The GUP effect on tunnelling of massive vector bosons from the 2+1 dimensional black hole

Ganim Gecim, Yusuf Sucu

1Department of Physics, Faculty of Science, Akdeniz University, 07058 Antalya, Turkey

Abstract In this study, we investigate the Generalized Uncertainty Principle (GUP) effect on the Hawking radiation from 2+1 dimensional New-type Black Hole by using the quantum tunnelling properties of a massive spin-1 particle, i.e. a massive vector boson. In this connection, we use the modified massive vector boson equation based on the GUP. Afterward, using Kerner-Mann’s quantum tunneling method, we calculate the tunnelling probabilities of the massive spin-1 vector particle and subsequently the Hawking temperature of the black hole. Then, due to the GUP effect, we observe that the Hawking temperature of the black hole depends on not only the black hole properties, but also the AdS radius defined as $L^2=\frac{1}{2\pi^2}m^2=\frac{1}{2\pi^2}\Lambda$ in terms of the $m$ graviton mass (or the $\Lambda$ cosmological constant) and the properties of the tunnelling massive vector boson, such as energy, mass, charge and total angular momentum etc. According to relation between these properties of the vector boson, we see that the Hawking temperature increase by the total angular momentum of the tunneled particle while it decreases by the energy and mass of the tunneled particle and the graviton mass.

On the point of this result, the radiation stemmed from the tunneled particle’s mass and energy and the graviton mass screens the black hole radiation.

1 Introduction

Classically, black holes are considered as a spacetime region where the gravitational field is so strong that does not emit any radiation. However, this consideration has changed under the quantum mechanical approach [1-3]. Meanwhile, by using the formulation of quantum field theory on curved spacetime, Hawking proved that a black hole emits thermal radiation, well known the Hawking radiation in the literature. Furthermore, the Hawking’s discovery has established a connection between general relativity, quantum field theory and thermodynamics. Hence, this discovery is very important to understand the black hole physics and the quantum gravity [8, 10]. Therefore, to compute the Hawking radiation, it has been developed many different methods. One of these methods is the Hamilton-Jacobi method based on the tunnelling probabilities for the classically forbidden trajectory from inside to outside of an horizon. The tunnelling probability, $\Gamma$, is related to the classical action of the particle, $S$, as $\Gamma = e^{-\frac{2\bar{\hbar}ImS}{S}}$. In this connection, the Hawking radiation as a quantum tunnelling process of a point elementary particle, such as spin-0 scalar particle described by Klein-Gordon equation, spin-1/2 Dirac particle described by Dirac equation [11-27] and spin-1 particle described by different equations: classically Proca equation [28-37] and, quantum mechanically, massive vector boson equation [38] that is derived by quantizing the classical Zitterbewegung model [39], was studied. In all of these studies, we see that Hawking radiation of a black hole was completely independent of the internal properties of a tunneled point particle, such as mass, angular (orbital+spin) momentum, energy, charge etc.

Alternative approaches related to quantum gravity predict the presence of a minimal observable length in Planck scale [40-47]. The existence of a minimal length leads to the generalized uncertainty principle (GUP) [41, 48, 49]. Given the GUP, the Hawking radiation of a black hole is related to the internal properties of a tunneled particle. This effect is known as "Quantum Gravity Correction" in the literature [50-53]. To include the quantum gravity effect in the Hawking radiation, the Klein-Gordon equation for spin-0 particle, Dirac equation for spin-1/2 particle, and massive $W^\pm$-boson equations derived from the Lagrangian given by the Glashow-Weinberg-Salam model for spin-1 particle are...
modified in the GUP framework [54–56]. Using these modified relativistic wave equations, the modified Hawking temperature of various black holes is calculated by quantum tunnelling process of the spinning particles. The calculations show that the quantum gravity correction term is related not only to the black hole’s properties but also to the tunnelled particle’s properties, such as mass, angular (orbital+spin) momentum, energy, charge etc. [54–68]. Hence, it is evident that the studies of the Hawking radiation by quantum tunnelling process of the relativistic quantum mechanical particles with various spin have an important situation in the context of the quantization of black hole. In this motivation, using the GUP relations, we will modified the massive vector boson equation which recently proposed to describe the behaviour of the spin-1 particle in 2 + 1 dimensional quantum electrodynamics (QED_{2+1}) [39].

In the context of the quantization of gravity, recently, many important studies have been carried out by focusing on 2 + 1 dimensional theories as a toy model [69–73]. Especially, the 2+1-dimensional massive gravity provides an active research area, both mathematically and physically, among these theories. For example, the New Massive Gravity is one of them [74]. In this theory, the graviton, which is the quantum particle of gravity in the standard model, gains a mass, topologically [74–79]. Furthermore, the theory has a black hole solution, for example, the new-type black hole [80]. And, the Hawking radiation of the black hole was discussed by using quantum tunneling process of point particles, such as massive spin-0, spin-1/2 and spin-1 particle [26, 27, 38]. All the point particles are tunnelled in the same way from the black hole because the Hawking radiation is only dependent of the black hole properties. To investigate the effect of the tunnelleing particle properties on the Hawking radiation, in this study, we aim to work out the GUP effect on the Hawking radiation of the 2+1 dimensional New-type black hole as a quantum tunnelling process of a massive spin-1 particle described by the 2 + 1 dimensional modified massive vector boson wave equation.

This paper is organized as follows. In the following section, we will modified the relativistic quantum mechanical vector boson equation under the GUP. After that, using Kerner-Mann’s quantum tunneling method, we calculate the Hawking temperature as a quantum tunnelning process of the massive vector boson from the New-type black hole with the GUP effect. In section 4, we summarize the results.

## 2 Modified Vector Boson Equation

The covariant form of the massive vector boson (massive spin-1 relativistic quantum particle) equation in a curved spacetime can be written as [39]

\[
iβ^μ(x)\nabla_μ Ψ(x) = \frac{m_0}{h}Ψ(x)
\]

(1)

where the \(m_0\) is mass of the vector boson and the \(β^μ(x)\),

\[
β^μ(x) = \nabla^μ(x) ⊗ I + I ⊗ \nabla^μ(x),
\]

(2)

are the 2+1 dimensional spacetime-dependent Kemmer matrices described by the 2+1 dimensional spacetime-dependent Dirac matrices as \(\nabla^μ(x) = (σ^1(x), iσ^1(x), iσ^2(x))\), on the other hand, they are related to the flat spacetime Kemmer matrices, \(β^{(i)}\), as \(β^μ(x) = e^{μ}_{(i)}β^{(i)}\), the covariant derivative, \(ν_{,μ}\), is defined by means of the spin connection coefficients for the spin-1 particle, \(Σ_{μ(ν)}\), as \(ν_{,μ} = Σ_{μ(ν)} + Σ_{(μ}ν_{,ν)}\), and \(Σ_{(μ}ν_{,ν)}\) can be written in terms of the spin-1/2 particle connections, \(Γ^μ_{(ν)}\),

\[
Σ_{μ(ν)} = Γ^μ_{(ν)} ⊗ I + I ⊗ Γ^μ_{(ν)}
\]

(3)

and the \(Γ^μ_{(ν)}\) is defined as [81]

\[
Γ^μ_{(ν)} = \frac{1}{4}\epsilon_{(ν,μ}e^{α}_{μ)}(Γ^α_{νμ} + Σ_{νμ})\lambda^ν(μ),
\]

(4)

where the \(e^{μ}_{(i)}\) are triads, the \(Γ^α_{νμ}\) are Christoffel symbols and \(Δ^ν(μ)\) is spin operator of the spin-1/2 particle and it is given by

\[
Δ^ν(μ) = \frac{1}{2}[[\nabla^μ(x), Ψ(x)],
\]

(5)

and, also, \(Ψ(x)\) is a wave function of the spin-1 particle and it has symmetric representation as \(Ψ(x) = (Ψ^+, Ψ^0, Ψ^-)^T\) [39] because of the quantization [82–87]. Also, in the vector boson equation, \(h\) is defined as \(h = h/2π\) in terms of the Planck constant, \(h\), and the speed of light in vacuum, \(c\), is taking as \(c = 1\) in throughout of the paper. In the rest frame, this equation has the particle, with the spin up state with positive energy, and antiparticle, with the spin down state with negative energy, solutions and is compatible with the Proca equation in the classical limit. This equation is directly derived as an excited state of the classical zitterbewegung model [39], which the symmetry and integrability properties of this model was discussed in 2+1 dimensions [88]. It is analogues in the 2+1 dimensional spacetimes that of the spin-1 dimensional spin-1 sector of the Duffin-Kemmer-Petiau (DKP) equation [82–87]. As discussing the Hawking radiation of the 2+1 dimensional New-type and Warped-AdS_{3} black holes by the quantum tunneling method of the massive vector boson as a point particle described by the vector boson equation [11, 38], we see that the spin-1 point particle tunnel as that of the scalar and Dirac point particles [26, 27].

On the other hand, to investigate the quantum gravity effect on the tunneling process of the massive vector boson, the massive vector boson wave equation must be rewritten in the framework of the GUP. Using the fact that in the existence of a minimal length at Planck scale, the standard Heisenberg uncertainty principle can be modified as follows [89–92].

\[
ΔxΔp ≥ \frac{h}{2}
\]

(6)
where $\alpha = \alpha_0/M_p^2$, the $M_p^2$ is the Planck mass and $\alpha_0$ is the dimensionless parameter. The Eq. (6) can be derived by using the modified commutation relation given as follows \cite{89,92}:

$$[x_i, p_j] = i\hbar \delta_{ij} \left[1 + \alpha p_0^2 \right],$$

where $x_i$ and $p_j$ are the modified position and the modified momentum operators defined by the standard position, $x_0$, and the standard momentum, $p_0 = -i\hbar \partial_j$, satisfying the standard commutation relation \([x_0, p_0] = i\hbar \delta_{ij}\), as,

$$x_i = x_0,$$
$$p_i = p_0 (1 + \alpha p_0^2),$$

respectively, and $p_0^2 = \Sigma p_0 p_0$ \cite{63,90,91}.

The modified energy relation is given by the following form \cite{55,93}:

$$\tilde{E} = E \left[1 - \alpha E^2 \right] = E \left[1 - \alpha \left( p^2 + m_0^2 \right) \right],$$

where the energy mass shell condition, $E^2 = p^2 + m_0^2$, is used. The square of the momentum operator can be expressed after neglecting the higher order terms of the $\alpha$ parameter as follows \cite{92}:

$$p^2 = p_0 p' \simeq -\hbar^2 \left[ \partial_i \partial^j - 2 \alpha (\partial_i \partial^j) (\partial'_j \partial^i) \right].$$

Hence, Using the Eq (8), Eq (9) and Eq (10), the modified massive vector boson equation is written as follows;

$$\left(i \beta^i (x) \partial_i - i \beta^j (x) \Sigma_{\mu} - \frac{m_0}{\hbar} \right) \left(1 + \alpha \hbar^2 \partial_i \partial^j - \alpha m_0^2 \right) \Psi$$
$$+ \beta^0 (x) \partial_0 \Psi = 0,$$

or its explicit form:

$$i \beta^0 \partial_0 \tilde{\Psi} + \left[i \beta^i (1 - \alpha m_0^2) \partial_i + i \alpha \hbar^2 \beta^j \partial_i (\partial'_j \partial^i) \right] \tilde{\Psi}$$
$$- \frac{m_0}{\hbar} \left(1 + \alpha \hbar^2 \partial_i \partial^j - \alpha m_0^2 \right) \Psi$$
$$- i \beta^0 \sum \left(1 + \alpha \hbar^2 \partial_i \partial^j - \alpha m_0^2 \right) \Psi = 0,$$

where the $\tilde{\Psi}$ is the modified wave function of a boson.

### 3 Vector boson particle tunnelling from New-type black hole

The New-type black hole is described as \cite{26,80}:

$$ds^2 = L^2 \left[ f(r) dr^2 - \frac{1}{f(r)} r^2 d\Omega^2 \right],$$

where $f(r) = r^2 + br + c$ with two constant parameters $b$ and $c$. The AdS$_3$ radius, $L$, is defined as $L^2 = \frac{1}{2m_0^2} = \frac{1}{8\pi G}$ in terms of the $m$ graviton mass or the $\Lambda$ cosmological constant. And, it has an outer and inner horizon at $r = \pm \frac{1}{b} \left( -b \pm \sqrt{b^2 + 4c} \right)$, respectively. Furthermore, the New-type black hole has six different types according to the signatures of the parameters $b$ and $c$ \cite{80} and the each type black hole has different mathematical and physical properties. For example, in case $b^2 = 4c$, the black hole becomes extremal. On the other hand, in the case $b = 0$ and $c < 0$, it is reduced to the non-rotating BTZ black hole \cite{80}.

To calculate the quantum gravity effect on the Hawking temperature of the black hole, firstly, we write the modified massive vector boson equation in the New-type black hole background by using the spin-$1$ matrices and spin connection coefficients calculated in \cite{80}, and secondly, we use the following ansatz for the modified wave function to compute the tunnelling probability of the vector boson \cite{39}.

$$\tilde{\Psi}(x) = \exp \left( \frac{i}{\hbar} S(t, r, \phi) \right) \begin{pmatrix} A(t, r, \phi) \\ B(t, r, \phi) \\ B(t, r, \phi) \\ D(t, r, \phi) \end{pmatrix}$$

where $A(t, r, \phi), B(t, r, \phi)$ and $D(t, r, \phi)$ are functions of the spacetime coordinates and $S(t, r, \phi)$ is the classical action function of the particle trajectory. Then, the modified massive vector boson equation can be reduced to the three coupled differential equations after neglecting the terms with $h$:

$$B \left[ \frac{L^2}{r} \frac{\partial S}{\partial \phi} + i \alpha \frac{f}{r} \left( \frac{\partial S}{\partial \phi} \right)^2 \frac{\partial^2 S}{2 \partial r} \right]$$
$$+ B \left[ \alpha L^2 m_0 f \frac{\partial S}{\partial r} - L^2 \sqrt{\frac{\partial S}{r}} - i \alpha \frac{m_0^2 L^2}{r} \frac{\partial S}{\partial \phi} \right]$$
$$+ B \left[ i \alpha \frac{f}{r} \left( \frac{\partial S}{\partial \phi} \right)^2 - \alpha \frac{m_0^2 L^2}{r} \frac{\partial S}{\partial \phi} \right]$$
$$+ A \left[ i \alpha \frac{Lm_0}{2r^2} \left( \frac{\partial S}{\partial \phi} \right)^2 + i \alpha \frac{Lm_0 f}{2} \left( \frac{\partial S}{\partial \phi} \right)^2 \right]$$
$$= 0,$$

$$A \left[ -L^2 \frac{\partial S}{\partial r} - i \alpha \frac{f}{r} \left( \frac{\partial S}{\partial \phi} \right) \frac{\partial S}{\partial r} \right]$$
$$+ A \left[ - \alpha \frac{\sqrt{f}}{r} \left( \frac{\partial S}{\partial \phi} \right)^2 - \alpha \frac{m_0^2 L^2}{r} \frac{\partial S}{\partial \phi} \right]$$
$$+ A \left[ - \alpha \frac{\sqrt{f}}{r} \left( \frac{\partial S}{\partial \phi} \right)^2 + \alpha \frac{m_0^2 L^2}{r} \frac{\partial S}{\partial \phi} \right]$$
$$+ D \left[ i \frac{L^2}{r} \frac{\partial S}{\partial \phi} - i \alpha \frac{m_0 L^2}{r} \frac{\partial S}{\partial \phi} + i \alpha \frac{f}{r} \left( \frac{\partial S}{\partial \phi} \right) \frac{\partial S}{\partial \phi} \right]$$
$$+ D \left[ -L^2 \frac{\partial S}{\partial r} - \alpha \frac{\sqrt{f}}{r} \left( \frac{\partial S}{\partial \phi} \right)^2 + \alpha \frac{m_0^2 L^2}{r} \frac{\partial S}{\partial \phi} \right]$$
$$+ D \left[ i \alpha \frac{1}{r} \left( \frac{\partial S}{\partial \phi} \right)^2 - \alpha \frac{\sqrt{f}}{r} \left( \frac{\partial S}{\partial \phi} \right)^2 \frac{\partial S}{\partial r} \right]$$
$$= 0.$$
where the condition, we get the Hamilton-Jacobi equation as Eq. (16) and the outgoing and incoming particles on the outer horizon, respectively. Using a contour that has a semicircle around the pole at the outer horizon, the integration in Eq. (15) are calculated as
\[ K_{\pm}(r) = \pm i \frac{\pi E}{(r_+ - r_-)} \left[ 1 + \alpha \Sigma \right]. \]

where \( \Sigma \) is
\[ \Sigma = \frac{(r_+ - r_-)^2 (9L^2m_0^2r_+^2 - 4f^2) + 16Ef^2r_+^2}{8L^2r_+^2 (r_+ - r_-)^2}. \]

On the other hand, the tunneling probabilities of particles crossing the outer horizon are given by
\[ P_{out} = \exp \left[ - \frac{2}{\hbar} \Im K_{+}(r) \right], \]
\[ P_{in} = \exp \left[ - \frac{2}{\hbar} \Im K_{-}(r) \right], \]
respectively. Furthermore, the tunneling probability for the classically forbidden trajectory from inside to outside of the black hole horizon is given by [24]
\[ \Gamma = e^{-\frac{2}{\hbar} \Im S_{(r, \phi)}}, \]

Moreover, the total imaginary part of the action is \( \Im S_{(r, \phi)} = \Im K_{+}(r) = \Im K_{+}(r) - \Im K_{-}(r) \) [93, 94]. Hence, using the fact that \( \Im K_{+}(r) = -\Im K_{-}(r) \), the probability of the vector boson tunneling from inside to outside of the outer event horizon of the black hole is calculated as
\[ \Gamma = e^{-\frac{2}{\hbar} \Im S_{(r, \phi)}} = \frac{P_{out}}{P_{in}} = \exp \left[ - \frac{4}{\hbar} \Im K_{+}(r) \right]. \]

If one expands the classical action in terms of the particle energy, then the modified Hawking temperature is obtained at the linear order. Thus, we can write the probability as
\[ \Gamma = e^{-\frac{2}{\hbar} \Im S_{(r, \phi)}} = e^{-\beta E}, \]
where \( \beta \) is the inverse temperature of the outer horizon. According to this, the modified Hawking temperature becomes
\[ T_{H} = \hbar \frac{(r_+ - r_-)}{4\pi} \left[ 1 + \alpha \Sigma \right]^{-1}. \]

If we expand the \( T_{H} \) in terms of the \( \alpha \) powers and neglect the higher order of the \( \alpha \) terms, then the modified Hawking temperature of the New-type black hole becomes as follows;
\[ T_{H} = T_{H} \left[ 1 - \alpha \Sigma \right], \]

where the \( T_{H} = \hbar \frac{(r_+ - r_-)}{4\pi} \) is the standard Hawking temperature of the black hole. From the \( T_{H} \) expression, we see that the modified Hawking temperature is related to not only the mass parameter of the black hole, but also to the AdS3 radius, \( L \), (and, hence, to the graviton mass) and the properties
of the tunnelled massive vector boson, such as angular momentum, energy and mass. On the other hand, in the case of $\alpha = 0$, the modified Hawking temperature reduces to the standard temperature obtained by quantum tunnelling process of the point particles with spin-0, spin-1/2 and spin-1, respectively [24, 33].

4 Summary and Conclusion

In this study, we investigate the quantum gravity effect on the tunnelled massive vector boson from New-type black hole in the context of 2+1 dimensional New Massive Gravity. For this, at first, using the GUP relations, we modify the massive vector bosons equation. Then, using the Kerner-Mann method, the tunnelling probabilities of the massive vector particle are derived, and subsequently, the corrected Hawking temperature of the black hole is calculated. And, we find that the modified Hawking temperature not only depends on the black hole’s properties, but also depends on the emitted spin-1 vector boson’s mass, energy, total angular momentum. Also, it is worth mention that the modified Hawking temperature depends on the mass of graviton in this context. As can be seen from Eqs. (24), the Hawking temperature increase by the total angular momentum of the tunnelled particle while it decreases by the energy and mass of the tunnelled particle and the graviton mass.

In addition, according to Eq. (24), we can summarize some important results as follows:

- If $9r_+^2(r_+ - r_-)^2 \frac{m_0}{m^2} + 16E^2 r_+^2 > 4j^2(r_+ - r_-)^2$, the modified Hawking temperature of the tunnelling vector boson is lower than the standard temperature. However, when $9r_+^2(r_+ - r_-)^2 \frac{m_0}{m^2} + 16E^2 r_+^2 < 4j^2(r_+ - r_-)^2$, the corrected temperature is higher than the standard temperature. If $9r_+^2(r_+ - r_-)^2 \frac{m_0}{m^2} + 16E^2 r_+^2 = 4j^2(r_+ - r_-)^2$, then the contribution of the GUP effect is canceled, and the modified temperature of the tunnelling vector boson reduce to the standard temperature.

- As described previously, the New type black hole is reduced to the static BTZ black hole in the case of $b = 0$ and $c < 0$. Hence, the Hawking temperature of the static BTZ black hole under the quantum gravity effect is

$$T_H = T_H \left[ 1 - \alpha m^2 \frac{|c|}{2m^2} - 4f^2 + 4E^2 \right],$$

where $r_+ = -r_- = \sqrt{|c|}$ is used and the $T_H = \hbar \frac{\sqrt{|c|}}{2\pi}$ is the standard Hawking temperature of the static BTZ black hole in the context of 2+1 dimensional New Massive Gravity theory [26]. In this case, the modified Hawking temperature is higher than standard Hawking temperature when $4E^2 + \frac{9m_0^2}{2m^2} |c| > 4j^2$. On the other hand, when $4E^2 + \frac{9m_0^2}{2m^2} |c| < 4j^2$, the modified Hawking temperature is lower than standard Hawking temperature.

- In the absence of the quantum gravity effect, i.e. $\alpha = 0$, the modified Hawking temperature is reduced to the standard temperature obtained by quantum tunnelling of the massive spin-0, spin-1/2 and spin-1 point particles [24, 33].

Acknowledgements This work was supported by Akdeniz University, Scientific Research Projects Unit. This study was supported by The Scientific and Technological Research Council of Turkey (TUBITAK) 1002-QSP (Project No: 116F329).

References

1. J. M. Greif, Junior thesis, Princeton University, (unpublished) (1969).
2. B. Carter, Nature, 238, 71 (1972).
3. J.D. Bekenstein, Phys. Rev. D 7, 2333 (1973).
4. S.W. Hawking, Nature, 248, 30 (1974).
5. S.W. Hawking, Commun. Mth. Phys. 43, 199 (1975).
6. S.W. Hawking, Phys. Rev. D 13, 191 (1976).
7. J. Zhang and Z. Zhao, Phys. Lett. B 43, 3292 (1974).
8. J.M. Bardeen, B. Carter and S.W. Hawking, Commun. math. Phys. 31, 161-170 (1973).
9. S.P. Robinson and F. Wilczek, Phys. Rev. Lett. 95, 011303 (2005).
10. S.B. Giddings, AIP Conf. Proc. 957, 69 (2007).
11. P. Kraus and F. Wilczek, Nucl. Phys. B 437, 231 (1995).
12. P. Kraus and F. Wilczek, Nucl. Phys. B 433, 403 (1995).
13. M.K. Parikh and F. Wilczek, Phys. Rev. Lett. 85, 5042 (2000).
14. R. Kerner and R.B. Mann, Phys. Rev. D 73, 104010 (2006).
15. R. Kerner and R.B. Mann, Class. Quantum Grav. 25, 095014 (2008).
16. D.Y. Chen,Q.Q Jian and X.T. Zu, Class. Quantum Grav. 25, 205022 (2008).
17. J. Zhang and Z. Zhao, Phys. Lett. B 638, 110-113 (2006).
18. J. Huang and W. Liu, Int. Journal of Theo. Phys. 49, 2621 (2010).
19. X.X. Zeng and Q. Li, Chinese Physics B 18, 11 (2009).
20. R.R. Criscienzo and L.L. Vanzo, Europhys. Lett. 82, 6001 (2008).
21. D.Y. Chen, Q.Q. Jiang and X.T. Zu, Phys. Lett. B 665, 106 (2008).
22. Q.Q. Jiang, Phys. Rev. D 78, 044009 (2008).
23. R. Li, J.K. Zhao and X.H. Wu, Commun. Theor. Phys. 66, 77 (2016).
24. R. Li and J.R. Ren, Phys. Lett. B 661, 370 (2008).
25. H.L. Li, S.Z. Yang, Q.Q. Jiang, and D.J. Qi, Phys. Lett. B 641, 139 (2006).
92. Z.H. Li, L.M. Zhang, Int. J. Theor. Phys. 55 401 (2016).
93. K. Lin and S.Z. Yang, Chin. Phys. B 20, 110403 (2011).
94. P. Mitra, Phys. Lett. B 648, 240 (2007).