Stability of the moving Bose glass phase at finite temperature

Y Fily, E Olive and JC Soret
LEMA, UMR 6157, Université F. Rabelais-CNRS-CEA, Parc de Grandmont, 37200 Tours, France
E-mail: yaouen.fily@etu.univ-tours.fr

Abstract. We present 3D numerical simulation results for driven vortex lattices in presence of weak random columnar disorder at finite temperature and high driving force. A moving Bose glass phase is expected theoretically, characterized by a flow along rough static channels and dynamical transverse Meissner effect, the moving flux lines being localized along the correlated disorder direction.

We find the moving Bose glass phase to be stable in a large range of temperature and velocity. Moreover, we confirm the existence of an effective static pinning potential in the transverse direction, previously observed in zero temperature simulations, and leading to a "kink" structure of the vortex lines beyond a critical transverse field.

Periodic structures, such as vortex lattices in type II superconductors, which perfect order is destroyed when put in a disordered medium, are known to reorder themselves when driven at high enough velocity. However, it has been shown [1] that the phase diagram of such systems is much more complex than the naive expectation of a moving vortex glass, due to the persistence of disorder in the direction transverse to motion at any velocity, which leads to several moving glass phases. In the case of 3D lattice and weak columnar disorder, two high velocity phases have been predicted [2]. At \( T = 0 \), a moving Bose glass phase (MBoG) is expected, characterized by dynamical transverse Meissner effect (DTME): the tilt response to transverse field vanishes below a critical value. Its existence has been confirmed by numerical simulations [3]. At \( T > 0 \), the MBoG persists below a critical velocity \( v_c \), while above \( v_c \) a correlated moving glass (CMG) is expected, which does not display DTME. \( v_c \) remains finite when \( T \to 0 \), i.e. at very high velocity DTME vanishes at any temperature. Both phases should melt into a vortex liquid upon increasing temperature.

In this paper, we perform 3D molecular dynamics simulations on vortices in a layered superconductor with random columnar pinning. We show that the MBoG phase persists at finite temperature and exhibits the same features as in the \( T = 0 \) case.

Following [3], we model a stack of \( N_z \) Josephson-coupled parallel superconducting planes of thickness \( d \) with interlayer spacing \( s \). Each layer in the \((x,y)\) plane contains \( N_v \) pancake vortices interacting with \( N_p \) columnar pins parallel to the \( z \) direction. The overdamped equation of motion of a pancake \( i \) at position \( \mathbf{r}_i \) reads

\[
\frac{d\mathbf{r}_i}{dt} = -\sum_{j \neq i} \nabla_j U^{vv}(\rho_{ij}, z_{ij}) - \sum_p \nabla_i U^{vp}(\rho_{ip}) + \mathbf{F}^{L}(t) + \mathbf{F}^{tilt}(z) + \mathbf{F}^{th}(t)
\]
where $\rho_{ij}$ and $z_{ij}$ are the components of $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ in cylindrical coordinates, $\rho_0$ is the in-plane distance between the pancake $i$ and a pinning site in the same layer at $\mathbf{r}_p$, and $\nabla_i$ is the 2D gradient operator acting in the $(x,y)$ plane. $\eta$ is the viscosity coefficient, $\mathbf{F}^L = F^L \hat{x}$ is the Lorentz driving force due to an applied current, $F_{i,th}^t$ is the thermal noise with $\langle F_{i,th}^t \rangle = 0$ and $\langle F_{i,th}^t(t) F_{j,th}^t(t') \rangle = 2\eta k_B T \delta_{i,j} \delta(\alpha - \beta) (t - t')$ where $\alpha, \beta = x, y$, and $k_B = 1$ is the Boltzmann constant. $\mathbf{F}^{tilt}(z)$ is the surface force due to the field tilting away from the $z$ axis in the $y$ direction. This force acts as a torque on each flux line, i.e. $\mathbf{F}^{tilt}(z = 0) = -\mathbf{F}^{tilt}(z = N_s s) = \mathbf{F}^{tilt} \hat{y}$ and $\mathbf{F}^{tilt}(z = 0) = 0$ for pancakes in the bulk. The tilting force modulus is defined by $F^{tilt} = e^2 \phi_0 H_y / 4\pi = 8\pi^2 \epsilon_0 \lambda_{ab}^2 H_y / \sqrt{3} \delta_{0}^2 H_z$, where $\epsilon_0 = (\phi_0 / 4\pi \lambda_{ab})^2$, $\lambda_{ab}$ is the in-plane magnetic penetration depth, $a_0$ is the average vortex distance, $H_y$ is the transverse field component, and $\epsilon$ is the anisotropy parameter. The intra-plane vortex-vortex repulsive interaction is given by a modified Bessel function $\mathbf{U}^{vv}(\rho_{ij}) = 2\epsilon_0 d K_0(\rho_{ij}/\lambda_{ab})$. The inter-plane attractive interaction between pancakes in adjacent layers of altitude $z$ and $(z + s)$ reads $\mathbf{U}^{vv}(\rho_{ij}, z_{ij} = s) = (2se_0 / \pi)[1 + ln(\lambda_{ab} / s)][(\rho_{ij} / 2s_g)^2 - 1]$ for $\rho_{ij} \leq 2s_g$ and $\mathbf{U}^{vv}(\rho_{ij}, z_{ij} = s) = (2se_0 / \pi)[1 + ln(\lambda_{ab} / s)][\rho_{ij} / r_g - 2]$ otherwise; in this expression $r_g = \xi_{ab} / \epsilon$, where $\xi_{ab}$ is the in-plane coherence length. Finally, the attractive pinning potential is given by $\mathbf{U}^{vp}(\rho_0) = -A_0 \epsilon^{-\lambda_{ab} / 2\eta d}$, where $A_0 = (\pi e_0 d / 2ln[1 + (R_p^2 / 2\eta d)^2])$ and $\alpha$ is a tunable parameter. All details about our method for computing the Bessel potential with periodic conditions can be found in [4]. We consider periodic boundary conditions of $(L_x, L_y)$ sizes in the $(x,y)$ plane while open boundaries are taken in the $z$ direction. Molecular dynamics simulation is used for $N_v = 30$ vortex lines in a rectangular basic cell $(L_x, L_y) = (5, 6\sqrt{3}/2) \lambda_{ab}$, and for a number of layers varying from $N_z = 19$ up to $N_z = 1999$. The number of columnar pins is set to $N_p = 30$. We consider the London limit $v = \lambda_{ab} / \xi_{ab} = 90$, with an average vortex distance $a_0 = \lambda_{ab}$, and $d = 2.83 \times 10^{-3} \lambda_{ab}$, $s = 8.33 \times 10^{-3} \lambda_{ab}$, $R_p = 0.22 \lambda_{ab}$, $\epsilon = 0.01$, $\eta = 1$. We choose the tunable pinning parameter $\alpha = 1/25$ so that the maximum pinning force is $F_{vp}^{\text{max}} \sim F_0 / 5$ where $F_0 = 2\epsilon_0 d / \lambda_{ab}$ is the unit force defined by the Bessel interaction. Temperature is chosen sufficiently low for an elastic flow to exist at high driving, and $\mathbf{F}^{L}$ is increased from 0 until this flow is obtained. The vortices then wander around the $T = 0$ channels, giving them a small thickness in the transverse direction.

Starting with the magnetic field along the $z$ direction, the transverse component $H_y$ is slowly increased. The average line inclination $\tan \theta_B = B_y / B_z$, where $B$ is the magnetic induction, is shown versus the magnetic field inclination $\tan \theta_H = H_y / H_z$ for several thicknesses $N_z$ in Fig.1a. At low $H_y$, transverse induction is reduced compared with the disorder free case, but not totally screened as expected, resulting in a non zero line inclination. However, $B_y / B_z$ decreases when the system thickness is increased, and vanishes in the $N_z \rightarrow \infty$ limit. When $H_y$ is increased above a critical value $H_y^{c}$, the response experiences a jump corresponding to an angular depinning transition of the lines. We conclude to the existence of DTME, therefore of a MBsO phase at finite temperature. At very low temperature ($T = 10^{-7}$), this behavior has been observed on more than 4 orders of magnitude, from $v \simeq 5 \times 10^{-4}$ (beginning of the elastic flow) up to $v \simeq 10$ (upper bound reachable because of simulation duration issues). At intermediate velocity ($v = 10^{-2}$), the MBsO has been observed up to $T = 10^{-4}$, temperature at which thermal broadening of the channels is of the same order as the distance between two channels (melting is expected at $T \geq 10^{-4}$).

We now take interest in the shape of the vortex lines. It is computed by averaging the transverse position over time. It means we also average over transverse displacements along channels due to their roughness; however, at high driving, these transverse displacements are very small compared to the distance between channels. Fig.1bc shows typical configurations of a line below and above the jump at $H_y^{c}$. Below $H_y^{c}$, lines are almost along $z$ in the bulk, generating a sharp extremum in the derivative, but curved at their extremities by the tilting range.
Figure 1. (a) Average vortex line inclination \( \tan \theta_B = B_y/B_z \) versus field inclination \( \tan \theta_H = H_y/H_z \) at \( T = 10^{-4} \) for thicknesses \( N_z = 19, 29, 49, 149 \) (open circles) compared with disorder free linear response (filled circles). (b) Projection in the \((y,z)\) plane of the vortex lines (top) and their derivatives \( \frac{dz}{dy} \) (bottom) at \( T = 10^{-4} \) and \( N_z = 49 \) before the jump \( (H_y/H_z \simeq 0.32) \). (c) Projection in the \((y,z)\) plane of the vortex lines (top) and their derivatives \( \frac{dz}{dy} \) (bottom) at \( T = 10^{-4} \) and \( N_z = 49 \) after the jump \( (H_y/H_z \simeq 0.45) \). In grey are materialized the pinned regions of the line, near the derivative minima.

force. This results in the non zero response observed in Fig.1a. Above \( H_y^c \), the lines show a modulated structure, leading to an almost periodical derivative \( \frac{dz}{dy} \). Fig.1c shows in grey the regions where the line inclination is smaller. Following [3], we call these regions "pinned". They are separated by regions where the inclination is bigger, called "unpinned" regions. The main point is that the vortex lines moving along the \( x \) direction see an effective "tin roof" shaped potential, depending only on the transverse position, which minima act as effective pins, generating the modulated structure. These pins are located where the static channels lie at \( T = 0 \) and \( H_y = 0 \). Although the value of line inclination extrema, and thus the depth of these effective pins is temperature dependant, the existence of the transverse potential has been observed at any temperature from \( T = 0 \) up to \( T = 10^{-4} \).

A model has been proposed in a previous study [3], based on this observation, which allows to understand many features of the system, including finite size effects, in terms of a single elastic line in a tin roof potential \( V_{eff} \). We phenomenologically apply this \( T = 0 \) model to the averaged lines at finite temperature. The energy \( E \) of a line of length \( L \) is given by (1), where \( u(z) \) is the one-dimensional displacement field in the \( y \) direction, \( c = \epsilon_2 \epsilon_0 \) is the elastic constant, and \( f \propto H_y \) is a surface force. Minimizing \( E \) with respect to \( u(z) \) while expanding \( V_{eff} \) quadratically near a minimum leads to (2).

\[
E(u) = \int_0^L dz \left( \frac{c}{2} \left( \frac{du}{dz} \right)^2 + V_{eff}(u) \right) + f (u(L) - u(0))
\]

\[
\lambda^2 \frac{d^2 u}{dz^2} - u = 0 \quad \text{where} \quad \lambda = \sqrt{c \left( \frac{dV_{eff}}{du^2} \right)^{-1}} \quad \text{at} \quad u = 0
\]

\[
\left( \frac{du}{dz} \right)_{z=L} = f \quad \text{at} \quad u = 0
\]

\[
u(z) = \frac{\lambda f}{c} \sinh \left( \frac{z-L/2}{\lambda} \right) \cosh \left( \frac{L}{2\lambda} \right)
\]
Solving (2) with boundary conditions (3) we find (4), where $\lambda$ characterizes the thickness of the region where the transverse field penetrates the sample near the surface, and the line inclination is given by $\tan \theta_B = [u(L) - u(0)] / L = \frac{2L}{L} \tanh \left( \frac{L}{2\lambda} \right)$. Hence, assuming that $\lambda$ is independant of both $N_z$ and $\theta_H$, the model predicts that at low $\theta_H$ (i.e. when $u(z)$ is small enough for the quadratic expansion to apply), $\tan \theta_B$ is a linear function of $\tan \theta_H$ and $\tan \theta_B \propto \tanh \left( \frac{L}{2\lambda} \right) / L$. If $L \gg \lambda$, $\tanh \left( \frac{L}{2\lambda} \right) \sim 1$ and the inclination should scale as $L^{-1}$, i.e. as $N_z^{-1}$. Note that the exact form of $V_{eff}$ is not needed, since we only use the existence of a quadratic expansion near minima.

Fig.2 shows how well the model fits the data. The line shape is precisely fitted by sinh function (Fig.2a). The penetration depth $\lambda$ is evaluated from these fits and confirmed to be $\theta_H$ and $N_z$ independant, which is coherent with, respectively, the linear response of line inclination versus field inclination (Fig.1a) and the size dependance of line inclination (Fig.2d) at low $\theta_H$. We find $\lambda = 6.1 \pm 0.1$ from the line shape fits and $\lambda = 5.9 \pm 0.1$ from the size effects fit.

**Figure 2.** (a) Projection in the $(y,z)$ plane of a vortex line for $T = 10^{-7}$, $N_z = 49$ and $H_y/H_z \simeq 0.03$ (circles), and sinh fit (line). (b) Penetration depth $\lambda$ versus $N_z$ at $H_y/H_z \simeq 0.03$ and $T = 10^{-7}$. (c) Penetration depth $\lambda$ versus $H_y/H_z$ at $N_z = 49$ and $T = 10^{-7}$. (d) $B_y/B_z$ at $H_y/H_z \simeq 0.03$ and $T = 10^{-7}$ versus $N_z$ (circles), $\tanh \left( \frac{L}{2\lambda} \right) / L$ fit (curved solid line) and power law limit at large $N_z$ (straight dashed line).

Finally, we adress the issue of how to identify a MBoG phase. A tricky feature of this phase is that size effects can make it look a lot like CMG phase. Among the several MBoG characteristics we have studied in this paper, we find that the behaviour of line inclination versus $N_z$ at low $\theta_H$ (Fig.2d) is the most efficient criterion to discriminate MBoG and CMG : in MBoG line inclination approaches 0 at large $N_z$, exhibiting a nice $N_z^{-1}$ power law, while in CMG it should approach a finite value.

To conclude, we find evidence of DTME under a critical transverse field, therefore of MBoG, in a large range of velocity and temperature. CMG has not been seen in the velocity range accessible to our simulations. We also confirm the existence of a transverse static tin roof potential, already observed in $T = 0$. Finally, we use a simple model to predict quantitatively vortex lines shape and size effects at low inclination.

**References**

[1] Le Doussal P and Giamarchi T 1998 Phys. Rev. B 57 11356
[2] Chauve P, Le Doussal P and Giamarchi T 2000 Phys. Rev. B 61 R11906
[3] Olive E, Soret J C, Doussal P L and Giamarchi T 2003 Phys. Rev. Lett. 91 037005
[4] Olive E and Brandt E H 1998 Phys. Rev. B 57 13861