VISIBILITY-BASED DEMODULATION OF RHESSI LIGHT CURVES

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ABSTRACT

The Reuven Ramaty High Energy Spectroscopic Solar Imager (RHESSI) uses the rotational modulation principle (Schnopper et al., 1968) to observe temporally, spatially, and spectrally resolved hard X ray and gamma ray images of solar flares. In order to track the flare evolution on time scales that are commensurate with modulation, the observed count rates must be demodulated at the expense of spatial information. The present paper describes improvements of an earlier demodulation algorithm, which decomposes the observed light curves into intrinsic source variability and instrumental modulation.

INTRODUCTION

The RHESSI (Lin et al. 2003) instrument is designed to explore particle acceleration in solar flares (e.g., Miller et al., 1997). It observes photons at energies from 3 keV to 17 MeV with spectral resolution up to \( E/\Delta E \sim 500 \). In order to obtain images, RHESSI is equipped with 9 pairs of X ray shadowing grids (9 ‘subcollimators’), which are fixed on the rotating spacecraft (spin period \( T_S \sim 4s \)). The instantaneous transmission probability of one subcollimator, projected onto the solar disc (heliocentric cartesian coordinates \( x \)), is called the ‘modulation pattern’ (Hurford et al., 2003a). It has the approximate form

\[
M^i(x, t) = a^i_0 + a^i_1 \cos \left( k^i(t) \cdot (x - P(t)) + \psi^i \right).
\]

The wave vectors \( k^i(t) \) correspond to periods (‘angular pitches’) of \( \sim 2.6 \cdot 3^{i/2} \) asec, and \( P(t) \) is the imaging (optical) axis. The angle \( \Phi^i_k(t) \) between \( k^i(t) \) and the heliocentric \( x \)-axis is given by \( \Phi^i_k(t) = \pi/2 - \Phi_{\text{roll}}(t) - \Phi_{\text{grid}}^i \), where \( \Phi_{\text{roll}}(t) \) is the roll angle and \( \Phi_{\text{grid}}^i \) is the grid orientation on the spacecraft. The triples \( (\Phi_{\text{roll}}(t), P(t)) \) are referred to as ‘aspect solution’, and are continuously monitored by the on-board aspect systems (Fivian and Zehnder, 2003; Hurford et al., 2003b). The coefficients \( a^i_0, a^i_1, \psi^i \) in eq. (1) depend on energy and (weakly) on \( x - P(t) \); they describe the X ray transmission of the grids. Above 100 keV the grids become increasingly transparent. In what follows, the energy band is fixed, and the energy dependence is omitted.

At any time, the 9 subcollimators respond to 9 different Fourier components of the solar brightness distribution. During half a RHESSI rotation, up to thousand Fourier components are collected, from which the spatial brightness distribution can be restored (Hurford et al., 2003a). The imaging information is thus encoded in the temporal modulation of the observed count rates. In order to track impulsive events (energy release, acceleration) on time scales below 2s, the observed count rates must be demodulated at the expense of spatial information.

The present article reports on further development of the demodulation method of Arzner (2003), which compared count rates with similar modulation patterns. This constraint was rather restrictive, since both the roll angle and the grid phase had to agree. The request for grid phase agreement can in fact be relaxed by the use of visibilities\(^1\); this is described in Section 2 below. In order to work properly, this approach

\(^1\)At each time, the two ‘visibilities’ are the projections of the instantaneous brightness distribution onto sine- and cosine
Fig. 1. Left: Motion of the imaging axis \( P(t) \) (circles) during \( N_P = 8 \) spacecraft revolutions at the Flare of August 23, 2002, 06:25 UT. The wave vectors (arrow indicates orientation of \( k^6 \) at final time) rigidly co-rotate with \( P(t) \). The average spin axis is marked by a cross. Right: change of modulation phase of subcollimator 3 per \( \Delta t \) at the average spin axis (\( \Delta \phi = \dot{\phi} \Delta t \)). The condition \( |\Delta \phi| \ll 2\pi \) is the basis of Eq. (6).

requires that the time binning is synchronized with the spacecraft rotation, i.e., that \( \Phi_{\text{roll}} \) (mod \( \pi \)) recurs after an integer number of time bins \( \Delta t \). On the other hand, \( \Delta t \) must be an integer multiple of a binary micro second, which is the hardware time resolution. The improved version chooses an optimum compromise between the two conflicting constraints, while keeping the number of counts per bin at a statistically useful level (\( \sim 10 \)). Section 3 shows benchmark simulations and the demodulation of real RHESSI data.

**DEMODULATION METHOD**

The goal of the method is to separate spatial and temporal variations of the solar brightness distribution. The basic observation is that the visibilities at equal grid orientation should be compatible if the brightness distribution did not change with time. Deviations are attributed to time dependence of the source distribution. Technically, this is done by a regularized maximum-likelihood fit with many (\( >10^2 \)) parameters describing both spatial and temporal source variability.

Let us start with a brief description of the spacecraft motion, which determines the time-dependent instrumental response (Eq. 1). During a few \( (N_p \lesssim 10) \) spin periods, the RHESSI motion is composed of an approximately uniform rotation and a slow translation (Figure 1 left). We formalize this by making the following assumptions (which are checked from the aspect solution): (i) the spin period \( T_S \) is constant to accuracy \( \Delta t \) over the time of interest \( (N_P T_S) \); (ii) the wave vectors \( k^i(t) \) rotate clockwise with spin period \( T_S \); (iii) the imaging axis \( P(t) \), which is generally not aligned with the spin axis\(^2 \) \( s(t) \), rotates (clockwise, period \( T_S \)) around \( s(t) \). (iv) the spin axis and \( |s(t)-P(t)| \) vary slowly due to precession \( (f_{\text{prec}} \sim 0.015 \text{ Hz}) \), inertia adjustment, and magnetic torquing (Lin et al., 2003).

We next set up a model for the solar brightness distribution. It is assumed that the true brightness can be represented as

\[
B(x,t) = \sum_k r_k(t) B_k(x)
\]

where \( B_k(x) \) are arbitrary non-negative normalized (\( \int B_k(x) dx = 1 \)) functions and \( r_k(t) \) (ct/s) is the instantaneous count rate. In principle, Eq. (2) describes all possible brightness distributions, but in practice we restrict ourselves to a few source components \( k \); perhaps, a gradual plus an impulsive component with different intrinsic time scales. Impulsive components could be associated with magnetic footpoints. It should be pointed out that Eq. (2) cannot account for continuously moving sources, and that non-solar background is neglected.

Combining the RHESSI response (Eq. 1) with the brightness model (Eq. 2) one can predict the expected counts in time bins \( \Delta t \). As we wish to resolve the source evolution it is assumed that \( r_k(t) \) varies slowly

\(^2\) Strictly speaking, \( s(t) \) denotes the penetration point of the instantaneous RHESSI spin axis through the solar disk.
during $\Delta t$. The expected number of counts in the $i$-th subcollimator and $t$-th time bin is then given by

$$\lambda_t^i = L_t^i \sum_{k=0}^{1} r_{k,t} m_{k,t}$$  \hfill (3)

where the livetime measure $L_t^i$ accounts for detector dead time (Smith et al., 2003) and data gaps$^3$, $r_{k,t} = (\Delta t)^{-1} \int_t^{t+\Delta t} dt' r_k(t')$ is the (discrete) count rate, and

$$m_{k,t}^i = \int_t^{t+\Delta t} dt' \int d\mathbf{x} M^i(\mathbf{x}, t') B_k(\mathbf{x}) .$$  \hfill (4)

We recall that $\mathbf{x}$ denotes heliocentric coordinates. While being natural, these coordinates are not well adapted to the observing geometry, and do not allow to separate fast (rotational) and slow (translational) contributions to modulation. Such a separation is, however, possible if we set $\mathbf{x} = \mathbf{x}' + \langle s \rangle$, where $\langle s \rangle$ is the average spin axis (Figure 1 right; average over $N_P$ spin periods). After expanding the cosine in Eq. (1), Eq. (4) becomes

$$m_{k,t}^i = \int_t^{t+\Delta t} dt' \int d\mathbf{x}' \left[ a_0^i(t') + a_1^i(t') \cos(k(t') \cdot \mathbf{x}') \cos \left\{ k^2(t') \cdot (\langle s \rangle - \mathbf{P}(t')) + \psi(t') \right\} \right]$$

$$- a_1^i(t') \sin(k(t') \cdot \mathbf{x}') \sin \left\{ k^2(t') \cdot (\langle s \rangle - \mathbf{P}(t')) + \psi(t') \right\} B_k(\mathbf{x}' + \langle s \rangle) .$$  \hfill (5)

Now, the term in curly brackets is the grid phase at the average spin axis, which is a slowly varying quantity. If also $a_0^i(t)$ and $a_1^i(t)$ are weakly varying, then Eq. (5) can be approximated by

$$m_{k,t}^i \simeq a_0^i \Delta t + a_1^i \int_t^{t+\Delta t} dt' \int d\mathbf{x}' \cos(k(t') \cdot \mathbf{x}') B_k(\mathbf{x}' + \langle s \rangle)$$

$$- a_1^i \sin \phi_1 \int_t^{t+\Delta t} dt' \int d\mathbf{x}' \sin(k(t') \cdot \mathbf{x}') B_k(\mathbf{x}' + \langle s \rangle)$$

$$\simeq a_0^i \Delta t + a_1^i \cos \phi_1 C_{k,t}^i - a_1^i \sin \phi_1 S_{k,t}^i$$  \hfill (6)

where $a_0^i$ and $a_1^i$ is a discrete version of $a_0^i(t)$ and $a_1^i(t)$, $\phi_1 = k^2 \cdot (\langle s \rangle - \mathbf{P}(t)) + \psi(t)$ is the discrete grid phase at average spin axis, and the last line defines the ($\Delta t$-integrated) visibilities $C_{k,t}^i$ and $S_{k,t}^i$. The visibilities satisfy $(C_{k,t}^i)^2 + (S_{k,t}^i)^2 \leq (\Delta t)^2$, $C_{k,j+N_S/2}^i = C_{k,j}^i$, and $S_{k,j+N_S/2}^i = -S_{k,j}^i$, where $N_S$ is the (even) number of time bins per spin period. The visibility approach (Eqs. 3 and 5) is valid if $|r_k/r| \Delta t \ll 1$ and

$$|\dot{a}_0^i/\dot{a}_0^i| \Delta t \ll 1, \quad |\dot{a}_1^i/\dot{a}_1^i| \Delta t \ll 1, \quad (2\pi)^{-1}|\dot{\psi}| \Delta t \ll 1,$$  \hfill (8)

which must hold during the whole time interval $N_P T_S$. A safe threshold for Eqs. 3 is 0.1, which is checked by the computer code. This is usually uncritical (Figure 1 right): from typical aspect data, one finds that time bins as large as $\Delta t = 0.3$s are admissible even for the finest subcollimator. It should be stressed that this is much larger than the modulation period – the modulation must not be resolved to predict similar ($\Delta t$-integrated) visibilities. Of course, integrating over modulation razes spatial information, but this does not affect the time profiles $r_{k,t}$.

At this point we have completed the forward model (Eqs. 3-7). It has parameters $\{C_{k,j}^i, S_{k,j}^i, r_{k,t}\}$, where $1 \leq j \leq N_S/2$, $1 \leq t \leq N_S \cdot N_P$, and $i$ is out of a subcollimator set satisfying Eq. (8). We turn now to the counting statistics. Let the observed light curve of subcollimator $i$ have $c_i^t$ counts in time bin $t$. These counts should scatter around the expectation value $\lambda_t^i$ (Eq. 3) according to Poisson statistics. The agreement between observation and prediction is therefore measured by the Poisson log likelihood ratio (Eadie et al., 1971) (‘C-statistic’)

$$\log L = \sum_{it} \left\{ - \lambda_t^i + c_i^t (1 + \ln \frac{\lambda_t^i}{c_i^t}) \right\} .$$  \hfill (9)

$^3$Data gaps occur independently in different subcollimators and have durations of milliseconds to seconds (Figure 1 right). They affect some 30% of all data, and are presumably caused by cosmic rays. The livetime measures $L_t^i$ are between zero and one.
The likelihood $L$ is proportional to the probability that $\{c_i\}$ is observed if $\{\lambda_i\}$ was true. Log $L$ is negative, and reduces to $-\frac{1}{2} \chi^2$ in the limit of high count rates. The asterisks in Eq. 9 indicates that the sum is restricted to count rates with livetimes $L_i^t \geq L_{\text{min}}$, with $L_{\text{min}}$ typically chosen as 0.5. This prevents from pile up and from redundant numerical operations, since $0 < L_i^t < L_{\text{min}}$ is rare but $L_i^t = 0$ is frequent ( $\gtrsim 25\%$ data gaps). The rejection of low livetimes is not expected to affect the result.

Since there are more model parameters $\{C_{k,j}, S_{k,j}, r_{k,t}\}$ than observations $\{c_i\}$, and since the model parameters are not independent of each other, the maximum-likelihood problem does not have a unique solution. In order to regularize the problem we assign an a priori probability $\exp\left\{- \frac{1}{2} \sum_{t} (r_{k,t+1} - r_{k,t})^2 \right\}$ to the $k$-th time profile $r_{k,t}$. This choice favors smooth time profiles, and allows an explicit trade-off between the smoothness of the demodulation and the likelihood. It also has the useful feature that it linearly interpolates across data gaps. Taking the a priori probability into account, the logarithm of the total (observational + a priori) probability $P$ becomes, according to Bayes’ rule,

$$\log P = \log L - \frac{1}{2} \sum_{kt} \alpha_k (r_{k,t} - r_{k,t-1})^2.$$  \hspace{1cm} (10)

The quantity $\log P$ possesses a unique maximum which is found by Newton/Marquardt type iterations. The coefficients $\alpha_k$ determine the smoothness of the solutions $r_{k,t}$. Small $\alpha_k$ yield small correlation times $\tau_k$ of $r_{t,k}$. While the exact relation between $\alpha_k$ and $\tau_k$ depends on the actual data set and on the definition of $\tau_k$, there exists a useful empirical estimate,

$$\tau_k \sim \max (\Delta t, \sqrt{\alpha_k \langle c \rangle / \langle a_0 L \rangle}).$$  \hspace{1cm} (11)

In Eq. 11, $\tau_k$ is defined via the normalized autocorrelation $A_k(t)$ of $r_{t,k}$ by $A_k(t) = 1 - \frac{1}{2}(t/\tau_k)^2 + O(t^4)$. The accuracy of Eq. 11 is within a factor two. The adjustment of $\alpha_k$ is done by the user. The Newton/Marquardt iterations have the property that two solutions $r_{t,k}$ agree if their $\alpha_k$ agree.

RESULTS AND DISCUSSION

As a benchmark, Figure 2a-c shows simulations where the model assumption (Eq. 2) is met. There are two sources at $(-715^\circ, 630^\circ)$ and $(-650^\circ, 690^\circ)$, of size 1" and 3", with impulsive (Figure 2a, solid line) and gradual (Figure 2b, dashed) time profiles. The small source sizes yield maximal modulation and therefore represent a worst-case test for demodulation and residual modulation. Fig 2c shows a close-up of the true impulsive time profile, which is to be compared with the estimates c)-e) obtained from simulated ‘observations’ with $\langle c_i \rangle / \Delta t = 6 \cdot 10^5$. The simulated aspect solution has the form $P(t) = v_{\text{prec}} t + r_{\text{cone}} (\cos(\delta - \Omega t), \sin(\delta - \Omega t))$ with $v_{\text{prec}} = (0.8, 1.3)$ asec/s the instantaneous precession rate, $\Omega = 2\pi/4s$ the RHESSI angular frequency, $r_{\text{cone}} = 120^\circ$ the coning radius, and $\delta$ a phase offset. Figure 2d shows the demodulation results (solid line) and averages over subcollimators 1-3 and time (dotted line) with similar nominal resolution. In Figure 2e, $\Delta t=5$ms, and the demodulation $\tau_0,1$+$\tau_1,$ (solid line) of subcollimators 1-9 is compared with a moving average $\langle c \rangle/\langle a_0 L \rangle$ (dotted) over subcollimators 1-2 and time interval $\tau_n=0.1$s. The autocorrelation time constant of the impulsive component $\tau_1$ is $\tau_1 = 0.09$s, while Eq. 11 gives $\tau_1 \sim 0.14$s. Numerically, the rms deviations from the true time profile are 5% (average) and 3% (demodulation), while the statistical error assigned to the $\Delta t$-binned true light curve would be about 6%. While the benefit from 3% to 5% is rather small and within the statistical error, the situation changes if data gaps are present. Figure 2f is similar to 2e, but includes simulated data gaps with $\langle L_i^t \rangle=0.7$. Simultaneous data gaps in subcollimators 1-3 with duration $>\tau_n$ cannot be interpolated with a moving average (dotted line), and the rms deviation of the moving average from the true time profile is 25% (whereas it is 4% for the demodulation). If one tries to improve the moving average by including coarser subcollimators one encounters the difficulty that their (at least, slowest) modulation commensurates with the time scale to be resolved. The moving average then fails even in the absence of data gaps (Figure 2: dotted). A possible way out is demodulation as described in this paper (Figure 2: solid). Here, both the demodulation (solid) and average (dotted) involve subcollimators 7-9. The rms deviation from the true time profile is 6% (demodulation) and 18% (average), respectively. A test was also made for violation of the model assumption (2). Figure 2 shows the demodulation of (simulated) counts from 5 sources with different time profiles. The demodulation (Figure 2 middle curve) uses subcollimators 1-9 and is based on the (wrong) assumption that $B(x, t)=r_0(t)B_0(x)+r_1(t)B_1(x)$. Nevertheless, it has slightly better
Fig. 2. Benchmark simulations: a) true time profiles of impulsive (solid) and gradual (dashed) sources; b) close-up; c) demodulation from subcollimators 1-9 (solid), compared to an average over subcollimators 1-3 and time $\tau_a=0.1s$ (dotted, shifted for better clarity); d) similar to c) including 30% simulated data gaps; e) demodulation (solid) and average (dotted, $\tau_a=0.1s$) of subcollimators 7-9 (no data gaps). f): robustness against violation of Eq. (2) - top to bottom curves: true unmodulated light curve of 5 independent components; two-component demodulation from subcollimators 1-9; average over subcollimators 1-3 and time $\tau_a=0.25s$. The middle and bottom curves are shifted for better clarity. g): iterative convergence of Eq. (10): $-\log P$ (boldface); $-\log L$ (solid); gradual ($k=0$, dotted) and impulsive ($k=1$, dashed) smoothness constraints.

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rms deviation from the true (spatially integrated) light curve (Figure 2f top) than a corresponding average (Figure 2f dotted line). Surprisingly, violation of the model assumption (2) does not degrade the result below the quality of a simple average. The iterative convergence to the maximum-P solution is demonstrated in Figure 2f, referring to simulation (2c). The total probability (boldface) is dominated by the log likelihood (solid line), indicating that the demodulation is primarily determined by the agreement with the observation.

The algorithm is then applied to RHESSI observations of solar flares. A flare with rich temporal fine structures occurred on August 23, 2002, 06:25 UT (Figure 3). The energy band under consideration is 6-25 keV, and the mean observed count rate is 1700/s/subcollimator. The time bins are 9.78ms, so that there are $\gtrsim 10$ counts per time bin. The flare center is at (920°,-160°) (Figure 3d-e). The time scale achievable in the demodulation is in the order of 0.1s, by the order-of-magnitude argument that for each (temporal+spatial) degree of freedom there should be some 100 photons. A first estimate on the true light curve is provided by the average $\langle c \rangle / \langle a_0 L \rangle$ over subcollimators 1,3,4 and time $\tau_a=0.08s$ (Figure 3a-b, dotted, shifted for better clarity). The demodulation (solid line) is obtained from all subcollimators except #2 and #5, which are excluded because of high background (#2) (Smith et al., 2003), and a distinctly inferior likelihood (#5) which is not fully understood at present. The decomposition into impulsive and gradual components (Eq.
one obtains the error bars shown in Figure 3b. The average error is 1400 ct/s. Large error bars are characteristic for times with data gaps in several subcollimators.

Let us finish this discussion with a last but important point. Although the formalism of Eq. (10) allows for arbitrarily small count rates, there is no gain in temporal resolution if \( \Delta t \) is made so small that only few (or fractions of) photons are contained in a time bin. At such low count rates, the Poisson error is large, and Eq. (10) becomes dominated by the smoothness constraint. Higher time resolution can only be achieved at higher count rates, or at the expense of statistical significance (\( \chi^2_{\text{red}} < 1 \)). As a conservative rule-of-thumb, a few \( 10^4 \) counts/s are needed to resolve 100 ms structures on the 5% level. It is not easy to beat this limit, not even for extremely transient and bright events, because their modulation is hard to estimate and dominates the Poisson error.

Fig. 3. Flare of August 23, 2002, 06:25 UT: a) average over subcollimators 1,3,4 and time \( \tau_0 = 0.08 \) s (dotted, shifted for better clarity), demodulation \( r_{0,t} + r_{1,t} \) (solid line) and gradual component \( r_{0,t} \) (smooth curve) from subcollimators 1,3,4,6,7,8,9; b) close-up of demodulation (solid) with errors (gray, see text) and moving average (dotted); c) example of observed (crosses) and predicted (gray line) counts for subcollimator 6. d), e): CLEAN images at total, gradual maxima (axes in heliocentric arc seconds).

2] removes data gaps, and also shows that the gradual component appears delayed (Figure 3a, smooth curve). This seems consistent with a spatial change of the brightness distribution (Figure 3c1-e), suggesting the emergence of a gradual (possibly thermal) source at \( \sim (924'', 155'' \) ). Figure 3 shows an example of observed counts (crosses), together with the predicted Poisson parameter (gray line). The data gaps have zero counts. The correlation time of \( r_{1,t} \) derived from the autocorrelation is \( \tau_1 = 0.163 \) s; the estimate (Eq. 11) gives 0.164 s. An important issue is the reliability of the demodulated fine structures. This can be tested by locally perturbing the maximum-P solution \( r_{1,t} \) until \( \log L \) changes by a given amount (Eadie et al. 1971). Adopting the conventional threshold \( \Delta \log L = \frac{1}{2} \), one obtains the error bars shown in Figure 3b. The average error is 1400 ct/s. Large error bars are characteristic for times with data gaps in several subcollimators.

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