A parameter survey of Sgr A* radiative models from GRMHD simulations with self-consistent electron heating

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ABSTRACT
The Galactic center black hole candidate Sgr A* is the best target for studies of low-luminosity accretion physics, including with near-infrared and submillimeter wavelength long baseline interferometry experiments. Here we compare images and spectra generated from a parameter survey of general relativistic MHD simulations to a set of radio to near-infrared observations of Sgr A*. Our models span the limits of weak and strong magnetization and use a range of sub-grid prescriptions for electron heating. We find two classes of scenarios can explain the broad shape of the submillimeter spectral peak and the highly variable near-infrared flaring emission. Weakly magnetized "disk-jet" models where most of the emission is produced near the jet wall, consistent with past work, as well as strongly magnetized (magnetically arrested disk) models where hot electrons are present everywhere. Disk-jet models are strongly depolarized at submillimeter wavelengths as a result of strong Faraday rotation, inconsistent with observations of Sgr A*. We instead favor the strongly magnetized models, which provide a good description of the median and highly variable linear polarization signal. The same models can also explain the observed mean Faraday rotation measure and potentially the polarization signals seen recently in Sgr A* near-infrared flares.

Key words: accretion, accretion discs — black hole physics — Galaxy: centre — MHD — polarization — radiative transfer

1 INTRODUCTION
The most extensively studied low-luminosity accretion flow is that onto the Galactic centre massive black hole candidate Sagittarius A* (Sgr A*). Detected in the radio (Balick & Brown 1974), Sgr A* shows an inverted spectrum up to a peak at submillimeter (submm) wavelengths (Serabyn et al. 1997; Falcke et al. 1998; Stone et al. 2016; von Fellenberg et al. 2018; Bower et al. 2019). The bolometric luminosity is $\approx 5 \times 10^{35}$ erg s$^{-1}$ (Bower et al. 2019), roughly 9 orders of magnitude sub-Eddington for the mass of 4 $\times$ 10$^{6}$ $M_\odot$ measured from the orbit of the star S2 (Ghez et al. 2008; Gillessen et al. 2009, 2017; Gravity Collaboration et al. 2018a, 2019; Do et al. 2019). Due to its proximity, Sgr A* is the largest black hole in angular size on the sky. It has long been a target of radio very long baseline interferometry (VLBI, Lo et al. 1975; Backer 1978; Krichbaum et al. 1998; Bower et al. 2006). With 1.3mm VLBI, the source size is as compact as $\approx 8 \times 10 r_g$ (Doeleman et al. 2008; Fish et al. 2011; Lu et al. 2018; Johnson et al. 2018), where $r_g = GM/c^2 \approx 6 \times 10^{11}$ cm is the gravitational radius.

Sgr A* shows large-amplitude “flares” simultaneously at near-infrared (NIR, Genzel et al. 2003; Ghez et al. 2004) and X-ray (Baganoff et al. 2001) wavelengths. They originate from transiently heated electrons, likely as a result of magnetic reconnection or shocks (Markoff et al. 2001) close to the black hole (Barrière et al. 2014; Haggard et al. 2019). The observed emission is due to synchrotron radiation at radio to NIR (and likely also X-ray, Dodds-Eden et al. 2009).
wavelengths and is strongly polarized from the submm to NIR (Aitken et al. 2000; Bower et al. 2003; Marrone et al. 2006; Eckart et al. 2006; Trippe et al. 2007).

The flares can now be spatially resolved with the second generation VLT Interferometer beam combiner instrument GRAVITY. In 2018, 3 flares were shown to continuously rotate clockwise at relativistic speeds (Gravity Collaboration et al. 2018b). The astrometric data are consistent with orbital motion around Sgr A* at a radius of $6 - 10 r_g$ (Gravity Collaboration et al. 2020). A matching polarization angle evolution suggests the presence of dynamically important magnetic fields in the emission region on event horizon scales. There is also evidence for ordered magnetic fields from 1.3mm VLBI (Johnson et al. 2015).

Theoretical models of the near horizon regions of low-luminosity accretion flows (Rees et al. 1982; Narayan et al. 1995; Yuan et al. 2003) are now commonly realized using general relativistic MHD (GRMHD) simulations (Gammie et al. 2003; De Villiers et al. 2003). These calculations capture the self-consistent evolution of the magnetic field, which drives accretion via the magnetorotational instability (Balbus & Hawley 1991) and extracts black hole spin energy to power relativistic jets (Blandford & Znajek 1977, BZ). Past models of Sgr A* based on such calculations have found submm spectra (e.g., Noble et al. 2007; Moscibrodzka et al. 2009; Shcherbakov et al. 2012; Moscibrodzka et al. 2014), variability (Dexter et al. 2009, 2010; Chan et al. 2015b), source sizes (Moscibrodzka et al. 2009; Dexter et al. 2010; Shcherbakov et al. 2012), and polarization (Shcherbakov et al. 2012; Gold et al. 2017) consistent with observations. Similar to analytic models (Falcke et al. 2000; Bromley et al. 2001; Broderick & Loeb 2006), they generically find that the submm and shorter wavelength emission originates near the event horizon, resulting in “crescent” shaped images.

Many of these calculations made two important, simplifying assumptions regarding the radiating electrons: i) that they are thermally distributed in energy, despite low densities implying a collisionless plasma (e.g., Mahadevan & Quataert 1997); and ii) that the electron to proton temperature ratio is a constant value (Goldston et al. 2005). Alternative, physically motivated prescriptions have instead put more of the available internal energy in electrons where the magnetic fields are stronger (Moscibrodzka & Falcke 2013; Chan et al. 2015a) or according to kinetic prescriptions taking into account anisotropic viscosity (Sharma et al. 2007; Shcherbakov et al. 2012). Anantua et al. (2020) recently studied a wide range of electron temperature prescriptions.

Those prescriptions were applied in post-processing to calculate the electron energy density (temperature) from single fluid MHD simulations. GRMHD algorithms can now evolve a separate electron fluid, which receive a fraction of the local dissipated energy according to a chosen subgrid prescription (Ressler et al. 2015). Such models self-consistently heat electrons, and can incorporate more directly results from kinetic calculations (e.g., Howe et al. 2010; Rowan et al. 2017; Werner et al. 2018; Kawazura et al. 2019; Zhinkin et al. 2019, H10, R17, W18, K19). Ressler et al. (2017) showed that “disk-jet” models are a natural outcome of turbulent heating prescriptions based on gyrokinetic theory, where the electrons are heated preferentially in strongly magnetized regions (H10). Chael et al. (2018) showed that electron heating is more uniform in alternative scenarios based on heating mediated by magnetic reconnection (R17).

Here we carry out a parameter survey, expanding the range of electron heating models and magnetic field strength compared to previous work (section 2). We constrain radiative models (section 3) using the updated submm to NIR polarized spectrum, total intensity r.m.s variability, and 86/230 GHz image size (section 4). We show (section 5) that two combinations of heating prescriptions and magnetic field strengths can explain the broad submm peak and large amplitude NIR variability of Sgr A*. Those correspond to disk-jet models considered in previous work (Moscibrodzka et al. 2014; Ressler et al. 2017), as well as magnetically arrested accretion flow scenarios (MADs, Shcherbakov & McKinney 2013; Gold et al. 2017). Our disk-jet models underproduce the observed submm linear polarization, and so are disfavored. We show that very long duration GRMHD simulations can produce the observed Faraday rotation measure of Sgr A*. Finally, we discuss the prospects of future observations and improvements to the theoretical models (section 6).

2 GRMHD SIMULATIONS WITH MULTIPLE ELECTRON HEATING PRESCRIPTIONS

We have carried out a set of 3D GRMHD black hole accretion simulations using the harmpi code (Tchekhovskoy 2019). harmpi is a 3D implementation of the HARM algorithm (Gammie et al. 2003; Noble et al. 2006) for conservative MHD in a fixed spacetime. Simulations were initialized from a Fishbone-Moncrief steady state hydrodynamic equilibrium torus in a Kerr spacetime with inner radius $r_a = 12 r_g$ and pressure maximum at $r_{max} = 25 r_g$. The torus was seeded with a single loop of poloidal magnetic field, whose radial profile is designed to supply either relatively modest (SANE) or maximal (MAD) magnetic flux. For each case, black hole spin values of $a = 0, 0.5, 0.9375$ were used. The simulations were carried out in modified spherical polar Kerr-Schild coordinates, with resolution concentrated towards the equatorial plane to resolve the accretion flow at small radius and towards the pole to resolve the jet at larger radius. The outer radial boundary was extended to $10^5 r_g$ using a super-exponential radial coordinate. The grid was chosen to provide a cylindrical innermost cell in polar angle (Tchekhovskoy et al. 2011; Ressler et al. 2017). This significantly increases the time step allowed by the Courant condition. The resolution used was $320 \times 256 \times 160$ cells, including the full $2\pi$ azimuthal domain. All simulations are evolved for at least $10^4 r_g/c$, and we analyze the period from $5 \times 10^3 - 10^4 r_g/c$, once the emission region has established inflow equilibrium. We also evolve one SANE $a = 0$ and one MAD $a = 0.9375$ simulation for a much longer time ($2 \times 10^5 r_g/c$ and $6 \times 10^4 r_g/c$, respectively), in order to allow larger radii ~ $100 r_g$ to reach inflow equilibrium. That location is thought to produce the bulk of the observed submm Faraday rotation from Sgr A* (Marrone et al. 2007; Ressler et al. 2018).

We use the version of harmpi with a separate electron fluid as implemented by Ressler et al. (2015). Magnetic and...
Table 1. Parameters and convergence criteria of GRMHD simulations averaged over $8 \times 10^3 r_g/c$.

| magnetic field | spin parameter | $\phi_{BH}$ | $\langle b_r b_r \rangle / \langle b_r b_\phi \rangle$ | $\langle Q^{(\theta)} \rangle$ | $\langle Q^{(\phi)} \rangle$ | $\langle \beta \rangle$ | $r_{eq} (r_g)$ |
|---------------|----------------|------------|--------------------------|----------------|----------------|----------------|---------------|
| SANE          | 0              | 7.3        | 0.14                     | 16.3          | 23.7          | 19.9          | 19.4          |
| SANE          | 0.5            | 7.9        | 0.16                     | 16.3          | 21.4          | 26.7          | 17.8          |
| SANE          | 0.9375         | 8.6        | 0.19                     | 19.6          | 24.4          | 27.1          | 19.8          |
| MAD           | 0              | 49.7       | 0.21                     | 55.6          | 56.4          | 7.9           | 31.1          |
| MAD           | 0.5            | 72.4       | 0.34                     | 87.9          | 67.0          | 8.0           | 25.6          |
| MAD           | 0.9375         | 56.5       | 0.41                     | 138.1         | 95.7          | 5.8           | 34.6          |

Table 2. Time evolution of convergence criteria, magnetic field strength, and inflow equilibrium radius for our long duration SANE $a = 0$ simulation.

| time ($10^3 r_g/c$) | $\phi_{BH}$ | $\langle b_r b_r \rangle / \langle b_r b_\phi \rangle$ | $\langle Q^{(\theta)} \rangle$ | $\langle Q^{(\phi)} \rangle$ | $\langle \beta \rangle$ | $r_{eq} (r_g)$ |
|---------------------|------------|--------------------------|----------------|----------------|----------------|---------------|
| 5–10                | 7.9        | 0.13                     | 15.2          | 21.5          | 23.0          | 17.8          |
| 10–25               | 6.0        | 0.16                     | 18.7          | 24.8          | 21.1          | 28.8          |
| 25–50               | 6.7        | 0.17                     | 22.6          | 29.8          | 17.8          | 41.0          |
| 50–100              | 8.0        | 0.18                     | 26.8          | 35.3          | 14.5          | 61.3          |
| 100–200             | 12.8       | 0.19                     | 30.1          | 38.8          | 13.3          | 89.0          |

Table 3. Time evolution of convergence criteria, magnetic field strength, and inflow equilibrium radius for our long duration MAD $a = 0.9375$ simulation.

| time ($10^3 r_g/c$) | $\phi_{BH}$ | $\langle b_r b_r \rangle / \langle b_r b_\phi \rangle$ | $\langle Q^{(\theta)} \rangle$ | $\langle Q^{(\phi)} \rangle$ | $\langle \beta \rangle$ | $r_{eq} (r_g)$ |
|---------------------|------------|--------------------------|----------------|----------------|----------------|---------------|
| 5–10                | 61.4       | 0.37                     | 112.5         | 91.3          | 6.4           | 13.4          |
| 10–20               | 59.2       | 0.37                     | 111.4         | 92.6          | 6.5           | 30.0          |
| 20–30               | 62.7       | 0.38                     | 125.9         | 100.2         | 7.8           | 52.5          |
| 30–40               | 62.7       | 0.34                     | 112.7         | 96.1          | 8.2           | 66.7          |
| 40–60               | 62.6       | 0.40                     | 141.8         | 103.9         | 7.5           | 65.9          |

Figure 1. Accretion rate and dimensionless magnetic flux on the black hole event horizon $\phi_{BH}$ as a function of time for the simulations used here. The SANE and MAD limits are reached as intended, with low and saturated dimensionless magnetic flux accumulated on the black hole. In the SANE case, the accretion rate remains steady or rises as the large initial torus drains. In the MAD case, strong magnetic fields lead to rapid accretion of the torus.
kinetic energy dissipated at the grid scale is recaptured as internal energy. A fraction is also assigned to a separate electron internal energy. This electron internal energy is evolved independently of the fluid dynamics, which allows us to incorporate multiple electron heating prescriptions within a single simulation. Some error is introduced in this approximation, since it effectively assumes that the electron and proton adiabatic indices are the same (e.g., see Sadowski et al. 2017; Ressler et al. 2017).

The electron heating mechanism is uncertain, as is its sub-grid implementation. We explore a total of 4 prescriptions based on 2 physical scenarios. We use fitting formulae derived from gyrokinetic linear theory (H10) and numerical simulations (K19) of heating in a turbulent cascade, as well as fitting formulae derived from particle-in-cell simulations of electron heating in magnetic reconnection (R17, W18).

We assess the convergence of our simulations using criteria from the literature (Hawley et al. 2011; Shiokawa et al. 2012; Hawley et al. 2013). We define the 1D radial, density-weighted profile of a quantity X as:

$$\langle X \rangle = \int \frac{d\theta d\phi \sqrt{-g} \rho X}{d\theta d\phi \sqrt{-g} \rho},$$

(1)

where \(\rho\) is the fluid mass density and \(\sqrt{-g}\) is the Jacobian. We define the shell-averaged plasma \(\beta = p_{\text{gas}}/p_B\) as,

$$\langle \beta \rangle = 8\pi \frac{\langle p_{\text{gas}} \rangle}{(b^2)}.$$

(2)

We evaluate simulation resolution quality using \(Q\) values (Hawley et al. 2011) calculated in the locally non-rotating frame (LNRF), \(Q^{(i)} = \lambda^{(i)}_{\text{MRI}}/\Delta x^{(i)}\) (Porth et al. 2019), where \(\lambda_{\text{MRI}}\) is the fastest growing MRI wavelength,

$$\lambda^{(i)}_{\text{MRI}} = \frac{2\pi b^i \mu^{(i)}}{\sqrt{\rho + \gamma u + b^2}}.$$

(3)

\(u\) is the fluid internal energy and \(\gamma\) is the adiabatic index. The tetrad vectors \(e^{(i)}_\mu\) describe the transformation to the LNRF (e.g., Takahashi 2008), and \(\Delta x^{(i)} = \Delta x^{(i)}_\mu e^{(i)}_\mu\) is the LNRF grid cell spacing. Finally we report on the relative strength of the radial and azimuthal field, here taken to be Kerr-Schild coordinate values of \(b_r b^r/b\phi b^\phi\).

3 RADIATIVE MODELS OF SGR A*

From each simulation and electron heating prescription, we compute radiative models of Sgr A*. The electron temperature is taken directly from the GRMHD electron internal energy density. We then scale the mass density until the
Figure 4. Radial profiles of density scale height and plasma $\beta$ averaged over early (red) to late (blue) time intervals for the long duration SANE (top) and MAD (bottom) simulations used here. Over time the average scale height and magnetization increase, at all radii for SANE simulations and at larger radii for MAD simulations.

median observed flux density at 230 GHz is $\simeq 3$ Jy (e.g., Dexter et al. 2014; Bower et al. 2015). We exclude emission from regions where $\sigma = b^2/\rho > 1$.

Observables are calculated using a ray tracing method with the public code grtrans (Dexter & Agol 2009; Dexter 2016). We follow Kerr photon geodesics corresponding to uniformly sampled camera pixels of a distant observer at a viewing orientation of $i = 25$, 45, and 60 degrees. The image resolution is 192 pixels over a $42 r_g$ (210 $\mu$as) field of view. Rays are sampled evenly in $1/r$ from an outer boundary $r_{\text{out}}$ until they either reach the black hole event horizon or return to $r_{\text{out}}$. Near radial turning points, we switch to sampling evenly in $\cos \theta$ to avoid taking large steps. Along each ray, we solve the full polarized radiative transfer equations for synchrotron emission and absorption and Faraday rotation and conversion, assuming a purely thermal electron distribution function. Coefficients are taken from Dexter (2016), with the Faraday rotation coefficient $\rho_V$ modified to correctly reproduce the non-relativistic limit (see appendix A). This is important for calculations of the Faraday rotation measure through the extended torus where the dimensionless electron temperature $\theta_e = kT_e/mc^2 \lesssim 1$. We do not include inverse Compton scattering, which allows comparisons to the observed X-ray luminosity (e.g., Dolence et al. 2009; Moscibrodzka et al. 2009). We calculate images at radio to NIR frequencies for all snapshots from $(5-10) \times 10^3 r_g/c$ spaced by $10 r_g/c$. Here we focus on results for time-averaged observables and their rms variability.

4 OBSERVATIONAL CONSTRAINTS

We compare our models to observational constraints derived from millimeter to NIR observations of Sgr A*.

4.1 Spectrum and variability

First, we consider the shape of the submm to NIR total intensity spectrum (e.g., Falcke et al. 1998; Bower et al. 2015; Stone et al. 2016; von Fellenberg et al. 2018; Bower et al. 2019; Schödel et al. 2011; Dodds-Eden et al. 2011; Witzel et al. 2018) and the rms variability fraction as a function
Figure 5. Snapshot azimuthal averages from the SANE $a=0.9375$ simulation of $\rho$, $T_{\text{gas}}$ (left, in units of $m_p c^2/k$), and $T_e$ for the four electron heating schemes (middle and right panels, H10, K19, W18, R17, in units of $m_e c^2/k$). The H10/K19 (“turbulence”) and W18/R17 (“reconnection”) pairs show similar behavior. The turbulence models heat electrons significantly only in polar jet regions where the magnetization is high, while the reconnection models also substantially heat the dense accretion flow near the midplane.

Figure 6. Snapshot azimuthal averages from the MAD $a=0.9375$ simulation of $\rho$, $T_{\text{gas}}$, and $T_e$ for the 4 electron heating schemes (H10, K19, W18, R17). Again the turbulence and reconnection pairs show very similar behavior. Here, there is strong electron heating near the midplane in both scenarios due to regions of high magnetization in the MAD accretion flow.

Quantitatively, we require a submm spectral index between 230 and 690 GHz of $-0.35$ to $-0.25$ (Marrone 2006; Bower et al. 2019) and an upper limit to the median NIR flux density of $< 1.4$ mJy (Dodds-Eden et al. 2011; Witzel et al. 2018, submitted), and a 230 GHz total flux density rms of 20 – 40% for a model to be considered viable.

4.2 86 and 230 GHz source sizes and NIR centroid motion

We also consider constraints on the image size at 86 and 230 GHz (Krichbaum et al. 1998; Doeleman et al. 2008; Fish et al. 2011; Johnson et al. 2018; Issaoun et al. 2019). We adopt semi-major axis size constraints of $a_{86} = 86 - 154 \, \mu\text{as}$ (Issaoun et al. 2019) and $a_{230} = 51 - 63 \, \mu\text{as}$ (Johnson et al. 2018). In the latter case, we have for simplicity assumed that the intrinsic source semi-major axis aligns with that of the interstellar scattering kernel. We further compare our results with the evolution seen in the NIR centroid (Gravity Collaboration et al. 2018b).
4.3 Polarization

We also compare to median submm observed linear and circular polarization fractions (Aitken et al. 2000; Eckart et al. 2006; Trippe et al. 2007; Muñoz et al. 2012; Liu et al. 2016; Bower et al. 2018; Gravity Collaboration et al. 2018b). Specifically, we enforce constraints on the median polarization fractions of LP = 2 – 8% (Bower et al. 2018) and |CP| = 0.5 – 2% (Muñoz et al. 2012).

Sgr A* also shows a dependence of electric vector position angle EVPA ≃ λ2 as expected for Faraday rotation “external” to the emission region (so that the polarized source has its EVPA coherently rotated, e.g. appears point-like on the Faraday screen). The rotation measure is RM ≃ −6 × 102 rad m−2 (Marrone et al. 2007), with a consistent sign in measurements over many years (e.g., Bower et al. 2003; Marrone et al. 2006, 2007; Bower et al. 2018). The RM is thought to result from the extended accretion flow, and has been used to constrain the accretion rate onto the black hole (Marrone et al. 2006). We do not use the RM to select models, since it originates outside of the region of inflow equilibrium in standard GRMHD models. We show that that an RM signature can be generated with approximately the right magnitude in radiative models from our long duration simulations.

5 RESULTS

We discuss accretion flow and convergence properties for our GRMHD simulations, compare their properties in radiative models of Sgr A* to the above observational constraints, and then study the properties of their multi-wavelength images and polarization maps.

5.1 Convergence and accretion flow properties

Our GRMHD simulations reach the MAD and SANE limits as expected. Figure 1 show the accretion rate histories and accumulated dimensionless magnetic flux on the horizon. SANE simulations have a relatively low net magnetic flux, ΦBH ≃ 5 – 10, while the flux saturates in the MAD case at a maximum value of roughly ΦBH ≃ 50–70 (Tchekhovskoy et al. 2011).

Table 1 lists average values of those quantities for all of our simulations. The magnetic flux ΦBH is measured on the horizon, while the Q values and magnetic field tilt angle are averaged over the region r = 6 – 15rg. MAD models are well resolved according to these criteria, while SANE models are more difficult to resolve due to their lower net vertical magnetic flux. Still, we find satisfactory convergence in all cases, generally defined as ⟨Q0⟩⟨Q0⟩ ≥ 200, tilt ≥ 0.15 – 0.20, and ⟨β⟩ ≥ 10 – 20 (radial profiles of β are shown in Figure 2). We define inflow equilibrium following Narayan et al. (2012) as the outermost radius where the elapsed time exceeds a viscous time, t(νr) / r(eq) ≳ 1. Inflow equilibrium at these early times only reaches out to r(eq) ≃ 20rg. This is sufficient to capture the submm to NIR emission (e.g., Mościbrodzka et al. 2009), but complicates our studies of linear polarization and Faraday rotation. All convergence criteria are readily satisfied for MAD simulations. The MAD models show lower equilibrium values of plasma β ≃ 5 – 10 and inflow equilibrium reaches larger radius r(eq) ≃ 25 – 30rg.

In SANE models the scale height is roughly constant in the region of inflow equilibrium (H/R ≃ 0.3 – 0.4), similar to past GRMHD (Shiokawa et al. 2012; Narayan et al. 2012) and recent pseudo-Newtonian (Dhang & Sharma 2019) results. In the MAD case, we find a strong decrease in the inner radii. This is due to magnetic pressure from the strong surrounding magnetosphere (McKinney et al. 2012). The azimuthal velocity profile is nearly Keplerian for SANE models, while MADs are sub-Keplerian in the region of inflow equilibrium due to magnetic pressure support (e.g., McKinney et al. 2012).

5.2 Long duration runs and inflow equilibrium

We have also run sample MAD and SANE simulations to long durations, similar to what was done by Narayan et al. (2012) and White et al. (2018b). Averaged properties of those simulations over various time intervals are listed in Table 2 and Table 3. Our SANE model evolves to a slightly larger scale height, lower median β, higher magnetic tilt angle, and larger Q(θ) and Q(θ) convergence values. Radial profiles averaged over different time intervals are shown in Figure 4. The inflow equilibrium radius r(eq) = 90rg reached is similar to that from Narayan et al. (2012) in the same elapsed coordinate time of 2 × 105rg/c. Our MAD model is run for much shorter (≈ 6 × 105rg/c) but nonetheless reaches inflow equilibrium out to r(eq) = 70rg.

5.3 Electron heating

Azimuthally averaged snapshots of electron temperature T_e are shown in Figure 5 and Figure 6. We find similar behavior for the various electron heating models as in previous work for SANE (Ressler et al. 2015, 2017; Chael et al. 2018; Ryan et al. 2018) and MAD (Chael et al. 2019) simulations. In the SANE case, the dense accretion flow near the midplane has relatively large plasma β ≃ 10 – 20. The electrons there are only significantly heated for the reconnection models. In the turbulent case, electrons in the disk body remains cold. For MAD solutions, the average plasma β is lower and electrons are heated efficiently everywhere in both scenarios. We find mild spin dependence, in the sense that for higher black hole spin the fluid and in turn the electrons have higher energies. More details on the time evolution and convergence of the electron heating solutions are provided in appendix B.

5.4 Spectrum and variability

Sample spectra are shown in Figure 7 for the four general classes of model considered. The spectra show median values over time at each frequency. The black hole spin is fixed at a = 0.5 and the inclination at i = 45°. We compare SANE (blue) with MAD (black), and turbulence (H10) and reconnection (W18) cases. Data are taken from the references listed in subsection 4.1. The combinations of SANE/reconnection (blue dashed) generically underproduce the observed THz emission from Sgr A*. They do not produce sufficiently hot electrons, and therefore show steep spectral breaks which are ruled out by recent Herschel and
Figure 7. Median spectra from sample SANE (blue) and MAD (black) models with $a = 0.5$ and $i = 45^\circ$ and the H10 and W18 electron heating models compared to Sgr A* mm to NIR data. The SANE/H10 and MAD/W18 models are consistent with the observed spectral shape. The SANE/W18 model does not produce sufficiently hot electrons and can’t explain the broad submm peak in Sgr A*. The MAD/H10 model produces too many hot electrons. It fails to match the submm peak and overproduces the observed NIR emission.

Figure 8. Linear polarization fraction (left) and rms variability amplitude (right) for sample SANE (blue) and MAD (black) simulations with $a = 0.5$ and $i = 45^\circ$ for the H10 and W18 electron heating models compared to Sgr A* mm to NIR data. The SANE/H10 (disk-jet) model is heavily depolarized as a result of Faraday rotation in the emission region. The MAD/W18 model is consistent with both constraints.

ALMA data. The combinations of MAD/turbulence by contrast strongly heat electrons to the degree that the spectral energy distribution (SED) peaks in the infrared. These models strongly overproduce the median NIR while underproducing the submm emission. The other two combinations, SANE/turbulence and MAD/reconnection, can match the Sgr A* SED shape at least for some combinations of inclination angle and black hole spin. The same general results hold for our full parameter survey. In particular, we have not found combinations of parameters for SANE/reconnection or MAD/turbulence models which reproduce the submm spectral index and NIR flux density upper limit.

Most models we consider produce submm variability consistent with that observed from Sgr A*. The SANE/turbulence and MAD/reconnection models show highly variable NIR emission, which can in principle account...
for the flaring emission seen from Sgr A*. In those cases, the frequency-dependent rms (Figure 8) is often in fairly good agreement with that observed, rising from the radio through the submm and NIR. A more detailed comparison to the NIR flux distribution is forthcoming (GRAVITY collaboration 2020, in prep).

The detailed results for submm spectral index and median NIR flux density are further separated into our 6 simulations (MAD/SANE at 3 values of black hole spin) and 3 electron heating models (H10/W18/R17) in Figure 9. The 3 points at each x-axis location correspond to 3 values of observer inclination (i = 25°, 45°, 60°). The gray bands show our allowed ranges for Sgr A*. The general trends from our chosen α = 0.5 models can be found there, but also with systematic trends of higher submm spectral index and median NIR flux density with increasing black hole spin and observer inclination angle. Both effects result in part from increased Doppler beaming at higher inclination.

The SANE/turbulence and MAD/reconnection models studied produce interesting NIR flaring events (large rms variability in Figure 9). MAD models show associated time-variable polarization and image photocenter (centroid) motion. Typical linear polarization fractions are ≳ 10 – 30%, consistent with observations of NIR flares from Sgr A* (Eckart et al. 2006; Trippe et al. 2007; Gravity Collaboration et al. 2018b). The rms centroid motion during flares is only ≳ 10μas, a factor of 2 – 3 smaller than seen from Sgr A*. Given the uncertainty in electron heating and distribution function, we nonetheless consider MAD accretion flows as promising for explaining the NIR (and X-ray) flares from Sgr A*.

5.5 86 and 230 GHz image sizes

Sample snapshot images and polarization maps corresponding to those same 4 model combinations are shown in Figure 10. The SANE/turbulence model shows a jet-like, elongated 86 GHz image, while the others are dominated by emission from the dense inflow in the midplane. The SANE/reconnection images have much higher optical depth and a larger source size, particularly at 86 GHz.

Figure 11 shows semi-major (blue) and semi-minor (orange, 86 GHz only) axis sizes for each model considered. Except when viewed at low inclination, SANE/turbulence models are disfavored due to their small semi-minor axis size (large axis ratio). Most other model combinations can satisfy both the 86 and 230 GHz semi-major axis size constraints.

We generally find increasing 86 GHz size with decreasing emission region electron temperature in the order H10, W18, R17. The trend is stronger in SANE than in MAD models. At 230 GHz, the emission region size systematically decreases with black hole spin for the reconnection heating models (W18, R17) although all spins considered can produce median values falling within the allowed range.

We also note that all models produce 86 GHz sizes on the smaller end of the allowed range. In particular, all sizes found are too small when compared to the model-fitting results of Johnson et al. (2018) at 86 GHz. To be viable, many or all models may require an additional non-thermal component to the electron distribution function which can produce mm-wavelength emission further from the black hole.

5.6 Polarization

The snapshot polarization maps show that all SANE models explored are highly depolarized as a result of strong Faraday rotation internal to the emission region. We also find that the outer torus (r ≳ 20rg) can significantly alter the linear polarization degree and polarization map structure for SANE models. That region is not in inflow equilibrium, and we omit it from the radiative transfer calculations presented here. Even excluding that material altogether, few SANE models can match the median observed linear polarization fraction of Sgr A*. Our MAD polarization maps frequently show signatures of strong poloidal field, leading to azimuthal EVPA structure (e.g., Gravity Collaboration et al. 2018b).

Figure 8 shows the median image-integrated linear po-
Figure 10. Sample snapshot 86 (left) and 230 (right) GHz linearly scaled false color images and polarization maps for $a = 0.5$ and $i = 45^\circ$. Polarization tick length is proportional to polarized flux. The models are ordered from top to bottom as SANE/H10, SANE/W18, MAD/H10, MAD/W18. All models at 230 GHz show a characteristic crescent morphology from the combination of Doppler beaming and light bending. The SANE/H10 model shows a “disk-jet” structure with prominent polar emission from the jet wall at 86 GHz. In the other cases, the emission is predominantly from close to the midplane. All models are substantially depolarized from Faraday rotation at 86 GHz, and the SANE models are also depolarized at 230 GHz. Images of the K19 and R17 models are similar to those of the H10 and W18 models, respectively.
Figure 11. Model image sizes at 86 (left) and 230 (right) GHz. The image second moments are shown along semi-major (blue) and semi-minor (orange, 86 GHz only) axes, multiplied by a factor of 2.35 for comparison with the Gaussian FWHM sizes reported in the literature (gray bands). In each model column, the three points correspond to three viewing inclinations and the error bars correspond to the rms scatter over time. Many models can satisfy both constraints, although they are generally small compared with the 86 GHz size. SANE/H10 models are too elliptical at 86 GHz for high viewing inclinations, and MAD/H10 models are too large at 230 GHz.

Figure 12. Model 230 GHz linear polarization fractions (left) and rms variability amplitudes (right). All SANE models except $a = 0.9375$ W18 show low linear polarization fractions as a result of Faraday rotation internal to the emission region. MAD models by contrast are frequently consistent with the range of median 230 GHz linear polarization seen from Sgr A*. All scenarios considered here can show 230 GHz variability with an rms amplitude $\sim 20 - 40\%$, consistent with that observed. The variability is the result of turbulence driven by the MRI.

The polarization fraction for our 4 sample models. The SANE models, and particularly SANE/turbulence, are too depolarized compared to observations of Sgr A*. MAD/reconnection models capture the frequency-dependent polarization fraction fairly well. In detail (Figure 12), MAD models show highly variable LP within the observed range of Sgr A*. The polarization fraction is lowest for the R17 model, and slightly higher at higher black hole spin values. Our SANE models are too depolarized to explain the relatively large 230 GHz LP from Sgr A*, except the W18 model at high spin. The depolarization is the result of Faraday rotation near the dense accretion flow midplane.

In all cases, the polarized emission is more time variable than the total intensity. This is due to a variable EVPA pattern over the images resulting from turbulence and/or Faraday rotation. Time-variable beam depolarization then drives large variability of the integrated Stokes parameters. One way to see this is to note that the integrated polarized flux $\sqrt{Q^2 + U^2}$ from each image pixel shows similar variability as in Stokes I, while both are much less variable than the net polarization integrated over images. This finding agrees with Sgr A* submm polarization observations (e.g., Marrone et al. 2006; Bower et al. 2018). By contrast, the NIR polarization degree seems much less variable (Eckart et al. 2008; Shahzamanian et al. 2015), likely as a result of a more compact emission region and negligible Faraday rotation at high frequency ($\nu / \nu_c \gtrsim 10^3$ with $\nu_c \sim T_{\text{e}}^2 B$ the critical synchrotron frequency).
Using our long duration SANE model with $a = 0.9375$ model with an inflow equilibrium radius of $r_{eq} \approx 50r_g$ ($t \approx 2.7 \times 10^5 r_g/c$) and a SANE $a = 0$ model with $r_{eq} \approx 100r_g$ ($t \approx 2 \times 10^5 r_g/c$). Both models are viewed at an inclination $i = 60^\circ$. The EVPA shows a clear linear trend with $\lambda^2$, particularly at longer wavelengths. We infer Faraday rotation measurements of $\approx 6 \times 10^5$ rad m$^{-2}$ (MAD) and $7 \times 10^6$ rad m$^{-2}$ (SANE). The typical MAD values are in good agreement with the observed Faraday rotation measurement of Sgr A* (e.g., Bower et al. 2003; Marrone et al. 2006, 2007; Bower et al. 2018). The external RM decreases to $\lesssim 10^5$ rad m$^{-2}$ for $i = 25^\circ$. We also find rapid changes in the sign of the RM at high inclination, while low inclinations can show a persistent sign (although our longest simulation only spans a few weeks for Sgr A*). Studies of the RM time variability, departures from $\lambda^2$, and spatially resolved maps are left to future work. Small RM values for the SANE long duration models are possible even when there is strong depolarization due to Faraday rotation internal to the emission region. For more details see appendix C.

5.7 Summary of comparison with observational constraints

Table 4 provides median observed and model values of the quantitative constraints used here: the submm spectral index, NIR flux density, 230 GHz LP and CP, image semi-major axis size at 230 and 86 GHz, the 230 GHz rms variability fraction. Comparing to the set of observed ranges, we produce a final pass/fail score for each model. Italicized entries indicate where models fail to match Sgr A* data.

Several MAD models can match all constraints considered, and several others fail in only one category. They also produce highly variable NIR emission, similar to that observed. While we have not exhaustively explored the parameter space of either the simulations or the radiative transfer models, we expect the results to hold for other viewing angles and black hole spin values. All SANE models are ruled out by multiple constraints. Matching the spectral shape with only thermal electrons requires disk-jet models where the jet wall electrons are heated and the accretion flow is cold. Those models are too strongly depolarized to explain the measured submm linear and circular polarization of Sgr A*.

5.8 Faraday rotation in long duration runs

Using our long duration SANE $a = 0$ model, we have also explored Faraday rotation and depolarization once inflow equilibrium has reached large radius $r \approx 100r_g$. For the turbulent electron heating, there is little difference. The accretion flow remains cold, and the emission from close to the black hole is depolarized to a maximum of $1 \sim 2\%$. For the reconnection heating models, at late times the large-scale accretion flow

can substantially heat. This increases the observed polarization fraction to values consistent with Sgr A* data.

For both long duration SANE and MAD models, we also find behavior consistent with external Faraday rotation, where the EVPA $\propto \lambda^2$ over a range of frequencies $\approx 200 \sim 300$ GHz. Figure 13 shows example fits to a MAD $a = 0$ model with an inflow equilibrium radius of $r_{eq} \approx 50r_g$ ($t \approx 2.7 \times 10^5 r_g/c$) and a SANE $a = 0$ model with $r_{eq} \approx 100r_g$ ($t \approx 2 \times 10^5 r_g/c$). Both models are viewed at an inclination $i = 60^\circ$. The EVPA shows a clear linear trend with $\lambda^2$, particularly at longer wavelength. We infer Faraday rotation measurements of $\approx 6 \times 10^5$ rad m$^{-2}$ (MAD) and $7 \times 10^6$ rad m$^{-2}$ (SANE). The typical MAD values are in good agreement with the observed Faraday rotation measurement of Sgr A* (e.g., Bower et al. 2003; Marrone et al. 2006, 2007; Bower et al. 2018). The external RM decreases to $\lesssim 10^5$ rad m$^{-2}$ for $i = 25^\circ$. We also find rapid changes in the sign of the RM at high inclination, while low inclinations can show a persistent sign (although our longest simulation only spans a few weeks for Sgr A*). Studies of the RM time variability, departures from $\lambda^2$, and spatially resolved maps are left to future work. Small RM values for the SANE long duration models are possible even when there is strong depolarization due to Faraday rotation internal to the emission region. For more details see appendix C.

6 DISCUSSION

We have carried out a parameter space survey of Sgr A* models using ray tracing radiative transfer calculations based on GRMHD simulation data output. We consider both low (SANE) and saturated (MAD) magnetic flux limits, and sub-grid prescriptions for dividing dissipated energy at the grid scale between electrons and protons.

Both prescriptions assign more dissipated heat to electrons in strongly magnetized regions, resulting in higher jet wall (outflow) than accretion flow (inflow) electron temperatures. The electron temperature contrast between jet wall and disk body is larger in the SANE case and for the turbulent heating prescription. This results in a relatively cold accretion flow ($\theta_e \lesssim 1$ for $r \gtrsim 30r_g$). For strongly magnetized MAD models, there are hot electrons everywhere. Our results for electron heating are consistent with those from recent work (Ressler et al. 2015, 2017; Chael et al. 2018, 2019).

We find two general paradigms for successfully reproducing the observed Sgr A* submm to NIR time-variable spectrum (including NIR flares), as well as 86 and 230 GHz resolved image sizes. Those combinations are SANE simulations with turbulent electron heating, and MAD simulations with reconnection electron heating. Other combinations underproduce or overproduce hot electrons for the black hole spin and inclination angles tried. The SANE/turbulence models have been proposed previously and result in a “disk-jet” morphology where jet emission becomes prominent at longer 3mm and 7mm wavelengths (Moscibrodzka et al. 2014; Ressler et al. 2017; Chael et al. 2018).

We have also calculated submm to NIR polarized images and spectra from our models. We find that all SANE/turbulence models are strongly depolarized due to Faraday rotation in the cold disk body, and cannot ex-
plain the net linear polarization observed from Sgr A* at 1.3mm. The SANE/turbulence models also show elongated
morphologies at 86 GHz due to extended jet structure, inconsistent with recent measurements except at low inclination (as found previously by Issaoun et al. 2019).

MAD/reconnection models can explain the highly time-variable net linear polarization of Sgr A* in the submm, and are only mildly depolarized. As a result, we favor MAD/reconnection models of Sgr A*. Within those models, the moderate black hole spin of $a = 0.5$ is more successful

| model          | $i$ (°) | $\alpha_{\text{submm}}$ | log $F_{\text{NIR}}$ (mJy) | LP | $|\text{CP}|$ | $a_{20} \ (\mu\text{as})$ | $a_{90} \ (\mu\text{as})$ | rms | summary |
|----------------|--------|------------------------|-----------------------------|----|-------------|---------------------|---------------------|-----|---------|
| SANE a=0.0 H10| 25     | -0.55                  | -0.7                         | 0.008 | 0.002 | 65.2 | 85.5 | 0.33 | fail    |
| SANE a=0.0 W18| 25     | -1.46                  | -5.6                         | 0.003 | -0.004 | 71.8 | 113.2 | 0.36 | fail    |
| SANE a=0.0 R17| 25     | -0.81                  | -4.1                         | 0.001 | -0.001 | 80.9 | 121.6 | 0.25 | fail    |
| SANE a=0.5 H10| 25     | -0.38                  | -0.0                         | 0.008 | 0.002 | 61.0 | 86.8 | 0.38 | fail    |
| SANE a=0.5 W18| 25     | -1.35                  | -5.4                         | 0.017 | -0.010 | 63.9 | 104.0 | 0.35 | fail    |
| SANE a=0.5 R17| 25     | -0.83                  | -4.5                         | 0.004 | -0.007 | 56.2 | 99.2 | 0.30 | fail    |
| SANE a=0.9375 H10| 25    | -0.43                  | -3.8                         | 0.003 | -0.005 | 51.3 | 94.4 | 0.27 | fail    |
| SANE a=0.9375 W18| 25   | -0.43                  | -4.6                         | 0.001 | -0.006 | 68.9 | 114.1 | 0.25 | fail    |
| SANE a=0.9375 R17| 25  | -0.34                  | -4.0                         | 0.001 | 0.001 | 62.6 | 109.6 | 0.23 | fail    |
| MAD a=0.0 H10 | 25     | 0.07                   | 1.2                          | 0.011 | 0.000 | 59.8 | 92.0 | 0.37 | fail    |
| MAD a=0.9375 W18| 25 | 0.44                   | 1.7                          | 0.003 | -0.000 | 61.9 | 102.6 | 0.34 | fail    |
| MAD a=0.9375 R17| 25  | 0.65                   | 2.0                          | 0.003 | -0.001 | 63.6 | 108.3 | 0.33 | fail    |
| MAD a=0.0 W18 | 25     | -0.47                  | 0.9                           | 0.041 | 0.007 | 78.4 | 100.6 | 0.12 | fail    |
| MAD a=0.9375 R17| 25  | -0.43                  | 1.0                           | 0.043 | 0.006 | 77.6 | 101.4 | 0.13 | fail    |
| MAD a=0.0 R17 | 25     | -0.56                  | -0.4                         | 0.043 | 0.011 | 62.8 | 91.6 | 0.24 | fail    |
| MAD a=0.5 H10 | 25     | -0.45                  | -0.3                         | 0.059 | 0.010 | 61.4 | 92.9 | 0.23 | fail    |
| MAD a=0.5 W18 | 25     | -0.43                  | -0.1                         | 0.031 | 0.018 | 63.6 | 96.1 | 0.18 | fail    |
| MAD a=0.5 R17 | 25     | -0.32                  | -0.1                         | 0.024 | 0.013 | 64.2 | 101.0 | 0.17 | fail    |
| MAD a=0.0 H10 | 25     | -0.20                  | 0.1                           | 0.007 | 0.009 | 63.9 | 102.3 | 0.18 | fail    |
| MAD a=0.5 H10 | 25     | -0.42                  | 1.2                           | 0.041 | 0.003 | 75.9 | 98.9 | 0.29 | fail    |
| MAD a=0.5 W18 | 25     | -0.37                  | 1.3                           | 0.058 | 0.003 | 72.2 | 96.8 | 0.28 | fail    |
| MAD a=0.5 R17 | 25     | -0.33                  | 1.4                           | 0.075 | 0.002 | 70.4 | 96.0 | 0.27 | fail    |
| MAD a=0.9375 H10| 25  | -0.45                  | 0.1                           | 0.040 | 0.006 | 60.1 | 90.4 | 0.33 | fail    |
| MAD a=0.9375 W18| 25  | -0.42                  | 0.2                           | 0.071 | 0.008 | 57.7 | 89.4 | 0.31 | fail    |
| MAD a=0.9375 R17| 25  | -0.34                  | 0.6                           | 0.039 | 0.012 | 56.0 | 86.4 | 0.33 | fail    |

Table 4. Comparison of the allowed ranges of various observational constraints considered with median values calculated for each model and a final pass/fail score.
at matching the observed properties than the high spin of \( a = 0.9375 \). At high black hole spin, the models tend to over-produce hot electrons, resulting in too much NIR emission relative to that in the submm. They are also too compact at 86 GHz.

### 6.1 Comparison to past work

Many groups have done similar studies in the last several years. Ressler et al. (2015) introduced the method used here for evolving multiple electron temperatures with sub-grid heating prescriptions. This method has been used in recent work on Sgr A* (Ressler et al. 2017; Chael et al. 2018) and radiation GRMHD models of M87 (Ryan et al. 2018; Chael et al. 2019). Compared to those studies, we have considered a larger parameter space with multiple electron heating models, varying black hole spin, and considering both MAD and SANE solutions.

Our SANE/reconnection models are similar to early GRMHD models (Noble et al. 2007; Moscibrodzka et al. 2009; Dexter et al. 2009, 2010; Drappeau et al. 2013) based on the assumption of constant proton-electron temperature ratio \( T_p/T_e \) everywhere. This behavior is also seen in R17 models at low and high spin from Chael et al. (2018). Our SANE/turbulence models are similar to “disk-jet” models realized by assuming highly magnetized regions receive more heat in post-processing (Moscibrodzka & Falcke 2013; Moscibrodzka et al. 2014; Chan et al. 2015a) or using self-consistent electron heating with the H10 model (Ressler et al. 2017). We have shown that such models, while otherwise promising, are highly depolarized as the result of Faraday rotation internal to the emission region (all inclination angles). Additionally they show very high Faraday rotation measure unless viewed at low inclination (here \( i = 25^\circ \)).

Previous MAD models of Sgr A* (Shcherbakov & McKinney 2013; Gold et al. 2017) have used different post-processing electron heating prescriptions. Some of those models are at least broadly consistent with the spatially resolved submm polarization of Sgr A* (Johnson et al. 2015).

We seem to find more coherent submm polarization maps than in those studies.

Average properties of our radiative models are given in Table 5. The accretion rate \( M \) is measured at the event horizon, while the others are intensity-weighted averages taken along each ray and over each pixel of the 230 GHz images. Viable models we identify have \( M = (1.0 - 2.1) \times 10^{-8} M_\odot \text{yr}^{-1} \), resulting in low radiative efficiencies \( \lesssim 0.1 \). The plasma parameters generally agree with one zone estimates (e.g., von Fellenberg et al. 2018; Bower et al. 2019) and physical conditions in previous analytic (e.g., Falcke & Markoff 2000; Özel et al. 2000; Yuan et al. 2003) and GRMHD (e.g., Moscibrodzka et al. 2009; Dexter et al. 2010) models.

### 6.2 Future measurements

The 230 GHz image polarization is primarily the result of relativistic effects of Doppler beaming and light bending, and does not depend strongly on the details of the magnetic field, black hole spin, or electron model (e.g., Kamruddin & Dexter 2013) when the emission region is optically thin.

Polarization encodes plasma and magnetic field properties (e.g., Shcherbakov et al. 2012; Dexter 2016; Moscibrodzka et al. 2017; Jiménez-Rosales & Dexter 2018). For the viable MAD models studied here, we find fairly coherent, azimuthal (“twisty”) EVPAs (Figure 10). The structure is due to significant poloidal magnetic field near the event horizon. Radial magnetic field and light bending both produce azimuthal EVPAs, which is balanced by vertical magnetic field which results in a preferred horizontal EVPAs (Gravity Collaboration et al. 2018b). The azimuthal EVPAs maps are far more evident at 345 GHz (or potentially in NIR flares) than at 230 GHz where external Faraday rotation can coherently rotate the EVPAs by \( \approx 20 - 40 \) degrees. Faraday rotation internal to the emission region produces additional disorder, but does not strongly depolarize the source or scramble the EVPAs map.

We have also studied the image-integrated polarization angle as a function of wavelength for the models run to late.
Figure 14. Sample snapshot 345 GHz linearly scaled false color images and linear polarization maps for $a = 0.5$ and $i = 45^\circ$. Polarization tick length is proportional to polarized flux. At 345 GHz, the MAD polarization maps show negligible scrambling from Faraday rotation. Instead, the polarization maps trace the underlying magnetic field configuration.

6.3 Limitations

The parameter survey presented here is both sparsely sampled and incomplete. We have used simplistic analytic models for sub-grid electron heating, based on a small number of recent kinetics calculations. As those calculations improve, so will the predictive power of our radiative models. In general, we expect that successful models of Sgr A* will need relatively high electron temperatures in the disk body, to avoid significant depolarization of the submm radiation. Within the prescriptions tried, that disfavors SANE/turbulence models. Successful models of the submm spectrum also require a range of electron temperatures, which disfavors the SANE/reconnection scenario. This qualitative understanding can be applied to future heating prescriptions as well.

MAD accretion flows produce regions of high magnetization throughout the simulation domain, where truncation errors can lead to negative internal energy which are corrected by imposing numerical floors. The convergence properties of MADs have been explored (White et al. 2019a), but remain less well understood than for the SANE case. For calculating radiative models of MADs, the treatment of high magnetization regions plays an important role (Chael et al. 2019). Here we follow Event Horizon Telescope Collaboration et al. (2019) and exclude emission from regions where $\sigma > 1$. That choice is arbitrary and for MAD models effects the NIR flux density (see appendix D).

We have also assumed a thermal electron distribution function, while non-thermal emission can broaden the submm spectrum (Özel et al. 2000; Yuan et al. 2003; Broderick & Loeb 2009) and increase the image size (Özel et al. 2000; Mao et al. 2017; Chael et al. 2017; Davelaar et al. 2018). In particular, it may be possible to find viable
SANE/reconnection models when non-thermal electrons are included. Their inclusion is also promising for comparing theoretical models with X-ray flare data (e.g., Ball et al. 2016). We have also assumed an accretion flow angular momentum axis aligned with that of the black hole spin. This may be a poor assumption in the Galactic center. The orientation of the stellar winds providing the extremely low accretion rate (Quataert 2004; Cuadra et al. 2006; Ressler et al. 2018) is unlikely to align with those of earlier accretion episodes. Disk tilt can change both the image morphology and spectrum (Dexter & Fragile 2013; White et al. 2020a; Chatterjee et al. 2020) as a result of shock heating (Frail & Blaes 2008; White et al. 2019b). We also neglect radiative cooling, which has found to be unimportant for the low accretion rate of Sgr A* (Dibi et al. 2012; Ryan et al. 2018).

With the above caveats, our results favor a strongly magnetized MAD accretion flow in Sgr A*. The resulting magnetic field structure is consistent with that inferred from the combined time-variable polarization and astrometric motions seen in NIF flares (Gravity Collaboration et al. 2018b). The MAD limit is associated with the strongest Poynting flux driven jets from black holes (Tchekhovskoy et al. 2020), and/or the black hole spin is small in magnitude (et al. 2007), and/or the black hole spin is small in magnitude due to significant near horizon poloidal fields and this pattern should be apparent especially at 345 GHz or higher frequency where Faraday rotation becomes negligible; while some of our detections, which led to an improved paper. J.D. and A.J.-R. were supported in part by a Sofja Kovalevskaja award from the Alexander von Humboldt foundation, by a CONACyT/DAAD grant (57255507), and by NASA Astrophysics Theory Program Grant 80NSSC20K0527. S.M.R. was supported by the Gordon and Betty Moore Foundation through Grant GBMF7392. The calculations presented here were carried out on the MPG supercomputers Hydra and Cobra hosted at MPCDF.

7 CONCLUSIONS

We have carried out a large parameter survey of GRMHD models of Sgr A* with self-consistent electron heating. We have considered a range of (prograde) black hole spin, vertical magnetic field strength (weak/SANE or strong/MAD), and sub-grid electron heating models (relatively uniform heating, “reconnection” or strongly \( \beta \)-dependent, “turbulent” heating). We have studied radiative models of the radio to NIR emission from Sgr A* based on our model survey. The main findings are as follows:

- Parameter combinations of magnetic fields and electron heating of SANE/turbulence and MAD/reconnection can explain the mm to NIR SED shape, variability including large-amplitude NIR flares, and mm/submm source sizes;

- SANE/turbulence models are heavily depolarized due to Faraday rotation effects, while some of our MAD/reconnection models remain viable for explaining a wide range of Sgr A* observations;

- MAD models show azimuthal (“twisty”) EVP maps due to significant near horizon poloidal fields and this pattern should be apparent especially at 345 GHz or higher frequency where Faraday rotation becomes negligible;

- limitations include uncertainty in the sub-grid electron heating schemes, the use of thermal electron distribution functions, the treatment of highly magnetized regions, particularly for NIR emission, and an accretion flow aligned with the spin axis of the central black hole.

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APPENDIX A: UPDATED FARADAY ROTATION COEFFICIENT FOR A THERMAL PLASMA

Dexter (2016) presented approximate, analytic forms for synchrotron and Faraday coefficients appropriate for relativistic electrons (θe ≥ 1), including for the case of purely thermal electrons. Their expression for the Faraday rotation coefficient (ρν)\textsubscript{e}, their equation B14) fails at low ν/ν\textsubscript{c} and θ\textsubscript{e} << 1. This can be seen from:

\[ρν = ρν, NR f(θ_e, ν/νc).\] (A1)

where ν\textsubscript{c} = (3eB/4πmc)^2 cos ø\textsubscript{B} is the critical synchrotron frequency and ø\textsubscript{B} is the angle between the emission and magnetic field directions in the fluid frame. The coefficient ρν, NR is the correct non-relativistic limit:

\[ρν, NR = \frac{2ne^3B cos ø\textsubscript{B}}{m^2V^2}.\] (A2)

related for example to the usual Faraday rotation measure. For ν/ν\textsubscript{c} ≫ 1, the function f(θ\textsubscript{e}, ν/ν\textsubscript{c}) = K\textsubscript{θ}(θ\textsubscript{e}^{-1})/K\textsubscript{2}(θ\textsubscript{e}^{-1}) (Shechterbakov 2008), while for general ν/ν\textsubscript{c} and θ\textsubscript{e} ≫ 1, there is an additive correction term (Jones & Hardee 1979) which Dexter (2016) included approximately with a fitting function ∆J\textsubscript{3}(ν/ν\textsubscript{c}):

\[ρν = ρν, NR \left(\frac{K\theta(\theta^{-1}) - ∆J\textsubscript{3}(ν/ν\textsubscript{c})}{K\textsubscript{2}(θ\textsubscript{e}^{-1})}\right).\] (A3)

For cold electrons θ\textsubscript{e} ≪ 1, the modified Bessel function K\textsubscript{θ}(x) reaches an asymptotic limit independent of n. Then the ratio K\textsubscript{θ}/K\textsubscript{2} → 1, correctly reproducing the non-relativistic limit. Adding the ∆J\textsubscript{3} term violates this limit. As a simple fix, in GRTRANS we now multiply the ∆J\textsubscript{3} term by a narrow step function at θ\textsubscript{e} = 1 so that it is suppressed in the non-relativistic limit. We additionally set f(θ\textsubscript{e}, ν/ν\textsubscript{c}) = 1 whenever θ\textsubscript{e} < 10\textsuperscript{-2} to avoid numerical errors in the ratio of modified Bessel functions of large argument. These changes are necessary for accurate calculations of Faraday rotation at larger radii where the electrons can be non-relativistic.

APPENDIX B: ELECTRON TEMPERATURE EVOLUTION AND CONVERGENCE

Our simulations evolve multiple electron energies starting from an initial condition with T\textsubscript{p}/T\textsubscript{e} = 10. The long duration simulations achieve equilibrium electron temperature profiles for roughly the last half of their duration (1–2 × 10\textsuperscript{5}r\textsubscript{g}/c for SANE and 4 – 6 × 10\textsuperscript{4}r\textsubscript{g}/c for MAD). Figure B1 shows shell-averaged radial profiles of the equilibrium electron temperature for the H10 and W18 models (solid lines) for the SANE simulation, compared to the same profiles for the time interval used for calculating the models in our parameter survey (dashed) and the initial condition (dotted). The electrons heat gradually and are too cold by a factor ≃ 1.5 everywhere at early times. For the SANE case at early times, the electron temperature has apparently not relaxed from its initial condition for r ≳ 20r\textsubscript{g}.

We mitigate this effect in our analysis by excluding material outside of 20r\textsubscript{g} in calculating the SANE models, since otherwise the cold electrons Faraday depolarize the emission region. We have also checked that polarization properties excluding r ≳ 20r\textsubscript{g} are similar to the full radiative transfer results from the long duration SANE model at late times, once the electron temperature distribution has converged. The effect of excluding emission (or running to very long durations) is large for the W18 model, where the electrons heat efficiently. It is modest for the H10 model, where the equilibrium temperature profile at those radii turns out to be similar to our assumed initial condition. In particular, Faraday rotation through an accretion flow is generally thought to be dominated by the location where θ\textsubscript{e} = 1 (e.g., Marrone et al. 2006). That location is at r = 30r\textsubscript{g} and 300r\textsubscript{g} for the H10 and W18 models. We also measure this directly in the simulation data (Figure B1), where we find contributions to the RM peaking at ≃ 30r\textsubscript{g} and ≃ 80r\textsubscript{g}. As a result of cold electrons near the midplane, our H10 models show large Faraday rotation measures and significant depolarization from cold electrons at relatively small radii.

APPENDIX C: INTERNAL AND EXTERNAL FARADAY ROTATION

The Faraday rotation measure (RM) is related to the Faraday optical depth τ\textsubscript{ν} = \int dl/ν by τ\textsubscript{ν} = 2RM/λ. For Sgr A*, the observed RM magnitude of 6×10\textsuperscript{5} rad m\textsuperscript{-2} at 230 GHz corresponds to τ\textsubscript{ν} ≲ 1, depending on the frequency bands used. The total EVPA rotation is still < π, insufficient to produce strong Faraday depolarization. We report much larger values of τ\textsubscript{ν} in Table 5, despite the fact that the models can show EVPA ≈ λ\textsuperscript{2} with realistic values of the RM magnitude Figure 13.

The RM is measured from the change in EVPA, which comes from the observed polarized flux. Strongly depolarized regions contribute little polarized flux. As a result, Faraday thick regions internal to the source do not necessarily lead to large RMs. Mościbrodzka et al. (2017) found that in submm models of M87 from SANE GRMHD simulations, the RM measured from the change in EVPA could be roughly constant, even as τ\textsubscript{ν} varied by several orders of magnitude.

Figure C1 shows sample polarization maps for a snapshot of our long duration SANE a = 0 simulation at late times using the H10 electron model and viewed at i = 25°. The full calculation shows a scrambled polarization map due to Faraday rotation internal to the emission region. The RM inferred for this snapshot is only ≃ 3 × 10\textsuperscript{4} rad m\textsuperscript{-2}. When we neglect Faraday rotation by setting ν\textsubscript{F} = 0, the polarization map appears ordered and the inferred RM drops to ≃ 0. The net linear polarization is also much higher when ν\textsubscript{F} = 0, ≃ 12% at 230 GHz compared to ≃ 2% in the full calculation. Evidently the depolarization in SANE H10 models is due to Faraday rotation, even when viewed at low inclination. Internal Faraday rotation can also be strong enough to substantially depolarize the image without showing up as a large RM as inferred by the change of EVPA with frequency.

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APPENDIX D: EFFECT OF EMISSION FROM HIGHLY MAGNETIZED REGIONS

In this work we follow Event Horizon Telescope Collaboration et al. (2019) and neglect emission from all regions where the magnetization $\sigma > 1$. Highly magnetized regions are difficult to evolve accurately in ideal MHD and may have mixed with artificially injected mass and energy (due to “floors”). Ressler et al. (2017) show that this choice makes little difference for SANE models, where most of the fluid is weakly magnetized. Highly magnetized regions are more prevalent in MAD models, and Chael et al. (2019) explored the effects of various cuts on $\sigma$ in their images and spectra of M87.

Figure D1 shows Sgr A* spectra and linear polarization fractions for two sample snapshots, one each from late times in our long duration SANE $\alpha = 0$ and MAD $\alpha = 0.9375$ simulations. We adopt the H10 (SANE) and W18 (MAD) electron heating models since those best describe the Sgr A* spectrum. In the SANE case, we confirm that high $\sigma$ material does not contribute significantly to the radio to NIR emission.

In the MAD case, $\sigma > 1$ plasma produces an increasing fraction of the emission at higher frequencies beyond the THz spectral peak and dominates the radiation in the NIR. Adopting a higher $\sigma$ cutoff value would therefore lead to higher NIR flux densities and slight changes to the submm spectral index. The choice of $\sigma$ cutoff value remains interesting to explore further in future work, but seems unlikely...
to be a major source of uncertainty in the analysis presented here.

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