Antenna Phase Error Compensation for Terahertz Coded-Aperture Imaging

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Abstract: Coded-aperture antenna plays an important role in terahertz coded-aperture imaging radar system. However, the performance of a system is inevitably affected by the phase errors introduced by the coded-aperture antenna elements. In this paper, we propose a phase error compensation method by deducing a formula to compute all element phase errors accurately. According to the formula, the phase errors can be calibrated by using a calibrator and can be used to compensate the imaging model of the system. Numerical simulations demonstrate that the proposed method can effectively improve the imaging quality when the elemental phase error exceeds 10°.

Keywords: terahertz; coded-aperture imaging; phase compensation

1. Introduction

As a promising imaging technology, terahertz coded-aperture imaging (TCAI) has the advantages of both terahertz imaging [1–3] and coded-aperture imaging [4,5]. TCAI uses a coded-aperture antenna to modulate the phase of incident terahertz wave to produce a spatiotemporal independent radiation field in the imaging area. By mathematically modeling the imaging system, the imaging equation can be obtained. The target can be reconstructed by using computational imaging algorithms [6,7]. Since terahertz wave has high frequency and short wavelength, TCAI system can realize high-resolution and high frame-rate imaging. In addition, terahertz wave has stronger penetration capability than light wave. TCAI is different from synthetic aperture radar (SAR) and inverse synthetic aperture radar (ISAR) [8,9], and it can realize forward-looking and staring imaging without relative motion between the platform and target. Furthermore, it does no harm to the human body. For TCAI systems, there are plenty of potential applications, such as autonomous driving, safety inspection, and missile terminal guidance, etc.

Over the past few years, a lot of effort has been devoted to developing TCAI. The authors in [10] proposed a single pixel terahertz imaging system using a series of random masks to achieve high-speed image acquisition applying the idea of coded aperture to terahertz imaging. A low-profile aperture capable of microwave imaging was demonstrated by leveraging metamaterials and compressive imaging [11]. This is also the first time to propose aperture-coded imaging algorithm based on metamaterials. In 2014, digital metamaterial was presented for the first time. The field-programmable gate array was used to control the digital metamaterial [12]. In the next year, digital metamaterials extended to terahertz domain [13]. Then a transmission-type coding metasurface was proposed and demonstrated which can bend normally incident terahertz beams in anomalous directions and generate nondiffractive Bessel beams in normal and oblique directions [14]. Furthermore, authors in [15] realized arbitrary multi-bit programmable phases. The development of digital metamaterials has greatly promoted the development of TCAI and provided device support for TCAI. In [16], the authors...
investigated and classified coding strategies based on different placement of coded aperture and coding objects. The authors in [17] proposed a high-resolution TCAI method for fast beam scanning of near-field 3D targets. With the continuous effort of researchers, TCAI has made great progress.

As we all know, phase information of the echo signal is an important part for radar imaging. Unfortunately, phase errors generally exist for most coded apertures. The existence of phase errors can make the established imaging model mismatch. Therefore, various studies have been presented on gain-phase errors. In 2015, the authors in [18] proposed the sparse auto-calibration method to compensate the gain-phase error in radar coincidence imaging. In the same year, an Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT)-based method was presented to estimate the gain-phase errors [19]. Furthermore, the problem of direction-of-arrival (DOA) estimation for monostatic MIMO radar with gain-phase errors was addressed [20]. In 2019 a novel online antenna array calibration method was presented for estimating DOA in the case of uncorrelated and coherent signals with unknown gain-phase errors [21]. Although these methods are efficient, there are still some challenges. Most of the methods above are based on eigenstructure, less sensitive to phase errors, or relatively dependent on prior information, have high sparsity requirements for targets.

The novel contribution of the work is to propose a phase error compensation method for TCAI by deducing a formula to compute all element phase errors. With the calibrator, we use appropriate coding sequences to deduce the imaging model aiming at calculating the phase errors accurately. The proposed method is robust and can significantly improve the imaging quality.

2. Principle of the Proposed Method

Consider a TCAI system with one transmitter, one receiver and one coded-aperture antenna at the transmitting end, as shown in Figure 1. The processor is used to control the signal generation module, antenna drive module and information acquisition module. It can also be used to process echo information to reconstruct the target. Signal generation module can generate the incident terahertz wave transmitted to coded-aperture antenna by the signal transmitter. Driven by the antenna driver, coded-aperture antenna element has only two distinct states ‘on’ and ‘off’. With appropriate coding sequences, coded aperture can randomly modulate the phase of incident terahertz wave to produce a spatiotemporal independent radiation field. After the target reflection, echo is received by signal receiver, then sampled by the information acquisition module, finally sent to the processor to reconstruct target scatters.

Figure 1. Terahertz coded-aperture imaging system.
Suppose the coded aperture contain $M$ elements. The imaging plane is divided into $K$ grid cells where the center position vector of each grid cell is $r_k$ and the strong scatters of target are assumed at the center of each grid cell exactly. $N$ is the number of samples. Assuming the TCAI system transmits a terahertz chirp signal

$$St(t) = A(t) \cdot \text{rect} \left( \frac{t}{T_p} \right) \cdot \exp \left[ j2\pi \left( f_c t + \frac{1}{2} \gamma t^2 \right) \right]$$

(1)

where $St(t)$ is the transmitting signal at time $t$, $A$ is the amplitude, $T_p$ is the chirp signal pulse width, $f_c$ is the center frequency of terahertz wave, $\gamma$ is the chirp rate of the signal, $\text{rect}(\frac{t}{T_p})$ is window function,

$$\text{rect}(\frac{t}{T_p}) = \begin{cases} 1, & |t| \leq T_p/2 \\ 0, & |t| > T_p/2 \end{cases}$$

(2)

The detection signal from the signal transmitter to the $m$-th coded-aperture element then to the $k$-th imaging grid cell and finally to the signal receiver has a time delay which can be expressed as

$$\tau_{mk} = \frac{|r_{tra} - d_m| + |d_m - r_k| + |r_k - r_{rec}|}{c}$$

(3)

where $c$ is the speed of light, $r_{tra}$, $d_m$, $r_k$ and $r_{rec}$ are position vectors of the signal transmitter, the $m$-th coded-aperture element, the $k$-th grid cell and the signal receiver, respectively. Then the detection signal arriving at coded-aperture antenna at time $t_n$ is

$$Sd(t_n, \tau_{mk}) = A(t_n) \cdot \text{rect} \left( \frac{t_n - \tau_{mk}}{T_p} \right) \cdot \exp \left[ j2\pi \left( t_n + \frac{1}{2} \gamma (t_n - \tau_{mk})^2 \right) \right]$$

(4)

In analogy to the digital circuit, coded-aperture element has only two distinct states ‘on’ and ‘off’. We use ‘0’ and ‘1’ to represent ‘off’ and ‘on’ respectively. With elaborately designed coding sequences, terahertz wave is randomly modulated then transmitted to the imaging plane. The reference signal corresponding to the $k$-th imaging plane grid cell at time $t_n$ is

$$S(t_n, r_k, \theta) = \sum_{m=1}^{M} Sd(t_n, \tau_{mk}) \cdot \exp \left[ j \cdot c_{t_n,m} \cdot (\varphi_{t_n,m} + \theta_m) \right]$$

(5)

where $c_{t_n} = \{ c_{t_n,m} \}_{m=1}^{M}$ stands for the coding sequence at time $t_n$. When $c_{t_n,m}=0$, the state of $m$-th coded-aperture element is ‘off’ with no phase error. Instead, when $c_{t_n,m}=1$, the state of $m$-th coded-aperture element is ‘on’ with phase error $\theta_m$. $\varphi_{t_n} = \{ \varphi_{t_n,m} \}_{m=1}^{M}$ are the phase modulation factors loaded on coded aperture at time $t_n$. Because the phase error of the coded-aperture element is the main factor affecting the target reconstruction, the effect of other factors are not considered temporarily. The echo signal at time $t_n$ can be denoted as

$$Sr(t_n) = \sum_{k=1}^{K} \beta_k S(t_n, r_k, \theta)$$

$$= \sum_{k=1}^{K} \sum_{m=1}^{M} Sd(t_n, \tau_{mk}) \cdot \exp \left[ j \cdot c_{t_n,m} \cdot (\varphi_{t_n,m} + \theta_m) \right] \cdot \beta_k$$

(6)
where $\theta = \{\theta_m\}_{m=1}^M$ is a vector that represents the phase errors, $\beta_k$ stands for the target corresponding to the $k$-th imaging plane grid cell. The matrix-imaging formula of Equation (6) can be written as

$$Sr(t_n) = \left[ \begin{array}{c} Sd(t_n, \tau_{11}) \\ \vdots \\ Sd(t_n, \tau_{m1}) \end{array} \right] \cdot \left[ \begin{array}{c} \beta_1 \\ \vdots \\ \beta_K \end{array} \right] \ast \exp \left[ j \cdot \frac{c_{t_n,1}}{\beta_1} \cdot \phi_{t_n,1} \\ \vdots \\ j \cdot \frac{c_{t_n,1}}{\beta_M} \cdot \phi_{t_n,M} \right] = \left[ \begin{array}{c} h_{1,1} \\ \vdots \\ h_{1,M} \end{array} \right] \cdot \exp \left[ j \cdot \frac{c_{t_n,1}}{\beta_1} \cdot \phi_{t_n,1} \\ \vdots \\ j \cdot \frac{c_{t_n,1}}{\beta_M} \cdot \phi_{t_n,M} \right]$$

where $\ast$ is hadamard product which is an operation of matrix. For convenience, we define two new functions

$$h_{i,j} = \exp \left( j \cdot \frac{c_{t_n,1}}{\beta_1} \cdot \phi_{t_n,1} \right)$$

So, imaging equations can be written as

$$\begin{align*}
Sr(t_1) &= h_{1,1} \cdot g(t_1, \theta_1) + h_{1,2} \cdot g(t_1, \theta_2) + \cdots + h_{1,M} \cdot g(t_1, \theta_M) \\
Sr(t_2) &= h_{2,1} \cdot g(t_2, \theta_1) + h_{2,2} \cdot g(t_2, \theta_2) + \cdots + h_{2,M} \cdot g(t_2, \theta_M) \\
&\vdots \\
Sr(t_N) &= h_{N,1} \cdot g(t_N, \theta_1) + h_{N,2} \cdot g(t_N, \theta_2) + \cdots + h_{N,M} \cdot g(t_N, \theta_M)
\end{align*}$$

(10)

Simplifying Equation (10), a general mathematical model about phase error can be obtained as follows

$$Sr = H \ast g(\theta)$$

(11)

where $H = \{h_{i,n,M}\}_{i=1}^N$, $g(\theta) = \{g(t_n, \theta)\}_{n=1}^N$. Equation (11) can be solved by least squares method. The signal processing is shown in Figure 2.

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**Figure 2.** Signal processing of the Antenna Phase Error Compensation.

To ensure that all phase errors of coded-aperture element can be obtained, the condition $M \leq N$ should be satisfied. Take one of the particular solutions as an example. Generate appropriate coding sequences to obtain a full-rank reference signal matrix $S$. In this way, the phase error of each coded-aperture element can be traversed, and the error can be solved accurately. In order to make it easy to operate, only one element’s state of coded-aperture antenna is ‘on’ at each sampling. The antenna element in ‘on’ will be not repeated throughout the process. So it is essential to make sure...
that $M = N$, $c_{ln} = \begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 \end{bmatrix}^T$. The scattering coefficient of calibrator is a constant set as $b$.

Calibrator can be placed at any grid cell of imaging plane. Then we can get

$$\theta_m = -j \ln \left( 1 - \frac{Sr(t_n) - \sum_{m=1}^{M} Sd(t_n, \tau_{mk}) \cdot b}{Sd(t_n, \tau_{mk}) \cdot b} \right)$$

(12)

After the phase error is obtained by using the calibrator and the appropriate coding sequences, we can bring the results calculated into Equation (5) to compensate the phase and correct the imaging model. The calculation process of the method is independent of the value of the calibrator, but we need to know the RCS of the calibration we are using. We believe that the source of the phase error is mainly the error of the coded antenna array element. In theory, the phase error derived from the proposed method is independent of the imaging distance. In fact, the process of calibration includes theoretical derivation and mathematical modeling. The process of echoes obtained by theoretical modeling and by real experiments will inevitably be affected by other factors. In order to ensure the accuracy of the proposed method, we believe that near-field conditions are better than far-field conditions, and indoor environment is better than outdoor environment.

3. Numerical Experiments and Discussions

In this section, first we confirmed that phase error of antenna element does make model mismatch and has bad effect on target reconstruction. Then the feasibility of the proposed method is verified by the simulation experiment. Finally, three algorithms are used to reconstruct targets. If the target is a sparse target, Orthogonal Matching Pursuit (OMP) algorithm [22] is more efficient. While the total variation (TV) regularization, such as the TVAL3 algorithm [23] and Sparse Bayesian Learning (SBL) algorithm [24] are worth considering when the target is extended target. The TVAL3 algorithm and SBL algorithm are also very effective for solving sparse targets. The main parameters of the experiment are shown in Table 1.

| Parameter                     | Value                        |
|-------------------------------|------------------------------|
| Center frequency              | 340 GHz                      |
| Band width                    | 10 GHz                       |
| Imaging distance              | 2 m                          |
| Size of coded aperture        | 0.25 m × 0.25 m              |
| Number of elements in coded aperture | 50 × 50                  |
| Size of imaging plane         | 0.42 m × 0.42 m              |
| Number of grid cells in the imaging plane | 60 × 60               |

3.1. Effect of Phase Error

Different coding sequences involve different phase errors. Each coding sequence corresponds to a row of the reference signal matrix. Thus, we analyze the effect of phase error on one row of the reference signal matrix. In Figure 3, we plot the amplitude ($|·|$) and phase ($·$) of one row of the reference signal matrix without phase error, one row of reference signal matrix with phase error and the difference under the same coding sequence. The phase error added to each element is between $(-20^\circ, 20^\circ)$ under the same coding sequences. As we can see from Figure 3a–f, the detection signal after randomly modulating by coded aperture forms a spatiotemporal independent radiation field both in amplitude and phase. From Figure 3a,b, the phase error influences amplitude, as confirmed by the error amplitude in Figure 3c. The highest amplitude difference reaches 0.25, a quarter of the original range. The phases shown in Figure 3d,e are affected more seriously. The phase difference of
Figure 3f highlights this point. The highest phase difference radian is 6 or \(-6\), two times the maximum in Figure 3d,e. From Figure 3, we can see that the phase error of coded-aperture element cannot be ignored in the imaging model and it is necessary to compensate the phase error.

![Figure 3](image)

**Figure 3.** Effect of phase error on amplitude and phase of the measurement matrix: (a) amplitude without phase error; (b) amplitude with phase error; (c) amplitude difference of the radiation field; (d) phase without phase error; (e) phase with phase error; (f) phase difference of the radiation field.

3.2. Performance for Different Targets with Different Algorithms

To verify the effectiveness of the proposed method, numerical experiment is carried out on two representative targets, i.e., sparse target and extended target. Imaging performance is measured by mean squared error (MSE). Its expression is:

\[
MSE = \frac{1}{PQ} \sum_{i=1}^{P} \sum_{j=1}^{Q} [x(i, j) - \hat{x}(i, j)]^2.
\]

The smaller the value of MSE is, the better the performance of the imaging result is.

To avoid the occurrence of accidental conditions, 20 Monte-Carlo trials were conducted. Each Monte-Carlo trial use a new target consist of 150–200 randomly distributed points. Thus, it proved that the method we proposed is not affected by the specific target shape. The average MSE obtained by the three algorithms are shown in Figure 4. Under the same algorithm, the MSE obtained after compensation is smaller than before, which proves that the proposed method can successfully modify the imaging model and thus improve the imaging quality. When the phase error is out of the range of \((-10^\circ, 10^\circ)\), target reconstruction results will be significantly affected. Figure 4 also shows that for randomly distributed points target, TVAL3 algorithm has the best imaging performance, followed by OMP, and then the SBL algorithm.
To study the specific influence of phase error on different targets with the increase of phase error, OMP, SBL, and TVAL3 are used to solve the imaging results and calculate the MSE before and after compensation. In this numerical experiment, the parameter settings of the TCAI system in Table 1 are adopted. Coding sequences satisfying the white Gaussian distribution are used to modulate the transmitted signals and $\text{SNR} = 20$ dB.

First, the RCS and the position of the calibrator are known. The real echo data of the calibrator can be obtained by experiment. Theoretically, we can obtain the theoretical echo data by deducing the reference signal matrix. The phase error can be obtained by solving the problem of optimal solution. After the phase error is obtained by using the calibrator and the appropriate coding sequences, we can bring the results calculated into Equation (5) to compensate the phase and correct the imaging model. Under the same experimental system, the echo data of the target shown in Figures 5 or 6 can be obtained. The target could be reconstructed by compressed sensing algorithm.

Figures 5a and 6a are original images of the sparse target and the extended target, respectively. Figure 5b–g are reconstruction results of sparse target with three algorithms before compensation and after compensation. When the phase error is $(-14^\circ, 14^\circ)$, the reconstruction result after compensation is obviously better than before compensation. Although results in Figure 5e–g have a few spurious weak scatters due to the influence of noise, strong scatters are obviously recovered in both of them and the edges of targets look clear. Figure 5b–d show that the weak scattering points are mixed with the background noise, affecting visual resolution. Figure 6b–g are reconstruction results of three algorithms when the phase error is $(-10^\circ, 10^\circ)$. Not only did Figure 6e–g reconstruct the basic outline of the extended target, but also the value of the strong scattering points was closer to the original target. In Figure 6b–d the background noise is obvious, and the details of the target are not recovered. By comparing Figures 5 and 6, we can see, under the same phase error condition, target reconstruction performs worse as the complexity of the target increases. For the sparse target, the phase error can be ignored within $(-14^\circ, 14^\circ)$, while for the extended target, the imaging quality of the reconstructed target decreases significantly when phase error exceeds $(-10^\circ, 10^\circ)$ where the proposed method can come in handy.
Figure 5. Reconstruction results for sparse target: (a) original sparse target; (b) OMP algorithm before compensation; (c) TVAL3 algorithm before compensation; (d) SBL algorithm before compensation; (e) OMP algorithm after compensation; (f) TVAL3 algorithm after compensation; (g) SBL algorithm after compensation.

Figure 6. Reconstruction results for extended target: (a) original extended target; (b) OMP algorithm before compensation; (c) TVAL3 algorithm before compensation; (d) SBL algorithm before compensation; (e) OMP algorithm after compensation; (f) TVAL3 algorithm after compensation; (g) SBL algorithm after compensation.
To further investigate the advantages of the proposed approach, we reconstructed the two targets under different SNRs. Figure 7a corresponds to the sparse target in Figure 5. Figure 7b corresponds to the extended target in Figure 6. No matter what kind of target it is, we can see from Figure 7 that the proposed method has the superiority for target reconstruction under different SNRs. In particular, TVAL3 algorithm after compensation has obvious improvement.

3.3. Performance with TVAL3 Algorithm under Different SNRs

The reconstruction performance based on the proposed method under different SNRs is studied. We added 5 dB Gaussian noise in Figure 8b,e, 15 dB Gaussian noise in Figure 8c,f, 25 dB Gaussian noise in Figure 8d,g, respectively. TVAL3 algorithm is used to reconstruct target and set the phase error \( \theta \in (-20^\circ, 20^\circ) \). The original extended target is shown in Figure 8a. The reconstructed results without compensation at SNR = 5 dB, SNR = 15 dB and SNR = 25 dB are shown in Figure 8b–d, respectively. Corresponding to the SNRs in Figure 8b–g are the reconstruction results after compensation. Under the same SNR condition, the result in Figure 8b failed to reconstruct while the reconstruction result in Figure 8e shows the contour of the target. Although the reconstructions in Figure 8c,d are successful, the results in Figure 8f,g are closer to the original target.
Figure 8. Reconstruction results for extended target with TVAL3 algorithm under different SNRs: (a) original extended target; (b–d) Imaging results before compensation at 5 dB, 15 dB and 25 dB; (e–g) Imaging results after compensation at 5 dB, 15 dB and 25 dB.

The MSE of the SNR reconstruction results from 0 dB to 30 dB is calculated, which is shown in Figure 9. It also demonstrates that the method proposed in this paper can improve the imaging quality under different SNRs, which is consistent with our previous conclusions. Bulleted lists look like this:

Figure 9. Influence of different SNRs with TVAL3 algorithm.
4. Conclusions

This paper analyzes the influence of phase error on the TCAI model and proposes an antenna phase error compensation method for TCAI method. From the theoretical derivation and experimental simulation, it is verified that the array element phase error compensation method can correct the imaging model and improve the imaging performance. Furthermore, the influence of phase error on sparse target and extended target is analyzed, and a general conclusion is drawn: the proposed method can effectively improve the imaging quality when the elemental phase error exceeds $10^\circ$. At the same time, the imaging performance of array element error compensation under different SNRs is analyzed, and it proved that the method has good robustness. Finally, the analysis and numerical results of this paper could be helpful for the design of code aperture antenna and TCAI system. In the future work, we will try to implement a TCAI experiment to verify the conclusions of this paper.

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