Constituent Quark Model versus Nonperturbative QCD.

Ariel R. Zhitnitsky

Physics Department, University of British Columbia, 6224 Agricultural Road, Vancouver, BC V6T 1Z1, Canada

Abstract

We discuss a few, apparently different (but actually, tightly related) problems:
1. The relation between QCD and valence quark model,
2. The asymptotic behavior of the nonperturbative pion wave function \( \psi(\vec{k}_1^2, x) \) at \( x \to 0 \), \( 1, \vec{k}_1^2 \to \infty \) and
3. The dimensional counting rules in the intermediate region of energy. The analysis is based on such general methods as dispersion relations, duality and PCAC. We calculate the asymptotic behavior of the wave function \( (wf) \) at the end-point region \( (x \to 1 \) and \( \vec{k}_1^2 \to \infty \) by analysing the corresponding large \( n \)-th moments in transverse \( \langle \vec{k}_1^{2n} \rangle \sim n! \) and longitudinal \( \langle (2x - 1)^n \rangle \sim 1/n^2 \) directions.

This information fixes the asymptotic behavior of \( wf \) at large \( \vec{k}_1^2 \) (which is turned out to be Gaussian commonly used in the phenomenological analyses). We discuss some applications of the obtained results. In particular, we calculate the nonleading “soft” contribution to the pion form factor at intermediate momentum transfer. We argue, that due to the specific properties of \( \psi(\vec{k}_1^2, x) \), the corresponding contribution can temporarily simulate the leading twist behavior in the extent region of \( Q^2 \). The same effect also takes place for the nucleon formfactor. Such a mechanism, if it is correct, would be an explanation of the phenomenological success of the dimensional counting rules at available, very modest energies for many different processes. We discuss some inclusive amplitudes also.

Talk given at the Workshop “Continuous Advances in QCD,96”.
Theoretical Physics Institute, University of Minnesota, Minneapolis, MN, March 28-31, 1996.

\(^1\) e-mail address: arz@physics.ubc.ca
1 Motivation. Definitions.

The problem of bound states in the relativistic quantum field theory with large coupling constant is an extremely difficult problem. To understand the structure of the bound state is a very ambitious goal which assumes the solution of a whole spectrum of tightly connected problems, such as confinement, chiral symmetry breaking phenomenon, and many others which are greatly important in the low energy region.

A less ambitious purpose is the study of the hadron wave function \( (w_f) \) with a minimal number of constituents\(^2\). As is known such a function gives the parametrically leading contributions to hard exclusive processes. The corresponding wave functions within QCD have been introduced to the theory in the late seventies and early eighties \([1]\) to describe the exclusive processes. We refer to the review papers \([2]\), \([3]\) on this subject for the details.

The main idea of the approach \([1]−[3]\) is the separation of the large and small distance physics. At small distances we can use the standard perturbative expansion due to the asymptotic freedom and smallness of the coupling constant. All nontrivial, large distance physics is hidden into the nonperturbative wave function \( (w_f) \) and can not be found by perturbative technique, but rather it should be extracted from elsewhere. The most powerful analytical nonperturbative method for such problems, I believe, is the QCD sum rules \([4]\), \([5]\).

The first application of QCD sum rules to the analysis of nonperturbative \( w_f \) was considered more than decade ago \([6]\). The information extracted for the few lowest moments, unambiguously shows the asymmetric form of the distribution amplitudes. At the same time, the applicability of the approach \([4]\) at experimentally accessible momentum transfers was questioned \([7]\). Since then this subject is a very controversial issue and we are not going to comment these quite opposite points in the present talk. Instead we would like to formulate the following question:

- If the asymptotically leading contribution can not provide the experimentally observable absolute values, than \emph{how can one explain the very good agreement between the experimental data and dimensional counting rules} which are supposed to be valid only in the region where the leading terms dominate?

It is clear, that the possible explanation can not be related to the specific

\(^2\) Such a study is the way to understand the valence quark model in the QCD terms. This remark explains the title of this talk.
amplitude, but instead, it should be connected, somehow, to the nonperturbative wave functions of the light hadrons \((\pi, \rho, N, ...)\) which enter the formulæ for exclusive processes. The analysis of the \(\pi\) meson and nucleon form factors, presented below supports this idea.

To anticipate the events we would like to formulate here the result of this analysis. The very unusual properties of a nonperturbative hadron wave function lead to the **temporarily simulation** of the dimensional counting rules by soft mechanism for the \(F_\pi Q^2, F_N Q^4\) in the extent range of intermediate momentum transfer: \(3 GeV^2 \leq Q^2 \leq 40 GeV^2\). In this region the soft contribution to the form factors does not fall-off, as naively one could expect.

- Therefore, our answer on the formulated question is the following. The light cone nonperturbative wave functions are very different from the ones, motivated by the naive quark model. In the former, QCD case, there is a dimensional parameter which determines all scales in the problem. This parameter is nothing, but a mean value of quark transverse momentum \(\langle \vec{k}^2 \rangle \simeq (330 MeV)^2\) which can be expressed in terms of QCD vacuum condensates. In the latter, valence quark model case, there is one more parameter, the constituent quark mass \(m\), which has the same order of magnitude. In spite of the fact that both these scales are numerically very close to each other, they are fundamentally different. The difference is due to the fact that \(m \simeq 330 MeV\) is a constant while \(0 \leq \vec{k}^2 \leq \infty\) is a variable with average \(\langle \vec{k}^2 \rangle \simeq (330 MeV)^2\).

Let me emphasize from the very beginning that the ideology and methods (unitarity, dispersion relations, duality) we use are motivated by QCD sum rules. However, we do not use the QCD sum rules in the common sense: we do not fit them to extract any information about lowest resonance (as people usually do in this approach), we do not use any numerical approximation or implicit assumption about higher states. Instead, we concentrate on analysis of the appropriate correlation functions themselves to extract the most general information.

The starting point is the definition of \(wf\) in terms of nonperturbative matrix elements. To be more specific, let us consider the \(\pi - \) meson axial wave function:

\[
i f_\pi q_\mu \phi_A(zq, z^2) = \langle 0|\bar{d}(z)\gamma_\mu\gamma_5 e^{ig\int_{-z}^z A_\mu dz'\nu} u(-z)|\pi(q)\rangle = \sum_n \frac{i^n}{n!}\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 (iz_\nu \bar{D}_\nu)^n u(0)|\pi(q)\rangle,
\]

where \(\bar{D}_\nu \equiv \bar{D}_\nu - \bar{D}_\nu\) and \(i\bar{D}_\mu = i\bar{\partial}_\mu + gA_\mu^a \lambda^a_2\) is the covariant derivative.

From its definition is clear that the set of different \(\pi\) meson matrix elements defines the nonperturbative wave function. Exactly this definition of the \(wf\)
as the set of different matrix elements we have in mind when we discuss the nonperturbative \( w_f \). Such a \( w_f \) may or may not satisfy some equations.

First of all, let us discuss the most important part (at asymptotically high \( q^2 \)) of the \( w_f \) which is related to the longitudinal distribution. In this case \( z^2 \approx 0 \) and the \( w_f \) depends on \( zq \) variable only. The corresponding Fourier transformed wave function will be denoted as \( \phi(\xi) \) and its \( n \)–th moment is given by the following local matrix element:

\[
\langle 0 | \bar{d} \gamma^5 (i \not{D} \mu) z \mu u | \pi(q) \rangle = i f_{\pi q} (zq \xi^n) = i f_{\pi q} (zq \xi^n) \int_{-1}^{1} d\xi \xi^n \phi(\xi) \quad (2)
\]

\[
- q^2 \to \infty, \quad zq \sim 1 \quad \xi = x_1 - x_2, \quad x_1 + x_2 = 1, \quad z^2 = 0. \quad (3)
\]

Therefore, if we knew all matrix elements (2) (which are well-defined) we could restore the whole distribution amplitude \( \phi(\xi) \).

Now we would like to analyse the similar moments, but in transverse direction. To do so, let us define the transverse vector \( t^\mu = (0, \vec{t}, 0) \) to be perpendicular the hadron momentum \( q^\mu = (q_0, 0_\perp, q_z) \). The vector \( t^\mu \) is an appropriate projector in transverse plane and plays the same role what \( z^\mu \) vector does in eq.(2) for longitudinal direction. We define the \( 2n \)–th moment for the transverse quark distribution in the following way:

\[
\langle 0 | \bar{d} \gamma^5 (i \not{D} \mu) t^2 u | \pi(q) \rangle = i f_{\pi q} (-t^2 \xi^n) \frac{(2n-1)!!}{(2n)!!} \langle \vec{k}_{\perp}^{2n} \rangle. \quad (4)
\]

where \( \not{D} \mu \) is the covariant derivative, acting on the one quark. The factor \( \frac{(2n-1)!!}{(2n)!!} \) is introduced to (4) to take into account the integration over \( \phi \) angle in the transverse plane: \( \int d\phi (\cos \phi)^{2n} / \int d\phi = (2n-1)!! / (2n)!! \).

We interpret the \( \langle \vec{k}_{\perp}^{2n} \rangle \) in this equation as a mean value square of the transverse quark momentum. Of course it is different from the naive, gauge dependent definition like \( \langle 0 | \bar{d} \gamma^5 \gamma^2 u | \pi(q) \rangle \), because the physical transverse gluon is participant of this definition. However, the expression (4) is the only possible way to define the moments \( \langle \vec{k}_{\perp}^{2n} \rangle \) in the gauge theory like QCD. We believe that such definition is the useful generalization of the transverse momentum conception for the interactive quark system.

The definition of the nonperturbative wave function is as follows: Let us assume that we know all longitudinal moments (2) as well as transverse moments (4). We introduce the wave function of two variables \( \psi(\vec{k}_{\perp}^{2n}, \xi) \), normalized to one

\[
\int d\vec{k}_{\perp}^{2n} \psi(\vec{k}_{\perp}^{2n}, \xi) = \phi(\xi), \quad \int_{-1}^{1} d\xi \phi(\xi) = 1 \quad (5)
\]
in such a way that it exactly reproduces the set of moments defined by the local matrix elements (2,4), The relations to Brodsky and Lepage notations \( \Psi_{BL}(x_1,\vec{k}_\perp) \) looks as follows:

\[
\Psi_{BL}(x_1,\vec{k}_\perp) = \frac{f_\pi 16\pi^2}{\sqrt{6}} \psi(\xi,\vec{k}_\perp), \quad \int d\vec{k}_\perp^2 d\xi \psi(\vec{k}_\perp^2, \xi) = 1 \quad (6)
\]

where \( f_\pi = 133 MeV \).

2 Constraints

As me mentioned earlier, the QCD sum rules is appropriate method to analyse the matrix elements given by the formula (2,4). We start our discussion from the analysis of the longitudinal moments.

The corresponding calculations have been done many years ago and a few first moments have been estimated\[6\]. However, this information is not enough to reconstruct the \( w_f \); the parametric behavior at \( \xi \to \pm 1 \) is the crucial issue in this reconstruction.

To extract the corresponding information, we use the following duality argument. Instead of consideration of the pion \( w_f \) itself, we study the following correlation function with pion quantum numbers:

\[
i \int dx e^{iqx} \langle 0 | T J_n^\parallel(x), J_0(0) | 0 \rangle = (zq)^{n+2} I_n(q^2), \quad J_n^\parallel = \bar{d} \gamma_\nu z_\nu \gamma_5 (i \not{D} \mu \not{z}_\mu)^n u \quad (7)
\]

and calculate its asymptotic behavior at large \( q^2 \). The result can be presented in the form of the dispersion integral, whose spectral density is determined by the pure perturbative one-loop diagram:

\[
\frac{1}{\pi} \int_0^\infty ds \frac{Im I_n^{pert}(s)}{s - q^2}, \quad Im I_n(s)^{pert} = \frac{3}{4\pi(n+1)(n+3)}. \quad (8)
\]

We assume that the \( \pi \) meson gives a nonzero contribution to the dispersion integral for arbitrary \( n \) and, in particular, for \( n \to \infty \). Formally, it can be written in the following way

\[
\frac{1}{\pi} \int_0^{S_n^{\parallel}} ds Im I(s)^{pert} = \frac{1}{\pi} \int_0^\infty ds Im I(s)^{\pi}, \quad (9)
\]

Our assumption means that \( S_n^{\parallel} \neq 0 \), where we specified the notation for the longitudinal distribution. In this case at \( q^2 \to \infty \) our assumption (9) leads to the following relation:

\[
f_\pi^2 \langle \xi^n \rangle(n \to \infty) \to \frac{3S_n^{\parallel}}{4\pi^2 n^2} \quad (10)
\]
It unambiguously implies the following behavior at the end-point region \[2\]:

\[
\langle \xi^n \rangle = \int_{-1}^{1} d\xi \xi^n \phi(\xi) \sim 1/n^2, \quad \phi(\xi \to \pm 1) \to (1 - \xi^2). \tag{11}
\]

Thus, our first constraint looks as follows:

1. \[\phi(\xi \to \pm 1) \to (1 - \xi^2).\]

We want to emphasize that we did not use any numerical approximation in this derivation. Therefore, the constraint (1) has very general origin and it should be considered as a direct consequence of QCD. Only dispersion relations, duality and very plausible assumption formulated above have been used in the derivation (1). We can repeat these arguments for the analysis of the transverse distribution. The result is \[3\]:

\[
f^2_\pi \langle \vec{k}_{\perp}^{2n} \rangle \sim S_{n+1}^{\perp} n! \Rightarrow f^2_\pi \langle \vec{k}_{\perp}^{2n} \rangle \sim n!, \quad n \to \infty. \tag{12}
\]

This behavior has been obtained in ref.\[8\] by analysing the perturbative series of the specific correlation function at large order. The dispersion relations and duality arguments translate this information into the formula (12).

The nice feature of (12) is its finiteness for arbitrary \(n\). It means that the higher moments

\[
\langle \vec{k}_{\perp}^{2n} \rangle = \int d\vec{k}_{\perp}^2 d\xi \vec{k}_{\perp}^{2n} \psi(\vec{k}_{\perp}^2, \xi) \tag{13}
\]

do exist. The existence of the arbitrary high moments \(\langle \vec{k}_{\perp}^{2n} \rangle\) means that the nonperturbative \(w f\), defined above, falls off at large transverse momentum \(\vec{k}_{\perp}^2\) faster than any power function. The relation (12) fixes the asymptotic behavior of \(w f\) at large \(\vec{k}_{\perp}^2\). Thus, we arrive to the following constraint:

2. \[\langle \vec{k}_{\perp}^{2n} \rangle = \int d\vec{k}_{\perp}^2 d\xi \vec{k}_{\perp}^{2n} \psi(\vec{k}_{\perp}^2, \xi) \sim n! \quad n \to \infty.\]

We can repeat our duality arguments again for an arbitrary number of transverse derivatives and large \((n \to \infty)\) number of longitudinal derivatives with the following result \[3\]:

3. \[\int d\vec{k}_{\perp}^2 \vec{k}_{\perp}^{2k} \psi(\vec{k}_{\perp}^2, \xi \to \pm 1) \sim (1 - \xi^2)^{k+1}.
\]

For \(k = 0\) we reproduce our previous formula for the \(\phi\) function: \(\phi(\xi \to \pm 1) = \int d\vec{k}_{\perp}^2 \psi(\vec{k}_{\perp}^2, \xi \to \pm 1) \sim (1 - \xi^2)^{1}\). The constraint (3) is extremely important and implies that the \(\vec{k}_{\perp}^2\) dependence of the \(\psi(\vec{k}_{\perp}^2, \xi)\) comes exclusively in the combination \(\vec{k}_{\perp}^2/(1 - \xi^2)\) at \(\xi \to \pm 1\). The byproduct of this constraint can be formulated as follows. The standard assumption on factorizability of the \(\psi(\vec{k}_{\perp}^2, \xi) = \psi(\vec{k}_{\perp}^2) \phi(\xi)\) does contradict to the very general

\[3\] Here and in what follows we ignore any mild (nonfactorial) \(n\)-dependence.
properties of the theory. Thus, the asymptotic behavior of the \( wf \) turns out to be Gaussian one with the very specific argument:

\[
\psi(k^2_1 \to \infty, \xi) \sim \exp(-\frac{k^2_1}{1-\xi^2}), \tag{14}
\]

Let us remark, that the same methods can be applied for the analysis of the asymptotical behavior of the nucleon \( wf \) as well. In obvious notations the asymptotic behavior for the nucleon \( wf \) takes the form:

\[
\psi_{\text{nucleon}}(k^2_{1i} \to \infty, x_i) \sim \exp(-\sum \frac{k^2_{1i}}{x_i}). \tag{15}
\]

### 3 QCD vs. valence Quark Model

We would like to make the following conjecture: The Gaussian \( wf \) reconstructed earlier from the QCD analysis (14, 15), not accidentally coincides with the harmonic oscillator \( wf \) from the constituent quark model. To make this conjecture more clear, let us recall few results from the constituent quark model.

It is well known [10] that the equal- time wave functions

\[
\psi_{CM}(q^2) \sim \exp(-q^2) \tag{16}
\]

of the harmonic oscillator in the rest frame give a very reasonable description of static meson properties. Together with Brodsky-Huang-Lepage prescription [11] connecting the equal-time and the light-cone wave functions of two constituents (with mass \( m \sim 300\, \text{MeV} \)) by identification

\[
q^2 \leftrightarrow \frac{k^2_1 + m^2}{4x(1-x)} - m^2, \quad \psi_{CM}(q^2) \leftrightarrow \psi_{LC}(\frac{k^2_1 + m^2}{4x(1-x)} - m^2), \tag{17}
\]

one can reproduce the Gaussian behavior (14) found from QCD. It means, first of all, that our identification of the moments (4) defined in QCD with the ones defined in quark model, is the reasonable conjecture.

However, there is a difference. In valence quark model we do have a parameter which describes the mass of constituents \( m \simeq 330\, \text{MeV} \). We have nothing like that in QCD. Indeed the presence of such a term in QCD, would mean the following behavior for the large moments in the longitudinal direction:

\[
\langle \xi^n \rangle = \int_{-1}^{1} d\xi \xi^n \phi(\xi) \sim \int_{-1}^{1} d\xi \xi^n \exp(-\frac{1}{1-\xi^2}) \sim \exp(-\sqrt{n}), \ n \to \infty. \tag{18}
\]
This is in contradiction with $1/n^2$ behavior (11), (1) found earlier. We do not see any possibilities to change this behavior from $1/n^2$ to $\exp(-\sqrt{n})$ within QCD. Thus, the massive term in the $wf$ motivated by valence quark model has no any justification from the QCD point of view. The same conclusion is also true for the nucleon $wf$. Again, we have no room for the mass term in the formula (13). This observation, as we shall see has very important impact on the phenomenological analysis.

4 Numerical Constraints.

In the previous sections we discussed some general $wf$ properties which should be satisfied for any QCD-based model. However, those constraints do not determine the scale of the problem; they do not give a dimensional factor which would govern the hadronic properties. In the present section we want to discuss some numerical (and therefore, less general) constraints on wave functions.

Thus, there is a big difference between constraints (•1 − •3) discussed above and the ones which follow. The first three constraints have very general origin. No numerical approximations have been made in the derivation of the corresponding formulae. The constraints which follow have absolutely different status. They are based on the QCD sum rules with their inevitable numerical assumptions about higher excited states in QCD. Thus, they must be treated as an approximate ones. The well-known constraint of such a kind is the second moment of the distribution amplitude in the longitudinal direction [3]:

\begin{equation}
\langle \xi^2 \rangle \equiv \int d\xi \phi(\xi)\xi^2 \simeq 0.4,
\end{equation}

(The asymptotic $wf$ corresponds to $\langle \xi^2 \rangle = 0.2$). Such a result was the reason to suggest the “two-hump” shape $wf$ [3] which meets the above requirement. The number cited as the constraint (•4) has been criticized in the literature. Thus, in what follows we shall discuss both possibilities: the narrow (asymptotic) $wf$ and the wider one (with larger $\langle \xi^2 \rangle > 0.2$).

The next “numerical” constraint is the second moment of the $wf$ in the transverse direction defined by equation (4) and calculated for the first time in [2] and independently (with quite different technique) in [12]. Both results are in a full agreement to each other:

\begin{equation}
\langle \vec{k}_\perp^2 \rangle = \frac{5}{36} \frac{\langle \bar{q}G_{\mu\nu}^2 q \rangle}{\langle \bar{q}\gamma^\mu q \rangle} \simeq \frac{5m_0^2}{36} \simeq 0.1 GeV^2, \quad m_0^2 \simeq 0.8 GeV^2.
\end{equation}

Essentially, the constraint (•5) defines the general scale of all nonperturbative phenomena for the pion. It is not accidentally coincides with $330 MeV$ which is the typical magnitude in the hadronic physics.

To study the fine properties of the transverse distribution it is desired to
know the next moment. The problem can be reduced to the analysis of the mixed vacuum condensates of dimension seven \[9\]:

\[
\langle \vec{k}_\perp^4 \rangle = \frac{1}{8} \left\{ \frac{-3\langle \vec{q}g^2\sigma_{\mu\nu}G_{\mu\nu}\sigma_{\lambda\sigma}q \rangle}{4\langle \vec{q}q \rangle} + \frac{13\langle \vec{q}g^2G_{\mu\nu}G_{\mu\nu}q \rangle}{9\langle \vec{q}q \rangle} \right\}, \tag{19}
\]

We analyzed the magnitudes for these vacuum condensates with the following result: the standard factorization hypothesis does not work in this case. The factor of nonfactorizability \( K \simeq 3.0 \div 3.5 \). The eq.(19) defines the new numerical constraint on the transverse distribution. We prefer to express this constraint not in terms of the absolute values, but rather, in terms of the dimensionless parameter \( R \) which is defined in the following way:

\[
R \equiv \frac{\langle \vec{k}_\perp^4 \rangle}{\langle \vec{k}_\perp^2 \rangle^2} \simeq 3K \cdot \frac{\langle g^2G^a_{\mu\nu}G^a_{\mu\nu} \rangle}{m_0^4} \simeq 5 \div 7, \quad m_0^2 \simeq 0.8 GeV^2,
\]

where we use the standard values for parameter \( m_0^2 \) and gluon condensate \[3\]. We would like to emphasize that the fluctuations of the transverse momentum are large enough. The quantitative characteristic of these fluctuations is parameter \( R \gg 1 \). In terms of the wave function this property means a very unhomogeneous distribution in transverse direction.

5 Applications.

Having found the constraints on the \( wf \) in the previous sections, one can model them and finally, one can apply them for the calculation of different amplitudes. The corresponding calculations he been carried out for the \( \pi \) meson in ref.[14] and for the nucleon in ref.[15]. We quote here some results of the corresponding calculations.

The first application is the pion form factor. The starting point is the Drell-Yan formula where \( F_\pi(Q^2) \) is expressed in terms of the wave functions:

\[
F_\pi(Q^2) = \int \frac{dx dq^2}{16\pi^3} \Psi_{BL}^*(x, \vec{k}_\perp + (1-x)\vec{q}_\perp) \Psi_{BL}(x, \vec{k}_\perp), \tag{20}
\]

where \( q^2 = -\vec{q}_\perp^2 = -Q^2 \) is the momentum transfer. In this formula, the \( \Psi_{BL}(x, \vec{k}_\perp) \) is the full wave function; the perturbative tail of \( \Psi_{BL}(x, \vec{k}_\perp) \) behaves as \( \alpha_s/\vec{k}_\perp^2 \) for large \( \vec{k}_\perp^2 \) and should be taken into account explicitly in the calculations. This gives the one-gluon-exchange (asymptotically leading) formula for the form factor in terms of distribution amplitude \( \phi(x) \)[4]. We recall, in passing, that the asymptotically leading contribution predicts that the combination \( Q^2F_\pi(Q^2) \) is a constant. Here the qualitative results of our
calculations of the “soft” contribution, which is asymptotically suppressed by power $1/Q^2$.

If we were started from the wave function motivated valence quark model (17) with mass parameter in it, than we could find the following general behavior: such a function gives a very reasonable magnitude for $Q^2 F_\pi(Q^2)$ in the intermediate region about few GeV$^2$ and it starts to fall off very quickly right after that. We expect that any reasonable, well localized, based on quark model wave function with the scale $\sim \langle \vec{k}_2^2 \rangle \sim m^2$ leads to the similar behavior.

Currently, much more interesting for us is the calculation of the soft contribution, based on the QCD motivated model with Gaussian behavior and without mass term in it, see (14). The corresponding calculations show the qualitative difference between quark model and QCD- motivated wave functions. Namely, the much slower fall off at large $Q^2$ is observed for the $wf$ motivated by QCD. The qualitative reason for that is the absence of the mass term, see discussion after the formula (18). Precisely this term was responsible for the very steep behavior in all previous calculations based on a quark model wave function. The declining of the form factor getting even slower if one takes into account the property of the broadening of $wf$ in transverse direction, see [14] for details. This property corresponds to the strong fluctuations in the transverse direction and quantitatively is related to the large parameter $R$ (6).

One more qualitative remark. The numerical magnitude for the form factor strongly depends on parameter $\langle \xi^2 \rangle$. The soft contribution is getting bigger when a wider (in longitudinal direction) wave function is used, see [14] for details. However, the most important observation that the QCD based $wfs$ simulate the leading twist behavior, where $Q^2 F_\pi(Q^2) \simeq const.$ remains approximately fulfilled. This constant, however, itself is very sensitive to the numerical parameter $\langle \xi^2 \rangle$. Whether this parameter is large (as QCD sum rules predict, see (14)) or small (close to the asymptotic value $\langle \xi^2 \rangle = 0.2$) is still a very controversial issue. Our present attitude is that the true value, as usual, is somewhere in the middle. In this case the soft contribution can be estimated as $Q^2 F(Q^2) \sim 0.2 \div 0.3.$, see [14] for details.

The precise fitting of the pion form factor was not among the goals of these calculations. Rather, we wanted to demonstrate how the qualitative properties of a nonperturbative $wf$, derived from the QCD analysis might significantly change its behavior.

Now we would like to extend our previous analysis to the nucleon form factor. The starting point, as before, is the fundamental constraints (11 – 13), which being applied to the nucleon wave function imply the Gaussian behavior with the specific argument (15). With these constraints in mind one
can model the nucleon \( w_f \) in the same way as we did for the pion. Having modeled the nucleon wave function, one can calculate a soft contribution to different nucleon amplitudes. The corresponding analysis was carried out in the ref. \[15\]. Here we quote some results from this paper.

The most important qualitative result of these calculations is similar to what we already observed previously in the \( \pi \)-meson case: namely, the combination \( Q^4 F_{\text{nuc.}}(Q^2) \) is almost constant in the extent region of \( Q^2 \) in spite of the fact that the corresponding “soft” contribution naively should be decreasing function of \( Q^2 \). The qualitative explanation of this phenomenon is the same as before and is related to the absence of the mass term in the QCD- motivated wave function.

The next observation which was made in ref.\[15\] is related to the longitudinal distribution and can be formulated as follows: A fit to the different experimental data leads to a wave function which has the same type of asymmetry which was found previously from the QCD sum rules. The asymmetry is however much more moderate numerically than QCD sum rules indicate. We already mentioned earlier that the similar conclusion is likely to take place for the pion wave function also.

Therefore, the general moral, based on already completed calculations, can be formulated in the following way. There is a standard viewpoint for the phenomenological success of the dimensional counting rules: it is based on the prejudice that the leading twist contribution plays the main role in most cases. This outlook, as we mentioned earlier, is based on the experimental data, where the dimensional counting rules work very well. We suggest here some different explanation for this phenomenological success. Our explanation of the slow falling off of the soft contribution with energy is due to the specific properties of nonperturbative \( w_f \). In particular, we argued that the absence of the of the mass parameter in the corresponding \( w_f \) is the strict QCD constraint. At the same time this property is responsible for the behavior mentioned above. Besides that, we found a new scale (\( \sim 1\text{GeV}^2 \)) in the problem, in addition to the standard low energy parameter \( \langle k^2_\perp \rangle \approx 0.1\text{GeV}^2 \). Both these phenomena lead to the temporarily simulation of the leading twist behavior in the extent region of \( Q^2 \). We believe that this is a new explanation of the phenomenological success of the dimensional counting rules at available, very modest energies.

Our next remark can be formulated as follows. Exclusive, as well as inclusive amplitudes can be expressed in terms of the one and the same particular hadron \( w_f \). Therefore, if our explanation (related to specific form of \( \psi(k^2_\perp,x) \)) of a temporary simulation of the leading twist behavior is considered as a reasonable one, then:

1. in the analysis of inclusive amplitudes one may expect the same effect
(it is our conjecture);
2. one may try to implement the intrinsic transverse momentum dependence into the inclusive calculations.

In particular, one may try to use the following prescription for the π meson distribution function (and analogously for nucleon, see (13)) at $x \to 1$:

$$G_{q/\pi}(x, Q^2) \Rightarrow \left\{ \int \frac{d^2k_\perp}{x(1-x)} \exp\left(-\frac{k_\perp^2}{x(1-x)}\right) \right\} G_{q/\pi}(x, Q^2)$$  \hspace{1cm} (21)

To support this conjecture, we would like to mention few inclusive processes where the intrinsic transverse distribution might be essential. First of all, it is Drell-Yan amplitude $\pi + N \to \mu^- \mu^+ + X$ which is parametrized as follows (for references and recent development see [16]):

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \sim 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi.$$  \hspace{1cm} (22)

Here $\theta, \phi$ are angles defined in the muon pair rest frame and $\lambda, \mu, \nu$ are coefficients. In the naive parton model the coefficient are $\lambda = 1, \mu = \nu = 0$. Experimental results do not support this naive prediction. Recently, some improvements have been made [16], but some problems are still remain. In particular, the Lam-Tung sum rule [16], $1 - \lambda - 2\nu = 0$ is violated by experimental data and the improved model [16] still can not explain the behavior $1 - \lambda - 2\nu$ as a function of $Q_\perp^2$ ($Q_\perp^2$ is the transverse momentum of the lepton pair).

Due to the fact that $Q^2$ is not large enough in this experiment one may expect that the intrinsic distribution (21) might be essential.

One may find many examples like that where the standard parton picture does not work well. We would like to mention here the recent analysis [18] of the direct photon production ($\pi + p \to \gamma + X$), with the result that perturbative QCD can not explain the data. Some nonperturbative broadening factor in transverse direction should be implemented. One may hope that formula (21) may improve the agreement with experiment.

Our last remark is the observation made in the recent preprint [15] that the valence quark distribution functions $u^p(x)$ and $d^p(x)$ at large $x$ can be fairly described in terms of the nonperturbative nucleon $w f$. Two properties of the $w f$ are important to provide such a fit: The absence of the mass term in the formula (13) (this leads to the correct power behavior at $x \to 1$) and a moderate asymmetry in the longitudinal direction (this provides an observed ratio for the $\frac{u^p(x)}{\bar{d}^p(x)}$) at $x \to 1$ in the contrast with the asymptotic formula prediction which gives value of 2 for the same ratio).
6 Conclusion

We believe that the most important result of the present analysis is the observation that due to the specific properties of $\psi(\vec{k}^2, x)$, the “soft” contribution can temporarily simulate the leading twist behavior in the extent region of $Q^2 : \ 3 GeV^2 \leq Q^2 \leq 40 GeV^2$. Such a mechanism, if it is correct, would be an explanation of the phenomenological success of the dimensional counting rules at available, very modest energies for many different processes.

References

[1] Brodsky S. and G.P. Lepage, Phys. Lett. B 87, 359 (1979).
Chernyak V. and A. Zhitnitsky, JETP Lett. 25, 510 (1977)
Dunkan A. and A.H. Mueller, Phys. Rev. D 21, 1636 (1980).
Efremov A.V. and A.V. Radyushkin, Phys. Lett. B 94, 245 (1980)
Farrar G. and D. Jackson, Phys. Rev. Lett. 43, 246 (1979).

[2] V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. 112, 173-318 (1984).

[3] S.J. Brodsky and G.P. Lepage, “Exclusive Processes in Quantum Chromodynamics” in Perturbative Quantum Chromodynamics edited by A.H. Mueller, World Scientific Publishing Co., 1989.

[4] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov Nucl. Phys. B 147, 385, 448, 519 (1979).

[5] M.A. Shifman, Vacuum Structure and QCD Sum Rules, North-Holland, 1992.

[6] V.L. Chernyak and A.R. Zhitnitsky, Nucl. Phys. B 201, 492 (1982).

[7] N. Isgur and C.H. Llewellyn Smith, Phys. Rev. Lett. 52, 1080 (1984); Phys. Lett. B 217, 535 (1989);

[8] A.R. Zhitnitsky Phys. Lett. B 357, 211 (1995).

[9] A.R. Zhitnitsky, Phys. Lett. B 329, 49 (1994).

[10] N. Isgur, in The New Aspects of Subnuclear Physics, edited by A. Zichichi (Plenum, New York, 1980);
J. Rosner, in Techniques and Concepts of High Energy Physics, edited by T. Ferbel (Plenum, New York, 1981).
[11] S.J. Brodsky, T. Huang and P. Lepage, in Particle and Fields, edited by A.Z. Capri and A.N. Kamal (Plenum Publishing Corporation, New York, 1983).

[12] V.A. Novikov et al. *Nucl. Phys. B* 237, 525 (1984).

[13] A.R. Zhitnitsky, *Phys. Lett. B* 325, 449 (1994).

[14] B. Chibisov and A.R. Zhitnitsky, *Phys. Rev. D* 52, 5273 (1995).

[15] J. Bolz and P. Kroll, preprint WU B 95-35, Wuppertal, February, 1996.

[16] A. Brandenburg, S. Brodsky, V. Khoze and D. Muller *Phys. Rev. Lett.* 73, 939 (1994).

[17] C.S. Lam and W.K. Tung, *Phys. Rev. D* 21, 2712 (1980).

[18] J. Huston et al., CTEQ-407, MSU-HEP-1027, hep-ph/9501230.