Hadronic Weak Decays of $\Lambda_b$ Baryon in the Covariant Oscillator Quark Model

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Cabibbo allowed two-body hadronic weak decays of $\Lambda_b$ baryons are analyzed in the factorization approximation. We use the covariant oscillator quark model to evaluate the heavy $\rightarrow$ heavy and heavy $\rightarrow$ light form factors. When applied in the heavy quark limit, our form factors satisfy all the constraints imposed by heavy quark symmetry. The decay rates and up-down asymmetries for $\Lambda_b$ baryon decaying into $\Lambda_c + P(V)$ are calculated. It is found that the up-down asymmetry is negative in all these decay modes. Furthermore, the prediction $\text{Br}(\Lambda_b \rightarrow \Lambda J/\psi) = 2.49 \times 10^{-4}$ is consistent with the recent experimental data. Finally it is pointed out that the CKM-Wolfenstein parameter $\rho^2 + \eta^2$, where $\eta$ is the CP phase, can be determined from the ratio of the widths $\Lambda_b \rightarrow \Lambda D^0$ and $\Lambda_b \rightarrow \Lambda J/\psi$, independent of the QCD parameter. The value of $(\rho^2 + \eta^2)^{1/2}$ calculated in our model agrees very well with the value recently predicted by Rosner.

§1. Introduction

Progress in understanding nonleptonic weak decays of bottom baryons has been very slow both theoretically and experimentally. While some new data of charmed baryon nonleptonic weak decays have become available, the experimental situation for hadronic weak decays of bottom baryons is meagre. The decay mode $\Lambda_b \rightarrow \Lambda J/\psi$ is the first successful measurement of the exclusive hadronic decay rate of bottom baryons. In the near future, one can expect new data on exclusive bottom baryon decays calling for a comprehensive theoretical analysis of these decay modes. These decay processes can provide useful information for QCD effects in weak decays and indirect CP asymmetry which involve the CKM-Wolfenstein parameters $\rho$ and $\eta$. However, a rigorous and reliable approach suitable for analyzing these decays does not exist at this time, and one must rely on some approximation to deal with them. The well-known factorization approach, which has been applied successfully to nonleptonic $B$ meson decays, can also be applicable to bottom baryon decays. In this approximation the effects of final state interactions (FSI) are neglected. These interactions are thought to be less important in bottom baryon decay, since the decay particles in the two-body final state are energetic and moving fast, allowing little time for significant final state interactions.
In previous papers, we made analyses of exclusive semi-leptonic and non-leptonic decays of $B$ mesons by using the covariant oscillator quark model (COQM), leading to satisfactory results. In this paper we discuss the nonleptonic decays $\Lambda_b \to \Lambda_c P(V)$ and $\Lambda_b \to \Lambda J/\psi$ using this model. One of the most important motives for COQM is to covariantly describe the centre-of-mass motion of hadrons, retaining the considerable successes of the non-relativistic quark model on the static properties of hadrons. As a result, in COQM we can treat both static and non-static problems simultaneously. A common systematic treatment of the systems with any quark flavor configuration is another feature of COQM. A key point in COQM for doing this is to treat directly the squared masses of hadrons in contrast to the mass itself, as is done in conventional approaches. This makes the covariant treatment simple. The COQM has been applied to various problems with satisfactory results.

Here we would like to note that, when applied to heavy-to-heavy baryon transitions, our model predictions satisfy the constraints imposed by the heavy quark symmetry, suggesting that COQM is reliable. Moreover, our model is also applicable for heavy-to-light transitions, which are beyond the scope of heavy quark effective theory (HQET).

We organise the paper as follows. Assuming factorization, in §2 we present the expressions for the decay rates and the up-down asymmetry parameter $\alpha$. We use the covariant oscillator quark model to evaluate the baryonic form factors. Results and discussion are presented in §3.

### §2. Methodology

#### 2.1. Effective weak Hamiltonian and factorized amplitude

Neglecting the penguin contribution, the effective Hamiltonian describing the decays under consideration is given by

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_i q_j} \left[ a_1 O_1 + a_2 O_2 \right]$$

(2.1)

with

$$O_1 = (\bar{q}_i q_j)^\mu (\bar{c} b)_\mu \quad \text{and} \quad O_2 = (\bar{q}_i b)^\mu (\bar{c} q_j)_\mu ,$$

(2.2)

where $a_1$ and $a_2$ correspond to the external and internal $W$ emission amplitudes. The quark current $(\bar{q}_i q_j)_\mu = \bar{q}_i \gamma_\mu (1 + \gamma_5) q_j$ denotes the usual $(V - A)$ current. $q_i$ and $q_j$ are two types of quark flavors that are hadronized to $P$ or $V$ mesons. Here Eq. (2.1) is to be understood as an effective Hamiltonian after considering the Fierz rearrangement.

In the factorization approximation, the baryon decay amplitude is factorized into a product of two matrix elements of the form $\langle B'| J_\mu | B \rangle$, which involves the form factors, and $\langle P(V)| J^\mu | 0 \rangle$, which is expressed by the meson decay constant. The factorized amplitude for the decay processes $\Lambda_b \to \Lambda_c P(V)$, which proceed via the
external $W$ emission is given as

$$\mathcal{M}(A_b \to A_c P(V)) = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_i q_j} a_1 \langle P(V) | (\bar{q}_i q_j) \mu | 0 \rangle \langle \Lambda_c | (\bar{c} \beta) | A_b \rangle . \quad (2.3)$$

Similarly, the amplitude for the decay mode $A_b \to \Lambda J/\psi$, which is described by the internal $W$ emission, is given by Eq. (2.3) in which $a_1$ is replaced by $a_2$. The current matrix elements between a pseudoscalar/vector/axialvector meson $(P/V/a_1)$ and the vacuum are related to the corresponding decay constants as

$$\langle P(p) | (\bar{q}_i q_j) \mu | 0 \rangle = -i f_P \not{p} \mu ,$$

$$\langle V(p, \epsilon) | (\bar{q}_i q_j) \mu | 0 \rangle = M_V f_V \epsilon \mu ,$$

$$\langle a_1(p, \epsilon) | (\bar{q}_i q_j) \mu | 0 \rangle = M_{a_1} f_{a_1} \epsilon \mu , \quad (2.4)$$

where $f_P$, $f_V$ and $f_{a_1}$ are the respective decay constants. To evaluate the baryon form factors, we use the covariant oscillator quark model, which is explicitly presented in the next section.

2.2. Model framework and the Hadronic form factors

The general treatment of COQM may be called the “boosted LS-coupling scheme,” and the wavefunctions, being tensors in $U(4) \times O(3,1)$-space, reduce to those in the $SU(2)_{\text{spin}} \times O(3)_{\text{orbit}}$-space in the nonrelativistic quark model in the hadron rest frame. The spinor and space-time portion of the wave functions separately satisfy the respective covariant equations, the Bargmann-Wigner (BW) equation for the former and the covariant oscillator equation for the latter. The model parameters and form of the wave function are determined completely through the analysis of mass spectra.

In COQM all the non-exotic $qqq$ baryons are described by tri-local fields $\Phi_{A_1 A_2 A_3}(x_1, x_2, x_3)$, where the $x_i$ are Lorentz four vectors representing the space time coordinates of constituent quarks, $A_1 = (a, \alpha)$, $A_2 = (b, \beta)$, $A_3 = (c, \gamma)$, describing the flavor and covariant spinor of the quarks. The baryon fields are assumed to satisfy wave equations of the Klein-Gordon type and expanded in terms of Fierz components (that is, the eigenfunctions of $\mathcal{M}^2$), and are written as

$$\Phi(X, r, \cdots) = \sum_{P,n} \left( e^{i P_n X} \psi_n^{(+)}(r, \cdots, P_n) + e^{-i P_n X} \psi_n^{(-)}(r, \cdots, P_n) \right) , \quad (2.5)$$

$$\mathcal{M}^2 (r_\mu \cdots, \partial / \partial r_\mu \cdots) \psi_n^{(\pm)}(r, \cdots, P_n) = M_n^2 \psi_n^{(\pm)}(r, \cdots, P_n) , \quad (2.6)$$

where $X_\mu(r, \cdots)$ is the centre-of-mass (relative) coordinate, and $\mathcal{M}^2$ is the squared mass operator depending on relative coordinate variables. The first (second) term of Eq. (2.3) corresponds to the positive (negative) frequency part of the centre-of-mass plane-wave motion with definite total four momentum $P_n$ and mass $M_n = \sqrt{-P_n^2}$.

The spin portions $U_n$ ($\tilde{U}_n$) of positive (negative) frequency internal wave functions $\psi_n^{(+)}(r, \cdots, P_n) \equiv U_n(P) f_n(r, \cdots, P_n)$ ($\psi_n^{(-)} \equiv \tilde{U}_n f_n$), satisfy the respective

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We also treat the decay $A_b \to A_c + a_1$, $a_1$ being the axial-vector meson.
Bargmann–Wigner (BW) equations. The BW spinor functions are reducible and decompose into the irreducible components as

\[
U_{n,ABC} = \frac{1}{\sqrt{2}} \left[ -\gamma_5 (1 + iv \cdot \gamma) C^{-1} \right]_{\alpha\beta} u_{n,abc;\gamma}^{(A)} + \frac{1}{\sqrt{2}} \left[ i\gamma_\mu (1 + iv \cdot \gamma) C^{-1} \right]_{\alpha\beta} 
\times \left[ u_{n,abc;\gamma,\mu}^{(S^\ast)} + \frac{1}{\sqrt{3}} \{ i\gamma_\mu + v_\mu \gamma_5 \} u_{n,abc;\gamma}^{(S)} \right],
\]

(2.7)

where \( v_\mu \) is the four velocity of the hadron \( (v_\mu = P_\mu/M) \) and \( C \) is the antiparticle conjugate matrix. The quantities \( u_{n}^{(A)} \), \( u_{n,\mu}^{(S^\ast)} \) and \( u_{n}^{(S)} \) denote the spin-1/2 ‘Dirac spinor’, spin-3/2 ‘Rarita-Schwinger vector-spinor’, and the spin-1/2 ‘Dirac spinor’ Fierz components, respectively. It may be noted that in the \( \Lambda_Q \)-type baryons, the two light quarks are in the flavor antisymmetric and spin-0 state. Thus the BW spinor function for \( \Lambda_Q \)-type baryons is given by the first term of Eq. (2.7).

The oscillator space-time wave function for the ground state baryons is given by

\[
f(P, \rho, \lambda) = f_\rho(P; \rho) f_\lambda(P; \lambda),
\]

(2.8)

where

\[
f_\rho(P; \rho) = \beta_\rho \pi \exp \left( -\frac{\beta_\rho}{2} \left( \frac{P \cdot \rho}{M^2} \right) \right),
\]

(2.9)

and

\[
f_\lambda(P; \lambda) = \beta_\lambda \pi \exp \left( -\frac{\beta_\lambda}{2} \left( \frac{P \cdot \lambda}{M^2} \right) \right),
\]

(2.10)

with

\[
\beta_\rho = \frac{\sqrt{3} m K}{4} \quad \text{and} \quad \beta_\lambda = \sqrt{\frac{m M K}{2m + M}}.
\]

(2.11)

In the above equation, \( m \) and \( M \) denote the mass of the light (heavy) quark, and \( K \) is the universal spring constant for all hadronic systems with the value \( K = 0.106 \) GeV\(^3\).[^1] The internal relative coordinates are defined as

\[
\rho_\mu = x_{2\mu} - x_{1\mu} \quad \text{and} \quad \lambda_\mu = x_{3\mu} - \frac{x_{1\mu} + x_{2\mu}}{2}.
\]

(2.12)

The effective action for weak interactions of baryons with \( W \)-bosons is given by

[^1]: In this paper we apply the pure-confining approximation, neglecting the effect of the one-gluon-exchange potential \( U_{OGE} \). This approximation is expected, aside from the spin-dependent structure, to be good for the light-light \( q\bar{q} \) and heavy-light \( Q\bar{q} \) meson systems, where the reduced mass of the system is small comparatively and the effect of the central potential out of \( U_{OGE} \) is expected to be not so large. A similar situation is also expected to be valid for the relevant \( Qqq \) (or \( qqq \)) baryon systems. The well-known phenomenological fact of a linearly-rising Regge trajectory for the \( q\bar{q} \) and \( qqq \) systems seems to support this conjecture. (See the papers referred to in Ref. [3], published in 1993 and 1994.)

[^2]: In this paper we apply the values of \( \beta \) corresponding to the case (A) in Ref. [7]. The values of the branching ratios with \( \beta \) corresponding to the case (B) generally become slightly smaller (by \( \lesssim 20\% \)).
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\[
S_W = \int d^4x_1 d^4x_2 d^4x_3 \langle \Phi_{F,P'}(x_1, x_2, x_3) C'BA \rangle i\gamma_\mu (1 + \gamma_5) \gamma^\gamma \times \Phi_{I,P}(x_1, x_2, x_3)_{AB} C_W \mu, q(x_3),
\]

where we have omitted the CKM matrix elements and the coupling constants. This is obtained from the consideration of covariance, assuming a quark current with the standard $V-A$ form. Here $\Phi_{I,P}$ ($\Phi_{F,P'}$) denotes the initial (final) baryons with definite four momentum $P_\mu$ ($P'_\mu$), and $q_\mu$ is the momentum of the $W$ boson. The notation $\langle \rangle$ represents the trace of Dirac spinor indices. Concentrating only on $\Lambda_Q(Qud) \rightarrow \Lambda_Q(Q'ud)$, the relevant effective currents $J_\mu(X)_{P', P}$ are obtained by identifying the above action with

\[
S_W = \int d^4X J_\mu(X)_{P', P} W_\mu(X) q.
\]

Then $J_\mu(X = 0)_{P', P} \equiv J_\mu$ is explicitly given as,

\[
I_{Q'Q} \mu = \frac{1}{w} \left( \frac{4\beta_\lambda \beta'_\lambda}{\beta_\lambda + \beta'_\lambda} \right) \frac{1}{\sqrt{C(w)}} \exp(-G(w)),
\]

where

\[
C(w) = (\beta_\lambda - \beta'_\lambda)^2 + 4\beta_\lambda \beta'_\lambda w^2,
\]

and

\[
G(w) = \frac{4m_Q^2(\beta_\lambda + \beta'_\lambda) w(w - 1)}{(\beta_\lambda - \beta'_\lambda)^2 + 4\beta_\lambda \beta'_\lambda w^2}.
\]

The form factor function $I_{ud}^{Q'Q}(w)$ for $\Lambda_b \rightarrow \Lambda_c$ decays corresponds to the baryonic Isgur-Wise function $\eta(w)$ in HQET. At the zero recoil point $w = 1$, the value of $I_{ud}^{Q'Q}(w)$ is given by

\[
I_{ud}^{Q'Q}(w = 1) = \frac{1}{w} \frac{4\beta_\lambda \beta'_\lambda}{(\beta_\lambda + \beta'_\lambda)^2}.
\]

In the heavy quark symmetry limit we have $\beta_\lambda = \beta'_\lambda$, so that Eq. (2.19) correctly reproduces the normalization condition of HQET, i.e., $\eta(w = 1) = 1$. However, HQET as it is predicts nothing about the Isgur-Wise function except for the zero recoil point, while in COQM the form factor functions can be derived at any kinematical point of interest. In addition, the functional form Eq. (2.16) of the COQM form factor $I_{ud}^{Q'Q}(w)$ also applies to the heavy-to-light transition processes, whereas HQET does not provide anything for this sector.
2.3. Decay rates and asymmetry parameters

After obtaining the effective current in the COQM with Eqs. (2.3) and (2.15), one can write the transition amplitude for the decay mode $A_b \rightarrow A_c P$ as

\[ \mathcal{M}(A_b(v) \rightarrow A_c(v')P(p)) = \frac{G_F}{2\sqrt{2}} V_{cb} V_{q_i q_j} a_1 f_P \ p^\mu I_{ud}^{cb}(w) (w + 1) \]

\[ \times \bar{u}_{A_c}^{\nu}(v') \gamma_\mu(1 + \gamma_5) \ u_{A_{bad}}(v), \quad (2.20) \]

and the corresponding decay width as

\[ \Gamma(A_b \rightarrow A_c P) = \frac{G_F^2}{32\pi M_{A_b}^2} |V_{cb} V_{q_i q_j}|^2 a_1^2 f_P^2 \ (I_{ud}^{cb}(w))^2 (w + 1)^2 |p| \]

\[ \times \left[ (M_{A_b}^2 - M_{A_c}^2)^2 - M_V^2 (M_{A_b}^2 + M_{A_c}^2) \right], \quad (2.21) \]

where $p$ is the c.m. momentum of the emitted particles in the rest frame of the parent $A_b$ baryon.

To obtain the asymmetry parameter $\alpha$, we write Eq. (2.20), substituting $p^\mu = M_{A_b} v^\mu - M_{A_c} v^\mu$ as

\[ \mathcal{M}(A_b \rightarrow A_c P) = f_P \ \bar{u}_{A_c}^{\nu}(v') (G_1 + \gamma_5 G_2) \ u_{A_{bad}}(v), \quad (2.22) \]

where

\[ G_1 = \lambda(M_{A_b} - M_{A_c}), \quad G_2 = -\lambda(M_{A_b} + M_{A_c}), \quad (2.23) \]

with

\[ \lambda = \frac{G_F}{2\sqrt{2}} V_{cb} V_{q_i q_j} a_1 I_{ud}^{cb}(w) (1 + w). \quad (2.24) \]

In terms of the new form factors $G_1$ and $G_2$, the asymmetry parameter $\alpha$ is given by

\[ \alpha = \frac{2G_1 G_2 |p|}{(E_{A_c} + M_{A_c}) G_1^2 + (E_{A_c} - M_{A_c}) G_2^2}. \quad (2.25) \]

To obtain the decay rate for the $A_b \rightarrow A_c V$ transition, we write the transition amplitude for the process as

\[ \mathcal{M}(A_b(v) \rightarrow A_c(v')V(p, \epsilon)) = i \frac{G_F}{2\sqrt{2}} V_{cb} V_{q_i q_j} a_1 f_V \ M_V \ e^\mu I_{ud}^{cb}(w) (w + 1) \]

\[ \times \bar{u}_{A_c}^{\nu}(v') \gamma_\mu(1 + \gamma_5) \ u_{A_{bad}}(v). \quad (2.26) \]

The corresponding decay width is given by

\[ \Gamma(A_b \rightarrow A_c V) = \frac{G_F^2}{32\pi M_{A_b}^2} |V_{cb} V_{q_i q_j}|^2 a_1^2 f_V^2 \ (I_{ud}^{cb}(w))^2 (w + 1)^2 |p| \]

\[ \times \left[ (M_{A_b}^2 - M_{A_c}^2)^2 + M_V^2 (M_{A_b}^2 + M_{A_c}^2 - 2M_V^2) \right]. \quad (2.27) \]

To obtain the asymmetry parameter, we compare Eq. (2.26) to the general form in the rest frame of $A_b$,

\[ \chi_{A_c}^T [S \sigma + P_1 \hat{p} + iP_2 (\hat{p} \times \sigma) + D(\sigma \cdot \hat{p}) \hat{p}] \cdot \epsilon \chi_{A_b}, \quad (2.28) \]
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with $\hat{p}$ now a unit vector in the direction of the $\Lambda_c$ baryon. The values for the four amplitudes $S$, $P_1$, $P_2$ and $D$ are given as

$$ S = i \frac{G_F}{\sqrt{2}} V_{cb} V_{q_i} a_1 f_V M_V (T_{i}^{q}(w))(1 + w) , $$  

(2.29)

$$ P_1/S = - \left( \frac{M_{\Lambda_b} + M_{\Lambda_c}}{E_{\Lambda_c} + M_{\Lambda_c}} \right) \left( \frac{|p|}{E_V} \right) , $$  

(2.30)

$$ P_2/S = \left( \frac{|p|}{E_{\Lambda_c} + M_{\Lambda_c}} \right) , $$  

(2.31)

$$ D/S = \left( \frac{|p|^2}{E_V (E_{\Lambda_c} + M_{\Lambda_c})} \right) . $$  

(2.32)

From these expressions, the up-down asymmetry $\alpha$ of the final $\Lambda_c$ with respect to $\Lambda_b$ polarization is given by

$$ \alpha = 2\text{Re} \left[ \frac{(1 + D/S)^* P_1/S + 2(P_2/S) M^2_{\Lambda_c}/E^2_V}{K} \right] , $$  

(2.33)

where

$$ K = [|1 + D/S|^2 + |P_1/S|^2 + 2(1 + |P_2/S|^2) M^2_{\Lambda_c}/E^2_V] . $$  

(2.34)

The decay rate for $\Lambda_b \to \Lambda J/\psi$ is given in analogy to Eq. (2.27) with $a_1$ replaced by $a_2$ and $f_V$ replaced by $f_{J/\psi}$, with the relevant CKM matrix elements as $|V_{cb}V_{cs}|^2$.

The COQM is applicable not only for heavy-to-heavy transitions but also for heavy-to-light transitions, where the final baryon moves extremely relativistically, as was mentioned in §1. Here we consider the case of the decay mode $\Lambda_b \to \Lambda \bar{D}^0$ to obtain the CKM-Wolfenstein parameter $(\rho^2 + \eta^2)$ from the ratio of the the decay widths $\Gamma(\Lambda_c \to \Lambda \bar{D}^0)/\Gamma(\Lambda_b \to \Lambda J/\psi)$. The decay width for the process $\Lambda_b \to \Lambda \bar{D}^0$ is given as

$$ \Gamma(\Lambda_b \to \Lambda \bar{D}^0) = \frac{G_F^2}{32\pi M_{\Lambda_b}} |V_{ub}V_{cs}|^2 a_2^2 f_D^2 (T_{i}^{s}(w))^2 (w + 1)^2 |p| \times \left[ (M^2_{\Lambda_b} - M^2_{\Lambda_c})^2 - M^2_D (M^2_{\Lambda_b} + M^2_{\Lambda_c}) \right] . $$  

(2.35)

§3. Results and conclusion

In order to make a numerical estimate, we use the following values of various quantities. The quark masses (in GeV) are taken as $m_u = m_d = 0.4$, $m_s = 0.51$, $m_c = 1.7$ and $m_b = 5$, which are determined from the analysis of meson mass spectra. The particle masses and lifetimes are taken from Ref. 14. The relevant CKM parameters used are $V_{cd} = 0.0395$, $V_{cs} = 1.04$, $V_{cd} = 0.224$, $V_{ud} = 0.974$ and $V_{us} = 0.2196$. The decay constants are taken as $f_{\pi} = 130.7$, $f_K = 159.8$; $f_{K^*} = 214$, $f_{\rho} = 210$, $f_D = 220$, $f_{D^*} = 230$, $f_{D_s} = 240$, $f_{D_{s^*}} = 260$ and
Table I. Branching ratios (in percent) for different nonleptonic \( \Lambda_b \) decay processes and their asymmetry parameters \( \alpha \).

| Decay processes          | \( \alpha \) | Br. ratio in % |
|--------------------------|-------------|--------------|
| \( \Lambda_b \rightarrow \Lambda^\circ \pi^- \) | -0.999     | 0.175        |
| \( \Lambda_b \rightarrow \Lambda^\circ K^- \) | -1.000     | 0.013        |
| \( \Lambda_b \rightarrow \Lambda^\circ D^- \) | -0.987     | 0.030        |
| \( \Lambda_b \rightarrow \Lambda^\circ D_s^- \) | -0.984     | 0.77         |
| \( \Lambda_b \rightarrow \Lambda^\circ \rho^- \) | -0.898     | 0.491        |
| \( \Lambda_b \rightarrow \Lambda^\circ K^{*-} \) | -0.865     | 0.027        |
| \( \Lambda_b \rightarrow \Lambda^\circ D_s^{*-} \) | -0.459     | 0.049        |
| \( \Lambda_b \rightarrow \Lambda^\circ \alpha \) | -0.758     | 0.532        |
| \( \Lambda_b \rightarrow \Lambda J/\psi \) | -0.208     | \( 2.55 \times 10^{-2} \) |

Table II. Branching ratios (in percent) for different \( \Lambda_b \) processes and comparison with other calculations.

| Decay processes | This work | Ref. [22] Large \( N_c \) | Ref. [23] Large \( N_c \) | Ref. [24] Large \( N_c \) | Ref. [25] Large \( N_c \) |
|-----------------|-----------|----------------------------|---------------------------|---------------------------|---------------------------|
| \( \Lambda_b \rightarrow \Lambda^\circ \pi^- \) | 0.175     | 0.38                       | -                         | 0.391                     | 0.503                     |
| \( \Lambda_b \rightarrow \Lambda^\circ K^- \) | 0.013     | -                          | -                         | -                         | -                         |
| \( \Lambda_b \rightarrow \Lambda^\circ D^- \) | 0.030     | -                          | -                         | -                         | -                         |
| \( \Lambda_b \rightarrow \Lambda^\circ D_s^- \) | 0.77      | 1.1                        | 2.23                      | 1.291                     | -                         |
| \( \Lambda_b \rightarrow \Lambda^\circ \rho^- \) | 0.491     | 0.54                       | -                         | 1.082                     | 0.723                     |
| \( \Lambda_b \rightarrow \Lambda^\circ K^{*-} \) | 0.027     | -                          | -                         | -                         | 0.037                     |
| \( \Lambda_b \rightarrow \Lambda^\circ D_s^{*-} \) | 0.049     | -                          | -                         | -                         | -                         |
| \( \Lambda_b \rightarrow \Lambda J/\psi \) | 2.49 \times 10^{-2} | 1.6 \times 10^{-2} | 6.037 \times 10^{-2} | -                         | -                         |

\( f_{a_1} = 205^{[12]} \) (in MeV). The decay constant \( f_{J/\psi} \) is determined from the value of \( \Gamma(J/\psi \rightarrow e^+ e^-) \):\[14\]

\[
f_{J/\psi} = \sqrt{\frac{9}{4} \left( \frac{3}{4 \pi \alpha^2} \right)} \Gamma(J/\psi \rightarrow e^+ e^-) M_{J/\psi} = 404.5 \text{ MeV}.
\]

The parameters \( a_1 \) and \( a_2 \) appearing in these decays have recently been extracted from the CLEO data and turn out to be \( a_1 = 1.05 \) and \( a_2 = 0.25 \).\[19\] Using these values we have obtained the values of the branching ratios and the asymmetry parameter \( \alpha \) for several nonleptonic \( \Lambda_b \) decays. These values are tabulated in Table I. The asymmetry parameter \( \alpha \) in all these decay modes is found to be negative, which indicates the \( V - A \) nature of the weak current. Our predicted branching ratio for the decay process \( \Lambda_b \rightarrow \Lambda J/\psi \) \( (2.55 \times 10^{-4}) \) is consistent with the recent experimental data\[14\] \( (4.7 \pm 2.7 \times 10^{-4}) \): Here it may be worthwhile to note that the final \( \Lambda \) in this process experiences relativistic motion with velocity \( v_A = 0.84c \), and the form factor function plays a significant role taking a value \( I(w) = 0.11 \).
Table III. Asymmetry parameter $\alpha$ for different $\Lambda_b$ processes and comparison with other calculations.

| Decay processes                  | This work | Ref. [22] | Ref. [23] | Ref. [25] |
|----------------------------------|-----------|-----------|-----------|-----------|
| $\Lambda_b \to \Lambda^+_b \pi^-$| $-0.999\pm0.02$ | $-0.99\pm0.02$ | -1.00     | -1.00     |
| $\Lambda_b \to \Lambda^+_b K^-$  | $-1.000\pm0.02$ | -       | -1.00     | -1.00     |
| $\Lambda_b \to \Lambda^+_b D^-$  | $-0.987\pm0.02$ | -       | -         | -         |
| $\Lambda_b \to \Lambda^+_b D_s^-$| $-0.984\pm0.02$ | $-0.99\pm0.02$ | $-0.98\pm0.02$ | -         |
| $\Lambda_b \to \Lambda^+_b \rho^-$| $-0.898\pm0.02$ | $-0.88\pm0.02$ | -         | $-0.885\pm0.02$ |
| $\Lambda_b \to \Lambda^+_b K^{*-}$| $-0.865\pm0.02$ | -       | -         | $-0.885\pm0.02$ |
| $\Lambda_b \to \Lambda^+_b D^{**}$| $-0.459\pm0.02$ | -       | -         | -         |
| $\Lambda_b \to \Lambda^+_b D_s^{**}$| $-0.419\pm0.02$ | $-0.36\pm0.02$ | $-0.40\pm0.02$ | -         |
| $\Lambda_b \to \Lambda_c \eta_1$  | $-0.758\pm0.02$ | -       | -         | -         |
| $\Lambda_b \to \Lambda J/\psi$   | $-0.208\pm0.02$ | $-0.1\pm0.02$ | $-0.18\pm0.02$ | -         |

From the ratio of the decay widths $\Lambda_b \to \Lambda D^0$ and $\Lambda_b \to \Lambda J/\psi$, we obtain

$$\frac{\Gamma(\Lambda_b \to \Lambda D^0)}{\Gamma(\Lambda_b \to \Lambda J/\psi)} = 10.235 \times 10^{-2} |V_{ub}/V_{cb}|^2 = 4.936 \times 10^{-3} (\rho^2 + \eta^2). \quad (3.2)$$

Substituting the value $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$, which is measured in charmless $b$ decays, into this relation, we obtain from Eq. (3.2)

$$(\rho^2 + \eta^2)^{1/2} = 0.364 \pm 0.091, \quad (3.3)$$

which is in excellent agreement with the recent prediction $\rho^2 + \eta^2)^{1/2} = 0.36 \pm 0.09$.

In this paper we have calculated the branching ratios of the exclusive nonleptonic decays of $\Lambda_b$ baryons using the covariant oscillator quark model, on the basis of the factorization approximation. The manifestly covariant weak currents are given by the overlapping integrals between the initial and final hadron wave functions. These currents are represented by a common form factor function, and for heavy $\to$ heavy baryon transitions this form factor function corresponds to the Isgur-Wise function of HQET and has similar properties at the zero recoil point. Using these currents we have derived the decay rates and up-down asymmetry parameter $\alpha$ for various nonleptonic weak decays of $\Lambda_b$ baryons. Our predicted result for the branching ratio $Br(\Lambda_b \to \Lambda J/\psi)$ is in agreement with the currently available experimental data. Recently, these decay processes have been studied using the quark model $\rho^2 + \eta^2)^{1/2}$ and HQET $\rho^2 + \eta^2)^{1/2}$ in the large $N_c$ limit. However, our predicted results for the branching ratios are smaller than the previous values, as can be seen from Table II. Future experimental data from the colliders are expected to verify and distinguish the various results. However, the values of the asymmetry parameter in all these calculations are nearly the same. Furthermore, the CKM-Wolfenstein parameter $\rho^2 + \eta^2)^{1/2}$ obtained in the framework of our model agrees very well with the recent prediction.
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