Quantum theory of light double-slit diffraction

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In this paper, we study the light double-slit diffraction experiment with quantum theory approach. Firstly, we calculate the light wave function in slits by quantum theory of photon. Secondly, we calculate the diffraction wave function with Kirchhoff’s law. Thirdly, we give the diffraction intensity of light double-slit diffraction, which is proportional to the square of diffraction wave function. Finally, we compare calculation result of quantum theory and classical electromagnetic theory with the experimental data. We find the quantum calculate result is accordance with the experiment data, and the classical calculation result with certain deviation. So, the quantum theory is more accurately approach for studying light diffraction.

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1. Introduction

In recent years, quantum information science has advanced rapidly, both at the level of fundamental research and technological development. For instance, quantum cryptography systems have become commercially available [1]. Classical optical lithography technology is facing its limit due to the diffraction effect of light. It is known that the nonclassical phenomena of two photon interference and two-photon ghost diffraction and imaging, have classical counterparts [2-3]. Two photon interference of classical light has been first discovered in the pioneering experiments by Hanbury Brown and Twiss and since then was observed with various sources, including true thermal ones, and coherent ones [4-7]. Somewhat later, ghost imaging with classical light has been demonstrated, both in the near-field and far-field domains [8-10]. The present optical imaging technologies, such as optical lithography, have reached a spatial resolution in the sub-micrometer range, which comes up against the diffraction limit due to the wavelength of light. However, the guiding principle of such technology is still based on the classical diffraction theory established by Fresnel, Kirchhoff and others more than a hundred years ago. Recently, the use of quantum-correlated photon pairs to overcome the classical diffraction limit was proposed and attracted much attention. Obviously, quantum theory approaches are necessary to explain the diffraction-interference of the quantum-correlated multi-photon state. As is well known, the classical optics with its standard wave-theoretical methods and approximations, such as Huygens’ and Kirchhoff’s theory, has been successfully applied to classical optics, and has yielded good agreement with many experiments. However, light interference and diffraction are quantum phenomena, and its full description needs quantum theory approach. In 1924, Epstein and Ehrenfest had firstly studied light diffraction with the old quantum theory, i.e., the quantum mechanics of correspondence principle, and obtained a identical result with the classical optics [11-17]. In this paper, we study the double-slit diffraction of light with the approach of relativistic quantum theory of photon. In view of quantum theory, the light has the nature of wave, and the wave is described by wave function. We calculate the light wave function in slits by quantum theory of photon, where the diffraction wave function can be calculated by the Kirchhoff’s law. The diffraction intensity is proportional to the square of diffraction wave function. We can obtain the diffraction intensity by calculating the light wave function distributing on display screen. We compare calculation results of quantum theory and classical electromagnetic theory with the experimental data. When the decoherence effects are considered, we find the quantum calculate result is in accordance

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with the experiment data, but the classical calculation result with certain deviation. In order to study the light double-slit diffraction more accurately, it should be applied the new approach of quantum theory.

2. Quantum approach of light diffraction

In an infinite plane, we consider a double-slit, its width $a$, length $b$ and the slit-to-slit distance $d$ are shown in Fig. 1. The $x$ axis is along the slit length $b$ and the $y$ axis is along the slit width $a$. We calculate the light wave function in the left slit with the light of the relativistic wave equation. At time $t$, we suppose that the incoming plane wave travels along the $z$ axis. It is

$$
\begin{align*}
\vec{\psi}_0(z, t) &= \vec{A} e^{i(\hat{p}z - \hat{E}t)} \\
&= \sum_j A_j \cdot e^{i(\hat{p}z - \hat{E}t)} \vec{e}_j \\
&= \sum_j \psi_0j \cdot e^{-i\hat{E}t} \vec{e}_j,
\end{align*}
$$

(1)

where $\psi_{0j} = A_j \cdot e^{i\hat{p}z}$, $j = x, y, z$ and $\vec{A}$ is a constant vector. The time-dependent relativistic wave equation of light is [12]

$$
\begin{align*}
i\hbar \frac{\partial}{\partial t} \vec{\psi}(\vec{r}, t) &= c\hbar \nabla \times \vec{\psi}(\vec{r}, t) + V \vec{\psi}(\vec{r}, t),
\end{align*}
$$

(2)

where $c$ is light velocity. From Eq. (2), we can find the light wave function $\vec{\psi}(\vec{r}, t) \to 0$ when $V(\vec{r}) \to \infty$. The potential energy of light in the left slit is

$$
V(x, y, z) = \begin{cases} 
0 & \text{if } 0 \leq x \leq b, -\frac{d}{2} - a \leq y \leq -\frac{d}{2}, 0 \leq z \leq c', \\
\infty & \text{otherwise},
\end{cases}
$$

(3)

where $c'$ is the slit thickness. We can get the time-dependent relativistic wave equation in the slit ($V(x, y, z) = 0$), it is

$$
\begin{align*}
i\hbar \frac{\partial}{\partial t} \vec{\psi}_1(\vec{r}, t) &= c\hbar \nabla \times \vec{\psi}_1(\vec{r}, t),
\end{align*}
$$

(4)

by derivation on Eq. (4) about the time $t$ and multiplying $i\hbar$ both sides, we have

$$
\begin{align*}
(i\hbar)^2 \frac{\partial^2}{\partial t^2} \vec{\psi}_1(\vec{r}, t) &= c\hbar \frac{\partial}{\partial t} \vec{\psi}_1(\vec{r}, t),
\end{align*}
$$

(5)
substituting Eq. (4) into (5), we have

\[
\frac{\partial^2}{\partial t^2} \vec{\psi}_1(\vec{r}, t) = -c^2 [\nabla (\nabla \cdot \vec{\psi}_1(\vec{r}, t)) - \nabla^2 \vec{\psi}_1(\vec{r}, t)],
\]

where the formula \( \nabla \times \nabla \times \vec{B} = \nabla (\nabla \cdot \vec{B}) - \nabla^2 \vec{B} \). From Ref. [11], the photon wave function is \( \vec{\psi}_1(\vec{r}, t) = \sqrt{\frac{m}{2\pi}(\vec{E}(\vec{r}, t) + i\sigma c\vec{B}(\vec{r}, t))} \), we have

\[
\nabla \cdot \vec{\psi}_1(\vec{r}, t) = 0,
\]

from Eq. (6) and (7), we have

\[
(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2) \vec{\psi}_1(\vec{r}, t) = 0.
\]

The Eq. (8) is the same as the classical wave equation of light. Here, it is a quantum wave equation of light, since it is obtained from the relativistic wave equation (2), and it satisfied the new quantum boundary condition: when \( \vec{\psi}_1(\vec{r}, t) \to 0, V(\vec{r}) \to \infty \). It is different from the classic boundary condition. When the photon wave function \( \vec{\psi}_1(\vec{r}, t) \) change with determinate frequency \( \omega \), the wave function of photon can be written as

\[
\vec{\psi}_1(\vec{r}, t) = \vec{\psi}_1(\vec{r})e^{-i\omega t},
\]

substituting Eq. (9) into (8), we can get

\[
\frac{\partial^2}{\partial x^2} \psi_{1x}(\vec{r}) + \frac{\partial^2}{\partial y^2} \psi_{1y}(\vec{r}) + \frac{\partial^2}{\partial z^2} \psi_{1z}(\vec{r}) + 4\pi^2 \frac{\lambda^2}{\lambda^2} \psi_1(\vec{r}) = 0,
\]

and the wave function satisfies boundary conditions

\[
\psi_1(0, y, z) = \psi_1(b, y, z) = 0,
\]

\[
\psi_1(x, -\frac{d}{2} - a, z) = \psi_1(x, -\frac{d}{2}, z) = 0,
\]

the photon wave function \( \vec{\psi}(\vec{r}) \) can be wrote

\[
\vec{\psi}(\vec{r}) = \psi_{1x}(\vec{r})\hat{e}_x + \psi_{1y}(\vec{r})\hat{e}_y + \psi_{1z}(\vec{r})\hat{e}_z = \sum_{j=x,y,z} \psi_{1j}(\vec{r})\hat{e}_j,
\]

where \( j \) is \( x, y \) or \( z \). Substituting Eq. (13) into (10), (11) and (12), we have the component equation

\[
\frac{\partial^2}{\partial x^2} \psi_{1x}(\vec{r}) + \frac{\partial^2}{\partial y^2} \psi_{1y}(\vec{r}) + \frac{\partial^2}{\partial z^2} \psi_{1z}(\vec{r}) + 4\pi^2 \frac{\lambda^2}{\lambda^2} \psi_{1j}(\vec{r}) = 0,
\]

\[
\psi_{1j}(0, y, z) = \psi_{1j}(b, y, z) = 0,
\]

\[
\psi_{1j}(x, -\frac{d}{2} - a, z) = \psi_{1j}(x, -\frac{d}{2}, z) = 0,
\]

the partial differential equation (14) can be solved by the method of separation of variable. By writing

\[
\psi_{1j}(x, y, z) = X_{1j}(x)Y_{1j}(y)Z_{1j}(z).
\]
From Eqs. (14-17), we can get the general solution of Eq. (14)

\[
\psi_{1j}(x, y, z) = \sum_{mn} \sin \frac{n\pi}{b} x \cdot (D_{mnj} \cos \frac{m\pi}{a} y + D'_{mnj} \sin \frac{m\pi}{a} y) \cdot \exp[i \sqrt{\frac{4\pi^2}{\lambda^2} - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \cdot z],
\]

(18)
since the wave functions are continuous at \(z = 0\), we have

\[
\psi_{0j}(x, y, z) \big|_{z=0} = \psi_{1j}(x, y, z) \big|_{z=0},
\]

(19)
or, equivalently,

\[
\psi_{0j}(x, y, z) \big|_{z=0} = \psi_{1j}(x, y, z) \big|_{z=0}.
\]

(20)
From Eq. (1), (18) and (20), we obtain the coefficient \(D_{mnj}\) by fourier transform

\[
D_{mnj} = \frac{4}{a \cdot b} \int_{0}^{b} \int_{-\frac{d}{2} - a}^{\frac{d}{2} - a} A_{1j} \cdot \sin \frac{n\pi}{b} x \cdot \cos \frac{m\pi}{a} y \cdot d_x \cdot d_y
\]

\[
-\frac{16A_{1j}}{(2m + 1) \cdot (2n + 1) \cdot \pi^2} \sin \left(\frac{2m + 1}{2a} \cdot \pi \cdot d\right),
\]

(21)

\[
D'_{mnj} = \frac{4}{a \cdot b} \int_{0}^{b} \int_{-\frac{d}{2} - a}^{\frac{d}{2} - a} A_{1j} \cdot \sin \frac{n\pi}{b} x \cdot \sin \frac{m\pi}{a} y \cdot d_x \cdot d_y
\]

\[
\frac{16A_{1j}}{(2m + 1) \cdot (2n + 1) \cdot \pi^2} \cos \left(\frac{2m + 1}{2a} \cdot \pi \cdot d\right),
\]

(22)
substituting Eq. (21) and (22) into (18), we have

\[
\psi_{1j}(x, y, z) = \sum_{j=x,y,z} \sum_{m,n=0}^{\infty} \frac{-16A_{1j}}{(2m + 1) \cdot (2n + 1) \cdot \pi^2} \cdot \sin \left(\frac{2m + 1}{2a} \cdot \pi x\right)
\]

\[
\cdot \left[\sin \left(\frac{2m + 1}{2a} \cdot \pi d\right) \cdot \cos \left(\frac{2m + 1}{2a} \cdot \pi y\right) + \cos \left(\frac{2m + 1}{2a} \cdot \pi d\right) \cdot \sin \left(\frac{2m + 1}{2a} \cdot \pi y\right)\right]
\]

\[
\cdot \exp[i \sqrt{\frac{4\pi^2}{\lambda^2} - \left(\frac{2m + 1}{2a} \cdot \pi\right)^2 - \left(\frac{2n + 1}{2b} \cdot \pi\right)^2} \cdot z],
\]

(23)
substituting Eq. (23) into (9) and (13), we can obtain the photon wave function \(\tilde{\psi}_1(x, y, z, t)\) in slit

\[
\tilde{\psi}_1(x, y, z, t) = \sum_{j=x,y,z} \sum_{m,n=0}^{\infty} \psi_{1j}(x, y, z, t) \cdot e^{i\epsilon_j}
\]

\[
= \sum_{j=x,y,z} \sum_{m,n=0}^{\infty} \frac{-16A_{1j}}{(2m + 1) \cdot (2n + 1) \cdot \pi^2} \cdot \sin \left(\frac{2m + 1}{2a} \cdot \pi x\right)
\]

\[
\cdot \left[\sin \left(\frac{2m + 1}{2a} \cdot \pi d\right) \cdot \cos \left(\frac{2m + 1}{2a} \cdot \pi y\right) + \cos \left(\frac{2m + 1}{2a} \cdot \pi d\right) \cdot \sin \left(\frac{2m + 1}{2a} \cdot \pi y\right)\right]
\]

\[
\cdot \exp[i \sqrt{\frac{4\pi^2}{\lambda^2} - \left(\frac{2m + 1}{2a} \cdot \pi\right)^2 - \left(\frac{2n + 1}{2b} \cdot \pi\right)^2} \cdot z] \cdot \exp[-i\omega t] \cdot e^{i\epsilon_j}.
\]

(24)
The potential energy of light in the right slit is

\[
V(x, y, z) = \begin{cases} 
0 & 0 \leq x \leq b, \frac{d}{2} \leq y \leq \frac{d}{2} + a, 0 \leq z \leq c', \\
\infty & \text{otherwise},
\end{cases}
\]

(25)
and the wave function satisfies boundary conditions
\[ \psi_2(0, y, z) = \psi_2(b, y, z) = 0, \]  
\[ \psi_2(x, \frac{d}{2}, z) = \psi_2(x, \frac{d}{2} + a, z) = 0, \]

similarly, we can obtain the light wave function \( \tilde{\psi}_2(x, y, z, t) \) in the right slit

\[
\tilde{\psi}_2(x, y, z, t) = \sum_{j=x,y,z} \sum_{m,n=0}^{\infty} \frac{-16A_{2j}}{(2m+1) \cdot (2n+1) \cdot \pi^2} \sin \left( \frac{(2m+1)\pi}{b} \right) \sin \left( \frac{(2n+1)\pi}{a} \right) \cdot \exp \left[ i \sqrt{\frac{4}{\lambda^2} - \left( \frac{(2m+1)\pi}{b} \right)^2 - \left( \frac{(2n+1)\pi}{a} \right)^2} \cdot z \right] \cdot \exp[-i\omega t] \varepsilon_j.
\]

3. The wave function of light diffraction

In the section 2, we have calculated the photon wave function in slit. In the following, we will calculate diffraction wave function. We can calculate the wave function in the diffraction area. From the slit wave function component \( \psi_j(\vec{r}, t) \), we can calculate its diffraction wave function component \( \Phi_j(\vec{r}, t) \) by Kirchhoff’s law. It can be calculated by the formula

\[
\Phi_j(\vec{r}, t) = -\frac{1}{4\pi} \int_{s_0} e^{iKR} \vec{n} \cdot \left[ \nabla' \psi_j + (ik - \frac{1}{r}) \vec{r} \right] ds,
\]

the total diffraction wave function is

\[
\Phi(\vec{r}, t) = \sum_{j=x,y,z} \Phi_j(\vec{r}, t) \varepsilon_j,
\]

in the following, we firstly calculate the diffraction wave function of the top slit, it is

\[
\Phi_{1j}(\vec{r}_1, t) = -\frac{1}{4\pi} \int_{s_1} e^{iKR} \vec{n} \cdot \left[ \nabla' \psi_{1j} + (ik - \frac{1}{r_1}) \vec{r}_1 \right] ds.
\]

The diffraction area is shown in Fig. 2, where \( k = \frac{2\pi}{\lambda} \), \( s_1 \) is the area of the top slit, \( \vec{r}_1 \) is the position of a point on the surface \( (z=c) \), \( P \) is an arbitrary point in the diffraction area, and \( \vec{n} \) is a unit vector, which is normal to the surface of the slit.

In Fig.2, we firstly consider the up slit, there are

\[
r_1 = R - \frac{\vec{R}}{R} \cdot \vec{r}_1 \approx R - \frac{\vec{r}_1}{r_1} \cdot \vec{r}_1
\]

\[
= R - \frac{k_1}{k} \cdot \vec{r}_1,
\]

and then,

\[
\frac{e^{ikr_1}}{r_1} = \frac{e^{ik(R - \frac{\vec{r}_1}{r_1} \cdot \vec{r}_1)}}{R - \frac{\vec{r}_1}{r_1} \cdot \vec{r}_1} \approx \frac{e^{ikR} e^{-ik_1 \cdot \vec{r}_1}}{R - \frac{\vec{r}_1}{r_1} \cdot \vec{r}_1},
\]

\[
(|\vec{r}_1| \ll R),
\]
with \( \vec{k}_1 = k\vec{r}_1 / r_1 \). Substituting Eq. (32) and (33) into Eq. (31), we can obtain

\[
\Phi_{1j}(x, y, z; t) = -e^{ikR}e^{-iwt} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{16A_{1j}}{(2m+1)(2n+1)\pi^2} \exp[i \sqrt{\frac{4\pi^2}{\lambda^2} - \left( \frac{(2m+1)\pi}{a} \right)^2 - \left( \frac{(2n+1)\pi}{b} \right)^2} \cdot c']
\]

\[
\times \left[ i \sqrt{\frac{4\pi^2}{\lambda^2} - \left( \frac{(2m+1)\pi}{a} \right)^2 - \left( \frac{(2n+1)\pi}{b} \right)^2 + i\vec{n} \cdot \vec{R} \right] \cdot \left[ \sin \left( \frac{(2m+1)\pi}{a} \right) \cos \left( \frac{(2n+1)\pi}{a} \right) + \cos \left( \frac{2m+1}{a} \pi \right) \sin \left( \frac{2n+1}{a} \pi \right) \right] \int_{s_1} \exp[-ik_1 \cdot \vec{r}' \cdot \vec{R}] \cdot \left[ \sin \left( \frac{(2m+1)\pi}{a} \right) \cos \left( \frac{(2n+1)\pi}{b} \right) \right] \int_{-d}^{-d} e^{-ik \sin \beta_1 \cdot y} \sin \left( \frac{(2m+1)\pi}{d} \right) \frac{dy}{dy}.
\]

(34)

For the second diffraction slit, we assume the angle between \( \vec{k}_1 \) and \( x \) axis (\( y \) axis) is \( \frac{\pi}{2} - \alpha (\frac{\pi}{2} - \beta_1) \), and \( \alpha(\beta_1) \) is the angle between \( \vec{k}_1 \) and the surface of \( yz \) (\( xz \)), then we have

\[
k_{1x} = k \sin \alpha, \quad k_{1y} = k \sin \beta_1,
\]

(35)

\[
\vec{n} \cdot \vec{k}_1 = k \cos \theta,
\]

(36)

where \( \theta \) is the angle between \( \vec{k}_1 \) and \( z \) axis, and the angles \( \theta, \alpha, \beta_1 \) satisfy the equation

\[
\cos^2 \theta + \cos^2 \left( \frac{\pi}{2} - \alpha \right) + \cos^2 \left( \frac{\pi}{2} - \beta_1 \right) = 1,
\]

(37)

with \( R = \sqrt{l^2 + s^2} \). Substituting Eqs. (35)-(37) into Eq. (34) yields

\[
\Phi_{1j}(x, y, z; t) = -e^{ikR}e^{-iwt}e^{-ik \cos \theta \cdot c'} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{16A_{1j}}{(2m+1)(2n+1)\pi^2} \exp[i \sqrt{\frac{4\pi^2}{\lambda^2} - \left( \frac{(2m+1)\pi}{a} \right)^2 - \left( \frac{(2n+1)\pi}{a} \right)^2} \cdot c']
\]

\[
\times \left[ i \sqrt{\frac{4\pi^2}{\lambda^2} - \left( \frac{(2m+1)\pi}{a} \right)^2 - \left( \frac{(2n+1)\pi}{a} \right)^2 + (ik - \frac{1}{R}) \cdot \sqrt{\cos^2 \alpha - \sin^2 \beta_1} \right] \cdot \left[ \sin \left( \frac{(2m+1)\pi}{a} \right) \cos \left( \frac{(2n+1)\pi}{b} \right) \right] \int_{0}^{b} e^{-ik \sin \alpha \cdot x} \sin \left( \frac{(2n+1)\pi}{b} \right) \sin \left( \frac{(2m+1)\pi}{d} \right) \frac{dy}{dy}.
\]

(38)
Substituting Eq. (38) into (30), we can get the diffraction function of the up slit

$$\Phi_1(x, y, z; t) = \sum_{j=x,y,z} \Phi_{1j}(x, y, z; t) c_j$$

$$= \frac{e^{i k R}}{4 \pi R} e^{-i \omega t} e^{-i k \cos \theta' c'} \sum_{j=x,y,z} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} 16 A_{1j} (2m+1)(2n+1) \pi^2 e^{i \sqrt{\frac{4 \pi^2}{\lambda^2} \left[ \frac{2(n+1) \pi}{b} - \frac{2(m+1) \pi}{a} \right]^2} c'$$

$$\cdot \left[ i \sqrt{\frac{4 \pi^2}{\lambda^2} - \frac{(2m+1) \pi}{b}^2} \frac{(2m+1) \pi}{a} + (ik - \frac{1}{R}) \cdot \sqrt{\cos^2 \alpha - \sin^2 \beta_1} \right]$$

$$\int_0^b e^{-ik \sin \alpha x} (2m+1) \pi b x dx \int_{-\frac{d}{2}}^{\frac{d}{2}} e^{-ik \sin \beta_1 y} \sin \frac{(2m+1) \pi}{a} (\frac{d}{2} + y) dy c_j$$

Similarly, the diffraction wave function of the down slit is

$$\Phi_2(x, y, z; t) = \frac{e^{i k R}}{4 \pi R} e^{-i \omega t} e^{-i k \cos \theta' c'} \sum_{j=x,y,z} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} 16 A_{2j} (2m+1)(2n+1) \pi^2 e^{i \sqrt{\frac{4 \pi^2}{\lambda^2} \left[ \frac{2(n+1) \pi}{b} - \frac{2(m+1) \pi}{a} \right]^2} c'$$

$$\cdot \left[ i \sqrt{\frac{4 \pi^2}{\lambda^2} - \frac{(2m+1) \pi}{b}^2} \frac{(2m+1) \pi}{a} + (ik - \frac{1}{R}) \cdot \sqrt{\cos^2 \alpha - \sin^2 \beta_2} \right]$$

$$\int_0^b e^{-ik \sin \alpha x} (2m+1) \pi b x dx \int_{-\frac{d}{2}}^{\frac{d}{2}} e^{-ik \sin \beta_2 y} \sin \frac{(2m+1) \pi}{a} (\frac{d}{2} - y) dy c_j$$

where $d$ is the two slit distance. The total diffraction wave function for the double-slit is

$$\Phi(x, y, z; t) = c_1 \Phi_1(x, y, z; t) + c_2 \Phi_2(x, y, z; t).$$

where $c_1$ and $c_2$ are superposition coefficients, and satisfy the equation

$$|c_1|^2 + |c_2|^2 = 1.$$ (42)

For the double-slit diffraction, we can obtain the relative diffraction intensity $I$ on the display screen

$$I \propto |\Phi(x, y, z; t)|^2.$$ (43)

4. The relative diffraction intensity $I$ on the display screen

Decoherence is introduced here using a simple phenomenological theoretical model that assumes an exponential damping of the interferences [19], i.e., the decoherence is the dynamic suppression of the interference terms owing to the interaction between system and environment. Eq. (41) describes the decoherence state coherence superposition, without considering the interaction of system with external environment. When we consider the effect of external environment, the total wave function of system and environment for the double-slit factorizes as [19]

$$\Phi(x, y, z; t) = c_1 \Phi_1(x, y, z; t) \otimes |E_1 >_t + c_2 \Phi_2(x, y, z; t) \otimes |E_2 >_t.$$ (44)

where $\otimes |E_1 >_t$ and $\otimes |E_2 >_t$ describe the state of the environment. Now, the diffraction intensity on the screen is given by [19]

$$I = (1 + |\alpha_t|^2)|c_1|^2 |\Phi_1|^2 + c_2|^2 |\Phi_2|^2 + 2c_1 c_2 \Lambda_t Re(\Phi_1^* \Phi_2^*),$$

where $\alpha_t =_{t} < E_2 | E_1 >_t$, and $\Lambda_t = \frac{2|\alpha_t|^2}{1 + |\alpha_t|^2}$. Thus, $\Lambda_t$ is defined as the quantum coherence degree. The fringe visibility of $n$ is defined as [19]

$$v = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}},$$ (46)
where $I_{\text{max}}$ and $I_{\text{min}}$ are the intensities corresponding to the central maximum and the first minimum next to it, respectively. The value for the fringe visibility of $\nu = 0.873$ is obtained in the experiment [18], and the quantum coherence degree $\Lambda_t \approx \nu$ [19]. Eq. (45) is the diffraction intensity of light double-slit diffraction including decoherence effects, and Eq. (43) is the diffraction intensity of light double-slit diffraction considering coherence superposition.

5. Numerical result

In this section, we report our numerical results of diffraction intensity for light double-slit diffraction. The theory result of quantum theory is from Eq. (45), and Eq. (47) is the theory result of classical electromagnetic theory. The Ref. [20] is the light double-slit diffraction experiment. In [20], two slit width are $a = 1.3 \times 10^{-4}m$, the distance between the two slit $d = 4 \times 10^{-4}m$, slit to the screen distance $l = 4m$, and the wavelength of the light $\lambda = 916 \times 10^{-9}m$. From FIG. 2, because $l \gg a + d$, we have $\beta_1 \approx \beta_2 = \beta$. In our calculation, we take the same experiment parameters above. The theory parameters are taken as: the slit length $b = 4.4 \times 10^{-3}m$, slit thickness $c = 8.5 \times 10^{-5}m$, $\alpha = 0$, $A_{1j} = 160.9$, $A_{2j} = 159.3$, $c_1 = 0.715$, $c_2 = 0.699$ ($|c_1^2| + |c_2^2| = 1$) and the quantum coherence degree $\nu = 0.873$. For the classical electromagnetic theory, the double-slit diffraction intensity is

$$I = 4I_0 \sin^2 \left( \frac{\pi a \sin \beta}{\lambda} \right) \cdot \cos^2 \left( \frac{\pi d \sin \beta}{\lambda} \right). \quad (47)$$

In FIG. 3, the point is the experimental data from Ref. [20]. The solid curve is the calculation result of quantum theory from Eq. (45), which include decoherence effects. We can find the quantum calculate results is in accordance with the experiment data. In Fig. 4, the point is the experimental data from Ref. [20]. The solid curve is the calculation result of classical theory from Eq. (47). We also find the theory results of classical electromagnetic have a certain deviation with the experimental data. The deviation mainly come from: (1) The theory curve intersect at the abscissa axis $\beta$, but experiment values have not intersection point with axis $\beta$. (2) The maximum values of calculation are less than the experiment date. So, the classical electromagnetic theory is an approximate approach to study light diffraction, and the more accurately approach is the quantum theory of light.
6. Conclusion

In conclusion, we have studied double-slit diffraction of light with the approaches of quantum theory and classical electromagnetic theory. In quantum theory, we give the relation among diffraction intensity and slit length, slit width, slit thickness, wave length of light and diffraction angle. In classical electromagnetic theory, only give the relation among diffraction intensity and slit width, wave length of light and diffraction angle. Obviously, the quantum theory include more diffraction information than the classical electromagnetic theory. By calculation, we find the classical electromagnetic theory result has a certain deviation with the experimental data, but the quantum calculate result is in accordance with the experiment data. So, the classical electromagnetic theory is an approximate approach, and the quantum theory is more accurately approach for studying light diffraction.

1. V. Scarani, Rev. Mod. Phys. 81, 1301 (2009)
2. D. V. Strekalov, A. V. Sergienko, D. N. Klyshko and Y. H. Shih, Phys. Rev. Lett. 74, 3600 (1995).
3. T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko, Phys. Rev. 52, R3429 (1995).
4. R. Hanbury Brown and R. Q. Twiss, Nature, 178, 1046 (1956).
5. A. B. Haner and N. R. Isenor, American Journal of Physics, 38, 748 (1970).
6. Y.-H. Zhai, X.-H. Chen, D. Zhang, and L.-A. Wu, Phys. Rev. 72, 043805 (2005).
7. O. Nairz, B. Brezger, M. Arndt, and A. Zeilinger, Phys. Rev. Lett. 87, 160401 (2001).
8. A. Gatti, E. Brambilla, M. Bache, and L. A. Lugiato, Phys. Rev. Lett. 93, 093260 (2004).
9. Kunze S, Dieckmann K, and Rempe G. Phys. Rev. Lett. 78, 2038 (1997).
10. G. Scarcelli, A. Valencia, and Y. Shih, Phys. Rev. 70, 051802 (2004).
11. Brian J Smith and M G Raymer, New J.phys. 9, 414 (2007).
12. C. Henkel, H. Wallis, N. Westbrook, C.I. Westbrook, A. Aspect, K. Sengstock, W. Ertmer, Applied Physics B 69, 277 (1999).
13. Nairz O, Arudt M, and Zeilinger A. J, Mod. Opt. 47, 2811 (2000).
14. B. Brezger, L. Hackermuller, S. Uttenthaler, J. Petschinka, M. Arndt, A. Zeilinger, Phys. Rev. Lett. 88, 100404 (2002).
15. A. del Campo, J. G. Muga, Journal of Physics A 39, 5897 (2006).
16. R. Tumulka, A. Viale, N. Zanghi, Physical Review A 75, 055602 (2007)
B. Brezger, M. Arndt, and A. Zeilinger, J. Opt. B: Quantum Semiclassical Opt. 5, S82 (2003).
Arndt M, Nairz O, Voss-Andreae J, Nature 401, 680 (1999)
S. A. Sanz, F. Borondo, J. M. Bastiaans, Physical Review A 71, 042103 (2005)
Angelo M D. Maria V. Chekhova M V, Physics Review. 87, 013602 (2001)