Test Flavor SU(3) symmetry in Exclusive Λc decays

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Flavor SU(3) symmetry is a powerful tool to analyze charmed baryon decays, however its applicability remains to be experimentally validated. Since there is not much data on Ξc decays, various exclusive Λc decays especially the ones into a neutron state are essential for the test of flavor symmetry. These decay modes are also helpful to investigate final state interactions in charmed baryon decays. In this work, we discuss the explicit roles of Λc decays into a neutron in testing the flavor symmetry and exploring final state interactions. The involved decay modes include semileptonic decays, two-body and three-body non-leptonic decays, but all of them have not been experimentally observed to date.

I. INTRODUCTION

Charmed baryon decays, in particular Λc and Ξc decays, are of great interest as they serve as a platform for the study of strong and weak interactions in heavy-to-light baryonic transitions. They can also provide the essential inputs for the Λb decay modes into a charmed baryon like Λc. On the experimental side, most available results on Λc decays are obtained using the old data until recently. In 2014, Belle collaboration provided an measurement of the branching fraction with a very small uncertainty [1],

\[ B(\Lambda_c^+ \rightarrow pK^-\pi^+)_{\text{Belle}} = (6.84 \pm 0.24^{+0.21}_{-0.27})\%, \] (1)

but the central value is much larger than the previous measurement by the CLEO-c collaboration [2]:

\[ B(\Lambda_c^+ \rightarrow pK^-\pi^+)_{\text{CLEO}} = (5.0 \pm 0.5 \pm 1.2)\%. \] (2)

Based on the large amount of data, Belle collaboration also started to study the doubly Cabibbo-suppressed processes [3]. Making use of the data collected in the \( e^+e^- \) collision at the center-of-mass energy of \( \sqrt{s} = 4.599 \) GeV and adopting the double-tag technique, BES-III collaboration has reported first measurements of absolute hadronic branching fractions of Cabibbo-favored decay
modes. In total, twelve $\Lambda_c$ decay modes were observed with the significant improvement on the branching fraction in particular for the $\Lambda_c \rightarrow pK^-\pi^+$:

$$B(\Lambda_c^+ \rightarrow pK^-\pi^+)_{\text{BESIII}} = (5.84 \pm 0.27 \pm 0.23)\%.$$  \hspace{1cm} (3)

While the uncertainties are comparable with the Belle results in Eq. (1), its central value is much smaller, which is closer to the central value of the CLEO results in Eq. (2). We believe this difference will be clarified in future since the experimental prospect on charmed baryon decays will be very promising.

Theoretical description of charmed baryon decays is mostly based on the factorization assumption together with the analysis of some non-factorizable contributions in nonperturbative explicit modes. However the factorization scheme does not seem to be supported by experiments, for instance the observed large branching fraction for decays like $\Lambda_c \rightarrow \Sigma^+\pi^0/\Xi^0K^+$, which are forbidden in the factorization scheme. An alternative and the model-independent approach is to make use of the flavor SU(3) symmetry, which has been argued to work better in charmed baryon decays and bottomed baryon decays.

As the experimental precision is gradually increasing, the time is ripe to validate/invalidate the applicability of the SU(3) symmetry to charmed baryon decays. The SU(3) transformation connects the $\Lambda_c$ with the $\Xi_c$. But at this stage and in the foreseeable future there is no experiment which will focus on the study on $\Xi_c$ decays. Thus the $\Lambda_c$ decays into various final states especially the ones into a neutron are of great value since they will be the only source for the test of the SU(3) symmetry in charmed-baryon decays. The motivation of this work is to discuss the roles of the $\Lambda_c$ decays into a neutron into the test of SU(3) symmetry and the exploration of final state interactions, including semileptonic decays, two-body and three-body nonleptonic decays. All these exclusive decay modes have not been experimentally measured yet.

This paper is organized as follows. In Sec. II, the semileptonic $\Lambda_c$ decays are studied. In Sec. III and Sec. IV we will explore the two-body and three-body nonleptonic decays of the $\Lambda_c$, respectively. The last section contains our summary.

## II. SEMILEPTONIC $\Lambda_c$ DECAYS

We start with the semileptonic $\Lambda_c$ decays. In the flavor SU(3) symmetry limit, the charmed baryons are classified according to the SU(3) irreducible representation, namely as multiplets of the light-quark system: $3 \otimes 3 = \bar{3} \oplus 6$. The $\Lambda_c$ and $\Xi_c$ forms the charmed-baryon anti-triplet in the initial state:

$$T^a = (\Xi^0_c, -\Sigma^+_c, \Lambda^+_c).$$ \hspace{1cm} (4)

For the light baryons, we focus on the SU(3) octet which is represented by the matrix:

$$B^a_6 = \left( \begin{array}{ccc} \frac{1}{\sqrt{6}}\Lambda^0 & \frac{1}{\sqrt{2}}\Sigma^0 & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{2/3}\Lambda^0 \end{array} \right).$$ \hspace{1cm} (5)
The operator responsible for the transition $c \rightarrow q e^+ \bar{\nu}_e$ is $[\bar{q} \gamma^\mu (1-\gamma_5)c][\bar{\nu}_e \gamma^\mu (1-\gamma_5)e]$ with $q = d, s$, which forms an SU(3) anti-triplet in the final state. Thus the effective Hamiltonian at hadron level is constructed as

$$H_{\text{eff}} = a H_a (3) T^b B^a_b \bar{\nu}_e e.$$  

(6)

An implication of the above Hamiltonian is obtained straightforwardly:

$$B(\Lambda_c \rightarrow ne^+ \nu_e) = \frac{3 |V_{cd}|^2}{2 |V_{cs}|^2} B(\Lambda_c \rightarrow \Lambda e^+ \nu_e).$$  

(7)

Measurements of the relevant branching fractions provide a most straightforward test of the flavor SU(3) symmetry in charmed baryon decays. With the most recent data from the BES-III collaboration [21]

$$B(\Lambda_c \rightarrow \Lambda e^+ \nu_e)_{\text{BESIII}} = (3.65 \pm 0.38 \pm 0.20)\%,$$  

(8)

we can obtain the following result:

$$B(\Lambda_c \rightarrow ne^+ \nu_e)_{\text{SU(3)}} = (2.93 \pm 0.34) \times 10^{-3},$$  

(9)

which might be accessible for BES-III and Belle-II collaborations [2, 4].

In semileptonic decays, the neutron can be produced together with a light pseudo-scalar meson. The lowest-lying pseudo-scalar meson can be written as

$$M^a_b = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\ \pi^- \\ -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\ K^- \\ K^0 \\ -\sqrt{2/3} \eta \end{pmatrix}.$$  

(10)

In this case, the effective hadronic interaction Hamiltonian is constructed as

$$H_{\text{eff}} = a [T^a H_a (3)] (B^a_b M^b_c) \bar{\nu}_e e + b [T^a B^b_a M^c_c H_c (3)] \bar{\nu}_e e + c [T^a M^b_a B^c_c H_c (3)] \bar{\nu}_e e,$$  

(11)

where the singlet contribution to $\eta$ has been neglected. The $a, b, c$ are nonperturbative coefficients. The above Hamiltonian leads to the expectation:

$$B(\Lambda_c \rightarrow nK^0 e^- e^+ \nu_e) = B(\Lambda_c \rightarrow pK^- e^+ \nu_e),$$  

(12)

which is testable in the near future. In fact, the above identity holds in the isospin symmetry, whose breaking effect is much smaller in the charm decays than that of the flavor SU(3) symmetry. In the semi-leptonic decays of $c \rightarrow se^+ \nu_e$, the isospins do not change, $\Delta I = 0$. It should be stressed here that this identity is applicable to both resonant and non-resonant contributions.

The branching fraction for the inclusive decay of the $\Lambda_c$ into an electron has been measured as [11]

$$B(\Lambda_c \rightarrow e^+ + X) = (4.5 \pm 1.7)\%.$$  

(13)

Combining the results for the $\Lambda_c \rightarrow \Lambda e^+ \nu_e$ in [8], we may expect:

$$B(\Lambda_c \rightarrow nK^0 e^+ \nu_e) = B(\Lambda_c \rightarrow pK^- e^+ \nu_e) \sim \mathcal{O}(10^{-3}).$$  

(14)
III. TWO-BODY NONLEPTONIC \( \Lambda_c \) DECAYS

For two-body nonleptonic decays of the \( \Lambda_c \), there is no Cabibbo allowed decay mode into a neutron. Two-body decays into a neutron are either singly Cabibbo suppressed,

\[
\Lambda_c \to n\pi^+, \quad \Lambda_c \to n\rho^+,
\]
or doubly Cabibbo suppressed,

\[
\Lambda_c \to nK^+, \quad \Lambda_c \to nK^{*+}. \tag{15}
\]

The nonleptonic \( \Lambda_c \) decays are induced by the operators \([\bar{s}c][\bar{u}d]\) for the Cabibbo-allowed mode and \([\bar{d}c][\bar{u}d]\) for the Cabibbo-suppressed mode. These operators can be decomposed into irreducible representations of flavor SU(3). For instance,

\[
(\bar{s}c)(\bar{u}d) = O_6 + O_{15}, \tag{16}
\]

with

\[
O_6 = \frac{1}{2}[(\bar{s}c)(\bar{u}d) - (\bar{uc})(\bar{sd})],
\]
\[
O_{15} = \frac{1}{2}[(\bar{s}c)(\bar{u}d) + (\bar{uc})(\bar{sd})]. \tag{17}
\]

Perturbative QCD corrections give rise to an enhancement of the coefficient for the \( O_6 \) over the coefficient for the \( O_{15} \) by \( \frac{22}{23} \frac{\alpha_s(m_b)}{\alpha_s(m_W)} \). \( \frac{18}{25} \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \) \( \sim 2.5. \tag{18} \)

If this is valid, then one has

\[
H_{\text{eff}} = eH_{ab}(6)T_{ac}\bar{B}_a^cM_b^d + fH_{ab}(6)T_{ac}M_a^c\bar{B}_b^d + gH_{ab}(6)\bar{B}_a^cM_b^d\bar{T}_{cd}, \tag{19}
\]

with \( H_{22}(6) = 1 \) for Cabibbo-allowed modes, \( H_{23}(6) = H_{32}(6) = -2\sin(\theta_C) \) for singly Cabibbo-suppressed modes, and \( H_{33}(6) = +2\sin(\theta_C)^2 \) for doubly-Cabibbo-suppressed modes, where \( \theta_C \) is the Cabibbo angle, and

\[
T_{ab} = \epsilon_{abc}T^c. \tag{20}
\]

The coefficients \( e, f, g \) are the nonperturbative amplitudes.

Using Eq. (19), we find that for the doubly-Cabibo-suppressed modes:

\[
\mathcal{B}(\Lambda_c \to nK^+) = \mathcal{B}(\Lambda_c \to pK^0). \tag{21}
\]

For the singly-Cabibo-suppressed modes, we have the decay amplitudes,

\[
\mathcal{A}(\Lambda_c \to n\pi^+) = \sqrt{2}\mathcal{A}(\Lambda_c \to p\pi^0) = (2f + 2g)\sin(\theta_C), \tag{22}
\]

\[
\mathcal{A}(\Lambda_c \to n\rho^+) = (2f - 2g)\sin(\theta_C), \tag{23}
\]

\[
\mathcal{A}(\Lambda_c \to nK^+) = (2f + 2g)\sin(\theta_C), \tag{24}
\]

\[
\mathcal{A}(\Lambda_c \to nK^{*+}) = (2f - 2g)\sin(\theta_C). \tag{25}
\]
which implies the relation:

$$B(\Lambda_c \to n\pi^+) = 2B(\Lambda_c \to p\pi^0).$$

(23)

Furthermore, we have the amplitudes for Cabibbo-allowed modes:

$$A(\Lambda_c \to \Lambda\pi^+) = \frac{1}{\sqrt{6}}(-2e - 2f - 2g),$$

(24)

$$A(\Lambda_c \to \Sigma^0\pi^+) = \frac{1}{\sqrt{2}}(-2e + 2f + 2g),$$

(25)

$$A(\Lambda_c \to p\bar{K}^0) = -2e.$$ 

(26)

Thus we can derive the sum rule that can be experimentally examined:

$$B(\Lambda_c \to n\pi^+) = \sin^2(\theta_C) \left[ 3B(\Lambda_c \to \Lambda\pi^+) + B(\Lambda_c \to \Sigma^0\pi^+) - B(\Lambda_c \to p\bar{K}^0) \right].$$

(27)

The recent BES-III data [4] implies

$$B(\Lambda_c \to n\pi^+) = \sin^2(\theta_C) \left[ 3 \times 1.24\% + 1.27\% - 3.04\% \right] \sim 0.9 \times 10^{-3}.$$ 

(28)

Measurements in future by BES-III will be able to validate/invalidate the dominance of the sextet assumption in the effective operator.

IV. THREE-BODY NONLEPTONIC $\Lambda_c$ DECAYs

Compared to two-body decays, three-body $\Lambda_c$ decays are more involved, since first they can proceed via quasi-two-body process and the non-resonant decays and secondly there are a number of independent amplitudes in SU(3) symmetry. In the following we consider the $NK\pi$ system in the isospin limit:

$$|p\bar{K}^0\pi^0\rangle = \frac{1}{\sqrt{2}} |1\rangle |1\rangle |10\rangle = |11\rangle |10\rangle = \frac{1}{\sqrt{2}} |21\rangle + \frac{1}{\sqrt{2}} |11\rangle^{(1)},$$

(29)

$$|pK^-\pi^+\rangle = \frac{1}{\sqrt{2}} |\frac{1}{2} \frac{1}{2} \rangle |\frac{1}{2} - \frac{1}{2} \rangle |11\rangle = \left( \frac{1}{\sqrt{2}} |10\rangle + \frac{1}{\sqrt{2}} |00\rangle \right) |11\rangle = \frac{1}{2} |21\rangle - \frac{1}{2} |11\rangle^{(1)} + \frac{1}{\sqrt{2}} |11\rangle^{(2)},$$

(30)

$$|n\bar{K}^0\pi^+\rangle = \frac{1}{\sqrt{2}} |\frac{1}{2} - \frac{1}{2} \rangle |\frac{1}{2} \frac{1}{2} \rangle |11\rangle = \left( \frac{1}{\sqrt{2}} |10\rangle - \frac{1}{\sqrt{2}} |00\rangle \right) |11\rangle = \frac{1}{2} |21\rangle - \frac{1}{2} |11\rangle^{(1)} - \frac{1}{\sqrt{2}} |11\rangle^{(2)},$$

(31)

where the superscripts (1) and (2) are isospin states from (1−1) and (0−1) couplings, respectively, which are independent with each other. Since the Hamiltonian of the $c \to \bar{s}d\bar{u}$ transition has $\Delta I = 1$, and the isospin of $\Lambda_c$ is zero, we can derive the decay amplitudes from the above decompositions:

$$A(\Lambda_c \to p\bar{K}^0\pi^0) = \frac{1}{\sqrt{2}} A^{(1)},$$

$$A(\Lambda_c \to pK^-\pi^+) = -\frac{1}{2} A^{(1)} + \frac{1}{\sqrt{2}} A^{(2)},$$

$$A(\Lambda_c \to n\bar{K}^0\pi^+) = -\frac{1}{2} A^{(1)} - \frac{1}{\sqrt{2}} A^{(2)}.$$ 

(32)
The above amplitudes lead to the sum rule
\[ \sqrt{2} A(\Lambda_c \to p\bar{K}^0 \pi^0) + A(\Lambda_c \to pK^-\pi^+) + A(\Lambda_c \to n\bar{K}^0 \pi^+) = 0. \] (33)

Note that the isospin amplitudes in eq. (32) can be changed if we firstly couple the \( K\pi \) states from eq. (29-31), but the sum rule in eq. (33) still holds.

Measurements of branching ratios of the three channels are able to determine the two amplitudes, and in particular investigate the relative strong phases between the two independent decay amplitudes. These phases arise from the final state interactions since if factorization works, the two independent amplitudes are real with vanishing phases at leading order. These amplitudes including phases can provide the essential inputs for the analysis of nonleptonic decays into other baryons like \( \Lambda \).

From eq. (32), we define the relative strong phase, \( \delta \), between \( A^{(1)} \) and \( A^{(2)} \):
\[ \frac{A^{(2)}}{A^{(1)}} = \frac{|A^{(2)}|}{|A^{(1)}|} e^{i\delta}. \] (34)

Then the branching fractions can be expressed as
\[ B(\Lambda_c \to p\bar{K}^0 \pi^0) = \frac{1}{4} |A^{(1)}|^2, \]
\[ B(\Lambda_c \to pK^-\pi^+) = \frac{1}{4} |A^{(1)}|^2 + \frac{1}{2} |A^{(2)}|^2 - \frac{1}{2\sqrt{2}} |A^{(1)}| |A^{(2)}| \cos \delta, \] (35)
\[ B(\Lambda_c \to n\bar{K}^0 \pi^+) = \frac{1}{4} |A^{(1)}|^2 + \frac{1}{2} |A^{(2)}|^2 + \frac{1}{2\sqrt{2}} |A^{(1)}| |A^{(2)}| \cos \delta, \]
where we consider the relative strong phase to understand the final state interaction, and neglect the phase spaces which are actually integrated in the three-body decays. Hence
\[ \cos \delta = \frac{B(n\bar{K}^0 \pi^+) - B(pK^-\pi^+)}{2\sqrt{B(p\bar{K}^0 \pi^0) \left( B(pK^-\pi^+) + B(n\bar{K}^0 \pi^+) - B(p\bar{K}^0 \pi^0) \right)}}. \] (36)

Defining
\[ R_p = \frac{B(\Lambda_c \to p\bar{K}^0 \pi^0)}{B(\Lambda_c \to pK^-\pi^+)}, \quad R_n = \frac{B(\Lambda_c \to n\bar{K}^0 \pi^+)}{B(\Lambda_c \to pK^-\pi^+)}, \] (37)
we have
\[ \cos \delta = \frac{R_n - 1}{2\sqrt{R_p(1 + R_n - R_p)}}. \] (38)

From the recent measurement by BESIII [4], \( R_p = 0.64 \pm 0.06 \). Then \( \cos \delta \) can be obtained once the \( R_n \) is measured. The relation between \( \cos \delta \) and \( R_n \) is shown in Fig. 1. Since \( -1 \leq \cos \delta \leq 1 \), we have \( 0.017 \leq R_n \leq 4.54 \), and then the branching fraction of \( \Lambda_c \to n\bar{K}^0 \pi^+ \) is obtained as,
\[ 0.04\% \leq B(\Lambda_c \to n\bar{K}^0 \pi^+)_{\text{Belle}} \leq 33\%, \] (39)
\[ 0.035\% \leq B(\Lambda_c \to n\bar{K}^0 \pi^+)_{\text{BESIII}} \leq 28\%. \] (40)

As we can see that this constraint is rather loose, thus the experimental measurements are requested.
V. SUMMARY

Unlike the bottom hadron decays where the momentum transfer is typically large enough to ensure the perturbation theory in QCD, charmed meson and baryon decays are very difficult to understand. Due to the limited energy release, the factorization scheme based on the expansion of $1/m_c$ and $1/E$ is not always valid. Flavor SU(3) symmetry is a powerful tool to analyze the charmed baryon decays, which has been argued to work better than in charmed meson decays, however its validity has to be experimentally examined. Since there is not much data on $\Xi_c$ decays, exclusive $\Lambda_c$ decays into a neutron are essential for the test of flavor symmetry and investigating final state interactions in charmed baryon decays.

In this work, we have discussed the roles of the exclusive $\Lambda_c$ decays into a neutron in testing the flavor symmetry and final state interactions. We found that the semileptonic decays into a neutron provide the most-straightforward way to explore the flavor SU(3) symmetry. Two-body nonleptonic decays are capable to examine the assumption of the sextet dominance mechanism. While three-body non-leptonic decays into a neutron are of great interest to explore the final state interactions in $\Lambda_c$ decays. All these decay modes have not been experimentally observed to date.

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