A possible explanation of the threshold enhancement in the process $e^+e^-\rightarrow\Lambda\bar{\Lambda}$

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Inspired by the recent measurement of the process $e^+e^-\rightarrow\Lambda\bar{\Lambda}$, we calculate the mass spectrum of the $\phi$ meson with the GI model. For the excited vector strangeonium states $\phi(3S, 4S, 5S, 6S)$ and $\phi(2D, 3D, 4D, 5D)$, we further investigate the electronic decay width with the Van Royen-Weisskopf formula, and the partial widths of the $\Lambda\Lambda, \Sigma^{+}\Xi^{-},$ and $\Sigma^{*+}\Sigma^{-}$ decay modes with the extended quark pair creation model. We find that the electronic decay width of the $D$-wave vector strangeonium is about $3 \sim 8$ times larger than that of the $S$-wave vector strangeonium. Around $2232$ MeV the partial decay width of the $\Lambda\Lambda$ mode can reach up to several MeV for $\phi(3S_1)$, while the partial $\Lambda\Lambda$ decay width of $\phi(2D_1)$ is of $10^{-3}$ keV. If the threshold enhancement reported by the BESIII Collaboration arises from the strangeonium meson, this state is very likely to be the $\phi(3S_1)$ state. We also note that the $\Lambda\Lambda$ and $\Sigma\Sigma$ partial decay widths of the states $\phi(3D_1)$ and $\phi(4S_1)$ are about several MeV, respectively, which are enough to be observed in future experiments.

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I. INTRODUCTION

Because the timelike electromagnetic form factors (FFs) provide a key to understand the strong interactions and inner structure of hadrons, there have been many measurements via the process $e^+e^-\rightarrow BB$ \[1,2\] (where $B$ stands for a spin-1/2 ground baryon state). The non-vanishing cross section in the near-threshold region has been observed \[3,4\]. Unusual behavior near threshold implies a more complicated underlying physical scenario and has driven many theoretical interpretations, including $BB$ bound states or meson-like resonances \[10–17\], final-state interactions \[18,19\] and an attractive Coulomb interaction \[20,21\].

Very recently, the BESIII Collaboration studied the process $e^+e^-\rightarrow\Lambda\bar{\Lambda}$ with improved precision \[22\]. The Born cross section at $\sqrt{s} = 2.2324$ GeV, which is $1.0$ MeV above the $\Lambda\bar{\Lambda}$ mass threshold, is measured to be $305 \pm 45^{+66}_{-36}$ pb. Is the unexpected feature in the near-threshold region due to an unobserved strangeonium meson resonance? In the present work, we will try to answer this question.

We will calculate the spectrum of the $s\bar{s}$ system in the framework of the Godfrey-Isgur (GI) model \[23\], which has achieved a good description of the known mesons and baryons \[23,25\]. After we obtain the masses of the higher excited strangeonium states, we further estimate the electronic decay width of the $J^{PC} = 1^{-}$ states $\phi(2D, 3D, 4D, 5D)$ and $\phi(3S, 4S, 5S, 6S)$ with the Van Royen-Weisskopf formula \[26\]. Meanwhile, we use the extended quark pair creation model \[27,28\] to calculate the partial $\Lambda\Lambda, \Sigma^{+}\Xi^{-},$ and $\Sigma^{*+}\Sigma^{-}$ decay widths of those vector states with the obtained spatial wave functions. Considering there existing many theoretical calculations of the two-body strong decays of the $s\bar{s}$ system with various models in the literature \[29–34\], in the present work we will emphasise on the baryon-antibaryon decay mode and electronic decay properties.

According to the theoretical predictions from various models, the masses of $\phi(3S_1)$ and $\phi(2D_1)$ mesons are about 2.2 GeV (see Table \[1\]). Therefore, we calculate the $e^+e^-\rightarrow\Lambda\bar{\Lambda}$ partial decay widths of the excited vector states $\phi(3S_1)$ and $\phi(2D_1)$. We find that the electronic decay width of $\phi(3S_1)$ is about 1/3 times smaller than that of $\phi(2D_1)$. However around $2232$ MeV the partial decay width of the $\Lambda\Lambda$ mode can reach up to several MeV for $\phi(3S_1)$, while the partial $\Lambda\Lambda$ decay width of the states $\phi(2D_1)$ is of $10^{-3}$ keV. The threshold enhancement in the process $e^+e^-\rightarrow\Lambda\bar{\Lambda}$ observed by the BESIII Collaboration \[22\] may be caused by $\phi(3S_1)$. We also notice that the $\Lambda\Lambda$ and $\Sigma\Sigma$ partial decay widths of the states $\phi(3D_1)$ and $\phi(4S_1)$ are about several MeV, respectively. These two states have a good potential to be observed in future experiments via their corresponding main baryon-antibaryon decay channel.

This paper is organized as follows. In Sec. II we give a brief introduction of the GI model and calculate the spectrum of the $s\bar{s}$ system. Then we present the Van Royen-Weisskopf formula and give the electronic decay properties in Sec. III. In Sec. IV we discuss the extended quark pair creation model and baryon-antibaryon decay results. We give a short summary in Sec. V.

II. MASS SPECTRUM

In this work, we employ the GI model to calculate the mass spectrum of the higher excited strangeonium. According to
the GI model [23], the Hamiltonian between the quark and antiquark reads

\[ H = \left( p^2 + m_i^2 \right)^{1/2} + \left( p^2 + m_2^2 \right)^{1/2} + V_{\text{eff}}(p, r), \]  

where \( m_i \) and \( p \) are the quark's mass and momentum in the center-of-mass frame. \( V_{\text{eff}}(p, r) \) is the potential between the quark and antiquark, which can be obtained by the on-shell scattering amplitude between the quark and antiquark in the center-of-mass frame. This Hamiltonian contains the short-range \( \gamma \gamma \otimes \gamma \gamma \) one-gluon-exchange (OGE) interaction and the \( 1 \otimes 1 \) linear confining interaction suggested by lattice QCD. In the nonrelativistic limit, it can reduce to the familiar Breit-Fermi interaction

\[ V_{\text{eff}}(p, r) = H_{12}^{\text{conf}} + H_{12}^{\text{hyp}} + H_{12}^{\text{so}}. \]

Here, \( H_{12}^{\text{conf}} \) is the spin-independent linear confinement and Coulomb-type interaction; \( H_{12}^{\text{hyp}} \) is the color-hyperfine interaction and \( H_{12}^{\text{so}} \) is the spin-orbit interaction.

To incorporate the relativistic effects, Godfrey and Isgur further built a semiquantitative model [23]. By introducing the smearing function for a meson \( q_i \bar{q}_j \)

\[ \rho_{ij}(r - r') = \frac{\sigma_i^j}{\pi^{3/2}} e^{- \sigma_i^j (r - r')^2}, \]

the OGE potential \( G(r) = -4a(r)/3r \) and confining potential

\[ M(\text{MeV}) \]

\[
\begin{array}{cccccccccccc}
\hline
\text{State} & \text{Mass} & \text{MGI} \ [29] & \text{GI} \ [23] & \text{RQM} \ [35] & \text{COQM} \ [36] & \text{Exp.} \ [37] \\
\hline
\phi(1^3S_1) & 1009 & 1030 & 1020 & 1038 & 1020 & 1020 \\
\phi(2^3S_1) & 1688 & 1687 & 1690 & 1698 & 1740 & 1680 \\
\phi(3^3S_1) & 2204 & 2149 & \ldots & 2119 & 2250 & \ldots \\
\phi(4^3S_1) & 2627 & 2498 & \ldots & 2472 & 2540 & \ldots \\
\phi(5^3S_1) & 2996 & \ldots & \ldots & 2782 & \ldots & \ldots \\
\phi(6^3S_1) & 3327 & \ldots & \ldots & \ldots & \ldots & \ldots \\
\phi(1^3D_1) & 1883 & 1869 & \ldots & 1845 & 1750 & \ldots \\
\phi(2^3D_1) & 2342 & 2276 & \ldots & 2258 & 2260 & \ldots \\
\phi(3^3D_1) & 2732 & 2593 & \ldots & 2607 & \ldots & \ldots \\
\phi(4^3D_1) & 3079 & \ldots & \ldots & \ldots & \ldots & \ldots \\
\phi(5^3D_1) & 3395 & \ldots & \ldots & \ldots & \ldots & \ldots \\
\hline
\end{array}
\]

FIG. 1: The spectrum of the \( s\bar{s} \) system.

TABLE I: The predicted masses (MeV) of the higher \( \phi \) mesons with \( J^{PC} = 1^{--} \) from various models.
$S(r) = br + c$ are smeared to $\tilde{G}(r)$ and $\tilde{S}(r)$ via
\[
\tilde{f}(r) = \int d^3r' \rho_1(r - r') f(r').
\]
Through the introduction of the momentum-dependent factors, the Coulomb term is modified according to
\[
\tilde{G}(r) \to \left(1 + \frac{p^2}{E_1E_2}\right)^{1/2} \tilde{G}(r) \left(1 + \frac{p^2}{E_1E_2}\right)^{1/2},
\]
and the contact, tensor, vector spin-orbit, and scalar spin-orbit potentials were modified according to
\[
\frac{\tilde{V}_c(r)}{m_1m_2} \left(\begin{array}{c} m_1m_2 \frac{1}{E_1E_2} \end{array}\right)^{1/2} + \tilde{S}(r),
\]
where $\epsilon_c$ corresponds to the contact (c), tensor (t), vector spin-orbit [so(v)], and scalar spin-orbit [so(s)].

With the notation
\[
f_{\alpha\beta}(r) = \left(\frac{m_\alpha m_\beta}{E_\alpha E_\beta}\right)^{1/2}\left(\frac{m_\alpha m_\beta}{E_\alpha E_\beta}\right)^{1/2},
\]
we have
\[
V_{\text{eff}}(p, r) = \tilde{H}_{12}^\text{conf} + \tilde{H}_{12}^\text{hyp} + \tilde{H}_{12}^\text{so},
\]
where
\[
\tilde{H}_{12}^\text{conf} = \left(1 + \frac{p^2}{E_1E_2}\right)^{1/2} \tilde{G}(r) \left(1 + \frac{p^2}{E_1E_2}\right)^{1/2} + \tilde{S}(r),
\]
\[
\tilde{H}_{12}^\text{so} = \left(\begin{array}{c} m_1m_2 \frac{1}{E_1E_2} \end{array}\right)^{1/2} \tilde{G}(r) \left(\begin{array}{c} m_1m_2 \frac{1}{E_1E_2} \end{array}\right)^{1/2} + \tilde{S}(r),
\]
\[
\tilde{H}_{12}^\text{hyp} = \frac{2S_1 \cdot S_2}{3m_1m_2} \nabla^2 \tilde{G}_{12} - \frac{S_1 \cdot (S_2 \cdot r - \frac{1}{2}S_1 \cdot S_2)}{m_1m_2} \left(\frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r}\right) \tilde{G}_{12}.
\]

The spin-orbit term $\tilde{H}_{12}^\text{so}$ can be decomposed into a symmetric part $\tilde{H}_{12}^\text{so}$ and an antisymmetric part $\tilde{H}_{12}^\text{so}$, while the $\tilde{H}_{12}^\text{so}$ vanishes when $m_1 = m_2$.

We adopt the free parameters in the original work of the GI model [23], and diagonalize the Hamiltonian in the simple harmonic oscillator bases $|nS^{2S+1}L_l\rangle$. The resulting mass spectrum of the strangeonium are shown in Fig. 1. Meanwhile, we compare our predicted mass of the higher vector $\phi$ mesons with various models predictions, as listed in Table II.

### III. THE ELECTRONIC DECAYS

With the Van Royen-Weisskopf formula [26, 38], the electronic decay width of the excited vector strangeonium states is given by
\[
\Gamma[\phi(nS) \to e^+e^-] \propto \frac{4\alpha^2 e^2}{M_{\phi}} |R_{nS}(0)|^2,
\]
\[
\Gamma[\phi(nD) \to e^+e^-] \propto \frac{25\alpha^2 e^2}{2M_{\phi}M_{\phi}^2} |R_{nD}(0)|^2.
\]
Here, $\alpha = \frac{e^2}{\hbar c}$ denotes the fine structure constant. $m_4 = 450$ MeV and $e = \frac{1}{2}$ are the strange quark constituent mass and charge in unit of electron charge, respectively. $M_{\phi}(M_{n\phi})$ is the mass for $\phi(nS)(\phi(nD))$. $R_{nS}(0)$ represents the radial $S$ wave function at the origin, and $R_{nD}(0)$ represents the second derivative of the radial $D$ wave function at the origin.

In the present calculation, we adopt the simple harmonic oscillator (SHO) wave functions for the space-wave functions of the initial meson. According to the wave functions obtained in mass spectrum calculations, we get the root mean square radius of the vector states. Then, we determine the value of harmonic oscillator strength $\beta_{\phi\phi}$ between the two strange quarks for the initial mesons (as listed in Table II).

According to PDG [37], the electronic decay branching ratio for $\phi(1S)$ is
\[
B[\phi(1S) \to e^+e^-] = (2.973 \pm 0.034) \times 10^{-4}.
\]

Combining this ratio with its total decay widths($\Gamma = 4.249 \pm 0.013$ MeV), the central value of the electronic decay width is $\Gamma[\phi(1S) \to e^+e^-] = 1.26$ keV. Then, from the formulas (12)-(13), we can obtain electronic decay width ratios of between the higher excited vector strangeonium states and the state $\phi(1S)$. Thus, we can get those states electronic decay widths, as shown in Table II.
From the table, the ratio $R$ is smaller than one. The electronic decay widths of the excited vector strangeonium states $\phi(3S, 4S, 5S, 6S)$ and $\phi(2D, 3D, 4D, 5D)$ are smaller than that of the state $\phi(1S)$. Meanwhile, the electronic decay width of the $D$-wave vector strangeonium is about $3 \sim 8$ times larger than that of the $S$-wave vector strangeonium. For the $S$-wave states, our predictions are in accordance with ref. [39], while for the $D$-wave states, our predictions are about $3$ times larger than those of ref. [39].

Considering the uncertainties of the predicted mass and harmonic oscillator strength $\beta_{th}$, we plot the variation of the electronic decay width ratio $R$ as a function of the mass with different values of $\beta = \beta_{th} + 20$ MeV, $\beta_{th}$, and $\beta_{th} - 20$ MeV, respectively, in Fig. 2. It is obvious that the ratio $R$ decreases with the mass with the same $\beta$ values.

### IV. DOUBLE BARYON DECAY MODE

#### A. The $^3P_0$ model

The quark pair creation ($^3P_0$) model was first proposed by Micu [40], Carlitz and Kislinger [41], and further developed by the Orsay group [42–44], which has been widely used to study the OZI-allowed two-body strong decays of hadrons. Very recently, the $^3P_0$ model was extended to study some OZI-allowed three-body strong decays [28] as well. In the framework of this model, the interaction Hamiltonian for one quark pair creation was described as [45–47]

$$H_{q\bar{q}} = \gamma \sum_f 2m_f \int d^3 \vec{x} \bar{\psi}_f \gamma_\mu \psi_f.$$  \hspace{1cm} (15)

Here, $\gamma$ is a dimensionless parameter and usually determined by fitting the experimental data. $m_f$ denotes the constituent quark mass of flavor $f$ and $\psi_f$ stands for a Dirac quark field.

FIG. 3: The strangeonium system decays into double baryons.
In our previous work \cite{27}, we extended the $^3P_0$ model to study the partial decay width of the $\Lambda_c\bar{\Lambda}_c$ mode for the charmonium system. In this work, we further use this model to study the process $\phi^*(A) \rightarrow B(B') \bar{B}(C)$, where $\phi^*$ denotes the excited strangeonium states. As pointed out in Ref. \cite{27}, two light quark pairs should be created for this type of reaction (as shown in Fig. 3), and the helicity amplitude $M^{M_{I_B}M_{I_C}M_{I}}$ reads

$$\delta^3(p_B - p_C)M^{M_{I_B}M_{I_C}M_{I}} \approx \frac{(BC|H_{q0}H_{q0}|A)}{2m_q}. \quad (16)$$

Here, $p_I (I = A, B, C)$ denotes the momentum of the hadron. $E_{A(k)}$ represents the energy of the initial(intermediate) state $A(k)$. Considering the quark-hadron duality \cite{48}, we simplify the calculations via taking $E_k - E_A$ as a constant, namely $E_k - E_A \approx 2m_q$. Here, $m_q$ is the constituent quark mass of the created quark. We adopt this crude approximation because the intermediate state differs from the initial state by two created additional quarks at the quark level \cite{27,28}. Thus, we can rewrite the Eq. (16) as

$$\delta^3(p_B - p_C)M^{M_{I_B}M_{I_C}M_{I}} \approx \frac{(BC|H_{q0}H_{q0}|A)}{2m_q}. \quad (17)$$

Then, the transition operator for the two quark pairs creation in the nonrelativistic limit reads

$$T = \frac{9\alpha^2}{2m_q} \int \int d^3p_1d^3p_2d^3p_3d^3p_4\delta^3(p_1 + p_4)\delta^3(p_2 + p_3)$$

$$\times \delta^3(p_1 - p_2)\delta^3(p_4 - p_3), \quad (18)$$

where $p_i$ (i=3, 4, 5, 6) stands for the three-vector momentum of the $i$th quark. $\varphi_0 = (iu + d\bar{d} + s\bar{s})/\sqrt{3}$ corresponds to the flavor function and $\omega_0 = \delta_{ij}$ represents the color singlet of the quark pairs created from vacuum. $\chi_{1\rightarrow m(r)}$ are the spin triplet states for the created quark pairs. The solid harmonic polynomial $\mathcal{Y}_{1m(r)}^m(p) = |p\rangle_{1m(r)}(\theta, \phi, \psi)$ denotes the $P$-wave quark pairs. $a^I_{i^I}b^I_{j^I}$ is the creation operator representing the quark pair creation in the vacuum.

Finally, the hadronic decay width $\Gamma$ in the relativistic phase space reads

$$\Gamma[A \rightarrow BC] = \pi^2 |p|^2 \frac{1}{M^2_A} \sum_{M_{I_B}M_{I_C}} |M^{M_{I_B}M_{I_C}}|^2, \quad (19)$$

Here, $p$ represents the momentum of the daughter baryon. $M_A$ and $J_A$ are the mass and total angular quantum number of the parent baryon $A$, respectively. In the center of mass frame of the parent baryon $A$, $p$ reads

$$|p| = \frac{\sqrt{(M^2_A - (M_B - M_C)^2)(M^2_A - (M_B + M_C)^2)}}{2M_A}. \quad (20)$$

|TABLE III: The parameters we used in this work \cite{57,49}. The unit is MeV except the $\gamma$, which has no unit.

| Mass of the final baryon | $\Lambda$ | 1115.68 |
| $\bar{\Lambda}$ | 1115.68 |
| $\Sigma^+$ | 1189.37 |
| $\Sigma^-$ | 1189.37 |
| $\Xi^-$ | 1321.71 |
| $\bar{\Xi}^+$ | 1321.71 |
| $\Sigma^{**}$ | 1382.80 |
| $\Sigma^{*+}$ | 1382.80 |
| $\Xi^{-*}$ | 1535.0 |

Constituent quark mass $m_u$ | 330 |
| $m_d$ | 330 |
| $m_s$ | 450 |

Harmonic oscillator parameter $\alpha$ | 400 |
| Strength of the quark pair $\gamma$ | 6.95 |

In our calculation, we take the standard constituent quark masses, namely $m_u=m_d=330$ MeV and $m_s=450$ MeV. The masses of the final baryons are taken from PDG \cite{57}, as listed in Table III. We adopt the simple harmonic oscillator (SHO) wave functions for the space-wave functions of the hadrons. The harmonic oscillator strength $\beta_\gamma$ between the two strange quarks for the initial mesons is determined by the spatial wave functions obtained in mass spectrum calculations (as listed in Table I). The harmonic oscillator strength between the two light quarks for final baryons is taken as $\alpha = 400$ MeV. As to the strength of the quark pair creation from the vacuum, we adopt the same value as in Ref. \cite{49}, $\gamma = 6.95$. The uncertainty of the strength $\gamma$ is about 30% \cite{47,50,52}, and the partial decay widths are proportional to $\gamma^\alpha$. Thus our predictions may bare a quite large uncertainty.

### B. $\Lambda \bar{\Lambda}$ decay mode

1. States around the $\Lambda \bar{\Lambda}$ threshold

In 2007, the BABAR Collaboration measured the cross section for $e^+e^- \rightarrow \Lambda \bar{\Lambda}$ from threshold up to 3 GeV \cite{29} and observed a possible nonvanishing cross section at threshold. Recently, the BESIII Collaboration published a measurement of the process $e^+e^- \rightarrow \Lambda \bar{\Lambda}$ \cite{28} with improved precision. The Born cross section at $\sqrt{s} = 2232.4$ MeV, which is 1.0 MeV above the $\Lambda \bar{\Lambda}$ mass threshold, is measured to be $305 \pm 45^{+66}_{-36}$ pb, which indicates an obvious threshold enhancement.

According to various model predictions (see Table I), there are two strangeonium meson resonances $\phi(3^3S_1)$ and $\phi(2^3D_1)$ with both masses around 2.2 GeV and $J^P = \frac{1}{2}^-$. As a possible source of the observed threshold enhancement, it is crucial to study the decay properties of the states $\phi(3^3S_1)$ and $\phi(2^3D_1)$.

We first explore the $\Lambda \bar{\Lambda}$ partial decay width of the state...
the mass in the range of $(2233-2300)\text{ MeV}$. Combining the predicted partial decay width with a mass of $2232\text{ MeV}$ (see Table IV), this partial decay width is large enough to be observed in experiments, and indicates that the observed threshold enhancement may arise from this state. Although the phase space is suppressed seriously around threshold, the transition amplitude for this decay mode is quite large. Hence, the partial decay width of the $\Lambda\Lambda$ mode for the state $\phi(3\text{$S$$1$})$ reaches several MeV. Considering the uncertainties of the predicted mass, we study the variation of the $\Lambda\Lambda$ decay width as a function of the mass of the state $\phi(3\text{$S$$1$})$. The decay width increases rapidly with the mass in the range of $(2233-2300)\text{ MeV}$.

Then, we investigate the decay properties of the state $\phi(2\text{$D$$1$})$. Fixing the mass at $M = 2232\text{ MeV}$, we get

$$\Gamma[\phi(2\text{$D$$1$})\rightarrow\Lambda\Lambda] \sim 3.90 \times 10^{-6}\text{ MeV}.\quad(22)$$

This width seems too small to be observed in experiments. Combining the predicted partial decay width of $\phi(3\text{$S$$1$})$, we further obtain

$$\Gamma[\phi(3\text{$S$$1$})\rightarrow\Lambda\Lambda]/\Gamma[\phi(2\text{$D$$1$})\rightarrow\Lambda\Lambda] \sim 1.50 \times 10^6.\quad(23)$$

The decay ratio of $\phi(3\text{$S$$1$})$ into the $\Lambda\Lambda$ channel is about $O(10^6)$ larger than that of $\phi(2\text{$D$$1$})$ into the $\Lambda\Lambda$ channel. Combining their electronic decay width we calculated in Sec. II, we obtain that if the threshold enhancement reported by the BESIII Collaboration in the process $e^+e^-\rightarrow\Lambda\Lambda$ were related to an unobserved strangeonium meson resonance, this state should most likely be $\phi(3\text{$S$$1$})$.

Besides the uncertainties coming from the predicted mass and harmonic oscillator strength $\beta_{th}$, the results of $\phi(3\text{$S$$1$})$ and $\phi(2\text{$D$$1$})$ may have large uncertainties due to their lower masses. At the hadron level, the energy of the intermediate states with the spin parity $J^{PC} = 1^{-}$, such as molecular states $KK_f(1270), K^*(892)K_f(700), K^*(892)K_f(1270)$, and $\phi(1020)\omega(980)$ and so on, is about 1.7 GeV.2 GeV. Thus the $E_{\Lambda} - E_{\Lambda}$ are small and sensitive to the masses of the intermediates state. In this case, taking $E_{\Lambda} - E_{\Lambda} = 2m_{\eta}$ as a constant will introduce a large uncertainty in this calculation.

### 2. higher states

Besides $\phi(3\text{$S$$1$})$ and $\phi(2\text{$D$$1$})$, we also analyze the decay properties of the S-wave states $\phi(4\text{$S$$1$}), \phi(3\text{$S$$1$}), \phi(5\text{$S$$1$})$ and the D-wave states $\phi(3\text{$D$$1$}), \phi(4\text{$D$$1$}), \phi(5\text{$D$$1$})$. The decay properties are collected in Table V. From the table, we get that the $\Lambda\Lambda$ partial decay widths of those states as functions of the masses in Fig. 4 with different values of $\beta = \beta_{th}+20\text{ MeV}, \beta_{th}$, and $\beta_{th}-20\text{ MeV}$, respectively. According to Fig. 4 for the state $\phi(3\text{$D$$1$})$, the variation curve like a bowl structure when the mass varies from 2550 MeV to 2850 MeV, and the partial width can reach up to $\Gamma \sim 3.7\text{ MeV}$ with $\beta = \beta_{th}$. The $\Lambda\Lambda$ partial decay width for $\phi(5\text{$D$$1$})$ is the smallest. The decay width is less than $\Gamma < 0.4\text{ MeV}$ with the mass in the range of $M = (3150 - 3450)\text{ MeV}$. As to $\phi(4\text{$S$$1$})$, its $\Lambda\Lambda$ decay width is very sensitive to the mass (see Fig. 3). When $\beta = \beta_{th}$, the width varies in the range of $\Gamma \sim (0.0-4.8)\text{ MeV}$ with the mass in the range of $M = (2450-2750)\text{ MeV}$. If the mass of $\phi(4\text{$S$$1$})$ lies in (2496-2590)MeV, the decay width of the $\Lambda\Lambda$ mode is less than one MeV. Most of the $\Lambda\Lambda$ partial decay widths for the other three states, $\phi(4\text{$D$$1$}), \phi(5\text{$S$$1$})$, and $\phi(6\text{$S$$1$})$, are less than one MeV (see Fig. 4). These partial widths seem to be sizeable as well.

#### C. Other double baryon decay modes

Besides $\Lambda\Lambda$ decay mode, we also investigate the $\Xi^{-}\Xi^{+}$ and $\Sigma^{+}\Sigma^{-}$ decay modes for the excited vector strangeonium. According to the predicted masses listed in Table III, the masses of the states $\phi(3\text{$S$$1$})$ and $\phi(2\text{$D$$1$})$ are the thresholds of $\Xi^{-}\Xi^{+}$ and $\Sigma^{+}\Sigma^{-}$, respectively. Thus, in this section, we focus on partial decay properties of the vector strangeonium states $\phi(4\text{$S$$1$}), 5\text{$S$$1$}, 6\text{$S$$1$})$ and $\phi(3\text{$D$$1$}, 4\text{$D$$1$}, 5\text{$D$$1$})$. Our predictions are collected in Table VI.

From the Table, we notice that the $\Sigma^{+}\Sigma^{-}$ partial decay width of $\phi(5\text{$S$$1$})$ and $\phi(3\text{$D$$1$})$ can reach up to $\Gamma \sim 2.9\text{ MeV}$ and $\Gamma \sim 1.5\text{ MeV}$, respectively, which are large enough to be observed in future experiments. Meanwhile, the $\Xi^{-}\Xi^{+}$ and $\Sigma^{+}\Sigma^{-}$ partial decay widths of the state $\phi(5\text{$S$$1$})$ are both larger than one MeV.

In addition, we also plot the decay properties of the states $\phi(4\text{$S$}, 5\text{$S$}, 6\text{$S$})$ and $\phi(3\text{$D$}, 4\text{$D$}, 5\text{$D$})$ as a function of the mass.
FIG. 4: The variation of the $\Lambda\bar{\Lambda}$ decay width with the mass of the S- and D-wave vector strangeonium. The red, black, and blue lines correspond to the predictions with different values of $\beta=\beta_{th}+20$ MeV, $\beta_{th}$, and $\beta_{th}-20$ MeV, respectively.

TABLE VI: The partial decay widths of the higher $\phi$ mesons with $J^{PC}=1^{-+}$. The unit is MeV.

| State     | Mass   | $\beta$ | $\Gamma[\Xi^{-}\bar{\Xi}^+]$ | $\Gamma[\Sigma^+\bar{\Sigma}^-]$ | $\Gamma[\Sigma^{++}\bar{\Sigma}^-]$ | $\Gamma[\Sigma^{++}\bar{\Sigma}^-]$ | $\Gamma[\Xi^{-}\bar{\Xi}^+]$ |
|-----------|--------|---------|-------------------------------|-----------------------------------|-------------------------------------|-------------------------------------|-------------------------------|
| $\phi(4^{3}S_{1})$ | 2627   | 351     | $\cdots$                      | 2.86                              | 0.91                                | $\cdots$                            | $\cdots$                      |
| $\phi(5^{3}S_{1})$ | 2996   | 341     | 1.28                           | 0.05                              | 0.62                                | 1.69                                | 0.03                          |
| $\phi(6^{3}S_{1})$ | 3327   | 334     | $2.26 \times 10^{-3}$          | 0.08                              | 0.06                                | $5.67 \times 10^{-3}$              | $7.63 \times 10^{-4}$          |
| $\phi(3^{3}D_{1})$ | 2732   | 355     | 0.16                           | 1.49                              | 0.41                                | $\cdots$                            | $\cdots$                      |
| $\phi(4^{3}D_{1})$ | 3079   | 344     | 0.11                           | 0.02                              | 0.07                                | 0.86                                | 0.01                          |
| $\phi(5^{3}D_{1})$ | 3395   | 336     | 0.02                           | 0.08                              | 0.03                                | 0.58                                | $9.55 \times 10^{-5}$          |

FIG. 5: The variation of the partial decay widths with the mass of the S- and D-wave vector strangeonium.
in Fig. [5]

To investigate the uncertainties of the parameter $\beta_{th}$, we further consider the partial decay properties with different $\beta_{th}$ values. The theoretical numerical results are not shown in the present work. According to our calculations, our main predictions hold in a reasonable range of the parameter $\beta_{th}$.

V. SUMMARY

In the present work, we have studied the mass spectrum of the strangeonium system with the GI model and further investigated the electronic decay width and $\Lambda \Lambda$, $\Xi^+(\Xi^+)$, and $\Sigma^+(\Sigma^+)$ double baryons decay widths of the excited vector strangeonium states $\phi(3S)$, $4S$, $5S$, $6S$) and $\phi(2D, 3D, 4D, 5D)$.

For the electronic decay widths, we obtain that the electronic decay widths of the excited vector strangeonium states $\phi(3S)$, $4S$, $5S$, $6S$) and $\phi(2D, 3D, 4D, 5D)$ are smaller than that of the state $\phi(1S)$. Meanwhile, the electronic decay width of the $D$-wave vector strangeonium is about $3 \sim 8$ times larger than that of the $S$-wave vector strangeonium.

For the double baryons decay widths, the partial decay width of the $\Lambda \Lambda$ mode can reach up to $\sim 5.84$ MeV for $\phi(3S)$, while the partial $\Lambda \Lambda$ decay width of the states $\phi(2D)$ is about $O(10^{-3})$ keV. Thus, the $\Lambda \Lambda$ decay width ratio between the states $\phi(3S)$ and $\phi(2D)$ is $O(10^6)$. If the threshold enhancement reported by the BESIII Collaboration in process $e^+e^- \rightarrow \Lambda \Lambda$ does arise from an unobserved strangeonium meson, the resonance is most likely to be the strangeonium state $\phi(3S)$. We also notice that the $\Lambda \Lambda$ and $\Sigma^+ \Sigma^-$ partial decay widths of the states $\phi(3D)$ and $\phi(4S)$ are about several MeV, respectively, which are enough to be observed in future experiments. The double baryons decay modes provide a unique probe of the excited vector strangeonium resonances, which may be produced and investigated at BESIII and BelleII.

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