Deciphering triply heavy baryons in terms of QCD sum rules

Jian-Rong Zhang and Ming-Qiu Huang
Department of Physics, National University of Defense Technology, Hunan 410073, China

The mass spectra of ground-state triply heavy baryons are systematically unscrambled and computed in QCD sum rules. With a tentative \((QQ) - (Q')\) configuration for \(QQQ'\), the interpolating currents representing the triply heavy baryons are proposed. Technically, contributions of the operators up to dimension six are included in operator product expansion (OPE). The numerical results are presented in comparison with other theoretical predictions.

PACS numbers: 14.20.-c, 11.55.Hx, 12.38.Lg

I. INTRODUCTION

The triply heavy baryon, wherever light quarks are absent, is well and truly not a new topic but with a history. As one refers to the studies on their properties, it can be traced back to two more decades ago \[1, 2\]. However, contrasted with the singly and doubly heavy baryons (such as Refs. \[3, 4\]), only infrequent attention has been paid to the triply heavy baryons. Whereas, the case for triply heavy baryon may be improved and several approaches have already appeared in recent years, such as effective field theory \[5\], lattice QCD \[6\], variational method \[7\], bag model \[8\], quark models \[9, 10, 11\] etc., for which is gradually becoming a exciting and remarkable theme nowadays. First, the field of heavy hadron spectroscopy is experiencing a rapid advancement, which is mainly propelled by the continuous discovery of hadronic resonances (for reviews, e.g., see \[12, 13\]). While experimentally reconstructing a candidate for \(\Omega_{ccc}\) is very difficult, it is not unthinkable according to Ref. \[2\]. Especially for the startup of Large Hadron Collider, it seems rather promising to establish triply heavy baryons in future \[14, 15, 16\]. Second, investigation of the triply heavy baryon is of great interest in understanding the dynamics of QCD at the hadronic scale. Although the statement that QCD is the correct theory underlying strong interaction has been commonly accepted and QCD is simple and elegant in its formulation, many questions concerning dynamics of the quarks and gluons at large distances remain unanswered or, at most, understood only at a qualitative level. The quantitative description of the hadronic properties runs into however arduous difficulties. For example, it is a great challenge to extract information on the spectrum from the rather simple Lagrangian of QCD. That’s because low energy QCD involves a regime where it is futile to attempt perturbative calculations and, inevitably, one has to treat a genuinely strong field in nonperturbative methods. Briefly recapitulating the second point, triply heavy baryons, free of light quark contamination, are ideal prototypes to refine one’s present understanding of heavy quark dynamics and may serve as a clean probe to the interplay between perturbative and nonperturbative QCD. Also stimulated by the above two aspects, it is interesting and significative to study their properties like masses through nonperturbative approaches, and the practitioner may resort to a vigorous and reliable working tool in hadron physics, the QCD sum rule \[17\], which is a nonperturbative analytic formalism firmly entrenched in QCD. On the sum rule analysis, the triply heavy baryon systems are analogous to the cases of charmonium and bottomonium, where light quarks are all absent. In fact, there have already been some works on calculating the charmonium and bottomonium masses in QCD sum rules, such as \[18\]. The \(c\) and \(b\) quark masses can also been determined from considering the two-point correlation function of the \(\bar{Q}\gamma_\mu Q\) current \((Q = c\) or \(b\), for instance in \[17, 19\], and some impressive progresses were made in updating the values of \(m_c\) and \(m_b\) later, including the \(O(\alpha_s^2)\) perturbative corrections \[20\]. In addition, the semileptonic decays of \(B_c\) have been investigated by three-point sum rules \[21\]. Thereby, it is feasible for QCD sum rules to study triply heavy baryons and we would like to carry out the sum rule calculations of their spectra in this work.

The paper is organized as follows. In Sec. \[II\] QCD sum rules for the triply heavy baryons are introduced,
and both the phenomenological representation and QCD side are derived, followed by the numerical analysis to extract the spectra and a comparison with other theoretical calculations in Sec. III. Section IV contains a brief summary and outlook.

II. TRIPLY HEAVY BARYON QCD SUM RULES

A generic QCD sum rule calculation consists of three main ingredients: an approximate description of the correlator in terms of intermediate states through the dispersion relation, a description of the same correlation function in terms of QCD degrees of freedom via an OPE, and a procedure for matching these two descriptions and extracting the parameters that characterize the hadronic state of interest. Concretely, coming down to the mass sum rules for triply heavy baryon $QQQ'$ (here $Q$ and $Q'$ can be the same or differently heavy quarks, $c$ or $b$), the starting point is the two-point correlation function

$$\Pi(q^2) = i \int d^4x e^{iq.x} \langle 0 | T [j(x) j(0)] | 0 \rangle.$$  \hfill (1)

Lorentz covariance implies that the correlation function (1) has the form

$$\Pi(q^2) = \lambda^2 e^{-M_H^2/M^2} = \int \frac{ds}{s - q^2} \rho_1(s) e^{-s/M^2}, \hfill (8)$$

where $M_H$ denotes the mass of the triply heavy baryon. In obtaining the above expression, the Dirac and Rarita-Schwinger spinor sum relations,

$$\sum_s^\frac{s}{2} N(q, s) \bar{N}(q, s) = \lambda + M_H,$$  \hfill (4)

for spin-$\frac{1}{2}$ baryon, and

$$\sum_s^\frac{s}{2} N_\mu(q, s) \bar{N}_\nu(q, s) = (\lambda + M_H)(g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu + \frac{q_\mu q_\nu - q_\nu q_\mu}{3M_H} - \frac{2q_\mu q_\nu}{3M_H^2}), \hfill (5)$$

for spin-$\frac{3}{2}$ baryon, have been used.

In the OPE side, the correlation function can be written in terms of a dispersion relation as

$$\Pi_i(q^2) = \int \frac{ds}{s - q^2} \rho_i(s) e^{-s/M^2}, \hfill (6)$$

where the spectral density is given by the imaginary part of the correlation function

$$\rho_i(s) = \frac{1}{\pi} \text{Im}\Pi_i^{\text{OPE}}(s). \hfill (7)$$

In detail, the spectral densities are calculated and embodied in Sec. III

After equating the two sides, assuming quark-hadron duality, and making a Borel transform, the sum rules can be written as

$$\lambda^2 e^{-M_H^2/M^2} = \int \frac{ds}{s - q^2} \rho_1(s) e^{-s/M^2}, \hfill (8)$$
\begin{equation}
\lambda_H^2 M_H e^{-M_H^2/M^2} = \int_{(2m_Q+m_{Q'})^2}^{s_0} ds \rho_1(s) \frac{s}{M^2} e^{-s/M^2}.
\end{equation}

To eliminate the baryon coupling constant \(\lambda_H\), one reckons the ratio of derivative of the sum rule and itself and yields

\begin{equation}
M_H^2 = \int_{(2m_Q+m_{Q'})^2}^{s_0} ds \rho_1(s) s e^{-s/M^2} / \int_{(2m_Q+m_{Q'})^2}^{s_0} ds \rho_1(s) e^{-s/M^2},
\end{equation}

\begin{equation}
M_H^2 = \int_{(2m_Q+m_{Q'})^2}^{s_0} ds \rho_2(s) s e^{-s/M^2} / \int_{(2m_Q+m_{Q'})^2}^{s_0} ds \rho_2(s) e^{-s/M^2}.
\end{equation}

### A. The interpolating currents

In a tentative picture for \(QQQ'\) system, the \(Q'\) orbits the bound \(QQ\) pair. The \((QQ) - (Q')\) structure may be described similar to \(QQ'\) mesons, where the \(QQ\) pair plays the same role of the antiquark \(Q\) in \(QQ'\). The study of such configuration can help one to adopt the appropriate interpolating currents. For the ground states, the currents are correlated with the spin-parity quantum numbers 0\(^+\) and yields currents may be determined according to the rules in \([22]\). For the baryon with \(T = \frac{3}{2}\), the pair of degenerate states. For the latter case, the \(QQ\) diquark has spin 1, and the spin of the third quark is either parallel, \(J^P = \frac{3}{2}^+\), or antiparallel, \(J^P = \frac{1}{2}^+\), to the diquark. The choice of \(\Gamma_k\) and \(\Gamma_k'\) matrices in baryonic currents may be determined according to the rules in \([22]\). For the baryon with \(J^P = \frac{3}{2}^+\), the current may be gained using \(SU(3)\) symmetry relations \([23]\). Consequently, following forms of currents are adopted

\begin{align}
\bar{J}_0_{QQQ} &= \epsilon_{abc}(Q'^T\bar{c}T_kQ_b)\Gamma_k'Q_c,
\bar{J}_{\Gamma QQQ'} &= \epsilon_{abc}(Q'^T\bar{c}T_kQ_b)\Gamma_k'Q_c',
\bar{J}_{\Gamma QQQ'} &= \epsilon_{abc}\frac{1}{\sqrt{3}}[2(Q'^T\bar{c}T_kQ_b)\Gamma_k'Q_c + (Q'^T\bar{c}T_kQ_b)\Gamma_k'Q_c'],
\bar{J}_{\Gamma QQQ'} &= \epsilon_{abc}(Q'^T\bar{c}T_kQ_b)\Gamma_k'Q_c'.
\end{align}

Here the index \(T\) means matrix transposition, \(C\) is the charge conjugation matrix, \(a, b, c\) are color indices, with \(Q\) and \(Q'\) denote heavy quarks. The categories of ground-state triply heavy baryons and the choice of \(\Gamma_k\) and \(\Gamma_k'\) matrices are listed in TABLE I.

### B. The spectra densities

Implementing the calculation of the OPE side, we work at leading order in \(\alpha_s\) and consider condensates up to dimension six. To keep the heavy-quark mass finite, one uses the momentum-space expression for the heavy-quark propagator. The final result is dimensionally regularized at \(D = 4\). It should be

| Baryon   | quark content | \(J^P\) | \(S_d\) | \(L_d\) | \(J_d^{P_d}\) | \(\Gamma_k\) | \(\Gamma_k'\) |
|----------|---------------|--------|--------|--------|-------------|-------------|-------------|
| \(\Omega_{QQQ}\) | \(QQ\) \(Q\) | \(\frac{3}{2}^+\) | 1      | 0      | 1\(^+\)     | \(\gamma_{\mu}\) | 1           |
| \(\Omega_{QQQ'}\) | \(QQ\) \(Q'\) | \(\frac{5}{2}^+\) | 1      | 0      | 1\(^+\)     | \(\gamma_{\mu}\) | \(\gamma_{\mu}\) |
| \(\Delta_{QQQ}\) | \(QQ\) \(Q'\) | \(\frac{3}{2}^+\) | 1      | 0      | 1\(^+\)     | \(\gamma_{\mu}\) | 1           |
| \(\Omega_{QQQ'}\) | \(QQ\) \(Q'\) | \(\frac{1}{2}^+\) | 0      | 0      | 0\(^+\)     | \(\gamma_{5}\) | 1           |

| TABLE I: The choice of \(\Gamma_k\) and \(\Gamma_k'\) matrices in baryonic currents. The index \(d\) in \(S_d\), \(L_d\), and \(J_d^{P_d}\) means diquark. \((QQ)\) denotes the diquark in the axial vector state and \([QQ]\) denotes diquark in the scalar state. |
distinguished for spectral densities of two sort triply heavy baryons, namely, containing the same heavy quark or differently. First, with

\[
\rho_1(s) = \frac{3}{2^4 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_0^\beta_1 d\beta + \int_{\beta_2}^{\beta_1} d\beta \right\} \frac{\alpha^2 \beta^2 (1 - \alpha \beta)^2}{(\alpha + \beta)^4} \left[ \alpha \beta (1 - \alpha \beta) s - (\alpha^2 + \beta^2 + \alpha \beta + 1) m_Q^2 \right] s \\
+ \frac{3^2}{2^4 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_0^\beta_1 d\beta + \int_{\beta_2}^{\beta_1} d\beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^2} \left[ \alpha \beta (1 - \alpha \beta) s - (\alpha^2 + \beta^2 + \alpha \beta + 1) m_Q^2 \right] \frac{1}{2} \\
+ \frac{3}{2^4 \pi^4} m_Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_0^\beta_1 d\beta + \int_{\beta_2}^{\beta_1} d\beta \right\} \alpha \beta \left[ \alpha \beta (1 - \alpha \beta) s - (\alpha^2 + \beta^2 + \alpha \beta + 1) m_Q^2 \right] \frac{1}{(\alpha + \beta)^2} \\
+ \frac{3(g^2 \tilde{G}_Q^2)}{2^5 \pi^4} m_Q \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_0^\beta_1 d\beta + \int_{\beta_2}^{\beta_1} d\beta \right\} \alpha^2 \beta^2 (1 - \alpha \beta) \frac{1}{(\alpha + \beta)^4} 
\]

for \( \Omega_{QQQ} \). The integration limits are given by

\[
\alpha_{\min} = \sqrt{(s^2 - 6m_Q^2 s - 3m_Q^4) - (s - m_Q^2) \sqrt{(s - m_Q^2)(s - 9m_Q^2)}}, \quad \frac{8m_Q^2 s}{s}, \\
\alpha_{\max} = \sqrt{(s^2 - 6m_Q^2 s - 3m_Q^4) + (s - m_Q^2) \sqrt{(s - m_Q^2)(s - 9m_Q^2)}}, \\
\beta_1 = \frac{\alpha (s - m_Q^2) - \sqrt{\alpha^2 s^2 - 6m_Q^2 \alpha^2 s - 4m_Q^4 - 3m_Q^4 \alpha^2 - 4m_Q^2 \alpha^4 s}}{2(\alpha^2 s + m_Q^2)}, \\
\beta_2 = \frac{\alpha (s - m_Q^2) + \sqrt{\alpha^2 s^2 - 6m_Q^2 \alpha^2 s - 4m_Q^4 - 3m_Q^4 \alpha^2 - 4m_Q^2 \alpha^4 s}}{2(\alpha^2 s + m_Q^2)}.
\]

Next, with

\[
\rho_1(s) = -\frac{3}{2^4 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_0^\beta_1 d\beta + \int_{\beta_2}^{\beta_1} d\beta \right\} \frac{\alpha^2 \beta^2 (1 - \alpha \beta)^2}{(\alpha + \beta)^4} \left[ \alpha \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_Q^2) \right] s \\
- \frac{3^2}{2^4 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_0^\beta_1 d\beta + \int_{\beta_2}^{\beta_1} d\beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^2} \left[ \alpha \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_Q^2) \right] \frac{1}{2} \\
- \frac{3}{2^4 \pi^4} m_Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_0^\beta_1 d\beta + \int_{\beta_2}^{\beta_1} d\beta \right\} \alpha \beta \left[ \alpha \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_Q^2) \right] \frac{1}{(\alpha + \beta)^2} \\
- \frac{3(g^2 \tilde{G}_Q^2)}{2^5 \pi^4} m_Q \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_0^\beta_1 d\beta + \int_{\beta_2}^{\beta_1} d\beta \right\} \alpha^2 \beta^2 (1 - \alpha \beta) \frac{1}{(\alpha + \beta)^4} 
\]

\[
\rho_2(s) = -\frac{3}{2^4 \pi^4} m_Q \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_0^\beta_1 d\beta + \int_{\beta_2}^{\beta_1} d\beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^2} \left[ \alpha \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_Q^2) \right] \frac{1}{(\alpha + \beta)^2} \\
- \frac{3}{2^4 \pi^4} m_Q \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_0^\beta_1 d\beta + \int_{\beta_2}^{\beta_1} d\beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^2} \left[ \alpha \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_Q^2) \right] \frac{1}{(\alpha + \beta)^2} 
\]}
\[- \frac{3}{4\pi^2} m^2 Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\alpha_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{1}{(\alpha + \beta)^2} \left[ \alpha \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right] \]

\[- \frac{\left( g^2 G^2 \right)}{2^3 \pi^4} m Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{(1 - \alpha \beta)(1 - \alpha \beta)^2 - \alpha \beta (1 - \alpha \beta)^2}{(\alpha + \beta)^4}, \]

for \( \Omega_{QQ'} \),

\[\rho_1(s) = \frac{1}{2^{1/4} \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right] \]

\[+ \frac{3}{2^{1/4} \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right]^2 \]

\[+ \frac{1}{2^{1/4} \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{\beta (1 - \alpha \beta)}{(\alpha + \beta)^2} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right]\]

\[+ \frac{1}{2^{1/4} \pi^2} m Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{1}{(\alpha + \beta)^2} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right]^2 \]

\[- \frac{\left( g^2 G^2 \right)}{3 \cdot 2^{1/4} \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{1}{(\alpha + \beta)^4} \left[ \beta^2 (1 - \alpha \beta) - \alpha (1 - \alpha \beta) \right] \]

\[- \frac{(1 - \alpha \beta)(1 - \alpha \beta)^2}{(\alpha + \beta)^4} \]

\[\rho_2(s) = \frac{\left( g^2 G^2 \right)}{2^3 \pi^4} m Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right] \]

\[+ \frac{(1 - \alpha \beta)(1 - \alpha \beta)^2}{(\alpha + \beta)^4} \]

\[\rho_1(s) = -\frac{3}{2^{1/4} \pi^2} m^2 Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right]^2 \]

\[+ \frac{3^2}{2^{1/4} \pi^2} m^2 Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right]^2 \]

\[- \frac{3}{2^{1/4} \pi^2} m^2 Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right] \]

\[- \frac{3}{2^{1/4} \pi^2} m^2 Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{1}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right] \]

for \( \Omega^*_{QQ'} \), and

\[\rho_1(s) = \frac{3}{2^{1/4} \pi^2} m Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right]^2 \]

\[- \frac{3^2}{2^{1/4} \pi^2} m Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right]^2 \]

\[- \frac{3}{2^{1/4} \pi^2} m Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{\alpha \beta (1 - \alpha \beta)}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right] \]

\[- \frac{3}{2^{1/4} \pi^2} m Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \, \mathrm{d} \alpha \left\{ \int_{0}^{\beta_{1}} \, \mathrm{d} \beta + \int_{\beta_{2}}^{\beta_{1}} \, \mathrm{d} \beta \right\} \frac{1}{(\alpha + \beta)^4} \left[ \beta (1 - \alpha \beta) s - ((\alpha + \beta)^2 m_Q^2 + (1 - \alpha \beta) m_{Q'}^2) \right] \]

for \( \Omega^*_{QQ'} \).
for triply heavy baryons are quite complicated and tedious as one has to tackle some three-loop massive windows are taken as for Ω

other comparisons can not be made for lack of lattice results at present.

proved to be true in the analysis for the singly heavy baryons (the radiative corrections to the perturbative

since a partial cancellation occurs in the ratio obtaining the mass sum rules (10) and (11). This has been

for Ω

can find that the central values of our results are lower than other predictions from potential models, in

In numerical analysis, not both but only the sum rule (11) will be numerically analyzed for brevity, and

and the input values are taken as $m_c = 1.25$ GeV, $m_b = 4.20$ GeV with $(g^2G^2) = 0.88$ GeV$^4$. Complying with the standard procedure of sum rule method, the threshold $s_0$ and Borel parameter $M^2$ are varied to find the optimal stability window, in which the perturbative contribution should be larger than the condensate contributions while the pole contribution larger than continuum contribution. Accordingly, the sum rule windows are taken as $\sqrt{s_0} = 5.5 \sim 6.5$ GeV, $M^2 = 6.5 \sim 8.0$ GeV$^2$ for $\Omega_{ccc}$, $\sqrt{s_0} = 14.0 \sim 15.0$ GeV, $M^2 = 14.0 \sim 15.5$ GeV$^2$ for $\Omega_{bbb}$, $\sqrt{s_0} = 8.0 \sim 9.0$ GeV, $M^2 = 8.5 \sim 10.0$ GeV$^2$ for $\Omega_{ccb}$, $\sqrt{s_0} = 8.5 \sim 9.5$ GeV, $M^2 = 8.5 \sim 10.0$ GeV$^2$ for $\Omega_{cbb}$, $\sqrt{s_0} = 11.0 \sim 12.0$ GeV, $M^2 = 10.0 \sim 11.5$ GeV$^2$ for $\Omega_{bcbc}$, $\sqrt{s_0} = 11.5 \sim 12.5$ GeV, $M^2 = 10.0 \sim 11.5$ GeV$^2$ for $\Omega_{bbb}$, $\sqrt{s_0} = 8.5 \sim 9.5$ GeV, $M^2 = 8.5 \sim 10.0$ GeV$^2$ for $\Omega_{cbb}$, and $\sqrt{s_0} = 11.5 \sim 12.5$ GeV, $M^2 = 10.0 \sim 11.5$ GeV$^2$ for $\Omega_{bcbc}$, respectively. The corresponding Borel curves are shown in Figs. 1-4. In Table I the numerical results are presented, together with the predictions from other theoretical approaches. It is worth noting that uncertainty in our results are merely owing to the sum rule windows, not involving the ones rooting in the variation of the quark masses and QCD parameters. Note that the QCD $O(\alpha_s)$ corrections are not included in this work, whose calculations for triply heavy baryons are quite complicated and tedious as one has to tackle some three-loop massive propagator diagrams. However, it is expected that the QCD $O(\alpha_s)$ corrections might be under control since a partial cancellation occurs in the ratio obtaining the mass sum rules (10) and (11). This has been proved to be true in the analysis for the singly heavy baryons (the radiative corrections to the perturbative terms increase the calculated baryon masses by about 10%) [24] and for the heavy mesons (the value of $f_D$ increases by 12% after the inclusion of the $O(\alpha_s)$ correction) [25]. After a detailed comparison, one can find that the central values of our results are lower than other predictions from potential models, in particular, for $\Omega_{bbb}$, slightly more than 1 GeV, whereas the relative discrepancy approximates to 10%. In addition, for $\Omega_{ccc}$, our result is in good agreement with the lattice QCD simulation in Ref. [4], but the other comparisons can not be made for lack of lattice results at present.
FIG. 1: The dependence on $M^2$ for the masses of $\Omega_{ccc}$ and $\Omega_{bbb}$ from sum rule (10). The continuum thresholds are taken as $\sqrt{s_0} = 5.5 \sim 6.5$ GeV and $\sqrt{s_0} = 14.0 \sim 15.0$ GeV.

FIG. 2: The dependence on $M^2$ for the masses of $\Omega_{ccb}$ and $\Omega_{∗_{ccb}}$ from sum rule (10). The continuum thresholds are taken as $\sqrt{s_0} = 8.0 \sim 9.0$ GeV and $\sqrt{s_0} = 8.5 \sim 9.5$ GeV.

FIG. 3: The dependence on $M^2$ for the masses of $\Omega_{bbc}$ and $\Omega_{∗_{bbc}}$ from sum rule (10). The continuum thresholds are taken as $\sqrt{s_0} = 11.0 \sim 12.0$ GeV and $\sqrt{s_0} = 11.5 \sim 12.5$ GeV.

FIG. 4: The dependence on $M^2$ for the masses of $\Omega_{∗_{ccb}}$ and $\Omega_{∗_{bbc}}$ from sum rule (10). The continuum thresholds are taken as $\sqrt{s_0} = 8.5 \sim 9.5$ GeV and $\sqrt{s_0} = 11.5 \sim 12.5$ GeV.
TABLE II: The mass spectra of triply heavy baryons (mass in unit of GeV).

| Baryon quark content | $J^P$ | $S_d$ | $L_d$ | $J^{P_d}$ | $^\prime$This work$^\prime$ | Ref. [1] | Ref. [2] | Ref. [6] | Ref. [7] | Ref. [8] | Ref. [10] |
|----------------------|-------|-------|-------|-----------|-----------------|---------|---------|---------|---------|---------|---------|
| $\Omega_{ccc}$      | {cc}c | $^\frac{1}{2}^+$ | 1     | 0        | $^1^+$           | 4.67 ± 0.15 | 4.79    | 4.925   | 4.681   | 4.76    | 4.777   | 4.803   |
| $\Omega_{bbk}$      | {bb}b | $^\frac{1}{2}^+$ | 1     | 0        | $^1^+$           | 13.28 ± 0.10 | 14.30   | 14.760  | 14.37   | 14.276  | 14.569  |
| $\Omega_{ccb}$      | {cc}b | $^\frac{1}{2}^+$ | 1     | 0        | $^1^+$           | 7.41 ± 0.13  |         |         | 7.984   | 8.018   |         |
| $\Omega_{*cc}$      | {cc}b | $^\frac{1}{2}^+$ | 1     | 0        | $^1^+$           | 7.45 ± 0.16  | 8.03    | 8.200   | 7.98    | 8.005   | 8.025   |
| $\Omega_{bbc}$      | {bb}c | $^\frac{1}{2}^+$ | 1     | 0        | $^1^+$           | 10.30 ± 0.10 |         |         | 11.139  | 11.280  |         |
| $\Omega_{*bc}$      | {bc}c | $^\frac{1}{2}^+$ | 1     | 0        | $^1^+$           | 10.54 ± 0.11 | 11.20   | 11.480  | 11.19   | 11.163  | 11.287  |
| $\Omega_{*bc}$      | {bc}b | $^\frac{1}{2}^+$ | 0     | 0        | $^0^+$           | 7.49 ± 0.10  |         |         |         |         |         |
| $\Omega_{*bb}$      | {bb}c | $^\frac{1}{2}^+$ | 0     | 0        | $^0^+$           | 10.35 ± 0.07 |         |         |         |         |         |

IV. SUMMARY AND OUTLOOK

In a tentative $(QQ) − (Q')$ configuration, the QCD sum rules have been employed to calculate the masses of triply heavy baryon $QQQ'$ ($Q = Q'$ or $Q \neq Q'$), including the contributions of the operators up to dimension six in OPE. The mass values extracted from the sum rules are collected in comparison with other theoretical predictions. The results in this work are lower than the predictions from potential models, nevertheless, the one for $\Omega_{ccc}$ is well compatible with the existing lattice study. Indubitably, there are still plenty of problems desiderated to resolve. Experimentally, the evidence on triply heavy baryons are expected to reveal nature of them, especially after the putting into operation of the Large Hadron Collider. In theory, in order to improve on the accuracy of the QCD sum rule analysis for triply heavy baryons, one certainly needs to take into account the QCD $O(\alpha_s)$ corrections to the sum rules in the further work. Additionally, it may be needed to carry out a comprehensive study on triply heavy baryon spectra from lattice QCD stimulations for the future.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Contract No.10675167.

[1] P. Hasenfratz, R. R. Horgan, J. Kuti, and J. M. Richard, Phys. Lett. B 94, 401 (1980).
[2] J. D. Bjorken, Preprint FERMILAB-Conf-85-069.
[3] E. Bagan, M. Chabab, H. G. Dosch, and S. Narison, Phys. Lett. B 278, 367 (1992); B 287, 176 (1992); B 301, 243 (1993); E. Bagan, M. Chabab, and S. Narison, Phys. Lett. B 306, 350 (1993).
[4] Y. B. Dai, C. S. Huang, C. Liu, and C. D. Lü, Phys. Lett. B 371, 99 (1996); F. O. Durães and M. Nielsen, Phys. Lett. B 658, 40 (2007); Z. G. Wang, Eur. Phys. J. C 54, 231 (2008); J. R. Zhang and M. Q. Huang, Phys. Rev. D 77, 094002 (2008); D 78, 094007 (2008); D 78, 094015 (2008).
[5] N. Brambilla, A. Vairo, and T. Rösch, Phys. Rev. D 72, 034021 (2005).
[6] T. W. Chiu and T. H. Hsieh, Nucl. Phys. A 755, 471c (2005).
[7] Y. Jia, JHEP 0610, 073 (2006).
[8] A. Berotas and V. Šimonis, arXiv:0808.1220.
[9] J. Vijande, H. Garcilazo, A. Valcarce, and F. Fernández, Phys. Rev. D 70, 054022 (2004).
[10] A. P. Martynenko, Phys. Lett. B 663, 317 (2008).
[11] B. Patel, A. Majethiya, and P. C. Vinodkumar, arXiv:0808.2880.
[12] E. S. Swanson, Phys. Rep. 429, 243 (2006).
[13] Particle Data Group, C. Amsler et al., Phys. Lett. B 667, 1 (2008).
[14] M. A. Gomshi Nobary, Phys. Lett. B 559, 239 (2003).
[15] M. A. Gomshi Nobary and R. Sepahvand, Phys. Rev. D 71, 034024 (2005); Nucl. Phys. B741, 34 (2006); Phys. Rev. D 76, 114006 (2007).
[16] M. A. Gomshi Nobary, B. Nikoobakht, and J. Naji, Nucl. Phys. A 789, 243 (2007).
[17] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Fortschr. Phys. 32, 585 (1984).
[18] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Nucl. Phys. B186, 109 (1981); Phys. Rep. 127, 1 (1985).
[19] C. A. Dominguez, G. R. Gluckman, and N. Paver, Phys. Lett. B 333, 184 (1994); S. Narison, Phys. Lett. B 341, 73 (1994).
[20] M. Eidemüller and M. Jamin, Phys. Lett. B 498, 203 (2001); G. Corcella and A. H. Hoang, Phys. Lett. B 554, 133 (2003); K. G. Chetyrkin, J. H. Kuhn, and M. Steinhauser, Nucl. Phys. B505, 40 (1997).
[21] E. Bagan et al., Z. Phys. C 64, 57 (1994); P. Colangelo, G. Nardulli, and N. Paver, Z. Phys. C 57, 43 (1993); V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Nucl. Phys. B569, 473 (2000).
[22] E. V. Shuryak, Nucl. Phys. B198, 83 (1982).
[23] B. L. Ioffe, Nucl. Phys. B188, 317 (1981); B191, 591(E) (1981); Z. Phys. C 18, 67 (1983).
[24] S. Groote, J. G. Körner, and O. I. Yakovlev, Phys. Rev. D 55, 3016 (1997).
[25] S. Narison, Phys. Lett. B 605, 319 (2005).