Visible Energy Alternative to Dark Energy

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Abstract

Quantum gravitational effects usually are assumed to be important on small scale (Planck scale), but actually these effects are also very significant on large (cosmological) scales. It is recognized that in curved spacetime, the existence of a minimal measurable momentum is inevitable. In this paper, we study thermodynamic properties of the late time universe in the presence of a minimal measurable momentum cutoff that encodes infra-red modification of the underlying field theory. In this regard, we consider a non-relativistic regime and show that the existence of a minimal measurable momentum in the very essence of the theory leads to accelerating expansion of the universe, which can be interpreted as an alternative to Dark Energy. The universe in this model has experienced the phantom line crossing in the near past.

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1 Introduction

It is well known that alternative scenarios to quantum gravity proposal demonstrate the existence of an inherent ultraviolet (UV) cutoff in nature [1, 2, 3, 4, 5, 6, 7]. The effects of this natural cutoff are not only noticeable at short distances (Planck scales). On the other hand, the gravitational effects are not only noticeable at short distances; they are also remarkable at large distances (cosmological scales). The curvature of the spacetime at large scales refers to an infrared (IR) cutoff [8]. It should be emphasized that gravity is a nonlinear field theory and the particle moving along a curved spacetime geodesic affects the geodesic itself. At the same time, the residual effects of gravity don’t allow the particle’s energy and therefore its momentum to be zero. Therefore, it is logically consistent to assume that there is a minimal uncertainty in momentum.

In this situation along with minimal momentum effects (which are inherent features of spacetime), many of the basic concepts of standard quantum mechanics lose their degree of credibility. There is even a question in this background: Is essentially spacetime discrete or continuous? This point has been widely studied in recent years [4, 8, 9, 10]. Indeed, the existence of an uncertainty gap in the configuration of spacetime causes the fabric of spacetime to be discrete. Therefore, position and momentum operators are also affected by this feature of spacetime so that to preserve the symmetry of the theory, the corresponding operators must be redefined. For this purpose, we use new position and momentum operators [11].

Another property of this background is that the plane wave is incomprehensible in curved spaces. This implies that there is a bound on the resolution of momenta which can be characterized [8, 12].
We will continue with the prediction that this can be represented as a nonzero minimal uncertainty in the momentum measurement, where we imply the ordinary description of the uncertainty in an observable. The mechanism of the existence of a minimal momentum causes the geometric corrections of spacetime structure. More precisely, it leads to noncommutative spacetime. Also, minimal uncertainty in the momentum can be considered as the regularization of the infra-red (IR) effect in the quantum field theory. In recent years, some attempts are dedicated to probing the role of quantum gravity at the cosmological scales and especially on the late time cosmological dynamics \cite{13, 14}. In this regard, we have investigated the thermodynamics of the universe in our previous work \cite{11} for particles with modified relativistic energy in the presence of natural cutoffs.

Observational data confirms that our universe is currently undergoing an accelerating phase of expansion. However, Einstein’s field equations with the standard matters can not describe this accelerating expansion. In this regard, the cosmologists have suggested that Einstein’s field equations should be modified. This modification can be performed in two major road maps. One way is to modify the energy-momentum side of the field equations by considering some sort of matter fields, such as scalar fields (as dark energy components) \cite{15}. Another way is to modify the geometric side of the field equations, like the $f(R)$ models (as dark geometry components) \cite{16}. There are also some other alternatives to explain late time cosmic speed up and large scale structure formation. For instance, the authors in Ref. \cite{17} have studied cosmology in a teleparallel modified gravity framework. In this regard, they have considered a different spacetime without curvature among a nonzero torsion in contrast to General Relativity. This model has been investigated by the Noether symmetry method and belongs to the second category. Also, in Ref. \cite{18}, the authors have studied nonlocal deformations of teleparallel gravity including its cosmological solutions. They have explained that the nonlocal deformations of teleparallel gravity are inspired by quantum gravitational effects. In addition, in Ref. \cite{19}, the authors have considered the effects of the uncertainty principle on the reliability of cosmological measurements. In this regard, they have used the relation between the Compton wavelength and the rest mass of a particle. The authors have supposed the reduced wavelength as a length measurement, and the definition of the rest mass as $m_0 = \frac{\Delta p}{c}$. In this way, by using the luminosity distance, they have found an equation for mass in terms of the reduced Planck constant, redshift, deceleration, and Hubble parameter. It has been shown that the uncertainty on the photon mass, can be the reason for the $H_0$ tension. In another work, the authors of Ref. \cite{20}, have considered the models based on the Integral Kernel Theories of Gravity, which is a non-local straightforward extension of $f(R)$ gravity. In these models, it is possible to interpret dark energy as a geometric contribution. The authors of this paper have studied the non-local gravity models by considering the so-called Noether Symmetry Approach and have found some interesting analytic cosmological solutions. Another aspect of non-local gravity has been studied in Ref. \cite{21}, where the authors have studied the large-scale structure. It has been shown that the logarithmic correction modifies the gravitational partition function and impacts on properties of the gravitational clustering.

Now, we want to describe the accelerating expansion of the universe by using a new distinct method without any dark component (neither dark energy nor dark geometry). We consider this fact that our universe contains two broad standard classes of particles. One class is relativistic particles, which is relevant to the early universe and the second class is non-relativistic particles, which concerns the late time universe (the thermodynamics of the universe infers that non-relativistic energies dominate the late time universe). Here, we focus on the non-relativistic class of the particles in the presence of a minimal measurable momentum at the late time universe. In this regard, the
study of the thermodynamics of the universe can enhance our understanding of the accelerating phase of the universe in this regime. In this paper, we are seeking to describe the expansion of the universe using thermodynamic approaches. Our motivation for proposing this scenario is that various theories of cosmology have been expanded mainly from a thermodynamic perspective. In this connection, we will answer the question that whether considering the infrared cutoff may be a new approach to explain the accelerating expansion of the universe. The important point in this work is the fact that we consider the quantum gravitational corrections at the low energy limit of the quantum gravity, corresponding to the late time universe. We show that considering this quantum correction, even in its weak field limit, is enough to explain the late time acceleration fascinatingly. In this perspective, there is no need for unknown, dark, and obscure sources to explain late time speed up. This is because that we are dealing with the energy of non-relativistic particles (standard model particles) which are known and transparent. The energy does not have the ambiguities of dark energy, but can be a simple and visible alternative for it. For this reason, in this article, we will use the phrase “Visible Energy” against “Dark Energy”. Note that, this perspective is studied for the first time in this context and we think it opens new windows and sheds light on a better understanding of how the universe works at the late time.

2 Gravitational effects at the large distances

Generically, there is no concept of a plane wave in the curved spaces at large distances. This consequence can be characterized as a property of spacetime. In other words, it can refer to noncommutative geometry. Also, it should be noticed that the fundamental properties of plane waves are eigenfunctions of the momentum operator and therefore the momentum operator definition should be modified in the noncommutative phase space. In fact, there exists a finite minimal uncertainty in momenta in curved space [22]. In Ref. [8], the author demonstrated IR regulari- zation regarding noncommutative geometries, which suggests the existence of minimal uncertainties in the momenta. From a technical perspective, the representation theory loses its credibility due to infrared regularization, such that we have to use distinct Hilbert space representations. The interrelations between the operators of noncommutative phase space have first been analyzed in Ref. [24]. Note that, by considering the correction, the momenta lose their credibility as generators of translation on the flat space. In specific circumstances, the momenta can generate translation on the curved space. In fact, there is a direct relationship between the existence of minimal uncertainty in the momenta and the nonexistence of the flat (plan) wave [8].

In this paper, we address noncommutative geometric corrections through the canonical commutation relation as follows [11]

\[
[x_i, p_j] = i\hbar(\delta_{ij} + \eta_{ijkl}x^kx^l + ...).
\]  (1)

In this noncommutative phase space, the position operators are allowed to commute with each other \([x_i, x_j] = 0\), and only the momenta are noncommutative \([p_i, p_j] \neq 0\). In this situation, the emergence of a nonzero minimal uncertainty in the momenta can regularize infrared divergencies. Another important issue about this discrete spacetime is that the position and momentum operators will find new definitions. Our suggestion for redefining these operators is as follows

\[
X_i = x_i, \quad P_i = p_i(1 + \eta x^2),
\]  (2)
where \( \eta \) is a small parameter that corresponds to the infrared effect of this hypothesis. Also, \( x \) and \( p \) are the position and momentum operators of the Heisenberg algebra of the standard quantum mechanics.

3 Quantum gravitational effects on the Maxwell-Boltzmann statistics of the late time universe

Observations confirm that our universe is in an accelerating expansion phase today. Since this accelerating expansion was discovered, several investigations have been done in this area, such as various types of models with scalar fields (dark energy models) [15] and \( f(R) \) gravity models (dark geometry models) [16]. In this paper, we seek to investigate this phase of accelerating expansion by considering the effects of quantum gravity. Besides, one of the important features of the physics of the universe is its thermodynamic properties. In light of this issue, we study the thermodynamic properties of the late time universe by considering the gravitational effects (in the form of an invariant infrared cutoff as a minimal measurable momentum), starting from statistical physics. In this regard, we consider the universe as a gaseous system and put it under the invariant infrared cutoff effect. We show that the presence of this natural cutoff leads to the accelerating expansion of the universe. It seems that the infrared cutoff can be responsible for the late time accelerating phase of the universe expansion.

As we know, the universe consists of different types of particles in distinct degrees of freedom. In this paper, we study the late time universe where the non-relativistic particles are dominant. Therefore, under statistical physics, the distribution function in the non-relativistic regime (for a thermodynamic system in equilibrium) is characterized as follows

\[
f(\vec{p}) = \frac{1}{e^{\frac{E-\mu}{T}}},
\]

Note that, since in the late time universe we have \( T \ll m \), we have used approximation \( e^{\frac{E-\mu}{T}} \approx 1 \approx e^{\frac{E-\mu}{T}} \) in equation (3) (we employ Maxwell-Boltzmann statistics, i.e., we discard the term \( \pm 1 \)). We also apply the non-relativistic prescription for particle energy \( E \). Henceforth, we call this energy “visible energy”. Our motivation for this naming is to show that it is tangible as known energy from known sources, which we will understand more about this case in the following. Now, we obtain the number density \( n \), energy density \( \rho \), and pressure \( P \) for various particles in the late time universe with the above distribution function. This means that particle distribution is performed as a function of phase space. Therefore, to get the number density \( n \), we integrate this quantity over the momentum \( p \)

\[
n = \frac{g}{(2\pi)^3} \int f(\vec{p}) d^3 p,
\]

where \( g \) express internal degrees of freedom and therefore the density of states in the phase space is \( \frac{g}{(2\pi)^3} \). To obtain the energy density \( \rho \), we require to measure each state by its energy \( E = m + \frac{p^2}{2m} \) such that we get

\[
\rho = \frac{g}{(2\pi)^3} \int E(\vec{p}) f(\vec{p}) d^3 p.
\]
Finally, the pressure $P$ is specified as

$$P = \frac{g}{(2\pi)^3} \int |\vec{p}|^2 f(\vec{p}) d^3p , \quad (6)$$

Now, we present some essential thermodynamic quantities which are important in interpreting the accelerating expansion of the universe. We will discuss them in subsequent sections. Accordingly, we explore the thermodynamics of the late time universe by incorporating quantum gravitational effects encoded in the Extended Uncertainty Principle (EUP) that recognizes a minimal measurable momentum. Since the late time universe is essentially an infrared regime, the hypothesis of the equilibrium is applicable. We focus on the thermodynamical quantities in the Maxwell-Boltzmann statistics context in the presence of an IR cutoff due to EUP in the form of

$$\Delta X \Delta P \geq \hbar \left( 1 + \eta (\Delta X)^2 \right) .$$

At this point, we calculate $n$, $\rho$, and $p$ for non-relativistic particles in the presence of infrared cutoff. We find the number density as follows

$$n = \frac{4\pi g}{(2\pi)^3} \int_{P_{\text{min}}}^{\infty} p^2 e^{-\frac{E}{T}} dp = \frac{g e^{-\frac{m}{T}}}{2\pi^2 \xi^3} \left[ \xi P_{\text{min}} e^{-\frac{1}{2} \xi^2 P_{\text{min}}^2} - \sqrt{\frac{\pi}{2}} \left( \text{erf} \left( \frac{\xi P_{\text{min}}}{\sqrt{2}} \right) - 1 \right) \right] , \quad (7)$$

where $\xi = \frac{1+2m^2}{\sqrt{mT}}$ and erf($x$) is the error function. Here it is necessary to point out again that, in Eq. (2), $x_i$ and $p_i$ are operators. In the definition of $\xi$, $x$ is not an operator, but it is an eigenvalue of the operator $x_i$. So, there is no inconsistency between the mentioned relations. Note that in equation (7), just for simplicity and economy, we have introduced $\xi$ which contains the eigenvalue $x$. The reason why we have not absorbed this additional term in $P_{\text{min}}$ is that we want $P_{\text{min}}$ to be an invariant quantity independent of $x$ and $T$. If we absorb this additional term in $P_{\text{min}}$, we would be faced with a minimal momentum that depends on temperature and position. This obviously breaks the invariance of $P_{\text{min}}$.

In this work, we neglect the chemical potential ($\mu = 0$), because at the late time and in the non-relativistic limit where $T \ll m$, this quantity is much smaller than the mass, that is, $\mu \ll m$ [25].

The important point in these calculations is that the integration range does not start from zero. In fact, due to the existence of the minimal measurable momentum constraints in the spacetime structure, the permissible range of these integrations is considered from $P_{\text{min}}$ to infinity.

To determine the energy density, we apply $E \approx m$ and identify from the definition in equation (5) that

$$\rho = \frac{4\pi g}{(2\pi)^3} \int_{P_{\text{min}}}^{\infty} p^2 E e^{-\frac{E}{T}} dp = \frac{mg e^{-\frac{m}{T}}}{2\pi^2 \xi^3} \left[ \xi P_{\text{min}} e^{-\frac{1}{2} \xi^2 P_{\text{min}}^2} - \sqrt{\frac{\pi}{2}} \left( \text{erf} \left( \frac{\xi P_{\text{min}}}{\sqrt{2}} \right) - 1 \right) \right] . \quad (8)$$

This equation is also consistent with $\rho = mn$, where $n$ is given by equation (7).

It should be noted that at the late time universe and in the nonrelativistic limit, $p \ll m$ and therefore we can apply the approximation $E \approx m$. Indeed, we apply this approximation just for coefficient of the exponential term. Since in the exponential term there is factor $\frac{E}{T}$, we are not allowed to apply the approximation $p \ll m$, because the temperature is concerned with the average kinetic energy as $T \approx \frac{p^2}{2m}$. Considering that in the late time universe the temperature has a small
value, the expression $\frac{p^2}{2mT}$ is not small anymore. Therefore, we cannot ignore this term in $e^{-E/T}$. That’s why we are using $E = m^2 + \frac{p^2}{2m}$ in the exponential term. Note also that if we discard the application of this approximation, the integrals cannot be solved analytically and only numerical solutions are possible (for example, the integral of Eq. (9)). In fact, without this approximation, it is impossible to obtain an analytical solution for the number density, the energy density, and the pressure as a function of temperature. To find more details about this issue, see Lecture 6 in Ref. [26].

Finally, from equation (6) we find the following expression for the pressure

$$P = \frac{4\pi g}{(2\pi)^3} \int_{P_{min}}^{\infty} p^4(1 + \eta x^2)^2 \frac{1}{3E} e^{-E/T} dp = \frac{gT}{2\pi^2} \left[ \xi P_{min}(1 + \frac{1}{2} \xi^2 P_{min}^2) \exp\left(-\frac{1}{2} \xi^2 P_{min}^2\right) \right. \left. - \sqrt{\frac{\pi}{2}} \left( \text{erf}\left(\frac{\xi P_{min}}{\sqrt{2}}\right) - 1 \right) \right] = nT + \frac{gT}{6\pi^2} P_{min}^3 \exp\left(-\frac{1}{2} \xi^2 P_{min}^2 - \frac{m}{T}\right). \quad (9)$$

The re-scaled pressure of the non-relativistic particles versus temperature, with a minimal momentum effect in the late time universe, is plotted in figure 1. As the figure shows, in our model, the effective pressure is negative which is an interesting and favorite result. A remarkable issue about this result is that we got a negative pressure without cosmological constant, scalar fields, or modified gravity. In fact, it can be interpreted that the natural infrared cutoff as a quantum gravitational effect itself has the potential to describe the late time accelerating expansion of the universe. We emphasize that this effect is achieved only with quantum effects and standard model particles that have a well-known, positive definite energy.

### 4 Cosmic Equation of State Parameter and Quantum Gravitational Effects

The equation of state parameter for the standard, non-interacting, non-relativistic particles (dust matter) is $w = 0$. However, as we have shown in previous sections, energy density and pressure
Figure 2: The equation of state parameter versus temperature $T$ (in Kelvin) in the presence of the minimal momentum for $\eta = -0.722 \times 10^{-47}$. In the right panel, we have zoomed out the left panel to show the phantom line crossing more clearly.

are modified in the presence of a minimal measurable momentum. Therefore, the equation of state parameter of the standard non-relativistic particle in the presence of quantum gravitational effect takes the following form

$$w = \frac{p}{\rho} = \frac{\left(1 + \frac{1}{3} \xi^2 \right) \xi P_{\text{min}} e^{-\frac{\xi^2 P_{\text{min}}^2}{2}} - \sqrt{\frac{\pi}{2}} \left[ \text{erf} \left( \frac{\xi P_{\text{min}}}{\sqrt{2}} \right) - 1 \right]}{\left( \xi P_{\text{min}} e^{-\frac{1}{3} \xi^2 P_{\text{min}}^2} - \sqrt{\frac{\pi}{2}} \left[ \text{erf} \left( \frac{\xi P_{\text{min}}}{\sqrt{2}} \right) - 1 \right] \right)} T.$$  \quad (10)

As indicated, when the effects of quantum gravity are ignored ($\eta = 0$), the equation of state parameter goes to $\frac{T}{m}$. Given that in the late time universe $T \ll m$, so $w$ tends to zero as in the standard model and as required. Moreover, the equation of state parameter in the standard model is not exactly zero, but just close to zero.

Now, we perform numerical analysis on the equation of state parameter. In this regard, we adopt some sample values for the $\eta$ parameter. Note that, both positive and negative values of $\eta$ are physically viable. In our model, the negative values of $\eta$ provide more interesting results. It is important to note that in various scenarios of GUP in the form $[x, p] = i\hbar(1 + \beta p^2)$, the parameter $\beta$ can be negative \cite{27}. In this work, we have $[x, p] = i\hbar(1 + \eta x^2)$, where $\eta = \frac{\eta_0}{\ell_{pl}^2}$. By considering the negative values of $\eta$, when $\eta_0 x^2 \to \ell_{pl}^2$, we get instantly $[x, p] \to 0$. In other words, the negative values of $\eta$ can indicate the existence of a classical universe. Strictly speaking, the classical trajectories in phase spaces of quantum systems (including the universe) are the most probable in essence. Accordingly, the negative values of $\eta$ allow us to have the classical modes as well.

Figure 2 shows the behavior of the equation of state parameter versus the temperature in the presence of the quantum gravitational effects as a minimal measurable momentum. Crossing the phantom divide line is also manifest in this plot for $\eta = -0.722 \times 10^{-47}$. For this value of $\eta$, the phantom line crossing occurs at $T = 3.4K$. Also, the current value of $w$ in our setup is consistent with observational data ($w = -1.03 \pm 0.03$ \cite{28}). In fact, the behavior of the equation of state
The phase space of $\eta$ and $T$ which leads to the observationally viable values of the equation of state parameter. The shaded pink region shows the allowed values of $\eta$ at the various times in cosmic history. The parameter shows the accelerating expansion of the universe in this model. Obviously, the effective role of the quantum corrections of gravity has led to this interesting result, without any vague justification. In other words, we were able to find an alternative to dark energy, except that it is not dark but visible alternative. In simple terms, visible energy described in this framework is a substitute to dark energy without any darkness.

In figure 3, the phase space of $\eta$ and $T$ leading to observationally viable values of the equation of state parameter is shown. The shaded pink region in the plot shows the allowed values of $\eta$ leading to $w = -1.03 \pm 0.03$.

5 Conclusion

Quantum gravity, though has not been well-formulated yet, phenomenologically requires that the fabric of spacetime inherently includes some natural cutoffs. As a result, the spacetime manifold finds a discrete and lattice structure in quantum gravity regime. Therefore, standard quantum mechanics under these natural cutoffs should be automatically modified. These natural cutoffs are more effectively depending on the scale of spacetime we are dealing with: at short distances (Planck scales) one has a minimal measurable length and at large distances (cosmological scales) one has a minimal measurable momentum. In this paper, we have studied quantum gravitational effects at large distances as a minimal measurable momentum (natural infrared cutoff).

With this motivation, we have investigated thermodynamics of the late time universe for non-relativistic particles in the presence of a minimal measurable momentum. In this regard, we have considered a gaseous system and applied certain quantum modifications to the energy of these particles. We have called this energy as “Visible Energy” because this is tangible as known energy from known sources. Then, consistent with statistical physics, the distribution function in a non-relativistic framework has been corrected. Accordingly, we have obtained some thermodynamic quantities, such as the number density, energy density and pressure in this IR-modified setup. A phenomenal issue about the modified pressure is that we have obtained a negative pressure without cosmological constant, scalar fields, or modified gravity. In fact, this result shows that the natural
infrared cutoff as a quantum gravitational effect can itself describe the late time accelerating expansion of the universe. We stress that this consequence is obtained only with quantum gravitational effects and standard model particles that have explicit and well-known energy. Finally, we have acquired a modified equation of state parameter as an important cosmic quantity that determines the behavior of the accelerating expansion of the universe. It is so interesting that this important result has been obtained just by taking quantum gravitation effects into account. In fact, this framework provides a safe alternative for dark energy without any darkness without recourse to some unknown components as dark energy. In other words, our Visible Energy is an alternative to Dark Energy without darkness. The strength of our model is that we have described the accelerating expansion of the universe and transition to the late time phantom phase with standard particles in the presence of infrared cutoff. Finally we note that the value of the temperature and the equation of state parameter in the time of transition to phantom phase are in very good agreement with the known values from dark energy models [29].

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