Narrowband frequency tunable light source of continuous quadrature entanglement

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We report the observation of non-classical quantum correlations of continuous light variables from a novel type of source. It is a frequency non-degenerate optical parametric oscillator below threshold, where signal and idler fields are separated by 740MHz corresponding to two free spectrum ranges of the parametric oscillator cavity. The degree of entanglement observed, -3.8 dB, is the highest to date for a narrowband tunable source suitable for atomic quantum memory and other applications in atomic physics. Finally we use the latter to visualize the Einstein-Podolsky-Rosen paradox.

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I. INTRODUCTION

Quantum interface between light and atomic systems requires narrowband frequency tunable sources of light possessing various non-classical features. Examples of sources which are in demand include narrowband tunable single photon sources and narrowband tunable sources of Einstein-Podolsky-Rosen entangled light. Both types of sources can be constructed using a non-degenerate optical parametric oscillator below threshold (ND-OPO). However, the vast majority of current experiments with entangled light are carried out with single pass downconversion sources with their inherent low spectral brightness. The first EPR-type experiment using a narrowband light source \cite{5,3} was also the first demonstration of entanglement for continuous variables. Recently another experiment demonstrating the EPR state for solitons has been carried out \cite{3} although in this experiment the bandwidth of the correlated fields was very wide. The first demonstration of a frequency tunable source of continuous light variables entanglement between the modes at 852 nm and 917 nm was reported in \cite{4}. A potentially tunable source of entangled beams at the same wavelength but in different spatial modes was used in the continuous variables teleportation experiment \cite{6}. In the latter experiment a degenerate OPO pumped in two opposite directions was utilized to generate two squeezed beams which were mixed on a beamsplitter to produce two entangled beams with the degree of entanglement around -1 dB. In the present paper we demonstrate the entanglement of a new type. Namely two symmetric longitudinal modes of the OPO cavity which are shown to possess the quadrature phase entanglement as suggested in the original paper by M. Reid \cite{6} where entanglement of the OPO output has been discussed for the first time.

\[ [\hat{a}_n(t), \hat{a}_m^\dagger(t)] = \delta_{nm}, \quad \{n, m\} = \{+, -\} \]  

A. Dynamics

The starting point for our treatment of the ND-OPO is the Hamiltonian

\[ H = H_{\text{free}} + H_{\text{int}} + H_{\text{res}} \]  

where free, int and res describes the free, noninteracting OPO cavity fields, the nonlinear interaction between the fields and the coupling of the fields to the external reservoir of vacuum modes respectively. The first two Hamiltonians take the form

\[ H_{\text{free}} = \hbar \omega_+ \hat{a}_+^\dagger \hat{a}_+ + \hbar \omega_- \hat{a}_-^\dagger \hat{a}_- \]  

and

\[ H_{\text{int}} = i\hbar g \left( \hat{a} e^{i\chi} \hat{a}_+^\dagger \hat{a}_- - \hat{a}^* e^{-i\chi} \hat{a}_+ \hat{a}_- \right) \]
where $g$ is the coupling strength, $\alpha = \alpha \exp(-2i\omega t)$ is the classical pump field and $\chi$ is the pump field phase. From (1), (2) and (3) we can derive the Heisenberg-Langevin equations of motion for the cavity fields

$$
\frac{d}{dt} \tilde{a}_\pm = -i\omega_\pm \tilde{a}_\pm + g\tilde{e}^{\pm i\gamma_\pm} \tilde{q}_\pm + \sqrt{2\kappa_\pm} \tilde{a}_\pm + 2(\gamma_\pm - \kappa_\pm) \tilde{b}_\pm^m \tag{5}
$$

where the $\tilde{a}_\pm^m$ fields describe the vacuum fluctuations leaking into the resonator through the main coupler and the $\tilde{b}_\pm^m$ fields enter the ND-OPO via residual leakage. $\kappa_\pm$ define the leak rate through the main coupler at frequencies $\omega_\pm$ and $\gamma_\pm$ define the overall leak rates. Our observables of interest are the slowly varying envelopes of the field quadratures

$$
q_\pm(\theta) = \tilde{a}_\pm e^{i(\omega t - \theta)} + \tilde{a}_\pm^† e^{-i(\omega t - \theta)} \tag{6}
$$

The pump field we parameterize in terms of its threshold value $\alpha = \varepsilon \alpha_{th}$, where $\alpha_{th} = \sqrt{\gamma^+ - \gamma^-}/g$. This results in the equations of motion for the subharmonic quadrature operators

$$
\frac{d}{dt} q_\pm(\theta) = \varepsilon \sqrt{\gamma^+ - \gamma^-} q_\pm(\theta - \gamma_\pm q_\pm(\theta)) + \sqrt{2\kappa_\pm} q_\pm^m(\theta) + 2(\gamma_\pm - \kappa_\pm)p_\pm^m(\theta) \tag{7}
$$

Here we have defined the slowly varying vacuum field quadratures $q_\pm^m(\theta) = \tilde{a}_\pm^m e^{i(\omega t - \theta)} + (\tilde{a}_\pm^m)^† e^{-i(\omega t - \theta)}$ and $p_\pm^m(\theta) = \tilde{b}_\pm^m e^{i(\omega t - \theta)} + (\tilde{b}_\pm^m)^† e^{-i(\omega t - \theta)}$. The phase dependence of $q_\pm^m$ and $p_\pm^m$ will be left out in the following since no phase can be assigned to the vacuum state and hence the properties of the ‘in’ quadrature phase operators is independent of the choice of $\theta$.

From (7) we see that only the quadratures with phases $\phi_\pm = \chi/2 \pm \theta$ will interact. Hence we find that the ND-OPO intracavity fields are described by a continuum of pairwise coupled equations of motion of the form

$$
\frac{d}{dt} q_\pm(\phi_\pm) = \varepsilon \sqrt{\gamma^+ - \gamma^-} q_\pm(\phi_\pm - \gamma_\pm q_\pm(\phi_\pm)) + \sqrt{2\kappa_\pm} q_\pm^m(\phi_\pm) + 2(\gamma_\pm - \kappa_\pm)p_\pm^m(\phi_\pm) \tag{8}
$$

Before moving on to solve (8) we make a few observations about the quadrature phase operators. Using (7) we find that these operators obey the following commutation relations

$$
[g_n(\phi), q_m(\phi')] = 2i \sin(\phi' - \phi) \delta_{nm}, \{m, n\} = \{+, -\} \tag{9}
$$

meaning that the combined signal/idler quadratures describing cross correlations of the ND-OPO output commute as

$$
[q_+(\phi_+) + q_-(\phi_-), q_+(\phi_+ + \psi) + q_-(\phi_- + \psi)] = [q_+(\phi_+) - q_-(\phi_-), q_+(\phi_+ + \psi) - q_-(\phi_- + \psi)] = 4i \sin \psi \tag{10}
$$

Hence the combinations $\{q_+(\phi_+) + q_-(\phi_-), q_+(\phi_+ + \pi/2) + q_-(\phi_- + \pi/2)\}$ and $\{q_+(\phi_+) - q_-(\phi_-), q_+(\phi_+ + \pi/2) - q_-(\phi_- + \pi/2)\}$ form conjugate observables. As we will see below the fluctuations of the quadrature difference $q_+(\phi_+) - q_-(\phi_-)$ can become very small in the ND-OPO output as compared to a mode coherent state output. This means that $q_+(\phi_+ + \pi/2) + q_-(\phi_- + \pi/2) = q_+(\phi_+') - q_-(\phi_-')$, with $\phi_+ = \chi/2 \pm (\theta + \pi/2)$, also can become a quiet observable since $\phi_\pm$ is defined for any arbitrary value of $\theta$. From the commutation relation (10) we then conclude that $q_+(\phi_+) + q_-(\phi_-)$ and $q_+(\phi_+ + \pi/2) - q_-(\phi_- + \pi/2)$ must become correspondingly noisy observables. This is illustrated in the phasor diagram of Fig. 4. In the parametric down conversion the phase is conserved, meaning that the signal and idler fields are created symmetrically around the axis making an angle of $\chi/2$ with the abscissa in phase space. This axis illustrates the pump phase. In the rotating frame oscillating at $\omega_p/2$ the signal field will rotate clockwise and the idler will rotate counterclockwise in such a way that their sum always will be parallel to the pump axis. Hence we find that the difference of projections of signal/idler fields onto the pump axis will tend to cancel on average. This difference is described by the cross correlation of quadratures $q_+(\phi_+) - q_-(\phi_-)$. Also the sum of projections onto the axis perpendicular to the pump axis will cancel. This observable is described by $q_+(\phi_+ + \pi/2) + q_-(\phi_- + \pi/2)$.

\section{B. Quantum fluctuations}

The solutions of (8) are found by Fourier transforming and solving the resulting algebraic equations

$$
Dq = \mathcal{A}q^m + \mathcal{B}p^m \tag{11}
$$

where we have defined

$$
D = \begin{pmatrix}
\gamma_+ - i\Omega & -\varepsilon \sqrt{\gamma^+ + \gamma^-} \\
-\varepsilon \sqrt{\gamma^+ + \gamma^-} & \gamma_+ - i\Omega
\end{pmatrix},
\mathcal{A} = \begin{pmatrix}
\sqrt{2\kappa_+} & 0 \\
0 & \sqrt{2\kappa_-}\end{pmatrix},
\mathcal{B} = \begin{pmatrix}
\sqrt{2(\gamma_+ - \kappa_+)} & 0 \\
0 & \sqrt{2(\gamma_- - \kappa_-)}
\end{pmatrix} \tag{12}
$$

\end{document}
FIG. 1: The parametric downconversion seen in a phasor diagram. In a frame rotating at the degeneracy frequency \( \Omega \) is rotating clockwise and \( a_- \) is rotating counterclockwise with relative phases such that the two phasors always add up to be parallel to the dashed line.

and

\[
\hat{q} = \begin{pmatrix} q_+ \phi_+ \\ q_- \phi_- \end{pmatrix}, \quad \hat{q}^n = \begin{pmatrix} q^+_n \\ q^-_n \end{pmatrix}, \quad \hat{p}^n = \begin{pmatrix} p^+_n \\ p^-_n \end{pmatrix}
\]

(13)

now are the Fourier transformed quadrature phase operators. From the solution of (11) we can find the fields escaping the ND-OPO through the main coupler via the boundary condition on the coupler

\[
\hat{q}^{out} = A\hat{q} - T \hat{q}^n
\]

(14)

These are given by

\[
\hat{q}^{out} = [AD^{-1}A - \mathcal{I}] \hat{q}^n + AD^{-1}B\hat{p}^n
\]

(15)

where \( \mathcal{I} \) is the 2×2 unit matrix. To describe the non-classical correlations between the out fields in (13), we define the general linear combinations of the signal and idler quadratures

\[
\hat{Q} = \begin{pmatrix} Q^+ \\ Q^- \end{pmatrix} = U\hat{q}, \quad U = \begin{pmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{pmatrix}
\]

(16)

Hence we find the outgoing linear combinations as

\[
\hat{Q}^{out} = U [AD^{-1}A - \mathcal{I}] U^T \hat{Q}^n + UAD^{-1}B\hat{p}^n
\]

(17)

where \( \hat{Q}^n = U\hat{q}^n \) and \( \hat{p}^n = U\hat{p}^n \). For the vacuum input fields we define the standard quantum limit (SQL) to be:

\[
\langle Q^\pm_n(-\Omega)Q^{\pm*}_n(\Omega) \rangle = \langle P^\pm_n(-\Omega)P^{\pm*}_n(\Omega) \rangle = \lambda(\psi)
\]

(18)

and all other correlations functions of the in fields are vanishing for a vacuum state input. Clearly \( \lambda \) must depend on our choice of \( \psi \) since for \( \psi = 0 \) \( Q^{out} \) describes the fluctuations of the individual signal and idler fields of the ND-OPO, whereas for \( \psi \neq 0 \) \( Q^{out} \) describes inter-beam correlations. As we shall see below in the case of symmetrical signal/idler losses and perfect inter-beam correlations we measure the strongest inter-beam correlations when choosing \( \psi = \pi/4 \). Hence we define \( \lambda(\psi = 0) = 1 \) and \( \lambda(\psi \neq 0) = 2 \), since in the latter case we must compare our inter-beam-correlations to a SQL given by the correlations between two independent coherent state fields. It is worth noting here that only very close to the ND-OPO threshold are the inter-beam quantum correlations perfect and the noise of the signal and idler can cancel each other exactly. However the single beam noise grows rapidly as the ND-OPO pump parameter \( \varepsilon \) grows. Hence due to imperfect quantum correlations between signal and idler fields it turns out that since this growing noise cannot be canceled, it is more advantageous to measure not the symmetric combination of the quadratures described by \( \psi = \pi/4 \), but rather a somewhat scaled combination with \( \psi = \pi/4 + \delta(\varepsilon) \).

The correlations of the outgoing quadratures \( \langle (Q^{out\pm}_n)^2 \rangle \) are found from (17) and (18) to be

\[
\langle (Q^{out\pm}_n)^2 \rangle = \lambda \left\{ 1 + 4\varepsilon\eta \left[ \frac{\sigma \cos^2 \psi}{\sin^2 \psi} + \frac{\sin^2 \psi}{\cos^2 \psi} \right] + \frac{(\Delta^2 + 1 + \varepsilon^2) \sin 2\psi \left( \begin{array}{c} 1 \\ -1 \end{array} \right)}{\Delta^2 + (E + \Lambda)^2} \right\}
\]

(19)

Here we have defined the normalized fluctuation frequency \( \Delta = \Omega/\sqrt{\gamma_+\gamma_-} \), the signal/idler escape efficiencies \( \eta_\pm = \kappa_\pm/\gamma_\pm \), the generalized OPO escape efficiency for the signal and idler fields \( \eta = \sqrt{\eta_+\eta_-} \), the escape efficiency asymmetry \( \sigma = \sqrt{\eta_+/\eta_-} \), the loss asymmetry function \( \Lambda = (\rho + \rho^{-1})/2 \), with \( \rho = \sqrt{\gamma_+/\gamma_-} \) and the effective pump parameter \( E = \varepsilon^2 + \Lambda^2 - 1 \). From (19) we find for \( \psi = 0 \) the single beam noise spectra

\[
\langle (q^{out\pm}_n)^2 \rangle = 1 + \eta_\pm \frac{8\varepsilon^2}{[\Delta^2 + (E + \Lambda)^2]|\Delta^2 + (E - \Lambda)^2|}
\]

(20)
These are independent on \( \theta \), meaning that the individual signal and idler fields contain phase insensitive excess noise relative to the standard quantum limit here being 1. As the threshold is approached \( E \rightarrow \Lambda \), and we find the noise to diverge at zero detuning. Naturally the linearized approach taken here breaks down in this limit, where we must take pump depletion into account as the ND-OPO gain becomes large and much energy is transferred from the pump to the fluctuations of the signal/idler fields.

By setting \( \psi = \pi/4 \) we find from (19) the inter-beam correlations

\[
\langle \langle Q_{\pm}^{\text{out}} \rangle^2 \rangle_{\pi/4} = 2 \left\{ 1 \pm 4\varepsilon \eta \frac{\Delta^2 + 1 + \varepsilon^2 \pm (\sigma + \sigma^{-1})\varepsilon}{(\sigma^2 + (E + \Lambda)^2)(\Delta^2 + (E - \Lambda)^2)} \right\}
\]

(21)

It is easy to check that in the case of symmetric losses \( \rho = \sigma = 1 \), \( \Lambda = 1 \) and \( E = \varepsilon \), we recover from (21) the well known result for the cross correlations [7]

\[
\langle \langle Q_{\pm}^{\text{out}} \rangle^2 \rangle_{\pi/4} = 2 \left\{ 1 \pm \eta \frac{4\varepsilon}{\Delta^2 + (\varepsilon + 1)^2} \right\}
\]

(22)

From (22) we find perfect inter-beam correlations at low frequencies as the threshold is approached \( \varepsilon \rightarrow 1 \), which reflects that \( \psi = \pi/4 \) corresponds to the natural signal/idler linear combination to measure in the case of symmetric losses for the two modes.

When allowing for non-equal losses of signal and idler modes, we see from Fig. 3 that a low frequency noise spike appears in the spectrum of fluctuations when \( \gamma_+ \) is increased relatively to \( \gamma_- \). With \( \rho = 3 \) we find that the inter-beam correlations are virtually lost. This is because the simple sums and differences of signal idler fields, corresponding to choosing \( \psi = \pi/4 \), no longer are the best combinations to measure in order to observe strong interbeam correlations. As we shall see by optimizing \( \psi \) we can recover at least some of the inter beam correlations around a given frequency.

A remark should be made about losses experienced by the signal and idler fields during propagation from the ND-OPO to the detectors. Any such losses including non-perfect detectors, limited local oscillator overlaps etc. for the signal/idler fields will reduce the quantum correlations on equal footing with non-unity escape efficiencies. Hence we define overall propagation efficiencies for the two field modes describing the survival probability for photons in these modes from birth in the \( \chi^{(2)} \) process until detection. We denote these efficiencies \( \xi_{\pm} \) for the two modes and for later reference we define them here as

\[
\xi_{\pm} = \eta_{\pm}^{\text{prop}} \eta_{\pm}^{\text{LO}} \eta_{\pm}^{\text{QE}}
\]

(23)

Here we have defined the signal/idler propagation efficiencies \( \eta_{\pm}^{\text{prop}} \), the overlap with the local oscillators of the heterodyne quadrature detections \( \eta_{\pm}^{\text{LO}} \) and the quantum efficiency of the detectors \( \eta_{\pm}^{\text{QE}} \).

In the realistic case of lossy propagation and non-perfect detection we should replace the signal/idler escape efficiencies in (19) with the overall efficiencies defined in (23).

As mentioned above we can optimize the inter-beam correlations at a given set of parameters by choosing to measure the optimized linear combination described by \( \psi = \psi_0 \), where \( \psi_0 \) is found from

\[
\tan 2\psi_0 = \frac{\Delta^2 + 1 + \varepsilon^2}{(\sigma - \sigma^{-1})\varepsilon}
\]

(24)

Here \( \Delta_0 \) is the frequency at which we wish to minimize fluctuations. From (24) it is again clear that in the symmetric limit where \( \sigma = 1 \) there is no advantage of deviating from measuring equally weighted \( q_{\pm}^{\text{opt}}(\phi_+) \) and

FIG. 2: Correlation spectra for \( \kappa_+ = \kappa_- \) and increasing loss asymmetry.

FIG. 3: Correlation spectra with increasing \( \gamma_+ \). Here \( \psi_0 \) has been used.
classical coherent field with frequency ω_+ but also classical fields. This means that if we inject a classical coherent field with frequency ω_+ and phase φ_+ into our ND-OPO, the nonlinear interaction will result in generation of an idler field of frequency ω_- and phase φ_- such that φ_+ + φ_- = χ, where χ is the pump field phase. This was utilized in the experiment to gain information about the pump phase of the ND-OPO.

q_{out}(φ_-) observables corresponding to ψ_0 = π/4. But in general we find a large variation of ψ_0 with σ for small values of Δ_0, hence there may be a big advantage of optimizing ψ when signal/idler losses become non-symmetric. As Δ_0 is increased the variation of ψ_0 gets smaller since at higher frequencies the impact of measuring unevenly weighted signal/idler amplitudes gets negligible.

By using ψ = ψ_0 in (19) and again varying γ_+ relative to γ_−, we find the inter beam correlation spectra shown in Fig. 4. These have been optimized at Δ_0 = 0. Here we see that the correlations at low frequencies persist despite the growing asymmetry of the losses, however since the escape efficiency η_+ decreases with increasing γ_+, we find the overall degree of inter-beam correlations to diminish when ρ deviates from 1. The importance of measuring scaled signal and idler fields is seen more clearly from Fig. 4 which corresponds to the somewhat unrealistic situation when ρ is increased, but σ and η are left unchanged. Still keeping ψ optimized we find no loss of inter beam correlations in an increasingly small frequency window around Δ = 0. Optimizing ψ around a nonzero frequency is not possible without a reduction of correlations and in general the gain in correlations by optimizing ψ is marginal when Δ_0 > 1.

As a conclusion to this section we remark that not only the quantum fields will show the behavior depicted in Fig. 4, but also classical fields. This means that if we inject a classical coherent field with frequency ω_+ and phase φ_+ into our ND-OPO, the nonlinear interaction will result in generation of an idler field of frequency ω_- and phase φ_- such that φ_+ + φ_- = χ, where χ is the pump field phase. This was utilized in the experiment to gain information about the pump phase of the ND-OPO.

III. EXPERIMENTAL DETAILS

We will now consider the experiment carried out to characterize the frequency ND-OPO described in the previous section. This was divided into three parts: First we simply measured the squeezing in the degenerate mode of the OPO. Then we investigated the signal/idler cross correlations of the ND-OPO output and finally we recorded the signal/idler phases to confirm the relationships derived above and to visualize the EPR paradox for our kind of quantum mechanically entangled light. But before considering the results we will just outline here some of the underlying experimental setup.

This was made up of a single ended optical ring resonator with a 10mm long Potassium Niobate crystal located in the smallest waist. Potassium Niobate is famous for its high (2) nonlinearity which can be up to 8 pm/volt. However it is equally notorious due to its nonlinear losses of which Blue Light Induced InfraRed Absorption (BLIIRA) is the most harmful (8, 9). In this experiment the nonlinear crystal had a pass nonlinearity of 0.015 ± 0.002 W⁻¹ measured by second harmonic generation. The ND-OPO output coupler had a power transmission of 0.122 around the signal/idler wavelength of 860 nm. Residual losses in the ND-OPO were 0.007 ± 0.001 excluding the BLIIRA. The cavity length was 81 cm resulting in a free spectrum range of 370 MHz. With a typical value of BLIIRA of 0.015 at 160 mW blue pump power we find the ND-OPO cavity bandwidth of 8.5 MHz FWHM.

The ND-OPO pump was produced by performing external cavity second harmonic generation (SHG) of a Titanium-Sapphire (Ti:S) laser operating at 860nm. This was done in a single ended ring resonator also with a 10 mm Potassium Niobate crystal placed in the waist. The cavity length was 70 cm long and the input coupler transmitted 0.078 at 860 nm. Residual internal losses were at the level of 0.009 again excluding BLIIRA. The cavity waist was 20 µm, close to the optimum for nonlinear conversion of 16 µm (10). However since the resonator was pumped with up to 800 mW of light from the Ti:S a slightly larger waist was chosen to compensate thermal lensing in the nonlinear medium. From the SHG we got up to 300 mW of blue light (430 nm) for pumping the OPO. The mode matching of the SHG Gaussian mode to the ND-OPO mode was typically 0.95 measured by the reference resonator shown in the setup on Fig. 2.

The signal and idler fields emerging from the ND-OPO to be analyzed were separated in oscillation frequency by two free spectrum ranges, that is 740 MHz. In order to separate the fields spatially we sent them onto a two ended ring resonator with a length of 20 cm and coupler transmissions of 0.078 at 860 nm. The length of this filtering cavity was stabilized to maintain resonance with ω_+, such that this frequency component was transmitted. The cavity linewidth was 38 MHz and the free spectrum range 1.5 GHz, meaning that less than 10⁻³ of the idler field was transmitted when the filtering cavity was reso-
A hybrid junction producing the sum and difference of fields. The two differential photocurrents were led into measurement of the quadrature phases of the signal/idler on four Silicon pin diodes and the photocurrents from detectors. The outputs from the beam splitters were detected with the ND-OPO outputs on separate 50/50 beam splitters to ensure a clean TEM mode and finally interfered 00 of the up-shifted light (ω_+). Mode matching and internal losses of the filtering cavity limited the transmission of the signal field to 0.95. This cavity as well as the SHG and OPO cavities were locked using the Pound-Drever-Hall FM technique. The mode cleaning cavities were locked to give a constant transmission at roughly 80% of their maximum transmission.

From the filtering cavity the signal and idler fields propagated in different directions each to be heterodyned with a frequency degenerate strong local oscillator and detected on a pair of detectors. The local oscillators were produced by sending part of the Ti:S output double pass through a pair of acousto optical modulators AOM(±). For the role of the miscellaneous cavities see text. The final photocurrents were Fourier transformed and recorded in the RF spectrum analyzers SA(±).

FIG. 5: The experimental setup. Long dashed lines are infrared beams (ω_+) and ω, short dashed lines are blue beams (2ω) and full lines are electrical cables. Frequency shifted local oscillators for the ω_+ heterodyne detectors were generated with the acousto optical modulators AOM(±). For the role of the miscellaneous cavities see text. The final photocurrents were Fourier transformed and recorded in the RF spectrum analyzers SA(±).

IV. RESULTS

In order to learn about the ability of our OPO to produce non classical quantum correlations we first analyzed the degenerate output mode. Here either increased or reduced quantum fluctuations should be observed depending on the relative phase difference between the heterodyne local oscillator and the OPO output. The phase of fluctuations in the latter was determined by the pump phase. Since the same mechanisms are responsible for squeezing as well as signal/idler cross correlations the squeezing results yielded valuable information about what could be expected when observing the nondegenerate OPO modes. Next employing the filtering cavity we analyzed the two nondegenerate modes closest to the degenerate mode. Here the cross correlations between signal and idler fields were measured while we tracked the local oscillator phases. This enabled us to visualize the EPR paradox now with frequency nondegenerate quantum correlated fields as opposed to the polarization nondegenerate fields of Ou et al. We will discuss this in more detail below.

A. Classical gain and squeezing

When observing squeezing we pumped the OPO with 160 mW of 430 nm light from the SHG stage. At this power we observed a BLIIRA level of 0.015±0.002 and by injecting a weak coherent field into the OPO via a high reflector we observed a maximum phase sensitive gain of 6.8. It can be shown that this gain is given by (1−ε)^−2, which gives us ε = 0.62. Taking into account the blue mode matching of 0.95 this gives us an OPO threshold power of 395 mW. From αth putting γ+ = γ− we find the power needed to drive the OPO above threshold to be

\[ P_{th} = \frac{(T + L + L_P)^2}{4E_{NL}} \]
TABLE I: The limitations to the degree of squeezing and their contributions.

| Loss source                          | $\eta$   |
|--------------------------------------|----------|
| OPO escape efficiency, $\eta$        | 0.847    |
| Heterodyning fringe visibility, $\eta^{LO}$ | (0.965)$^2$ |
| Detector quantum efficiency, $\eta^{QE}$ | 0.98    |
| Propagation efficiency, $\eta^{prop}$ | 0.98    |
| Overall detection efficiency, $\xi$  | 0.76 ± 0.02 |

TABLE II: The detection efficiencies for the ND-OPO signal (+) and idler (-) modes.

| Loss source | Signal | Idler |
|-------------|--------|-------|
| $\eta^E$   | 0.847  | 0.847 |
| $\eta^{LO}$ | (0.98)$^2$ | (0.975)$^2$ |
| $\eta^{QE}$ | 0.98   | 0.98  |
| $\eta^{prop}$ | 0.91  | 0.96  |
| $\xi_\pm$  | 0.73 ± 0.02 | 0.76 ± 0.02 |

FIG. 6: The quantum fluctuations of the degenerate OPO mode. The trace was taken with the local oscillator phase scanned. A minimum of -4.0 dB squeezing relative to the SQL was observed.

with our OPO characterized in terms of its ability to produce quadrature phase squeezed light we could now move on to investigate the quantum correlations between the frequency non degenerate OPO cavity modes adjacent to the degenerate mode. Since these modes only were shifted ±370 MHz relative to the degenerate mode it was safe to assume equal output coupler transmissions and internal OPO losses for both modes. However, since the signal and idler fields experienced different optical paths, the detection efficiencies were not equal for the two modes, meaning that $\sigma \neq 1$ in $\xi=\sqrt{\xi_+\xi_-}$, whereas we had $\rho = 1$. The limiting factors on the inter beam correlations are listed in Table I, from which we derive the parameters $\xi = \sqrt{\xi_+\xi_-}$ = 0.74 ± 0.01 and $\sigma = \sqrt{\xi_+}/\xi_- = 0.98 ± 0.01$. By using (24) we find the optimum phase choice $\psi_0 = -\pi/4 + 0.0086$ radians which inserted in (19) gives a predicted maximum degree of inter beam correlations of -4.9 dB. It should be mentioned that the correction to $\psi$ due to asymmetric losses was so small that we chose not to make any at all. This

\[
\langle q^2 \rangle = 1 - \xi \frac{4\epsilon}{\Delta^2 + (1 + \epsilon)^2} \tag{26}
\]

Inserting our parameters in (26) we find an expected degree of squeezing of $-5.2 \pm 0.3$ dB which we believe is in reasonable agreement with the observed value.
also applied to the correction of ψ due to a finite degree of correlations. Hence within our uncertainties we used ψ = π/4 throughout the experiment. The measured correlations are shown in Fig. 8, from which we find the noise reduction relative to two independent coherent state correlations of −3.8 ± 0.1 dB. Correcting again for electronic noise we get −4.3 ± 0.3 dB noise reduction due to the inter beam quantum correlations. Again this is in reasonable agreement with our theoretical predictions. Fig. 8 shows five traces: (a) The SQL obtained by blocking the ND-OPO pump, hence only the vacuum input to the heterodyne detectors was being probed yielding the quantum fluctuations from two uncorrelated vacuum states. (b) The SQL from a single heterodyne detector. This level was found by blocking the local oscillator of the other heterodyne detector as well as the ND-OPO pump. Unblocking now all beams we obtained the trace (c). As documented by the trace, phase fluctuations in our setup were relatively small, so that the local oscillator phases could be maintained constant on a timescale of hundreds of milliseconds in a way that maximum inter-beam quantum correlations were observed. The same procedure was used to obtain the level of maximum anti correlations (d). Next one of the local oscillators phases was allowed to drift freely while the other phase was scanned at a high rate. This resulted in the oscillatory trace (e) showing the transition between observing the correlated ⟨(Q_{+}^{\text{out}})^2⟩ and the anti correlated ⟨(Q_{-}^{\text{out}})^2⟩ defined in (26).

According to the criterion for state separability developed in refs. [14, 15] we must have

\[
\langle (q_+ - q_-)^2 \rangle + \langle (q_+ + \pi/2 - q_- + \pi/2)^2 \rangle < 2
\]

in order to be able to claim state inseparability for the signal and idler fields and hence entanglement. Inserting the measured values of interbeam correlations we get

\[
0.37 + 0.74 < 2
\]

due to a finite degree of entanglement. This was done using a standard dither and lock approach with an error signal derived from the spectrum analyzer output. Now the phase of the injected field was scanned linearly in time and the resulting interferences with the local oscillators were recorded. Before commenting on the results displayed in Fig. 8 we should note that when ϑ′ was varied in the negative direction for the ω− field, the change of ϑ′ would be in the positive direction for the ω+ field due to phase conservation in the difference frequency generation process. Hence we would expect the phase difference between the two interference fringes to be constant when the local oscillator phases were fixed to produce maximum or minimum quantum
correlations between the two heterodyne detectors. If this phase difference was constant it would correspond to having the local oscillator phases adding up to a constant value as illustrated in Fig. 8. We found the phase differences $\theta^L = \theta_+ - \theta_-$ and $\theta^{AC} = \theta_+ - \theta_-$ for the two frequencies in the cases of being locked to maximum correlations and maximum anti correlations respectively. This was done by fitting the somewhat noisy interference fringes to a sinusoidal function and the resulting phase variations as well as their differences are shown on the graphs of Fig. 8. We found $\theta^L - \theta^{L} = 0.53 \pm 0.11$ rad and $\theta^{AC} - \theta^{AC} = -2.52 \pm 0.06$ rad, and we note that $(\theta^L - \theta^{L}) - (\theta^{AC} - \theta^{AC}) = 3.05 \pm 0.13$ rad. Ideally the last result should be $\pi$ since the sum of the local oscillator phases should be shifted by this amount in order observe the change from measuring a quiet to a noisy quadrature combination. As a final remark here we note that our setup was one big Mach-Zender interferometer from which the systematic phaseshift of 0.53 radians seen in $\theta^L - \theta^{L}$ was originating. The procedure described here for establishing the local oscillator phases was used in the following experiment, where the EPR paradox was revived for a short while.

FIG. 8: The DC interference fringes revealing the relationship between the local oscillator phases and the ND-OPO pump field. The left column corresponds to the situation when the local oscillator phases were locked to produce maximum anti correlations in $(Q^{\pm})_{\psi}$, while the right column are the results when maximum correlations were observed. The upper row are the raw data and the lower row are the local oscillator phases and their differences inferred from sinusoidal fits to the raw data.

C. The EPR paradox revisited

It is well known that the quantum correlated field quadratures of the ND-OPO output can be used to visualize the EPR paradox in its original form put forward by Einstein, Podolsky and Rosen in 1935 [10]. This has been first demonstrated using a polarization non degenerate OPO [11, 12]. It is known that the non-negativity of the Wigner function describing the ND-OPO output prohibits us from violating local hidden variable theories when measuring quadrature phase amplitudes [13]. However an experimental realization of the EPR paradox is still interesting especially seen in the light of recent proposals suggesting that Bell inequalities generalized to continuous variables can be violated by measuring parity like observables [18, 19, 20]. Moreover when considering quantum information transfer with continuous variables using the ND-OPO, like quantum teleportation [21] one must have a fidelity larger than 2/3 for outperforming classical systems consisting of linear optics and coherent states in terms of reproducing the input state [21]. This can only be achieved when more than 50% noise reduction is present in the correlation between the signal and idler fields, which is just the condition required for realizing the EPR paradox [1]. We note that the 50% limit only applies when one uses $\psi = \pi/4$. For other values of $\psi$ it turns out that the EPR paradox can be realized for a smaller degree of correlations. However demonstration of the EPR paradox with our choice of $\psi$ in our ND-OPO is equivalent to demonstration of its usefulness as a resource for continuous variable quantum information transfer like cryptography [22, 23], teleportation [24], storage [25] etc.

We refer to refs. 22 and 24 for the detailed discussion of the EPR paradox. In brief we arrive at the paradox by inferring the idler quadratures $q_-(\phi_-)$ and $q_-(\phi_- + \pi/2)$ from measuring the correlated signal variables $q_+(\phi_+)$ and $q_+(\phi_+ + \pi/2)$. One can choose at will to measure either of the two variables, hence according to EPR they both should be "elements of objective reality". However according to quantum mechanics the non commuting observables $q_-(\phi_-)$ and $q_-(\phi_- + \pi/2)$ cannot have objective values simultaneously, hence the paradox, which is demonstrated if the product of the inferred uncertainties is $(q_-(\phi_-))^2_{inf} < 1$. Turning now to the experiment we sent the photocurrents from the two heterodyne detectors into a hybrid junction producing the sum and the difference of the photocurrents in its two outputs. These outputs were fed into two RF spectrum analyzers in order to establish the noise density around 1.15 MHz. During this experiment one local oscillator phase was kept constant while the other was scanned linearly. Like in the previous section we also here injected a weak signal into the OPO in order to keep track of the local oscillator phases. Depending on the phase of scanned local oscillator the outputs of the hybrid junction yielded the observables $q_+(\phi_+ + \theta_+) \pm q_-(\phi_-)$. That is, with $\theta_+ = 0$ we measured the non commut-
of light [27]. For this purpose two modes of light dis- ing observables $q_+(\phi_+)-q_-(\phi_-)$ in the (-) junction and $q_+(\phi_+)+q_-(\phi_-)=q_+(\phi_+ + \pi/2)-q_-(\phi_-+\pi/2)$ in the (+) junction (see Fig. 5). When $\theta_+=\pi$ we measured $-(q_+(\phi_+)+q_-(\phi_-))$ in the (-) junction and $-(q_+(\phi_+)-q_-(\phi_-))=-(q_+(\phi_+ + \pi/2)+q_-(\phi_-+\pi/2))$ in the (+) junction. From (22) we see that with $\theta_+=0$ we measured a quiet observable in the (-) junction and hence gained information about $q_-(\phi_-)$ while with $\theta_+=\pi$ we measured a quiet observable in the (+) junction and then learned about the conjugate idle observable $q_-(\phi_-+\pi/2)$. The result of a scan is shown in Fig. 6. Here the fluctuations of the two measurements are shown normalized to the vacuum noise level of a single detector. After averaging over 12 pairs of traces like the ones displayed in Fig. 6 we found the uncertainties $\langle (q_+(\phi_+)-q_-(\phi_-))^2 \rangle = -1.0 \pm 0.1$ dB and $\langle (q_-(\phi_-+\pi/2))^2 \rangle = -0.9 \pm 0.1$ dB both relative to the single beam SQL. These uncertainties resulted in an error of the same size in our estimation of the idler quadratures. Hence we could readily calculate the product of inferred variances in this experiment to be $\langle (q_-(\phi_-))^2 \rangle_{inf} \langle (q_-(\phi_-+\pi/2))^2 \rangle_{inf} = 0.65 \pm 0.02 < 1$ which was a clear demonstration of the paradox.

V. SUMMARY

We have demonstrated how the frequency non degenerate optical parametric oscillator can be used to generate spatially separated entangled optical fields. In this paper we have used the OPO cavity resonances adjacent to the frequency degenerate mode, resulting in entangled fields separated by 740 MHz corresponding to two OPO cavity free spectrum ranges. In principle one could select any pair of modes placed symmetrically around the degenerate mode, as long as the $\chi^{(2)}$ phase matching condition is fulfilled for this pair of modes. The entanglement for our source is present over a bandwidth of approximately 14 MHz FWHM depending on ND-OPO pump parameter $\varepsilon$. This results in a high spectral brightness making the source well suited for addressing atomic transitions.

In particular our source can be used for quantum memory readout via interspecies teleportation of a collective atomic spin state onto the polarization state of beam of light [24]. For this purpose two modes of light displaying entanglement in the Stokes parameters would be needed and only minor modifications to our source would be needed in order produce such states of light.

We found a degree of squeezing of -4.0 dB of the frequency degenerate mode and we found inter beam correlations of the nondegenerate modes of -3.8 dB. Both results could be improved by approximately 0.5 dB if the thermal noise of our detectors could be suppressed. In this case the agreement with our theoretical predictions was reasonable.

To gain information about the OPO pump phase we injected a weak coherent field with frequency $\omega_-$ and thereby produced a $\omega_+$ field via difference frequency generation. These fields interfered with the local oscillators yielding us information about the local oscillator phases relative to the OPO pump field. Here we found that the sum of local oscillator phases changed by 3.05 $\pm 0.13$ rad. when going from observing maximum inter beam correlation to observing maximum anti correlations. This was consistent with the theoretical value of $\pi$.

Finally we used the entangled fields of our ND-OPO to visualize the Einstein-Podolsky-Rosen paradox in its original form with continuous quantum variables. Here we found a product of inferred variances of $\langle (q_-(\phi_-))^2 \rangle_{inf} \langle (q_-(\phi_-+\pi/2))^2 \rangle_{inf} = 0.65 \pm 0.02$ which shows the paradox since this is significantly smaller than the Heisenberg limit of unity.
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