Quantum secure communication protocols based on entanglement swapping

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We present a quantum secure direct communication protocol and a multiparty quantum secret sharing protocol based on Einstein-Podolsky-Rosen pairs and entanglement swapping. The present quantum secure direct communication protocol makes use of the ideal of block transmission. We also point out that the sender can encode his or her secret message without ensuring the security of the quantum channel firstly. In the multiparty quantum secret sharing protocol, the communication parties adopt checking mode or encoding mode with a certain probability. It is not necessary for the protocol to perform local unitary operation. In both the protocols, one party transmits only one photon for each Einstein-Podolsky-Rosen pair to another party and the security for the transmitting photons is ensured by selecting Z-basis or X-basis randomly to measure the sampling photons.

PACS numbers: 03.67.Dd, 03.67.Hk

Quantum cryptography has been one of the most promising applications of quantum information science. It utilizes quantum effects to provide unconditionally secure information exchange. Quantum key distribution (QKD) which provides unconditionally secure key exchange has progressed quickly [1, 2, 3, 4]. In recent years, a good many of other quantum cryptography schemes have also been proposed and pursued, such as quantum secret sharing (QSS) [5, 6, 7, 8, 9, 10, 11, 12, 13], quantum secure direct communication (QSDC) [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. QSS is the generalization of classical secret sharing to quantum scenario and can share both classical and quantum messages among sharers. Many works have been carried out in both theoretical and experimental aspects after the pioneering QSS scheme proposed by M. Hillery et al. in 1999 (hereafter called HBB99) [5]. We can classify the QSS schemes as schemes using entanglement or schemes without entanglement. The HBB99 scheme is based on a three-particle entangled Greenberger-Horne-Zeilinger (GHZ) state. A. Karlsson et al. proposed a QSS scheme using two-particle Bell states [6]. Based on multi-particles entangled states and teleportation, we presented a multiparty QSS scheme of QSDC [8]. G. P. Guo and G. C. Guo presented a QSS scheme where only product states are employed [9]. Z. J. Zhang et al. [10] proposed a QSS scheme using single photons. QSDC’s object is to transmit the secret message directly without first establishing a key to encrypt them, which is different to QKD. QSDC can be used in some special environments which has been shown by K. Boström et al. and F. G. Deng et al. [11, 12]. Many researches have been carried out in QSDC. These works can also be divided into two types, one utilizes single photons [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26], the other utilizes entangled state [17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. F. G. Deng et al. proposed a QSDC scheme using batches of single photons which serves as one-time pad [17]. Q. Y. Cai et al. presented a deterministic secure direct communication scheme using single qubit in a mixed state [18]. We proposed a QSDC scheme based on order rearrangement of single photons [19]. The QSDC scheme using entanglement state is certainly the mainstream. K. Boström and T. Felbinger proposed a ”Ping-Pong” QSDC protocol which is quasi-secure for secure direct communication if perfect quantum channel is used [20]. Q. Y. Cai et al. pointed out that the ”Ping-Pong” Protocol is vulnerable to denial of service attack or joint horse attack with invisible photon [21, 22]. They also presented an improved protocol which doubled the capacity of the ”Ping-Pong” protocol [20]. F. G. Deng and G. L. Long put forward a two-step QSDC protocol using Einstein-Podolsky-Rosen (EPR) pairs [20]. We presented a QSDC scheme using EPR pairs and teleportation [21] and a multiparty controlled QSDC scheme using GHZ states [22].

Entanglement swapping can entangle two quantum systems that do not have direct interaction with each other. It plays an important role in quantum information. There are also many quantum cryptography schemes using entanglement swapping. Z. J. Zhang et al. presented a multiparty QSS scheme [12] and a QSDC scheme based on entanglement swapping and local unitary operations [23]. Y. M. Li put forward a multiparty QSS of quantum information by swapping quantum entanglement [13]. Based on entanglement swapping, T. Gao et al. proposed two QSDC schemes using GHZ states and Bell states, respectively [30, 31]. We first describe entanglement swapping simply. The four Bell states are

\[
|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle),
|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).
\] (1)

Suppose two distant parties, Alice and Bob, share \(|\phi_{12}^+\rangle\) and \(|\phi_{34}^+\rangle\) where Alice has qubits 1 and 4, and Bob pos-
serves 2 and 3. Note that
\begin{equation}
|\phi_{12} \otimes \phi_{34} \rangle = \frac{1}{2} (|\phi_{14}^+ \rangle |\phi_{23}^- \rangle + |\phi_{14}^- \rangle |\phi_{23}^+ \rangle ) \\
+ |\psi_{14}^+ \rangle |\psi_{23}^- \rangle + |\psi_{14}^- \rangle |\psi_{23}^+ \rangle .
\end{equation}

After Bell basis measurement on qubits 1 and 4, the state of the qubits 1, 2, 3, 4 collapses to $|\phi_{14}^+ \rangle |\phi_{23}^- \rangle$, $|\phi_{14}^- \rangle |\phi_{23}^+ \rangle$, $|\psi_{14}^+ \rangle |\psi_{23}^- \rangle$, and $|\psi_{14}^- \rangle |\psi_{23}^+ \rangle$ each with probability 1/4. If Alice and Bob share other Bell states, similar results can be achieved. We give the relations used in our paper, as

\begin{equation}
|\phi_{12} \otimes \phi_{34} \rangle = \frac{1}{2} (|\phi_{14}^+ \rangle |\phi_{23}^- \rangle + |\phi_{14}^- \rangle |\phi_{23}^+ \rangle ) \\
+ |\psi_{14}^+ \rangle |\psi_{23}^- \rangle + |\psi_{14}^- \rangle |\psi_{23}^+ \rangle ,
\end{equation}

\begin{equation}
|\psi_{12} \otimes \phi_{34} \rangle = \frac{1}{2} (|\phi_{14}^+ \rangle |\phi_{23}^- \rangle - |\phi_{14}^- \rangle |\phi_{23}^+ \rangle ) \\
+ |\psi_{14}^+ \rangle |\psi_{23}^- \rangle - |\psi_{14}^- \rangle |\psi_{23}^+ \rangle ,
\end{equation}

\begin{equation}
|\psi_{12} \otimes \phi_{34} \rangle = \frac{1}{2} (|\phi_{14}^+ \rangle |\phi_{23}^- \rangle - |\phi_{14}^- \rangle |\phi_{23}^+ \rangle ) \\
+ |\psi_{14}^+ \rangle |\psi_{23}^- \rangle - |\psi_{14}^- \rangle |\psi_{23}^+ \rangle .
\end{equation}

We then present a QSDC protocol using EPR pairs and entanglement swapping. Suppose the sender Alice wants to transmit her secret message directly to the receiver Bob.

(1) Alice prepares an ordered $N$ EPR pairs. Each of the EPR pairs is in the state $|\phi_{12}^+ \rangle$. We denote the ordered $N$ EPR pairs with $\{|P_1(1),P_1(2)|, \ldots, P_N(1),P_N(2)\}$, where the subscript indicates the order of each EPR pair in the sequence, and 1, 2 represents the two-photon of each pair. Alice takes one photon from each state to form an ordered partner photon sequence $\{P_1(1), P_2(1), \ldots, P_N(1)\}$, called $S_1$ sequence. The remaining partner photons compose $S_2$ sequence, $\{P_1(2), P_2(2), \ldots, P_N(2)\}$. Bob also prepares $N$ EPR pairs each of which is in the state $|\phi_{34}^- \rangle$. In the same way, Bob divides the $N$ EPR pairs into $S_3$ sequence, $\{P_1(3), P_2(3), \ldots, P_N(4)\}$ and $S_4$ sequence, $\{P_1(4), P_2(4), \ldots, P_N(4)\}$. Alice then sends the $S_2$ sequence to Bob. Bob sends the $S_4$ sequence to Alice at the same time.

(2) After confirming the two parties have received the photon sequence, Alice selects randomly a sufficiently large subset from the photon sequence for eavesdropping check. She chooses randomly one of the two measuring basis $Z$-basis $(|0\rangle, |1\rangle)$ and $X$-basis $(|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |\text{-}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle))$ to measure the photon 1. She then tells Bob the positions of the sampling photons, the measuring basis she has chosen and her measurement result. Bob measures the photon 2 in the same measuring basis as Alice and compares his result with Alice’s. He thus can evaluate the error rate of the transmission of the $S_2$ sequence. To ensure the security of the transmission of the $S_4$ sequence, Alice and Bob utilize the same method to check eavesdropping. If the error rate exceeds the threshold, they abort the protocol. Otherwise, they continue to the next step.

(3) After ensuring the security of the EPR pairs, Alice encodes her secret message on the photon 1. She performs one of the four unitary operations
\begin{equation}
I = |0\rangle \langle 0| + |1\rangle \langle 1|,
\end{equation}
\begin{equation}
\sigma_x = |0\rangle \langle 1| + |1\rangle \langle 0|,
\end{equation}
\begin{equation}
i\sigma_y = |0\rangle \langle 1| - |1\rangle \langle 0|,
\end{equation}
\begin{equation}
\sigma_z = |0\rangle \langle 0| - |1\rangle \langle 1|.
\end{equation}

on the photon 1, according to her secret message. The operations $I$, $\sigma_x$, $i\sigma_y$ and $\sigma_z$ denote secret message “00”, “01”, “10” and “11”, respectively. Alice then performs Bell basis measurement on the photons 1 and 4 and publishes her measurement results.

(4) Bob measures the photons 2 and 3 in the Bell basis. Because of the operation $I$, $\sigma_x$, $i\sigma_y$ and $\sigma_z$ performed on the photon 1, the state $|\phi_{12}^+ \rangle$ is changed to $|\phi_{12}^+ \rangle (|\psi_{12}^+ \rangle, |\psi_{12}^- \rangle, |\phi_{12}^- \rangle)$. Note that the equations 2 and 3 Bob can then deduce Alice’s secret according to his result and Alice’s result, as illustrated in Table 1. For example, if Alice’s result is $|\psi_{14}^- \rangle$ and Bob’s result is $|\phi_{23}^- \rangle$, Alice’s secret must be “01”. That is to say Alice performed $\sigma_x$ operation on the photon 1.

| Table 1: The recovery of Alice’s secret message |
|---|---|---|
| Alice’s secret | Alice’s result | Bob’s result |
| 00 (I) | $\{|\phi_{14}^+, \phi_{23}^+\}$ | $\{|\phi_{14}^+, \phi_{23}^+\}$ |
| & $\{|\phi_{14}^-, \phi_{23}^+\}$ | $\{|\phi_{14}^-, \phi_{23}^+\}$ |
| 01 ($\sigma_x$) | $\{|\phi_{14}^+, \phi_{23}^-\}$ | $\{|\phi_{14}^+, \phi_{23}^-\}$ |
| & $\{|\phi_{14}^-, \phi_{23}^-\}$ | $\{|\phi_{14}^-, \phi_{23}^-\}$ |
| 10 ($i\sigma_y$) | $\{|\phi_{14}^-, \phi_{23}^+\}$ | $\{|\phi_{14}^-, \phi_{23}^+\}$ |
| & $\{|\phi_{14}^+, \phi_{23}^+\}$ | $\{|\phi_{14}^+, \phi_{23}^+\}$ |
| 11 ($\sigma_z$) | $\{|\phi_{14}^+, \phi_{23}^-\}$ | $\{|\phi_{14}^+, \phi_{23}^-\}$ |
| & $\{|\phi_{14}^-, \phi_{23}^-\}$ | $\{|\phi_{14}^-, \phi_{23}^-\}$ |

The security for the present protocol is the same as that for BBM92 protocol. To ensure the security of the transmission of the $S_2$ and $S_4$ sequence, the communication parties perform $Z$-basis or $X$-basis measurements on the sampling photons, which is similar to that of BBM92 protocol. Only after confirming the security of the quantum channel could Alice encode her secret message on the photon 1 and announce her measurement result. Thus our protocol is unconditional secure.

Actually, in the present QSDC scheme, Alice can encode her secret message directly on the EPR pairs without ensuring the security of the quantum channel firstly. Alice prepares a batch of EPR pairs each of which is in one of the four Bell states according to her secret message. The states $|\phi^+ \rangle$, $|\phi^- \rangle$, $|\psi^+ \rangle$ and $|\psi^- \rangle$ represent the secret message “00”, “01”, “10” and “11”, respectively. Alice inserts randomly some sampling EPR pairs in the encoding sequence for eavesdropping check. Similar to the step 2 in the present protocol, Alice and Bob chooses randomly $Z$-basis or $X$-basis to measure the sampling photons. Only after confirming the security of the quantum channel could Alice publish her results of Bell basis.
measurement. After obtaining Alice’s results, Bob can then recover her secret message.

We then present a multiparty QSS protocol using EPR pairs and entanglement swapping. We first present a three-party QSS protocol and then generalize it to a multiparty QSS one. Suppose Alice want to share a random key with Bob and Charlie.

(1) Alice, Bob and Charlie agree that the four Bell states $|\phi^+\rangle$, $|\phi^-\rangle$, $|\psi^+\rangle$ and $|\psi^-\rangle$ represent the two-bit information “00”, “01”, “10” and “11”, respectively.

(2) Alice (Bob, Charlie) prepares an EPR pair in the state $|\phi_{12}^+\rangle (|\phi_{34}^+\rangle, |\phi_{56}^+\rangle)$. Alice (Bob, Charlie) send the photon 2 (4, 6) to Bob (Charlie, Alice).

(3) Alice chooses one of the two modes, checking mode with probability $p$ and encoding mode with probability $1 - p$. If Alice selects checking mode, the procedure goes to (C4), otherwise they perform the step (E4).

(C4) Bob performs Z-basis or X-basis measurement randomly on the photon 2 and informs Alice the measuring basis he has chosen and his measurement result. Alice then measures the photon 1 in the same measuring basis as Bob and compares her result with Bob’s. The method of eavesdropping check is similar to that of BBM92 protocol, which ensures the security of the transmission of the photon 2. If there is no eavesdropper, their results should be accordant. The same method is used to check the security of the transmission of the photon 6. To prevent a dishonest party’s attack and ensure the security of the transmission of the photon 4, Alice selects randomly Bob or Charlie to choose a random measuring basis (Z-basis or X-basis) to measure the photon and then publish his or her corresponding measurement result. If there is no eavesdropping, they return to the step 1. Otherwise, the protocol is aborted.

(E4) Alice (Bob, Charlie) performs Bell basis measurement on the photons 1 and 6 (2 and 3, 4 and 5). According to Eq. 2 and 3, we can obtain

$$|\phi_{12}^+\rangle \otimes |\phi_{34}^+\rangle \otimes |\phi_{56}^+\rangle = \frac{1}{4} \left( |\phi_{16}^+\rangle |\phi_{23}^+\rangle |\phi_{45}^+\rangle + |\phi_{16}^-\rangle |\phi_{23}^-\rangle |\phi_{45}^-\rangle + |\phi_{16}^+\rangle |\phi_{23}^-\rangle |\phi_{45}^+\rangle - |\phi_{16}^-\rangle |\phi_{23}^+\rangle |\phi_{45}^-\rangle \right).$$

After the three-party’s Bell basis measurements, the state of the photons 1, 2, 3, 4, 5, 6 collapses to one of the sixteen states in the Eq. 4 with probability 1/16. Alice can then share a random key with Bob and Charlie, as illustrate in Table 2. Suppose Bob’s result is $|\phi_{23}^+\rangle$ and Charlie’s result is $|\phi_{45}^+\rangle$. If Bob collaborates with Charlie, they can deduce that Alice’s result is $|\phi_{16}^+\rangle$ according to Eq. 5. Thus Alice shares two-bit secret “01” with Bob and Charlie.

| Alice’s result | Bob’s result, Charlie’s result | the sharing key |
|----------------|-----------------------------|---------------|
| $|\phi_{16}^+\rangle$ | $|\phi_{23}^+\rangle, |\phi_{45}^+\rangle$ | 00 |
| $|\phi_{16}^-\rangle$ | $|\phi_{23}^-\rangle, |\phi_{45}^-\rangle$ | 01 |
| $|\psi_{16}^+\rangle$ | $|\phi_{23}^+\rangle, |\phi_{45}^+\rangle$ | 10 |
| $|\psi_{16}^-\rangle$ | $|\phi_{23}^-\rangle, |\phi_{45}^-\rangle$ | 11 |

We then analyze the security of the three-party QSS protocol. Each of the communication parties transmits only one photon for each EPR pair. The communication parties selects randomly one of the two measuring basis (Z-basis and X-basis) to check eavesdropping. This method for eavesdropping check is similar to that of BBM92 protocol, which is proved to be unconditionally secure. As long as the security of the transmission of the photons 2, 4, 6 is ensured, the present protocol is secure. The attack of Eve or a dishonest parties will be detected in the checking mode.

The three-party QSS protocol can be easily generalized to a multiparty one. Suppose Alice want to share a random secret key with Bob, Charlie, Dick, · · · , York and Zach. Each of the communication parties prepares an EPR pair in the state $|\phi^+\rangle$ and sends one photon of the EPR pair to the next party. That is to say Alice send one photon to Bob, Bob send one photon to Charlie, · · · , York sends one photon to Zach and Zach send one to Alice. Similar to the three-party QSS protocol, Alice chooses checking mode and encoding mode with probability $p$ and $1 - p$, respectively. In the checking mode, the communication parties utilizes random Z-basis and X-basis measurement to ensure the security of the transmitting photons. In the encoding mode, each of the communication parties performs Bell basis measurement on their two photons. Thus if only Bob, Charlie, · · · , York and Zach collaborate, they can share a random key with Alice. The details of the multiparty QSS protocol is very similar to that of the three-party one. And the security for the multiparty QSS protocol is the same as that for three-party one.

So far we have presented a QSDC protocol and a multiparty QSS protocol using entanglement swapping. Both the protocols utilizes EPR pairs to achieve secure information exchange. The communication parties transmit only one photon for each EPR pair in the two protocols and utilize random Z-basis or X-basis measurement to ensure the security of the transmitting photon. Different to Ref. 29, both the parties prepares a batch of EPR pairs in our QSDC protocol. We also point out that the sender Alice can encode her secret message on the EPR.
pairs without ensuring the security of the quantum channel firstly. In our multiparty QSS protocol, different to Ref. [12], the parties share a random secret key without performing local unitary operations, which simplifies the protocol.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grant No. 60472032.

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