The Casimir effect at finite temperature in a six-dimensional vortex scenario

Hongbo Cheng*
Department of Physics, East China University of Science and Technology,
Shanghai 200237, China

Abstract

The Casimir effect for parallel plates satisfying the Dirichlet boundary condition in the context of effective QED coming from a six-dimensional Nielsen-Olesen vortex solution of the Abelian Higgs model with fermions coupled to gravity is studied at finite temperature. We find that the sign of the Casimir energy remains negative under the thermal influence. It is also shown that the Casimir force between plates will be weaker in the higher-temperature surroundings while keeps attractive. This Casimir effect involving the thermal influence is still inconsistent with the known experiments. We find that the thermal correction can not compensate or even reduce the modification from this kind of vortex model to make the Casimir force to be in less conflict with the measurements.

PACS number(s): 03.70.+k, 03.65.Ge

*E-mail address: hbcheng@ecust.edu.cn
I. Introduction

In order to unify the interactions in nature and solve the long-standing problems such as the hierarchy problem, the cosmological constant problem, etc. from particle physics and cosmology, various models of higher-dimensional spacetime were put forward and attract a lot of attention [1-13]. Within the frame of field theory for the approaches with more than four dimensions, the mechanisms for localization of the fields like scalar field, fermionic field and gauge field on the models must be needed, then these models can describe the matter fields and interactions [14-19]. It should be pointed out that an effective quantum electrodynamics in four-dimensional spacetime generated from a Higgs model with fermions coupled to gravity in the world with six dimensions lead the fermionic and gauge functions spread on the transverse direction in a small region around the core of a Nielsen-Olesen vortex [16]. In this issue the localization of gauge field was achieved and the Nielsen-Olesen vortex comes from the fluctuations of graviton and gauge field. It is significant that this construction admits the gravity and gauge field besides the scalar and fermionic fields.

The Casimir effect is essentially a direct consequence of quantum field theory subject to a change in the spacetime of vacuum oscillations under the inserted background field [20-26]. It is important to discuss the sign of the Casimir energy and the nature of the Casimir force for many subjects because the Casimir effect depends on various factors. The precision of the measurements has been greatly improved experimentally [27]. We can compare the theoretical results with the phenomena to explore the properties of various worlds. The Casimir effect has opened a new window to explore the topics. The Casimir effect for parallel plates in the spacetime with extra compactified dimensions was discussed [28-38]. The distinct influence from the fractal additional compactified dimension can be exhibited in the Casimir effect for parallel plates [39, 40]. The Casimir effect for the same system in the braneworld was also studied [41-50].

It is significant to research on the Casimir effect for parallel plates within the frame of the effective QED. The new QED developing from a six-dimensional Abelian Higgs model coupled to gravity in a Nielsen-Olesen vortex background with fermions is a mixture of the original six-dimensional metric and the vector potential [16]. The coupling constant and the size of the core of the six-dimensional vortex as the own features of this kind of effective gauged field theory modify the standard Casimir effect between parallel plates obviously [51]. The manifest deviations are not consistent with the known experiments, which making the phenomenological viability of the model less [51]. The works on the modified Casimir force can be used to explore the four-dimensional effective QED in a new direction [51].

The quantum field at finite temperature shares many effects. In many cases the thermal influence on the Casimir effect cannot be neglected, and its influence certainly modifies the effect. The Casimir effect for parallel plates under a nonzero temperature environment in the presence of additional compactified dimensions was considered and the magnitude of Casimir force as well as the sign of Casimir energy change with the temperature [53-55]. The Casimir energy for a rectangular cavity including thermal corrections was considered and the temperature controls the energy
The Casimir effect for a scalar field within two parallel plates under thermal influence in the bulk region of Randall-Sundrum models was also evaluated [57, 58]. In addition, the thermal modification to the Casimir effect for parallel plates involving massless Majorana fermions was analyzed [59].

It is also fundamental to probe the Casimir effect for parallel-plate system at finite temperature in the context of the effective QED. To our knowledge little contribution was made to this topic. Now we plan to study the Casimir effect for parallel plates in the background of a six-dimensional vortex scenario to generalize some of the conclusions of Ref. [51]. We wonder how the thermal influence modifies the Casimir effect in the vortex scenario and the difference between the effect limited by the higher-dimensional vortex and the standard one in particular. At first we derive the frequency of massless scalar field subject to the Dirichlet boundary conditions on the plates with thermal corrections in a Nielsen-Olesen vortex background by means of finite-temperature field theory. We regularize the frequency to obtain the Casimir energy density with the help of the zeta function technique and the Casimir force between the parallel plates further. We will compare our results with the standard Casimir effect to determine how the temperature change their difference. Our discussions and conclusions are listed at the end of this paper.

II. The Casimir effect at finite temperature in a six-dimensional vortex scenario

We make use of the imaginary time formalism to describe the Higgs field and gauge field in thermal equilibrium to introduce a partition function for a parallel-plate device [52],

\[
Z = N' \int_{\text{periodic}} DA_\mu D\Phi \exp\left[\int_0^\beta d\tau \int d^5x \sqrt{-g} L(\Phi, A_\mu, \partial_\mu \Phi, \partial_\mu A_\mu)\right]
\]

where \( L \) is the Lagrangian density for Higgs field and gauge field within the parallel-plate structure under consideration. \( N' \) is a constant and "periodic" means [52],

\[
\Phi(0, x^i) = \Phi(\tau = \beta, x^i)
\]

\[
A_\mu(0, x^i) = A_\mu(\tau = \beta, x^i)
\]

where \( i = 1, 2, 3, 4, 5 \). \( \beta = \frac{1}{T} \) is the inverse of the temperature and \( \tau = it \) The scalar field satisfies the Dirichlet boundary condition on the plates to lead the wave vector in the direction restricted by the plates to be \( k_N = \frac{\pi N}{L} \), where \( L \) is the plate separation. According to the solution to the field equations of effective QED [16, 51] and the boundary conditions on the fields, the generalized zeta function can be written as [52],

\[
\zeta(s, -\partial_E) = Tr(-\partial_E)^{-s} = \frac{1}{2\beta} \int \frac{d^2k_\perp dk_r}{(2\pi)^3} \sum_{n,N=1}^{\infty} \sum_{q=-\infty}^{\infty} \left[ k_\perp^2 + \frac{2e}{\pi^2} k_r^2 + \frac{\pi^2 N^2}{L^2} + \frac{2}{el^2} \frac{1}{n^2} + \left(\frac{2q\pi}{\beta}\right)^2\right]^{-s}
\]
where
\[
\partial E = \frac{\partial^2}{\partial r^2} - g^{ij}\partial_i\partial_j
\]  
(5)
leading the dispersion relation shown in Eq. (4) involving not only the terms from 4-dimensional spacetime but also the contributions from the additional compactified space and \(g_{\mu\nu}\) with \(\mu, \nu = 0, 1, 2, 3, 4, 5\) is the metric of six-dimensional vortex scenario [16, 18]. Here \(k_\perp\) denotes the two transverse components of the momentum. In the two-dimensional additional space the continuum variable \(k_r\) comes from the radial part and the other component coming from the vortex number appear as the term \(\frac{2}{\varepsilon l^2 n^2}\) with \(l = \frac{\kappa}{e}\) the ratio of the six-dimensional gravitational constant and the parameter of the Abelian Higgs model [51]. Here the parameter \(a\) standing for the size of the vortex’s core is due to the integration in \(k_r\) [51]. The factor \(p\) represents the possible polarization of the photon [51]. We make use of \(\varepsilon = -\frac{\partial}{\partial \beta}(\frac{\partial c(\kappa; -p_s)}{\partial s}|_{s=0})\) [52] to obtain the total energy density of the system with thermal corrections as follow,
\[
\varepsilon = \frac{1}{4\pi^2 a p} \sqrt{\frac{\varepsilon}{2 \varepsilon l^2}} \Gamma\left(\frac{3}{2}\right) \Gamma(-\frac{3}{2}) M_3(-\frac{3}{2}; \frac{2}{\varepsilon l^2}, -L^2, 2, 2, 2, -2)
\]  
(6)
where the multiple zeta functions with arbitrary exponents is [23, 24],
\[
M_N(s; a_1, a_2, \cdots, a_N; \alpha_1, \alpha_2, \cdots, \alpha_N) = \sum_{n_1, n_2, \cdots, n_N=1}^{\infty} (a_1 n_1^{\alpha_1} + a_2 n_2^{\alpha_2} + \cdots + a_N n_N^{\alpha_N})^{-s}
\]  
(7)
With the help of the zeta function technique [23, 24], we regularize the total energy density to obtain the Casimir energy per unit area for parallel plates at finite temperature,
\[
\varepsilon_C = \frac{\pi^{-\frac{5}{2}}}{32} a p \sqrt{\frac{\varepsilon}{2 \varepsilon l^2}} \Gamma\left(\frac{3}{2}\right) \Gamma(-\frac{3}{2}) \zeta(4) - \frac{\pi^{-3}}{32} a p \sqrt{\frac{\varepsilon}{2 \varepsilon l^2}} \sqrt{\frac{\varepsilon}{L}} \Gamma\left(\frac{3}{2}\right) \Gamma(-\frac{5}{2}) \zeta(5)
\]
\[
- \frac{\pi^{-3}}{8} a p \sqrt{\frac{\varepsilon}{2 \varepsilon l^2}} \sqrt{\frac{\varepsilon}{L}} L^{-\frac{5}{2}} \Gamma\left(\frac{3}{2}\right) \sum_{n_1, n_2=1}^{\infty} \left(\frac{1}{n_1 n_2}\right)^{\frac{1}{2}} K_{\frac{3}{2}}\left(2\sqrt{\frac{L}{n_1}}\right)
\]
\[
+ \frac{\pi^{-\frac{5}{2}}}{2} a p \sqrt{\frac{\varepsilon}{2 \varepsilon l^2}} \sqrt{\frac{\varepsilon}{L}} \Gamma\left(\frac{3}{2}\right) \sum_{n_1, n_2, n_3=1}^{\infty} n_1^{-1} \left(\frac{\pi^2}{L^2} n_2^2 + \frac{1}{\varepsilon l^2} n_3^2\right)
\]
\[
\times K_2(\beta n_1) \sqrt{\frac{\pi^2}{L^2} n_2^2 + \frac{1}{\varepsilon l^2} n_3^2}
\]
\[
+ \frac{\pi^{-\frac{5}{2}}}{2} a p \sqrt{\frac{\varepsilon}{2 \varepsilon l^2}} \sqrt{\frac{\varepsilon}{L}} \Gamma\left(\frac{3}{2}\right) \sum_{n_1, n_2, n_3=1}^{\infty} n_1^{-1} \left(\frac{\pi^2}{L^2} n_2^2 + \frac{1}{\varepsilon l^2} n_3^2\right)^{\frac{1}{2}}
\]
\[
\times [K_1(\beta n_1) \sqrt{\frac{\pi^2}{L^2} n_2^2 + \frac{1}{\varepsilon l^2} n_3^2} + K_3(\beta n_1) \sqrt{\frac{\pi^2}{L^2} n_2^2 + \frac{1}{\varepsilon l^2} n_3^2}]
\]
\[
< 0
\]  
(8)
where \(K_{\nu}(z)\) is the modified Bessel function of the second kind. In the expression of Casimir energy like Eq. (8) the terms with series converge very quickly and only the first several summands need to
be taken into account for numerical calculation to further discussion. If the temperature approaches zero, the Casimir energy will recover to be the results of Ref. [51], so will the Casimir force. The Casimir energy remains negative no matter how strong the thermal influence is.

In the six-dimensional vortex background the Casimir force on the plates is also obtained by the derivative of the Casimir energy with respect to the plate distance. The Casimir force per unit area on the plates due to the Dirichlet boundary condition becomes,

$$ f_C' = \frac{1}{32\pi^3} ap \sqrt{\frac{\epsilon}{2}} \left( \frac{2}{e^2} \right)^{\frac{3}{2}} \Gamma \left( \frac{3}{2} \right) \Gamma \left( \frac{-5}{2} \right) \zeta(5) + f_C $$

(9)

It should be pointed out that the first term in the expression above also has nothing to do with the plates gap, which means that there are two forces with the same magnitude and the opposite direction acting on the plates respectively, so the two forces will be compensated each other on every plate. We can remove this term to write the net Casimir force as follow,

$$ f_C = -\frac{3}{16\pi^3} \Gamma \left( \frac{3}{2} \right) ap \sqrt{\frac{\epsilon}{2}} \left( \frac{2}{e^2} \right)^{\frac{3}{2}} \frac{1}{L^3} \sum_{n_1,n_2=1}^{\infty} \left( \frac{1}{n_1n_2} \right)^{\frac{3}{2}} K_2 \left( 2 \sqrt{\frac{L n_1}{e^2 n_2}} \right) $$

$$ \times \left[ K_3 \left( 2 \sqrt{\frac{L n_1}{e^2 n_2}} \right) + K_2 \left( 2 \sqrt{\frac{L n_1}{e^2 n_2}} \right) \right] $$

$$ + \frac{2}{\pi^2} \Gamma \left( \frac{3}{2} \right) ap \sqrt{\frac{\epsilon}{2}} \frac{1}{\beta L^3} \sum_{n_1,n_2,n_3=1}^{\infty} \left( \frac{n_2}{n_1} \right)^2 K_2 \left( \beta n_1 \sqrt{\frac{\pi^2}{L^2 n_2^2} + \frac{2}{e^2 n_3^2}} \right) $$

$$ + \frac{1}{\pi^2} \Gamma \left( \frac{3}{2} \right) ap \sqrt{\frac{\epsilon}{2}} \frac{1}{\beta L^3} \sum_{n_1,n_2,n_3=1}^{\infty} \left( \frac{n_2}{n_1} \right)^2 \left( \frac{\pi^2}{L^2 n_2^2} + \frac{2}{e^2 n_3^2} \right)^{\frac{3}{2}} $$

$$ \times \left[ K_1 \left( \beta n_1 \sqrt{\frac{\pi^2}{L^2 n_2^2} + \frac{2}{e^2 n_3^2}} \right) + K_3 \left( \beta n_1 \sqrt{\frac{\pi^2}{L^2 n_2^2} + \frac{2}{e^2 n_3^2}} \right) \right] $$

$$ - \frac{1}{4\pi^2} \Gamma \left( \frac{3}{2} \right) ap \sqrt{\frac{\epsilon}{2}} \frac{1}{\beta L^3} \sum_{n_1,n_2,n_3=1}^{\infty} \left( \frac{n_2}{n_1} \right)^2 \left( \frac{\pi^2}{L^2 n_2^2} + \frac{2}{e^2 n_3^2} \right) $$

$$ \times \left[ K_0 \left( \beta n_1 \sqrt{\frac{\pi^2}{L^2 n_2^2} + \frac{2}{e^2 n_3^2}} \right) + 2K_2 \left( \beta n_1 \sqrt{\frac{\pi^2}{L^2 n_2^2} + \frac{2}{e^2 n_3^2}} \right) \right] $$

$$ + K_4 \left( \beta n_1 \sqrt{\frac{\pi^2}{L^2 n_2^2} + \frac{2}{e^2 n_3^2}} \right) $$

(10)

The nature of the net Casimir force appearing as the Casimir effect is attractive. Similarly, if the temperature vanishes, the Casimir pressure shown in Eq. (10) will be consistent with that of Ref. [51]. When the two plates are moved farther and farther away from each other, the Casimir effect will disappear gradually,

$$ \lim_{L \to \infty} f_C = 0 $$

(11)

It is also evident that the Casimir force is weaker and weaker with stronger thermal influence like,
\[ \lim_{T \to \infty} f_C = 0 \quad (12) \]

The asymptotic behaviours of the Casimir force in the cases of two parallel plates locating remotely each other or higher temperature respectively are favoured experimentally [27]. The Casimir energy and the Casimir force are modified by the parameters of six-dimensional vortex as well as the temperature, but the influence from the topological defect works as a multiplicative factor on the Casimir effect involving the thermal correction rather than only term added in the expressions, which is similar to the results in Ref. [51]. The terms associated with the temperature are also multiplied by the model variables and can not diminish the vortex correction independently. In the environment of six-dimensional vortex, the shapes of the net Casimir force between two parallel plates subject to the various temperature are similar although these thermal modifications are manifest. The factors showing the gauge coupling and the size of the vortex with six dimensions are dominant to the whole Casimir effect, but the evident deviations from these factors are inconsistent with the observational results [27, 51] and the thermal influence can not cancel and even reduce the experimentally exclusive deviations.

III. Discussion

In this paper the Casimir effect with thermal correction for parallel plates is investigated in the frame of six-dimensional Nielsen-Olesen vortex with fermions coupling to gravity. At first we obtain the Casimir energy for parallel-plate system satisfying the Dirichlet boundary condition under the vortex-controlled environment with more than four dimensions at finite temperature and find that the energy remains negative no matter how the temperature changes. It is also found that the magnitude of the Casimir force between two parallel plates with the same boundary conditions will be weaker as the surroundings are hotter while the Casimir force keeps attractive. As the temperature is extremely high, the Casimir force on the plates will vanish. The Casimir force between the plates will approach zero when the two plates are moved apart from each other much farther. It should be pointed out that the thermal influence modifies the Casimir effect such as the magnitude of the Casimir force and even makes the force nearly disappear at extremely high temperature, but the temperature can not greatly weaken or cancel the effect from the multiplicative factor of the effective QED describing the higher-dimensional Nielsen-Olesen vortex with fermions coupling to gravity. Although the Casimir effect we consider here contains the thermal corrections, the parameters of the effective QED control the shape of the Casimir effect, so the thermal influence can not improve the phenomenological viability of the model.

Acknowledge

This work is supported by NSFC No. 10875043.
References

[1] T. Kaluza, Sitz. Preuss. Akad. Wiss. Phys. Math. K1 1(1921)966

[2] O. Klein, Z. Phys. 37(1926)895

[3] V. A. Rubakov, M. E. Shaposhnikov, Phys. Lett. B125(1983)136

[4] M. Visser, Phys. Lett. B159(1985)22

[5] P. Horava, E. Witten, Nucl. Phys. B460(1996)506

[6] P. Horava, E. Witten, Nucl. Phys. B475(1996)94

[7] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, Phys. Lett. B429(1998)263

[8] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, Phys. Lett. B436(1998)257

[9] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, Phys. Rev. D59(1999)086004

[10] L. Randall, R. Sundrum, Phys. Rev. Lett. 83(1999)3370

[11] L. Randall, R. Sundrum, Phys. Rev. Lett. 83(1999)4690

[12] I. Brevik, K. A. Milton, S. Nojiri, S. D. Odintsov, Nucl. Phys. B599(2001)305

[13] E. Elizalde, S. Nojiri, S. D. Odintsov, Phys. Rev. D70(2004)043539

[14] T. Gherghetta, M. E. Shaposhnikov, Phys. Rev. Lett. 85(2000)041

[15] S. L. Dubovsky, V. A. Rubakov, P. G. Tinyakov, JHEP 0008(2000)041

[16] S. Randjbar-Daemi, M. Shaposhnikov, JHEP 0304(2003)016

[17] M. Giovannini, Phys. Rev. D66(2002)044016

[18] M. Giovannini, H. Meyer, M. E. Shaposhnikov, Nucl. Phys. B619(2001)615

[19] S. Randjbar-Daemi, M. Shaposhnikov, Nucl. Phys. B645(2002)188

[20] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. 51(1948)793

[21] G. Plunien, B. Muller, W. Greiner, Phys. Rep. 134(1986)87

[22] J. Ambjøen, S. Wolfram, Ann. Phys. (N. Y.) 147(1983)1

[23] E. Elizalde, S. D. Odintsov, A. Romeo, A. A. By senko, S. Zerbini, Zeta Regularization Techniques with Applications, World Scientific, Singapore, 1994

[24] E. Elizalde, Ten Physical Applications of Spectral Zeta Functions, Springer-Verlag, Berlin, 1995
[25] K. A. Milton, Physical Manifestation of Zero-Point Energy, World Scientific, Singapore, 2001
[26] V. M. Mostepanenko, N. N. Trunov, The Casimir Effect and its Applications, Oxford University Press, Oxford, 1997
M. Bordag, U. Mohideen, V. M. Mostepanenko, Phys. Rep. 353(2001)1
[27] U. Mohideen, A. Roy, Phys. Rev. Lett. 81(1998)4549
S. K. Lamoreaux, Rep. Prog. Phys. 68(2005)201
[28] K. Poppenhaeger, S. Hossenfelder, S. Hofmann, M. Bleicher, Phys. Lett. B582(2004)1
[29] H. Cheng, Mod. Phys. Lett. A21(2006)1957
[30] H. Cheng, Phys. Lett. B643(2006)311
[31] R. M. Cavalcanti, Phys. Rev. D69(2004)065015
[32] M. P. Hertzberg, R. L. Jaffe, M. Kardar, A. Scardicchio, Phys. Rev. Lett. 95(2005)250402
[33] A. Edery, Phys. Rev. D75(2007)105012
[34] A. Edery, N. Graham, I. MacDonald, Phys. Rev. D79(2009)125018
[35] H. Cheng, Phys. Lett. B668(2008)72
[36] K. Kirsten, S. A. Fulling, Phys. Lett. B671(2009)179
K. Kirsten, S. A. Fulling, Phys. Rev. D79(2009)065019
[37] K. A. Milton, J. Wagner, Phys. Rev. D80(2009)125028
[38] E. Elizalde, S. D. Odintsov, A. A. Saharian, Phys. Rev. D79(2009)065023
[39] H. Cheng, Int. J. Theor. Phys. 52(2013)3229
[40] H. Cheng, Commun. Theor. Phys. 58(2012)229
[41] A. Flachi, D. Toms, Nucl. Phys. B610(2001)144
[42] A. A. Saharian, M. R. Setare, Phys. Lett. B552(2003)119
[43] E. Elizalde, S. Nojiri, S. D. Odintsov, S. Ogushi, Phys. Rev. D67(2003)063515
[44] J. Garriga, A. Pomarol, Phys. Lett. B560(2003)91
[45] M. Frank, I. Turan, L. Ziegler, Phys. Rev. D76(2007)015008
[46] R. Linares, H. A. Morales-Tecotl, O. Pedraza, Phys. Rev. D77(2008)066012
[47] M. Frank, N. Saad, I. Turan, Phys. Rev. D78(2008)055014
[48] H. Cheng, Chin. Phys. Lett. 27(2010)031101
    H. Cheng, Commun. Theor. Phys. 53(2010)1125

[49] M. P. Hertzberg, R. L. Jaffe, M. Kardar, A. Scardicchio, Phys. Rev. D76(2007)045016

[50] A. Flachi, T. Tanaka, Phys. Rev. D80(2009)124022

[51] R. Linares, H. A. Morales-Tecotl, O. Pedraza, Phys. Lett. B633(2006)362

[52] D. Bailin, A. Love, Introduction to Gauge Field Theory, IOP Publishing Limited, 1986

[53] H. Cheng, Chin. Phys. Lett. 22(2005)3032

[54] L. P. Teo, Phys. Lett. B672(2009)190
    L. P. Teo, Nucl. Phys. B819(2009)431

[55] L. P. Teo, JHEP 0906(2009)076

[56] H. Cheng, J. Phys. A35(2002)2205

[57] M. Rypestol, I. Brevik, New J. Phys. 12(2010)013022

[58] H. Cheng, Chin. Phys. C35(2011)1084

[59] H. Cheng, Phys. Rev. D82(2010)045005