Can Long Range Anomalous Neutrino Interactions Account for the Measured Tritium Beta Decay Spectrum?

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Abstract

Recent experimental searches for neutrino mass in tritium beta decay yield a negative value for the neutrino (mass\textsuperscript{2}). If this effect is genuine, then it is hard to understand it using conventional particle physics ideas as embodied in the standard electroweak model or its simple extensions that have been widely discussed. We consider the possibility that there is a hidden anomalous long range interaction of neutrinos that is responsible for this effect and study the phenomenological consistency as well as tests of this idea. We also discuss how such interactions may arise in extensions of the standard model.
1 Introduction

Several high precision experiments measuring the tritium end point spectrum have been performed in order to put direct upper bounds on the mass of the electron neutrino, $m_{\nu_e}$. The best fits of all experiments indicate however a negative mass squared for the electron neutrino (i.e. $m_{\nu_e}^2 < 0$). Should this persist in the future and manifest also in the end point spectra of other nuclei, an explanation for this effect would clearly be called for. Needless to say that the physics reasons for such an effect have to be very dramatic.

It has been pointed out by Stevenson et al\cite{2} that the crossed process in which a $\nu_e$ is absorbed from a background of electron neutrinos

$$\nu_e + ^3H \rightarrow e^- + ^3He$$

leads to electrons in the anomalous endpoint region. However in order to compete with the decay process $^3H \rightarrow ^3He + e^- + \bar{\nu}_e$ in this region, the required density of the background $\nu_e$'s should be $n_{\nu_e} \simeq 10^{15}/cm^3$ or so i.e. of order $|m_{\nu_e}|^3$ for $m_{\nu_e} \simeq 5$ eV, the magnitude of the "imaginary-neutrino-best-fit-mass". This density vastly exceeds that for gravitationally clustered neutrinos with (real) mass $m_{\nu_e} \simeq 5$ eV\cite{3}. It could however be readily achieved if such neutrinos would experience a local potential well

$$U_{\nu_e} \simeq -|m_{\nu_e}| \simeq -(5 - 10) \text{ eV}$$

which fills up to a Fermi momentum $p_F \approx E_F \approx |U_{\nu_e}|$.

In the following, we will study various constraints on such new long range forces involving neutrinos. We then discuss particle physics implications and possible scenarios of new physics beyond the standard model that may accommodate such unconventional interactions.

2 Constraints on the new neutrino interactions

The new neutrino interactions $U_{\nu}$ cannot exist uniformly everywhere. A scalar uniform interaction simply shifts ($m_{\nu} \rightarrow m_{\nu} + U_{\nu}$) the neutrino mass whereas a uniform vector interaction would require a background of almost conserved charge.

Independently of this, an enhanced cosmological neutrino density of such large magnitude as $n_{\nu_e} \approx 10^{15}/cm^3$ is completely at odds with the standard big bang
scenario. The measured $2.7K$ photon background implies $n_{\nu}^{\text{cosm}} \approx 100/cm^3$. Thus the neutrino interaction energy and the attendant enhancement of neutrino density can occur only in isolated regions of typical size $R$, such that the total volume of these region comprises a small fraction of the total volume now:

$$\epsilon \leq \frac{n_{\nu}^{\text{cosm}}}{n_{\nu}^{\text{local}}} \approx 10^{-13} - 10^{-14}$$

(3)

The fraction of the Universe’s volume occupied by galaxies is more than $\epsilon$ and therefore the possibility $R = R_{\text{Galaxy}}$ is ruled out if all galaxies have the same enhanced density. Furthermore the assumed $\nu_e$ mass of 5 eV would lead to a dark mass density $\rho_e \approx 5 \times 10^6$ GeV or so which is some seven orders of magnitude larger than the allowed value from observations pertaining in particular to our galaxy. However since the $\nu_e$ mass anomaly manifests in experiments at vastly different locations on the Earth (Los-Alamos, Mainz etc), the new interaction as well as the enhanced density should at least extend over a region as large as the diameter of the Earth (i.e. $R \approx 10^9$ cm).

The local neutrino interaction potential can be generated as a sum of attractive pairwise potentials due to the exchange of a new, superlight, boson of mass $\mu$. These exchanges can occur between the particular $\nu_e$ in question located, say, at $r_{\nu_e} = 0$ and other matter particles (electrons, protons and neutrons) or with clustered neutrinos located at $r \approx r_i$:

$$U_{\nu}^{\text{local}}(r = 0) = \sum_i V(|r_i|)$$

(4)

The potential $V$ is taken to have the standard spin independent Yukawa form with a range $R \approx \mu^{-1}$ and strength factorizing to the coupling to neutrinos $g_{\nu_e}$ and the coupling to matter particles and neutrinos $g_{p,n,e,\nu_e}$. The exponential cut-off limits the number of particles ($N_p, N_n, N_e, N_{\nu}$) contributing to the above sum to those at locations $|r_i| \leq R \approx \mu^{-1}$. On general grounds of charge neutrality, we have $N_p = N_e$. Also from stellar $n/p$ ratio (called $\zeta$), $\zeta \approx 1/7$ cosmologically and in the solar system and $\zeta \approx 1.2$ terrestrially, we should have $\zeta \approx 0.14$ for $R \geq 1A.U. \approx 1.5 \times 10^{13}$ cm and $\zeta \approx 1.2$ for $1AU \geq R \geq R_E \approx 10^8$ cm. Under the natural assumption that the attracting particle distribution extends at least up to the range of the force, the sum $U_{\nu}^{\text{local}} = \sum_i g_{\nu_e} g_e \frac{e^{-\mu r_i}}{r_i}$ is dominated by $r_i \approx R \approx \mu^{-1}$. In order to generate an $U_{\nu}^{\text{local}} \approx 5 - 10eV$, we must therefore have,

$$\mu g_{\nu_e}[N_p(g_e + g_p + g_n) + (N_e \pm N_{\nu})] \geq 5 - 10eV$$

(5)
For neutrinos we may have antiparticles contributing with same(opposite) sign in the above equation depending on whether the interaction is scalar or vector.

If we assume that neutrino clustering is dominated by the attraction of normal matter, it will naturally occur near the Sun or the Earth as needed in order to explain the neutrino anomaly. The condition in Eq.(5) can then be written as

$$\mu N_p g_\nu g_m \geq 5 - 10 \text{ eV}$$

where $g_m$ denotes the effective matter coupling. Clearly clustering will be optimized if we use the the values of $R$ which maximize the number $N_p$ of matter particles.

One can contemplate two scenarios: (a) where clustering arises from interactions on solar scale or (b) from interactions on scale of the earth radius. In the former case $R \approx 10^{13}$ cm and $N_p \approx 10^{57}$ whereas in the latter case $R \approx 10^9$ cm and $N_p \approx 4 \times 10^{51}$.

From Eq. (6), we readily derive a lower bound on the strength of the couplings $g_i$:

$$g_m g_\nu \geq 0.8 \times 10^{-38} \quad \text{for case (a)}$$

$$g_m g_\nu \geq 0.5 \times 10^{-36} \quad \text{for case (b)}$$

It is important to point out that alongside neutrino matter interaction, the superlight boson exchange also mediates "diagonal" forces between ordinary matter particles

$$V_{mm} \approx g_m^2 e^{-\mu r}/r$$

and between neutrinos

$$V_{\nu\nu} = g_\nu^2 e^{-\mu r}/r$$

On distance scales $r \approx R \approx 1$ AU or $r \approx R \approx R_E$, the $V_{mm}$ interactions can compete with the ordinary gravitational interactions $V_{\text{grav}} \approx G N m_p^2 / r \approx 10^{-38} / r$. $V_{mm}$ can therefore spoil the equality of inertial and gravitational masses which has been verified to an accuracy of one part in $10^{11}$ for $R \approx 1$ A.U. (case (a)) and to one part in $10^9$ for $R \approx R_E$ (case (b))\[4\].

We can minimize the violation of the equivalence principle by artificially tuning $g_i = e, p, n$ to be proportional to the corresponding masses $m_i = e, p, n$. Even then the variation of nuclear binding energies

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1) We need not consider galactic scales for reasons stated earlier. Precisely for this reason, we also did not consider possible contribution to $U^\nu$ due to "wimps" or CDM particles etc.
leads to a deviation from the equality of inertial and gravitational masses at a level of $10^{-3} V_{inm} / V_{gr}$. From our previous discussion, we then conclude that

$$g_m^2 \leq 10^{-11+3} G_N m_p^2 \leq 10^{-46} \text{ or } g_m \leq 10^{-23}$$

(11)

for case (a) and

$$g_m^2 \leq 10^{-9+3} G_N m_p^2 \leq 10^{-44} \text{ or } g_m \leq 10^{-22}$$

(12)

for case (b). Combining this with Eq. (7) and (8), we conclude that

$$g_\nu \geq 10^{-15} \text{ case(a)}$$

(13)

$$g_\nu \geq 10^{-14} \text{ case(b)}$$

(14)

These lead to rather "strong" long range "diagonal" $\nu \nu$ interactions. Such forces manifest in various settings:

(1) For neutrinos of densities $n_\nu \approx \rho_\nu^3 \simeq U_\nu^3$ extending over large scales $R \approx \mu^{-1}$, mutual $V_{\nu \nu}$ interactions could dominate over $V_{\bar{\nu} \nu}$ and generate the requisite $U_\nu$ if

$$g_\nu^2 N_\nu / R \approx g_\nu^2 (RU_\nu)^3 / R \approx U_\nu$$

(15)

or

$$g_\nu \approx \frac{1}{RU_\nu} \approx \mu / U_\nu \approx 10^{-18} / (R \text{ in } AU)$$

(16)

However such self clustering of neutrino clouds could occur only for scalar exchange interactions which generate attraction between $\nu \nu$ as well as $\nu \bar{\nu}$ pairs. The vectorial interactions are inherently repulsive for any density of the relevant charges $\rho_\nu \equiv n_\nu - n_{\bar{\nu}}$. Indeed one can easily show that

$$U_{vect} = \frac{1}{2} \int d^3 r d^3 r' \rho(r) \rho(r') e^{-\mu|\mathbf{r} - \mathbf{r}'|} / |\mathbf{r} - \mathbf{r}'|$$

(17)

$$= \int d^3 q \tilde{\rho}(q) \tilde{\rho}(q) (\mu^2 + q^2)^{-1} \geq 0$$

where $\tilde{\rho}(q)$ and $(\mu^2 + q^2)^{-1}$ are the Fourier transforms of the neutrino density and the Yukawa potential.
Huge concentrations of neutrinos occur in the Supernovae during gravitational collapse. Since all ranges $R$ of the $V_{\nu\nu}$ and $V_{\nu m}$ considered exceed the supernova core radius ($R_{SN} \approx 10 - 30 \text{ K}m$), we expect a mutual $\nu\nu$ interaction of order:

$$W_{\nu\nu} \approx N_{\nu}^2 g_{\nu}^2 / 2R_{SN}$$ (18)

The neutrino densities and total numbers during the collapse are comparable to those of nucleons. Yet this $W_{\nu\nu}$ should not exceed the gravitational interaction during the collapse

$$W_{\text{grav}} \approx G_N m_p^2 N_{\nu}^2 / 2R_{SN}$$ (19)

in order not to disturb the standard supernova dynamics which agrees pretty well with observations. Since $N_p \approx N_{\nu}$, this would appear to lead to an independent bound on $g_{\nu}$

$$g_{\nu} \leq (G_N m_p^2)^{1/2} \approx 10^{-19}$$ (20)

At face value, this bound strongly conflicts with the minimal $g_{\nu}$ required, (see Eq.(13) and (14)). It turns out (as we show below) that the bound in Eq.(19) holds only for vectorial interactions but not for scalar interactions.

**Vector Interactions:**

For vectorial interactions, the $N_{\nu}$ in Eq.(17) should be replaced by $\Delta N_{\nu} \equiv N_{\nu} - N_{\bar{\nu}}$. The latter is roughly the total lepton number $N_L = N_e (= N_{\bar{\nu}})$ of the collapsing core since a fair fraction of the $N_L$ is trapped along with the $N_L = 0$ thermally generated neutrinos in the core. Thus it is clear that the above bound on $g_{\nu}$ applies in the case of vectorial interactions. [In passing we note that in the vectorial case, the ”turning on” of the repulsive interaction upon core collapse is naturally avoided if the almost massless vector boson couples to some conserved $U(1)$ charge. The conservation of this new $U(1)$ charge for reactions such as $e^- + p \rightarrow n + \nu_e$ responsible for neutrino production imply that the various $U(1)$ charges for the particles satisfy the relation$^2$

$$g_e + g_p = g_n + g_{\nu_e}$$ (21)

As a result, the total vector interaction energy is uneffected by the reaction $e^- + p \rightarrow n + \nu_e$ throughout supernova explosion process. The condition that the neutron star

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$^2$This is a consequence of Weinberg’s theorem extended to the case of $U(1)$ theories with almost massless photons.
not have substantial additional energy due to $W_{nn}$ interaction still implies then that $g_n \leq 10^{-19}$. This together with the constraint in Eq.(12) that $g_m \equiv g_p + g_n + g_e \leq 10^{-22}$ the condition in Eq.(20) implies that we cannot have $g_\nu \geq 10^{-15}$ also in this case.]

**Scalar Interaction:**

The situation is drastically different for scalar interactions for several reasons: first, the scalar couplings need not satisfy any conservation laws like in Eq.(20). More importantly, the estimate of the self interaction energy used above i.e. $W_{\nu\nu} \approx g_\nu^2 N^2_\nu / 2R$ is valid only for the case where the neutrinos are mildly relativistic (i.e. $p_\nu \leq U_\nu \leq m_\nu$) as in the putative neutrino cloud. However, for an extreme relativistic neutrino gas as in the supernova core, the above expression for $W_{\nu\nu}$ is invalid. Let us consider a scalar exchange potential between two neutrinos in the collapsing core. Because of their high energy ($E_{\nu_e} \geq 10 MeV \simeq 10^6 m_{\nu_e}$), the neutrinos are effectively helicity eigenstates. The scalar exchange always flips helicity. Therefore to retain coherence implicit in adding all pairwise interactions, we need to use the small $(m_{\nu_e}/E_{\nu_e})$ helicity admixture in the wave function of the relativistic neutrinos. One therefore finds that for relativistic neutrinos,

$$W_{\nu\nu}^{scalar} \approx \frac{1}{2} \left( \frac{N^2_\nu g_\nu^2}{R} \right) \left( \frac{m_\nu}{E_\nu} \right)^2$$

In the supernova core, $(m_\nu/E_\nu)^2 \approx 10^{-12}$ making the upper bound to $g_\nu \leq 10^{-13}$. This bound is much more stringent than direct bounds on $g_{\nu e}$ and $g_{\nu\mu}$ implied by considerations of possible distortions of the e spectrum in $\mu \rightarrow e\bar{\nu}_e\nu_\mu$ [5]; yet it allows for the anomalous long range interaction required for neutrino clustering.

We thus conclude that it is phenomenologically allowed to have a scalar interaction of neutrinos with strength $g_\nu \geq 10^{-15}$ that can explain the apparent negative $(mass)^2$ puzzle of the neutrino experiments. Let us therefore study possible particle physics implications of this idea.

### 3 Particle Physics Implications:

How likely is the possibility of such a scalar neutrino interaction from particle physics point of view? Since the force has a range of at least $10^9$ cm, this implies that
the mass of the scalar particle must be at most $\mu \approx 10^{-14}$ eV (or for $R = 1AU$, $\mu \approx 10^{-17}$ eV). Such small scalar masses are hard to understand since quantum corrections often introduce infinite corrections to them thereby requiring extreme finetuning in each order of perturbation theory to avoid large masses. The second problem for the case at hand are the small scalar couplings which also require a second fine tuning. To see the kind of fine tuning such small values for $g_\nu$ would require in a generic $\lambda \phi^4$ theory, let us note that we can retrieve the local potential due to a $\phi$ field as

$$U_\nu \approx g_\nu \phi_{local}$$

Thus $\phi_{local}$ is given in the static approximation by

$$\phi_{local} = g_\nu \Sigma e^{-\mu r_i}/r_i$$

The positive energy density in the $\phi$ field given by $\lambda \phi^4$ should not overwhelm the original negative energy $n_\nu U_\nu \approx m_\nu^3 U_\nu$ of the neutrinos. Using $U_\nu \approx m_\nu$, we find that

$$\lambda \phi^4_{local} = \lambda(U_\nu/g_\nu)^4 \leq m_\nu^3 U_\nu \approx U_\nu^4$$

or finally

$$\lambda \leq g_\nu^4$$

Even for $g_\nu$ saturating the supernova bounds, a very strong upper limit of $\lambda \leq 10^{-52}$ is implied. Clearly it calls for an extreme degree of fine tuning.

There are however field theories where these constraints on the masses and coupling constants may be met in a natural manner. We consider two examples below. In both cases, the scalar field is a pseudo-Goldstone boson which acquires scalar couplings as well as a mass due to the presence of CP-violation. The first example is a model proposed in Ref.\cite{6}, where it was shown that the specific Goldstone boson, the singlet Majoron\cite{7} which results from spontaneous breaking of global $B-L$ symmetry can in the presence of the QCD anomaly, acquire a mass. The majoron appears to have the right properties required for our purpose. The second example.

\footnote{It is amusing to note that $\lambda \simeq g_\nu^4$ is precisely the self coupling induced by box diagrams with four external $\phi$'s and four $\nu$ internal lines.}
is in the context of a general class of pseudo-Goldstone models discussed by Hill and Ross[8].

As is well-known, the Goldstone theorem requires that for a theory with the Nambu-Goldstone boson, $\phi$ the Lagrangian must be invariant under the transformation $\phi \rightarrow \phi + \alpha$ where $\alpha$ is a constant. This implies that the $\phi$ field must have zero mass and $\lambda = 0$. Unfortunately, the same invariance requirement also implies that the coupling of the $\phi$ field is derivative type so that it eliminates the possibility of having coherent $1/r$ type forces[9]. It actually leads to spin-dependent forces[7] in the non-relativistic limit. However, explicit symmetry breaking via QCD anomalies[6] not only generate small masses to make the force finite range but also induce spin-independent couplings. In particular, in the model of Ref.[6], it was noted that the scalar coupling of the Majoron to quarks is given by $g_Q \approx \theta m_u/F_\phi \approx 10^{-13}/(v_{B-L} \text{ in GeV})$, where $v_{B-L}$ is the $B-L$ symmetry breaking scale. The mass of the majoron is given by $m_\phi \approx g_Q \Lambda_{QCD}$. We note that if we choose $v_{B-L} \approx 10^9$ GeV, we satisfy the bound on $g_m$ derived in Sec.II and a mass $m_\phi$ of the right order (i.e. $m_\phi \approx 10^{-13}$ eV) that can lead to forces with range $R \approx R_E$ is obtained.

Let us now consider the coupling of the majoron (now massive due to QCD anomalies) to neutrinos. These couplings are given by $g_\nu \approx (m_\nu_e/M_N)$ where $M_N$ is the mass of the heavy right-handed neutrino. Choosing $m_\nu_e \approx 10$ eV a right-handed neutrino mass of $10^6$ or $10^7$ GeV is required to obtain $g_\nu \approx 10^{-14}$ or $10^{-15}$ as required to facilitate neutrino clustering. Since $M_N$ is related to $v_{B-L}$, it is interesting that they are numerically not too far from each other. In fact, if we choose a value for $\theta$ of about $10^{-12}$ (instead of its maximum value $\theta \leq 10^{-10}$ allowed by present neutron electric dipole moment searches) we would get $M_N = v_{B-L}$ making the model quite natural.

The second model is essentially an effective Lagrangian framework where one uses the Goldstone boson corresponding to the spontaneous breaking of of a chiral symmetry in conjunction with mass term for the Goldstone boson and CP violation to generate the long range force. The basic idea of the model can be demonstrated using left and right-handed neutrinos and the chiral lepton number symmetry as the broken symmetry. The effective Lagrangian can then be written as

$$L = L_0 + L_1$$ (27)
where

\[ L_0 = \bar{\nu}i\gamma^\mu \partial_\mu \nu + (m_{\nu L} \nu_{R} e^{i\phi/F} + h.c.) + 1/2(\partial \phi)^2 \]  
\hspace{1cm} (28)

and

\[ L_1 = \epsilon \bar{\nu}_L \nu_R + h.c. + \mu^2 F^2 \cos(\phi/F - \beta) \]  
\hspace{1cm} (29)

The effect of the \( \mu^2 \) term is to force \( \phi \) to have a vacuum expectation value. Defining \( \tilde{\phi} = \phi - \beta F \) such that \( \tilde{\phi} \) has zero vev, one can rewrite the Lagrangian in terms of \( \tilde{\phi} \). The resulting Lagrangian has a scalar coupling of \( \tilde{\phi} \) to the neutrinos with strength \( g_\nu = \epsilon \beta / F \) and a mass for \( \tilde{\phi} \) of \( \mu \) which is an arbitrary parameter. One can then choose the parameters \( \epsilon, \beta \) and \( F \) so as to get \( g_\nu \approx 10^{-14} \).

Let us now present a realization of this idea in a realistic extension of the standard model. For simplicity let us only work with one generation and extend the standard model by adding a right-handed neutrino, \( \nu_R \) and a heavy neutral leptons \( N_{L,R} \) as well as a complex scalar boson \( \Delta \) which is a singlet under the standard model gauge group. Let us assume that the model has a global \( U(1) \) symmetry under which \( \nu_R \) and \( N_L \) have charges +1 and -1 respectively and \( \Delta \) has charge +1. The rest of the fields are neutral under it. The Yukawa Lagrangian of this sub-sector of the theory is chosen to be

\[ L(\nu, N, \phi, H) = h \bar{\psi}_L H N_R + \bar{\nu}_R N_L \Delta^2 / M + f N_R \phi N_L \Delta + h.c. \]  
\hspace{1cm} (30)

where \( \psi \) denotes the lepton doublet (\( \nu, e^- \)) and \( H \) is the Higgs doublet of the standard model. The mass \( M \) correspond to unknown physics at a higher scale and is an unknown parameter for our model. It is clear that after the electroweak symmetry breaking and breaking of \( U(1) \) symmetry by the vev \( <\Delta> = F \) the \( \nu \) and \( N \) mix with each other. Writing the field \( \Delta = \frac{1}{\sqrt{2}} (F + \rho) e^{i\phi/F} \), we can obtain the coupling of the physical neutrino fields with the Goldstone boson \( \phi \) as follows:

\[ L_{\nu\nu\phi} \simeq m\bar{\nu}_R \nu_L e^{i\phi/F} \]  
\hspace{1cm} (31)

where \( m \simeq \frac{F^3}{f M^2} \). As in Hill and Ross\[8\], let us add to this theory the soft breaking terms in Eq.(26) which leads to the desired long range forces. The important point here is that due to the choice of our model, the soft breaking terms are all standard model singlets and therefore do not spoil the successes of the standard model. Moreover since the quarks or charged leptons do not connect to the field \( \Delta \), the light scalar has no coupling to quarks or charged leptons.
4 Conclusion and comments:

In conclusion, scalar long range interactions of neutrinos required to generate $U_\nu \sim 5 - 10$ eV and $n_\nu \simeq 10^{15}$ or $10^{16}$ cm$^{-3}$ on the solar system or Earth scale are not excluded by particle physics considerations. The bounds derived in this paper imply that neutrino self clustering will generally dominate over clustering due to attraction of normal particles in the Earth or the Sun. It is difficult to envision scenarios of capturing such neutrino clouds onto the Earth (or the solar system). However in a recent paper Stevenson et al. have pointed out that a "role reversal" can occur according to which primordial neutrino clouds can form first, once the temperature of the Universe drops below $m_\nu$, before the baryonic matter can cluster. These clouds can then act as nucleation sites for the solar system. This interesting idea deserves further investigation. It is however important to make the following point in this connection: it is generally believed that our solar system formed from a baryonic protocloud, larger by about a factor 100 than the present solar system. To efficiently assist in forming this protocloud, the "seed neutrino cloud" would have to be about this size. If its density is in the range considered above $\rho_\nu \simeq n_\nu m_\nu \approx 10^{16} - 10^{17}$ eV/cm$^3$, then the total mass of the neutrino cloud would range over $M_\nu \simeq (0.02 - 300) M_\odot$. Only a tiny portion of the neutrino cloud mass could lie within the solar system ($R \simeq 3.10^{14}$ cm) or at the inner planets ($R \leq R_E \simeq 1$ AU). These values of extra dark mass (about $0.2M_{\text{Earth}} - 3.10^{-4}M_{\text{Earth}}$) do not yet conflict with the recent Pioneer measurements and with precise orbit parameters for the inner planets found by radar ranging. However the effective total stellar masses seen by other stars would include the full mass of the neutrino cloud. Studies of star clusters could therefore exclude having $M_\nu \geq M_\odot$.

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References

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[1] See talks by J. Bonn, V. M. Lobashev and A. Swift in the proceedings of Neutrino’96, to be published by World Scientific.

[2] R. G. H. Robertson, T. J. Bowles, G. J. Stevenson Jr., D. L. Wark, J. F. Wilkerson and D. A. Knapp, Phys. Rev. Lett. 67, 957 (1991).

[3] S. Tremaine and J. E. Gunn, Phys. Rev. Lett. 42, 407 (1979)

[4] See E. Adelberger et al. Ann. Rev. Nucl. Part. Sc. 41, 269 (1991) for a review and earlier references.

[5] V. Barger, W. Y. Keung and S. Pakvasa, Phys. Rev. D25, 907 (1982); G. Gelmini, S. Nussinov and M. Roncadelli, Nucl. Phys. B209, 157 (1982).

[6] D. Chang, R. N. Mohapatra and S. Nussinov, Phys. Rev. Lett. 55, 2835 (1985).

[7] Y. Chikashige, R. N. Mohapatra and R. D. Peccei, Phys. Lett. 98B, 265 (1981).

[8] C. Hill and G. G. Ross, Nucl. Phys. B311, 253 (1988).

[9] G. Gelmini, S. Nussinov and T. Yanagida, Nucl. Phys. B219, 31 (1983).

[10] G. Stevenson Jr, T. Goldman and B. H. J. Mckeller, hep-ph/9603392

[11] V. L. Teplitz, private communication