ABSTRACT

Currently, one of the major theoretical problems concerning planet formation in close binary systems is whether the strong perturbation from the companion star can increase the relative velocities ($\Delta V$) of planetesimals around the primary and thus hinder their growth. According to previous studies, while gas drag can reduce the $\Delta V$ between bodies of the same size by forcing orbital alignment on planetesimals, it increases the $\Delta V$ among bodies of different sizes. In this paper, focusing on the $\gamma$ Cephei binary system, we propose a mechanism that can overcome this difficulty. We show that in a dissipating gas disk (with a typical dissipation timescale of $\sim 10^5 - 10^6$ yr), all the planetesimals eventually converge toward the same forced orbits regardless of their size, leading to much lower impact velocities. This process of decreasing $\Delta V$ progressively increases net mass accretion and can even trigger runaway growth for large bodies (radius above 15 km). The effect of the size distribution of planetesimals is discussed and is found to be one of the dominant factors determining the outcome of collisional evolution. In any case, it can be concluded that by including gas dissipation in the early stages of disk evolution, the conditions for planetesimal accretion become much better, and the progression from planetesimals to planet embryos can be accomplished in close binary systems such as $\gamma$ Cep.

Subject headings: methods: numerical — planetary systems: formation

Online material: color figures

1. INTRODUCTION

With the increasing number of planets being discovered in binary systems and the belief that a majority of solar-type stars reside in binary or multiple systems, the issue of planetary formation in binary systems has become a crucial one. Most of the discovered planet-bearing binaries are S-type systems (e.g., $\gamma$ Cep; see Hatzes et al. 2003), in which planets orbit the primary star with a companion star surrounding them on an outer orbit. According to the classical planetary formation scenario, planetesimals form in a protoplanetary disk of gas and dust orbiting a protostar. The formation process is usually treated in three stages (Lissauer 1993; Papaloizou & Terquem 2006; Armitage 2007): (1) formation of kilometer-sized planetesimals ($10^{18} - 10^{22}$ g) from sticking collisions of dust (Weidenschilling & Cuzzi 1993) or from gravitational fragmentation of a dense particle subdisk near the midplane of the protoplanetary disk (Goldreich & Ward 1973), on timescales on the order of $10^4$ yr; (2) accretion of planetesimals into planetary embryos ($10^{26} - 10^{27}$ g. Mercury to Mars size) through a phase of "runaway" and "oligarchic" growth on a timescale on the order of $10^4 - 10^5$ yr, depending on the initial planetesimal sizes, duration of the period of runaway growth, and possible transition to the oligarchic mode (Greenberg et al. 1978; Wetherill & Stewart 1989; Barge & Pellat 1993; Kokubo & Ida 1996, 1998, 2000; Rafikov 2003, 2004); and (3) giant impacts between embryos, producing full-sized ($10^{26} - 10^{28}$ g) terrestrial planets in about $10^7 - 10^8$ yr (Chambers & Wetherill 1998; Levison & Agnor 2003; Kokubo et al. 2006). Here we focus on the second stage to see the influence on the companion of the planetesimal accretion.

The companion star, especially if it is on a close orbit with high eccentricity, may prevent planetary formation by reducing the size of the accretion disk (Artymowicz & Lubow 1994) or exciting high relative velocities between colliding planetesimals (Heppenheimer 1978; Whitmire et al. 1998). The relative velocity ($\Delta V$) is a critical parameter, determining whether accretion or erosion dominates. As a result of perturbations by the companion, $\Delta V$ may exceed the planetesimal escape velocity ($V_{esc} \sim 100 R_p^{0.5}$ (100 km) m s$^{-1}$) and thus inhibit runaway growth. Furthermore, $\Delta V$ can even exceed the threshold velocity ($V_{run}$) at which erosion dominates accretion. Here $V_{run}$ is a few times larger than $V_{esc}$, depending on the prescription for collisions.

Since planetesimals orbit their star in a sub-Keplerian gas disk (Adachi et al. 1976), the presence of gas drag not only damps the companion's secular perturbation, it also forces a strong periastron alignment of planetesimal orbits. This alignment significantly reduces $\Delta V$ between equally sized bodies, favoring the accretion process (Marzari & Scholl 2000). Nevertheless, the alignment forced by gas drag induces another problem. As the alignment is size dependent, it can only reduce $\Delta V$ between planetesimals of the same size, and simultaneously it increases $\Delta V$ between planetesimals of different sizes. Thébault et al. (2006) found this differential orbital alignment to be very efficient, leading to a significant $\Delta V$ increase for any departure from the exact equal-size condition ($R_1 = R_2$, where $R_1$ and $R_2$ are the radii of the two colliding bodies).

Previous studies have adopted steady gas disks in which the dissipation process is neglected and the local gas density is constant. This assumption, which is valid only when the planetesimal accretion timescale (on the order of $10^4 - 10^5$ yr) is much shorter than the dissipation timescale of the local gas density, is violated under several conditions. (1) When the disk viscosity is high or photoevaporation from the external star exists (Hollenbach et al. 1994, 2000; Matsuyama et al. 2003), disks can dissipate very quickly and have short lifetimes, around a few times $10^5$ yr. (2) It has been suggested that the assumption of a single timescale for disk dissipation is not correct and that there could be a wide spread of disk lifetimes, with a large fraction of short-lived disks (Bouwman et al. 2006). As calculated by Matsuyama et al. (2003) and Alexander et al. (2006), even for a disk with a lifetime on the order of $10^6$ yr, the local density can decrease by as much as 2 orders of magnitude within the first few tens of thousands of years. From these two considerations, therefore, a model that...
includes gas dissipation is essential for studying planetesimal accretion.

In this paper, we consider a model in which the gas density progressively decreases, in order to see how the conditions for planetesimal accretion are affected by the gas dissipation process. As expected, the planetesimal growth conditions change to being accretion-friendly thanks to a dissipation-induced orbital convergence, which reduces the \( \Delta V \) between bodies of different sizes. We describe our numerical model and methods in § 2. In § 3, we first simply review the planetesimal dynamics under the coupled influence of the secular perturbation and gas drag and then present the results. Some related and important issues, such as the radial drift, impact rate, erosion conditions, and remnant gas, are discussed in § 4. Finally, in § 5 we summarize.

2. NUMERICAL MODEL AND METHODS

2.1. Gas Disk Model

Our gas model is similar to that of Thébault et al. (2004). Following Weidenschilling & Davis (1985), the gas drag is expressed as

\[
F = -K \nu r v ,
\]

(1)

where \( F \) is the force per unit mass, \( \nu \) is the relative velocity between the planetesimal and gas, \( v \) is its magnitude, and \( K \) is the drag parameter, defined as

\[
K = \frac{3 \rho_g C_d}{8 \rho_p R_p} ,
\]

(2)

\[
\rho_g = \rho_{g,0} T^{-n} , \quad T = \frac{I}{T_s + 1} ,
\]

(3)

Here \( \rho_g \) is the local gas density, with initial value \( \rho_{g,0} \), and \( \rho_p \) and \( R_p \) are the planetesimal density and radius, respectively. \( C_d \) is a dimensionless coefficient related to the shape of the body (\( \approx 0.4 \) for spherical bodies). The \( T^{-n} \) function, in which time \( T \) is scaled by \( T_s \), is used in order to include the gas dissipation and is based on the analytic similarity solutions given by Lynden-Bell & Pringle (1974). Taking typical parameters from Hartmann et al. (1998), where \( n = 3/2 \) and \( T_s = 10^5 \) yr, we plot in Figure 1 the evolution of the gas disk’s density versus time. The density is scaled by the minimum mass solar nebula (MMN) and has the same profile (Hayashi 1981). The initial gas density is 10 MMN, and the corresponding disk mass is about 100 Jupiter masses \( (M_J) \). As can be seen, the density rapidly decreases from 10 to 0.5 MMN within the first few tens of thousands of years and then undergoes a slow damping that lasts for a few million years. This dissipation model is consistent with current theoretical calculations (Matsuyama et al. 2003; Alexander et al. 2006a, 2006b) and with observations (Strom et al. 1993; Haisch et al. 2001; Chen & Kamp 2004), which suggest a typical disk age of 1 Myr with a large scatter from 0.1 to 10 Myr. Note that the effects of binarity on the dissipation of the gas disk are not taken into account, because the details of this issue are poorly known at present.

Our model implicitly assumes an axisymmetric gas disk with constant circular streamlines, and it follows a classical Hayashi (1981) power-law distribution. We are aware that this is a crude simplification for modeling gas disks in close binary systems. In reality, the gas disk around the primary also “feels” the companion’s perturbation, under the effect of which the disk structure will vary from the simplified gas model. For example, the companion’s perturbation can induce spiral structures within the disk (Artymowicz & Lubow 1994). To fully model the behavior of planetesimals in these complex gas disks, one would probably have to rely on hydrocode modeling of the gas in addition to \( N \)-body-type models for planetesimals. Such all-encompassing gas-plus-planetesimal modeling goes beyond the scope of our study in this paper, and it is certainly the direction for further binary disk studies. Therefore, taking a first step here, we prefer a simplified approach in which the gas drag force is given by equation (1). As discussed in some earlier studies (Thébault et al. 2006; Scholl et al. 2007), this kind of simplification, on the average, is reasonable at least for the dynamical evolution of kilometer-sized planetesimals.

2.2. Initial Conditions

We focus on the \( \gamma \) Cephei system, which is a close S-type binary planetary system and hence a good example for testing the influence of the companion on planetesimal accretion. Most of the parameters adopted in this paper are listed in Table 1. The initial gas disk has the same profile as the MMN but is denser by 10 times. We concentrate on planetesimals of four radii \( (R_p = 2.5, 5, 15, 50 \text{ km}) \). As observed by Thébault et al. (2006), for impacts between small bodies (1 km < \( R_p < 10 \text{ km} \)), the delivered kinetic energy peaks at roughly \( R_t \approx 1/R_2 \), where \( R_1 \) and \( R_2 \) are the radii of the two colliding bodies. For larger bodies, the \( R_t/R_2 \) ratio is somewhat smaller. Hence, the relative velocity \( \Delta V (2.5, 5) \) between bodies with \( R_p = 2.5 \text{ km} \) and \( R_p = 5 \text{ km} \) can serve as a representative value for small planetesimals, and \( \Delta V (15, 50) \) similarly for large ones. All the planetesimals initially have very small...
inclinations, based on the work of Hale (1994), which suggests that approximate coplanarity between the equatorial and orbital planes exists for solar-type binary systems with separations less than 30–40 AU. Since it is unrealistic that all planetesimals would have formed synchronously, some earlier-formed planetesimals may have been pumped up to eccentric orbits whereas some others have just formed. For this reason, the initial planetesimal orbits are assigned random eccentricities within the range from 0 to $e_{\text{max}}$, where $e_{\text{max}}$ is the maximum eccentricity induced by the companion. In the $\gamma$ Cep system, $e_{\text{max}}$ is about 0.1 at 2 AU from the primary.

One implicit initial condition in this paper is that, of course, kilometer-sized planetesimals have already formed when the disk begins dissipating. At present, with our poor knowledge of planetesimal formation in binary systems, whether this assumption is valid is not at all certain. According to the current limited knowledge about planetesimal formation around single stars, kilometer-sized planetesimal can form within $10^3$–$10^5$ yr through sticking collisions or as a result of gravitational instability after dust has settled down to the midplane (Goldreich & Ward 1973; Lissauer 1993; Weidenschilling 1997; Youdin & Shu 2002). In this case, the timescale for planetesimal formation could be much shorter than that of the gas disk’s dissipation (about $10^6$ yr is considered in this paper), and thus it is reasonable to assume that gas dissipation starts when a population of kilometer-sized planetesimals already exists in the system.

2.3. Numerical Methods

We performed two types of run. First, we numerically integrated the equations of motion for 1000 independent planetesimals with semimajor axes ranging from 1 to 4 AU. The focus is on the time evolution of orbital eccentricities and periastrons. Second, we concentrate on the time evolution of $\Delta h$ in a specific region near 2 AU from the primary star where a planet is detected. This is the configuration of the $\gamma$ Cep system. The planetesimals are initially distributed in a ring near 2 AU. Since the planetesimal sizes (on the order of kilometers) are very small compared with the system’s typical scale (order of AU), the planetesimals are “real” physical impacts among these objects (e.g., Brahic 1977; Charnoz et al. 2001; Charnoz & Brahic 2001; Lithwick & Chiang 2007). In such a case, we have to resort to the classical “inflated radius” assumption, which assigns an artificially increased radius to each particle (e.g., Brahic 1977; Thebault & Brahic 1998; Marzari & Scholl 2000). For the planetesimals considered here, an artificially increased radius (about $10^{-3}$ to $10^{-4}$ AU) 100 times larger than the “real” radius is adopted for each planetesimal.

In all the runs, we used the fourth-order Hermite integrator (Kokubo et al. 1998), including the gas drag force and the perturbation from the companion. As gas drag also forces an inward drift of planetesimals, we adopt the following boundary condition: bodies whose semimajor axes are less than $R_{\text{in}}$ are reset to $R_{\text{out}}$, and those greater than $R_{\text{out}}$ are set to $R_{\text{out}}$, where $R_{\text{in}}$ and $R_{\text{out}}$ are the inner and outer boundaries of the planetesimal belt, respectively. In this process, only the semimajor axes of the bodies are changed, while the other orbital elements are preserved.

3. RESULTS

3.1. Planetesimal Dynamics: The Secular Approximation

Before presenting the results, let us review the planetesimal dynamics in a perturbed system. Heppenheimer (1978) developed a simplified theory for the evolution of planetesimal eccentricity with time in binary systems. First, he defined two variables, $h$ and $k$, as

$$h = e_p \sin \varpi, \quad k = e_p \cos \varpi,$$

where $e_p$ is the planetesimal’s eccentricity and $\varpi$ is its periastron longitude, defined with respect to that of the companion star ($\varpi = \varpi_p - \varpi_B$, where $\varpi_p$ and $\varpi_B$ are the periastron longitudes of the companion and the planetesimal, respectively). Then, introducing the Lagrange planetary equations, he obtained the following equations for $h$ and $k$:

$$\frac{dh}{dt} = Ak - B, \quad \frac{dk}{dt} = -Ah,$$  

where the constants $A$ and $B$ are

$$A = \frac{3}{4} \frac{M_A}{n(1-e_B^6^{3/2})}, \quad B = \frac{15}{16} \frac{ae_B}{n(1-e_B^{5/2})}$$

with $e_B$ the eccentricity of the binary system and $M_A$ the mass of the primary star. Here $a$ and $n$ are the semimajor axis and mean motion of the planetesimal, respectively. The units of mass, distance, and time are normalized in such a way that the gravitational constant $G$ and the sum of the masses of the two stars are set equal to 1. The semimajor axis of the binary, $a_B$, is chosen as the unit of length, so that time is expressed in units of $T_B/(2\pi)$, where $T_B$ is the orbital period of the binary system.

In the $(h, k)$-plane, there is an equilibrium point (where $dh/dt = 0$ and $dk/dt = 0$) for these equations (eq. [5]), which we refer to as E0. At this point, $e_p = e_f$ and $\varpi = \varpi_f = \varpi_B$, where $e_f = B/A$ and $\varpi_f = \varpi_B$ are the forced eccentricity and periastron of the planetesimal, respectively. If a planetesimal reaches the equilibrium point E0, its eccentricity and periastron will be fixed at $B/A$ and $\varpi_B$ forever.

To compute the effect of gas drag on the variables $h$ and $k$, Marzari & Scholl (2000) modified equation (5) as follows:

$$\frac{dh}{dt} = Ak - B - Dh(h^2 + k^2)^{1/2},$$

$$\frac{dk}{dt} = -Ah - Dk(h^2 + k^2)^{1/2},$$

where $D$ is a coefficient to measure the gas drag force. According to these equations, for a specific $D$ the planetesimal orbit will quickly or slowly (depending on the D-value; a larger $D$ leads to a faster speed) reach another equilibrium point (different from E0), with an equilibrium eccentricity below $B/A$. Furthermore, and this is the crucial point, if $D$ slowly decreases (as a result of gas dissipation), planetesimals will eventually shift their orbits from this equilibrium point toward E0.

In Figure 2, we illustrate the process. In the no-gas case, the motion in the $(h, k)$-plane circulates around the equilibrium point E0 derived from equation (5). For the case with gas drag in our disk model, the motions divide into two phases: (1) the “no-dissipation phase,” for the first few thousand years, in which the gas disk does not significantly dissipate and planetesimals of different sizes quickly reach different equilibrium points depending on their size (point E1 for 50 km bodies, E2 for 20 km, etc.; see Fig. 2), and (2) the “dissipation phase,” in which the gas disk gradually dissipates and, at the same time, all the motions shift along the line $E4$–E0 and eventually fix on the same equilibrium point E0 regardless of their size. We also analyzed the effects of initial orbits on the dynamical behavior. As shown in Figure 2, bodies of the same size (5 km) but with different initial orbits (one is at 11, the other at 12) take different paths ($11$–$E4$, $12$–$E4$) to reach the same equilibrium point (E4). After that, they both
experience the same “dissipation phase” from E4 to E0. From this, we can see that how one chooses the initial planetesimal orbits does not affect the final results, which are based primarily on the latter dissipation phase.

The appearance of the dissipation phase and the dynamical behavior of planetesimal orbits during this phase are very important, because they provide channels to reduce the differential phasing induced by the size dependence of gas drag. Based on the above theoretical analysis, we can expect a decrease in relative velocity ($\Delta V$) from the convergence of all the planetesimal orbits. In the next two subsections, we numerically simulate this process.

3.2. Time Evolution of Eccentricity and Periastron

We first performed a simulation in which 1000 planetesimals (in four equal-number groups with $R_p = 2.5, 5, 15, 50$ km; mutual interactions were neglected) were initially distributed between 1 and 4 AU from the primary. Figure 3 shows the distributions of their eccentricities and periastrons versus semimajor axis at different epochs. Beyond 3 AU, the distributions are random, because the shorter period perturbation and mean motion resonances are dominant. Thus, hereafter only planetesimals within 3 AU are discussed. In Figure 3a (or 3b), every eccentricity (or periastron) reaches an equilibrium value at 5000 yr. These equilibrium values, as discussed in § 3.1 and also pointed out in previous studies (e.g., Thébault et al. 2006), depend on the balance between the perturbation from the companion and the gas drag force. Because of the size dependence of the drag force, bodies of different sizes reach different equilibrium eccentricities (or periastrons). The four gray lines in each panel correspond to the four different size groups ($R_p = 2.5, 5, 15, 50$ km). As the gas gradually dissipates, the equilibrium eccentricities (or periastrons) move to larger values, but at the same time the differences among them become smaller (see Figs. 3c and 3d). After a long time (4 Myr; see Figs. 3e and 3f), almost all eccentricities and periastrons have converged to $e_f$ and $\varpi_f$, respectively.

3.3. Time Evolution of the Relative Velocity

We performed another simulation to investigate the time evolution of $\Delta V$ in a specific place (2 AU from the primary). In this calculation, 1000 planetesimals were initially distributed with semimajor axes between 1.5 and 3 AU. This planetesimal ring is wide enough to trace most collisions at 2 AU.

![Fig. 2.— Phase diagram in the ($h$, $k$)-plane. Gas disks are included for all the cases, except for the one denoted by the large ellipse. E0 is the equilibrium point in the absence of gas, while E1, E2, E3, and E4 are the equilibrium points for bodies of 50, 20, 10, and 5 km, respectively. All the trajectories start from point I1($h = k = 0$), except for the black one, which originates at point I2. [See the electronic edition of the Journal for a color version of this figure.]

![Fig. 3.— Distributions of planetesimal eccentricities and periastrons vs. semimajor axis at three different epochs. The dotted (and dashed) lines in the left and right panels respectively denote $e_f$ (forced eccentricity) and $\varpi = 0$ (at which the planetesimal periastron $\varpi_p = \varpi_f$, the forced periastron). [See the electronic edition of the Journal for a color version of this figure.]]
The results are plotted in Figure 4. Figures 4b and 4c respectively show the average eccentricity and periastron of bodies at 2 AU as functions of time. As the disk gradually dissipates, the planetesimals converge toward the same forced orbits in which $\phi_p = \phi_f$ and $\omega_p = \omega_f$ (see also point E0 in Fig. 2). Figure 4a plots $\Delta V(R_1, R_2)$ as a function of time. It is evident that larger differences in the orbital elements result in larger values of $\Delta V$.

From Figure 4a, it appears that the $\Delta V$s between bodies of equal size is always small because of the orbital alignment. However, between bodies of different sizes the $\Delta V$s first quickly reach high values (e.g., 300–800 m s$^{-1}$) and then each experiences a relatively slow decrease. This is most efficient for large bodies. For 15 km and 50 km sized bodies, the relative velocity $\Delta V(15, 50) \sim 300$ m s$^{-1}$ is much larger than their escape velocities, $V_{esc} \sim 50$ m s$^{-1}$, at the beginning. After about $3 \times 10^5$ yr, $\Delta V(15, 50)$ becomes lower (about 40 m s$^{-1}$) than the escape velocities of the large planetesimals, so runaway growth can occur.

To compare with the case of dissipating gas drag shown in Figure 4, we performed one more simulation with constant gas drag. It shows (Fig. 5) that without gas dissipation, every $\Delta V$ is forced to a relatively high value determined by the equilibrium between the gas drag force and the secular perturbation. The main difference from the case of dissipating gas is that there is no late stage with size-independent orbital phasing and, thus, no decrease in $\Delta V$.

4. DISCUSSION

4.1. Impact Rate

As impacts of different types (between bodies of the same size or different sizes) have totally different values of $\Delta V$ and thus different outcomes (erosion, incomplete accretion, complete accretion, runaway growth), the conditions that govern which type of collision dominates become crucial for planetesimal growth. Figure 6 plots the distribution of impact rates for two cases: the standard case (Fig. 6a) and a randomized case (Fig. 6b). In both, we computed 1000 planetesimals whose radius distribution is assumed to be a Gaussian function centered at 8 km with dispersion $R = 7$ km. The only difference between the two is that the companion and gas drag are not included in the latter case. As shown in Figure 6, for the random case the distribution of impact rates depends only on the initial size distribution: impacts occur
more often where more planetesimals are distributed, for impacts between equal-sized bodies close to the center of the Gaussian. On the other hand, in the standard case the distribution is obviously size dependent: impacts mainly occur between bodies of different sizes. By comparing these two cases, it is clear that under the coupled effect of gas drag and the companion’s perturbation, impacts between bodies of different sizes are favored, while impacts between bodies of the same or similar sizes are hindered. This result can be understood as follows: for bodies of the same size, as they have the same forced orbits and radial drifts, one can only collide with another if their semimajor axes are very close; for bodies of different sizes, in contrast, given their different forced orbits and radial drifts, one can cross many more planetesimal orbits over a much larger region.

4.2. Accretion or Erosion

The key result of this paper, presented in § 3, is that as gas dissipates, all planetesimals will eventually converge toward the same forced orbits regardless of their size, leading to much lower values of $\Delta V$ than in the case of constant gas density. To further examine the effects of this process on planetesimal collisional evolution (accretion or erosion), we next performed a quantitative study.

Following Kortenkamp & Wetherill (2000), we adopt the disruption limit given by Love & Ahrens (1996) and compute the net mass accretion ratio (see Appendix for details) for every impact. Figure 7 shows the time evolution of the net mass accretion ratios ($A_t$) for impacts between different size groups. For impacts between bodies of the same size, the ratios are not plotted, since the $\Delta V$’s are always low enough for runaway growth in such cases. The results shown in Figure 7 can be summarized as follows: (1) for small bodies ($R_p < 5$ km), collisions always lead to erosion during the first $7 \times 10^5$ yr, after which accretion occurs with a progressively increasing $A_t$; (2) for intermediate bodies ($5$ km < $R_p < 15$ km), $A_t$ is initially modest ($75\%-80\%$) and progressively increases (to $90\%-95\%$) as the gas dissipates; (3) for large bodies ($R_p > 15$ km), $A_t$ is always very high ($\geq 95\%$); (4) for an impact between a large ($R_p > 15$) km and a small ($R_p < 5$) km planetesimal, while the $\Delta V$ is high and decreases slowly (see Fig. 4), $A_t$ is always high ($\geq 95\%$). Therefore, a full understanding of the details of collisions among a swarm of planetesimals requires complete information of the initial planetesimal size distribution—something that is, however, not at all clear with our current knowledge.

Here, for simplicity we just perform four tests, assuming for the planetesimal size distribution a Gaussian and three power-law functions, respectively. For the three power-law cases, the planetesimals have distributions given by $N/R = 1.7$ (Makino et al. 1998) with three radius ranges, namely, $1$–$50$, $2.5$–$50$, and $5$–$50$ km. For the Gaussian case, the radius distribution is taken to be a Gaussian function centered at $8$ km with dispersion $R = 7$ km.

![Fig. 6.—Distributions of impact rates in the $(R_1, R_2)$-plane, where $R_1$ and $R_2$ are the radii of the two colliding bodies. The impact rates are computed as the percentage of impacts that occur in areas of a given size in the $(R_1, R_2)$-plane. (a) The case similar to Fig. 4, in which gas drag and the companion’s perturbation are included. (b) A case for comparison, in which the planetesimal eccentricities and periapses are random and gas drag and the companion’s perturbation are not included (otherwise the orbital elements would not be random any more). For both cases, a Gaussian size distribution, centered on $R_p = 8$ km, is assumed for the planetesimals. [See the electronic edition of the Journal for a color version of this figure.]

![Fig. 7.—Time evolution of the net mass accretion ratio ($A_t$) for impacts between bodies of different sizes. The $A_t$ are defined and computed following the procedure described in the Appendix. [See the electronic edition of the Journal for a color version of this figure.]}

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Figure 8 plots the time evolution of the average $\Delta V$ and $A_r$ for these four cases and shows that for the first few thousand years (the no-dissipation phase), the conditions for accretion or erosion depend entirely on the initial size distribution of the planetesimals. In this phase, the average $\Delta V$ is pumped up by the size dependence of orbital alignment, and thus accretion is inefficient ($A_r \approx 75\%$) for one power-law case (5–50 km), precocious ($A_r \approx 30\%$) for the Gaussian case and another power-law case (2.5–50 km), and even completely suppressed for the remaining power-law case (1–50 km). However, after a few tens of thousands of years (the dissipating phase), all the $\Delta V$ reach low enough values and accretion is efficient ($A_r \geq 95\%$) in all cases, regardless of the initial size distribution of the planetesimals. Note that the smaller the bodies considered initially, the more time the system needs to become accretion-friendly. For the power-law case with minimum size of 1 km, it indeed takes about $6 \times 10^5$ yr before accretion is efficient. As discussed next, this long time span can worsen the radial drift problem.

4.3. Radial Drift

As they move in the gas disk, planetesimals face a headwind that drives them progressively inward (Adachi et al. 1976). In the runs above, we adopted the boundary condition described in § 2.3 to keep all the planetesimals within our computational zone (1.5–3 AU). This is reasonable only if the planetesimal disk is extended enough that planetesimals can flow into the computational zone from the outer disk. However, theoretical calculations of binary-disk interactions predict that companions might truncate circumstellar disks at an outer radius of 0.2–0.5 times the binary semimajor axis (Artymowicz & Lubow 1994). For the $\gamma$ Cep system $a_B = 18.5$ AU, so the truncated disk size would be about 3.7–9.3 AU. Therefore, there may not be enough material supplied from the outer disk, meaning that there need to be enough planetesimals that stay in the computational zone for at least a few times $10^5$ yr in order to form planets. For this reason, we performed a simulation without any boundary condition to compare with the results in Figure 4. We find that most large bodies, with $R_p = 15$ km or $R_p = 50$ km, stay in the computational zone, having $\Delta V$ curves similar to those in Figure 4, while almost all the small bodies, with $R_p = 2.5$ km and $R_p = 5$ km, are removed by gas drag. This problem of “too-fast migration” will become worse when even smaller bodies are considered, such as those with radii of 1–10 m. As shown in Figure 8, for the power-law case with 1–50 km, there is a $6 \times 10^5$ yr time span during which erosion dominates and planetesimals are thus transformed into small fragments that are quickly removed by inward drift.

Fast inward drift induced by gas drag is in fact a general problem in the classical planet formation model (Lissauer 1993; Papaloizou & Terquem 2006; Armitage 2007), and several ways have been proposed to address this issue. It is possible that large planetesimals ($R_p > 10$ km, large enough to overcome the inward drift) form directly by means of gravitational instability over a few thousand years (Goldreich & Ward 1973; Youdin & Shu 2002). In addition, radial drift may allow small bodies to pile up within the inner disk to form larger planetesimals (Youdin & Chiang 2004), and the presence of turbulence in the gas disk can also reduce radial drift (Haghighipour & Boss 2003; Rice et al. 2004; Durisen et al. 2005).

4.4. Remnant Gas for Planet Formation

There should also be enough remnant gas to form a massive gaseous planet, as required to fit the minimum mass ($\sim 2 M_J$) of the planet detected in the $\gamma$ Cep system. In this paper, for an initial gas disk of 10 MMN (about 100 $M_J$), after $5 \times 10^5$ yr, when most of the $\Delta V$’s have already decreased to low enough values, the remnant gas, according to Figure 1, is about 7 $M_J$. On the other hand, Kley & Nelson (2008) suggest that gas accretion onto a planet should be highly efficient in the $\gamma$ Cep system as a result of the large induced planetary orbital eccentricity. Their simulations indicate that a gas disk with only $\sim 3 M_J$ is needed in order to form a gaseous planet of $\sim 2 M_J$. Therefore, it is possible to form a massive gaseous planet in our dissipating-gas model.

5. SUMMARY

In this paper, focusing on the $\gamma$ Cephei system and concentrating on planetesimal impact velocities ($\Delta V$), we numerically investigated the conditions for planetesimal accretion in binary systems. This extends the studies of Thébault et al. (2004, 2006) by including the effect of a dissipating gas disk. We confirm some of their results, in that in a gas disk without dissipation, differential orbital alignment is very efficient and increases the $\Delta V$ between bodies of different sizes to high values that significantly inhibit planetesimal growth. Furthermore, we find that by including gas dissipation, the differential phasing effect induced by the size dependence of gas drag can be reduced. In this case, as the gas density decreases all planetesimals converge toward the same forced orbits, regardless of size. This orbital convergence induced by gas dissipation is most efficient for large bodies (15–50 km). Within $3 \times 10^5$ yr, $\Delta V(15, 50)$ decreases to low enough values (about 40 m s$^{-1}$, below the escape velocities of large bodies) for runaway growth to occur.

In order to obtain more information about the collisional evolution, we first discussed the impact rate distribution. For binary systems that include gas drag, collisions between bodies of different sizes predominate because of the differential orbital alignment.
and the size dependence of radial drift. Considering this result, our proposed mechanism, which can reduce the $\Delta V$ between bodies of different sizes, becomes much more important in the context of planetesimal growth.

Having defined the net mass accretion ratio ($A_r$), we then discussed the conditions for accretion or erosion of a swarm of planetesimals with different size distributions. We find that the size distribution is a crucial factor influencing the collisional evolution. For the case of constant gas density, it entirely dominates the growth of planetesimals and accretion is only efficient between equal-sized bodies. On the other hand, for the case of dissipating gas density the effect of the size distribution is dominant only at the beginning, and after a few times $10^5$ yr, accretion (or even runaway growth) is always favored, regardless of the initial size distribution of the planetesimals.

As a consequence of the companion’s perturbation in a binary system, the disk is truncated at a smaller radius and the planetesimals undergo a much faster inward drift. These effects may induce a problem of whether enough planetesimals can remain in the planet formation zone in the face of the inward migration. We performed some computations for this situation and find that most small bodies ($R_p < 10$ km) are removed within a few times $10^5$ yr, while large bodies ($R_p > 15$ km) are not significantly influenced. The problem of inward drift will be much more acute if the initial planetesimal population is composed mainly of small bodies ($R_p < 2.5$ km). In this case, erosion dominates for the first few tens of thousands of years, and planetesimals are transformed into small fragments that are quickly removed by the inward drift.

Finally, we estimated the remnant gas needed to form a gaseous planet. In our dissipating gas disk model, after $5 \times 10^5$ yr, when the $\Delta V$’s among most of the planetesimals have decreased to low enough values, the disk mass is about 7 Jupiter masses, enough to form a massive gaseous planet.

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APPENDIX

NET MASS ACCRETION RATIO

For the sake of simplicity, colliding planetesimals—both the target and the projectile—are normally considered to be nearly homogeneous and spherical bodies, and all collisions are treated as direct impacts. Given these assumptions, to describe a specific collision one needs only three input parameters: the mass of the target ($M_t$), the mass of the projectile ($M_p$), and the impact velocity ($V_{imp}$, namely, the $\Delta V$ derived from our simulations).

For a target and projectile of mass and radius ($M_t$, $R_t$) and ($M_p$, $R_p$), respectively, the surface escape velocity of the pair is

$$V_{esc}^2 = \frac{2G(M_t + M_p)}{(R_t + R_p)}$$

(A1)

where $G$ is the gravitational constant. The center-of-mass impact energy available for fragmentation is given by

$$Q_f = \frac{1}{2} k_1 V_{imp}^2 M_t M_p / (M_t + M_p),$$

(A2)

where the impact efficiency $k_1 = 0.5$ is the fraction of the impact energy not lost to heating. Assuming the crushing strength scaled from Love & Ahrens (1996)

$$Q_c = 24.2[R_t(cm)]^{1.13},$$

(A3)

where $R_t$ is the radius of the target, the mass of material fragmented by the impact is

$$M_f = Q_f / Q_c.$$  

(A4)

As some fragments fall back onto the target by virtue of gravity, the mass of material that escapes is only a fraction of $M_f$ and is given by

$$M_e = k_2 M_f V_{esc}^{-2.25}$$

(A5)

(Greenberg et al. 1978), where $k_2 = (3 \times 10^6 \text{ cm s}^{-1})^{2.25}$. In this paper, we define the ratio

$$A_r = 1 - M_e / M_p$$

(A6)

to measure the fraction of mass accreted onto the target. If the derived $M_e \geq M_p$, there is no growth of the target, and $A_r = 0$ is forced in such cases. Figure 9 maps the values of $A_r$ in the ($R_1$, $R_2$)-plane for four typical impact velocities: 100, 300, 600, and 1000 m s$^{-1}$. As can be seen, bodies with radii below 5 km hardly accrete onto each other; on the other hand, once one of the two colliding bodies has a radius larger than 15 km, accretion is always efficient.
Fig. 9.—Net mass accretion ratios distributed in the ($R_1$, $R_2$)-plane for four typical relative velocities: 100, 300, 600, and 1000 m s$^{-1}$. [See the electronic edition of the Journal for a color version of this figure.]

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