ENDPOINT ESTIMATES FOR COMMUTATORS OF INTRINSIC SQUARE FUNCTIONS IN MORREY TYPE SPACES

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Abstract. In this paper, the boundedness properties of commutators generated by $b$ and intrinsic square functions in the endpoint case are discussed, where $b \in BMO(\mathbb{R}^n)$. We first establish the weighted weak $L \log L$-type estimates for these commutator operators. Furthermore, we will prove endpoint estimates of commutators generated by $BMO(\mathbb{R}^n)$ functions and intrinsic square functions in Morrey type spaces. In particular, we can obtain endpoint estimates of these commutators in the weighted Morrey spaces $L^{1,\kappa}(w)$ for $0 < \kappa < 1$ and $w \in A_1$, and in the generalized Morrey spaces $L^{1,\Theta}$, where $\Theta$ is a growth function on $(0, +\infty)$ satisfying the doubling condition.

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