Implications of new CMB data for neutralino dark matter

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Abstract

The combination of new cosmic microwave background data with cosmological priors has determined the physical cold dark matter (cdm) density to be $\Omega_{\text{cdm}} h^2 = 0.13 \pm 0.01$. We find the corresponding regions of parameters in the minimal supergravity model for which the lightest neutralino is the cdm particle. We then compare with the muon anomalous magnetic moment ($a_\mu$) measurement, the mass of the lighter CP-even Higgs boson ($h^0$), the trilepton search at the upgraded Fermilab Tevatron collider, and a direct search for relic neutralinos. The intersection of $\Omega_{\text{cdm}} h^2$ and $a_\mu$ constraints selects $\tan \beta \approx 40 - 45$. 
Recent precision measurements of the Cosmic Microwave Background (CMB) anisotropy \cite{1,2} place restrictive constraints on the densities of matter and dark energy in the Universe. The results of parameter extraction from the CMB data made in conjunction with other cosmological priors (flat universe, supernovae, large-scale structure) give a baryon density $\Omega_b h^2 = 0.022 \pm 0.003$, which is very consistent with the predictions of Big Bang Nucleosynthesis theory. A cold dark matter density $\Omega_{\text{cdm}} h^2 = 0.13 \pm 0.01$ is extracted \cite{1,2}. This restrictive range for $\Omega_{\text{cdm}} h^2$ has significant impact on the allowed masses of cold dark matter particles of supersymmetry origin, which we study here.

The lightest supersymmetric particle (LSP) of a supersymmetry theory with a conserved $R$-parity \cite{3} is a preferred candidate for cold dark matter. In the minimal supergravity (mSUGRA) model \cite{4}, supersymmetry (SUSY) is broken in a hidden sector and SUSY breaking is communicated to the observable sector through gravitational interactions. For most of the mSUGRA parameter space, the LSP is the lightest neutralino ($\chi^0_1$), which is a linear combination of gauginos ($\tilde{B}, \tilde{W}_3$) and higgsinos ($\tilde{H}^0_1, \tilde{H}^0_2$) that are superpartners of gauge bosons ($B, W_3$) and Higgs bosons ($H_1, H_2$).

Neutralinos existed abundantly in the early Universe in thermal equilibrium with other particles; their pair annihilations were balanced by pair creation. As the Universe cooled, the neutralino density became Boltzmann-suppressed. Deviation from thermal equilibrium began when the temperature reached the freeze-out temperature $T_f \simeq m_{\chi^0_1}/20$. After the temperature dropped to $\sim \frac{1}{3} T_f$, the annihilation rate became equal to the expansion rate and $n_{\chi^0_1} = \dot{H}/\langle \sigma v \rangle$, where $\dot{H}$ is the Hubble expansion rate at that temperature. Here $\langle \sigma v \rangle$ is the thermally averaged cross section times neutralino velocity \cite{5}. The relic mass density became

$$\Omega_{\chi^0_1} h^2 = n_{\chi^0_1} m_{\chi^0_1}/\rho_c = H m_{\chi^0_1}/(\langle \sigma v \rangle \rho_c) \quad (1)$$

and the neutralinos remained as cold dark matter.

**mSUGRA Model.** The predicted $\Omega_{\chi^0_1}$ in the mSUGRA model increases with $m_{\chi^0_1}$, except where resonances of neutral Higgs bosons ($H^0, A^0$) enhance $\langle \sigma v \rangle$ and thereby suppress $\Omega_{\chi^0_1}$. Previous analyses made to limit the mSUGRA mass parameters allowed broad regions of the cdm density, typically $0.1 \lesssim \Omega_{\chi^0_1} h^2 \lesssim 0.5$ \cite{2}. With the very restrictive range now selected from the CMB data and priors, we can expect much tighter constraints on the mass spectrum of supersymmetric (SUSY) particles. In the present study we consider the $2\sigma$ cdm density range $0.11 \lesssim \Omega_{\chi^0_1} h^2 \lesssim 0.15$ determined by the CMB analyses. We evolve supersymmetry masses and couplings from the grand unified scale using two-loop renormalization equations \cite{6}. The mSUGRA parameters are a scalar mass ($m_0$), a gaugino mass ($m_{1/2}$), a

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\(^1\)The Hubble constant at the present time is expressed as $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$.

\(^2\)The relative density, $\Omega = \rho/\rho_c$, is the ratio of the density $\rho$ to the critical density $\rho_c$ which would close the Universe.

\(^3\)The Standard Model particles and Higgs bosons have $R = +1$ and their superpartners have $R = -1$. 

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trilepton coupling \( (A_0) \), the sign of a Higgs mixing parameter \( (\mu) \), and the ratio of Higgs field vacuum expectation values at the electroweak scale \( (\tan \beta = v_2/v_1) \). The value of \( A_0 \) only significantly affects results for high \( \tan \beta \); we initially take \( A_0 = 0 \) and study the \( A_0 \) dependence later. Moreover, we focus on \( \mu > 0 \), in the sign convention of Ref. [6], since positive \( \mu \) is preferred by mSUGRA calculations of the \( B \to s \gamma \) decay branching fraction [7,8] and required if the 2.6σ deviations of the muon anomalous magnetic moment \( a_\mu = \frac{1}{2}(g-2)_\mu \) from the Standard Model (SM) [9] are of supersymmetry origin [10–12].

**CDM Density** The dark blue bands in Figure 1 show regions in the \( m_0 \) versus \( m_{1/2} \) plane where the predicted \( \Omega_{\text{cdm}} h^2 \) is within the CMB 2σ range. The four panels of the figure display \( \tan \beta \) values of 3, 10, 35, and 50. The chargino search at LEP 2 [13] excludes \( m_{\chi^\pm_1} < 103 \text{ GeV} \), and the Higgs search at LEP 2 excludes \( 0.5 < \tan \beta < 2.4 \) [14]. At \( \tan \beta = 3, 10 \) and 35, there are pockets of allowed masses with \( m_0, m_{1/2} \lesssim 400 \text{ GeV} \). We note that the lightest neutralino mass is related to \( m_{1/2} \) by [15]

\[
m_{\chi^0_1} \approx 0.435 m_{1/2} - 2.8 \sin 2\beta - 10.4
\]

and the lightest chargino mass is given by \( m_{\chi^\pm_1} \approx 2 m_{\chi^0_1} \). The low mass pockets at \( \tan \beta = 3 \) and 10 allowed by the CMB range of \( \Omega_{\text{cdm}} h^2 \) have chargino and neutralino masses that are accessible at Run II of the Tevatron collider [3]. There is an additional narrow allowed band extending to high \( m_{1/2} \), where \( \chi^0_1 - \tilde{\tau}_1 \) coannihilation dominates [4] [16,17]; the region just below this band is excluded because the charged \( \tilde{\tau}_1 \) would be the LSP there. When \( \tan \beta \) is increased to 35, the allowed masses in the pocket region increase and also a new narrow window opens at high \( m_0 \), just below the theoretically excluded boundary from electroweak symmetry breaking. Then at \( \tan \beta = 50 \), the approximate maximum value for which the theory is perturbative, solutions exist only at high \( m_0 \) and \( m_{1/2} \) masses. The allowed bands and the empty regions in Fig. 1(d) can be qualitatively understood in terms of the \( s \)-channel Higgs resonance effects.

**Muon g − 2** The reported 2.6σ deviation of the muon anomalous magnetic moment [3] from its predicted SM value has triggered supersymmetry explanations (see e.g. Ref. [10,12]). Interestingly, the low mass pockets in Figs. 1(a,b,c) from our cdm analysis have considerable overlap with the 2σ region favored by the E821 measurement [12]. The dashed curves in Fig. 1 show the mSUGRA predictions for

\[
\Delta a_\mu = \left[ a_\mu - a_\mu(\text{SM}) \right] / 10^{-10}.
\]

The curves represent the central value of the measurement \( \Delta a_\mu(\text{exp}) = 43 \) and its 2σ uncertainties \( \Delta a_\mu(\text{exp}) \pm 2\sigma = 75, 11 \). At large \( \tan \beta \), the chargino-sneutrino loop diagram is the dominant SUSY contributor to \( \Delta a_\mu \).

**Lightest Higgs Boson** The LEP 2 collaborations found a tentative signal for a Higgs boson with mass of 115 GeV [14]. The contours of mSUGRA parameters for which this

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\(^4\) We use the calculations of coannihilation in Ref. [16].
mass is predicted are shown in Figure 1. We calculate the masses and couplings in the Higgs sector with one-loop corrections from the top, the bottom and the tau Yukawa interactions in the RGE-improved one-loop effective potential \cite{18} at the scale $Q = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$ \cite{19}. With this scale choice, the RGE improved one-loop corrections approximately reproduce the dominant two-loop perturbative calculation of the mass of the lighter CP-even Higgs scalar ($m_h$) \cite{21}. We note that pocket regions at $\tan \beta = 3, 10$ have $m_h \lesssim 100$ GeV, 115 GeV respectively, and are thus not favored by the LEP 2 Higgs search. However, at $\tan \beta = 10$, the coannihilation band overlaps with $m_h \gtrsim 115$ GeV.

**Trileptons at Run II** Systematic studies have been made of potential for supersymmetry discovery in Run II at the upgraded Fermilab Tevatron collider \cite{6}. The most promising signal is trileptons from chargino–neutralino ($\chi^\pm_1 \chi_0^2$) associated production. Figure 1 shows the $3\sigma$ signal contours with 30 fb$^{-1}$ luminosity \cite{21}. The CMB allowed mass regions at $\tan \beta < \sim 10$ have considerable overlap with the trilepton signal regions accessible at Run II.

**Direct Neutralino Detection.** In the mSUGRA model the $\chi^0_1$–nucleon scattering cross-section depends strongly on the values of $\tan \beta$ and $m_{\chi^0_1}$ \cite{5}, increasing with $\tan \beta$ but decreasing with $m_{\chi^0_1}$. The expected event rate in a $^{76}$Ge detector for $m_{\chi^0_1} \simeq 120$ GeV and $\tan \beta = 35$ is about 0.015 events/kg-day \cite{22}. The CDMS experiment in the Soudan mine expects to obtain a 2500 kg-day exposure on Germanium \cite{23}. The CDMS-Soudan experiment should observe at least 10 events due to neutralino scattering at $\tan \beta = 10$ with $m_{\chi^0_1} \lesssim 120$ GeV ($m_{1/2} \lesssim 300$ GeV) or at $\tan \beta = 35$ with $m_{\chi^0_1} \lesssim 160$ GeV ($m_{1/2} \lesssim 400$ GeV) \cite{22}.

**Indirect Neutralino Detection.** The CMB solutions at the largest $\tan \beta$ have neutralinos masses above 200 GeV. Large ice and water detectors (AMANDA, IceCube, ANTARES) can detect the high energy neutrinos from the annihilations of such heavy relic neutralinos that have gravitationally accumulated in the Sun \cite{24}.

**Intersection of Constraints.** The $2\sigma$ bands in Fig. 1 for $\Omega_{cdm} h^2$ overlap the $2\sigma$ ranges of the $\Delta a_\mu$ measurement, with different allowed $m_{1/2}$, $m_0$ mass regions in each $\tan \beta$ case. If we ask for a more restrictive intersection of the $\Omega_{cdm} h^2$ bands with the $1\sigma$ allowed ranges for $\Delta a_\mu$, high $\tan \beta$ values of order 40 to 45 are selected as shown in Fig. 2. A further constraint of $m_h = 115$ GeV gives $m_{1/2} \sim 300$ GeV and $m_0 \approx 350 - 550$ GeV for $\tan \beta \simeq 40 - 45$. The corresponding SUSY particle masses are listed in Table II.

**Infrared Fixed Point.** In SUGRA models with approximate $b – \tau$ Yukawa unification at the grand unified scale, the renormalization group equations have solutions dominated by the quasi-fixed point of the top Yukawa coupling, which can be realized at high $\tan \beta$ \cite{25,26}. Moreover, high $\tan \beta$ texture models for the quark and the lepton mass matrices \cite{27} require large values of $\tan \beta$.

**Summary.** We have determined the narrow regions of mSUGRA parameter space that can account for the restrictive cold dark matter density range inferred from recent CMB measurements. Our results are given in Figure 1 for four choices of $\tan \beta$. 

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i) At $\tan \beta < 10$, the $a_\mu$ measurement within $2\sigma$ can be explained and a trilepton signal is predicted in Run II at the Tevatron; however, $m_h \lesssim 110$ GeV at $\tan \beta = 3$ makes this value of $\tan \beta$ less likely.

ii) At $\tan \beta = 35$, both the measured $a_\mu$ within $2\sigma$ and the $m_h = 115$ GeV hint from LEP 2 are consistent with the CMB band; the trilepton signal here is marginal.

iii) At $\tan \beta = 50$, the allowed bands from $\Omega_{\text{cdm}}h^2$ and $\Delta a_\mu$ are overlapping near the $2\sigma$ limit of $\Delta a_\mu \sim 11$.

iv) Consistency of $\Omega_{\text{cdm}}h^2$ within the $1\sigma$ uncertainty range of the $a_\mu$ measurement selects $\tan \beta \approx 40 - 45$. We emphasize that this result does not depend on the Higgs mass. The dependence of the solution on the trilinear coupling ($A_0$) is illustrated in Table I.

v) With $m_h = 115$ GeV as an additional constraint, supersymmetric particle masses are approximately determined as given in Table II. The dependence on $\tan \beta$ is illustrated there. The predicted Higgs width is about 5 MeV, compared to the SM Higgs width of 3 MeV. A broader Higgs width would ease beam resolution requirements for muon collider Higgs factories [28].

Analysis of further data already taken on $a_\mu$ [9] will reduce its uncertainty by about a factor of 2 and allow even closer specification of mSUGRA parameters derived from the overlap of $\Delta a_\mu$ and the CMB favored neutralino relic density.

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| $A_0$ | $m_{\chi^0_1}$ | $m_{\chi^\pm_1}$ | $m_A$ | $m_h$ | $\Omega_{\chi^0_1} h^2$ | $\Delta a_\mu$ |
|---|---|---|---|---|---|---|
| −300 | 120 | 223 | 401 | 117 | 0.19 | 44 |
| −200 | 120 | 221 | 398 | 116 | 0.16 | 45 |
| −100 | 119 | 220 | 394 | 116 | 0.14 | 46 |
| 0 | 119 | 218 | 392 | 115 | 0.13 | 47 |
| +100 | 119 | 217 | 388 | 115 | 0.11 | 48 |
| +200 | 118 | 215 | 385 | 114 | 0.10 | 49 |
| +300 | 118 | 214 | 383 | 114 | 0.10 | 50 |

**TABLE I.** The relic density of neutralino dark matter ($\Omega_{\chi^0_1} h^2$), the SUSY contribution to the muon anomalous magnetic moment ($\Delta a_\mu$) [Eq. (3)], and masses of the lightest neutralino ($\chi^0_1$), the lighter chargino ($\chi^\pm_1$), the CP-odd Higgs pseudoscalar ($A_0$), and the lighter CP-even Higgs scalar ($h_0$), for $\tan \beta = 40$, $m_{1/2} = 300$ GeV, $m_0 = 350$ GeV, and several values of the trilinear coupling ($A_0$). The masses and the trilinear coupling $A_0$ are in the units of GeV.

| $A_0$ | $m_{\chi^0_1}$ | $m_{\chi^\pm_1}$ | $m_A$ | $m_h$ | $\Omega_{\chi^0_1} h^2$ | $\Delta a_\mu$ |
|---|---|---|---|---|---|---|
| −300 | 120 | 223 | 401 | 117 | 0.19 | 44 |
| −200 | 120 | 221 | 398 | 116 | 0.16 | 45 |
| −100 | 119 | 220 | 394 | 116 | 0.14 | 46 |
| 0 | 119 | 218 | 392 | 115 | 0.13 | 47 |
| +100 | 119 | 217 | 388 | 115 | 0.11 | 48 |
| +200 | 118 | 215 | 385 | 114 | 0.10 | 49 |
| +300 | 118 | 214 | 383 | 114 | 0.10 | 50 |

**TABLE II.** The relic density of neutralino dark matter ($\Omega_{\chi^0_1} h^2$), the SUSY contribution to the muon anomalous magnetic moment ($\Delta a_\mu$) [Eq. (3)], and mass of the CP-odd Higgs pseudoscalar ($A_0$) in GeV, for $m_{1/2} = 300$ GeV, $m_0 = 400$ GeV, $A_0 = 0$, and several values of $\tan \beta$. For all these values of $\tan \beta$, $m_{\chi^0_1} = 119$ GeV, $m_{\chi^\pm_1} = 218$ GeV, $m_h = 115$ GeV, and $\Gamma_h = 4.8$ MeV, where $\Gamma_h$ is the decay width of the lighter CP-even Higgs boson ($h_0$).
FIG. 1. Regions of relic density for neutralino dark matter satisfying the 2σ constraint from the new CMB data (dark blue bands) \((0.11 \leq \Omega_{\chi^0_1} h^2 \leq 0.15)\) in the \((m_{1/2}, m_0)\) plane of the mSUGRA model with \(\mu > 0\) and \(A_0 = 0\) for (a) \(\tan \beta = 3\), (b) \(\tan \beta = 10\), (c) \(\tan \beta = 35\) and (d) \(\tan \beta = 50\). Also shown are the contours in the parameter space for (i) SUSY contributions of \(\Delta a_\mu = 11, 43, 75\) as defined in Eq. (3), representing the measured value and its 2σ uncertainties, (ii) the mass of the lighter CP even Higgs boson \(m_h = 115\) GeV, and (iii) the 3σ observation potential for a trilepton signal at the upgraded Fermilab Tevatron. The hatched regions are excluded by theoretical requirements [having electroweak symmetry breaking, the correct vacuum (tachyon free) and \(\chi^0_1\) as the LSP] or by the chargino search at LEP 2 [13].
FIG. 2. Regions of the relic density allowed by the CMB data (dark blue bands) for (a) $\tan \beta = 40$ and (b) $\tan \beta = 45$; the SUSY contributions of $\Delta a_\mu = 27$, 43 and 59, represent the measured value and its 1$\sigma$ uncertainties.