A New Approach to Predict the Pit Depth Extreme Value of a Localized Corrosion Process

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1. Introduction

Pitting corrosion is an extremely dangerous form of localized corrosion since a perforation resulting from a single pit can cause complete failure in service of installations like water pipes, heat exchanger tubes or oil tank used for example in chemical industry plants or nuclear power stations.1–10 The most successful method of safety or reliability of industrial installations, statistical procedures must be constructed to assess the maximum pit depth to perform proper maintenance from limited inspection data.

This paper outlines a new methodology to predict accurately the pit depth extreme value related to a localized corrosion process independently of the nature of the unknown parent distribution of the experimental data. Based on computer calculations and simulations, this methodology combines the Generalized Lambda Distribution (GLD) and the Bootstrap statistical methods.

The GLD method was used in this study to determine a modeled distribution that fits the experimental frequency distribution of pit depths produced on a ferritic stainless steel sample during an accelerated corrosion test. This modeled distribution was used to generate, thanks to the Computer-Based Bootstrap Method (CBBM), simulated distributions of corrosion pit depths equivalent to the experimental one. An estimation of the mean with a 90% confidence interval of the maximum pit depth can be finally deduced not only for these simulated samples of equivalent surface size than the experimental one but also for a large scale installation.

KEY WORDS: stainless steel; pitting corrosion; extreme value statistics; Generalized Lambda Distribution; Computer-Based Bootstrap Method.

Depth of pits that propagate during a pitting corrosion process is an important characteristic of the damage of steels; the greater the depth, the more dramatic the damage. For evident reasons of safety and reliability of industrial installations, statistical procedures must be constructed to assess the maximum pit depth to perform proper maintenance from limited inspection data.

2. Experimental Procedure

2.1. Presentation of the Material and the Corrosion Test

The material considered in this study is a ferritic stainless steel AISI 409 used in automotive industry and pro-
duced by the society Arcelor (Isbergues, France). A sample of this material (area: 50 cm², thickness: 1.5 mm) was placed in an indoor equipment of a research center of this society to undergo an accelerated corrosion test that simulates automotive field conditions. This accelerated corrosion test, already described by Bourdeau et al., is basically composed of a salt fog spraying and cyclical wet and dry periods. Salt spray solution is a 1% NaCl at pH 4 lowered by sulfuric acid. Temperature around the test was maintained at 35°C and the total duration of the test was 2 weeks.

2.2. Characterization and Distribution of Pits Formed during the Corrosion Test

Firstly, optical macrography was used to render an account of the damage due to the accelerated corrosion test at a large scale. Figure 1 clearly reveals that the spatial distribution of the corrosion pits is inhomogeneous at the scale of the sample. During the test, corrosion pits probably initiate preferentially on sites affected by the droplets of the salt fog spraying where the chloride concentration is higher. Then the distribution of corrosion pits is assumed to be a consequence of the random impingement of the droplets with the tested sample surface.

Secondly, optical microscopy was used to determine the experimental frequency distribution of depths considering 195 corrosion pits (Fig. 2). This experimental histogram plotted seems to have the shape of a lognormal distribution. This visual finding is substantiated by the result of a Chi-square test used to assess how well a lognormal model can fit the experimental data of this study.

2.3. Limitations about the Application of the Gumbel Approach

The analytical expression of the Gumbel distribution called also the double exponential distribution has the following form:

\[
F_X(x) = \exp(-\exp(-(x-\lambda)/\alpha)) \quad \text{where } F_X(x) \text{ is the distribution function of the } x \text{ values that are possible for a random variable noted } X \text{ and the constants } \alpha \text{ and } \lambda \text{ are called the scale and location parameters, respectively.}
\]

One must remember that this analytical expression of the Gumbel distribution is in fact an asymptotic limit form of the largest values extracted from a parent distribution of exponential type. Hence, there are some mathematical requirements for the formal application of this analytical expression. The first one relates to the nature of the parent distribution and the second one to the number of largest values of this distribution that needs to be high. Concerning our experimental conditions, one may conclude that the first requirement is verified since the Chi-square test fails to reject the hypothesis of a difference between the lognormal distribution (which belongs to the exponential type) and the experimental frequency distribution of corrosion pit depths. However, the Chi-square test, as any statistical goodness-of-fit test, presents a major drawback: failure to reject does not mean the hypothesis is true; it could, for example, be that the dataset size is too small to detect differences that exist between the true and hypothesized models. Moreover, even if the expected lognormal model is accepted, there are only few large values in the experimental data set and the difference between this model and the unknown true distribution may be significant in this right tail region. In other words, the prediction in this right tail region, which corresponds to the extreme values, is far from being accurate for the experimental conditions of this study.

Beyond the particular limitations described previously, two general limitations have been outlined by several authors working in the corrosion field. The first limitation is the important property that the double exponential expression is unlimited, which implies a small but finite probability of observing a pit of exceptional depth. Up to now, this conflict between the unlimited nature of the distribution and the physical limits on pit growth kinetics has not been resolved. The second limitation is that a concrete criterion for the number and size of samples to obtain a reasonable accurate extreme value prediction is not available except making questionable assumptions. When the theory supporting the application of the Gumbel distribution was developed as the number of samples becomes infinite, Shibata mentioned that it might be not so plentiful in practical situations especially in engineering data because measurements are time consuming and expensive.

Because of these various limitations, an alternative approach combining the GLD and the Bootstrap methods is proposed hereafter to predict the maximum pit depth with a
90% confidence interval for an exposed surface of a given size.

3. Determination of the Pit Depths Distribution Using the GLD Method

3.1. Presentation of the GLD Method

Three ways can be usually considered to specify the chances of occurrence of the various values \( x \) that are possible for a random variable noted \( X \); the distribution function \( F_X(x) = \text{Pr}(X \leq x) \) which indicates the probability of \( X \) to be no larger than \( x \); the probability density function \( f_X(x) \) which is obtained by differentiating the distribution function; the inverse distribution function or percentile function \( Q_X(y) \) which, for each \( y \) between 0 and 1, tells us the value of \( x \) such that \( F_X(x) = y \).

The generalized lambda distribution (GLD) family is specified in terms of its percentile function (called also the inverse distribution function) with four parameters \( (\lambda_1, \lambda_2, \lambda_3, \lambda_4) \):

\[
Q_X(y; \lambda_1, \lambda_2, \lambda_3, \lambda_4) = \lambda_1 + \frac{(y^{\lambda_3} - (1-y)^{\lambda_4})}{\lambda_2} \quad (2)
\]

The parameters \( \lambda_1 \) and \( \lambda_2 \) are, respectively, the location and scale parameters, while \( \lambda_3 \) and \( \lambda_4 \) determine respectively the skewness and the kurtosis of the GLD.

The probability density function \( f_X(x) \) can then be easily expressed from the percentile function of the GLD:

\[
f_X(x) = \lambda_2 (\lambda_3 x^{\lambda_3-1} + \lambda_4 (1-y)^{\lambda_4-1}) \quad (3)
\]

The main problem is of course to estimate the parameters \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) in order to have the best fitting of the GLD with the experimental frequency distribution (of corrosion pit depths in this study). For this estimation, the method of percentiles was preferred to the method of moments that introduces numerical instabilities in the computer treatment of our experimental dataset. The method consists of solving the following system of equations estimating \( \rho_j \) value by \( \hat{\rho}_j \) with \( j \in \{1, 2, 3, 4\} \). Since these four equations are highly non-linear, it is easier to first solve the subsystem \( \hat{\rho}_j = \rho_j \) that involves only \( \lambda_1 \) and \( \lambda_2 \).

\[
\begin{align*}
\hat{\rho}_1 &= \hat{\pi}_{0,5} \quad (4) \\
\hat{\rho}_2 &= \hat{\pi}_{1-u} - \hat{\pi}_u \quad (5) \\
\hat{\rho}_3 &= \frac{\hat{\pi}_{0,5} - \hat{\pi}_u}{\hat{\pi}_{1-u} - \hat{\pi}_{0,5}} \quad (6) \\
\hat{\rho}_4 &= \frac{\hat{\pi}_{0,25} - \hat{\pi}_{0,25}}{\hat{\pi}_{1-u} - \hat{\pi}_u} \quad (7)
\end{align*}
\]

These statistics have the following interpretations:
- \( \hat{\rho}_1 \) represents a measure of the location of the distribution through the sample median,
- \( \hat{\rho}_2 \) represents a measure of the dispersion which range depends on the value of \( u \); when \( u = 0.25 \) this statistic is called the inter-quartile range,
- \( \hat{\rho}_3 \), the left-right tail-weight ratio, a measure of relative tail weights of the left tail to the right tail,
- \( \hat{\rho}_4 \) is the tail-weight factor or the ratio of the inter-quartile range to the dispersion; values close to 0 indicate the distribution has long tails.

From the definition of the GLD, Karian et al. express the GLD counterparts of the above statistics as follows:

\[
\begin{align*}
\hat{\rho}_1 &= \lambda_1 + \frac{0.5 \lambda_3 - 0.5 \lambda_4}{\lambda_2} \quad (8) \\
\hat{\rho}_2 &= \frac{(1-u)^{\lambda_3} - u^{\lambda_4} + (1-u)^{\lambda_4} - u^{\lambda_3}}{\lambda_2} \quad (9) \\
\hat{\rho}_3 &= \frac{(1-u)^{\lambda_3} - u^{\lambda_4} + 0.5 \lambda_3 - 0.5 \lambda_4}{(1-u)^{\lambda_4} - u^{\lambda_3} + 0.5 \lambda_4 - 0.5 \lambda_3} \quad (10) \\
\hat{\rho}_4 &= \frac{0.75 \lambda_3 - 0.25 \lambda_4 + 0.75 \lambda_4 - 0.25 \lambda_3}{(1-u)^{\lambda_3} - u^{\lambda_4} + (1-u)^{\lambda_4} - u^{\lambda_3}} \quad (11)
\end{align*}
\]

The fitting of a GLD \( (\lambda_1, \lambda_2, \lambda_3, \lambda_4) \) to the experimental dataset is done by solving this system of equations estimating \( \rho_j \) value by \( \hat{\rho}_j \) with \( j \in \{1, 2, 3, 4\} \). After these four equations are highly non-linear, it is easier to first solve the subsystem \( \hat{\rho}_j = \rho_j \) that involves only \( \lambda_1 \) and \( \lambda_2 \).

\[
\Psi(\lambda_3, \lambda_4) = \sum_{j=3}^{4} (\hat{\rho}_j - \rho_j)^2 \quad (12)
\]

When the pair \( (\lambda_3, \lambda_4) \) that minimizes the above equation is found, then by using Eqs. (9) and (8) successively will yield to \( \lambda_1 \) and \( \lambda_2 \).

3.2. Procedure and Results

An algorithm was written and computed using the Statistical Analyses System language to determine, by using the method of percentiles, the GLD and its related probability density function from the experimental dataset. The numerical results of the minimization process obtained with our computer algorithm are illustrated in Fig. 3. This figure presents both a 3D view and a contour plot of the values of the function \( \Psi(\lambda_3, \lambda_4) \) on which the gradient decreasing method processed when \( -2 < \lambda_1 < 1 \) and \( -2 < \lambda_2 < 2 \). The value of this function is found to be minimum for the pair \( (\lambda_3 = 0.0034, \lambda_4 = 0.055) \) which yields subsequently to the pair \( (\lambda_1 = 22.3, \lambda_2 = 0.0016) \).

Figure 4 presents the GLD and its related probability density function of corrosion pit depths obtained by replac-
ing the four previous values determined by using the method of percentiles in Eqs. (2) and (3).

4. Estimation of the Extreme Value Combining the GLD and the Bootstrap Methods

4.1. Presentation of the Computer-Based Bootstrap Method\textsuperscript{14,15)}

Efron introduced first the Computer-Based Bootstrap Method (CBBM) to avoid the risk of asserting wrong conclusions when analyzing experimental data that transgress inference assumptions of the traditional statistical theory. A main reason for making parametric assumptions is to facilitate the derivation from textbook formulae for standard errors. Unfortunately, the traditional statistical theory does not provide formulae to assess the accuracy of most statistic estimates other than the mean. Since no formulae are necessary using the CBBM in non-parametric mode, restrictive and sometimes-dangerous assumptions about the form of underlying populations can be avoided. Moreover, a standard error (thus an assessment of the accuracy) can be calculated for any computable statistic estimate using the constructed empirical probability density function. The CBBM does not work in isolation but rather its efficiency is emphasized when applied to other statistical procedure, as it will be shown hereafter for the GLD method.

Based on the mathematical resampling technique, the main principle of the CBBM consists in generating a high number \(B\) of simulated Bootstrap samples from the original data points using the power of a modern computer. The original dataset consists of either experimental or simulated points. A Bootstrap dataset of size \(n\), noted \((t_1^*, t_2^*, \ldots, t_n^*)\), is a collection of \(n\) values simply obtained by randomly sampling with replacement from the original data points \((t_1, t_2, \ldots, t_n)\), each of them with a probability of \(1/n\). The Bootstrap dataset consists thus of elements of the original data points; some appearing zero times, some appearing one, some appearing twice, \textit{etc.}

4.2. Procedure and Results

A second algorithm was computed using the Statistical Analyses System language to generate simulated Bootstrap datasets of size \(n/11005\). Each Bootstrap dataset is obtained using the GLD equation corresponding to the experimental dataset; i.e., the following equation:

\[
P(v) = 22.3 + (v^{0.0034} - (1-v)^{0.055})/0.0016 \ldots \ldots (13)
\]

where \(v\) is a uniform random number between 0 and 1 and \(P(v)\) is the related simulated corrosion pit depth. To illustrate the result of this procedure, two examples of simulated distributions related to two Bootstrap datasets are presented in Fig. 5. These distributions of corrosion pit depths obtained by the CBBM method simulate in fact two possible distributions equivalent to the experimental one.

The above procedure was repeated to obtain \(B = 1000000\) simulated Bootstrap datasets. The highest score, \textit{i.e.} the deepest corrosion pit, is then extracted from each simulated Bootstrap sample to obtain finally a dataset of 1 million extreme values which distribution is presented in Fig. 6. From this empirical probability density function, traditional statistic estimates like the mean and the 90% confidence interval (\textit{i.e.} the difference between the 95th and the 5th percentiles) can be then easily determined to assess respectively the central tendency and the dispersion. The calculated mean of the pit depth extreme values is 190 \(\mu m\) and 90% of these extreme values lie between 150 \(\mu m\) and 250 \(\mu m\).

Since one problem of practical importance is to determine the maximum pit depth that will be found in a large
scale installation by using a small number of samples with a small area, the CBBM was used again to assess the effect of the surface size on the evolution of the mean and the 90% confidence interval of the distribution of the pit depth extreme values. Assuming that the number of corrosion pits is proportional to the exposed surface size, the same methodology described previously can be executed to generate Bootstrap samples containing however 1 950, 19 500, 19 500... corrosion pits to simulate surfaces area 10, 100, 1 000... times greater than the experimental studied surface area. Figure 7 shows that the mean of the maximum pit depth increases with the exposed surface area. For a surface area 1 000 greater than the surface of the experimental studied sample, the mean of the maximum pit depth is 340 μm and 90% of the extreme values lie between 300 μm and 380 μm.

5. Conclusions

A new and alternative methodology to the Gumbel approach was presented in this paper in order to estimate accurately the maximum depth of pits that propagated on a ferritic stainless steel during an accelerated corrosion test. Based on the power of modern computer, this methodology combines the Generalized Lambda Distribution and the Bootstrap methods to simulate distributions of corrosion pit depths equivalent to the experimental one obtained from a small sample and to predict the mean with a 90% confidence interval of the maximum pit depth that can be found in a larger installation.

While corrosion pits have been generated in an atmospheric environment in this study, the alternative methodology presented here is thought to be also applicable for pits generated in aqueous solutions since it only necessitates the knowledge of the distribution of a morphological feature (i.e. pit depth). Such a methodology, which is a contribution to perform proper maintenance from limited inspection data in the field of corrosion, can be also applied to any kind of defects like inclusions or pores existing on engineering materials.

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