Google matrix of business process management

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Received: September 14, 2010

Abstract. Development of efficient business process models and determination of their characteristic properties are subject of intense interdisciplinary research. Here, we consider a business process model as a directed graph. Its nodes correspond to the units identified by the modeler and the link direction indicates the causal dependencies between units. It is of primary interest to obtain the stationary flow on such a directed graph, which corresponds to the steady-state of a firm during the business process. Following the ideas developed recently for the World Wide Web, we construct the Google matrix for our business process model and analyze its spectral properties. The importance of nodes is characterized by PageRank and recently proposed CheiRank and 2DRank, respectively. The results show that this two-dimensional ranking gives a significant information about the influence and communication properties of business model units. We argue that the Google matrix method, described here, provides a new efficient tool helping companies to make their decisions on how to evolve in the exceedingly dynamic global market.

PACS. 89.65.Gh Economics; econophysics, financial markets, business and management 89.75.Fb Structures and organization in complex systems – 89.20.Hh World Wide Web, Internet

1 Introduction

Business process models are dynamical systems that describe the interdependencies of functional units, or components, on a micro- or macroeconomic level. They depict the way a company works and eventually makes money with the strategy it uses. The efficiency of a model is primarily determined by the help a model can give for strategic decisions, e.g. if a reorientation of products or marketing is needed due to changes in the market or opportunities because of technological developments (see e.g. [12] and Refs. therein).

The building of a business model is a complicated task, because all important units in the company value production must be identified and properly linked at a certain level of modeling. This involves a cancellation of non-important unit, which might be even harder. What modelers do further is a qualitative identification if a unit positively or negatively stimulates a linked one (amplification or damping, respectively). This yields a directed graph, where the units of the model are linked and the direction reflects causality. The next step towards quantitative modeling is the prescription of a functional dependence of the units, which is basically a very heuristic procedure. Clearly, the functions have to be nonlinear, because a growth to plus/minus infinity is not allowed, so typical functions are of sigmoid-type, on the other hand minimal models are of predator-prey type, well known from biology. This reflects the modern point of view of a company as a quasi-organic, dynamical system.

In this work we introduce and analyze the Google Business Process Model (GBPM) of a real consulting company [3] whose major product is of intellectual nature. The detailed description of the original dynamical model can be found in [3] and thus we do not present it here. The model describes a dynamical workflow propagation (see e.g. [15]) which is simulated by certain dynamical equations.

In our approach we trace parallels and similarities between the directed graph of this model and the Google matrix approach used for the ranking of the World Wide Web (WWW) [6,7,8]. Thus, we investigate only the model graph and do not enter the subject of dynamical simulations, because we want to reveal the underlying structure of the stationary state of the model without using the quite heuristic functional dependencies which need to be further supported by statistical analysis and measurement. This is not to say that the latter is a wrong approach, however the determination of the stationary density by the application of the PageRank algorithm for the Google matrix, which is a variant of Frobenius–Perron operator [7], is a very powerful and well-established technique which gives fundamental results on the network without solving the dynamical equations and using a vast study of parameter variations.

Indeed, the construction of the Google matrix for the WWW and the determination of the stationary probability distribution over WWW network via the PageRank
algorithm has been proposed by Brin and Page \cite{6} and by now it became a powerful tool for classification of the
WWW nodes (see e.g. \cite{7,8,9,10}). The approach based on
the Google matrix construction for a directed network is
rather general and finds applications for various types of
networks including university WWW networks \cite{11}, Ulam
networks of dynamical maps \cite{12,13}, brain neural networks
\cite{14}, procedure call network of Linux kernel \cite{15,16}, hyper
link network of Wikipedia articles \cite{17}. PageRank finds
also applications in blog analysis \cite{18}, citation network of
Phys. Rev. \cite{19,20}, and food flow network between species
in ecosystems \cite{21}.

In this work we extend this approach to the network
of business management. How is the model built? Bas-
ically, one has identified major components of the com-
pany, which are refined in their dynamics in respective
subcomponents. By construction, the model is hierarchi-
cal, but links between components can be set according
to the needs of the modeler. We only mention here the
components and the nodes in the top component: managers,
consultants, ....; subcomponents are: top, consultants, prod-
ucts, proposals, customers, .... The full list of nodes and
links between them are given in Appendix. Depending
on the business process, one of the nodes is the most
important one, followed by others. This is the value of
our method: we identify without any bias the most im-
portant components in a model. This provides an extremely
helpful information. If these components are not the ones
wished by the shareholders or management, respectively,
the model has to be changed and adapted. Since the com-
putation is not very costly this gives a tool to simulate
small changes, e.g. by linking different nodes, and study-
ing their effect on the business process model. We consider
the GBPM as a first step in the application of the Google
matrix analysis to the business process management. Next
steps should extend this approach and take into account
actual workflow between nodes inside a company \cite{15}.

Our network is small in comparison of typical applica-
tions of Google Matrix, like the WWW \cite{9,10}, Linux ker-
nel network \cite{15,16} or Wikipedia network \cite{17}. It consists
of 175 nodes only and is graphically displayed in Fig. 1.
This size is comparable with the one of food network in
ecosystems \cite{21}. Our purpose is an elementary study of
the network properties using the spectral characteristics
of the Google matrix, PageRank and recently introduced
CheiRank and 2DRank such that the order $10^2$ is suf-
cient; the latter ranking algorithms are explained in detail
below. Most big business models are proprietary (for un-
derstandable reasons), and an application of the Google
matrix method is straightforward.

Let us have a look on the network in terms of con-
nectivity: the distribution of ingoing and outgoing links is
shown in Fig. 2. Of course, with only one decade available
it is useless to try to identify exact scaling behaviour; nev-
ertheless the global distribution is compatible with power
law scaling $f(d) \sim d^{-\nu}$ at $\nu \approx 3$. The exponent $\nu = 3$
is not so far from the exponent $\nu = 2.1$ and 2.7 found
for the WWW for ingoing and outgoing link distributions
respectively \cite{9,10}. It will be interesting to investigate the
generic scaling of business models in the future for net-
works of larger size.

2 Method

The Google matrix $G$ underlies the determination of Page-
Rank \cite{6}, which is a tool used by virtually every Internet
user when issuing an Internet search for some keywords.
This approach gives a powerful and general way to analyze
networks. For the construction of the Google matrix we
use the procedure described in \cite{9,10}:

\[
G_{ij} = \alpha S_{ij} + (1 - \alpha)/N ,
\]  

where $S_{ij}$ is the normalized adjacency matrix of the graph.
The elements of the adjacency matrix are zero (if there
is no link) or one (if there is a link). Due to the nor-
malization the sum of all elements inside one column is
equal to unity. Columns with zeros only are replaced by
$(1/N, \ldots, 1/N)$, with $N$ being the network size. Because

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{fig1.png}
\caption{Google Business Process Model with links taken from
\cite{6}. The network is structured into several subgraphs reflecting
the functionality of the model. The names (or meaning) of the
nodes and links between them are listed in the Appendix.}
\end{figure}
it is a full stochastic matrix of a Markov chain, the matrix $S$ has $N$ eigenvalues $\lambda_i$, $i = 1, \ldots, N$ which are generally complex. In agreement with the Perron-Frobenius theorem (see e.g. [7]) the largest eigenvalue is $\lambda_1 = 1$. The damping parameter $\alpha$ denotes the possibility for a random surfer on the graph to jump to any other node. Its effect is to bound away the eigenvalues with absolute value smaller than one: $|\lambda_i| \leq \alpha < 1$ for $i > 1$. A typical value, used as well for the WWW search, is $\alpha = 0.85$, however this choice can be varied without essential impact on the results presented below. The right eigenvectors, $\psi_i$, are defined by $G\psi_i = \lambda_i \psi_i$, cf. [7,11]. The PageRank vector is the one with $\lambda = 1$, and since $G$ is a Frobenius-Perron operator, the corresponding right eigenvector, $\psi_1 = (P(1), \ldots, P(N))^T$ gives the stationary probability density $P(i)$ that a random surfer is found at site $i$ with $\sum_i P(i) = 1$. Once it is found, the nodes are sorted according to decreasing $P(i)$, the node rank in this index, $K(i)$ corresponds to its relevance.

Other eigenvalues correspond to non-stationary, decaying modes. They are of transient nature and may play an important role in non-stationary considerations, because they may live for a long time before dying out. This is, however, not the focus of this work.

### 3 CheiRank versus PageRank

In a nutshell, the procedure uses the idea that a node is not only relevant if it is highly linked. One has also to take into account the relevance of the nodes pointing to it. Since this is an iterative procedure, the PageRank vector can be easily computed by the so-called power-iteration using consecutive multiplication of initial random vector on the Google matrix [7]. Of course, this vector is the most important one, because it represents the stationary distribution on the graph. The relaxation process to the steady-state given by the PageRank is affected by the eigenmodes with $|\lambda|$ close to $\alpha$. It is known that for the WWW there are many eigenvalues which are close or even equal to $\alpha$ (see e.g. [7,11]). The spectrum of the Google matrix $G$ of the GBPM is shown in the left panel of Fig. 3. The eigenvalue next after $\lambda = 1$ is $\lambda_2 = 0.706$ and other eigenvalues have $|\lambda| < 0.52$. There are only about 14% of eigenvalues with $|\lambda| > 0.1$ that gives an indication on a possibility of appearance of the fractal Weyl law for such type of networks of larger size $N$ in analogy with the Linux kernel network analyzed in [15,16]. The spectrum of the Google matrix $G^*$, obtained from the network with the inverted direction of links, is shown in the right panel of Fig. 3; its characteristics are similar to those of matrix $G$.

The PageRank probability $P(i)$ for our business model is shown in Fig. 3 (top panel) as a function of rank $K(i)$. Surprisingly, there is no dominant node, which means that this company is quite democratic - in terms of relevance. The first five nodes are: Identified Contact Loss (33), Identified Contacts (32), Projects (5), Consultants (2), Delivery Project Completion (87). The numbers in brackets denote the node indices, cf. the Appendix. Managers (node index 1) do not appear before rank 18. This is quite surprising, since the management is expected to be at least among top ten positions. How can one understand that behaviour? The management plays typically the role of coordinating projects and keeping all together, which means that they decide which points are most important and have many outgoing links related to orders given to others. However, the PageRank is proportional in average to the number of ingoing links [x]. This implies the management units are not most important according to the PageRank since they do not have a large number of ingoing links (not many units give order to managers). In the considered model of a consulting company the most relevant units are the customers, or contacts. Without them, no business is made, especially for consulting. The first two ranks can be explained by this. The following ranks are Projects and Consultants. Of course, without good projects and correspondingly good workers the firm will die, so this is of vital relevance. Rank 5 again involves projects, this time their delivery. This means that in this model the way the projects are completed is given a high importance. This might not be necessarily true in all cases, however for the model of the firm under consideration it is. We recognize that in this view the result makes perfect
sense: customers, products and consultants are the most relevant units in the model of a consulting firm. Such a firm can only survive when its consultants are top level and its products are alike - and if there are customers. The management is responsible only to get the firm running well. This result may be surprising, but reveals the power of the method. This means as well that the most attention for refinement of the model should be put on the top nodes given above. Nevertheless, one expects that the management plays somehow a very influential role.

It is interesting to note that a similar situation takes place for the procedure call network of the Linux kernel as it was shown in [15]. Indeed, for this network the PageRank gives at the top procedures which are often pointed on but which are not so much important for the code functionality. Thus it was proposed [15] to characterize the network also by the PageRank of the Google matrix obtained from the network with inverted link directions. The rank \( P^*(i) \) of this inverted matrix \( G^* \) named as the CheiRank [17], places on first positions rather influential code procedures. Hence, it is natural to use the CheiRank also for our model of business process management.

And indeed, using the CheiRank, introduced in [15] we obtain an adequate result. It corresponds to the stationary distribution, \( P^*(i) \), of the inverted flow, or the information returned from the nodes to their precedent ones. Thus, it describes the influence or communication ranking of the nodes. Again, the eigenvector with the eigenvalue 1 is computed and sorted according to the magnitude of the entries. This yields a new rank, \( K^*(i) \), the mentioned CheiRank. The result of the computation of \( P^*(i) \) vs. \( K^*(i) \) is displayed in Fig. 4 (bottom panel). Here, we can also give a tentative scaling \( P^*(i) \sim K^{1/(\nu-1)} \) which must be compared and verified, respectively, with other business models of larger size. While the distribution of \( P(i) \sim K^{1/(\nu-1)} \) is proportional to the distribution of incoming links, the distribution of \( P^*(i) \) is proportional to the distribution of outgoing links (see e.g. [7,8,11,15]). Due to a small size of our network we do not try to use different values of \( \nu \) for ingoing and outgoing links and for \( P \) and \( P^* \) respectively. According to the CheiRank the top nodes are: Principals (1), Projects (5), Consultants (2), Customers (6), Contacts (7). The management now has clearly first position in the ranking which is fully logical, since any management decision influences the whole company, while the management is not necessarily the most important component, as explained above.

Following [15] we also use the joint distribution of nodes in the plane of probabilities \( P(i), P^*(i) \) of PageRank and CheiRank shown in Fig. 5. That way, we see both ranks at once and can decide which emphasis to put, defining importance in a new way. In this sense, the most important nodes are indicated in Fig. 5. The distribution of all nodes in the plane of PageRank and CheiRank \((K, K^*)\) is shown in Fig. 6. In the plane \((K, K^*)\) the most important nodes are those with the smallest values of \( K \) and \( K^* \). The zoom of this region of the plane is shown in Fig. 7.

![Fig. 4. Top panel: probability of PageRank vector \( P(i) \) as a function of PageRank \( K(i) \) in log-log scale. Bottom panel: probability of CheiRank vector \( P^*(i) \) in log-log scale. The straight lines show the approximate power law dependence with the slope \( 1/(\nu - 1) = 1/2 \), corresponding the the average slope \( \nu = 3 \) shown in Fig. 2.](image)

Of course, nodes might be both relevant (well-known) and influential (communicative). This can be characterized by the correlator \( \kappa \) between PageRank and CheiRank which is defined as

\[
\kappa = N \sum_i P(i) P^*(i) - 1. \tag{2}
\]

For the WWW university networks [15] and Wikipedia network [17] it was found that the correlator is rather large with \( \kappa \approx 4 \) while for the Linux kernel network one has very small correlator \( \kappa \approx -0.05 \ll 1 \). For the GBPM we have \( \kappa = 0.164 \) showing that there is practically no correlations between nodes with large number of outgoing and incoming links. Thus the GBPM network has more similarities with the Linux kernel network in contrast to the WWW and Wikipedia networks which are characterized by high correlations between nodes which are highly known (high PageRank) and highly communicative (high CheiRank).
With the appearance of CheiRank all nodes are now distributed in a two-dimensional plane (see Figs. 5, 6, 7).

How can one combine both rankings in a way to find nodes which are both very relevant and influential? There are many ways to find such a single-valued one-dimensional ranking which combines $K$ and $K^*$: one can think of the distance $(K^2 + K^*^2)$, or the absolute value, or some other combination of $K$ and $K^*$. Since $P(K)$ and $P^*(K^*)$ are monotonic functions the plane $(K, K^*)$ is mapped into $(P, P^*)$ plane in a unique way.

A convenient way to order all nodes of the two-dimensional plane on a one-dimensional line was proposed in [17].

Fig. 5. (Color online) Distribution of nodes in the plane of probabilities of PageRank $P(i)$ and CheiRank $P^*(i)$. The marked nodes illustrate the first four nodes in Chei rank (Principals, Projects, Consultants, Customers), and the top node in PageRank (Identified Contacts Loss).

Fig. 6. (Color online) Distribution of nodes in the plane of PageRank $K$ and CheiRank $K^*$, size of circles and their color is proportional to their listing node index with large radius (red color) for small index and small radius (blue-rose) for large index.

Fig. 7. Zoom of the distribution of nodes in the plane of PageRank $K$ and CheiRank $K^*$ in the region of small $K, K^*$ values. Numbers near circles give the listing node index, grayness is proportional to 2DRank $K_2$ with black for minimum and light gray for maximum $K_2$ (see Appendix).

Fig. 8. Illustration of the 2DRank algorithm to find rank $K_2$ which combines PageRank $K$ and CheiRank $K^*$. Specific nodes are drawn in the $(K, K^*)$ plane when crawling through the squares, indicated by the grey lines, from small to large $(K, K^*)$ the nodes are labeled by $K_2$; numbers in brackets $(K_2(i), i)$ give the value of found 2DRank $K_2$ and the values of listing node index $i$. One recognizes that at most 2 nodes can be found on a square edge, and some edges might be empty.

for Wikipedia articles being named 2DRank $K_2$. This rank is described by the algorithm presented below; it is dubbed 2DRank $K_2$, since it combines the two ranks discussed above. Remember that a ranking is basically a list of pairs (rank and nodes index), in our case $K_2(i), i$, or simply $K_2(i)$. By $K_2$, we also use this ordering of nodes by the following, quite intuitive criterion: we look progressively if a point...
\((K, K^*)\) lies on the square \(j \times j\), where \(j\) is a running index starting at 1. Since the ordering is unique, there are only two possibilities for this to occur: either \(K = j\) or \(K^* = j\). It may happen, that neither \(K\) nor \(K^*\) lies on the square, then one increases \(j\) by one and compares again with \((K, K^*)\). The initial \(K_2\) list is empty. E.g. if there is no point with \(K = 1\) and \(K^* = 1\), then the first square \(1 \times 1\) has no point on it and the next square \(2 \times 2\) is considered. The algorithm works by setting \(j = 1\), then we look if \(K = j\), if yes, \(i(K, K^*)\) is determined and added to the list \(K_2(i)\) whose own running index is increased; then we apply this procedure to \(K^*\): if \(K^* = j\), the node index \(i(K, K^*)\) is determined and added to the list \(K_2(i)\). Since there are no more points to check, we step from \(j\) to \(j + 1\). The algorithm is finished if all nodes \(i\) have been visited.

We can deliberately choose if we first look for \(K\) or \(K^*\) (we have chosen first \(K\)). The procedure is illustrated for the first ten nodes in \(K_2\) ranking in Fig. S.

According to this 2DRank algorithm we find for the first five nodes in 2DRank \(K_2\): Projects (5), Consultants (2), Hire Rate (119), Principals (1), Required Delivery Proposal Effort (\#8). The principals are still not the most relevant node, but obviously this ranking gives a quite balanced characterization of the business process management under consideration.

Top 30 nodes ordered according to PageRank, CheiRank and 2DRank are given in Appendix. Ranking of all nodes is available at the website [22].

4 Discussion

We have presented a powerful method which quantitatively describes the business process management in terms of the Google matrix, its eigenvectors and eigenvalues. The application of the method yields the stationary distribution on the directed graph which describes the business process of a concrete company in the frame of our GBPM. Our results show that the importance and influence of the units of business process are well characterized by two-dimensional ranking in the plane defined by PageRank and CheiRank. These ranks show that certain units (e.g. Contacts) perform important tasks being highlighted by PageRank, while other units (e.g. Principals) realize influential communication processes highlighted by CheiRank. Thus the two-dimensional ranking described here establishes a broad and detailed characterization of main operational units of business process management. In contrast to the WWW university networks and Wikipedia network, the network of GBPM has rather small correlation between top units of PageRank and CheiRank that stresses a clear separation between communication and realization tasks of business process. In this respect the GBPM network is more similar to the procedure call network of Linux kernel which also has small correlation between these two ranks.

Of course, the approach developed here is in its initial stage and more advanced business process modeling will need weighted graphs with subgraphs for the flows of work, information, money, products, etc. These generalizations are straightforward and can be constructed at next more advanced stage. A study of changes in the model is quick and straightforward, such that systematic studies of future activities of a company are now feasible without sometimes very heuristic equations which can be used at a final modeling stage. But now one is relieved from the task to determine fine–tune parameters and equations each time a model is changed. We expect these results to have significant impact in econometry for the evaluation of small, middle-size and large-scale models of business process management. The application to macro-economy is straightforward, and global flows might be characterized by the GBPM procedure.

Acknowledgements

We acknowledge fruitful discussion with O. Grasl who kindly provided his model [3] to us and explained the basics of business process modeling.

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Appendix

List of Nodes (node number is followed by its name and comma):
1 Principals, 2 Consultants, 3 Value, 4 Products, 5 Projects, 6 Customers, 7 Contacts, 8 Heads Of Branch, 9 Total Principals, 10 Maximum Principal Proposal Effort, 11 Maximum Principal Hiring Effort, 12 Average Principal Work Effort, 13 Maximum Principal Work Effort, 14 Maximum Project Time Share, 15 Maximum Contact Maintenance Effort, 16 Maximum Product Effort, 17 Contact Maintenance Effort, 18 Maximum Contact Maintenance Time Share, 19 Maximum Principal Project Effort, 20 Contacting Effort, 21 Qualified Contacts, 22 Required Contact Maintenance Effort, 23 Qualified Contact Maintenance Effort, 24 Qualified Contact Lifetime, 25 Maximum Qualified Contacts, 26 Minimum Qualification Duration, 27 Qualification Fraction, 28 Contact Qualification Rate, 29 Qualified Contact Loss, 30 Maximum Qualification Rate, 31 Contact Identification, 32 Identified Contacts, 33 Identified Contact Loss, 34 New Customer Contact Potential, 35 Identification Duration, 36 Identified Contact Lifetime, 37 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Pressure, 66 Effect Of Delivery Project Per Principal, 67 Repeat Delivery Lead Success Fraction, 68 Repeat Delivery Proposal Success Fraction, 69 First Time Delivery Lead Generation, 70 First Time Delivery Leads, 71 First Time Delivery Proposals, 72 Delivery Projects Won, 73 First Time Delivery Lead Generation, 74 First Time Delivery Lead Success, 75 First Time Delivery Proposal Success, 76 First Time Delivery Lead Fraction, 77 First Time Delivery LeadLoss, 78 First Time Delivery Proposal Loss, 79 Delivery Proposal Closing Rate, 80 Delivery Lead Closing Duration, 81 First Time Delivery Proposal Success Fraction, 82 First Time Delivery Lead Success Fraction, 83 Average Time To Delivery Project Start, 84 Delivery Project Start, 85 Active Delivery Projects, 86 Delivery Project Effort, 87 Delivery Project Completion, 88 Delivery Project Completion Rate, 89 Principal Proposal Effort, 90 Active Delivery Projects, 91 Delivery Project Per Principal, 92 Total Consulting Staff, 93 Delivery 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**List of Links** (node number marked by dot is followed by numbers of nodes on which it points to, last node number or blank if empty is marked by comma):

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