Exclusive $J/\psi$ photoproduction and gluon polarization

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Abstract:

In exclusive $J/\psi$ production by polarized photons incident on polarized protons, a finite polarization asymmetry arises because of $c\bar{c}$ Fermi-motion and binding-energy effects. The asymmetry depends on the polarized nonforward gluon distribution of the proton and thus gives information on gluon polarization in the proton. The analyzing power, however, is rather small.

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Gluon polarization in the proton is a topic of current interest in hadronic physics (see e.g. [1, 2]). A useful probe of gluon distributions is the production of open or hidden charm, because charm quarks dominantly couple to gluons and not to light quarks in the proton. In this paper we discuss what the exclusive photoproduction of $J/\psi$ mesons could tell about the polarized gluon content of the proton.

In general, hard exclusive processes probe nonforward matrix elements of the target nucleon, i.e. matrix elements of QCD operators between nucleon states of different momenta [3, 4]. These matrix elements can be expressed in terms of nonforward parton distributions, which are functions of two momentum-fraction variables and generalize the usual (forward) parton distributions. Nonforward distributions reduce to forward distributions in the limit of equal proton momenta in the initial and final states. This limit cannot be realized in exclusive reactions because of simple kinematical reasons, but may be a good approximation at high energy [5, 6, 7]. Thus exclusive $J/\psi$ production in electron-proton collider experiments, e.g. at HERA, could yield information on the small $x$ behaviour of the usual gluon distribution of the proton, whereas fixed-target experiments will probe the nonforward parton distributions in a more general case.

The dependence of unpolarized exclusive $J/\psi$ cross sections on unpolarized gluon distributions has been discussed in Refs. [8, 9]. In principle, $J/\psi$ production by polarized beams on polarized targets should depend on polarized gluon distributions $\Delta G$. Unfortunately, contrary to earlier belief, the
production amplitude does not depend on $\Delta G$ in the first approximation (i.e. assuming a nonrelativistic $J/\psi$ bound state), as we showed in a recent paper [10].

However, as will be shown in detail in this paper, relativistic corrections to $J/\psi$ production will depend on $\Delta G$. Gluon polarization can therefore be accessed in $J/\psi$ production, although the analyzing power may be rather small.

The expansion of $J/\psi$ amplitudes around the nonrelativistic limit was first discussed by Keung and Muzinich [11]. Their approach is based on expanding the perturbative amplitudes for $c\bar{c}$ production in powers of the heavy quark relative momentum. Higher Fock states of the $J/\psi$ are neglected, and the calculation is therefore not gauge invariant. The issue of gauge invariance was discussed more recently by Khan and Hoodbhoy [12]. They pointed out that the gluonic contributions necessary to restore gauge invariance are proportional to $v^3$, where $v$ is the relative velocity of the charm quark and antiquark. Gluonic contributions are thus subleading as compared to the first relativistic corrections in the Keung–Muzinich approach, which are proportional to $v^2$.

We shall now calculate the $O(v^2)$ corrections to exclusive $J/\psi$ production following [11, 12]. We work in the photoproduction limit (photon virtuality $q^2 = -Q^2 = 0$) and consider only the case of collinear scattering.

In terms of nonforward parton distributions $G$ and $\Delta G$, the helicity am-
plitude for
\[
\gamma(q, \lambda) + p(p_1, S = \pm 1/2) \rightarrow J/\psi(K, \lambda' = \lambda) + p(p_2, S' = \pm 1/2)
\] (1)
reads \[10\]
\[
A_{\lambda\lambda \pm \pm} = \frac{1}{2} \int_{-1}^{1} du \frac{1}{(u - \xi + i\epsilon)(u + \xi - i\epsilon)} \times \sum_{\lambda_1} A_{\lambda\lambda \lambda_1 \lambda_1} \left[ G(u, \xi; \mu^2) \pm \lambda_1 \Delta G(u, \xi; \mu^2) \right],
\] (2)
where \( \xi = M^2/(2s - M^2) \) with \( s = (q + p_1)^2 \). The helicity amplitude \( A_{\lambda\lambda' \lambda_1 \lambda_2} \) for
\[
\gamma(q, \lambda) + g(k_1, \lambda_1) \rightarrow J/\psi(K, \lambda') + g(k_2, \lambda_2),
\] (3)
where
\[
k_1 = \frac{u + \xi}{1 + \xi} p_1,
\] (4)
is the convolution of a perturbative \( c\bar{c} \) production amplitude\[11\] \( H_{\lambda\lambda \lambda_1 \lambda_2} \) and a \( J/\psi \) matrix element (both are Dirac matrices):
\[
A_{\lambda\lambda' \lambda_1 \lambda_2}(q, K, k_1) = \text{Tr} \int \frac{d^4\ell}{(2\pi)^4} H_{\lambda\lambda_1 \lambda_2}(q, K, k_1; \ell) \int d^4 x e^{ix\cdot x} \left( K, \lambda' \right| \psi(x/2) \bar{\psi}(-x/2) \left| 0 \right>.
\] (5)
There are six terms in \( H_{\lambda\lambda_1 \lambda_2} \), corresponding to permutations of the photon and the two gluons on the charm quark line. A representative one, corresponding to the diagram in Fig. \[11\] is
\[
H_{\lambda\lambda_1 \lambda_2}(q, K, k_1; \ell)
\] (6)
\[
\sim \psi(q, \lambda) S_F(K/2 - q + \ell) \psi(k_1, \lambda_1) S_F(-K/2 - k_2 + \ell) \psi^*(k_2, \lambda_2),
\] (6)
where \(S_F(p) \equiv (\slashed{p} - m_c)^{-1}\). We have omitted normalization factors which cancel in a polarization asymmetry calculation. Expanding the hard amplitude to second order in \(\ell\), we obtain

\[
A_{\lambda_1 \lambda_2}(q, K, k_1)
= \text{Tr} \left[ H_{\lambda_1 \lambda_2}(q, K, k_1; \ell) \right] \left[ \psi(x/2) \bar{\psi}(-x/2) \right]_{x=0}
+ \frac{\partial}{\partial \ell^\alpha} H_{\lambda_1 \lambda_2}(q, K, k_1; \ell) \left[ \psi(x/2) \left( -i \bar{\gamma}_\alpha \right) \bar{\psi}(-x/2) \right]_{x=0}
+ \frac{1}{2} \frac{\partial^2}{\partial \ell^\alpha \partial \ell^\beta} H_{\lambda_1 \lambda_2}(q, K, k_1; \ell) \left[ \psi(x/2) \left( -i \bar{\gamma}_\alpha \right) \bar{\psi}(-x/2) \right]_{x=0}
\times \left[ \psi(x/2) \left( -i \bar{\gamma}_\beta \right) \bar{\psi}(-x/2) \right]_{x=0}.
\]

The \(J/\psi\) matrix elements can be expressed as

\[
\langle K, \lambda' | \psi(x/2) \bar{\psi}(-x/2) | 0 \rangle \bigg|_{x=0} = \frac{1}{2} M^{1/2} \left( \phi + \frac{\nabla^2 \phi}{M^2} \right) q^* \left( 1 + \frac{K}{M} \right) - \frac{1}{6} M^{1/2} \nabla^2 \phi q^* \left( 1 - \frac{K}{M} \right),
\]

\[
\langle K, \lambda' | \psi(x/2) \left( -i \bar{\gamma}_\alpha \right) \bar{\psi}(-x/2) | 0 \rangle \bigg|_{x=0} = -\frac{1}{3} M^{3/2} \nabla^2 \phi q^* \left( g_{\alpha \beta} + i \epsilon_{\alpha \beta \mu} \gamma^\mu \gamma_5 \frac{K_\nu}{M} \right),
\]

\[
\langle K, \lambda' | \psi(x/2) \left( -i \bar{\gamma}_\alpha \right) \left( -i \bar{\gamma}_\beta \right) \bar{\psi}(-x/2) | 0 \rangle \bigg|_{x=0} = \frac{1}{6} M^{5/2} \nabla^2 \phi q^* \left( g_{\alpha \beta} - \frac{K_\alpha K_\beta}{M^2} \right) \left( 1 + \frac{K}{M} \right).
\]

The amplitude given by eqs. (7-10) is gauge invariant at \(O(\nu^2)\). In eqs. (8-10), \(\phi\) and \(\nabla^2 \phi\) are the \(J/\psi\) meson’s Coulomb-gauge wavefunction and its Laplacian at the origin. For a wavefunction \(\phi \sim e^{-r/a}\), corresponding to a \(1/r\) heavy-quark potential appropriate for a heavy quarkonium state, \(\nabla^2 \phi\)
Figure 1: One of 6 Feynman diagrams which contribute to the amplitude (3). From Ref. [10].

is actually infinite at the origin. The quantity $\nabla^2 \phi$ in eqs. (8-10) has to be understood as representing the average of the Laplacian over a region of volume $\sim 1/m_c^3$ and thus regarded as a free parameter.

Further corrections of the same magnitude arise because the quark mass $m_c$ which appears in perturbative propagators is not exactly one half of the physical charmonium mass which appears in the matrix elements (5-9). These binding-energy corrections are obtained by expanding the amplitude to first order in $\epsilon_B = 2m_c - M$.

The parton-level amplitude simplifies to

$$A_{\lambda\lambda_1\lambda_2}(q, K, k_1) \sim \left(1 - \frac{\epsilon_B}{2M} + \frac{\nabla^2 \phi}{3M^2 \phi}\right)(\epsilon_1 \cdot \epsilon_2)(\epsilon_\gamma \cdot \epsilon_\psi)$$
\[ + \left( \frac{\epsilon_B}{2M} - \frac{\nabla^2 \phi}{M^2 \phi} \right) \frac{M^2}{\hat{s} - M^2} \left[ M^2(\epsilon_1 \cdot \epsilon_2^*)(\epsilon_\gamma \cdot \epsilon_\psi^*) \\
- \hat{s}(\epsilon_1 \cdot \epsilon_\psi^*)(\epsilon_\gamma \cdot \epsilon_2^*) + (\hat{s} - M^2)(\epsilon_1 \cdot \epsilon_\gamma)(\epsilon_\psi^* \cdot \epsilon_2^*) \right], \tag{11} \]

where \( \hat{s} = (q + k_1)^2 \) and \( \epsilon_1, \epsilon_2, \epsilon_\gamma, \epsilon_\psi \) are the usual transverse polarization vectors for the gluons, the photon and the \( J/\psi \). Inserting these into the hadron-level amplitude (2) gives

\[ A_{\lambda\lambda \pm \pm} \sim \int_{-1}^{1} du \left\{ \frac{1}{u - \xi + i\epsilon} - \frac{1}{u + \xi - i\epsilon} \right\} G(u, \xi; \mu^2)[1 + O(\eta)] \\
\pm \lambda \xi \eta \left\{ \frac{1}{(u - \xi + i\epsilon)^2} - \frac{1}{(u + \xi - i\epsilon)^2} \right\} \Delta G(u, \xi; \mu^2) \right\}, \tag{12} \]

where

\[ \eta = \frac{\epsilon_B}{M} - 2 \frac{\nabla^2 \phi}{M^2 \phi}. \tag{13} \]

In order to estimate the magnitude of the asymmetry by using model distributions, we shall employ the formalism of nonforward double distributions \( G(x, y) \) introduced by Radyushkin [4]. Using

\[ G(u, \xi) = \int_0^1 dx \int_0^{1-x} dy G(x, y) \delta(u - [x + (x + 2y - 1)\xi]) \tag{14} \]

and a similar relation for the polarized distribution, we can express the beam-target polarization asymmetry as

\[ \frac{d\sigma(\uparrow\uparrow) - d\sigma(\uparrow\downarrow)}{d\sigma(\uparrow\uparrow) + d\sigma(\uparrow\downarrow)} = \eta \frac{\text{Re}(I_G^* I_{\Delta G})}{|I_G|^2} \tag{15} \]

where

\[ \text{Re} I_G = 2 \int_0^1 dx \int_0^{1-x} dy \ln |(x + 2y)^2 - (\bar{\omega} x)^2| \frac{\partial G(x, y)}{\partial y}, \tag{16} \]
\[
\text{Re } I_{\Delta G} = -\int_0^1 dx \int_0^{1-x} dy \ln \left| \frac{x + 2y - \bar{\omega}x}{x + 2y + \bar{\omega}x} \right| \frac{\partial^2 \Delta G(x, y)}{\partial y^2}, \tag{17}
\]
\[
\text{Im } I_G = -2\pi \int_0^{2/(1+\bar{\omega})} dx G(x, y) \bigg|_{y=(\bar{\omega}-1)x/2}, \tag{18}
\]
\[
\text{Im } I_{\Delta G} = \pi \int_0^{2/(1+\bar{\omega})} dx \frac{\partial \Delta G(x, y)}{\partial y} \bigg|_{y=(\bar{\omega}-1)x/2} \tag{19}
\]

(we used the notation \( \bar{\omega} = 1/\xi \)). Thus the asymmetry is proportional to an unknown parameter \( \eta \) and depends on integrals of both \( G \) and \( \Delta G \). Derivatives of the distributions appear because we integrated by parts.

The values of the parameters \( \epsilon_B/M \) and \( \nabla^2 \phi/(M^2 \phi) \) were estimated in [12], where it was reported that the choice \( \epsilon_B/M = -0.076 \) and \( \nabla^2 \phi/(M^2 \phi) = -0.073 \) gives agreement with data on charmonium decays and inelastic \( J/\psi \) photoproduction. In accordance with this, we shall use \( \eta = 0.07 \) below.

We now evaluate the integrals (16-19) for simple model distributions and plot the resulting polarization asymmetry. Following the discussion of Ref. [14], we choose

\[
G(x, y) = \frac{30}{(1-x)^5} [y(1-x-y)]^2 x g(x), \tag{20}
\]
\[
\Delta G(x, y) = \frac{30}{(1-x)^5} [y(1-x-y)]^2 x \Delta g(x). \tag{21}
\]

We use the GRV-LO unpolarized gluon distribution \( g(x) \) [15] and the Gehrmann–Stirling A(LO) polarized distribution \( \Delta g(x) \) [16], evaluated at \( Q^2 = 4 \text{(GeV)}^2 \). Fig. 2 shows the asymmetry \( \eta \text{Re}(I_G^* I_{\Delta G})/|I_G|^2 \) for \( \eta = 0.07 \) as a function of photon energy in the proton rest frame. The asymmetry is of \( O(10^{-2}) \) in the photon energy range relevant for fixed-target energies. At photon energies relevant for e.g. the HERA collider experiments, the
asymmetry drops by an order of magnitude.

In summary, we have shown that a small but finite polarization asymmetry arises from charm quark Fermi-motion and binding-energy corrections to polarized exclusive $J/\psi$ photoproduction. The asymmetry depends on particular integrals of the polarized and unpolarized nonforward parton distributions of the proton. Using a simple model for parton distributions and previous estimates of $J/\psi$ parameters, we estimate that the asymmetry may be of $O(10^{-2})$ at fixed-target energies and $O(10^{-3})$ at collider energies.

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