Two-Loop Beta Functions Without Feynman Diagrams *

Peter E. Haagensen*, Kasper Olsena,b, and Ricardo Schiappa*

aCenter for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA
bThe Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen, DENMARK

(PIT-CTP#2641, hep-th/9705105

Starting from a consistency requirement between T-duality symmetry and renormalization group flows, the two-loop metric beta function is found for a $d = 2$ bosonic sigma model on a generic, torsionless background. The result is obtained without Feynman diagram calculations, and represents further evidence that duality symmetry severely constrains renormalization flows.

PACS number(s): 11.10.Hi, 11.10.Kk, 11.25.-w, 11.25.Db;
Keywords: string theory, sigma models, duality, perturbation theory.

I. INTRODUCTION

Since the time of its discovery [1], target space duality has been studied mostly as a symmetry of string backgrounds. That is to say, it is realized as a transformation taking one set of fields $\{g_{\mu\nu}, b_{\mu\nu}, \phi\}$ (respectively metric, antisymmetric tensor and dilaton) satisfying background field equations of motion, into another set $\{g'_{\mu\nu}, b'_{\mu\nu}, \phi'\}$ satisfying the same equations of motion. As such, it represents a parameter space symmetry of the associated sigma model at its conformal points only. It was recently observed, however, that it is also natural to impose it as a symmetry of the sigma model away from conformal points, throughout the entire parameter space [2]. This is expressed as the requirement (to be made precise below) that duality flows “covariantly” with the renormalization group (RG) evolution of the background fields. Because information about the RG flow is typically difficult to obtain, while a T-duality symmetry is considerably easier to identify, such an interplay between duality and RG flows can be of more than academic interest if it yields restrictions on the renormalization patterns of the theory.

At one-loop order ($O(\alpha')$), it was shown in [3] that indeed the requirement of duality symmetry away from conformal points of the 2d bosonic sigma model led to highly restrictive consistency conditions on the RG beta functions of the model. It was found that these conditions uniquely determine all beta functions at $O(\alpha')$. This is a particularly striking fact, in that essentially the only condition imposed is that of duality, a symmetry which is prima facie entirely unaware of the renormalization structure of the model. Similar (albeit weaker) restrictions have also been seen to follow from analogous consistency conditions in altogether different contexts, such as the 2d Ising and Potts models [3], and the quantum Hall system [4].

Naturally, for sigma models, this would probably be an inconsequential curiosity if such conditions only operated at $O(\alpha')$. This motivated two of us to further investigate the consistency conditions at two-loop order [5]. For a restricted, purely metric background, it was found that while both the beta functions and the duality transformations are modified by perturbative corrections, the ensuing consistency conditions (also modified) continue nonetheless to be satisfied. This indicates that, at least to $O(\alpha'^2)$, duality transformations mysteriously remain informed of the renormalization properties of the theory.

If this is so, one is led to inquire whether consistency conditions at $O(\alpha'^2)$ again allow for a determination of the beta functions at that order. The purpose of the present investigation is to show that indeed such a determination is possible.

After briefly reviewing the first nontrivial order, we will consider, as in [5], a restricted class of backgrounds in order to probe the consistency conditions at $O(\alpha'^2)$. In order to be self-contained we begin by deriving, from basic principles, the corrected duality transformations at $O(\alpha'^2)$ first presented in [5]. From these follow the $O(\alpha'^2)$ consistency conditions on the beta functions of the theory. We will then show that, out of the ten different tensor structures possibly appearing in the two-loop beta function, only the known, correct structure satisfies the consistency conditions. This represents a completely independent and diagram-free determination of the two-loop beta function of the purely metric 2d bosonic sigma model.

To be precise, with the restricted class of backgrounds we consider, this $O(\alpha'^2)$ beta function is only determined up to a global constant. However, it should be noted firstly that the beta function determined is valid for entirely generic metric backgrounds and, secondly, that the mechanism at work at $O(\alpha')$ indicates that, had we considered a more generic background at $O(\alpha'^2)$, even this global constant would have been determined.

---

*This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement #DF-FC02-94ER40818, and by NSF Grant PHY-92-06867. E-mail: haagensen, olsen @ctp.mit.edu, ricardos@mit.edu.
II. ORDER $\alpha'$

We consider a $d=2$ bosonic sigma model with a target abelian isometry ($\theta \to \theta + \text{constant}$):

$$S = \frac{1}{4\pi\alpha'} \int d^2 \sigma \left[ g_{00}(X) \partial_\alpha \theta \partial^\alpha \theta + 2g_{0i}(X) \partial_\alpha \theta \partial^\alpha X^i + i e^{\gamma} (2h_{0i}(X) \partial_\alpha \theta \partial_\beta X^i + b_{ij}(X) \partial_\alpha X^i \partial_\beta X^j) \right] .$$

The adapted target space coordinates are $X^\mu = (\theta, X^i)$, $i = 1, \ldots, D$, and the isometry is made manifest through the independence of background tensors on $\theta$. "Classical" duality transformations take a background to a new background $g$, $b$, $\phi$ such that $\bar{g}$, $\bar{b}$, $\bar{\phi}$ are determined by $g$, $b$, $\phi$ through:

$$\bar{g}_{00} = \frac{1}{g_{00}}, \quad \bar{g}_{ij} = \frac{g_{ij}}{g_{00}}, \quad \bar{b}_{ij} = \frac{g_{ij}}{g_{00}} ,$$

$$\bar{\phi} = \phi - \int d\phi,$$

On a curved worldsheet, another background coupling must be introduced, that of the dilaton $\phi(X)$. The RG flow of background couplings is given by their respective beta functions:

$$\beta^g_{\mu\nu} \equiv \frac{d}{d\mu} g_{\mu\nu} , \quad \beta^b_{\mu\nu} \equiv \frac{d}{d\mu} b_{\mu\nu} , \quad \beta^\phi \equiv \frac{d}{d\mu} \phi ,$$

while the trace of the stress energy tensor is found from the Weyl anomaly coefficients $\bar{T}$, $\bar{\beta}$:

$$\bar{T}^g_{\mu\nu} = \beta^g_{\mu\nu} + 2\alpha' \nabla_\mu \partial_\nu \phi ,$$

$$\bar{T}^b_{\mu\nu} = \beta^b_{\mu\nu} + \alpha' H_{\mu\nu} \partial_\lambda \phi ,$$

$$\bar{T}^\phi = \beta^\phi + \alpha' (\partial_\mu \phi) \partial^\mu \phi ,$$

Both the beta functions and the Weyl anomaly coefficients will satisfy the consistency conditions to be presented below. However, while the latter satisfy them exactly, the former satisfy them up to a target reparametrization. Since both encode essentially the same RG information, for simplicity we will consider RG motions as generated by the Weyl anomaly coefficients in what follows. We define (at any order) an operation $R$ on a generic functional $F[g, b, \phi]$ to be

$$RF[g, b, \phi] = \frac{\delta F}{\delta g_{\mu\nu}} \cdot \bar{T}^g_{\mu\nu} + \frac{\delta F}{\delta b_{\mu\nu}} \cdot \bar{T}^b_{\mu\nu} + \frac{\delta F}{\delta \phi} \cdot \bar{T}^\phi ,$$

and an operation $T$ affecting (at lowest order) the transformations through

$$TF[g, b, \phi] = F[\tilde{g}, \tilde{b}, \tilde{\phi}]$$

(where $\tilde{\phi}$ will be defined shortly). The requirement that duality flows "covariantly" with the RG is expressed as

$$[T, R] = 0 .$$

When applied to (2) this leads to the consistency conditions first presented in (3) for the Weyl anomaly coefficients

$$\bar{T}^g_{00} = -\frac{1}{g_{00}} \beta^g_{00} ,$$

$$\bar{T}^b_{0i} = -\frac{1}{g_{00}} (b_{0i} \beta^b_{00} - \beta^b_{0i} g_{00}) ,$$

$$\bar{T}^\phi = \beta^\phi - \frac{1}{g_{00}} (\beta^g_{00} g_{00} + \beta^b_{0i} g_{00} - \beta^b_{0j} b_{0j} - \beta^b_{0i} b_{0j}) ,$$

$$\beta^\phi = \beta^\phi + \frac{1}{g_{00}} (\beta^g_{00} b_{0j} + \beta^b_{0i} b_{0j} - \beta^b_{0i} g_{00} - \beta^b_{0j} g_{00}) .$$

At loop order $\ell$, the possible tensor structures $T_{\mu\nu}$ appearing in the beta function must scale as $T_{\mu\nu}(Ag, Ab) = A^{1-\ell} T_{\mu\nu}(g, b)$ under global scalings of the background fields $\bar{F}$). At $O(\alpha')$ one may then have

$$\beta^g_{\mu\nu} = \alpha' (A R_{\mu\nu} + B H_{\mu\lambda\nu} H^{\lambda\nu} + C g_{\mu\nu} R ,$$

$$+ D g_{\mu\nu} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} ) ,$$

$$\beta^b_{\mu\nu} = \alpha' (E \nabla^\lambda H_{\mu\nu\lambda} ) ,$$

with $A, B, C, D, E$ being determined from one-loop Feynman diagrams. As found in (3), requiring (8) to be satisfied, and choosing $A = 1$ determines $B = -1/4$, $E = -1/2$, and $C = D = 0$, independently of any diagram calculations. As it turns out, the consistency conditions (8) on $g_{\mu\nu}$ and $b_{\mu\nu}$ alone also allows for an independent determination of the dilaton transformation (or "shift") $\phi = \phi - \frac{1}{2} \ln g_{00}$. Applying (4) to this then yields the dilaton beta function (8).

III. ORDER $\alpha''$

At the next order $R$ is modified by the two-loop beta functions, and one must determine the appropriate modifications in $T$ such that $[T, R] = 0$ continues to hold. We work at this order with restricted backgrounds of the form

$$g_{\mu\nu} = \left( \begin{array}{cc} a & 0 \\ 0 & \bar{g}_{ij} \end{array} \right) ,$$

and $b_{\mu\nu} = 0$, so that no torsion appears in the dual background either. It is useful to define at this point the following two quantities: $a_i \equiv \partial_i \ln a$, and $q_{ij} \equiv \nabla_i a_j + \frac{1}{2} a_i a_j$, where barred quantities here and below
refer to the metric \( \bar{g}_{ij} \) (also, indices \( i, j, \ldots \), are contracted with the metric \( g_{ij} \)). Within this class of backgrounds classical duality transformations reduce to the operation \( a \to 1/a \), and it is simple to determine the possible corrections to \( T \) from a few basic requirements: i) \( \bar{g}_{ij} = g_{ij} = \bar{g}_{ij} \) does not get modified, as it corresponds to sigma model couplings entirely disconnected from the path integral dualization procedure (cf. \[3\]); ii) corrections should be \( D \)-dimensional generally covariant; iii) corrections to \( \bar{a} = 1/a \) must be proportional to \( a \):

\[
\ln \bar{a} = - \ln a + \alpha' m_i a^i, \quad m_i = m_i(a, \bar{g}_{ij}),
\]

(11) as it is simple to see that classical consistency conditions would be satisfied for \( a = \) constant; iv) dimensional analysis: \([\alpha'] = L^2 \) and \([a_i] = 1/L \), where \( L \) is a target length, so that \([m_i] = 1/L \); v) \( m_i \) should not contain nontrivial denominators, as the corrections should be finite for finite geometries; vi) because the duality group should still be \( \mathbb{Z}_2 \), by applying the transformations \([4\] twice one should re-obtain the original model. This constrains \( m_i \) to be odd under classical duality:

\[
m_i = \bar{m}_i(1/a, \bar{g}_{ij}) = -m_i(a, \bar{g}_{ij}).
\]

(12) All of the above then yields

\[
m_i = \lambda a_i,
\]

(13) with \( \lambda \) an undetermined real constant. As discussed in \([3\], moreover, we shall also require the measure factor \( \sqrt{g} \exp(-2\phi) \) to be invariant (so that \([T, R] = 0 \) implies invariance of the string background effective action), thus fixing also the correction on the dilaton transformation to be \( 1/4 \) that of \( g_{00} \). Altogether, for the backgrounds \([4\] the corrected duality transformations are:

\[
\ln \bar{a} = - \ln a + \lambda' \alpha' a_i a^i, \quad \bar{g}_{ij} = g_{ij} = \bar{g}_{ij}, \quad \phi = \phi - \frac{1}{2} \ln a + \frac{\lambda}{4} \alpha' a_i a^i.
\]

(14) The consistency conditions again follow by applying \( R \) to the above and using \([T, R] = 0 \) on the l.h.s.:

\[
\frac{1}{a^2} \bar{\beta}_{00} = - \frac{1}{a^2} \beta_{00} + 2 \lambda' \left[ a^i \partial_i \left( \frac{1}{a^2} \beta_{00} \right) - \frac{1}{2} a^i a^j \bar{\beta}_{ij} \right],
\]

\[
\bar{\beta}_{ij} = \beta_{ij},
\]

(15) The terms scaling correctly under \( g \to \Lambda g \) at this order, and thus possibly present in the beta function, are

\[
\beta_{ij}^{(2)} = A_1 \nabla_i \nabla_j \bar{R} + A_2 \nabla^2 R_{\mu\nu} + A_3 R_{\mu\nu\alpha\beta} R^{\alpha\beta} + A_4 R_{\mu\nu\alpha\beta} R_{ij}^{\alpha\beta} + A_5 R_{\mu\nu\alpha\beta} R_{ij}^{\alpha\beta} + A_6 R_{\mu\nu} \bar{R} + A_7 g_{ij} \nabla^2 \bar{R} + A_8 g_{ij} R_{\mu\nu} + A_9 g_{ij} R_{\alpha\beta} R^{\alpha\beta} + A_{10} R_{\mu\nu} R_{\alpha\beta} \nabla^2 \bar{R}^{\alpha\beta}
\]

(16) (we have used Bianchi identities to reduce from a larger set of tensor structures). It will suffice in fact to study the consistency conditions for the \((ij)\) components, \( \bar{\beta}_{ij} = \beta_{ij} \), in order to determine the only structure satisfying all the consistency conditions.

We write

\[
\bar{\beta}_{ij} = \alpha' \left( \beta_{ij}^{(1)} + 2\nabla_i \partial_j \phi \right) + \alpha'^2 \beta_{ij}^{(2)}
\]

(17) where \( \beta_{ij}^{(1)} = R_{ij} - \frac{1}{2} \phi \bar{g}_{ij} \) is the one-loop beta function, and perform the duality transformation \([4\] keeping terms to order \( O(\alpha'^2) \). Using the fact that the one-loop Weyl anomaly coefficient satisfies the one-loop consistency conditions \([6\], we arrive at

\[
\bar{\beta}_{ij}^{(2)} = \beta_{ij}^{(2)} - \frac{1}{4} \lambda a_i \partial_j (a_k a^k),
\]

(18) where the duality transformation of \( \beta_{ij}^{(2)} \) is given simply by \( a \to 1/a \) without \( \alpha' \) corrections, since this is already \( O(\alpha'^2) \). Separating the possible tensor structures into even and odd tensors under \( a \to 1/a \),

\[
\beta_{ij}^{(2)} = E_{ij} + O_{ij}, \quad \bar{E}_{ij} = E_{ij}, \quad \bar{O}_{ij} = -O_{ij},
\]

(19) the even structures drop out and we are left with

\[
O_{ij} = \frac{1}{8} \lambda a_i \partial_j (a_k a^k).
\]

(20) We now perform a standard Kaluza-Klein reduction on the ten terms in \([10\] to identify which if any satisfy this condition. The results can be obtained using the formulas in the Appendix of \([6\], and are as follows:

\[
\text{(1) : } \nabla_i \nabla_j R = \bar{\nabla}_i \bar{\nabla}_j (\bar{R} - q_{mn}^a),
\]

\[
\text{ (2) : } \nabla^2 R_{ij} = (\nabla^2 + \frac{1}{2} a_k g^{k(i}) (R_{ij} - \frac{1}{2} g_{ij}) - \frac{1}{4} a_i a_j g_{mn}^a
\]

\[
- \frac{1}{4} a_k a_{(i} (\bar{R}_{j)k} - \frac{1}{2} q_{jk})
\]

\[
\text{ (3) : } R_{\alpha ij} R^{\alpha\beta} = \frac{1}{4} q^{ij} q_{mn}^a + R_{ijmn} (\bar{q}_{nm}^m - \frac{1}{2} q_{nm}^m)
\]

\[
\text{ (4) : } R_{\alpha ij} R_{ij}^{\alpha\beta} = \frac{1}{2} q_{ik} q_{jk}^k + R_{ij} (\bar{q}_{km}^m - \frac{1}{2} q_{km}^m)
\]

\[
\text{ (5) : } R_{ij} R^{\alpha\beta} = \bar{R}_{ij} R^{\alpha\beta} = \frac{1}{2} q_{ik} q_{jk}^k + R_{ijmn} (\bar{q}_{km}^m - \frac{1}{2} q_{km}^m),
\]

\[
\text{ (6) : } \bar{R}_{ij} R = (\bar{R}_{ij} - \frac{1}{2} q_{ij}) (\bar{R} - q_{mn}^a)
\]

\[
\text{ (7) : } g_{ij} \nabla^2 \bar{R} = \bar{g}_{ij} \left[ \frac{1}{2} a_k \partial_k (\bar{R} - q_{mn}^a)
\]

\[
+ \bar{\nabla}^k \partial_k (\bar{R} - q_{mn}^a)
\]

\[
\text{ (8) : } g_{ij} R^{2} = \bar{g}_{ij} (\bar{R} - q_{mn}^a)^2
\]

\[
\text{ (9) : } g_{ij} R_{\alpha ij} R^{\alpha\beta} = \bar{g}_{ij} \left[ \frac{1}{4} (q_{mn}^a)^2 + (\bar{R}_{km} - \frac{1}{2} q_{km}^m)\right]^2
\]

\[
\text{ (10) : } g_{ij} R_{\alpha ij} R^{\alpha\beta} = \bar{g}_{ij} \left[ q_{km} q_{km}^m + \bar{R}_{km m} \bar{R}_{km m} \right].
\]
The respective odd parts are
\[
O_{ij}^{(1)} = -\nabla_i \nabla_j \nabla^a a^a,
\]
\[
O_{ij}^{(2)} = \frac{1}{2} a_k \nabla^k R_{ij} - \frac{1}{2} \nabla^2 \nabla^a a^a R_{ij} - \frac{1}{4} a_i a_j \nabla^k a^k,
\]
\[
O_{ij}^{(3)} = -\frac{1}{2} R_{\alpha \beta \gamma} a^\alpha \nabla^\beta a^\gamma + \frac{1}{8} a_n a^a \nabla^i a_j + \frac{1}{8} a_i a_j \nabla^a a^a,
\]
\[
O_{ij}^{(4)} = \frac{1}{4} a_k a_i (\nabla^j)k,
\]
\[
O_{ij}^{(5)} = -\frac{1}{2} \tilde{R}_{(i} \nabla_{j)} a^k + \frac{1}{8} a_k a_i (\nabla^j)k,
\]
\[
O_{ij}^{(6)} = -\frac{1}{2} \tilde{R} \nabla_i a_j - \tilde{R} \nabla^a a^a + \frac{1}{4} a_i a_j \nabla^a a^a
+ \frac{1}{4} a_n a^a \nabla^i a_j,
\]
\[
O_{ij}^{(7)} = \tilde{g}_{ij} \left[ \frac{1}{2} a^k \partial_k (\tilde{R} - \frac{1}{2} a^m a^m) - \nabla^k \partial_k (\nabla^a a^a) \right],
\]
\[
O_{ij}^{(8)} = \tilde{g}_{ij} \left[ -2(\tilde{k} a^k) \tilde{R} + (\tilde{k} a^k) a^m a^m \right],
\]
\[
O_{ij}^{(9)} = \tilde{g}_{ij} \left[ \frac{1}{4} (\tilde{k} a^k) a^m a^m - (\tilde{k} a^k) \tilde{R}^m k
+ \frac{1}{4} (\tilde{k} a^k) a^k a^m \right],
\]
\[
O_{ij}^{(10)} = \tilde{g}_{ij} (\nabla^k a^m a^k a^m).
\]

It is fortunate that none of these tensors contain purely even structures, since such structures are left unconstrained (and thus undetermined) by duality. The only odd term of the form \(g_{ab}\) comes from \(A_4 R_{\mu \alpha \beta \gamma} R_{\nu} \alpha^\beta^\gamma\), and a detailed inspection shows that no linear combination of the other terms gives rise to odd tensors generically of the form \(O(\alpha^2)\). This determines that, with the requirement of covariance of duality under the RG, the \(O(\alpha^2)\) term in the beta function is
\[
\beta^{(2)}_{\mu \nu} = \lambda R_{\mu \alpha \beta \gamma} R_{\nu} \alpha^\beta^\gamma.
\]

One should now check that the corresponding (00) component also satisfies its consistency condition. A straightforward computation shows that it does, and the determination of the two-loop beta function is thus complete.

Although we treated a restricted class of metric backgrounds, our result is valid for a generic metric, since none of the possible tensor structures are built out of the off-block-diagonal \(g_{ab}\) elements alone (in which case our consistency conditions would be blind to them, just as they are to the even terms \(E_{ij}\)).

Some final comments on scheme dependence are also in order: for a purely metric background, it is well-known \([13]\) that the two-loop beta function is scheme independent within the standard set of subtraction schemes determined by minimal and nonminimal subtractions of the one-loop divergent structure \(R_{\mu \nu}\). Under a broader definition of subtraction scheme, however, when other terms may also be subtracted, e.g., of the type \(g_{\mu \nu} \tilde{R}\), then the beta function becomes scheme dependent, and differs from \(O(\alpha^2)\). Our duality constraints have determined a beta function falling into the first (and standard) class of schemes, i.e., those in which the one-loop subtractions are of the form \((\text{const.} + 1/\epsilon)R_{\mu \nu}\). This is natural to expect, as these represent the subtraction of the inherent divergence of the theory. However, it raises the question of whether the duality constraints clash against the possibility of making more general subtractions. We have recently found \([13]\) that in fact there is no clash, since it is possible to explicitly determine the modification in the duality transformations themselves under a field redefinition, and they will be such as to preserve the consistency conditions w.r.t. the redefined beta functions. The statement \([\mathcal{T}, \mathcal{R}] = 0\) thus acquires a meaning beyond and independent of any field redefinition ambiguity.

Simply using the requirements that duality and the RG commute as motions in the parameter space of the sigma model, we have been able to determine the two-loop beta function to be
\[
\beta_{\mu \nu} = \alpha' R_{\mu \nu} + \alpha'^2 \lambda R_{\mu \alpha \beta \gamma} R_{\nu} \alpha^\beta^\gamma,
\]
for an entirely generic metric background, without any Feynman diagram calculations. Because we used an extremely restrictive class of backgrounds, it was not possible to determine the value of \(\lambda\) (the correct value is \(\lambda = \frac{1}{2}\)). However, we expect that, similarly to what happens at \(O(\alpha')\), once a more generic background is used in the consistency conditions, even this constant should be determined.

That duality symmetry should yield information on the renormalization structure of a theory is to us a striking fact, and one which we intend to further explore in the future.

ACKNOWLEDGMENTS

One of us (RS) is partially supported by the Praxis XXI grant BD-3372/94 (Portugal).

[1] K. Kikkawa and M. Yamasaki, Phys. Lett. 149B (1984) 357.
[2] P.E. Haagensen, Phys. Lett. 382B (1996) 356.
[3] P.H. Damgaard and P.E. Haagensen, J. Phys. A30 (1997) 4681.
[4] C.P. Burgess and C.A. Lütken, Phys. Rev. B294 (1987) 383.
[5] P.E. Haagensen and K. Olsen, hep-th/9704157.
[6] A.A. Tseytlin, Mod. Phys. Lett. A6 (1991) 1721.
[7] T.H. Buscher, Phys. Lett. 194B (1987) 59.
[8] A.A. Tseytlin, Phys. Lett. 178B (1986) 34; Nucl. Phys. B294 (1987) 383.
[9] L. Alvarez-Gaumé and D.Z. Freedman, Phys. Rev. D22 (1980) 846.
[10] See for instance, R.R. Metsaev and A.A. Tseytlin, *Nucl. Phys.* B293 (1987) 385, sec. 4.

[11] P.E. Haagensen, hep-th/9708110, “Duality and the Renormalization Group”, in the Proceedings of the NATO Workshop *New Trends in Quantum Field Theory*, Zakopane, Poland, 14-21 June 1997, ed. P.H. Damgaard (Plenum Press, 1997).