Crack Growth Prediction under Variable Amplitude Loading Considering Elastic-Plastic Stress Field ahead of Crack Tip

Abhishek Kumar\textsuperscript{a,b,*}, A. Ramachandra Murthy\textsuperscript{a,b} and Nagesh R. Iyer\textsuperscript{a}

\textsuperscript{a}CSIR – Structural Engineering Research Centre, Chennai-600113, India
\textsuperscript{b}Academy of Scientific and Innovative Research (AcSIR), Chennai-600113, India
\textsuperscript{*}E-mail ID: abhishek@serc.res.in

Abstract

Reliable prediction of fatigue life of the structural components under variable amplitude loading requires accurate computation of the residual stresses ahead of crack tip. In present study, a novel method of estimation of linear elastic stress field using concept of fictitious notch rounding is presented. The stress field is later used for evaluation of residual stress distribution. Corrective stress intensity factor is calculated from the residual stress field using modified weight function method. Corrective residual stress intensity factor is used to find the effective maximum stress intensity factor and effective stress intensity range. The values of root radius of aluminium alloys computed using proposed method is found to be closely matching with the values available in literature. Numerical investigation has been carried out to predict remaining life of plate panel with and without considering load interaction effects.

Keywords: Fatigue crack growth; Variable Amplitude Loading; Stress Intensity Factor; Residual Stress Intensity Factor, Notch root radius

Nomenclature

\begin{tabular}{ll}
\textit{a} & crack length \\
\textit{CAL} & constant amplitude loading \\
\textit{C}_p & fatigue crack growth coefficient \\
\textit{da/dN} & fatigue crack growth rate (FCGr) \\
\textit{FCG} & fatigue crack growth \\
\textit{E} & modulus of elasticity \\
\textit{FNR} & fictitious notch rounding \\
\textit{K} & stress intensity factor \\
\textit{K}' & cyclic strength coefficient \\
\end{tabular}
1. Introduction

Most components or structures experience a variety of cyclic stresses in its service. In the case of cyclic constant amplitude loading (CAL) the fatigue crack growth depends only on the crack length, the component geometry and the applied loading. For variable amplitude loading (VAL) it also depends on the preceding cyclic loading history. Various types of load sequence (overloads, under-loads, or combination of them) can cause either increase or reduction of the fatigue crack growth rate depending on relative magnitude of overloads.

The presence of interaction effects is evident from experimental results and fracture mechanics dealing with fatigue under variable amplitude loading (VAL) loading suggests that the load sequence has an important role on fatigue crack growth (FCG). For correctly predicting the crack growth under VAL, it is necessary to consider an interaction model with fatigue crack growth model for CAL [1]. Various load interaction modes have been developed by researches to predict crack growth under VAL. Most popular of them are yield zone models, crack closure models and strip yield models. The Willenborg model, Generalized Willenborg model and the Wheeler model are notable examples of early yield zone models [2-4]. In these models it is assumed that there is an increase in plastic zone size after an overload and which results in compressive residual stress around the crack tip (see Fig.1). These models assume retardation persist as long as the crack tip plastic zones for subsequent load cycles are within the overload plastic zone. The major limitation of Wheeler model is that it uses a fitting constant,\( m_1 \), shaping exponent, which is obtained through experiments. Rama Chandra Murthy et al. [5] proposed an improved Wheeler residual stress model where it is assumed that value of shaping exponent, \( m_1 \), depends on applied overload, crack size and width of the plate. Most of the available FCG models including those mentioned above are case specific and cannot predict crack growth for a general load history.
In this paper, an FCG model based on elastic-plastic stress analysis near crack tip is proposed which is capable of predicting fatigue crack growth rate for an arbitrary loading history. The model uses Creager and Paris solution for linear elastic stress field ahead of a blunt crack tip. A procedure to obtain root radius is proposed in this paper. Actual elastic-plastic crack tip strains and stresses are determined from the modified Neuber rule for which the linear elastic stress data acts as input. By using the weight function technique, the effect of the resultant residual stress field is presented in terms of the instantaneous residual stress intensity factor. Subsequently, residual stress intensity factor is included into effective fracture parameters that govern crack growth.

2. Linear-Elastic Stress Field Analysis of Stress Ahead of Crack Tip

It is generally accepted that the local stresses and strains near the crack tip control the fatigue crack growth process. The calculation of elastic-plastic strains and stresses at the crack tip requires solving the nonlinear boundary value problem of a cracked body. Analytical solutions of such problems for varied structural geometry are rarely attainable. Numerical Finite Element (FE) solutions are feasible but not very convenient in practice due to the complexity of the FE model and lengthy calculation in case of fatigue loading. Therefore, a simplified method based on the Neuber rule is used in the present study. The method requires a three-step approach, (i) determination notch root radius (ii) the linear-elastic stress–strain analysis and, (iii) the actual elastic–plastic crack tip strains and stresses are determined from the Neuber rule.

2.1 Estimation of Notch Root Radius

Local stresses and strain concentrators in structural components are usually modelled as cracks, whereas their simulation by notches would make calculations of the linear elastic stresses and strains induced by tensile loading are in essence reduced to the analysis of a notch of depth ‘a’ having the root radius, ‘ρ’[6]. The stress and stress concentration factor for a sharp notch obtained under linear-elastic condition is extremely high even for relatively small value of applied load. The considerable failure strength ahead of a sharp notch is explained using Neuber’s microsupport concept. The hypothesis states that for failure-relevant stress ahead of crack tip has to be obtained after averaging stress over a length, microstructural length, in the direction of crack propagation. To avoid integration for calculating averaged notch stress over microstructural length, Neuber [7] proposed the concept of fictitious notch rounding (FNR) (see Fig.2). To obtain stress field ahead of notch tip, FNR concept is utilized to avoid tedious procedure of averaging notch stresses obtained from linear elastic stress analysis.

2.1.1 Available Methods

Many methods for estimating ‘ρ*’ can be found in literature. The most popular method is based on threshold stress intensity factor. Glinka et al. [8] proposed eq. (1) for determining notch root radius using the fatigue limit stress for a crack length \(a\), \(\Delta \sigma_{\text{th}}^0\) and threshold stress intensity factor range, \(\Delta K_{\text{th}}\).

\[
\rho^* = \frac{1.633^2 \left( \frac{\Delta K_{\text{th}}}{\Delta \sigma_{\text{th}}^0} \right)}{2\pi}
\]  

(1)

Fig. 1. Schematic of a typical yield zone model.

Fig. 2. FNR for a blunt V-notch
A method based on near threshold fatigue crack growth experimental data is proposed by Nooroozi et al. [9]. The method can be applied only if sufficient experimental FCG data is available (i.e. CAL $da/dN$ data obtained at three stress ratios at least). The root radius, $r^*$ calculated is not unique when determined using this method. A need of simple method for calculating root radius which doesn’t need $\Delta K_{th}$ or FCG data is observed as experimental values of threshold stress intensity factor range is not reliable and FCG data might not be available for the desired material.

2.1.2 Proposed Method

The procedure to compute root radius is very similar to that of finding Irwin’s plastic zone size. It is known that due to plastic yielding of the material around the stress concentrator, the actual stresses near the notch are lower than the stress obtained by applying the linear elastic analysis which means that the local redistribution of stresses occur to satisfy the equilibrium condition in the body and this results in an increase in size of plastic zone. The two popular choices for redistribution of stress due to plastic yielding are constant yield and logarithmic distribution. In the proposed method, for calculation of root radius, constant distribution is assumed. In Fig. 3, Creager and Paris stress field solution (red solid line) and Westergaard stress field (grey dashed line) distributions are represented along x-axis are represented. Area I shows the stress transmitted due to redistribution of stresses due to plastic yielding and Area II depicts the difference between the stress transmitted by Westergaard stress field and Creager–Paris stress field. Area transmitted by two stress fields should be equal for these stress fields for being viable. This will be true when Area I equals Area II as Area III is common in both the stress distributions.

\[ \text{Area I} = \text{Area II} \]

A MATLAB code has been written for finding out root radius for a material. The procedure for finding root radius, $r^*$ is iterative, the value of $r^*$ is iterated till Area I is equal to Area II. Creager Paris and Westergaard stress fields obtained from MATLAB code are shown in Fig. 4. The values of root radius computed are in close agreement with the values available in literature.

2.2 LEA of Stresses Ahead of Crack Tip

Creager and Paris equation provides a closed form solution for stress and strain distribution ahead of blunt crack tip [10]. The blunt crack tip is represented by a slim parabolic notch and stress distribution mainly depends on the notch tip radius. The eq. (2) gives a nonsingular stress field at point arbitrary located at a radial distance $r'$ and making an angle $\varphi$ with respect to crack axis for crack whose tip is located at a distance $r'/2$ behind the notch root. Value of notch root radius, $r^*$ is obtained from procedure described in previous section.
\[\sigma_x = - \frac{K}{\sqrt{2\pi r}} \frac{\rho^*}{2r} \cos \frac{3\varphi}{2} + \frac{K}{\sqrt{2\pi r}} \cos \frac{\varphi}{2} \left[ 1 - \sin \frac{\varphi}{2} \sin \frac{3\varphi}{2} \right] + \ldots \]
\[\sigma_y = \frac{K}{\sqrt{2\pi r}} \frac{\rho^*}{2r} \cos \frac{3\varphi}{2} + \frac{K}{\sqrt{2\pi r}} \cos \frac{\varphi}{2} \left[ 1 + \sin \frac{\varphi}{2} \sin \frac{3\varphi}{2} \right] + \ldots \]
\[\tau_{xy} = - \frac{K}{\sqrt{2\pi r}} \frac{\rho^*}{2r} \sin \frac{3\varphi}{2} + \frac{K}{\sqrt{2\pi r}} \sin \frac{\varphi}{2} \frac{3\varphi}{2} + \ldots \] (2)

2.3 EPA of Stresses Ahead of Crack Tip

Elastic–plastic stress analysis is carried out using modified Neuber rule. Ramberg-Osgood relation, eq. (3), is used for describing stress-strain constitutive relation.

\[\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n'}\] (3)

The Neuber rule states the equivalence of the strain energy at the notch tip between the linear elastic and elastic-plastic behaviour of geometrically identical notched bodies subjected to identical external loading systems. The Neuber rule was originally derived for a uni-axial stress state (i.e. pure shear), eq. (4), later it was generalized for multi-axial proportional and non-proportional loading histories [11].

\[\sigma^a_{x,y} = \sigma^a_{y,x}\] (4)

For cracked bodies in plane stress the stress state near the crack tip is bi-axial. In the case of bodies in plane strain conditions the near tip stress state is tri-axial but the third principal stress is a function of the other two stress components and in both situations the modified bi-axial Neuber rule can be used. In addition, the elastic stress tensor used as the input does not rotate and all stress components change proportionally. Therefore, the Hencky equations of the total deformation theory of plasticity can be applied.

In the case of bi-axial stress state the combination of the Hencky stress-strain relationships, the Ramberg-Osgood stress-strain constitutive equation, eq. (3), and the multiaxial Neuber rule leads to the set of four equations, eq. (5) from which all maximum elastic-plastic crack tip stresses can be determined.

\[\varepsilon^a_{x,max} = \frac{1}{E} (\sigma^a_{x,max} - \nu \sigma^a_{y,max}) + \frac{1}{2} \frac{1}{K^\prime} \left( \sigma^a_{x,max} - \frac{1}{2} \sigma^a_{y,max} \right)^{1-n} \]
\[\varepsilon^a_{y,max} = \frac{1}{E} (\sigma^a_{y,max} - \nu \sigma^a_{x,max}) + \frac{1}{2} \frac{1}{K^\prime} \left( \sigma^a_{y,max} - \frac{1}{2} \sigma^a_{x,max} \right)^{1-n} \]
\[\sigma^a_{x,max} \varepsilon^a_{x,max} = \sigma^a_{y,max} \varepsilon^a_{y,max} \]
\[\sigma^a_{x,max} \varepsilon^a_{x,max} = \sigma^a_{y,max} \varepsilon^a_{y,max} \]
\[\sigma^a_{eq} = \sqrt{\sigma^a_{x,max}^2 + \sigma^a_{y,max}^2 - \sigma^a_{x,max} \sigma^a_{y,max}}\] (5)

The elastic-plastic stress-strain analysis discussed in this section assumes Masing type material behavior which is common for steel and aluminum alloys. However, if the material is a non-Masing type, another appropriate stress-strain model has to be chosen and implemented in order to determine local elastic-plastic stresses and strains in the crack tip region.

3. Proposed Methodology to Predict Crack Growth Rate

Residual stresses (i.e. stresses remaining in the material after an application of a loading cycle) can be calculated from eq.(5) using eq. (6).

\[\sigma_r = \sigma^a_{y,max} - \sigma^a_{y}\] (6)
In order to determine the residual stress distribution ahead of the crack tip, the procedure described above needs to be repeated for a sufficient number of points in the crack tip region.

Corrective residual stress intensity factor, $K_r$, is obtained from residual stress distribution, $\sigma_r(x)$ using weight function method given by eq. (7) [12]. The universal one dimensional weight function (eq. 8) is used in the analysis [13].

$$K_r = \int_0^a \sigma_r(x, a) \, dx$$  \hspace{1cm} (7)$$

$$m(x, a) = \frac{2P}{\sqrt{2\pi(a-x)}} \left[ 1 + M_1 \left(1 - \frac{x}{a}\right)^{\frac{1}{2}} + M_2 \left(1 - \frac{x}{a}\right) + M_3 \left(1 - \frac{x}{a}\right)^{\frac{3}{2}} \right]$$  \hspace{1cm} (8)$$

The two fracture parameters that governs crack growth, effective maximum stress intensity factor, $K_{\text{max,app}}$ and effective stress intensity range, $\Delta K_{\text{eff}}$, are obtained from eq. (9) and eq. (10) respectively.

$$K_{\text{max,eff}} = K_{\text{max,app}} - K_r$$  \hspace{1cm} (9)$$

$$\Delta K_{\text{eff}} = \Delta K_{\text{app}} - K_r$$  \hspace{1cm} (10)$$

4. Numerical Investigations

A plate with central through crack (see Fig. 5-a) made up of 350WTsteel is investigated. The material properties of 350WT steel used in analysis are given in Table 1 [14-16].

![Fig. 5](image-url) Details of the fatigue crack growth problem used for investigation (a) Central through crack specimen (b) CA loading history (c) VA loading history

Fig. 5-b gives the details of fatigue loading subjected on the plate. Overload has been applied after the specimen is subjected to 60,000 cycles (see Fig.5-c).

Fig. 6 shows the variation of fatigue crack growth rate (FCGr) against crack length. A retardation can be observed after application of overload. Delay cycles have been computed by using proposed approach, it is observed that the number of the cycles to failure under CAL is 145,232 and the number of the cycles to failure under VAL is 149,503 with delay cycles as 4,271.
Fig. 6 shows the variation of fatigue crack growth rate (FCGr) against crack length. A retardation can be observed after application of overload. Delay cycles have been computed by using proposed approach, it is observed that the number of the cycles to failure under CAL is 145,232 and the number of the cycles to failure under VAL is 149,503 with delay cycles as 4,271.

Further verification of the model for different geometries made up of different materials and various loading sequences is required. The proposed methodologies will be useful for damage tolerant design of structures subjected to fatigue loading.

### 5. Conclusions

A new procedure has been proposed to estimate the root radius, $\rho^*$, which was later used for describing linear elastic stress field ahead of crack tip. A methodology has been proposed for crack growth prediction under VAL which uses elastic-plastic stress field for modeling load interaction effects. The output obtained in the present study can be used (i) to carry fatigue crack growth analyses for variety of geometrical configurations under arbitrary variable amplitude loading sequence and (ii) design of structures based on damage tolerant concept.

### Table 1. Material properties of 350WT steel

| Material property | Value   | Unit   | Source               |
|-------------------|---------|--------|----------------------|
| $E$               | 1.191500 | MPa    | Chen et al. (2002)   |
| $\sigma_{ys}$     | 365     | MPa    |                      |
| $K_c$             | 638.01  | MPa    |                      |
| $n'$              | 0.1093  |        |                      |
| $\rho^*$          | $4.014 \times 10^{-5}$ | m | Present Study         |
| $\psi$            | 0.30    |        | Trask (1998)         |
| $C_p$             | $1.02 \times 10^{-8}$ | mm/cycle | Taheri et al (2003)  |
| $m_p$             | 2.94    |        |                      |

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Further verification of the model for different geometries made up of different materials and various loading sequences is required. The proposed methodologies will be useful for damage tolerant design of structures subjected to fatigue loading.
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