The study of attenuation of the shear stresses in a straight rod near the point of load application

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Abstract. For engineering calculations must be used the principle of Saint-Venant. Usually assume that the "perturbations" of the stress-strain state decay approximately at a distance of one transverse dimension of the rod from the point of application of load [1]. In a previous article [2] a study was conducted of the normal stresses in the cross sections rigidly clamped rod for various loads applied to its free end. At the present work is the study of shearing stresses in the zone of application of the loads and the near kinematic heterogeneities related to the conditions of attachment of a rod. The limits of the applicability of the Saint-Venant principle are investigated while using the LIRA-CAD app.

1. Research
In calculation No. 1 (Figure 1) a rigidly clamped bar with dimensions of 800x100x10 cm, loaded with a central tensile force $F = 1000$ kN, was considered.

![Figure 1. For calculation No. 1](image-url)
For such rods in the strength of materials it is assumed that in the sections perpendicular to the axis the tangential stresses are zero. In fact, we have an inhomogeneous perturbation of the tangential stresses, which rapidly decays in the zones far from the load. As near to the zone of load application shear stresses are present, such stress-strain state corresponds to the shear deformation of the layers relative to each other (in addition to the stretching caused by a longitudinal force $F$). In Figure 2 an presents the diagrams of the shear stresses, starting from the cross section 1-1 through 4-4 in increments of 20 cm. These diagrams are built in vertical cross-sections rotated by 90 degrees.

![Diagrams of shear stresses](image)

**Figure 2.** Diagrams of shearing stresses (to calculation No. 1)

As can be seen from the values in the presented diagrams shear stresses take values commensurate with the normal stresses:

- section 2-2: $\tau_{xy}^{\text{max}} = 9.4$ MPa; $\sigma_{x}^{\text{max}} = 32.4$ MPa;
- section 3-3: $\tau_{xy}^{\text{max}} = 4.3$ MPa; $\sigma_{x}^{\text{max}} = 17.6$ MPa;
- section 4-4: $\tau_{xy}^{\text{max}} = 2.2$ MPa; $\sigma_{x}^{\text{max}} = 13.0$ MPa.

Attenuation of the shear stress perturbation occurs approximately at a distance of 100 cm (one transverse dimension of the rod) from the end of the rod.

In calculation No. 2, the same rod was loaded with two tensile forces $F = 500$ kN, applied symmetrically at the free end (Figure 3).
Comparing the diagrams of tangential stresses near the load application area (Figure 4), it can be concluded that the shape of the tangential stresses over the sections is practically the same (excluding the cross section 1-1, which was taken directly from the points of loading), only the numbers change.

In accordance with the formulas of resistance of materials in the calculations of No. 1 and No. 2 shear stresses in the cross sections must be equal to zero. Two numerical calculations have shown the presence of tangential stresses near the place where the load is applied. Their values, which turn out to be comparable with the values of normal stresses, tend to zero as the cross section is removed from the loads by one transverse dimension of the rod.

In calculation No. 3 (Figure 5) were tested the same rigidly-clamped beam, but with a concentrated vertical force at the free end. Thickening of the finite element grid to 1x1x1 cm is caused by the need to clarify the values of the tangential stresses at the edges of the beam section. Sections was taken from 10 cm from the point of application of the load, in increments of 20 cm to eliminate the
phenomenon of concentration of shearing stresses at the point of application. In Figure 5 calculation No. 3 is shown with a grid of 10x10x10 cm to provide an overall picture of the distribution of tangential stresses in this beam.

![Figure 5](image)

**Figure 5.** To calculation No. 3 (fixing option No. 3.1)

The calculation No. 3 was carried out to verify the results obtained by the formula D Zhuravsky [3], according to which the tangential stresses in the transverse rectangular section of the beam are distributed according to the law of square parabola.

The discrepancy of less than 5% from the results obtained by the formula D Zhuravsky was observed, approximately, at the distance of 90 cm (0.9b, where b is the transverse dimension of the rod) from the free end of the beam (Figure 6).

It should be noted that the method of attachment of a rod depends on the distribution of shearing stresses in the local zone near it. The principle of Saint-Venant formulated to usually only to loads. It involves the rapid decay of the "perturbations" of the stress-strain state within the local zone, remote, about the size of one transverse dimension of the rod from the point of application of loads. In this paper, we have obtained results that allow us to generalize the Saint-Venant principle not only to static effects, but also to kinematic ones.

In calculation No. 3, rigid fixing was performed in the following way (fixing option No. 3.1). All the extreme left nodes were prohibited from moving horizontally along the X axis, and for the central node, there were prohibitions for movement in two directions (X and Z). Studies have shown (Figure 7) that there is a "perturbation" of tangential stresses near the central node. The discrepancy is less than 5% from the results obtained by the formula D Zhuravsky was observed at a distance of 56 cm from the fixation zone (0.56b, where b is the transverse size of the rod) (section 6-6). The section 7-7 is taken at a distance of 10 cm from the fixing zone (0,1b).

Other variants of boundary conditions were also investigated in the calculation of No. 3 when the cantilever beam was loaded with a vertical force (Figure 5).

In Figure 8 shows the results of calculation of shear stresses for the fixing option No. 3.2, when all the leftmost nodes were prohibited from moving along the X axis, and two extreme nodes added prohibitions on the movement axis Z.

In Figure 9, below, shows the results for the case when all the leftmost nodes were prohibited from moving in the horizontal direction along the X axis, and the bottommost node has been added prohibition on the movement along the Z axis.
Figure 6. Diagrams of shearing stresses (to calculation No. 3)

Figure 7. To calculation No. 3 (fixing option No. 3.1) – diagrams of shearing stresses in the sections 6-6 and 7-7 (near fixation)
Figure 8. The results of the calculation No. 3 (fixing option No. 3.2) and diagrams of shearing stresses at a distance of 0,1b from the zone of fixation

Figure 9. The results of the calculation No. 3 (fixing option No. 3.3) and diagrams of shearing stresses at a distance of 0,1b from the zone of fixation

In the fixing option No. 3.4 left nodes were prohibited from moving along the X axis, and the upper, central and lower node has been added prohibition on the movement along the Y axis (Figure 10).

Figure 10. The results of the calculation No. 3 (fixing option No. 3.4) and diagrams of shearing stresses at a distance of 0,1b from the zone of fixation
The thickening of the net of finite elements up to 1x1x1 cm was caused by the need to refine the tangential stresses near the fixing zone. In Figures 8, 9, and 10, the calculation is shown with a grid of 10x10x10 cm to provide an overall picture of the distribution of tangential stresses in the beam. In all calculations, it was found that the tangential stress concentration occurs near the fixing points along the Z axis.

In the calculation No. 4 to the rigidly-clamped beam at the end were attached two vertical concentrated forces. As in the case of calculation No. 3, the thickening of the finite element grid is caused by the need to refine the tangential stresses along the edges of the beam section. In Figure 11 shows the cross sections for which the diagrams of tangential stresses shown in Figure 12.

**Figure 11.** To calculation No.4

**Figure 12.** Diagrams of shearing stresses (to calculation No. 4)
As can be seen from the diagrams of the tangential stresses in the cross sections, the perturbation of the stress-strain state becomes less pronounced in comparison with the calculation of the No. 3. If in the previous numerical experiment, the fulfillment of the formula D Zhuravsky occurred at a distance of 0,9b from the load application area, in the case of calculation No. 4 this formula was fulfilled at a distance of 0,7b (b is the transverse dimension of the rod).

2. Conclusions
1. In all the examples considered, the Saint-Venant principle was confirmed, and the perturbation of the stress state associated with tangential stresses attenuated at a distance of approximately one transverse dimension.
2. At loading of the rod by concentrated forces acting along its axis, near places of their application arise comparable to normal stresses shear stresses, which in the strength of materials have values of zero. Since some building materials do not resist the shift, this fact should be taken into account in engineering calculations.
3. In all cases, the diagrams of the tangential stresses, which determine the perturbation of the stressed state, have a self-balanced nature, that is, they obey the condition

\[ Q_z = \int_A \tau_{xz} \, dA = 0. \]

4. Form component of the self-balanced shear stresses depend on the load type.
5. The present study can be continued with the aim of clarifying self-balanced components, the superposition of which will make it possible to clarify the solution of the strength of materials.
6. The perturbation of tangential stresses also occurs near the kinematic inhomogeneities along the binding boundary, the attenuation of which occurs also rapidly, as in the case of applying loads.
7. This study allows us to generalize the Saint-Venant principle not only to static but also to kinematic parameters, that is, a local rapidly damped disturbance of the stress-strain state occurs not only near the places of application of the loads, but also near the kinematic inhomogeneities associated with the rod fixation parameters.

References
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