Effect of Sudden Rise of Water in Stream on Adjacent Land

Abulla¹ A. Abo., Sarhang² M. Husain . Saud³ A. Hussein

1, 2 Department of Dams and water resources, College of Engineering, Salahaddin University, Erbil, Kurdistan Region, Iraq
3 Assistant lecturer, Department of Road Construction, Erbil Technical Institute, Erbil Polytechnic University, Hawler, Iraq.

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*Corresponding Author Contact Email:
abdulla.abo@su.edu.krd

ABSTRACT

This study uses both laboratory and mathematical models to compute water table through an embankment cross section due to sudden rise in stream flow level. The main aim for current study to proof whether a one-dimensional 1D mathematical model can capture the phreatic line through embankment. For this purpose four different identical laboratory sand homogeneous embankment models are constructed in a plywood box by using different sand particle sizes 2mm, 1mm, 0.5mm and 0.25mm to represent very coarse, coarse, medium and fine sand respectively. Experiments are conducted on each model by fixing the upstream water depth to 0.75m; in order steady state in water table prevail. The water table level is measured at different sections inside the model body at the steady state flow condition. The time required to achieve the steady state condition in each model is also recorded. For the mathematical model, the Darcy equation is solved considering the flow in one dimension through the embankment model. Then, the mathematical model is verified against the experimental and numerical results available in the literature. The comparisons show well agreement between the computed results of the mathematical model and the corresponding observed results gathered from the experimental and numerical models. The mathematical model derived in this study can be readily applied to predict water level in porous media of infinite length.

1. INTRODUCTION

The water table through adjacent land of a river is significantly influenced by the stream water depth. Structures built over such land might be affected by a sudden rise of water table elevation, which eventually may cause these structures to fail. This is because the seeping water considerably increases the uplift pressure under these structures due to the increase of water content in soil (Rushton and Redshaw 1979). Many special treatments, most importantly lining both sides of the river, are therefore required to avoid such structural failures, which increase the overall cost. Lining of rivers sometimes may not be feasible due to its cost effectiveness.
In addition, seepage through embankments rises the elevation of the phreatic line, which may result in dam failure due to sliding, piping and sloughing (Middlebrooks, 1953). Therefore, understanding the flow behavior through embankments is crucial to improve the design of such hydraulic structure, especially in terms of the elevation of phreatic line. This is because the maximum allowable wet height at the DS face of embankment dams is nearly one-third of its total height (Justin et al., 1944). This type of flow is analogue to the flow through porous media and is well known as seepage through embankments. In fact, Darcy’s law is commonly used to compute the water
surface profile for such flow. This study focuses on this kind of flow to find the phreatic line in an embankment dam. Rising of water table in adjacent land of a river mainly depends on the hydraulic conductivity of soil. The hydraulic conductivity in a relatively small area varies over a broad range as it depends on a number of factors such as water viscosity, particle size, and its distribution, density of the soils, size and continuity of cracks and joins (Cedergren 1988). Also flow through porous media, its paths length and travel time depend on type of soils, stratification and meteorological (climatological) conditions (Winter et al. 2002).

Stello (1987) developed design charts to compute the elevation of the phreatic line and amount of seepage passing through homogenous and zoned dams located on an impervious foundation using a computer program known as fragment method. The study showed that the results of this method were in close agreement with previously published results with an average percent error of about 18%. (Ching 1988) analyzes computation of a phreatic line numerically using the boundary element method (BEM). The numerical model is validated against the experimental results reported by (Kellogg 1941) which is considered a benchmark. The comparison showed a maximum variation of about 10%. The impact of hydraulic conductivity and its effect on fall of phreatic surface per various discharge rates that pass through dam body are studied analytically by (Rezk et al. 2010). Comparisons indicated that the phreatic line computed in the analytical method is agreed well with the phreatic line measured in the laboratory.

In the present study, a one dimensional 1D mathematical model is derived from Darcy’s law to calculate the phreatic line in homogenous embankment dam and other cases of identical conditions, and this is a limitation of current study. Also, an embankment model is constructed herein to verify the validity of the proposed equation.

1. MATHEMATICAL MODEL

The parameters required to express the flow through a typical embankment model are shown in Figure 1:

![Figure 1 Typical laboratory model.](image)

The flow through saturated porous media is governed by Darcy law:

\[ V = K I \]  

(1)

Where: \( V \) is the velocity of flow through porous media \( m/s \), \( K \) is hydraulic conductivity \( m/s \), \( I = \frac{dh}{ds} \) is hydraulic gradient, \( h \) is head of flow \( m \), and \( s \) is distance along the flow line \( m \).

Equation 1 can be considered only for laminar flow (Reynolds number less than 1).

\[ Re = \frac{V L}{\nu} \]  

(2)

Where: \( V \) is the velocity of flow through porous media \( m/s \), \( L \) is represents the characteristic length, \( \nu \) is viscosity of water \( m^2/s \).

Reynolds number in porous media is usually less than one. Hence, the flow through porous media can be considered laminar (Harr M. E. 1962). Darcy’s law for three-dimensional flow can be expressed as:

\[ u = K_x I_x = -K_x \frac{dh}{dx} \]

\[ v = K_y I_y = -K_y \frac{dh}{dy} \]

\[ w = K_z I_z = -K_z \frac{dh}{dz} \]  

(3)

(4)

(5)

Where \( u, v, w \) are velocity, \( K_x, K_y, K_z \) are hydraulic conductivity and \( I_x, I_y, I_z \) are hydraulic gradient in directions \( x, y, z \) respectively.

Applying continuity equation for incompressible flow in three dimensions yields:
\[
\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \quad (6)
\]

\[
\frac{\partial}{\partial x} \left( K_x \frac{dh}{dx} \right) + \frac{\partial}{\partial y} \left( K_y \frac{dh}{dy} \right) + \frac{\partial}{\partial z} \left( K_z \frac{dh}{dz} \right) = 0 \quad (7)
\]

Assuming that the soil is homogenous and isotropic produces:

\[
K_x = K_y = K_z = k \quad (8)
\]

Equation 7 can be re-written as below:

\[
\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (9)
\]

For unsteady state condition, equation 9 can be re-written as follows:

\[
\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{1}{\gamma} \frac{\partial h}{\partial t} \quad (10)
\]

And for one dimensional flow in x-direction, equation 10 is reduced to:

\[
\frac{\partial^2 h}{\partial x^2} = \frac{1}{\gamma} \frac{\partial h}{\partial t} \quad (11)
\]

The governing equation of unsteady (Cartesian form) flow through porous media and its boundary and initial conditions in one-dimensional can be set as below.

\[
\frac{\partial^2 h}{\partial x^2} = \frac{1}{\gamma} \frac{\partial h}{\partial t} \quad (12)
\]

\[
h(x, 0) = 0 \quad (13)
\]

\[
h(0, t) = h_0 \quad (14)
\]

\[
h(\infty, t) = 0 \quad (15)
\]

Where: \( h = h(x, t) \) is the rise in water level (Ground water head) caused by the change in stream level at an US face of the model, \( x \)-Distance from the US of the models embankment.

For unconfined aquifer \( \gamma = \frac{K}{S} \) (Gelhar, L.W. and Axness, C.L., 1983)

Where: \( \gamma \) is hydraulic diffusivity, \( K \) is hydraulic conductivity m/s, \( S \) is storage coefficient related to compressibility m\(^{-1}\).

Applying Laplace transform with respect to \( t \) for equations 12, 14, and 15, provides:

\[
L \left[ \frac{\partial^2 h}{\partial x^2} \right] = L \left[ \frac{1}{\gamma} \frac{\partial h}{\partial t} \right] \quad (16)
\]

\[
\frac{\partial^2 h}{\partial x^2} = \frac{1}{\gamma} \left[ s \hat{h} - h(x, 0) \right] \quad (17)
\]

\[
h(0, s) = \frac{h_0}{s} \quad (18)
\]

Where: \( s \cdot L \left[ h(x, t) \right] = h'(x, s) \)

\[
L \left[ h(\infty, t) \right] = L[0] \quad (19)
\]

\[
h'(\infty, s) = 0 \quad (20)
\]

Equation 16 is a second order differential equation with constant coefficients and can be solved by operator having a general solution:

\[
\hat{h}(x, s) = c_1 e^{-x/\sqrt{s/\gamma}} + c_2 e^{x/\sqrt{s/\gamma}} \quad (21)
\]

Where: \( L[\hat{h}(x, t)] = \hat{h}(x, s) \quad (22) \)

By applying boundary and initial conditions, particular solution for equation (21) can be obtained by firstly, applying boundary condition (14) in equation (15). Firstly, applying initial (18) in equation (19) yields:

\[
h'(\infty, s) = c_1 e^{-\infty} + c_2 e^{\infty} = 0 \quad (23)
\]

\[
0 + c_2 e^{\infty} = 0 \quad \text{but} \quad e^{\infty} \quad \text{not equal to zero, so} \quad c_2 = 0 \quad (24)
\]
Then equation (19) can be re-written as follows:

\[ \hat{h}(x, s) = c_1 e^{-x \sqrt{s/\gamma}} \]  
(20)

Secondly, applying boundary condition (17) in equation (20)

\[ \hat{h}(0, s) = \frac{h_0}{s} = c_1 e^0, \quad c_1 = \frac{h_0}{s} \]

so equation (9) can be re-written as follow:

\[ \hat{h}(x, s) = \frac{h_0}{s} e^{-x \sqrt{s/\gamma}} \]  
(21)

Now the inverse Laplace transform for equation (21) is:

\[ L^{-1}\left[ \hat{h}(x, s) \right] = h_0L^{-1}\left[ \frac{e^{-x \sqrt{s/\gamma}}}{s} \right] \]

\[ h(x, t) = h_0erfc\left( \frac{x}{\sqrt{4\gamma t}} \right) \]  
(22)

Where: erfc is complimentary error function which is expressed as below:

\[ erfc\left( x \right) = 1 - erf(x) \]  
(23)

So equation 11 can be re-written:

\[ h(x, t) = h_0\left[ 1 - erf\left( \frac{x}{\sqrt{4\gamma t}} \right) \right] \]  
(24)

Equation 24 defines the one dimensional phreatic line level along an embankment dam model.

2. MATHEMATICAL MODEL VALIDATION

Equation 24 obtained in section 2 is validated against the experimental data gathered from laboratory models of the current study and a homogenous laboratory dam of Kellogg (1941). The experimental data of Kellogg (1941) were collected on a physical dam of 38 cm high, 68 cm bottom width and identical US and DS side slopes of 1H:1.3V. The dam was constructed from coarse sand of hydraulic conductivity 0.1 cm/s (1*10-3 m/s). This dam configuration is also studied by Ching (1988) numerically using boundary element method (BEM). Figure 2 illustrates schematic diagram of the dam model constructed by Kellogg (1941).

Table 1 presents the results provided by the mathematical model of equation 24, those measured by Kellogg (1941) and those predicted by Ching (1988) in terms of the total head at different stations. The percentage of error due to the difference in results obtained from the experimental and mathematical models (column 5) and the numerical and mathematical models (column 6) for each station is also calculated and shown in Table1. These results are also plotted in Figure 3.

![Figure 2 Typical laboratory dam (Kellogg 1941).](image)

From Table 1 and Figure 3 it is clear the results provided by the mathematical model are closer than those computed by the numerical BEM model.

| Station (x) cm | Exp. Result Kellogg (1941) | Ching (1988) | Math. result | Error % | Error % |
|---------------|--------------------------|--------------|--------------|---------|---------|
| 0.0           | 38.0                     | 37.39        | 37.39        | 0.0     | 1.6     |
| 7.0           | 33.64                    | 34.64        | 34.64        | 1.9     | 0.0     |
| 20.0          | 26.79                    | 24.62        | 24.62        | 1.9     | 10.2    |
| 31.0          | 21.13                    | 22.74        | 22.74        | 2.7     | 4.7     |
| 35.0          | 20.09                    | 20.83        | 20.83        | 2.3     | 3.8     |
| 44.0          | 15.39                    | 16.16        | 16.16        | 1.5     | 5.0     |
| 48.0          | 13.76                    | 14.24        | 14.24        | 3.4     | 0.0     |
| 61.0          | 9.39                     | 8.38         | 8.38         | 2.6     | 8.4     |
3. RESULTS AND DISCUSSION

In this study experiments are conducted on an embankment dam to verify the validity of equation 24. The embankment was initially set horizontally in a plywood box as shown in Figure 4.

The shape of embankment is trapezoidal having dimensions of top width 1.5 m, base width 3.1 m, width 0.5 m, height 0.8 m and DS and US slopes of 1:1. The model is provided with two channels fixed at US and DS having vertical sidewalls and bed width of 0.25 m.

Six artificial screened piezometers are fixed at distances 0.05, 0.35, 0.65, 0.95, 1.25 and 1.55 m measured from the heel of the dam model.

Initially the water table through the embankment is maintained to 0.25 m by providing the water level at both sides of the model 0.25 m and kept constant until the water level reach constant at both sides channel and this is made by blocking the end sides of the model with 0.25 m height plywood.

Then suddenly the water level in the upstream is raised to 0.75 m and maintained constant until the water height in piezometers reach constant. Once the water level in the piezometers gets stable the height of water in piezometers are recorded by using a bar of circular cross-section with diameter 5 mm and length 1 m. Table 3 shows the water level in each of the six piezometers fixed at the distance mentioned above for the four type soils tested in this study. The time required to reach constant water level of each piezometer is also presented in Table 3.

For different types of soil are prepared according to the particle size distribution determined from sieving procedure. The properties of soil types examined in this study including particle size, hydraulic conductivity and storage coefficient are determined following Domenico and Schwartz (1990) and Morris and Jonhson (1967) as shown in Table 2

| Station (x) cm | Total head cm | Exp. Result Kellogg (1941) | Math. result | Ching (1988) | Error % | Error % |
|---------------|---------------|---------------------------|--------------|--------------|---------|---------|
| 0.0           | 38.0          | 38.0                      | 37.39        | 0.0          | 1.6     |
| 7.0           | 34.64         | 33.97                     | 34.64        | 1.9          | 0.0     |
| 20.0          | 27.42         | 26.79                     | 24.62        | 1.3          | 10.2    |
| 31.0          | 21.72         | 21.13                     | 22.74        | 2.7          | 4.7     |
| 35.0          | 20.09         | 19.63                     | 20.83        | 2.3          | 3.8     |
| 44.0          | 15.39         | 15.31                     | 16.16        | 0.5          | 5.0     |
| 48.0          | 14.24         | 13.76                     | 14.24        | 3.4          | 0.0     |
| 61.0          | 9.15          | 9.39                      | 8.38         | 2.6          | 8.4     |
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Table 3 Experimental results phreatic line depth.

| Models         | water level at different stations |
|----------------|----------------------------------|
|                | X=0    | X=0.05 | X=0.35 | X=0.65 | X=0.95 | X=1.25 | X=1.55 |
| Very coarse sand | 0.75   | 0.71   | 0.615  | 0.525  | 0.452  | 0.373  | 0.334  |
| Coarse sand    | 0.75   | 0.734  | 0.641  | 0.551  | 0.475  | 0.399  | 0.351  |
| Medium sand    | 0.75   | 0.74   | 0.662  | 0.573  | 0.502  | 0.426  | 0.382  |
| Fine sand      | 0.75   | 0.742  | 0.665  | 0.615  | 0.535  | 0.472  | 0.421  |

The mathematical model represented by equation 24 is applied to the same embankment model constructed in the present work to determine the water level in the model at the same positions where it measured experimentally. The results of the mathematical model are tabulated in Table 4.

Table 4 Explain detail of mathematical model.

| Models         | water level at different stations |
|----------------|----------------------------------|
|                | X=0    | X=0.05 | X=0.35 | X=0.65 | X=0.95 | X=1.25 | X=1.55 |
| Very coarse sand | 0.75   | 0.73   | 0.62   | 0.525  | 0.445  | 0.375  | 0.33   |
| Coarse sand    | 0.75   | 0.733  | 0.64   | 0.545  | 0.47   | 0.4    | 0.35   |
| Medium sand    | 0.75   | 0.733  | 0.65   | 0.57   | 0.495  | 0.43   | 0.375  |
| Fine sand      | 0.75   | 0.74   | 0.66   | 0.59   | 0.52   | 0.46   | 0.41   |

Table 5 presents the percentage error calculated as a result from the difference between the experimental results measured from each model of this study and the mathematical results at different stations. One can observe that the mathematical model adequately predicted the water level in the piezometers fixed within the embankment model.

Table 5 Comparison between experimental and mathematical results of the phreatic line for different soil types.

| Stations (m) | Experiment result (Exp. R.) | Analytical result (Ana. R.) | Error% |
|--------------|-----------------------------|-----------------------------|--------|
| Very coarse sand | 0.00 | 0.5 | 0.5 | 0 |
|               | 0.05 | 0.481 | 0.48 | 0.21 |
|               | 0.35 | 0.365 | 0.37 | 1.37 |
|               | 0.65 | 0.275 | 0.275 | 0 |
|               | 0.95 | 0.199 | 0.195 | 2.01 |
|               | 1.25 | 0.123 | 0.125 | 1.63 |
|               | 1.55 | 0.081 | 0.08 | 1.23 |
| Coarse sand   | 0.00 | 0.5 | 0.5 | 0 |
|               | 0.05 | 0.484 | 0.483 | 0.21 |
|               | 0.35 | 0.391 | 0.39 | 0.26 |
|               | 0.65 | 0.301 | 0.295 | 1.99 |
|               | 0.95 | 0.225 | 0.22 | 2.22 |
|               | 1.25 | 0.149 | 0.15 | 0.67 |
|               | 1.55 | 0.101 | 0.1 | 0.99 |
| Medium sand   | 0.00 | 0.5 | 0.5 | 0 |
|               | 0.05 | 0.490 | 0.483 | 1.43 |
|               | 0.35 | 0.412 | 0.4 | 2.91 |
|               | 0.65 | 0.323 | 0.32 | 0.93 |
|               | 0.95 | 0.252 | 0.245 | 2.78 |
|               | 1.25 | 0.176 | 0.18 | 2.27 |
|               | 1.55 | 0.128 | 0.125 | 2.34 |
| Fine sand     | 0.00 | 0.5 | 0.5 | 0 |
|               | 0.05 | 0.492 | 0.49 | 0.41 |
|               | 0.35 | 0.415 | 0.41 | 1.20 |
|               | 0.65 | 0.348 | 0.34 | 2.3 |
|               | 0.95 | 0.278 | 0.27 | 2.88 |
|               | 1.25 | 0.215 | 0.21 | 2.32 |
|               | 1.55 | 0.164 | 0.16 | 2.44 |

Also for each model the mathematical and experimental results are drawn in Figures 5, 6, 7 and 8. These figures clearly show that the agreement between these results is fairly good.
In the present study, the ability of mathematical model to compute water table elevation is investigated. The study is limited for homogenous embankment. The results demonstrate that a one-dimensional mathematical model is a suitable tool for simulating flow through porous media. The simulation of full-scale flow through porous media is possible using a mathematical modeling in this way, without the cost of producing physical models for a prototype, also with using a mathematical model the scale effect can be overcome.

The methodology used here relies on validation of the mathematical model for a similar flow case using physical experiment data. The study confirmed that the water table situation through porous media studied here could be extensively dependent in porous media particle size.

The results show that the maximum water table elevation can be obtained with greater particle size. The comparison between experimental and analytical result show good agreement, which is, can be considered in future with different media situations. The maximum variation between the analytical and experimental results are found 2.88, while with numerical method was found 10% and with fragment method was 18% as stated in literature.
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