Theoretical analysis of a pressure setting and control system with PWM direction control valve

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Abstract. The paper tackles theoretical aspects concerning an original automated system that sets and controls the pressure inside a tank chamber of fixed volume. The structure of the system integrates an original device developed and designed by the authors. The device digitally controls the one way flow of the working fluid using pulse width modulation, allowing the free flow in the other way. The purpose of this research stage was the theoretical establishing of the variation law of the pressure inside the controlled chamber.

1. Introduction
Pneumatic driving systems are nowadays used in a huge number of applications due to their recognized advantages such as robustness, simplicity of the construction, productivity, high reliability and not to be forgotten the lower price range.

An assessment of the field of pneumatic driving [1] leads to the identification of the main tendencies and perspectives, namely:
- strong development of pneumatics in the future; intensive fundamental and applied research that aims reducing the negative effects induced by the physical properties of the working fluid (such as reduced viscosity and high compressibility) is required;
- embedment of informatics in pneumatic driving systems and equipment, leading to the development of pneutronic systems;
- increase of reliability, functional accuracy and static and dynamic performance of actual pneumatic devices, as well as development and building of new types of high performance pneumatic equipment.

2. The developed system
The present paper rises to the challenges presented above by proposing a pneutronic system for automated pressure control that incorporates a proportional device of original construction. The system aims to set the pressure in a tank chamber of fixed volume $V$ at a target value $P_t$ and to maintain it constant. The functional scheme of the system is presented in figure 1.
The structure of the system includes the following components:
- the tank $R_z$;
- classical pneumatic direction control valve $DC$ 3/3, featuring preferred position and electric control;
- PWM controlled direction control valve $D_{IM}$ of special construction;
- pressure sensor $TP$;
- microcontroller $\mu C$.

Two stages are discerned during the functioning of the system if the initial pressure in the tank is assumed equal to the atmosphere pressure $P_0$:
- 1st stage: the classical direction control valve is set on position (1) (in the presence of the control signal $u_1$); the filling of the tank occurs through a flow section equal to the flow section $S_{DC}$ of the classical direction control valve; the pressure in the tank will consequently rise; when it equals an imposed threshold pressure $P_p$ the control signal $u_1$ stops and the valve $DC$ is set on preferred position (0);
- 2nd stage: if threshold pressure $P_p$ is reached, the PWM direction control valve is supplied till the target value $P_r$ is reached; this is the moment when the control signal $u_3$ stops.

The draining of the tank can be achieved if the direction control valve $DC$ is commuted on position (2) by generating the control signal $u_2$.

3. The original construction device
As already mentioned, the innovative device integrated in the structure of the system is in fact a PWM controlled direction control valve. It is a small sized device ($D_n = 2...3$ mm) of type 2/2 or 2/3, having a preferred position, and is electrically controlled using a magnet that can function at high working frequencies ($\approx 200$ Hz). An airflow is obtained this way through the consumer orifice (2), corresponding to the mean value of the real flow that passes through the internal circuit of the device. The device does not achieve a continuous control of the instantaneous flow, but controls the mean flow in direct proportion to the duty cycle. Among the advantages of the proposed solution are: reduced price compared to proportional pneumatic direction control valves, elimination of hysteresis and its bothersome effects, very good repeatability.

Various control techniques can be used [2, 3], based on the combining of different micro direction control valves of the same type, grouped together in so-called “banks” as well as on the adjustment of the opening and closing times of the valve section.

The authors have developed a number of constructive solutions of such pneumatic devices [4, 5]. For exemplification, figure 2 presents a longitudinal section and a 3D representation of such a device.
An experimental model was accordingly developed. The model was subsequently tested and integrated in the structure of the system previously described.

Figure 3 presents the principle of such a device with an innovative structure which is much simpler than the previous ones. Besides, the device allows the free flow of the fluid from (2) to (1).

The proposed device consists of a mobile subassembly formed by the mobile armature 3 and the spherical valve 2, the magnet 4 and the body 1 where the conical seat and the two nozzles are manufactured. The experimental model presented in figure 4 was designed and built starting from this principle scheme.

![Figure 2](image2). Version of pneumatic device with PWM direction control valve.

![Figure 3](image3). Principle scheme of the proposed device.

![Figure 4](image4). 3D representation of the built experimental model.
4. Mathematical model

The following notations are introduced in order to describe the mathematical model:

- \( V \) – fixed volume of the tank chamber;
- \( P_r \) – the target value of the pressure to be achieved in the chamber of fixed volume \( V \) (the set pressure);
- \( P \) – pressure in the chamber of fixed volume \( V \);
- \( P_o \) – supply pressure;
- \( P_p \) – threshold pressure; when is reached, the slide of the classical direction control valve \( DC \) moves back to the preferred position;
- \( P_o \) – atmosphere pressure;
- \( S_{\text{DC}} \) – nominal section of the classical direction control valve \( DC \);
- \( S_{\text{DIM}} \) – nominal section of the PWM controlled direction control valve \( DIM \);
- \( R \) – universal gas constant; \( R = 287.04 \text{ [m}^2/(\text{s}^2 \cdot \text{K})] \);
- \( T_o \) – temperature in the chamber \( V \);
- \( \chi \) – adiabatic constant; \( \chi = 1.4 \text{ [-]} \);
- \( \dot{m} \) - flow rate of the fluid that fills the chamber;
- \( T \) – period of the pulses;
- \( t_o \) – duration of the pulses.

The variation of pressure in the chamber of fixed volume \( V \) can be written:

\[
\frac{dP}{dt} = \frac{\chi \cdot R \cdot T_o}{V} \cdot \dot{m}
\]

where:

\[
\dot{m} = \frac{K \cdot S_{\text{DC} \rightarrow o} \cdot P_a}{\sqrt{T_o}} \cdot N \left( \frac{P}{P_a} \right)
\]

\[
K = \sqrt{\frac{\chi}{R} \left( \frac{2}{\chi + 1} \right)^{\frac{\chi - 1}{\chi + 1}}} \approx 0.04042 \text{ s} \sqrt{K/m}
\]

\( S_{\text{DC} \rightarrow o} \) denotes the flow section through the valve. In function of the type of valve that allows the flow of the fluid, it can be described as following:

\[
S_{\text{DC} \rightarrow o} = \begin{cases} 
S_{\text{DC}} & \text{if } P_o \leq P \leq P_p \\
S_{\text{DIM}} & \text{if } P_p \leq P < P_r
\end{cases}
\]

The flow section through the PWM controlled direction control valve can be written:

\[
S_{\text{DIM}} = \begin{cases} 
S_{\text{DIM}} & \text{if } \text{rem} \left( \frac{t - t_p}{T} \right) \leq t_o \\
0 & \text{else}
\end{cases}
\]

The function \( \text{rem} \) denotes the remainder of the division between the two values; \( t_p \) denotes the moment of time when the pressure \( P_p \) is reached (and the PWM controlled valve is actuated).

\( N \left( \frac{P}{P_a} \right) \) denotes the flow rate number.
The following notation is introduced: \( \frac{P}{P_a} = p \). Using this notation, the flow rate number \( N(p) \) is defined as follows:

\[
N(p) = \begin{cases} 
1 & \text{if } 0 < p \leq 0.528 \\
\frac{a}{\bigg[ p^{\frac{\chi}{2}} - p^{\frac{(\chi+1)}{2}} \bigg]^{1/2}} & \text{if } 0.528 < p \leq 1
\end{cases}
\]

(5)

\[
a = \sqrt{\frac{2}{\chi - 1} \left( \frac{\chi + 1}{2} \right)^{\frac{1}{\chi}}}, \quad a = 2.6143[-]
\]

Based on these relations, the analysis of the functioning of the automated system for pressure setting and control can be performed analytically as well as numerically.

5. Analytical calculation of the pressure in the fixed volume chamber

Analytical establishing of the variation law of the pressure \( P_a \) in the chamber of fixed volume \( V \) implies the separate analysis of three cases corresponding to the three functioning stages

1) \( 0 \leq p \leq 0.528 \); sonic conditions:

In this case \( S_{s-\rightarrow 0} = S_{aDC} \), equations (1) … (5) lead to:

\[
\frac{1}{P_a} \frac{dP}{dt} = \frac{\chi \cdot R \cdot \sqrt{T_a} \cdot K \cdot S_{aDC}}{V}
\]

(6)

\[
\frac{dp}{dt} = A, \quad A = \frac{\chi \cdot R \cdot \sqrt{T_a} \cdot K \cdot S_{aDC}}{V}
\]

(7)

The numerical values used for the theoretical study are the following: \( T_a = 293.15[K] \), \( S_{aDC} = 7.0686 \cdot 10^{-6}[m^2] \), \( S_{aIM} = 3.1416 \cdot 10^{-6}[m^2] \), \( V = 5 \cdot 10^{-4}[m^3] \). It results \( A = 0.393[N/(m^2 \cdot s)] \).

An imposed condition is that the initial value of the pressure inside the chamber (corresponding to \( t = 0 \)) would equal the atmosphere pressure \( P_0 \). It results accordingly \( P(t) = P_a \cdot A \cdot t + P_0 \).

If \( P_a = 6 \cdot 10^4[N/m^2] \) and it is considered that \( P_0 = 1.013 \cdot 10^5[N/m^2] \) it results that \( P(t) = (2.3587 \cdot t + 1.013) \cdot 10^5[N/m^2] \).

The moment of time when a pressure of \( 0.528 \cdot P_a \) is reached is denoted \( t_{sonic} \). For the above presented numerical values, \( t_{sonic} = 0.92[s] \).

2) \( 0.528 \leq p \leq p_p \), \( p_p = \frac{P_p}{P_a} \); subsonic conditions:

In this case it is also true that \( S_{s-\rightarrow 0} = S_{aDC} \). Equations (1) … (5) lead to:

\[
\frac{1}{P_a} \frac{dP}{dt} = \frac{\chi \cdot R \cdot \sqrt{T_a} \cdot K \cdot S_{aDC}}{V} \cdot a \left[ p^{\frac{\chi}{2}} - p^{\frac{(\chi+1)}{2}} \right]^{1/2}
\]

(8)

\[
\frac{dp}{dt} = B \left[ p^{\frac{\chi}{2}} - p^{\frac{(\chi+1)}{2}} \right]^{1/2}, \quad \text{where} \quad B = A \cdot a
\]

(9)

In the case under analysis \( B = 1.0277[N/(m^2 \cdot s)] \).
Equation (9) leads to:

\[ \frac{dp}{p^{\gamma/\chi} \cdot \sqrt{1 - p^{(x-1)/\chi}}} = B \cdot dt \]  

(10)

The following substitution is introduced:

\[ 1 - p^{(x-1)/\chi} = z^2 \]

(11)

\[ dz = \frac{X - 1}{2\chi} \cdot B \cdot dt \]

(12)

\[ z = -\frac{X - 1}{2\chi} \cdot B \cdot t + C_0 \]

(13)

The constant \( C_0 \) is computed putting the condition that \( p(t_{sonic}) = 0.528, C_0 = 0.5435[-] \). It results finally:

\[ P(t) = 6 \cdot 10^4 \left[ 1 - (0.5435 - 0.1468t)^2 \right]^{1.5} \]

(14)

The moment of time \( t_p \) when the threshold pressure \( P_p \) is reached results equal to \( t_p = 1.45[s] \).

3) \( p_p \leq p \leq p_r \), \( p_r = \frac{P}{P_a} \); subsonic conditions:

In this case \( S_{c_{1->0}} = S_{slm} \). Equations (1) ... (2) lead to:

\[
\frac{1}{P_a} \frac{dP}{dt} = \begin{cases} \frac{\chi \cdot R \sqrt{T_u \cdot K \cdot S_{slm}} \cdot a}{V} \left[ p^{\gamma/\chi} - p^{(x-1)/\chi} \right]^{1/2} & \text{if } \text{rem}\left(\frac{t - t_p}{T}\right) \leq t_a \\ 0 & \text{if } t_a \leq \text{rem}\left(\frac{t - t_p}{T}\right) \leq T \end{cases} 
\]

(15)

It is denoted \( A' = \frac{\chi \cdot R \sqrt{T_u \cdot K \cdot S_{slm}}}{V} \), \( B' = A' \cdot a \). The following values are obtained after replacements are made: \( A = 0.1747[N / (m^2 \cdot s)], B' = 0.4568[N / (m^2 \cdot s)] \).

If a similar procedure to the previous case is applied, it can be written that:

\[
\begin{cases} \sqrt{1 - p^{(x-1)/\chi}} = -\frac{X - 1}{2\chi} \cdot B' \cdot t + C' & \text{if } \text{rem}\left(\frac{t - t_p}{T}\right) \leq t_a \\ p = \text{constant} & \text{if } t_a \leq \text{rem}\left(\frac{t - t_p}{T}\right) \leq T \end{cases} 
\]

(16)

The value of the constant \( C' \) is modified at each moment of time \( t_a = t_p + kT, k \in N \) according to the relation:


\[ C^* = \sqrt{1 - p^\frac{1}{\chi}} + \frac{\chi - 1}{2\chi} \cdot B^* \cdot t_i \]  

(17)

The computation algorithm for this stage is presented in figure 5.

**Figure 5.** The computation algorithm to be used when the flow occurs through the PWM controlled valve.

The previously established dependency \( P = P(t) \) was implemented in a software program using MATLAB. The resulted variation law is presented in figure 6.

**Figure 6.** Characteristic diagram \( P = P(t) \) obtained by analytical calculation.
6. Mathematical model solving by numerical simulation

The functioning of the pressure setting and control system was numerically simulated using SIMULINK. The simulation started from the equations (1) … (5) presented above. Figure 7 presents the model of the subsystem built for the generation of the characteristic diagram of the pressure in the chamber of fixed volume $V$.

Figure 8 presents the characteristic diagram $P=P(t)$ obtained by numerical simulation. One can notice that the results obtained both by analytical study and by numerical simulation are very close.

![Figure 7. Numerical simulation model.](image)

![Figure 8. Characteristic diagram $P=P(t)$ obtained by numerical simulation.](image)
7. Conclusions
The proposed system allows the setting of a target pressure with an imposed error inside a tank. The structure of the system is simple, consisting mostly of module devices. A precise setting of the target pressure can be obtained using an original structure of PWM direction control valve. The construction of the proposed device is simple and does not require sophisticated manufacturing and mounting technologies. The first step required in order to perform the theoretical analysis of the system was the elaboration of the mathematical model. The obtained model was hereafter analytically integrated, leading to the establishment of the pressure variation law, and numerically simulated using the graphical programming environment Simulink. The results obtained by both methods are very close, except for small differences due to the integration method that was used. Actually the authors are concerned with the building of experimental models of the valve and of the system. The control unit will be structured around a microprocessor. The working software packages will implement various algorithms developed for setting and control. The obtained results will be presented in a further paper.

8. References
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