Dual solutions for nanofluid flow past a curved surface with nonlinear radiation, Soret and Dufour effects

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Abstract. Present work explores the numerical study of heat and mass transfer in nanofluid flow past a curve shaped stretchable (linearly/nonlinearly) geometry. The impact of Lorentz force caused by magnetic field, nonlinear radiation due to high temperature near the surface, frictional heating by virtue of viscous dissipation and cross diffusion by virtue of concentration and temperature differences are pondered while formulating the problem. The thermo physical properties of water and silver nanoparticles are used for calculations. Hamilton-Crosser model is considered for effective thermal conductivity of nanofluid. Shooting method and R.K. fourth order algorithm are adopted to solve the nonlinear coupled differential equations of the problem. The concentration, temperature and velocity fields are studied graphically for distinct values of flow parameters. Numerical values also calculated to know the impact of same parameters on mass and heat transfer coefficients. Results depict that mass and heat transfer performance in the flow via nonlinear curved surface is better when compared with the flow over linearly curved surface. As usually Eckert number and temperature ratio parameter boosts the fluid temperature in the present flow situation also.

Nomenclature:

\[ s \] : arc length coordinate along the surface
\[ r \] : normal to the tangent at any point of the surface
\[ a \] : radius of the circle
\[ u, v \] : velocity components in \( s, r \) directions respectively
\[ C, T \] : concentration and temperature of the fluid respectively
\[ p \] : pressure (dimensional)
\[ B_0 \] : applied magnetic field
\[ g \] : acceleration due to gravity
\[ T_s, T_\infty \] : temperatures near and far away the surface respectively
\[ C_u, C_\infty \] : concentration near and far away the surface respectively
\[ c_p \] : specific heat at constant pressure
\[ k \] : thermal conductivity
\[ q_r \] : radiative heat flux
\[ D_m \] : mass diffusivity
\(K_T\) : thermal diffusion ratio  
\(c_T\) : concentration susceptibility  
\(T_m\) : mean temperature  
\(m\) : power law stretching index  
\(b\) : constant related to stretching of the sheet  
\(F'\) : fluid velocity (dimensionless)  
\(P\) : pressure (dimensionless)  
\(K\) : curvature parameter  
\(M\) : magnetic field parameter  
\(Rd\) : radiation parameter  
\(Ek\) : Eckert number  
\(Pr\) : Prandtl number  
\(Du\) : Dufour number  
\(Sr\) : Soret number  
\(Sc\) : Schmidt number  
\(Ag\) : silver  
\(u_s(s)\) : stretching velocity of the sheet  
\(Sh_t\) : Sherwood number  
\(Nu_s\) : Nusselt number  
\(C_F\) : skin friction coefficient  
\(j_w\) : wall mass flux  
\(q_w\) : wall heat flux

Greek symbols:  
\(\rho\) : density  
\(\mu\) : dynamic viscosity  
\(\nu\) : kinematic viscosity  
\(\sigma\) : electrical conductivity  
\(\beta\) : thermal expansion coefficient  
\(\gamma'\) : chemical reaction parameter (dimensional)  
\(\gamma\) : chemical reaction parameter (dimensionless)  
\(\sigma'\) : Stefan-Boltzmann constant  
\(k'\) : mean absorption coefficient  
\(\phi\) : volume fraction of nanoparticles  
\(\xi\) : similarity variable  
\(\Theta\) : fluid temperature (dimensionless)  
\(\Phi\) : fluid concentration (dimensionless)  
\(\alpha\) : mixed convection parameter  
\(\theta_s\) : temperature ratio parameter  
\(\tau_{rs}\) : shear stress in rs-plane.

Suffixes:  
\(nf\) : nanofluid  
\(bf\) : basefluid
1. Introduction

Substantial attempts were made by many researchers to achieve higher heat transfer rate using the advancements in the fields of thermal engineering and thermo science. In 18th century there are some methods developed by Maxwell [1] to increase the heat transfer ability of fluids like water, oils etc by suspending micro/millimetre sized particles. But we come across some disadvantages with Maxwell’s idea. At the end of 19th century Choi and Eastman [2], who coined the word “nanofluids” (NFs) suggested that emission of solid nanoparticles in ordinary fluids dramatically enriches the heat transfer performance. They also proved that this procedure gives good results than the methods used earlier for the purpose of which nanofluids are used. The thermal scientists [3] of Argonne Laboratory (USA) discussed the applications of suspension of solid nanoparticles in industrial heat transfer fluids. They proposed that nanofluids will be very much useful in cooling technology. This technology can be applied in making of computer chips, coolant oils, vehicle engines, fiber-forming processes, X-rays etc. An experimental investigation was carried out by Xuan and Li [4] to know the heat transfer characteristics of nanofluid flow in a tube. They found that heat transfer ability of fluids enhances with the volume fraction of nanoparticles.

First time, Khan and Pop [5] described the laminar flow of nanofluids (NFs) caused by stretching of a surface. They found that Nusselt number have inverse relationship with thermophoresis and Brownian motion number. Meanwhile, Chamkha and Aly [6] used finite difference technique to examine the heat transfer in the flow of NFs with injection/suction. Their article says that the presence of nanoparticles in the base fluids alters the rate of mass transfer also. Xue [7] scrutinized various models for effective thermal conductivity of nanofluids available in the literature. He declared that Hamilton-Crosser model is the best one for effective thermal conductivity. Hence, this model was become more popular and also chosen by Reddy et al. [8] to inspect the flow of Ag-water and TiO$_2$-water NFs in a rotating disk with heat and mass transfer. Zhang et al. [9] formulated a problem to discuss the Brownian diffusion of copper nanoparticles in ethylene glycol with periodic heat flux. A comparative analysis was published by Hayat et al. [10] illustrating the effect of nonlinear radiation on mixed convective flow of Ag-water and Cu-water NFs along a curved geometry.

The magneto hydrodynamic flows over a stretching surface plays a vital role in petroleum engineering, casting, metal working, cooling of surface inside a containment vessel of nuclear reactors and agricultural processes. The concept of MHD is important in examining the interaction between the magnetic fluid particles in the blood stream. The MHD flow of dusty fluid between two parallel surfaces with variable thermal properties was studied by Makinde and Chinyoka [11]. The impact of heat transport in Cuo-water fluid flow in presence with the existence of Lorentz force was dealt by Sheikhholeslami et al. [12]. They showed that Nusselt number enriches with Lorentz force. Mabood et al. [13] investigated the Brownian motion of NFs past a nonlinear surface considering the effects of magnetic field and viscous dissipation. They observed a reduction in mass and heat transfer rates with the increase of nonlinearity in stretching sheet. Numerical solution for MHD flow of nanoliquid between two rotating surfaces was submitted by Sheikhholeslami [14] et al., by R.K. fourth order approach.

Dada and Disu [15] studied the radiative, convective and magneto hydrodynamic flow between vertical wavy surfaces. Recently, Reddy et al. [16] discussed the influence of nonlinear radiation on ferroliquid flow past a bidirectionally stretching sheet with temperature dependent viscosity. Khanafar et al. [17] focused on the study of nanofluid flow in an enclosure with heat transfer using various models.
available for physical properties of NFs. Heris et al. [18] published an article on convective heat transfer of NFs in a circular tube. The impact of nanoparticles size on convective heat transport of NFs was experimentally observed by Anoop et al. [19]. They proved that a fluid with nanoparticles of size 45nm effectively enhances the heat transfer when compared to that of with 150nm nanoparticles. Majeed et al. [20] gone into the study of viscous dissipation on time dependent flow of magnetic liquid above a stretching surface with magnetic dipole effect. The influence of frictional heat and unsteadiness on non-Newtonian nanofluid flow past a stretching sheet was addressed by Khan et al. [21].

Cortell [22] probed the domination of mass transfer on the motion of second grade fluid over a stretching surface. Alam and Rahman [23] make use of R.K. sixth order algorithm to peruse the impact of cross diffusion on convective flow past a porous sheet. They established that the effect of Dufour and Soret parameters on concentration and temperature fields is note worthy. Similar kind of research on Cu-water nanoliquid flow was done by Pal et al. [24]. Further, Dzulkifli et al. [25] extended the work of Pal et al. [24] by considering the effects of velocity slip and unsteadiness. Reddy et al. [26] researched the problem of three distinct MHD flows with cross diffusion by adopting Christov-Cattaneo heat flux model. Ullah et al. [27] scrutinized the unsteady flow of Casson fluid past a nonlinear stretching sheet with cross diffusion. They proved graphically that mass and heat transfer rates are dramatically affected by cross diffusion parameters. Venkateswarlu and Narayana [28] studied the mass transfer in nanofluids in a rotating system by taking first order chemical reaction in the governing equations. On the other hand, Rahman Lawatia [29] discussed the influence of higher order chemical reaction on micropolar fluid past a nonlinear stretching sheet with heat transfer. They detected that concentration field increases with the order of chemical reaction. Hayat et al. [30] dissected the motion of non-Newtonian fluids along a curved surface with heterogeneous and homogeneous chemical reactions. They stated that the strength in heterogeneous reaction elevates the fluid concentration.

The flow of Newtonian /non-Newtonian fluids over a stretching surface with heat and mass transfer is an important aspect in various industrial and engineering processes. The applications can be found in cooling of sheets, melt-spinning processes, spinning of fibres, casting etc. In every process the excellence of the finished products is dependent of heat transfer near the stretching sheet. The first paper published on flows over stretchable surfaces by Crane [31]. The boundary layer motion of NFs owing to nonlinear stretching of a sheet was investigated by Mabood et al. [32], numerically. They observed a decay in fluid velocity with rising values of nonlinear stretching parameter. The heat transfer in flows caused by stretching of a cylinder was reported by Rangi and Ahmad [33]. The motion of power-law fluid over a stretching wedge was analysed by Postelnicu and Pop [34]. The effects of heat and mass transfer on Fe₃O₄–water flow along a stretchable curved geometry were perceived by Imtiaz et al. [35] and stated that the friction near the surface enhances with curvature parameter. This work was continued by Hayat et al. [36] to find the impact of thermal relaxation time on Darcy-Forchheimer flow. Recently a few articles [37-38] were published to describe the flow over a nonlinearly stretching curved surface with heat and mass transfer. These articles established that Sherwood and Nusselt numbers magnifies with the power-law stretching index.

2. Mathematical formulation:

Let us contemplate the laminar flow of Ag-water nanofluid over a curve shaped sheet about the curvilinear coordinates \( s \) and \( r \). The flow is steady and incompressible. The sheet is taken in the direction of \( s \), which is the coordinate of arc length. \( r \)-axis is taken perpendicularly to any tangent on \( s \)-axis as depicted in Fig. 1. The nonlinear stretching velocity of the sheet is \( u_s(s) = bs^m \). The sheet is looped in a circle of radius \( a \). The varying magnetic field \( B(s) = B_0s^{(m-1)/2} \) is exerted parallel to \( r \)-direction. The influence of induced magnetic field is not accounted. Let \( T_\infty, C_\infty \) be temperature and concentration of the fluid near the sheet. The temperature and concentration far away from the sheet are \( T_s(<T_\infty), C_s(<C_\infty) \) respectively. The heat and mass transfer process is explored with the consideration of cross diffusion, dissipation, thermal radiation and chemical reaction. Based upon the value of \( m \), two
flow cases will exist. One is the flow along a linearly stretching \((m = 1)\) curved sheet and other is flow along a nonlinearly stretching \((m > 1)\) curved sheet.

\[
\begin{align*}
\mathbf{u}_w(s) &= b s^m \\
(s, u)
\end{align*}
\]

**Figure 1:** Flow model

With respect to above stated suppositions, the governing equations for nanofluid flow are (see Ref. \([10, 37]\)),

\[
((r+a)v)_r + a u_s = 0, \tag{1}
\]

\[
\rho_{nf} u^2 = (r+a) p_r, \tag{2}
\]

\[
\rho_{nf} \left( v u_s + \frac{a u}{r+a} u_s + \frac{u v}{r+a} \right) = -\frac{a}{r+a} p_s + \mu_{nf} \left( u_{ss} + \frac{u_r}{r+a} - \frac{u}{(r+a)^2} \right) - \sigma_{nf} B_0^2 u + g(\beta\rho)_{nf}(T - T_\infty), \tag{3}
\]

\[
(\rho c_p)_{nf} \left( \frac{a}{r+a} u T_s + v T_r \right) = k_{nf} \left( \frac{T_r}{r+a} + T_{rr} \right) + \mu_{nf} \left( u_r - \frac{u}{r+a} \right)^2 \tag{4}
\]

\[
-\frac{1}{r+a} ((r+a)q_r)_s + \frac{D_m K_r \rho_{nf}}{e_T} \left( C_r + \frac{C_r}{r+a} \right),
\]

\[
\left( \frac{a}{r+a} u C_s + v C_r \right) = D_m \left( \frac{C_r}{r+a} + C_{rr} \right) - \gamma' (C - C_\infty) + \frac{D_m K_T T_m}{T'} \left( T_r + \frac{T_r}{r+a} \right). \tag{5}
\]

with the following boundary conditions.

\[
u = bs^m, v = 0, T = T_w, C = C_w, \text{ at } r = 0, \tag{6}\]

\[
u \to 0, u \to 0, T \to T_\infty, C \to C_\infty, \text{ as } r \to \infty, \tag{7}\]
The thermophysical properties of nanofluids are listed below. (see Ref. [8, 16])

Density is \( \rho_{nf} = \rho_{bf} (1 - \phi) + \phi \rho_{np} \), dynamic viscosity is \( \mu_{nf} = \mu_{bf} (1 - \phi)^{-2.5} \), electrical conductivity is \( \sigma_{nf} = \sigma_{bf} \left( \frac{1 - 3(1 - \delta)\phi}{(2 + \delta) + \phi(1 - \delta)} \right) \) with \( \delta = \frac{\sigma_{np}}{\sigma_{bf}} \), heat capacity is \( (\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_{bf} + (\rho c_p)_{np} \phi \), thermal conductivity of nanoliquid is \( k_{nf} = \frac{(k_{bf} + (n_p - 1)k_{by}) - \phi(n_p - 1)(k_{by} - k_{bf})}{(k_{by} + (n_p - 1)k_{bf}) + \phi(k_{by} - k_{bf})} \), and thermal expansion coefficient of the nanoliquid is \( (\rho \beta)_{nf} = (1 - \phi)(\rho \beta)_{bf} + \phi(\rho \beta)_{np} \).

The radiative heat flux \( q_r \) using Rossenald approximation is given by,

\[
q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial r} = -\frac{16\sigma^*}{3k^*} T^3 \frac{\partial T}{\partial r},
\]

Here \( k^* \) is the mean absorption coefficient and \( \sigma^* \) is Stefan-Boltzmann constant.

We choose the following similarity transformations, which satisfy the equation of continuity.

\[
\xi = \left( b \frac{u}{v_{bf}} \right)^{\frac{1}{2}} \left( \frac{s}{r} \right)^{m-1} \frac{1}{r}\left[ \frac{(m+1)}{2} F(\xi) + \frac{(m-1)}{2} \xi F'_{\xi} \right],
\]

\[
u = bs^{m-1} F'_{\xi}, \rho = \rho_{bf} b^{2m-2} \rho C(\xi), \Theta(\xi) = \frac{T - T_c}{T_w - T_c}, \Phi(\xi) = \frac{C - C_w}{C_w - C_c}.
\]

In order to analyze the influence of nonlinear thermal radiation, we simplify \( \Theta(\xi) = \frac{T - T_c}{T_w - T_c} \) as,

\[
T = T_c (1 + (\theta - 1)\Theta),
\]

Where \( \theta = T_w / T_c \) is the temperature ratio parameter.

Exploiting eqs. (8)-(10) in the nonlinear system of PDEs (2)-(5) and their boundary conditions, we get

\[
A_i F_{\xi\xi} = (\xi + K) P_{\xi},
\]

\[
\frac{A_i}{A_1} \left( \frac{F_{\xi\xi}}{\xi + K} - \frac{F_{\xi}}{(\xi + K)^2} \right) - \frac{2mK + (m + 1)\xi}{2(\xi + K)} \frac{K(F_{\xi})^2}{2(\xi + K)^2} FF_{\xi\xi} = \frac{A_0}{A_1} M F_{\xi} - \frac{A_0}{A_1} \alpha\Theta - \frac{(m + 1)K}{2(\xi + K)^2} FF_{\xi} + \frac{1}{A_1} \left( \frac{2mK}{(\xi + K)} P + \frac{(m - 1)\xi K}{2(\xi + K)} P_{\xi} \right)
\]

\[
Rd \left( 3(\Theta_{\xi}^2) (1 + (\theta - 1)\Theta) (\theta - 1)(\xi + K) + (1 + (\theta - 1)\Theta)(\xi + K) \Theta_{\xi\xi} + \Theta_{\xi} \right)
+ A_3 \left( (\xi + K) \Theta_{\xi\xi} + \Theta_{\xi} \right) + \frac{Pr Ek A_i}{(\xi + K)} (\xi + K) F_{\xi\xi} - F_{\xi} \right) + A_3 (\xi + K) D u \Phi' = 0,
\]

\[
\left( \Phi_{\xi\xi} + \frac{\Phi_{\xi}}{(\xi + K)} \right) + Sc \left( \frac{m + 1}{2} \right) F \Phi_{\xi} - Sc \gamma F + Sr Sc \Theta_{\xi\xi} = 0.
\]
The non-dimensional form of boundary conditions is,

\begin{align*}
F &= 0, F' = 1, \Theta = 1, \Phi = 1, & \text{at } \xi &= 0, \\
F' &\to 0, F^* &\to 0, \Theta &\to 0, \Phi &\to 0, & \text{as } \xi &\to 0,
\end{align*}

(15)

Here,

\begin{align*}
K &= a \frac{b}{\sqrt{v_{bf} r}}, \quad E_k = \frac{b^2 s^{2m}}{(c_p)_{bf} (T_w - T_\infty)}, \quad R_d = \frac{16\sigma^2 T_w^3}{3 k_b f}, \quad a = \frac{Gr_\tau}{Re_\tau}, \quad Gr_\tau = \frac{s^3 (T_w - T_\infty) g \beta}{v_{bf}^2}, \quad Re_\tau = \frac{b s^2}{v_{bf}}, \\
Pr &= \frac{(c_p)_{bf} \mu_f}{k_b f}, \quad M = \frac{\sigma_{sf} B_0^2}{\rho_{bf} b}, \quad D_u = \frac{D_u K_f (C_w - C_\infty)}{v_{bf} c_f (c_p)_{bf} (T_w - T_\infty)}, \quad S_r = \frac{D_u K_f (T_w - T_\infty)}{v_{bf} T_m (C_w - C_\infty)}, \quad S_c = \frac{v_{bf}}{D_n}, \quad \gamma = \frac{\gamma'}{bs^m-1},
\end{align*}

and

\begin{align*}
A_1 &= (1 - \delta) + \frac{(\rho \beta)_{np}}{(\rho \beta)_{bf}} \phi, \quad A_2 = (1 - \phi) + \left(\frac{(\rho c_p)_{np}}{(\rho c_p)_{bf}}\right) \phi, \\
A_3 &= (1 - \phi)^{-2 \delta}, \quad A_4 = (1 - \phi) + \left(\frac{(\rho \beta)_{np}}{(\rho \beta)_{bf}}\right) \phi, \\
A_5 &= \frac{((n_p - 1) k_{bf} - \phi(n_p - 1) (k_{bf} - k_{np}))}{(k_{bf} + (n_p - 1) k_{np}) + \phi(k_{bf} - k_{np})}, \quad A_6 = \left(1 - \frac{3(1 - \delta) \phi}{(2 + \delta) + \phi(1 - \delta)}\right).
\end{align*}

Simplification of Eqs. (11) and (12) (eliminating $P$) gives,

\begin{align*}
A_1 \left(\left(\xi + K\right)^3 F_{\xi \xi \xi \xi} + 2 \left(\xi + K\right)^2 F_{\xi \xi \xi} - \left(\xi + K\right) F_{\xi \xi} + F_{\xi}\right) \\
- A_6 M \left(\left(\xi + K\right)^3 F_{\xi \xi} + \left(\xi + K\right)^2 F_{\xi} + A_1 \alpha \left(\left(\xi + K\right) \Theta + \left(\xi + K\right)^2 \Theta\right) + \left(\xi + K\right)^3 \Theta\right) \\
+ A_5 K \left[\frac{1 - 3m}{2} \left(\xi + K\right) F_{\xi}\left(F_{\xi}\right) + \left(\xi + K\right) F_{\xi \xi} - \frac{m + 1}{2} \left(\xi + K\right) F_{\xi \xi} \right] = 0
\end{align*}

(16)

The important applications of present problem in industrial and engineering processes are with Sherwood number ($Sh_w$), Nusselt number ($Nu_{w}$) and friction factor ($C_f$). These are defined as,

\begin{align*}
Sh_w &= \frac{s j_w}{D_n (C_w - C_\infty)}, \quad Nu_{w} = \frac{s q_w}{k_{bf} (T_w - T_\infty)}, \quad C_f = \frac{\tau_{ww}}{\rho_{bf} u_{w}^2},
\end{align*}

(17)

Here, the wall mass flux ($j_w$), wall heat flux ($q_w$) and shear stress ($\tau_{ww}$) are given by,

\begin{align*}
\dot{j}_w &= -D_m \left(\frac{\partial C}{\partial r}\right)_{r=0}, \quad q_w = -k_{bf} \left(\frac{\partial T}{\partial r}\right)_{r=0}, \quad \tau_{ww} = \frac{\mu_{bf}}{r + a} \left(\frac{\partial u}{\partial r} - \frac{u}{r + a}\right),
\end{align*}

(18)

Substituting Eqs. (9) and (18) in Eqn. (17), we get

\begin{align*}
\frac{Sh_w}{\sqrt{Re_s}} = -\Phi'(0), \quad \frac{Nu_{w}}{\sqrt{Re_s}} = -(A_s + R_d \theta_s) \Theta'(0), \quad C_f \sqrt{Re_s} = A_1 \left(F'(0) - \frac{F'(0)}{K}\right),
\end{align*}

(19)

3. Analysis of Results:
This section explores the changes in the flow, heat and mass transport phenomenon with the governing parameters of the problem. We considered Ag-water as nanofluid in the present study. The values of
thermo physical properties of Ag (silver) nanoparticles and water are given in table 1. The ODEs (13), (14) and (16) are solved according to the boundary conditions (15) by the consecutive application of shooting and R.K. algorithm. Graphs are drawn to examine the domination of problem parameters on the three usual flow distributions. Further, we analyzed the heat and mass transfer coefficients and friction factor with the help of numerical values given in table 2. All the results are attained using MATLAB bvp4c package with the values $\phi = 0.1$, $\Pr = 6.8$ (water), $K = 4$, $\alpha = 0.8$, $M = 3$, $\gamma = 0.6$, $Rd = 3$, $Ek = 0.1$, $Sc = 0.6$, $Du = 0.3$ and $Sr = 0.2$. These are used for the complete analysis of results. The changed values are specified in table 2 and graphs. Dual solutions are given by taking $m = 1$ for the flow via linearly stretching curved sheet and $m = 2(>1)$ for the flow via nonlinearly stretching sheet.

The curves for increasing values of $Du$ vs. $\Phi(\xi)$, $\Theta(\xi)$ and $F'(\xi)$ are evinced via Figures 2-4. Dufour number is the ratio of difference in temperature to that of concentration. By virtue of concentration gradient, it arises in the energy equation as the enthalpy flux. It indicates the ratio of difference in temperature to that of concentration. Ergo, strength in Dufour effect corresponds to inflation in the difference of temperature as well as deflation in the difference of concentration. Due to this logic, we see a hike in temperature and velocity distributions (Figures 3, 4) and diminution in concentration (Figure 2) distribution. Similarly the Soret number (the mass flux as a result of temperature gradient) signifies the ratio of difference in concentration to that of temperature. By same logic of Dufour effect, a hike in concentration distribution is noted via Figure 5 for increasing values of $Sr$. In general, increase in soret number suppresses the temperature distribution. But it is interesting to say from Figure 6 that the fluid temperature also rises with the same values of $Sr$. This may occur because of the domination of other physical parameters in the present problem.

Figures 7-8 respectively explore the variation in fluid temperature for heightening values of temperature ratio parameter ($\theta_0$) and Eckert number ($Ek$). These figures allow us to infer that temperature grows with the increase in $\theta_0$ or $Ek$. Increase in temperature ratio parameter causes a rise in the surface temperature. Also, Eckert number is the ratio of kinetic energy to enthalpy difference. Ergo, kinetic energy increases with $Ek$. Resultantly, we notice an increment in $\Theta(\xi)$ in both the figures. Figures 9-10 respectively elucidate the influence of radiation ($Rd$) on temperature and velocity distributions. We observe that uplifting values of $Rd$ exalts the fluid temperature due to the production of heat energy by radiative heat flux. The cause for this kind of outcomes is heat energy produced by the radiative heat flux. The same energy gives rise to a hike in motion of the fluid particles.

Figures 11-13 are constructed to study the nature of concentration, temperature and velocity fields for sundry values of curvature parameter $K$. From Figures 11 and 13, it is apparent that $\Phi$ and $F'$ are increasing functions of $K$. Higher values of $K$ corresponds to larger radius of the curved surface. Ergo, fluid moves faster along the surface. Fluid concentration also intensifies as $K$ is inversely proportional to kinematic viscosity. Figure 12 reveals that the temperature diminishes with an elevation in $K$.

Figures 14-16 display the impact of increase in the volume fraction of nanoparticles ($\phi$) on the curves of $\Phi(\xi), \Theta(\xi)$ and $F'(\xi)$. It is expected result i.e. fluid concentration increases (see Figures 14) with the volume fraction of silver nanoparticles. The thermal conductivity of the fluid will become strong with the increase of $\phi$. As a result we notice a gradual increment in fluid temperature. On the other hand, the fluid density will be increased due to the addition of silver nanoparticles. This results a small depletion in the fluid velocity.

The influence of chemical reaction rate ($\gamma$) on concentration and temperature fields is shown through Figures 17 and 18 respectively. We notice a decay in concentration and a hike in temperature with the strength in chemical reaction ($\gamma$). It is understood that the impact of $\gamma$ on fluid concentration is significant.

Figures 19-21 are drawn to know the nature of the curves $\Phi(\xi), \Theta(\xi)$ and $F'(\xi)$ for different values of mixed convection parameter $\alpha$. Usually, mixed convection in flows produces buoyancy forces,
which is responsible to develop the flow velocity and decay the concentration and temperature fields. So, we notice a growth in momentum boundary layer thickness (see Figures 19) for boosting values of $\delta$. With the same effect, we observe a deterioration in thermal and concentration boundary layer thicknesses.

Figures 22 and 23 respectively demonstrate the behaviour of the curves $\Theta(\xi)$ and $F'(\xi)$ for increase in Lorentz force. By examining the graphs, we clinch that temperature distribution upturns but velocity deteriorates for boosting values of $M$. We know that a swell in $M$ induces Lorentz force, which effectively obstruct the flow movement and promotes heat energy. Hence we notice the results of that sort.

Figures 24 depicts the variation in temperature distribution for ascending values of $Pr$. Actually, the thermal conductivity of fluids reduces as $Pr$ enhances. Hence, the fluid temperature decreases. Figures 25 and 26 explain the variation in concentration and temperature profiles for increasing values of Schmidt number ($Sc$). We know that Schmidt number ($Sc = \nu_f / D_m$) is the ratio of rate of viscous diffusion to rate of molecular diffusion. So an increase in $Sc$ lowers the rate of molecular diffusivity and causes the diffusion of higher density species in air. As a result, we glimpse a decrement in fluid concentration and increment in temperature from Figures 25 and 26 respectively.

**Figure 2:** Change in concentration field with $Du$

**Figure 3:** Change in temperature field with $Du$
**Figure 4:** Change in velocity field with $Du$

**Figure 5:** Change in concentration field with $Sr$

**Figure 6:** Change in temperature field with $Sr$
Figure 7: Change in temperature field with $\theta_w$.

Figure 8: Change in temperature field with $R_d$.

Figure 9: Change in temperature field with $R_d$. 
Figure 10: Change in velocity field with $Rd$

Figure 11: Change in concentration field with $K$

Figure 12: Change in temperature field with $K$
Figure 13: Change in velocity field with $K$

Figure 14: Change in concentration field with $\phi$

Figure 15: Change in temperature field with $\phi$
Figure 16: Change in velocity field with $\phi$

Figure 17: Change in concentration field with $\gamma$

Figure 18: Change in temperature field with $\gamma$
Figure 19: Change in concentration field with $\alpha$

Figure 20: Change in temperature field with $\alpha$

Figure 21: Change in velocity field with $\alpha$
Figure 22: Change in temperature field with $M$

Figure 23: Change in velocity field with $M$

Figure 24: Change in temperature field with $Pr$
Figure 25: Change in concentration field with $Sc$

Figure 26: Change in temperature field with $Sc$

Table 1. Thermo physical properties of basefluid and silver nanoparticles.

| Thermo physical properties | Water (H$_2$O) | Silver (Ag) |
|----------------------------|----------------|-------------|
| $k$ (W/mK)                | 0.612          | 425         |
| $\sigma$ (S/m)            | $55 \times 10^{-7}$ | $6.21 \times 10^{-7}$ |
| $\beta \times 10^3$ (K$^{-1}$) | 21             | 1.9         |
| $c_p$ (J/Kg K)            | 4181           | 234         |
| $\rho$ (Kg/m$^3$)         | 996.5          | 10500       |

Table 2. Computed values of $C_F$, $Nu_t$ and $Sh_t$

| Friction factor ($C_F$) | Heat transfer Coefficient ($Nu_t$) | Mass transfer Coefficient ($Sh_t$) |
|-------------------------|-----------------------------------|-----------------------------------|
| $m = 1$                 | $m = 2$                           | $m = 1$                           | $m = 2$                           | $m = 1$                           | $m = 2$                           |
| $M = 3$                 | -2.0699                           | 0.4050                            | 0.4283                            | 0.4940                            | 0.4982                            |
| $M = 4$                 | -2.6097                           | 0.3307                            | 0.3538                            | 0.4865                            | 0.4903                            |
| $M = 5$                 | -3.0610                           | 0.2750                            | 0.2975                            | 0.4818                            | 0.4853                            |
| $Rd = 1$                | -1.3398                           | 0.7136                            | 0.7796                            | 0.5117                            | 0.5219                            |
| $Rd = 2$                | -1.2899                           | 0.5863                            | 0.6352                            | 0.5197                            | 0.5310                            |
The computed values of heat and mass transfer coefficients and friction factor are bestowed in Table 2. The analysis of these results is important due to their salient applications in industrial and engineering processes. A careful observation of the entire table allows us to conclude that the increasing/decreasing nature of these physical quantities for both the cases (linear/nonlinear stretching of the sheet) is alike. We found that an increase in the temperature of the curved surface depletes both mass transfer rate and friction factor but uplifts the rate of heat transfer. The results are quite opposite if heat energy is produced in the flow due to radiative heat flux or viscous dissipation. The values of $Sh_r, Nu_r$, and $C_F$ reduces with increase in Lorentz force (caused by magnetic field) or Soret effect. But opposite results are observed for increasing values of mixed convection parameter. Curvature parameter has tendency to control the heat and mass transfer rates. The impact of chemical reaction and Dufour number is homogeneous. That is to say that increase in either of these parameters diminishes the rate of heat transfer but boosts the mass transfer rate and friction factor.

4. Conclusions:
Dual solutions are presented for the nonlinear radiative nanofluid flow over a curved stretching sheet. The effects of viscous dissipation, cross diffusion are accounted. The results are presented in the form of numerical values and graphs. The chief conclusions of governing problem are given below.

\[
\begin{array}{cccccccc}
Rd = 3 & -1.2607 & -1.3739 & 0.5169 & 0.5567 & 0.5246 & 0.5368 \\
K = 0.5 & -3.3910 & -3.3936 & 0.3266 & 0.3300 & 1.1411 & 1.1466 \\
K = 1.0 & -2.3986 & -2.4076 & 0.3728 & 0.3819 & 0.8755 & 0.8853 \\
K = 1.5 & -1.5699 & -1.6602 & 0.3619 & 0.3867 & 0.5579 & 0.5696 \\
\alpha = 0.1 & -2.1792 & -2.2981 & 0.0997 & 0.1291 & 0.7096 & 0.7211 \\
\alpha = 0.6 & -1.9569 & -2.0867 & 0.1593 & 0.1899 & 0.7117 & 0.7242 \\
\alpha = 1.4 & -1.7429 & -1.8839 & 0.2072 & 0.2382 & 0.7144 & 0.7277 \\
\phi = 0.05 & -1.7825 & -1.8283 & 0.3023 & 0.3292 & 0.8431 & 0.8491 \\
\phi = 0.10 & -1.9846 & -2.0408 & 0.1838 & 0.2082 & 0.8416 & 0.8461 \\
\phi = 0.15 & -2.2005 & -2.2671 & 0.0607 & 0.0826 & 0.8436 & 0.8470 \\
\gamma = 0.1 & -1.9738 & -2.1033 & 0.3773 & 0.4056 & 0.1804 & 0.1843 \\
\gamma = 0.1 & -1.9666 & -2.0966 & 0.3422 & 0.3716 & 0.3202 & 0.3232 \\
\gamma = 0.9 & -1.9598 & -2.0905 & 0.3034 & 0.3338 & 0.4678 & 0.4702 \\
Du = 0.1 & -2.1402 & -2.2619 & 0.3384 & 0.3656 & 0.3339 & 0.3474 \\
Du = 0.5 & -2.1275 & -2.2492 & 0.2387 & 0.2627 & 0.3485 & 0.3630 \\
Du = 0.9 & -2.1135 & -2.0235 & 0.1239 & 0.1445 & 0.3642 & 0.3793 \\
Sr = 0.2 & -3.0674 & -3.2731 & 0.2890 & 0.3164 & 1.0824 & 1.1468 \\
Sr = 0.8 & -3.1505 & -3.3000 & 0.2482 & 0.2793 & 1.0411 & 1.0724 \\
Sr = 1.4 & -3.2845 & -3.4364 & 0.1857 & 0.2232 & 1.0380 & 1.0171 \\
Sc = 1 & -3.0976 & -3.2024 & 0.3603 & 0.3843 & 0.7109 & 0.7201 \\
Sc = 2 & -3.1305 & -3.2101 & 0.2593 & 0.2735 & 1.0276 & 1.0927 \\
Sc = 3 & -3.2295 & -3.2781 & 0.1135 & 0.1099 & 1.3870 & 1.5054 \\
\theta_v = 1.2 & -1.7498 & -1.8222 & 0.1082 & 0.1219 & 1.0195 & 1.0255 \\
\theta_w = 1.7 & -1.7556 & -1.8266 & 0.2038 & 0.2148 & 0.9874 & 0.9965 \\
\theta_w = 2.5 & -1.7525 & -1.8226 & 0.2296 & 0.2384 & 0.9814 & 0.9929 \\
Ek = 1.0 & -2.8161 & -2.9106 & -0.3668 & -0.3549 & 0.7696 & 0.7851 \\
Ek = 1.5 & -2.8064 & -2.9011 & -0.6766 & -0.6753 & 0.7879 & 0.8040 \\
Ek = 2.0 & -2.7973 & -2.8922 & -0.9716 & -0.9805 & 0.8052 & 0.8220 \\
Pr = 6 & -2.8333 & -2.9269 & 0.2238 & 0.2517 & 0.7353 & 0.7499 \\
Pr = 7 & -2.8356 & -2.9297 & 0.2369 & 0.2689 & 0.7341 & 0.7482 \\
Pr = 8 & -2.8378 & -2.9324 & 0.2487 & 0.2845 & 0.7330 & 0.7465 \\
\end{array}
\]
An increase in the temperature near curved surface depletes the mass transfer rate and friction factor but uplifts the rate of heat transfer.

The above said result is opposite if heat energy is produced due to radiative heat flux or viscous dissipation.

Curvature parameter has tendency to enhance the performance of mass and heat transfer.

An increase in Eckert number/radiation/temperature ratio parameter improves the fluid temperature.

When compared both the cases, a better heat and mass transfer performance is noticed if the sheet stretches nonlinearly. Similarly, the concentration, temperature and velocity of the fluid is high in the case of linear stretching sheet.

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