Novel Josephson Effect in Triplet Superconductor - Ferromagnet - Triplet Superconductor Junctions

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We predict a novel type of Josephson effect to occur in triplet superconductor - ferromagnet - triplet superconductor Josephson junctions. We show that the Josephson current, \( I_J \), exhibits a rich dependence on the relative orientation between the ferromagnetic moment and the \( \mathbf{d} \)-vectors of the superconductors. This dependence can be used to build several types of Josephson current switches. Moreover, we predict an unconventional temperature dependence of \( I_J \) in which \( I_J \) changes sign with increasing temperature.

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Josephson junctions made of unconventional superconductors have attracted significant interest over the last few years due to their unconventional quantum transport properties. The latter are determined by the formation of low-energy Andreev bound states, which, for example, lead to a low-temperature Josephson effect in one-dimensional (1D) triplet superconductor - ferromagnet - triplet superconductor junctions. The latter are determined by the formation of low-energy Andreev bound states. In particular, we demonstrate a rich dependence of the Josephson current, \( I_J \), on the relative orientation between the ferromagnetic moment, \( \mathbf{M} \), in the barrier and the \( \mathbf{d}_{L,R} \)-vectors of the left and right triplet superconductor, as described by the angles \( \alpha \) and \( \theta \) (see Fig. 1). This dependence can be used to create Josephson current switches in which small changes of \( \alpha \) or \( \theta \) can tune the junction between two “current states”, in which \( I_J \) is either “on” (\( I_J \neq 0 \)) or “off” (\( I_J \approx 0 \)), or differs in its direction. These types of two-level systems are of great current interest in the field of quantum information technology. Moreover, we predict that a TSFTS junction leads to a qualitatively new temperature dependence of \( I_J \) such that in certain cases, \( I_J \) changes sign (i.e., its direction) with increasing temperature. Finally, we show that adiabatic changes in the orientation of \( \mathbf{d}_{L,R} \) yield a behavior of \( I_J \) which is significantly altered from its equilibrium form.

We take the 1D TSFTS junction to be aligned along the \( z \)-axis and be described by the Hamiltonian \( H = \int dz \, dz' H(z,z') \), where \( (h = 1) \)

\[
H(z,z') = \sum_\sigma \psi_\sigma^\dagger(z') \delta(z-z') \left[ -\frac{\partial^2}{2m} - \mu \right] \psi_\sigma(z) + \frac{\Delta(z,z')}{2} \left[ e^{-i\theta} \psi_\uparrow^\dagger(z') \psi_\uparrow(z) - e^{i\theta} \psi_\downarrow^\dagger(z') \psi_\downarrow(z) \right] + \text{h.c.} - \mathbf{M}(z,z') \cdot \sum_{\alpha,\beta} \psi_\alpha^\dagger(z') \sigma_{\alpha\beta} \psi_\beta(z), \tag{1}
\]

and \( \psi_\uparrow^\dagger(z) \) and \( \psi_\downarrow(z) \) are the fermionic creation and annihilation operators for a particle with spin \( \sigma \) at site \( z \), respectively. \( \sigma \) are the Pauli matrices, and \( \Delta(z,z') = -\Delta(z',z) \) is the superconducting gap. The \( \mathbf{d} \)-vector of the triplet superconductors on the left (\( j = L, z < 0 \)) and right (\( j = R, z > 0 \)) of the barrier is given by \( \mathbf{d}_L = (\cos \theta_j, \sin \theta_j, 0) \). We take \( \theta_L \equiv 0 \) such that \( \mathbf{d}_L \parallel \hat{x} \) and \( \theta_R = \theta \) with \( \mathbf{d}_R \) lying in the spin \( xy \)-plane. The ferromagnetic junction, located at \( z = 0 \), possesses a moment \( \mathbf{M}_0 \) and represents a magnetic scattering potential, described by the last term in Eq. (1), with \( \mathbf{M}(z,z') = \mathbf{M}_0 (\cos \alpha \sin \theta, 0, 0) \delta(z)(z') \).

We show below that two Andreev bound states with energies \( E_{a,b} \) are formed in the TSFTS junction. The Josephson current flows through these two states and...
\[ I_J = I_J^a + I_J^b = \frac{e^2}{\hbar} \sum_{i=a,b} \frac{\partial E_i}{\partial \Phi} \tanh \left( \frac{E_i}{2k_B T} \right). \]  

In order to obtain the energies of the Andreev states, we start from Eq. (1) and derive the Bogoliubov-de Gennes (BdG) equations \([12, 13]\) by introducing the unitary Bogoliubov transformation

\[
\Psi_j(z) = \sum_{\gamma=\pm} A_{j,\gamma} \left( \begin{array}{c} u_j(z) \\ v_j(z) \\ w_j(z) \\ x_j(z) \end{array} \right) e^{i\kappa k_F z}, \tag{3}
\]

where the sum runs over all eigenstates of the junction. For the wave functions of the localized Andreev states on the left and right side of the junction, we make the ansatz

\[
\Psi_j(z) = \left( \begin{array}{c} u_j(z) \\ v_j^*(z) \\ w_j(z) \\ x_j^*(z) \end{array} \right) = e^{i\kappa z} \sum_{\gamma=\pm} A_{j,\gamma} \left( \begin{array}{c} u_j(z) \\ v_j^*(z) \\ w_j(z) \\ x_j^*(z) \end{array} \right) e^{i\kappa k_F z},
\]

In the following we take \(\Delta_L = \Delta_0\) and \(\Delta_R = \Delta_0 e^{i\phi}\) for the superconducting gap on the left and right side of the junction. The above eigenvalue equation for \(E\) is subject to the boundary conditions \(\Psi(L) = \Psi(0)\) and

\[
\partial_z \Psi_R(0) - \partial_z \Psi_L(0) = -2mM_0 \left( \begin{array}{c} 0 \\ \hat{P} \end{array} \right) \Psi_R(0), \tag{4}
\]

where \(\hat{P} = \hat{\sigma}_3 \cos \alpha - i \hat{\sigma}_0 \sin \alpha\). The solution of the BdG equations yields two Andreev states with energies

\[
\frac{E_{a,b}}{\Delta_0} = k_F \sqrt{D \left( A + B - 2C \pm 2\sqrt{(A-C)(B-C)} \right)}, \tag{5}
\]

where

\[ A = \cos^2(\phi/2) \left[ 1 - D \sin^2(\theta/2) \right], \]
\[ B = \sin^2(\theta/2) \left[ 1 - D \cos^2(\phi/2) \right], \]
\[ C = (1-D) \left[ \cos^2(\phi/2) - \cos^2(\theta/2) \right] \cos^2(\alpha - \theta/2), \]

and \(D = [1 + g^2]^{-1}\) with \(g = mM_0/k_F\). The two Andreev states appear in the local density of states near the junction as two particle-like and two hole-like peaks at energies \(\mp E_{a,b}\), respectively.

We first consider \(d_L \parallel d_R\), and present in Fig. 2 the energies of the Andreev states and the resulting Josephson current at \(T = 0\) as a function of \(\phi\) for several values of \(\alpha\). Note that for \(T = 0\), only the negative energy branches of the bound states are populated, and thus contribute to \(I_J\). The dependence of \(E_{a,b}\) on \(\phi\) is qualitatively different for \(M \parallel d_{L,R}\) and \(M \perp d_{L,R}\). For \(M \parallel d_{L,R}\) (\(\alpha = \pi/2\)) the Andreev states are degenerate with \(E_{a,b}/\Delta_0 = k_F \sqrt{D \cos \phi/2}\), and possess well defined spin quantum numbers \(\sigma_a = \uparrow, \sigma_b = \downarrow\), since they are not coupled by the scattering at the ferromagnetic barrier. The zero energy level crossing at \(\phi_{LC} = (2n + 1)\pi\) (\(n = \text{integer}\)) occurs with \(\partial E_{a,b}/\partial \phi \neq 0\), resulting in discontinuous jumps of \(I_J\), as shown in Fig. 2(a). It is interesting to note that \(E_{a,b}\) is identical to that of Andreev states near a potential scattering barrier \([14]\). In contrast, for \(\alpha \neq \pi/2\) the ferromagnetic barrier couples the Andreev states, yielding a splitting of their energies, as shown in Fig. 2(a) for \(M \parallel d_{L,R}\) (\(\alpha = 0\)). This coupling yields \(\partial E_{a,b}/\partial \phi = 0\) at \(\phi_{LC}\), such that \(I_J\) evolves continuously with \(\phi\) [Fig. 2(b)]. While these results remain qualitatively unchanged with decreasing \(D\), the dependence of \(I_J\) on \(D\) changes with the orientation of \(M\) and \(d_{L,R}\). Specifically, for \(D \ll 1\) one finds, leading order in \(D\), \(I_J \sim D^{1/2}\) for \(\alpha \neq 0\), but \(I_J \sim D^{3/2}\) for \(\alpha = 0\). Finally, note that for \(\phi = 0\) and any \(\alpha\), the induced Andreev states with \(E_{a,b}/\Delta_0 = k_F \sqrt{D}\) are identical to the impurity (Shiba) states \([14]\) induced by a single (static) magnetic impurity in a 1D triplet superconductor.

The Josephson current in a TSFTS junction exhibits an unconventional temperature dependence in that \(I_J\) can change sign, and thus its direction, with increasing temperature, as shown in Fig. 3(a) for \(T = 0\) and \(\phi = \pi/2\) (we assumed a BCS temperature dependence of the superconducting gap). In order to understand this sign change, we consider the \(\phi\)-dependence of \(E_{a,b}\) shown in Fig. 3(b) and note that at \(T = 0\), only the branches indicated by 1 and 2, belonging to Andreev states \(a\) and \(b\), respectively, are occupied. Since the derivatives \(\partial E_{a,b}/\partial \phi\) possess opposite signs for the two Andreev states, the corresponding currents through them, \(I_J^a < 0\) and \(I_J^b > 0\),
flow in opposite directions with $I_{j}^{b} > |I_{j}^{a}|$. With increasing temperature, the occupation of branches 2 and 3 changes more rapidly than those of branches 1 and 4. As a result, the magnitude of $I_{j}^{b}$ decreases more quickly than that of $I_{j}^{a}$, and the total current, $I_{j}$, eventually changes sign. Moreover, the qualitative nature of the temperature dependence can be altered via a rotation of $\mathbf{M}$. Specifically, as $\alpha$ increases, the $T = 0$ value of $I_{j}^{a}$ decreases while $I_{j}^{b}$ remains practically unchanged. This leads to a qualitative change in the temperature dependence of $I_{j}$ such that for $\alpha > 0.21\pi$, $I_{j}$ does not undergo a sign change with increasing temperature [Fig. 4(a)]

The unique dependence of $I_{j}$ on $\phi$ and on the orientation of $\mathbf{M}$ can be used to build a Josephson current switch in which $I_{j}$ is turned “on” or “off” via the rotation of $\mathbf{M}$. This is shown in Fig. 4(a) where we present $I_{j}$ as a function of $\alpha$ for $\mathbf{d}_{L} \parallel \mathbf{d}_{R}$ at $T = 0$. Note that the

$\alpha$-dependence of $I_{j}$ changes significantly with increasing $\phi$. In particular, $I_{j}$ becomes sharply peaked around $\alpha = (2n + 1)\pi/2$ with integer $n$ (i.e., for $\mathbf{M} \perp \mathbf{d}_{L,R}$) as $\phi$ approaches $\pi$. As a result, small variations in $\alpha$ lead to large changes in the magnitude of the Josephson current, and can thus tune the junction between an “on”-state ($I_{j} \neq 0$) and an “off”-state ($I_{j} \approx 0$). This behavior of $I_{j}$ is directly reflected in the $\alpha$-dependence of the bound state energies, as shown in Fig. 4(b). While for $\phi = 0$ the Andreev states are $\alpha$-independent (see above), $E_{a,b}$ oscillate sinusoidally with $\alpha$ for $\phi = \pi/4$, as shown in Fig. 4(b). As $\phi$ approaches $\pi$ (see, e.g., $\phi = 0.9\pi$), the energies of the Andreev states alternate in being close to zero and almost $\alpha$-independent for half of the period and sinusoidal for the other half. Finally, for $\phi = \pi$ (not shown) the bound state energies are given by $E_{a}/\Delta_{0} = 2k_{F}\sqrt{D(1-D)}\cos \alpha$ and $E_{b} = 0$. Note that for $\phi \leq \pi$, the Andreev states exhibit no zero-energy crossings when $\mathbf{M}$ is rotated.

Another type of Josephson switch can be created by rotating $\mathbf{d}_{R}$ for fixed $\mathbf{M}$ and $\mathbf{d}_{L}$, as shown in Fig. 5 for $T = 0$. For $\alpha = 0$, $\phi = \pi/2$ and $D = 0.7$, $I_{j}$ exhibits

an almost perfect square wave form, is symmetric around $\theta = n\pi$, and is nearly $\theta$-independent between the discontinuous jumps, as shown in Fig. 5(a). For $\phi \neq \pi/2$, this symmetry is broken, and the ranges of $\theta$ over which $I_{j}$ is positive or negative are unequal, but the discontinuous jumps in $I_{j}$ persist. For $\alpha = \pi/4$, the Josephson current is skewed, and in some regions varies nearly linearly with $\theta$ [Fig. 5(b)]. For $\alpha = \pi/2$ and $\phi = \pi/2$, $I_{j}$ is again symmetric around $\theta = n\pi$ [Fig. 5(c)]. With decreasing $D$, the Josephson current becomes $\theta$-dependent between discontinuous jumps, even for $\alpha = 0$ and $\phi = \pi/2$ [Fig. 5(d)]. Thus small changes in the relative alignment of the $\mathbf{d}$-vectors can switch the junction between two current states with opposite direction of $I_{j}$.

A third type of Josephson switch can be formed via a correlated rotation of $\mathbf{M}$ and $\mathbf{d}_{R}$. Specifically, we note that $E_{a,b}$, Eq. 6, depends on $\alpha$ only via $\cos^{2}(\alpha - \theta/2)$. We therefore consider a simultaneous rotation of $\mathbf{d}_{R}$ and $\mathbf{M}$ such that $\alpha = \theta/2$, and present the resulting $I_{j}$ as a function of $\theta$ in Fig. 6(a). $I_{j}$ then exhibits square wave oscillations with discontinuous jumps from $I_{j} \approx 0$ to a negative (positive) value for $0 \leq \phi \leq \pi$ ($\pi \leq \phi \leq 2\pi$). To understand these sharp transitions, we plot the $\phi$-dependence of $-E_{a,b}$ in Fig. 6(b) (only these branches contribute to $I_{j}$ at $T = 0$). For $\theta = \pi$ the Andreev states are degenerate, with $\partial E_{a,b}/\partial \phi \leq 0(\geq 0)$ for $0 \leq \phi \leq \pi$ ($\pi \leq \phi \leq 2\pi$). In contrast, for $\theta = \pi/2$ the degeneracy of the Andreev states is lifted with $\partial E_{a,b}/\partial \phi \neq \partial E_{a,b}/\partial \phi$ in some range of $\phi$ (shown as dashed lines in Fig. 6(b)) such that $I_{j} \approx 0$, but $I_{j} \neq 0$ in other regions of $\phi$ (shown as solid lines). Note that as $\phi$ increases from zero, the.
adiabatic case changes continuously with $\theta$ in Fig. 7(b) remain occupied, while those shown as red of the Andreev states represented by the black solid lines thermodynamic equilibrium), then the energy branches $\theta = \phi$, and $\phi = 0.7$. (b) $\pm E_{a,b}$ as a function of $\theta$. The black solid lines represent the occupied states if $\theta$ is changed adiabatically.

Josephson current at $T = 0$ for an adiabatic rotation of $d_R$ and in the equilibrium case are shown in Fig. 7(a). To understand the *qualitative* form of $I_J$, we note that in the equilibrium case, only the negative energy branches of the Andreev states contribute to $I_J$. In contrast, if $\theta$ is adiabatically changed (from the state with $\theta = 0$ that is in thermodynamic equilibrium), then the energy branches of the Andreev states represented by the black solid lines in Fig. 7(b) remain occupied, while those shown as red dashed lines remain unoccupied. As a result, $I_J$ in the adiabatic case changes continuously with $\theta$ and does not exhibit the discontinuous jumps shown by the equilibrium $I_J$. Note that $I_J$ is $2\pi$-periodic in both cases. A special case arises for $\alpha = \pi/2$, when the periodicity of $I_J$ is changed to $4\pi$ (not shown). An adiabatic change of $\phi$ also leads to a $4\pi$-periodicity of $I_J$, in analogy to the fractional $ac$-Josephson effect discussed in Ref. [8].

Finally, scattering off the ferromagnetic barrier in general leads to a suppression of the superconducting order parameter near the barrier, which was not accounted for in the above approach. However, in nodeless superconductors, such as the one discussed above, the order parameter recovers its bulk value on a length scale set by $1/k_F$, which is assumed to be much shorter than the decay length of the Andreev bound state $\phi$. As a result, the spatial variation of the order parameter leads to only weak quantitative and no qualitative changes in the induced fermionic bound states $\epsilon$. We thus expect that the results presented above are unaffected by the inclusion of the order parameter suppression.

In summary, we predict a new type of Josephson effect in TSFTS junctions, in which $I_J$ exhibits a rich dependence on the relative orientation of $\mathbf{M}$ and $\mathbf{d}_{L,R}$. This dependence can be used to build several types of Josephson current switches in which small changes of $\alpha$ or $\theta$ can tune the junction between two current states. We predict an unconventional temperature dependence of $I_J$ such that for certain orientations of $\mathbf{M}$ and $\mathbf{d}_{L,R}$, $I_J$ changes sign with increasing temperature.

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