Multiple-attribute decision making based on single-valued neutrosophic Schweizer-Sklar prioritized aggregation operator

Peide Liu a,⇑, Qaisar Khan b,⇑, Tahir Mahmood b

a School of Management Science and Engineering, Shandong University of Finance and Economics, Jinan, Shandong 250014, China
b Department of Mathematics and Statistic, International Islamic University, Islamabad, Pakistan

Received 3 September 2018; received in revised form 11 October 2018; accepted 11 October 2018

Abstract

Single-valued neutrosophic (SVN) sets can successfully describe the uncertainty problems, and Schweizer-Sklar (SS) t-norm (TN) and t-conorm (TCN) can build the information aggregation process more flexible by a parameter. To fully consider the advantages of SVN and SS operations, in this article, we extend the SS TN and TCN to single-valued neutrosophic numbers (SVNN) and propose the SS operational laws for SVNNs. Then, we merge the prioritized aggregation (PRA) operator with SS operations, and develop the single-valued neutrosophic Schweizer-Sklar prioritized weighted averaging (SVNSSPRWA) operator, single-valued neutrosophic Schweizer-Sklar prioritized ordered weighted averaging (SVNSSPROWA) operator, single-valued neutrosophic Schweizer-Sklar prioritized weighted geometric (SVNSSPRWG) operator, and single-valued neutrosophic Schweizer-Sklar prioritized ordered weighted geometric (SVNSSPROWG) operator. Moreover, we study some useful characteristics of these proposed aggregation operators (AOs) and propose two decision making models to deal with multiple-attribute decision making (MADM) problems under SVN information based on the SVNSSPRWA and SVNSSPRWG operators. Lastly, an illustrative example about talent introduction is given to testify the effectiveness of the developed methods.

© 2018 Elsevier B.V. All rights reserved.

Keywords: Single-valued neutrosophic sets; Schweizer-Sklar operations; Prioritized aggregation operator; Multiple-attributte decision making

1. Introduction

The main purpose of MADM problems is to select the best alternative from the limited alternatives according to the preference values given by decision makers (DMs) with respect to the criteria. However, because of the complexity of decision environment, it is difficult for DMs to express the preference values by a single real number in practical problems. To deal with such situation, intuitionistic fuzzy set (IFS) proposed by Atanassov (1986) is one of the flourishing generalizations of fuzzy set (FS) introduced by Zadeh (1965) to express uncertain and imprecise information more accurately (Liu, Mahmood, & Khan, 2017; Xu and Yager, 2006; Xu, 2007). However, in some situations, only truth-membership degree (TMD) and falsity-membership degree (FMD) cannot describe the inconsistent information accurately. To deal with such situation, Smarandache (1999) proposed neutrosophic set (NS) which

https://doi.org/10.1016/j.cogsys.2018.10.005
1389-0417/© 2018 Elsevier B.V. All rights reserved.
describe the uncertain, imprecise and inconsistent information by TMD, indeterminacy-membership degree (IMD), and FMD. The three functions are independent and are standard or non-standard subsets \([0^-, 1^-]\). As the NS has the IMD, therefore it can describe the uncertain information more accurately than FS and IFS, and it is also more consistent with human natural feelings and judgement. But NS is hard to be used in real problem due to the contained non-standard subsets of \([0^-, 1^-]\). Therefore, in order to utilize NS easily in real problems, Wang, Smarandache, Zhang, and Sunderraman (2010) developed a SVNS, which is subclass of NS.

In real decision making, we need AOs to integrate the given information. In neutrosophic environment, many scholars have developed some AOs. For example, Ye (2014) firstly developed the operational rules for SVNNs and introduced the weighted averaging operator for SVNNs (SVNW A) and geometric average operator for SVNNs. Later on, Peng, Wang, Wang, Zhang, and Chen (2016) found out some limitations in the operational rules developed by Ye (2014), and introduced some improved operational laws for SVNNs, and proposed ordered weighted average for SVNNs (SVNOWA) and ordered weighted geometric operator for SVNNs (SVNOWG). Lu and Ye (2017) further developed some hybrid averaging and hybrid geometric operators for SVNNs and applied them to MADM. After these studies, several researchers developed different AOs, such as Liu, Chu, Li, and Chen (2014) proposed some generalized Hamacher AOs for NS and applied them to multiple-attribute group decision making (MAGDM). Wu, Wu, Zhou, Chen, & Guan (2018) defined some AOs based on Hamacher TN and TCN and applied them to deal with group decision making under SVN 2-tuple linguistic environment. Garg (2016) developed some AOs based on Frank TN and TCN and applied them to solve MADM problems under SVN environment. Zhang, Liu, and Shi (2016) extended TODIM to neutrosophic environment and applied it to MAGDM problem. Mandal and Basu (2018) developed some vector AOs for solving MADM problems under neutrosophic environment. Recently, Karasaslan and Hayat (2018) proposed some new operational laws for SVN matrices and give their application in MAGDM. Garg (2017) developed some parametric distance measures for SVNSs and give their applications in pattern recognition and medical diagnosis. Abdel-Basset, Mai, and Smarandache (2018), Abdel-Basset, Zhou, Mohamed, and Chang (2018) developed AHP-SWOT, ANP-TOPSIS and VIKOR for NSs and applied them to solve strategic planning, supplier selection and e-government website evaluation problems. Peng and Jingguo (2018) developed MABAC, TOPSIS and new similarity measure for SVNNs and proposed three approaches for MADM. Abdel-Basset, Mohamed, Zhou, and Hezam (2017), Abdel-Basset, Mai, Smarandache, and Chang (2018) developed MAGDM method based on neutrosophic analytic hierarchy process and neutrosophic association rule mining algorithm for big data analysis.

Abdel-Basset, Gunasekaran, Mohamed, and Smarandache (2018), Abdel-Basset, Manogaran, Gamal, and Smarandache (2018) developed a novel method for solving the full neutrosophic linear programming problems, and also developed a hybrid approach based on NSs and DEMATEL to solve supplier selection problems. Some other applications of NS were studied by researchers in (Abdel-Basset and Mai, 2018; Abdel-Basset, Mai, & Chang, 2018; Abdel-Basset, Gunasekaran, & Mai, 2018; Chang, Abdel-Basset, & Ramachandran, 2018).

All the above-stated operators are established based on the expectation that the aggregated input arguments are independent. But in some situations, it may be possible that there exists interaction between the decision making criteria under neutrosophic environment. To deal with such situation, Liu and Wang (2014) extended Bonferroni mean (BM) to neutrosophic environment and developed some normalized BM operators for SVNNs and applied them to MAGDM. Li, Liu, and Chen (2016) developed some Heronian mean (HM) operators for SVNNs and applied them to MAGDM problems under SVN environment. Yang and Li (2016) and Liu and Tang (2016) applied the power average operator to the neutrosophic environment and proposed a SVN power average operator and a generalized interval neutrosophic (IN) power averaging (GINPA) operator respectively, which have the property that they can remove the negative impact of the extreme evaluation values on the final ranking results. Wang, Yang, and Li (2016) introduced Maclaurin symmetric mean (MSM) operators to take the interrelationship among the aggregated arguments. Liu and You (2017) proposed Muirhead mean (MM) operator to deal with IN information. These existing AOs have not considered the situation in which the criteria have priority relationship among them. To solve this problem, Wu, Wang, Peng, and Chen (2016) extended prioritized aggregation (PA) operators (Yager, 2008) to SVN environment and proposed SVN prioritized weighted averaging (SVNPWA) and SVN prioritized weighted geometric aggregation (SVNPWG) operators, and applied them to MADM. Moreover, Liu and Wang (2016) developed some prioritized ordered weighted average/geometric operator to deal with neutrosophic information. Ji, Wang, and Zhang (2018) combined PA operators with BM operator and introduced some SVN prioritized BM operators by utilizing Frank operations. Recently, Wei and Wei (2018) proposed some PA operators based on Dombi TN and TCN and applied them to MADM.

From the above stated AOs, most of these AOs for NS or SVNS are based on algebraic, Hamacher, Frank and Dombi operational laws, which are special cases of...
Archimedean TN (ATN) and TCN (ATCN). Certainly, ATN and ATCN are the extensions of many TNs and TCNs, which have some special cases chosen to express the union and intersection of SVNSs (Liu, 2016). Schweizer-Sklar (SS) operations (Deschrijver and Kerre, 2002) are the special cases from ATN and ATCN, they are with a variable parameter, so they are more flexible and superior than the other operations. However, the most researches about SS mainly concentrated on the fundamental theory and characteristics of Schweizer-Sklar TN (SSTN) and TCN (SSTCN) (Deschrijver, 2009; Zhang, He, & Xu, 2006). Recently, Liu and Wang (2018), Zhang (2018) combine SS operations with interval-valued IFS (IVIFS) and IFS, and proposed power average/geometric operators and weighted averaging operators for IVIFSs and IFSs respectively.

From the above discussion, we can know (1) SVNNs are better to describe uncertain information by defining TMD, IMD and FMD than FSs and IFSs in solving the MADM problems; (2) The SS operations are more flexible and superior than the other operations by a variable parameter; (3) there are many MADM problems in which the criteria have priority relationship, and some existing AOs can consider this situation only when the criteria take the form of real numbers. Now, there are no such AOs to deal with MADM problems under SVN information based on SSTN and SSTCN, so, in this paper, we combine the ordinary PA operator with SS operations to deal with the information of SVNNs.

Based on the above research motivation, the goals and contributions of this article are shown as follows.

1. Developing single-valued neutrosophic Schweizer-Sklar prioritized weighted averaging (SVNSSPRWA) operator and single-valued neutrosophic Schweizer-Sklar prioritized ordered weighted averaging (SVNSSPRAOWA) operator.
2. Discussing properties and specific cases of these proposed AOs.
3. Proposing two MADM approaches based on the proposed AOs.
4. Verifying the effectiveness and practicality of the proposed approach.

To do so, the rest of this article is organized as follows. In Section 2, we initiated some basic ideas of SVNNs, PA operators, Schweizer-Sklar operations. In Section 3, we develop some Schweizer-Sklar operational laws for SVNNs. In Section 4, we propose SVNSSPRAWA and SVNSSPRAOWA operators, and discuss some properties and special cases of the proposed AOs. In Section 5, we propose SVNSSPRAWG and SVNSSPRAOGW operators, discuss some properties and special cases of the proposed AOs. In Section 6, we develop two MADM approaches based on these AOs. In Section 7, we solve a numerical example to show the validity and advantages of the proposed approach by comparing with other existing methods.

2. Preliminaries

2.1. Some concepts of SVNSs

In this subpart, we review some basic concepts about SVNSs, SVNNs, score and accuracy functions, and their operational rules.

Definition 1 ((Wang et al., 2010)). Let Ξ be the domain set, with a general element expressed by ψ. A SVNS $\tilde{S}V$ in Ξ is mathematically symbolized as

$$\tilde{S}V = \left\{ \left( \psi, \tilde{R}_V, \tilde{D}_V, \tilde{F}_V \right) | \psi \in \Xi \right\}$$

where $\tilde{R}_V, \tilde{D}_V, \tilde{F}_V$ represents truth-membership (TM) function, indeterminacy-membership (IM) function, and falsity-membership (FM) function, respectively, and they are single subsets of the standard unit interval $[0,1]$. That is, $\tilde{R}_V, \tilde{D}_V, \tilde{F}_V : \Xi \rightarrow [0,1]$, and the sum of these three functions must satisfy the condition $0 \leq \tilde{R}_V + \tilde{D}_V + \tilde{F}_V \leq 3$. The triplet $\left( \tilde{R}_V, \tilde{D}_V, \tilde{F}_V \right)$ is said to be a SVN number (SVNN). For computational simplicity, we shall denote a SVNN by $sv = \left( \tilde{R}_V, \tilde{D}_V, \tilde{F}_V \right)$ and the set of all SVNNs is designated by $\Theta$.

Definition 2 ((Peng et al., 2016)). Let $sv_1, sv_2, sv_3$ be any three SVNNs and $\xi > 0$. Then some operational laws for SVNNs are defined as follows:

1. $sv_1 \oplus sv_2 = \left( \tilde{R}_1 + \tilde{R}_2 - \tilde{R}_1 \tilde{R}_2, \tilde{D}_1 \tilde{D}_2, \tilde{F}_1 \tilde{F}_2 \right)$
2. $sv_1 \otimes sv_2 = \left( \tilde{R}_1 \tilde{R}_2, \tilde{D}_1 + \tilde{D}_2 - \tilde{D}_1 \tilde{D}_2, \tilde{F}_1 + \tilde{F}_2 - \tilde{F}_1 \tilde{F}_2 \right)$
3. $\xi sv = \left( 1 - \left( 1 - \tilde{R}_V \right)^{\xi}, \tilde{D}_V, \tilde{F}_V \right)$
4. $sv^{\xi} = \left( \tilde{R}_V^{\xi}, 1 - \left( 1 - \tilde{D}_V \right)^{\xi}, 1 - \left( 1 - \tilde{F}_V \right)^{\xi} \right)$
Definition 3 ((Peng et al., 2016)). Let \( sv = \langle \overrightarrow{TR}, \overrightarrow{TD}, \overrightarrow{FL} \rangle \) be a SVNN, a score function \( \tilde{SO} \) can be expressed as follows:

\[
\tilde{SO}(sv) = \frac{1}{3}(\overrightarrow{TR} + 2 - \overrightarrow{TD} - \overrightarrow{FL}) \geq 0, \quad \tilde{SO}(sv) \in [0, 1]
\] (6)

Definition 4 ((Peng et al., 2016)). Let \( sv = \langle \overrightarrow{TR}, \overrightarrow{TD}, \overrightarrow{FL} \rangle \) be a SVNN, an accuracy function \( \tilde{AR} \) can be expressed as follows:

\[
\tilde{AR}(sv) = \left( \overrightarrow{TR} - \overrightarrow{TD} \right), \quad \tilde{AC}(sv) \in [-1, 1]
\] (7)

Definition 5 ((Peng et al., 2016)). Let \( sv_1 = \langle \overrightarrow{TR}_1, \overrightarrow{TD}_1, \overrightarrow{FL}_1 \rangle \) and \( sv_2 = \langle \overrightarrow{TR}_2, \overrightarrow{TD}_2, \overrightarrow{FL}_2 \rangle \) be two SVNNs. Then the comparison rules of SVNNs are described as follows:

(1) If \( \tilde{SO}(sv_1) < \tilde{SO}(sv_2) \), then \( sv_2 \) is greater than \( sv_1 \), and is denoted as \( sv_2 > sv_1 \).

(2) If \( \tilde{SO}(sv_1) = \tilde{SO}(sv_2) \), and \( \tilde{AR}(sv_1) < \tilde{AR}(sv_2) \) then \( sv_2 \) is greater than \( sv_1 \), and is denoted as \( sv_2 > sv_1 \).

(3) If \( \tilde{SO}(sv_1) = \tilde{SO}(sv_2) \), and \( \tilde{AR}(sv_1) = \tilde{AR}(sv_2) \) then \( sv_1 \) is equal to \( sv_2 \), and is denoted as \( sv_1 = sv_2 \).

2.2. PA operator

Definition 6 ((Yager, 2008)). Let \( \Gamma = \left( \Gamma_1, \Gamma_2, \ldots, \Gamma_g \right) \) be set of attributes, and assure that there exist prioritization among the attributes expressed by a linear ordering \( \Gamma_1 > \Gamma_2 > \ldots > \Gamma_{g-1} > \Gamma_g \), which indicates that the criterion \( \Gamma_a \) has a higher priority than \( \Gamma_b \), if \( a < b \). \( \Gamma_a(s) \) is an evaluation value expressing the performance of the alternative \( s \) under the attribute \( \Gamma_a \) and satisfies \( \Gamma_a \in [0, 1] \). If \( PA\left( \Gamma_1(s), \Gamma_2(s), \ldots, \Gamma_g(s) \right) = \sum_{a=1}^{g} \omega_a \Gamma_a(s) \) (8)

Obviously, the PA operator has been effectively applied to the situation where the attributes are real values.

2.3. Schweizer-Sklar (SS) operations

The SS operations consist of the SS product and SS sum, which are special cases of ATT, respectively.

Definition 7 ((Wang et al., 2010)). Let \( \tilde{SV}_1 = \langle \overrightarrow{TR}_{SV_1}, \overrightarrow{TD}_{SV_1}, \overrightarrow{FL}_{SV_1} \rangle \) and \( \tilde{SV}_2 = \langle \overrightarrow{TR}_{SV_2}, \overrightarrow{TD}_{SV_2}, \overrightarrow{FL}_{SV_2} \rangle \) be two SVNNs. Then the generalized union and intersection are defined as follows:

\[
\tilde{SV}_1 \cup_{\tilde{T}, \tilde{T}} \tilde{SV}_2 = \left\{ \langle \psi, \tilde{T}^\ast \left( \overrightarrow{TR}_{SV_1} - \overrightarrow{TD}_{SV_1} \right), \tilde{T}^\ast \left( \overrightarrow{FL}_{SV_1} - \overrightarrow{TD}_{SV_2} \right) \rangle | \psi \in \mathbb{E} \right\}
\] (9)

\[
\tilde{SV}_1 \cap_{\tilde{T}, \tilde{T}} \tilde{SV}_2 = \left\{ \langle \psi, \tilde{T} \left( \overrightarrow{TR}_{SV_1} - \overrightarrow{TD}_{SV_2} \right), \tilde{T} \left( \overrightarrow{FL}_{SV_1} - \overrightarrow{TD}_{SV_2} \right) \rangle | \psi \in \mathbb{E} \right\}
\] (10)

where \( \tilde{T} \) and \( \tilde{T}^\ast \) respectively, express T-norm (TN) and T-conorm (TCN).

The Schweizer-Sklar TN and TCN (Deschrijver and Kerre, 2002) are defined as follows:

\[
\tilde{T}_{SS}(m, n) = (m^\rho + n^\rho - 1)\frac{1}{\rho} \quad (11)
\]

\[
\tilde{T}^\ast_{SS}(m, n) = 1 - ((1 - m)^\rho + (1 - n)^\rho - 1)\frac{1}{\rho} \quad (12)
\]

where \( \rho < 0, m, n \in [0, 1] \).

Additionally, when \( \rho = 0 \), we have \( \tilde{T}_{SS}(m, n) = mn \) and \( \tilde{T}^\ast_{SS}(m, n) = m + n - mn \). That is, SS TN and TCN reduce to algebraic TN and TCN.

Now, in the next section, based on TN \( \tilde{T}_{SS}(m, n) \) and TCN \( \tilde{T}^\ast_{SS}(m, n) \) of SS operations, we can give the following definition about SS operations of SVNNs.

3. Schweizer-Sklar operations for SVNNs

Definition 8. Assume \( sv_1 = \langle \overrightarrow{TR}_1, \overrightarrow{TD}_1, \overrightarrow{FL}_1 \rangle \) and \( sv_2 = \langle \overrightarrow{TR}_2, \overrightarrow{TD}_2, \overrightarrow{FL}_2 \rangle \) are any two SVNNs. Then based on SS operations, the generalized union and intersection are introduced as follows:

\[
sv_1 \oplus_{\tilde{T}, \tilde{T}} sv_2 = \langle \tilde{T} \left( \overrightarrow{TR}_1, \overrightarrow{TR}_2 \right), \tilde{T} \left( \overrightarrow{TD}_1, \overrightarrow{TD}_2 \right), \tilde{T} \left( \overrightarrow{FL}_1, \overrightarrow{FL}_2 \right) \rangle
\] (13)

\[
sv_1 \ominus_{\tilde{T}, \tilde{T}} sv_2 = \langle \tilde{T} \left( \overrightarrow{TR}_1, \overrightarrow{TR}_2 \right), \tilde{T}^\ast \left( \overrightarrow{TD}_1, \overrightarrow{TD}_2 \right), \tilde{T}^\ast \left( \overrightarrow{FL}_1, \overrightarrow{FL}_2 \right) \rangle
\] (14)
On the basis of Definition (7) and Eqs. (13), and (14), the SS operations of SVNNs are described as follows ($\rho < 0$):

\begin{align*}
(1) \quad sv_1 \otimes_{SS} sv_2 &= \left( \left( \overline{T}_{1} + \overline{T}_{2} - 1 \right)^{\frac{\lambda}{2}}, 1 - \left( \left( 1 - \overline{T}_{1} \right)^{\rho} + \left( 1 - \overline{T}_{2} \right)^{\rho} - 1 \right)^{\frac{\rho}{2}}, 1 - \left( \left( 1 - \overline{F}_{1} \right)^{\rho} + \left( 1 - \overline{F}_{2} \right)^{\rho} - 1 \right)^{\frac{\rho}{2}} \right) \\
(2) \quad sv_1 \otimes_{SS} sv_2 &= \left( 1 - \left( \left( 1 - \overline{T}_{1} \right)^{\rho} + \left( 1 - \overline{T}_{2} \right)^{\rho} - 1 \right)^{\frac{\rho}{2}}, \left( \overline{T}_{1} + \overline{T}_{2} - 1 \right)^{\frac{\lambda}{2}}, \left( \overline{F}_{1} + \overline{F}_{2} - 1 \right)^{\frac{\lambda}{2}} \right) \\
(3) \quad sv_1^k &= \left( \left( \overline{X}_{1} - (X - 1) \right)^{\frac{\lambda}{2}}, 1 - \left( \left( 1 - \overline{T}_{1} \right)^{\rho} - (X - 1) \right)^{\frac{\rho}{2}}, 1 - \left( \left( 1 - \overline{F}_{1} \right)^{\rho} - (X - 1) \right)^{\frac{\rho}{2}} \right) \\
(4) \quad \lambda sv_1 &= \left( 1 - \left( \left( 1 - \overline{T}_{1} \right)^{\rho} - (X - 1) \right)^{\frac{\rho}{2}}, \left( \overline{X}_{1} - (X - 1) \right)^{\frac{\lambda}{2}}, \left( \overline{F}_{1} - (X - 1) \right)^{\frac{\lambda}{2}} \right)
\end{align*}

**Theorem 1.** Let $sv_1 = \left( \overline{T}_{1}, \overline{T}_{1}, \overline{F}_{1} \right)$ and $sv_2 = \left( \overline{T}_{2}, \overline{T}_{2}, \overline{F}_{2} \right)$ be any two SVNNs, then

\begin{align*}
(1) \quad sv_1 \oplus_{SS} sv_2 &= sv_2 \oplus_{SS} sv_1, \\
(2) \quad sv_1 \otimes_{SS} sv_2 &= sv_2 \otimes_{SS} sv_1, \\
(3) \quad \lambda (sv_1 \otimes_{SS} sv_2) &= \lambda sv_1 \otimes_{SS} \lambda sv_2, \quad \lambda \geq 0; \\
(4) \quad \lambda (sv_1 \otimes_{SS} \lambda sv_2) &= (\lambda sv_1 \otimes_{SS} \lambda sv_2), \quad \lambda \geq 0; \\
(5) \quad sv_1 \otimes_{SS} \lambda sv_2 &= (sv_1 \otimes_{SS} sv_2), \quad \lambda \geq 0; \\
(6) \quad \lambda sv_1 \otimes_{SS} sv_2 &= (\lambda sv_1 \otimes_{SS} sv_2), \quad \lambda \geq 0.
\end{align*}

4. Single-valued neutrosophic Schweizer-Sklar prioritized weighted averaging (SVNSSPRAOWA) operator

In this part, for a group of SVNNs $sv_p = \left( \overline{T}_{p}, \overline{T}_{p}, \overline{F}_{p} \right), (p = 1, 2, ..., g)$ symbolized by $\Theta$, we introduce a few new PA operators for SVNNs, namely single valued neutrosophic Schweizer-Sklar prioritized weighted averaging (SVNSSPRAOWA) operator, and discuss some characteristics of these developed aggregation operators.

\begin{align*}
\text{SVNSSPRAOWA}(sv_1, sv_2, ..., sv_g) &= \left( 1 - \left( \sum_{p=1}^{g} \frac{\overline{T}_{p}}{T_{p}} \left( 1 - \overline{T}_{p} \right)^{\rho} - \sum_{p=1}^{g} \frac{\overline{T}_{p}}{T_{p}} \right) \right)^{\frac{\lambda}{2}}, \left( \sum_{p=1}^{g} \frac{\overline{T}_{p}}{T_{p}} \overline{F}_{p} - \sum_{p=1}^{g} \frac{\overline{T}_{p}}{T_{p}} \right)^{\frac{\rho}{2}}, \left( \sum_{p=1}^{g} \frac{\overline{T}_{p}}{T_{p}} \overline{F}_{p} - \sum_{p=1}^{g} \frac{\overline{T}_{p}}{T_{p}} \right)^{\frac{\rho}{2}}.
\end{align*}

**4.1. Single valued neutrosophic Schweizer-Sklar prioritized weighted averaging (SVNSSPRAOWA) operator**

**Definition 9.** A single valued neutrosophic Schweizer-Sklar prioritized weighted averaging (SVNSSPRAOWA) operator is a function $\text{SVNSSPRAOWA} : \Theta^g \rightarrow \Theta$, which is described as

\begin{align*}
\text{SVNSSPRAOWA}(sv_1, sv_2, ..., sv_g) &= \sum_{p=1}^{g} \frac{T_{p}}{T} sv_p
\end{align*}

where $T_{p} = 1$, and $T = \sum_{p=1}^{g} \overline{SO}(sv_h), (p = 2, 3, ..., g)$. Here, $\overline{SO}(sv_h)$ expresses the score value of SVNN $sv_h$.

**Theorem 2.** For a group of SVNNs $sv_p = \left( \overline{T}_{p}, \overline{T}_{p}, \overline{F}_{p} \right), (p = 1, 2, ..., g)$, the value aggregated by the developed SVNPRWA operator is still a SVNN and is specified by:

\begin{align*}
\text{SVNSSPRWA}(sv_1, sv_2, ..., sv_g) &= \left( \left( \sum_{p=1}^{g} \frac{T_{p}}{T} \left( 1 - \overline{T}_{p} \right)^{\rho} - \sum_{p=1}^{g} \frac{T_{p}}{T} \right) \right)^{\frac{\lambda}{2}}, \left( \sum_{p=1}^{g} \frac{T_{p}}{T} \overline{F}_{p} - \sum_{p=1}^{g} \frac{T_{p}}{T} \right)^{\frac{\rho}{2}}, \left( \sum_{p=1}^{g} \frac{T_{p}}{T} \overline{F}_{p} - \sum_{p=1}^{g} \frac{T_{p}}{T} \right)^{\frac{\rho}{2}}.
\end{align*}

**Proof.** We will prove Eq. (26) by utilizing mathematical induction (MI). The following steps of MI have been followed:

Step 1. For \( g = 2 \), we have

\[
SVNSSPRWA(s_{v_1}, s_{v_2}) = \frac{1}{p-1} \sum_{p=1}^{2} \left( \frac{T}{P_{p-1}} s_{v_p} \right),
\]

From the operational laws for SVNNs, proposed in Definition 8, we have

\[
\frac{T}{2} s_{v_1} = \left( 1 - \left( \frac{T}{2} (1 - TR_1) \right)^{\frac{1}{2}} - \left( \frac{T}{2} (1 - TR_2) \right)^{\frac{1}{2}} \right)^{\frac{1}{2}},
\]

and

\[
\frac{T}{2} s_{v_2} = \left( 1 - \left( \frac{T}{2} (1 - TR_2) \right)^{\frac{1}{2}} - \left( \frac{T}{2} (1 - TR_1) \right)^{\frac{1}{2}} \right)^{\frac{1}{2}},
\]

So, Eq. (27) becomes

\[
\frac{T}{2} s_{v_1} = \left( 1 - \left( \frac{T}{2} (1 - TR_1) \right)^{\frac{1}{2}} - \left( \frac{T}{2} (1 - TR_2) \right)^{\frac{1}{2}} \right)^{\frac{1}{2}},
\]

Then, for \( g = b + 1 \), according to the operational rules developed for SVNNs in Definition 8, we have

\[
SVNSSPRWA(s_{v_1}, s_{v_2}, \ldots, s_{v_b}) = \left( 1 - \left( \frac{T}{b+1} (1 - TR_p) \right)^{\frac{1}{b+1}} - \left( \frac{T}{b+1} (1 - TR_p) \right)^{\frac{1}{b+1}} \right)^{\frac{1}{b+1}}.
\]

Step 2. Assume that for \( g = b \), Eq. (26) is true. i.e.,

\[
SVNSSPRWA(s_{v_1}, s_{v_2}, \ldots, s_{v_b}) = \left( 1 - \left( \frac{T}{b} (1 - TR_p) \right)^{\frac{1}{b}} - \left( \frac{T}{b} (1 - TR_p) \right)^{\frac{1}{b}} + 1 \right)^{\frac{1}{b}}.
\]
\[
T_{b+1}\sum_{p=1}^{b+1} \frac{S_{b+1}}{\sum_{p=1}^{b+1} T^p} = \left( 1 - \left( \frac{T_{b+1}}{b+1} + \frac{T_p}{p-1} \right) \right)^\frac{1}{b+1} \left( 1 - \left( \frac{T_{b+1}}{b+1} \frac{T^p}{p-1} - \left( \frac{T_{b+1}}{b+1} - \left( \frac{T_{b+1}}{b+1} \frac{T^p}{p-1} - \left( \frac{T_{b+1}}{b+1} \right)^{b+1} - \left( \frac{T_{b+1}}{b+1} \right)^{b+1} \right) \right) \right) \right),
\]

and

\[
SVNSSPRWA(s_{v1}, s_{v2}, \ldots, s_{v_b}, s_{v_{b+1}}) = \left\langle 1 - \left( \frac{T_{b+1}}{b+1} + \frac{T_p}{p-1} \right) \right\rangle \left( 1 - \left( \frac{T_{b+1}}{b+1} \frac{T^p}{p-1} - \left( \frac{T_{b+1}}{b+1} \right)^{b+1} - \left( \frac{T_{b+1}}{b+1} \right)^{b+1} \right) \right) \right),
\]

(29)

So, when \( g = b + 1 \), Eq. (26) is true. Therefore, Eq. (26) is true for all \( g \).

When \( \frac{T_p}{p-1} \leq 0 \), such that \( \frac{g}{p-1} + \frac{T_p}{p-1} = 1 \), then, Eq. (26) degenerates into the following form:

\[
SVNSSPRWA(s_{v1}, s_{v2}, s_{v_3}) = \left\langle 1 - \left( \frac{T_{b+1}}{b+1} + \frac{T_p}{p-1} \right) \right\rangle \left( 1 - \left( \frac{T_{b+1}}{b+1} \frac{T^p}{p-1} - \left( \frac{T_{b+1}}{b+1} \right)^{b+1} - \left( \frac{T_{b+1}}{b+1} \right)^{b+1} \right) \right) \right),
\]

Example 1. Let \( s_{v1} = \langle 0.3, 0.4, 0.5 \rangle \), \( s_{v2} = \langle 0.4, 0.2, 0.1 \rangle \) and \( s_{v3} = \langle 0.6, 0.1, 0.2 \rangle \) be three SVNNs. Based on the score function of SVNNs, we get \( SO(s_{v1}) = 0.4667, SO(s_{v2}) = 0.7 \) and \( SO(s_{v3}) = 0.7667 \), and hence \( T_{v1} = 1, T_{v2} = 0.4667 \) and \( T_{v3} = 0.3267 \). By using this information (\( \rho = -2 \)), we can get

\[
SVNSSPRWA(s_{v1}, s_{v2}, s_{v_3}) = \left\langle 1 - \left( \frac{T_{b+1}}{b+1} + \frac{T_p}{p-1} \right) \right\rangle \left( 1 - \left( \frac{T_{b+1}}{b+1} \frac{T^p}{p-1} - \left( \frac{T_{b+1}}{b+1} \right)^{b+1} - \left( \frac{T_{b+1}}{b+1} \right)^{b+1} \right) \right) \right),
\]

(30)

\[
= \langle 0.4226, 0.1883, 0.1746 \rangle.
\]
Theorem 3. For a group of SVNNs \( sv_p = \langle \overline{TR}_p, \overline{D}_p, \overline{L}_p \rangle \), \( p = 1, 2, ..., g \), the SVNPRWA operator satisfies the following properties:

1. (Idempotency) If all \( sv_p(p = 1, 2, ..., g) \) are equal, i.e., 
   \[ sv_p = sv = \langle \overline{TR}, \overline{D}, \overline{L} \rangle \]; then 
   \[ SVNSSPRWA(sv_1, sv_2, ..., sv_g) = sv. \]

Proof. Since \( sv_p = sv = \langle \overline{TR}, \overline{D}, \overline{L} \rangle \): for all \( p \), so,

\[ SVNSSPRWA(sv_1, sv_2, ..., sv_g) = \left\langle 1 - \left( \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} \left( 1 - \overline{TR}_p \right)^{\alpha} - \frac{\sum_{p=1}^{g} T^p}{\sum_{p=1}^{g} \overline{T}^p} + 1 \right) \right\rangle, \]

\[ = \left\langle 1 - \left( \left( \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} \left( 1 - \overline{TR}_p \right)^{\alpha} \right) \right) \right\rangle \]

\[ = \left\langle \overline{TR}, \overline{D}, \overline{L} \right\rangle. \]

2. (Monotonicity) If \( sv'_p = \langle \overline{TR}'_p, \overline{D}'_p, \overline{L}'_p \rangle \) and 
   \( sv_p = \langle \overline{TR}_p, \overline{D}_p, \overline{L}_p \rangle \) are two groups of SVNNs, such 
   that \( sv'_p \geq sv_p \); i.e., 
   \( \overline{TR}'_p \geq \overline{TR}_p \), \( \overline{D}'_p \leq \overline{D}_p \), \( \overline{L}'_p \leq \overline{L}_p \) 
   and for all \( p \), then 
   \[ SVNSSPRWA(sv'_1, sv'_2, ..., sv'_g) \geq SVNSSPRWA(sv_1, sv_2, ..., sv_g). \]

Proof. Since \( sv'_p \geq sv_p \), which implies \( \overline{TR}'_p \geq \overline{TR}_p \) and so

\[ 1 - \overline{TR}'_p \leq 1 - \overline{TR}_p, \text{ and } 0 \leq \left( 1 - \overline{TR}'_p \right)^{\alpha} \leq \left( 1 - \overline{TR}_p \right)^{\alpha} \leq 1, \]

\[ \Rightarrow \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} \left( 1 - \overline{TR}_p \right)^{\alpha} \leq \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} \left( 1 - \overline{TR}'_p \right)^{\alpha} \]

\[ \Rightarrow \left( \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} \left( 1 - \overline{TR}_p \right)^{\alpha} - \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} + 1 \right) \leq \left( \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} \left( 1 - \overline{TR}_p \right)^{\alpha} - \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} + 1 \right)^{\frac{1}{2}} \]

\[ \Rightarrow 1 - \left( \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} \left( 1 - \overline{TR}_p \right)^{\alpha} - \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} + 1 \right)^{\frac{1}{2}} \geq 1 - \left( \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} \left( 1 - \overline{TR}'_p \right)^{\alpha} - \sum_{p=1}^{g} \frac{T^p}{\overline{T}^p} + 1 \right)^{\frac{1}{2}} \]

From Eqs. (32)–(34), we get

\[ SVNSSPRWA(sv'_1, sv'_2, ...., sv'_g) \geq SVNSSPRWA(sv_1, sv_2, ...., sv_g). \]
(3) (Boundedness) Let $sv_p = \langle \overline{TR}_p, \overline{ID}_p, \overline{FL}_p \rangle$ be a group of SVNNs, and $sv^+_p = \left\langle \max_{p=1}^{g} \overline{TR}_p, \min_{p=1}^{g} \overline{ID}_p, \min_{p=1}^{g} \overline{FL}_p \right\rangle$, $sv^-_p = \left\langle \min_{p=1}^{g} \overline{TR}_p, \max_{p=1}^{g} \overline{ID}_p, \max_{p=1}^{g} \overline{FL}_p \right\rangle$, then

$sv^- \leq SVNSSPRWA(sv_1, sv_2, ..., sv_g) \leq sv^+$. (35)

**Proof.** Since $\min_{p=1}^{g} \overline{TR}_p \leq \overline{TR}_p \leq \max_{p=1}^{g} \overline{TR}_p$, $\min_{p=1}^{g} \overline{ID}_p \leq \overline{ID}_p \leq \max_{p=1}^{g} \overline{ID}_p$, and $\min_{p=1}^{g} \overline{FL}_p \leq \overline{FL}_p \leq \max_{p=1}^{g} \overline{FL}_p$.

$SVNSSPRWA(sv_1, sv_2, ..., sv_g) = \left( 1 - \left( \frac{g}{p=1} \frac{\overline{TR}_p}{\overline{TR}_p} \right) \right) \cdot \left( \frac{g}{p=1} \frac{\overline{ID}_p}{\overline{ID}_p} \right) \cdot \left( \frac{g}{p=1} \frac{\overline{FL}_p}{\overline{FL}_p} \right) = (0.5327, 0.1256, 0.1568).$ (36)

**4.2. Single-valued neutrosophic Schweizer-Sklar prioritized ordered weighted averaging operator**

In this subpart, we consider ordered weighted average (OWA), and propose the SVNSSOWA operator.

**Definition 10.** A SVNPROWA operator is a function $SVNPROWA : \Theta^g \to \Theta$, described as follows:

$SVNSSPRWA(sv_1, sv_2, ..., sv_g) = \frac{\sum_{p=1}^{g} \overline{TR}_p}{\sum_{p=1}^{g} \overline{TR}_p}$ (37)

where $\sum_{p=1}^{g} \overline{TR}_p = 1$, and $\sum_{p=1}^{g} \overline{ID}_p = \sum_{p=1}^{g} \overline{FL}_p = 0$.

**Theorem 4.** For a group of SVNNs $sv_p = \langle \overline{TR}_p, \overline{ID}_p, \overline{FL}_p \rangle$, $(p = 1, 2, ..., g)$, the value aggregated by the developed SVNPROWA operator is still a SVNN and is specified by:

$SVNSSPRWA(sv_1, sv_2, ..., sv_g) = \left( 1 - \left( \frac{g}{p=1} \frac{\overline{TR}_p}{\overline{TR}_p} \right) \right) \cdot \left( \frac{g}{p=1} \frac{\overline{ID}_p}{\overline{ID}_p} \right) \cdot \left( \frac{g}{p=1} \frac{\overline{FL}_p}{\overline{FL}_p} \right) = (0.5327, 0.1256, 0.1568).$ (38)

**Proof.** Same as Theorem 2, it is omitted.

**Example 2.** Consider the SVNNs given in Example 1, we have $\sum_{p=1}^{g} \overline{TR}_p = 0.4667$ and $\sum_{p=1}^{g} \overline{ID}_p = 0.3267$. the score values are $SO(sv_1) = 0.4667, SO(sv_2) = 0.7$ and $SO(sv_3) = 0.7667$. So, we have $SO(sv_1) > SO(sv_2) > SO(sv_3)$, and hence, $sv_{\xi(1)} = sv_1, sv_{\xi(2)} = sv_2, sv_{\xi(3)} = sv_3$. By using this information $(p = -2)$, we can get

$SVNSSOWA_{p=0}(sv_1, sv_2, ..., sv_g) = \left( 1 - \frac{g}{p=1} \frac{\overline{TR}_p}{\overline{TR}_p} \right) \cdot \left( \frac{g}{p=1} \frac{\overline{ID}_p}{\overline{ID}_p} \right) \cdot \left( \frac{g}{p=1} \frac{\overline{FL}_p}{\overline{FL}_p} \right) = (0.5327, 0.1256, 0.1568).$ (36)

**Theorem 5.** For a group of SVNNs $sv_p = \langle \overline{TR}_p, \overline{ID}_p, \overline{FL}_p \rangle$, $(p = 1, 2, ..., g)$, the value aggregated by the SVNPROWA operator is still a SVNN and is specified by:

$SVNSSPRWA(sv_1, sv_2, ..., sv_g) = \left( 1 - \left( \frac{g}{p=1} \frac{\overline{TR}_p}{\overline{TR}_p} \right) \right) \cdot \left( \frac{g}{p=1} \frac{\overline{ID}_p}{\overline{ID}_p} \right) \cdot \left( \frac{g}{p=1} \frac{\overline{FL}_p}{\overline{FL}_p} \right) = (0.5327, 0.1256, 0.1568).$ (38)

(1) (Idempotency). If all $sv_p(p = 1, 2, ..., g)$ are equal, i.e.,

$sv_p = sv = \langle \overline{TR}_p, \overline{ID}_p, \overline{FL}_p \rangle$; then

$SVNSSPRWA(sv_1, sv_2, ..., sv_g) = sv$. (39)
(2) (Monotonicity). If \( SVNa_p = \left< \overline{TR}_p, \overline{FP}_p, \overline{FL}_p \right> \) and \( SVb_p = \left< \overline{TR}_p, \overline{FP}_p, \overline{FL}_p \right> \) are two groups of SVNNs, such that \( SVNa_p \geq SVb_p \), i.e., \( \overline{TR}_p \geq \overline{TR}_p, \overline{FP}_p \leq \overline{FP}_p, \overline{FL}_p \leq \overline{FL}_p \) for all \( p \), then

\[
SVNa \ominus SVb \geq SVNa \ominus SVb.
\]

(3) (Boundedness). Let \( SVa_p = \left< \overline{TR}_p, \overline{FP}_p, \overline{FL}_p \right> \) be a group of SVNNs, and \( SVa_p = \left< \max \overline{TR}_p, \min \overline{FP}_p, \max \overline{FL}_p \right> \), then

\[
SVNa \ominus SVb = \left< \min \overline{TR}_p, \max \overline{FP}_p, \max \overline{FL}_p \right>.
\]

5. Single-valued neutrosophic Schweizer-Sklar prioritized weighted geometric (SVNSSPRWG) operator

In this part, we develop single-valued neutrosophic Schweizer-Sklar prioritized weighted geometric (SVNSSPRWG) and single-valued neutrosophic Schweizer-Sklar prioritized ordered weighted geometric (SVNSSSPROWG) operators. We also discuss some characteristics of the developed aggregation operators.

5.1. Single-valued neutrosophic Schweizer-Sklar prioritized weighted geometric (SVNSSPRWG) operator

Definition 11. A single valued neutrosophic Schweizer-Sklar prioritized weighted geometric (SVNSSPRWG) operator is a function \( SVNSSPRWA : \Theta^p \rightarrow \Theta \), which is described as

\[
SVNSSPRWG\left(sv_1, sv_2, ..., sv_g\right) = \bigotimes_{p=1}^{g} sv_p^{\frac{1}{\sum}}
\]

Where \( T_1 = 1 \), and \( T_p = \bigotimes_{h=1}^{p-1} SO(sv_h) \), \( (p = 2, 3, ..., g) \).

Theorem 6. For a group of SVNNs \( sv_p = \left< \overline{TR}_p, \overline{FP}_p, \overline{FL}_p \right> \), \( (p = 1, 2, ..., g) \), the value aggregated by the developed SVNPRWG operator is still a SVNN and is specified by:

\[
SVNSSPRWG\left(sv_1, sv_2, ..., sv_g\right) = \left( \bigotimes_{p=1}^{g} \overline{TR}_p \right) \bigotimes_{p=1}^{g} \overline{FP}_p \bigotimes_{p=1}^{g} \overline{FL}_p
\]

Proof. We will prove Eq. (43) by utilizing MI. The following steps of MI have been followed:

Step 1. For \( g = 2 \), we have

\[
SVNSSPRWA\left(sv_1, sv_2\right) = sv_1^{\frac{1}{2}} \otimes sv_2^{\frac{1}{2}}
\]

From the operational laws for SVNNs, proposed in Definition 8, we have

\[
\frac{1}{2} sv_1^{\frac{1}{2}} \otimes sv_2^{\frac{1}{2}} = \left( \frac{T_2}{2} \otimes \overline{TR}_1 \right) - \left( \frac{T_2}{2} \otimes \overline{TR}_2 \right) - \left( \frac{T_2}{2} \otimes \overline{TR}_1 \right) - \left( \frac{T_2}{2} \otimes \overline{TR}_2 \right) - \left( \frac{T_2}{2} \otimes \overline{TR}_1 \right) - \left( \frac{T_2}{2} \otimes \overline{TR}_2 \right)
\]
and

\[
\frac{\rho_{p1} \otimes \rho_{p1}}{\rho_{p1} \otimes \rho_{p1}} = \left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\rho_{p1}} \cdot \frac{1}{\left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\rho_{p1}}} 
\]

So, Eq. (44) becomes

\[
\frac{\rho_{p1} \otimes \rho_{p1}}{\rho_{p1} \otimes \rho_{p1}} = \left( \left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\rho_{p1}} \cdot \frac{1}{\left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\rho_{p1}}} 
\]

\[
1 - \left( \left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\rho_{p1}} \cdot \frac{1}{\left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\rho_{p1}}} 
\]

i.e., when \( g = 2 \), Eq. (43) is true.

**Step 2.** Assume that for \( g = b \), Eq. (43) is true, i.e.,

\[
SVNSSPRWG(\alpha \delta \beta) = \left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\frac{\rho_{p1}}{\rho_{p1}}} \cdot \frac{1}{\left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\frac{\rho_{p1}}{\rho_{p1}}}}, \quad 1 - \left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\frac{\rho_{p1}}{\rho_{p1}}} \cdot \frac{1}{\left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\frac{\rho_{p1}}{\rho_{p1}}}} 
\]

Then, for \( g = b + 1 \), according to the operational rules developed for SVNNs in Definition 8, we have

\[
SVNSSPRWG(\alpha \delta \beta) = \left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\frac{\rho_{p1}}{\rho_{p1}}} \cdot \frac{1}{\left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\frac{\rho_{p1}}{\rho_{p1}}}}, \quad 1 - \left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\frac{\rho_{p1}}{\rho_{p1}}} \cdot \frac{1}{\left( \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} \right) - \left( \frac{T_p}{\rho_{p1} + \frac{T_p}{\rho_{p1}}} - 1 \right) \right)^{\frac{\rho_{p1}}{\rho_{p1}}}} 
\]
and
\[
SVNSSPRWG(s_{v1}, s_{v2}, \ldots, s_{v_b}, s_{v_{b+1}}) = SVNSSPRWG(s_{v1}, s_{v2}, \ldots, s_{v_b}) \otimes SV_{s_{v_{b+1}}}
\]

\[
= \left(\left(\left(\frac{b+1}{p=1} \frac{T_{s_{v_{b+1}}}}{p=1} \frac{T_{b+1}}{p=1} \frac{1}{b+1} \right) \right) + \left(\left(\frac{b+1}{p=1} \frac{T_{b+1}}{p=1} \frac{1}{b+1} \right) \right) \right) - 1.
\]

Example 3. Let \(sv_1 = (0.5,0.2,0.4)\), \(sv_2 = (0.7,0.3,0.4)\) and \(sv_3 = (0.3,0.4,0.6)\) be three SVNNs. Based on the score function of SVNNs, we get \(\tilde{SO}(sv_1) = 0.6333, \tilde{SO}(sv_2) = 0.6667\) and \(\tilde{SO}(sv_3) = 0.4333\), and hence \(T_{sv_1} = 1, T_{sv_2} = 0.6333\) and \(T_{sv_3} = 0.42222\). By using this information \((\rho = -2)\), we can obtain
\[
SVNSSPRWG(sv_1, sv_2, \ldots, sv_3) = \left(\left(\left(\frac{3}{p=1} \frac{b}{p=1} \frac{T_{b+1}}{p=1} \frac{1}{b+1} \right) \right) + \left(\left(\frac{3}{p=1} \frac{b}{p=1} \frac{1}{b+1} \right) \right) \right) - 1.
\]

So, when \(g = b + 1\), Eq. (43) is true. Therefore, Eq. (43) is true for all \(g\).

When \(\frac{T_{1}}{p=1} \geq 0\), such that \(\frac{g}{p=1} \frac{T_{1}}{p=1} = 1\), the Eq. (43) degenerates into the following form:

\[
SVNSSPRWG(sv_1, sv_2, \ldots, sv_G) = \left(\left(\left(\frac{g}{p=1} \frac{T_{p}}{p=1} \frac{1}{b+1} \right) \right) + \left(\left(\frac{g}{p=1} \frac{1}{b+1} \right) \right) \right) - 1.
\]
Theorem 7. For a group of SVNNs \( s_{vp} = \left( \overline{TR}_p, \overline{ID}_p, \overline{FP}_p \right) \), \((p = 1, 2, \ldots, g)\), the SVNPRWG operator satisfies the following properties:

1. (Idempotency). If all \( s_{vp}(p = 1, 2, \ldots, g) \) are equal, i.e.,
\[
sv_p = sv = \left( \overline{TR}, \overline{ID}, \overline{FP} \right);
\]

\( \text{SVNSSPRWG} (sv_1, sv_2,\ldots, sv_g) = sv \). \hspace{1cm} (47)

2. (Monotonicity). If \( sv_p' = \left( \overline{TR}_p', \overline{ID}_p', \overline{FP}_p' \right) \) and \( sv_p = \left( \overline{TR}_p, \overline{ID}_p, \overline{FP}_p \right) \) are two groups of SVNNs, such that

\( sv_p' \geq sv_p \), i.e., \( \overline{TR}_p \geq \overline{TR}_p', \overline{ID}_p \leq \overline{ID}_p', \overline{FP}_p \leq \overline{FP}_p' \) for all \( p \), then

\( \text{SVNSSPRWG} (sv_1', sv_2',\ldots, sv_g') \geq \text{SVNSSPRWG} (sv_1, sv_2,\ldots, sv_g) \). \hspace{1cm} (48)

3. (Boundedness). Let \( sv_p = \left( \overline{TR}_p, \overline{ID}_p, \overline{FP}_p \right) \) be a group of SVNNs, and \( sv_p' = \left( \max_{p=1}^{g} \overline{TR}_p, \min_{p=1}^{g} \overline{ID}_p, \max_{p=1}^{g} \overline{FP}_p \right) \),

\( sv_p' = \left( \min_{p=1}^{g} \overline{TR}_p, \max_{p=1}^{g} \overline{ID}_p, \min_{p=1}^{g} \overline{FP}_p \right) \),

\( \text{SVNSSPRWG} (sv_1, sv_2,\ldots, sv_g) = \left( \frac{1}{j} \left( \frac{\overline{TR}}{p=1}^{g} \overline{TR}_p - \frac{\overline{ID}}{p=1}^{g} \overline{ID}_p + 1 \right) \right)^{-1} - \left( \frac{1}{j} \left( \frac{1}{p=1}^{g} (1 - \frac{ID}{FP})^{p} - \frac{ID}{FP} \overline{ID}_p + 1 \right) \right)^{-1} - \left( \frac{1}{j} \left( \frac{1}{p=1}^{g} (1 - \frac{FP}{TR})^{p} - \frac{FP}{TR} \overline{FP}_p + 1 \right) \right)^{-1} \}

\( \text{SVNSSPRWG} (sv_1', sv_2',\ldots, sv_g') = \left( \frac{1}{j} \left( \frac{\overline{TR}}{p=1}^{g} \overline{TR}_p' - \frac{\overline{ID}}{p=1}^{g} \overline{ID}_p' + 1 \right) \right)^{-1} - \left( \frac{1}{j} \left( \frac{1}{p=1}^{g} (1 - \frac{ID}{FP})^{p} - \frac{ID}{FP} \overline{ID}_p' + 1 \right) \right)^{-1} - \left( \frac{1}{j} \left( \frac{1}{p=1}^{g} (1 - \frac{FP}{TR})^{p} - \frac{FP}{TR} \overline{FP}_p' + 1 \right) \right)^{-1} \}

\( (52) \)

When \( \rho = 0 \), the SVNPRASSWG operator reduces to the PG operator based on the algebraic operational laws for SVNNs. That is,

\( \text{SVNSSPRWG} (sv_1, sv_2,\ldots, sv_g) \leq \text{SVNSSPRWG} (sv_1', sv_2',\ldots, sv_g') \). \hspace{1cm} (49)

Definition 12. A SVNPROWG operator is a function \( \Theta ^{g} \rightarrow \Theta \), described as follows:

\( \text{SVNSSPRWG} (sv_1, sv_2,\ldots, sv_g) = \frac{g}{p=1}^{g} \text{SVNSSPRWG} (sv_p) \)

\( (51) \)

where \( T_0 = 1 \), and \( T_p = \frac{p-1}{\rho} \tilde{SO}(sv_p) \), \((p = 2, 3,\ldots, g)\), \( \zeta \) is a permutation of \((1, 2,\ldots, g)\) such that \( \zeta (p) \geq \zeta (p - 1) \) for \( p = 2, 3,\ldots, g \).

Theorem 8. For a group of SVNNs \( sv_p = \left( \overline{TR}_p, \overline{ID}_p, \overline{FP}_p \right) \), \((p = 1, 2,\ldots, g)\), the value aggregated by the developed SVNPRWOG operator is still a SVNN and is specified by:

\( \text{SVNSSPRWG} (sv_1, sv_2,\ldots, sv_g) = \left( \frac{1}{j} \left( \frac{\overline{TR}}{p=1}^{g} \overline{TR}_p - \frac{\overline{ID}}{p=1}^{g} \overline{ID}_p + 1 \right) \right)^{-1} - \left( \frac{1}{j} \left( \frac{1}{p=1}^{g} (1 - \frac{ID}{FP})^{p} - \frac{ID}{FP} \overline{ID}_p + 1 \right) \right)^{-1} - \left( \frac{1}{j} \left( \frac{1}{p=1}^{g} (1 - \frac{FP}{TR})^{p} - \frac{FP}{TR} \overline{FP}_p + 1 \right) \right)^{-1} \}

\( (50) \)

Proof. Same as Theorem 6, it is omitted.

Example 4. Consider the SVNNs given in Example 3, we have \( T_1 = 1, T_2 = 0.6333 \) and \( T_3 = 0.42222 \). The score values are \( \tilde{SO}(sv_1) = 0.6333, \tilde{SO}(sv_2) = 0.6667 \) and \( \tilde{SO}(sv_3) = 0.4333 \). So, we have \( \tilde{SO}(sv_2) > \tilde{SO}(sv_1) > \tilde{SO}(sv_3) \) and hence, \( sv_{\zeta(1)} = sv_2, sv_{\zeta(2)} = sv_1, sv_{\zeta(3)} = sv_3 \). By using this information \((\rho = -2)\), we can obtain
Theorem 9. For a group of SVNNs \( s_v = \langle \overline{TR}_p, \overline{ID}_p, \overline{FL}_p \rangle \), \((p = 1, 2, ..., g)\), the SVNPRWOG operator satisfies the following properties:

1. (Idempotency). If all \( s_v(p = 1, 2, ..., g) \) are equal, i.e., \( s_v = s_v = \langle \overline{TR}, \overline{ID}, \overline{FL} \rangle \), then \( SVNPRWOG(s_v, s_v, ..., s_v) = s_v \).

2. (Monotonicity). If \( s_v' = \langle \overline{TR}_p, \overline{ID}_p, \overline{FL}_p \rangle \) and \( s_v = \langle \overline{TR}_p, \overline{ID}_p, \overline{FL}_p \rangle \) are two groups of SVNNs, such that \( s_v' \geq s_v \), i.e., \( \overline{TR}_p \geq \overline{TR}_p, \overline{ID}_p \leq \overline{ID}_p, \overline{FL}_p \leq \overline{FL}_p \), for all \( p \), then \( SVNPRWOG(s_v', s_v, ..., s_v) \geq SVNPRWOG(s_v, s_v, ..., s_v) \).

3. (Boundedness). Let \( s_v = \langle \overline{TR}_p, \overline{ID}_p, \overline{FL}_p \rangle \) be a group of SVNNs, and \( s_v^+ = \langle \min_{p=1} g \overline{TR}_p, \min_{p=1} g \overline{ID}_p, \min_{p=1} g \overline{FL}_p \rangle \), \( s_v^- = \langle \max_{p=1} g \overline{TR}_p, \max_{p=1} g \overline{ID}_p, \max_{p=1} g \overline{FL}_p \rangle \), then

\[
s_v^- \leq SVNPRWOG(s_v, s_v, ..., s_v) \leq s_v^+. \tag{55}
\]

6. The MADM methods based on the proposed aggregation operators

In this part, we shall use the SVNPRWA and SVNPRWOG operators with SVNNs to solve the MADM problem. The following presumptions or notations are utilized to express the MADM problem. Let the discrete set of alternatives be expressed by \( \overline{H} = \{ \overline{H}_1, \overline{H}_2, ..., \overline{H}_h \} \), and the set of attributes be expressed by \( \overline{H} = \{ \overline{H}_1, \overline{H}_2, ..., \overline{H}_g \} \), and that there is a prioritization among the attributes represented by the linear-ordering \( \overline{H}_1 > \overline{H}_2 > ... > \overline{H}_{g-1} > \overline{H}_g \), the specified attribute \( \overline{H}_n \) has a higher priority than \( \overline{H}_m \) if \( n < m \). Assume that \( \overline{M} = (\overline{m}_{rs})_{h \times g} \) is the SVNN decision matrix, where \( \overline{TR}_r, \overline{ID}_r \) and \( \overline{FL}_r \) express the TM function, IF function and FM function respectively, such that \( \overline{TR}_r \in [0, 1], \overline{ID}_r \in [0, 1], \overline{FL}_r \in [0, 1], 0 \leq \overline{TR}_r + \overline{ID}_r + \overline{FL}_r \leq 3 \), \((r = 1, 2, ..., g, s = 1, 2, ..., h)\). The goal of this problem is to rank the alternatives.

6.1. The method based on SVNSSPRWA operator

In the following, a process for ranking and selecting the most preferable alternative(s) is provided as follows.

**Step 1.** Standardize the decision matrix.

First, the decision making information \( \overline{m}_{rs} \) in the matrix \( \overline{M} = (\overline{m}_{rs})_{h \times g} \) must be standardized. Consequently, the attribute can be grouped into the cost and benefit types. For benefit type attribute, the assessment information does not need to changed, but for cost type attribute, it must be modified with the complement set.

The decision matrix can be standardized by the following formula:

\[
\overline{m}_{rs} = \left\{ \begin{array}{ll}
\langle \overline{TR}_r, \overline{ID}_r, \overline{FL}_r \rangle & \text{for benefit type attribute } \overline{H}_r \\
\langle \overline{TR}_r, 1 - \overline{ID}_r, \overline{FL}_r \rangle & \text{for cost type attribute } \overline{H}_r
\end{array} \right.
\]

**Step 2.** Determine the values of \( T_r \) \((r = 1, 2, ..., h)\); \( s = 1, 2, ..., g) \) by using the following formula:

\[
T_r = \prod_{i=1}^{g} \overline{SO}(\overline{m}_{ri})(r = 1, 2, ..., h; s = 2, 3, ..., g) \tag{57}
\]

where \( T_r = 1 \) for \( r = 1, 2, ..., h \).

**Step 3.** Use the decision information from decision matrix \( \overline{M} = (\overline{m}_{rs})_{h \times g} \) and the SVNSSPRWA operator given in Eq. (26),

\[
\overline{m}_{rs} = \langle \overline{TR}_r, \overline{ID}_r, \overline{FL}_r \rangle = SVNSSPRWA(\overline{m}_{r1}, \overline{m}_{r2}, ..., \overline{m}_{rg}) \tag{58}
\]

to get the overall SVNN \( \overline{m}_r(r = 1, 2, ..., h) \).

**Step 4.** Determine the score values \( \overline{SO}(\overline{m}_r)(r = 1, 2, ..., h) \) of the overall SVNNs \( \overline{m}_r(r = 1, 2, ..., h) \) by Definition 3 to rank all the alternatives \( \overline{G}_r(r = 1, 2, ..., h) \).

**Step 5.** Rank all the alternatives \( \overline{G}_r(r = 1, 2, ..., h) \) and select best one utilizing Definition 5.

**Step 6.** End.
6.2. The method based on SVNSSPRWA operator

Steps 1 and 2 are same.

**Step 3.** Use the decision information permitted decision matrix $\overline{m} = (\overline{m}_r)_{k \times g}$ and the SVNSSPRWG operator given in Eq. (42)

$$\overline{m}_r = \left( \overline{m}_1, \overline{m}_2, ..., \overline{m}_g \right) = \text{SVNSSPRWG}(\overline{m}_1, \overline{m}_2, ..., \overline{m}_g) \quad (59)$$

To get the overall SVNN $\overline{m}_r (r = 1, 2, ..., h)$.

**Step 4.** Determine the score values $SO(\overline{m}_r) (r = 1, 2, ..., h)$ of the overall SVNNs $\overline{m}_r (r = 1, 2, ..., h)$ by Definition 3 to rank all the alternatives $\overline{G}_r (r = 1, 2, ..., h)$.

**Step 5.** Rank all the alternatives $\overline{G}_r (r = 1, 2, ..., h)$ and select best one utilizing Definition 5.

**Step 6.** End.

7. An illustrative example

In this part, we use a numerical example of selecting third-party logistics (TPL) providers with SVNNs (Ji et al., 2018) to show the effectiveness and advantages of the developed approach.

**Example 5.** An electronic commerce distributor expects to select a suitable TPL provider. Initially, four providers (alternatives) $\overline{G}_r (r = 1, 2, ..., 4)$ are available for selection and are evaluated by experts with respect to the following four attributes (1) customer satisfaction $\overline{H}_1$, (2) service cost $\overline{H}_2$, (3) market reputation $\overline{H}_3$, and (4) operational experience in the industry $\overline{H}_4$. The following priority relationship $\overline{H}_1 > \overline{H}_2 > \overline{H}_3 > \overline{H}_4$ among the four attributes is considered by the electronic commerce distributor. The assessment values of the four providers with respect to the four attributes are provided by expert in the form of SVNNs and listed in Table 1.

**Step 1.** Normalize the decision matrix. Since $\overline{H}_1$, $\overline{H}_3$, and $\overline{H}_4$ are of benefit type, and $\overline{H}_2$ is of cost type attribute. Hence, by using Eq. (56), the normalized decision matrix is given in Table 2.

**Step 2.** Determine the values of $\overline{T}_{rs} (r = 1, 2, ..., 4, s = 1, 2, ..., 4)$ by using the formula (57), and get

| $\overline{m}_1$ | $\overline{m}_2$ | $\overline{m}_3$ | $\overline{m}_4$ |
|----------------|----------------|----------------|----------------|
| (0.7, 0, 1, 0.2) | (0.3, 0, 0.9, 0.5) | (0.3, 0, 2, 0.1) | (0.5, 0, 1, 0.4) |
| (0.9, 0, 1, 0.1) | (0.3, 0, 0.8, 0.4) | (0.5, 0, 3, 0.2) | (0.3, 0, 2, 0.4) |
| (0.5, 0, 1, 0.4) | (0.1, 0, 8, 0.7) | (0.6, 0, 2, 0.2) | (0.8, 0, 1, 0.3) |
| (0.4, 0, 3, 0.2) | (0.2, 0, 9, 0.6) | (0.7, 0, 2, 0.1) | (0.2, 0, 2, 0.5) |

| $\overline{G}_1$ | $\overline{G}_2$ | $\overline{G}_3$ | $\overline{G}_4$ |
|----------------|----------------|----------------|----------------|
| $\overline{m}_1$ | (0.7, 0, 1, 0.2) | (0.5, 0, 1, 0.3) | (0.3, 0, 2, 0.1) | (0.5, 0, 1, 0.4) |
| $\overline{m}_2$ | (0.9, 0, 1, 0.1) | (0.4, 0, 2, 0.3) | (0.5, 0, 3, 0.2) | (0.3, 0, 2, 0.4) |
| $\overline{m}_3$ | (0.5, 0, 1, 0.4) | (0.7, 0, 2, 0.1) | (0.6, 0, 2, 0.2) | (0.8, 0, 1, 0.3) |
| $\overline{m}_4$ | (0.4, 0, 3, 0.2) | (0.6, 0, 1, 0.2) | (0.7, 0, 2, 0.1) | (0.2, 0, 2, 0.5) |

**Step 3.** Use the SVNSSPRWA given in Eq. (58) to get the overall SVNN $\overline{m}_r (r = 1, 2, ..., 4)$ (assume $\rho = -2$), and obtain

$$\overline{m}_r = \begin{bmatrix} 1 & 0.800 & 0.700 & 0.6667 \\ 1 & 0.900 & 0.6333 & 0.6667 \\ 1 & 0.6667 & 0.800 & 0.7333 \\ 1 & 0.6333 & 0.7667 & 0.8000 \end{bmatrix}$$

**Step 4.** Determine the score values $SO(\overline{m}_r) (r = 1, 2, ..., 4)$ of the overall SVNNs $\overline{m}_r (r = 1, 2, ..., 4)$ by Definition 3, and have

$$SO(\overline{m}_1) = 0.7682, SO(\overline{m}_2) = 0.7858, SO(\overline{m}_3) = 0.7984, SO(\overline{m}_4) = 0.7430.$$<ref>}}

So, we get $\overline{m}_3 > \overline{m}_2 > \overline{m}_1 > \overline{m}_4$.

**Step 5.** According to score values, ranking order of alternatives is $\overline{G}_3 > \overline{G}_2 > \overline{G}_1 > \overline{G}_4$. So the best provider is $\overline{G}_3$, while the worst one is $\overline{G}_4$.

Similarly, we solve the above Example 5 by utilizing SVNSSPWG operator:
Step 1 and step 2 are same.  

Step 3. Use the SVNSSPRWG operator to get the overall SVNN \( \overline{m}_r (r = 1, 2, \ldots, 4) \) (assume \( \rho = -2 \), and have  
\[
\begin{align*}
\overline{m}_1 &= (0.4493, 0.1253, 0.2681), \\
\overline{m}_2 &= (0.4364, 0.1980, 0.2643), \\
\overline{m}_3 &= (0.6055, 0.1502, 0.2900), \\
\overline{m}_4 &= (0.3289, 0.2208, 0.3088).
\end{align*}
\]

Step 4. Determine the score values \( \tilde{SO}(\overline{m}_r) (r = 1, 2, \ldots, 4) \) of the overall SVNNs \( \overline{m}_r (r = 1, 2, \ldots, 4) \), and have  
\[
\begin{align*}
\tilde{SO}(\overline{m}_1) &= 0.6853, \\
\tilde{SO}(\overline{m}_2) &= 0.6580, \\
\tilde{SO}(\overline{m}_3) &= 0.7218, \\
\tilde{SO}(\overline{m}_4) &= 0.5998.
\end{align*}
\]

So, \( \overline{m}_3 > \overline{m}_1 > \overline{m}_2 > \overline{m}_4 \).

Step 5. According to score values, ranking order of alternatives is \( \overline{G}_3 > \overline{G}_1 > \overline{G}_2 > \overline{G}_4 \). So, the best provider is \( \overline{G}_3 \), while the worst one is \( \overline{G}_4 \).

7.1. Effect of the parameter \( \rho \) on decision result of this example

In order to see the effect of the parameter \( \rho \) on the decision-making result, we set the distinct values for the parameter \( \rho \) in step 3, to rank the alternatives. The score values and ranking order are described in Tables 3 and 4.

As from Table 3, we can notice that the ranking orders by utilizing SVNSSSPWA operator are slightly different when the parameter \( \rho \) takes the distinct values. When the value of the parameter \( \rho \) tends to zero, the best choice is \( \overline{G}_3 \) and the worst choice is \( \overline{G}_2 \). When the value of the parameter \( \rho \) decreases from 0 then the best choice is \( \overline{G}_3 \) while the worst one is \( \overline{G}_4 \). We can also see from Table 3, when the value of the parameter decreases the score values become bigger and bigger.

From Table 4, we can see that the ranking orders by utilizing SVNSSPW operator do not change for different values of the parameter \( \rho \), the best choice is \( \overline{G}_3 \), while the worst one is \( \overline{G}_4 \). We can also notice from Table 4, when the value of the parameter \( \rho \) decreases, the score values become smaller and smaller. Generally, different DMs can set different values of the parameter \( \rho \) according to their actual need.

Example 6 ((Wei and Wei, 2018)). In order to reinforce the academic education, the school of management in a Chinese university wants to introduce excellent overseas teachers. This introduction caught much attention from the school, university president, dean of management school and human resource officer sets of a panel of decision makers who will take the whole responsibility for this introduction. The panel made strict assessment for five alternatives (candidates) \( \overline{G}_r (r = 1, 2, \ldots, 5) \) from four characteristics (attributes) namely, morality \( \overline{H}_1 \), research potential \( \overline{H}_2 \), skill of teaching \( \overline{H}_3 \), education background \( \overline{H}_4 \). The president of the university has absolute priority in decision making, and the dean of the school of management is next. In addition, this introduction will be in a strict

| Table 3 |
| --- |
| Score values and ranking order for different values of \( \rho \) utilizing SVNSSSPWA operator for example 5. |
| \( \rho \) & Score values & Ranking order |
| --- & --- & --- |
| \( \rho = 0 \) & \( \tilde{SO}(\overline{m}_1) = 0.7350, \tilde{SO}(\overline{m}_2) = 0.6853, \) & \( \overline{G}_3 > \overline{G}_1 > \overline{G}_4 > \overline{G}_2 \). |
| & \( \tilde{SO}(\overline{m}_3) = 0.7607, \tilde{SO}(\overline{m}_4) = 0.6950. \) &  |
| \( \rho = -1 \) & \( \tilde{SO}(\overline{m}_1) = 0.7526, \tilde{SO}(\overline{m}_2) = 0.7480, \) & \( \overline{G}_3 > \overline{G}_1 > \overline{G}_2 > \overline{G}_4 \). |
| & \( \tilde{SO}(\overline{m}_3) = 0.7806, \tilde{SO}(\overline{m}_4) = 0.7211. \) &  |
| \( \rho = -2 \) & \( \tilde{SO}(\overline{m}_1) = 0.7682, \tilde{SO}(\overline{m}_2) = 0.7858, \) & \( \overline{G}_3 > \overline{G}_1 > \overline{G}_2 > \overline{G}_4. \) |
| & \( \tilde{SO}(\overline{m}_3) = 0.7984, \tilde{SO}(\overline{m}_4) = 0.7430. \) &  |
| \( \rho = -7 \) & \( \tilde{SO}(\overline{m}_1) = 0.8070, \tilde{SO}(\overline{m}_2) = 0.8384, \) & \( \overline{G}_3 > \overline{G}_1 > \overline{G}_2 > \overline{G}_4. \) |
| & \( \tilde{SO}(\overline{m}_3) = 0.8404, \tilde{SO}(\overline{m}_4) = 0.7967. \) &  |
| \( \rho = -20 \) & \( \tilde{SO}(\overline{m}_1) = 0.8244, \tilde{SO}(\overline{m}_2) = 0.8571, \) & \( \overline{G}_3 > \overline{G}_1 > \overline{G}_2 > \overline{G}_4. \) |
| & \( \tilde{SO}(\overline{m}_3) = 0.8578, \tilde{SO}(\overline{m}_4) = 0.8207. \) &  |
| \( \rho = -100 \) & \( \tilde{SO}(\overline{m}_1) = 0.8316, \tilde{SO}(\overline{m}_2) = 0.8648, \) & \( \overline{G}_3 > \overline{G}_1 > \overline{G}_2 > \overline{G}_4. \) |
| & \( \tilde{SO}(\overline{m}_3) = 0.8649, \tilde{SO}(\overline{m}_4) = 0.8309. \) &  |
| \( \rho = -200 \) & \( \tilde{SO}(\overline{m}_1) = 0.8325, \tilde{SO}(\overline{m}_2) = 0.8657, \) & \( \overline{G}_3 > \overline{G}_1 > \overline{G}_2 > \overline{G}_4. \) |
| & \( \tilde{SO}(\overline{m}_3) = 0.8658, \tilde{SO}(\overline{m}_4) = 0.8321. \) &  |
accordance with the principle of combining ability with political integrity. The prioritization among the attributes is as follow, 

\[ g_3 > g_1 > g_2 > g_4. \]

The decision makers assess possible 5 alternatives \( g_r (r = 1, 2, ..., 5) \) with respect to the 4 attributes \( g_s (s = 1, 2, ..., 4) \) and construct the following SVN decision matrix given in Table 5.

**Step 1.** Normalize the decision matrices by using Eq. (56). Since all the attributes are of benefit type so there is no need to normalize it.

**Step 2.** Determine the values of \( T_{MN} (r = 1, 2, ..., 5; s = 1, 2, ..., 4) \) by using the formula (57), and get

\[
T_{MN} = \begin{bmatrix}
1 & 0.5333 & 0.3556 & 0.1011 \\
1 & 0.8000 & 0.6133 & 0.3435 \\
1 & 0.6333 & 0.3167 & 0.1404 \\
1 & 0.8000 & 0.5600 & 0.3435 \\
1 & 0.5333 & 0.2667 & 0.0948
\end{bmatrix}
\]

**Step 3.** Use the SVNSSPRWA given in Eq. (58) to get the overall SVNN \( \overline{g}_r (r = 1, 2, ..., 5) \) (assume \( \rho = -2 \)), and obtain

\[
\begin{align*}
\overline{g}_1 &= (0.5151, 0.4787, 0.1176), \\
\overline{g}_2 &= (0.7209, 0.2000, 0.1320), \\
\overline{g}_3 &= (0.5626, 0.4490, 0.1764), \\
\overline{g}_4 &= (0.7246, 0.1434, 0.1681), \\
\overline{g}_5 &= (0.5366, 0.5754, 0.1462).
\end{align*}
\]

**Step 4.** Determine the score values \( \overline{SO}(\overline{g}_r) (r = 1, 2, ..., 5) \) of the overall SVNNS \( \overline{g}_r (r = 1, 2, ..., 5) \) by using Definition (3), and have

\[ \overline{SO}(\overline{g}_1) = 0.6396, \overline{SO}(\overline{g}_2) = 0.7963, \overline{SO}(\overline{g}_3) = 0.6458, \overline{SO}(\overline{g}_4) = 0.8044, \overline{SO}(\overline{g}_5) = 0.6050. \]

So, \( \overline{g}_4 > \overline{g}_2 > \overline{g}_3 > \overline{g}_1 > \overline{g}_5 \). So, the best candidate is \( \overline{g}_4 \), while the worst one is \( \overline{g}_5 \).

Similarly, we solve the above Example 6 by the SVNSSPWG operator:

Step 1 and step 2 are same.

**Step 3.** Use the SVNSSPWG operator given Equation (Step 3) to get the overall SVNNS \( \overline{g}_r (r = 1, 2, ..., 5) \) (assume \( \rho = -2 \)), and have

\[
\begin{align*}
\overline{g}_1 &= (0.4498, 0.7400, 0.1745), \\
\overline{g}_2 &= (0.7104, 0.2000, 0.2269), \\
\overline{g}_3 &= (0.5477, 0.5821, 0.2233), \\
\overline{g}_4 &= (0.6554, 0.2051, 0.2265), \\
\overline{g}_5 &= (0.4790, 0.7022, 0.3042).
\end{align*}
\]
Step 4. Determine the score values $SO(\overline{m}_r)$ $(r = 1, 2, \ldots, 5)$ of the overall SVNNs $\overline{m}_r (r = 1, 2, \ldots, 5)$ by using Definition 3, and get

$$
SO(\overline{m}_1) = 0.5118, SO(\overline{m}_2) = 0.7612, SO(\overline{m}_3) \\
= 0.5808, SO(\overline{m}_4) = 0.7412, SO(\overline{m}_5) = 0.4909.
$$

So, $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$.

Step 5. According to score values ranking order of alternatives is $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$. So, the best candidate is $\overline{m}_2$, while the worst one is $\overline{m}_5$.

7.2. Effect of the parameter $\rho$ on decision result of this example 6

In order to see the effect of the parameter $\rho$ on the decision-making result, we set the distinct values for the parameter $\rho$ in step 3, to rank the alternatives. The score values and ranking order are described in Tables 6 and 7, and Figs. 1 and 2.

From Table 6, we can notice that the ranking orders by utilizing SVNSSPWA operator are slightly different when the parameter $\rho$ takes the distinct values. When the value of the parameter $\rho$ is -1 and tends to zero, the best choice is $\overline{m}_2$. When the value of the parameter $\rho$ decreases from -1 then the best choice is $\overline{m}_4$. We can also see from Table 6, when the value of the parameter decreases the score values become bigger and bigger.

From Table 7, we can see that the ranking orders by utilizing SVNSSPWG operator do not change for different values of the parameter $\rho$, the best choice is $\overline{m}_2$. We can also notice from Table 7, when the value of the parameter $\rho$ decreases, the score values become smaller and smaller. Generally, different DMs can set different values of the parameter $\rho$ according to their actual need.

### Table 6
Score values and ranking order for different values of $\rho$ utilizing SVNSSPWA operator for example 6.

| $\rho$ | Score values | Ranking order |
|-------|--------------|---------------|
| $\rho \rightarrow 0$ | $SO(\overline{m}_1) = 0.5935, SO(\overline{m}_2) = 0.7828, SO(\overline{m}_3) = 0.6195$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 1$ | $SO(\overline{m}_4) = 0.7716, SO(\overline{m}_5) = 0.5652.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 2$ | $SO(\overline{m}_1) = 0.6179, SO(\overline{m}_2) = 0.7908, SO(\overline{m}_3) = 0.6325, SO(\overline{m}_4) = 0.7987, SO(\overline{m}_5) = 0.5877.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 3$ | $SO(\overline{m}_4) = 0.6396, SO(\overline{m}_5) = 0.7963, SO(\overline{m}_1) = 0.6458, SO(\overline{m}_2) = 0.8044, SO(\overline{m}_3) = 0.6050.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 4$ | $SO(\overline{m}_1) = 0.6916, SO(\overline{m}_2) = 0.8110, SO(\overline{m}_3) = 0.6913, SO(\overline{m}_4) = 0.8433, SO(\overline{m}_5) = 0.6521.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 5$ | $SO(\overline{m}_1) = 0.7410, SO(\overline{m}_2) = 0.8248, SO(\overline{m}_3) = 0.7181, SO(\overline{m}_4) = 0.8588, SO(\overline{m}_5) = 0.6829.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 6$ | $SO(\overline{m}_1) = 0.7301, SO(\overline{m}_2) = 0.8317, SO(\overline{m}_3) = 0.7304, SO(\overline{m}_4) = 0.8651, SO(\overline{m}_5) = 0.6967.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 7$ | $SO(\overline{m}_1) = 0.7317, SO(\overline{m}_2) = 0.8325, SO(\overline{m}_3) = 0.7318, SO(\overline{m}_4) = 0.8659, SO(\overline{m}_5) = 0.6983.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |

### Table 7
Score values and ranking order for different values of $\rho$ utilizing SVNSSPWG operator for example 6.

| $\rho$ | Score values | Ranking order |
|-------|--------------|---------------|
| $\rho \rightarrow 0$ | $SO(\overline{m}_1) = 0.5463, SO(\overline{m}_2) = 0.7688, SO(\overline{m}_3) = 0.5985, SO(\overline{m}_4) = 0.7503, SO(\overline{m}_5) = 0.5241.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 1$ | $SO(\overline{m}_1) = 0.5271, SO(\overline{m}_2) = 0.7651, SO(\overline{m}_3) = 0.5894, SO(\overline{m}_4) = 0.7457, SO(\overline{m}_5) = 0.5067.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 2$ | $SO(\overline{m}_1) = 0.5118, SO(\overline{m}_2) = 0.7612, SO(\overline{m}_3) = 0.5808, SO(\overline{m}_4) = 0.7412, SO(\overline{m}_5) = 0.4909.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 3$ | $SO(\overline{m}_1) = 0.4651, SO(\overline{m}_2) = 0.7414, SO(\overline{m}_3) = 0.5514, SO(\overline{m}_4) = 0.7224, SO(\overline{m}_5) = 0.4470.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 4$ | $SO(\overline{m}_1) = 0.4268, SO(\overline{m}_2) = 0.7170, SO(\overline{m}_3) = 0.5255, SO(\overline{m}_4) = 0.6999, SO(\overline{m}_5) = 0.4188.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 5$ | $SO(\overline{m}_1) = 0.4053, SO(\overline{m}_2) = 0.7033, SO(\overline{m}_3) = 0.5053, SO(\overline{m}_4) = 0.6728, SO(\overline{m}_5) = 0.4037.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
| $\rho = 6$ | $SO(\overline{m}_1) = 0.4026, SO(\overline{m}_2) = 0.7017, SO(\overline{m}_3) = 0.5027, SO(\overline{m}_4) = 0.6697, SO(\overline{m}_5) = 0.4019.$ | $\overline{m}_2 > \overline{m}_3 > \overline{m}_4 > \overline{m}_5 > \overline{m}_1$ |
7.3. Comparison with the other methods

In order to further show the effectiveness of the proposed methods based on the proposed AOs, in this article, we solve Example 6 by seven existing methods based on different aggregation operators under SVN environment. SVN weighted averaging (SVNWA) operator proposed by Ye (2014), SVNWA operator proposed by Peng et al. (2016) based on improved operational laws for SVNNs, SVN-MABAC (Peng and Jingguo, 2018), SVN-TOPSIS (Peng and Jingguo, 2018), SVN prioritized weighted averaging (PRWA) operator developed by Wu et al. (2016), SVN Dombi prioritized weighted averaging (PRWA) operator developed by Wei and Wei (2018) and SVNN normalized BM (SVNNBM) operator developed by Liu and Wang (2014). The score values and ranking order are given in Table 8.

The weight vector of attributes for these methods is obtained using the PRA operator.

From Table 8, we can see that when value of the parameter \( q \) tends to zero, the ranking orders obtained by the proposed method based on the proposed aggregation operators are same with the other five methods. This shows that our method is valid. Further, when we set the parameter value \( q = -2 \), then, the ranking order is same as that obtained from the methods developed in (Peng and Jingguo, 2018) and (Wei and Wei, 2018) based on SVN-TOPSIS and SVN Dombi prioritized averaging operators.

Moreover, the comparison among our method with the existing seven methods can be pointed out as follows:

1. The methods developed by Ye (2014) and Peng et al. (2016) are based on SVNWA operators. These aggregation operators are based on algebraic operations, while the aggregation operators in this article are based on Schweizer-Sklar operations. Although the best alternative is same, however, when we change the value of the parameter \( \rho \) the best alternative changed. That’s why our method is more flexible and effective than Ye (2014) and Peng et al. (2016).
2. The method of Wu et al. (2016) is based on SVN prioritized weighted averaging operator. This is a special case of the developed aggregation operators, when the value of the parameter \( q \) tends to Zero.
Table 8
Score values and ranking orders with different methods.

| Methods                        | Score values                  | Ranking order |
|--------------------------------|--------------------------------|---------------|
| SVNWA operator (Ye, 2014)      | $SO(\mathbf{R}_1) = 0.5532$, $SO(\mathbf{S}_1) = 0.7698$, $SO(\mathbf{O}_1) = 0.6001$ | $G_2 > G_4 > G_3 > G_1 > G_5$ |
| SVNWA operator (Peng et al., 2016) | $SO(\mathbf{R}_1) = 0.5934$, $SO(\mathbf{S}_1) = 0.7828$, $SO(\mathbf{O}_1) = 0.6195$ | $G_2 > G_4 > G_3 > G_1 > G_5$ |
| SVNDPAW (Wei and Wei, 2018) $\rho = 2$ | $SO(\mathbf{R}_1) = 0.6540$, $SO(\mathbf{S}_1) = 0.7973$, $SO(\mathbf{O}_1) = 0.6259$ | $G_4 > G_2 > G_1 > G_3 > G_5$ |
| SVN-TOPSIS (Peng and Jingguo, 2018) | $C(\mathbf{R}_1) = -3.3557$, $C(\mathbf{S}_1) = -0.8123$, $C(\mathbf{O}_1) = -2.7509$ | $G_4 > G_2 > G_1 > G_3 > G_5$ |
| SVN-MABAC (Peng and Jingguo, 2018) | $D(\mathbf{R}_1) = 0.2637$, $D(\mathbf{S}_1) = 0.6122$, $D(\mathbf{O}_1) = 0.2176$ | $G_2 > G_4 > G_3 > G_1 > G_5$ |
| SVNPWA operator (Wu et al., 2016) | $SO(\mathbf{R}_1) = 0.5934$, $SO(\mathbf{S}_1) = 0.7828$, $SO(\mathbf{O}_1) = 0.6195$ | $G_2 > G_4 > G_3 > G_1 > G_5$ |
| SVNNBM operator (p=q=1) (Liu and Wang, 2014) | $SO(\mathbf{R}_1) = 0.56597$, $SO(\mathbf{S}_1) = 0.77429$, $SO(\mathbf{O}_1) = 0.60837$ | $G_2 > G_4 > G_3 > G_1 > G_5$ |
| SVNSSPRWA operator (in this article) ($\rho \to 0$) | $SO(\mathbf{R}_1) = 0.5935$, $SO(\mathbf{S}_1) = 0.7828$, $SO(\mathbf{O}_1) = 0.6195$ | $G_2 > G_4 > G_3 > G_1 > G_5$ |
| SVNSSPRWA operator (in this article) ($\rho = -2$) | $SO(\mathbf{R}_1) = 0.6396$, $SO(\mathbf{S}_1) = 0.7963$, $SO(\mathbf{O}_1) = 0.6458$ | $G_4 > G_2 > G_1 > G_3 > G_5$ |

(3) The method developed by Liu and Wang (2014) is based on the SVNNWB BM operator, to solve the same example, we set $p = q = 1$, then the ranking order is same as the one obtained by the developed aggregation operators, when the value of the parameter tends to zero. This shows the effectiveness of the proposed approach based on the developed aggregation operator. But the advantage of the developed method in this article is that it can deal with the situation in which the attributes are with the prioritized relationship.

(4) The methods developed by Peng and Jingguo (2018) is based on SVN-TOPSIS and SVN-MABAC method in which the weights of the attributes are obtained via gray system theory and cannot consider the prioritized relationship among the attributes.

(5) The method developed by Wei and Wei (2018) is based on Dombi prioritized aggregation for SVNSs. The Dombi prioritized aggregation operator also consists of parameter, but the decision makers can consider the parameter greater than zero, while in the proposed aggregation operators in this article the decision makers can consider the parameter values less than zero.

Certainly, the developed methods in this article are more general and flexible by the parameter, and are more advanced to be used in practical decision-making problems.

8. Conclusion

Since SVNNs are a better tool to define uncertain information more accurately than the FS and IFS. In this article, we investigated some Schweizer-Sklar prioritized aggregation operator based on SVNNs and proposed two methods to deal with single-valued neutrosophic information. First, we have developed some new aggregation operators and studied their desirable properties such as idempotency, monotonicity and boundedness. Moreover, we have analyzed some special cases of the developed operators, and have presented two MADM methods based on the proposed aggregation operators to deal with SVN information. Lastly, some practical examples are given to show the verification of the developed methods and to demonstrate the effectiveness and practicality of the developed approaches and a comparison analysis is also given to verify the developed methods.

In future we shall combine SSTN and STCN with several generalizations of NSs such as interval neutrosophic sets, Double-valued neutrosophic sets, multi-valued neutrosophic sets and develop different aggregation operators such as Bonferroni mean operators, Heronian mean operators, Maclaurin symmetric mean operators for SVNNs. In addition, we will also apply the proposed method to solve some real decision problems (Abdel-Basset and Mai, 2018; Abdel-Basset, Mai, et al., 2018; Abdel-Basset, Gunasekaran, et al., 2018; Chang et al., 2018; Guan, Zhao, & Du, 2017).

Acknowledgement

This paper is supported by National Natural Science Foundation of China (Nos.71771140, 71471172), the Special Funds of Taishan Scholars Project of Shandong Province (No. ts201511045), Shandong Provincial Social Science Planning Project (Nos. 17BGLJ04, 16CGLJ31 and 16CKJ27), and Key research and development program of Shandong Province (No. 2016GNC110016).

Please cite this article as: P. Liu, Q. Khan and T. Mahmood, Multiple-attribute decision making based on single-valued neutrosophic Schweizer-Sklar prioritized aggregation operator, Cognitive Systems Research, https://doi.org/10.1016/j.cogsys.2018.10.005
References

Abdel-Basset, M., Gunasekaran, M., & Mai, M. (2018). Internet of Things (IoT) and its impact on supply chain: A framework for building smart, secure and efficient systems. Future Generation Computer Systems, 86, 614–628.

Abdel-Basset, M., Gunasekaran, M., Mohamed, M., & Smarandache, F. (2018). A novel method for solving the fully neutrosophic linear programming problems. Neural Computing and Applications, https://doi.org/10.1007/s00521-018-3404-6.

Abdel-Basset, M., & Mai, M. (2018). The role of single valued neutrosophic sets and rough sets in smart city: Imperfect and incomplete information systems. Measurement, 124, 47–55.

Abdel-Basset, M., Mai, M., & Chang, V. (2018). NMCDA: A framework for evaluating cloud computing services. Future Generation Computer Systems, 86, 12–29.

Abdel-Basset, M., Mai, M., & Smarandache, F. (2018). An extension of neutrosophic AHP-SWOT analysis for strategic planning and decision-making. Symmetry, 10(4), 1–16.

Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. Journal of Intelligent & Fuzzy Systems, 33(6), 4055–4066.

Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2018). A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. Design Automation for Embedded Systems, 22(3), 257–278.

Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. Journal of Intelligent & Fuzzy Systems, 33(6), 4055–4066.

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87–96.

Chang, V., Abdel-Basset, M., & Ramachandran, M. (2018). Towards a reuse strategic decision pattern framework–from theories to practices. Information Systems Frontiers. https://doi.org/10.1007/s11796-018-9853-8.

Deschrijver, G. (2009). Generalized arithmetic operators and their relationship to t-norms in interval-valued fuzzy set theory. Fuzzy Sets and Systems, 160(21), 3080–3102.

Deschrijver, G., & Kerre, E. E. (2002). A generalization of operators on intuitionistic fuzzy sets using triangular norms and conorms. Notes on IFS, 8(1), 19–27.

Garg, H. (2016). Novel single-valued neutrosophic aggregated operators under frank norm operation and its application to decision-making process. International Journal for Uncertainty Quantification, 6(4).

Garg, H. (2017). Some new biparametric distance measures on single-valued neutrosophic sets with applications to pattern recognition and medical diagnosis. Information, 8(4), 162.

Guan, H., Zhao, A., & Du, J. (2017). Enterprise green technology innovation behaviour. Beijing: Economic Science Press.

Ji, P., Wang, J. Q., & Zhang, H. (2018). Frank prioritized Bonferroni mean operator with single-valued neutrosophic sets and its application in selecting third-party logistics providers. Neural Computing and Applications, 30(3), 799–823.

Karaaslan, F., & Hayat, K. (2018). Some new operations on single-valued neutrosophic matrices and their applications in multi-criteria group decision making. Applied Intelligence, 1–21.

Li, Y., Liu, P., & Chen, Y. (2016). Some single valued neutrosophic number heronian mean operators and their application in multiple attribute group decision making. Informatica, 27(1), 85–110.

Liu, P. (2016). The aggregation operators based on archimedean t-Conorm and t-Norm for single-valued neutrosophic numbers and their application to decision making. International Journal of Fuzzy Systems, 18(5), 849–863.

Liu, P., Chu, Y., Li, Y., & Chen, Y. (2014). Some generalized neutrosophic number hamacher aggregation operators and their application to group decision making. International Journal of Fuzzy Systems, 16(2), 242–255.

Liu, P., Mahmood, T., & Khan, Q. (2017). Multi-attribute decision-making based on prioritized aggregation operator under hesitant intuitionistic fuzzy linguistic environment. Symmetry, 9(11), 270.

Liu, P., & Tang, G. (2016). Some power generalized aggregation operators based on the interval neutrosophic sets and their application to decision making. Journal of Intelligent & Fuzzy Systems, 30(5), 2517–2528.

Liu, P., & Wang, Y. (2014). Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Neural Computing and Applications, 25(7–8), 2001–2010.

Liu, P., & Wang, Y. (2016). Interval neutrosophic prioritized OWA operator and its application to multiple attribute decision making. Journal of Systems Science and Complexity, 29(3), 681–697.

Liu, P., & Wang, P. (2018). Some interval-valued intuitionistic fuzzy Schweizer-Sklar power aggregation operators and their application to supplier selection. International Journal of Systems Science. https://doi.org/10.1080/00207721.2018.1442510.

Liu, P., & You, X. (2017). Interval neutrosophic Muirhead mean operators and their application in multiple attribute group decision-making. International Journal for Uncertainty Quantification, 7(4), 333–344.

Lu, Z., & Ye, J. (2017). Single-valued neutrosophic hybrid arithmetic and geometric aggregation operators and their decision-making method. Information, 8(3), 84.

Mandal, K., & Basu, K. (2018). Vector aggregation operator and score function to solve multi-criteria decision making problem in neutrosophic environment. International Journal of Machine Learning and Cybernetics. https://doi.org/10.1007/s13042-018-0819-4.

Peng, X., & Jingguo, D. (2018). Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function. Neural Computing and Applications, 29(10), 939–954.

Peng, J. J., Wang, J. Q., Wang, J., Zhang, H. Y., & Chen, X. H. (2016). Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. International Journal of Systems Science, 47(10), 2342–2358.

Smarandache, F. (1999). A unifying field in logics: Neutrosophic logic. In Wang, H., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2010). A generalized aggregation operator and score function to solve multi-criteria decision making problem in neutrosophic environment. International Journal of Fuzzy Systems, 12(4), 379–396.

Xu, Z., & Yager, R. R. (2006). Some geometric aggregation operators and their decision-making method. Information, 8(3), 84.
Yager, R. R. (2008). Prioritized aggregation operators. *International Journal of Approximate Reasoning, 48*(1), 263–274.

Yang, L., & Li, B. (2016). A multi-criteria decision-making method using power aggregation operators for single-valued neutrosophic sets. *Infinite Study*.

Ye, J. (2014). A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *Journal of Intelligent & Fuzzy Systems, 26*(5), 2459–2466.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*(3), 338–353.

Zhang, L. (2018). Intuitionistic fuzzy averaging Schweizer-Sklar operators based on interval-valued intuitionistic fuzzy numbers and its applications. *2018 Chinese Control and Decision Conference (CCDC)*. IEEE.

Zhang, X., He, H., & Xu, Y. (2006). A fuzzy logic system based on Schweizer-Sklar t-norm. *Science in China Series F: Information Sciences, 49*(2), 175–188.

Zhang, M., Liu, P., & Shi, L. (2016). An extended multiple attribute group decision-making TODIM method based on the neutrosophic numbers. *Journal of Intelligent & Fuzzy Systems, 30*(3), 1773–1781.