Vector inflation

Alexey Golovnev, Viatcheslav Mukhanov and Vitaly Vanchurin

ASC, Department für Physik, Ludwig-Maximilians-Universität München, Theresienstrasse 37, D-80333, Munich, Germany
E-mail: Alexey.Golovnev@physik.uni-muenchen.de, mukhanov@theorie.physik.uni-muenchen.de and vitaly@cosmos.phy.tufts.edu

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Abstract. We propose a scenario where inflation is driven by non-minimally coupled massive vector fields. In an isotropic homogeneous universe these fields behave in precisely the same way as a massive minimally coupled scalar field. Therefore our model is very similar to the model of chaotic inflation with a scalar field. For vector fields the isotropy of expansion is achieved either by considering a triplet of orthogonal vector fields or at the expense of $N$ randomly oriented vector fields. In the latter case the substantial anisotropy of the expansion of order $1/\sqrt{N}$ survives until the end of inflation. The lightest vector fields might also force the late time acceleration of the universe.

Keywords: dark energy theory, inflation

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1. Introduction

All successful inflationary scenarios are based on the use of classical scalar fields. Two main reasons for that are natural homogeneity and isotropy of such field and its ability to imitate a slowly decaying cosmological constant. This happens in models of chaotic inflation [1] and for $k$-inflation [2]. Although the higher spin bosonic fields can also form condensates they are usually overlooked, since they generically induce an anisotropy and because of the apparent difficulty of realizing the slow roll regime for them. For example, a model of vector inflation based on a potential $V(A_\alpha A^\alpha)$ was suggested in [3], where the potential does not change too much while $I = A_\alpha A^\alpha$ runs over an exponentially large range during inflation.

In this paper we show that both obstacles to realizing a successful vector inflation can be simultaneously surmounted in a natural way. In particular, isotropy of the vector field condensate can be achieved for the case of a triplet of mutually orthogonal vector fields [4, 5] (see also [6, 7] for exact isotropic solutions of the Einstein–Yang–Mills system based on the same idea) or by considering a large number of randomly oriented fields. (Another possibility is to consider purely time-like vector fields [8]–[11].) The problem of slow roll of the massive vector field can also be successfully solved by introducing a non-minimal coupling of this field to gravity. As a result, we obtain inflationary scenarios which are very similar to the simplest chaotic inflation with a massive scalar field [1] and $N$-flation [12]. However, in distinction from the $N$-flation case, in the case of $N$ vector fields an anisotropy of order $1/\sqrt{N}$ can survive until the end of inflation.

2. Equations

Let us consider a massive vector field which is non-minimally coupled to gravity, and has the action

$$S = \int d^4x \sqrt{-g} \left( -\frac{R}{16\pi} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \left( m^2 + \frac{R}{6} \right) A_\mu A^\mu \right),$$

where $F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$ and we use the Planck units ($G = 1$). Note that the non-minimal coupling of this vector field is very similar to conformal coupling for a scalar field. In the case of a scalar field this coupling converts massless scalar fields to
conformal invariant fields. As we will see, for the vector field the particular non-minimal coupling in (1) has an ‘opposite effect’, namely, it violates the conformal invariance of a massless vector field and forces it to behave in the same way as a minimally coupled scalar field. The variation of the action with respect to \( A^\mu \) yields the following equations of motion:

\[
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} F^{\mu\nu} \right) + \left( m^2 + \frac{R}{6} \right) A^\nu = 0.
\]

In the spatially flat Friedmann universe with the metric

\[
ds^2 = dt^2 - a^2(t) \delta_{ij} \, dx^i \, dx^j,
\]

these equations take the following form:

\[
-\frac{1}{a^2} \Delta A_0 + \left( m^2 + \frac{R}{6} \right) A_0 + \frac{1}{a^2} \partial_i A_i = 0,
\]

\[
\ddot{A}_i + \frac{\dot{a}}{a} A_i - \frac{1}{a^2} \Delta A_i + \left( m^2 + \frac{R}{6} \right) A_i - \partial_i A_0 - \frac{\dot{a}}{a} \partial_i A_0 + \frac{1}{a^2} \partial_i (\partial_k A_k) = 0,
\]

where \( \partial_i \equiv \partial/\partial x^i \), a dot denotes a derivative with respect to the physical time \( t \) and we assume the summation over repeated spatial indices. One of the quantities which characterize the strength of the vector field in a coordinate independent way is the scalar

\[
I = A^\alpha A_\alpha = A_0^2 - \frac{1}{a^2} A_i A_i,
\]

and this motivates us to introduce a new variable \( B_i \equiv A_i/a \) instead of \( A_i \) which is the ‘square root’ of the invariant \( I \). Considering the quasi-homogeneous vector field (\( \partial_i A_\alpha = 0 \)) we immediately infer from (2) that

\[ A_0 = 0, \]

and equation (3) rewritten in terms of the field strength \( B_i \) becomes

\[ \ddot{B}_i + 3H \dot{B}_i + m^2 B_i = 0, \]

where \( H \equiv \dot{a}/a \). This equation is similar to the equation for the massive minimally coupled scalar field, and when the Hubble constant \( H \) is larger than the mass \( m \) the fields \( B_i \) are ‘frozen’. Therefore one might expect that the potential \( -m^2 A^\mu A_\mu = m^2 B_i B_i \approx \) const could drive the quasi-de Sitter expansion analogously to the scalar field. To determine under which conditions this could happen we have to calculate the energy–momentum tensor for the vector field. Variation of the action (1) with respect to the metric gives

\[
T^\alpha_\beta = \frac{1}{4} \epsilon_{\gamma\delta} F^\gamma_\alpha F^\delta_\beta - F^\alpha_\gamma F^\gamma_\beta + \left( m^2 + \frac{R}{6} \right) A^\alpha A_\beta - \frac{1}{2} m^2 A^\gamma A_\gamma \delta^\alpha_\beta
\]

\[
+ \frac{1}{6} \left( R^\beta_\gamma \gamma^\beta_\alpha - \frac{1}{2} \delta^\alpha_\beta R \right) A^\gamma A_\alpha + \frac{1}{6} \left( \delta^\beta_\gamma - \nabla^\alpha \nabla_\beta \right) A^\gamma A_\alpha.
\]

For a homogeneous vector field in a flat Friedmann universe we obtain

\[
T^0_0 = \frac{1}{2} (\dot{B}_k^2 + m^2 B_k^2),
\]

\[
T^i_j = \left[ -\frac{5}{6} (\dot{B}_k^2 - m^2 B_k^2) - \frac{2}{3} H \dot{B}_k B_k - \frac{1}{3} (\dot{H} + 3H^2) B_k^2 \delta^i_j \right]
\]

\[
+ \ddot{B}_j \dot{B}_i + \dot{H} (\dot{B}_i B_j + \ddot{B}_i B_j) + (\dot{H} + 3H^2 - m^2) B_i B_j,
\]
where we assume summation over index $k$. The spatial part of the energy–momentum tensor contains off-diagonal components which are of the same order of magnitude as the diagonal components and therefore the isotropic Friedmann universe filled by the homogeneous vector field does not satisfy the Einstein equations. This is not surprising because the homogeneous vector field has a preferred direction. Therefore to make the energy–momentum tensor diagonal we have to consider several fields simultaneously.

3. Inflation

Let us first consider a triplet of mutually orthogonal vector fields $B_i^{(a)} [5]$, with the same magnitude $|B|$ each. Then from

$$\sum_i B_i^{(a)} B_i^{(b)} = |B|^2 \delta_a^b, \quad (7)$$

it follows that

$$\sum_a B_i^{(a)} B_j^{(a)} = |B|^2 \delta_i^j. \quad (7)$$

The last relation is easy to understand considering the components of the triplet as elements of a matrix; then condition (7) simply implies the orthogonality of this matrix. Using these relations we find from (5) and (6) that the total energy–momentum tensor of the vector fields is

$$T_{00} = \varepsilon = \frac{3}{2} (\dot{B}_k^2 + m^2 B_k^2),$$

$$T_{ij} = -p \delta_i^j = -\frac{3}{2} (\dot{B}_k^2 - m^2 B_k^2) \delta_i^j,$$

where $B_k$ are the components of any field from the triplet which satisfy

$$\ddot{B}_i + 3H \dot{B}_i + m^2 B_i = 0, \quad (8)$$

and $H$ is now given by

$$H^2 = 4\pi (\dot{B}_k^2 + m^2 B_k^2).$$

These equations are precisely the same as for the massive scalar field and their investigation literally repeats those for the scalar field [13]. In particular for $|B| > 1$ we have a slow roll regime ($\dot{B}_k^2 \ll m^2 B_k^2$) in which $p \approx -\varepsilon$ and the universe undergoes the stage of inflation. The inflation is over when the value of $|B|$ drops to the Planck value. Note that the vector field condensate can also imitate the ultra-hard equation of state $p = \varepsilon$ when its kinetic energy dominates.

Another way to resolve the issue of isotropy is to consider a large number of randomly oriented vector fields. For simplicity, let us first consider $N$ fields with equal masses $m$ assuming that they all have about the same magnitude of order $B$ initially. The components of these fields satisfy (8) and their total contribution to $T_{00}$ can be estimated as

$$T_{00} = \varepsilon \approx \frac{N}{2} (\dot{B}_k^2 + m^2 B_k^2).$$
To get an estimation of the spatial components of the energy–momentum tensor we note that

\[ \sum_{a=1}^{N} B_i^{(a)} B_j^{(a)} \simeq \frac{N}{3} B^2 \delta^i_j + O(1) \sqrt{N} B^2, \]

where \( B^2 = B_k^2 \) and the summation over \( k \) is assumed. The corrections proportional to \( \sqrt{N} \) are due to stochastic random distribution of directions of the fields and they do not vanish for \( i \neq j \) characterizing the typical magnitude of the off-diagonal spatial components of the energy–momentum tensor. It follows from (6) that during inflation a typical value of the off-diagonal spatial components is of order \( H^2 \sqrt{N} B^2 \). The isotropic inflationary solution is self-consistent only if these components are smaller than \( T_i^i \sim T_0^0 \sim H^2 \); hence this solution can be valid only for \( B < 1/N^{1/4} \). On the other hand, the vector fields are in the slow roll regime only if the ‘effective friction’ in (8), which is of order \( H \), exceeds their mass \( m \) and inflation is finished when \( H \simeq m \). Taking into account that in the inflationary stage

\[ H^2 = \frac{8\pi}{3} \epsilon \simeq \frac{4\pi}{3} Nm^2 B^2, \]  

(9)

we find that when the field drops to \( B \simeq 1/N^{1/2} \) it starts to oscillate and inflation is over. Thus the isotropic inflationary stage of expansion takes place when the fields \( B \) change within the interval

\[ \frac{1}{\sqrt{N}} > B > \frac{1}{\sqrt{N}}. \]

To estimate the number of e-folds during the inflationary stage we note that in the slow roll regime

\[ \dot{B} \simeq - \frac{m^2 B}{3H}, \]  

(10)

and then from (9) and (10) we find that during inflation the scale factor increases by

\[ \frac{a_f}{a_i} \simeq \exp(2\pi NB_{in}^2), \]

where \( B_{in} \) is the initial value of the vector fields. Taking \( B_{in} \simeq N^{-1/4} \) we find that in the case of \( N \) vector fields the maximum number of e-folds of isotropic inflation is about \( 2\pi \sqrt{N} \). For \( N \) of the order of a few hundreds, the duration of inflation is enough to explain the observed homogeneity. On the other hand, at the end of inflation there still survive off-diagonal spatial components of the energy–momentum tensor and their relative value compared to the diagonal components is about \( 1/\sqrt{N} \). They induce a global anisotropy which is of the same order of magnitude \( \simeq 1/\sqrt{N} \) at the end of inflation. In the case of a few hundred vector fields this anisotropy is about a few per cent.

The consideration above can easily be generalized to the case of many vector fields with different masses and initial amplitudes. The very light fields can remain frozen until today and thus serve as the observed dark energy which can even be anisotropic.

Moreover, instead of the mass term \( m^2 A_\alpha A^\alpha \) we can consider an arbitrary potential \( V(A_\alpha A^\alpha) \). In this case the vector fields satisfy the equation

\[ \ddot{B}_i + 3\frac{\dot{a}}{a} \dot{B}_i + V'(B^2) B_i = 0, \]
and it is clear that the slow roll regime can be realized for a wide class of potentials, thus providing us with a large variety of inflationary scenarios similarly to the case of the scalar field. The energy-momentum tensor is

\[ T^0_0 = \frac{1}{2}(\dot{B}_k^2 + V(B^2)), \]

\[ T^i_j = \left[ -\frac{5}{6}\dot{B}_k^2 + \frac{1}{2}V(B^2) - \frac{2}{3}H\dot{B}_kB_k - \frac{1}{2}(\dot{H} + 3H^2 - V'(B^2))B_k^2 \right] \delta^i_j + \dot{B}_i\dot{B}_j + H(\dot{B}_iB_j + \dot{B}_jB_i) + (\dot{H} + 3H^2 - V'(B^2))B_iB_j, \]

and after averaging over \( N \) fields we obtain

\[ T^i_j = -p\delta^i_j \simeq \frac{N}{2}(-\dot{B}_k^2 + V(B^2))\delta^i_j. \]

For a general potential \( V \) the exit from inflation occurs when \( V'B/V \sim \sqrt{N} \). Note that this allows us to make the anisotropy smaller because the inflation may end at small \( B_f \) and the corresponding anisotropy is then of order \( \sqrt{NB_f^2} \).

4. Conclusions

Scalar field inflation predicts a nearly completely isotropic universe. The anisotropy can be obtained only at the expense of initial conditions and fine-tuning the duration of the inflationary stage. In this paper, we have proposed a vector inflation, which could give us either a completely isotropic universe (with orthogonal triplet of vector fields) or a slightly anisotropic universe (with \( N \) randomly oriented vector fields). The degree of anisotropy is of order \( 1/\sqrt{N} \) at the end of inflation.

The model contains only minimal complexity and does not require any fine-tuning of the potential or initial conditions. To implement inflationary expansion with vector fields, two essential ingredients were added to the standard theory of massive vector fields. First, to obtain the slow roll regime for the vector fields we have coupled them to gravity in a non-minimal way. Second, to avoid a large anisotropy we have considered a large number of mutually uncoupled randomly oriented fields.

The same model can in principle explain the late time acceleration of the universe. In fact, this model naturally combines inflation and dark energy within the same framework. However, predictions of the unified model depend crucially on the distribution of masses of vector fields.

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