Velocity-induced Heavy Quarkonium Dissociation using the
gauge-gravity correspondence

Binoy Krishna Patra, Himanshu Khanchandani and Lata Thakur
Department of Physics, Indian Institute of Technology Roorkee, Roorkee 247 667, India

Abstract

Using the gauge-gravity duality we have obtained the potential between a heavy quark
and an antiquark pair, which is moving perpendicular to the direction of orientation, in a
strongly-coupled supersymmetric hot plasma. For the purpose we work on a metric in the
gravity side, \( \text{OKS-BH} \) geometry, whose dual in the gauge theory side runs with the
energy and hence proves to be a better background for thermal QCD. The potential obtained
has confining term both in vacuum and in medium, in addition to the Coulomb term alone,
usually reported in the literature. As the velocity of the pair is increased the screening of the
potential gets weakened, which may be understood by the decrease of effective temperature
with the increase of velocity. The crucial observation of our work is that beyond a critical
separation of the heavy quark pair, the potential develops an imaginary part which is
nowadays understood to be the main source of dissociation. The imaginary part is found
to vanish at small \( r \), thus agrees with the perturbative result. Finally we have estimated
the thermal width for the ground and first excited states and found that non-zero rapidities
lead to an increase of thermal width. This implies that the moving quarkonia dissociate
easier than the static ones, which agrees with other calculations. However, the width in our
case is larger than other calculations due to the presence of confining terms.

1 Introduction

The in-medium behaviour of heavy quark bound states is used to probe the state of matter in
quantum chromodynamics (QCD), where the screening of the potential results in the suppression
of the yields of heavy quarkonium states in relativistic heavy-ion collisions [1, 2]. The heavy
quarkonium bound states treated nonrelativistically possess the well separated energy scales,
\( \text{viz.} \) the heavy quark mass (\( m_Q \)) \( \gg \) the momentum transfer (\( 1/r \propto m_Q \alpha_s \)) \( \gg \) the binding energy
(\( E \propto m_Q \alpha_s^2 \)) [3, 4, 5, 6, 7]. In addition, the (thermal) medium introduces scales to the previous
ones, \( \text{viz.} \) \( T \), the Debye mass, \( m_D \) (\( gT \)) etc. which are also separated (\( T \gg m_D \)) in weak
coupling regime (\( g < 1 \))[8]. The above hierarchies facilitate to develop the sequence of effective
Theories namely NRQCD, pNRQCD etc. from the underlying theory of strong interaction (QCD) after integrating out the successive energy scales for both $T=0$ and $T \neq 0$ [5, 6, 7, 9, 3]. Now if the $Q\bar{Q}$ bound state moves with respect to the medium, the synthesis of effective theories becomes more complicated due to the additional scales associated with the motion of the pair. However, EFT have recently explored the in-medium modifications of heavy quarkonium states moving through a medium for two plausible situations: $m_Q \gg 1/r \gg T \gg E \gg m_D$ and $m_Q \gg T \gg 1/r, m_D \gg E$ [10], which are relevant for moderate temperatures and for studying dissociation, respectively. They found that the width decreases with the velocity for the former situation whereas for the latter regime the width increases monotonically with the velocity.

The aforesaid hierarchies assume the weak coupling limit and its identification with the relative velocity. Therefore one needs the framework in strong coupling regime, where the potential can be extracted from the Euclidean correlators calculated in the lattice but the limited sets of data in addition to the technical difficulties of lattice calculations, limits the reliability of the results. Thus some complementary methods for strong coupling are desirable, where the AdS/CFT conjecture provides such alternative [11, 12, 13] to calculate the potential for a class of non-abelian thermal gauge field theories. The expectation value of a particular time-like Wilson loop defines the potential between a static quark and antiquark at finite temperature. This is where the AdS/CFT correspondence makes the calculation easier by mapping the evaluation of a Wilson loop in a hot strongly interacting gauge theory plasma onto the much simpler problem of finding the extremal area of a classical string world sheet in a black hole background. The first calculation was done by Maldacena [14] for $\mathcal{N} = 4$ SYM for $T=0$ and was later extended to finite temperature in [15, 16].

Nowadays the dissociation of heavy quarkonia is understood to be not due to the Debye screening of the potential, but is rather overtaken by the thermal width obtained from the imaginary part of the potential [17, 8, 18]. There are mainly two processes in QCD, which contribute to the thermal width: the first process is the inelastic parton scattering mediated by the space-like gluons, known as Landau damping and the second one is the gluon-dissociation process which corresponds to the decay of a color singlet state into a color octet induced by a thermal gluon [19]. Recently two of us derived the imaginary component of the potential using thermal field theory, where the inclusion of confining string term makes the (magnitude) imaginary component larger, compared to the medium-modification of the perturbative term.
However the results referred above about the imaginary component of the potential are limited to the weak coupling techniques. Using holographic correspondence, authors in [23, 24] in strong coupling limit have obtained the imaginary part of the potential for $\mathcal{N} = 4$ supersymmetric gauge theory beyond a critical separation, $r_c$ of a static $Q\bar{Q}$ pair, by analytically continuing the string configurations into the complex plane. They suggested that the complex-valued saddle points beyond $r_c$ ($=0.87z_h$) may be interpreted as the quasi-classical configurations in the classically forbidden region of string coordinates, analogous to the method of complex trajectories used in quasi-classical approximations to quantum mechanics [25]. In an alternative approach in AdS/CFT framework, the imaginary part is calculated in a strongly coupled plasma through the thermal worldsheet fluctuation method [26]. In the aforesaid calculations, the dual geometry taken was the pure AdS black hole metric, hence their dual gauge theory is conformally invariant unlike QCD which depends on scale.

A resurrected interest in the properties of bound states moving in a thermal medium arose in recent years due to the advent of high energy heavy-ion colliders. Using holographic correspondence, the potential between two heavy quarks moving through the medium or, equivalently, two heavy quarks at rest in a moving medium is calculated in a pure AdS black hole background [27, 31, 32, 29, 30], where the solution above a critical separation of the pair is abandoned and is discussed in details in [28]. Recently the velocity dependence of the imaginary potential of the traveling bound states is also calculated [33], following the idea of thermal worldsheet fluctuation method [26, 34]. However in abovementioned calculations the metric used is AdS black hole, thus the dual gauge theory is conformal, unlike QCD.

Our aim is to improve the abovementioned calculations in two folds: first of all we improve the dual gravity which is somewhat closer to QCD than pure AdS geometry in the sense that it accounts for RG flow [35]. Secondly, unlike the world-sheet thermal fluctuations, we delve into the solution beyond the critical separation of the pair, which leads to the complex-valued string configurations and hence the potential turns out to be complex. Recently we have implemented both points in a calculation for a static pair [36], and in the present work, we wish to extend the calculation for a moving pair. Our paper is organised as follows: In Section 2, we have revisited the OKS-BH geometry in brief and in Section 3, we employ this geometry to obtain both the real and imaginary parts of the potential by the Nambu-Goto action. Thereafter we calculate the thermal width for the ground and excited states in Section 4. Finally we conclude in Section
2 Construction of dual geometry

In this section we discuss briefly about the OKS-BH geometry that we will use to calculate the potential. The discussion presented here closely follows [37, 35, 38] and the readers already aware of this geometry may go directly to next section to see the calculations.

A gauge theory is said to be conformal if it does not flow with the scale, i.e. has a trivial renormalisation group (RG) flow, and a conformal theory in a (3 + 1)-dimension can be conveniently described on the boundary of pure anti-de Sitter space [11]. But if the theory has a non-trivial RG flow like QCD, which is confining in IR while conformal in UV, we cannot describe the full theory on the boundary of some five dimensional space and hence need to look differently at running energy scales.

One way out is to embed the D branes in the geometry and as a result the corresponding gauge theory exhibit logarithmic RG flow. Such a construction has been done in the Klebanov-Strassler (KS) warped conifold geometry [39]. The gravity dual for KS model is warped deformed conifold with three-form type IIB fluxes and the corresponding gauge theory is confining in the far IR but is not free at UV. Other problems with KS model include the absence of quarks in the fundamental representation and the non applicability of model at non-zero temperature.

What we basically need for thermal QCD is a dual gravity theory that allows quarks in the fundamental representation at high temperature. The quarks in the fundamental representation can be introduced by inserting $N_f$ D7 branes in the KS geometry. The insertion is is a subtle issue and so far we only know how to insert coincident D7 branes in the Klebanov-Tseytlin background [39]. The resulting background is the Ouyang background discussed in [39] that has all the type IIB fluxes switched on, including the axio-dilaton. Now to switch on a non-zero temperature we need to insert a black hole into this background and the Hawking temperature will correspond to the gauge theory temperature. Combining all physics ingredients together the metric in OKS-BH geometry looks like

$$ds^2 = \frac{1}{\sqrt{h}} \left[ -g_1(u)dt^2 + dx^2 + dy^2 + dz^2 \right] + \sqrt{h} \left[ g_2^{-1}(u)du^2 + d\mathcal{M}_5^2 \right]$$

(1)
where $h$ is the warp factor, $g_i(u)$ are the black-hole factors, $u$ denotes the extra dimension and $dM_5^2$ is the metric of warped resolved-deformed conifold.

The above picture works well at IR. Now we need to make sure that the theory should become conformal and free at UV. In other words we want AdS-Schwarzschild geometry in asymptotic limit. It is clear that we cannot use the pure OKS-BH background and we need to add the appropriate UV cap to it. As a consequence, the above metric will receive corrections due to the UV cap as

$$ds^2 = \frac{1}{\sqrt{h}} \left[ -g(u)dt^2 + dx^2 + dy^2 + dz^2 \right] + \sqrt{h} \left[ g(u)^{-1} g_{uu} du^2 + g_{mn} dx^m dx^n \right]$$

(2)

where we have set black hole factors $g_1(u) = g_2(u) = g(u)$. The corrections $g_{uu}$ are of the form $u^{-n}$ and appear in the metric because the existence of axio-dilaton and the seven-brane sources tell us that the unwarped metric may not remain Ricci flat. Thus the corrections may be written as

$$g_{uu} = 1 + \sum_{i=0}^{\infty} \frac{a_{uu,i}}{u^i}$$

(3)

where $a_{uu,i}$ are the coefficients independent of the coordinate $u$ and can be solved for exactly as shown in [35]. The warp factor, $h$ can be obtained as

$$h = \frac{L^4}{u^4} \left[ 1 + \sum_{i=1}^{\infty} \frac{a_i}{u^i} \right]$$

where $a_i$ are coefficients, again independent of the coordinate $u$. These are of $O(g_s N_f)$ and can be solved for exactly as shown in [35] and $L$ denotes the curvature of space. This metric reduces to OKS-BH in IR and is asymptotically $AdS_5 \times M_5$ in the UV. It describes the geometry all the way from the IR to the UV.

With the change of coordinates $z = 1/u$, we can rewrite the metric (2) as

$$ds^2 = g_{\mu\nu} dX^\mu dX^\nu = A_n z^{-n-2} \left[ -g(z) dt^2 + d\vec{x}^2 \right] + \frac{B_i z^i}{A_n z^{m+2} g(z)} dz^2 + \frac{1}{A_n z^n} ds^2_{\mathcal{M}_5},$$

(4)

where $ds^2_{\mathcal{M}_5}$ is the metric of the internal space and $A_n$’s are the coefficients that can be extracted from the $a_i$’s as follows:

$$\frac{1}{\sqrt{h}} = \frac{1}{L^2 z^2 \sqrt{a_i z^i}} \equiv A_n z^{-n-2} = \frac{1}{L^2 z^2} \left[ a_0 - \frac{a_1 z}{2} + \left( \frac{3a_2^2}{8a_0} - \frac{a_2}{2} \right) z^2 + \cdots \right],$$

(5)

5
which gives $A_0 = \frac{a_0}{L^2}, A_1 = -\frac{a_1}{2L^2}, A_2 = \frac{1}{L^2} \left( \frac{3a_1^2}{8a_0^2} - \frac{a_2}{2} \right)$ and so on. Note that since $a_i$'s for $i \geq 1$ are of $\mathcal{O}(g_s N_f)$ and $L^2 \propto \sqrt{g_s N}$, so in the limit $g_s N_f \to 0$ and $N \to \infty$ all $A_i$'s for $i \geq 1$ are very small. The $u^{-n}$ corrections mentioned above in Eq. (2) are accommodated via $B_l z^l$ series which is given by

$$B_l z^l = 1 + a_{zz,i} z^i \quad (6)$$

In fact the complete picture can be divided into three regions. Region 1 is the IR region where we have pure OKS-BH geometry. Region 3 is the UV region where UV cap has been added. And region 2 is the interpolating region between UV and IR. The background for all these three regions and the process of adding UV cap has been described in full details in [35]. Also the RG flow associated with these regions and the corresponding field theory realizations have been discussed in [40]. We shall not go into the complete details here and will use the metric given in Eq. (4) in extremizing the action to calculate the potential in the next section.

### 3 Potential between a heavy quark and a heavy antiquark

The potential at finite temperature can be obtained from the Wilson loop’s expectation value evaluated in a thermal state of the gauge theory:

$$\lim_{T \to \infty} \langle W(C) \rangle \sim e^{i TV_{Q\bar{Q}}(r,T)} \quad (7)$$

According to the gauge/gravity prescription [11], the expectation value of $W(C)$ in a strongly coupled gauge theory dual to a theory of gravity is

$$\langle W(C) \rangle \sim Z_{str} \sim e^{i S_{str}} \quad (8)$$

where $S_{str}$ is the classical string action propagating in the bulk evaluated at an extremum. Hence the potential can be obtained by extremizing the world-sheet of open string attached to the heavy quark pair located at the boundary of AdS$_5$ space in the background of AdS$_5$ space and in the background of AdS$_5$ black-hole metric, for determining the potentials at $T = 0$ and $T \neq 0$, respectively.
We consider a quark-antiquark pair \((Q\bar{Q})\) moving along \(x_3\) direction with some velocity \(\vec{v}\). It is convenient to boost the system to the rest frame \((t', x'_3)\) of the quark-antiquark pair as
\[
\begin{align*}
dt &= dt' \cosh \eta - dx'_3 \sinh \eta, \\
dx_3 &= -dt' \sinh \eta + dx'_3 \cosh \eta,
\end{align*}
\]
with \(\tanh \eta = v\). The \(Q\bar{Q}\) dipole is now at rest but the quark-gluon plasma is moving with velocity \(\vec{v}\) in the negative \(x'_3\)-direction. Thus under the boost, the metric \((4)\) reduces to
\[
ds^2 = A_n z^{n-2} \left[ dt^2 \left( -1 + \frac{z^4}{z_h^4} \cosh^2 \eta \right) + dx_1^2 + dx_2^2 + dx_3^2 \left( 1 + \frac{z^4}{z_h^4} \sinh^2 \eta \right) - 2 dt \ dx_3 \ \sinh \eta \ \cosh \eta \ \frac{z^4}{z_h^4} + \frac{B_1 z^l}{A_m z^{m+2} g(z)} dz^2 + \frac{1}{A_n z^n} \ ds_M^2 \right]
\]
Now the Nambu-Goto action can be defined in terms of string coordinates \((\sigma, \tau)\):
\[
S_{NG} = \frac{1}{2\pi} \int d\sigma d\tau \sqrt{-\det \left[ (g_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi) \partial_\alpha X^\alpha \partial_\beta X^\beta \right]},
\]
where \(\phi\) is the background dilaton field, which is responsible for breaking of conformal symmetry of theory and is given by
\[
\phi = \log g_s - g_s D_{n+m_o} z^{n+m_o}
\]
We parametrize the two-dimensional world sheet as
\[
\tau = t, \quad \sigma = x_1 \in \left[-\frac{r}{2}, \frac{r}{2}\right], \quad x_2 = \text{const}, \quad x_3 = \text{const}, \quad z = z(x_1), \quad \partial_a = \frac{\partial}{\partial \tau}, \quad \partial_b = \frac{\partial}{\partial \sigma},
\]
thus the Nambu-Goto action can be rewritten as
\[
S_{NG} = \frac{T}{2\pi} \int_{-\frac{r}{2}}^{\frac{r}{2}} dx_1 \sqrt{(A_n z^n)^2 \left( \frac{1}{z^4} - \frac{1}{z_h^4} \cosh^2 \eta \right) + B_m z^m \frac{(z')^2}{z^4} \frac{(z_h^4 - z^4 \cosh^2 \eta)}{(z_h^4 - z^4)}},
\]
where we have used
\[
B_m z^m = B_1 z^l + 2 g(z) g_s^2 (n + m_o) D_{n+m_o}(l + m_o) D_{l+m_o} A_k \bar{c}^{k+n+l+2m_o}.
\]
The Nambu-Goto action can also be written as an integral over \(z\),
\[
S_{NG} = \frac{T}{\pi} \int_{0}^{z_{\text{max}}} \frac{dz}{z} \sqrt{(A_n z^n)^2 \left( \frac{1}{z^4} - \frac{1}{z_h^4} \cosh^2 \eta \right) + B_m z^m \frac{(z')^2}{z^4} \frac{(z_h^4 - z^4 \cosh^2 \eta)}{(z_h^4 - z^4)}}
\]
\]

7
Since this action does not depend explicitly on \( x \), so the corresponding Hamiltonian will be a constant of motion, i.e.,

\[
H = z' \frac{\partial L}{\partial z'} - L = C_0 \quad \text{(say)}
\]  

where the constant \( C_0 \) can be determined from the following equation:

\[
\frac{1}{L} \left( (z')^2 B_m \frac{z^m}{z^4} \frac{z^4 - z^4 \cosh^2 \eta}{(z_h^4 - z^4)} - L^2 \right) = - \left( \frac{1}{z^4} - \frac{1}{z_h^4} \cosh^2 \eta \right) \frac{(A_n z^n)^2}{L} = C_0
\]

Since at \( z = z_{\max} \), \( z' = 0 \), where \( z_{\max} \) is the maximum of the string coordinate along the fifth dimension, so we can find out the derivative \( z' \) by the simple algebra

\[
z' = \frac{dz}{dx} = \frac{(A_n z^n)^2 z_{\max}^2}{\sqrt{B_m z^m} z_{\max}^2 (A_n z_{\max}^n)} \sqrt{\frac{(z_h^4 - z^4)(z_h^4 - z^4 \cosh^2 \eta)}{(z_h^4 - z_{\max}^4 \cosh^2 \eta) (A_n z_{\max}^n)^2}} \times \left( 1 - \frac{(A_n z_{\max}^n)^2 (z_h^4 - z_{\max}^4 \cosh^2 \eta) z_{\max}^4}{(A_n z^n)^2 (z_h^4 - z^4 \cosh^2 \eta) z_{\max}^4} \right)^{\frac{1}{2}}
\]

Integrating both sides, the above equation yields the separation of the \( Q\bar{Q} \) pair, \( r \) as

\[
r = \frac{2z_h^2 \sqrt{z_h^4 - z_{\max}^4 \cosh^2 \eta} (A_n z_{\max}^n)}{z_{\max}^2} \int_0^{z_{\max}} dz \sqrt{\frac{z^2 \sqrt{B_m z^m}}{(z_h^4 - z^4)(z_h^4 - z^4 \cosh^2 \eta) (A_n z^n)^2}} \times \left( 1 - \frac{(z_h^4 - z_{\max}^4 \cosh^2 \eta) (A_n z_{\max}^n)^2 z_{\max}^4}{(z_h^4 - z^4 \cosh^2 \eta) (A_n z^n)^2 z_{\max}^4} \right)^{-\frac{1}{2}}
\]

Expanding the square root and keeping only the first term because the coefficients \( A_n \)'s are small, the separation \( r \) becomes

\[
r = \frac{2z_h^2 (A_n z_{\max}^n)}{z_{\max}^2} \sqrt{z_h^4 - z_{\max}^4 \cosh^2 \eta} I,
\]

where the integral \( I \) is defined by

\[
I = \int_0^{z_{\max}} dz \sqrt{\frac{z^2 \sqrt{B_m z^m}}{(z_h^4 - z^4)(z_h^4 - z^4 \cosh^2 \eta)}}
\]

Now we obtain the action

\[
S_{NG} = \frac{T}{\pi} \int_0^{z_{\max}} dz \frac{B_m z^m (z_h^4 - z^4 \cosh^2 \eta)}{z^2 (z_h^4 - z^4)} \left( 1 - \frac{(z_h^4 - z_{\max}^4 \cosh^2 \eta) (A_n z_{\max}^n)^2 z_{\max}^4}{(z_h^4 - z^4 \cosh^2 \eta) (A_n z^n)^2 z_{\max}^4} \right)^{-\frac{1}{2}}
\]
Expanding the square root and keeping only the first two terms (since the coefficients $A_i$'s are small), the above action can be written as

\[
S_{NG} = \frac{T}{\pi} \left[ \int_0^{z_{\text{max}}} dz \frac{B_m z^n (z^4_h - z^4) \cosh^2 \eta}{(z^4_h - z^4)} + \frac{1}{2} \frac{(z^4_h - z^4_{\text{max}}) \cosh^2 \eta (A_n z^n_{\text{max}})^2}{z^4_{\text{max}}} I \right],
\]

where $I$ is the same integral as in (20). Thus substituting the integral $I$ in terms of $r$, the action becomes

\[
S_{NG} = \frac{T}{\pi} \int_0^{z_{\text{max}}} dz \frac{B_m z^n (z^4_h - z^4) \cosh^2 \eta}{(z^4_h - z^4)} + \frac{T}{4\pi} \frac{z^4_h - z^4_{\text{max}} \cosh^2 \eta}{z^4_h - z^4_{\text{max}}} A_n z^n_{\text{max}} r
\]

\[
\equiv S^1 + S^2
\]

The first term in the action, $S^1$, diverges in the lower limit of the integration, so we regularize it by integrating from $\epsilon$ (instead of 0) to $z_{\text{max}}$ [35] and then identifying the divergent term in the integral

\[
S^1 = \frac{T}{\pi} \int_\epsilon^{z_{\text{max}}} dz \sqrt{B_m z^n} \sqrt{\frac{z^4_h - z^4 \cosh^2 \eta}{z^4_h - z^4}},
\]

Let us also assume for simplicity that the higher coefficients $A_i$ and $B_i$ are very small for $i \geq 3$ and can be neglected. This simplification reduces the series $A_n z^n$ to $1 + A_2 z^2$ and the other series $B_m z^m$ to $1 + B_2 z^2$, where we take $A_0 = 1$ and $A_1 = 0$ and similarly $B_0 = 1$ and $B_1 = 0$. This will also simplify all the expressions and helps us to keep an analytic control on the equations. Therefore, in the limit of small coefficients, $S^1$ becomes

\[
S^1 = \frac{T}{\pi} \int_\epsilon^{z_{\text{max}}} dz \sqrt{B_m z^n} \sqrt{\frac{z^4_h - z^4 \cosh^2 \eta}{z^4_h - z^4}} \equiv \frac{T}{\pi} \int_\epsilon^{z_{\text{max}}} dz F(z, v),
\]

where the integral, $F(z, v)$ can be expanded in the Taylor series in velocity:

\[
F(z, v) = F(z, 0) + \frac{\partial F}{\partial v} v + \frac{1}{2} \frac{\partial^2 F}{\partial v^2} v^2 + \cdots \cdots
\]

We can omit the dotted terms in the small velocity limit and can write $S^1$ as

\[
S^1 = \frac{T}{\pi} \left[ \int_\epsilon^{z_{\text{max}}} dz \left( \frac{1}{2} z^2 + \frac{B_2}{2} z^2 \right) - \frac{v^2}{2} \int_\epsilon^{z_{\text{max}}} dz \left( z^2 + \frac{B_2}{2} z^4 \right) \frac{1}{(z^4_h - z^4)} \right]
\]

After subtracting the divergent piece in the limit $\epsilon \to 0$, we get the renormalised Nambu-Goto action

\[
S_{NG}^{\text{ren}} = \frac{T}{\pi} \left[ \left( \frac{B_2}{2} z_{\text{max}} - \frac{1}{z_{\text{max}}} \right) - \frac{v^2}{16 z_h} \left( -4 B_2 z_h z_{\text{max}} + 2 (-2 + B_2 z_h^2) \tan^{-1} \left( \frac{z_{\text{max}}}{z_h} \right) \right) + \frac{2 + B_2 z_h^2}{z_h^4} \log \left( 1 + \frac{z_{\text{max}}}{z_h} \right) \right] + \frac{T}{4\pi} \frac{z^4_h - z^4_{\text{max}} \cosh^2 \eta}{z^4_h - z^4_{\text{max}}} A_n z^n_{\text{max}} r
\]
and hence the potential is obtained by

\[
V_{QQ} = \lim_{T \to \infty} \frac{S_{\text{ren}}}{T} = \frac{1}{\pi} \left[ \left( \frac{B_2}{2} \frac{z_{\text{max}}}{z_{\text{max}}} - \frac{1}{z_{\text{max}}} \right) - \frac{v^2}{16z_h} \left( -4B_2z_hz_{\text{max}} + 2(-2 + 2B_2z_h^2)\tan^{-1}\left( \frac{z_{\text{max}}}{z_h} \right) \right) + \left(2 + 2B_2z_h^2 \right) \log \left( \frac{z_h + z_{\text{max}}}{z_h - z_{\text{max}}} \right) \right] + \frac{1}{4\pi} \sqrt{\frac{z_h^2 - z_{\text{max}}^2}{z_h^2 z_{\text{max}}^2}} \cosh^2 \eta A_n z_{\text{max}}^n r .
\]  

(26)

The potential thus obtained are functions of both \(z_{\text{max}}\) and \(r\), which are redundant. To make the potential as a function of \(r\) only, we will express \(z_{\text{max}}\) as a function of \(r\) and then plug in to the above potential. To do that we first concentrate on the integral \(I\) which, keeping up to the second-order in both series in \(z\), reduces to:

\[
I = \int_{0}^{z_{\text{max}}} dz \frac{z^2\sqrt{1 + B_2z^2}}{\sqrt{(z_h^4 - z^4)(z_h^4 - z^4 \cosh^2 \eta)(1 + A_2z^2)^2}} \equiv \int_{0}^{z_{\text{max}}} dz \ 1'(z,v)
\]

(27)

In the limit of small velocity, we expand the integrand \(1'(z,v)\) in the Taylor series in velocity, \(v\) as:

\[
1'(z,v) = 1'(z,0) + \frac{\partial 1'}{\partial v} v + \frac{1}{2} \frac{\partial^2 1'}{\partial v^2} v^2 + \cdots
\]

and neglecting the higher order terms, the integral can be written as:

\[
I = \int_{0}^{z_{\text{max}}} dz \frac{z^2\sqrt{1 + B_2z^2}}{(z_h^4 - z^4)(1 + A_2z^2)^2} + \frac{v^2}{2} \int_{0}^{z_{\text{max}}} dz \frac{z^6\sqrt{1 + B_2z^2}}{(z_h^4 - z^4)^2(1 + A_2z^2)^2}
\]

We will now find the solution for the separation, \(r\) in two limits, namely \(z_{\text{max}} \gg z_h\) and \(z_{\text{max}} \ll z_h\). First for the \(z_{\text{max}} \gg z_h\) limit, the separation \(r\) from Eq.(19) becomes as a function of \(z_{\text{max}}\)

\[
\frac{r\sqrt{1 - v^2}}{2iz_h^2} = -(c_1 + ic_2 - \frac{v^2}{2}(c_3 + ic_4))(A_2z_{\text{max}}^2 + 1) + \frac{B_2(1 - \frac{v^2}{2})}{6A_2 z_{\text{max}}} + \frac{1}{2} \frac{z_h^4}{z_{\text{max}}^2}(c_1 + ic_2 - \frac{v^2}{2}(c_3 + ic_4)) + \frac{1}{30} \frac{A_2^3}{A_2^2 z_{\text{max}}^3} + \frac{1}{2} \frac{z_h^4}{z_{\text{max}}^2}(c_1 + ic_2 - \frac{v^2}{2}(c_3 + ic_4))
\]

(28)
where \( c_1, c_2, c_3 \) and \( c_4 \) are defined as

\[
\begin{align*}
c_1 &= \frac{\pi}{8} \left[ \frac{1}{z_h^2 (2 - z_h^2)} - \frac{6A_2 - B_2 + A_2^2 z_h^2(2A_2 - 3B_2)}{\sqrt{A_2(-1 + A_2^2 z_h^2)^2}} \right] \\
c_2 &= \frac{\pi}{8} \left[ \frac{2 + B_2 z_h^2}{z_h (2 - z_h^2 + 1)^2} - \frac{(6A_2 - B_2 + A_2^2 z_h^2(10A_2 - 7B_2))}{\sqrt{A_2(-1 + A_2^2 z_h^2)^3}} - \frac{(6 - 5B_2 z_h^2 + A_2 z_h^2(2 + B_2 z_h^2))}{4z_h (A_2 z_h^2 - 1)^3} \right] \\
c_3 &= \frac{\pi}{8} \left[ \frac{6 + 5B_2 z_h^2 + A_2 z_h^2(-2 + B_2 z_h^2)}{32z_h (2 - z_h^2 + 1)^3} \right]
\end{align*}
\]

We now invert the series (28) to obtain \( z_{\text{max}} \) in terms of \( r \) as,

\[
\begin{align*}
z_{\text{max}} &= \frac{1}{\sqrt{2A_2 z_h^2 (-ic_1 + c_2 + \frac{\pi^2}{2} (ic_3 - c_4))}} \sqrt{r \sqrt{(1 - v^2)}}
\end{align*}
\]

which shows that the string coordinates become complex\(^1\) and hence the potential from Eq. (26) becomes imaginary:

\[
\begin{align*}
V_{QQ} &= \frac{1}{\pi} \left( \frac{B_2}{2} z_{\text{max}} (1 + v^2) - \frac{\pi v^2}{16z_h} (-2 + B_2 z_h^2 - i(2 + B_2 z_h^2)) - \frac{1}{z_{\text{max}}} (1 + v^2) - \frac{v^2 z_h^2 B_2}{12 \sqrt{(1 - v^2)}} \right) \\
&- \frac{ir}{4\pi z_h^2 \sqrt{(1 - v^2)}} \left( A_2 z_{\text{max}}^2 + 1 - \frac{A_2 z_h^4 (1 - v^2)}{z_{\text{max}}^2} - \frac{1}{2} \frac{z_h^4 (1 - v^2)}{z_{\text{max}}^4} \right)
\end{align*}
\]

After substituting \( z_{\text{max}} \) in terms of \( r \) from Eq.(33), we obtain the real and imaginary parts of potential as

\[
\begin{align*}
\text{Re}[V(r, T, v)] &= \frac{r^2 (c_1 - \frac{\pi^2}{2} c_3)}{8\pi z_h^2 ((c_2 - \frac{\pi^2}{2} c_4)^2 + (c_1 - \frac{\pi^2}{2} c_3)^2)} + \frac{(10 - 5v^2 - 6v^4)}{24(1 - v^2)} \\
&\times \sqrt{\left( (c_2 - \frac{\pi^2}{2} c_4)^2 + (c_1 - \frac{\pi^2}{2} c_3)^2 \right)} + \frac{B_2}{\sqrt{2A_2 \pi}} \sqrt{r \sqrt{1 - v^2}} \\
&- \frac{v^2}{16z_h} (-2 + B_2 z_h^2) - \frac{(4A_2 + B_2) z_h (18 - 9v^2 - 10v^4)}{40(1 - v^2)} \\
&\times \sqrt{\left( (c_2 - \frac{\pi^2}{2} c_4)^2 + (c_1 - \frac{\pi^2}{2} c_3)^2 \right)} + \frac{1}{\sqrt{2A_2 \pi}} \sqrt{r \sqrt{1 - v^2}}
\end{align*}
\]

\(^1\) This solution was earlier abandoned in evaluating the potential for the heavy quark pair in a moving medium [28].
respectively. We have plotted the variation of the real and imaginary parts of potential with the separation \( r \) for different velocities. We find that the potential becomes stronger with the increase of velocity, which can be understood from the fact that the effective temperature gets reduced due to the recession of the medium and hence the screening becomes weaker. The imaginary part increases in magnitude as the velocity increases. We will now find the potential for the other extreme limit i.e., very small \( z_{\text{max}} \) and \( z_{\text{max}} \ll z_h \). In this limit, the separation \( r \) becomes

\[
r = \frac{2}{3} z_{\text{max}} + \frac{(3B_2 - 2A_2)}{15} z_{\text{max}}^3,
\]

which will eventually be inverted to express \( z_{\text{max}} \) in terms of \( r \) as

\[
z_{\text{max}} = \frac{3r}{2} + \frac{27}{80} (2A_2 - 3B_2) r^3 + O[r^5]
\]

Thus the potential in this limit reduces to

\[
\text{Re}[V_{QQ}(r, T)] \underset{z_{\text{max}} \ll z_h}{\simeq} -\frac{5}{9\pi r} + \frac{9(A_2 + B_2)}{20\pi} r + O[r^3]
\]

Hence up to first order in \( r \), the potential is independent of velocity in this limit. The first term here is like the coulombic term while the second term is the linear confining term.
4 Calculation of thermal width

The thermal width $\Gamma_{Q\bar{Q}}$ arises due to the imaginary component of the potential and is defined as

$$\Gamma_{Q\bar{Q}} = -\langle \psi | \text{Im} V_{Q\bar{Q}}(r, T, v) | \psi \rangle,$$

where the wave function for the ground state (1S) is given by

$$\psi = \frac{1}{\sqrt{\pi} a_0^{3/2}} e^{-r/a_0}$$

(41)

and for the first excited state

$$\psi = \frac{1}{4\sqrt{2\pi} a_0^{3/2}} \left( 2 - \frac{r}{a_0} \right) e^{-r/2a_0}$$

(42)

in a Coulomb-like potential, with $a_0 = (\mu e^2)^{-1}$ being the Bohr radius ($\mu$ is the reduced mass and $e^2$ is the square of the electric charge). Even though the real part of the potential is not purely Coulombic but the leading contribution for the potential for the deeply bound $Q\bar{Q}$ states in a conformal plasma and thus justifies the use of Coulomb-like wave functions to determine the
width. Thus we calculate the thermal width for the ground-state (1S) as
\[
\Gamma_{\bar{Q}Q}(1S) = -\frac{4}{a_0^3} \int_0^\infty dr r^2 e^{-2r/a_0} \text{Im} V_{\bar{Q}Q}(r,T) \\
= \frac{972\pi^5 T^4}{200 m_Q^2} \frac{(c_2 - \frac{v^2}{T} c_4)}{(c_2 - \frac{v^2}{T} c_4)^2 + (c_1 - \frac{v^2}{T} c_3)^2} - \frac{45\sqrt{2}\pi TB_2}{32 \sqrt{5} m_Q A_2} \frac{(1 - v^2)^{\frac{3}{2}}(10 - 5v^2 - 6v^4)}{24(1 - v^2)} \\
\times \sqrt{\frac{((c_2 - \frac{v^2}{T} c_4)^2 + (c_1 - \frac{v^2}{T} c_3)^2) - (c_2 - \frac{v^2}{T} c_4)}{2((c_2 - \frac{v^2}{T} c_4)^2 + (c_1 - \frac{v^2}{T} c_3)^2)}} \\
\] (43)
temperature and also increases slowly with the velocity. It can be understood from the fact that as the velocity of the medium increases, the flux of plasma passing through the $Q\bar{Q}$ pair increases and hence causes to broaden the thermal width. The same reason applies to the increase of imaginary part of potential with the velocity. As a consequence, the pair will be dissolved earlier into a moving medium compared to the static medium. Another observation is that the thermal width for bottomonium states (Fig. (4) and (5)) is much smaller than the charmonium states because bottomonium states are bound tighter than charmonium states.

5 Conclusions

In summary, we have obtained the inter-quark potential at finite temperature in a dual gravity closer to thermal QCD for a heavy quark and antiquark pair moving normal to its orientation. When the (hanging) string lying on the fifth dimension, $z_{\text{max}}$ is far above the horizon, i.e. $z_{\text{max}} << z_h$, the Nambu-Goto action gives rise to the similar form of Cornell potential, unlike the Coulomb term alone usually reported in the literature. On the other hand when the string reaches deep into the horizon ($z_{\text{max}} >> z_h$), the potential develops an imaginary component. Alternatively it can be stated that beyond a critical separation of $Q\bar{Q}$ pair, the string coordinates become imaginary and as a result, the potential becomes complex. The imaginary part vanishes in the limit of small separation which is also seen in the perturbative calculations. Furthermore as the pair starts moving, the screening of the potential becomes smaller, which may be understood
Figure 5: Same as Figure 3 but for the bottomonium state.

qualitatively by the decrease of effective temperature. However for a particular velocity of the pair, the screening becomes stronger with the increase of the temperature as observed in the potential for a static pair.

Since the quarkonium dissociation is presently thought mainly due to Landau damping induced thermal width, we have therefore obtained the thermal width for the ground and first excited states from the imaginary part of the potential. We found that the width not only increases with the increase of temperature, it also increases slowly with the velocity of the pair, which makes sense. The effect of enhancement of width is more pronounced to the charmonium states than the bottomonium states. Since the study of quarkonium bound states propagating in the QGP at finite velocity is nowadays relevant in heavy-ion collisions so our study gives an theoretical input in this regard. Indeed the PHENIX Collaboration [41] had found a substantial elliptic flow for heavy-flavor electrons, indicating a significant damping of heavy quarks while travelling across the medium [42].

Note added. When this article was being finished, we became aware of Ref.[43] which also discussed the effects of nonzero rapidity on the imaginary part of the heavy quark potential by the thermal worldsheet fluctuation method in a $\mathcal{N}=4$ supersymmetric plasma. In their work, they computed the imaginary potential for arbitrary orientations of the $Q\bar{Q}$ pair with respect to the hot plasma wind in pure AdS BH background. On the contrary we use OKS-BH geometry in the gravity side whose dual in the gauge theory side is closer to thermal QCD ie. runs with the energy scale similar to QCD, unlike scale-independent in $\mathcal{N}=4$ SYM theory. Moreover both the origin of complex nature of potential and renormalization subtraction are different from the above work [43] namely in our work $z_{\text{max}}$ (the maximum value of $z$ along the fifth direction, along which the string is hanging) becomes complex for a critical separation of the pair and leads to the complex-valued string coordinates [23, 24], hence the potential too becomes complex.
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