Modeling of Precision Machining for Crankshaft Follow-up Grinder

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Abstract. In this paper, follow-up grinding of crankshaft is studied and the mathematical model of follow-up grinding motion is established. Taking the double grinding wheel crankshaft follow-up grinder as the research object, the motion errors of the machine tool are obtained by analyzing the motion relations among the moving bodies. Based on the kinematics theory of multi-body system, the kinematics model of the whole machine tool is established. The position matrix of any cutting point is described according to the kinematic chain of "bed-workpiece" and "bed-tool", and the precise machining model of double grinding wheel follow-up grinding machine tool is established by using the coincidence principle of tool actual trajectory and tool path. The coordinate system of each moving body is established, the transformation matrix between moving bodies is obtained, and the precision machining model of double grinding wheel follow-up crankshaft CNC grinding machine is solved, which lays a foundation for error compensation of the machine.

Keywords: Crankshaft follow-up grinder; Precision machining modelling; MBS; Motion error

1. Introduction

Engine is the main component of automotive power device and often called the "heart" of automobiles. While, the crankshaft rotation is the power source of the engine, and its machining quality directly affects the performance of the vehicle. In addition, the last process of crankshaft precision machining is usually completed by crankshaft follow-up grinder. Therefore, considering the complexity of crankshaft machining and the composition and motion form of crankshaft follow-up grinder, it is very necessary to establish precision machining model of crankshaft follow-up grinder for improving the machining accuracy of crankshaft.

Over the past few decades, in the aspect of modeling methods of machine tools, scholars at home and abroad have carried out a lot of research work and obtained many research findings. Many different mathematical models have been created, such as geometric modeling, error matrix, quadratic relation model, mechanism modeling, rigid body kinematics and multi-body system theory. Wu et al. [1-2] described the motion relationship between the adjacent kinematic bodies of five-axis machine tools.
simply and conveniently based on homogeneous transform matrix (HTM) and multi-body system (MBS). Fan et al. [3] proposed a generalized kinematics error modeling method of machine tools based on MBS. Zhu et al. [4] established an integrated geometric error model of a five-axis machine tool based on MBS. Moreover, Research on crankshaft follow-up grinding has also developed to a mature stage. Tonshoff et al. [5] put forward continuous path grinding method of crankshaft. Jiang et al. [6] predicted the stability of crankshaft follow-up grinding process. By using a general parametric four-link motion model, Wei et al. [7] established a path generation model of crankshaft pin journal in NC grinding.

As can be observed in the above-mentioned research literatures, these modeling methods of machine tools are rarely aimed at crankshaft follow-up grinder. And these descriptions of crankshaft follow-up grinding methods are difficult to find the specific implementation methods and the key technical issues involved. Therefore, in this paper, the double grinding wheel crankshaft follow-up grinder produced by a machine tool factory is taken as the research object, and its precision machining model is established based on MBS.

2. The principle of crankshaft follow-up grinding

The grinding of crankshaft mainly includes main journal grinding and pin journal grinding. The grinding method of main journal is the same as that of common cylindrical grinding, which is not described here. As shown in Fig. 1, the follow-up grinding of pin journal is realized by controlling crankshaft variable speed rotating motion (C axis) and grinding wheel lateral feed motion (X axis) to keep the grinding wheel always tangent to the grinding point. And whenever the crankshaft rotates one cycle, the grinding wheel and grinding point rotate around the pin journal also rotate one cycle.

According to Fig.1, a schematic diagram of crankshaft follow-up grinding motion can be obtained, as shown in Fig.2. Where \( O, O_1, O_2 \) denote main journal center, grinding wheel center and pin journal center respectively; \( R_1, R_2, R_3 \) denote the eccentricity between the center of crankshaft pin journal and main journal, crankshaft pin journal radius and grinding wheel radius respectively; \( P \) denotes the grinding point; \( \theta, \phi, \phi \) denote rotation angle of crankshaft, rotation angle of grinding point around pin journal and rotation angle of grinding wheel respectively.

From Fig.2, by using Geometric relationship, the coordinates of grinding point \( P (x_{pw}, y_{pw})^T \) and \( (x_{pt}, y_{pt})^T \) can be obtained in the coordinate systems of X-O-Y and X-O-Y1 respectively, as shown in Eq.(1).
At the same time, the displacement $X$ of the grinding wheel along the X-axis can be given, as shown in Eq. (2).

$$X = R_x + R_y - R_3 \cos \theta - R_2 \sin \left( \arcsin \left( \frac{R_3 \sin \theta}{R_2 + R_3} \right) - R_3 \cos \left( \arcsin \left( \frac{R_3 \sin \theta}{R_2 + R_3} \right) \right) \right)$$

Therefore, the linkage model of X-axis and C-axis in crankshaft follow-up grinding is established, i.e., $(x \cdot \theta)$.

3. Motion error analysis of crankshaft follow-up grinder

In this study, a double grinding wheel crankshaft follow-up grinder is chosen as the research object, as shown in Fig.3. Where 0-6 denote bed, headframe, clamping chuck, workpiece, Z guide rail, X guide rail and grinding wheel, respectively.

![Fig. 3. Schematic diagram of double grinding wheel crankshaft follow-up grinder.](image)

It can be seen from Fig.3 that this machine tool is a special three-axis machine tool and mainly completes crankshaft machining through the linkage of two grinding wheel frame feed axes $X_1$, $X_2$ and workpiece rotary axis $C$ axis. In addition, the feed axes of grinding wheel frames are driven directly by linear servo motors and adopt closed hydrostatic guide rail structure, the sliding platform of grinding wheel frames is driven by AC servo-ball screw pair, and the workpiece rotary axis adopts the spindle structure of high precision rolling bearing, which can achieve extremely high rotary accuracy and positioning accuracy. The grinder has two branches, which can grind pin journals with different phases at the same time and grind main journal and pin journal separately. In the process of follow-up grinding, there are six position-dependent geometric errors (PDGEs) in each motion axis, including three linear errors and three angular errors, and there are also perpendicularity errors (i.e. position-independent geometric errors (PIGEs)) between motion axes [8]. Therefore, there are 29 errors in the machine tool, as shown in Table 1. Where $\varepsilon_{xc} = \sqrt{\varepsilon_{xc}^2 + \varepsilon_{yc}^2}$.
Table 1. Geometric error parameters of the machine tool.

| Axis | Linear error | Angular error |
|------|--------------|--------------|
|      | Along X      | Along Y      | Around X | Around Y | Around Z |
| PDGEs| δ₁(x₁)       | δ₁(x₁)       | ε₁(x₁)   | ε₁(x₁)   | ε₁(x₁)   |
|      | δ₂(x₂)       | δ₂(x₂)       | ε₂(x₂)   | ε₂(x₂)   | ε₂(x₂)   |
|      | δ₃(z)        | δ₃(z)        | ε₃(z)    | ε₃(z)    | ε₃(z)    |
|      | δ₄(C)        | δ₄(C)        | ε₄(C)    | ε₄(C)    | ε₄(C)    |
| PIGEs| ε₁₁₂         | ε₁₂₃         | ε₁₂c     | ε₁₂c     | ε₁₂c     |

4. Establishment of precision machining model for double grinding wheel crankshaft follow-up grinder based on mbs

4.1. Precision machining modelling

From Fig.2, it can be seen that there are only two kinds of single-degree-of-freedom motion between the moving bodies of double-wheel follow-up crankshaft CNC grinder: rotary motion and translational motion. So, based on MBS theory, the topological structure of the machine tool can be established, as shown in Fig.4.

As is seen in Fig. 4, there are three branches of the kinematic chain in the double grinding wheel crankshaft follow-up grinder, namely, the branch of "bed-workpiece" (B-W), the branch of "bed-tool 1" (B-T1) and the branch of "bed-tool 2" (B-T2). Two branches of "bed-tool" are similar in structure, so only the branch of "bed-tool 1" is selected to be analyzed relative to the branch of "bed-workpiece".

Assuming that the grinding point of crankshaft pin journal is denoted as P, the position matrix expressions of point P in the inertial body coordinate system are expressed as Eqs.(3) and (4) in the branches of "bed-workpiece" and "bed-tool 1" respectively.

\[
\{P_w\}_0 = [S01_p][S01_w][P01_1][S12_p][S12_w][S12_1][S12_2][S12_3][S12_4][S23_p][S23_w][S23_1][S23_2][r_w]_w
\] (3)

\[
\{P_t\}_0 = [S04_p][S04_w][S04_1][S04_2][S04_3][S45_p][S45_w][S45_1][S45_2][S45_3][S45_4][S45_5][S45_6][r_t]_w
\] (4)

Where \(\{r_w\}_w\) and \(\{r_t\}_w\) denote the position matrix expressions of point P in the workpiece coordinate system and the tool coordinate system respectively.

In the process of crankshaft grinding, the crankshaft contour is composed of relative motion between grinding wheel and crankshaft. In order to realize the precise machining of crankshaft, it is necessary to ensure that the actual trajectory of the tool center always coincides with the tool path. Therefore, the constraint equation for precision machining of crankshaft follow-up grinder with double grinding wheels can be obtained as follows.

\[
\{P_w\}_0 = \{P_t\}_0
\] (5)
Fig. 4. Topological structure diagram of the grinder.

4.2. Solution of precision machining model

In order to solve Eq.(5), the coordinate system of each moving body should first be determined as follows: (1) let the body reference coordinate system coincide with the motion reference coordinate system under the initial condition; (2) Let each moving body return to the absolute zero of the machine tool. Based on this initial condition, it is assumed that the body reference coordinate system origins of the bed, headframe, chuck, Z guide rail and X guide rail are all located at the end face center of the machine tool spindle (the point $O_2$ in Fig.2), the workpiece reference coordinate system is located at the intersection of the workpiece center and the spindle (the point $O_3$ in Fig.2), and the grinding wheel reference coordinate system is located at the center of the grinding wheel (the point $O_6$ in Fig.2); (3) Based on the direction of the machine tool coordinate system, which is taken as the reference direction, let the direction of the bed coordinate system and the motion reference coordinate system for the headframe and Z guide rail consistent with the direction of the reference coordinate system. Then, the direction of the motion reference coordinate system of the chuck is determined by rotating the reference coordinate system around axis X and axis Y at the perpendicularity of $\varepsilon_{cX}$ and $\varepsilon_{cY}$ respectively, and the direction of the motion reference coordinate system of X guide rail is determined by rotating the reference coordinate system around axis Y at the perpendicularity of $\varepsilon_{xY}$; (4) The direction of workpiece coordinate system is the same as that of motion reference coordinate system for the chuck, and the direction of grinding wheel coordinate system is the same as that of motion reference coordinate system for X guide rail.

Based on the above assumptions, the transformation matrix between the moving bodies is as follows:

\[
[S01]_{\mu} = E_{4*4}, [S01]_{\nu} = E_{4*4}, [S01]_{\sigma} = E_{4*4}, [S01]_{\tau} = E_{4*4}, [S12]_{\mu} = E_{4*4}, [S12]_{\nu} = E_{4*4}, [S12]_{\sigma} = E_{4*4}, [S12]_{\tau} = E_{4*4},
\]

\[
[S12] = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 & 0 \\
\sin \theta & \cos \theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix},
\]

\[
[S12]_{\mu} = \begin{bmatrix}
1 & -\varepsilon_x (C) & \varepsilon_y (C) & \delta_z (C) \\
-\varepsilon_x (C) & 1 & -\varepsilon_y (C) & \delta_z (C) \\
\varepsilon_y (C) & \varepsilon_x (C) & 1 & \delta_z (C) \\
\delta_z (C) & -\varepsilon_y (C) & \varepsilon_x (C) & 1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix},
\]
\[
[\mathbf{S}_{23}]_p = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & L/2 \\
0 & 0 & 0 & 1 \\
\end{bmatrix},
[\mathbf{S}_{23}]_w = E_{+4,4},
[\mathbf{S}_{23}]_w = E_{4,4},
[\mathbf{S}_{04}]_p = E_{+4,4},
[\mathbf{S}_{04}]_w = E_{4,4},
[\mathbf{S}_{45}]_p = E_{+4,4},
[\mathbf{S}_{45}]_w = E_{4,4},
[\mathbf{S}_{56}]_p = E_{+4,4},
[\mathbf{S}_{56}]_w = E_{4,4}.
\]

Where \( L \) denotes the length of tool, \((a, b, c)\) denotes the positional coordinate of point \( O_6 \) in inertial coordinate system.

According to Eq.(1), the following equation can be obtained.

\[
\{ \mathbf{r}_x \}_w = \begin{bmatrix}
R_1 \cos \theta + R_2 \cos \left( \arcsin \left( \frac{R_1 \sin \theta}{R_2 + R_3} \right) \right) \\
R_1 \sin \theta - R_2 \sin \left( \arcsin \left( \frac{R_1 \sin \theta}{R_2 + R_3} \right) \right) \\
\arcsin \left( \frac{R_1 \sin \theta}{R_2 + R_3} \right) \\
\end{bmatrix} \begin{bmatrix}
1 \\
0 \\
1 \\
\end{bmatrix}^	op
\]

\[
\{ \mathbf{r}_y \}_w = \begin{bmatrix}
-R_1 \sin \theta \left( \frac{R_1 \sin \theta}{R_2 + R_3} \right) \\
R_1 \cos \theta - R_2 \cos \left( \arcsin \left( \frac{R_1 \sin \theta}{R_2 + R_3} \right) \right) \\
\arcsin \left( \frac{R_1 \sin \theta}{R_2 + R_3} \right) \\
\end{bmatrix} \begin{bmatrix}
1 \\
0 \\
1 \\
\end{bmatrix}^	op
\]

Then, substituting Eqs.(6)-(7) into Eqs. (3)-(4) and ignoring the high order terms [9], Eq.(8) can be obtained.
\[
x_w = \left( e_{s_{le}} + e_s (c) \sin \theta + e_{s_{c}} (c) \cos \theta \right) \times L/2 + \delta_x (c) \cos \theta - \delta_y (c) \sin \theta - \left( e_{s_{bc}} (c) \sin \theta - \cos \theta \right)
\]
\[
(R_{x} \cos \theta + R_{c} \cos \left( \arcsin \left( \frac{R_{y} \sin \theta}{R_{y} + R_{c}} \right) \right)) - (\sin \theta - e_{s_{bc}} (c) \cos \theta) (R_{y} \sin \theta - R_{c} \sin \left( \arcsin \left( \frac{R_{y} \sin \theta}{R_{y} + R_{c}} \right) \right)) +
\]
\[
(\epsilon_{s_{bc}} (c) \sin \theta + \epsilon_{s_{c}} (c) \cos \theta) z_i
\]
\[
y_w = \left( e_{s_{c}} (c) \sin \theta - \epsilon_{s_{bc}} (c) \cos \theta \right) L/2 + \delta_y (c) \sin \theta + \delta_x (c) \cos \theta + (\sin \theta + \epsilon_{s_{bc}} (c) \cos \theta) (R_{c} \cos \theta +
\]
\[
+R_{c} \cos \left( \arcsin \left( \frac{R_{y} \sin \theta}{R_{y} + R_{c}} \right) \right)) - (\epsilon_{s_{c}} (c) \sin \theta - \cos \theta) (R_{y} \sin \theta - R_{c} \sin \left( \arcsin \left( \frac{R_{y} \sin \theta}{R_{y} + R_{c}} \right) \right)) +
\]
\[
(\epsilon_{s_{bc}} (c) \sin \theta + \epsilon_{s_{c}} (c) \cos \theta) \sin \theta + \epsilon_{s_{bc}} (c) \cos \theta) (R_{c} \cos \theta +
\]
\[
+R_{c} \cos \left( \arcsin \left( \frac{R_{y} \sin \theta}{R_{y} + R_{c}} \right) \right)) + (\epsilon_{s_{c}} (c) \sin \theta + \epsilon_{s_{bc}} (c) \cos \theta) (R_{y} \sin \theta - R_{c} \sin \left( \arcsin \left( \frac{R_{y} \sin \theta}{R_{y} + R_{c}} \right) \right)) + z_1
\]
(8)
\[
x_i = \delta_x (z) + x + a (e_{i} (x) - e_{i_{bc}} (z)) b + \delta_y (x) + \epsilon_{s_{bc}} (z) + \epsilon_{s_{c}} (x) c -
\]
\[
\frac{R_{y} \cos \left( \arcsin \left( \frac{R_{y} \sin \theta}{R_{y} + R_{c}} \right) \right)}{(\epsilon_{s_{c}} (z) + \epsilon_{s_{bc}} (x) (1)) (R_{y} \sin \theta - R_{c} \sin \left( \arcsin \left( \frac{R_{y} \sin \theta}{R_{y} + R_{c}} \right) \right))} -
\]
\[
(\epsilon_{s_{bc}} (z) + \epsilon_{s_{c}} (x) (1)) (R_{y} \sin \theta - R_{c} \sin \left( \arcsin \left( \frac{R_{y} \sin \theta}{R_{y} + R_{c}} \right) \right)) + z_2
\]
(9)

Based on Eq.(5), Eq.(9) can be obtained.

\[
x_w = x_i; y_w = y_i; z_w = z_i
\]

So far, precision machining model of crankshaft follow-up grinder is established.

After achieving the compensation NC code by using the proposed method in [1], the contour curve of crankshaft pin journal before and after compensation can be obtained, as shown in Fig.5. From Fig.5, the maximum contour error is 4.7μm before error compensation and 5.7μm after error compensation. Therefore, the validity and correctness of the established model in this paper are verified.

5. Conclusion

In this paper, the kinematics theory of multi-body system is applied to precision machining modeling of double grinding wheel crankshaft follow-up grinder, which lays a foundation for the follow-up study of this kind of machine tool. Therefore, some corresponding conclusions can be drawn:

(1) The theory of crankshaft follow-up grinding is studied. The mathematical model of X-axis and C-axis linkage in crankshaft follow-up grinding and the coordinate positions of grinding points in workpiece coordinate system and tool coordinate system are obtained, which lays a foundation for error modeling.
(2) Based on the kinematics theory of multi-body system, the topological structure of the machine tool is established, and the position matrix expressions of any cutting point in the kinematics chain of "bed-workpiece" and "bed-tool" are described.

(3) The coordinate system of each moving body is established, and the transformation matrix between moving bodies is obtained. Based on the coincidence principle of tool actual trajectory and tool path, the precision machining constraint equation of double grinding wheel follow-up grinding machine tool is established, and the essence of crankshaft precision machining is revealed.

Fig. 5. Contour curve of crankshaft pin journal.

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