Beyond universality in three-body recombination: An effective field theory treatment

C. Ji\(^{1(a)}\), D. R. Phillips\(^{1,2}\) and L. Platter\(^{3,4}\)

\(^{1}\)Department of Physics and Astronomy, Ohio University - Athens, OH 45701, USA
\(^{2}\)Helmholtz-Institut für Strahlen- und Kernphysik, Universität Bonn - D-53115, Bonn, Germany, EU
\(^{3}\)Department of Physics, Ohio State University - Columbus, OH 43120, USA
\(^{4}\)Institute for Nuclear Theory, University of Washington - Seattle, WA 98102, USA

received 17 May 2010; accepted in final form 21 September 2010
published online 29 October 2010

PACS 34.50.-s – Scattering of atoms and molecules
PACS 21.45.-v – Few-body systems
PACS 03.75.Nt – Other Bose-Einstein condensation phenomena

Abstract – We discuss the impact of a finite effective range on three-body systems interacting through a large two-body scattering length. By employing a perturbative analysis in an effective field theory well suited to this scale hierarchy we find that an additional three-body parameter is required for consistent renormalization once range corrections are considered. This allows us to extend previously discussed universal relations between different observables in the recombination of cold atoms to account for the presence of a finite effective range. We show that such range corrections allow us to simultaneously describe the positive and negative scattering-length loss features observed in recombination with \(^{7}\)Li atoms by the Bar-Ilan group. They do not, however, significantly reduce the disagreement between the universal relations and the data of the Rice group on \(^{7}\)Li recombination at positive and negative scattering lengths.

Introduction. – In the 1970s Vitaly Efimov showed that the non-relativistic three-body system with two-body scattering length, \(a\), much larger than the range of the underlying interaction displays universal properties that are independent of the details of the interaction \([1,2]\). In particular, in the unitary limit \(|a| \rightarrow \infty\), there exists a tower of three-body states (trimers) with a geometric spectrum,

\[ E_T^{(n)} = (e^{-2\pi/s_0})^n n \kappa^2/m, \]

where \(\kappa\) is the binding wave number of the trimer labeled by \(n\), \(m\) is the mass of the particles, and \(s_0 = 1.00624\). Equation (1) and other universal predictions have received significant attention recently in both atomic physics and nuclear physics. In the latter case, the three-nucleon system and certain halo nuclei display features reflecting their status as (effective) three-body systems which are near the unitary limit (for reviews see \([3,4]\)).

In atomic physics, the three-body recombination rate of ultracold atoms is an observable that displays the discrete-scale invariance of Efimov systems: it is proportional to \(a^4\) times a function that is log-periodic in \(a\) \([3]\). Several recent experiments have demonstrated the existence of such Efimov physics by measuring the recombination rate as a function of the scattering length. For example, ref. \([5]\) measured the three-body recombination rate for \(^{7}\)Li atoms. Gross \textit{et al.} found a recombination minimum at a scattering length \(a \equiv a_0 \approx 1160a_B\), as well as a rate enhancement when \(a = a' \approx -264(11)a_B\). (Here and below \(a_B\) denotes the Bohr radius.)

The ratio \(a'/a_0\) is very close to the “universal” result \([3,6]\):

\[ a'_0 = -\exp\left(\frac{(2n + 1)\pi}{2s_0}\right) a_0 \rightarrow -0.21a_0. \]

However, the interactions between \(^{7}\)Li atoms have a natural, van der Waals, length scale \(\sim 100a_B\), so significant corrections to this universal prediction for \(a'_0\) are expected. In particular, the impact of a finite effective range on the scattering-length dependence of Efimov physics has been discussed in two recent papers \([7,8]\). In contrast to those works, here we include the effect of a finite effective range perturbatively and analyze the...
consequences for universal predictions in the three-body sector. We do this by setting up a perturbation theory around the unitary limit, and thus organize the corrections that manifest the differences between different large–scattering-length systems. These are effects beyond universality, and their inclusion allows us to extend the reach of Efimov’s ideas. The small parameters in this effective field theory (EFT) expansion are \( \ell / a \) and \( r_s/a \), where \( k \) denotes the momentum scale of the problem under consideration, and is \( \sim 1 / |a| \) for the recombination processes that are our concern here. This “short-range EFT” applies to all non-relativistic systems in which \( |a| \) is much larger than the range of the underlying interaction \( \ell \). Part of the dynamics at scale \( \ell \) enters observables via the two-body effective range, \( r_s \), and can be straightforwardly accounted for. But, other effects due to the finite range of the inter-atomic force get encoded in new three-body EFT parameters. These must be included in the calculation if it is to contain all effects up to a given order in the EFT. In this way we systematically approximate the dynamics of any finite-range interaction that gives a large scattering length \( a \). The main result of our EFT analysis at next-to-leading order (NLO) is that, if (and only if) scattering-length–dependent observables are considered, one such additional three-body parameter must be included in the calculation in order to guarantee the accuracy of the EFT’s predictions. To demonstrate the implications and limitations of this result, we use this approach to calculate the three-body recombination rate of \(^7\text{Li}\) in the hyperfine states relevant to the experiments performed by Pollack et al. [9] and Gross et al. [5].

**Effective field theory.** – EFTs are a standard tool for calculating low-energy observables in systems with a separation of scales. Here we will consider EFT for particles interacting solely via short-range interactions. Another successful example of an EFT is chiral perturbation theory (χPT) that identifies pions as the Goldstone bosons of low-energy QCD (see ref. [10] for a recent review). At the heart of every EFT is a Lagrangian, which contains all possible operators allowed by the underlying symmetries. The short-range EFT applies to non-relativistic particles with a large scattering length and contains only contact interactions. In this EFT we have, at leading order in \( r_s / |a| \) (or, equivalently for our purposes, \( \ell / |a| \)) expansion

\[
\mathcal{L} = \bar{\psi} \left( i \partial_t - \frac{\nabla^2}{2m} \right) \psi - \frac{C_0}{2} (\bar{\psi} \psi)^2 - \frac{D_0}{6} (\bar{\psi} \psi)^3 + \ldots, \tag{3}
\]

with \( \psi \) our matter fields, and \( C_0 \) a two-body parameter fixed by the physical scale \( a \). At leading order the two-body amplitude is obtained by summing all two-body diagrams that contain only the four-boson operator proportional to \( C_0 \). Requiring that the two-body amplitude has the physical scattering length leads to a renormalization condition for \( C_0 \):

\[
\frac{1}{C_0} = \frac{m}{4\pi a} \frac{m\Lambda}{2\pi^2}, \tag{4}
\]

if cutoff regularization with a cutoff \( \Lambda \) is employed.

To go beyond leading order an ordering scheme derived from the requirement that each new order of the calculation gives contributions to observables that scale with powers of \( r_s / |a| \) or \( kr_s \) is required [11,12]. This extends eq. (3) to a predictive framework in which a well-defined class of terms has to be evaluated at every order in this small-parameter expansion.

**The three-body system.** – In refs. [13,14] Bedaque et al. showed that this EFT is well suited to describing the three-body problem near the unitary limit, and facilitates derivation of Efimov’s results. In particular, they showed that a three-body parameter \( D_0 \) must be present at LO in order to obtain renormalized quantities.

This result was obtained by using the Lagrangian above to derive an integral equation for atom-dimer scattering. The S-wave projected amplitude, \( A_0 \), for scattering from the relative-momentum state \( k \) into the relative-momentum state \( p \), at energy \( E \), is given by

\[
A_0(p, k; E) = \frac{8\pi}{apk} \left[ \ln \left( \frac{p^2 + pk + k^2 - mE}{p^2 - pk + k^2 - mE} \right) + 2H_0(\Lambda)pk \right] \\
+ \frac{2}{\pi} \int_0^\Lambda dq \frac{q}{p} \left[ \ln \left( \frac{p^2 + pq + q^2 - mE}{p^2 - pq + q^2 - mE} \right) + 2H_0(\Lambda)pq \right] \\
\times \frac{A_0(q, k; E)}{-1/a + \sqrt{3q^2/4 - mE - ie}} \tag{5}
\]

with \( e \) a positive infinitesimal. This equation (with \( H_0 = 0 \)) is known as the Skorniakov–Ter-Martirosian (STM) equation [15]. The additional term \( \sim H_0(\Lambda) \) can be thought of as a three-body force. It is proportional to \( D_0/C_0^2 \), and in general is a function of the integral-equation cutoff. The value of \( H_0 \) is fixed by reproducing the value of one three-body observable, e.g. the binding energy of a particular three-body bound state at some fixed scattering length. This relates \( H_0 \) to a physical scale in the three-body system. A common choice for this scale is \( \kappa_s \), which can be thought of as the binding momentum of one three-body bound state in the limit \( |a| \to \infty \). Once \( \kappa_s \) is fixed, LO predictions follow for all the other three-body observables, including observables at other scattering lengths that satisfy \( |a| > \ell \).

For \( p/|a|, p/k \gg 1 \), the asymptotic form of the amplitude \( A_0(p, k) \) is \( \sim p^{1/2} \kappa_s^{-1} \). Corrections are suppressed by powers of \( 1/(|a|p) \) or \( \ell p \) [16]:

\[
A_0(p, k) = \mathcal{N}(k/\Lambda) \left\{ \frac{1}{p} \sin \left[ s_0 \ln \left( \frac{p}{\kappa_s} \right) \right] + \frac{8|C_{-1}|}{ap^2} \ln \left[ s_0 \ln \left( \frac{p}{\kappa_s} \right) + \arg(C_{-1}) \right] + \ldots \right\}. \tag{6}
\]
Here $N(k/\Lambda)$ denotes a normalization factor and $C_{-1}$ is a complex number that can be obtained from the Mellin transform of the asymptotic form of the kernel of the integral equation in eq. (5) [16]. The need for a three-body contact interaction at leading order implies that—at fixed $a$—all observables in the three-body sector can be described by one-parameter correlations. For example, at fixed scattering length, particle-dimer scattering is correlated with the three-body binding energy. Mapping out such a correlation corresponds to varying $\kappa_s$, or $H_0$, keeping $C_0$ fixed. One can, however, also vary the two-body contact interaction and thus change $a$, keeping the three-body counterterm fixed. This is what is expected to happen in atomic systems close to a Feshbach resonance. There, a small variation in the magnetic field changes $a$.

In ref. [17] one term stemming from two-body scattering was considered. Experiments with ultracold atoms to a problem encountered in the application of $\chi$PT to nuclear physics. In this description of low-energy nuclear processes using only baryonic and pionic degrees of freedom, the small scale analogous to $1/a$ is the pion mass $m_{\pi}$, and operators proportional to powers of the pion mass are a mandatory part of expressions for low-energy nuclear observables. For example, at $O(m_{\pi}^2)$, the nucleon-nucleon ($NN$) potential calculated in $\chi$PT can be written as

$$V = [C_S + D_S m_{\pi}^2] + [C_T + D_T m_{\pi}^2] \sigma_1 \cdot \sigma_2 + V_{OPE} + V_{TPE}^{(2)},$$

where the first term comes from the NLO renormalization of the dimer field. Inserting eq. (6) into the integral in the first line of this expression makes it clear that the term $\sim H_1$ on the second line has to absorb two independent divergence structures, one of which is proportional to $1/a$. Therefore we must write

$$H_1(\Lambda) = H_{10}(\Lambda) + \frac{1}{a} H_{11}(\Lambda).$$

Considering only fixed scattering length does not introduce an additional three-body parameter, since the total $H_1(\Lambda)$ is fixed by the renormalization condition that the NLO amplitude has zero effect on the observable used for renormalization at leading order. However, if we desire predictions at NLO for observables as a function of $a$ we need to know the relative sizes of $H_{11}$ and $H_{10}$.

The existence of such an $a$-dependent three-body force can be understood by considering the Lagrangian shown in eq. (3). This Lagrangian is constructed by writing down all terms allowed by the symmetries of the system. This includes terms proportional to powers of the small scales inherent to the problem: in our case, $1/a$. Beyond LO the three-body term in eq. (3) must be augmented to

$$D_0(\psi^\dagger \psi)^2 + \frac{D_{11}}{a} (\psi^\dagger \psi)^3 + \frac{D_{22}}{a^2} (\psi^\dagger \psi)^4 + \cdots.$$  

This exemplifies the benefit of any EFT: all operator structures allowed by the underlying symmetries will contribute at an order in the small-parameter expansion that can be determined by careful analysis. It furthermore relates experiments with ultracold atoms to a problem encountered in the application of $\chi$PT to nuclear physics. In this description of low-energy nuclear processes using only baryonic and pionic degrees of freedom, the small scale analogous to $1/a$ is the pion mass $m_{\pi}$, and operators proportional to powers of the pion mass are a mandatory part of expressions for low-energy nuclear observables. For example, at $O(m_{\pi}^2)$, the nucleon-nucleon ($NN$) potential calculated in $\chi$PT can be written as

$$V = [C_S + D_S m_{\pi}^2] + [C_T + D_T m_{\pi}^2] \sigma_1 \cdot \sigma_2 + V_{OPE} + V_{TPE}^{(2)},$$  

where $V_{OPE}$ is one-pion exchange and $V_{TPE}^{(2)}$ is the “leading” two-pion exchange contribution to the $NN$ interaction. Both $C_{S/T}$ and $D_{S/T}$ are required for consistent renormalization [18]. The relative values of these two are not required for the calculation of observables measured in the laboratory, i.e. at fixed $m_{\pi}$. However, as lattice QCD attempts to predict and explain few-hadron properties, the determination of pion-mass–dependent coefficients becomes a crucial feature of future progress [19].

**Three-body recombination.**

The lithium-7 system. The new three-body counterterm $H_{11} (\equiv D_{11})$ is only relevant if systems with a variable scattering length are considered. Experiments with...
denotes an insertion of the NLO three-body force, $H_1$. The crossed double line denotes an insertion of the LO three-body amplitude, $A_0$, and the square denotes an insertion of the NLO three-body force, $H_1$.

Ultracold atoms are therefore the ideal place to explore its impact. Here we focus on $^7\text{Li}$, where we have information on how both the scattering length and the effective range vary with magnetic field, $B$. The dependence of $a$ and $r_s$ on the magnetic field as presented in ref. [5] is shown in fig. 2 for two different hyperfine states [20].

We will use our EFT, with this two-body input, to discuss two recent experiments which measured three-body recombination rates in systems of $^7\text{Li}$ atoms. First, Gross et al. have measured the three-body recombination of atoms in the $|F = 1\ m_F = 0\rangle$ hyperfine state [5]. They found a recombination minimum at $a > 0$ and a recombination maximum at $a < 0$ whose relative positions are quite well described by universal ($r_s = 0$) predictions. Second, Pollack et al. have measured the recombination rate of atoms in the $|F = 1\ m_F = 1\rangle$ state and found several recombination features associated with few-body universality [9]. In these data there was a systematic deviation by a factor of two from the universal prediction in the ratios of features on the positive and negative scattering-length sides of the Feshbach resonance.

Using the diagrammatics discussed above we perform an EFT calculation of the three-body recombination rate into dimers with binding energy $\sim 1/ma^2$, for positive scattering length. Since our calculation does not include dimers bound by $\sim 1/m\ell^2$, we cannot compute the recombination rate for negative scattering length. We can, however, determine the position of recombination maxima at $a < 0$, by calculating the scattering lengths for which the binding energy of a trimer becomes zero.

The recombination length, $\rho_3$, is obtained from the LO and NLO elastic atom-dimer scattering amplitudes, $A_0$ and $A_1$, via (in units where $\hbar = 1$):

$$\rho_3 = 2\sqrt{\frac{1}{\gamma}(A_0 + A_1)}\left(0, \frac{2\gamma}{\sqrt{3}}T \! 0\right), \quad (12)$$

with $\gamma = \sqrt{mB_2}$ and $B_2$ the binding energy of the atom-atom dimer ($\gamma = 1/a$ at LO). Figure 3 shows our results for $\rho_3$ as a function of the scattering length $a$ for the $|F = 1\ m_F = 0\rangle$ hyperfine state. The dashed line denotes the LO result renormalized to the recombination maximum $a'_0$ on the $a < 0$ side. This LO calculation predicts $a'_0 > 1200a_B$, c.f. the measured $a'_0 \approx 1160a_B$. Because, as discussed above, consistent renormalization at NLO requires us to choose two different three-body observables to fix the counterterms $H_{90} + H_{10}$ and $H_{11}$, we can describe both the position of this recombination minimum and that of the observed maximum at $a < 0$. The solid line displays that NLO result, renormalized to $a'_0$ and the recombination minimum $a''_0$ determined by Gross et al. [5]. The shaded area denotes the region where $|r_s/a| > 0.5$ and convergence of the EFT expansion is expected to be slow. The squares give the experimental data with the corresponding errors. We emphasize that we did not strive for a detailed reproduction of the experimental data here, since our predictions account for neither the effects of deep dimers nor those of finite temperature. The inclusion of such physics improves the overall agreement between experiment and theory. To demonstrate this we also present the LO result with deep-dimer effects included: it is the dot-dashed line in fig. 3. This shows that the positions of the loss features which we are focusing on in this letter are not affected by recombination into deep dimers. They are also not affected by finite temperature [21].

It is gratifying that we can explain ref. [5]'s measurements of both $a'$ and $a''_0$ and still obtain results consistent with the EFT expansion, but neither of these is an NLO prediction of the EFT. We now use our NLO calculation to predict the scattering length at which the atom-dimer resonance occurs, $a_*$, i.e. the atom-atom scattering length.

---

Fig. 1: Diagrams for the NLO amplitude of atom-dimer scattering. The crossed double line denotes an insertion of the range propagator, $D_1(|q|; q_0)$, the shaded blob denotes an insertion of the LO three-body amplitude, $A_0$, and the square denotes an insertion of the NLO three-body force, $H_1$.

Fig. 2: (Color online) Magnetic-field dependence of the scattering length and effective range in the hyperfine states relevant to the experiments by Gross et al. (upper panel) and Pollack et al. (lower panel). The solid lines denote the scattering length and the dashed lines denote the effective range.
The three-body recombination length of $^7\text{Li}$ in the $|F = 1\rangle$ magnetic substate is a function of the scattering length. The dashed line denotes the LO result where the three-body parameter is determined by the recombination maximum that has been measured experimentally in ref. [5]. The dot-dashed curve is the LO result including deep-dimer effects with fitting parameters as in ref. [5]. The solid line denotes our NLO result (which does not include deep dimers).

The dot-dashed curve is the LO result including deep-dimer effects with the fitting parameters as in ref. [5]. The solid line is our NLO result with the second three-body parameter adjusted to the second recombination maximum at $a^{(2)}_\ast = -298a_B$.

We have performed a similar analysis for the results obtained by the Rice group in ref. [9]. In fig. 4 we show our results for the recombination length of $^7\text{Li}$ atoms in the $|F = 1\rangle$ magnetic substate as a function of the scattering length. The LO result (dashed line) is renormalized to the first recombination maximum $a^{(1)}_\ast = -6301a_B$ on the $a < 0$ side. The solid line shows the NLO result that is additionally renormalized to the second measured recombination maximum $a^{(2)}_\ast = -298a_B$. Having adjusted both three-body parameters, we can use them to calculate other observables such as

$$\kappa_\ast = (0.242 \pm 0.030) \cdot 10^{-3} a_B^{-1},$$

and the position of the atom-dimer resonance

$$a_\ast = (355.8 \pm 55.5)a_B.$$  

In the latter case a shift upward from the LO prediction $a_\ast^{(LO)} = 295a_B$ is seen. We can also predict the position of the second recombination minimum in fig. 4 to be

$$a_{\ast 0} = (1348 \pm 151)a_B.$$  

The errors in eqs. (16)–(18) were obtained by propagating the systematic and statistical errors quoted in ref. [9]. The shift of the central value in (18) from the LO result is only $26a_B$, and so range effects $\sim r^2$ and higher are certainly much smaller than the uncertainty due to the experimental input. The EFT result (18) therefore disagrees with the experimental result obtained in ref. [9],

$$a_{\ast 0} = (2676 \pm 67 \pm 128)a_B,$$

by a factor of two, even after range corrections are included. (In eq. (19) the first error is statistical and the second due to a systematic uncertainty in the determination of the atom-atom scattering length.) Effects due to the finite effective range are therefore not responsible for the disagreement between the data of ref. [9] pertaining to different sides of the Feshbach resonance and the predictions of universality.

**Conclusion.** We have shown that at next-to-leading order in the EFT for systems with large scattering length an additional three-body input is required to describe physical observables as a function of the two-body scattering length. This parameter is needed for a complete
NLO description of such observables if $|r_s| \sim \ell$. It encodes the effect of dynamics at distances comparable to the range of the underlying two-body force on the scattering-length dependence of quantities in the three-body system. The necessity of this operator for renormalization exemplifies the general tenet of all EFTs, that all operators which are allowed by the symmetries of the system will appear in the EFT Lagrangian. An expansion of the short-distance physics around the unitary limit (i.e. in powers of $1/a$) makes clear that such an operator should be present at NLO in the short-range EFT. In this respect $1/a$ in this EFT plays a similar role to that of the pion mass in $\chi$PT.

Here we have analyzed the typical situation where the interaction range and the two-body effective range are of the same size: $\ell \sim |r_s|$. We expect the impact of our additional three-body counterterm to be smaller if $\ell \ll |r_s|$, i.e. the effective range is unnaturally large. This occurs, for example, close to a narrow Feshbach resonance. In that case the results of ref. [7] should hold. It is possible that the iteration of $r_s$ to all orders can be systematically justified in such systems. In that case a three-body counterterm would not be required at LO of the EFT calculation [22] and corrections to that “resummed LO” would scale as $r_s/a$ and $kr_s$.

We have shown that our approach can be applied to three-body recombination provided that the effective range is known. In the case of $^7$Li we calculated the shift in recombination features due to an effective range that is a function of the magnetic field. We found that deviations from universal predictions in the Bar-Ilan experiment can be explained by these effects but that they are not sufficient to explain the inconsistencies across the resonance in the experiment carried out by the Rice group.

The data of ref. [9] is not consistent with $\sim r_s$ corrections to universality in spite of the presence of the extra three-body parameter at that order in our calculation. It has been suggested that this apparent violation of universality is a result of the conditions in the Rice experiment. For positive $a$ the Rice group used a BEC, while for negative $a$ they used a thermal gas. In contrast, the Bar-Ilan group recently repeated the Rice $^7$Li experiment in the $|F = 1, m_F = 1\rangle$ state using a thermal gas on both sides of the Feshbach resonance. They found that features across the resonance were related by universality [23].

Our results show that effects proportional to $r_s$ correct the universal relations displayed, e.g. in ref. [3] in a way that improves the agreement with data in the case of $^7$Li atoms [5]. It would be very interesting to apply this framework to data on three-body recombination of $^{133}$Cs atoms. The existing data there, though, is rather different, since the features observed in experiments with $^{133}$Cs at $a > 0$ and $a < 0$ are connected through the region where $a = 0$ [24]. In the vicinity of this scattering-length zero $r_s/a$ diverges, and higher-order corrections to the short-range EFT cannot be reliably calculated. The connection between $a_s$ [25], $a'_s$, and recombination minima at $a > 1000a_B$ can, however, be addressed within EFT.

Given information on how the $^{133}$Cs two-body scattering length and effective range vary with magnetic field, we can make quantitative predictions for the impact of the range on three-body recombination features seen in experiments with cold gases of cesium atoms.

***

We thank E. Braaten for helpful discussions. We acknowledge the INT program “Simulations and Symmetries: Cold Atoms, QCD, and Few-Hadron Systems”, during which this work was completed. This research was supported in part by the DOE under grants DE-FG02-00ER41132, DE-FG02-93ER40756 and DE-FC02-07ER41457, by the NSF under grant PHY-0653312, and by the Mercator programme of the DFG.

REFERENCES

[1] Efimov V., Phys. Lett. B, 33 (1970) 563.
[2] Efimov V., Nucl. Phys. A, 210 (1973) 157.
[3] Braaten E. and Hammer H.-W., Phys. Rep., 428 (2006) 259.
[4] Platter L., Few-Body Syst., 46 (2009) 139.
[5] Gross N., Shotan Z., Kokkelmans S. and Khaykovich L., Phys. Rev. Lett., 103 (2009) 163202.
[6] Helfrich K., Hammer H.-W. and Petrov D. S., Phys. Rev. A, 81 (2010) 042715.
[7] Platter L., Ji C. and Phillips D. R., Phys. Rev. A, 79 (2009) 022702.
[8] Thogersen M., Fedorov D. V., Jensen A. S., Esry B. D. and Wang Y., Phys. Rev. A, 80 (2009) 013608.
[9] Pollack S. E., Dries D. and Hulet R. G., Science, 326 (2009) 1683.
[10] Bijnens J., Prog. Part. Nucl. Phys., 58 (2007) 521.
[11] Beane S. R., Bedaque P. F., Haxton W. C., Phillips D. R. and Savage M. J., in At The Frontier of Physics, edited by Shifman M. (World Scientific, Singapore) 2001, pp. 133–269, nucl-th/0008064 (2000).
[12] Bedaque P. F. and van Kolck U., Annu. Rev. Nucl. Part. Sci., 52 (2002) 339.
[13] Bedaque P. F., Hammer H.-W. and van Kolck U., Phys. Rev. Lett., 82 (1999) 463.
[14] Bedaque P. F., Hammer H.-W. and van Kolck U., Nucl. Phys. A, 646 (1999) 444.
[15] Skorniakov G. and Ter-Martirosian K., Sov. Phys. JETP, 4 (1957) 648.
[16] Bedaque P. F. et al., Nucl. Phys. A, 714 (2003) 589.
[17] Hammer H.-W. and Mehen T., Phys. Lett. B, 516 (2001) 353.
[18] Kaplan D. B., Savage M. J. and Wise M. B., Nucl. Phys. B, 478 (1996) 629.
[19] Beane S. R., Orginos K. and Savage M. J., Int. J. Mod. Phys. E, 17 (2008) 1157.
[20] Kokkelmans S., private communication.
[21] Braaten E., Kang D. and Platter L., Phys. Rev. A, 75 (2007) 052714.
[22] Petrov D. S., Phys. Rev. Lett., 93 (2004) 143201.
[23] Gross N., Shotan Z., Kokkelmans S. and Khaykovich L., Phys. Rev. Lett., 105 (2010) 103203.
[24] Kraemer T. et al., Nature, 440 (2006) 315.
[25] Knoop S. et al., Nat. Phys., 5 (2009) 227.