Quantum statistical model of nonlinear inverse bremsstrahlung absorption in strongly coupled plasmas

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A new approach for the calculation of collisional inverse bremsstrahlung absorption of laser light in dense plasmas is presented. Quantum statistical formalism used allows avoiding \textit{ad hoc} cutoffs that were necessary in classical approaches. Thus, the current method remains reliable for strong electron-ion interactions. In addition, both the dynamic, field dependent response and hard electron-ion collisions, are consistently incorporated. The latter were treated in an averaged manner as a stopping power that in turn was cast into a form of a friction force. Here, for the first time a link between the stopping power and the problem of collisional laser absorption is drawn. This allows the theories developed for the stopping power calculation, such as the quantum T-matrix approach, to be applied to the problem of collisional laser absorption. The new approach accommodates the low- and high-frequency limits explained in the text and is valid for arbitrary laser field intensities. A comparison with classical MD simulation is indicative of the validity of the new method in the wide parameter range tested.

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\section{I. INTRODUCTION}

Understanding the laser-matter interaction in strongly coupled plasmas is crucial for the design of the contemporary inertial confinement fusion (ICF) targets. In both the direct and indirect drive ignition schemes, the laser energy is deposited into the plasma of high-Z elements such as Au. The conditions imposed by the hydrodynamic instabilities on the spatial symmetry of the laser energy deposition, require the laser absorption to be carefully determined from the early stages of the laser-plasma interaction \cite{1}. In the case of the fast ignition \cite{2} it is important to model the laser absorption starting with the preformed plasma due to the unavoidable nanosecond prepulse. The critical plasma density corresponding to the third harmonic of the N\textsubscript{2}Y\textsubscript{ag} lasers, used in most inertial fusion designs is \( \sim 10^{22} \text{ cm}^{-3} \). In the limit of low temperatures (few eV) such plasmas are characterised by strongly coupled and degenerate electrons: \( \Gamma = (e^2/k_BT_e)(4\pi n_e/3)^{1/3} \sim 1 \) and \( n_e(2\pi\hbar^2/m_e k_BT_e)^{3/2} \sim 1 \) respectively.

The dominant mechanism of radiation absorption for lasers with intensities typical for ICF is the inverse bremsstrahlung. In this case, the radiation is absorbed via collisions between the plasma particles usually described in terms of the electron-ion collision frequency \( \bar{\epsilon} \). First calculation of the inverse bremsstrahlung in the high frequency limit for the lowest order in the laser field strength was reported by Dawson \& Oberman (DO) \textsuperscript{4}. Later Decker et al. \textsuperscript{5} have extended this result to arbitrary field strengths. A classical ballistic model was considered in Ref. \textsuperscript{6} in order to study the frequency-dependent electron-ion collisions in plasmas.

These approaches are formulated in the high-frequency limit, when the number of binary electron-ion collisions per laser cycle can be neglected. In this limit, the laser is coupled to the plasma via the induced polarisation current so that the electron-ion interaction has a collective rather than a binary character. In the low-frequency limit strong scattering due to the binary collisions dominate the process of the absorption and the induced polarisation becomes relatively small. At intermediate frequencies the effects of binary and collective scattering have to be considered simultaneously.

For a given laser frequency, plasma conditions can be such, that the rate of the binary collisions becomes comparable with the laser period. Considering 0.35 \textmu m lasers used in most applications, this occurs in a strongly coupled plasmas. The collisions in such plasmas have to be evaluated quantum mechanically. The first quantum treatment was reported in Ref. \textsuperscript{7}. The nonlinear absorption was determined using the first Born approximation in Refs. \textsuperscript{8, 9}. A quantum approach to calculate laser absorption in strong fields was also developed in Ref. \textsuperscript{10}. Semiclassical approach in the linear regime using a memory function kinetic formalism including lowest order quantum effects was developed in Ref. \textsuperscript{11}. A quantum statistical approach for dynamical conductivity in strongly coupled regime was developed in Refs. \textsuperscript{12, 13}. A quantum Vlasov approach for arbitrary field strength, similar to the classical approach developed by Decker et al. \textsuperscript{5} was presented in Ref. \textsuperscript{14}. Rigorous kinetic approach to the inverse bremsstrahlung absorption in strongly coupled plasmas using nonequilibrium Green’s function techniques was developed in a series of publications \textsuperscript{13, 14, 15}. The expression for the collision frequency derived by Bornath et al. \textsuperscript{16} using the latter approach is identical to the quantum Vlasov method.

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In fact, it can be obtained from the formula obtained by Decker et al. by replacing the classical dielectric function by the quantum Lindhard dielectric function. This indicates, that both approaches make similar approximation, neglecting the effect of strong scattering by binary collisions, which makes them applicable in the high-frequency limit only. The effect of the binary collisions was considered in a linear-response theory by using the Gould-DeWitt scheme in Ref. [14]. Inverse bremsstrahlung absorption for strongly coupled plasmas was also calculated using classical molecular dynamic (MD) simulations reported in Refs. [21, 22, 23].

In this paper, we present a description of collisional absorption that bridges the high- and low-frequency limits. It is shown that the interactions can be split into a weak collective interactions and hard collisions. The latter are treated as the stopping power of ions in the electron fluid that can be cast into the form of a friction between the electron and ion fluids. Thus, for the first time, the stopping power formalism is applied to the calculation of the collisional absorption, allowing one to use a well developed models of the stopping power (see, e.g., Refs. [23, 24, 25]) in the problem of laser absorption in plasmas. The description of the collective electron response can be kept almost unchanged from earlier approaches [3, 14]. Due to the use of full quantum mechanical formulation of the problem, no ad hoc cutoffs must be introduced and the theory stays reliable for strong electron-ion interactions and degenerate electrons. The few assumptions made are justified by the unprecedented agreement with molecular dynamic (MD) simulations [21, 22, 23] up to very high coupling strengths.

In the following chapters we develop a quantum formalism to describe the laser absorption in dense plasmas. In Sec. II a set of Vlasov-Poisson (VP) equations similar to the one developed by Kull and Plagne is introduced. The major improvement in the present model is the inclusion of the hard-collisions and its treatment using the stopping power formalism. In Sec. III the VP equations are solved in the Kramers-Henneberger (KH) frame that is calculated with respect to the effect of the hard-collisions on the electron fluid rest-frame. Results and discussion follow in Sec. IV.

II. QUANTUM VLASOV EQUATION WITH AN EFFECTIVE FRICITION FORCE

The motion of the electrons in a neutral plasma consisting of $N_i$ ions and $N_e = ZN_i$ electrons, where $Z$ is the average charge state is described by a one electron statistical operator $\hat{\rho}_1(t)$ whose evolution is governed by equation:

$$i\hbar \frac{\partial}{\partial t} \hat{\rho}_1 - \left[ \hat{H}_0, \hat{\rho}_1 \right] = \text{Tr} \left\{ N_e \hat{V}_{1,2}^{e e} \hat{\rho}_1 + N_i \hat{V}_{1,2}^{e i} \hat{\rho}_1 \right\},$$  \hspace{1cm} (1)

The ions are considered to be located at fixed positions $\bar{x}_j$, $j = 1, 2, \ldots, N_i$ distributed according to the temperature dependant ion-ion pair correlation function $g_{ii}(x)$. The Hamiltonian of the system is

$$\hat{H}_0 = \frac{p_i^2}{2m} + \hat{U}_{\text{ext}},$$  \hspace{1cm} (2)

where $\hat{U}_{\text{ext}}$ describes the externally applied field, $\hat{p}_i^2$ and $\hat{p}_i^2$ are the two particle electron-electron and electron-ion distribution functions respectively, $\hat{V}_{1,2}^{e e}$ and $\hat{V}_{1,2}^{e i}$ are the e-e and e-i interaction potentials. Next assuming that the two-particle density functions can be expressed as:

$$\hat{\rho}_{1,2}^{\beta} = \hat{\rho}_{1}^{\beta} + \hat{\gamma}_{1,2}^{\beta}$$  \hspace{1cm} (3)

The first term in the expansion of $\hat{\rho}_{1,2}^{\beta}$ on the r.h.s is the Hartree term, while all the higher order terms are contained in the the corresponding pair correlation function $\hat{\gamma}_{1,2}^{\beta}$. Using these definitions Eq. (1) can be written as:

$$i\hbar \frac{\partial}{\partial t} \hat{\rho}_1 - \left[ \hat{H}, \hat{\rho}_1 \right] = \text{Tr} \left\{ N_e \hat{V}_{1,2}^{e e} \hat{\rho}_1 + N_i \hat{V}_{1,2}^{e i} \hat{\rho}_1 \right\},$$  \hspace{1cm} (4)

where

$$\hat{H} = \hat{H}_0 - e\hat{\Phi},$$  \hspace{1cm} (5)

and the effective Hartree potential $\hat{\Phi}$ is:

$$- e\hat{\Phi} = \text{Tr} \left\{ N_e \hat{V}_{1,2}^{e e} \hat{\rho}_2 + N_i \hat{V}_{1,2}^{e i} \hat{\rho}_2 \right\}$$  \hspace{1cm} (6)

In Eq. (4) the hard, binary, collisions are grouped in the r.h.s. and weak collisions are treated in the framework of Hartree approximation as a collective average potential contributed by the system species. This effective potential is determined self-consistently from the charge densities of electrons and ions by the Poisson equation:

$$\Delta \hat{\Phi}(x, t) = 4\pi e \left( n_e(x, t) - Z \sum_{j=1}^{N_i} \delta(x - x_j) \right)$$  \hspace{1cm} (7)

where $n_e(x, t) = N_e \langle x | \hat{\rho}_1 | x \rangle$ is the electron density. The potential is calculated as a classical field $\Phi(\bar{x}, t)$ which is used as an operator $\hat{\Phi}(\bar{x}, t)$ in Eq. (4). Thus, assuming electrostatic interactions with a self consistent collective scattering potential $\Phi(\bar{x}, t)$ and an externally applied potential $\Phi_{\text{ext}} = -x \cdot E_{\text{ext}}(t)$, due to a time dependent laser field $E_{\text{ext}}(t) = E_0 \sin(\omega_0 t)$, the Hamiltonian can be written as

$$\hat{H} = \frac{p_i^2}{2m} - e \left[ \Phi(\bar{x}_1, t) + \Phi_{\text{ext}}(\bar{x}_1, t) \right]$$  \hspace{1cm} (8)

The heating rate of the plasma by the external laser field can be expressed using the effective electron-ion collision frequency $\tilde{\nu}$ given by the following expression:

$$\nu_{ei} = \frac{4\pi \omega_e^2}{\omega_p} \left( \frac{j}{E} \cdot \frac{E}{E} \right) = \frac{4\pi \omega_e^2}{\omega_p^2} \sigma.$$  \hspace{1cm} (9)
Here, the overline stands for averaging over one oscillation period and the angular brackets denote the expectation values of the quantum operators. In the last equation $\jmath$ is the electric current density operator: $\jmath = -(en_c/m)\hat{p}_1$. In order to determine the time dependence of the expectation values of the operators, one has to switch from the microscopic quantities given by the kinetic equations to the average macroscopic quantities via the statistical operators. Thus, multiplying by $\hat{p}_1$ and applying the trace to both sides of Eq. (11), the time change-rate of the expectation value of the momentum $\langle \hat{p}_1 \rangle$ is obtained

$$i\hbar \frac{\partial}{\partial t}\langle \hat{p}_1 \rangle - \left[\hat{p}_1, \hat{H}\right] = i\hbar \text{Tr} \tilde{\mathbb{T}}_{hc} \hat{p}_1, \quad (10)$$

where the hard-collision integral $\tilde{\mathbb{T}}_{hc}$ is defined by:

$$\tilde{\mathbb{T}}_{hc} = \text{Tr}\{N \hat{V}_{ei}(s) - \hat{V}_{ei}(s)N\}/i\hbar. \quad (11)$$

The integral on the right-hand side contains hard-collisions only and is related to the hard-collisions contribution to the stopping power by

$$\langle \frac{dE}{dx} \rangle = \text{Tr}\{\tilde{\mathbb{T}}_{hc}(t)\hat{p}_1\}. \quad (12)$$

In order to obtain an analytical solution of Eq. (9), it is useful to cast the effect of the stopping power as an average friction force between the electron and ion fluids $R \equiv -(dE/dx)/V$, where $V = (\hat{p}_1)/m$ is the average ensemble velocity and restrict the solution to small particle velocities, such that $V \lesssim v_{th}$, where $v_{th} \equiv kT_e/m$ is the electron thermal velocity, since in this case the friction coefficient is velocity independent 24.26. The dynamics of screening is of minor importance in the low velocity range, justifying the use of the stopping power data calculated assuming statically screened Coulomb interactions 24.26. In this limit, the rate equation for the average momentum becomes

$$\frac{d}{dt}\langle \hat{p}_1 \rangle = -e\left(\frac{E_0}{\omega_0} \sin (\omega_0 t) + \langle \hat{E} \rangle\right) - \nu_{hc} \langle \hat{p}_1 \rangle, \quad (13)$$

where the hard collision frequency $\nu_{hc}$ is defined as $\nu_{hc} \equiv R/m$ and the polarisation field is $\langle \hat{E} \rangle \equiv \langle d\hat{E}/d\hat{x} \rangle$.

Field amplitudes for which the assumption of velocity independent friction coefficient $R$ is valid are restricted by the condition $V_{max} < v_{th}$. Neglecting the contribution of the polarisation field $\langle \hat{E} \rangle$ in Eq. (13), the maximum average velocity can be estimated by

$$V_{max} = \frac{1 + \nu_{hc}}{1 + \nu_{hc}}v_0, \quad (14)$$

where $v_0 \equiv eE_0/m\omega_0$ is the free electron quiver velocity, and $\nu_{hc} \equiv \nu_{hc}/\omega_0$ is the normalised hard-collision frequency. Therefore, in the high-frequency limit: $\nu_{hc} \ll 1$ the fields amplitudes are restricted by $v_0 \lesssim v_{th}$; and in the low-frequency limit: $\nu_{hc} \gg 1$ the condition becomes $v_0 \lesssim \nu_{hc} v_{th}$. Thus, because of the strong damping of the velocity in the low-frequency limit, the region where $R = const$ is applicable can be extended to stronger fields.

Furthermore, the use of the stopping power formalism imposes additional limitation with respect to the laser frequency $\omega_0$. The stopping power treatment assumes that the typical collision time is greater than the period of plasma oscillation characterised by the plasma frequency $\omega_p \equiv \sqrt{4\pi e^2 n_e/m}$. Therefore, the analytical solution to be obtained is restricted to the laser frequency range of $\omega_0 \lesssim \omega_p$.

Multiplying both sides of Eq. (13) by $-en_c/m$ the current balance equation is obtained

$$\frac{d\langle \hat{j} \rangle}{dt} = \frac{\omega_p^2}{4\pi} \left[\frac{E_0}{\omega_0} \sin (\omega_0 t) + \langle \hat{E} \rangle\right] - \nu_{hc} \langle \hat{j} \rangle \quad (15)$$

Note, that in the low-frequency limit $\nu_{hc} \gg 1$, the polarisation field $\langle \hat{E} \rangle$ vanishes and Eq. (13) produces the well known Drude formula for the low-frequency conductivity:

$$4\pi\sigma_D = \frac{\omega_p^2}{\nu_{hc} - i\omega_0}. \quad (16)$$

A formal solution of Eq. (15) is

$$\langle \hat{j}(t) \rangle = -\frac{\omega_p^2}{4\pi} \frac{\gamma E_0}{\omega_0} \left[\cos (\omega_0 t) - \nu_{hc} \sin (\omega_0 t)\right] + \frac{\omega_p^2}{4\pi} \int_{-\infty}^{t} \langle \hat{E}(\tau) \rangle e^{\nu_{hc}(\tau-t)} d\tau \quad (17)$$

where $\gamma \equiv 1/(1 + \nu_{hc}^2)$. One can see that the current consists of two terms, namely the polarisation current represented by the second term on the r.h.s., and the free current, represented by the first term. The latter includes a contribution from the hard collisions that are in phase with the laser field. The collision rate is obtained by substituting Eq. (17) into Eq. (11):

$$\nu_{ei} = \gamma \nu_{hc} + \frac{2\omega_p^2}{E_0^2} \left[\frac{E_0}{\omega_0} \sin (\omega_0 t) \int_{-\infty}^{t} \langle \hat{E}(\tau) \rangle e^{\nu_{hc}(\tau-t)} d\tau\right] \quad (18)$$

Here, the first term is due to the the strong collisions. It has the same form as the real part of the Drude conductivity. The second term is due to the polarisation current.

To complete the derivation one needs to calculate the polarisation field $\langle \hat{E} \rangle$ to be used in the last equation. To do so the system of Eq. (11) and Eq. (17) has to be solved. The general solution of this problem is notoriously difficult however, it can be greatly simplified if one assumes that the same average friction force $-\nu_{hc} \langle \hat{p}_1 \rangle$ is acting on all the electrons irrespective of their direction, position
and velocity. This assumption is justified in the limit of 
\( V_{\text{max}} \gg v_{\text{th}} \), when the ions directed motion as a particle beam characterised by a single velocity with respect to the electrons can be considered. In such case, Eq. (13) can be cast into the form of the quantum Vlasov equation:

\[
\imath \hbar \frac{\partial}{\partial t} \hat{\rho}_1 - \left[ \hat{H}_{\text{eff}}, \hat{\rho}_1 \right] = 0, \tag{19}
\]

where the effective Hamiltonian \( \hat{H}_{\text{eff}} \) is:

\[
\hat{H}_{\text{eff}} = \frac{\hat{p}_1^2}{2m} - e \left( \hat{\Phi} + \hat{\Phi}_{\text{ext}} \right) + \nu_{\text{he}}(\hat{\rho}_1) \hat{x}_1 \tag{20}
\]

Eq. (19) produces the same current balance equation as Eq. (13). The set of two equations: Eq. (21) and Eq. (19) form a closed set of Vlasov-Poisson (VP) equations with the hard collision determined using the standard methods applied for the stopping power calculations.

### III. SOLUTION OF QUANTUM VP EQUATIONS

In this chapter we shall obtain the solution of the system of VP equations introduced above. The laser and the friction act both as an effective external force since they depend on \( \hat{x} \) only. Therefore, in the absence of the scattering field \( \hat{\Phi} \) the electrons would perform a quiver motion. It is useful to transform to Kramers-Henneberger (KH) reference frame – the rest frame of the electron fluid, since one can assume that in this frame the electrons are close to the equilibrium unperturbed state. The transformation to KH frame is given by

\[
\hat{\Phi}(k, \omega) = \frac{\Sigma_i(k, \omega)}{D(k, \omega)} \tag{23}
\]

where \( \Sigma_i(k, \omega) \) is the Fourier-Laplace transform of the ion potential in the KH frame. The Fourier transform assumes the form,

\[
\Sigma_i(k, t) = \Sigma_{i,0}(k) \times e^{ik \cdot \xi(t)} \tag{24}
\]

where

\[
\Sigma_{i,0}(k) = \frac{4\pi Z e}{k^2} \sum_j e^{-ik \cdot x_j} \tag{25}
\]

is a static part and

\[
e^{ik \cdot \xi(t)} = \sum_{n, m} (-1)^n (-i)^m J_n(\gamma z) J_m(\gamma \nu_{\text{he}} z) e^{i(n+m)\omega_0 t}, \tag{26}
\]

is a dynamic phase factor due to the quiver motion. Its Fourier coefficients are Bessel functions of the first kind \( J_n \) depending on the parameter \( z \equiv k \cdot e \). Applying the Laplace transformation to Eq. (24) yields

\[
\Sigma_i(k, \omega) = \Sigma_{i,0}(k) \sum_{n, m} (-1)^n (-i)^m J_n(\gamma z) J_m(\gamma \nu_{\text{he}} z) \frac{1}{-i(\omega + (n + m)\omega_0)} \tag{26}
\]

The electric potential generated by the ions in the KH frame is screened by the dielectric function \( D(k, \omega) \)

\[
D(k, \omega) = 1 + 4\pi \chi(k, \omega) \tag{27}
\]

which is known as the Lindhard dielectric function and the response function \( \chi(k, \omega) \) is given by

\[
\chi(k, \omega) = \frac{e^2}{k^2} \int d^3 u \frac{f(u + \hbar k/2m) - f(u - \hbar k/2m)}{\hbar(\omega - k \cdot u)} \tag{28}
\]

The perturbed charge density

\[
\tau_e(k, \omega) = -eN_e\rho^{(1)}(k, \omega) \tag{29}
\]

is related to the effective potential and the response function by
Finally, we apply the inverse Laplace transform to $\tau_r(k, \omega)$ and $\Phi(k, \omega)$ and obtain from the poles of $\Sigma_i(k, \omega)$ at the frequencies $\omega = -(n + m)\omega_0$ the asymptotic result,

\begin{align}
\Phi(k, t) &= \sum_{n,m} \Phi_{n,m}(k)e^{i(n+m)\omega_0 t} \\
\Phi(k, t) &= \sum_{n,m} \Phi_{n,m}(k)e^{i(n+m)\omega_0 t}
\end{align}

where the coefficients of these series are given by

\begin{align}
\Phi_{n,m}(k) &= \Sigma_{i,0}(k)\sum_{n,m} (-1)^n (-i)^m J_n(\gamma z) J_m(\gamma \nu_{he} z) \\
\tau_{n,m}(k) &= -k^2 \chi(k, -(n + m)\omega_0) \Phi_{n,m}(k)
\end{align}

These results allow us to obtain the expectation value of the electric field used in the calculation of the collision frequency in Eq. (31). In the momentum representation $E(k, t) = -ik\Phi(k, t)$ and using the transformations (A8b), (A15) from Ref. 14 we yield

$$\langle E(k, t) \rangle = -\frac{1}{eN_e} \int \frac{d^3 k}{(2\pi)^3} i e k \Phi^*(k, t) \tau_e(k, t)$$

Substituting this expression into Eq. (18) and using Eqs. (30) we get the collision frequency

$$\nu_{ei} = \frac{\gamma \nu_{he}}{\omega_p} - \sum_{n = -\infty}^{\infty} \alpha \int \frac{d^3 k}{k^2} \frac{J_n(\gamma z) J_m(\gamma \nu_{he} z)}{D(k, -(n + m)\omega_0)}$$

\begin{align}
&+ \sum_{s = -\infty}^{\infty} i^n J_{m+s}(\gamma \nu_{he} z) \left[ (n-s)(1 + i \nu_{he})J_{n-s}(\gamma z) \\
&- \nu_{he}(\gamma \nu_{he} z) J_{n-s-1}(\gamma z) \right] S_{ii}(k)
\end{align}

where, $S_{ii}(k)$ is the ion-ion structure factor, and

$$\alpha \equiv (2\pi^2)^{-1}(\omega_0/\omega_p)(Ze^2/m\nu_0^2)$$

Clearly, the known limiting cases can be readily retrieved. Decker’s result (Eq. 20 in Ref. 5) follows from Eq. (33) in the weak coupling limit $\nu_{he} \rightarrow 0$ and non-degenerate plasmas. In this case $D(k, \omega)$ becomes the classical dielectric function and the integral must be truncated at $k_{max}$ to avoid the divergence at small impact parameters (see Ref. [25] for the discussion of different cutoffs). Here, all integrals can be performed to infinity and no ad hoc cutoffs must be introduced as a result of the quantum mechanical treatment. The first term in line 1 dominates for small laser frequencies $\nu_{he} \rightarrow \infty$ giving a Drude-like expression.

IV. RESULTS AND DISCUSSION

In the previous sections we have presented a quantum mechanical formulation for the problem of laser absorption in dense plasmas. The formalism is similar to that developed by Kull and Plagne, but inherently includes the hard-collisions absent from this and other approaches [4, 12, 16]. This was achieved in three steps. Firstly, the the two-particle density function was split into: (i) the first order Hartree term, representing the weak interactions; (ii) the higher orders, representing the hard-collisions, which were collected into the pair correlation function (e.g. Eq. (34)). Secondly, the hard-collisions were cast into the form of the friction force using the stopping-power integral on the right-hand side of the momentum rate equation (10). At last, the collision frequency Eq. (9) was determined by finding the first order perturbation of the equilibrium density distribution as a result of the electron-ion scattering. The equilibrium electron distribution is set up in the electron KH rest frame determined by the external potential and the friction force due to the hard-collisions.

The use of the stopping power formalism restricts this approach by demanding that: (i) the interaction time is larger than or at least of the same order of magnitude as the typical plasma oscillation period, setting $\omega_0 \gtrsim \omega_p$; (ii) the field strength is sufficiently high to consider the ion motion relative to the electrons as that of a directed beam characterised by a single velocity, i.e. $V_{max} \gg v_{th}$. From the other hand, the analytical solution of Eq. (4) expressed in Eq. (34) is only valid in the low-velocity limit $V_{max} < v_{th}$, where the friction coefficient $R$ is constant. The value of the maximum relative velocity $V_{max}$ depends both on the laser field strength and $\nu_{he}$ according to Eq. (14).

The end result in Eq. (33) formally resembles the Gould-DeWitt ansatz [28] due to the splitting of the collision frequency into a sum of two contributions resulting from the strong and weak interactions. However, here no ad hoc assumption was made and the splitting to the hard and weak collision contributions in Eq. (33) was obtained as a result of the discussed solution process. Thus, the present approach might also hint on the region of applicability of the Gould-DeWitt scheme when used in other models.

The hard electron-ion collisions are incorporated in Eq. (33) via a friction force related to the stopping power of the ions in an electron gas that in turn sets up a more general KH frame instead of the freely oscillating one adopted in other works [4-6, 12, 16]. Many models have been developed for the stopping power [23], few include hard collisions. Within quantum statistical theory, they can be described by a T-matrix approach based on the quantum Boltzmann equation. The related cross sections are calculated from numerical solutions of the Schrödinger equation [24]. The full stopping power can be then determined applying the Gould-DeWitt scheme [28] or by velocity-dependent screening length [23, 20].
In conclusion, the multi-dimensional parameter space should also be examined along the direction of the laser frequency. This comparison is demonstrated in Fig. 2 for fixed electron temperature in the lower frame, and good agreement between the MD results and our approach is obtained for this wide data range. However, the contribution of hard-collision terms is rather small for the parameters presented in this parameter area. From this, the observed change in the slope sign is due to the turnover in the hard-collision contribution. The latter occurs because high coupling strengths.

Note, that different parameter sets are tested in the upper and lower frames of Fig. 1, the data running as a function of the increasing density in the upper and decreasing temperature in the lower frame, and good agreement between the MD results and our approach is obtained for this wide data range. However, the contribution of hard-collisions is rather small for the parameters presented in the upper frame, and therefore the advantages of the current approach are less pronounced in that case.

Degeneracy might obscure the comparison at low temperatures or high densities. However, degeneracy is neither included in the MD simulations nor in our calculation of the hard collision term which is based on solutions of two-particle Schrödinger equation. We therefore compare our data to the MD simulations on a similar level of approximation.

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density and the ratio of $v_0/v_{th} = 0.2$, where the conductivity is plotted as a function of the laser frequency. Here again we show the contributions of Eq. (33) separately. As expected, the contribution of the dynamic, polarization term vanishes at low frequencies $\omega_0 \ll \omega_p$, and the conductivity is dominated here by the Drude-like term due to the hard-collisions. At the intermediate frequency range $\omega_0 \sim \omega_p$, both terms contribute equally, with the dynamic term overtaking for lower values of $\Gamma$. As discussed earlier, the present approach is only valid for the laser frequencies that are of the same order of magnitude as the plasma frequency and lower, therefore we do not extend the comparison to the high laser frequencies. The breakdown of the present approach at high frequencies is already exhibited in Eq. (33). As follows from the latter, at the high frequency limit the hard-collision dominate the absorption, moreover $\nu_{ei} \sim \nu_{hc}$, which is obviously senseless.

In conclusion, a quantum mechanical approach for the calculation of collisional absorption of laser light in dense plasmas was presented. It consistently incorporates the dynamic, field dependent response and hard electron-ion collisions, in contrast to the earlier approaches that neglected the effect of the latter. The use of the quantum mechanical formulation allows avoiding the use of ad hoc cutoffs and thus the theory remains reliable for strong electron-ion interactions. The hard-collisions were introduced via the average friction force due to the stopping power. Therefore, for the first time a link between the stopping power and the problem of collisional laser absorption is drawn. It allows applying the many theories developed for the stopping power to the problem of collisional absorption. Although only results for the quasi-linear regime $v_0/v_{th} \leq 1$ were presented, the approach can be easily extended to higher field amplitudes, correlated ions, and multiple ionisation stages.

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