The solvent effect on the nonlinear dynamics of U-model microtubules

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Abstract. Microtubules are hollow cylindrical protein polymers composed of tubulin dimers and act as executives in cellular dynamics. The solvent effect on the microtubules dynamics of U-model is investigated. The focus of study is the effect of time-dependent damping coefficient on the system dynamics. The model of \( \gamma(t) \) is \( \gamma_1(t) = \gamma_0 + e^{-\beta t^2} \) and \( \gamma_2(t) = \gamma_0 - e^{-\beta t^2} \). The system dynamics is described by kink and anti-kink solutions obtained by using modified extended tanh-function method (METHF). The decrease of the damping coefficient will increase the kink and anti-kink velocities. The result obtained also shows that the solvent affects the process of disassembly and kinesin motion.

1. Introduction

Microtubules are hollow cylindrical protein polymers. The inner and outer diameters of the microtubule cylinder are 15 nm and 25 nm [1]. Generally, cylindrical walls of microtubules consist of 13 parallel arranged protofilaments. Each protofilament is composed of tubulin dimers. A dimer is polar and is a combination of \( \alpha \) monomer and \( \beta \) monomer with the weight of each monomer is 55 kdalton and 4nm in length [2]. Each tubulin monomer binds a guanine nucleotide, which is nonexchangeable when it is bound in the \( \alpha \) subunit, or N site, and exchangeable when bound in the \( \beta \) subunit, or E site [3]. Microtubules play an important role in cells and are considered as executives of cellular dynamics. To establish the function of living cell, microtubules assembly and disassembly spontaneously. GTP bound to \( \beta \) monomer is hydrolyzed immediately after the formation of the polymer. The energy released by GTP hydrolysis is used microtubule in its activity. Therefore, to understand how the energy is utilized then we need to understand the dynamics of microtubules.

U-model assumes only one degree of freedom of dimer motion in the protofilament. Based on in vitro experiment conducted [4], structural analysis shows that GTP hydrolysis is not required for sheet closure into a microtubule cylinder but only increases the probability of this event occurring, glycerol inhibites the process and the closure rate decreases. In addition, the interaction between the system and the surrounding environment leads to energy dissipation so that the system is no longer conservative and reversible [5]. Therefore, this study investigates U-model with solvent effect by adding viscous force \( F_d = -\gamma(t) \frac{\partial u_n}{\partial t} \) where \( \gamma(t) \) represents time-dependent damping coefficient.
2. The solvent effect on the microtubules dynamics of U-model

Based on the experiment [6], tubulin undergoes a conformational change induced by GTP-GDP hydrolysis in which one monomer shifts its orientation by 29° from the dimer’s vertical axis. Conformational change occurs because the mobile electron on each dimer (α monomer) move toward the β monomer so that the polarity changes. The U-model assumes a longitudinal displacement of a dimer at a position n denoted as \( u_n \) while the coordinate \( u_n \) is a projection of the top of the dimer on the direction of protofilament [7]. The overall effect of the surrounding dipoles on a chosen site n can be qualitatively described by the double-well quartic potential and a microtubule generates a nearly uniform intrinsic electric field parallel to its axis [8]. The Hamiltonian for one protofilament is

\[
H = \sum_{n=1}^{N} \left[ \frac{p_n^2}{2M} + \frac{1}{2} k (u_{n+1} - u_n)^2 - \frac{1}{2} A u_n^2 + \frac{1}{4} B u_n^4 - qE u_n \right].
\]  

(1)

The first term is kinetic energy of each dimer which has mass M and the second term is potential energy of interaction between adjacent dimer in one protofilament where k is stiffness parameter. Solvent can affect dynamics of microtubule so that the system is nonconservative. The viscosity effect can be shown by adding the damping force \( F_d = -\gamma(t) \frac{\partial u_n}{\partial t} \) to the equation of motion. An equation of motion for the system by using a continuum approximation is

\[
M \frac{\partial^2 u(x,t)}{\partial t^2} + \gamma(t) \frac{\partial u(x,t)}{\partial t} - k R_0^2 \frac{\partial^2 u(x,t)}{\partial x^2} - A u(x,t) + B u^3(x,t) - qE = 0,
\]  

(2)

where \( \gamma(t) \) is time-dependent damping coefficient. The model of \( \gamma(t) \) is \( \gamma_1(t) = \gamma_0 + e^{-\beta t^2} \) and \( \gamma_2(t) = \gamma_0 - e^{-\beta t^2} \). The value of damping coefficient is linear with solvent viscosity so that the value of damping coefficient indicates solvent viscosity. In this study, the damping coefficient value at physiological temperature 300K is \( \gamma_0 = 5.6 \times 10^{-11} \text{ kg/s} \) [8] and taken parameter value \( \beta \) is 2. Based on this parameter, the value of damping coefficient is positive and negative. For model \( \gamma_1(t) \), the value of damping coefficient is positive and decreases indicating that the viscosity solvent decreases. While the value of damping coefficient of the model \( \gamma_2(t) \) is negative and increase. The positive value of damping coefficient indicates stable system while negative damping coefficient value indicates unstable system [9]. The exact solution of motion equation is obtained by using the modified tanh-function method (METHF).

3. Kink and anti-kink propagations in dynamics of microtubules

Nonlinear phenomena plays an important role in physics and in particular, nonlinear waves that are encountered in numerous domains such as fluid mechanics, solid-state physics, plasma physics and chemical physics [10]. Nonlinear waves like kink and breather are the exact solutions of nonlinear classic field equations. A key step in investigating those problems is the derivation of traveling waves from the associated (one-dimensional) wave equation [11]. The solution of nonlinear wave equation can be obtained by using modified extended tanh-function method (METHF). The dynamics equation of U-model which is affected by solvent has the form of nonlinear partial differential equation. Therefore, the exact solution of the motion equation can be obtained by using METHF.

We look for localized waves of permanent profile of the form \( u(x,t) \rightarrow u(\xi) \) where \( \xi = x - vt \) is a single independent variable depending on v which is an arbitrary velocity of propagation. Substitution of x and t by transform \( \xi \) that equation (2) is ordinary differential equation (ODE) which can be written by

\[
(Mv^2 - k R_0^2) \frac{d^2 u(\xi)}{d\xi^2} - v \gamma(t) \frac{d u(\xi)}{d\xi} - A u(\xi) + B u^3(\xi) - qE = 0.
\]  

(3)

Then, we introduce normalized displacement field as

\[
u(\xi) = \sqrt{\frac{A}{B}} \phi(\xi),
\]  

(4)
so that the dimensionless equation of motion is
\[ \alpha^2 \frac{d^2 \psi(\xi)}{d\xi^2} (\xi) - \rho(t) \frac{d\psi(\xi)}{d\xi} (\xi) + \psi(\xi) - \psi(\xi) - \sigma = 0, \]
(5)
where
\[ \alpha = \left( \frac{Mv^2-kR_0^2}{A} \right), \quad \rho(t) = \frac{\nu}{A} \gamma(t), \quad \text{and} \quad \sigma = \frac{qE}{A} \sqrt{\frac{B}{A}}. \]
(6)
The next crucial step is that the solution we are looking for is expressed in the form [12]
\[ \psi(\xi) = a_0 + a_1 \varphi(\xi), \]
(7)
and
\[ \varphi'(\xi) = b + \varphi^2(\xi), \]
(8)
where b is a parameter to be determined \( \varphi(\xi) \) and \( \varphi'(\xi) = \frac{d\varphi(\xi)}{d\xi} \). By substituting equation (7) and (8) into equation (5) will get a system of algebraic equation for \( a_0, a_1, \text{ and } b \). All the coefficients of \( \varphi^i \) have to vanish and from which we find
\[ \alpha = -\frac{a_1^2}{2}, \]
(9)
\[ a_1 = \frac{\rho(t)}{3a_0}, \]
(10)
\[ b = \frac{1-3a_0^2}{2a}, \]
(11)
\[ a_0^3 - \frac{1}{4}a_0 + \frac{1}{8}\sigma = 0. \]
(12)
The solution of equation (12) is solved by using reference [13] and obtained three real solutions for \( \sigma < \sigma_0 \). For this case, we take one of the solution that is
\[ a_{01} = \frac{1}{2\sqrt{3}} \left[ \cos(F) + \sqrt{3}\sin(F) \right], \]
(13)
where \( F = \frac{1}{3} \cos^{-1} \left( \frac{\sigma}{a_0} \right) \) and \( \sigma_0 = \frac{2}{3\sqrt{3}} \).
(14)
The solution \( a_{01} \) corresponds with minimum energy of system. Based on the equations (9) to (12) we get the value of \( \alpha < 0 \) and the value of \( b < 0 \). The value of \( \alpha \) depends on the value of \( \sigma \) and \( \rho(t) \). The negative value of (9) suggests that the elastic term is bigger than the inertial one which indicates a small velocity of the wave and/or big \( k \), i.e. strong chemical bond between the neighbouring dimers in protofilament, as it can be seen from equation (6) [7]. The wave velocity of this case is
\[ v = \sqrt{\frac{kr_0^2}{m + \frac{(\gamma(t))^2}{18a_0^2}}} \]
(15)
The velocity of wave propagation v is the decreasing function of damping coefficient. While, the value of Riccati equation parameter is
\[ b = \frac{c}{\gamma^2(t)} + D, \]
(16)
where
\[ C = \frac{9a_0^2A^2m(3a_0^2-1)}{kR_0^2} \quad \text{and} \quad D = \frac{A(3a_0^2-1)}{2kR_0^2}. \]
(17)
The values of C and D are negative because the terms of the solution of this form for conditions \( \sigma < \sigma_0 \) which causes \( a_0^2 < \frac{1}{3} \) while the value \( y^2(t) \) is considered always positive so the value \( b < 0 \) then the solution of the Riccati equation is

\[
\varphi(\xi) = -\sqrt{-b} \tanh(\sqrt{-b}\xi).
\]  (18)

The final solution of equation (5) is

\[
\psi(\xi) = a_{01} - \frac{\nu(t)}{3a_{01}} y(t) \sqrt{-\left(\frac{C}{y^2(t)} + D\right)} \tanh \left(\sqrt{-\left(\frac{C}{y^2(t)} + D\right)}\xi\right).
\]  (19)

Solution \( \psi(\xi) \) depends on the damping coefficient and depends on the electric field \( E \) represented by \( a_{01} \).

![Figure 1](image1.jpg)

**Figure 1.** The soliton solution of equation (19) when \( t = 0 \). Anti-kink for model \( \gamma_1(t) \) (left) and kink for model \( \gamma_2(t) \) (right)

The solution is visualized and the parameters taken are \( M = 10^{-22} \text{ kg}, A = 200 \text{ Jm}^2, B = 10^{24} \text{ Jm}^4, q = 26\times36e, E = 10^5 \text{ Vm}^{-1}, \gamma_0 = 5.6\times10^{-11} \text{ kgs}^{-1}, R_0 = 8\times10^{-9} \text{ m} \) [8] while the parameters \( k = 1 \text{ Nm}^{-1} \) and \( \beta = 2 \). Based on figure 1, the solution for the \( \gamma_1(t) \) model is anti-kink while for \( \gamma_2(t) \) model is kink. Kink and anti-kink propagate with different directions. When \( \xi \) increases from \( -\xi \) to \( +\xi \) the pendulums rotate from \( 0 \) to \( 2\pi \) for kink while the anti-kink from \( -2\pi \) to \( 0 \) and kink propagates from right to left while the anti-kink propagates from left to right [14]. The kink soliton shows the system jumps from minimum energy to maximum energy while the anti-kink indicates the system falls from maximum energy to minimum energy.

![Figure 2](image2.jpg)

**Figure 2.** anti-kink propagation (left) and kink propagation (right)

Figure 2 shows that as time goes, kink and anti-kink propagate with the permanent form and in the same time interval, the change of position is greater. It shows that the velocity of soliton increases. The \( \gamma_1(t) \) model shows that the damping coefficient decreases which indicates the solvent viscosity reduces so that anti-kink propagation increases as shown in figure 2. Energy released by GTP hydrolysis is required for the kink soliton to be formed. It is well known that extra energy is required in order to create a kink, since kinks are not solutions of the equation of motion with the lowest energy [15]. When the GTP hydrolysis, energy is released then the system will jump to maximum energy then
return to the minimum energy immediately. The transition from maximum energy to minimum energy results in $u_n$ displacement. In the assembly process, GTP is hydrolyzed and releases energy to form a kink. Kink propagates along the microtubule from the positive end to the negative end. While the anti-kink is formed when the system jumps from the maximum energy state to the minimum energy state that propagates from negative end to the positive end and separates the adjacent dimer then microtubule disassembly.

4. Conclusion
This study investigates microtubules dynamics of U-model which is affected solvent with time-dependent damping coefficients. The kink and anti-kink solitons are obtained by using modified extended tanh-function method (METHF). The solution depends on $\gamma(t)$ and $\sigma$ which show that it depends on solvent viscosity and electric field $E$. When the value of damping coefficient decreases, the propagation velocity of kink and anti-kink increase. Assembly process and dynein motion are shown by the propagation of kink. While disassembly and kinesin motion are shown by the propagation of anti-kink. To accelerate the process of disassembly and kinesin motion can be adjusted by reducing the solvent viscosity. The damping coefficient model in this study yields a kink and anti-kink solution that can be linked to biological phenomena related to the dynamics of microtubules.

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