Cone in the Form of Function in the Pre-Service Mathematics Teacher Class Instruction of Tertiary Level

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Abstract. Mathematical learning carried out in the context of lesson study becomes an alternative to the professional development of teachers or lecturers. This paper is a lesson study of "cone as a function", which is the problem of formulation of a function formula was given. Students were demanded to determine the value of α (centre angle) in such a way that the value of volume V(α) will be maximum. The forty-three undergraduate students were involved in problem-based learning, and at least 50 domestic and international guests as WALS-2014 conference participants, witnessed the learning process that took place. The purpose of this research paper was to find out how many different strategies students could do in solving problems. Infact there are at least 5 different strategies in solving the problem. With more than 12 year experience as a coach in developing the professionalism of junior high school teachers using the lesson study format, this activity turned out to motivate the author himself to open up and organize open-lesson mathematics in tertiary level. From the results of the observations, it was shown that the diverse answers of students were influenced by the educational background and experience of students during college or while obtaining education at the high school level.

Keywords: Cone as a Function, Pre-Service Teacher Training, Lesson Study

1. Introduction

Experience in fostering lesson study in the field of mathematics for junior high school teachers since 2002-present ([1]; [2]; [3]; [4]) and reflecting on the courage of the teachers to become model teachers in a series of lesson studies [4] turned out to bring out the courage to open up in an open-lesson which was attended by no less than 50 national and international guests as a series from the 2014 WALS-2014 (World Association of Lesson Study) activities held in Bandung, Indonesia.

Mathematical learning in an undergraduate class in the School of Mathematics course (MT310) involving 43 students presented the topic "Functional-Cone". This was triggered by a doubt whether students were able to solve the "Functional-Cone" problem that had been designed. A sector with a fixed radius of S, has a central angle α with a large 0o ≤ α ≤ 360o, and α will be seen as an independent variable

Intuitively students have the idea that when the angle α = 360o, the cone will not form, because the radius of the base is S, but the cone height is 0. However, when the angle α is changed to 3400, a cone will form arguably flat, whose volume is still quite small. When the value of α is lowered to 3000, it appears that the cone is getting higher, even though the radius of the cone base is smaller, and intuitively it appears that the volume of the cone is enlarged. Furthermore, when the α angle decreases again, the cone height is relatively high, but the cone radius decreases, and the cone volume V (α) shrinks again. Here students are faced with problems which mean there is an angle situation α such
that the volume of cones reaches the highest value. Students are invited to determine the magnitude of the angle $\alpha$ which causes maximum cone volume.

The research question proposed in this research paper is "Can students formulate the proposed problem? How many different ways are students taken to solve the above problems? The ways they do are motivated by experience when taking a previous course or while in high school?"

2. Review of the Literature

The international study of PISA placed Indonesian students in almost the last rank of the world, as can be seen in the studies of 2000, 2003, 2006, 2009, 2012, and 2015 with the scores: 371, 360, 391, 371, 375, and 372 respectively with the international average of 500.

The low performance of Indonesian students internationally was still main issues in education as expressed by PISA-survey 2006 (Balitbang, 2011a). The rank of Indonesian students was 61st out of 65 participating countries with a mean score of 371, under the international average score of 500. Similarly, the TIMSS survey results indicated that Indonesia students achievement was also at the low level (Balitbang, 2011b). This condition motivated the teachers to always make an effort to improve their quality of teaching by providing lesson to the students more simple, challenging, attractive, and easier. Not only the teachers, the lecturer is also motivated to make an innovation to teach better for the students of undergraduate, in order they would have better way to teach to the students.

Some situations in mathematics teaching in Indonesian classroom consistent with studies of Silver [7] Romberg & Kaput [8], Senk & Thompson [9], and Ernest [10]. Romberg & Kaput [8] stated that mathematics classes mostly consisted of 3 segments: an initial segment where the previous day’s work is corrected. Next, the teacher presents new material, often working 1 or 2 new problems followed by a few students working similar problems at the chalkboard. The final segment involves students working on an assignment for the next day.

Regarding how to build mathematics lesson according to constructivist perspective, the authors of this article, as part of learning community of mathematics educator eager to propose teaching by constructing instead of teaching by telling. Excellent teachers of mathematics are purposeful in making a positive difference to the learning outcomes, both cognitive and affective of the students. They are sensitive and responsive to all aspects of the context in which they teach. This is reflected in the learning environments they establish, lessons they plan,...., their teaching practices, and the ways in which they assess and report on student learning (AAMT, cited in [16]).

The teacher who can present his/her mathematics lesson differently will give opportunity and chance for learners to think creatively. One of ideas is to give undergraduate students with an experience on how to introduce functional concepts using cone volume construction in the lesson study format. According to Fernandez & Yoshida [17], “Lesson study is a direct translation for the Japanese term jugyokenkyu, which is composed of two words: jugyo, which means lesson, and kenkyu, which means study or research” (p.7). As denoted by this term, lesson study consists of the study or examination of teaching practice. In the Confucian tradition, lesson study is said to be premised as “Seeing something once is better than hearing about it one hundred times” [18]. Its ultimate purposes to gain new ideas about the process of teaching and learning based on a better conceptual understanding of students’ thinking so the observation of actual research lessons is at the core of the LS process. Yet, the LS cycle encompasses much more than studying students’ responses while observing a lesson. It requires time to intensively and collaboratively investigate all aspects of the content to be taught and instructional materials available, and reviewing the reflection in the post-lesson review session [19].

Giving experience of lesson study to undergraduate students would make a foundation for the learners to have a professional teaching for the future. In relation to the collaboration among elements in the LS [20] explained that LS in Indonesia can be meant as a professional development model to enhance the quality of teaching through study of their lesson collaboratively and continuously based on the collegiality principles and mutual learning to build learning community (p. 14).

In this article, the author have a willingness to share our experiences in designing the lesson to the audiences, applying the lesson, and being given a reflection in those lesson for improvement.
3. Design Methodology

This lesson study was reported in the qualitative descriptive research with the aims “can the exemplary mathematic lesson which meet the criterion of mathematical investigation presented in the open lesson of lesson study?, How wer students’ action during the lesson in the open lesson of lesson study? What are the actions of students in learning mathematics that meet the investigation criteria of mathematics? To respon these questions, data were collected through design process, implementation of lesson, and evaluation of the program.

An instructional design was made to respons first question whether the students can find the value of $\alpha$ in such a way that the $V(\alpha)$ was maximum, whereas the $\alpha$ is the angle of sector and the $V(\alpha)$ is the volume of cone. And how the studenst designed the procedur to solve the problem in order the volume of cone was maximum.

A small dialog prior to the lesson was developed to find the relationship between the angle of $\alpha$ and the volume of cone $V(\alpha)$, in which the volume was $V(\alpha) = \frac{1}{3} \pi R^2 T$. In the real class according to the students the relationship between $\alpha$ and $h$ in onehand and $\alpha$ and $R$ on otherhand whereas $h$ is height of cone, and $R$ is radius of the base of cone can be constructed by the students in the classroom, whith the assumption that the generator line of the cone was constant as $S$.

The lesson would take place for 100 minutes.

For each minute the activities taking place were attaced in the following diagram

This was the hypothetical learning trajectory that be used for the teaching process with 100-minute lesson.

There was a prediction that it was not easy to contruct the relationship between $\alpha$ and $h$ or $R$
4. Implementation of Lesson

Prior to the classroom session, the dean of science made welcome and a brief session for the international guests to visit the lesson in the classroom. The model lecturer explained how the step lesson would be conducted. The lecturer adopted the constructivist perspective to present the lesson for students. The lecturer expected that students would construct their knowledge by themselves, instead of being fed by the lecturer. However, the lecturer (the author) was also worried if they could not solve the problem and arrive to the solution. “Let us see together in the classroom whether our lesson will arrive to the end of the lesson with productive results or will follow the traditional teaching approaches”. On the Friday, November 28th, 2014, about 09.00 lesson was started with no less than 50 domestic and international visitors. Some transcribe of the lesson would be presented in the following table:

| T | Good morning everybody (class), today’s lesson was unusual class, because we have a lot of guests from all over the world, seeing our lesson as usually been designed. First of all, I would inform you that today’s lesson would be a special occasion. I will ask you to find the measure of angle α of a sector so that the volume of cone produced by sector to be maximum. But please pay attention to my following explanation. Does anybody know what are the geometry shapes in the daily life? |
|---|---|
| S1 | Yes, I know, the cone of ice cream |
| T | Then what’s else? |
| S2 | “Tumpeng” |
| T | What was that? |
| S2 | The traditional food in our society, such as in Sunda tradition or Java tradition |
| T | Ok good, any body else? |
| S3 | Yes I know, … the cap of the farmer |
| T | Good, |
| S4 | Kindergarten kids’ hat or “haseupan” in bahasa Sunda or “kukusan” in bahasa Java. |
| T | Good |
| S4 | Kindergarten kids’ hat or “haseupan” in bahasa Sunda or “kukusan” in bahasa Java. |
| T | I think you all so familiar with the shape of cone. But how to construct the shape of cone? |
| Ss | Yes we know the cone, but we need to know how to construct the cone |

You need to know how to relate between α and volume of V(α). You have to think your self first, to make

![Diagram of cone with labels α, s, and h]

The arc length of the sector versus the circumference of the radius of circle S is equal to the center angle α of 3600, this formula to determine the relationship between variables. So that α/3600=AB/2πS atauanjangbusurAB = (2πS)/(α/360)..<(1). Because the sector is converted into a cone, then the length of AB will be equal to the circumference of the circle of cone base by 2πR, if the radius of the cone base is R. Therefore AB=2πR=(2πS)(α/360) this last form if simplified will be R = (α/3600) S …(2). Next we waited for students to arrive at R = (α/360) S. Because juring is converted into a cone,
we now see the relation S, R, and T which are respectively the length of the cone generator, the cone base radius, and the cone height, which is based on the basic geometrical knowledge we know as the Pythagorean theorem, with S is a constant in this case S = 30.

5. Results and Discussion

The mathematical problem proposed for students is about how much the central angle of a sector causes the cone to reach the maximum volume? Knowledge of the prerequisites that students should have is students (1) have knowledge of the meaning of cones, (2) have skills in how to describe cones, (3) have skills in making cones physically, (4) are able to see the relationship between the angles with a cone volume that is formed intuitively, (5) is able to declare the length of the arc into the radius (S) and the angle of the center juring (α), (6) able to connect between the length of the arc to the cone base circumference, (7) able to formulate cone volume. After the cone volume formula is formed in the form of a function that associates the magnitude of the angle α with volume V (α), or the volume formula in varying forms

\[ V(h) = \frac{1}{3} \pi R^2 h = \frac{1}{3} \pi (S^2 - h^2)h, \]
\[ V(R) = \frac{1}{3} \pi R h = \frac{1}{3} \pi R \sqrt{S^2 - R^2} \]
\[ V(\alpha) = \frac{1}{3} \pi \left( \frac{\alpha}{360^\circ} S \right)^2 \times S \sqrt{1 - \left( \frac{\alpha}{360^\circ} \right)^2} \]

(8) Students also able to find the value of α which make the V(α) to be maximum.

A group of students solved using (1) \[ V(h) = \frac{1}{3} \pi R^2 h = \frac{1}{3} \pi (S^2 - h^2)h. \] They knew to maximize the V it must be solve by finding h such that \( V'(h) = 0 \). First of all, they make α (corner of sector) combine with S the radius of sector and relate to circumstance of circle (the base cone, with the radius R).

It seems that students did not find any difficulties to find the relationship between r and α. The value of S is made fixed in the calculation in this class, the value S = 30. Now how to get the value of α so that the value of V (α) is maximum. It seems that there is a group of students using the Excel application tool to get the possible α values.

After giving the cone volume formula, which is obtained from the basic formula \[ V = \frac{1}{3} \pi r^2 h, \] it seems they are looking for the relationship between r, h, and S in the cone using the principle of the Pythagorean theorem. Then their diversity in solving the problem depends on how they arrange the function \( r^2 + h^2 = S^2 \)

If we compare the length of AB to the circumstance of total circle, it would be: AB: \( 2\pi S = \alpha: 360^\circ \)

so that

\[ AB = \left( \frac{\alpha}{360^\circ} \right) \times 2\pi S \]

(1)
When we modified the sector to be a cone, what was the arch of AB to be? This question was pointed to the students in the classroom. They responded that the arch of AB would be the circumstance of circle? Though in a group there was a variation to develop formula by using \( r^2 + h^2 = S^2 \), \( r^2 = S^2 - h^2 \), or \( h^2 = S^2 - r^2 \). These values were used to become the basis of formula \( V = \frac{1}{3} \pi r^2 h \). If \( r \) is an independent variable then it is obtained, 
\[
V = \frac{1}{3} \pi r^2 \sqrt{S^2 - r^2}
\] 
If \( h \) is an independent variable then it is obtained 
\[
V(\alpha) = \frac{1}{3} \pi \left( \frac{\alpha}{360^\circ} S \right)^2 \times S \times \sqrt{1 - \left( \frac{\alpha}{360^\circ} \right)^2}
\]
It is this diversity that is awaited by observers from domestic and international guests. One thing that was unexpected, it turns out that there were those who completed using Excel. When they finish using Excel means that they have found the third function formula, 
\[
V(\alpha) = \frac{1}{3} \pi \left( \frac{\alpha}{360^\circ} S \right)^2 \times S \times \sqrt{1 - \left( \frac{\alpha}{360^\circ} \right)^2}
\]
Their efforts to get the formula was written in Excel:

Then they enter the value \( S = 30 \) and the value \( \alpha \) with various values from 360\(^\circ\), 350\(^\circ\), 340\(^\circ\), and so on to 10\(^\circ\), then they input these values and use excel. Next they begin to observe the value of \( \alpha \) (in the table are the values in column A) which give the maximum volume. Looks they are somewhat suspicious of the value of 290\(^\circ\). For groups that use excel it turns out they have used calculus first and it turns out that the role of excel is only to check the truth of the calculations they do. There are other groups using maple, calculations using maple, apparently using differential roles. But they also have to construct this volume cone formula first. The group that uses maple builds a formula, there are two formulas:

First they use formulas 
\[
V(\alpha) = \frac{1}{3} \pi \left( \frac{\alpha}{12} \right)^2 \times \sqrt{900 - \left( \frac{\alpha}{12} \right)^2}
\] 
By using maple they get \( \alpha = 9.3 \) they see this is a failure. So they use alternative functions, namely functions in the form of \( r \) as follows 
\[
V(r) = \frac{1}{3} \pi \times r^2 \times \sqrt{900 - r^2}
\] 
Using maple software, they have got the differential function as the following: 
\[
V'(r) = \left( \frac{2}{3} \times \pi \times r \sqrt{(900 - r^2)} - \frac{1}{3} \times \pi \times r^3 \times \frac{1}{\sqrt{(900 - r^2)}} \right). 
\] 
For \( V'(r) = 0 \), we have \( r = 10\sqrt{6} \) and \( r = \alpha/12 \), so that \( \alpha = 12 \times 10\sqrt{6} = 120\sqrt{6} = 293,9388^\circ \)

In the manual solution using generator \( S=30 \), they solved the problem.

Our group uses manual methods. We make the function equation which is the volume in \( \alpha \). From the beginning we have set the value \( S = 30 \), so that \( S \) is a constant. For the cone equation \( V = \frac{1}{3} \pi \times r^2 \times h \)
Around, we get the relationship between the length of the arc and the circumference of the cone base. So that radius = \((\alpha/360) \times (2\pi S/2\pi) = (\alpha S/360)\)
The height is found using Pythagorean Theorem

\[
\text{Height} = \sqrt{(S^2 - r^2)} = S \sqrt{1 - \left(\frac{\alpha}{360}\right)^2}, \text{ then we come to the formula of Cone formula } V = \frac{1}{3} \pi r^2 h, \text{ so that }
\]

\[
V(\alpha) = \frac{1}{3} \times \pi \times \left(\frac{\alpha S}{360}\right)^2 \times S \times \left(1 - \left(\frac{\alpha}{360}\right)^2\right)
\]
From here the differential of the functions is

\[
V'(\alpha) = \left[\frac{1}{3} \times \pi \times S^3 \left(\frac{2\alpha}{360}^2\right) \times \left(1 - \left(\frac{\alpha}{360}\right)^2\right)\right]
+ \left[(1/3 \times \pi \times \left(\frac{\alpha}{360}\right)^2 \times S^3 \times (1/2)(1- \left(\frac{\alpha}{360}\right)^2)^{-1/2}(-2\alpha/360^2)\right]
\]
Jika \(V'(\alpha) = 0\) dan dengan menyederhanakan pekerjaan diperoleh bahwa \(\alpha^2 = (2x360^2)/3 = 86400\) dan \(\alpha = 293,9388^0\)

6. Conclusion
The Lesson Study in the WALS-2014 event was conducted. Students in several groups of 5 persons were able to solve the problems with five different way of solutions. After creating the formula of the cone volume, students solved the problem using (a) differential with involving the variable \(\alpha\) whereas the formula was \(V(\alpha) = \frac{1}{3} \times \pi \times S^3 \left(\frac{\alpha}{360}\right)^2 \times S \times \left(1 - \left(\frac{\alpha}{360}\right)^2\right)\), (b) differential using \(h\) variable \(V(h) = \frac{1}{3} \pi r^2 h\), (c) differential with involving the \(r\) variable using formula \(V(r) = \frac{1}{3} \times \pi \times r^2 \times \sqrt{(900 - r^2)}\), (d) using excel application, and (e) using maple software. Finally they arrive to the conclusion that the value of \(\alpha\) is 293,9388\(^0\). In order to make the cone was maximum they found the solution with 5 different ways.

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