Research on geometrical model and mechanism for metal deformation based on plastic flow

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Abstract. Starting with general conditions of metal plastic deformation, it analyses the relation between the percentage spread and geometric parameters of a forming body with typical machining process are studied. A geometrical model of deforming metal is set up according to the characteristic of a flowing metal particle. Starting from experimental results, the effect of technological parameters and friction between workpiece and dies on plastic deformation of a material were studied and a slippage deformation model of mass points within the material was proposed. Finally, the computing methods for strain and deformation energy and temperature rise are derived from homogeneous deformation. The results can be used to select technical parameters and compute physical quantities such as strain, deformation energy, and temperature rise.

1. Introduction
Metal deformation or machining is the process of shaping materials by either large plastic deformation or material removal through the relative motion of the tool and workpiece. There is a common nature for various craft processes that is the slipping of metallic material until cracks initiate, grow and fracture under extrusion. Past research on machining characteristics were used to set up the empirical formulas of the cutting force, the cutting temperature, and the cutter life by means of testing [1, 2]. To obtain the reasonable cutting parameters, a lot of experiments were needed to establish the cutting database, because of the variety of different of workpiece and cutter materials, as well as technical conditions. In fact, when using these cutting parameters they must still be modified to specific processing requirement: Besides, many empirical equations of cutting theory are the results of mathematical treatment of test data for a certain conditions. This information can guide understanding the law of metal deformation, but it is insufficient to predict the deformation mechanism. So this paper sets up the models of metal plastic deformation via studying its physical essential so as to probe into the mechanism of plastic deformation and influence of the geometrical factors. A reference to obtain some parameters at high speed can be offered for high efficiency.

2. Nature of metal plastic deformation

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Machining of metal depends on plastic deformation to get the required shape and dimension. Flow regularity of a metal particle is the most basic law. According to the relation equation between strain and displacement or geometric equation, the displacement field, and velocity field in the deforming body are set up to determine the strain value. It is helpful to see the deformation of an object and choose reasonable technical parameter for good processing quality. Meanwhile, it can provide a reference for the design and manufacture of tool and mold.

2.1. Metal flow law in plastic deformation

Keeping constant volume in plastic machining, the mass point has to move in the direction of the lowest resistance that is called the law of minimum resistance. In other words, the mass point will go the shortest path requiring the least energy. For this reason, we can control deformation by altering process conditions to adjust the magnitude of resistance in the certain direction [3]. The rectangle area of heavy line shown in Fig 1 is the geometrical model of moving masses point when a prism with rectangular cross-section is being compressed. Because of the effect of friction between the product and the forging dies, the resistance exerted on the mass point is proportional to the distance from side. Therefore, the mass points will certainly flow along the nearest normal direction of the boundary as material deforms. The rectangle shown in Fig 1 is divided into four areas of two triangles and two trapezoids marked with dot-dash line, in which mass points flow to the nearest border. It turns out that the rectangular cross-section becomes a polygon marked in double dot-dash line. The cross-section will finally become an oval that tends to minimize the perimeter of it. In the next change, all mass points flow to its radius direction. Circular cross section has the smallest perimeter in the same area of all shape section. So the law of the smallest resistance is equal to that of the smallest rim. It can be demonstrated in the following three models.

![Diagram of minimum rim law](image1)

![Deformation of square section](image2)

![Deformation of swaging rectangle section](image3)

- First is about the plastic deformation of square section material. According to the law of minimum rim, the deformation of square section material between up and down flat hammer tends to the imaginary line circle showed in figure 2.
- Next we consider the plastic deformation of a rectangular section of material. As the rectangular section of material is being elongated between anvils without regard to external impact, the transformation model relates to the relative ratio of feed (l) and width (a) of the material. When l>a most mass points transverse flow, the wide increases more than the long showed in figure 3(a). As l<a most mass points longitudinal flow, the long (l) elongates more the wide showed in figure 3(b). The area of two triangles marked in dot-dash line represents
number of mass points flowing along the length in figure 3. From above we can know that the percentage spread of a narrow board is greater than that of a wide board when rolling plate. Considering the impact of unshaped portion, displacement direction of mass points will change comparing with those mentioned above.

- The third example considered concerning the deformation of a rolling plate. Deformation of a rolling plate is being connected with roller diameter. If keeping the same rolling reduction and width, a large diameter roller has a bigger percentage spread than a small one. In Figs 4(a), 4(b), H and h are the thicknesses of the plate before and after rolling, respectively, the width before rolling is \( B_1 \). Because of the difference of diameters of the rollers, that is \( R_2 > R_1 \), the rolled width of the big diameter roller is larger than that of the smaller, i.e. \( B'_2 > B_2 \). Therefore, we can control percentage spread to use cluster mill with small diameter when finish rolling is performed.

![Figure 4. Effect of roller’s diameter on vertical and horizontal resistance.](image)

![Figure 5. Effect of friction on deforming and stress distribution of pressing cylinder.](image)

2.2. Effect of friction on plastic deformation

All factors acting on mass point flow will also incur plastic deformation [4].

- This concerns the impact of contact friction on the deformation of a solid. Under tool compression, the friction on the interface will have a certain effect on deformable body. The reason why cylindrical material changes into drum-type under pressure between up and down anvils is because of the friction effect. Friction forces make it more difficult to flow for the material nearby the contact interface than that near center section. The geometrical model showed in figure 5(a) has been divided into three different areas. It is the most difficult to deform in zone I due to the external frictional influence on the material while it is the easiest in zone II for the force acting at an angle of 45° with the material zone. The zone III is a free deformation circular area which is outside of the zone II. The stress state corresponding three areas are being showed in Fig 5(b). The zone I and II are three-way pressure stress state while the zone III is two-dimensional stress state of pulling stress and pressure stress. It shows that the friction affects not only uneven stress on the contact surface but also the deformation of the material. Such as the stress on an edge of the sample is equal to the yield limit \( \sigma_s \) of the material but it becomes bigger and bigger nearby the central area. Besides, the stress distributed from the interface to the middle of the body reduces [5-8].

- This concerns the impact of contact friction on deformation of ring unit. Friction will locally change the direction of flowing of a mass point when a ring body is pressed down. A ring body before pressing is being showed in figure 6(a), \( R_o \) is an external radius and \( R_i \) is an inner radius.

![Figure 6. Effect of friction on rolling deformation of a ring unit.](image)
radius, and H is a height of the annulus. It shows the geometrical model of the ring body being pressed in figure 6(b) when the friction coefficient (µ) is tiny. According to the condition of constant volume of plastic deformation, each material point flows outward along a radial direction. Then both inner diameter (ri) and outer diameter (ro) enlarge, that is, ro > Ro, ri > Ri. When the friction coefficient on interface increases (or µ≠0), the prevention of its force of friction to the flow of metal also increases. Furthermore, as the friction coefficient increases to a critical value, the resistance of the material points received near the inner diameter of ring body is so big that they change moving direction from outward to inward. Therefore, a shunt surface with radius (Rn) appears in an annulus so that the inner material flows inward, and the outer material flows outward. The inner diameter reduces and external diameter increases after deformation, thus, ro > Ro, ri > Ri. The radius (Rn) of the shunt surface increases with the friction coefficient on interface and it is less than the mid value of the wall thickness of the annulus. The reason of it is an inward flow of metal bringing about decrease of the inner diameter and increase of pressure stress, respectively. The direction of a radial component of the pressure stress fits that of the frictional resistance of shunt surface of the annulus.

2.3. Geometrical factors influencing plastic deformation
The interior material slips and extend when a pressure is being exerted on the metal. According to plastic yielding criteria, slip band is some orthogonal mesh line. Initially, a slip line is at an angle of 45° to the acting force. Slipping enlarges gradually as the rolling reduction increases. At this point, when a cylinder with height (H) and diameter (D) between hammer flatteners is forged or pressed, a model of the slip line is being showed in Fig 7. We make an isosceles right triangle with a bottom of diameter (D), and a height of the half bottom as a heavy line showed in Fig 7. The triangle is called a basic cone or a main cone, in which two sides of the triangle are at angle of 45° to the acting force. A plastic deformation firstly occurs near the main cone when the cylinder is forged because its shear stress at angle of 45° to force is the largest and easy to slip. With deformation going on, the slip appears inside and outside the main cone. Slip lines inside the main cone near the friction interface is hard to move and it consumes more energy to move because there is high hydrostatic pressure here. Although lying in the zone of low hydrostatic pressure, the slip line outside the main cone needs enough energy to extend toward deep and outward because the distance is big. It depends on the distance (h2) of the two-
cone tops for which slip line of inside and outside cone is dominant [9-12].

Figure 8. Effect of \( h_2 \) on slipping deformation.

- When \( h_2 < 0 \), \( H/D < 1 \), and its model is shown in Fig 8(a). Two cones insert each other, outside slip lines disturb severely and inside slip lines increase. Now slipping spreads the entire and deformation tends uniformly. Besides, the adhering zone of the contact surface will reduce greatly. It is possible to slide overall for small cylinder, but great external force is needed.

- When \( h_2 = 0 \), \( H/D = 1 \), its model is shown in Fig 8(b). Slip lines of both inside and outside cone will appear and focus nearby angle of 45° line. Slip lines outside the two cones disturb mutual and forms a single-drum shape obviously. Currently, adhering zone of the contract surface decreases and slip begins. So total deformation is relatively uniform, but more pressure is needed. It is reasonable to select these parameters for high cylindricity.

- When \( h_2 > 0 \), \( H/D > 1 \), its model is shown in Fig 8(c). The two-cone tops are out of contact but not far from. Now that there is better slip condition outside the main cone, more slip lines outside the main cone appear and also form obvious single-drum shape, which has been proved in test of forging short cylinder.

- When \( h_2 \gg 0 \), \( H/D \gg 1 \), its model is shown in Fig 8(d). The two main-cone tops are far from each other; the slip lines outside the main cone are liable to appear but not going deeply. Because the deformation occurs near the two main cones and contact surface, double-drum shape appears. Besides, the contact surface is severely adhered. This phenomenon often happens in forging long cylinder.

Analysis above models shows that the condition of the slip of material throughout the whole volume is \( H/D < 1 \). We can qualitatively explain the reason that deformation force increases with decreasing \( H/D \) by using these models.

3. Analysis of the mechanism of a plastic deformation on metal

Many factors act on metal plastic deformation, in which deformation is the result of coupling action of various factors. In fact, it is always expected that metal deformation is uniform and stable to keep high product quality and efficiency. To obtain the relations of some parameters, such as strain and energy consumption and temperature variation, we assume the deformation process is homogeneous, and the simplified model is showed in Fig 9.

3.1. Strain of uniform forging cylinder
Let’s assume that the cylinder material is isotropic and without initial stress, which is being forged between up and down rigid anvils. The anvils are smooth and parallel. The process of deforming is homogeneous. Besides reducing height of the material and increasing its diameter, we can keep round cross section and increase its diameter at the same speed in the course of forging. Suppose the ratio of initial height ($H_0$) to the diameter ($D_0$) is less than two without axial bend.

Natural strain increment ($d\varepsilon_z$) in the axial direction is given by

$$d\varepsilon_z = -dh / h$$

(1)

Where $h$ is a height of any cross section of the cylinder and $-dh$ is a tiny decrement of $h$ in compression process. The total strain of the material from $H_0$ to $H_1$ is

$$\varepsilon_z = \int_{H_0}^{H_1} (-dh / h) = \ln(H_0 / H_1)$$

(2)

The axial strain can be calculated by equation (2) so long as giving an initial and final height of the cylinder. For isotropy and incompressible material, we have $\varepsilon_r + \varepsilon_\theta + \varepsilon_z = 0$. Additionally, the radial strain equals the circumferential strain, and they are tension strains, then we get $\varepsilon_r = \varepsilon_\theta = -\varepsilon_z / 2$.

### 3.2. Energy consumed in forging cylinder material

Suppose that the axial height of metal material reduces a micro amount ($-dh$) when the plastic deformation occurs under acting force ($P$). Then the tiny consumed energy ($dE$) is

$$dE = P(-dh) = \sigma_z A(-dh)$$

(3)

Where $\sigma_z$ is an axial stress, and $A$ is any instant cross-sectional area.

When the material is compressed from an initial height ($H_0$) to a final height ($H_1$), the total energy consumed is

$$E = \int_{H_0}^{H_1} P(-dh) = \int_{H_0}^{H_1} \sigma_z A(-dh)$$

(4)

For incompressible material, the volume keeps invariable, then $A = V / h$. The energy consumed in compressing unit volume is

$$E / V = \int_{H_0}^{H_1} \sigma_z (-dh / h) = \int_{\varepsilon}^{\varepsilon} \sigma_z d\varepsilon_z$$

(5)

Where the integral ($\int_{\varepsilon}^{\varepsilon} \sigma_z d\varepsilon_z$) is obtained from an area surrounded by true stress-strain curve, the curve is decided by a test. The range of strain in practice is $0 \sim \varepsilon$.

- For ideal strong-plastic material, the one-way yield stress is $\sigma_s$. Then energy of per unit volume is

$$E / V = \int_{\varepsilon}^{\varepsilon} \sigma_s d\varepsilon_z = \sigma_s \int_{H_0}^{H_1} (-dh / h)$$

$$= \sigma_s \ln(H_0 / H_1) = \sigma_s \varepsilon_z$$

(6)

Equation (6) shows that the energy consumed in unit volume for axial yield stress is proportional to strain when we deform a rigid-plastic material.

- For strain-hardening material, relation of true stress to natural strain is approximate obey $\sigma = A \varepsilon^n$, and the energy consumed in unit volume deformation is
\[ E/V = \int_0^{\varepsilon_c} \sigma \cdot d\varepsilon_c = \int_0^{\varepsilon_c} A \varepsilon_c^n d\varepsilon_c = A \varepsilon_c^{n+1} / (n+1) \] 

(7)

Where both A and n in equation (7) are material constants.

For big plastic strain, an expression corresponding with test data of engineering material is \( \sigma = C(C + \varepsilon)^q \), the energy consumed per unit volume deformation is

\[ E/V = \int_0^{\varepsilon_c} C(D + \varepsilon)^q d\varepsilon = \left[ (D + \varepsilon)^{q+1} - D^{q+1} \right] C / l(q+1) \] 

(8)

Where q, C, D, and q in equation (8) are the given material constants, in which we have \( 0 \leq q \leq 1 \).

Above statement are about strain and consuming energy based on homogeneous deformation. A homogeneous deformation is one of the most effective means for forming metal, which can be used to estimate the value in other forming process.

3.3. Peak temperature rise of pressing cylinder material

In a compression process of material, most energy consumed in plastic deformation converts to quantity of heat, and causing temperature rise. Assume that the compression process is adiabatic. In this case, all plastic work turns into thermal energy without spreading away from the material. In line with the energy conservation law, an energy consumed per unit volume deformation can be expressed as

\[ E/V = J \rho c \Delta \theta \]

(9)

Where \( \Delta \theta \) is average temperature rise, \( \rho \) is a density of the material, c is a specific heat of the material, and J is the heat equivalent of the work [13, 14].

Taking advantage of equation (9), we can obtain the temperature rise of plastic deformation. But the actual temperature rise is less than the calculated value.

4. Conclusion

Plastic forming and machining of metal belongs to a large deformation. The process of deformation is extremely complex, and influence factors are numerous. In this paper, several geometrical models of plastic deformation have been set up by studying the flow characteristic of a mass point. The effect of primary geometrical parameters and function of friction have been discussed, the main conclusion is as follow:

- Besides invariant volume in plastic deformation, the movement of mass points follows the law of smallest resistance force, which means mass points move in the direction of least resistance.
- In the course of plastic deformation, each mass point moves in the direction of its nearest border so as to consume the least energy.
- Force of friction and geometric parameters of deformation have a great influence on the flow of the mass point. The force of friction has a significant influence on the deformation near the contact surface. The shorter is a deforming piece, the more homogeneous is the deformation.

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References

[1] An H P, Rui Z Y, Wang R F and Zhang Z M 2012 Model of shear slipping deformation and it’s cutting energy consumed for high-speed machining Advanced Materials Research 602 1967-70
[2] An H P, Rui Z Y, Wang R F and Zhang Z M 2014 Research on cutting-temperature field and
distribution of heat rates among a workpiece, cutter, and chip for high-speed cutting based on analytical and numerical methods *Strength of Materials* 46 289-95

[3] Peng D S 2004 *Metal Plastic Machining Theory* (Chang Sha: Press of Central South University)

[4] Li M J 2003 *Plastic Process Technology* (Beijing: China Machine Press)

[5] An H P 2014 Summary of equations used to solve primary stress by means of measuring strain under special plane stress state *Automation and Instrument* 7 181-4

[6] An H P 2009 The slip-line theory and its applications in the process of metal cutting *Chinese Journal of Construction Machinery* 7 308-11

[7] Sun J L 2007 *Friction and Lubrication of Material Formation* (Bei Jing: National Defense Industry Press)

[8] Shaw M C and Vyas A 1993 Chip formation in machining hardened steels *Annals of the CIRP* 42 29-32

[9] Shaw M C and Vyas A 1998 The mechanism of chip formation with hard turning steel *Annals of the CIRP* 47 72-82

[10] Komanduri R and Brown R 1981 On the mechanics of chip segmentation in machining *J. of Eng. for Ind. Trans. ASME* 103 33-51

[11] Barry J and Byme G 2002 The mechanisms of chip formation in machining hardened steels *Journal of Engineering* 124 528-35

[12] Filice L, Micari F, Rizzuti S and Umbrello D 2007 A critical analysis on the friction modeling in orthogonal machining *Journal of Machine Tools and Manufacture* 47 709-14

[13] Zhang X P, Gao E and Wand Liu C R 2009 Optimization of process parameter residual stresses for hard turned surfaces *Journal of Materials Processing Technology* 209 4286-91

[14] Benardos P G and Vasmiaskos G C 2003 Predicting surface roughness in machining: A review *International Journal of Machine Tool & Manufacture* 43 833-44