Free-fall accretion and emitting caustics in wind-fed X-ray sources

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ABSTRACT

In wind-fed X-ray binaries the accreting matter is Compton cooled and falls freely onto the compact object. The matter has a modest angular momentum \( l \) and accretion is quasi-spherical at large distances from the compact object. Initially small non-radial velocities grow in the converging supersonic flow and become substantial in the vicinity of the accretor. The streamlines with \( l > (GMa^2)^{1/2} \) (where \( M \) and \( a \) are the mass and radius of the compact object) intersect outside \( R_a \) and form a two-dimensional caustic which emits X-rays. The streamlines with low angular momentum, \( l < (GMa^2)^{1/2} \), run into the accretor. If the accretor is a neutron star, a large X-ray luminosity results. We show that the distribution of accretion rate/luminosity over the star surface is sensitive to the angular momentum distribution of the accreting matter. The apparent luminosity depends on the side from which the star is observed and can change periodically with the orbital phase of the binary. The accretor then appears as a ‘Moon-like’ X-ray source.

Key words: accretion, accretion discs — binaries: general — black hole physics — radiative mechanisms — stars: neutron — X-rays: stars

1 INTRODUCTION

Wind-fed accretion is believed to occur in massive X-ray binaries (see e.g. King 1995 for a review). The massive donor companion (OB star) produces a substantial wind, up to \( M_w \sim 10^{-6} M_\odot/\text{yr} \), which is partly captured by the compact companion. The wind material is captured from an accretion cylinder of radius

\[
R_a = \frac{2GM}{w^2}
\]  

where \( w \) is the wind velocity and \( M \) is the mass of the accretor (Hoyle & Lyttleton 1939, Bondi & Hoyle 1944). The typical \( w \approx 10^8 \text{ cm s}^{-1} \) for OB stars, so that \( R_a \approx 3 \times 10^{10}(M/M_\odot)(w/10^8)^{-2} \text{ cm} \). If the wind is isotropic then the accretion rate is \( \dot{M} \approx (1/4)(R_a/A)^2 M_w \) where \( A \) is the binary separation. The accretion rate is substantial in close binaries only, with orbital periods \( P \sim \text{ few days} \). The captured fraction, \( \dot{M}/M_w \), can be increased if the donor is a Be star which has a prominent slow equatorial wind (see e.g. van Paradijs & McClintock 1995) or if the wind is prefocused by the tidal effects (Blondin, Stevens & Kallman 1991).

Gas captured from the accretion cylinder falls many decades in radius down to the radius of the compact object \( R_a \) where the X-rays are produced. \( R_a \) equals \( r_g = 2GM/c^2 \approx 3 \times 10^{3}(M/M_\odot) \text{ cm} \) for a black hole (BH) and about \( 3r_g \) for a neutron star (NS). Owing to orbital rotation of the binary, the captured gas possesses a net angular momentum with respect to the accretor. The average angular momentum \( \bar{l} \) can be estimated (Illarionov & Sunyaev 1975; Shapiro & Lightman 1976). It is directed perpendicularly to the binary plane and equals \( \bar{l} = \zeta(1/4)\Omega R_a^2 \) where \( \Omega = 2\pi/P \) is the angular velocity of the binary and the numerical factor \( \zeta \approx 1 \) depends on adopted assumptions (see e.g. Wang 1981; Livio et al. 1986; Ruffert 1997, 1999). The angular momentum is small and the infall is radial at \( R \gg R_a \).

Compton cooling by the central X-ray source makes the inflow highly super-sonic inside the Compton radius, \( R_C \sim 10^{10} \text{ cm} < R_a \) (see Illarionov & Kompaneets 1990 and Section 2.2). Initially small non-radial velocities \( v_\perp = l/R \) grow in the freely falling flow and exceed the radial velocity component \( v_r \sim (GM/R)^{1/2} \) at \( R_d \sim l^2/GM \). Accretion can be assumed to be radial if \( R_d < R_c \), i.e. if \( l < l_\odot \) where

\[
l_\odot = (GMa^2)^{1/2}.
\]  

The deviations from the radial pattern are important if \( l \) is comparable to \( l_\odot \).

In BH and NS binaries, \( l_\odot \approx r_g c \) and \( l_\odot = (3/8) \approx 1.5 P^{-1}(M/M_\odot)(w/10^8)^{-4} \text{ cm} \) where the binary period \( P \) is measured in days. The observed \( P \) in massive (OB) X-ray binaries is typically a few days (White, Nagase & Parmar 1995; Tanaka & Lewin 1995). One thus concludes that \( l_\odot \) is about \( l \) in these systems. Note that the angular momentum of a particular streamline varies substantially around the average value. For instance, if the flow is in solid body rotation at \( R \gg R_a \) then \( l \) is highest for streamlines in the equatorial plane and vanishes on the polar axis.

Matter with \( l < l_\odot \) runs directly into the accretor before it reaches the equatorial plane. If the accretor is a neutron star then a strong shock results and X-rays are produced (Zeldovich &
This transformation induces a map \( \mathbf{S} \) (Shakura 1969; Shapiro & Salpeter 1975). The surface brightness of the star is determined by the distribution of the accretion rate over its surface, \( dM/dS \). In this paper, we find that \( dM/dS \) is sensitive to the angular momentum distribution in the flow. We assume a weakly magnetised NS (\( B \lesssim 10^8 \) G), so that the magnetic field does not affect the ballistic trajectories of the freely falling matter. The resulting surface brightness of the star is inhomogeneous and the apparent luminosity depends on the side from which the star is observed. The apparent luminosity can then change as the binary executes its orbital period (we dub it the ‘Moon’ effect).

By contrast, if the accretor is a black hole then matter with \( l < l_s \) plunges into the event horizon without producing substantial emission.

The streamlines with \( l > l_s \) intersect in the equatorial plane (the plane of symmetry) at \( R > R_a \). The loci of the intersections form a two-dimensional caustic. If the accretor is a black hole then the caustic is the only source of X-rays from the accretion flow.

The paper is organised as follows. In Section 2 we briefly review the pattern of wind-fed accretion on large scales, at distances \( \sim R_a \) from the accretor. In Section 3 we write down the equations of the freely falling flow inside the Compton radius. In Sections 4 and 5 we focus on the very vicinity of the compact object. We discuss asymmetric accretion onto the surface of a NS (Section 4) and then caustics outside the accretor (Section 5).

## 2 WIND-FED ACCRETION ON LARGE SCALES

### 2.1 The trapping of the wind matter

The radius of the accretion cylinder is small compared to the binary separation, \( R_a \lesssim 10^{11} \text{ cm} \ll A \sim 10^{12} \text{ cm} \), and hence the flow in the cylinder is nearly plane-parallel before it gets trapped by the gravitational field of the accretor. The wind reaches the compact companion on a time-scale \( A/w \sim 10^5 \) s which is much shorter than the orbital period \( P \sim 10^6 \) s. As a first approximation, one can assume the accretor to be at rest and the flow to be axisymmetric around the line connecting the two companions.

Let us introduce coordinates \((x, y, z)\) so that the \( y\)-axis is directed from the donor to the accretor and the \( z\)-axis is perpendicular to the binary plane, and choose the coordinate origin at the location of the accretor (see Fig. 1). Each streamline of the flow is specified by two impact parameters \((x_0, z_0)\) at \( R \gg R_a \). The initial angular momentum of a streamline \((x_0, z_0)\) is \( I = (x_0w, 0, x_0w)\) and its absolute value equals \( l = bw \) where \( b = (x_0^2 + z_0^2)^{1/2} \). The net integral over a ring \((b, b+db)\) vanishes, which is a consequence of the assumed symmetry around the \( y\)-axis.

For an initially super-sonic wind, a bow shock forms at distance \( \sim R_a \) from the accretor. Hunt (1971) first studied gas dynamics behind the shock and showed that a spherically symmetric inflow forms at \( R < R_a \) (see also Petrich et al. 1989; Ruffert 1997, 1999). Fig. 1 shows the picture of accretion. The transformation of the uniform plane-parallel flow into the isotropic spherical inflow can be described as follows. A streamline with an initial impact parameter \( b \) eventually infalls radially at some angle \( \beta \) with respect to the \(-y\) axis,

\[
\left( \frac{b}{R_a} \right)^2 = \frac{1 - \cos \beta}{2}, \quad b^2 = x_0^2 + z_0^2.
\]

This transformation induces a map \((x_0, z_0) \rightarrow (\theta, \varphi)\).

\[
\frac{x_0}{R_a} = \frac{\sin \theta \sin \varphi}{\sqrt{2(1 + \sin \theta \cos \varphi)}}, \quad \frac{z_0}{R_a} = \frac{\cos \theta}{\sqrt{2(1 + \sin \theta \cos \varphi)}}.
\]  

Here \( \theta \) is the polar angle measured from the \( z\)-axis and \( \varphi \) is the azimuthal angle measured in the \( xy \) plane from the \((-y)\)-axis. Note that the boundary of the accretion cylinder \( x_0^2 + z_0^2 = R_a^2 \) transforms into one point \( \theta = \pi/2, \varphi = \pi \).

The mapping (3) assumes that the flow is laminar behind the bow shock. Numerical simulations show that the flow is unstable if the bow shock is strong, with a high Mach number. However, in the case of modest Mach numbers, the fluctuations are weak and the flow is approximately laminar (e.g. Blondin et al. 1990; Ruffert 1997, 1999). This is the most likely case if accretion occurs in the radiation field of a luminous X-ray source (see below). Note also that the streamlines cross the shock nearly normally (Hunt 1971); therefore the shock does not generates vorticity in the flow (Landau & Lifshitz 1987).

### 2.2 Compton heating/cooling

If the central X-ray source is luminous, \( L > 10^{-2}L_E \) where \( L_E = 4\pi cGMm_p/\sigma_T \) is the Eddington luminosity, then Compton heating/cooling affects strongly the dynamics of accretion (Ostriker et al. 1976; Illarionov & Kompaneets 1990). We here discuss two effects: (i) On large scales, \( R \gtrsim R_a \), the wind matter is preheated and its Mach number is reduced to \( M \sim 1 \). (ii) On small scales, \( R < R_C \) (see eq. 7), the accreting matter is cooled by the X-rays and falls super-sonically (Zeldovich & Shakura 1969).

#### 2.2.1 Preheating of the wind

The initial temperature of the wind is \( T_0 \sim 10^7 \) K and the typical Mach number is \( M_0 = w/c_s \sim 10 \) where \( c_s = (10kT/3m_p)^{1/2} \) is the sound speed. One can evaluate how \( M \) decreases along a streamline when approaching \( R \sim R_a \). The Compton heating/cooling is dominant in the energy balance of the highly ionised
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The spherical inflow is given by equation (3). The corresponding transformation of angular momentum is not known. The analysis of small perturbations in a converging flow (e.g. Lai & Goldreich 2000 and references therein) shows that the rotational mode, \( v_{rot} \propto r^{-1} \), has the fastest growth inwards, while sonic modes are damped. The rotational mode is probably dominant on \( R_c \) and we restrict our consideration to this mode and the associated angular momentum.

A particular streamline with impact parameters \((x_0, z_0)\) contributes to the net trapped angular momentum proportionally to \( \Omega R^2 \) (see e.g. Shapiro & Lightman 1976). If this magnitude conserves along a streamline then mapping (3) also determines the distribution of the trapped \( l \) over angles \( \theta, \varphi \) at \( R \ll R_c \),

\[
l_s(\theta, \varphi) = l_0 \frac{\sin^2 \theta \sin^2 \varphi}{1 + \sin \theta \cos \varphi}, \quad \bar{l}_s = \frac{l_0}{2}.
\]

We assume that the inflow angular momentum is associated with rotation around the \( z \)-axis only, i.e. we assume \( v_r \gg v_\varphi \). Then the orbital angular momentum of a streamline is

\[
l = \frac{l_s}{\sin \theta}.
\]

The corresponding non-radial velocity is \( v_\perp = v_\varphi = l_s/R \). The distribution (8) is not axisymmetric, which leads to an essentially three-dimensional pattern of accretion in the vicinity of the compact object.

In the other limiting case, the trapped \( l \) is efficiently redistributed between the streamlines so that they come to solid body rotation around the \( z \)-axis with a common angular velocity \( \omega \). Then

\[
l_s(\theta, \varphi) = l_0 \sin^2 \theta, \quad \bar{l}_s = \frac{2}{3} l_0.
\]

The non-radial velocity is \( v_\perp = v_\varphi = \omega R \sin \theta = l_s/R \) and the inflow is axisymmetric.

3 FREE FALL

In this section, we write down the equations describing the matter free fall in Newtonian gravity (the relativistic equations are given in an accompanying paper, Beloborodov & Illarionov 2000). We consider sub-Eddington sources only, where radiation pressure does not affect the flow dynamics. We are interested in the part of the ballistic trajectories before the collision with the accretor or the intersection with the equatorial plane, the symmetry plane of the flow.

At \( R \ll R_c \), the free fall is nearly parabolic. A parabolic trajectory is described by the equation

\[
R(\psi) = \frac{l^2 R_s}{l^2(1 - \cos \psi)}, \quad l_s = \sqrt{GM R_s}
\]

where \( l \) is the orbital angular momentum of a streamline and \( \psi \) is the angle between the changing radius-vector of the streamline \( \mathbf{R} \) and its initial radius-vector at infinity \( \mathbf{R}_\infty \) (see Fig. 2). The label ‘\( \infty \)’ corresponds to distances \( R \sim R_c \).

The angular distribution of the accretion rate at a sphere \( S_R \) of radius \( R \) is different from the initial uniform distribution at \( S_\infty \): the rotation defocuses the inflow and makes it non-uniform. A streamline that starts at \( \theta_\infty, \varphi_\infty \) will cross \( S_R \) at \( \theta, \varphi \) which satisfy the relations (see Fig. 2)

\[
\cos \theta = \cos \theta_\infty \cos \psi, \quad \sin(\varphi - \varphi_\infty) = \frac{\sin \psi}{\sin \theta}\]

The upstream region, the wind matter is falling freely. From the angular momentum conservation we have

\[
\frac{dR^2}{dt} = \frac{v}{b/a} \frac{R^2}{\sin \alpha} \frac{\beta^2}{c^2} \frac{L}{L_c} \approx \frac{v}{b/a} \frac{R^2}{\sin \alpha} \frac{\beta^2}{c^2} \frac{L}{L_c} = \text{const}
\]

Taking \( R = b/a \sin \alpha \approx b/a \), we get the temperature

\[
T \approx T(R) = \frac{\sigma T_c^4 R^2}{3 \pi m_e c^2 b w}.
\]

As a result of the Compton preheating, \( \mathcal{M} \) decreases markedly below the initial \( \mathcal{M}_0 \) and, correspondingly, the strength of the bow shock is reduced. In bright hard sources, the heated wind may become subsonic at \( R \gtrsim R_a \) and then the shock disappears.

2.2.2 Compton radius

Compton heating leads to an inflow-outflow pattern of accretion with \( M \sim 1 \) down to the Compton radius (Illarionov & Kompaneets 1990; Igumenshchev, Illarionov, & Kompaneets 1993),

\[
R_c = \frac{G M m_p}{5 k T_c} \approx 3 \times 10^9 \left( \frac{M}{M_\odot} \right) \left( \frac{T_c}{10^8} \right)^{-1} \text{ cm}.
\]

Typically, \( R_c \) is \( \sim 10 \) times smaller than \( R_a \).

Inside \( R_c \), a spherical inflow forms. Its temperature exceeds \( T_c \), so that the gas is cooled by the X-rays rather than heated (see eq. 4). The cooling leads to a high Mach number of the spherical inflow, \( M \gg 1 \).

2.3 The trapped angular momentum

When the accretor orbital motion is taken into account, the flow pattern is no longer symmetric around the \( y \)-axis and the captured matter should have a small net angular momentum \( l_s \sim \Omega R_c^2 \) with respect to the accretor. One would like to know the distribution of \( l \) around \( l_s \) in the accretion flow. This distribution governs the flow dynamics in the vicinity of the accretor where deviations from the spherical pattern become substantial.

Assuming a weak bow shock, the flow is almost laminar all the way. Then the transformation of the accretion cylinder into
where $\cos \psi = 1 - l^2 R_e/l_0^2 R$ (from eq. 11). The accretion rate distribution at $S_R$ is determined by the Jacobian of the mapping $(\theta_\infty, \varphi_\infty) \rightarrow (\theta, \varphi)$. After some algebra we get the Jacobian
\begin{equation}
\Delta = \frac{\partial (\cos \theta, \varphi)}{\partial (\cos \theta_\infty, \varphi_\infty)} = \cos \psi - \frac{\partial \cos \psi}{\partial \theta_\infty} \frac{\partial \cos \psi}{\partial \varphi_\infty} \frac{\partial \cos \psi}{\partial \theta_\infty} \frac{\partial \cos \psi}{\partial \varphi_\infty} \tag{14}
\end{equation}

where
\begin{align*}
\frac{\partial \cos \psi}{\partial \theta_\infty} &= -\frac{2l_0}{l_0^2} \frac{\partial l}{\partial \theta_\infty}, \\
\frac{\partial \cos \psi}{\partial \varphi_\infty} &= -\frac{2l_0}{l_0^2} \frac{\partial l}{\partial \varphi_\infty}.
\end{align*}

The mapping is one-to-one if $\Delta > 0$ at any $\theta_\infty, \varphi_\infty$. Vanishing of the Jacobian implies intersection of the ballistic trajectories. The streamlines then approach each other and the pressure effects must switch on and prevent the streamlines from intersecting. If the velocity of the approaching is super-sonic then shocks must occur.

It is convenient to rewrite equation (14) as
\begin{equation}
\Delta = \left(1 - \lambda^2\right) \left(1 + q \frac{\partial \lambda}{\partial \varphi_\infty}\right) + \frac{\partial \theta_\infty}{\partial \varphi_\infty} \frac{\partial \lambda}{\partial \theta_\infty} \tag{15}
\end{equation}

where $\lambda = (l/l_0)(R_e/R)^{1/2}$ and $q = 2(2 - \lambda^2)^{-1/2} \sin^{-1} \theta_\infty$.

In the axisymmetric case ($l$ does not depend on $\varphi_\infty$) the Jacobian is positive if $dl/d\sin \theta_\infty > 0$, i.e. if $l(\theta_\infty)$ increases towards the equatorial plane. Then the ballistic approximation can be used down to the surface of the accretor (in the case $l < l_\ast$) or down to the equatorial plane (in the case $l > l_\ast$). The condition $dl/d\sin \theta_\infty > 0$ is satisfied for e.g. $l$-distribution (10).

With $\Delta > 0$ we have a simple expression for the angular distribution of the accretion rate $\dot{M}$ over a sphere of radius $R$

\begin{equation}
\frac{dM}{d\Omega} = \frac{1}{\Delta} \frac{dM}{d\Omega_\infty}, \quad d\Omega = d\cos \theta d\varphi. \tag{16}
\end{equation}
its minimum $-a l_0 \sin \theta_\infty$ (see eq. 18). Angular momentum determines the velocity of rotation in the $\phi-$direction. Like usual one-dimensional motion with position-dependent velocity, the flow with $\partial l/\partial \phi_\infty > 0$ diverges in the $\phi-$direction ($dM/d\phi$ decreases) while the flow with $\partial l/\partial \phi_\infty < 0$ converges ($dM/d\phi$ decreases). Therefore $dM/dS$ peaks at the trajectories with minimum $\partial l/\partial \phi_\infty$. These trajectories start at $\phi_\infty = 3\pi/2$ with angular momentum $l_0 \sin \theta_\infty$ and they collide with the star at $\sim 7\pi/4$.

The Moon effect is clearly seen from a simple analytical consideration. Assuming a small $l_0$ and neglecting the $\lambda^2$ terms in equations (13,15) one gets
\[
\varphi = \varphi_\infty + \frac{l_0}{l_s} \sqrt{2} (1 - a \cos \varphi_\infty),
\]
\[
\Delta = 1 + a \frac{l_0}{l_s} \sqrt{2} \sin \varphi_\infty.
\]

The dependences $\varphi(\varphi_\infty)$ and $\Delta(\varphi_\infty)$ describe a cycloid $\Delta(\varphi)$ in its standard parametric form. The peak of the accretion rate $dM/dS = (M_{\text{tot}}/4\pi R_s^2) [1 - a(l_0/l_*) \sqrt{2}]^{-1}$ is achieved at the meridian $\varphi = 3\pi/2 + (l_0/l_*) \sqrt{2}$. In the case $l_0 = 0.5 l_*$ it yields $\varphi = 270^\circ + 40^\circ$.

### 4.2 The observed luminosity

On the NS surface, the flux of accreting mass is converted into radiation with efficiency $GM/R_s c^2$. The emerging radiation flux is given by
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Figure 4. The apparent luminosity $L_{\text{obs}}$ normalised by the total true luminosity $L = G M \dot{M}_{\text{tot}} / R_\ast$. (a) $L_{\text{obs}}$ as a function of the binary inclination $i$ in the case of $\alpha = 0$, $l_0 = 0.5 l_\ast$ (see eq. 18). In this case, $L_{\text{obs}}$ does not depend on the orbital phase of the binary $\phi$ owing to the axial symmetry of the accretion flow. (b) $L_{\text{obs}}$ as a function of $\phi$ in the case of $\alpha = 0$, $l_0 = 0.5 l_\ast$. The solid, long-dashed, short-dashed, and dotted curves correspond to inclinations $i = \pi/2, \pi/3, \pi/6, \text{and } 0$, respectively. (c) Same as (b) but with $\alpha = 1$.

$$F(\theta, \phi) = \frac{G M dM}{R_\ast dS}. \quad (19)$$

The distribution $dM/dS$ shown in Fig. 3 also displays the brightness distribution as seen by an observer located at the polar axis (if one assumes an isotropic intensity of the produced radiation).

The apparent luminosity of the accreting star depends on the binary inclination $i$ and the orbital phase $\phi$. We choose $\phi = 0$ at the superior conjunction of the accretor, i.e. when the companion is between the observer and the accretor. Then the angular position of the observer as viewed from the accretor is given by $\theta_{\text{obs}} = i$ and $\varphi_{\text{obs}} = -\phi$.

Let $\Omega$ and $\Omega_{\text{obs}}$ be unit vectors corresponding to $(\theta, \phi)$ and $(\theta_{\text{obs}}, \varphi_{\text{obs}})$, respectively, and $\mu = (\Omega \cdot \Omega_{\text{obs}})$. The apparent luminosity is

$$L_{\text{obs}}(i, \phi) = 4 \int F(\theta, \phi) \mu H(\mu) \, dS$$

$$= \frac{G M \dot{M}_{\text{tot}}}{\pi R_\ast} \int \mu H(\mu) \, d\Omega_{\infty}. \quad (20)$$

Here $H(\mu)$ is the Heaviside step function. To compute the integral, we substitute $\mu = \cos \theta \cos i + \sin \theta \sin i \cos(\varphi + \phi)$ and take $\theta(R_\ast)$ and $\varphi(R_\ast)$ from equations (12,13).

Fig. 4 shows $L_{\text{obs}}$ found for accretion flows with $l_0 = 0.5 l_\ast$ and $\alpha = 0, 0.5, \text{and } 1$ (same cases as shown in Fig. 3). One sees strong variations of $L_{\text{obs}}$ with the orbital phase. The amplitude of the variability reaches $\sim 3$ at $i = \pi/2$ and vanishes at $i = 0$. The maximum at $\phi \sim 7\pi/4$ is produced by the bright spot on the surface of the star (see Fig. 3). Note that at large $i$ one should also take into account the eclipse by the donor.

5 CAUSTICS OUTSIDE THE ACCRETOR

5.1 Formation of caustics

We now address the case $l > l_\ast$. If the accretion flow is symmetric about the equatorial plane then a streamline with $l > l_\ast$ coming from above will collide in this plane with the symmetric streamline coming from below. The collision occurs at $r = (l/l_\ast)^2 R_\ast$. We study inflows with $\Delta > 0$ (see eq. 14), so that there are no intersections of the ballistic trajectories outside the equatorial plane.\footnote{In general, depending on the distribution $l(\theta_{\infty}, \varphi_{\infty})$, such ‘early’ intersections are possible. In that case, the increased pressure near $\Delta = 0$ would alter the trajectories. It would cause a relatively modest focusing effect on the streamlines, without substantial energy release. By contrast, the eventual collision in the symmetry plane liberates a large fraction of the infall kinetic energy.}

The collision in the equatorial plane is associated with a couple of shocks that envelope the caustic from above and below. The...
We thus have a mapping asymmetry of the caustic in the equatorial plane. As an illustration take the inflow (18). Then $l(\theta, \varphi) = \bar{l} \cos(\varphi) - \bar{l} \sin(\varphi)$, i.e., the plane of the figure corresponds to the equatorial plane, $\theta = \pi/2$. The dashed curve shows the radius $R_\ast$. The solid curves display the contours of constant $dM/dS$ with a logarithmic step 0.3. Dark regions correspond to high $dM/dS$. The assumed angular momentum of the accretion flow is given by equation (18) with $\bar{l}_0 = \sqrt{3} \bar{M}_0$. Plot (a) shows the axisymmetric case ($a = 0$) and plot (b) shows the case $a = 0.5$. The outer edge of the caustic is $r_0(\varphi) = 3 R_\ast (1 - a \sin \varphi)^2$.

Figure 5. The distribution of $dM/dS$ over the caustic surface ($r > R_\ast$) and the surface of the accretor ($r < R_\ast$). The view is taken from the top, i.e., the plane of the figure corresponds to the equatorial plane, $\theta = \pi/2$. The dashed curve shows the radius $R_\ast$. The solid curves display the contours of constant $dM/dS$ with a logarithmic step 0.3. Dark regions correspond to high $dM/dS$. The assumed angular momentum of the accretion flow is given by equation (18) with $\bar{l}_0 = \sqrt{3} \bar{M}_0$. Plot (a) shows the axisymmetric case ($a = 0$) and plot (b) shows the case $a = 0.5$. The outer edge of the caustic is $r_0(\varphi) = 3 R_\ast (1 - a \sin \varphi)^2$.

of the fact that the streamlines execute a 1/4 turn by the moment of collision. The caustic thus appears in the front of the accretor orbiting the donor star. If the trapped $\bar{l}_0$ had the opposite sign, the caustic would appear in the rear of the moving accretor.

(ii) The outer edge of the caustic is formed by the nearly equatorial streamlines, $\theta \rightarrow \pi/2$. The edge is sharp, with $dM/dS \rightarrow \infty$ if the $l$-distribution is smooth (differentiable) at $\theta_\infty = \pi/2$. 

\[ \partial l/\partial x \]

\[ \frac{\partial l}{\partial \theta} \]

\[ \frac{\partial l}{\partial \varphi} \]

\[ \frac{\partial l}{\partial \tau_0} \]

\[ \frac{\partial l}{\partial \varphi} \]
6 DISCUSSION

The regime of accretion studied in this paper applies to weakly magnetised accretors. In the case of accretion onto a strongly magnetised neutron star, the effective radius of the accretor is the Alfvénic radius, $R_A$, and therefore the characteristic $l_\star$ in that problem is $\sim (GMR_A)^{1/2}$.

In reality one expects wind-fed accretion flows to be time-dependent. Numerical simulations of the Compton heated subsonic region at $R > \sim R_C$ show that the flow is unsteady (Igumenshchev et al. 1993). Fluctuations at $R > \sim R_C$ imply variable boundary conditions for the free fall inside $R_C$. The typical time-scale of the variations is of order of the free-fall time at $R_C$. The accretion flow is thus expected to fluctuate on time-scales $\sim 10\text{ s}$.

Throughout the paper we assumed that the flow is symmetric with respect to the equatorial (binary) plane. If the symmetry is broken, a warped caustic may form near the accretor. The changes in the caustic shape are then driven by the ram pressure of the colliding gas. The warped caustic is likely to be unstable, leading to oscillations/fluctuations on time-scales $\sim (R_3^2/GM)^{1/2} \sim 0.1\text{ ms}$.

The pattern of accretion studied in this paper assumes that the shocks on the star surface/caustics are radiatively efficient and pinned to the star surface and/or the equatorial plane. At a low accretion rate (which implies low density) the protons heated in the shock may find it difficult to pass their energy to the electrons on the free-fall time-scale. Then the shocked gas cannot radiate the heat and a variable pressure-driven outflow is likely to form.

On the observational side, the possible modulation of X-ray emission with the binary period is especially interesting. Note that orbital modulations of soft X-rays are known to occur in wind-fed systems as a result of photoelectric absorption in the wind (e.g. Wen et al. 1999; Balucinska-Church et al. 2000). The study of orbital modulations in the hard X-ray band would be especially helpful since they can be caused by intrinsic anisotropy of the source.

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