Relativistic Description of $J/\psi$ Dissociation in Hot Matter (cq6346)

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Abstract

The mass spectra and binding radii of heavy quark bound states are studied on the basis of the reduced Bethe-Salpeter equation. The critical values of screening masses for $c\bar{c}$ and $b\bar{b}$ bound states at a finite temperature are obtained and compared with the previous results given by non-relativistic models.

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1 Introduction

One of the main aims of high energy nuclear collisions is to explore a new state of matter, the Quark-Gluon Plasma (QGP), through heavy ion collisions in the laboratory. It was theoretically proposed that a suppression of $J/\psi$ production in relativistic heavy ion collisions can serve as a clear signature for the formation of QGP [1]. Subsequently this suppression effect was observed by NA38 collaboration [2]. However, successive research pointed out that such suppression could also exist in hadronic matter (HM), even though caused by a completely different mechanism [3]. The anomalous $J/\psi$ suppression has recently been reported by the NA50 collaboration [4] and there have been a number of attempts to explain it [5-7]. Some authors believe that the data may implicate the possibility of the formation of a QGP [8]. For understanding the experimental data clearly, the dissociation mechanism of $J/\psi$ in hot QGP must also be studied carefully.

In QGP, quarks and gluons are deconfined and the confining force between quark and antiquark vanishes, the only interaction between quark and antiquark is the Coulomb-type color interaction. The color charge of a quark will be screened by the quark sea in the plasma. Due to Debye screening the final yields of $J/\psi$ will be suppressed. Binding and dissociation of $J/\psi$ at a finite temperature have been studied in the non-relativistic formalisms [9,10]. The $J/\psi$ was regarded as a non-relativistic bound state in those papers. However, generally speaking, the motion of a quark and an antiquark in a meson is relativistic, even for charmonium. As pointed out in Ref.[11], for the $c\bar{c}$ system, the kinetic energy is about 13% of the total energy and the ratio of relativistic corrections to the quark mass will not decrease with the increase of quark mass if the interaction is of Coulomb-type. As a result the bound state equation for $J/\psi$ in general should be relativistic. So it is an interesting task to discuss the binding and dissociation of charmonium in hot matter in a relativistic formalism. This is the main purpose of this paper.

It is well known that the Bethe-Salpeter (BS) equation [12] is the only effective relativistic equation of the two-body bound state problems. Because of its consistency with quantum field theory, the BS equation can be used for the study of the binding and dissociation of charmonium and bottomium which has seldom investigated in the framework of relativistic formalism in a thermal environment and of great interest. We expect that our calculations of observables could readily yield results at variance with those of non-
relativistic models. Furthermore, we are not only interested in the energy spectrum of mesons, which is an important source to study the interquark dynamics, but also in the wave functions which play a key role in the calculations of the root-mean-square (r.m.s) radii of $c\bar{c}$ and $b\bar{b}$ bound states.

In this paper, we shall discuss the binding and dissociation of heavy quark resonances in the hot matter within the context of the BS equation. In section 2, we focus our attention on the interaction between quark and antiquark in mesons and the properties of the BS equation. In section 3, we use the BS equation to calculate the mass spectra, r.m.s. radii and critical values of the screening masses for the $c\bar{c}$ and $b\bar{b}$ bound states and compare them with the previous results. The sensitivity of our results to the Lorentzian structure of the confining potential is also checked. In section 4, we discuss the results and conclude.

2 Formalism

It is well known that the BS equation is a proper tool for describing the relativistic two-body bound state problems[13]. The full bound state BS equation in momentum space, written in the two-sided notation, reads

$$\left(\eta_1 P + \not{p} - m_1\right)\chi_P(p)\left(\eta_2 P - \not{p} + m_2\right) = \frac{i}{(2\pi)^4} \int d^4 p' V(p, p'; P)\chi_P(p')$$

where $\eta_i = \frac{m_i}{m_1 + m_2}$ (i=1,2), $\chi_P$ is the momentum-space wave function for the quark-antiquark system with total four momentum $P$ in momentum space, $p$ is the relative four momentum. $V$ is the interaction kernel that acts on $\chi_P$ and formal products of $V\chi_P(p')$ in Eq.(1) take the form $V\chi_P(p') = V_s\chi_P(p') + \gamma_\mu \otimes \gamma_\mu V_v\chi_P(p')$, in which $V_s$ and $V_v$ are scalar, vector potential respectively. The short-distance behaviour of the $V$ can be calculated in QCD using perturbative theory. However, the long-distance behaviour of the $V$ involves non-perturbative effects, and the Lorentzian structure of the confining potential is not established theoretically in QCD. Consequently, we shall treat the form of $V$ in a partially phenomenological way. The parameters $m_1$, $m_2$ should be interpreted as effective constituent masses and similarly the whole propagator is an effective one.

Using the standard reduction and spin-independent treatment, one can get the spin-independent reduced Salpeter equation [14] for the three-dimensional equal-time BS wave
function

$$\phi(\vec{p}) = \int dp^0 \chi p(\vec{p}, \vec{p})$$ \tag{2}$$

$$(M - E_1 - E_2)\phi(\vec{p}) = \int \frac{d^3p'}{(2\pi)^3} \sum_{i=s,v} F^{si}_i(\vec{p}, \vec{p'}) V_i(|\vec{p} - \vec{p'}|)\phi(\vec{p'})$$ \tag{3}$$

where $M$ is the mass of a $q\bar{q}$ bound state, $E_i = (\vec{p}_i^2 + m_i^2)^{\frac{1}{2}}$, $i=1,2$ represent a quark and an antiquark, respectively. The functions $F^{si}_v$ and $F^{si}_s$ appearing in Eq.(3) are

$$F^{si}_v(\vec{p}, \vec{p'}) = \frac{1}{4E_1E_2}[(E_1 + m_1)(E_2 + m_2) + \vec{p}^2 + (E_1 + m_1)(E_2 + m_2)|\vec{p'}|^2$$

$$+ \frac{(\vec{p} \cdot \vec{p'})^2}{(E_1 + m_1)(E_2 + m_2)} + (\frac{1}{E_2 + m_2} + \frac{1}{E_1 + m_1} + \frac{1}{E_2 + m_2})\vec{p} \cdot \vec{p'}]$$ \tag{4}$$

and

$$F^{si}_s(\vec{p}, \vec{p'}) = \frac{1}{4E_1E_2}[(E_1 + m_1)(E_2 + m_2) - (\frac{1}{E_2 + m_2} + \frac{1}{E_1 + m_1} + \frac{1}{E_2 + m_2})\vec{p} \cdot \vec{p'} + \frac{(\vec{p} \cdot \vec{p'})^2}{(E_1 + m_1)(E_2 + m_2)}].$$ \tag{5}$$

Here $E_i' = (\vec{p}_i'^2 + m_i^2)^{\frac{1}{2}}$, $i=1,2$. Since $F^{si}_v$ and $F^{si}_s$ are spin-independent, the singlet and triplet are degenerate. There is no coupling between different orbital angular momenta in Eq.(3). We can therefore extract the angular dependence of $\phi(\vec{p})$ in a single spherical harmonic basis

$$\phi(\vec{p}) = \phi_{nL}(|\vec{p}|)Y_{LM}(\vec{p}).$$ \tag{6}$$

By using the following identity

$$\frac{4\pi}{2L + 1} \sum_{m} Y^*_{LM}(\vec{p})Y_{LM}(\vec{p'}) = P_L(cos\theta),$$ \tag{7}$$

where $\theta$ is the angle between the $\vec{p}$ and $\vec{p'}$, one can obtain the following equation for the radial wave function $\phi_{nL}(|\vec{p}|)$

$$(M_{nL} - E_1 - E_2)\phi_{nL}(|\vec{p}|) = \int \frac{d^3p'}{(2\pi)^3} \sum_{i=s,v} F^{si}_i(\vec{p}, \vec{p'}) V_i(|\vec{p} - \vec{p'}|)P_L(cos\theta)\phi_{nL}(|\vec{p'}|).$$ \tag{8}$$

Eq.(8) gives a well-defined eigenvalue problem for the masses of the $q\bar{q}$ bound states in momentum space. Here we would like to point out that the momentum dependence of the interaction is treated exactly in the above equation.
In the non-relativistic limit, Eq.(8) can be reduced to the usual Schrödinger equation

\[ (M_{nL} - E_1 - E_2)\phi_{nL}(|\vec{p}|) = \int \frac{d^3\vec{p}'}{(2\pi)^3} [V_s(|\vec{p} - \vec{p}'|) + V_v(|\vec{p} - \vec{p}'|)]P_L(\cos\theta)\phi_{nL}(|\vec{p}'|). \]  

(9)

It will be convenient in calculating the r.m.s radii of $q\bar{q}$ bound state to transform $\phi_{nL}(|\vec{p}|)$ to the position space. Making use of the following identity

\[ \exp(i\vec{p} \cdot \vec{r}) = 4\pi \sum_{l=0}^{\infty} i^lj_l(pr)Y_{lm}^*(\Theta, \Phi)Y_{lm}(\theta, \phi), \]  

(10)

where the direction of $\vec{p}$ and $\vec{r}$ is specified by the polar angles ($\Theta, \Phi$) and ($\theta, \phi$), respectively, we get

\[ \phi_{nLm}(\vec{r}) = \frac{1}{(2\pi)^2} \int \exp(i\vec{p} \cdot \vec{r})\phi_{nLm}(\vec{p})d^3\vec{p} \]  

\[ = \frac{iL}{2\pi^2} \int p^2 dp j_L(pr)\phi_{nL}(|\vec{p}|)Y_{Lm}(\theta, \phi). \]  

(11)

where we have used the orthogonal relation of spherical harmonic function and Eq.(7).

According to Eq.(10) one can get the radial wave function in the position space

\[ \phi_{nL}(|\vec{r}|) = \frac{iL}{2\pi^2} \int p^2 dp j_L(pr)\phi_{nL}(|\vec{p}|) \]  

(12)

in which the special functions $j_0(x) = \frac{\sin x}{x}$, $j_1(x) = \frac{\sin x}{x} - \cos x$. Solving Eq.(8) and Eq.(12), one can obtain the masses and corresponding r.m.s radii of the bound states.

To solve Eq.(8), one must have a good knowledge of the potential between quark and antiquark. At present, it is commonly accepted that the interaction between quark and antiquark consists of a short-range part describing the one-gluon-exchange(OGE) potential and an infinitely rising long-range part responsible for the confinement of the quarks. As it is well known that the OGE potential is a pure vector interaction. However the Lorentzian structure of the confining interaction is not clear. Wilson loop technique suggests that the confining potential should be taken purely scalar[15], but relativistic potential calculations [16-17] showed a need for some vector confinement. Therefore, we choose a confining potential to be a mixture of a scalar and a vector[18]. This leads to the following potential

\[ V(r) = V_s(r) + V_v(r), \]  

(13)

with

\[ V_s = (1 - x)\sigma r, \]  

\[ V_v = x\sigma r - \frac{4\alpha_s}{3} \frac{\alpha_s}{r}. \]
where $\sigma$ is the string tension, $\alpha_{\text{eff}} = -\frac{4}{3} \alpha_s$ the effective coupling constant and $x$ the vector-scalar mixing parameter obey the condition $0 \leq x \leq 1$. Note that the edge of the interval $x = 0$ corresponds to the case of pure scalar confinement.

In a thermodynamical environment of interacting light quarks and gluons at temperature $T$, quark binding becomes modified by color screening [1]

$$V(r, \mu) = \left[ \frac{x \sigma}{\mu} (1 - e^{-\mu r}) - \frac{4 \alpha_s}{3} \frac{e^{-\mu r}}{r} \right] + \frac{\sigma}{\mu}(1-x)(1-e^{-\mu r}). \tag{14}$$

Here $\mu$ is the Debye screening mass (which is assumed to be a function of temperature $T$) and the Debye screening length $r_D$ is defined as the inverse of the screening mass, $r_D = \frac{1}{\mu}$.

It is necessary to note that the factor $\exp(-\mu r)$ not only reflects the color screening effects but also avoids the infrared divergence. In fact, as pointed out in Ref.[19], the color screening effects are also required at $T=0$ in order to fit the experimental properties of quark systems. In this paper, we use the color screening potential (Eq.(14)) to study the binding and deconfinement of heavy quark resonances. In momentum space the potential can be written as

$$V_s(|\vec{p} - \vec{p}'|) = (1 - x) \left\{ \frac{\sigma}{\mu} \delta^3(\vec{p} - \vec{p}') - \frac{\sigma}{\pi^2} \frac{1}{((\vec{p} - \vec{p}')^2 + \mu^2)^2} \right\}, \tag{15}$$

and

$$V_v(|\vec{p} - \vec{p}'|) = -\frac{2}{3\pi^2} \frac{\alpha_s}{[(\vec{p} - \vec{p}')^2 + \mu^2]} + x \left( \frac{\sigma}{\mu} \delta^3(\vec{p} - \vec{p}') - \frac{\sigma}{\pi^2} \frac{1}{((\vec{p} - \vec{p}')^2 + \mu^2)^2} \right). \tag{16}$$

The constants $\sigma$, $\mu$, $x$ and $\alpha_s$ are the parameters characterizing the potential.

### 3 Calculations and Results

Based on the formula above, we first calculate the mass spectra and r.m.s radii of $c\bar{c}$ and $b\bar{b}$ bound states with the vector-scalar mixing parameter $x = 0$, which corresponds to the pure scalar confinement. The numerical results are listed in Table I. The data used in our studies consisted of the spin-averaged masses of $b\bar{b}$ and $c\bar{c}$ states and are given in the third column in Table I, with

$$\overline{M}_{nl} = \frac{1}{4(2l+1)} \sum_j (2j + 1) M(n, j, l, s). \tag{17}$$

Here we restrict ourselves to the first two radial excitations, corresponding to the (spin-averaged) $J/\psi$ and $\Upsilon$ for $n=1, l=0$, to the $\psi'$ and $\Upsilon$ for $n=2, l=0$, and to the $\chi_c$ and $\chi_b$ for
n=2, l=1. Therefore \( M_{nl} \) can be written explicitly as:

\[
M_{n0} = \frac{1}{4} [3M(n^3S_1) + M(n^1S_0)],
\]

\[
M_{n1} = \frac{1}{12} [5M(n^3P_2) + 3M(n^3P_1) + 3M(n^1P_1) + M(n^3P_0)]
\]

(n=0,1 corresponding to the first two radial excitations), where we recall the usual spectroscopic notation \( 2s+1L_J \) for a state with orbital angular momentum \( L \), spin \( s \), and total angular momentum \( J \); \( S, P, ... \) correspond to orbital angular momentum \( L=0,1, ... \), respectively. Since the spin-singlets \( \bar{b}b(1S_0) \) and \( \bar{b}b(1P_1) \) have not yet been unambiguously confirmed by experiment, we have therefore used the results of previous spin-dependent fits to the data to estimate the centers of gravity of the incomplete multiplets [14]. However, Ref.[10] compared their numerical results with the spin-triplet rather than the spin-averaged masses of \( \bar{b}b \) and \( c\bar{c} \) systems.

Table I. The mass spectra and rms radii of the \( c\bar{c} \) and \( b\bar{b} \) bound states.

| \( nl \) | Data | \( M_{nl} \) (GeV) | \( <r^2>^{1/2} \) (fm) |
|---|---|---|---|
| \( c\bar{c} \) | 1S \( J/\psi(3.068) \) | 3.0697 | 3.0700 | 3.067 | 0.4490 | 0.4453 | 0.2868 |
| | 2S \( \psi'(3.663) \) | 3.6978 | 3.6863 | 3.663 | 0.8655 | 0.9034 | 0.6317 |
| | 3S \( \psi''(4.025) \) | 4.1696 | 4.0806 | 4.019 | 1.2025 | 1.3765 | 0.8290 |
| | 1P \( \chi_c(3.525) \) | 3.5003 | 3.5054 | 3.526 | 0.6890 | 0.7000 | 0.5144 |
| \( b\bar{b} \) | 1S \( \Upsilon(9.436) \) | 9.4450 | 9.4310 | 9.436 | 0.2249 | 0.2211 | 0.1873 |
| | 2S \( \Upsilon'(10.013) \) | 10.0040 | 10.0083 | 10.013 | 0.5040 | 0.4998 | 0.4480 |
| | 3S \( \Upsilon''(10.341) \) | 10.3547 | 10.3564 | 10.343 | 0.7336 | 0.7457 | 0.6749 |
| | 1P \( \chi_b(9.899) \) | 9.8974 | 9.8981 | 9.901 | 0.4041 | 0.3982 | 0.3569 |

In our calculations, we have used the following parameters. \( \sigma=0.22 \) GeV\(^2 \), \( \mu_0=0.06 \) GeV, \( m_c=1.474 \) GeV, \( m_b=4.762 \) GeV, \( \alpha_s(c\bar{c})=0.47 \), and \( \alpha_s(b\bar{b})=0.38 \). All these parameters are within the scope of customary usage. According to the concept of a running gauge coupling constant, in the computations we allow different values of \( \alpha_s \) for charmonium (\( \alpha_s(c\bar{c}) \)) and bottomonium (\( \alpha_s(b\bar{b}) \)) [20]. The relative magnitude of \( \alpha_s \) is in accordance with the idea of asymptotic freedom as expected for the strong gauge coupling constant of quantum chromodynamics, that is, \( \alpha_s(b\bar{b})<\alpha_s(c\bar{c}) \).

In order to compare our results with those given in Refs.[9-10], we calculate the quantity \( \chi^2 \) which is defined as

\[
\chi^2 = \sum_{nl}(M_{nl}^{exp} - M_{nl}^{theory})^2 \frac{1}{N-1}
\]  

(18)

with \( N \) being the total number of \( (nl) \) state. We obtain \( \chi = 0.0066 \) GeV for our numerical
results. Comparing with $\chi = 0.0223$ GeV (for Ref.[10]) and $\chi = 0.0511$ (for the Ref.[9]), one can observe that the mass spectra obtained at present are more consistent with the experiment than the previous results.

As mentioned above, the wave functions play a key role in the calculation of r.m.s radii of $c\bar{c}$ and $b\bar{b}$ systems. The last column in Table I shows that the r.m.s radii of $c\bar{c}$ and $b\bar{b}$ bound states given by our calculations are smaller than those of Refs.[9,10]. This means that $J/\psi$ is more tightly bound in our case than estimated by non-relativistic models.

Next, we study the dissociation of $b\bar{b}$ and $c\bar{c}$ systems. A suitable quantity to observe the vanishing of bound states is the dissociation energy

$$E_{\text{dis}}^{nl}(\mu) = m_1 + m_2 + \frac{\sigma}{\mu} - M_{nl}(\mu). \quad (19)$$

The dissociation energy is positive for bound states and turns negative for the continuum. Thus

$$E_{\text{dis}}^{nl}(\mu_c) = 0 \quad (20)$$

defines the critical value of $\mu$ beyond which there is no bound state for the given quantum numbers. The calculated results of the $E_{\text{dis}}^{nl}(\mu_c)$ for the $c\bar{c}$ and $b\bar{b}$ systems are given in Fig.1 and Fig.2 respectively. The figures show that the dissociation energies of our calculations (solid lines) are shifted to larger $\mu$ regions in comparision with those (dotted lines) of Ref.[10]. The reason is that our calculations are based on the relativistic formula in which the momentum dependence is treated exactly, which is different from that used in Ref.[10].

The calculated critical values of Debye masses and $M_{nl}(\mu_c)$ for $c\bar{c}$ and $b\bar{b}$ resonances are given in Table II.

Table II. The calculated $\mu_c$ and $M_{nl}(\mu_c)$ for charmonium ($c\bar{c}$) and bottomium ($b\bar{b}$).

| state    | $\mu_c$(GeV) | $M_{nl}(\mu_c)$(GeV) |
|----------|--------------|----------------------|
|          | Ref.[9]      | Ref.[10]             | Ours | Ref.[9]      | Ref.[10] | Ours |
| Charmonium |             |                      |
| 1S       | 0.700        | 0.600                | 0.900 | 2.9145      | 2.8779   | 3.1911 |
| 2S       | 0.360        | 0.260                | 0.470 | 3.1725      | 3.2964   | 3.4160 |
| 1P       | 0.342        | 0.242                | 0.450 | 3.1982      | 3.3513   | 3.4363 |
| Bottomium |             |                      |
| 1S       | 1.560        | 1.500                | 1.640 | 9.6108      | 9.5379   | 9.6581 |
| 2S       | 0.660        | 0.560                | 0.690 | 9.7838      | 9.7528   | 9.8426 |
| 1P       | 0.578        | 0.460                | 0.640 | 9.8226      | 9.8274   | 9.8670 |
Table II shows that the critical values of the screening masses for $c\bar{c}$ and $b\bar{b}$ dissociations given by our calculations are larger than those given by Refs. [9,10]. This indicates that the results are dependent on the model. So a finer calculation of the screening masses for $c\bar{c}$ and $b\bar{b}$ dissociation is needed. Because the $J/\psi$ suppression is related to the colour screening, the dissociation of $J/\psi$ is more interesting at finite temperatures. According to our calculations, the critical value of the screening mass for $J/\psi$ dissociation is about $\mu_c = 0.900 \text{ GeV}$ (the corresponding screening length is 0.219 fm). This information is probably useful to the study and observation of $J/\psi$ production in high energy collisions.

In view of Table II, we would like to further note that the masses of all bound states are affected slightly by the change of $\mu$. As shown by Refs. [9,10], with the increment of $\mu$ the masses of the $c\bar{c}$ bound states and those of the higher $b\bar{b}$ bound states decrease, while the $\Upsilon$ mass increases, which is different from ours. In our case, the masses of the higher $c\bar{c}$ and $b\bar{b}$ bound states decrease with $\mu$, while the $J/\psi$ and $\Upsilon$ masses increase with $\mu$. This means that the positive string tension part of the potential dominates and is reduced as $\mu$ increases; only for the $\Upsilon$ and $J/\psi$ does the second term in the right hand side of Eq. (14) give the main contribution in the relativistic formalism.

One can also find in Table II that the relativistic correction for $c\bar{c}$ bound states is larger than that of $b\bar{b}$ ones. This is not surprising. Even before the detailed numerical calculations, this qualitative conclusion can be reached based on the following reasonable physical consideration.

As mentioned above, the Schrödinger equation is the non-relativistic limit of the BS equation. Usually, the $b$ quark is heavy enough compared to $\Lambda_{QCD}$, and one can expect that the relativistic correction is small for $b\bar{b}$ bound states. Nevertheless, the mass of $c$ quark, $m_c$, is not much larger than $\Lambda_{QCD}$. The relativistic correction may be large in the case of $c\bar{c}$ bound states, which is corroborated by the detailed numerical calculations listed in Table II.

Finally, we check the sensitivity of our results with respect to the vector-scalar mixing parameter $x$ appearing in the potential, the numerical results are listed in Table III.

On the basis of the analysis of the numerical results (see Table III), we come to the following conclusions:

For the systems containing two heavy quarks the sensitivity of the mass spectra to the Lorentz structure of the confining potential is rather moderate, especially in the $b\bar{b}$ system. Therefore, our results based on pure scalar confining potential are insensitive to
the particular Lorentz structure of confinement.

Table III. The dependence of $c\bar{c}$ and $b\bar{b}$ masses (GeV) on the mixing parameter $x$.

| states | $x = 0.0$ | $x = 0.1$ | $x = 0.3$ | $x = 0.5$ | $x = 0.7$ | $x = 0.9$ | $x = 1.0$ |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $c\bar{c}$ | 1S | 3.067 | 3.085 | 3.120 | 3.155 | 3.189 | 3.223 | 3.240 |
|        | 2S | 3.663 | 3.686 | 3.730 | 3.773 | 3.814 | 3.854 | 3.874 |
|        | 3S | 4.019 | 4.053 | 4.118 | 4.180 | 4.238 | 4.293 | 4.320 |
|        | 1P | 3.526 | 3.552 | 3.602 | 3.650 | 3.698 | 3.745 | 3.768 |
| $b\bar{b}$ | 1S | 9.436 | 9.440 | 9.448 | 9.455 | 9.462 | 9.470 | 9.473 |
|        | 2S | 10.013 | 10.017 | 10.027 | 10.036 | 10.047 | 10.053 | 10.058 |
|        | 3S | 10.343 | 10.350 | 10.363 | 10.376 | 10.390 | 10.402 | 10.408 |
|        | 1P | 9.901 | 9.905 | 9.915 | 9.924 | 9.933 | 9.943 | 9.947 |

4 Discussions and Conclusions

In summary, we have studied the mass spectra, root-mean-square radii and dissociation of $c\bar{c}$ and $b\bar{b}$ in the relativistic formalism. We would like to point out that a relativistic treatment of quark-antiquark bound states, by means of the reduced BS equation, does imply some improvement in the description of $c\bar{c}$ and $b\bar{b}$ meson mass spectra. This indicates that the relativistic description is necessary for a finer calculation of $J/\psi$ at finite temperatures. Because the $m_c$ is not heavy enough compared to $\Lambda_{QCD}$, the relativistic effect can not be neglected for the description of $J/\psi$ dissociation in hot matter. The critical values of the screening masses have also been calculated and compared with the previous results given by the non-relativistic models. The critical value of the screening mass for $J/\psi$ dissociation is $\mu_c=0.600$ GeV in Ref.[10] and $\mu_c=0.700$ GeV in Ref.[9], respectively. In the present calculations, however, $\mu_c=0.900$ GeV, which is larger than those from non-relativistic models. This means that the magnitude of the screening masses is model dependent. In order to get a finer evaluation of the screening mass of $J/\psi$, one must have a good knowledge the confining potential in addition to a relativistic description. Therefore the study of quark confining potential is of importance and should be further proceeded.

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**Figure Captions**

Fig.1 The dissociation energies for $c\bar{c}$ bound states. The solid lines indicate our results, the dotted ones are from Ref.[10].

Fig.2 The dissociation energies for $b\bar{b}$ bound states. The solid lines represent our results, the dotted ones are from Ref.[10].
Fig. 1
