Constraints on the cosmic neutrino background

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ABSTRACT

The radiative component of the Universe has a characteristic impact on both large scale structure (LSS) and the cosmic microwave background radiation (CMB). We use the recent WMAP data, together with previous CBI data and 2dF matter power spectrum, to constrain the effective number of neutrino species \( N_{\text{eff}} \) in a general Cosmology. We find that \( N_{\text{eff}} = 4.31 \) with a 95 per cent C.L. \( 1.6 \leq N_{\text{eff}} \leq 7.1 \). If we include the \( H_0 \) prior from the HST project we find the best fit \( N_{\text{eff}} = 4.08 \) and \( 1.90 \leq N_{\text{eff}} \leq 6.62 \) for 95 per cent C.L. The curvature we derive is still consistent with flat, but assuming a flat Universe from the beginning implies a bias toward lower \( N_{\text{eff}} \), as well as artificially smaller error bars. Adding the Supernovae constraint doesn’t improve the result. We analyze and discuss the degeneracies with other parameters, and point out that probes of the matter power spectrum on smaller scales, accurate independent \( \sigma_8 \) measurements, together with better independent measurement of \( H_0 \) would help in breaking the degeneracies.

Key words: gravitation – cosmology: theory – large-scale structure of Universe – cosmic microwave background – dark matter

1 INTRODUCTION

The quest for cosmological parameters is the major goal of cosmology, and we are experiencing an exciting era in which newer and better data allow for the precise determination of an increased number of parameters. In this paper we focus on one specific parameter, the amount of relativistic energy density at recombination/equlvalence, and we discuss the implications of the current CMB and LSS observations.

The relativistic energy density is often parametrized by the equivalent number of standard model neutrino species \( N_{\text{eff}} \). The standard model predicts three neutrino species, plus a correction imposed by neutrinos not being completely decoupled during electron-positron annihilation \((\Delta N_{\text{eff}} = 0.03)\) (Doğan et al. 1997, Gnedin & Gnedin 1998) and by the finite temperature QED correction to the electromagnetic plasma \((\Delta N_{\text{eff}} = 0.01)\) (Lopez et al. 1999, Mangano et al. 2002).

Departures from the standard model which imply a variation in \( N_{\text{eff}} \) may be due to decaying dark matter (Bonometto & Pierpaoli 1998, Hannestad 1998, Lopez et al. 1998, Kaplinghat & Turner 2001 or quintessence (Bean et al. 2001). In these scenarios \( N_{\text{eff}} \) can be either smaller or greater than 3.04, and the constraints on \( N_{\text{eff}} \) implied by CMB and LSS need not to agree with the ones implied by Big Bang Nucleosynthesis (Kaplinghat & Turner 2001). We will analyze and discuss the CMB+LSS results on \( N_{\text{eff}} \). Unlike previous work on this subject, we allow for a cosmology with general curvature. We will use the CMB data from the WMAP satellite (Bennet et al. 2003), and complement them with the CBI ones (Pearson et al. 2002). As for the LSS, we use the 2dF data (Percival et al 2002). We use a general cosmology in our analysis because at the present time we do not have an independent probe of the flatness of the Universe other than the CMB. It is therefore fictitious and misleading to restrict ourselves to flat cosmologies in the so called “precision cosmology” era. We will show that allowing for non-zero curvature changes quite significantly the results on \( N_{\text{eff}} \), despite the fact that the inferred constraints on curvature are still compatible with a flat Universe.

The paper is organized as follows: in section 2 we discuss the effects of \( N_{\text{eff}} \) on the matter and radiation power spectra, in section 3 we discuss the CMB+LSS constraints, while section (4) is dedicated to conclusions.

2 EFFECTS OF A RELATIVISTIC BACKGROUND ON THE MATTER AND RADIATION POWER SPECTRA

An excess of energy in relativistic particles with respect to the one predicted by the standard model has an impact on both the radiation and the matter power spectrum, for different reasons. The increased relativistic energy delays equivalence, and scale entering the horizon at equivalence is bigger. Since the amplitude of dark matter perturbations on smaller scales is frozen during the radiation dom-
in the material era, the matter power spectrum turnover is shifted toward lower $k$’s. The effect can be quantified via the shape parameter $\Gamma \simeq \Omega_m h^2 (g_*/3.65)^{1/2}$ (White, Gelmini & Silk 1995), where $g_*$ represents the relativistic degrees of freedom ($g_* = 2 + 0.454 N_{\text{eff}}$). For a given power at large scales, the small scale power $s$ therefore reduced, and a lower value of $\sigma_8$ is implied.

An increased radiation density implies a smaller conformal time and therefore a smaller sound horizon. As a consequence, the peaks of the CMB power spectrum, which reflects the peaks/valleys of the oscillations at recombination, are shifted toward smaller scales (bigger $\ell$’s) (Pierpaoli & Buometto 1999). In addition, since recombination occurs when the Universe is not completely matter dominated, the early Integrated Sachs-Wolfe (ISW) effect causes an enhanced first peak.

A description of parameter degeneracies in a flat Universe is given by Bowen et al. (2002), where they find that $N_{\text{eff}}$ is mainly degenerate with $\Omega_m$ and the spectral index $n$. They show that almost complete degeneracy can be obtained by keeping $z_{\text{re}}, \Omega_b h^2$ and $R$ (the position of the first acoustic peak with respect to a reference model) fixed while varying $N_{\text{eff}}$ produces almost degenerate power spectra. Here we extend the analysis to the real data and consider general curvature.

Allowing for possible curvature affects the radiation power spectrum. In particular, the position of the peaks is strongly dependent on curvature. A given comoving scale at the last scattering surface subverts an angle on the sky that is curvature-dependent. If the Universe is open ($\Omega_k > 0$) or closed ($\Omega_k < 0$) then the angle associated with a particular scale at last scattering is smaller [larger] than the corresponding flat Universe one. Consequently, the peaks/valleys of the CMB power spectrum will be located at larger [smaller] $\ell$’s. A slightly close Universe can compensate the shift of the peaks toward higher $\ell$’s implied by an increased radiation background. We therefore expect some degeneracy between curvature and $N_{\text{eff}}$, and if we aim to determine both from the CMB measurements, we ought to treat both of them as free parameters in the data analysis.

### 3 CMB AND LSS CONSTRAINTS

We used a modified version of the COSMOMC (Lewis & Bridle 2002) package to compute the likelihoods also including the WMAP new results (Verde et al. 2003). We considered only CDM adiabatic perturbations, and the following set of parameters, with usual definitions: $\Omega_m$, $\Omega_b$, $\Omega_k$, $H_0$, $N_{\nu}$, $n_s$, $A_s$, $z_{\text{re}}$ (reionization redshift). In our analysis we use the WMAP and CBI data together with the 2dF power spectrum, and we discuss the effect of adding other priors.

First we compare the results for a flat model ($\Omega_k = 0$) with those for a general Universe when $H_0$ to vary only in the range $64 \leq H_0 \leq 80$ corresponding to the 1σ allowed range of the HST result (Freedman et al. 2001). In fig.1 we present the likelihoods for each parameter, after marginalization over all the others, for flat and generally curved cases. For the number of effective neutrinos we find a best fit $N_{\text{eff}} = 4.08$ ($N_{\text{eff}} = 3.70$) for the general curvature (flat) case, with $2.23 \leq N_{\text{eff}} \leq 6.13$ ($1.82 \leq N_{\text{eff}} \leq 5.74$) at the 95% C.L. The assumption of a flat Universe biases the result toward low $N_{\text{eff}}$ and shrinks the inferred error bars. Our results are in agreement with what found by Crotty et al. (2003) with the assumption of flat geometry. Notice that the general curvature analysis is important at the present time because of the great improvement that the WMAP results have implied on the CMB power spectrum. Constraints from CMB and LSS on flat Universes previous to the WMAP were much weaker: $N_{\text{eff}} \leq 1.5$ at 95% C.L. (Hannestad 2001, see also Hansen et al., 2002).

In fig.1 the flat and curve case present the similar limit for $N_{\text{eff}}$ mainly because we imposed a tight top-hat prior on $H_0 (< 80)$. The likelihood indeed seems to prefer high values for $H_0$, which may allow for extra radiative component.

For the general curvature case, we now proceed to discuss the effect of various priors on the determination of $N_{\text{eff}}$.

In fig. 2 we present the results obtained allowing for a wider $H_0$ (and $z_{\text{re}}$) range. The best fit for $N_{\text{eff}}$ in this case is $N_{\text{eff}} = 4.31$, with a 95 per cent C.L. range : $1.6 \leq N_{\text{eff}} \leq 7.1$ from CMB and 2dF only, and $N_{\text{eff}} = 4.08$ with 1.9 $\leq N_{\text{eff}} \leq 6.62$ when the $H_0$ prior from the HST project is included.

As for the other parameters, we note that the inclusion of extra relativistic energy causes a higher $n_s$ values than in the standard case (Spergel et al 2003), a higher $z_{\text{re}}$ and a slightly closed Universe ($\Omega_k = -0.013 \pm 0.015$). We argue that, since a high $N_{\text{eff}}$ boosts the first peak through the early ISW effect, a higher $n_s$ combined with an early reionization of the Universe can still ensure a good fit to the CMB data. Matter power spectrum data on smaller scales than the ones probed by 2dF (e.g. Ly-$\alpha$ forest data) may be used in breaking this degeneracy, because the matter power spectrum is only sensitive to $n_s$ and not to $z_{\text{re}}$. Data of the matter power spectrum at very small scales would be incredibly sensitive to small increases in $n_s$. However, there is still much debate on small scale data, since their interpretation may be complicated by the non-linear growth of the fluctuations. We adopted here a conservative approach and chose not to include them in the analysis. In fig. 2 we note that the derived value of $\sigma_8$ is quite high (0.97 ± 0.1) yet because of the favoured high $n_s$ values. Actual quotations of the $\sigma_8$ value range between 0.75 and 1 (Pierpaoli et al. 2003, and references therein), so that a broad prior on $\sigma_8$ would already decrease the allowed $n_s$ and improve the errors on $N_{\text{eff}}$. Consistent 5 per cent accuracy measurements of $\sigma_8$ from both weak lensing and clusters would greatly improve the constraint on $N_{\text{eff}}$. As an example, in fig. 2 we plot the curves obtained with an hypothetical prior on $\sigma_8$ with the typical scaling derived from cluster and weak lensing: $\sigma_8 (\Omega_m/0.3)^{0.6} = 0.85 \pm 0.05$. Such prior would imply a lower $\Omega_m$ and $N_{\text{eff}} = 3.8 \pm 1.6$.

As for the other parameters, we note that $\Omega_m$ and $\Omega_\Lambda$ are well constrained in the range: $\Omega_m = 0.29 \pm 0.06$ and $\Omega_\Lambda = 0.72 \pm 0.05$ (1 σ error).

Hannestad (2003) while analyzing flat Universes uses a tight prior on $\Omega_m$ derived from the Supernova Cosmology Project (Perlmutter et al. 1999). As a consequence he derives a small $N_{\text{eff}}$ value with small error bars. We run a separate chain treating CMB+LSS+SN data in a general cosmology with COSMOMC, and found that the inclusion of the Supernovae in the analysis doesn’t particularly improve the results on $N_{\text{eff}}$ (see fig. 2). Including the SN in the anal-
The marginalized likelihoods for the parameters under consideration. We have assumed here a top–hat prior on $H_0$ corresponding to the 1σ interval allowed by the HST key project results. The solid line is for a general curvature, the dotted corresponds to the flat Universe case. The general curvature tends to push the constraints on $N_{\text{eff}}$ toward higher values. The same upper limits on $N_{\text{eff}}$ are probably due to the upper limit imposed on $H_0$ ($< 80$). Notice that $z_{\text{re}}$ in the range considered here is not constrained by the data.

In the case of general curvature, we explored a wider range in $H_0$ and $z_{\text{re}}$ and applied different priors. We find $1.6 \leq N_{\text{eff}} \leq 7.1$ (best fit $N_{\text{eff}} = 4.31$) at 95% C.L. from CMB and 2dF only, and $N_{\text{eff}} = 4.08$ with $1.9 \leq N_{\text{eff}} \leq 6.62$ when the $H_0$ prior from the HST project is included as a proper Gaussian prior. No significant modifications derive from the inclusion of the SN constraint.

We analyze the correlations between the various parameters and conclude that $N_{\text{eff}}$ is most degenerate with $\Omega_m h^2$ and $\Omega_k$. We argue that other independent measurements of the matter power spectrum, like precise determinations of $\sigma_8$ from clusters and lensing or probes at smaller scales from the Ly–$\alpha$ forest, would help in constraining the epoch of equivalence and therefore would improve the results on $N_{\text{eff}}$. Moreover, it would greatly improve the constraints on the large $n_s$ now allowed.

**Figure 1.** The marginalized likelihoods for the parameters under consideration. Note that the high $n_s$ and $z_{\text{re}}$ values. The long–dashed line is obtained adding an hypothetical prior on $\sigma_8$ with the typical scaling from clusters and weak lensing.

**Figure 2.** The marginalized likelihoods in the case of a general cosmology. The short–dashed line only consider CMB+2dF data, the dotted includes the $H_0$ prior from the HST project, and the solid also includes SN data. $N_{\text{eff}}$ is restricted to be $\leq 6.6$ at 95 per cent C.L., and $\Omega_\Lambda$ tends to be negative but is still consistent with flat. Note the high $n_s$ and $z_{\text{re}}$ values. The long–dashed line is obtained adding an hypothetical prior on $\sigma_8$ with the typical scaling from clusters and weak lensing.

4 CONCLUSIONS

The new CMB data (WMAP and CBI) together with the matter power spectrum derived by the 2dF galaxy survey can constrain the effective number of neutrino species much more precisely than previous experiments. Previous estimates of $N_{\text{eff}}$ were derived under the assumption of null curvature. Since we don’t have any independent confirmation of the flatness of the Universe other than the CMB itself, we argue that the curvature should be kept as a free parameter in the estimation of $N_{\text{eff}}$. We compare the results derived from the two different hypothesis. Applying a top hat prior for $H_0$ ($64 \leq H_0 \leq 80$), we find for a flat Universe $1.82 \leq N_{\text{eff}} \leq 5.74$ at 95 per cent C.L., with a best fit of $N_{\text{eff}} = 3.70$, while with general curvature $2.23 \leq N_{\text{eff}} \leq 6.13$ with a best fit $N_{\text{eff}} = 4.08$. Allowing for general curvature shifts the acceptance range of $N_{\text{eff}}$ toward higher values mainly because the curvature tends to compensate the effect of $N_{\text{eff}}$ on the peak locations in the CMB power spectrum.

By looking at the likelihood distribution for each parameter after marginalization over all the others we conclude that the inclusion of an extra relativistic component would suggest a higher expansion rate $H_0$, a higher spectral index $n_s$, and $\Omega_k$ slightly negative, if compared to the standard analysis with three neutrinos (Spergel et al. 2003).

We analyze the correlations between the various parameters and conclude that $N_{\text{eff}}$ is most degenerate with $\Omega_m h^2$ and $\Omega_k$. We argue that other independent measurements of the matter power spectrum, like precise determinations of $\sigma_8$ from clusters and lensing or probes at smaller scales from the Ly–$\alpha$ forest, would help in constraining the epoch of equivalence and therefore would improve the results on $N_{\text{eff}}$. Moreover, it would greatly improve the constraints on the large $n_s$ now allowed.
Table 1. Correlation matrix for the parameters in fig 1. We assume here a general curvature, and \( 64 \leq H_0 \leq 80 \).

|       | \( \Omega h^2 \) | \( \Omega_{dm} h^2 \) | \( H_0 \) | \( z_{re} \) | \( \Omega_k \) | \( N_{\nu} \) | \( n_s \) | \( A_s \) | \( \Omega_{\Lambda} \) | \( \text{Age} \) | \( \Omega_m \) |
|-------|-----------------|------------------|--------|--------|-------------|-----------|--------|--------|-------------|--------|--------|
| \( \Omega h^2 \) | 1.00       | -0.29            | 0.14   | 0.39   | 0.15        | -0.44     | 0.78   | 0.70   | 0.28        | 0.07   | -0.30  |
| \( \Omega_{dm} h^2 \) | -0.29       | 1.00              | 0.25   | -0.07  | -0.10       | 0.76      | -0.08  | -0.06  | -0.69       | -0.81  | 0.65   |
| \( H_0 \) | 0.14          | 0.25              | 1.00   | 0.11   | 0.48        | 0.17      | 0.22   | 0.16   | 0.46        | -0.75  | -0.57  |
| \( z_{re} \) | 0.39          | -0.07             | 0.11   | 1.00   | -0.38       | 0.18      | 0.71   | 0.87   | 0.27        | 0.01   | -0.12  |
| \( \Omega_k \) | 0.15          | -0.10             | 0.48   | -0.38  | 1.00        | -0.58     | -0.16  | -0.10  | 0.17        | -0.28  | -0.47  |
| \( N_{\nu} \) | -0.44         | 0.76              | 0.17   | 0.18   | -0.58       | 1.00      | -0.05  | -0.08  | -0.35       | -0.56  | 0.50   |
| \( n_s \) | 0.78          | -0.08             | 0.22   | 0.71   | -0.16       | -0.05     | 1.00   | 0.89   | 0.28        | -0.08  | -0.20  |
| \( A_s \) | 0.70          | -0.06             | 0.16   | 0.87   | -0.10       | -0.08     | 0.89   | 1.00   | 0.20        | -0.05  | -0.14  |
| \( \Omega_{\Lambda} \) | 0.28        | -0.69             | 0.46   | 0.27   | 0.17        | -0.35     | 0.28   | 0.20   | 0.10        | 0.22   | -0.95  |
| \( \text{Age} \) | 0.07         | -0.81             | -0.75  | 0.01   | -0.28       | -0.56     | -0.08  | -0.05  | 0.22        | 1.00   | -0.11  |
| \( \Omega_m \) | -0.30        | 0.65              | -0.57  | -0.12  | -0.47       | 0.50      | -0.20  | -0.14  | -0.95       | -0.11  | 1.00   |

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