An iterative identification algorithm and its convergence analysis of closed-loop power system based on ambient signals

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Abstract. Identification and control are important problems of closed-loop power system. At present, most studies are separate identification methods. This paper studies an online and real-time integrated identification method, which can solve the problems of model set and controller design of closed-loop power system. This paper investigates a new iterative identification algorithm and its convergence problem of closed-loop power system based on ambient signals. Firstly, the whole algorithm procedure is given. This algorithm uses the iterative process under the closed-loop condition, which combines system model identification with controller design. Then the complementary of model identification and control design has been realized. Secondly, because of the dynamic performance of the iterative identification algorithm, it has characteristics described from the perspective of a partitioned dynamic system. Regard each iterative identification step as a state node. In this situation, the algorithm guarantees all the state nodes converge to the Lyapunov stable equilibrium. Finally, the simulation results show the correctness and effectiveness of the proposed method through the simulation of a power system with four-machine-two-region.

1. Introduction
The power system model is important to help design a correct and effective controller and also to analyze the dynamic characteristics of the power system, the controller can help the power system to maintain stability[1]. Recently, more and more scholars have started to study the problem of identification of control[2]. Nie Yonghui et. al. proposed a new method to design wide area damping controller based on artificial immune system theory. The improved fireworks algorithm was used to optimize the parameters of immune controller, which could improve the stability of power system[3]. Li Peiping et al. proposed an adaptive wide-area coordinated damping controller based on the objective representation heuristic dynamic programming algorithm, which could suppress the low frequency oscillation in the weak damping interval of the system and improved the transient stability of the system[4]. Zhang Yan et. al. analyzed the damping characteristics of doubly fed induction generator (DFIG) grid-connected system based on energy method, and discussed the relationship between damping coefficient and system damping, which characterized oscillation energy dissipation, from the physical level[5]. Yang Jing et al. designed a subsynchronous oscillation robust damping controller based on the additional control of direct-drive wind turbine, which could effectively improve the adaptability of subsynchronous oscillation damping control[6]. In this paper, a new control method is designed based on iterative identification, which can solve the problems of model design and controller design that are difficult to
solve in current power systems by analyzing the convergence of algorithms for closed-loop power systems. The first chapter of this paper gives the introduction, and the second chapter gives the model to identify the environmental signals and the mathematical model of the closed-loop power system. A new control and iterative identification method is proposed in the third chapter. The simulation results and effect analysis of the identification are given in Section IV. Chapter 5 is the conclusion and outlook.

2. Problem formulation and preliminaries

2.1. Problem formulation

The closed-loop system environment signal model is shown in Figure 1, where we can add disturbance signals to the power system to better simulate the disturbance signals of random nature. The input signal in Figure 1 is \( u \) and \( i \) is the output signal. The random interference signals are \( e_{10} \) and \( e_{20} \), and their variances can be represented by \( \lambda_{10} \) and \( \lambda_{20} \) with a mean value of zero. Subject to an excitation signal with an external \( r \). The filters are \( H_{10} \) and \( H_{20} \), which are stable and reversible. \( W_0 \) is the feedback controller. In addition, Fig.1(b) represents the closed-loop identification model, \( y \) represents the system input signal, and \( u \) represents the output signal of the system. \( G(q, \theta) \) is the control system parameter of positive feedback, and \( H(q, \theta) \) represents the disturbance filtering model above the positive direction. From Fig.1(b) can Identify the mathematical equation as shown in 7[7]:

\[
y(t) = G(q, \theta)u(t) + H(q, \theta)e(t)
\]

\[
u(t) = -W(q, \theta)y(t) + r(t)
\]

where \( H(q, \theta) = \frac{F(q, \theta)}{E(q, \theta)}, G(q, \theta) = \frac{B(q, \theta)}{A(q, \theta)}, W(q, \theta) = \frac{R(q, \theta)}{P(q, \theta)} \)

the model used for recognition is represented:

\[
\begin{bmatrix}
y \\
u
\end{bmatrix} = \begin{bmatrix}
\frac{G}{1 + GW} & \frac{H}{1 + GW} \\
1 & \frac{W}{WH}
\end{bmatrix} \begin{bmatrix}
r \\
e
\end{bmatrix} = \Xi(G, H, W) \begin{bmatrix}
r \\
e
\end{bmatrix}
\]

(2)

The uncertainty set for error prediction for each iteration of the identification set from the identification model in the power system represented in Figure 1 is given by the following equation:

\[
\zeta_{re}(G_s, Y_s, Y_d, R) \equiv \left\{ G(q, \theta) \mid G(q, \theta) = \frac{G_s + Y_s \theta}{1 + Y_d \theta}, \theta \in U_{\theta} \right\}
\]

(3)

where \( \theta \in R^{k_{\text{sys}}} \) is a power system model parametrization, \( G_s \) is a known transfer function, \( R \in R^{k_{\text{sys}}} \) is the definite matrix of symmetric positive, the transfer function vectors are \( Y_s, Y_d \), and the estimated value is \( \hat{\theta} \). Uncertainty sets are easily obtained in closed-loop power systems using prediction error identification methods.
2.2. Preliminaries

Table 1 indicates a summary of some notations, and directed graphs are used to indicate dynamic networks with a graphical topology and dynamic networks with unknown random time delays.

Table 1  

| Symbols | Meaning |
|---------|---------|
| $\mathbb{R}^n$ | $n$-dimensional Euclidean space |
| $\mathbb{R}_{+}^n$ | nonnegative orthants of $\mathbb{R}^n$ |
| $\mathbb{R}_{++}^n$ | positive orthants of $\mathbb{R}^n$ |
| $x \geq 0, x \in \mathbb{R}^n$ | every component of $x$ is nonnegative |
| $x > 0, x \in \mathbb{R}^n$ | every component of $x$ is positive |
| $A \geq 0, A \in \mathbb{R}^{n \times n}$ | nonnegative matrix |
| $A > 0, A \in \mathbb{R}^{n \times n}$ | positive matrix |
| $A \geq 0, A \in \mathbb{R}^{n \times n}$ | nonnegative definite matrix |
| $\langle \cdot, \cdot \rangle_{\mathbb{R}}$ | matrix transpose |
| $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ | Drazin generalized inverse |
| $\mathcal{V} = \{1, \ldots, n\}$ | Euclidean vector norm |
| $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ | weighted directed graph |
| $\mathcal{A}_{i,j} = a_{i,j} > 0$ | set of vertices |
| $a_i$ | direction of information flow |
| $\mathcal{A}$ | weighted adjacency matrix |
| $\mathcal{T}$ | weight of each edge |

The basic definition of convergence analysis of iterative recognition algorithms is given below.

**Definition 1.** The dynamic network $\mathcal{G}$ with time delay can be expressed in the following form:

$$
\dot{\xi}(t) = f(\xi(t)) + f_d(\xi(t-\tau_1), \ldots, \xi(t-\tau_d))
$$

$$
\xi(\theta), \theta^r \leq \theta \leq 0, t \geq 0
$$

(4)

where $f_d : \mathbb{R}^n \times \ldots \times \mathbb{R}^n \rightarrow \mathbb{R}^m$ is locally Lipschitz continuous, $f_d(0, \ldots, 0) = 0$, $\tau^r = \max_{\tau_1, \ldots, \tau_d} \tau_i$, $i = 1, \ldots, n_d$ and the vector $\xi(t) = [\xi_1(t), \ldots, \xi_n(t)]^T$ is the state of algorithm nodes; $\Phi(\bullet) \in C = C([-\tau^r, 0], \mathbb{R}^m)$ is a continuous vector-valued function and satisfies the initial state of (4).

**Definition 2.** The (4) in a nonlinear time-delay dynamic system is a compartmentalized dynamic system if $F(\xi) = f(\xi) + f_d(\xi, \xi, \ldots, \xi)$ and $F(\bullet)$ is compartmental, where $f(\bullet)$ and $f_d = [f_{d1}, \ldots, f_{dn}]^T$ are given by
\[ f_i(\xi(t)) = -\sum_{j=1,j\neq i}^{n} a_{ij}(\xi(t)) \]
\[ f_{nt}(\xi(t-t_1),...,\xi(t-t_n)) = \sum_{j=1,j\neq i}^{n} a_{ij}(\xi(t-t_j)) \]  \hspace{1cm} (5)

\( \tau_1, i \neq j, i, j \in [1,\ldots,n] \) means the logistics transfer time from compartment \( j \) to \( i \), and \( a_{ij}(\xi(t)) \geq 0, \xi(t) \in \mathbb{C}, i \neq j, i, j \in [1,\ldots,n] \) means the instantaneous logistics rate from compartment \( j \) to \( i \).

From the above, the form of nonlinear dynamic network with \( g \) time lag can be written as

\[ u(t) = \xi(t) = f(\xi(t)) + \sum_{i=1}^{n_\phi} f_{ij}(\xi(t-t_j)) \]
\[ \xi(\theta) = \phi(\theta), -\tau^* \leq \theta \leq 0, t \geq 0 \] \hspace{1cm} (6)

where \( f: \mathbb{R}^n_{+} \rightarrow \mathbb{R}^n_+ \) is given by \( f(\xi) = [f(\xi_1),...,f(\xi_n)]^T \), \( f(0) = 0, f_{ij}: \mathbb{R}^n_+ \rightarrow \mathbb{R}^n_+, i = 1,\ldots,n_\phi \) and \( f_j(0) = 0 \).

Furthermore, \( f_j(\cdot), i = 1,\ldots,n \) and \( f_j(0) = 0 \).

3. Iterative identification algorithm and its convergence analysis

3.1. Iterative Identification Algorithm

In power system identification, how to evaluate the quality of the identification model is the main problem that we pay attention to. Currently, scholars believe that identification error is composed of two parts. The two components are bias error and variance, respectively. The variance is easily caused by the random noise of the observed data when performing system identification experiments. The bias error is brought about by the limited degree of model complexity[8], which can be described as follows:

\[ G_0(e^{i\omega}) - G(e^{i\omega}, \tilde{\theta}_G) = \underbrace{G_0(e^{i\omega}) - G(e^{i\omega}, \theta^*)}_{\text{bias error}} + \underbrace{G(e^{i\omega}, \theta^*) - G(e^{i\omega}, \tilde{\theta}_G)}_{\text{variance error}} \]  \hspace{1cm} (7)

where \( G_0(e^{i\omega}) \) is a real power system model, \( G(e^{i\omega}, \tilde{\theta}_G) \) is an identification model, \( \theta^* \) is the convergence point of \( \tilde{\theta}_G \), then \( G(e^{i\omega}, \theta^*) \) is the asymptotic convergence model of \( G(e^{i\omega}, \tilde{\theta}_G) \).

During the research of power system iterative identification, model uncertain sets are used to analyze the robust performance and control performance. If an uncertain model set is \( \zeta \), a real power system object is \( G_0 \in \zeta \) and the power system estimation model is \( \tilde{G} \), then the robust controller \( W \) must satisfy two characteristics. One is that the controller \( W \) can stabilize all the models within the set \( \zeta \). The other is that the controller \( W \) can meet the control performance requirements of set \( \zeta \). So the structure of power system uncertain model set is very important during the whole design process.

To get the identification model of the power system and to implement the algorithm for iterative identification, the whole process is summarized according to[9] as follows:

1. Fig.1(b) show the closed-loop power system, \( G_0 \) is the unknown model, \( Wi=0 \) is the initial controller, is chosen to satisfy the system stability;

2. In order to obtain the uncertainty set nominal model \( Gi \) can be obtained \((G, Wi)\) identification conditions.

3. Calculate \( \delta_w(G_j, \xi_j) \), and written as \( \rho \), and design controller \( W_{op} \);

4. Judgment condition if the condition is satisfied \( W_{op} \in W(G_j, \xi_j) \), then judge whether the performance indicator is satisfied, otherwise if find the optimal controller in \( Wi+1 \) from \( W(G_j, \xi_j) \);

5. If the program requirements are not met[4]. Let \( i = i + 1 \) and repeatedly do from the program 2 to 5.
3.2. Convergence analysis

Based on the dynamic behaviour of iterative identification algorithm, we use the compartmentalized system model defined in equation (6) to describe the iterative identification algorithm. In this section, we will prove that this dynamic system with time lags meets the requirement of semi-stability. First, we give some strict mathematical definitions of synchronization.

**Definition 3.** $D \subseteq \Omega$ is a nonempty subset, if $\xi^0(t) \in D$, make it possible for $\xi(t) \in \Omega$ for all $t \geq 0$, and

$$\lim_{t \to \infty} \|\xi(t) - \xi_0(t)\| = 0, 1 \leq i \leq n$$

$\mathcal{G}$ is a dynamic network illustrating that network synchronization is achieved and $D \times \cdots \times D$ is called the synchronization region of the dynamic network.

Note that the stability of the zero solution needs to be slightly corrected for the normal stability definition given in [10-11] to deal with time delay negative systems. In practice, A subset of equilibrium solutions of the system is defined in the literature [12]. The following definitions generalize non negative concepts to vector fields.

**Definition 4** [12]. $f$ is nonnegative if $f_i(\xi) \geq 0$ for all $i = 1, \ldots, n$ and $\xi \in \mathbb{R}^n$, such that $\xi(t) = 0$, where $f = [f_1, \ldots, f_n] : \mathcal{H} \to \mathbb{R}^n$.

**Definition 5** [13]. According to the nonlinear delay system in (4). If $f(\cdot)$ is nonnegative, $f_d(\cdot)$ is nonnegative, so we get $\phi(\cdot) \in \mathcal{C}_\phi, \mathcal{C}_\phi \triangleq \{\phi(\theta) \in \mathcal{C} : \phi(\theta) \geq 0, \theta \in [-\tau^*, 0]\}$, and solve $\xi(t)$ with $t \geq 0$ and $\mathcal{G}$ are non-negative.

**Lemma** [13]. If $e^T(\xi(\tau) + \sum_{i=1}^n f_d(\xi)) = 0$, there exist nonnegative diagonal matrices $Q_i \in \mathbb{R}^{a_i \times a_i}$, such that

$$Q_i^T Q_i f_d(\xi) = f_d(\xi), \xi \in \mathbb{R}^n, i = 1, \ldots, n_d$$

and since the positive orbit $(\xi(t))_{\tau}$ of (6) is delimited, where $(\xi(t))_{\tau} \in \mathcal{C}_\phi$, we can get (11). According to the [10], we know that $\lim_{t \to \infty} \xi(t) = \mathcal{M}$, where $\mathcal{M}$ indicate the largest invariant set contained in $\mathcal{P}$. Since $e^T(\xi(\tau) + \sum_{i=1}^n f_d(\xi)) = 0, \xi \in \mathbb{R}^n$.
\[ \mathcal{P} \subset \hat{\mathcal{P}} \subset \{ \zeta(\theta) \in \mathcal{C}_n : f(\zeta(0)) + \sum_{i=1}^{n} f_{\tau_i}(\xi(-\tau_i)) = 0 \} = \{ \zeta(\theta) \in \mathcal{C}_n : \zeta(\theta) = M \cdot \theta \in [-\tau,0], M \geq 0 \}, \]

which implies that

\[ \lim_{t \to \infty} \zeta(t) = \hat{\mathcal{P}}. \]

Then, we consider the following function \( W : C_n \to \mathbb{R} \) by

\[ W(\zeta(\theta)) = e^{t} \zeta(0) + \sum_{i=1}^{n} e^{t} f_{\tau_i}(\zeta(\theta)) d \theta. \]

Thus, for all \( t \geq 0 \), along the trajectory of (6) is:

\[ W(\zeta(t)) = W(\phi(\theta)) = e^{t} \phi(0) + \sum_{i=1}^{n} e^{t} f_{\tau_i}(\phi(\theta)) d \theta \quad (13) \]

which the imply that \( \zeta(t) \to \hat{\mathcal{P}} \cap \mathcal{T} \), where \( \mathcal{T} \triangleq \{ \zeta(\theta) \in \mathcal{C}_n : W(\zeta(\theta)) = W(\phi(\theta)) \} \). Hence, \( \hat{\mathcal{P}} \cap \mathcal{T} = \{ M^* \cdot e \} \) and \( \lim_{t \to \infty} \zeta(t) = M^* \cdot e \), where \( M^* \) satisfie (11). Finally, we use the Lyapunov functional as follows

\[ V(\zeta(\theta)) = -\sum_{n=1}^{n} \zeta_{(0)} Q_{n,i} (f_{n}(\mathcal{P}) - f_{n}(M)) d \mathcal{P} + \sum_{n=1}^{n} \int_{-\tau}^{0} \left[ f_{\tau_i}(\zeta(\theta)) - f_{\tau_i}(M) \right] Q_{n,i} (f_{\tau_i}(\zeta(\theta)) - f_{\tau_i}(M)) d \theta \]

and noting that \( V(\zeta) \geq -\sum_{n=1}^{n} Q_{n,i} (f_{n}(M) - f_{n}(M + \delta(\zeta(0) - M))) \zeta_{(0)}(0) - M) > 0 \), for all \( \zeta_i(0) \neq M \), where \( 0 < \delta < 1 \). So \( M \cdot e, M \geq 0 \) is a semistable equilibrium point of (6).

4. Results and Discussion

We can verify the rationality of our algorithm through the previous sections, and the following simulation is verified through four motors with two regions:

\[ G(q) = \frac{q^4 + 0.708q^3 + 0.46q^2 + 0.248q + 0.16}{q^4 - 0.83q^3 + 0.66q^2 + 0.285q + 0.216q - 0.34} \]

\[ H(q) = \frac{q^4 - 1.782q^3 + 1.423q^2 - 0.593q + 0.08348}{q^4 - 1.8q^3 + 0.35q^2 + 1.016q - 0.685} \]

\[ W(q) = \frac{-0.24 + 0.58q}{-0.6 + q} \quad (14) \]

We can see that fig 2 represents the interference curve and output curve of the fourth identification. The system sampling time of \( T = 5s \) was chosen so that an iterative identification procedure could be designed and a model for identification could be obtained, and the correctness of the method was verified in the simulation.

\[ G_2(q) = \frac{1.04q^4 + 0.652q^3 - 0.23q^2 + 0.148q + 0.23}{0.805q^4 - 1.932q^3 + 0.36q^2 + 0.175q^2 + 0.6q - 0.1805} \]

\[ W_1(q) = \frac{0.675q(q - 0.4)}{(q + 0.7)(q - 0.1)} \quad (15) \]

Fig.2 The fourth identification output in the power system
The performance of the real system, and the step responses curve of the closed-loop power system are represented in Fig 3. From figure, it can be obtained that after the fourth iteration of identification, \([G, W_3]\) is closed to the ture power system.

Fig.3 Closed loops step response curves of different types of power systems

The simulation result in Fig.3 tell us that the iterative identification mechanism is very superior to other methods and can make the power system stability improve and remain stable.

5. Conclusions
In this paper, we propose an iterative identification algorithm based on environmental signals, which can identify closed-loop power systems and solve the research work of setting and modeling the controller of the power system. This paper starts with an uncertain power system model and gives specific conditions for stability proof, from which Lyapunov asymptotic stability is found as the equilibrium of the dynamic network, proving that the force iterative identification algorithm is in a convergent state. The validity and reasonableness of the method proposed in this paper is verified by simulating the four-stage two-drive system. In the next step of the study, the time delay problem can be added and the effect of time lag on the power system controller.

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References
[1] Ninness B, Goodwin G C. (1995) Estimation of model quality.Automatica. J ,31(12),1771–1797.
[2] Gevers M. (2007) A personal view of the development of system identification: A 30-year journey through an exciting field. IEEE Control Systems Magazine, 26(6), 93-105.
[3] Yonghui N, Hantong X, Guowei C, et. al. (2020) Design of Wide-area Damping Controller Based on Immune System. J Power System Technology, 44(12): 4713-4721.

[4] Peiping L, Lingkang Z, Jianfeng W, et. al. (2020) Adaptive Wide-area Coordinated Damping Control Considering Bidirectional Random Communication Time-delay Compensation. J Power System Technology, 44(10): 56-65.

[5] Yan Z, Meng Z, Han G. (2021) Damping Control for Grid Connected DFIG System Based on Dissipated Energy Coefficient. J Power System Technology, 45(07): 2781-2795.

[6] Jing Y, Tong W, Jingtian B, et. al. (2020) Robust Damping Control of Subsynchronous Oscillation in Power System with Direct-drive Wind Turbines. J Automation of Electric Power Systems, 44(03): 56-65.

[7] Miao Y, Chao L. (2012) A new optimized method of excitation signal for closed-loop identification of power system based on ambient signals. J Applied Mechanics and Materials, 157-158, 277-285.

[8] Hjalmarsson H. (2015) Experiment design and identification for control. Springer London; pp 396-405.

[9] Miao Y, Chao L. (2015) A new iterative identification and control method of closed-loop power system based on ambient signals. J Applied Mechanics and Materials, 798, 261-265.

[10] Hale J K, Lunel S V. (1993) Introduction to functional differential equations, Springer-Verlag, New York.

[11] Ebadat A, Valenzuela P, et. al. (2014) Applications oriented input design for closed-loop system identification: A Graph-Theory Approach. IEEE conference on Decision and Control; pp.15-17.

[12] Haddad W M, Chellaboina V. (2005) Stability and dissipativity theory for nonnegative dynamical systems: A unified analysis framework for biological and physiological systems. J Nonlinear Analysis: Real World Applications, 6(1), 35-65.

[13] Haddad W M, Chellaboina V. (2004) Stability theory for nonnegative and compartmental dynamical systems with time delay. System and Control Letters, 51(5), 355-361.