Gravitino, axino, and Kaluza–Klein graviton warm and mixed dark matter and reionization

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Abstract. Stable particle dark matter may well originate during the decay of long-lived relic particles, as recently extensively examined in the cases of the axino, gravitino, and higher dimensional Kaluza–Klein (KK) graviton. It is shown that in much of the viable parameter space such dark matter emerges naturally warm/hot or mixed. In particular, decay produced gravitinos (KK gravitons) may only be considered cold for the mass of the decaying particle in the several TeV range, unless the decaying particle and the dark matter particle are almost degenerate. Such dark matter candidates are thus subject to a host of cosmological constraints on warm and mixed dark matter, such as limits from a proper reionization of the Universe, the Lyman-\(\alpha\) forest, and the abundance of clusters of galaxies. It is shown that constraints from an early reionization epoch, such as are indicated by recent observations, may potentially limit such warm/hot components to contributing only a very small fraction to the dark matter.

Keywords: dark matter, cosmology of theories beyond the SM, power spectrum

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The nature of the ubiquitous dark matter is still unknown. Dark matter in the form of fundamental, and as yet experimentally undiscovered, stable particles predicted to exist in extensions of the standard model of particle physics may be particularly promising. For a scale of the new physics around 1 TeV, as preferred by theoretical arguments, such dark matter abundances produced either during freeze-out of stable particles from equilibrium, or freeze-out of metastable particles and their subsequent decay into stable particles, may come tantalizingly close to the abundance required by cosmology. For this reason, a number of candidate dark matter particles, including also stable axinos produced in decays of next-to-lightest supersymmetric particles (NLSP), binos or staus [1]–[4], stable gravitinos produced via NLSP bino, stau, or sneutrino decays [5]–[9], or stable KK gravitons produced via the decay of KK $U(1)$ hypercharge gauge bosons [5], have been recently considered/reconsidered. For details and the theoretical motivation for the possible existence of such particles we refer the reader to the original literature.

In this paper we consider the fact that particle dark matter (DM) produced by the decay of a relic population of metastable particles is often warm or even hot. This has been known for some time [10], but has escaped entering into the conclusions of some recent studies. This effect is also known to exist in the context of dark matter production by cosmic string evaporation [11]. Nevertheless, very recently the very same observation has been rediscovered in the studies by Cembranos et al [12] and Kaplinghat [13]. These latter papers concentrate on the possible resolution of a number of alleged deviations between the observed substructure of galactic haloes and structure of dwarf galaxies and that predicted in structure formation with a purely cold DM particle, whereas our study focuses mostly on constraining such warm or mixed dark matter models by reionization. In any case, for a complete view the reader is referred to the above studies as well.

Decay produced particle dark matter is often warm/hot, i.e. is endowed with primordial free-streaming velocities leading to the early erasure of small-scale perturbations [14], due to the kinetic energy imparted on the decay product during the decay process itself. Since axinos, gravitinos, and KK gravitons are superweakly interacting, they will not thermalize after decay, and inherit as kinetic energy a good fraction of the rest mass energy of the mother particle. For the decay $\chi \rightarrow \gamma + \tilde{G}$ of a massive particle $\chi$ to an essentially massless particle $\gamma$ (with $m_\gamma \ll m_\chi$) and the dark matter particle $\tilde{G}$, with $m_\chi > m_{\tilde{G}}$, one finds for the instantaneous post-decay momentum of $\tilde{G}$

$$p_{\tilde{G}}^i = \frac{(m_\chi^2 - m_{\tilde{G}}^2)}{2m_\chi}. \quad (1)$$

The momentum of particles of arbitrary relativity redshifts with the scale factor of the Universe, and insisting on the particle being non-relativistic at the present epoch, one may compute the present free-streaming velocity of the DM particle

$$v_{0\text{DM}} = \left(\frac{m_\chi^2 - m_{\tilde{G}}^2}{2m_\chi m_{\tilde{G}}}\right)T_0\left(\frac{g_0}{g_d}\right)^{1/3}, \quad (2)$$

where $T$ and $g$ are cosmic temperature and statistical weight of the plasma at the present ‘0’ and particle decay ‘d’ epochs, respectively. It has been shown that a sudden decay approximation, approximating all particles to decay at the same $T_d$, is excellent [3], provided one uses the relation $t_d \simeq \Gamma_d^{-1}$ with $t = (2H)^{-1}$ in the early radiation dominated
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Universes, where $t$ is cosmic time, $\Gamma_d$ is the decay width of the particle, and $H$ is the Hubble constant. Using the above one finds

$$v_{\text{DM}}^0 \approx 4.57 \times 10^{-5} \text{km s}^{-1} \left(\frac{m^2 - m_G^2}{2m_x m_G^2}\right) g_d^{-1/12} \left(\frac{t_d}{18}\right)^{1/2}. \quad (3)$$

Free-streaming velocities may become appreciable for either light DM particles (i.e. relativistically freezing out gravitino, leading thereby to a well-specified relation on the mass of a hypothetical gravitino DM particle. Such limits assume a Lyman-$\alpha$ KK graviton DM parameter space.

Cosmological constraints on warm dark matter deriving, for example, from the Lyman-$\alpha$ forest or cosmological reionization are often formulated as lower limits on the mass of a hypothetical gravitino DM particle. Such limits assume a relativistically freezing out gravitino, leading thereby to a well-specified relation\(^3\) $v_{\text{rms}} \approx 0.044 \text{ km s}^{-1} (\Omega_G h^2/0.15)^{0.34} (m_G/1 \text{ keV})^{-1.34}$ between the gravitino mass and root mean square present day gravitino velocity (cf. e.g. [15, 16]). However, it is rather the latter quantity, $v_{\text{rms}}$, which is, without further assumptions about the nature and production mechanism of the dark matter, constrained by cosmology. Free-streaming particles erase primordial dark matter perturbations between very early times and the epoch of matter–radiation equality (EQ), after which further erasure becomes inefficient. Up to which scale perturbations have been erased than depends essentially only on $v_{\text{rms}}$ and the time of EQ (determined itself by $\Omega_m$, the total non-relativistic matter density), such that one finds [16]

$$R_c^0 \approx 0.226 \text{ Mpc} \left(\frac{\Omega_m}{0.15}\right)^{-0.14} \left(\frac{v_{\text{rms}}}{0.05 \text{ km s}^{-1}}\right)^{0.86}, \quad (4)$$

by detailed Boltzmann equation simulations [17]. Here $R_c^0 \equiv 1/k_c$ defines the wavevector $k_c$ for which the primordial power spectrum is suppressed by a factor two\(^4\) when compared to the same cosmological model, but with cold dark matter. Armed with a relation between $v_{\text{rms}}$ and gravitino mass, as well as equation (3), we may now translate limits existing in the literature on relativistically freezing out gravitino warm dark matter into limits on warm dark matter generated by particle decay. Here we employ decay widths as calculated in the original literature.\(^5\)

We note here that the equivalence between traditional warm dark matter, described by a Fermi–Dirac distribution, and metastable particle decay produced dark matter, with a velocity distribution given by exponential decay, is not entirely perfect. This is due to the differing velocity distributions. Kaplinghat [13] computes the transfer function for decay produced dark matter and finds differences in damping tails between decay produced dark matter and warm dark matter. Nevertheless, given current measurement and theoretical uncertainties in cosmology, for example in the determination of the epoch

\(^3\)Here $\Omega_G$ is the gravitino contribution to the present critical density and $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$.

\(^4\)The linear power spectrum for warm dark matter (WDM) is given by the power spectrum for cold dark matter (CDM) multiplied by a transfer function $T_{\text{WDM}} = (1 + (c k R_c^0)^{2\nu} - \eta/\nu)$ which accounts for the additional free streaming in WDM scenarios. In the above, $k$ is the wavevector and the values $\eta = 5$, $\nu = 1.2$, and $\epsilon = 0.361$ are frequently used.

\(^5\)For sleptons decaying into leptons and gravitinos $\tilde{l} \rightarrow l + \tilde{G}$ we employ equation (6) of [8], whereas for binos (KK $U(1)$ gauge bosons) decaying into photons (Z bosons) and gravitinos (KK gravitons) $\tilde{B} \rightarrow \gamma(Z) + \tilde{G}$ ($B^1 \rightarrow \gamma(Z) + G^1$) we employ equation (1) (equation (2)) of [5].
of reionization, the two types of DM should be considered as having equivalent effects on structure formation as long as they have an identical second moment of the distribution, i.e. an identical root mean square velocity.

There exist a large number of observable cosmological differences between scenarios with warm (WDM) and cold dark matter (CDM). It has even been argued that WDM has phenomenological advantages over CDM, potentially resolving possible difficulties of CDM scenarios in explaining a scarce of substructure in Milky Way-type haloes or the existence of cores in dwarf galaxies. Nevertheless, counter-arguments in favour of CDM have also been presented, such that the situation is not resolved. We will here only focus on three cosmological implications of WDM or mixed dark matter (MDM) scenarios, namely the optical depth in the Lyman-α forest of mildly non-linear fluctuations on smaller scales \( \sim 1 \text{ Mpc} \) [18], the successful reionization of the Universe at high redshift (probing perturbations on the smallest scales \( \sim 10-100 \text{ kpc} \)) [16,19], and for the case of MDM, the abundance of clusters at close to the present epoch [20]. A recent analysis of the matter power spectrum as implied by observations of the Lyman-α forest and the cosmic microwave background anisotropies (CMBR) by the WMAP mission has yielded a 2\( \sigma \) lower limit on the WDM gravitino mass of 550 eV [18]. Using the above this may be translated to a limit \( v_{\text{rms}}^0 \lesssim 0.1 \text{ km s}^{-1} \). One year of observations of polarization of the CMBR by the WMAP satellite have revealed a large optical depth \( \tau \approx 0.17 \pm 0.04 \) [21] for Thomson scatterings of CMBR photons on electrons. The recently presented three-year WMAP analysis has led to a downward revision of this value to \( \tau \approx 0.09 \pm 0.03 \) [22], where error bars are one sigma and a \( \Lambda \)CDM concordance model has been assumed. This implies a fairly complete reionization of the Universe at redshift \( 8.5 \lesssim z \lesssim 15 \) (at the 95% confidence level) seemingly consistent with CDM scenarios. Such scenarios predict an early reionization of the Universe due to the early formation of subgalactic haloes and massive stars therein. The situation is different in WDM scenarios due to the lack of small-scale power and the concomitant late formation of the first stars. If the fairly high optical depth is indeed due to the first stars, rather than due to some ‘exotic’ mechanism, reionization places very stringent limits on the warmness of the DM. In particular, Yoshida et al [19] have shown that even for WDM with \( m_{\tilde{G}} \approx 10 \text{ keV} \) reionization is far from substantial at redshift \( z \sim 17 \) (appropriate to the \( \tau \) central value of the one-year WMAP analysis). This corresponds to \( v_{\text{rms}}^0 \lesssim 0.002 \text{ km s}^{-1} \). In the numerically expansive study of Yoshida et al WDM scenarios with \( m_{\tilde{G}} > 10 \text{ keV} \) have not been considered. As the case \( m_{\tilde{G}} = 10 \text{ keV} \) fails, even larger \( m_{\tilde{G}} \) could potentially fail. Nevertheless, uncertainties in these calculations exist due to the modelling of the physics of gas cooling, radiation transport, and star formation. We thus regard such limits as preliminary.

Scenarios of DM due to decaying metastable particles often may come in the flavour of mixed dark matter when only a component of the DM is due to decay, with the other component possibly due to thermal scatterings at high temperature. Such scenarios of MDM scenarios may also be constrained by the abundance of clusters of galaxies [20]. Nevertheless, MDM may only be constrained by these means, in the case where the warm component is warm/hot enough to erase primordial perturbations on the scale of a cluster of galaxies. For \( \lambda_c \equiv 2\pi/k \approx 20 \text{ Mpc} \) we find that a half-wavelength roughly encompasses \( 10^{14} M_\odot \), the approximate mass of a typical cluster. In order to erase perturbations on the scale \( \lambda_c \) the present day DM velocity has to exceed \( v_{\text{rms}}^0 \gtrsim 1 \text{ km s}^{-1} \). If this is the
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\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Contour plots of constant present-day free-streaming velocities in a variety of scenarios where the dark matter $\tilde{G}$ is generated by the late decay of a non-relativistic primary $\chi \to \tilde{G} + \gamma$ (cf. equation (1)). Results are shown in the plane of $m_{\tilde{G}}$ and $\Delta m \equiv m_{\chi} - m_{\tilde{G}}$ (all in GeV). The scenarios are bino decay into gravitinos (red solid), slepton decay into gravitinos (blue dotted), and $B^1$ decays into KK gravitons (green dashed). Shown are the contours of velocities (from top to bottom) $v = 0.002$, 0.1, and 1 km s$^{-1}$, respectively. Also shown, by the thin dotted lines, are the contours where the effects of small-scale suppression due to the coupling of a charged slepton to the CMBR decaying later into a gravitino are similar to those of warm dark matter with $v = 0.002$, 0.1, and 1 km s$^{-1}$, respectively (see the text for details).}
\end{figure}

case, however, only a fraction between 10 and 20% of all the DM may be warm/hot [20] (cf. also [18] for similar limits from the Lyman-$\alpha$ forest).

In figure 1 WDM limits on gravitino DM produced by metastable particle decay are shown in the parameter space spanned by the mass splitting between the gravitino and the NLSP $\Delta m \equiv m_{\text{NLSP}} - m_{\tilde{G}}$ and the gravitino mass itself. The respective limits of $v_{\text{rms}}^0 = 0.002, 0.1,$ and 1 km s$^{-1}$, as discussed above, are indicated as heavy lines for the cases when either the bino (long-dashed) or the stau or sneutrino (solid) is the NLSP. The order of the lines is such that $v_{\text{rms}}^0$ exceeds the limit in the parameter space below the lines, with lines at higher $\Delta m$ corresponding to a limit with lower $v_{\text{rms}}^0$. It is seen that most of the parameter space results in WDM, potentially already in conflict with, at least, the high optical depth as inferred by WMAP. In particular, only for excessively large $m_{\text{NLSP}} \gtrsim 5$ TeV may decay produced gravitino dark matter be considered almost cold. The limit may be less stringent when only a small component $\lesssim 10\%$ of the gravitino DM is due to decay, as then masses $m_{\text{NLSP}} \gtrsim 100$ GeV ensure that the decay produced component is not too hot to delay the formation of clusters of galaxies.

We note here, however, that for gravitino masses not too small ($m_{\tilde{G}} \lesssim 0.1$ GeV for a bino NLSP and $m_{\tilde{G}} \lesssim 10$ GeV for a stau NLSP [7, 8]) very stringent constraints on such scenarios apply from a disruption of Big Bang nucleosynthesis (BBN) with smaller mass splittings $\Delta m$ preferentially constrained by the effects of electromagnetic energy injection after the epoch of BBN and distortion of the CMBR black body spectrum and larger $\Delta m$ by hadronic three-body decays during and after BBN. In particular, $^{6}\text{Li}$ [23].
decay widths of the literature \[\text{production as well as significant perturbations of the } ^3\text{He}/\text{D} [24] \text{ ratio potentially rule out much of the parameter space at larger } m_{\tilde{G}}. \] If such constraints are combined with the requirement of having the supersymmetric potential bounded from below, only very little parameter space remains, at least in the constrained minimal supersymmetric standard model (CMSSM) and for small trilinear couplings \(A\) [25]. These limits are not shown in figure 1.

Figure 1 shows also analogous results for a KK \(B^1\) decaying into a photon and KK graviton, with constraints indicated by the dotted line. It is seen that constraints from the warmness are not quite as stringent as in the gravitino case, yet, for \(m_{G^1}\) around the weak scale, \(m_{B^1} \gtrsim 1 \text{ TeV}\) is still required for a successful reionization by star formation. Only for much smaller \(m_{G^1} \lesssim 1 \text{ GeV}\) may decay of lighter \(m_{B^1} \lesssim 300 \text{ GeV}\) result in CDM. However, it is rather expected for the mass difference between the KK \(B^1\) and KK graviton to be small, since it should be only due to radiative corrections. Almost degenerate \(B^1\) and \(G^1\) are then constrained by limits from BBN. It has recently been pointed out that a generation of dark matter by particle decay may lead to an additional suppression of small-scale power provided the decaying particle is charged [26]. Since the charged ‘primary’ is coupled to the CMBR, subhorizon perturbations in the primary-photon fluid are below the Jeans mass. Perturbations may thus only start growing by the gravitational instability when decoupled from the CMBR, i.e. after the decay of the primary. In particular, it was found in [26, 27] that for a charged particle decay time of \(\tau_d = 3.5 \text{ yr}\) the wavevector \(k_d\) where the power is reduced by half, when compared to a CDM scenario, is approximately \(k_d \approx 3 \text{ Mpc}^{-1}\) (assuming \(h = 0.7\)). Comparing this to equation (4) one may infer a velocity \(v_{\text{rms}}^0 \approx 0.078 \text{ km s}^{-1}\) which would yield a similar erasure of small-scale power due to finite DM velocities. An analogy between the net results of these two physically different small-scale power suppression mechanisms may be established by noting that the damping scale due to charged particle decay is approximately the horizon scale at decay [26], \(\lambda_d \approx 0.265 \text{ Mpc}(\tau/\text{yrs})^{1/2}\). Comparing this scale to equation (4), the above limits of \(v_{\text{rms}}^0 \approx 0.002, 0.1, \text{ and } 1 \text{ km s}^{-1}\) translate to decay times \(\tau_d \approx 3 \times 10^{-3}, 2, \text{ and } 125 \text{ yr}\), respectively. In figure 1 the light lines are lines of constant decay time for the decay \(\tilde{\tau} \rightarrow \tau + \tilde{G}\) with values as given above. Here shorter decay times are at higher \(\Delta m\). It is seen that in the scenarios considered here the effects of free streaming are generally more important than those of charged particle decay, though this may be different when other scenarios are considered [27].

We now investigate the resulting free-streaming velocities in axino DM generation due to stau \((\tilde{\tau} \rightarrow \tau + \tilde{a})\) and bino \((\tilde{B} \rightarrow \gamma + \tilde{a})\) decays as proposed by [1]–[4]. Taking the decay widths of the literature\(^6\) we find present root mean square velocities of

\[
v_0 \approx 14.7 \text{ km} \text{s}^{-1} \left( \frac{m_{\tilde{\tau}}}{1 \text{ MeV}} \right)^{-1} \left( \frac{m_{\tilde{\tau}}}{100 \text{ GeV}} \right)^{1/2} \left( \frac{m_{\tilde{B}}}{100 \text{ GeV}} \right)^{-1} \left( \frac{f_{\tilde{a}}}{10^{11} \text{ GeV}} \right)
\]

(5)

\[
v_0 \approx 1.68 \text{ km} \text{s}^{-1} \left( \frac{m_{\tilde{\tau}}}{1 \text{ MeV}} \right)^{-1} \left( \frac{m_{\tilde{B}}}{100 \text{ GeV}} \right)^{-1} \left( \frac{f_{\tilde{a}}}{10^{11} \text{ GeV}} \right)
\]

(6)

for stau equation (5) and bino equation (6) decay, respectively. Here \(m\) denote the masses for the axino \(\tilde{a}\), stau \(\tilde{\tau}\), and bino \(\tilde{B}\), and \(f_{\tilde{a}}\) required to be in the range

\(^6\) For decay of binos into photons and axinos \(\tilde{B} \rightarrow \gamma + \tilde{a}\) we use equation (4.3) from [3] and for decay of right-handed staus into taus and axinos \(\tilde{\tau}_R \rightarrow \tau + \tilde{a}\) we use equation (3) of [28].
$10^{10}$ GeV $\lesssim f_\alpha \lesssim 10^{12}$ GeV, is the scale where the chiral $U(1)$ symmetry, related to the Peccei–Quinn symmetry, is spontaneously broken. In the above we have neglected numbers of order unity and a very weak dependence on the statistical weight during decay. Velocities become excessively large for small $m_{\tilde{a}}$. It is evident that axinos generated by decay having masses $m_{\tilde{a}} \lesssim 10^2$ (1) GeV should be considered warm and may be in conflict with an early reionization by stars. However, since the decay contribution to the axino density $\Omega_{\tilde{a}}^\text{decay} = \Omega_{\tilde{a},\ast} m_{\tilde{a}} / m_{\tilde{a},\ast}$ becomes typically small when $m_{\tilde{a}}$ is small and moreover efficient production of ‘thermal’ axinos during reheating is possible at low temperatures for small $m_{\tilde{a}}$ [29], axino dark matter should be naturally mixed, rather than warm, with a small component of axinos with large velocities, and the rest essentially cold. Note that due to the relatively large decay width, suppression of small-scale power due to existence of a charged particle primary [26] is generally unimportant for axinos.

We have observed that constraints on warm dark matter from cosmic reionization may become very stringent, particularly if an early reionization epoch as indicated by the excessive low multipole CMBR polarization in the WMAP data [21] is confirmed. On the other hand, gravitino and axino dark matter scenarios often come as mixed dark matter scenarios with only a smaller fraction of the dark matter warm/hot. As we will see, even in this case future constraints could be very stringent.

The Universe is believed to have been reionized mostly due to UV radiation by the first stars, with probably only a small contribution due to the UV radiation emitted by quasars [16]. It has been shown [30] that the collapse of the smallest haloes and the stars which form therein actually impede further star formation as the principal cooling agent $H_2$ is photodissociated. Only when harder radiation $\sim 1$ keV is emitted at the same time is cooling of baryons in small haloes still possible. Efficient star formation and reionization thus generally may only occur when larger haloes, with virial temperatures $T_{\text{vir}} \gtrsim 10^4$ K, may collapse, as in such haloes cooling is possible due to atomic hydrogen. The typical UV radiation production for a star cluster at low metallicity and for an initial mass function close to that observed locally is estimated as around $N_\gamma = 4000$ ionizing photons per stellar proton. Since normally only a small fraction $f_\ast \sim 10\%$ of the gas forms stars and moreover only a small fraction $f_{\text{esc}} \sim 10\%$ is capable of leaving the host galaxy, one may estimate around 40 ionizing photons per collapsed and cooled baryon. This implies that the fraction of haloes $F(T_{\text{vir}} > 10^4)$ with virial temperature larger than $10^4$ K has to be rather large, $1 / (N_\gamma f_\ast f_{\text{esc}})$ = $2.5 \times 10^{-2}$. Note that this conservatively neglects further recombinations. A virial temperature of $10^4$ K corresponds to a mass scale of $M_\ast \approx 3 \times 10^7 M_\odot (1 + z)/11^{-3/2}$ where $z$ is the redshift.

We have utilized CMBFAST in order to find the transfer function in models of mixed dark matter with varying warm/hot dark matter fractions $\Omega_\chi$ and root mean square velocity $v_\chi$. The transfer function allowed us to compute the collapsed mass fraction $F(M)$ as a function of redshift. Our models assume $\Omega_\chi + \Omega_\gamma + \Omega_\delta = 0.3$, where $\Omega_\chi$ is the cold dark matter density parameter and $\Omega_\delta$ is the baryonic density parameter, and we assumed $\Omega_\delta = 0.04$ and a Hubble parameter of $h = 0.65$. Most of our calculations assume a red spectral index for scalar adiabatic perturbations $n_\delta = 0.951$ corresponding to the central value of the three years WMAP results, according to which $n_\delta = 0.951^{+0.015}_{-0.019}$ within the $\Lambda$CDM concordance model. This model assumes a negligible contribution of primordial gravitational waves to the CMBR anisotropies. Gravitational waves due to an inflationary epoch mostly contribute to the CMBR anisotropies on large scales. If present,
they may therefore also describe the WMAP data well, albeit with a lower normalization of scalar perturbations on the largest scales and a larger spectral index $n_s$. As the two-sigma upper limit on $n_s$ from a combination of WMAP observations and large-scale structure surveys (and in the absence of a running spectral index) is close to 1, we have also utilized a Harrison–Zeldovich spectrum $n_s = 1$ for one particular computation. This may be indicative of variations of our results with respect to the assumed cosmological model. All models are normalized on the present horizon scale (taking COBE-DMR results, i.e. equation (30) in [31]), irrespective of whether gravitational waves are present or not. Finally, the warm component was simulated by adding a massive ‘neutrino’ to the fluid with a root mean square velocity of $v_x$, while keeping also the three massless standard model neutrinos.

In figure 2 we show the redshifts at which in a particular mixed model (defined by $\Omega_x$ and $v_x$) a fraction $>10^{-2}$ of all baryons have collapsed within haloes exceeding a
virial temperature of $10^4$ K. Assuming an essentially instantaneous baryon cooling and star formation\(^7\), and given the required $F(T_{\text{vir}} > 10^4) > 2.5 \times 10^{-2}$ as inferred in a ‘standard’ reionization scenario, this redshift $z$ may be approximately identified with the epoch of reionization. It is seen that even for warm/hot dark matter fractions as small as $\Omega_x \sim 10^{-3} – 10^{-2}$ reionization with standard efficiencies may be problematic for redshifts larger than $z > 10$. This should be compared to the current estimate of $z_{\text{reion}} \approx 8.5–15$ as inferred by WMAP, indicating the potentially stringent nature of limits that one could potentially derive on very small warm/hot dark matter fractions from reionization. Such limits, if held up against future observational tests, could severely constrain the nature of particle dark matter, by requiring it to be essentially exclusively cold.

However, it needs to be stressed here that the results as shown in figure 2 (as well as in the figures which follow) are not intended by the authors to be employed to derive limits on mixed dark matter models. This is due to a variety of factors, such as uncertainties in the reionization process and cosmological model (see below), but also due to the fact that we have not marginalized over all cosmological parameters entering the analysis. Furthermore, different constraints, not shown in the figures, may be of importance. For example, models with relatively large $v_x$ and $\Omega_x$, corresponding to a significant hot dark matter component, may potentially be ruled out simply by a relative mismatch between the predicted and observed powers on $8 \text{ Mpc} h^{-1}$ and the present horizon scale $3000 \text{ Mpc} h^{-1}$, respectively. This is due to the hot component erasing power on $8 \text{ Mpc} h^{-1}$ due to free streaming.

\(^7\) We have further conservatively neglected a potential delay in halo formation due to an increase of the effective dark matter Jeans mass [16].
Intrinsic uncertainties about the reionization process during the dark ages are still large enough to evade drastic conclusions. It may be that either the efficiency of star forming regions for producing UV radiation (via a top-heavy initial mass function, for example) or the escape fraction/star formation efficiency is substantially larger at early times than at the current epoch. Most of such evolutionary effects should be testable, by, for example searches for very high redshift galaxies or heavy element abundances in the high redshift gas, by the NGST (next-generation space telescope). In figure 3 we show the redshifts where a much smaller fraction of $10^{-4}$ of the gas was able to collapse in virial haloes with $T_{\text{vir}} > 10^4$ K. This corresponds to a factor 100 increase in ‘reionization’ efficiency. It is seen that fairly early $z \sim 16$ reionization epochs are possible except when $\Omega_x$ becomes large, $\gtrsim 10^{-2}$, and free-streaming velocities are not too small, $v_x \gtrsim 0.1 \text{ km s}^{-1}$. It is evident that even with a factor $\sim 100$ higher efficiency reionization than expected, and provided the reionization redshift is found to be larger than $z > 12$, limits from reionization on warm/mixed dark matter may be more stringent than those derived from the CMBR Lyman-α forest. This conclusion is also in accord with the numerical simulations recently performed in [19].

Both plots show, by the star and the lines, also the recently derived limit [18] on a small thermal warm/hot gravitino component from a combination of WMAP and CMBR data (probing perturbations on larger scales, $\sim 10-3000$ Mpc) and the Lyman-α forest (probing perturbations on smaller scales, $\sim 1-40$ Mpc). The models considered in [18] are lying on the diagonal dotted line and are ruled out because of a too severe suppression of the power spectrum at small scales, at odds with observations of the Lyman-α forest. Models with the same $\Omega_x$ but even larger $v_x$ than that indicated by the star, moving the power spectrum suppression to even larger scales, should also be at odds with the...
observations. We have therefore added a vertical dotted line, such that the region in the upper right-hand corner of the star (bounded by these dotted lines) should be ruled out.

Similarly, uncertainties in the prediction of the reionization redshift in mixed dark matter models are also due to uncertainties in the cosmological model and/or scalar spectral index $n_s$. In figure 4 we show the reionization redshift in our simple reionization model for the same required collapse fraction and virial halo temperatures as in figure 2, but for a model which has a Harrison–Zeldovich $n_s$ spectral index. It is seen that the requirement of early reionization is more easily satisfied for $n_s = 1$ than for $n_s = 0.951$. Nevertheless, the change is not big and amounts only to $\Delta z_{\text{reion}} \approx 2$. We have also varied the COBE normalization, moving it approximately two sigma upwards (resulting in a factor 1.14 larger perturbations) and have obtained results very similar to those shown in figure 4, i.e. $\Delta z_{\text{reion}} \approx 2$. We conclude that the proper reionization of the Universe seems a promising ally in constraining warm and mixed dark matter scenarios. Finally we note that the potentially strong constraints on warm/mixed dark matter possible from a determination of the reionization redshift have also been noted in the two very recent papers [12, 13].

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