Modified EOQ Model for Refrigerated Display’s Shelf-Space Allocation Problem

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Abstract. This study modifies the economic order quantity (EOQ) model for the problem of allocating refrigerated rack space in retail. The purpose of this study is to obtain the amount of space allocation for each product that can maximize retail profits. In the modified EOQ model, there are three components of storage costs, specifically storage costs, energy costs and quality costs. Energy costs and quality costs are adjusted to the ratio of cooled performance (i.e. Coefficient of Performance) and quality deterioration rate, respectively. This study adapts Water’s algorithm for EOQ orders with constraints on storage space in determining the amount of space allocation for each product. The Water's algorithm uses an Additional Cost approach that is inherent in each unit of space used. The results show the cooling temperature is an important parameter in determining energy costs and quality costs. Through numerical examples, it is determined the right storage temperature and space allocation which gives maximum profit.

1. Introduction
Nowadays, grocery retails are not only selling packaged/processed food products, but also fresh food products. Freshness is a parameter for the quality of fresh food products. Quality of fresh food products can evaluate by three components, biological, chemical, and physical. In terms of biology, the growth of microorganisms, especially dangerous ones such as pathogenic bacteria, is one parameter in assessing the quality of fresh food products. In terms of chemistry, the nutrient content in the product can be lost during storage due to reactions with the environment. While in physical terms, changes in color and texture are the most recognizable parameters. Controlling the environment is one way to slow down the quality deterioration rate [1]–[5]. Thus, refrigerated facility become main handling equipment for storage fresh food products.

Retail has limited resources, which are divided into shop floor and shelf-space. Meanwhile, retailers sell various types of products and brands. Those two things are contradictory. Thus, space management becomes a quite critical retail operational decision.

The problem of shelf-space allocation (SSAP) is the development of EOQ with constraints on limited space. The objective of SSAP is profit maximization by determining the number of space used on each product until it is within acceptable limit [6]. Nogales and Gomez Suarez [7] mention that
SSAP is a challenging problem for retail industry as an increasing number of brands for one product type.

Some retail industries are trying to expand business opportunities by having their own brands. The decision to allocate space for private brands with national brands has been made by Nogales and Gomez Suarez [7] and Amrouche and Zaccour [8]. Spatial allocation decisions are not the sole concentration of retail, some researchers try to solve joint optimization between space allocation decisions with other operational decisions such as pricing decision [9], [10] and replenishment policies [11], [12]. The complexity of SSAP provides an opportunity for researchers to develop methods of solving including using a dynamic program approach [13]–[15]; heuristic and metaheuristic [16]–[21]; data mining approaches [22]–[24]; and game theory [25].

The best of our knowledge, SSAP that considers food products [26];[27];[28] as objects has not much discussed. Moreover, which combines the use of refrigerated facilities. Therefore, through this research, we modify the EOQ model to complete the SSAP for refrigerated facility. This study aims to get maximum retail’s profit by determine the right number of refrigerated display’s shelf space allocation for each product.

2. Related model

2.1. EOQ model
The inventory model that is still used today is the economic order quantity or EOQ model. By knowing the demand rate D, RC ordering costs, and HC handling costs, the optimal number of \( Q_0 \) orders can be obtained. There are several assumptions that must be fulfilled so that the resulting order value is the optimal value, that is not permitted by stock out and backorder. Mathematically the EOQ model can be formulated as follows [29]:

\[
Q_0 = \left( \frac{2 \times RC \times D}{HC} \right)^{1/2}
\]

(1)

2.2. Quality deterioration model
Although it has been stored in cold conditions, the fruits and vegetables are still ripening until the end of their shelf life. Ripening makes the colour and texture change. These parameters (colour and texture) can indicate the fruit quality level. Ripening is a natural process that is inevitable. In general, a quality deterioration depends on the time \( t \) and temperature of storage \( T \), and other environmental conditions (humidity, light intensity, etc.). Some quality deterioration models involve those two parameters [30], [31]. With the Arrhenius kinetic equation, the quality deterioration rate \( \sigma \) can be determined by:

\[
\sigma = k_A e^{\frac{-E_A}{RT_d}}
\]

(2)

With \( k_A \) is quality rate constants, \( E_A \) is the energy of activation for the reaction that controls quality loss, R is ideal gas constants, and \( T_d \) display’s temperature.

2.3. Multi-temperature performance model
The performance of energy efforts in changing the air temperature in the cooling system is measurable. It is introduced by Rong, et al. [32] and commonly known as the Coefficient of Performance (COP). Furthermore, based on COP value, Zanoni and Zavanella [30] decides the storage requirement between entities in cold supply chain (producer, distributor, and retail). First, they determining the ratio of COP to the lower temperature and the higher temperature. Then, the ratio used
as a comparison of costs as a form of energy needed to store at a certain temperature compared to the ideal temperature.

By adapting the model, the authors tried to synchronize the ideal storage temperature between products in one storage facility with consideration of the quality expected to be fulfilled. The energy needed between one product and another can be different. The performance of energy efforts between one product and another product will be formulated as follows:

\[
\text{COP}_{\text{cooling}} = \frac{Q_{\text{cold}}}{Q_{\text{hot}}} = \frac{T_{\text{cold}}}{T_{\text{hot}}T_{\text{col}}}
\]  

With COP is cooling system’s performance measurement, \( Q_{\text{cold}} \) is the cool (low) temperature, \( Q_{\text{hot}} \) is the hot (high) temperature or ambient temperature, \( \rho_{T_i} \) is the \( COP \) ratio of item \( i \) consider facility’s absolute temperature, \( T_{\text{ref}} \) is ideal storage temperature of item \( i \).

3. Method

3.1. Modified EOQ model

The storage cost component in cold inventory for fresh food products is different from others. The cost of using refrigerated facilities can be considered through the calculation of energy costs (ec), while consideration of product quality levels is calculated through quality costs (qc). Both parameters are adjusted to ratio of COP, and quality deterioration rate, respectively. So, in this study, the handling cost is not only a single parameter (hc), but also quality cost and energy cost. Then, the total handling cost (HC) is:

\[
HC = hc + (ec \times \rho_{T_i}) + (qc \times \sigma)
\]  

Based on Eq. (1), the optimal order quantity for single item by substituted HC with Eq. (5):

\[
Q_o = \sqrt{\frac{2 \times RC \times D}{hc + (ec \times \rho_{T_i}) + (qc \times \sigma)}}
\]  

Then, the variable cost (VC_o) for equation (6) is:

\[
VC_o = HC \times Q_o
\]  

If it is assumed that all the order quantity will sold, the profit (F_o) becomes:

\[
F_o = SP \times Q_o - VC_o - UC \times Q_o - RC
\]
Storage facilities have limited capacity. To save costs, one facility is used to store several types of products. The previous formula (EOQ) is no longer in accordance with this condition. Procedures of reducing the stock until it meets the capacity are needed. Water used one approach on the same kind problem. Water puts an additional cost (AC) on space used \( (S_i) \). Notation \( i \) is respect to each item \( (i = 1, 2, 3, \ldots n) \).

Actually, in practice this additional costs are usually not included in the calculation of total costs. By adding this cost, the equation (6) becomes equation (8) for each item:

\[
Q_i = \left( \frac{2 \times RC \times D_i}{hC_i + \left( \frac{hC_i \times \rho T_i}{2} \right) + \left( qC_i \times s_i \right) + (S_i \times AC)} \right)^{1/2}
\]  

Subject to:

\[
\sum_{i=1}^{n} \frac{Q_i}{2} \times S_i \leq W
\]  

Equation (10) guarantees that the space allocated to each product does not exceed the display capacity.

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**Figure 1.** Additional cost on space algorithm (Water’s algorithm)

To obtain \( Q_{opt} \), adjust the AC value to achieve the acceptable proportion of space and not exceeding capacity’s limit. The stages of completion can be seen in Figure 1. Then the average allocation space for each product in display’s facility is:

\[
AS_i = \frac{Q_i}{2} \times S_i
\]  

Then, the variable cost for Eq. (8) becomes:
Thus, the retail’s profit is:

\[
VC_i = RC \times D_i / Q_i + HC_i \times Q_i
\]

(12)

\[
F_i = SP \times Q_i - VC_i - UC \times Q_i
\]

(13)

4. Results and discussion

4.1. Numerical example

Consider a retailer that has refrigerated display facilities to store their fresh fruit. It is assumed there are five variations of fresh fruit products sold by retailers. Each product has a different size (see Table 1). In this study, the five products are assumed to be divided into three product groups based on their storage temperature requirements, e.g. products with low temperatures (-1 °C - 3°C), products with high temperatures (7 °C - 11°C), and mixed temperature products (-1 °C - 21 °C). Meanwhile, the temperature of the cooling facility for chilling needs is 2-8 °C.

| Item | Space Needed (space/unit) | Demand Rate (unit) | Unit Cost (Rp/unit) | Selling Price (Rp/unit) |
|------|---------------------------|--------------------|--------------------|------------------------|
| A    | 0.25                      | 300                | 18,000.00          | 27,000.00              |
| B    | 0.3                       | 410                | 1,350.00           | 2,025.00               |
| C    | 4                         | 90                 | 25,000.00          | 37,500.00              |
| D    | 0.12                      | 250                | 5,600.00           | 8,400.00               |
| E    | 0.22                      | 500                | 2,300.00           | 3,450.00               |

It is assumed that the holding cost is 20 percent of the unit cost, with an energy cost of Rp 15, - / unit and a quality cost of 5 percent of the unit cost. Each item is replenish independently, joint replenishments are not considered. Stock-outs and back orders are not permitted. (-1 °C – 21 °C).

4.2. Results and discussions

This study observe multi temperature products that must be storage in a single facility with single temperature set. Figure 2 show the average saving for high temperature product groups in different facility temperature set.

![Figure 2. Energy cost saving on high temperature products](image-url)
If retail uses the economic order quantity for all items in an inventory, the resulting total stock might exceed the available capacity. By using the Water’s algorithm that show in Figure 1, the number of each product can be reduced until it does not violate facility capacity limits. In the temperature mix product group, the temperature of the facility that provides the highest total profit is 3 °C, with allocations for each item as shown in Table II. As for the low and high temperature product groups, the temperature of the facilities that provide the highest total profit for each group is 5 °C and 6 °C, respectively. Comparison of the total profit generated from each storage temperature can be seen in Figure 3. It is known that there was an increase in total profit of 4,355% between the standard EOQ (Q_o) and revised EOQ (Q_i).

5. Conclusion
Among three product groups, the high temperature product group provides the highest profit value Rp 2,326,069.71, with the product composition i.e. A 162 units; B 315 units; C 56 units; D 199 units and E 441 units. Temperature has an important role in cost savings and retail’s profit by determining the display shelf-space allocation number. Store the product at a lower temperature than the ideal temperature, giving an average energy cost savings of 55%. However, this needs to be reviewed in
relation to the characteristics of fruit products that can experience chilling injuries if stored low temperature (extreme conditions).

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