Vector Galileon and inflationary magnetogenesis

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Abstract. Cosmological inflation provides the initial conditions for the structure formation. However, the origin of large-scale magnetic fields can not be addressed in this framework. The key issue for this long-standing problem is the conformal invariance of the electromagnetic (EM) field in 4-D. While many approaches have been proposed in the literature for breaking conformal invariance of the EM action, here, we provide a completely new way of looking at the modifications to the EM action and generation of primordial magnetic fields during inflation. We explicitly construct a higher derivative EM action that breaks conformal invariance by demanding three conditions — theory be described by vector potential $A^\mu$ and its derivatives, Gauge invariance be satisfied, and equations of motion be linear in second derivatives of vector potential. The unique feature of our model is that appreciable magnetic fields are generated at small wavelengths while tiny magnetic fields are generated at large wavelengths that are consistent with current observations.

Keywords: inflation, magnetic fields, modified gravity, primordial magnetic fields

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1 Introduction

Since the early days of quantum electrodynamics, higher derivative field theories [1, 2] have been proposed to improve the divergence structure. However, higher-derivative theories lead to extra degrees of freedom in the system which, in consequence, make the Hamiltonian linear in extra momentum term. Thus, the Hamiltonian or the energy of the system is unbounded and such systems suffer from Ostrogradsky instability [3, 4]. These negative energy states can be traded by negative norm states leading to non-unitary theories [5–7]. The effect of extra degrees of freedom can also be seen from the equations of motion as the number of degrees of freedom matches the number of initial conditions needed to solve the equations. For example, in case of linear second order theory, the number of degrees of freedom in phase-space is two and in configuration space, order of equation of motion is two. In case of higher derivative theory, we obtain higher derivative (i.e., more than 2) equation(s) of motion. Therefore, one needs extra initial conditions to solve the differential equations which is again can be though of as an effect of extra degrees of freedom.

Recently, it has been realized that it is possible to construct scalar field theories whose action can have higher derivatives, however, the equations of motion are still second order, thus automatically avoiding the instability [8–14]. These models have been constructed by imposing the Galilean symmetry in the field space, i.e.,

$$\phi \rightarrow \phi + b_\mu x^\mu + c,$$

where, $b, c$ are constants and the equations of motion are invariant under the symmetry. Nicolis et al. [9] first introduced this model in flat space-time which later was extended to generalized curved space-time in [10, 11], which also coincidentally matches with Horndeski [8]. These are referred to as Galilean models and do not suffer from Ostrogradsky instabilities [8–14]. In order to construct Galileon models in curved space-time, one needs to add
non-minimal couplings between the matter and the gravity to vector Galileon action in flat space-time. However, most recently, it has been discovered that a ‘simpler’ Galileon model without the non-minimal coupling terms exists in curved space-time where, equations are third order in nature but due to an hidden second order constraint, all equations can be reduced to second order [15–18].

Scalar Galilean theories have a lot in common with Lovelock theories of gravity [19, 20]. Lovelock theories are obtained by imposing three conditions — gravity must be described by metric and its derivatives, diffeomorphism invariance and equations of motion be quasi-linear. Using these conditions, it can be shown that Einstein’s gravity is unique in 4-D. In higher dimensions, \( R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd} \) also lead to quasi-linear equations of motions. Lovelock extensions of Einstein gravity are shown to be free of ghost and evade problems of Unitarity [21, 22].

Horndeski, first provided us the generalized vector-tensor theory in curved space-time [23]. Recently, in a similar way, vector Galileons with three degrees of freedom, which is often referred to as generalized Proca theory and generalized vector Galileons have been constructed [24–30]. But, the action for all these aforementioned models contains linear time derivatives of the fields. Unfortunately, in the literature, there is no higher derivative vector Galileon model. Unlike scalar Galileons, there also exists a no-go theorem [31] that states that, higher derivative vector Galileons cannot be constructed in flat space-time. This possess the following question: Can we construct a higher derivative Electromagnetic (EM) field action by demanding following three conditions: theory be described by vector potential \( A^\mu \) and its derivatives, \( U(1) \) Gauge invariance is satisfied, i.e., \( A^\mu \rightarrow A^\mu + \partial^\mu \pi \) and equations of motion be second order?

We also show that the solution to the above question may also provide a solution to the problem of generation of the primordial magnetic field during inflation. Observations indicate that magnetic fields in galaxies which are coherent on scales of several kpc have strengths of order \( 10^{-6} \) [32–34]. Recent FERMI measurement of gamma-rays emitted by blazars seem to provide lower bound of the order of \( 10^{-16} \) G in voids [35, 36].

Several mechanisms have been proposed to explain the origin of these magnetic fields. These can be broadly classified into two: top-down and bottom-up scenarios [37–41]. In the bottom-up scenario, magnetic fields are first produced in stars and propagate outwards to galaxies and intergalactic space. In the top-down scenario, primordial magnetic field is generated in the early-universe and the accretion of matter within stars and galaxies amplifies the primordial magnetic field [37]. Both the scenarios have problems. For instance, the top-down scenario can not generate required magnetic field strength, while the bottom-up can not generate fields with the required coherence length. The lower bound of the magnetic fields in the voids favors top-down scenario.

During inflation, the conductivity becomes negligible thus allowing a generation of large magnetic fields during this phase [42, 43]. However, the problem is that the standard electromagnetic (EM) action in 4-D space-time:

\[
S_{SEM} = - \int \frac{d^4x}{4} \sqrt{-g} F_{\mu\nu}F^{\mu\nu}; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu
\]  

is conformally invariant and the equations of motion of the magnetic field in FRW space-time are time independent [37]:

\[
(\partial^2 - \nabla^2) (a^2 B) = 0.
\]
Thus, to generate sufficient magnetic fields during inflation, it is necessary to break conformal invariance of the EM action. Starting from Turner and Widrow [42, 44–54], several authors have suggested many ways to break the conformal invariance of the electromagnetic field by introducing (i) coupling of the electromagnetic field with the Ricci/Riemann tensors, (ii) non-minimal coupling of the electromagnetic field with scalar/axion/fermionic field and (iii) compactification from higher dimensional space-time.

As mentioned earlier, in this work, we show that obtaining a higher-derivative electromagnetic field action that preserves gauge-invariance and equations of motion is second order leads to a viable candidate for primordial magnetogenesis. Thus, our approach provides a new way of looking at the modifications to the EM action and the primordial magnetic field generation during inflation.

The work is divided into two parts. In the first part, we explicitly construct a higher derivative electromagnetic action. In doing so, we have used non-minimal covariantization method. In addition to that, we also rely upon to a specific yet simple line-element, i.e., FRW metric with arbitrary Lapse function to simplify equations as for generalized curved background, it is very difficult to evaluate such equations. We also obtain our desire first higher derivative vector Galileon in FRW background. We explicitly show that in flat background, it identically vanishes and being consistent with the no-go theorem [31].

In the second part of our work, we implement our newly constructed vector Galileon model in power-law and slow-roll inflationary scenarios. We show that, for slow-roll inflation, $E$ and $B'$-part of the spectrum of the energy density vanishes quickly and the $B$-part of the spectrum of the energy density remains significant. To obtain positive energy during this time, we fix the sign of the arbitrary constant. The immediate consequence of the fixing the sign of the constant is that: kinetic part in the action becomes negative. The spectrum has large blue tilt with a special feature: denominator contains $(1 - \epsilon)$ term which diverges at the end of the inflation, providing necessary seed magnetic field. In this work, we use $(-, +, +, +)$ metric signature and natural units $\hbar = c = 1/(4\pi\epsilon_0) = 1$.

2 Part I: new vector Galileon — the model

In this section, we briefly discuss procedure to construct scalar Galileons and then use the method to construct vector Galileon.

2.1 Brief discussion about constructing scalar Galileons

In case of single scalar field theory in flat space-time, if the Lagrangian contains second order time derivative, equation of motion contains, in general, fourth order time derivative. However, it can easily be shown that using Levi-Civita tensor/completely anti-symmetric tensor, higher derivative terms are suppressed. Consider the example:

$$L \sim f(\partial \phi, \varphi) \epsilon^{\mu \nu} \epsilon_{\alpha \beta} \partial_{\mu \alpha} \phi \partial_{\nu \beta} \phi,$$  \hspace{1cm} (2.1)

where $\epsilon^{\mu \nu}$ is an anti-symmetric tensor. As one can see, in flat space-time, higher derivative terms in the equation of motion vanishes as the contraction between symmetric and anti-symmetric tensor vanishes. Moreover, the action preserves local Lorentz invariance. In fact, the above Lagrangian matches with the third kind of Lagrangian in ref. [9] if the anti-symmetric tensor part $\epsilon^{\mu \nu} \epsilon_{\alpha \beta}$ is replaced in terms of the flat metric, $\eta^{\mu \nu}$ as $\eta^{\mu \alpha} \eta^{\nu \beta} - \eta^{\mu \beta} \eta^{\nu \alpha}$. However, in curved space-time, to maintain Lorentz invariance, ordinary partial derivative $\partial$
is replaced by covariant partial derivative $\nabla$. In that case, because of connection terms in covariant derivative, higher derivative terms appear in the equations of motion. In order to compensate those terms, non-minimal couplings with the curvature term are added and by appropriately fixing the coefficients of those added terms, higher derivative terms in the equation of motion successfully omitted and Galilean symmetry preserved [10]. This method of adding and fixing non-minimal coupling terms is referred to as non-minimal covariantization. In a similar way, higher order Galileons can be constructed in flat as well as curved space-time.

We use the similar method to construct the new higher derivative vector Galileon in the next section.

2.2 Constructing vector Galileon

As previously mentioned in (2.1), using the similar technique we consider the following additional term to the standard EM action (1.1):

$$S_{VG} = \lambda \int d^4x \sqrt{-g} \epsilon^{\alpha\gamma\nu} \epsilon^{\mu\beta} \nabla_\alpha \nabla_\beta A_\gamma \nabla_\mu A_\eta$$

(2.2)

where $\nabla$ is covariant derivative, $\lambda$ — whose dimension is inverse mass square — is the coupling constant that determines the effect of the higher-derivative terms in the propagation of the EM field and $\epsilon^{\alpha\beta\gamma}$ is any anti-symmetric tensor. Note that there is a small change in the action compared to (2.1). There is no $f(A_\mu)$ function in the front of the action as our goal is to preserve U(1) symmetry, i.e., $A_\mu \rightarrow A_\mu + \partial_\mu \pi$. In any dimension ($\geq 3$), the product of two anti-symmetric tensors can be expressed as

$$\epsilon^{\alpha\gamma\nu} \epsilon^{\mu\beta} = \frac{\left| \begin{array}{ccc} g^{\alpha\mu} & g^{\alpha\eta} & g^{\alpha\beta} \\ g^{\gamma\mu} & g^{\gamma\eta} & g^{\gamma\beta} \\ g^{\nu\mu} & g^{\nu\eta} & g^{\nu\beta} \end{array} \right|}{g^{\alpha\beta}}.$$  

(2.3)

Thus, the action (2.2) is a scalar. Before we proceed, it is important to understand how the above action behaves in the flat Minkowski space-time:

1. The contraction between the first and third indices of the anti-symmetric tensors and the derivative of the vector potential $\epsilon^{\alpha\gamma\nu} \epsilon^{\mu\beta} \nabla_\alpha \nabla_\beta A_\gamma \nabla_\mu A_\eta$ ensures no higher derivative terms in the equations of motion.

2. The covariant derivatives $\nabla$ are replaced by partial derivatives $\partial$. Hence, in flat space-time, contraction between first two indices of the anti-symmetric tensor and derivative of the vector potential $\epsilon^{\alpha\gamma\nu} \partial_\alpha A_\gamma$ preserves the gauge-invariance.

3. It can also be shown that in flat space-time, the action (2.2) vanishes identically. Hence, even with the precise construction of the action that preserve U(1) symmetry and the equations are of quadratic order as well, the action does not contribute in flat space-time. This is the no-go theorem by Deffayet et al. [31].

In curved space-time, however, action (2.2) does not vanish due to the covariant derivatives of the vector potential which lead to extra terms in the action. This leads to dire consequences as

1. Connection terms are not gauge-invariant, i.e., U(1) symmetry is broken.

2. These additional terms can also lead to higher-derivative terms in the equation of motion.
The fact that, by construction, the action (2.2) is gauge-invariant in flat space-time implies that we need to include non-minimal coupling terms of the electromagnetic potential and its derivatives with the Riemann/Ricci tensors and Ricci scalars (non-minimal covariantization) as performed in case of scalar Galileons in curved space-time. However, ab initio we do not know the non-minimal coupling terms and, we need to consider all possible terms. The following modification to the electromagnetic action becomes by adding twelve non-minimal couplings become:

\[ S_{\text{VEC}} = S_{\text{VG}} + \lambda \sum_{i=1}^{12} S_i , \]  

where \( S_i \)'s are given by

\[
\begin{align*}
S_1 &= D_1 \int d^4x \sqrt{-g} g^{\mu\nu} g^{\alpha\beta} g^{\gamma\delta} R_{\mu\nu} \nabla_{\alpha} A_{\gamma} \nabla_{\beta} A_{\delta} , \\
S_2 &= D_2 \int d^4x \sqrt{-g} g^{\mu\alpha} g^{\nu\beta} g^{\gamma\delta} R_{\mu\nu} \nabla_{\alpha} A_{\gamma} \nabla_{\beta} A_{\delta} , \\
S_3 &= D_3 \int d^4x \sqrt{-g} g^{\mu\nu} g^{\alpha\beta} g^{\gamma\delta} R_{\mu\nu} \nabla_{\alpha} A_{\beta} \nabla_{\gamma} A_{\delta} , \\
S_4 &= D_4 \int d^4x \sqrt{-g} g^{\mu\nu} g^{\alpha\beta} g^{\gamma\delta} R_{\mu\nu} \nabla_{\alpha} A_{\beta} \nabla_{\gamma} A_{\delta} , \\
S_5 &= D_5 \int d^4x \sqrt{-g} g^{\mu\nu} g^{\alpha\beta} g^{\gamma\delta} g^{\eta\xi} R_{\mu\nu} \nabla_{\alpha} A_{\beta} \nabla_{\gamma} A_{\delta} \nabla_{\eta} A_{\xi} , \\
S_6 &= D_6 \int d^4x \sqrt{-g} g^{\mu\alpha} g^{\nu\beta} g^{\gamma\delta} g^{\eta\xi} R_{\mu\nu} \nabla_{\alpha} A_{\beta} \nabla_{\gamma} A_{\delta} \nabla_{\eta} A_{\xi} , \\
S_7 &= D_7 \int d^4x \sqrt{-g} g^{\mu\alpha} g^{\nu\beta} g^{\gamma\delta} R_{\alpha\beta} \nabla_{\mu} A_{\nu} \nabla_{\gamma} A_{\delta} , \\
S_8 &= D_8 \int d^4x \sqrt{-g} g^{\mu\alpha} g^{\nu\beta} g^{\gamma\delta} R_{\alpha\beta} \nabla_{\mu} A_{\nu} \nabla_{\gamma} A_{\delta} , \\
S_9 &= D_9 \int d^4x \sqrt{-g} g^{\alpha\beta} g^{\gamma\delta} g^{\mu\nu} R_{\alpha\beta} R_{\gamma\delta} A_{\mu} A_{\nu} , \\
S_{10} &= D_{10} \int d^4x \sqrt{-g} g^{\alpha\beta} g^{\mu\nu} g^{\gamma\delta} R_{\alpha\beta} R_{\gamma\delta} A_{\mu} A_{\nu} , \\
S_{11} &= D_{11} \int d^4x \sqrt{-g} g^{\alpha\gamma} g^{\beta\delta} g^{\mu\nu} R_{\alpha\beta} R_{\gamma\delta} A_{\mu} A_{\nu} , \\
S_{12} &= D_{12} \int d^4x \sqrt{-g} g^{\alpha\gamma} g^{\beta\delta} g^{\mu\nu} R_{\alpha\beta} R_{\gamma\delta} A_{\mu} A_{\nu} .
\end{align*}
\]

\( D_i \)'s are the twelve unknown dimensionless coefficients and \( \lambda \) is the coupling constant in action (2.2). Notice that the last four terms in the (2.5) look like gauge-dependent part. The reason behind adding the gauge-dependent terms is as follows: covariant derivative contains gauge-dependent par as well connection term. Hence, the two covariant derivatives (as can be seen in (2.2) as well as the first eight terms in the (2.5)) have the same structure. In order to compensate those terms, we separately add the last four terms.
2.3 Fixing the coefficients

Demanding that the above action is gauge-invariant in curved space-time and that the equations of motion do not contain higher order terms, the coefficients $D_i$’s can be fixed uniquely. However, it is extremely difficult to evaluate such equations in general curved space-time. Hence, for simplicity, we consider FRW line element $ds^2 = a(\eta)^2 (\text{d} \eta^2 + \text{d} \mathbf{x}^2)$. Also for the time being, we drop all the spatial derivatives and only concentrate on time derivatives.

Using the FRW line element in conformal time and after performing integration by-parts, action (2.4) becomes,

\begin{equation}
S^{\text{FRW}}_{\text{VEC}} = \lambda \int \text{d}^4 x \left( E_1 \frac{a'}{a} A_0 A_0^2 + E_2 \frac{a''}{a^3} \delta^{ij} A_i A_j + E_3 \frac{a''}{a^5} A_0 A_0^2 + E_4 \frac{a''}{a^5} \delta^{ij} A_i A_j \\
+ E_5 \frac{a''}{a^4} A_0 A_0^2 + E_6 \frac{a''}{a^4} \delta^{ij} A_i A_j' + E_7 \frac{a''}{a^4} \delta^{ij} A_i A_j + E_8 \frac{a''}{a^4} A_0 A_0' + E_9 \frac{a''}{a^4} A_0 A_0' + E_{10} \frac{a''}{a^4} \delta^{ij} A_i A_j' + E_{11} \frac{a''}{a^3} A_0 A_0^2 + E_{12} \frac{a''}{a^3} \delta^{ij} A_i A_j' \right),
\end{equation}

where the coefficients, $E_i$’s are linear functions of $D_i$’s and are given by the relations,

\begin{equation}
\begin{aligned}
E_1 &= 6 + 15 D_2 + 12 D_5 + 15 D_6 + 2 D_8 - 12 D_{11} - 9 D_{12}, \\
E_2 &= 22 - 13 D_2 - 3 D_6 - 5 D_8 + 12 D_{11} + D_{12}, \\
E_3 &= -12 + 24 D_1 - 3 D_2 + 24 D_3 + 24 D_4 - 3 D_6 - 3 D_8 + 18 D_{10} + 12 D_{11} + 18 D_{12}, \\
E_4 &= -16 - 12 D_1 + 5 D_2 - 12 D_4 - D_6 + 6 D_{10} - 12 D_{11} + 2 D_{12}, \\
E_5 &= -3 D_2 - 3 D_5 - 3 D_{10}, \\
E_6 &= -4 + D_7 + D_8 + 3 D_9, \\
E_7 &= -4 + 12 D_1 + 6 D_2 + 12 D_4 + 4 D_6 + 2 D_8, \\
E_8 &= -12 D_1 - 6 D_2 + 24 D_3 - 12 D_4 + 6 D_5 - 6 D_6 - 6 D_8, \\
E_9 &= 6 - 36 D_9 - 18 D_{10} - 12 D_{11} - 9 D_{12}, \\
E_{10} &= 36 D_9 + 6 D_{10} + 12 D_{11} + D_{12}, \\
E_{11} &= 6 D_1 + 2 D_2 + 6 D_3 + 6 D_4 + 3 D_5 + 3 D_6, \\
E_{12} &= 2 - 6 D_1 - 3 D_2 - D_7 - D_8,
\end{aligned}
\end{equation}

Let us focus on the action (2.6). Since we have only considered time derivatives and dropped all spatial derivatives, it may not be easy to identify the gauge-invariant terms. However, by looking at the action (2.6), it is apparent that, except fifth and sixth terms inside the right hand side, all other terms are either gauge-dependent terms or may lead to higher-derivative terms in the equations of motion. Hence, at this stage, we can safely avoid the gauge-dependent terms and the terms that lead to higher derivative terms in the equations of motion by setting all $E_i$’s (except $E_5$ and $E_6$) to zero and solve equations (2.7).
This leads to
\[
\begin{align*}
D_2 &= 2, \\
D_4 &= -D_1 - D_3, \\
D_6 &= -2 - D_5 \\
D_7 &= -4 - 6D_1 - 6D_3 - 2D_5, \\
D_8 &= 6D_3 + 2D_5, \\
D_9 &= \frac{1}{12} - \frac{D_3}{2} - \frac{D_5}{12}, \\
D_{10} &= -\frac{1}{6}, \\
D_{11} &= -\frac{1}{4} + \frac{3D_3}{2} + \frac{D_5}{4}, \\
D_{12} &= 1.
\end{align*}
\]  

(2.8)

As it is apparent now, we have solved \(D_i\)'s using the ten equations and out of these equations, nine are independent. Hence, out of twelve coefficients, only nine can be fixed. Moreover, it can also be seen that, \(E_5\) automatically satisfies the solution and vanishes, and only \(E_6\) survives in the action as

\[
S_{\text{FRW}} = -\lambda (1 + 3D_1) \int d^4x \frac{a^2}{a^2} \delta^{ij} A_i' A_j'.
\]  

(2.9)

It is also interesting to see that, \(D_3\) and \(D_5\) are arbitrary parameters, and though they do not vanish, \(S_3\) and \(S_5\) do not contribute to the action and the action (2.9) only depends on the parameter \(D_1\).

Till now, we have not considered any spatial derivative. In order to make it relatively more generalized, now we consider the action (2.4) not only with special derivatives but also with FRW line element which includes arbitrary Lapse function \(N(\eta)\) i.e.,

\[
ds^2 = -N(\eta)^2 d\eta^2 + a(\eta)^2 dx^2
\]  

(2.10)

and by imposing the two necessary and sufficient conditions: gauge-invariance and quadratic equations of motion, we try to see whether this can constrain the parameters further. This can lead to one of three consequences:

1. It leads to extra constraint equations which exceeds number of unfixed parameters.
   In this case, constructing vector Galileon in curved space-time is not possible and the no-go theorem [31] may be also extended to curved space-time.

2. It leads to extra constraint conditions but does not exceed the number of unfixed parameters. In that case, we may again fix some of the unfixed parameters and we can construct vector Galileon in FRW space-time.

3. It leads to no extra conditions, hence the solution set (2.8) remains same.

The action (2.4) in FRW metric with arbitrary Lapse function is evaluated in appendix A. We repeat the same procedure by imposing the desired conditions and we find that, it does not lead to extra conditions and the action remains gauge-invariant as well as it leads to quadratic equations of motion. This means that, even using more generalized
scenario with spacial derivatives and arbitrary Lapse function, the solution set for $D_i$’s (2.8) remains same and the action contains one arbitrary parameter, $D \equiv \lambda(1 + 3 D_1)$. Because of this reason, although we have chosen a maximally symmetric metric, we strongly expect the relations (2.8) holds true for arbitrary curved background as well. This is the first key result in our work.

3 Part II: magnetogenesis

Having constructed the gauge-invariant Electromagnetic action, our next step is to study the phenomenological consequences of this model. The modified electromagnetic action is

$$S_{\text{EM}} = \int d^4x \sqrt{-g} \left( -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + A_\mu J^\mu \right) + S_{\text{VEC}}. \quad (3.1)$$

The first two terms correspond to the standard electromagnetic action while the last term is given by eq. (2.4). $S_{\text{VEC}}$ has one unknown coupling parameter $D$ that can be fixed from observations.

As we discussed earlier, in the flat space-time, $S_{\text{VEC}}$ vanishes. Thus, the above action reduces to standard electromagnetic action in flat space-time. From the equation of motion of $A_0$ in the FRW background (2.10), the scalar potential is given by:

$$\Phi \equiv -A_0 = \frac{1}{4\pi(1 - 4DH^2)} \frac{\rho(r_0)}{r} \quad (3.2)$$

where $D \equiv \lambda(1 + 3 D_1)$. Thus, the effect of action (3.1) is to change the permittivity to $\epsilon \equiv (1 - 4DH^2)$ where $H$ is the Hubble constant. The electrostatic potential still goes as inverse of the distance. Permittivity being positive provides a condition on the value of $D$. If $D$ is negative all values are allowed, however, if $D$ is positive, then $4DH^2 < 1$. Thus, the modified action do not have any observable consequence in the terrestrial experiments. However, as we will show in the rest of this work, the above modified action has important consequence in the early Universe.

3.1 Breaking of conformal invariance and inflationary magnetogenesis

Having discussed the model and the effect on the Coulomb potential, let us now look at the effects in the early Universe. Since FRW background is conformally flat, the background gravitational field does not produce particles in the case of standard electromagnetic action (1.1) [55]. However, the modified action (2.4) explicitly breaks conformal invariance and thus may lead to production of magnetic fields and can have significant contribution in the early Universe. Since we are interested in the particle production and not in the vacuum polarization, we henceforth only consider the new term (2.4) and ignore the standard EM action. As we will show this is consistent.

Since action (2.4) is gauge-invariant, we choose Coulomb gauge ($A_0 = 0$) for rest of the calculations. In the FRW background (2.10), action (2.4) becomes:

$$S_{\text{VEC}} = D \int d^4x \left[ -2 \frac{a'^2}{N^3 a} A_i'^2 + 2 \frac{a''}{N a^2} (\partial_i A_j)'^2 - 2 \frac{a'}{N^2 a} (\partial_i A_j)'^2 \right]. \quad (3.3)$$
We can evaluate the vector potential at late times by fixing the initial state of the electromagnetic field. The first term is the energy density of the Electric field \( \rho_{E} \). Secondly, during most part of the evolution of the Universe, electrical conductivity is high \cite{42}, hence, electric fields decay and do not contribute to the energy density. This implies that \( D > 0 \).

Varying the above action with respect to \( A_i \) and setting \( N(\eta) = a(\eta) \), i.e., for conformal time, it leads to the following equations of motion:

\[
A''_i + 2 \frac{J^i}{J} A'_i - \frac{(aJ')^i}{(aJ)^2} \nabla^2 A_i = 0, \quad \text{where} \quad J = \frac{\mathcal{H}}{a}. \tag{3.4}
\]

Fourier decomposing the vector potential \( A_i \) \cite{40}, we get

\[
\hat{A}_i(\eta, \mathbf{x}) = \sqrt{4\pi} \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{\Lambda=1}^{2} \epsilon_{\Lambda i}(\mathbf{k}) \left[ \hat{i}_k^\Lambda A^\Lambda_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{i}^\Lambda_k A^\Lambda_*(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}} \right], \tag{3.5}
\]

where \( \Lambda \) corresponds to two orthonormal transverse polarizations and \( \epsilon_{\Lambda i} \) are the polarization vectors. Substituting (3.5) in (3.4), we get

\[
A''_k + 2 \frac{J^i}{J} A'_k + k^2 \frac{(aJ')^i}{(aJ)^2} A_k = 0. \tag{3.6}
\]

We can evaluate the vector potential at late times by fixing the initial state of the electromagnetic field.

To compare with the observations, we need to evaluate the energy density \cite{50}. 0-0 component of the energy momentum tensor \( T_{\mu\nu} \) in the FRW background (2.10) is

\[
T_{00} = -\frac{N^2}{a^3} \frac{\delta \mathcal{L}}{\delta \dot{N}}.
\]

The energy density in conformal coordinates is:

\[
\rho \equiv -T^0_0 = -6 D \frac{\mathcal{H}^2}{a^6} \delta^{ij} A'_i A_j' + 4 D \frac{\mathcal{H}^2}{a^6} \delta^{ij} \delta^{ij} \partial_i A_j \partial_j A_i + 4 D \frac{\mathcal{H}}{a^3} \delta^{ij} A'_i \nabla^2 A_j.
\]

The first term is the energy density of the Electric field (\( \rho_E \)). Second and the third terms are the energy densities of the magnetic field (\( \rho_B \)) and (\( \rho_{PB,B'} \)), respectively.

Using the decomposition (3.5), the electric, magnetic part of the perturbation spectrum per logarithmic interval can be written as:

\[
\mathcal{P}_B(k) = \frac{d}{d\ln k} \langle 0 | \rho_B | 0 \rangle = \frac{16 D \mathcal{H}^2 k^5}{\pi} \frac{k^5}{a^6} |A_k|^2, \tag{3.7}
\]

\[
\mathcal{P}_E(k) = \frac{d}{d\ln k} \langle 0 | \rho_E | 0 \rangle = -\frac{24 D \mathcal{H}^2 k^3}{\pi} \frac{k^3}{a^6} |A_k|^2, \tag{3.8}
\]

\[
\mathcal{P}_{B,B'}(k) = \frac{d}{d\ln k} \langle 0 | \rho_{B,B'} | 0 \rangle = -\frac{16 D \mathcal{H} k^5}{\pi} \frac{k^5}{a^6} A'_k A^*_k. \tag{3.9}
\]

It is important to note the following: first, in the standard electromagnetic action, the energy density is always positive and can be written as \( (B_i B'^i + E_i E'^i) \). However, in our case, it is given by \( D (H^2 B_i B'^i - H^2 E_i E'^i - HB'_i B_i) \) and hence, it is not positive definite for arbitrary background, however, depending on the model, it may become positive-definite. The result may be identified as the nature of Galileon models \cite{56}. This is the second key result of our work. Secondly, during most part of the evolution of the Universe, electrical conductivity is high \cite{42}, hence, electric fields decay and do not contribute to the energy density. This implies that \( D > 0 \).
Until now the analysis has been general and can be applied at any stage of the Universe evolution. In the rest of this work, we calculate the energy density of the electromagnetic field during inflation. We assume that the inflation is driven a scalar field and that the energy density of the electromagnetic field do not contribute to the accelerated expansion during inflation. In other words, we treat the electromagnetic field as a test field and obtain the power spectrum.

Let us first consider power-law inflation i.e. $a(t) = a_0 t^p; a(\eta) = a_0 (-\eta)^{1+\beta}$, where $p > 1; \beta \leq -2$. Note that $\beta = -2$ corresponds to de Sitter. Substituting $a(\eta)$ in (3.6), we have:

$$A"_k + \left(c^2 k^2 - \frac{(2+\beta)(3+\beta)}{\eta^2}\right) A_k = 0$$

(3.10)

where $A_k \equiv J(\eta) A_k$ and $c_s \equiv -\frac{1}{1+\beta} > 0$. This is the third key result of this work. The electromagnetic perturbations do not propagate at the speed of light. This is not unusual, as the scalar perturbations in Galileon inflation also propagate less than the speed of light [57, 58], however, the two speeds are not the same.

During power-law inflation, speed of sound, $c_s$ is a constant and the solution to the above differential equation is given by:

$$A_k = \sqrt{-\eta} \left[C_1 J_{\beta+\frac{5}{2}}(-c_s \kappa \eta) + C_2 J_{-\beta-\frac{5}{2}}(-c_s \kappa \eta)\right].$$

(3.11)

Imposing the initial condition in the sub-Hubble scales ($-k \eta \to \infty$) that the field is in vacuum state corresponds to $A_k \to \frac{1}{\sqrt{2c_s \kappa}} e^{-ic_s \kappa \eta}$. This leads to:

$$C_1 = \sqrt{\frac{\pi}{4 \cos(\beta \pi)}} \frac{e^{i(\beta+\frac{5}{2}) \frac{\pi}{2}}}{2^2 \beta + 5}, \quad C_2 = \sqrt{\frac{\pi}{4 \cos(\beta \pi)}} \frac{e^{-i(\beta+\frac{5}{2}) \frac{\pi}{2}}}{2^2 \beta + 5}. \quad (3.12)$$

It is important to note that for $\beta \leq -5/2$, $J_{\beta+5/2}$ dominates, however, $J_{-(\beta+5/2)}$ dominates for $\beta \geq -5/2$.

From (3.11), we obtain the spectra of the energy-densities (3.7), (3.8) and (3.9) at the crossing of the sound horizon ($c_s k_s = a_s H_s = \frac{1+\beta}{\eta}$). The magnetic part of the energy density is (Electric and $B.B'$ part of the energy density are provided in appendix B):

$$P_B = \frac{16 D}{\pi c_s^{1+2\beta}} \mathcal{F}_1(\beta) H^4 \left(\frac{k}{k_s}\right)^{10+2\beta} \quad \text{for} \quad \beta < -\frac{5}{2}, \quad \mathcal{F}_1(\beta) = \frac{|C_1|^2}{2^{2\beta+5} (\Gamma(\beta+7/2))^2}$$

$$= \frac{16 D}{\pi c_s^{1-2\beta}} \mathcal{F}_2(\beta) H^4 \left(\frac{k}{k_s}\right)^{-2\beta} \quad \text{for} \quad \beta > -\frac{5}{2}, \quad \mathcal{F}_2(\beta) = \frac{|C_2|^2}{2^{-2\beta-5} (\Gamma(-\beta-3/2))^2}. \quad (3.13)$$

This is the fourth key result regarding which we would like to stress the following points: first, for $\beta = -5$, the magnetic spectra is scale invariant. However, for $\beta = -5$, the electric field energy density diverges. Hence, $\beta = -5$ is ruled out as that will lead to negative energy density (since $D > 0$). Second, for $\beta \simeq -2$, the spectra is highly blue-titled [59]. To go about understanding the consequence of the same, the energy spectra during the slow-roll inflation [43] is given by (see appendix C):

$$P_B = \frac{8 D}{\pi c_s^4} H^4 \left(\frac{k}{k_s}\right)^4 ; \quad c_s = 1 - \epsilon_1$$

(3.14)
where $\epsilon_1$ is the first slow-roll parameter [43]. It is interesting to note that in the beginning of the inflation, $\epsilon_1 \ll 1$ and the speed of the EM perturbations is close to unity. However, during inflation, as $\epsilon_1$ increases, the speed of perturbations decrease, hence, leading to larger value of the energy spectrum and near the exit of inflation with $\epsilon \rightarrow 1$, large magnetic fields are produced which may be sufficient for the galactic dynamo condition. Finally, it is important to note that the power-spectrum in our model has the same blue-tilt as that of the vacuum polarization power-spectrum in the standard electromagnetic action. However, the power-spectrum evaluated here is due to particle production during inflation and depends on $D$ and $c_s$ [55, 60]. To fix these values and compare with observations, we need to evolve magnetic fields to the current epoch.

3.2 Post inflationary evolution

Reheating is expected to convert the energy in inflaton field to radiation [43] and Universe for most cosmic history has been good conductor ($\sigma \gg 1$). Assuming instantaneous reheating, the equation of motion of $A_i$ for large wavelength modes is [38]:

$$\ddot{A}_i + \frac{\sigma + H \left(1 - 8D\dot{H} - 4D\dot{H}^2\right)}{1 - 4D\dot{H}^2} \dot{A}_i = 0$$

(3.15)

where $J_i = -g^{ij} \sigma \dot{A}_j$. At late times, using eq. (3.2), we have $DH^2 \ll 1$. Hence, the above equation reduces to:

$$\ddot{A}_i + \sigma \dot{A}_i = 0 \Rightarrow A_i = C_1(x)t^{-\sigma t} + C_2(x),$$

(3.16)

which is same as standard EM action (1.1). Thus, the vector potential $A_i$ is constant in time implying that the electric field vanishes and magnetic field decays as $a^{-2}$. During Radiation-dominated era, $H \propto a^{-2}$, and the energy density corresponding to $S_{\text{VEC}}$ decays as $a^{-6}$. However, the energy density of the standard EM action goes as $a^{-4}$. At late times, only EM action (1.1) contributes.

3.3 Constraints from observations

To compare whether the generated magnetic field (3.14) has the right magnitude needed to seed galactic fields, we need to compare $\rho_B$ with radiation background energy density $\rho_\gamma \propto T^4$. This is because, the magnetic field generated during inflation evolve as $\rho_B \propto a^{-4}$ [38, 43, 60] which is same as $\rho_\gamma$. Hence, the dimensionless quantity $r \equiv \rho_B/\rho_\gamma$ remains approximately constant and provides a convenient method to constrain the primordial magnetic field [60]. From eq. (3.14), we get,

$$r \sim \frac{D}{c_s} \frac{10^{-104}}{\lambda_{\text{Mpc}}^2 eV^2}.$$  

(3.17)

Note that $D$ has dimensions of inverse mass square. The field strength required to seed galactic fields with an efficient galactic dynamo translates to $r \sim 10^{-34}$ [38, 60]. For length scales of 1Mpc, this translates to $D/c_s \sim 10^{70}$. Using the fact that permittivity has to be positive, from eq. (3.2), we get $D \sim 10^{-36} eV^{-2}$. Thus, near the exit of inflation, $c_s \sim 10^{-116}$. 

– 11 –
This is the last key result of this work and we would like to stress the following points: first, at the early epoch of inflation $\epsilon_1 \ll 1$, implying that, $c_s \sim 1$. Hence, the energy density of the magnetic fields generated at the early epoch of inflation is tiny and the magnetic fields, present at decoupling and homogeneous on scales larger than the horizon at that time is much less than the current limit of $B \leq 10 \, nG$ [37]. Second, appreciable seed magnetic fields are generated only close to the exit of inflation. Thus, our model naturally generates appreciable magnetic field at Mpc scale as the modes that leave the horizon close to the exit of inflation re-enter early during radiation epoch and an efficient dynamo mechanism can generate the observed magnetic field. Thus, our model generates appreciable magnetic fields only for smaller wavelength modes. This is the key unique feature of our model compared to other proposed models for magnetogenesis.

4 Conclusions and discussions

In this work — by demanding that the theory be described by vector potential $A^\mu$ and its derivatives, Gauge invariance be satisfied, and equations of motion be linear in second derivatives of vector potential — we have constructed a higher derivative electromagnetic action that does not have ghosts and preserve U(1) gauge invariance. This is the first higher derivative vector Galileon model constructed as all other models in the literature contain linear derivatives of the vector fields. We have shown that the higher order terms vanish in the flat space-time and hence, consistent with the no-go theorem by Deffayet et al. [31].

We have shown that the model breaks conformal invariance and generate magnetic field during inflation. In doing so, we encountered an important aspect of our model: the energy density is not positive definite for arbitrary background and depending upon the model, it can become positive definite. The magnetic fields generated have two key features: first, the modes generated propagate less than the speed of light and the speed of propagation depends on the slow-roll parameter (3.14). Second, the model generates appreciable magnetic field for small wavelength modes ($\sim$ Mpc) while the model generates tiny magnetic fields for large wavelength modes. This is an unique feature of our model compared to other models that generate magnetic field during inflation. The energy density of the magnetic field is appreciable only at the end of inflation and hence, our model does not lead to any back-reaction. Since the vector Galileon field does not couple directly with inflaton [61], our model ensures sufficient number of e-folds during inflation and at the same time, significant magnetic fields are generated very close to the exit of inflation. Therefore, by using this new and unique model, the tight constraint given in [61] is avoided while generating magnetic fields.

For the inflation to exit, $\epsilon_1 = 1$. Our model can generate appreciable magnetic field near the exit and, in principle, can provide a dynamical mechanism for the exit of inflation. This is under investigation.

The magnetic field spectra generated in our model is blue tilted. This should be contrasted with other models where the spectra can be fine tuned [50]. Recently, Kalnshvili et al. [59], have done a detailed analysis to place constraints on the primordial magnetic field from the cosmological data including models that have blue tilt. It is interesting to investigate how the mass dispersion $\sigma(M, z)$ behaves and its effect on the structure formation. This is currently under investigation.

We showed that the action is gauge invariant and does not contain any higher order derivatives for the FRW background. The assumption is mainly driven for simplicity and applications to cosmology. Also, when constructing the model, we initially chose a simpler
situation with no spatial derivative and then we considered more general situation with arbitrary Lapse function and spatial derivatives of the gauge field. In both the situations, the solution remained the same and therefore, the model is locally Lorentz invariant, we highly expect the model should be applicable for arbitrary curved background as well.

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A Action in FRW space-time with arbitrary Lapse function

Evaluating the equations of motion for an arbitrary metric is hard and also non-transparent. Hence, to calculate equations of motion and thus to fix the coefficients, we consider FRW background

\[ ds^2 = -N(\eta)^2 d\eta^2 + a^2 dx^2 \]

where \( N(\eta) \) is the Lapse function. The action becomes

\[
\begin{align*}
L_{SW} &= 4\delta^{ij} A_i \partial_0 a J_j N(-4) a^2 d(-4) - 2\delta^{ij} A_i A_j \delta_{ij} N(-4) a^2 d(-6) - 4\delta^{ij} A_i A_j a'' N(-4) a^2 d(-5) + 8\delta^{ij} A_i \partial_0 a J_j N(-5) a^2 d(-4) + 4\delta^{ij} A_i A_j N(-5) a^2 d(-5) - 4\delta^{ij} \partial_0 A_i \partial_0 a J_j N(-4) a d(-3) + 4\delta^{ij} \\
&+ 4\delta^{ij} A_i \partial_0 a J_j N(-4) a d(-3) + 2\delta^{ij} \partial_0 A_i \partial_0 a J_j N(-4) a^2 d(-4) + 2\delta^{ij} A_i A_j N(-4) a^2 d(-4) + 4\delta^{ij} \partial_0 A_i \partial_0 A_j a'' N(-4) a d(-4) - 4\delta^{ij} \partial_0 A_i \partial_0 A_j N(-4) a^2 d(-4) + 8\delta^{ij} \\
&- 8\delta^{ij} \partial_0 A_i N'' a J_j \partial_0 A_0 N(-5) a d(-3) + 4\delta^{ij} N' a J_j \partial_0 A_0 N(-5) a d(-3) - 2\delta^{ij} \delta^{kl} \partial_0 A_k \partial_0 A_j N(-2) a d(-4) + 4\delta^{ij} N' a J_j \partial_0 A_0 N(-5) a d(-3) - 4A_0 \delta^{ij} a'' \partial_0 A_i N(-4) a d(-3) + 6\delta^{ij} \delta^{kl} a J_i \partial_0 A_k \partial_0 A_j N(-2) a d(-5) + 6A_0^2 N(-8) N(2) a d(-2) + 6A_0^2 N(-6) a^2 d(-2) - \ldots
\end{align*}
\]

A.1
\[\mathcal{L}_1 = 6a^2 N^{(6)} \partial_0 A_0^2 a^{-1} - 6N a' N^{(-7)} \partial_0 A_0^2 a^{-1} + 6N^{(6)} \partial_0 A_0^2 a^2 a^{-2} - \\
6 \partial_0^2 A_0 \partial_0 A_0 N^{(-4)} a^{(3)} - 6 \partial_0^2 A_0 \partial_0 A_0 N^{(-5)} a^{(4)} - 6 \partial_0^2 A_0 \partial_0 A_0 N^{(-6)} a^{(5)} - 6 \partial_0^2 A_0 \partial_0 A_0 N^{(-7)} a^{(6)} + \\
6 \partial_0^2 A_0 \partial_0 A_0 N^{(-8)} a^{(7)} - 6 \partial_0^2 A_0 \partial_0 A_0 N^{(-9)} a^{(8)} + \\
12 \partial_0 \partial_0 A_0 N^a'' N^{(-7)} a^{(1)} - 12 \partial_0 \partial_0 A_0 N^a'' N^{(-7)} a^{(2)} + \\
6 \partial_0^3 A_0 \partial_0 A_0 N^{(-4)} a^{(4)} - 6 \partial_0^3 A_0 \partial_0 A_0 N^{(-5)} a^{(5)} + \\
6 \partial_0^3 A_0 \partial_0 A_0 N^{(-6)} a^{(6)} - 6 \partial_0^3 A_0 \partial_0 A_0 N^{(-7)} a^{(7)} + \\
6 \partial_0^3 A_0 \partial_0 A_0 N^{(-8)} a^{(8)} - 6 \partial_0^3 A_0 \partial_0 A_0 N^{(-9)} a^{(9)} + \\
6 a_0^2 N_a^{(8)} N^{(-2)} a^{(-2)} - 6 a_0^2 N_a^{(9)} N^{(-3)} a^{(-3)} + 18 a_0^2 N_a^{(6)} a^{(-4)} + \\
A_0 (2.2) \]

\[\mathcal{L}_2 = 3a^2 N^{(6)} \partial_0 A_0^2 a^{-1} - 3N a' N^{(-7)} \partial_0 A_0^2 a^{-1} - 3 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-4)} a^{(-3)} + \\
3 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-5)} a^{(-4)} + 3 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-6)} a^{(-5)} - 3 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-7)} a^{(-6)} + \\
3 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-8)} a^{(-7)} + 3 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-9)} a^{(-8)} + \\
\partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-5)} a^{(-5)} + \partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-6)} a^{(-6)} + \\
\partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-7)} a^{(-7)} + \partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-8)} a^{(-8)} + \\
\partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-9)} a^{(-9)} + \partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-10)} a^{(-10)} + \\
3 a_0^2 A_0^2 N^{(8)} N^{(-2)} a^{(-2)} - 3 a_0^2 A_0^2 N^{(9)} N^{(-3)} a^{(-3)} + 3 A_0 A_0 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-4)} a^{(-4)} + \\
3 A_0 A_0 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-5)} a^{(-5)} + 3 A_0 A_0 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-6)} a^{(-6)} + \\
3 A_0 A_0 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-7)} a^{(-7)} + 3 A_0 A_0 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-8)} a^{(-8)} + \\
3 A_0 A_0 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-9)} a^{(-9)} + 3 A_0 A_0 \partial_0^2 \partial_0 A_0 A_0 N^a'' N^{(-10)} a^{(-10)} + \\
A_0 \partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-5)} a^{(-5)} + A_0 \partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-6)} a^{(-6)} + \\
A_0 \partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-7)} a^{(-7)} + A_0 \partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-8)} a^{(-8)} + \\
A_0 \partial_0^2 \partial_0 \partial_0 A_0 A_0 N^a'' N^{(-9)} a^{(-9)} + 3 N a' A_0^2 N^{(-7)} a^{(-3)} + 6 a_0^2 A_0 N^{(-6)} a^{(-4)} + \\
3 a_0^2 A_0^2 N^{(6)} a^{(-2)} a^{(-2)} \]

\[\mathcal{L}_3 = 6a^2 N^{(6)} \partial_0 A_0^2 a^{-1} - 6N a' N^{(-7)} \partial_0 A_0^2 a^{-1} + 6N^{(6)} \partial_0 A_0^2 a^2 a^{-2} - \\
12 \partial_0^2 \partial_0 A_0 A_0 N^{(-4)} a^{(3)} + 12 \partial_0^2 \partial_0 A_0 A_0 N^{(-5)} a^{(4)} - 12 \partial_0^2 \partial_0 A_0 A_0 N^{(-6)} a^{(5)} - 12 \partial_0^2 \partial_0 A_0 A_0 N^{(-7)} a^{(6)} + \\
6 \partial_0^3 \partial_0 A_0 A_0 N^{(-2)} a^{(5)} - 6 \partial_0^3 \partial_0 A_0 A_0 N^{(-3)} a^{(6)} + 6 \partial_0^3 \partial_0 A_0 A_0 N^{(-4)} a^{(7)} - \\
12 \partial_0 \partial_0 A_0 N^a'' N^{(-7)} a^{(1)} - 12 \partial_0 \partial_0 A_0 N^a'' N^{(-7)} a^{(2)} - 48 \partial_0 \partial_0 A_0 N^a'' N^{(-8)} a^{(3)} a^{(-2)} + \\
12 \partial_0 \partial_0 \partial_0 A_0 A_0 N^{(-2)} a^{(3)} - 12 \partial_0 \partial_0 \partial_0 A_0 A_0 N^{(-3)} a^{(4)} + 36 A_0 \partial_0 \partial_0 A_0 a'' N^{(-6)} a^{(-2)} - 36 A_0 \partial_0 \partial_0 A_0 a'' N^{(-7)} a^{(-3)} + \\
36 A_0 \partial_0 \partial_0 A_0 a'' N^{(-8)} a^{(-4)} - 6 a_0^2 A_0 N^a'' a^{(-2)} a^{(-2)} - 6 a_0^2 A_0 N^a'' a^{(-3)} a^{(-3)} + \\
42 A_0^2 N^{(-8)} N^{(-2)} a^{(-2)} - 36 N a' a'' A_0^2 N^{(-7)} a^{(-2)} - 90 N a' a'' A_0^2 N^{(-7)} a^{(-3)} + \\
54 a_0^2 N^{(-6)} a^{(-2)} a^{(-3)} + 54 a_0^2 N^{(-6)} a^{(-4)} a^{(-4)} \]

(A.2)
\( \mathcal{L}_4 = 6a'' N^{(-6)} \partial_0 A_0 a^{-1} - 6N' a' N^{(-7)} \partial_0 A_0 a^{-1} + 6N^{(-6)} \partial_0 A_0 a^2 a^{(-2)} - \\
12\delta^i \partial_0 A_0 \partial_0 a'' N^{(-4)} a^{-3} + 12\delta^i \partial_0 A_0 N' a' \partial_0 A_0 N^{(-5)} a^{-3} - 12\delta^i \partial_0 A_0 \partial_0 A_0 N^{(-4)} a^2 a^{(-4)} + \\
6\delta^i \delta^j \partial_0 \partial_0 A_0 A_0'' N^{(-2)} a^{-5} - 6\delta^i \delta^j \delta^k N' \partial_0 A_0 \partial_0 A_0 N^{(-3)} a^{-5} + 6\delta^i \delta^j \delta^k \partial_0 \partial_0 A_0 N^{(-2)} a^{-6} - \\
12\delta^0 \partial_0 A_0 N'' a^{-7} a^{-1} + 12\delta^0 \partial_0 A_0 N'' a^{-8} N'' a^{-2} + \\
6A_j \delta^i \partial_0 A_0 N^{(-4)} a^{-4} - 6A_j \delta^i \delta^j N' \partial_0 A_0 N^{(-5)} a^2 a^{-4} + 6A_j \delta^i \delta^j \partial_0 A_0 N^{(-4)} a^3 a^{-5} + \\
6A_j \delta^i \delta^j \partial_0 A_0 a'' N^{(-4)} a^{-4} - 6A_j \delta^i \delta^j N' \partial_0 A_0 N^{(-5)} a^2 a^{-4} + 6A_j \delta^i \delta^j \partial_0 A_0 N^{(-4)} a^3 a^{-5} + \\
6A_j \delta^i \delta^j a'' \partial_0 A_0'' N^{(-3)} a^{-3} - 6A_j \delta^i \delta^j N' \partial_0 A_0 N^{(-4)} a^2 a^{-4} + 6A_j \delta^i \delta^j \partial_0 A_0 N^{(-3)} a^3 a^{-5} + \\
6a'' A_0 a^{-2} N^{(-8)} a^{-1} - 6a' A_0 N^{(-9)} a^{-1} + 6A_0^0 N^{(-8)} a^{-2} a^{-2} - \\
6A_1 \delta^i \delta^j a'' N^{(-4)} a^{-4} - 6A_1 \delta^i \delta^j N' \partial_0 A_0 N^{(-5)} a^2 a^{-4} + 6A_1 \delta^i \delta^j \partial_0 A_0 N^{(-4)} a^3 a^{-5} + \\
6A_1 \delta^i \delta^j a'' \partial_0 A_0'' N^{(-3)} a^{-3} - 6A_1 \delta^i \delta^j N' \partial_0 A_0 N^{(-4)} a^2 a^{-4} + 6A_1 \delta^i \delta^j \partial_0 A_0 N^{(-3)} a^3 a^{-5} + \\
18a'' A_0 a^{-2} N^{(-6)} a^3 a^{-3}\ A_0 a^{-2} N^{(-7)} a^3 a^{-3} + 18A_0 a^{-2} N^{(-6)} a^4 a^{-4} \quad (A.5) \)

\( \mathcal{L}_5 = 3a'' N^{(-6)} \partial_0 A_0 a^{-1} - 3N' a' N^{(-7)} \partial_0 A_0 a^{-1} + \delta^i \partial_0 A_0 N' a' \partial_j A_0 N^{(-5)} a^{-3} \) \\
45\delta^i \partial_0 \partial_0 A_0 N^{(-4)} a^2 a^{-4} - 6\delta^0 \partial_0 \partial_0 A_0 a'' N^{(-4)} a^{-3} + 24\delta^i \partial_0 \partial_0 A_0 A_0'' N^{(-4)} a^2 a^{-4} + \\
3\delta^0 \partial_0 \partial_0 A_0 a'' N^{(-4)} a^{-3} + 3\delta^0 \partial_0 \partial_0 A_0 N' a' \partial_0 A_0 N^{(-5)} a^{-3} - 6\delta^0 \delta^i \delta^k N' a' \partial_0 A_0 A_0 N^{(-3)} a^{-5} + \\
45\delta^0 \delta^i \delta^j \partial_0 \partial_0 A_0 N^{(-2)} a^2 a^{-6} + 6\delta^0 \delta^i \delta^k \partial_0 \partial_0 A_0 a'' N^{(-2)} a^{-5} - 2\delta^0 \delta^i \delta^k \partial_0 \partial_0 A_0 A_0'' N^{(-2)} a^{-6} - \\
6A_0 \partial_0 A_0 N'' a'' N^{(-7)} a^{-1} + 6A_0 \partial_0 A_0 a'' N^{(-8)} N'' a^{-1} + 3A_0 \delta^0 \delta^i N' \partial_0 A_0 a'' N^{(-5)} a^{-3} - \\
3A_0 \delta^0 \delta^i a' \partial_j A_0 N^{(-6)} a^{-2} a^{-3} - 12A_0 \partial_0 A_0 N'' a^{-7} a^2 a^{-2} - 6A_0 \partial_0 A_0 N^{(-6)} a^3 a^{-3} + \\
12A_0 \partial_0 a'' N^{(-4)} a^{-4} - 4A_0 \delta^0 N'' \partial_0 A_0 N^{(-5)} a^2 a^{-4} - 3A_0 \delta^0 \delta^i a' \partial_j A_0 N^{(-4)} a^{-4} - \\
A_0 \delta^0 \delta^i a' \partial_j A_0 N^{(-6)} a^2 a^{-3} + 7A_0 \delta^0 N'' \partial_0 A_0 N^{(-5)} a^2 a^{-4} + 4A_0 \delta^0 \delta^i N' \partial_0 A_0 a'' N^{(-5)} a^{-3} + \\
3a'' A_0 a^{-2} N^{(-8)} a^{-1} - 3a' A_0 a^{2} N^{(-9)} N'' a^{-1} + 12A_0 a^{2} N^{(-8)} N'' a^{-2} a^{-2} - \\
15A_0 a^{2} N^{(-7)} a^3 a^{-3} - 12N' a'' a'' a'' N^{(-2)} a^{-2} - 12A_0 \delta^i \delta^j \partial_0 A_0 N^{(-4)} a^4 a^{-4} - \\
3A_0 \delta^0 \delta^i a' \partial_j A_0 a'' N^{(-4)} a^{-4} + 18A_0 a^{2} N^{(-6)} a^4 a^{-4} + 9a'' A_0 a^{2} N^{(-6)} a^2 a^{-3} \quad (A.6) \)

\( \mathcal{L}_6 = 3a'' N^{(-6)} \partial_0 A_0 a^{-1} - 3N' a' N^{(-7)} \partial_0 A_0 a^{-1} - 35\delta^i \partial_0 A_0 \partial_0 A_0 a'' N^{(-4)} a^{-3} + \\
3\delta^0 \partial_0 a'' A_0 N^{(-5)} a^{-3} + \delta^0 \partial_0 a' \partial_j A_0 N^{(-5)} a^{-3} - 4\delta^0 \partial_0 a' \partial_0 A_0 N^{(-4)} a^2 a^{-4} - \\
\delta^0 \partial_0 \partial_0 A_0 a'' N^{(-4)} a^{-3} - \delta^0 \delta^i \delta^k N' a' \partial_0 A_0 A_0 N^{(-3)} a^{-5} + \\
4\delta^0 \delta^i \delta^j \partial_0 \partial_0 A_0 A_0 N^{(-2)} a^2 a^{-6} + \delta^0 \delta^i \delta^k \partial_0 \partial_0 A_0 a'' N^{(-2)} a^{-5} - 2\delta^0 \delta^i \delta^k \partial_0 \partial_0 A_0 A_0'' N^{(-2)} a^{-6} - \\
6A_0 \partial_0 A_0 N'' a'' a^{-7} a^{-1} + 6A_0 \partial_0 A_0 N'' N'' a^{-2} a^{-1} - A_j \delta^j N' \partial_0 A_0 N^{(-5)} a^2 a^{-4} + \\
2A_j \delta^j \partial_0 A_0 a'' N^{(-4)} a^{-4} - 3A_j \delta^j a' \partial_0 A_0 a'' N^{(-4)} a^{-4} + 3A_j \delta^j \partial_0 A_0 a'' a'' N^{(-4)} a^{-4} - \\
3A_j \delta^j \partial_0 A_0 N^{(-5)} a^2 a^{-4} + 2A_0 \delta^0 \delta^i N'' \partial_0 A_0 N^{(-5)} a^2 a^{-4} + 3a'' A_0 a^2 N^{(-8)} N'' a^{-2} a^{-1} - \\
3a'' A_0 a^2 N^{(-9)} a^3 a^{-3} - 4A_0 \delta^0 a'' N'' a'' a'' N^{(-4)} a^{-4} + 4A_0 \delta^0 \delta^i N'' a'' a'' N^{(-4)} a^{-4} - \\
4A_0 \delta^0 \delta^i \delta^j \partial_0 A_0 a'' N^{(-4)} a^3 a^{-5} - 3A_0 \delta^0 \delta^i a' \partial_j A_0 a'' a'' N^{(-4)} a^{-4} - 3N' A_0 a^{2} N^{(-7)} a^3 a^{-3} + \\
6A_0 a^{2} N^{(-6)} a^4 a^{-4} + 3a'' A_0 a^{2} N^{(-6)} a^2 a^{-3} \quad (A.7) \)
\[ S_i = D_i \int d^4 x \sqrt{-g} \mathcal{L}_i = D_i \int d^4 x N a^3 \mathcal{L}_i \]
B Spectrum of electric and $B.B'$

Electric part and $B.B'$ part of the energy density at sound horizon become
\begin{align}
\mathcal{P}_E &= -\frac{24 D}{\pi c_s^{1+2\beta}} G_1(\beta) H_1^s \left( \frac{k}{k_s} \right)^{2\beta+8}, \quad \beta \langle -\frac{5}{2}, \quad G_1(\beta) = \frac{|C_1|^2}{2^{2\beta+3} (\Gamma(\beta+5/2))^2} \\
-\frac{24 D}{\pi c_s^{1-2\beta}} G_2(\beta) H_1^s \left( \frac{k}{k_s} \right)^{-2-2\beta}, \quad \beta \langle -\frac{5}{2}, \quad G_2(\beta) = \frac{|C_2|^2}{2^{-2\beta-3} (\Gamma(-\beta-1/2))^2}
\end{align}

\begin{align}
\mathcal{P}_{B.B'} &= -\frac{16 D}{\pi c_s^{10+2\beta}} J_1(\beta) H_1^s \left( \frac{k}{k_s} \right)^{2\beta+10}, \quad \beta \langle -\frac{5}{2}, \quad J_1(\beta) = \frac{|C_1|^2}{2^{2\beta+4} (-\beta-5/2) (\Gamma(\beta+5/2))^2} \\
-\frac{16 D}{\pi c_s^{2-2\beta}} J_2(\beta) H_1^s \left( \frac{k}{k_s} \right)^{-2-2\beta}, \quad \beta \langle -\frac{5}{2}, \quad J_2(\beta) = \frac{|C_2|^2}{2^{-2\beta-4} (-\beta-3/2) (\Gamma(-\beta-3/2))^2}
\end{align}

(C.1)

(C.2)

(C.3)

(C.4)

(C.5)

(C.6)

(C.7)

(C.8)

(C.9)
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