Dynamics of the oscillatory system of the near-bit slurry grinder

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Abstract. The article proposes a near-bit device for vibrodynamic grinding of sludge in a horizontal section of a well – a near-bit sludge shredder, its advantages are given. Theoretical studies of the dynamics of the near-bit sludge grinder oscillatory system depending on the main geometric design parameters are presented. Based on the solution of the mathematical model of the operation of the oscillatory system of the slurry grinder, the law of the relative rotation of the drum-mesh and its angular velocity of rotation relative to the shaft of the turbodrill (bottom of the drilling tool) is established. According to the results of solving the equation of motion of the working elements near-bit slurry grinder, the dependence of the amplitude of the angular velocity of the relative vibrational rotation of the drum on the stiffness of the springs and on the moment of inertia of the drum is established.

1. Introduction

The bottoms of oil and gas wells during drilling, as a rule, become clogged with large drill cuttings, which leads to a deterioration in the process of rock destruction and a decrease in the performance of bits [1]. With an increase in drilling depth, more and more sediment remains in the face, consisting mainly of large-sized sludge [2]. The removal of sludge is complicated when drilling deep vertical, deviated and horizontal wells, as well as when drilling shallow wells, when the circulation of the flushing fluid does not create a sufficient upward flow for the removal of sludge. The most urgent is the problem of cleaning the horizontal wellbore from cuttings. Flushing a horizontal wellbore is characterized in that the flushing fluid flow velocity vector is perpendicular to the slurry particle deposition rate vector. The absence of a vertical component of the flow velocity vector causes sludge to settle on the bottom wall of the well [3–6]. If sludge particles are not removed efficiently enough, they will settle around the string, usually in the area of the bottom assembly of the drill string, blocking pipes and clamping the string until sticking occurs.

In horizontal drilling, all drill cuttings tend to sink to the bottom wall of the well. Two sludge transport mechanisms are possible:

1) At a sufficiently high flow velocity, turbulent pulsations (vortices) pick up sludge particles and transfer them to the region of high flow velocities. When the vortex decays, the particles begin to settle until they are picked up by the next vortex. Thus, the transport of particles in suspension.
2) If the average value of the transverse velocity pulsations is less than the sedimentation rate of solid particles, the solid phase will begin to settle on the bottom wall of the well, forming sediments. Settled particles could move over the surface of the sediment if the force exerted on the particle by the liquid is greater than the forces of resistance to the movement of the particle. Such a mechanism for moving a particle is called moving in an entrained state [7].

To prevent sedimentation of sediment and the formation of sediment, it is necessary that the particles of the cuttings are in a suspended rather than entrained state. But, an increase in the supply and an increase in the density of the drilling fluid can cause undesirable consequences: the mud of the well bottom; increase in excess pressure on permeable formations, up to their hydraulic fracturing; erosion of the walls of the well during drilling of unstable rocks [3, 8].

The condition for the transport of sludge particles along the bottom of the channel by a turbulent flow [2]:

\[
Q = U \cdot S_{Kn} \geq \sqrt{\frac{5.3\Delta \rho \cdot d \cdot f \cdot (1 + 0.46e^{2e})}{\lambda \cdot \rho}} \cdot S_{kn},
\]

where \( Q \) is the flow rate of flushing fluid, \( U \) is the upward velocity of flushing fluid in the annular space, \( S_{Kn} \) is the largest area of the annular space, \( \Delta \rho \) is the pressure drop in the eccentric annular space, \( d \) is the equivalent diameter of the solid particle, \( f \) is the coefficient of resistance to movement, \( e \) is the relative the eccentricity of the drill string, \( \lambda \) is the coefficient of hydraulic resistance, \( \rho \) is the density of the fluid.

Calculations show that it is possible to transport particles of sludge in size of not more than 1-2 mm in suspension. Larger particles, even with a turbulent flow regime, will settle on the bottom wall of the well and can only move in the entrained state [9, 10].

A new technical solution for cleaning vertical, deviated and horizontal wells from cuttings using a slurry grinder is proposed. A near-bit design was developed for crushing, sieving and slurry removal from the bottom when drilling vertical, deviated and horizontal wells (Figure 1).

Figure 1. Structural diagram of a near-bit slurry grinder
The device is installed during rotary drilling (rotary method, downhole motors) above the bit. When drilling a well, the sub 2 rotates, drives the screw 3 and the rotor 4. The rotating sub 2 and the drum connected by springs 13 and 14 form an oscillating system. The auger directs the flushing liquid with slurry into the internal cavity of the device. The auger directs the liquid with slurry in the axial direction and in the radial direction, while the slurry is partially crushed due to impacts, small particles are sifted through the grid (mesh holes 2–3 mm), large particles fall into the rotor and are thrown radially to the reflector, further grinding of the slurry occurs. The final grinding occurs when large particles enter the channels 16 and crush them with the end face of the rotor. Purification of the solution is facilitated by torsional vibrations of the drum (mesh), which occur under the influence of the inertia of the drum, the fluid ejected by the rotor onto the eccentric reflector, and spring forces [9].

The device is designed for drilling soft and medium hard formations. Grinding sludge using a near-bit device will help prevent accumulation of sludge dune on the bottom wall of a horizontal well.

2. Methods and materials
To determine the rational geometrical dimensions of the near-bit sludge grinder elements that ensure the stable operation of the device, theoretical and experimental studies are necessary.

To determine the law of motion of the drum relative to the rotating shaft, analytical studies of the processes under study were carried out using the classical principles of modern theoretical mechanics.

3. Results
The rotating shaft of the turbo-drill and the drum, connected by springs, form an oscillating system. When the turbo-drill shaft rotates due to the presence of springs, torsional vibrations of the drum are created, which ensures grinding and sieving of the sludge.

The rotating drum receives rotation from the shaft through an elastic spring of stiffness “с”. The rotation of the drum is coaxial with the shaft, on which the torque \( M_{\text{ap}} \) acts. We believe that the axial moments of inertia of the drum \( J_6 \) and the shaft \( J_r \) relative to the axis of rotation \( O \). We neglect the spring mass.

The mechanical system (shaft, drum, spring) begins to work from a state of rest under the action of a torque \( M_{\text{ap}} \). It is assumed that when the shaft rotates under the action of spring elasticity, the drum will oscillate rotationally with respect to the shaft. The nature of these oscillations will depend on the torque acting on the shaft, \( M_{\text{ap}}=M_{\text{ap}}(t) \). The task is to determine the law of torsional vibrations of the drum relative to the rotating shaft.

We believe that as an assumption, and based on the principle of superposition of vibrations, in this case we will consider torsional vibrations without taking into account their influence on longitudinal vibrations, i.e. longitudinal and torsional vibrations do not affect each other (manifest independently of each other).

Consider the dynamic transmission of rotation from the leading link of the mechanism (shaft) to the driven link (drum) through an elastic spring (Figure 2).

When the shaft begins to rotate under the influence of the torque \( M_{\text{ap}} \) , the spring first twists (moving point A of the spring with the shaft) relative to the stationary drum (point B of the spring with the drum); in this case, an internal elastic moment \( M_f \) (counteracting the moment \( M_{\text{ap}} \)), arises in the spring, which increases in proportion to the spring angle \( \varphi_{\text{ap}} \). After overcoming the inertia of the rest of the drum, the last one begins to turn in the direction of \( M_{\text{ap}} \) by the value of the angle \( \varphi_6 \) (moving – turning point B). Therefore, at a certain point in time \( t \), the angle of rotation of the shaft (relative to the fixed space XOY) \( \varphi_s \) will be determined by the sum of the angle of rotation of the drum \( \varphi_6 \) and the angle of rotation of the spring \( \varphi_{\text{ap}} \), i.e. \( \varphi_s=\varphi_6+\varphi_{\text{ap}} \). It follows that initially the rotation of the drum will lag behind the rotation of the shaft by the value of the angle of rotation of the spring \( \varphi_{\text{ap}} \), and this is essentially the circular displacement of the drum relative to the shaft. The function \( \varphi_{\text{ap}}(t) \) describes the relative rotation of the drum; and the function \( \varphi_s(t) \) is the portable rotation of the drum (together with the shaft). Thus, the task is to determine the relative rotation of the drum, that is, the function of changing the angle of rotation of the spring \( \varphi_{\text{ap}}(t) \) depending on the actual torque \( M_{\text{ap}} \) on the shaft.
The dynamic transmission of rotation from the leading link of the mechanism (shaft) to the driven link (drum) through an elastic spring

The problem is solved on the basis of Lagrange equations of the second kind. The system has two degrees of freedom. The generalized coordinates $q_1$ and $q_2$ of the mechanical system are selected (independent parameters that uniquely determine the position of all elements of the mechanical system at any time.)

Accept:

- $q_1 = \varphi_\text{v}(t)$ is the angle of rotation of the shaft;
- $q_2 = \varphi_\text{пр}(t)$ is the angle of twisting of the spring (relative angle of rotation of the drum).

In the future, for the convenience of judgment, we take $\varphi_\text{пр}(t) = \varphi_\text{бr}(t)$ – the angle of rotation of the drum relative to the shaft (relative rotation of the drum).

Thus, we accept:

- $q_1 = \varphi_\text{v}(t)$;
- $q_2 = \varphi_\text{бr}(t)$.

The initial Lagrange equations in accordance with the degrees of freedom:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_1} \right) - \frac{\partial T}{\partial q_1} = Q_1; \quad \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_2} \right) - \frac{\partial T}{\partial q_2} = Q_2,$$

where $T$ is the kinetic energy of the entire mechanical system; $Q_1$, $Q_2$ are the generalized forces of a mechanical system corresponding to each generalized coordinate.

Next is the kinetic energy of the mechanical system created by the rotating shaft and drum (the spring is neglected due to the relatively small mass), taking into account that $\varphi_\text{пр}(t) = \varphi_\text{бr}(t)$.

$$T = T_\text{v} + T_\text{б} = \frac{1}{2} J_\text{v} \ddot{\varphi}_\text{v}^2 + \frac{1}{2} J_\text{б} \ddot{\varphi}_\text{бr}^2 =$$

$$= \frac{1}{2} \left[ (J_\text{v} + J_\text{б}) \dot{\varphi}_\text{v}^2 + J_\text{б} \dot{\varphi}_\text{бr}^2 - 2 J_\text{б} \dot{\varphi}_\text{v} \dot{\varphi}_\text{бr} \right],$$

where $T_\text{v}$ and $T_\text{б}$ are the kinetic energy of the shaft and drum, respectively, and $J_\text{v}$, $J_\text{б}$ are the axial moments of inertia of the shaft and drum.

The expression (2) determines the values of the derivatives for the Lagrange equation (1). Next, we find the expressions of the generalized forces $Q_1$ and $Q_2$ for differential equations (1). To determine the expressions $Q_1$ and $Q_2$, we consider a mechanical system as a system with ideal bonds (without friction in the supports). Therefore, the possible work will be performed by the torque $M_\varphi$ on the rotor shaft and the elastic moment $M_\varphi$ of the spring.

Based on this

$$Q_1 = \frac{\partial A_1}{\partial q_1} = \frac{\partial A_1}{\partial \varphi_\text{v}} = M_\varphi; \quad Q_2 = \frac{\partial A_2}{\partial q_2} = \frac{\partial A_2}{\partial \varphi_\text{бr}} = -c \varphi_\text{бr},$$

where $A$ is work; $c$ – spring stiffness.
Taking into account expressions (2) and (3), the initial differential equations (1) take the form

\[
\begin{align*}
\left( J_u + J_o \right) \dddot{\phi}_u - J_o \cdot \dddot{\phi}_{or} &= M_{\text{ap}}; \\
J_o \left( \dddot{\phi}_{or} - \dddot{\phi}_u \right) &= M_{\text{ap}}.
\end{align*}
\]

(4)

From the system of Lagrange equations (4) we obtain the differential equation of relative rotation of the drum

\[
\dddot{\phi}_{or} + k^2 \cdot \dddot{\phi}_{or} = aM_{\text{ap}},
\]

(5)

where \( k = \sqrt{\frac{1}{J_u} + \frac{1}{J_o}} \) and \( a = \frac{1}{J_u} \).

For torsional vibrations, the expression of torque for the general case has the form

\[
M_{\text{ap}} = M_{\alpha} + A \sin \alpha \tau,
\]

where \( M_{\alpha} \) is the average value of the torque, \( A \) is the amplitude of the momentum oscillations \( M_{\alpha p} \), \( A \), \( \omega \) are constants.

In this case, differential equation (5) takes the form

\[
\dddot{\phi}_{or} + k^2 \dddot{\phi}_{or} = M + N \sin \omega t,
\]

where \( M = aM_{\alpha} = \frac{M_{\alpha}}{J_u} = \text{const} \), \( N = aA = \frac{A}{J_u} = \text{const} \).

The solution to the obtained differential equation of forced torsional vibrations of the drum (5) is found at zero initial conditions: at \( t=0 \), \( \phi_{or}=0 \).

We obtain expressions for determining the relative rotation of the drum \( \phi_{or} \) and its relative angular velocity of rotation \( \omega_{or}(t) \):

\[
\phi_{or} = \frac{-N\omega}{k(k^2 - \omega^2)} \sin kt + \frac{M}{k^2} \cos kt + \frac{N}{k^2 - \omega^2} \sin \omega t,
\]

(6)

where \( k \) is time.

The first two terms of expression (6) determine vibrations with a natural frequency \( k \) (natural vibrations), and the third term defines forced vibrations of a drum with a frequency \( \omega \) of torque.

At the same time, we find the angular velocity of the relative vibrational rotation of the drum

\[
\omega_{or} = \frac{-N\omega}{k^2 - \omega^2} \cos kt + \frac{M}{k} \sin kt + \frac{N\omega}{k^2 - \omega^2} \cos \omega t,
\]

(7)

where \( r \) is time.

When calculating the angular velocity of the drum relative to the shaft \( \omega_{or} \), the optimality criterion is the maximum swing of the angular velocity of the drum relative to the shaft \( \Delta \omega_{or} \):

\[
\omega_{or\text{max}} - \omega_{or\text{min}} = \Delta \omega_{or}.
\]

(8)

From expression (6), one can obtain the equation (law) of the relative torsional vibrations of the drum for simpler expressions of the torque \( M_{\text{ap}} \) (special cases).

In the case of \( M_{\text{ap}} = A \sin \omega t \).

\[
\omega_{or} = \frac{-N\omega}{k(k^2 - \omega^2)} \cos kt + \frac{N\omega}{k^2 - \omega^2} \cos \omega t,
\]

(9)

In this case, the drum performs its own torsional vibrations with a frequency \( k \) (first term) and forced oscillations with a frequency \( \omega \) (second term).

In the case of \( M_{\text{ap}} = M_{\theta} = \text{const} \).

\[
\phi_{or} = \frac{M}{k^2}(1 - \cos kt),
\]

(10)
those, the drum will only perform its own torsional vibrations (with frequency k) relative to the rotating shaft.

\[ \omega_{\omega} = \frac{\dot{\varphi}_\omega}{\varphi_\omega} = \frac{M}{k} \sin kt. \]

It is of interest to determine the law (equation) of rotation of the shaft \( \varphi_\omega(t) \) for a given expression of the moment of rotation \( M_\omega \). Having obtained the law of the relative rotational motion of the drum \( \varphi_\beta(t) \), we can find the law of rotation of the shaft \( \varphi_\omega(t) \).

In the case of \( M_\omega = M_0 = \text{const} \), the angle of rotation of the shaft is the sum of the angle of rotation of the drum together with the shaft (first term) and the relative rotation of the drum \( \varphi_\beta \), i.e.

\[ \varphi_\omega(t) = \frac{J_\beta}{J_\omega + J_\beta} \left[ \frac{1}{2} M t^2 + \varphi_\omega(t) \right] \]

Find the angular velocity of the shaft

\[ \omega_\omega = \frac{\dot{\varphi}_\omega}{\varphi_\omega} = \frac{J_\beta}{J_\omega + J_\beta} \left[ \frac{M t + \dot{\varphi}_\omega}{J_\omega + J_\beta} \right] = \frac{J_\beta}{J_\omega + J_\beta} \left[ M \cdot t + \frac{M}{k} \sin kt \right] \]

In the case of \( M_\omega = A \sin \omega t \), the angle of rotation of the shaft is

\[ \varphi_\omega = \frac{A}{(J_\omega + J_\beta) \omega} \left( t - \frac{1}{\omega} \sin \omega t \right) + \frac{J_\beta}{(J_\omega + J_\beta)} \varphi_\beta. \]

Angular shaft speed

\[ \omega_\omega = \varphi_\omega = -\frac{d\varphi_\omega}{dt} = \frac{A}{(J_\omega + J_\beta) \omega} \left( 1 - \cos \omega t \right) + \frac{J_\beta}{(J_\omega + J_\beta)} \varphi_\beta. \]

By solving expression (8), we find the values \( \Delta \omega_\omega \) for springs with different stiffness. The problem was solved under the following conditions: a bit of type 215,9T3-ГН15, a turbodrill 3TCIII1-195, an average flow rate of flushing fluid of 0.023–0.028 m³/s, an average axial load on a bit of 180 kN, \( \omega = 6.28 \) rad/s – the frequency of change in torque, \( J_\omega = 4.5 \text{ kg} \cdot \text{m}^2 \) – the moment of inertia of the bit with the turbo-drill rotor and the near-bit shaft of the device, \( J_\beta = 0.346 \text{ kg} \cdot \text{m}^2 \), \( M_\omega = 1131 \text{ N} \cdot \text{m} \) – the average value of the torque, \( A = 246 \text{ N} \cdot \text{m} \) – the vibration amplitude \( M_\omega \).

Based on the obtained values, a graph of the change in the magnitude of the angular velocity of the drum relative to the rotating shaft is constructed (Figure 3). With an increase in spring stiffness from 400 to 1000 N/m, the oscillation range decreases from 14 to 9.5 rad/s.

Let us determine the dependence of \( \Delta \omega_\omega \) on \( J_\beta \). By solution (8) we find the values \( \Delta \omega_\omega \) for various values of \( J_\beta \). The problem was solved under the following conditions: a bit of type 215,9T3-ГН15, a turbodrill 3TCIII1-195, an average flow rate of flushing fluid of 0.023–0.028 m³/s, an average axial load on a bit of 180 kN, \( \omega = 6.28 \) rad/s – the frequency of rotation moment, \( J_\omega = 4.5 \text{ kg} \cdot \text{mN/m} \) – spring stiffness – moment of inertia of a bit with a turbodrill rotor and near-bit device shaft, \( c = 400 \text{ N/m} \) – spring stiffness, \( M_0 = 1131 \text{ N} \cdot \text{m} \) – average torque value, \( A = 246 \text{ N} \cdot \text{m} \) – the amplitude of the oscillations \( M_\omega \).
Magnitude of the angular velocity of the drum relative to the rotating shaft, rad/sec

Stiffness of the spring, N/m

**Figure 3.** The dependence of the magnitude of the angular velocity of the drum relative to the rotating shaft from the stiffness of the springs

Magnitude of the angular velocity of the drum relative to the rotating shaft, rad/sec

Inertia momentum of the drum, kg m²

**Figure 4.** The dependence of the magnitude of the angular velocity of the drum relative to the rotating shaft from the moment of inertia of the drum

The dependence of the magnitude of the angular velocity of the drum relative to the shaft on the moment of inertia of the drum is shown in Figure 4. With an increase in the moment of inertia of the drum from 0.173 kg ∙ m² to 0.43 kg ∙ m², the amplitude of the relative vibrational rotation of the drum increases from 10.4 to 16.2 rad/s.

4. **Conclusion**

Based on the study of the developed mathematical model of the oscillatory motion of the drum near-bit slurry grinder, the dependencies of the relative angular velocity of the drum on the parameters of the oscillatory system – spring stiffness and drum inertia – are obtained. It was found that for a bit with a diameter of 215.9 mm, an increase in the moment of inertia of the drum from 0.173 to 0.43 kg ∙ m² increases the magnitude of the angular velocity of the drum from 10.4 to 16.2 rad/s, while increasing the stiffness of the springs from 400 to 1000 N/m, the magnitude of the angular velocity decreases from 14 to 9.5 rad/s.
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