Gauge hierarchy problem, supersymmetry and fermionic symmetry

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November 11, 2013

Abstract

We reconsider the gauge hierarchy problem from the viewpoint of effective field theories and a high-energy physics, motivated by the alternative scenario that the standard model holds up to a high-energy scale such as the Planck scale. The problem is restated as the question whether it is possible to construct a low-energy effective theory and the interaction with heavy particles, without spoiling the structure of a high-energy physics supported by an excellent concept. Based on this reinterpretation, we give a conjecture that theories with hidden fermionic symmetries can be free from the gauge hierarchy problem and become candidates of the physics beyond and/or behind the standard model, and present a prototype model for the grand unification.

1 Introduction

Recent experimental results at the Large Hadron Collider (LHC) would revisit the gauge hierarchy problem [1,2], because the Higgs boson has been found with $m_h \approx 126$ GeV [3,4], and evidences from new physics such as spacetime supersymmetry (SUSY), compositeness and extra dimensions have not yet been discovered.

The gauge hierarchy problem is related to the feature that an effective field theory becomes unnatural, because fine tuning is required to obtain the weak scale and/or to stabilize it against radiative corrections, if there is a high-energy physics such as a grand unified theory (GUT) relevant to the standard model (SM). It is summarized as the following questions. What generates the weak scale (or the Higgs boson mass)? How is it stabilized? For example, logarithmic divergences in radiative corrections due to heavy particles can ruin the stability of the weak scale.

There are at least three possibilities for the problem. First one is that there is a new physics at the terascale with a new concept such as SUSY, compositeness and/or extra

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dimensions, to solve the problem completely. Second one is that there is a new physics at the terascale to derive the weak scale, and the scale is stabilized by some excellent mechanism and/or symmetry at a high-energy scale $M_U$ such as the Planck scale $M_{Pl}$. Third one is that there is no new physics concerning the Higgs boson mass at the terascale, and a high-energy physics solves the problem completely.

In this paper, we reconsider one side of the problem “*how is the weak scale stabilized?*”, from the viewpoint of effective field theories and a high-energy physics. Our study is motivated by the alternative scenario that the SM (modified with massive neutrinos) holds up to $M_{Pl}$ [5, 6] and the guiding principle that the gauge hierarchy is stabilized by a symmetry that should be unbroken in the SM [7]. Based on a reinterpretation of the problem from the perspective of a high-energy physics, we give a conjecture that theories with fermionic symmetries different from spacetime SUSY can be free from the gauge hierarchy problem and become candidates of the physics beyond and/or behind the SM, and present a prototype model to explain the unification of the SM gauge coupling constants, the triplet-doublet splitting of Higgs boson, and the longevity of proton.

The outline of this paper is as follows. In the next section, we review the gauge hierarchy problem and discuss it in relation to masslessness. We reconsider the problem from the viewpoint of a high-energy physics, point out that specific fermionic symmetries can play the important role to stabilize a mass hierarchy, and propose a grand unification scenario based on the conjecture, in Sect. 3. In the last section, we give conclusions and discussions.

## 2 Gauge hierarchy problem and masslessness

### 2.1 Gauge hierarchy problem

We discuss a fine tuning among parameters, from the viewpoint of effective field theories.

After subtracting the quadratic divergences on scalar mass squareds, radiative corrections on parameters $a_i$, up to finite corrections, are given by

$$
\delta a_i = \sum_j \frac{c_{ij}}{(4\pi)^2} a_j \ln \frac{\Lambda^2}{\mu^2}, \quad (i, j = 1, \cdots, n),
$$

where $c_{ij}$ are functions of parameters, $\Lambda$ is a cutoff scale, and $\mu$ is a renormalization point. From the feature that $\delta a_i \to 0$ in the limit of $a_i \to 0$, the smallness of $a_i$ is understood, if the magnitude of $a_i$ is small enough. If physical parameters are determined without fine tuning, the condition $c_{ij} a_j \leq O(a_i)$ is roughly imposed on $a_i$.

Fine tuning is, in general, necessary, if there is a physics relevant to the SM at a higher energy scale beyond the terascale. For instance, in the presence of heavy particles with masses $M_I$ and some SM gauge quantum numbers, the radiative corrections on the Higgs mass squared are given by

$$
\delta m_h^2 = \tilde{c}_h \Lambda^2 + c'_h m_h^2 \ln \frac{\Lambda^2}{m_h^2} + \sum_I \tilde{c}'_{hI} M_I^2 \ln \frac{\Lambda^2}{M_I^2} + \cdots,
$$

where
where $\hat{c}_h$, $c'_h$ and $c''_h$ are functions of parameters. From (2), we find that the fine tuning is indispensable for $c''_h M_I^2 \gg m^2_h$ due to the appearance of the quadratic terms of $M_I$ (part of the logarithmic divergences), even if the quadratic divergence $\hat{c}_h \Lambda^2$ is removed and unless some miraculous cancellation mechanism works among corrections due to heavy particles. This induces the technical side of the gauge hierarchy problem [1, 2], i.e., *an unnatural fine tuning is required to stabilize the weak scale against radiative corrections, if there is a high-energy physics such as a GUT relevant to the SM.*

### 2.2 Possible solutions

If nature dislikes fine tuning among parameters, there must exist a reasonable explanation about the absence of fine tuning. Here, we introduce some possibilities.

1. The concept of the first one is “compositeness”, i.e., some particles are not elementary but composite, made of a more fundamental constituents. We assume that there is a new dynamics at the terascale, to compose some SM particles. The typical example is a model that the Higgs doublet is made of new fermions [8, 9, 10]. The existence of new particles and strong dynamics among them is predicted at the terascale.

2. The concept of the second one is “symmetry”, that protects physical parameters against large radiative corrections. As a new symmetry appearing at the terascale, we enumerate three candidates.

   2.1 Supersymmetry [11, 12]. The SUSY must be realized in a broken phase if exists. In the presence of soft SUSY breaking terms, $\delta m^2_h$ is given by

   $$\delta m^2_h = \hat{c}'_h m^2_h \ln \frac{\Lambda^2}{m^2_h} + \hat{c}''_h m^2_{\text{soft}} \ln \frac{\Lambda^2}{m^2_{\text{soft}}} + \cdots,$$

   where $\hat{c}'_h$ and $\hat{c}''_h$ are functions of parameters, and $m_{\text{soft}}$ is a typical mass parameter representing the soft SUSY breaking. The magnitude of $m_{\text{soft}}$ is a same order of masses of superpartners for the SM particles. The absence of fine tuning requires roughly $m_{\text{soft}} \leq O(1)\text{TeV}$, and then the existence of superpartners concerning the SM particles is predicted at the terascale.

   2.2 Global symmetry. The Higgs boson can be a pseudo Nambu-Goldstone boson relating a spontaneous breakdown of a global symmetry [13]. The smallness and the stability of the Higgs boson mass and the smallness of Yukawa coupling constants arise from the nature of Nambu-Goldstone particle.

   2.3 Conformal symmetry [14]. The quantum conformal invariance in collaboration with finiteness, which is called “conformality”, can solve the problem. In the appearance of new particles at the terascale, the theory becomes scale invariant with the vanishing $\beta$ functions. Then, physical parameters do not run beyond the scale, and the concept of scale becomes vague.

   3. The concept of the third one is “extra dimensions”, i.e., there exists extra spatial dimensions other than 4-dimensional spacetime. We assume that there is a fundamental theory at the terascale with a fundamental mass parameter of $O(1)\text{TeV}$, concerning extra dimensions. The typical examples are models with large extra dimensions [15, 16].

   The combination of “extra dimensions” and “symmetry” produces a new solution, which is called “gauge-higgs unification” [17, 18]. The extra-dimensional component
\((A_y)\) of gauge field is massless at the tree level due to the gauge invariance, and receives a finite correction on its mass upon compactification [19]. By the identification of \(A_y\) with the Higgs boson, \(m_h\) becomes a natural parameter because gauge symmetry enhances on the higher-dimensional one in the limit of \(m_h \rightarrow 0\), and the weak scale is stabilized in the case with a large compactification scale of \(O(1)\) TeV\(^{-1}\).

The stabilization of the extra-dimensional space is crucial for the solution to the gauge hierarchy problem in theories on a higher-dimensional space-time, including the Randall-Sundrum model [20].

(4) There is a new physics at a higher energy scale \(M_I\) than the terascale, but the interaction with the SM is extremely weak. Or large radiative corrections are not induced, if the mixing among parameters of the physics at \(M_I\) and the SM is tiny enough such that \(c''_{hI} \leq O(m_h^2/M_I^2)\).

(5) The SM (or an extension of the SM with new particles around the terascale) holds up to a high energy scale \(M_U\), without a new concept to stabilize the Higgs boson mass at the terascale. We assume that a new physics appears at \(M_U\), which is described by an ultimate theory, the initial value of \(m_h\) is fixed by the new physics, and some mechanism and/or symmetry protects \(m_h^2\) against large radiative corrections [7, 21, 22].

There is a possibility that a new physics and/or concept is hidden behind the SM, too.

### 2.3 Masslessness and finiteness

Before we reexamine the gauge hierarchy problem from a different angle, we discuss its related topics on a basis of an ultimate theory.

First, we assume that the physics at \(M_U\) is described by an ultimate theory, which has “finiteness”, i.e., physical quantities are calculated as finite values. At a rough guess, the magnitude of quantities with mass dimension \(d\) is estimated as \(O(M_U^d)\), and natural initial conditions for masses of particles in low-energy physics would be given by

\[
m_i(M_U) = 0,
\]

as far as a mechanism to generate a tiny value does not work in the ultimate theory. We refer to the relations \(m_i(M_U) = 0\) as “masslessness”\(^1\). Non-zero masses and scales are expected to be dynamically generated by quantum effects, in the effective field theory. In other words, it might be a natural choice that every particle in a low-energy theory is massless at \(M_U\), the effective theory has the classical conformal symmetry in addition to chiral symmetry and gauge symmetry, and masses are induced after the breakdown of relevant symmetries by some dynamics. The typical examples are the Higgs mechanism in electroweak theory and the dimensional transmutation in quantum chromodynamics.

At this stage, the following questions arise. **What is the origin of masslessness and finiteness? How is masslessness protected against quantum effects?**

\(^1\) Masslessness could be related to the vanishment of bare Higgs boson mass around \(M_{Pl}\) [23].
We need to specify an ultimate theory, in order to answer the above questions. Here, we take string theory as a possible candidate. In string theory, the world-sheet conformal invariance induces the massless string states, and the world-sheet modular invariance guarantees finiteness of physical quantities. Hence, it can be said that the world-sheet modular invariance is responsible for the protection of masslessness against quantum corrections.

Concretely, from the world-sheet modular invariance for the closed string, the correction $\delta m^2_\phi$ (radiative corrections of the scalar mass squared including contributions from innumerable string states) should be given by

$$\delta m^2_\phi = \int_{\mathcal{F}} \frac{d^2 \tau}{\tau^2} G(\tau),$$

where $\tau = \tau_1 + i \tau_2$ is a modular parameter, $G(\tau)$ is a worldsheet modular invariant function, i.e., $G(\tau) = G(\tau + 1)$ and $G(\tau) = G(-1/\tau)$, and $\mathcal{F}$ stands for the fundamental region defined by $\mathcal{F} = \{ \tau : |\text{Re}\tau| \leq 1/2, 1 \leq |\tau| \}$. In cases where SUSY holds exactly, $G(\tau)$ vanishes, and then $\delta m^2_\phi = 0$. Even if SUSY is broken down, there is a possibility that $G(\tau)$ vanishes in conspiracy with infinite towers of massive particles, as suggested in Ref. [7].

From the viewpoint of effective field theories, symmetries relevant to naturalness such as chiral symmetry, gauge symmetry and conformal symmetry become useful tools for realistic model-building, that is, naturalness becomes a powerful guiding principle to construct an effective theory [10]. The relation of naturalness and conformal symmetry has been reexamined by Bardeen [25].

On the other hand, from the viewpoint of an ultimate theory, masslessness is more essential than naturalness or symmetries that make parameters natural. In other words, naturalness or the relevant symmetry is regarded as a secondary concept, originated from masslessness. Also, there is a possibility that an ultimate theory provides constraints on its effective theory. For instance, the ultimate theory possesses a duality like the world-sheet modular invariance. If this symmetry or its remnant could be imposed on the effective theory, only logarithmic divergent parts might be picked out and the Higgs boson mass could become a natural parameter, as discussed in [35].

For the gauge hierarchy problem, the fine tuning of order $(m_\phi/M_{\text{Pl}})^2$ is required for the scalar mass squared $m^2_\phi (\ll M^2_{\text{Pl}})$ from the viewpoint of effective field theories. In the ultimate theory, there must be symmetries such as world-sheet conformal symmetry, modular invariance and SUSY in string theory, to generate masslessness and protect it against radiative corrections of $O(M^2_{\text{Pl}})$. Hence, we expect that such fine tuning might be also an artifact in its effective field theory, and it could be improved if features of the ultimate theory are taken in and the ingredients of the effective theory are enriched.

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2 As a candidate of field theory version, the theories called finite unified theories, which have a large predictive power, have been proposed [24]. They are based on the finiteness and the principle of reduction of coupling constants.

3 Extensions of the SM have been proposed by adopting the classical conformal invariance as a guiding principle [26, 27, 28, 29, 30, 31, 32, 33, 34].
3 Supersymmetry and fermionic symmetry

Let us reconsider the technical side of the gauge hierarchy problem, based on the last possibility presented in Sect. 2.2, because it is plausible on the basis of recent experimental results at LHC. Evidences from SUSY, compositeness and extra dimensions have not yet been discovered, and a definite discrepancy has not yet been observed between the predictions in the SM (modified with massive neutrinos) and experimental results. These suggest that, even if new particles and/or new dynamics exist, those effects must be adequately suppressed.

Our consideration is based on the following assumptions, relating a physics beyond and/or behind the SM.

(a) There is an ultimate theory at a high-energy scale $M_U$, and it contains particles with masses of $O(M_U)$ and massless ones. The physical sector of massless particles is described by the SM (or an extension of the SM with new particles around the terascale). We denote it by $\text{SM} + \alpha$. This model holds up to $M_U$, and the Higgs boson is described as an elementary particle.

(b) There exists a new physics with a new concept $\mathcal{X}$ beyond and/or behind $\text{SM} + \alpha$, which is one of characteristics in the ultimate theory. The new physics can be formulated by an effective field theory possessing $\mathcal{X}$.

(c) The full effective theory consists of three parts, the part $X_{\text{heavy}}$ describing heavy particles with masses of $O(M_U)$, the part $X_{\text{light}}$ ($\supset \text{SM} + \alpha$) including light (or massless at $M_U$) particles that survive at a lower-energy scale, and the part $X_{\text{mix}}$ describing the interactions between particles in $X_{\text{high}}$ and those in $X_{\text{light}}$. The gauge hierarchy problem does not occur, and the physical sector can be described by an effective theory without $X_{\text{heavy}}$ and $X_{\text{mix}}$. This feature consists of the following ingredients.

(c1) The physical parameters in $\text{SM} + \alpha$ do not receive large quantum corrections, in the presence of $X_{\text{heavy}}$ and $X_{\text{mix}}$.

(c2) The concept $\mathcal{X}$ is preserved in the full effective theory, independent of the behavior of the particles in $\text{SM} + \alpha$.

In the following, we search for $\mathcal{X}$ to realize (a) – (c), and explore theories with $\mathcal{X}$ beyond and/or behind $\text{SM} + \alpha$.

3.1 Fragility of supersymmetry and gauge hierarchy problem

For the sake of completeness, we examine whether spacetime SUSY is suitable as $\mathcal{X}$ or not, by making clear the strong and weak points of SUSY, using a toy model.

Let us begin with the Lagrangian density given by

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{(0)} + \mathcal{L}_{(1,2)} + \mathcal{L}_{\text{mix}},$$

$$\mathcal{L}_{(0)} = \left| \partial_\mu \phi_0 \right|^2 + \overline{\psi}_0 \sigma^{\mu \nu} \partial_\mu \psi_0,$$

$$\mathcal{L}_{(1,2)} = \left| \partial_\mu \phi_1 \right|^2 - M^2 \left| \phi_1 \right|^2 + \left| \partial_\mu \phi_2 \right|^2 - M^2 \left| \phi_2 \right|^2 + \overline{\psi}_1 \sigma^{\mu \nu} \partial_\mu \psi_1 + \overline{\psi}_2 \sigma^{\mu \nu} \partial_\mu \psi_2 - M \psi_1 \psi_2 - M \overline{\psi}_1 \overline{\psi}_2,$$

$$\mathcal{L}_{\text{mix}} = - f^2 \left| \phi_0 \right|^2 \left( \left| \phi_1 \right|^2 + \left| \phi_2 \right|^2 \right) - f^2 \left| \phi_1 \right|^2 \left| \phi_2 \right|^2 - f M \left( \phi_0 + \phi_0^* \right) \left( \left| \phi_1 \right|^2 + \left| \phi_2 \right|^2 \right) - f \phi_0 \psi_1 \psi_2 - f \psi_0 \phi_1 \psi_2 - f \psi_0 \psi_1 \phi_2 - f \phi_0 \overline{\psi}_1 \overline{\psi}_2 - f \overline{\psi}_0 \overline{\psi}_1 \overline{\psi}_2 - f \overline{\psi}_0 \psi_1 \phi_2^* + \phi_1^* \phi_2^*.$$
where \( \phi_k \) and \( \psi_k \) \((k = 0, 1, 2)\) are complex scalar bosons and Weyl fermions, respectively, and parameters \( M \) and \( f \) are chosen as real, for simplicity. The SUSY invariance of \( \mathcal{L}_{\text{SUSY}} \) is understood from the rewritten version,

\[
\mathcal{L}_{\text{SUSY}} = \sum_k \left( |\partial_\mu \phi_k|^2 + \overline{\psi}_k i \sigma^\mu \partial_\mu \psi_k - |\partial_\mu W|^2 \right) - \left( \frac{1}{2} \sum_{k,l} \frac{\partial^2 W}{\partial \phi_k \partial \phi_l} \psi_k \psi_l + \text{h.c.} \right),
\]

where \( W = M\phi_1\phi_2 + f\phi_0\phi_1\phi_2 \) and h.c. represents the hermitian conjugate.

From (7) and (8), we find that, at the tree level, both \( \phi_0 \) and \( \psi_0 \) are massless, both \( \phi_1 \) and \( \phi_2 \) have a mass \( M \), and \( \psi_1 \) and \( \psi_2 \) form a Dirac fermion with a mass \( M \).

Let us suppose that the parts described by \( \mathcal{L}_{(0)} \), \( \mathcal{L}_{(1,2)} \) and \( \mathcal{L}_{\text{mix}} \) correspond to a SUSY extension of SM + \( \alpha \), \( X_{\text{heavy}} \) and \( X_{\text{mix}} \), respectively. Although the second term in \( \mathcal{L}_{\text{mix}} \) contains only heavy fields, we assume that it belongs to \( X_{\text{mix}} \) because it originates from the mixing of light and heavy fields in \( W \).

The non-renormalization theorem states that both \( M \) and \( f \) do not receive any radiative corrections perturbatively, and hence the mass spectrum remains unchanged and the hierarchical structure holds at the quantum level. This is the strong point of SUSY, and SUSY extensions of GUTs become candidates of a theory with \( X \) [36, 37].

However, SUSY has not yet been found in particle physics. Hence, if SUSY exists in nature, it is realized in a broken form. There are at least two possibilities to explain the current status. One is that superpartners of the SM particles exist, but they are too heavy to observe through the present collider experiments. Then, naturalness of \( m_h \) would be viewed with suspicion because of the necessity for a (mild) fine tuning, as superpartners become heavier [38, 39]. The other one is that there are no superpartners of the SM particles at all. Then, a weakness of SUSY would become apparent (as will be shown below) because of the missing of SUSY in the low-energy physics (after the reduction of SUSY occurs and the elimination of superpartners), even if it exists at an ultimate theory.

In the following, we hit a sensitive point of SUSY by answering the question what happens if superpartners are missing, considering the case that \( \psi_0 \) is eliminated in the toy model. Or we show that the parameters receive radiative corrections in the absence of \( \psi_0 \), and the mass hierarchy is spoiled. Concretely, at the one-loop level, the mass squared of \( \phi_0 \), \( m_{\phi_0}^2 \), does not receive any radiative corrections. This is due to the fact that \( \phi_0 \) couples with heavy fields in a SUSY invariant form. The mass squareds of \( \phi_1 \) and \( \phi_2 \) receive the radiative corrections of \( O(M^2) \), even if quadratic divergences are removed. On the other hand, \( \psi_1 \) and \( \psi_2 \) do not receive mass corrections at the one-loop level, and the mass degeneracy of heavy fields is lost. Hence \( m_{\psi_0}^2 \) receives large radiative corrections of \( O(M^2) \) at the two-loop level, and the mass hierarchy is destroyed.

Although we have considered the toy model where the system with \( \mathcal{L}_{(0)}' = |\partial_\mu \phi_0|^2 \) corresponds to SM + \( \alpha \), it is easily understood that a similar thing happens in any SUSY extensions of the SM and the gauge hierarchy problem occurs [4]. In this way, SUSY makes a strong showing in the presence of superpartners, but it is fragile if missing.

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4 In SUSY extensions of the SM, quadratic divergences appear in radiative corrections on scalar masses in the absence of (part of) superpartners, even if we neglect the contributions from heavy particles. To avoid such a complication and extract effects of heavy particles, the toy model is considered.
We rethink what happened, from the viewpoint of heavy particles. Let light fields introduce without their superpartners into a SUSY invariant system including heavy fields. The structure of SUSY invariant system is broken down through the coupling to the system without superpartners, and it induces large radiative corrections on masses of light scalar fields. This is a root or might be an essence of the gauge hierarchy problem. In other words, the gauge hierarchy problem can be restated as "without upsetting the structure of a high-energy physics under cover of an excellent concept, is it possible to formulate a low-energy effective theory and the interaction with heavy particles?"

3.2 Fermionic symmetry with a charmed life

First, we speculate a theory whose structure is stabilized by some symmetry \( \mathcal{X} \). The strong point of SUSY provides a useful hint. It is that the cancellation on radiative corrections works very well due to contributions from particles with different statistics, that form supermultiplets, if SUSY holds exactly.

The spacetime SUSY pairs every particle with its superpartner whose helicity is one-half different from, because SUSY charges \( (Q_\alpha, \bar{Q}_{\dot{\alpha}}) \) have helicity \( \pm 1/2 \) and satisfy the relation \( \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^{\mu}_{\alpha\dot{\alpha}} P_\mu \). Here, we consider \( N = 1 \) SUSY for simplicity. Note that \( Q_\alpha \) and \( \bar{Q}_{\dot{\alpha}} \)-singlets satisfying \( Q_\alpha \psi(x) = 0 \) and \( \bar{Q}_{\dot{\alpha}} \psi(x) = 0 \) are not allowed because of \( P_\mu \psi(x) = i\partial_\mu \psi(x) \neq 0 \). This feature leads to a weak point of SUSY in case that some superpartners are missing.

From this observation, we anticipate that something quite interesting can happen, if there is a symmetry such that the SM particles are singlets and heavy particles form doublets under the transformation group, and quantum corrections from each component in the doublet are canceled out each other. Then, a possible candidate of \( \mathcal{X} \) is a fermionic symmetry that transforms an ordinary particle into its ghost partner. Here, an ordinary particle means a particle that obeys the spin-statistics theorem. The ghost partner has same spacetime and internal quantum numbers with the corresponding ordinary particle, but yields a different statistics.

We explore theories with ghost fields, in the expectation that the fermionic symmetry concerning ghost fields can play the vital role to stabilize the Higgs boson mass, although it is hidden behind the SM.

Let us consider a toy model with a complex scalar particle \( \phi \) with a light mass \( m_\phi \) and a pair of complex scalar particles \( (\phi, c_\phi) \) with a heavy mass \( M_\phi \). Here, \( \phi \) is an ordinary scalar field yielding the commutation relations and \( c_\phi \) is its ghost partner yielding the anti-commutation relations. If both \( \phi \) and \( c_\phi \) interacts with \( \phi \) in the same way, radiative corrections on \( m_\phi \) due to heavy fields would vanish because of the cancellation between contributions from \( \phi \) and \( c_\phi \). Note that the extra minus sign appears for the virtual ghost field running in the loop. Furthermore, the mass of \( c_\phi \) would receive the same size of radiative corrections with that of \( \phi \) through the interactions with \( \phi \). Hence we expect that the mass hierarchy is stabilized, unless the fermionic symmetry is broken at the quantum level.

Next, we embody our speculation, using the Lagrangian density,

\[
\mathcal{L}_1 = \mathcal{L}_\phi + \mathcal{L}_{\phi, c} + \mathcal{L}_{\text{mix}},
\]
\[ \mathcal{L}_\phi = \partial_\mu \phi^\dagger \partial^\mu \phi - m_\phi^2 \phi^\dagger \phi - \lambda_\phi \left( \phi^\dagger \phi \right)^2, \]  
(12)  
\[ \mathcal{L}_{\phi,c} = \partial_\mu \phi^\dagger \partial^\mu \phi + \partial_\mu c_\phi^{\dagger} \partial^\mu c_\phi - M_\phi^2 \left( \phi^\dagger \phi + c_\phi^{\dagger} c_\phi \right) \]  
- \lambda_\phi \left( \phi^\dagger \phi + c_\phi^{\dagger} c_\phi \right) \star \left( \phi^\dagger \phi + c_\phi^{\dagger} c_\phi \right), \]  
(13)  
\[ \mathcal{L}_{\text{mix}} = -\lambda' \phi^\dagger \phi \left( \phi^\dagger \phi + c_\phi^{\dagger} c_\phi \right), \]  
(14)

where \( \lambda_\phi \) is the quartic self-coupling constant of \( \phi \), \( \lambda_\phi \) and \( \lambda' \) are other quartic coupling constants, and the star product (\( \star \)) represents a non-local interaction. The self-interactions of heavy fields are indispensable, because they are induced radiatively via the couplings between light and heavy fields. Features of interaction terms are given in the Appendix A. The parts with \( \mathcal{L}_\phi \), \( \mathcal{L}_{\phi,c} \) and \( \mathcal{L}_{\text{mix}} \) correspond to SM + \( \alpha \), \( X_{\text{heavy}} \) and \( X_{\text{mix}} \), respectively.

Here, we outline radiative corrections on parameters. Details are presented in the Appendix A. At the one-loop level, the mass squared of \( \phi \) does not receive any radiative corrections from heavy fields, because the contributions from \( \phi \) and \( c_\phi \) are exactly canceled out. On the other hand, the parameters \( M_\phi, \lambda_\phi \) and \( \lambda' \) receive radiative corrections through \( \mathcal{L}_{\text{mix}} \) and the interactions in \( \mathcal{L}_{\phi,c} \). If both \( \phi \) and \( c_\phi \) receive exactly the same size of contributions, the structure of \( \mathcal{L}_{\phi,c} \) and \( \mathcal{L}_{\text{mix}} \) remain unchanged. This can be shown at the one-loop level if interaction terms satisfy some features. If the stabilization of \( \mathcal{L}_{\phi,c} \) and \( \mathcal{L}_{\text{mix}} \) hold at the all order of perturbation and the quadratic divergence in the mass squared of \( \phi \), originated from the self-interaction of \( \phi \), is subtracted, the system is free from fine tuning. Then, the mass hierarchy can be stabilized against quantum corrections.

Now, we pursue a characteristics \( \mathcal{X} \) to stabilize the theory. From (13) and (14), we guess that the quadratic form \( \mathcal{I} = \phi^\dagger \phi + c_\phi^{\dagger} c_\phi \) is a key and a symmetry relating transformations which leave \( \mathcal{I} \) invariant can be \( \mathcal{X} \). It is equivalent to OSp(2|2)\footnote{The OSp(2|2) is the group whose elements are generators of transformations (corresponding (a), (b) and (d)) which leave the inner product of two vectors such as \( x_1 x_2 + y_1 y_2 + (\theta_1 \overline{\theta}_2 - \overline{\theta}_1 \theta_2)/2 \) invariant, where \( x_i \) and \( y_i \) (\( i = 1, 2 \)) are bosonic variables, and \( \theta_i \) and \( \overline{\theta}_i \) are fermionic ones. Note that the inner product is given by \( x^2 + y^2 + \partial \overline{\partial} (= |z|^2 + \partial \overline{\partial}) \) for a same vector, where \( z = x + i y \).}.

The transformations are classified into following four types.

(a) \text{U(1) transformation relating a particle number}:

\[ \delta_o \phi = i \epsilon_o \phi , \quad \delta_o \phi^\dagger = -i \epsilon_o \phi^\dagger , \quad \delta_o c_\phi = 0 , \quad \delta_o c_\phi^{\dagger} = 0 , \]  
(15)

where \( \epsilon_o \) is an infinitesimal real number.

(b) \text{U(1) transformation relating a ghost number}:

\[ \delta_g \phi = 0 , \quad \delta_g \phi^\dagger = 0 , \quad \delta_g c_\phi = i \epsilon_g c_\phi , \quad \delta_g c_\phi^{\dagger} = -i \epsilon_g c_\phi^{\dagger} , \]  
(16)

where \( \epsilon_g \) is an infinitesimal real number.

(c) Transformation that \( c_\phi \) changes into \( c_\phi^{\dagger} \) and its hermitian conjugation:

\[ \delta_c \phi = 0 , \quad \delta_c \phi^\dagger = 0 , \quad \delta_c c_\phi = \epsilon_c c_\phi^{\dagger} , \quad \delta_c c_\phi^{\dagger} = 0 , \]  
(17)
\[ \delta^+_\varphi = 0, \quad \delta^+_c \varphi = 0, \quad \delta^+_c c_\varphi = 0, \quad \delta^+_c c_\varphi = \epsilon^+_c c_\varphi, \] (18)

where \( \epsilon^+_c \) and \( \epsilon^+_c \) are Grassman numbers with ghost number 2 and \(-2\), respectively.

(d) Fermionic transformations:

\[
\begin{align*}
\delta_F \varphi &= -\zeta c_\varphi, \quad \delta_F \varphi^+ = 0, \quad \delta_F c_\varphi = 0, \quad \delta_F c_\varphi^+ = \zeta \varphi^+, \\
\delta_F^+ \varphi &= 0, \quad \delta_F^+ \varphi^+ = \zeta^+ c_\varphi^+, \quad \delta_F^+ c_\varphi^+ = \zeta^+ \varphi^+, \quad \delta_F^+ c_\varphi = 0, \\
\end{align*}
\] (19)

\[
\begin{align*}
\delta^+_F \varphi &= 0, \quad \delta^+_F \varphi^+ = \zeta^+ c_\varphi^+, \quad \delta^+_F c_\varphi^+ = \zeta^+ \varphi^+, \quad \delta^+_F c_\varphi = 0, \\
\end{align*}
\] (20)

where \( \zeta \) and \( \zeta^+ \) are Grassman numbers with ghost number \(-1\) and \(1\), respectively. Note that \( \delta_F \) is not generated by a hermitian operator, different from the generator of the BRST transformation in systems with first class constraints [40] and that of the topological symmetry [41][42].

From the above transformation properties, we see that \( \delta_c, \delta^+_c, \delta_F \) and \( \delta^+_F \) are nilpotent, i.e., \( \delta^2_c = 0, \delta^2_\varphi = 0, \delta^2_F = 0 \) and \( \delta^2_F = 0 \), where \( \delta_c, \delta^+_c, \delta_F \) and \( \delta^+_F \) are defined by \( \delta_c = \epsilon_c \delta_c, \delta^+_c = \epsilon^+_c \delta^+_c, \delta_F = \zeta \delta_F \) and \( \delta^+_F = \zeta^+ \delta^+_F \), respectively. Furthermore, we find the algebraic relations,

\[
\{Q_c, Q^+_c\} = Q_g, \quad \{Q_F, Q^+_F\} = Q_o + Q_g \equiv N_D, \quad (21)
\]

where \( Q_o, Q_g, Q_c, Q^+_c, Q_F \) and \( Q^+_F \) are the corresponding generators (charges) given by

\[
\begin{align*}
\delta_o \varphi &= i [\epsilon_o Q_o, \varphi], \quad \delta_g \varphi = i [\epsilon_g Q_g, \varphi], \quad \delta_c \varphi = i [\epsilon_c Q_c, \varphi], \\
\delta^+_c \varphi &= i [\epsilon^+_c Q^+_c, \varphi], \quad \delta_F \varphi = i [\zeta Q_F, \varphi], \quad \delta^+_F \varphi = i [Q^+_F \zeta, \varphi].
\end{align*}
\] (22)

It is easily understood that \( \varphi^+_\varphi + c^+_\varphi c_\varphi \) is invariant under the transformations \( \delta_F \) and \( \delta_F^+ \) and the relations,

\[
\begin{align*}
\varphi^+_\varphi + c^+_\varphi c_\varphi &= \delta_F \left( c^+_\varphi \varphi \right) = \delta_F^+ \left( \varphi^+_\varphi \right) = \delta_F^+ \left( \varphi^+_c \varphi \right) = \delta_F^+ \left( \varphi^+_c c_\varphi \right) = -\delta^+_F \delta_F \left( \varphi^+_\varphi \right) \\
&= -\delta^+_F \delta_F \left( c^+_\varphi c_\varphi \right) = \delta^+_F \delta_F \left( c^+_\varphi c_\varphi \right). \\
\end{align*}
\] (23)

Using them, the Lagrangian density \( \mathcal{L}_T \) is rewritten as

\[
\mathcal{L}_T = \mathcal{L}_\varphi + \delta_F \delta_F^+ \left[ \partial_\mu \varphi^+ \partial^\mu \varphi - M^2 \varphi^+_\varphi \varphi - \frac{\lambda^\varphi}{2} \left( \varphi^+_\varphi \varphi^+_\varphi \varphi + c^+_\varphi c_\varphi c^+_\varphi c_\varphi \right) \right].
\] (24)

The theory is specified by the fermionic charges \( Q_F \) and \( Q^+_F \) and the number operator \( N_D \) of the doublet \( (\varphi, c_\varphi) \) with the relation \( \{Q_F, Q^+_F\} = N_D \). In the case that \( \varphi \) is invariant under \( OSp(2|2) \) transformation, i.e., \( \varphi \) is \( Q_F \) and \( Q^+_F \)-singlet satisfying \( \delta_F \varphi = 0 \) and \( \delta^+_F \varphi = 0 \), the full system described by \( \mathcal{L}_T \) has \( OSp(2|2) \) invariance. Note that \( Q_F \) and \( Q^+_F \)-singlets are allowed, because \( N_D \) is irrelevant to spacetime symmetries, different from the case of spacetime SUSY.
To formulate our model in a consistent manner, we use a feature that a conserved charge can, in general, be set to be zero as an auxiliary condition. We impose the following subsidiary conditions on states to select physical states $|\text{phys}\rangle$ can be selected\(^6\)

\[
Q_\phi|\text{phys}\rangle = 0, \quad Q_{\phi}^\dagger|\text{phys}\rangle = 0, \quad N_D|\text{phys}\rangle = 0,
\]

and then heavy fields $\phi$ and $c_\phi$ are expected to be unphysical and not to give any quantum effects on the light field $\phi$. This is regarded as a field theoretical version of the Parisi-Sourlas mechanism\(^{45}\). Hence, there is a possibility that $\phi$ and $c_\phi$ are not dangerous for the fermionic symmetries with a charmed life.

If we take $\phi^\dagger \phi \phi^\dagger \phi$ in place of $\phi^\dagger \phi \ast \phi^\dagger \phi$ in \((24)\), the self-interactions of $Q_\phi$-doublet $\lambda_{\phi} \left( \phi^\dagger \phi + c_\phi^\dagger \phi \right) \ast \left( \phi^\dagger \phi + c_\phi^\dagger \phi \right)$ in $\mathcal{L}_{\phi,c}$ is replaced by

\[
\lambda_{\phi} \left( \phi^\dagger \phi \phi^\dagger \phi + \phi^\dagger \phi c_\phi^\dagger \phi + c_\phi^\dagger \phi \phi^\dagger \phi + \phi^\dagger \phi \ast c_\phi^\dagger \phi \ast c_\phi^\dagger \phi \ast c_\phi^\dagger \phi \right). \tag{26}
\]

Hereafter, we do not consider self-interactions containing both local and non-local ones such as \((26)\), because the form of these interactions could not be stable against radiative corrections in the framework of effective field theory.

In the following, we consider theories with fermionic symmetries (whose generators are denoted by $Q_\phi$ and $Q_{\phi}^\dagger$) that relate particles to their ghost partners and the bosonic symmetry relating the number operator $N_D$ of the doublets (pairs of particles and their ghost partners) with the relation $\{Q_\phi, Q_{\phi}^\dagger\} = N_D$, and assume that those symmetries are not broken down at the quantum level, and ghost fields are unphysical and harmless. Then we arrive at the conjecture that the gauge hierarchy problem does not occur and the physical low-energy theory can be described by SM $+$ $\alpha$, if a full effective theory has fermionic symmetries with an eternal life, the SM particles and some extra light fields are $Q_\phi$-singlets and others including heavy fields are $Q_\phi$-doublets. Furthermore, a theory beyond and behind the SM is expected to be expressed as

\[
\mathcal{L}_{\text{BSM}} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}} + \mathcal{L}_{\text{mix}}, \quad \mathcal{L}_{\text{light}} = \mathcal{L}_{\text{SM}+\alpha} + \delta_\phi \delta_{\phi}^\dagger \Delta \mathcal{L}, \tag{27}
\]

where $\mathcal{L}_{\text{light}}$, $\mathcal{L}_{\text{heavy}}$ and $\mathcal{L}_{\text{mix}}$ stand for the Lagrangian densities of the parts $X_{\text{light}}$, $X_{\text{heavy}}$ and $X_{\text{mix}}$, respectively. The $\mathcal{L}_{\text{SM}+\alpha}$ represents the Lagrangian densities of SM $+$ $\alpha$, and $\delta_\phi \delta_{\phi}^\dagger \Delta \mathcal{L}$ contains light $Q_\phi$-doublet fields. Both $\mathcal{L}_{\text{heavy}}$ and $\mathcal{L}_{\text{mix}}$ are also written in the $\delta_\phi$-exact form, for instance,

\[
\mathcal{L}_{\text{heavy}} = \delta_\phi \left[ \sum_k \left( c_{Lk}^\dagger \sigma^\mu D_\mu \psi_{Lk} + c_{Rk}^\dagger i \sigma^\mu D_\mu \psi_{Rk} - M_k c_{Lk}^\dagger \psi_{Rk} - M_k c_{Rk}^\dagger \psi_{Lk} \right) + \sum_l \left( D^\mu c_l \right) \left( D_\mu \varphi_l \right) - M_l^2 c_l^\dagger \varphi_l + \cdots \right].
\]

\(^6\) The conditions \((25)\) are interpreted as counterparts of the Kugo-Ojima subsidiary condition in BRST quantization\(^{43}\). It is shown that the system containing both free ordinary fields and their ghost partners is quantized consistently, though it becomes empty leaving the vacuum state alone\(^{44}\).
\[
\delta F = \delta F \left[ \sum_k \left( \psi_{Lk}^\dagger i\sigma^\mu D_\mu \psi_{Lk} + \psi_{Rk}^\dagger i\sigma^\mu D_\mu \psi_{Rk} - M_k \psi_{Lk}^\dagger \psi_{Rk} - M_k \psi_{Rk}^\dagger \psi_{Lk} \right) \\
+ \sum_l \left( (D_\mu \varphi_l)^\dagger (D_\mu \varphi_l) - M_l^2 \varphi_l^\dagger \varphi_l \right) + \cdots \right],
\]

\[
\mathcal{L}_{\text{mix}} = \delta F \left[ -\sum_l \lambda_l H^\dagger H \varphi_l^\dagger \varphi_l + \cdots \right] = \delta F \left[ -\sum_l \lambda_l H^\dagger H \varphi_l^\dagger \varphi_l + \cdots \right],
\]

where \((\psi_{Lk}, c_{Lk})\) and \((\psi_{Rk}, c_{Rk})\) are heavy Weyl spinor \(Q_F\)-doublets, and \((\varphi_l, \bar{c}_l)\) are complex scalar \(Q_F\)-doublets, and \(H\) is the Higgs boson in the SM that is a \(Q_F\)-singlet. Note that both \(\mathcal{L}_{\text{heavy}}\) and \(\mathcal{L}_{\text{mix}}\) are also expressed in the \(\delta F\)-exact form.

More exotic fermionic symmetries might be needed to construct a realistic model. Here and hereafter, we use the symmetries concerning \((Q_F, Q_F^\dagger, N_D)\) as an illustrative example, because we do not know an underlying symmetry that an ultimate theory possesses.

### 3.3 Grand unification and fermionic symmetry

First, let us start with a conjecture relating an ultimate theory. The ultimate theory must explain the birth of our physical world as follows. *Our world comes into existence from “nothing”*. Here, *nothing* means not an empty but a world whose constituents are unphysical objects and/or fundamental objects including only gauge bosons (and their superpartners, i.e., gauginos), that form multiplets of a large gauge symmetry. “Beings” including matter fields are generated at \(M_U\), after reducing the large symmetry into a smaller one, by some mechanism. The constituents after the reduction are massless particles and massive unphysical ones. The massless particles contain GUT multiplets and incomplete ones. Parts of the GUT multiplets become unphysical in collaboration with ghost partners belonging to incomplete ones. After all, the SM particles and extra particles survive as physical ones or “beings”, in a lower-energy world.\(^7\) Note that, in our scenario, extra components of GUT multiplets can remain massless and decouple to the SM particles because they become unphysical, which is different from the ordinary case that they decouple to the SM particles because they become heavy on the breakdown of GUT symmetry.

Next, we discuss the verifiability and predictions of our conjecture. Although unphysical particles do not give any dynamical effects on the physical sector, there are at least two predictions, which can be indirect proofs of unphysical sector. First one is that physical quantities calculated in the SM + \(\alpha\) should precisely match with the experimental values at the terascale, up to any gravitational effects, because radiative corrections from unphysical particles are canceled out. Second one is that parameters in the SM + \(\alpha\) should satisfy specific relations at \(M_U\), reflecting on a large symmetry realized in the ultimate theory.

In the following subsections, we explain that the second prediction can be realized, in the appearance of new particles \((Q_F\text{-singlets})\) around the terascale and light \(Q_F\text{-doublets},\)

\(^7\) Based on this conjecture, a toy model has been proposed that physical modes originate from unobservable fields.\(^{46}\)
under a situation with following features.
(i) An ultimate theory has a large gauge symmetry potentially. Gauge bosons originate from an object such as $D$-brane in string theory.
(ii) Other particles including matter fields appear with changing the structure of space-time and/or object at a high-energy scale $M_U$. All massive fields form $Q_F$-doublets and become unphysical. Massless fields consist of ordinary fields and ghost fields. Most ordinary fields including the gauge bosons form multiplets of a gauge group $G_o$, other ordinary fields form multiplets of a smaller gauge group $G'_o$ and ghost fields form multiplets of a gauge group $G_g$. The gauge symmetry of the system increases from $G'_o$ or $G_g$ to $G_o$, if other ordinary fields and massless ghost fields were removed, i.e., $G_o \supset G'_o, G_g$.
(iii) The system survives in a consistent manner, thanks to fermionic symmetries. The fermionic symmetries are unbroken at the quantum level, and all ghost fields are unphysical and harmless.

3.3.1 Symmetry reduction with ghost administration

Let us demonstrate that the gauge symmetry is reduced in the appearance of incomplete multiplets at $M_U$, using toy models with the $SU(2)$ Yang-Mills field.

We assume that the ultimate theory possesses many solutions corresponding universe such as the string landscape and some solutions contain ghost fields. Their low-energy effective field theories are constructed from the massless spectra. In the following, we write down the Lagrangian densities with fermionic symmetries if ghost fields exist, in several cases for a given set of massless particles.

First we consider an ordinary case that there are no ghost fields.

(A) Case with $Q_F$ singlets matter fields
Let the set $(A^{a \mu}_\mu, \phi, \psi_L, \psi_R)$ be given as the massless ones. Here, $A^{a \mu}_\mu$ are $SU(2)$ gauge bosons ($a = 1, 2, 3$), $\phi = (\phi^1, \phi^2)^T$ is a scalar field of $SU(2)$ doublet (the superscript $T$ represents the operation of transposition), $\psi_L = (\psi^{1L}_L, \psi^{2L}_L)^T$ and $\psi_R = (\psi^{1R}_R, \psi^{2R}_R)^T$ are left-handed and right-handed chiral fermions of $SU(2)$ doublets, respectively. From the $SU(2)$ gauge invariance, the Lagrangian density is given by

$$\mathcal{L}_{SU(2)}^{(A)} = -\frac{1}{4} F^{a \mu \nu} F_{a \mu \nu} + \mathcal{L}_m, \quad (30)$$

$$\mathcal{L}_m = (D_{\mu} \phi)^\dagger (D^\mu \phi) + \psi_L^\dagger \sigma^\mu D_{\mu} \psi_L + \psi_R^\dagger i \sigma^\mu D_{\mu} \psi_R, \quad (31)$$

where $F^{a \mu \nu} = \partial_{\mu} A^{a \nu}_\nu - \partial_{\nu} A^{a \mu}_\mu - g \epsilon^{abc} A^{b}_\mu A^{c}_\nu$, $D_{\mu} = \partial_{\mu} + ig A^a_{\mu} \tau^a/2$, and $g$ is a gauge coupling constant, $\tau^a$ are Pauli matrices. For simplicity, we omit interactions other than gauge interactions. The system is described by an ordinary $SU(2)$ Yang-Mills theory with a complex scalar field and two Weyl spinors (a Dirac spinor).

Next we consider the extremal case that all matter fields company with their ghost partners.

(B) Case with $Q_F$ doublets matter fields
Let the set $(A^{a \mu}_\mu; \phi, c_\phi; \psi_L, c_L; \psi_R, c_R)$ be given as the massless ones. Here, $c_\phi$ is the ghost partner of $\phi$, and $c_L$ and $c_R$ are the ghost partners of $\psi_L$ and $\psi_R$, respectively. To formulate a theory consistently, we require the $SU(2)$ gauge invariance and the invariance...
under the fermionic transformations,
\[
\delta_F \phi = -c_\phi, \quad \delta_F \phi^\dagger = 0, \quad \delta_F c_\phi = 0, \quad \delta_F c_\phi^\dagger = \phi^\dagger, \quad \delta_F \psi^\dagger_L = c_L, \quad \delta_F \psi^\dagger_R = 0,
\]
\[
\delta_F c_L = 0, \quad \delta_F c_L^\dagger = \psi^\dagger_L, \quad \delta_F c_R = c_R, \quad \delta_F \psi^\dagger_R = 0, \quad \delta_F c_R^\dagger = \psi^\dagger_R, \quad \delta_F A^a_\mu = 0 \tag{32}
\]
and
\[
\delta_F \phi^\dagger = 0, \quad \delta_F \phi^\dagger = c_\phi^\dagger, \quad \delta_F c_\phi = \phi, \quad \delta_F c_\phi^\dagger = 0, \quad \delta_F \psi^\dagger_L = 0, \quad \delta_F \psi^\dagger_R = c_L^\dagger, \quad \delta_F c_L = -\psi_L, \quad \delta_F c_L^\dagger = 0, \quad \delta_F \psi^\dagger_R = c_R, \quad \delta_F c_R^\dagger = -\psi_R, \quad \delta_F c_R^\dagger = 0, \quad \delta_F A^a_\mu = 0 \tag{33}
\]
Then, we obtain the Lagrangian density
\[
\mathcal{L}^{(B)}_{SU(2)} = -\frac{1}{4} F^{a\mu\nu}_{\mu\nu} + \mathcal{L}_m + \mathcal{L}^{(B)}_{gh} = \frac{1}{4} F^{a\mu\nu}_{\mu\nu} + \delta_F \delta^{(B)}_{\mu\nu} \mathcal{L}^{(B)}_{m}, \tag{34}
\]
\[
\mathcal{L}^{(B)}_{gh} = (D_\mu c_\phi^\dagger)(D^\mu c_\phi) + c_L^\dagger \sigma^{\mu\nu} D_\mu c_L + c_R^\dagger \sigma^{\mu\nu} D_\mu c_R. \tag{35}
\]
For simplicity, we omit interactions other than gauge interactions. The system is essentially identical to that described by the pure SU(2) Yang-Mills theory, because $Q_F$ doublets are unphysical under the subsidiary conditions [25].

Here, we give a comment on a SUSY extension of the system. Let the set $(A^a_\mu, \lambda^a, c^a)$ be given as the massless ones. Here, $\lambda^a$ are SU(2) gauginos and $c^a$ are their ghost partners. We obtain the Lagrangian density
\[
\mathcal{L}^{(B')}_{SU(2)} = -\frac{1}{4} F^{a\mu\nu}_{\mu\nu} + \frac{1}{2} \lambda^a \gamma^\mu (D_\mu \lambda)^a + \frac{1}{2} c^a \gamma^\mu (D_\mu c)^a
\]
\[
-\frac{1}{4} F^{a\mu\nu}_{\mu\nu} + \delta_F \delta^{(B')}_{\mu\nu} \left( \frac{1}{2} \lambda^a \gamma^\mu (D_\mu \lambda)^a \right). \tag{36}
\]
This system is also identical to that described by the pure SU(2) Yang-Mills theory, under the subsidiary conditions [25].

Finally, we consider an exotic case such that incomplete ghost fields exist.

(C) Case with incomplete $Q_F$ singlets matter fields
Let us obtain the set of particles $A^a_\mu, \phi, \psi_L, \psi_R$ and the ghost fields which do not form SU(2) multiplets such as $c^1, c^2, c^3_\Phi, c^4_L$ and $c^4_R$, as the massless ones. The gauge quantum numbers of ghost fields are same as those of $A^+\mu = (A^1_\mu - i A^2_\mu) / \sqrt{2}, A^-\mu = (A^1_\mu + i A^2_\mu) / \sqrt{2}, \phi^1, \psi^1_L$, and $\psi^1_R$, respectively, but they obey statistics different from ordinary counterparts. To formulate a theory, we require the $U(1)$ gauge invariance and the invariance under the fermionic transformations,
\[
\delta_F \phi = -c^1_\phi, \quad \delta_F \phi^\dagger = 0, \quad \delta_F c^1_\phi = 0, \quad \delta_F c^1_\phi^\dagger = \phi^\dagger, \quad \delta_F \psi^\dagger_L = c^1_L, \quad \delta_F \psi^\dagger_R = 0,
\]
\[
\delta_F c^1_L = 0, \quad \delta_F c^1_L^\dagger = \psi^\dagger_L, \quad \delta_F c^1_R = c^1_R, \quad \delta_F c^1_R^\dagger = 0, \quad \delta_F c^1_R^\dagger = \psi^\dagger_R, \quad \delta_F A^+_\mu = -c^1_\mu, \quad \delta_F A^-_\mu = 0, \quad \delta_F c^1_\mu = 0, \quad \delta_F c^1_\mu^\dagger = \phi^\dagger, \quad \delta_F \psi^2_L = 0, \quad \delta_F \psi^2_R = 0, \quad \delta_F A^3_\mu = 0 \tag{37}
\]
and
\[
\delta_F \phi = 0, \quad \delta_F \phi^\dagger = c^1_\phi, \quad \delta_F c^1_\phi^\dagger = 0, \quad \delta_F c^1_\phi = \phi^\dagger, \quad \delta_F \psi^\dagger_L = 0, \quad \delta_F \psi^\dagger_R = 0, \quad \delta_F c^1_L = 0.
\]
\[
\begin{align*}
\delta^{\dagger} c_{L}^{1} &= -\psi_{L}^{1}, \quad \delta^{\dagger} c_{L}^{1\dagger} = 0, \quad \delta^{\dagger} \psi_{R}^{1} = 0, \quad \delta^{\dagger} \psi_{R}^{1\dagger} = c_{R}^{1}, \quad \delta^{\dagger} c_{R}^{1} = -\psi_{R}^{1}, \quad \delta^{\dagger} c_{R}^{1\dagger} = 0, \\
\delta^{\dagger} c_{F}^{A^{+}} = 0, \quad \delta^{\dagger} c_{F}^{A^{-}} = c_{F}^{A^{-}}, \quad \delta^{\dagger} c_{F}^{+} = c_{F}^{+}, \quad \delta^{\dagger} c_{F}^{-} = 0, \quad \delta^{\dagger} \phi_{R}^{2} = 0, \quad \delta^{\dagger} \phi_{R}^{2\dagger} = 0, \\
\delta^{\dagger} \psi_{L}^{2} = 0, \quad \delta^{\dagger} \psi_{L}^{2\dagger} = 0, \quad \delta^{\dagger} \psi_{R}^{2} = 0, \quad \delta^{\dagger} \psi_{R}^{2\dagger} = 0, \quad \delta^{\dagger} A_{\mu}^{3} = 0.
\end{align*}
\]

(38)

Then, we obtain the Lagrangian density,
\[
\mathcal{L}_{SU(2)}^{(C)} = -\frac{1}{4} [F_{\mu\nu}^{a} F^{a\mu\nu}]_{*} + \mathcal{L}_{m} + \mathcal{L}_{gh}^{(C)} + \mathcal{L}_{\text{int}}^{(C)},
\]

(39)

where \([F_{\mu\nu}^{a} F^{a\mu\nu}]_{*}\) is the gauge kinetic term that \((g^{2}/4)(A_{\mu}^{+} A_{\mu}^{-} - A_{\mu}^{-} A_{\mu}^{+}) (A^{-\nu} A^{\mu} - A^{\mu} A^{-\nu})\) is replaced by the non-local one \((g^{2}/4)(A_{\mu}^{+} A_{\mu}^{-} - A_{\mu}^{-} A_{\mu}^{+}) \ast (A^{-\nu} A^{\mu} - A^{\mu} A^{-\nu})\) in \(F_{\mu\nu}^{a} F^{a\mu\nu}\), and \(\mathcal{L}_{gh}^{(C)}\) and \(\mathcal{L}_{\text{int}}^{(C)}\) are given by

\[
\begin{align*}
\mathcal{L}_{gh}^{(C)} &= -(D_{\mu}^{C} c_{\nu}^{A^{+}}) (D^{\mu} c_{\nu}^{A^{-}}) + (D_{\mu}^{C} c_{\nu}^{A^{-}}) (D^{\mu} c_{\nu}^{A^{+}}) + (D_{\mu}^{C} \phi_{R}^{2}) (D^{\mu} \phi_{R}^{2}) \\
&+ c_{L}^{1} \overline{\sigma}^{\mu} D_{\mu}^{C} c_{L}^{1} + c_{R}^{1} i \sigma^{\mu} D_{\mu}^{C} c_{R}^{1},
\end{align*}
\]

(40)

\[
\begin{align*}
\mathcal{L}_{\text{int}}^{(C)} &= -\frac{i g}{2} \left( \partial_{\nu} A_{\mu}^{3} - \partial_{\mu} A_{\nu}^{3} \right) \left( C_{\nu} C_{\mu}^{+} - C_{\mu} C_{\nu}^{+} \right) \\
&+ \frac{g^{2}}{2} \left( A_{\nu}^{+} A_{\mu}^{-} - A_{\mu}^{+} A_{\nu}^{-} \right) \ast \left( C_{\nu} C_{\mu}^{+} - C_{\mu} C_{\nu}^{+} \right) \\
&+ \frac{g^{2}}{2} \left( C_{\nu}^{+} C_{\mu}^{+} - C_{\mu}^{+} C_{\nu}^{+} \right) \ast \left( C_{\nu}^{+} C_{\mu}^{+} - C_{\mu}^{+} C_{\nu}^{+} \right) \\
&+ \frac{g^{2}}{2} \left( -\phi_{R}^{2} \phi_{R}^{2} + \phi_{R}^{2} \phi_{R}^{2} \right) \\
&+ \frac{g^{2}}{2} c_{\phi}^{1} \left( A_{\mu}^{+} - C_{\mu}^{+} \right) c_{\phi}^{1} + \frac{i g}{\sqrt{2}} (D_{\mu}^{C} \phi_{R}^{2}) \phi_{R}^{2} - \frac{i g}{\sqrt{2}} \phi_{R}^{2} C_{\mu}^{+} (D^{\mu} \phi_{R}^{2}) \\
&+ \frac{i g}{\sqrt{2}} (D_{\mu}^{C} \phi_{R}^{2}) C_{\mu}^{+} c_{\phi}^{1} - \frac{i g}{\sqrt{2}} c_{\phi}^{1} C_{\mu}^{+} (D^{\mu} \phi_{R}^{2}) + \frac{g}{\sqrt{2}} c_{L}^{1} \overline{\sigma}^{\mu} C_{\mu}^{+} \phi_{L}^{2} \\
&+ \frac{g}{\sqrt{2}} \psi_{L}^{2} \overline{\sigma}^{\mu} C_{\mu}^{+} c_{L}^{1} + \frac{g}{\sqrt{2}} c_{R}^{1} \overline{\sigma}^{\mu} C_{\mu}^{+} \psi_{R}^{2} + \frac{g}{\sqrt{2}} \psi_{R}^{2} \overline{\sigma}^{\mu} C_{\mu}^{+} c_{R}^{1},
\end{align*}
\]

(41)

where \(D_{\mu}^{C} = \partial_{\mu} + i g A_{\mu}^{3} \tau^{3}\) (\(T^{3}\) is the third component of \(su(2)\) algebra), \(\mathcal{L}_{gh}^{(C)}\) are kinetic terms of ghost fields including a minimal coupling with the \(U(1)\) gauge boson \(A_{\mu}^{3}\), and \(\mathcal{L}_{\text{int}}\) contains interactions between ordinary matters and ghosts.

The total Lagrangian density is rewritten as

\[
\mathcal{L}_{SU(2)}^{(C)} = -\frac{1}{4} [F_{\mu\nu}^{a} F^{a\mu\nu}]_{*} + \mathcal{L}_{m} + \mathcal{L}_{gh}^{(C)} + \mathcal{L}_{\text{int}}^{(C)} = \mathcal{L}_{U(1)} + \delta \mathcal{L}_{U(1)}^{(C)},
\]

(42)

where \(\mathcal{L}_{U(1)}\) and \(\Delta \mathcal{L}^{(C)}\) are given by

\[
\begin{align*}
\mathcal{L}_{U(1)} &= -\frac{1}{4} \left( \partial_{\nu} A_{\mu}^{3} - \partial_{\mu} A_{\nu}^{3} \right) \left( \partial^{\phi} A_{3\nu}^{\phi} - \partial^{\phi} A_{3\mu}^{\phi} \right) + (D_{\mu}^{C} \phi_{R}^{2}) (D^{\mu} \phi_{R}^{2}) \\
&+ \psi_{L}^{2} \overline{\sigma}^{\mu} D_{\mu}^{C} \psi_{L}^{2} + \psi_{R}^{2} \overline{\sigma}^{\mu} D_{\mu}^{C} \psi_{R}^{2},
\end{align*}
\]

(43)

\[
\Delta \mathcal{L}^{(C)} = -(D_{\mu}^{C} A_{\nu}^{3}) (D^{\mu} A^{3\nu}) + (D_{\mu}^{C} A_{\nu}^{3}) (D^{\nu} A^{\mu})
\]

15
Massless states consist of three types of constituents, ordinary fields (collectively denoted by $\Phi$) including the gauge bosons which belong to multiplets of a unified gauge group $G_U$, and massive states. All massive states form doublets of massive states. The gauge coupling constants precisely measured at the Large Electron-Positron collider (LEP) suggest that the SM gauge interactions are unified at a high-energy scale. We take the following viewpoint and scenario for a physics beyond and behind the SM.

### 3.3.2 Grand unification scenario

We take the following viewpoint and scenario for a physics beyond and behind the SM. The gauge coupling constants precisely measured at the Large Electron-Positron collider (LEP) suggest that the SM gauge interactions are unified at $M_U$ in SM $+ \alpha$. An ultimate theory has a grand unified gauge symmetry potentially, and contains both massless and massive states. All massive states form doublets of $Q_F$, and they become unphysical. Massless states consist of three types of constituents, ordinary fields (collectively denoted by $\Phi_U$) including the gauge bosons which belong to multiplets of a unified gauge group $G_U$, ordinary fields (collectively denoted by $\Phi_0$) which belong to those of a smaller $U(1)_Y$, and they become unphysical.

From (42), we find that $SU(2)$ gauge symmetry is hidden in the form that it emerges after removing ghost fields and replacing the non-local self-interactions among $A_\mu^\pm$ by the local ones. The $A_\mu^+$ and $A_\mu^-$ behave as charged matters and change their phase under the $U(1)$ gauge transformation. The time-components of $A_\mu^\pm$ generate negative norm states, but they can be unphysical and harmless with the help of those of $C_\mu^\pm$. Hence the theory would not encounter inconsistency, so far as the $U(1)$ gauge invariance and fermionic symmetries are respected. To treat the non-local interactions and formulate the system consistently, the framework beyond the effective field theory might be necessary.

Furthermore, (42) can be regarded as a matching condition between a system with $SU(2)$ gauge bosons and that with the reduced $U(1)$ symmetry at a high-energy scale $M_U$, where matters and ghosts are administrated. Hence, we expect that specific relations among physical parameters, reflecting a larger gauge symmetry, are revived at $M_U$, and they are tested by analyzing renormalization group flows of parameters in a system with a reduced gauge symmetry.

The system is essentially identical to the $U(1)$ gauge theory described by $\mathcal{L}_{U(1)}$ under the subsidiary conditions.

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gauge symmetry $G'_o$, and ghost fields (collectively denoted by $\Phi_g$) which belong to those of a gauge symmetry $G_g$. The physics of $\Phi_U$ is effectively described by a GUT. If $G'_o$ and/or $G_g$ is the gauge group of SM + $\alpha$, the GUT symmetry is broken down into the SM + $\alpha$ one at $M_U$, in the presence of $\Phi'_o$ and $\Phi_g$. Then the theory turns out to be SM + $\alpha$ with specific relations among parameters reflecting on the unified symmetry, at $M_U$. Or specific initial conditions are imposed on parameters of SM + $\alpha$, at $M_U$. Note that there are no contributions such as threshold corrections due to heavy particles, in case that they are unphysical and do not give any quantum effects.

With the help of the toy model in Sect. 3.3.1, our scenario is summarized as

$$\mathcal{L}_{\text{light}} = \mathcal{L}_{\text{GUT}} + \mathcal{L}'_o + \mathcal{L}_{\text{gh}} + \mathcal{L}_{\text{int}} = \mathcal{L}_{\text{SM} + \alpha} + \delta F \Delta \mathcal{L} \bigg|_{M_U},$$  \hspace{1cm} (45)

where $\mathcal{L}_{\text{GUT}}$ is the Lagrangian density describing the GUT concerning $\Phi_U$, $\mathcal{L}'_o$ and $\mathcal{L}_{\text{gh}}$ contain kinetic terms of $\Phi'_o$ and $\Phi_g$ including minimal couplings with the gauge bosons in SM + $\alpha$, and $\mathcal{L}_{\text{int}}$ contains interactions between ordinary particles and ghosts. We present a prototype model describing the grand unification, in the Appendix B.

The theory has following excellent features.

- The Lagrangian density in SM + $\alpha$ is obtained with the following conditions for gauge coupling constants,

$$g_3 = g_2 = g_1 = g_U \bigg|_{M_U}, \quad \left(g_1 = \sqrt{\frac{5}{3}} g' \right),$$ \hspace{1cm} (46)

where $g_3$, $g_2$ and $g'$ are the gauge coupling constants for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$, respectively, and $g_U$ is the unified gauge coupling constant.

- The triplet-doublet splitting of Higgs boson is realized, if extra colored components are unphysical with the advent of their ghost partners.

- The SM gauge interactions are unified under a large gauge group, but the proton can be stabilized if extra colored particles such as $X$ gauge bosons are unphysical, in the presence of their ghost partners, and do not give any quantum effects on physical particles.

Furthermore, new particles around the terascale in SM + $\alpha$ can provide useful hints to the physics such as the grand unification and SUSY at $M_U$. For instance, if (part of) new particles form hypermultiplets as remnants of SUSY, it can be an evidence of (the reduction of) $N = 2$ SUSY through the analysis of renormalization group evolutions of parameters [22].

8 The basic idea of our scenario is same as those in Refs. [48, 49].
4 Conclusions

We have reconsidered the gauge hierarchy problem from the viewpoint of effective field theories and a high-energy physics, motivated by the alternative scenario that the SM (modified with massive neutrinos) holds up to a high-energy scale such as the Planck scale and the principle that the hierarchy is stabilized by a symmetry that should be unbroken in the SM. We have given a conjecture that theories with specific internal fermionic symmetries can be free from the gauge hierarchy problem and become candidates of the physics beyond and/or behind the SM, and presented a grand unification scenario and its prototype model.

Our consideration is based on the reinterpretation of the gauge hierarchy problem such that "without spoiling the structure of a high-energy physics supported by an excellent concept, is it possible to construct a low-energy effective theory and the interaction with heavy particles?"

It is also based on following thoughts. A large symmetry is, in general, broken down to the smaller one, if two systems with different size of symmetries interact with each other. The spacetime SUSY is no exception. A requirement of large and manifest symmetries causes strict laws of physics, and often leads to an unrealistic system. Diversity of nature might be a result of a partial breakdown or reduction of such symmetries, keeping its inner beauty. It would be attractive that the SM particles behave liberally to the extent permitted by the laws of physics including hidden symmetries.

In this way, spacetime SUSY seems not to be within reach of our direct measurements, because it is too beautiful and prominent. However, it does not mean that SUSY is absent in our world, at all. It is just contrary, and SUSY must exist at an ultimate level, because it achieves the unification of bosons and fermions, and it is deeply connected to the consistency of the theory such as superstring theory. Then, it can be said that the existence of fermions is a proof for SUSY. It is also possible to gain information on SUSY realized at $M_U$ from new particles around the terascale \[22\].

A magical ability would be required to keep an inner beauty eternally. If the fermionic symmetries relating ghost fields remain unbroken, the SM particles could behave liberally as singlets. In this situation, even if the SM gauge interactions are unified under a large gauge group, proton can be stable because extra colored particles such as $X$ gauge bosons become unphysical. In other words, proton acquires an eternal life as a result of the fact that extra colored particles sell their souls to the ghosts.

Furthermore, a definite discrepancy has not yet been observed between the predictions in the SM (modified with massive neutrinos) and experimental results, and this fact might be a proof for the existence of hidden fermionic symmetries and its related unphysical particles. The theory can be tested indirectly, using features of symmetries. In particular, physical quantities calculated in the SM + $\alpha$ should precisely match with the experimental values at the terascale, because radiative corrections from unphysical particles are canceled out. Parameters in the SM + $\alpha$ should satisfy specific relations at $M_U$, reflecting on a large symmetry realized in the ultimate theory.

Our scenario offers a system where the vacuum energy vanishes at $M_U$, because contributions from heavy particles are canceled out and those from massless particles turn out to be zero after the quartic divergences are removed. Our scenario could also have
a long life if consistent, because both spacetime SUSY and internal fermionic symmetry can coexist. That is, in case that superpartners are discovered, they can be treated as new particles in SM + α. If some of superpartners were absent, the fermionic symmetry would have a chance to show up.

In our formulation, there appear non-local interactions among unphysical particles. Though they could induce the breakdown of causality, it occurs in the unphysical sector and our physical sector is expected to maintain the causality. They must be properly formulated in an ultimate theory. It would be interesting to reconsider CPT theorem in this framework.

Even if our fermionic symmetries have a weak point, our conjecture would be survive that a magical symmetry can play the central role to solve the gauge hierarchy problem, if the SM particles are singlets and heavy particles belong to non-singlets of the transformation group, and the symmetry is unbroken and hidden in the low-energy theory.

It is important to examine whether theories with internal fermionic symmetries are consistently formulated in a manner to satisfy unitarity. It is also challenging to study the structure of ultimate theory and to derive its low-energy effective theory. If our world originated from only unphysical objects, larger fermionic symmetries would be needed to formulate unphysical theories including gauge bosons and gravitons, and the concept of orbifold grand unification [50, 51] would be useful on the reduction of relevant symmetries.

Acknowledgments

This work was supported in part by scientific grants from the Ministry of Education, Culture, Sports, Science and Technology under Grant Nos. 22540272 and 21244036.

A Non-local interactions and radiative corrections

We study interactions among unphysical particles, and radiative corrections on parameters, using a toy model described by the Lagrangian density,

\[ \mathcal{L}_T = \mathcal{L}_\phi + \mathcal{L}_{\phi,c} + \mathcal{L}_{\text{mix}}, \]

\[ \mathcal{L}_\phi = \partial_\mu \phi^\dagger \partial^\mu \phi - m_{\phi}^2 \phi^\dagger \phi - \lambda_{\phi} \left( \phi^\dagger \phi \right)^2, \tag{48} \]

\[ \mathcal{L}_{\phi,c} = \partial_\mu \phi^\dagger \partial^\mu \phi + \partial_\mu c_{\phi}^\dagger \partial^\mu c_{\phi} - M_{\phi}^2 \phi^\dagger \phi - M_{c_{\phi}}^2 c_{\phi}^\dagger c_{\phi} - \lambda_{c_{\phi}} \left( \phi^\dagger \phi \right) \left( c_{\phi}^\dagger c_{\phi} \right) - \lambda_{\phi c_{\phi}} \phi^\dagger \phi \left( c_{\phi}^\dagger \phi \right) - \lambda_{c_{\phi} \phi} \left( c_{\phi}^\dagger \phi \right) c_{\phi}^\dagger c_{\phi}, \tag{49} \]

\[ \mathcal{L}_{\text{mix}} = -\lambda_{\phi c_{\phi}} \phi^\dagger \phi \left( c_{\phi}^\dagger \phi \right) - \lambda_{c_{\phi} \phi} \phi^\dagger \phi \left( c_{\phi}^\dagger \phi \right), \tag{50} \]

where \( \lambda_{\phi}, \lambda_{\phi}^{(\phi)} \) and \( \lambda_{\phi}^{(c)} \) are the quartic self-coupling constants of \( \phi, \phi \) and \( c_{\phi} \), respectively, and \( \lambda_{\phi c_{\phi}}^{(\phi, c)}, \lambda_{\phi}^{(c)} \) and \( \lambda_{\phi c_{\phi}}^{(c)} \) are the quartic coupling constants among \( \phi, c_{\phi} \) and \( \phi \).

In the form of action integral, the star product (\( \star \)) represents a non-local interaction
such that $-\lambda^{(q)}_q \varphi^\dagger \varphi \star \varphi^\dagger \varphi$ stands for

$$
- \int \lambda^{(q)}_q v(x_1, x_2) \varphi^\dagger (x_1) \varphi(x_1) \varphi^\dagger (x_2) \varphi(x_2) d^4x_1 d^4x_2 ,
$$

(51)

where, $v(x_1, x_2)$ is a function of two spacelike points $x_1$ and $x_2$ and is common to other interactions in (49). We assume that $v(x, x) = 0$ and $v(x_1, x_2)$ can take non-zero values for $(x_1 - x_2)^2 = O(\ell^2)$ ($\ell$ is a fundamental length). The vertex representing the non-local interaction is depicted in Fig. 1 Here, the factor such as $4 v(x_1, x_2)$ for $\lambda^{(q)}_q \varphi^\dagger \varphi \star \varphi^\dagger \varphi$ is omitted, for simplicity. The same applies hereafter.

In the case that $v(x_1, x_2) = \delta^4(x_1 - x_2 + \xi)$ with $\xi^2 = O(\ell^2)$, the non-local interaction (51) becomes

$$
- \int \lambda^{(q)}_q \varphi^\dagger (x_1) \varphi(x_1) \varphi^\dagger (x_1 + \xi) \varphi(x_1 + \xi) d^4x_1 .
$$

(52)

If $\xi^\mu = 0$, the interaction becomes local. Then the self-interaction of $c_\varphi$ vanishes such that $-\lambda^{(c)}_q c_\varphi^\dagger c_\varphi c_\varphi^\dagger c_\varphi := 0$ because of $c_\varphi^2 = 0$. However, the self-interaction of $c_\varphi$ is indispensable, because it is induced radiatively through the coupling between light and heavy fields and it contains infinities that should be removed through the renormalization of relevant coupling constant. This is the reason why we introduce non-local self-interactions.

$\tilde{\mathcal{L}}_1$ has $OSp(2|2)$ invariance, when the following relations among parameters hold

$$
M^{(q)2}_q = M^{(c)2}_q , \quad \lambda^{(q)}_q = \lambda^{(q, c)}_q = \lambda^{(c)}_q , \quad \lambda^{(q)}_q = \lambda^{(c)}_q .
$$

(53)

Let us study radiative corrections on parameters at the one-loop level, without specifying the form of $v(x_1, x_2)$, and examine whether $OSp(2|2)$ invariance holds at the quantum level.

First, we consider radiative corrections on $m^2_\varphi$. The one-loop diagrams concerning $\delta m^2_\varphi$ are given in Fig. 2

The contributions from the first two diagrams are canceled each other for the case with (53), because the statistics of particles running in the loop is different from each other. In this case, $\delta m^2_\varphi$ is given by

$$
\delta m^2_\varphi = \frac{-\lambda_\varphi}{4\pi^2} m^2_\varphi \ln \frac{\Lambda^2}{m^2_\varphi} ,
$$

(54)
where we subtract the quadratic divergence. In the same way, radiative corrections on \( \lambda_\phi \) come from only the self-interaction of \( \phi \), because the contributions from \( \varphi \) and \( c_\varphi \) are exactly canceled out.

Next, we study radiative corrections on \( M_{\varphi}^{(\varphi)2} \) and \( M_{\varphi}^{(c)2} \). The one-loop diagrams of \( \delta M_{\varphi}^{(\varphi)2} \) are given in Fig. 3.

Those of \( \delta M_{\varphi}^{(c)2} \) are given by exchanging \( \varphi \) for \( c_\varphi \), \( \lambda_\varphi^{(\varphi)} \) for \( \lambda_\varphi^{(c)} \) and \( \lambda^{(\varphi)} \) for \( \lambda^{(c)} \). The contributions from the first two diagrams and the fifth one are canceled for the case with \( 53 \). In this case, after the subtraction of quadratic divergence, \( \delta M_{\varphi}^{(\varphi)2} \) is given by

\[
\delta M_{\varphi}^{(\varphi)2} = -\frac{\lambda^{(\varphi)}}{4\pi^2} m_\phi^2 \ln \frac{\Lambda^2}{m_\phi^2} + 2\lambda^{(\varphi)} J_\varphi ,
\]

(55)

where \( J_\varphi \) represents the sum of contributions from the third and fourth diagrams. In the same way, \( \delta M_{\varphi}^{(c)2} \) is given by

\[
\delta M_{\varphi}^{(c)2} = -\frac{\lambda^{(c)}}{4\pi^2} m_\phi^2 \ln \frac{\Lambda^2}{m_\phi^2} + 2\lambda^{(c)} J_{c_\varphi} ,
\]

(56)

where \( J_{c_\varphi} \) represents the counterpart to \( J_\varphi \). From (55) and (56), we find that \( \delta M_{\varphi}^{(\varphi)2} = \delta M_{\varphi}^{(c)2} \) for the case with \( 53 \) and \( J_\varphi = J_{c_\varphi} \). The equality \( J_\varphi = J_{c_\varphi} \) holds if the following reasoning is correct. In the third and fourth diagrams, \( \varphi (c_\varphi) \) does not form the closed
line by itself (the loop is composed of $\phi$ ($c_\phi$) and the dashed line representing non-local interactions), and hence an extra minus sign is not required for the propagation of $c_\phi$. In this case, the same size of contributions are expected for $J_\phi$ and $J_{c_\phi}$.

Finally, we study radiative corrections on $\lambda^{(q)}_\phi$, $\lambda^{(c)}_\phi$ and $\lambda^{(q,c)}_\phi$. The one-loop diagrams of $\delta(\lambda^{(q)}_\phi \nu(x_1, x_2))$ are given in Fig. 4.

Figure 4: The one-loop diagrams of $\delta(\lambda^{(q)}_\phi \nu(x_1, x_2))$.

Here, the factor $4 \times$ comes from the fact that there are four ways to contract external lines with non-local interaction points. Those of $\delta(\lambda^{(c)}_\phi \nu(x_1, x_2))$ are given by exchanging $\phi$ for $c_\phi$, $\lambda^{(q)}_\phi$ for $\lambda^{(c)}_\phi$ and $\lambda^{(q)}_\phi$ for $\lambda^{(c)}_\phi$. The contributions from the first and fifth diagrams are canceled for the case with (53). The one-loop diagrams of $\delta(\lambda^{(q,c)}_\phi \nu(x_1, x_2))$ are given in Fig. 5.

Here, the factor $2 \times$ stems from the fact that there are two ways to contract external lines with non-local interaction points. The contributions from the first and fourth diagrams are canceled for the case with (53). Then we find that $\delta(\lambda^{(q)}_\phi \nu(x_1, x_2)) = \delta(\lambda^{(c)}_\phi \nu(x_1, x_2)) = \delta(\lambda^{(q,c)}_\phi \nu(x_1, x_2))$, when the relations (53) hold, and an extra minus sign is not required for the propagation of $c_\phi$ in case that the loop is not composed of $c_\phi$ alone.

In this way, it is shown that, if the relations (53) and the above feature relating $c_\phi$ hold and quadratic divergences are removed, the light field $\phi$ receives neither any quantum corrections from heavy fields $\phi$ and $c_\phi$ nor large corrections from the self-interaction, $\phi$ and $c_\phi$ receive exactly the same size of radiative corrections, and hence the mass hierarchy is stabilized against quantum corrections at the one-loop level.

B Prototype model for grand unification

Let us present a prototype model of a grand unified theory in a hidden form, composed of massless fields including incomplete matter fields and ghost ones, by reference to the
model (C) in Sect. 3.3.1 The Lagrangian density possessing SU(5) gauge invariance is given by,

\[ \mathcal{L}_{\text{GUT}} = \mathcal{L}_{\text{GUT}}^{(1)} + \mathcal{L}_{\text{GUT}}^{(2)}, \]

\[ \mathcal{L}_{\text{GUT}}^{(1)} = -\frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} + (D_\mu H)^\dagger (D^\mu H) - \lambda_H (H^\dagger H)^2, \]

\[ \mathcal{L}_{\text{GUT}}^{(2)} = \sum_k \Psi_{Lk}^\dagger \overline{\sigma}_\mu D_\mu \Psi_{Lk} + \sum_{k'} \Psi_{Rk'}^\dagger i\sigma_\mu D_\mu \Psi_{Rk'} + \cdots, \]

where \( D_\mu = \partial_\mu + ig_U A_\mu^a T^a \), \( H = (H_C, H_W)^T \) is the Higgs boson with the fundamental representation 5, and \( \Psi_{Lk} \) and \( \Psi_{Rk} \) are left-handed and right-handed Weyl fermions. \( A_\mu^a (\alpha = 1, \cdots, 24) \) are SU(5) gauge bosons, and \( g_U \) is the unified gauge coupling constant. \( H_C \) and \( H_W \) stand for the colored components and the weak ones in \( H \), respectively. The ellipsis stands for terms such as Yukawa interactions. We do not consider them, because it depends on the origin of matter fields.

In the introduction of ghost fields \( C_\mu \) and \( c_{HC} \), whose gauge quantum numbers are same as those of \( X_\mu \) and \( H_C \), the following Lagrangian density can be added,

\[ \mathcal{L}_{\text{gh}}^{(1)} = -(D_\mu C_\nu)^\dagger (D^\mu C^\nu) + (D_\mu c_{HC})^\dagger (D^\mu c_{HC}) + (D_\mu X_\nu)^\dagger (D^\mu X^\nu), \]

\[ \mathcal{L}_{\text{int}}^{(1)} = -\frac{ig_U}{2} F_{\mu \nu}^a \left( C_\nu^\dagger C^\mu - C_\mu^\dagger C^\nu \right)^a + \frac{g_{2}}{2} \left[ X_\nu^\dagger X_\mu - X_\mu^\dagger X_\nu \right] \left( C_\nu^\dagger C^\mu - C_\mu^\dagger C^\nu \right)^a + \frac{g_{2}}{4} \left( C_\nu^\dagger C_\mu - C_\mu^\dagger C_\nu \right)^a \left( C_\nu^\dagger C^\mu - C_\mu^\dagger C^\nu \right)^a + \frac{g_{2}}{2} H_W^\dagger C_\mu C^\mu H_W - \frac{g_{1}}{2} H_C^\dagger C_\mu C^\mu H_C + \frac{g_{1}}{2} c_{HC}^\dagger X_\mu^\dagger - C_\mu C^\mu \right) c_{HC} + \frac{ig_{U}}{\sqrt{2}} (D_\mu c_{HC})^\dagger C^\mu H_W - \frac{ig_{U}}{\sqrt{2}} H_W^\dagger C_\mu (D_\mu c_{HC}) - \frac{ig_{U}}{\sqrt{2}} (D_\mu H_W)^\dagger C_\mu c_{HC} \]
where \( D'_{\mu} = \partial_{\mu} + i g U A^{a}_{\mu} T^{a} \), and \( F^{a}_{\mu\nu} \) is the field strength of the SM gauge bosons \( A^{a}_{\mu} (a = 1, 2, \cdots, 8, 21, \cdots, 24) \). Using (58), (60) and (61), we obtain the relation

\[
\mathcal{L}^{(1)}_{\text{light}} = \mathcal{L}^{(1)}_{\text{GUT}*} + \mathcal{L}^{(1)}_{\text{gh}} + \mathcal{L}^{(1)}_{\text{int}} = \mathcal{L}^{(1)}_{\text{SM}*} + \delta_{F} \delta_{V} \Delta \mathcal{L}^{(1)} \bigg|_{\mu} ,
\]

where \( \mathcal{L}^{(1)}_{\text{GUT}*} \) is the Lagrangian density that the self-interactions of \( X_{\mu} \) and \( H \) are replaced by the non-local ones in (59), \( \mathcal{L}^{(1)}_{\text{SM}*} \) and \( \Delta \mathcal{L}^{(1)} \) are given by

\[
\mathcal{L}^{(1)}_{\text{SM}*} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + (D'_{\mu} H_{W})^\dagger (D'_{\mu} H_{W}) - \lambda_{H} H_{W}^\dagger H_{W} \star H_{W}^\dagger H_{W} ,
\]

\[
\Delta \mathcal{L}^{(1)} = -(D'_{\mu} X_{\nu})^\dagger (D'_{\mu} X_{\nu}) + (D'_{\mu} X_{\nu})^\dagger (D'_{\nu} X_{\mu}) - \frac{ig_{U}}{2} F^{a}_{\mu\nu} \left( X_{\nu}^{\dagger} X_{\mu}^\dagger - X_{\mu}^{\dagger} X_{\nu} \right)^{a} + \frac{g_{U}^{2}}{8} \left( C_{V}^{\dagger} C_{\mu} - C_{\mu}^{\dagger} C_{V} \right)^{a} \left( C_{\nu}^{\dagger} C_{\mu} - C_{\mu}^{\dagger} C_{\nu} \right)^{a} + (D'_{\mu} H_{C})^\dagger (D'_{\mu} H_{C}) + \frac{g_{U}^{2}}{4} \left( H_{C}^{\dagger} C_{\mu} X_{\mu}^\dagger H_{C} + C_{\mu}^{\dagger} C_{\mu} C_{\mu}^\dagger H_{C} \right) + \frac{g_{U}^{2}}{2} H_{W}^\dagger X_{\mu}^\dagger H_{W} + \frac{ig_{U}}{\sqrt{2}} (D'_{\mu} H_{C})^\dagger X_{\mu}^\dagger H_{W} - \frac{ig_{U}}{\sqrt{2}} H_{W}^\dagger X_{\mu}^\dagger (D'_{\mu} H_{C}) - \frac{\lambda_{H}}{2} \left( H_{C}^{\dagger} H_{C} \star H_{C}^{\dagger} H_{C} - c_{H_{C}}^\dagger c_{H_{C}} \star c_{H_{C}}^\dagger c_{H_{C}} \right) - 2 \lambda_{H} H_{C}^{\dagger} H_{C} \star H_{C}^{\dagger} H_{C} .
\]

Note that the self-interaction of the weak Higgs boson \( H_{W} \) is given as the non-local one in (63), but it is regarded as the local one if the fundamental length \( \ell \) is small enough.

The \( \mathcal{L}^{(1)}_{\text{light}} \) is invariant under the fermionic transformations,

\[
\delta_{F} H_{C} = -c_{H_{C}} , \quad \delta_{F} H_{C}^\dagger = 0 , \quad \delta_{F} c_{H_{C}} = 0 , \quad \delta_{F} \bar{c}_{H_{C}} = H_{C}^{\dagger} , \quad \delta_{F} X_{\mu} = -C_{\mu} , \quad \delta_{F} X_{\mu}^\dagger = 0 , \quad \delta_{F} C_{\mu} = 0 , \quad \delta_{F} C_{\mu}^\dagger = X_{\mu}^\dagger , \quad \delta_{F} H_{W} = 0 , \quad \delta_{F} H_{W}^\dagger = 0 , \quad \delta_{F} A^{a}_{\mu} = 0
\]

and

\[
\delta_{F} H_{C} = 0 , \quad \delta_{F} H_{C}^\dagger = c_{H_{C}} , \quad \delta_{F} c_{H_{C}} = H_{C} , \quad \delta_{F} \bar{c}_{H_{C}} = 0 , \quad \delta_{F} X_{\mu} = 0 , \quad \delta_{F} X_{\mu}^\dagger = C_{\mu} , \quad \delta_{F} C_{\mu} = X_{\mu} , \quad \delta_{F} C_{\mu}^\dagger = 0 , \quad \delta_{F} H_{W} = 0 , \quad \delta_{F} H_{W}^\dagger = 0 , \quad \delta_{F} A^{a}_{\mu} = 0 .
\]

Next, we consider the matter part \( \mathcal{L}^{(2)}_{\text{GUT}} \), taking three types of matter multiplets such as \( \Psi_{5_{L}} \), \( \Psi_{5_{R}} \), and \( \Psi_{10_{L}} \).
(a) Starting from $\Psi_{bL} = (d_{L}^{c}, l_{L}^{c})^{T}$, after introducing the ghost partner $c_{L}^{c}$ of $l_{L}$, the kinetic term of down type $SU(2)_{L}$-singlet quark $d_{L}^{c}$ is derived as follows,

$$\Psi_{bL}^{\dagger} i \sigma^{\mu} D_{\mu} \Psi_{bL} + c_{L}^{c \dagger} i \sigma^{\mu} D_{\mu}^{c} c_{L} + \frac{gU}{\sqrt{2}} d_{L}^{c \dagger} \sigma^{\mu} C_{\mu}^{c} c_{L} + \frac{gU}{\sqrt{2}} c_{L}^{c \dagger} \sigma^{\mu} c_{L}^{c T} d_{L}^{c}$$

$$= d_{L}^{c \dagger} i \sigma^{\mu} D_{\mu}^{c} d_{L}^{c} + \delta_{F} \delta_{F}^{\dagger} \left( l_{L}^{c \dagger} i \sigma^{\mu} D_{\mu}^{l} l_{L} + \frac{gU}{\sqrt{2}} d_{L}^{c \dagger} \sigma^{\mu} X_{\mu} l_{L} - \frac{gU}{\sqrt{2}} l_{L}^{c \dagger} \sigma^{\mu} X_{\mu} d_{L}^{c} \right) \right|_{M_{U}}, (67)$$

where the superscripts $c$ and $*$ represent the operation of charge conjugation and complex conjugation, respectively, and the fermionic transformations are given by

$$\delta_{F} l_{L} = c_{L}^{c}, \quad \delta_{F} l_{L}^{c} = 0, \quad \delta_{F} c_{L} = 0, \quad \delta_{F} c_{L}^{c} = l_{L}^{c}, \quad \delta_{F} X_{\mu} = -X_{\mu}^{T}, \quad \delta_{F} X_{\mu}^{*} = 0,$$

$$\delta_{F} C_{\mu} = 0, \quad \delta_{F} C_{\mu}^{c} = X_{\mu}^{c}, \quad \delta_{F} d_{L}^{c} = 0, \quad \delta_{F} d_{L}^{c T} = 0 \quad (68)$$

and

$$\delta_{F}^{c} l_{L} = 0, \quad \delta_{F}^{c} l_{L}^{c} = c_{L}^{c}, \quad \delta_{F}^{c} c_{L} = -l_{L}, \quad \delta_{F}^{c} c_{L}^{c} = 0, \quad \delta_{F}^{c} X_{\mu} = 0, \quad \delta_{F}^{c} X_{\mu}^{c} = C_{\mu}^{c},$$

$$\delta_{F}^{c} C_{\mu}^{c} = X_{\mu}^{c}, \quad \delta_{F}^{c} C_{\mu}^{c T} = 0, \quad \delta_{F}^{c} d_{L}^{c} = 0, \quad \delta_{F}^{c} d_{L}^{c T} = 0 \quad (69)$$

(b) From the content with $\Psi_{sR} = (d_{R}^{c}, l_{R}^{c})^{T}$ and the ghost partner $c_{R}^{c}$ of $d_{R}$, the kinetic term of $SU(2)_{L}$-doublet lepton $l_{L} = (l_{R}^{c})^{c}$ is derived as (after the charge conjugation is performed),

$$\Psi_{sR}^{\dagger} i \sigma^{\mu} D_{\mu} \Psi_{sR} + c_{R}^{c \dagger} i \sigma^{\mu} D_{\mu}^{c} c_{R}^{c} + \frac{gU}{\sqrt{2}} c_{R}^{c \dagger} \sigma^{\mu} C_{\mu} \sigma_{R}^{c} + \frac{gU}{\sqrt{2}} c_{R}^{c \dagger} \sigma^{\mu} C_{\mu} \sigma_{R}^{c} c_{R}^{c}$$

$$= l_{R}^{c \dagger} i \sigma^{\mu} D_{\mu}^{l} l_{R}^{c} + \delta_{F} \delta_{F}^{\dagger} \left( d_{R}^{c \dagger} i \sigma^{\mu} D_{\mu}^{d} d_{R} + \frac{gU}{\sqrt{2}} d_{R}^{c \dagger} \sigma^{\mu} X_{\mu} l_{R}^{c} + \frac{gU}{\sqrt{2}} l_{R}^{c \dagger} \sigma^{\mu} X_{\mu} d_{R}^{c} \right) \right|_{M_{U}}, (70)$$

where the fermionic transformations are given by

$$\delta_{F} d_{R} = -d_{R}^{c}, \quad \delta_{F} d_{R}^{c} = 0, \quad \delta_{F} c_{R} = 0, \quad \delta_{F} c_{R}^{c} = d_{R}^{c}, \quad \delta_{F} l_{R}^{c} = 0, \quad \delta_{F} l_{R}^{c T} = 0 \quad (71)$$

and

$$\delta_{F}^{c} d_{R} = 0, \quad \delta_{F}^{c} d_{R}^{c} = d_{R}^{c}, \quad \delta_{F}^{c} c_{R} = -d_{R}^{c}, \quad \delta_{F}^{c} c_{R}^{c} = 0, \quad \delta_{F}^{c} l_{R}^{c} = 0, \quad \delta_{F}^{c} l_{R}^{c T} = 0 \quad (72)$$

(c) From the content with $\Psi_{10_{L}} = (q_{L}, u_{L}^{c}, e_{L}^{c})^{T}$ and the ghost partner $c_{L}^{c}$ of $u_{L}^{c}$ and $c_{L}^{c}$ of $e_{L}^{c}$, the kinetic term of $SU(2)_{L}$-doublet quark $q_{L}$ is derived as

$$\Psi_{10_{L}}^{\dagger} i \sigma^{\mu} D_{\mu} \Psi_{10_{L}} + c_{L}^{c \dagger} i \sigma^{\mu} D_{\mu}^{c} c_{L}^{c} + c_{L}^{c \dagger} i \sigma^{\mu} D_{\mu}^{c} c_{L}^{c}$$

$$+ \frac{gU}{\sqrt{2}} c_{L}^{c \dagger} \sigma^{\mu} C_{\mu} q_{L} - \frac{gU}{\sqrt{2}} q_{L}^{c \dagger} \sigma^{\mu} C_{\mu} q_{L}^{c} + \frac{gU}{\sqrt{2}} q_{L}^{c \dagger} \sigma^{\mu} C_{\mu} e_{L} - \frac{gU}{\sqrt{2}} c_{L}^{c \dagger} \sigma^{\mu} C_{\mu} q_{L}$$

$$= q_{L}^{c \dagger} i \sigma^{\mu} D_{\mu}^{q} q_{L}^{c} + \delta_{F} \delta_{F}^{\dagger} \left( u_{L}^{c \dagger} i \sigma^{\mu} D_{\mu}^{u} u_{L}^{c} + e_{R}^{c \dagger} i \sigma^{\mu} D_{\mu}^{e} e_{R} \right)$$
\[
- \frac{g_U}{\sqrt{2}} u_L^c \sigma^\mu X_\mu q_L + \frac{g_U}{\sqrt{2}} q_L^c \sigma^\mu X_\mu u_L^c - \frac{g_U}{\sqrt{2}} q_L^c \sigma^\mu X_\mu e_L^c + \frac{g_U}{\sqrt{2}} e_L^c \sigma^\mu X_\mu q_L \bigg|_{M_U}, \quad (73)
\]

where the fermionic transformations are given by

\[
\delta_F u_L^c = c_{u_L^c}, \quad \delta_F u_L^{c\dagger} = 0, \quad \delta_F c_{u_L^c} = u_L^c, \quad \delta_F c_{u_L^{c\dagger}} = 0, \quad \delta_F e_L^c = e_L^c, \quad \delta_F e_L^{c\dagger} = 0, \quad \delta_F q_L = 0, \quad \delta_F q_L^{\dagger} = 0 \quad (74)
\]

and

\[
\delta_F^\dagger u_L^c = 0, \quad \delta_F^\dagger u_L^{c\dagger} = c_{u_L^c}, \quad \delta_F^\dagger c_{u_L^c} = -u_L^c, \quad \delta_F^\dagger c_{u_L^{c\dagger}} = 0, \quad \delta_F^\dagger e_L^c = e_L^c, \quad \delta_F^\dagger e_L^{c\dagger} = -e_L^c, \quad \delta_F^\dagger q_L = 0, \quad \delta_F^\dagger q_L^{\dagger} = 0. \quad (75)
\]

The kinetic terms of \( u_L^c \) and \( e_L^c \) should be added, by introducing \( u_L^c \) and \( e_L^c \) as \( \Phi'_0 \) (ordinary fields belonging to multiplets of the SM gauge group).

This model has following excellent features. The unification of the SM gauge coupling constants occurs such that \( g_3 = g_2 = g_1 = g_U \) at \( M_U \). The triplet-doublet splitting of Higgs boson is realized in the form that \( H_W \) becomes the Higgs doublet in the SM and the ghost \( c_H \) makes \( H_C \) unphysical. The longevity of proton is fully guaranteed because both \( X \) gauge bosons and their ghost partners are unphysical.

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