Observation of Topological Structures in Photonic Quantum Walks

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Phases of matter with non-trivial topological order are predicted to exhibit a variety of exotic phenomena, such as robust localized bound states in 1D systems, and edge states in 2D systems, which are expected to display spin-helicity, immunity to back-scattering, and weak anti-localization. In this Letter, we present an experimental observation of topological structures generated via the controlled implementation of two consecutive non-commuting rotations in photonic discrete-time quantum walks. The second rotation introduces valley-like Dirac points in the system, allowing to create the non-trivial topological pattern. By choosing specific values for the rotations, it is possible to coherently drive the system between topological sectors characterized by different topological invariants. We probe the full topological landscape, demonstrating the emergence of localized bound states hosted at the topological boundaries, and the existence of extremely localized or delocalized non-Gaussian quantum states. Our results pave the way for the study of valley-polarization and applications of topological mechanisms in robust optical-device engineering.

PACS numbers: 03.65.Yz, 05.40.Fb, 71.23.-k, 71.55.Jv, 37.10.Jk, 05.30.Rt

Phase transitions play a fundamental role in science, and in physics in particular. While classical phase transitions are typically driven by thermal noise, quantum phase transitions are triggered by quantum fluctuations \cite{1}. Quantum phase transitions have received increasing attention within the realm of ultra-cold atom in optical lattices \cite{2,3}. Standard phase transitions, both classical and quantum, follow the, so called, Landau scenario, and consist in spontaneous symmetry breaking. The ordered phase can then be described by a local order parameter.

A different kind of quantum phase transitions occurs in systems characterized by a, so called, topological order. Such systems have generically degenerate ground states, which cannot be described by a local order parameter; they exhibit localized edge states, that are protected against noise by underlying symmetries. Paradigm examples of topological phases appear in quantum Hall effect (QHE) \cite{4,5}, fractional QHE \cite{6,7}, and in spin QHE, or generally speaking in topological insulators (TIs), predicted in \cite{8,9} and realized experimentally in \cite{10,11}.

Topological edge states characterizing TIs have recently been simulated in a number of different systems ranging from ultra-cold atoms in optical lattices \cite{15}, to photonic networks of coupled resonators in silicon platforms \cite{16}. Furthermore, it has recently become apparent that discrete-time quantum walks (DTQWs) \cite{17} offer a versatile platform for the exploration of a wide range of non-trivial topological effects (experiment) \cite{18,20}, and (theory) \cite{21,23}. Further, QWs are robust platforms for modelling a variety of dynamical processes from excitation transfer in spin chains \cite{26,27} to energy transport in biological complexes \cite{28}. They enable to study multi-path quantum interference phenomena \cite{29,32}, and can provide for a route to validation of quantum complexity \cite{33,34}, and universal quantum computing \cite{35}. Moreover, multi-particle QWs warrant a powerful tool for encoding information in an exponentially larger space, and for quantum simulations of biological, chemical and physical systems, in 1D and 2D geometries \cite{19,36,38}.

In this Letter, we present a novel theoretical and experimental scheme for generation of topological structure in 1D time-multiplexed DTQW architectures \cite{39,40}. The novelty relies on the introduction of two consecutive non-commuting rotations along the walk (Fig. 1 (a)). The second rotation allows to close the quasi-energy gap at additional points, and creates the non-trivial topological structure (Fig. 1 (b)). In contrast to previous work \cite{18}, the additional Dirac points are located at non-zero quasi-momentum, emulating valley-Dirac points in graphene, or related materials (Fig. 1 (c)). For graphene, symmetry allowed energy bands form two pairs of cones (or valleys) located at quasi-momenta $k = \pm \pi/2$. As a crystal momentum $K$ separates the two valleys, the valley degrees-of-freedom (DOF) are robust against slowly varying potentials and scattering and have promising applications in valleytronics, in addition to be considered an
The modified dispersion relation becomes \( \cos(E_{\theta,\phi}(k)) = \cos(k) \cos(\phi) + \sin(k) \sin(\phi) \sin(\theta) \), where we recover the Dirac-like dispersion relation for \( \phi = 0 \). We stress that since the dispersion relation characterizing our system differs from the homogenous case [15], the bound states observed here have not previously been observed. Moreover, for specific rotation parameters our scheme allows for the direct observation of extremely (de)localized quantum states.

The basic step in the standard DTQW is given by a unitary evolution operator \( U(\theta) = TR_x(\theta) \), where \( R_x(\theta) \) is a rotation along an arbitrary direction \( \vec{r} = (n_x, n_y, n_z) \), given by \( R_y(\theta) = \left( \begin{array}{c} \cos(\theta) - in_z \sin(\theta) \\ i(n_x - n_y) \sin(\theta) \\ i(n_x + n_y) \sin(\theta) \cos(\theta) + in_z \sin(\theta) \end{array} \right) \), in the Pauli basis [18]. In this basis, the y-rotation is defined by a coin operator of the form \( R_y(\theta) = \left( \begin{array}{c} \cos(\theta) - \sin(\theta) \\ \sin(\theta) \cos(\theta) \end{array} \right) \).

This is followed by a spin- or polarization-dependent translation \( T \) given by \( T = \sum_{\vec{r}} |\vec{r} + 1\rangle \otimes |\vec{r}\rangle + |\vec{r} - 1\rangle \otimes |\vec{r}\rangle \otimes |V\rangle |V\rangle \), where \( H = (1,0)^T \) and \( V = (0,1)^T \) (Fig. 1 (a)). The evolution operator for a discrete-time step is equivalent to that generated by a Hamiltonian \( H(\theta) \), such that \( U(\theta) = e^{-iH(\theta)} (h = 1) \), with \( H(\theta) = \int d\vec{k} |E_0(\vec{k})\vec{n}(\vec{k})\rangle \otimes \langle \vec{k}| |k\rangle \) and \( \vec{r} \) the Pauli matrices, which readily reveals the spin-orbit coupling mechanism in the system. The quantum walk described by \( U(\theta) \) has been realized experimentally in a number of systems [19]–[21], and has been shown to posess chiral symmetry, and display Dirac-like dispersion relation given by \( \cos(E_0(k)) = \cos(k) \cos(\theta) \). Here, we present localization effects given by the introduction of a second rotation along the x-direction by an angle \( \phi \), such that the unitarity step becomes \( U(\theta, \phi) = TR_x(\phi)R_y(\theta) \), where \( R_x(\phi) \) is given, in the same basis, by [18]:

\[
R_x(\phi) = \left( \begin{array}{c} \cos(\phi) \\ i \sin(\phi) \\ i \sin(\phi) \cos(\phi) \end{array} \right).
\]

The modified dispersion relation becomes \( \cos(E_{\theta,\phi}(k)) = \cos(k) \cos(\phi) + \sin(k) \sin(\phi) \sin(\theta) \), where we recover the Dirac-like dispersion relation for \( \phi = 0 \).

FIG. 1. (a) Poincaré sphere representation, (b) phase diagram for parameter values within \(-\pi \leq \theta, \phi \leq \pi\). (c) Band structure in first Brillouin zone for rotation parameters \(-\pi/2 \leq \theta, \phi \leq \pi/2\). Red (blue) lines correspond \( \phi = 0 (\pi/2) \). \( \phi = \pm \pi/2 \) allows to close the quasi-energy gap at additional zero quasi-energy points for quasi-momentum \( k = \pm \pi/2 \). Such Dirac points separated by the crystal momentum \( K \) are analogous to valley-Dirac points in graphene. (d) Numerically simulated benchmark states after \( N = 7 \) steps. Circle: delocalized (non-Gaussian) quantum state \( (\theta = \pi/4, \phi = 0) \), for Hadamard QW; Square: Localized bound state at topological boundary \( (\theta = \pi/4, \phi = \pi/4) \); Romboid: Extremely delocalized (non-Gaussian) quantum state at edge of Brilliouin zone \( (\theta = -\pi, \phi = \pi) \); Filled circles: extremely localized state at center of topological sector \( (\theta = 0, \phi = \pi/2) \).
values for the topological invariants ($Q_0, Q_\pi$) (where the subscript refers to the quasi-energy $E$), which correspond to the parity (even=0 or odd=1) of the number of times the gap closes at quasi-energy $E = (0, \pi)$ along a straight line in parameters space, starting from a fixed point in the zone (0,0) [24], indicated by a star. Note that such guideline does not refer to the actual trajectory followed by the quantum walker. By choosing specific values for the rotation parameters ($\theta, \phi$) it is possible to drive the system across topological sectors characterized by different topological invariants $Q_{0,\pi}$. At the boundary of each topological sector the emergence of localized bound states is predicted.

The experimental scheme for time-multiplexed DTQWs with non-commuting coin operations is based on Ref. [40] (see Fig. 2 (a)). Our scheme allows to implement an arbitrarily large step-number in a compact architecture, in combination with detector gating and suitable ND filters. Equivalent single-photon states are generated with an attenuated pulsed diode laser centered at 810 nm and with 111 kHz repetition rate (RR). The initial state of the photons is controlled via half-wave plates (HWP)s and quarter-wave plates (QWP)s, to produce eigen-states of chirality $|\psi^\pm_0 \rangle = |0\rangle \otimes 1/\sqrt{2}(|H\rangle \pm i|V\rangle)$. Inside the loop, the first rotation ($R_y(\theta)$) is implemented by a HWP with its optical axis oriented at an angle $\alpha = \theta/2$. The rotation along the x-axis ($R_x(\phi)$) is implemented by a combination of two QWP$s$ with axes oriented horizontally(vertically), characterized by Jones matrices of the form $\begin{pmatrix} 1 & 0 \\ 0 & (-1)^i \end{pmatrix}$ (Fig. 2 (b)). In between the QWP$s$, a HWP oriented at $\beta = \phi/2$ determines the angle for the x-rotation. The spin-dependent translation is realized in the time domain via a polarizing beam splitter (PBS) and a fiber delay line, in which horizontally polarized light follows a longer path. The resulting temporal difference between both polarization components corresponds to a step in the spatial domain ($x \pm 1$). Polarization controllers (PC) are introduced to compensate for arbitrary polarization rotations in the fibers. After implementing the time-delay the time-bins are recombined in a single spatial mode by means of a second PBS and are re-routed into the fiber loops. After a full evolution the photon wave-packet is distributed over several discrete positions, or time-bins. The detection is realized by coupling the photons out of the loop by a beam sampler (BS) with a probability of 5% per step. Compensation HWPs (CHWPs) are introduced to correct for dichroism at the beam samplers (BS). We employ two avalanche photodiodes (APDs) to measure the photon arrival time and polarization properties. The probability that a photon undergoes a full round-trip is given by the overall coupling efficiency (> 70%) and the overall losses in the setup resulting in $\eta = 0.50$. The average photon number per pulse is controlled via neutral density filters and is below $\langle n \rangle < 0.003$ for the relevant iteration steps ($N = 7$) to ensure negligible contribution from multi-photon events.

We characterized the round-trip time (RTT=750 ns) and the time-bin distance (TBD=52 ns) with a fast Os-
cilloscope (Lecroy 640ZI, 4GHz). The RTT, and the laser 
RR determine the maximum number of steps that can be 
observed in our system \( N_{\text{max}} = 12 \). Therefore \( N_{\text{max}} \) 
can be easily increased by adjusting these two design 
parameters. Figure 2 (c), shows typical time-bin traces 
obtained from time-delay histogram recorded with 72 ps 
resolution. The actual number of counts was obtained by 
integrating over a narrow window. We first implemented 
the Hadamard quantum walk, by setting \( \theta = \pi/4 \) and 
\( \phi = 0 \). This is shown in Fig. 3 (a) for the first \( N = 7 \) 
steps with no numerical corrections for systematic errors, 
after background subtraction. We compare the theoret-
ical and experimental probability distributions via the 
similarly \( S = |\sum_{x} \sqrt{P_{\text{theo}}(x)P_{\text{exp}}(x)}|^2 \), with \( S = 0(1) \) 
for orthogonal(identical) distributions [38], typically ob-
taining \( S \approx 0.85 \). The difference between raw data and 
theory are displayed in Fig. 3 (b). Experimental errors 
can be explained in terms of asymmetric coupling, im-
perfect polarization-rotation compensation in the fibers, 
unequal efficiency in the detectors, and other sources of 
polarization dependent losses, in addition to shot-noise.

Uncontrolled reflections are a main source of error. We 
removed this by subtracting the counts of the two APDs, 
and filtering peaks located at positions different from the 
RTT and the TBD during data analysis.

Next, we probed the topological landscape (Fig. 4 
inset) for two input states \( |\psi_0^+\rangle \) of well defined chiral-
ity obtaining equivalent results. Experimental results 
are only displayed for \( |\psi_0^+\rangle \). Figure 4 (a), (b), (c) and 
(d) show experimentally reconstructed probability dis-
tributions (blue bars) for a trajectory characterized by 
fixed \( \theta = -\pi/4 \) and \( 0 \leq \phi \leq \pi \) after \( N = 7 \) steps, 
by tracing over polarization DOF. Red empty bars are 
numerical simulations, error bars are statistical. (a), 
(b) Delocalized quantum state for \( (\phi = 0, \pi) \), display-
ing characteristic non-Gaussian distribution corresponding 
to Hadamard QW; (c), (d), localized states for \( (\phi = 
\pi/4, 3\pi/4) \) at topological boundaries. Fig. 4 (e), (f) and 
(g) display theoretical prediction, experimental results, 
and difference, respectively. In order to quantify the de-
gree of localization we define a localization parameter 
(\( S_L \)) as the difference between outer and inner proba-
bility peaks \(-1/2 \leq S_L = P_{\text{outer}} - P_{\text{inner}} \leq 1/2 \), lo-
cated at positions \( |x_{\text{outer}}| = 5 \) and \( |x_{\text{inner}}| = 1 \), for the 
\( N = 7 \) step. We note that \( S_L \) is not an order parameter 
[1], rather it is a simple way of characterizing the shape of 
the probability distributions. Localized(delocalized) 
quantum states are characterized by a negative(positive) 
parameter \( S_L < (>0) \). Experimentally reconstructed 
localization parameters (\( S_L \)) (blue bars) for three full 
trajectories in parameter space (\( \alpha, \beta, \gamma \)) are dis-
played in Fig. 4 (h), (i) and (j), respectively. Red empty 
bars correspond to theoretically expected values. Tra-
jectories \( \alpha, \beta \) are symmetric as expected, and show 
localized states at the topological boundaries (squares), 
or delocalized non-Gaussian quantum states (circles) in 
the outer regions. Additionally, we demonstrate the ex-
istence of extremely (de)localized states saturating the bound of \( S_L \) 
(trajectory \( \gamma \)), for parameters \( (\theta = \pm \pi, \phi = \pi, \text{romboids}) \), and 
\( (\theta = (0, \pm \pi/2), \phi = (0, \pm \pi/2), \text{filled circles}) \).
structures in 1D photonic DTQWs by tailoring two successive non-commuting coin operations along the walk, experimentally confirming the existence of topological boundaries, localized bound states and extremely delocalized non-Gaussian quantum states. Localization effects are displayed after $N = 7$ steps. This number could be increased by adjusting the round-trip time and the laser repetition rate. Our scheme can be implemented for the study of valley-polarization and topological protection \cite{50}. The results presented here can be generalized to 2D architectures \cite{38}, and can find relevant applications in robust optical device engineering \cite{49}, and entanglement topological protection \cite{50}.

Acknowledgements.- The authors gratefully acknowledge A. Schreiber, P. Neumann, F. Reinhard, Y. Shikano, J. Asbóth, J. P. Torres and S. Moulieras for useful discussions and technical support. We acknowledge financial support by the Max-Planck-society, EU (Squtec), Darpa (Quasar), BMBF (CHIST-ERA), ERC (Quagatua, Osysris), and contract research of the Baden-Württemberg foundation.

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