Exploring Parameter Constraints on Quintessential Dark Energy: the Albrecht-Skordis model

Michael Barnard, Augusta Abrahamse, Andreas Albrecht, Brandon Bozek, and Mark Yashar
University of California, Davis.

We consider the effect of future dark energy experiments on “Albrecht-Skordis” (AS) models of scalar field dark energy using the Monte-Carlo Markov chain method. We deal with the issues of parameterization of these models, and have included spatial curvature as a parameter, finding it to be important. We use the Dark Energy Task Force (DETF) simulated data to represent future experiments and report our results in the form of likelihood contours in the chosen parameter space. Simulated data is produced for cases where the background cosmology has a cosmological constant, as well as cases where the dark energy is provided by the AS model. The latter helps us demonstrate the power of DETF Stage 4 data in the context of this specific model. Though the AS model can produce equations of state functions very different from what is possible with the $w_0-w_a$ parametrization used by the DETF, our results are consistent with those reported by the DETF.

I. INTRODUCTION

Current astronomical observations are best fit by cosmologies containing a significant density of energy with negative pressure. This has been dubbed dark energy, and its exact nature remains a mystery. Though the properties of the dark energy are poorly constrained by current data, a number of proposed experiments will have the reach to probe them. Most assessments of proposed experiments use abstract parameters such as the “$w_0-w_a$” parametrization of the dark energy equation of state used by the Dark Energy Task Force (DETF) [1], or generalized forms of scalar field potentials [2, 3, 4]. In order to fully evaluate future experiments it is useful to understand the impact and discriminating power they will have on specific proposed models of dark energy. In this paper, we will focus on one scalar field (or “quintessence”) model of dark energy, the so-called “Albrecht-Skordis” (AS) model [5, 6].

The AS model is interesting for a number of reasons. Fields of the form needed for the AS model can arise as radius moduli of curled extra dimensions [7], and, while these moduli may be treated as scalar fields in gravitational calculations, they are not subject to the quantum corrections that can create serious problems for most quintessence models [8, 9]. The phenomenology of these models is also interesting because, unlike the majority of dark energy models, the AS quintessence field can contribute a significant fraction of the total cosmic energy density throughout the history of the universe.

Central to this work are the projected data sets (or “data models”) created by the DETF [1]. We use the supernova, weak lensing, baryon oscillation, and Planck data sets (though not the cluster data sets, for technical reasons similar to those outlined in [10]), to show how these data sets would impact the range of possible parameters for quintessence models. To this end, we use Monte-Carlo-Markov chains (MCMC) on these data sets to analyze dark energy and cosmological parameters, rather than the fisher matrix methods used by the DETF. In addition to using “standard” simulated data based on a universe with a cosmological constant, we create projected data sets based on an AS model universe, and run the MCMC around these to highlight the discriminating power of the projected data sets. This paper is one in a series of papers that consider different scalar field models using similar methods [11]. (Technical information about our data models and MCMC methods will be presented in an appendix of another paper in the series [11].)

We show that the stage-by-stage improvement in parameter constraints for the Albrecht-Skordis model is similar to the relative improvement in the $w_0-w_a$ constraints seen by the DETF. We do the same for data sets generated around a specific AS model which shows small deviations from $w(a) = -1$ in the present epoch. This AS model is chosen to illustrate a case where if the real universe is described by an AS model, pure cosmological constant dark energy can be ruled out by a large margin using good Stage 4 data. We also note some features in the AS parameter contours when fitting to cosmological constant based data that reflect the fact that the AS models we consider can only ever duplicate a cosmological constant in an approximate manner.

II. PARAMETER SPACE OF THE ALBRECHT-SKORDIS MODEL

Scalar field models of dark energy, or quintessence, are considered in the framework of an FRW cosmology, with and equation of motion of

$$\ddot{\phi} + 2\frac{a}{a} \dot{\phi} + a^2 \frac{\partial V}{\partial \phi} = 0$$  \hspace{1cm} (1)

where $a$ is the scale factor, $\phi$ is the homogeneous scalar field, and $V$ its potential. The background is assumed to be the homogeneous Friedmann equation

$$H^2 = \frac{8\pi G}{3} (\rho_r + \rho_m + \rho_\phi) - \frac{k}{a^2}$$  \hspace{1cm} (2)
The equation of state parameter of the scalar field is

\[ w = \frac{p_\phi}{\rho_\phi} \]  

(3)

where \( p_\phi \) and \( \rho_\phi \) are the pressure and density of the scalar field. Non-relativistic matter has \( w = 0 \), radiation has \( w = 1/3 \), and a cosmological constant has \( w = -1 \); a homogeneous scalar field can have any behavior in this range, depending on the actual time evolution \( \phi(t) \). The Albrecht-Skordis model[5] postulates a scalar field with an “exponential-with-pre-factor” potential,

\[ V(\phi) = V_0 \left[ (\phi - B)^2 + A \right] e^{-\lambda \phi} \]  

(4)

One of the attractions of this model is that realistic cosmologies can be achieved for cases where the parameters in this potential are all roughly of order unity in Planck units. Specifically, with such parameter choices the potential can have a local minimum with a height consistent with the dark energy density observed today. It is assumed that the field starts high up on the potential. The field will then rapidly approach an attractor solution where the field mimics the equation of state of the dominant energy. It continues this tracking behavior until the field reaches the (approximately) quadratic shaped local minimum, at which point it begins a damped oscillation around that minimum, giving its equation of state a wave like variation as the field becomes the dominant energy. Figure 1 shows \( w(a) \) throughout the history of the universe for a typical AS model.

The potential of the AS model has in principle four degrees of freedom important to cosmology, most of which are complicated combinations of the various parameters. Figure 2 illustrates some of the parameter dependences of the potential. Perhaps the most important degree of freedom, because it determines the scalar field fraction of the total energy density during the tracking behavior, is the logarithmic slope, \( (\ln V)' \) or \( \frac{1}{V'} \); this is almost entirely controlled by \( \lambda \). The height of the local minimum sets the dark energy density today. The curvature of the potential at its minimum is related to the height of the local maximum. The height of the local maximum determines whether the field stops in the minimum or rolls off to infinity, and the curvature at the minimum sets the frequency of the late time oscillations. We have found that the oscillation frequency is not measurable by any of the simulated data sets we considered. Also, the parameter space for models that roll off to infinity but still give realistic cosmic acceleration is a only an exponentially thin and extremely finely tuned region[12]. We choose to ignore this exotic behavior and focus completely on parameters for which the field (classically) stays in the local minimum.

It will be helpful to re-express Eqn. 3 in the following parameters:

\[ V(\phi) = V_0 \left[ \chi (\phi - \beta)^2 + \delta \right] e^{-\lambda \phi} \]  

(5)

FIG. 1: The equation of state \( w \) for the scalar field of a typical AS model. Note that here the \( a \) scale is logarithmic in order to show behavior on all time scales. When the background energy density drops to a level approaching the initial energy of the field during radiation domination, the energy density goes through a transient before mimicking the equation of state of radiation \( (w = 1/3) \). At matter domination, the field again goes through a transient, but the field approaches the local minimum in the potential and becomes the dominant form of energy \( (w \to -1) \) before it can stabilize with a matter-like equation of state \( (w = 0) \).

FIG. 2: The effects of the original parameters (from Eqn. 4) on the potential minimum. All measures are in Planck units. Note that very small changes in \( \lambda \) or \( B \) cause substantial changes in the height of the minimum.

Because the number of parameters is greater than the number of parameters measured by our simulated data the AS model has a number of degenerate directions. Along these degenerate directions, the parameters can change without significantly affecting the observables. The most apparent of these directions are the ways that
FIG. 3: Under the new parameterization, $V_0$ maintains its previous effects, while the new parameter $\beta$ becomes the primary parameter for the AS model. $\beta$ sets the exponential slope as well as the minimum curvature.

the height of the local minimum of the potential can be changed. This height is exponentially sensitive to the product of the exponential factor $\lambda$, and $\beta$, which primarily controls the location of the minimum. This degeneracy can be stabilized by fixing the product of $\lambda$ and $\beta$, so that $\lambda = \frac{27}{\beta^2}$. There is also a dependence of the minimum on $\delta$ and $\frac{\chi}{\lambda^2}$. To work around these difficulties, we fix $\delta$, and vary $\chi$ as a function of $\beta$ so that the overall constant $V_0$ is the only parameter that adjusts the height of the minimum. This necessarily excludes the portion of the parameter space where there is no minimum (with essentially no loss of generality, as discussed above). Also, these parameter constraints do not allow the oscillation frequency near the local minimum to vary independently of the other parameters. This feature is required to successfully run the MCMC calculations since none of the data sets are sensitive to the oscillation frequency. Figure 3 shows how $\beta$ affects the new parameterization.

Aside from degeneracy in the specifics of the potential, there are broad patches of parameter space that are, for most data sets, not detectably different from a cosmological constant. This happens when the exponential slope is steep, and the field slides very early down to the local minimum and oscillates in a shallow and rapid manner, resulting in an equation of state very nearly $-1$ at the times of interest.

The starting value of the scalar field is mostly unimportant in the AS model, but it should be noted that there are values of $\beta$ and the initial $\phi$ that can cause problems. For large values of $\beta$, if the scalar field starts at a point higher on its potential than the radiation density during nucleosynthesis, then the scalar field will form a significant fraction of the energy density at that epoch. Such regions of parameter space would be ruled out by observations of primordial element abundances, though our algorithm does not contain such considerations.

Other related problems with the algorithm specific to the AS model include the last scattering surface. The algorithm does not calculate the size of perturbations at the time of last scattering, but takes them as an input, so the effect of the scalar field on that size is not taken into account by the algorithm even when the scalar field energy density is significant. With respect to the growth of linear density perturbations, the algorithm does not use the scalar field in its calculation until a redshift of ten. The effect of these issues is that some constraining power has been ignored. Albrecht and Skordis investigated this issue to some degree, but while we do not believe this affects the conclusions of this paper, this matter remains interesting and worthy of further exploration.

In our parameterization, the amplitude of the oscillations in the equation of state at late times is primarily determined by the value of $\beta$. For small values of $\beta$ the oscillation amplitudes are very small, but become large for values of $\beta$ approaching 100. This somewhat unintuitive behavior can be explained by noting that, when the dominant form of energy is scaling as $a^{-n}$, $\rho_\phi/\rho_c = \frac{n}{n}$ (where $\rho_c$ is the critical density). Thus, the smaller $\beta$ is (and the larger $\lambda$ is), the further the field energy is below the matter density during matter domination, and the more Hubble friction will slow the field. The closer the scalar field energy density is to the matter density during matter domination, the more kinetic energy remains in the field when it becomes dominant, and thus free to oscillate.
FIG. 5: Likelihood contours for the AS model in $\beta$ and $V_0$ for Stage 2 (top) and Stage 3 photometric, optimistic (bottom), for data generated from a cosmological constant model. Though constants in $V_0$ improve greatly, the AS model has little observable difference from a cosmological constant for values of $\beta$ below 50.

III. CONSTRAINING THE AS MODEL AROUND COSMOLOGICAL CONSTANT MODEL DATA SETS

Having chosen to explore the scalar field parameter space in terms of the parameters $\beta$, $V_0$, and the initial value of the scalar field (with the other parameters constrained), we then performed an MCMC analysis on simulated data sets generated around a cosmology with a cosmological constant. The technical details of this work are presented in the appendix of [11]. These data sets are consistent with those used by the DETF, using simulated Stage 2, Stage 3, and Stage 4 supernova, weak lensing, baryon oscillation, and CMB observations. We did not use the cluster data because of the difficulty of adapting the DETF construction to a quintessence cosmology, nor have we included possible improvements from cross-correlations [14, 15]. In the DETF language Stage 2 encompasses projects that are fully funded and underway, Stage 3 models medium term, medium cost proposals, and Stage 4 are the larger projects, such as a large ground-based survey or a new space telescope.

Though there are three parameters controlling the state of the scalar field, the initial value of the scalar field is generally washed out by the tracking behavior, and the variance in height of the minimum is mostly a reflection of the uncertainty in the Hubble parameter. This leaves us with one important parameter, $\beta$, that mostly controls the unique properties of the AS model. We include a $V_0$ axis in the contour plots for convenience. One should take care, though, to remember that $V_0$ is not a parameter that can be said to have a significant effect on the

FIG. 6: Likelihood contours for the AS model in $\beta$ and $V_0$ for Stage 4 space (top), and Stage 4 ground LST (bottom), both optimistic cases, for data generated from a cosmological constant model. Though constants in $V_0$ improve greatly, the AS model has little observable difference from a cosmological constant for values of $\beta$ below around 50. The different regions represent 68.27%, 95.44% and 99.73% confidence regions. This convention is used for all contour plots in this paper.
equation of state of the dark energy.

Figure 5 shows the likelihood contours for DETF Stage 2 and Stage 3 (optimistic) data. From the point of view of these data sets the behavior of the AS model can be thought of as trending toward a cosmological constant as $\beta$ goes toward zero. While these plots show that the constraints on $V_0$, which corresponds to the dark energy density now, improve greatly with stage number, the critical consideration here is $\beta$. The constraints on $\beta$ do not appear to improve in as dramatic a manner. This is because there is a substantial range of $\beta$ where the these data sets cannot distinguish the AS model from a cosmological constant. As seen in Fig. 4 at $\beta = 50$ already one must look back to $a = .2 (z = 4)$ to see a large deviation from $w = -1$. To some extent this feature will affect any attempt to evaluate the AS model using data from a universe with a cosmological constant. In that low $\beta$ region, the small oscillations of the field persist at late times, and the Stage 4 data sets may be strong enough to pick up on some effects of these oscillations. Figure 6 shows likelihood contours for DETF Stage 4 space and ground (optimistic) data. There is a kink in the Stage 4 space contour plot which corresponds to a range of $\beta$ where the residual oscillations at late times peak in amplitude. As the oscillations represent an energy density above the amount corresponding to the minimum of the potential, $V_0$ must be lower to accommodate regions with larger late time amplitudes.

Figure 7 shows contours in $\omega_k - \beta$ space ($\omega_k$, the “curvature density” is a measure of the curvature of the universe). There are well known degeneracies associated with determining curvature and the dark energy equation of state simultaneously. Here these degeneracies show up by allowing “best fits” to data from a flat universe using non-zero curvature for certain values of $\beta$, as can be seen in contours for the Stage 2 data. The Stage 4 data set shown in the lower panel clamps down on this behavior considerably.

IV. MCMC OF THE AS MODEL AROUND AS MODEL-BASED DATA SETS

As a way of further exploring the power of the advanced data sets, a sample AS model was chosen as a new fiducial model around which the data sets were generated. This model was chosen to be marginally consistent with the cosmological constant-based Stage 2 data, but different enough from a cosmological constant that a cosmological constant would be strongly ruled out with Stage 4 data (the point being to illustrate the power of Stage 4). The fiducial model chosen here has a $\beta = 80$, as depicted in Fig. 4. Because the Stage 2 data allowed larger values of $\beta$ when curvature was present, the sample AS model was chosen with non-zero spatial curvature to make it more consistent with a flat cosmological constant model for this value of $\beta$. The results of the calculation, as seen in Fig. 8 and 9, show that, for a universe described by this specific model, the Stage 4 experiments will rule out a cosmological constant by several sigma.

V. DISCUSSION AND CONCLUSIONS

We have analyzed the impact of the DETF simulated data sets in the context of the Albrecht-Skordis scalar field model of dark energy. We find that the effect of the DETF data sets on the parameter space of the AS model is very similar to the DETF findings. There is substantial improvement in the constraining power of each successive stage of experiments. We presented likelihood contour plots in AS parameter space for a few key combinations of DETF simulated data. We used these plots to demonstrate the broad agreement with the DETF re-
FIG. 8: Likelihood contours for the AS model in $\beta$ and $V_0$ for Stage 2 (top) and Stage 3 photometric, optimistic (bottom), for data generated from the $\beta = 80$ AS model.

FIG. 9: Likelihood contours for the AS model in $\beta$ and $V_0$ for Stage 4 space (top), and Stage 4 ground LST, both optimistic (bottom) cases, for data generated from the selected AS model.

results and also to point out more subtle effects such as degeneracies between the dark energy equation of state and curvature and some peculiarities of the way the AS model can approximate a cosmological constant. In the course of this work we have produced similar contour plots for many more instances of the DETF simulated data (taking individual techniques separately for example, and using both the DETF optimistic and pessimistic data models). We find our overall conclusions and detailed points apply quite generally across the full range of DETF simulated data. We also find that as one moves to better experiments, the improved discriminating power will definitely rule out interesting portions of the AS model space and could completely rule out the cosmological constant if the AS model is correct.

The DETF reported a figure of merit in terms of the inverse of the area inside of likelihood contours in their model space, $w_0$ and $w_a$. This figure of merit showed an improvement of a factor of three going from Stage 2 to Stage 3, and a factor of ten going from Stage 2 to Stage 4, assuming good Stage 3 and Stage 4 projects. In the parameter space we used for the AS model we have only one parameter that strongly affects the equation of state. The constraints on this parameter, $\beta$, show improvement by roughly the square roots of these factors, appropriate for the reduced dimensionality in the AS parameter space. We also see some subtle differences between the constraining power of ground and space Stage 4 data models in that the other variable plotted, $V_0$, is somewhat more strongly constrained by the Stage 4 ground data set than for the Stage 4 space. Other scalar field models [11, 17] display the opposite behavior. We are currently investigating this effect further, which may lead to interesting insights into the complementarity of ground and space-based Stage 4 experiments.

It is interesting to consider the relationship between this work and recent work by one of us and G. Bernstein [10]. There the same DETF data sets were studied in the context of an abstract dark energy model where the
equation of state was modeled by many more than the two parameters used by the DETF. One conclusion of [10] was that high quality data can make good measurements of significantly more than two equation of state parameters. We have seen in this paper that the main equation of state parameter $\beta$ of the AS model is constrained by DETF data sets to a similar degree as the DETF $w_0 - w_a$ parameters, even though they describe very different functions $w(a)$. As discussed in [18], we believe that this ability to constrain a wide variety of functions $w(a)$ is another manifestation of the rich constraining power demonstrated in [10].

A good way to think about this may be to consider the expansion of the AS family of $w(a)$ function as well as the $w_0 - w_a$ family of functions in terms of the independently measured orthogonal functions $w_i(a)$ from [10]. The fact that we have seen here (and also in [11, 17]) that a variety of different $w(a)$ functions can be constrained as well as the DETF $w_0 - w_a$ functions appears to reflect the fact that fundamentally many more functions $w(a)$ are measured than are contained in any one of these families alone.

An upshot of this and the companion work in [11, 17] is that modeling the impact of future experiments using the DETF parameters appears to give a good indication of the impact on quintessence dark energy models with a similar number of parameters. An advantage of the methods used here is that one can see explicitly how future data can constrain real dark energy models in a significant way, and can even eliminate some models entirely.

Acknowledgments

We wish to acknowledge Lloyd Knox, Jason Dick, and Michael Schneider for conversations and consultation that contributed to this paper. We would also like to thank David Ring for finding an error in our code, and Tony Tyson and his group (especially Perry Gee and Hu Zhan) for use of their computational resources. We thank Gary Bernstein for providing us with Fischer matrices suitable for adapting the DETF weak lensing data models to our methods. This work was supported in part by DOE grant DE-FG03-91ER40674 and NSF grant AST-0632901.

[1] A. Albrecht et al. (2006), astro-ph/0609591.
[2] D. Huterer and M. S. Turner, Phys. Rev. D 60, 081301 (1999).
[3] D. Huterer and M. S. Turner, Phys. Rev. D 64, 123527 (2001).
[4] D. Huterer and H. V. Peiris, Phy. Rev. D 75, 083503 (2007).
[5] A. Albrecht and C. Skordis, Phys. Rev. Lett. 84, 2076 (2000), astro-ph/9908085.
[6] C. Skordis and A. Albrecht, Phys. Rev. D66, 043523 (2002), astro-ph/0012195.
[7] A. Albrecht, C. F. Burgess, F. Ravndal, and C. Skordis, Phys. Rev. D65, 123507 (2002), astro-ph/0107573.
[8] S. M. Carroll, Phys. Rev. Lett. 81, 3067 (1998).
[9] N. Kaloper and L. Sorbo, JCAP 0604, 007 (2006), astro-ph/0511543.
[10] A. Albrecht and G. Bernstein, Phy. Rev. D 75, 103003 (2007).
[11] A. Abrahamse, A. Albrecht, M. Barnard, and B. Bozek (2007), arXiv:0712.2879 [astro-ph].
[12] J. Barrow, R. Bean, and J. Magueijo, Mon. Not. Roy. Astron. Soc. 316, L41 (2000), astro-ph/0004321.
[13] P. G. Ferreira and M. Joyce, Phys. Rev. D58, 023503 (1998), astro-ph/9711102.
[14] M. Schneider, L. Knox, H. Zhan, and A. Connolly, Astrophys. J. 651, 14 (2006), astro-ph/0606098.
[15] H. Zhan, JCAP 0608, 008 (2006), astro-ph/0605696.
[16] L. Knox, Phy. Rev. D 73, 023503 (2006).
[17] B. Bozek, A. Abrahamse, A. Albrecht, and M. Barnard (2007), arXiv:0712.2884 [astro-ph].
[18] A. Albrecht, AIP Conf. Proc. 957, 3 (2007), arXiv:0710.0867 [astro-ph].