Temporal Oscillations of Nonlinear Faraday Rotation in Coherently Driven Media

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(December 10, 2001)

Abstract

New phenomenon of temporal oscillations of nonlinear Faraday rotation in a driven four-level system is predicted. We show that in this system with one upper level, under the conditions of electromagnetically induced transparency created by a strong coupling field, the polarization rotation of weak probe light exhibits slowly damped oscillations with a frequency proportional to the strength of an applied magnetic field. This opens up an alternate way to sensitive magnetometric measurements. Applications in low-light nonlinear optics such as photon entanglement are feasible.

PACS numbers: 42.50.Gy, 07.55.Ge

In the last years there has been growing interest in the study of nonlinear Faraday rotation (NFR) in coherent atomic media. Both the regime of weak fields, low atomic coherence and low optical density and the regime of strong field, large long-lived coherence and optically dense medium have been extensively investigated. If in the first case the ultranarrow magneto-optic resonances were observed with relaxation rate of $2\pi \times 1\text{Hz}$, in high light-power regime the large atomic coherence giving rise to electromagnetically induced transparency (EIT) allows to enhance essentially the nonlinear Faraday signal in
optically thick medium. These experiments [4, 6], as well as the theoretical estimations [7], demonstrate the significant potential of both techniques to prove substantially the sensitivity of NFR-based magnetometers to small magnetic fields.

An important feature of coherent atomic medium is its extremely high dispersion near sharp atomic resonance lines such that a large NFR occurs even at very weak magnetic fields. The simplest system displaying this feature is a three-level Λ-atom with ground-state Zeeman sublevels interacting with the right (RCP) and left (LCP) circularly polarized components of optical light in the presence of a static magnetic field. In usual terms of induced-coherence treatment, the RCP and LCP components of the light can be represented as playing the role of coupling and probe fields with respect to each other. Having equal intensities they excite a maximal Zeeman coherence in high-power regime. On the other hand, the steep dispersion at probe frequency in more general case of strong coupling and weak probe fields [8, 9] has been used earlier [10] to detect small magnetic fields by measuring the probe phase shift via optical interferometry. These investigations were aimed at improvement of sensitivity to magnetic fields compared to conventional optical pumping magnetometers [11, 12].

We study here, for the first time, the magneto-optical rotation in a strongly driven coherent medium. The model we are considering is a four-level system (Fig.1) where, as before, the RCP and LCP components of weak probe light $E_p$ of frequency $\omega_p$, tuned to the $|1>\rightarrow |3>$ and $|2>\rightarrow |3>$ transitions respectively, create a Zeeman coherence, while a strong coupling field $E_c$ with frequency $\omega_c$ drives the atom on the auxiliary transition $|0>\rightarrow |3>$. It is obvious, that in Faraday geometry the Zeeman sublevels $|1,2>$and the state $|0>$belong to different hyperfine components of the ground state. Moreover, the coupling field $E_c$ should be circularly polarized. Our motivation for the present work is related to the novel features of Faraday rotation arising in this system in the EIT-regime. We show here that the Faraday rotation angle oscillates in time with a slowly damped amplitude and a frequency proportional to strength of magnetic field. For weak probe light and small magnetic fields the damping rate of the oscillations is determined by the lifetime of Zeeman coherence,
which can be made very long compared to the decay time of excited state. This opens up a promising possibility to detect the small magnetic fields via the measurements of Faraday signal oscillations with a sensitivity not less than already reached one.

The underlying mechanism of the long-lived oscillations of Faraday signal is an existence of stable coherent superposition of two ground-state sublevels $|1\rangle$ and $|2\rangle$ with different energies $\delta_{1,2} = \mp \delta$, where $\delta = g \mu_B B / \hbar$ is the Zeeman level shift induced by a magnetic field $B$ ($g$ is a Lande factor). In the absence of magnetic field the strong coupling field drives the atoms into population-trapped (or noncoupled) states $(|i\rangle - \frac{\Omega_c}{\Omega_c} |0\rangle)^{-} |i\rangle$, $i=1,2$ ($\Omega_c$ and $\Omega_p$ are the Rabi frequencies of the coupling and probe fields: $\Omega_c >> \Omega_p$), which are mixed with each other due to two-photon interaction with the circular components of the probe light. This occurs at the time scale of lifetime of upper state $|3\rangle$. For all time thereafter the atom is in a steady-state and does not interact with the fields. When a weak magnetic field is applied, the system evolves in the same scenario with except that now the states $|1\rangle$ and $|2\rangle$ acquire different phases due to their different energies. This means that in the EIT-regime the coherent superposition of the states $|1\rangle \exp(-i\delta_1 t)$ and $|2\rangle \exp(-i\delta_2 t)$ is eventually created with the constant probabilities except for a very slow decay due to Zeeman coherence damping. These superposition states exhibit the quantum beats in time evolution of the system, which are revealed in NFR in the form of slowly damped temporal oscillations. Thus, the key point is that to a good approximation the time behavior of Zeeman coherence is the oscillatory free evolution, but at the same time the susceptibility of probe transition, although small, is different from zero that makes it possible to detect these oscillations in Faraday signal.

We now turn to the analytical calculations of this effect in the scheme depicted in Fig.1. The total interaction-picture Hamiltonian corresponding to this system has the form

$$H = \hbar \Delta_1 |1\rangle \langle 1| + \hbar \Delta_2 |2\rangle \langle 2| + \hbar \Delta_3 |3\rangle \langle 3| - \hbar (\Omega_c |3\rangle \langle 0| + \Omega_1 |3\rangle \langle 1| + \Omega_2 |3\rangle \langle 2| + H.c.)$$

where the Rabi frequencies of coupling field and polarization components of probe light are defined as $\Omega_c = \mu_{30} E_c / \hbar$ and $\Omega_{1,2} = \Omega_p \exp(i\Phi_{1,2})$, $\Omega_p = \mu E_p / \hbar$, $\mu_{ik}$ is the dipole moment of
the i-k transition, \( \mu = \mu_{31} = \mu_{32} \), the amplitudes \( E_{c,p} \) are considered real and \( \Phi_{1,2} \) are the phases of probe polarization components. The detunings

\[
\Delta_c = \omega_{30} - \omega_c - \delta_0, \quad \Delta_i = \omega_{3i} - \omega_p - \delta_i, \quad i = 1, 2,
\]

(2)

include the frequency shifts of atomic transitions \( \delta_{1,2} \) and \( \delta_0 = g_0 \mu_B B m_0 / \hbar \) caused by a magnetic field, where \( g_0 \) and \( m_0 \) are the giromagnetic factor and magnetic quantum number (momentum projection) of the level \( |0\rangle \). The thermal motion of the atoms with velocity \( v \) in the propagation direction is taken into account by incorporating into the detunings the Doppler shifts \( \omega_{c,p} v / c \), which are practically the same for closely spaced hyperfine components of the ground atomic state.

The time evolution of the system’s density matrix obeys the equation

\[
\frac{d}{dt} \rho = -\frac{i}{\hbar} [H, \rho] + L \rho
\]

(3)

where the operator \( L \) accounts for all relaxation processes. In our model they are determined by the spontaneous emission from the upper level \( |3\rangle \) to the lower levels and the damping of lower-level coherence. For a simplicity, all optical decay rates are assumed to be the same: \( \gamma_{3i} = \gamma \), and the coherence decay rates of the lower levels are taken equal \( \gamma_{ij} = \gamma_0 << \gamma \), \( i, j = 0, 1, 2 \).

In general case the Eq.(3) cannot be solved exactly. However, an analytical solution can be found in symmetric case, when the coupling field is tuned to resonance with \( |0\rangle \rightarrow |3\rangle \) transition in the presence of magnetic field, i.e. \( \Delta_c = 0 \), while for the probe field the condition \( \omega_{3i} - \omega_p = 0 \), \( i = 1, 2 \) holds, so that \( \Delta_{1,2} = -\delta_{1,2} = \pm \delta \). We consider the case of strong coupling laser and weak magnetic fields such that the EIT conditions are satisfied:

\[
\Omega_c^2 >> \delta \Delta_D, \gamma_0 (\gamma + \Delta_D),
\]

(4)

where \( \Delta_D \) is the Doppler width of the optical transitions. Then, with the assumptions of \( \Omega_p, \delta << \Omega_c, \gamma \), we are able to derive the solution for the Zeeman coherence and the absorption \( \alpha_{1,2} \) and dispersion \( \beta_{1,2} \) coefficients of probe field in the lowest order of small
parameters $\Omega_p/\Omega_c$ and $\delta/\Omega_c$. With equal initial populations in the Zeeman sublevels $\rho_{11}(0) = \rho_{22}(0) = 0.5$ this solution for time $t$ much larger than the upper level inverse decay rate: $t \gg \gamma^{-1}$, has the form:

$$\rho_{21} = -\frac{\Omega_1 \Omega_2^*}{2 \Omega_c^2} \{1 + \exp(-\gamma_d t) \exp[i(\delta_1 - \delta_2)t]\} + 0(e^{-\gamma t}) \quad (5)$$

$$\alpha_i(t) = \frac{1}{l_0} \text{Im}\left(\frac{\rho_{3i} \Gamma}{\Omega_i}\right), \quad \beta_i(t) = \frac{1}{2l_0} \text{Re}\left(\frac{\rho_{3i} \Gamma}{\Omega_i}\right), \quad i = 1, 2 \quad (6)$$

where

$$\rho_{31} = \frac{\Omega_1 \delta_1}{2 \Omega_c^4}(\Omega_2^2 + i \Gamma \delta_1) - \frac{\Omega_1 |\Omega_2|^2 \delta_2}{2 \Omega_c^4} \exp(-\gamma_d t) \exp[i(\delta_1 - \delta_2)t] + 0(e^{-\gamma t}) \quad (7)$$

$$\rho_{32} = \rho_{31}(1 \leftrightarrow 2), \quad (8)$$

$$\gamma_d = \gamma_0 + 2\Gamma \Omega_p^2 \delta^2 / \Omega_c^4 \quad (9)$$

Here $l_0 = \frac{1}{\sigma N}$, $\sigma = 4\pi\omega_p\mu^2/hc\Gamma$ is resonant absorption cross-section, $\Gamma = 3\gamma/2$ is optical dephasing linewidth and $N$ is the atomic density. From the symmetry of the system with respect to two circular components of probe light it follows that $\alpha_1(t) = \alpha_2(t) = \alpha(t)$ and $\beta_1(t) = -\beta_2(t)$, as is apparent from Eqs.(6-8). Note also that due to the condition $\Omega_p \ll \Gamma$ nearly all of the atomic population remains in the states $|1\rangle$ and $|2\rangle$, so that the depletion of coupling field can be neglected.

The first terms in Eqs.(5) and (7,8) represent the results of EIT regime in $\Lambda$-subsystems (0-3-1) and (0-3-2) and, of course, coincide with that obtained in phase-sensitive model \cite{4} up to the factor 1/2 emerged from initial conditions. The immediate interaction between the two polarization components of probe field in three-level configuration (1-3-2) is responsible for the second terms, which reflect the quantum beats in the system. It is worth noting that in the limit $B \to 0$ the two ways yield the same contribution to Zeeman coherence $\rho_{21}$. We recognize from Eqs.(5-9) that in this limit there is no oscillation in the system. At the same time it is seen that the frequency of the oscillations is linear in the magnetic
field, while their damping rate $\gamma_d$ has square dependence on $B$. As it follows from Eq.(9), for $2\Gamma\Omega^2_p\delta^2/\Omega_c^4<<\gamma_0$ the damping of oscillations is determined solely by Zeeman decoherence rate $\gamma_0$. In Fig.2 we show the time evolution of the real part of $\rho_{21}$ obtained by numerical integration of Eq.(3) for two different values of magnetic field with the initial conditions $\rho_{11}(0)=\rho_{22}(0)=0.5$ and the other $\rho_{ij}=0$ ($i,j=0$-3). As expected, the oscillations occur in time region $t>>\Gamma^{-1}$. Note that they always start with same value $\text{Re}(\rho_{21})\sim -|\Omega_1\Omega_2|/(\Omega_c^2 + 2\Omega_p^2)$, but according to Eq.(9) the amplitude of the oscillations is damped fast as the strength of magnetic field increases. In the absence of magnetic field $\rho_{21}$ is purely real and reaches very quickly the steady-state value, which is two times larger (when disregarding the sign) than time averaged value of $\text{Re}(\rho_{21})$ at $B\neq 0$ (see Eq.(5)). It is not difficult to be convinced, that the analytical solution Eq.(5) is represented graphically by the same curves with insignificant deviation. For comparison we plot in Fig.2 the Zeeman coherence in the case of $\Delta_c = -\delta_0$, when the symmetry between the two circular components of probe field is violated. However, we observe that this has a negligible effect, if magnetic field is small. This analysis allows us to conclude that for sub-Gauss magnetic fields our analytical results represent the quantitative physics quite well.

The equations of motion for the propagation of the intensity $I_p \sim \Omega_p^2$ and phases $\Phi_{1,2}$ of the RCP and LCP components of probe field read

$$ \left( \frac{d}{dz} + \frac{1}{v_g} \frac{d}{dt} \right) \Omega_p^2 = -\alpha(t) \Omega_p^2, \quad \text{(10)} $$

$$ \left( \frac{d}{dz} + \frac{1}{v_g} \frac{d}{dt} \right) \Phi_{1,2} = \mp\beta_2(t), \quad \text{(11)} $$

where the group velocities $v_g$ expressed as

$$ v_g = c[1 + c\frac{\partial\beta_1}{\partial \delta_i}]^{-1} \approx 4\Omega_c^2\eta_0\Gamma^{-1} \quad \text{(12)} $$

are equal for two polarization components.

The most transparent results are obtained in the case of optically thin media, where $\alpha L << 1, L$ is the atomic cell length. Using the Eqs.(6) and (7) this condition can be written as
In this case the integration of Eqs.(10) and (11) yields for the probe transmitted power and rotation angle $\Phi = (\Phi_2 - \Phi_1)/2$

$$\Omega_p^2(L, t) = \Omega_p^2(0, t - L/v_g) = \Omega_p^2(0)$$

$$\Phi(t) = \int_0^L \beta_2(t - z/v_g) dz = \frac{\delta L}{v_g} + \frac{\Omega_p^2(0)}{\Omega_c^2} \sin\left(\frac{\delta L}{v_g}\right) \cos(2\delta t) \exp\left(-\gamma_d t\right)$$

where $\Omega_p(0)$ corresponds to the input probe field. Here we have used the Eq.(12) for $v_g$ and have taken into account that $\gamma_d << \delta$.

The Eq.(15) shows an interesting variation of oscillating term on parameter $\eta = \delta L/v_g$, which is the ratio of probe propagation time $L/v_g$ to the atomic oscillation characteristic time $\delta^{-1}$. This modulation can be understood rather simply. When $\eta << 1$, the atoms on propagation line are in the same phase of ground-state oscillation during all passing of probe field through the medium. This leads to total accumulation of polarization rotation and to the maximal relative contribution of oscillating term in output rotation angle. In opposite case of $\eta >> 1$ the probe pulse moves very slowly and sees the atoms in different phases of oscillation. As a result, the polarization rotation varies periodically with distance travelled that eventually leads to the sinusoidal dependence of the total rotation angle on the cell length. In this case, of course, the relative contribution of oscillating term is reduced, even if $\sin\eta = 1$. By the same reasons, one can conclude that in high density case ($\alpha L >> 1$) the oscillating term in $\Phi(t)$ is suppressed even more.

The estimations with the help of Eq.(15) show that in the case of D$_2$ line in Rb the polarization rotation as high as 0.1 rad can be achieved for sub-Gauss magnetic fields ($B \approx 0.7$G), atomic density $N \approx 10^{10}$cm$^{-3}$, $L=3$cm and $\Omega_c = \Gamma (\alpha L^{-0.1})$ with the amplitude of oscillations $\Omega_p^2/\Omega_c^2 \approx 0.1$. For large values of $\eta$, we should make sure of compatibility of the conditions $\eta >> 1$ and $\alpha L << 1$. Combining the parameters $\alpha, \eta$ and using the expression of $v_g$ from Eq.(12) the latter condition can be written in the form $2\eta \delta \Gamma/\Omega_c^2 << 1$, which can
be fulfilled in a wide range of $\eta$, if one takes into account that $\delta << \Omega_c$. The predicted effect is much easy to be observed in a cooled atomic ensemble, where the Doppler broadening can be neglected that allows to avoid the strict restrictions of Eq.(4) on the magnetic field.

It is worth noting that the present technique can be used to fully entangle a pair of very weak optical fields. There have been several studies [13,14] predicting an unprecedented strength of nonlinear interaction between two low-power light pulses. The attempts to apply these schemes to generation of photon entangled states [15] have confronted with the problem of the mismatch between the slow group velocities of propagating weak fields that restricts their effective interaction time [16]. To overcome this difficulty it was proposed in [13] to use the mixture of different isotopes of alkali atoms and to apply an appropriate magnetic field. To demonstrate how the efficient interaction of two weak pulses arises in our scheme we consider the case of two uncorrelated weak beams with equal frequencies instead of two circular components of linearly polarized probe light and suppose that the frequency of the fields is chosen such that, say, the first field is in exact resonance with the $|1\rangle \rightarrow |3\rangle$ transition in the presence of magnetic field, i.e. $\Delta_1 = 0$, while $\Delta_2 = 2\delta$. Then, when putting in Eq.(7) $\delta_1 = 0$ and $\delta_2 = 2\delta$, we find that the phase shift and absorption of the first field are proportional to the intensity of the second one that creates the cross-modulation between weak beams. The main advantage of our scheme is that the problem of group velocity matching does not arise here, since, as it was shown above, the two beams have the same group velocities and, hence, the interaction between them is maintained for a very long time.

In summary, we have described new phenomenon of long-lived temporal oscillations of NFR, which arise when the conventional Faraday configuration is driven by a strong laser field, and have shown that these oscillations reflect the quantum beats in coherent superposition of ground-state Zeeman sublevels. We have found the analytical solution for total angle of Faraday rotation, which allows to understand quantitatively the main features of the effect. Our study predicts also the ability of the proposed scheme to achieve an efficient creation of two-photon entanglement. This question will be discussed in detail elsewhere.
REFERENCES

[1] See review W.Gawlik, in Modern Nonlinear Optics, by M.Evans and S.Kieliich, eds., Vol. LXXXV of Advances in Chemical Physics Series (Wiley, New York, 1994).

[2] D.Budker, V.Yashchuk, and M.Zolotorev, Phys. Rev. Lett. 81, 5788 (1998).

[3] D.Budker, D.F.Kimball, S.M.Rochester, and V.V.Yashchuk, Phys. Rev. Lett. 83, 1767 (1999).

[4] V.A.Sautenkov, M.D.Lukin, C.J.Bednar, I.Novikova, E.Mikhailov, M.Fleischhauer, V.L.Velichansky, G.R.Welch, and M.O.Scully, Phys.Rev. A 62, 023810 (2000); I.Novikova, A.B.Matsko, and G.R.Welch, Optics Letters 26, 1016 (2001); Y.Dancheva, G.Alzetta, S.Cartaleva, M.Taslokov, and C.Andreeva, Opt.Commun. 178, 103 (2000).

[5] S.E.Harris, Phys.Today, 50, No. 7, 36 (1997).

[6] D.Budker, D.F.Kimball, S.M.Rochester, V.V.Yashchuk, and M.Zolotorev, Phys. Rev. A 62, 043403 (2000); I.Novikova, A.B.Matsko, V.A.Sautenkov, V.L.Velichansky, G.R.Welch, and M.O.Scully, Optics Letters 25, 1651 (2000).

[7] H.Lee, M.Fleischhauer, and M.O.Scully, Phys. Rev. A 58, 2587 (1998); M.Fleischhauer, A.B.Matsko, and M.O.Scully, Phys. Rev. A 62, 013808 (2000).

[8] S.E.Harris, J.E.Field, and A.Kasapi, Phys.Rev. A 46, R29 (1992); A.Kasapi, M.Lain, G.Y.Yin, and S.Harris, Phys.Rev.Lett. 74, 2447 (1995).

[9] M.Xiao, Y.-Q.Li, S.Z.Jin, and J.Gea-Banacloche, Phys.Rev.Lett. 74,666 (1995).

[10] M.O.Scully and M.Fleischhauer, Phys.Rev.Lett. 69, 1360 (1992); M.Fleischhauer and M.O.Scully, Phys.Rev. A 49, 1973 (1994);

[11] C.Cohen-Tannoudji, J.Dupont-Roc, S.Haroche, and F.Laloe, Phys.Rev.Lett. 22, 758 (1969).
[12] E.B.Alexandrov and V.A.Bonch-Bruevich, Opt.Eng. (Bellingham) 31, 711 (1992) (and references therein).

[13] H.Schmidt and A.Imamoglu, Opt.Lett. 21, 1936 (1996).

[14] S.E.Harris and Y.Yamamoto, Phys.Rev.Lett. 81, 3611 (1998).

[15] M.D. Lukin and A.Imamoglu, Phys.Rev.Lett. 84, 1419 (2000).

[16] S.E.Harris and L.V.Hau, Phys.Rev.Lett. 82, 4611 (1999).

**Figure captions**

FIG.1. Four-level model for observation of oscillating Faraday rotation. Ωc and Ω1,2 are the Rabi frequencies of coupling field and σ± components of probe light, respectively. ∆2,1 are the shifts of m=±1 levels induced by magnetic field.

FIG.2. Numerical results for time evolution of Zeeman coherence Re(ρ21) in the case of Ωc=Γ, Ω1,2=0.3Γ, γ0=10^{-4}Γ, ∆c = 0 and for different values of magnetic field B: 1) δ = 0, 2) δ=0.03Γ and 2) δ=0.1Γ. The dotted line corresponds to the case of ∆c = δ=0.03Γ with the same values of the rest parameters. The Doppler broadening is ignored.
FIG. 1

\[ \Omega_c \]

\[ \Delta_1 \]

\[ \Omega_1 \]

\[ \Omega_2 \]

\[ \Delta_2 \]

\[ 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \]
