Brane Cosmology With Generalized Chaplygin Gas in The Bulk

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Abstract

We find exact solution of the Einstein equations in the context of the brane world scenario. We have supposed a generalized chaplygin gas equation of state for bulk. This study display a constant energy density and pressure for bulk in late time. It is shown that our assumptions impose a specific equation of state on brane. In this work, we have obtained a decelerate universe in early time and late time. In the end, it is shown that under some assumption we have equation of state of cosmological constant for bulk.

Keywords: Brane Cosmology; generalized Chaplygin gas;

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1 Introduction

There is no experimental evidence that support our universe might possess any spacetime dimensions beyond the four dimensions, it is a rather remarkable fact that we actually know of no compelling reason as to the number of spacetime dimension should be four. Consequently, it is legitimated to consider the possible existence of additional dimensions beyond four. In fact attempts at a higher-dimensional unification were initiated a long time ago. Investigation on the possibility of extra dimensions began with Kaluza-Klein type theory (see e.g. [1]). The Randall-Sundrum model [2], [3] suggests the universe is identified with a four dimensional hypersurface (a three-brane) in a five dimensional bulk with negative cosmological constant (AdS space). In their first model, our universe is assumed to be negative tension brane; however, Shiromizu et al. [4] have shown that, in this model, the gravitation is repulsive. In their second model they put a single positive tension and suggest the extra dimension need not be compact. It has been shown that [5], generically, the equations governing the cosmological evolution of the brane will be different from standard cosmology. The important difference is related to the quadratical dependence of new Friedmann equation to the brane energy density. For law energy densities on the brane, and a pure AdS, we can recover the standard Freidmann-Robertson-Walker cosmology (for review, see [6], [7], [8]). The brane evolution can be discussed either in two system of coordinates. The first coordinate system, namely a GN coordinate, is very useful for a "brane-based" point of view [5], [9]. the second coordinate system is known as the five dimensional Schwarzschild-Anti de Sitter metric, which relies on a "bulk-based" point of view [10], [11].

Although in string theory the bulk field should correspond in some way to the string theory fields, from a phenomenological view point, matter in the bulk can be anything [12]. In this work, we have tried to solve Einstein equation in the brane world scenario for a perfect fluid, that is called generalized chaplygin gas[13]. In the standard cosmology, generalized chaplygin gas model describe a transition from a universe with dust-like matter to an accelerated expanding stage. So, in this paper, we have investigated the evolution of a homogeneous isotropic brane world that embedded in the bulk filled with generalized chaplygin gas.

The plan of this paper is the following. Section 2 is preliminary, so we present some basic equations that are useful for our investigation. In section 3, we impose a generalized chaplygin gas model on bulk, and we have obtained infinite bulk energy density in early time an a constant value for bulk energy density in late time. Also, the assumption imposed a specific equa-
tion of state on brane. Our investigation showed that we have a decelerate universe in early time and deceleration universe in late time. In section 4, it is assumed that the energy density of bulk depends only on time, and the coefficient of fifth coordinate in metric, be independence on time. It is shown that, under this assumption, the cosmological constant equation of state is only equation of state which can describe bulk.

2 Preliminary

In this section, we introduce some basic equation for our work. Suppose that five-dimensional spacetime metric is characterized as:

\[ ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\delta_{ij}dx^i dx^j + b^2(t,y)dy^2, \]  

(1)

where \( \delta_{ij} \) is a maximally symmetric 3-dimensional metric (\( k = -1,0,1 \) will parameterize the spatial curvature), and \( y \) is the fifth coordinate. The 5-D Einstein equations are as usual form:

\[ G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}R_{\alpha\beta} = \kappa^2 T_{\alpha\beta}, \]  

(2)

(note that \( \alpha,\beta = 0,1,2,3,5 \)), where \( R_{\alpha\beta} \) is 5-D Ricci tensor and \( R \) is scalar curvature and \( \kappa \) is related to five-dimensional Newton’s constant, \( G^{(5)} \). \( T_{\alpha\beta} \) is the total energy-momentum tensor. We distinguish it into two kind of source:

i) the energy-momentum of bulk:

\[ T^{\alpha\beta}|_B = \text{diag}(\rho_B, P_B, P_B, P_B, P_5), \]  

(3)

where the energy density \( \rho_B \) and pressure \( P_B \) and \( P_5 \) are independent of fifth coordinate.

ii) the energy-momentum of brane. We suppose only homogeneous and isotropic geometry inside the brane,

\[ T^{\alpha\beta}|_{br} = \frac{\delta(y)}{b} \text{diag}(-\rho_b, p_b, p_b, p_b, 0), \]  

(4)

where energy density \( \rho_b \) and pressure \( p_b \) are independent of the position inside the brane. In here, we clearly see that, by this assumption, \( T_{05} = 0 \); it means that there is no flow of energy along the fifth dimension. Using Eq.(1) and(2), we get
\[ G_{00} = 3 \left\{ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{n^2}{b^2} \left( \frac{a''}{a} + \frac{a'}{a} \left( \frac{a'}{a} - \frac{b'}{b} \right) \right) + k \frac{n^2}{a^2} \right\}, \]  

(5)

\[ G_{ij} = \frac{a^2}{b^2} \gamma_{ij} \left\{ \frac{a'}{a} \left( \frac{a'}{a} + 2 \frac{n'}{n} \right) - \frac{b'}{b} \left( \frac{n'}{n} + 2 \frac{a'}{a} \right) + 2 \frac{a''}{a} + \frac{n''}{n} \right\} \]

\[ + \frac{a^2}{n^2} \gamma_{ij} \left\{ \frac{\dot{a}}{a} \left( - \frac{\dot{a}}{a} + 2 \frac{\dot{n}}{n} \right) - 2 \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \left( - 2 \frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) - \frac{\dot{b}}{b} \right\} - k \gamma_{ij}, \]  

(6)

\[ G_{05} = 3 \left( \frac{n' \dot{a}}{n a} + \frac{a' \dot{b}}{a b} - \frac{\dot{a'}}{a} \right), \]  

(7)

\[ G_{55} = 3 \left\{ \frac{a'}{a} \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{b^2}{n^2} \left( \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) \right) - k \frac{b^2}{a^2} \right\}, \]  

(8)

where \( \dot{\chi} = \frac{d \chi}{dt} \) and \( \chi' = \frac{d \chi}{d y} \). For having a well define geometry, the metric must be continuous across the brane, but its derivative with respect to fifth coordinate can be discontinues on the brane. This will entail the presence of Dirac delta function in the second derivative metric with respect to fifth coordinate. So according to [5], one can obtain:

\[ \frac{[a']}{b_0 a_0} = - \frac{\kappa^2}{3} \rho_b, \]  

(9)

\[ \frac{[n']}{b_0 n_0} = \frac{\kappa^2}{3} (3p_b + 2\rho_b), \]  

(10)

the subscript 0 implies their value on the brane. By taking the jump of (7) and using Eqs.(9) and (10), we derive the conservation equation on the brane as follow:

\[ \dot{\rho}_b + 3 \frac{\dot{a}_0}{a_0} (\rho_b + p_b) = 0. \]  

(11)

By imposing \( Z_2 \)-symmetry on the average value of (8), one can arrive at [5]

\[ \frac{\ddot{a}}{a_0} + \frac{\ddot{a}_0}{a_0} = - \frac{\kappa^4}{36} \rho_b (\rho_b + 3p_b) - \frac{\kappa^2}{3b_0^2} P_5 - k \frac{\dot{a}_0}{a_0}. \]  

(12)

There is chosen \( n_0 = 1 \), this is possible by suitable time transformation. Eq.(12) has important difference with standard cosmology, there is quadratic energy density of the brane. This term can be very important in early
universe. One can rewrite 5-D field equations (5) and (8) in the compact form as [5]:

$$\psi' = -\frac{2}{3}a'a^3\kappa^2\rho_B,$$  (13)

$$\dot{\psi} = \frac{2}{3}\dot{a}a^3\kappa^2 P_5,$$  (14)

where $\psi$ is a function of time and fifth coordinate,

$$\psi(t, y) \equiv \frac{(a'a)^2}{b^2} - \frac{\dot{a}a^2}{n^2} - ka^2.$$  (15)

Now, we equate the time derivative of (13) with the fifth coordinate derivative of (14), to get

$$a'a\dot{\rho}_B + \dot{a}P'_5 + (\rho_B + P_5)\left( \dot{a'} + 3\frac{\dot{a}a'}{a} \right) = 0,$$  (16)

also, the constraint $\nabla_\alpha G^{\alpha 0} = 0$ gives

$$\dot{\rho}_B + 3\frac{\dot{a}}{a}(\rho_B + P_B) + \frac{\dot{b}}{b}(\rho_B + P_5) = 0.$$  (17)

Eqs. (16) and (17) are the conservation relation on the bulk.

By imposing the $Z_2$-symmetry and junction condition (9) and the result of average value of (15), one can obtain the generalized Friedmann equation on the brane as follow:

$$\frac{\dot{a}_0^2}{a_0^4} = \frac{\kappa^4}{36\rho_B^2} - \frac{\psi_0(t)}{a_0^3} - \frac{k}{a_0^5}.$$  (18)

3 Generalized Chaplygin Gas Model For Bulk

In this work we choose a generalized chaplygin gas for bulk, with the equation of state as

$$P_B = P_5 = A\rho_B - \frac{B}{\dot{\rho}_B},$$  (19)

where $A$ and $B$ are positive constant, and $0 < \alpha \leq 1$. By substituting this equation of state in the conservation equation (17), one can obtain

$$\rho_B = \left\{ \frac{1}{1 + A} \left( \frac{\rho_{B1}}{\dot{b}a^3} \right)^{(1+\alpha)(1+\alpha)} + B \right\}^{\frac{1}{1+\alpha}},$$  (20)
where $\rho_{B1}$ is an integration constant which can be depends on $y$. According to the equation of state

$$P_B = P_0 = A \left\{ \frac{1}{1 + A} \left( \frac{(\rho_{B1})^{(1+A)(1+\alpha)}}{ba^3} + B \right) \right\} \frac{1}{1+\alpha} - \frac{B}{\left\{ \frac{1}{1 + A} \left( \frac{(\rho_{B1})^{(1+A)(1+\alpha)}}{ba^3} + B \right) \right\} \frac{\alpha}{1+\alpha}}. \quad (21)$$

Since, we have assume that $\rho_B$ is independence on $y$, so $\frac{(\rho_{B1})^{(1+A)(1+\alpha)}}{ba^3}$ must be independence on $y$. With the help of bulk conservation equations (16) and (17), we arrive at this result that

$$\frac{\dot{b}}{b} = \frac{\dot{a}'}{a'}. \quad (22)$$

One can integrate of (22) and find out a relation between $b(t, y)$ and $a(t, y)$ as follow

$$b(t, y) = \beta(y)a'(t, y), \quad (23)$$

where $\beta(y)$ is a integration constant which can be dependence on $y$. Also, by using (22) in (7), one can realize that $n(t, y)$ is independence on $y$; namely

$$n'(t, y) = 0. \quad (24)$$

So, from the junction condition (10), we obtain an equation of state for brane as

$$p_b = -\frac{2}{3} \rho_b. \quad (25)$$

It is clearly seen that our assumptions impose a specific equation of state on brane. From the brane conservation equation (11), it can be found out a relation between brane energy density, $\rho_b$, and brane scale factor, $a_0(t)$, as follow

$$\rho_b = \frac{\rho_0}{a_0(t)}, \quad (26)$$

where $\rho_0$ is an integration constant. From another junction condition (9) and also with the help of $\mathbb{Z}_2$-symmetry we can have a specific value for $\beta(y)$ in the $y = 0$

$$\beta^{-1}(0) = -\frac{\kappa^2}{6} \rho_0. \quad (27)$$
In the continuance of this work we suppose that, it is possible to write \( a(t,y) \) as a separate function of time and fifth coordinate; namely

\[
a(t,y) = f(y)a_0(t),
\]

where \( f(y) \) is an arbitrary function of fifth coordinate, and \( a_0(t) \) is our brane scale factor, also it is only a function of time. Because of \( a(t,y = 0) \) is brane scale factor, then as \( y \to 0, f(y) \) must tends to 1. From the (23), \( b(t,y) \) can be written as a separate function as well,

\[
b(t,y) = \beta(y)f'(y)a_0(t) = g(y)b_0(t),
\]

for having nonzero value of \( b(t,y = 0) \), \( f'(0) \) must not be vanished. According to the (29), it is seen that, the time behavior of \( b(t,y) \) is like to the \( a(t,y) \). In here, we can predict a function for \( \rho_{B1} \) to give \( \rho_B \) in which be independence on \( y \), as

\[
\rho_{B1} = c\beta(y)f'(y)f^3(y),
\]

where \( c \) is a coupling constant. Now, we rearrange the energy density and pressure of bulk:

\[
\rho_B = \left\{ \frac{1}{1 + A} \left( \frac{c}{a_0^{4(1+A)(1+\alpha)}} + B \right) \right\}^{\frac{1}{1+\alpha}},
\]

and

\[
P_B = P_5 = A\left\{ \frac{1}{1 + A} \left( \frac{c}{a_0^{4(1+A)(1+\alpha)}} + B \right) \right\}^{\frac{1}{1+\alpha}} - \frac{1}{B} \left\{ \frac{1}{1 + A} \left( \frac{c}{a_0^{4(1+A)(1+\alpha)}} + B \right) \right\}^{\frac{1}{1+\alpha}}.
\]

By using above assumption we arrive at this result that, for every value of \( y \) we have

\[
\frac{\dot{a}(t,y)}{a(t,y)} = \frac{\dot{b}(t,y)}{b(t,y)} = \frac{\dot{a}_0(t)}{a_0(t)}.
\]

For investigation the evolution of universe, at first we need to recognized \( \psi(t,y) \). From (13), because \( \rho_B \) is independence on \( y \), we have

\[
\psi(t,y) = -\frac{\kappa^2}{6}\rho_B a^4 + \zeta(t),
\]
where $\zeta$ is a constant of integration which depends on time. With equating the time derivative of (34) to (14), and with the help of (33), we realize that $\dot{\zeta}(t) = 0$, so $\zeta$ is a constant value. Now, from (34), (12) and (18) one can rewrite the evolution equations of brane as:

$$
\dot{a}_0^2 = \frac{\kappa^4}{36} \rho_0^2 + \frac{\kappa^2}{6} \rho_B a_0^2 - \frac{\zeta}{a_0^2} - k, \quad (35)
$$

and

$$
\ddot{a}_0 = -\frac{\kappa^2}{3\beta^2(0) f^2(0)} \frac{P_5}{a_0(t)} - \frac{\kappa^2}{6} \rho_B a_0 + \frac{\zeta}{a_0^3}. \quad (36)
$$

According to these two evolution equations, in early time when $a_0 \rightarrow 0$, $\rho_B$ tends to infinite; also $\dot{a}_0^2 \rightarrow +\infty$ and $\ddot{a}_0 \rightarrow -\infty$. Then we have a decelerate universe in early time. In late time as $a_0$ tends to infinite, $\rho_B \rightarrow \left(\frac{P_5}{1 + \alpha}\right)^{\frac{1}{1+\alpha}}$. It means we arrive at a constant value for bulk energy density and bulk pressure in late time. In this era $\dot{a}_0^2 \rightarrow +\infty$ and $\ddot{a}_0 \rightarrow -\infty$. So we have a decelerate universe in late time.

4 Equivalence With The Cosmological Constant

In this section we show that, with the help of some assumption which can be seen in some works (e.g. [5]), one can arrive at equation of state of cosmological constant for bulk. We suppose that the energy density and pressure of bulk which are specified by $\rho_B$ and $P_B = P_5$ respectively, depend only on time, and also the coefficient of fifth coordinate in metric; namely $b^2(t, y)$, be independence on time. With the help of conservation relations of bulk (16), (17) and above assumption one can write

$$
\dot{\rho}_B + (\rho_B + P_5) \left(\frac{\dot{a}'}{a'} + 3\frac{\ddot{a}}{a}\right) = 0, \quad (37)
$$

and

$$
\dot{\rho}_B + 3\frac{\ddot{a}}{a} (\rho_B + P_B) = 0. \quad (38)
$$

By using (38) in (37), we have

$$
(\rho_B + P_5) \frac{\dot{a}'}{a'} = 0. \quad (39)
$$

This relation is satisfied in every time. Here, we have two situation:
• If \((\rho_B + P_5) = 0\).
then we arrive at this result that
\[ P_5 = P_B = -\rho_B. \] (40)
This is the equation of state of cosmological constant. By substituting this relation in (38) we realize that, \(\rho_b\) is independence on time as well, and we get a constant value for bulk energy density.

• If \(\dot{a}' = 0\).
In this situation with the help of (7), we realized that \(n' = 0\), so from junction condition (10), we get a specific equation of state for brane; namely
\[ p_b = -\frac{2}{3}\rho_b. \] (41)
Whereas, \(\dot{a}' = 0\) therefore
\[ a(t, y) = X(t) + Y(y), \] (42)
this form is an unusual form for coefficient of metric. From conservation equation (38), for generalized chaplygin gas equation of state, we have
\[ \rho_B = \left\{ \frac{1}{1 + A} \left( \frac{\rho_1}{a} \right)^{3(1+A)(1+\alpha)} + B \right\}^{1/(1+\alpha)}, \] (43)
where \(\rho_1\) is a constant of integration which can be depended on \(y\). Since, \(\rho_B\) is independence on \(y\), then \(\left(\frac{\rho_1}{a}\right)\) must be independence on \(y\); namely
\[ \frac{d}{dy} \left( \frac{\rho_1(y)}{a(t, y)} \right) = 0. \] (44)
From this relation, we should find out a function for \(\rho_1\). With the help of (44), we arrive at
\[ \frac{\rho_1'}{\rho_1} = \frac{Y'}{Y + X}. \]
By some working, we find out that, \(\rho_1 = Y(y)\). Now, derivative of \(\left(\frac{\rho_1}{a}\right)\) with respect to \(y\) shows that \(Y' = 0\). From (9), one can realize that, it means \(\rho_b = 0\), while this result is incompatible with the experiment. So, this case is not suitable.
5 Conclusion

Finally, we see that in early time the bulk energy density and bulk pressure are infinite; however, in late time they arrive at a constant value. Also, it was shown that, our assumption imposed a specific equation of state on the brane, and we can not give any other equation of state to brane. We have shown that in this model we have a decelerate universe in early time and late time. Also, it was seen that, when we supposed that, as well as previous assumption, the coefficient of fifth coordinate in metric, namely $b(t,y)$, be independence on time, there is equation of state of cosmological constant on the bulk. So, we have a constant value for bulk energy density.

References

[1] D. Bailin, A. Love, Rep. Prog. Phys. 50 (1987), 1087.
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, (1999), 3370.
[3] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, (1999), 4690.
[4] T. Shiromizu, K. Maeda and M. Sasaki, Phys. Rev. D 62, (2000), 024012.
[5] P. Binetruy, C. Deffayet and D. Langlois, Nucl. Phys. B 565, (2000), 269;
   P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B 477, (2000), 285.
[6] R. Maartens, Living Rev. Rel. 7, (2004), 7.
[7] E. Papantonopoulos, Lect. Notes Phys. 592, (2002), 458.
[8] P. Brax, C. van de Bruck, Class. Quantum Gravity 20, (2003), R201-R232.
[9] C. Csaki, M. Graesser, C. F. Kolda and J. Terning, Phys. Lett. B 462, (1999) 34;
   M. R. Setare, Phys. Lett. B 642, (2006), 421;
   M. R. Setare and E. N. Saridakis, JCAP 0903, (2009), 002;
   J. M. Cline, C. Grojean and G. Servant, Phys. Rev. Lett. 83, (1999), 4245.
[10] P. Kraus, JHEP 9912, (1999), 011.
[11] D. Ida, JHEP 0009, (2000), 014.

[12] R. Maartens (private communication); A. Mazumdar, R. N. Mohapatra, and A. Perez-Lorenzana, JCAP 0406, (2004), 004; C. Bogdanos and K. Tamvakis, Phys. Lett. B 646, (2007), 39.

[13] M. R. Setare, Phys. Lett. B 654, (2007), 1-6; M. R. Setare, Phys. Lett. B 648, (2007), 329.