T-odd Asymmetry in W+jet Events at the LHC

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W bosons produced at high transverse momentum in hadron collisions can have polarization along the direction perpendicular to the production plane, which is odd under naïve-T-reversal where both the three-momenta and angular momenta are reversed. Perturbative QCD predicts non-zero polarization at the one-loop level, which can be measured as parity-odd components in the angular distribution of charged leptons from the decay of W bosons. We perform a detector-level simulation with the generator MadGraph5_AMC@NLO, and demonstrate that the asymmetry can be observed at the 8 TeV LHC with 20 fb−1 of data. If confirmed, it will be the first experimental measurement of the sign of one-loop QCD amplitudes.

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Naïve-T-reversal is a unitary transformation in which we impose time-reversal on the initial and final states respectively, but do not reverse the time direction from the initial to the final state. In CP-conserving theories like perturbative QCD, asymmetry under naïve-T-reversal appears through the absorptive part of the scattering amplitudes1,2, and hence offers a non-trivial test of perturbative QCD at one and higher-loop levels. Various tests have been proposed in the past, including asymmetries in \( Y \) decay into 3 jets3,4, \( ee^{-} \) annihilation production of 3 jets5,6, neutrino (electro) production of 2 jets7,8, Drell-Yan production of high-\( q_T \) W-boson at hadron collisions9-12, Z-boson decay into 3 jets13, and top-quark radiative decays14. Although the predictions deserve much interests as probes of the absorptive part of the loop-level QCD amplitudes, no experimental confirmation has been made so far.

In this paper, we consider the W+jet production at the LHC,

\[ p p \rightarrow W^{+}(\rightarrow l^{+} \nu_l) + \text{jets}, \]

where \( l \) denotes \( e \) or \( \mu \), in which T-odd effects that flip sign under naïve-T-reversal arise in the parity-odd (P-odd) angular distributions of \( l \) in the decay of the W boson3-6. The following subprocesses contribute to the above process in the leading order (LO): \( u g \rightarrow W^{+}d, \ u\bar{d} \rightarrow W^{+}g, \ \bar{d} g \rightarrow W^{+} \bar{u} \). The differential cross section for the process can be expressed as,

\[
\frac{d\sigma}{dq_T^2 d\cos \theta d\cos \phi d\phi} = F_1(1 + \cos^2 \theta) + F_2(1 - 3 \cos^2 \theta)
+ F_3 \sin 2\theta \cos \phi + F_4 \sin^2 \theta \cos 2\phi + F_5 \cos \theta
+ F_6 \sin \theta \cos \phi + F_7 \sin \theta \sin \phi + F_8 \sin 2\theta \sin \phi
+ F_9 \sin^2 \theta \sin 2\phi.
\]

(2)

Here \( (\theta, \phi) \) measures the direction of the \( l^+ \) three-momentum in the W-boson rest-frame whose \( y \)-axis is taken perpendicular to the scattering plane1. \( q_T \) denotes the transverse momentum of the W boson, and \( \theta \) denotes the scattering angle of the W boson in the W+jet center-of-mass frame. The structure functions \( F_{1-9} \), which are functions of \( q_T \) and \( \cos \theta \), are described by the polarization density matrix for W+jet production. The \( F_1 \) term governs the overall normalization, while the other eight terms affect the lepton angular distributions. The LO analytical expressions for \( F_{1-6} \) at \( O(\alpha_s) \) are found in Ref. [10], and the next-to-leading order (NLO) corrections have been analyzed in Ref. [11]. \( F_{7-9} \) terms represent the P-odd and T-odd components of the lepton angular distribution, because under parity transformation or naïve-T-reversal, \( \phi \) flips sign while \( \theta \) and \( \phi \) remain unchanged. The LO contribution to these terms comes from the absorptive part of the one-loop amplitudes at \( O(\alpha_s^2) \), whose analytical expressions are found in Ref. [5,6]. Experimentally, some of the P-even azimuthal angular distributions have been measured in W+jet events at the Tevatron12, in good agreement with the NLO QCD prediction11. At the LHC, only the polar angular distributions have been measured13,14, which confirm the helicity fraction of W bosons predicted in QCD15.

In the rest of this work, we focus on the \( F_7 \) term, owing to the fact that this has the largest size of asymmetry among the three terms5,6.

Although a simulation study at the parton-level indicates that the Tevatron has enough potential to observe the T-odd terms6, no experimental measurement

1 The z-axis can be chosen along the direction of the W momentum in the laboratory frame (the helicity frame), or along the direction which makes the same angle with the two beam momenta (the Collins-Soper frame9). The results in this report do not depend on the choice of the z-axis.
2 The contributions from CP-violating terms in the Standard Model are negligibly small.
has been reported so far. One of the reasons for the difficulty of the measurement might be that loop-level effects, such as T-odd asymmetries of the amplitudes, were not available in the LO event generators which are commonly used to simulate detector responses by experimentalists. In this Letter, we study how the T-odd effects are included in the multi-purpose NLO event generator MadGraph5_AMC@NLO \cite{4, 5}, which has been made public very recently. Furthermore, we demonstrate how the effects of QCD initial-state/final-state radiation (ISR/FSR) and those of finite detector resolution affect the measurements. In order to study systems of higher order QCD corrections, we prepare two types of event samples: one is generated by MadGraph5_AMC@NLO \cite{4, 5} where the W+jet events are calculated at the NLO+PS (parton shower) level, and the other is generated by a hand-made event generator which we call LOMC where all the $F_{1-9}$ structure functions are implemented at the LO with the help of BASES/SPRING code \cite{6}. We stress that, although the MadGraph5_AMC@NLO code generates events with NLO accuracy, the T-odd observables constructed from these events are accurate at LO because these observables receive contributions only at the one-loop level and beyond.

We remind the reader that all contributions to NLO calculations are completely automated in the MadGraph5_AMC@NLO code: the virtual corrections are computed in the MadLoop module \cite{7}, which is based on the OPP integrand-reduction method \cite{8} (as implemented in CutTools \cite{9}) and the OpenLoops technique \cite{10}; the factorization of the infra-red singularities is achieved by adopting the FKS method \cite{11}, as implemented in the MadFKS module \cite{12}; and the consistent matching to parton showers is obtained by using the MC@NLO technique \cite{13}.

For the MadGraph5_AMC@NLO simulation, we generate the $p p \to \mu^+ \nu\mu_j$ process\footnote{We have confirmed by the stand-alone matrix-element calculation that the T-odd terms completely agree with the analytic expressions in Ref. \cite{15} at arbitrary phase-space points.} at the NLO. CTEQ6M parton distribution functions (PDFs) \cite{16, 17} are used, and the factorization and renormalization scales are set to $\mu_F = \mu_R = q_T$. Phase-space cuts are applied at the generation-level, which are $q_T > 25$ GeV, $p_{T\mu} > 25$ GeV, $p_{T\nu} > 22$ GeV in the regions of $|\eta_\mu| < 2.5$, and $p_{T\nu} > 10$ GeV, where $p_{T\mu}$ and $\eta_\mu$ are the transverse momentum and pseudo-rapidity of a particle $i$, respectively. Parton showering and hadronization are simulated with Herwig6 \cite{18}, and detector simulation is performed with PGS4 \cite{19}. Jets are reconstructed using the anti-$k_T$ jet clustering \cite{20} with $\Delta R = 0.4$.

We generate net about 100M of events with MadGraph5_AMC@NLO as a difference between positive weight events and negative weight events. The scale variation can be estimated at no extra computational cost \cite{21}. For the LOMC, we perform the simulation in a similar setup to that for MadGraph5_AMC@NLO, but with CTEQ6L PDFs and LO matching with parton showers. For each of the three choices of the scales, $\mu = q_T, q_T/2$ and $2q_T$, we generate 100M of only positive weight events.

For the generated events, we apply the following selection cuts. Denoting the missing transverse momentum by $\vec{\not{p}}_T$ and defining the transverse mass as $M_T \equiv \sqrt{2(p_{T\mu} q_T - \vec{p}_{T\mu} \cdot \vec{q}_T)}$, we require (a) one $\mu^\pm$ with $p_T > 25$ GeV and $|\eta| < 2.4$; (b) $p_T > 25$ GeV; (c) $q_T \equiv |p_{T\mu} + \vec{q}_T| > 30$ GeV; (d) $M_T > 60$ GeV; (e) the leading jet satisfies $p_{T\mu} > 30$ GeV and $|\eta| < 4.4$. After these selection cuts, the cross section is about 200 pb at the NLO. We note that these cuts are similar to those applied in the earlier W boson observation at the LHC \cite{22, 23}, where a good signal-to-background ratio has been achieved.

To observe the $F_7$ contribution, we have to measure $\sin \theta \sin \phi$ and $\cos \theta$, event by event, because $F_7$ is an odd function of $\cos \theta$. We define the charged-lepton momentum component perpendicular to the scattering plane as

$$p_T^\perp = \frac{\vec{p}_{\mu T} \times \vec{q}_T \cdot \vec{p}_T}{|\vec{p}_{\mu T} \times \vec{q}_T|},$$ \hspace{1cm} (3)$$

where $\vec{p}_{\mu T}$, $\vec{q}_T$ and $\vec{p}_T$ are the right-moving proton momentum, the W transverse momentum and the lepton momentum, respectively, all in the laboratory frame. In terms of $p_T^\perp$, $\sin \theta \sin \phi$ of eq. \cite{24} can be observed as,

$$\sin \theta \sin \phi = \frac{p_T^\perp}{(m_W/2)} \equiv x^\perp,$$ \hspace{1cm} (4)$$

in the narrow width limit of the W boson. On the other hand, the measurement of $\cos \theta$ is affected by the twofold ambiguity in determining the neutrino longitudinal momentum, or the W-boson rest frame. Instead, we use the pseudo-rapidity difference between the charged lepton and the leading hard jet, $\Delta \eta \equiv \eta_\mu - \eta_j$, which has strong correlation with $\cos \theta$ \cite{25}.

The determination of $x^\perp$ is affected by the uncertainty in the $\vec{p}_T$ measurement, because the scattering plane is determined by the W transverse momentum, which is the vector sum of the lepton and missing transverse momenta. To reduce the impact of this uncertainty, we select events with large $|x^\perp|$ and simply focus on difference in the numbers of events for $x^\perp > 0$ and $x^\perp < 0$, which we call the left-right asymmetry. To pin down an appropriate selection cut on $|x^\perp|$, we investigate the distribution of $x^\perp$. In Fig. \ref{fig:4}, we show the $x^\perp$ distribution after the selection cuts (a-c) and a cut of $\Delta \eta > 1.0$ at the parton level, where an outgoing parton is identified with a hard jet. By selecting events with large $|x^\perp|$, we can reduce the smearing of the asymmetric distribution without loss of statistics.
be induced due to the use of different set of PDFs and

\[ P \text{ the} \]

\[ 1.5 \text{ for smaller} \]

\[ \text{is consistent with our na¨ıve expectation. This is because} \]

\[ \text{The results by the two simulations are very similar, which} \]

\[ \text{central scale choice, the} \]

\[ K \]

\[ \text{at the 8 TeV LHC, in the leading-order calculation at the} \]

\[ \text{parton-level. Predictions for} x_l^+ > 0 \text{ and} x_l^+ < 0 \text{ regions} \]

\[ \text{are separately plotted in the red solid and blue dashed lines,} \]

\[ \text{respectively.} \]

\[ \text{Now that we have established the size of the asymmetry at the} \]

\[ \text{parton level, we present our main results} \]

\[ \text{in Fig. 2} \]

\[ \text{In this figure, we show our simulated cross sections at the detector-level after the selection cuts (a-e) and a cut of} \]

\[ \Delta \eta > 0.6 \]

\[ \text{The left, middle and right panels show the} \]

\[ \Delta \eta \]

\[ \text{distributions for the cross section, the left-right difference of the cross sections defined as} \]

\[ \sigma(x_l^+ > 0) - \sigma(x_l^+ < 0), \text{ and the left-right asymmetry of the cross sections defined as} \]

\[ A \equiv \frac{\sigma(x_l^+ > 0) - \sigma(x_l^+ < 0)}{\sigma(x_l^+ > 0) + \sigma(x_l^+ < 0)}, \]

\[ \text{respectively. Results obtained by} \]

\[ \text{MadGraph5_AMC@NLO and LOMC simulations are} \]

\[ \text{shown in the dark-colored large-hatched histograms and} \]

\[ \text{light-colored small-hatched histograms, respectively.} \]

\[ \text{Histograms are normalized to the expected number of} \]

\[ \text{events per bin at the 8 TeV LHC with 20 fb}^{-1} \text{ of data} \]

\[ \text{after the selection cuts (a-e) and a cut of} |x_l^+| > 0.6 \text{ are} \]

\[ \text{applied. The vertical widths of the histograms indicate the} \]

\[ \text{scale uncertainty in the simulation.} \]

\[ \text{As seen in the left panel, there is a difference between the} \]

\[ \text{predicted cross sections for} \]

\[ \text{MadGraph5_AMC@NLO and LOMC. This comes from nothing but the NLO correction to the total cross section, which is included in the} \]

\[ \text{MadGraph5_AMC@NLO but not in the LOMC. For our} \]

\[ \text{central scale choice, the} \]

\[ K \text{-factor is found to be around} 1.5 \text{ for smaller} |\Delta \eta|, \text{but above 2 for larger} |\Delta \eta|. \]

\[ \text{In the middle panel, the left-right difference of the cross sections is} \]

\[ \text{found consistent with the behavior of the} F_7 \text{ terms.} \]

\[ \text{The results by the two simulations are very similar, which} \]

\[ \text{is consistent with our na¨ıve expectation. This is because} \]

\[ \text{both the generators contain the leading} \mathcal{O}(\alpha_s^2) \text{ terms for the} \]

\[ \text{P-odd contributions. In principle, differences can be induced due to the use of different set of PDFs and} \]

\[ \text{the different treatment in the parton-shower simulation} \]

\[ \text{at the NLO and LO. However, our results suggest that} \]

\[ \text{these effects are negligibly small. In the right panel, we} \]

\[ \text{find that an order of 5-10% left-right asymmetry is} \]

\[ \text{predicted and that the asymmetry is robust even after the} \]

\[ \text{inclusion of QCD ISR/FSR and the detector smearing.} \]

\[ \text{We point out that a smaller left-right asymmetry is predicted by} \]

\[ \text{MadGraph5_AMC@NLO than LOMC, due to the} \]

\[ \text{large enhancement of the total cross section which} \]

\[ \text{enters in the denominator of the asymmetry.} \]

\[ \text{The scale uncertainties in our simulations deserve extra} \]

\[ \text{attention. In the LOMC simulation, there are two sources of} \]

\[ \text{scale uncertainty; one is the choice of the scales at} \]

\[ \text{the parton-level calculation, namely, the scales in strong} \]

\[ \text{coupling constant and in the PDFs, and the other is the} \]

\[ \text{choice of the initial scale in the parton showering.} \]

\[ \text{Variation of the choice of the former scales affects the cross} \]

\[ \text{section and the lepton distribution at the parton-level.} \]

\[ \text{Since the} P \text{-even (} P \text{-odd) part of the cross section is} \]

\[ \mathcal{O}(\alpha_s) \left[ \mathcal{O}(\alpha_s^2) \right], \text{ we expect an overall scale dependence of} \]

\[ \mathcal{O}(10\%) \left[ \mathcal{O}(20\%) \right] \text{ by the scale variation of} \]

\[ \alpha_s. \]

\[ \text{Variation of the scale in the parton showering affects the number} \]

\[ \text{and distribution of the ISR/FSR jets. In our event} \]

\[ \text{analysis, it affects the probability that the leading jet is} \]

\[ \text{misidentified by an ISR jet, which results in the error of} \]

\[ \Delta \eta \]. \text{If the scale is taken higher, more jets are produced} \]

\[ \text{via parton showering, and the misidentification probability} \]

\[ \text{increases. This causes a significant scale dependence in} \]

\[ \text{the cross sections for large} |\Delta \eta|, \text{because ISR jets tend to} \]

\[ \text{appear at large} |\eta_j|. \text{ The total scale uncertainty in the} \]

\[ \text{cross section is not very large because the increase in} \]

\[ \text{the number of} W+\text{jet events due to ISR jets is partially} \]

\[ \text{canceled by the smaller} \alpha_s \text{ at the higher scale. For the} \]

\[ \text{left-right difference of the cross section, the shower-scale} \]

\[ \text{variation does not cause significant shift in any} \]

\[ \Delta \eta \text{ regions, because the sum of the left-right difference over the entire} \Delta \eta \text{ range is zero. Its scale dependence is only} \]

\[ \text{governed by the overall} \alpha_s^2 \text{ factor. Overall, the scale uncertainty in the left-right asymmetry is estimated to be about} 20\% \text{ (30\%) in the small (large) } |\Delta \eta| \text{ regions.} \]

\[ \text{In the} \]

\[ \text{MadGraph5_AMC@NLO simulation, there is a} \]

\[ \text{cancellation in the dependence on the parton shower starting scale and the Monte Carlo subtraction terms} \]

\[ \text{leading to a negligible uncertainty coming from this scale for the observables studied here. Therefore, the total scale uncertainty for the left-right asymmetry, which is about 10\% in any region of} |\Delta \eta| \text{ is significantly reduced from that in the LOMC results.} \]

\[ \text{The difference in magnitude of the left-right asymmetry between the two simulations can be understood by the} \]

\[ K \text{-factor for the total cross section in the} \]

\[ \text{MadGraph5_AMC@NLO result, entering only in the denominator of eq.}\]

\[ \text{1. The LOMC predictions do not have this apparent mismatch, since both numerator and denominator are computed at LO accuracy. Since the difference between the} \]

\[ \text{MadGraph5_AMC@NLO and LOMC simulations is larger than the accuracy of either one of} \]

\[ \text{these} \]
codes, we regard this difference as an additional source of uncertainty for this observable. To improve the situation, the NLO corrections also to the numerator of eq. (9) are needed, however, they are currently not known.

Before closing, we present several comments. We estimate the expected statistical error as $\delta A = \sqrt{(1 - A^2)/N_{\text{evt}}}$, and find that with 20 fb$^{-1}$ of data, $\delta A$ is about (1.1, 1.5, 2.5, 4.5)×10$^{-3}$ for $|\Delta \eta| = (0, 1, 1.2, 1.5, 3.4)$ bins, respectively. Therefore, the data collected at the LHC should be enough to measure the asymmetry. When a cut of $|x_T^+| > 0.8$ is applied, the asymmetry is enlarged by 10-20%, while the statistical error also grows by about 30%. We comment on background events from the $W^+ \rightarrow \tau^+ \nu$ decay followed by the $\tau^-$ decay into $\mu^-$. We find that such events do not exceed 2% of the $W^+ \rightarrow \mu^+ \nu$ events in each bin of $\Delta \eta$ after selection cuts (a-e) and a cut on $|x_T^+| > 0.6$ are applied. Hence non-zero value of the left-right asymmetry is still observable in the presence of the $W^+ \rightarrow \tau^+ \nu$ background.

To summarize, we have examined the possibility of observing T-odd asymmetry in $W$+jet events at the LHC. The asymmetry arises from the absorptive part of the scattering amplitudes in perturbative QCD, and manifests itself as a difference in the parity-odd distributions in the lepton decay angle. We have demonstrated by a simple detector-level analysis that the difference due to the T-odd term remains detectable after the inclusion of ISR/FSR radiation and detector resolution. The prediction by the next-to-leading order event generator MadGraph5_AMC@NLO contains relatively small scale uncertainties due to the matching to the parton shower at the NLO accuracy. On the other hand, the size of the asymmetry may be under-predicted, because the as-yet unavailable NLO corrections to the T-odd cross section could be as large as those to the T-even cross section.

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FIG. 2. $\Delta \eta$ distributions for the cross section (left), left-right difference of the cross section (middle), and the left-right asymmetry (right) at the 8 TeV LHC after the selection cuts and a cut of $|x_T^+| > 0.6$. Results by the MadGraph5_AMC@NLO and LOMC simulations are shown in dark-colored and light-colored histograms, respectively, with scale uncertainties.
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