Single-mode squeezing in arbitrary spatial modes

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Abstract: As the generation of squeezed states of light has become a standard technique in laboratories, attention is increasingly directed towards adapting the optical parameters of squeezed beams to the specific requirements of individual applications. It is known that imaging, metrology, and quantum information may benefit from using squeezed light with a tailored transverse spatial mode. However, experiments have so far been limited to generating only a few squeezed spatial modes within a given setup. Here, we present the generation of single-mode squeezing in Laguerre-Gauss and Bessel-Gauss modes, as well as an arbitrary intensity pattern, all from a single setup using a spatial light modulator (SLM). The degree of squeezing obtained is limited mainly by the initial squeezing and diffractive losses introduced by the SLM, while no excess noise from the SLM is detectable at the measured sideband. The experiment illustrates the single-mode concept in quantum optics and demonstrates the viability of current SLMs as flexible tools for the spatial reshaping of squeezed light.

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OCIS codes: (270.6570) Squeezed states;(230.6120) Spatial light modulators.

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1. Introduction

Squeezed states of the electromagnetic field [1, 2, 3, 4] have been the subject of intense theoretical and experimental study over the past four decades. As a generalisation of Glauber’s coherent states [5] to minimum uncertainty states [6, 7], their experimental realisation [8] has provided a striking confirmation of the quantum theory of light. From an applied point of view, squeezed states allow the limitations imposed by quantum uncertainties on the accuracy of optical measurements to be overcome. It was pointed out early on by Caves [9] that the sensitivity of gravitational wave interferometers can be enhanced by squeezing the vacuum state entering the interferometer’s unused port. In recent years, this idea became reality in the GEO 600 interferometer, where squeezing is currently used to enhance long-term sensitivity by 2.0 dB [10]. In quantum information science, squeezed states are relevant as a resource for continuous-variable (CV) entanglement [11], as well as CV quantum key distribution protocols [12, 13, 14].

Meanwhile, the recognition of light’s transverse spatial degrees of freedom as an information carrier [15] has made spatial modes relevant for optical implementations of quantum information protocols [16, 17, 18]. The consideration of the effects of quantum noise on the spatial statistics of photons also led to the field of quantum imaging [19, 20], and it was found that the displacement and tilt of a laser beam can be measured more accurately by interfering squeezed light with higher-order spatial modes [21]. Finally, it was shown that thermal noise from mirror coatings in gravitational wave interferometers can be reduced by using higher-order Laguerre-Gauss (LG) modes instead of the fundamental mode, allowing for higher optical powers and thus an improved signal-to-noise ratio [22, 23]. Combining this technique with squeezed light within the stringent parameter regime required by gravitational wave interferometry presents a formidable challenge, but would allow the phase sensitivity to increase even further.

Here, we present a proof-of-principle experiment to generate amplitude squeezing in light beams with arbitrary spatial intensity patterns. As a demonstration of the setup’s versatility, we generate squeezed LG and Bessel-Gauss (BG) beams of different orders, as well as a complex pattern containing high spatial frequencies. All modes are generated without modifications to the setup. The presented experiment showcases the possibility of generating practically arbitrary two-dimensional spatial modes that are single-mode squeezed.

Before detailing the experimental setup, we first review the relation between spatial modes and quantum states of light, and describe some of the existing approaches for spatial mode squeezing.

1.1. Quantum states of light and transverse spatial modes

In order to make more precise the relationship between spatial modes of the electromagnetic field and photon statistics, we recall the canonical quantisation of the transverse electromagnetic field, which considers the modes of a finite volume with either periodic or reflecting boundaries (eventually to be taken to infinity). The electric field operator can be expanded as

$$\hat{E}(\vec{r}, t) = \sum_{k} \frac{\hbar \omega_k}{2} \left( \hat{a}_k(\vec{r}, t)\hat{a}_k^\dagger(\vec{r}, t) - \hat{a}_k^\dagger(\vec{r}, t)\hat{a}_k(\vec{r}, t) \right),$$

where $\hat{a}$ and $\hat{a}^\dagger$ are bosonic ladder operators satisfying $[\hat{a}_i, \hat{a}^\dagger_j] = \delta_{ij}$, and the functions $\tilde{u}_k$ denote mutually orthogonal solutions of the Helmholtz equation that describe transverse oscillation of the transverse electric fields and can be directly derived from classical Maxwell equations.
However, the generalised definition of an optical mode permits superpositions of such solutions to be treated as a single-mode excitation as long as the field possesses first-order coherence \[24, 25, 26, 27\]. Thus, as long as the condition of first-order coherence is met, a beam with a complicated spatial structure may be treated as a single-mode excitation of the field.

When a squeezed beam interacts with a diffractive optical element (such as an SLM in the present work), the resulting mode pattern can be determined from the classical theory of diffraction by considering the plane wave spectrum. Diffraction does not affect the quantum statistics of the mode per se. Rather, we observe a reduction of the degree of squeezing since we are no longer able to integrate over the full plane wave spectrum with our detector (i.e., high diffraction orders result in losses) \[28\]. The effect of such losses on the single-mode squeezing can then be found from the beam splitter relation to be \(\text{Var}_{\text{out}} = \eta \cdot \text{Var}_{\text{in}} + (1 - \eta) \cdot \text{Var}_{\text{vac}}\). Here, \(\eta\) is the efficiency, \(\text{Var}_{\text{in}}\) and \(\text{Var}_{\text{out}}\) represent the variances of the input and output beam of the squeezed quadrature, and \(\text{Var}_{\text{vac}}\) is the vacuum variance (shot-noise). Apart from losses induced by imperfect reflectivity and absorption one also has to take into account possible sources of additional classical noise. If the diffractive element were to impose an unwanted temporal modulation at a frequency \(f_N\) (for example, due to electronic flicker noise in the case of a liquid crystal SLM), the quantum noise of the detected light mode would be masked by excess noise at the optical sidebands at \(f_0 \pm f_N\), rapidly degrading the observable squeezing.

There are various studies of nonclassical beams in a multimode setting \[29, 30, 31, 32, 33, 34\]. Here, we concentrate on single-mode squeezing, which is particularly suited for applications such as quantum-enhanced interferometry.

1.2. Existing experimental approaches

Two main approaches to squeezing a single spatial mode have been demonstrated so far:

1. **Reshaping**, where a squeezed fundamental TEM\(_{00}\) mode is generated first and subsequently converted into the desired spatial mode. Any conversion loss necessarily reduces the squeezing from the initial value. This has been achieved with phase plates \[21, 35, 36\], where the wavelength and designated mode are fixed, or with special-purpose liquid crystal devices \[37\], which allow more flexibility in the choice of wavelength and mode parameters. Another approach uses programmable adaptive optics, for which, although capable in principle of generating any mode, squeezing has so far only been demonstrated in 1D with Hermite-Gauss HG\(_{0n}\) modes \[38\].

2. **Direct squeezing**, where the nonlinear medium is either resonant for the desired spatial mode, or transmissive, as in the case of a traveling-wave, or single-pass scheme, a squeezed spatial mode can be generated directly. Examples include misaligned OPO cavities \[39, 40\] and photonic crystal fibers \[41\]. The multi-mode squeezing mentioned above can be achieved with this approach when the nonlinear medium does not enforce a particular spatial mode.

In this work we take the reshaping approach, using an asymmetric fiber Sagnac interferometer as a squeezing source for TEM\(_{00}\) modes \[42\] and a spatial light modulator for the subsequent mode conversion.

2. Experimental setup

2.1. Squeezing

Our light source is a shot-noise limited laser (Origami, Onefive GmbH) emitting linearly polarised light in 220fs pulses, centered at a wavelength of \(\lambda_0 = 1558\) nm. Fig. 1 shows the asymmetric Sagnac interferometer used to generate amplitude squeezed light in the initial Gaussian
mode. The laser beam is split on an asymmetric beam splitter with a splitting ratio of 90:10. This results in a strong and a weak pulse counter-propagating in the polarisation-maintaining single-mode fiber (FS PM 7811 by 3M). Due to the fiber’s nonlinear Kerr effect, a quadrature squeezing is achieved in the bright pulse that, by means of the counter-propagating weak pulse, is adjusted to occur in the amplitude quadrature [42].

2.2. Mode Conversion

The squeezed Gaussian beam, having a waist of \( w_0 = 1.32 \text{ mm} \), is converted into a higher-order spatial mode by a reflective liquid-crystal-on-silicon spatial light modulator (LCoS-SLM, Pluto, Holoeye Photonics AG, 1920x1080 pixels, display optimised for 1550nm, no anti-reflection coating). This SLM is designed for phase-only modulation and does not directly modulate the amplitude. The local refractive index is modulated due to the preferential alignment of the rod-shaped LC molecules with the electric field at each pixel. This affects only the polarization component along the long axis of the LC molecules, leaving the orthogonal polarization component unmodulated.

We program our SLM with phase patterns consisting of four contributions: First, the transverse phase pattern of the theoretical mode function of the desired mode as described later in this section. Second, a blazed grating phase which diffracts the modulated beam away from the zeroth order and transfers the energy mostly into the first diffraction order. This step is required to spatially separate the modulated light from the approximately 20% of incoming light which the SLM effectively does not modulate due to its limited diffraction efficiency. The grating period of \( 35 \text{ px} \times 8 \mu\text{m/px} \approx 180 \lambda_0 \) is chosen empirically to maximise diffraction into the first order while enabling sufficient transverse separation from other diffraction orders in the detection plane at a distance of 45cm from the SLM (corresponding to 1/8th of the Rayleigh length before conversion). Additionally, a lens phase is added to the hologram. And finally, a binary circular aperture pattern is multiplied to the entire hologram, restricting modulation to

![Fig. 1: Experimental setup.](image)

A femtosecond laser emits pulses of 220fs duration centered at \( \lambda_0 = 1558 \text{ nm} \). For squeezed light generation, the pulses are split up on a 90:10 beam splitter and launched into a Kerr fiber (\( \chi^{(3)} \) nonlinearity) of length 3.8m in a counter-propagating configuration. The exiting pulses typically exhibit \(-3.0 \text{ dB}\) of amplitude squeezing prior to the SLM. A pair of folding mirrors (FM\(_a\), FM\(_b\)) allow the squeezer to be bypassed to obtain a coherent shot noise reference for squeezing measurements. The beam impinges on a reflective SLM. An iris aperture selects the 1. diffraction order (see text for details). Another folding mirror (FM\(_c\)) is used to direct the beam either at a InGaAs camera for mode inspection or at a detector, whose 9 MHz sideband fluctuations and DC amplitude are respectively recorded by an electronic spectrum analyser and a volt meter.
Fig. 2: Example phase patterns. Basic phase patterns for generating (a) a Laguerre-Gauss beam and (b) a Bessel-Gauss beam. In addition, a blazed grating, kinoform lens and aperture are added to each pattern (not shown, see text for details).

An SLM’s important advantage is that it allows for the generation of arbitrary patterns, i.e. superpositions of very many basis modes with almost any combination of coefficients. To show the versatility of the setup, we generate amplitude squeezed Laguerre-Gauss beams, Bessel-Gauss beams as well as an arbitrary pattern. Laguerre-Gauss (LG) modes are of particular interest as they represent a natural basis for optical orbital angular momentum [43]. Fig. 2(a) shows the phase pattern required to generate an LG beam with radial index \( p = 1 \) and helical index \( l = 1 \), where each photon carries an orbital angular momentum of \( \hbar \). Bessel beams, too, exhibit remarkable features: they are non-diffractive and self-healing [44]. These ideal beams extend transversely to infinity and contain an infinite amount of energy, similar to plane waves. In a real setting it is hence only possible to generate Bessel-Gauss (BG) beams, for which the ideal mode function is multiplied by a Gaussian envelope, while, however, retaining some of its favourable properties. Fig. 2(b) displays the phase pattern used to generate a BG beam of order \( n = 1 \).

The two-dimensional spatial Fourier transform of the desired beam is used as the phase pattern on the SLM. The patterns employed to generate both BG and LG beams are dominated by their mode functions’ defining polynomials, i.e. the generalized Laguerre polynomial [15] and the \( n \)th-order Bessel function of the first kind [44]. Every zero in the radial direction of the polynomial defining the modes results in a phase discontinuity of \( \pi \) of the phase mask (see Fig. 2). The azimuthal phase consists in a repeated continuous gradient from 0 to \( 2\pi \).

2.3. Measurements

The generated modes are analysed with respect to the quality of the spatial modes and the quantum noise reduction. The transverse intensity distributions of the experimentally generated modes are recorded with a Xenics XS-1.7-320 InGaAs camera. As a measure of mode quality, the intensity distribution as inferred from the theoretical mode function is fitted to a line section of the measured mode.

The amplitude squeezing is measured by direct photodetection at a sideband frequency of 9 MHz using an electronic spectrum analyser with resolution bandwidth 1 MHz and video bandwidth 3 kHz. The shot-noise reference level is determined by measuring the fluctuations of a coherent beam of the same continuous-wave equivalent optical power in the same spatial mode. The final squeezing figure is determined by forming the difference between the respective time-averaged values.

For each mode pattern, squeezing is measured in this way both prior to the mode conversion and in the final converted mode. The mode conversion efficiency \( \eta \) is estimated by comparing
the continuous-wave equivalent power of each diffracted beam behind the SLM to that of the squeezed Gaussian input beam before the SLM. The observed reduction in squeezing is compared to the expected reduction from the variance relation in Sec. 1.1. Agreement of the two values within their experimental error indicates that any excess noise power added by the SLM at the measured sideband lies below the detectable threshold.

3. Results and Discussion

3.1. Quality of the generated spatial modes

The observed intensity distributions for LG modes with $p, l \in \{1, 2, 3\}$ and BG modes with $n = 0, 1, 2$ are shown in Fig. 3(a) and (b), respectively. The intensity distributions are in very good agreement with the expected beam profiles. For the same modes, Fig. 4(a) and (b) show the cross-sectional intensity distributions (black dots) and corresponding fitted curves (red line). The fitted curves, derived from the theoretical mode functions, generally agree well with the observed line sections. However, for LG modes with larger values of $p$ the overlap decreases.
due to the order of the radial Laguerre polynomial growing with \( p \) and becoming increasingly hard to approximate by phase-only modulation. A method for amplitude and phase modulation via a single phase-only SLM has been demonstrated by Bolduc et al. [45], based on careful spatial modulation of the blazed grating depth while compensating the resulting phase aberrations. However, the technique introduces some additional loss, so that the suitable trade-off between mode quality and squeezing must be found in accordance with the requirements of a given application. More generally, it may be possible to achieve an improvement by measuring and compensating for any mechanical distortions in the silicon backplane of the SLM.

### 3.2. Optical conversion losses

We find \( \eta_d = 0.90(3) \) for the diffraction efficiency and \( \eta_r = 0.61(2) \) for the reflectivity of the SLM. The grating efficiency was found to be \( \eta_g = 0.91(3) \) in the first order, leading to a total efficiency of \( \eta = \eta_d \eta_r \eta_g = 0.50(3) \) with no phase pattern applied. As shown in tables [1] and [2], the total efficiency is reduced further by a few percent for modes with high orders. For the arbitrary pattern, the efficiency was 0.15.

### 3.3. Squeezing in the generated higher-order modes

Before the SLM, we typically observe \( \text{Var}_m = (-3.0 \pm 0.3) \text{ dB} \) of amplitude squeezing in the fundamental Gauss beam. The procedure described in [2] to quantify squeezing is demonstrated by example of an LG\(_1^1\) mode in Fig. 5. Here, a noise reduction of \( (-1.30 \pm 0.30) \text{ dB} \) below the shot-noise level can be seen. Tables [1] and [2] show a complete list of squeezing values for the LG and BG beams, respectively. For the arbitrary mode pattern, a noise reduction of \( (-0.4 \pm 0.3) \text{ dB} \) below shot noise was observed. Tables [1] and [2] show that for lower-order modes the efficiencies are much higher (\( \approx 50\% \)), allowing a typical squeezing of \( (-1.3 \pm 0.3) \text{ dB} \). All quoted squeezing figures were verified by attenuation measurements to result from quantum

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Fig. 4: Mode quality. Measured cross-sectional intensity distributions (dots) with fitted theoretical curves (lines) for (a) Laguerre-Gauss beams and (b) Bessel-Gauss beams.
Fig. 5: *Amplitude squeezing*. Noise trace showing typical amplitude squeezing at 9 MHz in the LG\textsuperscript{1} beam. Shaded areas correspond to one standard deviation.

noise reduction. For each mode, the measured squeezing matches the expected value corresponding to the measured total efficiency $\eta$ of the SLM for the respective mode to within the experimental accuracy. In other words, the upper bound for excess noise at this frequency is lower than the error bars of the measurement. We conclude that no detectable excess noise was added by the SLM in the mode conversion process at the 9 MHz sideband.

| Radial index | Helical index | Efficiency | Squeezing |
|--------------|---------------|------------|------------|
| 1            | 1             | 52.0%      | $(-1.34 \pm 0.32)\, \text{dB}$ |
| 1            | 2             | 51.3%      | $(-1.32 \pm 0.31)\, \text{dB}$ |
| 1            | 3             | 50.1%      | $(-1.34 \pm 0.31)\, \text{dB}$ |
| 2            | 1             | 51.2%      | $(-1.34 \pm 0.30)\, \text{dB}$ |
| 2            | 2             | 51.2%      | $(-1.31 \pm 0.32)\, \text{dB}$ |
| 2            | 3             | 50.1%      | $(-1.27 \pm 0.32)\, \text{dB}$ |
| 3            | 1             | 50.3%      | $(-1.30 \pm 0.30)\, \text{dB}$ |
| 3            | 2             | 50.0%      | $(-1.30 \pm 0.31)\, \text{dB}$ |
| 3            | 3             | 50.0%      | $(-1.28 \pm 0.31)\, \text{dB}$ |

Table 1: Experimental conversion efficiency and measured squeezing in LG\textsuperscript{p} beams.

| Order | Efficiency | Squeezing |
|-------|------------|------------|
| 0     | 47.0%      | $(-1.17 \pm 0.31)\, \text{dB}$ |
| 1     | 47.3%      | $(-1.17 \pm 0.31)\, \text{dB}$ |
| 2     | 47.5%      | $(-1.17 \pm 0.31)\, \text{dB}$ |

Table 2: Experimental conversion efficiency and measured squeezing in BG beams.
4. Summary

We have shown that a commercially available SLM can be used to transfer squeezing from the fundamental transverse mode of an optical field into arbitrary higher-order modes. With this approach, different spatial modes can be generated simply by applying a different phase pattern to the SLM with no further modifications to the setup. In principle, the range of achievable spatial modes is unlimited (up to the resolution of the SLM), but there is a trade-off between mode quality and conversion efficiency, which ultimately affects the observable squeezing in the output mode. In all cases, the observed reduction of squeezing was consistent with linear losses, ruling out excess noise from the SLM at the 9 MHz sideband investigated. Our work provides a direct illustration of the generalised single-mode concept in quantum optics and shows that applications requiring squeezed light in tailored spatial modes are within reach of commercially available technology.