Unavoidable losses in neutron quantum experiments

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Abstract. Quantum optics experiments contribute to a deeper understanding of quantum theory. Neutron phase-echo and spin rotation experiments have shown that, in all cases of an interaction, parasitic beams are produced which cannot be recombined with the original beam in an ideal way. This means that a complete reconstruction of the original state would, in principle, be impossible. Thus a kind of intrinsic irreversibility occurs, even when the original quantum state survives to a very high extent. When plane waves are used, completely reversible situations can be constructed in certain cases, but in any physical situation, wave packets have to be used which do not permit complete reversibility. Even small interaction potentials can have huge effects when they are arranged periodically and resonance effects appear. This gives various constraints for repetitive measurements and prevents a complete freezing of a quantum state in Zeno-like experiments. Additionally, a spectral change occurs, due to the dispersive action of the interaction, which has to be taken into account when many repetitive measurements are considered. A dedicated proposal for a repetitive neutron spin rotation experiment within a perfect crystal resonator will be analyzed in detail. Unavoidable losses due to quantum phenomena can be separated from losses caused by experimental imperfections.

1. Introduction
Neutron interferometry has been used for many test experiments in quantum physics. They have been summarized in a recent book (Rauch and Werner 2000). The curious dual nature of the neutron, sometimes a particle, sometimes a wave, is wonderfully manifested in the various non-local interference effects observed in neutron interferometry. Due to the extremely low phase space density of neutron beams ($10^{-14}$), one observes pure single particle interferences. Coherent beam separations up to several centimeters can be achieved by perfect silicon interferometers and these beams can be influenced by nuclear, magnetic, and gravitational interactions, but also by topological effects. It can be stated in a rather general form that the wave function behind the interferometer contains much more information than can be extracted in any standard experiment. In most cases one focuses on one parameter, but there are always additional effects which may be of interest in the discussion over basic quantum irreversibility and decoherencing effects. All related experiments show unavoidable quantum losses. The question of decoherence becomes an essential issue for the understanding of quantum mechanics and especially of the quantum measurement process. Several books and review articles deal with this topic (Giulini et al 1996, Namiki et al 1997, Mensky 2000, Rauch 2001). Here we will show that not only dissipative interactions cause an irreversible change of the wavefunction, but that deterministic ones can cause such a change too.
The well-known Zeno-phenomenon (paradoxon) can be taken as a characteristic example of a temporal or spatial evolution of a quantum system which is kept under frequent observation and which becomes frozen in the initial state (Misra and Sudershan 1977, Fonda et al 1978, Home and Whitaker 1993, Joos 1996). It is also an example of the fact that simplified considerations show a completely different effect than a realistic physical one (Nakazato et al 1995, Venugopalan and Ghosh 1995, Pati 1996). Proposals for related experiments and some realizations of them exist for various decay processes (Itano et al 1990) for light polarization rotation experiments (Peres 1980, Kwiat et al 1995) and for neutron polarization rotation experiments (Pascazio et al 1993, Nakazato et al 1995, Machida et al 1999, Hradil et al 1998), which will be discussed in more detail in this paper. The topic is closely related to the non-exponential decay of quantum states for very short times where a quadratic time dependence is expected and to so-called "interaction-free" measurements (Elitzur and Vaidman 1993, Kwiat et al 1995). Here we discuss several neutron optical experiments which show unavoidable losses due to parasitic reflection effects or due to the wave packet structure of realistic wave functions. It will also be shown that a general coupling in phase space exists which prohibits the view onto one parameter space of the system only.

2. Basic neutron interferometry relations
The wave function for the beam in forward direction (0) behind a Mach-Zehnder interferometer is given by the superposition of wave functions arising from the right and the left beam paths. They are transmitted-reflected-transmitted (trr) and reflected-reflected-transmitted (rrt), respectively, and they are equal in amplitude and phase due to symmetry reasons \( \psi_{trr} = \psi_{rrt} \).

When parts of the beams are exposed to an interaction, a phase shift occurs

\[
\chi = \int k_c ds ,
\]

(1)

where \( k_c \) denotes the canonical momentum of the neutrons along the beam paths. Conservative interactions cause a change of the kinematical momentum \( k \), which can be described by an index of refraction, e.g., Rauch and Werner 2000

\[
n = \frac{k}{k_0} = \sqrt{1 - \frac{\bar{V}}{E}} .
\]

(2)

\( \bar{V} \) denotes the mean interaction potential within the phase shifter which amounts to \( \bar{V} = 2\pi\hbar^2N_b/m \) for nuclear interaction and to \( \bar{V} = \pm\mu B \) for magnetic interaction. \( m \) and \( \mu \) denote the mass and the magnetic moment of the neutron, \( N \) the particle density and \( b_c \) the coherent neutron scattering length of the phase shifter. When a purely magnetic interaction is considered, (2) describes the longitudinal Zeeman splitting. The different kinematical momentum of the beam inside the phase shifter also causes a spatial shift \( \Delta \) of the wave packets and the phase shift can also be written as

\[
\chi = (1 - n) kD = \Delta \cdot k .
\]

(3)
A realistic description of a neutron beam can only be achieved by a wave packet formalism, which, for a stationary and one-dimensional situation, is

$$\psi(r) \propto \int a(k) e^{ikr} dk ,$$  

(4)

where $a(k)$ denotes the amplitude function, which is related to the momentum distribution function as $g(k) = |a(k)|^2$. The interference pattern follows as

$$I(\Delta) \propto \left| \psi_{rrr} + \psi_{rrl} e^{i(\Delta k)} \right|^2 = |\psi(0) + \psi(\Delta)|^2 = 1 + |\Gamma(\Delta) \cos(\Delta \cdot k)| ,$$  

(5)

where $\Gamma(\Delta)$ denotes the coherence function which is given by the autocorrelation function of the wavefunction, e.g., Mandel and Wolf 1995

$$|\Gamma(\Delta)| = |\langle \psi(0) \psi(\Delta) \rangle| \propto \int g(k) e^{ik\Delta} dk .$$  

(6)

$|\Gamma(\Delta)|$ can be measured by the interference order dependent visibility of the interference pattern at large phase shifts (Rauch et al 1996). For Gaussian wave packets the characteristic widths $\Delta_c$ of the coherence function and the momentum spread of the packets $\delta k$ fulfil the minimum Heisenberg uncertainty relation ($\Delta_c \delta k = 1/2$) and

$$|\Gamma(\Delta)| = \exp\left[ -(\Delta \cdot \delta k)^2 / 2 \right] .$$  

(7)

$\chi$ and $\Delta$ are additive quantities (1) and (3), which has been demonstrated in a dedicated phase echo experiment (Clothier et al 1991).

Figure 1: Phase-echo experiment using phase shifters with positive (Bi) and negative (Ti) coherent scattering lengths, and causing positive and negative phase shifts. The retrieval of the contrast becomes visible.
\[ \chi = \chi_1 + \chi_2 \quad \text{or} \quad \Delta = \Delta_1 + \Delta_2. \] (8)

In this experiment a large phase shift, i.e. larger than the coherence length, has been applied by a Bi and alternatively by a Ti phase shifter. The visibility of the interference pattern nearly disappeared completely, in agreement with (5) and (8) as shown in figure 1. Bi and Ti cause an opposite phase shift due to a positive and negative coherent scattering length and, therefore, the interference pattern can be retrieved when both phase shifters are applied simultaneously and \( \chi_1 + \chi_2 \equiv 0 \) (Clothier et al 1991). Even more ideal situations can be envisaged when magnetic fields are used which do not cause any absorption or incoherent scattering effects. The analogy of the phase-echo concept to the spin-echo concept should be emphasized (Mezei 1972, Badurek et al 1980).

**REVERSIBILITY - IRREVERSIBILITY**

Figure 2: Approximate and complete wavefunctions when differently shaped phase shifters are used. One notices that the complete wavefunction has much more components than the simple ones (above) which are often used for implicit.
The question arises whether a complete retrieval of the interference pattern is feasible or not. First of all it should be mentioned that any phase shift is caused by an interaction which influences the momentum and such interaction potentials do not only cause a phase shift but also a back-reflection and/or back- and forth-reflections of parts of the wavefunction, as shown schematically in figure 2. A complete reconstruction of the original wavefunction in principle seems to be impossible. One notices additionally that the phase shifts of the direct transmitted beam are additive whereas all other partial waves have much larger phase shifts and the amplitudes vary as well. Back- and forth-reflections can vanish for certain plane waves, but never for wave packets with a finite momentum spread (figure 3). The parasitic beams may be very weak because $E \gg \sqrt{V}$, but they exist for positive and negative potentials regardless of what the ratio $\sqrt{V}/E$ is.

![Figure 3: Various combinations of different potential barriers causing quite different wavefunction behind the barriers.](image)

In the case of neutrons, absorption-free potentials can be realized by means of magnetic fields. The related transmissions $T$ and reflectivities $R$ can be calculated from the time-independent Schrödinger equation and are given for a square potential of height $V$, width $d$ and for neutrons with an energy $E = (\hbar^2k^2/2m) > \sqrt{V}$ as, e.g. Cohen-Tannoudji et al 1977

$$R = 1 - T = 1 - \frac{4E(e-\sqrt{V})}{4E(E-\sqrt{V})+\sqrt{V}^2 \sin^2 kL} \equiv \frac{1}{2} \left( \frac{\sqrt{V}}{2E} \right)^2 \left( 1 - \cos^2 kL \right)$$
where, for the last expression, \( E >> V \) has been assumed as in the neutron case. The oscillating term vanishes when the barrier produces phase shifts larger than the coherence length of the beam (see (5))

\[
\frac{\bar{R}}{\bar{R}} \equiv \frac{1}{2} \left( \frac{V}{2E} \right)^2,
\]

which is on the order of \( 10^{-10} \) for thermal neutrons and reasonable material or magnetic potentials. The transmission and the reflectivity of arbitrarily shaped potentials can be calculated in a similar way leaving the general conclusions unchanged, e.g. Cohen-Tannoudji et al. 1977. For multiple potential barriers the formulas become rather complicated because multiple interference effects have to be taken into account. Figure 3 shows that different wave functions arise for different arrangements of the same potential. Thus the wave function behind the interaction (and in front of it) contains the full information about the structure and strength of the interaction region.

Figure 4: Parasitic waves from a three-fold barrier and how they can be measured in principle.

Figure 4 shows how such contributions arise, and a method for a quantum complete measurement. A non-monotonic behaviour of T (and R) as a function of E, d and the distances between the barriers is expected, yielding high (near to 1) reflectivities when resonance (Bragg) conditions are fulfilled. Multilayer systems and supermirrors (Mezei 1976, Saxena and Schoenborn 1977, Schelten and Mika 1979, Sears 1983, Majkrzak and Passel 1985) are typical
examples where high reflectivity (low transmission) systems are realized. Band structures, Bragg diffraction and dynamical diffraction effects are typical features of periodic structures (Cohen-Tannoudji et al 1977). All single and multiple barrier situations show that for wave packets unavoidable losses occur, indicating that \( T < 1 \) even when \( E >> V_0 \). Reflectionless potentials can be defined by inverse scattering methods, but, due to singularities, such potentials can only be approximatively realized (Chadan and Sabatier 1989, Perelomov and Zel’dovich 1998).

When the neutron polarization lies perpendicular to the magnetic field \( B \) of length \( \ell \), the \( \vert + \rangle \) and \( \vert - \rangle \) component feel a different potential \( \pm \mu B \) and experience, therefore, slightly different transmission and reflections. Additionally, a Larmor rotation occurs whose angle is given as

\[
\varphi_L = \omega_L t = \frac{2 \mu B \ell}{\hbar} \frac{\ell}{v},
\]

where again, one notices a velocity (momentum) dependence resulting in different rotation angles for different components of the neutron wave packet. The behaviour of such wave packets within magnetic fields has also been investigated by means of a Wigner function representation (Rauch and Suda 1998).

3. Neutron Postselection Experiments

Equation (5) shows that the interference pattern at high interference order disappears when the phase shift becomes larger than the coherence length. When the interference in ordinary space disappears, a modulation of the momentum distribution appears. This is shown schematically in figure 5 (Rauch 1993).

![Figure 5: Schematic contrast reduction at higher interference order (middle) and wave packets in ordinary and momentum space at 0th, 50th and 100th interference order.](image_url)
In light optics (Mandel and Wolf 1965, Agarwal and James 1993) and more recently in neutron optics (Jacobson et al 1994), it has been shown that interference can be revived when a momentum postselection is applied. In this case, the wave packets in ordinary space become enlarged and overlap with the reference packet (figure 6). This figure also shows that interferences can be observed with beam detectors No. 1 ... n, whereas a simultaneous measurement of the total count rate does not show any interference pattern. In terms of a wave-particle dualism, several authors describe the wave properties by the square of the fringe visibility ($V^2$) and the particle properties by the diagonal terms of the density operator ($P_D$, which can also be seen as path distinguishability), (Greenberger and Yasin 1988, Englert 1996, Ghose 1999)

$$P_D^2 + V^2 = 1.$$  \hspace{1cm} (12)

Figure 6: Scheme of momentum postselection in neutron interferometry and a simultaneous measurement of the total count rate.

An experiment, as shown in figure 6, creates an interesting situation when the interfering filtered beams, where the visibilities $V \neq 0$, and the total count rate are measured, whereas the total beam shows a visibility $V \equiv 0$. In this case it looks like a simultaneous beam path and interference detection. It should be mentioned that postselection experiments need an additional filtering of the beam as indicated in figure 6, which can only be done by an additional interaction, causing several parasitic and unavoidable losses, as discussed in the previous section.

4. Zeno-Effect Experiments with Neutrons
The quantum Zeno-effect describes the hindrance of the transition between quantum states when a large number of successive measurements are performed to check whether the system is still in the initial state or not (Misra and Sudershan 1977, Fonda et al 1978, Home and Whitaker 1993, Joos 1996, Whitaker 1998). The quadratic behaviour of the survival probability for short times
also exists for the upper (or lower) Zeeman level when a magnetic field perpendicular to the polarization direction of a neutron beam is applied, e.g. Mezei 1972,

\[ P_+ = \cos^2 \left( \frac{\omega \ell}{2v} \right). \] (13)

\( \ell/v \) denotes the time the neutron spends within such a field of length \( \ell \) and \( \omega = 2 \mu |B/h \) is the Larmor rotation frequency. When the thickness of the rotation coil fulfills the condition \( \ell = \ell_0 = (2m+1) \pi v/\omega_L \), complete spin reversal occurs (\( P_+ = 0, m = 1, 2, ... \)), which is used in many polarized neutron experiments (Williams 1988). The same relation holds when the coil is separated into \( n \) shorter coils \( \ell_0 = n.\ell \) and no magnetic fields act between the coils, as shown in figure 7. Similarly, more rotation stages (or more transitions through one coil) with correspondingly lower magnetic fields can be used.

Figure 7: Arrangement for successive spin rotation by several DC-coils (above) and arrangement for the observation of the neutron Zeno-phenomenon.

A completely different situation exists when, behind each coil, a filtering process takes place where only spin-up neutrons can pass through whereas spin-down neutrons are detected. A polarized He-3 filter could fulfill these requirements because it acts simultaneously as filter and detector for flipped neutrons, e.g. Surkau et al 1997. When one uses the same field configuration as discussed for the case without filters, one gets

\[ P_+ = \left[ \cos^2 \left( \frac{\omega \ell}{2v} \right) \right]^n = \left[ \cos^2 \frac{\pi}{2n} \right]^n \overset{n \to \infty}{\longrightarrow} 1, \] (14)
i.e. no transition will occur and the initial state seems to be frozen (figure 8). This relation also holds when wave packets are used to describe the spatial part of the wave function. There may be deviations when the coherence length $\Delta_c$ exceeds the dimension $\ell$ of the field regions because then passage time and evolution time become different (Nakazato et al 1995, Venugopalan and Ghosh 1995, Pati 1996, Pan et al 1998).

Figure 8: Survival probability in the upper spin state in spin-flip ($n = 1$) and Zeno-like situations.

As in the case of multiple barrier transmission, one should notice that each rotation stage also causes at least small parasitic beams which can add up in the case of large $n$, or when resonance conditions exist, to an essential change of the situation discussed above. These losses are velocity (momentum) dependent and change (narrow) the initial spectrum, which causes longer coherence lengths and an enforced collective action of the rotation stages. Thus, a situation may exist where the spin state does not change because the transition probability goes to zero (14) but the intensity changes due to parasitic quantum losses, which means that a perfect Zeno-effect is in principle unphysical. When $n$ independent successive barriers are taken, the intensity of the $+\text{ polarized}$ beam becomes reduced at least as (10)

$$I_+ = P_+ T^n = P_+ (1 - R)^n = P_+ \left[ 1 - \frac{1}{2} \left( \frac{V}{2E} \right) \right]^n$$

$$\equiv 1 - \frac{n}{2} \left( \frac{V}{2E} \right)^2 + \ldots \text{ as } n \to \infty$$

and an even enhanced reduction is expected when the collective action of the barriers is taken into account. Thus, a state variation is expected in any case. Nevertheless, there is no question about the existence of the Zeno-like phenomena for reasonably low $n$ values, but the $n \to \infty$ limit
is still under discussion. In this respect, an intrinsic contradiction concerning the existence of the classical Zeno-effect exists, which justifies new experimental work.

Such an arrangement has been tested at the pulsed ISIS spallation source where we are using the features of our newly developed perfect crystal neutron resonator (figure 9) (Schuster et al 1990, Jericha et al 2000, Jaekel et al 2005). This instrument allows a multiple (up to 4000 times) transmission of neutrons through a spin rotation coil and a continuous measurement of the negative spin state due to the energy-dependent transmission of the exit perfect crystal. The spin rotation angle in the flipper coil can be controlled by the amplitude of the oscillating flipper field.

Figure 9: Perfect crystal resonator system adapted for a neutron Zeno-effect experiment. The neutron magnetic resonance system causes an energy change $\Delta E = \pm \mu B$, is just adequate that those neutrons are back-reflected from the perfect crystal plates.

5. Discussion
It has been shown that any interaction of a neutron beam with a potential produces unavoidable parasitic beams which are characteristic for this kind of interaction. Multiple potential barriers do not only produce an additive effect of losses, but also show enhanced losses due to various resonance effects. Therefore, the Zeno-phenomenon, which is based on a repetitive interaction and observation of a quantum system, has to be discussed in a new light including unavoidable quantum losses. In this respect, the imperfect Zeno-phenomenon and the limit of a non-completely frozen state for an infinite number of stages can be seen as a fingerprint of quantum mechanics. In the limiting case, when the coherence length exceeds the dimension of the spin rotation stages ($\Delta_c \geq d$), a deviation from (14) can be expected when the evolution time ($\ell/v$) has to be replaced by the passage time of the packet ($\Delta_c/v$). In this case, the survival probability would also tend to zero. This limit is still being debated in the literature (Nakazato et al 1995, Venugopalan and Ghosh 1995, Pati 1996, Pan et al 1998). Related experiments with a perfect crystal neutron resonator are in progress.

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