Fractal characteristics of dendrite and cellular structure in nickel-based superalloy at intermediate cooling rate

Aimin Yang\textsuperscript{a,b,*}, Yuhua Xiong\textsuperscript{a}, Lin Liu\textsuperscript{a}

\textsuperscript{a}Department of Applied Physics, Northwestern Polytechnical University, Xi’an 710072, People’s Republic of China
\textsuperscript{b}Department of Mechanical Engineering, Xi’an Petroleum Institute, Xi’an 710065, People’s Republic of China

Received 14 June 1999

Abstract

The fractal characteristics of dendrite and cellular structure of nickel-based superalloy K5 are investigated under directional solidification. Results show that the fractal dimension of the dendrite increases from 1.228 to 1.418 as the withdraw speeds change from 40 to 264 μm/s, whereas the fractal dimension of the cellular changes little as the withdraw speeds from 600 to 952 μm/s. The physical significance of the fractal dimension is analyzed by fractal theory. Based on this, a new idea is proposed that both the fractal dimension and the dendrite arm spacing or cellular spacing be considered to describe the evaluation of the solidification structure completely and integrally. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Dendrite; Fractal dimension; Non-linear

1. Introduction

Fractal theory has been applied successfully to describe the morphologies of materials [1–3]. Numerous studies have shown that the solidification process is a self-organized process that is far removed from equilibrium [4,5], i.e. the solidification system develops to the dissipative structure through the exchange of energy. The liquid–solid interface in directional solidification appears as the fractal feature. From this, a good experiment basis has been built for deep research on solidification structure with fractal theory. Although previous work described the fractal feature at slower cooling rates [6,7], it is difficult to understand the action of the fractal dimension in solidification. The present paper deals with the authors’ own observation of the fractal feature for two main structures, dendrite and cellular, by investigating the morphology evolution process under various cooling conditions.

2. Experimental

Nickel-based superalloy K5 was used in this study, its composition being 0.137%C, 10.2%Co, 10.3%Cr, 5.54%Al, 2.62%Ti, 3.96%Mo, 5.1%W and Ni balance. Samples were prepared using a zone melting and liquid metal cooling (ZMLMC) device under vacuum conditions. Different cooling rates were achieved by changing the withdraw speed, ranging from 40 to 952 μm/s. The fractal dimension was measured with a IBAS-KAT386 image analyzer according to $N = Fr^{-D}$ [8], where $N$ is the square lattice number which covers the surface border, $F$ a constant, $D$ the fractal dimension and $r$ the dimension size in measurement. The smaller is the $r$ value, the more accurate is the surface structure. When set $r$ approaches zero, $D$ represents the corresponding fractal dimension.

3. Results

Dendrite and cellular have different fractal features in the various cooling conditions of nickel-based superalloy. Dendrite was obtained at a relatively lower cooling rate ranging from 40 to 264 μm/s (Fig. 1). At higher withdraw speed of from 600 to 952 μm/s, cellular structure appears (Fig. 2). The fractal dimension of the dendrite increases from 1.228 to 1.365 if the withdraw rates change from 40 to 123 to 264 μm/s. Fig. 3 shows the fractal dimension at the rate 40 μm/s. In contrast, the fractal dimension of the cellular keeps almost constant at 1.011, 1.015 and 1.013 at different withdraw speeds from 600, 760 to 925 μm/s (see Table 1). Therefore, the fractal dimension of the dendrite is
apparently affected by the solidification rate and increases with the increase of the rate.

4. Discussion

The physical mechanism of fracture structure is the non-linear, random and dissipation of the systems, the fractal dimension showing these features [9,10]. The non-linear is caused by interaction within the system. A large fractal dimension indicates more intensive non-linear interaction. In directional solidification, the atomic concentration at the front of the liquid–solid interface becomes higher with the increase of the cooling rate and the interactions between atoms are more intensive. Furthermore, the increase of undercooling results in more atomic clusters of large size to transform synergistically into non-equilibrium structures within a short time, which requires more intensive interactions and makes the fractal dimension increase with the increase of the rates. On the other hand, the fractal dimension exhibits the extent of the energy exchange between the system and the environment, i.e. dissipation. Based on self-structure theory, a dissipative structure depends on the production of super-entropy in an open system [11]:

\[
\frac{d}{dt} \left( \frac{1}{2} \delta^2 S \right) = - \frac{1}{T} \int dV \left[ \sum \delta J_i \left( \nabla \mu_i / T \right) + \sum \delta U_i \left( \Phi / T \right) \right]
\]

(1)

where \( J_i \) is the diffusion flow, \( \nabla (\mu_i / T) \) the chemical potential, \( U_i \), the rate of chemical reaction, \( \Phi / T \) the chemical force and \( d/dt(1/2(\delta^2 S)) \) the negative entropy flow, i.e. the producing of the super-entropy. In the solidification process, increase of the cooling rate leads to the increase of both \( \delta J_i \) and \( \nabla (\mu_i / T) \), which makes the negative entropy flow increasing, so that the entropy becomes smaller and smaller. Set \( S_i(\delta) \) is entropy and \( \delta \) is the measuring size, so that according to the definition of the fractal dimension:

\[
D_k = \frac{S_i(\delta)}{\log(\delta)}
\]

The fractal dimension can be expressed as the entropy value, which makes the meaning of fractal dimension clearer in the solidification. The fractal dimension not only reveals the ability of forming a new phase on the L–S interface by the increase of the negative flow, but also reflects the extent of chaos of non-linear interaction among atoms at the interface, i.e. the ability of the formation of complicated pattern. Therefore, the fractal dimension contains more abundant significance than the entropy for describing structure evolution in the solidification process.

The above analysis is suitable for the process of dendrite growth. For cellular structure, its fractal dimension does not reflect the effect of the cooling rate. In fact, the interaction in the system and the energy exchange can be shown in two ways: one is by changing the morphology of a single state, another is by changing the whole numbers. The variation of fractal dimension shows the former, while the dendrite arm spacing reflects the latter. Apparently, during dendrite growth, most energy is consumed in forming the complicated morphology (Fig. 1). For cellular growth, there is only little morphology change, especially at high rates (Fig. 2). Its energy is consumed to the change of the numbers. Therefore, the fractal dimension in the solidification reflects actually the extent of complication for the pattern and can

| Structure       | Withdraw speeds (µm/s) | Fractal dimensions |
|-----------------|------------------------|-------------------|
| Dendrite        | 40                     | 1.228             |
| 123             |                        | 1.365             |
| 264             |                        | 1.418             |
| Cellular        | 600                    | 1.011             |
| 760             |                        | 1.015             |
| 952             |                        | 1.013             |
describe the morphological feature to some extent. In general, it can be used partially to describe the process of solidification. To describe completely and integrally the evaluation of the solidification structure, both the fractal dimension and the dendrite arm spacing or cellular arm spacing have to be considered. Actually, this idea has been applied in production [12–14]. Only when the dendrite has sufficient growth or the cellular has stable morphology, the fractal dimension and the dendrite arm spacing represent, respectively, and independently the feature of the solidification.

5. Conclusion

1. Dendrite and cellular structure have different fractal features for various cooling conditions of nickel-based superalloy. The fractal dimension of dendrite is larger than that of cellular.
2. The fractal dimension of dendrite is apparently affected by the solidification rate and increases with a higher rate, while the fractal dimension of the cellular changes only slightly with the rate.
3. The fractal dimension exhibits non-linearity and dissipation in solidification by changing the morphology of the structure. Therefore, the conception of fractal can be used to describe the solidification structure only when the most energy is consumed in the change of morphology, such as dendrite grows at slow cooling rate. In general, both the fractal dimension and the dendrite arm spacing or cellular arm spacing are considered in order to describe completely and integrally the evaluation of the solidification organization.

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