Optimal Design of Mufflers Equipped with Pod and Concentric Acoustical Rings using the SA Method

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Abstract. There has been, of recently, much research on mufflers lined with sound absorbing material. In dealing with high frequency noise emitted from venting systems, a sound absorbing material inside a tube covered with a perforated plate has been used. However, the acoustical performance of mufflers equipped with one-layer of sound absorbing material has still proved to be insufficient. In order to promote acoustic performance, a design and theoretical assessment on pod mufflers hybridized with multi-layer concentric sound-absorbing rings is proposed. By using the acoustical lumped-method, a four-pole system matrix for evaluating the acoustic performance (STL) will emerge. During the optimization process, the simulated annealing (SA) method, which is a robust scheme utilized to search for the global optimum by imitating a physical annealing process, is used. Before dealing with a broadband noise, the STL’s maximization relative to a one-tone noise (1000Hz) is offered to confirm the SA method’s reliability. To appreciate the influence of acoustical efficiency with respect to the design parameters, an analysis of design parameters (dh: the diameter of a perforated hole; W: the width of an air channel; R: acoustic flow resistivity of the acoustic fiber; σ: the porosity of the perforated plate; Df: the thickness of the acoustic fiber; L1: the horizontal length of the sound absorbing ring) is completed. Subsequently, to bring into focus the acoustical interaction with respect to the number of air-channels (between the sound absorbing rings), three types of mufflers (mufflers A~C) hybridized with one, two, and three air-channels have been surveyed. Results reveal that for a rectangular muffler internally conjugated with one air-channel, the maximal STL is located at the desired tone, and the acoustical performance of a cone muffler will increase if the number of air-channels internally conjugated within the cone muffler increases. Consequently, the approach used for the optimal design of the STL on pod mufflers internally equipped with multiple air-channels within a space-constrained situation can be easily assessed.

1. Nomenclature
This paper is constructed on the basis of the following notations:

c_o : sound speed (m s⁻¹)
dh : the diameter of a perforated hole on the front perforated plate (m)
D_f : the thickness of the acoustic fiber
L_i : diameter of the i-th horizontal segment of the muffler (m)
D_i : diameter of the concentric sound absorbing ring (m)
Introduction

Morse [1] started research on mufflers that reduced high frequency noise using a dissipative duct (a duct lined with sound absorbing material) in 1939. Scott [2] predicted the sound transmission of circular and rectangular mufflers internally lined with porous material using a bulk reactive model. Later, in 1975, Ko [3] analyzed the sound transmission loss in acoustically lined flow ducts separated by porous splitters. These researchers [1-3] focused on the sound attenuation of an infinite duct. Cummings and Chang [4] then, in 1988, investigated a finite length dissipative flow duct silencer with an internal mean flow in the absorbent channel using a modal method. Peat [5], in 1991, considering the volume modulus, adopted a transfer matrix for evaluating the acoustical performance of an absorption silencer element. Subsequently, Selamet et al. [6, 7], adopting a one-dimensional analytical method and a three-dimensional boundary element method (BEM), explored the acoustical attenuation...
for perforated concentric absorbing silencers and hybrid silencers. In 2003, Munjal [8] analyzed pod silencers using a four-pole transfer matrix. In 2004, Xu et al. [9] proposed a characteristic equation to calculate the sound attenuation in dissipative expansion chambers. However, an exploration of a muffler’s optimal design within a limited space was never approached. Therefore, Chiu [10] has presented shaped optimization of a circular muffler internally lined with sound absorbing material within a constrained space. Later, Chiu [11] assessed circular mufflers internally equipped with multi-channel splitters using neural networks in conjunction the boundary element method and a genetic algorithm. However, the study is limited to a one-layer air channel muffler.

Here, then, in order to improve the acoustical performance of space-constrained cylindrical pod mufflers, a mathematical model of a cylindrical pod muffler internally equipped with multi-layer concentric sound- absorbing rings is proposed using an acoustical lump analysis. Three types of cylindrical pod mufflers equipped with multi-layer concentric sound- absorbing rings (muffler A: a muffler equipped with one air channel; muffler B: a muffler equipped with two air channels; muffler C: a muffler equipped with three air channels) are presented. Also note: the acoustical lump method used to form a four-pole matrix is in line with the simulated annealing method.

3. Theoretical background

Three kinds of multi-channel pod mufflers have been adopted for noise elimination in the Nitrogen emergent venting device shown in Fig. 1. Before being analyzed, the acoustical fields and related outline dimensions of the mufflers are shown in Figs. 2(A), (B), and (C). Individual transfer matrices with respect to one-channel, two-channel, and three-channel pod mufflers are described below.

![Figure 1. The gas emergent venting device.](image-url)
3.1. Muffler A (a one-channel pod muffler)

As indicated in Fig. 2(A) and Fig. 3(A), the horizontal length of a muffler with 1 layer \((n=1)\) of sound-absorbing rings is divided into \(k\) segments, and the cross section of the muffler is divided into 2 sections \((m=2)\). Concerning the acoustical field of the one-segment muffler \(A\) shown in Fig. 2(A), the inner perforated plate is partitioned as \(L_{11}\) and the outer perforated plate is partitioned as \(L_{12}\). Based on the plane wave theory, the cutoff frequency \((f_c)\) is \(c_0/L_{\text{max}}\) where \(L_{\text{max}}\) is the maximum value of \((L_{11}, L_{12})\) in the muffler. The related matrix form between node 000 and node 001 is

\[
\begin{bmatrix}
\dot{p}_{001} \\
\dot{u}_{001}
\end{bmatrix} = \begin{bmatrix}
\cos[K_{\text{fiber}}(H_1)] & jZ_{\text{fiber}} \sin[K_{\text{fiber}}(H_1)] \\
\sin[K_{\text{fiber}}(H_1)] & \cos[K_{\text{fiber}}(H_1)]
\end{bmatrix} \begin{bmatrix}
\dot{p}_{000} \\
\dot{u}_{000}
\end{bmatrix}
\tag{1}
\]

Developing Eq.(1) yields

\[
Z_{001} = \frac{\dot{p}_{001}}{\dot{u}_{001}} = \frac{\dot{p}_{000} \cos[K_{\text{fiber}}(H_1)] + j\dot{u}_{000} Z_{\text{fiber}} \sin[K_{\text{fiber}}(H_1)]}{\dot{p}_{000} \sin[K_{\text{fiber}}(H_1)] + j\dot{u}_{000} \cos[K_{\text{fiber}}(H_1)]}
\]

\[
= \frac{\dot{p}_{000} \cos[K_{\text{fiber}}(H_1)] + jZ_{\text{fiber}} \sin[K_{\text{fiber}}(H_1)]}{\dot{u}_{000} \sin[K_{\text{fiber}}(H_1)] + \cos[K_{\text{fiber}}(H_1)]}
\tag{2}
\]

Because the acoustical particle velocity is zero at the solid boundary, the acoustical impedance at node 000 is

\[
Z_{000} = \frac{\dot{p}_{000}}{\dot{u}_{000}} = \infty
\tag{3}
\]

Plugging Eq. (3) into Eq.(2) yields

\[
Z_{001} = \frac{\cos[K_{\text{fiber}}(H_1)]}{j \sin[K_{\text{fiber}}(H_1)]} Z_{\text{fiber}}
\tag{4}
\]

where \(Z_{\text{fiber}} = R_{\text{fiber}} + jX_{\text{fiber}}\) and \(K_{\text{fiber}} = K_{014} + jK_{024}\)

Using the wool’s acoustical impedance formula from Delany and Bazley [12] yields

\[
K_{014} = \left[ \frac{\omega H_1}{c_o} \right] + 0.978 \left( \frac{\rho_{\text{f}}}{R_f} \right)^{-0.706} ; \quad K_{024} = \left[ \frac{\omega H_1}{c_o} \right] - 0.189 \left( \frac{\rho_{\text{f}}}{R_f} \right)^{-0.595}
\tag{6a}
\]

\[
R_{\text{fiber}} = \rho_{\text{f}} c_o \left[ 1 + 0.571 \left( \frac{\rho_{\text{f}}}{R_f} \right)^{-0.754} \right] ; \quad X_{\text{fiber}} = \rho_{\text{f}} c_o \left[ -0.087 \left( \frac{\rho_{\text{f}}}{R_f} \right)^{-0.732} \right]
\tag{6b}
\]

Here, \(R_f\) is the flow resistance of the wool.
Assuming that the sound wave quickly passes through node 2 from node 001, the acoustical pressure between node 001 and node 2 can be regarded as the same. Also, adopting the continuity equation yields

\[ p_{001} = p_2; u_{001} = u_2 \]  

(7)

According to the definition of acoustical impedance at node 2 and using Eq.(7) yields

\[ p_2 = Z_{p0}u_{001} + p_{001} \]  

(8)

Combining Eq.(7) with Eq.(8), the matrix form between node 001 and node 2a is

\[
\begin{pmatrix}
Z_{p0} & p_{001} \\
0 & u_{001}
\end{pmatrix} = \begin{pmatrix} 1 & Z_{p0} \\ 0 & 1 \end{pmatrix}
\]  

(9)

Developing Eq.(9) yields

\[ Z_{2a} = Z_{001} + Z_{p0} \]  

(10)

Using the perforated plate’s acoustical impedance formula from Beranek and Ver [13] yields

\[ Z_{p0} = \frac{\rho_c}{pp_l} \sqrt{8\pi \omega \left[ 1 + \frac{1}{2d_{ai}} \right]} + j \frac{\omega p_{00}}{pp_l} \sqrt{\frac{8\pi \omega}{\omega}} \left[ 1 + \frac{1}{2d_{ai}} \right] + t_{p1} + \delta_1 \]  

(11a)

\[ \delta_1 = 0.85(2d_{ai}) \left( 1 - 1.47 \sqrt{pp_l} + 0.47 \sqrt{pp_l}^3 \right) \]  

(11b)

Therefore, the acoustical impedance at node 2a is

\[ Z_{2a} = -j \cos[K_{fiber}(H_i)] Z_{fiber} + \frac{\rho_c}{pp_l} \sqrt{8\pi \omega \left[ 1 + \frac{1}{2d_{ai}} \right]} + j \frac{\omega p_{00}}{pp_l} \sqrt{\frac{8\pi \omega}{\omega}} \left[ 1 + \frac{1}{2d_{ai}} \right] + t_{p1} + \delta_1 \]  

(12)

Similarly, the acoustical impedance at node 2b is

\[ Z_{2a} = -j \cos[K_{fiber}(H_i)] Z_{fiber} + \frac{\rho_c}{pp_l} \sqrt{8\pi \omega \left[ 1 + \frac{1}{2d_{ai}} \right]} + j \frac{\omega p_{00}}{pp_l} \sqrt{\frac{8\pi \omega}{\omega}} \left[ 1 + \frac{1}{2d_{ai}} \right] + t_{p1} + \delta_1 \]  

(13)

For node 3a, the matrix form between node 010 and node 011 inside the inner perforated plate can be expressed as

\[
\begin{pmatrix}
Z_{p0} & p_{010} \\
\sin[K_{fiber}(H_i)] & Z_{fiber}
\end{pmatrix} = \begin{pmatrix} 1 & Z_{p0} \\ \sin[K_{fiber}(H_i)] & Z_{fiber} \end{pmatrix}
\]  

(14)

Developing Eq.(14) yields

\[ Z_{010} = \frac{p_{010}}{u_{010}} = \frac{\cos[K_{fiber}(H_i)] Z_{fiber} + j \mu_{010} Z_{fiber} \cos[K_{fiber}(H_i)]}{\sin[K_{fiber}(H_i)] + \mu_{010} \cos[K_{fiber}(H_i)]} \]  

(15)

Because the acoustical particle velocity is zero at the solid boundary, the acoustical impedance at node 010 is

\[ Z_{010} = \frac{p_{010}}{u_{010}} = \infty \]  

(16)

Plugging Eq. (16) into Eq.(15) yields

\[ Z_{010} = \frac{\cos[K_{fiber}(H_i)] Z_{fiber} + j \mu_{010} Z_{fiber} \cos[K_{fiber}(H_i)]}{\sin[K_{fiber}(H_i)] + \mu_{010} \cos[K_{fiber}(H_i)]} \]  

(17)
where \( Z_{\text{ fiber}} = R_{\text{ fiber}} + jX_{\text{ fiber}} \); \( K_{\text{ fiber}} = K_{11,4} + jK_{12,4} \) 

\[
K_{11,4} = \left[ \frac{\omega \cdot H_1}{c_0} \left[ 1 + 0.0978 \left( \frac{\rho_f c_f}{R_f} \right)^{0.1016} \right] \right] ; \quad K_{12,4} = \left[ \frac{\omega \cdot H_2}{c_0} \left[ 1 - 0.189 \left( \frac{\rho_f c_f}{R_f} \right)^{-0.321} \right] \right] 
\]

\[
R_{\text{ fiber}} = \rho_f c_f \left[ 1 + 0.0571 \left( \frac{\rho_f c_f}{R_f} \right)^{0.744} \right] ; \quad X_{\text{ fiber}} = \rho_f c_f \left[ -0.087 \left( \frac{\rho_f c_f}{R_f} \right)^{-0.732} \right] 
\]

Similarly, using the wool’s acoustical impedance formula from Delany and Bazley [12] yields

\[
\begin{cases}
\begin{aligned}
P_{12} & = \rho_0 c_0 \left[ 1 + \frac{1}{8 \pi \nu_0} \left( \frac{\nu_0 c_0}{pp} \left( \frac{1}{2} + \frac{t_{12}}{2d_{12}} \right) + \delta_1 \right) \right] \\
\delta_1 & = 0.85(2d_{12})^2 \left[ 1 - 1.47 \frac{pp}{\nu_0} + 0.47 \left( \frac{pp}{\nu_0} \right)^2 \right] 
\end{aligned}
\end{cases} 
\]

Therefore, the acoustical impedance at node 3a is

\[
Z_{3a} = Z_{a11} + Z_{p2} 
\]

Similarly, using the perforated plate’s acoustical impedance formula from Beranek and Ver [13] yields

\[
Z_{p2} = \frac{\rho_0}{pp} \left[ \frac{8\pi\nu_0}{1 + \frac{t_{22}}{2d_{22}}} + j \frac{\nu_0}{pp} \left[ \frac{8\pi\nu_0}{1 + \frac{t_{22}}{2d_{22}}} + \frac{t_{22} + \delta_2}{2} \right] \right] 
\]

As indicated in Fig. 2, a sound wave propagates from the left side to the right side. According to the continuity equation and geometric data

\[
\begin{cases}
\begin{aligned}
p_1 & = p_{2a} = p_{2b} = p_{3a} = p_4 ; \\
v_1 & = v_{2a} + v_{2b} + v_{3a} + v_4 \\
S_i & = \frac{\pi}{4} (D_i^2 - D_j^2) \cdot \frac{1}{M} = S_M ; \\
S_{2a} & = S_{2b} = L_1 * L_{11} \cdot pp_i ; \\
S_{3a} & = S_{3b} = L_3 * L_{33} \cdot pp_i ; \\
\rho_p S_i u_i & = \rho_p S_{2a} u_{2a} + \rho_p S_{2b} u_{2b} + \rho_p S_{3a} u_{3a} + \rho_p S_{3b} u_{3b} + \rho_p S_{4a} u_{4} 
\end{aligned}
\end{cases} 
\]

Based on the definition of acoustical impedance

\[
Z_{2a} = \frac{P_{2a}}{u_{2a}} ; Z_{2b} = \frac{P_{2b}}{u_{2b}} ; Z_{3a} = \frac{P_{3a}}{u_{3a}} ; Z_{3b} = \frac{P_{3b}}{u_{3b}} 
\]

Or in the form of
\[ u_{3a} = \frac{p_{2a}}{Z_{2a}} \cdot u_{3b} = \frac{p_{2b}}{Z_{2b}} \cdot u_{3a} = \frac{p_{1a}}{Z_{1a}} \cdot u_{3b} = \frac{p_{b}}{Z_{b}} \]  

(29)

Here, \( Z_{2a} = Z_{2b} \) and \( Z_{3a} = Z_{3b} \).

Summing up Eqs. (26)-(29) yields

\[
\begin{bmatrix}
p_1 \\
p_c u_1
\end{bmatrix} = \begin{bmatrix}
p_4 \\
p_c u_4
\end{bmatrix}
\]

(30)

\[ p_1 = p_4 \]

Eq. (30) can be expressed as

\[
\begin{bmatrix}
p_1 \\
p_c u_1
\end{bmatrix} = \begin{bmatrix}
p_4 \\
p_c u_4
\end{bmatrix}
\]

(31a)

\[ \rho_c c_o u_1 = (\rho_c c_o u_4 + \rho_c c_o u_4 \cdot \frac{S_{2a}}{S_M} Z_{2a} + \rho_c c_o \cdot 2 \frac{S_{3a}}{S_M} Z_{3a}) \]

(31b)

The STL between node 1 and node 4 is

\[ STL = 10 \log \left[ \frac{\sqrt{P_{11} + T_{12} + T_{21} + T_{22}}}{4} \right] \]

(32)

Similarly, for a cylindrical one-channel muffler with the section number \( m=M \) (with the segment number \( k = 1 \) and the layer number \( n = 1 \)), the matrix form between node 1 and node 4 is

\[
\begin{bmatrix}
p_1 \\
p_c u_1
\end{bmatrix} = \begin{bmatrix}
p_4 \\
p_c u_4
\end{bmatrix}
\]

(33a)

\[ \rho_c c_o u_1 = \rho_c c_o \left( M \frac{S_{2a}}{S_M} + M \frac{S_{3a}}{Z_{3a}} \right) \]

(33b)

The STL between node 1 and node 4 is

\[ STL = 10 \log \left[ \frac{\sqrt{P_{11} + T_{12} + T_{21} + T_{22}}}{4} \right] \]

(34)

Furthermore, as indicated in Fig. 3(A), for a cylindrical one-channel muffler horizontally partitioned into \( k \) segments (with the section number \( m=M \) and the layer number \( n = 1 \)), the matrix form from node 1 and node 2 to node \( k \) and node \( k+1 \) is

\[
\begin{bmatrix}
p_1^* \\
p_c u_1^*
\end{bmatrix} = \begin{bmatrix}
p_2^* \\
p_c u_2^*
\end{bmatrix}
\]

(35a)

\[
\begin{bmatrix}
p_2^* \\
p_c u_2^*
\end{bmatrix} = \begin{bmatrix}
p_3^* \\
p_c u_3^*
\end{bmatrix}
\]

(35b)

\[
\begin{bmatrix}
p_k^* \\
p_c u_k^*
\end{bmatrix} = \begin{bmatrix}
p_{k+1}^* \\
p_c u_{k+1}^*
\end{bmatrix}
\]

(35c)

The total transfer matrix (between node 1 and node \( k+1 \)) assembled by multiplication is

\[
\begin{bmatrix}
p_1^* \\
p_c u_1^*
\end{bmatrix} = \begin{bmatrix}
p_{k+1}^* \\
p_c u_{k+1}^*
\end{bmatrix}
\]

(36)

The STL between node 1 and node \( k+1 \) yields
$$STL_i = 10 \log \left[ \frac{P_{T1} + T_{T2} + T_{T1} + T_{T2}}{4} \right]$$  \hspace{1cm} (37)$$

![Diagram of muffler A and acoustic field](image)

(A) Dimensions of muffler A

(B) Acoustical field of a segment of muffler A (section number: m=2, segment number: k = 1, layer number: n = 2)

![Diagram of acoustic field](image)

number of layer (n) = 1

Figure 3. The dimensions and acoustical field of muffler A.

3.2. Muffler B (a two-channel pod muffler)

The acoustical field of a segment of a cylindrical two-channel muffler (section number: m=2, segment number: k = 1, layer number: n = 2) is shown in Fig. 2(B) and Fig. 4 where Z is the acoustic impedance, p is the acoustic pressure, and u is the particle velocity. As indicated in Fig. 4, the inner perforated plate is partitioned as L1*L1-11, and the outer perforated plate is partitioned as L1*L1-22. Based on the plane wave theory, the cutoff frequency (fc) is $c_0/2L_{max}$ where $L_{max}$ is the maximum value of (L1*L1-11) and (L1*L1-22).

Similarly, the acoustical impedances of node 2-1a, node 2-1b, node 3-1a, and node 3-1b are

$$Z_{2-1a} = -j \cot [K_{p_{1b}}(H_{1,1})] \left[ Z_{p_{1b}} + \frac{\rho_s}{pp_{1b}} \sqrt{8\pi\nu\omega}(1 + \frac{t_{s1-1}}{2d_{s1-1}}) + j \frac{\varrho_{s}}{pp_{1b}} \left[ \frac{8\pi\nu\omega}{\omega}(1 + \frac{t_{s1-1}}{2d_{s1-1}}) + t_{s1-1} + \delta_{s1-1} \right] \right]$$  \hspace{1cm} (38a)$$

$$Z_{2-1b} = -j \cot [K_{p_{1b}}(H_{1,2})] \left[ Z_{p_{1b}} + \frac{\rho_s}{pp_{1b}} \sqrt{8\pi\nu\omega}(1 + \frac{t_{s2-1}}{2d_{s2-1}}) + j \frac{\varrho_{s}}{pp_{1b}} \left[ \frac{8\pi\nu\omega}{\omega}(1 + \frac{t_{s2-1}}{2d_{s2-1}}) + t_{s2-1} + \delta_{s2-1} \right] \right]$$  \hspace{1cm} (38b)$$

$$Z_{3-1a} = -j \cot [K_{p_{1b}}(H_{1,1})] \left[ Z_{p_{1b}} + \frac{\rho_s}{pp_{1b}} \sqrt{8\pi\nu\omega}(1 + \frac{t_{s1-2}}{2d_{s1-2}}) + j \frac{\varrho_{s}}{pp_{1b}} \left[ \frac{8\pi\nu\omega}{\omega}(1 + \frac{t_{s1-2}}{2d_{s1-2}}) + t_{s1-2} + \delta_{s1-2} \right] \right]$$  \hspace{1cm} (38c)$$

$$Z_{3-1b} = -j \cot [K_{p_{1b}}(H_{1,2})] \left[ Z_{p_{1b}} + \frac{\rho_s}{pp_{1b}} \sqrt{8\pi\nu\omega}(1 + \frac{t_{s2-2}}{2d_{s2-2}}) + j \frac{\varrho_{s}}{pp_{1b}} \left[ \frac{8\pi\nu\omega}{\omega}(1 + \frac{t_{s2-2}}{2d_{s2-2}}) + t_{s2-2} + \delta_{s2-2} \right] \right]$$  \hspace{1cm} (38d)$$

Also, the acoustical impedances of node 2-2a, node 2-2b, node 3-2a, and node 3-2b are

$$Z_{2-2a} = -j \cot [K_{p_{1b}}(H_{2,1})] \left[ Z_{p_{1b}} + \frac{\rho_s}{pp_{2b}} \sqrt{8\pi\nu\omega}(1 + \frac{t_{s2-1}}{2d_{s2-1}}) + j \frac{\varrho_{s}}{pp_{2b}} \left[ \frac{8\pi\nu\omega}{\omega}(1 + \frac{t_{s2-1}}{2d_{s2-1}}) + t_{s2-1} + \delta_{s2-1} \right] \right]$$  \hspace{1cm} (39a)$$

$$Z_{2-2b} = -j \cot [K_{p_{1b}}(H_{2,2})] \left[ Z_{p_{1b}} + \frac{\rho_s}{pp_{2b}} \sqrt{8\pi\nu\omega}(1 + \frac{t_{s2-2}}{2d_{s2-2}}) + j \frac{\varrho_{s}}{pp_{2b}} \left[ \frac{8\pi\nu\omega}{\omega}(1 + \frac{t_{s2-2}}{2d_{s2-2}}) + t_{s2-2} + \delta_{s2-2} \right] \right]$$  \hspace{1cm} (39b)
As indicated in Fig. 4, a sound wave propagates from the left side to the right side. According to the continuity equation and geometric data

\[ p_1 = p_2 = p_{3-2a} = p_{3-1a} = p_{3-1b} = p_{2-2a} = p_{2-2b} = p_{3-2b} = p_4; \]  
(40a)

\[ v_1 = v_{2-1a} + v_{2-1b} + v_{3-1a} + v_{3-1b} + v_{2-2a} + v_{2-2b} + v_{3-2a} + v_{3-2b}; \]  
(40b)

\[ S_1 = S_4 = \frac{\pi}{4} \left( \frac{D_1^2 - D_2^2}{2} \right) \frac{1}{M} = S_M; \]  
(40c)

\[ S_{2-1a} = S_{2-1b} = L_1 \ast L_{2-1a} \ast p_{p2-1a}; \]  
(40d)

\[ S_{2-1b} = S_{2-2b} = L_1 \ast L_{2-1b} \ast p_{p2-1b}; \]  
(40e)

\[ S_{3-1a} = S_{3-2a} = L_1 \ast L_{2-1a} \ast p_{p2-1a}; \]  
(40f)

\[ S_{3-1b} = S_{3-2b} = L_1 \ast L_{2-1b} \ast p_{p2-1b}; \]  
(40g)

For the mass conservation theory \( \rho_S S_t u_1 = \rho_S S_{2-1a} u_{2-1a} + \rho_S S_{2-1b} u_{2-1b} + \rho_S S_{3-1a} u_{3-1a} + \rho_S S_{3-1b} u_{3-1b} + \rho_S S_{2-2a} u_{2-2a} + \rho_S S_{2-2b} u_{2-2b} + \rho_S S_{3-2a} u_{3-2a} + \rho_S S_{3-2b} u_{3-2b} + \rho_S S_M u_4 \)  
(41)

Based on the definition of acoustical impedance

\[ Z_{2-1a} = \frac{p_{2-1a}}{u_{2-1a}}; \quad Z_{2-1b} = \frac{p_{2-1b}}{u_{2-1b}}; \quad Z_{3-1a} = \frac{p_{3-1a}}{u_{3-1a}}; \quad Z_{3-1b} = \frac{p_{3-1b}}{u_{3-1b}}; \quad Z_{2-2a} = \frac{p_{2-2a}}{u_{2-2a}}; \quad Z_{2-2b} = \frac{p_{2-2b}}{u_{2-2b}}; \quad Z_{3-2a} = \frac{p_{3-2a}}{u_{3-2a}}; \quad Z_{3-2b} = \frac{p_{3-2b}}{u_{3-2b}}; \]  

\[ u_{2-1a} = \frac{p_{2-1a}}{Z_{2-1a}}; \quad u_{2-1b} = \frac{p_{2-1b}}{Z_{2-1b}}; \quad u_{3-1a} = \frac{p_{3-1a}}{Z_{3-1a}}; \quad u_{3-1b} = \frac{p_{3-1b}}{Z_{3-1b}}; \quad u_{2-2a} = \frac{p_{2-2a}}{Z_{2-2a}}; \quad u_{2-2b} = \frac{p_{2-2b}}{Z_{2-2b}}; \quad u_{3-2a} = \frac{p_{3-2a}}{Z_{3-2a}}; \quad u_{3-2b} = \frac{p_{3-2b}}{Z_{3-2b}}; \]  

\[ \text{where } Z_{2a} = Z_{2b}; \quad Z_{3a} = Z_{3b}. \]  

Summing up the above formulas yields

\[ \begin{bmatrix} p_1 \end{bmatrix} = \begin{bmatrix} \rho_S c_o u_1 = \rho_S c_o u_4 + (\rho_S c_o \cdot 2 \cdot \frac{S_{2-1a}}{S_M} + \rho_S c_o \cdot 2 \cdot \frac{S_{2-1b}}{S_M}) p_4 \\ \rho_S c_o \cdot 2 \cdot \frac{S_{2-1a}}{S_M} \cdot \frac{1}{Z_{2-1a}} + \rho_S c_o \cdot 2 \cdot \frac{S_{3-1b}}{S_M} \cdot \frac{1}{Z_{3-1b}} \end{bmatrix} \]  
(43)

which can be expressed in a matrix form as

\[ \begin{bmatrix} p_1 \\ \rho_S c_o u_1 \end{bmatrix} = \begin{bmatrix} 2 \cdot \frac{S_{2-1a}}{S_M} \cdot \frac{1}{Z_{2-1a}} + 2 \cdot \frac{S_{3-1a}}{S_M} \cdot \frac{1}{Z_{3-1a}} + 1 \\ 2 \cdot \frac{S_{2-1b}}{S_M} \cdot \frac{1}{Z_{2-1b}} + 2 \cdot \frac{S_{3-1b}}{S_M} \cdot \frac{1}{Z_{3-1b}} \end{bmatrix} \begin{bmatrix} p_4 \\ \rho_S c_o u_4 \end{bmatrix} \]  
(44a)

Or in the form of

\[ \begin{bmatrix} p_1 \\ \rho_S c_o u_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_4 \\ \rho_S c_o u_4 \end{bmatrix} \]  
(44b)

The STL between nodes 1 and 4 is

\[ STL = 10 \log \left[ \frac{T_{11} + T_{12} + T_{21} + T_{22}}{4} \right] \]  
(45)
Furthermore, as indicated in Fig. 2(B), for a cylindrical two-channel muffler horizontally partitioned as k segments (with a section number \( m = M \) and a layer number \( n = 2 \)), the matrix form from node 1 and node 2 to node k and node K+1 is

\[
\begin{bmatrix}
\mathbf{p}_1^* \\
\rho_c \mathbf{e}_c \mathbf{u}_1^*
\end{bmatrix} = \mathbf{\rho}_c \mathbf{e}_c \begin{bmatrix}
\frac{1}{S_{2-1a}} + M \frac{1}{S_{2-1b}} \\
+ M \frac{1}{S_{2-1a}} + M \frac{1}{S_{2-1b}} \\
\end{bmatrix} \begin{bmatrix}
\mathbf{p}_2^* \\
\rho_c \mathbf{e}_c \mathbf{u}_2^*
\end{bmatrix}
\]

\[
(46a)
\]

Similarly, as indicated in Fig. 2(C), for a cylindrical three-channel muffler horizontally partitioned as k segments (with a section number \( m = M \) and a layer number \( n = 3 \)), the matrix form from node 1 and node 2 to node k and node K+1 is

\[
\begin{bmatrix}
\mathbf{p}_1^* \\
\rho_c \mathbf{e}_c \mathbf{u}_1^*
\end{bmatrix} = \mathbf{\rho}_c \mathbf{e}_c \begin{bmatrix}
\frac{1}{S_{2-1a}} + M \frac{1}{S_{2-1b}} \\
+ M \frac{1}{S_{2-1a}} + M \frac{1}{S_{2-1b}} \\
\end{bmatrix} \begin{bmatrix}
\mathbf{p}_2^* \\
\rho_c \mathbf{e}_c \mathbf{u}_2^*
\end{bmatrix}
\]

\[
(46b)
\]

The system matrix between node 1 and node k+1 is

\[
\begin{bmatrix}
\mathbf{p}_1^* \\
\rho_c \mathbf{e}_c \mathbf{u}_1^*
\end{bmatrix} = \begin{bmatrix}
\mathbf{T}_{11} & \mathbf{T}_{12} \\
\mathbf{T}_{21} & \mathbf{T}_{22}
\end{bmatrix} \begin{bmatrix}
\mathbf{p}_{k+1}^* \\
\rho_c \mathbf{e}_c \mathbf{u}_{k+1}^*
\end{bmatrix}
\]

\[
(47)
\]

The STL between node 1 and node k+1 is

\[
STL_2 = 10 \log \left[ \frac{T_{11} + T_{12} + T_{21} + T_{22}}{4} \right]
\]

\[
(48)
\]

**Figure 4.** The acoustical number of a segment of muffler B (section number: \( m = 2 \), segment number: \( k = 1 \), layer number: \( n = 2 \)).

### 3.3. Muffler C (a three-channel pod muffler)

Similarly, as indicated in Fig. 2(C), for a cylindrical three-channel muffler horizontally partitioned as k segments (with a section number \( m = M \) and a layer number \( n = 3 \)), the matrix form from node 1 and node 2 to node k and node K+1 is

\[
\begin{bmatrix}
\mathbf{p}_1^* \\
\rho_c \mathbf{e}_c \mathbf{u}_1^*
\end{bmatrix} = \mathbf{\rho}_c \mathbf{e}_c \left[ \sum_{n=1}^{3} \left( M \frac{1}{S_{(n+1)-1a}} + M \frac{1}{S_{(n+1)-1b}} \right) \right] \begin{bmatrix}
\mathbf{p}_2^* \\
\rho_c \mathbf{e}_c \mathbf{u}_2^*
\end{bmatrix}
\]

\[
(49a)
\]
\[
\begin{bmatrix}
  p_2^* \\
  p_3^* \\
  \vdots \\
  p_K^*
\end{bmatrix} = \left[ \rho c_A \left( \sum_{n=0}^{3} \left( M \cdot \frac{S_{n+1}-S_n}{S_M} \cdot \frac{1}{Z_{n+1}-Z_n} + M \cdot \frac{S_{n+1}-S_n}{S_M} \cdot \frac{1}{Z_{n+1}-Z_n} \right) \right) \right]^{0.5} \left[ \begin{bmatrix}
  p_1^* \\
  p_2^* \\
  \vdots \\
  p_K^*
\end{bmatrix}
\right]
\]

...(49b)

\[
\begin{bmatrix}
  p_{K-1}^* \\
  p_K^*
\end{bmatrix} = \left[ \rho c_A \left( \sum_{n=0}^{3} \left( M \cdot \frac{S_{n+1}-S_n}{S_M} \cdot \frac{1}{Z_{n+1}-Z_n} + M \cdot \frac{S_{n+1}-S_n}{S_M} \cdot \frac{1}{Z_{n+1}-Z_n} \right) \right) \right]^{0.5} \left[ \begin{bmatrix}
  p_{K-1}^* \\
  p_K^*
\end{bmatrix}
\right]
\]

...(49c)

\[
\begin{bmatrix}
  p_k^* \\
  p_{k+1}^*
\end{bmatrix} = \left[ \rho c_A \left( \sum_{n=0}^{3} \left( M \cdot \frac{S_{n+1}-S_n}{S_M} \cdot \frac{1}{Z_{n+1}-Z_n} + M \cdot \frac{S_{n+1}-S_n}{S_M} \cdot \frac{1}{Z_{n+1}-Z_n} \right) \right) \right]^{0.5} \left[ \begin{bmatrix}
  p_k^* \\
  p_{k+1}^*
\end{bmatrix}
\right]
\]

...(49d)

The resulting system matrix between node 1 and node \( k+1 \) can be expressed as

\[
\begin{bmatrix}
  T_{11} & T_{12} \\
  T_{21} & T_{22}
\end{bmatrix} = \left[ \begin{bmatrix}
  p_{1}^* \\
  p_{k+1}^*
\end{bmatrix}
\right]
\]

...(50)

The STL between node 1 and node \( k+1 \) is

\[
STL_{1k} = 10 \log \left( \frac{p_{1}^*}{p_{k+1}^*} \right)
\]

...(51)

3.4 Overall sound power level

The silenced octave sound power level emitted from the \( k \)-th muffler’s outlet is

\[
SWL_{k} = SWLO(f_{k}) - STL_{k}(f_{k})
\]

where \( SWLO(f_{k}) \) is the original SWL at the inlet of a muffler (or pipe outlet), and \( f_{k} \) is the relative octave band frequency; \( STL_{k}(f_{k}) \) is the \( k \)-th muffler’s STL with respect to the relative octave band frequency \( (f_{k}) \); \( SWL_{k} \) is the silenced SWL at the outlet of the \( k \)-th muffler with respect to the relative octave band frequency.

Finally, the overall \( SWT \) silenced by the \( k \)-channel pad muffler at the outlet is

\[
SWT = 10 \log \left( \sum_{k=1}^{10} \frac{SWL_{k}}{10} \right)
\]

...(53)

3.5. Objective function. STL maximization for a tone (\( f \)) noise

The objective function for maximizing the STL of muffler A at a pure tone \((f)\) is

\[
OBJ_{1} = STL_{1}(f, RT_1, RT_2, RT_3, RT_4, RT_5, RT_6, RT_7)
\]

...(54a)

Similarly, the objective function for maximizing the STL of muffler B at a pure tone \((f)\) is

\[
OBJ_{2} = \left( Q, f, RT_1^{*}, RT_2^{*}, RT_3^{*}, RT_4^{*}, RT_5^{*}, RT_6^{*}, RT_7^{*} \right)
\]

...(55a)

Also, the objective function for maximizing the STL of muffler C at a pure tone \((f)\) is

\[
OBJ_{3} = \left( Q, f, RT_1^{**}, RT_2^{**}, RT_3^{**}, RT_4^{**}, RT_5^{**}, RT_6^{**}, RT_7^{**} \right)
\]

...(56a)

The related ranges of the parameters are shown in Table 1.
Table 1. The corresponding space constraints and the ranges of design parameters for the mufflers.

| Range of design parameters |
|-----------------------------|
| Muffler A | $Q=0.1 \text{ (m}^3/\text{s})$; $L_1=0.6 \text{ (m)}$; $D_1=0.05 \text{ (m)}$; $RT_1: [5, 40]$; $RT_2: [0.003, 0.015]$; $RT_3: [0.5, 0.7]$; $RT_4: [0.5, 0.7]$; $RT_5: [0.2, 0.99]$; $RT_6: [0.2, 0.99]$; $RT_7: [3000, 20000]$ |
| Muffler B | $Q=0.1 \text{ (m}^3/\text{s})$; $L_1=0.6 \text{ (m)}$; $D_1=0.05 \text{ (m)}$; $RT_1^*: [5, 40]$; $RT_2^*: [0.003, 0.015]$; $RT_3^*: [0.025, 0.075]$; $RT_4^*: [0.025, 0.075]$; $RT_5^*: [0.025, 0.075]$; $RT_6^*: [0.2, 0.99]$; $RT_7^*: [0.2, 0.99]$; $RT_8^*: [0.2, 0.99]$; $RT_9^*: [3000, 20000]$ |
| Muffler C | $Q=0.1 \text{ (m}^3/\text{s})$; $L_1=0.6 \text{ (m)}$; $D_1=0.05 \text{ (m)}$; $RT_1^*: [5, 40]$; $RT_2^*: [0.003, 0.015]$; $RT_3^*: [0.025, 0.05]$; $RT_4^*: [0.025, 0.05]$; $RT_5^*: [0.025, 0.05]$; $RT_6^*: [0.2, 0.99]$; $RT_7^*: [0.2, 0.99]$; $RT_8^*: [0.2, 0.99]$; $RT_9^*: [0.2, 0.99]$; $RT_{10}^*: [0.2, 0.99]$; $RT_{11}^*: [3000, 20000]$ |

3.6. SWL minimization for a broadband noise

To minimize the overall SWL, the objective functions are

$$OBJ_{11} = SWL_{11}(f, RT_1, RT_2, RT_3, RT_4, RT_5, RT_6)$$

$$OBJ_{12} = SWL_{12}(f, RT_1^*, RT_2^*, RT_3^*, RT_4^*, RT_5^*, RT_6^*)$$

$$OBJ_{13} = SWL_{13}(f, RT_1^**, RT_2^**, RT_3^**, RT_4^**, RT_5^**, RT_6^**)$$

(57a) \hspace{1cm} (57b) \hspace{1cm} (57c)

4. Case studies

A space-constrained muffler for a nitrogen venting system shown in Fig. 1 is connected with a high pressure nitrogen blow-down (with a mass flow rate(W) of 1000 kg/H) system for the purpose of eliminating noise. The upstream pressure ($P_{oo}$) and temperature ($T_{oo}$) of the safety valve are 20kg/cm²G and 20°C. The diameters of the valve throat and the downstream pipe are 1 inch each. The unsilenced sound power level ($SWL$) produced by the N₂ vents and blow-down is [14]

$$L_{w} = 10\log(P_{oo}) + 10\log(A_o) + 20\log(T_{oo}) + 20\log\left(\frac{1}{SG}\right) + 94 + CCC_i(f,m)$$

where the peak frequency ($f_m = \frac{0.2(V_j)}{dm}$) is a function of the jet’s velocity ($V_j$) and the valve size ($dm$).

The spectrum correction ($CCC_i(f,m)$) is a function of the pipe’s downstream diameter. $A_o$ is the section area of the valve’s throat. SG, the specific gravity of N₂, is 0.969. The maximal volume flow rate ($Q$) during venting is given as 0.1 (m³/s). The calculated SWL at various octave frequencies is listed in Table 2.

To efficiently reduce the sound energy, a cylindrical multi-channel pod muffler is adopted. As shown in Fig. 1, the available space for a cylindrical muffler is 1.0 m in diameter. Three kinds of available lengths (1.5m, 2.5m, and 3.5 m) will be assessed during the muffler’s shape optimization process. Moreover, to simplify the optimization, the thickness of the inner perforated plate ($t=0.008(m)$) is preset in advance. Before the minimization of the broadband noise is performed, a SA function check using the maximization of the STL at the targeted pure tone (1000 Hz) has been performed. The corresponding OBJ functions are summarized in Eqs.(54)–(57).

Table 2. Unsilenced SWL of a Nitrogen venting noise inside a duct outlet.

| Octave Band Frequencies (Hz) | overall |
|-----------------------------|---------|
| 125                         | 140.8   |
| 250                         | 140.8   |
| 500                         | 140.8   |
| 1000                        | 140.8   |
| 2000                        | 140.8   |
5. Influence of design parameters

In order to achieve an optimally shaped muffler, the selection of appropriate design parameters is essential. An investigation of the acoustical influence with respect to the design parameters in muffler A is exemplified. The resulting $STL$s with respect to design parameters pp% (porosity of the perforated plate), dh (diameter of the perforated hole), H (thickness of the sound absorbing ring), W (width of the air way), Df (thickness of the sound absorbing wool), R (flow resistivity of the sound absorbing wool), L1 (length of the sound absorbing rings), and tw (thickness of the perforated plate) are assessed and shown in Figs. 6 and 12. As indicated in Figs. 6 and 12, the $STL$ is influenced by the design parameters $RT1$~$RT7$. Similarly, the acoustical influence with respect to the design parameters in muffler B and muffler C also indicate that the $STL$ is influenced by the design parameters $RT1^*$$RT6^*$ and $RT1^{**}$$RT11^{**}$. Therefore, the design parameters $RT1$~$RT7$ in muffler A, the design parameters $RT1^*$$RT6^*$ in muffler B, and the design parameters $RT1^{**}$$RT11^{**}$ in muffler C will be selected as the optimization parameters in section 6.

| Spectrum correction - CCC<sub>i</sub> (fm=5664 (Hz)) | -27 | -19 | -12 | -7 | -5 |
|-----------------------------------------------------|-----|-----|-----|----|----|
| A-weighted                                          | -16 | -9  | -3  | +0 | +1 |
| $SWLO – dB(A)$                                      | 97.8| 112.8| 125.8| 133.8| 136.8| 138.8|

**Figure 5.** The resulting $STL$ with respect to the design parameter - dh (diameter of a perforated hole).

**Figure 6.** The resulting $STL$ with respect to the design parameter - pp% (porosity of perforated plate).

**Figure 7.** The resulting $STL$ with respect to the design parameter - H (thickness of the sound absorbing ring).

**Figure 8.** The resulting $STL$ with respect to the design parameter - W (width of the air way).
6. Simulated annealing method

Because there is no need to choose starting data that is necessary for the classical gradient methods of EPFM, IPFM and FDM [15], the Simulated Annealing (SA), which is one of the best stochastic search method and does not need a good starting point for global searching, is convenient for the numerical assessment.

The simulated annealing (SA) was first introduced by Metropolis et al. [16] and further developed by Kirkpatrick et al. [17]. According to the philosophy of SA, the main issue is to bring the system from an arbitrary initial state to a state with minimum possible energy, which is the desired objective. Annealing is a slow cooling process that stabilizes a metal’s temperature. Because of this, the particles remain close to the minimum energy state. In order to emulate the SA’s evolution, a new random solution (X') is chosen from the neighborhood of the current solution. If, possibly, there is a negative change in the objective function (ΔF ≤ 0), the new solution will be recognized as the new current solution. If, conversely, there is not a negative change, the probability (pb(X')) of a transition to the new state will be calculated using the Boltzmann factor (pb(X') = exp(-ΔF / CT)) where C and T are the Boltzmann constant and the current temperature.

\[
pb(X') = \begin{cases} 
1, & \Delta F \leq 0 \\
\exp(-\frac{\Delta F}{CT}), & \Delta F > 0
\end{cases}
\]

\[
\Delta F = F(X') - F(X)
\]
For the purpose of escaping from the local optimum, SA also permits movement that results in inferior solutions (uphill movement). Hence, if the transition property \( \varphi(X') \) is greater than a random number of \( \text{rand}(0,1) \), the new inferior solution which results in a higher energy condition will be accepted; if not, it will be rejected. Each successful substitution of the new current solution will conduct to the decay of the current temperature as

\[
T_{\text{new}} = k \times T_{\text{old}}
\]

where \( k \) is the cooling rate.

This process is repeated until the preset (\( \text{iter} \)) of the outer loop is reached [18, 19, 20, 21, 22, 23].

7. Results and discussion

7.1. Results

To achieve good optimization, two kinds of SA parameters including \( k \) (cooling rate) and \( \text{iter} \) (maximum iteration) are varied step by step:

\( k \) (0.91, 0.93, 0.95, 0.97, 0.99); \( \text{iter} \) (50, 100, 500, 1000, 5000, 10000).

Two results of optimization (one, pure tone noises used for SA’s accuracy check; and the other, a broadband noise occurring in a venting outlet) are described below.

7.2. Pure tone noise optimization

Before dealing with a broadband noise for muffler A–C, the STL’s maximization for muffler A with respect to a one-tone noise (1000Hz) is introduced for a reliability check using the SA method. By using Eq. (54), the maximization of the STL with respect to muffler A (a one-channel pod muffler) at the specified pure tone (1000Hz) and limited length of \( L_0=0.6 \) (m) was performed first. As indicated in Table 3, ten sets of SA parameters were tried during the muffler’s optimization. Obviously, the optimal design data can be obtained with the tenth set of SA parameters at \((k, \text{iter}) = (0.99, 10000)\).

Using the optimal design in a theoretical calculation, the optimal STL curves with respect to various SA parameters \((k, \text{iter})\) are plotted and depicted in Figs. 13 and 14. As revealed in Figs. 13 and 14, the STL is precisely maximized at the desired frequency of 1000 Hz. Consequently, the SA optimizer is reliable in the optimization process. Using the same SA parameters \((k, \text{iter})\) in mufflers A and C, the optimal STL curves obtained are shown in Table 4 and plotted in Fig. 15. As illustrated in Table 3, the STLS of mufflers A–C at 1000 Hz reach 12.3 dB, 6.3 dB, and 8.4 dB, respectively.

Table 3. Optimal STL for muffler A (equipped with two layers of sound absorbing rings) at various SA parameters (targeted tone of 1000 Hz and \( L_0=0.6 \) (m)).

| Item | SA Control Parameter | Design parameters | STL (dB) |
|------|---------------------|------------------|----------|
|      |                     | \( k \) | \( \text{iter} \) | RT1 | RT2 | RT3 | RT4 | RT5 | RT6 | RT7 | RT8 | RT9 | RT10 | RT11 | RT12 |
| 1    | 0.91                | 50    | 35.64 | 0.0135 | 0.675 | 0.675 | 0.8915 | 0.8915 | 17880 | 3.47 |
| 2    | 0.93                | 39.94 | 0.01498 | 0.6997 | 0.6997 | 0.9886 | 0.9886 | 19970 | 4.93 |
| 3    | 0.95                | 36.98 | 0.01397 | 0.6828 | 0.6828 | 0.9219 | 0.9219 | 18530 | 5.73 |
| 4    | 0.97                | 39.3  | 0.01476 | 0.696  | 0.696  | 0.9741 | 0.9741 | 19660 | 6.07 |
| 5    | 0.99                | 39.89 | 0.01496 | 0.6994 | 0.6994 | 0.9874 | 0.9874 | 19950 | 6.52 |
| 6    | 0.99                | 39.52 | 0.01483 | 0.6972 | 0.6972 | 0.9791 | 0.9791 | 19770 | 7.23 |
| 7    | 0.99                | 39.4  | 0.01479 | 0.6965 | 0.6965 | 0.9764 | 0.9764 | 19710 | 7.29 |
| 8    | 0.99                | 39.39 | 0.01479 | 0.6965 | 0.6965 | 0.9762 | 0.9762 | 19700 | 7.40 |
| 9    | 0.99                | 39.41 | 0.01480 | 0.6967 | 0.6967 | 0.9768 | 0.9768 | 19720 | 7.54 |
| 10   | 0.99                | 35.8  | 0.01356 | 0.6760 | 0.6760 | 0.8953 | 0.8953 | 17960 | 12.25 |
Table 4. Optimal design data for three kinds of mufflers (mufflers A~C) (at a targeted tone of 1000 Hz and Lo=0.6 (m)) (kk=0.99, iter=10000).

| Muffler  | Design parameters | STL (dB) |
|----------|-------------------|----------|
| A        | RT1   35.8 |
|          | RT2   0.01356 |
|          | RT3   0.6760 |
|          | RT4   0.6760 |
|          | RT5   0.8953 |
|          | RT6   0.8953 |
|          | RT7   17960 |
| B        | RT1*  39.25 |
|          | RT2*  0.01474 |
|          | RT3*  0.07392 |
|          | RT4*  0.07392 |
|          | RT5*  0.9730 |
|          | RT6*  0.9730 |
|          | RT7*  0.9730 |
|          | RT8*  19630 |
| C        | RT1** 39.97 |
|          | RT2** 0.01499 |
|          | RT3** 0.04998 |
|          | RT4** 0.04998 |
|          | RT5** 0.04998 |
|          | RT6** 0.04998 |
|          | RT7** 0.9893 |
|          | RT8** 0.9893 |
|          | RT9** 0.9893 |
|          | RT10** 0.9893 |
|          | RT11** 19990 |

Table 5. Optimal design data for three kinds of mufflers (mufflers A~C) at Lo=1.5 (m) (broadband noise) (kk=0.96, iter=2000).

| Muffler  | Design parameters | SWL (dB) |
|----------|-------------------|----------|
| A        | RT1   37.44 |
|          | RT2   0.01412 |
|          | RT3   0.6854 |
|          | RT4   0.6854 |
|          | RT5   0.9322 |
|          | RT6   0.9322 |
|          | RT7   18760 |
| B        | RT1*  39.32 |
|          | RT2*  0.01477 |
|          | RT3*  0.07403 |
|          | RT4*  0.07403 |
|          | RT5*  0.9746 |
|          | RT6*  0.9746 |
|          | RT7*  0.9746 |
|          | RT8*  19670 |
| C        | RT1** 39.07 |
|          | RT2** 0.01468 |
|          | RT3** 0.04933 |
|          | RT4** 0.04933 |
|          | RT5** 0.04933 |
|          | RT6** 0.04933 |
|          | RT7** 0.9690 |
|          | RT8** 0.9690 |
|          | RT9** 0.9690 |
|          | RT10** 0.9690 |
|          | RT11** 19550 |

Adopting the same SA parameters of (kk, iter) = (0.99, 10000) and doing the minimization of the broadband noise at SWL_{T-1}, SWL_{T-2}, and SWL_{T-3} (mufflers A–C) at a limited length of Lo=1.5 (m), the resulting design parameters are obtained and shown in Table 5. Using the optimal design parameters in a theoretical calculation, the optimal STL curves with respect to various mufflers A–C are plotted and depicted in Fig. 16.

Similarly, using the same SA parameters of (kk, iter) = (0.99, 10000) and minimizing the broadband noise at SWL_{T-1}, SWL_{T-2}, and SWL_{T-3} (mufflers A–C) at a limited length of Lo=2.5 (m), the resulting design parameters are obtained and shown in Table 6. Using the optimal design parameters in a theoretical calculation, the optimal STL curves with respect to various mufflers A–C are plotted and depicted in Fig. 17.

Likewise, using the same SA parameters of (kk, iter) = (0.99, 10000) and minimizing the broadband noise at SWL_{T-1}, SWL_{T-2}, and SWL_{T-3} (mufflers A–C) at a limited length of Lo=3.5 (m), the resulting design parameters are obtained and shown in Table 7. Using the optimal design parameters in a theoretical calculation, the optimal STL curves with respect to various mufflers A–C are plotted and depicted in Fig. 18.

Table 5. Optimal design data for three kinds of mufflers (mufflers A–C) at Lo=1.5 (m) (broadband noise) (kk=0.96, iter=2000).

| Muffler  | Design parameters | SWL (dB) |
|----------|-------------------|----------|
| A        | RT1   37.44 |
|          | RT2   0.01412 |
|          | RT3   0.6854 |
|          | RT4   0.6854 |
|          | RT5   0.9322 |
|          | RT6   0.9322 |
|          | RT7   18760 |
| B        | RT1*  39.32 |
|          | RT2*  0.01477 |
|          | RT3*  0.07403 |
|          | RT4*  0.07403 |
|          | RT5*  0.9746 |
|          | RT6*  0.9746 |
|          | RT7*  0.9746 |
|          | RT8*  19670 |
| C        | RT1** 39.07 |
|          | RT2** 0.01468 |
|          | RT3** 0.04933 |
|          | RT4** 0.04933 |
|          | RT5** 0.04933 |
|          | RT6** 0.04933 |
|          | RT7** 0.9690 |
|          | RT8** 0.9690 |
|          | RT9** 0.9690 |
|          | RT10** 0.9690 |
|          | RT11** 19550 |
Table 6. Optimal design data for three kinds of mufflers (mufflers A–C) at Lo=2.5 (m) (broadband noise) (kk=0.96, iter=2000).

| Design parameters | SWL (dB) |
|-------------------|----------|
| Muffler A         |          |
| RT1               | 37.42    |
| RT2               | 0.01411  |
| RT3               | 0.6852   |
| RT4               | 0.9317   |
| RT5               | 18750    |
| RT6               |          |
| RT7               |          |
| Muffler B         |          |
| RT1*              | 39.32    |
| RT2*              | 0.01477  |
| RT3*              | 0.07404  |
| RT4*              | 0.07404  |
| RT5*              | 0.9748   |
| RT6*              | 0.9748   |
| RT7*              | 0.9748   |
| RT8*              | 0.9748   |
| RT9*              | 0.9748   |
| Muffler C         |          |
| RT1**             | 39.14    |
| RT2**             | 0.04938  |
| RT3**             | 0.04938  |
| RT4**             | 0.04938  |
| RT5**             | 0.04938  |
| RT6**             | 0.04938  |
| RT7**             | 0.04938  |
| RT8**             | 0.04938  |
| RT9**             | 0.04938  |
| RT10**            | 0.04938  |
| RT11**            | 0.04938  |

Table 7. Optimal design data for three kinds of mufflers (mufflers A–C) at Lo=3.5 (m) (broadband noise) (kk=0.96, iter=2000).

| Design parameters | SWL (dB) |
|-------------------|----------|
| Muffler A         |          |
| RT1               | 38.16    |
| RT2               | 0.01437  |
| RT3               | 0.6895   |
| RT4               | 0.9486   |
| RT5               | 0.9486   |
| RT6               | 19110    |
| RT7               |          |
| Muffler B         |          |
| RT1*              | 39.32    |
| RT2*              | 0.01477  |
| RT3*              | 0.07404  |
| RT4*              | 0.07404  |
| RT5*              | 0.9748   |
| RT6*              | 0.9748   |
| RT7*              | 0.9748   |
| RT8*              | 0.9748   |
| RT9*              | 0.9748   |
| Muffler C         |          |
| RT1**             | 39.19    |
| RT2**             | 0.04942  |
| RT3**             | 0.04942  |
| RT4**             | 0.04942  |
| RT5**             | 0.04942  |
| RT6**             | 0.04942  |
| RT7**             | 0.04942  |
| RT8**             | 0.04942  |
| RT9**             | 0.04942  |
| RT10**            | 0.04942  |
| RT11**            | 0.04942  |

7.4. Discussion
In order to achieve an optimally shaped muffler, the selection of design parameters is essential. An investigation of the design parameters’ influence on muffler A’s acoustical performance (STL) is introduced. The acoustical influence with respect to design parameters RT1~RT7 is shown in Figs. 6 and 12. Figs. 6 and 12 indicate that the STL is closely related to the design parameters RT1~RT7. Therefore, the design parameters RT1~RT7 in muffler A, the design parameters RT1*~RT7* in muffler B, and the design parameters RT1**~RT7** in muffler C are selected as the optimization parameters during the SA optimization.

To achieve a sufficient optimization, the selection of the appropriate SA parameter set is essential. As indicated in Table 3, the best SA set for muffler A at the targeted pure tone of 1000 Hz was found in
the eight set. The related STL curves with respect to various SA parameters are plotted in Figs. 13 and 14. Figs. 13 and 14 reveal that the predicted maximal value of the STL is precisely located at the desired frequency. Similarly, in dealing with pure tone noise (1000 Hz) in muffler B and muffler C, the profiles shown in Fig. 15 indicate that the maximum STLS of the mufflers are also located at the specified frequency.

In dealing with the broadband noise, three kinds of mufflers at three lengths (Lo = 1.5(m), 2.5(m), and 3.5 (m)) are adopted in the optimal process. As illustrated in Table 5 and Fig. 16, the resultant sound power levels with respect to three kinds of mufflers at Lo=1.5 (m) have been reduced from 138.8 dB(A) to 127.3 dB(A), 125.6 dB(A), and 124.3 dB(A). Also, Table 6 and Fig. 17 indicate that the resultant sound power levels with respect to three kinds of mufflers at Lo=2.5 (m) have been reduced from 138.8 dB(A) to 124.9 dB(A), 122.1 dB(A), and 120.6 dB(A). Likewise, Table 7 and Fig. 18 reveal that the resultant sound power levels with respect to three kinds of mufflers at Lo=3.5 (m) have been reduced from 138.8 dB(A) to 121.9 dB(A), 119.6 dB(A), and 118.1 dB(A). The results mentioned above indicate that the acoustical performance of muffler C is superior to the other mufflers. This means that the overall sound transmission loss will increase when the number of the sound absorbing layer increases.

An investigation of the effect of length (Lo) on the acoustical performance (STL) is also explored and shown in Figs. 19~21. As illustrated in Fig. 19, the STL of muffler A will increase when the length of Lo increases. In addition, as indicated in Fig. 20, the STL profile of muffler B will be promoted if the length of Lo increases. Consequently, Fig. 21 also reveals that the STL profile of muffler C will be promoted if the length of Lo increases.

8. Conclusion

It has been shown that pod mufflers hybridized with multiple layers of concentric sound-absorbing rings can be efficiently optimized within a limited space by using the acoustical lump analysis, a four-pole transfer matrix, and a SA optimizer. As indicated in Table 3, Figs. 13, and 14, two kinds of SA parameters (kk and iter) play essential roles in the solution’s accuracy during SA optimization. Figs. 13, 14, and 15 indicate that the tuning ability established by adjusting design parameters in mufflers A–C is reliable. Moreover, the appropriate design parameters of three kinds of mufflers hybridized with multiple layers of concentric sound-absorbing rings (mufflers A–C) have been assessed. Consequently, as indicated in Table 5, the resultant noise reductions with respect to mufflers A–C at Lo = 1.5 (m) are 11.5 dB, 13.2 dB, and 14.5 dB. Also, as indicated in Table 6, the resultant noise reductions with respect to mufflers A–C at Lo = 2.5 (m) are 13.9 dB, 16.7 dB, and 18.2 dB. Likewise, in Table 7, the resultant noise reductions with respect to mufflers A–C at Lo = 3.5 (m) are 16.9 dB, 19.2 dB, and 20.7 dB. Obviously, the muffler hybridized with more layers of concentric sound
absorbing rings is superior to mufflers equipped with fewer layers of concentric sound absorbing rings. Also, the longer muffler(length (Lo)) is superior to shorter mufflers(length (Lo)).

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9. References
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