Minimum Roman Dominating Distance Energy of a Graph

Lakshmanan. R
Department of Mathematics
Central University of Tamil Nadu
Tiruvarur-610005, Tamil Nadu, India
Email: laksh1983ram@gmail.com

N. Annamalai
Central University of Tamil Nadu
Tiruvarur-610005, Tamil Nadu, India
Email: algebra.annamalai@gmail.com

Abstract

In this correspondence, we introduced the concept of minimum roman dominating distance energy $E_{RDd}(G)$ of a graph $G$ and computed minimum roman dominating distance energy of some standard graphs. Also, we discussed the properties of eigenvalues of a minimum roman dominating distance matrix $A_{RDd}(G)$. Finally, we derived the upper and lower bounds for $E_{RDd}(G)$.

Keywords: Energy of a graph, Roman dominating function, Roman domination.

2010 Mathematical Subject Classification: Primary 05C50, 05C69
1 Introduction

In 1978, I. Gutman was introduced the concept of energy of a graph \[7\]. The graph \( G = (V, E) \) mean a simple connected graph with \( n \) vertices and \( m \) edges. The distance between two vertices \( u \) and \( v \) is the length shortest distance between \( u \) and \( v \). The Wiener index \( W(G) \) of \( G \), is the sum of the lengths of the shortest paths between all pairs of vertices. Let \( A = (a_{ij}) \) be the adjacency matrix of a graph \( G \). Then the energy \( E(G) \) of a graph \( G \) is defined by the sum of absolute value of all eigenvalues of \( A \). For more details about energy of a graph\[1\].

The union of two simple graphs \( G_1 = (V_1, E_1) \) and \( G_2 = (V_2, E_2) \) is the simple graph with vertex set \( V_1 \cup V_2 \) and edge set \( E_1 \cup E_2 \). The union of \( G_1 \) and \( G_2 \) is denoted by \( G_1 \cup G_2 \). A crown graph \( S^0_n \) is a bipartite graph with two sets of vertices \( \{a_1, a_2, \ldots, a_k\} \) and \( \{b_1, b_2, \ldots, b_k\} \) and with an edge from \( a_i \) to \( b_j \) whenever \( i \neq j \). For a positive integer \( n \geq 2 \), a healthy spider is a star \( K^*_{1,n-1} \) with all of its edges subdivided\[8\].

The distance matrix \( A_d = (d_{ij}) \) of \( G \) is a symmetric matrix of order \( n \) where \( d_{ij} \) is the distance between \( i^{th} \) and \( j^{th} \) vertices of a graph. The distance energy \( E_d(G) \) of the graph \( G \) is defined by the sum of absolute value of all eigenvalues of \( A_d \). The distance matrix of an undirected graph has been widely studied in the literature, see \[2\] \[4\] \[5\] \[6\].

A set \( S \subseteq V \) is a dominating set if every vertex of \( V \setminus S \) is adjacent to at least one vertex in \( S \). The domination number \( \gamma(G) \) is the minimum cardinality of a dominating set in \( G \), and a dominating set \( S \) of minimum cardinality is called a \( \gamma \)-set of \( G \). E.J. Cockayne et al.(2004) introduce the concept of roman domination in graphs\[3\]. A Roman dominating function on a graph \( G = (V, E) \) is a function \( f : V \rightarrow \{0, 1, 2\} \) satisfying the condition that every vertex \( u \) for which \( f(u) = 0 \) is adjacent to at least one vertex \( v \) for which \( f(v) = 2 \).

Let \( (V_0, V_1, V_2) \) be the ordered partition of \( V \) induced by \( f \), where \( V_i = \{v \in V \mid f(v) = i\} \) and \( |V_i| = n_i \), for \( i = 0, 1, 2 \). Note that there exists a one-one correspondence between the functions \( f : V \rightarrow \{0, 1, 2\} \) and the ordered partitions \( (V_0, V_1, V_2) \) of \( V \). Thus, we will write \( f = (V_0, V_1, V_2) \). A function \( f = (V_0, V_1, V_2) \) is a Roman dominating function (RDF) if \( V_2 \geq V_0 \), where \( \geq \) means that the set \( V_2 \) dominates the set \( V_0 \). The weight of \( f \) is \( f(V) = \sum_{v \in V} f(v) = 2n_2 + n_1 \).

The Roman domination number, denoted \( \gamma_R(G) \), equals the minimum weight of an RDF of \( G \), and we say that a function \( f = (V_0, V_1, V_2) \) is a \( \gamma_R \)-function if it is an RDF and \( f(V) = \gamma_R(G) \).

**Theorem 1.1.** \[3\] For any graph \( G \), \( \gamma(G) \leq \gamma_R(G) \leq 2\gamma(G) \).

M.R. Rajesh kannan et al.(2013) studied the minimum covering distance energy of a graph\[8\] and also they were studied Laplacian minimum dominating energy of a
M.R. Rajesh kannan et al. (2014) introduced the concept of the minimum dominating distance energy of a graph. Let $G$ be the graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$. Let $D$ be a minimum dominating set of a graph $G$. The minimum dominating distance matrix of $G$ is the square matrix $A_{Dd}(G) := (d_{ij})$ where

$$d_{ij} = \begin{cases} 
1 & \text{if } i = j \text{ and } v_i \in D \\
d(v_i, v_j) & \text{otherwise}
\end{cases}$$

Let $\delta_1, \delta_2, \ldots, \delta_n$ be the eigenvalues of $A_{Dd}(G)$. Then the minimum dominating energy $E_{Dd}(G)$ of $G$ is

$$E_{Dd}(G) = \sum_{j=1}^{n} |\delta_j|.$$ 

In this paper, we introduce the concept of minimum roman dominating distance energy of a graph in section 2. In Section 3, we find the minimum roman dominating distance energy of some standard graphs. In section 4, we discussed the properties of eigenvalues of a minimum roman dominating distance matrix $A_{RDd}(G)$. We derived the upper and lower bounds for $E_{RDd}(G)$ in section 5.

## 2 The Minimum Roman Dominating Energy

In this section, we introduce the concept of minimum roman dominating energy of a graph.

Let $f = (V_0, V_1, V_2)$ be a $\gamma_R$-function of a graph $G$. The minimum roman dominating distance matrix $A_{RDd}(G)$ of $G$ is defined as $A_{RDd}(G) := (\bar{d}_{ij})$ where

$$\bar{d}_{ij} = \begin{cases} 
2 & \text{if } i = j \text{ and } v_i \in V_2 \\
1 & \text{if } i = j \text{ and } v_i \in V_1 \\
d(v_i, v_j) & \text{otherwise}
\end{cases}$$

Let $\rho_1, \rho_2, \ldots, \rho_n$ be the eigenvalues of $A_{RDd}(G)$. Then the minimum roman dominating distance energy $E_{RDd}(G)$ of $G$ is defined as

$$E_{RDd}(G) = \sum_{k=1}^{n} |\rho_k|.$$ 

Note that $tr(A_{RDd}(G)) = \gamma_R(G)$.

**Example 2.1.** The minimum roman dominating function of the following graph $G$
is \( f = (V_0, V_1, V_2) \) where \( V_2 = \{ v_1 \}, V_1 = \{ v_7, v_9 \} \) and \( V_0 = \{ v_2, v_3, v_4, v_5, v_6, v_8 \} \). Then the minimum roman dominating distance matrix is

\[
A_{\text{RDd}}(G) = \begin{pmatrix}
2 & 1 & 1 & 1 & 1 & 2 & 1 & 2 \\
1 & 0 & 1 & 2 & 2 & 3 & 2 & 3 \\
1 & 1 & 0 & 2 & 2 & 3 & 2 & 3 \\
1 & 2 & 2 & 0 & 1 & 2 & 3 & 2 \\
1 & 2 & 2 & 1 & 0 & 2 & 3 & 2 \\
1 & 2 & 2 & 2 & 0 & 1 & 2 & 3 \\
2 & 3 & 3 & 3 & 3 & 1 & 1 & 3 \\
1 & 2 & 2 & 2 & 2 & 3 & 0 & 1 \\
2 & 3 & 3 & 3 & 3 & 4 & 1 & 1 \\
\end{pmatrix}_{9 \times 9}
\]

Then the characteristic equation of \( A_{\text{RDd}}(G) \) is

\[
\rho^9 - 4\rho^8 - 171\rho^7 - 1034\rho^6 - 2339\rho^5 - 1284\rho^4 + 2659\rho^3 + 4438\rho^2 + 2410\rho + 444 = 0
\]

and the eigenvalues are \( \rho_1 = -3, \rho_2 = -1, \rho_3 = -1, \rho_4 \approx -4.5615, \rho_5 \approx -0.4384, \rho_6 \approx -3.9721, \rho_7 \approx -0.8397, \rho_8 \approx 1.2642 \), and \( \rho_9 \approx 17.5476 \). Hence the minimum roman dominating energy of \( G \) is \( E_{\text{RDd}}(G) \approx 33.6237 \).

Note that this graph has unique minimum roman dominating function.

### 3 Minimum Roman Dominating Distance Energy of Some Standard Graphs

In this section, we studied the minimum roman dominating distance energy of complete, complete bipartite, crown, star and healthy spider graphs.

Denote \( J_n \) is an \( n \times n \) all ones matrix, \( I_n \) is an \( n \times n \) identity matrix, \( D_k \) is a diagonal matrix whose \( k^{th} \) diagonal entry is zero and other diagonal entries are two, \( a_n \) is a \( 1 \times n \) row vector \([a, a, \ldots, a]\) and \( a_n' \) is the transpose of \( a_n \).

**Theorem 3.1.** For any integer \( n \geq 3 \), \( E_{\text{RDd}}(K_n) = 2n - 2 \).

**Proof.** For a complete graph \( K_n \), the minimum roman dominating function is \( f = (V_0, V_1, V_2) \) where \( V_2 = \{ v_i \} \) for any \( i \), \( V_1 = \emptyset \) and \( V_0 = V \setminus V_2 \). Then the minimum
roman dominating distance matrix \( A_{RDd}(K_n) = (a_{ij}) \) where

\[
a_{ij} = \begin{cases} 
2 & \text{if } i = j \text{ and } v_i \in V_2 \\
0 & \text{if } i = j \text{ and } v_i \in V_0 \\
1 & \text{otherwise}
\end{cases}
\]

One can easily show that the characteristic polynomial of \( A_{RDd}(K_n) \) is \((\rho + 1)^{n-2}(\rho^2 - n\rho + n - 3)\). Hence the eigenvalues are -1 with multiplicity \( n - 2 \) and \( \frac{n \pm \sqrt{n^2 - 4n + 12}}{2} \). Since for \( n \geq 3 \), \( n \geq \sqrt{n^2 - 4n + 12} \). Therefore, the eigenvalues \( \frac{n + \sqrt{n^2 - 4n + 12}}{2} \) and \( \frac{n - \sqrt{n^2 - 4n + 12}}{2} \) are positive. Hence the sum of absolute values of all eigenvalues is \( 2n - 2 \). That is, \( E_{RDd}(K_n) = 2n - 2 \).

\textbf{Corollary 3.2.} For any integer \( n \geq 3 \), \( E_{RDd}(K_n) = E_d(K_n) = E(K_n) \).

\textit{Proof.} Let \( n \geq 2 \) be an integer. The adjacency matrix of a complete graph \( K_n \) is \( J_n - I_n \) and the eigenvalues are \( n - 1 \) and \( -1 \) with multiplicity \( n - 1 \), the energy of \( K_n \) is \( 2n - 2 \).

\textbf{Theorem 3.3.} For any integer \( r \geq 2 \),

\[ E_{RDd}(K_{r,r}) = 2(2r - 4) + \sqrt{(r - 2)^2 + 8 + \sqrt{(3r - 2)^2 + 24}}. \]

\textit{Proof.} Let \( X = \{v_1, v_2, \ldots, v_r\} \) and \( Y = \{w_1, w_2, \ldots, w_r\} \) be a partition of the vertex set of a complete bipartite graph \( K_{r,r} \). Then the minimum roman dominating function is \( f = (V_0, V_1, V_2) \) where \( V_2 = \{v_i, w_j\} \) for any \( 1 \leq i, j \leq r \), \( V_1 = \emptyset \) and \( V_0 = V \setminus V_2 \). Then the minimum roman dominating distance matrix is

\[
A_{RDd}(K_{r,r}) = \begin{bmatrix}
2J_r - D_r & J_r \\
J_r & 2J_r - D_j
\end{bmatrix}_{2r \times 2r}
\]

One can easily show that the characteristic equation of \( A_{RDd}(K_{r,r}) \) is \((\rho + 2)^{2r-4}(\rho^2 - (r - 2)\rho - 2)(\rho^2 - (3r - 2)\rho - 6) = 0 \). Then the eigenvalues are \(-2\) with multiplicity \( 2r - 4 \), \( \frac{(r - 2) \pm \sqrt{(r - 2)^2 + 8}}{2} \) and \( \frac{(3r - 2) \pm \sqrt{(3r - 2)^2 + 24}}{2} \). Therefore, the eigenvalues \( \frac{(r - 2) - \sqrt{(r - 2)^2 + 8}}{2} \) and \( \frac{(3r - 2) + \sqrt{(3r - 2)^2 + 24}}{2} \) are negative and the eigenvalues \( \frac{(r - 2) + \sqrt{(r - 2)^2 + 8}}{2} \), \( \frac{(3r - 2) + \sqrt{(3r - 2)^2 + 24}}{2} \) are positive. Hence the sum of absolute values of all eigenvalues is \( 2(2r - 4) + \sqrt{(r - 2)^2 + 8 + \sqrt{(3r - 2)^2 + 24}} \). That is, \( E_{RDd}(K_{r,r}) = 2(2r - 4) + \sqrt{(r - 2)^2 + 8 + \sqrt{(3r - 2)^2 + 24}} \).

\textbf{Theorem 3.4.} For any \( n \geq 3 \), \( E_{RDd}(K_{1,n-1}) = 4n - 6 \).
Proof. Consider the star graph $K_{1,n-1}$ with vertex set $V = \{v_0, v_1, v_2, \ldots, v_{n-1}\}$, where $\text{deg}(v_0) = n - 1$. The minimum roman dominating distance function is $f = (V_0, V_1, V_2)$ where $V_2 = \{v_0\}$, $V_1 = \emptyset$ and $V_0 = V \setminus V_2 = \{v_1, v_2, \ldots, v_{n-1}\}$. Then the minimum roman dominating distance matrix is

$$A_{RDd}(K_{1,n-1}) = \begin{bmatrix} 2 & 1_{n-1} \\ 1'_{n-1} & 2J_{n-1} - 2I_{n-1} \end{bmatrix}_{n \times n}$$

The characteristic equation is $(\rho + 2)^{n-2}(\rho^2 - (2n - 2)\rho + 3n - 7) = 0$. Then the eigenvalues are $-2$ with multiplicity $n - 2$, $(n - 1) \pm \sqrt{n^2 - 5n + 8}$. Hence the sum of absolute values of all eigenvalues is $4n - 6$. That is, $E_{RDd}(K_{1,n-1}) = 4n - 6$.

**Theorem 3.5.** For an odd integer $n \geq 4$,

$$11n - 19 \leq E_{RDd}(K_{1,n-1}^*) \leq 6n^2 - 4n - 16.$$  

Proof. Consider the healthy spider graph $K_{1,n-1}^*$ with vertex set $V = \{v_0, v_1, v_2, \ldots, v_{n-1}, u_1, u_2, \ldots, u_{n-1}\}$. The vertex $v_0$ is adjacent with $v_1, v_2, \ldots, v_{n-1}$ and for $1 \leq i \leq n - 1$, $u_i$ is adjacent with $v_i$. Then the minimum roman dominating function is $f = (V_0, V_1, V_2)$ where $V_2 = \{v_0\}$, $V_1 = \{u_1, u_2, \ldots, u_{n-1}\}$ and $V_0 = \{v_1, v_2, \ldots, v_{n-1}\}$.

Then the minimum roman dominating distance matrix is

$$A_{RDd}(K_{1,n-1}^*) = \begin{bmatrix} 2 & 1_{n-1} \\ 1'_{n-1} & 2J_{n-1} - 2I_{n-1} \\ 2'_{n-1} & 3J_{n-1} - 3I_{n-1} \end{bmatrix}_{2n-1 \times 2n-1}$$

The characteristic equation of $A_{RDd}(K_{1,n-1}^*)$ is

$$(\rho^2 + 5\rho + 2)^{n-2}[\rho^3 - (6n - 9)\rho^2 - (n^2 - 7n + 14)\rho + 2n^2 - 3n - 3] = 0.$$  

The sum of absolute values of the roots of $(\rho^2 + 5\rho + 2)^2 = 0$ is $5n - 10$ and the sum of absolute values of roots of the $\rho^3 - (6n - 9)\rho^2 - (n^2 - 7n + 14)\rho + 2n^2 - 3n - 3 = 0$ is greater than or equal to $6n - 9$. Therefore,

$$E_{RDd}(K_{1,n-1}^*) \geq 11n - 19.$$
By Cauchy’s bound for roots of a polynomial, all the roots of \( \rho^3 - (6n - 9)\rho^2 - (n^2 - 7n + 14)\rho + 2n^2 - 3n - 3 = 0 \) lie in the closed interval \([- (M + 1), M + 1]\) where \( M = 2n^2 - 3n - 3 \). Therefore, the sum of absolute values of these roots is bounded above by \( 3(2n^2 - 3n - 2) = 6n^2 - 9n - 6 \). Thus,

\[
E_{RDd}(K^*_1, n-1) \leq 5n - 10 + 6n^2 - 9n - 6 = 6n^2 - 4n - 16.
\]

**Theorem 3.6.** For any integer \( k \geq 2 \), \( E_{RDd}(S^0_k) = 7k - 6 + \sqrt{k^2 - 4k + 12} \).

**Proof.** Consider the crown graph \( S^0_k \) with vertex set \( V = X \cup Y \) where \( X = \{v_1, v_2, \ldots, v_k\} \) and \( Y = \{w_1, w_2, \ldots, w_k\} \). The minimum roman dominating distance function is \( f = (V_0, V_1, V_2) \) where \( V_0 = V, V_1 = \emptyset \) and \( V_2 = \{v_i, w_i\} \) for any \( 1 \leq i \leq k \). Then the minimum roman dominating distance matrix is

\[
A_{RDd}(S^0_k) = \begin{bmatrix}
2J_k - D_i & J_k + 2I_k \\
J_k + 2I_k & 2J_k - D_i
\end{bmatrix}_{2k \times 2k}
\]

The Characteristic equation of \( A_{RDd}(S^0_k) \) is

\[
\rho^{2k-2}(\rho + 4)^{2k-2}[\rho^2 - (3k + 2)\rho + 6(k - 1)][(\rho^2 + (6 - k)\rho - 2k + 6) = 0.
\]

Then the eigenvalues are \(-4\) with multiplicity \( k-2 \), \(0\) with multiplicity \( k-2 \), \((-6+k)\pm\sqrt{k^2 - 4k + 12}\) and \((-3k+2)\pm\sqrt{9k^2 - 12k + 2k}\). Hence the sum of absolute values of all eigenvalues is \( 7k - 6 + \sqrt{k^2 - 4k + 12} \). That is, \( E_{RDd}(S^0_k) = 7k - 6 + \sqrt{k^2 - 4k + 12} \). \( \square \)

**Theorem 3.7.** Let \( G \) and \( H \) be two disjoint graphs. Then \( E_{RDd}(G \cup H) = E_{RDd}(G) + E_{RDd}(H) \).

**Proof.** Let \( A \) and \( B \) be the minimum roman dominating distance matrix of \( G \) and \( H \), respectively. Then the minimum roman dominating distance matrix of \( G \cup H \) is

\[
A_{RDd}(G \cup H) = \begin{bmatrix}
A & 0 \\
0 & B
\end{bmatrix}.
\]

The characteristic polynomial of \( A_{RDd}(G \cup H) \) is the product of characteristic polynomial of \( A \) and \( B \). Therefore, \( E_{RDd}(G \cup H) = E_{RDd}(G) + E_{RDd}(H) \). \( \square \)

### 4 Properties of Eigenvalues of Minimum Roman Dominating Distance Matrix \( A_{RDd}(G) \)

In this section, we discussed the relation between the eigenvalues of the minimum roman dominating distance matrix \( A_{RDd}(G) \) and the minimum roman dominating energy \( \gamma_R \) of \( G \).
Theorem 4.1. Let $G = (V, E)$ be a graph and let $f = (V_0, V_1, V_2)$ be a $\gamma_R$-function. If $\rho_1, \rho_2, \ldots, \rho_n$ are the eigenvalues of minimum roman dominating distance matrix $A_{RDd}(G)$, then

(i) $\sum_{i=1}^{n} \rho_i = \gamma_R(G)$

(ii) $\sum_{i=1}^{n} \rho_i^2 = \gamma_R(G) + 2m + 2M$ where $M = \sum_{i<j, d(v_i, v_j)\neq 1} d(v_i, v_j)^2$ and $m = |E|$.

Proof. (i) The sum of the eigenvalues of $A_{RDd}(G)$ is the trace of $A_{RDd}(G)$. Therefore,

$$\sum_{i=1}^{n} \rho_i = \sum_{i=1}^{n} d(v_i, v_i) = 2|V_2| + |V_1| = \gamma_R(G).$$

(ii) The sum of squares of the eigenvalues of $A_{RDd}(G)$ is trace of $(A_{RDd}(G))^2$. Therefore,

$$\sum_{i=1}^{n} \rho_i^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} d(v_i, v_j)d(v_j, v_i)
= \sum_{i=1}^{n} d(v_i, v_i)^2 + \sum_{i\neq j} d(v_i, v_j)d(v_j, v_i)
= \sum_{i=1}^{n} d(v_i, v_i)^2 + \sum_{i<j} d(v_i, v_j)^2
= \gamma_R(G) + 2\sum_{i<j} d(v_i, v_j)^2
= \gamma_R(G) + 2m + 2M.$$  

Corollary 4.2. Let $G$ be a graph with diameter 2 and let $f = (V_0, V_1, V_2)$ be a $\gamma_R$-function. If $\rho_1, \rho_2, \ldots, \rho_n$ are eigenvalues of minimum roman dominating distance matrix $A_{RDd}(G)$, then

$$\sum_{i=1}^{n} \rho_i^2 = \gamma_R(G) + 2(2n^2 - 2n - 3m).$$

Proof. We know that in $A_{RDd}(G)$ there are $2m$ elements with 1 and $n(n - 1) - 2m$ elements with 2 and hence corollary follows from the above theorem. \qed

5 Bounds for Minimum Roman Dominating Energy

In this section, we discussed the bounds for minimum roman dominating energy.

The proofs of the following Theorems are similar to the proofs in [9].
Theorem 5.1. Let $G$ be a graph. If $f = (V_0, V_1, V_2)$ is a $\gamma_R$-function and $P = |\det(A_{RDd}(G))|$, then

$$\sqrt{(2m + 2M + \gamma_R) + n(n-1)P^2} \leq E_{RDd}(G) \leq \sqrt{n(2m + 2M + \gamma_R(G))}$$

where $\gamma_R$ is a roman domination number.

By Theorem 1.1, we have the following Corollary

Corollary 5.2. Let $G$ be a graph. If $f = (V_0, V_1, V_2)$ is a $\gamma_R$-function and $P = |\det(A_{RDd}(G))|$, then

$$\sqrt{(2m + 2M + \gamma) + n(n-1)P^2} \leq E_{RDd}(G) \leq \sqrt{n(2m + 2M + 2\gamma(G))}$$

where $\gamma$ is a minimum domination number of $G$.

Remark 5.3. In 3.5, for the healthy spider graph $K_{1,n-1}$, $m = 2n-2$, $M = (n-1)(19n-6)$ and $\gamma_R(K_{1,n-1}) = n+1$. Hence $\sqrt{n(2m + 2M + \gamma_R(G))} = \sqrt{(2n-1)(38n^2 + 31n - 3)} > 6n^2 - 4n - 16$.

Theorem 5.4. If $\rho_1(G)$ is the largest eigenvalue of a minimum roman dominating distance matrix $A_{RDd}(G)$, then

$$\rho_1(G) \geq \frac{2W(G) + \gamma_R(G)}{n}$$

where $W(G)$ is the Wiener index of $G$.

Theorem 5.5. Let $G$ be a graph of diameter 2 and $\rho_1(G)$ is the largest eigenvalue of a minimum roman dominating distance matrix $A_{RDd}(G)$, then

$$\rho_1(G) \geq \frac{2n^2 - 2m - 2n + \gamma_R(G)}{n}$$

Conclusion

In this paper, we introduced the concept of minimum roman dominating distance energy $E_{RDd}(G)$ of a graph $G$ and computed minimum roman dominating distance energy of complete, complete bipartite, crown, star and healthy spider graphs. Also, we discussed the properties of eigenvalues of a minimum roman dominating distance matrix $A_{RDd}(G)$. Finally, we derived the Upper and lower bounds for $E_{RDd}(G)$. 
References

[1] R. B. Bapat, Graphs and Matrices, Hindustan Book Agency, 2011.

[2] R. Bapat, S. J. Kirkland, and M. Neumann, On distance matrices and Laplacians, Linear Algebra Appl. 401 (2005), 193–209. MR 2133282

[3] E.J. Cockayne, P.A. Dreyer Jr., S.M. Hedetniemi and S.T. Hedetniemi, Roman domination in graphs, Discrete Mathematics, 278 (2004) 11–22.

[4] R. L. Graham, A. J. Hoffman, and H. Hosoya, On the distance matrix of a directed graph, J. Graph Theory 1 (1977), no. 1, 85–88. MR 505769.

[5] R. L. Graham and L. Lovász, Distance matrix polynomials of trees, Adv. in Math. 29 (1978), no. 1, 60–88. MR 480119.

[6] R. L. Graham and H. O. Pollak, On the addressing problem for loop switching, Bell System Tech. J. 50 (1971), 2495–2519. MR 289210.

[7] I. Gutman, The energy of a graph, Ber. Math-Statist.Sekt.Forschungsz.Graz, 103(1978), 1-22.

[8] M.R. Rajesh Kanna, B. N. Dharmendra, and R. Pradeep Kumar, Minimum Covering Distance Energy of a Graph, Applied Mathematical Sciences, 7:111 (2013), 5525 - 5536.

[9] M.R. Rajesh Kanana, B.N Dharmendra, and G. Sridhara. Minimum dominating distance energy of a graph, Journal of Indonesian Mathematical Society, 20(1):19-29, 2014.

[10] M. R. Rajesh Kanna, B. N. Dharmendra, and G. Sridhara, Laplacian Minimum Dominating Energy of a Graph, International Journal of Pure and Applied Mathematics, 89:4 (2013), 565-581.