Non-leptonic decays of the $B_c$ into tensor mesons

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Abstract

We have computed the branching ratios of the exclusive pseudoscalar (vector) + tensor modes that are allowed in the decays of the $B_c$ meson. The dominant spectator and annihilation contributions in those decays are evaluated using the factorization hypothesis. We find that some of these decay channels, such as $B_c^- \to (\rho^-, D^*_s- , D^*_s^-) \chi_{c2}$ and $B_c^- \to \pi^- D^{*0}_{s2}$, have branching ratios of the order of $10^{-4}$, which seems to be at the reach of forthcoming experiments at the LHC. The inclusive branching fraction of the two-body $B_c$ decays involving tensor particles is approximately $1.28 \times 10^{-3}$. At the dynamical level, it is interesting to observe that the exclusive decays $B_c^- \to K^- (\pi^-) D^{*0}_{s2}$, $\pi^0 D^*_2$, $\eta' D^{*+}_{s2}$, are dominated by the annihilation contributions.

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Studies of the $B_c$ meson decays are important for several reasons. The mass of the $\mathcal{J}^P = 0^-$ meson composed of two heavy quarks ($b\bar{c}$) lies between the masses of the corresponding $bq$ ($q = u, d, s$) and $b\bar{b}$ systems. While the light quark in $B_{u,d}$ mesons plays a marginal role as an spectator in the dominant decays, both $b$ and $\bar{b}$ quarks fully participate in the decays of beauty quarkonium states. The beauty-charmed meson share two interesting features of those systems. Firstly, the phase-space in $B_c$ meson decays is large enough to allow decays where not only the charm antiquark, but also the $b$ quark can play the role of spectator. Secondly, in some decays of the $B_c$ mesons we can expect that both processes, namely those containing a $c$ quark as spectator (a $b \rightarrow cW^*$ transition) and the $b\bar{c} \rightarrow W^*$ annihilation diagrams, play similar important roles since they can have the same Cabibbo-Kobayashi-Maskawa mixing weights. Thus, $B_c$ meson decays can allow a unique place for testing the interplay of both dynamical processes which are not easily accessible in the decays of $B_{u,d}$ mesons. Furthermore, as also is explored in this paper, some $B_c$ decays have $W$-annihilation contributions which largely dominate some of the decay amplitudes.

The semileptonic and non-leptonic decays of the $B_c$ meson that are expected to be the dominant ones, have been considered previously by other authors \cite{1}. Here, we focus on the two-body non-leptonic $B_c$ decays that contain a $\mathcal{J}^P = 2^+$ tensor meson in the final state, i.e. $B_c \rightarrow P(V)T$, where $P(V)$ denotes a pseudoscalar (vector) and $T$ a tensor meson. The analogous decays of the $B_{u,d}$ mesons have been considered in refs. \cite{2, 3} (see also \cite{4}). Our study is exhaustive in the sense that we make predictions for all the decays that are allowed by the kinematical constraints and the leading order dynamical factorizable contributions. As in our previous works, we have used the non-relativistic quark model of Isgur et al (ISGW-model) \cite{5} as input for the hadronic matrix elements $\langle T|J_\mu|B_c \rangle$ and $\langle PT|J_\mu|0 \rangle$ that are required in our factorization approximation for the spectator and annihilation contributions, respectively. Note that factorizable amplitudes proportional to the matrix element $\langle T|J_\mu|0 \rangle$ do not contribute because this matrix element vanishes identically from Lorentz covariance considerations. As a final remark, let us mention that the annihilation amplitudes will be more important in $B_c$ than in $B_u$ decays because they depend upon the same CKM mixing factors (namely $V_{cb}$) as the spectator $b \rightarrow c$ contributions to $B_c$ decays.
The specific interest of $B \to XT$ decays is two-fold. First, we would like to test how quark models work in describing matrix elements that involve orbital excitations of the $q\bar{q}$ system (the tensor meson). Second, we are interested in the evaluation of the fraction of non-leptonic decays provided by modes containing tensor mesons. This is important in order to test how different exclusive channels contribute to the inclusive rates which can in principle be computed using QCD and the quark-hadron duality [6]. In addition, the study of the $B_c$ decays involving spin-2 (tensor) mesons can provide complementary tests for the quark models predictions of the orbital pieces of the meson wavefunctions, which are not accessible with purely lowest lying $P$ and $V$ states.

Our results show that some branching ratios of $B_c \to P(V)T$ decays, turn out to be of the order of $10^{-4}$. Those fractions seems to be at the reach of future experiments at the LHC where it is expected to produce $2.1 \times 10^8 B_c$ mesons with an integrated luminosity of $100 \text{ fb}^{-1}$ and cuts of $P_T(B_c) > 20 \text{ GeV}$, $|y(B_c)| < 2.5$ [7]. With a further increase in the luminosity one could reach $10^{10} B_c$ mesons per year [8]. In fact, with a fragmentation ratio of $3.8 \times 10^{-4}$ ($5.4 \times 10^{-4}$) for the $B_c$ ($B_c^*$) meson [9], $10^8$-$10^9 B'_c$'s can be available from a rather conservative estimate [10]. These samples of $B_c$ mesons, would allow a detailed study of a large diversity of properties and decay modes of these mesons.

Let us start our discussion of $B \to XT$ decays with the effective weak Hamiltonian that involve a $b$ quark decay (namely a $\bar{c}$ as spectator) [11]:

$$H_{eff}(\Delta b = 1) = \frac{G_F}{\sqrt{2}} \sum_{q',Q,q} V_{q'b}V_{Q'q}^* \left[ a_1(\overline{q}b)(\overline{Q}Q') + a_2(\overline{q}b)(\overline{q}Q') \right] + h.c.$$  \hspace{1cm} (1)

The corresponding operator for $B_c \to XT$ transitions that proceed through the $c$-antiquark decay ($b$ as spectator) in the $B_c$ meson is given by:

$$H_{eff}(\Delta c = 1) = \frac{G_F}{\sqrt{2}} \sum_{Q',q,q} V_{cq}V_{u'q}^* \left[ a_1(\overline{Q}c)(\overline{p}q) + a_2(\overline{Q}c)(\overline{q}p) \right] + h.c.$$  \hspace{1cm} (2)

In the above expressions $(\overline{q}q')$ is used for the $V-A$ current, $Q'$, $q'$ = $u$, $c$ and $Q$, $q$ = $d$, $s$, $G_F$ denotes the Fermi constant, $V_{ij}$ are the CKM mixing factors, and $a_{1,2}$ denote the QCD coefficients. The effective Hamiltonian that give rises to annihilation amplitudes would be

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considered below (see Eq. (4)).

The contribution from $a_1$ corresponds to the $W$-external emission diagram at the tree level (class I decays [12]) and the contribution from $W$-internal emission (class II decays [12]) decays is given by $a_2$. We also distinguish two types of suppressed decays according to the occurrence of the CKM factors [3]: in the type-I(II) decays the suppression occurs in the vertex where the boson $W$ is produced (annihilated). It is important to note that for the $b$-spectator $B_c$ decays these two kinds of suppression are at the same order in contrast to the $c$-spectator $B_c$ decays where the type-I suppression is stronger than in the type-II case.

In order to provide numerical values of the branching ratios we use the expressions (9) and (11) for the decay rates given in Ref. [2] and the following values of the CKM elements [13]: $|V_{cb}| = 0.0402$, $|V_{ud}| = 0.9735$, $|V_{cs}| = 0.9749$, $|V_{us}| = 0.2196$, $|V_{cd}| = 0.224$, and $|V_{ub}| = 3.3 \times 10^{-3}$. The values for the lifetime and the mass of $B_c$ were taken from the experimental measurements of Ref. [14] and the value of the masses of the tensor mesons $B_2^*$ and $B_{s2}^*$ are 5.733 and 5.844 GeV, respectively [15]. All the other masses required are taken from [13]. The values used for the QCD coefficients are $a_1 = 1.132$ [16] and $a_2 = 0.29$ (which is obtained using the relation $|a_2/a_1| = 0.26$ of Ref. [17]).

The decay constants of pseudoscalar mesons $f_P$ (in GeV units) have the following central values: $f_{\pi^-} = 0.131$ [13], $f_{\pi^0} = 0.130$ [13], $f_\eta = 0.131$ [12], $f_{\eta'} = 0.118$ [12], $f_{D_s} = 0.280$ [18], $f_D = 0.252$, $f_{\eta_c} = 0.393$ [19], and $f_K = 0.159$ [13]. The decay constant $f_D$ was obtained using the theoretical prediction $f_D/f_{D_s} = 0.90$ [20] and the value for $f_{D_s}$ quoted above. On the other hand, the central values for the dimensionless decay constants of vector mesons $f_V$ are $f_\rho = 0.281$, $f_\omega = 0.249$, $f_\phi = 0.232$, $f_{D_s^*} = 0.128$, $f_{D^*} = 0.124$, $f_{J/\psi} = 0.1307$, and $f_{K^{*+}} = 0.248$.

We have also used the following expressions for the physical states of the mixing between octet and singlet states of SU(3) with $I = 0$:
\[ \eta = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \sin \phi_P - (s\bar{s}) \cos \phi_P, \]

\[ \eta' = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \cos \phi_P + (s\bar{s}) \sin \phi_P, \]

\[ \omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \cos \phi_V + (s\bar{s}) \sin \phi_V, \]

\[ \phi = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \sin \phi_V - (s\bar{s}) \cos \phi_V, \]  

(3)

where the mixing angle is given by \( \phi_i = \arctan \left( \frac{1}{\sqrt{2}} \right) - \theta_i \) \( (i = P \text{ or } V) \) and the experimental values of \( \theta_i \) are given by \(-20^0\) and \(39^0\) \[13\] for pseudoscalar \((\eta, \eta')\) and vector \((\omega, \phi)\) mesons, respectively.

With the above convention and numerical values\[4], we have computed the decay amplitudes for \( B_c \to XT \) modes. They are shown in the second column of Tables 1 and 2 (second column in Table 3), respectively, for the \( PT \) and \( VT \) channels when the \( c \)-antiquark (the \( b \)-quark) plays the role of spectator in the \( B_c \) decay. We do not calculate the \( b \)-spectator \( B_c \to VT \) decays because they are completely forbidden by kinematics. We can observe that the amplitudes of \( B_c \to P(V)T \) are proportional to only one of the QCD coefficients \( a_i \) at the time. The expressions for the factors \( F^{i \to f} \) and \( F^{i \to f}_{\mu \nu} \) in the amplitudes and the properties of the symmetric polarization tensor \( \epsilon_{\mu \nu} \) describing the tensor meson can be found in Ref. \[2\].

We have computed the branching ratios of \( B_c \to PT \) and \( B_c \to VT \) (last column in tables 1–3) using the non-relativistic constituent quark model ISGW \[3\], which can be reliable to describe the \( B_c \) meson composed of two different heavy quarks \[21\]. We restrict ourselves to this model because the predictions for the required form factors are not available in a systematic way in other quark model approaches. The branching ratios obtained for the \( B_c^- \to (\rho^-, D_s^{*-}, D_s^-)\chi_{c2} \) and \( B_c^+ \to \pi^+ B_s^{*0} \) decays (which are proportional to the QCD coefficient \( a_1 \)) turn out to be of the order of \( 10^{-4} \), which seems to be at the reach of future

\(^1\text{Since we are interested in getting estimated values for the branching fractions, we do not quote their corresponding error bars which at present are dominated by the uncertainties in the mass and lifetime of the } B_c \text{ meson.}\)
$B_c$ samples at the LHC [7].

The decays $B_c \rightarrow P(V)T$ with the largest branching fractions are of the same order that some $B_c \rightarrow PP, PV, VV$ modes [1]. Note that the decay $B^+_c \rightarrow \pi^+ B^{*0}_{s2}$, which proceeds having the $b$-quark as an spectator, has a similar branching ratio than $B^-_c \rightarrow (\rho^-, D^{*-}_s, D^{-}_s)\chi_{c2}$ because it is favored by the CKM factor although it is suppressed by phase space. As a matter of fact, if this decay and someone of $B^-_c \rightarrow (\rho^-, D^{*-}_s, D^{-}_s)\chi_{c2}$ were measured, they would provide a good test for the flavor independence of the QCD coefficient $a_1$ since in $B_c$ decays with $b$- or $c$ quarks, they are assumed to be the same.

Let us mention that some of the branching ratios of the type-I suppressed decays $B_c \rightarrow VT$ (rows 5-10 in table 2) turn out to be of the same order than $B(B^+_c \rightarrow \rho^+(K^{*-})\gamma)$ ($\approx 10^{-8} - 10^{-9}$) [22]. Observe also that the ratio $B(B_c \rightarrow VT)/B(B_c \rightarrow PT) \approx 3$ when the contribution arises from the $a_1$ coefficient, and $B(B^-_c \rightarrow D^{*0}D^{*-}_2)/B(B^-_c \rightarrow D^{*0}D^{*-}_2)$ is different from unity (see the second ant the last columns in table 2) because they arise from the transitions $b \rightarrow c$ and $b \rightarrow u$, respectively.

In a recent paper [16], the authors have computed all the $c$-spectator modes $B_c \rightarrow P(V)\chi_{c2}$ using the so-called generalized instantaneous approximation [23]. In order to compare their results and ours we show their numerical values within parenthesis in the third column of tables 1 and 2. Our results in these decays are in some cases of the same order of magnitude than the results of Ref. [16] and in the other cases are smaller by an order of magnitude except in the decay $B^-_c \rightarrow K^{*-}\chi_{c2}$.

Now we will focus on the study of the annihilation contributions to $B_c \rightarrow PT$ decays. The evaluation of the corresponding contributions to $B_c \rightarrow VT$ would require the knowledge of the $\langle VT|J_{\mu}|0\rangle$ matrix element, which has not been computed in the literature. As is well known, the annihilation amplitudes for the two-body non-leptonic $B_{u,d}$ decays are helicity-suppressed [12, 24]. In fact, as it has been proved in Refs. [25], the $B_{u,d} \rightarrow P_1 P_2$ decays that proceed only via a single W-exchange or W-annihilation quark diagram have negligible branching ratios, although some of them can be at the reach of B-factories. On the
other hand, in some exclusive processes where the non-annihilation contribution are highly suppressed, the annihilation contributions may become important \[24, 26\].

Ali \textit{et al.} \[24\] have shown explicitly that the annihilation amplitude in \(B_{u,d} \to P_1 P_2\) is suppressed by a hefty factor with respect to the non-annihilation contribution, and have suggested that in the decays \(B_{u,d} \to PV, VV\) the situation may be different because the annihilation quark-diagram may enhance the decay rates. We expect that the annihilation contributions to some exclusive \(B_c \to PT\) decays, where the non-annihilation contribution is too suppressed, may become more important than in the \(B_{u,d}\) decays. This happens because the \(B_c\) meson is composed of two heavy quarks and the annihilation amplitude is proportional to the CKM factor \(V_{cb}\) instead of \(V_{ub}\) as in \(B_{u,d}\) decays.

The effective weak Hamiltonian for the annihilation contributions to \(B_c \to PT\) is \[11\]:

\[
\mathcal{H}_{a}^{eff} = \frac{G_F}{\sqrt{2}} \sum_{q' q} V_{cb} V_{q' q}^{*} a_1 (\bar{t}b)(\bar{q'}Q') + \text{h.c.,}
\]

(4)

where the subscript \(a\) denotes \textit{annihilation}, \(Q' = u, c\) and \(q = d, s\). The \(a_1\) contribution is associated to the \(W\)-annihilation for charged mesons. It is clear that in this case there is not an \(a_2\) contribution corresponding to the \(W\)-exchange for neutral mesons.

The annihilation amplitude can be written as \[24\]

\[
\mathcal{M}_a(B_c \to PT) = \frac{i G_F}{\sqrt{2}} V_{cb} V_{q' q}^{*} a_1<TP|J_\mu|0>_{a}<0|J^\mu|B_c>,
\]

(5)

where \(<0|J_\mu|B_c> = if_{B_c}(P_{B_c})^\mu\) and \(<TP|J_\mu|0>_{a}\) is taken from the ISGW-model \[8\], using the crossing symmetry of the \(<T|J_\mu|P>\) amplitude. Thus, we write the annihilation amplitude as:

\[
\mathcal{M}_a(B_c \to PT) = \frac{G_F}{\sqrt{2}} V_{cb} V_{q' q}^{*} a_1 f_{B_c} \epsilon_{\mu\nu}^{*}(P_P)^\mu(P_P)^\nu \mathcal{F}^{PT}(m_{B_c}^2),
\]

(6)

where \(\epsilon_{\mu\nu}^{*}\) is the polarization of the tensor meson, \(\mathcal{F}^{PT}(m_{B_c}^2) = k_a + (m_P^2 - m_T^2)(b_+)_a + m_{B_c}^2 (b_-)_a\), with \(k_a\), \((b_\pm)_a\) being the \(P \to T\) form factors evaluated in \(q^2 = m_{B_c}^2\).
The decay rate for those processes which are produced only by the annihilation contribution is

$$\Gamma_a(B_c \to PT) = |A_a|^2 \left(\frac{m_{B_c}}{m_T}\right)^2 \frac{|P_T|^5}{12\pi m_T^2},$$

(7)

with $A_a$ given by

$$A_a = \frac{G_F}{\sqrt{2}} V_{cb} V_{q'd}^* a_1 f_{B_c} F_{PT}(m_{B_c}^2).$$

(8)

To get an idea about the order of magnitude of the annihilation contribution to $B_c \to PT$ we can compare the expressions given by the Eq. (9) in Ref. [2], and our Eq. (7) shown above. If a given decay has $W$-emission ($t$) and annihilation ($a$) contributions, we get the following ratio among them:

$$R \equiv \frac{\Gamma_t(B_c \to PT)}{\Gamma_a(B_c \to PT)} = \left[\frac{V_t^* V_t a_t f_{P} F_{B_c \to T}(m_{P}^2)}{V_a^* V_a a_1 f_{B_c} F_{PT}(m_{B_c}^2)}\right]^2.$$  (9)

In table 4 we list the ratio $R$ for some exclusive $B_c \to PT$ decays that receive both of these contributions. In our numerical evaluations we have used the value $f_{B_c} = 0.48$ GeV from Ref. [25]. If we had assumed that the form factors for the annihilation and the direct diagrams are of the same order, we would see that the annihilation contribution becomes much bigger than the $W$-emission contribution. For example, in the $B_c^- \to K^- D_s^{\ast 0}$ decay, the annihilation contribution is almost $10^5$ times the tree level one, and for the decay $B_c \to \eta' D_{s2}^{\ast -}$ it becomes approximately $10^6$ times the direct contribution. These hefty factors arise basically from the CKM factors. Thus, we expect that the annihilation contribution in these decays, which have a highly suppressed non-annihilation amplitude, may become important.

In summary, in this paper we have computed the branching ratios of the two-body non-leptonic $B_c$ decays involving tensor mesons. The measurement of these decays would provide
additional tests for the quark models used to compute the hadronic matrix elements that involve orbital excitations of the $q\bar{q}'$ system (as is the case of tensor mesons). The decays with the largest branching ratios –of order $10^{-4}$– are $B_c^- \rightarrow (\rho^-, D_s^{*-}, D^-)\chi_{C2}$ and $B_c^+ \rightarrow \pi^+ B_{S2}^{*0}$, which proceed dominantly through $c$ and $b$ quarks as spectators, respectively. The fractions for these decays seems to be at the reach of future experiments at the LHC.

On the other hand, we have found that annihilation contributions to $B_c \rightarrow PT$ decays may become important in the exclusive channels $B_c^- \rightarrow K^-(\pi^-)\overline{D}^{*0}_2, \pi^0 D_{s2}^{*-}, \eta' D_{s2}^{*-}$, which have highly suppressed non-annihilation contributions. Taking into account all the results for the estimated exclusive modes given in Tables 1-3, we observe that the inclusive production of tensor mesons in $B_c$ meson decays have a branching ratio of $B(B_c \rightarrow XT) = 1.28 \times 10^{-3}$. This result is dominated by the modes having a $c$-quark as an spectator.

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| Process | Amplitude | $B(B_c \to PT)$ |
|---------|-----------|----------------|
| $B_c^- \to \pi^- \chi_{c2}$ | $V_{cb} V_{ud} a_1 f_{\pi^-} F_{B_c \to \chi_{c2}} (m_{\pi^-}^2)$ | $7.5 \times 10^{-5} (2.48 \times 10^{-4})$ |
| $B_c^- \to D^0 D_s^{*-}$ | $V_{cb} V_{ud} a_2 f_{D^0} F_{B_c \to D_s^{*-}} (m_{D_s}^2)$ | $6.26 \times 10^{-8}$ |
| $B_c^- \to D_s^- \chi_{c2}$ | $V_{cb} V_{cs} a_1 f_{D_s^-} F_{B_c \to \chi_{c2}} (m_{D_s^-}^2)$ | $1.54 \times 10^{-4} (4.54 \times 10^{-4})$ |
| $B_c^- \to \eta_c D_s^{*-}$ | $V_{cb} V_{cs} a_2 f_{\eta_c} F_{B_c \to D_s^{*-}} (m_{\eta_c}^2)$ | $1.4 \times 10^{-6}$ |
| $B_c^- \to \pi^- D_2^{*-}$ | $V_{ub} V_{ud} a_1 f_{\pi^-} F_{B_c \to D_2^{*-}} (m_{\pi^-}^2)$ | $1.79 \times 10^{-9}$ |
| $B_c^- \to \pi^0 D_s^{*-}$ | $V_{ub} V_{ud} a_2 f_{\pi^0} F_{B_c \to D_s^{*-}} (m_{\pi^0}^2)/\sqrt{2}$ | $5.81 \times 10^{-11}$ |
| $B_c^- \to \eta D_2^{*-}$ | $V_{ub} V_{ud} a_2 f_{\eta} F_{B_c \to D_2^{*-}} (m_{\eta}^2)/\sqrt{2}$ | $7.46 \times 10^{-12}$ |
| $B_c^- \to \eta' D_2^{*-}$ | $V_{ub} V_{ud} a_2 f_{\eta'} F_{B_c \to D_2^{*-}} (m_{\eta'}^2)/\sqrt{2}$ | $5.4 \times 10^{-11}$ |
| $B_c^- \to D_s^- D_s^{*-}$ | $V_{ub} V_{cs} a_1 f_{D_s^-} F_{B_c \to D_s^{*-}} (m_{D_s}^2)$ | $2.18 \times 10^{-8}$ |
| $B_c^- \to \overline{D}^0 D_s^{*-}$ | $V_{ub} V_{cs} a_2 f_{\overline{D}^0} F_{B_c \to \overline{D}^0 D_s^{*-}} (m_{\overline{D}^0}^2)$ | $5.66 \times 10^{-9}$ |
| $B_c^- \to K^- \chi_{c2}$ | $V_{cb} V_{us} a_1 f_{K^-} F_{B_c \to \chi_{c2}} (m_{K^-}^2)$ | $5.49 \times 10^{-6} (1.78 \times 10^{-6})$ |
| $B_c^- \to D^0 D_s^{*-}$ | $V_{cb} V_{us} a_2 f_{D^0} F_{B_c \to D_s^{*-}} (m_{D_s}^2)$ | $1.94 \times 10^{-8}$ |
| $B_c^- \to D^- \chi_{c2}$ | $V_{cb} V_{ud} a_1 f_{D^-} F_{B_c \to \chi_{c2}} (m_{D^-}^2)$ | $7.56 \times 10^{-6} (1.86 \times 10^{-5})$ |
| $B_c^- \to \eta_c D_s^{*-}$ | $V_{cb} V_{cs} a_2 f_{\eta_c} F_{B_c \to D_s^{*-}} (m_{\eta_c}^2)$ | $1.92 \times 10^{-8}$ |
| $B_c^- \to K^- D_2^{*-}$ | $V_{ub} V_{us} a_1 f_{K^-} F_{B_c \to D_2^{*-}} (m_{K^-}^2)$ | $1.43 \times 10^{-10}$ |
| $B_c^- \to \pi^0 D_s^{*-}$ | $V_{ub} V_{us} a_2 f_{\pi^0} F_{B_c \to D_s^{*-}} (m_{\pi^0}^2)/\sqrt{2}$ | $1.99 \times 10^{-11}$ |
| $B_c^- \to \eta D_s^{*-}$ | $V_{ub} V_{us} a_2 f_{\eta} F_{B_c \to D_s^{*-}} (m_{\eta}^2)/\sqrt{2}$ | $2.51 \times 10^{-12}$ |
| $B_c^- \to \eta' D_s^{*-}$ | $V_{ub} V_{us} a_2 f_{\eta'} F_{B_c \to D_s^{*-}} (m_{\eta'}^2)/\sqrt{2}$ | $1.74 \times 10^{-11}$ |
| $B_c^- \to D^- D_2^{*-}$ | $V_{ub} V_{cd} a_1 f_{D^-} F_{B_c \to D_2^{*-}} (m_{D^-}^2)$ | $8.57 \times 10^{-10}$ |
| $B_c^- \to \overline{D}^0 D_2^{*-}$ | $V_{ub} V_{cd} a_2 f_{\overline{D}^0} F_{B_c \to \overline{D}^0 D_2^{*-}} (m_{\overline{D}^0}^2)$ | $5.6 \times 10^{-11}$ |

Table 1. Decay amplitudes and branching ratios for the c-spectator $B_c \to PT$ decays (the amplitudes must be multiplied by $(iG_F/\sqrt{2})\varepsilon^{\mu}_{\nu} p_{B_c}^\mu p_{B_c}^\nu$). The values within parenthesis in the third column are taken from the Ref. [16].
| Process | Amplitude | $B(B_c \rightarrow VT)$ |
|---------|-----------|-------------------------|
| $B_c^- \rightarrow \rho^-\chi_{c2}$ | $V_{cb}V_{ub}^*a_1f_\rho m_{\rho}^2 F_{\rho \mu \nu}^{B_c \rightarrow \chi_{c2}}(m_{\rho}^2)$ | $2.38 \times 10^{-4}$ $(5.18 \times 10^{-4})$ |
| $B_c^- \rightarrow D^0 D_{s}^{-}\chi_{c2}$ | $V_{cb}V_{ud}^*a_2 f_{D^0} m_{D^0}^2 F_{D^0 \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{D^0}^2)$ | $3.42 \times 10^{-7}$ |
| $B_c^- \rightarrow D_s^{-}\chi_{c2}$ | $V_{cb}V_{cs}^*a_1 f_{D_s^{-}} m_{D_s^{-}}^2 F_{D_s^{-} \mu \nu}^{B_c \rightarrow \chi_{c2}}(m_{D_s^{-}}^2)$ | $5.25 \times 10^{-4}$ $(2.4 \times 10^{-3})$ |
| $B_c^- \rightarrow J/\psi D_{s}^{-}\chi_{c2}$ | $V_{cb}V_{cs}^*a_2 f_{J/\psi} m_{J/\psi}^2 F_{J/\psi \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{J/\psi}^2)$ | $2.06 \times 10^{-5}$ |
| $B_c^- \rightarrow \rho^{-}D_{s}^{0}$ | $V_{ub}V_{ud}^*a_1 f_\rho m_{\rho}^2 F_{\rho \mu \nu}^{B_c \rightarrow D_s^{0}}(m_{\rho}^2)$ | $7.43 \times 10^{-9}$ |
| $B_c^- \rightarrow \rho^{0}D_{s}^{-}$ | $V_{ub}V_{ud}^*a_2 f_\rho m_{\rho}^2 F_{\rho \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{\rho}^2)/\sqrt{2}$ | $2.44 \times 10^{-10}$ |
| $B_c^- \rightarrow \omega D_{s}^{-}$ | $V_{ub}V_{ud}^*a_2 f_\omega m_{\omega}^2 \cos \phi_F F_{\omega \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{\omega}^2)/\sqrt{2}$ | $1.21 \times 10^{-10}$ |
| $B_c^- \rightarrow \phi D_{s}^{-}$ | $V_{ub}V_{ud}^*a_2 f_\phi m_{\phi}^2 \sin \phi_F F_{\phi \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{\phi}^2)/\sqrt{2}$ | $1.55 \times 10^{-10}$ |
| $B_c^- \rightarrow D^{*-} D_{s}^{0}$ | $V_{ub}V_{cs}^*a_1 f_{D^{*-}} m_{D^{*-}}^2 F_{D^{*-} \mu \nu}^{B_c \rightarrow D_s^{0}}(m_{D^{*-}}^2)$ | $8.72 \times 10^{-8}$ |
| $B_c^- \rightarrow D^{*-} D_{s}^{0}$ | $V_{ub}V_{cs}^*a_2 f_{D^{*-}} m_{D^{*-}}^2 F_{D^{*-} \mu \nu}^{B_c \rightarrow D_s^{0}}(m_{D^{*-}}^2)$ | $1.89 \times 10^{-8}$ |
| $B_c^- \rightarrow K^{*-}\chi_{c2}$ | $V_{cb}V_{us}^*a_1 f_{K^{*-}} m_{K^{*-}}^2 F_{K^{*-} \mu \nu}^{B_c \rightarrow \chi_{c2}}(m_{K^{*-}}^2)$ | $1.33 \times 10^{-5}$ $(3.12 \times 10^{-6})$ |
| $B_c^- \rightarrow D^{0} D_{s}^{-}$ | $V_{cb}V_{us}^*a_2 f_{D^{0}} m_{D^{0}}^2 F_{D^{0} \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{D^{0}}^2)$ | $7.63 \times 10^{-8}$ |
| $B_c^- \rightarrow D^{*-} D_{s}^{0}$ | $V_{cb}V_{us}^*a_2 f_{D^{*-}} m_{D^{*-}}^2 F_{D^{*-} \mu \nu}^{B_c \rightarrow D_s^{0}}(m_{D^{*-}}^2)$ | $2.42 \times 10^{-5}$ $(8.66 \times 10^{-5})$ |
| $B_c^- \rightarrow J/\psi D_{s}^{-}$ | $V_{cb}V_{cs}^*a_2 f_{J/\psi} m_{J/\psi}^2 F_{J/\psi \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{J/\psi}^2)$ | $4.22 \times 10^{-7}$ |
| $B_c^- \rightarrow K^{*} D_{s}^{-}$ | $V_{ub}V_{us}^*a_1 f_{K^{*}} m_{K^{*}}^2 F_{K^{*} \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{K^{*}}^2)$ | $4.52 \times 10^{-10}$ |
| $B_c^- \rightarrow \rho^{0} D_{s}^{-}$ | $V_{ub}V_{us}^*a_2 f_{\rho} m_{\rho}^2 F_{\rho \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{\rho}^2)$ | $7.91 \times 10^{-11}$ |
| $B_c^- \rightarrow \omega D_{s}^{-}$ | $V_{ub}V_{us}^*a_2 f_{\omega} m_{\omega}^2 \cos \phi_F F_{\omega \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{\omega}^2)/\sqrt{2}$ | $3.94 \times 10^{-11}$ |
| $B_c^- \rightarrow \phi D_{s}^{-}$ | $V_{ub}V_{us}^*a_2 f_{\phi} m_{\phi}^2 \sin \phi_F F_{\phi \mu \nu}^{B_c \rightarrow D_s^{-}}(m_{\phi}^2)/\sqrt{2}$ | $4.81 \times 10^{-11}$ |
| $B_c^- \rightarrow D^{*-} D_{s}^{0}$ | $V_{ub}V_{cs}^*a_1 f_{D^{*-}} m_{D^{*-}}^2 F_{D^{*-} \mu \nu}^{B_c \rightarrow D_s^{0}}(m_{D^{*-}}^2)$ | $3.26 \times 10^{-9}$ |
| $B_c^- \rightarrow \rho^{0} D_{s}^{-}$ | $V_{ub}V_{cs}^*a_2 f_{\rho} m_{\rho}^2 F_{\rho \mu \nu}^{B_c \rightarrow D_s^{0}}(m_{\rho}^2)$ | $2.12 \times 10^{-10}$ |

Table 2. Decay amplitudes and branching ratios for the c-spectator $B_c \rightarrow VT$ decays (the amplitudes must be multiplied by $(G_F/\sqrt{2})\varepsilon_{\mu\nu}^*$. The values within parenthesis in the third column are taken from the Ref. [10].
Table 3. Decay amplitudes and branching ratios for the $b$-spectator $B_c \to PT$ decays. The amplitudes must be multiplied by $\left(iG_F/\sqrt{2}\right)\epsilon^*_\mu\epsilon^*_\nu p_{B_c}^\mu p_{B_c}^\nu$.

| Process                | Amplitude                                                                 | $B(B_c \to PT)$ |
|------------------------|---------------------------------------------------------------------------|-----------------|
| $B_c^+ \to \pi^+ B_{s2}^{*0}$ | $V_{cs}V_{ud}a_1 f_{\pi^+} F_{B_c \to B_{s2}^{*0}}(m_{\pi^+}^2)$          | $2.01 \times 10^{-4}$ |
| $B_c^+ \to K^0 B_{2}^{*+}$     | $V_{cs}V_{ud}a_2 f_{K^0} F_{B_c \to B_{2}^{*+}}(m_{K^0}^2)$              | $4.22 \times 10^{-6}$ |
| $B_c^+ \to \pi^0 B_{2}^{*0}$     | $V_{cd}V_{ud}a_2 f_{\pi^0} F_{B_c \to B_{2}^{*0}}(m_{\pi^0}^2)$          | $1.18 \times 10^{-5}$ |
| $B_c^+ \to \pi^0 B_{2}^{*+}$     | $V_{cd}V_{ud}a_2 f_{\pi^0} F_{B_c \to B_{2}^{*+}}(m_{\pi^0}^2)/\sqrt{2}$| $3.86 \times 10^{-7}$ |
| $B_c^+ \to K^+ B_{s2}^{*0}$     | $V_{cs}V_{us}a_1 f_{K^+} F_{B_c \to B_{s2}^{*0}}(m_{K^+}^2)$            | $5.03 \times 10^{-7}$ |
| $B_c^+ \to \eta B_{2}^{*+}$     | $-\cos \phi_F V_{cs}V_{us}a_2 f_{\eta} F_{B_c \to B_{2}^{*+}}(m_{\eta}^2)$ | $6.46 \times 10^{-8}$ |
| $B_c^+ \to K^+ B_{2}^{*0}$     | $V_{cd}V_{us}a_1 f_{K^+} F_{B_c \to B_{2}^{*0}}(m_{K^+}^2)$            | $1.90 \times 10^{-7}$ |
| $B_c^+ \to K^0 B_{2}^{*+}$     | $V_{cd}V_{us}a_2 f_{K^0} F_{B_c \to B_{2}^{*+}}(m_{K^0}^2)$            | $1.19 \times 10^{-8}$ |

Table 4. Relation between the tree level and annihilation contributions to $B_c^- \to PT$. The values in the second column must be multiplied by $\left[F_{B_c \to T}(m_P^2)/F_{PT}(m_{B_c}^2)\right]^2$.

| $B_c \to PT$ | $R = \Gamma_t/\Gamma_a$ |
|--------------|-------------------------|
| $B_c^- \to K^- D_{2}^{*0}$ | $3.8 \times 10^{-5}$ |
| $B_c^- \to \eta' D_{s2}^{*+}$ | $1.01 \times 10^{-6}$ |
| $B_c^- \to \pi^- D_{2}^{*0}$ | $9.8 \times 10^{-3}$ |
| $B_c^- \to \pi^0 D_{2}^{*0}$ | $3.24 \times 10^{-4}$ |