Next-to-leading-order time-like pion form factors in $k_T$ factorization

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We calculate the time-like pion-photon transition form factor and the pion electromagnetic form factor up to next-to-leading order (NLO) of the strong coupling constant in the leading-twist $k_T$ factorization formalism. It is found that the NLO corrections to the magnitude (phase) are lower than 30% (30°) for the former, and lower than 25% (10°) for the latter at large invariant mass squared $Q^2 > 30$ GeV$^2$ of the virtual photons. The increase of the strong phases with $Q^2$ is obtained, consistent with the tendency indicated by experimental data. This behavior is attributed to the inclusion of parton transverse momenta $k_T$, implying that the $k_T$ factorization is an appropriate framework for analyzing complex time-like form factors. Potential extensions of our formalism to two-body and three-body hadronic $B$ meson decays are pointed out.

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I. INTRODUCTION

The $k_T$ factorization formalism [1,2] has been applied to next-to-leading-order (NLO) analysis of several space-like form factors, such as the pion-photon transition form factor [7,8], the pion electromagnetic (EM) form factor [9], and the $B \to \pi$ transition form factors [10]. The calculations are nontrivial, because partons off-shell by $k^2_T$ are considered in both QCD quark diagrams and effective diagrams for meson wave functions. The gauge invariance of hard kernels, derived from the difference of the above two sets of diagrams, needs to be verified. The regularization of the light-cone singularity in the effective diagrams generates double logarithms, which should be summed to all orders. It has been found that the NLO corrections, after the above treatments, are negligible in the pion transition form factor, but reach 30% in the latter two cases. In this Letter we shall extend the NLO $k_T$ factorization formalism to the time-like pion transition and EM form factors.

One of the widely adopted theoretical frameworks for two-body hadronic $B$ meson decays is the perturbative QCD (PQCD) approach [11] based on the $k_T$ factorization. It has been shown that factorizable contributions to these decays can be computed in PQCD without the ambiguity from the end-point singularity. These computations indicated that sizable strong phases are produced from penguin annihilation amplitudes, with which the direct CP asymmetry in the $B^\pm \to K^\pm \pi^\mp$ decays was successfully predicted. It is then a concern whether PQCD predictions for strong phases are stable against radiative corrections. The factorizable penguin annihilation amplitudes involve time-like scalar form factors. Before completing NLO calculations for two-body hadronic $B$ meson decays, it is possible to acquire an answer to the above concern by studying the time-like pion EM form factor. Besides, the PQCD formalism for three-body $B$ meson decays [12] has demanded the introduction of two-meson wave functions [13], whose parametrization also involves time-like form factors associated with various currents. If PQCD results for complex time-like form factors are reliable, a theoretical framework for three-body $B$ meson decays can be constructed.

NLO corrections to time-like form factors are derived easily from those to space-like ones by suitable analytic continuation from $-Q^2$ to $Q^2$, with $Q^2$ denoting the momentum transfer squared. We shall present the $k_T$ factorization formulas for the time-like pion transition and EM form factors up to NLO at leading twist. Following the prescription proposed in [1,11], both the renormalization and factorization scales are set to the virtuality of internal particles. With this scale choice, it will be demonstrated that the NLO corrections to the time-like pion transition and EM form factors are under control at leading twist. It implies that PQCD predictions for strong phases of factorizable annihilation amplitudes in two-body hadronic $B$ meson decays may be stable against radiative corrections. Moreover, we observe the increase of the strong phases of the above form factors with $Q^2$, consistent with the tendency indicated by experimental data. It will be explained that this behavior is attributed to the inclusion of the parton transverse momenta $k_T$. This consistency supports the $k_T$ factorization as a potential framework for studying complex time-like form factors and three-body $B$ meson decays.
II. PION-PHOTON TRANSITION FORM FACTOR

In this section we present the leading-twist NLO factorization formula for the time-like pion-photon transition form factor. The leading-order (LO) QCD quark diagram describing $\gamma^*(q) \rightarrow \pi(P_1) \gamma(P_2)$ is displayed in Fig. 1(a), where the momentum $P_1$ of the pion and the momentum $P_2$ of the outgoing on-shell photon are chosen as

$$P_1 = (P_1^+, 0, 0_T), \quad P_2 = (0, P_2^-, 0_T), \quad P_1^+ = P_2^- = Q/\sqrt{2},$$

with $Q^2 = q^2 = (P_1 + P_2)^2 > 0$ being the invariant mass squared of the virtual photon $\gamma^*$. Figure 1(a) leads to the LO hard kernel

$$H_{\pi\gamma}^{(LO)}(x, Q^2, k_T) = -i N_c \frac{N_c}{2N_c} \text{Tr} \left[ \gamma^5 \gamma^5 \gamma^5 \right] = -i \sqrt{N_c} \frac{N_c}{2} \frac{N_c}{2} \left[ \frac{k_T^2 - xQ^2 - i\epsilon}{k_T^2 - xQ^2 - i\epsilon} \right],$$

where $N_c = 3$ is the number of colors, $\epsilon$ is the polarization vector of the outgoing photon, $k = (xP_1^+, 0, 0_T)$ is the momentum carried by the valence quark, $\gamma^5 P_1^\mu/\sqrt{2N_c}$ is the leading-twist spin projector of the pion, and the subscript $\mu$ associated with the virtual photon vertex is implicit on the left-hand side. In the previous works on the space-like transition form factor [7, 14–16], the internal quark remains off-shell by $(P_2 - k)^2 = -(xQ^2 + k_T^2) < 0$ as indicated in Fig. 1(b). For the time-like case, the internal quark may go on mass shell, and an imaginary part is generated in the hard kernel according to the principal-value prescription

$$\frac{1}{k_T^2 - xQ^2 - i\epsilon} = \text{Pr} \frac{1}{k_T^2 - xQ^2} + i\pi \delta(k_T^2 - xQ^2).$$

Fourier transforming Eq. 2 into the impact-parameter $b$ space, we derive the LO pion transition form factor

$$F_{\pi\gamma}^{(LO)}(Q^2) = i\pi \frac{\sqrt{2f_\pi}}{6} \int_0^1 dx \int_0^\infty db \phi_\pi(x) \exp[-S(x, b, Q, \mu)] H_{\pi\gamma}^{(1)}(\sqrt{x}Qb),$$

with the pion decay constant $f_\pi$, the renormalization and factorization scale $\mu$, the Hankel function of the first kind $H_{\pi\gamma}^{(1)}$, and the twist-2 pion distribution amplitude (DA) $\phi_\pi$. Here we shall not consider the potential intrinsic $k_T$ dependence of the pion wave function [17], because its inclusion would introduce additional model dependence, which is not the focus of this work. For example, the intrinsic $k_T$ dependence has been parameterized into the different Gaussian and power forms in [18]. The Sudakov factor $e^{-S}$ sums the double logarithm $\alpha_s \ln^2 k_T$ to all orders, and takes the same expression for both the space-like and time-like form factors [19], since it is part of the universal meson wave function. For its explicit expression, refer to [5, 20, 21]. Note that Eq. 3 can be obtained from the LO space-like transition form factor in [13] by substituting $i\pi H_{\pi\gamma}^{(1)}/2$ for the Bessel function $K_0$, as a consequence of the analytic continuation $q^2 = -Q^2 \rightarrow (Q^2 + i\epsilon)$ in the hard kernel.

As stated in the Introduction, the NLO hard kernel is derived by taking the difference of the $O(\alpha_s)$ quark diagrams and the $O(\alpha_s)$ effective diagrams for meson wave functions. The ultraviolet divergences in loops are absorbed into
the renormalized strong coupling constant $\alpha_s(\mu)$, and the infrared divergences are subtracted by the nonperturbative meson wave functions. The above derivation has been demonstrated explicitly in [7] for the space-like pion transition form factor. We repeat a similar calculation for the time-like pion transition factor, and derive the NLO hard kernel

$$H_{\pi\gamma}^{(NLO)}(x, Q^2, k_T, \mu) = h_{\pi\gamma}(x, Q^2, k_T, \mu)H_{\pi\gamma}^{(LO)}(x, Q^2, k_T),$$

(5)

with the NLO correction function

$$h_{\pi\gamma}(x, Q^2, k_T, \mu) = \frac{\alpha_s(\mu)C_F}{4\pi} \left\{ -3 \ln \frac{\mu^2}{Q^2} - \ln \left[ \frac{k_T^2 - xQ^2}{Q^2} \right] + 2 \left[ 1 - i\pi - i\pi \Theta \left( k_T^2 - xQ^2 \right) \right] \ln \left[ \frac{k_T^2 - xQ^2}{Q^2} \right] - 2 \ln x + \left( 4\pi^2 - i\pi \right) \Theta \left( k_T^2 - xQ^2 \right) - 3 - i5\pi \right\},$$

(6)

$C_F$ being the color factor. The imaginary pieces proportional to the step function $\Theta$ are generated from the $O(\alpha_s)$ quark diagrams. For the evaluation of the $O(\alpha_s)$ effective diagrams, we have chosen the direction $\pi^\mu$ of the Wilson lines the same as in [7] in order to respect the universality of the meson wave function. Equation (5) can also be achieved by substituting $(Q^2 + i\epsilon)$ for the virtuality of the external photon, and $(xQ^2 - k_T^2 + i\epsilon)$ for the internal quark in [7], and then employing the relations $\ln(-Q^2 - i\epsilon) = \ln Q^2 - i\pi$ and $\ln(-(xQ^2 + k_T^2 - i\epsilon)) = \ln |xQ^2 - k_T^2| - i\pi \Theta(xQ^2 - k_T^2)$.

Fourier transforming Eq. (5) to the $b$ space, we arrive at the NLO $k_T$ factorization formula for the time-like pion transition factor

$$F_{\pi\gamma}^{(NLO)}(Q^2) = i\pi \sqrt{\frac{2}{3}} \int_0^1 dx \int_0^\infty db \phi_\pi(x) \exp[-S(x, b, Q, \mu)] \times \frac{\alpha_s(\mu)C_F}{4\pi} \left[ \tilde{h}_{\pi\gamma}(x, Q^2, k_T, \mu)H_{\pi\gamma}^{(1)}(\sqrt{x}Qb) + H_{\pi\gamma}^{(1)\prime}(\sqrt{x}Qb) \right],$$

(7)

with

$$\tilde{h}_{\pi\gamma}(x, Q^2, k_T, \mu) = -3 \ln \frac{\mu^2}{Q^2} - \frac{1}{4} \ln \frac{4x}{Q^2 b^2} + \left( 1 + \gamma_E - \frac{3\pi}{2} \right) \ln \frac{4x}{Q^2 b^2} - 2 \ln x + \frac{17\pi^2}{12} + \pi - 3 - 2\gamma_E - \gamma_E^2 - i(4 - 3\gamma_E),$$

(8)

$\gamma_E$ being the Euler constant. The function

$$H_{\pi\gamma}^{(1)\prime\prime}(\rho) \equiv \left[ \frac{\partial^2}{\partial s^2} H_{\pi\gamma}^{(1)}(\rho) \right]_{s=0},$$

(9)

where $Q$ denotes the order parameter of the Hankel function, comes from the Fourier transformation of $\ln^2(-xQ^2 + k_T^2 - i\epsilon)$ in Eq. (6). For a small argument $\rho = \sqrt{x}Qb \rightarrow 0$, its magnitude behaves as $|H_{\pi\gamma}^{(1)\prime\prime}(\rho)| \sim (1/3) \ln^3 \rho |H_{\pi\gamma}^{(1)}(\rho)|$, which represents a double-logarithmic correction essentially. The perturbative expansion could be improved by summing the double logarithm $\alpha_s \ln^2(x/(Q^2b^2))$ in Eq. (8), which arises from the Fourier transformation of the term $\alpha_s \ln^2(k_T^2 - xQ^2)/Q^2$ in Eq. (5). Strictly speaking, it differs from the threshold resummation of $\alpha_s \ln^2 x$ performed in [20], and deserves a separate study. Besides, there is no end-point enhancement involved in the present calculation, so we shall not perform the resummation here for simplicity. Equations (5) and (6) will be investigated numerically in Sec. IV.

### III. PIon ELECTROMAGNETIC FORM FACTOR

We then derive the NLO, i.e., $O(\alpha_s^2)$ contribution to the time-like pion EM form factor at leading twist. An LO quark diagram for the corresponding scattering $\gamma^* (q) \rightarrow \pi^+(P_1) \pi^-(P_2)$ is depicted in Fig. 2(a). We choose light-cone coordinates, such that the momenta $P_1$ and $P_2$ are parameterized the same as in Eq. (4) with $Q^2 = q^2 = (P_1 + P_2)^2 > 0$. The valence quark carries the momentum $k_1 = (x_1 P_1^+, 0, k_{1T})$ and the valence anti-quark carries $k_2 = (0, x_2 P_2^-, k_{2T})$. The LO hard kernel reads

$$H_{\pi\gamma}^{(LO)}(x_1, k_{1T}, x_2, k_{2T}, Q^2) = i4\pi\alpha_s C_F \frac{\left| x_1 \text{Tr} [P_2 H_1 \gamma_\mu H_1] \right|}{(x_1 Q^2 - k_{1T}^2 + i\epsilon)(x_1 x_2 Q^2 - |k_{1T} + k_{2T}|^2 + i\epsilon)},$$

(10)

1 Compared to [3], three effective diagrams for the self-energy corrections to the Wilson lines have been included in Eq. (5).
FIG. 2: LO quark diagrams for time-like and space-like pion electromagnetic form factors.

where the denominators \((x_1 Q^2 - k_{1T}^2)\) and \((x_1 x_2 Q^2 - |k_{1T} + k_{2T}|^2)\) are the virtuality of the internal quark and gluon, respectively. The subscript II denotes that the \(k_T\)-dependent terms in both the internal quark and gluon propagators are retained. When one of the internal particle propagators goes on mass shell, an imaginary part is produced according to the principle-value prescription in Eq. (3).

Fourier transforming Eq. (10) from the transverse-momentum space \((k_{1T}, k_{2T})\) to the impact-parameter space \((b_1, b_2)\), we obtain a double-\(b\) convolution for the LO time-like pion EM form factor \(\mathcal{F}_{EM}^{(b)}\) from the space-like form factor in Eq. (10) reduces to

\[
\mathcal{F}_{EM}^{(LO)}(Q^2) = \frac{\pi^3 f_T^2 C_F}{2N_c} Q^2 \int_0^1 dx_1 dx_2 \int_0^\infty db_1 db_2 b_1 b_2 \alpha_s(\mu) x_1 \phi_1(\mu) \phi_2(\mu) \exp\left[-S_{II}(x_1, b_1, x_2, b_2, Q, \mu)\right]
\]

\[
\times H_0^{(1)}(\sqrt{x_1 x_2 Q b_2}) H_0^{(1)}(\sqrt{x_1 Q b_1}) J_0(\sqrt{x_1 Q b_2}) [(b_1 - b_2) + H_0^{(1)}(\sqrt{x_1 Q b_2}) J_0(\sqrt{x_1 Q b_1}) \Theta(b_2 - b_1)],
\]

with the Bessel function of the first kind \(J_0\), and the Sudakov exponent \(S_{II}(x_1, b_1, x_2, b_2, Q, \mu) = S(x_1, b_1, Q, \mu) + S(x_2, b_2, Q, \mu)\). The above expression can also be obtained via analytical continuation of the space-like form factor in Fig. 2(b) to the time-like region.

The NLO hard kernel for the space-like pion EM form factor has been computed as the difference between the one-loop QCD quark diagrams and effective diagrams in [9]. To simplify the calculation, the hierarchy \(x_1 Q^2, x_2 Q^2 \gg x_1 x_2 Q^2, k_T^2\) has been postulated, since the \(k_T\) factorization applies to processes dominated by small-x contributions. Ignoring the transverse momenta of the internal quarks, the LO hard kernel in Eq. (10) reduces to

\[
H_1^{(LO)}(x_1, k_{1T}, x_2, k_{2T}, Q^2) = i 4 \pi \alpha_s C_F \frac{\text{Tr}[H_2 P_1 \gamma_5 H_3]}{Q^2(x_1 x_2 Q^2 - |k_{1T} + k_{2T}|^2 + i \epsilon)}.
\]

The Fourier transformation of the above expression leads to a single-\(b\) convolution

\[
F_1^{(LO)}(Q^2) = i \frac{\pi^3 f_T^2 C_F}{2N_c} \int_0^1 dx_1 dx_2 \int_0^\infty db b \alpha_s(\mu) \phi_1(\mu) \phi_2(\mu) \exp\left[-S_1(x_1, x_2, b, Q, \mu)\right] H_0^{(1)}(\sqrt{x_1 x_2 Q b}),
\]

with the simplified Sudakov exponent \(S_1(x_1, x_2, b, Q, \mu) \equiv S_{II}(x_1, b, x_2, b, Q, \mu)\). Comparing the outcomes from Eqs. (11) and (13), we can justify the proposed hierarchical relation, and tell which particle propagator, the internal quark or the internal gluon, provides the major source of the strong phase.

Substituting \((Q^2 + i \epsilon)\) for the virtuality of the external photon, and \((x_1 x_2 Q^2 - |k_{1T} + k_{2T}|^2 + i \epsilon)\) for the internal gluon in [9], we have the NLO hard kernel for the time-like pion EM form factor

\[
H_{EM}^{(NLO)}(x_1, k_{1T}, x_2, k_{2T}, Q^2, \mu) = h_{EM}(x_1, x_2, \delta_{12}, Q, \mu) H_1^{(LO)}(x_1, k_{1T}, x_2, k_{2T}, Q^2),
\]

with the NLO correction function

\[
h_{EM}(x_1, x_2, \delta_{12}, Q, \mu) = \frac{\alpha_s(\mu) C_F}{4 \pi} \left[-\frac{3}{4} \ln^2 \frac{\mu^2}{Q^2} - \frac{17}{4} \ln x_1 + \frac{27}{8} \ln x_1 \ln x_2 - \frac{13}{8} \ln x_1 + \frac{31}{16} \ln x_2 - \ln^2 \delta_{12} + \left(\frac{17}{4} \ln x_1 + \frac{23}{8} + i 2 \pi\right) \ln \delta_{12} + \frac{\pi^2}{12} + \frac{3}{2} \ln 2 + \frac{53}{4} - \frac{3 \pi}{4}\right],
\]
and the notation
\[
\ln \delta_{12} \equiv \ln \left( \frac{|k_{1T} + q_{2T}|^2 - x_1 x_2 Q^2}{Q^2} \right) + i \pi \Theta \left( |k_{1T} + q_{2T}|^2 - x_1 x_2 Q^2 \right).
\] (16)

Fourier transforming Eq. (14), we derive the \( k_T \) factorization formula for the NLO contribution at leading twist
\[
\begin{align*}
F_{\text{EM}}^{(\text{NLO})}(Q^2) &= i \frac{\pi f_F^2 C_F^2}{4 N_c} \int_0^1 dx_1 dx_2 \int_0^\infty db \alpha_s^2(x_1) \phi_\pi(x_1) \phi_\pi(x_2) \exp \left[-S_1(x_1, x_2, b, Q, \mu)\right] \\
&\quad \times \left[ \bar{h}_{\text{EM}}(x_1, x_2, b, Q, \mu) H_0^{(1)}(\sqrt{x_1 x_2 Q b}) + H_0^{(1)''}(\sqrt{x_1 x_2 Q b}) \right],
\end{align*}
\] (17)
with the function
\[
\begin{align*}
\bar{h}_{\text{EM}}(x_1, x_2, b, Q, \mu) &= -\frac{3}{4} \log \frac{\mu^2}{Q^2} - \frac{1}{4} \log x_1 x_2 \frac{4 x_1 x_2}{Q^2 b^2} + \left( \frac{17}{8} \log x_1 + \frac{23}{16} + \gamma_E + i \frac{\pi}{2} \right) \log x_1 x_2 \\
&\quad - \frac{17}{4} \log x_1 + \frac{27}{8} \log x_1 x_2 - \left( \frac{13}{8} + \frac{17 \gamma_E}{4} - i \frac{17 \pi}{8} \right) \log x_1 x_2 \\
&\quad - \frac{\pi^2}{2} + \left( 1 - 2 \gamma_E \pi \right) \log x_1 x_2 + \frac{5}{2} \log x_1 x_2 \\
&\quad - \frac{23}{8} \log x_1 x_2 + \frac{31}{16} \log x_2 \\
&\quad - \frac{23}{8} \log x_1 x_2 + i \left( \frac{171}{16} + \gamma_E \pi \right). \quad (18)
\end{align*}
\]

The perturbative expansion could be improved by organizing the double logarithm \( \alpha_s \log^2 x_1 \) in Eq. (13) into the threshold resummation factor \( S_1(x_1, Q^2) \) [21]. This double logarithm, the same as analyzed in [20], appears in the loop correction to the virtual photon vertex under the hierarchical relation \( x_1 Q^2 \gg k_T^2 \) [9]. Because there is no end-point enhancement involved at leading twist, we shall not perform the threshold resummation here. However, the end-point enhancement exists in the two-parton twist-3 contribution, for which \( S_1 \) will play a crucial role, and be implemented in Sec. IV. We shall investigate the NLO effect at leading twist in the time-like pion EM form factor based on Eqs. (11) and (17).

IV. NUMERICAL ANALYSIS

The numerical analysis is performed in this section, for which we adopt the standard two-loop QCD running coupling constant \( \alpha_s(\mu) \) with the QCD scale \( \Lambda_{\text{QCD}} = 0.2 \) GeV, the pion decay constant \( f_\pi = 0.131 \) GeV, the nonasymptotic two-parton twist-2 pion DA
\[
\phi_\pi(x) = 6x(1-x) \left[ 1 + a_2 C_2^{3/2} (1 - 2x) \right],
\] (19)
with the Gegenbauer coefficient \( a_2 = 0.2 \) being fixed by lattice QCD [23], and the Gegenbauer polynomial \( C_2^{3/2}(u) = (3/2)(5u^2 - 1) \).

We compute the LO and NLO contributions to the time-like pion-photon transition form factor at leading twist via Eqs. (13) and (17), with the renormalization and factorization scale \( \mu \) being set to the virtuality of the internal quark \( \mu = \max(\sqrt{x Q}, 1/b) \). The behavior of \( Q^2 F_{\pi\gamma}(Q^2) \) for \( Q^2 \leq 20 \) GeV\(^2\) displayed in Fig. 4 reflects the oscillatory nature of the LO hard kernel in the \( b \) space. The LO time-like pion transition form factor exhibits an asymptotic magnitude, \( Q^2 F_{\pi\gamma}(Q^2) \approx 0.225 \) GeV at large \( Q^2 \). Recall that an asymptotic scaling is known as \( Q^2 F_{\pi\gamma}(Q^2) \rightarrow \sqrt{2} f_\pi = 0.185 \) GeV for the space-like pion transition form factor at large \( -q^2 = Q^2 \) [25]. The larger asymptotic value for the former is expected in the \( k_T \) factorization, because the internal quark may go on mass shell for a time-like momentum transfer \( q \), but it is always off-shell for a space-like \( q \). The ratio between the asymptotic values of the above two transition form factors is roughly 1.22, comparable to the data 1.14 from the \( \eta - \gamma \) transition form factors for \( Q^2 > 10 \) GeV\(^2\) [20]. Note that the time-like and space-like transition form factors would have equal magnitudes in the LO collinear factorization without including the parton transverse momentum \( k_T \). The NLO contribution to the time-like pion transition form factor is also displayed in Fig. 5 which decreases with \( Q^2 \) as expected in PQCD. Compared to the LO result, the NLO correction to the magnitude is about 30% at \( Q^2 = 30 \) GeV\(^2\), and less than 20% for \( Q^2 > 50 \) GeV\(^2\).

For the phase, the LO result arises with \( Q^2 \), and approaches an asymptotic value close to 180° as shown in Fig. 6. It is obvious that the variation with \( Q^2 \) is also attributed to the inclusion of the parton transverse momentum \( k_T \). If \( k_T \) in Eq. (24) was dropped, the LO hard kernel reduces to the traditional expression in the collinear factorization, which always leads to a real \( F_{\pi\gamma} \). A quantitative understanding can be attained via Eq. (13): the contributions from the two
terms in Eq. (13) are comparable at low $Q^2$, such that the time-like pion transition form factor acquires a nontrivial phase. At high $Q^2 > 20$ GeV$^2$, the phase is dominated by the first term in Eq. (13), since it is unlikely to have a large parton $k_T^2 = xQ^2$ demanded by the second term. That is, the tiny deviation (less than 5°) of the asymptotic phase from $180°$ is caused by the power-suppressed $k_T^2/Q^2$ effect. The NLO correction to the phase is about $30°$ at $Q^2 = 30$ GeV$^2$, and fewer than $20°$ for $Q^2 > 50$ GeV$^2$. The above investigation implies that higher-order corrections to the complex time-like transition form factors are under control in the $k_T$ factorization. As stated before, the perturbative expansion could be improved by resumming the double logarithm $\alpha_s \ln^2 x$ in Eq. (13).

For the analysis of the time-like pion EM form factor, we first identify the major source of the strong phase by comparing the results from Eqs. (11) and (13) in Fig. 3. The renormalization and factorization scale $\mu$ is set to $\mu = \max (\sqrt{x_1 Q, 1/b_1, 1/b_2})$ [20, 22], associated with the virtuality of the internal particles. The curve from Eq. (11) implies that the magnitude of the time-like pion EM form factor has an asymptotic behavior $Q^2 |F_{\text{EM}}(Q^2)| \to 0.14$ GeV$^2$ as $Q \to \infty$. Similar to the case of the transition form factor, this asymptotic value is larger than that of the space-like pion EM form factor [2], because of the inclusion of the parton transverse momenta $k_T$. The inclusion of $k_T$ also leads to the variation of the phase with $Q^2$, which arises from the first quadrant, and then approaches an asymptotic value close to $165°$. For $Q^2 > 15$ GeV$^2$, the difference between using the single-$b$ and double-$b$ convolutions is insignificant in both magnitude and phase, verifying the hierarchical relation $x_1 Q^2 > x_2 Q^2$, and the major source for the strong phase as the internal gluon propagator. We then investigate the NLO effect in the time-like pion EM form factor based on Eqs. (11) and (17), which is also shown in Fig. 3. For $Q^2 > 10$ GeV$^2$, the observed NLO correction is roughly 25% for the magnitude, and less than 10° for the phase. That is, the perturbative evaluation of the time-like pion EM form factor is stable against radiative corrections at leading twist.

At last, we include another piece of subleading effects, the LO two-parton twist-3 contribution [27], for completeness. Following the same derivation of the twist-2 contribution in Sec. III the $k_T$ factorization formula for the LO two-parton twist-3 contribution to the time-like pion EM form factor was obtained in [22], where an explicit double-$b$ convolution expression similar to Eq. (11) can be found. The hard kernel in the impact-parameter space is identical to the one in Eq. (11). We employ the asymptotic two-parton twist-3 DAs,

$$\phi^{\text{EM}}_{\pi}(x) = 1, \quad \phi^{T}_{\pi}(x) = 1 - 2x,$$

with the associated chiral scale $\mu_\pi = 1.3$ GeV. The threshold resummation factor $S_t(x, Q)$ with a shape parameter $c = 0.4$ is included, since the important double logarithm $\alpha_s \ln^2 x$ at small $x$ needs to be summed [20]. The numerical outcomes for the time-like pion EM form factor are presented in Fig. 4 where the available experimental data [28, 29] are displayed for comparison. It is known that the pion EM form factor is dominated by the two-parton twist-3 contribution, instead of by the twist-2 one at currently accessible energies, because of the end-point enhancement developed by the above DAs [22, 31, 31]. This enhancement is understood easily as follows: the virtual quark and gluon propagators behave like $1/x_1$ and $1/(x_1 x_2)$, respectively, as indicated in Eq. (10). The twist-2 pion DA is proportional to $\phi_\pi(x) \sim O(x)$, but the twist-3 pion DAs remain constant $\phi^{T}_{\pi}(x) \sim O(1)$ at small $x$, which then enhance the end-point contribution dramatically. This enhancement was also observed in perturbative evaluation of the $B \to \pi$ transition form factors [32], and confirmed by the light-cone sum-rule analysis [33]. The relative phase between the twist-2 and two-parton twist-3 pieces is about $70°$ as indicated by Figs. 4 and 5, so the magnitude of the
In this Letter we have calculated the time-like pion-photon transition and EM form factors up to NLO in the $k_T$ factorization formalism. The corresponding NLO hard kernels were derived by analytically continuing the space-like ones to the time-like region of the momentum transfer squared $Q^2$. We have identified the $k_T$-dependent internal gluon propagator as the major source for the strong phase of the time-like pion EM form factor, which increases with $Q^2$, and approaches an asymptotic value for $Q^2 > 3$ GeV$^2$. The magnitude of the time-like form factors is hardly affected by the former. However, the twist-2 contribution does have a sizable effect on the phase as illustrated in Fig. 5.

The predictions for the magnitude of the time-like pion EM form factor from the $k_T$ factorization can accommodate the data for $Q^2 > 4$ GeV$^2$, an observation consistent with that from the LO analysis. We point out that the measured magnitude of the time-like pion EM form factor is larger than the space-like one, and simultaneous accommodation of both data is possible in the $k_T$ factorization, but not in the collinear factorization. Though the perturbative calculations may not be justified for small $Q^2 < 4$ GeV$^2$, it is interesting to see the coincidence between the increases of the phase with $Q^2$ from the $k_T$ factorization and from the data for $Q^2 < 1.3$ GeV$^2$. In a Breit-Wigner picture, the observed phase increase could be attributed to a resonant $\rho$ meson propagator. It happens that the parton transverse momentum $k_T$ plays the role of the $\rho$ meson mass, such that the two curves in Fig. 5 exhibit the similar tendency, and begin to merge for $Q^2 > 1$ GeV$^2$. Again, this coincidence cannot be achieved in the collinear factorization, which does not generate a significant phase shift.

The consistency between the present analysis and the data supports the $k_T$ factorization formalism as an appropriate framework for studying complex time-like form factors. It has been understood that the complex penguin annihilation contribution is essential for explaining direct CP asymmetries in two-body hadronic $B$ meson decays. This contribution involves time-like scalar form factors, which can be calculated in the same $k_T$ factorization formalism. It has been observed that the phase of the $S$-wave component in $\pi\pi$ scattering shows a similar $Q^2$ dependence to that of the $P$-wave. Therefore, the PQCD predictions for the above direct CP asymmetries are expected to be reliable. The formalism for three-body hadronic $B$ meson decays has required the introduction of two-meson wave functions, whose parametrization also involves time-like form factors of various currents. Stimulated by our work, we have the confidence on computing these complex time-like form factors directly in the PQCD approach.

V. CONCLUSIONS

FIG. 4: Magnitude and phase of the time-like pion EM form factor at leading twist. Contributions from LO with the single-$b$ convolution (dotted), LO (dashed), and LO+NLO (solid) are shown.
FIG. 5: Contributions to the time-like pion EM form factor from two-parton twist-3 LO (dotted), two-parton twist-3 LO plus twist-2 LO (dashed), and two-parton twist-3 LO plus twist-2 up to NLO (solid).

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[1] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. B 242, 97 (1990); Nucl. Phys. B 366, 135 (1991).
[2] J.C. Collins and R.K. Ellis, Nucl. Phys. B 360, 3 (1991).
[3] E.M. Levin, M.G. Ryskin, Yu.M. Shabelski, and A.G. Shuvaev, Sov. J. Nucl. Phys. 53, 657 (1991).
[4] J. Botts and G. Sterman, Nucl. Phys. B 325, 62 (1989).
[5] H.-n. Li and G. Sterman, Nucl. Phys. B 381, 129 (1992).
[6] T. Huang and Q.X. Shen, Z. Phys. C 50, 139 (1991); J.P. Ralston and B. Pire, Phys. Rev. Lett. 65, 2343 (1990).
[7] S. Nandi and H.-n. Li, Phys. Rev. D 76, 034008 (2007).
[8] H.-n. Li and S. Mishima, Phys. Rev. D 80, 074024 (2009).
[9] H.-n. Li, Y.-L. Shen, Y.-M. Wang, and H. Zou, Phys. Rev. D 83, 054029 (2011).
[10] Y.Y. Keum, H-n. Li, and A.I. Sanda, Phys. Lett. B 504, 6 (2001); Phys. Rev. D 63, 054008 (2001); Y.Y. Keum and H.-n. Li, Phys. Rev. D 63, 074006 (2001); C.D. Lu, K. Ukai and M.Z. Yang, Phys. Rev. D 63, 074009 (2001).
[11] D. Muller et al., Fortschr. Physik. 42, 101 (1994); M. Diehl, T. Gousset, B. Pire, and O. Teryaev, Phys. Rev. Lett. 81, 1782 (1998); M.V. Polyakov, Nucl. Phys. B555, 231 (1999).
[12] R. Jakob and P. Kroll, Phys. Lett. B 315, 463 (1993); B 319, 545 (1993).
[13] S. J. Brodsky, T. Huang, and G. P. Lepage, SLAC-PUB-2540; S.J. Brodsky, P. Hoyer, C. Peterson, and N. Sakai, Phys. Lett. B 93, 451 (1980); S.J. Brodsky, C. Peterson, and N. Sakai, Phys. Rev. D 23, 2745 (1981).
[14] U. Raha and H. Kohyama, Phys. Rev. D 82, 114012 (2010).
[15] H.-n. Li, Phys. Rev. D 66, 094010 (2002); K. Ukai and H.-n. Li, Phys. Lett. B 555, 197 (2003).
[16] H.-n. Li, Phys. Rev. D 52, 3958 (1995).
[17] J.-W. Chen, H. Kohyama, K. Ohnishi, U. Raha, and Y.-L. Shen, Phys. Lett. B 693, 102 (2010).
[18] T. Gousset and B. Pire, Phys. Rev. D 51, 15 (1995).
[19] V. Braun, M. Gockeler, R. Horsley, H. Perlt, D. Pleiter, et al., Phys. Rev. D 74, 074501 (2006).
[20] S.J. Brodsky and G.P. Lepage, Phys. Rev. D 24, 1808 (1981).
[21] Bernard Aubert et al. Phys. Rev. D 74, 012002 (2006).
[22] Bernard Aubert et al. Phys. Rev. D 74, 012002 (2006).
[23] M. Nagashima and H.-n. Li, Eur. Phys. J. C 40, 395 (2005).
[24] M.R. Whalley, J. Phys. G 29, A1 (2003); J. Milana, S. Nussinov, and M.G. Olsson, Phys. Rev. Lett. 71, 2533 (1993); T.K. Pedlar et al., Phys. Rev. Lett. 95, 261803 (2005).
[25] S.D. Protopopescu, M. Alston-Garnjost, A. Barbaro-Galtieri et al., Phys. Rev. D 7, 1279 (1973).
[26] F-g. Cao, Y.-b. Dai, and C.-s. Huang, Eur. Phys. J. C 11, 501 (1999).
[27] T. Huang and X.-G. Wu, Phys. Rev. D 70, 093013 (2004).
[32] T. Kurimoto, H.-n. Li, and A. I. Sanda, Phys. Rev. D 65, 014007 (2002).
[33] A. Khodjamirian, R. Ruckl, and C. W. Winnhart, Phys. Rev. D 58, 054013 (1998).
[34] B. Ananthnarayan, I. Caprini and I. S. Imsong, Phys. Rev. D 85, 096006 (2012).