The toroidal momentum pinch velocity

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In this letter a pinch velocity of toroidal momentum is shown to exist for the first time. Using the gyro-kinetic equations in the frame moving with the equilibrium toroidal velocity, it is shown that the physics effect can be elegantly formulated through the “Coriolis” drift. A fluid model is used to highlight the main coupling mechanisms between the density and temperature perturbations on the one hand and the perturbed parallel flow on the other. Gyro-kinetic calculations are used to accurately assess the magnitude of the pinch. The pinch velocity leads to a radial gradient of the toroidal velocity profile even in the absence of a torque on the plasma. It is shown to be sizeable in the plasmas of the International Thermonuclear Experimental Reactor (ITER) leading to a moderately peaked rotation profile. Finally, the pinch also affects the interpretation of current experiments.

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In a tokamak the total toroidal angular momentum is a conserved quantity in the absence of an external source. Transport phenomena determine the rotation profile which is of interest because a radial gradient in the toroidal rotation is connected with an ExB shearing that can stabilise micro-instabilities and, hence, improve confinement. Furthermore, a toroidal rotation of sufficient magnitude can stabilise the resistive wall mode. In present day experiments the rotation is often determined by the toroidal torque on the plasma that results from the neutral beam heating. Such a torque will be largely absent in a reactor and it is generally assumed that the rotation, and hence its positive influence, will be small. The novel pinch velocity described in this letter, however, may generate a sizeable toroidal velocity gradient in the confinement region even in the absence of a torque.

We will focus on the Ion Temperature Gradient (ITG) mode, which is expected to be the dominant instability governing the ion heat channel in a reactor plasma. The equations are formulated using the gyro-kinetic framework, which has been proven successful in explaining many observed transport phenomena. Because of the rotation, the background electric field cannot be ordered small, and the starting point for the derivation is a set of equations for the time evolution of the guiding centre and the parallel (to the magnetic field) velocity component in the co-moving system (with background velocity \( u_0 \)) obtained from Ref.

\[
\frac{dX}{dt} = v_\parallel b + \frac{b}{eB_\parallel} \times (e\nabla \phi + \mu B + m u_0^* \cdot \nabla u_0^*),
\]

(1)

\[
\frac{dv_\parallel}{dt} = -\frac{B^*}{mB_\parallel} \cdot (e\nabla \phi + \mu B + m u_0^* \cdot \nabla u_0^*).
\]

(2)

Here \( b = B/B \) is the unit vector in the direction of the magnetic field \( B \), \( \phi \) is the perturbed gyro-averaged potential (i.e. the part not connected with the background rotation), \( \mu \) the magnetic moment, \( m \) the particle mass (charge), and \( u_0^* = u_0 + v_\parallel b \). For the background velocity \( u_0 \) we assume a constant rigid body toroidal rotation with angular frequency \( \Omega \) (this is an equilibrium solution see, for instance, Refs. 20, 21, 22)

\[
u_0 = \Omega \times X = R^2 \Omega \nabla \phi,
\]

(3)

where \( \phi \) is the toroidal angle. We briefly outline the derivation of the final equations here. More details can be found in 29. The background velocity \( u_0 \) will be assumed smaller than the thermal velocity, and only the terms linear in \( u_0 \) will be retained. This eliminates the centrifugal forces but retains the Coriolis force. Furthermore, the low beta approximation is used for the equilibrium magnetic field (i.e. \( b \cdot \nabla B \approx \nabla \perp B/B \) where \( \perp \) indicates the component perpendicular to the magnetic field). With these assumptions

\[
u_0^* \cdot \nabla u_0^* \approx \frac{v_\parallel^2}{\nabla \perp B} + 2v_\parallel \Omega \times b.
\]

(4)

Using the definition of \( B^* \) (see Ref. 20) and expanding up to first order in the normalised Larmor radius \( \rho^* = \rho/R \), where \( R \) is the major radius, one obtains

\[
B^* = B + \frac{B}{\omega_e} \nabla \times u_0^* = B \left[ b + \frac{2\Omega}{\omega_e} + \frac{v_\parallel}{\omega_e} \frac{B \times \nabla B}{B^2} \right]
\]

(5)

and \( B^* = b \cdot B^* = B(1 + 2\Omega/\omega_e) \) (\( \omega_e = eB/m \) is the gyro-frequency). Expanding now the equations of motion retaining only terms up to first order in \( \rho^* \) yields

\[
\frac{dX}{dt} = v_\parallel b + v_\parallel \frac{B \times \nabla \phi}{\omega_e} + \frac{v_\parallel^2 + v_\parallel^2/2}{B^2} B \times \nabla B + 2v_\parallel \Omega \nabla \perp
\]

(6)

The terms in this equation are from left to right, the parallel motion \( (v_\parallel b) \), the ExB velocity \( v_E \), the combination of curvature and grad-B drift \( v_d \), and an additional term proportional to \( \Omega \). An interpretation of this term can be found if one uses the standard expression for a drift.
velocity \( \mathbf{v}_D \) due to a force \( \mathbf{F} \) perpendicular to the magnetic field \( \mathbf{v}_D = \mathbf{F} \times \mathbf{B} / eB^2 \). Substituting the Coriolis force \( \mathbf{F}_C = 2m\mathbf{v} \times \mathbf{B} / \Omega \), and taking for the velocity \( \mathbf{v} \) the lowest order (parallel) velocity one obtains

\[
\mathbf{v}_{dc} = \frac{\mathbf{F}_C \times \mathbf{B}}{eB^2} = \frac{2v_{\parallel}}{\omega_c} \mathbf{\Omega}_\perp \tag{7}
\]

The last term in Eq. (6) is therefore the Coriolis drift. Expanding the terms in the equation for the parallel velocity to first order in \( \rho \) one can derive

\[
mv_{\parallel} \frac{dv_{\parallel}}{dt} = -e \mathbf{dX}/dt \cdot \nabla \phi - \mu e \mathbf{dX}/dt \cdot \nabla B \tag{8}
\]

where \( \mathbf{dX}/dt \) is given by Eq. (6). The derived equations are similar to the non-rotating system, with the difference being the additional Coriolis drift. It follows that this Coriolis drift appears in a completely symmetric way compared with the curvature and grad-B drift.

In this letter the approximation that assumes circular surfaces and small inverse aspect ratio \( e \) is used. In this case the Coriolis drift adds to the curvature and grad-B drift

\[
\mathbf{v}_d + \mathbf{v}_{dc} \approx \frac{v_{\parallel}^2 + 2v_{\parallel}v_{\perp} + v_{\perp}^2 / 2}{\omega_c R} \mathbf{e}_z \tag{9}
\]

where \( \mathbf{e}_z \) is in the direction of the symmetry axis of the tokamak. The linear gyro-kinetic equation is solved using the ballooning transform \( \mathbf{R} \). The equations, except from the Coriolis drift are standard and can be found in, for instance, Ref. [31]. In the following \( u' \equiv -R\nabla R \Omega / v_{th} \) and \( u \equiv R\Omega / v_{th} \). Unless explicitly stated otherwise all quantities will be made dimensionless using the major radius \( R \), the thermal velocity \( v_{th} \equiv \sqrt{2T/m_i} \), and the ion mass \( m_i \). Densities will be normalised with the electron density. The toroidal momentum flux is approximated by the flux of parallel momentum \( (\Gamma_\phi) \) which is sometimes normalised with the total ion heat flow \( (Q_t) \)

\[
(\Gamma_\phi, Q_t) = \left\langle \mathbf{v}_E \int \mathbf{d}^3 \mathbf{v} \left( m v_{\parallel}^2 / 2m v^2 \right) f \right\rangle , \tag{10}
\]

where \( f \) is the (fluctuating) distribution function and the brackets denote the flux surface average.

Before turning to the gyro-kinetic calculations, the first implications of the Coriolis drift will be investigated using a simple fluid model (more extended models have been published in Refs. [32, 33]). A (low field side) slab like geometry will be assumed with all plasma parameters being a function of the x-coordinate only. The magnetic field is \( \mathbf{B} = Be_y, \nabla B = -B/R \mathbf{e}_z \). The model can be build by taking moments of the gyro-kinetic equation in \( \mathbf{v}_\parallel, v_\perp, \mathbf{\Omega}_\perp \) coordinates

\[
\frac{\partial f}{\partial t} + (\mathbf{v}_d + \mathbf{v}_{dc}) \cdot \nabla f = -\mathbf{v}_E \cdot \nabla F_M - \frac{eF_M}{T} (\mathbf{v}_d + \mathbf{v}_{dc}) \cdot \nabla \phi , \tag{11}
\]

where \( F_M \) is the Maxwell distribution. Note that translation symmetry in the z-direction is assumed, eliminating the parallel dynamics. Building moments of these equations neglecting the heat fluxes (this is a clear simplification, see for instance [34, 35, 36, 37]), and taking the space and time dependence of the perturbed quantities as \( \exp[\text{i}(k_R z - \omega t)] \), one arrives at the following equations for the perturbed density \( (n) \) normalised to the background density \( (n_0) \), the perturbed parallel velocity \( (u) \) normalised with the thermal velocity, and the perturbed ion temperature \( (T) \) normalised with the background ion temperature \( (T_0) \)

\[
\omega n + 2(n + T) + 4uu = \left[ \frac{R}{L_N} - 2 \right] \phi , \tag{12}
\]

\[
\omega u + 4w + 2un + 2uT = [u' - 2u] \phi , \tag{13}
\]

\[
\omega T + 4n + 14T + 8uu = \left[ \frac{R}{L_T} - 4 \right] \phi . \tag{14}
\]

Here \( R/L_N \equiv -R\nabla n_0 / n_0, R/L_T \equiv -R\nabla T_0 / T_0 \), the potential \( \phi \) is normalised to \( T_0/\epsilon \), and the frequency is normalised with the drift frequency \( \omega_D = -k_BT_0/eBR \). The Coriolis drift (all terms proportional to \( u \)) introduces the perturbed velocity in the equations for the perturbed density, and temperature. However, since \( u \ll 1 \) the influence of the Coriolis drift on the “pure” ITG (with \( u = 0 \)) is relatively small. The Coriolis drift generates a coupling between \( w \) and the density, temperature as well as potential fluctuations. Note that for \( u = 0 \) the perturbed velocity is directly related to the gradient \( u' \), resulting in a purely diffusive flux. For finite rotation \( (u \neq 0) \) the ITG will generate a perturbed parallel velocity \( w \), which is then transported by the perturbed ExB velocity. If the perturbed temperature is kept the expressions for the momentum flux become rather lengthy and are, therefore, reported elsewhere [29]. Retaining only the coupling with the perturbed density and potential, and assuming an adiabatic electron response \( (n = \phi/\tau \) with \( \tau = T_e/T_0 \) being the electron to ion temperature ratio) one can derive

\[
\Gamma_\phi = \frac{1}{4} k_0 e \text{Im}[\phi^* w] = \chi_\phi \left[ u' - \frac{2 + 2\tau}{\tau} u \right] , \tag{15}
\]

with

\[
\chi_\phi = \frac{1}{4} k_0 \rho \frac{\gamma}{\omega_R + 4} = \frac{1}{4} k_0 \rho \left[ \frac{\gamma}{\omega_R + 4} + \gamma^2 \right] \phi^2 . \tag{16}
\]

Here, the dagger denotes the complex conjugate, \( \omega_R \) is the real part of the frequency, and \( \gamma \) the growth rate of the mode. Note that \( \chi_\phi \) is positive since \( \omega_R (\gamma) \) are normalised to \( \omega_D = -k_BT_0/eBR \). The second term between the square brackets of Eq. (15) represents an inward pinch of the toroidal velocity (the word pinch is
used here because the flux is proportional to $u$, unlike off-diagonal contributions that are due to pressure and temperature gradients [38, 39]. If one assume no torque, i.e. $\Gamma_\phi = 0$ it can be seen that the pinch can lead to a sizeable gradient length $R/L_u \equiv R\dot{V}u/u = 4$ (for $\tau = 1$). The peaking is in roughly the same range as the expected density peaking [40].

The diagonal part has been calculated previously using fluid [43, 44, 45, 46, 47] as well as gyro-kinetic theory [48, 49]. The pinching velocity is negative (inward) for positive $u$ such that it enhances the gradient. It changes sign with $u$ such that for negative velocities it will make $u'$ more negative, i.e. the pinch always enhances the absolute value of the velocity gradient in agreement with the results from the fluid theory. Fig. 1 also shows that the pinch decreases with $k_\theta \rho_i$. It is noted here that also $\chi_\phi$ in becomes smaller for smaller $k_\theta \rho_i$ [39].

Fig. 1 shows the parallel momentum flux as a function of various parameters. The magnetic shear and the density gradient have a rather large impact. Note that both due to $\hat{s}$, as well as due to $q$, $R/L_N$ and $\epsilon$, the pinch velocity is expected to be small in the inner core, but sizeable in the confinement region.

The novel pinch velocity described in this letter has several important consequences. It can explain a gradient of the toroidal velocity in the confinement region of the plasma without momentum input. A spin up of the plasma column without torque has indeed been observed [51, 52, 53, 54, 55, 56]. Although a consistent description ordering the different observations is still lacking, the calculations of this letter show that the pinch velocity is expected to play an important role. This finite gradient without torque is especially important for a tokamak reactor in which the torque will be relatively small. From the calculations shown above, and for typical parameters in the confinement region of a reactor plasma, one obtains a gradient length $R/L_u = u'/u$ in the range 2-4 representing a moderate peaking of the toroidal velocity profile similar to that of the density. Unfortunately, the current calculation only yields the normalised toroidal velocity gradient. In order to determine the velocity gradient one would need to know the edge rotation velocity. This situation is similar to that of the ion temperature [50].

The existence of a pinch can resolve the discrepancy between the calculated $\chi_\phi$ and the experimentally obtained effective diffusivity ($\chi_{\text{eff}} = \Gamma_\phi/u'$). The latter is often found to decrease with increasing minor radius and to be smaller than the theoretical value of $\chi_\phi$ in the outer region of the plasma [57, 58, 59]. The pinch indeed leads
to a decrease of $\chi_{\text{eff}}$

$$\chi_{\text{eff}} = \chi_{\phi} \left[ 1 + \frac{RV_{\phi}}{\chi_{\phi}} \frac{1}{R/L_u} \right]. \quad (18)$$

The calculations in this letter show that the second term in the brackets can be of the order -1, leading to $\chi_{\text{eff}} < \chi_i$.

[1] R.E. Waltz et al., Phys. Plasmas 1, 2229 (1994)
[2] T.S. Hahm, K.H. Burrell, Phys. Plasmas 2, 1648 (1995)
[3] H. Biglary et al., Phys. Fluids B 2, 1 (1990)
[4] A. Bondeson et al., Phys. Rev. Lett. 72, 2709 (1994).
[5] E.J. Strait et al., Phys. Rev. Lett. 74, 2483 (1995).
[6] H. Reimerdes et al., Phys. Plasmas 13, 056107 (2006)
[7] E.A. Friedman et al., Phys. Fluids 25, 502 (1982)
[8] D.H.E. Dubin et al., Phys. Fluids B 21, 1 (1983)
[9] W.W. Lee, J. Comput. Phys. 72, 243 (1987)
[10] T.S. Hahm, Phys. Fluids 31, 2670 (1988)
[11] C. Bourdelle et al., Nucl. Fusion 42, 892 (2002)
[12] E.J. Synakowski et al., Plasma Phys. Contr. Fusion 44, A165 (2002)
[13] J. Candy et al., Phys. Rev. Lett. 91, 045001 (2003)
[14] X. Garbet et al., Nucl. Fusion 43, 975 (2003)
[15] D.R. Ernst et al., Phys. Plasmas 11, 2637 (2004)
[16] M. Romanelli et al., Phys. Plasmas 11, 3845 (2004)
[17] J.E. Kinsey et al., Nucl. Fusion 45, 450 (2005)
[18] A.G. Peeters et al., Phys. Plasmas 12, 022505 (2005)
[19] F. Jenko et al., Plasma Phys. Contr. Fusion 47, B195 (2005)
[20] C. Angioni et al., Phys. Plasmas 12, 112310 (2005)
[21] A.G. Peeters et al., Nucl. Fusion 45, 1140 (2005)
[22] A. Bottino et al., Plasma Phys. Contr. Fusion 48, 215 (2006)
[23] T.S. Hahm, Phys. Fluids B4, 2801 (1992)
[24] M. Artun, Phys. Plasmas 1, 2682 (1994)
[25] T.S. Hahm, Phys. Plasmas 3, 4658 (1996)
[26] A.J. Brizard, Phys. Plasmas 2, 459 (1995)
[27] S.P. Hirshman et al., Nucl. Fusion 21, 1079 (1981).
[28] A.G. Peeters, Phys. Plasmas 5, 763 (1998)
[29] A.G. Peeters et al., *The toroidal momentum pinch*, to be submitted to Phys. Plasmas (2007)
[30] J.W. Connor et al., Phys. Rev. Lett. 40, 396 (1978).
[31] M. Kotschenreuther et al., Comput. Phys. Commun. 88, 128 (1995).
[32] J. Weiland, et al., Nucl. Fusion 29, 1810 (1989)

[33] R.E. Waltz et al., Phys. Plasmas 4, 2482 (1997)
[34] W. Dorland et al., Phys. Fluids B 5, 812 (1993)
[35] G.W. Hammett et al., Plasma Physics Contr. Fusion 35, 973 (1993)
[36] M.A. Beer et al., Phys. Plasmas 3, 4046 (1996)
[37] Bruce D. Scott, Phys. Plasmas 12, 102307 (2005)
[38] B. Coppi, Nucl. Fusion 42, 1 (2002)
[39] A.G. Peeters et al., Plasma Phys. Contr. Fusion 48, B413 (2006).
[40] C. Angioni et al., Phys. Rev. Lett. 90, 205003 (2003)
[41] A.G. Peeters et al., Phys. Plasmas 11, 3748 (2004)
[42] R.E. Waltz et al., Phys. Plasmas 2, 2409 (1995)
[43] N. Mattor et al., Phys. Fluids 31, 1181 (1988)
[44] S.-I Itoh, Phys. Fluid B 4, 796 (1992)
[45] R.R. Dominguez et al., Phys Fluid B 5, 3876 (1993)
[46] P.H. Diamond et al., Proceedings of the 15th IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research. (Sevilla 1994) p. 323 (IAEA Vienna 1994)
[47] X. Garbet et al., Phys. Plasmas 9, 3893 (2002)
[48] A.G. Peeters et al., Phys. Plasmas 12, 072515 (2005).
[49] J.E. Kinsey et al., Phys. Plasmas 12, 062302 (2005)
[50] M. Kotschenreuther et al., Phys. Plasmas 2, 2381 (1995)
[51] L.G. Eriksson et al., Plasma Phys. Contr. Fusion 39, 27 (1997)
[52] J.E. Rice et al., Nucl. Fusion 39, 1175 (1999)
[53] I.H. Hutchinson et al., Phys. Rev. Lett. 84, 3330 (2000)
[54] J.E. Rice et al., Nucl. Fusion 44, 370 (2004)
[55] J.S. deGrassie et al., Phys. Plasmas 11, 4323 (2004)
[56] A. Scarabosio et al., Plasma Phys. Contr. Fusion 48, 663 (2006)
[57] D. Nishijima et al., Plasma Phys. Contr. Fusion 47, 89 (2005)
[58] P.C. de Vries et al., Plasma Phys. Control. Fusion 48, 1993 (2006)
[59] C. Angioni et al., "Theoretical understanding of core transport phenomena in ASDEX Upgrade", to be submitted to Nucl. Fusion.