Dissolved-gas influence on the Francis part-load oscillation

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Abstract. A forced oscillation of system pressure occurs when a Francis turbine operates at partial load, typically between 50-85% of best-efficiency flow. Its cause is the precession of the draft tube vortex; due to cavitation of the vortex, the oscillation may enter in resonance. Predicting such a resonance from reduced-scale model tests is a long-standing goal of hydraulic research. Key parameter controlling the natural frequency of the underlying eigenmode is the compressibility of the cavitation zone. With the vortex cavity at vapor pressure, the natural frequency would scale by the same factor as the runner speed, thus enabling simple resonance prediction based on the model test. Based on experimental evidence, the actual pressure inside the vortex cavity is higher than vapor pressure, and the non-condensable gas content has significant influence. Due to uncontrolled variation of cavity pressure, the natural frequency in model tests may deviate by as much as 30% under nominally identical conditions. A 1D mathematical model can represent the influence of non-condensable gas on the cavity volume and compliance; even quite moderate cavity pressures explain the observed discrepancies.

1. Introduction
The part-load pulsation is the most common flow-excited oscillation in Francis turbines and other single-regulated reaction turbines. At partial load, the flow from the runner has a swirl component and the resulting vortex takes a corkscrew shape that rotates about the draft tube axis [1]. In an elbow-shaped draft tube, the interaction between the vortex and the elbow boundary produces a near-periodic axial forcing momentum, and therefore also a pulsation of turbine discharge and shaft power.

The submergence of the turbine can normally not suppress cavitation at the vortex core; cavitation renders the flow in the draft tube cone highly compressible and thus creates a low-frequency eigenvalue [2]. Its natural frequency and the forcing frequency are of the same order of magnitude; resonance is not uncommon and may produce intense pressure and power swings.

1.1. Requirements of similarity for reduced-scale model tests
To predict such conditions for a project should be an objective of the customary pressure fluctuation tests on the reduced-scale model [3]. The experimental approach is, however, faced with practical difficulties. Straightforward prediction by applying scaling factors requires that (a) all important laws of similarity are fulfilled and, in addition, (b) all important boundary conditions are reproduced. Geometric similarity of the flow-carrying components, as well as kinematic similarity of steady-state flow is already required for correct representation of steady-state performance. Friction effects have been shown to be uncritical [4] because for Reynolds numbers in excess of 80,000 the main oscillation parameters become Re-independent while model tests are normally done at much higher Re. The crucial similarity problem is connected with the cavitation phenomenon. The fluid volume $V_C$...
displaced by cavitation and its response to flow and pressure fluctuations at the runner exit have important influence on the pulsation amplitudes. The shape and size of the cavitation zone depends on the surrounding flow pattern and on the Thoma number $\sigma$. The Thoma criterion is only fulfilled over any large vertical extent if Froude similarity is respected at the same time. The same scale factor should thus apply for runner diameter $D$ and for net head $H$, or specific energy $E$. It is also usually expected that reproducing the Thoma and Froude number preserves the relative natural frequencies; but this assumption is valid only if the internal pressure of the cavity persists at vapor pressure.

1.2. Dealing with boundary conditions

The IEC standard for model testing [3] cautions that amplitudes must not be expected to be transposable from model to prototype if the pulsation interacts with the hydraulic system. This warning is perfectly justified because such interaction is not an exception but actually unavoidable. The ‘synchronous’ part of the pulsation – the one that is prone to resonance – inevitably depends on interaction with both the upstream and downstream water column. The dynamic response of those parts of the test rig, in particular the upstream part, is almost never equivalent to the conditions in the power plant. Once this has been recognized in the early 1980s [10], the logical consequence was to identify physically meaningful and scalable parameters of the problem from the model test, transpose them into a hydraulic transients model of the power plant and simulate the behavior of the prototype.

1.3. The forgotten parameter - non-condensable gas content

Equal Thoma cavitation number $\sigma$ between otherwise similar machines provides similar extent of cavitation if the internal pressure of the cavities is the vapor pressure. Cavity pressure in the draft tube vortex of Francis models has been measured around 1970 [5][6][7]; it was found to be about 10-50kPa, one order of magnitude above vapor pressure. No models for the part-load pulsation existed at that time, hence there was no possible application for the test results. A 1D vortex model that could accommodate such data was proposed in 2016 [8], but in the meantime the cavity pressure was no longer given attention. Nevertheless, recent research [9] revealed considerable and variable air content in a cavitating propeller tip vortex. The present paper describes phenomena that apparently can only be explained by increased cavity pressure due to non-condensable gas out of the working fluid.

2. 1D Vortex Model

In the simplest possible model, the compressibility of the cavitation zone can be described by a single parameter, the cavitation compliance

$$C_C = - \frac{\partial V_C}{\partial p_C} \quad (1)$$

$C_C$ decreases monotonously over the surrounding pressure $p_C$; it also depends on the turbine discharge coefficient $\varphi$ (or $Q_{in}$) controlling the swirl ratio. If the upstream fluid-dynamic impedance is not too low, then $C_C$ together with the inertia $I_{DT}$ of the water mass in the draft tube

$$I_{DT} = \rho g \int ds / A(s) \quad (2)$$

acts as a mass-spring oscillator with the natural frequency

$$\omega_0 = (C_C I_{DT})^{1/2} \quad (3)$$

Due to the variation of $C_C$, this ‘natural frequency’ of the downstream subsystem depends both on the operating conditions and on the draft tube pressure level. If it coincides with the forcing frequency, then the synchronous pulsation enters in resonance. As shown in [8] the dependency between $C_C$ and the cavitation number $\sigma$ can often be approximated by an exponential law

$$C_C = a \exp(b \sigma) \quad (4)$$

In particular, this holds for the models in subsections 3.1 and 3.2, as also shown in [8]. It follows from (1) and (4) that $V_C$ is also an exponential function of $\sigma$ with exponent $b$. If two similar machines (for
instance model and prototype) fulfill both Thoma and Froude similarity (together with similar geometry and operating conditions), then theoretically $C_c$ and $V_c$ should scale like $D^3/E$ and $D^3$ because the size and compliance of the vortex depend on centrifugal and gravity forces only. In case of significant gas content, however, this similarity is lost because the pressure and bulk modulus of the gas content do not scale accordingly. For the volume $V_c$, the gas pressure $p_g$ acts like a downward shift of $\sigma$ by $\Delta\sigma_G=p_g/\rho E$; it increases the cavitation volume by a factor $\exp(-b\cdot\Delta\sigma_G)$ while the inverse cavitation compliance $1/C_c$ now has a contribution from the bulk modulus of the gas content [8]

$$1/C_c = 1/C_V + n\cdot p_g/V_c$$  \hspace{1cm}(5a)$$

$$C_c = C_0\cdot\exp(-b\cdot\Delta\sigma_G)/(1-n\cdot b\cdot\Delta\sigma_G)$$ \hspace{1cm}(5b)$$

In equation (5), $C_V$ is the theoretical compliance pertaining to the actual $V_c$ due to centrifugal action only - in absence of non-condensable gas but with $\sigma$ reduced by $\Delta\sigma_G$. $C_0$ would hold without gas content and at actual $\sigma$. The two terms in (5b) counteract each other but beyond some threshold of $p_g$, the exponential term begins to increase $C_c$ and reduce the natural frequency. The natural frequency shift caused by gas pressure $p_g$ according to equ. (5) depends very much on the test head. Figure 1(a) shows the influence on natural frequency according to equations (4), (5), for a test condition PL1 described in subsection 3.2 and in ref. [8]. The calculation assumes an exponent $b=-37.3$ for the gas-free compliance. The prototype, with much higher head, has negligible $\Delta\sigma_G$; therefore the bias due to gas content occurs at the model only, see Figure 1 (b).

3. Case studies
This section presents a selection of otherwise unexplained irregularities which must be ascribed to variations of the uncontrolled cavity pressure. Table 1 gives the turbine data and test conditions.

| Section | D (m) | n (s⁻¹) | E (J/kg) | $Q_{AD,BEP}$ (-) | $\sigma_{AD}$ (-) | Ref. |
|---------|--------|---------|---------|-----------------|-----------------|-----|
| 3.1     | 0.30   | 21.45   | 392     | 0.59            | 0.92+var        | [10][11] |
| 3.2     | 0.35   | 13.34/8.42 | 263/113.8 | 0.70            | 1.31+var        | [8][13] |
| 3.2     | 5.40   | 2.143   | 1755    | 0.70            | 1.31            | [12][13] |
| 3.3     | 0.31   | 13.33   | 175     | 0.72            | 1.45+var        | n.a. |

3.1. Test history influencing pulsation results
A series of test points with constant speed, head and guide vane opening corresponding to 67% of best efficiency flow was conducted on a Francis model with medium specific speed, and evaluated in order to establish a theory of the part-load pulsation [10][11]. The draft tube pressure level was reduced stepwise, and afterwards increased stepwise, and the pressure fluctuation in several locations was
recorded for every pressure level. This experiment yielded the very first data on cavitation compliance for a draft tube vortex. A hysteresis-like behavior of the cavitation compliance and the synchronous pulsation was observed quite clearly, see Figure 2 (a); for example, at \( \sigma_{pl} (=0.097) \) the compliance found in the two directions differed by 20%. It was suspected [10] that delay effects on the transient mass balance of the cavity were the cause of this difference. In the following equations, \( S_C \) is the surface area, and \( A_C \) is the downstream cross section of the cavity. \( p_G \) and \( p_{G,S} \) are the gas pressure and its value at equilibrium with the gas in the fluid while \( c_m \) is the axial velocity at the tail of the vortex.

\[
\text{Mass supplied} \quad \frac{dm_1}{dt} = k_1 \cdot S_C \cdot (p_{G,S} - p_G) \quad (6a)
\]

\[
\text{Mass entrained} \quad \frac{dm_2}{dt} = k_2 \cdot A_C \cdot c_m \cdot p_G \quad (6b)
\]

The test sequence had been recorded [10], see Figure 2 (b). A simulation of steady-state vortex data based on equ. (4) through (6) yields the results in Figure 2 (c). Over the examined range of cavitation number \( \sigma \), the gas pressure \( p_G \) assumed values between 15 and 40kPa, or 50-60% of the saturation value \( p_{G,S} \) which was assumed equal to draft tube pressure \( p_C \). The very small mass \( m_G \) of the gas content remained fairly constant, between 0.46 and 0.75g. It appears that the process controlling the mass balance of the cavity (equ. 6) is too fast for causing the presumed delay effect. The likely cause is a slow change (in this case an increase) of the concentration of air dissolved in the working fluid.

![Figure 2 Measured (a) and simulated (c) cavitation compliance, test schedule (b)](image)

3.2. Resonance shifted between model and prototype

A comprehensive series of studies has been dedicated to the hydraulic pulsations of the model and prototype of units 1&2 of BC Hydro’s Mica plant [14]. At the reduced-scale model, natural frequency and resonance conditions in partial load have been established by various methods and under various test conditions including Thoma and Froude similarity. Contrary to expectations, the field test at unit 2 resulted in significantly higher natural frequency \( f_0/n \) [13]. As shown in Figure 3 (a) and (b), natural frequency at the model was lacking up to 36%, corresponding to a cavitation compliance \( C_C \) overestimated by 150%. Resonance occurs at different load conditions at the prototype and the model [14]. In [13],[14] this discrepancy has been ascribed to a deviating local cavitation index at the draft tube cone – despite of Froude and Thoma similarity; the deviation was attributed to different Reynolds number and wall roughness; according to [13] it corresponds to an excess of 0.35bar in prototype draft tube pressure.
Non-similarity of cavity pressure between model and prototype offers an alternative explanation for the difference. Figure 3(c) shows the cavity pressure for the model which would correspond to the observed deviation of natural frequency.

Figure 3 (a), (b) Predicted vs. measured prototype frequencies [12][13], (c) Dissolved-gas influence

For the operating condition PL1, at 80% of best-efficiency flow, the estimated dependency of the natural frequency from both test head and gas content is shown in Figure 1(a); this estimate is based on the compliance model described in Section 2 and ref. [8], taking the exponent b from the measured natural frequencies for the function $V_{\text{V}}=f(\sigma)$. The plot Figure 1(a) shows that the influence of gas pressure is as important as the Froude number influence.

3.3. Largely differing natural frequencies

During the development of a medium specific speed pump turbine, several test series with almost identical hydraulic conditions in turbine mode were conducted, comparing slightly different runner versions for optimizing the pump performance. The swirl-free discharge was the same for all versions, and no other components of the model and test rig were modified. Accordingly all the basic properties of the part-load vortex remained unchanged between runner versions ‘A’ and ‘B’: the precession frequency $f_{V}$, Figure 4 (b), the asynchronous pulsation (a) and the cavitation-free pressure source $p_{\text{EX}}$ (a).

Figure 4 Shifted natural frequency $f_{0}$ (b) causing differing pulsation amplitudes (c)
Despite of these concordant properties and the nominally identical test conditions, the natural frequencies found in the consecutive test series differed a lot, see graph (b). As a consequence, resonance occurred at different discharges Q_{ad}, as indicated by the arrows in Figure 4, and therefore the synchronous pulsation amplitudes were much different for the actually equivalent runner versions A and B (Figure 4 (c)). It is likely that version A was tested with better de-aerated water; unfortunately dissolved oxygen content (DO) was not measured.

4. Conclusions
For the direct transposition of pulsation parameters from model to prototype, the conventional assumption of saturated vapor pressure inside the vortex cavity is a necessary condition. It is usually taken for granted but not confirmed by experiments. Unexpected variation of pulsation amplitudes and natural frequencies have occurred both within reduced-scale model tests and between homologous model tests and the corresponding prototype. The analytical model for the influence of cavitation number and gas content on the cavity volume, cavitation compliance and natural frequency provides an explanation for these seemingly anomalous findings.

The reliability of predictions based on model tests could be established by measuring the cavity pressure and considering its influence. It would also be desirable to set up a generic model for estimating this pressure (based on DO content), in order to keep the model tests as simple as today; anyway p_G can easily be measured. It is likely that the reported discrepancies are mainly due to properties of the model test, and that vortex cavitation at the prototypes is less sensitive to gas content.

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