Synthetic Aperture Radar Tomography in Urban Area Based on Compressive MUSIC

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Abstract: Synthetic aperture radar (SAR) tomography (TomoSAR) extends the synthetic aperture principle into the elevation for three-dimensional (3-D) imaging. Spectral estimation is the conventional TomoSAR imaging method, such as multiple signal classification (MUSIC). For the sparse elevation distribution, compressive sensing (CS) is a favorable technique for the elevation reconstruction. Compressive sensing multiple signal classification algorithm (CS-MUSIC) is a combination of CS and MUSIC algorithm. It takes the advantage of both conventional spectral estimation and CS technology, and hence overcomes the drawbacks of existing methods and obtains the super-resolution ability. In this paper, the effectiveness of CS-MUSIC in TomoSAR imaging of urban area has been validated by simulated and real TerraSAR-X data. CS-MUSIC algorithm can effectively solve the problem of acquiring high-resolution TomoSAR imaging with small amount of data.

1. Introduction

Traditional two-dimensional synthetic aperture radar (SAR) imaging can only obtain slant range-azimuth information. Synthetic aperture radar tomography (TomoSAR) releases this restriction, which extends the synthetic aperture principle into the elevation direction. TomoSAR imaging methods mainly include spectrum estimation and compressive sensing (CS) [1]-[3]. In 2000, Reigber and Moreira proposed the airborne TomoSAR model for the first time, introduced the principle of TomoSAR and recovered the elevation information using beamforming (BF) algorithm [1]. BF is the first method used to solve the overlap problem, but does not have super-resolution capability. Compared with BF algorithm, CAPON can effectively enhance elevation resolution and suppress sidelobes [4]. However, the radiometric measurements of capon algorithm are inaccurate, the intensity of the reconstructed elevation reflectance function does not match the actual one. In 2003, Fornaro et.al. proposed a singular value decomposition (SVD)-based [5] TomoSAR imaging method. Multiple signal classification (MUSIC) is a method based on matrix eigenspace decomposition. It was introduced to TomoSAR in 2002 by Gini [6] [7]. In general, the reconstruction results of MUSIC algorithm have higher resolution and lower sidelobes compared to BF and CAPON. But the radiation measurement of MUSIC algorithm is also incorrect. Although the above TomoSAR imaging methods can obtain higher resolution. The elevation resolution is mainly decided by the aperture length and the number of baselines. If we want to
improve the quality of the reconstructed elevation, we need more baselines. In TomoSAR, the massive baselines make it becomes an expensive and difficult task. Therefore, using limited data to improve the quality of elevation recovery is worth researching.

Compressive sensing (CS) is an important spare signal processing technique which was proposed by Donohoo et.al. in 2006 [8]. It can break through the limitations of Shannon-Nyquist sampling theorem [9] [10], recover the sparse original signal from less samples. In 2009, Zhu and Budillon introduced CS into TomoSAR imaging [11] [12]. Compared to spectrum estimation methods, CS theory not only possess super-resolution capabilities, but also enables to realize to reconstruct the elevation with a lower number of samples than that required by sampling theory. In urban environment, the elevation distribution of the artificial target is always sparse. Thus CS is a favorable technique for TomoSAR imaging. Compressive sensing multiple signal classification (CS-MUSIC) is a novel algorithm, which combines CS and generalized MUSIC together [13] [14]. CS-MUSIC divides the support set into two parts. It identifies the parts of support using CS, after which the remaining supports are estimated by a generalized MUSIC algorithm. Consider the advantages of CS-MUSIC, we introduce it to TomoSAR imaging of urban area. It inherits the advantages of traditional spectrum estimation algorithm and CS, which solves the drawbacks of existing methods and achieves the superior recovery ability. In addition, CS-MUSIC algorithm can reduce the number of baselines required and achieve the performance of super-resolution.

The rest of this paper is organized as follows. Section II presents the TomoSAR imaging model and process to CS-MUSIC. Section III presents the results of simulation experiment with different TomoSAR algorithms. Section IV shows the experimental results and performance analysis based on the X-band TerraSAR data. Conclusions are provided in V.

2. TomoSAR imaging model and Compressive MUSIC recovery

2.1. TomoSAR model

TomoSAR uses aperture synthesis to achieve three-dimensional (3-D) imaging of the target area. The basic principle of TomoSAR is shown in Figure 1. The focused complex value measurement $y_n$ of an azimuth-range pixel of the $n$th acquisition is

$$y_n = \int_{\Delta s} \gamma(s) \exp(-j \cdot 2\pi \xi_n s) \, ds, \quad n = 1, 2, ..., N$$

(1)

$\gamma(s)$ is the complex reflection function along the elevation direction $s$, $\xi_n = -2b_n/\lambda r_0$ is the spatial (elevation) frequency, $\Delta s$ is the elevation extent, $\Delta b$ is the elevation aperture size, $\lambda$ is wavelength of emission pulse, $r_0$ is the instantaneous slope distance. We use $s_i$ to discrete the complex reflection function $\gamma(s)$ in elevation direction along $s$. So the model in (1) can be expressed as

$$y_n \approx \delta_s \sum_{i=1}^L \gamma(s_i) \exp(-j2\pi \xi s_i) \, ds$$

(2)

$L$ is the number of points discrete elevation, $\delta_s$ is the elevation discretization interval, which equals to $\Delta s/(L-1)$. Let $\gamma = [y_1, y_2, ..., y_n]^T$ and $\gamma = [\gamma(s_1), \gamma(s_2), ..., \gamma(s_L)]^T$ represent the data and discrete complex reflection function at the studied azimuth-range resolution unit. So the imaging model can be rewritten as

$$y_{n \times 1} = \Phi_{n \times L} y_{L \times 1}$$

(3)

$\Phi \in \mathbb{C}^{M \times L}$ is observation matrix based on TomoSAR imaging geometry with $\Phi(n, l) = e^{j\pi b_n s_l}$. So the equation can be considered as a complicated reflection function against the elevation of the irregular. A SAR measurement value can be regarded as a spectral parameter of the target complicated reflection function along the elevation direction. For nonparametric spectral analysis, the theoretical resolution $\rho_s$
in the elevation direction depends on the size of the elevation aperture \( \Delta b \). In the case of sufficiently dense sampling of the elevation toward the aperture, \( \rho_s \) can be calculated by the following equation.

\[
\rho_s = \frac{\lambda r}{2\Delta b}
\]  

(4)

Figure 1. Principles of tomographic SAR imaging

2.2. Compressive MUSIC for TomoSAR

Instead of considering CS or spectral estimation to solve TomoSAR imaging problems, the main aim of CS-MUSIC is considering the advantage of both methods. Let \( \|X\|_0 = r \) with \( X \) being the support. We first estimated the \( k - r \) active set of coefficients of \( \text{supp}X \) using SOMP or 2-thresholding algorithms, and then estimate the remaining unknown support based on the generalized MUSIC algorithm [16]. CS-MUSIC algorithm can guarantee the recovery result of the spare signal, which detailed steps are given below.

**Step1**: Use CS algorithm such as SOMP or 2-thresholding to estimate \( k - r \) active set of coefficients of \( \text{supp}X \).

**Step2**: Use the subscript set in step1 to determine the column vector set of the corresponding flow matrix \( A_{k-r} \), to acquire the projection space \( \mathcal{R}(QQ^HA_{k-N}) \). Construct the noise subspace of CS-MUSIC algorithm \( \mathcal{R}(P_R(QQ^HA_{k-r}) - P_R(QQ^HA_{k-r})). \)

**Step3**: Calculate the quantities \( \eta(j) = a_j^* [P_R(Q) - P_R(QQ^HA_{k-r})] a_j \) for all \( j \notin I_{k-r} \). Make an ascending ordering of \( \eta(j), j \notin I_{k-r} \), choose indices that correspond to the first \( r \) element, and put these indices into \( S \).

CS-MUSIC makes good use of CS to recover accurately in the case of under-sampling, and also ensures that MUSIC algorithm can recover when the number of targets is known, which greatly reduces the possibility of false peaks.

3. Experiment based on simulated data

In this section, the simulated data is used for validate the proposed method. The elevation aperture is 90 m. The wavelength is 0.8 m. The reference slant range is 6000 m. The number of baselines is 10. The number of snapshots is 1. Two scattering points are set in the elevation direction. We first set the distance between two scatterers to be 40 m, then reduce it to 20 m, 10 m, respectively. At this time, the elevation resolution is 26.67 m. When the distance between two scatterers is 40 m. IST and CS-MUSIC can
accurately distinguish the position of the two scatters. There is a deviation in the position of the scatter estimated by the BF algorithm and the MUSIC algorithm. When the distance between the two scatterers is 20 m, traditional BF algorithm has completely failed. While the position of the scatterers can be estimated by MUSIC, CS, and CS-MUSIC. When the distance between two scatterers is reduced to 10 m, it is found that only CS-MUSIC algorithm can accurately estimate the position of two scatters.

(a) Elevation direction recovery results by BF

(b) Elevation direction recovery results by CAPON

(c) Elevation direction recovery results by MUSIC

(d) Elevation direction recovery results by CS
4. Experiment based on real TerraSAR-X data

In order to further verify the effectiveness of the CS-MUSIC algorithm in TomoSAR imaging, TerraSAR-X dataset include 19 complex images are used. The data is collected from November 2014 to January 2018 in part of Shenzhen, China. The elevation aperture size is 475.99 m. We selected a building in Baoan District, Shenzhen for 3-D reconstruction, whose optical image is shown in Figure 3. In TomoSAR, overlapping phenomena is very common. When recovering the elevation toward the result, there will be two location peaks, one of which is the ground and the other is the actual height. To judge the performance of the used TomoSAR imaging algorithm, the ability to accurately resolve the overlapping phenomena is one of the key indicators. CS-MUSIC, BF and MUSIC algorithms are used to reconstruct the building. As shown in the Figure 4, firstly, when the number of baselines is 19, all three methods can recover the building well. Then when we reduce the number of baselines to 4, it is seen that the quality of elevation recovery is significantly reduced. Only CS-MUSIC can distinguish the ground and actual height values. Due to the limited number of baselines, it is found that MUSIC makes errors in recovering the elevation towards the roof part of the building, and CS-MUSIC algorithm can clearly distinguish the roof from the ground. The actual data processing results show that when the number of baselines is 19, MUSIC, BF and CS-MUSIC can all separate the ground and the top of the building. When the number of baselines is reduced to 4, MUSIC and BF can no longer find the scattering centers of the ground and roof areas, and some false targets appeared. While CS-MUSIC algorithm can still clearly distinguish them with fewer errors.

Figure 2. Elevation recovery results by different methods. The distance between two scatters is 40m, 20m, and 10m, respectively.

Figure 3. Selected building in Shenzhen, China
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(a) Elevation recovery results by different methods with 19 complex image data

(b) Elevation recovery results by different methods with 4 complex image data

Figure 4. Elevation recovery results

5. Conclusions
In this paper, CS-MUSIC algorithm is introduced to TomoSAR imaging of urban area. Compared with traditional spectral estimation and CS based methods, CS-MUSIC can accurately estimate the position of scatter with less baselines along the elevation direction, reduce the false alarm targets, and has high-resolution ability. Experimental results based on real dataset show that CS-MUSIC can achieve high-resolution TomoSAR imaging from small amount of acquired 2-D complex image data, and obtain the 3-D image of observed target with less sidelobes and ambiguities. In the future, we will further introduce CS-MUSIC for large-scale 3-D and 4-D reconstruction to obtain high-resolution 3-D image and deformation information of the observed urban area.

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