Phenomenological constraints on the Jaffe-Wilczek model of pentaquarks

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A model recently introduced by Jaffe and Wilczek based on the quarks being dynamically bound into diquarks has been used to predict that the recently observed exotic baryons (pentaquarks) fall into a nearly ideally mixed combination of the $8$ and $\overline{10}$ representations of SU(3) flavor. The model predicts two states with nucleon quantum numbers which have tentatively been identified with the $N^*(1440)$ and the $N^*(1710)$. This paper examines the viability of this model by focusing on the decay width of the nucleon members of the multiplet. An inequality relating the partial widths of these nucleon states in the $\pi$+nucleon channel to the width of the $\theta^+$ is derived for this model under the assumption ideal mixing and that the only significant exact SU(3) symmetry violations are the result of ideal mixing, threshold effects and the masses of pseudo-Goldstone bosons. This inequality is badly violated if the states in the multiplet are the $N^*(1440)$ and the $N^*(1710)$ and if the recent bounds extracted for the $\theta^+$ width are reliable. Thus, the model appears to require a scenario with the existence of at least one presently unknown resonance with nucleon quantum numbers.

I. INTRODUCTION

There is considerable ferment in the field of hadronic physics. A narrow baryon resonance with a strangeness of $+1$ (the $\theta^+$) has been identified by a number of experimental groups. This state is clearly exotic in the sense that it cannot be described by a simple quark model. There is also experimental evidence for an additional exotic state with $S = -2$ and $Q = -2$. Almost as remarkable as the exotic nature of these states is their extremely small widths. All measurements of the widths are consistent with experimental resolution of $\sim 20$ MeV and comparisons with previous experiments bound the width of the $\theta^+$ to be very small; recent analyses of these experiments suggest that the width is less than 1-2 MeV. The question of how to interpret such states has become pressing.

Of course, in one sense the interpretation is simple—such states must emerge directly from QCD. Early attempts at doing this via lattice methods have been made. Whether the state of lattice technology is presently good enough so that the various extrapolations in quark mass, lattice spacing and lattice volume are under control for observables associated with these exotic states are reliable remains to be seen. In any event, it is plausible that the lattice will ultimately be capable of determining the $\theta^+$ mass reliably. However, even if one ultimately has a reliable lattice determination of the $\theta^+$ mass it cannot truly be said that one understands the nature of the exotic states; one merely knows that the state arises from QCD. Understanding requires some intuitive picture which captures the essence of the physics, i.e., a model, along with some understanding of how the model is connected with the underlying theory. In this sense, the simple interpretation that the state emerges from QCD, though correct, is incomplete. An analogy to condensed matter physics is useful: the BCS theory of superconductivity is intrinsically limited but it allows one to abstract the essential physics far more readily than does a full microscopic many-body theory.

Thus, there have been multiple attempts to find simple models which capture the core of the physics associated with exotic baryons. One class of models are the chiral solitons which have the virtue of being connected with large $N_c$ QCD and to approximate chiral symmetry in a manner similar to that of QCD. Unfortunately most of the analyses using such models has been based on rigid-rotor quantization which is highly controversial. It is the view of this author that the arguments in refs. are compelling and that rigid rotor quantization is almost certainly wrong in that it has no justification from large $N_c$ QCD. Objections to this view expressed in ref. were fully rebutted in ref. Thus, all of these treatments of exotic baryons in chiral soliton models cannot be considered valid. There has been an attempt to describe the exotic states in a consistent manner in these models using the Callen-Klebanov method. However, such an attempt is highly dependent on details of the interaction which are hard to constrain from other phenomenology; therefore such an approach remains largely unpredictive.

Another class of models are variants of constituent quark models. Perhaps most prominent among these is a model introduced by Jaffe and Wilczek which describes the exotic states as pentaquarks composed of two strongly bound scalar diquarks interacting with an anti-quark. The model predicts that the lowest-lying pentaquark states are a nearly ideally mixed combination of an SU(3) flavor octet and antidecuplet. One key feature of this model is the prediction of two pentaquark states with nucleon quantum numbers: one with a mass of $\sim 1450$ MeV and the other with a mass of $\sim 1700$ MeV. These states have been tentatively identified with the $N^*(1440)$ (also referred to as the Roper resonance) and the $N^*(1710)$. The purpose of the present paper is to explore the phenomenology of the Jaffe-Wilczek model. In particular, it focuses...
on the widths of the $N^*$ states with emphasis on determining whether the model is consistent with presently known phenomenological constraints. It will be shown that the identification of the two nucleon states as the Roper resonance and the $N^*(1710)$ is inconsistent with the phenomenology of the widths given the assumptions underlyiing the model and recent bounds on the $\theta^+$ width.

The reason for this inconsistency in the identification is easy to see. Given the assumption of nearly ideal mixing, the two physical states with nucleon quantum numbers each have substantial contributions from both the octet and anti-decuplet multiplets. The $\theta^+$, which has no octet contribution, is extremely narrow. Hence the coupling of the anti-decuplet part of the two states to the pion plus nucleon final state is quite small. In contrast, the Roper resonance is known to be very broad with a large coupling to the pion plus nucleon state. If the Roper resonance is identified with the lighter of the two $N^*$ states, one can deduce that the coupling to the octet component must be large. A large coupling to the octet, in turn, implies a large partial width for the higher of the two $N^*$ states. However, the partial width for the $N^*(1710)$ into pion plus nucleon is, in fact, quite small.

In this paper, the simple argument presented above will be quantified. In particular, an inequality will be derived relating the partial widths for the decay of the two $N^*$ states into $\pi + N$ with the width of the $\theta^+$. The only assumptions entering the inequality are that the physical states are ideally mixed, and that the only sources of SU(3) violation are ideal mixing, threshold effects and masses of pseudo-Goldstone bosons. These assumptions are not quite correct: the Jaffe-Wilczek model is based on nearly ideal mixing and other sources of SU(3) violation are present. Thus the inequality may be modestly violated even if the model is essentially correct; however, gross violations of the inequality are simply not compatible with the model. Identifying the pentaquark $N^*$ states as the Roper resonance and the $N^*(1710)$ and putting in the various widths, one finds that the inequality is grossly violated provided the $\theta^+$ width is as narrow as reported in refs. 1-4.

If one accepts the validity of the model and the reported bounds on the $\theta^+$ width then one of three scenarios must be adopted: i) the $N^*(1710)$ state is, in fact, a nearly ideally mixed pentaquark and it has a previously undiscovered narrow partner with a mass in the neighborhood of 1450 MeV; ii) the Roper resonance is a nearly ideally mixed pentaquark and it has a previously undiscovered broad partner with a mass in the neighborhood of 1700 MeV; iii) neither the lower the lower nor the upper $N^*$ states have been detected.

This paper is organized as follows: In the following section the Jaffe-Wilczek model is discussed. Section III derives an inequality for the partial widths of the $N^*$ states based on the model. The following section is devoted to phenomenology. It is shown that the inequality in sec. III is badly violated if the Roper resonance and $N^*(1710)$ are both identified as ideally mixed pentaquarks. The various scenarios enumerated above are discussed with a particular emphasis on the possibility of verifying or ruling out particular scenarios via the study of pion-nucleon scattering.

II. THE JAFFE-WILCZEK MODEL

The model is based on the following dynamical assumptions: a) The states can be well described in terms of a constituent quark model with four quarks and one anti-quark. b) There is a strong quark-quark interaction that binds the quarks into scalar diquarks which are antitriplets in both color and flavor. To be more precise, this means that the wave function of the pentaquark is completely dominated by configurations in which the quarks are grouped into some particular lowest diquark configuration: this lowest mass diquark is a spatial scalar and has 3 quantum numbers in both flavor and color. c) In the absence of SU(3) flavor violations the lowest-lying states are an SU(3) flavor octet and a decuplet which are nearly degenerate; all other states are well separated. These multiplets are both (1/2)^+ states. d) The dynamics is such that the SU(3) symmetry breaking Hamiltonian acting among these states is given to good approximation by

$$H_I = (a(n_s + n_\tau) + b_{n_s}(2m_s - m_u - m_d)),$$

which leads to a nearly ideal mixing in the sense that the eigenstates of the Hamiltonian have nearly well-defined expectation values for the number of strange quarks and strange anti-quarks separately. Note the form of this interaction is slightly different from that given in ref. 10. This different form, although equivalent to that in ref. 10, is useful as it manifests the fact that interaction only mixes the two multiplets via explicit SU(3) symmetry breaking.

It is worth remarking at the outset that any of these assumptions may be questioned. The intent of the present paper is not to try to understand the theoretical underpinnings of the model or test whether they can ultimately be justified. Rather, the approach is to take the model seriously and see whether the phenomenology that emerges is qualitatively consistent with what is observed in nature.

The focus here is on the ideally mixed states with nucleon quantum numbers. The first step in analyzing the decays of these states is the construction of the two states in the context of the model. We first accept assumptions a) and b) as stated above and write model wave functions for a system with two scalar diquarks (in a 3 flavor representation and a 3 color representation) and one anti-quark. The analysis begins in the exact SU(3) limit; the form of ideal mixing as given by the model will be computed. In general the wave function has color, flavor, spin and spatial pieces. Explicit color will be suppressed since all states are fully anti-symmetric with respect to color.
The flavor wave functions are expressed in terms of the 3 diquarks which couple in the same way as anti-quarks and are denoted in the obvious way:

\[
\begin{align*}
\mathcal{U} &\equiv (ds) \\
\mathcal{D} &\equiv (us)
\end{align*}
\]

The possible flavor wave functions with \( I = 1/2 \) and \( I_3 = +1/2 \) are given by

\[
\begin{align*}
\langle 10 \rangle_{\text{flavor}} &= \sqrt{\frac{1}{3}} \left( \langle S \bar{D} \bar{s} \rangle + \langle D \bar{S} \bar{s} \rangle + \langle S \bar{S} \bar{d} \rangle \right) \\
\langle 8, \text{SA} \rangle_{\text{flavor}} &= \sqrt{\frac{1}{6}} \left( \langle S \bar{D} \bar{s} \rangle + \langle D \bar{S} \bar{s} \rangle - 2 \langle S \bar{S} \bar{d} \rangle \right) \\
\langle 8, \text{AS} \rangle_{\text{flavor}} &= \sqrt{\frac{1}{2}} \left( \langle S \bar{D} \bar{s} \rangle - \langle D \bar{S} \bar{s} \rangle \right)
\end{align*}
\]

where we have distinguished the two possible ways of forming an octet: symmetric-antisymmetric (SA) where the diquarks are symmetric with each other and antisymmetric with the antiquark, and antisymmetric-symmetric (AS) where the diquarks are antisymmetric with each other and symmetric with the antiquark. The most general form for the full wave functions with nucleon quantum numbers are

\[
\begin{align*}
|10\rangle &= |10\rangle_{\text{flavor}} \otimes |10\rangle_{\text{space-spin}} \\
|8\rangle &= C_{\text{SA}} |8, \text{SA} \rangle_{\text{flavor}} \otimes |8 \rangle_{\text{space-spin}} + C_{\text{AS}} |8, \text{AS} \rangle_{\text{flavor}} \otimes |8 \rangle_{\text{space-spin}} \quad \text{with } |C_{\text{SA}}|^2 - |C_{\text{AS}}|^2 = 1.
\end{align*}
\]

The spin-space wave functions are labeled by the flavor configuration with which they are associated. Note that \(|10\rangle_{\text{space-spin}}\) and \(|8, \text{SA} \rangle_{\text{space-spin}}\) must have an antisymmetric relative spatial wave function between the two diquarks, presumably a p-wave. The reason for this is simply that the diquarks are bosons and their relative wave function must be fully symmetric. The color part is antisymmetric implying that the spin-flavor part is also anti-symmetric. For the decuplet and the SA octet, the flavor is anti-symmetric forcing the space to be anti-symmetric. The AS octet, however, has a symmetric spatial wave function between the diquarks which in turn implies that the anti-quark is in a p-wave relative to them.

It is immediately apparent that all the matrix elements of the number operator for strange quarks \( n_s \) are equal to the number operator for anti-strange quarks \( n\bar{s} \) when evaluated between states with nucleon quantum numbers. Ideal mixing implies the states are eigenstates of the strange quark number operators with eigenvalues of zero or unity. These states are to be constructed from linear combinations of the states in eq. (2.4). Consider the linear combination with eigenvalue zero:

\[
0 = n_s \left( \alpha |10\rangle + \beta |8\rangle \right)
\]

\[
= \left( |S \bar{D} \bar{s} \rangle + |D \bar{S} \bar{s} \rangle \right) \otimes \left( \alpha \sqrt{\frac{1}{3}} |10\rangle_{\text{space-spin}} + \beta C_{\text{SA}} \sqrt{\frac{1}{6}} |8, \text{SA} \rangle_{\text{space-spin}} \right)
\]

\[
+ \beta C_{\text{AS}} \left( |S \bar{D} \bar{s} \rangle - |D \bar{S} \bar{s} \rangle \right) \otimes |8, \text{AS} \rangle_{\text{space-spin}}
\]

The conditions in eqs. (2.7) and (2.8) imply that ideal mixing requires the space-spin wave functions in the two states be identical. This is a very strong condition on the underlying dynamics. These conditions will be of some significance in the phenomenological discussions in sect. (IV). Note that any substantial deviation from this condition such as a large mixing of the octet pentaquark

\[
\bar{S} \equiv (ud).
\]
component with ordinary three quark states will spoil ideal mixing.

It is easy to see that only way to construct such states from a superposition of the states in eq. (2.4) is:

\[ |n_s = 0\rangle = \sqrt{\frac{1}{3}} |\pi^0\rangle + \sqrt{\frac{2}{3}} |\pi^8\rangle \]
\[ |n_s = 1\rangle = \sqrt{\frac{2}{3}} |\pi^0\rangle - \sqrt{\frac{1}{3}} |\pi^8\rangle \]  

subject to the constraints in eqs. (2.7) and (2.8). The state with \( n_s = 0 \) is identified as the lower of the two \( N^* \) states while the \( n = 1 \) state is higher. The construction of the explicit states allows for the derivation of an inequality for the decay widths. This is derived in the following section.

III. AN INEQUALITY FOR DECAY WIDTHS

Approximate SU(3) flavor symmetry has played an essential role in our understanding of hadronic physics. Typically the symmetry works quite well and violations can be expected to be less than \( \sim 30\% \) for generic quantities. However, there are cases where the violations of SU(3) are characteristically much larger. These include: i) masses of pseudo-Goldstone bosons; ii) SU(3) multiplets with nearly degenerate masses as occurs in cases of ideal mixing; iii) decay widths near threshold (even when couplings respect SU(3) symmetry to good approximation, phase space factors near threshold vary rapidly with small changes in mass and can yield quite different widths). Fortunately, these cases are well understood. Apart from these well-established effects SU(3) works quite well phenomenologically.

The key to establishing an inequality for pentaquark decay widths in the Jaffe-Wilczek model are the assumptions that all SU(3) violating effects are the three enumerated above, and that the physical \( N^* \) pentaquark states are exact ideal mixtures of the octet and the anti-decuplet. Clearly, neither of these assumptions is strictly correct. Thus the inequality derived is approximate. Small violation of the inequality can arise either from small deviations from ideal mixing or from garden variety SU(3) violations. Large violations, however, indicate either large deviations from ideal mixing in contradiction to the Jaffe-Wilczek model or a new and previously unknown mechanism for large effects due to SU(3) breaking.

Since the latter is quite implausible, large violations of the inequality would strongly suggest that the model is untenable.

The only additional assumption about the dynamics which goes into the derivation is that the decay of an unstable particle can be cleanly treated via the coupling of the unstable particle to the continuum of open channels with no significant interference from background effects. In this circumstance, the partial width for the two-body decay of a \((1/2)^+\) baryon \((B')\) into another baryon \((B)\) plus a pseudo-scalar meson \((P)\) may be computed by combining a coupling constant parameterizing the strength with appropriate kinematic factors which incorporates both the phase space and the p-wave nature of the coupling of a pseudo-scalar meson to \((1/2)^+\) states.

\[
\Gamma_{B'\to BP} = |g_{PB'B}|^2 \kappa_{PB'B} \\
\kappa_{PB'B} = \frac{q}{4\pi M_B'} \left( \sqrt{q^2 + M_B'^2} - M_B \right) 
\]

where the kinematical factor \( \kappa_{PB'B} \) depends on the momentum of the decaying fragments in the rest frame of \( B' \):

\[
M_{B'} = \sqrt{q^2 + M_B'^2} \quad M_B = \sqrt{q^2 + M_B^2} 
\]

Here we relate the decays of three baryons, the \( \theta^+ \) (into a kaon plus a nucleon) and the two ideally mixed pentaquarks with nucleon quantum numbers (into a pion plus a nucleon). The ideally mixed states will be labeled \( N_0 \) and \( N_1 \) where the subscript indicates the number of strange quarks (which equals the number of anti-strange quarks in the state) so that with the tentative identification of Jaffe and Wilczek, the \( N_0 \) is the Roper resonance and the \( N_1 \) is the \( N^*(1710) \). Given the assumption that the only sources of SU(3) violation are due to phase space, ideal mixing, and the masses of pseudo-scalar mesons, it follows from eq. (2.10) that

\[
g_{\pi N_1 N} = -\sqrt{\frac{1}{3}} g_{\pi N_0 N} + \sqrt{\frac{2}{3}} g_{\pi N_{10} N} \\
g_{\pi N_0 N} = \sqrt{\frac{2}{3}} g_{\pi N_0 N} + \sqrt{\frac{1}{3}} g_{\pi N_{10} N} 
\]

where \( g_{\pi N_0 N} \) and \( g_{\pi N_{10} N} \) are the coupling constants for the unmixed pure SU(3) states with nucleon quantum numbers. Simple algebra yields the inequality:

\[
|g_{\pi N_0 N}|^2 - 2|g_{\pi N_1 N}|^2 + |g_{\pi N_{10} N}|^2 \leq 2\sqrt{2}|g_{\pi N_{10} N}| \sqrt{|g_{\pi N_0 N}|^2 + |g_{\pi N_1 N}|^2} - |g_{\pi N_{10} N}|^2, 
\]

where (3.4) is an inequality rather than equality due to unknown relative phases between the coupling constants. The important thing is that this inequality is of a form which relates the \( N_0 \) and \( N_1 \) couplings to the \( N_{10} \) coupling, which
by SU(3) symmetry, is related to the coupling constant for the decay of the \( \theta^+ \) into kaon plus nucleon:

\[
 g_{\pi N^{\theta^+}} = \frac{1}{2} g_{K^\theta^+ N}
\]

where the factor of 1/2 is an SU(3) isoscalar factor.\(^{12}\) Combining eqns. (3.1), (3.3) and (3.5) yields

\[
\left| \frac{\Gamma_{\pi N_0 N}}{\kappa_{\pi N_0 N}} - \frac{2\Gamma_{\pi N_1 N}}{\kappa_{\pi N_1 N}} + \frac{\Gamma_{K^\theta^+ N}}{4\kappa_{K^\theta^+ N}} \right| \leq \sqrt{\left( \frac{2\Gamma_{K^\theta^+ N}}{\kappa_{K^\theta^+ N}} \right)^2 + \frac{\Gamma_{\pi N_0 N}}{\kappa_{\pi N_0 N}} + \frac{\Gamma_{\pi N_1 N}}{\kappa_{\pi N_1 N}} - \frac{\Gamma_{K^\theta^+ N}}{4\kappa_{K^\theta^+ N}}},
\]

where the \( \kappa \) are the kinematic factors defined in eq. (3.6).

Inequality (3.6) is the principal result of this work. It constrains the possible widths of the various decays and depends only upon the assumptions outlined above.

**IV. PHENOMENOLOGICAL IMPLICATIONS**

Inequality (3.6) can be used to rule out the possibility that \( N_0 \) is the \( N^* (1440) \) (i.e., the Roper resonance) and \( N_1 \) is the \( N^* (1710) \). It can be seen by taking the masses and partial widths of these states from the particle data table\(^{20}\). As always, there is some ambiguity in extracting these from experiment. The quoted values for the mass of the \( N^* (1440) \) is in a range from 1430-1470 MeV with a best estimate of \( \approx 1440 \) MeV; the total width is estimated to be in the range from 250-450 MeV with a best estimate of \( \approx 350 \) MeV; and the branching fraction to \( \pi N \) is 60%-70%. The quoted values for the mass of the \( N^* (1440) \) is in a range from 1680-1740 MeV with a best estimate of \( \approx 1710 \) MeV; the total width is estimated to be in the range from 50-250 MeV with a best estimate of \( \approx 100 \) MeV; and the branching fraction to \( \pi N \) is 10%-20%.

As a first test, associate the Roper resonance with \( N_0 \), the \( N^* (1710) \) with \( N_1 \) and take the best estimate values for the masses and partial widths: \( M_{N_1} = 1440 \) MeV, \( \Gamma_{\pi N_1 N} = 227.5 \) MeV, \( M_{N_0} = 1710 \) MeV, \( \Gamma_{\pi N_0 N} = 15 \) MeV. Taking these values and using the direct experimental bound of 20 MeV for \( \Gamma_{K^\theta^+ N} \), one sees that the left-hand side of the inequality (3.6) is 71.8 while the right-hand side is 61.8. This violates the inequality, but only modestly; as noted above, modest violations can be understood as arising for ordinary SU(3) mixing or small deviations from ideal mixing. However, if the indirect bound on the width of the \( \theta^+ \) of 2 MeV from ref. \( \text{[2, 3]} \) is used as the \( \theta^+ \) width, the left-hand side of the inequality is 77.1 MeV while the right-hand side is 19.7. The inequality is violated by nearly a factor of 4. This is a gross violation and clearly indicates that something is wrong. Of course, the 2 MeV is a bound; the actual width could be smaller. If, for example, the width were 1 MeV, the violation would be even worse with the left-hand side a factor of nearly 6.

The violations of the inequality reported in the previous paragraph were based on best estimate values for
effectively counts the total strength under the $K^+ N$ resonance peak which is proportional to the width. However, in $K^+ D$ there is no observed structure in the vicinity of the $\theta^+$ allowing one to bound the width. A rather crude limit of 6 MeV for the width was reported based on a study of a rather limited set of data. More complete data reduces the bound to 1.5 MeV. This is consistent with the bound of 1-2 MeV found from a full re-analysis of the $K^+$ scattering data base. A similar analysis of the xenon experiment produces a bound of 1 MeV, although the nuclear structure uncertainties with Xe are clearly larger than with the deuteron. Together, these analyses lead to a fairly conservative estimate of 2 MeV as an upper bound for the width.

It is clear that if the $\theta^+$ width is as narrow as reported in refs. 1, 2, then the Jaffe-Wilczek model with the mixed states identified as the Roper resonance and the $N^*(1710)$ is simply not viable. One possibility, of course, is that the Jaffe-Wilczek model is wrong. The other is that the identification of the two ideally mixed states with the Roper resonance and the $N^*(1710)$ is not correct. Let us explore this second possibility in some detail.

There are three scenarios to consider: i) the $N^*(1710)$ state is, in fact, a nearly ideally mixed pentaquark and it has a previously undiscovered narrow partner with a mass of approximately 1450 MeV; ii) the Roper resonance is a nearly ideally mixed pentaquark and it has a previously undiscovered broad partner with a mass of approximately 1700 MeV; iii) neither the lower nor the upper $N^*$ states have been detected.

First consider scenario i). If one takes the $N^*(1710)$ as the $N_1$ and assumes that the there exists a previously undiscovered $N^*$ state around 1450 MeV, then inequality (3.6) can be used to get at least crudely bound its width. Taking the best estimate values for the $N^*(1710)$, a $\theta^+$ width of 2 MeV, and assuming the inequality is not violated gives an approximate upper bound for the partial width of this undetected state into pion plus nucleon of 245 MeV. The interest-

There is another theoretical issue which makes scenario ii) rather problematic, namely, the existence of two widely different width scales in the problem, a very wide octet pentaquark, and a very narrow anti-decuplet. Note that this same issue was confronted in ref. 16 where an attempt was made to reconcile the narrow $\theta^+$ with the wide Roper resonance. Moreover, with the recent determination that the $\theta^+$ width is far smaller than the experimental bound, the problem has become even more acute. Jaffe and Wilczek argued in ref. 10 that the internal and flavor structure in the Roper resonance were quite different from that of the $\theta^+$ and that this difference could account for the differing widths. In fact, this is not completely correct. As shown in sec. 11 ideal mixing can occur only if the pentaquark wave function for the octet and the anti-decuplet are identical except for their flavor structure; they must have the same internal wave functions in terms of spin and space. At least naively one would think it unlikely that the octet would have a very large coupling to the continuum while an octet state of essentially the same internal structure has a very small coupling.

It is not clear how seriously one ought to regard this problem. The key issue is whether the problem is serious enough to rule out scenario ii). Glozman has taken the position that the disparate widths for the $\theta^+$ and the Roper resonance rule out the possibility that the Roper resonance is a pentaquark. This position is probably too strong. It certainly does seem perverse to suggest that both the widest and the narrowest baryon resonances known in this region of the baryon spectrum are states with virtually identical spin-space wave functions. However, there appears to be no mathematically rigorous argument which rules out this possibility. Nevertheless the widely differing widths make this scenario at the very least a somewhat unattractive possibility.
The third scenario is that neither of the two states has been seen heretofore. If this is true then both states are very narrow. The inequality requires either both states be wide or both be narrow so the possibility of two broad states near 1450 MeV (the Roper resonance and a new state) can be ruled out. In some ways this scenario is unattractive since it depends on the existence of two states at low energies which have evaded detection until now. However, this possibility cannot be totally disregarded because the N*(1710) is not completely well established (it is listed as a three-star resonance in the particle data book, and not seen at all in a recent re-analysis of the pion-nucleon scattering data [23] while possible candidates for narrow resonances were seen at 1680 MeV and 1730 MeV [21]).

It is worth noting that these conclusions hold rather more generally than for the Jaffe-Wilczek model. Any model which predicts nearly ideal mixing and positive parity pentaquarks will face the same constraints as the Jaffe-Wilczek model. All such models will similarly face the problem that the physical states in the theory with nucleon quantum number cannot be identified with the nucleon states as the Roper resonance and the N*(1440) and N*(1710). Thus, any such model will require the existence of at least one presently unknown nucleon resonance in a region which has been well explored in the past.

To summarize: the extreme narrowness of the θ+ as extracted from the absence of a detectable resonance kaon-deuteron scattering, strongly constrains the phenomenology of the Jaffe-Wilczek model. Given this narrowness, the identification of the nearly ideally mixed nucleon states as the Roper resonance and the N*(1710) is not tenable. To be viable the model requires that either both ideally mixed states be narrow or both wide, and there is at least a theoretical prejudice that the scenario where they are both narrow is preferred. A re-analysis of pion-nucleon scattering in the region around 1450 MeV for evidence of a previously missed narrow resonance may shed light on the situation.

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