Azimuthal polarization of polarized top quark in noncommutative standard model

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Abstract Top quark decays before hadronization and its information transfer to final products. The top quark decays via the weak interaction, hence the azimuthal polarized decay rate is zero in the leading order of SM. Therefore the study of top quark could be taken into consideration as useful tools to search for new physics. In this paper, we calculate the decay of polarized top quark $t(↑) \rightarrow bW^+ \rightarrow bℓ^+ν_ℓ$, in the framework of the noncommutative standard model. We investigate the contribution of noncommutative space-time on the azimuthal polarized decay rate and spin analysing power. Also we depict the energy spectrum for the charged lepton and the variation of the total differential decay rate with respect to the azimuthal angle $φ$ which receive some correction in noncommutative space-time.

Keywords single top · azimuthal polarization · noncommutativity

1 Introduction

Top quark produces as single top and $t\bar{t}$ pair at tevatron and LHC. It provides a suitable situation to study the top quark properties. The top quark is the heaviest known elementary particle having the mass $173.21 \pm 0.51 \pm 0.71$ GeV [1]. The large mass leads to a very short lifetime for top quark [2], in the standard model (SM). The top quark decays via electro-weak interaction before hadronization because its lifetime is an order of magnitude smaller than typical (QCD) hadronization time scale ($\frac{1}{Λ_{QCD}} \sim 30 \times 10^{-25}$) [2]. Therefore study of top quark could help to probe the electroweak symmetry breaking mechanism. Also it provides the appropriate circumstance to search for any deviation from the Standard Model and to test the New physics beyond the Standard Model [3]. Using the productions of single top decay and $t\bar{t}$ pair annihilation, one could obtains some information about the polarization and spin properties of top quark. The decay before hadronization cause the spin and the polarization properties transfer to the final products. By studying the polarization properties of the products, the polarization and spin information of the primary particle could be determined. When the primary particle’s beams that produce the top quarks are polarized, some information could be obtained from them. In hadron colliders, this opportunity don’t exist, because the polarization of the primary beams are complicated. But this is possible for future lepton colliders as $e^-e^+$ and $μ^-μ^+$. We could obtain information from the initial beams as the final products. There are many studies in this subject in hadron colliders [4] and also for $e^-e^+$ colliders [5,6,7].

Top quark properties are interested from various points of view, the new decay modes, the flavour changing neutral current, the anomalous coupling in the twb vertex, the helicity of $W$ boson, the top quark polarization and the spin correlations [8-11]. We discuss the single top quark polarization in this paper.

In SM, the top quark decays almost completely into a $b$-quark and a $W$-boson with the branching ratio $BR(t \rightarrow bW^+) \sim 0.998$ [1]. According to the possible leptonic and hadronic decay modes of the $W^+$-boson [12], the dominant semi-leptonic decays of the top quark are $t \rightarrow bW^+ \rightarrow b(ℓ^+ν_ℓ)$ in the SM [13]. There are many studies on the top quark decay properties in SM. We summarize some of them in the following:

- The spin-dependent energy distribution of B-hadrons from polarized top decays is studied by considering the azimuthal correlation rate [14].
- The QED and QCD radiative corrections to the charged lepton energy distributions are calculated in the dominant semi-leptonic decays of the unpolarized top quark \( t \to bW^+ \to b(\ell^+\nu_\ell) \) in the SM. The QCD corrections are calculated in the leading and next-to-leading logarithmic approximations, but the QED is considered in the leading logarithmic approximation only \([13,15]\).

- \( \Gamma_A^0, \Gamma_B^0 \) and \( \Gamma_C^0 \) denoting the unpolarized decay rate(UPDR), polarized decay rate (PPDR) and azimuthal polarized decay rate(APDR), that are the polarization observables, respectively, are studied for the polarized top quark in the semi-leptonic decay, \( t(\uparrow) \to X_b + \ell^+ + \nu_\ell \). \( \Gamma_A^0 \) vanishes due to the left chiral \((V-A)\) nature of the weak currents \([16]\). There are some other examples have been caused from this effect(see\([16]\) and references there in). Therefore in the leading order (LO) of SM, the APDR is zero. The NLO and NLO QCD corrections had been calculated for longitudinal polarized top quark \([17,10]\). The NLO QCD corrections is carried out to the \( \Gamma_C^0 \) for \( m_b = 0 \) and the results is compared to the corresponding contribution of a non-standard-model(N-SM), right-chiral quark current, for \( m_b \neq 0 \). For \( m_b \neq 0 \), it is possible to have the interference contribution from the interference of the left and right chiral quark currents when squaring the full matrix element whereas it is not possible when \( m_b = 0 \). NLO and N-SM right-chiral corrections are \( \left| \Gamma_C^{(1)} \right| = 2.4 \times 10^{-3} \Gamma_A^0 \) and \( \left| \Gamma_C^{(R)} + \Gamma_C^{(int)} \right| \leq 4.7 \times 10^{-5} \Gamma_A^0 \), respectively \([18,19]\) where \( \Gamma_C^{(1)} \) indicates the APDR, caused by NLO QCD corrections, \( \Gamma_C^{(R)} \) is the right chiral quark current contribution and \( \Gamma_C^{(int)} \) display the interference between the SM and right chiral contributions. The absence of the rate \( \Gamma_C^0 \) in the LO of the SM is a consequence of the left-chiral \((V-A)(V-A)\) nature of the current-current interaction in the SM \([18]\).

As it is concluded from \([18,19]\), nonzero contributions to the rate \( \Gamma_C^0 \) can either arise from N-SM effects or from higher order SM radiative corrections. Many studies is done on the top quark features via N-SM effects. The nonstandard effects on the full top width have been investigated in the minimal super-symmetric standard model and in the technicolor model \([20,21,22]\). Also the effects of anomalous \( tWb \) couplings is investigated on the top width and some constraints have been applied on the anomalous couplings \([23,24,25,3]\). The noncommutative space-time (NCST) is one of the features beyond the SM. We intend to investigate the polarized top quark decay in NCST.

The non-commutativity in space-time is a possible generalization of the usual quantum mechanics and quantum field theory to describe the physics at very short distance of the order of the Plank length, since the nature of the space-time changes at these distances. The properties of the top quark can be studied in noncommutative standard model (NCSM). Some before work are as following:

- At High energy collider experiments we can refer for example to forbidden decays in standard model (SM) such as \( Z \to \gamma\gamma \) \([26]\), top quark decays \([27,28,2]\), compton scattering \([29]\) which have been investigated in NCST. In the experiment which has been done by OPAL collaboration, NC bound from \( e^-e^+ \) scattering at 95% CL is \( \Lambda_{NC} > 141 GeV \) \([30]\). Also there are some researches in electron-proton colliders and hadron colliders to probe the NCST \([31,32]\).

- Some studies is done on the bosons and fermions \([33,34,35,36,37,38]\) and on the forbidden decays \([39,40]\) in the NCST.

- The bounds on the NC scale \( \Lambda_{NC} \) are estimated using the measurements of the unpolarized top quark two-body decay width, \( t \to Wb \), and the W-boson polarization in top pair events from CDF experiment at tevatron. The bounds on \( \Lambda_{NC} \) from the measured top quark width and W polarization are \( \Lambda_{NC} \geq 625 GeV \) and \( \Lambda_{NC} \geq 1550 GeV \), respectively \([24]\).

- Decay \( t \to Wb \) is studied in the NCSM and lowest contribution to the decay comes from the terms quadratic in the matrix describing the NC effects. The NC effects are found to be significant only for low values of the NC characteristics scale \( (\Lambda_{NC} \sim 140 GeV) \) \([2]\).

- The lepton spectrum from the decay of a polarized top quark is studied in the NCST, \( t(\uparrow) \to b\ell^+\nu_\ell \), where the hadron vertex \( t \to Wb \) is used up to the order \( \theta^2 \). The \( \ell^+ \) spin correlation coefficient has a remarkable deviation from the SM for \( \Lambda \leq 1 TeV \). Also the total decay rate of polarized top quark is shown with respect to (w.r.t) NC scale where the NC effects is negligible for \( \Lambda_{NC} \geq 625 GeV \), but \( \Gamma_A, \Gamma_B \) and \( \Gamma_C \) are not calculated \([25]\). Also the NC corrected vertex is used just in the hadronic vertex in \([25]\). In this article we study the decay \( t(\uparrow) \to b\ell^+\nu_\ell \) in the NCST that SM leptonic and hadronic vertices can be replaced by NC corrected vertices. Therefore we calculate \( \Gamma_A, \Gamma_B \) and \( \Gamma_C \) in NCST up to the order \( \theta^2 \). Considering the top quark polarization, \( \Gamma_C \) could be measured in the future colliders.

In the short and mid-term future, top quark studies will be mainly driven by the LHC experiments. However, detection of top quark properties will be an integral part of particle physics studies at any future facility. A possible future lepton collider such as \( e^-e^+ \) would be enable to do the unique top physics program far beyond the capabilities of the LHC. A \( e^-e^+ \) collider
will allow us to study electroweak production of \( tt \) pairs with no concurring QCD background. Therefore, precise measurements of top quark properties such as \( \Gamma_t \) become possible. It should be noted that some studies have been done on polarized top quark to search in future \( e^-e^+ \) colliders \cite{11,42,43}. The organization of the paper is as follows: In sec. 3 we give a short introduction of NCSM. In sec. 4 the UPDR, PPDR and APDR for \( t(\uparrow) \rightarrow b \ell^+ \nu_\ell \) are calculated in NCSM. The results of sec. 4 are expressed and discussed in sec. 5. Finally, in sec. 5 we summarize our results and give our conclusion.

2 Noncommutative space time (NCST)

The shortcomings of existing theories in ordinary spacetime in the expression of the some phenomena and even the a slight deviation of some others from precise laboratory results is leaded to search for new physics beyond the SM to justify these cases. One of the new theories, which began inclusive research on it since 1999, is theory of the field in NCST. It come back to the string theory where the end points of an open string show the noncommutative behavior. In NCST, the commutation relation between space-time coordinates versus ordinary space-time is a nonzero quantity \cite{44}:

\[
\{x^\mu, x^\nu\} = i\theta^{\mu\nu}
\]

where \( \theta^{\mu\nu} \) is called NC parameter and is a constant, real and antisymmetric tensor.

Eq. (1) enters in string theory through the Weyl-Moyal star product \cite{44,45,46}:

\[
(f \ast g)(x) = \exp\left(\frac{i}{2} \theta^{\mu\nu} \partial_\mu f \partial_\nu g\right) f(y)g(z)_{y=z=x} = f(x)g(x) + \frac{i}{2} \theta^{\mu\nu} (\partial_\mu f(x)) (\partial_\nu g(x)) + O(\theta^2)
\]

Writing the theory of the field in NCST from this way causes problems such as the charge quantization and etc \cite{47,48}. To solve these problems, two solutions are proposed. The first solution known as the Seiberg-Witten map \cite{49}, the NC fields are expanded in terms of ordinary fields \cite{50} and the symmetry group is similar the SM one. In the second solution, the symmetry group is larger than the SM one and using two higgs mechanisms, the symmetry of the group reduces to the SM symmetry group \cite{51}. In this paper, we use the Seiberg-Witten map. One of the important features of the theory of NC field is that in addition to the correction of the Feynman rules related to the existing vertices, it predicts new vertices that does not exist in the SM. In the following, we study the decay of the polarized top quark into a b-quark, anti-lepton and neutrino related to it, using the W-boson channel. To study this decay, we needed to corrected vertex in NCST for fermions with W-boson using Seiberg-Witten map, which is \cite{53}:

\[
\frac{\nu}{2\sqrt{2} \sin \theta_\omega} \left( \frac{V^{yy}_{f(i)}(\omega)}{V^{\mu\nu}_{f(i)}(\omega)} \right) \{ \left( \gamma^\mu - \frac{i}{2} \theta_{\mu\rho} k^\rho \right) (1 - \gamma^5) - \frac{i}{2} \theta_{\mu\rho} \left( \frac{m_f^{(y)}}{m_{\mu}^{(y)}} \right) p^n_m (1 - \gamma^5) - \left( \frac{m_f^{(y)}}{m_{\mu}^{(y)}} \right) p^n_{out} (1 + \gamma^5) \}
\]

where \( m \) is the fermion mass and \( \theta_{\mu\rho} \) is defined as follows:

\[
\theta_{\mu\rho} = \theta_{\mu\nu} \gamma^\nu + \theta_{\rho\nu} \gamma^\nu + \theta_{\nu\rho} \gamma^\nu
\]

In the Eq. (3), \( V^{yy}_{f(i)} \) is CKM matrix element for quarks and is equal to 1 for leptons \cite{52,53}.

In this paper, by replacing the Eq. (3) in the hadron and lepton vertices, we calculate the invariant amplitude of decay in NCST up to the order \( \theta^2 \). It should be mentioned that the following identities are used in the calculations \cite{25}:

\[
A_\alpha \theta^{\alpha\beta} B_\beta = A \theta B = \theta (A \times B)
\]

\[
A_\mu \theta^{\mu\nu} \theta^\nu_{\alpha} B_\alpha = |\theta|^2 (A \times B) - (A \cdot \theta)(B \cdot \theta)
\]

The NC field theories are not unitary for \( \theta^{\alpha\beta} \neq 0 \) , therefore we choose \( \theta^{\alpha\beta} = 0 \) in our calculations \cite{54}.

3 Polarized top quark decay in noncommutative space-time

The top quark has short lifetime because of its large mass. This quark decays before hadronization and its spin information can be deduced from its decay products. If polarized top quark decays in the form of semileptonic so that b quark and \( W^+ \)- boson to be decay products and \( W^+ \) decays into a charged lepton \( \ell^+ \) and a neutrino \( \nu_\ell \), so we have \( t(p_1) \rightarrow b(p_2) W^+(q) \rightarrow b(p_2) \ell^+(p_3) \nu_\ell(p_4) \). Here \( t, b, \ell^+ \) and \( \nu_\ell \) particles have been tagged with 1, 2, 3 and 4 numbers, respectively. \( W^+ \)- boson four-vector is \( q = p_3 + p_4 \). Considering helicity system shown in Fig. \( \Box \) that the event plane defines the \((x,z)\) plane and the \( z \)-axis be in the direction of the \( \ell^+ \) momentum and also considering top quark polarization vector, \( P_\ell \), so that makes \( \theta_\rho \)
The definition of the polar angle $\theta_F$ and the azimuthal angle $\phi$ in the rest frame decay of a polarized top quark. The event plane defines the $(x,z)$ plane with $p_T \parallel z$.

polar angle with $\ell^+$ momentum and $\phi$ azimuthal angle with the event plane(Figure 1 in Ref.[15]), we can write $t(p_1) \rightarrow b(p_2) W^+(q) \rightarrow b(p_2) \ell^+(p_3) \nu_L(p_4)$ decay amplitude for LO in NCSM by using the Seiberge-Witten map as follows:

$$M^{NC} = (\frac{g_w}{\sqrt{2}})^2 V_{tb}$$
$$\times \bar{u}(p_2, s_2) \left\{ \frac{1}{2} \gamma_{\mu} (1 - 2s) + \frac{i}{2} \theta_{\mu \rho \sigma \rho} q^\rho p_1^\sigma \left( \frac{1}{2} - s \right) \right\} u(p_1, s_1)$$
$$\times \left( -i \frac{g_{\mu \nu} - \frac{m_W^2}{q^2}}{q^2 - m_W^2 + i\epsilon} \right)$$
$$\times \bar{u}(p_4, s_4) \left\{ \frac{1}{2} \gamma_{\nu} (1 - 2s) - \frac{i}{2} \theta_{\nu \beta \sigma \rho} q^\rho p_3^\sigma \left( \frac{1}{2} - s \right) \right\} v(p_3, s_3)$$

(7)

LO in NCST means that we calculate the amplitude in the LO of SM while vertices are replaced by the corrected one in the NCST. To obtain the amplitude(Eq.(12)), we have used the SM $W^+$ propagator (because propagators don’t receive any correction in NCST) and Eq.(11) for the corrected fermion-fermion-W boson in NCST. Also lepton masses and b quark mass are neglected.

We write W-propagator as [19]

$$-i \frac{g_{\mu \nu}}{q^2 - m_W^2 + i\epsilon} \rightarrow -i \frac{g_{\mu \nu}}{q^2 - m_W^2 + m_W T_W}$$

$$- \frac{i g_{\mu \nu}}{m_W (y^2 - 1 + i\gamma)}$$

(8)

Eq.(12) could be written as:

$$M^{NC} = i \frac{G_F V_{tb}}{\sqrt{2}(\frac{m_W^2}{y^2} - 1 + i\gamma)} H^{NC}_{\mu} L^{\mu}_{NC}$$

(9)

where $y = \sqrt{\frac{m_T^2}{m_W^2}}$, $\tilde{y} = \frac{m_T}{m_W}$ and $q$ and $T_W$ are $W^+$ boson momentum and resonance width. The hadronic current $H^{NC}_{\mu}$ and leptonic current $L^{NC}_{\mu}$ in NCST and $G_F$ Fermi coupling constant are defined as follows:

$$H^{NC}_{\mu} = \bar{u}(p_2, s_2) \left\{ \gamma_{\mu} (1 - 2s) + \frac{i}{2} \theta_{\mu \rho \sigma \rho} q^\rho p_1^\sigma \left( \frac{1}{2} - s \right) \right\} u(p_1, s_1)$$

(10)

$$L^{NC}_{\mu} = \bar{u}(p_4, s_4) \left\{ \frac{1}{2} \gamma_{\nu} (1 - 2s) - \frac{i}{2} \theta_{\nu \beta \sigma \rho} q^\rho p_3^\sigma \left( \frac{1}{2} - s \right) \right\} v(p_3, s_3)$$

(11)

$$G_F = \frac{g_w^2}{4\sqrt{2} m_W}$$

(12)

We need the average of squared amplitude to obtain the decay rate. Therefore with averaging over initial spins, sum over final spins and using Casimir’s trick we calculate it ($\langle |M^{NC}|^2 \rangle$). Also we have defined NC vector, $\theta$, in the $(x,z)$ plane $(\theta_1 = 0)$ and $\theta = \frac{\gamma}{\sqrt{2}} (1, 0, 0)$ where $|\theta| = \theta_0 \sim \frac{1}{\Lambda_{NC}}$ ($A_{NC}$ is the NC scale). We have

$$\langle |M^{NC}|^2 \rangle = \frac{G_F^2 V_{tb}^2}{21 (1 - \frac{3}{2} y^2 + y^2)}$$

$$\times \{ 128 (\bar{p}_1, p_3) \left( p_2, p_4 \right) - 16 \left( q, \theta, \theta, q \right) \left( \bar{p}_1, p_1 \right) \left( p_2, p_3, p_4 \right) + 32 \left( \bar{p}_1, p_1 \right) \left( p_2, p_3, p_4 \right) \}$$

$$+ 32 \left( \bar{p}_1, p_2 \right) \left( p_3, p_4 \right) + 32 \left( \bar{p}_1, p_2 \right) \left( p_3, p_4 \right) \}$$

(13)

In Eq.(13), the momentum four-vectors and the top quark polarization vector($s^p$) in the rest frame of top quark are

$$p_1^\mu = p_1^\mu = m_t (1; 0, 0, 0),$$

$$p_2^\mu = p_2^\mu = \frac{m_1}{2} (1 - y^2) (1; \sin \theta, 0, \cos \theta),$$

(13)
\[ p^\mu_3 = p^\mu_4 = \frac{m_1 + \overline{p}}{2} x(1; 0, 0, 1), \]
\[ s^\mu_1 = S^\mu_2 = (0; \sin \theta_P \cos \phi, \sin \theta_P \sin \phi, \cos \theta_P), \]
\[ \overline{p}^\mu = p^\mu_1 - m_1 s^\mu_1 \]

where
\[
\cos \theta_v = \frac{x(1 - x + y^2) - 2y^2}{x(1 - x + y^2)}, \\
\cos \theta_b = \frac{2y^2 - x(1 + y^2)}{x(1 - x + y^2)}. 
\]

Using 4-vectors and Eqs. (5)-(10) we can calculate Eq. (14).

For the contraction of the lepton and hadron tensors in NCST, we have
\[
H^{NC}_{\nu}\nu L^{\mu\nu}_{NC} = 32m_1^2 [M^A_{NC}(x, y)] + M^B_{NC}(x, y) \cos \theta_P + M^C_{NC}(x, y) \sin \theta_P \cos \phi 
\]

Where \( M^A_{NC}(x, y) \), \( M^B_{NC}(x, y) \) and \( M^C_{NC}(x, y) \) are the unpolarized, polar polarized and azimuthal polarized amplitudes, respectively and we calculate them in the following. It should be noted that the scattered lepton energy \( x \) is equal to \( x = \frac{2E}{m_1} \).

For the unpolarized amplitude, we have
\[
M^A_{NC}(x, y) = x(1 - x) + \frac{1}{8\sqrt{2}} \theta_2 m_1^2 \left( x(1 + y^2) - 2y^2 \right) \left( 2 \times \frac{\sqrt{y^2(x - x^2 - y^2 + y^2)} x}{y} \right) \]
\[
= \frac{1}{128} \theta_2^m m_1^4 y^2 (1 - y^2)^2 \left( 2y^2 - x(1 + y^2) \right) \left( 4 \times \frac{\sqrt{y^2(x - x^2 - y^2 + y^2)}}{y} \right). 
\]

First term in Eq. (15) shows the ordinary space-time and other terms of order \( \theta \) and \( \theta^2 \) are the corrections caused by noncommutativity.

The polarized amplitude is
\[
M^B_{NC}(x, y) = x(1 - x) + \frac{1}{8\sqrt{2}} \theta_2 m_1^2 \left( 4y^2(x - x^2 - y^2 + y^2) \right) \]
\[
= \frac{1}{128} \theta_2^m m_1^4 y^2 (1 - y^2)^2 \left( 2y^2 - x(1 + y^2) \right) \left( 4 \times \frac{\sqrt{y^2(x - x^2 - y^2 + y^2)}}{y} \right). 
\]

Eq. (16) expresses the contributions of the corrections caused by noncommutativity in order \( \theta \) and \( \theta^2 \) that they are added to the ordinary space-time contribution when \( \theta = 0 \). Eventually for the azimuthal polarized amplitude, we have
\[
M^C_{NC}(x, y) = \frac{1}{4\sqrt{2}} \theta_2 m_1^2 y^2 (1 - x) \left( 2 \times \frac{\sqrt{y^2(x - x^2 - y^2 + y^2)}}{y} \right) \\
= \frac{1}{128} \theta_2^m m_1^4 y^2 (1 - y^2)^2 \left( 2 \times \frac{\sqrt{y^2(x - x^2 - y^2 + y^2)}}{y} \right). 
\]

As mentioned in the introduction the azimuthal polarized amplitude is zero in the LO of SM \( [M^C_{NC} = 0 \) see Refs. [15-19], therefore Eq. (17) shows the NC contribution of order \( \theta \) and \( \theta^2 \). The differential rate is
\[
d\Gamma^{NC} = \frac{1}{2m_1(2\pi)^3} \left[ \frac{d^2p}{2E} \frac{d^2p}{2E} \frac{d^2p}{2E} \frac{d^2p}{2E} \right] \\
\times (2\pi)^3 \delta^4(p_1 - p_2 - p_3 - p_4) |M^{NC}|^2. 
\]

Substituting the Eq. (14) in the Eq. (18), the differential decay rate is given by
\[
\frac{d\Gamma^{NC}}{dx \cos \theta_P \sin \phi} = \frac{1}{4\pi} \Gamma_F \left( \frac{12}{(1 - x)^2} + 2 \right) dy^2 \left[ M^A_{NC}(x) \right] \\
+ M^B_{NC}(x) \cos \theta_P + M^C_{NC}(x) \sin \theta_P \cos \phi \right] 
\]

The general form of the differential decay rate is:
\[
\frac{d\Gamma^{NC}}{dx \cos \theta_P \cos \phi} = \frac{1}{4\pi} \left[ \frac{d\Gamma^{NC}}{dx} \right] + \frac{d\Gamma^{NC}}{dx} \cos \theta_P \cos \phi 
\]

By inserting Eqs. (19) and (20) and using the narrow-width approximation for the W-propagator (see sec. 1), we obtain the differential rates in NCST, i.e. the unpolarized \( \frac{d\Gamma^{NC}}{dx} \), the polarized \( \frac{d\Gamma^{NC}}{dx} \) and the azimuthal polarized \( \frac{d\Gamma^{NC}}{dx} \) differential rates. The important point is that the differential rates dependence on \( y \) disappears. They are only depend on \( x \), as in ordinary space-time. Therefore the unpolarized differential rate (UPDIR) is
\[
\frac{d\Gamma^{NC}}{dx} = \frac{8\pi m_1^3 \overline{m}_3}{16\pi^2 \Gamma_W} V_{tb}^2 \left[ F_0(x) + \frac{1}{A^2} F_1(x) + \frac{1}{A^3} F_2(x) \right] 
\]

(21)
Where
\[ F_0(x) = x(1 - x), \]
\[ F_1(x) = \frac{1}{8\sqrt{2}}m_1^2(4\hat{y}^2 - x^2 + (1 + \hat{y}^2)x - \hat{y}^2) \]
\[ + 2\hat{y}(x(1 + \hat{y}^2) - 2\hat{y}^2)\sqrt{-x^2 + (1 + \hat{y}^2)x - \hat{y}^2}, \]
\[ F_2(x) = \frac{1}{128}m_1^4(-\hat{y}^2(1 - \hat{y}^2)^3) \]
\[ + 4\hat{y}(1 - \hat{y}^2)(2\hat{y}^2 - x(1 + \hat{y}^2))\sqrt{-x^2 + (1 + \hat{y}^2)x - \hat{y}^2}. \]

(22)

First term in Eq. (21), \( F_0(x), \) is ordinary space-time contribution and the other terms of order \( \theta \sim \frac{1}{A^2} \) and \( \theta^2 \sim \frac{1}{A^4}, \) \( F_1(x) \) and \( F_2(x), \) are NCST contributions.

The polar polarized differential rate (PPDiR) is
\[ \frac{d\Gamma_{NC}^{PPDir}}{dx} = \frac{G_F^2m_1^3m_W}{16\pi^3I_W}V_{ub}^2\{G_0(x) + \frac{1}{A^2}G_1(x) + \frac{1}{A^4}G_2(x)\} \]

(23)

Where
\[ G_0(x) = x(1 - x), \]
\[ G_1(x) = \frac{1}{8\sqrt{2}}m_1^2(4\hat{y}^2 - x^2 + (1 + \hat{y}^2)x - \hat{y}^2) \]
\[ + 2\hat{y}(x(1 + \hat{y}^2) - 2\hat{y}^2)\sqrt{-x^2 + (1 + \hat{y}^2)x - \hat{y}^2}, \]
\[ G_2(x) = \frac{1}{128}m_1^4(-\hat{y}^2(1 - \hat{y}^2)^3) \]
\[ + 4\hat{y}(1 - \hat{y}^2)(2\hat{y}^2 - x(1 + \hat{y}^2))\sqrt{-x^2 + (1 + \hat{y}^2)x - \hat{y}^2}. \]

(24)

Like the unpolarized case, Eq. (23) contains the ordinary space-time contribution in order of \( \theta \sim \frac{1}{A^2} \) and NCST ones in order of \( \theta \sim \frac{1}{A^4} \) and \( \theta^2 \sim \frac{1}{A^6}. \)

The azimuthal polarized differential rate (APDiR) in NCST is
\[ \frac{d\Gamma_{NC}^{APDir}}{dx} = \frac{G_F^2m_1^3m_W}{16\pi^3I_W}V_{ub}^2\{\frac{1}{A^2}H_1(x) + \frac{1}{A^4}H_2(x)\} \]

(25)

Where
\[ H_1(x) = \frac{1}{4\sqrt{2}}m_1^2(-2\hat{y}^3(1 - x)\sqrt{-x^2 + (1 + \hat{y}^2)x - \hat{y}^2}) \]
\[ -\hat{y}^2(1 - x)(1 + \hat{y}^2)x - 2\hat{y}^2), \]
\[ H_2(x) = \frac{1}{128}m_1^4[2\hat{y}^3(1 - \hat{y}^2)^2 \times \sqrt{-x^2 + (1 + \hat{y}^2)x - \hat{y}^2}] \]
\[ - 8\hat{y}^2(1 + \hat{y}^2)x - 2\hat{y}^2)((-x^2 + (1 + \hat{y}^2)x - \hat{y}^2)]. \]

(26)

As mentioned before, the LO SM contribution for APDiR is zero and Eq. (26) contains the NCST contributions only. The integrated rates are calculated in the interval \( \hat{y}^2 < x \leq 1. \) We have:

\[ \Gamma_{NC}^{APDir} = G_F^2m_1^3m_W^2V_{ub}^2\left\{\frac{1}{2}(1 - \hat{y}^2)^2(1 + 2\hat{y}^2) \right\} \]
\[ + \frac{1}{A^2}\left[\frac{1}{8\sqrt{2}}m_1^2(2\hat{y}^2(1 - \hat{y}^4) + 8\hat{y}^4\ln \hat{y} \right] \]
\[ - \frac{1}{A^4}\hat{y}(1 + \hat{y}^2)(1 - \hat{y}^2)^2 + 2\hat{y}^4(1 - \hat{y}^2)^2 \\pi \}
\[ + \frac{1}{A^4}\left[\frac{1}{128}m_1^4(-\hat{y}^2(1 - \hat{y}^2)^4 + 2\hat{y}^3(1 - \hat{y}^2)^2(1 - \hat{y}^2)^4)\right] \}

(27)

for the unpolarized rate,
\[ \Gamma_{NC}^{APDir} = G_F^2m_1^3m_W^2V_{ub}^2\left\{\frac{1}{2}(1 - \hat{y}^2)^2(1 + 2\hat{y}^2) \right\} \]
\[ + \frac{1}{A^2}\left[\frac{1}{8\sqrt{2}}m_1^2(2\hat{y}^2(1 - \hat{y}^4) + 8\hat{y}^4\ln \hat{y} \right] \]
\[ - \frac{1}{A^4}\hat{y}(1 + \hat{y}^2)(1 - \hat{y}^2)^2 + 2\hat{y}^4(1 - \hat{y}^2)^2 \\pi \}
\[ + \frac{1}{A^4}\left[\frac{1}{128}m_1^4(-4\hat{y}^4(1 - \hat{y}^2)^2)\ln \hat{y} \right] \]
\[ - \frac{1}{A^4}\hat{y}^2((1 - \hat{y}^2)^4 + 4\hat{y}^2(1 - \hat{y}^2) - 4\hat{y}^2\ln \hat{y} \right] \]
\[ + \frac{1}{A^4}\left[\frac{1}{128}m_1^4(-\hat{y}^2(1 - \hat{y}^2)^2(1 - \hat{y}^2)^2 - 24\hat{y}^4(1 - \hat{y}^4) \}
\[ - 16\hat{y}^4(1 + 4\hat{y}^2 + \hat{y}^4)\ln \hat{y} \right] \}

(28)

for the azimuthal polarized rate.

One can takes the variables as \( m_1 = 173.21 \) GeV, \( m_W = 80.385 \) GeV, \( \hat{y} = \frac{m_W}{m_1}, \) \( I_W = 2.085 \) GeV, \( G_F = 1.1663787 \times 10^{-5} \) (GeV)^{-2} and \( V_{ub} = 0.999 \) \( \frac{1}{A}. \) We obtain Eqs. (27) \( \sim \) (29) w.r.t NC scale, by replacing the mentioned variables, as follows:

The unpolarized rate:
\[ \Gamma_{NC}^{APDir} = \Gamma_{APDir}^{(0)} + \Gamma_{APDir}^{nc} = 1.11308(0.146803 + \frac{88.9289}{A^2} - \frac{288.195}{A^4}) \]

(30)

\( \Gamma_{APDir}^{(0)} \) is the unpolarized rate in LO of SM and \( \Gamma_{APDir}^{nc} \) is the contribution of NCST in order of \( \frac{1}{A^2} \) and \( \frac{1}{A^4}. \)
The unpolarized differential rates w.r.t. the charged lepton energy (charged lepton spectra).

The polar polarized rate:

$$\Gamma^{NC} = \Gamma_B^{(0)} + \Gamma_B^{nc} = 1.11308(0.146803 + \frac{88.9289}{A^2} + \frac{1189.41}{A^4})$$  \hfill (31)

Where $$\Gamma_B^{(0)}$$ is the polar polarized rate in LO of SM and $$\Gamma_B^{nc}$$ is the contribution of NCST in order of $$\frac{1}{\Lambda^2}$$ and $$\frac{1}{\Lambda^4}$$. The azimuthal polarized rate:

$$\Gamma_C^{NC} = 1.11308(\frac{164.045}{A^2} - \frac{211731}{A^4})$$  \hfill (32)

The contribution of azimuthal polarized rate in LO of SM is zero and Eq. (32) shows the azimuthal polarized rate in NCST of order $$\frac{1}{\Lambda^2}$$ and $$\frac{1}{\Lambda^4}$$. It has the non-zero value.

According to theoretical calculations of rates, if an experiment’s central values exactly agree with the SM prediction for $$\Gamma_A$$, $$\Gamma_B$$ and $$\Gamma_C$$ etc. and also the anticipated errors are small and ignorable, then obtained limits in next section are acceptable. Otherwise, these limits depend on the amount of experimental errors.

4 Results and Discussions

The NC effects on the decay of polarized top quark has been investigated in sec. 3 by inserting the Eq. (3) in the hadronic and leptonic vertices. The corrections of the NCST has been calculated up to LO in terms of the NC parameter, $$\theta$$. The UPDiR has been calculated up to the order of $$\frac{\theta^2}{A^2}$$ in Eq. (24). The UPDiR are depicted in Fig. 2. The value of UPDiR is changing by variation of NC scale. For $$\Lambda \geq 1200$$ GeV the variation is not recognizable with its SM value.

The PPDiR has been calculated in NCST up to the order of $$\frac{\theta^2}{A^2}$$ in Eq. (25). The azimuthal polarized rate up to the order of $$\frac{\theta^2}{A^2}$$ has been achieved in Eq. (31). The dependence of PPDR to the NC scale can be seen in Fig. 6. The deviation of the partial decay width from the SM expectations is more obvious for $$\Lambda \leq 2$$ TeV.
According to the first term in Eqs. (30) and (31), $\Gamma_A^{(0)}$ and $\Gamma_B^{(0)}$, it is clear that the UPDR and PPDR are equal in the LO of SM (see Ref. [19]). But they are not equal in NCST. It is noteworthy that two rates are equal up to the order of $\theta \sim 1$ but are not equal in the order of $\theta^2 \sim 1$. The difference term is positive for the PPDR and negative for the UPDR, therefore the PPDR is bigger than the UPDR in the NCST ($\Gamma_B^{NC} > \Gamma_A^{NC}$), but it is insignificant and can be ignored.

The APDR has been achieved in Eq. (32). It is just the contribution of NC space-time. The result of NC corrections on both leptonic and hadronic vertices for the azimuthal polarized rate has been shown in Fig 8. According to the plot, the deviation of the partial width decay from SM expectations occurs when $\Lambda \leq 1.5$ TeV and the results of NC tend toward the SM when $\Lambda \geq 1.8$ TeV. Of course, the reader should pay attention that as it is described at the end of the previous section, the specified limits in this section depend on the experimental values and its errors.

Now, we are going to describe some angular distribution results of Eq. (20):

1) To obtain the total differential rate in the NCST, $\Gamma^{NC}_{total}$, we have used Eq. (20). By getting the uniform angular distribution, i.e. $0 \leq \theta_P \leq \pi$ and $0 \leq \phi \leq 2\pi$, and integrating on these intervals, we will have the following equality:

$$d\Gamma^{NC}_{total} = d\Gamma^A_{NC}$$

as a result

$$\Gamma^{NC} = \Gamma^{NC}_{total} = \Gamma^A_{NC}$$

These equations show that the uniform angular distribution leads to the equality of the total differential decay rate and the UPDiR and eventually to the equality of the total decay rate and the UPDR. Therefore Figs 2 and 5 display total differential decay rate and total decay rate, too.

Fig 9 shows the charged lepton energy spectra in the decay of unpolarized top quark by applying corrections on both leptonic and hadronic vertices.
Fig. 9 The energy spectra for the charged lepton in the polarized top quark semileptonic decay.

Fig. 10 The total rate for \( t(\uparrow) \to b\ell^+\nu_\ell \) decay w.r.t NC scale in the logarithmic scale.

Fig. 11 The spin analyzing power \( \alpha_\ell(x) \) w.r.t NC scale in the logarithmic scale.

Fig. 12 The dependence of total differential rate to the \( \phi \) azimuthal angle and NC scale.

The spin analyzing power decreases by increasing both the lepton energy and NC scale. The deviation from the SM is obvious for \( \Lambda \leq 1 \text{ TeV} \) and the NC effect is ignorable for \( \Lambda > 500 \text{ GeV} \).

2) By inserting Eq. (37) in Eq. (20) and integrating on the \( \phi \) azimuthal angle, we have

\[
\frac{d\Gamma^{NC}_{\ell}}{dx} = \frac{d\Gamma^{NC}_{\ell} \cos \theta_P}{dx} \times \frac{1}{2} \left[ 1 + \alpha_\ell(x) \cos \theta_P \right] \tag{35}
\]

and \( \alpha_\ell(x) \), correction coefficient or spin analyzing power, is

\[
\alpha_\ell(x) = \frac{\frac{d\Gamma^{NC}_{\ell}}{dx}}{\frac{d\Gamma^{NC}_{\ell}}{dx} \cos \theta_P} = \frac{G_0(x) + \frac{1}{\Lambda} G_1(x) + \frac{1}{\Lambda^2} G_2(x)}{F_0(x) + \frac{1}{\Lambda} F_1(x) + \frac{1}{\Lambda^2} F_2(x)} \tag{36}
\]

where \( F_0(x), F_1(x), F_2(x), G_0(x), G_1(x) \) and \( G_2(x) \) have been defined in Eqs. (22) and (24). The deviation becomes small. 

3) Eq. (37) denotes the total differential rate \( \frac{d\Gamma^{NC}_{\ell}}{dx} \) for \( t(\uparrow) \to b\ell^+\nu_\ell \).

\[
\frac{d\Gamma^{NC}_{\ell}}{dx} = \frac{1}{4\pi} \left[ 2\Gamma^{NC}_A - \frac{x}{2} \Gamma^{NC}_B \cos \phi \right] = \frac{131308}{4\pi} \left[ 2(0.146803 - 0.288195 + 0.889289) - \frac{\pi}{2} \left( -\frac{211731}{\Lambda^4} + 164.045 \right) \cos \phi \right] \tag{37}
\]

where \( \Gamma^{NC}_A \) and \( \Gamma^{NC}_B \) has been placed from Eqs. (30) and (32). It is depicted in Fig 12 at various values of \( \Lambda \). It shows the total differential rate vs the \( \phi \) azimuthal angle deviates from the SM. By increasing the NC scale, the deviation becomes small.

4) Eq. (20) has been used to obtain the polar analyzing power or polar asymmetry in NCST, \( \frac{\Gamma^{NC}_{\ell}}{\Gamma^{NC}_{\ell}} \). We
The polar analyzing power in LO of SM is equal 1 and the polar analyzing power in NCST is between 0 and 1. However it could be understood that the correction of NCST to the polar analyzing power is less than 0.003%. It means that for this value of \( \Lambda \) the deviation of energy spectra for charged lepton is depicted in Fig 9. It shows that in the future colliders with access to high energies and the production of top quark in such energies, these may produce more charged lepton than the SM, because of NC effects. The dependence of total differential rate to the production of top quark in such energies, these may produce more charged lepton than the SM, because of NC effects.

5 Conclusion

We have calculated the effects of NCST on the semileptonic decay of polarized top quark. This decay has been investigated in the helicity system where the event plane defines the \((x,z)\) plane and the \(z\)-axis is in the direction of the \(\ell^+\) momentum. The calculations show that the unpolarized, polarized and azimuthal polarized rates, the total differential rate, the spin analyzing power and the energy spectra of charged lepton receive some corrections from NCST.

The unpolarized and polarized (differential) rates receive small corrections from NCST, see Figs [5][6][7][8][9][10] (see Figs 23, the azimuthal polarized (differential) rate that is zero in LO of SM, is given the non-zero value in NCST, see Fig 8 see Fig 11). It is equal to the contribution of NLO QCD when \( \Lambda \simeq 680 \text{ GeV} \). The total decay rate, Fig 11 also get a bit correction. The spin analyzing power is sensitive to the NC scale. It is 1 in the LO of SM whereas it is bigger than 1 when \( \Lambda \simeq 1 \text{ TeV} \) (Fig 11). The deviation of energy spectra for charged lepton is depicted in Fig 9. It shows that in the future colliders with access to high energies and the production of top quark in such energies, these may produce more charged lepton than the SM, because of NC effects. The dependence of total differential rate to the production of top quark in such energies, these may produce more charged lepton than the SM, because of NC effects.

By increasing the \( \Lambda \), this ratio becomes smaller. For 500 GeV \( \leq \Lambda \leq 900 \text{ GeV} \) it is co-order with the value of NLO QCD \( |\Gamma^{NLO}_C| = 2.4 \times 10^{-3} \Gamma^{(0)}_A \) in Ref. [13][19]). The NC effect on \( \Gamma^{NC}_C \) is equal to the NLO QCD when \( \Lambda \simeq 680 \text{ GeV} \). It means that for this value of \( \Lambda = 680 \text{ GeV} \), the correction of NC and NLO are the same.

\[
\Lambda \simeq 680 \text{ GeV}, \quad \Gamma^{NC}_C = 2.4 \times 10^{-3} \Gamma^{(0)}_A
\]  

Note that the NLO contribution is negative and the NC contribution is positive, therefore the magnitude of NLO contribution is used.

\[\text{References}\]

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\[
\begin{array}{|c|c|c|c|c|}
\hline
\Lambda (\text{GeV}) & 200 & 500 & 900 & 1200 \\
\hline \frac{\Gamma^{NC}_C}{\Gamma^{(0)}_A} & 1.0012 & 1.0003 & 1.000068 & 1.0000095 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\Lambda (\text{GeV}) & \frac{\Gamma^{NC}_C}{\Gamma^{(0)}_A} \\
\hline
200 & 27.03 \times 10^{-3} \Gamma^{(0)}_A \\
500 & 4.45 \times 10^{-3} \Gamma^{(0)}_A \\
900 & 1.38 \times 10^{-3} \Gamma^{(0)}_A \\
1200 & 0.76 \times 10^{-3} \Gamma^{(0)}_A \\
\hline
\end{array}
\]
