Axial anomaly and hadronic properties in a nuclear medium

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GF & A. Hosaka, Phys. Rev. D 95, 116011 (2017)
GF & A. Hosaka, Phys. Rev. D 98, 036009 (2018)
Motivation

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AXIAL ANOMALY OF QCD:

- **$U_A(1)$ anomaly**: anomalous breaking of the $U_A(1)$ subgroup of $U_L(N_f) \times U_R(N_f)$ chiral symmetry
  - vacuum-to-vacuum topological fluctuations (instantons)

$$\partial_\mu j^\mu_A = -\frac{g^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} [T^a F_{\mu\nu} F_{\rho\sigma}]$$

- $U_A(1)$ breaking interactions depend on instanton density
  - suppressed at high $T^1$ (valid beyond $T_c$)
  - is the anomaly present at the phase transition?

- Very little is known at finite baryochemical potential $(\mu_B)^2$
  - sign problem in lattice simulations
  - effective models have not been extensively explored

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1. R. D. Pisarski, and L. G. Yaffe, Phys. Lett. B97, 110 (1980).
2. T. Schaefer, Phys. Rev. D57, 3950 (1998).
Motivation

$\eta'$ - NUCLEON BOUND STATE:

- Effective models at finite $T$ and/or density:
  $\rightarrow$ mean field calculations (NJL$^3$, linear sigma models$^4$) predict a $\sim 150$ MeV drop in $m_{\eta'}$ at finite $\mu_B$

- Effective description of the mass drop:
  $\rightarrow$ attractive potential in medium $\Rightarrow \eta'N$ bound state
  $\rightarrow$ Analogous to $\Lambda(1405) \sim \bar{K}N$ bound state

$^3$P. Costa, M. C. Ruivo & Yu. L. Kalinovsky, Phys. Lett. B 560, 171 (2003).
$^4$S. Sakai & D. Jido, Phys. Rev. C88, 064906 (2013).
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\( \eta' \) - NUCLEON BOUND STATE:

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- Problem with mean field calculations: they treat model parameters as environment independent constants
  \( \rightarrow \) „\( A \cdot \nu \)“ type of terms decrease (\( A \)-constant, \( \nu \)-decreases)
  \( \rightarrow \) evolution of the „\( A \)“ anomaly at finite \( T \) and \( \mu_B \)?

- What is the role of fluctuations?

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Fluctuation effects in a quantum system is encoded in the effective action

**Partition function** and **effective action** in field theory:
[S: classical action, \(\phi\): dynamical variable, \(\bar{\phi}\): mean field, \(J\): source field]

\[
Z[J] = \int D\phi e^{-\left(S[\phi] + \int J\phi\right)}, \quad \Gamma[\bar{\phi}] = -\log Z[J] - \int J\bar{\phi}
\]

\(\Gamma\) contains the truncated 1PI *n-point functions*

How to calculate the effective action? \(\Rightarrow\) **perturbation theory!**

\(-\rightarrow\) find a **small parameter** in \(S\) and Taylor expand

\(-\rightarrow\) fails in QCD & eff. models are not weakly coupled either

Non-perturbative methods are necessary:
**Functional Renormalization Group (FRG)**

\[\text{C. Wetterich, Phys. Lett. B301, 90 (1993)}\]

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FRG generalizes the idea of the Wilsonian RG: fluctuations are taken into account at the level of the quantum effective action. Introduce a flow parameter $k$ and include fluctuations for which $q \gtrsim k$:

$$Z_k[J] = \int D\phi e^{-\left(S[\phi] + \int J\phi\right)} \times e^{-\frac{1}{2} \int \phi R_k \phi}$$

→ regulator: mom. dep. mass term suppressing low modes
Functional Renormalization Group

- FRG generalizes the idea of the Wilsonian RG: fluctuations are taken into account at the level of the quantum effective action.
- Introduce a flow parameter $k$ and include fluctuations for which $q \gtrsim k$

$$Z_k[J] = \int \mathcal{D}\phi e^{-\left(S[\phi] + \int J\phi\right)} \times e^{-\frac{1}{2} \int \phi R_k \phi}$$

→ regulator: mom. dep. mass term suppressing low modes.

- Scale dependent effective potential and its flow equation:

$$\Gamma_k[\tilde{\phi}] = -\log Z_k[J] - \int J\tilde{\phi} - \frac{1}{2} \int \tilde{\phi} R_k \tilde{\phi}$$

$$\partial_k \Gamma_k = \frac{1}{2} \int_{q,p} \left( \Gamma_k^{(2)} + R_k \right)^{-1}(p, q) = \frac{1}{2}$$
The scale dependent effective action \( \Gamma_k \) is an average action from fluctuations with wavelengths \( \lambda \sim k^{-1} \) are integrated out

\[ \rightarrow k \rightarrow \infty: \text{no fluctuations} \quad \Rightarrow \quad \Gamma_{k \rightarrow \infty}[\bar{\phi}] = S[\bar{\phi}] \]

\[ \rightarrow k = 0: \text{all fluctuations} \quad \Rightarrow \quad \Gamma_{k=0}[\bar{\phi}] = \Gamma[\bar{\phi}] \]

The scale-dependent effective action interpolates between classical- and quantum effective actions

The trajectory depends on \( R_k \) but the endpoint does not

Choice of \( R_k \leftrightarrow \) choice of scheme
3 FLAVOR CHIRAL NUCLEON-MESON MODEL:

- Effective model of chiral symmetry breaking: order par. $M$
  [excitations of $M$: $\pi, K, \eta, \eta'$ and $a_0, \kappa, f_0, \sigma$]

\[
\mathcal{L}_M = \text{Tr} \left[ \partial_i M^\dagger \partial_i M \right] - \text{Tr} \left[ H (M^\dagger + M) \right] + V_{ch}(M) + A \cdot (\det M^\dagger + \det M)
\]

\[
\mathcal{L}_{\omega+N} = \frac{1}{4} (\partial_i \omega_j - \partial_j \omega_i)^2 + \frac{1}{2} m_\omega \omega_i^2 + \bar{N}(\bar{\phi} - \mu_B \gamma_0) N,
\]

\[
\mathcal{L}_{\text{Yuk}} = \bar{N} (g_Y \tilde{M}_5 - ig_\omega \varphi) N
\]

$\rightarrow$ nucleon mass: entirely from Yukawa coupling

- Fluctuation effects are calculated in the mesonic potentials:

\[
V_k = V_{ch,k}(M) + A_k(M) \cdot (\det M^\dagger + \det M)
\]

$\rightarrow$ solve a set of functional differential equations on a grid
Baryon Silver Blaze property:

\[ \rightarrow \text{no change in the effective action for } T = 0 \text{ if } \mu_B < m_N - B \equiv \mu_{B,c} \]
Baryon Silver Blaze property:

\[ \mu_B < m_N - B \equiv \mu_{B,c} \]

At \( \mu_B = \mu_{B,c} \):\(^6\)

\[ \rightarrow \text{1st order phase transition from nuclear gas to liquid} \]
\[ \rightarrow \text{nuclear density jumps from zero to } n_0 \approx 0.17 \text{ fm}^{-3} \]
\[ \rightarrow \text{non-strange chiral condensate jumps from } f_\pi \text{ to } v_{ns,\text{nucl}} \]
\[ (\text{Landau mass } M_L \approx 0.8m_N \Rightarrow v_{ns,\text{nucl}} \approx 69.5 \text{ MeV}) \]

\(^6\)M. Drews and W. Weise, Prog. Part. Nucl. Phys. 93, 69 (2017).
Baryon **Silver Blaze** property:

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At $\mu_B = \mu_{B,c}$:

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$\rightarrow$ nuclear density jumps from zero to $n_0 \approx 0.17 \text{ fm}^{-3}$

$\rightarrow$ non-strange chiral condensate jumps from $f_\pi$ to $\nu_{ns,nucl}$

(Landau mass $M_L \approx 0.8m_N \Rightarrow \nu_{ns,nucl} \approx 69.5 \text{ MeV}$)

The first order transition is related to the condensation of the **timelike component** of the $\omega$ vector particle

$\omega$ couples to $\nu_{ns}$ that couples to $\nu_s$

$\rightarrow$ jump in all order parameters

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The model consists of the following parameters:

\[ V(M) : m^2, g_1, g_2, b_i \ (i = 1..4) \ [b_i \text{ are non-renormalizable interactions!}] \]

\[ \text{explicit breaking, anomaly: } h_0, h_8, A \]

\[ \omega + N: g_\omega^2/m_\omega^2, g_Y \]

12 parameters in total. Input:

\[ \text{masses in the vacuum: } m_\pi, m_K, m_\eta, m_\eta', m_{a_0}, m_N \]

\[ \text{normal nuclear density: } n_0 \]

\[ \text{critical chemical potential: } \mu_{B,c} \]

\[ \text{nucleon mass drop in the medium: } \Delta m_N \]

\[ 2 \text{ PCAC relations (decay constants } f_\pi, f_K \text{)} \]

\[ \text{temperature of the critical endpoint } T_{CEP} \]

[Compression modulus: prediction! \( K = \frac{9n_0}{\partial n_0/\partial \mu_B} \approx 287 \text{ MeV} \) ]
Numerical results

\[ V_{\text{eff}}(\nu_{\text{ns}}, \mu_B) \]

- \( \mu_B = 915 \text{ MeV} \)
- \( \mu_B = 922.7 \text{ MeV} \)
- \( \mu_B = 930 \text{ MeV} \)

\( T = 0 \text{ MeV} \)
Numerical results

$T = 18 \text{ MeV}$

$\mu_B = 914 \text{ MeV}$

$\mu_B = 905.85 \text{ MeV}$

$\mu_B = 898 \text{ MeV}$
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Numerical results

$V_s$

$V_{ns}$

condensates [MeV]

$\mu_B$ [MeV]

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Axial anomaly and hadronic properties in a nuclear medium
Numerical results

![Graph showing phase transitions in a nuclear medium with and without mesonic fluctuations. The graph plots temperature ($T$) versus chemical potential ($\mu_B$). The phase transitions are indicated by different colors and markers.](image-url)

- **Gas** phase at lower temperatures.
- **Liquid** phase at higher temperatures.

**Legend:**
- Purple line: with mesonic fluctuations.
- Green line: without mesonic fluctuations.
Numerical results

\[ A_{k=0} \text{ [GeV]} \]

\[ \sqrt{I_1} \text{ [MeV]} \]

\[ \rightarrow I_1 = \left( v_{ns}^2 + v_s^2 \right)/2 \]
Numerical results

\[ I_1 = \left( \nu_{ns}^2 + \nu_s^2 \right) / 2 \]

\[ A_{k=0} [\text{GeV}] \]

\[ T = 0 \]

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Numerical results

\[ \Delta |A| (\mu_B; T) \text{ [MeV]} \]

\[ \mu_B - \mu_{B,c}(T) \text{ [MeV]} \]

\( T = 0 \text{ MeV} \)
\( T = 12 \text{ MeV} \)
\( T = 18 \text{ MeV} \)
Numerical results

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\[
\begin{array}{cccc}
\text{masses [MeV]} & f_0 & \kappa & a_0' \\
\text{\(\mu_B\) [MeV]} & \eta' & N & \sigma \\
\end{array}
\]

Axial anomaly and hadronic properties in a nuclear medium
Numerical results

- Conventional wisdom is that the axial anomaly should decrease as the chiral condensate drops
  ➔ How can we obtain the opposite effect?
- Earlier perturbative calculations are based on a high-$T$ expansion and take into account instanton effects
  ➔ these calculations are valid way above $T_c$ and definitely not for $T \lesssim T_c$
Conventional wisdom is that the axial anomaly should decrease as the chiral condensate drops.

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Current effect: mesonic quantum fluctuations, not instanton contributions.

→ backreaction of the anomaly on itself

→ mean field theory is questionable

Even the bare anomaly coefficient $A$ can depend explicitly on $T$ and $\mu_B$!
Numerical results

- Conventional wisdom is that the axial anomaly should decrease as the chiral condensate drops
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- Current effect: mesonic quantum fluctuations, not instanton contributions
  \[\rightarrow\] backreaction of the anomaly on itself
  \[\rightarrow\] mean field theory is questionable

- Even the bare anomaly coefficient $A$ can depend explicitly on $T$ and $\mu_B$
  \[\rightarrow\] competition between instantons and mesonic loop effects
  \[\rightarrow\] extension: assume a form of $A = A(T, \mu_B)$
Summary

- Mesonic and nucleon fluctuations effects on chiral symmetry, axial anomaly and mesonic spectrum in a nuclear medium using the Functional Renormalization Group (FRG) approach

Findings:
- Mesonic fluctuations make the anomaly coefficient $\Delta |A|$ condensate dependent
- (partial) restoration of chiral symmetry seems to increase the anomaly ($\Delta |A| \gtrsim 15\%$ relative difference)
- Nuclear transition: $\sim 20\%$ drop in (n.s.) chiral cond.
- $\eta'$ mass is smooth at the transition point

Important:
- No instanton effects have been included!
- Environment dependence of the bare anomaly coefficient could be relevant!

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