Mathematical modeling of the interaction of magnetic fields of red blood cells

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Abstract. The article discusses the movement of red blood cells along narrow capillaries with a diameter less than the diameter of the red blood cell. The erythrocyte membrane in the narrow capillary performs a tank-treating motion with a frequency reaching several tens of revolutions per second. Electric charges located on the surface of an erythrocyte move together with the erythrocyte membrane and create a magnetic field in the surrounding space. A two-dimensional model of the movement of red blood cells along narrow capillaries is developed. Various options were considered: either neighboring red blood cells rotate in one direction, or in different directions. With different options, different distributions of the magnetic field strength are obtained. Calculations of the magnetic field were carried out at various distances between red blood cells and the speed of rotation of the erythrocyte membrane. It was shown that at distances between red blood cells less than two capillary diameters, the influence of neighboring red blood cells can be neglected (the difference is less than 3%).

1. Introduction

If the red blood cell moves along a narrow capillary, then its membrane performs an tank-treating motion with a frequency reaching several tens of revolutions per second [1-8]. The erythrocyte charge is negative, and amounts to about 20 million elementary charges, that is equal to 0.32 $10^{-15}$ C [9]. The calculations were carried out on a computer in the range from $10^{-15}$ to $10^{-12}$ C. Electric charges are located uniformly on the surface of the red blood cell, rotate with the erythrocyte membrane and generate a magnetic field in the surrounding space. The magnetic field affects the content of red blood cells. It is known that at sufficiently large distances, for example, from a rotating charged sphere, the magnetic field is similar to the field created by a magnetic dipole, and at distances that are not too large; the magnetic field differs from the magnetic dipole field. Therefore, it is necessary to carry out calculations on a computer of the magnetic field at small distances from the erythrocyte membrane [10].

2. Mathematical model

We believe that the erythrocyte volume is about 94 $\mu$m$^3$, the surface area of the red blood cell is 135 $\mu$m$^2$, the erythrocyte charge varies in the range from $10^{-15}$ to $10^{-12}$ C and, therefore, the charge density on the erythrocyte membrane is from 0.74 $10^{-17}$ to 0.74 $10^{-14}$ C/$\mu$m$^2$. The charges on the membrane are grouped around the molecules and form negatively charged formations.

The magnetic field strength of a moving charged particle
\[ H = \frac{qV \sin \alpha}{4\pi r^2}, \]

where \( q \) is the particle charge, \( V \) is the particle velocity, \( r \) is the distance from the particle to the point at which the magnetic field strength \( H \) is determined, \( \alpha \) is the angle between the direction of particle velocity and the straight line connecting the particle and the point at which the intensity \( H \) is determined [11], [12].

The total magnetic field strength of several moving charged particles is defined as the vector sum of the magnetic field strengths of individual moving charged particles.

In the two-dimensional model, it is assumed that the red blood cells that move along the capillary are squares, and the erythrocyte membrane is the sides of the square. Discrete charges are located on the sides of the square and move either clockwise or counterclockwise. When two adjacent red blood cells move, the following options are possible. Their membranes either rotate in the same direction, or in opposite directions. The calculations were carried out for both options and at different speeds of rotation of the erythrocyte membranes (from 0 to 50 revolutions per second).

3. Results and discussion

The calculations were performed on a computer for various diameters of narrow capillaries (from 3 to 5 μm), for various speeds (from 0 to 50 revolutions per second) and directions of rotation (clockwise and counterclockwise). So, in figure 1 shows the change in the intensity \( H \) (A/m) of the magnetic field strength \( H \) (A/m) along the axis of the capillary (μm).

![Figure 1](image-url)

**Figure 1.** Change in the magnetic field strength \( H \) (A/m) (ordinate axis) along the capillary axis (μm) (abscissa axis).

Red blood cells are assumed to be squares with a side equal to 4 μm. The charges are located uniformly on the erythrocyte membrane (sides of the square), the number of charges is 16 and each of the charges is 1/16 of the charge of the red blood cell, i.e., 2 \( 10^{-13} \) C. The charges together with the membrane (the faces of the square) move clockwise along the faces of the square, the rotation speed is
10 revolutions per second; the distance between red blood cells is 2 μm, which corresponds to half the diameter of the capillary.

Figure 2 shows the change in the magnetic field strength $H$ (A/m) along the axis of the capillary (μm). The difference from that shown in figure 1 is that the distance between red blood cells is 8 μm, which corresponds to two capillary diameters.

![Figure 2. Change in the magnetic field strength $H$ (A/m) (ordinate axis) along the capillary axis (μm) (abscissa axis).](image)

From the figures it can be seen that at distances between red blood cells equal to half the diameter of the capillary, the mutual influence of magnetic fields is significant. At distances between red blood cells equal to two or more diameters of the capillary, the mutual influence is not so significant, the difference is less than 3% and, therefore, it can be neglected.

This can be explained by the fact that from the above formula it is seen that the magnetic field strength decreases in proportion to the inverse square of the distance from the mobile charge. Thus, when constructing mathematical models of red blood cells in narrow capillaries, it makes sense to take into account the magnetic interaction of red blood cells when the distances between red blood cells are not very large, i.e., they are less than one capillary diameter.
**Figure 3.** Change in the magnetic field strength $H$ (A/m) (ordinate axis) along the capillary axis ($\mu$m) (abscissa axis).

**Figure 4.** Change in the magnetic field strength $H$ (A/m) (ordinate axis) along the capillary axis ($\mu$m) (abscissa axis).
The magnetic field generated by charges located on the surface of the red blood cell propagates both inside and outside the red blood cells. The magnetic field affects the contents of the red blood cell (hemoglobin molecule) and can lead to their movement inside the red blood cell, since they have charges and contain iron.

4. Conclusion
When constructing mathematical models of red blood cells moving along capillaries whose diameter is less than the diameter of the red blood cell, the magnetic field cannot be taken into account if the distance between red blood cells is more than two capillary diameters. If the distance is less, or the red blood cells are located at distances shorter than the diameter of the capillary, then the mutual influence of the magnetic fields is essential and must be taken into account. At large distances from the red blood cell rotating in the capillary, the magnetic field generated by it is similar to the field of a magnetic dipole, but at short distances it is not like that and it must be calculated on a computer, since the magnetic field is large and the mutual influence of the magnetic fields of closely spaced red blood cells is significant and cannot be neglected. The study of the magnetic fields of red blood cells and electromagnetic irradiation of blood can be useful in the diagnosis and treatment of, for example, diseases such as methemoglobinemia and the like.

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