For the introduction, you can use:

**INTRODUCTION**

In the field of motion dynamics, there are generally two methods used, Kinematics and Kinetics. Kinematics is the study of motion, describing how a hierarchical skeletal structure move irrespective of the real world forces that may or may not have an effective impact on the motion gaits. Whereas, the study involving the effect of real world forces and torques on the dynamic motion characteristics of a structure is called Kinetics. In the field of computer animation both principles are used extensively to produce realistic dynamic motions of characters. However in this paper the focus is on the use of kinematics in motion dynamics.

The research on kinematics and kinetics is done in various fields from Medical science (Apriantono, et al., 2006, Roepstorff, et al., 1999 and Jonathan, et al., 2008), biomechanics (Robin, et al., 2006 and Bruce, et al., 2003), aviation (Stirling, et al., 2009), robotics (Kulpa, et al., 2005, Kuffner, et al., 2005, Tejomurtula, et al., 1999, Xia, et al., 2005 and Li, et al., 2001) and computer animation (Grochow, et al., 2004 and Rose III, et al., 2001). The common use of kinematics in the field of computer animation is for manipulating character’s limbs which are defined through hierarchal tree of rotational joints connected with each other through rigid links (Parent, 2008). Two types of kinematics based approaches are used in positioning each hierarchy or rotational joint limbs, one is known as Forward Kinematics (FK) and the other is Inverse Kinematics (IK).
1.2 Inverse Kinematics (IK)

In IK system the animator simply specifies the final desired position of the limb using an end-effector and all the joints rotation are calculated automatically to place them at desired location. With IK move the last child in the hierarchy and all its parent joints will rotate, in Inverse motion or kinematics. The IK system in contrast to FK, uses a much direct approach in this case as shown in (Fig. 2). The IK end-effector is directly translated to its desired place, in Figure 2 at the glass and all the joint rotations are computed and adjusted automatically according to the placement of the IK end-effector.

![Fig. 2: IK Translation of Arm towards the glass on the table.](image)

It may seem that IK system is appropriate for moving arm or leg limbs but in situations where the character is falling, or just swinging its limbs then the IK proves to yield incorrect tangent and arcs for realistic motion, in such cases FK system is required. Thus in a 3D production environment both IK and FK approaches are used immensely and are equally important.

1.3 The problem with current systems.

The problem with using both these systems is that they cannot be used simultaneously or concurrently as IK systems needs explicit control over the joint structure, therefore once the joint have IK attached to them, they no longer are able to rotate with plain FK system and similarly, when FK joint rotations are required, then the joint cannot have IK.

One way to solve this problem is to use a 3 joint hierarchy structure of each limb. The first joint chain is controlled by FK system, the second joint chain is controlled by IK system and the third joint chain is attached to the mesh for deformation. Both IK and FK joint chains are constrained to the third joint chain and a switch is used to specify which control system the animator wants to use (Allen, et al., 2008). The problem with this methodology is that being a complex joint structure, the switching between FK or IK system creates a popping or jumping effect. For example, when the animator translates the arm using IK, the FK is disabled and the arm is moved to its desired place, but when the animator switches back to FK, the third joint jumps back the original position of FK joint chain. Then when the animator again switches to IK, the FK joint becomes inactive and the third joint chain jumps to its previous IK position. Thus this approach is inappropriate and not feasible.

1.4 The proposed system

In this research work the system of single joint chain with both IK and FK system is implemented on same hierarchy. The user selects the system to be used and internally the switch is made preserving the initial and final location for both IK and FK without creating the popping or jumping effect, producing a seamless transition between the two systems. The (Fig. 3) shows the overview of the approach. We use Jacobian to determine the joint rotations and location between the IK and FK switch automatically and this way moving the corresponding kinematic joint system to the desired place automatically eliminating the jumping/popping effect and reducing the complexity of the system.

![Fig. 3: Overview of the IK / FK matching system](image)

2. MATERIAL AND METHOD

Our technique uses a single joint chain driven by both FK and IK system with a switch parameter to determine the currently active system as shown in
Fig. 3, both FK and IK are built on the same joint chain. When FK system is active the IK is disabled automatically and when the IK system is used then the FK system becomes inactive by the system. When the system is switched from FK to IK, we simply determine the position of the end of the joint or the last joint in the system, and move the IK end-effector to the same coordinates in 3D space. Then the IK solver is enabled and the joint remain in the same position without popping effect. When the system is switched from the IK to FK, now the real problem occurs, as the joints were controlled by the position of end-effector as illustrated in (Fig. 4). When the end-effector becomes inactive, the joint will go back to their original FK position. To solve this we used Jacobian transpose mechanism to determine the joint positions and then rotate each joint automatically to their new positions relevant to the IK effector, thus creating a seamless match and solving the problem.

3. COMPUTING FORWARD KINEMATICS

In FK the joints are always rotated towards their desired goal manually by the animator. The traversal of the joint chain follows a depth-first pattern from the root to leaf node. The process of computing FK motion in world space coordination is based on joint Degree Of Freedom (DOF) values as shown in (Fig. 5).

As we already have \( e = [ e_1, e_2, ..., e_N ] \) from the FK system, and we want \( \theta = [ \theta_1, \theta_2, ..., \theta_M ] \), therefore to determine the joint angle inversely by using equation (2).

\[
\theta = f^{-1}(e)
\]  

4. COMPUTING INVERSE KINEMATICS

To determine the position of upper joints of the hierarchy when the user switches the IK system to FK mode with end-effector at a specific goal position, we use the Jacobian which involves forming the matrix of partial derivatives (Parent, R., 2008). The concept of inverse kinematics is to compute the vector of joint DOFs that will cause the end effector to reach some desired goal state as shown in (Fig. 6).

![Fig. 4: IK chain with base and end-effector](image-url)

![Fig. 5: A joint structure using FK system for motion in world space.](image-url)

![Fig. 6: IK end-effector moved towards a goal.](image-url)

![Fig. 7: FK and IK trajectory](image-url)
\[ J = \frac{de}{d\theta} \]

\[ d\theta = J d\theta \]

\[ \theta = [\theta_1 \ \theta_2 \ \ldots \ \theta_M]^T \]

\[ e = [e_x \ e_y \ e_z]^T \]

\[ \theta = f(e) \]

\[ J = \begin{bmatrix}
\frac{\partial e_x}{\partial \theta_1} & \frac{\partial e_x}{\partial \theta_2} & \ldots & \frac{\partial e_x}{\partial \theta_M} \\
\frac{\partial e_y}{\partial \theta_1} & \frac{\partial e_y}{\partial \theta_2} & \ldots & \frac{\partial e_y}{\partial \theta_M} \\
\frac{\partial e_z}{\partial \theta_1} & \frac{\partial e_z}{\partial \theta_2} & \ldots & \frac{\partial e_z}{\partial \theta_M}
\end{bmatrix} \]

The Jacobian Matrix thus is obtained as

\[ \frac{de}{\theta} \approx \begin{bmatrix}
\frac{\partial e_x}{\partial \theta_1} & \frac{\partial e_y}{\partial \theta_1} & \frac{\partial e_z}{\partial \theta_1} \\
\frac{\partial e_x}{\partial \theta_2} & \frac{\partial e_y}{\partial \theta_2} & \frac{\partial e_z}{\partial \theta_2} \\
\frac{\partial e_x}{\partial \theta_M} & \frac{\partial e_y}{\partial \theta_M} & \frac{\partial e_z}{\partial \theta_M}
\end{bmatrix}^T \]

Recall that \( \theta = f^{-1}(e) \) and \( de = J d\theta \), thus we obtained \( d\theta = J^{-1} de \).

4.1 Computing Jacobian numerically

Let us examine one column of the Jacobian Matrix

\[ \frac{de}{\partial \theta_1} \approx \begin{bmatrix}
\frac{\partial e_x}{\partial \theta_1} \\
\frac{\partial e_y}{\partial \theta_1} \\
\frac{\partial e_z}{\partial \theta_1}
\end{bmatrix} \]

We can add a small \( \Delta \theta \) to \( \theta \), then we can calculate how the end effector moves: \( \Delta e = e' - e \). Now we have equation (5) which can be used to fill the Jacobian Matrix.

4.2 Inverting the Jacobian Pseudoinverse

We then used the pseudo inverse to find a matrix that effectively inverts a non-square matrix by further solving the equation 3 as we already have computed Jacobian

\[ \text{de} = J \cdot d\theta \]

\[ J^T \cdot \text{de} = (J^T J) \cdot d\theta \]

\[ (J^T J)^{-1} J^T \cdot \text{de} = d\theta \]

\[ J^* \cdot \text{de} = d\Delta \theta \]

\[ J^* = (J^T J)^{-1} J^T \]

Equation (6) now represents the pseudoinverse of \( J \) and it maps the desired motion of the end effector to the required changes of the joint angles.

4.3 Inverting the Jacobian—Jacobian Transpose

Another technique is just to use the transpose of the Jacobian matrix. As the Jacobian is already an approximation to \( f() \) it is much faster, but the quality of the resultant is found to be inaccurate and thus for our solution we preferred quality over performance, therefore pseudo inverse method was implemented to produce higher quality results.

4.4 Solving IK—Incremental Changes

Forward Kinematics is a nonlinear type of motion which implies that the Jacobian can only be used as an approximation that is valid near the current configuration, so the process of computing the Jacobian must be repeated while taking small iterative steps towards the target goal until we reach the desired location. The final algorithm for the said process is given in (Fig 8).

Algorithm: Solving IK—Algorithm of the Jacobian Method

1. \textbf{while} (e is too far from g) {
2. \hspace{1em} compute the Jacobian matrix J
3. \hspace{1em} compute the pseudoinverse of the Jacobian matrix \( J^* \)
4. \hspace{1em} compute the change in joint DOFs:
5. \hspace{2em} \( \Delta \theta = J^* \cdot \Delta e \)
6. \hspace{1em} apply the changes to DOFs, move a small step of \( \alpha \Delta \theta \): \( \theta = \theta + \alpha \Delta \theta \)
7. \textbf{end while}
}

Fig 8: Algorithm of Solving IK using Jacobian Method

5. RESULTS

The implementation of our system provides an efficient and practically feasible system of switching between an inverse kinematics and forward kinematics based joint rig of any 3D virtual characters. The initial biped control rig was created using the procedural mechanism described in our previous work (Bhatti, et al., 2012). As our system uses single joint chain the complexity of creating the rig is reduced and consequently the time tradeoff in computing the motion of the rig is also automatically decreased, hence improving the overall performance of the virtual character during animation process. According to our system the user initially can start working with the character rig in either IK or FK mode. Then all that is required is to change a single attribute in the system ‘Enable IK’ to either ‘On’ for IK mode or set this variable to ‘Off’ for FK mode of motion as shown in (Fig. 9(a-g)).
In Fig. 9-A the arm rig is in standard rest pose, then the ‘switch IK’ attribute is turned on to move the arm using inverse kinematics as shown in Figure 9-B. Then the ‘Switch IK’ attribute is turned off and the rig controls change to forward kinematics without altering the position of the arm joint as shown in Figure 9-C. From here the animator can easily continue to animate the arm using forward kinematics as shown in figure 9-D and 9-E where joint ae rotated through forward kinematics. Finally the ‘Switch IK’ attribute is again turned on to transpose the rig control back to inverse kinematics and thus by doing so the rig again remains exactly at the same place without altering its location, and the IK controls appear and the animator can safely animate the rig now using the IK movement. This process can be repeated and the animator can easily animate and choose between various types of kinematics movements based on the requirements without worrying to alter the rig.

6. CONCLUSION

In this paper we attempted to solve the real world problem of IK to FK matching of joint chain for a 3D animated character without creating the popping or jumping effect. We used the mechanism of finding the Jacobian matrix $J$ and then computing its pseudoinverse $J^\dagger$ to map the orientation values of IK joint chain to FK joint and similarly from FK to IK joints. The procedure allowed us to obtain the automation desired to do the seamless matching between the two types. As mathematical procedure is already proved its application in this situation gave us the solution and proved to be efficient also. The working algorithm was developed to implement the said technique in the animation of 3D articulated characters with both IK and FK system simultaneously. The work in future can be extended to use a full-body inverse and forward kinematics system with easy switching mechanism. The complexity of the algorithm can be further explored and minimized. The switching mechanism can also be implemented in quadruped joint structure with advance controls.

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