Research on braking dynamics of multi-axle vehicle

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Abstract. Braking dynamics is an important part of longitudinal dynamics. Through the analysis of braking performance of multi-axle vehicles, we can deepen our understanding of longitudinal dynamics. Starting from the braking dynamics analysis of the whole vehicle, this paper proposes to establish the braking dynamics model of multi-axle vehicle by using the suspension deformation coordination equation, so as to calculate the general calculation formula of ground reaction force of multi-axle vehicle when braking. The brake force distribution of 4-axle brake is analyzed to verify its rationality and provide basis for multi-axle brake design.

1. Introduction
The high maneuverability and safety of multi-axle special vehicles make them widely used in national defense and military industry, providing transport media and carrying platform for large equipment[1]. The dynamic characteristics of the vehicle determine the operability of the vehicle and the safety of personnel and equipment, in which the braking performance is the basic guarantee for the safety of the vehicle[2]. Due to the characteristics of multi-axle vehicle such as axle load, large number of axles and through-connection between axles and shafts, the detection of braking performance of multi-axle vehicle is more complicated than that of single-axle vehicle. At present, there is little research on multi-axle vehicle braking dynamics model and whole vehicle braking performance in domestic and international publications.

Chen [3] studied the approximate braking difference of multi-axle vehicle by using zero-differential braking test stand with asynchronous drive. Cheng[4] established a refined vehicle dynamic model including brake system, suspension system and steering system by analyzing its highly coupled mechanical system. Wan[5] put forward a general multi-axle vehicle linear braking dynamic model with suspension deformation coordination equation, and deduced the general calculation formulas of ground braking force, braking deceleration and braking distance.

Starting from the braking dynamics analysis of the whole vehicle, this paper determines the force condition of the whole vehicle. Based on the statically indeterminate problem of multi-axle vehicle braking according to classical model, a dynamic model of multi-axle vehicle braking is put forward to calculate the ground reaction force of multi-axle vehicle when braking, and then the ground braking force is obtained. To analyze brake force distribution of multi-axle vehicle and verify its rationality

2. Analysis of vehicle braking performance
Braking is the basic performance of a car. The completion value of vehicle braking is that under the action of external force, the vehicle speed gradually decreases from a certain speed to 0. In the
longitudinal direction, the braking dynamics equation expression of electric vehicle is shown in equation 1:

\[ m\delta \frac{dv}{dt} = -F_{xb} - F_f - F_w - F_i \]  

(1)

In the above formula, the acceleration is \( \frac{dv}{dt} \), the total mass of the whole vehicle is \( m \), the ground braking force is \( F_{xb} \), and the rolling resistance is \( F_f \); Air resistance is \( F_w \); Gradient resistance is \( F_i \); The rotating mass coefficient of the vehicle is \( \delta \).

In practice, the passive resistance is generally ignored in the dynamic analysis of the whole vehicle braking process, and only the braking force generated by the vehicle itself is considered. The braking process of electric vehicle is jointly completed by regenerative braking and friction braking. The braking force equation of the vehicle is shown in equation 2:

\[ F_{xb} = F_{\mu 1} + F_{\mu 2} + F_r \]  

(2)

Where, the front wheel friction braking force is \( F_{\mu 1} \); The friction braking force of the rear wheel is \( F_{\mu 2} \); Regenerative braking force is \( F_r \).

The braking force expression of the brake is shown in equation 3:

\[ F_{\mu} = \frac{T_{\mu}}{r} \]  

(3)

Where, \( r \) is the radius of the wheel and the friction torque generated by the brake is \( T_{\mu} \).

When the electric vehicle is braking, in order to ensure that the vehicle obtains sufficient ground braking force, two conditions need to be met at the same time: the ground can provide sufficient adhesion and the brake can provide sufficient braking force[6].

3. Dynamic calculation of multi axle vehicle braking

3.1. Braking dynamics model of multi axle vehicle

In the force calculation of multi axle vehicle, the classical model simplifies the mechanics of multi axle vehicle too much, which often presents the problem of static uncertainty, resulting in the failure to correctly calculate the force of multi axle vehicle. Therefore, in order to effectively calculate the stress of multi axle vehicles, necessary mechanical constraints should be retained to ensure the smooth completion of stress calculation in the process of analyzing multi axle vehicles. During the force analysis of multi axle vehicle, in order to retain the constraint characteristics of suspension, the dynamic model of suspension deformation coordination is introduced (as shown in Figure 1), in which the body, wheel and frame are simplified as rigid bodies; At the same time, the suspension is simplified as a spring, the stiffness of the spring is the equivalent stiffness of the suspension and tire, the deformation of the suspension is constrained by the frame (as shown in Figure 2), and the connection point between the suspension and the frame is always kept in a straight line. The passive resistance of multi axle vehicle is generally ignored in the overall force analysis of multi axle vehicle.
3.2. Calculation of ground normal reaction force during braking of multi axle vehicle

According to the classical braking model, when a multi axle vehicle brakes in a straight line on a horizontal road, an independent force balance equation can be obtained, as shown in equation 4:

\[
\sum_{i=1}^{n} F_{zi} = mg \quad i = 1, 2, 3 \ldots n
\]  

(4)

Where, the ground normal reaction force of each axis is \( F_{zi} \); Vehicle quality \( m = 30000 \) kg; The gravitational acceleration is \( g = 9.8 \) m/s\(^2\).

Take the first axis center as the reference point to obtain an independent expression of torque balance equation, as shown in equation 5:

\[
\sum_{i=1}^{n} F_{zi}l_i = mgl_i - mah_g \quad i = 1, 2, 3 \ldots n
\]  

(5)

Where, \( l_i \) is the horizontal distance from the center of the first axis to the center of the first axis, where \( l_i = 0 \); \( l_c \) is the horizontal distance from the mass center of the whole vehicle to the center of the first axle, taking \( l_c = 3175 \) mm; \( h_g \) is the height from the mass center of the whole vehicle to the ground, taking \( h_g = 1200 \) mm; Braking deceleration is \( a \), where \( a = zg \). For multi axle vehicles, equations 4 and 5 are still valid.

According to the model established in this paper, a mutually independent suspension force equation can be obtained, and its expression is shown in equation 6:

\[
F_{zi} = k_i \Delta z_i \quad i = 1, 2, 3, \ldots, n
\]  

(6)

Where, \( k_i \) is the suspension equivalent stiffness of each axle; \( \Delta z_i \) is the suspension deformation of each axle.
According to the schematic diagram of linear constraint (as shown in Figure 2), n-2 mutually independent suspension deformation coordination equations can be obtained, and the expression is shown in equation 7:

\[
(l_j - l_i)(\Delta z_j - \Delta z_i) = (l_j - l_i)(\Delta z_2 - \Delta z_1)
\]

\[
\Delta z_j = \Delta z_1, \quad l_j = l_i, \quad j = 3, 4, \ldots, n
\]  

(7)

By combining Eq. 4, Eq. 5, Eq. 6 and Eq. 7, 2n independent equations can be obtained. By combining these equations, the magnitude of the ground normal reaction force of each axle under any braking deceleration can be determined. Formula 8 can be obtained by combining Formula 4 and 5:

\[
AF_z = C
\]

(8)

\[
F_z = [F_{z1} \ F_{z2} \ \ldots \ \ F_{zn}]^T; \quad C = \begin{bmatrix}
mg \\
mg_l - mahl
\end{bmatrix}; \quad A = \begin{bmatrix}
e_{n} \\
l
\end{bmatrix}
\]

\[
e_n = [1 \ 1 \ 1 \ \ldots \ 1]_{n \times 1}; \quad l = [l_1 \ l_2 \ l_3 \ \ldots \ l_n]
\]

Formula 9 is obtained from Formula 6:

\[
F_z = K\Delta z
\]

(9)

\[
\Delta z = \begin{bmatrix}
\Delta z_1 \\
\Delta z_2 \\
\Delta z_3 \\
\cdots \\
\Delta z_n
\end{bmatrix}, \quad K = \text{diag}(k_1, k_2, k_3, \ldots, k_n)
\]

Formula 10 can be obtained from Formula 7:

\[
X\Delta z = [x_{ab}]_{(n-2) \times n} \Delta z = 0
\]

(10)

\[
x_{ab} = \begin{cases}
\frac{l_{a+2}}{l_2} - 1 & b = 1 \\
\frac{l_{a+2}}{l_2} & b = 2 \\
1 & b = a + 2 \\
0 & \text{other}
\end{cases}
\]

Formula 11 can be obtained by combining Formula 9 and Formula 10:

\[
XK^{-1}F_z = 0
\]

(11)

Matrix Formula 12 can be obtained by combining Formula 8 and 11:

\[
F_z = \begin{bmatrix}
A \\
\begin{bmatrix}
XK^{-1}
\end{bmatrix}
\end{bmatrix}^{-1}C
\]

(12)

In Formula 12, make \(W = \begin{bmatrix}
A \\
\begin{bmatrix}
XK^{-1}
\end{bmatrix}
\end{bmatrix}^{-1}\), Formula 12 variable to Formula 13:

\[
F_z = \begin{bmatrix}
W_{11} & W_{12} & \cdots & W_{1n} \\
W_{21} & W_{22} & \cdots & W_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
W_{n1} & W_{n2} & \cdots & W_{nn}
\end{bmatrix} \begin{bmatrix}
mg \\
mg_l - mahl
\end{bmatrix}
\]

(13)

\[
W = \begin{bmatrix}
\begin{bmatrix}
W_{11} & W_{12} & \cdots & W_{1n}
\end{bmatrix} \\
\begin{bmatrix}
W_{21} & W_{22} & \cdots & W_{2n}
\end{bmatrix} \\
\vdots \\
\begin{bmatrix}
W_{n1} & W_{n2} & \cdots & W_{nn}
\end{bmatrix}
\end{bmatrix}
\]

(13)
Finally, an expression of the normal ground reaction force for each axis can be obtained, as shown in Eq. 14:

\[ F_{zi} = W_{i1}mg + W_{i2}mgl - W_{i2}mah \]

(14)

\[ W_{i1} = \frac{k_j \sum_{j=1}^{n} l_j k_y (l_j - l_i)}{\sum_{j=1}^{n} k_j k_y (l_j - l_j) (l_j - l_i)} \]

\[ W_{i2} = \frac{k_j \sum_{j=1}^{n} k_j (l_j - l_i)}{\sum_{j=1}^{n} k_j k_y (l_j - l_j) (l_j - l_i)} \]

The research object of this paper is 4-axle vehicle. By using the above formula and substituting the specific parameters (as shown in Table 1), the ground normal reaction force of each axle under different braking strength can be calculated. The calculation results are shown in Table 2.

| Table 1. Relevant parameters of 4-axle electric wheel vehicle |
|-------------|----------------|
| **Type**     | **Value**      |
| \( l_1 \) (mm) | 0              |
| \( l_2 \) (mm) | 1524           |
| \( l_3 \) (mm) | 5334           |
| \( l_4 \) (mm) | 6858           |
| \( l_c \) (mm) | 3175           |
| \( k_1 \) (N/mm) | 881            |
| \( k_2 \) (N/mm) | 881            |
| \( k_3 \) (N/mm) | 733            |
| \( h_g \) (mm) | 1200           |

| Table 2. Result |
|-----------------|----------------|
| **Braking deceleration** | **\( F_{z1} \) (kN)** | **\( F_{z2} \) (kN)** | **\( F_{z3} \) (kN)** | **\( F_{z4} \) (kN)** |
| 0 | 80.556 | 80.405 | 66.583 | 66.457 |
| 0.1g | 84.573 | 82.499 | 64.327 | 62.602 |
| 0.2g | 88.589 | 84.593 | 62.071 | 58.746 |
| 0.3g | 92.606 | 86.688 | 59.815 | 54.891 |
| 0.4g | 96.623 | 88.782 | 57.559 | 51.036 |
| 0.5g | 100.639 | 90.877 | 55.303 | 47.181 |
| 0.6g | 104.655 | 92.971 | 53.048 | 43.326 |
| 0.7g | 108.673 | 95.065 | 50.792 | 39.470 |
| 0.8g | 112.689 | 97.160 | 48.536 | 35.615 |

4. Analysis of braking force distribution of multi axle vehicle

The ground braking force expression of the non locked shaft between the nth and N + 1 locking points is shown in equation 15:

\[ F_{zi} = \beta_i F_{\mu} \]

(15)

Where, \( F_{\mu} \) is the total braking force of the brake; \( \beta_i \) is the brake force distribution coefficient of each axle; The ground braking force expression of locked shaft is shown in equation 16:

\[ F_{zi} = \phi F_{zi} \]

(16)
Where, $F_{zi}$ is the ground normal reaction force of each axis; $\varphi$ is the ground adhesion coefficient.

In this paper, the brake force distribution coefficient of each axle of four axle vehicle is set as $\beta_1=0.35$, $\beta_2=0.3$, $\beta_3=0.2$, $\beta_4=0.15$; In this section, the rationality of braking force distribution of four axle vehicle is verified by using the graphical method of adhesion coefficient. The main steps are as follows:(1)The above method is used to calculate the ground normal reaction force of each axis;(2)Calculate the utilization adhesion coefficient of each axis;(3)Calculate and draw the relationship between the adhesion coefficient and braking strength of each axle, and attach the relevant curves of ECE braking regulations;(4)The rationality of braking force distribution of multi axle vehicle is analyzed. Using adhesion coefficient $\varphi_i$, I can be calculated from formula 17 ~ 20:

$$\varphi_i = \frac{F_{xbi}}{F_{zi}}$$  \hspace{1cm} (17)

$$F_{xbi} = F_{\mu i} = \beta G z$$  \hspace{1cm} (18)

$$\beta_i = \frac{F_{\mu i}}{\sum_{i=1}^{4} F_{\mu i}}$$  \hspace{1cm} (19)

$$F_{xbi} + F_{xb2} + F_{xb3} + F_{xb4} = zG$$  \hspace{1cm} (20)

According to the above calculation formula of ground normal reaction force, it can be obtained as shown in equations 21~ 24:

$$F_{z1} = 40989.8 z + 80556$$  \hspace{1cm} (21)

$$F_{z2} = 21367.3 z + 80405$$  \hspace{1cm} (22)

$$F_{z3} = -23020.4 z + 66583$$  \hspace{1cm} (23)

$$F_{z4} = -39336.7 z + 66457$$  \hspace{1cm} (24)

Where $z$ is the braking strength. When the formula 17 ~ 24 is combined, the $\varphi_i = \varphi(z)$As shown in figure 3.

![Figure 3: Using the relationship curve between adhesion coefficient and braking strength](image)

It can be seen from Figure 3 that each axis is close to each other by using the adhesion coefficient curve $\varphi = z$ line, with high adhesion utilization rate; And the braking strength $z$ is between 0.15~0.3, and the utilization adhesion coefficient of each axle is $\varphi = z \pm 0.08$ curves, and $\varphi_1 > \varphi_2 > \varphi_3 > \varphi_4$. It meets the requirements of wheel locking sequence and ensures the directional stability of vehicle braking; And, $\varphi$ Between 0.2~0.8, $z \geq 0.1 + 0.85(\varphi - 0.2)$; When $z \geq 0.3$, it also conforms to $z \geq 0.3 + 0.74(\varphi - 0.38)$. Therefore, the braking force distribution coefficient of each axle set in this paper meets the...
requirements of ECE braking regulations and meets the directional stability during braking. Therefore, the braking force distribution method of 4-axle vehicle in this paper is reasonable.

5. Conclusions
Based on the braking performance of the vehicle, the braking performance of the vehicle is analyzed in detail. The braking dynamics model of multi-axle vehicle was proposed, and the ground braking force was calculated, and the ground braking force of each axle was calculated. The braking force distribution of multi-axle vehicle brakes was analyzed, which was in line with ECE braking regulations and distribution principles. This provides an idea for the research on braking performance and dynamic model of multi-axle vehicle.

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