International Conference on Canonical Metrics
and Nonlinear PDEs in Geometry

2021.03.19 - 03.21

Sponsors
School of Mathematics and Statistics, Wuhan Univ.
School of Mathematical Sciences, USTC
Tianyuan Mathematical Center in Central China
Collaborative Innovation Center of Mathematics, Wuhan Univ.
1. About the Conference

The International Conference on Canonical Metrics and Nonlinear PDEs in Geometry will be held from March 19-March 21, 2021. The aim of this conference is to promote the academic research of geometric analysis, show the current frontier and development trend of geometric analysis, strengthen the cooperation and exchange between relevant experts and scholars, and produce new ideas and new achievements. The conference will be held online because of the novel coronavirus.

Time: 19-21 March, 2021
Zoom meeting ID: 671 6335 5768
Password: 0319

Organizing Committee

Hua Chen  Co-chair, Wuhan Univ.  Qun Chen  Wuhan Univ.
Jia-Yu Li  Co-chair, USTC  Xi-Nan Ma  USTC
Xi Zhang  USTC

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2. Schedule

19 March, 2021 Friday

✧ 08:20-08:30  Opening Chair: Hua Chen, Jia-Yu Li
✧ **Chair: Fu-Quan Fang**
  ➢ 08:30-09:20  Gang Tian (Peking Univ.)  
      *Progress on Ricci flow*
  ➢ 09:20-10:10  Jie Qing (UC Santa Cruz)  
      *On Huber’s type theorems in general dimensions*
  10:10-10:20  Break
✧ **Chair: Hua Chen**
  ➢ 10:20-11:10  Yan-Yan Li (Rutgers Univ.)  
      *On the σ^k-Nirenberg problem and related topics*
  ➢ 11:10-12:00  Xiao-Hua Zhu (Peking Univ.)  
      *Kähler-Ricci flow with varied complex structures*
  12:00-14:00  Lunch
✧ **Chair: Zi-Zhou Tang**
  ➢ 14:00-14:50  Hai-Zhong Li (Tsinghua Univ.)  
      *Curvature flows for hypersurfaces in hyperbolic space and their geometric applications*
  ➢ 14:50-15:40  Jun-Bin Li (Sun Yat-Sen Univ.)  
      *Some mathematical problems in gravitational collapse*
  15:40-15:50  Break
✧ **Chair: Jia-Yu Li**
  ➢ 15:50-16:40  Juergen Jost (MPI MIS at Leipzig)  
      *Recent progress in Dirac-harmonic maps*
  ➢ 16:40-17:30  Guo-Fang Wang (Freiburg Univ.)  
      *A canonical metric on a Riemannian foliation*
20 March, 2021 Saturday

✧ **Chair: Xi-Ping Zhu**
  - 08:30-09:20 Wei-Ping Zhang (Nankai Univ.)
    *Positive scalar curvature on manifolds and foliations*
  - 09:20-10:10 Qing Han (Univ. of Notre Dame)
    *On the Negativity of Ricci Curvatures of Complete Conformal Metrics*

  **10:10-10:20 Break**

✧ **Chair: Yu-Guang Shi**
  - 10:20-11:10 Peng Lu (Univ. of Oregon)
    *Ancient solutions for Andrews’ hypersurface flow*
  - 11:10-12:00 Qi S. Zhang (UC Riverside)
    *Gradient and Eigenvalue Estimates on the canonical bundle of Kähler manifolds*

  **12:00-14:00 Lunch**

✧ **Chair: Guang-Han Li**
  - 14:00-14:50 Ji-Xiang Fu (Fudan Univ.)
    *A Hermitian mean curvature flow*
  - 14:50-15:40 Bing-Long Chen (Sun Yat-Sen Univ.)
    洛伦兹几何及爱因斯坦场方程

  **15:40-15:50 Break**

✧ **Chair: Xi-Nan Ma**
  - 15:50-16:40 Yu-Xin Dong (Fudan Univ.)
    *Prescribed Webster scalar curvatures on compact pseudo-Hermitian manifolds*
  - 16:40-17:30 Yu-Xiang Li (Tsinghua Univ.)
    *A surface whose curvature is representable by a measure*
21 March, 2021 Sunday

✧ **Chair: An-Min Li**

- 08:30-09:20 Peng-Fei Guan (McGill Univ.)
  
  *The prescribed curvature measure problem*

- 09:20-10:10 Jing-Yi Chen (UBC)
  
  *Hamiltonian Stationary Manifolds: Regularity and Compactness*

  **10:10-10:20 Break**

✧ **Chair: Qun Chen**

- 10:20-11:10 Qing-Ming Cheng (Fukuoka Univ.)
  
  *Complete self-shrinkers in $\mathbb{R}^4$*

- 11:10-12:00 Xi Zhang (USTC)
  
  *Poisson metrics on projectively flat bundles*
3. Abstract

Day 1 (19 March, 2021)

Gang Tian (Peking Univ.)
Title: Progress on Ricci flow
Abstract: Ricci flow was introduced by R. Hamilton in the 80s and has found many applications to geometry. In this talk, I will discuss some recent progress.

Jie Qing (UC Santa Cruz)
Title: On Huber’s type theorems in general dimensions
Abstract: In this talk I will report some recent joint work with Shiguang Ma on Huber’s type theorem in general dimensions our approach relies on our earlier work on the extension of Arsove-Huber in general dimensions. We also extend the injectivity theorem of Schoen-Yau for manifolds that admit conformal immersions into the round sphere and satisfy integral bounds on curvature.

Yan-Yan Li (Rutgers Univ.)
Title: On the $\sigma_k$-Nirenberg problem and related topics
Abstract: In this talk we will present some recent works on the $\sigma_k$-Nirenberg problem and related topics.

The talk is based on joint works with Luc Nguyen and Bo Wang, and joint works with Han Lu and Siyuan Lu.
Xiao-Hua Zhu (Peking Univ.)
Title: Kähler-Ricci flow with varied complex structures
Abstract: We will discuss a new version of formula for the second variation of Perelaman’s entropy in the space of Kähler-metrics near a Kähler-Ricci soliton. Then by establishing Lojasiewicz’s typed inequality for Perelman’s entropy in the space of Kähler metrics in $2\pi c_1 (M)$, we study the convergence of Kähler-Ricci flow with varied complex structures. As an application, we prove Yau-Tian-Donaldson’s conjecture for the existence of KR solitons for the deformation of complex structures in finite dimensional progress. We also prove the uniqueness of Kähler-Ricci solitons in sense of diffeomorphism orbits by using method of Kähler-Ricci flow.

Hai-Zhong Li (Tsinghua Univ.)
Title: Curvature flows for hypersurfaces in hyperbolic space and their geometric applications
Abstract: In this talk, we discuss various curvature flows for hypersurfaces in hyperbolic space and their applications to geometric inequalities.

Jun-Bin Li (Sun Yat-Sen Univ.)
Title: Some mathematical problems in gravitational collapse
Abstract: I will give a brief review on the mathematical theories of gravitational collapse in general relativity, and talk about some problems and recent developments, including my works on the formation of trapped surfaces, black holes and instability of naked singularities.
Juergen Jost (MPI MIS at Leipzig)

Title: Recent progress in Dirac-harmonic maps

Abstract: Dirac-harmonic maps are critical points of an action functional that couples a map between Riemannian manifolds with a spinor field along that map. They are motivated by the supersymmetric non-linear sigma model of quantum field theory, and they were introduced in collaboration with Chen Qun, Li Jiayu and Wang Guofang. These Dirac-harmonic maps are analytically and geometrically challenging, and they have therefore inspired a couple of new techniques in geometric analysis. Since the action functional is not bounded from below, they cannot be obtained via variational minimization. Instead, we have introduced a mixed elliptic-parabolic system, hoping to obtain them as asymptotic limits of the corresponding flow. I shall report recent results in this direction obtained with Liu Lei, Zhu Miaomiao, and Zhu Jingyong.

Guo-Fang Wang (Freiburg Univ.)

Title: A canonical metric on a Riemannian foliation

Abstract: We introduce a Yamabe type problem on Riemannian foliations. This is a joint work with Yongbing Zhang.
Wei-Ping Zhang (Nankai Univ.)
Title: Positive scalar curvature on manifolds and foliations
Abstract: We discuss some advances arising from the famous Lichnerowicz vanishing theorem which states that the A-hat genus of a closed spin manifold vanishes if the underlying spin manifold carries a Riemannian metric of positive scalar curvature.

Qing Han (Univ. of Notre Dame)
Title: On the Negativity of Ricci Curvatures of Complete Conformal Metrics
Abstract: A version of the singular Yamabe problem in bounded domains yields complete conformal metrics with negative constant scalar curvatures. In this talk, we study whether these metrics have negative Ricci curvatures. The positive mass theorem and the polyhomogeneous expansions for solutions of the Yamabe equation play an important role in the study.

Peng Lu (Univ. of Oregon)
Title: Ancient solutions for Andrews’ hypersurface flow
Abstract: We construct the ancient solutions of the hypersurface flows in Euclidean spaces studied by B. Andrews in 1994. As time $t \to 0^-$ the solutions collapse to a round point where 0 is the singular time. But as $t \to -\infty$ the solutions become more and more oval. Near the center the appropriately-rescaled pointed Cheeger-Gromov limits are round cylinder solutions $S^J \times \mathbb{R}^{n-J}$, $1 \leq J \leq n-1$. These results are the analog of the corresponding results in Ricci flow ($J = n - 1$) and mean curvature flow.
This is a joint with Jiuru Zhou at Yangzhou University.

Qi S. Zhang (UC Riverside)
Title: Gradient and Eigenvalue Estimates on the canonical bundle of Kähler manifolds
Abstract: In this joint work with Zhiqin Lu and Zhu Meng, we prove certain gradient and eigenvalue estimates, as well as the heat kernel estimates, for the Hodge Laplacian on \((m, 0)\) forms, i.e., sections of the canonical bundle of Kähler manifolds, where \(m\) is the complex dimension of the manifold. Instead of the usual dependence on curvature tensor, our condition depends only on the Ricci curvature bound. The proof is based on a new Bochner type formula.

Ji-Xiang Fu (Fudan Univ.)
Title: A Hermitian mean curvature flow
Abstract: A Hermitian curvature flow is proposed and a regularity result is discussed.

Bing-Long Chen (Sun Yat-Sen Univ.)
Title: 洛伦兹几何及爱因斯坦场方程
Abstract: 在本报告中，我们将综述几何分析方法在洛伦兹几何及爱因斯坦场方程方面的部分进展。

Yu-Xin Dong (Fudan Univ.)
Title: Prescribed Webster scalar curvatures on compact pseudo-Hermitian manifolds
Abstract: In this talk, we will discuss the problem of prescribing Webster scalar curvatures on compact strictly pseudoconvex CR manifolds. In terms of the method of upper and lower solutions and the perturbation theory of self-adjoint operators, we are able to describe the sets of Webster scalar curvature functions which are associated with pointwise CR conformal and CR conformally equivalent pseudo-Hermitian structures respectively. This is a joint work with Yibin Ren and Weike Yu.

Yu-Xiang Li (Tsinghua Univ.)

Title: A surface whose curvature is representable by a measure

Abstract: Let $\Sigma$ be a Riemann surface with a Borel measurable metric $g$. Let $\mu_g$ be a Radon measure. We assume in local complex coordinates, $g = e^{2u}g_{eu}$ with $u \in W^{1,1}$, and

$$\int \nabla u \nabla \varphi = \int \varphi \mu_g$$

for any compactly supported smooth function $\varphi$. We will study the behaviors of $g$ near the points with $\mu_g(p) \neq 0$ and the convergence of a sequence $g_k$ with bounded $|\mu_{g_k}|(\Sigma)$. This is joint work with Jingyi Chen.
Peng-Fei Guan (McGill Univ.)

Title: The prescribed curvature measure problem

Abstract: Abstract: Alexandrov initiated the prescribed curvature measure problem by considering 0-th curvature measure for convex bodies in Euclidean space, as a counterpart of the Minkowski problem related to n-th area measure. The problem for prescribing general k-th curvature measure in Euclidean space was considered and solved in 2010’s. The recent work by Fengrui Yang settled the corresponding problem in hyperbolic space. We will discuss the associated fully nonlinear PDE and the challenges of the problem in general space form.

Jing-Yi Chen (UBC)

Title: Hamiltonian Stationary Manifolds: Regularity and Compactness

Abstract: We will discuss recent advances on compactness of the space of Hamiltonian stationary Lagrangian submanifolds in the complex Euclidean space, by using a regularity theory developed with Micah Warren, and compactness in a Kähler surface by a different approach with John M.S. Ma.

Qing-Ming Cheng (Fukuoka Univ.)

Title: Complete self-shrinkers in $\mathbb{R}^4$

Abstract: In this talk, we consider classifications of complete self-shrinkers in $\mathbb{R}^{n+1}$ with constant squared norm of the second fundamental form. The following conjecture is well-known and very important:
Conjecture. A n-dimensional complete self-shrinker $X: M \to \mathbb{R}^{n+1}$ in $\mathbb{R}^{n+1}$ with constant squared norm of the second fundamental form is isometric to one of the following:

1. $\mathbb{R}^n$,
2. $S^k(\sqrt{k}) \times \mathbb{R}^{n-k}$, $k=1, 2, \ldots, n-1$,
3. $S^n(\sqrt{n})$.

For $n = 2$, Cheng and Ogata have solved this conjecture affirmatively. For the higher dimension $n$, it is hard to classify complete self-shrinkers in Euclidean space with constant squared norm $S$.

Recently, Cheng, Li and Wei (arXiv: 2003.11464, to appear in Math. Z.), under the assumption that $f_4$ is constant, we have solved this conjecture for $n = 3$ affirmatively. Theorem. Let $X: M^3 \to \mathbb{R}^4$ be a 3-dimensional complete self-shrinker in $\mathbb{R}^4$. If the squared norm $S$ of the second fundamental form and $f_4$ are constant, then $X: M^3 \to \mathbb{R}^4$ is isometric to one of

1. $\mathbb{R}^3$,
2. $S^1(\sqrt{1}) \times \mathbb{R}^2$,
3. $S^2(\sqrt{2}) \times \mathbb{R}^1$,
4. $S^3(\sqrt{3})$.

Xi Zhang (USTC)

Title: Poisson metrics on projectively flat bundles

Abstract: In this talk, we introduce our recent works on the existence of Poisson metric on projectively flat bundle over non-Kähler manifold and its applications. These works are joint with Changpeng Pan and Chuanjing Zhang.