Neutrino and scalar boson mass in algebraic quantum field theory

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Abstract. - The hypothesis is explored that fermion rest mass is due entirely to self-interaction via virtual excitation of gauge bosons. This requires revising the standard model to treat both chiral projections of a fermion field as SU(2) doublets, which precludes Yukawa coupling to a scalar (Higgs) boson field. The estimated self-interaction mass of the electron neutrino is $0.291 \times 10^{-5} m_e$. The implied self-interaction mass of the Higgs boson itself is very small, comparable to the neutrino. Because there is no direct coupling to fermions, only to the $Z^0$ gauge boson, this can be reconciled with failure to detect low-mass Higgs bosons. This argument eliminates many undetermined parameters of the standard model, but requires an ad hoc Lagrangian term to account for neutral current asymmetries. The proposed algebraic formalism is consistent with fermion generations defined by distinct eigenvalues of a self-interaction mass operator.

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The standard model revisited. – The standard model (SM) [1,2], based on conventional quantum field theory [3], explains qualitatively and in many cases quantitatively almost all established experimental data for elementary particles and fields other than gravity. A striking exception is neutrino mass, empirically small but nonzero [4]. Although nonvanishing neutrino mass appears to be empirically established, the electroweak theory of Weinberg and Salam [1,2,5] constrains neutrino mass to vanish by excluding right-handed chiral neutrino fields. SM assumes left-chiral fermion projections to be SU(2) doublets, while right-chiral projections are SU(2) singlets. Equivalently, chiral projectors modify gauge boson Feynman vertices.

The theory is assumed to descend by successive symmetry-breaking from a renormalizable grand-unified theory (GUT) of higher symmetry. This excludes bare fermion mass, so that bare fermion fields are chiral eigenstates. Hence chiral projection operators do not affect the parental GUT. Their introduction in SM justifies fermion mass generation by Yukawa coupling of fermion fields to the SU(2) doublet Higgs scalar boson. Such coupling terms in the Lagrangian are invariants only for fermion current densities constructed from an SU(2) doublet field and a distinct SU(2) singlet field [6].

Since neutrino mass requires some modification of the theory, it is of interest to see whether the large number of arbitrary parameters required by SM might be reduced, without invoking unknown new physics or departing from quantum field theory. Both chiral projections of fermion fields are assumed here to be SU(2) doublets, eliminating inequivalent field projections. This assumption excludes Higgs-fermion Yukawa coupling terms from the Lagrangian, because they cannot be SU(2) invariants. In accord with the logical principle of ‘Occam’s razor’, removing chiral projection operators eliminates arbitrary mass parameters. This can be done without affecting the success of SM electroweak theory for processes involving only leptons. The very small mass of neutrinos makes chiral projection redundant. The modified theory can be reconciled with neutral current asymmetries by inserting into the Lagrangian density an ad hoc term that does not add arbitrary parameters.

Self-interaction mass results from virtual emission and reabsorption of gauge field quanta [3,7–9]. Depending on the same operators as spontaneous emission, these processes cannot be eliminated without selective chiral projections. Otherwise, it is inconsistent to postulate neutrino mass exactly zero [10]. For lepton/neutrino processes with intermediate charged $W^\pm$ gauge bosons it will be shown here that chiral projectors can be omitted without affecting the established V-A phenomenology. This justifies a calculation of the electron-neutrino self-interaction mass consistent with current empirical data.

An interacting massless bare fermion is dressed by vir-
tual gauge fields to become a quasiparticle. Stability of the dressed field requires diagonalization of a mass operator defined by gauge field coupling. The Lagrange multiplier required for such secular stability is a spacetime invariant mass. Mass diagonalization defines a canonical multiplier required for such secular stability is a spacetime vector defined by gauge field coupling. The Lagrange multiplier defined by electromagnetic interaction. The ultimate self-contained theory must include a cutoff mechanism that implies correct masses for all fermions.

Extending QED to nonabelian gauge symmetry \[11\], electroweak theory \[1, 2, 5\] incorporates neutrinos, quarks and SU(2) weak gauge fields. The mass of the Maxwell field is forced to zero by decoupling from the Higgs scalar boson field. SM postulates Yukawa coupling of fermions to the Higgs field, introducing coupling parameters adjusted to physical fermion masses. If masses are determined by self-interaction induced by coupling to the gauge fields, parametrized Higgs coupling is not needed. Since neutrinos are coupled only to the weak gauge fields, their self-interaction mass must be small.

A significant implication of this analysis is that for fermions the self-interaction mass operator might very well have several eigenvalues that correspond to discrete states pushed down below successive overlapping continua. This could explain the existence of higher generations of fermions, such as heavy leptons and their corresponding neutrinos.

Induced self-interaction is considered here for a scalar boson field. In fact, Lagrangian terms implied by SU(2) gauge covariance can replace the parametrized Higgs self-interaction, while retaining the essential structure of electroweak theory. If the Higgs mass results solely from its coupling to the weak gauge fields, it might be very small.

Renormalizable field theory with no bare mass is covariant under a local scaling transformation. If gravitational theory is modified to incorporate this same conformal symmetry \[12\], it identifies the imaginary mass term in the scalar field equation with the Ricci scalar, a measure of global spacetime curvature. The self-interaction mass considered here is not inconsistent with this gravitational theory. It augments the extremely small but nonzero implied curvature mass by a much larger dynamical self-interaction, still much smaller than the currently anticipated multi-GeV Higgs mass \[6, 13, 14\].

The algebraic formalism is applied here to consider the mass of the electron, of the electron-neutrino, and of the scalar (Higgs) boson. This analysis reproduces the Feynman electron self-energy at the lowest level of approximation. It justifies a calculation that results in a neutrino mass consistent with current empirical limits. If similar logic is valid for the scalar boson, its mass would be comparable to that of the neutrino. This conflicts with current expectations, but might merit reexamination of empirical data if a multi-GeV Higgs boson is not found \[6\]. A neutral boson of very small mass, not coupled to fermions, might not be directly observable. A \(Z^0\)-induced Higgs cascade might be analogous to bremsstrahlung.

**Postulates of the theory.** That a physical fermion field is a quasiparticle which acquires mass from its self-interaction is consistent with the structure of the Dirac equation: a mass parameter couples field components of opposite chirality whose energy values have opposite sign. The field equation for a bare fermion coupled to gauge fields can be rewritten so that its mass is an eigenvalue of a scalar self-interaction mass operator. Diagonalization in the Fock space of the interacting system defines a canonical transformation from bare fermions to dressed quasiparticles, identified as physical fermions. An algebraic formulation is developed here in which this transformation, which breaks chiral symmetry as it produces nonvanishing mass, is explicit.

The constraint of fixed normalization for timelike displacements determines quasiparticle mass as an invariant Lagrange multiplier. If a quasiparticle state is embedded in a continuum of virtual excitations, the implied entity is a resonance, with an inherent finite lifetime. The secular stability of a quasiparticle constructed by diagonalization is global in spacetime. Matrix elements relevant to decay into constituent fields are removed by construction. This means in particular that a dressed electron, built from chiral bare fields of positive and negative energy, cannot decompose by interactions, short of a canonical transformation of the renormalized vacuum. Thus one cannot consider massive chiral fermions to be elementary fields.

Quantum field theory is based on two distinct postulates. The first is the dynamical postulate that action integral \(W = \int \mathcal{L} dx\), defined by Lagrangian density \(\mathcal{L}\) over a specified space-time region, is stationary with respect to variations of the independent fields, subject to homogeneous boundary conditions. The second postulate, which requires the fields to be operators in a Fock space, modifies and constrains classical field theory. Field operators act on state vectors defined by virtual excitations of a vacuum state. Field equations for interacting field operators imply coupled algebraic equations when projected onto the state vectors.

For Dirac field \(\psi\) and Maxwell field \(A_\mu\), defining \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\), the QED Lagrangian density in units such that \(\hbar = c = 1\) is

\[
\mathcal{L} = -(1/4)F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu \psi.
\] (1)

Coupling to the electromagnetic 4-potential \(A_\mu\) occurs
through the covariant derivative
\[ D_\mu = \partial_\mu - ieA_\mu, \]  
where \(-e\) is the renormalized electronic charge. The notation used here defines covariant 4-vectors
\[ x_\mu = (t, -\mathbf{r}), \quad \partial_\mu = (\partial/t, \nabla), \]
\[ A_\mu = (\phi, -A). \]

The metric tensor \( g_{\mu\nu} \) is diagonal, with elements (1, -1, -1, -1). Dirac matrices are represented in a form appropriate to a 2-component fermion theory, in which chirality \( \gamma^5 \) is diagonal for mass-zero fermions,
\[ \gamma^\mu = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}. \]

Classical fields \( \psi_\mu, A_\mu^x \) that satisfy homogeneous field equations define quantum field operators \( A^\mu(x) = \sum_k A^x_k(x) a_k + \psi(x) = \sum_j \psi_j(x) n_j \). Operators \( a_k, n_j \) and fermion mass parameter \( m_0 \) define vacuum state \( |0\rangle \) such that \( a_k|0\rangle = 0 \) and \( n_j|0\rangle = 0 \), where \( n_j^\dagger = n_j^\dagger \) if \( \epsilon_j > 0 \), \( n_j^\dagger = n_j \) if \( \epsilon_j < 0 \). \( A^x_k(x) = u^k_k e^{-i k \cdot x} \) is the free photon field for 4-momentum \( k \). Dirac free-wave functions \( \psi(x) \) are defined for \( \epsilon_p = p_0 > 0 \) by \( \psi_p(x) = \psi^p(x) = V^{-2} e^{-i p \cdot x} u_p(p - m_0) u_p \). Reversing all components of 4-momentum for \( \epsilon_p = -p_0 < 0, \psi^\dagger p(x) = V^{-2} e^{i p \cdot x} \psi_p(p + m_0) u_p \).

The standard model extends this formalism to include SU(2) gauge fields. Chiral projection operators, which take a finite chirality, are defined by \( L_\mu = \frac{1}{2}(1 - \gamma^5)\gamma^\mu \psi^\dagger \psi \). Defining chiral massless Weyl spinors into 4-component Dirac spinors,
\[ \chi = \frac{1}{2}(1 - \gamma^5)\gamma^\mu \psi^\dagger \psi. \]

Without chiral projection, \( c_V = -\cos 2\theta, c_A = 0 \). This can be converted to the SM form by the current density difference \( \Delta j^{\mu}_{NC} = \psi \gamma^\mu \Delta (c_V - c_A \gamma^5) \psi \) where \( \Delta (c_V - c_A \gamma^5) = (-\frac{1}{2}I + 2 \sin^2 \theta \cos 2\theta + \frac{1}{2} \gamma^5) = \frac{1}{2}(I + \gamma^5) \). The incremental Lagrangian density is \( \Delta L_{IZ} = \frac{1}{2} \sin 2\theta \Delta j^{\mu}_{NC} Z_\mu^0 \). This ad hoc correction has no undetermined parameters. It preserves lepton-W± coupling for neutrino self-interaction.

**Algebraic formalism: mass as an eigenvalue.** – The dynamical postulate implies field equations,
\[ \partial^{\mu} F_{\mu\nu} = j_\nu = -e \psi^\dagger \gamma^0 \gamma^\nu \psi, \quad i \gamma^\mu D_\mu \psi = 0. \]
Because the current density satisfies \( \partial_\mu j^\mu = 0 \), the inhomogeneous Maxwell equation cannot change gauge condition \( \partial_\mu A^\mu_5 = 0 \), if assumed for the photon fields. Separating \( A_\mu = A_\mu^{int} + A_\mu^{ext} \) into self-interaction and external subfields, and using notation \( A_\mu(x) = \gamma^\mu A_\mu(x) \), the fermion field equation
\[ \{i \gamma^\mu \partial_\mu + e A^{ext}_\mu \} \psi = -e A^{int}_\mu \psi = \hat{m} \psi \]
defines a self-interaction mass operator \( \hat{m} = -e A^{int}_\mu \). For the quantized Maxwell field, the field amplitude operators have only transition matrix elements. Hence \( \hat{m} \) is purely nondiagonal, associated with virtual excitations of the radiation field. The mass operator acts in a Fock space defined by products of creation and annihilation operators. There is no classical analog, hence no valid classical model of elementary fermion mass.

What is proposed here is to rewrite Eq.(6) as a renormalized Dirac equation and a mass-eigenvalue equation, related by consistency condition \( m_0 = m \):
\[ \{i \gamma^\mu \partial_\mu + e A^{ext}_\mu - m_0 \} \psi = 0, \quad \{\hat{m} - m \} \psi = 0. \]

The second equation is solved in a Fock space defined by mass parameter \( m_0 \), which is to be adjusted iteratively to satisfy the consistency condition. \( m_0 = 0 \) for bare fermions, but otherwise is a free parameter. The computed eigenvalue can be identified with \( \delta m(m_0) \) in standard perturbation theory. The consistency condition \( m_0 = \delta m(m_0) \) forces the two equations to be equivalent to the original field equation in renormalized Fock space.

Eqs.(7) exhibit the algebraic structure implied by renormalization. A canonical transformation of field operators and the vacuum state diagonalizes the mass operator and determines a c-number mass. This transformation mixes field components of positive and negative energy, breaking chiral symmetry. The resulting Dirac equation combines chiral massless Weyl spinors into 4-component Dirac spinors.

On a spacelike surface indexed by parameter \( \tau \), excitation operator \( \chi^\dagger(\tau) \) defines state \( \chi^\dagger(\tau)|0\rangle \) and surface action integral \( W(\tau) = \int_0^\tau d^3x \sqrt{g} \mathcal{L} \chi^\dagger|0\rangle \). Condition \( \chi^\dagger|0\rangle = 0 \) required for a stable pseudostate, may imply a canonical transformation of the vacuum. Defining
The gauge field equations determine coefficients 
explore the true limit of this expansion, derivations here 
an infinite sum of vanishing terms, may be finite, implying 
that \( \sum \chi^\dagger_\mu c_\mu(\tau) \) as a sum of invariant excitation operators, 
subject to \( \chi(\tau)|0\rangle = 0 \). If field Hamiltonian operator \( \hat{H}(\tau) \) is defined on spacelike surface \( \tau \), \( \partial_\tau \chi^\dagger = i[H, \chi^\dagger] \) implies linear algebraic equations for the coefficients \( c_\mu(\tau) \). A field state is determined only if field and algebraic equations 
are both satisfied. Lagrange multiplier \( m \), distinct from 
renormalization parameter \( m_0 \), is required to enforce the normalization constraint \( \sum_\mu |c_\mu|^2 = \text{const.} \). \( \tau \)-dependent phase factors are not determined, but can be adjusted to 
remove diagonal elements of the algebraic equations. This 
reduces them to the form of mass-eigenvalue equations.

Defining \( W(\tau) \) on a spacelike surface clarifies several aspects of the theory. Past and future are well-defined. A 
stable pseudostate constructed on surface \( \tau \) by superimposing 
global spacetime field solutions requires consistency with 
past creation events (retarded potentials) and with 
future annihilation events (advanced potentials). This is built into Feynman propagators [8].

For eigenvalue equations dominated by a particular excitation 
operator \( \chi^\dagger_0 \), it is convenient to normalize the state 
eigenfunction so that \( \langle \chi^\dagger_0 | \chi_0 \rangle = 1 \). The basis set of elements 
equation operators can be augmented systematically, 
adding terms which affect higher orders in a perturbation 
expansion. The algebraic equations have a leading term 
\( m = \sum_a (0) \tilde{m}(a) c_a \), exact for nondiagonal \( \tilde{m} \) if all 
nonzero matrix elements are included. Using a formal solution 
for the coefficients \( c_a \), \( a \neq 0 \), 
\( m = -\sum_{a,b} (0) \tilde{m}(a) [\tilde{m} - m]_{a,b}^{-1} (b) \tilde{m}(0) \).

This is a sum of terms that all vanish as \( m \to \infty \). The limit, 
an infinite sum of vanishing terms, may be finite, implying 
a cutoff inherent in the algebraic equations. Rather than 
explore the true limit of this expansion, derivations here 
will be limited to the mass formula implied by the leading 
algebraic equation, which acts as a mean value expression.

The gauge field equations determine coefficients \( c_a \) consistent with 
perturbation theory, which would expand the inverse matrix here in powers of a coupling constant.

### Neutrino mass.

The self-interaction mass of the neutrino involves two distinct weak-interaction processes. 
The first, virtual excitation of an electron/positron and a 
\( W^\pm \) gauge boson, dominates the second, virtual excitation of a neutrino and \( Z^0 \), because interaction mass is proportional to intermediate fermion mass. SU(2) symmetry can be assumed for the intermediate \( W^\pm \) process, eventually 
broken by the self-interaction masses of the \( \nu, \bar{e} \) doublet. 
The SU(2) Lagrangian density [11] is 
\[
\mathcal{L} = -\frac{1}{4} W^{\mu
u} \cdot W_{\mu
u} + i \bar{\psi} \gamma^\mu D_\mu \psi,
\]
where \( W^{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g W_\mu \times W_\nu \). Covariant derivative \( D_\mu = \partial_\mu + i \gamma^\mu \tau \cdot W_\mu \) ensures SU(2) gauge invariance. \( \tau \) is a 3-vector of \( 2 \times 2 \) matrices identical to the 
Pauli matrices \( \sigma \).

In the classical inhomogeneous SU(2) field equation, the fermion source current density is \( j_\psi^{\psi\nu} = \frac{1}{2} g \bar{\psi} \gamma^\nu \tau \psi \). The nonabelian gauge field contributes an additional self-interaction to a conserved total weak current density [11]. It is assumed here that this self-interaction can be combined with that induced by the Higgs boson to produce a field mass \( M \) in an effective inhomogeneous weak field equation \( (\Box^2 + M^2) W^{\mu
u} = j_\psi^{\psi\mu\nu} \). The Green function defined such that \( W^{\mu\nu}(x,y) = \int d^4 y W^{\mu\nu}(y,x) j_\psi^{\psi\mu\nu}(y) \) has the Fourier transform \( \tilde{G}^{\psi\mu\nu}(k) = g \epsilon^{\mu\nu}(k^2 + M^2) \).

The neutrino self-interaction mass operator is \( m_\nu = \frac{G}{2} \gamma^\mu \tau \cdot W_\mu^{\psi\nu} \). If \( m_\nu + m_\psi \psi \) = 0 and \( m_{\nu\tau} = m_\nu \), the 
renormalized neutrino field equation is 
\[ \{ i \gamma^\mu \partial_\mu - \frac{G}{2} \gamma^\mu \tau \cdot W_\mu^{\psi\nu} + m_\nu \} \psi = 0. \]
In analogy to the electron self-interaction, the mass 
value can be estimated by the second-order Feynman con-
voltage integral
\[ m_V = \frac{\hbar^2}{2m_e} \int \frac{d^4k}{(2\pi)^4} \epsilon_p \epsilon_q \gamma_\mu G_D(k) \gamma_\nu G_D(p - k) \gamma_\nu \epsilon_q. \]
Feynman propagator \( G_D \) refers to an electron or positron, with mass \( m_e \), accompanied by gauge field \( W^\pm \), restricted to \((t_y < t_x < t_z)\), respectively. An intermediate neutrino plus \( Z^0 \) implies an integral proportional to neutrino mass, which can be neglected. Spinors \( \epsilon_p \cdots \epsilon_q \) project onto the renormalized neutrino state. Neglecting the neutrino mass parameter, \( \rho \approx 0 \) simplifies the integral.

The gauge field requires a Klein-Gordon propagator for \( M = M_W \). The Feynman 4-momentum cutoff for convergent integrals replaces the photon factor
\[ C(k^2) = 1 - \frac{k^2 - M^2}{k^2 - M^2 - \Lambda^2}, \]
by
\[ C_M(k^2) = 1 - \frac{k^2 - M^2 - \Lambda^2}{k^2 - M^2 - \Lambda^2}. \]

With these assumptions, using standard \( \gamma \)-matrix algebra, the integral to be evaluated is
\[ m_V(m_e) = \frac{2\hbar}{\pi} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^2 - M^2 - \Lambda^2} \epsilon_p \gamma_\mu \epsilon_q \gamma_\nu \epsilon_q \gamma_\nu \epsilon_q. \]
The denominators here can be combined using
\[ \frac{1}{ab} = \int_0^1 dy \int_0^1 dx \frac{2y}{(ax + (1-x)y + c(1-y))^2}. \]
Setting \( k^2 = k^2 - p \cdot p = (1-y) \), and using \( \phi \) zero at \( t_y = 0 \) in the numerator of the integral for \( m_e \). Its final factor is effectively replaced by \( 4m_e/(k^2 - m_e^2) \). Then
\[ m_V(m_e) = \frac{i g^2 m_e^2 \Lambda^2}{16 \pi^2} \int \frac{d^4k}{(2\pi)^4} \frac{k^2 + M^2}{k^2 - M^2 - \Lambda^2}. \]

The integrand has poles at \( k_0 \approx \pm \sqrt{2\Lambda^2 + L^2} \). Expanding \( k_0^2 = k^2 + L^2 \), and \( \zeta = 2\sqrt{k^2 + L^2} \), a contour integral enclosing the positive pole (displaced below the real axis) implies
\[ \int \frac{d^4k}{(2\pi)^4} (k_0^2 - k_0^2)^{-1} = -i \operatorname{Res}_{k_0 \to 0} \epsilon_p \epsilon_q \gamma_\mu \epsilon_q \gamma_\nu \epsilon_q \gamma_\nu \epsilon_q = \frac{\hbar}{16\pi}. \]

For \( a^2 = k^2 + L^2 \),
\[ J_3 = \frac{1}{16\pi^2} \int_0^1 dy \int_0^1 \frac{k^2 + M^2}{k^2 - M^2 - \Lambda^2}. \]
From this analysis,
\[ m_V \approx \frac{\hbar}{16\pi^2} \int_0^1 dy \frac{1}{\gamma_\mu \gamma_\nu \gamma_\mu \gamma_\nu} \left[ \frac{1}{2\Lambda^2} \right]. \]

Unlike the photon, \( W^\pm \) is unstable. Its energy width \( \Gamma \) suppresses the neutrino mass integrand in coordinate space by \( e^{-\Gamma (t_z - t_y)} \), establishing a dynamical scale for Feynman cutoff factor \( C_M(k^2) \). \( C_M(k^2) \) is
\[ \frac{\hbar^2}{2m_e^2} \int G(\Lambda) d\Lambda = 1. \]

If \( G(\Lambda) = \frac{\sqrt{2\Lambda^2 + L^2}}{16\pi^2} \), then
\[ \int_0^\infty G(\Lambda) d\Lambda = 2\Lambda. \]
\[ \Lambda \ll M, \quad \frac{\Lambda^2}{M^2} \approx 1, \quad \frac{\Lambda^4}{M^4} \approx 1. \]

The algebraic theory considered here supports the speculation that fermion generations correspond to excitation operators in the sequence \( \eta^1, \eta^1 \eta \psi, \eta \eta \psi \eta \psi \), acting on the renormalized vacuum. For example, the muon might correspond to a \( \ell \ell \) pair stabilized by an external electron, while the \( \nu \)-neutrino is such a pair stabilized by an external e-neutrino. Decay mechanisms would agree with well-established muon decay. Observed neutrino mixing implies very similar masses for the three generations of neutrinos [4]. Transition matrix elements coupled to gauge fields would be the same for all generations, so that all neutrinos might have comparable self-interaction mass. For charged leptons, the intermediate virtual state would be the charged lepton itself plus a photon. This depends on the still unknown mechanism for 4-momentum cutoff, which might differ substantially for different lepton generations.

The SU(2) scalar field. - Weinberg-Salam electroweak theory includes a term of the form \([2,5]\)
\[ \Delta L = -\frac{\varphi^2}{2} \Phi \Phi = -\lambda(\Phi^2) - \varphi^2 \]
in the postulated Lagrangian density \( L^e \) of the Higgs scalar \( SU(2) \) doublet \( \Phi = (\Phi_+ + \Phi_0), \quad \Phi = (0, \varphi), \) determines a stable neutral vacuum ground state. Setting parameter \( \lambda = \frac{\varphi^2}{2^2} > 0 \),
\[ \Delta L = -\frac{\varphi^2}{2} \Phi (\Phi + \varphi^2 \Phi - \varphi^2 (\Phi^2)). \]

The imaginary mass term \( w^2 \Phi \) in \( \Delta L \) would by itself destabilize the vacuum state. It must be counteracted in any viable theory. This is accomplished by the biquadratic term \( -\frac{w^2}{2^2} (\Phi^2) \). \( L^e \) is
\[ \Delta L = \frac{(\Phi^2)}{2} \Phi \Phi + \Delta L, \]
but including gauge coupling, the field equation is \( (\varphi^2 - w^2) = -\Phi^2 (\Phi^2 - \varphi^2). \) Electroweak theory considers an exact particular solution \( \Phi \) such that \( \varphi^2 - \Phi \) = 0 and \( \Phi = \varphi, \) \( \Phi = \varphi, \) and \( \Phi = \varphi^2 \), constant in spacetime. \( \Phi = (0, \varphi, H/\sqrt{2}) \) defines Higgs field \( H \). When \( h = \sqrt{2}w \), the field equation reduces to \( \varphi^2 + m_H^2 \) \( H = -\frac{\varphi^2}{2} \Phi \Phi \). Covariant derivative
\[ D_\mu = \partial_\mu + i \frac{2}{3^2} B_\mu + i \frac{2}{3^2} \tau \cdot W_\mu \]
implies mass proportional to \( \varphi^2 \) for each gauge field coupled to \( \Phi \). The particular linear combination of \( B_\mu \), \( W^\mu \) that decouples from \( \Phi \) defines \( A_\mu \), massless if \( \Phi = 0 \).

In gauge-invariant \( L_\Phi = (\Phi^2) \) \( D_\mu \Phi \), the covariant derivatives couple isospin doublet components of an SU(2) scalar field to the weak gauge fields. Implied self-interaction terms can replace a separately postulated \( \Delta L \). The quadratic form of derivatives provides an imaginary mass term, counteracted by virtual gauge field emission and reabsorption. This replaces parameters \( u^2 \) and \( \lambda \) by interaction terms. The renormalized scalar field equation is equivalent to that assumed in electroweak theory.

In \( U(1) \times SU(2) \) gauge theory, covariant derivative
\[ D_\mu = \partial_\mu + i \frac{1}{4} (g_1 y B_\mu + g_2 \tau \cdot W_\mu), \]
acts on gauge-dependent fields. For real gauge fields \( U_\mu \) such that \( \partial_\mu U_\mu = 0 \), Lagrangian density \( L^e = (\Phi^2) \) \( D_\mu \Phi \) for scalar field \( \Phi \) implies the field equation
\( (\varphi^2 - w^2) \Phi(y) = -i (g_1 y B_\mu + g_2 \tau \cdot W_\mu) \Phi(y) \),
where
\[ w^2 = \frac{1}{4} (g_1 y B_\mu + g_2 \tau \cdot W_\mu) (g_1 y B^\mu + g_2 \tau \cdot W^\mu). \]
Self-interaction due to virtual intermediate weak field quanta arises from the weak current density \( j_\mu = \frac{\partial L^e}{\partial \Phi^\dagger} \), coupled to \( U^\mu \). \( w^2 \) can be replaced in \( \Delta L \) by an eigenvalue.
or mean value of $\ddot{w}^2$ computed for the virtual gauge fields generated by this current density. For $j_{\mu} \sim \Phi^I \partial_\mu \Phi$, the 2nd order self-interaction is operationally equivalent to a term $-\lambda (\Phi^4 \Phi^2)^2$ in $\Delta L$. Coupled only to the weak gauge field $Z_\mu^0$, $\Phi_0$ should have a self-interaction mass comparable to the neutrino.

**Relation to gravitational theory.** – The SU(2) scalar field provides a possible link between the standard model and gravitational theory [12]. Invariant action integral $I_a = \int d^4x \sqrt{-g}L_a$ determines covariant energy-momentum tensor $T^\mu_\nu = -\frac{28I_a}{\sqrt{-g}g_{\mu\nu}}$. For gravitational

\[ L_g = \frac{\kappa}{2} \sum_a T^\mu_\nu_g \] implies field equation

\[ W^\mu_\nu_g = \frac{8\pi G_N}{\sqrt{-g}}g_{\mu\nu} \] in standard Einstein/Hilbert theory, for Newton constant $G_N$ and Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$.

\[ L_g = -\frac{R}{16\pi G_N} \] implies field equation

\[ R^\mu_\nu - \frac{1}{2}g^{\mu\nu}R = G^\mu_\nu = 8\pi G_N \sum_a T^\mu_\nu_g \] in conformal gravitational theory [12].

\[ L_g = -2\alpha_g (R^{\mu\nu}R_{\mu\nu} - \frac{1}{4}R^2) = -2\alpha_g (L_2 - \frac{1}{2}L_1) \]

The field equation is $-2\alpha_g W^\mu_\nu = -2\alpha_g (W^\mu_\nu - \frac{1}{4}W^\mu_\nu) = \frac{1}{2} \sum_a T^\mu_\nu_a$. For a complex scalar field, $L_\Phi = (\partial_\mu \Phi^\dagger \partial^\mu \Phi + w^2 \Phi^\dagger \Phi - \lambda (\Phi^4 \Phi^2)^2$. $I_\Phi$ is scale invariant if $R^{\mu\nu} = -3w^2g^{\mu\nu}$, or $w^2 = -\frac{R}{12}$. The standard model set $\lambda = \frac{w^2}{v^2}$ and postulates a particular solution $\Phi = \phi_0$. This determines a global solution of the coupled equations such that $T^{\mu\nu}_\Phi = -\frac{1}{6} \phi_0^2 G^\mu_\nu + g^{\mu\nu} \lambda \phi_0^4$ [12].

In a geometry (Robertson-Walker) such that $W^\mu_\nu_g = 0$, averaged uniform matter produces $T^\mu_\nu_k$, and the field equation reduces to $T^{\mu\nu}_\Phi + T^{\mu\nu}_k = 0$. Defining $\Lambda = 6\lambda \phi_0^4 = 3w^2G = 6/\phi_0^2$, the field equation becomes [12]

\[ G^{\mu\nu} - \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu}_k \]

This has the same form as the Einstein field equation. Effective coupling constant $G$ and cosmological constant $\Lambda$ are determined by the scalar field parameters $\phi_0$ and $w$. In a universe with negligible curvature, the cosmological (Friedmann) equation [16] for Hubble constant $H_0$ is $H_0^2 = \frac{8\pi G}{3} \rho_m + \frac{1}{a^2}$. This defines empirical parameters $\Omega_m = \frac{8\pi G \rho_m}{3H_0^2}$ and $\Omega_\Lambda = \frac{1}{a^2}$.

Conformal theory implies $\Omega_\Lambda = \frac{6\lambda \phi_0^4}{3H_0^2} = \frac{w^2}{v^2}$. Without self-interaction, empirical values \[ \Omega_\Lambda \approx 0.732 \text{ and } H_0 \approx 70.9 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{, with } \phi_0 = 180 \text{ GeV}, \text{ imply Higgs mass } m_H = \sqrt{2w} \approx 10^{-33} \text{ eV, if all quantum field action integrals are scale-invariant (conformal). Thus in the standard model, which omits self-interaction, only an extremely small } m_H \text{ is compatible with the empirical Hubble constant. The present argument indicates that virtual excitation of } Z^0_\mu \text{ could determine the otherwise undetermined parameter } \lambda, \text{ and might supplement the bare Lagrangian term } w^2 = -R/12 \text{ by a much larger induced self-interaction, still very small compared with the induced dynamical mass of charged fermions or the gauge bosons.}

**Implications and conclusions.** – If fermions are SU(2) doublets, regardless of chiral projection, this excludes direct Higgs-fermion interaction. Fermion mass must arise from self-interaction. This is shown here to explain the finite but small mass of neutrinos, while providing a rationale for the existence of fermion generations distinguished only by mass. Applied to a scalar (Higgs) boson in an SU(2) manifold, the present analysis implies that its self-interaction mass could arise solely from weak interactions, and might be very small, analogous to neutrino mass. Such small mass is contrary to lower bounds deduced from experimental data [6], assuming Yukawa coupling of the Higgs boson to fermions. Since such Yukawa coupling is not consistent with the theoretical model used here, implications regarding small Higgs mass should be reexamined.

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