Reduced-Dimension DOA and Polarization Parameters Joint Estimation Method for Electromagnetic Vector Sensor Array

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Abstract. MUSIC algorithm is a very typical algorithm in the joint estimation of DOA and polarization parameters. But the two spectral peak searches of two-dimensional parameter space in the algorithm cause a damaging effect on calculation, which limits the application of the algorithm in practice. Therefore, this paper proposes a joint estimation algorithm of dimensionality reduction DOA and polarization parameters based on successive MUSIC in the case of uniform linear array arrangement of the electromagnetic vector sensor. The proposed algorithm improves the classical MUSIC algorithm from two two-dimensional parameter searches to four one-dimensional parameter searches in the joint estimation of DOA and polarization parameters, which greatly reduces the computational complexity of the algorithm and can lead to the automatic matching of DOA and polarization parameters. Simulation results show that the computational complexity of the proposed algorithm is reduced by 10^4 orders of magnitude compared to the classical MUSIC algorithm. And the performance of the polarization parameter estimation of this algorithm is about 72%~85% higher than that of the ESPRIT algorithm. Therefore, the algorithm is more applicable.

1. Introduction
In the field of joint estimation of DOA and polarization parameters, scholars have proposed many effective algorithms, including estimation of signal parameters via rotational invariance technique (ESPRIT) algorithm, multiple signal classification (MUSIC) algorithm, cumulant algorithm, cyclostationarity-based algorithm, as well as others [1-4]. In addition, literature [5] obtains an initial estimation of DOA, then estimates more accurate DOA through a one-dimensional (1-D) local searching according to the initial estimation of DOA, and finally obtains polarization parameter estimation via the estimated polarization steering vectors. Literature [6] built a half-quaternions model to remove the redundant information. Then, the directions of arrival (DOAs) are estimated via the root-MUSIC algorithm, and the polarizations were estimated by generalized eigenvalue method. In literature [7], a fourth-order cumulant-based ESPRIT algorithm is proposed, for joint estimation of DOA and polarization based on a uniform circular electromagnetic vector sensor array. Literature [8-10] studies the DOA and polarization parameters estimation under sparse arrays. Compared with traditional full arrays, sparse arrays expand the array aperture and reduce the mutual coupling effect of array elements. Improved DOA and polarization parameter estimation methods are proposed under the array to improve measurement accuracy and algorithm complexity.

It can be seen from previous studies that MUSIC algorithm and ESPRIT algorithm are the most commonly used joint estimation methods of DOA and polarization parameters, but the classic MUSIC algorithm has a problem of too much computation, and the ESPRIT algorithm has poor accuracy in...
polarization parameter estimation. In order to avoid these problems, this paper proposes a joint estimation method for reduced-dimensional DOA and polarization parameters based on electromagnetic vector sensor array. The four-dimensional search is optimized to four one-dimensional searches, which greatly reduces the algorithm’s the complexity.

2. Signal Model

Supposing that $K$ non-coherent narrowband signals are incident on the line array, the $k$-th source has a pitch angle of $\theta_k$, an azimuth angle of $\phi_k$, a polarization angle of $\gamma_k$, and a polarization phase difference of $\eta_k$, and $0 \leq \theta_k \leq \pi$, $-\pi \leq \phi_k \leq \pi$, $0 \leq \gamma_k \leq \pi/2$, $-\pi \leq \eta_k \leq \pi$. Considering the additive noise independent of the signal source statistics, the array output signal vector is represented as

$$X(n) = \sum_{i=1}^{N} (b(\theta_i, \phi_i) \otimes \Omega(\theta_i, \phi_i, \gamma_i, \eta_i)) s_i(n) + N(n) = AS(n) + N(n)$$

(1)

where $b(\theta_i, \phi_i) \triangleq [1, u_i, u_i^2, \ldots, u_i^{M-1}]$ is the signal $M \times 1$-dimensional spatial domain steering vector, with $u_i = \exp(2\pi d \sin \theta_i \sin \phi_i)$. And $\Omega(\theta_i, \phi_i, \gamma_i, \eta_i)$ is the signal $6 \times 1$-dimensional polarization-angle domain steering vector,

$$\Omega(\theta_i, \phi_i, \gamma_i, \eta_i) \triangleq a(\theta_i, \phi_i) \cdot e(\gamma_i, \eta_i) = \begin{bmatrix} -\sin \phi_i \cos \gamma_i + \cos \theta_i \cos \phi_i \sin \gamma_i e^{in} \\ \cos \phi_i \cos \gamma_i + \cos \theta_i \sin \phi_i \sin \gamma_i e^{in} \\ -\sin \theta_i \sin \gamma_i e^{in} \\ -\sin \phi_i \sin \gamma_i e^{in} - \cos \theta_i \cos \phi_i \cos \gamma_i \\ \cos \phi_i \sin \gamma_i e^{in} - \cos \theta_i \sin \phi_i \cos \gamma_i \\ \sin \theta_i \cos \gamma_i \end{bmatrix}$$

(2)

where

$$a(\theta_i, \phi_i) = \begin{bmatrix} \cos \theta_i \cos \phi_i & -\sin \phi_i \\ \cos \theta_i \sin \phi_i & \cos \phi_i \\ -\sin \theta_i & 0 \\ -\sin \phi_i & -\cos \theta_i \cos \phi_i \\ \cos \phi_i & -\cos \theta_i \sin \phi_i \\ 0 & \sin \theta_i \end{bmatrix}$$

and

$$e(\gamma_i, \eta_i) = \begin{bmatrix} \sin \gamma_i e^{in} \\ \cos \gamma_i \end{bmatrix}.$$  

$A$ is an $6M \times K$-dimensional polarization-angle domain mixed steering vector matrix, $A = [b_1 \otimes \Omega_1, b_2 \otimes \Omega_2, \ldots, b_K \otimes \Omega_K]$. $S(n)$ is the signal vector matrix, $S(n) = [s_1(n), s_2(n), \ldots, s_K(n)]^T$. $N(n)$ is the array noise vector, respectively.

The autocorrelation matrix of the received signal vector can be expressed as

$$R = E[X(n)X^H(n)]$$

(3)

The autocorrelation matrix can be estimated from $N$ observation samples, i.e.,

$$\hat{R} = \frac{1}{N} \sum_{i=1}^{N} X(i)X^H(i)$$

(4)

Using the estimation matrix $\hat{R}$ of the correlation matrix instead of $R$ for eigen decomposition can be obtained as

$$\hat{R} = E\hat{D}_{e}E_{\theta}^H + E\hat{D}_{n}E_{\theta}^H$$

(5)

where $D_{\theta}$ is a diagonal matrix consisting of $K$ largest eigenvalues, $E_{\theta}$ is a signal subspace, consisting of eigenvectors corresponding to the $K$ largest eigenvalues; $D_{n}$ is also a diagonal matrix, consisting of smaller $6M - K$ eigenvalues, $E_{n}$ is a noise subspace composed of the feature vectors corresponding to the smaller $6M - K$ eigenvalues, respectively.
3. MUSIC algorithm for joint estimation of two-dimensional DOA and polarization parameters

According to the above model, a four-dimensional scanning function jointly estimated by DOA and polarization parameters can be constructed according to the principle of MUSIC algorithm, i.e.,

\[ f_{\text{MUSIC}}(\phi, \theta, \gamma, \eta) = \frac{1}{A^\dagger(\phi, \theta, \gamma, \eta)E_nE_n^\dagger A(\phi, \theta, \gamma, \eta)} \]

As a result of the four-dimensional spectral peak search in the spatial domain and the polarization domain parameters is very complicated, the four-dimensional search needs to be reduced to two two-dimensional searches in the spatial domain and the polarization domain, respectively.

In literature [11], it is analyzed in detail how to use the matrix rank-loss principle to reduce the four-dimensional spectral peak search into two two-dimensional search. According to the principle of rank-loss MUSIC algorithm, the incident signal received by the electromagnetic vector array does not need to consider the polarization parameters when constructing the spatial spectrum. Therefore, in the peak search, it is still only a two-dimensional search, and only the values of the azimuth and pitch angles are searched. After estimating the values of the azimuth and pitch angles through a two-dimensional search, then through the spectral function of the MUSIC algorithm jointly estimated by DOA and polarization parameters, the values of the polarization angle and the polarization phase difference can be estimated by a two-dimensional search.

Therefore, the two-dimensional spatial spectral function can be expressed as

\[ f_{\text{MUSIC}}(\phi, \theta) = \frac{1}{[b(\phi, \theta) \otimes a(\phi, \theta)]^H E_nE_n^\dagger [b(\phi, \theta) \otimes a(\phi, \theta)]} \]

In the interval \( \theta \in [0, \pi] \) and \( \phi \in [-\pi, \pi] \), the peak search is performed on the above formula, and the \( K \) largest peaks are the estimated values \( \hat{\theta} \) and \( \hat{\phi} \) of the pitch and azimuth of the source, respectively.

4. Joint estimation of two-dimensional DOA and polarization parameters based on successive MUSIC

In this section, we demonstrate the specific steps of the algorithm proposed in this paper. We first perform initial estimation parameter based on the single electromagnetic vector receiver, so that one initial estimation parameter is determined in the two-dimensional search and the other parameter is searched in a smaller range.

Then, we transform the two-dimensional search into four one-dimensional search. Finally, we compare the complexity of the algorithm with the traditional MUSIC algorithm.

4.1. Initial estimate

The signal \( x(t) \) obtained by the single electromagnetic vector sensor and the signal \( x(t + \tau) \) obtained after the delay, then the output vector \( Z(t) \) can be expressed as

\[ Z(t) = \begin{bmatrix} x(t) \\ x(t + \tau) \end{bmatrix} \]

where \( x(t) = \sum_{k=1}^{K} \Omega(\theta_k, \phi_k, \gamma_k, \eta_k) \cdot s_k(t) + N(t) = \Omega(\theta, \phi, \gamma, \eta)S(t) + N(t) \). Then

\[ Z(t) = \begin{bmatrix} x(t) \\ x(t + \tau) \end{bmatrix} \begin{bmatrix} \Omega \\ \Omega \Phi \end{bmatrix} S(t) + \begin{bmatrix} N_x(t) \\ N_x(t) \end{bmatrix} = \tilde{A}S(t) + N_x(t) \]

where \( \Phi \) is a diagonal matrix of \( K \times K \)-dimensional, \( \Phi = \text{diag}\{e^{i2\pi f_1\tau}, e^{i2\pi f_2\tau}, ..., e^{i2\pi f_{K}\tau} \} \).

From \( N \) observation samples, the autocorrelation matrix \( Z(t) \) can be expressed as

\[ \hat{R}_z = \frac{1}{N} \sum_{t=1}^{N} Z(t)Z^\dagger(t) \]

And there must be a unique non-singular matrix \( T \), then the signal subspace \( U_s \) of \( Z(t) \)'s autocorrelation matrix \( \hat{R}_z \) can be written as
Let $\Psi = T^{-1} \Phi T$, so $\Psi$ and $\Phi$ have the same eigenvalue and have $U_k = U_i \Psi$.
Then the flow vector of a single electromagnetic vector sensor can be estimated as
$$\hat{U}_k = \left[ U_i \right] = \left[ \Omega T \right]$$

(11)

From matrix $\hat{\Omega}$ we can obtain $\hat{P}$ by
$$\hat{P} = \left[ \hat{\omega}_i (1:3), \hat{\omega}_i, \hat{\omega}_j, \ldots, \hat{\omega}_k \right] = U_i \Omega \Phi T^{\dagger}$$

(12)

Then the initial value estimate $\hat{\theta}_m_i = \arccos\left( \hat{\omega}_i \right)$ and $\hat{\theta}_m = \arctan\left( \hat{\theta}_i / \hat{\theta}_j \right)$ can be estimated. So the initial value $\hat{\theta}_m = \arctan\left( \hat{\omega}_i (1) / \hat{\omega}_i (2) \right)$ and $\hat{\theta}_m = \arctan\left( \hat{\omega}_i (1) \right)$ can be estimated, respectively.

4.2. Successive MUSIC algorithm

In order to make a more accurate estimation of the DOA information pitch angle $\theta_i$ and azimuth $\phi_i$, and convert the two-dimensional search into a one-dimensional search, the initial estimate $\hat{\theta}_m$ is brought into the one-dimensional MUSIC spatial spectrum estimation of $\hat{\theta}$. Then
$$\hat{\theta} = \arg \max_{\hat{\theta} \notin \left[ \hat{\theta}_m - \Delta \theta, \hat{\theta}_m + \Delta \theta \right]} \left\{ \left[ b(\hat{\theta}, \hat{\phi}, a(\hat{\theta}, \hat{\phi})) \right]^H E_n^H b(\hat{\theta}, \hat{\phi}, a(\hat{\theta}, \hat{\phi})) \right\}$$

(13)

By searching for $\hat{\theta}$ in the interval $\left[ \hat{\theta}_m - \Delta \theta, \hat{\theta}_m + \Delta \theta \right]$, a more accurate $\hat{\theta}$ is obtained. Similarly, $\phi$ is searched for within a smaller interval $[\hat{\phi}_m - \Delta \phi, \hat{\phi}_m + \Delta \phi]$, and $\hat{\phi}$ is estimated with a one-dimensional MUSIC spatial spectrum. $\hat{\phi}$ can be expressed as
$$\hat{\phi} = \arg \max_{\hat{\phi} \notin \left[ \hat{\phi}_m - \Delta \phi, \hat{\phi}_m + \Delta \phi \right]} \left\{ \left[ b(\hat{\phi}, \hat{\theta}, a(\hat{\phi}, \hat{\theta})) \right]^H E_n^H b(\hat{\phi}, \hat{\theta}, a(\hat{\phi}, \hat{\theta})) \right\}$$

(14)

After estimating the more accurate $\hat{\theta}$ and $\hat{\phi}$, bring them into two one-dimensional MUSIC spatial spectral estimates of the polarization amplitude $\gamma$ and the polarization phase difference $\eta$, respectively.

Then we bring the initial estimate $\hat{\gamma}_m$ into the one-dimensional spatial spectrum estimate of $\hat{\gamma}$, and search for $\gamma$ in the interval $\left[ \hat{\gamma}_m - \Delta \gamma, \hat{\gamma}_m + \Delta \gamma \right]$. Then the more accurate $\hat{\gamma}$ can be written as
$$\hat{\gamma} = \arg \max_{\hat{\gamma} \notin \left[ \hat{\gamma}_m - \Delta \gamma, \hat{\gamma}_m + \Delta \gamma \right]} \left\{ \left[ b(\hat{\phi}, \hat{\theta}, \hat{\gamma}, \hat{\eta}) \right] \right\}^H E_n^H b(\hat{\phi}, \hat{\theta}, \hat{\gamma}, \hat{\eta})$$

(15)

Similarly, there is
$$\hat{\eta} = \arg \max_{\hat{\eta} \notin \left[ \hat{\gamma}_m - \Delta \gamma, \hat{\gamma}_m + \Delta \gamma \right]} \left\{ \left[ b(\hat{\phi}, \hat{\theta}, \hat{\gamma}, \hat{\eta}) \right] \right\}^H E_n^H b(\hat{\phi}, \hat{\theta}, \hat{\gamma}, \hat{\eta})$$

(16)

The main steps of the joint estimation algorithm for DOA and polarization parameters based on successive MUSIC are as follows:

**Step 1**: Estimate the autocorrelation matrix $\hat{R}_i$ of the signal received by the polarization sensitive array, and perform eigenvalue decomposition to obtain the noise subspace $E_n$. The computational complexity of this step is $O(|6M^2| + N + (6M)^2)$.

**Step 2**: Construct a new signal from the signal received by the single electromagnetic vector sensor and its delay signal, estimate its autocorrelation matrix $\hat{R}_e$, decompose it into eigenvalues to obtain
signal subspace $U_s$, and decompose $U_s$ into $U_i \cup U_z$ according to the formula, the parameter estimation matrix $\hat{P}$ is obtained, thereby obtaining initial estimates $\hat{\phi}_m$, $\hat{\phi}_i$, $\hat{\gamma}_m$, respectively. The computational complexity of this step is $O\{12^2 \times N + 12^3 + 6K + K^3\}$.

**Step 3:** Fix $\hat{\phi}_m$, according to the formula (14), obtain a more accurate estimate $\hat{\Theta}$ by the one-dimensional MUSIC algorithm, search for the number of steps $p_1$. Then $\hat{\Theta}$ will be fixed, and a more accurate estimate $\hat{\phi}$ will be obtained by the one-dimensional MUSIC algorithm according to the formula (15), and the number of search steps is $p_2$. The computational complexity of this step is $O\{(6M)^2 \times (6M - K) + K \times (6M)^3 + 6K \times 6M \times (p_1 + p_2)\}$.

**Step 4:** Bring the obtained $\hat{\Theta}$ and $\hat{\phi}$ into the estimation of the polarization parameter. Similar to step 3, two one-dimensional MUSIC search can be used to obtain more accurate estimates of the polarization parameters $\hat{\gamma}$ and $\hat{\eta}$ according to the formula (16) and formula (17). The number of steps in the two searches is $p_3$ and $p_4$, respectively. The computational complexity of this step is $O\{(6M)^3 \times (6M - K) + (6M)^2 \times (6M)^2 + (6M)^2 + 6M \times (p_3 + p_4)\}$.

It can also be seen from the above analysis that the proposed algorithm can implement automatic matching of two-dimensional DOA parameters and polarization parameters.

### 4.3. Algorithm complexity analysis

The traditional MUSIC algorithm based DOA and polarization parameters joint estimation, carried out two two-dimensional search, the algorithm complexity is mainly

$$O\{N \times (6M)^3 + (6M)^3 + ((6M)^3 + K \times (6M)^3) \times q_1 \times q_2 + (6M)^2 \times (6M - K) + (6M)^2 + 6M \times q_1 \times q_2\}$$

Where $q_1$, $q_2$ represent steps number of the two-dimensional search in the global range in the DOA estimation, and $q_3$, $q_4$ represent steps number of the two-dimensional search in the global range in the polarization parameter estimation, and $q_1 \gg p_1$, $q_2 \gg p_2$, $q_3 \gg p_3$, $q_4 \gg p_4$, respectively.

Compare the complexity of the two algorithms under different parameters, the results are shown in Figure 1.

![Figure 1](image-url)

(a) Comparison of the complexity of the algorithm and MUSIC algorithm under different parameters. (a) Comparison of the complexity of the proposed algorithm and MUSIC algorithm under different electromagnetic vector sensors numbers $M$; (b) Comparison of the complexity of the algorithm and MUSIC algorithm under different snapshots $N$.

It can be clearly seen from the results in Figure 1 that the complexity of the proposed algorithm is much lower than the classical MUSIC algorithm when performing estimation of DOA and polarization parameters. And the computational complexity of the proposed algorithm is reduced by $10^4$ orders of magnitude compared to the classical MUSIC algorithm.
5. Algorithm performance

The following is a simulation experiment using the successive MUSIC algorithm to implement the joint estimation of signal DOA and polarization parameters. Present a hypothesis there are two narrow-band signals that illuminate a polarization-sensitive array uniformly arranged by \( M \) array elements. The true DOA and polarization parameters are \( \phi_1 = 45^\circ, \ \theta_1 = 50^\circ, \ \gamma = 35^\circ, \ \eta_1 = 60^\circ, \ \phi_2 = 135^\circ, \ \theta_2 = 150^\circ, \ \gamma_2 = 60^\circ, \ \eta_2 = -120^\circ \), respectively. Define the root mean square error as

\[
\sigma_{RMSE} = \sqrt{\frac{1}{TK} \sum_{t=1}^{T} \sum_{k=1}^{K} (\hat{\omega}_{k,t} - \omega_k)^2}
\]

where \( T \) and \( K \) represent the number of simulations and the number of sources, respectively; \( \hat{\omega}_{k,t} \) represents the estimated value of the parameters (pitch angle, azimuth, polarization or polarization phase difference) of the \( k \) th source in the \( t \) th simulation; \( \omega_k \) represents the actual value of the parameter (pitch angle, azimuth angle, polarization amplitude or polarization phase difference) of the \( k \) th source.

In Figure 2, we perform 200 Monte Carlo simulations with a signal-to-noise ratio of 10 dB. It can be seen from the results in the figure that the proposed algorithm can effectively estimate the DOA and polarization parameters of the two sources.

![Figure 2](image.png)

Figure 2. The scatter diagram of the proposed algorithm for joint estimation of DOA and polarization parameters (\( N = 700; M = 8; K = 2 \)); (a) DOA estimation, (b) Polarization parameter estimation.

It can be seen from Figure 2 that the algorithm can effectively estimate the DOA and polarization parameters of the two sources when the signal-to-noise ratio is 10 dB.

Figure 3 compares the performance of the proposed algorithm and the traditional MUSIC algorithm and ESPRIT algorithm for each parameter. It can be seen from the results in the figure that the performance of the proposed algorithm is not as good as the MUSIC algorithm, but it is obviously better than the ESPRIT algorithm. And the performance of the polarization parameter estimation of this algorithm is about 72%~85% higher than that of the ESPRIT algorithm.
6. Conclusion

In this paper, a joint estimation algorithm based on successive MUSIC for dimensionality reduction two-dimensional DOA and polarization parameters is proposed. The proposed algorithm uses the successive idea to reduce the two-dimensional search of the classical MUSIC algorithm in the joint estimation of DOA and polarization parameters into four one-dimensional search, which greatly reduces the computational complexity compared with the classical MUSIC algorithm. In the proposed algorithm, since the signal subspace and the noise subspace are fully utilized, the parameter estimation performance is better than the ESPRIT algorithm. At the same time, the algorithm can achieve automatic pairing of signal DOA parameters and polarization parameters. The proposed algorithm has more practical engineering application value in the joint estimation of DOA and polarization parameters based on electromagnetic vector sensor, which greatly reduces the computational complexity, can quickly acquire the signal parameters of enemy radiation sources, and better adapt to the battlefield environment.

References

[1] Li J, Compton R T. Angle and polarization estimation using ESPRIT with a polarization sensitive array[J]. IEEE Transactions on Antennas and Propagation, 1991, 39(9):1376-1383.

[2] Yuan Q, Chen Q, Sawaya K. MUSIC based DOA finding and polarization estimation using USV with polarization sensitive array antenna[C]// Radio and Wireless Symposium, 2006 IEEE. IEEE, 2006.

[3] You-Gen X U, Zhi-Wen L. Cumulant-Based Two-Dimensional DOA and Polarization Estimation with a Polarization Sensitive Array Comprising a Spatially Stretched Tripole[J]. Acta Electronica Sinica, 2004.

[4] Thompson, J.N. (1984) Insect Diversity and the Trophic Structure of Communities. In: Ecological Entomology. New York. pp. 165-178.

[4] Zhang Q, Wang L, Wang Y, et al. Cyclostationarity-Based DOA and Polarization Estimation for Multipath Signals with a Uniform Linear Array of Electromagnetic Vector Sensors[C]// International Conference on Machine Learning & Cybernetics. IEEE, 2009.
[5] Zhang X, Chen C, Jianfeng Li. Blind DOA and polarization estimation for polarization-sensitive array using dimension reduction MUSIC[J]. Multidimensional Systems and Signal Processing, 2014, 25(1):67-82.

[6] Dong W, Diao M, Gao L, et al. A Low-Complexity DOA and Polarization Method of Polarization-Sensitive Array[J]. Sensors, 2017, 17(5):1170-.

[7] Na W, Zhiyu Q, Weijian S, et al. DOA and Polarization Estimation Using an Electromagnetic Vector Sensor Uniform Circular Array Based on the ESPRIT Algorithm[J]. Sensors, 2016, 16(12):2109-.

[8] Tao J W, Liu L, Lin Z Y. Joint DOA, Range, and Polarization Estimation in the Fresnel Region[J]. IEEE Transactions on Aerospace & Electronic Systems, 2011, 47(4):2657-2672.

[9] Tian Y, Xu H. DOA, power and polarization angle estimation using sparse signal reconstruction with a COLD array[J]. AEU - International Journal of Electronics and Communications, 2015, 69(11):1606-1612.

[10] Guo M, Zhang Y D, Chen T. DOA Estimation Using Compressed Sparse Array[J]. IEEE Transactions on Signal Processing, 2018, 66(15):4133-4146.

[11] Zeng Fuhong, Qu Zhigan, Si Weijian, Dimension-reduction for DOA and polarization estimation based on polarization sensitive array[J]. Applied Science and Technology, 2017, 44(03):39-42+90.