1. Theoretical Motivation of RTFM

**Theorem 1.1** (Expected Separability Between Abnormal and Normal Videos). *Assuming that \( \mathbb{E}[\|x^+\|_2] \geq \mathbb{E}[\|x^-\|_2] \), where \( X^+ \) has \( \mu \) abnormal samples and \( (T - \mu) \) normal samples, where \( \mu \in [1, T] \), and \( X^- \) has \( T \) normal samples. Let \( D_{\theta,k}(\cdot, \cdot) \) be the random variable from which the separability scores \( d_{\theta,k}(\cdot, \cdot) \) of Eq. 3 in the main paper are drawn [2].

1. If \( 0 < k < \mu \), then
   \[
   0 \leq \mathbb{E}[D_{\theta,k}(X^+, X^-)] \leq \mathbb{E}[D_{\theta,k+1}(X^+, X^-)].
   \]

2. For a finite \( \mu \), then
   \[
   \lim_{k \to \infty} \mathbb{E}[D_{\theta,k}(X^+, X^-)] = 0.
   \]

**Proof.**

\[
\mathbb{E}[D_{\theta,k}(X^+, X^-)] = \mathbb{E}[g_{\theta,k}(X^+)] - \mathbb{E}[g_{\theta,k}(X^-)]
\]
\[
= p_k^+(X^+)\mathbb{E}[\|x^+\|_2] + p_k^-(X^-)\mathbb{E}[\|x^-\|_2] - \mathbb{E}[\|x^+\|_2] - \mathbb{E}[\|x^-\|_2] \tag{1}
\]

1. Trivial given that \( \mathbb{E}[\|x^+\|_2] \geq \mathbb{E}[\|x^-\|_2] \) and that \( p_{k+1}^+(X^+) > p_k^+(X^+) \) for \( 0 < k < \mu \).

2. Trivial given that as \( \mu \) is finite, \( \lim_{k \to \infty} p_k^+(X^+) = 0. \)

**Intuition of feature magnitude:** Assuming the expected magnitude of abnormal samples is larger than of normal samples, we can derive Thm. 3.1 that proves that the expected feature magnitude-based separability score between normal and abnormal videos grows for \( 0 < k < \mu \) and reduces to zero for \( k \to \infty \). Hence, to use Thm. 3.1, we need to enforce larger magnitude for abnormal features using our proposed RTFM. The similarity between the theoretical and empirical curves in Fig. 2(left) is evidence of the soundness of Thm. 3.1.

2. Multi-scale Temporal Feature Learning

Our proposed multi-scale temporal network (MTN) captures the multi-resolution local temporal dependencies and the global temporal dependencies between video snippets, as displayed in Fig. 1.

3. Computational Efficiency

We investigate if our system can run in real time. During inference, our method processes a 16-frame clip in 0.76 seconds on a Nvidia 2080Ti—this time includes the I3D extraction time. This indicates that our system can achieve
good real-time detection in real-world applications.

4. Temporal Dependency

Temporal Dependency has been explored in [1, 3–5, 7, 8, 10]. In anomaly detection, traditional methods [1, 8] convert consecutive frames into handcrafted motion trajectories to capture the local consistency between neighbouring frames. Diverse temporal dependency modelling methods have been used in deep anomaly detection approaches, such as stacked RNN [5], temporal consistency in future frame prediction [4], and convolution LSTM [3]. However, these methods capture short-range fixed-order temporal correlations only with single temporal scale, ignoring the long-range dependency from all possible temporal locations and the events with varying temporal length. GCN-based methods are explored in [7, 10] to capture the long-range dependency from snippets features, but they are inefficient and hard to train. By contrast, our proposed module combines PDC [9] and TSA [6] on the temporal dimension to seamlessly and efficiently incorporate both the long and short-range temporal dependencies into our temporal feature ranking loss.

![Figure 2. AUC w.r.t. top-k (Left) and the margin m (Right).](image)

5. Ablations for $k$ and $m$

We show the AUC results as a function of top-$k$ and margin $m$ values on ShanghaiTech in Fig.2. Consistent to our theoretical analysis, the performance of our model peaks at a sufficiently large $k$, flattens at around $k = \mu$ and then drops with increasing $k$ (Fig.2(left)). It is also robust to a large range of $m \in [50, 1200]$ with a stable AUC in [93%, 96%] (Fig.2(right)).

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