Why does steady-state magnetic reconnection have a maximum local rate of order 0.1?

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Simulations suggest collisionless steady-state magnetic reconnection of Harris-type current sheets proceeds with a rate of order 0.1, independent of dissipation mechanism. We argue this long-standing puzzle is a result of constraints at the magnetohydrodynamic (MHD) scale. We perform a scaling analysis of the reconnection rate as a function of the opening angle made by the upstream magnetic fields, finding a maximum reconnection rate close to 0.2. The predictions compare favorably to particle-in-cell simulations of relativistic electron-positron and non-relativistic electron-proton reconnection. The fact that simulated reconnection rates are close to the predicted maximum suggests reconnection proceeds near the most efficient state allowed at the MHD-scale. The rate near the maximum is relatively insensitive to the opening angle, potentially explaining why reconnection has a similar fast rate in differing models.

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Introduction—Magnetic energy is abruptly released in solar and stellar flares [13], substorms in magnetotails of Earth and other planets [4–6], disruptions and the sawtooth crash in magnetically confined fusion devices [6], laboratory experiments [7], and numerous high energy astrophysical systems [8,9]. Magnetic reconnection, where a change in topology of the magnetic field allows a rapid release of magnetic energy into thermal and kinetic energy, is a likely cause. The reconnection electric field parallel to the X-line (where magnetic field lines break) not only determines the rate that reconnection proceeds, but can also be crucial for accelerating energetic superthermal particles. It was estimated that a normalized reconnection rate of \( \sim 0.1 \) is required to explain time scales of flares and substorms [10].

Reconnection rates have been studied observationally, experimentally, theoretically, and numerically. Measurements can be in situ, such as in the magnetosphere and lab, or remote, as in solar and astrophysical contexts. Reconnection rates from these different vantage points can be the same but need not be; for example, flux ropes in the corona have macroscopic forces that can influence the evolution of current sheets where reconnection occurs. Therefore, it is important to distinguish between system scales. We define global-scale as system-size scale of magnetic domains. The local-scale is a smaller MHD-scale region where the magnetic field and plasma parameters achieve relatively uniform conditions upstream of the diffusion region. The micro-scale is the scale of the diffusion region. Here we focus on reconnection rates at the local- and micro-scales; coupling to global scales is beyond the scope of this paper.

The original model for the local reconnection rate was the Sweet-Parker model [11,12], but it was too slow to explain observed time scales of flares and substorms [13]. The collisional diffusion region is long and thin (i.e., the upstream magnetic fields have a small opening angle), developing a bottleneck that keeps the inflow speed small. The Petschek model [13] was much faster by producing an open outflow region (i.e., a larger opening angle), but is not a self-consistent model [15,16].

The collisionless limit is more appropriate for many systems of interest. Two-dimensional (2D) local simulations of isolated, thin, Harris-type current sheets reveal that the steady-state reconnection has a fast rate of 0.1 [17] when normalized by the magnetic field and Alfvén speed at the local-scale. This rate is independent of simulated electron mass [18,19] and system size [17,19]. In particular, the GEM challenge study [20] showed that the rate is comparable in Hall-MHD, hybrid, and particle-in-cell (PIC) simulations. Consequently, it was argued that the Hall term, the minimal non-ideal-MHD term in all three models, is the key physics for producing the fast rate [21,22]. However, further studies have raised important questions. One gets similar fast rates in electron-positron plasmas, for which the Hall term vanishes [23–25], and in the strong out-of-plane (guide) magnetic field regime [26,27] for which the Hall term is inactive. Even within resistive-MHD, the same 0.1 rate arises when a localized resistivity is employed [16]. This evidence calls into question whether the Hall term is the critical effect. It was suggested that the appearance of secondary islands could provide a universal mechanism for limiting the length of the diffusion region [21,20], but this model is also not satisfying since the same rate is obtained even when islands are absent [22,28].

In situ magnetospheric observations reveal (local) reconnection rates near 0.1 [31,32]. Solar observations suggest (global) reconnection rates can be this high as well [33,35], or somewhat lower [36,37]. Therefore, ob-
The observations suggest the local rate is 0.1, and the global rate can be at or below 0.1. This also has numerical support; in island coalescence, the global rate can be lower than 0.1 [38, 40], while the local rate remains close to 0.1 [38].

What causes the local reconnection rate to be \(\sim 0.1\) across different systems remains an open question [e.g., Ref. 11]. In this paper, we offer a new approach to this long-standing problem. We propose that the local rate has a maximum as a result of constraints at MHD scales (rather than physics at the diffusion-region-scale as is typically discussed). We perform a scaling analysis to derive the maximum local rate for low-\(\beta\) plasmas, which we find is \(O(0.1)\). The fact that local simulations produce rates close to this maximum value suggests that steady reconnection proceeds at a rate nearly as fast as possible. We show the predictions are consistent with existing simulations of a relativistic electron-positron plasma and a non-relativistic electron-proton plasma.

Simple model—Let the thickness and length of the (micro-scale) diffusion region be \(\delta\) and \(L\), respectively. For collisionless reconnection, \(\delta\) is controlled by inertial or gyro-radius scales [42]. If the opening angle made by the upstream magnetic field is small, the diffusion region is long and thin. Reconnection in this case is very slow, as in Sweet-Parker reconnection [11, 12]. As the opening angle increases, reconnection becomes faster. This is true to a point, but cannot continue for all angles for two reasons. First, in order to satisfy force balance, the upstream region develops structures over a larger scale, as in the classical Petschek-type analyses [14, 43]; this is what we define as the local-scale. Since the diffusion region thickness continues to be controlled by micro-scales, the diffusion region becomes embedded in a wider structure of local-scale \(\Delta z\), where the magnetic field and plasma parameters achieve relatively uniform upstream conditions. The magnetic field \(B_{zm}\) immediately upstream of the diffusion region becomes smaller than the asymptotic magnetic field \(B_{x0}\). (The subscript "0" indicates asymptotic quantities at the local-scale and "m" indicates quantities at the micro-scale.) This is crucial because it is \(B_{zm}\) that drives the outflow from the diffusion region; as it becomes smaller, reconnection proceeds more slowly.

The second reason reconnection does not become faster without bound is that the \(J \times B\) force of the reconnected field becomes smaller as the opening angle increases [40]. In the limit where the separatrices are at a right angle, the tension force driving the outflow is canceled by the magnetic pressure force, so reconnection does not spontaneously occur.

These observations suggest the following: the reconnection rate has a maximal value for an intermediate opening angle which is large enough to avoid the bottleneck for extremely thin current layers, but is not too large to weaken the reconnection drive. We present a scaling analysis simply capturing these main aspects using only the reconnection geometry and force balance. We consider low-\(\beta\) systems in the relativistic limit; a more general derivation should be future work.

The inflow region is illustrated in Fig. 1(a). With the diffusion region at the micro-scale, the asymptotic (local) magnetic field (at the top) must bend as it weakens toward the diffusion region (at the bottom). In the \(\beta \ll 1\) limit, thermal pressure is negligible, so to remain near equilibrium the inward-directed magnetic pressure gradient force \(-\nabla B^2/(8\pi)\) must be almost perfectly balanced by outward-directed magnetic tension \(B \cdot \nabla B/4\pi\). Evaluating these at point 1 marked in Fig. 1(b) gives

\[
\frac{B_{x0}^2 - B_{zm}^2}{8\pi \Delta z} \approx \left(\frac{B_{x0} + B_{zm}}{2}\right)\frac{2B_{zh}}{4\pi \Delta x},
\]

where \(B_{zh}\) is evaluated at the upstream field line near the separatrix.

We make the reasonable assumption the opening angle made by the upstream field at the local-scale, \(\theta \equiv \tan^{-1}(\Delta z/\Delta x)\), matches the opening angle of the micro-scale field at the corner of the diffusion region, \(\phi \equiv \tan^{-1}(B_{zm}/B_{zm})\). Then, from geometry, we get \(B_{zh}/(B_{x0} + B_{zm})/2 \approx \Delta z/\Delta x \approx B_{zm}/B_{zm}\). Eliminating \(B_{zh}\) and solving for \(B_{zm}/B_{x0}\) gives

\[
\frac{B_{zm}}{B_{x0}} \approx \frac{1 - (\Delta z/\Delta x)^2}{1 + (\Delta z/\Delta x)^2},
\]

For small opening angles, \(B_{zm} \approx B_{x0}\); for large opening angles approaching 45\(^\circ\), \(B_{zm} \ll B_{x0}\), and embedding is significant.

![Diagram](image-url)
To estimate the outflow speed, we employ force balance in the $x$-direction at point 2 in Fig. 1(c). In the relativistic limit \cite{Lundquist1953}, $n'm_iU_{out}^2/2L + B_{zm}^2/8\pi L \simeq (B_{zm}/2)(B_{xm}/2)/4\pi \delta$, where $n'$ is the density measured in the fluid rest frame, $m_i$ is the ion mass, $U_{out}$ is the outflow speed, and $\delta$ is the opening angle for intermediate values. This may explain why reconnection rates in disparate physical systems are so similar.

The dotted curve in Fig. 2(a) shows the non-relativistic prediction if $V_{out,m}$ is taken to be identically $V_{Am}$ in Eq. 3. This comparison indicates that the correction to the outflow speed in Eq. 3 does not significantly alter $R_0$, although it does impact $R_m$ as $\Delta z/\Delta x$ approaches 1. Thus, the most significant effect limiting the local rate with a increasing opening angle is the embedding. We plot $R_0$, $R_m$ and $V_{in,m}/c$ in Fig. 2(b) as functions of $B_{zm}/B_{z0}$ to facilitate a comparison with simulations. A similar plot is shown in Fig. 2(c) for the relativistic limit, specifically with $\sigma_z = 89$. The peak $R_0$ is 0.3, and it does not change with increasing $\sigma_z$. This bounds rates seen in relativistic simulations \cite{Pritchett2012, Pritchett2013, Pritchett2015}.

We point out that there are similarities between the present model and the classical Petschek model \cite{Petschek1964}. However, there are a number of important differences. For example, the Petschek model assumes a value of 0.5 for what we call $B_{zm}/B_{z0}$, whereas we estimate it self-consistently. Furthermore, the way Petschek obtained the upstream condition, strictly speaking, only works for the small opening angle limit, while our result is valid for any opening angle. Finally, and most importantly, the weak dependence on reconnection rate reported by Petschek has a logarithmic dependence on Lundquist number, so the normalized reconnection rate is not bounded by 0 and 1 as it must be on physical grounds. In the present work, the reconnection rate is manifestly bounded between 0 and 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.pdf}
\caption{Predictions for the non-relativistic limit as functions of (a) $\Delta z/\Delta x$ and (b) $B_{zm}/B_{z0}$. (c) Predictions for relativistic limit ($\sigma_z = 89$).}
\end{figure}
The model presented here is completely independent of dissipation mechanism. The only ingredients are MHD-scale considerations and that the diffusion region remains at micro-scales when the opening angle increases. The fact that the simulated fast rates in disparate physical models are all similar to the predicted maximum rate of order 0.1 suggests that MHD-scheme constraints on magnetic energy release determine the fast rate. The obvious counterpoint to this is reconnection in MHD simulations with a uniform resistivity, which does not proceed at this rate. Even when the Lundquist number is large enough to produce magnetic

FIG. 3: Local-scale structure around the X-line in electron-positron reconnection with $\sigma_{x0} = 89$ at $t = 600/\omega_{pi}$. (a) $V_{iz}$ and its cut at $x = 0$. (b) $|B_z|$ and a cut of $B_z$ at $x = 0$. Contours of in-plane magnetic flux are overlaid. The color table in (b) has an upper limit $B_{x0}$.

FIG. 4: Time evolution of the measured local reconnection rate $R_0$, micro-scale rate $R_m$, micro-scale inflow speed $V_{in,m}/c$ and $B_{xm}/B_{x0}$. The blue circle marks the deviation of $R_m$ from $R_0$. The orange vertical line marks the time plotted in Fig. 3.
islands [15, 57], the reconnection rate is an order of magnitude smaller [58, 60]. This indicates that considerations at MHD scales are not sufficient to explain fast reconnection; the micro-scale dissipation/localization mechanism must be able to support the desired opening angle at the local-scale. However, if the diffusion region can support a larger opening angle, the local rate of order \( O(0.1) \) is not strongly sensitive to the opening angle over a wide range of values. The micro-scale rate \( R_m \) is sensitive to the opening angle, resulting in the large difference between \( R_m \) and \( R_0 \) observed in the relativistic limit [45].

The present model is not complete in that it does not include some physics that may affect the reconnection rate. As discussed earlier, the self-generated pressure anisotropy in the exhaust can reduce the outflow speed [55, 56]. The plasma pressure gradient force in the outflow direction can also affect the outflow speed [55, 56]. Self-generated upstream temperature anisotropies [62] may modify the embedding. Relaxing the low-\( \beta \) assumption is important. However, we note that the reduction of \( B_{rm} \) also occurs in simulations with \( \beta \sim O(1) \) in Ref. [54]. Finally, this model does not take into account the conversion of upstream energy into heat and accelerated particles, which undoubtedly impacts the energy conversion process and is important in maintaining the intense current sheet during reconnection [46, 63]. Nevertheless, this simple model offers a new approach to the long-standing fast reconnection rate problem, which is broadly relevant in basic plasma physics, fusion science, solar and space physics, and astrophysics, and potentially provides an avenue for understanding the important link between the micro- and global-scales.

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