Stochastic Modelling of the Hydraulic Anisotropy of Ash Impoundment Sediment

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Abstract. In the case reported here the impoundments of a 400 MW coal heated power plant with an annual production of about 1.5 million tons of fuel ash are of the cross-valley type, operated by the simple and cheap „upstream method“. The aim of the research was to determine overall and local values of the permeability in horizontal as well as in vertical direction and the anisotropy of the thin-layered sedimented ash. The coal ashes are hydraulically transported through pipelines in form of a slurry and periodically floated on the beach of the impoundment. The ashes are deposited in the form of a thin-layered sediment, with random alternation of layers with a coarser or finer granularity. The ash impoundment sediment is anthropogenic sediment with horizontally laminated texture. Therefore, the sediment is anisotropic from the viewpoint of water seepage. The knowledge of the permeability and the seepage anisotropy of the sediment is a basic requirement for the design of an appropriate dewatering system. The seepage anisotropy of the ash sediment has been checked by means of stochastic modelling, based on the correlation between the effective grain diameter and the coefficient of permeability of the ash: the effective grain diameter and the thickness of individual layers have been proposed to be random events.

1. Introduction

The method of floating ashes to impoundment results in formation of approximately laminated sediment containing randomly alternating positions with coarse and fine granularities. The respective quasi-homogeneous positions are several millimetres to decimetres thick, often covering considerable area. Nature of lamination of the ash sediment produced by hydraulic transportation of ashes to the accumulation area of the impoundment is shown in Figure 1.

The sediment formed inside the impoundment is anisotropic in terms of water seepage. Permeability of the layered ash sediment in horizontal direction is significantly higher than in vertical direction. Literature reports [1], [2] that in ash impoundments, hydraulic anisotropy of ash sediment determined by ratio of vertical and horizontal permeability can range between 1 – 100. The magnitude of hydraulic anisotropy of ash sediment determined by this range is virtually unusable. Besides the determining values of permeability coefficients in horizontal and vertical direction, value of hydraulic anisotropy should represent another important input information for economic design of the impoundment’s drainage systems. Therefore, determination of the actual value of hydraulic anisotropy of ash sediment is practical for use in design.
2. Hydraulic anisotropy of ash sediment

Hydraulic anisotropy of ash sediment can be considered from two points of view [3]. The total seepage through settled ash is determined by the average value of anisotropy, derived from the overall height of the sediment. This anisotropy value can be referred to as an average or total anisotropy. After embedding drains, which capture water from relatively narrow zone, the flow into the drains will be determined by anisotropy at the level of the zone (1.0 – 1.5 m). This value can be referred to as a local anisotropy. Both values, total and local, are of practical meaning.

For seeping porous environments, hydraulics uses the following equations:

average value of permeability coefficient parallel to the layers

\[
k_x = \frac{\sum p d_i k_i}{\sum i d_i} \text{ [m. s}^{-1}]\]

(1)

average value of permeability coefficient perpendicular to the layers

\[
k_z = \frac{\sum k_i d_i}{\sum k_i d_i} \text{ [m. s}^{-1}]\]

(2)

hydraulic anisotropy of multi-layer consisting of n layers
where \( d_i \) [m] is thickness of \( i^{th} \) layer and \( k_i \) [m.s\(^{-1}\)] is permeability coefficient of the \( i^{th} \) layer.

Ash sediment has a very high number of layers, while each layer has its own respective values of \( d_i \) and \( k_i \), which cannot be determined experimentally. Therefore, this problem was solved by mathematical modelling.

2.1 Assumptions for creating mathematical model

Assumptions used in building the mathematical model for solving hydraulic anisotropy of ash sediment were as follows [4], [5]:

- The thicknesses of homogeneous layers of settled ash \( d_i \) vary randomly within the margins of \( d_{\text{min}} \) and \( d_{\text{max}} \).
- The correlation (4) Figure 2 between grain diameter \( d_{10} \) and filtration coefficient \( k \) of the ashes applies, based on evaluation of a large set of archive measurements of permeability coefficients for ashes of variable granular composition, complemented with our own laboratory measurements.

\[
k = 8.9 \times 10^{-4} \cdot 0.27^{0.49 \log d_{10}} \text{ [m.s}^{-1}] \text{; } d_{10} \text{ [mm]} \tag{4}
\]

Figure 2. The correlation of grain diameter \( d_{10} \) with permeability coefficient \( k \) of ashes

- Probability of occurrence of diameter \( d_{10} \) in ash sediment can be approximated by logarithmic-normal distribution of grain diameters \( d_{10} \), derived from evaluation of a large set of granularity values of ashes. Probability distribution given by function (5) with relative frequency histogram is shown in the Figure 3a, b.

\[
f(ln d_{10}) = 0.389 \cdot e^{-\left(\frac{ln d_{10}+3.727}{1.026}\right)^2}
\tag{5}
Figure 3. a) The relative frequency histogram and b) probability distribution of grain diameter $d_{10}$

2.2 Input parameters for mathematical model

The previous assumptions imply the following inputs for the mathematical model:

- Based on visual evaluation of the walls of excavation performed at impoundments of Nováky brown coal power plant located in Zemianske Kostoľany, layer thicknesses were determined, ranging between $d_{\text{min}} = 1$ mm and $d_{\text{max}} = 100$ mm.
- The requirement of random changes in layer thickness $d_i$ within boundaries of $d_{\text{min}}$ and $d_{\text{max}}$ was solved by assignment of numeric values of uniformly distributed random numbers within the boundaries of the minimum and maximum layer thickness.
- For a given ash granularity, characterized by grain diameter $d_{10}$, permeability coefficient was expressed based on (4) as $k = f(d_{10})$.
- For distribution of probability density for occurrence of grain $d_{10}$ in ash sediment, as described by the function (5), respective diameters $d_{10}$ can be assigned random values with a logarithmic-normal distribution.

Mathematical model based on the aforementioned assumptions combines two random quantities – layer with any granularity ($d_{10}$) and, therefore, any permeability ($k$) can have any thickness ($d_i$). If the chosen number of layers corresponds to the height of ash sediment above the subsoil or if it is so high that it can be considered a representative statistical sample, then evaluation of the multi-layer, using equations (1), (2) and (3), will necessarily yield actual values of $k_x$, $k_z$ and $\lambda$.

The representative statistical set incorporated 300 layers divided into:

- Case a) 10 groups of equal numbers of layers per group (30);
- Case b) 10 groups with approximately equal thickness of the group multi-layers (1.5 m);
- Case c) 30 groups with approximately equal thickness of the group multi-layers (0.5 m).

2.3 Results of mathematical model

Based on the results of the mathematical modelling, average values $k_x$, $k_z$ and $\lambda$ were processed according to the equations (1), (2) and (3) for respective groups of layers as local average values and by gradual integration of the respective groups as the total average values for the overall thickness of the ash sediment. The average local and cumulated values $k_x$, $k_z$ and $\lambda$ are clearly graphically represented in Figure 4 a,b,c,d for case a), Figure 5a,b,c,d for case b) and Figure 6a,b,c,d for case c).
Figure 4 a, b - Case a) Average local values of permeability coefficients and hydraulic anisotropy for 300 layers divided into 10 groups with equal numbers of layers per group (30)

Figure 4 c, d - Case a) Cumulated values of permeability coefficients and hydraulic anisotropy for 300 layers divided into 10 groups with equal numbers of layers per group (30)

Figure 5 a, b - Case b) Average local values of permeability coefficients and hydraulic anisotropy for 300 layers divided into 10 groups with approximately equal thickness of the group multi-layer (1.5 m)
Figure 5 c, d - Case b) Cumulated values of permeability coefficients and hydraulic anisotropy for 300 layers divided into 10 groups with approximately equal thickness of the group multi-layer (1.5 m)

Figure 6 a, b - Case c) Average local values of permeability coefficients and hydraulic anisotropy for 300 layers divided into 30 groups with approximately equal thickness of the group multi-layer (0.5 m)

Figure 6 c, d - Case c) Cumulated values of permeability coefficients and hydraulic anisotropy for 300 layers divided into 30 groups with approximately equal thickness of the group multi-layer (0.5 m)
Results of calculations, organized in clear graphs, show that local average values exhibit lower or higher fluctuations, depending on the specific place. Ranges of average values for respective cases are listed in Table 1. Comparison of results confirmed the expectation that the lower the evaluated number of layers (lower thickness of the ash sediment), the higher the fluctuation of average values is.

Table 1. Range of average values of the permeability coefficients and of hydraulic anisotropy for the solved Cases a), b), c)

| Solved case | Thickness of the group multi-layer [m] | Permeability coefficient $k_x$ [m.s$^{-1}$] | Permeability coefficient $k_z$ [m.s$^{-1}$] | Hydraulic anisotropy $\lambda$ [-] |
|-------------|----------------------------------------|-----------------------------------|-----------------------------------|---------------------|
| a)          | 1.328 – 1.594                          | 1.4.10$^{-5}$ – 3.8.10$^{-5}$      | 1.6.10$^{-6}$ – 1.2.10$^{-5}$      | 2.9 – 10.0          |
| b)          | 1.465 – 1.515                          | 1.5.10$^{-5}$ – 3.7.10$^{-5}$      | 1.7.10$^{-6}$ – 6.8.10$^{-6}$      | 4.3 – 10.6          |
| c)          | 0.454 – 0.538                          | 8.4.10$^{-6}$ – 4.5.10$^{-5}$      | 7.1.10$^{-7}$ – 2.2.10$^{-5}$      | 1.8 – 22.9          |

Cumulative values of permeability coefficients and of hydraulic anisotropy expressed by gradual integration of respective layers show gradual stabilization with increasing multi-layer thickness. These determined values can be considered overall average permeability coefficients and hydraulic anisotropy for ash sediment consisting of 300 layers with overall thickness of approximately 15 m. Stabilized overall averages for the cases solved are listed in Table 2. Stability of values represents evidence of representativeness of the statistical set and shows that the results obtained are realistic for given assumptions of the mathematical model.

Table 2. Cumulative values of the permeability coefficients and of hydraulic anisotropy expressed by gradual integration of respective layers

| Solved case | Thickness of the multi-layer [m] | Permeability coefficient $k_x$ [m.s$^{-1}$] | Permeability coefficient $k_z$ [m.s$^{-1}$] | Hydraulic anisotropy $\lambda$ [-] |
|-------------|----------------------------------|----------------------------------|----------------------------------|---------------------|
| a)          | 14.996                           | 2.4.10$^{-5}$                    | 3.2.10$^{-6}$                    | 7.5                 |
| b)          | 14.992                           |                                  |                                  |                     |
| c)          | 15.078                           |                                  |                                  |                     |

3. Hydraulic anisotropy of ash sediment

To conclude, it can be declared that actual overall averages for layered ash sediment are, most likely, as follows:

- permeability coefficient in horizontal direction $k_x = 2.4.10^{-5}$ [m.s$^{-1}$];
- permeability coefficient in vertical direction $k_z = 3.2.10^{-6}$ [m.s$^{-1}$];
- value of hydraulic anisotropy $\lambda = 7.5$ [-];

with ranges of local fluctuations:

- for the permeability coefficient in horizontal direction $k_x = 1.4.10^{-5} – 3.8.10^{-5}$ [m.s$^{-1}$];
- for the permeability coefficient in vertical direction $k_z = 1.6.10^{-6} – 1.2.10^{-5}$ [m.s$^{-1}$];
- for the value of hydraulic anisotropy $\lambda = 2.9 – 10.6$ [-].

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