SUNGLASS: a new weak-lensing simulation pipeline

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ABSTRACT

A new cosmic shear analysis pipeline SUNGLASS (Simulated UNiverses for Gravitational Lensing Analysis and Shear Surveys) is introduced. SUNGLASS is a pipeline that rapidly generates simulated universes for weak-lensing and cosmic shear analysis. The pipeline forms suites of cosmological N-body simulations and performs tomographic cosmic shear analysis using line-of-sight integration through these simulations while saving the particle lightcone information. Galaxy shear and convergence catalogues with realistic 3D galaxy redshift distributions are produced for the purposes of testing weak-lensing analysis techniques and generating covariance matrices for data analysis and cosmological parameter estimation. We present a suite of fast medium-resolution simulations with shear and convergence maps for a generic 100 deg² survey out to a redshift of $z = 1.5$, with angular power spectra agreeing with the theory to better than a few per cent accuracy up to $\ell = 10^3$ for all source redshifts up to $z = 1.5$ and wavenumbers up to $\ell = 2000$ for the source redshifts $z \geq 1.1$. At higher wavenumbers, there is a failure of the theoretical lensing power spectrum reflecting the known discrepancy of the Smith et al. fitting formula at high physical wavenumbers. A two-parameter Gaussian likelihood analysis of $\sigma_8$ and $\Omega_m$ is also performed on the suite of simulations, demonstrating that the cosmological parameters are recovered from the simulations and the covariance matrices are stable for data analysis. We find that any residual bias in our calculation is below our uncertainty within the 1σ confidence limits. The SUNGLASS pipeline should be an invaluable tool in weak-lensing analysis.

Key words: gravitational lensing: weak – methods: numerical – large-scale structure of Universe.

1 INTRODUCTION

Cosmic shear analysis is an excellent method for probing the dark Universe (for reviews, see Mellier 1999; Bartelmann & Schneider 2001; Refregier 2003; Schneider 2006; Munshi et al. 2008; Massey, Kitching & Richard 2010, and references therein). It is also a reasonably new field of research with cosmic shear first being observed just 10 years ago (Bacon, Refregier & Ellis 2000; Kaiser, Wilson & Luppino 2000; Van Waerbeke et al. 2000; Wittman et al. 2000). Weak gravitational lensing effects on a cosmic scale are a mere 1 per cent change in shape, and observational systematics make the measurement of these changes challenging. However, the combination of the well-understood underlying physics and the expected precision of cosmological parameter estimation makes the effort worthwhile.

Next generation telescope surveys will observe more of the sky than ever before, and the volume of data they will produce is unprecedented. Future surveys promise to determine the equation of state of dark energy to 1 per cent as well as probing the possibilities of extra dimensional gravity models and alternative cosmologies. The first Pan-STARRS¹ telescope is currently undertaking a cosmic shear survey of the entire visible sky from its location in Hawaii, and new projects such as VST-KIDS,² DES,³ HALO,⁴ Euclid⁵ and LSST⁶ are planned to perform wide field cosmic shear surveys, measuring both large, linear scales, and small, non-linear scales.

Given that this field is relatively young, techniques are still being developed to exploit the weak-lensing data from these surveys to provide further understanding on the nature of the Universe. To realize the potential of these new telescope surveys and to test new weak-lensing analysis techniques, challenges must be met. To

¹ The Panoramic Survey Telescope & Rapid Response System, Pan-STARRS: http://pan-starrs.ifa.hawaii.edu/
² The Very Large Telescope Survey Telescope Kilo-Degree Survey, VST-KIDS: http://www.astro-wise.org/projects/KIDS/
³ The Dark Energy Survey, DES: http://www.darkenergysurvey.org/
⁴ The High Altitude Lensing Observatory, HALO: Rhodes et al. (in preparation).
⁵ Euclid: http://sci.esa.int/euclid
⁶ The Large Synoptic Survey Telescope, LSST: http://www.lsst.org/
achieve the small statistical errors required, experiments require full end-to-end simulations of huge volumes which also probe the non-linear regime to assist in understanding the limitations of the analysis techniques. Simulations offer data sets with known parameters which are essential when testing analysis pipelines. Simulations can also include effects which may be difficult to model theoretically, such as source clustering and galaxy alignments, as well as other systematics and real-world effects. An additional role for simulations is in accurate estimation of the covariance of observable quantities. This is needed for the analysis of surveys as analytic approximations can be wholly inadequate (e.g. Semboloni et al. 2007). Monte Carlo analysis can be performed with simulations to provide covariance matrices that are required for data analysis and cosmological parameter estimation. Simulations are also required for rigorous testing and development, so all analysis methods can be analysed blindly before the same techniques are applied to real data. To address these challenges, the SUNGLASS (Simulated UNiverses for Gravitational Lensing Analysis and Shear Surveys) pipeline has been developed to produce simulations and mock shear and convergence catalogues rapidly for weak-lensing and cosmic shear analysis. The purpose of this paper is to introduce SUNGLASS and show rigorous testing of its outputs.

Many weak-lensing studies use simulations with very high resolution to run their analysis (e.g. Fosalba et al. 2008; Hilbert et al. 2009; Teyssier et al. 2009; Schrabback et al. 2010). The computational cost of running these simulations is high and consequently there is often only a single realization available. However, it is very important to ensure that covariance matrices calculated from these simulations are not contaminated by correlations in the simulations (Hartlap, Simon & Schneider 2007). In order to ensure uncorrelated data, a Monte Carlo suite of simulations should be used to determine the covariance matrix (Sato et al. 2009). In this work, 100 independent simulations were constructed using SUNGLASS.

To date, there are still reasonably few weak-lensing simulations available. Of the few that are available, many implement a ray-tracing technique where light rays are propagated from an observer to a lensing source plane (e.g. Jain, Seljak & White 2000; Vale & White 2003; Forero-Romero et al. 2007; Hilbert et al. 2009; Teyssier et al. 2009; Sato et al. 2009; Dietrich & Hartlap 2010; Vafaei et al. 2010). Some alternative ray-tracing methods do not use lensing source planes and instead calculate the full 3D gravitational potential (e.g. Couchman, Barber & Thomas 1999; Carbone et al. 2008). Ray-tracing is computationally intensive and time consuming when solving the full ray-tracing equations. If the Born approximation is used in the ray-tracing, the time to run the analysis is reduced but the process is still computationally intensive and the simulation data are still binned in three dimensions to perform the calculations. An alternative to ray-tracing is line-of-sight integration, which uses the Born approximation to calculate rapidly the weak-lensing signal through a lightcone (e.g. White & Hu 2000; Fosalba et al. 2008). This method is not suitable in the strong-lensing regime, but in the weak-lensing regime it is rapid and requires fewer computational resources than ray-tracing techniques. In this paper, a new line-of-sight integration technique, implemented in the SUNGLASS pipeline, for measuring convergences in an N-body simulation is introduced. This new method is rapid and after the N-body simulations have been run on a modest computer cluster, the weak-lensing analysis can be run on a single processor of a desktop computer. In contrast to ray-tracing, the method does not bin in the radial direction, using all of the redshift information available. Although the catalogues are suitable for real-space analysis, SUNGLASS analyses and tests our mock weak-lensing surveys in Fourier space, using power spectra, as it is possible to cleanly distinguish between linear and non-linear regimes in Fourier space. We are also able to easily identify scales where the simulations are reliable by determining the region of the power spectrum in Fourier space that lies between the size of the simulated volume at low wavenumbers and shot-noise due to particle discreteness and pixelization effects at high wavenumbers.

The outline of this paper is as follows. Section 2 introduces the SUNGLASS pipeline. Details of the simulations are given in Section 2.1, and the line-of-sight integration method for determining shear and convergence without radial binning is described in Section 2.2. Section 2.3 presents the shear and convergence power spectrum analysis, and Section 2.4 deals with the generation of the mock galaxy shear catalogues. An application of the mock catalogues is discussed in Section 3 where Gaussian likelihood estimates of $\Omega_m$ and $\sigma_8$ are performed. A summary of the pipeline and methods concludes the paper in Section 4.

2 DETAILS OF THE SUNGLASS PIPELINE

SUNGLASS is a pipeline that generates cosmic shear and convergence catalogues using N-body simulations. The pipeline creates mock galaxy shear catalogues that can be used to test the cosmic shear analysis software used on telescope survey data sets. The nature of the pipeline also allows many simulation realizations to be generated rapidly to produce covariance matrices for data analysis and cosmological parameter estimation. The pipeline begins by creating a suite of cosmological N-body simulations. Lightcones are generated through the simulations and tomographic shear, and convergence maps are determined using line-of-sight integrations at multiple lensing source redshifts. Finally, mock galaxy catalogues with fully 3D shear and convergence information and galaxy redshift distributions are assembled from the lightcones and the tomographic shear and convergence planes. The following sections detail each step of the SUNGLASS pipeline.

2.1 The N-body simulations

All of the simulations presented in this work were run on a modest Xeon cluster, using four nodes with dual Xeon E5520 2.27-GHz quad-core processors per node and 24-GB shared memory per node. The simulations were run using the cosmological structure formation software package GADGET2 (Springel 2005). GADGET2 represents bodies by a large number, $N$, in this work we use 512$^3$, particles. Each particle is ‘tagged’ with its own unique kinematic and physical properties that evolve with the particle over time. GADGET2 models the dynamics of dark matter particles using a Tree-PM scheme, and for the purposes of this work only dark matter particles were considered.

The pre-initial particle distribution for the simulations used in this work is a glass which has sub-Poissonian noise properties (White 1994). This distribution has no preferred direction with forces on each particle being close to zero. If a glass is used as the initial condition in a standard integrator, structures do not evolve. Particle displacements are imposed manually as an initial step to enable structure formation. The initial power spectrum was imposed on the particles using the parallel initial conditions generator N-GENIC that was provided by Volker Springel. The initial particle displacement field is formed by using the Zel’dovich approximation (Zel’dovich 1970) to perturb the particles, imposing an Eisenstein & Hu (1998) matter power spectrum on the particles, and giving each particle an initial velocity.
Multiple medium-resolution simulations were run with 512$^3$ dark matter particles, in a box of $L = 512\, h^{-1} \, \text{Mpc}$ comoving side length with periodic boundary conditions. The following cosmological parameters were used for a flat concordance $\Lambda$ cold dark matter ($\Lambda$CDM) model consistent with the Wilkinson Microwave Anisotropy Probe (WMAP) 7-yr results (Jarosik et al. 2011): $\Omega_{\Lambda} = 0.73$, $\Omega_m = 0.27$, $\Omega_b = 0.045$, $n_s = 0.96$, $\sigma_8 = 0.8$ and $h = 0.71$ in units of $100 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$. The particle mass is $7.5 \times 10^{10} \, M_{\odot}$ and the softening length is $33\, h^{-1} \, \text{kpc}$. The simulations were all started from a redshift of $z = 60$ and allowed to evolve to the present.

The simulation data were stored at 26 output times corresponding to a $128\, h^{-1} \, \text{Mpc}$ comoving separation, between $z = 1.5$ and the present. These snapshots were chosen to fall within the photometric redshift error of $\sigma_z < 0.05(1+z)$ corresponding to a displacement of $\approx 147 \, h^{-1} \, \text{Mpc}$ at $z = 1$. In a $512^3$ particle simulation, this amounts to 100 GB data per simulation and takes approximately 21 h to run on the Xeon cluster’s 32 processors.

2.2 Shear and convergence map generation

We begin by determining the shear and convergence for a source plane at fixed comoving distance $r_s$. We consider a distribution of sources in Section 2.4.

The effects of weak gravitational lensing on a source can be described by two fields: the spin-2 shear, $\gamma$, which describes the stretching or compression of an image, and a scalar convergence, $\kappa$, which describes its change in angular size. These can be related to a lensing potential field, $\phi$, by

$$\kappa = \frac{1}{2} \partial^2 \phi,$$

$$\gamma = \gamma_1 + i \gamma_2 = \frac{1}{2} \partial \phi,$$  

where $\gamma_1$ and $\gamma_2$ are the orthogonal components of the shear distortion, and $\partial = \partial_1 + i \partial_2$ is a complex derivative on the sky.

We want to generate shear and convergence maps along a lightcone through the simulation. Instead of using ray-tracing to determine the lightcone (e.g. Wambsganss, Cen & Ostriker 1998; Jain, Seljak & White 2000; Teyssier et al. 2009; Hilbert et al. 2009), a line-of-sight integration was implemented using the Born approximation where one integrates along an unperturbed path (e.g. Cooray & Hu 2002; Vale & White 2003). Fosalba et al. (2008) build their convergence calculation in a fine angular grid while allowing them to keep their radial coordinate.

Rewriting equation (4) we find the average convergence in an angular pixel, with no radial binning, is given by

$$\bar{\kappa}_p(r_s) = \sum_k \frac{K(r_s, r_k)}{\Delta \Omega_p \bar{n}(r_k) r_k^2} \int_0^r dr K(r, r_k),$$  

where $\Delta \Omega_p = \Delta \theta_1 \Delta \delta_1$ is the pixel area, $r_k$ is the comoving radial distance of each individual particle $k$ in the lightcone and $K(r, r_k)$ is the scaled lensing kernel:

$$K(r, r_k) = \frac{3 H_0^2 \Omega_m (r - r_k)}{2c^2},$$

Hereafter, we drop the overline and assume all fields are averaged over an angular pixel. A derivation of equation (5) is given in Appendix A. In practice, equation (5) can be calculated by a running summation so that it is not necessary to recalculate the convergence from scratch for each source redshift:

$$\sum_k \frac{K(r_k, r_s)}{\Delta \Omega_p \bar{n}(r_k) r_k^2} = \frac{1}{\Delta \Omega_p} \left( \frac{3 H_0^2 \Omega_m}{2c^2} \right) \int \left[ \sum_k \frac{1}{r_k a(r_k) \bar{n}(r_k)} - \frac{1}{r_k} \sum_k \frac{1}{a(r_k) \bar{n}(r_k)} \right].$$

Each of the sums in the square brackets can be calculated as running sums to calculate the first term in equation (5). The second term in equation (5) is constant for each source redshift, but can be calculated in the same fashion by splitting the integral into two parts:

$$\int_0^r \frac{dr}{r} K(r, r_s) = \frac{3 H_0^2 \Omega_m}{2c^2} \times \left[ \int_0^r \frac{dr}{r} \frac{\rho}{a(r)} - \frac{1}{r_s} \int_0^r \frac{dr}{a(r)} \right].$$

The convergence maps are generated by adding the particles that fall within the lightcone to the line-of-sight integration. To show evolution through the lightcone, the simulation volumes are split into $128\, h^{-1} \, \text{Mpc}$ sections. The first $128\, h^{-1} \, \text{Mpc}$ of the first $(z = 0)$ snapshot is used, the second $128\, h^{-1} \, \text{Mpc}$ of the second $(z > 0)$ snapshot and so on until the end of the simulation box volume is reached at snapshot 4 as shown in Fig. 1. The centroid of the next simulation box is then shifted and the simulation box is rotated randomly to try to avoid repeated structures along the line of sight (e.g. White & Hu 2000; Vale & White 2003). The boxes are always periodic in the transverse direction. This continues through all of the snapshots out to a redshift of $z = 1.5$. The source redshifts have been placed at $\Delta z = 0.1$ intervals (Table 1), because the change in convergence between these redshifts is small enough that desired redshift values in between can be accurately determined by interpolation.

Once the convergences have been calculated at each of the source redshifts, the shear values can be determined on a flat-sky. The flat-sky shear and convergence Fourier coefficients are related by

$$\gamma_1(\ell) = \kappa(\ell) \left( \frac{\ell^2}{\ell^2 + \ell_s^2} \right),$$

$$\gamma_2(\ell) = \kappa(\ell) \left( \frac{2 \ell \ell_s}{\ell^2 + \ell_s^2} \right),$$

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where $\kappa(\ell)$ is the Fourier transform of the convergence, and $\ell_x$ and $\ell_y$ are the Fourier variables. The Fast Fourier transform used throughout this paper is the Fastest Fourier Transform in the West (FFTW). The nature of FFTW is such that it has periodic boundary conditions. In this work, the lightcone is not periodic and the convergence is calculated with a 1 per cent larger field than the final 100 deg$^2$. After Fourier transforming this convergence field to determine the shear, the extra is trimmed away to compensate for any periodic structures in the resulting shear field. To test the algorithm, we also estimated B modes by calculating the unphysical imaginary part of the convergence $\beta = \text{imag}(\kappa)$, from the shear,

$$
(11)
$$

theoretical prediction for the shear and convergence power spectrum for sources at redshift $z$ is given by

$$
C^{\gamma\gamma}_\ell(z) = C^{\kappa\kappa}_\ell(z) = \frac{9H_0^3\Omega_m^2}{4\pi^3} \int_0^{r_s} dr P \left( \frac{\ell}{r_s} \right) \left[ \frac{r_s(z) - r}{r_s(z)} \right]^2
$$

Table 1. Table of parameters for the source redshift plane lightcones used in this paper. $N_{sims}$ is the number of independent lightcones, $n_{pix}$ is the number of angular pixels on the sky, $N_{C\ell}$ is the number of bins in the angular power spectrum, $z_{\text{max}}$ is the maximum redshift in the lightcone and $n_{\text{plane}}$ is the number of source redshift planes in the lightcone.

| $N_{sims}$ | Area (deg$^2$) | $n_{pix}$ | $N_{C\ell}$ | $z_{\text{max}}$ | $n_{\text{plane}}$ |
|------------|---------------|-----------|-------------|-----------------|------------------|
| 100        | 100           | 2048$^2$  | 32          | 1.5             | 15               |

Fig. 2 is an example of a convergence and shear map for a field that is 100 deg$^2$ at a source redshift of $z = 0.8$. There are 2048 bins in each transverse direction and no binning in the radial direction. The background of the map shows the integrated convergence along the lightcone up to $z = 0.8$, and the white ticks show the shear at this source redshift. The length of the ticks has been multiplied by an arbitrary constant to make them visible as the magnitude of the shear is at the per cent level. The red patches show areas of the highest convergence, and the shear ticks clearly trace these regions tangentially. These maps can be generated for the standard simulations at multiple source redshifts quite rapidly once the simulations have been run. The most time consuming module in this code is reading in the snapshots due to their reasonably large size of 100 GB. This module can be optimized by using the fastest available data transfer rates on the drive where the snapshot data are stored.

2.3 Shear and convergence power spectra

In order to verify the accuracy of the shear and convergence maps, the shear and convergence power spectra are determined for each source redshift. From equation (4), the discretized convergence power spectrum for sources at redshift $z$ is given by

$$
C^{\kappa\kappa}_\ell(z) = \frac{9H_0^3\Omega_m^2}{4\pi^3} \int_0^{r_s} dr P \left( \frac{\ell}{r_s} \right) \left[ \frac{r_s(z) - r}{r_s(z)} \right]^2
$$

(Munshi et al. 2008), where $P(\ell/r; r)$ is the 3D matter density power spectrum at a redshift $z$.

From the simulations, it is possible to determine an angle-averaged power spectrum from the convergence and shear calculated in the lightcones. When taking into consideration the conventions used in FFTW, the discretized convergence power spectrum for a slice in redshift is given as the sum over logarithmic shells in $\ell$-space as

$$
\frac{(\ell + 1)C^{\kappa\kappa}_\ell(z)}{2\pi} = \sum_{\ell \in \text{shell}} \frac{|\kappa(\ell, z)|^2}{n_{pix} \Delta \ln \ell},
$$

where $n_{pix}$ is the total number of bins in the Fourier transform (2048$^2$ in this case), $\Delta \ln \ell$ represents the thickness of the shell in $\ell$-space and $C^{\kappa\kappa}_\ell$ is the estimated power. Similarly, the shear power is

% http://www.fftw.org/
estimated by
\[ \frac{\ell(\ell + 1)C_{\ell}^{gg}(z)}{2\pi} = \sum_{\ell' \in \text{shell}} \frac{|\gamma_1(\ell, z)|^2 + |\gamma_2(\ell, z)|^2}{n_{\ell \Delta \ln \ell}}. \] (14)

The B-mode power is estimated in the same way as the convergence.

The modes in this power spectrum are arranged on a square grid, which causes discretization errors when binned in annuli at small \( \ell \). To correct for this, the power is scaled by the ratio of the measured number of modes to the expected number of modes,

\[ N_{\exp} = g \pi (\ell_{\max}^2 - \ell_{\min}^2), \] (15)

where \( g = (L/2\pi)^2 \) is the density of states, \( L \) is the size of the field in radians, \( \ell_{\max} \) and \( \ell_{\min} \) are the maximum and minimum wavenumbers in this shell. The effect of this normalization correction is about 10 per cent at the lower wavenumbers, while the higher wavenumbers remain largely unaffected. The discretization correction is not perfect which is why the same slight zig-zag of the power spectrum is evident in all of the source redshift planes at wavenumbers \( \ell < 100 \).

We can compare our simulated shear and convergence power spectra with the theoretical expectation [as demonstrated in Joachimi & Schneider (2008, 2009, 2010)], and extensively tested against ICOSMO\(^8\) (Refregier et al. 2011)]. This code uses the method of Smith et al. (2003) for the non-linear power spectrum, the matter transfer function of Eisenstein & Hu (1998) and a numerically calculated linear growth factor.

Due to the discrete number of particles in an N-body simulation, the measured power spectrum will be the combined real shear and convergence power plus a shot-noise power contribution:

\[ \tilde{C}_{\ell}^{\chi\chi} = C_{\ell}^{\chi\chi} + C_{\ell}^{\chi\chi \text{SN}}, \] (16)

where \( \tilde{C}_{\ell}^{\chi\chi} \) is the power estimated from the simulation. The shot-noise power can be derived from equation (12) using a white-noise power spectrum, \( P_{\text{SN}}(k, r) = 1/\bar{n}_s(r) \), where \( \bar{n}_s(r) \) is the 3D mean comoving number density of particles in the simulation. The shot-noise power for the shear and convergence is then given by

\[ C_{\ell}^{\chi\chi \text{SN}} = \frac{9H_0^4\Omega_m^2}{4c^3} \int_0^{\theta_{\text{max}}/2} \frac{dr}{\bar{n}_s(r) r^2 \pi^2 (z^2 + r^2)^2}. \] (17)

Usually, for simulated particles, \( \bar{n}_s \) will be a constant in comoving coordinates.

Fig. 3 shows the mean, normalized 2D shear power spectra estimated from 100 independent simulations (black points and line), with the error bars showing the scatter on the estimated mean. The figures show the shear power for sources at redshifts of \( z = 0.3, 0.6, 0.8, 1.0, 1.3 \) and 1.5. The smooth (red) line shows the theoretical prediction for the ensemble-averaged shear power spectrum, while the diagonal (dotted blue) lines show the shot-noise power for each source redshift. The (dot-dashed light blue) curve between the simulated data and the theory curve shows the mean power spectrum with the expected shot-noise subtracted, and the lower (dashed magenta) curve shows the estimated B-mode power spectrum.

The bottom panel of each figure shows the percentage difference between the measured shear power spectrum and the ensemble-average theory prediction (upper black points), while the lower (light blue) points show the shot-noise subtracted shear power spectrum. Overall, the mean shear power agrees well, to within a few per cent, with the ensemble-averaged theoretical model over the \( \ell \)-range \( \ell < 1000 \) for all source redshifts. The difference of a few per cent is due to the fact that the theory 3D matter density power spectrum is a few per cent lower than the measured data power spectrum. Calculating the highly non-linear power spectrum is currently not accurate to a few per cent, and many calculations of this theory curve do not agree with each other to within a few per cent. The Joachimi theory curve was the closest fit to the simulations and was used for all subsequent calculations. At low \( \ell \) the measured signal drops as we reach the size of the simulation box, while at high \( \ell \) the estimated mean shear power becomes shot-noise dominated before reaching the highest mode allowed by the resolution of the angular pixels beyond \( \ell = 1/\theta_{\text{pix}} \sim 10^4 \).

Before reaching pixel resolution, the measured shear power at high-\( \ell \) agrees well with the predicted shot-noise. This agreement suggests that the shot-noise model works reasonably well in this regime, even though the initial particle distribution is a glass, however we do not expect to be able to model the shot-noise to the per cent level accuracies required for future telescope missions (see Baugh, Gaztanaga & Efstathiou 1995, for a discussion). This suggests an improved (but not perfect) estimate of the mean power can be found by subtracting off the shot-noise contribution. As expected, the shot-noise subtracted shear power does not follow the ensemble-averaged theoretical power estimated from the theory code. It is likely that, as well as not modelling the shot-noise absolutely accurately, this is also a failure of the theoretical model for the matter density power spectrum – on small scales, the Smith et al. (2003) non-linear correction formula is known to underestimate the matter density power spectrum, \( P(k) \), by up to 10 per cent at wavenumbers of \( k < 1 \) and as great as 50 per cent at \( k = 10 \text{Mpc}^{-1} \) (Giocoli, private communication) and hence has been shown to underestimate the shear and convergence power spectrum by up to 30 per cent on scales of \( \ell < 10^4 \) (Hilbert et al. 2009).

As a test, we also calculated 2D shear power spectrum theory curves generated from a measurement of the 3D matter density power spectrum from these simulations. The resulting power spectrum was within 1 per cent of the theory calculation in the wavenumbers that were already being recovered well in the 2D power spectrum, which is a good validation of the theoretical prediction in this range. At higher wavenumbers, the spectra are affected significantly by shot-noise, which is problematic to model accurately (Baugh et al. 1995). For higher resolution simulations, using the numerical power spectrum may provide more accurate fits than the fitting formulae of Smith et al. (2003).

In subsequent analysis in this paper, we will restrict our analysis to the region of the measured power spectrum that agrees with the theoretical prediction.

Fig. 3 also shows the estimated B-mode power spectra. When galaxies trace the shear signal, we expect the B-mode power to pick up a shot-noise dependence. But here the shear signal is a pixelized field which would be continuous in the limit of infinite pixels or periodic boundary conditions. Therefore, we do not expect there to be a noise-generated B mode. However, B modes can still be generated due to leakage of power from the convergence field caused by the finite window function introduced by the small buffer we use when we generate the shear field from equation (10). As a consequence, the induced B mode has the shape of the shear power, but suppressed by around 3 orders of magnitude, showing that we can recover the B-mode signal but giving a signal small enough to not affect any measurements made in the rest of this work.

In this section we have shown that the SUNGLASS algorithm for calculating the shear and convergence maps and the power spectra

\(^8\)http://www.icosmo.org

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Figure 3. Simulated slices of the shear power spectra for $N$-body particle data at source redshifts of $z = 0.3, 0.6, 0.8, 1.0, 1.3$ and $1.5$. The smooth (red) line shows the theoretical predictions, the straight diagonal (dotted blue) line is the predicted shot-noise at each source redshift. The black points are the mean power spectrum of the simulated data for the 100 realizations with errors on the mean shown, and the (dot–dashed light blue) curve under these points is the simulation data with the shot-noise subtracted. The sub-shot-noise (dashed magenta) curve is the estimated induced B mode. The lower panel shows the fractional percentage difference between the simulated shear power and the theoretical prediction with upper black points representing the simulated data and the lower light blue points representing the shot-noise subtracted simulation data.
in redshift slices is accurate to a few per cent over a wide range of scales and redshifts. Wavenumbers up to 1500 can be recovered for the source redshifts \( z \geq 1.1 \) with this simulation resolution. For shot-noise subtracted power spectra, the recovered modes increase before the angular pixel resolution cuts off the power.

### 2.4 Mock 3D weak-lensing galaxy catalogues

Real, 3D weak-lensing data analysis is applied to a galaxy catalogue where galaxy angular positions and redshifts are added to estimated shears for each galaxy. For a 2D analysis, individual redshifts are ignored, and the theory uses only the redshift distribution. It is straightforward to generate a simple 3D mock weak-lensing galaxy catalogue with the information in the lightcones we have generated from the simulations. Shear and convergence maps are generated for each lensing source redshift and then each particle in the simulation is assigned a shear and convergence by interpolating between adjacent planes. The error introduced by linearly interpolating the shear and convergence between source redshift planes separated by \( \Delta z = 0.1 \) was estimated by comparing with much higher redshift-sampled planes and found to be substantially below the theoretical prediction \((\Delta [\ell(\ell+1)C_\gamma^\gamma/2\pi] < 10^{-5}\) except at angular wavenumbers where shot-noise becomes dominant. With the interpolated shear and convergence assigned to each particle, we now have a fully sampled 3D mock weak-lensing galaxy catalogue, which can be downsampled to generate realistic weak-lensing surveys.

To downsample the full 3D weak-lensing simulated lightcone to construct a realistic 3D weak-lensing galaxy catalogue, we use a galaxy redshift distribution (Efstathiou et al. 1991):

\[
n(z) \propto z^\alpha \exp\left[-\frac{z}{z_0}\beta\right],
\]

(18)

where \( z_0, \alpha \) and \( \beta \) set the depth, low-redshift slope and high-redshift cut-off for a given galaxy survey. We take \( \alpha = 2, \beta = 2 \) and \( z_0 = 0.78 \), yielding a median redshift of \( z_m = 0.82 \), similar to the Canada–France–Hawaii Telescope Legacy Survey (CFHTLS).

As the particles in our simulation are in comoving coordinates, we transform this redshift distribution to a probability distribution for the particle to enter our catalogue given its comoving radial distance:

\[
p(r) \propto r^\alpha \left(\frac{dr}{dz}\right) \exp\left[-\left(\frac{z(r)}{z_0}\right)^\beta\right],
\]

(19)

where

\[
\frac{dr}{dz} = \frac{c}{H(z)},
\]

(20)

and

\[
H(z) = \frac{H_0}{[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda]^{1/2}},
\]

(21)

where \( H_0 \) is the current Hubble value, \( \Omega_m \) is the current matter density, \( \Omega_k \) is the current dark energy density and \( \Omega_\Lambda \) is the curvature parameter. Throughout we have assumed a flat, \( \Omega_k = 0 \), cosmology for our simulations. We sample the particle distribution so our final galaxy catalogue has a surface density of around 15 galaxies per arcmin\(^2\), with a maximum redshift cut-off at \( z = 1.5 \).

The left-hand panel of Fig. 4 is an example of a redshift distribution taken from the full particle lightcone. The red line shows the theoretical distribution from equation (19), normalized to the number of particles selected, from which the simulation particles were drawn. The clustered nature of the particles in the distribution is apparent as the peaks and troughs around the theoretical curve can be seen.

Our 3D weak-lensing catalogue currently assumes that the redshift to each galaxy is accurately known. This would be appropriate for a spectroscopic redshift survey, but with such large surveys we can expect most weak-lensing catalogues will contain photometric redshift errors for each galaxy. To account for photometric redshift errors, we randomly sample the measured redshift from the true redshift using a Gaussian distribution with uncertainty

\[
\sigma_z = \sigma_0(1 + z),
\]

(22)

where \( z_0 \) is the true redshift of the particle. For the purposes of this work, we assume a fixed \( \sigma_0 = 0.05 \). The right-hand panel of Fig. 4 shows what the distribution on the left looks like with photometric redshift errors. The structures are smoothed out and the distribution becomes featureless. The photometric redshift errors were implemented by specifying a Gaussian error.

Fig. 5 shows the ensemble-averaged 2D shear power spectrum estimated from 100 mock weak-lensing surveys in the top panel (black line) with errors on the mean and compares the theoretical prediction (long smooth red line) and the ensemble-averaged B-mode power (dashed magenta line). The (blue dotted) diagonal line shows the shot-noise prediction for these galaxy redshift distributed lightcones. The shot-noise was determined by running the SUNGLASS analysis on a number of simulation box volumes filled with randomly distributed particles. The power spectrum of these lightcones represents the shot-noise estimate for the simulations and is a remarkably straight power law. The (dot–dashed light blue) curve between the shot-noise and the measured power spectrum is the shot-noise subtracted power spectrum. The bottom panel shows the fractional difference between the average of the mock surveys and the theory curve, with the error on the mean (upper, black) and the shot-noise subtracted points below (lower, light blue). This shows that the mock weak-lensing survey agrees with the theoretical expectation from wavenumbers of \( \ell = 200 \) to 2000, whereas the disagreement with theory can be ascribed to the uncertainty on the theory curve, and the rise of shot-noise. The shot-noise subtraction in this case is a few per cent lower than the theoretical prediction and the reason for this is not well understood and is the subject of ongoing investigation. The analyses in this paper will use the measured simulation power spectrum only. The B-mode power appears to follow a shot-noise profile which is consistent...
averaged power and its scatter measured. Here we want to use the mock surveys to test a maximum likelihood cosmological parameter estimation analysis, typically used to extract parameters from weak-lensing surveys. Here we try and recover the amplitude of the matter clustering, \( \sigma_8 \), and density parameter, \( \Omega_m \), from a 2D weak-lensing survey.

In Section 2.4 we showed that our simulations could produce unbiased estimates of the shear power from a mock survey over a range of \( \ell \) modes from 200 to 2000. For parameter estimation, we need to know the conditional probability distribution of shear power, \( p(\tilde{\gamma}^{\gamma\gamma}_\ell \mid \sigma_8, \Omega_m) \), for the likelihood function, where we have fixed all other parameters to their fiducial values. This is usually assumed to be Gaussian [although see Hartlap et al. (2009), who study non-Gaussian likelihoods]. Here we test this assumption on our mock catalogues. Fig. 6 shows the distribution of variations about the mean of the \( \tilde{C}_\ell \), \( \Delta \tilde{C}_\ell^{\gamma\gamma} \), divided by the ensemble-averaged scatter in the power, \( \sigma(\tilde{C}_\ell^{\gamma\gamma}) \). If the distribution is Gaussian, these distributions should all lie on the unit-variance Gaussian. The left-hand panel shows a histogram of the distribution of points for modes of \( \ell < 400 \) which is close to the linear region of the power spectrum. The middle panel shows the distribution of \( C^{\gamma\gamma}_\ell \) for modes of \( 400 < \ell < 1300 \) which represents the non-linear region of the power spectrum. The final panel shows the distribution for modes \( \ell > 1300 \) which is the shot-noise dominated regime. The smooth (red) line in each of the panels is a normalized unit-Gaussian curve. In each of the panels, the histogram of points is peaked slightly to the left of the Gaussian peak which indicates a slight non-Gaussianity in the distribution of points. This slight non-Gaussianity may bias the Gaussian likelihood analysis, but the dominant effect is currently the inaccurate fitting of the matter power spectrum by the Smith et al. (2003) formula at high \( k \) (Giocoli et al. 2010).

The cosmological parameters of the simulations were estimated using Gaussian likelihood analysis where the likelihood is given by

$$ L \left( \tilde{\gamma}^{\gamma\gamma}_\ell \mid \sigma_8, \Omega_m \right) = \frac{1}{(2\pi)^{N/2}(\det M_{\ell\ell})^{1/2}} \exp \left[ -\frac{\chi^2}{2} \right], \quad (23) $$

where

$$ \chi^2 = \sum_{\ell \ell'} \left( \tilde{C}_\ell^{\gamma\gamma} - \langle C^{\gamma\gamma}_\ell \rangle \right) M_{\ell\ell'}^{-1} \left( \tilde{C}_{\ell'}^{\gamma\gamma} - \langle C^{\gamma\gamma}_{\ell'} \rangle \right), \quad (24) $$

and \( M_{\ell\ell'} \) is the covariance matrix of the shear power spectra given by

$$ M_{\ell\ell'} = \langle \Delta C^{\gamma\gamma}_{\ell} \Delta C^{\gamma\gamma}_{\ell'} \rangle. \quad (25) $$

The inverse covariance matrix was determined by performing a singular value decomposition on the covariance matrix (Press et al. 1992). The resulting inverse covariance matrix is, however, biased due to noise in the covariance matrix, Hartlap et al. (2007) propose a correction for this bias by multiplying the inverse covariance matrix by an extra factor.
Figure 6. Histogram of the distribution of power spectra for the suite of lightcones with the \(n(z)\) particle distribution. The left-hand panel shows the distribution of the \(C_\ell^{\gamma\gamma}\)s less than \(\ell = 400\), the middle panel shows the \(C_\ell^{\gamma\gamma}\) distribution from 400 < \(\ell\) < 1300, and the right-hand panel shows the distribution from \(\ell > 1300\).

by a factor:

\[
\hat{M}_{\ell\ell'}^{-1} = \frac{N_S - N_p - 2}{N_S - 1} M_{\ell\ell'}^{-1},
\]

(26)

where \(N_S\) is the number of simulations used to determine the covariance matrix, \(N_p\) is the number of bins in the power spectrum and \(\hat{M}_{\ell\ell'}^{-1}\) is the unbiased covariance matrix.

The likelihood analysis relies on accurate estimation of the covariance matrix to show the degree of correlations. The correlation coefficients are

\[
r_{\ell\ell'} = \frac{M_{\ell\ell'}}{\sqrt{M_{\ell\ell} M_{\ell'\ell'}}} \]

(27)

The correlation coefficient matrix is equal to one along the diagonal and the off-diagonal components will show how correlated the \(\ell\) modes are, with numbers close to zero indicating low correlation and numbers close to (minus) one indicating high (anti-)correlation.

Fig. 7 shows the correlation coefficient matrix for the \(\ell\) modes being considered between 100 < \(\ell\) < 2500. The modes with a low correlation are represented in black and dark blues, and the modes with a high correlation shown in yellows and reds. This shows that the bandpowers at low \(\ell\) have very little correlation between them, as we would expect, since for an all-sky survey the linear power is uncorrelated. At higher \(\ell\) bandpower, the modes become more correlated, due to cross-talk between different scales due to non-linear clustering in the matter power spectrum. The variations in this coefficient matrix indicate an error of around 10 per cent which is suitable for the studies in this paper. This error can be reduced by introducing more realizations into the calculations. In our analysis we shall consider modes up to \(\ell = 1500\), where the correlation coefficient is around \(r_{\ell\ell'} \approx 0.6\).

Fig. 8 shows the \(\chi^2\)-distribution in the \(\sigma_8 - \Omega_m\) plane for our ensemble of simulations. These calculations assume that all other cosmological parameters are known. The black lines represent the \(\chi^2\) two-parameter, 1\(\sigma\), 2\(\sigma\) and 3\(\sigma\) (which should contain 68.3, 95.4 and 99.7 per cent of the points assuming a bivariate Gaussian distribution), contours of parameter space for the cosmological parameters. However, this clearly is not a bivariate Gaussian distribution. The contours shown are representative and come from the simulation that had the best-fitting parameters that were closest to the true input parameters (the point shown by the red polygon). The blue triangles represent the best-fitting points for each of the 100 realizations. With this distribution, 68 per cent of the points lie within the 1\(\sigma\) contour, 93 per cent within the 2\(\sigma\) contour and 97 per cent within the 3\(\sigma\) contour. The black diamond represents the best fit for the combined \(\chi^2\) estimate as discussed below.

The results from this analysis give us very encouraging results for the parameter estimation. Fig. 9 shows the results of combining the likelihoods for all 100 realizations, as if we have 100 independent 100 deg\(^2\) surveys. Even for this test we see the maximum likelihood recovered parameter values lie within the 1\(\sigma\) confidence contour. The marginalized error on the measured parameters for the combined 100 surveys is \(\Delta \Omega_m = 0.012\) and \(\Delta \sigma_8 = 0.022\). Any residual bias is below our uncertainty within the 1\(\sigma\) confidence limits. This implies that the inaccuracies in the simulations or theoretical predictions are smaller than the statistical fluctuations over all realizations.

Figure 7. Correlation coefficient matrix. This figure shows the correlation between the bandpower \(\ell\) modes in the covariance matrix. The higher \(\ell\) bandpowers are strongly correlated (shown in reds), while the lower bandpowers are only weakly correlated (shown in blues).
4 DISCUSSION AND CONCLUSIONS

This work introduces SUNGLASS, a new, rapid pipeline that generates cosmological N-body simulations with GADGET2. It computes weak-lensing effects along a lightcone using line-of-sight integrations with no radial binning and the Born approximation to determine the convergence and shear at multiple source redshifts. This information is interpolated back on to the particles in the lightcone to generate mock shear catalogues in 3D for testing weak-lensing observational analysis techniques.

In this work, SUNGLASS was used to generate 100 simulations with 512^3 particles, a box length of 512 $h^{-1}$ Mpc and a WMAP7 concordance cosmology. The corresponding mock shear catalogues were 100 deg^2 with a source redshift distribution with median $z_m = 0.82$ and 15 galaxies per arcmin^2. The parameters are easily changed within the SUNGLASS pipeline so that the mock shear catalogues match the survey of interest.

To show the reliability of the lightcones generated with SUNGLASS, E- and B-mode power spectra were shown at multiple source redshifts. The results show that at low redshifts, the signal becomes dominated by shot-noise at reasonably low $\ell$. With increasing source redshift, the power spectrum recovers the theoretical prediction over a wider range of modes, $\ell < 2500$.

Given that the measured power spectrum of the simulations appears to follow the predicted shot-noise at higher modes, the shot-noise was subtracted from the power spectra to increase the recovered range. The theoretical prediction is expected to underpredict the power spectrum around the turnover and consequently the simulations could be recovering the power spectrum up to around $\ell = 5 \times 10^4$ at the highest redshift planes.

The multiple source redshift plane shear and convergence was interpolated on to the particles in the lightcone to generate a mock shear catalogue. A redshift sampling was also imposed on the lightcone to mimic an observed shear catalogue. Binning this distribution too finely resulted in empty bins which had the effect of suppressing the power spectrum. This has implications for observations where the number of objects per arcmin^2 should be taken into account, as well as the density of the binning, when determining the accuracy of the power spectrum.

The mock shear catalogues were used to determine a covariance matrix which is essential for both parameter estimation and data analysis. A strength of SUNGLASS is the ability to rapidly produce Monte Carlo realizations of these catalogues, ensuring independent mock data sets for the generation of the covariance matrices.

The mock catalogues were also used to perform a simple parameter estimation using Gaussian likelihood analysis. The distribution of power spectra was shown to be reasonably Gaussian and the resulting parameter estimation contours for a single realization showed a good agreement with the input parameters within the two-parameter $1\sigma$, $2\sigma$ and $3\sigma$ error contours.

The combined likelihood from the 100 simulations shows narrow likelihood contours and accurate parameter recovery within the expected errors, and any residual bias is below our uncertainty within the $1\sigma$ confidence limits.

Current and future telescope surveys promise to provide an enormous amount of data for weak-lensing analysis. Weak lensing is still a young field and analysis techniques are still being developed. It is essential that the strengths and weaknesses of these techniques are fully understood before using them on real data with unknown parameters. Using the simulations, lightcones and mock shear catalogues provided by the SUNGLASS pipeline, demonstrated in this paper, is an excellent way to test these observational weak-lensing analysis techniques. The outputs of this pipeline have been rigorously tested and are well understood, making them ideal for generating covariance matrices that are critical to many observational analysis techniques.
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APPENDIX A: DERIVATION OF THE LINE-OF-SIGHT CONVERGENCE WITH NO RADIAL BINNING

This appendix shows how the line-of-sight convergence shown in equation (5) was derived.

We start with the general equation for the convergence:

$$\kappa = \int d^3r \cdot K(r, r) \delta(r),$$  \hspace{1cm} (A1)

where $r_1$ is the lensing source redshift, $\delta(r)$ is the fractional matter overdensity and $K(r, r_1)$ is the kernel:

$$K(r, r_1) = \frac{(r_k - r) r_k^3}{2c^2} \frac{3H_0^2 \Omega_m}{r_k a(r)}.$$  \hspace{1cm} (A2)

The overdensity $\delta(r)$ is given by

$$\delta(r) = \frac{n(r)}{\bar{n}(r)} - 1,$$  \hspace{1cm} (A3)

where $\bar{n}(r)$ is the average density at the comoving radial distance $r$ and is constant in comoving coordinates.

The particle number density, $n(r)$, is given by a sum of 3D delta functions:

$$n(r) = \sum_{i=\text{part}} \delta^3(r - r_i) = \sum_{i} \frac{\delta^3(r - r_i)}{r^2} \delta^2(\theta - \theta_i),$$  \hspace{1cm} (A4)

where ‘part’ are the particles in the pixel with $r_i \leq r$. Substituting this sum of delta functions into equation (A1) yields the average convergence per pixel on the sky, $\bar{\kappa}$, with no radial binning:

$$\bar{\kappa}_p = \frac{1}{\Delta \Omega_p} \int d^2\theta \kappa = \sum_{i} \frac{k(r_i, r_c)}{\Delta \Omega_p \bar{n}(r_i)} \int_0^{r_c} d r k(r, r_i),$$  \hspace{1cm} (A5)

where $\Delta \Omega_p = \Delta \theta_i \Delta \theta_i$.

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