Ultrafast strain-induced charge transport in semiconductor superlattices

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We investigate the effect of hypersonic (> 1 GHz) acoustic phonon wavepackets on electron transport in a semiconductor superlattice. Our quantum mechanical simulations demonstrate that a GHz train of picosecond deformation strain pulses propagating through a superlattice can generate current oscillations whose frequency is several times higher than that of the strain pulse train. The shape and polarity of the calculated current pulses agree well with experimentally measured electric signals. The calculations also explain and accurately reproduce the measured variation of the induced current pulse magnitude with the strain pulse amplitude and applied bias voltage.

Our results open a route to developing acoustically-driven semiconductor superlattices as sources of millimetre and sub-millimetre electromagnetic waves.

I. INTRODUCTION

The development of picosecond techniques for generating ultra-fast coherent phonons\textsuperscript{1}\textsuperscript{2} has provided researchers with new efficient acoustoelectric tools for controlling charge transport in semiconductor nanostructures (see\textsuperscript{3} for review). For example, acoustoelectric effects have been proposed for the generation\textsuperscript{4}\textsuperscript{7}, heterodyne-mixing\textsuperscript{8}, and detection\textsuperscript{9}\textsuperscript{10} of high-frequency electromagnetic waves. Acousto-electronic pumps have also been used to control quantum phenomena in nanostructures and quantum dots\textsuperscript{11}\textsuperscript{16}, with recent experiments demonstrating that ultrafast acoustic pulses can dramatically enhance the emission output of a quantum-dot cavity\textsuperscript{17}\textsuperscript{18}. Also, semiconductor nanostructures can serve as active media for transforming THz acoustic waves into sub-mm electromagnetic radiation\textsuperscript{19}\textsuperscript{20}. It has also been recently shown that GHz-strain pulses can be rectified by a semiconductor superlattice (SL) producing a DC response similar to that due to rectification of microwaves\textsuperscript{21}. Despite this progress in understanding the GHz-THz acoustoelectric phenomena, the induced high-frequency charge transport and the corresponding electronic response to changing the parameters of the strain pulses have not yet been widely considered. In particular, it is still unclear how the coherent acoustic/strain stimuli affect the quantum tunneling of the charge and the resulting current flow dynamics.

In this paper, we present a quantum-mechanical theory of electron transport in a biased weakly-coupled SL driven by a coherent phonon wavepacket in the form of a train of ultrafast (picosecond) strain pulses. We solve numerically the Schrödinger equation with a time-dependent potential generated by propagating deformation pulses. Our simulations show that the polarity and the magnitude of the generated current response depend on the amplitude of the strain pulses and on the applied bias voltage. The results of our numerical modelling agree well with the experimental measurements of the electrical signals from the SL that we present. We also discuss how the high-frequency electronic response of the SL to coherent acoustic phonons differs fundamentally from the effects of electromagnetic radiation on the SL. This difference is due to the finite propagation time of the phonons through the SL. Finally, our theoretical analysis predicts that the acoustic stimuli can induce current oscillations in the device, with frequencies several times higher than that of the strain pulse train itself. The high-frequency oscillations of the current reflect those of the drifting electrons within the alternating acoustic potential propagating along the SL.

The paper has the following structure. In Section\textsuperscript{11} we present our quantum model and numerical simulations of electron transport in a voltage-biased SL driven by strain pulses. We present the results and related discussion in Section\textsuperscript{11} Section\textsuperscript{11} provides conclusions and outlook.

II. THEORETICAL MODEL

In our study, we model the experiment, described in detail in Ref.\textsuperscript{21}. The device is schematically pictured in Fig.\textsuperscript{1} (a). The SL region of length \( L = 490 \) nm is formed by 50 GaAs and AlAs layers with thicknesses of 5.9 and 3.9 nm respectively, giving a lattice period of \( d = 9.8 \) nm. The bias voltage \( V_0 \) is applied to the ohmic contacts. The strain of train pulses is generated by femtosecond laser pulses incident on an Al film with thickness 30 nm deposited on the reverse to the SL GaAs substrate. We estimate the strain amplitude value using previous experimental measurements\textsuperscript{22}, where it has been shown that the maximum strain amplitude \( \epsilon_a = 10^{-4} \) corresponds to a laser pulse energy density of \( \approx 1 \) mJ cm\textsuperscript{-2}.\textsuperscript{22}
The time-dependent current \( I(t) \) was measured using a digitizing oscilloscope with a 12.5 GHz bandwidth. All measurements were performed at a lattice temperature of \( T = 5 \) K.

To calculate charge transport in the SL we solve numerically the time-dependent Schrödinger equation

\[
-i \hbar \frac{\partial \psi(x,t)}{\partial t} = \hat{H} \psi(x,t),
\]

for the electron wavefunction \( \psi(x,t) \), taking the Hamiltonian to be

\[
\hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U_{\text{SL}}(x) - eV_0x + U_a(x,t),
\]

where \( m = 0.067m_e \) (\( m_e \) is the electron mass) is the electronic effective mass in GaAs, \( e \) is the electron charge, \( V_0 \) is the bias voltage applied to the SL, \( U_{\text{SL}}(x) \) is the SL potential, and \( U_a(x,t) \) is the time-dependent potential energy field generated by the deformation pulses [see inset in Fig. 1(b)], which includes reflections of the acoustic wave from the boundaries and is given, together with the corresponding parameters, by Eq. (7) in [21]. This potential propagates along the SL with the speed of sound in GaAs/AlAs layers \( v_s \approx 5000 \text{ m/s} \). To integrate Eq. (1) we use the Crank-Nicolson method, adapted for General Purpose computing on Graphics Processing Units (GPGPU) using the CUDA platform. From \( \psi(x,t) \), we calculate the expectation value of the electron displacement \( \langle x \rangle \). With no acoustic wave (\( U_a = 0 \)), a non-zero bias \( V_0 \neq 0 \) induces Bloch oscillations with a frequency proportional to \( V_0 \) and an amplitude inversely proportional to \( V_0 \) [23].

To calculate the DC current, \( I_0 \), in the SL with no acoustic stimulus applied, i.e. \( U_a = 0 \), we introduce the drift velocity \( v_d \) of the electrons using the Esaki-Tsu formalism [24]

\[
v_d = \delta \int_0^\infty \frac{dt}{\tau} \langle v(t) \rangle e^{-t/\tau},
\]

which takes into account the effect of electron scattering. Here, \( \tau = \tau_e \delta = 0.47 \) ps and \( \delta = \sqrt{\tau_e/\tau_i + \tau_e} = 12.9 \) are parameters that depend on the inelastic, \( \tau_e \) and elastic, \( \tau_i \), scattering times [25]. The DC electric current in the SL is \( I_0 = \pi r^2 n_e e v_d \), where \( r = 50 \mu m \) is the radius of the SL mesa and \( n_e = 10^{17} \text{ cm}^{-3} \) is the doping density. The effect of the ohmic contacts is taken into account by setting \( V_0 = V_{\text{SL}} - I_0 R_c \), where \( V_{\text{SL}} \) is the voltage drop across the SL and \( R_c = 54.7 \) \( \Omega \) is the contact resistance.

The time-dependent current \( I(t) = \pi r^2 n_e e v_d(t) \) generated by the voltage-biased SL driven by the strain pulses is calculated using the time-dependent drift velocity [26]

\[
v_d(t) = \int_{-\infty}^t \frac{dt_0}{\tau} \langle v(t) \rangle e^{-(t-t_0)/\tau}.
\]

To study the response of the SL to strain stimuli, we analyse the incremental current \( \delta I(t) = I(t) - I_0 \).

III. RESULTS AND DISCUSSION

The solid black curve in Fig. 1(b) shows the current-voltage characteristic, \( I_0(V_0) \), measured for the SL used in our experiments, and whose parameters (see Fig. 1(a)}
and Section II were used in the model (1)-(3) and resulting numerical simulations. For $V_0$ between -0.2 and +0.4 V, $I_0$ grows monotonically with increasing $V_0$. However, when $V_0 > 0.4$ V an electric instability results from the onset of negative differential conductivity (NDC) [27], which induces propagating charge domains and associated current self-oscillations [28, 29]. In order to avoid these propagating charge domains affecting and complicating our measurements and analysis, we ensure that $|V_0| \leq 300$ mV. The measured $I_0(V_0)$ characteristic was used to determine $\tau$, and $\delta$ (see Section II) by fitting the corresponding calculated curve to it. As shown in Fig. 1(b), the model (1)-(2) (empty circles) reproduces the measured $I_0(V_0)$ curve accurately.

Typical $\delta I(t)$ current pulses calculated numerically for $\epsilon_a = 1.5 \times 10^{-4}$ and $V_0 = 100$ mV, 0 mV and -100 mV are shown by the gray solid curves in Figs. 2 (a), (b) and (c), respectively. Our calculations reveal fast oscillations with a frequency that is several ($\sim 5$ ns) times larger than the 46 GHz frequency of the acoustic oscillations. Notably, our results are qualitatively different from the case of a SL driven by an electromagnetic wave [27, 30], which responds only to the fundamental frequency (and harmonics) of the AC driving field. Figure 2 (d) shows the Fourier spectra calculated for the signals in Fig. 2 (a-c). All three spectra reveal a sharp peak at a frequency (190 GHz) that is far higher than the frequency of the picosecond pulse train (46 GHz). Our quantum simulations show that this originates from slow propagation of the strain pulses, which, in contrast to the case of electromagnetic driving (see Appendix), cannot be neglected.

Figure 2 (e) illustrates the dynamics of the wavepacket $|\psi^2(x,t)|$ as it travels through the SL. Initially, it is accelerated by the acoustic wave incident on the SL, but after $\sim 1.5$ ns the wavepacket slows due to the decay of the acoustic pulse (see inset in Fig. 1 (b)). However, for $t \leq 1.5$ ns, the wavepacket travels with almost constant mean speed ($\approx 17.8$ m/s), whilst also demonstrating small superimposed high-frequency oscillations, see the averaged electron trajectory $\langle x(t) \rangle$ in Fig. 2 (e) and its inset.

Similar behavior was previously predicted in [6], where it was also shown that electron miniband transport induced by a plane acoustic wave can up-convert the frequency of the wave, into faster electron oscillations, by the factor

$$\beta = (\epsilon_a D \Delta_{SL}/\pi)^{1/2}(d/h\nu_a), \tag{4}$$

where $\Delta_{SL}$ is the width of the SL miniband and $D \approx 10$ eV is the deformation potential for GaAs [31, 32]. Within a semiclassical picture, these fast oscillations can be associated with pendulum-like motion of electrons in the propagating acoustic (strain) potential [7]. For our experimental SL, we estimate $\Delta_{SL} \approx 3.4$ meV, which yields $\beta = 3.79$. This value agrees well with the 4.1 up-conversion factor demonstrated in Fig. 2 (d).

This theoretical analysis is supported by our experiments. Comparison of the time-dependent incremental current $\delta I(t)$ calculated numerically using Eqs. (1)-(3) and measured experimentally for $\epsilon_a = 1.5 \times 10^{-4}$ is presented in Fig. 3. The green, blue and red empty cir-
FIG. 3. Typical current pulses $\delta I(t)$ calculated numerically (open circles, right axis) and experimentally measured (solid curves, left axis) for $V_0 = +100$ mV (red), -100 mV (green) and 0 (blue), $\epsilon_a = 1.5 \times 10^{-4}$.

The positive or negative values of $\delta I_a$ reflect the polarity of the current pulses, see also Fig. 3. Increasing the bias magnitude $|V_0|$ leads to proportional change in $\delta I_a$. The polarity of the response current changes at $V_0 \approx 50$ mV, reflecting the fact that even in the absence of an applied bias, i.e. $V_0 = 0$, the acoustic excitation generates a current response with negative polarity (see Fig. 3). The open circles in Fig. 4(b) show the numerically calculated $\delta I_a(\epsilon_a)$ dependence for $V_0 = 300$ mV. These calculations show that the magnitude of the current pulses grows non-linearly and monotonically as the strain stimuli become stronger, and preserves the polarity of the current pulse defined by the bias $V_0$.

Next, we study how the peak deformation stain $\epsilon_a$ and bias voltage $V_0$, applied to the SL, affect the amplitude $\delta I_a$ of the generated current pulses $\delta I(t)$, where $\delta I_a$ is defined as the maximum/minimum of $\delta I(t)$ in the case of positive/negative polarity of the pulse (see Fig. 3). Figure 4 summarises our analysis of the current response to variation of $V_0$ and $\epsilon_a$, where the calculated $\delta I_a$ values are shown after filtering with a bandwidth of 12.5 GHz. The empty circles in Figs. 4(a) show $\delta I_a$ values calculated numerically for $\epsilon_a = 1.27 \times 10^{-4}$ and various $V_0$ values.

The experimentally measured $\delta I_a(V_0)$ and $\delta I_a(\epsilon_a)$ variations are presented in Fig. 4(a) and (b) by solid lines. In all cases, the numerically calculated values demonstrate excellent agreement with the measured characteristics, thus confirming the validity of our numerical model.

Finally, we compare the response of the SL to the acoustic excitations with known results on the rectification and detection of electromagnetic waves by semiconductor tunneling devices [27, 30, 39], based on a theoretical framework that we discuss briefly in the Appendix. To facilitate this comparison, we calculated the amplitude of the time-dependent response of the SL to electromagnetic irradiation (photons), which produces a potential with the same frequency and amplitude as the deformation pulses (coherent phonons) in our study (see Eq. (6) in the Appendix). We note that since the electromagnetic waves propagate through the lattice at the speed of light, and their wavelength in GHz range significantly exceeds the SL length $L$, propagation effects and any spatial dependence of the potential are neglected in this alternative model. Plots of $\delta I_a(V_0)$ and $\delta I_a(\epsilon_a)$ calculated using analytical formula (6) are shown by crosses in Figs. 4(a) and (b). These figures reveal that the values of $\delta I_a$ estimated from (6) significantly deviate both from the experimental and numerical results – compare the positions of the
circles, solid line and crosses in the figure. This divergence is not only quantitative, with a scaling factor of at least ±20, but also qualitative, since the dependence of the theoretical and experimental curves on \( V_0 \) and \( \epsilon_a \) are significantly different.

This difference highlights that quantum tunneling due to strain waves can only be understood if the propagation of the wave through the material is explicitly included.

IV. CONCLUSION AND OUTLOOK

In conclusion, we have investigated the DC and time-dependent response of a weakly coupled SL to a hypersonic strain pulse train. In theory and experiment, we have shown that increasing either the applied bias voltage or the amplitude of the acoustic stimuli changes the magnitude of the electric current pulses induced in the SL. Our quantum model, based on the time-dependent Schrödinger equation with a propagating potential generated by strain stimuli, reproduces the shape and polarity of current pulses generated in the SL for various intensities of acoustic wave packets and a range of applied bias voltage. Remarkably, the simulations predict current oscillations with a frequency much higher than that of the strain pulse train. The quantitative agreement that we report between the theoretical and experimental behavior of the current pulses supports our prediction of these high-frequency oscillations. Our results highlight the potential of SL devices as tunable acoustoelectric transducers for generating high-frequency (sub-THz/THz) electromagnetic waves.

Our quantum model is also applicable to a range of other device architectures, materials, and acoustic wave parameters, and can therefore be applied widely to study high-frequency acoustoelectric effects in heterostructures including novel devices based on 2D and van der Waals materials \[36, 37\], and, even more generally, to cold atom transport in periodic optical lattices driven by a propagating potential \[38\].

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V. APPENDIX: TUNNEL CURRENT IN SUPERLATTICES UNDER PHOTON/PHONON EXCITATION

We gain further insight into our results by deriving an analytical model adapted from the theory of nonlinear single-particle tunnel junctions under electromagnetic irradiation \[39\]. Previously, we demonstrated that the coherent acoustic waves can be rectified in a SL via a similar process to the rectification of electromagnetic waves in such structures \[21\]. Here, we extend the theoretical framework used in \[21\] to estimate the magnitude of the electric pulses induced by a strain wave packet in the biased SL.

In a weakly coupled SL, electrons propagate through the lattice via quantum mechanical tunneling between neighboring quantum wells. Interaction of a SL electron with a coherent strain wave, with frequency \( \omega \), will cause the electron to absorb a phonon of energy \( h\omega \). Therefore, assuming tunneling between the quantum wells is independent, i.e. affected only by the properties of adjacent wells, the tunneling rate will be affected only by the energy of the excitation, rather than its phase. This assumption neglects any effects caused by changes of the phase of the acoustic wave as it propagates along the SL. Then the acoustic wave can be described by an effective oscillating electric potential \( V_{ac}(t) = (\epsilon_a DL)/(ed \sin \omega t) \)

\[21, 39\], where \( L \) is the SL length.

If the conductance of the SL varies slowly on the voltage scale \( h\omega/e \), and \( (\epsilon_a DL)/(e\omega d) \) is small enough, we can estimate the DC and time-dependent current response classically by considering the detection and rectification of an effective voltage signal \( V_0 + (\epsilon_a DL)/(ed \sin \omega t) \) by a device with a current-voltage characteristic \( I_0(V_0) \). In this case, the amplitude of the time-dependent current component \( \delta I_a \) can be estimated as \[21, 30, 39\]

\[
\delta I_a(V_0, \epsilon_a) = \left( \frac{\epsilon_a DL}{ed} \right) \frac{dI_0(V_0)}{dV_0}.
\]

Remarkably, the shape of the \( I_0(V_0) \) curve is accurately described by a cubic polynomial \( I_0(V_0) = aV_0 - bV_0^3 \), which is widely used to approximate the \( N \)-shaped current-voltage characteristics of tunnel devices \[40\]. A good fit to the data is obtained using the least-squares method [see crosses in Fig. 1(b)], which gives \( a = 1.65 \times 10^{-2} \text{ S}^{-1} \) and \( b = 9.1 \times 10^{-3} \text{ V}^{-2} \).

Substituting the polynomial approximation of the current-voltage characteristics, \( I_0(V_0) = aV_0 - bV_0^3 \), into Eq. (5) gives

\[
\delta I_a(V_0, \epsilon_a) = \epsilon_a DL/(ed)(a - 3bV_0^2).
\]

The corresponding dependencies of \( \delta I \) on \( V_0 \) and \( \epsilon_a \) are shown by crosses in Fig. 4.
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