A Stator Flux Calculation Method for Permanent Magnet Synchronous Motor in 60º Coordinate System

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Abstract. The direct torque control of permanent magnet synchronous motor is with a greater concern. The space vector modulation mode algorithm can greatly improve the torque ripple of traditional direct torque control. The 60º coordinate system used in the space vector pulse width modulation algorithm can make it simpler. The accurate observation of stator flux is the premise of direct torque control. A simple and effective method is to adopt the voltage model based on low-pass filter. In order to ensure the consistency of the algorithm, the form of calculating the stator flux based on low pass filter is deduced in 60º coordinate system.

1. INTRODUCTION

Permanent magnet synchronous motor direct torque control (PMSM-DTC) has received more and more attention [1-4]. The traditional direct torque control has simple structure, but with the drawbacks of large torque ripple and changing switching frequency. To solve the problems, the SVPWM mode DTC (SVM-DTC) structure is a better choice [5-7]. But the SVM-DTC system becomes more complicated. The authors previously studied SVPWM algorithm and its application in PMSM-DTC in 60º coordinate system. The 60º coordinate system is very suitable for SVPWM algorithm due to its characteristics. And then the system is simplified to some extent [8].

In order to ensure the consistency of the algorithm and in-depth study of 60º coordinate system application in DTC, this paper will analyze the representation of a PMSM stator flux observation method in 60º coordinate system.

2. 60º COORDINATE SYSTEM AND COORDINATE TRANSFORMATION

The 60º coordinate system is called gh coordinate system and its relationship with the two-phase stationary vertical αβ coordinate system is shown as below.

![Figure 1. αβ coordinate system and 60º coordinate system](image-url)
The following expressions can be obtained from figure 1.
\[
\begin{align*}
V_m &= V_{rg} + V_{rh} \cos 60^\circ \\
V_{\phi} &= V_{rh} \sin 60^\circ
\end{align*}
\]
(1)

That is
\[
\begin{align*}
V_m &= V_{rg} + \frac{1}{2} V_{rh} \\
V_{\phi} &= \frac{\sqrt{3}}{2} V_{rh}
\end{align*}
\]
(2)

Equation (2) can be changed as
\[
\begin{align*}
V_{rg} &= V_m - \frac{1}{\sqrt{3}} V_{\phi} \\
V_{rh} &= \frac{2}{\sqrt{3}} V_{\phi}
\end{align*}
\]
(3)

Equation (3) is written in matrix form as
\[
\begin{bmatrix}
V_{rg} \\
V_{rh}
\end{bmatrix} =
\begin{bmatrix}
1 & -\frac{1}{\sqrt{3}} \\
0 & \frac{2}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
V_m \\
V_{\phi}
\end{bmatrix} = C \cdot \begin{bmatrix}
V_m \\
V_{\phi}
\end{bmatrix}
\]
(4)

where \(C\) is the transformation matrix from the \(a\beta\) coordinate system to the \(gh\) coordinate system. If the three-phase coordinate system is transformed directly to the \(gh\) coordinate system and named the transformation matrix as \(D\), the following equation could be gotten:
\[
\begin{bmatrix}
V_{rg} \\
V_{rh}
\end{bmatrix} = D \cdot \begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} = C \cdot C_{32} \begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
\]
(5)

where \(C_{32}\) is the transformation matrix from the three-phase coordinate system to the \(a\beta\) coordinate system. That is
\[
C_{32} = \frac{2}{3}
\begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1
\end{bmatrix}
\]
(6)

3. THE OBSERVATION METHOD OF STA-TOR FLUX BASED ON LOW-PASS FILTER IN 60° COORDINATE SYSTEM

3.1 Voltage model stator flux observation

There are two commonly used stator flux observation methods for PMSM which named “voltage model” and “current model” respectively. The parameters of the motor are more in the current model, and the rotor position information is needed in order to make the coordinate transformation. Compared
with the current model stator flux observation method, the voltage model involves less motor parameters, which is done in stationary coordinate system and does not need to do coordinate transformation. The voltage model is as

\[
\begin{align*}
\psi_\alpha &= \int (U_\alpha - I_\alpha R) dt \\
\psi_\beta &= \int (U_\beta - I_\beta R) dt
\end{align*}
\]  

(7)

where \(\psi_\alpha\) and \(\psi_\beta\) are the stator flux components, \(U_\alpha\) and \(U_\beta\) are the stator voltage components, \(I_\alpha\) and \(I_\beta\) are the stator current components, \(R\) is the stator resistance.

### 3.2 Voltage model stator flux observation based on low-pass filter

Because of the initial value problem and the error accumulation problem, the pure integral voltage model could not be used in practical applications. A simple and effective solution is to replace the integrator with a low-pass filter (LPF) in the voltage model [9]. The transfer function of low-pass filter \(G_{LPF}\) is as

\[
G_{LPF} = \frac{1}{s + \omega_c}
\]  

(8)

where the \(\omega_c\) is the cut-off frequency of the LPF. The stator flux observer based on LPF is shown as the following figure.

![Figure 2. Stator flux observer based on low-pass filter](image)

3.3 The compensation for low-pass filter

The LPF can solve the problem of pure integrator in the practical application, but it also leads to amplitude and phase errors of stator flux observation with the actual flux. The errors will affect the performance of the motor. It is needed to compensate for the LPF in order to make the observed flux is consistent with the actual value. There are many kinds of compensation methods, and the idea is to make the stator flux observation results after compensation are consistent with the results of pure integration. A compensation method is proposed to process the input signal of the LPF in literature [10]. The results are as follows:

\[
\begin{align*}
E_\alpha' &= E_\alpha + \frac{\omega_c}{\omega} E_\beta' \\
E_\beta' &= E_\beta - \frac{\omega_c}{\omega} E_\alpha'
\end{align*}
\]  

(9)

where the \(E_\alpha'\) and \(E_\beta'\) are the input components of the LPF without compensation processing which equal the stator voltage component \(U_{sa}\) and \(U_{sb}\) minus the value of the stator resistance voltage drop respectively. The \(E_\alpha\) and \(E_\beta\) are the modified input components of the LPF with compensation. The stator flux is calculated as

\[
\begin{align*}
\psi_\alpha &= \left[(U_\alpha - I_\alpha R) + \frac{\omega_c}{\omega} (U_\beta - I_\beta R)\right] \cdot \frac{1}{s + \omega_c} \\
\psi_\beta &= \left[(U_\beta - I_\beta R) - \frac{\omega_c}{\omega} (U_\alpha - I_\alpha R)\right] \cdot \frac{1}{s + \omega_c}
\end{align*}
\]  

(10)

It will cause the stator flux with compensation to be too large at the beginning of motor starting due to the small angular velocity. It is needed to make amplitude limiting. It is often to make \(|\omega_c/\omega|<3\) in the above equations.
### 3.4 Representation of the stator flux observation in 60° coordinate system

First, the voltage and current components in the three-phase coordinate system are transformed to the expressions in 60° coordinate system according to the transformation rules. Then the flux expressions will be changed to the representation in 60° coordinate system. According to (3), (10) will be changed as (11).

\[
\begin{align*}
\psi_g + \frac{1}{2} \psi_h &= \left( \left( U_g + \frac{1}{2} U_h \right) - \left( I_g + \frac{1}{2} I_h \right) R \right) + \frac{\omega}{\omega} \left( \frac{\sqrt{3}}{2} U_h - \frac{\sqrt{3}}{2} I_h R \right) \cdot \frac{1}{s + \omega_c} \\
\frac{\sqrt{3}}{2} \psi_h &= \left( \frac{\sqrt{3}}{2} U_h - \frac{\sqrt{3}}{2} I_h R \right) - \frac{\omega}{\omega} \left( U_g + \frac{1}{2} U_h \right) - \left( I_g + \frac{1}{2} I_h \right) R \cdot \frac{1}{s + \omega_c}
\end{align*}
\]

(11)

Then the stator flux components in 60° coordinate system are as (12).

\[
\begin{align*}
\psi_g &= \left( U_g - I_g R \right) + \frac{\omega}{\omega} \left( \frac{1}{\sqrt{3}} (U_g - I_g R) + \frac{2}{\sqrt{3}} (U_h - I_h R) \right) \cdot \frac{1}{s + \omega_c} \\
\psi_h &= \left( U_h - I_h R \right) - \frac{\omega}{\omega} \left( \frac{2}{\sqrt{3}} (U_g - I_g R) + \frac{1}{\sqrt{3}} (U_h - I_h R) \right) \cdot \frac{1}{s + \omega_c}
\end{align*}
\]

(12)

### 4. SIMULATION RESULTS

The stator flux observation method in 60° coordinate system is verified by simulation based on MATLAB. The parameters of the PMSM and control in the simulation model are shown in Table 1.

| Parameter                     | Value       |
|-------------------------------|-------------|
| DC-bus voltage                | $U_{dc}$    | 200V        |
| Reference stator flux         | $\psi_{ref}$ | 0.12Wb      |
| Permanent magnet flux         | $\psi_f$    | 0.1 Wb      |
| Number of pole pairs          | $p$         | 4           |
| Stator resistance             | $R$         | 0.7 Ω       |
| d-axis inductance             | $L_d$       | 1.5mH       |
| q-axis inductance             | $L_q$       | 1.5mH       |
| Sampling period               | $T_s$       | 100 μs      |

Table 1. Parameters of the PMSM and Control.

The simulation results are as follows.
Figure 3. Stator flux based on LPF without compensation and pure integration

Figure 4. Stator flux based on LPF with compensation and pure integration

Figure 5. Stator flux in 60° coordinate system

The curves are the $\alpha$-components of stator flux in Fig.3 in which the Curve 1 is based on LPF without compensation and the Curve 2 is based on pure integration. It can be seen there are amplitude and phase errors between the outputs of the two kinds of stator flux observations. The errors disappear in Fig.4 in which the LPF is with compensation. Because of the incomplete compensation, the two curves are not consistent in the beginning cycles due to the limit value of $|\omega_c/\omega|$ in (9). The $\beta$-components of stator flux are with the same conclusions as in Fig.3 and Fig.4.

The waveforms in Fig.5 are the $g$-components of stator flux, the above one is the changed result from the $\alpha$-component and $\beta$-component of stator flux, the nether one is calculated according to (12). The two curves are completely consistent.

5. CONCLUSION
In order to apply the 60° coordinate system in the PMSM-DTC, this paper analyzes the representation of voltage model flux observation method based on low-pass filter in 60° coordinate system. The correctness of stator flux observation method is illustrated through theoretical analysis and simulation.
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