Proton to pion ratio at RHIC from dynamical quark recombination

Alejandro Ayala*, Mauricio Martínez†, Guy Paić* and G. Toledo Sánchez**

*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apartado Postal 70-543, México Distrito Federal 04510, Mexico
†Frankfurt Institute for Advanced Studies, Johann Wolfgang Goethe University, Ruth-Moufang-Str. 1 60438 Frankfurt am Main, Germany
**Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20-364, México Distrito Federal 01000, Mexico

Abstract.

We propose an scenario to study, from a dynamical point of view, the thermal recombination of quarks in the midsts of a relativistic heavy-ion collision. We coin the term dynamical quark recombination to refer to the process of quark-antiquark and three-quark clustering, to form mesons and baryons, respectively, as a function of energy density. Using the string-flip model we show that the probabilities to form such clusters differ. We apply these ideas to the calculation of the proton and pion spectra in a Bjorken-like scenario that incorporates the evolution of these probabilities with proper time and compute the proton to pion ratio, comparing to recent RHIC data at the highest energy. We show that for a standard choice of parameters, this ratio reaches one, though the maximum is very sensitive to the initial evolution proper time.

Keywords: Relativistic heavy-ion collisions, dynamical quark recombination
PACS: 25.75.-q

INTRODUCTION

Recently, it has been recognized that thermal recombination of quarks plays an important role for hadron production at intermediate $p_t$ in relativistic heavy-ion collisions. This idea, first studied in Refs. [1,2], explains the formation of low to intermediate $p_t$ hadrons from the bounding of quarks in a densely populated phase space, assigning appropriate degeneracy factors for mesons and baryons An implicit assumption is that hadronization happens at a single temperature. However, it is known that hadronization is not an instantaneous process but rather that it spans a window of temperatures and densities. For instance lattice calculations [3] show that the phase transition from a deconfined state of quarks and gluons to a hadron gas is, as a function of temperature, not sharp. Motivated by these shortcomings of the original recombination scenario, here we set out to explore to what extent the probability to recombine quarks into mesons and baryons depends on density and temperature and whether this probability differs for hadrons with two and three constituents, that is to say, whether the relative population of baryons and mesons can be attributed not only to the degeneracy factors but rather to the dynamical properties of quark clustering in a varying density environment.

A detailed answer to the above question stemming from first principles can only be found by means of non-perturbative QCD. Nevertheless, in order to get a simpler but still quantitative answer, here we address such question by resorting to the so called string-
flip model [4] which has proven to be successful in the study of quark/hadron matter as a function of density [5, 6, 7]. In this proceedings contribution, we only outline the main features of the calculation and refer the interested reader to Ref. [8] for further details. Other approaches toward a dynamical description of recombination, in the context of fluctuations in heavy-ion collisions, have been recently formulated in terms of the qMD model [9].

**THERMAL PARTICLE SPECTRA**

In the recombination model, the phase space particle density is taken as the convolution of the product of Wigner functions for each hadron’s constituent quark at a given temperature and the constituent quark wave function inside the hadron. For instance, the meson phase space distribution is given by

$$F^M(x, P) = \sum_{a, b} \int_0^1 dz |\Psi^M_{ab}(z)|^2 w_a(x, zP^+) \bar{w}_b(x, (1-z)P^+),$$

(1)

where $P^+$ is the light-cone momentum, $\Psi^M_{ab}(z)$ is the meson wave function and $a, b$ represent the quantum numbers (color, spin, flavor) of the constituent quark and antiquark in the meson, respectively. An analogous equation can also be written for baryons. When each constituent quark’s Wigner function is approximated as a Boltzmann distribution and momentum conservation is used, the product of Wigner functions is given by a Boltzmann-like factor that depends only on the light-cone momentum of the hadron [2]. For instance, in the case of mesons

$$w_a(x, zP^+) \bar{w}_b(x, (1-z)P^+) \sim e^{-zP^+ / T} e^{-(1-z)P^+ / T} = e^{-P^+ / T}.$$  

(2)

In this approximation, the product of parton distributions is independent of the parton momentum fraction and the integration of the wave function over $z$ is trivially found by normalization. There can be corrections from a dependence of each constituent quark Wigner function on momentum components that are not additive because energy is not conserved in this scenario [10]. An important feature to keep in mind is that in this formalism, the QCD dynamics between quarks inside the hadron is encoded in the wave function.

In order to allow for a more realistic dynamical recombination scenario let us take the above description as a guide, modifying the ingredients that account for the QCD dynamics of parton recombination. Let us assume that the phase space occupation can be factorized into the product of a term containing the thermal occupation number, including the effects of a possible flow velocity, and another term containing the system energy density $\varepsilon$ driven probability $\mathcal{P}(\varepsilon)$ of the coalescence of partons into a given hadron. We thus write the analog of Eq. (1) as

$$F(x, P) = e^{-P \cdot v(x) / T} \mathcal{P}(\varepsilon),$$

(3)

where $v(x)$ is the flow velocity. In order to compute the probability $\mathcal{P}(\varepsilon)$ we explicitly consider a model that is able to provide information about the likelihood of clustering of
constituent quarks to form hadrons from an effective quark-quark interaction, the string-flip model, which we proceed to describe.

**STRING FLIP MODEL AND HADRON RECOMBINATION PROBABILITY**

The String Flip Model is formulated incorporating a many-body quark potential able to confine quarks within color-singlet clusters [4]. At low densities, the model describes a given system of quarks as isolated hadrons while at high densities, this system becomes a free Fermi gas of quarks. For our purposes, we consider up and down flavors and three colors (anticolors) quantum numbers. Our approach is very close to that described in Refs. [5] and [6], where we refer the reader for an extensive discussion of the model details.

The many-body potential $V$ is defined as the optimal clustering of quarks into color-singlet objects, that is, the configuration that minimizes the potential energy. In our approach, the interaction between quarks is pair-wise. Therefore, the optimal clustering is achieved by finding the optimal pairing between two given sets of quarks of different color for all possible color charges. The minimization procedure is performed over all possible permutations of the quarks and the interaction between quarks is assumed to be harmonic with a spring constant $k$. Through this procedure, we can distinguish two types of hadrons:

i) **Meson-like.** In this case the pairing is imposed to be between color and anticolors and the many-body potential of the system made up of mesons is given by:

$$V_\pi = V_{B\bar{B}} + V_{G\bar{G}} + V_{R\bar{R}}$$

where $R(\bar{R})$, $B(\bar{B})$ and $G(\bar{G})$ are the labels for red, blue and green color (anticolor) respectively. Note that this potential can only build pairs.

ii) **Baryon-like.** In this case the pairing is imposed to be between the different colors in all the possible combinations. In this manner, the many-body potential is:

$$V_p = V_{RB} + V_{BG} + V_{RG}$$

which can build colorless clusters by linking 3(RBG), 6(RBGRBG),... etc., quarks. Since the interaction is pair-wise, the 3-quark clusters are of the delta (triangular) shape.

The formed hadrons should interact weakly due to the short-range nature of the hadron-hadron interaction. This is partially accomplished by the possibility of a quark flipping from one cluster to another. At high energy density, asymptotic freedom demands that quarks must interact weakly. This behavior is obtained once the average inter-quark separation is smaller than the typical confining scale.

We study the meson and baryon like hadrons independently. Therefore, $V = V_\pi$ or $V_p$, depending on the type of hadrons we wish to describe. We use a variational Monte Carlo approach to describe the evolution of a system of $N$ quarks as a function of the particle density. We consider the quarks moving in a three-dimensional box whose sides have length $a$ and the system described by a variational wave function of the form:

$$\Psi_\lambda(x_1, \ldots, x_N) = e^{-\lambda V(x_1, \ldots, x_N)}\Phi_{FG}(x_1, \ldots, x_N),$$
where $\lambda$ is the single variational parameter, $V(x_1, \ldots, x_N)$ is the many-body potential either for mesons or baryons and $\Phi_{FG}(x_1, \ldots, x_N)$ is the Fermi-gas wave function given by a product of Slater determinants, one for each color-flavor combination of quarks. These are built up from single-particle wave functions describing a free particle in a box [6].

The variational parameter has definite values for the extreme density cases. At very low density it must correspond to the wave function solution of an isolated hadron. For example, the non-relativistic quark model for a hadron consisting of 2 and 3 quarks, bound by a harmonic potential, predicts, in units where $k = m = 1$, that $\lambda_\pi \rightarrow \lambda_{0\pi} = \sqrt{1/2}$ and $\lambda_p \rightarrow \lambda_{0p} = \sqrt{1/3}$ respectively; at very high densities the value of $\lambda$ must vanish for both cases.

Since the simulation was performed taking $m = k = 1$, to convert to physical units we consider each case separately.

**Baryons:** To fix the energy unit we first notice that in a 3-body system the energy per particle, including its mass, is given by (with $m = k = 1$):

$$
\frac{E}{3} = \sqrt{3} + 1.
$$

(7)

If we identify the state as the proton of mass $M_p = 938$ MeV, then the correspondence is

$$
\sqrt{3} + 1 \rightarrow 312.7 \text{ MeV}.
$$

(8)

To fix the length unit we use the mean square radius, which for a 3-body system is:

$$
\sqrt{\langle r^2 \rangle} = (3)^{1/4}.
$$

The experimental value for the proton is

$$
\sqrt{\langle r^2 \rangle} = 0.880 \pm 0.015 \text{ fm}.
$$

(9)

Then the correspondence is: $(3)^{1/4} \rightarrow 0.88 \text{ fm}$.

**Mesons:** In a similar fashion we obtain for mesons (taking the pion as the representative 2-body particle): Energy: $\frac{3}{2\sqrt{2}} + 1 \rightarrow 70 \text{ MeV}$, length: $2^{1/4} \rightarrow 0.764 \text{ fm}$.

Our results come from simulation done with 384 particles, 192 quarks and 192 antiquarks, corresponding to having 32 $u$ ($\bar{u}$) plus 32 $d$ ($\bar{d}$) quarks (antiquarks) in the three color charges (anti-charges). To determine the variational parameter as a function of density we first select the value of the particle density $\rho$ in the box, which, for a fixed number of particles, means changing the box size. Then we compute the energy of the system as a function of the variational parameter using a Monte Carlo Method. The minimum of the energy determines the optimal variational parameter. We repeat the procedure for a set of values of the particle densities in the region of interest.

The information contained in the variational parameter is global, in the sense that it only gives an approximate idea about the average size of the inter-particle distance at a given density, which is not necessarily the same for quarks in a single cluster. This is reflected in the behavior of the variational parameter $\lambda_p$ for the case of baryons which goes above 1 for energies close to where the sudden drop in the parameter happens. We interpret this behavior as as a consequence of the procedure we employ to produce colorless clusters for baryons, which, as opposed to the case to form mesons, allows...
the formation of clusters with a number of quarks greater than 3. When including these latter clusters, the information on their size is also contained in $\lambda$. To correct for this, we compute the likelihood to find clusters of 3 quarks $P_3$. Recall that for $3N$ quarks in the system, the total number of clusters of 3 quarks that can be made is equal to $N$. However this is not always the case as the density changes, given that the potential allows the formation of clusters with a higher number of quarks. $P_3$ is defined as the ratio between the number of clusters of 3 quarks found at a given density, with respect to $N$.

Therefore, within our approach, we can define the probability of forming a baryon as the product of the $\lambda/\lambda_0p$ parameter times $P_3$, namely

$$P_p = \frac{\lambda}{\lambda_0p} \times P_3.$$  \hspace{1cm} (10)

For the case of mesons, since the procedure only takes into account the formation of colorless quark-antiquark pairs, we simply define the probability of forming a meson as the value of the corresponding normalized variational parameter, namely

$$P_\pi = \frac{\lambda}{\lambda_0\pi}.$$  \hspace{1cm} (11)

The probabilities $P_p$ and $P_\pi$ as a function of the energy density are displayed in Fig. 1. Notice the qualitative differences between these probabilities. In the case of baryons, the sudden drop found in the behavior of the variational parameter is preserved at an energy density around $\epsilon = 0.7$ GeV/fm$^3$ whereas in the case of mesons, this probability is smooth, indicating a difference in the production of baryons and mesons with energy density.

**PROTON TO PION RATIO**

In order to quantify how the different probabilities to produce sets of three quarks (protons) as compared to sets of two quarks (pions) affect these particle’s yields as
the energy density changes during hadronization, we need to resort to a model for the space-time evolution of the collision. For the present purposes, we will omit describing the effect of radial flow and take Bjorken’s scenario which incorporates the fact that initially, expansion is longitudinal, that is, along the beam direction which we take as the \( \hat{z} \) axis. In this 1+1 expansion scenario, the relation between the temperature \( T \) and the 1+1 proper-time \( \tau \) is given by

\[
T = T_0 \left( \frac{\tau_0}{\tau} \right)^{v_s^2},
\]

(12)

where \( \tau = \sqrt{t^2 - z^2} \). Equation (12) assumes that the speed of sound \( v_s \) changes slowly with temperature. A lattice estimate of the speed of sound in quenched QCD [11] shows that \( v_s^2 \) increases monotonically from about half the ideal gas limit for \( T \sim 1.5 T_c \) and approaches this limit only for \( T > 4 T_c \), where \( T_c \) is the critical temperature for the phase transition. No reliable lattice results exist for the value of the speed of sound in the hadronic phase though general arguments indicate that the equation of state might become stiffer below \( T_c \) and eventually softens as the temperature approaches zero. For the ease of the argument, here we take \( v_s \) as a constant equal to the ideal gas limit \( v_s^2 = 1/3 \).

We also consider that hadronization takes place on hypersurfaces \( \Sigma \) characterized by a constant value of \( \tau \) and therefore

\[
d\Sigma = \tau \rho \, d\rho \, d\phi \, d\eta,
\]

(13)

where \( \eta \) is the spatial rapidity and \( \rho, \phi \) are the polar transverse coordinates. Thus, the transverse spectrum for a hadron species \( H \) is given as the average over the hadronization interval, namely

\[
E \frac{dN^H}{d^3P} = \frac{g}{\Delta \tau} \int_{\tau_0}^{\tau_f} d\tau \int_{\Sigma} d\Sigma \frac{P \cdot u(x)}{(2\pi)^3} F^H(x, P),
\]

(14)

where \( \Delta \tau = \tau_f - \tau_0 \).

To find the relation between the energy density \( \varepsilon \) – that the probability \( \mathcal{P} \) depends upon – and \( T \), we resort to lattice simulations. For the case of two flavors, a fair representation of the data [3] is given by the analytic expression

\[
\varepsilon / T^4 = a \left[ 1 + \tanh \left( \frac{T - T_c}{b T_c} \right) \right],
\]

(15)

with \( a = 4.82 \) and \( b = 0.132 \). We take \( T_c = 175 \text{ MeV} \). For a purely longitudinal expansion, the flow four-velocity vector \( v^\mu \) and the normal to the freeze-out hypersurfaces of constant \( \tau \), \( u^\mu \), coincide and are given by \( v^\mu = u^\mu = (\cosh \eta, 0, 0, \sinh \eta) \), therefore, the products \( P \cdot u \) and \( P \cdot v \) appearing in Eq. (14) can be written as

\[
P \cdot v = P \cdot u = m_t \cosh(\eta - y),
\]

(16)

where \( m_t = \sqrt{m_H^2 + p_T^2} \) is the transverse mass of the hadron and \( y \) is the rapidity.
Considering the situation of central collisions and looking only at the case of central rapidity, \( y = 0 \), the final expression for the hadron’s transverse distribution is given by

\[
E \frac{dN_H}{d^3P} = \frac{g}{(2\pi)^3} \frac{2m_t A}{\Delta \tau} \int_{\tau_0}^{\tau_f} d\tau \tau K_1 \left[ \frac{m_t}{T(\tau)} \right] \mathcal{P}[\varepsilon(\tau)].
\]  

(17)

To obtain the pion and proton distributions, we use the values \( \tau_0 = 0.75 \) fm and \( \tau_f = 3.5 \) fm and an initial temperature \( T_0 = 200 \) MeV. From Eq. (12), this corresponds to a final freeze-out temperature of \( \sim 120 \) MeV. For protons we take a degeneracy factor \( g = 2 \) whereas for pions \( g = 1 \), to account for the spin degrees of freedom. Figure 2 shows the proton to pion ratio for three different values of the initial evolution proper time \( \tau_0 = 0.5, 0.75 \) and 1 fm and the same final freeze-out proper-time \( \tau_f = 3.5 \) fm, compared to data for this ratio for Au + Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV from PHENIX [12]. We notice that the maximum height reached by this ratio is sensitive to the choice of the initial evolution time. We also notice that the \( p_t \) value for which the maximum is reached is displaced to larger values than what the experimental values indicate. This result is to be expected since the model assumptions leading to Eq. (17) do not include the effects of radial flow that, for a common flow velocity, are known to be larger for protons than for pions, and which will produce the displacement of the ratio toward lower \( p_t \) values.

**SUMMARY AND CONCLUSIONS**

In conclusion, we have used the string-flip model to introduce a dynamical quark recombination scenario that accounts for the evolution of the probability to form a meson or a baryon as a function of the energy density during the collision of a heavy-ion sys-
tem. We have used the model variational parameter as a measure of the probability to form colorless clusters of three quarks (baryons) or of quark-antiquark (mesons). We have shown that these probabilities differ; whereas the probability to form a pion transits smoothly from the high to the low energy density domains, the probability to form a baryon changes abruptly at a given critical energy density. We attribute this difference to the way the energy is distributed during the formation of clusters: whereas for mesons the clustering happens only for quark-antiquark pairs, for baryons the energy can be minimized by also forming sets of three, six, etc., quarks in (colorless) clusters. These produces competing minima in the energy that do not reach each other smoothly. We interpret this behavior as a signal for a qualitative difference in the probability to form mesons and a baryons during the collision evolution.

We have incorporated these different probabilities to compute the proton and pion spectra in a thermal model for a Bjorken-like scenario. We use these spectra to compute the proton to pion ratio as a function of transverse momentum and compare to experimental data at the highest RHIC energies. We argue that the ratio computed from the model is able to reach a height similar to the one shown by data, although the maximum is displaced to larger $p_t$ values. This could be understood by recalling that the model does not include the effects of radial flow which is known to be stronger for protons (higher mass particles) than pions. The inclusion of these effects is the subject of current research that will be reported elsewhere.

ACKNOWLEDGMENTS

Support for this work has been received by PAPIIT-UNAM under grant number IN116008 and CONACyT under grant number 40025-F. M. Martinez was supported by DGEP-UNAM.

REFERENCES

1. R. C. Hwa and C. B. Yang, Phys. Rev. C 67, 034902 (2003); V. Greco, C. M. Ko, and P. Lévai, Phys. Rev. Lett. 90, 202302 (2003).
2. R.J. Fries, B. Müller, C. Nonaka and S.A. Bass, Phys. Rev. Lett. 90, 202303 (2003).
3. F. Karsch, E. Laermann and a Peikert, Phys. Lett. B478, 447 (2000); F. Karsch, Lect. Notes in Phys. 583, 209 (2002).
4. C.J. Horowitz, E.J. Moniz and J.W. Negele, Phys. Rev. D 31, 1689 (1985).
5. C. Horowitz and J. Piekarewicz, Nucl. Phys. A536, 669-696 (1992).
6. G. Toledo Sánchez and J. Piekarewicz, Phys. Rev. C 65, 045208 (2002).
7. G. Toledo Sánchez and J. Piekarewicz, Phys. Rev. C 70, 035206 (2004).
8. A. Ayala, M. Martinez, G. Paic and G. Toledo Sánchez, Dynamical quark recombination in ultrarelativistic heavy-ion collisions and the proton to pion ratio, arXiv:0710.3629 [hep].
9. S. Haussler, S. Scherer and M. Bleicher, The effect of dynamical parton recombination on event-by-event observables, hep-ph/0702188.
10. R.J. Fries, B. Müller, C. Nonaka and S.A. Bass, Phys. Rev. C 68, 044902 (2003).
11. S. Gupta, Pramana 61, 877 (2003).
12. S.S. Adler et al. (PHENIX Collaboration), Phys. Rev. C 69, 034909 (2004).