Rotational Dependence of Large-scale Dynamo in Strongly-stratified Convection: What Causes It?

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ABSTRACT

In a rigidly-rotating magnetohydrodynamic (MHD) system with convective turbulences, a large-scale dynamo, categorized as the $\alpha^2$-type, can be excited when the spin rate is large enough. In this paper, the rotational dependence of the $\alpha^2$-type dynamo and the cause of it are explored by mean-field (MF) dynamo models coupled with direct numerical simulations (DNSs) of MHD convections. Bearing the application to the solar/stellar dynamo in mind, we adopt a strongly-stratified polytrope as a model of the convective atmosphere. Our DNS models show that the $\alpha^2$-type dynamo is excited when $R_\text{O} \lesssim 0.1$ where $R_\text{O}$ is the Rossby number defined with the volume-averaged mean convective velocity. From the corresponding MF models, we demonstrate that the rotational dependence of the $\alpha^2$-type dynamo is mainly due to the change in the magnitude of the turbulent magnetic diffusion. With increasing the spin rate, the turbulent magnetic diffusion weakens while the $\alpha$-effect remains essentially unchanged over the convection zone, providing the critical point for the excitation of the large-scale dynamo. The $R_\text{O}$-dependence of the stellar magnetic activity observable in the cool star is also discussed from the view point of the rotational dependence of the turbulent electro-motive force. Overall our results suggest that, to get a better grasp of the stellar dynamo activity and its $R_\text{O}$-dependence, it should be quantified how the convection velocity changes with the stellar spin rate with taking account of the rotational quenching and the Lorentz force feedback from the magnetic field on the convective turbulence.

Keywords: convection – magnetohydrodynamics (MHD) – Stars: magnetism

INTRODUCTION

Almost all of the late-type main-sequence (MS) stars, including our Sun, emit X-rays from a magnetically-confined plasma known as a corona. Since the stellar coronae are thought to be heated by the release of magnetic energy generated by a magnetic dynamo process, the stellar X-ray luminosity is considered as an important proxy for the stellar magnetic activity. A lesson of stellar observations is that the stellar X-ray luminosity, i.e., the level of the magnetic activity, in solar and late-type stars is well-regulated by the spin period of the stars (e.g., Pallavicini et al. 1981; Noyes et al. 1984; Pizzolato et al. 2003; Wright et al. 2011). Qualitatively, the younger stars with the shorter spin period, the more magnetically-active it becomes. As the star ages and spins down due to the rotational braking (e.g., Skumanich 1972), the stellar magnetic activity becomes weaker. Actually, our Sun is an older star with a longer spin period compared to the other solar-type stars, having a relatively lower level of the magnetic activity.

A key quantity to understand the physics lying behind the stellar rotation–activity relationship should be the “Rossby number”, $R_\text{O} \equiv P_{\text{rot}}/\tau$, the ratio of the stellar rotation period, $P_{\text{rot}}$, and the convective turnover time, $\tau$ (Noyes et al. 1984; Pizzolato et al. 2003). This is inferred by statistical studies on a large number of stellar samples with different masses, spin periods and X-ray luminosities. For example, Wright et al. (2011) shows, from a sample of 824 solar and late-type stars, that the power-law slope $L_X/L_{\text{bol}} \propto R_\text{O}^{\beta}$ is well-fitted by $\beta = -2.7$ in the range $R_\text{O} \gtrsim R_{\text{O,sat}} \approx 0.1$, while $\beta \simeq 0.0$, due to the saturation of $L_X$, for very fast rotators with $R_\text{O} \lesssim R_{\text{O,sat}}$, where $L_X/L_{\text{bol}}$ is the ratio of the stellar X-ray to bolometric luminosities. Note that the mass-dependent convective turnover time, based on the mixing-length theory (e.g., Kippenhahn & Weigert 1990), is adopted for the $R_\text{O}$ in these observational studies.

Not only for G-type and F-type dwarfs, composed of convective envelope and radiative core like the Sun, recent observations indicate that the same rotation–activity relationship holds even for fully-convective M-type dwarfs (Wright &
Drake 2016; Wright et al. 2018) and evolved stars in post-MS phases (Lehtinen et al. 2020; Godoy-Rivera et al. 2021). This implies that there exists a universal dynamo scaling and the underlying dynamo mechanism should be common to late-type stars irrespective of their mass or evolutionary stage and the resulting different internal structures. However, theoretical dynamo models still do not have the ability to predict the rotational dependence of the magnetic activity quantitatively.

After the first successful global simulation, presented by Ghizaru et al. (2010), that reproduced the solar-like cyclic large-scale magnetic field, the numerical modeling of the solar and stellar dynamos has made significant progress in the past decade. See Brun & Browning (2017) and Charbonneau (2020) for reviews. While most of recent numerical models have examined the solar dynamo process in greater detail as a prototypical model for stellar dynamos (e.g., Käpylä et al. 2012; Masada et al. 2013; Nelson et al. 2013; Fan & Fang 2014; Augustson et al. 2015; Guerrero et al. 2016; Hotta et al. 2016; Käpylä et al. 2017), few of them have focused on the rotational dependence of the convective dynamo process.

Mabuchi et al. (2015) found that, in the regime $\text{Ro} \lesssim 1$, the stellar differential rotation becomes weaker and the mean kinetic helicity (net helicity) decreases with the Ro, while the convective dynamo becomes more active as Ro decreases. Strugarek et al. (2017) found that from a set of turbulent global simulations that the cycle period of the large-scale magnetic field scales as $\text{Ro}^{-1}$, which is compatible with the “rotation–activity cycle” relationship of the Sun and the other solar-type stars (see also, Strugarek et al. 2018; Warnecke et al. 2018). With focusing on the configuration of the stellar magnetic field, Viviani et al. (2018) found numerically that an axisymmetric (non-axisymmetric) dynamo mode dominates at slow (fast) rotation and the transition between them occurs at around $C_0 \simeq 3.0$, where $C_0 \propto \text{Ro}^{-1}$ is the Coriolis number. From the viewpoint of the dynamo mechanism, Warnecke & Käpylä (2020) found that the trace of turbulent $\alpha$ tensor, which induces an inductive effect, increases with $C_0^{0.5}$ for moderate rotation, and levels off for rapid rotation.

The rotational effect on the convective dynamo has been studied in a Cartesian system as well. A pioneering work is Käpylä et al. (2009) in which they demonstrated, for the first time, that a rigidly-rotating convection can drive a large-scale dynamo even in the absence of the differential rotation. They also found that the turbulent diffusivity decreases monotonically with the spin rate while the turbulent $\alpha$-effect is almost independent of the rotation. As a result, the large-scale dynamo is excited only when the system rotates rapidly enough. See, e.g., Käpylä et al. (2013), Masada & Sano (2014a,b) (hereafter, MS14a,b), and Bushby et al. (2018) for the details of the large-scale dynamo in rigidly-rotating convections.

Although significant progress has been made in the theoretical study of the solar and stellar dynamos, we still do not have a solid explanation on the physics lying behind the stellar rotation–activity relationship. To reveal it, we need to further understand how the rotation affects the property of the convective dynamo. We study, in this paper, the rotational dependence of the large-scale dynamo in rigidly rotating convections. Bearing the application to the solar/stellar dynamo in mind, as with our previous work (Masada & Sano 2016, hereafter MS16), we adopt a strongly-stratified polytrope as a model of the convective atmosphere, that is the main difference from the earlier studies in which the weakly-stratified atmosphere is focused (Käpylä et al. 2009, MS14a,b). A unique approach in our study is to utilize the mean-field (MF) dynamo models coupled with direct numerical simulations (DNSs) of MHD convections to explore the cause of the rotational dependence of the large-scale dynamo. See MS14b for the MF dynamo model linked to the corresponding DNS.

The outline of this paper is as follows. In § 2, a numerical setup and the model of the convective atmosphere are introduced. The results obtained from our DNS runs are presented in § 3. We construct, in § 4, a MF dynamo model, using the DNS results as profiles of the turbulent electromotive force (EMF). Then, we try to pin down the physics which dominates the onset and rotational dependence of the large-scale dynamo. After discussing the stellar rotation–activity relationship from the viewpoint of the rotational dependence of the turbulent EMF, we summarize our findings in § 5.

2. NUMERICAL SETUP

Convective MHD dynamo system is solved numerically in Cartesian domain. Our model covers only the convection zone (CZ) of depth $d$ ($0 \leq z \leq d$), where $x$- and $y$-axes are taken to be horizontal, and $z$-axis is pointing downward. We set the width of the domain to be $W = 4d$. See Figure 1(a) for the setup of the simulation domain.

The following set of equations for fully-compressible MHD is solved in the rotating reference frame with the angular velocity $\Omega = -\Omega_0 e_z$,

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho u) ,$$

$$\frac{\partial u}{\partial t} = -\nabla P + \frac{J \times B}{\rho} - 2\Omega \times u + \nabla \cdot \Pi + g ,$$

$$\frac{\partial \Pi}{\partial t} = -\rho \nabla \cdot u + Q_{\text{heat}} ,$$

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B - \eta_0 J) ,$$

where $D/Dt$ is the total derivative, $\epsilon = c_v T$ is the specific internal energy, $J = \nabla \times B/\mu_0$ is the current density with the vacuum permeability $\mu_0$, and $g = g_0 e_z$ is the gravity of the constant $g_0$. The viscosity, magnetic diffusivity, and thermal conductivity are represented by $\nu_0$, $\eta_0$, and $\kappa_0$, respectively. The magnitude of the angular velocity, $\Omega$, is the control parameter in this study.

The viscous stress $\Pi$ is written by $\Pi = 2\nu_0 S$ with the strain rate tensor

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij} \frac{\partial u_k}{\partial x_k} \right) ,$$

The heating term $Q_{\text{heat}}$ consists of the thermal conduction, viscous and Joule heatings,

$$Q_{\text{heat}} = \gamma \nabla \cdot (\kappa_0 \nabla \epsilon) + 2\nu_0 S^2 + \frac{\mu_0 \eta_0 J^2}{\rho} ,$$

with $\epsilon = c_v T$ and $\gamma = c_p / c_v$ the adiabatic index.
We adopt a perfect gas law $P = (\gamma - 1) \rho e$ with $\gamma = 5/3$ to close the equations.

We assume an initial hydrostatic equilibrium with a polytropic stratification given by
\[
\epsilon = \epsilon_0 + \frac{g_0 z}{(\gamma - 1)(m + 1)}, \quad \text{and} \quad \rho = \rho_0 (\epsilon/\epsilon_0)^m,
\]
where $\epsilon_0$ and $\rho_0$ are the internal initial energy and density at the CZ surface. The polytropic index $m = 1.49$ is adopted in our models, providing the super-adiabaticity of $\delta \equiv \nabla - \nabla_{ad} = 1.6 \times 10^{-3}$, where $\nabla_{ad} = 1 - 1/\gamma$ and $\nabla = (\partial \ln T/\partial \ln P)$. Since the condition $\delta > 0$ is satisfied in whole the domain, the convection in our system is essentially driven by the local entropy gradient, differing from the cooling-driven convection, qualitatively. See, Yokoi et al. (2022) for the difference between local and non-local (cooling) driven convections in turbulent transports (see also, e.g., Spruit 1997; Cossette & Rast 2016).

Normalization quantities are defined by setting $g_0 = \rho_0 = \mu_0 = \epsilon_p = d/2 = 1$. The normalized pressure scale-height at the surface, defined by $\xi = H_p/d_{cz} = (\gamma - 1)\epsilon_0/(g_0 d_{cz})$, controls the stratification level and is chosen here as $\xi = 0.01$, yielding a strong stratification with the density contrast between top and bottom CZs about 700.

Solid lines in Figures 1(b) and 1(c) show the profiles of the density and temperature for the initial setup of our simulation model. The density and temperature profiles in the range $0.71 \leq r/R_\odot \leq 0.991$ of the standard solar model (SSM) are also shown by dashed lines in such a way as to fit the computational domain. As the SSM, we adopt “Model S” in Christensen-Dalsgaard et al. (1996). Our model has a stratification almost encompassing the solar convection zone except its uppermost part.

All the physical variables are assumed to be periodic in horizontal directions. Stress-free boundary conditions are used in the vertical direction for the velocity, perfect conductor and vertical field conditions are used for the magnetic field at the bottom and top boundaries, i.e.,
\[
\begin{align*}
\partial_z u_x &= \partial_z u_y = u_z = 0 \quad (\text{top \& bottom BCs}) , \\
B_x &= B_y = \partial_z B_z = 0 \quad (\text{top BC}) , \\
\partial_z B_x &= \partial_z B_y = B_z = 0 \quad (\text{bottom BC}) .
\end{align*}
\]

A constant energy flux which prompts thermal convection is imposed on the bottom boundary, while the specific internal energy is fixed at the top boundary,
\[
\epsilon = \epsilon_0 \quad (\text{top BC}) , \\
\partial_z \epsilon = \text{const.} \quad (\text{bottom BC}) ,
\]

The fundamental equations are solved by the second-order Godunov-type finite-difference scheme that employs an approximate MHD Riemann solver (Sano et al. 1998). The magnetic field evolves with the Consistent MoC-CT method (Evans & Hawley 1988; Clarke 1996). Non-dimensional parameters $Pr = 20$, $Pm = 2$, $Ra = 6 \times 10^7$, and the spatial resolution of $(N_x, N_y, N_z) = (256, 256, 256)$ are adopted for all the models, where the Prandtl, magnetic Prandtl, and Rayleigh numbers are defined by
\[
Pr = \frac{v_0}{(\kappa_0/\rho c_p)} , \quad Pm = \frac{v_0}{\eta_0} , \quad Ra = \frac{g_0 d^4}{\chi_0 \nu_0 \beta H_p} ,
\]
where $\rho$, $\beta$ (super-adiabaticity), and $H_p$ are evaluated at the mid-part of the CZ, $z = d/2$.

In the following, the volume-, and horizontal-averages are denoted by single angular brackets with subscripts “v”, and “H”, respectively. The time-average of each spatial mean is denoted by additional angular brackets. The convective turn-over time and the equipartition field strength are defined by $\tau_{cv} \equiv 1/(ucv k_L)$ and $B_{eq}(z) \equiv \sqrt{(\mu B^2)_{hv}}$, where $ucv \equiv \sqrt{\langle \langle u_z^2 \rangle \rangle}$ is the mean convective velocity at the saturated state and $k_L \equiv 2\pi/d$ is the estimation on the scale.
of the energy-carrying eddies. Note that $B_{eq}(z)$ is evaluated locally from the convective kinetic energy, and thus has a depth-dependence.

Since the sound speed in the deep CZ becomes very large in the strongly-stratified model and imposes a strict limit on computational time-step, a long thermal relaxation time is required for our fully-compressible simulations. To alleviate it, we first construct a progenitor model, in which the convection reaches a fully-developed state and the system becomes thermally-relaxed, by evolving a non-rotating hydrodynamic run for $800\,\tau_{cv}$. See, Figures 1b and 1c in MS16, as for our progenitor model. By imposing the rotation, which is the control parameter in this study, and then adding a seed magnetic field to the progenitor model, the DNS run of the convective dynamo is started. The imposed angular velocities are $\Omega = 0.05$, $0.1$, $0.25$, and $0.5$ for models 1, 2, 3, and 4, respectively.

3. RESULTS OF THE DNS

3.1. Rotational dependence of convective motion

After the relaxation phase of $t \lesssim 100\tau_{cv}$, each model reaches a saturated state. Note that the convective turn-over time is different among models and is evaluated, at the saturated state, as $\tau_{cv} = 31, 34, 44, 54$ for models 1–4. (e) Two-dimensional spectra of the vertical components of the convection velocity for the models with different $\Omega$.

Figure 2. (a)–(f) Distributions of $u_z$ on the CZ surface at the saturated stage for the models with different $\Omega$. As for the comparison, $u_z$ for the progenitor model is shown in the top left panel. The saturated state is achieved after the relaxation phase of $t \lesssim 100\tau_{cv}$, where $\tau_{cv} = 31, 34, 44, 54$ for the models 1–4. (e) Two-dimensional spectra of the vertical components of the convection velocity for the models with different $\Omega$. The distribution of $u_z$ on the horizontal $x$–$y$ plane at the CZ surface is shown in Figure 2 for the saturated state of each model. The panels (a)–(d) are corresponding to the models 1–4 with $\Omega = 0.05$, $0.1$, $0.25$, and $0.5$, respectively. The darker and lighter tones denote the downward and upward velocities. The convective motion has a strong up-down asymmetry due to the strongly-stratified atmosphere: the broader and slower upflow-dominant cells surrounded by narrower and faster downflow networks (e.g., Hurlburt et al. 1984; Stein & Nordlund 1998; Miesch et al. 2000; Brummell et al. 2002). Since the rotation gives rise to the Coriolis force acting on the convective motion, the size of the convective cell shrinks and thus the scale-separation becomes larger with the increase of $\Omega$ (see, e.g., Brummell et al. 2002; Miesch 2005).

The rotational quenching on the convective motion can be seen more quantitatively in the Fourier space. Shown in Figure 2(e) is the two-dimensional spectra of the vertical component of the convection velocity for the models with different $\Omega$. The distribution of $u_z$ on the horizontal $x$–$y$ plane at the CZ surface is shown in Figure 2 for the saturated state of each model. The upper left panel demonstrates that for the progenitor run (see, MS16). The panels (a)–(d) are corresponding to the models 1–4 with $\Omega = 0.05, 0.1, 0.25$, and $0.5$, respectively. The darker and lighter tones denote the downward and upward velocities.
ponent of the velocity for the models with different $\Omega$. The horizontal axis is normalized by $k_L = 2\pi / d$. Note that the spectrum of $\sqrt{u_z^2}$ at the each depth is projected onto a one-dimensional wavenumber $k^2 = k_x^2 + k_y^2$ and then is averaged over the whole CZ in $z$-direction and given time span ($\approx 20 t_{cv}$) at the saturated stage (e.g., Käpylä et al. 2009). The purple, green, cyan, and orange lines correspond to the models 1–4 with $\Omega = 0.05$, 0.1, 0.25, and 0.5, respectively. The peak amplitude of the spectrum becomes lower and the wavenumber which gives the maximum, $k_{\text{max}}$, becomes larger with the increase of $\Omega$.

As seen in the spectra, the higher the spin rate of the system, the lower the convection velocity becomes. Shown in Figure 3(a) is the dependence of $u_{cv}$ on $\Omega$, where $u_{cv} \equiv \langle u_z \rangle_{x,y}$ is the mean convective velocity. The vertical axis normalized by that of the progenitor model, i.e., $u_{cv,0} \approx 0.013$. We can find that the convection velocity becomes lower with the increase of $\Omega$, following a scaling law of $u_{cv} \propto -0.15 \log \Omega$ with $\beta = 0.15$. Since the compressible convection is accompanied by contraction and expansion of fluid elements due to the change of the ambient density, the fluid motion is, even when the rotational axis and gravity vector are aligned, more strongly constrained by the Coriolis force as $\Omega$ increases. This results in the lower convection velocity at the higher $\Omega$ (e.g., Brummell et al. 2002; Miesch 2005). Actually, we have not figured out why $u_{cv}$ decreases in proportion with $\log \Omega$, such a dependence may be important to understand the why the “saturation” of the magnetic activity occurs in the rapidly-rotating regime of late-type stars (e.g., Wright et al. 2011; Wright & Drake 2016). We discuss the possible relationship between $\Omega$-dependence of $u_{cv}$ and the saturation of the stellar magnetic activity in § 5.3.

The effect of the rotation, i.e., the rotational constraint imposed by the Coriolis force, on the typical size of the convective cell is quantified. Two kinds of the Rossby number are defined here in the following manner:

$$\text{Ro} = \frac{u_{cv} k_L}{2 \Omega}, \quad \text{Ro}_p = \frac{u_{\text{peak}} k_L}{2 \Omega},$$

where $k_L = 2\pi / d$, and $u_{\text{peak}}$ is the convection velocity at the peak of each spectrum in Fig.2(e). Note that $\text{Ro}$ can be evaluated as 0.33, 0.15, 0.045, 0.019 for the models 1–4. We put a special focus on this form of $\text{Ro}$ in the following section.

In Figure 3(b), the dependence of $k_{\text{max}}$ on the inverse of the Rossby number is shown, where $k_{\text{max}}$ is the wavenumber which gives the maximum of the spectrum for each model in Fig.2(e). The red (blue) broken line with circles (diamonds) denotes the dependence on $\text{Ro}$ ($\text{Ro}_p$). The gray dashed lines are reference power-law slopes with $\text{Ro}^{-1/3}$ and $\text{Ro}_p^{-1/3}$.

$k_{\text{max}}$ is found to change according to the scaling relation of $k_{\text{max}} \propto \text{Ro}^{-1/3}$ or $\text{Ro}_p^{-1/3}$, which is consistent with the theoretically expected “$-1/3$” scaling for rotating hydrodynamic convection near onset. See Chandrasekhar (1961) for the linear theory. While the dependence of $k_{\text{max}}$ on $\text{Ro}$ obtained in our DNS runs is compatible with that in the other numerical models (e.g., Schmitz & Tilgner 2010; Stellmach et al. 2014; Viviani et al. 2018; Guervilly et al. 2019), Viviani et al. (2018) argue that the slope becomes steeper ($\propto \text{Ro}^{-1/2}$) in higher-resolution simulations at a rapid rotation regime (see also, Featherstone & Hindman 2016). This implies that the size of convective cells might be more drastically reduced if increasing the resolution and/or the spin rate even in our simulation models.

### 3.2. Rotational dependence of large-scale dynamo

In MS16, we found that cyclic large-scale magnetic components are spontaneously organized in a strongly stratified rotating convection. The model 4 with $\Omega = 0.5$ in this study
is corresponding one examined in MS16. When decreasing \( \Omega \), we can see that the large-scale dynamo becomes gradually inactive and, is finally disappeared in the high \( \Ro \) regime.

Figure 4 depicts the time-depth diagram of \( \langle B_x \rangle_h \) for each model. The red (blue) tone denotes the positive (negative) \( \langle B_x \rangle_x \) in units of \( \B_{eq}(z) \), where \( \B_{eq}(z) = \langle \rho(z)u(z)^2 \rangle_h \) is the equipartition field strength. The time is normalized by \( \tau_{cv} \) for each model. Since turbulent components of the magnetic field are averaged out by taking horizontal mean, only the large-scale magnetic component and its time-space evolution can be observable in this figure. Note that \( \langle B_y \rangle_h \) also shows a similar cyclic behavior with \( \langle B_x \rangle_h \), but with a phase delay of \( \pi/2 \), suggesting that an “\( \alpha^2 \) -type” dynamo is excited in our DNS models (see, e.g., Baryshnikova & Shukurov 1987; Raedler & Braeuer 1987; Brandenburg et al. 2009, MS14a,b). See MS16 for the organization of the magnetic structure at the CZ surface in the case of strongly-stratified atmosphere.

In Figure 4, we can see two kinds of space-time evolution patterns: in the slowly rotating models with (a) \( \Omega = 0.05 \) and (b) 0.1 (models 1 & 2 with \( \Ro = 0.33 \) & 0.15), the large-scale magnetic component starts to grow, but gradually stalls as time passes and finally disappear. On the contrary to them, in the rapidly rotating models with (c) \( \Omega = 0.25 \) and (d) 0.5
(models 3 & 4 with Ro = 0.045 & = 0.019), the oscillatory large-scale magnetic field is spontaneously organized.

Commonly in the successful dynamo models, \( \langle B_x \rangle_h \) has a peak in the mid-part of the CZ and migrates from there to top and bottom Czs. The large-scale magnetic component becomes stronger with the increase of \( \Omega \) (decrease of \( \text{Ro} \)), reaching the super-equipartition in the fastest spinning model. The cycle period of the large-scale magnetic component can be evaluated as \( P_{\text{cyc}} \approx 100 \tau_{\text{cv}} \) for model 3 and \( \approx 150 \tau_{\text{cv}} \) for model 4, implying that \( P_{\text{rot}} / P_{\text{cyc}} \) is an increasing function of \( \text{Ro} \) (decreasing function of \( \text{Co} \)). This seems to be compatible with the correlation between \( P_{\text{rot}} / P_{\text{cyc}} \) and \( \text{Ro} \) found in the global model of the convective dynamo (c.f., Warnecke 2018, and references therein). We will discuss the Ro-dependence of the dynamo cycle in § 5.2.

Since the property of the large-scale dynamo changes between models 2 and 3, the critical Rossby number that separates the success or failure of the large-scale dynamo seems to exist, within the range 0.045 \( \lesssim \text{Ro} \lesssim 0.15 \), at around

\[ \text{Ro}_{\text{crit}} \approx 0.1. \]  

(12)

Intriguingly, the \( \text{Ro}_{\text{crit}} \), which is estimated from our DNS models in a Cartesian geometry, is compatible with that inferred from the convective dynamo simulation in a rotating spherical-shell performed by, e.g., Käpylä et al. (2012) and Warnecke (2018) if we convert our Rossby number to their definition of the Coriolis number, despite the difference of the geometry. This implies that the presence of the large-scale flow, such as the differential rotation and meridional flow, would not have a strong impact on the “excitation” of the large-scale dynamo. Specifically in § 5.1, we compare the \( \text{Ro}_{\text{crit}} \) (or \( \text{Co}_{\text{crit}} \)) between our DNS models and the other global convective dynamo simulations.

4. ANALYSIS WITH MEAN-FIELD DYNAMO MODEL

4.1. Governing Equation and Link to DNS

In the previous section, we verified that the spin rate of the system is essential for exciting the large-scale dynamo. The critical Rossby number that separates the success or failure of the large-scale dynamo, exists at \( \text{Ro}_{\text{crit}} \approx 0.1 \). Then, a question naturally arises, “How does the difference of the Rossby number impact on the physics of the convective dynamo and then change its property?” To explore the answer to it, we utilize one-dimensional mean-field (MF) dynamo model, wherein the turbulent electromotive force (EMF) is determined based on the velocity and helicity profiles directly extracted from the DNS models.

Our MF dynamo model is constructed in a similar manner to Masada & Sano (2014b). The \( \alpha \)-effect is solely responsible for the inductive effect in our MF model, while the \( \Omega \)-effect is ignored there. This is because the angular momentum transport does not occur due to the alignment of the rotation and gravity axes, and thus the differential rotation is not developed in our DNS setup. If the MF model linked to the DNS can reproduce the rotational dependence of the large-scale dynamo observed in the DNS runs, it would be convincing evidence that the essential ingredient for it is manifested in the turbulent EMF. Our aim is to confirm it.

By dividing the variables into the horizontal mean and fluctuating components, \( u = u_m + u' \) and \( B = B_m + B' \), the MF equation for the \( \alpha^2 \)-type dynamo is obtained, from the induction equation, as

\[ \frac{\partial B_m}{\partial t} = \nabla \times [\mathcal{E} - \eta_t \nabla \times B_m], \]  

(13)

with

\[ \mathcal{E} = \alpha B_m - \eta_t \nabla \times B_m, \]  

(14)

where \( \eta_t \) is the microscopic magnetic diffusivity, \( B_m = (B_m^x, B_m^y) \) is the mean horizontal field, and \( \mathcal{E} \) is the turbulent EMF (e.g., Parker 1979; Krause & Raedler 1980; Ossendri- jver et al. 2002; Brandenburg & Subramanian 2005). Note that \( B_m \) has dependences on time and depth. All the terms related to \( u_m \) and \( B_m^x \) can be ignored in considering the symmetry of the system. The MF coefficients \( \alpha \) and \( \eta_t \) represent the turbulent \( \alpha \)-effect and the turbulent magnetic diffusivity. The turbulent pumping effect is ignored in this study for the simplicity. See, e.g., MS14b for the EMF with the turbulent pumping effect.

The mean magnetic component generated by the large-scale dynamo produces a Lorentz force that tends to quench the turbulent motions, and thus control the nonlinear saturation of the system. Since there is no definitive model to describe the quenching effect as yet, we adopt the prototypical model, an algebraic \( \alpha \)-quenching, formulated as

\[ \alpha = \frac{\alpha_k}{1 + \langle B_h \rangle_k^2 / B_{\text{eq}}^2}, \]  

(15)
Figure 6. Time-depth diagrams of $B^m_x$ for the MF models corresponding to the DNS models with (a) $\Omega = 0$ ($Ro = 0.05$), (b) $\Omega = 0.1$ ($Ro = 0.15$), (c) $\Omega = 0.25$ ($Ro = 0.045$), and (d) $\Omega = 0.5$ ($Ro = 0.019$). The red (blue) tone denotes the positive (negative) value of $B^m_x$. The normalization unit for $B^m_x$ and $t$ are $B_{eq}(z)$ and $\tau_{cv}$ evaluated from each DNS model.

(see, e.g., Brandenburg & Subramanian 2005, for the review of the quenching models). Here, the subscript “$k$” refers to the unquenched $\alpha$ coefficient, which is calculated directly from DNS results of the saturated convective turbulence. The characteristic wavenumber $k_c$ and the equipartition field strength at the saturated state, $B_{eq, sat}$, are determined directly from DNS runs as

$$k_c(z) = \frac{2\pi}{H_d(z)}, \quad B_{eq, sat}(z) = \sqrt{\langle\rho u^2\rangle_h},$$

(16)

where $H_d(z) = -dz/d\ln(\langle\rho\rangle_h)$ is the density scale height of the stratified atmosphere. With the first-order smoothing approximation (FOSA), the MF coefficients $\alpha_k$ and $\eta_t$ are determined, in anisotropic forms, by (e.g., Käpylä et al. 2006, 2009),

$$\alpha_k(z) = -\tau_{cor} [\langle u_z \delta_x u_y \rangle_h + \langle u_z \delta_y u_z \rangle_h] \equiv -\tau_{cor} H_{eff},$$

$$\eta_t(z) = \tau_{cor} \langle u_z^2 \rangle_h,$$

(17)

where $H_{eff}$ is the effective helicity, and $\tau_{cor}$ is the correlation time of the convective turbulence. With assuming the Strouhal number is an order of unity in the CZ ($St = \tau_{cor} \sqrt{\langle u_z^2 \rangle_h k_c} \approx 1$), $\tau_{cor}$ is determined, in the form with the depth dependence, by

$$\tau_{cor} = \tau_{cor}(z) = \frac{1}{\sqrt{\langle u_z^2 \rangle_h k_c}},$$

(18)
The vertical profiles of $\alpha_k(z)$ and $\eta_t(z)$, which are determined from the DNS results of the saturated state, are shown in Figure 5 for each model. The purple, green, cyan, and orange curves denote the models 1–4 with $\Omega = 0.05, 0.1, 0.25,$ and $0.5$ ($Ro = 0.33, 0.15, 0.045,$ and $0.019$), respectively. The time average spans in the range $220\tau_{cv} \leq t \leq 250\tau_{cv}$ for each model. Note that the profiles remain almost the same even if we change the range of time average.

It is intriguing that, with our definitions in eq (17), the profiles of $\alpha_k$ or $\eta_t$ on the spin rate is compatible with that seen in the model with the weakly-stratified convection (Käpylä et al. 2009). When seeing the dependence of effective kinetic helicity $H_{eff}$ on $\Omega$, the larger the spin rate, the smaller the $H_{eff}$ becomes, throughout the CZ except near the surface. On the other hand, the correlation time becomes larger with the decrease of $Ro$ ($\eta_t$ decreases with the decrease of $Ro$). $\alpha_k$ has a maximum at around the CZ surface and decreases deeper down with the sign change around the mid-part of the CZ. See, e.g., Brummell et al. (1998) and Miesch et al. (2000) for the depth-dependence of the kinetic helicity induced by rotating convective turbulence. Qualitatively, the dependence of $\alpha_k$ or $\eta_t$ on the spin rate is compatible with that seen in the model with the weakly-stratified convection (Käpylä et al. 2009).

When seeing the dependence of effective kinetic helicity $H_{eff}$ on $\Omega$, the larger the spin rate, the smaller the $H_{eff}$ becomes, throughout the CZ except near the surface. On the other hand, the correlation time becomes larger with the increase of $\Omega$ because $\langle \langle u_z^2 \rangle_h \rangle$ is a decreasing function of $\Omega$ (see Fig.3(a)). As a result of cancellation of the $\Omega$-dependence between $H_{eff}$ and $\langle \langle u_z^2 \rangle_h \rangle$, the profile of $\alpha_k$ becomes almost independent of $\Omega$ in the most part of the CZ.

At the top and bottom boundaries, we can anticipate that the convective turbulence is not fully developed, and thus $\alpha_k = \eta_t = 0$. Should be satisfied there. To smoothly vanish the turbulent EMF at the boundaries, we drop $\alpha_k$ and $\eta_t$ around there by extrapolating their profiles by $\alpha_k(z) = f(z)\alpha_k(z)$ and $\eta_t(z) = f(z)\eta_t(z)$ artificially, with complementary error function,

$$f(z) = \frac{1}{4} \text{erfc} \left( \frac{z_b - z}{w_b} \right) \text{erfc} \left( \frac{z - z_t}{w_t} \right),$$

where $z_i (i = t, b)$ represents the depth which separates the regions with and without fully developed turbulence, and $w_i$ ($i = t, b$) is the width of the transition layer between them. We define $z_t$ and $z_b$ as the depth where $\alpha_k$ achieves the maximum and minimum values, respectively. The width of the transition layer $w_i$ is an arbitrary parameter and determined here so that the amplitude of $\alpha_k$ drops fast enough and becomes absolutely zero at top and bottom boundaries. By solving coupled equations (13)–(18) with substituting $\alpha_k'(z)$ and $\eta_t'(k)$ for $\alpha_k(z)$ and $\eta_t(k)$, we conduct a series of one-dimensional simulation of the MF dynamo corresponding to each DNS model. The boundary conditions for the magnetic field in the MF dynamo model are same as those adopted in the DNS runs (see, eq.(8)).

4.2. Results of MF dynamo model

Shown in Figure 6 is the the time-depth diagram of $B_{x}^{m}/B_{eq}(z)$ for the MF model corresponding to each DNS model. Panels (a)–(d) are the MF counterparts of the DNS models 1–4 with $\Omega = 0.05, 0.1, 0.25,$ and $0.5$ ($Ro = 0.33, 0.15, 0.045,$ and $0.019$). The red (blue) tone denotes the positive (negative) $B_{x}^{m}$. Note that the normalization units $B_{eq}(z)$ and $\tau_{cv}$ are extracted from each DNS model. Note that $B_{x}^{m}$ also shows a similar cyclic behavior with $B_{x}$, but with a phase delay of $\pi/2$ as in the DNS models.

The property of the large-scale dynamo observed in the DNS models are reproduced, at least qualitatively, even in the simple MF dynamo model: in the MF counterparts to the slowly rotating DNS models, $B_m$ initially grows, but gradually decays as time passes, and finally disappear (see panels (a) and (b)). In contrast, in the MF models corresponding to the rapidly rotating DNS models, $B_m$ is maintained with a cyclic periodicity (see panels (c) and (d)). The oscillation period of $B_m$ in the MF model is also similar to that of $\langle B_h \rangle_h$ in the DNS model.

A remarkable difference between the MF and DNS models is the amplitude of the mean magnetic component in the fastest spinning model: while the field strength reaches maximally $\sim 1.75B_{eq}$ in the case of the DNS model at the mid part of the CZ, it shows $\sim 1.2B_{eq}$ in the MF model. On the other hand, when the saturation level is sub-equipartition (i.e., models 1–3), the amplitude of the large-scale magnetic component is almost the same between MF and DNS models, regardless of whether the large-scale dynamo is maintained. This implies that some physics might be missing in the quenching model, which controls the nonlinear saturation of the system, especially in realizing the regime of the super-equipartition field.

Overall our results obtained by a series of one-dimensional simulation of the MF dynamo indicate that the space-time evolution of the large-scale magnetic field in our setup can be understood as a manifestation of the propagation of “dynamo wave” due to the $\alpha^2$-type dynamo mode (e.g., Baryshnikova & Shukurov 1987; Raedler & Braeuer 1987). Furthermore, recalling the rotational dependence of the turbulent EMF, i.e., $\alpha_k$ and $\eta_t$ in Fig.5, our results suggest that the $Ro$-dependence of the large-scale dynamo in the rigidly-rotating convection is mainly due to the change in the magnitude of the turbulent magnetic diffusivity: with increasing the spin rate, the turbulent magnetic diffusion becomes weaker while the inductive $\alpha$-effect remains essentially unchanged over the CZ except near the surface, providing the $Ro_{crit}$ for the excitation of the large-scale dynamo.

5. DISCUSSION
5.1. Comparison of $Ro_{\text{crit}}$ with Global Simulations

In § 3.2, we found from the DNS results that there exists a critical Rossby number $Ro_{\text{crit}}$, which separates the success or failure of the large-scale dynamo. Here we compare the $Ro_{\text{crit}}$, with that obtained in the other convective dynamo simulations. As an example, we focus on the global dynamo simulations performed with PENCIL CODE (Pencil Code Collaboration et al. 2021).

From the time-latitude diagram of the convective dynamo achieved in the global simulations with PENCIL CODE, we read that the critical Coriolis number, $Co_{\text{crit}}$, that separates the success or failure of the large-scale dynamo exists in the range $4.7 \lesssim Co_{\text{crit}} \lesssim 7.6$ for Käpylä et al. (2012) and $4.5 \lesssim Co_{\text{crit}} \lesssim 6.6$ for Warnecke (2018), where the $Co$ is defined there as,

$$Co = \frac{2\Omega}{u_{\text{rms}}k_f}.$$  \hspace{1cm} (20)

with $u_{\text{rms}} = \sqrt{\langle u_x^2 + u_y^2 \rangle_v}$. When we convert our Rossby number, $Ro$, to their definition of the Coriolis number, with the relation $u_{\text{rms}} \simeq \sqrt{\langle u_x^2 \rangle_v} = \sqrt{3}u_{cv}$, our DNS models provides,

$$3.8 \lesssim Co_{\text{crit}} \lesssim 12.8.$$  \hspace{1cm} (21)

The condition (21) for $Co_{\text{crit}}$ largely overlaps with that suggested from the results in Käpylä et al. (2012) and Warnecke (2018), despite the difference in the geometry of the simulation models. It is also interesting that, according to the result of Käpylä et al. (2009), even in the large-scale dynamo operated in a weakly-stratified rigidly-rotating convection and meridional flow, as well as the stratification level, would not have a strong impact on the “excitation” of the large-scale dynamo, whereas the turbulent $\alpha$-effect would be essential for it. Furthermore, this may imply that the $Ro_{\text{crit}}$ (or $Co_{\text{crit}}$) is universal in all the astrophysical dynamo activities (e.g., Masada et al. 2022).

5.2. Ro-dependence of $P_{\text{cyc}}/P_{\text{rot}}$

In § 3.2, we roughly estimated the cycle period of the large-scale dynamo as $P_{\text{cyc}} \simeq 100\tau_{cv}$ for model 3 and $\simeq 150\tau_{cv}$ for model 4. Here we evaluate $P_{\text{rot}}/P_{\text{cyc}}$, where

$$\frac{P_{\text{rot}}}{P_{\text{cyc}}} = \left( \frac{2\pi}{\Omega} \right) \left( \frac{1}{P_{\text{cyc}}} \right),$$  \hspace{1cm} (22)

and then compare it between our models and the other models of the global convective dynamo.

Shown in Figure 7 is Ro-dependence of $P_{\text{rot}}/P_{\text{cyc}}$ for our DNS models. The red circle denotes the model with the successful large-scale dynamo (models 3&4), and the blue cross is the model with no dynamo solution (models 1&2). For the reference, the low (high) Ro regime for the successful (unsuccessful) large-scale dynamo is red (blue) shaded.

It seems that $P_{\text{rot}}/P_{\text{cyc}}$ for the successful dynamo model follows a scaling relation, $P_{\text{rot}}/P_{\text{cyc}} \propto Ro^\beta$ with $\beta \simeq 1$. This interestingly suggests that, despite the difference in the geometry of the computational models, the scaling with respect to $Ro$ shows a similar trend between our model and the global dynamo model (Warnecke 2018). However, our result is not statistically significant because, in our simulations, in addition to the fact that only two models have periodic variations, both models have only been able to calculate up to the early part of the cycle. To confirm the result, a larger sample of successful dynamo models and longer term calculations are required. That is beyond the scope of this paper, and should be a target of our future work.

Figure 7. Dependence of $P_{\text{rot}}/P_{\text{cyc}}$ on $Ro$ for the models with $\Omega = 0.05, 0.1, 0.25$, and 0.5 ($Ro = 0.33, 0.15, 0.045$, and 0.019). The filled red circle presents $P_{\text{rot}}/P_{\text{cyc}}$ for the model with the successful large-scale dynamo (models 3&4), while the blue cross is for the model with no dynamo solution (models 1&2). The red shaded region denotes the low $Ro$ regime ($Ro < Ro_{\text{crit}}$) for the successful large-scale dynamo, while the blue shaded region denotes the high $Ro$ regime where the large-scale dynamo fails to be excited.

5.3. A Clue to the Saturation of Stellar Magnetic Activity?

In § 3.1, we found that the RMS value of the vertical component of the velocity, $u_{cv}$, becomes lower when increasing of $\Omega$ with a scaling law of $u_{cv} \propto -\log \Omega$. With leaving aside whether this functional form on $\Omega$ of $u_{cv}$ is accurate, the response of the convection velocity to $\Omega$ seen in Fig.3(a) may rather imply a sign that the convection velocity bottoms out in high $\Omega$ (low $Ro$) regime. We will discuss here the possible relationship between the presence of the floor value of $u_{cv}$ and the saturation of the stellar magnetic activity via the dynamo number $N_D$ in the framework of the MF dynamo model.
With supposing the saturation of the convection velocity, we consider, as a demonstration, a simple piecewise function
\[
\tilde{u}_{cv}(\Omega) = \begin{cases} \Omega^{-\delta} & (\Omega \leq \Omega_c) \\ \tilde{u}_{cv,b} (= \text{const.}) & (\Omega_c < \Omega) \end{cases},
\]
(23)
where \(\tilde{u}_{cv}\) and \(\Omega\) are the convection velocity and angular velocity normalized by arbitrary values, \(\Omega_c\) is an arbitrary value of the angular velocity at which the dependence on \(\Omega\) changes, \(\tilde{u}_{cv,b}\) is an arbitrary floor value of the convection velocity. As a measure of the buoyancy force with respect to the Coriolis force, we use the “convective Rossby number” defined by
\[
\text{Ro}_{\text{conv}} = \tilde{u}_{\text{MLT}}/\Omega
\]
(24)
where \(\tilde{u}_{\text{MLT}}\) is the (normalized) typical convection velocity expected only from the stellar internal structure. Note that \(u_{\text{MLT}}\) is usually evaluated based on the mixing-length theory (MLT) in which the impact of the rotational quenching on the convective motion is not taken account of.

When recalling the scalings \(\eta_t \propto \tau_{\text{corr}} u_{cv}^2\) and \(\tau_{\text{corr}} \propto u_{cv}^{-1}\) (see eqs. (17) and (18)), the normalized turbulent magnetic diffusivity \(\tilde{\eta}_t\) should be related to \(\tilde{u}_{cv}\) as
\[
\tilde{\eta}_t(\Omega) = \tilde{u}_{cv}^{-1}(\Omega)\,
\]
(25)
where the normalization unit for \(\eta_t\) is arbitrarily chosen. When taking account of our insight that the inductive \(\alpha\)-effect is almost independent of \(\Omega\), the dynamo number for the \(\alpha^2\)-type dynamo mode is given
\[
N_D(\Omega) = \frac{\bar{\alpha} \bar{d}}{\tilde{\eta}_t(\Omega)},
\]
(26)
where \(\bar{\alpha}\) is a constant \(\alpha\)-effect and \(\bar{d}\) is the length scale normalized by an arbitrary value. We finally relate the luminosity due to the stellar magnetic activity \(L_{\text{mag}}\) to \(N_D\) via the functional form \(L_{\text{mag}} = N_D^{\chi}\).

Then, with eqs.(23)–(26), we show the dependence of \(L_{\text{mag}}\) on \(\text{Ro}_{\text{conv}}\) as a function of \(\chi\) in Figure 8. Here, as arbitrary values, \(\Omega_c = \tilde{u}_{cv,b} = \tilde{u}_{\text{MLT}} = \bar{\alpha} = \bar{d} = 1\) are chosen for the simplicity. As the power-law index \(\delta\) in eq (23), we choose \(\delta = 0.2\) which is evaluated from the fitting of the convection velocity for models 1 and 2 (unsuccessful dynamo models). The different lines are corresponding to the cases with different values of \(\chi\) (\(\chi = 1, 2, 4, 6, 8, 10\)).

As seen in this figure, with our toy model, the stellar magnetic activity possibly shows a power-law dependence \(L_{\text{mag}} = \text{Ro}_{\text{conv}}^{-2.7}\) in the high \(\text{Ro}_{\text{conv}}\) regime, while it saturates in the low \(\text{Ro}_{\text{conv}}\) regime. This implies that there exists a possible relationship between the presence of the floor value of \(\tilde{u}_{cv}\) and the saturation of the stellar magnetic activity. On the other hand, our model additionally suggests that, to explain the observed power-law slope \(\sim \text{Ro}_{\text{conv}}^{-2.7}\) (e.g., Wright et al. 2011; Wright & Drake 2016), \(L_{\text{mag}}\) should have an extremely strong dependence on \(N_D\), i.e., \(\chi \gtrsim 10\).

Although \(N_D\) determines the kinematic cycle period and growth rate, it may not determine the nonlinear cycle period nor the saturated magnetic field strength (e.g., Tobias 1998; Blackman & Thomas 2015). It should be an important issue to understand how the strength of the dynamo-generated field and the level of the magnetic activity depend on \(N_D\). To validate the effectiveness of our toy model and to get a better grasp of the stellar dynamo activity and its Ro-dependence, it should be quantified how the convection velocity changes with the stellar spin rate with taking account of the rotational quenching and the Lorentz force feedback from both the small-scale (e.g., Hotta & Kusano 2021) and large-scale magnetic fields on the convective turbulence.

6. SUMMARY

In this paper, we studied the rotational dependence of the large-scale dynamo in the rigidly rotating convection. Aiming at the application to the solar and stellar dynamos, a strongly-stratified polytrope is adopted as the model of the convective atmosphere. Additionally to the DNS models, we took a unique approach to utilizing the MF model linked to the DNS to pin down the cause of the rotational dependence of the large-scale dynamo. Our main findings are summarized as follows:

1. Due to the rotational constraint on the convective motion, the convection velocity decreases with the increase of \(\Omega\). In addition, the typical size of the convective cell \(l\) is found to change according to the scaling relation of \(l \propto \text{Ro}^{1/3}\), consistent with the theoretically expected scaling for rotating hydrodynamic convection near onset.
2. There exist two kinds of time-space evolution patterns in the large-scale magnetic field in our DNS models: in the slowly rotating models, the large-scale magnetic component starts to grow, but gradually decays as time passes and finally disappear. In contrast, in the rapidly rotating models, the oscillatory large-scale magnetic component is spontaneously organized. It becomes stronger with the decrease of $\text{Ro}$, reaching the super-equipartition in the fastest rotating model. The Rossby number for the successful large-scale dynamo is $\text{Ro} \lesssim \text{Ro}_{\text{crit}} \approx 0.1$. From the rough estimation on the dynamo cycle, a scaling relation $P_{\text{rot}}/P_{\text{cyc}} \propto \text{Ro}$ can be found.

3. The depth dependence of turbulent $\alpha$-effect, extracted directly from each DNS model, is similar despite the difference of $\text{Ro}$ except the region around the CZ surface. It peaks at around the CZ surface, decreasing deeper down with the sign change at around the mid-part of the CZ. In contrast, while the vertical profile of the turbulent magnetic diffusion is similar between models, the amplitude is suppressed with the decrease of $\text{Ro}$.

4. Overall properties of the large-scale dynamo seen in the DNS models were reproduced in our simplified MF models at least qualitatively. This suggests that the $\text{Ro}$-dependence of the convective dynamo is mainly due to the change in the magnitude of the turbulent magnetic diffusion: with decreasing $\text{Ro}$, the turbulent magnetic diffusion weakens while the inductive $\alpha$-effect remains essentially unchanged over the CZ, providing $\text{Ro}_{\text{crit}}$ for the excitation of the large-scale dynamo and the $\text{Ro}$-dependence of dynamo activities.

Our finding, which brings up the possibility that the $\alpha$-effect is independent of the stellar spin rate, is suggestive and is widely applicable. For instance, when modeling the stellar dynamo activity with a sort of mean-field model, a major issue in the conventional approach is how to give the amplitude and profile of the $\alpha$-effect. However, if the $\alpha$-effect is independent of the stellar spin rate, there may be no need to worry about it. Once the profile of $\alpha$-effect as shown in Fig.5(a) is given in the polar region and its latitudinal dependence is complemented by a certain function (probably $\cos \theta$, with co-latitude $\theta$) (e.g., Käpylä et al. 2006), it may give a universal function of the $\alpha$-effect applicable to all the G-type stars including the Sun. The next target of our research is to construct the global MF model for the solar and stellar dynamos with $\alpha$-effect and turbulent magnetic diffusivity extracted from our DNS models studied here.

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