Optimal use of Information for Measuring $M_t$
in Lepton+jets $t\bar{t}$ Events.

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The observation of the top ($t$) quark served as one of the major confirmations of the validity of the standard model (SM) of particle interactions [1,2]. Through radiative corrections of the SM, the mass of the top quark, along with that of the W boson [3], provide the best indication for the value of the mass of the hypothesized Higgs boson [4]. The mass of the W is known to a precision of $<0.1\%$, while the uncertainty on the mass of the top quark is at the $4\%$ level [5]. Improvements in both measurements are required to limit the range of mass that the Higgs boson can assume in the SM, and, of course, to check whether that agrees with expectation. It is therefore important to develop techniques for extracting the mass of the top quark that can provide the sharpest values possible.

Measurement of $M_t$

We report on a new preliminary measurement of the mass of the top quark from $t\bar{t}$ data in lepton+jets channels accumulated by the DØ experiment in Run-1 of the Tevatron. The luminosity corresponds to 125 events/pb, and this analysis is based on the same data sample that was used to extract the mass of the top quark in our previous publication [5]. Information pertaining to the detector and to the older analysis can be found in Refs. [6] and [5], respectively.

After offline selections on lepton transverse energies ($E_T > 20\text{ GeV}$) and angles ($|\eta_\mu| < 1.7$, and $|\eta_e| < 2.0$, for muon and electron channels, respectively), on jet transverse energies ($E_T > 15\text{ GeV}$) and angles ($|\eta| < 2.0$), imbalance in transverse momentum (missing-$E_T > 20\text{ GeV}$), and after applying several less important criteria [5], the event sample consisted of 91 events with one isolated lepton and four or more jets. (Unlike the previous analysis, we do not distinguish between events that have or lack a muon associated with one of the jets, signifying the possible presence of a $b$-quark jet in the final state.) The new analysis involves a comparison of the data with a leading-order matrix element for the production and decay process, and to minimize the effect of higher-order corrections we therefore restrict the study to events containing only four jets. This requirement reduces data sample to 71 events.

In the previous analysis, the four jets with highest $E_T$ were assumed to represent the four quarks in the events (two $b$ quarks from the initial
t \to W + b$ decays, and the $q'\bar{q}$ decays of one of the $W$ bosons). These, along with the lepton and the missing $\nu$ (from the decay of the other $W$ boson) were fitted to the kinematic hypothesis $p\bar{p} \to t\bar{t} \to WWb\bar{b}$, subject to the constraints of overall momentum-energy conservation, the known mass of the $W$ boson, and the fact that the unknown mass of the top quark ($M_t$) had to be identical for the top and antitop quarks in the event. With 12 ways to permute the jets, there were 12 possible fits, and the solution with the lowest $\chi^2$ (but $<10$) was chosen as the best hypothesis, thereby defining the fitted mass $m_{fit}$ for the event, as well as the longitudinal momentum of the $\nu$. The same procedure was used for generating templates in variables of interest, as a function of input mass of the top quark. This was based on the Monte Carlo (MC) HERWIG program [7], which was used to generate events that were passed through full event-reconstruction in the detector [8]. The background, which consisted mainly of all-jet production (20%) and $W$+jets (80%) was also processed in a similar manner. The background from all-jet production was based on data, and the background from $W$+jets was based on events generated with VECBOS [9]. A discriminant was formed based on differences in distributions in four variables predicted for $t\bar{t}$ signal and background, and this defined the probability that any event represented signal as opposed to background. A probability density was defined as a

![Discriminator plot](image-url)

**Fig. 1.** Discriminator $D = P_{signal} / (P_{signal} + P_{background})$ for the new analysis, and (inset) for the two versions of the analysis included in the previous DØ publication [4].
function of the discriminant \((D)\) and \(m_{\text{fit}}\), and a comparison of data and MC via a likelihood was used to determine the most likely mass of the top quark.

The new analysis is also based on a likelihood, but this likelihood is a function of all measured variables in the event, with the exception of the unclustered energy in the calorimeter that is used to define the missing \(E_T\). Our method is similar to that suggested for \(t\bar{t}\) dilepton decay channels [10], and used in previous mass analyses of dilepton events [11]. We define a probability for background purely in terms of the matrix element contained in VECBOS, and for signal in terms of the leading-order matrix element for \(t\bar{t}\) production and decay. This is convoluted with a transfer function that relates objects at the parton level to the objects observed (fully reconstructed) in the detector. We assume that the observed electrons correspond to the produced electrons, and that the muons are smeared with their known resolution. The angles of the jets are assumed to reflect the angles of the partons in the final state, and we ignore any transverse momentum for the incident partons. We use a large MC sample of \(t\bar{t}\) events (generated with masses between 140–200 GeV in HERWIG, and processed through the DØ detector-simulation package) to determine a set of ten parameters that correlate any jet \(E_T\) with its parton value. The parameters used for \(b\) quarks are different than for the lighter quarks, and there are therefore 20 parameters in all.

We can write the probability for \(t\bar{t}\) production in terms of the following elements:

\[
P_{tt} = \int d\rho_1 dm_1^2 dM_1^2 dm_2^2 dM_2^2 \sum_{\text{perm, } \nu} |M|^2 \frac{f(q_1)f(q_2)}{|q_1||q_2|} \Phi_6 W(x, y) \tag{1}
\]

where the sum is over all 12 permutations of the jets, and all possible values of \(\nu\) momenta. \(|M|^2\) is the matrix element for the process, \(f(q_1)\) and \(f(q_2)\) are the parton distribution function (PDF) for the incident particles, \(\Phi_6\) is the phase-space factor for the 6-object final state and \(W(x, y)\) correspond to a function that parameterizes the mapping between parton level quantities \(y\) and detector measurements \(x\). With two incident parton energies (we take these partons to be quarks, and ignore the \(\approx 10\%\) contribution from gluon fusion), and six objects in the final state, the integrations over the essentially fifteen sharp variables (3 components of lepton momentum, eight jet angles, and four \(\delta\)-functions representing energy-momentum conservation), leaves five integrals that must be performed to obtain the probability that any event represents \(t\bar{t}\) production for some specified value of top mass \(m_t\). Four of the variables chosen for the remaining integrations, namely the masses of the \(W\) bosons and of the top quarks in the event, are economical in CPU time, because the value of \(|M|^2\) is essentially negligible except at the peaks of the four Breit-Wigner terms in the matrix element.
Fig. 2. Distribution for background probability calculated for the 71 4-jets $t\bar{t}$ candidates (data points). Only the events to the left of the vertical line ($\ln[P_{\text{bkg}}] < -11, 0$) are considered for further analysis. The data is compared with the expected results from MC-simulated samples (dashed histogram) formed with a mixture of 16 signal (left-hatched histogram) and 55 background (right-hatched histogram) events.

Introducing an analogous expression for the background, the likelihood as a function of $m_t$ can be written as:

$$-\ln L(\alpha) = -\sum_{i=1}^{N} \ln[c_1 P_{t\bar{t}}(x_i; \alpha) + c_2 P_{\text{bkg}}(x_i)]$$

$$+ N \int A(x)[c_1 P_{t\bar{t}}(x; \alpha) + c_2 P_{\text{bkg}}(x)]dx$$

where $A(x)$ is the acceptance of the detector in terms of the parton variables (and contains the transfer function), and the sum over the probabilities for the individual events reflects the product of the probabilities for final sample of $N$ of events. The best values of $\alpha$, representing $m_t$, and the parameters $c_i$, are all defined by the most probable value of the likelihood.

The main difference between this method and the previous analysis is that each event now has its individual probability as a function of the mass parameter. This probability, reflecting both signal and background, depends on all measured variables in the event (excepting unclustered energy), with well-measured events contributing more sharply to the extraction of the mass of the top quark than those poorly measured. The fact that so much of the
information in each event is retained in the probability provides a far better separation of signal and background. Figure 4 displays the effective difference in the discriminating power for the parameter $D$ in the current and in the previous analyses. In all these analyses, this parameter is not used to select regions of high signal-probability, but the comparison is instructive in showing the greater power of the present method.

![Graph of log likelihood vs. top mass](image)

**Fig. 3.** a) Negative of the log of the likelihood as a function of the mass of the top quark for the 22 $t\bar{t}$ candidates in our final sample. b) Probability distribution determined from the likelihood, with the hatched area corresponding to the 68% probability interval, from which we determine $M_t = 179.9 \pm 3.6$ GeV. A 0.5 GeV upward bias correction has been included based on the results of MC ensemble tests.

Studies on samples of HERWIG MC events used in the previous analysis indicate that the expected uncertainty using the new method would be capable of yielding almost a factor of two reduction in the uncertainty of the extracted top mass. However, the method appears to be sensitive to a systematic shift of the mass (of about 2 GeV) as a function of the a priori sample purity, whose true value cannot be determined in the analysis. To minimize the impact of this limiting systematic uncertainty, we proceed as follows. Figure 2 shows the probability that each event corresponds to background (VECBOS), and we compare that to a sample of 71 MC events corresponding to the mixture of 16 signal and 55 background events found in the data sample. From MC studies we find that eliminating from further consideration in the analysis those events that have a high probability of being background, reduces markedly the dependence on the extracted mass on sample purity. We consequently retain for further study only those events that have a poor probability of being background. This selection is shown in Fig 2 and its imposition leaves a sample of only 22 events, 12 of which are
signal and 10 background, after the recalculation of the new minimum in the likelihood, the result of which is shown in Fig. 3.

Another aspect of this analysis is that it provides a natural method for measuring any other parameter in the $t\bar{t}$ differential cross section. As an example we show in Fig. 4 the likelihood as a function of the mass of the $W$ boson ($M_W$) for the same sample of events. This suggests the very interesting possibility of checking the jet energy scale (JES) in this events, which corresponds to the largest contribution to the systematic uncertainty in the current and previous analyses. The next step in this analysis is, in fact, to utilize the information on JES contained in these events. However, in this preliminary result, we only estimate this uncertainty using the previous analysis [5], where the JES calibration from $\gamma + jet$ events was used to match the energy scale in the experiment to that in the MC simulation. Here we estimate the uncertainty in $M_t$ coming from JES as follows. The analysis is performed before and after applying this correction, and the difference defined as the systematic uncertainty due to JES. Table 1 summarizes all the systematic uncertainties in this preliminary result.

![Fig. 4.](image-url) a) Negative of the log of the likelihood as a function of the mass of the $W$ boson for the 22 $t\bar{t}$ candidates in our final sample. b) Probability distribution determined from the likelihood, with the hatched area corresponding to the 68% probability interval.

**Conclusion**

We have presented a new preliminary measurement of the mass of the top quark using a method that compares each individual event with the differential cross section for $t\bar{t}$ production and decay. We obtain a significant improvement over the statistical uncertainty of the previous measurement [5].
that is equivalent to having a factor of 2.4 more data. The new preliminary result is:

\[ M_t = 179.9 \pm 3.6 \text{(stat)} \pm 6.0 \text{(sys)} \text{ GeV} \]  

The possibility of checking the value of the W mass in the same events offers the possibility of redoing the largest systematic uncertainty, of the same events provides a new handle on controlling the largest systematic, namely the jet energy scale.

| Table 1. Table of systematic uncertainties for the measurement of $M_t$ |
|---------------------------------------------------------------|
| Model for $t\bar{t}$                                    | 1.5 GeV |
| Model for background($W$+jets) | 1.0 GeV |
| Noise and multiple interactions  | 1.3 GeV |
| Jet energy scale                                  | 5.6 GeV |
| Parton distribution function                      | 0.2 GeV |
| Acceptance correction                             | 0.5 GeV |
| **Total**                                           | **6.0 GeV** |

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