Interacting cosmic fluids and phase transitions under
a holographic modeling for dark energy

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We discuss the consequences of possible sign changes of the Q-function which measures the energy
transference between dark energy and dark matter. We investigate this scenario from a holographic
perspective to modeling the dark energy by a linear-parametrization of the equation of state pa-
rameter denoted by ω. By imposing the strong constraint of the second law of thermodynamics,
we show that sign changes of Q due to the cosmic evolution imply changes in the temperatures of
dark energy and dark matter, respectively. We also discuss the phase transitions, in the past and
future, experienced by dark energy and dark matter (or, equivalently, the sign changes of their heat
capacities).

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I. INTRODUCTION

Certain observational evidence seems to indicate that dark energy and dark matter, considered as dominant com-
ponents of the cosmic fluid, can interact [1] or perhaps an interaction between dynamical vacuum energy and matter
it is also possible [2]. Appears unnatural to think that cosmic fluids coexist and do not interact with each other as is
usual in the standard ΛCDM model (both components evolve independently and satisfy usual energy conservation
equations). As we know, ΛCDM, despite their successes, is theoretically unappealing because of several well known
shortcomings, among other, a negative energy density for the dark energy at z = 2.34 [3] and the case ω < −1
(phantom dark energy, which is not ruled out by the observational data), an interesting issue which it seems very
difficult to understand within the ΛCDM-framework.

On the other hand, dark energy and dark matter, both, seen as ordinary fluids (perfect fluids), is a simple assumption
which is consistent to describe them. In other words, here, dark energy and dark matter are described under a
representation of perfect fluids and in the literature we see nothing against this assumption. Any case, it is an
interesting approach if we are thinking that a perfect fluid is something that we know how to handle.

Components under interaction lead to a new perspective for describing/visualizing the cosmic evolution. There is a
rich literature on the subject and for describing that interaction, for instance, various Ansatzes for Q (defined before)
are used. In other words, the Q function is put by hand given that the field equations so require. That is a first
approximation to describe the interaction required by the observational data, but there is not formalism that allow
us to obtain Q from first principles.

In this work, we analyze thermodynamical aspects from the aforementioned interaction in the framework of two
interacting fluids. And as already said, we will use the holographic philosophy for modeling the dark energy where
the emphasis will put on the temperature evolution, during the cosmic evolution, from each fluid (dark energy and
dark matter. Phase transitions (seen as sign changes in the heat capacity) will be also discussed. 8πG = c = 1 units
will be used.

The organization of the paper is as follows: in Sec. II we present a brief description of two non-interacting fluids and
we revise the behaviour of its temperatures through the cosmic evolution. In Section III we incorporate interaction and
we revise the involved thermodynamics bearing in mind the second law. In Section IV we discuss the interaction
under holographic considerations and we show the presence of phase transitions (sign changes in its heat capacities)
experienced by both fluids during the evolution.

Finally, Sec. V is devoted to conclusions. We have added an Appendix in order to show explicitly the sign changes
in the heat capacities of the interacting fluids.

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II. NON-INTERACTING FLUIDS AND THERMODYNAMICS

We consider, in the framework of General Relativity (GR), the non-interacting flat-FLRW scheme between two components

\[ 3H^2 = \rho_{de} + \rho_{dm}, \]  

(1)

and

\[
\begin{align*}
\dot{\rho}_{de} + 3H(1 + \omega_{de})\rho_{de} &= 0, \\
\dot{\rho}_{dm} + 3H(1 + \omega_{dm})\rho_{dm} &= 0,
\end{align*}
\]

(2)

where \( \rho_{de} \) denotes the dark energy density, \( \rho_{dm} \) denotes the dark matter density, \( H \) is the Hubble parameter and \( \dot{ } \) denotes derivative with respect to the cosmic time. We assume that both components can be amenable to study by using a description under the rigorous scope of the thermodynamic laws. Then, by using the equation \( TdS = d(\rho V) + p \, dV \), for a generic fluid, where \( T \) is the temperature, \( S \) the entropy, \( \rho \) the energy density, \( p \) its pressure and \( V \) the physical volume, besides the integrability condition \( \partial^2 S/\partial T \partial V = \partial^2 S/\partial V \partial T \), we can obtain the following equation for the temperature during the cosmic evolution \[ \frac{dT}{T} = - \frac{dV}{V} \left( \frac{\partial p}{\partial \rho} \right)_V = -3Hdt \left( \frac{\partial p}{\partial \rho} \right)_a, \]

\[
\rightarrow T(z) = T(0) \exp \left( 3 \int \frac{dz}{1+z} \omega(z) \right),
\]

(3)

we have used \( p = \omega \rho \), \( 1 + z = a_0/a \) with \( z \) being the redshift parameter and \( a \) the cosmic scale factor with \( a_0 = a(0) \). Since \( V = \Omega_0 a^3 \) (for a spatially flat section \( \Omega_0 = 4\pi/3 \), see [7]), we obtain \( dV/V = 3da/a = 3Hdt \). By considering for dark energy \( \omega_{de}(z) \approx -1 \) and for non-relativistic dark matter \( \omega_{dm} \approx -1 \), we can obtain \( T_{dm}(z) \approx (1 + z)^2 \) after using (3). So,

\[
T_{de}(z) = \frac{T_{de}(0)(1+z)^{-3}}{T_{dm}(0)(1+z)^{2}},
\]

(4)

i.e., \( T_{de}(z) \) grows and \( T_{dm}(z) \) decreases when \( z \) goes to the future. This fact appears to be unusual if we want to think in energy transference from dark energy to dark matter, at least from some \( z_e \) in the past, and \( Q(0) > 0 \) as is suggested by the observational data [1]. This point will be discussed later. Equilibrium? We do

\[
T_{de}(z_e) = T_{dm}(z_e),
\]

\[
\implies z_e = [T_{de}(0)/T_{dm}(0)]^{1/3} - 1,
\]

\[
\rightarrow \begin{cases} 
  z_e \in \text{past} \implies T_{de}(0)/T_{dm}(0) > 1, \\
  z_e \in \text{future} \implies T_{de}(0)/T_{dm}(0) < 1.
\end{cases}
\]

(5)

Nowadays we would expect that \( T_{de}(0) > T_{dm}(0) \) (see later) and in this case \( z_e \) belongs to the past. But the problem is \( z_e \) at the future. If we do not have equilibrium through the evolution, we can think in two options: only one sign of \( Q \) and then only one option for the inequality between the temperatures, \( T_{de}(z) > T_{dm}(z) \) or \( T_{de}(z) < T_{dm}(z) \). Nevertheless, there are holographic arguments in order to justify, at least in a theoretical scope, both options for the sign of \( Q \). As a second option we can imagine that even when there are sign changes in \( Q \), the amount of energy transferred is not enough for generating changes in the relation between \( T_{de}(z) \) and \( T_{dm}(z) \). But the thermodynamical constraint given by the second law rejects the latter option.

By following the reference [4], we ascribe a temperature (Gibbs integrability condition) to the dark energy in the form \( T_{de} \sim \rho_{de}^{\omega/(\omega+1)} \), and we see that if \( -1 < \omega < 0 \), \( T_{de} \) increases when \( \rho_{de} \) decreases and vice versa. If we assume that the given relationship between temperature and energy density is valid for \( \omega \rightarrow -1 \) (phantom dark energy, not ruled out by the observational data) we have that \( T_{de} \) and \( \rho_{de} \), both increase with the time and dark energy increasing in the future appears to be not only a conjecture.

On other hand, if we write for dark matter \( T_{dm} \sim \rho_{dm}^{\omega/(\omega+1)} \), we can see that \( \omega = 0 \) (dust) drives to \( T_{dm} = \text{const} \) and if this were the case, we would have, from a thermodynamical point of view, a fluid without the ability of interacting with other and, in this sense, we could accept the idea developed in the reference [8]: there is interaction, but \( Q = 0 \). So, if we accept that both temperatures \( T_{de}(\omega = -1) = 0 \) and \( T_{dm} = 0 \), we repeat, the above mentioned idea appears to be interesting.

In the next Sections we introduce the interaction and discuss its consequences for the evolution of the temperatures.
III. INTERACTING FLUIDS AND THERMODYNAMICS

By using the interacting scheme

\[ \dot{\rho}_{de} + 3H (1 + \omega_{de}) \rho_{de} = -Q, \]

\[ \dot{\rho}_{dm} + 3H (1 + \omega_{dm}) \rho_{dm} = Q, \]

or, equivalently

\[ \dot{\rho}_{de} + 3H (1 + \omega^{eff}_{de}) \rho_{de} = 0, \]

\[ \dot{\rho}_{dm} + 3H (1 + \omega^{eff}_{dm}) \rho_{dm} = 0, \]

where

\[ \omega^{eff}_{de} = \omega_{de} + \frac{Q}{3H \rho_{de}} \quad \text{and} \quad \omega^{eff}_{dm} = \omega_{dm} - \frac{Q}{3H \rho_{dm}}, \]

the temperatures are, respectively,

\[ T_{de} (z) \sim \exp \left( 3 \int \frac{dz}{1 + z} \omega^{eff}_{de} \right) \]

\[ = \exp \left( 3 \int \frac{dz}{1 + z} \omega_{de} \right) \exp \left( 3 \int \frac{dz}{1 + z} H \rho_{de} \right), \]

and

\[ T_{dm} (z) \sim \exp \left( 3 \int \frac{dz}{1 + z} \omega^{eff}_{dm} \right) \]

\[ = \exp \left( 3 \int \frac{dz}{1 + z} \omega_{dm} \right) \exp \left( -3 \int \frac{dz}{1 + z} H \rho_{dm} \right), \]

and here we have considered a generic \( \omega_{dm} \) for dark matter without any special commitment with it. In Section IV we will use \( \omega_{dm} = 0 \) (dust).

On the other hand, in presence of interaction, the entropy production associated to the interaction is

\[ \dot{S}_{de} (t) + \dot{S}_{dm} (t) = \left( \frac{Q (t)}{T_{dm} (t)} - \frac{-Q (t)}{T_{de} (t)} \right) V \geq 0, \]

\[ \forall \ t, \]

or, by using the redshift parameter

\[ \forall \ z, \]

\[ - (1 + z) H (z) \frac{d}{dz} [S_{de} (z) + S_{dm} (z)] = \left( \frac{1}{T_{dm} (z)} - \frac{1}{T_{de} (z)} \right) V Q (z) \geq 0, \]

\[ \forall \ z, \]

I.e. \( T_{de} (z) > T_{dm} (z) \) at late times \( \Rightarrow \) \( Q (z) > 0 \). However, at early times \( T_{de} (z) < T_{dm} (z) \) (a reasonable assumption), then we should have \( Q (z) < 0 \) in order to satisfy the second law. As was said before, it may seem strange that today \( T_{de} (z) \) grows with \( z \) if dark energy is transferring energy to dark matter \( (Q (z) > 0) \) at least from some \( z_e \) [5]. Can be a signal of a negative heat capacity for the dark energy? As we will see in the next Section, the incorporation of interaction shows explicitly this fact. Moreover, the dark matter shows also phase transitions, in the past and in the future. Sign changes of \( Q \) (one in the past and another in the future) can be visualized in reference [8] where a holographic modeling for the dark energy was used beside the linear-parametrization \( \omega_{de} (z) = \omega_{de} (0) + \sigma \star z \), and \( \sigma = \text{const.} \) [10].

We end this Section by making a consistency check. By using the usual concepts of thermodynamics, we deduce the equation for the evolution of the temperature for a generic fluid denoted by \( \rho \). We start by setting \( \rho = \rho (V, T) \) and \( p = p (V, T) \), a reasonable setup. So,

\[ \dot{\rho} = aH \left( \frac{\partial p}{\partial a} \right)_T + \left( \frac{\partial p}{\partial T} \right)_a \dot{T}, \]

\[ \text{(13)} \]
and $V = (4\pi/3)a^3$ so that $\dot{V} = 3VH$ and $(\partial \rho/\partial V)_T = (a/3V)(\partial \rho/\partial a)_T$. On the other hand, from the second law besides the integrability condition given before, it is straightforward to obtain the expression

$$\left(\frac{\partial \rho}{\partial a}\right)_T = \frac{3T}{a} \left[ (\frac{\partial \rho}{\partial T})_a - \frac{p + \rho}{T} \right]. \quad (14)$$

By replacing this last expression in (13) and by using the non-conservation equation

$$\dot{\rho} = Q_f - 3H (\rho + p), \quad (15)$$

where $Q_f = -Q$ for dark energy and $Q_f = Q$ for dark matter, we can obtain

$$\frac{\dot{T}}{T} = -3H \left( \frac{\partial p}{\partial \rho} \right)_a + \left( \frac{\partial \rho}{\partial T} \right)_a^{-1} Q_f, \quad (16)$$

and by using the redshift parameter, we write the last equation in the form

$$\frac{dT}{T} = \frac{dz}{1 + z} \left[ 3 \left( \frac{\partial p}{\partial \rho} \right)_z - \frac{Q_f}{H} \left( \frac{\partial \rho}{\partial T} \right)_z^{-1} \frac{1}{T} \right], \quad (17)$$

so that, for dark energy

$$\frac{dT_{de}}{T_{de}} = \frac{dz}{1 + z} \left[ 3 \left( \frac{\partial \rho_{de}}{\partial \rho_{de}} \right)_z + \frac{Q}{H} \left( \frac{\partial \rho_{de}}{\partial T_{de}} \right)_z^{-1} \frac{1}{T_{de}} \right], \quad (18)$$

and for dark matter

$$\frac{dT_{dm}}{T_{dm}} = \frac{dz}{1 + z} \left[ 3 \left( \frac{\partial \rho_{dm}}{\partial \rho_{dm}} \right)_z - \frac{Q}{H} \left( \frac{\partial \rho_{dm}}{\partial T_{dm}} \right)_z^{-1} \frac{1}{T_{dm}} \right]. \quad (19)$$

Now, we compare the expressions given in (2) and (18), i. e.,

$$\frac{dT_{de}}{T_{de}} = \frac{dz}{1 + z} \left[ 3\omega_{de}(z) + \frac{Q}{H} \frac{1}{\rho_{de}} \right], \quad (20)$$

and

$$\frac{dT_{de}}{T_{de}} = \frac{dz}{1 + z} \left[ 3\omega_{de}(z) + \frac{Q}{H} \left( \frac{\partial \rho_{de}}{\partial T_{de}} \right)_z^{-1} \frac{1}{T_{de}} \right], \quad (21)$$

where we have done $(\partial \rho_{de}/\partial \rho_{de})_z = \omega_{de}(z)$. The consistency between (20) and (21) indicates that

$$\rho_{de} = \left( \frac{\partial \rho_{de}}{\partial T_{de}} \right)_z T_{de}, \quad (22)$$

and the same occurs if we compare (19) and (15)

$$\rho_{dm} = \left( \frac{\partial \rho_{dm}}{\partial T_{dm}} \right)_z T_{dm}, \quad (23)$$

as it is expected (see [11]).

### IV. SIGN CHANGE OF Q AND HOLOGRAPHY

One approach for treating the $Q$-function is by considering the following Ansatz

$$Q = 3H (\lambda_1 \rho_{de} + \lambda_2 \rho_{dm}), \quad (24)$$

where
where $\lambda_1$ and $\lambda_2$ are both constant parameters to be determined by observations. According to observational settings, both parameters have equal sign and so there is not sign change in $Q$ [12].

The second approach, which we will use from now on, is based on a holographic model

$$\rho_{de} (z) = 3H^2 (z) \left[ \alpha - \frac{\beta}{2} (1 + z) \frac{d\ln H^2 (z)}{dz} \right], \tag{25}$$

where $\alpha$ and $\beta$ are both positive parameters which are well confined by the observational data: $\beta < \alpha < 1$ [13].

The infrared cut-off given for $\rho_{de}$ [13] can be understood as a generalization of the model $\rho_{de} \sim -R$ [13], where $R$ is the Ricci scalar given by $R = -6 (2H^2 + \dot{H} + k/a^2)$ or, as an extension of the holographic model $\rho_{de} = 3\alpha H^2$ proposed by M. Li [15]. In the last case, the key idea is to use the holographic principle [16] and its possible role in cosmology. This approach is an open issue and, under this philosophy, the model given in (25) it is an interesting start point in order to visualize what we mean by dark energy. This is a crucial fact if we are thinking (the usual) in cosmology. This approach is an open issue and, under this philosophy, the model given in (25) it is an interesting start point in order to visualize what we mean by dark energy. This is a crucial fact if we are thinking (the usual) in dark energy seen as a cosmological constant, although the observational data would indicate that $\rho_{de}$ is not necessarily a constant [13]. In this sense, the $\Lambda CDM$ model could be questioned, despite their successes.

By using (25) besides (1) and (6), it is possible to write

$$\rho_{de} (z) = 3H^2 (z) \left( \frac{2\alpha - 3\beta}{2 + 3\beta \omega_{de} (z)} \right), \tag{26}$$

and so

$$\rho_{dm} (z) = 3H^2 (z) \left( \frac{2 (1 - \alpha) + 3\beta (1 + \omega_{de} (z))}{2 + 3\beta \omega_{de} (z)} \right), \tag{27}$$

so that we can obtain an explicit expression for $Q$

$$\frac{Q}{9H^3} (z) = - (2\alpha - 3\beta) \left[ \frac{2 (1 - \alpha) + 3\beta (1 + \omega_{de})}{2 + 3\beta \omega_{de}} \right] \omega_{de} + \frac{\beta (1 + z)}{2 + 3\beta \omega_{de}} d\omega_{de}/dz. \tag{28}$$

The interaction sign can change through the evolution as can be seen from the Ansatz $\omega_{de} (z) = \omega_{de} (0) + \sigma z$ (see later). Therefore, according to (25) and by using the Ansatz given before, $Q (z)$ experiences two sign changes, one in the past and another in the future, as can be seen in [3]. But, what does it mean a sign change of $Q$ at late times?, would we have dominion of dark matter again?, the current accelerated expansion would be transient previous to a possible future collapse? [18].

On the other hand, and under an entropic philosophy (entropic cosmology) in which we have an amount of nonconservation energy, we can observe sign changes in it and, in particular, that sign is mainly dependent of the equation of state parameter ($\omega$) in each stage of the cosmic evolution. [13]. But, models based in entropic considerations appear to be somewhat inconsistent with the observational data [9].

And, under a holographic philosophy also, if we are considering an interaction between the bulk and the boundary of the spacetime, we can see sign changes in $Q (z)$ [20].

Now, by using (25) besides (27) and (20) into (9), we write

$$\omega^{eff}_{de} = \left( \frac{2\alpha - 3\beta}{2 + 3\beta \omega_{de}} \right) \left[ \omega_{de} - \frac{\beta (1 + z)}{2\alpha - 3\beta} d\omega_{de}/dz \right], \tag{29}$$

and

$$\omega^{eff}_{dm} = \omega_{dm} + \left( \frac{2\alpha - 3\beta}{2 + 3\beta \omega_{de}} \right) \left[ \omega_{de} + \frac{\beta (1 + z)}{2 (1 - \alpha) + 3\beta (1 + \omega_{de})} d\omega_{de}/dz \right], \tag{30}$$

and $Q (z)$ can be written in the form

$$\frac{Q}{9H^3} (z) = - \frac{1}{3\beta^2 \sigma} (2\alpha - 3\beta) \frac{(z - 1.86) (z + 0.2)}{(A + z)^2}, \tag{31}$$

and the temperatures are given, respectively, by

$$T_{de} (z) = T_{de} (0) (1 + z)^a \left( 1 + \frac{z}{A} \right)^b, \tag{32}$$
\begin{equation}
T_{dm} (z) = T_{dm} (0) (1 + z)^a \left[1 + \frac{z}{A}\right]^{b+1} \left[1 + \frac{z}{B}\right]^c,
\end{equation}

where
\begin{align*}
a &= \left(\frac{2\alpha - 3\beta}{\beta \sigma}\right) \left(\sigma + \frac{\omega (0) - \sigma A}{A - 1}\right), \\
b &= -1 - \left(\frac{2\alpha - 3\beta}{\beta \sigma}\right) \left(\frac{\omega (0) - \sigma A}{A - 1}\right), \\
c &= 2\alpha - 3\beta; \\
A &= \frac{2 + 3\beta \omega (0)}{3\beta \sigma}, \\
B &= A - \left(\frac{2\alpha - 3\beta}{3\beta \sigma}\right),
\end{align*}

and we have considered $\omega_{dm} = 0$ (dust). The parameters involved are: $\omega_{de} (0) = -1.29$, $\sigma = 0.47$, $\alpha = 0.73$ and $\beta = 0.38$. All these parameters were estimated by using an adjustment with type I Supernovae (Union 2) \cite{8,21} and from them we have $Q (1.86) = Q (-0.2) = 0$, and $a \approx 264.00$, $b \approx -264.15$, $c \approx 0.32$; $A \approx 0.98$ and $B \approx 0.39$. In order to have a good vision of the Figures shown below, the following limits can be obtained from (32-33)

\begin{align*}
T_{de} (z \to -A) &\to \infty, \\
T_{dm} (z \to -B) &\to 0, \\
T_{de} (z \to \infty) &\to 0, \\
T_{dm} (z \to \infty) &\to 0,
\end{align*}

and
\begin{align*}
\frac{T_{de}}{T_{dm}} (z \to -B) &\to \infty, \\
\frac{T_{de}}{T_{dm}} (z \to \infty) &\to 0.
\end{align*}

According to \cite{22,39}, we note that if $\omega_{de} = \text{const.}$ and $\omega_{dm} = 0$ both temperatures are equal. This last situation does not appear to be consistent, is this a signal that $\omega_{de} \neq \text{const.}$ through the evolution?

So, sign changes in $Q$ imply changes in the temperatures of dark energy and dark matter, as dictates the second law. Additionally, we can visualize phase transitions, sign changes in its heat capacities, for both energy densities. See Appendix for details.

Clearly, the observational data eventually will tell us if these changes will definitely occur. Today, the observational data is only just showing signals of the presence of $Q$, but as we stated above, appears unnatural to think that cosmic fluids coexist and do not interact with each other. Additionally, nothing we can say about values of $T_{de} (0)$ and $T_{dm} (0)$: future observations could elucidate this point.

And roughly speaking, the sign change in the cosmic acceleration ($z \sim 0.6$) is located "inside" the zone where $Q (z) > 0$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{The behaviour of $Q$ and its two sign changes: for $-0.2 < z < 1.86$ we have $Q > 0$, and $Q < 0$ otherwise.}
\end{figure}
FIG. 2: We see that $T_{de} > T_{dm}$ in the range $-0.2 < z < 1.86$, i.e., when $Q > 0$ and $T_{de} < T_{dm}$ for $z > 1.86$, i.e., when $Q < 0$, in accord to the second law. However, in the range $-0.39 < z < -0.2$, when $Q < 0$, the quotient $T_{de}/T_{dm}$ diverges.

FIG. 3: We see that $T_{de}$ always grows and the dark energy exhibits a negative heat capacity from $z \approx 1.86$ to $z \approx -0.2$ (dark energy is heated while being delivered energy to dark matter). Out the indicated range, we have a positive heat capacity. And $T_{de}$ diverges when $z$ goes to $-0.98$.

V. FINAL REMARKS

Consistently with the second law of thermodynamics, we have studied the behaviour of the temperatures of two interacting fluids (dark energy and dark matter) and its relationship with the sign changes of $Q$ through the cosmic evolution. We have investigated the phase transitions (sign changes of its heat capacities) experienced for both, dark energy and dark matter. We have used a holographic model for the dark energy and, as usual, we have considered a pressureless fluid (dust) for dark matter.

Finally, the presence of $Q$ is a fact already confirmed by observations, the validity of our results are full dependent from that in the sense of possible changes in $Q$ that can be observed in future observations. If this is so, very interesting consequences we should have for the late cosmology, in particular, if definitely we are heating or cooling in the sense of “dark” (radiation background: we are cooling).
FIG. 4: We see that $T_{dm}(z)$ grows from $\infty$ to $z = -0.33$ (maximum of $T_{dm}$) and goes to zero when $z = -0.39$. The dark matter heat capacity experiences a sign change, that is, from $z = \infty$ to $z = 1.86$ we have a negative heat capacity (dark matter is heated while being delivered energy to dark energy), from $z = 1.86$ to $z = -0.2$ the heat capacity is positive, from $z = -0.2$ to $z = -0.33$ its heat capacity is negative (dark matter cools while receiving energy) and from $z = -0.33$ to $z = -0.38$ the heat capacity is positive.

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Appendix: Heat capacities

We write

$$C = \frac{\Delta U}{\Delta T},$$

and by doing $\Delta U = Q$ and recalling

$$\dot{\rho}_{de} + 3H (1 + \omega_{de}) \rho_{de} = -Q \quad \text{and} \quad \dot{\rho}_{dm} + 3H \rho_{dm} = Q,$$

we write

$$C_{de} = \frac{-Q}{\Delta T_{de}} < 0, \quad \text{if} \quad Q > 0 \quad \text{and} \quad \Delta T_{de} > 0,$$

and

$$C_{dm} = \frac{+Q}{\Delta T_{dm}} > 0, \quad \text{if} \quad Q > 0 \quad \text{and} \quad \Delta T_{dm} > 0.$$

The temperatures. We see that $T_{de}(z)$ always grows with $z$ ($\Delta T_{de}(z) > 0$) in the range $-0.2 < z < 1.86$, see Fig. 3. Then, the dark energy heat capacity is

$$C_{de} (-0.2 < z < 1.86) = \frac{-Q}{\Delta T_{de}} < 0,$$

and out this range $C_{de} = +Q/\Delta T_{de} > 0$.

We see also that $T_{dm}$ grows with $z$ from $\infty$ to $-0.33$ (maximum of $T_{dm}$) and goes to zero when $z = -0.39$. In this range $\Delta T_{dm}(z) > 0$, but in the range $-0.39 < z < -0.33$ we have $\Delta T_{dm}(z) < 0$. Then, the dark matter heat capacity changes are

$$C_{dm} (1.86 < z < \infty) = \frac{-Q}{\Delta T_{dm}} < 0 \quad \text{and} \quad \Delta T_{dm} > 0,$$
\[ C_{dm} (-0.2 < z < 1.86) = \frac{+Q}{\Delta T_{dm}} > 0 \text{ and } \Delta T_{dm} > 0, \]
\[ C_{dm} (-0.2 < z < -0.33) = \frac{-Q}{\Delta T_{dm}} < 0 \text{ and } \Delta T_{dm} > 0, \]
\[ C_{dm} (-0.33 < z < -0.39) = \frac{-Q}{\Delta T_{dm}} > 0 \text{ and } \Delta T_{dm} < 0. \]

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