Experimental realization of secure multi-party computation in an entanglement access network

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To construct a quantum network with many end users, it is critical to have a cost-efficient way to distribute entanglement over different network ends. We demonstrate an entanglement access network, where the expensive resource, the entangled photon source at the telecom wavelength and the core communication channel, is shared by many end users. Using this cost-efficient entanglement access network, we report experimental demonstration of a secure multiparty computation protocol, the privacy-preserving secure sum problem, based on the network quantum cryptography.

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The peculiar quantum correlation, entanglement, provides the crucial resource for quantum information processing [1–8]. Generation of remote entanglement is a key step for quantum communication and distributed computation [4–5]. Distribution of entanglement in a multiple-end quantum network is costly through the point-to-point protocol as one needs to establish an entanglement source and a quantum communication channel between each end [4–10]. We demonstrate a cost-efficient entanglement access network where multiple end users share the same entanglement source and the core communication channel. We verify entanglement and quantum nonlocality between different end users. Using this entanglement access network, we report the experimental demonstration of a secure multi-party computation protocol where several end users compute cooperatively to solve a joint problem without revealing the information of their actual inputs [11–13]. This demonstration opens up the prospect of applying the cost-efficient entanglement access network for achieving secure multi-party computation that protects the privacy of end users.

With popularity of the internet, distributed computation becomes increasingly more important, where a number of end users need to work cooperatively to solve a common problem. The answer to the problem typically depends on the inputs of all the end users, which need to be communicated in the network. At the same time, the users need to protect their privacy and do not want to reveal their information to the others. Secure multi-party computation is a branch of cryptography and computer science that studies this kind of problems [11–13]. A well-known primitive example of this field is the millionaire problem, first introduced by Yao [11], in which two millionaires want to know which of them is richer without revealing their actual wealth. Secure multi-party computation has many applications in e-commerce and data mining where people need to compare numbers which are confidential [11,12]. Classically, the secure multi-party computation protocols typically rely on computationally hard mathematical problems, which require specific assumptions and are subject to security loopholes in particular under attack by a quantum computer [13].

In this paper, we demonstrate an entanglement access network which offers an alternative route for secure multi-party computation based on the network quantum cryptography. Quantum access network is a concept introduced in a recent paper where the expensive quantum resource is shared by many end users [8,9]. For quantum key distribution based on the BB84 protocol [14], the photon detector is the relatively expensive part of its implementation and thus shared in the quantum access network demonstrated in Ref. [8]. Here, we realize an entanglement access network to efficiently distribute entanglement between network end users. In this network, the entanglement source is the most expensive part of the implementation and thus shared between many end users. Entanglement provides the crucial quantum resource for achieving device-independent quantum cryptography, the most secure way for cryptographic communication [15–20]. Entanglement is also the critical resource for certified generation of shared randomness [21] and for quantum communication and multi-party computation through the entanglement-based schemes [3, 22]. We demonstrate an entanglement access network which efficiently distributes entanglement between a number of end users connected through fibers of more than 20 km length by sharing a single entangled photon source at the telecom frequency. By use of this entanglement access network, we report experimental demonstration of a secure multi-party computation protocol, the secure sum problem, in which several millionaires want to know how much money they have in total but none of them is willing to reveal his wealth to others [11,12]. This demonstration shows that the entanglement access network provides a cost-efficient way to realize a quantum network with shared resource, opening up the prospect for its applications in secure multi-party cryptography and distributed quantum computation.

The entangled photon source shown in Fig. 1(a) is generated by a type-II BBO crystal pumped by ultrafast laser pulses (with the pulse duration less than 150 fs and a repetition rate of 76 MHz) at the wavelength of 775.
nm from a Ti:Sapphire laser. The spontaneous parametric down conversion in the BBO crystal produces entangled photon pairs at the telecom wavelength of 1550 nm, which, in the ideal case, are in the polarization entangled state \(|\Psi\rangle = (|H\rangle + e^{i\varphi}|V\rangle)/\sqrt{2}\), where \(|H\rangle\) and \(|V\rangle\) denote respectively the horizontal and the vertical polarization state and \(\varphi\) is a controllable phase \([23]\). We have verified that the experimentally generated state \(\rho\) has an entanglement fidelity \(F_s\) of \((95.12 \pm 0.25)\%\) with respect to this ideal state \(|\Psi\rangle\) through the quantum state tomography \([24]\).

The entangled photons at the telecom frequency are coupled into optical fibers representing the core communication channel with the total distance about 20 km. The output photons from the fibers are fed into a 1×8 optical switch at each side (called Alice’s and Bob’s side) which is electrically controlled by the computer to deliver the photons to one of the eight output ports according to the electric control signal. This forms an entanglement access network with \(8 \times 8\) possible choices of pairs of end users who can share entangled photons with the entanglement distance more than 20 km, as shown in Fig. 1(b). All the end users share the same entanglement source and the core communication channel. The polarization states of photons are subject to tension and temperature dependent rotations in the optical fibers which are carefully compensated afterwards through the fiber polarization controllers.

To demonstrate entanglement between different end users, we randomly choose two users \(A_1\) and \(A_2\) at Alice’s side and two users \(B_1\) and \(B_2\) at Bob’s side. For the four pairs of end users \(A_iB_j\) \((i,j = 1, 2)\), we measure their entanglement by detecting the photon coincidences in different polarization bases. To rotate the polarization basis, we use a fast switchable electric optical modular (EOM), which induces a polarization transformation \(|H\rangle \rightarrow \cos \theta |H\rangle - i \sin \theta |V\rangle\) and \(|V\rangle \rightarrow -i \sin \theta |H\rangle + \cos \theta |V\rangle\) with the angle \(\theta\) determined by the computer controlled electric signal. The output \(H\) and \(V\) polarized photons are split by a polarization beam splitter (PBS) and then detected through single photon counters. In the Z (or X) basis detection, we register the photon coincidence counts by fixing the angle \(\theta_A\) of \(A_i\) at 0° (or 45°) and rotating the angle \(\theta_B\) of \(B_j\). The resulting coincidence counts as functions of the angle \(\theta_B\) are shown in Fig. 2 for different pairs \(A_iB_j\). The big contrast of these oscillations is a clear demonstration of quantum entanglement. Quantitatively, we calculate the visibilities of these oscillation curves \(V_x\) and \(V_z\) in the Z and the X basis, respectively, and the entanglement fidelity \(F_s\) is then bounded by \(F_s \geq (V_x + V_z)/2\) \([25]\). The visibilities and the corresponding fidelity bounds are listed in Fig. 3. All the pairs have the entanglement fidelity higher than 90%. The small decrease of the entanglement fidelity compared with the fidelity \(F_s\) of the entangled photon source is due to the imperfection in compensation of the polarization rotation in the optical fibers.

The shared entanglement allows demonstration of quantum key distribution between any pair of the end users in this network using the Ekert protocol \([2]\). For this purpose, we need to randomly choose the angle \(\theta_A\) from the set \(\{0^\circ, 22.5^\circ, 45^\circ\}\) and \(\theta_B\) from the set \(\{22.5^\circ, 45^\circ, 67.5^\circ\}\), and record the individual measurement outcomes for each coincidence count \([26]\). From

![FIG. 1. The experimental setup for implementation of an entanglement access network.](image)
or 45°, the coincidence counts when we fix the rotation angle of the electric optical modulator (EOM) of one end at 0° (for the Z-basis) or 45° (for the X-basis) while rotating the angle of the EOM at the other end. Different sub-figures correspond to different pairs of the end users (A1 and B1, A1 and B2, A2 and B1, A2 and B2). The visibilities of the oscillations in the X and the Z bases together bound the entanglement fidelity of the photonic states shared between the corresponding parties.

FIG. 2. The entanglement demonstrated between different pairs of the end users. The figures show the coincidence counts for the angles \(\theta_A = \{0°, 45°\}\) and \(\theta_B = \{22.5°, 67.5°\}\), we calculate the expectation value of the Clauser-Horne-Shimony-Holt (CHSH) observable \(\langle S \rangle = \langle 0°, 22.5° \rangle + \langle 45°, 22.5° \rangle + \langle 45°, 67.5° \rangle - \langle 0°, 67.5° \rangle \) \[\text{[27]}\], where \((\theta_A, \theta_B)\) denotes the photon coincidence counts with \(\theta_A, \theta_B\) at the specified values. The CHSH values are listed in Fig. 3 for different pairs \(A_i, B_j\). The CHSH inequality \(\langle S \rangle \leq 2\) for any classical correlation is clearly violated, demonstrating quantum nonlocality. The significant violation of the CHSH inequality for each pair of the end users is a guarantee of the security of the entangled-based QKD protocol \[\text{[15–20]}\]. To generate quantum key, the measurement outcomes at \(\theta_A = \theta_B = \{22.5°, 45°\}\) are kept, which yield the sifted keys \[\text{[20]}\]. For each pair of the parties, we obtain roughly \(1.2 \times 10^4\) sifted keys. We randomly choose 20% of the keys to estimate the quantum bit error rate (QBER). For our data, the QBER \(\gamma\) is about 5%, smaller than the security threshold of 11% \[\text{[21]}\]. We use the low density parity check (LDPC) code \[\text{[22, 20]}\] to do the error correction for the sifted keys, which is more efficient compared with the conventional CASCADE protocol \[\text{[23]}\]. We obtain about 8000 error-free shared keys, which are then purified by the privacy amplification protocol via a universal-2-class hash function \[\text{[31]}\], yielding about 1800 shared secret final keys. The residual information available to any potential eavesdroppers is reduced by a factor of \(2^{-300}\) during the privacy amplification and thus much less than one bit.

We now use the entanglement access network to demonstrate a secure multi-party computation protocol: the privacy-preserving secure sum problem \[\text{[12]}\]. The problem can be illustrated with the following example: assume several millionaires want to know how much money they have in total, but none of them want to reveal his actual wealth to others. To be concrete, we consider the case of four parties \(A_1, A_2, B_1, B_2\). Denote by \(a_1, a_2, b_1, b_2\) the input from \(A_1, A_2, B_1, B_2\), respectively. We want to calculate the sum \(T = a_1 + a_2 + b_1 + b_2\) without revealing the inputs \(a_1, a_2, b_1, b_2\). The quantum protocol to accomplish this task using the entanglement access network goes as follows:

**Step 1.**—Using the entanglement-based network QKD, \(A_1, B_1, A_2, B_2\), and publicly announces \(X_1, X_2\) to others; \(B_1\) calculates \(X_2 = b_1 + R_{a_2b_1} - R_{a_1b_1}\) and publicly announces \(X_2\) to others; \(A_2\) calculates \(X_3 = a_2 + R_{a_2b_2} - R_{a_2b_1}\) and publicly announces \(X_3\) to others; \(B_2\) calculates \(X_4 = b_2 + R_{a_2b_2} - R_{a_1b_2}\) and publicly announces \(X_4\) to others.

**Step 2.**—\(A_1\) calculates \(X_1 = a_1 + R_{a_1b_1} - R_{a_1b_1}\) and publicly announces \(X_1\) to others; \(B_1\) calculates \(X_2 = b_1 + R_{a_1b_1} - R_{a_1b_1}\) and publicly announces \(X_2\) to others; \(A_2\) calculates \(X_3 = a_2 + R_{a_2b_2} - R_{a_2b_1}\) and publicly announces \(X_3\) to others; \(B_2\) calculates \(X_4 = b_2 + R_{a_1b_2} - R_{a_1b_2}\) and publicly announces \(X_4\) to others.

**Step 3.**—All of them know the sum \(T\) by calculating \(T = X_1 + X_2 + X_3 + X_4\). At the same time, the inputs \(a_1, a_2, b_1, b_2\) remain confidential to the other parties as the public information \(X_i (i = 1, 2, 3, 4)\) has no correlation with the inputs due to the one-time pad theorem. For security of this secure sum protocol, we have assumed that different parties do not collaborate to steal the input information of the other parties. With the sum \(T\), when a few parties collaborate, it is always possible to reveal the input information of the other party.

In the experimental demonstration, as an example,

| Z basis visibility | X basis visibility | Fidelity bound | CHSH Value |
|-------------------|-------------------|---------------|------------|
| A,B₁               | 91.68±0.21%       | 88.76±0.32%   | 90.22±0.26% | 2.428±0.020 |
| A,B₂               | 92.59±0.19%       | 88.11±0.30%   | 90.35±0.24% | 2.449±0.019 |
| A,B₁               | 94.10±0.18%       | 87.91±0.36%   | 91.01±0.27% | 2.453±0.020 |
| A,B₂               | 93.54±0.16%       | 86.73±0.32%   | 90.14±0.24% | 2.458±0.021 |

FIG. 3. The experimental data to show entanglement and quantum nonlocality shared between different parties. The table lists the oscillation visibilities of the coincidence counts in the Z and the X bases, whose average gives a lower bound to the entanglement fidelity (the fourth column). The error bars are obtained by assuming a Poissonian distribution for the photon counts and propagated from the measured coincidence counts to the quantities listed in the table through exact Monte Carlo simulation. The last column of the table shows the measured value for the CHSH observable, and a larger-than-2 value indicates violation of the Bell inequality, demonstrating quantum nonlocality shared between the corresponding parties.
we take randomly generated numbers $a_1 = 55406$, $b_1 = 116559$, $a_2 = 988150$ and $b_2 = 2839885$. We run the above implementation of the secure sum protocol 30 times, and for each time we use different keys $R_{a_1b_1}, R_{a_2b_2}, R_{a_3b_3}, R_{a_4b_4}$ with the number of bits $n = 25$ bits to encode the public information $X_1, X_2, X_3, X_4$. For these 30 experimental runs, the publicly announced numbers $X_1, X_2, X_3, X_4$ are shown in Fig. 4, which look completely random from trial to trial and reveal no information of the inputs $a_1, a_2, b_1, b_2$. However, for each run, their sum is always fixed to be $4 \times 10^6$, which gives the correct calculation result for $T = a_1 + a_2 + b_1 + b_2$.

Similar to the quantum access network realized recently [8], we expect that the entanglement access network demonstrated in this paper provides a source-efficient way for network cryptography and secure multi-party computation. In particular, the shared entanglement between each ends of the network opens up the possibility to realize device independent quantum cryptography [15–19], which allows most secure communication by closing the practical security loopholes in conventional quantum cryptography [20]. Apart from the example demonstrated in this paper, the entanglement access network may allow realization of a number of secure multi-party computation problems [13]. The entanglement shared in the network could also find applications for certified generation of random numbers [21] and for implementation of distributed quantum computation and multiparty quantum cryptography protocols [1, 3].

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\[ T = \sum_{i=1}^{4} 4 \times 10^6 X_i \]

\[ X_i = a_i - b_i \]

\[ T = a_1 + a_2 + b_1 + b_2 \]

![Figure 4](https://example.com/figure4.png)

**FIG. 4.** The experimental demonstration of the four-party secure-sum protocol through the entanglement access network. The top four sub-figures show the publicly announced data from each of the four parties for 30 rounds of experiments for demonstration of the same secure-sum problem with the same input from each party. The randomly distributed data over different experimental runs is an indication that the announced data reveal no information of the input from each party. The last sub-figure shows the secure sum calculated by each party from the publically announced data, which is identical for different experimental runs and always equal to the true value of the sum for the underlying problem.

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