Managing mobile production-inventory systems influenced by a modulation process

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Abstract
The objective of this paper is to investigate the potential added value of being able to relocate production capacity, relative to fixed production capacity, in a network of multiple, geographically distributed manufacturing sites. There is a growing number of examples of production capacity that can be geographically relocated with a modest amount of effort; e.g., 3D printers, bioreactors for cell and gene manufacturing, and modular units for pharmaceutical intermediates. Such a capability shows promise for enabling the fast fulfillment of a distributed network with a reduction in the total inventory and total production capacity of a distributed network with fixed production capacity without sacrificing customer service levels or total system resilience. Allowing also for transshipment, we model a production-inventory system with L production sites and Y units of relocatable production capacity, develop efficient and effective heuristic solution methods for dynamic relocation and multi-location inventory control, and analyze the potential added value and implementation challenges of being able to relocate production capacity. We describe the (L, Y) problem as a problem of sequential decision making under uncertainty to determine transshipment, mobile production capacity relocation, and replenishment decisions at each decision epoch. To enhance model realism, we use a partially observed stochastic process, the modulation process, to model the exogenous and partially observable forces (e.g., the macro-economy) that affect demand. We then model the (L, Y) problem as a partially observed Markov decision process. Due to the considerable computational challenges of solving this model exactly, we propose two efficient, high quality heuristics. We show for an instance set with five locations that production capacity mobility and transshipment, relative to the fixed production capacity case, can improve systems performance by as much as 41% on average over the no-flexibility case and that production capacity mobility can yield as much as 10% more savings compared to when only transshipment is permitted.

Keywords Decentralized control · Mobile production capacity · Inventory transshipment · Markov-modulated demand · Partially observed modulation process

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1 Introduction

The aim of this paper is to better understand the impact of being able to relocate easily relocatable production capacity in a distributed production-inventory system subject to time varying demand uncertainty. In achieving this aim, we develop efficient methods that can be useful in addressing such questions as: (i) When, how much, and to where inventory should be transshipped and/or transportable production capacity should be relocated? (ii) How should replenishment decisions be made in coordination with this capability to transship inventory and/or relocate production capacity? We believe that the ability to relocate production capacity can enable the fast fulfillment of a distributed network with a reduction in the total inventory and total production capacity of a distributed network with fixed production capacity without sacrificing customer service levels or total system resilience, and the results of this paper investigate this belief. Although we recognize that seamlessly operating mobile production capacity at various locations depends on agile logistics, for modeling and computational simplicity, we will not consider the logistics implications of production capacity relocation in this paper.

In examining the potential value added of relocatable production capacity, we investigate a multi-period, distributed production-inventory system under stochastic demand that allows backlogging, assumes instantaneous replenishment, and has the capability to relocate transportable production units and/or transship inventory between locations. Historically, transshipment has been a tool to reposition inventory in order to improve supply chain performance. We now add the capability of repositioning production capacity to further aid in improving the performance of a supply chain. Transportable production units, which we refer to as modules, have recently generated significant interest in manufacturing (Geek Wire 2018; Bayer Technology Services GMBH 2014; Pfizer 2015; MIT News 2016); we remark that manufacturing and/or storing the final, or near-final product close to demand can enable fast fulfillment. Relatedly, we further remark that relocatable storage capacity has also become of interest recently to the parcel express industry, as described in Verlinde et al. (2014) and elsewhere.

We model this problem as a specially structured, large-scale, partially observed Markov decision process (POMDP) in order to determine replenishment decisions, when to transship and/or relocate production capacity, and hence determine the value of having the capability to transship inventory and relocate production capacity. Our model of data-driven demand forecasting assumes the existence of a stochastic process, the modulation process, that affects demand. The modulation process is governed by an action independent Markov chain and is partially observed by the demand process and another process, the additional observation data (AOD) process. The modulation process can model exogenous factors, such as current macro-economic conditions, the weather, and seasonal effects that can affect the demand process. The AOD process models all available data beyond demand data, e.g., interest rates, unemployment rates, consumer price indices, that might be useful in improving demand forecast accuracy. Including the AOD process in our model reflects the facts that supply chains are becoming increasingly data driven to support improved supply chain performance and industry has major interest in using increased data availability velocity, volume, and variety to improve demand forecasts. We will show that the current belief function of the modulation process can significantly influence the current demand forecast. The problem objective is to minimize the expected total discounted cost criterion composed of backorder, holding, transshipment, and module relocation costs.
A complexity analysis indicates the need to develop good, tractable heuristics (i.e., sub-optimal policies) for solution determination. We approach the development of heuristics in two ways: a centralized approach and a decentralized approach. We investigate the quality and the computational characteristics of the heuristics developed. A desirable, but not guaranteed, feature of a (sub-optimal) heuristic is that the heuristic will improve the system’s performance with improved observation accuracy. We present a preliminary numerical study that indicates the heuristics under consideration share this feature with high likelihood. With regard to the value of being able to relocate production capacity, we analyze the value of transshipment without module relocation, module relocation without transshipment, and both transshipment and module relocation. We find that in certain cases, the value added due to resource mobility can be significant, indicating the potential importance of resource mobility in the design of next generation supply chains.

More specifically, we consider a distributed production - inventory system with \(L\) locations and \(Y\) transportable production modules. None, one, or more up to a maximum number of modules can be located at each of the locations. At each decision epoch, we assume the (centralized) decision maker (DM) knows the current demand forecast, inventory level, and production capacity at each location. This production capacity is made up of fixed capacity and transportable capacity. The DM decides how the current inventory and transportable production capacity should be relocated. We assume these relocations occur instantaneously. Once the inventory and transportable production capacity have been relocated, the DM determines the replenishment decisions at each location based on current demand forecasts, the new inventory levels, and the new production capacities at the locations. Replenishment is instantaneous. Once replenishment is complete, demands at the locations are realized. Based on these realizations and possibly other data, the demand forecast is updated just before the next decision epoch.

We remark that how often modules would be relocated and the lead time for the relocation would in reality be situation dependent. For 3D printers, bioreactors, and smart locker modules (a form of mobile storage capacity), relocation decisions might be made monthly and take a day or two, with other decisions (e.g., reagent replenishment for the bioreactors) made hourly, daily, or weekly, where reagent replenishment may have a two-week lead time. When disruptions occur, these decisions may be event driven. Relocation decisions for modular production units for pharmaceutical intermediates might be made quarterly and require several weeks of lead time if the unit is to be moved across a national border. However, if the relocatable module is a smart locker or 3D printer on a truck or trailer (Geek Wire 2018; Verlinde et al. 2014), then relocation decisions might be made once, possibly twice, daily and require less than an hour or two of lead time. Since the aim of the research presented in this paper is to understand the potential impact of being able to relocate production capacity, for modeling and computational simplicity, we will assume both replenishment and relocation decisions are made instantaneously at each decision epoch with full knowledge that a specific application would require a more realistic, and a more computationally challenging, model.

### 1.1 Literature review

The problem considered in this paper involves inventory transshipment, mobile production capacity relocation, fixed production capacity of each single location production facility, and a centralized controller determining transshipment, module relocation, and replenishment decisions. Numerous innovative developments in manufacturing, such as containerized production for pharmaceutical manufacturing processes (Bayer Technology Services GMBH
2014; Pfizer 2015; MIT News 2016) and on-demand mobile production (Geek Wire 2018) necessitate the planning of logistics for flexible production and inventory systems that are characterized by resource mobility, interconnectivity, sharing, and decentralization (Marcotte and Montreuil 2016). Malladi et al. (2020) investigate the dynamic mobile production and inventory problem without the option of inventory transshipment under stationary and independent demands and have proposed heuristic approaches to solve the problem. A value addition of more than 10% over in-the-ground production systems was determined for a system of twenty locations. Wörsdörfer et al. (2017) present a real options pricing based method of evaluating the value added by mobile containerized production systems. Other research that address the operational logistics of mobile facilities can be found in Halper and Raghavan (2011), Qiu and Sharkey (2013). The problem of managing mobile production capacity under deterministic demands may be viewed as a dynamic facility location problem with multiple facilities at the each location that may be opened and closed (Ghiani et al. 2002; Melo et al. 2005; Jena et al. 2015; Wörsdörfer and Lier 2017). However, inventory is generally not managed jointly with capacity allocation in these problems. Solving an expanded mixed integer program, which is often the solution approach proposed in literature, will not be tractable under uncertainty and inventory control in tandem. Additionally, a mixed integer programming approach may not even be able to incorporate complex demand processes with a large number of potential demand outcomes, such as the one addressed in the current paper.

Regarding multi-location inventory management with transshipment, Karmarkar (1979, 1981) considers the multi-location inventory control problem over a single period and multiple periods, respectively, under uncertain demands. It is proved that when the inventory addition and subtraction matrix has a Leontief structure, there exists a base stock policy that is optimal when attainable. In Karmarkar (1987), a restricted Lagrangean dual-based lower bound and a dual relaxation based upper bound on the optimal cost of the multi-location problem are presented. The upper bound assumes the post ordering and shipment inventory position does not fall below the initial inventory position. Rudi et al. (2001) indicate that localized transshipment strategies are outperformed by centralized strategies. Axsäter et al. (2002) propose heuristics for a problem that considers inventory held at a warehouse and allocated for distribution to various locations in a centralized fashion. Herer et al. (2006) prove the optimality of order-up-to policies at each location in a multi-location inventory control system with reactive transshipment for a long-run average cost criterion and present a heuristic for computation. The authors consider only replenishment decisions that result in non-negative inventory positions post replenishment at each location. Lien et al. (2011) present a comparison of chain and group configurations of transshipment network design building on the ideas of manufacturing process flexibility (Jordan and Graves 1995) and restricted connectivity in a transshipment network (Herer et al. 2002). Wee and Dada (2005) consider a multi-retailer, one warehouse framework that allows reactive transshipment either from the warehouse to the retailers and/or between retailers. The authors prove that it is optimal to adopt either retailer only, warehouse only, retailer first, or warehouse first protocols, when considering transshipment. Various cost parameter thresholds based intervals are presented to indicate the system that is optimal in each regime.

The literature on the single location inventory problem is vast and varied (Malladi et al. 2018; Katehakis et al. 2015; Cheung and Simchi-Levi 2019; Godfrey and Powell 2001; Bernstein and DeCroix 2006). We consider the data-driven online learning demand model presented in Malladi et al. (2018) and adopt it for the multi-location problem in this paper. Malladi et al. (2018) analyze a single location, infinite capacity inventory control problem with demand and additional observation data influenced solely by a Markov modulation process. The modulation process is intended to model forces that may be partially observed,
influence the demand process, but are not affected by actions taken by the DM (e.g., the macro-economy, air currents, tides). Demand realizations and other data (e.g., housing starts, consumer spending) represent observations of the modulation process. What is known to the DM about the modulation process is provided by the belief function, which is updated with new data using Bayes’ Rule. A base stock policy, having a base stock level dependent on the belief function, is proved to be optimal for the infinite horizon problem when an attainability assumption holds. The modulation process can be used to model the correlation between demands at different locations.

We consider approximate dynamic programming approaches that do not rely on maintaining the cost function’s lookup table over the entire horizon to find good heuristic solutions to the multi-location mobile capacity and inventory control problem (Powell 2007, 2012; Ryzhov et al. 2012; Secomandi 2001; Goodson et al. 2017; Burnetas and Katehakis 1997). In particular, we are interested in rollout based heuristics which are known to perform well on dynamic systems with stochasticity as suggested in Secomandi (2001) for solving the vehicle routing problem with stochastic demands. Goodson et al. (2017) provide a systematic classification-aimed analysis of rollout policies. Additionally, the literature suggests that centralized control is expected to perform better than decentralized control from a solution-quality perspective; however, there is an inherent tradeoff between solution quality and computational expense (Kouvelis and Gutierrez 1997; Bernstein and Federgruen 2005; Bernstein and DeCroix 2006). In the current paper, we propose and analyze a decentralized control policy that performs comparably with a more computationally intensive centralized control policy. Needed foundational results can be found in the appendices.

1.2 Paper outline

This paper is organized as follows. In Sect. 2, we state the problem, model it as a POMDP, present several preliminary results for the POMDP, and examine the tractability challenges of this model. These challenges indicate the need for heuristic approaches. In Sect. 3, we develop an approximation of the value function for the general $L$ production facility model, based on the value function of the least computationally demanding, single production facility problem. We also discuss the challenges of solving the $L = 1$ case. We then determine two approximations for the $L = 1$ problem in Sect. 4. In Sect. 5, we present five heuristics for solving the general problem, based on these two approximations. Section 6 then presents the results of a computational study of these five heuristics. We observe performance improvement when production capacity is mobile as high as 26% in some instances, relative to systems with no mobility, irrespective of the presence of transshipment flexibility. Also, we note that non-stationary modeling of demand when demand is non-stationary, rather than using a stationary approximation, can result in as much as a 6% increase in performance and that complete observability of the modulation process can increase the value addition of mobility by 5–27% on the instances considered. Additionally, we infer that although joint control results in slightly lower costs, decentralized control heuristics require significantly less computational time. Conclusions are presented in Sect. 7. “Appendix A1” contains significant foundational results for the $L = 1$ case that are needed for the analysis of the heuristics, “Appendix A2” presents the proof of a key result, “Appendix A3” presents a computationally useful heuristic, and “Appendix A4” presents additional numerical results that complement results in Sect. 6.2.
2 Problem statement and preliminary results

We now define the general $L$ location, $Y$ module problem statement in Sect. 2.1 and present the POMDP model of this problem and general results for the model in Sect. 2.2. In Sect. 3, we will examine the simplest case ($L = 1$, $Y = 0$) and use its solution as the basis for the development of heuristics for the general problem.

2.1 Problem statement

Consider a distributed production-inventory system with $L$ locations and $Y$ portable manufacturing modules. At each decision epoch $t$ we assume the decision-maker (DM) knows:

- $s(t) = \{s_l(t), l = 1, \ldots, L\}$, where $s_l(t)$ is the inventory level at location $l$,
- $u(t) = \{u_l(t), l = 1, \ldots, L\}$, where $u_l(t) \in \{0, 1, \ldots, Y_l'\}$ is the number of modules positioned at location $l$ and $Y_l'$ is the maximum number of modules that location $l$ can support (e.g., due to space limitations),
- $I(t) = \{d(t), \ldots, d(1), z(t), \ldots, z(1), x(0)\}$, where:
  - $d_l(t)$ is the demand realized during period $(t - 1, t)$ that location $l$ is required to fulfill (or back order) and $d(t) = \{d_l(t), l = 1, \ldots, L\}$
  - $z(t)$ represents additional observation data, in addition to the realization of demand, that might be of use to the DM,
  - $x(0)$ is an a priori probability vector defined below.

We assume the demand process $\{d(t), t = 1, 2, \ldots\}$ and the additional observation data (AOD) process $\{z(t), t = 1, 2, \ldots\}$ are linked to the modulation process $\{\mu(t), t = 0, 1, \ldots\}$ through the given conditional probability $P(d(t + 1), z(t + 1), \mu(t + 1) \mid \mu(t))$, where $x(0) = \{x_i(0), \mid i \in \{1, \ldots, N\}\}$ where $x_i(0) = P(\mu(0) = \mu_i)$ for each of $N$ modulation states. A discussion of this general description of data-driven demand and learning and how it generalizes and extends the Markov-modulated demand and Bayesian updating literatures can be found in Malladi et al. (2018).

The chronology of events within period $(t, t + 1)$ is as follows:

**Step 1:** Given $I(t)$, $s(t)$, and $u(t)$, the DM relocates inventory and modules to reach the post-relocation state $(s'(t), u'(t))$, where we assume $\sum_{l=1}^{L} s'_l(t) = \sum_{l=1}^{L} s_l(t)$ and $\sum_{l=1}^{L} u'_l(t) = \sum_{l=1}^{L} u_l(t)$. Necessarily, $-(s_l(t)+) - \Delta_l^S(t) \leq \Delta_l^S(t) \leq \sum_{k=1, k \neq l} (s_k(t))^+$ for each location $l$, where $\Delta_l^S(t)$ is the amount of inventory relocated to location $l$. Thus, $s'_l(t) = s_l(t) + \Delta_l^S(t)$ for all $l$ and hence $s'(t) = s(t) + \Delta^S(t)$, where $\Delta^S(t) = \{\Delta_l^S(t), l = 1, \ldots, L\}$. The decision variables are $\Delta_l^S(t)$ and $u'(t)$ for Step 1.

**Step 2:** Given $I(t)$, $s'(t)$, and $u'(t)$, the DM determines $q(t) = \{q_l(t), l = 1, \ldots, L\}$, where $q_l(t)$ is the replenishment decision at location $l$. Necessarily, $0 \leq q_l(t) \leq U_l + u_l'(t)G$, where $U_l$ is the fixed amount of capacity at location $l$ and $G$ is the capacity of each module. Let $y_l(t) = s'_l(t) + q_l(t)$, the inventory level at location $l$ after inventory and module relocation and replenishment but before demand realization, and assume $y(t) = \{y_l(t), l = 1, \ldots, L\}$. The decision variables are therefore $q(t)$, or equivalently $y(t)$, for Step 2, where necessarily, $s'_l(t) \leq y_l(t) \leq s'_l(t) + U_l + u_l'(t)G$ for all $l$.

**Step 3:** The realizations of the random variables $d(t + 1)$ and $z(t + 1)$ become known and unfulfilled demands are backordered, $I(t + 1) = \{d(t + 1), z(t + 1), I(t)\}$, $s(t + 1) = y(t) - d(t + 1)$, and $u(t + 1) = u'(t)$.

**Step 4:** $t = t + 1$. 
We assume that for location $l$, $c_l(y_l(t), d_l(t + 1)) = b_l(d_l(t + 1) - y_l(t))^+ + h_l(y_l(t) - d_l(t + 1))^+ \geq 0$ is the single period cost accrued between $t$ and $t + 1$, where $b_l$ and $h_l$ are respectively the backorder and holding cost per unit per period and for all $d_l$, $c_l(y_l, d_l)$ is convex in $y_l$ and $\lim_{|y_l| \to \infty} c_l(y_l, d_l) = \infty$.

We assume that the modulation and the observation state spaces are finite and that for each location, the demand state space is finite and the inventory state space is countable.

Let the single period $(t, t + 1)$ cost be:

\[
\sum_{l=1}^{L} \left( K^S_1(\Delta^S(t))^+ + K^S_1(-\Delta^S(t))^+ \right) \\
+ K^M \sum_{l=1}^{L} |u_l(t) - u'_l(t)|/2 + \sum_{l=1}^{L} c_l(y_l(t), d_l(t + 1)),
\]

where $K^S_1^+$ ($K^S_1^-$) is the cost of moving a unit of inventory to (from) location $l$, and $K^M$ is the cost of moving a module from one location to another. A feasible policy determines $(q(t), \Delta^S(t), u'(t))$ based on $\mathcal{I}(t)$, $s(t)$, and $u(t)$ for all $t$.

The problem criterion is the expected total discounted cost over the infinite horizon, where $\beta \in [0, 1)$ is the discount factor. The problem is to determine a feasible policy that minimizes the criterion with respect to the set of all feasible policies.

We remark that we have used the model of transshipment cost described above due to its modeling simplicity and note that for some transshipment problems (e.g., between retail stores within an urban area) this model might be reasonably suitable. However, in general the cost of transshipment will be different for different origin-destination pairs, and hence a specific real-world example of a distributed production-inventory problem may require a more realistic model of transshipment costs.

### 2.2 POMDP model and general results

This problem can be recast as a partially observed POMDP as follows. Results in Smallwood and Sondik (1973) and Sondik (1978) imply that $(x(t), s(t), u(t))$ is a sufficient statistic, where the belief function $x(t) = \{x_i(t), \forall i = 1, \ldots, N\}$ is such that $x_i(t) = P(\mu(t) = \mu_i | \mathcal{I}(t))$ and $x(t) \in X = \{x \geq 0 : \sum_{i=1}^{N} x_i = 1\}$. Let

\[P_{ij}(d, z)\]

\[= P(d(t + 1) = d, z(t + 1) = z, \mu(t + 1) = j | \mu(t) = i)\]

\[\forall i, j \in 1, \ldots, N,\]

\[\sigma(d, z, x) = x \sum_{j=1}^{N} P_{ij}(d, z)\]

\[= x \sum_{j=1}^{N} \sum_{i=1}^{N} P_{ij}(d, z),\]

\[\lambda(d, z, x) = \{\lambda_j(d, z, x), \forall j = 1, \ldots, N\}\]

\[= x \sum_{j=1}^{N} \sigma(d, z, x)/\sigma(d, z, x), \sigma(d, z, x) \neq 0, \text{ and}\]

\[\mathcal{L}(x, y) = E[c(y, d)]\]

\[= c(y, d) = \sum_{j=1}^{L} c_j(y_j, d_j).
\]
Thus, if $x$ is the prior belief function, then $\lambda(d, z, x)$ is the posterior belief function, given realizations $(d, z)$, and $\sigma(d, z, x)$ is the probability that $(d, z)$ will be the demand and observation realizations, given prior $x$. Define the operator $H$ as follows:

$$[Hv](x, s, u) = \min_{\Delta^S, u', y} \{G(x, u, y, \Delta^S, u', v)\}, \tag{1}$$

where $G(x, u, y, \Delta^S, u', v)$

$$= \sum_{l=1}^{L} \left( K^S_l (\Delta^S_l)^+ + K^S_l (-\Delta^S_l)^+ \right)$$

$$+ K^M \sum_{l=1}^{L} |u_l - u'_l|/2 + L(x, y)$$

$$+ \beta \sum_{d, z} \sigma(d, z, x)v(\lambda(d, z, x), y - d, u'),$$

and where the minimization is with respect to

$$\sum_{l=1}^{L} u'_l = Y, \quad \quad 0 \leq u'_l \leq Y'_l, \quad \forall \ l \in \{1, \ldots, L\}, \quad \quad \sum_{l} \Delta^S_l = 0, \quad \quad -(s_l)^+ \leq \Delta^S_l \leq \sum_{k \neq l} (s_k)^+, \quad \forall \ l \in \{1, \ldots, L\}, \quad \quad (s_l + \Delta^S_l) \leq y_l \leq (s_l + \Delta^S_l) + U_l + u'_l G, \quad \forall \ l \in \{1, \ldots, L\}, \quad \text{and} \quad u'_l, \Delta^S_l, y_l \in \mathbb{Z}, \quad \forall \ l \in \{1, \ldots, L\}.$$

Results in Puterman (1994) guarantee that there exists a unique $v^*$ such that $v^* = Hv^*$ and that this fixed point is the minimum expected total discounted cost over the infinite horizon. Further, a policy that causes the minimum in (1) to be attained is an optimal policy and is decision epoch invariant. For any given bounded function $v_0$, let $\{v_n\}$ be such that $v_{n+1} = Hv_n$. Then, $\lim_{n \to \infty} ||v^* - v_n|| = 0$, where $||.||$ is the sup-norm.

Results in Smallwood and Sondik (1973) guarantee that $v_n(x, s, u)$ is piecewise linear and concave in $x$ for fixed $(s, u)$ for all $n$, assuming $v_0(x, s, u)$ is also piecewise linear and concave in $x$ for fixed $(s, u)$. In the limit, $v^*(x, s, u)$ may no longer be piecewise linear in $x$ for fixed $(s, u)$; however, concavity will be preserved. The cardinality of the state space of this POMDP is infinite since the belief vector belongs to a continuous $N$-dimensional real space. Even when dealing with the i.i.d. case, since the number of inventory and module count combinations is exponential in the number of locations $L$, solving this POMDP exactly becomes intractable for even relatively small values of $L$. Therefore, we seek good suboptimal approaches that significantly reduce this computational burden.
3 Bounds and approximate value function based on $L = 1$ case

Throughout this paper, we will base the development of heuristics on the most tractable problem, the single location inventory control problem, i.e., the $L = 1, Y = 0$ case. Solving each of the $L$ local replenishment problems for the i.i.d. case requires $|S_I|^2 |A_I|$ multiplications per successive approximation iteration, and $L$ of these are required. For $L = 10$ and $|S_I| = |A_I| = 50$, $L |S_I|^2 |A_I|$ is on the order of $10^5$, which is a large but computationally manageable problem. The operator $H$ simplifies to $H^F_l$ for location $l$ with fixed capacity, where

$$
[H^F_l v^F_l](x, s_l, u_l) = \min \{ G^F_l(x, u_l, y_l, v^F_l) \},
$$

$$
G^F_l(x, u_l, y_l, v^F_l) = L^F_l(x, y_l) + \beta \sum_{d,z} \sigma(d, z, x) v^F_l(\lambda(d, z, x), y_l - d_l, u_l),
$$

where $L^F_l(x, y_l) = \sum_{d,z} \sigma(d, z, x) c_l(y_l, d_l), \forall l \in \{1, \ldots, L\}$, (2)

and where the minimization in (2) is with respect to $s_l \leq y_l \leq s_l + U_l + u_l G$. Proposition 4 in “Appendix A1” guarantees that the fixed point of $H^F_l, v^F_l$, is non-decreasing in capacity for fixed $(x, s_l)$. This monotonicity result implies

$$
\sum_{l=1}^{L} v^F_l(x, s_l, Y'_l) \leq v(x, s, u).
$$

At this point, it is important to note that the arguments for the function $G$ in (1) are different from the arguments for the function $G^F_l$ in (2). The arguments of $G$ contain the additional terms $A^S$ and $u'$, which are the relocation decision variables in (1). Implicit in these terms being absent in the arguments of $G^F_l$ is the assumption that for the single location case, transshipment and/or module relocation are assumed not to occur in the future. Hence,

$$
v(x, s, u) \leq \sum_{l=1}^{L} v^F_l(x, s_l, u_l).
$$

Thus, the solutions of the local replenishment problems provide upper and lower bounds on the cost function of the initial problem.

We now approximate the optimal cost-to-go function of the POMDP presented in (1). Let $\theta \in [0, 1]$, be such that

$$
v^F_{l, \theta}(x, s_l, u_l) = (1 - \theta)v^F_l(x, s_l, Y'_l) + \theta v^F_l(x, s_l, u_l)
$$

$\forall l \in \{1, \ldots, L\}$ and

$$
\tilde{v}^\theta(x, s, u) = \sum_{l=1}^{L} v^F_{l, \theta}(x, s_l, u_l).
$$

Hence, $\tilde{v}^\theta(x, s, u)$ is an approximation of $v(x, s, u)$ that relies solely on the solution of the single location ($L = 1, Y = 0$) problem. Then,
\[ [H \hat{v}^\theta](x, s, u) = \min_{A^S, u'} \left\{ \sum_{l=1}^{L} (K^+_l (\Delta^+_l) + K^-_l (-\Delta^-_l)) + K^M \sum_{l=1}^{L} |u_l - u'_l|/2 \right\} + \min_y \left\{ \mathcal{L}(x, y) + \beta \sum_{d, z} \sigma(d, z, x) \hat{v}^\theta(\lambda(d, x, y) - d, u') \right\}. \]

In (3), the inner minimization is over all \( y_l \) such that \( s_l + \Delta^+_l \leq y_l \leq s_l + \Delta^-_l + U_l + u'_l G \). We note that the coefficient \( \theta \) may be location-dependent depending on the nature of the instances.

4 Approximating the value function for the single location problem

Two of the heuristics presented in Sect. 5 make use of the value function of the single location problem. We remark that when \( L = 1 \), the (local) decision maker assumes there will be no inventory and/or module relocation in the future and does not attempt to coordinate its decisions with either the controller determining the inventory and/or module relocation decisions or the replenishment decision makers at the other locations. Foundational results for the \( L = 1 \) problem presented in “Appendix A1” imply that there exists an optimal replenishment policy that is a base stock policy, the optimal base stock value is non-increasing in capacity, but an optimal base stock policy is myopic only when production capacity is sufficiently large (Proposition 7 of Section A1). Computational complexity and the likelihood of intractability for the case where demand is not i.i.d., even for the \( L = 1 \) problem, increases substantially when an optimal policy is not myopic. For computational reasons, we now present two approximations of the value function for the single location problem, the static belief function approximation and the piecewise linear approximation based on the convexity of the value function in \( s \) and \( u \).

4.1 The static belief function approximation

Assume that \( x(t + 1) = x(t) \) for all \( t \). Then the \( L = 1 \) operator becomes

\[ \left[ \hat{H}^F \hat{v}^F \right](x, s_l, u_l) = \min_{s_l \leq y_l \leq s_l + U_l + G u_l} \left\{ \sum_{d_l} \sum_{i=1}^{N} x_i \Pr(d_l | i) \left[ c_l(y_l, d_l) + \beta \hat{v}^F(x, y_l - d_l, u_l) \right] \right\}. \]

which for given \( x \) and \( u_l \), requires essentially the same number of operations per successive approximations step as required in the i.i.d. case. Since the case where \( x(t + 1) = x(t) \) for all \( t \) is a special case of the general problem, there exists an optimal policy that is a base stock policy, an optimal base stock level is non-increasing in capacity, and the optimal value function is non-increasing in capacity and convex in inventory level (see “Appendix A1”). Thus, the resulting approximation \( \hat{v}^F \) shares the same structural properties of \( v^F \). We now present a result that bounds the gap between \( \hat{v}^F \) and \( v^F \) that will prove useful in our computational study; proof is presented in “Appendix A2”.

Proposition 1 We have \( v^F(x, s_l, u_l) \geq \hat{v}^F(x, s_l, u_l) - \rho/(1 - \beta) \) for all \( x, s_l, \) and \( u_l \) for \( l \in \{1, \ldots, L \} \) where \( \rho = \sum_d k(d_l) c_1(y_l, d_l) \) and \( k(d_l) = \left( \max_k \Pr(d_l | k) - \min_k \Pr(d_l | k) \right) \).
4.2 A piecewise linear and convex approximation of the value function of $L = 1$
static fixed problem

We use the following approximation of the optimal cost of the single location static fixed
problem $\hat{v}_t^F$, drawing inspiration from the approximation of the cost-to-go function in the
lookahead of fixed future (LAF) heuristic in Malladi et al. (2020):

$$\hat{v}_t^F (x(t + 1), s_l(t + 1), u_l(t + 1)) \approx \left( \hat{v}_t^F (x(t + 1), \tilde{s}_l(t + 1), u_l(t)) + \hat{v}_t^F (x(t + 1), s_l(t), u_l(t + 1)) \right)/2, \forall l \in \{1, \ldots, L\},$$

where $\tilde{s}_l(t + 1) = y_l(t) - \left[ E[D_l(t)] \right]$ and $\lfloor a \rfloor$ denotes the nearest integer to which $a$ is rounded.

Since $v_t^F (x, s_l, u_l)$ is piecewise linear and convex in $s_l$ when $u_l$ is held constant and in
$u_l$ when $s_l$ is held constant (from Propositions 3 and 6 in “Appendix A1”) and $\hat{v}_t^F (x, s_l, u_l)$
iherits these properties as it is a stationary special case, the latter can be represented as
$max\{\gamma_j^l s_l + \hat{\gamma}_j^l : (\gamma_j^l, \tilde{\gamma}_j^l) \in \Gamma_t^l (u_l)\}, \forall l \in \{1, \ldots, L\}$ and as $max\{\theta_j^l u_l + \hat{\theta}_j^l : (\theta_j^l, \tilde{\theta}_j^l) \in \Theta_t^l (s_l)\}, \forall l \in \{1, \ldots, L\}$. The set $\Gamma_t^l (u_l)$ ($\Theta_t^l (s_l)$) is the set of coefficients describing the
facets of the piecewise linear and convex function $\hat{v}_t^F (x, s_l, u_l)$, when $u_l (s_l)$ is held constant
at time $t$. Thus, the following expression is the approximation:

$$\hat{v}_t^F (x(t + 1), s_l(t + 1), u_l(t + 1)) \approx \left( \max\{\gamma_j^l s_l + \hat{\gamma}_j^l : (\gamma_j^l, \tilde{\gamma}_j^l) \in \Gamma_t^l (u_l(t))\} + \max\{\theta_j^l u_l + \hat{\theta}_j^l : (\theta_j^l, \tilde{\theta}_j^l) \in \Theta_t^l (s_l(t))\} \right)/2, \forall l \in \{1, \ldots, L\}. \quad (5)$$

5 Heuristics

As the cardinality of our state space is exponential in the number of locations (see Malladi
et al. 2020), we pursue approximate dynamic programming methods (Bertsekas et al. 1997;
Powell 2007) to design policies instead of obtaining a representation of the entire lookup
table of the optimal cost function. We first present the Myopic Policy (MP) in Sect. 5.1 to
determine dynamic inventory and relocation decisions myopically, followed by two policies
in Sect. 5.2, Myopic No-Flexibility (MNF) and Dynamic No-Flexibility (DNF), that do not
allow inventory and module relocation. DNF serves as our computational benchmark policy
against which we compare the quality of the remaining heuristics. We consider a class of
heuristic policies known as lookahead policies, which use an approximate cost-to-go term in
the optimality equations at every decision epoch. We employ rollout policies that determine
actions at every epoch by solving a forward pass of the optimality equation with the cost-to-
go approximated as the expected cost of a given policy under a specified set of conditions
from the next decision epoch onward (Goodson et al. 2017; Secomandi 2001). Specifically,
we propose two policies which assume at every decision epoch that from the next epoch
onward, mobility of production capacity and transshipment capability are not available and
demand distributions remain stationary at the current belief-mixed distributions. In Sect. 5.3,
we consider a rollout policy, Rollout of Stationary Future (RSF), that determines module
and inventory relocation decisions as well as production decisions at each epoch, with the described future conditions beginning from the next decision epoch. We present a policy, Lookahead of Stationary Future (LSF), that uses the same idea as RSF but with a piecewise linear approximation of the cost-to-go term in Sect. 5.4. In Sect. 5.5, we propose a second rollout policy, Rollout for Relocation Only (RRO), that determines only the module and inventory relocation decisions at each epoch, with the described future conditions beginning before the production event in the current period.

Here, we present additional notation that will be useful in this section. Let: \( P_{ij} = \sum_{d, z} P_{ij}(d, z) = P(\mu(t + 1) = j \mid \mu(t) = i) \) for all \( i, j \in \{1, \ldots, N\} \), \( P = \{P_{ij}\} \), and \( \pi \) satisfy \( \pi = \pi P \). Thus, \( P \) is the transition matrix of the modulation process, and \( \pi \) is a stationary probability vector, which we will assume is unique in \( X \) and hence has interpretation as the distribution of the modulation process. Further, let \( O_{ij}(d, z) = P(d(t + 1) = d, z(t + 1) = z \mid \mu(t) = j, \mu(t + 1) = i) = P_{ij}(d, z) / P_{ij} \), or equivalently, \( O_{ij}(d, z) = P(d(t + 1) = d, z(t + 1) = z \mid \mu(t + 1) = j, \mu(t) = i) / P_{ij} \). Thus, \( O_{ij}(d, z) \) describes the relationship between the modulation process and the demand and the AOD observations of the modulation process.

5.1 Myopic policy (MP)

For the Myopic Policy (MP), the decision-maker optimizes over the one period cost function to determine relocation and replenishment decisions. At every decision epoch with current state \((x, s, u)\), we therefore solve the following integer program:

\[
\text{MP: } \min_{\Delta S, u', \gamma} \sum_{l=1}^{L} \left\{ (K_S^{S^+} \Delta S^+ + K_S^{S^-} \Delta S^-) + K_M |u_l - u'_l|/2 + \sum_{n=1}^{M} \sigma(d^n_l, x) [h_l r^n_l + b_l o^n_l] \right\}
\]

subject to

\[
\sum_{l=1}^{L} u'_l = Y, \\
0 \leq u'_l \leq Y'_l, \quad \forall \ l \in \{1, \ldots, L\}, \\
\sum_{l=1}^{L} \Delta S^+_l = \sum_{l=1}^{L} \Delta S^-_l, \\
0 \leq \Delta S^+_l \leq \sum_{k \neq l} (s_k)^+, \quad 0 \leq \Delta S^-_l \leq -(s_l)^+, \\
\forall \ l \in \{1, \ldots, L\}, \\
(s_l + \Delta S^+_l - \Delta S^-_l) \leq y_l \leq (s_l + \Delta S^+_l - \Delta S^-_l) + U_l + u'_l G, \quad \forall \ l \in \{1, \ldots, L\}, \\
r^n_l \geq y_l - d^n_l, \quad o^n_l \geq d^n_l - y_l, \\
\forall \ l \in \{1, \ldots, L\}, n \in \{1, \ldots, M\}, \\
r^n_l, \ o^n_l \in \mathbb{Z}^+, \ u'_l, \ \Delta S^+_l, \ \Delta S^-_l, \ y_l \in \mathbb{Z}, \\
\eta_l, \ \zeta_l \in \mathbb{R} \quad \forall \ l \in \{1, \ldots, L\}. \quad (6)
\]
where $M$ is the number of demand outcomes at any location $l$ and where we have assumed $O_{ij}(d, z)$ is independent of $i$ and $z$,

$$P(d(t + 1) | \mu(t + 1)) = \prod_l P(d_l(t + 1) | \mu(t + 1)),$$

and $d^n_l$ is the $n$th realization of the random variable $d_l$. MP accounts for transshipment quantities entering and leaving each location $l$ as $\Delta^S_{l}^{+}$ and $\Delta^S_{l}^{-}$ respectively, the post module movement capacity count as $u'_l$, the post-replenishment inventory position as $y_l$, and the held and backlogged inventory quantities as $r^n_l$, and $o^n_l$ for the $n$th demand scenario. The flow balance constraints for modules and inventory are followed by the inventory accounting constraints. We will find later that the computational quality of MP is poor, emphasizing the need for policies that enable dynamic optimization. We do not use MP as a benchmark as the computational analysis in Malladi et al. (2020) indicates that the quality of the myopic policy is influenced by the number of locations in the system. Thus, in the following subsection, we pursue benchmark policies that do not allow resource mobility.

5.2 No-flexibility policies

In this section, we present two No-Flexibility policies that provide an upper bound on the optimal solution of the $L$ location, $Y$ module problem.

5.2.1 Myopic no-flexibility (MNF) policy

We remark that a natural and easily computed and implemented sub-optimal policy for the finite capacity $L = 1$ problem is to order either the difference between the optimal base stock value for the infinite capacity case and the current inventory level or to order the capacity of the production system, whichever is smaller. More specifically, the local order up to level at each location $l$ is given by

$$\hat{y}_l = \min \left\{ \max\{s^*_l(x), s_l + \Delta^S_l\}, s_l + \Delta^S_l + U_l + u'_l G \right\},$$

(7)

where $s^*_l(x)$ is an optimal myopic base stock level for the infinite capacity problem, as proposed by Malladi et al. (2018).

The MNF policy does not permit inventory and module relocation, assumes that local replenishment is based on the policy presented in (7), and assumes that the fixed, static production capacities at the locations are selected in order to minimize multi-location expected total cost with stationary belief distribution $\pi$. We have initially considered its use as a benchmark policy owing to its performance in the single location problem and its computational simplicity; however we find that it is outperformed as an upper bound by a dynamic policy for the no-flexibility system proposed in the next subsection.

5.2.2 Dynamic no-flexibility (DNF) policy: the benchmark policy

The Dynamic No-Flexibility (DNF) policy is a dynamic policy of executing inventory control at each location for a fixed production module configuration, disallowing both stock and module relocations. The following integer program that accounts for the future cost must be solved at every decision epoch to implement the DNF policy. We make use of the static belief approximation $\hat{v}_F^l$ of the $L = 1$ subproblems’ solutions (from Sect. 4.1) in the future cost term. Additionally, we assume that for every $l \in \{1, \ldots, L\}$, the local controller approximates
\( \hat{\lambda}(d, z, x) \) as \( \hat{\lambda}(d^n, x) \) by using the data sourced locally (i.e., \( d^n \), the demand realized at location \( l \)). This assumption allows the decomposition of the cost-to-go term by location.

We note that the DNF policy is independent of the coefficient \( \theta \), making it a suitable benchmark.

\[
\text{DNF: } \min_{l=1}^{L} \sum_{q=0}^{\tilde{u}_l G} w(l, q) \left\{ \sum_{n=1}^{M} \sum_{i=1}^{N} P_{ij} O_{nj}^l \left[ h_l r^n_l + b_l o^n_l + \beta \hat{v}_{F}^l (\hat{\lambda}(d^n, x), s_l + \Delta^S + q - d^n_l, u_l + \Delta^M) \right] \right\},
\]

subject to

\[
\begin{align*}
& r^n_l \geq s_l + \sum_{\Delta^S, \Delta^M, q} w(l, \Delta^S, \Delta^M, q) (\Delta^S + q) - d^n_l, \\
& \forall l \in \{1, \ldots, L\}, n \in \{1, \ldots, M\}, \\
& o^n_l \geq d^n_l - s_l - \sum_{\Delta^S, \Delta^M, q} w(l, \Delta^S, \Delta^M, q) (\Delta^S + q), \\
& \forall l \in \{1, \ldots, L\}, n \in \{1, \ldots, M\}, \\
& r^n_l, o^n_l \in \mathbb{Z}^+ \forall n \in \{1, \ldots, M\}, \\
& w(l, q) \in \{0, 1\}, \forall q \in \{0, \ldots, U_l + \tilde{u}_l G\}, \\
& \text{for } l \in \{1, \ldots, L\}, \tag{8}
\end{align*}
\]

where \( M \) is the number of demand outcomes at any location \( l \). For this integer program, as the future cost term is obtained from a lookup table and is a nonlinear expression, binary variables \( w(l, q) \) are used to choose the production decisions at the current epoch. The constraints account for inventory flows, namely, of held \( (r^n_l) \) and backordered \( (o^n_l) \) quantities. The value function approximation used for this policy remains relevant for the cases with flexibility as well.

### 5.3 Rollout of stationary future (RSF)

The Rollout of Stationary Future (RSF) heuristic policy is based on the approximation presented in (3). In RSF, at each decision epoch with current state \( (x, s, u) \), we require the integer program RSF given below be solved. The resulting policy utilizes the same value function approximation as DNF by assuming the local data sourcing assumption holds. For this integer program, since the future cost term \( \hat{v}_{F}^l \) is obtained from a lookup table and is a nonlinear expression, we adopt the following formulation that uses binary variables \( w(l, \Delta^S, \Delta^M, q) \) to choose the actions at the current epoch: transshipment quantity \( \Delta^S \) entering location \( l \), the number of modules \( u \) entering location \( l \), and the production quantity \( a \) at location \( l \). These binary variables enable suitable selection of \( v_{1}^l \theta \) from lookup tables in the integer program:

\[
\text{RSF: } \min_{l=1}^{L} \sum_{\Delta^S = -(s_l)^+}^{\Delta^S} \sum_{\Delta^M = -u_l}^{\Delta^M} \sum_{q=0}^{U_l + \tilde{u}_l G} w(l, \Delta^S, \Delta^M, q)
\]

\( \sum_{l=1}^{L} \sum_{i=1}^{N} P_{ij} O_{nj}^l \left[ h_l r^n_l + b_l o^n_l + \beta \hat{v}_{F}^l (\hat{\lambda}(d^n, x), s_l + \Delta^S + q - d^n_l, u_l + \Delta^M) \right] \}

subject to

\[
\begin{align*}
& r^n_l \geq s_l + \sum_{\Delta^S, \Delta^M, q} w(l, \Delta^S, \Delta^M, q) (\Delta^S + q) - d^n_l, \\
& \forall l \in \{1, \ldots, L\}, n \in \{1, \ldots, M\}, \\
& o^n_l \geq d^n_l - s_l - \sum_{\Delta^S, \Delta^M, q} w(l, \Delta^S, \Delta^M, q) (\Delta^S + q), \\
& \forall l \in \{1, \ldots, L\}, n \in \{1, \ldots, M\}, \\
& r^n_l, o^n_l \in \mathbb{Z}^+ \forall n \in \{1, \ldots, M\}, \\
& w(l, q) \in \{0, 1\}, \forall q \in \{0, \ldots, U_l + \tilde{u}_l G\}, \\
& \text{for } l \in \{1, \ldots, L\}, \tag{8}
\end{align*}
\]
\[
\left\{ K^S_+ (\Delta S)^+ + K^S_- (\Delta S)^- + K^M |\Delta M|/2 \\
+ \sum_{n=1}^N \sum_{l=1}^L x_l \sum_{j=1}^N P_{lj} O_{lj}^n \left[ h_l r^n_l + b_l o^n_l \\
+ \beta v^F,\theta (\lambda (d^n_l, x_l), s_l + \Delta S + q - d^n_l, u_l + \Delta M) \right] \right\},
\]

subject to
\[
\begin{align*}
& L \sum_{l=1}^L \sum_{k \neq l} (s_k)^+ Y'_{l} - u_l U_l + (u_l + \Delta M) G \\
& \sum_{q=0} w(l, \Delta S, \Delta M, q) \Delta M = 0, \\
& L \sum_{l=1}^L \sum_{k \neq l} (s_k)^+ Y'_{l} - u_l U_l + (u_l + \Delta M) G \\
& \sum_{q=0} w(l, \Delta S, \Delta M, q) \Delta S = 0, \\
& r^n_l \geq s_l + \sum_{\Delta S, \Delta M, q} w(l, \Delta S, \Delta M, q) (\Delta S + q) - d^n_l, \\
& \forall l \in \{1, \ldots, L\}, \ n \in \{1, \ldots, M\}, \\
& o^n_l \geq d^n_l - s_l - \sum_{\Delta S, \Delta M, q} w(l, \Delta S, \Delta M, q) (\Delta S + q), \\
& \forall l \in \{1, \ldots, L\}, \ n \in \{1, \ldots, M\}, \\
& r^n_l, \ o^n_l \in \mathbb{Z}^+, \ \forall l \in \{1, \ldots, L\}, \ n \in \{1, \ldots, M\}, \\
& w(l, \Delta S, \Delta M, q) \in \{0, 1\}, \\
& \forall \Delta S \in -(s_l)^+, \ldots, \sum_{k \neq l} (s_k)^+, \text{ and} \\
& \Delta M \in \{-u_l, \ldots, Y'_l - u_l\}, \\
& q \in \{0, \ldots, U_l + (u_l + \Delta M) G\}, \ l \in \{1, \ldots, L\}. 
\end{align*}
\] (9)

The first two constraints ensure the balance of module flows and transshipped inventory flows between locations. The next two sets of constraints help determine held and backordered quantities at each location.

In this approach, the number of binary variables required to solve the one period problem at every epoch grows linearly in \(L\) and quadratically in the total number of modules \(Y\). Hence, we present a lookahead approach in Sect. 5.4 to improve the computational efficiency of the joint controller’s strategy using the piecewise linear and convex approximation of \(v^F,\theta\) presented in Sect. 4.2 that reduces the number of binary variables used.

### 5.4 Lookahead of stationary future (LSF)

The mixed integer program LSF, presented below, makes use of the piecewise linear and convex approximation of the single location capacitated inventory control system’s cost-to-
go function presented in Sect. 4.2 in order to reduce the computational effort required for RSF. Using this functional approximation of the cost-to-go function reduces the number of integer variables by $O(GY^2 LI)$ where $G$, $Y$, $L$, and $I$ are, respectively, the capacity per module, the total number of production modules, the number of locations, and the available storage capacity at each location:

**LSF:**

$$\begin{align*}
\text{minimize} \quad & \sum_{l=1}^{L} \left\{ (K_l^{S+} \Delta_l^{S+} + K_l^{S-} \Delta_l^{S-}) + K^M |u_l - u'_l|/2 \\
& + \sum_{n=1}^{M} \sigma(d^n_l, x) \left[ h_l r^n_l + b_l o^n_l + \beta(\zeta_l + \eta_l)/2 \right] \right\}, \\
\text{subject to} \quad & \zeta_l \geq \gamma_j^l(y_l - \left[ E[D_l(t)] \right]) + \hat{\gamma}_j^l \forall (\gamma_j^l, \hat{\gamma}_j^l) \in \Gamma_{i+1}(u_l) \\
& \eta_l \geq \theta^l_j u'_l + \hat{\theta}^l_j \forall (\theta^l_j, \hat{\theta}^l_j) \in \Theta_{i+1}(s_l) \forall l \in \{1, \ldots, L\}, \\
& \sum_{l=1}^{L} u'_l = Y, \\
& 0 \leq u'_l \leq Y'_l \forall l \in \{1, \ldots, L\}, \\
& \sum_{l=1}^{L} \Delta_l^{S+} = \sum_{l=1}^{L} \Delta_l^{S-}, \\
& 0 \leq \Delta_l^{S+} \leq \sum_{k \neq l}^{S_k}^+, \quad 0 \leq \Delta_l^{S-} \leq -(S_l)^+, \\
& \forall l \in \{1, \ldots, L\}, \\
& (s_l + \Delta_l^{S+} - \Delta_l^{S-}) \leq y_l \leq (s_l + \Delta_l^{S+} - \Delta_l^{S-}) + U_l + u'_l G, \\
& \forall l \in \{1, \ldots, L\}, \\
& r^n_l \geq y_l - d^n_l, \quad o^n_l \geq d^n_l - y_l, \forall l \in \{1, \ldots, L\}, \\
& n \in \{1, \ldots, M\}, \text{ and} \\
& r^n_l, o^n_l \in \mathbb{Z}^+ \forall n \in \{1, \ldots, M\}, u'_l, \Delta_l^{S+}, \Delta_l^{S-}, y_l \in \mathbb{Z}, \\
& \eta_l, \zeta_l \in \mathbb{R} \forall l \in \{1, \ldots, L\}. \quad (10)
\end{align*}$$

This heuristic utilizes significantly fewer integer variables compared to the integer program in (9). Additionally, we have the following result that shows LSF can be solved as a linear program to obtain an optimal solution when module capacity equals 1. This result improves the speed of implementation dramatically in such instances, in comparison with RSF.

**Proposition 2** LSF can be solved exactly by relaxing all the integrality constraints when module capacity $G = 1$.
Proof of this result follows the proof of Proposition 2 Malladi et al. (2020). We remark that the numerical results in Sect. 6 will justify the robustness of the $G = 1$ assumption.

5.5 Rollout for relocation only (RRO)

We now consider a distributed decision-making structure in which all the relocation decisions are made using the heuristic Rollout for Relocation Only (RRO) while replenishment decisions are made at the individual locations. In (3), consider the inner minimization and note the terms in the inner brackets are bounded below by

\[
\sum_{l=1}^{L} \left[ \left(1 - \theta \right) \min_{y_l} \left\{ \mathcal{L}_l(x, y_l) \right\} \right.
+ \beta \sum_{d,z} \sigma(d, z, x)v_l^F(\lambda(d, z, x), y_l - d_l, Y_l')
+ \theta \min_{y_l} \left\{ \mathcal{L}_l(x, y_l) \right\}
+ \beta \sum_{d,z} \sigma(d, z, x)v_l^F(\lambda(d, z, x), y_l - d_l, u_l') \left. \right]\right] ,
\]

(11)

where the first minimization is now relaxed to operate over all $y_l$ such that $s_l + \Delta S_l \leq y_l \leq s_l + \Delta I_l + U_l + Y_l'G$ and the second minimization is over all $y_l$ such that $s_l + \Delta S_l \leq y_l \leq s_l + \Delta I_l + U_l + u_l'G$. We note that the terms in (11) equal

\[
\sum_{l=1}^{L} \left[ \left(1 - \theta \right)v_l^F(x, s_l + \Delta S_l, Y_l') + \theta v_l^F(x, s_l + \Delta I_l, u_l') \right].
\]

The value function approximation from Sect. 4.1 is employed here for tractability of calculations. Then, for the RRO heuristic, at every decision epoch with beginning state $(x, s, u)$, we first solve

1. the following integer program to determine the transport decisions, i.e., the amount of inventory $\Delta S$ received at every location $l$ and the number of production modules $\Delta M$ received at every location $l$:

RRO:

\[
\begin{align*}
\min \sum_{l=1}^{L} \sum_{d \neq l} Y_l' - u_l \
& \quad \sum_{\Delta S = -(s_l)}^{+} \sum_{\Delta M = -u_l}^{+} w(l, \Delta S, \Delta M) \left\{ K_l^{S+}(\Delta S)^+ + K_l^{S-}(-\Delta S)^+ + K_l^{M} |\Delta M|/2 + v_l^{F,\theta}(x, s_l + \Delta S, u_l + \Delta M) \right\},
\end{align*}
\]

subject to

\[
\begin{align*}
& \sum_{l=1}^{L} \sum_{\Delta S = -(s_l)}^{+} \sum_{\Delta M = -u_l}^{+} w(l, \Delta S, \Delta M) \Delta M = 0, \\
& \sum_{l=1}^{L} \sum_{\Delta S = -(s_l)}^{+} \sum_{\Delta M = -u_l}^{+} w(l, \Delta S, \Delta M) \Delta S = 0, \quad \text{and}
\end{align*}
\]
\[ w(l, \Delta^S, \Delta^M) \in [0, 1], \forall \Delta^S \in \{-(s_l)^+, \ldots, \sum_{k \neq l}^\Delta s_k^+\}, \]
\[ \Delta^M \in \{-u_l, \ldots, Y'_l - u_l\}, \ l \in \{1, \ldots, L\}. \]  

\[ w(l, \Delta^S, \Delta^M) \in [0, 1], \forall \Delta^S \in \{-(s_l)^+, \ldots, \sum_{k \neq l}^\Delta s_k^+\}, \]
\[ \Delta^M \in \{-u_l, \ldots, Y'_l - u_l\}, \ l \in \{1, \ldots, L\}. \]  

2. We then determine the local controllers’ replenishment decisions through the location-wise order-up-to-policy presented in (7), in which the quantity transshipped to any location \( l \) will be obtained using the solution of the above integer program RRO as
\[ \Delta^S_l = \sum_{k \neq l}^\Delta s_k^+ + \Delta^S = -(s_l)^+ + \sum_{k \neq l}^\Delta s_k^+ \]
\[ \Delta^M = -(u_l), \ldots, Y'_l - u_l \] for all locations \( l \).

6 Computational study and results

In Sect. 6.1, we present the experimental design of generating instances that would allow us to study the variation of heuristic quality and the value added due to mobility as a function of the number of modulation states \( N \), the probability of not transitioning away from any modulation state \( \phi \), the number of locations \( L \), the module capacity \( G \), the movement cost per unit of inventory between any pair of locations \( K^S \), and the movement cost per production module \( K^M \). On each instance of the generated instance sets, we implement the heuristic policies proposed in Sect. 5 on fifty sampled trajectories to obtain a sample average cost of performance for each policy. All the policies are then compared against the selected benchmark policy, DNF. We then present an analysis of our computational findings in Sects. 6.2 and 6.3.

6.1 Instance design

We generated two sets of instances in the following manner.

6.1.1 Set A

We fix the following parameters: length of the horizon \( T = 30 \), number of locations \( L = 5 \), and total number of modules \( Y = \lceil \frac{1}{4} L \rceil \). We vary the module capacity \( G \in \{1, 2, 5\} \), fixing the number of demand outcomes \( M = 2G + 1 \) (allowing all integer outcomes between 0 and \( 2G \)) at each location. We consider three different values for the number of modulation states \( N \in \{2, 3, 4\} \). The underlying Markov chain’s transition structure is presented in Fig. 1. We vary the probability of not leaving any modulation state, which we refer to as the staying probability, \( \phi \in \{0.75, 0.95\} \). We randomly obtain a multi-location discrete demand distribution for each combination of the parameters listed so far, with demand outcomes \( \{0, \ldots, 2G\} \) such that the probabilities are randomly generated ensuring that exactly one of the \( N \) expected demands at each location lies in each of following the intervals

- \([0, G)\) and \([G, 2G]\) if \( N = 2 \)
- \([0, 0.6G), [0.6G, 1.4G), [1.4G, 2G]\) if \( N = 3 \), and
- \([0, 0.5G), [0.5G, G), [G, 1.5G], [1.5G, 2G]\) if \( N = 4 \).

We fix the backorder cost \( b \) to 2 and the holding cost to 1. We pair each combination of transshipment cost \( K^S \in \{0, 1, 2, 2.5, 10000\} \) and module movement cost \( K^M \in \{0, 1.5, 2, 2.5, 10000\} \) with the demand instances created above. There are a total of \( 3 \times 3 \times 2 \times 25 = 450 \) randomly generated instances, with 18 underlying demand instances.
6.1.2 Set B

In this instance set, we focus on varying the number of locations \( L \in \{2, 5, 10, 15, 20, 25\} \) and the movements costs \( K^S \) and \( K^M \in \{0, 1.5, 2, 2.5, 1000\} \). We fix the remaining parameters as follows: length of the horizon \( T = 30 \), total number of modules \( Y = \lceil \frac{4}{3} L \rceil \), module capacity \( G = 1 \), number of modulation states \( N = 3 \), staying probability \( \phi = 0.95 \), number of demand outcomes \( M = 2G + 1 \), backorder cost \( b = 2 \), and holding cost \( h = 1 \). A multi-location discrete demand distribution for each combination of these parameters are randomly generated ensuring that exactly one of the \( N \) expected demands at each location lies in each of the intervals \((0, 0.6G)\), \((0.6G, 1.4G)\), and \((1.4G, 2G)\). This procedure results in a total of \( 6 \times 25 = 150 \) instances, with 6 underlying demand instances (Fig. 2).

Without loss of generality, we set the production cost \( c_l \) at all locations to zero in the instances of both sets.

We note that making the cost parameters location-dependent would make the experiments more realistic. However, since we aim to demonstrate the value of mobile production capacity in interaction with inventory transshipment as a proof of concept even when all the locations are identical, we have generated our instances thus.

6.2 Quality of heuristics

We evaluated the heuristic policies, RRO, RSF, LSF, and MP on fifty sample trajectories of the instance set, obtained by Monte Carlo simulation, for five values of the coefficient \( \theta \in \{0, 0.2, 0.5, 0.8, 1\} \) where relevant. We compared their performance against the benchmark policy DNF and also juxtapose MNF against DNF. For each instance, we computed the
Table 1  Variation of average savings due to proposed heuristics over DNF (no flexibility case) with varying $G$ when $\theta = 0.2$ for Instance Set A

| $G$ | MNF (%) | MP (%) | RRO (%) | LSF (%) | RSF (%) |
|-----|---------|--------|---------|---------|---------|
| 1   | -9      | -5     | 42      | 43      | 44      |
| 2   | -10     | 13     | 38      | 38      | -       |
| 5   | -11     | 19     | 44      | 44      | -       |
| Overall | -10 | 9      | 41      | 42      | -       |

approximate value function of the $L = 1, Y = 0$ problem with the various capacities and determined the minimum total fixed cost among all configurations. We then generated 50 sample demand trajectories at each epoch based on the current simulated modulation state. For each trajectory, the beginning state is the zero inventory position at all locations and the module configuration that minimizes the sum of the fixed expected total cost of the single location problems with the steady state belief-based distribution of demand as the epoch-invariant demand distribution at each location. We computed the upper bound $\hat{v}_l(x, s_l, u_l)$ for $x \in X', u \in \{0, \ldots, Y\}$, $\forall s_l, \forall l$ in a one time offline pre-computation step. We approximated the belief space $X = \{x : \sum_{i=1}^{N} x_i = 1, x_i \geq 0\}$ with its non-empty, fixed, finite subset $X' = \{x : \sum_{i=1}^{N} x_i = 1, x_i \in \{0, 1/3, 2/3, 1\}\} \cup \pi$, for $\pi$ such that $\pi = \pi \hat{P}$ when it exists Lovejoy (1991).

We performed a forward dynamic programming pass or a forward rollout implementing the decision-making proposed by each method at each epoch. We obtained the average performance of each heuristic over the 50 simulated trajectories of each instance to analyze various resultant trends in comparison to DNF.

We compared heuristic performance across values of the coefficient $\theta$ (in Tables 10, 11, and 12 in “Appendix A4”) and found the best performance usually at $\theta = 0.2$ for all the heuristics. Table 1 presents the comparison of the performance of all heuristics at $\theta = 0.2$. We find that although LSF at $\theta = 0.2$ is the best performer, for other values of $\theta > 0$, RRO outperforms it. For $\theta = 0$, the cost of RRO is very negative as the inventory holding and backordering components are not considered in the policy’s immediate cost. We note that the cost of the naive policy MP is worse than that of DNF for $G = 1$. However, for higher $G$, MP results in significant savings. This observation establishes the need for intelligent, dynamic heuristics that account for future costs, especially when $G = 1$. The proposed heuristics provide about a $38\% - 44\%$ average reduction in cost compared to DNF, in effect extracting $38\% - 44\%$ improvement in system performance from the two forms of mobile flexibility.

We note that heuristic quality is almost identical between RRO, LSF, and RSF.

We repeat the experiments on Set A with a shorter horizon $T = 10$ instead of $T = 30$ (Table 13 in “Appendix A4”) and find that LSF (which mimics RSF) outperforms RRO on average by $2\% - 3\%$. The strength of RRO is its unique usefulness while managing instances where different locations are coupled (or correlated) not only through the modulation process. RSF and LSF rely on the assumption that the demands at different locations are mutually independent, conditional on the belief state.

We now consider the computational efficiency of the heuristics. Table 2 presents the time taken to compute the policy on a single trajectory using MP, RRO, LSF, and RSF and the time taken to compute $\hat{v}_l^F$ for all locations per instance. We note that MP is the fastest while RRO and LSF are significantly faster than RSF. Between RRO and LSF, RRO is faster, with a clear edge for $G > 1$.  

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Table 2: Variation of average runtime in seconds with respect to $G$ for Instance Set A

| $G$ | MP Per trajectory | RRO | LSF | RSF | $\hat{v}_l^F \forall l$ Per instance |
|-----|-------------------|------|------|-----|----------------------------------|
| 1   | 0.89              | 1.46 | 2.17 | 2081| 41                               |
| 2   | 1.31              | 1.97 | 3.94 | –   | 211                              |
| 5   | 2.61              | 3.64 | 7.59 | –   | 2384                             |

Table 3: Variation of average runtime per trajectory in seconds with respect to $L$ for Instance Set B

| $L$ | RRO/MP | LSF/MP |
|-----|--------|--------|
| 2   | 0.66   | 2.14   |
| 5   | 1.44   | 2.26   |
| 10  | 2.97   | 2.05   |
| 15  | 6.2    | 1.94   |
| 20  | 8.2    | 1.9    |
| 25  | 11.8   | 1.7    |

Table 4: Variation of average runtime in seconds with respect to $N$ for Instance Set A

| $N$ | MP Per trajectory | RRO | LSF | $\hat{v}_l^F \forall l$ Per instance |
|-----|-------------------|------|------|----------------------------------|
| 2   | 0.93              | 2.43 | 3.56 | 294                             |
| 3   | 1.47              | 2.23 | 4.35 | 539                             |
| 4   | 2.41              | 2.41 | 5.79 | 1802                            |

LSF is computationally faster than RRO on Set B (Table 3) that contains only $G = 1$ instances as LSF can be solved as a linear program for $G = 1$.

Table 4 shows that the computational effort of computing $\hat{v}_l^F \forall l$ increases significantly when the number of belief states considered ($N$) is increased.

Thus, we find that LSF works best for for $G = 1$ and RRO for $G > 1$ when considering both speed and performance.

6.3 Value of mobility

We now study the trends of value addition due to mobility of production capacity. Table 1 indicates that with increase in $G$, the percentage of savings does not show a clear trend. This behavior might be partly attributed to the fact that the movement cost per unit of capacity is lower when $G$ is higher for the same movement cost per module and the amount of free capacity per location does not scale properly with problem size.

When the number of modulation states $N$ is varied, the average savings over DNF due to LSF do not exhibit a clear trend but we note that the configuration of the other parameters, such as staying probability $\phi$ and module capacity $G$, affect the influence of $N$ (Table 5) on the amount of savings. It is interesting to note the profit potential in certain configurations: when $G = 5$, $N = 3$, and $\phi = 0.95$, mobility extracts about 65% savings. With respect to the probability of not leaving in any modulation state $\phi$, we find that when the dynamics of the world are such that $\phi$ is closer to 1, about 17% higher average savings are observed (Table 6) than when it is farther.
Table 5  Variation of average savings due to LSF with \( \theta = 0.2 \) over DNF (no flexibility case) with varying \( N \) for Instance Set A

| \( N \) | Overall (%) | \( \phi = 0.95 \) (%) |
|-------|-------------|------------------|
| 2     | 44          | 44               |
| 3     | 35          | 52               |
| 4     | 46          | 54               |
| Overall | 42         | 50               |

Table 6  Variation of average savings over DNF (no flexibility case) with varying \( \phi \) for Instance Set A

| \( \phi \) | MP (%) | RRO (\( \theta = 0.2 \)) (%) | LSF (\( \theta = 0.2 \)) (%) |
|-----------|--------|------------------------------|------------------------------|
| 0.75      | 1      | 33                           | 33                           |
| 0.95      | 17     | 48                           | 50                           |

Table 7  Value of mobility represented as % savings using the heuristic LSF over DNF (no flexibility case) with \( \theta = 0.2 \) across varying \( K_S \) and \( K_M \) for Instance Set A

| Module movement cost \( K^M \) | Transshipment cost \( K^S \) |
|-------------------------------|--------------------------|
|                               | 0 (%) | 1.5 (%) | 2 (%) | 2.5 (%) | 1000 (%) |
| Transshipment cost \( K^S \) | 0     | 49      | 48    | 46      | 50       |
|                                | 1.5   | 48      | 41    | 40      | 42       |
|                                | 2     | 49      | 41    | 41      | 41       |
|                                | 2.5   | 47      | 43    | 41      | 39       |
|                                | 1000  | 50      | 44    | 43      | 40       |

Table 8  Variation of average savings due to LSF with \( \theta = 0.2 \) over DNF (no flexibility case) with varying \( L \) for Instance Set B

| \( L \) | 2 | 5 | 10 | 15 | 20 | 25 | Overall |
|--------|---|---|----|----|----|----|---------|
| LAJ    | 57% | 44% | 32% | 61% | 64% | 48% | 51%     |

Table 7 presents the value of mobility expressed as percentage savings due to LSF over TableDNF as a function of movement costs \( K_S \) and \( K_M \). We note that both forms of flexibility offer significantly high savings even when operated independently as seen from the row/column with the cost set to 1000. We note that for all the considered combinations of movement costs (except 1000 for both), significantly high savings, to the tune of 40%, are observed. We also make note that production capacity mobility independently (\( K_S = 1000 \)) extracts 3–5% higher savings than transshipment operated independently (\( K_M = 1000 \)), emphasizing the value of production capacity mobility in comparison to transshipment. For the subset of Set A with \( G = 5 \), we note that the independent savings from production capacity mobility are about 10–13% higher than those from inventory mobility; these quantities are almost twice those at \( G = 1 \) (Tables 14 and 15 in “Appendix A4”).

From Table 8 that presents a trend of average savings from LSF in Set B containing only \( G = 1 \) instances, we note that as the number of locations \( L \) increases, the average value addition due to resource mobility over DNF is very high (30-65%) generally. Once again, certain configurations extract very high savings. Although an increasing trend is expected as seen in Malladi et al. (2020), we do not see it clearly in these averages.
Table 9  Comparison of average savings LSF with $\theta = 0.2$ over DNF (no flexibility case) when a) the DM models epoch-invariant steady state (SS) demand distributions when a partially observed (PO) modulation process is acting, b) the DM partially observes the modulation process, and c) the DM completely observes (CO) the modulation process for Instance Set A

| $G$ | SS (%) | PO (%) | CO (%) |
|-----|--------|--------|--------|
| 1   | 44     | 43     | 44     |
| 2   | 38     | 38     | 39     |
| 5   | 44     | 44     | 44     |
| Overall | 42 | 42     | 42     |

Since the efficiency of the heuristics decreases with increase in the number of modulation states $N$ in partially observed (PO) decision-making, we consider the case where the decision maker uses the steady state (SS) distribution $\pi$ (when it exists) as the *epoch-invariant* belief state without dynamics. From Table 9, we note that LSF with SS and LSF with PO decision-making perform almost identically. However, on shorter horizons, the additional savings from PO over SS are about 2-3% higher (Table 13 of Section A4). This observation reinforces the value addition of non-stationary demand modeling over short horizons although over long horizons, accurate stationary demand distributions would yield similar savings. We also compare PO with the case where the modulation process is completely observed and find that complete observability of the modulation process improves overall savings by only 1%.

Thus, we conclude our computational analysis by emphasizing the significant impact of module mobility, the value of module capacity, and nature of epoch-variance of demands on the performance of production-inventory systems under epoch-variant demands and while indicating the comparative advantages of the heuristics LSF and RRO over RSF. In particular, LSF for $G = 1$ and RRO for $G > 1$ are efficient as well as effective.

7 Conclusion

We have modeled a multi-location production-inventory system under stochastic demand with geographically relocatable production capacity and have developed computationally efficient heuristic methods for this problem. We have shown that the heuristics LSF and RRO are computationally efficient and improve in solution quality as the system size and uncertainty increase for the $L$ location, $Y$ module problem. We have observed the value of mobility of production capacity to be around 41% on average, relative to systems with no production capacity mobility, irrespective of the presence of transshipment flexibility. We have noted that making decisions assuming a stationary belief state set to the steady state distribution $\pi$ performs comparably with decision-making with partially observed Markov-modulated demands. Complete observability of the modulation state does not appear to add significant value in the current context. Additionally, we infer that although centralized control (RSF and LSF) results in slightly lower costs, decentralized control (RRO) heuristics perform significantly faster for $G > 1$.

We conclude that data-driven production capacity relocation represents a promising new supply chain design and operations feature, and we have presented effective heuristics to determine inventory replenishment and production capacity relocation, with or without inventory transshipment. Our results serve to justify more detailed analyses for specific cases having more realistic assumptions regarding frequency of decision epochs, lead times, and
transshipment costs and that this analysis could use the heuristics developed in this paper for giving guidance to simulation models.

**Appendix**

A1 provides the foundational results for the $L = 1$ case on which bounds presented in Sect. 4 for the general $(L,Y)$ case are based. A2 presents a proof of Proposition 1, A3 presents a heuristic that is analogous to the heuristic LSF, and A4 presents additional tables of computational results.

**A1 Analysis for the $L = 1$ Case**

Assume $v_0 = 0$, $v_{n+1} = H v_n$, define $G_n(x,y) = G(x,y,v_n)$ for all $n$, and let $y^*_n(x,C)$ be the smallest value that minimizes $G_n(x,y)$ with respect to $y$. We remark that

$$v_{n+1}(x,s,C) = \begin{cases} G_n(x,s) & \text{if } s \geq y^*_n(x,C) \\ G_n(x,s+C) & \text{if } s \leq y^*_n(x,C) - C \\ G_n(x,y^*_n(x,C)) & \text{otherwise.} \end{cases}$$

We now present claims for structured results with respect to $G_n$, $v_n$, and $y^*_n$ based on results in Federgruen and Zipkin (1986) and Malladi et al. (2018).

**Proposition 3** For all $n$, $x$, and $C$,

(i) $G_n(x,y)$ is convex in $y$

(ii) $v_n(x,s,C)$ is:

   (a) convex in $s$,
   (b) non-decreasing for $s \geq y^*_n(x,C)$,
   (c) non-increasing for $s \leq y^*_n(x,C) - C$,
   (d) equal to $v_n(x,y^*_n(x,C),C)$ otherwise

(iii) $v_{n+1}(x,s,C) \geq v_n(x,s,C)$ for all $s$.

**Proof of Proposition 3** The convexity of $G_0(x,y)$ in $y$ for all $x$ follows from the definitions and assumptions. Assume $G_n(x,y)$ is convex in $y$ for all $x$. It is then straightforward to show that item (ii) holds for $n = n + 1$ and all $(x,C)$. We remark that the function $g(y) = w(f(y))$ is convex and non-decreasing (non-increasing) if $w$ is convex and non-decreasing (non-increasing) and if $f$ is linear and non-decreasing. Hence, $G_{n+1}(x,y)$ is convex in $y$ for all $x$, and item (i) and item (ii) hold for all $n$ by induction. Since $v_1(x,s,C) \geq v_0(x,s,C)$, a standard induction argument guarantees that item (iii) holds.

Let $v_n(x,s) = v_n(x,s,C)$, $v'_n(x,s) = v_n(x,s,C')$, $G_n(x,y) = G(x,y,v_n)$, and $G'_n(x,y) = G(x,y,v'_n)$.

**Proposition 4** Assume $C \leq C'$, and that $y^*_n(x,C) - d \leq y^*_n(\lambda(d,z,x),C)$ for all $n$ and all $(d,z,x)$. Then for all $n$, $x$, and $s$,

(i) $v'_n(x,s,C) \leq v_n(x,s,C)$

(ii) If $y \leq y' \leq y^*_n(x,C)$, then $G_n(x,y') - G_n(x,y) \leq G'_n(x,y') - G'_n(x,y)$
(iii) If $s \leq s' \leq y^*_n(x, C)$, then $v_{n+1}(x, s', C) - v_{n+1}(x, s) \leq v'_{n+1}(x, s', C) - v'_{n+1}(x, s, C)$.

(iv) $y^*_n(x, C') \leq y^*_n(x, C)$.

**Proof of Proposition 4** Proof of item (i) is straightforward. Regarding item (ii)-item (iv), note item (ii) holds for $n = 0$; assume item (ii) holds for $n$. Then item (iv) also holds for $n$. We now outline the proof that item (iii) holds for $n = n + 1$. Recall

$$v_{n+1}(x, s, C) = \begin{cases} G_n(x, s + C) & \text{if } s \leq y_n - C \\ G_n(x, s) & \text{if } s \geq y_n \\ G_n(x, y_n) & \text{otherwise,} \end{cases}$$

where $y_n = y^*_n(x, C)$, and

$$v'_{n+1}(x, s, C) = \begin{cases} G'_n(x, s + C') & \text{if } s \leq y'_n - C' \\ G'_n(x, s) & \text{if } s \geq y'_n \\ G'_n(x, y'_n) & \text{otherwise,} \end{cases}$$

where $y'_n = y^*_n(x, C')$. Similar to the proof of Proposition 5 and the proof of [Federgruen and Zipkin (1986), Theorem 3], there are two cases: (1) $y_n - C \leq y'_n$, (2) $y'_n \leq y_n - C$, which are more completely described as

$$y' - C' \leq y_n - C \leq y'_n \leq y_n,$$

$$y'_n - C' \leq y'_n \leq y_n - C \leq y_n,$$

respectively. For each case, there are 10 different sets of inequalities that the pair $(s, s')$ can satisfy. Showing that item (iii) holds when $n = n + 1$ for each of the 20 sets of inequalities is tedious but straightforward. We now show that for $s \leq s'$,

$$v_{n+1}(x, s', C) - v_{n+1}(x, s, C) \leq v'_{n+1}(x, s', C) - v'_{n+1}(x, s, C)$$

implies that for $y \leq y' \leq y_n, G_{n+1}(x, y') - G_{n+1}(x, y) \leq G'_{n+1}(x, y') - G'_{n+1}(x, y)$. Note

$$v_{n+1}(\lambda(d, z, x), y' - d, C) - v_{n+1}(\lambda(d, z, x), y - d, C) \leq v'_{n+1}(\lambda(d, z, x), y' - d, C) - v'_{n+1}(\lambda(d, z, x), y - d, C),$$

for $y - d \leq y' - d \leq y^*_n(\lambda(d, z, x), C)$, which implies

$$G_{n+1}(x, y') - G_{n+1}(x, y) \leq G'_{n+1}(x, y') - G'_{n+1}(x, y)$$

for all $y \leq y' \leq y^*_n(\lambda(d, z, x), C)$ assuming $y^*_n(\lambda(d, z, x), C) - d \leq y^*_n(\lambda(d, z, x), C)$ for all $(d, z, x)$. A standard induction argument completes the proof. $\square$

**Proposition 5** Assume $y^*_n(x, C) - d_l \leq y^*_n(\lambda(d, z, x), C)$ for all $n$ and all $(d, z, x)$. Then for all $n$, $s \leq s' \leq y^*_n(x, C)$ implies:

(i) $v_n(x, s', C) - v_n(x, s, C) \geq v_{n+1}(x, s', C) - v_{n+1}(x, s, C)$,

(ii) $G_n(x, s') - G_n(x, s) \geq G'_{n+1}(x, s') - G'_{n+1}(x, s)$,

(iii) $y^*_n(x, C) \leq y^*_n(x, C)$.

**Proof of Proposition 5** We note item (i) holds when $n = 0$. Assume item (i) holds for $n = n - 1$. Let $y \leq y' \leq y^*_n(\lambda(d, z, x), C)$ implying that $y - d \leq y' - d \leq y^*_n(\lambda(d, z, x), C) - d$ for all $(d, z, x)$. Hence,

$$v_{n-1}(\lambda(d, z, x), y' - d, C) - v_{n-1}(\lambda(d, z, x), y - d, C)$$
For real-valued and continuous $v_n(\lambda(d, z, x), y' - d, C) - v_n(\lambda(d, z, x), y - d, C),$

and thus item (ii) holds for $n = n - 1$ for all $y \leq y' \leq y^*_n(x, C)$. Letting $y' = y^*_n(x, C)$, we observe

$$0 \geq G_{n-1}(x, y^*_n(x, C)) - G_{n-1}(x, y)$$

$$\geq G_n(x, y^*_n(x, C)) - G_n(x, y);$$

hence, item (iii) holds for $n = n - 1$.

We now outline a proof that $v_n(x, s') - v_n(x, s) \geq v_{n+1}(x, s') - v_{n+1}(x, s).$ (13)

Following an argument in the proof of Federgruen and Zipkin (1986, Theorem 2) we consider two general cases: (1) $y^*_n(x, C) - C \leq y^*_n(x, C)$ and (2) $y^*_n(x, C) \leq y^*_n(x, C) - C$. Letting the dependence on $(x, C)$ be implicit, cases (1) and (2) are more completely described as

$$y^*_n - C \leq y^*_n - C \leq y^*_n - C \leq y^*_n,$$

respectively. For each case, there are 10 different sets of inequalities that the pair $(s, s')$ can satisfy. The values $v_n(x, s'), v_n(x, s), v_{n+1}(x, s'),$ and $v_{n+1}(x, s)$ are well defined for each of these inequalities in terms of $G_{n-1}$ and $G_n$. Showing that (13) holds for each of these 20 different sets of inequalities is again tedious but straightforward.

A standard induction argument completes the proof of the proposition. \hfill \Box

We now claim that $v(x, s, C)$ is convex in $C$.

**Proposition 6** (i) If $y \in A(s, C)$ and $y' \in A(s, C')$, then $\lambda y + (1 - \lambda)y' \in A(s, \lambda C + (1 - \lambda)C')$.

(ii) If $\xi \in A(s, \lambda C + (1 - \lambda)C')$, then there is a $y \in A(s, C)$ and a $y' \in A(s, C')$ such that $\xi = \lambda y + (1 - \lambda)y'$.

(iii) For real-valued and continuous $v$,

$$\min\{v(\xi) : \xi \in A(s, \lambda C + (1 - \lambda)C')\}$$

$$= \min\{v(\lambda y + (1 - \lambda)y') : y \in A(s, C)$$

and $y' \in A(s, C')\}.

(iv) For all $(x, s)$ and $n$, $v_n(x, s, C)$ is convex in $C$.

**Proof of Proposition 6** (i) $y \in A(s, C)$ and $y' \in A(s, C')$ imply $\lambda s \leq \lambda y \leq \lambda(s + C)$ and $(1 - \lambda)s \leq (1 - \lambda)y' \leq (1 - \lambda)(s + C')$; summing terms implies the result.

(ii) Let $X = (\lambda C + (1 - \lambda)C' + s)$ and $\Delta^S = (X - \xi)/(X - s).$ Note $\Delta^S \in [0, 1]$ and $\Delta = \Delta^S s + (1 - \Delta^S)X.$ Let $y = \Delta^S s + (1 - \Delta^S)(s + C)$ and $y' = \Delta^S s + (1 - \Delta^S)(s + C').$

Then, $y \in A(s, C)$, $y' \in A(s, C')$, and $\lambda y + (1 - \lambda)y' = \xi$.

(iii) Proof by contradiction follows from items (i) and (ii).

(iv) From item (iii) and the convexity of $G_n(x, y)$ in $y$ for all $n$ and $y$ (by Proposition 3 item (i)), it follows that

$$v_n(x, s, \lambda C + (1 - \lambda)C')$$

$$= \min\{G_n(x, \lambda y + (1 - \lambda)y') : y \in A(s, C), y' \in A(s, C')\}$$
\[
\begin{align*}
&\leq \min\{\lambda g_n(x, y) + (1 - \lambda) g_n(x, y') : \\
y \in A(s, C), y' \in A(s, C')\} \\
&= \lambda v_n(x, s, C) + (1 - \lambda) v_n(x, s, C').
\end{align*}
\]

Clearly, the assumption that \(y_n^*(x, C) - d_l \leq y_n^*(\lambda(d, z, x), C)\) for all \(n\) and all \((d, z, x)\) is in general a challenge to verify \textit{a priori}. Arguments in Federgruen and Zipkin (1986) suggest that as \(n\) gets large, \(y_n^*(x, C)\) may converge in some sense to a function \(y_\infty^*(x, C)\). From Malladi et al. (2018), \(y_0^*(x, C)\) is straightforward to determine. Let \(\hat{y}(x, C) \geq y_\infty^*(x, C) \geq y_n^*(x, C)\) for all \(n\) and \(x\). Then \(\hat{y}(x, C) - d_l \leq y_0^*(\lambda(d, z, x), C)\) for all \((d, z, x)\) implies the above assumption holds. Determination of a function \(\hat{y}\) for the general case is a topic for future research. We present a special case where \(y_0^* = y_n^*\) for all \(n\) in the appendix section.

We point out two key differences between the infinite capacity and the finite capacity cases when the reorder cost, \(K' = 0\). First, when \(C\) is infinite, the smallest optimal base stock level \(y_n^*(x)\) is independent of the number of successive approximation steps, making it (relatively) easy to determine. Unfortunately, this result may not hold when \(C\) is finite except for the situation considered below in Proposition 7. This fact has implementation implications for the controllers at the locations; e.g., determining the base stock levels for the capacitated case will in general be more difficult than for the infinite capacity case.

Second, Propositions 4 and 6 state that \(v(x, s, C)\) is non-decreasing and convex in \(C\). We also know that \(v(x, s, C)\) is convex in \(s\) (from Proposition 3, which is also true for the infinite capacity case) and concave and possibly piecewise linear in \(x\) (from earlier cited results, which is also true for the infinite capacity case). We showed in Sect. 6.2 that these structural results can be computationally useful in determining solutions to the stock and production module relocation problem. The relocation problem for determining \((\Delta^S, \sigma, u')\) given \((x, s, u)\), requires knowing \(v_l(x, s', u')\) for all \(l\). We now consider approaches to compute or approximate \(v(x, s, C)\), following the presentation of a special case where \(y_0^* = y_n^*\) for all \(n\).

**Proposition 7** Assume that for all \((d, z, x)\), \(y_0^*(\lambda(d, z, x), C) - C \leq y_0^*(x, C) - d \leq y_0^*(\lambda(d, z, x), C)\). Then, \(y_n^*(x, C) = y_0^*(x, C)\) for all \(n\).

We remark that the left inequality in Proposition 7 essentially implies that although capacity may be finite, it is always sufficient to insure the inventory level after replenishment can be \(y_0^*(x, C)\).

**Proof of Proposition 7** By induction. Assume \(y_n^*(x, C) = y_0^*(x, C)\). Note therefore,

\[
v_{n+1}(x, s, C) = \begin{cases} 
G_n(x, s + C), & s \leq y_0^*(x, C) - C \\
G_n(x, s), & s \geq y_0^*(x, C) \\
G_n(x, y_0^*(x, C)) & \text{otherwise.}
\end{cases}
\]

Note

(i) \(\min_y G_{n+1}(x, y) \leq G_{n+1}(x, y_0^*(x, C))\)

(ii) \(\min_y G_{n+1}(x, y) \geq \min_y L(x, y) + \beta \sum_{d,z} \sigma(d, z, x) \min_y v_{n+1}(\lambda(d, z, x), y - d)\).

The minimum with respect to \(y\) \(v_{n+1}(\lambda(d, z, x), y - d, C)\) is such that \(y_0^*(\lambda(d, z, x), C) - C \leq y - d \leq y_0^*(\lambda(d, z, x), C)\). By assumption, \(y = y_0^*(x, C)\) satisfies these inequalities. Thus,

\[
\min_y G_{n+1}(x, y) \geq L(x, y_0^*(x, y_0^*(x, C)))
\]

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\[ + \beta \sum_{d,z} \sigma(d, z, x) v_{n+1} \left( \lambda(d, z, x), y_0^*(x, C) - d, C \right) \]

\[ = \mathcal{G}_{n+1}(x, y_0^*(x, C)), \]

and hence \( y_{n+1}^*(x, C) = y_0^*(x, C). \)

### A2 Proof of Proposition 1

**Proof** Let \( v_0(x, s, C) = \hat{v}_0(x, s, C) = 0. \) Consider \( d = (d_l, d_{j \neq l}), \) where \( d_{j \neq l} \) can be considered as additional observation data \( z. \) Let \( \sum_z \sigma(d_l, z, x) = \sigma(d_l, x). \)

\[ v_1(x, s, C) \]

\[ = \min_{s \leq y \leq s+\delta} \left\{ \sum_{d_l} \sigma(d_l, x) \left[ c(y, d_l) \right] \right\} \]

\[ = \min_{s \leq y \leq s+\delta} \left\{ \sum_{d_l} \sum_i x_i \sum_j \Pr(j \mid i) \Pr(d_l \mid j) \left[ c(y, d_l) \right] \right\} \]

\[ = \min_{s \leq y \leq s+\delta} \left\{ \sum_{d_l} \sum_i x_i \left( \Pr(d_l \mid i) \right) \right\} \]

\[ \geq \min_{s \leq y \leq s+\delta} \left\{ \sum_{d_l} \sum_i x_i \left( \Pr(d_l \mid i) - \max_k \Pr(d_l \mid k) \right) \right\} \]

\[ \geq \min_{s \leq y \leq s+\delta} \left\{ \sum_{d_l} \sum_i x_i \Pr(d_l \mid i) c(y, d_l) \right\} \]

\[ \geq \min_{s \leq y \leq s+\delta} \left\{ \sum_{d_l} \sum_i x_i \Pr(d_l \mid i) c(y, d_l) \right\} \]

\[ + \min_{s \leq y \leq s+\delta} \left\{ - \sum_{d_l} \left( \max_k \Pr(d_l \mid k) - \min_k \Pr(d_l \mid k) \right) c(y, d_l) \right\} \]

\[ = \hat{v}_1(x, s, C) + \min_{s \leq y \leq s+\delta} \left\{ - \sum_{d_l} k(d_l) c(y, d_l) \right\} \]

\[ = \hat{v}_1(x, s, C) - \max_{s \leq y \leq s+\delta} \left\{ \sum_{d_l} k(d_l) c(y, d_l) \right\} \]

\[ = \hat{v}_1(x, s, C) - \sum_{d_l} k(d_l) c(\hat{x}, d_l) \]
\[ = \hat{v}_1(x, s, C) - u, \text{ where } u = \sum_{d_i} k(d_i) c(\hat{y}, d_i) \]

and \( \hat{y} \in \{s, s + C\} \) due to convexity of \( c(y, d_i) \)
\( \forall \ y, d_i \), where \( k(d_i) = \left( \max_k \Pr(d_i \mid k) - \min_k \Pr(d_i \mid k) \right) \).

By induction and infinite summation,
\[ v_n(x, s, C) \geq \hat{v}_n(x, s, C) - u (1 + \beta + \cdots + \beta^n); \]
\[ v(x, s, C) \geq \hat{v}(x, s, C) - u / (1 - \beta). \]

A3 The Heuristic LARRO

We now present a heuristic for large instances with low computational overhead. LARRO stands for lookahead of rollout for relocations only.

**LARRO:**

\[
\begin{align*}
\min_{A^s, u', y} & \sum_l \left\{ (K^S_l + \Delta^S_l + \hat{K}^S_l - \Delta^S_l - \hat{K}^S_l) \\
& + K^M \sum_l |u_l - u'_l|/2 + (\xi_l + \eta_l)/2 \right\}, \\
\text{subject to} & \\
\xi_l & \geq \gamma^l_j(s_l + \Delta^S_l - \Delta^S_l) + \hat{\gamma}^l_j \forall (\gamma^l_j, \hat{\gamma}^l_j) \in \Gamma^l_{i+1}(u_l) \forall l \\
\eta_l & \geq \theta^l_j u'_l + \hat{\theta}^l_j \forall (\theta^l_j, \hat{\theta}^l_j) \in \Theta^l_{i+1}(s_l) \forall l \\
\sum_l u'_l & = Y \\
\sum_l \Delta^S_l & = \sum_l \Delta^S_{l}, \\
0 & \leq u'_l \leq Y'_l, \forall l \\
0 & \leq \Delta^S_l \leq \sum_{k \neq l} (s_k)^+, \forall l \\
0 & \leq \Delta^S_l \leq -(s_l)^+, \forall l \\
u'_l, \Delta^S_l, \Delta^S_{l} & \in \mathbb{Z}, \ \eta_l, \ \xi_l \in \mathbb{R} \ \forall l
\end{align*}
\]

**Proposition 8**  LARRO can be solved exactly by relaxing the integrality constraints.

A4 Results: Additional Tables

We now present additional numerical results that complement Sect. 6.2. See Tables 10, 11, 12, 13, 14 and 15.
Table 10  Variation of average savings due to RRO over DNF with varying \( \theta \) for Instance Set A

| \( G \setminus \theta \) | 0 (%) | 0.2 (%) | 0.5 (%) | 0.8 (%) | 1 (%) |
|--------------------------|-------|---------|---------|---------|-------|
| 1                        | -131  | 42      | 40      | 39      | 38    |
| 2                        | -133  | 38      | 36      | 35      | 35    |
| 5                        | -141  | 44      | 42      | 40      | 39    |
| Overall                  | -135  | 41      | 39      | 38      | 37    |

Table 11  Variation of average savings due to LSF over DNF across \( \theta \) for Instance Set A

| \( G \setminus \theta \) | 0 (%) | 0.2 (%) | 0.5 (%) | 0.8 (%) | 1 (%) |
|--------------------------|-------|---------|---------|---------|-------|
| 1                        | 34    | 43      | 37      | 33      | 31    |
| 2                        | 38    | 38      | 33      | 29      | 27    |
| 5                        | 44    | 44      | 37      | 33      | 31    |
| Overall                  | 39    | 42      | 36      | 32      | 30    |

Table 12  Variation of average savings due to RSF over DNF across \( \theta \) for Instance Set A

| \( G \setminus \theta \) | \( \text{Overall} \) |
|--------------------------|---------------------|
| 1                        | 37%                 |
| 2                        | 44%                 |
| 5                        | 39%                 |
| Overall                  | 35%                 |
|                         | 32%                 |

Table 13  Variation of average savings due to heuristics over DNF across \( G \) for \( \theta = 0.2 \) on a shorter horizon \( T = 10 \) instead of \( T = 30 \) for Instance Set A

| \( G \) | MNF (%) | MP (%) | RRO (%) | LSF (%) | LSF-SS (%) | LSF-CO (%) |
|---------|---------|--------|---------|---------|------------|------------|
| 1       | -3      | -2     | 24      | 28      | 26         | 28         |
| 2       | -4      | 6      | 22      | 24      | 23         | 25         |
| 5       | -5      | 14     | 27      | 29      | 27         | 29         |
| Overall | -4      | 6      | 24      | 27      | 25         | 28         |

Table 14  Value of mobility (% savings over DNF) using LSF with \( \theta = 0.2 \) across varying \( K^S \) and \( K^M \) for \( G = 1 \) instances of Instance Set A

| Module movement cost \( K^M \) | Transshipment cost \( K^S \) |
|-------------------------------|-----------------------------|
|                               | 0  | 1.5 | 2  | 2.5 | 1000 |
| Transshipment cost \( K^S \)  | 0  | 53  | 50 | 49  | 52   | 55   |
|                               | 1.5| 50  | 44 | 38  | 42   | 46   |
|                               | 2  | 54  | 42 | 44  | 42   | 43   |
|                               | 2.5| 48  | 44 | 40  | 36   | 40   |
|                               | 1000| 52  | 43 | 45  | 38   | -3   |
Table 15  Value of mobility (% savings over DNF) using LSF with θ = 0.2 across varying $K^S$ and $K^M$ for $G = 5$ instances of Instance Set A

| Transshipment cost $K^S$ | Module movement cost $K^M$ | 0 | 1.5 | 2 | 2.5 | 1000 |
|--------------------------|----------------------------|----------------|----------------|----------------|----------------|----------------|
|                          | 0                          | 48            | 49            | 47            | 53            | 50             |
|                          | 1.5                        | 49            | 45            | 44            | 46            | 39             |
|                          | 2                          | 50            | 44            | 41            | 44            | 37             |
|                          | 2.5                        | 49            | 46            | 45            | 45            | 34             |
|                          | 1000                       | 50            | 48            | 50            | 44            | −5             |

References

Axsäter, S., Marklund, J., & Silver, E. A. (2002). Heuristic methods for centralized control of one-warehouse, N-retailer inventory systems. *Manufacturing & Service Operations Management, 4*(1), 75–97. https://doi.org/10.1287/msom.4.1.75.291.

Bayer Technology Services GMBH. (2014). Flexible, fast and future production processes. Tech. rep.

Bernstein, F., & DeCroix, G. A. (2006). Inventory policies in a decentralized assembly system. *Operations Research, 54*(2), 324–336. https://doi.org/10.1287/opre.1050.0256.

Bernstein, F., & Federgruen, A. (2005). Decentralized supply chains with competing retailers under demand uncertainty. *Management Science, 51*(1), 18–29. https://doi.org/10.1287/mnsc.1040.0218.

Bertsekas, D. P., Tsitsiklis, J. N., & Wu, C. (1997). Rollout algorithms for combinatorial optimization. *Journal of Heuristics, 3*, 245–262.

Burnetas, A. N., & Katehakis, M. N. (1997). Optimal adaptive policies for Markov decision processes. *Mathematics of Operations Research, 22*(1), 222–225. https://doi.org/10.1287/moor.22.1.222.

Cheung, W. C., & Simchi-Levi, D. (2019). Sampling-based approximation schemes for capacitated stochastic inventory control models. *Mathematics of Operations Research, 44*(2), 668–692. https://doi.org/10.1287/moor.2018.0940.

Federgruen, A., & Zipkin, P. (1986). An inventory model with limited production capacity and uncertain demands II: The discounted-cost criterion. *Mathematics of Operations Research, 11*(2), 208–215. https://doi.org/10.2307/3689804.

Geek Wire. (2018). Amazon finally wins a patent for 3-D printing on demand, for pickup or delivery.

Ghiani, G., Guerriero, F., & Musmanno, R. (2002). The capacitated plant location problem with multiple facilities in the same site. *Computers & Operations Research, 29*(13), 1903–1912.

Godfrey, G. A., & Powell, W. B. (2001). An adaptive, distribution-free algorithm for the newsvendor problem with censored demands, with applications to inventory and distribution. *Management Science, 47*(8), 1101–1112.

Goodson, J. C., Thomas, B. W., & Ohlmann, J. W. (2017). A rollout algorithm framework for heuristic solutions to finite-horizon stochastic dynamic programs. *European Journal of Operational Research, 258*(1), 216–229. https://doi.org/10.1016/j.ejor.2016.09.040.

Halper, R., & Raghavan, S. (2011). The mobile facility routing problem. *Transportation Science, 45*(3), 413–434. https://doi.org/10.1287/trsc.1100.0335.

Herer, Y. T., Tzur, M., & Yücesan, E. (2002). Transshipments: An emerging inventory recourse to achieve supply chain leagility. *International Journal of Production Economics, 80*(3), 201–212. https://doi.org/10.1016/S0925-5273(02)00254-2.

Herer, Y. T., Tzur, M., & Yücesan, E. (2006). The multilocation transshipment problem. *IIE Transactions (Institute of Industrial Engineers), 38*(3), 185–200. https://doi.org/10.1080/07408170500434539.

Jena, S. D., Cordeau, J. F., & Gendron, B. (2015). Dynamic facility location with generalized modular capacities. *Transportation Science, 49*(3), 489–499.

Jordan, W. C., & Graves, S. C. (1995). Principles on the benefits of manufacturing process flexibility. *Management Science, 41*(4), 577–594. https://doi.org/10.1287/mnsc.41.4.577.

Karmarkar, U. S. (1979). Convex/Stochastic programming and multilocation inventory problems. *Naval Research Logistics, 26*(1), 1–19. https://doi.org/10.1002/nav.3800260102.

Karmarkar, U. S. (1981). The multiperiod multilocation inventory problem. *Operations Research, 29*(2), 215–228.
Karmarkar, U. S. (1987). The multilocation multiperiod inventory problem: Bounds and approximations. *Management Science, 33*(1), 86–94. https://doi.org/10.1287/mnsc.33.1.86.

Katehakis, M. N., Melamed, B., & Shi, J. J. (2015). Optimal replenishment rate for inventory systems with compound Poisson demands and lost sales: A direct treatment of time-average cost. *Annals of Operations Research*. https://doi.org/10.1007/s10479-015-1998-y.

Kouvelis, P., & Gutierrez, G. J. (1997). The newsvendor problem in a global market: Optimal centralized and decentralized control policies for a two-market stochastic inventory system. *Management Science, 43*(5), 571–585. https://doi.org/10.1287/mnsc.43.5.571.

Lien, R. W., Iravani, S. M., Smilowitz, K., & Tzur, M. (2011). An efficient and robust design for transshipment networks. *Production and Operations Management, 20*(5), 699–713. https://doi.org/10.1111/j.1937-5956.2010.01198.x.

Lovejoy, W. S. (1991). Computationally feasible bounds for partially observed Markov decision processes. *Operations Research, 39*(1), 162–175.

Malladi, S. S., Erera, A. L., & White, C. C., III. (2018). Inventory control with modulated demand and a partially observed modulation process. arXiv.

Malladi, S. S., Erera, A. L., & White, C. C., III. (2020). A dynamic mobile production capacity and inventory control problem. *IIE Transactions, 52*(8), 926–943. https://doi.org/10.1080/24725854.2019.1693709.

Marcotte, S., & Montreuil, B. (2016). Introducing the concept of hyperconnected mobile production. In *Progress in material handling research*.

Melo, M. T., Nickel, S., & da Gama, F. (2005). Dynamic multi-commodity capacitated facility location: A mathematical modeling framework for strategic supply chain planning. *Computers & Operations Research, 33*(1), 181–208.

MIT News. (2016). Pharmacy on demand.

Pfizer. (2015). Pfizer announces collaboration with GSK on next-generation design of portable, continuous, miniature and modular (PCMM) oral solid dose development and manufacturing units.

Powell, W. B. (2007). *Approximate dynamic programming: Solving the curses of dimensionality* (Wiley Series in Probability and Statistics) (2 ed.). Wiley. https://doi.org/10.1080/00401706.1995.10484354.

Rudi, N., Kapur, S., & Pyke, D. F. (2001). A two-location inventory model with transshipment and local decision making. *Management Science, 47*(12), 1668–1680. https://doi.org/10.1287/mnsc.47.12.1668.10235.

Ryzhov, I. O., Powell, W. B., & Frazier, P. I. (2012). The knowledge gradient algorithm for a general class of online learning problems. *Operations Research, 60*(1), 180–195. https://doi.org/10.1287/opre.1110.0999.

Secomandi, N. (2001). A rollout policy for the vehicle routing problem with stochastic demands. *Operations Research, 49*(5), 796–802. https://doi.org/10.1287/opre.49.5.796.10608.

Smallwood, R. D., & Sondik, E. J. (1973). The optimal control of partially observable Markov processes over a finite horizon. *Operations Research, 21*(5), 1071–1088. https://doi.org/10.1287/opre.21.5.1071.

Sondik, E. J. (1978). The optimal control of partially observable Markov processes over the infinite horizon: Discounted costs. *Operations Research, 26*(2), 282–304. https://doi.org/10.1287/opre.26.2.282.

Verlinde, S., Macharis, C., Milan, L., & Kin, B. (2014). Does a mobile depot make urban deliveries faster, more sustainable and more economically viable: Results of a pilot test in Brussels. In *Transportation Research Procedia: Mobil. TUM 2014 “Sustainable Mobility in Metropolitan Regions”*, May 19–20, 2014. https://doi.org/10.1016/j.trpro.2014.11.027.

Wee, K. E., & Dada, M. (2005). Optimal policies for transshipping inventory in a retail network. *Management Science, 51*(10), 1519–1533. https://doi.org/10.1287/mnsc.1050.0441.

Wörsdörfer, D., & Lier, S. (2017). Optimized modular production networks in the process industry. In *Operations Research Proceedings 2015*. Springer International Publishing.

Wörsdörfer, D., Lier, S., & Crasselt, N. (2017). Real options-based evaluation model for transformable plant designs in the process industry. *Journal of Manufacturing Systems, 42*, 29–43. https://doi.org/10.1016/j.jmsy.2016.11.001.

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