Observations of spatiotemporal instabilities in the strong-driving regime of an AC-driven nonlinear Schrödinger system

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Localized dissipative structures (LDS) have been predicted to display a rich array of instabilities, yet systematic experimental studies have remained scarce. We have used a synchronously-driven optical fiber ring resonator to experimentally study LDS instabilities in the strong-driving regime of the AC-driven nonlinear Schrödinger equation (also known as the Lugiato-Lefever model). Through continuous variation of a single control parameter, we have observed a string of theoretically predicted instability modes, including irregular oscillations and chaotic collapses. Beyond a critical point, we observe behaviour reminiscent of a phase transition: LDSs trigger localized domains of spatiotemporal chaos that invade the surrounding homogeneous state. Our findings directly confirm a number of theoretical predictions, and they highlight that complex LDS instabilities can play a role in experimental systems.

The AC-driven nonlinear Schrödinger equation (NLSE) is a prototypical model equation for pattern formation. It has relevance to many physical systems, including RF-driven plasma, long Josephson junctions, and easy-axis ferromagnets in external magnetic fields. In particular, it provides for the canonical description of Kerr nonlinear optical cavities. In that context, the model is commonly referred to as the “Lugiato-Lefever” equation, and it has been successfully applied to various distinct configurations, including spatially diffractive cavities, fiber ring resonators, and monolithic microresonators.

The AC-driven NLSE has fixed-point solutions corresponding to localized dissipative structures (LDSs). These are wave packets that sit on top of a homogeneous background, with potential applications as coherent optical frequency combs, or as bits in all-optical buffers. Besides stable stationary LDSs, theoretical analyses have revealed several dynamic instabilities by considering the following dimensionless equation: \[ S = 1 + i|\psi|^2 - |\Delta| + i \frac{\partial^2}{\partial x^2} \psi + S. \] (1)

Here \( \psi(t, x) \) represents a field amplitude, whilst \( S \) describes the strength of the homogeneous driver and \( \Delta \) its frequency. Complex LDS instabilities have been predicted for \( X = |S|^2 \gtrsim 25 \). In Fig. 1 we illustrate general bifurcation characteristics for a driving strength similar to the experiments that will follow (\( X = 30 \)). We show the peak amplitudes \( |\psi|_{\text{max}}^2 \) of the coexisting homogenous (\( H \), black curves) and LDS solutions (red curves) as a function of \( \Delta \). Consider-
Accordingly, the up-switching point $\Delta_{\uparrow}$ marks the coherent background on top of which the LDSs sit. Referring to the inset in Fig. 1(a), LDSs in the unstable regime (I) oscillate and eventually decay to a homogeneous state (II); LDSs that trigger spatiotemporal chaos (III). (b-d) Examples of numerically simulated dynamics for different regimes. (b) Regime I, $\Delta = 8$; (c) Regime II, $\Delta = 7.45$; (d) Regime III, $\Delta = 6$.

We have experimentally investigated LDS instabilities using a high-finesse optical fiber ring resonator that is coherently driven with an external laser. Prior studies in the weak-driving regime ($X < 10$) have demonstrated that such a system is described by Eq. (1), and that it can support LDSs in the form of “temporal cavity solitons”: persisting pulses of light that recirculate in the resonator [12–14, 39]. In reference to Eq. (1), the “spatial” $x$ dimension represents a “fast time” variable ($x \rightarrow \tau$) that is defined in a reference frame moving with the LDSs, whilst $t$ is a “slow time” variable that describes the evolution of the optical field inside the resonator over several roundtrips. The dimensionless timescales and the field amplitude in Eq. (1) are related to the corresponding dimensional variables (indicated with subscript $D$) through the following normalisation [12, 39]: $t = \alpha t_D/t_R$; $\tau = \tau_D \sqrt{2\alpha/|\beta_2| L}$; $\psi = \psi_D \sqrt{\gamma L/\alpha}$. Here, $t_R$ is the cavity roundtrip time, $\alpha$ is equal to half of the percentage power dissipated over one round trip, $L$ is the length of the resonator, and $\beta_2(< 0)$ and $\gamma$ are the fiber group-velocity dispersion and nonlinearity coefficients, respectively. The normalised driving strength $S = (P_m \gamma L/\alpha^3)^{1/2}$, where $\theta$ is the intensity transmission coefficient of the coupler used to inject the coherent driving laser, with power $P_m$, into the resonator. Finally, $\Delta$ characterizes the frequency detuning of the driving laser at $\omega$ from the closest cavity resonance at $\omega_0$, with $\Delta \approx t_R(\omega_0 - \omega)/\alpha$.

Figure 2 shows our specific experimental configuration. The fiber ring cavity consists of 100-meters of standard single-mode optical fiber with $\beta_2 = -21.4$ ps$^2$km$^{-1}$ and $\gamma = 1.2$ W$^{-1}$km$^{-1}$, closed on itself with a 90/10 fiber cou-
The cavity output passes through a narrow (0.6 nm) bandpass filter (BPF) centered at 1551 nm. This removes the homogeneous background at 1550 nm, thereby improving the signal-to-noise ratio of the measurement [12]. The finesse of the cavity is $F = \pi/\alpha \approx 20$.

Similar cavities have previously been employed to investigate stable LDSs [13, 14, 39]. In those studies, however, the cavity was driven with continuous wave (cw) laser light, which does not readily allow access to the strong driving regime required to study LDS instabilities. To overcome this issue, in our experiments we drive the cavity with approximately 20 ns flat-top pulses (see inset in Fig. 2) at a duty cycle of 25:1. Such quasi-cw pulses can easily be amplified to sufficiently high strengths, and we note that a similar pulsed pumping scheme was recently employed in a study of competing Faraday and Turing instabilities [40]. We obtain our pump pulses by using a Mach–Zehnder modulator (MZM) to modulate a 1550 nm narrow linewidth cw laser. The MZM is driven with an electronic bit pattern generator at a frequency corresponding to an integer multiple of the cavity free-spectral range $t_R^{-1}$, thus ensuring synchronous driving [41]. Before the pump pulses are coupled into the cavity through the 90/10 coupler ($\theta = 0.1$), they are amplified with an erbium-doped fiber amplifier (EDFA) to a quasi-cw power of about 6 W (normalised driving strength $X \sim 30$), and passed through a BPF centered at 1550 nm to remove amplified spontaneous emission.

To investigate different LDS regimes, we systematically control $\Delta$ by using a function generator to adjust the optical carrier frequency of the pump laser. Moreover, we monitor $\Delta$ by leveraging the non-ideal extinction of the MZM used to create the pump pulses: the linear resonance associated with the low-power cw background that exists in between the pump pulses provides for a $\Delta = 0$ reference. We estimate the error in $\Delta$ obtained in this way to be about ±0.2. At this point, we emphasise that the 20 ns width of our quasi-cw pump pulses is about 4 orders of magnitude larger than the sub-picosecond widths of the LDSs that exist for our experimental parameters. The LDSs thus experience the driving as effectively homogeneous.

Referring to Fig. 1 we start the experiment with the pump laser blue-detuned from a cavity resonance ($\Delta < 0$), and then slowly increase $\Delta$ to $\Delta \approx 27$ by reducing the laser frequency. This results in the spontaneous excitation of LDSs [20, 22]. We then reverse the direction of the frequency scan, and continuously reduce $\Delta$ at a rate of about $d\Delta/dt \approx -0.019 \mu s^{-1}$. (Similar forward + backward tuning has recently been used to control the number of stable LDSs in microresonators [23].) Because the cavity photon lifetime $t_{ph} \approx 1.6 \mu s$, the intracavity field reacts almost adiabatically to the frequency scan. Accordingly, by recording a long time trace at the cavity output, we are able to examine LDS behaviour as a function of $\Delta$. To facilitate visualisation, we divide the experimentally measured time trace into segments spanning one roundtrip, and concatenate the resulting sequences on top of each other. Figure 5 shows typical results obtained from such a measurement: the density maps depict the spatiotemporal field evolution (for clarity we only show a 2 ns segment of the full 20 ns pump profile), while the line plots capture the evolution of the integrated energy around a single LDS. The full measurement encompasses $\Delta$-values continuously reducing from about 27 to 5, but we only show snapshots around four regions that highlight the main dynamical behaviours.

The results are in remarkable agreement with the predicted bifurcation characteristics [see Fig. 1(a)]. We first observe 6 stable LDSs until $\Delta$ passes the theoretically predicted Hopf threshold at about $\Delta \approx 11.3$, beyond which simple oscillatory behaviour ensues [region I in Fig. 1(a)]. For $\Delta < 8$, we witness irregular oscillations that lead to the spontaneous collapse of particular structures [region II]. Owing to the chaotic nature of the dynamics, and the fact that $\Delta$ is continuously reduced, some LDSs avoid a collapse and survive to the second regime of stable oscillations that manifests itself for $\Delta$ below the collapse region. Finally, around $\Delta \approx 6$ the few remaining LDSs transform into localized chaotic domains [region III], which are quickly engulfed by the full destabilization of the coherent background at about $\Delta \approx 5.8$. This point coincides almost exactly with the theoretically predicted up-switching point $\Delta^- = 5.75$.

Before proceeding, we comment on two aspects of our measurement. First, energy variations of oscillating LDSs are exaggerated in our experiment. This is due to the offset BPF in the detection path of our setup (see Fig. 2): oscillating LDSs exhibit variations in their spectral bandwidth, which can impact strongly on the energy transmitted through the BPF. Second, due to the 50 ps response time of our detection electronics, we are
not able to directly discern the sub-picosecond profiles of the LDSs, or the proliferation into numerous closely-packed structures that are predicted in the chaotic regime [see Fig. 1(d)]. Nevertheless, the abrupt increase in integrated energy (and the spread of the LDS envelope) around $\Delta \approx 6$ provides convincing evidence for such behaviour.

The results in Fig. 3 show clear evidence of oscillating and collapsing LDSs. However, due to the rate at which the driver frequency $\Delta$ was varied, the transitions to spatiotemporal chaos are not satisfactorily captured. We have therefore performed another experiment, where we significantly reduced the laser scan rate after reaching the region of interest ($\Delta \approx 6$). Figure 4(a) shows the detected signal over the full 20 ns-width of the pump pulses when $\Delta$ is slowly reduced around $\Delta \approx 6$ over 1 ms (we estimate that $\Delta$ reduces from about 6.2 to 5.8 during the measurement). We observe 6 oscillating LDSs (highlighted with arrows), some of which abruptly transition into expanding fronts. Figure 4(b) shows a zoom on a particular event [white arrow in Fig. 3(a)], illustrating how the oscillating LDS triggers a chaotic domain that invades the surrounding homogeneous state at an almost constant rate. These observations are in good agreement with corresponding simulations of Eq. 1, shown in Fig. 4(c). The simulations use experimental parameters, aside from $\Delta$ which is reduced from 6.75 to 5.75 during the simulation (the values are nevertheless in reasonable agreement with experimental estimates). To facilitate comparison with experiments, the simulation results have been post-processed to mimic the effect of the offset filter and limited detection bandwidth in our experiment; the raw theoretical data is qualitatively similar to that shown in Fig. 1(d), consisting of chaotically oscillating, rapidly proliferating LDSs. In this context, besides results shown in Fig. 4(c), we have carefully verified that all of our experimental observations are in good agreement with numerical simulations of both the AC-driven NLSE and a more rigorous Ikeda-like cavity map [18].
hbiting irregular oscillations, collapses and transitions into expanding domains of chaos. In this way, the findings directly confirm theoretical predictions that have been drawn across various physical contexts over the past three decades \[1 4 10\]. By implication, our results confirm that the AC-driven NLSE remains a valid predictor of Kerr nonlinear cavity dynamics in the strong-driving regime relevant to e.g. many microresonator frequency comb experiments \[20 43 45\]. We accordingly predict that complex LDS instabilities may play a role in such systems. More generally, our work demonstrates that synchronously pumped fiber cavities permit systematic interrogation of complex LDS instabilities and their bifurcations. Given the universal nature of LDSs and related phenomenologies \[46–50\], the ability to perform detailed and controlled laboratory experiments is expected to have wide impact across numerous nonlinear dissipative systems. For example, we note in closing that instabilities similar to those reported here have also been predicted for LDSs of the parametrically-driven NLSE \[51 52\], whose many applications include strongly coupled pendula \[53\], vertically driven fluids \[54 55\], nonlinear lattices \[56\], and optical parametric oscillators \[57\].

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