The nuclear binding and the EMC effect in the deuteron

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Abstract

Influence of the nuclear binding in the deuteron deep inelastic structure function \( F_2(x, Q^2) \) is investigated. The description is based on the Bethe-Salpeter formalism in the ladder approximation and the operator product expansion within an effective meson-nucleon theory. It is shown that the binding in a fully relativistic treatment can be simply parametrised and its origin can be understood in terms of conventional nuclear degrees of freedom. A numerical estimate of the EMC effect in the deuteron is given.

1 Introduction

Ever since nuclear effects in deep inelastic lepton scattering on hadron targets have clearly manifested itself over the last decade, both particle and nuclear theorists obtained a profound insight into physics of the EMC effect. However a majority of models of the EMC effect provided by a fully relativistic hadronic theory resorts to the Relativistic impulse approximation (RIA) with the off-mass-shell kinematics, for example see Refs. \[1, 2\].

It seems to be evident that the free nucleon structure function (SF) \( F_2^N(x, Q^2) \) is somehow affected by the nuclear medium. Here \( Q^2 \) and \( x \) are the four momentum transfer squared and the Bjorken scaling variable \( x = Q^2/(2m\nu) \), with \( \nu \) being the energy transfer in the laboratory frame. Conventional nuclear physics models, for a review see Ref. \[3\], assume that properties of nuclear constituents, nucleons and mesons, are insensitive to the nuclear environment. Yet the hadronic current, which is poorly understood object so far, may depend on dynamics of nucleons inside a nucleus and their interactions. The common assumption in this case is to treat the hadronic current as the sum of currents of single (quasi)free nucleons, whereas the relation between the momentum and energy of the virtual nucleon is specified in a relevant treatment of the relativistic bound state. Such a proviso substantiates derivations of the convolution formula for the nuclear SF with a consideration of relativistic kinematics.

The approach we take in this paper follows a rigorous method introduced in Refs. \[4, 5\] to investigate deep inelastic scattering on light nuclei. The method relays on the leading order operator product expansion (OPE) within the effective meson-nucleon theory. It allows one to derive the convolution formula for the nuclear SF which accounts for the role of the Fermi motion and binding effects in a new and unambiguous form. Such a formulation yields the \( n \)th moment of the nuclear SF as a product of the moments of the isolated-hadron SF’s and the moments of the effective hadron momentum distributions. The latter are given by matrix elements of twist-two operators sandwiched between the ground state vector of the target.

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Using the Hamiltonian formalism relevant operators in the OPE are found in the lowest order approximation on the meson-nucleon coupling constant and resulting operators involve the meson degrees of freedom. Finally the matrix elements of these operators are calculated over the ground state of the target in the non-relativistic (NR) limit. The coefficient functions together with meson cloud contributions are substituted by the moments of the isolated-hadron SF.

In this paper we apply the method proposed in Refs. [4, 5] in order to describe the deuteron SF \( F_2^D(x, Q^2) \) where the system is viewed as the relativistic bound state in the Bethe-Salpeter (BS) formalism. To this end expressions for the twist-two operators in the OPE are found by making use of the relativistic Hamiltonian. Explicit expressions for the moments of the deuteron SF are found through actual calculation of the matrix elements in terms of the BS vertex function. At the first step we assume that the off-shell effect on quark and anti-quark momentum distributions in hadrons may be neglected\(^2\). In this way the Mellin transform reconstructs the deuteron SF as a sum of convolution terms describing the Fermi motion and the binding in the deuteron due to meson exchanges. Since meson exchange currents (MEC) are associated with exchange mechanisms responsible for the binding in the deuteron, momentum distribution for the constituent mesons reveal itself as well.

Advantage of this relativistic approach is that it differentiates the pure binding effects from the off-shell effects as both have an unlike origin. The magnitude of the former is given in terms of well-understood quantities such as the kinetic energy of a nucleon and the binding energy of the deuteron. Besides dealing with the deuteron we can obtain all required formulae analytically within well-defined approximations and subject the problem to intense scrutiny.

The paper is divided into five sections. In Sec. II the basic formalism is reviewed. The twist-two operators in the OPE of the Compton amplitude is specified. In Sec. III the moments of the fractional momentum distributions of nucleons and mesons in the deuteron are calculated. The NR limit of the energy-momentum sum rule is performed. Computation of \( F_2^D(x, Q^2) \) as a function of \( x \) is carried out in Sec. IV. And Sec. V contains a summary of principle results and conclusions.

## 2 Structure functions of the deuteron in the BS approach

### 2.1 The relativistic vertex function of the deuteron

Consider a system of two nucleons interacting via a scalar, pseudo-scalar and vector meson exchanges. The Lagrangian density for meson-nucleon couplings is

\[
\mathcal{L}_I = -g_V \bar{N} \gamma_\mu N V^\mu - g_S \bar{N} S N S - ig_\pi \bar{N} \gamma_5 \tau_i N \varphi_i + \ldots
\]  

with \( N \) being the nucleon and \( S, \varphi_i, V_\mu \) meson fields.

Assuming that the NN interaction to proceed through the meson exchange only, we start with the one-boson-exchange (OBE) model in application to the BS equation (BSE) in the bound state region of the deuteron. Therefore the BS amplitude, \( \chi(p; P) = \langle 0| T N_{p_1} N_{p_2} | P \rangle \),

\(^2\)Though it may well be incorporated in this line of considerations.
and equivalently the BS vertex function \( \Gamma(p; P) = S^{-1}(p_1, p_2)\chi(p; P) \), satisfy the BSE in the momentum space as

\[
\Gamma(p; P) = i \int \frac{d^4p'}{(2\pi)^4} V_{\text{OBE}}(p - p')\chi(p'; P),
\]

where \( S(p_1, p_2) \) is the free two-nucleon Green’s function and \( V_{\text{OBE}} \) is the OBE interaction kernel. The BSE (2) is written in terms of the relative momenta \( p, p' \) of the nucleons and the center-of-mass (c.m.) momentum \( P \) and \( p_{1,2} = P/2 \pm p \). The interaction kernel \( V_{\text{OBE}} \) is the sum of one-particle exchange amplitudes of certain bosons with given masses and couplings generated by \( \mathcal{L}_I \) in Eq. (1).

Since the BSE is homogeneous, it cannot determine a multiplicative constant of \( \chi(p; P) \). In order to normalise the BS amplitude, condition based on the electromagnetic (EM) current conservation at zero momentum transfer can be used. In the IA the normalisation condition reads

\[
-\frac{i}{N} \int d^4p \bar{\chi}(p; P) \left[ \frac{\partial}{\partial P_0} S^{-1}(p_1, p_2) \right] \chi(p; P) = P_0,
\]

where \( P_0 = \sqrt{P^2 + M^2} \) and \( \bar{\chi} \) stands for the BS amplitude conjugate of \( \chi \). An appropriate normalisation constant in Eq. (3) is denoted as \( N \). In the Euclidean space the amplitude \( \chi \) and its conjugate \( \bar{\chi} \) are related as follows: \( \bar{\chi}(p_4, p; P) = -[\chi(-p_4, p; P)]^{*} \gamma_0(1) \gamma_0(2) \), where \( p_4 \) lies on the imaginary axis.

The Lagrangian (1) and the BSE (2) define an effective meson-nucleon theory for the description of the bound state of the deuteron which is meaningful in a low-order approximation. Such a theory is not fundamental one in a sense that interactions in Eq. (1) are viewed as effective ones which are modified by vertex form factors. If the BSE is applied in the ladder approximation as done in Eq. (2), all further calculation of matrix elements of EM currents expressed in terms of the nucleon and meson fields over the deuteron state have to be made up to the second order in the strong coupling. This procedure is justified in describing the nuclear structure effects in electron scattering.

### 2.2 Operator product expansion

The structure functions \( F_{1,2} \) can be calculated via the OPE of the amplitude for forward Compton scattering of virtual photons off a hadronic target \([6, 7]\). The expansion separates characteristic hadronic and nuclear scales and formulates this particular analysis of the nuclear SF as a calculation of matrix elements of certain operators within low-energy physics.

The spin-averaged Compton amplitude is expanded in terms of local operators \( O \) and coefficient functions \( E \) in the form

\[
T_{\mu\nu}(P, q) \propto -g_{\mu\nu} \sum_{n=\text{even}} \left( \frac{2}{Q^2} \right)^n q_{\mu_1} \cdots q_{\mu_n} \sum_{a=N,B} E_{1,a}^n(Q^2, g) \langle P|O_{\mu_1\cdots\mu_n}|P\rangle + 2g_{\mu_1}g_{\nu\mu_2} \sum_{n=\text{even}} \left( \frac{2}{Q^2} \right)^{n-1} q_{\mu_3} \cdots q_{\mu_n} \sum_{a=N,B} E_{2,a}^n(Q^2, g) \langle P|O_{\mu_1^*\cdots\mu_n}|P\rangle,
\]

where \( \langle P|O_{\mu_1^*\cdots\mu_n}|P\rangle \) represents the matrix element of the operators \( O_{\mu_1^*\cdots\mu_n} \) in the deuteron state.
where \( O_{a}^{\mu_{1}...\mu_{n}} \) are series of operators having twist two at the level of naive dimension counting and \( a \) tags the fundamental fields of the theory, i.e. nucleons (N) and bosons (B). Each operator is appropriately symmetrised and trace-subtracted. In this procedure the deuteron SF \( F_{2} \) is expressed in terms of invariant diagonal matrix elements \( \mu_{n}^{a/D} \)

\[
\langle P | O_{a}^{\mu_{1}...\mu_{n}} | P \rangle = \{ P^{\mu_{1}}...P^{\mu_{n}} \} \mu_{n}^{a/D},
\]

and the coefficient functions \( E_{2,a}^{n} \) by

\[
M_{n-1}(F_{2}^{D}) = \sum_{a=N,B} E_{a,n}^{(2)}(Q^{2},g) \mu_{n}^{a/D},
\]

where \( M_{n}(F) \) defines the \( n \)th moment of \( F \), \( M_{n}(F) = \int_{0}^{1} dx x^{n-1} F(x, Q^{2}) \). In Eq. (5) we neglect the effects of nucleon and target masses corrections which can be taken into account in a closed analytic form. The explicit line of our calculation proceeds as follows: (1) Define a basis of the twist-two operators in OPE (4). (2) Parametrise the coefficient functions in terms of moments \( E_{n}^{a} \) of \( n \)th moment of \( n \) operators in the expansion (4) for the meson-nucleon theory. (3) Obtain the moments of the deuteron SF by calculating the matrix elements \( \mu_{n}^{a/D} \) in Eq. (5). (4) Use the Mellin transform to invert the moments and exhibit SF’s.

Let us determine explicitly the operators in the expansion (4) for the meson-nucleon theory. For a nucleon field and a pseudo-scalar (and scalar) field interacting via coupling given by (4) there are only two series of symmetric bilinear operators

\[
O_{N}^{\mu_{1}...\mu_{n}} = \left( \frac{i}{2} \right) n^{-1} S\{ N_{\lambda \mu_{1}}^{\nu_{1}} \bar{\partial}^{\nu_{1}} N_{\lambda \mu_{2}}^{\nu_{2}} \cdots \bar{\partial}^{\nu_{n}} N \},
\]

\[
O_{\pi}^{\mu_{1}...\mu_{n}} = \left( \frac{i}{2} \right) n^{-1} S\{ \varphi_{i}^{\mu_{1}} \bar{\partial}^{\nu_{1}} \cdots \bar{\partial}^{\mu_{n}} \varphi_{i} \},
\]

where \( S \) implies complete symmetrization in the Lorentz indices \( \mu_{1} \ldots \mu_{n} \) and removes all traces with respect to each pair of indices.

If a theory involves fundamental massive vector bosons, matters become more complicated by the fact that the vector field itself has twist zero and some subsidiary condition is needed to introduce the set of relevant operators. Two sets of the operators can be formed.

Set (a):

\[
O_{N}^{\mu_{1}...\mu_{n}} = \left( \frac{i}{2} \right) n^{-1} S\{ N_{\lambda \mu_{1}}^{\nu_{1}} \bar{\partial}^{\nu_{1}} \cdots \bar{\partial}^{\nu_{n}} N \},
\]

\[
O_{F}^{\mu_{1}...\mu_{n}} = \left( \frac{i}{2} \right) n^{-2} S\{ F^{\lambda \mu_{1}}_{\nu_{1}} \bar{\partial}^{\nu_{1}} \cdots \bar{\partial}^{\nu_{n-1}} F_{\lambda}^{\mu_{n}} \},
\]

\[
O_{V}^{\mu_{1}...\mu_{n}} = \left( \frac{i}{2} \right) n^{-2} S\{ V_{\lambda \mu_{1}}^{\nu_{1}} \bar{\partial}^{\nu_{1}} \cdots \bar{\partial}^{\nu_{n-1}} V_{\mu_{n}} \},
\]

and set (b):

\[
O_{N}^{\mu_{1}...\mu_{n}} = \left( \frac{i}{2} \right) n^{-1} S\{ \bar{N}_{\lambda \mu_{1}}^{\nu_{1}} \bar{\partial}^{\nu_{1}} \cdots \bar{\partial}^{\nu_{n}} N \},
\]

\[
O_{V}^{\mu_{1}...\mu_{n}} = \left( \frac{i}{2} \right) n^{-1} S\{ V_{\lambda \mu_{1}}^{\nu_{1}} \bar{\partial}^{\nu_{1}} \cdots \bar{\partial}^{\nu_{n}} V_{\lambda} \}. \]
Set (a) is introduced on the ground of gauge-invariance [8]. The first two operators labelled by \(N\) and \(F\) are invariant under the transformation \(V_{\mu}(x) \rightarrow V_{\mu}(x) + g_{V} \partial_{\mu} \Lambda(x)\) and \(N(x) \rightarrow \exp(ig_{V} \Lambda(x))N(x)\), although the mass term in the free Lagrangian for the vector field \(-\frac{1}{2}g_{V}^{2}V^{2}\) breaks this gauge symmetry; and the subsidiary condition is used [9] that the symmetrised form of the energy-momentum tensor for the massive vector bosons is restored by the operators when \(n = 2\):

\[
\Theta^{\mu \nu} = F^{\lambda \mu}F_{\lambda}^{\nu} + \mu_{V}^{2}V^{\mu}V^{\nu} - \frac{1}{2}g_{V}(\bar{N}\gamma_{\mu}NV^{\nu} + \bar{N}\gamma_{\nu}NV_{\mu}) - g^{\mu \nu}L. \tag{14}
\]

The operators \(O_{V}^{\mu_{1} \cdots \mu_{n}}\) (13) guarantee that. On the other hand set (b) is chosen on the basis of that the Lagrangian of the vector field is taken in the form of a covariant sum of four Lagrangians each separately corresponding to the components of \(A_{\mu}\) and imposing the invariant subsidiary condition \(\partial \cdot A = 0\). Thus the operators depending explicitly on the interaction and composed out of the tensor \(F_{\mu \nu}\) are excluded from the set. In this case the energy-momentum tensor is restored in the form

\[
\Theta^{\mu \nu} = \partial^{\mu}A_{\lambda}\partial^{\nu}A^{\lambda} - g^{\mu \nu}L'. \tag{15}
\]

and the operators (12) and (13) preserve the momentum sum rule for the \(F_{2}(x)\), i.e. \(M_{1}(F_{2}) = 1\).

3 The moments of the deuteron structure function

3.1 Nucleon contribution

The set (a) was considered in Ref. [9] in a calculation of the nuclear SF within the Walecka model. The result is that the exchange by vector meson is incompatible with the observed magnitude of the EMC effect. The nucleon part of the nuclear SF, because of the final-state interaction with the vector meson field of a nucleus, is found to be twice as large as the experimentally observed one. Moreover the meson part peaks at small values of \(x\), which is either not observed.

We consider the twist-two operators that belong to the set (b). In this set operators for the vector and scalar meson fields are taken into account on equal basis. Taking the Bjorken limit, defined by \(Q^{2}, \nu \rightarrow \infty\) with \(x\) fixed, and choosing a coordinate system so that \(q = (\nu, 0_{\perp}, q_{t})\), where \(q_{t} = -\sqrt{Q^{2} + \nu^{2}}\), leads to the following. The mathematical part is to compute the \(n\)th \(\partial_{\perp}\) derivatives, \(q \cdot \partial = \nu\partial_{\perp}\), in Eqs. (7), (8) and (12) and (13) by making use of the equation of motion for the coupled meson and nucleon fields. Consequently the resulting operators involve the meson degrees of freedom. The matrix elements of the output operators over the deuteron state can be calculated by the Mandelstam method [10].

A representation of the moments \(M_{n}(F_{2}^{D})\) in terms of diagrams is shown on the Fig. 1, where the inner solid lines denote the nucleon propagators and the wavy lines denote the meson propagators. The closed triangle and square indicates the corresponding moments of the isolated-hadron SF. The moments of effective nucleon distributions, \(\mu_{n}^{N/D}\), are described by Fig. 1(a) and (b) and can be written down as

\[
\mu_{n}^{N/D} = \mu_{n}^{N/D}(\text{RIA}) + \mu_{n}^{N/D}(\text{int}), \tag{16}
\]
where we call the first term in Eq. (16) as the RIA with the on-mass-shell kinematics:

$$\mu_n^{N/D}(\text{RIA}) = -\frac{i}{P_+} \int d^4p \bar{\chi}(p; P)\gamma^+(p'; P)k \left(\frac{\bar{p}_+}{P_+}\right)^{n-1},$$

(17)

where $S^{-1}(p_2) = m - \hat{p}_2$ and the $+$-component of a vector defined as the sum of the time and longitudinal components originates from $p \cdot q \approx \nu p_+ + \tilde{p}$; $\tilde{p}$ is on-mass-shell momentum, $\tilde{p}^2 = m^2$.

The form of Eq. (17) is close to that of Ref. [11]. Although the nucleon of the upper-half circle at Figs. 1(a) and 1(b) is confined kinematically to the mass-shell, its propagation is described in a covariant way as given in Eq. (17).

The contribution $\mu_n^{N/D}(\text{int})$ in Eq. (16) is due to the strong interaction between two nucleons in the deuteron and is described by Fig. 1(b). It is obtained through the calculation procedure of the operators (9) in the Hamiltonian approach:

$$\mu_n^{N/D}(\text{int}) =$$

$$\frac{1}{P_+} \int d^4p d^4p' (2\pi)^4 \bar{\chi}(p; P)\gamma^+(p'; P)\gamma^0 V_{\text{OBEP}}(k) \left(\frac{\hat{p}_+ + \hat{p}'_+ + k_+}{2}\right)^{n-1} \bar{\chi}(p'; P) \left(\frac{\hat{p}_+ + \hat{p}'_+ + k_+}{2}\right)^{n-1},$$

(18)

where $k = p' - p$ and $\hat{p}_+, \hat{p}'_+$ are the on-mass-shell nucleon momenta.

The first two moments of $\mu_n^{N/D}$ have a physical interpretation. For $n = 1$ we find that $\mu_1^{N/D}(\text{int}) = 0$ and as a result the baryon number associated with the nucleons fields is preserved. The normalisation condition (3) implies that $\mu_1^{N/D}(\text{RIA}) = 1$. For $n = 2$ Eq. (18) contributes to the energy-momentum sum rule. The fraction of the total momentum of the deuteron carried by nucleons is

$$\frac{2m}{M} \langle z \rangle = \mu_2^{N/D}(\text{RIA}) + \mu_2^{N/D}(\text{int}),$$

(19)

3Below the normalisation constant $N$ is included in the definition of the BS amplitude.
where at the deuteron rest frame, \( P_+ = M \), we have

\[
\mu_2^{N/D} (\text{RIA}) = -\frac{i}{M^2} \int d^4p \tilde{\bar{p}}_+ \bar{\chi}(p; P)\gamma_+^{(1)} S^{-1}(p_2)\chi(p; P),
\]

\[
\mu_2^{N/D} (\text{int}) = -\frac{1}{M^2} \int \frac{d^4p d^4k}{(2\pi)^4} \bar{\chi}(p; P) \frac{[\gamma_+\gamma^0]^{(1)} S^{-1}(p_1) + S^{-1}(p_1)\gamma^0\gamma_+^{(1)}}{2} \chi(p + k; P).
\]

In order to estimate contribution given by Eq. (21) we can manipulate with the BSE (2) and find

\[
\mu_2^{N/D} (\text{int}) = -\frac{i}{M^2} \int d^4p \tilde{\bar{p}}_+ \bar{\chi}(p; P) \frac{[\gamma_+\gamma^0 S^{-1}(p_1) + S^{-1}(p_1)\gamma^0\gamma_+^{(1)}}{2} S^{-1}(p_2)\chi(p; P).
\]

The nucleon contribution to the deuteron momentum can be written as \( \langle z \rangle = 1 - \Delta \), where \( \Delta \) is the missing fraction of the deuteron momentum carried by exchange mesons. Notice that Eq. (22) suggests that the magnitude of the binding in the deuteron does not explicitly depend on the interaction kernel.

### 3.2 Meson exchange currents

The meson part of the nuclear SF is concentrated at small values of \( x \). We expect that the bulk of the relevant meson effects in deep inelastic scattering on the deuteron to be associated with pion exchange. Therefore, we consider the role of pions only in this section. Although other kinds of meson exchanges which appear in the OBE kernel of Eq. (2) are taken into account in this approach, it is impossible to estimate their contribution to deuteron SF. There is no data on the free SF’s for these mesons. We can also expect that contribution due to the scalar and vector exchange in the deuteron cancels, if both kinds are considered in the manner given by the operators in Eqs. (8) and (13).

In the impulse approximation, the contribution of the mesons according to Fig. 1(c) is summed up to (here we write down expression for \( \pi \)-mesons only):

\[
\mu_2^{\pi/D} = \int d^4k n_\pi(k; P) \frac{k_+}{\Omega_k} \left( \frac{k_+}{P_+} \right)^{n-1},
\]

where \( \Omega_k^2 = k^2 - \mu_\pi^2 \) and function \( n_\pi(k) \) can be interpreted as the excess pion 4-momentum distribution associated with the one-pion exchange in the deuteron

\[
n_\pi(k; P) = \frac{1}{P_+} \int \frac{d^4p}{(2\pi)^4} \tilde{\bar{p}}_+ \frac{\bar{\chi}(p; P)}{\Omega_k} \chi(p + k; P),
\]

where the one-pion exchange kernel \( \mathcal{V}_\pi \) appearing in Eq. (24) is used in solving the BSE (2).

The first two moments of \( \mu_2^{\pi/D} \) are interpreted as the mean number of the excess pions in deep inelastic scattering, \( \mu_1^{\pi/D} = \langle N_\pi \rangle \), and the missing fraction of the deuteron momentum carried by the excess pions, \( \mu_2^{\pi/D} = \langle y \rangle_\pi \), respectively. Since the twist-two operators from the set (b) for \( n = 2 \) are related to the energy-momentum tensor, the deuteron SF obeys the momentum sum rule (4.4):

\[
\int_0^1 F_2^D(x)dx = \frac{1}{P_0} \langle P|\Theta^{00} + \Theta^{33}|P \rangle = 1,
\]
where the components of the energy-momentum tensor $\Theta^{\mu\nu}$ correspond to those in Eq. (13). Eq. (27) ensures that the MEC carry away the missing part of the deuteron momentum.

### 3.3 Non-relativistic limit

In order to find the NR of the theory, though it may seem to be dubious, we need to take into account only the positive-energy states in the BS amplitude. The negative-energy components do not survive in the limit $|\mathbf{p}|/m \ll 1$.

We write down the BS amplitudes in the rest frame of the deuteron as decomposition in terms of the product of Dirac spinor of nucleons:

$$\chi(p; P_{(0)}) = u(p)u(-p)\{\mathcal{Y}_{L=1}^{01}(\hat{\mathbf{p}})u(p_0, |\mathbf{p}|) + \mathcal{Y}_{L=1}^{21}(\hat{\mathbf{p}})w(p_0, |\mathbf{p}|)\}, \quad (26)$$

where $u(p)$ is the positive-energy Dirac spinor. We rewrite Eq. (26) equivalently as

$$\chi(p; P_{(0)}) = u(p)u(-p)\Psi(p_0, \mathbf{p}), \quad (27)$$

$$\tilde{\chi}(p; P_{(0)}) = -\bar{u}(p)\bar{u}(-p)\Psi^*(-p_0, \mathbf{p}),$$

where the radial wave functions $u(p_0, |\mathbf{p}|)$ and $w(p_0, |\mathbf{p}|)$ can be expressed in terms of the BS vertex functions $\Gamma_L$ with $L = 0, 2$ as follows ($E_p = \sqrt{\mathbf{p}^2 + m^2}$)

$$u(p_0, |\mathbf{p}|) = \frac{1}{\sqrt{N}} \frac{\Gamma_0(0, |\mathbf{p}|)\xi_0(p_0, |\mathbf{p}|)}{[(\frac{M}{2} - E_p)^2 - p_0^2]}, \quad w(p_0, |\mathbf{p}|) = \frac{1}{\sqrt{N}} \frac{\Gamma_2(0, |\mathbf{p}|)\xi_2(p_0, |\mathbf{p}|)}{[(\frac{M}{2} - E_p)^2 - p_0^2]} \quad (28)$$

with a smooth function $\xi_L(p_0, |\mathbf{p}|)$ reflecting the dependence on the relative energy $p_0$ of $\Gamma_L$.

Let us consider Eq. (17) for the positive-energy components only. We find that

$$\mu_{n/D}^{N/D} (\text{RIA}) = \frac{i}{M} \int d^4p \left| \Psi(p) \right|^2 (1 + \frac{p_2}{E_p})(E_p - p_0^0)(\frac{E_p + p_2}{M})^{n-1}, \quad (29)$$

where $\tilde{\mu}_{n/D}^L = (E_p, \mathbf{p})$ and $p_2^{\mu} = (M/2 - p_0, -\mathbf{p})$. Our next step is to integrate over the relative energy picking up the pole in the 4-quarter:

$$\int_{-\infty}^{+\infty} \frac{dp_0}{2\pi i(E_p - p_0 + i\epsilon)(\frac{M}{2} - E_p + p_0 + i\epsilon)^2} = \frac{\xi_L(p)\xi_L^*(p)}{(\frac{M}{2} - E_p - p_0)^2} \bigg|_{p_0 = E_p - \frac{M}{2}}, \quad (30)$$

Keeping in mind Eq. (30) we reduce Eq. (29) to the form

$$\mu_{n/D}^{N/D} (\text{IA}) = \int d\mathbf{p} n(\mathbf{p}) (1 + \frac{p_2}{E_p})(\frac{E_p + p_2}{M})^{n-1}, \quad (31)$$

with the nucleon density $n(\mathbf{p}) = |\Psi|^2$ depending on radial wave functions of the $L = 0, 2$ angular momentum states ($\xi \approx 1$)

$$\sqrt{2\pi} \frac{\Gamma_L(0, |\mathbf{p}|)}{N \sqrt{M(2E_p - M)}}, \quad (32)$$
which in the NR limit are usually identified with the conventional wave functions in $^3S_1$- and $^3D_1$-states. Eq. (31) is the impulse approximation with a hit nucleon on its mass-shell.

The other important quantity is $\langle z \rangle$, because its magnitude is well-known from NR studies. First of all we integrate over $p_0$ in Eq. (22) in the same manner as done above. We get

$$\mu^{N/D}_{(\text{int})} = \frac{1}{M} \int d\mathbf{p} n(\mathbf{p})(M - 2E_\mathbf{p}) = \frac{2}{M} (M - 2\langle E_\mathbf{p} \rangle).$$

(33)

Using Eq. (31) for $n = 2$ we obtain

$$\mu^{N/D}_{(\text{RIA})} = \frac{2}{M} (\langle E_\mathbf{p} \rangle + \langle p_z^2 \rangle / E_\mathbf{p}).$$

(34)

It is clear that the proton and neutron in the deuteron carry the fraction of the total momentum of the deuteron given by

$$\langle z \rangle = \frac{1}{m} (M - \langle E_\mathbf{p} \rangle + \langle p_z^2 \rangle / E_\mathbf{p}) \approx 1 + \frac{5}{6m} \langle T \rangle_{\text{IA}} + m \langle V \rangle_{\text{int}},$$

(35)

where $\langle T \rangle = \langle p_z^2 \rangle / m$ is the kinetic energy, $\langle V \rangle = -\langle T \rangle + \varepsilon_d$ is the potential energy of two nucleons in the deuteron and $\varepsilon_d = M - 2m$.

4 Results and discussions

Owing to Eq. (6) the SF $F_D^2(x, Q^2)$ is written as a sum of convolutions of the SF of the unbound nucleon and pion and the momentum distribution functions of the nucleons and pions inside the deuteron (we assume that the SF $F_{N,\pi}^2$ do not carry any dependence on the relative 4-momentum). The contribution of the interaction corrections to the RIA given by Eq. (18) can be approximated in a very plain form yielding

$$F_D^2(x, Q^2) = F_{N/D}^2(x, Q^2) - \frac{\langle V \rangle}{m} \frac{\partial}{\partial \ln x} F_{N}^2(x, Q^2) + \delta F_{\pi/D}^2(x, Q^2),$$

(36)

where $\langle V \rangle = m\mu^{N/D}_{(\text{int})}$ and

$$F_{N/D}^2(x, Q^2) = \int_{z \geq x} f_{N/D}^2(z) F_{N}^2(x/z, Q^2) dz,$$

(37)

$$\delta F_{\pi/D}^2(x, Q^2) = \int_{y \geq x} f_{\pi/D}^2(y) F_{\pi}^2(x/y, Q^2) dy.$$

(38)

The distribution functions $f_{N/D}^2(z)$ and $f_{\pi/D}^2(y)$ describe the momentum distributions of the on-mass-shell nucleons and off-mass-shell pions carrying the fractional longitudinal momenta of the deuteron $y$ and $z$, respectively. We have at the rest frame:

$$f_{N/D}^2(z) = \frac{1}{M} \sum_\alpha \int d^4p |\phi_\alpha(p; P(0))|^2 \left(1 + \frac{p_z}{E_\mathbf{p}}\right) \delta \left(z - \frac{p_z}{M}\right),$$

(39)

$$f_{\pi/D}^2(y) = \int d^4k n_\pi(k; P(0)) \frac{k_z}{M} \delta \left(y - \frac{k_z}{M}\right).$$

(40)
where \( \tilde{p} = (\rho E_p, \mathbf{p}) \) is the momentum of the struck nucleon in the partial state \( \alpha = 2S + 1L\); \( \phi_\alpha \) denotes a partial amplitude.

Both functions \( f^{N/D}(z) \) and \( f^{\pi/D}(y) \) specify the number densities of the nucleons and excess pions and satisfy the normalisation integrals

\[
\int f^{N/D}(z) dz = 2, \quad \langle N_\pi \rangle = \int f^{\pi/D}(y) dy. \tag{41}
\]

The SF \( F_2 \) probes the momentum distribution of the constituents and thus conservation of the energy-momentum requires that

\[
\int f^{N/D}(z) zdz + \int f^{\pi/D}(y) ydy + \int f^{B/D}(y) ydy = 1, \tag{42}
\]

where \( f^{B/D}(y) \) the momentum distributions of some other mesons in \( V_{\text{OBEP}} \).

For the sake of a simple analytical analysis of the nucleon density in Eq. (39) and leaving no doubts of accuracy of theoretical predictions, we employ the BS vertex function which is the solution of the homogeneous BSE for two spin-\( \frac{1}{2} \) particles interacting through a covariant, separable potential \cite{12}. The relevance of such an approximation to the expressions with the original BS vertex function of the OBE potential could be justified. Indeed the final formulae for \( \mu_2^{N/D}(\text{int}), \langle z \rangle \) and \( \langle N_B \rangle \) (v. i.) do not contain the OBE potential. On the other hand discarding of the negative-energy components in the vertex function is also justified by the conclusion in Ref. \cite{12}.

The magnitude of the binding in the deuteron can be calculated in a straightforward way by making use of Eq. (22). Computation of \( \langle z \rangle \) in the energy-momentum sum rule gives: the on-mass-shell nucleons carry \( \langle z \rangle_{\text{RIA}} = 1.0153 \) which breaks down the sum rule \cite{12}. The interaction term \cite{18} takes away a fraction of the longitudinal momentum, \( \mu_2(\text{int}) = -0.0188 \), and reduces \( \langle z \rangle_{\text{RIA}} \) yielding the final value, \( \langle z \rangle = 0.9965 \).

Unfortunately the total contribution of the mesons cannot be calculated explicitly without running into formidable computation. Yet we find by means of Eq. (42) that the fraction of the momentum carried by the exchange mesons is \( \Delta = 0.0035 \). The first moment \( \mu_1^{B/D} \) is the mean number of the exchange mesons associated with the reaction. The exact expression being similar to Eq. (23) can be approximated by the formula

\[
\langle N_B \rangle \approx \frac{i}{2P_+} \int d^4p \bar{\chi}(p; P) \frac{\partial}{\partial p_-} S^{-1}(p_1, p_2) \chi(p; P), \tag{43}
\]

where \( p_- = p_0 - p_z \). It is expected that \( \langle N_B \rangle \) is totally accounted for by the pions, since the part due to heavier meson exchanges is suppressed in average by their larger masses in the propagator \( \Omega_k^2 \) and their effective couplings. If we pretend not to take interest in a detail structure of the \( f^{\pi/D}(y) \) that emerges from Eq. (40), we can tentatively estimate the momentum distribution of the pions for not sufficiently large values of \( z \) by exploiting results of Ref. \cite{13}.

The quantitative analysis shows that results are not sensitive to the precise functional shape of \( f^{\pi/D}(y) \) but controlled mainly by \( \langle N_\pi \rangle \) and \( \langle y \rangle \). Thus \( f^{\pi/D}(y) \) could be parametrised imposing the balance between \( \langle N_\pi \rangle \) and \( \langle y \rangle \) in the simplest manner.

10
Figure 2: The ratio of the deuteron and isoscalar nucleon structure functions. Solid curves: curve 1 denotes the relativistic impulse approximation (RIA) with on-mass-shell kinematics; curve 2 denotes the RIA with taking into account the nuclear binding; curve 3 denotes the sum contribution of the RIA, the binding and meson exchange currents. The dashed curve corresponds to the solid curve 3 with a different parametrisation of $F^2_N(x, Q^2)$. Experimental data are taken from Ref. [14]. The error bars are combined statistical and systematic errors.

4.1 Comparison with data

Now we are in a position to perform numerical analysis. Fig. 2 shows the EMC effect in the deuteron at $Q^2 = 25$ GeV$^2$ given by the ratio $R(x, Q^2) = F^2_D(x, Q^2)/F^2_N(x, Q^2)$ in the present approach. All computations of the nucleon momentum distribution $f^{N/D}(z)$ and $\mu^2(int)$ are carried out using the BS vertex function in the framework with multi-rank separable interaction [12]. The free nucleon $F^2_N = (F^p_N + F^n_N)/2$ and pion $F^\pi_2(x, Q \approx 25$ GeV$^2)$ SF’s obtained from the combined proton and deuteron data of the BCDMS collaboration [15] and data on massive lepton pair production of the CERN NA10 collaboration [16], respectively, are taken from Refs. [5] and [13].

At small $x$, $x \leq 0.2$, the MEC increase the deuteron SF relative to the nucleon one. At exactly $x = 0$ the ratio has maximum, $R(0) = 1 + \langle N_\pi \rangle F^\pi_2(0)/F^2_N(0)$, where $\langle N_\pi \rangle \approx 0.015$, and falls off roughly linearly as $x$ grows. The value of $F^\pi_2(0)/F^2_N(0)$ is known from the parametrisation of the SF’s. As can be seen, the nucleon contribution $F^{N/D}(x, Q^2)$ to $R(x, Q^2)$ is not important in this range of $x$.

At intermediate $x$, $0.2 < x < 0.6$, the MEC are negligible as long as $f^{\pi/D}(y)$ vanishes at $y \to 1$, but the correction due to strong interaction between nucleons, v. s. Eq. (36), becomes responsible for the dip. This depletion is totally controlled by the value of $\mu^2(int)$. In a more realistic case we still may have contribution from deep inelastic scattering on pions in this region, because the argument $x/y$ of $F^2_\pi$ in the convolution formula, see Eq. (38), will run over broader range in $f^{\pi/D}(y)$.

The rise of the ratio $R(x)$ at large $x$, $x > 0.6$, is associated with the relativistic Fermi motion. The tendency is that the harder “tail” of $F^2_D(x)$ in the vicinity of the boundary for the single-nucleon kinematics, the harder the momentum distribution $f^{N/D}(z)$ in Eq. (37).

Unfortunately pertinent data on the ratio $R(x, Q^2)$ are not still available. However in Ref. [17] it is suggested that the A-dependence of the $F^A_2(x)$ should be determined by the local
properties of nuclear matter, i.e. the average nuclear density $\rho(A)$. The naive extrapolation to the deuteron case leads to $R(x, Q^2) \approx 1 + (F_A^2/F_D^2 - 1)/[(\rho(A)/\rho(D) - 1)$, although this (may) overestimate the effect due to the isoscalarity of the deuteron. The model-dependent value of the ratio $F_D^2/F_N^2$ estimated from averaging over measured data on various nuclei $F_A^2/F_D^2$ in SLAC-E139 experiment is presented on Fig. 2. Within this hypothesis the deuteron has a significant EMC effect which contradicts to our theoretical predictions together with the parametrisation of the neutron SF [5]: a deviation of $R(x, Q^2)$ from unity at intermediate $x$ is extremely large, about 4% versus the theoretically predicted 2%, and centers of data points continue to decrease up to $x = 0.7$ which is the feature of data for heavy nuclei.

Our computation analysis shows that $R(x, Q^2)$ does not exhibit substantial dependence on $Q^2$ in the broad range and on a specific parametrisation of the neutron SF (both effects are of the second order relative to the binding). The ratio is more sensitive to the behaviour of the free nucleon SF as $x \to 1$, since this region is enhanced by the nucleon momentum distribution.

5 Concluding remarks

In this paper the deuteron SF $F_2(x, Q^2)$ is examined in the theoretical approach based on a picture in which nucleons and mesons are the relevant degrees of freedom. The deuteron is treated as two spin-$\frac{1}{2}$ particle bound state through a consideration of the Bethe-Salpeter equation in the ladder approximation. In the model the deuteron SF given by Eq. (36) is obtained as a sum of three terms. The first term known as the relativistic impulse approximation is the convolution of the the free nucleon SF with the fractional momentum distribution function of the nucleons projected to their mass-shell. The second term in Eq. (36) accounts for the binding correction to the relativistic impulse approximation. Similarly to the first term the last piece is the convolution of the free pion SF with the excess pion fractional momentum distribution in the deuteron. The momentum distributions of the deuteron constituents are derived from the matrix elements of certain twist-two operators in the operator product expansion method and expressed in terms of the BS vertex function.

The principal result of the paper is that the binding in the deuteron in the approach incorporating the relativistic dynamics in terms of the conventional degrees of freedom could be revealed in an unambiguous way. The relativistic impulse approximation is obtained in the form that avoids the explicit dependence of the momentum distribution on the relative energy of nucleons and thus we are able to present accurate estimates in a manner similar to the non-relativistic treatment of Ref. [4].

The magnitude of the nuclear binding and Fermi motion correction together with a neutron SF $F_2^N$ shape $x$-dependence of the EMC ratio for the deuteron. High precision measurements of the nucleon SF at large $x$ combined with a reliable theoretical treatment of the deuteron may help to resolve the question of the extraction of the neutron SF from experimental data on a deuterium target and a heavy nuclei target [18].

The open questions the model suggests are that an additional work to be done in order to establish connection between the off-shell and on-shell SF $F_2^N$ in this framework; next to what extent the negative-energy states in the BS vertex function impact on the momentum distributions of the nucleons; and finally the exact functional shape of the momentum distribution of the excess pions associated with the one pion exchange in the deuteron is not of the least
interest.

To sum up, we can conclude that the point of view developed in this paper might help to discriminate the contribution of the pure binding effects in the nuclear SF and regard corrections due to the relativistic Fermi motion in a transparent way.

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