Closed String Brane-Like States, Brane Bound States and Noncommutative Branes

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Abstract

We study the mass and different RR charge distributions of the BPS $(p,p-2)$-brane bound states in the closed string brane-like $\sigma$-model. We show that such brane bound states can be realized by introducing a constant B field in the closed string theory. In addition we show that the worldvolume coordinates of these brane bound states turn out to be noncommutative.

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1 Introduction

It is believed that D-branes play a crucial role in the dynamics of type II string theories at strong couplings. In the usual treatment, D-branes are described perturbatively by open strings satisfying Dirichlet boundary conditions for the transverse directions \[1\]. In an alternative approach the non-perturbative dynamics of D-branes in string theory is described in terms of exotic massless vertex operators, brane-like states. In the open string sector one of these operators is described by the five form BRST invariant vertex\[2\]:

\[
V^{(-3)}_{m_1 \ldots m_5} = \oint e^{-3\phi} \psi_{m_1} \ldots \psi_{m_5} \frac{dz}{2\pi i} + \text{ghosts},
\]

where \(\psi_m\) are ten dimensional NSR worldsheet fermions, \(\phi\) is bosonized superconformal ghost and we have skipped the purely ghost terms which play no role in correlation functions but which are in principle necessary to insure the BRST invariance. The remarkable property of these vertex operators is that they exist at non-zero ghost pictures only. These vertex operators also appear as central terms in the picture changed space-time superalgebra in the NSR string theory, i.e. due to the no-go theorem they should be related to the dynamics of extended objects, such as D-branes. In the closed string sector the corresponding 5-form BRST invariant propagating brane-like vertex is given by\[3\]:

\[
V_5(p) = \lambda_5(p) \epsilon_{a_1 \ldots a_4} \int d^2z e^{-3\phi} \psi_{a_1} \ldots \psi_{a_4} \bar{\psi}_t \bar{\partial} X^t e^{ip \cdot x^a}. \tag{2}
\]

The space-time indices in the \(V_5\) vertex are split in the \((4+6)\) way, \(a_1, ..., a_4 = 0, 1, 2, 3 \quad t = 4 \ldots 9\). It is crucial that the BRST invariance condition confines the propagation of this vertex to four longitudinal dimensions. Moreover the condition of worldsheet conformal invariance restricts the momentum dependence of the four dimensional \(\lambda_5\) field as:

\[
\lambda_5(p) = \frac{\lambda_0}{p^4}, \tag{3}
\]

where \(\lambda_0\) is some constant presumably related to mass or RR charge of the D3-brane. It is shown that the NSR sigma model with the \(V_5\) vertex in flat space-time is equivalent to GS superstring theory in the background of the D3-brane, namely the \(AdS_5 \times S^5\), and reproduces the correlation functions of the large \(N\) \(\mathcal{N} = 4\), \(D = 4\) SYM theory\[4\]. The \(V_5\) vertex is therefore presumed to play an important geometrical role, accounting for, in particular, the dynamical compactification of the flat ten dimensional space-time to another maximally supersymmetric \(AdS_5 \times S^5\) vacuum. In the formula (3) the longitudinal 4-dimensional indices of the \(V_5\) operator correspond to the worldvolume of D3-brane or the boundary of \(AdS_5\).
Apart from individual branes one can realize the non-marginal bound states of the branes, i.e. bound states of p, p-2, p-4, ...-branes or bound states of D-branes with fundamental strings. In the open string theory language these bound states can be introduced by the open strings with mixed boundary conditions at the ends parallel to branes and Dirichlet boundary conditions for the transverse coordinates [6, 7]:

\[
\begin{align*}
\left\{ \begin{array}{l}
g_{\mu \nu}(\partial - \bar{\partial})X^\nu + B_{\mu \nu}(\partial + \bar{\partial})X^\nu|_{z = \bar{z} = 0} = 0 \quad \mu, \nu = 0, \ldots, p \\
(\partial + \bar{\partial})X^\mu = 0 \quad \mu = p + 1, \ldots, 9.
\end{array} \right.
\end{align*}
\]

(4)

The electric mixing, \( B_{0i} \neq 0 \), leads to brane-string bound state while the magnetic mixing, \( B_{ij} \neq 0 \), produces the brane-brane bound states. Quantizing the open strings with the above boundary conditions reveals the noncommutative structure of the brane worldvolume coordinates [8, 9].

In this paper we will study the closed string \( \sigma \)-model with the \( V_5 \) brane-like state in the presence of constant non-zero B field. In the section 2, we will show that the mass density of the D3-brane in the presence of the B field is reproduced by the scattering amplitude (correlation function) of two \( V_5 \) operators (that describe the D3-brane dynamics), which is in exact agreement with the well-known results. In order to elucidate the corresponding D3-D1 bound state, we analyze the scattering of RR fields in the above mentioned \( \sigma \)-model. We find that the structure of the relevant four point correlation functions, \( < V_5 V_5^p V_{RR}^p V_{RR}^{p'} > \), shows that our \( \sigma \)-model has the correct RR charge distributions of the corresponding D3-D1 bound state. In the section 3, studying the two point \( X^a X^b \) correlation functions in the above closed string \( \sigma \)-model, we show that the noncommutative structure of the brane bound state worldvolume appears just like in the open string case. Namely, we will argue that the regularization of the closed string amplitudes with the \( V_5 \) insertions leads to the well-known open string propagators in the presence of B field, however this result appears to be regularization dependent. In the concluding section we discuss possible connections between brane like \( \sigma \)-models, and NCSYM theories and the related interesting questions.

## 2 Brane bound states and brane-like states

In this section we study the partition function of the closed string theory with inserted \( V_5 \) state and constant B field:
\[ Z[\lambda_5, B] = \int DX e^{\int g_{\mu\nu} \partial X^\mu \partial X^\nu + B_{\mu\nu} \partial X^\mu \partial X^\nu + \lambda_5 V_5 + \lambda_5 \bar{V}_5} \]  

In order to explore the resulting brane structure we study the partition function perturba-
tively in \( \lambda_0 \) and B field.

\[ Z = \sum_{n=0}^{\infty} \frac{1}{n!} <V_5 V_5 V_B^m> + O(\lambda^3), \]  

where \( V_B = B_{\mu\nu} \partial X^\mu \partial X^\nu \) and we use the usual OPE’s for closed strings. The first non-trivial

contribution to the above sum comes from \( V_2 B \), which in the proper ghost picture is

\[ <V_5 V_5 V_B V_B> = \lambda_0^2 \text{det}(g) \text{Tr}(B^2), \]  

and for the same reason we can find all the contributions to (6). Summing up all of those
terms, up to second order in \( \lambda_0 \) we find

\[ Z[\lambda_5, B] = Z[\lambda_5, B = 0] \frac{\text{det}(g + B)}{\text{det}g}, \]  

As we discussed in the introduction \( \lambda_5 \) shows the RR charge and hence the mass density of the BPS D3-brane. In the same manner the (2) shows that upon introducing the B field, the BPS mass density of the related extended object is given by, \( \text{D3 - mass} \times \sqrt{\text{det}(g + B)}. \)

Since the constant B field term in the (5) is quadratic in \( X \)'s, the above calculations can be understood and performed by replacing \( g_{ij} \) by \( g_{ij} + B_{ij} \) in the usual closed string propagators (both in bosonic and fermionic parts), i.e.

\[ :X_i(z_1)X_j(z_2) :|B = (g_{ij} + B_{ij})\log|z_1 - z_2|. \]  

Hence \( Z = <V_5 V_5 > |B + O(\lambda^3) \), which clearly reproduces the above results.

To show that turning on the B-field in the brane-like \( \sigma \)-model (5) leads to the bound

D3-D1 state, one has to study the scattering of probe RR-fields off the \( V_5 \)-state (which
corresponds to D3-brane in our model) in the presence of the B-term. The bound D3-D1 state (its appearance is the effect of switching on the B-field) manifests itself through the presence of the appropriate Ramond-Ramond charge distributions in the system. The relevant correlation function is given by:

\[ A^{V_{5}-RR}(z_1, \bar{z}_1, \ldots, z_4, \bar{z}_4) = < V_5^{(1,0)}(z_1, \bar{z}_1)V_5^{(-3,0)}(z_2, \bar{z}_2)V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z_3, \bar{z}_3)V_{RR}^{(\frac{1}{2}, -\frac{3}{2})}(z_4, \bar{z}_4) > \]

\[ V_5^{(1,0)}(z_1, \bar{z}_1, \bar{z}_1) = e^{\phi}\psi_0\psi_1\psi_2\psi_3\psi_4\bar{\partial}X^{11}e^{ik_1}X(z_1, \bar{z}_1) \]

\[ V_5^{(-3,0)}(z_1, \bar{z}_1, \bar{z}_2) = e^{-3\phi}\psi_0\psi_1\psi_2\psi_3\psi_4\bar{\partial}X^{12}e^{ik_2}X(z_2, \bar{z}_2) \]

\[ V_{RR}(z_3, \bar{z}_3) = e^{\frac{\phi}{2} - \frac{3\bar{\phi}}{2}}\Sigma_{\alpha_1}\Gamma^{m_1 \ldots m_5}S_{\beta_1}e^{ip_1}X(z_3, \bar{z}_3)F_{m_1 \ldots m_5}^{RR}(p_1) \]

\[ V_{RR}(z_4, \bar{z}_4) = e^{\frac{\phi}{2} - \frac{3\bar{\phi}}{2}}\Sigma_{\alpha_2}\Gamma^{m_2 \ldots m_3}S_{\beta_2}e^{ip_2}X(z_4, \bar{z}_4)F_{m_1 \ldots m_3}^{RR}(p_2) \]  

\[ (11) \]

The upper indices in brackets refer to the left and right ghost numbers of vertex operators, \( \Sigma \) are space-time spin operators, the momenta \( k_1, k_2 \) of the \( V_5 \)-vertices are four-dimensional (polarized along the 0,1,2,3 longitudinal directions), while the momenta of the 3-form and 5-form RR-vertices are ten-dimensional. The presence of the constant B-field results in the modification of the O.P.E. between spin fields and fermions. First of all, the two-point function of the longitudinal worldsheet fermions (parallel to the D3-brane worldvolume) is given by:

\[ < \psi^a(z)\psi^b(w) > = \frac{\eta^{ab} + B_{ab}}{z - w} \]  

\[ (12) \]

while the correlators of the transverse fermions are unchanged. For simplicity, we will consider the B-field with the only non-vanishing \( B_{23} \) component elsewhere in this paper. Performing the usual construction of space-time spinors out of worldsheet fermions with taking into account the modification related to the B-field, we find that the relevant modified O.P.E.s are given by:

\[ \left\{ \begin{array}{l}
\Sigma_\alpha(z)\Sigma_\beta(w) \sim \frac{\tilde{\epsilon}_{\alpha\beta}}{(z-w)^2} + \sum_p \frac{\Gamma_\alpha^{m_1 \ldots m_p}\psi_{m_1} \ldots \psi_{m_p}}{(z-w)^{\frac{3}{2}-p}} \\
\Sigma_\alpha(z)\psi^b(w) \sim \frac{\tilde{\Gamma}_\alpha^{b\beta}\Sigma_\beta}{(z-w)^{\frac{3}{2}}} + \ldots \\
\tilde{\Gamma}^{m_1 \ldots m_p} \equiv \Gamma^{m_1 \ldots m_p} + \Gamma^{23}B_{23}\Gamma^{m_1 \ldots m_p} \\
\tilde{\epsilon}_{\alpha\beta} \equiv \epsilon_{\alpha\beta} + \Gamma^{23}_{\alpha\beta}B_{23}
\end{array} \right. \]  

\[ (13) \]

The part of the correlation function, consisting of holomorphic ghosts, fermions and spin
The evaluation of the bosonic $X$-dependent part gives:

\[
< e^{\phi} \psi_0 ... \psi_3 \psi_4 (z_1) e^{-3\phi} \psi_0 ... \psi_3 (z_2) e^{-\frac{\phi}{2}} \Sigma_{a_1} (z_3) e^{\frac{\phi}{2}} \Sigma_{a_2} (z_4) >
\]

\[
= \frac{\eta^{12} \xi_{a_1 a_2} (z_1 - z_4)}{(z_1 - z_2)^2 (z_1 - z_3) (z_1 - z_4)} + \frac{\Gamma_{a_1 a_2}^{12}}{(z_1 - z_2) (z_1 - z_3)^2}
\]

\[
\left( \frac{\Gamma_{0123}^{12} \Gamma_{0123}^{12}}{(z_1 - z_2)^2 (z_1 - z_3)^2 (z_1 - z_4)^2} + \frac{\Gamma_{12}^{12} \Gamma_{01}^{12}}{(z_1 - z_2) (z_1 - z_3) (z_1 - z_4)^2} + \frac{\Gamma_{01}^{12} \Gamma_{12}^{12}}{(z_1 - z_2) (z_1 - z_3)^2 (z_1 - z_4)} \right)
\]

\[
\left( \frac{\Gamma_{0123}^{12} \Gamma_{0123}^{12}}{(z_1 - z_2)^2 (z_1 - z_3)^2 (z_1 - z_4)^2} + \frac{\Gamma_{12}^{12} \Gamma_{01}^{12}}{(z_1 - z_2) (z_1 - z_3) (z_1 - z_4)^2} + \frac{\Gamma_{01}^{12} \Gamma_{12}^{12}}{(z_1 - z_2) (z_1 - z_3)^2 (z_1 - z_4)} \right)
\]

The antiholomorphic fermionic part of the correlator is given by:

\[
< e^{-\frac{\phi}{2}} \Sigma_{\beta_1} (\bar{z}_3) e^{-\frac{\phi}{2}} \Sigma_{\beta_2} (\bar{z}_4) > = \frac{\bar{\epsilon}_{\beta_1 \beta_2}}{(\bar{z}_3 - \bar{z}_4)^2}
\]

The evaluation of the bosonic $X$-dependent part gives:

\[
< \bar{\partial} X^1 e^{ik_1 X} (z_1, \bar{z}_1) \partial X^3 e^{ik_2 X} (z_2, \bar{z}_2) e^{ip_1 X} (z_3, \bar{z}_3) \partial X_m e^{ip_2 X} (z_4, \bar{z}_4) >
\]

\[
= |z_1 - z_2|^{-2(k_1 + k_2)} |z_1 - z_3|^{-2(k_1 + p_1)} |z_1 - z_4|^{-2(k_1 + p_2)}
\]

\[
|z_2 - z_3|^{-2(k_2 p_1)} |z_2 - z_4|^{-2(k_2 p_2)} |z_3 - z_4|^{-2p_1 p_2}
\]

\[
\left\{ \frac{\eta^{12} p_1}{(z_1 - z_2)^2} + \frac{\eta^{12} p_2}{(z_1 - z_2)(z_1 - z_3)} + \frac{\eta^{12} p_3}{(z_1 - z_2)(z_1 - z_4)} + \frac{\eta^{12} p_4}{(z_1 - z_3)(z_1 - z_4)} \right\}
\]

The “B-covariant” momentum $\tilde{p}^m$ is related to the usual one as

\[
\left\{ \begin{array}{l}
\tilde{p}^2 = p^2 + B_{23} p^3 \\
\tilde{p}^3 = p^3 - B_{23} p^2 \\
\tilde{p}^m \equiv p^m, m = 0, 1, 4, ..., 9.
\end{array} \right.
\]

whereas the “B-covariant” scalar product is defined as

\[
\{p_1 p_2\} = (p_1)_m (p_2)_m + B_{23} (p_2^3 p_1^3 - p_1^3 p_2^3)
\]
Finally, putting together all the pieces of the correlator \( A^{V_5-RR}(z_1, \bar{z}_1, \ldots, z_4, \bar{z}_4) \), fixing the \( SL(2,C) \) remnant conformal symmetry by setting \( z_2 \to 1, z_3 \to 0, z_4 \to \infty \) and integrating over \( z_1 \) with the appropriate Koba-Nielsen’s measure, we obtain the following expression for the scattering amplitude:

\[
A^{V_5-RR}(k_1, k_2, p_1, p_2) = \int d^2 z_1 |z_2 - z_3|^2 |z_2 - z_4|^2 |z_3 - z_4|^2 A^{V_5-RR}(z_1, \bar{z}_1, \ldots, z_4, \bar{z}_4)
\]

\[
= 3(Tr(CT^{n_1 \ldots n_3} \Gamma^m \Gamma^{m_1 \ldots m_5}) + Tr(CT^{n_1 \ldots n_3} \Gamma^2 \Gamma^3 \Gamma^{m} \Gamma^{m_1 \ldots m_5})B_{23})
\]

\[
\times \int d^2 z_1 |1 - z_1|^{-2(k_1 k_2 + 2)} |z_1|^{-2(k_1 p_1)}
\]

It is remarkable that, after fixing the global \( SL(2,C) \) conformal invariance the only terms that survive in the amplitude are those linear in \( B_{23} \) (in the kinematic factor). All the contributions to the kinematic factor which are non-linear in the B-field, go away as we take the limit \( z_4 \to \infty \). Performing the integration over \( z_1 \), we get:

\[
A^{V_5-RR}(k_1, k_2, p_1, p_2) = 3\lambda_5(k_1)\lambda_5(k_2)F_{m_1 \ldots m_5}(p_1)F_{n_1 \ldots n_3}(p_2)
\]

\[
(Tr(CT^{n_1 \ldots n_3} \Gamma^m \Gamma^{m_1 \ldots m_5}) + Tr(CT^{n_1 \ldots n_3} \Gamma^2 \Gamma^3 \Gamma^{m} \Gamma^{m_1 \ldots m_5})B_{23})(p_1)_m
\]

\[
\times \frac{\Gamma(1+\frac{1}{2}(k_1 k_2))\Gamma(1-\frac{1}{2}(k_1 k_2))\Gamma(1/2(k_1 k_2)1/2(k_1 k_2))}{\Gamma(1-\frac{1}{2}(k_1(k_2 + p_1)))\Gamma(1-\frac{1}{2}(k_1 k_2))\Gamma(1/2(k_1 p_1))}
\]

This expression has yet to be integrated over the four-dimensional momenta \( k_1 \) and \( k_2 \) of the \( V_5 \)-vertices. The integration similar to the one done in \( \mathbb{I} \), taking into account the momentum conservation \( p_2 = -p_1 - k_1 - k_2 \) along with the condition (3), gives:

\[
A = \int d^4 k_1 \int d^4 k_2 A^{V_5-RR}(k_1, k_2, p_1, p_2) = 3(\lambda_0)^2 F_{m_1 \ldots m_5}(p_1)F_{n_1 \ldots n_3}(p_2)
\]

\[
(p_1)_m(Tr(CT^{n_1 \ldots n_3} \Gamma^m \Gamma^{m_1 \ldots m_5}) + Tr(CT^{n_1 \ldots n_3} \Gamma^2 \Gamma^3 \Gamma^{m} \Gamma^{m_1 \ldots m_5})B_{23})log(p_1||)^2 + ...
\]

where \( p_1|| \) is the projection of the ten-dimensional momentum of the RR-vertex to the four longitudinal dimensions, and we have skipped the contributions analytic in \( p_1|| \) (which do not play any role upon the Fourier transformation back to the position space). Finally, taking traces of the gamma-matrices, we find that the amplitude is proportional to:

\[
A \sim \lambda_0^2 F_{01234}F_{001t}B_{23}
\]

corresponding to the attraction force between the D3-brane and D-string in the \( D3 - D1 \) bound state.
Analogously, one can consider turning on the $B_{01}$ component of the B-field, along with $B_{23}$. In this case, the effect of the B-field in the D3-brane worldvolume will result in appearance of the D-instanton-D-string-D3-brane bound state, in addition to the D1-D3 one. The related string-theoretic $V^{RR-V_5}$ scattering amplitude will be quadratic in the B-field, being proportional to $A \sim \lambda_5^2 F_0 F_{0123} B_{01} B_{23}$, where $F_i$ is the RR field strength, corresponding to the D-instanton. Analogously, one can observe the appearance of other various D-brane bound states as a result of switching on the B-field, in the context of the brane-like $\sigma$-model.

3 Noncommutativity from closed string $\sigma$-model

In this section we will consider the propagators of two bosonic closed string $X$ fields in the presence of the B field and the $V_5$ operator. The correlation function of interest is given by

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5}.$$ 

As it follows from the $\sigma$-model action (5) that the first non-trivial contribution to this correlator (of order of $\lambda_5^2$) is proportional to

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

The integration over $k$ is proportional to $\sim \int \frac{d^4k}{k^4} k^a k^b$ which gives $g^{ab}$ after the appropriate regularization. The above correlation function is then given by

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

$$<\partial X^a(z_1)\bar{\partial}X^b\bar{(\bar{z}_2)}|_{B,V_5} \sim \lambda_5^2 \int \frac{d^4k}{k^4} \int d^2w_1 d^2w_2$$

Let us consider the first integral given by

$$I(z_1, \bar{z}_2) = \int \frac{d^2w_1 d^2w_2}{|w_1 - w_2|^4} \times \frac{1}{(z_1 - w_1)(\bar{z}_2 - \bar{w}_1)}.$$

Writing $w_1 = x + iy$, we have

$$I(z_1, \bar{z}_2) = \int d^2w_2 \int_\infty^\infty dy \int_\infty^\infty dx \frac{1}{(x + iy - w_2)^2(x + iy - \bar{w}_2)^2(z_1 - x - iy)(\bar{z}_2 - x + iy)}.$$
For the sake of certainty let us consider the case $\text{Im}(z_1), \text{Im}(z_2) \geq 0$, so that both of $z_1$ and $z_2$ are in the upper half plane. Now in order to elucidate the open string structure of the propagator we propose to consider not all the poles in the above integral, but only those located in the upper half plane and then to retain only the terms surviving the translational invariance along the $x$ axis. Evaluating residues of those poles in $x$ and then in $y$ we find

$$I(z_1, \bar{z}_2) = 2 \partial_{z_1} \partial_{\bar{z}_2} \int d^2w \frac{1}{(z_1 - w_2)(\bar{z}_2 - \bar{w}_2)}. \quad (27)$$

Our procedure for distinguishing the upper half plane poles contains few subtleties; see below for more explicit explanation. Again we can perform the $w_2$ integration by writing it as $w_2 = u + iv$, so that the integral can be written as

$$I(z_1, \bar{z}_2) = 2 \partial_{z_1} \partial_{\bar{z}_2} \int_0^\infty dv \int_{-\infty}^{\infty} du \frac{1}{(z_1 - u - iv)(\bar{z}_2 - u + iv)}. \quad (28)$$

The essential point in our computation is that we only take into account the poles located in the upper half plane (this may be achieved e.g. by including the exponential factor $\sim e^{i\alpha x}, \alpha > 0$ in the complex integral over $x$ and taking the limit $\alpha \to 0$ afterwards).

$$I(z_1, \bar{z}_2) = 2 \partial_{z_1} \partial_{\bar{z}_2} \int_0^\infty dv \frac{1}{(\bar{z}_2 - z_1 + 2iv)}. \quad (29)$$

Now we shall stress that in the integration over $v$, the integration must be taken from zero to $\infty$, rather than from $-\infty$ to $\infty$, in order to exclude the terms breaking the translational invariance in the direction of the real axis. Finally, regularizing the above integral and differentiating over $z_1$ and $\bar{z}_2$ we find

$$I(z_1, \bar{z}_2) = \frac{1}{(z_1 - \bar{z}_2)^2}. \quad (30)$$

The corresponding contribution to the $XX$ propagator is given by

$$(g + B)g^{-1}(g + B)\log(z_1 - \bar{z}_2)$$

. Adding the necessary complex conjugate part coming from

$$< \bar{\partial}X^a(z_1)\partial X^b(\bar{z}_2) > |_{B,V_5} = (g - B)g^{-1}(g - B) \frac{1}{(\bar{z}_1 - \bar{z}_2)^2},$$

we get the corresponding part of the $XX$ propagator:

$$\left( (g + B)g^{-1}(g + B) \right)_S \log|z_1 - \bar{z}_2|^2 + \left( (g + B)g^{-1}(g + B) \right)_A \log \frac{z_1 - \bar{z}_2}{\bar{z}_1 - z_2} \quad (31)$$
where the $S, A$ show the symmetric and antisymmetric parts of that matrix. To find the full expression for the propagator we also have to take into account the contributions coming from $< \partial X^a(z_1) \partial X^b(\bar{z}_2) > |_{B,V_5}$ and $< \bar{\partial} X^a(z_1) \bar{\partial} X^b(\bar{z}_2) > |_{B,V_5}$. Adding all of these contributions together up to second order in $\lambda_0^2$ we have

$$< X^a(z_1) X^b(z_2) > \sim -g_{ab}(\log|z_1 - z_2| - \log|z_1 - \bar{z}_2|)$$

$$-(G_{ab} + \theta_{ab})\log(z_1 - \bar{z}_2) - (G_{ab} - \theta_{ab})\log(\bar{z}_1 - z_2),$$

where $G$ and $\theta$ are the the symmetric and antisymmetric parts of $(g + B)^{-1}_{ab}$. This is in agreement with the expression for open string $X X$ propagator in the constant B field background giving rise to the noncommutative structure of the brane worldvolume coordinates [10].

In order to make the full correspondence with the open string picture, we should also work out the closed string propagators for the transverse coordinates and show that they result in the open string propagators with Dirichlet boundary conditions. To do this we have to take into account the transverse degrees of freedom of the D3-brane which in the closed string language correspond to the vertex operators $V_3$

$$V_3 = \frac{\lambda_0}{(p_t p^t)^3} \epsilon_{a_1...a_4} \int d^2 z \partial \{ e^{i p^t X^t} \psi_{a_1} \psi_{a_2} \psi_{a_3} \psi_{a_4} \} + \text{ghosts},$$

where $p_t$ is the transverse momentum. The relevant correlator is given by $<X^t(z, \bar{z})X^t(z', \bar{z}') > |_{V_3,V_5,B}$. Again instead of the $X$'s we consider the correlators of their derivatives first. Up to the second order in $\lambda_0$ the relevant correlation functions are given by $< \partial X^t(z, \bar{z}) \bar{\partial} X^t(z', \bar{z}') V_3 V_5 >$ and $< \partial X^t(z, \bar{z}) \bar{\partial} X^t(z', \bar{z}') V_5 V_5 >$ and their complex conjugate which, as previously, should be integrated over the coordinates and the momenta of the brane-like vertices. The computation totally similar to the one performed above gives the result

$$< X^t(z, \bar{z}) X^t(z', \bar{z}') > |_{V_5,V_5,B} \sim \lambda_0^2 \text{det}(g + B)(\log|z - z'| - \log|z - \bar{z}'|),$$

which coincides with the well-known expression for the open string propagator with Dirichlet boundary conditions.

4 Discussions and Remarks

In this paper we have studied the adding of a constant B field background to the closed string brane-like $\sigma$-model. We argued that since the B term is quadratic in $X$ and only contains
the parallel $X$ components, the contractions between $V_B$ and brane-like states vanish and therefore the constant $B$ field effects can be totally absorbed in the closed string propagators. Calculating the mass density and different RR charge densities we have shown that turning on the $B$ field in the brane-like $\sigma$-model gives rise to the non-marginal $D3$-$D1$ bound state. In addition we have shown that the closed string propagators in the presence of $B$ field reveal the noncommutativity of the brane coordinates. All the above results can be checked and clarified in the brane-brane scattering processes \cite{6,7}. In order to study such scatterings we should be able to identify the location of $D_p$-brane in the $9-p$ dimensional transverse space. This can be done by adding a delta function of the brane location, $Y^t$, to the $V_5$ state:

$$V_5(k_{||}, k_t) = (\frac{\lambda_0}{k_{||}})\varepsilon_{a_1...a_4} \int d^2z e^{-3\phi}\psi_{a_1}\psi_{a_4}\psi_t\bar{\psi}X^{t}\bar{e}^{ik_{||}X^t}e^{ik_t(X^t-Y^t)}.$$  

We should note that adding such a term will not destroy the superconformal and the BRST invariance.

As it has been argued in \cite{3,4}, adding the brane-like states to the closed strings in the flat background leads to strings effectively living in the related supergravity backgrounds, namely $AdS_5 \times S^5$. So we expect that the same ideology should work for the case we have at hand, the brane bound states. However, in the presence of the $B$ field, the related supergravity solution is not $AdS$ anymore. It has been argued in \cite{11,12} that one can find a limit in which the corresponding NCSYM theory (described in terms of properly scaled parameters) decouples from the bulk gravity. So it seems that we should be able to build some more concrete relations between string theory in the above mentioned brane bound state backgrounds and the deformed gauge theories on the boundary. In particular one can find the four point function of NCSYM theory, $<F_{NC}^2 F_{NC}^2>$ from the dilaton scattering off these bound states.

The more interesting question we can address here is the Wilson loops. Since it is believed that the $V_5$ operator (accounting for dynamical compactification $AdS_5 \times S^5$) compensates for the zig-zag non-invariance of the gravity part and since the $B$ term is zig-zag invariant, Wilson loops of the NCSYM theory calculated by means of the gravity/NCSYM correspondence should be zig-zag invariant.

It is well-known that the conformal invariance should be broken in the confining phase. In the gauge theory language that means that we are just living apart from the UV fixed point. From the string theory point of view breaking the conformal symmetry in the four dimensional space-time usually corresponds to turning on the gravity in transverse dimensions (e.g. the radial coordinate of $AdS_5$) \cite{13}. However the presence of the transverse graviton
modes is expected to break the zig-zag symmetry, necessary to relate the string theory with
the gauge-theoretic large N limit. The alternative approach, which preserves the zig-zag
symmetry, is based on including the axionic B term to the functional for the Wilson loop.
In this case one can expect this functional both to obey the large N loop equation and to
exhibit the area law behaviour, necessary for the confinement.

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