Detecting fibered strongly quasi-positive links

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Abstract

We prove that an \( n \)-component fibered link \( L \) in \( S^3 \) is strongly quasi-positive if and only if \( \tau(L) = g_3(L) + n - 1 \), where \( g_3(L) \) denotes the Seifert genus and \( \tau \) is the Ozsváth-Szabó concordance invariant. We also provide a table which contains a list of some fibered prime links with at most 9 crossings; and we explicitly determine the ones that are strongly quasi-positive and their maximal self-linking number.

1 Introduction

It is a very known result in low dimensional topology that every contact 3-manifold can be presented by an open book decomposition; this is a pair \((L, \pi)\) where \( L \) is a smooth link in \( M \) and \( \pi: M \setminus L \to S^1 \) is a locally trivial fibration whose fiber’s closure is a compact surface \( F \) with \( \partial F = L \). Such an \( L \) is then called a fibered link in \( M \) and \( F \) is a fibered surface for \( L \).

In this paper we are only interested in the case where the ambient manifold \( M \) is the 3-sphere. Fibered links in \( S^3 \) possess special properties; in fact, those links have unique fibered surfaces up to isotopy and, once such a surface is fixed, the fibration \( \pi \) is also completely determined. It follows that in \( S^3 \) we can define fibered links as the links whose complement fibers over the circle.

Since an open open book decomposition \((L, \pi)\) always determines a contact structure, we obtain a partition of fibered links in \( S^3 \) by saying that \( L \) carries the structure \( \xi_L \). In particular, it is a consequence of classical results of Harer and Stallings [9, 22], and the Giroux correspondence [8], that the subset of fibered links, inducing the unique tight structure \( \xi_{st} \) on \( S^3 \), coincides with the set of strongly quasi-positive links. We recall that a link is said strongly quasi-positive if it can be written as closure of the composition of \( d \)-braids of the form

\[
(\sigma_i \cdots \sigma_{j-2})\sigma_{j-1}(\sigma_i \cdots \sigma_{j-2})^{-1}
\]

for some \( d \geq j \geq i + 2 \geq 3 \)

or

\[
\sigma_i \quad \text{for} \quad i = 1, \ldots, d - 1,
\]

where \( \sigma_1, \ldots, \sigma_{d-1} \) are the Artin generators of the \( d \)-braids group.

Remark 1.1. Fibered and strongly quasi-positive links in \( S^3 \) are connected transverse \( \mathbb{C} \)-links in the sense of [3].

The main result of our paper is a criterion for detecting exactly which fibered links in \( S^3 \) are strongly quasi-positive, and then carry the contact structure \( \xi_{st} \), generalizing results of Hedden [11] (for knots) and Boileau, Boyer and Gordon [1] (for non-split alternating links). We recall that Ozsváth, Stipsicz
and Szabó defined the $\tau$-set (of $2^{n-1}$ integers) of an $n$-component link in [17], using the link Floer homology group $cHFL^-$, and they denote with $\tau_{\text{max}}$ and $\tau_{\text{min}}$ its maximum and minimum. Moreover, the $\tau$-set contains $\tau$, the concordance invariant defined in [2] by the author.

**Theorem 1.2.** A fibered link $L$ in $S^3$ is strongly quasi-positive if and only if $\tau(L) = g_3(L) + n - 1$ and, in this case, one also has $\tau(L) = \tau_{\text{max}}(L)$.

Combining Theorem 1.2 with [6] Theorem 1.1 and [3] Theorem 1.4 we easily obtain the following corollary.

**Corollary 1.3.** For a fibered link $L$ the fact that $\tau(L) = g_3(L) + n - 1$ immediately implies $SL(L) = 2\tau(L) - n$, where $SL(L)$ denotes the maximal self-linking number of $L$.

Furthermore, we give a table where we list all prime alternating fibered links with crossing number at most 9, together with the prime non-alternating fibered ones with crossing number at most 7, and we denote whether they are strongly quasi-positive. We plan to expand the table in the future, but unfortunately at the moment we do not have knowledge of any program to compute $\tau$ for links.

The paper is organized as follows: in Section 2 we recall the basics properties of link Floer homology and fibered surfaces in the 3-sphere; moreover, we prove Theorem 1.2. Finally, in Section 3 we present our table of small prime fibered links and we explain how to read it.

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## 2 Proof of the main theorem

### 2.1 Link Floer homology

In [21] Ozsváth and Szabó describe how to construct the chain complex $\left(\widehat{CF}_L(D), \partial \right)$ from a Heegaard diagram $D = (\Sigma, \alpha, \beta, w, z)$, where the sets of basepoints both contain $n$ elements. They show that, under some conditions, the diagram $D$ represents an $n$-component link $L$.

If we ignore the information given by $z$ then $D$ is just a (multi-pointed) Heegaard diagram for $S^3$ and, using the same procedure, we obtain the complex $\left(\widehat{CF}_s(D), \partial \right)$, see [13]. When $n = 1$ the homology of the latter complex is denoted by $\widehat{HF}_s(S^3) \cong \mathbb{F}_0$, where $\mathbb{F}$ is the field with two elements. In general, we have that

$$H_* \left(\widehat{CF}(D)\right) \cong \widehat{HF}_s(S^3) \otimes (\mathbb{F}_{-1} \oplus \mathbb{F}_0)^{\otimes n-1};$$

hence, if $D_1$ and $D_2$ have the same number of basepoints then $\widehat{CF}_s(D_1)$ is chain homotopy equivalent to $\widehat{CF}_s(D_2)$.

The homology of $\widehat{CF}_s(D)$, following the notation of the author in [2], is denoted by $\widehat{HF}_L(L)$, but for what we said before this group is graded isomorphic to $H_* \left(\widehat{CF}(D)\right)$. The difference, in the case of links, is given by the fact that the basepoints in $z$ define the Alexander (collapsed) increasing filtration $A^s \widehat{CF}_L(D)$ with $s \in \mathbb{Z}$. Such a filtration descends into homology in the following way: consider the quotient projection $\pi_d : \text{Ker} \partial_d \to \widehat{HF}_d(L)$, where $\partial_d : \widehat{CF}_d(D) \to \widehat{CF}_{d-1}(D)$ is the restriciton of $\partial$ to $\widehat{CF}_d(D)$, and define

$$\text{Ker} \partial_{d,s} = \text{Ker} \partial_d \cap A^s \widehat{CF}_d(D).$$

Then we say that

$$A^s \widehat{HF}_d(L) = \pi_d(\text{Ker} \partial_{d,s}) \subset \widehat{HF}_d(L).$$
for any \( d \in \mathbb{Z} \). It is important to observe that \( A^{s}HF\hat{L}_{s}(L) \) is not the same homology group obtained by just taking the homology of the Alexander level \( s \) of \( CF\hat{L}(D) \), which is instead denoted with \( H_{s}\left(A^{s}CF\hat{L}(D)\right) \); more specifically, the latter group is obtained by first restricting \( CF\hat{L}_{d}(D) \) and \( \hat{\partial}_{d} \) to the Alexander level \( s \) and then extracting the homology. In particular, note that \( A^{s}HF\hat{L}_{s}(L) \) is, by definition, always a subgroup of \( H_{s}\left(A^{s}CF\hat{L}(D)\right) \), while this needs not be true for \( H_{s}\left(A^{s}CF\hat{L}(D)\right) \).

In addition, we write \( HFL_{s,s}(L) \) for the homology of the graded object associated to \( ACF\hat{L}_{s}(D) \), which is the bigraded complex \( (\text{gr}_{s,s}(D),\text{gr}(\hat{\partial})) \) defined as follows, see [17]: we have that \( \text{gr}_{d,s}(D) \) is the subspace of \( A^{s}CF\hat{L}_{d}(D) \) spanned by generators that are not in \( A^{s-1}CF\hat{L}_{d}(D) \), while \( \text{gr}(\hat{\partial}) \) is the component of \( \hat{\partial} \) which preserves the Alexander grading, so that \( \text{gr}_{d,s}(\hat{\partial}) := \text{gr}(\hat{\partial})|_{\text{gr}_{d,s}(D)} : \text{gr}_{d,s}(D) \rightarrow \text{gr}_{d-1,s}(D) \).

We conclude this subsection by recalling that in [2] Theorem 1.3 the invariant \( \tau_{\min}(L) \) was shown by the author to coincide with the minimal integer \( s \) such that \( A^{s}HF\hat{L}_{s}(L) \) is non-zero.

### 2.2 Fibered surfaces and the Thurston norm

It is a result of Ghiggini [7] and Ni [15] that link Floer homology detects fibered links in the 3-sphere.

**Theorem 2.1** (Ghiggini-Ni). A link \( L \hookrightarrow S^{3} \) with \( n \)-components is fibered if and only if \( \dim HFL_{s,s_{\text{top}}}(L) = 1 \), where \( s_{\text{top}} \) is the maximal \( s \in \mathbb{Z} \) such that the group \( HFL_{s,s}(L) \) is non-zero.

We recall that \( \|L\|_{T} \) denotes the evaluation of the Thurston semi-norm [23] at the homology class represented by the Seifert surfaces of \( L \); and it is defined as

\[
\|L\|_{T} = o(L) - \max \{\chi(\Sigma)\},
\]

where \( \Sigma \) is a compact and oriented surface in \( S^{3} \), such that \( \partial\Sigma = L \), and \( o(L) \) is the number of unknotted unknots in \( L \).

Hence, it follows from [16] that, for an \( L \) as in Theorem 2.1, one has \( 2s_{\text{top}} = \|L\|_{T} + n \), assuming that \( L \) is not the unknot. Moreover, we have the following result.

**Proposition 2.2.** If \( L \) is a fibered \( n \)-component link in \( S^{3} \) then \( s_{\text{top}} = g_{3}(L) + n - 1 \).

**Proof.** The statement is clearly true for the unknot and it is a classical result, see [13] Chapters 4 and 5, that fibered surfaces are always connected and minimize the Thurston norm of \( L \). Therefore, we have

\[
s_{\text{top}} = \frac{-\chi(F) + n}{2} = \frac{-\left(2 - 2g(F) - n\right) + n}{2} = g(F) + n - 1,
\]

where \( F \) is a fibered surface for \( L \).

Now, if \( L \) admits a Seifert surface \( T \), with \( g(T) < g(F) \), then this contradicts the fact that \( F \) minimizes \( \|L\|_{T} \) and the claim follows. \( \square \)

### 2.3 The contact invariant \( c(L) \)

In order to prove Theorem 1.2 we need to recall the original definition of the contact invariant \( \tilde{c}(\xi) \in HF(S^{3}) \) from [19] and adapt it to our setting. In [19] Ozsváth and Szabó, for a given fibered knot \( K \hookrightarrow S^{3} \), constructed a Heegaard diagram \( D_{K} \), representing the mirror image of the knot \( K \), with a distinguished intersection point \( c(K) \) such that

\[
A(c(K)) = -g_{3}(K) \quad \text{and} \quad A(y) > -g_{3}(K)
\]

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for every other intersection point \( y \) in \( \mathcal{D}_K \). In particular, this gives that
\[
H_* \left( A^{-g_3(K)} \text{CF}_L(\mathcal{D}_K) \right) \cong \mathbb{F}
\]
and
\[
H_* \left( A^s \text{CF}_L(\mathcal{D}_K) \right) \cong \{0\} \quad \text{for any } s < -g_3(K).
\]
This construction works exactly in the same way for an \( n \)-component fibered link \( L \): only that now one has \( A(c(L)) = -g \) with \( g \in \mathbb{Z} \) and \( c(L) \) is still the unique minimal Alexander grading intersection point in \( \mathcal{D}_L \).

**Lemma 2.3.** Suppose that \( L, c(L) \) and \( g \) are as before. Then we have that \( g = g_3(L) + n - 1 \).

**Proof.** The element \( c(L) \) is necessarily a cycle and \([c(L)]\) has to be non-zero in \( \widehat{HF}_L(L^*) \); hence, we have that \( A(c(L)) \) is the minimal integer \( s \) such that \( \widehat{HL}_{s,s}(L^*) \) is non-zero. Because of the symmetries of \( \widehat{HL}(L) \), see [20], and Proposition 2.2 this means that \( A(c(L)) = -g_3(L) - n + 1 = -g \).

If we consider the inclusion \( i_{L^*} : A^{-g} \text{CF}_L(\mathcal{D}_L) \hookrightarrow \text{CF}(\mathcal{D}_L) \) then we can see \( c(L) \) as a cycle in \( \text{CF}(\mathcal{D}_L) \) by dropping the information about the Alexander grading.

The element \( c(L) \) can then be identified with the contact element defined in [4], using multi-pointed Legendrian Heegaard diagrams, generalizing the presentation of Honda, Kazez and Matić, see [12]. This means that
\[
[c(L)] = \tilde{c}(\xi_L) \otimes e_{1-n} \in \widehat{HF}(S^3) \otimes (\mathbb{F}_{-1} \oplus \mathbb{F}_0)^{\otimes n-1},
\]
where \( e_{1-n} \) is the unique generator of \( (\mathbb{F}_{-1} \oplus \mathbb{F}_0)^{\otimes n-1} \) in grading \( 1 - n \). In particular, we can state the following lemma.

**Lemma 2.4.** Suppose that \( L \) is a fibered link and \( \xi_L \) is the contact structure on \( S^3 \) carried by \( L \). Then \( c(L) \) is zero in homology whenever \( \xi_L \) is overtwisted.

**Proof.** From [19] one has \( \tilde{c}(\xi_L) = [0] \) when \( \xi_L \) is overtwisted. Hence, the homology class \([c(L)]\) is equal to zero for Equation (2.1).

From the definition of \( c(L) \) we observe that \( c(L) \) is non-zero in homology if and only if \(-g \) is the minimal \( t \) such that the map
\[
i_{L^*} : H_* \left( A^t \text{CF}_L(\mathcal{D}_L) \right) \rightarrow \widehat{HF}_L(L^*) \cong \widehat{HF}(S^3) \otimes (\mathbb{F}_{-1} \oplus \mathbb{F}_0)^{\otimes n-1}
\]
induced by the inclusion \( i_{L^*} \) is non-trivial.

**Proof of Theorem 1.2.** If \( L \) is strongly quasi-positive then from [3] Theorem 1.4] one has \( \tau(L) = \tau_{\max}(L) = g_3(L) + n - 1 \). Suppose now that \( \tau_{\max}(L) = g_3(L) + n - 1 = g \), see Lemma 2.3 we want to prove that \( L \) is strongly quasi-positive.

We have that
\[
-g = -\tau_{\max}(L) = \tau_{\min}(L^*) = \min_{s \in \mathbb{Z}} \left\{ A^s \widehat{HL}_L(L^*) \text{ is non-zero} \right\} = \min_{s \in \mathbb{Z}} \left\{ i_{L,s} \text{ is non-trivial} \right\}
\]
from the definition of \( \tau \)-set given in [2]. Since \( L \) is fibered, the combination of Equations (2.2) and (2.3) tells us precisely that \( c(L) \) is non-zero in homology and, applying Lemma 2.4, that \( \xi_L = \xi_{\text{st}} \).

At this point, we conclude by recalling that a fibered link \( L \) carries the tight structure on \( S^3 \) if and only if is strongly quasi-positive, as we observed in the introduction.
3 Table of small prime fibered strongly quasi-positive links

Using Theorem [12] and some computation we can detect exactly which prime links are fibered and, among these, which ones are strongly quasi-positive. Those data are enclosed in Table [1]. We denote the links following the notation on Linkinfo database [13]. Therefore, the number inside brackets identifies the relative orientation of the link. Clearly, being fibered and strongly quasi-positive does depend on relative orientations. Moreover, a link $L$ is fibered if and only if $L^*$ is fibered; while when in Table [1] we write that $L$ is strongly quasi-positive, we mean that at least one between $L$ and $L^*$ is. In fact, the following result holds.

**Theorem 3.1.** Suppose that $L$ is a strongly quasi-positive link in $S^3$. Then its mirror image $L^*$ is also strongly quasi-positive if and only if $L$ is an unlink.

Theorem 3.1 follows from a more general result of Hayden [10], but we can prove it directly using only link Floer homology. Let us start from the following lemma.

**Lemma 3.2.** If $L$ is strongly quasi-positive then the same its true for $L'$, where $L = L' \sqcup \bigcirc o(L)$ and $\bigcirc e$ is the $e$-unlink. Furthermore, one has $\|L\|_T = \|L'\|_T$.

**Proof.** The link $L$ bounds a quasi-positive surface $\Sigma$. It is known that quasi-positive surfaces always minimize the Thurston norm; hence, since one has $\|L\|_T = \|L'\|_T$ from the additivity of $\|\cdot\|_T$, we have that $\Sigma$ is the disjoint union of $\Sigma'$ with some disks, where $\partial \Sigma' = L'$. This implies that $L'$ is strongly quasi-positive. \qed

At this point, in order to prove Theorem 3.1 we only need Lemma 3.3 which solves Exercise 8.4.9 (a) in [17].

**Lemma 3.3.** For every $n$-component link $L$ we have

$$0 \leq \tau_{\text{max}}(L) - \tau_{\text{min}}(L) \leq n - 1.$$ 

**Proof.** It follows from [5] because $\tau_{\text{min}}(L) = -\tau_{\text{max}}(L^*)$, for the symmetries of $cHFL^-(L)$, and $\tau_{\text{max}}(L)$ is a slice-torus concordance invariant. To show this we can apply [5, Lemma 2.1]; then the only non-trivial fact to prove is that $\tau_{\text{max}}$ coincides with $\tau$ for torus links, but this follows from [3, Theorem 1.4]. \qed

In particular, this result implies that $\tau(L) + \tau(L^*) \leq n - 1$.

**Proof of Theorem 3.1** From Lemma 3.2 we can suppose that $o(L) = 0$. By using [3, Theorem 1.4] again, the fact that $L$ and $L^*$ are both strongly quasi-positive gives $2\tau(L) - n = 2\tau(L^*) - n$, since $\|L\|_T = \|L^*\|_T$. Therefore, from Lemma 3.3 one has

$$2\tau(L) \leq n - 1 \quad \text{and then} \quad \|L\|_T \leq -1,$$

but this is impossible. \qed

The following easier criterion can be used for alternating links. We recall that fibered links are always non-split and we denote by $\sigma(L)$ the signature and by $\nabla_L$ the Conway polynomial of $L$.

**Proposition 3.4.** A quasi-alternating $n$-component link is fibered and strongly quasi-positive if and only if $\nabla_L(z) = z^{-\sigma(L)} + f(z)$, where $f(z)$ has degree strictly smaller than $-\sigma(L)$.

**Proof.** It follows from Theorem 2.1 and the facts that $\widehat{HFL}(L)$ categorifies the polynomial $(t^{\frac{1}{2}} - t^{-\frac{1}{2}})^{n-1} \cdot \nabla_L \left( t^{\frac{1}{2}} - t^{-\frac{1}{2}} \right)$ and quasi-alternating links are $\widehat{HFL}$-thin, see [17]. \qed

This result agrees with [1], where is shown that, for a non-split alternating link $L$, being strongly quasi-positive is equivalent to being positive and this happens if and only if $2g_3(L) = 1 - n - \sigma(L)$. In fact, we also recall that for such a link $L$ one has $2\tau(L) = n - 1 - \sigma(L)$. 

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| name             | strongly QP (SL) | name             | strongly QP (SL) | name             | strongly QP (SL) |
|------------------|------------------|------------------|------------------|------------------|------------------|
| L2a1{0}          | Y (0)            | L8a10{1}         | N                | L9a41{0}         | N                |
| L2a1{1}          | Y (0)            | L8a14{0}         | Y (6)            | L9a42{0}         | N                |
| L4a1{1}          | Y (2)            | L8a15{0,0}       | N                | L9a42{1}         | N                |
| L5a1{0}          | N                | L8a16{0,0}       | N                | L9a43{1,0}       | N                |
| L5a1{1}          | N                | L8a16{1,1}       | N                | L9a43{0,1}       | N                |
| L6a1{0}          | N                | L8a17{1,1}       | N                | L9a43{1,1}       | N                |
| L6a3{0}          | Y (4)            | L8a18{0,0}       | N                | L9a44{0,0}       | N                |
| L6a{0,0}         | N                | L8a18{1,1}       | N                | L9a44{1,0}       | N                |
| L6a1{1,0}        | N                | L8a19{0,0}       | N                | L9a44{0,1}       | N                |
| L6a1{0,1}        | N                | L8a19{1,1}       | N                | L9a46{0,0}       | N                |
| L6a1{1,1}        | N                | L8a20{0,0}       | N                | L9a46{1,0}       | N                |
| L6a5{1,0}        | N                | L8a20{1,0}       | N                | L9a46{0,1}       | N                |
| L6a5{0,1}        | N                | L8a21{1,0,0}     | N                | L9a46{1,1}       | N                |
| L6a1{1,1}        | N                | L8a21{0,1,0}     | N                | L9a47{0,0}       | N                |
| L6a1{0,0}        | N                | L8a21{0,0,1}     | N                | L9a47{1,0}       | N                |
| L6n1{1,0}        | N                | L8a21{1,0,1}     | N                | L9a47{1,1}       | N                |
| L6n1{0,1}        | Y (3)            | L8a21{0,1,1}     | N                | L9a48{0,0}       | N                |
| L6n1{1,1}        | N                | L8a21{1,1,1}     | N                | L9a49{1,0}       | N                |
| L7a1{0}          | N                | L9a2{0}          | N                | L9a49{0,1}       | N                |
| L7a1{1}          | N                | L9a2{1}          | N                | L9a50{0,0}       | N                |
| L7a2{1}          | N                | L9a5{0}          | N                | L9a50{1,0}       | N                |
| L7a3{0}          | N                | L9a6{1}          | N                | L9a50{1,1}       | N                |
| L7a3{1}          | N                | L9a8{0}          | N                | L9a51{0,0}       | N                |
| L7a5{1}          | N                | L9a8{1}          | N                | L9a51{1,0}       | N                |
| L7a6{0}          | N                | L9a9{0}          | N                | L9a51{1,1}       | N                |
| L7a7{0,0}        | N                | L9a9{1}          | N                | L9a52{0,0}       | N                |
| L7a7{1,0}        | N                | L9a11{0}         | N                | L9a52{1,1}       | N                |
| L7a7{0,1}        | N                | L9a12{1}         | N                | L9a53{0,0}       | N                |
| L7n1{0}          | Y (4)            | L9a14{0}         | N                | L9a53{1,0}       | N                |
| L7n1{1}          | N                | L9a14{1}         | N                | L9a53{0,1}       | N                |
| L7n2{0}          | N                | L9a16{0}         | N                | L9a53{1,1}       | N                |
| L7n2{1}          | N                | L9a20{0}         | N                | L9a54{0,0}       | N                |
| L8a1{0}          | N                | L9a21{0}         | N                | L9a54{1,0}       | N                |
| L8a1{1}          | N                | L9a22{0}         | N                | L9a54{0,1}       | N                |
| L8a2{0}          | N                | L9a24{1}         | N                | L9a54{1,1}       | N                |
| L8a2{1}          | N                | L9a26{1}         | N                | L9a55{0,0,0}     | N                |
| L8a3{1}          | N                | L9a27{0}         | N                | L9a55{0,1,0}     | N                |
| L8a4{0}          | N                | L9a28{0}         | N                | L9a55{0,0,1}     | N                |
| L8a4{1}          | N                | L9a29{0}         | N                | L9a55{1,0,1}     | N                |
| L8a5{0}          | N                | L9a31{0}         | N                |                  |                  |
| L8a7{1}          | N                | L9a32{1}         | N                |                  |                  |
| L8a8{0}          | N                | L9a33{0}         | N                |                  |                  |
| L8a8{1}          | N                | L9a36{0}         | N                |                  |                  |
| L8a9{0}          | N                | L9a38{0}         | N                |                  |                  |
| L8a9{1}          | N                | L9a39{0}         | N                |                  |                  |

Table 1: Some fibered prime links with 9 or fewer crossings. In the second column we write whether or not the corresponding fibered link (or its mirror image) is strongly quasi-positive. In the latter case, the maximal self-linking number is denoted between brackets.
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