Finite-temperature Hubbard local field corrections on
electron mobility in strictly 2D electron gas

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Abstract. Low-temperature electron mobility of a strictly two-dimensional electron gas (2DEG) in a modulation delta-doped Al$_x$Ga$_{1-x}$As/GaAs/Al$_x$Ga$_{1-x}$As heterostructure has been obtained by using Boltzmann equation and relaxation time approximation. At low temperatures, the electron mobility is governed by the electron-ionized impurity scattering in which the electrons screen electron-impurity interaction through the dielectric function of the system. We have investigated the effect of ionized impurities on the low-temperature electron mobility beyond the random phase approximation by including the local field corrections. We have used Hubbard and finite-temperature Hubbard local field corrections to calculate the electron mobility as a function of temperature. We have found that the Hubbard local field corrections enhance the electron mobility at low temperatures in comparison with RPA results and less enhancement is obtained by using temperature dependent Hubbard approximation.
1. Introduction
Investigating the mobility of 2DEG due to its wide applications in low-dimensional semiconductor technology has directed the aim of many scientific works for last two decades [1-13]. Among these researches, calculating the low-temperature electron mobility of 2DEG due to the importance of low-temperature regime has attracted the attention of many authors [6-12]. It is known that at low temperatures, the ionized impurities scattering plays the dominant role in the electron mobility [14].

Many attempts have been done to calculate the electron mobility by using more precise approximations such as Hubbard approximation [10-12].

We investigated the low-temperature electron mobility of a modulation δ-doped AlxGa1-xAs/GaAs/AlxGa1-xAs heterostructure in a strictly 2D GaAs well [9]. The dielectric response function of the mentioned system has been obtained within random phase approximation (RPA).

In this work, we have calculated the low-temperature electron mobility of the mentioned heterostructure by using Hubbard approximation (HA) and finite-temperature Hubbard approximation (T-HA) in the dielectric function of the system as a function of temperature and spacer width. The final results are compared with RPA mobility curves.

This paper is organized as follows. In Sec. 2 we present our theoretical approach for low-temperature electron mobility. Sec. 3 considers the screening effects in 2DEG system through Hubbard local field corrections and its temperature-dependent formalism. In Sec. 4 the numerical results are given and discussed. Finally, a summary and conclusion is given in Sec. 5.

2. Low-Temperature Mobility
The dominant scattering mechanism which affects electrons at low temperatures is ionized impurity scattering. So the low-temperature mobility must be calculated by considering electron-ionized impurity coulomb interaction. The mobility is given by [5]:

$$\mu = \frac{e}{m} \int dE \tau(E) E \left( -\frac{\partial f^0}{\partial E} \right) \int dE \left( -\frac{\partial f^0}{\partial E} \right)$$  \hspace{1cm} (1)

Here $\tau(E)$ and $f^0(E)$ are the relaxation time and the Fermi-Dirac distribution function respectively. The relaxation time due to ionized impurity scattering in the first Born approximation is given by [14]:

$$\frac{1}{\tau(E)} = \frac{2\pi}{h} \sum_{k'} \int_{-\infty}^{\infty} dz \; N_i(z) v_s(k-k',z) \left( 1 - \cos \theta_{kk'} \right) \delta \left( \frac{h^2}{2m}(k^2 - k'^2) \right)$$  \hspace{1cm} (2)

where $k$ and $k'$ are 2D wave vectors and $\theta_{kk'}$ is the scattering angle between $k$ and $k'$. The presence of delta function in equation (2) guarantees the energy conservation for this elastic scattering. $v_s(k-k',z)$ is the screened electron-impurity interaction potential and $N_i(z)$ is the 3D ionized impurity concentration due to modulation doping or remote impurities. The screened electron-impurity potential is governed by:

$$v_s(q;z) = \frac{v_f(q;z)}{e(q)}$$  \hspace{1cm} (3)
where \( v_I(q, z) \) is the bare Coulomb electron-ionized impurity potential, \( \varepsilon(q) \) is the static dielectric response function which screens the bare potential and \( |\mathbf{k} - \mathbf{k}'| = q \). The bare potential is denoted by:

\[
v_I(q, z) = \frac{2\pi Z e^2}{\varepsilon_S q} e^{-qd}
\]

(4)

Here \( Z, \varepsilon_S \) and \( d \) are the ionized impurity charge strength, static dielectric constant and the spacer width, respectively. It should be mentioned that the factor \( e^{-qd} \) is the inter-potential form factor for the strictly 2D layer [9].

3. Screening

The only unrevealed part in section 2 is the screening which is introduced by the dielectric response function [15]:

\[
\varepsilon(q, \omega) = 1 - \frac{\nu(q) \chi^0(q, \omega)}{1 + \nu(q) \chi^0(q, \omega) G(q)}
\]

(5)

where \( \nu(q) \) is the 2D electron-electron Coulomb interaction; \( \chi^0(q, \omega) \) is the finite-temperature 2D Lindhard polarizability function and \( G(q) \) denotes local field corrections (LFC).

The method of calculating the polarizability function has been explained in ref. [9] in details.

It is worth mentioning that in Eq. (5) the RPA dielectric function which is valid at high 2D electron density, \( n_{2D} \), or at low density parameter \( r_s = (a_B^2 \pi n_{2D})^{-1/2} \) (\( a_B \) is the effective Bohr radius) can be obtained by setting \( G(q) = 0 \).

The dielectric function at low electron densities \( (r_s > 1) \) can be calculated more precise with replacing RPA by HA LFC. The HA LFC in 2D is denoted by [16]:

\[
G(q) = \frac{1}{g_d} \frac{q}{\sqrt{q^2 + k_F^2}}
\]

(6)

where \( k_F, g_d \) are Fermi wave vector and spin-valley degeneracy, respectively.

As it is observed this LFC is temperature independent. It is possible to include the temperature effect in HA LFC by using temperature dependent Fermi wave vector, \( k_0(T) \), which is defined as follows:

\[
k_0^2(T) = 2mk_B T \ln\left[ e^{\left( \frac{k_F^2}{g_d} \right)} - 1 \right]
\]

(7)

Therefore, the LFC has been obtained by:

\[
G(q) = \frac{1}{g_d} \frac{q}{\sqrt{q^2 + k_0^2(T)}}
\]

(8)

4. Results
Using above formulae, the electron mobility has been calculated within RPA, HA and T-HA. The electron mobility in a strictly 2D layer with 300 Å spacer width is plotted versus temperature in Fig. 1 for a density parameter $r_s=3$.

As it is expected, Fig. 1 shows that the mobility increases with temperature at low temperatures. Our calculated mobility shows a rather good agreement with previous results where the numerical results for the modulated doped GaAs/Al$_x$Ga$_{1-x}$As heterostructure have been compared with experimental data [13]. It is important to be noticed that our calculations have taken into account only the effect of remote ionized impurity on mobility, thus our result can not be compared completely with the measured mobility data which includes all scattering mechanisms.

The behavior of mobility at low temperature can be easily understood because as the temperature increases, electrons would move faster and consequently they would be less deflected by scattering centers. Considering Fig. 1, it is clear that using HA LFC enhances the electron mobility. It can be explained by this fact that the HA LFC introduces the stronger screening effect compared to RPA, so the electron response will be easier and faster. Another interesting result is that the mobility values calculated by T-HA is smaller than HA values. As it is shown in Fig. 1 the difference between these two mobility curves augments with increasing the temperature.

**Fig. 1.** Temperature dependence of the electron mobility in a strictly 2D layer with 300 Å spacer width, for $r_s=3$, using RPA, HA and T-HA.
Fig. 2 illustrates the electron mobility versus spacer width at 15K and for \( r_s=3 \). It should be emphasized that the carrier concentration remains constant during the spacer width augmentation. It is expected that the mobility increases with spacer width because the ionized donors placed farther from 2DEG and thus the scattering strength reduces considerably. Increase in spacer width is not always useful; if the spacer width becomes larger than a critical value, the charged carriers lose the most of their kinetic energies while they are transferring from doped region to the undoped well.

As it is shown in Fig. 2, all three curves suit the main concept of using spacers. Here similar to Fig. 1, using LFCs for calculating the dielectric function enhances the electron mobility compared to RPA mobility results. The T-HA mobility curve predicts the mobility almost ten times smaller than HA curve after 200 Å, which is due to the definition of the Fermi wave vector.

**Fig. 2.** The electron mobility in a strictly 2D layer with as a function of spacer width at \( T=15K \), and \( r_s=3 \) using RPA, HA and T-HA.

5. **Summary and Conclusion**

In this work the low temperature electron mobility of a strictly 2DEG in a modulation delta-doped Al\(_x\)Ga\(_1-x\)As/GaAs/Al\(_x\)Ga\(_1-x\)As heterostructure is calculated by using Boltzmann equation and relaxation
time approximation. At low temperatures, the only dominant scattering mechanism is ionized impurity scattering. It should be noted that the electrons screen electron-impurity interaction through the dielectric function of the system. In the current work the effect of ionized impurities on the low-temperature electron mobility has been investigated beyond the random phase approximation by including the HA and T-HA local field corrections. First, the electron mobility has been calculated as a function of temperature. As it is expected the mobility increases with temperature at low temperatures because as the temperature increases, electrons becomes faster and then the effect of ionized donors on electrons reduces. It should be mentioned that the HA and T-HA curves show the higher electron mobility than RPA. It shows that the HA LFC increases the screening effect in the system compared to RPA. Also, the T-HA mobility curve placed lower than HA and the difference between two LFCs augments with temperature. Then the electron mobility has been calculated versus spacer width within HA and T-HA. It should be emphasized that the electron concentration is considered fixed during the spacer width increasing. As it is expected the mobility increases with spacer width because the ionized donors are farther from 2DEG and therefore, the scattering strength reduces considerably. Here again, including LFCs in the dielectric function enhances electron mobility in comparison with RPA results. The HA and T-HA mobility curves are close to each other at first but after a definite spacer width, the T-HA mobility curve predicts the mobility values almost ten times smaller than HA values which is due to the definition of the Fermi wave vector.

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