Drell–Yan lepton-pair production: 
$q_T$ resummation at $N^3\text{LL}$ accuracy 
and fiducial cross sections at $N^3\text{LO}$

Stefano Camarda\textsuperscript{(a)}, Leandro Cieri\textsuperscript{(b)} and Giancarlo Ferrera\textsuperscript{(c)}

\textsuperscript{(a)} CERN, CH-1211 Geneva, Switzerland
\textsuperscript{(b)} INFN, Sezione di Firenze, I-50019 Sesto Fiorentino, Florence, Italy
\textsuperscript{(c)} Dipartimento di Fisica, Università di Milano and 
INFN, Sezione di Milano, I-20133 Milan, Italy

Abstract

We present high-accuracy QCD predictions for the transverse-momentum ($q_T$) distribution and fiducial cross sections of Drell–Yan lepton pairs produced in hadronic collisions. At small value of $q_T$ we resum to all perturbative orders the logarithmically enhanced contributions up to next-to-next-to-next-to-leading logarithmic ($N^3\text{LL}$) accuracy, including all the next-to-next-to-next-to-leading order ($N^3\text{LO}$) (i.e. $O(\alpha_S^3)$) terms. Our resummed calculation has been implemented in the public numerical program \textsc{DYTurbo}, which produces fast and precise predictions with the full dependence on the final-state leptons kinematics. We consistently combine our resummed results with the known $O(\alpha_S^3)$ fixed-order predictions at large values of $q_T$ thus obtaining full $N^3\text{LO}$ accuracy also for fiducial cross sections. We show numerical results at LHC energies discussing the reduction of the perturbative uncertainty with respect to lower-order calculations.
After the successful operation of the first two runs of the Large Hadron Collider (LHC) at CERN and the discovery of the long sought Higgs boson, a major task of the high-energy physics community has become a direct investigation of the electroweak symmetry breaking mechanism. In the absence of clear direct signals of new physics phenomena, precision studies give us a unique opportunity to search for possible deviations from Standard Model (SM) predictions. In this scenario it is clear that theoretical predictions for SM cross sections and associated distributions at an unprecedented level of accuracy are indispensable to fully exploit the discovery potential provided by the collected and forthcoming collider data.

The electroweak (EW) vector boson production, through the Drell–Yan (DY) mechanism \[1, 2\], is the most “classical” hard-scattering process in hadronic collisions. The large production rates and clear experimental signatures make this processes important for detector calibration, as luminosity monitor and to probe underlying event. Moreover it plays a fundamental role in the contest of SM precision studies \[3, 4, 5, 6, 7\] and for searches of physics signals beyond the SM \[8, 9, 10, 11\]. It is thus essential to provide accurate theoretical predictions, through detailed computations of the higher-order radiative corrections in QCD and in the EW theory, for vector boson production cross sections and related kinematical distributions. Among the various kinematical distributions the vector boson transverse-momentum \(q_T\) spectrum plays a special role. Precise knowledge of the \(Z\) boson \(q_T\) distribution gives important information on the \(W\) boson spectrum which in turn directly affects the measurement of the \(W\) boson mass \[12, 13, 14\].

The next-to-next-to-leading order (NNLO) corrections in the QCD coupling \(\alpha_s\) have been computed for the total cross section \[15, 16\], the rapidity distribution \[17\] and at fully differential level including the leptonic decay of the vector boson \[18, 19, 20, 21\]. More recently next-to-next-to-next-to-leading order (N3LO) QCD calculations of the total cross section have been performed in Refs. \[22, 23\]. The next-to-leading order (NLO) EW corrections, the mixed QCD-EW and QCD-QED corrections have also been computed \[24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43\]. In the large-\(q_T\) region, where \(q_T\) is of the order of the invariant mass of the lepton pair \(M\), the fixed-order QCD corrections for the \(q_T\) distribution are known up to \(\mathcal{O}(\alpha_s^2)\) in analytic form \[44, 45, 46, 47, 48\] and up to \(\mathcal{O}(\alpha_s^3)\) numerically through the fully exclusive NNLO calculation of vector boson production in association with jets \[49, 50, 51, 52, 53\]. However the bulk of the vector boson cross section lies in the small-\(q_T\) region \((q_T \ll M)\) where the reliability of the fixed-order expansion is spoiled by the presence of large logarithmic corrections of the type \(\ln(M^2/q_T^2)\) due to the initial-state radiation of soft and/or collinear partons. In order to obtain reliable perturbative QCD predictions, the enhanced-logarithmic terms have to be evaluated and systematically resummed to all orders in perturbation theory \[54, 55, 56, 57, 58, 59\]. Resummed calculations at different levels of theoretical accuracy have been performed in Refs. \[60, 61, 62, 63, 64, 65, 66, 67, 68\] also applying methods from Soft Collinear Effective Theory \[69, 70, 71, 72, 73, 74, 75\] and transverse-momentum dependent factorisation \[76, 77, 78, 79, 80, 81\].

In this Letter we apply the QCD transverse-momentum resummation formalism of Refs. \[57, 61, 64\] for the case of \(Z/\gamma^*\) boson production up to N3LL accuracy. We analytically include all the N3LO terms at small-\(q_T\) reaching full N3LL+N3LO accuracy in the small-\(q_T\) region\[\textsuperscript{1}\]. We implement our resummed calculation in the public numerical program DYTurbo \[82\] which provides fast and numerically precise predictions both for resummed and fixed-order QCD calculations including the full kinematical dependence of the decaying lepton pair with the corresponding spin

\[\textsuperscript{1}\] Sometimes in the literature this is referred as N3LL\(^\prime\) accuracy.
correlations and the finite value of the $Z$ boson width. We consistently match our resummed predictions with the NNLO numerical results at large-$q_T$ calculated in Refs.\cite{50,53} and reported in Ref.\cite{67} thus including the $\mathcal{O}(\alpha_s^3)$ corrections for the entire spectrum of $q_T$. By using the connection between the $q_T$ resummation and the $q_T$ subtraction formalism \cite{83} for fixed-order calculations we analytically \cite{84} expanded the resummed results thus providing predictions for fiducial cross section of the Drell–Yan process both at N$^3$LL+N$^3$LO and at N$^3$LO which, to our knowledge, have never appeared in the literature. Higher-order calculations beyond NLO QCD are definitely an hard task and are based on forefront and highly-specialized computations and numerical codes. The calculation presented in this paper is released as public software \cite{85} with the aim of facilitating an efficient and wide spread of the results to the theoretical and experimental communities.

We briefly review the resummation formalism developed in Refs.\cite{57,61,64} highlighting the main aspect relevant for our calculation. We consider the process

\[ h_1 + h_2 \rightarrow V + X \rightarrow l_3 + l_4 + X, \]  

where $V$ denotes the vector boson\cite{[1]} produced by the colliding hadrons $h_1$ and $h_2$ with a centre–of–mass energy $s$, while $l_3$ and $l_4$ are the final state leptons produced by the $V$ decay.

The hadronic cross section, fully differential in the lepton kinematics, is completely specified in terms of the transverse-momentum $q_T$ (with $q_T = \sqrt{q_T^2}$), the rapidity $y$ and the invariant mass $M$ of the lepton pair, and by two additional variables $\Omega$ that specify the angular distribution of the leptons with respect to the vector boson momentum. The differential hadronic cross section can be written as

\[ \frac{d\sigma_{h_1 h_2 \rightarrow l_3 l_4}}{d^2 q_T dM^2 dy d\Omega}(q_T, M^2, y, s, \Omega) = \sum_{a_1, a_2} \int_0^1 dx_1 \int_0^1 dx_2 f_{a_1/h_1}(x_1, \mu_F^2) f_{a_2/h_2}(x_2, \mu_F^2) \times \frac{d\hat{\sigma}_{a_1 a_2 \rightarrow l_3 l_4}}{d^2 q_T dM^2 dy d\Omega}(q_T, M, \hat{y}, \hat{s}, \Omega; \alpha_S, \mu_R^2, \mu_F^2), \]  

where $f_{a/h}(x, \mu_F^2)$ ($a = q_f, \bar{q}_f, g$) are the parton densities of the colliding hadron $h$, $\hat{s} = x_1 x_2 s$ is the square of the partonic centre–of–mass energy, $\hat{y} = y - \ln x_1/x_2$ is the vector boson rapidity with respect to the colliding partons, $\mu_R$ and $\mu_F$ are the renormalization and factorization scales. The last factor in the right-hand side of Eq. (2) is multi-differential partonic cross sections, computable in perturbative QCD as a series expansion in the strong coupling $\alpha_S = \alpha_S(\mu_R)$, which will be denoted in the following by the shorthand notation $[d\hat{\sigma}_{a_1 a_2 \rightarrow l_3 l_4}]$.

The partonic cross section can be decomposed as

\[ [d\hat{\sigma}_{a_1 a_2 \rightarrow l_3 l_4}] = [d\hat{\sigma}_{a_1 a_2 \rightarrow l_3 l_4}^{(\text{res.})}] + [d\hat{\sigma}_{a_1 a_2 \rightarrow l_3 l_4}^{(\text{fin.})}], \]  

where the first term on the right-hand side of Eq. (3) is the resummed component which contains all the logarithmically-enhanced contributions of the type $\alpha_S^n M^2/q_T^m \ln^m(M^2/q_T^2)$ (with $0 \leq m \leq 2n - 1$) that have to be resummed to all orders, while the second term is the finite component which can be evaluated at fixed order in perturbation theory.

\[ \text{[In this paper we explicitly consider the case } V = Z/\gamma^*, \text{ however our analytic results and the ensuing numerical implementation can be extended for the generic case of the production of colourless high-mass systems.} ] \]
We perform the resummation in the impact-parameter space \( b \) \(^{53}\). The resummed component can then be written as

\[
\left[ d\sigma^{\text{res.}}_{a_1 a_2 \to l_3 l_4} \right] = \sum_{b_1, b_2 = q, \bar{q}} \frac{d\hat{\sigma}^{(0)}_{b_1 b_2 \to l_3 l_4}}{d\Omega} \frac{1}{\hat{s}} \int_0^\infty \frac{db}{2\pi} b J_0(b q_T) \mathcal{W}_{a_1 a_2, b_1 b_2 \to V}(b, M, \hat{y}, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2) ,
\]

where \( J_0(x) \) is the 0th-order Bessel function, the factor \( d\hat{\sigma}^{(0)}_{b_1 b_2 \to l_3 l_4} \) is the Born level differential cross section for the partonic subprocess \( q\bar{q} \to V \to l_3 l_4 \).

The function \( \mathcal{W}_V(b, M, \hat{y}, \hat{s}) \) can be expressed in an exponential form by considering the ‘double’ \( \langle N_1, N_2 \rangle \) Mellin moments with respect to the variables \( z_1 = e^{\gamma_E} M/\sqrt{\hat{s}} \) and \( z_2 = e^{-\gamma_E} M/\sqrt{\hat{s}} \) at fixed \( M \) \(^{57, \ 86}\).

\[
\mathcal{W}_V(b, M; \alpha_S, \mu_R^2, \mu_F^2) = \mathcal{H}_V(M; \alpha_S, M/\mu_R, M/\mu_F, M/Q) \times \exp\{ G(\alpha_S, \tilde{L}; M/\mu_R, M/Q) \} ,
\]

where we have introduced the logarithmic expansion parameter \( \tilde{L} \equiv \ln(Q^2 b_0^2/b_0^2 + 1) \) with \( b_0 = 2e^{-\gamma_E} \) (\( \gamma_E = 0.5772... \) is the Euler number). The scale \( Q \sim M \) is the resummation scale \(^{87}\), which parameterizes the arbitrariness in the resummation procedure.

The process dependent function \( \mathcal{H}_V \) \(^{88, \ 89}\) includes the hard-collinear contributions and it can be expanded in powers of \( \alpha_S \) as

\[
\mathcal{H}_V(M; \alpha_S) = 1 + \frac{\alpha_S}{\pi} \mathcal{H}_V^{(1)} + \left( \frac{\alpha_S}{\pi} \right)^2 \mathcal{H}_V^{(2)} + \left( \frac{\alpha_S}{\pi} \right)^3 \mathcal{H}_V^{(3)} + \ldots .
\]

The universal (process independent) form factor \( \exp\{ G \} \) in the right-hand side of Eq. \( \mathcal{H}_V \) contains all the terms that order-by-order in \( \alpha_S \) are logarithmically divergent as \( b \to \infty \) (i.e. \( q_T \to 0 \)). The resummed logarithmic expansion of \( G \) reads

\[
G(\alpha_S, \tilde{L}) = \tilde{L} g^{(1)}(\alpha_S \tilde{L}) + g^{(2)}(\alpha_S \tilde{L}) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S \tilde{L}) + \left( \frac{\alpha_S}{\pi} \right)^2 g^{(4)}(\alpha_S \tilde{L}) + \ldots ,
\]

where the functions \( g^{(n)} \) control and resum the \( \alpha_S^k \tilde{L}^k \) (with \( k \geq 1 \)) logarithmic terms in the exponent of Eq. \( \mathcal{H}_V \) due to soft and collinear radiation. At NLL+NLO we include the functions \( g^{(1)}, g^{(2)} \) and \( \mathcal{H}_V^{(1)} \), at NNLL+NNLO we also include the functions \( g^{(3)} \) and \( \mathcal{H}_V^{(2)} \) \(^{90, \ 91}\). In order to reach full N\(^3\)LL+N\(^3\)LO accuracy in the small-\( q_T \) region (i.e. including all the \( \mathcal{O}(\alpha_S^3) \) terms) we have included the functions \( g^{(4)} \) \(^{92, \ 93, \ 94}\) and \( \mathcal{H}_V^{(3)} \). The function \( \mathcal{H}_V^{(3)} \) has been determined by exploiting its relation with the matching coefficients of the transverse-momentum dependent parton densities (TMD) calculated in Refs. \(^{95, \ 96}\) (see also Refs. \(^{97, \ 98, \ 99}\)). The Mellin moments of the function \( \mathcal{H}_V \) have been calculated using the method of Ref. \(^{100}\), and the \textsc{form} \(^{101}\) packages \textsc{summer} \(^{102}\) and \textsc{harmpol} \(^{103}\). The evolution of parton densities in Mellin space, and the Mellin moments of the splitting functions are calculated with the package \textsc{QCD-PEGASUS} \(^{104}\), the Mellin inversion and the Fourier–Bessel inverse transform from the impact-parameter space are performed numerically as discussed in Ref. \(^{82}\).

The function \( G \) is singular when \( \alpha_S \tilde{L} = \pi/\beta_0 \) (where \( \beta_0 \) is the one-loop coefficient of the QCD \( \beta \) function) which corresponds to the region of transverse-momenta of the order of the scale of

\(^{\dagger}\)For the sake of simplicity the explicit dependence on parton indices (which are relevant for the exponentiation in the multiflavour space) and the Mellin indices are understood. The interested reader can find the details in Ref. \(^{57}\) (in particular Appendix A) and Ref. \(^{86}\).
the Landau pole of the QCD coupling or $b^{-1} \sim \Lambda_{QCD}$. This signals that a truly non-perturbative (NP) region is approached and perturbative results (including resummed ones) are not reliable. In this region a model for NP QCD effects, which has to include a regularization of the singularity of the function $G$, is necessary. In our calculation we explicitly implemented the so-called Minimal Prescription \cite{105 106 107} which has the advantage to regularize the Landau singularity in resummed calculations \textit{without} introducing higher-twist power-suppressed contributions of the type $O(\Lambda_{QCD}/Q)$. Power-suppressed contributions can certainly be relevant at very small transverse-momentum ($q_T \sim \Lambda_{QCD}$) and should be eventually included, taking into account the delicate interplay with the leading-twist term, in order to correctly describe the experimental data in that region. In this Letter we have included the NP contribution in the form of a NP form factor in that region. In this Letter we have included the NP contribution in the form of a NP form factor in that region.

We have performed the analytic expansion of the resummed component Eq. (4) up to $O(\alpha_S^3)$ while the fixed-order cross section at large $q_T$ (formally at $q_T > 0$) can be obtained from the the fully-exclusive computation of vector boson production in association with a jet at LO, NLO \cite{109} and NNLO \cite{49, 50, 51, 52, 53}. We observe that both the fixed-order cross section and the expansion of the resummed part are separately divergent with the same small-$q_T$ limit and the finite component formally satisfies the equation \cite{82}

$$\lim_{q_T \to 0} q_T d\sigma_{\text{fin.}}^{(\text{fin.})} = 0.$$  

We have checked that our analytic expression for the expansion of the resummed part agrees in the small-$q_T$ limit with the NNLO fixed-order results reported in Ref. \cite{67} at permille level down to $q_T \sim 4$ GeV\footnote{This value is of the same order of the ones typically fitted in the literature, see e.g. Refs. \cite{107 108 63}.}

In the following we consider $Z/\gamma^* \rightarrow \ell^+\ell^-$ production and leptonic decay at the LHC. We present resummed predictions at NLL+NLO, NNLL+NNLO and $N^3LL+N^3LO$ accuracy, matching our computation with the fixed-order results at large-$q_T$ respectively at LO, NLO and NNLO. The hadronic cross section is obtained by convoluting the partonic cross section in Eq. (3) with the parton densities functions (PDFs) from the NNPDF3.1 set \cite{110} at NNLO with $\alpha_S(m_Z^2) = 0.118$ where we have evaluated $\alpha_S(\mu_R^2)$ at $(n+1)$-loop order at $N^nLL+N^nLO$ accuracy. In the case of $Z$ production, because of the axial coupling, additional Feynman diagrams with quark loops contribute to the cross-section at $O(\alpha_S^2)$ and $O(\alpha_S^3)$. Their contribution cancels out for each isospin multiplet when massless quarks are considered. The effect of a finite top-quark mass in the third generation has been considered and found extremely small at $O(\alpha_S^2)$ \cite{111, 114} while the finite mass top-quark contribution at $O(\alpha_S^3)$ remains to be derived \cite{112}. Therefore these contributions have currently been neglected in our calculation. We use the so called $G_\mu$ scheme for EW couplings.
with input parameters \( G_F = 1.1663787 \times 10^{-5} \) GeV\(^{-2} \), \( m_Z = 91.1876 \) GeV, \( \Gamma_Z = 2.4952 \) GeV, \( m_W = 80.379 \) GeV. Our calculation implements the leptonic decays \( Z/\gamma^* \rightarrow l^+l^- \) and we include the effects of the \( Z/\gamma^* \) interference and of the finite width \( \Gamma_Z \) of the \( Z \) boson with the corresponding spin correlations and the full dependence on the kinematical variables of final state leptons. This allows us to take into account the typical kinematical cuts on final state leptons that are considered in the experimental analysis. The resummed calculation at fixed lepton momenta requires a \( q_T \)-recoil procedure. We implement the general procedure described in Ref. \[64\] which is equivalent to compute the Born level distribution \( d\sigma^{(0)} \) of Eq. (4) in the Collins–Soper rest frame \[113\].

We have applied the resummation formalism to the production of \( l^+l^- \) pairs from \( Z/\gamma^* \) decay at the LHC (\( \sqrt{s} = 13 \) TeV) with the following fiducial cuts: the leptons are required to have transverse momentum \( p_T > 25 \) GeV, pseudo-rapidity \( |\eta| < 2.5 \) while the lepton pair system, is required to have invariant mass \( 66 < M < 116 \) GeV and transverse momentum \( q_T < 100 \) GeV\[^\dagger\].

In Fig. 1 we show the resummed component (see Eq. (9)) of the transverse-momentum distribution in the small-\( q_T \) region. In order to estimate the size of yet uncalculated higher-order terms and the ensuing perturbative uncertainties we present the dependence of the resummed component on the auxiliary scales \( \mu_F, \mu_R \) and \( Q \). The scale dependence band is obtained through independent variations of \( \mu_F, \mu_R \) and \( Q \) in the range \( M/2 \leq \{\mu_F,\mu_R,2Q\} \leq 2M \) with the constraints \( 0.5 \leq \{\mu_F/\mu_R,Q/\mu_R,Q/\mu_F\} \leq 2^{**} \). The lower panel shows the ratio of the distribution with

\[^\dagger\]In order to match with the NNLO numerical results at large-\( q_T \) we follow the kinematical selection cuts applied in Ref. \[67\].

\[^{**}\]In order to estimate the \( Q \) scale dependence of the resummed component we set the logarithmic expansion parameter to be \( L = \ln(Q^2b^2/b_0^2) \) which is equivalent to \( \tilde{L} \) in the small-\( q_T \) region.
Figure 2: The $q_T$ spectrum of $Z/\gamma^*$ bosons with lepton selection cuts at the LHC ($\sqrt{s} = 13$ TeV) at various perturbative orders. Full matched results between resummed and finite part of the hadronic cross section at central values of the scales.

respect to the $N^3LL+N^3LO$ prediction at the central value of the scales $\mu_F = \mu_R = 2Q = M$. We observe that the NLL+NLO and NNLL+NNLO scale dependence bands do not overlap thus showing that the NLL+NLO scale variation underestimates the true perturbative uncertainty. This feature was observed and discussed in Ref.[64]. Conversely the NNLL+NNLO and $N^3LL+N^3LO$ scale variation bands do overlap in the entire region $q_T < 30$ GeV (except that they nearly overlap in the window $1 < q_T < 4$ GeV) thus suggesting that, from NNLL+NLO, missing higher order corrections are correctly estimated by scale variations. We also observe that the scale dependence is reduced by a factor of 2 (or more) going from NNLL+NNLO to $N^3LL+N^3LO$: the scale variation at $N^3LL+N^3LO$ (NNLL+NNLO) is around ±0.8% (±2.5%) at the peak ($q_T \sim 4$ GeV), then it reduces at ±0.3% (±0.8%) level at $q_T \sim 12$ GeV and increase up to ±0.4% (±1.4%) level at $q_T \sim 25$ GeV. Finally, we note that in the low $q_T$ region non-perturbative effects are expected to become important. In particular by considering variations of the NP parameter in the range $0.3 \leq g_{NP} \leq 0.9$ GeV$^2$ we obtain the following additional uncertainties for the $N^3LL+N^3LO$ resummed prediction in Fig.[1] ±0.3% at $q_T \sim 25$ GeV, ±0.6% at $q_T \sim 12$ GeV and ±0.7% at $q_T \sim 4$ GeV. For $q_T \lesssim 4$ GeV the NP uncertainties rapidly increase at few percent level. These NP uncertainties have a delicate interplay with the non-perturbative parton densities uncertainties which deserve a careful analysis.

In Fig.2 we show the resummed $q_T$ distribution matched with the finite part at LO, NLO and NNLO. The auxiliary scales have been fixed to their central values $\mu_F = \mu_R = 2Q = M^{[1]}$. The

---

$^{[1]}$ Central scales for the fixed-order result have been set to $\mu_F = \mu_R = \sqrt{M^2 + q_T^2}$. The calculation of the scale variation band of the matched distribution would require the knowledge of the NNLO fixed-order result at $q_T > 0$ for different values of $\mu_F$ and $\mu_R$. 
Figure 3: The $q_T$ spectrum of $Z/\gamma^*$ bosons with lepton selection cuts at the LHC ($\sqrt{s} = 13$ TeV). Contribution of the finite part of the hadronic cross section for central values of the scales at $O(\alpha_S)$, $O(\alpha_S^2)$ and $O(\alpha_S^3)$ (see Eq. 7).

The lower panel shows the $K$-factors $K_{N^n\text{LO}}$ defined as the ratio of between the $N^n\text{LL}+N^n\text{LO}$ and the $N^{n-1}\text{LL}+N^{n-1}\text{LO}$ predictions (with $n = 2, 3$). By looking at the $K$-factors we observe that the impact of the $N^3\text{LL}+N^3\text{LO}$ ($\text{NNLL}+\text{NNLO}$) corrections with respect to the previous order is around $-4\%$ ($-19\%$) at the peak, then it becomes $-0.1\%$ ($+22\%$) at $q_T \sim 30$ GeV and increase to $+3\%$ ($+55\%$) at $q_T \sim 90$ GeV.

| Order                          | NLO          | NNLO         | $N^3\text{LO}$ |
|-------------------------------|-------------|-------------|---------------|
| $\sigma(pp \rightarrow Z/\gamma^* \rightarrow l^+l^-)$ [pb] | 766.3 ± 1   | 757.4 ± 2   | 746.1 ± 2.5   |
| Order                         | NLL+NLO     | NNLL+NNLO   | $N^3\text{LL}+N^3\text{LO}$ |
| $\sigma(pp \rightarrow Z/\gamma^* \rightarrow l^+l^-)$ [pb] | 773.7 ± 1   | 759.8 ± 2   | 749.6 ± 2.5   |

Table 1: Fiducial cross sections at the LHC ($\sqrt{s} = 13$ TeV): fixed-order results and corresponding resummation results obtained with the DYTurbo numerical program. The uncertainties on the values of the cross sections plots refer to an estimate of the numerical uncertainties in the integration.

By exploiting the connection between the $q_T$ resummation and the $q_T$ subtraction formalism \cite{83} we are able to provide fixed-order results for fiducial cross sections up to $N^3\text{LO}$\footnote{A fully consistent $N^3\text{LO}$ calculation for hadronic cross sections would require PDFs at the corresponding order which are currently not available. Uncertainties from missing higher order PDFs have been studied in Refs. \cite{114, 115, 116}.}. In Table 1 we report the predictions for the cross section in the fiducial region at NLO, NNLO and $N^3\text{LO}$,
NLL+NLO, NNLL+NNLO, N^3LL+N^3LO fixing the auxiliary scales to their central values. The N^3LO corrections decrease the NNLO cross-section at −1.5% level and the resummation effects further enhance the N^3LO result by +0.5%. We observe that the K-factor between the N^3LO and NNLO results is 0.985 which is comparable with results reported in Table I of Refs. [22, 23].

Generally speaking scale dependence cannot be regarded as a consistent estimate of the perturbative uncertainty because the effect due to uncalculated higher-order terms is typically larger than conventional scale dependence (see e.g. the comments on the scale variation bands of Fig. 1). A more realistic uncertainty estimate of the “true” perturbative uncertainty can be obtained, for instance, by comparing two subsequent orders of the expansion at central values of the scales and using half of the difference between them to assign the perturbative uncertainty [117]. This procedure leads (see Table 1) to an uncertainty of about ±2% (±4%) at NLO (NLL+NLO), ±0.7% (±1%) at NNLO (NNLL+NNLO) and ±0.8% (±0.7%) at N^3LO (N^3LL+N^3LO). As expected (see comments below on fiducial cuts) this procedure shows a better convergence on the uncertainties of the resummed perturbative expansion.

In order to judge the numerical stability of the matching procedure and the effects of the power-suppressed terms we consider the contribution of the finite part of the cross section. We show in Fig. 3 the finite part of the cross section for central values of the scales at \( O(\alpha_S) \), \( O(\alpha_S^2) \) and \( O(\alpha_S^3) \). The effect of the finite component smoothly vanishes as \( q_T \to 0 \) (see Eq. 9) and gives a small contribution to the matched result in the small \( q_T \) region: the integral over the ranges \( 4 < q_T < 20 \text{ GeV} \) and \( 1 < q_T < 4 \text{ GeV} \) of the LO finite component represents respectively the 1.5% and 0.12% of the NLL+NLO fiducial cross section in Table 1, the \( O(\alpha_S^2) \) correction in the same ranges is respectively the 0.10% and −0.04% of the NNLL+NNLO result while the \( O(\alpha_S^3) \) correction in the range \( 4 < q_T < 20 \text{ GeV} \) the 0.16% of the N^3LL+N^3LO. As previously observed below \( q_T \sim 4 \text{ GeV} \) the agreement with the \( O(\alpha_S^3) \) results of Ref. [67] (see Fig. 2) deteriorates. However the finite component gives a tiny contribution in the small \( q_T \) region and we thus avoided to include in our results the finite part at \( O(\alpha_S^3) \) for \( q_T < 4 \text{ GeV} \).

The results in Table 1 have been obtained applying the symmetric lepton \( p_T \) cuts previously defined. It is well known [118, 117] that in the case of symmetric cuts fixed-order calculations are affected by perturbative (soft-gluon) instabilities at higher orders. The results in Table 1 are obtained with a lower integration limit for the finite part of the cross section fixed to \( q_{T\text{cut}} = 0.5 \text{ GeV} \) and the numerical uncertainties include an estimate of the corresponding systematic uncertainty. More accurate fixed-order results and an estimate of such uncertainty can be obtained by evaluating the \( q_{T\text{cut}} \to 0 \) extrapolation or by a direct calculation of perturbative power corrections of the type \( \mathcal{O}((q_{T\text{cut}}/M)^p) \) with \( p > 0 \) which are neglected for \( q_{T\text{cut}} > 0 \) [119, 120, 121, 122, 73, 123]. We stress however that the inclusion of such contributions cannot improve the physical predictivity of the fixed-order results in case of symmetric cuts which are affected by sizable theoretical instabilities produced by the soft-gluon effects.

We have performed the implementation of both the \( q_T \) resummation and \( q_T \) subtraction formalism for Drell–Yan processes up to N^3LL+N^3LO and N^3LO in the DYTurbo numerical program. In this Letter we have illustrated the first numerical results for the case of Z/\gamma^* production and leptonic decay at the LHC.
Acknowledgments. We gratefully acknowledge Stefano Catani for useful discussions and comments on the manuscript and Ludovica Aperio Bella for extensive tests of the numerical code. This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement number 754496 and under European Research Council grant agreement number 740006.

References

[1] S. D. Drell and T.-M. Yan, “Massive Lepton Pair Production in Hadron-Hadron Collisions at High-Energies,” Phys. Rev. Lett. 25 (1970) 316–320 [Erratum: Phys.Rev.Lett. 25, 902 (1970)].

[2] J. H. Christenson, G. S. Hicks, L. M. Lederman, P. J. Limon, B. G. Pope, and E. Zavattini, “Observation of massive muon pairs in hadron collisions,” Phys. Rev. Lett. 25 (Nov, 1970) 1523–1526.

[3] CMS Collaboration, “Measurement of the weak mixing angle with the Drell-Yan process in proton-proton collisions at the LHC,” Phys. Rev. D 84 (2011) 112002, arXiv:1110.2682 [hep-ex].

[4] ATLAS Collaboration, “Measurement of the forward-backward asymmetry of electron and muon pair-production in pp collisions at $\sqrt{s}=7$ TeV with the ATLAS detector,” JHEP 09 (2015) 049, arXiv:1503.03709 [hep-ex].

[5] CMS Collaboration, “Measurement of the differential cross section and charge asymmetry for inclusive pp → W± + X production at $\sqrt{s}=8$ TeV,” Eur. Phys. J. C 76 no. 8, (2016) 469, arXiv:1603.01803 [hep-ex].

[6] ATLAS Collaboration, “Precision measurement and interpretation of inclusive $W^+$, $W^-$ and $Z/\gamma^*$ production cross sections with the ATLAS detector,” Eur. Phys. J. C 77 no. 6, (2017) 367, arXiv:1612.03016 [hep-ex].

[7] S. Camarda, J. Cuth, and M. Schott, “Determination of the muonic branching ratio of the $W$ boson and its total width via cross-section measurements at the Tevatron and LHC,” Eur. Phys. J. C 76 no. 11, (2016) 613, arXiv:1607.05084 [hep-ex].

[8] CMS Collaboration, “Search for narrow resonances in dilepton mass spectra in proton-proton collisions at $\sqrt{s}=13$ TeV and combination with 8 TeV data,” Phys. Lett. B 768 (2017) 57–80, arXiv:1609.05391 [hep-ex].

[9] CMS Collaboration, “Search for heavy gauge $W'$ boson in events with an energetic lepton and large missing transverse momentum at $\sqrt{s}=13$ TeV,” Phys. Lett. B 770 (2017) 278–301, arXiv:1612.09274 [hep-ex].

[10] ATLAS Collaboration, “Search for high-mass new phenomena in the dilepton final state using proton-proton collisions at $\sqrt{s}=13$ TeV with the ATLAS detector,” Phys. Lett. B 761 (2016) 372–392, arXiv:1607.03669 [hep-ex].
[11] ATLAS Collaboration, “Search for new resonances in events with one lepton and missing transverse momentum in $pp$ collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector,” *Phys. Lett. B* 762 (2016) 334–352, arXiv:1606.03977 [hep-ex]

[12] D0 Collaboration, V. M. Abazov et al., “Measurement of the W Boson Mass with the D0 Detector,” *Phys. Rev. Lett.* 108 (2012) 151804, arXiv:1203.0293 [hep-ex]

[13] CDF Collaboration, T. Aaltonen et al., “Precise measurement of the $W$-boson mass with the CDF II detector,” *Phys. Rev. Lett.* 108 (2012) 151803, arXiv:1203.0275 [hep-ex]

[14] ATLAS Collaboration, “Measurement of the $W$-boson mass in $pp$ collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector,” *Eur. Phys. J. C* 78 no. 2, (2018) 110, arXiv:1701.07240 [hep-ex]

[15] R. Hamberg, W. L. van Neerven, and T. Matsuura, “A complete calculation of the order $\alpha_s^2$ correction to the Drell-Yan $K$ factor,” *Nucl. Phys. B* 359 (1991) 343–405. [Erratum: Nucl. Phys. B644,403(2002)].

[16] R. V. Harlander and W. B. Kilgore, “Next-to-next-to-leading order Higgs production at hadron colliders,” *Phys. Rev. Lett.* 88 (2002) 201801, arXiv:hep-ph/0201206 [hep-ph]

[17] C. Anastasiou, L. J. Dixon, K. Melnikov, and F. Petriello, “High precision QCD at hadron colliders: Electroweak gauge boson rapidity distributions at NNLO,” *Phys. Rev. D* 69 (2004) 094008, arXiv:hep-ph/0312266 [hep-ph]

[18] K. Melnikov and F. Petriello, “The $W$ boson production cross section at the LHC through $O(\alpha_s^2)$,” *Phys. Rev. Lett.* 96 (2006) 231803, arXiv:hep-ph/0603182 [hep-ph]

[19] K. Melnikov and F. Petriello, “Electroweak gauge boson production at hadron colliders through $O(\alpha_s^3)$,” *Phys. Rev. D* 74 (2006) 114017, arXiv:hep-ph/0609070 [hep-ph]

[20] S. Catani, L. Cieri, G. Ferrera, D. de Florian, and M. Grazzini, “Vector boson production at hadron colliders: a fully exclusive QCD calculation at NNLO,” *Phys. Rev. Lett.* 103 (2009) 082001, arXiv:0903.2120 [hep-ph]

[21] S. Catani, G. Ferrera, and M. Grazzini, “$W$ Boson Production at Hadron Colliders: The Lepton Charge Asymmetry in NNLO QCD,” *JHEP* 05 (2010) 006, arXiv:1002.3115 [hep-ph]

[22] C. Duhr, F. Dulat, and B. Mistlberger, “Drell-Yan Cross Section to Third Order in the Strong Coupling Constant,” *Phys. Rev. Lett.* 125 no. 17, (2020) 172001, arXiv:2001.07717 [hep-ph]

[23] C. Duhr, F. Dulat, and B. Mistlberger, “Charged current Drell-Yan production at $N^3$LO,” *JHEP* 11 (2020) 143, arXiv:2007.13313 [hep-ph]

[24] S. Dittmaier and M. Krämer, “Electroweak radiative corrections to $W$ boson production at hadron colliders,” *Phys. Rev. D* 65 (2002) 073007, arXiv:hep-ph/0109062 [hep-ph]

[25] U. Baur and D. Wackeroth, “Electroweak radiative corrections to $p\bar{p} \to W^\pm \to \ell^\pm \nu$ beyond the pole approximation,” *Phys. Rev. D* 70 (2004) 073015, arXiv:hep-ph/0405191 [hep-ph]
[26] V. A. Zykunov, “Radiative corrections to the Drell-Yan process at large dilepton invariant masses,” Phys. Atom. Nucl. 69 (2006) 1522 [Yad. Fiz.69,1557(2006)].

[27] A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, G. Nanava, and R. Sadykov, “One-loop corrections to the Drell-Yan process in SANC. I. The Charged current case,” Eur. Phys. J. C 46 (2006) 407–412, arXiv:hep-ph/0506110 [hep-ph] [Erratum: Eur. Phys. J.C50,505(2007)].

[28] C. M. Carloni Calame, G. Montagna, O. Nicrosini, and A. Vicini, “Precision electroweak calculation of the charged current Drell-Yan process,” JHEP 12 (2006) 016, arXiv:hep-ph/0609170 [hep-ph].

[29] U. Baur, O. Brein, W. Hollik, C. Schappacher, and D. Wackeroth, “Electroweak radiative corrections to neutral current Drell-Yan processes at hadron colliders,” Phys. Rev. D 65 (2002) 033007, arXiv:hep-ph/0108274 [hep-ph].

[30] V. A. Zykunov, “Weak radiative corrections to Drell-Yan process for large invariant mass of di-lepton pair,” Phys. Rev. D 75 (2007) 073019, arXiv:hep-ph/0509315 [hep-ph].

[31] C. M. Carloni Calame, G. Montagna, O. Nicrosini, and A. Vicini, “Precision electroweak calculation of the production of a high transverse-momentum lepton pair at hadron colliders,” JHEP 10 (2007) 109, arXiv:0710.1722 [hep-ph].

[32] A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, G. Nanava, and R. Sadykov, “One-loop corrections to the Drell–Yan process in SANC. (II). The Neutral current case,” Eur. Phys. J. C 54 (2008) 451–460, arXiv:0711.0625 [hep-ph].

[33] A. Kotikov, J. H. Kuhn, and O. Veretin, “Two-Loop Formfactors in Theories with Mass Gap and Z-Boson Production,” Nucl. Phys. B 788 (2008) 47–62, arXiv:hep-ph/0703013 [HEP-PH].

[34] W. B. Kilgore and C. Sturm, “Two-Loop Virtual Corrections to Drell-Yan Production at order $\alpha_s \alpha^3$,” Phys. Rev. D 85 (2012) 033005, arXiv:1107.4798 [hep-ph].

[35] S. Dittmaier, A. Huss, and C. Schwinn, “Mixed QCD-electroweak $O(\alpha_s \alpha)$ corrections to Drell-Yan processes in the resonance region: pole approximation and non-factorizable corrections,” Nucl. Phys. B 885 (2014) 318–372, arXiv:1403.3216 [hep-ph].

[36] R. Bonciani, F. Buccioni, R. Mondini, and A. Vicini, “Double-real corrections at $O(\alpha \alpha_s)$ to single gauge boson production,” Eur. Phys. J. C 77 no. 3, (2017) 187, arXiv:1611.00645 [hep-ph].

[37] R. Bonciani, F. Buccioni, N. Rana, I. Triscari, and A. Vicini, “NNLO QCD×EW corrections to Z production in the $q\bar{q}$ channel,” Phys. Rev. D 101 no. 3, (2020) 031301, arXiv:1911.06200 [hep-ph].

[38] R. Bonciani, F. Buccioni, N. Rana, and A. Vicini, “Next-to-Next-to-Leading Order Mixed QCD-Electroweak Corrections to on-Shell Z Production,” Phys. Rev. Lett. 125 no. 23, (2020) 232004, arXiv:2007.06518 [hep-ph].
[39] L. Cieri, G. Ferrera, and G. F. R. Sborlini, “Combining QED and QCD transverse-momentum resummation for Z boson production at hadron colliders,” *JHEP* **08** (2018) 165, arXiv:1805.11948 [hep-ph]

[40] D. de Florian, M. Der, and I. Fabre, “QCD⊕QED NNLO corrections to Drell Yan production,” *Phys. Rev. D* **98** no. 9, (2018) 094008, arXiv:1805.12214 [hep-ph]

[41] M. Delto, M. Jaquier, K. Melnikov, and R. Röntsch, “Mixed QCD⊗QED corrections to on-shell Z boson production at the LHC,” arXiv:1909.08428 [hep-ph]

[42] L. Cieri, D. de Florian, M. Der, and J. Mazzitelli, “Mixed QCD⊗QED corrections to exclusive Drell Yan production using the $q_T$-subtraction method,” *JHEP* **09** (2020) 155, arXiv:2005.01315 [hep-ph]

[43] L. Buonocore, M. Grazzini, S. Kallweit, C. Savoini, and F. Tramontano, “Mixed QCD-EW corrections to $pp\to\ell\nu_\ell+X$ at the LHC,” arXiv:2102.12539 [hep-ph]

[44] R. K. Ellis, G. Martinelli, and R. Petronzio, “Lepton Pair Production at Large Transverse Momentum in Second Order QCD,” *Nucl. Phys. B* **211** (1983) 106–138.

[45] P. B. Arnold and M. H. Reno, “The Complete Computation of High $p_T$ W and Z Production in 2nd Order QCD,” *Nucl. Phys. B* **319** (1989) 37–71 [Erratum: Nucl. Phys.B330,284(1990)].

[46] R. J. Gonsalves, J. Pawlowski, and C.-F. Wai, “{QCD} Radiative Corrections to Electroweak Boson Production at Large Transverse Momentum in Hadron Collisions,” *Phys. Rev. D* **40** (1989) 2245.

[47] E. Mirkes, “Angular decay distribution of leptons from W bosons at NLO in hadronic collisions,” *Nucl. Phys. B* **387** (1992) 3–85.

[48] E. Mirkes and J. Ohnemus, “Angular distributions of Drell-Yan lepton pairs at the Tevatron: Order $\alpha – s^2$ corrections and Monte Carlo studies,” *Phys. Rev. D* **51** (1995) 4891–4904, arXiv:hep-ph/9412289

[49] R. Boughezal, C. Focke, X. Liu, and F. Petriello, “W-boson production in association with a jet at next-to-next-to-leading order in perturbative QCD,” *Phys. Rev. Lett.* **115** no. 6, (2015) 062002, arXiv:1504.02131 [hep-ph]

[50] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss, and T. A. Morgan, “Precise QCD predictions for the production of a Z boson in association with a hadronic jet,” *Phys. Rev. Lett.* **117** no. 2, (2016) 022001, arXiv:1507.02850 [hep-ph]

[51] R. Boughezal, J. M. Campbell, R. K. Ellis, C. Focke, W. T. Giele, X. Liu, and F. Petriello, “Z-boson production in association with a jet at next-to-next-to-leading order in perturbative QCD,” *Phys. Rev. Lett.* **116** no. 15, (2016) 152001, arXiv:1512.01291 [hep-ph]

[52] R. Boughezal, X. Liu, and F. Petriello, “W-boson plus jet differential distributions at NNLO in QCD,” *Phys. Rev. D* **94** no. 11, (2016) 113009, arXiv:1602.06965 [hep-ph]
[53] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss, and D. M. Walker, “Next-to-Next-to-Leading-Order QCD Corrections to the Transverse Momentum Distribution of Weak Gauge Bosons,” Phys. Rev. Lett. **120** no. 12, (2018) 122001, arXiv:1712.07543 [hep-ph].

[54] Y. L. Dokshitzer, D. Diakonov, and S. I. Troian, “On the Transverse Momentum Distribution of Massive Lepton Pairs,” Phys. Lett. B **79** (1978) 269–272.

[55] G. Parisi and R. Petronzio, “Small Transverse Momentum Distributions in Hard Processes,” Nucl. Phys. B **154** (1979) 427–440.

[56] J. C. Collins, D. E. Soper, and G. F. Sterman, “Transverse Momentum Distribution in Drell-Yan Pair and W and Z Boson Production,” Nucl. Phys. B **250** (1985) 199–224.

[57] G. Bozzi, S. Catani, D. de Florian, and M. Grazzini, “Transverse-momentum resummation and the spectrum of the Higgs boson at the LHC,” Nucl. Phys. B **737** (2006) 73–120, arXiv:hep-ph/0508068 [hep-ph].

[58] S. Catani and M. Grazzini, “QCD transverse-momentum resummation in gluon fusion processes,” Nucl. Phys. B **845** (2011) 297–323, arXiv:1011.3918 [hep-ph].

[59] P. F. Monni, E. Re, and P. Torrielli, “Higgs Transverse-Momentum Resummation in Direct Space,” Phys. Rev. Lett. **116** no. 24, (2016) 242001, arXiv:1604.02198 [hep-ph].

[60] G. Bozzi, S. Catani, G. Ferrera, D. de Florian, and M. Grazzini, “Transverse-momentum resummation: A Perturbative study of Z production at the Tevatron,” Nucl. Phys. B **815** (2009) 174–197, arXiv:0812.2862 [hep-ph].

[61] G. Bozzi, S. Catani, G. Ferrera, D. de Florian, and M. Grazzini, “Production of Drell-Yan lepton pairs in hadron collisions: Transverse-momentum resummation at next-to-next-to-leading logarithmic accuracy,” Phys. Lett. B **696** (2011) 207–213, arXiv:1007.2351 [hep-ph].

[62] A. Banfi, M. Dasgupta, S. Marzani, and L. Tomlinson, “Predictions for Drell-Yan φ* and QT observables at the LHC,” Phys. Lett. B **715** (2012) 152–156, arXiv:1205.4760 [hep-ph].

[63] M. Guzzi, P. M. Nadolsky, and B. Wang, “Nonperturbative contributions to a resummed leptonic angular distribution in inclusive neutral vector boson production,” Phys. Rev. D **90** no. 1, (2014) 014030, arXiv:1309.1393 [hep-ph].

[64] S. Catani, D. de Florian, G. Ferrera, and M. Grazzini, “Vector boson production at hadron colliders: transverse-momentum resummation and leptonic decay,” JHEP **12** (2015) 047, arXiv:1507.06937 [hep-ph].

[65] F. Coradeschi and T. Cridge, “reSolve — A transverse momentum resummation tool,” Comput. Phys. Commun. **238** (2019) 262–294, arXiv:1711.02083 [hep-ph].

[66] W. Bizoń, X. Chen, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, P. F. Monni, E. Re, L. Rottoli, and P. Torrielli, “Fiducial distributions in Higgs and Drell-Yan production at N^3LL+NNLO,” JHEP **12** (2018) 132, arXiv:1805.05916 [hep-ph].
[67] W. Bizon, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, P. F. Monni, E. Re, L. Rottoli, and D. M. Walker, “The transverse momentum spectrum of weak gauge bosons at N^3LL+NNLO,” arXiv:1905.05171 [hep-ph].

[68] S. Alioli, C. W. Bauer, A. Broggio, A. Gavardi, S. Kallweit, M. A. Lim, R. Nagar, D. Napoletano, and L. Rottoli, “Matching NNLO to parton shower using N^3LL colour-singlet transverse momentum resummation in GENEVA,” arXiv:2102.08390 [hep-ph].

[69] T. Becher and M. Neubert, “Drell-Yan Production at Small q_T, Transverse Parton Distributions and the Collinear Anomaly,” Eur. Phys. J. C 71 (2011) 1665, arXiv:1007.4005 [hep-ph].

[70] T. Becher, M. Neubert, and D. Wilhelm, “Electroweak Gauge-Boson Production at Small q_T: Infrared Safety from the Collinear Anomaly,” JHEP 02 (2012) 124, arXiv:1109.6027 [hep-ph].

[71] M. A. Ebert and F. J. Tackmann, “Resummation of Transverse Momentum Distributions in Distribution Space,” JHEP 02 (2017) 110, arXiv:1611.08610 [hep-ph].

[72] T. Becher and M. Hager, “Event-Based Transverse Momentum Resummation,” arXiv:1904.08325 [hep-ph].

[73] M. A. Ebert, J. K. L. Michel, I. W. Stewart, and F. J. Tackmann, “Drell-Yan q_T Resummation of Fiducial Power Corrections at N^3LL,” arXiv:2006.11382 [hep-ph].

[74] T. Becher and T. Neumann, “Fiducial q_T resummation of color-singlet processes at N^3LL+NNLO,” arXiv:2009.11437 [hep-ph].

[75] G. Billis, B. Dehnadi, M. A. Ebert, J. K. L. Michel, and F. J. Tackmann, “The Higgs p_T Spectrum and Total Cross Section with Fiducial Cuts at N^3LL’+N^3LO,” arXiv:2102.08039 [hep-ph].

[76] J. Collins, “Foundations of perturbative QCD,” Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 32 (2011) 1–624.

[77] J. C. Collins and T. C. Rogers, “Equality of Two Definitions for Transverse Momentum Dependent Parton Distribution Functions,” Phys. Rev. D 87 no. 3, (2013) 034018, arXiv:1210.2100 [hep-ph].

[78] J. Collins and T. Rogers, “Understanding the large-distance behavior of transverse-momentum-dependent parton densities and the Collins-Soper evolution kernel,” Phys. Rev. D 91 no. 7, (2015) 074020 arXiv:1412.3820 [hep-ph].

[79] I. Scimemi and A. Vladimirov, “Analysis of vector boson production within TMD factorization,” Eur. Phys. J. C 78 no. 2, (2018) 89 arXiv:1706.01473 [hep-ph].

[80] V. Bertone, I. Scimemi, and A. Vladimirov, “Extraction of unpolarized quark transverse momentum dependent parton distributions from Drell-Yan/Z-boson production,” JHEP 06 (2019) 028 arXiv:1902.08474 [hep-ph].
[81] A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, F. Delcarro, F. Piacenza, and M. Radici, “Transverse-momentum-dependent parton distributions up to N^{3}LL from Drell-Yan data,” JHEP 07 (2020) 117, arXiv:1912.07550 [hep-ph].

[82] S. Camarda et al., “DYTurbo: Fast predictions for Drell-Yan processes,” Eur. Phys. J. C 80 no. 3, (2020) 251, arXiv:1910.07049 [hep-ph] [Erratum: Eur.Phys.J.C 80, 440 (2020)].

[83] S. Catani and M. Grazzini, “An NNLO subtraction formalism in hadron collisions and its application to Higgs boson production at the LHC,” Phys. Rev. Lett. 98 (2007) 222002, arXiv:hep-ph/0703012 [hep-ph].

[84] J. Ablinger, J. Blümlein, M. Round, and C. Schneider, “Numerical Implementation of Harmonic Polylogarithms to Weight w = 8,” Comput. Phys. Commun. 240 (2019) 189–201, arXiv:1809.07084 [hep-ph].

[85] S. Camarda et al., “DYTurbo.” https://dyturbo.hepforge.org/

[86] G. Bozzi, S. Catani, D. de Florian, and M. Grazzini, “Higgs boson production at the LHC: Transverse-momentum resummation and rapidity dependence,” Nucl. Phys. B 791 (2008) 1–19, arXiv:0705.3887 [hep-ph].

[87] G. Bozzi, S. Catani, D. de Florian, and M. Grazzini, “The q(T) spectrum of the Higgs boson at the LHC in QCD perturbation theory,” Phys. Lett. B 564 (2003) 65–72, arXiv:hep-ph/0302104.

[88] S. Catani, D. de Florian, and M. Grazzini, “Universality of nonleading logarithmic contributions in transverse momentum distributions,” Nucl. Phys. B 596 (2001) 299–312, arXiv:hep-ph/0008184 [hep-ph].

[89] S. Catani, L. Cieri, D. de Florian, G. Ferrera, and M. Grazzini, “Universality of transverse-momentum resummation and hard factors at the NNLO,” Nucl. Phys. B 881 (2014) 414–443, arXiv:1311.1654 [hep-ph].

[90] S. Catani, L. Cieri, D. de Florian, G. Ferrera, and M. Grazzini, “Vector boson production at hadron colliders: hard-collinear coefficients at the NNLO,” Eur. Phys. J. C 72 (2012) 2195, arXiv:1209.0158 [hep-ph].

[91] T. Gehrmann, T. Lubbert, and L. L. Yang, “Transverse parton distribution functions at next-to-next-to-leading order: the quark-to-quark case,” Phys. Rev. Lett. 109 (2012) 242003, arXiv:1209.0682 [hep-ph].

[92] Y. Li and H. X. Zhu, “Bootstrapping Rapidity Anomalous Dimensions for Transverse-Momentum Resummation,” Phys. Rev. Lett. 118 no. 2, (2017) 022004, arXiv:1604.01404 [hep-ph].

[93] J. M. Henn, G. P. Korchemsky, and B. Mistlberger, “The full four-loop cusp anomalous dimension in $\mathcal{N} = 4$ super Yang-Mills and QCD,” JHEP 04 (2020) 018, arXiv:1911.10174 [hep-th].
A. von Manteuffel, E. Panzer, and R. M. Schabinger, “Cusp and collinear anomalous dimensions in four-loop QCD from form factors,” Phys. Rev. Lett. 124 no. 16, (2020) 162001, arXiv:2002.04617 [hep-ph].

M.-x. Luo, T.-Z. Yang, H. X. Zhu, and Y. J. Zhu, “Quark Transverse Parton Distribution at the Next-to-Next-to-Next-to-Leading Order,” Phys. Rev. Lett. 124 no. 9, (2020) 092001, arXiv:1912.05778 [hep-ph].

M. A. Ebert, B. Mistlberger, and G. Vita, “Transverse momentum dependent PDFs at N^3LO,” JHEP 09 (2020) 146 arXiv:2006.05329 [hep-ph].

P. A. Baikov, K. G. Chetyrkin, A. V. Smirnov, V. A. Smirnov, and M. Steinhauser, “Quark and gluon form factors to three loops,” Phys. Rev. Lett. 102 (2009) 212002, arXiv:0902.3519 [hep-ph].

R. N. Lee, A. V. Smirnov, and V. A. Smirnov, “Analytic Results for Massless Three-Loop Form Factors,” JHEP 04 (2010) 020, arXiv:1001.2887 [hep-ph].

T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli, and C. Studerus, “Calculation of the quark and gluon form factors to three loops in QCD,” JHEP 06 (2010) 094, arXiv:1004.3653 [hep-ph].

S. Albino, “Analytic Continuation of Harmonic Sums,” Phys. Lett. B 674 (2009) 41–48 arXiv:0902.2148 [hep-ph].

J. Kuipers, T. Ueda, J. A. M. Vermaseren, and J. Vollinga, “FORM version 4.0,” Comput. Phys. Commun. 184 (2013) 1453–1467, arXiv:1203.6543 [cs.SC].

J. A. M. Vermaseren, “Harmonic sums, Mellin transforms and integrals,” Int. J. Mod. Phys. A 14 (1999) 2037–2076 arXiv:hep-ph/9806280.

E. Remiddi and J. A. M. Vermaseren, “Harmonic polylogarithms,” Int. J. Mod. Phys. A 15 (2000) 725–754 arXiv:hep-ph/9905237.

A. Vogt, “Efficient evolution of unpolarized and polarized parton distributions with QCD-PEGASUS,” Comput. Phys. Commun. 170 (2005) 65–92, arXiv:hep-ph/0408244.

S. Catani, M. L. Mangano, P. Nason, and L. Trentadue, “The Resummation of soft gluons in hadronic collisions,” Nucl. Phys. B 478 (1996) 273–310, arXiv:hep-ph/9604351.

E. Laenen, G. F. Sterman, and W. Vogelsang, “Higher order QCD corrections in prompt photon production,” Phys. Rev. Lett. 84 (2000) 4296–4299, arXiv:hep-ph/0002078.

A. Kulesza, G. F. Sterman, and W. Vogelsang, “Joint resummation in electroweak boson production,” Phys. Rev. D 66 (2002) 014011, arXiv:hep-ph/0202251.

A. V. Konychev and P. M. Nadolsky, “Universality of the Collins-Soper-Sterman nonperturbative function in gauge boson production,” Phys. Lett. B 633 (2006) 710–714 arXiv:hep-ph/0506225 [hep-ph].

J. M. Campbell and R. K. Ellis, “MCFM for the Tevatron and the LHC,” Nucl. Phys. Proc. Suppl. 205-206 (2010) 10–15 arXiv:1007.3492 [hep-ph].
[10] **NNPDF** Collaboration, R. D. Ball *et al.*, “Parton distributions from high-precision collider data,” *Eur. Phys. J. C* 77 no. 10, (2017) 663, arXiv:1706.00428 [hep-ph].

[11] D. A. Dicus and S. S. D. Willenbrock, “Radiative Corrections to the Ratio of Z and W Boson Production,” *Phys. Rev. D* 34 (1986) 148.

[12] T. Gehrmann and A. Primo, “The three-loop singlet contribution to the massless axial-vector quark form factor,” *Phys. Lett. B* 816 (2021) 136223, arXiv:2102.12880 [hep-ph].

[13] J. C. Collins and D. E. Soper, “Angular Distribution of Dileptons in High-Energy Hadron Collisions,” *Phys. Rev. D* 16 (1977) 2219.

[14] S. Forte, A. Isgrò, and G. Vita, “Do we need N3LO Parton Distributions?,” *Phys. Lett. B* 731 (2014) 136–140, arXiv:1312.6688 [hep-ph].

[15] S. Moch, B. Ruijl, T. Ueda, J. A. M. Vermaseren, and A. Vogt, “Four-Loop Non-Singlet Splitting Functions in the Planar Limit and Beyond,” *JHEP* 10 (2017) 041, arXiv:1707.08315 [hep-ph].

[16] S. Moch, B. Ruijl, T. Ueda, J. A. M. Vermaseren, and A. Vogt, “On quartic colour factors in splitting functions and the gluon cusp anomalous dimension,” *Phys. Lett. B* 782 (2018) 627–632, arXiv:1805.09638 [hep-ph].

[17] S. Catani, L. Cieri, D. de Florian, G. Ferrera, and M. Grazzini, “Diphoton production at the LHC: a QCD study up to NNLO,” *JHEP* 04 (2018) 142, arXiv:1802.02095 [hep-ph].

[18] S. Catani and B. R. Webber, “Infrared safe but infinite: Soft gluon divergences inside the physical region,” *JHEP* 10 (1997) 005, arXiv:hep-ph/9710333.

[19] M. A. Ebert, I. Moult, I. W. Stewart, F. J. Tackmann, G. Vita, and H. X. Zhu, “Subleading power rapidity divergences and power corrections for $q_T$,” *JHEP* 04 (2019) 123, arXiv:1812.08189 [hep-ph].

[20] L. Cieri, C. Oleari, and M. Rocco, “Higher-order power corrections in a transverse-momentum cut for colour-singlet production at NLO,” *Eur. Phys. J. C* 79 no. 10, (2019) 852, arXiv:1906.09044 [hep-ph].

[21] M. A. Ebert and F. J. Tackmann, “Impact of isolation and fiducial cuts on $q_T$ and N-jettiness subtractions,” *JHEP* 03 (2020) 158, arXiv:1911.08486 [hep-ph].

[22] L. Buonocore, M. Grazzini, and F. Tramontano, “The $q_T$ subtraction method: electroweak corrections and power suppressed contributions,” *Eur. Phys. J. C* 80 no. 3, (2020) 254, arXiv:1911.10166 [hep-ph].

[23] C. Oleari and M. Rocco, “Power corrections in a transverse-momentum cut for vector-boson production at NNLO: the $qg$-initiated real-virtual contribution,” *Eur. Phys. J. C* 81 no. 2, (2021) 183, arXiv:2012.10538 [hep-ph].