HALO ASPHERICITY AND THE SHEAR THREE-POINT FUNCTION

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ABSTRACT

We demonstrate through the use of a simple toy model that asphericity in dark matter halos has a measurable effect on the configuration dependence of the weak-lensing shear three-point function at small scales. In principle, this sensitivity provides a way to measure the shapes of dark matter halos. The distribution of halo ellipticities should be included in models aiming at a high-fidelity prediction of n-point shear correlation statistics.

Subject headings: dark matter — galaxies: halos — gravitational lensing

1. INTRODUCTION

Gravitational lensing (for reviews, see Mellier 1999; Bartelmann & Schneider 2001; Wittman 2002) provides us with a unique opportunity to probe the matter distribution of the universe and, in combination with galaxy surveys, the galaxy-halo connection. In particular, the small angular scale structure of the lensing shear field is sensitive to the density distribution of dark matter halos over a range of mass scales, as has been emphasized by Takada & Jain (2003b).

In this brief report we show, using a simple toy model, that asphericity in dark matter halos has a measurable effect on the configuration dependence of the weak-lensing shear three-point function at small scales. In principle, this sensitivity provides a way to measure the shapes of dark matter halos. The distribution of halo ellipticities should be included in models aiming at a high-fidelity prediction of n-point shear correlation statistics.

2. THE HALO MODEL

Our calculations are all performed within the halo model paradigm (see Cooray & Sheth 2002). Specifically, we assume that all the mass in the universe is contained within virialized halos with a range of sizes and shapes. Under this assumption, the correlation function and higher order moments of the shear can be related to integrals over the (projected) density profiles of the halos (multiplied by geometrical factors) and terms involving the clustering of the halos (Takada & Jain 2003a; Zaldarriaga & Scoccimarro 2003). If we work on small (arcminute) angular scales, then the dominant contribution to the shear three-point function will arise when all three points sample the mass from within the same halo (Takada & Jain 2003a). This term is independent of the clustering of the halos.

To describe the three-point function, we use conventions similar to those used by Zaldarriaga & Scoccimarro (2003) and Schneider & Lombardi (2003), with $\theta_i$ the vector to vertex $i$ of the triangle, the origin of the coordinate system set to the center of the triangle, and $\mathbf{n}$ as the center of the halo we are considering. The geometry is shown in Figure 1.

We construct scalar three-point correlation functions by contracting the three observed shears with spin-2 quantities (Mellier 1999; Bartelmann & Schneider 2001; Wittman 2002; Zaldarriaga & Scoccimarro 2003):

$$P_\pm = \left(\theta_i^2 - \theta_j^2, 2\theta_i \theta_j\right) \theta^{-2} = (\cos 2\phi, \sin 2\phi),$$

$$P_x = \left(-2\theta_i \theta_j, \theta_x^2 - \theta^2\right) \theta^{-2} = (-\sin 2\phi, \cos 2\phi),$$

which have opposite parity. When we contract the above quantities with the shear components $\gamma_1$ and $\gamma_2$, we get $\gamma_+$ and $\gamma_-$. Thus, we can define our three-point function as

$$\zeta_{\alpha \beta \chi} = \left\langle \gamma_1 \gamma_2 \gamma_3 \right\rangle = P^\alpha_\alpha P^\beta_\beta P^\chi_\chi \left\langle \gamma_1 \gamma_2 \gamma_3 \right\rangle,$$

where $\alpha$, $\beta$, and $\chi$ are either + or $\times$.

The contribution to the small-angle $\zeta$ from a single halo of projected density $\Sigma$ is then

$$\zeta_{\alpha \beta \gamma} = \int d^2\mathbf{u} e^\alpha \Sigma(\mathbf{1} - \mathbf{u}) e^{\beta} \Sigma(\mathbf{2} - \mathbf{u}) e^{\gamma} \Sigma(\mathbf{3} - \mathbf{u}),$$

where the halo is centered at $\mathbf{u}$ and the $e^\ell$ are phase factors that depend on the geometry of the halo.

It has been standard in the literature to use a “spherically averaged” halo profile in the computation of the higher order moments of the shear field. In this limit, the shear is tangential to the halo center and the phase factors are simply

$$e^+ = \cos 2\alpha, \quad e^- = \sin 2\alpha,$$

where $\alpha$ is the angle between $\mathbf{n}$ and $\theta$. However, we know from numerical simulations (Frenk et al. 1988; Dubinski & Carlberg 1991) that dark matter halos are triaxial, so we relax the spherical assumption.

In order to highlight the effects of halo asphericity on the three-point function with minimal complications, we employ a simple toy model. First, we work entirely in terms of a single halo; we do not integrate over the entire mass distribution. Second, we work throughout with projected quantities. Specifically, we modify the spherical (projected) density profile $\Sigma(r)$ to $\tilde{\Sigma}(s)$, where $s = \left[ x'^2 + (y')^2 \right]^{1/2}$, and $x'$ and $y'$ are sky coordinates, possibly rotated with respect to the axes defining the shear triangle. For $q < 1$ the profile is elliptical, elongated along the $x'$-axis. Meneghetti, Bartelmann, & Moscardini (2003) found $q \approx 0.7$ characterized the ellipticity of dark
matter halos found in $N$-body simulations, and we use this value later as an indicator of the potential size of ellipticity effects. Since our profile is now elliptical, the phase factors $\epsilon$ become

$$\epsilon^+ = \cos 2(\alpha - \beta), \quad \epsilon^x = \sin 2(\alpha - \beta)$$

where $\beta$ is the angle between $\theta$ and the normal to the tangent at $\theta$. The definitions of the variables can be easily understood from Figure 1. Thus, for an NFW halo (Navarro, Frenk, & White 1996) our expression for $\zeta_{+++}$ becomes

$$\zeta_{+++} = \int d\phi \int d^2u \prod_{i=1}^3 \cos 2(\alpha_i - \beta_i)\Sigma(s_i),$$

where

$$s_i = \sqrt{x_i^2 + (qy_i^2)},$$

$$x_i = (\theta_i^x - u_x) \cos \phi + (\theta_i^y - u_y) \sin \phi,$$

$$y_i = -(\theta_i^x - u_x) \sin \phi + (\theta_i^y - u_y) \cos \phi.$$  

$\alpha_i$ and $\beta_i$ are functions of $\theta_i$ and $q$, $\int d\phi$ is an integral over the orientation of the halo, and $\Sigma$ is the projected profile, which we take to be (Bartelmann 1996)

$$\Sigma^2(s) = \left(\frac{1}{s^2 - 1}\right) \left(1 - \frac{2}{|s^2 - 1|^{1/2}} \tanh^{-1}\left|\frac{s-1}{s+1}\right|^{1/2}\right).$$

where $\tanh^{-1}(x) = \tan^{-1}(x)$ when $x > 1$ and $\tanh^{-1}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x}\right)$ when $x < 1$. Similar expressions hold for the other components of $\zeta$.

### 3. RESULTS AND DISCUSSIONS

It has been argued that halo ellipticity does not affect the higher order correlations of the shear, since different orientations "wash out" the effect. Our results show that this argument is too simplistic: the physical observables do depend on $q$. One reason for this is the change in the projection of the shear at $\theta_i$ onto the $+$ and $x$ components. The other is simply that the orientation average of $\rho^3$ is not equal to the cube of the orientation-averaged $\rho$, an inequality that becomes stronger as we go to higher orders.

Since $\zeta_{+++}$ has the strongest signal among all of the three-point functions, we have used it in Figure 2 to illustrate the effect of halo asphericity. The most noticeable property of Figure 2 is the peak shift and amplitude change when $q < 1$. The peak first moves to the right from $-2\pi/3$, and then moves to the left. It may be possible to see such a shift in Figure 10 of Takada & Jain (2003b), which compares the analytic theory (assuming spherical halos) to numerical results. We can try to understand the shape of $\zeta_{+++}$ and the peak shift by starting from a simple case when the halo is spherical and the triangle is equilateral. Let us consider the halo center to be inside the triangle, which minimizes the distances to the vertices and thus maximizes the signal, and assume that the triangle fits entirely within the halo. For $q = 1$, the configuration that best matches the shear orientations to $\gamma_{++}$ at each vertex can easily be seen to be an equilateral triangle. For a wide range of halo-center positions, one obtains a positive contribution to $\zeta_{+++}$. If the halo has $q < 0.7$, then triangles with $\phi < 2\pi/3$ have some orientations that allow more of the high-density region of the halo to fall within the triangle and to better match the orientations of the $+$ shear components, enhancing the signal at slightly smaller angles than the $q = 1$ case. For completeness, we also show two of the seven other three-point functions in Figures 3 and 4. To avoid clutter, we only show results for $q = 1, 0.8,$ and $0.7$. These
figures show clearly that halo asphericity has an effect on all of the (nonzero) shear three-point functions.

We have shown in this brief report that aspherical dark matter halos give rise to a different configuration dependence of the weak-lensing shear three-point function than spherical halos. The positions and amplitudes of the peaks in the three-point function shift as the typical halo is made more aspherical. In principle, this effect provides us a probe of dark matter halo shapes, which have long been predicted by numerical simulations to be aspherical. It should also be included in semianalytic models aiming at a high-fidelity prediction of higher order correlations in the shear field.

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REFERENCES

Bartelmann, M. 1996, A&A, 313, 697
Bartelmann, M., & Schneider, P. 2001, Phys. Rep., 340, 291
Cooray, A., & Sheth, R. 2002, Phys. Rep., 372, 1
Dubinski, J., & Carlberg, R. 1991, ApJ, 378, 496
Frenk, C. S., White, S. D. M., Davis, M., & Efstathiou, G. 1988, ApJ, 327, 507
Mellier, Y. 1999, ARA&A, 37, 127
Meneghetti, M., Bartelmann, M., & Moscardini, L. 2003, MNRAS, 340, 105
Navarro, J., Frenk, C., & White, S. D. M. 1996, ApJ, 462, 563
Schneider, P., & Lombardi, M. 2003, A&A, 397, 809
Takada, M., & Jain, B. 2003a, MNRAS, 340, 580
———. 2003b, MNRAS, 344, 857
Wittman, D. 2002, in Gravitational Lensing: An Astrophysical Tool, ed. F. Courbin & D. Minniti (Berlin: Springer), 55
Zaldarriaga, M., & Scoccimarro, R. 2003, ApJ, 584, 559