Here’s looking at you, fireball

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Plasmas screen electromagnetic waves of frequency less than the plasma frequency, \( \omega_p \), with a skin depth, \( \delta \), specified by the electrical conductivity, \( \sigma \). Current estimates of the transport properties of the QCD plasma indicates that photons with energy less than 250–500 MeV would be screened with a skin depth of 2–4 fm. In a hadron gas, a little below \( T_c \), screening occurs for much softer photons, of energy less than approximately 50 MeV, albeit with similar values of \( \delta \). If the QCD plasma is indeed strongly interacting, then it can be proven by merely looking at the brightness of a fireball from different angles.

With clear signatures of dense matter formation in heavy-ion collisions at the RHIC [1], attention must shift to the characterization of this matter. One fact is self-evident—this matter consists of mobile electrically charged constituents, and is therefore an electromagnetic plasma. All such plasmas screen photons. The photon skin depth, \( \delta \), is related to the electrical conductivity of the plasma, \( \sigma \). Photons of frequency \( \omega \) are screened for \( \omega < \omega_p \), where \( \omega_p \) is the plasma frequency. There is evidence that the plasma formed at RHIC is strongly interacting. If so, the skin depth should be fairly small, and soft photon emission should be approximately a surface effect, leading to strong variations of intensity when the fireball is observed from different angles.

The hot phase: Photon spectra were first computed in weak-coupling theory almost two decades ago, and the state of the art has improved continuously [2]. There was even a computation of variations of intensity when the fireball is observed from different angles. Nevertheless, agreement between phenomenological analyses of data and lattice QCD results is emerging: it has become clear that the high-temperature phase of QCD (\( T > T_c \), where \( T_c \) is the crossover temperature for QCD matter) is strongly dissipative. There is strong evidence for short thermalization times, which requires rapid dissipation [3]. A recent lattice computation was able to extract the shear viscosity, \( \eta \), from RHIC data on particle spectra and flow [4]. This gives \( \eta/S \approx 0.14 \), where \( S \) is the entropy density in the plasma. A recent lattice computation in quenched QCD implies that \( \eta/S \approx 0.2 \) [5]. It has been conjectured that the universal strong coupling limit of this quantity is \( \eta/S \geq 1/4\pi \) [4], implying that QCD is close to this limit.

In the plasma phase of QCD, the electric charge carriers are quarks, and the momentum carriers are dominantly gluons. As a result, the electrical conductivity, \( \sigma \propto \tau_q \) whereas the shear viscosity, \( \eta \propto \tau_g \), where \( \tau_{q,g} \) are effective mean-free times for quarks and gluons [6]. However, the two interaction strengths are related by the gauge symmetry of the theory. As a result, the (dimensional) ratio \( \sigma/\eta \), should be completely specified by the temperature and chemical potentials in the QCD plasma. To put this another way, two different estimates of \( \eta \) in a certain ratio to each other would give rise to two estimates of \( \sigma \) in the same ratio.

An estimate of \( \sigma \) can be directly used to compute the skin depth for a soft photon by plugging the constitutive

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relation, \( j = \sigma E \), i.e., Ohm’s law, into Maxwell’s equations \([13]\) to give

\[
\delta = \sqrt{\frac{2}{\omega_p \sigma}}. \tag{1}
\]

A lattice computation estimated \( \sigma \approx T/7 \) and \( \omega_p \approx 0.94T \) \([8]\). This implies that the skin depth is rather small, with \( \delta \approx 2 \text{ fm} \) at \( \omega_p \). One can scale this for the value of \( \eta \) estimated in \([8]\), to obtain \( \delta \approx 2.5 \text{ fm} \). Finally, if the bound in \([8]\) is saturated by the QCD plasma, then one should find a somewhat larger skin depth— \( \delta \approx 3.5 \text{ fm} \). In this last case, the plasma frequency would be \( \omega_p \approx 500 \text{ MeV} \).

**The cold phase:** The cold phase of this hadronic matter exists in a narrow band of temperature— between \( T_c \) and the freeze out temperature \( T_f \). Current estimates of \( T_f \) are bounded by \( m_\pi < T_f < 2m_\pi \) \([14]\), as a result of which thermal pair production of charged pions is extremely unlikely. In fact,

\[
\frac{1}{\exp\left(\frac{E_\pi}{T}\right) - 1} = e^{-m_\pi/T} e^{-m_\pi v^2 /2T} + \mathcal{O}\left(e^{-2m_\pi/T}\right), \tag{2}
\]
and hence the hadron gas may be considered to be a classical non-relativistic plasma within an accuracy of about 30%. The next lightest particle is the kaon. Since $m_K/m_\pi > 2$, this same argument suffices to show that its contribution to transport coefficients is negligible at this level of approximation. Thus, as far as the transport properties are concerned, the cold phase can be considered approximately as a classical non-relativistic two component electromagnetic plasma, with charge carriers $\pi^\pm$ mixed in with the uncharged $\pi^0$.

In a Maxwell-Boltzmann plasma, light is screened in the medium with a skin depth of the order of the inverse plasma frequency, i.e.,

$$\delta = 1/\omega_p, \quad \text{for} \quad \omega < \omega_p = \left(\frac{8\pi \alpha n_\pi}{m_\pi}\right)^{1/2},$$

where $\alpha$ is the fine structure constant and $n_\pi$ is the number density of each species of charge pions. We have taken into account the fact that there are two oppositely charged species of pions. There are numerically small corrections to this estimate, due to the change in the wavenumber from medium effects, leading to the estimate

$$\delta = \frac{1}{\omega_p}\left[\left(\frac{2T}{\pi m_\pi}\right)^{1/2} \frac{\omega_p}{\omega}\right]^{1/3}.$$  \hspace{1cm} (4)

for $\omega < \omega_p$. For pions to be the appropriate degrees of freedom, there can be no more than one pion in every box of side equal to a pion Compton wavelength. Since $n_\pi$ above counts only one species of pions, by this argument it should equal 1/3 per pion Compton wavelength. In this high density limit one finds screening for photon energies less than 50 MeV with a skin depth of 3.9 fm or more. As $n_\pi$ decreases $\omega_p$ decreases as $\sqrt{n_\pi}$ and simultaneously the skin depth increases. Thus, for more realistic charged pion densities, screening would occur at even smaller energies, and with larger skin depth. Note that there is no particular relation between momentum and charge transport time scales for a pion gas. As a result, the AdS/CFT limit is not an upper bound to $\delta$.

Collective effects may become strong as $n_\pi$ approaches the limit of one pion per Compton wavelength. If this were to happen at low temperature then one might expect condensation phenomena of various kinds which could tend to lower the pion mass and lead to a decrease in $\omega_p$. However, $T_c$ is not small enough for condensation to occur. Instead, the limit $n_\pi \to 1$ becomes strongly interacting, and possibly leads to the universal limit of $\delta$. This scenario preserves the holy cow of continuity at the QCD crossover, $T_c$.

Finally, we give an estimate of the error due to the neglect of the heavier mesons. If one takes into account the full hadronic spectrum, then there is a contribution to $\omega_p^2$ of $4\pi\alpha e^2 n_h/m_h$ from each hadron $h$. Since the density of heavier hadrons is smaller at any $T \simeq m_\pi$, these give exponentially small corrections in $m_h/m_\pi$.

**Experimental signatures:** As we have estimated above, very soft photons, of energy at most $\omega_p = 250–500$ MeV, would be required to see the skin depth of a plasma. This poses a technical challenge of rejecting all the photons which are produced by decays after free streaming of hadrons sets in. This is a problem I leave to people who know the detectors best. The question I ask here is— if this separation can be done reliably, then what could one observe?

Let us define a quantity called the contrast which takes values between zero and unity—

$$\kappa(\delta) = \frac{N(\hat{k};\delta) - N(\hat{k}';\delta)}{N(\hat{k};\delta) + N(\hat{k}';\delta)}\bigg|_{\max},$$  \hspace{1cm} (5)

where $N(\hat{k};\delta)$ is the photon intensity observed in the direction $\hat{k}$ (specified by the pseudo-rapidity $\eta$ and azimuthal angle $\phi$) when the skin depth is $\delta$, and the ratio on the right is maximized by varying over two directions, $\hat{k}$ and $\hat{k}'$, independently within the limits of the experimental acceptance. The photon intensity entering the definition above can be computed by the formula—

$$N(\hat{k};\delta) = \int d^4x\frac{d^4N(T)}{dx^4}e^{-\ell(x\hat{k})/\delta},$$  \hspace{1cm} (6)

where the integral is over the 4-volume of the fireball, the photon emission rate depends implicitly on the point of emission, $x$, through the local temperature, $T(x)$, and the path-length of the photon through the fireball is a purely geometric quantity which is completely specified by the point of origin, $x$, and the direction of propagation, $\hat{k}$. At the level of this approximation, all the interaction effects are taken into $\delta$. It would be appropriate then to take $d^4N(T)/dx^4$ as that for a blackbody at temperature $T$.

Two limits can be obtained analytically. First, for photon energies large enough that the electrical conductivity of the plasma can be neglected, the skin depth can be set to infinity. Then the exponential factor in eq. (6) is unity, and
the full volume is observable from any angle. The contrast vanishes in this limit—$\kappa(\infty) = 0$. This is exactly what happens for $\omega \gg \omega_p$.

In the opposite limit, where the conductivity is very large, i.e., in the limit of vanishing $\delta$, the exponential factor becomes a delta function at the freezeout surface. Since the photon emission rate is completely specified by the temperature, it can be pulled out of the integral, which then becomes a product of the surface area and a surface brightness. The latter factor is a function only of $T$, and hence is constant over the freezeout surface. As a result, the contrast becomes a purely geometric quantity, being proportional to the surface area visible from the detector element in the direction $k$. It is clear that if the surface is a sphere then the contrast vanishes—the detector sits macroscopically far away, and therefore, from any angle sees one hemisphere, i.e., one half of the total surface area. This generalizes. If the freezeout surface has an inversion symmetry about the center and is strictly convex, then from any viewpoint one sees half the surface \cite{16}. In this limit again, the contrast vanishes—$\kappa(0) = 0$.

The contrast does not vanish in general for intermediate values of $\delta$. A simple example is a translucent chapati (thin disk)—face on the full volume may be visible, but edge on only a small fraction is visible. However, even in this case, some extremely symmetric shapes such as a sphere or an infinitely long tube give vanishing contrast.

As we have argued above, when the skin depth is finite, freezeout is no longer confined to a surface. However, the notion of isothermal surfaces in the fireball (of which the freezeout surface is an example) still exists and is useful. We have presented the arguments above for space-like freezeout isotherms, but a generalization to the time-like case is straightforward. It also becomes clear from the above discussion why it is necessary to distinguish photons emitted before decoupling from photons produced by decays of particles after decoupling. An analogous problem in astronomy is to separate fluctuations of the primordial radiation from later events like stars or nearby gas clouds.

Due to the high degree of symmetry, central collisions yield the smallest contrast. The contrast can be increased by selecting non-central collisions. Since there seems to be a range of impact parameters where the initial energy density changes little, it is possible to improve the contrast without sacrificing the initial energy density, and hence the signal which depends on the contrast. At the RHIC (and the LHC) the skin depth could be 30–50% of the fireball dimensions \cite{17}. As a result, some contrast should be visible. If it is, then eq. \ref{eq:contrast} can be used in conjunction with realistic models such as a full hydro code (preferably with dissipation built in, at least to first order) to extract the value of $\delta$.

It is interesting to note that the contrast may be boosted by using asymmetric ion combinations, say S-Au or Pb-U. In this case, the fireball no longer has inversion symmetry, and one can have $\kappa(0) > 0$ (think of the difference between an egg and an ellipsoid). Since $\kappa(\infty) = 0$ independent of the shape of the fireball, the qualitative fact of a skin-depth smaller than the fireball dimension can be proven (or ruled out) by using an asymmetric ion combinations. No fit or detailed modelling is needed.

There is one other point that requires comment. In the discussion of the contrast, $\kappa$, we have assumed that the freezeout surface has uniform properties. However, if a jet punches through, then it makes spurious contributions to the contrast. These events should be removed from the sample which is analyzed. In the present fairly advanced stage of jet physics at the RHIC, it should be possible to do this efficiently.

The point of this paper is the following—in the high-temperature phase of QCD gauge symmetry relates the reaction rates which are relevant to different transport coefficients such as the shear viscosity and the electrical conductivity. Since there is growing evidence that the viscosities and other dissipative time scales in QCD matter are short, this has implications for the electrical conductivity, and hence observational consequences as a small skin depth for photons of frequency less than or equal to the plasma frequency. I have suggested one method for the observation of the skin depth here, through a study of the contrast. However, the underlying physics is important enough that one should explore other ways of investigating it even if the contrast is not easily visible—varying the ion species used and the beam energies, or using asymmetric ion combinations, are all ideas worth exploring.

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Very close to $T_c$, lattice computations imply that the assumption that there are gluon-like quasi-particles in the plasma is false \cite{11}. The conjecture that the plasma is full of coloured composites \cite{12} would give rise to interesting transport phenomena, including quantitative predictions of the electrical conductivity, which can then be compared to lattice results.

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