Research Article

Efficient Hierarchical Authentication Protocol for Multiserver Architecture

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Received 23 August 2019; Revised 14 November 2019; Accepted 22 January 2020; Published 23 March 2020

Academic Editor: José Maria de Fuentes

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The multiserver architecture authentication (MSAA) protocol plays a significant role in achieving secure communications between devices. In recent years, researchers proposed many new MSAA protocols to gain more functionality and security. However, in the existing studies, registered users can access to all registered service providers in the system without any limitation. To ensure that the system can restrict users that are at different levels and can access to different levels of service providers, we propose a new lightweight hierarchical authentication protocol for multiserver architecture using a Merkle tree to verify user’s authentication right. The proposed protocol has hierarchical authentication functionality, high security, and reasonable computation and communication costs. Moreover, the security analysis demonstrates that the proposed protocol satisfies the security requirements in practical applications, and the proposed protocol is provably secure in the general security model.

1. Introduction

Rapid advances in wireless communication technologies bring convenience to our lives. With an increasing number of users and services, the single-server architecture authentication protocols can no longer meet people’s various requirements [1]. Multiserver architecture authentication (MSAA) protocols have emerged and been widely used in the Internet of things, wireless sensor networks, smart grid, cloud computing, and mobile payment. Because MSAA protocols have better properties than single-server architecture authentication protocols [2, 3], it becomes a hot spot in current research.

However, due to the openness of the multiserver environment, an adversary can easily control communication channels and carries out many types of attacks such as intercept, modify, replay, and delay messages between multiple parties. For defending these attacks, researchers proposed many authentication protocols for the multiserver architecture that are using cryptographic methods to secure communication between different parties. New protocols also have lower computation and communication costs than the previous protocols. Currently, MSAA protocols can be divided into two types by whether it involves the registration center (RC) at the authentication phase. The first type is MSAA protocols with the RC involving at the authentication phase (MSAA1) [1, 2, 4–11], and the second type is MSAA protocols without RC involving at the authentication phase (MSAA2) [12–23]. In MSAA1 protocols, RC verifies every mutual authentication process, which makes it a bottleneck in MSAA1 protocols. The communication cost in MSAA1 protocols is significantly increased compared to MSAA2 protocols. To address these drawbacks, researchers proposed many MSAA2 protocols which have more efficiency and security than the existing MSAA1 protocols [14].

Currently, hierarchical authentication functionality is missing in the existing MSAA2 protocols. When a user registered at RC, he/she can authenticate with all registered service providers in this system, and the user’s level is different from each other, and low-level users should not successfully authenticate with high-level service providers and access to their services. Besides, there should be some high-level service providers only providing service for some particular users such as VIP users. In general, the MSAA protocols with the hierarchical authentication functionality will have flexibility in managing
user authentication rights and access capabilities. The hierarchical authentication functionality has been achieved in the MSAA1 protocol [2, 4]. In the MSAA1 protocol, an RC is required at the authentication phase. Therefore, RC can verify the user’s authentication rights to determine whether he/she can access to service providers that are at a particular level. However, MSAA1 protocols have several drawbacks such as unreasonable communication cost that we showed earlier, making the whole system inefficient. Suppose we apply the existing MSAA2 protocols to the above environment; there should be multiple RCs to manage users and service providers at different levels. Users and service providers need to store various certificates from different RCs. The missing of hierarchical authentication functionality in the existing MSAA2 protocols motivates us to design a new lightweight hierarchical authentication protocol for multiserver architecture.

The proposed protocol uses a self-constructed Merkle tree to achieve hierarchical authentication functionality. In the proposed protocol, a session key is established between service providers and users without involving RC; this significantly reduces communication cost and makes the authentication process faster. The proposed protocol can meet the security requirements of the multiserver architecture and is provably secure in general security model.

The remainder of this paper is organized as follows. Section 2 discusses the related work. Section 3 describes preliminaries. In Section 4, we show the details of the proposed protocol. Section 5 gives out the formal security proof of the proposed protocol. Section 6 presents a comparison of the proposed protocol with other related protocols on security, computation, and communication costs. Section 7 concludes the paper.

2. Related Work

Li et al. [1] proposed a new multiserver architecture authentication (MSAA) protocol for cloud computing based on the identity-based model. Shao and Chin [4] proposed an authentication protocol for multiserver architecture but failed to resist the server spoofing and the impersonation attacks. He and Wang [5] constructed the first genuinely three-factor authentication protocol for the multiserver architecture, but their protocol is vulnerable to the known session-specific temporary information attack and impersonation attack. Odelu et al. [6] proposed an improved protocol to solve the security drawbacks in [5]. Xie et al. [7] proposed a two-factor authentication protocol. However, Xie’s protocol cannot resist the lost smart card attack and the offline dictionary guessing attack. To address the drawbacks, Chandrakar and Om [8] proposed a new security-enhanced three-factor protocol. Feng et al. [9] proposed an enhanced biometrics-based authentication protocol that can provide user anonymity. Amin et al. [10] proposed a lightweight authentication protocol that has lower computational and communication costs. Cui et al. [11] proposed an efficient protocol that only uses nonce, exclusive-OR operation, and one-way hash function; their protocol greatly reduces the computation cost. However, in this kind of protocol, they all need the help of an online registration center to achieve mutual authentication, which increases the communication cost.

In order to solve the drawbacks in the first type protocol, Choi et al. [12] proposed the first MSAA protocol without the online registration center. Tseng et al. [13] proposed a list-free ID-based authentication protocol using bilinear pairings for the multiserver architecture. However, Tseng et al. [13] cannot provide confidential and privacy. Recently, Odelu et al. [14] and He et al. [15] proposed new protocols that reduce the computation and communication costs. Irshad et al. [16] found protocol in [17] cannot achieve desired security goals. Therefore, they proposed an improved multiserver authentication protocol for distributed mobile cloud computing services. Afterward, Xiong et al. [18] found protocol in [16] has unreasonable computation cost, so they proposed an enhanced protocol for distributed mobile cloud. At the same time, Xiong et al. [19] proposed a new lightweight anonymous authentication protocol to reduce computation and communication costs. Barman et al. [20] used fuzzy commitment approach to secure the information stored on personal device. Kumari et al. [21] proposed a concept of the fuzzy extractor to provide the proper matching of biometric patterns. Xu et al. [22] proposed a new protocol that provides untraceability. Jiang et al. [23] performed a security analysis to the protocol in [17], pointing out that it is vulnerable to the impersonation attack. Chatterjee et al. [24] proposed a biometric-based protocol using the chaotic map and enhanced the security for multiserver architecture. We summarize techniques, advantages, and disadvantages that the existing protocols used in Table 1.

3. Preliminaries

In this section, we introduce preliminaries of the proposed protocol.

Let $G_1$ and $G_2$ be an additive cyclic group and a multiplicative cyclic group, both of them have a large prime order $q$. Let $\tilde{\circ}: G_1 \times G_1 \rightarrow G_2$ denote a bilinear map. Suppose $P$ is a generator of $G_1$, $g$ is a generator of $G_2$. A bilinear map $\tilde{\circ}$ has the following properties:

(i) Bilinearity: for all $P, Q \in G_1$, and for all $a, b \in Z_q^*$, $\tilde{\circ}(aP, bQ) = \tilde{\circ}(P, Q)^{ab}$

(ii) Computability: there exists an algorithm that can successfully compute $\tilde{\circ}(P, Q)$ for all $P, Q \in G_1$

(iii) Nondegeneracy: there exists $P, Q \in G_1$ such that $\tilde{\circ}(P, Q) \neq 1$, where 1 is the identity element of $G_2$.

We list the hard problems that we used in the proposed protocol as follows:

(i) Discrete logarithm (DL) problem: given an element $x \in G_2$, it is hard to compute $a \in Z_q^*$ such that $x = g^a$

(ii) Computational Diffie–Hellman (CDH) problem: given two elements $g^a, g^b \in G_2$, it is hard to compute $g^{ab} \in G_2$, where $a$ and $b$ are unknown and randomly chosen from $Z_q^*$
### Table 1: Related work summaries.

| Protocol       | Technique       | Advantage                               | Disadvantage                                           |
|----------------|-----------------|-----------------------------------------|--------------------------------------------------------|
| [1]            | Identity-based  | Lightweight and efficient               | Cannot provide user anonymity                          |
| [4]            | Identity-based  | Provides user anonymity, resists server spoofing attack and impersonation attack, etc. | Cannot resist server spoofing attack and impersonation attacks |
| [5]            | Biometrics-based| First truly three-factor authenticated scheme | Cannot resist known session-specific temporary attack and the impersonation attack |
| [6]            | Biometrics-based| Provides secure authentication and resists passive and active attacks | Needs registration center online for authentication |
| [7]            | Identity-based  | Security enhanced and supports smart card revocation and password update without centralized storage | Cannot resist the lost smart card attack and the offline dictionary guessing attack |
| [8]            | Biometrics-based| Efficient in terms of computation cost, communication cost, and resists smart card storage cost | High maintenance cost |
| [9]            | Biometrics-based| Incurs low overhead, suitable for deployment at mobile devices | Needs registration center online for authentication |
| [10]           | Two-factor-based| Security enhanced, lightweight, and efficient | Needs registration center online for authentication |
| [11]           | Identity-based  | Resists the server spoofing attack       | Cannot provide hierarchical authentication |
| [12]           | Identity-based  | Does not need registration center online for authentication | Cannot provide credentials privacy and untraceability |
| [13]           | Identity-based  | Provides black/white list-free and simple revocation mechanism | Cannot provide hierarchical authentication |
| [14]           | Identity-based  | Provides SK-security and strong credentials' privacy | Cannot provide hierarchical authentication |
| [15]           | Identity-based  | Uses the self-certified public key cryptography and has lower computation and communication costs | Cannot provide hierarchical authentication |
| [16]           | Two-factor-based| Resists server spoofing attack, desynchronization attack, and denial-of-service attack | Cannot provide hierarchical authentication |
| [17]           | Two-factor-based| Reduces authentication processing time required by communication and computation between cloud service providers and traditional trusted third-party service | Cannot resist service provider impersonation attack and has no user revocation facility |
| [18]           | Biometrics-based| Provides three-factor security, user revocation, and reregistration | Cannot provide hierarchical authentication |
| [19]           | Biometrics-based| User anonymity, perfect forward secrecy, and resistance to desynchronization attack | Cannot provide hierarchical authentication |
| [21]           | Biometric-based | Provides user untraceability and perfect forward security | Cannot provide hierarchical authentication |
| [23]           | Biometric-based | Uses chaotic map to improve efficiency  | Cannot provide hierarchical authentication |

(iii) Modified Bilinear Inverse Diffie–Hellman with k value (k-nBIDH) problem [12]: given k elements \(\{a_1, a_2, \ldots, a_k\} \subset \mathbb{Z}_q^*\) and k + 2 elements \(\{r \cdot P, \eta \cdot P, (1/(r + a_1)) \cdot P, (1/(r + a_2)) \cdot P, \ldots, (1/(r + a_k)) \cdot P\}\) each of them is in \(G_1\), it is hard to compute \(\tilde{e}(P, P)^{\eta/(r+\alpha)}\), where \(\alpha \notin \{a_1, a_2, \ldots, a_k\}\) and \(r, \eta\) are two unknown elements in \(\mathbb{Z}_q^*\).

A security system parameter generator used in the proposed protocol is introduced below.

Gen(\cdot)\): the system parameter generator takes a security parameter \(n\) and outputs system parameters, a bilinear map, an elliptic curve, a multiplicative group, etc. Intuitively, the system parameters will be publicly known.

The notations used in the proposed protocol are listed in Table 2.

### 4. The Proposed Protocol

#### 4.1. RC Initialization Phase.
Registration center runs the generation function Gen(\(1^n\)) which takes a security parameter \(n \in \mathbb{Z}^+\) and outputs parameters as follows:

1. RC chooses two bilinear map groups \(G_1\) and \(G_2\) with a prime order \(q\), the generator \(P \in G_1\) and \(g = \tilde{e}(P, P) \in G_2\), where \(\tilde{e}: G_1 \times G_1 \rightarrow G_2\) is a bilinear map.

2. RC chooses cryptographic hash functions \(H_1: \{0, 1\}^* \rightarrow \mathbb{Z}_q^*, H_2: \mathbb{Z}_2 \rightarrow \mathbb{Z}_q^*, H_3: \{0, 1\}^* \rightarrow \{0, 1\}^n\).

3. RC chooses a random number \(s \in \mathbb{Z}_q^*\) as the master key, computes the corresponding public key \(P_{pub} = sP \in G_1\), and constructs an authentication right tree \(T\) as Figure 1. The detail of the tree will be described in Section 4.5. Finally, RC publishes \(\{G_1, G_2, q, \tilde{e}, P, P_{pub}, g, H_1, H_2, H_3, H_4\}\).

#### 4.2. User Registration Phase.
If a user \(U_i\) wants to register with the registration center RC, the following steps will be executed. The main steps are provided in Table 3.

1. \(U_i\) sends his/her identity \(ID_{U_i}\) to RC via a secure channel.
Table 2: Notations used in the proposed protocol.

| Notation | Description |
|----------|-------------|
| RC       | The registration center |
| $U_i$    | The $i$th user |
| $S_j$    | The $j$th service provider |
| $\sigma$ | The adversary |
| ID$s_i$ | The identity of $i$th user |
| ID$s_j$ | The identity of $j$th service provider |
| $\sigma_i$ | The biometric key |
| $e$ | The private key expire parameter |
| $\tau_1, \tau_2$ | Random numbers from $Z_q^*$ |
| sk | The session key |
| $\mathcal{T}$ | The authentication right tree |
| $a_i, \tilde{a}_i$ | The authentication right parameters |
| $KR_i$ | The $i$th node of the authentication right tree |
| $L_i$ | The subnode of $KR_i$ |
| $f(\cdot)$ | The fuzzy-extractor generation procedure |
| $f(\cdot)^{-1}$ | The deterministic reproduction procedure |
| $H(\cdot)$ | Secure one-way hash functions |
| IDRL | ID revocation list |

(2) RC selects an authentication parameter $KR_i \in \mathcal{T}$ and computes the $U_i$’s private key $d_{U_i} = (1/(s + H_1(ID\_U_i || e || KR_i))) \cdot P$, where $e$ is the expire date of the private key and $\mathcal{T}$ is an authentication right tree. RC chooses a parameter $a_i \in \mathcal{T}$ and sends $\{d_{U_i}, a_i\}$ to $U_i$ via a secure channel.

(3) $U_i$ computes $(\sigma, \theta) \leftarrow f(b_i)$ using the fuzzy-extractor generation procedure $f(\cdot)$ [26], where $\sigma_i$ is a biometric key, $\theta_i$ is a public reproduction parameter, and $b_i$ is his/her personal biometrics. $U_i$ computes $A = d_{U_i} \oplus H_3(pw||\sigma_i)$ and uses the widely-implemented fuzzy-verifier technique [27, 28] to compute $B = H_4(H_1(ID\_U_i || H_2(pw||\sigma_i)) \mod n_0)$, where $pw$ is his/her password and $n_0$ is the integer that defines in [27]. Finally, $U_i$ stores $\{a_i, \sigma_i, A, e, f(\cdot), f^{-1}(\cdot), t, H_1, H_2, H_3, H_4\}$ on its mobile device, where $t$ is the threshold in fuzzy extractor, $f(\cdot)$ is the probabilistic generation procedure for outputting $\sigma_i$, and $\theta_i$. $f^{-1}(\cdot)$ is the deterministic reproduction procedure that can recover $\sigma_i$ and $\theta_i$ from a new personal biometrics input.

4.3. Service Provider Registration Phase. If a service provider $S_j$ wants to register with the RC, the following steps will be executed. The main steps are provided in Table 4.

| Step | Description |
|------|-------------|
| (1) | $S_j$ sends his/her identity ID$s_j$ to RC via a secure channel. |
| (2) | RC computes the private key $d_{S_j} = (1/(s + H_1(ID$is_j))) \cdot P$ for $S_j$ and sends $\{d_{S_j}, \tilde{T}\}$ to him via a secure channel, where $\tilde{T}$ is an authentication right tree for service provider. We will describe the detail of $\tilde{T}$ in Section 4.5. |

4.4. User and Service Provider Authentication Phase. In this part, we show the mutual authentication phase between a user and a service provider without involving RC. The main steps are provided in Table 5.

| Step | Description |
|------|-------------|
| (1) | First, $U_i$ inputs his/her biometrics $b_i$, identity ID$s_U$, and password $pw$ into his/her mobile device. Mobile device computes $\sigma_i = f^{-1}(\theta_i, b_i)$ and $B^* = H_4((H_4(ID\_U_i)||H_2(pw||\sigma_i)) \mod n_0)$ and verifies the validity of inputted biometrics and password by computing $B^* = B$. If it holds, mobile device retrieves $U_i$’s private key by computing $d_{U_i} = A \oplus H_3(pw||\sigma_i)$ and temporarily saves ID$L_i$. Then, $U_i$ selects a temporary session secret $\tau_1$, calculates $r_1 = H_1(\tau_1 || d_{U_i})$, and computes $g_1 = g^{r_1}$, $C = r_1 \cdot (H_1(ID\_S_j)^P + P_{pub})$ using the identity of $S_j$. Next, $U_i$ computes $D = H_1(ID\_U_i || ID\_S_j || g_1)$, $E = (r_1 + D) \cdot d_{U_i}$, and $F = (a_i || ID\_U_i || ID\_S_j) \oplus H_4(g_1^{r_1})$. Finally, $U_i$ sends the login message $[C, E, F]$ to $S_j$. |
| (2) | After receiving $[C, E, F]$, $S_j$ checks whether ID$L_j$ is in IDRL; if ID$L_j$ is not in IDRL, $S_j$ retrieves $g_{i,j}$ using his/her private key $d_{S_j}$ as $g_{i,j} = g^{t_{i,j}} = \overline{e}(C, d_{S_j})$. $S_j$ retrieves $a_i, D, ID\_U_i, e, ID\_S_j$ by computing $F \oplus H_4(g_1)^{r_2}$. $S_j$ computes $KR_i = H_4(KR_{i,j} || L_i)$, where $L_i = H_4(a_i || \tilde{a}_i)$, and verifies $\overline{e}(E, H_1(ID\_U_i || e || KR_i)) \cdot P + P_{pub} = g_1^{r_2} \cdot g_i^{t_{i,j}}$. If both are equal, $U_i$ is allowed to authenticate with $S_j$. Then, $S_j$ selects a temporary session secret $\tau_2$, then he/she calculates $r_2 = H_1(\tau_2 || d_{S_j})$ and computes $g_2 = g^{r_2}$ and session key is set as $sk = H_2(g_1^{r_2}) = H_2(g^{r_2})$. Finally, $S_j$ calculates $G = H_4(sk || g_1 || g_2 || ID\_S_j || C)$ and sends the message $[g_{2,j}, G]$ to $U_j$. |
| (3) | Upon receiving $[g_{2,j}, G]$, $U_i$ computes $sk = H_2(g_1^{r_2}) = H_2(g^{r_2})$ and $G^* = H_4(sk || g_1 || g_2 || ID\_S_j || C)$ and checks whether $G$ and $G^*$ are equal. If both are not equal, $U_i$ aborts the session. Otherwise, $U_i$ confirms $S_j$ as a valid service provider and sets $sk$ as session key between $U_i$ and $S_j$. |
4.5. Tree Construction and Verification. The proposed protocol uses an authentication right tree $\mathcal{T}$ to store user’s and service provider’s hierarchical authentication information. $\mathcal{T}$ is a Merkle hash tree that was introduced by Merkle [29] in 1998. Merkle hash tree is a digital signature scheme that only uses a conventional encryption function to compute the digital signature, making it extremely efficient. In 2009, Satoshi proposed a peer-to-peer electronic cash system, as known as bitcoin [30]. Bitcoin system stores transactions in a secure channel, which saves disk space, and this method can be used to verify transactions in each block. Therefore, we use a Merkle hash tree to construct our authentication right tree. In this part, we will show how we construct an authentication tree and how to verify a user’s authentication right based on the rules of Merkle hash tree.

### 4.5.1. Tree Construction

First, we introduce the construction of the authentication right tree $\mathcal{T}$ as Figure 1. An authentication tree $\mathcal{T}$ contains the information of $n$ different levels. The first level is the lowest level in the system, and the $n_{th}$ level is the highest level in the system. Node $KR_i$ denotes a user that has the authentication right which is from the first level to $i_{th}$ level. Value stored in node $KR_i$ is computed from the hash values of its left child node $KR_{i-1}$ and right child node $L_i$ as $KR_i = H_1(KR_{i-1} \| L_i)$, and the calculation of node $KR_i$ as $KR_i = H_1(L_i)$ is different from other KR nodes. If a user is at the first level, $KR_i$ is embed in his/her private key, and he/she can only access to first level service providers in this system. If user is at $i_{th}$ level, $KR_i$ is embed in his/her private key, and he/she can access to service providers which are from first to $i_{th}$ levels. The right child node $K_{R_{i+1}}$ and the left child node $L_i$ are recursively computed.

#### Example:

Consider a user at the first level has an authentication right $KR_1$ is embedded in his/her private key. The first level is the lowest level in the system, and the $n_{th}$ level is the highest level in the system, and $KR_{n-1}$ is the root node of the tree.

- **Level 1:** $KR_1 = H_1(ID_U \| \text{pw})\| KR_0)$
- **Level 2:** $KR_2 = H_1(KR_1 \| \text{pw})\| KR_1)$
- **Level 3:** ... $KR_n = H_1(KR_{n-1} \| \text{pw})\| KR_{n-1})$

The root node of the tree is $KR_1$, and the hash values of the root node are computed recursively from the child nodes. If a user is at the first level, $KR_1$ is embed in his/her private key, and he/she can only access to first level service providers in this system. If user is at $i_{th}$ level, $KR_i$ is embed in his/her private key, and he/she can access to service providers which are from first to $i_{th}$ levels.

### 4.5.2. Tree Verification

The verification of the authentication right tree $\mathcal{T}$ is performed by service providers and the service provider’s hierarchical authentication information. The verification process is performed recursively from the root node to the leaf node. If a user is at the first level, $KR_1$ is embed in his/her private key, and he/she can only access to first level service providers in this system. If user is at $i_{th}$ level, $KR_i$ is embed in his/her private key, and he/she can access to service providers which are from first to $i_{th}$ levels.
child node $L_i$ is an intermediate variable, which prepares for calculating $KR_i$. Value stored in node $L_i$ is computed from the hash values of its left leaf node $a_i$ and right leaf node $\tilde{a}_i$ as $L_i = H_4(a_i||\tilde{a}_i)$. Leaf node $a_i$ and $\tilde{a}_i$ are two 160-bit random strings that are stored on user and service provider separately. If a user is at $i_{th}$ level, number $i$ is stored in the last $\log_2^i$ bits, and the first $(160 - \log_2^i)$ bits should be a random string.

4.5.2. Tree Stored on Service Provider. The authentication right tree $T$ stored on the service provider has a little different from $T$. Service provider uses $T$ to verify user’s authentication right. The scale of $T$ stored on each service provider is based on the level of service provider. As we mentioned above, $n_{th}$ level is the highest level in the system. If service provider is at $n_{th}$ level, he/she only provides service for $n_{th}$ level user, so he/she only needs to save the authentication right tree $T$ as Figure 2. If service provider is at the $j_{th}$ level, he/she can provide service for user that is from $j_{th}$ to $n_{th}$ level. Therefore, he/she needs to save $\tilde{a}_j$ to $\tilde{a}_n$ and $KR_{j-1}$ to $KR_n$ as Figure 3, where the symbol “?” denotes the node that service provider does not have.

4.5.3. Authentication Right Verification. When an $i_{th}$ level user wants to access a $j_{th}$ level service provider, where $i \geq j$, user sends $a_i$ to service provider, and when service provider received authentication parameter $a_i$, he/she checks the last $\log_2^i$ bits of $a_i$ to get user’s level and finds $\tilde{a}_i$. Service provider computes the value of $L_i$ as $L_i = H_4(a_i||\tilde{a}_i)$ and the value of $KR_i$ as $KR_i = H_4(KR_{i-1}||L_i)$. Then service provider verifies user’s authentication right by calculating $\bar{v}(E, H_1(ID_U||e||KR_j) \cdot P + P_{pub}) = g_1 \cdot g^D$. If the equation holds, service provider continues. Otherwise, he/she aborts the session. For instance, if a $10_{th}$ level user wants to access to a $5_{th}$ level service provider, user sends $a_{10}$ to service provider, and when service provider received authentication parameter $a_{10}$, he/she checks the last $\log_2^i$ bits of $a_{10}$ to get user’s level and finds $\tilde{a}_{10}$. Service provider computes $L_{10} = H_4(a_{10}||\tilde{a}_{10})$ and $KR_{10} = H_4(KR_9||L_{10})$. Service provider verifies user’s authentication right by calculating $\bar{v}(E, H_1(ID_U||e||KR_{10}) \cdot P + P_{pub}) = g_1 \cdot g^D$. If the equation holds, service provider continues. Otherwise, he/she aborts the authentication.

4.6. The User Revocation and Reregistration Phase. Revocation and reregistration has been used in many protocols [31, 32]. In this part, we describe user revocation and reregistration. When a user $U_i$ lost his/her smart card, he/she needs to reregister. $U_i$ submits his/her personal information to RC, and then, RC verifies $U_i$’s personal information and checks the expire date of the private key $d_{U_i}$. If $d_{U_i}$ has already expired, RC issues $U_i$ a new private key with a new expire date. If $d_{U_i}$ has not expired, RC issues $U_i$ a new ID and a new private key with the same expire date as the lost smart card. RC adds the lost ID$_{U_i}$ to its ID revocation list (IDRL) and board casts ID$_{U_i}$ to all service providers. After received ID$_{U_i}$, service providers save it into their local storage.

5. Security Proof

In this section, we analyze the security of our protocol. First, we present a security model for our protocol, which is based on Bellare–Rogaway (BR) model [33] and CK-adversary model [34], and we use Zipf’s law [35] to enhance the security of the base model. Second, we show that the security of the proposed protocol is based on the hardness of mathematical problems. Third, we show that our protocol satisfies security requirements.

5.1. Security Model. We propose a security model for the proposed protocol based on literature studies [5, 15, 27, 33, 36]. There are $U_i$ and $S_j$ at the authentication phase of the proposed protocol. The security of the proposed protocol is defined by a game played between an adversary $A$ and a challenger $C$. Let $\Pi^i_\Lambda$ denote the $i_{th}$ instance of the participant of $\Lambda \in \{U_i, S_j\}$, respectively. In this game, we describe the capabilities of $A$ that is defined in the literature [27] as follows:

(i) $A$ can enumerate offline all the items in the Cartesian product $D_{pw} \times D_{id}$ within polynomial time, where $D_{pw}$ and $D_{id}$ denote the password space and the identity space, respectively

(ii) $A$ has the capability of somehow learning the victim’s identity when evaluating security strength (but not privacy provisions) of the protocol

(iii) $A$ is in full control of the communication channel between the protocol participants

(iv) $A$ may either (i) learn the password of a legitimate user via malicious card reader or (ii) extract the sensitive parameters in the card memory by side-channel attacks, but cannot achieve both
can learn previous session keys

(vi) \( \mathcal{A} \) has the capability of learning server’s longtime private keys only when evaluating the resistance to eventual failure of the server (e.g., forward secrecy)

\( \mathcal{A} \) can issue queries to \( C \) and get answers from it as follows:

(i) \( H_i(q_i) \): at any time, \( \mathcal{A} \) issues query \( q_i \) where \( q_i \) can be any string, and \( C \) picks a random number \( r_j \in Z_q^* \) and stores \( \langle q_i, r_j \rangle \) into list \( \Gamma_{\text{list}} \), where \( i \in \{1, 2, 3, 4 \} \) and \( j \in \{ \text{poly}(n) \} \). Finally, \( C \) sends \( r_j \) to \( \mathcal{A} \).

(ii) ExtractUID (ID\(_U\)): \( \mathcal{A} \) issues queries of user identity ID\(_U\), and \( C \) generates users private key \( d_{U_i} \), and stores \( \langle \text{ID\(_U\)}, d_{U_i} \rangle \) into list \( \mathcal{E}_{\text{list}} \).

(iii) ExtractSID (ID\(_S\)): \( \mathcal{A} \) issues queries of service provider’s identity ID\(_S\), and \( C \) generates service provider’s private key \( d_{S_j} \) and stores \( \langle \text{ID\(_S\)}, d_{S_j} \rangle \) into list \( \mathcal{E}_{\text{list}} \).

(iv) Send \( (\Pi_U^*): \mathcal{A} \) issues query of the message \( m \), and \( C \) runs the protocol and returns the result to \( \mathcal{A} \).

(v) SKReveal \( (\Pi_S^*): \mathcal{A} \) issues query, and \( C \) returns the session key produced in \( \Pi_S^* \) to \( \mathcal{A} \).

(vi) CorruptUID (ID\(_U\)): \( \mathcal{A} \) issues the query of user’s identity ID\(_U\), and \( C \) returns user’s private key \( d_{U_i} \) to \( \mathcal{A} \).

(vii) CorruptSID (ID\(_S\)): \( \mathcal{A} \) issues the query of service provider’s identity ID\(_S\), and \( C \) returns service provider’s private key \( d_{S_j} \) to \( \mathcal{A} \).

(viii) Test \( (\Pi_U^*): \) when \( \mathcal{A} \) issues the query, \( C \) flips a random coin \( b \in \{0, 1\} \). If \( b = 1 \), \( C \) sends session key in \( \Pi_U^* \) to \( \mathcal{A} \); otherwise, \( \mathcal{A} \) outputs \( b' \), where \( b' \) is about the coin \( b \) produced in Test \( (\Pi_U^*): \mathcal{A} \) violates the authentication key agreement (AKA) of the proposed protocol \( \Sigma \), if \( \mathcal{A} \) can guess \( b \) correctly. We define \( \mathcal{A} \)'s advantage in attacking the proposed protocol \( \Sigma \) as

$$\text{Adv}_{\text{AKA}}^\Sigma (\mathcal{A}) = 2\text{Pr}[b = b'] - 1.$$

Definition 1 (AKA-Secure). The proposed protocol \( \Sigma \) is authentication key agreement secure (AKA-Secure) if \( \text{Adv}_{\text{AKA}}^\Sigma (\mathcal{A}) = 2\text{Pr}[b = b'] - 1 \) is negligible for any polynomial-time adversary \( \mathcal{A} \).

\( \mathcal{A} \) violates the mutual authentication of the proposed protocol \( \Sigma \), if \( \mathcal{A} \) can generate a legal login message or a legal response message. Let \( E_{U;S} \) and \( E_{S;U} \) denote the events that \( \mathcal{A} \) generates a legal login message and a legal response message. We define the advantage of \( \mathcal{A} \) attacking the mutual authentication of the proposed protocol \( \Sigma \) as

$$\text{Adv}_{\text{MA}}^\Sigma (\mathcal{A}) = \text{Pr}[E_{U;S}] + \text{Pr}[E_{S;U}].$$

Definition 2 (MA-Secure). The proposed protocol \( \Sigma \) is mutual authentication secure (MA-Secure) if \( \text{Adv}_{\text{MA}}^\Sigma (\mathcal{A}) \) is negligible for any polynomial-time adversary \( \mathcal{A} \).

5.2. Proof of Security. In this part, we show the proposed protocol \( \Sigma \) for multiserver architecture is AKA-secure and MA-secure in the security model we described above.

Lemma 1. No polynomial-time adversary \( \mathcal{A} \) can forge a legal login message with a nonnegligible probability \( \epsilon \).

Proof. Suppose the adversary \( \mathcal{A} \) forges a legal login message with a nonnegligible probability \( \epsilon \). We show there is a challenger \( C \) who can solve the discrete logarithm (DL) problem with a nonnegligible probability.

Given an instance \( (g, g^s) \) of the DL problem, the aim of challenger \( C \) is to compute \( s \in Z_q^* \), and \( C \) sends the system parameters \( \{g, q, \tilde{c}, p, \tilde{P}, g, H_1, H_2, H_3, H_4\} \) to \( \mathcal{A} \). \( C \) randomly selects ID\(_U\) and answers \( \mathcal{A} \)'s queries according to the following description:

(i) \( H_i(q_i) \): \( C \) maintains a list \( \mathcal{E}_{\text{list}} \) initialized empty. Upon receiving the query \( q_i \), \( C \) checks if \( \langle q_i, r_j \rangle \) exists in \( \mathcal{E}_{\text{list}} \). If yes, \( C \) sends \( r_j \) to \( \mathcal{A} \); otherwise, \( C \) randomly picks \( r_j \in Z_q^* \) and stores \( \langle q_i, r_j \rangle \) in \( \mathcal{E}_{\text{list}} \) then sends \( r_j \) to \( \mathcal{A} \), where \( j \in \{ \text{poly}(n) \} \).

(ii) ExtractUID (ID\(_U\)): \( C \) maintains a list \( \mathcal{E}_{\text{list}} \) initialized empty. When receiving the query ID\(_U\), \( C \) checks if \( \langle \text{ID\(_U\)}, d_{U_i} \rangle \) exists in \( \mathcal{E}_{\text{list}} \) then \( C \) send \( d_{U_i} \) to \( \mathcal{A} \); otherwise, \( C \) executes the operations as follows:

(1) If \( \text{ID}_{U_i} \neq \text{ID}_{U_i} \) \( C \) sets \( H_1(\text{ID}_{U_i}) \longrightarrow \alpha, \) \( d_{U_i} \longrightarrow 1 \), and stores \( \langle \text{ID}_{U_i}, \alpha \rangle \) into \( H_{\text{list}}^1 \), \( \langle \text{ID}_{U_i}, d_{U_i} \rangle \) into \( \mathcal{E}_{\text{list}} \).

(2) Otherwise if \( \text{ID}_{U_i} \neq \text{ID}_{U_i} \) \( C \) randomly selects \( \alpha_i \in \{\alpha_1, \alpha_2, \ldots, \alpha_n\} \), \( \alpha_i = 1 \) \( H_1(\text{ID}_{U_i}) \longrightarrow \alpha_i, d_{U_i} \longrightarrow (1/(s + \alpha_i)) \cdot P \), and stores \( \langle \text{ID}_{U_i}, \alpha_i \rangle \) in \( H_{\text{list}}^1 \), \( \langle \text{ID}_{U_i}, d_{U_i} \rangle \) into \( \mathcal{E}_{\text{list}} \).

(iii) ExtractSID (ID\(_S\)): \( C \) maintains a list \( \mathcal{E}_{\text{list}} \) initialized empty. When receiving the query ID\(_S\), \( C \) checks if \( \langle \text{ID}_{S}, d_{S_j} \rangle \) exists in \( \mathcal{E}_{\text{list}} \). If yes, \( C \) send \( d_{S_j} \) to \( \mathcal{A} \); otherwise, \( C \) randomly picks \( r_{d_{S_j}} \in Z_q^* \) and computes \( d_{S_j} = (1/(s + r_{d_{S_j}})) \cdot P \). \( C \) stores \( \langle \text{ID}_{S_j}, d_{S_j} \rangle \) into \( \mathcal{E}_{\text{list}} \), \( \langle \text{ID}_{S_j}, r_{d_{S_j}} \rangle \) into \( H_{\text{list}}^1 \) and sends \( d_{S_j} \) to \( \mathcal{A} \).

(iv) Send \( (\Pi_U^*): \) \( C \) checks if \( \Lambda \) and \( U_{\text{pub}} \) are equal; if yes, \( C \) aborts the game; otherwise, \( C \) operates according to protocol \( \Sigma \).

(v) SKReveal \( (\Pi_S^*): \) after receiving the query, \( C \) sends session key produced in \( \Pi_S^* \) to \( \mathcal{A} \).

(vi) CorruptUID (ID\(_U\)): \( C \) searches for \( \langle \text{ID}_{U_i}, d_{U_i} \rangle \) in \( \mathcal{E}_{\text{list}} \) and returns \( d_{U_i} \) to \( \mathcal{A} \).

(vii) CorruptSID (ID\(_S\)): \( C \) searches for \( \langle \text{ID}_{S_i}, d_{S_i} \rangle \) in \( \mathcal{E}_{\text{list}} \) and returns \( d_{S_i} \) to \( \mathcal{A} \).

(viii) Test \( (\Pi_U^*): \) \( C \) randomly picks a number with the same length of session key and sends it to \( \mathcal{A} \).

At last, \( \mathcal{A} \) outputs a legal login message \( \{C, E, F\} \) corresponding to user’s identity ID\(_U\). If \( \text{ID}_{U_i} \neq \text{ID}_{U_i} \), \( C \) aborts the game. Based on the forking lemma [37], \( \mathcal{A} \) can output
another legal login message \((C, Et, Ft)\). Because the login messages is legal, we get the following two equations, \(g_{U_i}\) is computed by rising \(g\) to the power of a random number chose by \(\mathcal{A}\):

\[
\begin{align*}
\bar{e}(E, H_1(1D_{U_i}\|e\|KR)) \cdot P + P_{pub} &= g_{U_i} \cdot g^{D_i}, \\
\bar{e}(E', H_1(1D_{U_i}\|e\|KR)) \cdot P + P_{pub} &= g_{U_i} \cdot g^{D'_i}. \\
\end{align*}
\]

(1)

Based on the two equations above, we get the following equations:

\[
\begin{align*}
\frac{\bar{e}(E, H_1(1D_{U_i}\|e\|KR)) \cdot P + P_{pub}}{\bar{e}(E', H_1(1D_{U_i}\|e\|KR)) \cdot P + P_{pub}} &= \frac{g_{U_i} \cdot g^{D_i}}{g_{U_i} \cdot g^{D'_i}} = g^{D-D'_i}, \\
\end{align*}
\]

(2)

\(\mathcal{C}\) outputs \((D - D')^{-1} \cdot (E - E')\) as the solution to the given DL problem. The probability that \(\mathcal{C}\) can solve the DL problem is described as follows:

(i) \(E_1\): \(\mathcal{C}\) dose not abort in the any Send-queries

(ii) \(E_2\): \(\mathcal{A}\) outputs a legal login request

(iii) \(E_3\): \(ID_{U_i}\) and \(ID_{U_j}\) are equal

Let \(l\) denote the number of bits in biometric data, \(q_{Send}\) and \(q_{H}\) denote the number of Send-queries and \(H_1\)-queries executed in the game, \(C'\) and \(s'\) are the Zipf’s parameters [35], and \(l\) is the length of biometric information. We can get \(\Pr[E_1] \geq (1 - (1/(q_{Send} + 1)))^{q_{Send}}, \Pr[E_2] \geq \varepsilon, \text{ and } \Pr[E_3] \geq (1/(q_{H_i})).

Therefore, the nonnegligible probability that \(\mathcal{C}\) can solve the DL problem is given by

\[
\begin{align*}
\Pr[E_1 \land E_2 \land E_3] &= \Pr[E_3] \cdot \Pr[E_2] \cdot \Pr[E_1] \\
&\geq \frac{1 - (1/(q_{Send} + 1)))^{q_{Send}} - \varepsilon}{q_{H_i}}, \\
&\text{max}\left\{C' \cdot \frac{s'}{q_{Send} \cdot 2^l}\right\}.
\end{align*}
\]

(3)

This contradicts with the hardness of the DL problem. Therefore, we get that no polynomial-time adversary against the proposed MSAA protocol can forge a legal login message with a nonnegligible probability. \(\square\)

**Lemma 2.** No polynomial-time adversary \(\mathcal{A}\) can forge a legal response message with a nonnegligible probability.

**Proof.** Suppose the adversary \(\mathcal{A}\) forges a legal response message with a nonnegligible probability \(\varepsilon\). We show there is a challenger \(\mathcal{C}\) who can solve the \(k\)-mBIDH problem with a nonnegligible probability.

Given an instance \((P, y \cdot P, z \cdot P, (1/(y + a_i)) \cdot P, (1/(y + a_j)) \cdot P, \ldots, (1/(y + a_k)) \cdot P) \in G_1\) of the \(k\)-mBIDH problem, the aim of challenger \(\mathcal{C}\) is to compute \(\bar{e}(P, P)^{y+a}\); he picks a random number \(x \in Z^*_2\) and computes \(x \cdot P, y \cdot P, \) and sends the system parameters \((G_1, G_2, q, e, P, P_{pub}, g, H_1, H_2, H_3, H_4)\) to \(\mathcal{A}\). \(\mathcal{A}\) randomly selects \(ID_{U_i}\) and answers \(\mathcal{A}'s\) queries according to the following description:

(i) \(H_1(q_j): \mathcal{C}\) maintains a list \(H^\text{list}\) initialized empty. Upon receiving the query \(q_j\), \(\mathcal{C}\) checks if \((q_j, r_j)\) exists in \(H^\text{list}\). If yes, \(\mathcal{C}\) sends \(r_j\) to \(\mathcal{A}\); otherwise, \(\mathcal{C}\) randomly picks \(r_j \in Z^*_2\) and stores \((q_j, r_j)\) in \(H^\text{list}\) then sends \(r_j\) to \(\mathcal{A}\), where \(j \in \{\text{poly}(n)\}\).

(ii) ExtractUID (\(ID_{U_i}\)): \(\mathcal{C}\) maintains a list \(\mathcal{L}_{\text{list}}^{ID_{U_i}}\) initialized empty. When receiving the query \(ID_{U_i}\), \(\mathcal{C}\) checks if \((ID_{U_i}, d_{U_i})\) exists in \(\mathcal{L}_{\text{list}}^{ID_{U_i}}\) and then \(\mathcal{C}\) sends \(d_{U_i}\) to \(\mathcal{A}\); otherwise, \(\mathcal{C}\) randomly picks \(r_{d_{U_i}} \in Z^*_2\) and computes \(d_{U_i} = (1/(s + r_{d_{U_i}})) \cdot P\). \(\mathcal{C}\) stores \((ID_{U_i}, d_{U_i})\) in \(\mathcal{L}_{\text{list}}^{ID_{U_i}, d_{U_i}}\) in \(H^\text{list}\) and sends \(d_{U_i}\) to \(\mathcal{A}\).

(iii) ExtractSID (\(ID_{S_j}\)): \(\mathcal{C}\) maintains a list \(\mathcal{L}_{\text{list}}^{ID_{S_j}}\) initialized empty. When receiving the query \(ID_{S_j}\), \(\mathcal{C}\) checks if \((ID_{S_j}, d_{S_j})\) exists in \(\mathcal{L}_{\text{list}}^{ID_{S_j}}\) and then \(\mathcal{C}\) sends \(d_{S_j}\) to \(\mathcal{A}\); otherwise, \(\mathcal{C}\) executes the operations as follows:

(1) If \(ID_{S_j} = ID_{S_{\alpha}}\), \(\mathcal{C}\) sets \(H_1(1D_{U_i})\) to \(\alpha\) and stores \((\mathcal{L}_{\text{list}}^{ID_{S_j}}), \mathcal{A}\) into \(H^\text{list}\) into \(\mathcal{L}_{\text{list}}^{ID_{S_j}}\).

(2) Otherwise if \(ID_{S_j} \neq ID_{S_{\alpha}}\), \(\mathcal{C}\) randomly selects \(a_i \in \{a_1, a_2, \ldots, a_k\}\), sets \(H_1(1D_{S_j})\) to \(\alpha\), and stores \((\mathcal{L}_{\text{list}}^{ID_{S_j}}), \mathcal{A}\) in \(H^\text{list}\) to \((\mathcal{L}_{\text{list}}^{ID_{S_j}}, \mathcal{A}_{(1/(s + a_j))} \cdot P)\) in \(\mathcal{L}_{\text{list}}^{ID_{S_j}}\).

(iv) Send (\(\mathcal{L}_{\text{list}}\)): \(\mathcal{C}\) checks if \(A\) and \(S_{\alpha}\) are equal; if yes, \(\mathcal{C}\) aborts the game; otherwise, \(\mathcal{C}\) operates according to the proposed protocol \(\Sigma\).

Finally, \(\mathcal{A}\) outputs a response message corresponding to identity \(ID_{S_j}\). \(\mathcal{C}\) outputs \(g_1\) as the solution of \(k\)-mBIDH problem. The probability that \(\mathcal{C}\) can solve the \(k\)-mBIDH problem is described as follows:

(i) \(E_1\): \(\mathcal{C}\) dose not abort in any Send-queries

(ii) \(E_2\): \(\mathcal{C}\) outputs a legal response message

(iii) \(E_3\): \(ID_{S_j}\) and \(ID_{S_{\alpha}}\) are equal

Let \(q_{Send}\), \(q_{H_1}\), and \(q_{H_2}\) denote the number of Response-query, \(H_1\)-query, and \(H_2\)-query in the game. We can get \(\Pr[E_1] \geq (1 - (1/(q_{Send} + 1)))^{q_{Send}}, \Pr[E_2] \geq \varepsilon, \text{ and } \Pr[E_3] \geq (1/(q_{H_1} \cdot q_{H_2})).\) Therefore, the nonnegligible probability that \(\mathcal{C}\) can solve the \(k\)-mBIDH problem is given by

\[
\begin{align*}
\Pr[E_1 \land E_2 \land E_3] &= \Pr[E_1] \cdot \Pr[E_2] \cdot \Pr[E_3] \\
&\geq \frac{(1 - (1/(q_{Send} + 1)))^{q_{Send}} - \varepsilon}{q_{H_1} \cdot q_{H_2}}.
\end{align*}
\]

(4)

This contradicts with the hardness of the \(k\)-mBIDH problem. Therefore, we get that no polynomial-time adversary against the proposed MSAA protocol can forge a legal response message with a nonnegligible probability. \(\square\)

**Theorem 1.** The proposed protocol is MA-secure if the DL problem and the \(k\)-mBIDH problem are hard.
Proof. Based on Lemmas 1 and 2, we get no polynomial-time adversary can forge a legal login message or a legal response message if the DL problem and the k-mBIDH problem are hard. Therefore, we get the proposed protocol is MA-secure. □

**Theorem 2.** The proposed protocol is AKA-secure if the CDH problem is hard.

**Proof.** Suppose $\mathcal{A}$ guesses $b$ correctly in Test-query with a nonnegligible probability $\epsilon$, then $\mathcal{C}$ can solve the CDH problem with a nonnegligible probability.

Let $E_{sk}$ denote the event that $\mathcal{A}$ gets the correct session key. Since the probability that $\mathcal{A}$ correctly guesses the value $b$ is at least $1/2$, we can get $\Pr[E_{sk}] \geq (\epsilon/2)$.

Let $E_{TU}$ and $E_{TS}$ denote the events that $\mathcal{A}$ uses in the Test-query to a user’s instance and a service provider’s instance, respectively. Let $E_{(U,S)}$ denote the event that $\mathcal{A}$ can violate the user to service provider authentication. We get the following two equations:

\[
\frac{\epsilon}{2} \leq \Pr[E_{sk}] = \Pr[E_{sk} \land E_{TU}] + \Pr[E_{sk} \land E_{TS}]
\land E_{(U,S)} + \Pr[E_{sk} \land E_{TS} \land \neg E_{(U,S)}] \leq \Pr[E_{sk} \land E_{TU}] + \Pr[E_{(U,S)}] + \Pr[E_{sk} \land E_{TS} \land \neg E_{(U,S)}].
\]

(5)

\[
\Pr[E_{sk} \land E_{TU}] + \Pr[E_{sk} \land E_{TS} \land \neg E_{(U,S)}] \geq \frac{\epsilon}{2} - \Pr[E_{(U,S)}].
\]

(6)

Since $\Pr[E_{TS} \land E_{(U,S)}] = \Pr[E_{TU}]$, we get

\[
\Pr[E_{TS} \land E_{(U,S)}] \geq \frac{\epsilon}{4} - \frac{\Pr[E_{(U,S)}]}{2}.
\]

(7)

We get the probability as follows:

\[
\Pr[sk = H_2(g^{r_1z_2}) \mid r_1, r_2 \in Z_q^*] \geq \frac{\epsilon}{4} - \frac{\Pr[E_{(U,S)}]}{2}.
\]

According to the proof of Lemma 1, we get $\Pr[E_{(U,S)}]$ is negligible. However, $(\epsilon/4) - ((\Pr[E_{(U,S)}])/2)$ is nonnegligible, suppose that $x = g^\alpha$, $y = g^\beta$ where $r_1, r_2 \in Z_q^*$. Given an instance $(x, y)$ of the CDH problem, $\mathcal{A}$ computes $z = g^{r_1z} \text{ with } a \text{ a nonnegligible probability } (\epsilon/4) - ((\Pr[E_{(U,S)}])/2)$. Therefore, $\mathcal{C}$ can use $\mathcal{A}$ to solve the CDH problem with a nonnegligible probability. This contradicts with the hardness of the CDH problem. Therefore, we can conclude that the proposed protocol is AKA-secure if the CDH problem is hard. □

5.3. **Security Requirements Analysis.** We briefly show the proposed protocol satisfies the security requirements as follows:

(i) Single registration: according to the specification of the proposed protocol, a user registers at the registration center once, and he/she can log into registered service providers, which is at a specific level. Therefore, the proposed protocol can provide single registration.

(ii) Mutual authentication: two lemmas described above show that the adversary against the proposed protocol cannot produce a valid login or response message. Then, $ID_{U}$ and $ID_{S}$ can authenticate with the participant by checking the legality of the received response message and login message, respectively. Therefore, the proposed protocol can support mutual authentication.

(iii) User anonymity: according to the proposed protocol, the user’s identity $ID_{U}$ is only in the message $F = (D || ID_{U} || ID_{S}) \oplus H_2(g^\alpha)$. To get $ID_{U}$, adversary needs to compute $g^\beta$ from $C = r_1 \cdot (H_1(ID_{S}) \cdot P + P_{pub})$, and it turns out the adversary need to solve the k-mBIDH problem. Then, we know that the proposed protocol can provide user anonymity as long as k-mBIDH problem is hard.

(iv) Untraceability: according to the proposed protocol, user generates a new random number $r_1 \in Z_q^*$ to compute $C = r_1 \cdot (H_1(ID_{S}) \cdot P + P_{pub})$, $g^{\alpha}$, $F = (a || D || ID_{U} || ID_{S}) \oplus H_2(g^\alpha)$. Due to the randomness of $r_1$, adversary cannot find any relation of messages sent by the user and cannot trace the user’s behavior. Therefore, the proposed protocol can provide untraceability.

(v) Session key agreement: according to the proposed protocol, both two participants calculate session key $sk = H_2(g^{r_1z_2})$, which can be used in future communications. Therefore, the proposed protocol can provide session key agreement.

(vi) Perfect forward secrecy: assume the adversary steals both private keys of the user and the service provider. We also assume that the adversary intercepts $C, E, F, G, g_2$ sent between the user and the service provider. Using the service provider’s private key, the adversary can compute $g_1 = \hat{e}(C, d_S) = g^\alpha$. To get session key $sk = H_2(g^{r_1z_2})$, the adversary must to compute $g_2 = g_1^{r_2} = g^{r_1z_2}$ where $g_1 = g^\alpha$, $g_2 = g^\beta$. Thus, adversary must solve the CDH problem. Then, the proposed protocol can provide the perfect forward secrecy, since the CDH problem is hard.

(vii) No smart card lose attack [20]: assume the adversary steals the user’s device. By using the side-
channel attack, the adversary can extract the data 
\( B = H_4(IID_U||H_4((pw||\sigma)), \quad A = d_{Uj} \oplus H_3((pw||\sigma)). \)
The adversary can guess password \( pw \), but he/she cannot verify its correctness because we have implemented the fuzzy-verifier technique [27, 28]. The adversary cannot get the user’s password, so he cannot get the user’s private key \( d_{Uj} \). Therefore, adversaries cannot impersonate the user to the service provider, and the proposed protocol can resist smart card lose attack.

(viii) No password verifier table: according to the proposed protocol, both two participants need to store their private keys, and the registration center needs no password verifier table. Thus, the proposed protocol provides no password verifier table.

(ix) No online registration center: according to the proposed protocol, two participants can authenticate with each other without the help of the registration center. Thus, the proposed protocol does not need an online registration center.

(x) Hierarchical authentication: this security requirement only satisfied by the proposed protocol. The hierarchical information \( KR_i \) is embedded in the user’s private key \( d_{Uj} = (1/(s + H_1(IID_U||e[KR_i])) \cdot P \), when authentication is with a service provider, service provider will check the user’s authentication right by computing whether the equation

\[ \tilde{\varepsilon}(E, H_1(ID_U||e[KR_r]) \cdot P + P_{pub}) = g_1 \cdot g^D \] holds.

(xi) The resistance of various attacks: the proposed protocol can resist the insider attack, the replay attack, the man-in-the-middle attack, etc. We briefly describe it as follows:

(1) Temporary information attack: if the adversary gets the temporary information \( \tilde{r}_1, \tilde{r}_2 \), he/she has no ability to derive the user’s secret key from \( (r_1 + D) \cdot d_{Uj} \) because the exponential of \( g \) is composed of two values: one is session temporary secret \( \tilde{r}_1 \) and other is the private key of the user \( d_{Uj} \). Therefore, the proposed protocol is secure against temporary information attack.

(2) Insider attack: suppose an insider in the system gets the user’s information \( IID_U, H_1((pw||\sigma)) \). The adversary can guess a password \( pw \). However, he/she cannot verify its correctness because user’s password is protected by the secure hash function and the biometric key \( \sigma_1 \). Thus, the insider cannot get user’s password, and the proposed protocol withstands the insider attack.

(3) User impersonation attack [38, 39]: according to the proof of Lemma 1, we conclude that no adversary can forge a legal login message without the user’s private key. Thus, the service provider can find out about the attack by verifying the validity of the received login message. Therefore, the proposed protocol can resist the server impersonation attack.

(4) Server spoofing attack: according to the proof of Lemma 2, we know that no adversary can generate a legal response message without the service provider’s private key. Therefore, users can find out about the attack by verifying the validity of the received response message. Therefore, the proposed protocol can resist the server spoofing attack.

(5) Modification attack: according to the proof of Lemma 1, we know that \( C, E, F \) is a digital signature of the login message and no polynomial-time adversary can forge a legal one. The service provider can find any modification by checking if the equation

\[ g_1 = g^s = \tilde{\varepsilon}(C, d_5) \quad \text{and} \quad \tilde{\varepsilon}(E, H_1(ID_U||e[KR_r]) \cdot P + P_{pub}) = g_1 \cdot g^D \] holds. Besides \( G \) is the message authentication code of the response message \( \{C, E, F\} \) under the key \( g_1 = \tilde{\varepsilon}(C, d_5) \). The user can find out about any modification of the response message because the hash function \( H_4(\cdot) \) is secure. Therefore, the proposed protocol can resist the modification attack.

(6) Replay attack: according to the proposed protocol, both two participants generate new random number \( r_1, r_2 \in Z^\ast_q \) and \( g_1 = g^{r_1}, g_3 = g^{r_2} \), which are involved in the login message and the response message. Due to the freshness of \( g_1, g_2 \), the user and the service provider can find the replay of messages by checking the validity of the received message. Therefore, the proposed protocol resists the replay attack.

(7) Man-in-the-middle attack: based on the above description, we conclude that the proposed protocol provides mutual authentication between two participants. Therefore, the proposed protocol can resist the man-in-the-middle attack.

5.4. Security Comparison. In this part, we compare the security of the proposed protocol with other multiserver architecture protocols in Table 6. We introduce a new independent criterion, which is based on a widely adopted standard [40, 41] as follows:

C1 (no password verifier table): the server does not need to maintain a database for storing user passwords or some derived values of user passwords.

C2 (password friendly): the password is memorable and can be chosen freely and changed locally by the user.

C3 (no password exposure): the password cannot be derived by the privileged administrator of the server.

C4 (no smart card loss attack): the scheme is free from smart card loss attack, i.e., unauthorized users getting a victim’s card should not be able to easily change the password of the smart card, recover the victim’s password by using online, offline or hybrid guessing
| Protocol          | Computation cost         | Communication cost | Storage cost | Security requirements |
|-------------------|--------------------------|--------------------|--------------|-----------------------|
|                   |                         | User side          | Server side  |                       |
| Odulu et al. [14] | $T_{mtp} + 4T_h + 3T_{sm}$ + $2T_{exp} = 78.519$ |                      |              | ✓ ✓ ✓ ✕ ✕ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ |
| He et al. [15]    | $2T_{sm} + T_{pa} + 2T_{exp} + 8T_h = 31.837$ |                      |              | ✓ ✓ ✓ ✕ ✕ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✕ |
| Ours              | $3T_{sm} + 2T_{exp} + T_{pa}$ + $6T_h = 45.354$ |                      |              | ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ |

Table 6: Performance comparison.
attacks, or impersonate the user to login to the system, even if the smart card is obtained and/or secret data in the smart card are revealed.

C5 (resistance to known attacks): the scheme resists various kinds of basic/sophisticated attacks, including offline password guessing attack, replay attack, parallel session attack, desynchronization attack, stolen verifier attack, impersonation attack, key control, unknown key share attack, and known key attack.

C6 (sound repairability): the scheme provides smart card revocation with good repairability, i.e., a user can revoke her card without changing her identity.

C7 (provision of key agreement): the client and the server can establish a common session key for secure data communications during the authentication process.

C8 (no clock synchronization): the scheme is not prone to the problems of clock synchronization and time delay, i.e., the server needs not to synchronize its time clock with these time clocks of all input devices used by smart cards, and vice versa.

C9 (timely typo detection): the user will be timely notified if she inputs a wrong password by mistake when login.

C10 (mutual authentication): the user and server can verify the authenticity of each other.

C11 (user anonymity): the scheme can protect user identity and prevent user activities from being traced.

C12 (forward secrecy): the scheme provides the property of perfect forward secrecy.

C13 (hierarchical authentication): when a server authenticates with a user, it checks the user’s authentication right. If the user authentication right belongs to the server authentication level, the authentication is successful. Otherwise, the authentication request is denied.

6. Performance Comparison

We show the computation and communication costs of the proposed protocol. We compare its performance with other protocol. For the purpose of getting a trusted security level (1024-bit RSA algorithm), an Ate pairing: \( \overline{e} : G_1 \times G_1 \rightarrow G_2 \) is used. \( G_1 \) with order \( q \) is a 512-bit prime number. \( G_2 \) is generated by a point on a supersingular elliptic curve \( E(F_p) \): \( y^2 = x^3 + 1 \) which is defined on the finite field \( F_p \). Order \( q \) is a 160-bit prime number and \( p \) is a 512-bit prime number.

6.1. Computation Cost Comparison. We give the running time of various operations performed in the proposed protocol, and we compare the results with He et al. [15] and Odelu et al. [14]. In this section, we use the following notations for the following running times in this paper:

(i) \( T_{\text{exp}} \): the running time of a scalar multiplication operation

(ii) \( T_{\text{exp}} \): the running time of a map-to-point hash function in \( G_1 \)

(iii) \( T_{\text{mul}} \): the running time of a point addition operation in \( G_1 \)

(iv) \( T_{\text{mul}} \): the running time of an exponentiation operation in \( G_2 \)

(v) \( T_{\text{mul}} \): the running time of a multiplication operation in \( G_2 \)

(vi) \( T_{\text{mul}} \): the running time of a general hash operation

Table 7: The running time of related operations (milliseconds).

|             | \( T_{\text{exp}} \) | \( T_{\text{exp}} \) | \( T_{\text{exp}} \) | \( T_{\text{exp}} \) | \( T_{\text{exp}} \) | \( T_{\text{exp}} \) |
|-------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| User        | 33.582               | 32.713               | 13.405               | 0.081                | 2.249                | 0.008                | 0.006                |
| Server      | 4.174                | 1.665                | 1.665                | 0.011                | 0.260                | 0.001                | 0.006                |

6.2. Communication Cost Comparison. According to the description of the trusted security level, \( q \) is a 160-bit prime number and \( p \) is a 512-bit prime number. The size of an element in \( G_1, G_2 \) is 1024 bits. The size of the hash function’s output is 160 bits, and the identity and the expire parameter are both 32 bits. In our protocol, we only have two rounds of communication for establishing a session key. On client side, the messages \( C, E, F \) require \( 320 + 320 + 256 = 896 \) bits, and on service provider side, messages \( G, g \) require \( 160 + 512 = 672 \) bits. The total communication costs are 1568 bits. In He et al.’s [15] protocol, on client side, messages \( R_{i,j}, C_{i,j} \) require \( 1024 + 32 + 1024 + 160 = 2240 \) bits, and on server side, messages \( y, a, \psi \) require \( 1024 + 160 + 1184 \) bits. In Odelu et al.’s [14] protocol, on client side, messages \( M_1, M_3 \) require \( 320 + 512 = 832 \) bits, and on server side, message \( M_2 \) require 672 bits. The comparison of communication costs is shown in Table 6.

6.3. Storage Cost Analysis. Because the mobile devices are limited to storage spaces, we therefore analyze the storage cost on the user side to show the proposed protocol has reasonable storage cost. In Odelu et al.’s protocol, a user needs to store \( \{ E_{i,j}, v_i, \theta_j, t, e, L_{t,i}, P, P_{\text{pub}}, g, \theta \} \) in his/her device, which costs 1674 bits. In He et al.’s protocol, user needs to store \( \{ R_{i,j}, y, a, \theta_j, \psi, v_i, b_i \} \), which costs 3230
bits. In our protocol, user needs to store \( \{A, B, \theta, a, c, P, P_{pub}, g, q\} \), which costs 1834 bits.

7. Conclusion

In this paper, we have proposed a hierarchical authentication protocol for the multiserver environment. The significant contribution of this paper is that we have built an authentication right tree based on the Merkle hash tree, which can be used to verify the authentication right of a user when he/she is authenticating with the service provider. The extended hierarchical authentication feature has added more flexibility and security to multiserver architecture. The security proof has demonstrated that our protocol is provably secure under the random oracle model. Our protocol has reasonable computation and communication costs, which could be suitable for multiserver architecture.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Acknowledgments

This work was funded by the Chengdu Science and Technology Bureau (no. 2016-XT00-00015-GX) and the Civil Aviation Administration of China (no. PSDA201802).

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