Research Article

An Algorithm for Detection of DVB-T Signals Based on Their Second-Order Statistics

Pierre Jallon

CEA-LETI, MINATEC, 17 Rue des Martyrs, 38054 Grenoble Cedex 09, France

Correspondence should be addressed to Pierre Jallon, pierre.jallon@cea.fr

Received 1 June 2007; Revised 5 October 2007; Accepted 26 November 2007

Recommended by F. K. Jondral

We propose in this paper a detection algorithm based on a cost function that jointly tests the correlation induced by the cyclic prefix and the fact that this correlation is time-periodic. In the first part of the paper, the cost function is introduced and some analytical results are given. In particular, the noise and multipath channel impacts on its values are theoretically analysed. In a second part of the paper, some asymptotic results are derived. A first exploitation of these results is used to build a detection test based on the false alarm probability. These results are also used to evaluate the impact of the number of cycle frequencies taken into account in the cost function on the detection performances. Thanks to numerical estimations, we have been able to estimate that the proposed algorithm detects DVB-T signals with an SNR of $-12$ dB. As a comparison, and in the same context, the detection algorithm proposed by the 802.22 WG in 2006 is able to detect these signals with an SNR of $-8$ dB.

Copyright © 2008 Pierre Jallon. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

1. INTRODUCTION

The cognitive radio concept, introduced by Mitola [1], defines a class of terminals able to modify their transmission parameters according to their surroundings. Among the set of possible applications, the one dealing with a better usage of spectrum resources has given rise to many contributions. These contributions can be sorted in two classes: the one addressing the issue of identifying unused spectral resources (the term of opportunistic radio is also used for these applications), and the one addressing the issue of a better exploitation of the free bands (see [2] and the reference therein for more details). In this paper, we focus on the first class of problems and more precisely on the opportunistic access to DVB-T bands.

The DVB-T signals are transmitted in some UHF bands. Several studies have shown the under-exploitation of these spectral resources [3, 4], and the American regulatory body (FCC) [5] has proposed to open these UHF bands for an unlicensed use. The IEEE 802.22 WG has thus been created to develop an air interface based on an opportunistic access in the TV bands. According to their results (see [6, 7] for more details), an opportunistic terminal can set a communication in a DVB-T band only if no DVB-T signal is present with an SNR higher than $-10$ dB. This threshold gives us an estimate of the required performances of the DVB-T signals detection algorithms for an opportunistic usage of their bands.

We address in this paper the detection of OFDM signals and more particularly the detection of DVB-T signals. As we expect to detect OFDM signals with an SNR close to $-10$ dB, the energy detector algorithms [8] are not efficient in these contexts and we had rather focused on cyclostationary-based detectors. General studies on the detection of cyclostationary signals can be found in [9–12]. In [13], the authors particularize these studies for detection of linear modulations of symbols and OFDM signals. To detect OFDM signals, the authors propose to perform a detection test of the cyclostationarity induced by the cyclic prefix. Another approach has been proposed in [14], inspired from blind detection techniques [15, 16], which consists in detecting the (time-averaged) correlation induced by the cyclic prefix.

In this contribution, we propose an algorithm that jointly exploits the correlation induced by the cyclic prefix and the fact that this correlation is time-periodic, that is, the fact that the OFDM signal is a so-called cyclostationary signal. We therefore introduce a cost function to test this property and give some theoretical results on its behavior in general contexts in Section 2. In Section 3, we explain how to use this function to perform the detection test based on asymptotic results. These asymptotic results are also...
exploited in Section 4 to give some indications on the impact of the number of cycle frequencies taken into account in the cost function on the detection algorithm performances. We conclude this paper with some numerical simulations in Section 5.

2. A COST FUNCTION FOR DETECTION OF OFDM SIGNALS

The time continuous version of an OFDM signal writes

\[ s_a(t) = \sqrt{E_s} \sum_{k \in \mathbb{Z}} \sum_{n=0}^{N-1} a_{kN+n} e^{j2\pi(n/N)(t-NT_c-k(N+D)T_c)} g_a(t-k(N+D)T_c), \]

(1)

where \( 1/T_c \) is the sample rate, \( N \) is the number of carriers, \( D \) is the length of the cyclic prefix, \( \{a_k\}_{k \in \mathbb{Z}} \) are the transmitted symbols assumed to be i.i.d. (independant and identically distributed) with variance 1, and \( g_a(t) \) is the function equal to 1 if \( 0 \leq t < (N + D)T_c \) and 0 otherwise.

For each OFDM symbol, defined by one term of the argument of the sum over \( k \) in (1), a part of its end is copied at its beginning. This part is the so-called cyclic prefix and is used to facilitate the equalization of the received OFDM signal at the receiver. It also induces a correlation between the OFDM signal and its time-shifted version since

\[ s_a(k(N+D)T_c + t + NT_c) = s_a(k(N+D)T_c + t), \]

\[ \forall k \in \mathbb{Z}, \forall t \in [0, DT_c]. \]

(2)

2.1. Noiseless Gaussian channel case

We first assume that the channel between the transmitter and the receiver is a noiseless Gaussian channel. This assumption is of course unrealistic; in the next section, we will use these first results to provide a study on the impact of noisy multipath fading channels on the proposed cost function.

Sampled at a rate \( T_c \), the received signal \( y(u) = \sqrt{E_s} s_a(uT_c) \) writes

\[ y(u) = \sqrt{E_s} \sum_{k \in \mathbb{Z}} \sum_{n=0}^{N-1} a_{kN+n} e^{j2\pi(n/N)(u-ND-k(N+D))} g(u-k(N+D)), \]

(3)

where \( g(u) = g_a(uT_c) \) and \( E_s \) is the transmitted signal power. Its autocorrelation function \( R_y(u, m) = E \{ y(u + m)y^*(u) \} \) equals

\[ R_y(u, m) = \frac{E_s}{N} \sum_{k \in \mathbb{Z}} \sum_{n=0}^{N-1} E \left| a_{kN+n} \right|^2 e^{j2\pi(nm/N)} \times g(u + m - k(N + D))g^*(u - k(N + D)). \]

(4)

If all carriers are used to transmit data, that is, \( E|a_{kN+n}|^2 = 1 \), for all \((k, n)\), \( R_y(u, m) \) is simplified to

\[ R_y(u, m) = R_y(u, 0)\delta(m) + R_y(u, N)\delta(m - N) + R_y(u, -N)\delta(m + N). \]

(5)

The terms \( R_y(u, N) \) and \( R_y(u, -N) \) correspond to the correlation induced by the cyclic prefix (see (2)). Note that if some carriers are unused, some additional terms appear in (5). Nevertheless, as these terms have a very limited impact on the results of this paper, we do not mention them in what follows.

The first r.h.s. term of (5) is the power of the received signal. With the power detector algorithm being unable to detect signal with very low SNR, we focus only on the last two terms of (5) to build a cost function. The first one, \( R_y(u, N) \), is simplified to \( \sum_{k \in \mathbb{Z}} E \sum_{n \in \mathbb{Z}} g(u + N - k(N + D))g^*(u - k(N + D)) \), a periodic function of \( u \) of period \( N = N + D \). As this function depends on \( u \) in a periodic way, the signal \( y \) is not a stationary signal but a cyclostationary one. Its autocorrelation function can be written as a Fourier series:

\[ R_y(u, N) = R_y^{(0)}(N) + \sum_{k \neq 0} R_y^{(kN)}(N)e^{j2\pi k nu}. \]

(6)

\( R_y^{(kN)}(N) \) is the so-called cycle correlation at cycle frequency \( k\alpha \) and at time lag \( N \):

\[ R_y^{(kN)}(N) = \lim_{U \to \infty} \frac{1}{U} \sum_{u=0}^{U-1} E \{ y(u + N)y^*(u) \} e^{j2\pi k nu}, \]

(7)

and it can be estimated as

\[ \hat{R}_y^{(kN)}(N) = \frac{1}{U} \sum_{u=0}^{U-1} y(u + N)y^*(u)e^{-j2\pi k nu}, \]

(8)

where \( U \) is the observation time.

Exploiting this decomposition has already been proposed in several contributions [14]: proposes to only exploit the term \( R_y^{(0)}(N) \) to perform the detection. In [13], the proposed cost function is based on one term of the sum in (6), \( R_y^{(kN)}(N) \), \( k \neq 0 \).

The cost function proposed in this paper jointly exploits both terms of (6):

\[ J_y(N_b) = \frac{1}{2N_b + 1} \sum_{k=-N_b}^{N_b} \left| R_y^{(kN)}(N) \right|^2. \]

(9)

The parameter \( N_b \) stands for the number of positive cycle frequencies taken into account to compute the cost function \( J_y(N_b) \). Its choice is discussed in Section 4.

Remark 1. The third term \( R_y(u, -N) \) in (5) is not taken into account in \( J_y(N_b) \) since for any signal \( x(n) \), the following equalities hold:

(i) \( R_y^{(kN)}(N) = (R_y^{(-kN)}(-N))^*, \) for all \( k \),

(ii) \( \hat{R}_y^{(kN)}(N) = (\hat{R}_y^{(-kN)}(-N))^*, \) for all \( k \).
For any signal \( y \) (noise or OFDM signal + noise), the function 
\[
(1/(2Nb+1)) \sum_{k=0}^{Nb} \left| R_y^{(k\alpha)}(N) \right|^2 + \left| R_y^{(k\alpha)}(-N) \right|^2
\]
is hence equal to \( 2J_y(N_b) \). (This equality also holds for the estimated versions.)

### 2.2. Noisy multipath fading channel case

In what follows, we drop the assumption that the channel is a Gaussian channel, and we consider a noisy multipath fading channel. We denote in this context \( z(n) \) as the received signal after the sampling operation (at a rate \( T_c \)):

\[
z(u) = \left( \sum_{l=0}^{L-1} h(l) y(u - l) \right) e^{(2\pi\delta f u)} + \sigma w(u),
\]

where \( \delta f \) is the frequency carrier offset, \( h(l) \) is the impulse response of the channel, and \( L \) is its delay spread.

**Theorem 1.** The criterion \( J_z(N_b) \) does not depend on the frequency offset \( \delta f \) or on the noise signal \( \sigma w(u) \).

The proof is straightforward. In what follows, we assume that \( \delta f = 0 \).

We evaluate the impact of the impulse response of the propagation channel on the criterion \( J_z \) through its impact on the cycle coefficients.

**Theorem 2.** As long as all the carriers are used to transmit data, the cycle coefficients of the signal \( z(n) \) are given by

\[
R_z^{(k\alpha)}(N) = R_y^{(k\alpha)}(N) \left( \sum_{l=0}^{L-1} \left| h(l) \right|^2 e^{-2\pi\delta f k\alpha} \right),
\]

\[
\forall k \in \left\{ -\frac{N + D}{2}, \ldots, \frac{N + D}{2} - 1 \right\}.
\]

The proof is given in the appendix.

**Remark 2.** Note that if the condition, that *all the carriers* are used to transmit data, is not satisfied, some additional terms appear in (5) and the demonstration is no more valid. Nevertheless, with these terms being numerically small in regard to \( R_y(u, N) \), their impacts on the result of Theorem 2 can be neglected.

The criterion \( J_z \) is a random variable of the channel whose expectation is given by

\[
E_h \{ J_z(N_b) \} = \frac{1}{2Nb + 1} \sum_{k=-Nb}^{Nb} \left| R_y^{(k\alpha)}(N) \right|^2 E_h \left( \sum_{l=0}^{L-1} \left| h(l) \right|^2 e^{-2\pi\delta f k\alpha} \right)^2.
\]

\[
(12)
\]

To go further into the evaluation of the impact of the channel impulse response on \( E_h \{ J_z \} \), it is necessary to use a channel model. We hence particularize our results to the detection of DVB-T signals and we consider the DVB-T discrete time channel described in [17] to evaluate its impact on \( J_z \).

**Theorem 3.** For DVB-T channels, as long as \( N_b < N/D \), for all \( k \leq Nb \), \( E_h \left( \sum_{l=0}^{L-1} \left| h(l) \right|^2 e^{-2\pi\delta f k\alpha} \right)^2 \) tends to a constant \( A_0 \) when \( N \) and \( L \) grow to infinity and \( D/N = \eta \). For DVB-T signals and channels where \( N = 8192 \) and \( L \rightarrow \infty \), \( E_h \{ J(N_b) \} \) can thus be written as

\[
E_h \{ J_z(N_b) \} = A_0 J_y(N_b) + o(1).
\]

The proof is given in the appendix. Note that the condition \( N_b < N/D \) will be discussed in what follows, but it is not a restrictive condition. As the expectation of \( J_z(N_b) \) tends to be proportional to \( J_y(N_b) \), we will focus in what follows on \( J_y(N_b) \).

### 3. DETECTION ALGORITHM

The detection problem objective is to determine which of the following assumptions is the most likely:

\[
H_0: y(u) = \sigma w(u), \quad H_1: y(u) = \sqrt{E_s\alpha}(uT_c) + \sigma w(u).
\]

If \( H_0 \) holds, \( J_y(N_b) = 0 \), and if \( H_1 \) holds, \( J_y(N_b) > 0 \). This result gives the test to be performed on the value reached by \( J_y \) to determine whether an OFDM signal is present or not. In practice, \( J_y \) cannot be computed and the algorithm is based on its estimate \( \hat{J}_y \) given by

\[
\hat{J}_y(N_b) = \frac{1}{2Nb + 1} \sum_{k=-Nb}^{Nb} \left| \hat{R}_y^{(k\alpha)}(N) \right|^2,
\]

where \( \hat{R}_y^{(k\alpha)}(N) \) is an estimate of \( R_y^{(k\alpha)}(N) \) given by (8).

In general, when \( H_0 \) holds, \( \hat{J}_y(N_b) \) does not vanish and in order to determine if \( H_0 \) is less likely than \( H_1 \), \( \hat{J}_y \) has to be compared to a positive threshold which depends on its statistical behavior. In this section, we give some asymptotic results on the statistical behavior of \( \hat{J}_y \) under both assumptions and we propose a detection test based on the false alarm probability. This kind of test has already been proposed in [12, 13] with whitened cost functions.

#### 3.1. Asymptotic probability density function of \( J_y(N_b) \) when \( H_0 \) holds

We first assume that the assumption \( H_0 \) holds; that is, the received signal \( y(u) \) equals \( \sigma w(u) \). The asymptotic behavior of \( J_y(N_b) \) is based on this preliminary result.

**Theorem 4.** If the assumption \( H_0 \) holds, the cycle coefficients of the received signal are asymptotically normal with mean 0 and variance \( \sigma^2/U \). Furthermore, these cycle coefficients are asymptotically uncorrelated, and hence mutually independent.

The proof is given in the appendix. As the cycle coefficients are asymptotically uncorrelated, the probability density function of \( J_y(N_b) \) can be estimated without whitening these coefficients. Note that to reach the asymptotic regime, \( U \) has to be higher than the inverse of the smallest cycle frequency.

Theorem 4 leads to the following corollary.
Corollary 1. If the assumption \( H_0 \) holds, the distribution law of \( J_y(N_b) \) converges in distribution to a \( \chi^2 \) distribution given by
\[
\mathcal{P}^{(\infty)}(\hat{J}_y(N_b) \mid H_0) = \frac{U}{\sigma^4} (2N_b + 1) 2^{2N_b} \int_0^\infty \! \left( 2N_b + 1 \right) \hat{J}_y(N_b) \frac{U}{\sigma^4} e^{-\left(2N_b + 1\right) \hat{J}_y(N_b)/(U/\sigma^4)} dt.
\]

The proof is given in the proof of Theorem 4.

3.2. Asymptotic probability density function of \( J_y(N_b) \) when \( H_1 \) holds

If \( H_1 \) holds, the signal \( y(u) = \sqrt{E_s} a(u) + \sigma w(u) \). The following result holds.

Theorem 5. If the assumption \( H_1 \) holds, \( \hat{R}_y^{(\alpha)'}(N) \) is asymptotically normal with mean \( R_y^{(\alpha)}(N) \) and a variance proportional to \( 1/U \).

The proof is included in the proof of Theorem 6 given in the appendix.

Thanks to this result, we can deduce that \( \hat{J}_y(N_b) \) is asymptotically normal with mean \( J_y(N_b) \). This probability cannot be estimated in the considered context (since \( J_y(N_b) \) depends at least on the received signal power) and cannot be used to perform the detection test.

3.3. Application of these results to build a detection test

As only the asymptotic probability density function of \( \hat{J}_y(N_b) \) can be estimated when \( H_0 \) holds, we focus on the alarm probability. We therefore consider the constant \( \lambda \) defined as
\[
\mathcal{P}^{(\infty)}(\hat{J}_y(N_b) \geq \lambda \mid H_0) = P_{fa},
\]
where \( P_{fa} \) is the fixed false alarm probability. Thanks to the result of Corollary 1, the function \( \mathcal{P}^{(\infty)}(\hat{J}_y(N_b) \geq \lambda \mid H_0) \) is simplified to \( y(2N_b + 1, (2N_b + 1)\lambda) \), where
\[
y(2N_b + 1, x) = \frac{1}{(2N_b)!} \int_0^\infty \! t^{2N_b} e^{-t} dt.
\]

As this function grows with \( \lambda \), the following test can hence be performed to decide between \( H_0 \) and \( H_1 \):

(i) if \( 1 - y(2N_b + 1, \hat{J}_y(N_b)/(U/\sigma^4)) \geq P_{fa} \), then \( H_0 \) is decided,

(ii) if \( 1 - y(2N_b + 1, \hat{J}_y(N_b)/(U/\sigma^4)) \leq P_{fa} \), then \( H_1 \) is decided.

4. SOME INDICATIONS ON THE CHOICE OF \( N_b \)

The asymptotic results on the behavior of the function \( \hat{J}_y(N_b) \) can also be used to give some indications on the choice of \( N_b \). We hence evaluate in this section the impact of this parameter on the mean and on the variance of \( \hat{J}_y(N_b) \) under both assumptions.

Thanks to the result of Corollary 1, we can deduce the following result.

Corollary 2. The asymptotical mean and variance of \( \hat{J}_y \), when \( H_0 \) holds, write
\[
\lim_{U \to \infty} U \mathbb{E} [\hat{J}_y] = \sigma^4,

\lim_{U \to \infty} U^2 \mathbb{E} [\hat{J}_y^2] - \mathbb{E} [\hat{J}_y^2] = \frac{\sigma^8}{2N_b + 1}.
\]

The proof is given in the proof of Theorem 4.

When assumption \( H_1 \) holds, the following result is satisfied.

Theorem 6. The asymptotical mean of \( \hat{J}_y \), when \( H_1 \) holds, writes
\[
\mathbb{E} [\hat{J}_y] = J_y + \sigma^4 \left( \frac{1}{U} \right).
\]

And for very low SNR, that is, \( E_s \ll \sigma^2 \), the asymptotical variance writes
\[
\lim_{U \to \infty} U \mathbb{E} [\hat{J}_y^2] - \mathbb{E} [\hat{J}_y^2] = \frac{\beta}{2N_b + 1},
\]

where \( \beta \) does not depend on \( U \) and \( N_b \).

The proof of this theorem is given in the appendix.

The difference between the asymptotical means of \( \hat{J}_y \) under both assumptions is equal to \( J_y + o(1/U) \). To estimate the variation of \( J_y \) in terms of \( N_b \), we first evaluate the variation of the cycle coefficients with \( k \).

Theorem 7. The cycle correlation coefficients are given by
\[
| R_y^{(\alpha)}(N) |^2 = \left| \frac{1}{N + D} \sin \left( \pi k (D/(N + D)) \right) \right| ^2, \quad \forall k.
\]

The proof is given in the appendix.

Hence, \( J_y(N_b) \) equals
\[
J_y(N_b) = \frac{1}{2N_b + 1} \sum_{k=-N_b}^{N_b} \left| \frac{1}{N + D} \sin \left( \pi k (D/(N + D)) \right) \right|^2.
\]

The values reached by the cycle correlation coefficients are in the first lob of the function when \( k < N/D \). When \( k > N/D \), the values taken by the cycle coefficients are small compared to the values taken by the terms around \( k = 0 \). The number of cycle frequencies \( N_b \) taken into account for the criterion \( J_y(N_b) \) should hence be smaller than \( N/D \). In this interval, \( J_y \) decreases with \( N_b \). This parameter has hence to be chosen as such to ensure a good compromise between the value of \( J_y \) and the values of the asymptotic variances.

5. NUMERICAL EVALUATION OF THE PERFORMANCES OF THE PROPOSED ALGORITHMS

We now give some numerical estimation of the performances of the DVB-T signals detection algorithm. These performances have been estimated in several contexts leading to the
simulation of many realizations. Before describing these contexts, we describe one realization.

We have generated OFDM signals with the same modulation parameters as DVB-T signals [18]. We used the 8k mode, corresponding to $N = 8192$ carriers where only the first 6818 carriers are used to transmit data and pilots. According to [18], the sample rate is equal to $T_c = 1/8$ microsecond, and we have generated signals of 50 milliseconds. Two cases have been considered for the cycle prefix length, corresponding to $\eta = 1/4$ and $\eta = 1/32$.

For each realization, a simulated DVB-T discrete time signal is passed through the DVB-T channel model described in [17], and an i.i.d. centered Gaussian noise is added to the output of this filter. The resulting signal is used as an input to the detection algorithm.

Each context is defined by an SNR value and a value of $N_b$. We have evaluated the performances of the proposed algorithm as the percentage of realizations where the criterion excited by the simulated DVB-T signals satisfies the detection test proposed in Section 3 with $P_{fa} = 2\%$ over 1000 realizations.

The estimated good detection probabilities of the algorithm are illustrated in Figure 1 for $\eta = 1/4$ and in Figure 2 for $\eta = 1/32$. Several choices of $N_b$ have been tested to illustrate the impact of this parameter on the performances of the algorithm.

In both figures, the results show that whatever the value of $\eta$ is, the detection algorithm performances are improved as long as $N_b$ is chosen to be lower than $1/\eta$. Despite the loss on the asymptotic mean value of the criterion, taking into account the cycle frequencies leads to a significant improvement on the detection performances.

Note that without taking into account the cycle frequencies, the performances of the cyclic prefix detector proposed by [14] do not fit with the requirement of the 802.22 WG when $\eta = 1/32$ since the good detection probability when the SNR is close to $-10$ dB is close to 70%. Thanks to our algorithm, when taking into account 15 cycle frequencies ($N_b = 7$), the good detection probability remains close to 1 up to SNR of $-12$ dB.

6. CONCLUSION

In this paper, we have proposed a detection algorithm based on a cost function testing the cyclostationary property of the OFDM signals. The noise and multipath channel impacts on the proposed cost function have been theoretically analyzed. Thanks to asymptotic results, a detection test has also been proposed based on the false alarm probability, and some indications on the choice of the $N_b$ have been given.

The evaluated performances of our detection algorithm are illustrated in Figures 1 and 2. As shown, the proposed detection algorithm has a good detection probability close to 1 with an SNR of $-12$ dB when $\eta = D/N = 1/32$. In the same context, the detection algorithm proposed by [14] has a good detection probability close to 1 with an SNR of $-8$ dB. When $\eta = 1/4$, the proposed algorithm has a gain of $2$ dB compared to the algorithm of [14].

APPENDICES

A. PROOF OF THEOREM 2

As $\delta f$ has no impact on the cost function, it can be neglected. The signal $z(u)$ then writes

$$z(u) = \sum_{l=0}^{L-1} h(l)y(u - l) + \sigma w(u). \quad (A.1)$$
Its cycle correlation coefficients are given by
\[ R_z^{(k_0)}(N) = \lim_{U \to \infty} \frac{1}{U} \sum_{u=0}^{U-1} \mathbb{E} \{ z(u+N)z^*(u) \} e^{-2\pi i k_0 u}. \] (A.2)

Using the mutual independence between the noise signal \( w(u) \) and the signal of interest \( y(u) \), and the fact that the noise signal is white, this coefficient is simplified to
\[ R_z^{(k_0)}(N) = \lim_{U \to \infty} \frac{1}{U} \sum_{u=0}^{U-1} \sum_{l=0}^{L-1} \sum_{k=0}^{N} h(l)h^*(l) \mathbb{E} \{ y(u+N-l) \} y^*(u-l) e^{-2\pi i k_0 u}. \] (A.3)

The correlation function of \( y(n) \) is given by (5), and assuming that the channel impulse response satisfies \( 2L < N \), \( R_y(u-l, N+l-l) \) is simplified to \( R_y(u-l, N) \delta(l-l) \).

**Remark 3.** The condition \( 2L < N \) is not satisfied for the DVB-T channel model given in [17]. Nevertheless, the coefficients vanish in an exponential way and can be neglected when \( L > D = N/2 \), the cyclic prefix size of DVB-T signals.

\[ R_z^{(k_0)}(N) \] then writes
\[ R_z^{(k_0)}(N) = \lim_{U \to \infty} \frac{1}{U} \sum_{u=0}^{U-1} \sum_{l=0}^{L-1} |h(l)|^2 R_y(u-l, N) e^{-2\pi i k_0 l}. \] (A.4)

Thanks to the Fourier decomposition of \( R_y(u-l, N) \) (see (6)), \( R_z^{(k_0)}(N) \) also equals
\[ R_z^{(k_0)}(N) = \lim_{U \to \infty} \frac{1}{U} \sum_{u=0}^{U-1} \sum_{k=0}^{(N+D)/2} |h(k)|^2 R_z^{(k_0)}(N) e^{2\pi i k_0 u} e^{-2\pi i k u}. \] (A.5)

As \( |k_2-k| k_0 < 1 \), we get the expected result:
\[ R_z^{(k_0)}(N) = R_z^{(k_0)}(N) \sum_{l=0}^{L-1} |h(l)|^2 e^{-2\pi i k_0 l}. \] (A.6)

**B. PROOF OF THEOREM 3**

This proof is based on the model of the DVB-T channels given by [17]. The channel coefficients are randomly chosen and uncorrelated, that is, \( \mathbb{E} \{ h(l)h^*(k) \} = 0 \), for all \( l \neq k \). Each channel coefficient \( h_t \) is randomly chosen according to a zero-mean complex Gaussian distribution with the variance given by
\[ \mathbb{E} \{ |h(0)|^2 \} = c_0 + c_1 (1 - e^{-T_c/\tau}), \] (B.1)
\[ \mathbb{E} \{ |h(k)|^2 \} = c_1 (1 - e^{-T_c/\tau}) e^{-k(T_c/\tau)}, \quad \forall k \geq 1. \]

\( c_0 \) and \( c_1 \) are randomly chosen coefficients. \( c_0 + c_1 \) defines the channel attenuation or channel power, and the ratio \( c_0/c_1 \) is referred to as the \( K \) factor. When \( K \) grows to infinity, the channel impulse response corresponds to an LOS scenario (flat fading channel). Otherwise, \( K \) is the ratio between the direct path and the other one. For the NLOS scenarios, \( K \) in dB takes values in the set \( (-\infty; 8) \) dB, \( K \) is the delay spread and takes values in the set \( [0, 1, 0.8] \) microsecond. We remind that for DVB-T signals, \( T_c \) equates 0.125 microsecond, making the ratio \( T_c/\tau \) close to 1.

For each value of \( k \), the term \( \mathbb{E}_h \{ \sum_{l=0}^{L-1} |h(l)|^2 e^{-2\pi i k_0 l} \} \) writes
\[ \mathbb{E}_h \{ \sum_{l=0}^{L-1} |h(l)|^2 e^{-2\pi i k_0 l} \} = \sum_{l=0}^{L-1} \mathbb{E}_h \{ |h(l)|^2 |h(l)|^2 \} e^{-2\pi i (l-l) k_0}. \] (B.2)

In terms of cumulants, since \( h[l] \) is circular, \( \mathbb{E}_h \{ |h(l)|^2 |h(l)|^2 \} \) writes
\[ \mathbb{E}_h \{ |h(l)|^2 |h(l)|^2 \} = \text{cum}(h(l), h^*(l), h(l), h^*(l)) + \mathbb{E}_h \{ |h(l)|^2 \} \mathbb{E}_h \{ |h(l)|^2 \} + \mathbb{E}_h \{ |h(l)|^2 |h(l)|^2 \}^2. \] (B.3)

As \( h(l) \) is Gaussian, the fourth-order cumulant vanishes. With the channel coefficient being uncorrelated, \( \mathbb{E}_h \{ |h(l)|^2 |h(l)|^2 \} = (\mathbb{E}_h \{ |h(l)|^2 \})^2 \mathbb{E}_h \{ |h(l)|^2 \} \),
\[ \mathbb{E}_h \{ \sum_{l=0}^{L-1} |h(l)|^2 e^{-2\pi i k_0 l} \} \text{ then writes in terms of the second-order moment as} \]
\[ \mathbb{E}_h \{ \sum_{l=0}^{L-1} |h(l)|^2 e^{-2\pi i k_0 l} \}^2 \]
\[ = \sum_{l=0}^{L-1} \mathbb{E}_h \{ |h(l)|^2 e^{-2\pi i k_0 l} \}^2 + \sum_{l=0}^{L-1} \mathbb{E}_h \{ |h(l)|^2 \}^2. \] (B.4)

Thanks to (B.1), the first r.h.s. term writes
\[ \sum_{l=0}^{L-1} \mathbb{E}_h \{ |h(l)|^2 e^{-2\pi i k_0 l} \}^2 \]
\[ = \left( c_0 + c_1 \left( 1 - e^{-T_c/\tau} \right) \right) \left( 1 - e^{-2\pi i k_0 - L(T_c/\tau)} \right). \] (B.5)

When \( L \) is large enough, \( 1 - e^{-2\pi i k_0 - L(T_c/\tau)} \rightarrow 1 \). Concerning the term \( 1 - e^{-2\pi i k_0 - L(T_c/\tau)} \), as \( \alpha_0 = 1/(N + D) \) and \( k \leq N \), \( \alpha_0 = N/D = 1/\eta, k \alpha_0 < 1/2 \eta N \). Hence, when \( N \) grows to infinity, \( 1 - e^{-2\pi i k_0 - L(T_c/\tau)} \rightarrow 1 - e^{-T_c/\tau} \). When \( N \) and \( L \) grow to infinity, the first r.h.s. term of (B.4) tends to \( \sum_{l=0}^{L-1} \mathbb{E}_h \{ |h(l)|^2 \}^2 \).
These results led to the expected result: when $N$ and $L$ grow to infinity, (B.4) tends to
\[
E_h \left( \sum_{l=0}^{L-1} |h(l)|^2 e^{-2\pi i k \alpha u} \right)^2 \rightarrow \Lambda_k = \left( \sum_{l=0}^{L-1} E_h |h(l)|^2 \right)^2 + \sum_{l=0}^{L-1} E_h |h(l)|^2 \right)^2.
\] (B.6)

C. PROOF OF THEOREM 4

If $H_0$ holds, $y(n) = \sigma^2 w(n)$ is a centered i.i.d. Gaussian noise. The estimates of its cycle coefficients are given by
\[
\hat{R}_y^{(k\alpha)}(N) = \frac{1}{U} \sum_{u=0}^{U-1} y(u+N) y^*(u) e^{-2\pi i k \alpha u}, \tag{C.1}
\]
where $U$ is the observation time. Thanks to the law of large number, $\hat{R}_y^{(k\alpha)}(N)$ is asymptotically normal. Its mean is given by
\[
E \{ \hat{R}_y^{(k\alpha)}(N) \} = \frac{1}{U} \sum_{u=0}^{U-1} E \{ y(u+N) y^*(u) \} e^{-2\pi i k \alpha u} = R_y^{(k\alpha)}(N).
\] (C.2)

As $y$ is a Gaussian i.i.d. signal, $E \{ \hat{R}_y^{(k\alpha)}(N) \} = 0$. We now focus on the asymptotic covariance:
\[
E \{ \hat{R}_y^{(k\alpha)}(N) (\hat{R}_y^{(k\alpha)}(N))^* \} = \frac{1}{U^2} \sum_{u_1, u_2=0}^{U-1} E \{ y(u_1+N) y^*(u_1) y^*(u_2+N) y(u_2) \} e^{-2\pi i k \alpha (u_1-u_2)}.
\] (C.3)

The fourth-order moment is written in terms of the cumulant as
\[
E \{ y(u_1+N) y^*(u_1) y^*(u_2+N) y(u_2) \} = \text{cum}(y(u_1+N), y^*(u_1), y^*(u_2+N), y(u_2)) + E \{ y(u_1+N) y^*(u_1) \} E \{ y^*(u_2+N) y(u_2) \} + E \{ y(u_1+N) y(u_2) \} E \{ y^*(u_2+N) y^*(u_1) \} + E \{ y(u_1+N) y^*(u_2+N) \} E \{ y^*(u_1) y(u_2) \}.
\] (C.4)

As the noise is Gaussian, the fourth-order cumulant vanishes. The second term equals $R_y(u_1, N)(R_y(u_2, N))^*$ which also vanishes since the signal is i.i.d. The third term equals 0 since the signal is circular at order 2. The asymptotic covariance depends hence only on the third term and is simplified to
\[
E \{ \hat{R}_y^{(k\alpha)}(N)(\hat{R}_y^{(k\alpha)}(N))^* \} = \frac{1}{U^2} \sum_{u_1, u_2=0}^{U-1} E \{ y(u_1+N) y^*(u_2+N) \} \times E \{ y^*(u_1) y(u_2) \} e^{-2\pi i k \alpha (k_1-u_2)}.
\] (C.5)

Using the i.i.d. property of the noise signal, this expression vanishes if $u_1 \neq u_2$. If $u_1 = u_2$, $E \{ y(u_1+N) y^*(u_2+N) \} = E \{ y^*(u_1) y(u_2) \} = \sigma^2$. We get
\[
E \{ \hat{R}_y^{(k\alpha)}(N)(\hat{R}_y^{(k\alpha)}(N))^* \} = \sigma^4 \frac{1}{U^2} \sum_{u=0}^{U-1} e^{-2\pi i k \alpha (u_1-u_2)}.
\] (C.6)

The asymptotic variance $E \{ \hat{R}_y^{(k\alpha)}(N) \}^2$ is then equivalent to $\sigma^4 / U$. When $k_1 \neq k_2$, the asymptotic covariance $E \{ \hat{R}_y^{(k\alpha)}(N)(\hat{R}_y^{(k\alpha)}(N))^* \}$ is equivalent to
\[
E \{ \hat{R}_y^{(k\alpha)}(N)(\hat{R}_y^{(k\alpha)}(N))^* \} = \frac{\sigma^4}{U^2} e^{-i\pi (i k_1 - k_2)} \frac{\sin(\pi a_0 (k_1 - k_2))}{\sin(\pi a_0 (k_1 - k_2))}.
\] (C.7)

When $U$ grows to infinity, $U E \{ \hat{R}_y^{(k\alpha)}(N)(\hat{R}_y^{(k\alpha)}(N))^* \}$ tends to 0. Note that the $\hat{R}_y^{(k\alpha)}(N)$ and $\hat{R}_y^{(k\alpha)}(N)$ can be considered as uncorrelated only if $U > 1/|k_1 - k_2|$.

C.1. Proof of Corollary 1

With the estimate of the cycle correlation coefficients being asymptotically uncorrelated Gaussian variables, the probability density function of
\[
(2N_b + 1) \frac{\sigma^4}{U} \hat{J}(N_b) = \sum_{k=-N_b}^{N_b} \frac{U}{\sigma^4} \left| \hat{R}_y^{(k\alpha)}(N) \right|^2
\] (C.8)
is a $\chi^2$ law with $2(2N_b + 1)$ degrees of freedom. The expected result can then be deduced.

C.2. Proof of Corollary 2

Thanks to the previous results, we also know that
\[
E \{ (2N_b + 1) \frac{\sigma^4}{U} \hat{J}(N_b) \} = (2N_b + 1) \tag{C.9}
\]
or equivalently that $E \{ \hat{J}(N_b) \} = U / \sigma^4$. Concerning the asymptotical covariance, we get
\[
E \left\{ \left( 2N_b + 1 \right) \frac{\sigma^4}{U} \hat{J}(N_b) - E \left\{ 2N_b + 1 \right\} \right\}^2 = 2N_b + 1 \tag{C.10}
\]
or equivalently $E \{ \hat{J}(N_b) - E \{ \hat{J}(N_b) \} \}^2 = \sigma^4 / (U^2 (2N_b + 1))$.

D. PROOF OF THEOREM 6

To give some results on the statistical behavior of $\hat{J}(N_b)$, we first give some results on the behavior of the cycle coefficients estimator $\hat{R}_y^{(k\alpha)}(N)$.

D.1. Statistical behavior of $\hat{R}_y^{(k\alpha)}(N)$

If $H_1$ holds, $y(n) = \sqrt{E_s} s_n(nT_b) + \sigma w(n)$ is a centered i.i.d. Gaussian noise. To evaluate the statistical behavior of $J_b(N_b)$, we first introduce the vector $R_y$ defined as
\[
R_y = \left[ R_y^{(-N\alpha)}(N) , \ldots , R_y^{(0)}(N) , \ldots , R_y^{(N\alpha)}(N) \right]^T \tag{D.1}
\]
and \( \hat{R}_y \) as its estimate. Thanks to the law of large number, \( \hat{R}_y \) is asymptotically normal. For each component, its mean is given by

\[
\mathbb{E} \left[ \hat{R}_y^{(kn_0)} (N) \right] = \frac{1}{U} \sum_{u=0}^{U-1} \mathbb{E} \left[ y(u + N) y^*(u) \right] e^{-2i\pi n_0 u} = R_y^{(kn_0)} (N). \tag{D.2}
\]

Only the OFDM signal \( z_c(t) \) contributes to this term which does not vanish. To compute the estimator variance, we introduce the covariance matrix as

\[
\Gamma = \lim_{U \to \infty} U \mathbb{E} \{(\hat{R}_y - R_y)(\hat{R}_y - R_y)^T\}. \tag{D.3}
\]

Its coefficients are given by

\[
[\Gamma]_{k,l} = \lim_{U} U \mathbb{E} \left\{ \hat{R}_y^{(-N_0+kn_0)}(N) \hat{R}_y^{(-N_0+ln_0)}(N)^* \right\} - UR_y^{(-N_0+kn_0)}(N) R_y^{(-N_0+ln_0)}(N)^*.
\]

Similarly to the previous proof, we get after some calculations

\[
\mathbb{E} \left[ \hat{R}_y^{(-N_0+kn_0)}(N) \hat{R}_y^{(-N_0+ln_0)}(N)^* \right] = R_y^{(-N_0+kn_0)}(N) R_y^{(-N_0+ln_0)}(N)^* + \frac{1}{U^2} \sum_{u,v} R_y(u+N,v) R_y^*(u,v) e^{-2i\pi n_0 (k-l) u}.
\]

Hence,

\[
[\Gamma]_{k,l} = \lim_{U} \frac{1}{U^2} \sum_{u,v} R_y(u+N,v) R_y^*(u,v) e^{-2i\pi n_0 (k-l) u} \tag{D.4} \]

\( R_y(u,v) \) vanishes when \( v \neq 0 \) and \( v \neq \pm N \). If \( v = 0 \), \( R_y^*(u,0) \) does not depend on \( u \). Hence,

\[
\lim_{U} \frac{1}{U} \sum_u R_y(u+N,0) R_y^*(u,0) e^{-2i\pi n_0 (k-l) u} = (E_x + \sigma^2)^2 \delta(k-l). \tag{D.5}
\]

When \( v \) is equal to \( \pm N \), the expression is more complex. We will write it as

\[
\lim_{U} \frac{1}{U} \sum_u R_y(u+N,\pm N) R_y^*(u,\pm N) e^{-2i\pi n_0 ((k-l) u - k(\pm N))} = \mathcal{O}(E_x^2). \tag{D.6}
\]

The matrix \( \Gamma \) has then the following form:

\[
\Gamma = [\mathcal{O}(E_x^2)] + \begin{bmatrix}
(E_x + \sigma^2)^2 + \mathcal{O}(E_x^2) & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & (E_x + \sigma^2)^2 + \mathcal{O}(E_x^2)
\end{bmatrix} \tag{D.7}
\]

\[
\mathbb{E} |\hat{J}_y - J_y|^2 = \frac{\beta}{2N_b + 1}. \tag{D.15}
\]
E. PROOF OF THEOREM 7

The cycle coefficients of the signal \( y(n) = \sqrt{\mathbb{E} s_0(uT_c)} + \sigma w(u) \) are given by

\[
R_y^{(k\alpha_0)}(N) = \lim_{U \to \infty} \frac{1}{U} \sum_{u=0}^{U-1} \mathbb{E} \{ y(u + N) y^*(u) \} e^{-2\pi i k \alpha_0 u}.
\]

(E.1)

As \( \alpha_0 = 1/(N+D) \), the r.h.s term is simplified to the expected result:

\[
R_y^{(k\alpha_0)}(N) = e^{-i\pi (k/(N+D))} \frac{\sin \left( \pi (D/(N+D)) \right)}{\sin \left( \pi (k/(N+D)) \right)}.
\]

(E.2)

REFERENCES

[1] J. Mitola, Cognitive radio: an integrated agent architecture for software defined radio, Ph.D. thesis, Royal Institute of Technology, Stockholm, Sweden, 2000.
[2] R. Etkin, A. Parekh, and D. Tse, “Spectrum sharing for unlicensed bands,” IEEE Journal on Selected Areas in Communications, vol. 25, no. 3, pp. 517–528, 2007.
[3] Shared Spectrum Company, “Spectrum occupancy report for new york city during the republican convention August-September, 2004,” in ICC, 2006.
[4] P. Kolodzy, “Spectrum policy task force: findings and recommendations,” in Proceedings of the International Symposium on Advanced Radio Technologies (ISART ’03), Boulder, Colo, USA, March 2003.
[5] Federal Communications Commission, “FCC 04-113—unlicensed operation in the TV broadcast bands,” in ET Docket No. 04-186, May 2004.
[6] G. Chouinard, “IEEE P802.22 wireless RANs—WRAN keep-out region,” June 2006.
[7] C. Cordeiro, K. Challapali, D. Birru, and N. Sai Shankar, “IEEE 802.22: an introduction to the first wireless standard based on cognitive radios,” Journal of Communications, vol. 1, no. 1, pp. 38–47, 2006.
[8] H. Urkowitz, “Energy detection of unknown deterministic signals,” Proceedings of the IEEE, vol. 55, no. 4, pp. 523–531, 1967.
[9] S. V. Schell, “An overview of sensor-array processing for cyclostationary signals,” in Cyclostationarity in Communication and Signal Processing, W. A. Gardner, Ed., pp. 168–239, IEEE Press, New York, NY, USA, 1994.
[10] D. Dehay and H. L. Hurd, “Representation and estimation for periodically and almost periodically correlated random processes,” in Cyclostationarity in Communications and Signal Processing, W. A. Gardner, Ed., IEEE Press, New York, NY, USA, 1993.
[11] H. L. Van Trees, Detection, Estimation, and Modulation Theory: Radar-Sonar Signal Processing and Gaussian Signals in Noise—Part III, John Wiley & Sons, New York, NY, USA, 2nd edition, 2001.
[12] A. V. Dandawate and G. B. Giannakis, “Statistical tests for presence of cyclostationarity,” IEEE Transactions on Signal Processing, vol. 42, no. 9, pp. 2355–2369, 1994.
[13] M. ¨Oner and F. Jondral, “On the extraction of the channel allocation information in spectrum pooling systems,” IEEE Journal on Selected Areas in Communications, vol. 25, no. 3, pp. 558–565, 2007.
[14] Huawei Technologies UESTC, “IEEE P802.22 wireless RANs—sensing scheme for DVB-T,” November 2006.
[15] P. Liu, B.-B. Li, Z.-Y. Lu, and F.-K. Gong, “A blind time-parameters estimation scheme for OFDM in multi-path channel,” in Proceedings of International Conference on Wireless Communications, Networking and Mobile Computing (WCNM ’05), vol. 1, pp. 242–247, Wuhan, China, September 2005.
[16] H. Ishii and G. W. Wornell, “OFDM blind parameter identification in cognitive radios,” in Proceedings of the 16th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC ’05), vol. 1, pp. 700–705, Berlin, Germany, September 2005.
[17] E. Sofer and G. Chouinard, “IEEE P802.22 wireless RANs—WRAN channel modeling,” September 2005.
[18] ETSI, “Digital video broadcasting (DVB) framing structure, channel coding and modulation for digital terrestrial television,” November 2004.
[19] R. A. Davis and P. J. Brockwell, Time Series: Theory and Methods, Springer Series in Statistics, Springer, New York, NY, USA, 1998.