Cosmic Acceleration from Elementary Interactions

R. Aldrovandi\textsuperscript{1*}, R. R. Cuzinatto\textsuperscript{1†} and L. G. Medeiros\textsuperscript{1‡}

\textsuperscript{1} Instituto de Física Teórica, Universidade Estadual Paulista. Rua Pamplona 145, CEP 01405-000, São Paulo, SP, Brazil

It is possible to generate an accelerated period of expansion from reasonable potentials acting between the universe particle constituents. The pressure of primordial nucleons interacting via a simple nuclear potential is obtained via Mayer’s cluster expansion technique. The attractive part of the potential engenders a negative pressure and may therefore be responsible for the cosmic acceleration.

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I. INTRODUCTION

Cosmic primordial acceleration is usually derived from a scalar field. The idea of using a scalar field arose in the context of GUT’s \cite{1}, at first with the Higgs boson field in mind. This attribution has been dropped in the long run, as the Higgs field did not reproduce all necessary features required by the inflationary period \cite{2}. As a consequence the inflaton – an \textit{ad hoc} scalar field leading to the desired properties – was introduced. This arbitrary field, however, has no simple interpretation in terms of fundamental physical phenomena — which justifies the search for alternative explanations.

We intend here to show that primeval acceleration may come from the strong interactions in the constituents’ equation of state. Such short range interactions are taken into account via the well–known and systematic method of cluster expansions \cite{3}.

An application to cosmology of a toy-model equation of state (EOS) for systems with interaction has been made in \cite{4}, where the elementary constituents of the universe were taken to be hard-spheres. A recent paper \cite{5} has proposed an explanation for the inflationary period through a short range interaction, just what we shall do. Nevertheless, the modification on the equation for the pressure was not obtained through the Mayer method here employed, which brings to light a time scale determined by the temperature and leads to a natural passage from the accelerated period to the decelerated one.

In section \textbf{II} we synthesize the main characteristics of our model, specifying the relevant particles of the matter content. Section \textbf{III} deals with the construction of the equation of state for the interacting media using Mayer’s approach of virial cluster expansion. The nuclear interaction is modeled by the effective potentials presented in section \textbf{IV} The results for our simplistic model point to a period of accelerated expansion and are discussed in section \textbf{V}.

\textsuperscript{*}Electronic address: ra@ift.unesp.br
\textsuperscript{†}Electronic address: rodrigo@ift.unesp.br
\textsuperscript{‡}Electronic address: leo@ift.unesp.br

II. ON THE PRE-NUCLEOSYNTHESIS PERIOD

The pre-nucleosynthesis period is the stage immediately preceding the cosmological formation of the light elements (deuteron, He\textsuperscript{3}, etc). This formation only started when the universe mean energy attained values around the deuteron binding energy ($E_B \simeq 2.23 MeV$). So, we roughly characterize this period by the red-shift $z \gtrsim 2 \times 10^{12}$ or, in terms of energy, $kT \gtrsim 4 MeV$.

Radiation dominates the universe content in this phase. Photons generate a panoply of particles via pair production, with populations dependent on the energy of the thermalized system \cite{6,7}. In particular, at hundreds of $MeV$ nucleons produced by this process are in chemical equilibrium \cite{8}. At these energies the numerical density $n_N$ of nucleons deriving from the radiation is much superior to that ($n_b$) of the present-day nucleons, which can be consequently neglected.

A realistic scenario takes into account, besides the nucleons, light particles appearing even before the nucleons in the pair production process. Examples are pions, electron-positron pairs, muons. For simplicity, we will adopt as relevant content photons, nucleons and anti-nucleons (protons and anti-protons, neutrons and antineutrons). We will neglect electrons and positrons, pions, neutrinos, muons and anti-muons.

The presence of charged particles would require the consideration of the electromagnetic interaction among the constituents as a source for the gravitational field. However, $n_N \gg n_b$ implies symmetry between matter and anti-matter. We shall suppose overall Debye screening: in large enough scales, the system will be electrically neutral. Once the electromagnetic interaction is neglected, we are left with the strong and weak interactions. Obviously, the weak nuclear interaction is much less intense compared with the strong one: the strength of the former is $10^{-13}$ orders of magnitude of the last \textbf{10}. This justifies the approximation we will adopt: our equation of state for the pre-nucleosynthesis period considers only the strong nuclear interaction. In short, our model $(\gamma + N\bar{N})$ is constituted by photons plus interacting nucleons treated as classical non-relativistic particles obeying Boltzmann statistics.
III. EOS OF A SYSTEM UNDER INTERACTION

With the hypothesis introduced above, the equation of state of the system composed by photons (γ) and nucleons/antinucleons (N/¯N) will have the form:

\[ p(kT, n_N) = p_{\gamma}(kT) + p_N(kT, n_N), \]  

\[ (1) \]

where \( kT \) is the energy and \( n_N \) is the nucleon+antinucleon numerical density.

We shall approach the calculation of the strong interaction between nucleons by Mayer’s method [3]. In this formalism, the pressure of a classic gas of interacting particles is calculated through the virial expansion:

\[ p_N(kT, n_N) = n_N kT \sum_{l=1}^{\infty} a_l(T) \left( \frac{n_N \lambda^3}{g} \right)^{l-1}, \]  

\[ (2) \]

\( a_l(T) \) being the virial coefficients, viz.

\[ a_1 = 1; \quad a_2 = -2\pi \int_0^{\infty} (e^{-u(r)/kT} - 1)r^2 dr; \]  

\[ (3a) \]

\[ a_3 = -\frac{1}{3\lambda^6} \int_0^{\infty} f_{12}f_{13}f_{23}d^3r_{12}d^3r_{13}; \]  

\[ (3b) \]

and so on, given in terms of the mean thermal wavelength \( \lambda = \hbar/\sqrt{2\pi m_N kT} \) of the nucleons, and of the two-particle Mayer function \( f_{ij} = e^{-\beta u_{ij}} - 1 \) which depends on \( \beta = 1/kT \) and on the inter-particle potential \( u_{ij} = u(|\vec{r}_i - \vec{r}_j|) \). The second virial coefficient \( a_2 \) in (3a) reflects the interaction of the particles two by two (the 2-cluster); and \( a_3 \) accounts for the interaction among three particles. \( g \) is the number of possible values for the internal degrees of freedom. Four species are to be taken into account (protons, anti-proton, neutron and antinucleon). Besides, our species have spin \( \frac{1}{2} \); \( g = 4 \times 2 = 8 \).

Under our assumptions, only short-range nuclear forces will be at work. It is consequently reasonable to admit that interaction occurs only between few particles, and that \( p_N \) as given by (2) can be conveniently approximated by its first three terms. This hypothesis is reasonable as long as the density is not too high. Substituting (2) in the equality (1) leads to the EOS

\[ p(n_N, kT) = \frac{\pi^2}{45 \lambda^6} (kT)^4 + n_N kT + \right. \]

\[ + n_N kT \left. \left[ a_2(kT) \left( \frac{n_N \lambda^3}{g} \right) + a_3(kT) \left( \frac{n_N \lambda^3}{g} \right)^2 \right] \right]. \]  

\[ (4) \]

The first term, \( p_{\gamma}(kT) \), comes from the black-body thermodynamics.

The natural time parameter along the thermalized period predating recombination is just \( kT \). To examine the history of Eq. (4), it would be convenient to express it in terms of this parameter. This requires the determination of \( n_N = n_N(kT) \). Supposing equality between the nucleon and antinucleon concentrations and masses, the numerical density will be

\[ n_N(kT) = \frac{g}{\lambda^3} \left[ e^{\frac{2\pi}{\lambda^3}} - 2a_2 e^{\frac{2\pi}{\lambda^3}} + \right. \]

\[ + \left. 3 \left( \frac{4a_2^2 - a_3}{2} \right) e^{\frac{4\pi}{\lambda}} \right], \]  

\[ (5) \]

where \( \mu_N \) is the nucleon chemical potential.

The system in equilibrium must assure the formation of the nucleons, i.e., \( \mu_N = -m_N c^2 \).

IV. GRAPHICS OF \( p(kT) \) FOR PHENOMENOLOGICAL POTENTIALS

In a simplistic manner, we describe the strong interaction by a nuclear potential of the square-well+hard-core type shown in Figure 1. The choice of this potential is justified by two facts: (i) it shows a behavior qualitatively similar to some successful phenomenological nuclear potentials, e.g., those discussed in [11, 12]; (ii) it makes it possible to calculate analytically the corrections to the ideal case.

The potential parameters are determined from the deuteron binding energy, from its mean square radius — which constrain the well width \( b - c = 1.3 \) fm and its depth \( V_0 = 75.6 \) MeV — and from nucleons scattering data at high energies — which determine the hard core extension \( c = 0.4 \) fm (cf. reference [8]).

Function \( p(kT) \) is obtained by substituting (6) in (4) and using the \( a_2(kT) \) and \( a_3(kT) \) explicit expressions. Its aspect is shown in Figure 2. It exhibits an increasing behavior at low energies (\( kT < 330 \) MeV). This is only natural, since at these energies the nucleons density is still too small to cause a relevant interaction effect. A
sudden change occurs, however, at around 375 MeV. The pressure starts decreasing as the energy rises, becoming negative at around 420 MeV. This surprising behavior is due to the action of the interaction term \(a_2(kT)\) which dominates in [4]. To be more specific: it is the attractive part (negative sector) of the potential in \(a_2(kT)\) that surpasses all the other terms.

It is worth noticing that for energies until 490 MeV the two by two interaction term \((a_2(kT)n_N\lambda^3)\) between nucleons is at least five times larger than the three by three interaction term \((a_3(kT)n_N^2\lambda^3)\). This is a point in favor of truncating the series [2] in the first terms.

V. FINAL REMARKS

As the pressure, the energy density \(\rho_N(kT, n_N)\) may be expressed in terms of a series of type [2]. The method is the same presented in section III and the result is:

\[
\rho(kT, n_N) = \frac{\pi^2}{15\hbar^2c^4}(kT)^4 + n_N \left( m_N c^2 + \frac{3kT^2}{2} \right) + \frac{3}{2} kT^3 n_N^2 \lambda^3 \left( a_2 - \frac{2T}{3} \frac{\partial a_2}{\partial T} \right) + \left( a_3 - \frac{T}{3} \frac{\partial a_3}{\partial T} \right) \frac{n_N^2 \lambda^3}{g},
\]

\[\lambda^3\text{ being, we recall, the nucleon mass.}\]

With the pressure equation [4] and the equality [8], we can rewrite the Friedmann equation

\[
\frac{\dot{a}}{a} = -\frac{4\pi G}{3c^2} (\rho + 3p)
\]

in terms of \(kT\). The behavior of \(\rho + 3p\) is given in Figure 3.

Combined with [7], it shows that positive acceleration occurs from 450 MeV on. A detailed analysis of the Friedmann equations brings forth a discussion in the thermal history of the model with interactions in four distinct periods:

Further improvement would come from taking into account the neglected particles. The nuclear interaction could also be approached in a more sophisticated way, e.g., by using the phenomenological potentials given in refs. [11, 12], introducing quantum corrections a la Bethe-Uhlenbeck [3], adding relativistic corrections, etc.

The idea that an attractive potential between the constituents can lead to accelerated expansion can be tentatively applied to the present day observed acceleration. The scenario is then that of matter interacting through the gravitational potential. The latter being attractive, the expected global result would be a decreasing in the value of the pressure towards negative values, eventually causing the acceleration. Around the same period, but locally, the gravitational potential would become sufficiently effective to engender the large structures in the universe. This is consistent with the observational data, which indicate that both the accelerated expansion [15] and the large scale structure formation [14] have begun recently.

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