Latitude and power characteristics of solar activity in the end of the Maunder minimum

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Abstract

Two important sources of information about sunspots in the Maunder minimum are the Spörer catalog [1] and observations of the Paris observatory [2], which cover in total the last quarter of the 17th and the first two decades of the 18th century.

These data, in particular, contain information about sunspot latitudes. As we showed in [3, 4], dispersions of sunspot latitude distributions are tightly related to sunspot indices, so we can estimate the level of solar activity in this epoch by a method which is not based on direct calculation of sunspots and is weakly affected by loss of observational data.

The latitude distributions of sunspots in the time of transition from the Maunder minimum to the common regime of solar activity proved to be wide enough. It gives evidences in favor of, first, not very low cycle No. −3 (1712–1723) with the Wolf number in maximum W = 100 ± 50, and, second, nonzero activity in the maximum of cycle No. −4 (1700–1711) W = 60 ± 45.

Therefore, the latitude distributions in the end of the Maunder minimum are in better agreement with the traditional Wolf number and new revisited indices of activity SN and GN [5, 6] than with the GSN [7]; the latter provide much lower level of activity in this epoch.

1 Introduction

The epoch of the Maunder minimum (MM) [8] lasted, as it is traditionally believed, since the middle of the 17th to the beginning of the third decade of the 18th century. It was very special in a low level of solar activity as well as its eminent hemisphere asymmetry. Now nobody doubts that the activity of the Sun in this epoch was low; however, it is still discussed how low it was
Figure 1: The “Maunder butterflies” for the Spörer catalog [1] (empty circles) and the observations of the Paris observatory [2] (filled circles).

(see, e.g., [9, 10]). The related question is when the solar activity turned back to its normal regime. The answers of these questions are tangled by the fact that the observations of sunspots in this epoch are incomplete. That is why the problem of valid estimations of solar activity indices on the base of fragmentary observational data is of special importance.

Two important sources of information about sunspot group during the MM are the Spörer catalog of sunspots [1] and the observations of the Paris observatory [2], which in total cover the most part of the epoch of grand minimum (1672–1719). These sources include information not only on numbers, but also on heliolatitudes of sunspots. As we showed in [3, 4], there is a high correlation between the latitude dispersions of sunspots and power of solar activity. Therefore, we can make independent estimates of the activity level in this epoch.

2 Data and method

We used latitudes of sunspots from the paper of Spörer [1] (64 observations) and observations of the Paris observatory (213 observations), which were digitized and compiled to a single catalog in [11]. The “Maunder butterflies” diagram for
these catalogs are plotted in Fig. 1. For these data we calculated the “index of sunspot groups” G, which is equal to yearly averaged number of daily observed groups, and yearly dispersions of absolute values of heliographic latitudes of sunspots $\sigma^2_\phi$.

In these data it is usually unclear whether a single sunspot or a sunspot group was observed, and we will treat all observations as groups. Treating them in opposite way, i.e. as individual sunspot, would affect G but not $\sigma^2_\phi$, and it is the latter values that are of primary importance for us.

We also calculated indices G and $\sigma^2_\phi$ for the extended Greenwich/NOAA catalog (GC) (1875–2015) [12].

In Fig. 2 the dependence G — $\sigma^2_\phi$ for GC is presented. We do not take into account years of cyclic minimums and adjacent years (the empty circles in Fig. 1), because in that times wings of the “Maunder butterflies” tend to overlap, and, therefore, $\sigma_\phi$ can be overestimated. The rest of data (the filled
circles) are described well (with the correlation coefficient \( r = 0.88 \)) by the linear regression
\[
\sigma_\phi^2 = a + b G, \tag{1}
\]
where \( a = 13.6 \pm 1.0 \text{ deg}^2 \) and \( b = 3.09 \pm 0.16 \text{ deg}^2 \).

This relation is quite stable to loss of data. For example, if to choose randomly only 2% of sunspot groups observations from GC (hereafter we will refer to the ratio of the number of the resiudary observations of sunspot groups to their total number as “the loss ratio” \( q \); in this case \( q = 0.02 \)), the errors raise, but the coefficients of the regression, within the error limits, do not change: \( a = 14.5 \pm 1.9 \text{ deg}^2 \) and \( b = 2.98 \pm 0.26 \text{ deg}^2 \) \((r = 0.75)\) (Fig. 2b). (The dependence of the coefficients on \( q \) was in more details discussed in [4].) One can use regression (1) to obtain estimates of \( G \) by \( \sigma_\phi^2 \). The standard errors of the obtained \( G \) can be estimated as \( \delta G = \Delta/b \), where
\[
\Delta = \sqrt{\frac{1}{N} \sum_{i=1}^N \left( (\sigma_\phi^2)_i - a - b G_i \right)^2} \tag{2}
\]
is the rms of the regression residuals (here the subscripts \( i \) is the number of the year for which indices are calculated). Strictly speaking, one should calculate the residuals in (2) for a given interval of \( G \), but they are weakly dependent on \( G \) (see Fig. 2) and we can look for the estimate of errors summing over the total set of indices \( i \). The dependence of \( \delta G \) on the loss ratio \( q \) is shown in Fig. 3, where each point for the given \( q \) was calculated as a mean of 12 random runs.

Having reconstructed \( G \) by known \( \sigma_\phi \), we can estimate the loss ratio \( q \) as \( G_0/G \), where \( G_0 \) is (generally speaking, underestimated) “index of sunspot
groups”, calculated by a fragmentary observational data. After that we can find
the error of reconstruction $\delta G$ for the given $q$ using the empirical dependence
shown in Fig. 3.

3 Results and discussion

Until the beginning of the 18th century the number of observation in the catalog
under investigation is too small to estimate the latitude dispersion correctly.
Therefore, we apply the described method to the data starting from 1700. The
estimates are made only for years with four or more observations and for the
cases when it leads to positive values. Besides, in the first cycle we have taken
into account that sunspots existed in the south hemisphere only and the estimate
evaluated by the regression must be divided by 2.

We will compare the estimates with other indices of activity known for this
epoch: W, GSN and their recently revised versions SN and GN [5, 6, 13]. To
make the comparison more transparent it is convenient to renormalize the ob-
tained G, introducing $G_w = 11.9 G$, where the coefficient is selected to minimize
the rms difference between W and $G_w$ for the epoch 1875–1976. The same pro-
Table 1: Amplitudes and moments (in brackets) of the 11-year cycles of solar activity Nos. −4 and −3 in different sunspot indices.

| No. of cycle | Amplitudes of cycles and years of maxima | Gw | W | GSN | SN | GNw |
|--------------|------------------------------------------|----|---|-----|----|-----|
| −4           |                                          | 58 ± 44 (1703) | 58 (1706) | 5.5 (1705) | 97 (1705) | 70 ± 12 (1705) |
| −3           |                                          | 101 ± 53 (1716) | 63 (1717) | 34 (1719) | 105 (1717) | 93 ± 16 (1716)* |

Procedure is made for GN, leading to GNw = 13.2 GN.

The correlation coefficient of yearly indices W, Gw, GSN and GNw for the Greenwich epoch 1875–1976 is higher than 0.98 and their rms difference is less than 10 units. Therefore, for rough estimates of activity in MM we do not make difference between these four indices, expecting them to give approximately the same, by the order of magnitude, level of activity.

In Fig. 4 we compare these indices and our estimates Gw for years 1700–1719 (cycles Nos. −3 and −4 in Wolf’s numeration). Comparison of amplitudes and moments of maxima of cycles is made in Table 1 (the asterisk marks the year of the first of two maximums of GNw in cycle −3).

One can see that our estimates of amplitudes are, in spite of large uncertainties, in fair agreement with three indices (W, SN, GNw) and in significantly less agreement with GSN. The latter index is lower for both cycles and its difference from Gw is more than 1.2 standard deviations; it means that Gw > GSN with probability about 90%. The moment of the sunspot latitude dispersion maximum in cycle −3 also agrees with other data. For cycle −4 it is shifted three years to the past, which can be a result of loss of data in years 1704–1706.

Of course, the obtained estimates are correct under assumptions that a) the latitudes of sunspots in the catalogs do not contain systematic errors, and b) the linear regression found for “common” epoch was the same in epochs of grand minimums. Under these assumptions the latitude distribution of sunspots, in agreement with the Wolf number and new revisited indices of activity SN and GN, gives independent evidences in favor of not extremely low cycles −3 and −4. Thus, the classical Wolf numbers, evidently, describe solar activity in the end of the Maunder minimum more correct than GSN do. The latitudinal data also confirm the conclusion (see [6]) that the MM ended in the very beginning of the 18th century rather than in 1720s.

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