Modeling For Helicopter Mode of a Tilt Tri-Rotor UAV

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Abstract. This paper developed a new structural configuration of tilt tri-rotor UAV. Based on the establishment of the coordinate system, the force in the helicopter mode was analyzed and the nonlinear six-degree-of-freedom equation of the aircraft was obtained. Finally, the model was established and the simulation of the open loop system was carried out.

1. Introduction

With functions of helicopter and fixed wing, the tiltrotor UAV are gradually being used in military and civilian applications. The tiltrotor UAV is equipped with tilting rotors on the wings on both sides of the fixed-wing aircraft to provide lift during taking off and landing and flying forward. The tiltrotor UAV has the characteristics of taking off vertically, flying at a high speed, enabling free conversion of fixed wing mode, transition mode and helicopter mode. Currently, the US Osprey V22 has been used in military field, but this aircraft often has an accident [1]. In 2010, Chiba University of Japan developed a Quad Rotors QTW-UAV with Tilt Wing [2], but it has a lot of resistance when flying forward. This paper developed a new structural configuration of tilt tri-rotor UAV, depicted in Figure 1. There are two motors in front of the aircraft and one motor in the back. The first two motors can be tilted to implement conversion between the three modes- fixed wing mode, transition mode and helicopter mode.

Figure 1. Tilt tri-rotor prototype

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The total weight of the aircraft is 4.6kg. In the helicopter mode, the pitch motion of the aircraft is realized by the different speed of the front and rear motors. The roll motion of the aircraft is realized by different speed of the left and right motor. The yaw motion of the aircraft is realized by the different steering angle of the steering gear. In the following, the dynamics and kinematics analysis of helicopter mode for the aircraft will be introduced which lays the foundation for designing the control system.

2. Dynamic model of the helicopter mode

The mathematical model of the aircraft is the basis for the design, simulation and flight performance analysis of the flight control system. This section uses the Newton-Euler method to establish a six-DOF model of the aircraft.

2.1. Defining the coordinate system

As presented in figure 2, the x-axis of the body coordinate system $X_{Body}$ is parallel to the theoretical longitudinal axis of the aircraft and points to the head, y-axis points to the right and z-axis points down. The x-axis of the ground coordinate system $X_{Earth}$ is in the ground plane and points in a certain direction, z-axis points to the center of the earth and the y-axis is determined according to the right hand rule.

![Figure 2. Coordinate system diagram](image)

The conversion relationship between the ground coordinate system and the body coordinate system [3] is given by

$$X_{Body} = R_{eb}X_{Earth}$$

(1)

$$R_{eb} = \begin{bmatrix}
sin \phi \cos \psi & c \theta \cos \psi & -c \theta \\

-c \phi \cos \psi & s \theta \cos \phi + c \psi \phi & c \psi \\

-s \phi \cos \psi & s \theta \sin \phi - c \psi \phi & c \psi \end{bmatrix}$$

(2)

Where, $c$ is equivalent to $\cos$, $s$ is equivalent to $\sin$. $\phi$, $\theta$, $\psi$ are Euler angels—roll angel, pitch angle, yaw angle.

2.2. Force analysis of helicopter mode

According to the blade element theory [4], to calculate the tension and torque, introduce the inflow ratio coefficient $\lambda$ and the forward ratio coefficient $\mu$ that characterize the rotor operating state.

$$\lambda = \frac{v_0 - V \sin \alpha_e}{\omega r_f}$$

$$\mu = \frac{V \cos \alpha_e}{\omega r_f}$$

(3)
Where, \( \alpha \) is the angle of attack of propeller blade. \( r \) is the radial position where the blade element is located. \( \omega \) is the rotor’s speed. \( v_0 \) a is the induction speed at the paddle. \( V \) is airspeed of the aircraft.

When flying the rotor is affected by the lift \( F \), the counter torque \( Q \), the roll moment \( L \) and the resistance \( D \). Their relationship to the square of the rotor’s speed is as follows

\[
F = \frac{1}{2} \rho AC_p R^2 \omega^2
\]

\[
Q = \frac{1}{2} \rho AC_q R^2 \omega^2
\]

\[
L = \frac{1}{2} \rho AC_L R^2 \omega^2
\]

\[
D = \frac{1}{2} \rho AC_D R^2 \omega^2
\]

Where, \( \rho \) is atmospheric density. \( A, R \) are the area and radius of the propeller paddle. The lift coefficient, torque coefficient, roll coefficient and resistance coefficient of the rotor \( C_F, C_Q, C_L, C_D \) are given by

\[
\frac{C_F}{2g_{\theta}} = \left( \frac{1}{6} + \frac{1}{4} \mu^2 \right) \theta_s - \frac{1}{4} \lambda
\]

\[
\frac{C_Q}{2g_{\theta}} = \frac{1}{8} \left( 1 + \mu^2 \right) \overline{C_{\mu}} + \lambda \left( \frac{1}{6} \theta_s - \frac{1}{4} \lambda \right)
\]

\[
\frac{C_L}{2g_{\theta}} = \frac{1}{4} \mu \overline{C_{\mu}} + \frac{1}{4} \lambda \mu \theta_s
\]

\[
\frac{C_D}{2g_{\theta}} = \mu \left( \frac{1}{6} \theta_s - \frac{1}{8} \lambda \right)
\]

Where, \( \theta \) is the real degree. \( \theta_s \) is the geometric installation angle of the blade element.

When the aircraft is hovering or flying at low speed, the speed is small, and its airspeed \( V \) is considered to be approximately zero. We can get the lift and torque from equation (2)-(11)

\[
F = b \omega^2
\]

\[
Q = d \omega^2
\]

Where, rotor tension factor \( b = \frac{1}{6} \theta a \theta_s \rho AR^2 \). Rotor torque factor \( d = \frac{1}{8} \theta C_d \rho AR^2 \).

When the rotor rotates at a high speed around its axis of rotation, it will also tilt forward and backward under the driving of the tilting steering gear. In this process, the gyroscopic torque [5] will be generated.

For the front two motors, there are two sources of gyroscopic torque: generated by the fuselage movement in the air; generated by the rotor tilting driven by steering machine. Gyro moment is given by

\[
Q_g = J_m \omega \times (\omega_B + \omega_C)
\]
Where, \( I_B \) is the moment of inertia of the rotating part including motor and propeller. \( \omega_a \) is the tilting angular velocity of rotor. \( \omega_B \) is the rotational angular velocity of the aircraft in the ground coordinate system.

The gyroscopic torque of rear motor is only generated by the fuselage movement in the air. The torque that the rotor is subjected to is given by

\[
Q = r \times F + Q
\]

(15)

Where, \( r \) is position vector of the rotor lift point.

The direction of gravity always points to the center of the earth. Assuming that the change in gravitational acceleration \( g \) is negligible, the expression of gravity in the body coordinate system is given by

\[
G_b = \begin{bmatrix}
G_x \\
G_y \\
G_z \\
G_w
\end{bmatrix} = \begin{bmatrix}
-mg \sin \theta \\
mg \cos \theta \sin \Phi \\
mg \cos \theta \cos \Phi
\end{bmatrix}
\]

(16)

Where, \( m \) is the weight of the aircraft.

From equation 1 and 12-16, we can obtain the total force \( F_T \) and \( Q_T \) total torque received by the aircraft in the Body coordinate system.

2.3. Kinematics and dynamics analysis

The dynamics of the aircraft is extremely complex. Before establishing the equation of motion, it is assumed that the aircraft is a rigid body and the center of mass is unchanged; The aircraft is symmetric about the OXZ plane, then there is \( I_{xy} = I_{yx} = I_{yz} = I_{zy} = 0 \). The aircraft has six degrees of freedom: line motion— forward motion, lifting motion, lateral motion; angular motion—pitch motion, yaw motion, rolling motion. Applying Newton’s second law in the inertial system, we can obtain that

\[
F = \frac{d(mV)}{dt}
\]

(17)

\[
Q = \frac{d(J\omega)}{dt}
\]

(18)

The above formula can be expressed in the body coordinate system as [6]

\[
F_T = m\dot{V} + \omega \times (mV)
\]

(19)

\[
M_T = J\dot{\omega} + \omega \times (J\omega)
\]

(20)

Where, \( J \) is the moment of inertia matrix of the aircraft.

Substituting \( V=[u \ v \ w]^T \) and \( \omega=[u \ v \ w]^T \) into the above equation, we can obtain the equation of force and moment of the aircraft.

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix} = \begin{bmatrix}
v r - w q \\
-w r + u p \\
u q - v p
\end{bmatrix} + \begin{bmatrix}
\frac{F_{Tx}}{m} \\
F_{Ty} \\
F_{Tz}
\end{bmatrix}
\]

(21)

\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
\frac{Q_{Tx} + q r (l_y - l_z)}{l_x} \\
\frac{Q_{Ty} + p r (l_x - l_z)}{l_y} \\
\frac{Q_{Tz} + p q (l_x - l_y)}{l_z}
\end{bmatrix}
\]

(22)
Where, \( p, q, r \) are the components of the rotational angular velocity \( \omega \) on each axis of the body coordinate system. \( u, v, w \) are the component of airspeed \( V \) on each axis of the body coordinate system.

The angular motion equations and the linear motion equations can be obtained by the relationship between the body coordinate system and the ground coordinate system.

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi \sec \theta & \cos \phi \sec \theta
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\tag{23}
\]

\[
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix} = R_{eb}^T
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix}
\tag{24}
\]

Where, \( x, y, z \) represent the displacement motion of the aircraft in the ground coordinate system.

#### 3. Open Loop System Simulation

In order to study the flight characteristics of the aircraft and design control system, the six-degree-of-freedom model of the aircraft is analyzed:

1) The torque equation shows that the moment received by the aircraft causes the change of angular velocity \( [p \quad q \quad r]^T \);

2) The angular motion equation shows that the change of angular velocity causes the change of attitude angle \( [\phi \quad \theta \quad \psi]^T \);

3) The force equations show that the angular velocity, the attitude angle and the force cause the change of the velocity \( [u \quad v \quad w]^T \);

4) The linear motion equation shows that the position \( [x \quad y \quad z]^T \) of the aircraft changes under the joint action of the attitude angle and the velocity.

It can be seen from the above analysis that there is a series relationship between the four quantities of the aircraft. On the basis of the 6-DOF system, the open-loop system of the aircraft is modeled in MATLAB/Simulink. Under the premise of ignoring the gyro moment, the equilibrium point is obtained using the numerical algorithm. The initial values set are shown in Table 1.

| Parameter | \( \omega_1/\text{rad/s} \) | \( \omega_2/\text{rad/s} \) | \( \omega_3/\text{rad/s} \) | \( \alpha_1/\degree \) | \( \alpha_2/\degree \) | \( x \quad y \quad z \)^T | \( \phi \quad \theta \quad \psi \)^T |
|-----------|----------------------------|----------------------------|----------------------------|----------------|----------------|------------------|------------------|
| Value     | 572.4                      | 571.1                      | 464.1                      | 6.2477        | -6.2475        | [0 0 0]^T        | [0 0 0]^T        |

Where, \( \alpha_1, \alpha_2 \) are the tilt angles of the left and right motors respectively. The front two motors have an initial angle to counteract the reverse torque of the tail motor. The simulation results are shown in Figure 3.
As can be seen from the above figure, the position of three axes has been increasing, the pitch angle and the yaw angle have been oscillating at high speed and the roll angle is increasing rapidly. Actually the aircraft has already crashed. Therefore, a well-designed flight control system plays an important role in the stable flight of the aircraft.

4. Conclusion
A tilt tri-rotor UAV with helicopter and fixed wing function was developed. The 6-DOF model was established and open loop system was simulated. Next, the design of the control system will be carried out.

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