How to realize a negative refractive index material at the atomic level in an optical frequency range

Jian-Qi Shen\textsuperscript{1*}, Zhi-Chao Ruan\textsuperscript{1}, and Sailing He\textsuperscript{1,2†}

\textsuperscript{1} Centre for Optical and Electromagnetic Research, Joint Research Centre of Photonics of the Royal Institute of Technology (Sweden) and Zhejiang University, Zhejiang University, Hangzhou Yuquan 310027, P. R. China

\textsuperscript{2} Laboratory of Photonics and Microwave Engineering, Department of Microelectronics and Information Technology, Royal Institute of Technology, Electrum 229, SE-164 40 Kista, Sweden

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The theoretical mechanism for realizing the negative refractive index with electromagnetically induced transparency (EIT) is studied. It is shown that in a three-level dense atomic gas, there is a frequency band in which the EIT medium will exhibit simultaneously negative electric permittivity and magnetic permeability in the optical frequency range, and the atomic gas thus becomes a left-handed material. The expressions for the electric permittivity and the magnetic permeability for the probe frequency is presented. One of the remarkable features of the present novel scheme is such that the obtained EIT left-handed material is isotropic and may therefore have some potentially important applications in the development of new techniques in quantum optics.

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\section{I. INTRODUCTION}

More recently, a kind of artificial composite metamaterials (so-called \textit{left-handed media} or \textit{negative refractive index media}) having a frequency band where the electric permittivity and the magnetic permeability are simultaneously negative attracts considerable attention of many researchers in various fields such as materials science, condensed matter physics, optics and classical applied electromagnetism [1–6]. It can be readily verified that the left-handed media exhibit a number of peculiar electromagnetic and optical properties, including reversals of both the Doppler shift and Cherenkov radiation, anomalous refraction and amplification of evanescent wave. In experiments, a combination of two structures (\textit{array of long metallic wires} and \textit{split ring resonators} [4,5]) yields such a type of artificial negative refractive index media [3]. Although Veselago’s original paper [1] and most of the recent theoretical works investigated mainly the electromagnetic and optical properties in the \textit{isotropic} left-handed media [7], up to now, the left-handed media that have been prepared successfully experimentally are actually \textit{anisotropic} in nature, and it may be very difficult to prepare an isotropic left-handed medium [2,6,8]. In the previous experimental and theoretical work, investigators used classical optical approaches to design and fabricate the negative refractive index materials [3–5,9,10]. In the present paper, we will suggest an alternative method (quantum optical approach) to realize the negative refractive index: specifically, under certain conditions, the electric-dipole and magnetic-dipole transitions in a multilevel EIT (electromagnetically induced transparency) atomic system will exhibit simultaneously negative permittivity and negative permeability. Recently, many theoretical and experimental investigations show that the control of phase coherence in multilevel atomic ensembles will give rise to many novel and striking quantum optical phenomena in the wave propagation of near-resonant light [11–13]. One of the most interesting phenomena is electromagnetically induced transparency [11]. More recently, some unusual physical effects associated with EIT observed experimentally include the ultraslow light pulse propagation, superluminal light propagation, light storage in atomic vapor and atomic ground state cooling, some of which are believed to be useful for the development of new techniques in quantum optics [11,14,15].

\*E-mail address: jqshen@coer.zju.edu.cn

\†E-mail address: sailing@kth.se
In the following, we will consider a new property of atomic media, i.e., the possibility for the EIT media to become the left-handed media under some conditions. Consider a $\Lambda$-type three-level atomic ensemble with one upper level $|a\rangle$ and two lower levels $|b\rangle$ and $|c\rangle$ (see Fig. 1). Such an atomic system interacts with two optical fields, i.e., the coupling laser and the probe laser, which couple the level pairs $|a\rangle$-$|c\rangle$ and $|a\rangle$-$|b\rangle$, respectively. Here we assume that the coupling laser is in resonance with the $|a\rangle$-$|c\rangle$ transition, while the probe laser has a frequency detuning $\Delta$ that is defined by $\Delta = \omega_{ab} - \omega$, where $\omega_{ab}$ and $\omega$ denote the $|a\rangle$-$|b\rangle$ transition frequency and the probe mode frequency, respectively. Note that since the level pairs $|a\rangle$-$|c\rangle$ and $|a\rangle$-$|b\rangle$ can be coupled to two laser fields, the parity of level $|a\rangle$ is different from both $|b\rangle$ and $|c\rangle$. Thus, the electric dipole matrix elements $\vec{\rho}_{ab} = \langle a|e\vec{r}|b\rangle \neq 0$ and $\vec{\rho}_{cb} = \langle c|e\vec{r}|b\rangle = 0$, and the magnetic dipole matrix elements $\vec{m}_{ab} = \langle c|(e/2m_e)(\vec{L} + 2\vec{S})|b\rangle \neq 0$ and $\vec{m}_{ab} = \langle a|(e/2m_e)(\vec{L} + 2\vec{S})|b\rangle = 0$, where $\vec{L}$ and $\vec{S}$ denote the operators of the orbital angular momentum and spin of electrons, respectively. So, it is possible for the nearly resonant probe laser to couple the electric-dipole transition between levels $|a\rangle$ and $|b\rangle$, and the magnetic-dipole transition between levels $|c\rangle$ and $|b\rangle$ in the three-level atomic medium. The electric-dipole transition ($|a\rangle$-$|b\rangle$) and the magnetic-dipole transition ($|c\rangle$-$|b\rangle$) will yield the electric polarizability and the magnetic susceptibility at probe frequency, respectively. In general, the dimensionless ratio $|\vec{m}_{cb}| / (\rho_{abc}) \approx 10^{-2}$ in an atomic system, where $c$ denotes the speed of light in vacuum. For this reason, the magnetic-dipole transition may not be considered in the treatment for the wave propagation in an artificially electromagnetic material. However, in a three-level EIT medium where the intensity of coupling laser is much larger than that of the probe light, the population in level $|c\rangle$ is much greater than that in the upper level $|a\rangle$. In other words, the stronger coupling laser enhances the probability amplitude of level $|c\rangle$. Thus, the order of magnitude of the density matrix element $\rho_{cb}$ may be larger than that of $\rho_{ab}$. This, therefore, means that the magnetic dipole moment $\langle 2m_{cb}\rho_{cb}\rangle$ may possibly have the same order of magnitude of the electric dipole moment $\langle 2e\vec{\rho}_{ab}\rho_{ab}\rangle$. Further analysis shows that such an EIT medium may exhibit negative permittivity and permeability, and will therefore become an ideal candidate for realizing isotropic left-handed media.

II. PERMITTIVITY AND PERMEABILITY IN A THREE-LEVEL EIT MEDIUM

We should first consider the steady density matrix elements of this EIT. The density matrix elements, $\rho_{ab}$ and $\rho_{cb}$, of such a three-level system can be rewritten as $\rho_{ab} = \tilde{\rho}_{ab}\exp (-i\omega t)$ and $\rho_{cb} = \tilde{\rho}_{cb}\exp (-i(\omega + \omega_{ab})t)$. Here, we assume that the intensity of the probe laser is sufficiently weak and therefore nearly all the atoms remain in the ground state, i.e., the atomic population in level $|b\rangle$ is unity. Under this assumption, $\tilde{\rho}_{ab}$ and $\tilde{\rho}_{cb}$ satisfy the following matrix equation [16]

$$\frac{\partial}{\partial t} \begin{pmatrix} \tilde{\rho}_{ab} \\ \tilde{\rho}_{cb} \end{pmatrix} = \begin{pmatrix} -(\gamma_1 + i\Delta) & \frac{i}{2}\Omega_c \\ \frac{i}{2}\Omega_c^* & -(\gamma_3 + i\Delta) \end{pmatrix} \begin{pmatrix} \tilde{\rho}_{ab} \\ \tilde{\rho}_{cb} \end{pmatrix} + \begin{pmatrix} \frac{i\varphi_{ab}\mathcal{E}}{2h} \\ 0 \end{pmatrix},$$

(1)

where $\gamma_1$ and $\gamma_3$ represent the spontaneous decay rate of level $|a\rangle$ and the dephasing rate (nonradiative decay rate) of $|c\rangle$, respectively. $\varphi_{ab}$, $\mathcal{E}$ and $\Omega_c$ denote the electric dipole matrix element, the probe field envelope and the Rabi frequency of coupling laser ($\Omega_c = \varphi_{ac}E_c/h$ with $E_c$ being the electric field strength of the coupling laser). It can be readily verified that the steady solution of Eq. (1) takes the following form

$$\tilde{\rho}_{ab} = \frac{i\varphi_{ab}\mathcal{E}(\gamma_3 + i\Delta)}{2h[(\gamma_1 + i\Delta)(\gamma_3 + i\Delta) + \Omega_c^2/4]}, \quad \tilde{\rho}_{cb} = -\frac{\varphi_{ab}\mathcal{E}\Omega_c^*}{4h[(\gamma_1 + i\Delta)(\gamma_3 + i\Delta) + \Omega_c^2/4]}.$$

(2)

Apparently, there exists a relation between $\tilde{\rho}_{cb}$ and $\tilde{\rho}_{ab}$, i.e.,

$$\tilde{\rho}_{cb} = \frac{i}{2} \left( \frac{\Omega_c^*}{\gamma_3 + i\Delta} \right) \tilde{\rho}_{ab}.$$

(3)

As to the problem of local field correction, here, in the simplest case (valid for gases), we can take the local field to be the same as the macroscopic field (average field in the sample). So, we need not consider the Clausius-Mossotti-Lorentz relation [17]. In a three-level atomic system, the electric polarizability $\chi_e$ and the magnetic susceptibility $\chi_m$ are of the form $\chi_e = 2N\varphi_{ab}\rho_{ab}/(\varepsilon_0\mathcal{E})$ and $\chi_m = 2N\varphi_{cb}\rho_{cb}/\mathcal{H}$, respectively, where $N$ denotes the atomic density (total number of atoms per volume). Thus, with the help of the steady solution (2), one can obtain the relative permittivity $\varepsilon_r = 1 + \chi_e$ and the relative permeability $\mu_r = 1 + \chi_m$ of the above atomic system. By using the relation $\mathcal{H} = \sqrt{\varepsilon_r\varepsilon_0/\mu_r\mu_0}\mathcal{E}$ between the envelopes of magnetic and electric fields, we have the expressions for the electric polarizability and the magnetic susceptibility, i.e.,

2
Since the dephasing rate $\gamma_3$ is small compared with the spontaneous decay rate $\gamma_1$ ($\gamma_3$ is in general two or three orders of magnitude less than $\gamma_1$) [14], in the following analysis we will ignore $\gamma_3$.

With the help of Eqs. (4), one can obtain

$$
\begin{align*}
\epsilon_r &= 1 + \chi_e = 1 - \frac{N|\varphi_{ab}|^2 \Delta}{\epsilon_0 h} \left( \frac{\Omega_c^* \Omega_c}{4} + \Delta^2 \gamma_1 \right), \\
\mu_r^\pm &= 1 + \frac{\Delta^2 \gamma_1}{2 \Omega_c^* \Omega_c} \left( \varsigma = \frac{m_e^*}{\epsilon_0 c} \Omega_c^* \sqrt{\varsigma \chi_e} \right)
\end{align*}
$$

In an EIT left-handed medium, if $\chi_e$ has a very small imaginary part, the parameter $\varsigma$ will be an imaginary number. It follows from Eqs. (5) that if $\varsigma^2 < -4$, the magnetic permeability $\mu_r$ will have a negative real part and a nearly vanishing imaginary part. Further analysis shows that both the negative and the positive roots $(\chi_m^\pm = (\varsigma \pm \sqrt{\varsigma^2 + 4\gamma_1^2})/2)$ of the magnetic susceptibility are valid for Eqs. (4).

In the following section, we will find out a frequency band in which the three-level EIT atomic gas will exhibit simultaneously negative electric permittivity and magnetic permeability.

### III. EXISTENCE OF NEGATIVE PERMITTIVITY AND PERMEABILITY

In order to find out the conditions for realizing the negative optical indices, we should first analyze the real and imaginary parts of the electric polarizability $\chi_e$ in Eqs. (5). Set $\chi_e = \chi_e' + i\chi_e''$. Then by the aid of Eq. (4), one can arrive at

$$
\chi_e' = \frac{N|\varphi_{ab}|^2 \Delta \left( \Delta^2 - \frac{\Omega_c^* \Omega_c}{4} \right)}{\epsilon_0 h \left( \Delta^2 - \frac{\Omega_c^* \Omega_c}{4} \right)^2 + \Delta^2 \gamma_1^2}, \quad \chi_e'' = \frac{N|\varphi_{ab}|^2 \Delta^2 \gamma_1}{\epsilon_0 h \left( \Delta^2 - \frac{\Omega_c^* \Omega_c}{4} \right)^2 + \Delta^2 \gamma_1^2}.
$$

For the present, we focus our attention only on the low-absorption EIT medium. This requires that the imaginary part $\chi_e''$ is negligibly small (or $\chi_e'' \ll |1 + \chi_e'|$). In an ordinary EIT experiment, the Rabi frequency, $\Omega_c$, of the coupling laser often has the same order of magnitude of $\gamma_1$, e.g., $10^7 \sim 10^8$ s$^{-1}$ [14,15]. Thus, it follows from Eqs. (6) that for a low-absorption EIT medium, the frequency detuning of the probe field should be much less than the Rabi frequency of the coupling laser, i.e., $\Delta \ll \Omega_c$. To achieve the negative electric permittivity, i.e., $\chi_e' < -1$, according to Eqs. (6), the quantities $\Delta$, $\Omega_c$, $\gamma_1$, $\varphi_{ab}$ and $N$ should agree with the following restriction condition

$$
\varsigma \Delta \left( \Delta^2 - \frac{\Omega_c^* \Omega_c}{4} \right) + \left( \Delta^2 - \frac{\Omega_c^* \Omega_c}{4} \right)^2 + \Delta^2 \gamma_1^2 < 0
$$

with $\varsigma = N|\varphi_{ab}|^2/ (\epsilon_0 h)$. Since $\Delta \ll \Omega_c$, one can ignore the $\Delta^2$ term in inequality (7), and then obtain $\Delta > \Omega_c^* \Omega_c/ (4\varsigma)$ from the inequality (7). Thus, the requirement for the realization of negative permittivity $(\epsilon_r \approx 1 + \chi_e' < 0)$ near the probe frequency in a three-level atomic system is $\Omega_c \gg \Delta > \Omega_c^* \Omega_c/ (4\varsigma)$. It should be noted that the condition $\Omega_c \gg \Omega_c^* \Omega_c/ (4\varsigma)$ imposes a restriction on the atomic number density $N$ of the EIT medium, i.e., $N \gg \epsilon_0 h |\Omega_c^*|^2 / |\varphi_{ab}|^2$. This, therefore, means that in order to realize the low-absorption negative permittivity, one should choose the dense gas EIT media. For a rough discussion, we choose the values for the dipole matrix elements and decay rates, which are typical for transitions in hyperfine-split Na D lines [18]: $\varphi_{ab} = 1.2 \times 10^{-31}$ D, $\gamma_1 = 1.2 \times 10^8$ s$^{-1}$. In this case, $\varsigma \approx 10^{-17} N$ m$^3$s$^{-1}$. If the Rabi frequency of the coupling laser takes the value of $10^8$ s$^{-1}$ [14,15], the detuning $\Delta$ of the probe laser should be larger than $10^{33}/N$ m$^{-3}$s$^{-1}$ in accordance with the condition $\Delta > \Omega_c^* \Omega_c/ (4\varsigma)$, and then the EIT medium will exhibit a negative permittivity. As far as the low absorption is concerned, the atomic number density $N$ should be much larger than $10^{24}$ m$^{-3}$, according to the requirement $N \gg \epsilon_0 h |\Omega_c^*|^2 / |\varphi_{ab}|^2$. It follows that the larger is the atomic density, the more negative is the permittivity obtained at nearly resonant frequency with low absorption.

As for the treatment for the negative permeability in the EIT medium, the only task left to us is to discuss the parameter $\varsigma^2$. As an illustrative example, here we are concerned only with the case of the large negative permeability (i.e., $\chi_e' \ll -1$). In this case, $\varsigma \rightarrow i (m_e^* / \varphi_{ab} c) \sqrt{\Omega_c^* / \Omega_c} \sqrt{\varsigma / \Delta}$. This, therefore, means that the restriction for
realizing the negative permeability can be rewritten as \( (\frac{m^*_c}{\nu_c^*}) \) \( (\frac{\Omega_c^*}{\Omega_c}) \) \( \frac{\zeta}{\Delta} \gg 4 \). Note that in an atomic system, \( m^*_c/ (\nu_c^* \approx 10^{-2} \). If we choose \( \Omega_c^* = \Omega_c \), for a tentative consideration, the above condition can be rewritten as \( \Delta \ll \zeta/(4 \times 10^4) \), i.e., \( \Delta \ll 10^{-21} N \text{ m}^3\text{s}^{-1} \).

In conclusion, for a typical EIT experiment in which \( \gamma_1 \sim \Omega_c \approx 10^8 \text{ s}^{-1} \), in order to obtain the negative permittivity, the frequency detuning \( \Delta \) of the probe laser should satisfy \( \Delta > 10^{33}/N \text{ m}^3\text{s}^{-1} \); in order to obtain the negative permeability, \( \Delta \) should satisfy \( \Delta \ll 10^{-21} N \text{ m}^3\text{s}^{-1} \); in order to obtain the low absorption for the probe light, the two requirements \( \Delta \ll \Omega_c \) and \( N \gg 10^{24} \text{ m}^{-3} \) should be satisfied. Thus, it is shown that only when the three-level atomic system is dense gas, the atomic density \( N \) of which is larger than \( 10^{27} \text{ m}^{-3} \) and the frequency detuning \( \Delta \) is approximately or less than \( 10^6 \text{ s}^{-1} \) will such a three-level EIT medium exhibit a low-absorption negative optical refractive index. The behaviors of permittivity, permeability and absorption for the probe laser are shown in Figs. 2 and 3.

IV. PHYSICAL SIGNIFICANCE OF THE THREE-LEVEL EIT-BASED REALIZATION OF NEGATIVE REFRACTIVE INDEX

We proposed a novel scheme of EIT-based realization of left-handed media. Such a scenario of achieving the negative refractive index with EIT atomic gas may have three main advantages:

(i) can realize the isotropic left-handed media;

(ii) can lead to a controllable manipulation of negative refractive index by an external field (the coupling laser);

(iii) can obtain the visible and infrared frequency band where the permittivity and the permeability are simultaneously negative. For these reasons, we think that the present scheme for realizing the EIT left-handed media deserves further investigation both theoretically and experimentally.

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FIGURE CAPTIONS

Fig. 1. The schematic diagram of the Λ-type three-level EIT atomic system. The electric-dipole transition (|a⟩→|b⟩) and the magnetic-dipole transition (|c⟩→|b⟩) will possibly yield the negative electric permittivity and the magnetic permeability at probe frequency, respectively.

Fig. 2. The real and imaginary parts of the permittivity and permeability. It is shown that in the case of small frequency detuning, the permittivity has a negative real part and a very small imaginary part if the frequency detuning ∆ is greater than $2.3 \times 10^{-3} \gamma_1$. When the frequency detuning ranges from $2.3 \times 10^{-3} \gamma_1$ to $7.7 \times 10^{-3} \gamma_1$, the dense EIT atomic gas will exhibit a negative permeability. In the band of frequency detuning $[2.3 \times 10^{-3} \gamma_1, 4.6 \times 10^{-3} \gamma_1]$, the imaginary part of permeability is negligibly small. In the detuning range $[4.6 \times 10^{-3} \gamma_1, 7.7 \times 10^{-3} \gamma_1]$, however, the imaginary part of permeability increases significantly. Here, $N = 10^{27} \text{ m}^{-3}$, $\gamma_1 = \Omega_c \simeq 10^8 \text{ s}^{-1}$, $\psi_{ab} = 1.2 \times 10^{-31} \text{D}$ and $m^*/(\psi_{ab} c) = 10^{-2}$.

Fig. 3. The absorption coefficient ($\alpha = -2\pi \text{Im} \{\sqrt{\epsilon_r \mu_r}\}$). It is shown that the probe laser will experience a small amplification in the detuning range $[0, 2.3 \times 10^{-3} \gamma_1]$, where both the permittivity and permeability are positive numbers. However, in the detuning range $[2.3 \times 10^{-3} \gamma_1, 7.7 \times 10^{-3} \gamma_1]$ for the EIT atomic gas to be a left-handed medium, such an EIT medium is an absorptive material. For the case of $\Delta < 4.6 \times 10^{-3} \gamma_1$, it is a low-absorptive medium, while for the case of $\Delta > 4.6 \times 10^{-3} \gamma_1$, it is a high-absorptive one, for the permeability has a large imaginary part.

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