Statistical Analysis and Trend Detection of the Hydrological Extremes of the Danube River at Bratislava

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Abstract. The territory of the Danube River Basin is one of the most flood-endangered regions in Europe. The flow regime conditions of the Danube River are continually changing. These changes are the result of natural processes and anthropogenic activities. In the present study, we focused on the statistical analysis and trend detection of the hydrological extremes of the Danube River at Bratislava. This paper firstly analyses the changes in correlation between water levels of the Danube River at Bratislava and Kienstock. Studied period of 1991–2013 included one or three hour measured water levels of the Danube River at Bratislava and Kienstock and shorter periods (1991–1995, 1999–2002, and 2004–2013) were selected for identification of the water level changes at Bratislava. One of the factors that recall the necessity to establish empirical-regression relationships was increasing of water levels of the Danube River at Bratislava (due to sediments accumulation at Bratislava). The results of the analysis indicated an increasing of water levels corresponding to the same flood discharges observed in the past. We also can say that travel time of the Danube floods between Kienstock and Bratislava did not change significantly during the analysed period. In the second part of the paper, we have identified changes in commonly used hydrological characteristics of annual maximum discharges, annual discharges and daily discharges of the Danube River at Bratislava during the period of 1876–2019. We examined whether there is a significant trend in discharges of the Danube River at Bratislava.

1. Introduction
Three types of hydrological changes may to result from regulation of rivers: increased frequency of high flows; redistribution of water from periods of base flow to periods of storm flow, and increased daily variation in stream flow. These changes do not necessarily occur in all regulated rivers, but they are common and need to be addressed as part of any comprehensive effort to rehabilitate regulated rivers.

The estimation of hydrological characteristics cannot be considered as closed and computing methods (models) have to be constantly updated on the latest conditions and current situation in the river. We focused on the Danube River with respect to the Danube River basin as an area with high economical and water management importance. These anthropogenic activities have a direct or indirect effect on river regime. Then, we can say, that the water level in the Danube is a result of the discharge, depth and shape of the riverbed, including the flood-plain area, which is restricted to the area between flood protection dikes since the last century. Construction of dams in the upper section of the Danube in the 20th century gradually reduced the volume of transported sediments to the middle section of the Danube. For example, Rákoczí [1] analysed regime of the sediments of the Danube River during the period of 1956–1985. The effect of the river bed sedimentation on water level changes of the Danube at
Bratislava was also analysed in work of [2] and [3]. Author [3] calculated that the water level with the peak of \( Q_{100} \) at Bratislava would be 20–25 cm higher in 1998 due to the changed geometry of the river bed compared to 1996. The analysis of the changes in the cross profile (bed), flow velocities changes and changes of rating curves of the Danube at Bratislava profile showed that the river barrages causes characteristic changes in the grain-size distribution of both, the suspended sediments and the riverbed materials can cause changes in hydrological regime of the river [4]. The detailed description of the Danube River basin for understanding the occurrence of floods in the basin with: a historical overview of individual floods, analysis of homogeneity, cyclicality and long-term trends, seasonality, extreme discharges of selected water stations, coincidence of flood waves in the main river basin and major side tributaries, least and not last theoretical design hydrographs and regionalization of river basin flood regimes you can find in monography of Pekárová [5]. General summary of floods on tributaries of the Upper Danube and their impact on society and the population is published in [6].

The objective of this paper is to analyse and present changes of the water levels of the Danube River at Bratislava profile. The simple empirical regression relationships were used to illustrate changes of the water levels on the Danube at Bratislava profile. This method allows assigning the water levels from one or more upper stations to water level of the lower station. The period of study included the measured three-hour or one-hour water levels of the Danube River at Kienstock and at Bratislava over the period of 1991-2013. These relationships are necessary to be updated after each major flood situation. One of the factors that recall the necessity to establish empirical - regression relationships was increasing of water levels of the Danube River at Bratislava (due to sediments accumulation at Bratislava). The second part of the paper is focused on the identification of the changes in commonly used hydrological characteristics (annual maximum discharges \( Q_{\text{max}} \), annual discharges \( Q_{\text{a}} \) and daily discharges \( Q_{\text{d}} \)) of the Danube River at Bratislava during the period of 1876–2019. We examined whether there is a significant trend in discharges of the Danube River at Bratislava.

The results of the statistical analysis and trend detection in the hydrological characteristics of the Danube River at Bratislava are illustrated in figures, listed in tables and summarized in the conclusions of the paper.

2. Study area

The Danube River is the second largest river in Europe after the Volga. The basin covers an area of 817 000 km². The length of the river is 2 872 km. The river originates from the Black Forest in Germany at the confluence of the Brigach and the Breg streams. It discharges into the Black Sea via the Danube delta, which lies in Romania and Ukraine (figure 1). The Danube River is an important hydrologic and hydrographic system, which is formed by several significant tributaries. The maximum runoff occurs in June in the upper Danube basin. Over 120 major rivers directly confluence with the Danube and a majority of them has its own significant tributaries. The Slovak part of the Danube River is situated from river-km 1 708.2 to river-km 1 880.2.

| River | Station | River km | Period | Country | Area [km²] | LAT | LONG | Elevation [m a.s.l] | Qamax [m³s⁻¹] |
|-------|---------|----------|--------|---------|------------|-----|------|-------------------|-------------|
| Danube | Bratislava | 1 868.8 | 1876–2019 | SK | 131 338 | 48.14 | 17.1 | 132.86 | 5 884 |
3. Methods
In our study we used the method of corresponding water levels. This method is based on the calculation of the movement of a flood wave through a riverbed and allows the water level in one or more upper stations to be assigned a water level in the lower station. The values of the water level in the lower profile can then be written in the form:

$$H_{d,t+\tau} = H_{h,t} + \Delta H_t,$$

(1)

where: $H_{d,t+\tau}$ - water level in lower station in $t+\tau$; $H_{h,t}$ - water level in upper station in time $t$ and $\Delta H_t$ - increase of the water level between upper and lower stations, which is achieved in time $\tau$.

The increase in the water level $\Delta H_t$ is influenced by several factors: water from smaller tributaries; runoff of groundwater; water in the basin; transformation of the flood wave during the progress in the riverbed. These factors can be neglected on short sections and after adjustment equation (1) takes form:

$$Q_{d,t+\tau} = (1 + \beta)Q_{h,t},$$

(2)

where: $\beta$ - is the coefficient of water levels, and we assume $\Delta H_t/H_{h,t} \approx \text{const}=\beta$.

The determination of the function of water levels depending on the travel time $\tau$ is based on the measured values of water levels in the upper and lower profile and the determined travel times.

$$\tau_t = t_{i,H_h} - t_{i,H_d},$$

(3)

where: $t_{i,H_h}$ - time of occurrence of the characteristics points of the water level in upper station, $t_{i,H_d}$ - time of occurrence of the characteristics points of the water level in lower station.

The Mann-Kendall nonparametric test (M-K test) was used as tool to identify significant trends in hydrological characteristics. The Mann-Kendall nonparametric test (M-K test) is one of the most widely used non-parametric tests for significant trends detection in time series. The nonparametric tests are more suitable for the detection of trends in hydrological time series, which are usually irregular, with many extremes ([7]; [8] and [9]). By M-K test, we test the null hypothesis $H_0$ of no trend, i.e. the observation $x_i$ is randomly ordered in time, against the alternative hypothesis $H_1$, where there is an increasing or decreasing monotonic trend. The M-K test detects trends at four levels of significance: $\alpha = 0.001$, $0.01$, $0.05$ and $\alpha = 0.1$. Significance level of 0.001 means that there is a 0.1% probability that the value of $x_i$ is from a random distribution and are likely to make a mistake if we reject the hypothesis.
H0; Significance level of 0.1 means that there is a 10% probability that we make a mistake if we reject the hypothesis H0. If the absolute value of Z is less than the level of significance, there is no trend.

4. Results

4.1. Changes in correlation between water levels of the Danube River at Bratislava and Kienstock

We analysed changes in correlation between water levels of the Danube River at Bratislava and Kienstock. Studied period of 1991–2013 included one-hour or three-hour measured water levels of the Danube River at Bratislava and Kienstock. Station Kienstock is situated in sufficient distance from Bratislava and already captured significant Alpine tributaries of the Danube River (Ybbs, Enns, Traun). Number of 68 maximum water levels over 300 cm (simultaneously at Kienstock and Bratislava) was selected. Shorter periods (I. 1991–1995, II. 1999–2002 and III. 2004–2013) were used for identification of water level changes of the Danube River at Bratislava. Selected periods were chosen with respect to availability of data, anthropogenic activities and large flood situations. Derived relations (equations) showed the difference in relationships between water levels at Kienstock and Bratislava during the selected period of 1991–2013. The dependencies between water levels of given profiles is illustrated in figure 2a. The figure 2b shows rating curves of the Danube River at Bratislava profile valid until 1903, valid in 1965, 1991 and valid in March 2002 and August 2002. The maximum values of the water level and peak discharge reached at Bratislava during the Danube flood occurred in June 2013 also illustrated figure 2b. The water level value of 1 034 cm is the highest value of the water level for a given discharge value of 10 640 m$^3$/s of the Danube at Bratislava profile. For example, the culmination discharge value of 10 870 m$^3$/s with maximum water level of 970 cm during the Danube flood in 1899. According to author’s analysis, the difference between peak water level of the floods in 2013 and 1954 was about +70 cm at Bratislava gauge, while the peak discharge of both floods was almost the same (10 640 m$^3$/s and 10 400 m$^3$/s, respectively).

Figure 2. a) Relationship between water levels of the Danube River at Bratislava and water levels at Kienstock (periods: I. 1991–1995, II. 1999–2002 and III. 2004–2013) and b) the rating curves of the Danube River at Bratislava (dark red point – maximum values of the flood in June 2013).

Culmination of the March 2002 flood (8 556 m$^3$/s, 870 cm) would reach water level value of 808 cm according to rating curve valid in year 1991. Derived empirical relationships were used to simulate the water levels of two major floods in the new millennium: August 2002, and June 2013. In a previous study [10] authors showed that the travel times of medium floods between Kienstock and Bratislava (period of 1991–2002) are shorter by about 41% compared to period of 1923–1966, and about 11% compared to period of 1975–1991. During the 1954 flood the travel time between Kienstock and Bratislava was 61 hours and in 2013 the travel time was only about 41 hours in the same section (figure 3a). In the second part of the paper the three rising limbs of flood waves occurred in August 1991, March 2002, and August 2002 were analysed in more detail to derive the time of culmination occurrence. Flood of June 2013 on the Danube River was used for verification of derived empirical equations. The travel times of the last four biggest Danube floods between Kienstock and Bratislava did not change.
significantly (figure 3b). The travel time of the Danube flood peaks over 991 cm varies between 36 and 47 hours from Kienstock to Bratislava.

Figure 3. a) travel time of the peaks of selected floods and b) relationship between some points of the rising limb for selected floods.

4.1.1. Simulation of the two floods occurred on the Danube river in August 2002 and June 2013
Derived empirical relationships were used to simulate the water levels and travel time of two major Danube floods in the new millennium: August 2002, and June 2013. The regression equations of all three periods I. 1991-1995, II. 1999-2002, and III. 2004-2013 were used to simulate water levels at Bratislava from water levels at Kienstock, respectively. Water levels of all floods simulated using the empirical regression derived from period I. were underestimated (figures 4a, 4a'). Water levels of the August 2002 flood simulated by the empirical regression equations derived from periods II. and III. were overestimated (figures 4b-c, 4b'-c'). Simulation of the water levels of the Danube River at Bratislava for flood in June 2013 by the regression III. (2004–2013) shows sufficiently accurately water levels of the flood. The accuracy of the estimation of the water levels at Bratislava station according derived relationships were evaluated by mean error and mean percentage absolute error estimation (table 2). Table 3 presents comparison between measured and simulated maximum water levels of the Danube River at Bratislava based on water levels of the Danube River at Kienstock.

Table 2. Values of the estimation error (ME-mean error, MAPE mean absolute percentage error) of the Danube River at Bratislava based on water levels of the Danube River at Kienstock.

| Flood        | ME [cm] | Period I.1991–1995 | Period II.1999–2002 | Period III.2004–2013 |
|--------------|---------|---------------------|----------------------|----------------------|
| August 2002  | 65      | 25.5                | 12.6                 |
| June 2013    | 97      | 59                  | 32                   |
| Flood        | MAPE [%] |                     |                      |
| August 2002  | 9.6     | 3.7                 | 1.8                  |
| June 2013    | 12      | 7.3                 | 5.2                  |
| R [-]        |         |                     |                      |
| August 2002  | 0.88    | 0.89                | 0.87                 |
| June 2013    | 0.80    | 0.83                | 0.86                 |

Table 3. Comparison between measured and simulated maximum water levels of the Danube River at Bratislava based on water levels of the Danube River at Kienstock.

| Year         | Measured Hmax [cm] | Simulated Hmax [cm] |
|--------------|--------------------|---------------------|
|              | 1.1991–1995        | Period II.1999–2002 | Period III.2004–2013 |
| August 2002  | 991                | 973                 | 1007                 | 1034                  |
| June 2013    | 1034               | 967                 | 1001                 | 1029                  |
Figure 4. Simulation of water levels of the Danube River at Bratislava for flood occurred in August 2002 and in June 2013: a) a regression 1991–1996, b) regression 1999–2002 and c) regression 2004–2013.

4.2. Trend analysis of the of annual maximum discharges, annual discharge and daily discharges of the Danube River at Bratislava

The second part of present paper, is focused on the statistical analysis of changes in some selected hydrological characteristics like annual maximum discharges, annual discharges and daily discharges of the Danube River at Bratislava during the period of 1876–2019. The time series of annual discharges were calculated from daily mean discharges. Course of the annual discharges and decadal averages, the Danube River at Bratislava are illustrated in figure 5. The annual maximum discharges of the Danube River at Bratislava show slightly increasing long-term linear trend during the selected period of 1876–2019. There were also occurred some extreme floods in 1899, 1954, 2002 or 2013 (figure 5a). The annual discharge of the Danube River at Bratislava show slightly decreasing long-term linear trend during the selected period of 1876–2019. The daily discharge of the Danube River at Bratislava do not
show long-term linear trend during the selected period of 1876–2019. To assess the significance of the
detect trends we used one non-parametric test for trend analysis the Mann-Kendall test. Results of the
Man-Kendall test parameters are listed in table 4.

**Figure 5.** Course of the a) maximum annual discharges linear trend and 5-year moving average, b) annual discharges and 5-year moving average and c) daily discharges, linear trend and 4-year moving average of the Danube River at Bratislava station (1876–2019).

**Figure 6.** Long-term trend - conclusions of Mann-Kendall trend test for annual maximum discharges Qmax and annual discharges Qa of the Danube River at Bratislava for period 1876–2019.
Table 4. Conclusions of Mann-Kendall trend test for annual maximum discharges Qmax and annual discharges Qa of the Danube River at Bratislava. (A, B are parameters of the linear trend line $y = A*x+B$, A is slope of the trend line)

| Time series | Mann-Kendall trend | Sen's slope estimate |
|-------------|--------------------|----------------------|
|             | First year | Last Year | n | Test Z | Significant | A | B |
| Qmax [m$^3$/s] | 1876 | 2019 | 144 | 1.49 | no | -0.436 | 1999.46 |
| Qa [m$^3$/s] | 1876 | 2019 | 144 | -0.36 | no | 7.909 | 5368.64 |

According to the IPCC’s report of 2001, the consequences of climate change may result in altered distribution curves of climate elements (air temperature, precipitations, dis-charge rates, etc.), thus time series of other periods will likely change also. Therefore, we analysed the long-term average annual regime (365 days in a year) of the average daily discharges for two periods 1876–1947 and 1948–2019. The long-term average daily discharges for two periods 1876–1947 and 1948–2019 are illustrated in figure 7a). The red circles point to the most significant differences between the periods in question. Table 5 summarizes some basic statistical characteristics of the average daily flows for the Danube River taking into account both periods.

The basic statistical characteristics of the two data sets do not indicate any significant changes; the long-term average discharge differs only by 35 m$^3$/s, which is negligible. The analysis of long-term average daily discharges showed their decrease in the months of May-November in the period 1948-2019 in comparison with the period 1876–947.

In the middle of September, the discharges were reported to have dropped by 413 m$^3$/s in comparison with the previous 72-year period. In contrast, in late March, long-term average daily values from the period 1948–2019 exceeded the value from the 1876–1947 period by maximum value 285 m$^3$/s and in late December by 442 m$^3$/s. The differences between long-term daily discharges of the periods 1876–1947 and 1948–2019 are illustrated in figure 7b).

Table 5. Basic statistical characteristics of the average daily flow series of the Danube River at Bratislava, for periods: 1876–1947, 1948–2019.

| Statistical characteristic | Average | Standard Dev. | Median | Min. | Max. | Kurtosis | Skewnes | Range |
|---------------------------|---------|---------------|--------|------|------|----------|---------|-------|
| 1876–1947                 | 2 067.5 | 993           | 1 860  | 580  | 10 810 | 4.8      | 1.6     | 10 230|
| 1948–2019                 | 2 031.9 | 960           | 1 828  | 133  | 10 520 | 6.7      | 1.9     | 10 387|

Figure 7. a) The course of the long-term average daily discharges and b) differences of the long-term average daily discharges of Danube at Bratislava for periods: 1876–1947.
5. Conclusions

The intensive gravel excavation below Bratislava was stopped after 1980. The Danube dam Gabčíkovo near Cunovo caused increasing the Danube water level at Bratislava after November 1992. Decrease of flow velocities and stream turbulence caused the sedimentation of bed load (originally transported by the stream into lower profiles) in the Danube channel in Bratislava. Such interventions caused acceleration of the travel time of flood waves or decreasing of transformation capacity of the channel and floodplains. It can also cause increasing of water levels corresponding to the same flood flow observed in the past. In conclusion we can say that travel time of the Danube floods between Kienstock and Bratislava did not change significantly during the period, but water levels tend to increase their culmination level at the same flow rate. In conclusion, we can also conclude that the derived regressions simulate water levels of the flood waves of the Danube at Bratislava sufficiently accurate. Underestimation of the water levels above 900 cm according to regression I. (1991–1995) may be due to the fact that in this period there was no flood wave above 9500 m$^3$s$^{-1}$. There still applies quote "Let the flood be however large, there always comes up even larger one in the future", as it follows from the theory of extremes and experience confirms this.

In the present paper we also analyzed changes in the characteristics of annual maximum discharges, annual discharge and daily discharges of the Danube River at Bratislava during the period of 1876–2019. Generally, in Bratislava in Danube discharge series there doesn't exist (from the statistical point of view), significant trend of discharges within the used 144-years period 1876–2019. The analysis of the two shorter periods (1876–1947 and 1948–2019 did not confirm significant changes in total distribution of the annual discharges or maximum annual discharges. On the other hand, a time change in the long-term average daily discharges was identified at the end of March and end of December, when these increased over the period 1948–2019. This increase can be attributed to the onset of snow-melt in the Danube basin due to the increased atmospheric temperature. In contrast, the long-term daily discharges dropped in the summer months comparison with the period of 1876–1947. It seems that the amount of water discharged in March is missing in the river outflow in September.

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