D-wave $B_c$ meson production at LHC

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Abstract. Hadronic production of $D$ wave states of $B_c$ are discussed. Preliminary estimations of $D$-wave $B_c$ yield are presented for LHC conditions.

1. Introduction

Recently $2S$ excitations of $B_c$ mesons have been discovered by the CMS [1, 2, 3] in $B_c\pi^+\pi^-$ spectrum. The observation has been confirmed by the LHCb experiment [4]. These excellent experimental results and the old theoretical prediction, that $D$ states can also decay to $B_c\pi^+\pi^-$ with probability $\sim 20\%$ [5], stimulate us to estimate the cross section of $D$ wave $B_c$ states in hadronic interactions. It is worth to note that the relative $B_c(2S)$ yield $\sigma(B_c(2S))/\sigma(B_c)$ published by CMS ($\sim 8\%$) is in a good agreement with our prediction ($\sim 10\%$) [6, 7]. This why we hope that our prediction for $D$ wave states relative yield obtained within analogous technique will fairly good describe the future experimental observation of the discussed states.

2. Calculation technique

To estimate the production amplitude of $D$ wave $B_c$ states we use the analogous technique as for $S$ and $P$ waves, namely, the color singlet model and the $\delta$-approximation within NRQCD (see, for example [8]):

$$A \sim \int d^3q \Psi^*(\vec{q}) \left\{ T(p_i, \vec{q})|_{\vec{q}=0} + q^\alpha \frac{\partial}{\partial q^\beta} T(p_i, \vec{q})|_{\vec{q}=0} + \frac{1}{2} q^\alpha q^\beta \frac{\partial^2}{\partial q^\alpha \partial q^\beta} T(p_i, \vec{q})|_{\vec{q}=0} + \cdots \right\}$$

(1)

where $T$ is the amplitude of four heavy quark gluonic production with momenta $p_i$ in LO approximation (36 diagrams), $\vec{q}$ is a quark three momentum in the $B_c$ meson rest frame, and $\Psi^*(\vec{q})$ is the $B_c$ meson wave function.

For $D$ wave states two first terms in (1) are equal to zero, and therefore an amplitude in that case is proportional to second derivative of the wave function at origin $R''(0)$ and second derivatives of $T$ over $\vec{q}$. The amplitudes for the spin singlet $A^{J_3^s}$ ($J = 2$, $j_z = l_z$) and for the spin triplet $A^{J_3^t}$ ($J = 1, 2, 3$; $j_z = s_z + l_z$) can be expressed as follows:

$$A^{J_3^s} = \frac{1}{2} \sqrt{\frac{15}{8\pi}} R''(0) e^{\alpha\beta}(j_z) \frac{\partial^2 M(q)}{\partial q^\alpha \partial q^\beta}|_{q=0} ,$$

(2)

$$A^{J_3^t} = \frac{1}{2} \sqrt{\frac{15}{8\pi}} R''(0) \Pi^{\alpha\beta\rho}(j_z) \frac{\partial^2 M_\mu(q)}{\partial q^\alpha \partial q^\beta}|_{q=0} ,$$

(3)
where $\epsilon^p$ and $\epsilon^{a\beta}$ are vector and tensor polarizations and

$$\Pi^{J, \alpha\beta \rho}(j_z) = \sum_{l_z, s_z} \epsilon^{a\beta}(l_z) \epsilon^\rho(s_z) \cdot C^{Jj_z}_{s_z l_z},$$

(4)

where $C^{Jj_z}_{s_z l_z}$ are Clebsch-Gordan coefficients.

The states with definite spin value are constructed by operators

$$\mathcal{P}(0, 0) = \frac{1}{\sqrt{2}} \{ v_+(p_b + k) \bar{u}_+(p_c - k) - v_-(p_b + k) \bar{u}_-(p_c - k) \},$$

(5)

and

$$\mathcal{P}(1, S_z) = \begin{cases} \mathcal{P}(1, 1) = v_-(p_b + k) \bar{u}_+(p_c - k) \\ \mathcal{P}(1, 0) = \frac{1}{\sqrt{2}} \{ v_+(p_b + k) \bar{u}_+(p_c - k) + v_-(p_b + k) \bar{u}_-(p_c - k) \} \\ \mathcal{P}(1, -1) = v_+(p_b + k) \bar{u}_-(p_c - k) \end{cases}$$

(6)

The spinors in (5) and (6) are expressed as follows:

$$v_{\lambda_1}(p_b + k) = (1 - \frac{k}{2m_b}) v_{\lambda_1}(p_b),$$

(7)

$$\bar{u}_{\lambda_2}(p_c - k) = (1 - \frac{k}{2m_c}) \bar{u}_{\lambda_2}(p_c),$$

(8)

where $p_b = \frac{m_b}{m_b + m_c} P_{B_c}$, $p_c = \frac{m_c}{m_b + m_c} P_{B_c}$ and $k(\bar{q})$ is a Lorentz boost of $(0, \bar{q})$ to the system where $B_c$ momentum is equal to $\bar{P}_{B_c}$.

Amplitudes and there derivatives have been calculated numerically as below:

$$\frac{\partial^2 M}{\partial q^2_{\pi x}} = \frac{M(p_1, \bar{q}_x) + M(p_1, -\bar{q}_x) - 2M(p_1, 0)}{\Delta^2},$$

(9)

and

$$\frac{\partial^2 M}{\partial q_{\pi x} \partial q_y} = \frac{M(p_1, \bar{q}_x + \bar{q}_y) + M(p_1, 0) - M(p_1, \bar{q}_x) - M(p_1, \bar{q}_y)}{\Delta^2}$$

(10)

To simplify calculations we square and summarize amplitude, keeping only spin value $S = 0$ or $S = 1$. Amplitude squared for spin $S = 0$ ($1^3D_1$) is given by the following equation:

$$|A_{S=0}|^2 = \left( \frac{5}{16\pi} \right) |R_D''(0)|^2 \times \left[ \left( \frac{\partial^2 M_{S=0}}{\partial q^2_{\pi x}} \right)^2 + \left( \frac{\partial^2 M_{S=0}}{\partial q^2_{\pi y}} \right)^2 + \left( \frac{\partial^2 M_{S=0}}{\partial q_{\pi x} \partial q_y} \right)^2 \right]$$

$$+ 3 \left( \frac{\partial^2 M_{S=0}}{\partial q_{\pi x} \partial q_y} \right)^2 + \left( \frac{\partial^2 M_{S=0}}{\partial q_{\pi y} \partial q_x} \right)^2 + \left( \frac{\partial^2 M_{S=0}}{\partial q_{\pi x} \partial q_{\pi y}} \right)^2$$

$$- \operatorname{Re} \left( \frac{\partial^2 M_{S=0} \partial^2 M_{S=0}^{*}}{\partial q^2_{\pi x} \partial q^2_{\pi y}} + \frac{\partial^2 M_{S=0} \partial^2 M_{S=0}}{\partial q^2_{\pi y} \partial q^2_{\pi x}} + \frac{\partial^2 M_{S=1, s=1} \partial^2 M_{S=0}^{*}}{\partial q^2_{\pi y} \partial q^2_{\pi x}} + \frac{\partial^2 M_{S=1, s=j} \partial^2 M_{S=0}^{*}}{\partial q^2_{\pi x} \partial q^2_{\pi y}} \right).$$

(11)

Sum of amplitudes squared for states with spin $S = 1$ ($1^3D_1$, $1^3D_2$, $1^3D_3$) is presented below:
Table 1. Gluonic cross section of $1S$ and $1D$ $B_c$ states for different gluonic energies. The following parameter values have been chosen for calculations: $\alpha_s = 0.2$, $|R^d(0)(D)|^2 = 0.1$ GeV$^7$, $|R(0)(S)|^2 = 1.4$ GeV$^3$.

| energy, $\sqrt{\sigma_{gg}}$, GeV | $\sigma_{gg}$, pb |
|----------------------------------|----------------|
| 20                               | 21.9 0.21     |
| 30                               | 32.2 0.51     |
| 50                               | 29.1 0.60     |
| 100                              | 16.1 0.39     |
| 200                              | 7.7 0.11      |

![Figure 1. Transverse momentum dependence of the gluonic cross section at 100 GeV for $D$ and $S$ states. Red lines: $S = 1$, blue lines: $S = 0$, solid lines: $D$ waves, dashes lines: $S$ waves scaled by 0.01.](image)

3. Cross section estimations

We predict that relative yield of $B_c(D)$ comparing to $B_c(1S)$ is about $1 \pm 2\%$ (see table 1, where the cross section values of gluonic production are presented the different gluonic energies). It is worth to note, that the predicted yield ratio of states with spin $S = 1$ ($1^3D_1$, $1^3D_2$, $1^3D_3$) v.s. states with spin $S = 0$ is in accordance with a simple spin counting rule:

$$\frac{\sigma(1^3D_1 + 1^3D_2 + 1^3D_3)}{\sigma(1^1D_1)} \sim 3.$$  \hspace{1cm} (13)

As it is shown in figure 1 the kinematical distributions for $D$ wave states are quite similar to the distributions for $S$ wave states.

4. To do

We plan to convolute the obtained gluonic cross section values with the PDF to obtain the hadronic production cross section LHC conditions. However, it can be already concluded from Table 1 that relative yield of $D$ wave states in hadronic interaction is about $1 \pm 2\%$. Also prediction uncertainties will be carefully estimated.
5. Conclusions

$B_c(2S)$ excitations have been observed at LHC in $B_c \pi^+\pi^-$ spectrum [1, 2, 3, 4], and this stimulated us to estimate possibilities to search for $B_c(D)$ excitations in the same spectrum. We estimate $B_c(D)$ states yield in hadronic production as $\sim 1 - 2\%$. Our estimations for $D$-wave states of $B_c$ in hadronic production do not contradict the analogous estimations for $e^+e^-$ annihilation [9].

We propose to search for the $D$ wave excitations at LHC at large statics. However it is quite challenging experimental task due to the small relative yield of such states.

Acknowledgments

We would like to thank V. Galkin for help and useful discussions. Also we acknowledge the support from RFBR (Grant 20-02-00154 A).

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