Flow reversals in turbulent convection via vortex reconnections

Mani Chandra and Mahendra K. Verma

1 Department of Physics, Indian Institute of Technology - Kanpur 208016, India

We employ detailed numerical simulations to probe the mechanism of flow reversals in two-dimensional turbulent convection. We show that the reversals occur via vortex reconnection of two attracting corner rolls having same sign of vorticity, thus leading to major restructuring of the flow. Large fluctuations in heat transport are observed during the reversal due to this flow reconfiguration. The flow configurations during the reversals have been analyzed quantitatively using large-scale modes. Using these tools, we also show why flow reversals occur for a restricted range of Rayleigh and Prandtl numbers.

PACS numbers: 47.55.P-, 47.27.De

Several experiments and numerical simulations on turbulent convection exhibit “flow reversals” in which the probes near the lateral walls of the container show random reversals (also see review articles [12]). These reversals have certain similarities with magnetic field reversals in dynamo and Kolmogorov flow [7]. Researchers typically study convection in a controlled setup called “Rayleigh-Bénard convection” in which a fluid confined between two plates is heated from below and cooled at the top. The two nondimensional numbers used to characterize the flow are the Rayleigh number (Ra), which is the ratio of the buoyancy term and the diffusive term, and the Prandtl number (Pr), which is the ratio of the kinematic viscosity and the thermal diffusivity. Cioni et al. [1], Niemela et al. [2], Brown and Ahlers [3], and Xi et al. [4] performed convection experiments on mercury, water, and helium gas in a cylindrical geometry and observed reversals for Ra > 10^9. Sugiyama et al. [5] and Vasiliev and Frick [6] studied reversals in a rectangular box with water. No reversal was observed for a cubical box (aspect ratio 1), but a quasi-two-dimensional box (aspect ratio < 0.2) exhibits reversals for a band of Rayleigh and Prandtl numbers [6]. Surprisingly, a cubical box containing mercury shows reversals [7], indicating a strong role played by the Prandtl number and geometry in the reversal dynamics.

Several theoretical models have been invoked to explain flow reversals. Benzi and Verzicco [8] and Sreenivasan et al. [11, 14] used stochastic resonance, while Arajuo et al. [12] employed low-dimensional models with noise to explain reversals. Brown and Ahlers [3] and Mishra et al. [11] showed that in a cylindrical geometry, the flow reversals are induced by a rotation or cessation of large-scale flow structures. For two-dimensional box geometry, Sugiyama et al. [8] relate the flow reversal to the growth of the corner rolls due to the plume detachments from the boundary layers. Chandra and Verma [12] studied the reversals quantitatively by representing flow structures as Fourier modes and showed that during the reversals, the amplitude of the first Fourier mode (k_x = 1, k_y = 1) becomes very small, while the Fourier mode (k_x = 2, k_y = 2) gains strength. The growth of the secondary modes at the expense of the primary modes is akin to the cessation-led reversals reported by Brown and Ahlers [3] and Mishra et al. [11], and to the emergence of quadrupolar mode in dynamo reversals [7].

One important question that persists is why do flow reversals occur in a restricted parameter regime? In this letter, we explain this through a quantitative investigation of the convective flow structure for a range of parameters using Fourier basis decomposition. This scheme allows for accurate representation of the flow [12] and lets us quantify when reversals occur in terms of amplitudes of the modes. In addition we show that flow reversals occur via vortex reconnections, and connect them to vortex dynamics. We investigate in detail the heat transport in the system during a reversal and find large fluctuations in it as a result of the flow reorganization during the reversals. We restrict our study to 2D flow reversals whose flow structures are well represented by experiments conducted in quasi two-dimensional geometry [8].

We solve the equations governing Rayleigh-Bénard convection under Boussinesq approximation [12] in a 2D box of aspect ratio 1 with no-slip walls on all sides. The side walls are insulating, while the top and the bottom walls are maintained at constant temperatures. The simulations are performed using the open-source code NEK5000 [10] that employs spectral element method. We use 28 × 28 spectral element with 7th order polynomial, thus we have an effective grid of 196 × 196 points. The grid density is higher at the boundaries in order to resolve the boundary layers. We perform simulations for Pr = 1 and Rayleigh numbers ranging from 10^5 to 10^9.

To quantify the flow structures, we project the nondimensionalized horizontal velocity (u), vertical velocity (v), and temperature (T) onto the following basis:

\[ u = \sum_{m,n} \hat{u}_{m,n} \sin(m \pi x) \cos(n \pi y) \]

\[ (v, T) = \sum_{m,n} (\hat{v}_{m,n}, \hat{T}_{m,n}) \cos(m \pi x) \sin(n \pi y) \]

It has been shown that the above basis is a good representative of the flow field in a box geometry [12]. We...
choose the above formalism over proper orthogonal decomposition (POD) in order to study the temporal evolution of the flow structures. The mode with wavenumber $(m, n)$ corresponds to a flow structure with $m$ rolls in the $x$ direction and $n$ rolls in the $y$ direction. In the following discussion we will use the above basis to analyze the mechanism of flow reversals as well as the range of Pr and Ra for which reversals take place.

An important puzzle in 2D turbulent convection is why flow reversals take place only for a range of Prandtl and Rayleigh numbers. It has been shown that the corner rolls (represented by $(m = 2, n = 2)$ mode) are crucial for the occurrence of reversals [8, 12]. In particular, the relative strength of the $(2,2)$ mode with respect to the dominant mode determines if a reversal will occur or not. We observe three distinct flow structures for the range of Rayleigh numbers. It has been shown that the corner rolls become weaker compared to the $(1,1)$ roll for $Ra \approx 10^5$, a single roll with the dominant $(m=1, n=1)$ mode, (b) for $10^6 \leq Ra \leq 10^7$, two rolls stacked on top of each other corresponding to the mode $(1,2)$, and (c) for $Ra \geq 2 \times 10^7$, two corner rolls with a dominant roll aligned along the 45 degree diagonal, a configuration dominated by the modes $(1,1)$ and $(2,2)$.

![FIG. 1. Steady state flow structures at different Rayleigh numbers: (a) for $Ra \leq 10^5$, a single roll with the dominant $(m = 1, n = 1)$ mode, (b) for $10^6 \leq Ra \leq 10^7$, two rolls stacked on top of each other corresponding to the mode $(1,2)$, and (c) for $Ra \geq 2 \times 10^7$, two corner rolls with a dominant roll aligned along the 45 degree diagonal, a configuration dominated by the modes $(1,1)$ and $(2,2)$.](image)

Reversals are observed for a narrow band near $Ra = 2 \times 10^7$. The average value of the ratio $\hat{v}_{2,2}/\hat{v}_{1,1}$ decreases from 0.45 to 0.10 as $Ra$ increases from $2 \times 10^7$ to $10^9$. Reversals are observed for a narrow band near $Ra = 2 \times 10^7$ where $\hat{v}_{2,2}$ is strong.

![FIG. 2. Plot of the ratio $|\hat{v}_{2,2}|/|\hat{v}_{1,1}|$ vs. Ra. The mode $\hat{v}_{2,2}$, which quantifies the strength of the corner rolls, is born only near $Ra = 2 \times 10^7$. The average value of the ratio $|\hat{v}_{2,2}|/|\hat{v}_{1,1}|$ decreases from 0.45 to 0.10 as Ra increases from $2 \times 10^7$ to $10^9$.](image)

Now we probe in detail the process of flow reversal by studying the flow structures of six snapshots during one of the reversals (see Fig. 3). A movie of the flow reversal can be downloaded from [16]. The starting point of the reversal is the stable configuration shown in Fig. 3(c). At first, the mode $(2,2)$ (or the corner rolls) grows at the expense of the mode $(1,1)$, as evident from the subfigures 3(a,b). The top-left and bottom-right corner rolls have vorticity in the same direction, hence they attract each other and come close due to the vortex dynamics in 2D [17]. Since the velocities of the streamlines are directed in opposite directions, they reconnect, thus converting two corner rolls into a single roll (see Fig. 3(c)).

that Pr = 1, consistent with the results of Sugiyama et al. [8].
FIG. 3. Velocity and temperature profiles for six snapshots during the reversal: (a,b) Growth of corner rolls; (c) The two corner rolls at the upper-left and bottom-right corners reconnect to form a large single roll. The streamlines, represented by black curves, combine via vortex reconnections; (d,e) The flow reconfigures itself via rotation of the large roll formed after the reconnection. There is a strong nonlinear interaction among the modes (1,1), (2,2), (1,3), and (3,1) during these events; (f) The flow stabilizes to a quasi-steady state configuration.

As a result of the reconnection, the new large roll has the cumulative vorticity in the same direction. The reconnection event leads to a change in the flow topology, and is similar to 2D magnetic field reconnections in magnetohydrodynamics [18]. The new large roll aligned along the -45 degree diagonal (Fig. 3(c)) has vorticity opposite to that of the large roll before the reversal (Fig. 1(c)).

The strength of the new roll grows along with an emergence of two new secondary modes (1,3) and (3,1), which are generated as a result of triad interaction with the condition $k = p + q$ [12]. Note that $(1,3) = (2,2) + (-1,1)$ and $(3,1) = (2,2) + (1,-1)$. The subsequent rotation of the newly formed roll leads to flow configurations of Figs. 1(d) and (e), which are dominated by (1,1), (2,2), (1,3) and (3,1) modes. Fig. 3(d) illustrates three horizontally stacked rolls corresponding to the (1,3) mode. After around 0.01 thermal diffusive time units, the system attains a quasi-steady state with the roll aligned along −45 degree diagonal with an opposite vorticity (Fig. 3(f)) compared to the original one (Fig. 1(c)). The process would repeat for the next reversal with a difference that the bottom-left and the top-right corner rolls would reconnect during the next reversal.

The modes (1,1) and (2,2) are the most dominant ones for the quasi-steady configurations, with the (1,1) mode switching sign between reversals. The sign of the (2,2) mode or the sense of rotation of the corner rolls however remains unchanged after the reversal, as shown through symmetry arguments [12]. Another surprising observation is that the amplitude of the (2,2) mode is always positive. As a result, the hot plume of the corner rolls (at the bottom plate) always ascends via the vertical walls, and the cold plume descends via the vertical walls (see Figs. 1(c) and 3(f)). This seems to be a generic feature of many convection simulations and experiments, but its mathematical justification eludes us at present.

We now discuss heat transport during the reversal. The global Nusselt number $\text{Nu} = \int \left[ \frac{dT(y)}{dy} + vT \right] dx$ exhibits large fluctuations including negative values during a short time interval (see Fig. 4(A)). Here, the nondimensionalized $\tilde{T}(y)$ is the averaged temperature over the cross-section at height $y$. The negative Nusselt number occurs for the flow configurations resembling Fig. 3(e) in which the hot fluid parcel in the middle of the box descends while the cold parcel ascends, contrary to generic situations when the hot parcels ascend and the cold ones descend. Negative $\text{Nu}$ may appear contradictory, but it could be understood in terms of the heat flux through the cross-section at height $y$.

$$\text{Nu}(y) = \frac{dT(y)/dy + \int_x v(x,y)T(x,y)dx}{\int_x v(x,y)dx}; \quad \text{here} \int_x \text{is the integral over the line at height} \ y.$$ We compute this quantity for various horizontal cross-sections for the flow configurations corresponding to Fig. 3(a-i). As illustrated in Fig. 4(B), $\text{Nu}(y)$ fluctuates significantly in the bulk for flow configurations (c,d,e) during the reversals, with strong positive values for (d), to strong negative values for (e). Note however that $\text{Nu}(y)$ is positive near the top and bottom plates for all cases. The large fluctuations in $\text{Nu}(y)$ in the bulk is a result of the rotation of the central role formed after reconnection. We remark that the neg-
FIG. 4. (A) Time series of the global Nusselt number. The six snapshots of Fig. 3 are marked as (a-f) in this plot. (B) Plot of Nu(y) (the normalized heat transport for the cross-section at height y) vs. y during the reversal. The six curves represent the six snapshots of Fig. 3. The flow reconfigurations during the reversal lead to large fluctuations in Nu(y) in the bulk, ranging from strong positive values for (d) to strong negative values for (e). Note however that the Nu(y) is positive near both the plates (y → 0, 1).

ative Nu for the two-dimensional convective flow is due to strong geometrical constraints faced by the rolls during the reversals, and may not be present in cylindrical convection.

In summary, we show that flow reversals are caused by vortex reconnections of two attracting rolls. The flow reconfigurations during the reversals are due to the nonlinear interactions among the large-scale modes (1,1), (2,2), (1,3), and (3,1). We find large fluctuations in heat transport during the reversals, which are due to the above restructuring of the flow. The (2,2) mode, critical for the dynamics of flow reversals, is born after Ra = 2 \times 10^7 (for Pr = 1), and its strength relative to the (1,1) mode decreases monotonically afterwards. This is the reason why flow reversals are observed only for a range of parameter values.

The role of the large-scale modes described in the present letter is analogous to the “cessation-led reversal” observed in turbulent convection in cylinder [11] as well as in dynamo reversals [7], where the quadrupolar mode (equivalent to (2,2) mode) dominates the dipolar mode (equivalent to (1,1) mode) during the reversal. Future work for different geometries, and Prandtl and Rayleigh numbers would provide valuable insights that will help us build a comprehensive theory of reversals in convection and dynamo.

We are grateful to Paul Fischer and other developers of Nek5000 for opensourcing Nek5000 as well for providing valuable assistance during our work. We thank Annick Pouquet, Stephan Fauve, and Jörg Schumacher for very useful discussions. We also thank the Centre for Development of Advanced Computing (CDAC) and the Computer Center of IIT Kanpur for providing us computing time. Part of this work was supported by Swarnajayanti fellowship to MKV, and BRNS grant BRNS/PHY/20090310.

---

[1] S. Cioni, S. Ciliberto, and J. Sommeria, J. Fluid Mech. 335, 111 (1997).
[2] J. J. Niemela, L. Skrbek, K. R. Sreenivasan, and R. J. Donnelly, J. Fluid Mech. 449, 169 (2001).
[3] E. Brown, A. Nikolaenko, and G. Ahlers, Phys. Rev. Lett. 95, 084503 (2005); J. Fluid Mech. 568, 351 (2006); E. Brown and G. Ahlers, J. Fluid Mech. 568, 351 (2006); E. Brown and G. Ahlers, Phys. Rev. Lett. 98, 134501 (2007).
[4] H. Xi and K. Xia, Phys. Rev. E 75, 066307 (2007).
[5] A. Vasiliev and P. Frick, J Phys.: Conf. Ser., 318, 082013 (2011).
[6] T. Yanagisawa et al., Phys. Rev. E 82, 016320 (2010).
[7] B. Gallet et al., Geophys. and Astrophys. Fluid Dyn. (2012).
[8] K. Sugiyama et al., Phys. Rev. Lett. 105, 034503 (2010).
[9] R. Benzi and R. Verzicco, Euro. Phys. Lett. 81, 64008 (2008).
[10] M. Breuer and U. Hansen, Euro. Phys. Lett. 86, 24004 (2009).
[11] P. K. Mishra, A. K. De, M. K. Verma, and V. Eswaran, J. Fluid Mech. 688, 480 (2011).
[12] M. Chandra and M. K. Verma, Phys. Rev. E 83, 067303 (2011).
[13] G. Ahlers, S. Grossmann, and D. Lohse, Rev. Mod. Phys. 81, 503 (2009); D. Lohse and K. Q. Xia, Ann. Rev. Fluid Mech. 42, 335 (2010).
[14] K. R. Sreenivasan, A. Bershadskii, and J. J. Niemela, Phys. Rev. E 65, 056306 (2002).
[15] F. F. Araujo, S. Grossmann, and D. Lohse, Phys. Rev. Lett. 95, 084502 (2005).
[16] http://turbulence.phy.iitk.ac.in/animations/convection (video-1)
[17] H. Aref and I. Zawadzki, in New Approaches and Concepts in Turbulence, Eds. T. Dracos and A. Tsinober, Birkhäuser Basel (1993).
[18] E. Priest and T. Forbes, Magnetic Reconnection: MHD Theory and Applications, Cambridge University Press (2000).
[19] P. F. Fischer, J. Comp. Phys. 133, 84 (1997); http://http://nek5000.mcs.anl.gov/