Space Dependence of Entangled States and Franson-type EPR Experiments

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Abstract

We analyze some aspects of recently performed Franson-type experiments with entangled photon pairs aimed to test Bell’s inequalities. We point out that quantum theory leads to the coincidence rate between detectors which includes in fact a dependence on the distance. We study this dependence and obtain that for large distances the correlation function vanishes. Therefore with taking into account the space parts of wave functions of photons for large distances quantum mechanical predictions are consistent with Bell’s inequalities. We propose an experimental study of space dependence of correlation functions in Bell-type experiments.

1 Introduction

Violation of Bell’s inequalities is the subject of numerous theoretical and experimental investigations [1, 2]. Most of EPR-type experiments used optical photons and requires the use of polarizers. But Franson [3, 4] and others researchers [5 – 7, 12] pointed out that we have the possibility of experimental testing of Bell’s inequalities by alternative ways. One can get a violation of Bell’s inequalities as a result of interference between the probability amplitudes for a pair of photons emitted at various times by an excited atom. The quantum-mechanical uncertainly in the position of a particle or the time of its emission is shown to produce observable effects that are inconsistent with any local hidden-variable theory if one neglects the space part of the wave function.

This experiment is based on optical interference. However the transmission of photons in all the types of EPR-experiment is a real physical process in space and time. Therefore it is important to study the spacetime dependence of correlation functions. The importance of consideration of the spacetime dependence in Bell’s type computations in quantum theory was pointed out in [9 – 11]. In this paper we consider the spacetime dependence of correlation functions in Franson-type experiments for entangled states.
Consider a pair of spin one-half particles in the singlet spin state. If we neglect the spacetime dependence of the wave function the quantum mechanical correlation of two spins is

\[ E_{\text{spin}}(a, b) = \langle \psi_{\text{spin}} | \sigma \cdot a \otimes \sigma \cdot b | \psi_{\text{spin}} \rangle = -a \cdot b \] (1)

Here \( a \) and \( b \) are two unit vectors in space and \( \sigma = (\sigma_1, \sigma_2, \sigma_3) \) are the Pauli matrices. Bell’s theorem states that this cannot be represented in the form of classical correlation function \( P(a, b) \). Bell-Clauser-Horn-Shimony-Holt (CHSH) form of Bell’s inequality of two stochastic processes is

\[ |P(a, b) - P(a, b') + P(a', b) + P(a', b')| \leq 2 \] (2)

From the other hand if \( ab = a'b = ab' = \sqrt{2}/2 \) then

\[ |E_{\text{spin}}(a, b) - E_{\text{spin}}(a, b') + E_{\text{spin}}(a', b) + E_{\text{spin}}(a', b')| = 2\sqrt{2} \]

and Bell’s inequality is violated.

Now we suppose that detectors are located in two separated regions \( O_1 \) and \( O_2 \) respectively and perform localized observations. In this case the quantum correlation is

\[ E(a, O_1, b, O_2) = \langle \psi | \sigma \cdot a P_{O_1} \otimes \sigma \cdot b P_{O_2} | \psi \rangle \] (3)

Here \( P_O \) is the projection onto the region \( O \). If the wave function has a special form of the product of the spin part and the space part, \( \psi = \psi_{\text{spin}} \phi(r_1, r_2) \) then

\[ E(a, O_1, b, O_2) = g(O_1, O_2) E_{\text{spin}}(a, b) \] (4)

where the function

\[ g(O_1, O_2) = \int_{O_1 \times O_2} |\phi(r_1, r_2)|^2 dr_1 dr_2 \]

describes correlation of particles in space. The factor \( g(O_1, O_2) \) is important. We obtain that Bell’s inequalities can be violated only if

\[ g(O_1, O_2) > 1/\sqrt{2} \]

The factor \( g(O_1, O_2) \) deserves an experimental studying. It was shown in [11] that at large distances any quantum state becomes disentangled.

That’s why in the present paper we analyze the Franson test of Bell’s inequalities in a particularly simple configuration consisting of two separated interferometers driven by photons from the source. Our goal is to investigate the space dependence of quantum correlation function in this case. We will try to connect the value of coincidence rate \( R_0 \) and a possibility of violation of Bell’s inequality. The decrease of value of \( R_0 \) with the increase of distances between detectors and the source of photons is a crucial point for violation of Bell’s inequalities.
The interference experiment can be described in the following way (from work of Franson [3]). At time $t = 0$ an atom is assumed to be excited into the upper state $\psi_1$, which has a relatively long lifetime $\tau_1$. After emission of a photon $\gamma_1$ with wavelength $\lambda_1$ the atom will be in the intermediate state $\psi_2$, which has a relatively short lifetime $\tau_2 \ll \tau_1$. Thus a second photon $\gamma_2$ with wavelength $\lambda_2$ will be emitted very soon after $\gamma_1$ and a coincidence counting experiment would show a very narrow peak with a width $\sim \tau_2$. Photons $\gamma_1$ and $\gamma_2$ are collimated by lenses $L_1$ and $L_2$ into beams which propagate toward distant detectors $D_1$ and $D_2$, respectively. The coincidence counting rate will simply show a narrow peak indicating that $\gamma_1$ and $\gamma_2$ were emitted at times which were the same to within a small uncertainty $\sim \tau_2$.

The quantum-mechanical description of this process is highly nonlocal in space since the time at which either photon was emitted was initially uncertain over a much larger time interval $\sim \tau_1$. As a result, the two photons must initially be described by wave packets in which their time of emission and thus their position is relatively uncertain. The detection of one of the photons, say $\gamma_1$, immediately determines the time of emission of the other photon and thus its position to within a much smaller uncertainty, which must be reflected by a nonlocal change in the wave function describing the other photon. This nonlocal reduction of the wave function is analogous to that which occurs in the polarization measurements of Bell’s original theorem. The relative phases of the photons play the role of polarization angles in Bell’s type experiments.

2 Correlation function

Let us investigate the dependence on the distance in the coincidence experiment with the half-silvered mirrors in place. To calculate the coincidence rates one introduced [3] the field operator

$$\psi(r) = \int e^{ik \cdot r} a(k) dk$$

where $a(k)$ annihilates a particle with momentum $k$ and $r$ is a distance from the source. The time dependence of the field operator is given by

$$\psi(r, t) = e^{itH/\hbar} \psi(r) e^{-itH/\hbar}$$

where $H$ is the Hamiltonian of the system.

The field at detector $D_1$ is given by

$$\psi(r_1, t) = \frac{1}{2} \psi_0(r_1, t) + \frac{1}{2} e^{i\phi_1} \psi_0(r_1, t - \Delta T)$$

Here $\psi_0(r, t)$ is the field operator with the half-silvered mirrors removed and $\Delta T$ is the difference between in the transit times via the longer and shorter paths, $\phi_1, \phi_2$ (see below) are phase shifts into the two beams.
The field at the detector $D_2$ is

$$
\psi(r_2, t) = \frac{1}{2} \psi_0(r_2, t) + \frac{1}{2} e^{i\phi_2} \psi_0(r_2, t - \Delta T)
$$

The coincidence rate $R_c$ between $D_1$ and $D_2$ with the mirrors inserted can now be calculated from

$$
R_c = \eta_1 \eta_2 \langle 0 | \psi^\dagger(r_1, t) \psi^\dagger(r_2, t) \psi(r_2, t) \psi(r_1, t) | 0 \rangle
$$

where $\eta_1$ and $\eta_2$ are the detection efficiencies of $D_1$ and $D_2$ and $|0\rangle$ is the vacuum state. One can compute $R_c$ to be of the form

$$
R_c = \frac{1}{4} R_0 \cos^2 \left( \phi_1' - \phi_2' \right)
$$

(5)

where

$$
R_0 = \langle 0 | \psi^\dagger_0(r_1, t) \psi^\dagger_0(r_2, t) \psi_0(r_2, t) \psi_0(r_1, t) | 0 \rangle
$$

is the coincidence rate with the half-silvered mirrors removed.

Let us study the dependence $R_0$ on the distances $r_1$ and $r_2$ in a simple model.

3 Model

Let us consider the quantum model with the Hamiltonian

$$
H = H_0 + V = \int dk \omega(k) a^\dagger(k) a(k) + \int dk (f(k) a(k) + f(k) a^\dagger(k))
$$

Here $f(k)$ is a formfactor (test function) and $\omega(k)$ is a dispersion low. The field operators (in coordinate representation) are

$$
\psi_0(r) = \int dp a(p) e^{i pr}
$$

In the Heisenberg representation the states remain constant while the operators evolve in time so that

$$
\psi_0(r, t) = e^{itH} \psi_0(r) e^{-itH} = \int dp e^{ipr} f_p(t)
$$

Here

$$
f_p(t) = e^{itH} a(p) e^{-itH} = a(p) e^{-i\omega(p)t} + \frac{f(p)}{\omega(p)} (e^{-i\omega(p)t} - 1)
$$

Our goal is to study the quantity

$$
R_0 = \langle 0 | \psi^\dagger_0(r_1, t) \psi^\dagger_0(r_2, t) \psi_0(r_2, t) \psi_0(r_1, t) | 0 \rangle
$$
Using (3) one immediately gets

\[ R_0 = \int dp_1 dp_2 dp'_1 dp'_2 e^{-ip_1 r_1 - ip_2 r_2 + ip'_1 r_1 + ip'_2 r_2} \langle 0 | f^\dagger_{p_1}(t) f_{p_2}(t) f_{p'_2}(t) f_{p'_1}(t) | 0 \rangle \]

\[ = \int dp_1 dp_2 dp'_1 dp'_2 e^{-ip_1 r_1 - ip_2 r_2 + ip'_1 r_1 + ip'_2 r_2} \frac{\mathcal{T}(p_1)}{\omega(p_1)}(e^{i\omega(p_1)t} - 1) \frac{\mathcal{T}(p_2)}{\omega(p_2)}(e^{i\omega(p_2)t} - 1) \]

\[ \times \frac{f(p'_1)}{\omega(p'_1)}(e^{-i\omega(p'_1)t} - 1) \frac{f(p'_2)}{\omega(p'_2)}(e^{-i\omega(p'_2)t} - 1) = |\varphi(r_1, t)|^2 |\varphi(r_2, t)|^2 \]

Here

\[ \varphi(r, t) = \int dp e^{ipr} \frac{f(p)}{\omega(p)}(e^{-i\omega(p)t} - 1) \]

Let us consider two cases. The first case the formfactor \( f(p) \) has an exponential form and the second case the formfactor has the form of a simple cut-off.

In order to investigate the behavior of \( \varphi(r, t) \) as \( |r| \to \infty \) let us consider some concrete examples of formfactor and dispersion law. Suppose that our Hamiltonian describes massless particles, i.e. \( \omega(p) = |p|, \ p \in \mathbb{R}^3 \). We will consider two types of formfactor - the step-function and a gaussian formfactor.

In both cases formfactor depends only on \( |p| \). Therefore for \( \varphi(r, t) \) one has

\[ \varphi(r, t) = \frac{2\pi}{i|r|} \int_0^\infty dp f(|p|)(e^{-i|p|t} - 1)(e^{i|p||r|} - e^{-i|p||r|}) \]

\[ = \frac{2\pi}{i|r|} (I(t - |r|) - I(t + |r|) + I(|r|) - I(-|r|)) \]

Here we denoted

\[ I(\alpha) = \int_0^\infty dx f(x)e^{-i\alpha x} \]

Now we consider the case \( f(|p|) = \theta(|p| - A) \) where \( \theta \) is the step function for some positive constant \( A \). We have

\[ I(\alpha) = i\frac{e^{-iA\alpha} - 1}{\alpha} \]

This means that \( |\varphi(r, t)|^2 \) vanishes for large \( |r| \) at least as \( 1/|r|^2 \).

For the gaussian formfactor \( f(|p|) = e^{-|p|^2/A} \) the asymptotics has the same form \( |\varphi(r, t)|^2 \sim 1/|r|^2 \).

This means that \( R_0 \) polynomially decreases at large distances between detectors.

Moreover one can slightly modify the gaussian formfactor in some neighborhood of \( p = 0 \) in such a way that \( R_0 \) will decrease faster then any polynomial. To obtain such a behaviour just take \( f(x) \) to be a test function with a compact support on the positive semiaxis.
4 Conclusions

In this paper the space dependence of correlation function in Franson-type experiments is discussed. The role of the spacetime dependence of the correlation function in Bell-type experiments was considered earlier in [9–11]. The Franson-type experiment that is taken as a basis of this paper is a new variant of EPR-type experiments (according to [3–7] and others researchers) and the form of $R_e$ in Eq. (5) is identical to that obtained in earlier experiments based upon Bell theorem where $\phi_1'$ and $\phi_2'$ correspond instead to the orientation of distant polarizers [3].

In our case the space dependent coefficient $R_0$ can play the role of the function $g(O_1, O_2)$ in Eq (4). Although in work of W. Tittel et al.[6] correlation coefficient $E$ is independent of $R_0$ ($R_0$ is canceled out in process of calculation of $E$) but Larsson et al. [8] suggest that Franson-type experiments can not demonstrate violation of Bell’s inequality. Authors notes that only half of the events will belong to coincidence class and if all that are taken into account standard Bell inequalities are not violated. They suggested the quantum model predicted probability that equal 1/2 of the probability of photon coincidence from [6]. Hence 1/2 is a coefficient in left part of Bell inequality and that’s why it is not violated. In [12] also one showed the dependence of probability and visibility value on number of registered events.

In [6, 7] one proposed to use the visibility $V$ as a coefficient before the function of cosine of phase differences. If $V \leq 1/\sqrt{2}$ then there is no violation of the Bell’s inequalities and therefore there is no violation of locality in the corresponding state of wave function. Authors notes that visibility decreased compared to short distance experiments but without obvious visibility dependence on distance. In [6] the maximal visibility is a function of counts ($C, A$) were $C$ is the number of detected coincidences and $A$ is the number of accidental ones.

Therefore we need a careful analysis of number of discarded events because this is a crucial point for detecting a possible violation of Bell’s inequality. But in real EPR-type experiments this number depends on a distance between two separated regions $O_1$ and $O_2$ where detectors are located. In our case the increasing of the distance between detectors and the source of photons leads to the decreasing of the coefficient $R_0$ that plays the role of space part of correlation function. We suggest that there is a connection between the space depended function $g(O_1, O_2)$ and the coefficient $R_0$. When $g(O_1, O_2)$ becomes smaller than $1/\sqrt{2}$ Bell’s inequality is not violated and therefore for large distances quantum mechanical predictions are consistent with Bell’s inequalities. That’s why it is possible that the value $R_0$ plays a key role in this line of Franson-type experiments and really we must perform a study of the dependence of $R_0$ on the distance. This question requires further theoretical and experimental investigations.
5 Acknowledgments

This work is partially supported by RFFI 02-01-01084, by INTAS 99-00545, the grant for leading scientific schools RFFI 00-15-96073 and also by the INTAS 01/1-200 for A.P.

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