Forecasting fuel combustion-related CO₂ emissions by a novel continuous fractional nonlinear grey Bernoulli model with grey wolf optimizer

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Abstract
Foresight of CO₂ emissions from fuel combustion is essential for policy-makers to identify ready targets for effective reduction plans and to further improve energy policies and plans. A new method for forecasting the future development of China’s CO₂ emissions from fuel combustion is proposed in this paper by using grey forecasting theory. Although the existing fractional nonlinear grey Bernoulli model (denoted as FNGBM(1,1)) has been theoretically proven to enhance the adaptability to diverse sequences, its fixed integer-order differential derivative still impairs the performance to some extent. To this end, a varying-order differential derivative is introduced into the existing differential equation to enable a more flexible structure, thus improving the prediction ability of FNGBM(1,1). Specifically, because of the advantages of conformable fractional accumulation, the traditional differential derivative is first replaced by the conformable fractional differential derivative. As a consequence, the continuous conformable fractional nonlinear grey Bernoulli model (hereinafter referred to as CCFNGBM(1,1)) is proposed. To further increase the validity of the model, a metaheuristic algorithm, namely Grey Wolf Optimizer (GWO), is then applied to search for the optimal emerging coefficients for the proposed model. Two real examples and China’s CO₂ emissions from fuel combustion are considered to verify the effectiveness of the newly proposed model, the experimental results show that the newly proposed model outperforms other benchmark models in terms of forecasting accuracy. The proposed model is finally employed to forecast the future China’s CO₂ emissions from fuel combustion by 2023, accounting for 10,039.80 million tons. Based on the forecasts, several policy suggestions are provided to curb CO₂ emissions.

Keywords CO₂ emissions · Fuel combustion · Conformable fractional differential derivative · Nonlinear grey Bernoulli model

Introduction

The topic of carbon emission mitigation (Fang and Chen 2019; Home-Ortiz et al. 2019) has been a focus of ongoing concern across countries since CO₂ is one of the foremost greenhouse gases in the atmosphere that cause the global warming effect. Moreover, CO₂ from energy represents approximately 60% of the anthropogenic greenhouse gas emissions among global emissions. To control carbon emissions (Rocco et al. 2020), numerous efforts have been made by international communities, such as the UNFCCC (United Nations Framework Convention on Climate Change), the Kyoto Protocol and the Paris Agreement. At the present stage, as the largest emitter of CO₂, China has promulgated a continuously growing array of policies to curb carbon emissions. Particularly, the Chinese government has repeatedly stated that China is committed to reducing carbon intensity by 40–45% by 2020 and 60–65% by 2030, compared to the 2005 level. As a consequence, China promises to reach the peak of CO₂ by 2030. In the prevailing low-carbon economy, China is under substantial pressure to reduce CO₂ emissions. As many studies reveal, CO₂ emissions from fuel combustion represent the largest share of the total. It is necessary to forecast fuel combustion-based CO₂ emissions, which could...
provide a solid basis for decision-makers to prepare and implement reasonable plans and policies.

In effect, diverse factors have uncertain and complicated impacts on final carbon emissions, making the forecast of CO₂ emissions from fuel combustion more arduous. To address this problem, many published papers have striven to predict CO₂ emissions by sector. For example, Köne and Büke (2010) made projections of CO₂ emissions from fuel combustion in selected countries by trend analysis. Pérez-Suárez and López-Menéndez (2015) combined environmental Kuznets curves with logistic growth models, to forecast CO₂ emissions for different countries. To forecast province-level CO₂ emissions, Sun et al. (2017) put forward a novel particle swarm optimization-based extreme learning machine (PSO-ELM) by incorporating factor analysis. Ding et al. (2017) estimated the future energy-related CO₂ emissions by a novel discrete grey multivariate model. From a conceptual perspective, these approaches can be classified into three categories - statistical models, machine learning methods and grey models.

Statistical models have since been widely used in various disciplines due to the advantages of a strong explanatory power. To predict CO₂ emissions, Zhao and Du (2015) constructed an empirical panel regression model that can be regarded as an extension of the environmental Kuznets curve (EKC) literature, and they projected the total emissions of sample countries in four scenarios according to the country’s emissions per capita. With the help of multiple linear regression (MLR) and multiple polynomial regression (MPR), Hosseini et al. (2019) explored the carbon emissions by 2030 under assumptions of business-as-usual and the Sixth Development Plan. Malik et al. (2020) used the autoregressive integrated moving average (ARIMA) model to forecast Pakistan’s CO₂ emissions in the China-Pakistan Economic Corridor (CPEC) scenario. Belbute and Pereira (2020) employed an autoregressive fractionally integrated moving average (ARFIMA) model to forecast fossil fuel combustion and cement production-related CO₂ emissions for Portugal. These models can evidently obtain a satisfactory result and exhibit desirable explanatory power given sufficient observations. However, the major limitation lies in their requirement of expert knowledge to identify relationships between variables, and the prediction accuracy heavily relies on parameter estimation.

As a type of advanced intelligent algorithm, machine learning methods can solve CO₂ emissions issues due to their advantage of conducting complicated calculations. Qiao et al. (2020) considered the stability neglected in the previous literature, and proposed a novel hybrid algorithm by combining a genetic algorithm and lion swarm optimization to improve the performance of the traditional least squares, the results of forecasting carbon dioxide emissions of developed countries, showed the merits of a stronger global optimization ability and higher accuracy. By taking the multiple influential factors into consideration, Zhao et al. (2018) put forward a hybrid approach of the salp swarm algorithm (SSA) and least squares support vector machine (LSSVM), further optimizing the model parameters by PSO and revealing that the results of this model enhanced the predictive capacity and reliability. Wen and Cao (2020) proposed a novel method for forecasting energy-related carbon emissions in Shanghai, in which they combined a support vector machine, improved chicken swarm optimization with embedded chaotic mutation and a nonlinear weight index, and principal component analysis. To forecast the national carbon emissions in the building sector by region, Hong et al. (2018) studied an optimized gene expression programming model based on metaheuristic algorithms. Ameyaw et al. (2019) designed a long short-term memory (LSTM) algorithm to forecast CO₂ emissions solely from fossil fuel combustion, and the model was proven to be effective in the absence of exogenous variables and assumption requirement. The main drawback of machine learning methods is the requirement of a considerable amount of data. In contrast, if the sample size is small, the performance will be poor. Moreover, the internal operation mechanism is unknown.

As introduced by Ofosu-Adarkwa et al. (2020), it is better to focus on a time sequence that involves the most relevant shock, when attempting to modeling a time series impacted by shocks. In short, we should narrow the sequence closest to the current time to capture the shocks, specially for China, which has been under economic restructuring and upgrade since joining the World Trade Organization (WTO) in 2001. In this background, adequate information about annual carbon emissions by sector is unavailable. The grey forecasting model is known as an effective approach for forecasting the development trend of an uncertain system with sparse data. Thus, the essential motivation of this study is to explore the future development trend of CO₂ emissions from fuel combustion by the grey forecasting system.

In reality, there are many studies on forecasting carbon emissions that are grey model based. For example, Wu et al. (2015) established a multivariate grey model based on the rolling mechanism to predict CO₂ emissions for Brazil, Russia, India, China and South Africa (the BRICS countries). Wang and Li (2019) described the relationship between CO₂ emissions and economic growth in China by presenting a PSO-based grey Vehulst model. More recently, Ding et al. (2020) took nonlinear effects of the influential factors on emissions into considerations; furthermore, Ding and colleagues developed a novel discrete grey power model by plugging grey power indices into the model structure. In the same vein, Ofosu-Adarkwa et al. (2020) combined the grey Vehulst model and GM(1,N) model, and
as a result, a hybrid model was proposed for forecasting CO\textsubscript{2} emissions from China’s cement production. Wu et al. (2020) studied the properties of the conformable fractional grey model and then applied this model to forecast CO\textsubscript{2} emissions in the BRICS countries. Admittedly, these models achieved satisfactory results as a whole. However, they are not without limitations in some certain situations. In consequence, this fact provides insights into further improving the prediction performance in forecasting CO\textsubscript{2} emissions from fuel combustion.

Among these grey models, the nonlinear grey Bernoulli model (NGBM(1,1)) is commonly considered to be an effective approach to capturing nonlinear trends and achieving accurate projections (Chen et al. 2008). Later, considering the background-value limitation, Wang (2013) proposed a Nash NGBM(1,1) model and (Lu et al. 2016) put forward an improved NGBM(1,1) by combining it with a new condition. Most recently, Wu et al. (2020) and Xiao and Duan (2020) plugged \(b(x^{(1)} + c)^{\gamma}\) and \(b(x^{(1)})^{\gamma} + c\), respectively, into the original differential equation of NGBM(1,1), and as a result, the nonhomogeneous nonlinear grey Bernoulli model and grey Riccati model were proposed. On the other hand, due to the advantages of the fractional accumulating generation operator, Wu et al. (2019) proposed an improved NGBM(1,1) model with fractional order accumulation (denoted as FNGBM(1,1)). On this basis, considering the background-value limitation, Şahin (2020a) and Şahin (2020b) further optimized the fractional NGBM(1,1) model based on a dynamic background value. Admittedly, these models can enhance the prediction performance of NGBM(1,1) to some extent. However, regarding the modeling mechanism of the above models, their differential derivatives are all integer-order, the fixed structure impairs their flexibility regarding diverse data sets, and there are rare studies on this issue, therefore, inspired by Khalil and Abu-Shaab (2015), the conformable fractional accumulation (Wu et al. 2013) is discontinuous, therefore, inspired by Khalil and Abu-Shaab (2015), the new differential derivative in the current study is based on the conformable fractional accumulation, which is also an innovation of this paper. On the other hand, although the studies focusing on CO\textsubscript{2} emissions have increased over the last years, up-to-date and accurate projections merit further research, which will provide a solid basis for decision-makers to prepare and frame reasonable plans and policies.

In view of the claims mentioned above, aiming to attain more accurate forecasts of CO\textsubscript{2} emissions from fuel combustion, the primary contributions of this paper are summarized as follows. First, the fixed integer-order differential derivative of FNGBM(1,1) is replaced with a conformable fractional differential derivative. Second, for efficacy purposes, the emerging coefficients are determined by the grey wolf optimization (GWO)(Mirjalili et al. 2014) algorithm based on a comparison with the ant lion optimizer (ALO)(Wang et al. 2018), whale optimization algorithm (WOA)(Mirjalili and Lewis 2016) and PSO(Kennedy and Eberhart 1995). Third, two real examples are used for verifying the validity of the newly proposed model. The proposed model is found to be more suitable for forecasting the future development trend of China’s CO\textsubscript{2} emissions from fuel combustion from 2020 to 2023. Finally, based on the forecasts of this paper and the current situation of the Chinese economy, several suggestions are proposed for curbing CO\textsubscript{2} emissions in China.

The rest of this paper is organized as follows. “Methodology” elaborates the modeling procedure of the newly proposed model. “Verification of CCFNGBM(1,1)” verifies the effectiveness of the proposed model. Application of the newly proposed model to predicting CO\textsubscript{2} emissions from fuel combustion is conducted in “Application” and “Conclusion” concludes.

**Methodology**

**Traditional FNGBM(1,1) model**

The FNGBM(1,1) is aimed at expanding the application scope of the NGBM(1,1) model by incorporating the fractional accumulating generation operator. The modeling steps can be summarized as follows.

Assume \(X^{(0)} = (x^{(0)}(0), x^{(0)}(1), \cdots, x^{(0)}(n)), n \geq 4\) to be a nonnegative sequence; then, the \(r\)-order fractional accumulating generation operation \((r\text{-FAGO})\) sequence is

\[
X^{(r)} = \left(x^{(r)}(1), x^{(r)}(2), \cdots, x^{(r)}(n)\right)
\]

where

\[
x^{(r)}(k) = \sum_{i=1}^{k} \binom{k-i+r-1}{k-i} x^{(0)}(i)
\]

with \(\binom{m}{0} = 1, \binom{m}{k} = 0\) and \(\binom{k-i+r-1}{k-i} = \frac{(k-1)!}{(k-i)!r!}\).

Then, the differential equation of the FNGBM(1,1) model, based on \(X^{(r)}\), is defined as

\[
\frac{dx^{(r)}}{dt} + ax^{(r)} = b \left(x^{(r)}\right)^{\gamma}
\]

where \(a\), \(b\) and \(\gamma\) represent the development coefficient, grey action quantity and power index.

The discrete form of Eq. (3) is obtained as

\[
x^{(r)}(k) = x^{(r)}(k-1) + az^{(r)}(k) = b \left(z^{(r)}\right)^{\gamma}
\]

where \(z^{(r)}\) is the background value, and \(z^{(r)}(k) = 0.5 \times (x^{(r)}(k) + x^{(r)}(k-1))\).
The model parameters \( \hat{\psi} = (a, b)^T \) can be calculated by the least square method:

\[
\hat{\psi} = (a, b)^T = \left( B^T B \right)^{-1} B^T Y
\]

where

\[
B = \begin{bmatrix}
-\zeta^{(r)}(2) & \left( \zeta^{(r)}(2) \right)^\gamma \\
-\zeta^{(r)}(3) & \left( \zeta^{(r)}(3) \right)^\gamma \\
\vdots & \vdots \\
-\zeta^{(r)}(n) & \left( \zeta^{(r)}(n) \right)^\gamma
\end{bmatrix},
Y = \begin{bmatrix}
x^{(r)}(2) - x^{(r)}(1) \\
x^{(r)}(3) - x^{(r)}(2) \\
\vdots \\
x^{(r)}(n) - x^{(r)}(n-1)
\end{bmatrix}
\]

Then, the time response function of Eq. (3) is calculated as

\[
\hat{x}^{(r)}(k) = \left( \left( (b^{(0)}(1) \right)^{1-\gamma} - \frac{b}{a} \right) \times \exp \left( -a(1-\gamma)(k-1) + \frac{b}{a} \right) \right)^{1/\gamma}
\]

Consequently, the restored values of \( \hat{X}^{(0)} \) can be obtained after an inverse calculation:

\[
\hat{x}^{(0)}(k) = a^{\gamma} \hat{x}^{(r)}(k) = \hat{x}^{(r)}(k) - \hat{x}^{(r)}(k-1)
\]

**The proposed CCFNGBM(1,1) model**

Admittedly, the fractional accumulating generation operator improves the prediction performance of NGBM(1,1). However, in Eq. (3), the differential derivative is still of integer-order, and the fixed order impairs the accuracy of the model to some extent (elaborated on in "Introduction"). To this end, the conformable fractional derivative is introduced to establish the continuous conformable fractional NGBM(1,1) model (denoted as CCFNGBM(1,1) for short), aiming to improve the adaptability to diverse sequences.

**Conformable fractional accumulation and difference**

Inspired by Khalil and Abu-Shaab (2015), Ma et al. (2020) proposed the conformable accumulation operator and its difference operator, their calculation formulas are as follows.

**Definition 1** Given a differential function \( f : [0, \infty) \to R \), the conformable fractional accumulating generation operator (CFAGO) of \( f \) with \( \alpha \) order is

\[
\begin{align*}
\hat{\gamma}^\alpha f(k) &= \gamma \left( \frac{f^{(k)}}{k!} \right) = \sum_{j=0}^{\lfloor \alpha \rfloor - 1} \left( \frac{\lfloor k \rfloor}{j!} \right) \frac{\gamma^j}{\alpha^j} k \in N^+, \alpha \in (0, 1], n \in N \\
\hat{\gamma}^\alpha f(k) &= \gamma^{\alpha} \left( \frac{f^{(k)}}{k!} \right) \\
n \in N^+, \alpha \in (n, n+1], n \in N
\end{align*}
\]

where \( [\cdot] \) is the ceil function, for example, \( [\alpha] \) is the smallest integer no larger than \( \alpha \). Without loss of generality, \( \gamma^{\alpha} \) denotes the conformable fractional accumulating generation operator.

**Definition 2** The conformable fractional difference operator (CFDO) of \( f \) with \( \alpha \) order is

\[
\begin{align*}
\Delta^\alpha f(k) &= k^{1-\alpha} f(k) \\
\Delta^\alpha f(k) &= k^{(a-\alpha) \Delta^{n+1} f(k)} \\
k \in N^+, \alpha \in (0, 1], n \in N
\end{align*}
\]

Without loss of generality, \( \Delta^\alpha \) denotes the conformable fractional difference operator.

**Model establishment**

Suppose that \( X^{(0)} \) and \( X^{(r)} \) are the same as those mentioned in "Traditional FNGBM(1,1) model"; the differential equation of CCFNGBM(1,1) is expressed as

\[
d^\alpha x^{(r)} + ax^{(r)} = b \left( x^{(r)} \right)^\gamma \\
r \in R^+, \alpha \in (0, 1], \gamma \in R
\]

where \( a \) and \( b \left( x^{(r)} \right)^\gamma \) represent the development coefficient and grey action quantity, respectively. \( r, \alpha \) and \( \gamma \) indicate the fractional order, conformable fractional order and power index, respectively. Obviously, when \( \alpha = 1 \) and \( r = 1 \), CCFNGBM(1,1) reduces to NGBM(1,1); when \( \alpha = r = 1 \) and \( r = 2 \), CCFNGBM(1,1) is equivalent to the grey Vehulst model (Evans 2014).

**Theorem 1** The time response function of CCFNGBM(1,1) is given as

\[
\hat{x}^{(r)}(k) = \left( \left( (b^{(0)}(1) \right)^{1-\gamma} - \frac{b}{a} \right) \times \exp \left( -a(1-\gamma)(k-1) + \frac{b}{a} \right) \right)^{1/\gamma}
\]

**Proof** See Appendix.

With the aim of calculating the model parameters, the discrete formula of Eq. (10) should be derived. First, integrating the both sides of Eq. (10) yields

\[
\int \int \cdots \int_{k-1}^k \frac{d^\alpha x^{(r)}}{dt^\alpha} dt^\alpha + a \int \int \cdots \int_{k-1}^k x^{(r)} dt^\alpha \\
= b \int \int \cdots \int_{k-1}^k \left( x^{(r)} \right)^\gamma dt^\alpha
\]

In accordance with the two-point trapezoidal formula, each term of Eq. (10) becomes

\[
\begin{align*}
\int \int \cdots \int_{k-1}^k \frac{d^\alpha x^{(r)}}{dt^\alpha} dt^\alpha &\approx \gamma^{\alpha} x^{(r)}(k) \\
ax^{(r)} &\approx a \frac{\gamma x^{(r)}(k)+x^{(r)}(k-1)}{2} = a\zeta^{(r)}(k) \\
bt \int_{k-1}^k \left( x^{(r)} \right)^\gamma dt^\alpha &\approx b \left( x^{(r)}(k) \right)^\gamma
\end{align*}
\]

Substituting Eq. (13) into Eq. (12), one can write

\[
x^{(r-\alpha)}(k) + a\zeta^{(r)}(k) = b \left( x^{(r)}(k) \right)^\gamma
\]
On this basis, the model parameters \( \hat{\psi} = (a, b)^T \) can be obtained by the least squares method:

\[
\hat{\psi} = (a, b)^T = \left( \vartheta^T \vartheta \right)^{-1} \vartheta^T \kappa
\]  

(15)

where

\[
\vartheta = \begin{bmatrix}
-\varepsilon(r)(2) (\varepsilon(r)(2))^y \\
-\varepsilon(r)(3) (\varepsilon(r)(3))^y \\
\vdots \\
-\varepsilon(r)(n) (\varepsilon(r)(n))^y
\end{bmatrix}, \quad \kappa = \begin{bmatrix}
x^{(r-a)}(2) \\
x^{(r-a)}(3) \\
\vdots \\
x^{(r-a)}(n)
\end{bmatrix}
\]

After a simple inverse calculation according to Definition 2, the predicted values of \( x^{(0)}(k) \), \( k = 2, 3, \ldots \) can be acquired.

**Determination of emerging coefficients for CCFNGBM(1,1)**

The emerging coefficients, namely, fractional order \( r \), conformable fractional order \( \alpha \), and power index \( \gamma \), need to be estimated. The estimation is also a crucial step in the modeling procedure of the newly proposed model since the quality of the emerging coefficients has a great impact on the final prediction performance. With the goal of searching for the proper coefficients for the newly proposed model, we construct a simple optimization problem with constraints for this goal; the calculation formula is as follows.

\[
\arg \min_{r, \alpha, \gamma} f(r, \alpha, \gamma) = \frac{1}{n} \sum_{k=1}^{n} \left[ \frac{x^{0}(k) - x^{0}(k)}{\max(x^{0}(k))} \right] \times 100\%
\]

s.t.

\[
\hat{\vartheta} = (a, b)^T = (\vartheta^T \vartheta)^{-1} \vartheta^T \kappa
\]

\[
\hat{\vartheta}^{(r)}(k) = \left[ \left( \tilde{x}^{(0)}(1) \right)^{1-r} - \frac{a}{r} \right] \times \exp \left( -\frac{a}{r} (1 - r) (k^n - 1) + \frac{a}{r} \right) \quad \sum
\]

(16)

where \( x^{0}(k) \) and \( \tilde{x}^{0}(k) \) represent the actual value and the corresponding prediction, respectively. However, it is difficult to solve Eq. (16) by the ordinary method due to its nonlinear characteristics. To address this problem, a metaheuristic algorithm, namely GWO, is introduced to search for the proper emerging coefficients for CCFNGBM(1,1) in the current study.

GWO, proposed by Mirjalili et al. (2014), stems from the imitation of the social hierarchy and hunting behaviors of grey wolves. In GWO, the wolves are classified into four levels, namely \( \alpha, \beta, \sigma \), and \( \omega \), according to their social hierarchies, in which \( \alpha \) denotes the fittest solution (or, best candidate agent), followed by \( \beta, \sigma \), and \( \omega \). Generally, the hunting is guided by \( \alpha \), \( \beta \), and \( \sigma \). The distance of the wolves is calculated as

\[
\hat{D} = \left| \hat{C} \tilde{x}_p(t) - \tilde{x}(t) \right|
\]

(17)

\[
\tilde{x}(t + 1) = \hat{C}_p(t) - \hat{A} \hat{D}
\]

(18)

where \( \tilde{x}_p \) refers to the position of the prey, and \( \tilde{x} \) denotes the position of a grey wolf. In addition, \( \hat{A} \) and \( \hat{C} \) are coefficient vectors, and the relevant calculation formulas are as follows.

\[
\hat{A} = 2 \tilde{a} \tilde{r}_1 - \tilde{a}
\]

\[
\hat{C} = 2 \tilde{r}_2
\]

(19)

(20)

In Eqs. (19)–(20), components of \( \tilde{a} \) linearly decline from 2 to 0 for each iteration, and both \( \tilde{r}_1 \) and \( \tilde{r}_2 \) are random vectors generated within the interval [0,1].

After saving the first three best solutions obtained thus far and having other search agents update their positions in accordance with the best search agent, the relevant calculation formula is given as

\[
\tilde{x}(t + 1) = \frac{\tilde{x}(t)}{3}
\]

(21)

For clarity, the pseudocode of the GWO algorithm-based CCFNGBM(1,1) model is summarized in Algorithm 1.

**Algorithm 1** The algorithm of GWO to seek the optimal coefficients \( r, \alpha, \gamma \).

**Input:** Set the fitness function and initialize the parameters for GWO

- The original series \( x^{0} \)

**Output:** The optimal coefficients \( r, \alpha, \gamma \)

1: for \( i < T \) do

- Substitute \( (r, \alpha, \gamma) \) into Eq. 15 to obtain parameters \( \hat{\psi} \)

- Substitute parameters into Eq. 11 to calculate the simulative values \( \tilde{x}^{(r)} \)

- Calculate the restored values \( \hat{x}^{(0)} \) in Eq. 9

- Calculate the MAPE value Eq. 16

2: end for

- Update the minimum value MAPE

- return the best coefficients \( r, \alpha, \gamma \).

**Model evaluation metric**

To evaluate the degree of the prediction accuracy of the newly proposed model, three commonly used indicators, namely, absolute percentage error (APE), mean absolute percentage error (MAPE), and root mean square error (RMSE) are selected as the judgement metrics, and their calculation formulas are as follows.

\[
\text{APE} = \left| \frac{\hat{x}^{(0)}(k) - x^{0}(k)}{x^{0}(k)} \right| \times 100\%
\]

(22)

\[
\text{MAPE} = \frac{1}{n - 1} \sum_{k=2}^{n} \left| \frac{\hat{x}^{(0)}(k) - x^{0}(k)}{x^{0}(k)} \right| \times 100\%
\]

(23)

\[
\text{RMSE} = \sqrt{\frac{1}{n - 1} \sum_{k=2}^{n} (\hat{x}^{(0)}(k) - x^{0}(k))^2}
\]

(24)
Verification of CCFNGBM(1,1)

In this section, two numerical examples are taken to verify the validity of the newly proposed model by comparing it with other relevant forecasting models, namely GM(1,1), DGM(1,1), NGBM(1,1) and FNGBM(1,1). The unknown emerging coefficients of the newly proposed model are determined by a range of algorithms, namely ALO, WOA, and PSO, to demonstrate the reasonability of GWO.

**Case 1.** (Forecasting diesel fuel consumption in Shanghai [unit: 10^4 tons]) In this case, diesel fuel consumption in Shanghai from 2000 to 2017 is taken as an example to verify the validity of the proposed model. The diesel fuel consumption information is collected from the National Bureau of Statistics of China, and the data from 2000 to 2015 are chosen to calibrate the model, while the remaining two years of data are used to examine the accuracy. In regard to the emerging coefficients for CCFNGBM(1,1), they are determined by the four algorithms, and the corresponding best fitness values are listed in Table 1, while the track of the search process for these algorithms is graphed in Fig. 1. After comprehensive comparisons with other algorithms, GWO is shown to be more effective in terms of the fitness value and time complexity, and the emerging coefficients are thus obtained as \( r = 0.6660, \alpha = 0.0958, \) and \( \gamma = 6.0956 \). As Table 2 shows, the prediction performance of the newly proposed model is evidently the best in either the simulative stage or prediction stage, thus indicating that the proposed CCNGBM(1,1) model is better than the others.

**Case 2.** (Forecasting total CO₂ emissions of Germany [unit: million tons]) In this case, the total CO₂ emissions of Germany from 2008 to 2018 are employed to validate the predictive capacity of the newly proposed model, which are collected from the *BP Statistical Review of World Energy 2019*. Similar to Case 1, the raw data are categorized into two groups, of which the first 8 samples are used for model calibration, and the remaining two samples are used for detecting the accuracy. Table 3 exhibits the overall performance of the four algorithms, and Fig. 2 depicts the search process for the four algorithms. On this basis, the optimal emerging coefficients are \( r = 1.6713, \alpha = 0.3242 \) and \( \gamma = 0.9512 \). The prediction performance of the five competitive models is shown in Table 4. In regard to the simulation stage, although the MAPE and RMSE values are not best, they are very close to the best values produced by NGBM(1,1). In particular, the MAPE is much smaller than that of the suboptimal model in the prediction stage. This fact leads to the conclusion that the proposed model outperforms other competitive models in this case.

Application

In view of the superiority of the proposed CCFNGBM(1,1) model mentioned above, it is applied to project CO₂ emissions from fuel combustion in China. Therefore, this section is divided into four subsections, including data description, model calibration, comparative analysis and future emissions forecasting. The flowchart of forecasting fuel combustion-related CO₂ emissions with the newly proposed model is graphed in Fig. 3.

**Data description**

We consider annual data for CO₂ emissions from fuel combustion in China for the period between 2001 and 2019. This is driven by the fact that the Chinese economy and CO₂ emissions increased slowly before 2000, while they grew quickly after 2000. An important reason explaining this growth is that China joining the WTO in 2001, becoming a market economy country instead of a planned economy country. Information before 2001 has little reference value according to the principle of the priority of new information. In addition, Ofosu-Adarkwa et al. (2020) noted that in modeling a sequence impacted by shocks, it is better to focus on the most relevant shocks, and thus narrow the series closest to the current time to capture the pertinent shocks. As
Table 2  Results of five grey models in forecasting diesel fuel consumption in Shanghai [unit: 10^4 tons]

| Year | Data  | GM   | DGM  | NGBM | FNGBM | CCFNGBM |
|------|-------|------|------|------|-------|---------|
| 2000 | 176.44| 176.44| 176.44| 176.44| 176.44| 176.44  |
| 2001 | 231.80| 276.89| 277.48| 196.26| 205.12| 210.14  |
| 2002 | 236.78| 293.53| 294.10| 245.81| 236.78| 251.08  |
| 2003 | 288.32| 311.16| 311.72| 288.32| 270.97| 288.31  |
| 2004 | 346.56| 329.86| 330.39| 325.89| 306.96| 322.94  |
| 2005 | 329.60| 349.68| 350.18| 359.67| 343.70| 355.65  |
| 2006 | 370.98| 370.70| 371.16| 390.40| 380.00| 386.83  |
| 2007 | 370.98| 370.70| 371.16| 390.40| 380.00| 386.83  |
| 2008 | 370.98| 370.70| 371.16| 390.40| 380.00| 386.83  |
| 2009 | 370.98| 370.70| 371.16| 390.40| 380.00| 386.83  |
| 2010 | 370.98| 370.70| 371.16| 390.40| 380.00| 386.83  |
| 2011 | 370.98| 370.70| 371.16| 390.40| 380.00| 386.83  |
| 2012 | 370.98| 370.70| 371.16| 390.40| 380.00| 386.83  |
| 2013 | 370.98| 370.70| 371.16| 390.40| 380.00| 386.83  |
| 2014 | 370.98| 370.70| 371.16| 390.40| 380.00| 386.83  |
| 2015 | 370.98| 370.70| 371.16| 390.40| 380.00| 386.83  |
| MAPE (%) | 8.09 | 8.11 | 4.61 | 3.81 | 3.66 |
| RMSE | 10.17 | 10.23 | 5.88 | 4.62 |
| 2016 | 562.20| 664.42| 664.09| 597.81| 571.75| 562.10  |
| 2017 | 550.38| 704.35| 703.87| 611.94| 576.20| 546.77  |
| MAPE (%) | 23.08 | 23.00 | 8.76 | 3.19 | 0.34 |
| RMSE | 23.59 | 23.51 | 9.00 | 3.46 |

such, the above data sequence is suitable for making future projections of CO₂ emissions from fuel combustion.

We obtained emissions between 2001 and 2019 from the Statistical Review of World Energy, in which, specifically, the carbon emissions reflect only those from consumption of oil, gas and coal for combustion-related activities. In addition, we take CO₂ emissions from fuel combustion reported by the International Energy Agency (IEA) into consideration, which are divided into four components - coal, oil, gas, and international marine bunkers and aviation bunkers; CO₂ emissions from fuel combustion by sector are also recorded. As Fig. 4 shows, allowing for statistical bias, the two series are basically consistent for the period they overlap, making them suitable for making future projections in this study.

Figure 4 shows that the CO₂ emissions from fuel combustion in China have experienced an extensive increase, accounting for 3593.1 Mt in 2001 and increasing to 9920.5 million tons in 2019, which is over 2.5 times the original emissions. With reference to Fig. 5, there are obvious discrepancies between different sectors; for example, electricity and heat production occupy the largest share with 50%, followed by manufacturing industries and construction, and transport. The sum of the other sectors

Table 3  Performance of the four algorithms in Case 2

| Algorithm | ALO | GWO | WOA | PSO |
|-----------|-----|-----|-----|-----|
| Fitness value | 1.6856 | 1.516 | 1.4760 | 1.5583 |
| Mean run time (s) | 5.2964 | 5.3122 | 7.4948 | 12.2336 |

Fig. 2  Track of search for the emerging coefficients with the four algorithms
### Table 4  Results of five grey models in forecasting total CO₂ emissions of Germany [unit: million tons]

| Year | Data   | GM  | DGM | NGBM | FNGBM | CCFNGBM |
|------|--------|-----|-----|------|-------|---------|
| 2008 | 806.5  | 806.50 | 806.50 | 806.50 | 806.50 | 806.50 |
| 2009 | 751.0  | 751.00 | 767.63 | 751.00 | 751.00 | 752.18 |
| 2010 | 780.6  | 780.60 | 767.03 | 780.60 | 780.60 | 758.17 |
| 2011 | 761.0  | 761.00 | 766.44 | 761.00 | 761.00 | 765.19 |
| 2012 | 770.3  | 770.30 | 765.85 | 770.30 | 770.30 | 770.13 |
| 2013 | 794.6  | 794.60 | 765.25 | 794.60 | 794.60 | 771.67 |
| 2014 | 748.4  | 748.40 | 764.40 | 748.40 | 748.40 | 770.16 |
| 2015 | 751.9  | 751.90 | 764.07 | 751.90 | 751.90 | 766.18 |
| 2016 | 766.6  | 766.60 | 763.48 | 766.60 | 766.60 | 760.22 |
|      | MAPE (%) | 1.64 | 1.64 | 1.51 | 1.36 | 1.52 |
|      | RMSE     | 1.93 | 1.93 | 1.76 | 1.83 | 1.92 |
| 2017 | 762.6  | 762.60 | 762.88 | 762.60 | 762.60 | 752.70 |
| 2018 | 725.7  | 725.70 | 725.29 | 725.70 | 725.70 | 743.96 |
|      | MAPE (%) | 2.54 | 2.56 | 2.19 | 3.39 | 1.91 |
|      | RMSE     | 3.58 | 3.55 | 2.45 | 4.33 | 2.00 |

**Fig. 3** Structural flowchart of the newly proposed model for forecasting CO₂ emissions from fuel combustion in China
Fig. 4  Statistical data of CO₂ emissions from fuel combustion in China (e.g. residential, commercial and public services, and other energy industry own use) comprises a 10% share. CO₂ emissions from fuel combustion in China are closely related to the energy structure. Therefore, by observing China’s CO₂ emissions from fuel combustion from the perspective of the energy structure, as shown in Fig. 6, coal combustion.

Fig. 5  CO₂ emissions from fuel combustion by sector
is found to be a dominant driver contributing to China’s CO2 emissions from fuel combustion, reaching 2689.1 million tons in 2001 and 7469.9 million tons in 2017, with a share of approximately 80% of total emissions.

**Model calibration for CCFNGBM(1,1)**

For the purpose of demonstrating the superiority of the newly proposed model, in addition to the grey-based models (specified in “Verification of CCFNGBM(1,1)”), non-grey-based benchmark approaches, namely, statistical models and machine learning methods, are also used for comparison in the current work. The polynomial regression (PR) (Akhlaghi et al. 2019) and ARIMA (Singh et al. 2019) are among the building blocks of statistical models. The artificial neural network (ANN) (Afram et al. 2017) and support vector machine (SVM) (Richhariya and Tanveer 2018) are considered representative of machine learning methods. By comparison with these competitive models, the effectiveness and superiority of the newly proposed model can be verified comprehensively. In particular, the data from 2001 to 2017 are used for model calibration, and the remaining two observations are chosen to verify the accuracy. Here, we take the proposed model as an example; its computational steps can be summarized as follows.

**Step one.** Obtain the original sequence $X^{(0)} = (3593.1, 3910.6, \cdots, 9396.9)$, for initialization purposes, both the $r$-FAGO sequence $X^{(r)}$ and the background value $z^{(r)}$ are then obtained.

**Step two.** Initialize $\alpha = 1$ and $\gamma = 1$ with the aid of the least squares method, and the model parameters can be calculated by $\hat{\psi} = (a, b)^T = (\vartheta^T \vartheta)^{-1} \vartheta^T \kappa$, which yields $\hat{a} = -0.0439$ \quad $\hat{b} = 5136.16$.

**Step three.** Calculate the time response function:

$$\hat{x}^{(r)}(k) = \left\{3593.10^{1-\gamma} + 117030.89 \times \exp(0.0439(1-\gamma)(k-1)) - 117030.89\right\}^{1/\gamma}$$

**Step four.** Optimize the model parameters $\hat{\psi}$ by searching for the optimal value for the emerging coefficients with the following fitness function:

$$\text{fitness}(r, a, \gamma) = \frac{1}{n-1} \sum_{i=2}^{n} \left|\frac{\hat{x}^{(0)}(k) - x^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\% \quad (25)$$

Taking the efficacy of GWO into consideration (elaborated on in “Verification of CCFNGBM(1,1)”), the optimal emerging coefficients of the proposed model are
determined by GWO. With the help of the GWO algorithm, the optimal emerging coefficients $r$, $\alpha$, and $\gamma$ that correspond to the best fitness value can be searched and found to be $(0.938, 0.3037, 1.3164)^T$. The track of the search process for GWO over 100 trials is shown in Fig. 7.

### Step five

Substituting the model parameters into Steps 1-3, the time response function of the newly proposed model is given by

$$
\hat{x}(k) = (0.0613 \times \exp\left(-0.3686\left(k^{0.3037} - 1\right)\right))^{3.1605}, k = 2, 3, \cdots
$$

| Year | Data | ARIMA(1,2,0) | PR(2) | ANN | SVM | GM | DGM | NGBM | FNGBM | CCFNGBM |
|------|------|-------------|-------|-----|-----|----|-----|------|-------|---------|
| 2001 | 3593.1 | 3591.49 | 3306.58 | 3451.82 | 4593.15 | 3593.10 | 3593.10 | 3593.10 | 3593.10 | 3593.10 |
| 2002 | 3910.6 | 3914.71 | 4083.19 | 3969.77 | 4988.34 | 5411.73 | 5422.82 | 3654.78 | 3910.51 | 3912.12 |
| 2003 | 4603.4 | 4236.88 | 4806.87 | 4571.96 | 5383.53 | 5654.53 | 5665.04 | 4603.98 | 4667.74 | 4680.63 |
| 2004 | 5413.4 | 5215.51 | 5477.62 | 5235.31 | 5778.71 | 5908.22 | 5918.07 | 5413.41 | 5412.98 | 5378.46 |
| 2005 | 6174.0 | 6198.20 | 6095.44 | 5923.98 | 6173.90 | 6173.29 | 6182.42 | 6109.25 | 6093.64 | 6007.43 |
| 2006 | 6757.2 | 6945.22 | 6660.33 | 6596.32 | 6569.09 | 6450.25 | 6458.56 | 6709.18 | 6698.80 | 6573.08 |
| 2007 | 7325.4 | 7378.54 | 7172.28 | 7214.46 | 6964.28 | 6739.63 | 6747.04 | 7226.33 | 7228.15 | 7080.43 |
| 2008 | 7457.4 | 7896.82 | 7631.31 | 7752.17 | 7354.96 | 7042.00 | 7048.41 | 7671.02 | 7685.18 | 7533.91 |
| 2009 | 7796.6 | 7683.18 | 8037.40 | 8197.86 | 7754.65 | 7357.94 | 7363.24 | 8051.68 | 8074.93 | 7937.51 |
| 2010 | 8231.7 | 8091.25 | 8390.57 | 8552.71 | 8149.84 | 7688.05 | 7692.12 | 8375.38 | 8403.00 | 8294.91 |
| 2011 | 8916.3 | 8646.18 | 8690.80 | 8826.30 | 8545.03 | 8032.97 | 8035.70 | 8648.18 | 8675.06 | 8609.54 |
| 2012 | 9090.0 | 9547.26 | 8938.10 | 9032.06 | 8940.21 | 8393.37 | 8394.63 | 8875.33 | 8896.69 | 8884.59 |
| 2013 | 9335.5 | 9373.54 | 9132.47 | 9183.93 | 9335.40 | 8769.93 | 8769.59 | 9061.44 | 9073.15 | 9123.09 |
| 2014 | 9329.6 | 9565.56 | 9273.91 | 9294.48 | 9730.59 | 9163.39 | 9161.30 | 9210.60 | 9209.38 | 9327.86 |
| 2015 | 9276.5 | 9377.75 | 9362.41 | 9374.14 | 10157.8 | 9574.50 | 9570.50 | 9326.45 | 9309.97 | 9501.56 |
| 2016 | 9230.3 | 9233.55 | 9397.99 | 9431.12 | 10520.96 | 10004.05 | 9997.98 | 9371.31 | 9379.09 | 9646.68 |
| 2017 | 9396.9 | 9182.62 | 9380.63 | 9471.67 | 10916.15 | 10452.88 | 10444.55 | 9470.96 | 9420.56 | 9765.55 |
| 2018 | 9606.6 | 9517.75 | 9310.35 | 9500.41 | 11311.34 | 10921.84 | 10911.07 | 9505.20 | 9437.86 | 9860.35 |
| 2019 | 9920.5 | 9648.43 | 9187.13 | 9520.74 | 11706.53 | 11411.85 | 11398.43 | 9517.40 | 9434.11 | 9933.12 |
Comparative analysis

In view of the implementation of the above procedure, the prediction results of the newly proposed model and eight benchmark models can be calculated, as shown in Tables 5 and 6.

The APEs of the newly proposed model and FNGBM(1,1) vary in a small range, [0%, 5%] for the entire sequence, while those of others models have wider fluctuations of at least [0%, 6%], indicating that fractional order accumulation is an effective approach for improving the prediction performance significantly. In particular, the APE range of APEs with [0.03%, 2.54%] for the newly proposed model is narrower for the prediction stage, which confirms that the newly proposed model outperforms competitive models.

For the MAPE values, those obtained by these nine models are less than 10%, meaning that these models are appropriate for modeling CO₂ emissions from fuel combustion according to the Lewis standard (Lewis 1982). Nevertheless, the newly proposed model exhibits the lowest MAPE value of 1.33% for the prediction stage, suggesting that this model has an outstanding ability to forecast CO₂ emissions from fuel combustion compared to the other benchmark models. In regard to RMSE values, they are basically in line with the corresponding MAPE values. Analyzing the modeling procedure of these models, NGBM(1,1) captures nonlinear characteristics and thus obtains a more desirable outcome relative to linear models, namely GM(1,1) and DGM(1,1). In addition, the combination of fractional accumulation and the differential derivative make the NGBM(1,1) model better. As machine learning methods that require a large sample size, the ANN and SVM do not acquire the best results, similar to the statistical models (i.e. ARIMA(2,1,0) and PR(2)), and this is primarily due to the small data sequence.

Figures 8 and 9 visibly display the line graph and error distribution of these competitors. They support the above findings that the newly proposed model can well match the prediction values to the observations and produce a lower prediction derivation from a graphic perspective.

Fuel combustion-based CO₂ emission forecasts

In view of the superiority of the newly proposed model, there is a pressing need to apply this model to forecast the future emissions from fuel combustion, which could provide a basis for decision-makers to prepare suitable

| Year | ARIMA(1,2,0) | PR(2) | ANN | SVM | GM | DGM | NGBM | FNGBM | CCFNGBM |
|------|--------------|-------|-----|-----|----|-----|------|-------|---------|
| 2001 | 0.04         | 7.97  | 3.93| 27.83| 0.00| 0.00| 0.00 | 0.00  | 0.00    |
| 2002 | 0.11         | 4.41  | 1.51| 27.56| 38.39| 38.67| 6.54 | 0.00  | 0.04    |
| 2003 | 7.96         | 4.42  | 0.68| 16.95| 22.83| 23.06| 0.01 | 1.40  | 1.68    |
| 2004 | 3.66         | 1.19  | 3.29| 6.75 | 9.14 | 9.32 | 0.00 | 0.01  | 0.65    |
| 2005 | 0.39         | 1.27  | 4.05| 0.00 | 0.01 | 0.14 | 1.05 | 1.30  | 2.70    |
| 2006 | 2.78         | 1.43  | 2.38| 2.78 | 4.54 | 4.42 | 0.71 | 0.86  | 2.72    |
| 2007 | 0.73         | 2.09  | 1.51| 4.93 | 8.00 | 7.90 | 1.35 | 1.33  | 3.34    |
| 2008 | 5.89         | 2.33  | 3.95| 1.31 | 5.57 | 5.48 | 2.86 | 3.05  | 1.03    |
| 2009 | 1.45         | 3.90  | 5.15| 0.54 | 5.63 | 5.56 | 3.27 | 3.57  | 1.81    |
| 2010 | 1.71         | 1.93  | 3.90| 0.99 | 6.60 | 6.55 | 1.75 | 2.08  | 0.77    |
| 2011 | 3.03         | 2.53  | 1.01| 4.16 | 9.91 | 9.88 | 3.01 | 2.71  | 3.44    |
| 2012 | 5.03         | 1.67  | 0.64| 1.65 | 7.66 | 7.65 | 2.36 | 2.13  | 2.26    |
| 2013 | 0.41         | 2.17  | 1.62| 0.00 | 6.06 | 6.06 | 2.94 | 2.81  | 2.28    |
| 2014 | 2.53         | 0.60  | 0.38| 4.30 | 1.78 | 1.80 | 1.28 | 1.29  | 0.02    |
| 2015 | 1.09         | 0.93  | 1.05| 9.16 | 3.21 | 3.17 | 0.54 | 0.36  | 2.43    |
| 2016 | 0.04         | 1.82  | 2.18| 13.98| 8.38 | 8.32 | 1.97 | 1.61  | 4.51    |
| 2017 | 2.28         | 0.17  | 0.80| 16.17| 11.24| 11.15| 0.79 | 0.25  | 3.92    |
| MAPE (%) | 2.30 | 2.35 | 2.24 | 8.18 | 9.31 | 9.32 | 1.90 | 1.55  | 2.11    |
| RMSE | 3.18         | 3.00  | 2.67| 12.08| 12.93| 13.00| 2.47 | 1.88  | 2.47    |
| 2018 | 0.92         | 2.96  | 1.06| 17.05| 13.15| 13.04| 1.01 | 1.69  | 2.54    |
| 2019 | 2.74         | 7.33  | 4.00| 17.86| 14.91| 14.78| 4.03 | 4.86  | 0.13    |
| MAPE (%) | 1.80 | 5.15 | 2.53| 17.45| 14.03| 13.91| 2.52 | 3.28  | 1.33    |
| RMSE | 2.02         | 5.60  | 2.92| 17.46| 14.06| 13.94| 2.94 | 3.64  | 1.80    |
plans and policies. Driven by the CCFNGBM(1,1) model as the above subsection introduces, CO₂ emissions from fuel combustion from 2020 to 2023 are drawn in Fig. 10. There is an upward trend by 2023, accounting for 9860.35 Mt in 2020 and 10,039.80 million tons in 2023, supporting the findings that energy-related CO₂ emissions may reach 9936 million tons. Our results are relatively reasonable, as seen by referring to the official database reported by the International Energy Agency (IEA, 2017) (10,205 million tons) and the Economic Forecasting Division of the State Information Center (EFDSI, 2016) (9,860 million tons). In addition, the growth rate of fuel combustion-based CO₂ emissions will experience a downward trend in the future years, basically below 1%, and this increasing trend is aligned with the target promised by the Chinese government, indicating the potential to achieve the target of peaking CO₂ emissions by 2030 or earlier.

Conclusion

Fuel combustion-based carbon emission forecasting is crucial for framing and implementing reasonable plans and policies, and this is primarily due to diverse national energy structures. To accurately make projections of CO₂ emissions from fuel combustion in China by 2023, this paper develops a novel continuous fractional NGBM(1,1) model, namely, CCFNGBM(1,1), by simultaneously incorporating conformable fractional accumulation and derivative into the traditional NGBM(1,1) model that is capable of capturing the nonlinear characteristics hidden in sequences. To further improve the predictive capacity of the newly proposed model, this study employs GWO for determining the emerging coefficients. This model can not only perfect the grey forecasting model by replacing the integer-order derivative with the fractional derivative but also provide relatively reliable forecasts for decision-makers.
Two examples already in place are employed to verify the superiority of the newly proposed model in comparison with other grey model-based approaches by introducing the MAPE. Subsequently, this model is applied to model and predict CO$_2$ emissions from fuel combustion in China in comparison with other benchmark models, including grey model-based and non-grey-model-based approaches. Then, the predictive capacity of the newly proposed model is verified again. The future value in 2023 is expected to reach 10,039.80 Mt, while the growth rate falls significantly, which could provide a solid basis for decision-makers to prepare the following plans and polices.

Based on the analysis in “Application”, combined with the forecasts of this study and the current situation in China, several suggestions on curbing CO$_2$ emissions from fuel combustion are presented as follows.

- Developing low-carbon technologies. As Fig. 6 suggests, the largest share of CO$_2$ emissions from fuel combustion is electricity and heat production, and this process produces approximately 50% of total carbon emissions of China. Accelerating the progress of low-carbon technology is essential for green economic growth of China. Low-carbon technologies involve power, transportation, construction, metallurgy, chemical, petrochemical and other sectors, as well as developments in renewable energy and new energy, clean and efficient use of coal, exploration and development of oil and gas resources and coalbed methane, and carbon dioxide capture and storage. New technologies can effectively control greenhouse gas emissions.

- Accelerating the promotion of the national carbon market. In addition to electricity and heat production, the manufacturing industry is also high energy consumer,
making economic restructuring and industrial upgrading a difficult task. Coal consumption still accounts for more than 50% of China’s energy production, and the intensity of carbon dioxide emissions per energy unit is 30% higher than the world average. Energy consumption per unit of GDP is still very high, approximately 1.5 times the world average and 2 to 3 times that of developed countries. Framing an action plan for peak carbon dioxide emissions and accelerating the promotion of the national carbon market are important. China must make greater efforts than developed countries to achieve carbon neutrality.

- Strengthening citizens’ environmental awareness. On the basis of establishing citizens environmental rights, further expanding their vision of environmental ethics and raising awareness of environmental issues are necessary. We must mobilize all aspects of society to give full play to the role of organizations at all levels, including agencies, media, schools, units, and communities, to reach the masses. Vigorously publicizing the significance of environmental protection can make people aware of the hazards of environmental problems, and enhance a sense of responsibility and mission of environmental protection.

Of course the proposed method is not without limitations. For example, the results we forecast by using the newly proposed model are conservative, and this is primarily due to the sparse historical records we consider with the mechanism of grey forecasting models. On the other hand, the ease of implementation is achieved at the cost of ignoring the potential factors related to fuel combustion-based carbon emissions, such as the urbanization level, the industrialization level and energy structure, and so forth, and these issues need to be addressed in the following research.

**Appendix A. Proof of Theorem 1**

**Proof** Multiplying both sides of Eq. (10) by \((x^{(r)}(t))^γ\), we have

\[
dαx^{(r)}(t)\over dt^α \left( x^{(r)}(t) \right)^γ + a (x^{(r)}(t))^{1-γ} = b \tag{26}
\]

Setting \(y^{(r)}(t) = (x^{(r)}(t))^{1-γ}\), Eq. (26) can be rewritten as

\[
dαy^{(r)}(t)\over dt^α + a (1-γ) y^{(r)}(t) = b (1-γ) \tag{27}
\]

According to the features of the conformable fractional derivative (\(\), \(T_αf(t) = t^{1-α} df(t)\over dt\)), where \(T_αf(t)\) and \(df(t)\over dt\) represent the \(α\)–order conformable fractional derivative and the integer-order derivative in Riemannian geometry, respectively. Equation (27) becomes

\[
\left( y^{(r)}(t) \right)^{1-α} \frac{dy^{(r)}(t)}{dt} + a (1-γ) y^{(r)}(t) = b (1-γ) \tag{28}
\]

Solving Eq. (28) yields that

\[
y^{(r)}(t) = \exp \left( - \int (1-γ) \frac{a}{t^{1-α}} dt \right) \left\{ \int (1-γ) \frac{b}{t^{1-α}} dt + C \right\} \tag{29}
\]
Further,

\[ x^{(r)}(t) = \left\{ \exp \left(-\int (1 - \gamma) \frac{a}{t^{1-\alpha}} \, dt \right) \int (1 - \gamma) \frac{b}{t^{1-\alpha}} \, dt \right\}^{\gamma} \]  

Assuming that \( x^{(r)}(1) = x^{(0)}(1) \), we obtain

\[ C = \left( x^{(0)}(1)^{1-\gamma} - \frac{b}{a} \right) \times \exp \frac{a}{a} (1 - \gamma) \]  

Substituting Eq. (31) into Eq. (30), we have

\[ x^{(r)}(t) = \left( x^{(0)}(1)^{1-\gamma} - \frac{b}{a} \right) \times \exp \left( -\frac{a}{a} (1 - \gamma) \right)^{\gamma} \]  

Setting \( t = k \), this completes the proof.

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Declarations

Competing interests The authors declare competing interests.

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