Implementation of the Peak Stress Method for the automated FEM-assisted design of welded joints subjected to constant amplitude multiaxial fatigue loads

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Abstract. The Peak Stress Method (PSM) is a local approach to the fatigue strength assessment of welded structures. Starting from FE-calculated opening, in-plane shear and out-of-plane shear peak stresses at the weld toe and weld root, the PSM defines an equivalent peak stress for estimating the fatigue failure location and fatigue life of welded structures, in compliance with properly defined design curves. An interactive tool has been developed in Ansys\textsuperscript{®} Mechanical to automate all calculation tasks required to apply the PSM to generic welded structures. The developed application allows to identify and analyse all weld toe and weld root lines of the structure in a fully-automated way, performing fatigue life estimation on each analysed node. Finally, fatigue life results can be visualised directly on the model’s geometry through dedicated contour plots. In this work, some joint geometries taken from the literature and related to steel welded joints subjected to multiaxial fatigue loads are re-analysed taking advantage of two design procedures: (i) manual application of the PSM, (ii) automated implementation of the PSM. Taking advantage of the tool developed in Ansys\textsuperscript{®}, a remarkable reduction in analysis time and effort can be achieved, with respect to manual PSM analyses.

1. Introduction

According to the local fatigue approach based on the NSIF-parameters [1–3] the weld toe and weld root of an arc-welded joint can be modelled and studied as sharp V-notches having null tip radius \( \rho = 0 \) and notch opening angles equal to 135° and 0° respectively, as highlighted in next figure 1. Assuming a linear-elastic behavior for the material of the welded structure, external loads produce singular stress fields in the local regions enclosing weld toes and roots. The NSIF-parameters allow to quantify the intensity of the singular stress distributions, according to equation (1).

\[
K_i = \sqrt{2\pi} \lim_{r \to 0} \left( \sigma_{jk} \right)_{\theta = 0} \cdot r^{1-\lambda_i} \quad \text{where} \quad i = 1, 2, 3 \quad \text{and} \quad \sigma_{jk} = \sigma_{00}, \tau_{00}, \tau_{0z}
\]  

(1)

Where the linear elastic stresses \( \sigma_{00}, \tau_{00} \) and \( \tau_{0z} \) are calculated close to the notch tip \( (r \to 0) \), along the notch bisector direction \( (\theta = 0) \), as shown in next figure 2. Parameters \( \lambda_i \), i.e. stress singularity exponents, are function of the notch opening angle \( 2\alpha \) of the considered V-notch site, according to [4,5].

The strain energy density (SED) averaged over a structural volume surrounding the notch tip has been adopted as a fatigue strength criterion by Lazzarin and collaborators [6–8] in order to assess weld
toe and weld root fatigue failures. The closed-form expression of the averaged SED as a function of the relevant NSIF-parameters tied to mode I, mode II and mode III loadings is reported in equation (2).

$$\Delta \bar{W} = \left\{ \frac{c_{w1}}{E} \left[ \frac{\Delta K_1}{R_0^{1-\lambda_1}} \right]^2 + \frac{c_{w2}}{E} \left[ \frac{\Delta K_2}{R_0^{1-\lambda_2}} \right]^2 + \frac{c_{w3}}{E} \left[ \frac{\Delta K_3}{R_0^{1-\lambda_3}} \right]^2 \right\}$$

(2)

$R_0$ represents the size of the structural volume and equals 0.28 mm for welded joints made of structural steels. $E$ represents the material Young’s modulus of elasticity while $e_i$ ($i = 1, 2$ and $3$) are coefficients dependent on the notch opening angle $2\alpha$ and on material, in terms of Poisson’s ratio $\nu$. Moreover, $\Delta K_1$, $\Delta K_2$ and $\Delta K_3$ are the range values of the NSIFs tied to mode I, mode II and mode III, respectively. Finally, $c_{wi}$ account for the mean stress effect and depend on the load ratio $R_i$ of the $i$-th loading mode ($i = 1, 2$ or $3$), according to equation (3).

$$c_{wi}(R_i) = \begin{cases} 
\frac{1+R_i^2}{(1-R_i)^2} & \text{if stress-relieved and } -1 \leq R_i \leq 0 \\
\frac{1-R_i^2}{(1-R_i)^2} & \text{if stress-relieved and } 0 \leq R_i < 1 \\
1 & \text{if as-welded for any } R \text{ value}
\end{cases}$$

(3)

It is worth noting that $c_{wi} = 1$ should be adopted in the case of welded joints tested in as-welded conditions, being them almost insensitive to mean stresses, according to design standards [9].

![Figure 1](attachment:image1.png)

**Figure 1.** Details of the NSIF-based and averaged SED approaches with respect to a partial-penetration tube-to-flange welded joint under multiaxial fatigue loading. (a) Cylindrical reference system $(r, \theta, z)$ centred at the weld toe and local elastic stresses. (b) Structural volume of size $R_0$ centred at the weld toe and weld root, according to the averaged SED approach.
2. The Peak Stress Method (PSM)

The Peak Stress Method (PSM) is a rapid, FE-oriented tool to estimate the NSIF-terms $K_1$, $K_2$ and $K_3$, taking advantage of the local peak stresses, i.e. opening, in-plane shear and out-of-plane shear peak stresses, calculated from a linear elastic FE analysis [10]. One of the main advantages of the PSM is the possibility to employ coarse meshes with respect to those required by other local approaches, such as the NSIF approach. Moreover, only the linear elastic peak stresses calculated at the notch tip suffice to be considered. The estimated NSIFs values can be evaluated from the following expressions [11–13]:

\begin{align}
(a) \quad K_1 &= K_{FE}^* \cdot \sigma_{\theta=0,0,\text{peak}} \cdot d^{1/2}; \\
(b) \quad K_2 &= K_{FE}^{**} \cdot \tau_{\theta=0,0,\text{peak}} \cdot d^{1/2}; \\
(c) \quad K_3 &= K_{FE}^{***} \cdot \tau_{0z,0,\text{peak}} \cdot d^{1/2} 
\end{align}

The terms $\sigma_{\theta=0,0,\text{peak}}$, $\tau_{\theta=0,0,\text{peak}}$ and $\tau_{0z,0,\text{peak}}$ in equation (4) refer to the peak stresses defined in a local cylindrical coordinate system centered at the node at the V-notch tip, having $z$-direction tangential to the notch tip edge, $r$-direction defined by the notch bisector line and $\theta$ as the radial direction (see next figure 3). Each peak stress is evaluated with respect to $\theta = 0$, which means that, as an example, the opening stress $\sigma_{\theta=0,0,\text{peak}}$ acts in the normal direction with respect to the notch bisector. Parameter $d$ in equation (4) refers to the average size of the finite elements to be given as input to the free mesh generation algorithm of the FE software. Parameters $K_{FE}^*$, $K_{FE}^{**}$ and $K_{FE}^{***}$ in equation (4) depend on: (i) element type and formulation; (ii) FE mesh pattern; (iii) procedure employed by the FE code to extrapolate stress values at nodes. The reader is referred to the recently published state-of-the-art review of the PSM [10] for additional detail about the method.

2.1. The Peak Stress Method with tetrahedral finite elements

The main advantage in adopting tetrahedral finite elements resides in the possibility to easily discretize either simple and complex 3D geometries without the need for sub-modeling techniques. Dealing with the application of the PSM with tetrahedral elements, an average peak stress formulation [10,14] has been introduced according to equation (5) in order to compensate the variability of peak stress values along the notch tip profile due to the intrinsic irregularity of the free tetra mesh pattern. More in detail, equation (5) performs the moving average of the peak stresses calculated on three adjacent vertex nodes, i.e. $n = k-1$, $n = k$ and $n = k+1$ and assigns the calculated average stress at the central node ($n = k$).

\[ \bar{\sigma}_{ij,\text{peak},n=k} = \frac{\sigma_{ij,\text{peak},n=k-1} + \sigma_{ij,\text{peak},n=k-1} + \sigma_{ij,\text{peak},n=k+1}}{3} \quad (n=\text{node}) \]

Moreover, peak stresses evaluated at the notch tip nodes belonging to free surfaces of the analysed structure can be affected by distorted mesh patterns and are therefore to be excluded. Furthermore, only peak stresses calculated at vertex nodes located along the notch tip line must be used, while peak stress values related to mid-side nodes are to be neglected. PSM parameters $K_{FE}^*$, $K_{FE}^{**}$ and $K_{FE}^{***}$ have previously been calibrated using either 10-node [14] and 4-node [15] tetra elements (see table 1). The calibration of parameters $K_{FE}^*$, $K_{FE}^{**}$ and $K_{FE}^{***}$ for tetra element types has been performed under the following conditions, which are more widely discussed in [10]:

a. A range of notch opening angle $2\alpha$ values is supported. As reported in table 1, in the case of mode I and mode III loadings, the range $2\alpha = 0^\circ - 135^\circ$ has been investigated, being $2\alpha = 0^\circ$ and $2\alpha = 135^\circ$ the typical notch opening angles at weld root and weld toe, respectively. However, weld toe profiles having opening angles up to $150^\circ$ have been analysed adopting the same calibration constants [16]. In the case of mode II loading, only $2\alpha = 0^\circ$ [14,15] and $2\alpha = 90^\circ$ [17] have been investigated, being the typical cases of the weld root without or with a gap.

b. A minimum mesh density ratio $(a/d)_{\text{min}}$ must be fulfilled when defining the free mesh pattern at the notch tip according to the PSM, $a$ being the characteristic size of the considered notch, defined in
As reported in Table 1, the value of \((a/d)_{\text{min}}\) depends on: (i) adopted FE type, (ii) considered loading mode and (iii) opening angle \(2\alpha\) of the analysed notch.

Table 1: Summary of parameters \(K_{\text{FE}}^\star\), \(K_{\text{FE}}^{\star\star}\) and \(K_{\text{FE}}^{\star\star\star}\) and mesh density \(a/d\) requirements to apply the Peak Stress Method with 10-node SOLID 187 tetra elements in Ansys® FE software [10].

| Loading   | PSM parameters | \(2\alpha = 0^\circ\) | \(2\alpha = 90^\circ\) | \(2\alpha = 120^\circ\) | \(2\alpha = 135^\circ\) | \(a\) root side | \(a\) toe side |
|-----------|----------------|------------------------|------------------------|------------------------|------------------------|----------------|-------------|
| Mode I    | \(K_{\text{FE}}^\star\) \((a/d)_{\text{min}}\) | 1.05±15%               | 1.05±15%               | 1.05±15%               | 1.21±10%               | min\(\{l, z\}\) | \(t\)        |
| Eq. (1a)  |                | 3                      | 3                      | 3                      | 1                      |                 |             |
| Mode II   | \(K_{\text{FE}}^{\star\star}\) \((a/d)_{\text{min}}\) | 1.63±20%               | 2.65±10%               | n.a.                   | n.a.                   | min\(\{l, z\}\) | \(t\)        |
| Eq. (1b)  |                | 1                      | 1                      | n.a.                   | n.a.                   |                 |             |
| Mode III  | \(K_{\text{FE}}^{\star\star\star}\) \((a/d)_{\text{min}}\) | 1.37±15%               | 1.37±15%               | 1.70±10%               | 1.70±10%               | \(t\)          |             |
| Eq. (1c)  |                | 3                      | 3                      | 3                      | 3                      |                 |             |

*‘Full graphics’ option must be activated when calculating peak stresses according to PSM 3D

2.2. Fatigue design according to the PSM

The averaged SED can be expressed as a function of the relevant NSIF-parameters \(K_1\), \(K_2\) and \(K_3\), according to equation (2). Moreover, the PSM allows to rapidly estimate the NSIFs-terms as a function of the relevant FE-calculated peak stresses, according to equations (4a)-(4c). As a result, the averaged SED itself can be re-written as a function of the relevant peak stresses. By introducing the SED expression for an equivalent uniaxial strain state (i.e. \(W=\left(1-\nu^2\right)\sigma_{\text{eq,peak}}^2/2E\)), an equivalent peak stress term can be defined according to equation (6) [10] and can be referred as a design stress quantity for the fatigue strength assessment of welded joints.

\[ \Delta \sigma_{\text{eq,peak}} = \sqrt{c_{w1}f_{w1}^2\Delta \tau_{0,0,0=0,\text{peak}}^2 + c_{w2}f_{w2}^2\Delta \tau_{0,0,0=0,\text{peak}}^2 + c_{w3}f_{w3}^2\Delta \tau_{0,0,0=0,\text{peak}}^2} \]  

Equation (6) is valid for 4-node as well as 10-node tetra elements and the averaged peak stress terms \(\Delta \tau_{0,0,0=0,\text{peak}}\), \(\Delta \tau_{0,0,0=0,\text{peak}}\) and \(\Delta \tau_{0,0,0=0,\text{peak}}\) are defined according to equation (5). Parameters \(f_{wi}\) \((i = 1, 2 \text{ and } 3, \text{ according to the loading mode})\) account for peak stress averaging inside the material structural volume having size \(R_0\) (see figure 1) [10] and are defined as in following equation (7).

\[ f_{wi} = K_{\text{FE}}: \sqrt{\frac{2e_i}{1-\nu^2}} \left( \frac{d}{R_0} \right)^{1-\nu_i} \]  

where \(i = 1, 2, 3\).

The FE-calculated peak stresses involved in equations (5)-(6), as well as \(f_{wi}\) parameters in equation (7), are functions of the average FE size \(d\) adopted in the FE model (see next figure 3); however, the equivalent peak stress defined in equation (6) is ultimately independent from the FE size \(d\) owing to the multiplication of the peak stresses by the relevant \(f_{wi}\) parameters [10].

Moreover, dealing with the fatigue assessment of welded joints subjected to multiaxial loads, an approximate approach originally proposed in [18] has been adopted in the current work. The strain energy contribution tied to each separate load case \(j\) must be accounted and evaluated individually. Afterwards, all averaged SED contributions are to be combined in order to determine the total averaged SED value, according to equation (8).
\[ \Delta \bar{W} = \sum_j \Delta \bar{W}_j \]  

(8)

Introducing the SED expression for an equivalent uniaxial strain state (i.e. \( W = (1-v^2)\sigma_{\text{eq,peak}}^2/2E \)) as before, the equivalent peak stress can be re-formulated according to the following equation (9).

\[ \Delta \sigma_{\text{eq,peak}} = \sqrt{\sum_j c_{w,j} \left( f_{w1,j}^2 \Delta \sigma_{00,0=0,\text{peak}}^2 + f_{w2,j}^2 \Delta \sigma_{00,0=0,\text{peak}}^2 + f_{w3,j}^2 \Delta \sigma_{0z,0=0,\text{peak}}^2 \right)} \]  

(9)

Assuming for instance the case of a partial-penetration tube-to-flange welded joint under combined bending-torsion loading (see next figure 2), the averaged SED contributions tied to the bending load \((M_b)\) and to the torsion load \((M_t)\) are to be accounted separately and furtherly combined according to equation (8). In order to do so, each separate load case must be considered and solved individually and. Finally, the equivalent peak stress tied to the combined bending-torsion loading condition must be calculated according to equation (9), where \(j\) identifies the single accounted load case, i.e. \(j = M_b, M_t\).

The equivalent peak stress can be adopted to estimate the fatigue life of welded joints, in compliance with a proper PSM-based design curve. A local biaxiality ratio \(\lambda\) has been formulated as the ratio between the SED contributions due to mode II/III shear and mode I stresses, in order to guide the selection of the correct design curve to address. \(\lambda\) can be expressed as a function of the relevant averaged peak stresses according to equation (10), which is valid or 4-node as well as 10-node tetra elements.

\[ \lambda = \frac{c_{w2} f_{w2}^2 \Delta \sigma_{0z,0=0,\text{peak}}^2 + c_{w3} f_{w3}^2 \Delta \sigma_{0z,0=0,\text{peak}}^2}{c_{w1} f_{w1}^2 \Delta \sigma_{00,0=0,\text{peak}}^2} \]  

(10)

The proper PSM-based design curve can be selected according to the criteria proposed in [10] and reported in table 2 for the case of joints made of structural steels, where \(\Delta \sigma_{\text{eq,peak,A,50\%}}\) is the fatigue class at \(2\cdot10^6\) cycles (for a survival probability of 50%) and \(\lambda\) is the slope of the fatigue design curve. The case \(\lambda = 0\) corresponds to a pure local mode I stress state, while \(\lambda > 0\) implies a mixed mode opening-shear stress condition [10].

Finally, fatigue life can be estimated with respect to the desired survival probability by comparing the equivalent peak stress with the proper PSM-based fatigue design curve (see next figure 4).

**Table 2.** Criterion of selection for the PSM-based fatigue design curve for steel arc-welded joints [10].

| Thickness, \(T\) [mm] | \(\lambda\) Eq. (10) | \(\Delta \sigma_{\text{eq,peak,A,50\%}}\) [MPa] | \(k\) | \(T_d\) |
|------------------------|----------------------|------------------------|------|------|
| \(T \geq 2\) mm        | \(\lambda = 0\)      | 214                    | 3.0  | 1.90 |
| \(T \geq 2\) mm        | \(\lambda > 0\)      | 354                    | 5.0  | 1.90 |

2.3. Step-by-step manual application of the PSM for the fatigue assessment of welded joints

The current paragraph reports all the elaboration steps required to manually apply the PSM for the fatigue assessment of a generic arc-welded structure. The reader is referred to the recently published state-of-the-art review of the PSM [10] and to [17] for a comprehensive insight on the application of the method.

a) Identify weld toes and weld roots in the structure. For each considered notch site, define the notch opening angle \(2\alpha\) and the characteristic notch size \(a\), as indicated in next figure 2. Moreover, identify the thickness \(T\) characterizing the welded detail under exam.
b) Adopt a proper global element size $d$ in order to satisfy the $(a/d)_{\text{min}}$ PSM requirements (as a function of the opening angle $2\alpha$ and loading mode, as reported in table 1) and generate a free FE mesh. Solve the FE model afterwards.

Figure 2. Tube-to-flange welded joint under combined bending-torsion loading. Criterion for the selection of the characteristic notch size $a$, according to the PSM.

Once solution is done, for each node on the considered notch tip line (weld toe or weld root), iterate the following steps (c-h):

c) Define a proper local coordinate system $(r,\theta,z)$ centred on the considered node, having directions as indicated in next figure 3. Retrieve the relevant peak stress components with respect to the local coordinate system (see figure 3).

d) Perform the moving average calculation on adjacent vertex nodes, in order to calculate the average peak stress components according to equation (5) and figure 3.

e) Evaluate the stress singularity exponents $\lambda_i$ and the averaged SED coefficients $e_i$. Recently, some polynomial expressions were proposed in [17,19] to achieve rapid evaluation of $\lambda_i$ exponents as a function of the notch opening angle $2\alpha$. Accordingly, two-variables polynomial expressions were also proposed in [17,19] to achieve rapid evaluation of $e_i$ coefficients as a function of either the notch opening angle $2\alpha$ and the considered material, in terms of Poisson’s ratio $\nu$.

f) Evaluate the mean stress correction factors $c_{wi}$ according to equation (3), which account for the mean stress effect and depend on the load ratio $R_i$ of the $i$-th loading mode ($i = 1, 2$ and $3$). Evaluate the PSM parameters $f_{wi} (i = 1, 2$ and $3)$ according to equation (7), which account for peak stress averaging inside the material structural volume having size $R_0$ (see figure 1).

g) Calculate the equivalent peak stress, according to equation (6). Moreover, dealing with the fatigue assessment of the welded joints subjected to multiaxial loads, each different load case must be simulated separately and the equivalent peak stress tied to the combined load case must be evaluated using equation (9), according to the approximate approach described in paragraph 2.2 and originally proposed in [18].
**Figure 3.** FE model of a tube-to-flange welded joint under combined bending-torsion loading. Definition of local coordinate systems and extraction of the peak stresses according to the PSM.

h) Calculate the local biaxiality ratio \( \lambda \) according to equation (10), in order to address the proper PSM-based design curve. Finally, perform fatigue life estimations with respect to the desired survival probability by comparing the equivalent peak stress calculated at (g) with the proper PSM-based fatigue design curve, according to figure 4.

\[
\Delta \sigma_{\text{eq}, \text{peak}} = \Delta \sigma_{\text{eq}, \text{peak}, \lambda = 0} + \frac{\Delta \sigma_{\text{eq}, \text{peak}, \lambda > 0} - \Delta \sigma_{\text{eq}, \text{peak}, \lambda = 0}}{2} \times \lambda
\]

**Figure 4.** Fatigue strength assessment of weld toe and weld root failures according to the PSM in the case of structural steel arc-welded joints having thickness \( T \geq 2 \) mm and \( \lambda \geq 0 \) (see table 2).
3. Automated implementation of the PSM for the fatigue design of welded joints

An interactive application has been developed in Ansys® Mechanical APDL [17,19], in order to automate the fatigue strength assessment of welded structures subjected to either uniaxial and multiaxial loading conditions, according to the PSM. In this perspective, a more powerful and efficient tool has been recently developed for Ansys® Mechanical, the FE environment of Ansys® Workbench, taking advantage of Ansys® Customization Toolkit (Ansys® ACT). ACT is a programming interface, designed to grant support and integration between Ansys® FE code and modern programming languages (i.e. IronPython, Microsoft® C#, Microsoft® Jscript, XML, HTML) allowing the user to develop applications and integrate them within the different CAD/FE environments available in Ansys® Workbench, in order to automate different modeling, simulation and post-processing tasks. The application presented in this work is designed to provide the analyst a user-friendly set of tools tailored to automate each implementation task required to apply the PSM to a generic FE model (see paragraph 2.2). Therefore, two different graphical interfaces have been developed: (i) an ACT Wizard, namely an interactive application designed to guide the analyst throughout the automated PSM analysis and (ii) a dedicated toolbar, which provides different post-processing features that allow visualization and consultation of analysis results through contour plots, charts and numerical reports.

More in detail, the developed tool is designed to automate the following PSM application tasks:

a. Perform compatibility checks on the FE model according to PSM requirements, in terms of mesh density \(a/d\) with respect to \((a/d)_{\text{min}}\) for the adopted element type (see table 1), depending on user-inputs on \(a\), \(d\) and \(T_{\text{min}}\) (i.e. the minimum welded thickness in the structure).

b. Automatically identify weld toes and weld roots among all the edges of the structure, by locally evaluating the notch opening angle \(2\alpha\) along model edges (see also figure 1 and figure 2).

c. Calculate PSM-related parameters, i.e. stress singularity exponents and SED coefficients. Stress singularity exponents \(\lambda_i\) and SED coefficients \(e_i\) are computed taking advantage of the polynomial expressions proposed in [17,19], as function of the opening angle \(2\alpha\) and material Poisson’s ratio \(\nu\).

d. Define and orientate a local coordinate system on each analysed peak node in order to retrieve the relevant peak stresses tied to loading mode I, mode II and mode III, according to figure 3.

e. Perform the moving average operation on adjacent vertex nodes, according to figure 3.

f. Calculate PSM parameters \(c_{\omega i}\) and \(f_{\omega i}\) \((i = 1, 2\) and \(3\)) according to equations (3, 7) and combine the calculated average peak stresses into the equivalent peak stress on each analysed node, according to equation (6).

g. Calculate the local biaxiality ratio \(\lambda\) on each analysed node, according to equation (10).

h. Estimate fatigue life on each analysed node, referencing to the proper PSM-based design curve among those proposed in [10] according to material type, local biaxiality ratio and survival probability for fatigue life estimations (see figure 4).

The developed tool also provides post-processing features that allow the analyst to access and visualize the analysis results with different grades of detail.

a. **Through contour plots.** Fatigue life, equivalent peak stress and local biaxiality ratio results can be plotted directly along analysed edges on a wireframe view of the geometry, as shown in next figure 5 and figure 9. Coloured contour plots allow to rapidly single out and compare at a glance the different competing crack initiation sites of the analysed structure.

b. **Through results charts.** Fatigue life, equivalent peak stress and local biaxiality ratio results can be consulted taking advantage of dedicated comparison charts, which can be generated directly within Ansys® Mechanical. Comparison charts allow the analyst to simultaneously plot and compare results distributions on one or more analysed edges.

c. **Through numerical reports.** A comprehensive results report is generated during the automated analysis (see figure 5). All numerical values related to analysed edges and nodes, i.e. edge id, nodal coordinates, relevant peak stresses, average peak stresses, local biaxiality ratio as well as equivalent peak stress and fatigue life values for each analysed node are collected and available in a log file, which can be directly opened and consulted in Microsoft® Excel from the dedicated shortcut.
Figure 5. The automated PSM tool developed in Ansys® Mechanical. Illustration and description of the automated analysis workflow for the FE analyst.
4. Steel joints under multiaxial loads: geometries and FE analyses according to the PSM

Some experimental data available in literature, related to fatigue failures at the weld toe and weld root in steel welded joints subjected to either uniaxial and multiaxial fatigue loading have been considered and re-analysed according to the PSM [20]. The analysed joint geometries and loading conditions are reported in table 3. Previous PSM analyses were conducted in [21] on the considered geometries, taking advantage of either free meshes generated using 2D 4-node plane harmonic elements and mapped meshes generated using 3D 8-node brick elements (PLANE 25 and SOLID 185 of the Ansys® element library, respectively). The joints characterized by an axis-symmetric geometry were simulated using PLANE 25 plane harmonic elements, while the remaining geometries were simulated using SOLID 185 brick elements (K-option 2 set to 3), taking advantage of the sub-modeling technique available in Ansys® Mechanical APDL to isolate and analyze the critical areas of the joint at the weld toe and weld root.

Table 3. Arc-welded joint geometries and load cases re-analysed according to the PSM [20].
All geometries reported in table 3 have been re-analysed by means of the following assessment approaches: (i) manual PSM application in Ansys® Mechanical APDL, as described in paragraph 2.2, and (ii) automated PSM implementation, taking advantage of the developed tool available in Ansys® Mechanical (see paragraph 3). A free mesh pattern of 3D 10-node tetrahedral elements (SOLID187 of the Ansys® element library) has been generated for each considered geometry, taking advantage of the free meshing algorithms provided either by Ansys® Mechanical APDL and Ansys® Mechanical, in the case of the manual and automated PSM analyses, respectively. Loads and constraints, as well as the adopted global element size $d$, have been applied likewise to the FE models, either in Ansys® Mechanical APDL and Ansys® Mechanical. Moreover, differently from the analyses conducted in [21], no sub-modeling techniques were required due to the higher 3D discretization capabilities offered by 10-node tetra elements with respect to plane and brick elements.

In this paper, a set of welded joints has been considered among all analysed geometries of table 3 in order to provide a deeper insight on the analysis procedures and achieved results. Information related to material properties, as well as involved welding process and testing conditions for the considered geometries is reported in table 4, while next figures 6–8 report more details on the FE analyses performed according to the PSM.

### Table 4. Material properties, welding process and testing conditions of the considered steel joints [21].

| Reference                  | Material | Yield strength [MPa] | Ultimate strength [MPa] | Welding process | Testing conditions |
|----------------------------|----------|----------------------|-------------------------|-----------------|--------------------|
| Yousefi et al. [23]        | P 460    | 520                  | 670                     | MIG             | Stress-relieved    |
| Frendo and Bertini [29]    | S355 JR  | 360                  | 520                     | -               | As-welded          |
| Takahashi et al. [30,31]   | JIS SM400B | 283              | 432                     | MAG             | As-welded          |

All joints have un-machined welds. Joint geometries and loading conditions are reported in figures 6–8.

#### 4.1. Yousefi et al. [23], tube-to-flange joints

The tube-to-flange joints under exam are characterized by a weld toe and a weld root with partial penetration and have been fatigue-tested under reversed ($R = -1$) as well as pulsating ($R = 0$) pure bending, pure torsion and combined bending-torsion loadings [23], as reported in figure 6 and table 5. Multiaxial loadings were applied both in-phase ($\phi = 0^\circ$) and out-of-phase ($\phi = 90^\circ$) by adopting a nominal biaxiality ratio $\Lambda = \sigma_{nom}/\tau_{nom} = 1$ [21]. The original experimental results are expressed in terms of nominal stress range (maximum minus minimum value) with respect to the tube and the accounted number of cycles refers to the generation of a through-the-thickness fatigue crack.

The geometry has been modelled as 1/4 of the entire welded joint, taking advantage of the axisymmetry, and has been simulated taking advantage of a 10-node tetra free-generated 3D mesh either in Ansys® Mechanical APDL and Ansys® Mechanical, as shown in figure 6. The element size is prescribed by the mesh density requirement $a/d \geq 3$. The tube thickness is assumed as characteristic size at weld toe, namely $a = 8$ mm. Consequently, $d$ must be at least equal to $8/3=2.67$ mm according to the requirements in table 1, in order to achieve compatibility with respect to either mode I and mode III loadings at weld toe. A global element size of 2 mm has been finally adopted to generate the mesh pattern. Moreover, a mesh refinement was performed at the weld root, in order to achieve a local element size of $\approx 0.3$ mm (see figure 6) and guarantee PSM applicability. In this case indeed $a = 1$ mm, namely equal to the weld root length, and $d$ must be at least equal to $1/3 = 0.33$ mm, in order to achieve compatibility with respect to mode I, mode II and mode III loadings at the weld root.

All welded joints were tested under stress-relieved conditions; therefore, the nominal load ratio $R$ has been taken into account by means of equation (3) and the equivalent peak stress has been calculated on each analysed node, either in the manual PSM analysis and by the automated one. As reported in table 5, both the manual and automated PSM analyses allow to correctly estimate the crack initiation site, which always occurred at the weld toe [23].
4.2. Frendo and Bertini [29], tube-to-flange joints

The tube-to-flange fillet-welded joints under exam (see figure 7) were fatigue-tested under reversed (R = -1), as well as pulsating (R = 0), in-phase (φ = 0°) and out-of-phase (φ = 90°) combined bending-torsion loading. Nominal biaxiality ratios $\Lambda = \sigma_{nom}/\tau_{nom} = 0.88$ and $\Lambda = 3.25$ were adopted during the tests. The number of cycles to break-through reported in the original contribution [29] were expressed in terms of bending and torsion nominal stresses, according to Navier’s formula and Bredt’s formula respectively.

The geometry has been modelled as 1/4 of the entire welded joint, taking advantage of the axisymmetry, and has been simulated by means of a 10-node tetra free-generated 3D mesh either in Ansys® Mechanical APDL and Ansys® Mechanical, as reported in figure 7. In order to guarantee PSM applicability at either weld toe and weld root with respect to mode I, mode II (at the weld root only) and mode III loadings, the mesh density requirement $a/d \geq 3$ needs to be enforced. The characteristic notch size equals the tube thickness at weld toes, namely $a = 10$ mm, which also corresponds to weld roots length and to weld legs length. Consequently, $d$ must be equal to or lower than $10/3 = 3.33$ mm. Finally, a 10-node tetra mesh of global element size 3 mm has been generated (see figure 7), achieving PSM applicability at both weld toes and weld roots simultaneously.

All welded joints were tested in as-welded conditions and the equivalent peak stress has been calculated on each analysed node either in the manual PSM analysis and by the automated one. As reported in table 5, both the manual and automated PSM analyses allow to correctly estimate the crack initiation site, which always occurred at the weld toe of the outer weld bead [29].
4.3. Takahashi et al. [30,31], plates with box-fillet-welded (wrap-around) joint

The geometry under exam is composed by two steel plates having a box-fillet-welded (wrap-around) joint (see figure 8). The joints were tested under pulsating biaxial fatigue loadings, either in-phase and out-of-phase, by adopting the nominal biaxiality ratios $\Lambda$ reported in table 5. The number of cycles provided by the original contributions [30,31] refer to: (i) the first technically detectable crack, corresponding to a crack depth of $1 \div 2$ mm; (ii) the number of cycles to complete failure of the welded joint, which will be considered by the re-analysis proposed in the current work.

The geometry has been modelled as 1/8 of the entire welded joint, taking advantage of the three symmetry planes characterizing the model, and has been simulated by means of a 10-node tetra free-generated 3D mesh either in Ansys® Mechanical APDL and Ansys® Mechanical, as reported in figure 8. In order to guarantee PSM applicability at either weld toe and weld root with respect to mode I, mode II (at the weld root only) and mode III loadings, the mesh density requirement $a/d \geq 3$ needs to be enforced. The characteristic notch size equals half the plates thickness at weld toes, namely $a = 6$ mm, which also corresponds to the weld leg length. Consequently, $d$ must be at least equal to $6/3 = 2$ mm. Finally, a 10-node tetra mesh of global element size 1 mm has been generated in order to provide a larger number of nodes for the calculations, achieving PSM applicability at both the weld toe and weld root simultaneously.

All welded joints were tested in as-welded conditions and the equivalent peak stress has been calculated on each analysed node either in the manual PSM analysis and by the automated one. Dealing with the experimental data, Takahashi et al [30,31] reported that fatigue failures always occurred at the weld toe with respect to the critical point A, with one exception where the fatigue crack initiated at the critical point B (see figure 8 and table 5).
5. Manual and automated assessment of weld toe and weld root fatigue failures

The original fatigue results, provided in terms of nominal stress range and corresponding number of cycles to failure for each one of the considered geometries, have been re-evaluated in terms of equivalent peak stress range at the crack initiation site by means of both manual and automated PSM analyses for each geometry.

Dealing with welded joints subjected to multiaxial loading conditions, each load case has been simulated separately by means of a FE analysis, according to the approximate approach described in paragraphs 2.2-2.3. $\sigma_{\text{nom}} = 1$ MPa has been enforced for axial and bending load cases; $\tau_{\text{nom}} = 1$ MPa has been enforced for torsional load cases. Afterwards, the equivalent peak stress range tied to each combined load case has been evaluated according to equation (9), taking account of the corresponding nominal load ratio $\Lambda = \sigma_{\text{nom}}/\tau_{\text{nom}}$ (see table 5). In order to convert the experimental data from nominal stress range to equivalent peak stress range, each nominal stress range given in the experimental fatigue tests has been multiplied by the equivalent peak stress calculated at the estimated crack initiation point from the FE model.

Figure 8. Plates with box-fillet-welded (wrap-around) joint geometry by Takahashi et al. [30,31]. Loading conditions and details of the FE analyses performed according to the PSM. FE mesh generated in Ansys® Mechanical FE software.
Next figures 10-11 show good agreement between the experimental fatigue results and the fatigue life estimations provided by the PSM, dealing with fatigue failures at the weld toe and weld root of the considered joints subjected to either uniaxial and multiaxial loading conditions. Moreover, figures 10-11 allow to compare the results coming from the manual PSM application with the ones generated by the automated PSM analysis. It has been noted that percent variations between the equivalent peak stress values obtained from the two approaches are always below 10% and account for: (i) the different mesh patterns generated by the two FE programs, i.e. Ansys® Mechanical APDL and Ansys® Mechanical, given the joint geometry; (ii) the different procedures adopted to define the local coordinate systems and extrapolate the relevant peak stresses on the analysed nodes. The automated algorithm performs indeed a numerical estimation of the notch opening angle 2α at each peak node and refers to nodal coordinates in order to define local coordinate systems and retrieve the peak stresses. Therefore, the automated procedure is affected by a degree of approximation which is tied to FE mesh density.

| Reference | Load case | Rr (R2x) | Rr (R2y) | Phase | A | Failure location | Manual PSM | Automated PSM | Δσeq,peak [MPa] | Δτnom [MPa] | Δτnom [MPa] |
|-----------|-----------|-----------|-----------|--------|---|-----------------|------------|--------------|----------------|------------|------------|
| Yousefi et al. [23] | B | -1 | - | - | ∞ | Toe 1 (16) | 1.703 | 1.771 | 207±825 | - | - |
| | B | 0 | - | - | ∞ | Toe 1 (9) | 2.408 | 2.505 | 187±261 | - | - |
| | T | -1 | -1 | - | 0 | Toe 1 (9) | 2.019 | 2.121 | - | 242±401 | - |
| | T | 0 | 0 | - | 0 | Toe 1 (9) | 2.855 | 2.999 | - | 209±304 | - |
| | B+T | -1 | -1 | 0 | 1 | Toe 1 (7) | 2.644 | 2.744 | 145±312 | 145±312 | - |
| | B+T | 0 | 0 | 0 | 1 | Toe 1 (7) | 3.740 | 3.880 | 144±358 | 144±358 | - |
| | B+T | -1 | -1 | 90 | 1 | Toe 1 (8) | 2.644 | 2.744 | 140±220 | 140±220 | - |
| | B+T | 0 | 0 | 90 | 1 | Toe 1 (9) | 3.740 | 3.880 | 145±312 | 145±312 | - |
| Frendo and Bertini [29] | B+T | -1 | -1 | 90 | 3.25 | Toe 1 (5) | 16.028 | 16.724 | 98±187 | 30±57 | - |
| | B+T | 0 | 0 | 90 | 3.25 | Toe 1 (5) | 16.028 | 16.724 | 78±178 | 24±55 | - |
| | B+T | -1 | -1 | 90 | 0.88 | Root (5) | 6.242 | 6.282 | 56±100 | 64±114 | - |
| | B+T | 0 | 0 | 90 | 0.88 | Root (5) | 6.242 | 6.282 | 50±90 | 57±103 | - |
| Takahashi et al. [30,31] | A | 0 | 0 | 180 | 1.02 | Toe 1 A (5) | 2.994 | 2.916 | 79±132 | 78±130 | - |
| | A | 0 | 0 | 180 | 0.85 | Toe 1 A (1) | 2.549 | 2.482 | 111 | 130 | - |
| | A | 0 | 0 | 180 | 0.68 | Toe 1 A (1) | 2.116 | 2.061 | 87 | 130 | - |
| | A | 0 | 0 | 180 | 0.34 | Toe 1 B (1) | 1.599 | 1.695 | 45 | 131 | - |
| | A | 0 | 0 | 180 | 0 | Toe 1 A (1) | 4.341 | 4.227 | 130 | -85 | - |
| | A | 0 | 0 | 180 | 0.25 | Toe 1 A (1) | 3.611 | 3.517 | 106 | -84 | - |
| | A | 0 | 0 | 180 | 1.02 | Toe 1 A (1) | 2.998 | 2.920 | 87 | -84 | - |
| | A | 0 | 0 | 180 | 0 | Toe 1 A (1) | 2.554 | 2.488 | 73 | -85 | - |

* A = axial, B = bending, T = torsion
* Nominal biaxiality ratio Λ = σnom/τnom
* Calculated with Δσnom = 1 MPa and/or Δτnom = 1 MPa, Λ as indicated. Reported values are at the crack initiation point.
6. Discussion and conclusions

The Peak Stress Method (PSM) grants the possibility to assess fatigue failures in welded joints subjected to either uniaxial and multiaxial, in-phase and out-of-phase loading conditions. In compliance with a proper PSM-based design curve, the equivalent peak stress provides a reliable design quantity able to single out the fatigue failure location in presence of multiple competing crack initiation sites (see also figure 9).

In this paper, three of the nine welded joint geometries proposed in table 3 have been considered for a deeper insight (see figures 6-8); however, the complete re-analysis work [20] considered all joint geometries proposed in table 3, for a total of 306 re-analysed experimental data.
Figure 10. Fatigue assessment of weld toe failures according to the PSM. Comparison between the fatigue design scatter band (λ = 0) proposed in [10] and experimental fatigue data by [23] and [30,31].

A = axial load; B = bending load; T = torsion load; AW = as-welded; SR = stress-relieved.

Figure 11. Fatigue assessment of weld toe and weld root failures according to the PSM. Comparison between the fatigue design scatter band (λ > 0) proposed in [10] and experimental fatigue data by [23] and [29]. B = bending load; T = torsion load; AW = as-welded; SR = stress-relieved.
The PSM allowed to successfully assess the fatigue crack initiation site in approximately 270 fatigue failure cases occurring at the weld toe or weld root, i.e. ~90% of the considered cases. Moreover, about 180 of the total 306 experimental data, i.e. ~60%, fall within the 2.3%-97.7% PSM-based design scatter bands, proving a good agreement with the proposed fatigue design curves, while the remaining data, i.e. ~40%, are on the safe side. Finally, the possibility to take advantage of the new tool developed in Ansys® Mechanical to automate PSM analyses on generic welded joint geometries allowed to achieve a remarkable reduction in calculation effort and time, with respect to the ones required to manually apply the method and elaborate the results of the FE analyses. More in detail, table 6 shares some estimations on “manual versus automated” analysis time, required to apply the PSM on each competing weld toe and weld root characterizing the geometries proposed in figures 6-8. Thanks to the coarse meshes required by the PSM to perform compatible FE analyses and the full-automated implementation in Ansys® Mechanical, the newly developed PSM application might reveal itself a useful tool in every-day design practice when dealing with the fatigue assessment of welded joints and structures.

Table 6. Analysis time comparison between manual and automated PSM approach.

| Reference                        | Analysed nodes | Manual PSM analysis \(^a\) estimated time [s] | Automated PSM analysis \(^b\) estimated time [s] |
|----------------------------------|----------------|---------------------------------------------|-----------------------------------------------|
| Yousefi et al. [23]              | 226            | ~9000 (~2 h 30 minutes)                      | ~40 seconds\(^c\)                             |
| Frendo and Bertini [29]          | 62             | ~2400 (~40 minutes)                          | ~20 seconds                                  |
| Takahashi et al. [30,31]         | 124            | ~3600 (~60 minutes)                          | ~40 seconds                                  |

All time estimations are given for a single PSM analysis on all weld toes and weld roots of the geometry

\(^a\) Time required to: (i) define and orientate local coordinate systems; (ii) retrieve relevant peak stresses

\(^b\) Time required to perform a complete automated PSM analysis, up to fatigue life estimation

\(^c\) Separate analyses required at the weld toe and weld root due to the adopted element sizes (see paragraph 4.1)

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