XFEM Analysis of the Effect of Fiber Volume Fraction on Crack Propagation in Fiber Reinforced Ceramic Matrix Composites

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Abstract. The mechanism of crack propagation near the interphase plays an important role in the failure procedure of ceramic matrix composites. The aim of this paper is to examine the effect of fiber volume fraction on the crack propagation. A three-phase unit-cell model was used to describe the micro-structure of fiber reinforced ceramic matrix composites. The interphase between fiber and matrix was modelled as a finite-thickness cylinder around the fiber. The extended finite element method (XFEM) was applied to simulate the crack initiation and propagation in the unit-cell. Without the definition of crack path priori, the approach based on XFEM can simulate the arbitrary path of cracks growing in the composites. The crack patterns and stress-strain responses for different fiber volume fraction were compared. In particular, the penetration/deflection mechanism when the matrix cracks grow to the interphase was analysed and compared. The results show that the initiation and propagation of the secondary crack/cracks were significantly influenced by the fiber volume fraction.

1. Introduction

Owing to the excellent thermo-mechanical properties, the continuous fiber reinforced ceramic matrix composites (CMCs) are promising structural candidates for advanced high-performance structure, such as turbine engine, hypersonic aircraft, gas cooled fast reactor, and fusion reactor[1-3]. In these applications, the components made of CMCs should have ability to operate at high-temperature and maintain their strength and toughness. To this end, the CMCs consist of a ceramic matrix and a reinforcing fiber and this combination can lead to a composite with superior properties to the constituents. Therefore, the understanding of the connection between composite behaviors and constituents characteristics has an important role in tailoring the properties of CMCs components.

It is now well demonstrated that the mechanical behavior of fiber reinforced CMCs is strongly dependent on the properties of the fiber/matrix interface [4-6]. In a properly designed CMCs, interphase can arrest, deflect and branch the propagating cracks that have initiated at outer or void surfaces of the matrix. The most common interphase is pyrolytic carbon (PyC) or boron nitride (BN). During further loading process, the deflected cracks in the interphase enables energy dissipation through interface sliding and consequently allows a pseudo-ductile failure behavior in CMCs [7].
Therefore, it is necessary to study the crack propagation in the interphase of CMCs. Many researches had been performed in order to capture the mechanism of crack deflection in CMCs.

Previous authors had studied the case where the crack deflects at an interface between fiber and matrix using micromechanics. Cook et al. [8] predicted the crack deflection at interfaces using a stress criterion considering the interfacial debonding caused by the approaching crack tip into consideration. Gupta et al. [9] determined the crack propagation using the ratio of maximum normal stress for the deflected and penetrated crack. He et al. [10] developed a deflection criterion by comparing the energy release rates for the deflected and penetrated crack. Within these studies, conditions for crack deflection were established by comparing a toughness or a strength ratio to a critical ratio depending on the elastic mismatch between the two constituents. Most of these studies cannot consider the structure and properties of the interphase with a finite thickness, they treated the interphase as an interface with zero thickness.

A few attempts aimed at modeling the propagation of a matrix crack through numerical technology. By numerical method, the thickness of interphase between fiber and matrix can be considered. Carrere et al. [11] computed strain energy release rates using the finite element method (FEM) at the penetrated or deflected crack tip of CMCs with a finite interphase thickness. The extension of a penetrated or deflected crack was pre-defined. Liu et al. [12] combined the virtual crack closure technique (VCCT) and a finite element model to predict the competition between the matrix crack deflection and penetration in C/C composites. They numerically investigated the effect of interphase by considering the thickness of interphase or not. Therefore, a few numerical studies based on FEM have discussed the influence of a finite interphase on the crack propagation of composites. But, it is still a great challenge to model the deflected and branched cracks in the interphase of CMCs.

In general, the approaches based on classical finite element method such as VCCT, are powerful numerical tools for cracking simulation [13]. These approaches usually possess high requirement of modeling skills and computing resource because of mesh refinement at the crack tip and pre-defined crack path [14]. To avoid these disadvantages, the extended finite element method (XFEM) has been developed and revealed to be a useful numerical tool for strength and fracturing problems. In the past decades, the XFEM has been combined with the level-set method (LSM) and provided another methodology for simulation of crack propagation involving interface of dissimilar materials. Several works [15-17] had been performed to study numerically the cracking process of bi-material interfaces in composites using XFEM. Recently, Braginsky et al. [18] simulated the crack deflection in CMCs using the extended finite element method (XFEM). The XFEM are widely used to simulate the crack propagation along an arbitrary path. They discussed the effects of relative stiffness and strength of interphase on the crack propagation. It is obviously that a similar methodology can be used to analysis the influence of other characteristics of microstructure.

As mentioned, phenomena of vital importance in CMCs are the mechanism of crack propagation, particularly the alternative of penetration/deflection at the interface between fiber and matrix. It has been well demonstrated that the mechanical behaviors of CMCs are strongly dependent on the interface debond process, i.e., the deflection of matrix crack at interface. Theoretical analyses dealing with the crack propagation near the fiber/matrix interface are essential to understanding how and what extent the mesoscopic characteristic, such as macro-geometry and constituents properties, influence the macroscopic structural performance of the CMCs.

After reviewing the main approaches for the classical problem of crack deflection at fiber/matrix interface in CMCs, we intend to develop a simulation method: (1) considering the actual thickness of interphase, and (2) allowing the initiation of crack. To this end, the XFEM was applied to simulate the crack propagation in a unit-cell of unidirectional fiber reinforced CMCs with a finite interphase thickness. The propagation process of a primary matrix crack were simulated and the effect of fiber volume fraction, $V_f$, is discussed. A clear understanding of this mechanism of crack propagation at the interphase and the influence of related factor, e.g. $V_f$, is necessary for the optimal designs of CMCs.
2. Analysis methodology

According to the concept of representative volume element (RVE), the unit-cell model composed of a single fiber composite cylinder with axisymmetric boundary conditions is often employed to study the mechanical behavior of fiber/matrix interface. Here, this unit-cell was used to model a representative SiC/SiC composites with BN interphase. Consider a unit-cell with cylindrical reinforcements of circular cross-section (fiber) surrounded by cylindrical interphase and embedded in a matrix material (Figure 1). Table 1 lists the basic properties, e.g. elastic modulus and Poisson's ratio, of each constituent. The matrix, fiber and interphase are characterized as isotropic elastic constitutive law. An annulus pre-crack normal to the fiber/matrix interface was incorporated into the matrix in the unit-cell. Multiple matrix cracking is not taken into account here as the present study consider the early stage of crack propagation. To simulate failure procedure under tensile in the fiber direction, strain-controlled loading with uniform displacements was applied at the both ends of cylindrical unit-cell. Considering the unit-cell as an axisymmetric problem, the loading and boundary conditions are illustrated in Figure 1.

![Figure 1. Geometry and axisymmetric model of the representative SiC/SiC composites.](image)

All of the simulations were performed using the commercially software ABAQUS. The response of a pre-crack unit-cell model under quasi-static tensile loading was analyzed by the XFEM implemented in this software. Axisymmetric elements were chosen for this type of analysis. Table 1 lists the basic parameters, initial stress and fracture energy, for crack simulations. In ABAQUS, the additional parameter damage stabilization and tolerance is necessary to aid convergence of the XFEM solution. These parameters of each constituent are also listed in Table 1 according to references [18] and [19]. To setup the simulation using XFEM, a maximum principal stress criterion is chosen to predict the crack initiation, and a cohesive law is applied to govern the crack propagation. Accordingly, the values of this initiation stress (Tables 1) are referred to as the strength of each constituent. For the cohesive law, the cohesive stiffness defined by the fracture energy is degraded once the initiation criterion is met.

In general, the approaches based on classical finite element method such as VCCT, are powerful numerical tools for cracking simulation. However, their use in the case where secondary cracks initiates and propagates is very difficult. The apparent lack of such studies is possibly due to numerical challenges associated with the prediction of crack path. The FEM has been widely used to model
cracks with pre-defined locations and lengths. However, modeling crack initiation is challenging due to the fact that the FEM does not efficiently handle moving interfaces and discontinuities (such as cracks) because of the need for re-meshing. The approaches based on VCCT has been used to simulate crack propagation by inserting cohesive elements along potential crack paths. Therefore, the problem of re-meshing may be avoided if the crack path is assumed to be known a priori, but in general, the simulation of crack propagation remains a challenge. According to the present study, the propagation of primary crack and the initiation of secondary cracks can be simulated by using XFEM. Using this technology, there is no need to re-mesh the model after crack propagation and the crack can initiate and propagate along an arbitrary path.

**Table 1.** Properties of a representative SiC/SiC composites with BN interphase.

| Property                  | Matrix (SiC) | Interphase (BN) | Fiber (SiC) |
|---------------------------|--------------|-----------------|-------------|
| Elastic modulus, $E$ (GPa) | 360          | 10              | 380         |
| Poisson's ratio, $\nu$    | 0.185        | 0.05            | 0.185       |
| Initiation stress, $\sigma_I$ (MPa) | 800      | 75              | 2600        |
| Fracture energy, $\Gamma$ (J/m²) | 36       | 5               | 50          |
| Tolerance                 | 0.05         | 0.05            | 0.05        |
| Damage stabilization      | 0.005        | 0.01            | 0.005       |

It is clear that the pre-define propagation path for cracking is no longer necessary for the simulation based on XFEM. So, the crack deflection mechanism can be modeled without assumptions of deflect position. Therefore, the XFEM is suitable for the simulation of crack propagation within CMCs which contains complex mechanism. But, the implementation of XFEM in ABAQUS employed here does not allow for crack coalescence. This is a major limitation when the matrix crack and secondary cracks attempt to interact and coalesce. For all the simulations performed here, results were only considered up to the moment when the cracks grew to within two elements of each other just prior to pending crack coalescence.

3. **Analysis methodology**

This work concentrates on the influence of fiber volume fraction on the crack propagation in fiber reinforced CMCs. Several numerical studies were performed with different unit-cells. Table 2 lists the geometry parameters of the unit-cells for the studied SiC/SiC composites. The length of unit-cell is chosen as $L=10R_f$ to ensure a uniform remote axial strain. The thickness of BN interphase is assumed to be $0.5\mu m$ as the interphase thickness is usually $<100$ nm for multilayer interphase and $<1\mu m$ for monolayer interphase. In order to study the effect of fiber volume fraction on the crack propagation, a set of values for the fiber volume fraction, $V_f$, is selected between 10% and 50%. The radius of unit-cell, $R_c$, can be calculated accordingly.

**Table 2.** Geometry parameters of the unit-cell.

| Parameters                  | Data                  |
|-----------------------------|-----------------------|
| Length of the unit-cell, $L_c$ (μm) | 65                    |
| Radius of fiber, $R_f$ (μm) | 6.5                   |
| Length of the pre-crack, $a_c$ (μm) | 2                     |
| Thickness of interphase, $t_i$ (μm) | 0.5                   |
| Fiber volume fraction, $V_f$ | 10%, 15%, 20%, 30%, 40%, 50% |

3.1. **Simulation of crack patterns**

Figure 2 shows a typical procedure of crack propagation in the unit-cell of SiC/SiC composites with a $V_f=20\%$. As can be seen, the primary matrix crack grown toward the interphase as the increasing load.
In the meantime, the stress-strain response of the unit-cell behaves linearity. As the load increases further, several cracks initiated inside the interphase when the primary matrix crack propagate to a finite distance from the matrix/interphase (M/I) interface. And the stress-strain curve begins to deflect. These cracks is indicated as secondary crack/cracks. The mechanism of secondary cracks in CMCs had been observed and discussed by several studies [11]. They believed that the stress concentration induced ahead of the matrix crack can initiate a secondary crack along the fiber/matrix interface. The secondary crack or cracks may grow within the interphase. As the applied displacement at the end of unit-cell increased continuously, the load carrying capacity may decrease appearing as the decline of stress. This is probably caused by the propagation of the primary matrix crack and the secondary crack/cracks. The primary crack propagates in the matrix along the radial direction of unit-cell towards M/I interface. In the same time, the secondary crack/cracks propagate in the interphase along the fiber direction. These two cracks will tend to interact and coalesce when they grow to the vicinity of each other. As mentioned forward, the coalescence of cracks is not numerically possible in this study. So the present simulations were interrupted artificially just before the moment of crack coalescence and the crack patterns at the last moment were also plotted in Figure 2.

![Figure 2](image2.png)

**Figure 2.** A typical procedure of crack propagation in a SiC/SiC unit-cell with BN interphase of fiber volume fraction, \( V_f = 20\% \).

![Figure 3](image3.png)

**Figure 3.** Stress components of the stress field inside the interphase before the secondary cracking in the interphase with matrix crack tip at a distance away from the interphase/matrix interface for \( V_f = 20\% \); (a) \( \sigma_r \) in the interphase, (b) \( \sigma_{rz} \) in the interphase, (c) \( \sigma_{zz} \) in the matrix and interphase.
Figure 3 shows the stress states in the interphase when the primary crack tip is removed some finite distance from the M/I interface for the unit-cell of a medium fiber volume fraction, $V_f = 20\%$. We analyzed the stress contours as shown in Figure 3 according to the stress components. The stress distributions varied greatly for different components. It can be concluded that the tensile stresses in radial direction are a major damage pattern when the matrix crack grows toward the interphase. When the crack tip grows near the M/I interface, the radial stress drives one or multiple secondary crack initiation along fiber direction. In most cases when the secondary crack initiated before the primary crack intersected the M/I interface, it is observed that the secondary and primary crack continued to grow.

The competition between crack deflection and penetration at the fiber/matrix interface is the basis of toughening for CMCs. It has been found that the matrix crack tends to deflect inside the interphase or at the M/I interface in the most of CMCs reinforced with treated fibers (indicating a strong fiber/interphase bond). According to further studies, a deflected crack in the interphase may initiate before the primary matrix crack grows into the interphase just like the simulated results of this study. Numerical simulations dealing with the crack propagation near the fiber/matrix interface are essential to understanding how and what extent the characteristics of microstructure influence the macroscopic performance of the CMCs. To this end, variations in the fiber volume fraction on the initiation of secondary cracks will be discussed in the following part.

### 3.2. Effects of the fiber volume fraction

As mentioned, the characteristics of microstructure, such as fiber volume fraction and interphase thickness, have important influences on the macroscopic behavior of CMCs. In this part, the effect of fiber volume fraction on the crack propagation in CMCs will be discussed. The fiber volume fraction of the CMC can be controlled by adjusting the prepregging conditions. In this study, the range of fiber volume fraction varied from 10\% to 50\%.

Recognizing the significance of the stress concentration at the crack tip near a fiber/matrix interface, increasing efforts are being directed towards employing numerical methods. The use of the XFEM in particular allows amore effective simulation of the crack propagation in practical composites containing a weak interphase. In addition, the specific micro-geometries, i.e. the fiber volume fraction, as well as the interphase of finite thickness can be properly taken into account.

![Variety of crack patterns](image)

**Figure 4.** Variety of crack patterns after the initiation of secondary cracks in the interphase of SiC/SiC unit-cell with different fiber volume fractions.
Figure 4 shows the variety of crack patterns after the initiation of secondary cracks in the interphase for different $V_f$. As can be seen, secondary crack or cracks initiated in the interphase before the primary crack arriving at the interface of matrix/interphase in all cases. As the fiber volume fraction increased, the initiated location of the secondary cracks varied. For unit-cell with large fiber volume fraction, the secondary cracks prefer to initiate near the M/I interface. In the other hand, the secondary cracks tend to initiate near F/I interface when the fiber volume fraction is low. What’s more, multiple cracks initiated in the interphase simultaneously for a medium fiber volume fraction (30%). The variety of crack patterns is properly a result of local microstructures. To a large extent, the fibers in unit-cells with lower fiber volume fraction would carry more load after matrix cracking. The fiber loading for each unit-cell was approximated based on the ratio of cross-sectional area between fiber and unit-cell.

The distance of the primary crack tip from the M/I interface and the stress capacity of the unit-cell, immediately prior to secondary crack initiation in the interphase, were plotted as a function of fiber volume fraction in Figure 5. As can be seen, the distance of the primary crack tip from the M/I interface decreased as the fiber volume fraction increasing. Meanwhile, the stress capacity of the unit-cell also decreased. As a displacement loading was applied, the constraint reaction at the end of the unit-cell was extracted to calculate the stress capacity. Examination of this figure shows that the dependence of these parameters on the fiber volume fraction indicate that both parameters are clearly influenced by the microstructure of CMCs.

![Figure 5](image-url)

**Figure 5.** Distance of the primary crack tip from the interphase/matrix interface and the stress capacity of the unit-cell, immediately prior to secondary crack initiation in the interphase as a function of fiber volume fraction.

Figure 6 shows that the fiber volume fraction strongly modifies the propagation of the secondary crack in the SiC/SiC composites. As shown previously, the crack patterns are with great diversity for different fiber volume fraction. In the case of a low fiber volume fraction, the secondary crack/cracks tend to initiate within the interphase close to the fiber. In the other hand, the cracks prefer to initiate near the matrix within a high fiber volume fraction. With further loading, the secondary cracks propagated in the interphase along fiber direction while the primary crack grew towards the interphase. At the end of the simulation due to numerical difficulties, the final crack patterns before the intersection of secondary cracks and primary crack were compared in Figure 6.

To examine the dependence of the propagation of secondary cracks on the fiber volume fraction, the length of secondary cracks at the time just prior to the intersection with primary crack was plotted...
as a function of fiber volume fraction. As can be seen, the secondary crack length decreases with the increase of fiber volume fraction.

![Variety of crack patterns before the intersection of secondary cracks and primary crack.](image)

**Figure 6.** Variety of crack patterns before the intersection of secondary cracks and primary crack.

![Secondary crack length at the time just prior to the intersection with primary crack versus fiber volume fraction.](image)

**Figure 7.** Secondary crack length at the time just prior to the intersection with primary crack versus fiber volume fraction.

A comparison between different cracking patterns were illustrated in Figure 4 and 6. In the view of micromechanics, mechanical behavior of a material or component is directly influenced by its microstructure. Accordingly, these competing crack modes are a result of the differences between the stress states in the interphase induced by the variety of microstructure, i.e. the unit-cells with different fiber volume fraction. To capture the difference of stress states, the stress contours in the interphase in front of the primary crack tip at the time before secondary crack occurs for $V_f=10\%$ were plotted in Figure 8. Comparing to the stress contours for for $V_f=20\%$ (as shown in Figure 3), we can see that the stress distributions are very different versus fiber volume fraction. The major similarity is that the normal stress in the radial direction, $\sigma_r$, dominates the stress field in the interphase for the both cases when the primary crack tip is at some distance from the interphase. This is the reason why crack
deflection occurred in all of the simulations. Specifically, the distribution of stress along fiber direction and radial direction are both of great differences for different fiber volume fractions. Obviously, these differences explain the variety of initial locations of the secondary cracks.

Figure 8. Stress components of the stress field inside the interphase before the secondary cracking in the interphase with matrix crack tip at a distance away from the interphase/matrix interface for $V_f=10\%$: (a) $\sigma_{r}$ in the interphase, (b) $\sigma_{rz}$ in the interphase, (c) $\sigma_{zz}$ in the matrix and interphase.

4. Conclusions

In this paper, a numerical study of the classical problem of crack deflection in the interphase between fiber and matrix of CMCs has been conducted. Unlike approaches based on classical fracture mechanics or numerical methods based on traditional FEM, the present method considered the actual thickness of interphase and allowed for crack initiation and propagation without pre-defined path. This should own to the capability of cracking simulation by using the approaches based on XFEM. A series of numerical simulations has been performed to examine the effect of the fiber volume fraction on the crack propagation in fiber reinforced CMCs. The findings from the numerical results can be summarized as follows:

1. For the studied SiC/SiC composites with BN interphase, the crack deflection mechanism at fiber/matrix interface is dominated by secondary crack initiation inside the interphase before the arrival of the matrix crack’s tip. Moreover, at the moment when the secondary cracks initiated, the distance from the primary crack tip to the interphase increase with the fiber volume fraction.

2. After the initiation of secondary cracks, they would propagate together with the primary matrix crack. Nevertheless, the primary crack grows toward the interphase while the secondary cracks propagate in interphase along the fiber direction. At last, these two cracks would interact and coalesce with each other. The fiber volume fraction was found to have a great influence on the propagation length prior to the crack coalescence.

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