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COMBINING SIMULATION WITH GENETIC ALGORITHM FOR SOLVING STOCHASTIC MULTI-PRODUCT INVENTORY OPTIMIZATION PROBLEM

Abstract. All companies are challenged to match supply and demand, and the way the company tackles this challenge has a tremendous impact on its profitability. Due to the fact that markets are rapidly evolving and becoming more complex, flexible, and information-intensive, notorious bingeing-and-purging approach is inappropriate. Such an approach, in which product is, firstly, overpurchased or overproduced in order to prepare for expected demand spikes and then discarded by sharp decline in price. Thus, in order to tailor inventory control to urgent industrial needs, the discrete-event simulation model is proposed. The model is stochastic and operates with multiple products under constrained total inventory capacity. Besides that, the model under consideration is distinguished by uncertain replenishment lags and lost-sales. The paper contains both mathematical description and algorithmic implementation. Besides that, an optimization framework based on genetic algorithm is proposed for deriving an optimal control policy. The proposed approach contributes to the field of industrial engineering by providing a simple and flexible way to compute nearly-optimal inventory control parameters.

Key words: stochastic inventory control, constrained optimization, simulation-optimization, genetic algorithm.
Комбинация имитационного моделирования и генетического алгоритма для решения задач оптимизации стохастической многопродуктовой системы управления запасами

Аннотация. Каждая компания сталкивается с необходимостью синхронизации спроса и предложения и то, насколько компания справляется с данной задачей, оказывает огромное влияние на ее прибыльность. В связи с тем, что рынки быстро развиваются и становятся все более сложными, гибкими, волатильными и информационно насыщенными, устаревшие подходы, предполагающие перепроизводство, сменяемое резким снижением цены для стимулирования продажи излишней продукции, являются неприемлемыми. В этой связи, данная статья преследует цель – разработать реалистичную дискретно-событийную имитационную модель системы управления запасами для нахождения оптимальных параметров контроля. Описываемая модель является стохастической и работает с несколькими продуктами при ограниченной совокупной вместимости. Кроме того, рассматриваемая модель отличается неопределенными временными лагами между поставками продукции. Статья содержит как математическое описание, так и алгоритмическую реализацию. Кроме того, для нахождения оптимальной политики управления предлагается подход на основе генетического алгоритма. Описанный метод является простым и гибким способом вычисления около-оптимальных параметров для систем управления запасами. Предложенный подход вносит свой вклад в область промышленного проектирования, предоставляя простой, но все же эффективный способ вычисления почти оптимальных параметров запасов с учетом политики риска и надежности. Кроме того, метод может быть применен в автоматизированных системах заказа.

Ключевые слова: стохастическое управление запасами, оптимизация с ограничениями, симуляция-оптимизация, генетический алгоритм.

Introduction

Modern markets are extremely competitive. Businesses are facing unceasingly growing pressure on both prices and quality. Besides that, the company is required to swiftly respond to stochastic market conditions. Incorrect inventory policy leads not only to corporate losses, but also to overproduction (Altiok, 2012). In this regard, traditional inventory policies are not appropriate anymore. Moreover, overproduction causes serious environmental problems, depleting natural resources and polluting the atmosphere.

The real-world inventory optimization is commonly characterized by the large-scale size and the necessity for nearly-optimal solutions in feasible computing times (Angun, 2011). That is why, the metaheuristics in general and genetic algorithms in particular are used so widely to define an optimal inventory policy. The world is full of uncertainty, which frequently makes classical deterministic approaches unsuitable due to excessive simplicity.

As it is mentioned in the recent research (Juan et al., 2015), real-life stochastic combinatorial optimization problems may be reformulated as a simulation in a natural way. Thus, the hybridization of metaheuristics and simulation techniques promises to be an efficient solution of stochastic inventory optimization and inventory control problems. First and foremost, the combination of simulation and metaheuristics is focused on efficiency taking into account stochastic components that may be contained either in the objective function or in the constraints. Such approaches are conventionally called simulation–based optimization or “simheuristics”. The method aims to utilize a simulation instead of an objective function in traditional form and apply the genetic algorithm to find such simulation adjustments that would lead to the optimal output. In the proposed method, the iterative searching process of the genetic algorithm has to assess the quality of individual solutions, highlighting the promising ones. Besides, real-world stochasticity may be modelled throughout the best-fit probability distribution. The distribution may be either theoretical or empirical, without the need to be approximated to normal or exponential.

Nowadays discrete-event simulation is the most dominant simulation paradigm for simulation-optimization frameworks (Gosavi, 2015). The first simulation-based optimization of inventory control system dates back to Fu and Hill (1997). The model assumes zero replenishment lead time and periodic review. The cost function comprises holding, purchasing, transportation and backlogging.
Among modern papers metaheuristic in general and genetic algorithms in particular are distinguished. For instance, researchers considered a stochastic supply chain management problem (Peirleitner et al., 2016). The problem is stated as bi-objective optimization problem. Such that overall supply chain costs are subject to minimization, while service level must be maximized. Such optimal control parameters as reorder points and lot sizes are derived by combining genetic algorithm with discrete-event simulation. In the same year discrete-rate simulation paradigm is used as a core to solve single-product inventory control problem (Zvirkzdina and Toluje, 2016). In this study the model is developed in ExtendSim using inbuilt genetic algorithm to find optimal control parameters. The recent research focuses on spare part inventory control for an industrial plant. Assuming that the demand is driven by maintenance requirements, spare part provision for a single-line conveyor-like system is considered (Zahedi-Hosseini, 2018).

Average cost per unit time is taken as the optimality criterion and optimization is conducted using SimRunner’s inbuilt genetic algorithm.

This paper describes a possible combination of discrete-event simulation and genetic algorithm to define the optimal inventory policy in stochastic multi-product inventory systems. The discrete-event model under consideration corresponds to the just-in-time inventory control system with a floating reorder point. The system operates under stochastic demand and replenishment lead time. The utilized genetic algorithm is distinguished by a non-binary chromosome encoding, uniform crossover and two mutation operators. The proposed approach contributes to the field of industrial engineering by providing a simple, but still efficient way to compute nearly-optimal inventory parameters with regard to risk and reliability policy. Besides, the method may be applied in automated ordering systems.

Materials and methods

First of all, the method requires designing a simulation that corresponds to the real system with a high degree of accuracy. As it is already mentioned, such a simulation will play the role of an objective function. Thus, an optimization process will be reduced to the search of the best simulation adjustments. The inventory theory at its current stage has developed a significant mathematical foundation for solving problems related to the determination of the optimal inventory policy (Zipkin, 2000). The most suitable model among considered is the model of Hopp and Spearman (2008). It is also worth noting that several distinguishing features were taken from “lost sales (r, Q) inventory control model” (Kouki et al., 2015). The considered model makes several assumptions:

- Unfulfilled demands are defined as a lost opportunity and no backlog shall be fulfilled late;
- Demand size, demand frequency and replenishment lead time are continuous random variables;
- Product of a particular type is replenished by an individual supplier.

Discrete-event simulation paradigm is chosen in order to take into account random components without a dramatic increase in system complexity at the computational level. Unlike in continuous simulation, system dynamics is not unceasingly tracked during the simulation time. Discrete-event simulation contains a list of events, such that each event takes place at a particular instant of time altering the state of the system. It is important to emphasize that there are no changes in the system between consecutive events. That is why, the simulation laps in time from previous event to the next one and runs much faster saving precious computational resources. Each event is scheduled according to preliminary generated time $t_s$ and executes sequentially. Generated time is appended to a time vector $T = (t_p, t_1, ..., t_n)$, which may be interpreted as a time-counter. The total inventory assortment corresponds to the set of products $P$, such that each product $p \in P$. The storage capacity allocation is the first priority task. Presuming that $I_{max}$ is the total storage capacity, we declare $B$ as a vector of individual storage capacities assigned for products:

$$\sum_{i=1}^{P} b_i = I_{max}; \; \forall b_i \in B \; \text{(1)}$$

The simulation begins with an initial inventory level of $I_p$ at $t_p$. During the simulation, emerging demands $x_{p,t}$ are satisfied and the stock level declines gradually. If the stock level falls below a reorder point $r_{p,t}$, the inventory places a new order $y_{p,t}$ to refill the stock. Therefore, an inventory level at a particular moment of time equals to an inventory level in previous moment subtracting received demand and adding an order that was placed at $t - L$ Equation 2. Where $L_p$ is the replenishment lead time for a product $p$. It is also worth noting that such a model aims to represent the inventory under some sort of just-in-time policy, thus, the order size $y_{p,t}$ equals to the corresponding maximal inventory capacity $b_p$ subtracting the difference between the
current inventory level $I_{p,t}$ and adjusted safety-stock $SS_p$, Equation 3. In the proposed model, a new reorder point $r_{p,t}$ is recalculated after each replenishment Equation 4. Where $m_{p,[t-t_{l}]}$ stands for a mean demand during the replenishment lead time and $SS_p$ is a value of the corresponding safety-stock. Based on that, the number of arisen backorders for product $p$ at time $t$ may be determined as the step function:

$$ I_{p,t+1} = I_{p,t} - x_{p,t} + y_{p,t-L} $$  (2)

$$ y_{p,t} = \begin{cases} b_p & \text{if } I_{p,t} > SS_p \\ b_p & \text{if } I_{p,t} \leq SS_p \\ \end{cases} $$  (3)

$$ r_{p,t} = m_{p,[t-t_{l}]} + SS_p $$  (4)

$$ O_{p,t} = \begin{cases} 0, & \text{if } x_{p,t} \leq I_{p,t} \\ x_{p,t} - I_{p,t}, & \text{if } x_{p,t} > I_{p,t} \end{cases} $$  (5)

Discrete-event simulation of such models is simple enough and can be performed by the iterative algorithm (Figure 1).

The number of backorders for product $p$ at time $t$ may be determined as the step function:

$$ T_{C_{p}} = I_{p} \sum_{i=0}^{t} x_{p,t} + h_{p} \sum_{i=0}^{t} I_{p,t} + o_{p} \sum_{i=0}^{t} O_{p,t} $$  (6)

In such settings, an overflow may occur:

$$ F_{t} = \begin{cases} 0, & \text{if } \sum_{i=1}^{t} I_{i} \leq \sum_{i=1}^{t} b_{i} \\ F_{t}, & \text{if } \sum_{i=1}^{t} I_{i} > \sum_{i=1}^{t} b_{i} \end{cases} $$  (7)

Such a case may be taken into account by declaring a specific cost $s$ related to the unit overflow and tracing the overflow level. In real world, such a cost corresponds to the warehouse outsourcing or reverse logistics (Bijvankand Vis I, 2011).

$$ T_{C} = \sum_{p=1}^{P} T_{C_{p}} + s \sum_{i=0}^{t} F_{i} $$  (9)

The genetic algorithm was invented and firstly introduced by Holland (1975). To date, genetic algorithms have been successfully implemented in logistics and supply chain management (Yeh and Chuang, 2011). The motivation for combining genetic algorithm with simulation is that in real-life inventory problems, it is highly preferable to obtain a nearly-optimal solution for a precisely accurate model than the absolutely optimal solution for an oversimplified deterministic model. Genetic algorithms are totally different in comparison with the conventional search techniques. The optimization procedure starts with an initial set of randomly generated solutions that are called population. Each individual solution in the population is called a chromosome. The chromosomes undergo changes through sequential iterations. Such iterations are called generations. The chromosomes within the generation are evaluated, according to a fitness function. The next generation is composed by a set of new chromosomes, called offspring. Offspring, in its turn, is mainly formed by the fittest chromosomes, partially altered by either crossover or mutation operators.

In order to apply genetic algorithm, the following initial parameters are required:

- Population size ($N$) – the number of chromosomes in each generation;
- Crossover rate ($P_c$) – the probability of executing a crossover operator;
- Mixing ratio ($P_u$) – the probability for each attribute to be exchanged;

Each product in an assortment has a different market price and thus a different backorder cost $o_{p}$. Likewise, unit costs of storage and shipping, $h_{p}$ and $l_{p}$ respectively, vary depending on product’s properties and subtleties of handling. Thereby, the total cost function for each product is the sum of the products of unit costs on number of units shipped, stored or backordered respectively:

![Figure 1 – The logic behind the simulation](image)
– Mutation rate ($P_{m_1}$) – the probability of executing a mutation operator 1;
– Mutation rate ($P_{m_2}$) – the probability of executing a mutation operator 2;
– Mutation step (delta) – the gene-multiplier used by the mutation operator 2;
– Tournament size ($t$).

Practically, genetic algorithm is quite efficient in cases of large search space with lack of knowledge on the structure of the fitness function. The stochastic inventory optimization problem undoubtedly belongs to this domain. Moreover, in cases of high stochasticity, it becomes difficult to apply some traditional optimization techniques.

Genetic algorithm is quite famous as a problem-independent approach, nevertheless, the chromosome representation is a critical issue. Applying genetic algorithm to the inventory optimization problem under consideration, we are looking for such adjustments to simulation parameters: storage-resources allocation $\beta$ and corresponding safety-stock levels $SS$ that lead to the best fitness. The chromosome may be encoded as a $|P|$ size list of integers $v = (b_1, SS_1, b_2, SS_2, \ldots, b_{|P|}, SS_{|P|})$. In such a list each odd element stands for the inventory capacity allocated to each product $p$ and each even element represents adjusted safety-stock level for the corresponding product $p$ (Figure 2).

![Figure 2 – Chromosome representation](image)

In such a simulation-driven approach, fitness function is evaluated by sequential runs of several simulations. In this case, fitness is the mean value of total costs calculated in several sequential simulation’s runs, with the same parameters. We are looking for such parameters that lead to the minimal mean value of the total cost function satisfying the constraints:

$$\min_{\alpha \in \mathcal{E}} E\left[\sum_{p=1}^{P} TC_p(\alpha)\right]$$  \hspace{1cm} (10)

$$\sum_{i=1}^{P} b_i \leq I_{\text{max}}; \forall i = 1,2,3,\ldots,|P|$$  \hspace{1cm} (11)

$$SS_i \leq b_i; \forall i = 1,2,3,\ldots,|P|$$  \hspace{1cm} (12)

In case the solution does not satisfy constraints, the fitness will take extremely high values, due to infeasibility of such a solution. During the optimization procedure, such individuals (candidate solutions) will have only an insignificant chance to pass to the next generation.

It is pointing out that a suitable chromosome representation for the particular problem domain is an extremely important task, since a good choice will make the search faster and easier by restricting the search space. However, it is tremendously important to keep in mind that the crossover and mutation operators must take into account the design of the chromosome. It is important to emphasize that in the considered problem a non-binary chromosome representation was chosen. The main reason why binary representation is the most frequent is the simplicity to implement and popularity in academic papers (Davis, 1991). Moreover, binary chromosome representation is usually space-efficient, that is why it was so popular in times, when memory was a serious problem. However, in real-world problems, it becomes common to create a genotype representation that corresponds to the considered problem with a high degree of accuracy. Crossover is the distinguishing operator of the genetic algorithm. Basically, it is a process of taking two parent solutions and producing of offspring solutions in order to get a new, potentially better one. Crossover is used to vary chromosomes from one generation to the next.

In order to solve the stochastic inventory problem, the uniform crossover is proposed (Figure 3).

![Figure 3 – Uniform crossover representation](image)
to different simulation parameters \( SS \) and \( B \), we seek a way to keep odd and even genes separated. Secondly, the uniform crossover is an efficient way to avoid the premature convergence (Michalewicz, 1996).

\[
P_m \leftarrow \text{probability of swapping values } \\quad \mathbf{w} \leftarrow \text{first vector } <v_1, v_2, ..., v_n> \\quad \mathbf{w} \leftarrow \text{second vector } <w_1, w_2, ..., w_n> \\quad \text{for } i \text{ in } (1, \text{length of vector}) \text{do} \\quad \text{if } P_m \geq \text{random number} \text{ then} \quad \text{return } \mathbf{v} \text{ and } \mathbf{w}
\]

Besides, genetic algorithm requires a mutation operator to perform the optimization. Taking into account the particularities of chromosome encoding, it is proposed to apply two different mutation operators (“mild” and “radical”). The radical mutation is applied in order to prevent the premature convergence (otherwise population may get stuck in local optima). In radical mutation we replace gens in the chromosome by a new integer number in a feasible range \((0, I_{\text{max}})\) with the probability \( P_{m1} \).

\[
P_m \leftarrow \text{probability of replacing } \quad \mathbf{v} \leftarrow \text{vector} \\quad \text{for } i \text{ in } (1, \text{length of } \mathbf{v}) \text{ do} \\quad \text{if } P_m \geq \text{random number} \text{ then} \quad \mathbf{v}[i] \leftarrow \text{random integer} \quad \text{return } \mathbf{v}
\]

On the other hand, mild mutation is applied to accelerate convergence. The mild-mutation operator alters genes in the chromosome with the probability \( P_{m2} \), by multiplying them on the step \( \Delta \) rounding to the nearest integer after that.

\[
P_m \leftarrow \text{probability of altering value } \quad \mathbf{v} \leftarrow \text{vector} \\quad \text{for } i \text{ in } (1, \text{length of } \mathbf{v}) \text{ do} \quad \text{if } P_m \geq \text{random number} \text{ then} \quad \mathbf{v}[i] \leftarrow \text{round}(\mathbf{v}[i] \times \Delta) \quad \text{return } \mathbf{v}
\]

It is concluded that tournament selection is an efficient and robust mechanism for working with imperfect fitness functions (Miller and Goldberg, 1995). Tournament selection runs several “tournaments” among \( t \) individuals (chromosomes) randomly chosen from the population. The fittest individual in each tournament is selected for the following crossover. Since weak individuals have relatively a small chance to be selected in large tournaments, it is quite important to find the optimal tournament size \( t \). Tournament Selection can be programmed by the extremely simple algorithm:

\[
P \leftarrow \text{population} \quad t \leftarrow \text{tournament size, } t \geq 2
\]

Results and Discussion

Consider an example of the six-product inventory control system that operates under just-in-time policy. There is a retailer selling products of 6 types that are replenished by an individual supplier. Products of all six types share a common storage with a limited capacity of 150 pallets \( \sum_1^6 \leq 150 \). Each type of product has a unique triangular distribution for both demand size and replenishment lead time and exponential distribution for demand interarrivals.

We apply given adjustments and execute 60-days simulation of the inventory control system. As the result, the algorithm has successfully converged at the optimum in 122 generations. The optimal solution is represented by the chromosome \( \mathbf{v} = (30, 4, 17, 2, 24, 4, 41, 5, 15, 1, 13, 1) \) with the expected total costs of \( E(\sum_{i=1}^{6} T_C(v)) = 2828835 \text{ USD} \). The fittest chromosome \( \mathbf{v} \) stands for the following simulation adjustments: storage-resources allocation \( B = (30, 17, 24, 41, 15, 13) \) pallets with the corresponding safety-stock levels \( SS = (4, 2, 4, 5, 1, 1) \) pallets. The pivotal advantage of the involved discrete-event simulation is a possibility to perform risk and reliability analysis. Additionally, such an approach allows the researcher to plot the inventory dynamics in details and easily spot existing bottlenecks or system vulnerabilities.

Since the parameters of a genetic algorithm (especially a population size \( N \) and a tournament size \( t \)) have tremendous impact on convergence speed and a probability of premature convergence, it is very important to find a balance between search speed and premature convergence prevention.

Parameters with such a balance may be found empirically. We also can conclude that even relatively insignificant alterations in parameters of the genetic algorithm noticeably affect the convergence speed. Furthermore, some unsuccessful settings may result a premature convergence.
Conclusion

In conclusion, the proposed optimization technique is a simple to design and computationally efficient approach to find nearly-optimal inventory policy in stochastic multi-product inventory systems. Additionally, the combination of discrete-event simulation and genetic algorithm provides a flexible method to solve complex problems with lack of knowledge on the structure of the objective function. Besides, the key advantage of such a simulation-driven approach is the possibility to trace inventory dynamics in details. It is supposed that the method may be applied in automated ordering systems by retail companies.

The research also concludes with a statement that the non-binary chromosome encoding in combination with uniform crossover and two mutation operators provide a fine balance between convergence speed and likelihood of premature convergence. There are still several minor problems to solve, such as the program-optimization of both the simulation and genetic algorithm. Moreover, it is crucially important to test the proposed approach on problems with higher dimension and compare it to alternative metaheuristic techniques. These issues are waiting to be deeply explored in a future research.

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