Spin and charge optical conductivities in spin-orbit coupled systems

Jesús A. Maytorena, Catalina López-Bastidas, and Francisco Mireles

Centro de Ciencias de la Materia Condensada,
Universidad Nacional Autónoma de México,
Apdo. Postal 2681, 22800 Ensenada, Baja California, México

(Dated: March 23, 2022)

Abstract

We study the frequency dependent spin- and charge- conductivity tensors of a two-dimensional electron gas (2DEG) with Rashba and Dresselhaus spin-orbit interaction. We show that the angular anisotropy of the spin-splitting energy induced by the interplay between the Rashba and Dresselhaus couplings gives rise to a characteristic spectral behavior of the spin and charge response which is significantly different from that of pure Rashba or Dresselhaus case. Such new spectral structures open the possibility for control of the optical response by applying an external bias and/or by adjusting the light frequency. In addition, it is shown that the relative strength of the spin-orbit coupling parameters can be obtained through optical probing.

PACS numbers: 73.63.Kv, 72.25.-b, 73.21.La, 72.25.Dc
Spin-orbit interaction (SOI) in systems lacking inversion symmetry is a phenomenon with great potential in the development of spintronic-based devices. Since the celebrated proposal of a spin-FET relying in the tunability of the Rashba SOI strength through electrical gating, there has been a remarkable attention in the search for new ways of manipulating electron spins without employing ferromagnetic materials and/or external magnetic fields. For instance, a spin Hall effect in which a transverse spin current is driven by a dc electric field (without a net charge current) was predicted to arise in low-dimensional systems with a substantial SOI. Spin (Hall) accumulation has been observed through optical measurements and very recently the first observation, purely electrical, of the spin Hall effect in a metallic conductor was reported.

The spin splitting $\Delta$, caused by SOI in electron systems, opens the possibility of resonant effects as a response to alternating electric fields due to transitions between the spin-split states. For instance, the absorption of linearly polarized infrared radiation has been recognized as a mechanism to inject pure spin currents in quantum wells by inter-spin-split-subband transitions. An ac spin current generation by the time variation of the Rashba coupling through time modulated gate voltages has also been proposed. In a recent study it has been suggested that an intense ac probing field can be used to control the spin-Hall current in 2DEGs with Rashba SOI. It has also been shown that in the THz range of frequencies, the cancellation of the intrinsic spin Hall effect due to impurity scattering is no longer perfect, and in principle the effect could be present. Moreover, for finite frequencies at the window of $\Delta > h\omega > h\tau^{-1}$, in which $\tau^{-1}$ is the impurity scattering rate, the spin Hall conductivity converges to its universal intrinsic value of $e/8\pi$. Another resonant phenomenon is the electric-dipole-induced spin resonance in which the electron spins may be manipulated via SO coupling by means of time-dependent electric fields (rather than resonant magnetic fields) as is shown to occur in both clean and disordered systems.

Other recent studies have also emphasized the importance of the dynamical regime, investigating several relevant physical aspects, such as the relation between the spin Hall conductivity and the spin susceptibility or the dielectric function, the presence of electron-electron interactions and plasmon modes, the study of the optical absorption spectrum and the renormalization of the Rashba parameter and the spin-splitting energy. More recently, effects of strains and of the electron-phonon
interaction\[20\] on the spin Hall currents have been also explored. All these studies considered only the Rashba type of SOI. However, both experimental\[31, 32\] and theoretical work\[24, 25, 33, 34, 35, 36\] have pointed out the importance of the (linear) Dresselhaus type of SOI contribution and its interplay with the Rashba coupling.

In this work, we study the charge- and spin-current conductivity tensors of a 2DEG with Rashba and Dresselhaus SOI as a linear response to a frequency dependent (spatially homogeneous) weak electric field. We find that the angular anisotropy of the energy spin-splitting introduced by the interplay between both coupling strengths yields a finite-frequency response with spectral features that are significantly different from that of a pure Rashba (Dresselhaus) coupling case. As a consequence, an optically modulable spin and charge current response is then achievable in such systems. It is noticed as well that the inter-spin-split subband excitations via photon absorption are now possible in a wider frequency range, depending explicitly upon the Fermi wave vector and on the SOI parameters. Furthermore, it is shown that such effect may be used to extract the ratio between the SOI coupling parameters via optical and/or transport experiments.

We consider a 2D free electron system lying at \( z = 0 \) plane, with a Hamiltonian given by

\[
H = \frac{\hbar^2 k^2}{2m^*} + H_{so},
\]

where the spin-orbit interaction is

\[
H_{so} = \alpha (k_x \sigma_y - k_y \sigma_x) + \beta (k_x \sigma_x - k_y \sigma_y),
\]  

(1)

The first term corresponds to the Rashba SO coupling which originates from any source of structural inversion asymmetry of the confining potential. The second term is the linear Dresselhaus coupling which results from bulk-induced inversion asymmetry. The spectral properties of this Hamiltonian are well known.\[24, 33, 34\] The eigenstates \(|k\lambda\rangle\) for the in-plane motion are specified by the wave vector \( k = (k_x, k_y) = k (\cos \theta, \sin \theta) \) and chirality \( \lambda = \pm 1 \) of the spin branches. The double sign corresponds to the upper (+) and lower (−) parts of the energy spectrum given by 

\[
\varepsilon_\lambda(k, \theta) = \frac{\hbar^2 k^2}{2m^*} + \lambda k\Delta(\theta),
\]

where \( \Delta(\theta) = \sqrt{\alpha^2 + \beta^2 - 2\alpha\beta \sin 2\theta} \) describes the angular anisotropy of the spin splitting. At zero temperature, the two spin-split subbands are filled up to the same (positive) Fermi energy level \( \varepsilon_F \) but with different Fermi wave vectors \( q_\lambda(\theta) = \sqrt{2m^* \varepsilon_F / \hbar^2 + k_{so}^2(\theta)} - \lambda k_{so}(\theta) \), determined from the equations \( \varepsilon_\lambda(q_\lambda(\theta), \theta) = \varepsilon_F \). Here, \( k_{so}(\theta) = m^* \Delta(\theta) / \hbar^2 \) is the characteristic SO momentum, \( \varepsilon_F = \hbar^2 (k_0^2 - 2q_{so}^2) / 2m^* \) with \( k_0 = \sqrt{2\pi n} \) being the Fermi wave vector of a spin-degenerate 2DEG with density \( n \), and \( q_{so} = m^* \sqrt{\alpha^2 + \beta^2} / \hbar^2 \). Because of the SOI,
the Fermi line splits into two curves with radii given by \( q_\lambda (\theta) \) which, as the energy surfaces \( \varepsilon_\lambda (k) \), are symmetric with respect to the (1,1) and (-1,1) directions in \( k \)-space (Fig. 1). When \( \alpha \) or \( \beta \) is null, the dispersions are isotropic and the Fermi contours are concentric circles.

The 2DEG is excited by a uniform electric field \( E \) oscillating at frequency \( \omega \) along the \( y \)-direction. The driven electric current is described by the charge current conductivity tensor \( \sigma_{ij}(\omega) = \delta_{ij}\sigma_D(\omega) + \sigma_{ij}^s(\omega) \), \( i, j = x, y \), in which \( \sigma_D(\omega) = in\epsilon^2/m^*\omega \) is the Drude conductivity, and \( \sigma_{ij}^s(\omega) \) is the contribution due to inter-spin-split induced transitions. Within the linear response Kubo formalism this SO contribution is

\[
\sigma_{ij}^s(\omega) = \frac{1}{\hbar(\omega + i\eta)} \int_0^\infty dt \, e^{i(\omega + i\eta)t} \langle [j_i(t), j_j(0)] \rangle ,
\]

the symbol \( \langle [A(t), B(0)] \rangle = \sum_\lambda J(\omega, B(0)) \langle k\lambda | [A(t), B(0)] | k\lambda \rangle \) indicates quantum and thermal averaging of the commutator of the operators \( A \) and \( B \) in the interaction picture, \( f(\varepsilon) \) is the Fermi distribution function, and \( \eta \to 0^+ \). The prime on the integral indicates that integration is restricted to the region between the Fermi contours, \( q_+(\theta) < k < q_-(\theta) \), for which \( \varepsilon_-(k) < \varepsilon_F < \varepsilon_+(k) \), (Fig. 1). Here, \( j_i = ev_i \), with \( i = x, y \), is the electric charge current operator, where \( v_i \) is a component of the velocity operator \( \mathbf{v}(k) = \nabla_k H/\hbar = \hbar k/m^* + \hat{\mathbf{x}} (\beta \sigma_x + \alpha \sigma_y)/\hbar - \hat{\mathbf{y}} (\alpha \sigma_x + \beta \sigma_y)/\hbar \).

There is a connection between \( \sigma_{ij}^s(\omega) \) and the spin current response. The spin conductivity describing a \( z \)-polarized-spin current flowing in the \( i \)-direction as a response to the field \( E\hat{\mathbf{y}} \) is given by

\[
\Sigma_{iy}^z(\omega) = \frac{1}{\hbar(\omega + i\eta)} \int_0^\infty dt \, e^{i(\omega + i\eta)t} \langle [\mathcal{F}^z_i(t), j_y(0)] \rangle ,
\]

where \( \mathcal{F}^z_i = (\hbar/4)(\sigma_z v_i + v_i \sigma_z) \) is the spin current operator, with \( i \) indicating the transport direction; \( \sigma_{x,y,z} \) are the Pauli matrices.

Using the equation of motion for \( j_y(t) \) to relate \( dj_y(t)/dt \) and \( \mathcal{F}^z_i(t) \) [17] a relation between the above conductivities can be obtained. In particular, the spin Hall conductivity \( \Sigma_{xy}^z \) and the diagonal charge conductivity \( \sigma_{yy}^s \) are related through the expression

\[
\frac{\Sigma_{xy}^z(\omega)}{(e/8\pi)} = \frac{\hbar \omega}{\varepsilon_R - \varepsilon_D} \frac{i \sigma_{yy}^s(\omega)}{(e^2/2\pi\hbar)} ,
\]

where \( \varepsilon_R = m^* \alpha^2/\hbar^2 \) and \( \varepsilon_D = m^* \beta^2/\hbar^2 \) are the SO characteristic energy scales for the Rashba and Dresselhaus coupling.
FIG. 1: Fermi contours $q_{\lambda}(\theta)$ and the constant-energy-difference curve $C_r(\omega)$ defined by $\varepsilon_+(k) - \varepsilon_-(k) = \hbar\omega$, shown for two values of the photon energy $\omega_1 > \omega_2$. $C_r(\omega)$ is a rotated ellipse with semi-axis of lengths $k_a(\omega) = \hbar\omega/2|\alpha - \beta|$ and $k_b(\omega) = \hbar\omega/2|\alpha + \beta|$ oriented along the $(1,1)$ and $(-1,1)$ directions respectively. The sample parameters used here are $n = 5 \times 10^{11}\text{cm}^{-2}$, $\alpha = 1.6 \times 10^{-9}\text{eV cm}$, $\beta = 0.5\alpha$ and $m^* = 0.055m$.

We evaluate these formulas in the limit of vanishing temperature and in the absence of impurity scattering. Given that the optical absorption spectrum is determined by the imaginary part of the dielectric function $\varepsilon_{ij}(\omega) \propto i\sigma_{ij}(\omega)/\omega$ [39], in Fig. 2(a) we show $\text{Re}\sigma_{yy}(\omega)$. The result is markedly different from that of the isotropic splitting case $\beta = 0, \alpha \neq 0$, for which $\text{Re}\sigma_{ij}^I(\omega) = \delta_{ij}\sigma_R$ for $2\alpha k_+ \leq \hbar\omega \leq 2\alpha k_-$, where $\sigma_R = e^2/16\hbar$ and $k_\lambda = q_\lambda(\beta = 0)$; the absorption band width is thus determined only by the coupling strength $\alpha$, yielding an spectrum of optical absorption essentially featureless. [10, 29].

In contrast, when both coupling strengths $\alpha$ and $\beta$ are present, the spectrum becomes wider and highly asymmetric, and new spectral features appear. To understand this structure we note, from Eq. (2), that $\text{Re}\sigma_{ij}^s(\omega) = \int d^2k M_{ij}(k)\delta(\varepsilon_+(k) - \varepsilon_-(k) - \hbar\omega)$ with $M_{ij}(k) = (e^2/4\pi\omega)(-|v_i(k)|+:|v_j(k)|-)$.

This expression shows that the response may be understood in terms of optical transitions and energy absorption; such expression could also be derived from a golden rule calculation. From the spin splitting of the Fermi
line, it can be seen that the only transitions allowed between spin-split subbands $\varepsilon_\lambda$ due to photon absorption at energy $\hbar\omega$ are those for which $\hbar\Omega_+(\theta) \leq \hbar\omega \leq \hbar\Omega_-(\theta)$ with $\hbar\Omega_+(\theta) = \varepsilon_F - \varepsilon_-(q_+(\theta), \theta)$ and $\hbar\Omega_-(\theta) = \varepsilon_+(q_-(\theta), \theta) - \varepsilon_F$. That is, for a given $\omega$ only those angular regions in $\mathbf{k}$-space satisfying this condition are available for optical transitions (see Fig. 2(c)). Thus, the charge conductivity tensor can be recasted as

$$\text{Re} \sigma_{ij}^s(\omega) = \frac{e^2(\alpha^2 - \beta^2)^2}{32\pi\hbar} \int d\theta \frac{\delta_{ij} - (1 - \delta_{ij})\sin^2\theta}{\Delta^4(\theta)} \Theta[(\hbar\omega - \hbar\Omega_+(\theta))][\Theta[\hbar\Omega_-(\theta) - \hbar\omega]],$$

(5)

where the unit step functions $\Theta(x)$ impose the above mentioned condition. This is different to the pure Rashba (or Dresselhaus) case, where the whole interval $[0, 2\pi]$ contributes to the integral for each allowed photon energy. Interestingly, the non-isotropic spin-splitting originated by the simultaneous presence of both coupling strengths, forces the optical excitation to be $\mathbf{k}$-selective. We also notice that for $\alpha^2 = \beta^2$ the SO contribution $\sigma_{ij}^s(\omega)$ vanishes and the charge conductivity tensor becomes isotropic. This is due to the particular form acquired by the eigenstates and dispersion relations exactly at that case. \[33, 34\]

The minimum (maximum) photon energy $\hbar\omega_+ (\hbar\omega_-)$ required to induce optical transitions between the initial $\lambda = -1$ and the final $\lambda = +1$ subband corresponds to the excitation of an electron with wave vector lying on the $q_+$ ($q_-$) Fermi line at $\theta_+ = \pi/4$ or $5\pi/4$ ($\theta_- = 3\pi/4$ or $7\pi/4$), giving $\hbar\omega_\pm = \hbar\Omega_\pm(\theta_\pm) = 2k_0|\alpha \mp \beta| \mp 2m^*(\alpha \mp \beta)^2/\hbar^2$. The absorption edges in the spectrum of Fig. 2 correspond exactly to $\hbar\omega_\pm$. The real part of $\sigma_{ij}^s(\omega)$ can also be written as a line integral to be performed along the arcs of the resonant curve $C_\tau(\omega)$ lying within the region enclosed by the Fermi lines $q_\pm(\theta)$ (Fig. 1). The peaks observed in Fig. 2(a) correspond to electronic excitations involving states with allowed wave vectors on $C_\tau(\omega)$ such that the velocity $|\nabla_{\mathbf{k}}(\varepsilon_+ - \varepsilon_-)/\hbar|$ takes its minimum value. The first (second) peak is at a photon energy $\hbar\omega_a (\hbar\omega_b)$ for which the major (minor) semi-axis of the ellipse $C_\tau(\omega)$ (Fig. 1) coincides with the Fermi line $q_+(\theta_+)$ ($q_-(\theta_-)$), hence $\hbar\omega_a = \hbar\Omega_-(\theta_+) = 2k_0|\alpha - \beta| + 2m^*(\alpha - \beta)^2/\hbar^2$ and $\hbar\omega_b = \hbar\Omega_+(\theta_-) = 2k_0|\alpha + \beta| - 2m^*(\alpha + \beta)^2/\hbar^2$. This resembles the presence of critical points or van Hove singularities which are sources of structure in the joint density of states (JDOS) and in optical constants. The JDOS is displayed in Fig. 2(b). The unequal splitting at the Fermi level along the symmetry $(1, 1)$ and $(-1, 1)$ directions is thus responsible for the absorption and high density peaks at photon energies $\hbar\omega_a$ and $\hbar\omega_b$ respectively, giving meaning to the structure of the spectra. The overall magnitude and the asymmetric shape of the spectrum are due to the factor $(\alpha^2 - \beta^2)/\Delta^4(\theta)$ in Eq. (5). The results for several values
FIG. 2: (c) Angular region (shaded) in k-space available for direct transitions as a function of photon energy. Only the shaded region contribute to the optical absorption [eq. (5)]. The energy boundaries are given by $\hbar \Omega_{\pm}(\theta) = 2q_{\pm}(\theta)\Delta(\theta)$. (b) The joint density of states. (a) SO contribution to the charge conductivity, $\text{Re} \sigma_{yy}(\omega)$. For the frequencies $\omega_+ = \Omega_+(\pi/4)$, $\omega_a = \Omega_-(\pi/4)$, $\omega_b = \Omega_+(3\pi/4)$, $\omega_- = \Omega_-(3\pi/4)$, see the text. The sample parameters are the same as in Fig. 1.
FIG. 3: Charge conductivity $\text{Re} \sigma_{yy}(\omega)$ for several values of the ratio $\beta/\alpha$. Other parameters are as those used in Fig.1.

of $\beta/\alpha$ are shown in Fig.3. The value $\text{Re} \sigma_{xy}(\omega)$ (not shown) displays a similar spectral behavior as the diagonal component $\sigma_{yy}$.

The absorption bandwidth $\Delta \mathcal{E} = \hbar \omega_- - \hbar \omega_+$ is independent of the frequency and is given by

$$
\Delta \mathcal{E}(\alpha, \beta; n) = 4k_0[\beta\Theta(\alpha - \beta) + \alpha\Theta(\beta - \alpha)] + 4(\varepsilon_R + \varepsilon_D),
$$

(6)

assuming $\left(k_{so}(\theta)/k_0\right)^2 \ll 1$. We notice that the two first terms can be about an order of magnitude larger than the Rashba result, $\Delta \mathcal{E}_R = 4\varepsilon_R$, as the spectra in Fig.2 clearly illustrates for typical cases. Moreover, $\Delta \mathcal{E}$ becomes explicitly dependent on the electron density $n$ (through the Fermi wave vector $k_0$), which is also in contrast to the $\beta = 0$ case.29

Thus, as a result of the interplay between the Rashba and Dresselhaus interactions, another manipulable parameter ($n$) appears to control the optical spin-split response in addition to the tunable coupling strength $\alpha$. The expression for $\Delta \mathcal{E}$ suggests that its variation with $\alpha$ can be about an order of magnitude larger if $\alpha < \beta$ than for the opposite case. This can be useful to determine the sign of $\alpha - \beta$.

Moreover, an expression for the ratio $\beta/\alpha$ can also be derived in terms of the relevant
FIG. 4: Spin Hall conductivity tensor vs. photon energy, for $\beta/\alpha = 0.5, 0.25, 0.1$ (solid, dashed and dotted respectively). The rest of the parameters are the same as in Fig. 1.

frequencies of the absorption spectrum (assuming $\alpha > \beta > 0$),

$$\frac{\beta}{\alpha} = \frac{(\omega_- - \omega_+) + (\omega_b - \omega_a)}{(\omega_- + \omega_+) + (\omega_b + \omega_a)}$$

which may be useful in optical experiments for extracting information of the relative SOI strengths.

As for the spin Hall conductivity tensor $\text{Re}\Sigma_{iy}^z(\omega)$ we obtain from Eq. (3)

$$\text{Re} \Sigma_{iy}^z(\omega) = \Sigma_{iy}^z(0) + \frac{e}{8\pi} \frac{\hbar \omega}{\varepsilon_R - \varepsilon_D} I_i(\omega)$$
where explicitly,

\[ I_i(\omega) = \frac{(\alpha^2 - \beta^2)^2}{8\pi} \int_0^{2\pi} d\theta \, g_i(\theta) \log \left| \frac{\omega + \Omega_+(\theta)}{\omega - \Omega_-(\theta)} \right| \]

with \( g_i(\theta) = [\delta_{iz} \cos^2 \theta + \delta_{iy} \sin \theta \cos \theta] / \Delta^4(\theta) \).

The static values are \( \Sigma_{iyy}^z(0) = (e/8\pi) \text{sgn}(\alpha^2 - \beta^2) \) and \( \Sigma_{yy}^z(0) = (e/8\pi)(\beta/\alpha) \Theta(\alpha^2 - \beta^2) - (\alpha/\beta) \Theta(\beta^2 - \alpha^2) \), in coincidence with Refs. 35 and 36. For \( \beta = 0 \), \( I_y(\omega) = 0 \) and \( \Sigma_{yy}^z(\omega) = 0 \). As a result, for photon energies \( \hbar \omega_a \leq \hbar \omega \leq \hbar \omega_b \), the tensor \( \text{Re} \Sigma_{iy}^z(\omega) \) takes a constant value, which is zero for the transverse component \( (i = x) \) and \(- (e/8\pi)(\alpha^2 - \beta^2)/2\alpha \beta \) for the longitudinal component \( (i = y) \), provided \( \alpha \neq \beta \neq 0 \). This is a consequence of the particular angular symmetry acquired by the integrand in Eq. (9) for such frequencies. The spectra in Fig. 4 show that the magnitude and the direction (sign) of the spin Hall conductivity \( \Sigma_{xy}^z(\omega) \) depend on the frequency and the SO coupling strengths \( \alpha, \beta \), and therefore could be manipulated via electrical gating and/or by adjusting the light frequency. The latter offers new possibilities of control of spin currents in electron systems with competing Rashba and Dresselhaus SOI.

In the above calculations we have not included any type of relaxation mechanism. The parameter \( \eta \to 0^+ \) means, as usually, that the perturbation is turned on adiabatically, ensuring a causal response. In the following, in the line of Ref. 40, we obtain the static value of the spin conductivity for a finite damping parameter \( \eta > 0 \). This parameter accounts phenomenologically for dissipation effects due to impurity scattering and it is considered here as a momentum relaxation rate.

The substitution of \( \omega \to i\eta \) in eqs. (8) and (9), leads to

\[
\Sigma_{iy}^z(0; \eta) = \Sigma_{iy}^z(0; \eta = 0) - \frac{e}{8\pi} \frac{\hbar \eta}{\varepsilon_R - \varepsilon_D} F_i(\eta) \quad (10)
\]

where \( \Sigma_{iy}^z(0; \eta = 0) \) is the zero frequency value for \( \eta \to 0^+ \) mentioned above, and

\[
F_i(\eta) = \frac{(\alpha^2 - \beta^2)^2}{4\pi} \int_0^{2\pi} d\theta \, g_i(\theta) \arctan \left[ \frac{4\varepsilon_{so}(\theta) / \hbar \eta}{1 + 8\varepsilon_F \varepsilon_{so}(\theta) / (\hbar \eta)^2} \right] \quad (11)
\]

with \( \varepsilon_{so}(\theta) = m^* \Delta^2(\theta)/\hbar^2 \). It can be shown that if \( m^*(\alpha + \beta)^2/\hbar^2 < \hbar \eta \), eq. (10) leads to

(for \( \alpha \neq \beta \neq 0 \))

\[
\Sigma_{xy}^z(0; \eta) \approx \frac{e \varepsilon_F}{\pi \hbar \eta} \left( \frac{\varepsilon_R - \varepsilon_D}{\hbar \eta} \right) - \frac{8e}{\pi} \left( \frac{\varepsilon_R + \varepsilon_D}{\hbar \eta} \right) \left( \frac{\varepsilon_F}{\hbar \eta} \right)^2 \left( \frac{\varepsilon_R - \varepsilon_D}{\hbar \eta} \right)^2, \quad (12)
\]

\[
\Sigma_{yy}^z(0; \eta) \approx \frac{8e}{\pi} \left( \frac{\sqrt{\varepsilon_R \varepsilon_D}}{\hbar \eta} \right) \left( \frac{\varepsilon_F}{\hbar \eta} \right)^2 \left( \frac{\varepsilon_R - \varepsilon_D}{\hbar \eta} \right)^2. \quad (13)
\]
These expressions show that the static limits of the transversal and longitudinal spin conductivities vanish to different orders in the parameter $|\varepsilon_R - \varepsilon_D|/\hbar\eta$. For $\beta = 0$ equations (10) and (12) agree with the results of Ref. 40.

In summary, we have shown that the coexistence of Rashba and Dresselhaus SOI in 2DEGs induces an anisotropic spin-splitting which gives rise to a characteristic frequency dependence of the charge- and spin Hall-conductivities of 2DEG systems. Such response provides us a ‘fingerprint signature’ of the presence of a competing Rashba and Dresselhaus SO mechanism. This also suggests the possibility of the optical manipulation of charge and spin (Hall) currents in addition to the control obtained through external bias. Probing the SO coupling strengths through optical spectroscopy and/or transport measurements could also be considered.

This work was supported by CONACyT-Mexico grants J40521F, J41113F, and by DGAPA-UNAM IN114403-3.

[1] S. Datta and B. Das, Appl. Phys. Lett. 56, 665 (1990).
[2] S. Murakami, N. Nagaosa, S.-C. Zhang, Science 301, 1348 (2003).
[3] J. Sinova, D. Culcer, Q. Niu, N.A. Sinitsyn, T. Jungwirth, and A.H. MacDonald, Phys. Rev. Lett. 92, 126603 (2004).
[4] J. Sinova, S. Murakami, S.-Q. Shen, and M.-S. Choi, Solid State Commun. 138, 214 (2006).
[5] J. Schliemann, Int. J. Mod. Phys. B 20, 1015 (2006).
[6] J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. 94, 047204 (2005).
[7] Y.K. Kato, R.C. Myers A.C. Gossard, and D.D. Awschalom, Science 306, 1910 (2004).
[8] V. Sih, R.C. Myers, Y.K. Kato, W.H. Lau, A.C. Gossard, and D.D. Awschalom, Nature 1, 31 (2005).
[9] S.O. Valenzuela and M. Tinkham, Nature 442, 176 (2006).
[10] L.I. Magarill, A.V. Chaplik, and M.V. Éntin, JETP 92, 153 (2001).
[11] E.I. Rashba, Phys. Rev. B 70 161201(R) (2004).
[12] E. Ya. Sherman, A. Najmaie, and J.E. Sipe, Appl. Phys. Lett. 86, 122103 (2005).
[13] A.G. Mal’shukov, C.S. Tang, C.S. Chu, and K.A. Chao, Phys. Rev. B 68, 233307 (2003).
[14] M. Governale, F. Taddei, and Rosario Fazio, Phys. Rev. 68, 155324 (2003).
[15] C.M. Wang, S. Y. Liu, and X.L. Lei, Phys. Rev. B 73, 035333 (2006).
A. Khaetskii, Phys. Rev. Lett. 96, 056602 (2006) and references therein.

A. Shekhter, M. Khodas, A.M. Finkel’stein, Phys. Rev. B 71, 165329 (2005).

E.G. Mishchenko, A.V. Shytov, and B.I. Halperin, Phys. Rev. Lett. 93, 226602 (2004).

O. Chalaev and D. Loss, Phys. Rev. B 71, 245318 (2005).

C. Grimaldi, E. Cappelluti, and F. Marsiglio, Phys. Rev. Lett. 97, 066601 (2006).

E.I. Rashba and A.L. Efros, Appl. Phys. Lett. 83, 5295 (2003).

M. Duckheim and D. Loss, Nature Phys. 2, 195 (2006).

C. Zhang and Z. Ma, Phys. Rev. B 71, 121307(R) (2005).

E.G. Mishchenko and B.I. Halperin, Phys. Rev. B 68, 045317 (2003).

S.I. Erlingsson, J. Schliemann, and Daniel Loss, Phys. Rev. B 71, 035319 (2005).

W. Xu, Appl. Phys. Lett. 82, 724 (2003).

X.F. Wang, Phys. Rev. B 72, 085317 (2005).

M. Pletyukhov and V. Gritsev, Phys. Rev. B 74, 045307 (2006).

D.W. Yuan, W. Xu, Z. Zeng, and F. Lu, Phys. Rev. B 72, 033320 (2005).

T.O. Cheche and E. Barna, Appl. Phys. Lett. 89, 042116 (2006).

S.D. Ganichev, V.V. Bel’kov, L.E. Golub, E.L. Ivchenko, P. Schneider, S. Gligiberger, J. Eroms, J. De Boeck, G. Borghs, W. Wegscheider, D. Weiss, and W. Prettl, Phys. Rev. Lett. 92, 256601 (2004).

J.B. Miller, D.M. Zumbühl, C.M. Marcus, Y.B. Lyanda-Geller, D. Goldhaber-Gordon, K. Campman, and A.C. Gossard, Phys. Rev. Lett. 90, 076807 (2003).

J. Schliemann and D. Loss, Phys. Rev. B 68, 165311 (2003).

J. Schliemann, J.C. Egues, and D. Loss, Phys. Rev. Lett. 90, 146801 (2003).

N.A. Sinitsyn, E.M. Hankiewicz, Winfried Teizer, and Jairo Sinova, Phys. Rev. B 70 081312(R) (2004).

Shun-Qing Shen, Phys. Rev. B 70 081311(R) (2004).

The spectrum resembles the \((\omega, \mathbf{q})\)-dependent spin susceptibility describing the response of a 2DEG with finite Rashba SOI to a space and time-dependent electric field studied by Rashba [E.I. Rashba, J. Supercond. 18, 137 (2005)].

In Ref. 25 \(\Sigma_{ij}(\omega)\) was obtained to lowest order in \(g_{so}/k_0 \ll 1\), showing resonances at \(2k_0|\alpha \pm \beta|\). This is equivalent however, to neglecting the terms \(2m^*(\alpha \pm \beta)^2/\hbar^2\) in the resonances \(\hbar \omega_a\) and \(\hbar \omega_b\). They can give non-negligible contributions as our calculation illustrates.
[39] H. Haug and S.W. Koch, *Quantum Theory of the Optical and Electronic Properties of Semiconductors*, Third Ed. (World Scientific, Singapore, 1994).

[40] J. Schliemann and D. Loss, Phys. Rev. B 69, 165315 (2004).