Forecasting China's per capita energy consumption using dynamic grey model based on ARIMA model

Liang Liu¹, Jiahao Cao¹, Xi Wang Xiang¹ and Peng Zhang¹, ²*

¹School of science, Southwest University of Science and Technology, Mianyang, Sichuan, 621010, China
²School of Information and Software Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, 610054, China
*Corresponding author’s e-mail: zhangpeng@swust.edu.cn

Abstract. Energy is the cornerstone of social progress, and energy consumption is an important reference data when the government makes energy policies. Therefore, prediction of energy consumption can provide a basis for the formulation of policies. In the traditional grey system theory, the grey input is always regarded as an invariant after determined by the least square method. However, the grey input would be changed along with the change of the internal information in the grey system. As a result, the performance and accuracy of the grey model would be affected by it. Therefore, this paper proposes the ARIMA-b-GM (1,1) model, which uses the ARIMA prediction model to fit and predict the grey input and realize the dynamic change of the grey input. Then the time response sequence of the ARIMA-b-GM (1,1) model is deduced by changing the form of grey differential equation. Finally, the ARIMA-b-GM (1,1) model is applied to predict the per capita energy consumption in China. The results exhibit that ARIMA-b-GM (1,1) model has higher prediction accuracy than GM (1,1) model, NGM (1,1) model and SAIGM (1,1) model.

1. Introduction

Energy problems have always been the focus of the world's attention, for all activities of human society and the development of all walks of life are inseparable from energy support. But the development of human society has consumed large amounts of energy in recent centuries. Nowadays, global energy resources are exhausted, and serious pollution is caused to the environment in the process of energy utilization. Therefore, how to use energy scientifically and reasonably is what the human need to study. The prediction of energy consumption can better analyze the energy supply and demand situation in the future and support the formulation of energy policies. At the same time, on the basis of the balance of energy supply and demand, necessary feedback should be made on social and economic development and environmental impact so as to realize the goal of sustainable development of human society.

Since grey system theory was created in 1982, it has been studied by many scholars. In particular, the research on grey prediction model has made great progress [1-3]. Among the numerous research literatures on grey prediction model, the improvement of the model has focused by many scholars. Wang et al. [4] proposed an improved gray prediction model with unequal spacing AUGM (1,1). Ning et al. [5] corrected the modeling deviation of GM (1,1) model and established an unbiased GM (1,1) model based on the metabolic theory. Wu et al. [6] improved the GM (1,1) model for three times to reduce the prediction error for fire accidents with strong randomness. On the optimization of grey
input. Qian et al. [7] changed the grey input $b$ to $bt^a + c$. And Cui et al. [8] changed the grey input $b$ to $bt$. Then Chen et al. [9] changed the grey input $b$ to $bt + c$.

In summary, there is great significance to change the invariance of grey input. In our research work, it is found that when the grey input changes, it can be regarded as the change of time series. As we all know, ARIMA model is an important time series prediction method. Therefore, this paper utilizes ARIMA model to make the grey input have dynamic characteristics, on the basis of which the form of grey differential equation is changed, and the ARIMA-b-GM (1,1) model is proposed. Then the model is applied to the prediction of energy consumption. The comparison results illustrate that the ARIMA-b-GM (1,1) model has good prediction performance, which can provide help for the rational utilization of energy resources.

2. Overview of the GM (1,1) model and the ARIMA model

2.1 GM(1,1) model

GM (1,1) model is the most representative grey prediction model in the field of grey system. Let the original sequence be $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$. Its accumulated generating operation is $X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n))$, where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, 3, \ldots, n$. Let $z^{(1)}(k) = 0.5(x^{(1)}(k) + x^{(1)}(k-1))$, $k = 2, 3, \ldots, n$, the sequence of means immediately adjacent of $X^{(1)}$ is obtained as follows: $Z^{(0)} = (z^{(0)}(1), z^{(0)}(2), \ldots, z^{(0)}(n))$.

The grey differential equation of GM (1,1) model is

$$x^{(0)}(k) + az^{(0)}(k) = b, \quad k = 2, 3, \ldots, n.$$  \hspace{1cm} (1)

Set parameter column $P = [a, b]^T$. Then equation (1) can be written as $FP = E$, where $E = [x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)]^T, F = \begin{bmatrix} -1 & -1 & \cdots & -1 \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$. So the least squares solution to the parameter column $P$ is $P^* = (F^TF)^{-1}F^TE$. The corresponding white differential equation of formula (1) is:

$$\frac{dx^{(1)}(i)}{dt} + ax^{(1)}(i) = b.$$  \hspace{1cm} (2)

So the time response sequence of GM (1,1) model can be obtained by solving the white differential equation as follows:

$$z^{(1)}(k+1) = \left( x^{(0)}(1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a}, \quad k = 1, 2, \ldots, n, \ldots.$$  \hspace{1cm} (3)

2.2 ARIMA model

ARIMA model is a well-known time series prediction model. It can model and predict not only stationary time series, but also non-stationary time series. Through experiments, the author found that the grey input is sometimes an unstable time series, but also non-stationary time series. Therefore, this paper chooses ARIMA model to make the grey input have dynamic characteristics, making the model proposed in this paper more applicable.

In fact, ARIMA $(p, d, q)$ model is a model that combines ARMA $(p, q)$ model with the difference method, where $d$ is the order of the difference. Assuming that there is a sequence $Y_t = (y_1, y_2, \ldots, y_t), t = 1, 2, \ldots, n$. $Y_t$ makes $d$ differentials and gets a stationary time series $Y^*_t = (y^*_1, y^*_2, \ldots, y^*_m), t = 1, 2, \ldots, m$. Then the expression of ARIMA $(p, d, q)$ model is as follows [10]:
\[ y_i^* = \mu + \sum_{j=1}^{i} y_{t-j}^* + \varepsilon_i + \sum_{j=1}^{i} \theta \varepsilon_{t-j} \]  
(4)

3. ARIMA-b-GM (1,1) model

In the GM (1,1) model, the development coefficient and grey input in the grey differential equation regarded as invariable constants after they are calculated, and then substituted into the white differential equation to solve the response sequence. However, there are two problems in this approach. The first problem is that the internal information of the grey system has changed during the process of solving the problem, which leads to the change of the grey function. The second improper point is that grey differential equation is a discrete equation and white differential equation is a continuous equation, and it is not reasonable to jump directly from discrete to continuous [11]. Therefore, this paper uses the discrete ARIMA model to make the grey input dynamic, and then changes the form of grey differential equation to derive a new grey prediction model. The process of ARIMA-b-GM (1,1) model is as follows:

Step 1: Let the original sequence be

\[ x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)), \quad x^{(0)}(k) \geq 0, k = 1, 2, 3, \ldots, n. \]

According to the treatment in GM (1,1) model, the following can be obtained: \( X^{(1)} \) and \( Z^{(1)} \).

Step 2: Since the new model proposed in this paper is based on the dynamic nature of grey action, the development coefficient needs to be a numerical constant, just like the traditional GM(1,1) model. So the value of development coefficient \( a \) can be obtained according to the treatment in GM (1,1) model.

Step 3: In the introduction of this paper, some improvements on grey input are mentioned. But after their improvement, the newly defined grey action quantity is calculated by least square method. However, when the grey action amount fluctuates greatly, the credibility of the result is not always satisfactory. Therefore, in this paper, the discrete sequence of grey input is calculated by grey differential equation, and the discrete sequence is used to reflect the dynamic change of grey input. Then, the discrete sequence is fitted and predicted by ARIMA model, which makes the new model proposed in this paper more applicable.

Definition 1: The equation

\[ x^{(0)}(k) + az^{(1)}(k) = b_{k-1}, \quad k = 2, 3, \ldots, n \]  
(5)

is called the grey differential equation of ARIMA-b-GM (1,1) model.

When \( k = 2, 3, \ldots, n \), we can get

\[
\begin{align*}
    b_1 &= x^{(0)}(2) + az^{(1)}(2), \quad k = 2 \\
    b_2 &= x^{(0)}(3) + az^{(1)}(3), \quad k = 3 \\
    \vdots
    b_{n-1} &= x^{(0)}(n) + az^{(1)}(n), \quad k = n
\end{align*}
\]  
(6)

The values of \( (x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)) \) and \( (z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)) \) can be obtained from the step 1, and the value of development coefficient \( a \) can be obtained from the step 2, so the time series \( B = (b_1, b_2, b_3, \ldots, b_{n-1}) \) of grey input can be obtained according to equation (6).
Step 4: The ARIMA model is used to fit and predict the sequence $B$ obtained in the step 3, and $\hat{B} = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_{n-1}, \hat{b}_n, \hat{b}_{n+1}, \ldots)$ is obtained. $\hat{b}_i (i = 1, 2, 3, \ldots, n-1)$ is the fitting data of ARIMA model, and $\hat{b}_j (j = n, n+1, n+2, \ldots)$ is the prediction data of ARIMA model.

Step 5: From the step 1 we can get

$$x^{(0)}(k) = x^{(i)}(k) - x^{(i)}(k-1),$$

$$as^{(i)}(k) = 0.5a \left( x^{(i)}(k) + x^{(i)}(k-1) \right).$$

Substitute equations (7) and (8) into equation (5), and change the grey input $b$ in equation (5) into the grey input $\hat{b}$ obtained in step 4, and we can get

$$x^{(i)}(k) - x^{(i)}(k-1) + 0.5a \left[ x^{(i)}(k) + x^{(i)}(k-1) \right] = \hat{b}_{k-1}.$$

Combine the common factors and get

$$(1 + 0.5a)x^{(i)}(k) + (0.5a - 1)x^{(i)}(k-1) = \hat{b}_{k-1},$$

$$x^{(i)}(k) = \frac{1 - 0.5a}{1 + 0.5a} x^{(i)}(k-1) + \frac{1}{1 + 0.5a} \hat{b}_{k-1}.$$ (11)

Let

$$\mu_1 = \frac{1 - 0.5a}{1 + 0.5a}, \quad \mu_2 = \frac{1}{1 + 0.5a},$$

one obtains:

$$x^{(i)}(k) = \mu_1 x^{(i)}(k-1) + \mu_2 \hat{b}_{k-1}, \quad k = 2, 3, \ldots, n-1.$$ (13)

Definition 2: The equation

$$\hat{x}^{(i)}(k) = \mu_1 \hat{x}^{(i)}(k-1) + \mu_2 \hat{b}_{k-1},$$

is called as the ARIMA-b-GM (1,1) model.

Continue to do some work on the expression of the ARIMA-b-GM (1,1) model. According to equation (14), the following equation can be obtained:

$$\hat{x}^{(i)}(k+1) = \mu_1 \hat{x}^{(i)}(k) + \mu_2 \hat{b}_k$$

$$= \mu_1 \left( \hat{b}_1 \hat{x}^{(i)}(k-1) + \mu_2 \hat{b}_{k-1} \right) + \mu_2 \hat{b}_k$$

$$= \mu_1^{k+1} \hat{x}^{(i)}(1) + \mu_2 \left( \mu_1^{k+1} \hat{b}_1 + \mu_1^k \hat{b}_2 + \cdots + \mu_1 \hat{b}_{k-1} + \hat{b}_k \right)$$

$$= \mu_1^{k+1} \hat{x}^{(i)}(1) + \mu_2 \left( \hat{b}_k + \sum_{i=1}^{k-1} \mu_1^{i+1} \hat{b}_i \right), \quad k = 1, 2, \ldots, n-1, \ldots.$$ (15)

(It is stipulated that when $k = 1$, $\sum_{i=1}^{k-1} \mu_1^{i+1} \hat{b}_i = 0$).

Similarly, the time response sequence of ARIMA-b-GM (1,1) model can be obtained as follows:
\[ x^{(1)}(k+1) = \mu_k x^{(0)}(1) + \mu_i \left( \hat{b}_k + \sum_{i=1}^{k-1} \mu_i b_i \right), \quad k=1,2,\ldots,n-1,\ldots \]  

(16)

**Step 6:** From the step 5, the restored value of ARIMA-b-GM (1,1) model can be obtained as follows:

\[ \hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k). \]  

(17)

### 4. Model application and comparison

In this paper, China's per capita energy consumption from 2000 to 2010 was used as the fitting modeling data, and the per capita energy consumption from 2011 to 2014 was used as the verification data of the model prediction. The data of per capita energy consumption from 2000 to 2014 are shown in table 1. These data are from China statistical yearbook.

| Year | Raw data | GM | NGM | SAIGM | ARIMA-b-GM |
|------|----------|----|-----|-------|-------------|
| 2000 | 1152.600 | 1152.600 | 1152.600 | 1152.600 | 1152.600 |
| 2001 | 1182.600 | 1266.077 | 599.5248 | 1118.812 | 1212.358 |
| 2002 | 1245.200 | 1367.989 | 1029.781 | 1305.472 | 1291.380 |
| 2003 | 1426.500 | 1478.104 | 1361.180 | 1480.628 | 1370.755 |
| 2004 | 1646.900 | 1597.083 | 1616.438 | 1644.989 | 1542.276 |
| 2005 | 1810.200 | 1725.639 | 1813.047 | 1799.222 | 1746.094 |
| 2006 | 1973.100 | 1864.544 | 1964.483 | 1943.948 | 1909.854 |
| 2007 | 2128.500 | 2014.629 | 2079.755 | 2076.578 |
| 2008 | 2200.200 | 2176.795 | 2207.193 | 2241.173 |
| 2009 | 2429.100 | 2541.339 | 2293.468 | 2479.423 |
| 2010 | 3121.0 | 3463.798 | 2438.989 | 3361.372 |

MAPE (%): 4.262628, 7.772898, 1.842483, 2.862057

Then, the ARIMA-b-GM(1,1) model was constructed to obtain the fitting value and prediction value of China's per capita energy consumption from 2000 to 2014, and compared with GM(1,1) model, NGM(1,1) model and SAIGM(1,1) model. The results are shown in table 2 and table 3. Where, MAPE represents the average absolute percentage error of the data, and its calculation equation is as follows:

\[
MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{observed_i - predicted_i}{observed_i} \right| \times 100
\]

| Year | Raw data | GM | NGM | SAIGM | ARIMA-b-GM |
|------|----------|----|-----|-------|-------------|
| 2011 | 2589.0 | 2745.902 | 2334.523 | 2544.288 | 2634.146 |
| 2012 | 2678.0 | 2966.932 | 2366.144 | 2643.096 | 2861.244 |
| 2013 | 3071.0 | 3205.753 | 2170.968 | 2207.193 | 3012.960 |
| 2014 | 3121.0 | 3246.343 | 2438.989 | 3361.372 |

MAPE (%): 8.055245, 16.60950, 5.874724, 4.332203
It can be seen from table 2 that the data fitting error of ARIMA-b-GM (1,1) model is only 2.862%, which is significantly lower than that of the classical GM(1,1) model and NGM(1,1) model. Although slightly higher than that of SAIGM(1,1) model, it still has a good fitting performance. As can be seen from table 3, the prediction error of ARIMA-b-GM (1,1) model is 4.332%, which is significantly lower than the other three models, indicating that the model proposed in this paper has good prediction performance. Figure 1 below is a visual comparison of table 2 and table 3.

5. Conclusions
In the traditional grey prediction model, the grey input is regarded as a constant, which has a certain impact on the prediction accuracy. In this paper, starting from the dynamic grey input, ARIMA model and GM (1,1) model are combined to build the ARIMA-b-GM (1,1) model. This model is applied to predict China's per capita energy consumption, and the results are compared not only with GM(1,1) model, but also with NGM(1,1) and SAIGM(1,1) model. It can be seen from table 2, table 3 and figure 1 that ARIMA-b-GM (1,1) model has good fitting performance and fitting performance. So ARIMA-b-GM (1,1) model can better predict energy consumption and provide theoretical support for government to make energy policies.

Acknowledgements
We are very grateful to the reviewers and our teachers for their pertinent comments on this paper.

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