Probing the charm-Higgs Yukawa coupling via Higgs boson decay to $h_c$ plus a photon

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(Dated: March 18, 2022)

Abstract

In this paper, we investigate the decay of Higgs boson to $h_c$ plus a photon in the NRQCD theoretical framework. Comparing with the Higgs decay to $J/\psi$ plus a photon channel, this process has not indirect contribution, can be used to detect the Yukawa coupling of Higgs and charm quarks. The results show that the decay branch ratio of this process is about $10^{-8}$. If we takes into account the $10^{-3}$ efficiency in the $h_c$ detection, no events will be available even in the case of 30$ab^{-1}$ luminosity at FCC-pp with 100 TeV center of mass energy. However, if the detection efficiency of $h_c$ is greatly improved in the future, this process will play an important role at linear $e^+e^-$ future colliders and at LHCb. Moreover, this process should be also play an important role when the anomalous charm Yukawa couplings are larger and direct sensitivity.

PACS numbers: 11.15.-q, 13.38.-b, 14.40.Lb, 14.80.Bn

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I. INTRODUCTION

The discovery of the Higgs boson is a triumph of the LHC [1–4] and it is also a success for the Standard Model (SM) with its minimal Higgs sector of electroweak (EW) symmetry breaking (EWSB). After discovery of the Higgs boson, one of the most tasks is to determine its properties, such as its spin, CP, width, and couplings. Up to now, all measurements of the Higgs boson properties are so far indicating that the observations are compatible with SM Higgs. For the main Higgs discovery modes $\gamma\gamma$, $ZZ$ and $WW$, the couplings to gauge bosons are measured directly, which are fixed through the well measured diboson decays of the Higgs determined at the $20 \sim 30\%$ level. Direct evidence for the Yukawa coupling of the Higgs boson to the top [5] and bottom [6, 7] quarks was recently obtained. Measurements of the Yukawa coupling of the Higgs boson to the first- and second-generation quarks are need to do in the near future.

The Standard Model Higgs boson direct decaying to a pair of charm quarks, through associated production of the Higgs and Z bosons, in the decay mode $HZ \rightarrow l^+l^-c\bar{c}$ is studied by the ATLAS experiment at the LHC in Ref.[8]. Charm jets are particularly challenging to tag because c-hadrons have shorter lifetimes and decay to fewer charged particles than b-hadrons. The largest uncertainty is due to the normalization of the dominant $Z$+jets background. Therefore, the charm quark Yukawa coupling are hard to accurate measurement in hadron colliders through the direct $H \rightarrow c\bar{c}$ decay, owing to large QCD backgrounds, and challenges in jet flavor identification [9, 10].

Heavy quarkonium $J/\psi$ is a $c\bar{c}$ bound state and can decay to $e^+e^-$ or $\mu^+\mu^-$. These leptonic decay modes are clean channels in experiments and suppress large QCD backgrounds. In Ref.[11], The authors showed that the exclusive decays of the Higgs boson to vector mesons can probe the Yukawa couplings of first- and second-generation quarks and serve as searching New Physics (NP) beyond SM at future runs of the LHC. Then, Higgs rare decay to a vector quarkonium ($J/\psi$, $\Upsilon$) received considerable attention [12–15]. The relativistic correction for Higgs boson decay to an S-wave vector quarkonium plus a photon have been calculated in Ref. [16]. In Ref. [17], the authors evaluated the NLO corrections to $H \rightarrow J/\psi + \gamma$ and find that the direct contribution are greatly reduced by the NLO QCD correction. A search for Higgs and Z bosons decaying to $J/\psi$ and $\Upsilon$ is performed in integrated luminosities $20.3 fb^{-1}$ with the ATLAS detector at 8 TeV LHC. No significant excess of events is observed above
expected backgrounds and 95% CL upper limits are placed on the branching fractions. 

There is another problem needs to be solved via Higgs decay to $J/\psi \gamma$ to study the charm quark Yukawa couplings. In this decay channel, there are two part contributions for total decay width: one come from the direct contribution, which is related to charm quark Yukawa coupling, and the other part come from the indirect contribution, which arises from $H \rightarrow \gamma^* \gamma$ with virtual $\gamma$ substantially converting into $J/\psi$, and not related to charm quark Yukawa coupling. However, the width of indirect decay is much larger than that of direct decay. Suppressing the indirect contribution becomes an important and unavoidable question. Previously, all the people focus on Higgs decay to S state charmonium $J/\psi$. In fact, if the P state charmonium is selected as a candidate, like $h_c$, which has quantum numbers $J^P C = 1^{+-}$, due to the CP invariance of Quantum Electrodynamics (QED), virtual $\gamma$ converting into $h_c$ is forbidden, the contribution of the indirect decay can be completely removed.

$h_c$ meson is the lowest spin-singlet P-wave charmonium, which is first found via the process $p\bar{p} \rightarrow h_c \rightarrow J/\psi \pi^0$ at Fermilab E760 experiment in 1992. Then, the $h_c$ state is measured by Fermilab E835, CLEO-c, BESIII experiments. Its C-parity was established by radiative decay $h_c \rightarrow \eta_c \gamma$. In recent years, the production and decay of $h_c$ has been studied at $e^+e^-$ and hadron colliders. In this paper, we will investigate the process $H \rightarrow h_c + \gamma$ within the non-relativistic QCD (NRQCD) framework by applying the covariant projection method.

The paper is organized as follows: we present the details of the calculation strategies in Sec.II. The numerical results are given in Sec.III. Finally, a short summary and discussions are given.

II. CALCULATION DESCRIPTIONS

In this section, we present the calculation for the decay process $H \rightarrow h_c + \gamma$. There are two Feynman diagrams for $H \rightarrow h_c + \gamma$ at parton level in leading order (LO), which are drawn in Fig.1. We calculate the amplitudes by making use of the standard methods of NRQCD factorization, which provides a rigorous theoretical framework for the description of heavy quarkonium production and decay. In the NRQCD, the idea of perturbative factorization is applied, the process of production and decay of heavy quarkonium is separated into two
parts: short distance part, which allows the intermediate $Q\bar{Q}$ pair with quantum numbers different from those of the physical quarkonium state, and the long distance matrix elements (LDMEs), which can be extracted from experiments. NRQCD is an effective factorization method and has become the standard tool for theoretical calculations for heavy quarkonium [33].

The partonic process $H \to c\bar{c} + \gamma$ at LO is denoted as:

$$H(p_1) \to c(p_2)\bar{c}(p_3) + \gamma(p_4).$$

The amplitudes for these two diagrams are given by

$$iM_{i1} = \bar{u}_{si}(p_2) \cdot \frac{-ie^2m_c}{2m_ws_W} \cdot \frac{i}{p_1 - p_2 - m_c} \cdot i\frac{e^2\gamma^\mu}{3} \cdot v_{s'j}(p_3)\epsilon^*_\mu(p_4),$$

$$iM_{i2} = \bar{u}_{si}(p_2) \cdot i\frac{2^2e^2\gamma^\mu}{3} \cdot \frac{i}{p_1 - p_3 - m_c} \cdot \frac{-ie^2m_c}{2m_ws_W} \cdot v_{s'j}(p_3)\epsilon^*_\mu(p_4).$$

where $s$ and $s'$ are spin indices, $i$ and $j$ are color indices of the outgoing $c$ quark and $\bar{c}$ quark, respectively. The relative momentum between the $c$ and $\bar{c}$ is defined as $q = (p_2 - p_3)/2$, and the total momentum of the bound state $c\bar{c}$ is defined as $p = p_2 + p_3$. Then, we obtain the following relations among the momenta:

$$p_2 = \frac{1}{2}p + q, \quad p_3 = \frac{1}{2}p - q, \quad p \cdot q = 0,$$

$$p_2^2 = p_3^2 = m_c^2, \quad p^2 = E^2, \quad q^2 = m_c^2 - E^2/4 = -m_c^2v^2.$$

In the $c\bar{c}$ rest frame, $p = (E, 0)$ and $q = (0, q)$. In the non-relativistic $v = 0$ limit, $p^2 = 4m_c^2$, $q^2 = 0$. In order to produce a $h_c$, the $c\bar{c}$ pair must be produced in a spin-singlet, color-singlet fock state with orbital angular momentum $L = 1$. The short-distance amplitudes are
obtained by differentiating the spin-singlet, colour-singlet projected amplitudes with respect to the momentum \( q \) of the heavy quark in the \( c\bar{c} \) rest frame, and then setting relative momentum \( q \) to zero. As following the notations in Ref. [31], the short-distance amplitudes are expressed as:

\[
\mathcal{M}_{1P_0^{(1)}} = \epsilon_\beta \frac{d}{dq} \text{Tr} \left[ C_1 \Pi_0 \mathcal{M} \right] \bigg|_{q=0},
\]

where \( \epsilon_\beta \) is the polarization vector of \( 1^P_1 \) state, and the spin-singlet projector is given by

\[
\Pi_0 = \frac{1}{\sqrt{8m_c^2}} \left( \frac{\hat{p}}{2} - \hat{q} - m_c \right) \gamma_5 \left( \frac{\hat{p}}{2} + \hat{q} + m_c \right) .
\] (4)

The colour singlet state will be projected out with the following operator:

\[
C_1 = \frac{\delta_{ij}}{\sqrt{N_c}}
\] (5)

The amplitude \( \mathcal{M} \) is obtained by truncating the external spinor \( \bar{u}(p_2) \) and \( v(p_3) \) in Fig. 1. The trace is sum over all the Lorenz and colour indices. The selection of the appropriate total angular momentum quantum number is done by performing the proper polarization sum. Here, we define:

\[
\Pi_{\alpha\beta} \equiv -g_{\alpha\beta} + \frac{p_\alpha p_\beta}{M^2} ,
\] (6)

where \( M = 2m_c \).

After the application of this set of rules, the short-distance contribution to the differential decay width for \( H \rightarrow \bar{c}c[1^P_1] + \gamma \) process reads:

\[
d\hat{\Gamma}(H \rightarrow \bar{c}c[1^P_1] + \gamma) = \frac{1}{32\pi^2} |\mathcal{M}_{1P_0^{(1)}}|^2 \frac{|p|}{m_H^2} d\Omega ,
\] (7)

where \( |p| = \frac{m_H^2 - m_{bc}^2}{2m_H} \) and \( m_H \) represent the Higgs boson mass. \( d\Omega = d\phi d(\cos\theta) \) is the solid angle of particle \( h_c \).

\[
|\mathcal{M}_{1P_0^{(1)}}|^2 = \frac{256\pi^2 \alpha^2 m_c(\mu)^2}{3m_c^2 m_W^2 s_W^2} \]

(8)

where \( m_c(\mu) \) appeared in charm quark Yukawa coupling is the running mass of charm quark [34]. In the modified minimal subtraction or \( \overline{MS} \) scheme, the relation between the
pole masses and the running masses at the scale of the pole mass, \( \overline{m}_c(m_c) \), can be expressed as

\[
\overline{m}_c(m_c) = m_c [1 - \frac{4}{3} \frac{\alpha_s(m_c)}{\pi} + (1.0414N_f - 14.3323) \frac{\alpha_s^2(m_c)}{\pi^2} \\
+ (-0.65269N_f^2 + 26.9239N_f - 198.7068) \frac{\alpha_s^3(m_c)}{\pi^2})]
\]

(9)

where \( \alpha_s \) is the \( \overline{MS} \) strong coupling constant evaluated at the scale of the pole mass \( \mu = m_c \). The evolution of \( \overline{m}_c \) from \( m_c \) upward to a renormalization scale \( \mu \) is

\[
\overline{m}_c(\mu) = \overline{m}_c(m_c) \frac{c[\alpha_s(\mu)/\pi]}{c[\alpha_s(m_c)/\pi]}
\]

(10)

with the function \( c \), up to three-loop order, given by

\[
c(x) = (25x/6)^{12/25}[1 + 1.014x + 1.389x^2 + 1.091x^3] \text{ for } m_c < \mu < m_b
\]

\[
c(x) = (23x/6)^{12/23}[1 + 1.175x + 1.501x^2 + 0.1725x^3] \text{ for } m_b < \mu < m_t
\]

\[
c(x) = (7x/2)^{4/7}[1 + 1.398x + 1.793x^2 - 0.6834x^3] \text{ for } m_t < \mu
\]

(11)

Then, the total decay width is

\[
\Gamma(H \to h_c + \gamma) = \hat{\Gamma}(H \to c\bar{c}(1P_{1}^{(1)}) + \gamma) \frac{<O^{h_c(1P_{1}^{(1)})}>}{2N_cN_{col}N_{pol}},
\]

(12)

where \( N_{col} \) and \( N_{pol} \) refer to the number of colors and polarization states of the \( c\bar{c} \) pair produced. The color-singlet states \( N_{col} = 1 \), and \( N_{pol} = 3 \) for polarization vector in 4 dimensions. \( <O^{h_c(1P_{1}^{(1)})}> \) is the vacuum expectation value of the operator \( \mathcal{O}^{h_c(1P_{1}^{(1)})} \), \( 2N_c \) is due to the difference between the conventions in Ref. 31 and Ref. 32.

### III. NUMERICAL RESULTS AND DISCUSSION

In this section, we present our numerical results for the \( H \to h_c + \gamma \) decay. The relevant input parameters are set as follows 35:

\[
\alpha^{-1} = 137.036, \ m_H = 125.09 \text{ GeV}, \ m_Z = 91.1876 \text{ GeV}, \ m_W = 80.385 \text{ GeV}, \ (13)
\]

\[
m_c = 1.64 \text{ GeV}, \ s_W^2 = 1 - m_W^2/m_Z^2, \ m_{hc} = 3.76 \text{ GeV}, \ (14)
\]
\[ \mu \quad M_H/2 \quad M_H \quad 2M_H \]

| mc(\mu)(GeV) | 0.66 0.62 0.51 |
| ----------------- |----------------|
| \(\Gamma(H \to h_c + \gamma) \times 10^{11}\) | 0.86 0.76 0.51 |

TABLE I: The renormalization scale \(\mu\) dependence of the decay widths for the process \(H \to h_c + \gamma\)

The LDME of \(\langle \mathcal{O}^{h_c[1P_1]} \rangle\) can be expressed in terms of radial derivative of the wave function of quarkonium at the origin \(\langle \mathcal{O}^{h_c[1P_1]} \rangle = \frac{27}{2\pi} |R'_P(0)|^2\), where \(|R'_P(0)|^2 = 0.075 GeV^5\) from the potential model calculations has been used in our calculation. The value of color-singlet Long Distance Matrix Elements (LDME) is set as \(\langle \mathcal{O}^{h_c[1P_1]} \rangle = 0.32 GeV^5\) [29, 36, 37].

Since the mass of charm quark in the charm Yukawa coupling is dependent on the renormalization scale, the strength of the charm Yukawa coupling is also dependent on the renormalization scale. We take the Higgs mass as the central value of the renormalization scale for processes \(H \to h_c + \gamma\), and the short distance theoretical uncertainty is estimated by the renormalization scale range from \(1/2m_H\) to \(2m_H\). In Table II we list the running charm mass and decay width at different renormalization scales, where \(m_c\) is taken as the pole mass 1.64 GeV except charm mass in the charm Yukawa coupling, \(m_{hc}\) is taken as 3.76 GeV.

The total width of a 125 GeV SM Higgs boson is \(\Gamma(H) = 4.07 \times 10^{-3} GeV\), with a relative uncertainty of \(+4.0\% \quad -3.9\%\) [35, 38]. Using this width of Higgs boson decays, we obtain the following results for the branching fraction in the SM:

\[ \mathcal{B}(H \to h_c + \gamma) = 0.187 \times 10^{-8} \quad (15) \]

If there is new physics beyond the Standard Model (SM), charm Yukawa coupling strength may be different from that in SM. In order to consider the theoretical uncertainty, we assume that the Yukawa coupling strength of charm quark and Higgs boson is deviation from the coupling in SM. The deviations from the SM are implemented as scale factors \((\kappa^2)\) of Higgs couplings relative to their SM values, and it is defined as:

\[ g_{Hc\bar{c}} = \kappa \cdot g_{Hc\bar{c}}^{SM} \quad (16) \]

such that \(\kappa = 1\) in SM. In Table III we illustrate the parameter \(\kappa^2\) dependence of the decay widths for the process \(H \to h_c + \gamma\). The mass of \(h_c\) is set as 3.76 GeV, \(m_c = 1.64 GeV\), the
TABLE II: The parameter $\kappa^2$ dependence of the decay widths for the process $H \to h_c + \gamma$

| $\kappa^2$ | 0.1 | 0.2 | 0.5 | 1   | 2   | 5   | 10  |
|------------|-----|-----|-----|-----|-----|-----|-----|
| $\Gamma(H \to h_c + \gamma) \times 10^{11}$ | 0.076 | 0.152 | 0.38 | 0.76 | 3.8 | 7.6 |

renormalization scale is set as $\mu = m_H$. When $\kappa^2$ running from 0.1 to 10, the decay widths vary from $0.076 \times 10^{-11}$ GeV to $7.6 \times 10^{-11}$ GeV for the processes $H \to h_c + \gamma$, respectively.

In experiment, $h_c$ is detected mainly by the following three decay channels:

\[
\begin{aligned}
  h_c & \to \pi_0 J/\psi \to l^+ l^- \gamma \gamma, \\
  h_c & \to \eta_c \gamma \to p \bar{p} \gamma, \\
  h_c & \to \eta_c \gamma \to \gamma \gamma \gamma.
\end{aligned}
\]

The branching ratio of $h_c \to \pi_0 J/\psi$ and $h_c \to \eta_c \gamma$ are estimated to be about 0.5% \cite{39} and 50% \cite{40, 43} in theory, respectively. In these decay chains, the branching ratio of $J/\psi$ decaying into $l^+ l^-$ is about 12% \cite{35}; $\pi^0$ almost completely decays into $\gamma \gamma$, the branching ratio of $\eta_c$ decaying into $p \bar{p}$ is about 0.13%, and into $\gamma \gamma$ with a ratio of 0.024% \cite{35}. The total cross section of Higgs production at 14 TeV LHC is about 62 pb. If the integral luminosity of LHC reaches $3000 \text{ fb}^{-1}$, it will accumulate about $2 \times 10^8$ Higgs events. Considering the decay branching ratio of $H \to h_c + \gamma$, there will be about 0.4 events of $h_c + \gamma$ decaying from Higgs boson. If we takes into account the $10^{-3}$ efficiency in the $h_c$ detection in hadron colliders, no events will be available even in the case of $30 ab^{-1}$ luminosity at FCC-pp with 100 TeV center of mass energy. However, if the detection efficiency of $h_c$ is greatly improved in the future, this process will play an important role at linear $e^+ e^-$ future colliders and at LHCb. Moreover, like $h \to J/\psi + \gamma$, this process should be also play an important role when the anomalous (large $\kappa$) charm Yukawa couplings are larger and direct sensitivity.

IV. SUMMARY

Compared to the process of Higgs decay to $J/\psi$ plus a photon, the process of Higgs decaying to $h_c$ plus a photon can greatly reduce the indirect contribution and can be used to directly detect the coupling of Higgs and charm quarks. In this paper, we calculated the decay width and decay branch ratio of Higgs decay to $h_c$ plus a photon in the theoretical framework of NRQCD. We found that the branch ratio is about $0.187 \times 10^{-8}$, and there will
be no enough events to produce with integrated luminosity $3000 fb^{-1}$ at the 14 TeV LHC. If the detection efficiency of $h_c$ is taken into consideration, it is difficult to observe it on the LHC. However, if the detection efficiency of $h_c$ is greatly improved in the future, this process will play an important role at linear $e^+e^-$ future colliders and at LHCb and it also will play an important role when the anomalous (large $\kappa$) charm Yukawa couplings are larger and direct sensitivity.

V. ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China (No.11305001, No.11575002, No.11805001) and the Key Research Foundation of Education Ministry of Anhui Province of China under Grant (No.KJ2017A032).

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