Two-component liquid model for the quark-gluon plasma†

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(Dated: July 13, 2010)

We consider a two-component-liquid model, a la Landau, for the quark-gluon plasma. Qualitatively, the model fits well some crucial observations concerning the plasma properties. Dynamically, the model assumes the existence of an effective scalar field which is condensed. The existence of such a condensate is supported by lattice data. We indicate a possible crucial test of the model by lattice simulations.

I. INTRODUCTION

The discovery of the strongly interacting quark-gluon plasma at RHIC1 made a great impact on the landscape of theoretical papers devoted to quantum chromodynamics. There emerged a new problem of explaining the exotic properties of the plasma. It is as fundamental and interesting as the confinement problem and in fact the two problems are to be considered in conjunction with each other. Moreover, there is renewed interest in relativistic hydrodynamics, superfluidity and, more generally, in applying the holographic methods to condensed-matter systems.2

In this paper we consider the possibility that a variation of the famous two-component model of superfluidity applies directly to the quark-gluon plasma. Let us remind the reader, very briefly, the model itself. The main point is that a volume element of the liquid cannot be characterized any longer by a single 4-velocity $u^\mu$ with $(u^\mu)^2 = -1$. Instead there are two substances, or motions with (normal) density $\rho_n$ and superfluid density $\rho_s$ and with independent 4-velocities, $u^\mu$ and $v^\mu$, respectively. The total density is the sum of the two components:

$$\rho_{tot} = \rho_n + \rho_s. \quad (1)$$

The superfluid component is described in terms of a scalar field $\phi$. Which in the non-relativistic limit becomes the phase of the condensate wave function.

Formally, the current and energy-momentum tensor are written as follows:

$$j^\nu = \rho_n u^\nu + \rho_s v^\nu \quad (2)$$

$$T^{\nu\sigma} = (\epsilon + P)u^\nu u^\sigma + P\eta^{\nu\sigma} + \mu \rho_s v^\nu v^\sigma, \quad (3)$$

where $\eta^{\nu\sigma} = (-1,1,1,1)$, $\epsilon$ is the energy density of the normal component, $P$ is the pressure and $\mu$ is the chemical potential. Note also that for simplicity we approximated the energy-momentum tensor by the case of an ideal liquid. The equations of motion are then

$$\partial_{\nu} T^{\nu\sigma} = 0, \quad \partial_{\nu} j^{\nu} = 0, \quad v^\nu v_{\nu} = -1. \quad (3)$$

The most important point about the model is, of course, our assumption of two independent motions taking place at the same point. We will comment on this later, in the context of field theory.

The outline of the paper is as follows. First, in Sec. 2 we discuss the qualitative features of the plasma and argue that the two-component model fits the data qualitatively. Next, in Sec. 3 we consider the issue of scalar fields in Yang-Mills theories. The existence of scalar fields with certain properties is dynamically a prerequisite for the validity of the model. We conclude that the lattice data rather support the existence of (effective) scalar fields. In Sec. 4 we propose a crucial test of the model through measuring the correlator of components of the energy-momentum tensor.

II. QUALITATIVE FEATURES

It might be useful (for the purpose of model building) to reduce the plasma properties to three points, namely, equation of state, viscosity and the role of quantum effects.

A. The existence of the plasma was conjectured long time ago. Moreover the equation of state of the plasma...
has been known also since long since it was established via numerical experiments within the lattice formulation of QCD, for references see, e.g., [1]. It turns out that the equation of state is close to that of an ideal gas of quark and gluons:

$$[\epsilon(T)]_{\text{plasma}} \approx (1 - \delta) [\epsilon(T)]_{\text{ideal gas}}, \quad (4)$$

where the correction $\delta \approx 0.15$, $\epsilon(T)$ is the energy density as function of temperature and $[\epsilon(T)]_{\text{ideal gas}}$ is the energy density for non-interacting quarks and gluons.

Thus, the equation of state indicates that the plasma is close to an ideal gas.

B. The observation (4) produces the illusion of simplicity of the properties of the plasma. However, analysis of the data obtained at RHIC led to the conclusion that the plasma possesses the lowest viscosity $\eta$ among all the substances known so far:

$$\frac{\eta}{s}_{\text{plasma}} \approx \frac{1}{4\pi}, \quad (5)$$

where $s$ is the entropy density (introduced to measure the viscosity in dimensionless units). The value of $1/4\pi$ is somewhat symbolical. The actual value of $\eta$ might be larger, say $\eta/s \sim 0.4$ [1] or even lower, see [2]. The value $\eta/s = 1/4\pi$ represents the conjectured lower limit [3].

Anyhow, the viscosity observed for the plasma is the lowest one among all the known liquids [1]. Thus, measurements of the viscosity indicate that the plasma is close to an ideal liquid (which is defined as having $\eta = 0$). Note that for the ideal gas the viscosity tends to infinity,

$$\frac{\eta}{s}_{\text{ideal gas}} \rightarrow \infty. \quad (6)$$

More precisely, this ratio is inverse proportional to the coupling constant squared $\eta/s \sim 1/\alpha^2$.

C. As a kind of variation of point B, one argues [6] that such a low value of viscosity implies that quantum effects are crucial and that the liquid cannot be, rigorously speaking, treated classically. Indeed, based on estimates common in kinetics one readily finds that

$$\eta \sim \tau_{\text{relaxation}} \epsilon,$$

while for the entropy density one can use $s \sim k_B n$ where $k_B$ is the Boltzmann constant, $\tau$ is the relaxation time, $\epsilon$ is the energy density and $n$ is the density of particles. The central point is that from the uncertainty principle the product of energy of a particle, $\epsilon/n$ times its life time, $\tau$ cannot be smaller that the Planck constant. Thus:

$$\frac{\eta}{s} \sim \frac{\tau_{\text{relaxation}}}{\tau_{\text{quantum}}}, \quad (7)$$

where the ”quantum time” $\tau_{\text{quantum}} \sim \hbar/k_B T$. Then the observation [2] implies quantum nature of the quark–gluon plasma.

It is a challenge to theory to explain all three observations, [4], [5], [7] which are apparently pointing in opposite directions. Indeed, one starts at point A with the idea that the plasma is an ideal gas and ends up at point C with a kind of a proof that the plasma is in fact a quantum liquid.

It is amusing that it is quite straightforward to suggest a model which allows – on a qualitative level – to unify all the would-be contradictory features of the plasma [2]. We have in mind the two-component model of superfluidity a la Landau.

Indeed, what is “special” about the viscosity? How is it possible to have an equation of state close to that of the ideal gas and still a nearly vanishing viscosity? Let us imagine that we are dealing with a two-component substance. One of the components occupies a larger phase space, $c_1$ and is responsible for the equation of state. The other one has a smaller phase space, $c_2$ but very small viscosity. Then the total viscosity can still be small since, at least naively, to evaluate the total viscosity one adds inverse powers of the partial viscosities:

$$\frac{1}{\eta_{\text{tot}}} = \frac{c_1}{\eta_1} + \frac{c_2}{\eta_2}, \quad (8)$$

where $c_{1,2}$ are normalized by $c_1 + c_2 = 1$. Indeed, the meaning of the viscosity $\eta$ is similar to that of resistance and if we have two independent motions then we would apply the rule\footnote{Equation [8] can be found in, e.g., in old books on classical solutions [8]. In more modern terms, the example of the superfluidity itself might serve as the best illustration to [8]. Indeed, the superfluid fraction can be small while the whole liquid is superfluid. On more detailed level, some care should be exercised since one has distinguish between viscosity with respect to a capillary motion and with respect to rotations, for a recent exposition see, e.g., [8].} [8].

Thus, the two-component model accommodates naturally points A, B above. Assuming one of the components be superfluid explains, as a bonus, the point C as well.

Another point is worth emphasizing. In the non-relativistic case the superfluid component evaporates at finite temperature $T_c$. The physics behind this is readily understood. Indeed, at $T = 0$ the superfluid component is related to the condensate of particles with momentum $p = 0$. At non-vanishing temperature the particles are excited by temperature. Because of the conservation of the number of particles in the non-relativistic case, the superfluid component disappears at finite temperature.

In the relativistic case, that is in the absence of conservation of particles, the theoretical constraints on the phase space occupied by the superfluid component are weaker. Indeed, even at $T \rightarrow \infty$ the non-perturbative component in case of Yang-Mills theories vanishes only logarithmically:

$$\lim_{T \rightarrow \infty} c_2(T) \sim g_2^4(T) \sim \frac{1}{(\ln T)^3}, \quad (9)$$
where $g^2(T)$ is the coupling of the original 4d theory.

Finally, we come to discuss the point C above. Well, it is quite clear that if we assume superfluidity then the bounds like (10) could be violated. Indeed, in the superfluid case we have a condensate and the whole counting of degrees of freedom in terms of the density $n$ breaks down, generally speaking.

To summarize, the two-liquid model explains very naturally all the three points A–C which superficially look self-contradictory.

III. SCALAR CONDENSATE

A. General constraints

Dynamically, the validity of a superfluidity scenario depends crucially on the existence of an (effective) scalar $\phi$, see the basic equations (2). This degree of freedom is kept in the hydrodynamic approximation and is to be light, therefore. Moreover, in field-theoretic language the only way to ensure lightness of a scalar is to have spontaneous symmetry breaking, described by a condensate

$$\langle \phi \rangle_{\text{ground state}} \neq 0 \ .$$

(10)

The phase of this condensate corresponds then to a new light degree of freedom.

The condition (10) looks very restrictive and, in more detail, assumes in fact a number of constraints:

a. The field $\phi$ is a complex field.

b. Nevertheless the condensate (10) should not violate conservation of any known quantum number, like charge.

c. In case of superfluidity, one is to think rather in terms of a three-dimensional field $\varphi(r) \equiv \arg \phi(r)$ while its time derivative in the rest frame of the normal part of the fluid is determined by the chemical potential $\mu$:

$$\partial_t \varphi = \mu \ .$$

(11)

Generalizations of (11) to the case of relativistic plasma are mentioned in the Introduction [the third relation in Eq. (3)]. It is not clear which charge could be associated with the chemical potential $\mu$.

B. Thermal scalar

If we consider the conditions A–C above in an abstract form, they look very difficult to satisfy. It is then even more amusing that a 3d field with similar properties arises naturally (9) within the string approach to the deconfinement phase transition and is commonly called thermal scalar, for a concise review and further insights see (10). The reservation is that the thermal scalar refers to the temperatures below $T_c$ while our prime interest is $T > T_c$.

One considers temperatures $T$ below and close to the temperature of the Hagedorn transition $T_H$ that in critical string dimension $d = 26$ coincides with the critical temperature $T_c$. Below we neglect the difference between $T_H$ and $T_c$. In the string picture $\beta_H \equiv 1/T_H = 1/\alpha'$ where $(2\pi \alpha' \equiv l_s^{-2}$ is the string tension. At $T = T_H$ the statistical sum over the states diverges. The main observation is that at small $|T - T_H|$ the sum is dominated by the contribution of a single degree of freedom, that is a scalar field with mass

$$m^2_{\beta} \simeq \frac{\beta \beta_H}{2\pi^2(\alpha')^2} \ ,$$

(12)

In other words, at $T = T_H$ the mass is becoming tachyonic.

In more detail, it is convenient to use the polymer approach to field theory of a scalar particle (see, e.g. (11)) so that the action associated with a trajectory of length $L$ is $S = L \cdot M$ where $M$ is the bare mass. The trajectories are random walks with renormalized mass. The free energy of the thermal scalar can be represented as a sum over random walks and the final expression reduces to:

$$F = \beta \ln Z = \beta \int L \exp(-\frac{m^2_{\beta} l_s L}{l_s L}) \ ,$$

(13)

where $d$ is the number of spatial coordinates, in our case $d = 3$. Expression (13) is quite generic to the polymer approach. A specific feature of (13) is that $l_s$ plays the role of the length of the links and is fixed in terms of the string tension.

The crucial point is that the free energy of the thermal scalar is exactly the partition function for a single static string with tension $1/(2\pi \alpha')$. Moreover, the single string dominates the free energy of a gas of strings.

C. Scalar particles at $T > T_c$

What happens to the thermal scalar at $T > T_c$ is an open question. Consider first the case of a second order phase transition which is relevant to the SU(2) gauge group. Then we would expect that the thermal scalar is condensed at $T > T_c$. Such a scenario is typical for the percolation picture, which is a realization of the second-order-phase-transition scenario, see, e.g. (12). The basic features can be understood from Eq. (13). At $m^2_{\beta} = 0$ the exponential suppression of very large lengths $L$ disappears. However, the integral over $L$ is still divergent in the ultraviolet, not in the infrared. This means that small clusters with $L \sim l_s$ dominate. The probability of having infinite length is suppressed by a power of $L$ at $L \to \infty$. For a tachyonic mass there emerges an infinite
cluster. However, its density is suppressed as a power of $m^2_\phi$ and small for temperatures above and close to $T_c$. In field theoretic language appearance of the infinite cluster means condensation of the field, $\langle \phi \rangle \neq 0$.

Imagine that the thermal scalar is indeed condensed at $T > T_c$. Then, remarkably enough, the conditions we formulated above are satisfied. Indeed,

a) The thermal scalar is a complex field. It is encoded in the fact that the integration in Eq. (13) is over closed loops which means a complex field in the polymer language.

b) The thermal scalar is associated with topological quantum number which is a wrapping around the compactified time direction (due to finite temperature).

c) The thermal scalar is a 3d scalar field, as it follows from the representation [13].

d) Concerning the chemical potential $\mu$. As is emphasized in [10], near the phase transition all string configurations are time-oriented. In terms of random-walk formalism for a scalar particle time orientation of the walk means chemical potential.

Nowadays, it is common to consider dual models of Yang-Mills theories in terms of strings living in extra dimensions with non-trivial geometry. The thermal scalar at temperatures below and close to $T_c$ is generic to such models as well, see [10] and references therein. One would not claim, however, that the most naive version of the condensation of the thermal scalar is realized within this scenario. Rather, the phase transition is a change of geometry in the extra dimensions.

However, the scalar fields at $T > T_c$ are resurrected in another disguise. Namely, one predicts existence of defects of various dimensions, see in particular [13]. At $T > T_c$ the models predict existence of time-oriented strings. Their 3d projection then looks as trajectories and correspond indeed to scalar 3d particles. There are independent lattice data which seem to support the validity of this prediction [14].

To summarize, there is strong evidence that at $T > T_c$ there exists an effective 3d scalar field condensed in the thermal vacuum of QCD. The existence of such a scalar is a necessary condition for the validity of the two-component model.

IV. POSSIBLE CRUCIAL TEST OF THE MODEL

The considerations given above demonstrate that the two-component model of the quark-gluon plasma does not contradict existing data. One cannot claim, however, that the model is indeed validated by the data.

A crucial test of the model could performed through lattice measurements of a correlator of components of the energy-momentum tensor $T^{t_i}$, $i = 1, 2, 3$, where the index $t$ stands for the Euclidean time direction. In more detail, consider the retarded Green’s function defined as:

$$G^{t_i, t_i}_R(k) = \left. i \int d^4xe^{-ikx} \theta(t) \langle [T^{t_i}(x), T^{t_i}(0)] \rangle \right. .$$  (14)

Moreover, concentrate on the case of vanishing frequency, $k_\omega = 0$. There are two independent form factors, corresponding to transverse and longitudinal waves,

$$G^{t_j, t_i}_R(0, k) = \frac{k^i k^j}{k^2} G^{L}_R(k) + \left( \delta^{ij} - \frac{k^i k^j}{k^2} \right) G^T_R(k).$$  (15)

Contribution of the superfluid component to the form factors $G^{L, T}_R$ has been discussed in many papers and textbooks. Here, we quote the result of the paper [4] which includes also relativistic corrections:

$$\lim_{k_\omega \to 0} G^{L}_R(k) = -(sT + \mu s),$$  (16)

$$\lim_{k_\omega \to 0} G^{T}_R(k) = -(sT + \mu s),$$

where $s$ is the entropy density, $T$ is the temperature, $\mu$ is the chemical potential, $\mu_{s} = \rho_n + \rho_s$ is the total density (1), while $\rho_n$ and $\rho_s$ are the densities of the normal and superfluid components, respectively.

Equations (15) and (16) lead to the following result for zeroth Matsubara frequency of the correlator (14)

$$\lim_{k_\omega \to 0} G^{t_j, t_i}_R(0, k) = -\delta^{ij} (sT + \mu s) + \mu s \frac{k^i k^j}{k^2}.$$  (17)

It is only the superfluid component that leads to the non-analyticity in the limit of small spatial momenta $k$. And it is only the superfluid component that leads to appearance of the off-diagonal terms in the correlator (17).

Thus, we propose to test the possible presence of the superfluid component by evaluating the off-diagonal components of the correlator (17). Note that the proposed crucial test of the two-component model (17) refers to static quantities, corresponding to exactly zero temporal momentum in Minkowski space, $k_0 = 0$, and, consequently, to the zero Matsubara frequency, $\omega_n = 0$, on the lattice. Since there is no time (or frequency) dependence, the continuation from the Euclidean to Minkowski space is straightforward, and no analytical continuation in the low–frequency region is required. Thus, the prediction of the model, $\rho_s \neq 0$, can directly be tested on the lattice.

V. CONCLUSIONS

It is amusing that the known qualitative features of the quark-gluon plasma seem to favor a two-component model of superfluidity for the plasma. In terms of field theory, the model implies the condensation of an effective 3d scalar field. This consequence of the model seems to be qualitatively supported by the lattice data as well.
A crucial test of the model could be performed through the search for the non-analyticity in the spatial off-diagonal components of the correlator of the energy-momentum tensor on the lattice. This property can be tested directly in lattice simulations of Yang-Mills theories.

Acknowledgments

The work of MNC has been partially supported by the French Agence Nationale de la Recherche project ANR-09-JCJC “HYPERMAG”.

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