Relation between the x-dependence of Higher Twist Contribution to $F_3$ and $g_1^n - g_1^p$ in the Light of the Recent Experimental Data

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We compare the recent results on the higher twist (HT) contribution to the nonsinglet combination $g_1^n - g_1^p$ of the polarized proton and neutron structure functions with that one to the unpolarized structure function $F_3$ using the assumption that the HT contributions to the Gross-Llewellyn Smith and the Bjorken sum rules are similar. We have found, that the relation $\frac{g_1^n}{g_1^p}(x) \approx \frac{1}{2} \frac{h_{2T}}{h_1}$ is valid for $x \geq 0.2$ in the case of NLO QCD approximation for the leading term parts of the structure functions.

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Presently the structure functions in deep inelastic lepton nucleon scattering are a subject of intensive experimental and theoretical investigations. While the leading twist (LT) part of the structure functions related with the parton distributions and their $Q^2$-evolution is studied in detail in pQCD, the higher twist corrections ($\sim 1/Q^2$) are of a big interest and an intensive study in the last years. From the very beginning the x-dependence of the HT contribution was determined from analyses of the data on the unpolarized structure functions $F_2$ \cite{1, 2, 3}, $F_3$ \cite{4}, $F_1$ \cite{5} and $F_L$ \cite{6, 7}.

The most of the precise experimental data on polarized structure functions (JLAB, HERMES, SLAC) are in the region of $Q^2 \sim 1$ GeV$^2$. While in the determination of the parton densities (PD) in the unpolarized case we can cut the low $Q^2$ and $W^2$ data in order to eliminate the less known non-perturbative HT effects, it is impossible to perform such a procedure for the present data on the spin-dependent structure functions without loosing too much information. That is why the higher twist effects are especially important in the case of polarized structure functions and should be taken into account in the QCD analysis. Since the first results on HT contribution to the $g_1$ structure function \cite{8} the accuracy of the x-dependence of the HT in $g_1^n$ and $g_1^p$ extracted directly from the data has been considerably improved \cite{8}.

Are there any relations between the HT contributions to the different structure functions? We continue to discuss this question based on the new results of the paper \cite{8} on the x-dependence of the HT in $g_1$. It was obtained using the recent very precise CLAS \cite{9} and COMPASS \cite{10} inclusive polarized DIS data. In this note we continue our study \cite{12} on the relation between the HT contribution to the unpolarized structure function $F_3$ and $g_1^n - g_1^p$ which are pure non-singlets.

As it was shown in the paper \cite{13} the $Q^2$-evolutions of the $F_3$ and the nonsinglet part of the $g_1$ structure functions are identical up to NLO. Moreover, the x-shapes of the $F_3$ and nonsinglet part of $g_1$ are also similar. By analogy, one could suppose that the HT contributions to $F_3$ and $g_1^n - g_1^p$ are similar too. Such an assumption was used for the first moments of the HT corrections in the Gross-Llewellyn Smith and Bjorken sum rules in the infrared renormalons approach \cite{14}:

\begin{align}
GLS(Q^2) &= \int_0^1 dx F_3(x, Q^2) = 3(\text{GLS}_{pQCD} - \frac{\langle\langle O_1\rangle\rangle}{Q^2}) \\
Bjp(Q^2) &= \int_0^1 dx [g_1^n(x, Q^2) - g_1^p(x, Q^2)] = \frac{g_A}{6} (Bjp_{pQCD} - \frac{\langle\langle O_2\rangle\rangle}{Q^2})
\end{align}

where

\begin{equation}
\langle\langle O_1\rangle\rangle \approx \langle\langle O_2\rangle\rangle \tag{3}
\end{equation}

Here $GLS_{pQCD}$ and $Bjp_{pQCD}$ are the leading twist contribution to corresponding sum rules:

\begin{align}
GLS_{LO} = Bjp_{LO} &= 1 \tag{4} \\
GLS_{NLO} = Bjp_{NLO} &= 1 - \frac{\alpha_S(Q^2)}{\pi} \tag{5}
\end{align}

In this note we are going to verify if the relation (3) between the lowest moments of the HT contribution can...
be generalized for the higher twists themselves, namely:

\[ \frac{1}{3x} h^{xF_3}(x) \approx \frac{6}{g_A} h^{g^p,g^n}(x) \]  

(6)

To test this relation we will use the values of HT obtained in the QCD analysis of the corresponding structure functions in model independent way. In the QCD analysis of DIS data when the higher twist corrections are taken into account, the structure functions are given by:

\[ x F_3(x,Q^2) = x F_3(x,Q^2)_{LT} + h^{xF_3}(x)/Q^2 \]
\[ g_1^{p(n)}(x,Q^2) = g_1^{p(n)}(x,Q^2)_{LT} + h^{g_1^{p(n)}}(x)/Q^2 \]  

(7)

In Fig. 1, \( h^{xF_3}(x) \), \( h^{g_1^{p}}(x) \) and \( h^{g_1^{n}}(x) \) are the dynamical higher twists corrections to \( x F_3 \), \( g_1^{p} \) and \( g_1^{n} \), which are related to multi-parton correlations in the nucleon. They are non-perturbative effects and can not be calculated without using models. The target mass corrections, which are also corrections of inverse powers of \( Q^2 \), are calculable \([15, 16]\) and effectively belong to the leading twist term. A model independent determination of \( h^{xF_3}(x) \) was done in \([17]\) on the basis of the analysis of CCFR-NuTev (anti-)neutrino deep–inelastic scattering data \([18]\) at \( Q^2 > 5 \text{ GeV}^2 \). The values of \( h^{g_1^{p}}(x) \) and \( h^{g_1^{n}}(x) \) in NLO(MS) are given in \([19]\), where the results of the analysis of the world data on polarized structure function \( g_1 \) at \( Q^2 > 1 \text{ GeV}^2 \), are presented including the precise CLAS \([19]\) and COMPASS \([20]\) \( g_1/F_1 \) data. This analysis provides more precise and detailed results on \( h^{g_1^{p}}(x) \) and \( h^{g_1^{n}}(x) \). In particular, the x-range is split into 7 bins instead of 5, as used in the previous analyses \([7, 11]\). Using these new results and taking into account the coefficients in \([1] \) and \([2]\) one could construct the l.h.s. and r.h.s. of Eq. \([4]\).

In Fig. 1 we present the HT contributions to \( F_3 \) structure functions and to the nonsinglet combination \( g_1^n - g_1^p \). One can see the large difference of the scales for HT contribution to polarized (small central values) and unpolarized (large central values) structure functions.

In Fig. 2 the coefficients in \([1] \) and \([2]\) are taking into account. As seen from Fig. 2 the equality \([5]\) is approximately valid for \( x \geq 0.2 \). It means that the higher Mellin moments of the both parts of equation \([4]\) should approximately coincide:

\[ \int_0^1 dx \ x^N \frac{1}{3x} h^{xF_3}(x) \approx \int_0^1 dx \ x^N \frac{6}{g_A} h^{g_1^n - g_1^p}(x), \]

\[ N - \text{large}. \]  

(8)

In order to show the similarity of the functions \( \frac{1}{3x} h^{xF_3}(x) \) and \( \frac{6}{g_A} h^{g_1^n - g_1^p}(x) \), we have parametrised them in the region \( x > 0.2 \) by linear function: \( A + Bx \). The results of parametrisation are in a good agreement for the both constants, A and B \([9, 10]\).
We would like to mention, that equality (3) is suggested in the framework of the infrared renormalon approach, so the violation of equality (4), which is shown in Fig. 2 at $x < 0.2$, could be due to the contribution of the dynamical higher twists connected with the non-perturbative structure of the nucleon in this $x$ region.

Finally, we would like to note, that there are additional sources of uncertainties which should be taken into account in a more detailed test of Eq. (6): the contribution of $O(1/Q^4)$; the separation of the twist-3 contribution in the polarized case, which is effectively included in $h^g(x)$; the $Q^2$ dependence of the functions $h(x)$, etc.

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