintained and the decrease of the cross section for energies above 8 MeV is retrieved both for the nuclear as well as for the total cross sections.

A comparison of the final calculated cross section based on the RQTBA and the experimental results for $^{124}$Sn is presented in Fig. 6. In order to compare to the differential cross section measured in the experiment the calculations have been normalized to the corresponding solid angle. Also given are the corresponding $B_{em}(E1)$ distributions. In both cases the strongest states are about a factor of ten stronger in the RQTBA compared to the experiment. This difference has the following reason. By construction, the fragmentation in the conventional RQTBA is not sufficient for a state-by-state description of the dipole strength in this energy region and coupling to higher configurations have to be included. The overall 800 keV shift of the RQTBA strength distributions has the same origin: all the strength in this energy region consists of fragments of the RQRPA pygmy mode located at about 9 MeV, and higher-order correlations are

![FIG. 3: (Color online) Ratios between the inelastic cross sections, calculated by putting to zero the energies of the states, to their corresponding $B(EL)$. The two panels show the results for the nuclear (panel (a)) and Coulomb (panel(b)) excitations. The four black lines correspond to the four dipole states in fig. 1 (a) with the bigger response to the isoscalar probe.](image1)

![FIG. 4: (Color online) Deflection function for the system $\alpha + ^{124}$Sn at $E_\alpha=136$ MeV. The horizontal dashed lines delimit the experimental $\alpha$ scattering angle range. The vertical solid line determine the corresponding impact parameters ranges.](image2)

![FIG. 5: (Color online) Same as fig. 2, in this case only the impact parameters intervals whose corresponding trajectories terminate to the measured scattering angle range are taken into account.](image3)
expected to reinforce the fragmentation to lower energies. In addition, the bunching procedure described above results in the strength distributions of 40 keV bins rather than of the single states. However, the similar enhanced factor of about ten that we have for the cross sections and the \( B_{\text{em}}(E1) \) values for the individual states shows that the calculations for the cross sections are consistent and the under-prediction of the fragmentation has no influence on the gross features of the two distributions. Integrating the total cross section up to 8.7 MeV results in 43.6 mb/sr for the RQTBA and 44.1(5) mb/sr for the experiment when including also the contribution of the continuum (see [19], Fig. 3). The cumulative sums of these quantities are displayed in Fig. 7 showing that the experimental and theoretical total cross section, aside from the global energy shift, are in excellent agreement, which further supports the validity of our calculation on the absolute scale. However, one has to keep in mind that some details of the calculations may depend on the uncertainties that are connected with the use of the semiclassical model and the associated parameters as well as of the M3Y-Reid potential.

Looking on the dependence of the strength distribution on excitation energy a good agreement of data and calculation is observed, i.e. in both cases the \( \alpha \)-scattering cross section is strongly reduced at higher excitation energies compared to the isovector channel. This is in agreement to the qualitative comparison previously given in Ref. [19]. The proper calculation of the \( \alpha \)-scattering excitation cross section, thus, confirms the good agreement between the experimentally observed splitting of the low-lying \( E1 \) strength and the results of the RQTBA calculation. However, we are aware of the fact that the NL3 interaction used in the microscopic structure calculations produce a neutron skin which is bigger with respect to the experimental data. How this fact can affect the calculated cross section is not easy to find out and it is out of the scope of this work. Further work has to be dedicated to this delicate problem. However, our calculations show, beyond any doubt, that the same separation is found in the cross section calculations making in this way the comparison between experiment and theory straightforward.

In summary, we have presented the calculation of \( \alpha \)-scattering excitation cross sections for \( J^\pi = 1^- \) states based on the results of microscopic calculation in the RQTBA model using a semiclassical framework for the reaction. The aim of this work is to verify that the excitation cross section due to an isoscalar probe reproduces the structure of the theoretical isoscalar \( B(E1) \), whose comparison with the experimental data was done in a previous work [19] only on a qualitative level. We have shown that the comparison between experimental and theoretical cross sections confirms the different behaviour of the population of the low lying dipole states with different probes. In the calculations nuclear-Coulomb interference is included and the resulting cross sections are sensitive to the character of the transition densities as expected. The calculations allow for a direct comparison of the RQTBA model to experimental results obtained in inelastic \( \alpha \)-scattering on an absolute scale. The good agreement to the data confirms the accurate description of the corresponding transition densities of the low-lying \( E1 \) strength in the RQTBA model, which shows for the low-lying group of \( E1 \) excitations enhanced neutron contribution on the surface of the nucleus and an isoscalar behavior in the interior. The combination of the presented calculations and the experimental data, thus, provides a first clear identification of this signature often associated to the pygmy dipole resonance.

**Acknowledgments**

E.L. acknowledges support from the US-NSF grant PHY-1204486 and from NSCL; D.S. acknowledges support by the Alliance Program of the Helmholtz Association (HA216/EMMI); E.G.L. and A.V. acknowledge support by the italian PRIN (grant 2009TWL3MX).
[1] N. Paar, D. Vretenar, E. Khan, and G. Colò, Rep. Prog. Phys. 70, 691 (2007).
[2] D. Savran, T. Aumann, and A. Zilges, Prog. Part. Nucl. Phys. 70, 210 (2013).
[3] J. Piekarewicz, Phys. Rev. C 73, 044325 (2006).
[4] A. Klimkiewicz, N. Paar, P. Adrich, M. Fallot, K. Boretzky, T. Aumann, D. Cortina-Gil, U. D. Pramanik, T. W. Elze, H. Emling, et al. (LAND Collaboration), Phys. Rev. C 76, 051603 (2007).
[5] N. Tsoneva and H. Lenske, Phys. Rev. C 77, 024321 (2008).
[6] N. Paar, Y. F. Niu, D. Vretenar, and J. Meng, Phys. Rev. Lett. 103, 032502 (2009).
[7] A. Carbone, G. Colò, A. Bracco, L.-G. Cao, P. F. Bortignon, F. Camera, and O. Wieland, Phys. Rev. C 81, 041301 (2010).
[8] J. Piekarewicz, Phys. Rev. C 83, 034319 (2011).
[9] D. Vretenar, Y. F. Niu, N. Paar, and J. Meng, Phys. Rev. C 85, 044317 (2012).
[10] H. Nakada, T. Inakura, and H. Sawai, Phys. Rev. C 87, 034302 (2013).
[11] A. Repko, P.-G. Reinhard, V. O. Nesterenko, and J. Kvasil, Phys. Rev. C 87, 024305 (2013).
[12] P.-G. Reinhard and W. Nazarewicz, Phys. Rev. C 81, 051303 (2010).
[13] A. Tamii, I. Poltoratska, P. von Neumann-Cosel, Y. Fujita, T. Adachi, C. A. Bertulani, J. Carter, M. Dozono, H. Fujita, K. Fujita, et al., Phys. Rev. Lett. 107, 062502 (2011).
[14] J. Piekarewicz, B. K. Agrawal, G. Colò, W. Nazarewicz, N. Paar, P.-G. Reinhard, X. Rocca-Maza, and D. Vretenar, Phys. Rev. C 85, 041302 (2012).
[15] S. Goriely, in Nuclear Astrophysics, edited by M. Buballa, W. Nörenberg, J. Wambach, and A. Wirzba (GSI, Darmstadt, 1998), pp. 320-325.
[16] E. Litvinova, H.P. Loens, K. Langanke, G. Martinez-Pinedo, T. Rauscher, P. Ring, F.-K. Thielemann, and V. Tselyaev, Nucl. Phys. 823, 26 (2009).
[17] D. Savran, A. M. van den Berg, M. N. Harakeh, J. Hasper, A. Matić, H. J. Wörthc, and A. Zilges, Phys. Rev. Lett. 97, 172502 (2006).
[18] J. Endres, D. Savran, A. M. van den Berg, P. Dendooven, M. Fritzschc, M. N. Harakeh, J. Hasper, H. J. Wörthc, and A. Zilges, Phys. Rev. C 80, 034302 (2009).
[19] J. Endres, E. Litvinova, D. Savran, P. A. Butler, M. N. Harakeh, S. Harissopulos, R.-D. Herzberg, R. Krücken, A. Lagoyannis, N. Pietralla, et al., Phys. Rev. Lett. 105, 212503 (2010).
[20] J. Endres, D. Savran, P. A. Butler, M. N. Harakeh, S. Harissopulos, R.-D. Herzberg, R. Krücken, A. Lagoyannis, E. Litvinova, N. Pietralla, et al., Phys. Rev. C 85, 064331 (2012).
[21] V. Derya, J. Endres, M. Elvers, M. Harakeh, N. Pietralla, C. Romig, D. Savran, M. Scheck, F. Siebenhüner, V. Stoica, et al., Nucl. Phys. 906, 94 (2013).
[22] D. Savran, A. M. van den Berg, M. N. Harakeh, K. Rambeck, H. J. Wörthc, and A. Zilges, Nucl. Instr. and Meth. Phys. Res. A 564, 267 (2006).
[23] A. Bracco and F. C. L. Crespi, EPJ Web of Conferences 38, 03001 (2013).
[24] X. Roca-Maza, G. Pozzi, M. Brenna, K. Mizuyama, and G. Colò, Phys. Rev. C 85, 024601 (2012).
[25] A. Vitturi, E. G. Lanza, M. Andrés, F. Catara, and D. Gambacurta, PRAMANA 75, 73 (2010).
[26] A. Vitturi, E. G. Lanza, M. Andrés, F. Catara, and D. Gambacurta, J. Phys.: Conf. Ser. 267, 012006 (2011).
[27] E. G. Lanza, A. Vitturi, M. V. Andrés, F. Catara, and D. Gambacurta, Nuclear Theory, Vol. 29, 162 (2010), ISSN 1313-2822, edited by A. Georgieva and N. Minkov (Heron Press, Sofia, 2010).
[28] E. G. Lanza, A. Vitturi, M. V. Andrés, F. Catara, and D. Gambacurta, Phys. Rev. C 84, 064602 (2011).
[29] E. Litvinova, P. Ring, and V. Tselyaev, Phys. Rev. C 78, 014312 (2008).
[30] E. Litvinova, P. Ring, V. Tselyaev, and K. Langanke, Phys. Rev. C 79, 054312 (2009).
[31] G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C 55, 540 (1997).
[32] N. Paar, P. Ring, T. Nikšíć, and D. Vretenar, Phys. Rev. C 67, 034312 (2003).
[33] V. I. Tselyaev, Phys. Rev. C 88, 054301 (2013).
[34] E. Litvinova, P. Ring, and V. Tselyaev, Phys. Rev. C 75, 064308 (2007).
[35] G. R. Satchler and W. G. Love, Phys. Rep. 55, 183 (1979).
[36] G. Bertsch, J. Borysowicz, H. McManus, and W. G. Love, Nucl. Phys. A284, 399 (1977).
[37] F. Catara, E. G. Lanza, M. A. Nagarajan, and A. Vitturi, Nucl. Phys. A624, 449 (1997).
[38] K. Govaert, F. Bauwens, J. Bryssinck, D. De Frenne, E. Jacobs, W. Moneelaers, L. Govor, and V. Y. Ponomarev, Phys. Rev. C 57, 2229 (1998).