Understanding the nuclear initial state with an electron ion collider

Tobias Toll
Brookhaven National Laboratory, Upton, NY, USA
E-mail: ttoll@bnl.gov

Abstract. In these proceedings I describe how a future electron-ion collider will allow us to directly measure the initial spatial distribution of gluons in heavy ions, as well as its variance (“lumpiness”) in exclusive diffraction. I show the feasibility of such a measurement by means of simulated data from the novel event generator Sartre.

1. Introduction
Measurements over the last decade in heavy ion collision experiments at RHIC indicate the formation of a strongly coupled plasma of quarks and gluons (sQGP). This sQGP appears to behave like a “near-perfect liquid” with a ratio of the shear viscosity to entropy density ($\eta/s$) approaching $1/4\pi$ [1, 2, 3, 4]. Despite the significant insight that the sQGP is a strongly correlated nearly perfect liquid, little is understood about how the system is created. The largest uncertainty in our understanding of the evolution of a heavy-ion collision comes from our limited knowledge of the initial conditions, such as the spatial distribution of gluons in the nuclei as well as how it fluctuates around its mean. For example, different assumptions about the nuclear initial distributions give differences up to factors of two for the obtained $\eta/s$ value [5, 6]. Measurements of the initial gluon distributions with existing machines is only possible indirectly and with large uncertainties. The study of diffractive events in electron-ion collisions would allow one to directly measure the initial condition of the colliding ions providing its spatial gluon distributions as well as the underlying fluctuations (“lumpiness”). In fact, exclusive diffractive $eA$ events are the only way to study the initial spatial distributions and shed light on these fundamental questions.

Much of the content in these proceedings are discussed in more detail in the EIC white paper [7] and in [8].

2. Diffraction in $eA$
When an electron and an ion collide, they do so electromagnetically via the exchange of a virtual photon, which interacts with a constituent parton in the ion. At a given collision energy $s$, such collisions are characterized by the virtuality of the photon $Q^2$ and the longitudinal momentum fraction of the participating parton $x$. In a diffractive $eA$ event, the electron collides with the ion leaving the nucleus intact, via exchange of a color neutral virtual pomeron as well as the virtual photon. This gives a couple of more characteristic kinematic variables, namely $t = (p - p')^2$, which is the square four-momentum difference between the incoming and outgoing ions, and $x_P$, which is the longitudinal momentum fraction of the ion carried by the pomeron. Experimentally,
these colorless exchanges produce a rapidity gap, i.e. an angular region in the detector without hadronic activity. The interaction with the nucleus is either elastic or inelastic, and in the latter case the nucleus subsequently radiates a photon or breaks up into color neutral fragments. When it is elastic, the event is called coherent and when it is inelastic, the event is called incoherent.

3. Imaging gluons in space

When light is scattered inelastically against a target, the scattered light shows a diffractive pattern with regions of more and less intense light. These regions are distributed in angle such that the angles of intensity minima $\theta_i \propto 1/kR$, where $k$ is the wave number and $R$ is the size of the target (see Fig. 1). There is thus direct information about the size of the target in the scattered light. Similarly, the spectrum of the elastic cross-section with respect to the hadronic momentum transfer $t$ in diffractive $eA$ events is related to the transverse spatial distribution of the gluons in the ion through a Fourier transform. According to the Good-Walker picture [9], the incoherent cross-section is a direct measure of the lumpiness of the gluons in the ion: $\frac{d\sigma}{dt} \propto \langle |A|^2 \rangle - |\langle A \rangle|^2$, where $A$ is the quantum mechanical amplitude of the event, and the average is taken over spatial distributions of gluons in the nucleus. Note that the second term on the R.H.S. is the coherent part of the cross-section and the first term is the total (incoherent and coherent) cross-section.

In order to access $t$ in these events, the complete final state has to be measured, which is in general experimentally impossible since some of the ion remnants will always be lost down the beam-pipe. It is only possible to measure $t$ in exclusive processes, such as vector-meson production or Deeply Virtual Compton Scattering (DVCS), where $t$ can be reconstructed by the four-momenta of the produced particle, as well as the scattered electron.

3.1. The dipole model and Sartre

A novel Monte Carlo event generator called Sartre [8] has been constructed to simulate exclusive vector meson and DVCS events. It is based on an extension of the dipole model bSat [10, 11] from $ep$ to $eA$. In the dipole model, the virtual photon splits up into a quark-antiquark pair – a color dipole. This dipole then interacts with the ion via one or many color-less two gluon exchanges. The dipole fragments into a vector meson, or annihilates into a real photon (DVCS). The bSat model contains an explicit dependence on the impact parameter $b$. The amplitude in bSat contains two main parts: 1. The wave-overlap, i.e. the overlap of the wave-functions of the incoming virtual photon and the outgoing vector meson. 2. The dipole cross-section, i.e. the hard interaction between the dipole and the ion. These parts are modelled in coordinate space,
and in order to express the amplitude in momentum space, which corresponds to what can be measured, they have to be Fourier transformed. The dipole cross-section for eA in bSat is:

\[
\frac{d\sigma^{(A)}_{qq}(x, r, b)}{d^2 b} = 2 \left[ 1 - \exp \left( -\frac{\pi^2}{2N_C} r^2 \alpha_s(\mu^2) x g(x, \mu^2) \sum_{i=1}^{A} T(|b - b_i|) \right) \right].
\]

where \( x g(x, \mu^2) \) is the DGLAP gluon density taken at a scale \( \mu^2 = 4/r^2 + \mu_0^2 \), where \( r \) is the dipole size, \( \mu_0^2 \) is the starting scale in the evolution, and \( N_C = 3 \) is the number of colours. The function \( T(b) \) is a Gaussian profile of a nucleon in impact parameter space, and \( b_i \) is the position of each nucleon. \( \sum_{i=1}^{A} T(|b - b_i|) \), is thus the sum of the contributions from all nucleons in the ion at a given impact parameter \( b \). The positions \( b_i \) are distributed according to the Woods-Saxon potential. Many of these configurations of nucleons are then averaged over for the first and second moments of the amplitude such that both the incoherent and coherent parts of the cross-section can be calculated.

There is also a linearized version of bSat, called bNonSat\cite{10} implemented in Sartre. The dipole cross-section in bNonSat is:

\[
\frac{d\sigma^{(A)}_{qq}}{d^2 b} = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) x g(x, \mu^2) \sum_{i=1}^{A} T(|b - b_i|)
\]

In bSat there are numerous pomerons exchanges between the dipole and the nucleus, which causes the cross-section to saturate for small \( x \) and large dipole sizes. In bNonSat there is only one pomeron exchanged, giving no such saturation.

The amplitude of the event is a Fourier transform from coordinate- to momentum space \( t \), of the convolution of the wave overlap and dipole cross-section. By squaring this amplitude one obtain the cross-section as a function of \( t \). By looking at its mean and variance with respect to the spatial gluon distribution we can access its coherent and incoherent parts.

### 3.2. Resulting cross-sections

In Fig. 2 the differential cross-section with respect to \( t \), \( d\sigma/dt \) is shown, for both \( J/\psi \)- and \( \phi \)-meson production, for both dipole models. The coherent cross-section clearly exhibits the typical diffractive pattern, and the incoherent cross-section, which is proportional to the lumpiness of the nucleus, is also shown.

![Figure 2](image-url) Differential distributions with respect to \( t \) for exclusive \( J/\psi \) (left) and \( \phi \) (right) for coherent and incoherent events. Both bSat and bNonSat models are shown.

Following [12], one can regain the impact-parameter distribution of gluons by performing a Fourier transform on the amplitude. The amplitude can be obtained by taking the square root...
of the cross-section, switching its sign in every second minimum. The Fourier transform of this modified amplitude \( \sqrt{d\sigma_{\text{coherent}}/dt}_{\text{mod}} \) is:

\[
F(b) = \frac{1}{2\pi} \int_0^\infty d\Delta \Delta J_0(\Delta b) \sqrt{d\sigma_{\text{coherent}}/dt}_{\text{mod}},
\]

where \( \Delta = \sqrt{-t} \) and \( J_0 \) is a Bessel function. The function \( F(b) \) is a function of impact-parameter only. For bNonSat, \( F(b) \) is directly proportional to the input Woods-Saxon density function, while for bSat the relation is more complex.

\[\text{Figure 3.}\] The Fourier transforms obtained from the distributions in Fig. 2 for \( J/\psi \)-mesons in (a) and (b) and \( \phi \)-mesons in (c) and (d). Both bSat and bNonSat results are shown with error bands (too small to be seen). The input Woods-Saxon distribution is shown as a reference.

Figure 3 shows the resulting Fourier transforms of the coherent curves in Fig. 2, using the range where \( |t| < 0.36 \text{ GeV}^2 \). The obtained distributions have been normalized to unity. As a reference the original input distribution is shown, which is the Woods-Saxon function normalized to unity. The bNonSat curves for \( \phi \)- and \( J/\psi \)-meson production reproduce the shape of the input distribution perfectly as is expected since the bNonSat amplitude is directly proportional to the input distribution. For bSat, the shape of the \( J/\psi \) curve also reproduces the input distribution, while the \( \phi \) curve does not. This is not surprising, as the size of the \( J/\psi \) meson’s wave function is much smaller than the \( \phi \) meson’s. Therefore, the \( J/\psi \) is better suited for probing the transverse structure of the nucleus. As a sign of the robustness of the method, the resulting curves include error-bands (too small to see) coming from the statistical uncertainties in Fig. 2, and they do not require a large range in \( t \); \( |t| < 0.2 \) gives a good convergence for the shape.

The results in Figure 3 provide a strong indication that the EIC and the LHeC will be able to obtain the nuclear spatial gluon distribution from the measured coherent \( t \)-spectrum from exclusive \( J/\psi \) production in \( eA \), in a model independent fashion.

4. References

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