Numerical Simulation of the Normal Glow Discharge for Conditions of the Experimental Research at Low Distance between Plane Electrodes

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Abstract. Two-dimensional drift-diffusion calculation model of charged particles in a normal glow discharge between plane disc electrodes is presented. The verification of the created computational model has been performed using the experimental data on a normal glow discharge between flat electrodes with distance \( H = 1 \) cm. Results of numerical simulation and initial experimental data are presented for the pressure of \( p = 5 \) Torr and emf of power supply in the region of \( 1000 \div 1750 \) V. The sensitive of the computational models to its internal half-empirical parameters are analysed on the example of volt-ampere characteristics.

1. Introduction
The study of the structure of normal glow discharge is of considerable interest not only from the point of view of applied physics of gas discharges but also from the point of view of fundamental physics of gas-discharge phenomena as well as the processes of self-organization in partially ionized rarefied gases. The fact of fulfilling the law of “normal current density” and the formation of a current column of a glow discharge in free space is a vivid manifestation of the self-organization of the structure of the gas-discharge plasma channel. Here the plasma is supported by the external electric field, and the free boundary of the plasma-conducting region is formed due to the mobility and diffusion of charged particles.

Experimental studies of a normal glow discharge were started about 100 years ago [1] in connection with observations of current structures in gas discharge devices and the behavior of cathode and anode spots on electrodes. Recently, experimental studies have been performed on a large-scale experimental device with plate electrodes specially created for this purpose [2]. In [2] a normal glow discharge in the inter-electrode gas discharge gap of height \( H = 2 \) cm at pressures \( p = 1 \div 10 \) Torr was studied. The glow discharge, in this case, was characterized by the presence of a sufficiently long positive column (about 1 cm) and near-electrodes layers.
Recently authors of [2] performed a series of experiments in a gas-discharge gap at \( H = 1 \) cm and \( p = 1 \div 10 \) Torr (see the paper in a given volume of JPCS). In this case, at low pressures, the positive column is practically absent, and the cathode layer immediately passes into the anode layer. The experimental data on current-voltage characteristics and visual observations obtained make it possible to compare with the results of numerical modeling of the structure of a glow discharge.

Systematic research of the normal glow discharge showed that its structure is well described by the diffusion-drift model [3, 4]. However, this model belongs to the class of phenomenological models, in which there are a number of physical constants that are borrowed from experiment or calculations using kinetic plasma models. These constants include the coefficient of secondary ion-electron emission from the cathode and the ionization coefficient of the gas by electron impact. When attempting to compare the calculated and experimental data, it turned out that the successful calculated prediction of the experimental values of the current-voltage characteristic largely depends on the choice of these values.

This paper presents the results of calculations of a two-dimensional structure of a normal glow discharge in molecular nitrogen in the pressure range \( p = 3 \div 10 \) Torr in a gas discharge gap of height \( H =1 \) cm. Comparison with recently obtained experimental data is discussed. The analysis of the structural features of a normal glow discharge was carried out with various initial data.

2. The drift-diffusion model

As was shown in [3, 4], the equations of the drift-diffusion model which are used to simulate a normal glow discharge were obtained from the system of equations for a multi-fluid and multi-temperature partially ionized gas mixture. These are:

\[
\text{div} \mathbf{E} = -\varepsilon \left( n_+ - n_e \right),
\]

\[
\frac{\partial n_e}{\partial t} + \text{div} \Gamma_e = \alpha |\Gamma_e| - \beta n_e n_e, \quad (2)
\]

\[
\frac{\partial n_+}{\partial t} + \text{div} \Gamma_+ = \alpha |\Gamma_+| - \beta n_e n_e, \quad (3)
\]

where: \( n, n_e \) are the concentration of ions and electrons; \( \mathbf{E} \) is the vector of electric field strength with electric potential \( \varphi \):

\[
\mathbf{E} = -\text{grad}\varphi.
\]

The vectors of electron and ion flux densities \( \Gamma_e, \Gamma_+ \) are defined as following
\[
\Gamma_e = -n_e \mu_e E - D_e \nabla n_e, \quad \Gamma_i = -n_i \mu_i E - D_i \nabla n_i,
\]  
\hspace{1cm} (5)

where: \(\mu_e, \mu_i\) are the mobilities of electrons and ions; \(D_e, D_i\) are the diffusion coefficients of electrons and ions; \(\alpha, \beta\) are the coefficient of impact ionization of a neutral gas by electrons (the 1st Townsend coefficient) and the recombination coefficient; \(\varepsilon = 4\pi\varepsilon_0 = 1.81 \times 10^{-6} \text{ V/cm}\).

Boundary conditions for equations (1) – (3) are the following:

\[
x = 0: \quad \Gamma_{e,x} = \beta \Gamma_{i,x}, \quad \frac{\partial n_e}{\partial x} = 0, \quad \varphi = 0, \\
x = H: \quad \frac{\partial n_e}{\partial x} = 0, \quad n_e = 0, \quad \varphi = V, \\
r = 0: \quad \frac{\partial n_e}{\partial r} = \frac{\partial n_i}{\partial r} = \frac{\partial \varphi}{\partial r} = 0, \\
r = R_e: \quad \frac{\partial n_e}{\partial r} = \frac{\partial n_i}{\partial r} = \frac{\partial \varphi}{\partial r} = 0
\]  
\hspace{1cm} (6)

where: \(V\) is the voltage drop at the electrodes, determined from the electric circuit equation:

\[
\frac{E - V}{e R_0} = 2\pi \int_0^R \Gamma_{e,n_e} r dr
\]  
\hspace{1cm} (7)

The integral in (7) should be calculated along the surface of the electrode with the projection of the electron flux density \(\Gamma_{e,n_e}\) on the local normal to the surface \(n_{s,a}\).

Condition (7) determines the electric current through the anode. For a stationary glow discharge, the total electric current through the gas-discharge gap is a constant value and can also be determined from the following equation:

\[
\frac{E - V}{e R_0} = 2\pi \int_0^R \Gamma_{e,n_e} r dr
\]  
\hspace{1cm} (7)

where \(n_{s,c}\) is the local normal to surface of cathode \(n_{s,c}\).

We also note that when analysing the transient processes of establishing a stationary state, one should take into account the effect of accumulation of charges on electrodes (see [4] for more details).

The closing relations for the problem being solved are given for molecular nitrogen [2]:

\[
\mu_e p = 4.4 \times 10^5, \quad \mu_i p = 1.45 \times 10^3 \text{ Torr} \times \text{cm}^2/\text{V} \times \text{s}, \quad \beta = 2 \times 10^{-7}, \text{ cm}^3/\text{s} \\
D_e = \mu_e T_e, \quad D_i = \mu_i T_i, \quad T_e = 1 \text{ eV}, \quad T_i = 0.026 \text{ eV}
\]

Two approximations were used for the 1-st Townsend coefficient

\[
\frac{\alpha}{p} = A \exp \left( - \frac{B}{E/p} \right), \quad 1/(\text{cm} \times \text{Torr}),
\]

- In the case of the one-mode approximation the following approximation coefficients

\[
A = 12, \quad (\text{cm} \times \text{Torr})^{-1}, \quad B = 342, \quad \text{V/(cm} \times \text{Torr)}
\]

were used for any value of \(E/p\).

- In the case of the two-mode approximation values of \(A\) and \(B\) were different for two regions of variation of the reduced field \(E/p\): if \(E/p > 200 \text{ V/(cm} \times \text{Torr)}\), then \(A = 12, \quad (\text{cm} \times \text{Torr})^{-1},\)
B=342, V/(cm×Torr); if $E/p < 200$ V/(cm×Torr), then $A=8.8$, (cm×Torr)$^{-1}$, $B=275$, V/(cm×Torr).

3. **Numerical simulation results**

Calculations of the normal glow discharge were performed for the electric circuit shown in figure 1. The resistive resistance of the external circuit in all cases was $R_0 = 300$ kΩ, the height of the electric charging gap $H = 1$ sm. The best conditions for the existence of the normal glow discharge were realized at $p = 5$ Torr. At lower pressures, as will be shown below, the electric current column significantly increased its radial dimensions and the discharge approached the anomalous discharge. At high pressures, the radial dimensions of the current column decreased so much that the discharge passed into the subnormal combustion mode.

A systematic study of the normal glow discharge was also performed at $p = 3$ Torr. In addition, the calculations were performed for different values of the secondary electron emission coefficient $\gamma = 0.05; 0.1$ and $0.33$, as well as for two approximation dependences of the 1-st Townsend coefficient.

Electrodynamic fields, presented in figures 2–12 correspond to the one-mode approximation of the first Townsend coefficient.

![Diagram 1](image1.png)

**Figure 2.** Normal glow discharge at $p=5$ Torr, $\gamma=0.33$, $E=1$ kV:

- a) Electric potential $F_l = \varphi/E$ (to left) and reduced strength of the electric field $EDP = E_l/p$, V/(cm×Torr) (to right);
- b) Volume concentration of electrons (to left) and ions (to right);

$N_e, N_i = (n_e, n_i)/10^9$ cm$^{-3}$

Figures 2–4 show the fields of electrodynamic functions in the normal glow discharge with $p = 5$ Torr; $\gamma = 0.33$ for three values of emf: $E = 1000$, $2500$, and $4000$ V. Clearly seen how the radial dimensions of the electric current column increase. However, the absolute values of the reduced field and the concentrations of ions and electrons in the vicinity of the axis of symmetry change slightly.

A comparison of the axial distributions of the concentrations of ions and electrons is given in figure 5 for emf: $E = 1000$ V and $4000$ V. The distribution of current density at the cathode and anode, corresponding to different values of emf, indicates good compliance with the law of normal current density, especially for relatively large emf.
Figure 3. Normal glow discharge at $p=5$ Torr, $\gamma=0.33$, $E=2.5$ kV:

a) Electric potential $F_i = \varphi/E$ (to left) and reduced strength of the electric field $EDP = E_i/p \cdot V/(\text{cm} \cdot \text{Torr})$ (to right);

b) Volume concentration of electrons (to left) and ions (to right);

$$N_e, N_i = (n_e, n_i) / 10^9 \text{ cm}^{-3}$$

Figure 4. Normal glow discharge at $p=5$ Torr, $\gamma=0.33$, $E=4.0$ kV:

a) Electric potential $F_i = \varphi/E$ (to left) and reduced strength of the electric field $EDP = E_i/p \cdot V/(\text{cm} \cdot \text{Torr})$ (to right);

b) Volume concentration of electrons (to left) and ions (to right);

$$N_e, N_i = (n_e, n_i) / 10^9 \text{ cm}^{-3}$$
Figure 5. Normal glow discharge at \( p = 5 \) Torr, \( \gamma = 0.33 \), \( E = 1.0 \) kV and 4.0 kV. Distributions of volume concentrations of electrons and ions along axis of symmetry.

Figure 6. Distributions of electric current density on the cathode and anode at \( E = 1.0 \) kV (curves 1), \( E = 1.5 \) kV (curves 2), \( E = 2.0 \) kV (curves 3), \( E = 2.5 \) kV (curves 4), \( E = 3.0 \) kV (curves 5), \( E = 3.5 \) kV (curves 6), \( E = 4.0 \) kV (curves 7).

Normal glow discharge at \( p = 5 \) Torr, \( \gamma = 0.33 \) (a) and \( \gamma = 0.053 \) (b).

The distribution of current densities is shown in figure 6.a for \( \gamma = 0.33 \) and in figure 6.b for \( \gamma = 0.05 \). From this, it is seen that a decrease in \( \gamma \) leads to a noticeable decrease in current density at the cathode, but a slight decrease in the anode. This significantly increases the radial dimensions of the current column.

In figures 7 ÷ 11 shows the distribution of electric potential, reduced field and concentrations of charged particles at \( p = 3 \) Torr; \( \gamma = 0.05 \) for three values of emf: \( E = 1000 \), 2500, and 4000 V.

We note a significant increase in the radial dimensions of the electric current columns in comparison with the previous calculated series (figures 2 ÷ 4). In the case under consideration, the radial dimensions of the cathode and anode spots practically coincided, which follows from the spatial distributions of functions in figures 7 ÷ 9 and from the current density distributions on the cathode and anode shown in figure 10. In this case, the axial distribution of the concentrations of charged particles in the axial regions changes slightly. We also pay attention to the effect of increasing the current density at the boundary of the electric current column.
Figure 7. Normal glow discharge at $p=5$ Torr, $\gamma=0.05$, $E=1.0$ kV:

a) Electric potential $F_i = \varphi / E$ (to left) and reduced strength of the electric field $EDP = E_i / p , V/(cm\cdot Torr)$ (to right);

b) Volume concentration of electrons (to left) and ions (to right); $N_e, N_i = (n_e, n_i) / 10^9$ cm$^{-3}$

Figure 8. Normal glow discharge at $p=5$ Torr, $\gamma=0.05$, $E=2.5$ kV:

a) Electric potential $F_i = \varphi / E$ (to left) and reduced strength of the electric field $EDP = E_i / p , V/(cm\cdot Torr)$ (to right);

b) Volume concentration of electrons (to left) and ions (to right); $N_e, N_i = (n_e, n_i) / 10^9$ cm$^{-3}$

Comparison of the calculation results with the experimental data of the current-voltage characteristic is given in figure 12. From this figure, there is a noticeable effect on the volt-ampere characteristics of the method of approximation of the 1-st Townsend coefficient and the secondary electron emission coefficient. For $\gamma = 0.1$ and $p = 5$ Torr, good agreement between the calculated and experimental data was obtained.
Figure 9. Normal glow discharge at \( p = 5 \) Torr, \( \gamma = 0.05 \), \( E = 4.0 \) kV:

a) Electric potential \( \Phi = \phi / E \) (to left) and reduced strength of the electric field \( \text{EDP} = E_r / p, \text{V/(cm\cdotTorr)} \) (to right);

b) Volume concentration of electrons (to left) and ions (to right);

\[ N_e, N_i = (n_e, n_i) / 10^9 \ \text{cm}^{-3} \]

Figure 10. Distributions of electric current density on the cathode and anode at \( E = 1.0 \) kV (curves 1), \( E = 1.5 \) kV (curves 2), \( E = 2.0 \) kV (curves 3), \( E = 2.5 \) kV (curves 4), \( E = 3.0 \) kV (curves 5), \( E = 3.5 \) kV (curves 6), \( E = 4.0 \) kV (curves 7). Normal glow discharge at \( p = 3 \) Torr, \( \gamma = 0.05 \)
Figure 11. Normal glow discharge at $p=5$ Torr, $\gamma=0.05$, $E=1.0$ kV and 4.0 kV. Distributions of volume concentrations of electrons and ions along axis of symmetry.

Figure 12. Volt-ampere characteristics of the normal glow discharge at $\gamma=0.33$ and $\gamma=0.1$. Solid curves correspond to the use of the 2-mode approximation of the 1st Townsends coefficient, dotted lines correspond to the use of the 1-mode approximation of the 1st Townsends coefficient; circles - are the experimental data.
4. Conclusion
A numerical simulation of the two-dimensional structure of a normal glow discharge in molecular nitrogen in the pressure range \( p = 3 \) Torr and \( p = 5 \) Torr was performed.

It is shown that two empirical coefficients (ionization coefficients and second electron emission coefficients) included in the diffusion-drift model have a significant effect on the parameters of the current-voltage characteristic.

An example of a good calculated description of experimental data is given.

Acknowledgments
The work was supported by the Russian Science Foundation grant # 16-11-10275-II.

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