Static quantities of the $W$ boson in the $SU_L(3) \times U_X(1)$ model with right-handed neutrinos

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The static electromagnetic properties of the $W$ boson, $\Delta \kappa$ and $\Delta Q$, are calculated in the $SU_L(3) \times U_X(1)$ model with right-handed neutrinos. The new contributions from this model arise from the gauge and scalar sectors. In the gauge sector there is a new contribution from a complex neutral gauge boson $Y^0$ and a singly-charged gauge boson $Y^\pm$. The mass of these gauge bosons, called bileptons, is expected to be in the range of a few hundreds of GeV according to the current bounds from experimental data. If the bilepton masses are of the order of 200 GeV, the size of their contribution is similar to that obtained in other weakly coupled theories. However the contributions to both $\Delta Q$ and $\Delta \kappa$ are negligible for very heavy or degenerate bileptons. As for the scalar sector, an scenario is examined in which the contribution to the $W$ form factors is identical to that of a two-Higgs-doublet model. It is found that this sector would not give large corrections to $\Delta \kappa$ and $\Delta Q$.

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I. INTRODUCTION

The experimental scrutiny of the Yang-Mills sector is essential to test the standard model (SM). In particular, the trilinear gauge boson couplings (TGBCs) offer a unique opportunity to find evidences of new physics through the study of their one-loop corrections. Hopefully, the TNGBCs will be tested with unprecedent accuracy beyond the tree level at hadronic and leptonic colliders in the near future [1]. Particular emphasis has been given to the study of the static quantities of the $W$ boson. The CP-even electromagnetic properties of the $W$ boson are characterized by two form factors, $\Delta \kappa$ and $\Delta Q$, which are the coefficients of Lorentz tensors of canonical dimension 4 and 6, respectively [2]. Both form factors can only arise at the one-loop level within the SM and other renormalizable theories, thereby being sensitive to sizeable new physics effects. It has been argued that $\Delta Q$ is not sensitive to nondecoupling effects and thus it could only be useful to search for effects of new physics near the Fermi scale [3]. On the contrary $\Delta \kappa$ might be sensitive to heavy physics effects due to its nondecoupling properties [3]. The one-loop contributions to $\Delta Q$ and $\Delta \kappa$ were long ago studied in the SM [4, 5] and more recently in the context of several specific theories [6, 7, 8]. Also, a model-independent study of the $WW\gamma$ vertex via the effective Lagrangian approach was presented in [9].

In a recent work [8] two of us studied the static quantities of the $W$ boson in the context of the minimal $SU_L(3) \times U_X(1)$ model, dubbed as 331 model [10]. The main attraction of this model is the unique mechanism of anomaly cancellation, which is achieved provided that all of the fermion families are summed over rather than within each fermion family, as occurs in the SM. As a consequence, the number of fermion families must be a multiple of the quark color number, which offers a possible solution to the flavor problem. The 331 model has been the source of considerable interest recently [11]. In this work we will focus on the contributions to $\Delta \kappa$ and $\Delta Q$ from both the gauge and scalar sectors of the 331 model with right-handed neutrinos [12]. This version is attractive because, in order to achieve the mechanism of spontaneous symmetry breaking (SSB) and generate the gauge boson and fermion masses, it requires a Higgs sector which is more economic than that of the minimal version [12]. Evidently the features of the 331 model with right-handed neutrinos are rather different than those of the minimal version, and so are the contributions to the static quantities of the $W$ boson. It is thus worth evaluating the behavior of $\Delta \kappa$ and $\Delta Q$ in the new scenario raised by this model. Special attention will be paid to discuss the contribution arising from the gauge sector because it is the one which has the more interesting features. As will be shown below, the contribution from

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the scalar sector is similar to that arising in a two-Higgs-doublet model (THDM) \( \Phi \).

A peculiarity of 331 models is that they predict a pair of massive gauge bosons arranged in a doublet of the electroweak group, which emerge when \( SU_L(3) \times U_X(1) \) is broken into \( SU_L(2) \times U_Y(1) \). While the minimal 331 model predicts a pair of singly-charged and a pair of doubly-charged gauge bosons, the model with right-handed neutrinos predicts a pair of neutral self-conjugate gauge bosons \( Y^0 \) instead of the doubly-charged ones. These new gauge bosons are called bileptons since they carry lepton number \( L = \pm 2 \), and thus are responsible for lepton-number violating interactions \( \Phi \). It has been pointed out that the neutral bilepton is a promising candidate in accelerator experiments since it may be the source of neutrino oscillations \( \Phi \). Very interestingly, the couplings between the SM gauge bosons and the bileptons do not involve any mixing angle and are similar in strength to the couplings existing between the SM gauge bosons themselves. Current bounds establish that the bilepton masses may be in the range of a few hundred of GeVs \( \Phi \). It is then feasible that these bileptons may show up through their virtual effects in low-energy processes. This is an important reason to investigate the effect of these particles on the \( WW \gamma \) vertex function. It is also interesting that, due to the SSB hierarchy, the splitting between the bilepton masses \( m_Y^\pm \) and \( m_Y^0 \) is bounded by \( m_Y \), so the bileptons would be almost degenerate since their masses are expected to be heavier than the \( W \) mass. Therefore, the gauge boson contribution to the static quantities of the \( W \) boson would depend on only one free parameter. As far as the scalar sector is concerned, this model predicts the existence of ten physical scalar bosons \( \Phi \): four neutral CP-even, two neutral CP-odd and four charged ones. From these scalar bosons, only three of them, two neutral CP-even and a charged one, couple with the \( W \) boson at the tree level because they are the only ones which get their masses at the Fermi scale. Furthermore, in order to reproduce the SM at low energies, we will concentrate on a scenario in which one of the neutral Higgs bosons coincides with the SM Higgs boson.

The rest of the paper is organized as follows. Sec. \( \Phi \) is devoted to a brief description of the 331 model with right-handed neutrinos. In Sec. \( \Phi \) we present the calculation of the static properties of the \( W \) boson. The numerical results are analyzed in Sec. \( \Phi \) and the conclusions are presented in Sec. \( \Phi \).

**II. BRIEF REVIEW OF THE 331 MODEL WITH RIGHT-HANDED NEUTRINOS**

The features of the 331 model with right-handed neutrinos has been discussed to a large extent in Ref. \( \Phi \). Here we will only review those aspects which are relevant for the present discussion. The fermion spectrum of the model is shown in Table I. The three lepton families are arranged in triplets of \( SU_L(3) \), whereas in the quark sector it is necessary to introduce three exotic quarks \( D_1, D_2, \) and \( T \). In order to cancel the \( SU_L(3) \) anomaly, two quark families must transform as \( SU_L(3) \) antitriplets and the remaining one as a triplet. It is customary to arrange the first two quark families in antitriplets and the third one in a triplet. This choice is meant to distinguish the possible new dynamics effects arising in the third family.

| Leptons | First two quark families | Third quark family |
|---------|--------------------------|-------------------|
| \( f_L = \begin{pmatrix} v_L^1 \\ e_L^1 \\ (v_L^2)^c \end{pmatrix} \sim (1, 3, -1/3) \) | \( Q_L = \begin{pmatrix} d_L^1 \\ -u_L^1 \end{pmatrix} \sim (3, 3, 0) \) | \( Q_L^3 = \begin{pmatrix} u_L^3 \\ d_L^3 \end{pmatrix} \sim (3, 3, 1/3) \) |
| \( \tilde{e}_R \sim (1, 1, -1) \) | \( u_R^1 \sim (3, 1, 2/3) \) | \( u_R^3 \sim (3, 1, 2/3) \) |
| \( d_R^1 \sim (3, 1, -1/3) \) | \( d_R^3 \sim (3, 1, -1/3) \) | \( T_R \sim (3, 1, 2/3) \) |

**TABLE I: Fermion spectrum of the 331 model with right-handed neutrinos, along with the quantum number assignments.**

The electric charges of the exotic quarks are \( Q_{D_1, D_2} = -1/3 e \) and \( Q_T = 2/3 e \). This is to be contrasted with the three new quarks, \( D, S \) and \( T \), predicted by the minimal 331 model, whose charge is indeed exotic, namely \( Q_{D, S} = -4/3 e \) and \( Q_T = 5/3 e \).

As already mentioned, the 331 model with right-handed neutrinos has the advantage that it requires a Higgs sector simpler than that introduced in the minimal version. In fact, only three triplets of \( SU_L(3) \) are needed to reproduce the known physics at the Fermi scale:

\[
\chi = \begin{pmatrix} \Phi_3^+ \\ \chi_0 \end{pmatrix} \sim (1, 3, -1/3), \quad \rho = \begin{pmatrix} \Phi_1^- \\ \rho_+ \end{pmatrix} \sim (1, 3, 2/3), \quad \eta = \begin{pmatrix} \Phi_2^- \\ \eta_0 \end{pmatrix} \sim (1, 3, -1/3),
\]

where \( \Phi_1^+ = (\rho^-, \rho^0), \Phi_2^+ = (\eta^0, \eta^+), \Phi_3^+ = (\chi^0, \chi^+) \) are \( SU_L(2) \times U_Y(1) \) doublets with hypercharge +1, −1, and −1, respectively. This is to be contrasted again with the minimal 331 model, which requires the presence of
three triplets and one sextet [10]. The vacuum expectation value (VEV) \( <\chi> = (0, 0, w/\sqrt{2}) \) breaks down the \( SU_L(3) \times U_N(1) \) group into \( SU_L(2) \times U_Y(1) \). In this first stage of SSB, the new gauge bosons and quarks, as well as some physical scalars, get their masses. At the Fermi scale all the known SM particles and some physical scalar bosons are endowed with masses through the VEV \( <\Phi_1> = (0, v_1/\sqrt{2}) \) and \( <\Phi_2> = (0, v_1/\sqrt{2}) \). In this way, the \( \Phi_1 \) and \( \Phi_2 \) doublets break the \( SU_L(2) \times U_Y(1) \) group into \( U_Y(1) \).

In addition to the three exotic quarks, the model predicts the existence of five new gauge bosons: two singly charged \( Y^\pm \), two neutral no self-conjugate \( Y^0 \), and a neutral self-conjugate \( Z' \). The \( Y^\pm \) and \( Y^0 \) gauge bosons are called bileptons because they carry two units of lepton number [13]. All the new particles acquire their masses at the \( w \) scale. At this stage of SSB, the exotic quarks together with the \( Z' \) boson emerge as singlets of the electroweak group. Consequently, these particles cannot interact with the \( W \) boson at the tree level. It follows that there is no contribution from the exotic quarks to the static electromagnetic properties of the \( W \) boson at the lowest order. As for the \( Z' \) boson, it couples to the \( W \) boson via the \( Z' - Z \) mixing induced at the Fermi scale, which means that the respective contribution to the \( WW\gamma \) vertex is expected to be strongly suppressed since it is proportional to the corresponding mixing angle, which is expected to be negligibly small [12]. On the other hand, the dynamical behavior of the bileptons is different since they arise as a doublet of the electroweak group at the \( w \) scale and thus have nontrivial couplings with the SM gauge bosons. Due to the fact that the \( SU_L(2) \) group is completely embedded in \( SU_L(3) \), the bileptons couple with the SM gauge bosons with a strength similar to that of the couplings existing between the SM gauge bosons. In particular, these new couplings are entirely determined by the coupling constant associated with \( SU_L(2) \) and the weak angle \( \theta_W \). When \( SU_L(2) \times U_Y(1) \) is broken down to \( U_Y(1) \), the masses of the bileptons receive new contributions. In the gauge sector, the mass eigenstates arise from the Higgs kinetic-energy term, which is given by

\[
\mathcal{L} = (D_\mu \chi)^\dagger (D^\mu \chi) + (D_\mu \rho)^\dagger (D^\mu \rho) + (D_\mu \eta)^\dagger (D^\mu \eta),
\]

where \( D_\mu \) is the covariant derivative associated with the \( SU_L(3) \times U_N(1) \) group, which in the fundamental representation is given by

\[
D_\mu = \partial_\mu - g^a \frac{\lambda^a}{2} A^a_\mu - ig \lambda^9 N_\mu,
\]

with \( \lambda^a \) \( (a = 1, 2, \ldots, 8) \) being the Gell-man matrices and \( \lambda^9 = \sqrt{2/3} \text{diag}(1, 1, 1) \). Once this sector is diagonalized, there emerge the following mass-eigenstate fields:

\[
Y^0_\mu = \frac{1}{\sqrt{2}} (A^4_\mu - iA^5_\mu),
\]

\[
Y^-_\mu = \frac{1}{\sqrt{2}} (A^6_\mu - iA^7_\mu),
\]

\[
W^+_\mu = \frac{1}{\sqrt{2}} (A^1_\mu - iA^2_\mu),
\]

with masses \( m^2_{Y^0} = g^2(w^2 + v_2^2)/4 \), \( m^2_{Y^-} = g^2(w^2 + v_1^2)/4 \), and \( m^2_{W^+} = g^2(v_1^2 + v_2^2)/4 \). From these expressions, it is easy to see that there is an upper bound on the splitting between the bilepton masses:

\[
|m^2_{Y^0} - m^2_{Y^-}| \leq m^2_{W^+}.
\]

The remaining three gauge fields \( A^3_\mu \), \( A^8_\mu \) and \( N_\mu \) define the self-conjugate mass eigenstates \( A^\mu, Z_\mu \), and \( Z'^\mu \) [12]. As far as the Yang-Mills sector of the model is concerned, it is given by

\[
\mathcal{L}_{YM} = -\frac{1}{4} F^{\alpha}_{\mu\nu} F^{\mu\nu}_\alpha - \frac{1}{2} N_{\mu\nu} N^{\mu\nu},
\]

where \( F^{\alpha}_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + f^{abc} A^b_\mu A^c_\nu \) and \( N_{\mu\nu} = \partial_\mu N_\nu - \partial_\nu N_\mu \), being \( f^{abc} \) the structure constants associated with \( SU_L(3) \). After this Lagrangian is expressed in terms of mass eigenstate fields, it can be split into three \( SU_L(2) \times U_Y(1) \)-invariant pieces:

\[
\mathcal{L}_{YM} = \mathcal{L}^{SM}_{YM} + \mathcal{L}^{SM - NP}_{YM} + \mathcal{L}^{NP}_{YM},
\]
where the first term represents the well known Yang-Mills sector associated with the electroweak group, whereas \( L_{YM}^{SM-NP} \) represents the interactions between the SM gauge fields and the heavy ones:

\[
L_{YM}^{SM-NP} = -\frac{1}{2} (D_\mu Y_\nu - D_\nu Y_\mu) (D^\mu Y^\nu - D^\nu Y^\mu) - i g Y_\mu (g F_{\mu\nu} + g' B_{\mu\nu}) Y_\nu
\]

\[
- \frac{ig}{2} \sqrt{\frac{3-4s_W^2}{c_W}} Z_\mu \left( Y_\nu^\dagger (D^\mu Y^\nu - D^\nu Y^\mu) - (D^\mu Y^\nu - D^\nu Y^\mu)^\dagger Y_\nu \right),
\]

(10)

where \( Y_\mu = (Y^0, Y^+ \pm) \) is a doublet of the electroweak group with hypercharge \(-1\), and \( D_\mu = \partial_\mu - ig A_\mu + ig B_\mu \) is the covariant derivative associated with this group. We have introduced the definitions \( F_{\mu\nu} = \sigma^i F_{\mu\nu}^i/2, A_\mu = \sigma^i A_\mu^i/2, \) and \( B_\mu = Y B_\mu/2, \) with \( \sigma^i \) the Pauli matrices. Finally, the last term in Eq. (9) is also invariant under the electroweak group and comprises the interactions between the heavy gauge fields. There are no contributions to the \( WW\gamma \) vertex arising from this Lagrangian and we refrain from presenting the respective expression here.

In the unitary gauge, the contributions to the \( WW\gamma \) vertex arise from the first two terms of the Lagrangian \( L_{YM}^{SM-NP} \). These contributions are given by the vertices \( W^\pm Y^+ Y^0, W^\pm W^\mp \gamma, Y^\pm W^+ Y^0 \gamma, W^\pm W^\mp Y^\pm \gamma, \) and \( W^\pm Y^+ Y^0 \gamma \). The corresponding Feynman rules are represented in Fig. 1.

![FIG. 1: Unitary gauge Feynman rules for the vertices which contribute to the on-shell \( WW\gamma \) vertex in the gauge sector of the 331 model with right-handed neutrinos. The arrows represent the flow of the 4-momenta. S_α_λ_ρ_ = (p - k_2)_λ g_ρ_α_ + (k_2 - k_1)_α g_λ_ρ_ + (k_1 - p)_ρ g_α_λ_, T_μ_λ_ρ_ = (k_2 - k_1)_μ g_λ_ρ_ + (q - k_2)_λ g_μ_ρ_ + (k_1 - q)_ρ g_μ_λ_, U_α_β_λ_ρ_ = 2 g_α_ρ g_λ_β_ - g_α_β g_λ_ρ_ - g_α_λ g_β_ρ_ - g_β_λ g_α_ρ_, and ν_μ_α_λ_ρ_ = g_α_λ δ_μ_ρ_ - 2 g_α_ρ δ_μ_λ_ + g_α_ρ δ_μ_λ_.](image)

As far as the scalar sector is concerned, it was analyzed in detail in Refs. [12, 16]. Although the most general Higgs potential is very cumbersome, it gets simplified to a large extent if one assumes the discrete symmetry \( \chi \rightarrow - \chi \). Under this assumption, the scalar potential can be written in the following way

\[
V(\chi, \rho, \eta) = \mu_1^2 (\eta^\dagger \eta) + \mu_2^2 (\rho^\dagger \rho) + \mu_3^2 (\chi^\dagger \chi) + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\chi^\dagger \chi)^2
\]

\[
+ (\eta^\dagger \eta) [\lambda_4 (\rho^\dagger \rho) + \lambda_5 (\chi^\dagger \chi)] + \lambda_6 (\rho^\dagger \rho) (\chi^\dagger \chi) + \lambda_7 (\rho^\dagger \eta) (\eta^\dagger \rho)
\]

\[
+ \lambda_8 (\chi^\dagger \eta) (\eta^\dagger \chi) + \lambda_9 (\rho^\dagger \rho) (\chi^\dagger \rho) + \lambda_{10} (\chi^\dagger \eta + \eta^\dagger \chi)^2.
\]

(11)

It is worthwhile to analyze the behavior of the scalar potential at the first stage of SSB. To this end we split \( V(\chi, \rho, \eta) \) into the following two terms

\[
V(\chi, \rho, \eta) = V(\Phi_1, \Phi_2) + V_w,
\]

(12)
with
\[ V(\Phi_1, \Phi_2) = \mu_1^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_4 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_7 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1), \]

and \( V_w \) an intricate function which includes all those terms not appearing in \( V(\Phi_1, \Phi_2) \). \( V_w \) is not relevant for the present discussion, so we will refrain from presenting its explicit form here. We will content ourselves with mentioning that this term generates the heavy Higgs boson masses, \( i.e. \) those which are proportional to the \( w \) scale, whereas the SM gauge bosons and the remaining physical scalar bosons receive their masses from \( V(\Phi_1, \Phi_2) \) at a relatively light scale. Note that \( V(\Phi_1, \Phi_2) \) corresponds to the scalar potential of a THDM and so there are five Higgs bosons, which are relatively light. In fact, the explicit diagonalization of \( V(\chi, \rho, \eta) \) leads to five light and five heavy scalar bosons [10]. The light scalar bosons are, in the notation of [10], two neutral CP-even Higgs bosons, \( H_1 \) and \( H_2 \), a pair of charged ones, \( H^\pm_5 \), and a massless neutral CP-odd Higgs boson, \( A_2 \). The last one would receive its mass through radiative corrections. As for the heavy Higgs spectrum, it is comprised by two neutral CP-even scalar bosons, \( H_3 \) and \( H'_1 \), a neutral CP-odd one, \( A_1 \), and a pair of charged ones, \( H^\pm_5 \). As pointed out in Ref. [10], the neutral CP-even scalar boson \( H_2 \) coincides with the SM one provided that \( \lambda_4 = \lambda_5 \). For the purpose of this work, it is enough to consider this scenario. For the sake of clarity, we will only present the expressions which relate the gauge states to the mass eigenstates of the light sector. The real part of the \( \rho^0 \) and \( \eta^0 \) neutral components of \( \Phi_1 \) and \( \Phi_2 \) define the CP-even states \( H_1 \) and \( H_2 \) via the following rotation
\[ H_1 = c_\beta \eta^0 - s_\beta \rho^0, \quad H_2 = s_\beta \eta^0 + c_\beta \rho^0, \]

where \( \beta \) is defined by \( \tan \beta = (v_2/v_1) \), \( c_\beta = \cos \beta \), \( s_\beta = \sin \beta \), and the subscript \( r \) denotes the real part of the field. In the charged sector, the \( \rho^+ \) and \( \eta^+ \) components of \( \Phi_1 \) and \( \Phi_2 \) define the charged \( H^+_5 \) Higgs boson and the pseudo-Goldstone boson associated with the \( W \) gauge boson \( G_W^+ \):
\[ H^+_5 = c_\beta \eta^+ + s_\beta \rho^+, \quad G^+_W = -s_\beta \eta^+ + c_\beta \rho^+. \]

Finally, the imaginary part of \( \rho^0 \) and \( \eta^0 \) define the pseudo-Goldstone boson associated with the \( Z \) gauge boson \( G_Z \) and the massless CP-odd scalar boson \( A_2 \).

Once the light Higgs mass eigenstates are defined, from the Higgs-kinetic term it is straightforward to obtain those couplings involving the \( W \) gauge boson. In order to analyze the behavior of the Higgs sector at the Fermi scale in the scenario with \( \lambda_4 = \lambda_5 \), we will present the full Lagrangian involving the couplings of the \( W \) boson to the neutral and charged Higgs bosons. It can be written as
\[ \mathcal{L} = \left( m_W^2 + g m_W H_2 + \frac{g^2}{4} (H_1^2 + H_2^2 + 2H^+_5 H^-_5) \right) W^-_\mu W^{\mu +}. \]

There is a similar expression involving the \( Z \) boson. It is also interesting to note that there is no trilinear self-coupling of the \( H_1 \) Higgs boson. From this Lagrangian it is evident that the couplings of \( H_2 \) to the SM gauge bosons are SM-like, which means that it should be identified with the SM Higgs boson. So its contribution to the \( WW\gamma \) vertex should be considered as a part of the SM [4] rather than a new physics effect. In fact, the only contribution which can be considered as a new physics effect is that induced by the \( H^+_5 H^-_5 WW \) vertex. The model also induces the trilinear \( W^\pm H^+_5 H^-_5 \) and quartic \( \gamma W^\pm H^+_5 H^-_5 \) vertices, which also can contribute to the \( WW\gamma \) coupling. The corresponding Lagrangian for these terms can be written as
\[ \mathcal{L} = \frac{i g}{2} \left( W^+_\mu \left( H_1 \partial^\mu H^-_5 - H^-_5 \partial^\mu H_1 \right) - W^-_\mu \left( H_1 \partial^\mu H^+_5 - H^+_5 \partial^\mu H_1 \right) \right) + \frac{e g}{2} H_A \mu \left( W^\mu H^-_5 + W^- H^+_5 \right). \]

There is no analogous Lagrangian for \( H_2 \), which is in agreement with the fact that \( H_2 \) plays the role of the SM Higgs boson. It is worth noting that all of the couplings which contribute to the \( WW\gamma \) vertex are determined entirely by the coupling constant \( g \), in contrast with the case of the most general THDM potential, which involves mixing angles. This fact will simplify considerably the analysis of the \( \Delta \kappa \) and \( \Delta Q \) form factors as they will depend only on the \( H_1 \) and \( H^+_5 \) masses, which resembles the situation arising in the gauge sector, where the form factors depend only on the bilepton masses. In particular, since both \( H_1 \) and \( H^+_5 \) receive their masses at the Fermi scale, it is also reasonable to analyze the scenario in which they are degenerate. From the above Lagrangians it is straightforward to obtain the Feynman rules necessary for our calculation. For the sake of completeness they are shown in Fig. [4].
III. STATIC QUANTITIES OF THE W BOSON

In the usual notation, the most general CP-even on-shell $WW \gamma$ vertex can be written as:

$$\Gamma^{\mu\alpha\beta} = i e \left\{ A \left[ 2 p^\beta g^{\alpha\beta} + 4 \left( Q^\beta g^{\mu\alpha} - Q^\alpha g^{\mu\beta} \right) \right] + \Delta \kappa \left( Q^\beta g^{\mu\alpha} - Q^\alpha g^{\mu\beta} \right) + \frac{4 \Delta Q}{m_W^2} p^\mu Q^\alpha Q^\beta \right\},$$

where the momenta of the particles are denoted as follows. $(p-Q)_{\alpha}$ and $(P+Q)_{\beta}$ are the momenta of the incoming and outgoing $W$ boson, respectively, and $2Q_{\mu}$ is that of the photon. We have dropped the CP-odd terms since they do not arise in the 331 model with right-handed neutrinos. This class of terms can be generated for instance in models in which the $W$ boson couples to both left- and right-handed fermions simultaneously.[17] Both $\Delta \kappa$ and $\Delta Q$ vanish at the tree level in the SM, and the one-loop corrections from the fermion, gauge, and scalar sectors are all of the order of $\alpha/\pi$.[4] These form factors define the magnetic dipole moment ($\mu_W$) and the electric quadrupole moment ($Q_W$) of the $W$ boson, which are given by

$$\mu_W = \frac{e}{2 m_W} \left( 2 + \Delta \kappa \right),$$

$$Q_W = -\frac{e}{m_W^2} \left( 1 + \Delta \kappa + \Delta Q \right).$$

It is interesting to note that the gauge invariant form is obtained only after adding up the full contributions of a particular sector of any specific model. Gauge invariance along with the cancellation of ultraviolet divergences are thus a test to check the correctness of the result.

In this work we are interested in the contribution to $\Delta \kappa$ and $\Delta Q$ from the 331 model with right-handed neutrinos. As already explained, the exotic quarks do not contribute to the $WW \gamma$ vertex, whereas the extra neutral boson $Z'$ contribution arises from $Z-Z'$ mixing and it is expected to be negligibly small. The only contributions to $\Delta \kappa$ and $\Delta Q$
arise from the gauge and scalar sectors. In the former, the static properties of the $W$ boson receive contributions from both the neutral and singly-charged bileptons. As far as the scalar sector is concerned, it contributes via the neutral and singly charged Higgs bosons. Before presenting the results for these contributions, we would like to comment briefly on our calculation scheme, which has been already discussed in Refs. \[8, 18\].

Rather than solving the loop integrals by the Feynman parameters technique, one alternative is to use the Passarino-Veltman method \[19\] to reduce the tensor integrals down to scalar functions. However, this scheme cannot be applied together with the kinematic condition $Q^2 = 0$ since it requires the inversion of a matrix whose Gram determinant is directly proportional to $Q^2$. Nevertheless, the Passarino-Veltman reduction scheme can be safely applied for $Q^2 \neq 0$, and the limit $Q^2 \to 0$ can be taken at the end of the calculation, which usually requires the application of l'Hôpital rule: $\lim_{Q^2 \to 0} f(Q^2)/Q^2 = f'(0)$. This means that the result is given in terms of scalar functions and its derivatives. It was shown in Ref. \[18\] that any $N$-point scalar function and its derivatives with respect to any of its arguments can be expressed in terms of a set of $(N - 1)$-point scalar functions when the kinematic Gram determinant vanishes. It follows that one can express the three-point scalar function $C_0$ appearing in the calculation and its derivative with respect to $Q^2$ in terms of two-point scalar functions $B_0$. The explicit reduction was presented in Ref. \[18\]. It is then straightforward to obtain the limit $Q^2 \to 0$. The advantages of this method are twofold: it can be implemented in a computer program \[20\], which avoids the risk of any error, and the two-point scalar functions can be readily solved analytically or numerically \[21\]. This calculation scheme is suited to solve loop diagrams carrying vector bosons, which may give rise to some cumbersome tensor integrals.

### A. Gauge boson contribution

We turn now to the contributions to $\Delta Q$ and $\Delta \kappa$ from the Feynman diagrams of Fig. 3. The amplitudes for these diagrams can be written down with the help of the Feynman rules shown in Fig. 1. After applying the calculation scheme described earlier and taking into account Eq. (20) we are left with

$$\Delta Q^V = a \left( \frac{12 \eta}{2 \xi} - \chi \right) \left( f_0(\xi, \eta) + f_1(\xi, \eta) \log \left( \frac{\eta}{\xi} \right) + f_2(\xi, \eta) \arccot \left( \frac{\xi + \eta - 1}{\chi} \right) \right),$$

and

$$\Delta \kappa^V = \frac{a}{4 \xi^2} \left( g_0(\xi, \eta) + g_1(\xi, \eta) \log \left( \frac{\eta}{\xi} \right) + g_2(\xi, \eta) \arccot \left( \frac{\xi + \eta - 1}{\chi} \right) \right),$$

where we have introduced the definitions $a = g^2/(96 \pi^2)$, $\xi = m_{W^\pm}^2/m_W^2$, $\eta = m_{Y_0}^2/m_Y^2$, and $\chi^2 = 4 \xi \eta - (\xi + \eta - 1)^2$. The $f_i$ and $g_i$ functions read

$$f_0(\xi, \eta) = -\frac{2}{3} - 2 (\xi - \eta)^2 + 3 \xi - \eta.$$
The result for this contribution was obtained long ago \cite{4, 5}. As for the Feynman diagram of the right-hand side of Fig. 4, it is in fact a SM effect rather than a new physics contribution to the electromagnetic form factors. Since the neutral Higgs boson coincides with the SM Higgs boson, so the left-hand triangle contribution belongs to the SM and will not be considered a new physics effect.

In the scenario described in the text, \( H_2 \) coincides with the SM Higgs boson, so the left-hand triangle contribution belongs to the SM and will not be considered a new physics effect.

One interesting scenario is that in which the bileptons are degenerate, i.e. \( m_\pm = m_0 = m_Y \), which is actually a good assumption for \( m_{Y^{\pm}} \) much larger than \( m_{W^\pm} \) because of the mass splitting \cite{7}. In this scenario we obtain

\[
\Delta Q^Y = \frac{a}{\zeta} \left( 1 + 4 \zeta (3 \zeta - 1) \right) \left( 1 - \frac{1}{3 \zeta} - \frac{2 \zeta - 1}{\sqrt{4 \zeta - 1}} \arccot \left( \frac{2 \zeta - 1}{\sqrt{4 \zeta - 1}} \right) \right),
\]

and

\[
\Delta \kappa^Y = \frac{a}{2 \zeta^2} \left( 12 + \zeta (19 - 36 \zeta (1 + \zeta)) + \frac{1}{2 \zeta} + \frac{6 \zeta - 1}{\sqrt{4 \zeta - 1}} \arccot \left( \frac{2 \zeta - 1}{\sqrt{4 \zeta - 1}} \right) \right),
\]

with \( \zeta = m_{\gamma^Y}^2/m_{W^\pm}^2 \). It can be shown that in the heavy bilepton limit both \eqref{eq:deltaQY} and \eqref{eq:deltaKappaY} behave as \( m_{W^\pm}^2/m_{Y^0}^2 \) at the leading order in \( m_{Y^0} \). It is then evident that both form factors are insensitive to nondecoupling effects of heavy bileptons.

### B. Scalar contribution

In the scenario discussed earlier, the \( W \) electromagnetic form factors are induced by the charged scalar \( H_5^\pm \) and the neutral scalar bosons \( H_1 \) and \( H_2 \), which give rise to the Feynman diagrams shown in Fig. 4. The Feynman rules necessary for this calculation are shown in Fig. 4. We would like to mention that the neutral \( H_1 \) and the charged scalar \( H_5^\pm \) bosons induce three additional two-point diagrams, but they are not shown in Fig. 4 as give no contribution to the electromagnetic form factors. Since the neutral Higgs boson \( H_2 \) coincides with the SM one, the contribution from the triangle diagram of the left-hand side of Fig. 4 is in fact a SM effect rather than a new physics effect. The result for this contribution was obtained long ago \cite{4, 5}. As for the Feynman diagram of the right-hand...
side, it yields a contribution similar to that arising in the THDM, as can be inferred from the Feynman rules given in Fig. 2. Although this contribution was already obtained in terms of Feynman-parameter integrals [6], we would like to present an alternative result in terms of elementary functions. The calculation scheme described above yields

$$\Delta Q^\mu = a \left( \tilde{f}_0(\lambda, \lambda_+) + \tilde{f}_1(\lambda, \lambda_+) \log \left( \frac{\lambda_+}{\lambda} \right) + \tilde{f}_2(\lambda, \lambda_+) \arccot \left( \frac{\lambda + \lambda_+ - 1}{\chi} \right) \right),$$  \hspace{1cm} (33)

and

$$\Delta \kappa^\mu = a \left( \tilde{g}_0(\lambda, \lambda_+) + \tilde{g}_1(\lambda, \lambda_+) \log \left( \frac{\lambda_+}{\lambda} \right) + \tilde{g}_2(\lambda, \lambda_+) \arccot \left( \frac{\lambda + \lambda_+ - 1}{\chi} \right) \right),$$  \hspace{1cm} (34)

with

$$\tilde{f}_0(\lambda, \lambda_+) = \frac{2}{3} + \lambda_+ (2 \lambda_+ - 3) + \lambda - 4 \lambda_+ \lambda + 2 \lambda^2,$$  \hspace{1cm} (35)

$$\tilde{f}_1(\lambda, \lambda_+) = -\lambda_+ (\lambda_+ - 1)^2 + \lambda_+ \lambda (3 \lambda_+ - 2) - 3 \lambda_+ \lambda^2 + \lambda^3,$$  \hspace{1cm} (36)

$$\tilde{f}_2(\lambda, \lambda_+) = \frac{2}{\lambda} \left( \lambda_+ (\lambda_+ - 1)^3 - \lambda_+ \lambda (\lambda_+ - 1) (4 \lambda_+ - 1) + \lambda_+ (6 \lambda_+ - 1) \lambda^2 - (1 + 4 \lambda_+) \lambda^3 + \lambda^4 \right),$$  \hspace{1cm} (37)

$$\tilde{g}_0(\lambda, \lambda_+) = 2 - \lambda_+ + 4 \lambda,$$  \hspace{1cm} (38)

$$\tilde{g}_1(\lambda, \lambda_+) = \frac{1}{2} \left( 1 + (\lambda_+ - 2) \lambda_+ + \lambda - 5 \lambda_+ \lambda + 4 \lambda^2 \right),$$  \hspace{1cm} (39)

$$\tilde{g}_2(\lambda, \lambda_+) = -\frac{1}{\lambda} \left( (\lambda_+ - 1)^3 - 6 (\lambda_+ - 1) \lambda_+ \lambda + 3 (1 + 3 \lambda_+) \lambda^2 - 4 \lambda^3 \right).$$  \hspace{1cm} (40)

In these equations, $\lambda, \lambda_+$ and $\chi$ are given as follows. $\lambda = m_{H_1}^2 / m_W^2$, $\lambda_+ = m_{H_\pm}^2 / m_W^2$, and $\chi^2 = 4 \lambda \lambda_+ - (\lambda + \lambda_+ - 1)^2$.

When both the neutral and the charged Higgs bosons are mass degenerate ($\lambda_+ = \lambda = \tilde{\zeta}$) Eqs. (33) and (34) yield

$$\Delta Q^\mu = \frac{2 a}{3} \left( 1 - 3 \tilde{\zeta} + \frac{3 \tilde{\zeta} (2 \tilde{\zeta} - 1)}{\sqrt{4 \lambda - 1}} \arccot \left( \frac{2 \lambda - 1}{\sqrt{4 \lambda - 1}} \right) \right),$$  \hspace{1cm} (41)

and

$$\Delta \kappa^\mu = a \left( 2 + 3 \tilde{\zeta} + \frac{(1 - 3 \tilde{\zeta} (1 + 2 \tilde{\zeta}))}{\sqrt{4 \lambda - 1}} \arccot \left( \frac{2 \lambda - 1}{\sqrt{4 \lambda - 1}} \right) \right).$$  \hspace{1cm} (42)

We have numerically evaluated the above results and found agreement with those obtained in Ref. [6].

**IV. NUMERICAL EVALUATION**

We will now analyze the behavior of the form factors for some range of values of the bilepton and the Higgs boson masses. These are the only free parameters which enter into $\Delta Q$ and $\Delta \kappa$. We will analyze separately each contribution.
A. Gauge boson contribution

To begin with, it is worth analyzing the current bounds on the bilepton masses from both theoretical and experimental grounds. First of all, it is interesting to note that the matching of the gauge couplings constants at the SU_L(3) × U_Y(1) breaking leads to 4 sinθ_W ≤ 1 in the minimal 331 model [10], from which an upper bound on the bilepton masses can be derived, namely m_Y ≲ 1 TeV. Therefore, this model would be confirmed or ruled out by collider experiments in a near future. The version with right-handed neutrinos requires however that 4 sinθ_W ≤ 3, which yields no useful constraint on m_Y. As already mentioned, because of the symmetry breaking hierarchy, the splitting |m^Y± - m^Y0| is bounded by the W boson mass. Therefore m^Y0 and m^Y± are not arbitrary at all. One cannot, for instance, make large m^Y± while keeping fixed m^Y0 or viceversa. In fact, when m^Y0 ≫ m_W, the charged and neutral bileptons would become degenerate. As far as the lower bounds on the bilepton masses are concerned, in Ref. [12] it was argued that the data from neutrino neutral current elastic scattering give a lower bound on the mass and neutral bileptons would become degenerate. As far as the lower bounds on the bilepton masses are concerned, in those scenarios in which ∆κ give rise to nondecoupling effects. Since the fermions acquire their masses from Yukawa couplings, a large fermion splitting which yields no useful constraint on m_Y. This is to be contrasted with the case of a fermion pair accommodated in a SU_L(2) doublet, which are known to be in agreement with that obtained from the latest BNL measurement on the muon anomaly [12, 13].

According to the above discussion, we deem it interesting to evaluate the form factors in the range 100 GeV ≤ m^Y0 ≤ 1000 GeV, which will be useful to illustrate their behavior and get an idea about their size. At this point it is important to mention that to cross-check our results, the form factors were obtained independently by the Feynman parameters method. The integrals were evaluated numerically and the result was compared with the one obtained by the Passarino-Veltman method. A perfect agreement was observed. We refrain from presenting the results in terms of parametric integrals since the closed expressions [23] and [24] can be handled more easily.

The ∆κ and ∆Q Y form factors are shown in Fig. 5 and Fig. 6 as a function of the neutral bilepton mass. There are two curves in each plot, which correspond to the extremal values of m^Y±, namely m^Y± = m^Y0 - m^W and m^Y± = m^Y0 + m^W. The form factors are restricted to lie in the area surrounded by the two extremal lines. In Fig. 6 it is clear that the bileptons can give a negative or positive contribution to ∆κ Y, which depends on which bilepton is the heaviest. Also, we can observe that ∆κ Y is sensitive to the value of the splitting and has a larger size for nondegenerate bileptons than for degenerate bileptons. The ∆κ Y form factor in the latter scenario is displayed in Fig. 14. In this plot we can observe that, when one of the bilepton masses is close to m_W and the splitting is maximal, ∆κ Y can have a size of about one order of magnitude above the one obtained when there is degeneracy of the bilepton masses. On the contrary, ∆Q Y is less sensitive to the mass splitting and the extremal values of m^Y± yield values of the same order of magnitude than the one observed in the degenerate case, which is also shown in Fig. 6. From these plots we can conclude that the size of the contribution to the form factors from the gauge sector of the 331 model with right-handed neutrinos is about of the same order of magnitude than the one obtained in the case of the bilepton contribution in the minimal 331 model and in the case of the contributions of other SM extensions. The larger absolute values are obtained for lighter bileptons and when the charged bilepton mass reaches its maximal allowed value. It is interesting to note that all weakly coupled theories studied up to now give a contribution to the W form factors of similar size [26, 7, 8].

In Fig. 7 we can clearly see that both ∆κ Y and ∆Q Y are insensitive to heavy physics effects and approach to zero very quickly as the bilepton masses increase. The only scenario which may give rise to nondecoupling effects is that in which one bilepton mass is kept fixed while the other is made very large, which of course is forbidden by the mass splitting constraint [7]. In Ref. 25 we already discussed a similar situation arising in the minimal 331 model, with a doubly charged bilepton playing the role of the neutral one. This case also resembles the one discussed in Ref. 22 for a scalar doublet which acquires mass from a bare parameter. The reason why there is no decoupling effects is not surprising since a large bilepton mass implies a large VEV which is heavier than the electroweak scale. On the contrary, the splitting between the bilepton masses arises from VEVs which are of the size of the electroweak scale. This is to be contrasted with the case of a fermion pair accommodated in a SU_L(2) doublet, which are known to give rise to nondecoupling effects. Since the fermions acquire their masses from Yukawa couplings, a large fermion mass implies a large coupling, whereas a heavy bilepton mass implies a large VEV instead of a large coupling. The former scenario is the one which is known to break down the decoupling theorem. 23. Very interestingly, even in those scenarios in which ∆κ Y is sensitive to heavy physics effects, ∆Q Y is not 22. The decoupling theorem establishes that only those terms arising from renormalizable operators may be sensitive to nondecoupling effects, whereas those terms induced by nonrenormalizable operators are suppressed by inverse powers of the heavy mass 22. Thus ∆Q Y always decouples when one particle circulating in the loop is made large since it is generated by a nonrenormalizable dimension-six operator, but ∆κ Y may be sensitive to nondecoupling effects as it is induced by a dimension-four operator.
FIG. 5: Gauge boson contribution to $\Delta\kappa$ in the 331 model with right-handed neutrinos as a function of the mass of the neutral bilepton when the charged bilepton mass is maximal (solid line) and minimal (dashed line). According to the mass splitting, the extremal values are given by $m^2_{Y} = m^2_{Y,0} \pm m^2_{W}$. The form factor is restricted to lie in the area enclosed by the lines.

FIG. 6: The same as in Fig. 5 for the $\Delta Q$ form factor.

B. Scalar contribution

The scalar contribution to the $\Delta\kappa$ and $\Delta Q$ form factors is shown in Figs. 8 and 9 as a function of the charged Higgs boson mass and for different values of the neutral Higgs boson mass. We would like to emphasize that the values shown in those plots correspond to the contribution from new physics only. From these Figs., we can observe that both $\Delta\kappa^H$ and $\Delta Q^H$ decreases rapidly for increasing $m^H_{\pm}$. In fact, the latter goes to zero quickly as either $m_{H_1}$ or $m_{H_2}$ increase. Although $\Delta\kappa^H$ seems to increase with increasing $m_{H_1}$ for a relatively light $m_{H_2}$, it approaches the limiting value $\Delta\kappa^H = a$ for very large $m_{H_1}$. We can also observe that when the scalar boson masses are of the same size than those of the bilepton gauge bosons, the contribution from the Higgs sector is about one order of magnitude below than that of the gauge sector. In fact, if the scalar boson masses are degenerate, the respective contribution to

FIG. 7: Bilepton contribution to the $\Delta\kappa$ (solid line) and $\Delta Q$ (dashed line) form factors, in units of $a$, in the 331 model with right-handed neutrinos when the bileptons are degenerate and have a mass $m_Y$. 
FIG. 8: Scalar contribution to $\Delta \kappa$ in the 331 model with right-handed neutrinos as a function of $m_{H^\pm}$ for different values of the mass of the neutral Higgs boson $m_{H_1}$: 115 GeV (solid line), 250 GeV (dashed line) and 500 GeV (dashed-dotted line).

FIG. 9: The same as in Fig. 8 for the $\Delta Q$ form factor.

$\Delta \kappa$ and $\Delta Q$ is very small, as shown in Fig. 10. As pointed out in Ref. 6, this is a reflect from the fact that the Higgs boson is not strongly interacting. Thus, for the Higgs sector to give a large correction to the $W$ form factors, it would be necessary to have the contributions from an unrealistic number of Higgs bosons. Although we have restricted to a particular form of the scalar potential, we can conclude that we cannot expect large contributions from this sector even in the most general case.

It is interesting to analyze the behavior of Eqs. (33) and (34) in the decoupling limit. It turns out that $\Delta Q^H$ always vanishes no matter which one of $m_{H_1}$ or $m_{H^\pm}$ is made large. On the other hand, $\Delta \kappa^H$ do may give rise to nondecoupling effects. If both $m_{H_1}$ and $m_{H^\pm}$ become simultaneously large, $\Delta \kappa^H$ vanishes, but when $m_{H_1}$ becomes infinite and $m_{H^\pm}$ remains finite, it approaches the constant value $\Delta \kappa^H = a$; when the situation is reversed, $\Delta \kappa^H \to -a/2$. This is in accordance the previous discussion on the decoupling properties of the $W$ form factors.

Finally, we would like to compare the size of the new contributions with those of the SM, which is known to give

FIG. 10: Scalar contribution to the $\Delta \kappa$ (solid line) and $\Delta Q$ (dashed line) form factors, in units of $a$, in the 331 model with right-handed neutrinos when $H_1$ and $H^\pm_5$ are degenerate and have a mass $m_S$. 
the following one-loop corrections to $\Delta \kappa$ and $\Delta Q$ [4]: $\Delta \kappa_{\text{max}}^{\text{SM}} = 30 \, a$ and $\Delta Q_{\text{max}}^{\text{SM}} = 5 \, a$. The contribution from the 331 model with right-handed neutrinos is thus only a few percent that of the SM. From all the studies presented in the literature [5], it can be inferred that only those models in which there are contributions from a large number of particles would have the chance of giving large corrections to the $W$ form factors.

V. FINAL REMARKS

In this work we have calculated the static quantities of the $W$ boson in the framework of the 331 model with right-handed neutrinos. Apart from the usual SM contributions, there is new contributions from the gauge and the scalar sectors. In the former there is a new contribution induced by a singly charged $Y^\pm$ and a complex neutral gauge boson $Y^0$, called bileptons. In the scalar sector there is the contribution from a singly charged Higgs boson $H^\pm_2$ and two neutral scalar bosons $H_1$ and $H_2$, but $H_2$ coincides with the SM Higgs boson and its contribution should be identified with a SM effect rather than with new physics. Although the model predicts three exotic quarks and an extra neutral gauge boson $Z'$, these particles give no contribution to $\Delta Q$ and $\Delta \kappa$. It turns out that the exotic quarks do not couple to the $W$ boson as they are $SU_L(2)$ singlets, whereas $Z'$ can only contribute through $Z - Z'$ mixing and its contribution is expected to be negligibly small. Analytical expressions were presented for both nondegenerate and degenerate masses of the bileptons and the Higgs bosons. The loop integrals were worked out by a modified version of the Passarino-Veltman reduction scheme. To cross-check our results, the form factors were obtained independently by the Feynman parameter technique and the resulting integrals were numerically evaluated and compared with the results obtained through the Passarino-Veltman method. It was found that the new contributions can be of the same order of magnitude as those arising in other weakly coupled renormalizable theories. It is interesting to note that the contribution from the scalar sector is similar to that of a THDM. This means that the form factors will not help to discriminate between different theories. Instead the on-shell $WW\gamma$ vertex would be useful to test the particular theory realized in nature with high precision once all the free parameters of the theory are known. In the scenario in which the non SM particles circulating in the loops (bileptons or Higgs bosons) are degenerate, the form factors are smaller than in the case in which they are nondegenerate. It was also found that in the scenario in which the bilepton and scalar boson masses are of the same order of magnitude, the gauge sector gives dominant contribution to the $W$ form factors. The nondecoupling properties of the $\Delta \kappa$ and $\Delta Q$ form factors were analyzed. It was found that $\Delta Q$ is always of decoupling nature, whereas $\Delta \kappa$ is sensitive to heavy Higgs bosons but insensitive to heavy bileptons. In fact, the numerical analysis shows that the contribution from a heavy bilepton with mass of the order of 1 TeV is negligibly small.

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