Effects for atmospheric neutrino experiments from electron neutrino oscillations

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Abstract

The minimal interpretation of the atmospheric neutrino data suggests that the muon neutrino oscillates into another species with a mixing angle close to the maximal $\pi/4$. In the Exact Parity Symmetric Model, both the muon and electron neutrinos are expected to be maximally mixed with essentially sterile partners ($\nu'_\mu$ and $\nu'_e$ respectively). We examine the impact of maximal $\nu_e - \nu'_e$ oscillations on the atmospheric neutrino experiments. We estimate that maximal $\nu_e - \nu'_e$ oscillations will have effects on atmospheric neutrino data for $|\delta m^2 (\nu_e - \nu'_e)| > 7 \times 10^{-5} \text{ eV}^2$. For $\delta m^2$ in this range, a slight but distinctive rise in the ratio of muon-like to electron-like events is predicted for the low-energy sample. Furthermore, the ratio of low-energy electron-like events with zenith angles less than 90 deg to those with zenith angles greater than 90 deg should be greater than 1.
There are three main experimental indications that neutrinos have mass and oscillate: the atmospheric neutrino anomaly\cite{1, 2, 3}, the solar neutrino problem\cite{4} and the LSND experiment\cite{5}. In the atmospheric neutrino experiments, the fluxes of electron and muon neutrinos resulting from cosmic ray interactions with the atmosphere are measured. The results of these experiments are usually expressed in terms of the quantity $R$, where

$$R \equiv \frac{(n_\mu/n_e)_{data}}{(n_\mu/n_e)_{MC}}$$

and $n_\mu$ and $n_e$ are the numbers of $\nu_\mu$ and $\nu_e$ induced events respectively. The ratio $(n_\mu/n_e)_{MC}$ is obtained from Monte Carlo simulated events based on theoretical calculations of the fluxes\cite{6}. The experimental results are summarized in Table 1 for the low energy (sub GeV) data. Observe that while no anomaly has been observed by the Frejus\cite{7} or Nusex collaborations\cite{8}, the errors for these experiments are significantly larger than the errors quoted by the Kamiokande\cite{1} and IMB\cite{2} collaborations. In this paper we assume that the anomaly is real and represents evidence for neutrino oscillations.

In this context, the atmospheric neutrino anomaly suggests that the muon neutrino oscillates maximally (or close to maximally) with another as yet unidentified species\cite{1}. The solar neutrino problem\cite{4}, on the other hand suggests that either the electron neutrino oscillates maximally (or near maximally) with another species\cite{4, 11, 12, 13, 14} or there are small angle oscillations which are enhanced due to matter effects in the sun\cite{15}. It is tempting to assume that both the atmospheric and solar neutrino anomalies are due to essentially the same mechanism, which then suggests that they are both solved by large angle or maximal neutrino oscillations.

From a model building perspective, several simple ideas have been put forward to explain both the atmospheric and solar neutrino problems via large angle oscillations, including,

(i) The electron and muon neutrinos oscillate into each other with near maximal mixing\cite{11}. This solution is also compatible with the LSND experiment\cite{16}.

(ii) The three known neutrinos are maximally mixed with each other\cite{12, 17}. This solution can explain the atmospheric and solar neutrino problems but is incompatible with LSND.

(iii) All three of the neutrinos are maximally mixed with a sterile species (we denote the three sterile neutrinos by the notation $\nu'_e, \nu'_\mu, \nu'_\tau$)\cite{10, 18, 19}. This scenario is also compatible with the LSND experiment.

The purpose of this paper is to study some implications of this last possibility for current atmospheric neutrino experiments.

Although the solution (iii) is non-minimal, it can be theoretically well motivated. Our
interest in the above scheme comes from the observation that it is naturally realized in gauge models with exact (i.e. unbroken) parity symmetry[18]. Exact parity symmetry is possible if the particle content of the standard model is doubled (the doubling occurs because each of the known particles has a mirror image which is a distinct particle). If neutrinos have mass, and mass mixing between ordinary and mirror neutrinos occurs, then each of the known neutrinos is necessarily a maximal mixture of two states (assuming that the mixing between generations is small)[18]. There are also other interesting schemes which suggest that the ordinary neutrinos oscillate maximally into sterile states[19].

Maximal $\nu_e - \nu'_e$ oscillations reduce the flux of solar neutrinos by an energy independent factor of 2 for the large range of parameters

$$3 \times 10^{-10} \lesssim |\delta m^2_{ee}|/eV^2 \lesssim 7.5 \times 10^{-3},$$

where the upper bound is the most recent experimental bound[20]. (Note that the MSW matter effects due to neutrino propagation through the sun can be ignored if the electron neutrino oscillates maximally[10]). Maximal $\nu_\mu - \nu'_\mu$ oscillations can explain the atmospheric neutrino anomaly provided that $10^{-3} \lesssim |\delta m^2_{\mu\mu}|/eV^2 \lesssim 10^{-1}$[1, 21, 22]. The best fit (obtained from a fit to the zenith angle dependent multi-GeV neutrino data)[1] occurs for $|\delta m^2_{\mu\mu}| \approx 1.6 \times 10^{-2} eV^2$. Note however that the atmospheric neutrino experiments are sensitive to both $\nu_\mu - \nu'_\mu$ and $\nu_e - \nu'_e$ oscillations in principle. Assuming that the oscillations are exactly maximal, we will study the constraints on the parameters $\delta m^2_{ee}$ and $\delta m^2_{\mu\mu}$ suggested by the existing atmospheric neutrino experiments.

For maximal $\nu_\alpha - \nu'_\alpha$ oscillations (with $\alpha = e$ for $\nu_e - \nu'_e$ oscillations and $\alpha = \mu$ for $\nu_\mu - \nu'_\mu$ oscillations), the probability that a weak eigenstate neutrino $\nu_\alpha$ of energy $E_\nu$ remains a weak eigenstate after travelling a distance $L$ is in general[15],

$$P(\nu_\alpha \rightarrow \nu_\alpha, L, E_\nu) = 1 - \frac{\sin^2 \delta_m}{f},$$

where $\delta_m$ is given by

$$\delta_m \approx 1.27 \left[ \frac{\delta m^2_{\alpha\alpha}}{eV^2} \right] \left[ \frac{L}{km} \right] \left[ \frac{GeV}{E_\nu} \right] \sqrt{f}. $$

The quantity $f$ contains the matter effects. For oscillations in vacuum, $f = 1$. In general, $f$ is given by,

$$f = 1 + a^2.$$
interactions. For $\alpha = e$, i.e. $\nu_e - \nu'_e$ oscillations$^{[23]}$, 

$$a = -\sqrt{2} G_F (2N_e - N_n) E_{\nu} \over \delta m_{ee'}^2 ,$$

(6)

where $G_F$ is the Fermi constant and $N_e$ and $N_n$ are the number densities of electrons and neutrons of the medium through which the neutrinos propagate. Note that equal number densities for electrons and protons has been assumed in Eq.(6). For neutrino propagation through the Earth, $N_e = N_p \simeq N_n$ where $N_p$ is the number density of protons. Hence $N_e$ is related to the density $\rho$ and the proton mass $m_p$ by $N_e \simeq 0.5 \rho/m_p$.

Of course, in a detailed analysis the probability $P(\nu_\alpha \to \nu_\alpha, L, E_{\nu})$ must be averaged over $L$ and $E_{\nu}$ with an appropriate weighting factor, which takes into account the energy spectrum of the neutrinos, the cross section and so on. Such an analysis is quite difficult without detailed knowledge of the experiments (for example, the lepton detection efficiency function is not given in Ref.$^{[1]}$) and is most easily performed by the experimentalists themselves. However, much can still be learned without doing a rigorous analysis.

We define the useful quantity $D_{\alpha\alpha'}$ by the distance for which $|\delta m| = \pi/4$, which corresponds to a survival probability of $1/2$ (if $f \simeq 1$). From Eq.(4),

$$D_{\alpha\alpha'} \simeq \left[ E_{\nu} \over \text{GeV} \right] \left[ E_{\nu}^2 \over \delta m_{2\alpha\alpha'}^2 \right] \left[ 1 \over \sqrt{f} \right] \left[ 1 \over 4 \right] \left[ 1 \over 1.27 \right] \text{km.}$$

(7)

For distances $L \sim D_{\alpha\alpha'}/2$, the oscillation length is too large to have a significant effect. For $L \simeq D_{\alpha\alpha'}$, the oscillations are significant, and should deplete the number of neutrinos by a factor of about 2 (if $f \simeq 1$) after suitable averaging is performed. In the intermediate regime, $D_{\alpha\alpha'}/2 \sim L \sim D_{\alpha\alpha'}$, an $L-$ dependent depletion occurs.

In the absence of $\nu_e$ oscillations, the zenith angle dependent multi GeV atmospheric neutrino data suggest that the muon neutrino oscillates maximally with $|\delta m_{2\mu\mu'}^2| \sim 10^{-2} eV^2$. In terms of the parameter $D_{\mu\mu'}$, this corresponds to $D_{\mu\mu'} \sim 40$ km for $E_{\nu} \sim 0.6$ GeV (sub GeV data) and $D_{\mu\mu'} \simeq 400$ km for $E_{\nu} \sim 6$ GeV (multi GeV data). For the sub GeV data, therefore, there should not be much zenith angle dependence since most of the neutrinos travel distances greater than 40 km. There may be some effect for neutrinos coming close to vertically down, however at low energies the correlation between the lepton direction and the incident neutrino direction is quite weak.

Qualitatively it is clear that the atmospheric neutrino anomaly suggests that if the electron neutrino oscillates maximally with a sterile neutrino then $D_{ee'} \gg D_{\mu\mu'}$, otherwise there would not be a significant decrease in the ratio $R$. The effect of $\nu_e - \nu'_e$ oscillations will be most important for the low energy neutrino data, since for low energies, the length $D_{ee'}$ is reduced. Let us define the quantities $R_\pm$ where $R_+$ is the contribution to $R$ from neutrinos with $\cos \theta > 0$ and $R_-$ is the contribution to $R$ from neutrinos with $\cos \theta < 0$ ($\theta$
is the zenith angle, with \( \theta = 0 \) corresponding to downward travelling neutrinos). Neutrinos with \( \cos \theta > 0 \) travel through the atmosphere (where matter effects can be neglected) for distances \( 20 \lesssim L/km \lesssim 500 \), while neutrinos with \( \cos \theta < 0 \) travel in matter for distances \( 500 \lesssim L/km \lesssim 13000 \). In the absence of oscillations, \( R_+ \) and \( R_- \) should each contain approximately half of the interactions.

Note that matter effects will be important when \( a \gtrsim 1 \). Furthermore, for \( a \gtrsim \sqrt{2} \), the effects of the \( \nu_e - \nu'_e \) oscillations become suppressed and can be approximately neglected. From Eq.(3), the quantity \( a \) can be expressed as

\[
a \simeq 1.5 \left[ \frac{10^{-4} \text{ eV}^2}{\delta m_{ee}^2} \right] \left[ \frac{\rho}{4 \text{ g/cm}^3} \right] \left[ \frac{E_{\nu}}{\text{GeV}} \right].
\]

Assuming that \( \rho \simeq 4 \text{ g/cm}^3 \), the condition \( a \gtrsim \sqrt{2} \) implies that

\[
|\delta m_{ee}^2| \gtrsim 7 \times 10^{-5} \text{ eV}^2 \quad \text{for } E \simeq 0.6 \text{ GeV}.
\]

Thus for the above range of parameters the matter effects ensure that the \( \nu_e - \nu'_e \) oscillations can be approximately neglected for \( \nu_e - \nu'_e \) oscillations through the Earth. The \( \nu_e - \nu'_e \) oscillations during propagation through the atmosphere can also be neglected if \( |\delta m_{ee}^2| \lesssim 7 \times 10^{-5} \text{ eV}^2 \), since for this range of \( |\delta m_{ee}^2| \), \( D_{ee'} \gtrsim 4000 \text{ km} \gg 500 \text{ km} \).

Observe that for \( |\delta m_{ee}^2| \gtrsim 7 \times 10^{-5} \text{ eV}^2 \), the length \( D_{ee'} \gtrsim 4000 \text{ km} \) for the sub GeV neutrinos. Thus, for \( \delta m_{ee}^2 \) in this range, the \( \nu_e - \nu'_e \) oscillations will be important and will reduce the number of electron neutrinos. This will increase \( \langle R \rangle \), where the brackets \( \langle ... \rangle \) denote the average over all zenith angles. As \( |\delta m_{ee}^2| \) increases, \( D_{ee'} \) decreases and \( \langle R \rangle \) increases (and matter effects quickly become negligible for the sub-GeV neutrinos). For \( E_{\nu} \simeq 0.6 \text{ GeV} \), the value \( |\delta m_{ee}^2| \simeq 10^{-3} \text{ eV}^2 \) corresponds to \( D_{ee'} \simeq 500 \text{ km} \) which is the distance to the horizon (that is, the \( \theta = 90 \text{ deg line} \)). In order to obtain insight into the increase of \( \langle R \rangle \) with \( |\delta m_{ee}^2| \), it is useful to explicitly calculate \( \langle R \rangle \) for this point. This is because neutrinos coming from the hemisphere with \( 0 \leq \theta \leq \pi/2 \) travel distances less than \( D_{ee'} \), whereas those from the \( \pi/2 \leq \theta \leq \pi \) hemisphere travel distances greater than \( D_{ee'} \). So, \( R_- \) should be approximately equal to the standard model value since both \( \nu_e \) and \( \nu_\mu \) fluxes are reduced by a factor of 2, while \( R_+ \) will be about half of the standard model value. This leads to \( \langle R \rangle \simeq 0.67 \) for \( |\delta m_{ee}^2| \simeq 10^{-3} \text{ eV}^2 \). Clearly \( |\delta m_{ee}^2| \gtrsim 10^{-3} \text{ eV}^2 \) implies that \( \langle R \rangle \gtrsim 0.67 \), with \( R_- \simeq 1 \) and \( 0.5 \lesssim R_+ \lesssim 1 \). For some value of \( |\delta m_{ee}^2| \), \( \langle R \rangle \) becomes so large that it is disfavoured by the data that actually suggest an anomaly. Note that the current laboratory bound \( |\delta m_{ee}^2| < 7.5 \times 10^{-3} \text{ eV}^2 \)) is probably in the range where \( \langle R \rangle \) crosses over into disfavoured values. In summary then, for \( \delta m_{ee}^2 \) in the range

\[
|\delta m_{ee}^2| \gtrsim 7 \times 10^{-5} \text{ eV}^2
\]
the effects of the maximal $\nu_e - \nu'_e$ oscillations should be significant for the low energy data. The effect of the $\nu_e - \nu'_e$ oscillations with $\delta m^2_{ee'}$ in the range Eq. (I) will be to increase $\langle R \rangle$.

As well as increasing $\langle R \rangle$, the $\nu_e - \nu'_e$ oscillations will also make the flux of electron neutrinos zenith angle dependent for the low energy data. The zenith angle dependence should manifest itself by an increase in $R$ for decreasing values of $\cos \theta$. Such a result would not be expected if the anomaly is interpreted assuming only $\nu_\mu - \nu'_\mu$ oscillations and this should provide a distinctive signature for $\nu_e - \nu'_e$ oscillations. However, since the angular correlation between the neutrinos and the produced charged leptons is quite poor at low energies (r.m.s. $\sim 60^\circ$), this effect will be quite difficult to measure. Although no evidence for zenith angle dependence in the low energy data has been found by the existing experiments, the sensitivity should be greatly improved in the near future with the data expected from Superkamiokande. In this context, we remark that a sensitive way to test for $\nu_e - \nu'_e$ oscillations is to compare the measured number of electron events with $\cos \Theta > 0$, $n^+_e$, with the number of electron events with $\cos \Theta < 0$, $n^-_e$. (Here $\Theta$ is the zenith angle of the detected electron). If electron neutrino oscillations are negligible then $R_e \equiv n^+_e/n^-_e \simeq 1$. If $\nu_e$ oscillations occur with $\delta m^2_{ee'}$ in the range Eq. (I), then $R_e > 1$. Note that $R_e$ should be almost free of systematic uncertainties. With the improved statistics expected from the superKamiokande experiment it should be possible to measure $R_e$ quite accurately. Thus the hypothesis that $\nu_e$ oscillates maximally with a sterile $\nu'_e$ can be tested, provided that $\delta m^2_{ee'}$ is in the range Eq. (I).

Finally, it only remains to comment on the impact of $\nu_e - \nu'_e$ oscillations on the multi-GeV data. For $|\delta m^2_{ee'}| \lesssim 7 \times 10^{-4}$ eV$^2$, $a \lesssim \sqrt{2}$ for $E_\nu \sim 6$ GeV [see Eq. (I)], which means that the matter effects will suppress the $\nu_e - \nu'_e$ oscillations through the Earth. Thus, for much of the parameter space of interest, Eq. (I), $\nu_e - \nu'_e$ oscillations will not modify the expectations for the multi-GeV atmospheric neutrino data. There may be some effects for $\nu_e - \nu'_e$ oscillations with $\delta m^2_{ee'}$ in the range $|\delta m^2_{ee'}| \simeq 7 \times 10^{-4}$ eV$^2$. The effect of $\delta m^2_{ee'}$ in this range would be to increase $R$ for $\cos \theta \sim -1$.

In conclusion we have examined the implications of the hypothesis that both the $\nu_\mu$ and $\nu_e$ neutrinos are maximally mixed with sterile partners for the atmospheric neutrino experiments. The assumption of maximal mixing means that the oscillations can be described by just two parameters, $\delta m^2_{ee'}$ and $\delta m^2_{\mu\mu'}$. We have shown that the maximal $\nu_e - \nu'_e$ oscillations should not significantly affect the atmospheric neutrino experiments if $\delta m^2_{ee'}$ is in the range $|\delta m^2_{ee'}| \lesssim 7 \times 10^{-5}$ eV$^2$. For $\delta m^2_{ee'}$ in the remaining range, $|\delta m^2_{ee'}| \simeq 7 \times 10^{-5}$ eV$^2$, the effects of the $\nu_e - \nu'_e$ oscillations will be significant. They should lead to the distinctive signature of an increasing value of $R$ for lower values of $\cos \theta$ for the low-energy neutrino sample. This prediction may be tested by the new data expected from Superkamiokande. A related effect is that the value of $R$ averaged over zenith angle, $\langle R \rangle$, should be somewhat higher than the expected value in the absence of $\nu_e - \nu'_e$ oscillations (again for the low-energy sample). Further data, especially from Superkamiokande, should clarify these issues.
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mantle (that is $\rho \simeq 4 \text{ g/cm}^3$) since the neutrinos which pass through the core must also pass through at least 6000 km of mantle. In the region of interest where $a \lesssim \sqrt{2}$ in the mantle, $D_{ee'} \lesssim 4000 \text{ km} \leq 6000 \text{ km}$. Thus any suppression of the neutrino oscillations while the neutrinos are propagating through the core does not matter.
Table Captions

Table 1: Summary of the current low energy atmospheric neutrino experiments ($E_\nu \sim 0.6$ GeV). Note however that the Frejus result includes both contained and semicontained events.
Table 1

| Experiment      | \((R)\)                                      |
|-----------------|----------------------------------------------|
| Kamiokande¹     | \(0.60^{+0.06}_{-0.05}(\text{stat.}) \pm 0.05(\text{syst.})\) |
| IMB²            | \(0.54 \pm 0.05(\text{stat.}) \pm 0.12(\text{syst.})\) |
| Soudan2³        | \(0.72 \pm 0.19(\text{stat.})^{+0.05}_{-0.07}(\text{syst.})\) |
| Frejus⁷         | \(1.00 \pm 0.15(\text{stat.}) \pm 0.08(\text{syst.})\) |
| Nusex⁸          | \(1.04^{+0.28}_{-0.32}\)                     |