Effect of Fock terms on nuclear symmetry energy based on Lorentz-covariant decomposition of nucleon self-energies

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Abstract. Using the Lorentz-covariant decomposition of nucleon self-energies with relativistic mean-field approximation, we study the effect of Fock terms on the density dependence of nuclear symmetry energy, \( E_{\text{sym}} \). It is found that the Fock contribution suppresses the potential part of \( E_{\text{sym}} \) at higher densities, and the constraint from the heavy-ion collision data is in favor of the present result including exchange terms with the cutoff parameters given by the CD-Bonn potential. In addition, not only the isovector-vector (\( \rho \)) meson but also the isoscalar (\( \sigma, \omega \)) and \( \pi \) mesons give influence on the potential part of \( E_{\text{sym}} \) through the exchange diagram.

1. Introduction
The nuclear symmetry energy, \( E_{\text{sym}} \), is a significant physical quantity in nuclear physics and in astrophysics [1, 2]. It plays an important role not only in the properties of isospin-asymmetric nuclear matter but also in the equation of state (EoS) for a neutron star. Since the recent astrophysical measurements of massive neutron stars [3, 4] and the gravitational wave from binary neutron-star merger give stringent constraints on the EoS [5, 6], the study of \( E_{\text{sym}} \) is in particular required to understand the observations by multi-messenger astronomy. Although many theoretical calculations on \( E_{\text{sym}} \) have been currently performed, various results for its density dependence have been reported so far and it is undetermined yet [7].

The relativistic mean-field (RMF) models based on Quantum Hadrodynamics (QHD) have been often applied to study the astrophysical phenomena as well as the properties of nuclear matter [8, 9]. At present, most of the models are calculated within relativistic Hartree (RH) approximation, in which only the direct diagram is considered [10, 11]. Recently, several EoSs for neutron stars are suggested by the improved many-body calculations, for instance the relativistic Hartree-Fock (RHF) approximation [12, 13, 14] or the Dirac-Brueckner-Hartree-Fock (DBHF) approach [15, 16], in which the exchange contribution as well as the direct contribution are taken.
into account. However, it has not yet been investigated in detail how the exchange terms affect the properties of dense matter.

In the present study, using the Lorentz-covariant decomposition of nucleon self-energies based on the Hugenholtz–Van Hove (HVH) theorem [17], we investigate the effect of exchange diagrams on $E_{\text{sym}}$ in RHF approximation.

2. Relativistic Mean-Field Lagrangian Density

For the description of uniform nuclear matter, we present the relativistic formulation based on the QHD model. The total Lagrangian density is written as

$$\mathcal{L} = \sum_{N=p,n} \bar{\psi}_N (\gamma^\mu \partial_\mu - M_N) \psi_N + \mathcal{L}_M + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{NL}},$$

(1)

where $\psi_N$ is the nucleon ($N = p, n$) field with the mass in vacuum, $M_N = 939$ MeV, and $\mathcal{L}_M$ is for the meson terms [12, 13, 14, 18]. For simply, we here consider the isoscalar ($\sigma$ and $\omega$) and isovector ($\rho$ and $\pi$) mesons in the interaction Lagrangian density,

$$\mathcal{L}_{\text{int}} = \sum_{N=p,n} \bar{\psi}_N \left(g_{\sigma} \sigma - g_{\omega} \gamma_{\mu} \omega^\mu - g_{\rho} \gamma_{\mu} p^\mu \cdot \tau_N + \frac{f_\rho}{2M_N} \sigma_{\mu\nu} \rho^\mu \cdot \tau_N - \frac{f_\pi}{M_N} \gamma_{\sigma} \gamma_{\mu} \rho^\mu \pi \cdot \tau_N\right) \psi_N.$$  

(2)

The $\sigma$, $\omega$, $\rho$, and $\pi$ coupling constants are respectively denoted by $g_{\sigma}$, $g_{\omega}$, $g_\rho$, and $f_\pi$, while $f_\rho$ is the $\rho$-N tensor coupling constant. In addition, the nonlinear potential for $\sigma$ meson, $\mathcal{L}_{\text{NL}} = g_\sigma \sigma^3/3 + g_3 \sigma^4/4$, is considered in $\mathcal{L}$. It is possible to include the tensor coupling for $\rho$ meson and the pseudo-vector coupling for $\pi$ meson through the exchange term. Furthermore, the momentum dependence of the self-energies comes from the exchange terms as well.

The coupling constants, $g_{\sigma}$, $g_{\omega}$, $g_\rho$, $g_\pi$, and $g_3$, are determined so as to fit the properties of nuclear matter at the saturation density, $\rho_0 = 0.16$ fm$^{-3}$, namely the saturation energy ($-160$ MeV), the effective nucleon mass ($M^*_N/M_N = 0.70$), the incompressibility $K_0 = (250$ MeV), and the currently estimated value of $E_{\text{sym}}$ ($32.5$ MeV). The coupling constants, $f_\rho$ and $f_\pi$, take the empirical values, $f_\rho/g_\rho = 6.0$ and $f_\pi^2/4\pi = 0.08$. For comparison, we present the RH result, which is calibrated so as to adjust the same values of $M^*_N$ and $K_0$ in the RHF calculation.

3. Nucleon self-energy and Schrödinger-equivalent potential

In uniform matter, the nucleon self-energy can be generally written as [8, 19]

$$\Sigma_N(k) = \Sigma^s_N(k) - \gamma_0 \Sigma^0_N(k) + (\gamma \cdot \hat{k}) \Sigma^\tau_N(k),$$

(3)

with $\hat{k}$ being the unit vector along the (three) nucleon momentum, $k$. It is divided into the scalar ($s$), time ($0$), and space ($\tau$) components, which provide the effective nucleon mass, four momentum, and energy in matter:

$$M^*_N(k) = M_N + \Sigma^s_N(k),$$

(4)

$$k^\mu_N \equiv \left(k^0, k^\tau_N \right) = \left(k^0 + \Sigma^0_N(k), k + \hat{k} \Sigma^\tau_N(k)\right),$$

(5)

$$E^*_N(k) = \left(k^2 + M^2_N(k)\right)^{1/2}.$$  

(6)

In order to clarify the effect of Fock terms on the matter properties, it is important to study the momentum dependence of nucleon self-energies, $\Sigma^{s,0,\tau}_N$. We consider the so-called Schrödinger-equivalent potential (SEP) based on the Dirac equation with Lorentz-covariant scalar and vector self-energies for nucleon [20],

$$U^{\text{SEP}}_N(k, \epsilon_k) = \Sigma^s_N(k) - \frac{E^*_N(k)}{M_N} \Sigma^0_N(k) + \frac{1}{2M_N} \left(\left[\Sigma^s_N(k)\right]^2 - \left[\Sigma^0_N(k)\right]^2\right),$$

(7)
Figure 1. Energy dependence of $U_{N}^{\text{SEP}}$ in symmetric nuclear matter at $\rho_0$. We also show the results of the nucleon-optical-model potential extracted from analyzing nucleon-nucleus scattering data [22] and the Schrödinger-equivalent potential obtained by Dirac phenomenology for elastic proton-nucleus scattering data [23], which are respectively denoted by X.-H. Li et al. and Hama et al.

where the nucleon kinetic energy, $\epsilon_k$, reads $\epsilon_k = E_N - M_N$ with $E_N$ being the single-particle energy for nucleon.

The energy dependence of single-nucleon potential (or nucleon optical potential), $U_{N}^{\text{SEP}}$, is depicted in Fig. 1. We here show the results of the RH and RHF calculations. In the RHF1 case, we employ the cutoff parameters given by the CD-Bonn potential [21], while those in the RHF2 case are adjusted so as to cover the scattering data [22, 23]. Due to the momentum dependence of $\Sigma_{s,0,v}^{N}$, $U_{N}^{\text{SEP}}$ depends on $\epsilon_k$ non-linearly in both RHF1 and RHF2 cases. Meanwhile, $U_{N}^{\text{SEP}}$ is proportional to $\epsilon_k$ in the RH case because of the constant $\Sigma_{s,0,v}^{N}$. We note that $U_{N}^{\text{SEP}}$ strongly depends on the effective nucleon mass [24].

4. Lorentz-covariant decomposition of nuclear symmetry energy

According to the HVH theorem [25, 26], the nucleon chemical potential in asymmetric nuclear matter should be equal to its Fermi energy. Thus, the single-particle energy at Fermi surface is generally given by $E_N(k_{F_N}) = d(\rho_B E_B)/d\rho_B$, where $k_{F_N}$ is the Fermi momentum for nucleon, $E_B$ is the nuclear binding energy per nucleon, and the total baryon density, $\rho_B$, reads $\rho_B = \rho_p + \rho_n$ with $\rho_p$ ($\rho_n$) being the proton (neutron) density. Therefore, $E_{\text{sym}}$ can be written as

$$E_{\text{sym}} = \frac{1}{8} \rho_B \left( \frac{\partial}{\partial \rho_p} - \frac{\partial}{\partial \rho_n} \right) \left[ E_p(k_{F_p}) - E_n(k_{F_n}) \right]_{\rho_p=\rho_n}. \quad (8)$$

With the self-consistent calculations of $\Sigma_{s,0,v}^{N}$ under the conditions given in Eqs. (4)–(6), $E_{\text{sym}}$
Figure 2. Contents of $E_{\text{sym}}$ at $\rho_0$. The black and blue bands show respectively the total and kinetic symmetry energy, $E_{\text{sym}}$ and $E_{\text{sym}}^{\text{kin}}$. The $E_{\text{sym}}^{\text{pot}}$ is given by the Lorentz-covariant components, $E_{\text{sym}}^s$, $E_{\text{sym}}^0$, and $E_{\text{sym}}^v$.

is divided into the kinetic and potential terms as $E_{\text{sym}} = E_{\text{sym}}^{\text{kin}} + E_{\text{sym}}^{\text{pot}}$.

\begin{equation}
E_{\text{sym}}^{\text{kin}} = \frac{1}{6} \frac{k_F^s}{E_F^s} k_F^s,
\end{equation}

\begin{equation}
E_{\text{sym}}^{\text{pot}} = \frac{1}{8} \rho_B \left( \frac{M^s}{E_F^s} \partial \Sigma_{\text{sym}}^s - \partial \Sigma_{\text{sym}}^0 + \frac{k_F^s}{E_F^s} \partial \Sigma_{\text{sym}}^v \right),
\end{equation}

with $k_F = k_{F_p} = k_{F_n}$, $E_F^s = \sqrt{k_F^s + M_N^s}$, and $\partial \Sigma_{\text{sym}}^s[v]\equiv \left( \frac{\partial}{\partial \rho_p} - \frac{\partial}{\partial \rho_n} \right) \left( \Sigma_{\text{sym}}^s[v] - \Sigma_{\text{sym}}^0 \right)_{\rho_p=\rho_n}$.

We find that $E_{\text{sym}}^{\text{pot}}$ can be divided into the scalar ($E_{\text{sym}}^s$), time ($E_{\text{sym}}^0$), and space ($E_{\text{sym}}^v$) components, $E_{\text{sym}}^{\text{pot}} = E_{\text{sym}}^s + E_{\text{sym}}^0 + E_{\text{sym}}^v$, based on the Lorentz-covariant structure of $\Sigma_{\text{sym}}^s$.

In Fig. 2, we present the detail of $E_{\text{sym}}$ at $\rho_0$. We cannot see any large difference in $E_{\text{sym}}^{\text{kin}}$ between the RH and RHF calculations, while $E_{\text{sym}}^{\text{pot}}$ strongly depends on the exchange contribution. Since $E_{\text{sym}}^s$ and $E_{\text{sym}}^v$ do not affect $E_{\text{sym}}$ in the RH case, $E_{\text{sym}}^{\text{pot}}$ is given by only the direct term through $E_{\text{sym}}^{\text{pot}} = g_\rho^2 \rho_B/(2m_\rho^2)$ with $m_\rho$ being the free mass of $\rho$ meson. Meanwhile, in the RHF1 and RHF2 cases, $E_{\text{sym}}^{\text{pot}}$ is mainly composed by the Fock contribution, which is calculated by a cancellation of large positive and negative values of $E_{\text{sym}}^0$ and $E_{\text{sym}}^s$, respectively. The direct contribution in the RHF2 case is somewhat larger than that in the RHF1 case, but they are smaller than that in the RH case. It is also found that the space component, $E_{\text{sym}}^v$, is negligible at $\rho_0$ in both RHF cases.

In the left panel of Fig. 3, we present the density dependence of $E_{\text{sym}}$, $E_{\text{sym}}^{\text{kin}}$ and $E_{\text{sym}}^{\text{pot}}$. The present results of $E_{\text{sym}}$ are consistent with the constraints calculated by the isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU04) transport model ($E_{\text{sym}} = 31.6(\rho_B/\rho_0)^x$ with $x = 0.69$–1.05) [27, 28] and by the improved quantum molecular dynamics (ImQMD) transport model ($E_{\text{sym}} = 12.5(\rho_B/\rho_0)^{2/3} + 17.6(\rho_B/\rho_0)^\gamma$ with $\gamma = 0.7^{+0.35}_{-0.3}$) [29]. In the middle panel, we show...
Figure 3. Density dependence of $E_{\text{sym}}$, $E_{\text{kin}}^{\text{sym}}$ and $E_{\text{pot}}^{\text{sym}}$ (left panel) and meson contributions to the Lorentz-covariant components of $E_{\text{sym}}^{\text{ex}}$ in the RHF1 case (right panel).

$E_{\text{kin}}^{\text{sym}}$ in the RHF1, RHF2, and RH cases as well as the free case, in which the interactions are ignored. Owing to the relativistic many-body interactions, $E_{\text{kin}}^{\text{sym}}$ in the RHF1, RHF2, and RH cases is much larger than that in the free case. In the bottom panel, $E_{\text{pot}}^{\text{sym}}$ is presented with the constraint from the analysis of heavy-ion collision (HIC) data using the ImQMD transport model [29]. We find that the Fock contribution suppresses $E_{\text{pot}}^{\text{sym}}$ in the RHF1 case at high densities, and it is consistent with the result of the ImQMD transport model with 2σ confidence region. In contrast, as the density increases, $E_{\text{pot}}^{\text{sym}}$ in the RH and RHF2 cases becomes larger than that in the RHF1 case. As shown in Fig. 2, this is because $E_{\text{pot}}^{\text{sym}}$ in the RH and RHF2 cases is affected by the larger direct contribution.

The $E_{\text{sym}}^{\text{pot}}$ is also expressed in terms of the direct and exchange contributions, $E_{\text{sym}}^{\text{pot}} = E_{\text{sym}}^{\text{dir}} + E_{\text{sym}}^{\text{ex}}$. In the right panel of Fig. 3, we show the meson ($M = \sigma, \omega, \rho, \pi$) contributions to the Lorentz-covariant components of $E_{\text{sym}}^{\text{ex}}$ ($= E_{\text{sym}}^{\text{ex}} + E_{\text{sym}}^{\text{dir},\text{ex}} + E_{\text{sym}}^{\text{v},\text{ex}}$) in the RHF1 case. In RH approximation, only $\rho$ meson affects $E_{\text{sym}}^{\text{pot}}$ through the direct diagram. In contrast, in the RHF calculations, not only $\rho$ meson but also $\sigma$, $\omega$, and $\pi$ mesons give influence on $E_{\text{sym}}^{\text{pot},\text{ex}}$. It is thus of great interest that the $\sigma$ and $\omega$ mesons play an important role in $E_{\text{sym}}^{\text{pot},\text{ex}}$. On the other hand, the contribution due to the $\rho$ and $\pi$ mesons is extremely small even at high densities. Although $\Sigma_N$ and thus $E_{\text{sym}}^{\text{pot},\nu}$ are often ignored in relativistic calculations, it is no longer negligible at high densities [15, 16].
5. Summary

We have studied the effect of exchange terms on $E_{\text{sym}}$ in RHF approximation. Using the HVH theorem, $E_{\text{sym}}$ is expressed as $E_{\text{kin}}^{\text{sym}}$ and $E_{\text{pot}}^{\text{sym}}$, and $E_{\text{pot}}^{\text{sym}}$ is also composed by three components, $E_0^{\text{sym}}$, $E_0^{\text{sym}}$, and $E_0^{\text{sym}}$, based on the Lorentz-covariant structure of $\Sigma_A$.

It is found that the Fock terms do not give any impact on $E_{\text{kin}}^{\text{sym}}$ at $\rho_0$, while it strongly affects $E_{\text{pot}}^{\text{sym}}$. In particular, due to the large exchange contribution to $E_{\text{pot}}^{\text{sym}}$, the direct contribution in the RHF case is quite smaller than that in the RH case. In addition, we have found that the Fock contribution suppresses $E_{\text{sym}}$ at higher densities, and the constraint from the HIC data is in favor of the present result in the RHF1 case. Furthermore, not only the $\rho$ meson but also the $\sigma$, $\omega$, and $\pi$ mesons give significant influence on $E_{\text{sym}}$ through the exchange diagram.

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