A Variational Step for Reduction of Mixed Gaussian-Impulse Noise from Images

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Abstract—Reduction of mixed noise is an ill posed problem for the occurrence of contrasting distributions of noise in the image. The mixed noise that is usually encountered is the simultaneous presence of additive white Gaussian noise (AWGN) and impulse noise (IN). A standard approach to denoise an image with such corruption is to apply a rank order filter (ROF) followed by an efficient linear filter to remove the residual noise. However, ROF cannot completely remove the heavy tail of the noise distribution originating from the IN and thus the denoising performance can be suboptimal. In this paper, we present a variational step to remove the heavy tail of the noise distribution. Through experiments, it is shown that this approach can significantly improve the denoising performance of mixed AWGN-IN using well-established methods.

Index Terms—\(l_1\) norm, mixed noise removal, variational approach

I. INTRODUCTION

Image denoising is a fundamental problem in image processing. The physical properties of imaging devices as well as faulty sensors and transmission equipments are primarily responsible for noise in images [1]. Two types of noise which are often encountered in practice are the additive white Gaussian noise (AWGN) and the impulse noise (IN). AWGN is commonly introduced by the temperature of the sensor and the level of illumination in the environment that corrupts every pixels, whereas IN is caused by faulty sensor triggers or transmissions corrupting a certain image pixels [2]. Due to the common origin, these noises often occur simultaneously in practice which includes digital photography [3], tomography [4] and thermal imaging [5]. This mixed noise removal has been studied fairly well in the past [6]–[13].

Removing mixed AWGN-IN is relatively difficult due to the unique nature of the AWGN and IN. In general, the order statistics filters are effective in reducing impulse noise whereas the additive filters are successful in reducing the AWGN [6]. Thus the common approach to tackle the mixed AWGN-IN is to employ an order statistics filter which detects and removes the IN and then use a smoothing algorithm to remove the remaining residual noise. It is natural to consider the residual noise as Gaussian like and use a AWGN denoiser to remove the residual noise. A common benchmark in the literature uses a rank order filter (ROF) such as adaptive median filter (AMF) [14] for salt and pepper impulse noise, followed by a Gaussian denoiser such as block matching and 3D filtering (BM3D) AWGN denoiser [15]. In a similar fashion, Xiong et al. [10] recommended a robust outlyingness ratio (ROR) statistics of neighboring pixels to detect IN, and then employed non local means to remove mixed noise from images. The next class of methods use rank order filter to detect and remove impulse noise and then employ an optimization technique to obtain noise free images. The optimization techniques explored under this framework include \(l_1\) norm [7], [9] and total variation norm [8]. Non-local regularization, in the second phase, has generated considerable success.

In order to unify the framework of impulse detection and noise reduction, Jiang et al. [11] employed weighted encoding of the mixed noise to denoise the images. The method requires initialization of the estimated noise free image, which is performed by employing a ROF on the noisy image. In a similar framework, Huang et al. [12] employed Laplacian scale mixtures modeling to fit the IN and employed non-local low rank regularizer to reduce mixed AWGN-IN. Recently, variational impulse removal
followed by CNN has been employed in [13].

From the discussion, it is evident that the methods generally require impulse detection and removal step before it can be tackled for overall noise removal. This impulse removal step, generally, can be considered as a “Gaussianization” step, which removes the heavy tail caused by the IN. Since, the AWGN removal process is pretty well established, if efficient algorithms to remove the heavy tail are available, it will be possible to increase the overall denoising performance. With this motivation, in this paper, a variational approach with local regularization is proposed to reduce the heavy tail of the noise. We show that performance of the existing methods can be significantly improved using our proposed approach by experimenting on several well established methods. As a result, this method can be employed as an important step in mixed AWGN-IN removal algorithm.

II. PROBLEM FORMULATION

Under the mixed AWGN-IN degradation model, the observation \( x_n \in \mathbb{R}^{M \times N} \) of the noisy image can be modeled as a function of its noise free version \( x \in \mathbb{R}^{M \times N} \) as [11]

\[
x_n = f(x)
\]

where \( f(\cdot) \) is the degradation function. In this paper two types of degradation are considered: 1) mixed AWGN and salt and pepper impulse noise (SPIN) and 2) mixed AWGN, SPIN and random valued impulse noise (RVIN). Let a pixel of the noisy image be denoted as \( x_n(i,j) \). For the case when an image is corrupted by AWGN alone, the noisy pixel is given by,

\[
x_n(i,j) = x(i,j) + \nu(i,j)
\]

where \( \nu(i,j) \) is a sample of i.i.d. zero-mean Gaussian distribution with standard deviation \( \sigma \). Let the dynamic rage of the image pixels be within the range \([d_{\text{max}}, d_{\text{min}}]\). In such case, the SPIN originates when an image pixel is stuck either in the maximum pixel value \( d_{\text{max}} \) with probability \( p/2 \) or the minimum pixel value \( d_{\text{min}} \) with probability \( p/2 \), where \( p \leq 1 \). In a mixed AWGN+SPIN scenario, the pixel is contaminated by AWGN with a probability \( (1-p) \). Similarly, an image pixel \( x_n(i,j) \) is corrupted with RVIN when it gets stuck to a random value \( d(i,j) \) with probability \( r \) \( (r \leq 1) \). The value \( d(i,j) \) is uniformly distributed in the dynamic range \([d_{\text{min}}, d_{\text{max}}]\). Using this definition of SPIN and RVIN, the general case of an image pixel \( x_n(i,j) \) corrupted by mixed AWGN-IN can be given by

\[
x_n(i,j) = \begin{cases} 
  d_{\text{min}} & \text{with probability } p/2 \\
  d_{\text{max}} & \text{with probability } p/2 \\
  d(i,j) & \text{with probability } r(1-p) \\
  x(i,j) + \nu(i,j) & \text{with probability } (1-p)(1-r)
\end{cases}
\]

If \( r = 0 \), the noise is mixed AWGN+SPIN, otherwise the noise is mixed AWGN+SPIN+RVIN. The purpose of denoising is to estimate the noise free image \( x \) from the corresponding noisy observation \( x_n \).

III. PROPOSED METHOD

In the proposed method, we focus on removing impulse noise to remove the heavy tail caused by it. In this light, we took inspiration from the impulse removal and detection approach provided in [9] and employed an \( l_1 \)-norm based regularization approach to lessen the effect of impulse noise.

Let \( z \in \mathbb{R}^{M \times N} \) be the image obtained by the rank order filtering operation on the mixed noise corrupted image \( x_n \). The filtered image \( z \) is employed to determine the noise candidates by

\[
\mathcal{N} = \{(i,j) \in \mathcal{A} | z(i,j) \neq x_n(i,j)\}
\]

where \( \mathcal{N} \) is the set of pixels locations corrupted by impulse noise, \( \mathcal{A} \) is the set of all observed pixels locations. A proper choice of impulse detector should detect most of the noisy pixels successfully. For SPIN, the set \( \mathcal{N} \) can be the set of locations of \( d_{\text{max}} \) and \( d_{\text{min}} \). Thus, locations of the pixels that are uncorrupted by impulse noise are defined as

\[
\mathcal{U} = \mathcal{A} \setminus \mathcal{N}
\]

The optimization is performed only on these uncorrupted image pixels. The resultant ill-posed inverse problem is solved by using a variational method and requires minimization of the convex function given by [9]

\[
\sum_{(i,j)\in \mathcal{A}} \chi(i,j)|x(i,j) - x_n(i,j)| + \beta \sum_{(i,j)\in \mathcal{A}} \sum_{(k,l)\in \mathcal{V}_{i,j}} |x(i,j) - x(k,l)|
\]

where the first term in the function is the \( l_1 \) norm, the second term is an edge preserving local regularizer, \( \chi \) is the characteristic function of the set \( \mathcal{U} \), \( \beta \) is the regularizing parameter and \( \mathcal{V}_{i,j} \) is the set of four neighboring
pixels of the pixel at \((i, j)\) for local regularization. The function \(\chi\) is given by
\[
\chi(i, j) = \begin{cases} 
1 & \text{if } (i, j) \in \mathcal{U} \\
0 & \text{otherwise}
\end{cases}
\] (7)

Following [16], the function can be optimized using
d fixed point iteration by introducing a weak smooth regularizer given by
\[
\mathcal{L}(x) = \sum_{(i,j) \in A} \sqrt{\chi(i,j)[x(i,j) - x_n(i,j)]^2 + \eta} \\
+ \beta \sum_{(i,j) \in A} \sum_{(k,l) \in \mathcal{V}_{i,j}} \sqrt{|x(i,j) - x(k,l)|^2 + \eta}
\] (8)
which can be differentiated and equated to zero to obtain a solution. Given the solution of \(x\) at \((p-1)\)th iteration, the solution at \(p\)th iteration can be computed by solving the following linear equation [9]
\[
\chi \circ [x^p - x_n] + \beta G^* \sqrt{[Gx^p]^2 + \eta} = 0
\] (9)
where \(\circ\), \([\cdot]^2\), and \(\cdot\) are elementwise multiplication, square and division, respectively. \(G\) is the difference matrix such that \(Gx(ij, kl) = x(i,j) - x(k,l)\) for \((i,j) \in \mathcal{A}\) and \((k,l) \in \mathcal{V}_{i,j}\), and \(G^*\) is the adjoint matrix of \(G\). A good choice of \(\beta\) makes it an efficient process to remove the heavy tail.

Fig. 1 shows the residual noise of the Lena image in four scenarios: the image is corrupted by AWGN only, it is corrupted by mixed AWGN-IN, the IN removal process using AMF and the proposed variational step, for different noise parameters. The proposed method is employed by \(\beta = 0.0002\) and a tolerance of \(0.001\) in the numerical solution. It can be observed from the figure that the process effectively reduces the heavy tail of the residual noise. The effect can be clearly observed from the logarithmic scale of the distributions.

A typical mixed AWGN-IN removal algorithm has two primary steps. First, the IN is removed using an ROF and then, the residual noise is removed using another algorithm. Under the proposed method, the ROF is followed by this variational step and then the denoiser follows.

IV. EXPERIMENTS AND RESULTS

In order to conduct experiments, three commonly referred methods have been chosen. The methods are: ROF+BM3D [15], WESNR [11] and LSM-NLR [12]. The codes of these methods and the variational optimizer [9] have been downloaded from respective authors’ website. These methods use ROF as the mandatory first step. For experimentation regarding the proposed approach, the ROF step of these methods have been replaced by ROF followed by the variational step. The codes of the experiments have been made available in [17]. In order to differentiate between the original method and the proposed modification, an \(M\) is added as a prefix in the modified methods’ name. The ROF used for mixed AWGN+SPIN removal is AMF, and for mixed AWGN+SPIN+RVIN the ROF is AMF followed by adaptive center weighted median filter (ACWMF) as...
in [12]. The value of $\beta$ is set to 0.0002 when only SPIN is present and 0.002 when RVIN is present in the noise. The performance of the algorithms are measured using the peak signal to noise ratio (PSNR) and the structural similarity (SSIM) indices [18].

Table I shows results of mixed AWGN+SPIN denoising on three typical images, Lena, an image with smooth details, House, an image with repetitive structures and Boat, an image with higher details and with many edges in different directions. The experiment is conducted in two scenarios where the AWGN noise parameters $\sigma$ is set to 25 and the SPIN noise parameter $p$ is varied between 30% and 50%. Each of the methods has been run five times and the average metric is reported in the table. It can be observed from the table that the proposed modification certainly facilitates the overall denoising performance. The Gaussian denoising algorithm BM3D achieves the largest increase of performance in terms of both metrics. The WESNR and LSM-NLR method, tailored for mixed noise, also achieve significant performance boost. It can be seen that as the SPIN noise is increased, keeping the AWGN noise fixed, the percentage increase of performance also increases, which shows the proposed method is an inevitable step for high quality denoising performance for these methods. Similarly, table II shows the result of mixed AWGN+SPIN+RVIN removal for $\sigma$, $p$, and $r$ set to 10, 25% and 5%, respectively. It can be observed that the variational step improves the denoising performance in all of the considered methods.

Fig. 2 shows the visual comparison of the denoising performance of the methods for noise parameters $\sigma = 25$ and $p = 30\%$ using both default and proposed modification using the variational step. It can be observed from the figures that the proposed modification improves the overall quality of the images for the considered methods. The over all consistency of the images and sharpness of the edges have been improved using the proposed variational step in mixed noise removal.

V. CONCLUSION

The removal of mixed AWGN-IN is a challenge as the two types of noise are contrasting. The IN in the distribution causes a heavy tail which is hard to capture using the denoising algorithms. In order to reduce this heavy tail, in this paper, a variational method based
approach has been taken. It has been shown that by introducing a variational denoising step, the heavy tail caused by the IN can be efficiently reduced. By conducting experiments, it has been observed that the proposed variational step to modify the existing methods can be employed to reduce mixed Gaussian-impulse noise with improved denoising performance. Furthermore, this approach can be adopted as an integral step in any mixed AWGN-IN removal method.

**REFERENCES**

[1] T. Rabie, “Adaptive hybrid mean and median filtering of high-ISO long-exposure sensor noise for digital photography,” *J. Electronic Imaging*, vol. 13, no. 2, pp. 264–277, 2004.

[2] R. C. Gonzalez and R. E. Woods, *Digital Image Processing*, 3rd ed. NJ: Prentice Hall, 2008.

[3] R. Grou-Szabo and T. Shibata, “Random-valued impulse noise detector for switching median filters using edge detectors,” in *Proc. 2009 3rd Int. Conf. Signal Processing and Communication Systems*. Omaha, NE, USA: IEEE, 2009, pp. 1–4.

[4] Y. Norose, K. Mizutani, N. Wakatsuki, and T. Ebihara, “Noise reduction in ultrasonic computerized tomography by preprocessing for projection data,” *Japanese Journal of Applied Physics*, vol. 54, no. 7S1, pp. 1–4, 2015.

[5] C. Qi, Z. Wang, J. Han, and S. Qi, “Wavelet threshold denoising of thermal image from transmission joints,” in *Int. Conf. Information Technology, Computer Engineering and Management Sciences (ICM)*, vol. 3. Nanjing, Jiangsu, China: IEEE, 2011, pp. 108–111.

[6] S. Peng and L. Lucke, “Fuzzy filtering for mixed noise removal during image processing,” in *Proc. IEEE Int. Conf. Fuzzy Systems*, Orlando, FL, 1994, pp. 89–93.

[7] J.-F. Cai, R. H. Chan, and M. Nikolova, “Two-phase approach for deblurring images corrupted by impulse plus Gaussian noise,” *Inverse Problems and Imaging*, vol. 2, no. 2, pp. 187–204, 2008.

[8] Y.-M. Huang, M. K. Ng, and Y.-W. Wen, “Fast image restoration methods for impulse and Gaussian noises removal,” *IEEE Signal Processing Letters*, vol. 16, no. 6, pp. 457–460, 2009.

[9] J.-F. Cai, R. H. Chan, and M. Nikolova, “Fast two-phase image deblurring under impulse noise,” *J. Mathematical Imaging and Vision*, vol. 36, no. 1, pp. 46–53, 2010.

[10] B. Xiong and Z. Yin, “A universal denoising framework with a new impulse detector and nonlocal means,” *IEEE Trans. Image Processing*, vol. 21, no. 4, pp. 1663–1675, 2012.

[11] J. Jiang, L. Zhang, and J. Yang, “Mixed noise removal by weighted encoding with sparse nonlocal regularization,” *IEEE Trans. Image Processing*, vol. 23, no. 6, pp. 2651–2662, 2014.

[12] T. Huang, W. Dong, X. Xie, G. Shi, and X. Bai, “Mixed noise removal via Laplacian scale mixture modeling and nonlocal low-rank approximation,” *IEEE Trans. Image Processing*, vol. 26, no. 7, pp. 3171–3186, 2017.

[13] M. T. Islam, S. M. M. Rahman, M. O. Ahmad, and M. N. S. Swamy, “Mixed Gaussian-impulse noise reduction from images using convolutional neural network,” *Signal Processing: Image Communication*, vol. 68, pp. 26–41, 2018.
algorithms and results,” *IEEE Trans. Image Processing*, vol. 4, no. 4, pp. 499–502, 1995.

[15] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, “Image denoising by sparse 3-D transform-domain collaborative filtering,” *IEEE Trans. Image Processing*, vol. 16, no. 8, pp. 2080–2095, 2007.

[16] C. R. Vogel and M. E. Oman, “Iterative methods for total variation denoising,” *SIAM Journal on Scientific Computing*, vol. 17, no. 1, pp. 227–238, 1996.

[17] Codes and images. [Online]. Available: https://github.com/tariqul-islam/Variational-Step-for-Mixed-AWGN-IN-Removal-form-Images

[18] Z. Wang, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli, “Image quality assessment: From error visibility to structural similarity,” *IEEE Trans. Image Processing*, vol. 13, no. 4, pp. 600–612, 2004.