Creation of the de sitter space from nothing in Eddington-inspired Born-Infeld theory

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Abstract. In this paper, we revisit Vilenkin’s ‘‘creation of universes from nothing’’ in the framework of the Eddington-inspired Born-Infeld (EiBI) theory of gravity. In Vilenkin solution, a de Sitter space universe is created by quantum tunneling process from nothing. We incorporate the EiBI theory into this model. We use two kinds of cosmological constants, one from the EiBI theory and another from the effective potential. The resulting action has a slight modification than the original one.

1. Introduction

A modified theory of gravity was proposed by Banados and Ferreira in Ref. [1]. This new modified theory of gravity, called the Edington-inspired Born-Infeld (EiBI) theory, is built based on the original Eddington’s proposal [2] by means of Born-Infeld-type of gravity [3]. This theory is constructed with Palatini formalism, where the metric and connection are independent with each other [4]. The formalism avoids various problems that plague the pure metric formalism, such as the ghost-like instabilities. With the absence of matter, the EiBI reduces to General Relativity. EiBI theory has been studied in wide variety of astrophysical and cosmological conditions, like in neutron stars [5], collapsing dust [6], black hole solutions [7, 8], and global monopole [9].

One major advantage of the EiBI theory is its ability to resolve the initial singularity of the universe [1]. If we consider the homogeneous and isotropic universe, we will find that there is a minimum length at early times. In classical sense, this describes a universe without an initial singularity.

Even though this result leads to a non-singular description of the universe, this does not necessarily mean that it does not have an origin point. In General Relativity, there are many proposals where the universe could be created by semi-classical method, such by quantum tunneling. One notable proposal is by Vilenkin [10, 11, 12]. In [10, 11] Vilenkin proposes that the universe is created from nothing to the de Sitter (dS) space by quantum tunneling using the Euclidean method. In the following paper [12], Vilenkin updates his result by proposing a different solution which uses Wheeler-DeWitt equations.

In this article, we would like to Vilenkin’s first proposal into the EiBI theory. We would like to show that such a Vilenkin’s proposal of creation from nothing using the Euclidean method is feasible within the EiBI framework. This result will further improve our understanding of the tunneling process using instanton in EiBI theory, thus this will help us when we want to explore
another tunneling proposal. Here we work with the reduced Planck units $8\pi G = 1$ and set the speed of light $c = 1$, with an exception for section 3.

2. The Eddington-inspired born-infeld theory

The action in EiBI theory of gravity is given by [1],

$$ S = \frac{1}{\kappa} \int d^4x \left( \sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}|} - \lambda \sqrt{|g|} \right) + S_M [g_{\mu\nu}, \Phi_M]. \tag{1} $$

$\lambda \equiv 1 + \kappa \Lambda$, where $\kappa$ is the so-called Eddington parameter and $\Lambda$ is the cosmological constant. $S_M$ is the matter Lagrangian as a function of the metric $g_{\mu\nu}$ and the field $\Phi_M$, not necessarily a scalar. The theory is constructed using the Palatini formalism, where the metric $g_{\mu\nu}$ and the connection $\Gamma$ are treated as independent variables. As the result, the Ricci tensor $R_{\mu\nu}$ is purely a function of $\Gamma$. Only the symmetric part of the Ricci tensor $R_{\mu\nu}$ is considered.

Varying the action with respect to $g_{\mu\nu}$ and $\Gamma$, we obtain the field equations,

$$ q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}, \tag{2} $$

$$ \sqrt{-q} q^{\mu\nu} = \lambda \sqrt{-g} g^{\mu\nu} - 8\pi \kappa \sqrt{-q} \Gamma^{\mu\nu}. \tag{3} $$

In the above equation $T_{\mu\nu}$ is defined as the energy-momentum tensor and $q_{\mu\nu}$ is an auxiliary metric. It should be noted that $q^{\mu\nu}$ is the inverse of $q_{\mu\nu}$, while raising and lowering index of $T_{\mu\nu}$ should be done with the physical metric $g_{\mu\nu}$.

The connection $\Gamma$ is defined as,

$$ \Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} q^{\mu\nu} (q_{\nu\alpha,\beta} + q_{\nu\beta,\alpha} - q_{\alpha\beta,\nu}). \tag{4} $$

From above equation it should be clear that the connection $\Gamma$ is not explicitly a function of the physical metric $g_{\mu\nu}$, but purely a function of the auxiliary metric $q_{\mu\nu}$.

3. Vilenkin’s creation of the universe from nothing proposal

In [10, 11], Vilenkin gives a cosmological model in which a de Sitter space universe is created by quantum tunneling process from nothing. In this model, we assume the universe starts in the symmetric vacuum state that has nonzero energy density $\rho_v$. The spacetime metric is given by

$$ ds^2 = dt^2 - a^2(t) d\Omega_3^2, \tag{5} $$

where $a(t)$ is the scale factor and $d\Omega_3^2$ is the metric on a three-sphere. Solution of $a(t)$ can be found from the evolution equation,

$$ a^2 + 1 = \frac{8}{3} \pi G \rho_v a^2, \tag{6} $$

where $\dot{a} = da/dt$. The solution of above equation gives us the de Sitter space,

$$ a(t) = H^{-1} \cosh(Ht), \tag{7} $$

where $H = (8\pi \rho_v/3)^{1/2}$.

This solution describes a universe which is contracting at $t < 0$ until it reaches minimum size ($a_{\text{min}} = H^{-1}$) at $t = 0$, and is expanding at $t > 0$. This scenario is very similar to the particle’s quantum tunneling scenario. Here we see $a$ as the particle coordinate, in which this particle
bounces at $a_{\text{min}}$. This gives us a hint that the creation of the universe (in this case, only the de Sitter space universe) might be created from quantum tunneling event.

To discuss this cosmic tunneling event, we move to the Euclidean framework by Wick-rotating the time coordinate, $t \rightarrow -i\tau$. As a result, Eq. (6) becomes $-\dot{a}^2 + 1 = H^2 a^2$. Now the dot indicates the derivative with respect to $\tau$. The solution of this equation is

$$a(\tau) = H^{-1} \cos(H\tau),$$

(8)

This solution also bounces at the classical turning point $a_{\text{min}}$. However, this solution did not approach any initial state at $\tau \rightarrow \pm\infty$, as now the solution is defined only for $|\tau| \leq \pi/2H$.

The tunneling process is illustrated in Fig. 1. The area below x-axis ($t < 0$) describes the universe before the nucleation. In this area, the universe is still 'living' in the Euclidean space. At $t = 0$, The universe undergoes a tunneling process from the Euclidean space to the Lorentzian dS space with $a = a_{\text{min}}$. Afterward, the universe continued to expand, illustrated by the area above x-axis ($t > 0$). We can interpret the tunneling process as a tunneling to dS from nothing, where 'nothing' means a state without classical Lorentzian spacetime.

We can further describe this tunneling process by calculate the action. The action of the de Sitter instanton is given by,

$$S_E = -\frac{1}{16\pi G} \int d^4x R\sqrt{g} + \int d^4x \sqrt{g} \left[ \frac{1}{2} (\partial_\mu \phi)^2 + V(\phi) \right],$$

(9)

Where $V(\phi) = \rho_v$. We assume a slowly changing field $\phi \approx \text{constant}$, thus removing the dynamics of $\phi$. Using the solution of $a$, we can find the curvature of the instanton solution $R = 12H^2$.

Under normal condition, we integrate the time from $-\infty$ to $\infty$. But we will only integrate from $-\pi/2H$ to $\pi/2H$, as the solution is only defined for that range. We find,

$$S_E = -\frac{3}{8G^2\rho_v},$$

(10)
We can use our previous analogy to interpret the quantity \( \exp(-S_E) \) to be something proportional to the tunneling probability. Then, if the potential has several maxima, most universes will be created at the maximum with the smallest value.

4. Creation of De Sitter Space from nothing in EiBI theory

We will use the result from [13], where they already calculate the Euclidean EiBI action for general case. We use the mini superspace metrics given by,

\[
\begin{align*}
ds_g^2 &= d\tau^2 + a(\tau)^2d\Omega_3, \\
ds_q^2 &= M(\tau)^2d\tau^2 + b(\tau)^2d\Omega_3,
\end{align*}
\]

(11)

(12)

\( a \) and \( b \) are the scale factors for \( g_{\mu\nu} \) and \( q_{\mu\nu} \) respectively, while \( M \) is the lapse function. With this metrics, the Euclidean EiBI action is,

\[
S_E = \frac{2\pi^2}{\kappa} \int d\tau a^3 \left( \lambda + \kappa V(\phi) + \frac{\kappa\dot{\phi}^2}{2} \right) \left( 1 - \frac{b^2}{a^2} \right),
\]

(13)

with the equations of motion,

\[
\begin{align*}
\ddot{b} &= \frac{b}{3\kappa} + \frac{\dot{b}M}{M} - \frac{bM^2}{3\kappa}, \\
\frac{\dot{b}^2}{M^2} &= 1 + \frac{a^2}{2\kappa} - \frac{b^2}{3\kappa} - \frac{b^2}{6\kappa M^2}, \\
\frac{b^3}{a^3M} &= \lambda + \kappa V(\phi) - \frac{\kappa\dot{\phi}^2}{2}, \\
\frac{bM}{a} &= \lambda + \kappa V(\phi) + \frac{\kappa\dot{\phi}^2}{2}, \\
0 &= \ddot{\phi} + \dot{\phi}^2 \frac{3a}{a} - \frac{dV(\phi)}{d\phi}.
\end{align*}
\]

(14)

(15)

(16)

(17)

(18)

These are the general results of [13] where we set \( N(\tau) = 1 \) (here \( N(\tau) \) is the lapse function for \( g_{\mu\nu} \)).

Before we proceed, we can see that there are two kinds of the cosmological constants in this action, one coming from the minima of the potential, \( V(\phi) \), and another one from the EiBI theory, embedded in \( \lambda = 1 + \kappa\Lambda \). We will use these two simultaneously. Now we follow Vilenkin’s approach as described in section 3. First, we set the scalar field \( \phi \) to a constant value, thus removing the dynamics of the scalar field, \( \dot{\phi} = 0 \).

From the equations of motion we obtain

\[
M^2 = \frac{b^2}{a^2} = \pm(\lambda + \kappa V(\phi)).
\]

(19)

Note that the last part of above equation is a constant, as we assume that \( \dot{\phi} = 0 \) and \( \lambda \) is not dependent to \( \tau \). The other equations of motion will become,

\[
\begin{align*}
\dot{a}^2 &= 1 + \frac{a^2}{3\kappa}(1 - (\lambda + \kappa V(\phi))), \\
\ddot{a} &= \frac{a^2}{3\kappa}(1 - (\lambda + \kappa V(\phi))).
\end{align*}
\]

(20)

(21)
The solution for these equations is,

\[ a = \sqrt{\frac{3}{V(\phi) + \Lambda}} \cos \left( \sqrt{\frac{V(\phi) + \Lambda}{3}} \tau \right). \]  

(22)

As in the Vilenkin proposal, this solution is defined only for \(|\tau| \leq (\pi/2)\sqrt{3/(V(\phi) + \Lambda)}\).

Now we can calculate the Euclidean EiBI action. We find,

\[
S_E = \frac{2\pi^2}{\kappa} \int d\tau a^3(\lambda + \kappa V(\phi)) \left( 1 - \frac{b^2}{a^2} \right),
\]

\[
= -2\pi^2(\lambda + \kappa V(\phi))(V(\phi) + \Lambda) \left( \frac{3}{V(\phi) + \Lambda} \right)^{3/2},
\]

\[
\times \int_{-\pi/2 \sqrt{V(\phi) + \Lambda}}^{\pi/2 \sqrt{V(\phi) + \Lambda}} d\tau \cos^3 \left( \sqrt{\frac{V(\phi) + \Lambda}{3}} \tau \right);
\]

\[
= -8\pi^2(\lambda + \kappa V(\phi)) \frac{3}{(V(\phi) + \Lambda)},
\]

\[
= -24\pi^2 \left( \frac{1}{(V(\phi) + \Lambda) + \kappa} \right). \tag{23}
\]

Note that we work with the reduced Planck units \(8\pi G = 1\). This result only differs from the result in the previous section by the extra \(\kappa\) term and the combined cosmological constant.

The value of this action could be changed by using a different value of \(\kappa\). By using a positive value of \(\kappa\), the action will be more negative than the general relativity counterpart. The opposite situation happens if we use a negative value of \(\kappa\), the action will be more positive. Also, the action could cross \(S_E = 0\) and change sign to a positive value for the bigger value of \((V(\phi) + \Lambda)\) when we use a negative value of \(\kappa\).

Another point here, as there are two types of cosmological constants we can choose to use only one of them by setting the other to zero. The resulting action would be same, but it can have a different initial condition. If we choose to set \(V(\phi) = 0\), the action will be independent of \(\phi\). Surprisingly, even if we remove the contribution from the scalar field and the potential, we still arrive at the same action. Hence, the resulting action comes purely from gravity.

In [1], Bañados and Ferreira found that for \(\kappa > 0\), the universe loiters for a very long time before it expands to the usual cosmological evolution. If we rewind back the time, the scale factor will need an infinitely long time before reaching the minimum length \(a_B\). This behavior describes an universe without an initial singularity, because there will be no initial point for the universe. But it should be noted that all calculation in [1] is done classically. The universe might be created by a semi-classical process. Let us assume that at the moment of creation, the universe tunneled directly to, or near, the minimum length, \(a_B \sim 10^{-32}(\kappa)^{1/4}a_0\), where \(a_0\) is the scale factor today. Taking the amplitude of (22) as \(a_B\), we can find the Euclidean action for tunneling to \(a_B\) as \(-24\pi^2 \left(10^{-64}\kappa^{1/2}a_0^3/3 + \kappa\right)\). This gives a possible explanation for a universe with non-singular beginning in EiBI theory.

5. Conclusion
In this article, we incorporate Vilenkin’s creation of the universes from nothing into EiBI theory. We show that such proposal is feasible within the EiBI framework; i.e., the universe governed by the EiBI gravity could have been created from nothing by means of quantum tunneling. We calculate the corresponding Euclidean action. If we use a positive value of Eddington parameter
\( \kappa \), the Euclidean action is decreased. Otherwise, it is increased. We also find that with EiBI theory, the Euclidean action can be achieved without the scalar field contribution, resulting in action that comes only from gravity.

In our future works, we would like to explore another Instanton solution and incorporate it into EiBI theory. Also we want to try the feasibility of Vilenkin’s another proposal [12] that uses Wheeler-DeWitt equations in EiBI theory.

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