Entangler via electromagnetically induced transparency with an atomic ensemble

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Quantum entanglement plays an essential role in quantum information processing and quantum networks. One of the commonly-used methods to generate multiple entangled fields is to employ polarizing beam splitters. However, nonclassical input light fields are required and the generated entangled fields are always degenerate in such case. Here, we present a proof-of-principle demonstration of an efficient and convenient way to entangle multiple light fields via electromagnetically induced transparency (EIT) in an atomic ensemble. The atomic spin wave, produced through EIT in the A-type atomic system, can be described by a Bose operator and can act as an entangler. With such an entangler, any desired number of nondegenerate narrow-band continuous-variable entangled fields, in principle, can be generated through stimulated Raman scattering processes. This scheme holds great promise for applications in scalable quantum communication and quantum networks. PACS: 42.50.Gy, 03.67.Bg, 42.50.Dv, 42.65.Lm.

Quantum state exchange between light and matter is a basic component for quantum interface in quantum information processing. As is well known, light is the best long-distance quantum information carrier and the atomic ensembles can provide the promising tools for quantum information manipulation and storage. In facilitating quantum information processing and quantum networks, generations of light-light, atom-atom, and atom-light multipartite entanglements play essential roles in the implementations of quantum information protocols. So far, the majority studies on entanglement have dealt with the generations of multiple entangled light fields. Apart from the conventional way of generating multipartite entanglement by mixing squeezed fields created through parametric down-conversion processes in nonlinear optical crystals with linear optical elements, i.e., polarizing beam splitters (PBS) as entanglers, the atomic ensembles provide an alternative avenue to the generation of multi-entangled fields due to the virtue of narrow bandwidth, nondegenerate frequencies, and long correlation time. Based on the seminal proposal of Duan et al., the electromagnetically induced transparency (EIT)-based double-A-type atomic system has been actively implemented for efficiently creating nondegenerate entangled twin fields through either nondegenerate four-wave mixing (FWM) or Raman scattering processes. The multicolor multipartite continuous-variable (CV) entanglement has also been achieved by using the multi-order coherent Raman scattering or multiple nondegenerate FWM processes. Moreover, generations of entanglements between an atomic ensemble and light fields, as well as between two atomic ensembles, have also been realized, which are vital for storage and processing of quantum information.

In this study, we propose an efficient and convenient scheme for quantum entangler via EIT in an atomic ensemble. The atomic spin wave, which can be described by a Bose operator and acts as the entangler, is produced through EIT in the A-type atomic configuration. Through stimulated Raman scattering processes, nondegenerate narrow-band multi-entangled fields up to any desired number can, in principle, be achieved via such an entangler. This proposed entangler is quite distinct as compared to the conventional PBS entangler, since only coherent input light fields are needed for generating multipartite entanglement in the present scheme, whereas nonclassical input light fields are required when using the PBS entangler. Moreover, the present proposal is different from the previously proposed ones as given in Refs. 13, 14, 18. Here, the entanglement features between the atomic ensemble and the generated fields are investigated, and it is shown that under the EIT condition the generated atomic spin wave can serve as an entangler. Also, this proposal is different from the one proposed in Ref. 8, since, in principle, any desired number of entangled fields can be produced through this entangler.

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Received 12 August 2013
Accepted 26 November 2013
Published 11 December 2013

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**Results**

**Mechanism of excitations.** The considered model is shown in Fig. 1a, where energy levels |1⟩, |2⟩, and |3⟩, forming the three-level A-type system, correspond, respectively, to the ground-state hyperfine levels 5S_{1/2} (F = 2), 5S_{1/2} (F = 3), and the excited state 5P_{1/2} in D_1 line of the ^85Rb atom with the ground-state hyperfine splitting of 3.036 GHz. A strong coupling field E_c (with frequency ω_c and Rabi frequency Ω_c) and a relatively weak probe field E_p (with frequency ω_p and Rabi frequency Ω_p) are tuned to be on resonance with the transitions |2⟩–|3⟩ and |1⟩–|3⟩, respectively. By applying a third mixing field E_m1 (with frequency ω_{m1} and Rabi frequency Ω_{m1}), off-resonantly coupled to levels |1⟩ and |3⟩, a Stokes field E_S can be created through the nondegenerate FWM process^{13,14}. In fact, the produced Stokes field can be equivalently considered as a result of scattering the mixing field E_m1 off the atomic spin wave (S) pre-built by the strong on-resonant E_c and E_p fields in the corresponding Stokes fields (E_2, E_3, …E_N) generated through multiple FWM processes. (b) The equivalent configuration of (a) with the two lower states driven by the atomic spin wave S pre-built by the strong on-resonant E_c and E_p fields in the A-type EIT configuration.

**Generation of bipartite entanglement.** We first investigate the bipartite entanglement between the atomic spin wave S and the generated Stokes field E_S. As shown in Fig. 1b, we assume that the generated Stokes field is very weak as compared to the scattering field, thus, the scattering field is treated classically, whereas the Stokes field E_S and the atomic spin field S are treated quantum mechanically. After adiabatic elimination of the upper excited state, the effective Hamiltonian of this system in the interaction picture can be written as^{27}

\[ H_I = i\hbar(k_1 a_1^+ S^+ - k_1 a_1 S), \]

where \(a_1\) is the bosonic annihilation operator for the Stokes field E_S, and \(k_1 = g_{21} \sqrt{N_1} / \Delta_1\) with \(\Delta_1 = \omega_{m1} - \omega_{21}\) being the detuning of the mixing field E_m1 from the resonant transition |1⟩–|3⟩, and \(g_{23}\) being the coupling coefficient between the Stokes field and the respective atomic states. According to the realistic experimental parameters^{13,14}, \(k_1\) is in the order of several cm^{−1}; in such case, the estimated timescale for \(k_1 t = 1\) corresponds to about tens of picoseconds. By solving the Heisenberg equations of motion for the operators \(S(t)\) and \(a_1(t)\), one can get the solutions for the two operators as functions of their initial values:

\[ S(t) = \cos(k_1 \sqrt{a} t) S(0) + \sqrt{a} \sinh(k_1 \sqrt{a} t) a_1^+ (0), \]

\[ a_1(t) = \frac{1}{\sqrt{a}} \sinh(k_1 \sqrt{a} t) S^+ (0) + \cosh(k_1 \sqrt{a} t) a_1 (0), \]

where \(a = \cos^2 \theta - \sin^2 \theta\) with \(tg \theta = \Omega_c / \Omega_p\). As seen in Fig. 1b, the collective atomic state is initially in a coherent superposition state, and the Stokes field is initially in vacuum, so the initial state of the atom-field system can be written as |

| \(|\phi_0\rangle = (1 / \sqrt{N_0}) \sum_{i=1}^{N_0} |i\rangle \mid (\cos \theta |1\rangle + \sin \theta |2\rangle)_{\uparrow} (0)\rangle\), where |1⟩|0⟩, |(2)⟩|0⟩⟩ represents an atom in state |1⟩ (|2⟩) and the Stokes field in vacuum. We use the criterion \(V = (\Delta u)^2 + (\Delta v)^2 < 4\) proposed by Duan et al.\(^{28}\) to verify the bipartite entanglement between the atomic spin wave S and the generated Stokes field E_S, where \(u = x_1 - x_3\) and \(v = p_3 - p_5\) with \(x_1 = (a_1 + a_3^+)\) and \(p_3 = -i (a_1 - a_3^+)\). Smaller the correlation V is, stronger the degree of the bipartite entanglement becomes.

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**Figure 1** | (a) The A-type system of the D_1 transitions in ^85Rb atom coupled by the coupling (E_c), probe (E_p), and mixing (E_{m1}, E_{m2}…E_{mN}) fields and the corresponding Stokes fields (E_1, E_2, …E_N) generated through multiple FWM processes. (b) The equivalent configuration of (a) with the two lower states driven by the atomic spin wave S pre-built by the strong on-resonant E_c and E_p fields in the A-type EIT configuration.
Figure 2 displays the interaction time evolutions of $V$ under different ratios of $\Omega_\gamma/\Omega_\zeta$ with $k_1 = 1$. Obviously, it can be seen in Figs. 2a–2d that bipartite entanglement can be created as soon as the interaction becomes nonzero; under different ratios of $\Omega_\gamma/\Omega_\zeta$, $V$, with the initial value of about 4, evolves with interaction time and becomes less than 4, which sufficiently indicates the generation of genuine bipartite entanglement between the Stokes field $E_1$ and the atomic spin-wave field $S$. In addition, there exists a range of interaction time that $V$ nearly approaches to zero, which means that the generated Stokes field is perfectly quantum correlated with the atomic spin wave. With an increase of the ratio $\Omega_\gamma/\Omega_\zeta$ in a moderate range, the interaction time range for near-zero $V$ values would decrease and the degree of bipartite entanglement becomes weakened.

The stimulated Raman scattering process described above for generating bipartite entanglement between the Stokes field $E_1$ and the atomic spin-wave $S$ has certain similar features as the parametric down-conversion process in a nonlinear crystal. The generated Stokes field $E_1$ can be viewed as a result of frequency down-conversion process through mixing the scattering field $E_{m1}$ with the atomic spin-wave field $S$ pre-established by the coupling and probe fields. As the generation of a Stokes photon is always accompanied by an atomic spin-wave excitation, the down-converted frequency component (i.e., Stokes field) is strongly quantum correlated with the atomic spin wave. This proposed scheme can be further tested by acting as an entangler. This idea can be generalized to the case with more laser fields $E_{m2}$, $E_{m3}$, ..., $E_{mN}$, tuned to the vicinity of the transition $|1\rangle$–$|3\rangle$ to mix with the same atomic spin wave, then all of the generated Stokes fields $E_1$, $E_2$, ..., $E_N$ would be entangled with the atomic spin wave, and therefore entangled with each other. In such a situation any desired number of entangled light fields, in principle, can be created with the atomic spin wave acting as an entangler. This proposed scheme can be further tested by applying another laser field $E_{m2}$, scattering off the atomic spin wave to generate the tripartite entanglement.

**Generation of tripartite entanglement.** In the following, we consider the case of generating the Stokes fields $E_1$ and $E_2$ through scattering the laser fields $E_{m1}$ and $E_{m2}$ off the same atomic spin wave.

![Figure 2](https://www.nature.com/scientificreports/article-figures/03479/Figure2.png)

**Figure 2** | The evolutions of $V$ as a function of interaction time in terms of the normalized time $k_1 t$ under different ratios of $\Omega_\gamma/\Omega_\zeta$, with $k_1 = 1$. (a) $\Omega_\gamma/\Omega_\zeta = 1/50$, (b) $\Omega_\gamma/\Omega_\zeta = 1/20$, (c) $\Omega_\gamma/\Omega_\zeta = 1/10$, and (d) $\Omega_\gamma/\Omega_\zeta = 1/5$. respectively, as shown in Fig. 1b. The effective Hamiltonian of the system in the interaction picture has the form:

$$H_I = i\hbar k_2 [a_1^+ (a_1^+ S^+ - a_1 S) + k_2 (a_2^+ (a_2^+ S^+ - a_2 S))],$$

where $k_2 = \omega_{m2}/(\hbar \Delta_1)$ with $\Delta_1 = \omega_{m2} - \omega_{m1}$ being the frequency detuning of the mixing field $E_{m2}$ from the transition $|1\rangle$–$|3\rangle$. In the similar way as calculating the bipartite entanglement generation, we solve the Heisenberg equations of motion for the operators $S(t)$, $a_1(t)$, and $a_2(t)$, and the solutions for the three operators as functions of their initial values can be expressed as:

$$S(t) = \cos (\beta t) S(0) + \frac{k_2 a}{\beta} \sinh (\beta t) a_1^+(0) + \frac{k_2 a}{\beta} \sinh (\beta t) a_2^-(0),$$

$$a_1(t) = \frac{k_2}{\beta} \sinh (\beta t) S^+(0) + \frac{k_2 a}{\beta^2} \left( \cosh (\beta t) - 1 \right) a_1(0) + \frac{k_2 k_2 a}{\beta^2} \left( \cosh (\beta t) - 1 \right) a_1(0)^2,$$

$$a_2(t) = \frac{k_2}{\beta} \sinh (\beta t) S^+(0) + \frac{k_2 k_2 a}{\beta^2} \left( \cosh (\beta t) - 1 \right) a_1(0) + \frac{k_2 k_2 a}{\beta^2} \left( \cosh (\beta t) - 1 \right) a_1(0)^2,$$

where $\beta = \sqrt{(k_1^2 + k_2^2)}$. We demonstrate the tripartite entanglements of the generated Stokes fields $E_1$, $E_2$, and atomic spin wave $S$ by employing the criterion proposed by van Lock-Furusawa (VLF) with inequalities as follows:

$$V_{12} = V(x_1 - x_2) + V(p_1 + p_2 + g_1 p_3) < 4,$$

$$V_{15} = V(x_1 - x_5) + V(p_1 + p_2 + p_3) < 4,$$

$$V_{25} = V(x_2 - x_5) + V(p_1 + p_2 + p_3) < 4,$$

where $V(A) = \langle A^2 \rangle - \langle A \rangle^2$ and $g_i$ ($i = 1, 2, 3$) are arbitrary real numbers. In a similar way as that used in Ref. 29, we set $g_1 = -\langle (p_p p_2) + (p_p p_3) \rangle$, $g_2 = -\langle (p_p p_2) + (p_p p_3) \rangle$, and $g_3 = -\langle (p_p p_3) + (p_p p_3) \rangle$. Satisfying any pair of these three inequalities is a sufficient condition of realizing the tripartite entanglement, and also smaller the correlations $V_{12}$, $V_{15}$, and $V_{25}$, are, higher the degree of tripartite entanglement will be.

Figures 3a–3d depict the evolutions of the VLF correlations as a function of interaction time with $k_1 = 1$ and $\Omega_\gamma/\Omega_\zeta = 1/20$ under different $k_2$ values. It can be seen that when $k_2$ is far smaller than $k_1$ (see Fig. 3a), there only exists a very limited range of the interaction time within which the inequalities for $V_{12}$ and $V_{25}$ are fulfilled, whereas $V_{15}$, with the minimal value nearly to zero is smaller than 4 in a wide range of the interaction time; this means that strong entanglement between the generated Stokes field $E_1$ and the atomic spin wave is obtained, whereas the generated Stokes field $E_2$ is weakly entangled with both the Stokes field $E_1$ and atomic spin wave $S$. With an increase of $k_2$, the minimal values of $V_{12}$ and $V_{25}$ would decrease and the ranges of interaction time with both $V_{12}$ and $V_{25}$ being smaller than 4 would increase, whereas $V_{15}$ displays an opposite behavior, that is, the degree of bipartite entanglement between the generated Stokes field $E_1$ and atomic spin wave is weakened and that between the Stokes field $E_2$ and Stokes field $E_1$ (or atomic spin wave) is strengthened. When $k_1 = k_2 = 1$ (see Fig. 3c), $V_{12}$, approaching to 2 after certain interaction time, is smaller than 4 over the entire interaction time range, and there exists a broad range of interaction...
The influences to entanglement due to those realistic imperfections can be resolved and better understood. Above analysis, we did not take into account these effects, so that the atomic medium, there will be realistic imperfections, such as finite detunings so that coupling between different scattering fields can be neglected, and the atomic spin wave should be strong enough to ensure that different scattering fields have little influence on it, which can be realized by using substantially strong probe and coupling fields as compared to the scattering fields. Moreover, the probe field should be relatively weak as compared to the coupling field, so that the produced atomic spin wave can be treated as a bosonic field, and serve as an entangler, and the quantum state of the atomic ensemble can be transferred to that of the generated Stokes field. Also, the generated anti-Stokes fields are relatively weak as compared to the Stokes fields, so the entanglement properties between the Stokes and anti-Stokes fields do not need to be taken into account. However, as discussed in Refs. 13, 14, strong coupling and probe fields with nearly equal Rabi frequencies are employed in the previous scheme in order to enhance the production efficiencies of both the Stokes and anti-Stokes fields and promote entanglement between them.

Note also that, as compared to the routinely-employed method to produce multi-field entanglement by using PBS entanglers, where the interferometric stabilization of the optical paths is required, and the entangled multi-fields are degenerate and suffer from short correlation time (~ps), the present scheme can be employed to generate narrow-band multicolor entangled CV fields with a long correlation time (~ms or even ~s) by using a single entangler, where the correlation time is determined by the coherence decay time between the two lower atomic states due to the finite interaction time between atomic and light fields. Furthermore, the proposed entangler is different from and superior to the PBS entangler in that the generation of multipartite entanglement using PBS entanglers is a linear process, whereas it is a nonlinear one for the present entangler, so as nonclassical input light fields are required for using PBS entanglers to generate multipartite entanglement, only coherent input light fields are needed for using the present entangler to create multipartite entanglement, which will greatly simplify its practical implementation.

In conclusion, we have proposed an efficient and convenient scheme for realizing an entangler from an EIT-based atomic ensemble. The entangler has the virtue of being able to generate, in principle, arbitrary number of nondegenerate and narrow-band CV-entangled fields with a long correlation time, which can satisfy all the essential requirements for constructing a scalable quantum repeater for the realization of long-distance quantum communication. In addition, only coherent input light fields are required for generating multipartite entanglement. Together with the entanglement generations between two atomic ensembles and entanglement swapping, light-light, atom-atom, and atom-light multipartite entanglements can be achieved, which will find promising applications in quantum information processing and quantum networks.

**Methods**

**Calculation of time-dependent operators.** According to the Heisenberg equation of motion $i\hbar \frac{d}{dt} a(t) = [a(t), H]$, one can get the solutions for the field operators as functions of their initial values by substituting the effective Hamiltonian $H_e$ into the equations of motion. We assume that the atomic spin wave is strong enough, so different scattering fields will have little influence on it. We also assume that the generated Stokes fields are very weak as compared to the corresponding scattering fields, so the approximations of undepleted coupling and probe fields as well as scattering fields can be employed in the above calculations. Similar hyperbolic solutions for generating tripartite entanglement by applying cascaded or concurrent nonlinearities have been examined by Olsen et al.\(^{27}\).

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Acknowledgments
This work is supported by NBRPC (Nos. 2012CB921804 and 2011CB900205), the National Natural Science Foundation of China (Nos. 11274225, 10974132, 50932003, and 11021403), and Innovation Program of Shanghai Municipal Education Commission (No. 10YZ10). Yang’s e-mail is yangxihua@yahoo.com or yangxih@yahoo.com.cn; M. Xiao’s e-mail is mxiao@uark.edu.

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X.Y. and M.X. conceived the idea for this project. X.Y. and Y.Z. performed the calculations and prepared figures 1–3. X.Y. and M.X. wrote the main manuscript text. All authors reviewed the manuscript.

Additional information
Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Yang, X.H., Zhou, Y.Y. & Xiao, M. Entangler via electromagnetically induced transparency and light storage in an atomic mott insulator. Phys. Rev. Lett. 103, 033003 (2009).

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