Spinor fields in spherically symmetric space-time

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Within the scope of a spherically symmetric space-time we study the role of a nonlinear spinor field in the formation of different configurations with spherical symmetries. The presence of the non-diagonal components of energy-momentum tensor of the spinor field leads to some severe restrictions on the spinor field itself. Since spinor field is the source of the gravitational one, the metric functions also changes in accordance with it. The system as a whole possesses solutions only in case of some additional conditions on metric functions.

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1. INTRODUCTION

In the recent past spinor description of matter and dark energy was used to draw the picture of the evolution of the Universe within the scope of Bianchi type anisotropic cosmological models [1–4]. It was found that the approach in question gives rise to a variety of solutions depending on the choice of spinor field nonlinearity. Thanks to its sensitivity to gravitational field spinor field brings some unexpected nuances in the behavior of both the spinor and the gravitational fields. Taking this in mind in this paper we consider the nonlinear spinor field within the framework of spherically symmetric gravitational field. Since a variety of astrophysical systems such as stars, black holes are described by spherically symmetric configurations, the use of spinor field in this area might be very promising.

2. BASIC EQUATION

Let us consider a system of nonlinear spinor and spherically symmetric gravitational fields. The corresponding action we choose in the form

\[ \mathcal{S}(g; \psi, \bar{\psi}) = \int (L_g + L_{sp}) \sqrt{-g} d\Omega \]  

(2.1)
Here $L_g$ corresponds to the gravitational field

$$L_g = \frac{R}{2\kappa},$$

where $R$ is the scalar curvature, $\kappa = 8\pi G$ with $G$ being Newton's gravitational constant and $L_{sp}$ is the spinor field Lagrangian which we take in the form

$$L_{sp} = \frac{1}{2} \left[ \bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi \right] - m \bar{\psi} \psi - F,$$

with the nonlinear term $F = F(K)$ and $K$ taking one of the following expressions: $\{I, J, I + J, I - J\}$. Here $I = \bar{\psi} \psi$ and $J = i \bar{\psi} \gamma^5 \psi$. Here $m$ is the spinor mass.

The spinor field equations corresponding to the spinor field Lagrangian (2.3) are

$$i \gamma^\mu \nabla_\mu \psi - m \psi - D \psi - i \bar{\psi} \gamma^\mu \psi = 0,$$  

(2.4a)

$$i \nabla_\mu \bar{\psi} \gamma^\mu + m \bar{\psi} + D \bar{\psi} + i \bar{\psi} \gamma^\mu \bar{\psi} = 0,$$  

(2.4b)

where we denote $D = 2SF_KK_I$ and $G = 2PF_KK_J$, with $F_K = dF/dK$, $K_I = dK/dI$ and $K_J = dK/dJ$.

In view of (2.4) it can be shown that

$$L_{sp} = 2KF_K - F.$$

In the above expressions $\nabla_\mu \psi = \partial_\mu \psi - \Gamma_\mu \psi$ and $\nabla_\mu \bar{\psi} = \partial_\mu \bar{\psi} + \bar{\psi} \Gamma_\mu$ with $\Gamma_\mu$ being the spinor affine connection.

The spherically-symmetric metric we choose in the form

$$ds^2 = e^{2\mu} dt^2 - e^{2\alpha} dr^2 - e^{2\beta} \left( d\vartheta^2 + \sin^2 \vartheta d\varphi^2 \right),$$

(2.6)

where the metric functions $\mu, \alpha, \beta$ depend on the spatial variable $r$ only.

The spinor affine connection matrices are defined as

$$\Gamma_\mu(x) = \frac{1}{4} \bar{\gamma}_\rho \sigma(x) \left( \partial_\mu \varepsilon^b \varepsilon^\rho - \Gamma^\rho_\mu \delta \right) \gamma^\sigma \gamma^\delta,$$

(2.7)

where the tetrad $\varepsilon^\rho_b$ correspond to the metric (2.6) we choose as follows:

$$e^{(0)}_0 = e^\mu, \quad e^{(1)}_1 = e^\alpha, \quad e^{(2)}_2 = e^\beta, \quad e^{(3)}_3 = e^\beta \sin \theta.$$

(2.8)

The flat $\gamma$ matrices we choose in the from

$$\bar{\gamma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \bar{\gamma}^1 = \begin{pmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{pmatrix},$$

$$\bar{\gamma}^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}, \quad \bar{\gamma}^3 = \begin{pmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{pmatrix},$$

where

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} \cos \vartheta & \sin \vartheta e^{-i\varphi} \\ \sin \vartheta & -\cos \vartheta \end{pmatrix},$$

$$\sigma^2 = \begin{pmatrix} -\sin \vartheta & \cos \vartheta e^{-i\varphi} \\ \cos \vartheta & \sin \vartheta \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 0 & ie^{-i\varphi} \\ -ie^{i\varphi} & 0 \end{pmatrix}.$$

(2.9)
Defining $\gamma^5$ as follows:

\[
\gamma^5 = -\frac{i}{4} E_{\mu\nu\sigma\rho} \gamma^\mu \gamma^\nu \gamma^\sigma \gamma^\rho, \quad E_{\mu\nu\sigma\rho} = \sqrt{-g} \epsilon_{\mu\nu\sigma\rho}, \quad \epsilon_{0123} = 1,
\]

\[
\gamma^5 = -i\sqrt{-g} \gamma^0 \gamma^1 \gamma^2 \gamma^3 = -i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = \tilde{\gamma}^5,
\]

we obtain

\[
\tilde{\gamma}^5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}.
\]

Taking into account that the functions $\alpha$, $\beta$ and $\mu$ depend on only $r (x^1)$ from (2.7) we find

\[
\Gamma_0 = -\frac{1}{2} \mu' e^{(\mu - \alpha)} \tilde{\gamma}^0 \tilde{\gamma}^1, \quad (2.10a)
\]

\[
\Gamma_0 = 0, \quad (2.10b)
\]

\[
\Gamma_2 = \frac{1}{2} \beta' e^{(\beta - \alpha)} \tilde{\gamma}^2 \tilde{\gamma}^1, \quad (2.10c)
\]

\[
\Gamma_3 = \frac{1}{2} \beta' \sin \theta e^{(\beta - \alpha)} \tilde{\gamma}^3 \tilde{\gamma}^1 + \frac{1}{2} \cos \theta \tilde{\gamma}^3 \tilde{\gamma}^2. \quad (2.10d)
\]

Let us now find the energy-momentum tensor of the spinor field which is given by

\[
T^\rho_\mu = \frac{i}{4} s^{\rho \nu} \left( \bar{\psi} \gamma'_\nu \nabla_\nu \psi + \bar{\psi} \gamma'_\nu \nabla_\nu \psi - \nabla_\mu \bar{\psi} \gamma'_\nu \psi - \nabla_\nu \bar{\psi} \gamma'_\mu \psi \right) - \delta^\rho_\mu L_{sp}
\]

\[
= \frac{i}{4} s^{\rho \nu} \left( \bar{\psi} \gamma'_\nu \partial_\nu \psi + \bar{\psi} \gamma'_\nu \partial_\nu \psi - \partial_\mu \bar{\psi} \gamma'_\nu \psi - \partial_\nu \bar{\psi} \gamma'_\mu \psi \right)
\]

\[
- \frac{i}{4} s^{\rho \nu} \bar{\psi} \left( \gamma'_\mu \Gamma'_\nu + \Gamma'_\nu \gamma'_\mu + i \theta \Gamma'_\nu + \Gamma'_\mu \gamma'_\nu \right) \psi - \delta^\rho_\mu L_{sp}. \quad (2.11)
\]

On account of spinor field equations one finds the following non-trivial and linearly independent terms of the energy-momentum tensor

\[
T_0^0 = F(K) - 2KF_K, \quad (2.12a)
\]

\[
T_1^1 = mS + F(K), \quad (2.12b)
\]

\[
T_2^2 = F(K) - 2KF_K, \quad (2.12c)
\]

\[
T_3^3 = F(K) - 2KF_K, \quad (2.12d)
\]

\[
T_0^1 = \frac{1}{4} \cos \theta e^{(\alpha - \mu - \beta)} A^3, \quad (2.12e)
\]

\[
T_2^0 = -\frac{1}{4} \left( \mu' - \beta' \right) e^{(\beta - \alpha - \mu)} A^3, \quad (2.12f)
\]

\[
T_3^0 = \frac{1}{4} \left( \mu' - \beta' \right) e^{(\beta - \alpha - \mu)} \sin \theta A^2 + \frac{1}{4} e^{-\mu} \cos \theta A^1. \quad (2.12g)
\]

We consider the case when the spinor field depends on $r$ only. Then in view of (2.10) we have

\[
te^{-\alpha} \bar{\gamma}^1 \psi' + \frac{i}{2} \left( \mu' + 2\beta' \right) e^{-\alpha} \bar{\gamma}^1 \psi' + \frac{i}{2} \cos \theta e^{-\beta} \bar{\gamma}^2 \psi' - m \psi - \nabla \psi - i \partial \gamma^5 \psi' = 0, \quad (2.13a)
\]

\[
te^{-\alpha} \bar{\psi}' \gamma^1 + \frac{i}{2} \left( \mu' + 2\beta' \right) e^{-\alpha} \bar{\psi}' \gamma^1 + \frac{i}{2} \cos \theta e^{-\beta} \bar{\psi}' \gamma^2 + m \bar{\psi} + \nabla \bar{\psi} + i \partial \bar{\psi} \gamma^5 = 0. \quad (2.13b)
\]
For the invariants of spinor field from (2.13) we find

\[ S' + (\mu' + 2\beta')S - 2e^{\alpha\beta}A^1 = 0, \quad (2.14a) \]
\[ P' + (\mu' + 2\beta')P + 2e^{\alpha}(m + D)A^1 = 0, \quad (2.14b) \]
\[ A^1' + (\mu' + 2\beta')A^1 + \frac{\cos \theta}{\sin \theta}e^{\alpha-\beta}A^2 + 2e^{\alpha}(m + D)P + 2e^{\alpha}D S = 0, \quad (2.14c) \]
\[ A^2' + (\mu' + 2\beta')A^2 - \frac{\cos \theta}{\sin \theta}e^{\alpha-\beta}A^1 = 0, \quad (2.14d) \]

where \( A^\mu = \bar{\psi}\gamma^\mu \gamma^\mu \psi \) is the pseudovector. The foregoing system gives

\[ \left( SS' - PP' + A^1A^1' + A^2A^2' \right) + (\mu' + 2\beta') \left( S^2 - P^2 + A^{12} + A^{22} \right) = 0, \quad (2.15) \]

with the relation

\[ \left( S^2 - P^2 + A^{12} + A^{22} \right) = C_0 e^{-2(\mu + 2\beta)}, \quad (2.16) \]

On account of (2.12) we find the following system of Einstein equations

\[ (2\mu' \beta' + \beta'^2) - e^{2(\alpha-\beta)} = -\kappa e^{2\alpha} (mS + F(K)), \quad (2.17a) \]
\[ (\mu'^2 + \mu' \beta' - \mu' \alpha' + \beta'^2 - \beta' \alpha' + \mu'' + \beta'') = -\kappa e^{2\alpha} (F(K) - 2KF_K), \quad (2.17b) \]
\[ (3\beta'^2 - 2\beta' \alpha' + 2\beta'') - e^{2(\alpha-\beta)} = -\kappa e^{2\alpha} (F(K) - 2KF_K), \quad (2.17c) \]
\[ 0 = \frac{\cos \theta}{\sin \theta} e^{(\alpha-\mu-\beta)}A^3, \quad (2.17d) \]
\[ 0 = (\mu' - \beta') \left[ e^{(\beta-\alpha-\mu)}A^3 \right], \quad (2.17e) \]
\[ 0 = \left[ (\mu' - \beta') e^{(\beta-\alpha)}A^2 + \frac{\cos \theta}{\sin \theta}A^1 \right]. \quad (2.17f) \]

From (2.17d) we obtain \( A^3 = 0 \) at least everywhere expect \( \theta = \pi/2 \). Hence (2.17e) fulfills identically.

Inserting \( \frac{\cos \theta}{\sin \theta} e^{(\alpha-\beta)}A^1 = - (\mu' - \beta')A^2 \) from (2.17f) into (2.14d) for \( A^2 \) we find

\[ A^2' + (2\mu' + \beta')A^2 = 0, \quad (2.18) \]

with the solution

\[ A^2 = z_2 e^{-(2\mu + \beta)}, \quad (2.19) \]

with \( z_2 \) being some integration constant.

On account of (2.12) from Bianchi identity \( \Gamma^\nu_{\mu;\nu} = 0 \) i.e.,

\[ T^\nu_{\mu;\nu} = T^\nu_{\mu;\nu} + \Gamma^\nu_{\alpha\nu} T^\alpha_{\mu} - \Gamma^\alpha_{\mu\nu} T^\nu_{\alpha} = 0 \quad (2.20) \]

we find

\[ (mS + F)' + (\mu' + 2\beta') (mS + 2KF_K) = 0. \quad (2.21) \]
Let us now consider two different cases. In case of $K = I = S^2$, on account of $F' = 2SF\kappa S'$ from (2.21) we find

\[(m + 2SF\kappa) S' + (\mu' + 2\beta') (mS + 2S^2F\kappa) = 0,\]  
\[(2.22)\]

which leads to

\[S' + (\mu' + 2\beta') S = 0,\]  
\[(2.23)\]

with

\[S = c_1 e^{-(\mu + 2\beta)}.\]  
\[(2.24)\]

In case of $K$ one of \{\(J, I + J, I - J\)\} we consider the massless spinor field, as it was done in cosmology [1, 2]. Then on account of $F' = F\kappa K'$ we rewrite (2.21) as

\[F\kappa K' + 2(\mu' + 2\beta') KF\kappa = 0,\]  
\[(2.25)\]

which leads to

\[K' + 2(\mu' + 2\beta') K = 0,\]  
\[(2.26)\]

with

\[K = c_1^2 e^{-2(\mu + 2\beta)}.\]  
\[(2.27)\]

Thus we conclude that the relations (2.27) holds for a massless spinor field if $K$ takes one of \{\(I, J, I + J, I - J\)\}, whereas it is true for a non-trivial spinor mass, only if $K = I$.

Now we can deal with the zero component of $A^\mu$. From Fierz identity we know

\[S^2 + P^2 = -A_\mu A^\mu = -\left(A^{02} + A^{12} + A^{22} + A^{32}\right),\]  
\[(2.28)\]

Subtraction of (2.28) from (2.16) in view of $A^3 = 0$ leads to

\[A^{02} = -C_0 e^{-2(\mu + 2\beta)} - 2P^2,\]  
\[(2.29)\]

whereas their addition yields

\[A^{02} = C_0 e^{-2(\mu + 2\beta)} - 2\left(S^2 + A^{12} + A^{22}\right).\]  
\[(2.30)\]

Hence all the components of $A^\mu$ can be expressed in terms of metric functions.

Thus the non-diagonal Einstein equations together with the equations for invariants of spinor field give us valuable information about the spinor field. On the other hand Bianchi identity relates the invariants with metric functions. Now we have only three diagonal Einstein equations left.

Before dealing with diagonal Einstein equations let us go back to spinor field equations. As far as the spinor field equation (2.4a) is concerned, denoting $\phi = \psi e^{(\mu + 2\beta)/2}$ it can be rewritten in the following matrix form

\[\phi' = B\phi,\]  
\[(2.31)\]

where
\[
B = \begin{pmatrix}
Y_3 \sigma^1 + iY_1 \sigma^3 & Y_2 \sigma^1 \\
-2Y_2 \sigma^1 & -Y_3 \sigma^1 + iY_1 \sigma^3
\end{pmatrix}, \quad \phi = \col(\phi_1, \phi_2, \phi_3, \phi_4),
\]

(2.32)

and \(Y_1 = (\cos \vartheta / 2 \sin \vartheta) \exp[\alpha - \beta], \ Y_2 = i [m + \omega] \exp \alpha, \) and \(Y_3 = \omega \exp \alpha.\)

As one sees, \(\det B = -Y_2^1 + Y_2^2 - Y_3^2 \neq 0.\) The solution to the equation (2.31) can be written in general form.

Let us now go back to Einstein field equations. In order to solve these equations we need to consider some special cases. In what follows we consider a few of them.

**Case I** Let us assume that

\[
\alpha = \mu + 2\beta.
\]

(2.33)

This type of assumption was consider in a number of papers [5, 6] and known as harmonic condition.

Denoting \(\beta'^2 + \beta' \mu' = U\) we rewrite the diagonal equations of Einstein system, i.e. (2.17a), (2.17b) and (2.17c) as follows:

\[
e^{-2\alpha}U - e^{-2\beta} = -\kappa (mS + F)
\]

(2.34a)

\[
e^{-2\alpha} (\mu'' + \beta'' - U) = \kappa (2KF K - F)
\]

(2.34b)

\[
e^{-2\alpha} (2\beta'' - U) - e^{-2\beta} = \kappa (2KF K - F)
\]

(2.34c)

Subtraction of (2.34c) from (2.34b) gives

\[
\mu'' - \beta'' + e^{2(\mu + \beta)} = 0,
\]

(2.35)

Subtraction of (2.34c) from (2.34a) gives

\[
\beta'' - \beta'^2 - \beta' \mu' = \frac{\kappa}{2} e^{2(\mu + 2\beta)} (mS + 2KF K)
\]

(2.36)

As far as \(F\) is concerned, we may choose it in the form \(F = \lambda K^n,\) where \(\lambda\) is the self-coupling constant. In case of \(K = I = S^2\) we consider a massive spinor field, otherwise massless one.

\(K = I = S^2\) we find the following system of equations

\[
\mu'' - \beta'' = -e^{2(\mu + \beta)}
\]

(2.37a)

\[
\beta'' - \beta'^2 - \beta' \mu' = \frac{\kappa}{2} (c_1 m e^{(\mu + 2\beta)} + 2\lambda n c_1^2 e^{2(1-n)(\mu + 2\beta)})
\]

(2.37b)

The foregoing system we solved numerically. In doing so we have give some concrete value of problem parameters as well as initial conditions. For simplicity we have chosen \(\lambda = 1, n = 1, m = 1, \kappa = 1, c_1 = 1.\) As initial conditions we have set \(\mu(0) = 1, \beta(0) = 1, \mu'(0) = 0, \beta'(0) = 0.\) Here our main aim was to find some solutions which we can use in our following detailed studies. In Figs. 1, 2 and 3 we have plotted the metric functions \(\mu(r), \beta(r)\) and \(\alpha(r),\) respectively.
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FIG. 1: Behavior of $\mu(r)$ for $\lambda = 1, n = 1, m = 1, \kappa = 1, c_1 = 1$ with the initial conditions $\mu(0) = 1, \beta(0) = 1, \mu'(0) = 0, \beta'(0) = 0$

FIG. 2: Behavior of $\beta(r)$ for $\lambda = 1, n = 1, m = 1, \kappa = 1, c_1 = 1$ with the initial conditions $\mu(0) = 1, \beta(0) = 1, \mu'(0) = 0, \beta'(0) = 0$

**Case II** As a second case we consider the widely used model setting $\beta = \ln r$. In this case the diagonal components of Einstein system takes the form

\[
e^{-2\alpha} \left( \frac{2\mu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = -\kappa (mS + F)
\]

(2.38a)

\[
e^{-2\alpha} \left( \mu'^2 + \frac{\mu'}{r} - \mu'\alpha' - \frac{\alpha'}{r} + \mu'' \right) = \kappa (2KF_K - F)
\]

(2.38b)

\[
e^{-2\alpha} \left( \frac{1}{r^2} - 2\frac{\alpha'}{r} \right) = \kappa (2KF_K - F)
\]

(2.38c)

Subtracting (2.38c) from (2.38b) one finds

\[
e^{-2\alpha} (\mu' + \alpha') = -\frac{\kappa r}{2} (mS + 2KF_K).
\]

(2.39)

Inserting (2.39) into (2.38b) we find
FIG. 3: Behavior of $\alpha(r)$ for $\lambda = 1$, $n = 1$, $\kappa = 1$, $c_1 = 1$ with the initial conditions $\mu(0) = 1$, $\beta(0) = 1$, $\mu'(0) = 0$, $\beta'(0) = 0$.

\[
e^{-2\alpha} \left( 2\mu'^2 + 2\frac{\mu'}{r} + \mu'' \right) = \kappa (2KF_{\kappa} - F) - \frac{\kappa r}{2} \left( \mu' + \frac{1}{r} \right) (mS + 2KF_{\kappa}). \tag{2.40}
\]

It should be noted that in this case $S = \left( \frac{c_1}{r^2} \right) e^{-\mu}$. As far as nonlinear term is concerned, as in previous case we consider the massive spinor field with $F = \lambda I^n = \lambda S^{2n}$.

Inserting $F = \lambda I^n = \lambda S^{2n}$ into the equations finally we find the following system

\[
e^{-2\alpha} (\mu' + \alpha') = -\frac{\kappa r}{2} \left( \frac{mc_1}{r^2} e^{-\mu} + \frac{2\lambda nc_1^2}{r^{2n}} e^{-2\mu} \right) \tag{2.41a}
\]

\[
e^{-2\alpha} \left( 2\mu'^2 + 2\frac{\mu'}{r} + \mu'' \right) = \kappa \left( \frac{(2n - 1)\lambda c_1^2}{r^{2n}} e^{-2\mu} \right)
- \frac{\kappa r}{2} \left( \mu' + \frac{1}{r} \right) \left( \frac{mc_1}{r^2} e^{-\mu} + \frac{2\lambda nc_1^2}{r^{2n}} e^{-2\mu} \right). \tag{2.41b}
\]

This system can also be solved numerically to find $\mu$ and $\alpha$. Since in this case the point $r = 0$ leads to singularity, we have to set the initial value at any point except this one. Like in the previous we consider the same problem parameters with the following initial conditions: $\alpha(0.1) = 0.3$, $\mu(0.1) = 0.5$, and $\mu'(0.1) = 0.2$. The behavior of $\mu(r)$ and $\alpha(r)$ are given in the Figs. 4 and 5, respectively. It should be noted that this case was studied in [7].

3. CONCLUSION

Within the scope of spherically symmetric gravitational the the role of nonlinear spinor field in the formation of different configuration is studied. Earlier it was found that the spinor field play a very important rope in the evolution of the Universe. This study can be taken as an attempt to exploit the spinor field in astrophysics. We hope to use these experience and results for the well studied astrophysical objects such as compact stars, black holes, wormholes etc. We plan to discuss some aspects of modern astrophysics exploiting the spinor description of matter in our coming papers.
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FIG. 4: Behavior of $\mu(r)$ for $\lambda = 1, n = 1, m = 1, \kappa = 1, c_1 = 1$ with the initial conditions $\mu(0.1) = 0.5, \alpha(0.1) = 1, \mu'(0.1) = 0.2$

FIG. 5: Behavior of $\alpha(r)$ for $\lambda = 1, n = 1, m = 1, \kappa = 1, c_1 = 1$ with the initial conditions $\mu(0.1) = 0.5, \alpha(0.1) = 1, \mu'(0.1) = 0.2$

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