In the present paper the two and three point functions, which occur at the study of the various physical processes are considered. The investigation is done in the framework of the perturbation theory at the one loop level. The general analytical and asymptotic expressions for these functions are obtained.
1 INTRODUCTION.

With the increasing role which place the perturbative approach within the field theory, the evolution of the effects beyond tree approximation in the elementary particle physics becoming of great interests. Such effects play an important role e.g. in the investigation of rare decays of hadrons, leptons, quarks, meson oscillation etc. Within the framework of standard theory of electro weak interactions.

During the study of physical processes beyond the tree approximation within the perturbative approach image so called loop integrals. The study of loop integrals and the investigation of the behavior of this integrals for the various values of cinematic parameters is of great interest. Such investigation has been cared out in references [1-5]. However for the practical point of view the utilization of the results obtained in these paper encounters the difficulties. In the present work the two and tree point functions which appear during the study of the physical processes at the one loop level are investigated. The general and asymptotic expressions for these functions for the different values of cinematic parameters are obtained.

2 INTEGRAL EXPRESSIONS OF THE TWO AND TREE POINT FUNCTIONS.

During the study physical processes at one loop level there exist integrals of the following types:

\[
\{ I_0; I_\alpha \}(p^2, m_1^2, m_2^2) = \int \frac{d^4q}{(2\pi)^4} \frac{(1, q_0)}{[(q - p)^2 - m_1^2 + i\varepsilon][(q^2 - m_2^2 + i\varepsilon)]}, \tag{1}
\]

\[
\{ R_0, R_\alpha, R_{\alpha\beta} \}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_2^2, m_3^2) = \int \frac{d^4q}{(2\pi)^4} \frac{(1, q_0, q_\alpha q_\beta)}{[(p_1 - q)^2 - m_1^2 + i\varepsilon][(p_2 - q)^2 - m_2^2 + i\varepsilon][(q^2 - m_3^2 + i\varepsilon)]}. \tag{2}
\]

The Lorentz decomposition of these integrals (I_\alpha, R_\alpha and R_{\alpha\beta}) have following forms:

\[
I_\alpha(p^2, m_1^2, m_2^2) = p_\alpha I_1(p^2, m_1^2, m_2^2),
\]

\[
R_\alpha(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_2^2, m_3^2) = p_{1\alpha} R_{10}(p_1^2, p_2^2, (p_1 - p_2)^2 m_1^2, m_2^2, m_3^2) + p_{2\alpha} R_{01}(p_1^2, p_2^2, (p_1 - p_2)^2 m_1^2, m_2^2, m_3^2),
\]

\[
R_{\alpha\beta}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_2^2, m_3^2) = g_{\alpha\beta} R_{00}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_2^2, m_3^2) + p_{1\alpha} p_{1\beta} R_{20}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_2^2, m_3^2) + p_{2\alpha} p_{2\beta} R_{02}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_2^2, m_3^2) + (p_{1\alpha} p_{2\beta} + p_{2\alpha} p_{1\beta}) R_{11}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_2^2, m_3^2). \tag{3}
\]
Some of these integralas contaain divergences. In the dimenshinal regularization scheem it is possible to single out the divergent part in the following form:

\[ I_0(p^2, m^2_1, m^2_2) = \frac{1}{\epsilon} + \bar{I}_0(p^2, m^2_1, m^2_2) \]

\[ I_\alpha(p^2, m^2_1, m^2_2) = p_\alpha I_1(p^2, m^2_1, m^2_2) = p_\alpha \left( \frac{1}{2\epsilon} + \bar{I}_1(p^2, m^2_1, m^2_2) \right) \]

\[ R_{00}(p^2_1, p^2_2, (p_1 - p_2)^2, m^2_1, m^2_2, m^2_3) = \frac{1}{4\epsilon} + \bar{R}_{00}(p^2_1, p^2_2, (p_1 - p_2)^2, m^2_1, m^2_2, m^2_3) \]

where \( 1/\epsilon = \frac{i}{(4\pi)^2} [1/\epsilon - \gamma + \ell n4\pi] \), \( \gamma \) is eiler maskeroni constant, \( \gamma = 0.5772157... \), \( 2\epsilon = 4 - n \), \( n \) is dimenshin of space-time. In the framework of Feinman parametrization the finite parts of these integrals have respectively the form:

\[ \bar{I}_0(p^2, m^2_1, m^2_2) = -\frac{i}{(4\pi)^2} \int_0^1 dx \ln \frac{m^2_1(1-x) + m^2_2x}{\mu^2} \]

\[ \bar{I}_1(p^2, m^2_1, m^2_2) = -\frac{i}{(4\pi)^2} \int_0^1 dx (1-x) \ln \frac{m^2_1(1-x) + m^2_2x}{\mu^2} \]

\[ \bar{R}_{00}(p^2_1, p^2_2, (p_1 - p_2)^2, m^2_1, m^2_2, m^2_3) = -\frac{i}{(4\pi)^2} \int_0^1 dx \int_0^1 dy \frac{x^3y^j(1-y)^i}{L(p^2_1, p^2_2, (p_1 - p_2)^2, m^2_1, m^2_2, m^2_3, x, y)} \]

\[ R_{ij}(p^2_1, p^2_2, (p_1 - p_2)^2, m^2_1, m^2_2, m^2_3) = -\frac{i}{(4\pi)^2} \int_0^1 dx \int_0^1 dy \frac{x^2y^{1-i}(1-y)^{1-j}}{L(p^2_1, p^2_2, (p_1 - p_2)^2, m^2_1, m^2_2, m^2_3, x, y)} \]

where \( \mu^2 \) is arbitrary mass scale parameter and we have introduce the following notations:

\[ L(p^2_1, p^2_2, (p_1 - p_2)^2, m^2_1, m^2_2, m^2_3, x, y) = Ax^2 + Bx + m_2^2 \]

\[ A = [p_2 + y(p_1 - p_2)]^2, \]

\[ B = (m_1^2 - i\epsilon)y + (m_2^2 - i\epsilon)(1-y) - m_3^2 + i\epsilon - p_1^2y - p_2^2(1-y). \]
3 TWO POINT FUNCTIONS.

Integrals \( \tilde{I}_0(p^2, m_1^2, m_2^2) \) and \( \tilde{I}_1(p^2, m_1^2, m_2^2) \), after integration by part can take in the following form:

\[
\tilde{I}_0(p^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2} \left\{ \ln \frac{m_2^2}{\mu^2} - \int_0^1 dx \frac{m_2^2 - m_1^2 - p^2 + 2p^2 x}{m_1^2(1-x) + m_2^2 x - p^2 x(1-x) - i\epsilon} \right\}, \tag{11}
\]

\[
\tilde{I}_1(p^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2} \left\{ \frac{1}{2} \ln \frac{m_2^2}{\mu^2} - \frac{3}{2} \frac{m_2^2 - m_1^2 + p^2}{2p^2} + \frac{(m_2^2 - m_1^2 - p^2)^2 + 2p^2(m_2^2 - 2m_1^2 - p^2)}{4p^4} \ln \frac{m_2^2}{m_1^2} + m_2^2 - m_1^2 - 3p^2 \sqrt{-\lambda(m_1^2, m_2^2, p^2)} \arctg \frac{\sqrt{-\lambda(m_1^2, m_2^2, p^2)}}{m_2^2 + m_1^2 - p^2} \right\}, \tag{12}
\]

if \((m_1 - m_2)^2 < p^2 < (m_1 + m_2)^2\); here \(\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz\),

\[
\tilde{I}_0(p^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2} \left\{ \ln \frac{m_2^2}{\mu^2} - i\pi S_0 + \frac{m_2^2 - m_1^2 - p^2}{2p^2} \ln \frac{m_2^2}{m_1^2} - 2 - \frac{\sqrt{\lambda(m_1^2, m_2^2, p^2)}}{2p^2} \ln \frac{m_1^2 + m_2^2 - p^2 + \sqrt{\lambda(m_1^2, m_2^2, p^2)^2}}{4m_1^2 m_2^2} \right\}, \tag{13}
\]

\[
\tilde{I}_1(p^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2} \left\{ \frac{1}{2} \ln \frac{m_2^2}{\mu^2} - i\pi S_1 + \frac{m_1^2 - m_2^2 - p^2}{2p^2} - \frac{1}{2} \frac{(m_1^2 - m_2^2 - p^2)^2 - 2p^2 m_2^2}{4p^4} \ln \frac{m_2^2}{m_1^2} + \frac{m_1^2 - m_2^2 - p^2 \sqrt{\lambda(m_1^2, m_2^2, p^2)}}{4p^4} \ln \frac{m_1^2 + m_2^2 - p^2 + \sqrt{\lambda(m_1^2, m_2^2, p^2)^2}}{4m_1^2 m_2^2} \right\}, \tag{14}
\]

if \(p^2 \leq (m_1 - m_2)^2 \cup p^2 \geq (m_1 + m_2)^2\).
In the formula (14) \(S_0\) and \(S_1\) are the contribution due the pole of the integrals (11),(12):

\[
S_0 = \begin{cases} 
\sqrt{\lambda(m_1^2, m_2^2, p^2)/p^2}; & x_1 \in [0, 1], \ x_2 \in [0, 1], \\
(m_1^2 - m_2^2 + p^2 + \sqrt{\lambda(m_1^2, m_2^2, p^2)})/2p^2; & x_1 \in [0, 1], \ x_2 \not\in [0, 1], \\
(m_2^2 - m_1^2 - p^2 + \sqrt{\lambda(m_1^2, m_2^2, p^2)})/2p^2; & x_1 \not\in [0, 1], \ x_2 \in [0, 1], \\
0; & x_1 \not\in [0, 1], \ x_2 \not\in [0, 1],
\end{cases}
\]

\[
S_1 = \begin{cases} 
(m_2^2 - m_1^2 + p^2)\sqrt{\lambda(m_1^2, m_2^2, p^2)/2p^4}; & x_1 \in [0, 1], \ x_2 \in [0, 1], \\
\frac{1}{4\pi}[(m_1^2 - m_2^2)^2 + p^4 - 2m_1^2p^2 - (m_1^2 - m_2^2)^2 + p^2\lambda(m_1^2, m_2^2, p^2)]; & x_1 \in [0, 1], \ x_2 \not\in [0, 1], \\
\frac{1}{4\pi}[-(m_1^2 - m_2^2)^2 - p^4 + 2m_1^2p^2 - (m_1^2 - m_2^2)^2 + p^2\lambda(m_1^2, m_2^2, p^2)]; & x_1 \not\in [0, 1], \ x_2 \in [0, 1], \\
0; & x_1 \not\in [0, 1], \ x_2 \not\in [0, 1],
\end{cases}
\]

where \(x_1\) and \(x_2\) are the roots of quadratic polynomial \(p^2 x^2 + (m_2^2 - m_1^2 - p^2) x + m_1^2\). It should be mentioned that in integrals (11) and (12), when the cinematic \(p^2 \leq (m_1 - m_2)^2\) \(\cup\) \(p^2 \geq (m_1 + m_2)^2\), it appears poles in deficit of following parameters \(m_1^2, m_2^2, p^2\). During the investigation of physical processes, when one or more parameters are significantly large in compare with others, it is more conveniently to use approximately expressions of integrals \(\tilde{I}_0\) and \(\tilde{I}_1\). We will show below approximately expressions of these integrals for frequently arising values of kinematic parameters \((p^2 < m_1^2\) and/or \(p^2 < m_2^2\)):

\[
\tilde{I}_0(p^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2}\{\ln\frac{m_1^2}{\mu^2} + a_1(x) + \frac{p^2}{m_1^2}a_2(x) + \frac{p^4}{m_1^4}a_3(x)\},
\]

\[
\tilde{I}_1(p^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2}\{\frac{1}{2}\ln\frac{m_1^2}{\mu^2} + b_1(x) + \frac{p^2}{m_1^2}b_2(x) + \frac{p^4}{m_1^4}b_3(x)\},
\]

Where we apply following notation:

\[
k_1(x) = \frac{x - 1 - x\ln x}{1 - x}, \quad k_2(x) = \frac{x^2 - 1 - 2x\ln x}{2(1 - x)^3}, \quad k_3(x) = \frac{x^3 + 9x^2 - 9x - 1 + 6x(1 + x)\ln x}{6(1 - x)^5},
\]

\[
l_1(x) = \frac{-3x^2 + 4x - 1 + 2x^2\ln x}{4(1 - x)^2}, \quad l_2(x) = \frac{-2x^3 - 3x^2 + 6x - 1 + 6x^2\ln x}{6(1 - x)^4},
\]

\[
l_3(x) = \frac{-3x^4 - 44x^3 + 36x^2 + 12x - 1 + 12(3x^2 + 2x^3)\ln x}{24(1 - x)^6}, \quad x = \frac{m_2^2}{m_1^2}.
\]

In the framework of large momentum \((p^2 - m_1^2 > m_2^2)\), the approximative expressions of \(\tilde{I}_0\) and \(\tilde{I}_1\) are obtained the following form consequently:

\[
\tilde{I}_0(p^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2}\{\ln\frac{m_2^2}{\mu^2} - i\pi[1 - \frac{m_1^2}{p^2} - \frac{m_2^2(p^2 + m_1^2)}{p^2(p^2 - m_1^2)}] - 2 +
\]

5
Let us go to the more detail investigation of the tree point function $s$

After solving the system (19) we get following expressions for the t ree point functions

$$
\frac{m_2^2 - p^2 + m_1^2}{p^2 - m_1^2} \ln \frac{m_1^2}{m_1^2} + \left[1 - \frac{m_1^2}{p^2} - \frac{m_3^2(p^2 + m_1^2)}{p^2(p^2 - m_1^2)} \right] \ln \left(\frac{p^2}{m_1^2} - 1\right)
$$

$$\tilde{I}_1(p^2, m_1^2, m_2^2) = -\frac{i}{4\pi^2} \left\{ -\frac{1}{2} \ln \frac{m_1^2}{\mu^2} + \frac{m_1^2}{p^2} \ln \frac{m_1^2}{\mu^2} + \frac{i\pi}{2} \left[1 - 2 \frac{m_1^2}{p^2} \left(1 + \frac{m_1^2}{p^2 - m_1^2}\right)\right] \right\}
$$

$$\tilde{I}_1(p^2, m_1^2, m_2^2) = \frac{m_1^4}{p^4} \left[1 + \frac{2m_2^2}{p^2 - m_1^2}\right] + 1 - \frac{3m_1^2}{2p^2} + \frac{3m_2^2}{2p^2} \frac{m_2^2}{\mu^2} \ln \frac{m_2^2}{\mu^2}
$$

$$\frac{1}{2} \left[1 - 2 \frac{m_1^2}{p^2} \left(1 + \frac{m_2^2}{p^2 - m_1^2}\right) + \frac{m_1^4}{p^4} \left[1 + \frac{2m_2^2}{p^2 - m_1^2}\right] \ln \left(\frac{p^2}{m_1^2} - 1\right)\right] \right\}
$$

(18)

It should be pointed out that at the zero value of the cinematic parameters the formulae (14), (17) and (18) should be understood as limited expressions.

Below we give usefull relations for the two point functions

$$I_0(p^2, m_1^2, m_2^2) = I_0(p^2, m_1^2, m_2^2),$$

$$I_1(p^2, m_1^2, m_2^2) = I_0(p^2, m_2^2, m_1^2) - I_1(p^2, m_2^2, m_1^2).$$

We have the same relations for the functions $\tilde{I}_0$ and $\tilde{I}_1$.

4 TREE POINT FUNCTIONS.

Let us go to the more detail investigation of the tree point functions $R_0, R_\alpha$ and $R_{\alpha\beta}$. Multiplying the second expression in the formulae (3) on $p_{1\alpha}$ and $p_{2\alpha}$ we obtain the system of equations for the functions $R_{10}$ and $R_{01}$:

$$
\begin{align*}
2p_1^2 R_{10} + 2(p_1 p_2) R_{01} &= I_0((p_1 - p_2)^2, m_1^2, m_2^2) - I_0(p_2^2, m_2^2, m_3^2) + \\
&\quad (m_3^2 - m_1^2 + p_1^2) R_0, \\
2(p_1 p_2) R_{10} - 2p_2^2 R_{01} &= I_0((p_1 - p_2)^2, m_1^2, m_2^2) - I_0(p_2^2, m_2^2, m_3^2) + \\
&\quad (m_3^2 - m_2^2 + p_2^2) R_0.
\end{align*}
$$

(19)

After solving the system (19) we get following expressions for the tree point functions $R_{10}$ and $R_{01}$:

$$R_{10} = \frac{1}{2[p_{10}^2 - (p_{10} p_2)^2]} \left\{ p_{10}^2 I_0(p_2^2, m_2^2, m_3^2) - I_0((p_1 - p_2)^2, m_1^2, m_2^2) \right\} +$$

$$\left(p_1 p_2\right) I_0((p_1 - p_2)^2, m_1^2, m_2^2) - I_0(p_1^2, m_1^2, m_3^2) +$$

$$\left[p_2^2(m_1^2 - m_3^2 - p_1^2) - (p_1 p_2)(m_2^2 - m_3^2 - p_2^2)\right] R_0,$$

$$R_{01} = \frac{1}{2[p_{10}^2 - (p_{10} p_2)^2]} \left\{ p_{10}^2 I_0(p_1^2, m_1^2, m_3^2) - I_0((p_1 - p_2)^2, m_1^2, m_2^2) \right\} +$$

$$\left(p_1 p_2\right) I_0((p_1 - p_2)^2, m_1^2, m_2^2) - I_0(p_2^2, m_2^2, m_3^2) +$$

$$\left[p_2^2(m_1^2 - m_3^2 - p_1^2) - (p_1 p_2)(m_2^2 - m_3^2 - p_2^2)\right] R_0.$$
\[ [p_1^2(m_2^2 - m_3^2 - p_2^2) - (p_1p_2)(m_1^2 - m_2^2 - p_1^2)]R_0 \}\]

In order to obtain explicit expressions for the functions \( R_{00}, R_{20}, R_{02}, R_{11} \) let us multiply the third term of formula (3) on \( p_{1a} \) and \( p_{2a} \). In results we get the system of linear equations, whose determinant is equal zero. We can add one independent equation obtained via the multiplying of the Lorentz decomposition of \( R_{\alpha\beta} \) from formula (3) on \( g_{\alpha\beta} \). The new system can be explicitly solved with respect to the functions \( R_{00}, R_{20}, R_{02}, R_{11} \) and has the form:

\[
\begin{align*}
4R_{00} + p_1^2R_{20} + 2(p_1p_2)R_{11} + p_2^2R_{02} &= m_3^2R_0 + I_0((p_1 - p_2)^2, m_1^2, m_2^2) \equiv a_1 \\
R_{00} + p_1^2R_{20} + (p_1p_2)R_{11} &= -\frac{1}{2}((m_1^2 - m_2^2 - p_1^2)R_{10} - I_1((p_1 - p_2)^2, m_1^2, m_2^2)) \equiv a_2 \\
p_1^2R_{11} + (p_1p_2)R_{02} &= -\frac{1}{2}((m_2^2 - m_3^2 - p_1^2)R_{01} + I_1((p_1 - p_2)^2, m_1^2, m_2^2)) - I_0((p_1 - p_2)^2, m_1^2, m_2^2) + I_1(p_1^2, m_1^2, m_2^2) \equiv a_3 \\
(p_1p_2)R_{20} + p_2^2R_{11} &= -\frac{1}{2}((m_2^2 - m_3^2 - p_2^2)R_{10} - I_1((p_1 - p_2)^2, m_1^2, m_2^2)) + I_1(p_1^2, m_1^2, m_2^2) \equiv a_4.
\end{align*}
\]

The solution of the given above system of equation has the following form:

\[
\begin{align*}
R_{00} &= \frac{1}{2}\{a_1 - 2a_2 - \frac{p_2^2}{(p_1p_2)}a_3 + \frac{p_1^2}{(p_1p_2)}a_4\}, \\
R_{20} &= \frac{a_1p_2^2(p_1p_2) - 4a_2p_1^2(p_1p_2) - a_3p_1^4 + a_4(2(p_1p_2)^2 + p_1^2p_2^2)}{2[(p_1p_2)^2 - p_1^2p_2^2]}, \\
R_{11} &= \frac{(p_1p_2)(-a_1 + 4a_2) + p_2^2a_3 - 3p_1^2a_4}{2[(p_1p_2)^2 - p_1^2p_2^2]}, \\
R_{02} &= \frac{(a_1 - 4a_2)p_1^2(p_1p_2) + a_3[2(p_1p_2)^2 - 3p_1^2p_2^2] + 3a_4p_1^4}{2[(p_1p_2)^2 - p_1^2p_2^2]}.
\end{align*}
\]

From the expressions (20), (22), given above one observes that all three point functions can be expressed via the two point functions \( I_0, I_1 \) (whose explicit expressions are given in section (3) (see formulae (13) and (14)) and the three point function \( R_0 \). Bellow we shall obtained the explicit expressions for the three point function \( R_0 \).

5 \hspace{1cm} \textbf{CALCULATION OF THE THREE POINT FUNCTION} \hspace{1cm} R_0

Let us consider the three point function \( R_0 \):

\[
R_0(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_2^2, m_3^2) =
\]
\[
\int \frac{d^4q}{(2\pi)^4} \frac{(1, q_\alpha, q_\beta)}{[(p_1 - q)^2 - m_1^2 + i\epsilon][(p_2 - q)^2 - m_2^2 + i\epsilon](q^2 - m_3^2 + i\epsilon)}. \tag{23}
\]

After using the Feinman parametrization the three point function (23) is written in the following form:

\[
R_0 = \frac{i}{16\pi^2} \int_0^1 dx \int_0^x \frac{1}{\{ax^2 + by^2 + cxy + dx + ey + f\}} \cdot \ln(by^2 + (c + e)y + d + a + f) - \ln(by_1^2 + (c + e)y_1 + d + a + f)] - \int_0^1 dy \frac{1 - \alpha}{(2ab + c)y(1 - \alpha) + d + e\alpha}[\ln((a + b + c)y^2 + (e + d)y + f) - \ln((a + b + c)y_2^2 + (e + d)y_2 + f)] - \int_0^1 dy \frac{\alpha}{-(2ab + c)y\alpha + d + e\alpha}[\ln(ay^2 + dy + f) - \ln(ay_3^2 + dy_3 + f)] \}, \tag{25}
\]

where \(\alpha\) is the root of the quadratic polynomial \(b\alpha^2 + c\alpha + a\) and \(y_1, y_2, y_3\) are defined in the following way:

\[
y_0 = -\frac{d + e\alpha}{c + 2b\alpha}, \quad y_1 = y_0 + \alpha, \quad y_2 = \frac{y_0}{1 - \alpha}, \quad y_3 = -\frac{y_0}{\alpha}. \tag{26}
\]

From (25) we see that all three integrals have the similar form. Let us consider the first of them:

\[
F = \frac{i}{16\pi^2(2b\alpha + c)} \int_0^1 dy \frac{1}{y - y_1} \left\{ \ln(by^2 + (c + e)y + d + a + f) - \ln(by_1^2 + (c + e)y_1 + d + a + f) \right\}. \tag{27}
\]

The function \(F\) which is defined by formula (27) is expressed via the Spens functions [7-8]:

\[
F = \frac{i}{16\pi^2(2b\alpha + c)} \left\{ Sp\left(\frac{y_1}{y_1 - y_1}\right) - Sp\left(\frac{y_1 - 1}{y_1 - y_1}\right) + Sp\left(\frac{y_1}{y_1 - y_2}\right) - Sp\left(\frac{y_1 - 1}{y_1 - y_2}\right) \right\} = \ldots
\]
\[
\frac{i}{16\pi^2(2\alpha + c)} \sum_{i=1}^{2} \left\{ Sp\left( \frac{y_1}{y_i - y_i'} \right) - Sp\left( \frac{y_1 - 1}{y_i - y_i'} \right) \right\}
\]

Using the expressions (27) and (25) the three point function \( R \) takes the final form:

\[
R_0 = \frac{i}{16\pi^2(2\alpha + c)} \sum_{i=1}^{3} \sum_{\sigma = \pm} (-1)^{1+j} \left\{ Sp\left( \frac{y_i}{y_i - y_i'} \right) - Sp\left( \frac{y_i - 1}{y_i - y_i'} \right) \right\},
\]

where \( y_i^\pm \) are the roots of the following quadratic polynomials respectively:

\[
(p_1 - p_2)^2 y^2 + (m_1^2 - m_2^2 - (p_1 - p_2)^2)y + m_2^2 - i\epsilon,
\]

\[
p_1^2 y^2 + (m_1^2 - m_3^2 - p_1^2)y + m_3^2 - i\epsilon,
\]

\[
p_2^2 y^2 + (m_2^2 - m_3^2 - p_2^2)y + m_3^2 - i\epsilon.
\]

It should be pointed out that with the using formulae (11)-(14),(20),(22),(29) all three point functions can be analyzed analytically as well as numerically.

In the particular case, when one of the mass parameter significantly exit ales, it is more convenient to use the following expressions for the integrals:

\[
R_0(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_2^2, m_3^2) = -\frac{i}{(4\pi)^2 m_1^2} \left\{ c_1(x) + \frac{p_1^2 + p_2^2}{m_1^2} c_2(x) + \frac{(p_1 - p_2)^2}{m_1^2} c_3(x) \right\}
\]

\[
R_{10}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2 m_1^2} \left\{ d_1(x) + \frac{p_1^2 + 2p_2^2}{m_1^2} d_2(x) + \frac{(p_1 - p_2)^2}{m_1^2} d_3(x) \right\}
\]

\[
R_{01}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_2^2, m_2^2) = R_{10}(p_2^2, p_1^2, (p_1 - p_2)^2, m_1^2, m_1^2, m_2^2)
\]

\[
\tilde{R}_{00}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2} \left\{ \frac{1}{4} \ell n \frac{m_1^2}{\mu^2} + e_1(x) + \frac{p_1^2 + p_2^2}{m_1^2} c_2(x) + \frac{(p_1 - p_2)^2}{m_1^2} c_3(x) \right\}
\]

\[
R_{20}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2 m_1^2} \left\{ f_1(x) + \frac{p_1^2 + 3p_2^2}{m_1^2} f_2(x) + \frac{(p_1 - p_2)^2}{m_1^2} f_3(x) \right\}
\]

\[
R_{11}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_1^2, m_2^2) = -\frac{i}{(4\pi)^2 m_1^2} \left\{ \frac{1}{2} f_1(x) + \frac{p_1^2 + p_2^2}{m_1^2} f_2(x) + \frac{2(p_1 - p_2)^2}{3} f_3(x) \right\}
\]

\[
R_{02}(p_1^2, p_2^2, (p_1 - p_2)^2, m_1^2, m_1^2, m_2^2) = R_{20}(p_2^2, p_1^2, (p_1 - p_2)^2, m_1^2, m_1^2, m_2^2)
\]

Where we have introduced the following notation:

\[
c_1(x) = \frac{1 - x + x\ell n x}{(1 - x)^2}, \quad c_2(x) = \frac{1 + 4x - 5x^2 + 2x(2 + x)\ell n x}{4(1 - x)^4}, \quad c_3(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2\ell n x}{12(1 - x)^4},
\]

\[
d_1(x) = \frac{1 - 4x + 3x^2 - 2x^2\ell n x}{4(1 - x)^3}, \quad d_2(x) = \frac{1 - 9x - 9x^2 + 17x^3 - 6x^2(3 + x)\ell n x}{36(1 - x)^5}, \quad d_3(x) = \frac{1 - 6x + 18x^2 - 10x^3 - 3x^4 + 12x^3\ell n x}{36(1 - x)^5}.
\]
\[ e_1(x) = \frac{-1 + 4x - 3x^2 + 2x^2\ln x}{8(1 - x)^2}, \quad e_2(x) = -\frac{1}{2}c_3(x), \quad e_3(x) = \frac{-2 + 9x - 18x^2 + 11x^3 - 6x^3\ln x}{72(1 - x)^4}, \]

\[ f_1(x) = -4e_3(x), \quad f_2(x) = \frac{1 - 8x + 36x^2 + 8x^3 - 37x^4 + 12x^3(4 + x)\ln x}{144(1 - x)^6}, \]

\[ f_3(x) = \frac{3 - 20x + 60x^2 - 120x^3 + 65x^4 + 12x^5 - 60x^4\ln x}{240(1 - x)^6}, \quad x = \frac{m_2^2}{m_1^2} \quad (31) \]

It should be pointed out that the expressions (17') and (31) at x=1 are considered as the limiting expressions for \( x \to 1 \).

## 6 SUMMARY.

In the given work the general expressions for the two and three point functions are obtained. The asymptotical behavior with respect to the values of the external and internal particle momenta is investigated. Different asymptotic representation for these functions (17), (18), (30) are given. It is convenient to use the different representations for these functions depending upon the concrete physical beyond investigated. Some particular cases it is possible to use integral representation (5)-(10), (22),(25) to written down answer in terms of one integral and them calculate this integral analytically or numerically. In other cases it may be more convenient to use the expressions (13)-(14), (22), (29). However, if the cinematic of the process beyond investigated allows us to use the definite approximations [9-10], then it is convenient from the practical point of view to use the asymptotical expressions for the two and three point functions (17),(18),(30).

The authors express their deep gratitude to J. Gegelia and G. Japaridze for useful discussions.
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