Quantum spread spectrum multiple access

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We describe a quantum multiple access scheme that can take separate single photon channels and combine them in the same path. We propose an add-drop multiplexer that can insert or extract a single photon into an optical fibre carrying the qubits of all the other users. The system follows the principle of code division multiple access, a spread spectrum technique widely used in cellular networks.

I. MULTIPLE ACCESS IN PHOTONIC COMMUNICATION CHANNELS

Quantum communication protocols such as quantum key distribution [1, 2], dense coding [3] or quantum teleportation [4] expand the possibilities of classical data transmission. The quantum states of light are the natural choice to implement most of these protocols and there are successful optical implementations of quantum key distribution systems that send single photons through commercial optical fibre links [5].

As the number of users grows, there appears the problem of channel access. Many different users might want to use the same resources at the same time. Classical communication networks solve this problem with a variety of multiple access schemes [6, 7]. Quantum networks have already made use of wavelength and frequency division multiple access techniques where each user sends data at different frequencies [8–10], as well as time division multiple access where users wait for their turn [11]. In our previous work, we have also proposed different multiple access schemes using coherent states [12] and the orbital angular momentum of single photons [13, 14].

In this paper, we propose a multiple access scheme based on spread spectrum techniques. The scheme is described for optical fibre, but could be easily adapted to free-space transmission. Our system is built from widely used optical elements. It sends the photons of multiple users through the same optical fibre so that they share their path, frequency band and time window. Previous spread spectrum multiple access techniques for quantum optical communication introduce heavy losses when they combine and separate the data from each user [15–17]. Our scheme can, in principle, achieve almost deterministic operation. It follows an add-drop architecture with simple combination and extraction points and it has been designed to adapt classical spread spectrum methods directly. The system can reuse existing code families and most of the classical techniques. The users only need to add the multiplexer and demultiplexer systems we describe.

II. SPREAD SPECTRUM

In cellular communication networks, spread spectrum techniques are often used to share the channel. In spread spectrum, a modulated data signal \( d(t) \) of bandwidth \( W \) is transformed so that it becomes a spread signal \( s(t) \) of a larger bandwidth \( SW \), where \( S \) is called the spreading factor [18].

We are going to discuss a method based on direct-sequence spread spectrum technologies and their application to Code Division Multiple Access, CDMA. In CDMA, each user \( U_i \) from a group of \( N \) users, \( U_1, U_2, \ldots, U_N \), is assigned a code \( c_i \). We describe codes \( c_i \) as vectors of elements 1 and -1. The codes are chosen to be orthogonal, with \( c_i \cdot c_j = \delta_{ij} \), or nearly orthogonal, with \( c_i \cdot c_j \leq n \) for \( i \neq j \) and an integer value of \( n \) as small as possible.

Figure 1 shows the spreading and despreading processes. We can basically consider spreading as a second modulation where the signal is multiplied by \( c_i \). Spreading can be undone at the receiver if we multiply again the received signal and \( c_i \).

![Figure 1: Example of a direct-sequence spread spectrum multiple access system.](image)

If the codes are chosen well, the signals from all the
users can be clearly separated even though they have shared the whole bandwidth at the same time. Furthermore, spreading provides improved results against noise and allows to increase the number of users beyond the perfect separation limit given by the finite number of orthogonal codes of a certain length. In spread spectrum there is a “gentle degradation” where each additional user above the limit appears only as a low level noise.

Direct-sequence spread spectrum methods can be directly applied to single photons [10]. The photon’s wavefunction can be spread through an extended bandwidth and later be recovered at the receiver. The despreading procedure takes the wavefunction back to its original bandwidth and, at the same time, spreads the noise which is then filtered.

In this paper, we show how to take advantage of the properties of spread spectrum multiple access in photonic channels.

III. BUILDING BLOCKS

Our system uses three standard optical fibre elements: optical modulators, circulators and fibre Bragg gratings. Electro-optic modulators alter the waveform of a signal according to a control signal [20]. This operation can be extended to the quantum regime [21]. We require an optical modulator that acts on the photon’s phase. We can follow [10] and introduce a phase shift $\pi$ for -1. The total effect is $\frac{\pi}{2}$ or 0/π, except for an unimportant global phase π.

We now need a method to combine the spreaded photons of all the users in the same optical fibre. We use two elements: circulators and fibre Bragg gratings, FBG. Circulators are non-reciprocal optical three- (or more) port devices that reroute incoming signals to the next output port (see Figure 2).

Fibre Bragg gratings are frequency specific reflectors that, in their most common form, let most of the incoming signal pass unaffected while a specific frequency band is reflected (see Figure 3).

FIG. 2: Symbol of the circulator and the correspondence between input and output ports. The element is not reciprocal. Light entering port 1 goes out port 2, but light coming into port 2 moves on to port 3 instead of going back to port 1.

**FIG. 3:** Fibre Bragg grating. The power spectral density of an incoming signal is divided into two parts. The grating acts as a band-stop filter that reflects the part of the spectrum around a frequency $f_B$ and transmits the rest of the input signal.

IV. SIGNALS

Our data signals are qubits that can have different encodings. Frequency modulation is common in classical spread spectrum systems. We can follow that model and use frequency qubits, where states $|0\rangle$ and $|1\rangle$ correspond to wavefunctions at different frequencies [22] or use phase modulation on the sidebands of a strong carrier [23].

Alternatively, we can use time-bin encoding and send a photon with a wavefunction restricted to a time window $(0, T_0)$ to encode state $|0\rangle$ or introduce a delay to move the wavefunction to time window $(T_0, 2T_0)$ to encode state $|1\rangle$ [24]. Time-bin qubits are particularly attractive for optical fibre transmission and this is the preferred encoding in many experimental quantum key distribution systems. In this case, the codes must be designed for a time period of length $T_0$ and be applied twice. Otherwise, two orthogonal codes with the same first or second half could produce the same spreaded signals for different users and the signals can interfere.

V. MULTIPLEXING

First, we discuss how the transmitter of each user can add its signal to a superposition state $|\psi_S\rangle$ that carries the photons of all the previous users. Figure 4 shows the block diagram of the multiplexer. The multiplexing and demultiplexing systems follow the ideas of add-drop multiplexers that are extensively used in optical fibre networks to combine channels of different frequencies. We use them to combine single photons.

Most classical methods that add two signals do no work well with individual photon states. Quantum operations must be reversible and two photons cannot be directly combined like in an optical Y-junction which takes two
inputs into the same output. This problem limits previous quantum spread spectrum multiple access methods \cite{15,17}. We propose an architecture that minimizes lost photons.

In our system, new photons are added in the multiplexer of Figure 4. When the input superposition reaches the circulator and is directed to a second modulator, we can recover the photons. The new superposition is then put into the same optical fibre.

![Multiplexer](image)

**FIG. 4:** Multiplexer: The new user \( U_i \) wants to add photon \( P_i \) to the input superposition coming from the common optical fibre. The central element is the fibre Bragg grating where the user’s photon \( P_i \) and the input superposition meet. Photon \( P_i \) comes from port 1 of the circulator. The input superposition has been previously multiplied by \( c_i \). The grating reflects the band of the spectrum which contains the whole wavefunction of \( P_i \). The photons of the input superposition only have a fraction \( 1/S \) of their wavefunctions in that part of the spectrum. The joint signal which contains the new and old photons comes into port 2 of the circulator and goes out port 3 where a second modulator restores the old photons to their original state and spreads \( P_i \). The new superposition is then put into the same optical fibre.

At the same time, we send the modulated, but not yet spreaded, data signal \( d_i(t) \) of the new user (photon \( P_i \)) through a circulator so that it reaches the FBG at the same time as the superposition signal but in the opposite direction. We assume the system has a precise enough synchronization to obtain an output signal that includes the new photon and most of the spreaded superposition to the circulator. The grating reflects the new photon and a residual part of the spreaded superposition back to the fibre it came from.

The signal with the old photons and the new photon reaches the circulator and is directed to a second modulator in port 3 which applies once more the code \( c_i \). The input superposition is now back to its original form (except for a small loss) and the photon from user \( U_i \) is spreaded. The resulting output is a superposition of the previous wavefunctions plus the new photon. The photons do not interact. The new term can be thought of as the tensor product of orthogonal wavefunctions sharing the same frequency band.

The fibre Bragg grating reflects a small part of the wavefunction of the incoming photons. If the spreading factor \( S \) is large enough, the probability that a photon is reflected back and lost, \( 1/S \), is small. This can be considered as a small channel loss. This loss is cumulative and can limit the total number of users. At each multiplexer, the wavefunction of the incoming photons is spreaded with a different code. The parts of the wavefunction that end in the reflected part of the spectrum is different for each added photon and, when the signal is restored at the second modulation, we can consider the total effect as a uniform loss.

VI. DEMULTIPLEXING

If we repeat the multiplexing procedure for the \( N \) users, we end with a fibre carrying \( N \) photons with the qubits of all the users. We can separate them at each of the \( N \) intended receivers with the demultiplexing optical circuit of Figure 5. Notice that, as the circulator is a non-reciprocal element, we cannot just use the multiplexing circuit in reverse. We need a slight modification in the order of the elements.

![Demultiplexer](image)

**FIG. 5:** Demultiplexer: A receiver extracts from a fibre carrying multiple photons the qubit coming from user \( U_i \). The input superposition is first multiplied by code \( c_i \). This operation despreads photon \( P_i \), which is now confined to the part of the spectrum that the fibre Bragg grating reflects. The other photons remain spreaded. The circulator takes this signal to the grating, which reflects \( P_i \) and forwards the other photons, up to a loss \( 1/S \), to a second modulator which restores their original state. These photons go back to the common optical fibre while the reflected photon \( P_i \) can be recovered from port 3 of the circulator.

In the demultiplexer, the incoming photon superposition is directed to a modulator which spreads the signal with a code \( c_i \). The modulator re-spreads all the photons but \( P_i \), which is despreaded. Its wavefunction is now concentrated in its original band of the spectrum \( W \).

The new signal is then sent through the circulator into a fibre Bragg grating. The grating reflects the desired photon back to the circulator which directs it to its intended recipient (along with a residual noise signal with a fraction \( 1/S \) of the wavefunction of the other photons). The largest part of the wavefunctions of the previous photons goes through the grating and meets a second modulator which multiplies the signal again by \( c_i \) and restores the original superposition minus the extracted photon. We can repeat the procedure \( N \) times to extract all of the incoming photons.
VII. DISCUSSION

We have proposed a multiple access scheme that brings code division multiple access into the quantum realm. We take $N$ qubits encoded into $N$ separate photons and send them together using the same optical fibre.

The wavefunction of each photon is spreaded using a code unique to each user. If the code has $S$ distinct elements ($S$ chips), a wavefunction of bandwidth $W$ is stretched to a bandwidth $SW$.

At each multiplexing stage, the qubit of a new user is added to the photons already in the channel. During the procedure, the “old” photons have a probability $1/S$ of being lost, even for ideal elements. The same happens at each demultiplexing stage. If $N$ users share the channel, a photon can at most undergo $2N - 2$ lossy multiplexing/demultiplexing stages. There is a maximum probability of photon loss $\frac{2N-2}{S}$, which could be reduced if the qubits of each user are added and extracted at the right points. We can, for instance, require that the first photon in is the first photon out. In practice, the optical circuits of Figures 4 and 5 will introduce coupling losses that should also be taken into account.

In any case, the system seems adequate for Quantum Key Distribution in its present form. All the necessary elements are already used in optical fibre systems and the codes can be assigned using existing CDMA schemes. Ideally, a large value of $S$ is desirable. It both reduces the unavoidable $1/S$ loss at each element and increases the number of potential users. For $N$ users with $N \leq S$, there are enough orthogonal codes to allow for perfect separation, but it is possible to go above $S$ users using nearly orthogonal codes with a small overlap. A recent experiment on spread spectrum modulation of a single photon shows that a value $S = 50$ is technically feasible \cite{19}. Coupling losses at the optical elements are likely to be the major limitation. We can, anyway, limit the number of users and still use as large an $S$ as permitted by the modulation (the speed at which we can change the phase) to reduce the $1/S$ losses and keep the processing gain against noise.

An alternative way to limit losses is using integrated optical elements. For instance, microring structures can replace the fibre Bragg gratings and the circulators and act as frequency selective filters and routing devices \cite{22, 26}. The whole multiplexing and demultiplexing subsystems can thus be integrated into one compact optical element. A detailed analysis of this microring-based solution with a full simulation of losses will be presented elsewhere.

As a final note, we would like to remark that, although the presented multiple access scheme has been designed with single photon channels in mind, both the discrete elements and integrated microring versions of the system are also an interesting alternative to classical multiple access techniques. In particular, the low losses of the scheme can eliminate or at least mitigate the need for amplifiers.

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