AXISYMMETRIC CIRCUMSTELLAR INTERACTION IN SUPERNOVAE

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ABSTRACT

Multiwavelength observations of Type II supernovae have shown evidence of the interaction between supernovae and the dense slow winds from the red supergiant progenitor stars. Observations of planetary nebulae and the nebula around SN 1987A show that the slow winds from extended stars frequently have an axisymmetric structure with a high density in the equatorial plane. We have carried out numerical calculations of the interaction of a supernova with such an axisymmetric density distribution. For small values of the angular density gradient at the pole, the asymmetry in the interaction shell is greater than, but close to, that expected from purely radial motion. If the angular density gradient is above a moderate value, then the flow qualitatively changes and a protrusion emerges along the axis. For a power-law supernova density profile, the flow approaches a self-similar state in which the protrusion length is 2–4 times the radius of the main shell. The critical density gradient is larger for steeper density profiles of the ejecta. Most of our calculations are axisymmetric, but we have carried out a three-dimensional calculation to show that the protrusion is not a numerical artifact along the symmetry axis. For typical supernova parameters, the protrusions take ≥ several years to develop. The appearance of the shell with protrusions is similar to that observed in VLBI radio images of the remnant 41.9 + 58 in M82 and possibly of SN 1986J. We also considered the possibility of asymmetric ejecta and found that it had a relatively small effect on the asymmetry of the interaction region.

Subject headings: circumstellar matter — hydrodynamics — ISM: structure — shock waves — supernova: remnants — supernovae: general

1. INTRODUCTION

Previous models of supernovae (SNe) of Types Ib, Ic, and II interacting with circumstellar gas have relied on the assumption that the circumstellar medium (CSM), created by the stellar wind of the progenitor, has a spherically symmetric density distribution (e.g., Chevalier 1982; Chevalier & Fransson 1994). For most SNs this has proven to be a good assumption from which to explain the observations (Chevalier 1984; Chevalier & Fransson 1994), but there is now a growing consensus that at least for some SNe, nonspherical symmetry may be important.

The ring around SN 1987A (e.g., Jakobsen et al. 1991) and the bipolar nebula connected to this ring (Wampler et al. 1991; Wang & Wampler 1992; Burrows et al. 1995; Plait et al. 1995) probably constitute the clearest evidence for nonspherical symmetry. To explain the observed structure Luo & McCray (1991), Wang & Mazzali (1991), Blondin & Lundqvist (1993), Martin & Arnett (1995), and Chevalier & Dwarkadas (1995) used the assumption that the progenitor had a spherically symmetric blue supergiant (BSG) wind that was interacting with a previously lost nonspherically symmetric red supergiant (RSG) wind. The hydrodynamical models by Blondin & Lundqvist (1993) show that if the structure was formed in this way, a high ratio (≥ 20) of equatorial-to-polar mass loss during the RSG stage is necessary. The interacting winds model has gained support from imaging observations by Crotts, Kunkel, & Heathcote (1995), which show an equatorial ring and a structure connected to this, as in the model. Other models for the formation of the CSM around SN 1987A invoke a nonspherically symmetric BSG wind interacting with a spherically symmetric RSG wind (see, for a review, Blondin 1994), but these scenarios are more speculative. Regardless of which model turns out to be closest to the real situation, SN 1987A will start interacting with its CSM in a nonspherically symmetric fashion no later than the point at which it collides with the ring (e.g., Luo, McCray, & Slavin 1994). At least in the model with an asymmetric RSG wind it is clear that the circumstellar interaction would have been quite different from spherical from the start had the SN exploded while still an RSG.

VLBI radio observations of SNe are an especially powerful technique for investigating asymmetric interaction because it is possible to image emission from the interaction region. Marciaud et al. (1995a, 1995b) have produced a series of images of SN 1993J over the age range 6–18 months and found that the image is remarkably symmetric, although there are brightness variations around the shell and the radio structure appears to change with time. An asymmetric CSM around SN 1993J may be indicated by the high degree of polarization observed (Smith 1993; Trammell, Hines, & Wheeler 1993), although this may be owing to an asymmetric explosion. The other two SNe for which there is VLBI imaging, SN 1986J in NGC 891 and 41.9 + 58 in M82, show large asymmetries. SN 1986J, which is thought to have occurred in 1982, shows three protrusions from a shell (Bartel et al. 1991). The remnant 41.9 + 58, which is thought to have occurred in the mid-1950s, shows a shell with two protrusions on opposite sides (Bartel et al. 1987; Wilkinson & de Bruyn 1990). The images
are poorly resolved, but the outer radii of the protrusions are \( \sim 2-3 \) times the shell radius. Chevalier & Blondin (1995) investigated the possibility that the protrusions are the result of Rayleigh-Taylor instabilities in the decelerating thin shell of ejecta, which has radiatively cooled. In fact, the unstable fingers did not distort the outer shock front, so the influence of inhomogeneous density structure was indicated for the protrusions.

Optical line emission can also be an indicator of asymmetric or inhomogeneous structure. Chuigai & Danziger (1994) studied the narrow-line emission from SN 1988Z and suggested that the CSM could either be clumpy or have a high density in the equatorial plane. SN remnants also show some evidence of being relics of SNe interacting with nonspherically symmetric surroundings. In an analysis similar to ours, Igumenschev, Tutukov, & Shustov (1992) find that the asphericity present for \( \sim 30\% \) of remnants observed using the Einstein Observatory (Seward 1990) can be explained if the progenitor had stronger mass loss in the equatorial plane than along the poles. In an attempt to explain the evolution of the optical structure of the synchrotron emission from the Crab Nebula, Fesen, Martin, & Shull (1992) suggested that the Crab progenitor may have also experienced asymmetric mass loss shortly before the explosion.

The fact that some SNe are expected to interact with an asymmetric CSM is not surprising from the structure observed around some RSGs and red giants. In particular, \( \mu \) Cep (e.g., Mauron & Querci 1990) shows evidence of asymmetry, and in the study of the four red giants R Aql, V Hya, g Her, and R Leoby by Plez & Lambert (1994) it was found that all four had a highly asymmetrical distribution of K \( \lambda 7699 \) emission. In the VLA study by Bowers, Johnston, & De Vegt (1989), R Aql and other Mira variables, as well as the supergiant S Per, show evidence for asymmetry. In addition, Trammell, Dinerstein, & Goodrich (1994) find in their study of postasymptotic giant branch stars that a majority of the stars displayed intrinsic polarization, presumably owing to an aspherical distribution of circumstellar matter. These findings may support the currently most popular model for the shaping of planetary nebulae (PNe), in which the fast wind from the central star interacts with a previously lost cylindrically symmetric red giant wind (Kwok, Burton, & Fitzgerald 1978; Kahn & West 1985); i.e., they are very similar to the interacting winds model for the CSM of SN 1987A. If this model is correct, then nonspherically symmetric red giant (and presumably RSG) winds should be common. In fact, Zuckerman & Aller (1986) find that out of 108 PNs, \( \sim 50\% \) displayed bipolar symmetry.

Thus, there is both observational and theoretical support for studying the interaction of SNs with a nonspherically symmetric CSM. It is of particular interest to study the stability of spherically symmetric circumstellar interaction against asymmetry, as well as how different polar density distributions of the circumstellar matter affect the SN/wind interaction. We present our numerical model for the SN/CSM interaction in § 2. In § 3, we discuss the results of our numerical simulations, including the dependence on the asymmetry of the CSM, the dependence on the ejecta density profile, the effect of enforcing axisymmetry, and the effect of asymmetric ejecta. The implications of these results concerning observed SNe are discussed in § 4.

2. MODEL

The numerical model used to study the asphericity of the SN/wind interaction is similar to that used in Chevalier, Blondin, & Emmering (1992): two-dimensional hydrodynamic simulations of a self-similar driven wave (SSDW) using the numerical hydrodynamics code VH-1, and a multidimensional code based on the PPM algorithm of Colella & Woodward (1984). A spherical SSDW was initialized on a two-dimensional numerical grid spanning one quadrant of a sphere, i.e., assuming axisymmetry about the polar axis and reflection symmetry about the equator. The grid was expanded in the radial direction as the SSDW evolved, so that the interaction could be followed for many expansion times. The gas was assumed to be adiabatic, with a ratio of specific heats \( \gamma = 5/3 \). The SSDW was initialized with an outer shock velocity of \( 10^5 \text{c}m^{-1} \) at a radius of \( 10^{13} \text{cm} \). Unless specified otherwise, all of the models shown here have an ejecta density profile described by a power law, \( \rho_e \propto r^{-\alpha} \), and a CSM with a power law in density, \( \rho_o \propto r^{-\beta} \), i.e., \( n = 7 \) and \( s = 2 \) in the SSDW model of Chevalier (1982). The CSM also possessed a variation in density with polar angle (from Blondin & Lundqvist 1993)

\[
\rho(\theta) = C \langle \rho \rangle \left[ 1 - A \frac{\exp \left( -2b \cos^2 \theta \right) - 1}{\exp \left( -2b \right) - 1} \right],
\]

where \( \langle \rho \rangle = M/4\pi r^2 v_e \) is the angle-averaged mass-loss rate. The parameter \( A \) is the value of the wind asymmetry, such that \( \alpha = 1/(1 - A) \) is the ratio of density at the equator to density at the pole, and \( \beta \) is a steepness parameter. The angular dependence of the mass-loss rate given by this function for the values of \( \beta \) used in the simulations is shown in Figure 1. For values of \( \beta \leq 1 \), the density varies gradually from pole to equator, while for \( \beta > 1 \), the density is uniform over most of the circumstellar region but ramps up quickly to the maximum value in a very narrow region near the equator. Thus, large values of \( \alpha \) and \( \beta \) would resemble a dense circumstellar disk, whereas a more gradual density variation is close to what Asida & Tuchman (1995) find for simulations of rotating red giant variables. The normalization constant \( C \) ensures the same total mass-loss rate independent of asymmetry values.

3. RESULTS

An extensive series of simulations were run on a grid of 340 radial zones by 128 angular zones with \( n = 7 \) and \( s = 2 \).
The asymmetry parameters were varied, with $\alpha$ ranging from 2 to 16 and $\beta$ ranging from 0.1 to 8. Each simulation was run for at least 20,000 time steps, corresponding to an expansion of over 8 orders of magnitude in radius.

3.1. Typical Evolution

In all the simulations that we computed, the dense shell of shocked gas near the back of the interaction region was subject to the Rayleigh-Taylor (RT) instability described in detail in Chevalier et al. (1992). This instability reaches saturation at a time of order $100t_0$, or approximately five doubling times. (Here $t_0$ is the age of the SN when the simulation was started with a spherically symmetric SSDW.) Therefore, the structure of the unstable region remains self-similar in an averaged sense. In the case of no asymmetry, the RT fingers reach a length of approximately one-half the width of the interaction region, with relatively little effect on the forward shock. This result is not always the case in the presence of an asymmetry in the ambient wind.

In many of our simulations, the RT fingers push all the way out to the forward shock, producing a local bulging of the shock front. This effect is attributable to the obliquity of both the forward and reverse shocks. In the case of an asymmetric SSDW, the forward shock is typically propagating into the CSM at an oblique angle, while the supersonic ejecta may be impinging the reverse shock at an oblique angle. The post-shock flow behind both shocks is then not purely radial, and the resulting tangential flow may substantially aid the growth of RT fingers. In particular, when the forward/reverse shock pair forms an inflection point, such as at the polar tip or near the cusp seen in high-$\beta$ models, the obliquity of the shocks tends to create a pair of opposing vortices. The updraft (flow in the positive radial direction) created between these paired vortices will enhance the growth of RT fingers (which are normally accompanied by similar but much weaker pairs of vortices), leading to RT fingers that extend all the way to the forward shock. Once the finger pushes on the forward shock, the bulge of the shock front increases the obliquity of the shock. This enhances the strength of the vortices, leading to a virtually permanent RT finger. This is in contrast to the spherical case, in which the RT fingers extend out, fall over, and advect back into the thin shell of shocked ejecta, never affecting the forward shock.

Figure 2 shows the result of a calculation for a high-$\beta$ wind: the wind is spherically symmetric except for a narrow region close to the equatorial plane. There is an indentation in the equatorial plane, as expected, but there are also two slight bulges in the forward shock, caused by unusually large RT fingers. One of these protrusions lies just above the indentation near the equatorial plane and is created by the obliquity of the forward and reverse shocks (resulting from the asymmetric CSM), as described above. The strong vorticity of the post-shock flow is evident in this plot, with the updraft side of each vortex moving in the direction of growth of the RT finger that reaches most of the way to the forward shock.

The other protrusion lies on the polar axis, and although it has the characteristics described above, we believe that it is an artifact of the axial symmetry in the two-dimensional calculation and would not occur in a three-dimensional calculation. The origin of this protrusion lies in the fact that an RT finger on the symmetry axis does not have a chance to bend and fall over, as in other parts of the shell. As a result, it can grow substantially longer than if it were not on the numerical axis. We have seen this effect in two-dimensional axisymmetric simulations of spherical blast waves, and for that reason it did not include the polar axis in the simulations described in Chevalier et al. (1992). Furthermore, the ambient density in the simulation shown in Figure 2 is very nearly spherically symmetric in the vicinity of the polar axis, yet Chevalier et al. (1992) did not see any such protrusions for spherical SSDWs. Finally, we ran three-dimensional simulations with a spherical CSM and did not see any protrusions.

However, as described in § 3.2, we have found that an angular density gradient on the axis can greatly enhance the growth of a protrusion on the axis. We performed a three-

![Figure 2](image_url)

**Fig. 2**—Density (right) and velocity (left) of an evolved SSDW propagating into an asymmetric CSM, described by $\beta = 8$, $\alpha = 2$. Density in the right image is displayed with both logarithmic contours and shading, although the shading is only used within the interaction region to highlight the shell of dense shocked ejecta and the Rayleigh-Taylor fingers that emanate from this shell. Velocity vectors in the left image are plotted in the expanding self-similar frame, and the locations of the forward and reverse shocks are marked with heavy solid lines. No vectors are plotted for the undisturbed CSM or ejecta.
3.2. Dependence on Asymmetry

The asymmetry of the SSDW is a monotonic function of the asymmetry in the circumstellar density distribution. However, the asymmetry of the SSDW is not easy to quantify because it varies significantly with time. In virtually all cases, the SSDW does not approach a truly constant shape because of subsonic flow within the interaction region, e.g., the Rayleigh-Taylor instability. To account for the temporal variations in shape while attempting to illustrate the asymmetry in the SSDW, we have plotted the time history of the aspect ratio, $R_p/R_e$, of the SSDWs in Figures 3 and 4. Here $R_p(R_e)$ is the distance from the center of the blast wave to the outer shock in the polar (equatorial) direction. The ratio $R_p/R_e$ is plotted as a function of $t/t_0$, where $t_0$ is the age of the SSDW when the simulation was started from spherically symmetric initial conditions. Note that because of the problems with the polar axis described above, low aspect ratios ($R_p/R_e \sim 1.1$) are at least partially an artifact of the two-dimensional axisymmetry. The interior flow leads to many bumps and wiggles in these plots of aspect ratio, but the ratio tends to hover around a well-defined value.

For comparison, we have plotted the aspect ratio expected under the assumption of purely radial flow. This would be expected if the sound speed in the interaction region was much slower than the expansion velocity of the SSDW. Alternatively, one might try to solve for the shape of

![Figure 3](image-url)  
**Fig. 3.**—Evolution of the aspect ratio ($R_p/R_e$) of an SSDW propagating into an asymmetric CSM, described by a value of $\beta = 8$ and a value of $z$, as labeled. Dashed lines are the values of the aspect ratio predicted in the limit of no tangential flow for the same values of $z$.

![Figure 4](image-url)  
**Fig. 4.**—Evolution of the aspect ratio of an SSDW propagating into an asymmetric CSM. Each plot shows the evolution of four simulations corresponding to $\beta = 0.1, 1, 4,$ and $8$ ($\alpha = 16$ does not have a curve for $\beta = 0.1$). In each case, the simulation with the largest aspect ratio corresponds to the smallest value of $\beta$. 

...
the forward shock under the assumption that there is uniform pressure within the interaction region, i.e., in the limit that the sound speed is much larger than the expansion velocity. However, in this problem the expansion velocity is given by the shock velocity of the forward shock, \( v_{sh} \), and the sound speed immediately behind a strong adiabatic shock is given by

\[
c^2 = \gamma \frac{2(y - 1)}{(y + 1)^2} v_{sh}^2.
\]

Thus, the sound speed in the intershock region is always of order the expansion velocity, and neither approximation will be valid. Nonetheless, we would expect that the assumption of purely radial flow will give a lower limit to the aspect ratio. The tangential flow behind the shock will be driven by the high pressure at the equator relative to the pole, creating a flow that will decrease the postshock pressure at the equator and increase the postshock pressure at the pole, similar to what is seen in the simulations by Blondin & Lundqvist (1993). This, in turn, will drive the aspect ratio further from unity. From dimensional analysis, the radius of the SSDW under the assumption of radial flow is \( R \propto \rho(0)^{1/(\alpha - \beta)} \) (Chevalier 1982). A lower limit on the aspect ratio is therefore \( R_{ss} / R_o \propto \alpha^{1/(\alpha - \beta)} \).

As seen in Figure 3, the aspect ratio of the SSDW increases uniformly with increasing asymmetry. The strong variations in the aspect ratio over time are attributed to the growth and motion of Rayleigh-Taylor fingers emanating from the shell of shocked ejecta. In particular, the forward shock is modified by two such fingers: one at the polar axis and one at the cusp formed just above the equator in high-\( \beta \) models (see Fig. 2 corresponding to the run in Fig. 3 with \( \alpha = 2 \)). As the SSDW evolves, the strong vortex flow within the interaction region tends to drag smaller RT fingers toward these two sustained fingers. When these smaller features merge with one of the larger fingers, the sustained finger pushes out a little further on the forward shock, leading to a change in the overall aspect ratio. In particular, the sharp rises in the aspect ratio for \( \alpha = 2 \) seen in Figure 3 are associated with the mergers of two RT fingers with the RT finger fixed on the polar axis (see Fig. 2).

As the value of \( \beta \) is decreased, the aspect ratio stays relatively constant until \( \beta \) reaches a critical value, at which the aspect ratio suddenly increases. This effect is illustrated in the series of plots shown in Figure 4. For all values of \( \alpha \) used in our simulations, low values of \( \beta \) produced aspect ratios far greater than the apparently constant aspect ratio seen at high values of \( \beta \). Furthermore, there appears to be only two self-similar values of the aspect ratio for a given value of \( \alpha \): a low value (consistent with expectations) for high values of \( \beta \), and an anomalously high value for low values of \( \beta \) (see, e.g., \( \alpha = 8 \) in Fig. 4). This critical value of \( \beta \), below which the aspect ratio is anomalously high, is slightly dependent on the value of \( \alpha \). For \( \alpha = 2 \), the SSDW with \( \beta = 1 \) was similar to the high-\( \beta \) simulations with an aspect ratio of about 1.2, while the SSDW with \( \beta = 0.1 \) produced an aspect ratio of almost 2. In contrast, for \( \alpha = 16 \) an anomalously large aspect ratio was achieved, with a value of \( \beta \) as high as 1. Note also that the value of this anomalously high aspect ratio increases monotonically with \( \alpha \).

These large aspect ratios in the case of small values of \( \beta \) are attributed to the formation of a "protrusion" extending out along the symmetry axis of the circumstellar density distribution—i.e., the SSDW appears to "poke through" the low-density hole in the CSM. The formation of such a protrusion is illustrated in Figure 5, where we have displayed the results from a model with \( \alpha = 4 \) and \( \beta = 0.1 \) at several epochs. This particular run was computed on a grid of 640 radial zones by 300 angular zones in order to better resolve the flow in the narrow protrusion and to resolve the flow in the interaction region, which becomes a relatively small fraction of the total radial extent of the grid by the time the protrusion reaches a self-similar state. Furthermore, the angular zones were unevenly spaced, in such a way that roughly half of the zones are within \( \pi/8 \) of the polar axis. The overall shape and evolution of this model does not differ significantly from a similar model run on our standard low-resolution grid.

The growth of the protrusion is driven by a tangential flow within the shock interaction region that is heading toward the polar axis. This flow is already evident in the first epoch shown in Figure 5, and it persists throughout the
growth of the protrusion. This flow is driven primarily by a pressure gradient set up in the shock interaction region by the slightly nonspherical forward shock. Because the CSM near the pole is less dense than that near the equator, the shock radius extends out slightly further in the polar direction. Since the postshock pressure scales as \( P \propto \rho v^2 \propto \rho_0(\theta)(R^{-2})(R^2) \propto \rho(\theta) \), the postshock pressure is lower behind the forward shock at the pole compared to the equator, and this pressure gradient in the \( \theta \) direction drives the tangential flow. The slight obliquity of the forward shock also deflects the immediate postshock flow, but in this case it does so away from the polar axis. The deceleration of this shock-deflected flow increases the \( \theta \) pressure gradient and sets up a vortex flow within the shock interaction region. Note that the reverse shock is also nonspherical and it deflects its postshock flow toward the pole. However, the velocity of the reverse shock is much lower than the forward shock, so the postshock flow velocity and the postshock sound speed are relatively small. As a result, the tangential flow within the shocked ejecta is only about one-fifth of the tangential flow velocity within the shocked CSM.

The tangential flow in the shocked CSM is quite strong in the first two frames shown in Figure 5, yet the protrusion of the forward shock has not begun to form. Instead, the protrusion forms only after a significant amount of shocked ejecta has advected to the polar axis. This flow of shocked ejecta is created in part by the obliquity of the reverse shock, but it appears to be dominated by the advection of RT fingers by the shocked CSM. As RT fingers begin to grow from the contact discontinuity, as seen in the second frame of Figure 5, the tangential flow of shocked CSM starts to drag these fingers toward the pole. Once these RT fingers of shocked ejecta reach the pole, they coalesce to form one large RT finger aligned with the polar axis. This gas has nowhere to go but up, and a protrusion is formed. Without the RT fingers of shocked ejecta, the shocked CSM might simply turn around (owing to its high sound speed compared to the shocked ejecta), completing a large vortex within the shock interaction region. However, even this flow pattern would eventually drag the shocked ejecta toward the axis as a result of the Kelvin-Helmholtz instability along the contact between shocked CSM and shocked ejecta.

Once a significant protrusion is formed, the flow changes character in that the tangential flow through the interaction region becomes less important compared to the strong vortex generated by the protruding forward shock. This new flow pattern is beginning to appear in the last frame of Figure 5, and it becomes more dominant as the protrusion grows. The protrusion eventually reaches a self-similar structural pattern along the polar axis for \( \beta = 0.1 \) and \( \alpha = 4 \). Right-hand image displays the density, as in Fig. 2. Left-hand image displays the entropy of the gas, as in Fig. 5.

Fig. 6.—Late time structure of the protrusion along the polar axis for \( \beta = 0.1 \) and \( \alpha = 4 \). Right-hand image displays the density, as in Fig. 2. Left-hand image displays the entropy of the gas, as in Fig. 5.
state, shown in Figure 6, and the aspect ratio becomes relatively constant. At this point, the protrusion appears to be a local (in $\theta$) phenomenon and does not depend on the pressure-driven flow of gas from the vicinity of the equator. We have run simulations with the same asymmetry parameters, but with the asymmetry function applied only for $\theta < 0.1\pi$, and the SN evolves to the same self-similar state. The large value of $R_p/R_e$ is driven by the concentration of a relatively large amount of ejecta into a small region of the decelerating flow.

The flow pattern in the protrusion is illustrated in more detail in Figure 7. In this self-similar state, the CSM strikes the forward shock of the polar protrusion at a very oblique angle. The shocked CSM is therefore directed almost straight down, parallel to the polar axis. The supersonic (in the self-similar frame) downdraft along the outside of the protrusion is decelerated in a strong shock at the base of the protrusion. The high gas pressure between the nominal forward and reverse shocks squeezes the flow toward and up the polar axis. The vortex ring created by this redirection of the flow forms a de Laval nozzle on the axis, which accelerates the flow up the polar axis to Mach numbers of order 5. The updraft is further collimated by a series of weak incident shocks, as seen in the plot of gas pressure in Figure 7. By the time the flow reaches the head of the protrusion, it is confined to a very narrow “jet” with a width of only 1/10 the length of the protrusion. At the top of the SN, this updraft is decelerated in another strong shock, similar to the working surface associated with supersonic jet outflows.

The parameter determining the presence of a polar protrusion appears to be the gradient in the circumstellar density distribution as a function of angle near the pole. For the circumstellar distribution given by equation (1), the gradient is zero at the pole; therefore, we consider

$$\frac{d \ln \rho}{d \ln \theta} = -(\alpha - 1) \left( \frac{e^{-2\beta}}{e^{-\beta} - 1} \right) 4\beta.$$  

For the two models that appear just on the edge of an anomalous solution ($\alpha = 4$, $\beta = 1$ and $\alpha = 2$, $\beta = 0.1$), this quantity is 1.8. Given that the polar protrusion depends only on the density distribution within $\theta \sim \pi/8$ of the polar axis, we note that the density at the pole need only be $\sim 15\%$ smaller than the rest of the CSM (see Fig. 1). For comparison, the asymmetry function used in Luo & McCray (1991) gives

$$\frac{d \ln \rho}{d \ln \theta} = 2(\alpha - 1).$$

Thus, for $\alpha \gtrsim 2$ the Luo & McCray function should produce SSDWs with an anomalously large aspect ratio. For $\alpha = 2$, our simulation using the Luo & McCray function produced a protrusion, resulting in an SSDW with an aspect ratio of $\sim 2$.

We note that our results are nearly unaffected by radiative cooling of the shocked ejecta. Using the numerical methods of Chevalier & Blondin (1995) for treating cooling of the shocked ejecta, we computed a simulation with $\alpha = 4$ and $\beta = 1$. These parameters were chosen because they are close to the transition from the expected asymmetry to the anomalous solution. The resulting evolution was similar to the purely adiabatic case, forming a protrusion only after extended evolution (see, for the adiabatic case, Fig. 4).

3.3. Three-dimensional Simulation

A significant limitation of the two-dimensional simulations described above is the assumption of axisymmetry. This assumption is particularly questionable in the low-$\beta$ cases when a protrusion extends up the symmetry axis. The requirement of reflection symmetry on the axis means that this protrusion cannot be deflected off to one side or the other; it will just continue to grow along the axis. To check the dependence of the anomalous aspect ratio on the forced axisymmetry in the two-dimensional simulation, we ran a three-dimensional simulation with $\alpha = 10$ and $\beta = 1$.

The three-dimensional simulation was run on a numerical grid of 235 (r) $\times$ 128 ($\theta$) $\times$ 128 ($\phi$) zones. The grid was laid out with the polar axis of the spherical grid in the equatorial plane of the circumstellar material, in such a way so that the symmetry axis of the circumstellar density distribution did not coincide with any unique axis on the numerical grid. Thus, $\theta$ and $\phi$ both ranged from 0 to $\pi$. Note that this leads to a spatial resolution in the angular direction of only one-half that used in the two-dimensional models. To avoid severe constraints on the time step as a result of vanishingly small zone sizes in the $\phi$ direction near the polar axis of the grid, the density, pressure, and $\phi$ veloci-

Fig. 7.—A close up of the polar protrusion from the model shown in Fig. 6. Shading on the right corresponds to the logarithm of the gas pressure, spanning a range of a factor of 2 in the logarithm. For reference, the velocity vectors are plotted every fourth numerical zone in the angular direction, and every eighth zone in the radial direction.
ties were smoothed over within a small cone about the axis. While this produces a noticeable effect in the simulation, it only affects a small region in the equatorial plane of the SSDW and therefore does not affect the goal of the simulation—to quantify the aspect ratio of the SSDW and the stability of the polar protrusion.

The three-dimensional model evolved almost identically to the corresponding two-dimensional simulation. Figure 8 shows a slice of the three-dimensional model next to a plot from a two-dimensional simulation with identical parameters and at roughly the same evolutionary time. Note that the SSDW has not yet reached the steady state value of the aspect ratio, which for this case should be $\sim 3.8$. The shape and flow patterns in the two simulations appear qualitatively similar. In particular, the three-dimensional model has a similar polar protrusion to that seen in the two-dimensional models. The flow of material up the polar axis is not entirely straight because of the influence of the Kelvin-Helmholtz instability expected in the presence of the strong shear flow between the downdrafts and updrafts within the protrusion. Although the flow is not exactly axisymmetric, it appears that the assumption of axisymmetry is reasonable.

The strong similarity between the two- and three-dimensional simulations suggests that the forced axisymmetry in two dimensions does not affect the SSDW for large aspect ratios. In particular, the small ($R_p/R_0 \sim 1.1$) polar protrusion that we attributed to an artifact of the axisymmetry in two-dimensional simulations with high values of $\beta$ does not affect the aspect ratio for low-$\beta$ models in which $R_p/R_0 > 1$.

To test the stability of the vortex tube and associated polar protrusion, we evolved the three-dimensional SSDW into an asymmetric circumstellar medium. A cloud with 3 times the ambient density was placed on one side of the blast wave in an attempt to “knock” the protrusion over. This experiment failed to produce any significant effects, suggesting that this unusual flow pattern is relatively robust.

### 3.4. Dependence on Ejecta Density Profile

The effect of an axisymmetric CSM is expected to be more pronounced for smaller values of $n$ because of the stronger dependence of the expansion velocity on the circumstellar density in such cases (Chevalier 1982). To examine the effect of varying $n$, we have run simulations with steeper ejecta density profiles ($n = 10$ and 15).

For simulations with large $\beta$, where the anomalous aspect ratio does not come into play, the aspect ratio behaves as expected. Figure 9 shows the evolution of the aspect ratio for runs with $\alpha = 4$ and $\beta = 4$. At late times, the aspect ratio is comparable to the analytic estimate of $\alpha^{1/(n-s)}$.

Given the smaller aspect ratio of higher $n$ SSDWs, one would also expect the need for a larger density gradient in order to stimulate the growth of a polar protrusion. Figure 9 also shows the aspect ratio for simulations with $\beta = 0.1$, which in the case of $n = 7$ produced an anomalous aspect ratio of $\sim 2.5$. For $n = 10$, a protrusion begins to form at the pole, but it is much smaller than the $n = 7$ SSDW. For $n = 15$, there is an RT finger that pushes on the forward shock at the pole, but it is not able to generate a large protrusion, as with lower $n$ simulations. If, on the other hand, the asymmetry in the CSM is increased to produce the same canonical aspect ratio in an $n = 10$ SSDW as in an SSDW with $\alpha = 4$ and $n = 7$ (i.e., $\alpha = 10$ for $n = 10$), then the polar protrusion is again unusually large and is, in fact, very similar to that in the $n = 7$ simulation. The critical density gradient in the CSM needed to produce a polar protrusion is therefore larger for larger values of the ejecta exponent, $n$.

### 3.5. Axisymmetric Ejecta

Finally, we consider the complementary problem of axisymmetric stellar ejecta driving an SSDW into a spherically symmetric CSM. We use the same asymmetry function (eq. [1]) to describe the density distribution of the stellar ejecta, and we evolve the SSDW into a spherically symmetric CSM. The result is a much smaller effect on the aspect ratio of the SSDW. For $\alpha = 4$ and $\beta = 0.1$, the aspect

![Figure 8](image_url)  
Fig. 8.—Two-dimensional slice of the three-dimensional model (left) appears qualitatively similar to an axisymmetric two-dimensional model (right) with the same parameters (in this case, $\beta = 1$, $\alpha = 10$). Note that these simulations have not yet reached the maximum aspect ratio expected for these parameters.
ratio of the remnant is only $\sim 0.9$, with the equatorial radius larger than the polar radius. The assumption of purely radial flow predicts an aspect ratio of 0.76.

This difference in the resulting aspect ratio can be attributed to at least two effects: changes in the pressure of the intershock region caused by changes in shock radius, and the direction of tangential flow. In the case of an asymmetric CSM, the higher density CSM pinches in the SSDW, which tends to increase the postshock pressure. Any tangential flow generated by this increased pressure will be away from the high-density CSM and will therefore increase the aspect ratio. In the case of asymmetric ejecta, the higher density ejecta pushes out on the SSDW. While the higher density ejecta will increase the pressure of the intershock region, pushing the shock outward will lower this pressure (as opposed to the asymmetric CSM, which increases the pressure when it moves the shock inward). Furthermore, in the asymmetric ejecta case, any tangential flow generated by the overpressure region will serve to drive the aspect ratio back toward unity.

4. DISCUSSION

The most novel aspect of our calculations is the development of protrusions for moderate values of the angular density gradient at the poles. Only a mild asymmetry ($\rho_p/\rho_e \sim 2$) is needed to form the protrusion, as long as there is a moderate angular density gradient at the pole. Protrusions that are possibly related have been found in models of the interacting winds scenario for planetary nebulae and other objects. Icke et al. (1992) found the growth of a jetlike feature along the polar axis if they set up a low density in the slow external wind along the polar axis. The angular dependence of density that they chose in the slow wind is the inverse of that used here and tends to create a channel along the polar axis. In both cases, the backflow down the sides of the narrow outflowing gas helps to keep the protrusion in place. In the planetary nebula case, the collimation is aided by the fact that the termination shock of the fast, central wind is elongated in the polar direction in such a way that the newly shocked gas receives a velocity component directed toward the pole. This effect does not appear to be especially significant in our calculations (see the reverse shock front in Figs. 6 and 7). What is significant is the presence of a reverse shock relatively close to the forward shock, so that the backflow is confined and redirected back up the polar axis. In addition, the deceleration of the interaction shell leads to Rayleigh-Taylor instabilities and vortical motion in the shell, which aid in the formation of the protrusion. In essence, the vortex surrounding the polar axis becomes highly elongated.

The clearest application of the calculations of protrusion development is to the remnant 41.9 + 58 in M82. The radio image shows a compact shell (so that circumstellar interaction is plausible) with two oppositely directed protrusions (Bartel et al. 1987; Wilkinson & de Bruyn 1990). As noted by these authors, the morphology is unusual for an SN remnant. We would identify the long axis with the symmetry axis of an axisymmetric dense presupernova wind.

Fig. 9.—Aspect ratio of SSDWs as a function of the power-law exponent of the ejecta density profile, $n$. For large values of $\beta$ (left), the aspect ratio decreases with increasing $n$, in agreement with analytic prediction (dashed lines). For small values of $\beta$ (right), the polar protrusion is prominent for low $n$, present, but it is small for $n = 10$, and essentially absent for $n = 15$. Asymmetry in all of these runs is $a = 4$. 

$\beta = 4.0$

$\beta = 0.1$

$\frac{R_p}{R_e}$ vs $\log (t/t_\text{sh})$
remaining problem is whether there is sufficient time for the protrusions to develop, because Figure 4 shows that a large increase in time can be necessary to approach the self-similar state. The presence of a dense wind implies that the SN progenitor was red supergiant star. If the initial stellar parameters are similar state. The presence of a dense wind implies that the reverse shock is radiative in a particular supernova, then the formation of protrusions can be expected to influence the optical emission-line profiles in supernova spectra. However, the heating and ionization of the gas is a complicated problem that is beyond the scope of this paper. Considering the widespread evidence for asymmetric structure in dense winds, this problem is worthy of attention.

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REFERENCES

Asida, S. M., & Tuchman, Y. 1995, ApJ, 455, 286
Bartel, N., et al. 1987, ApJ, 323, 505
Bartel, N., Rupen, M. P., Shapiro, I. I., Preston, R. A., & Rius, A. 1991, ApJ, 379, L63
Blondin, J. M. 1994, in Circumstellar Media in the Lates Stages of Stellar evolution, ed. R. E. S. Clegg, W. P. S. Meikle, & I. R. Stevens (Cambridge: Cambridge Univ. Press), 139
Blondin, J. M., & Lundqvist, P. 1993, ApJ, 405, 337
Bowers, P. F., Johnston, K. J., & De Vegt, C. 1989, ApJ, 340, 479
Burrows, C. J., et al. 1995, ApJ, 452, 680
Chevalier, R. A. 1982, ApJ, 258, 790
Chevalier, R. A., & Blondin, J. M. 1995, ApJ, 444, 312
Chevalier, R. A., Blondin, J. M., & Emmering, R. T. 1992, ApJ, 392, 58.
Chugai, N. N. 1993, ApJ, 414, L101
Chugai, N. N., & Danziger, I. J. 1994, MNRAS, 268, 173
Colella, P., & Woodward, P. R. 1984, J. Comput. Phys., 54, 174
Crotts, A. P. S., Kunkel, W. E., & Heathcote, S. R. 1995, ApJ, 438, 724
Fesen, R. A., Martin, C. L., & Shull, J. M. 1992, ApJ, 399, 599
Icke, V., Meliema, G., Balick, B., Eulderink, F., & Frank, A. 1992, Nature, 355, 524
Igumenshchev, I. V., Tutukov, A. V., & Shustov, B. M. 1992, Soviet Astron., 36, 241

The other SN with asymmetric VLBI structure, SN 1986J, is a less promising case for comparison with our theory because the image shows three protrusions at roughly 120° intervals (Bartel et al. 1991). This is not compatible with the axisymmetric structure that we have assumed here. However, there is evidence that the CSM around SN 1986J is inhomogeneous (Chugai 1993), and there is the possibility that inhomogeneities are able to trigger the growth of protrusions.

Unfortunately, radio imaging is possible for only a small number of supernovae. If the reverse shock is radiative in a particular supernova, then the formation of protrusions can be expected to influence the optical emission-line profiles in supernova spectra. However, the heating and ionization of the gas is a complicated problem that is beyond the scope of this paper. Considering the widespread evidence for asymmetric structure in dense winds, this problem is worthy of attention.

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REFERENCES

Asida, S. M., & Tuchman, Y. 1995, ApJ, 455, 286
Bartel, N., et al. 1987, ApJ, 323, 505
Bartel, N., Rupen, M. P., Shapiro, I. I., Preston, R. A., & Rius, A. 1991, ApJ, 379, L63
Blondin, J. M. 1994, in Circumstellar Media in the Lates Stages of Stellar evolution, ed. R. E. S. Clegg, W. P. S. Meikle, & I. R. Stevens (Cambridge: Cambridge Univ. Press), 139
Blondin, J. M., & Lundqvist, P. 1993, ApJ, 405, 337
Bowers, P. F., Johnston, K. J., & De Vegt, C. 1989, ApJ, 340, 479
Burrows, C. J., et al. 1995, ApJ, 452, 680
Chevalier, R. A. 1982, ApJ, 258, 790
Chevalier, R. A., & Blondin, J. M. 1995, ApJ, 444, 312
Chevalier, R. A., Blondin, J. M., & Emmering, R. T. 1992, ApJ, 392, 118
Chevalier, R. A., & Dwarkadas, V. V. 1995, ApJ, 452, L45
Chevalier, R. A., & Fransson, C. 1994, ApJ, 420, 268
Chugai, N. N. 1993, ApJ, 414, L101
Chugai, N. N., & Danziger, I. J. 1994, MNRAS, 268, 173
Colella, P., & Woodward, P. R. 1984, J. Comput. Phys., 54, 174
Crotts, A. P. S., Kunkel, W. E., & Heathcote, S. R. 1995, ApJ, 438, 724
Fesen, R. A., Martin, C. L., & Shull, J. M. 1992, ApJ, 399, 599
Icke, V., Meliema, G., Balick, B., Eulderink, F., & Frank, A. 1992, Nature, 355, 524
Igumenshchev, I. V., Tutukov, A. V., & Shustov, B. M. 1992, Soviet Astron., 36, 241

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