New Types of Accessible Environmental Influence on Neutrino Oscillation

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Abstract

To describe possible effects of medium, besides the hamiltonian introduced by Mikheev, Smirnov and Wolfenstein (MSW), in principle, there may exist new terms in the evolution equation for the neutrino oscillation due to the environment. Considering a two-energy-level quantum system: $\nu_1 - \nu_2$, two-species neutrino system which is embedded in a matter environment, we solve the neutrino evolution equation precisely with the possible extra terms, especially, those cause alternations between pure and mixed states of the neutrino quantum system. We obtain some remarkable features for the neutrino oscillations, which are different from that of MSW and accessible to be observed in solar neutrino observation and/or in the planned long-baseline neutrino experiments.

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To solve the solar neutrino deficit problem \[1\], there have been many models which all assume neutrinos to be massive \[2\]. And it has been realized that the solar medium influence on the neutrino oscillation may be crucial to solving the problem. Due to the influence of the solar medium, the behavior of oscillation can be drastically modified and there can be a resonance in matter if \(\sin \theta_m \sim 1\), even though the mixing angle in vacuum \(\theta_v\) is very small. This is the well-known Mikheev-Smirnov-Wolfenstein (MSW) mechanism \[3, 4\]. Besides the solar matter effect, the Earth matter effect may also play a certain role in the neutrino oscillation problem, i.e. the recently reported atmospheric neutrino puzzle \[5\] could be attributed to the medium effects either.

The key point of the oscillation scenario may be illustrated by a two-level quantum system \((\nu_e, \nu_\mu)\) or \((\nu_1, \nu_2)\). The former are eigenstates of weak-interaction and the later are that of mass matrix respectively. In the scenario, the relevant quantities derived from quantum field theory are transformed into a Hamiltonian of quantum mechanics (QM). The species of neutrino produced by nuclear reactions is \(\nu_e\) which can be decomposed with respect to the basis of \(\nu_1\) and \(\nu_2\). Thus, without the medium effects, its evolution is

\[
|\nu_e(t)\rangle = \cos \theta_v e^{-iE_1 t} |\nu_1\rangle + \sin \theta_v e^{-iE_2 t} |\nu_2\rangle,
\]

where \(\theta_v\) is the mixing angle in vacuum and \(E_1, E_2\) are energies of \(\nu_1\) and \(\nu_2\). As neutrinos are light, we have \(E_i \approx k + m_i^2/2k\) \((i = 1, 2)\), \(k = |k|\) and \(k\) is the three-momentum of the neutrinos.

When MSW medium-effects are taken into account, the Hamiltonian should be modified from \(H\) to \(H'\) as

\[
H = k + \begin{pmatrix} \frac{m_1^2}{2k} & 0 \\ 0 & \frac{m_2^2}{2k} \end{pmatrix} \implies H' = k + \begin{pmatrix} \frac{m_1^2}{2k} - GN_e \cos^2 \theta_v & -GN_e \sin \theta_v \cos \theta_v \\ -GN_e \sin \theta_v \cos \theta_v & \frac{m_2^2}{2k} - GN_e \sin^2 \theta_v \end{pmatrix}
\]

(2)

where \(GN_e\) is related to the electron density in a uniform matter environment.

By solving the Schrödinger equation for the neutrino evolution

\[
\frac{d}{dt} \begin{pmatrix} \nu_1' \\ \nu_2' \end{pmatrix} = H' \begin{pmatrix} \nu_1' \\ \nu_2' \end{pmatrix}
\]

(3)

one can see that a resonant structure of the mixing angle in medium \(\theta_m\) is resulted in and the oscillation form becomes \[3, 4\]

\[
|\langle \nu_\mu | \nu_e(t) \rangle|^2 = \frac{1}{2} \sin^2(2\theta_v)(\frac{l_m}{l_v})^2[1 - \cos \frac{2\pi t}{l_m}],
\]

(4)
where
\[ l_m = l_v(k)[1 + \left( \frac{l_v(k)}{l_0} \right)^2 - 2 \cos(2\theta_v)\left( \frac{l_v(k)}{l_0} \right)]^{-1/2}, \]  
and
\[ l_0 = \frac{2\pi}{GN_e}, \quad l_v = \frac{4\pi k}{m_1^2 - m_2^2}. \]

The mixing angle and period of the neutrino oscillation are modified accordingly.

Recently, some authors have studied various extensions of quantum mechanics (QM) (or sometimes called violation of QM), such as due to existence of ‘micro black holes’, a non-hermitian Hamiltonian is induced, thus a quantum system of pure state would turn into a mixed state, and CP, even CPT violation may be resulted\(^\text{[6, 7]}\). From a different motivation, Reznik studied a quantum system submerged in a ‘proper surrounding environment’ and he found that instead of a non-reversible evolution of the quantum system from a pure state to a mixed, there may be a periodic transition between pure and mixed states\(^\text{[9]}\).

In fact, neutrino propagating in the dense medium of the Sun may be such a system. The oscillation between two species of the neutrinos, in principle, may be affected by the environment on its way to the surface of the Sun. First of all, we would like to consider the ‘extension’ by Reznik, and following him we write down a generalized Hamiltonian and find the solution of the evolution equation. One can note that it is indeed an extension of the Mikheev-Smirnov-Wolfenstein’s result.

In the following, we briefly recall Reznik’s work and present our derivation of a modified expression for \( |\langle \nu_\mu |\nu_e(t) >|^2 \), as well as demonstrate how the modification practically affects the oscillation rate.

\textit{Formulation} (i) Unitary evolution between pure and mixed states for a quantum system embedded in a matter environment.

The quantum system can be described by a density matrix \( \rho \) in general. If the system is in a pure state, one has \( Tr\rho^2 = Tr\rho = 1 \), instead, when it is in a mixed state, \( Tr\rho^2 < 1 \). In quantum mechanics, the Schrödinger equation is
\[ i\frac{d}{dt}\rho = [H, \rho]. \]
In order to consider the environmental influence due to matter in very general cases, especially a possible unitary evolution of the system between pure and mixed states, Reznik proposed an operator $\hat{\rho}$ which relates to $\rho$, the reduced density matrix of the system, as

$$\rho \equiv \hat{\rho}\hat{\rho}^{\dagger}, \quad (7)$$

whereas $\hat{\rho}^{\dagger}\hat{\rho}$ (in general $\neq \rho$) stands for the density matrix of the environment. At absence of an extra QM extension term, $\hat{\rho}$ still satisfies the same Liouville equation as (6).

At the time $t = 0$, the system is supposed to stay in a pure state, so

$$\rho(t = 0) \equiv |\psi_0\rangle < \psi_0| = \hat{\rho} \equiv \hat{\rho}^{\dagger}. \quad (8)$$

Reznik shows that the equation (3) can be generalized to

$$i\frac{d}{dt}\hat{\rho} = [H, \hat{\rho}] + L\hat{\rho} + \hat{\rho}R + g_{ij}K_i\hat{\rho}K_j^{\dagger}, \quad (9)$$

where $L, R, K_i$ and $K_j^{\dagger}$ all are hermitian operators, $g_{ij}$ are real parameters.

It is noted that as long as the $K_j$ in eq.(9) is not a unit matrix ($\sigma_0$), i.e. it is not a trivial case, there cannot be a simple equation for the density matrix $\rho$. Thus people sometimes call the last term in eq.(9) as an extension of QM, whereas being different in nature, a corresponding extra term is named as the violation term of QM in the case of ref.[6]. In the present case, rewriting eq.(9), we have

$$i\frac{d}{dt}\rho = [H + L, \rho] + g_{ij}(K_i\hat{\rho}K_j^{\dagger}\hat{\rho}^{\dagger} - h.c), \quad (10)$$

and in the last part of the equation, $\rho$ does not appear as an independent variable directly at all (see below for more details). We can also easily prove that a resultant $\rho \equiv \hat{\rho}\hat{\rho}^{\dagger}$ describes a mixed state as $Tr(\rho^2) < 1$ in the case. In fact, if there is no the last extension term, $d/dt(Tr\rho^2) = 0$ always holds, namely, the system remains in either pure or mixed state forever. It never transits from a pure state to a mixed or vice versa. Whereas when the ‘extra’ term exists, it is easy to prove $d/dt(Tr\rho^2) \neq 0$, and it indicates that an alternation[3] between pure and mixed states occurs.

(ii) Let us consider a system, that only two species of neutrinos are involved, thus they can be treated as a two-energy-level quantum system.

1 Throughout the paper we adopt the word ‘alternation’ here instead of ‘oscillation’ adopted by [3], to avoid possible confution with the ‘oscillation’ for neutrinos.
Following ref.[9] and considering the situation for neutrinos, we may write down the term $H$ appearing in eq.(9) directly as

$$H = k + \left( \begin{array}{cc} m_1^2 & 0 \\ 0 & m_2^2 \end{array} \right) \tag{11}$$

in the basis of $|\nu_1\rangle$ and $|\nu_2\rangle$. The trivial part $k$ is a unit matrix and can be omitted in our later calculations. After a transformation $\hat{\rho} \to \hat{\rho} \, U$ with

$$U = \exp[-i \int^t (R - H) dt'],$$

according to ref.[9] the eq.(9) is turned to

$$i \frac{d}{dt} \hat{\rho} = \tilde{H} \hat{\rho} + g_{ij} K_i \hat{\rho} \tilde{K}_j^\prime \tag{12}$$

and

$$\tilde{H} = H + L.$$

The extra Hamiltonian $L$ is hermitian which is induced by the environment, therefore can be absorbed into the original Hamiltonian $H$.

Comparing eq.(3) and eq.(12), $\tilde{H}$ can be identified as $H'$ and one can easily believe that the extra $L$ is nothing new, but the Hamiltonian derived by Mikheev, Smirnov and Wolfenstein. Thus

$$L = \left( \begin{array}{cc} -G N_e \cos^2 \theta_v & -G N_e \sin \theta_v \cos \theta_v \\ -G N_e \sin \theta_v \cos \theta_v & -G N_e \sin^2 \theta_v \end{array} \right). \tag{13}$$

Therefore $\tilde{H} \equiv H + L$ is exactly the hamiltonian $H'$ in eq. (3).

However, one can notice that there is one more extra term in eq.(12), i.e. $g_{ij} K_i \hat{\rho} \tilde{K}_j^\prime$ which violates the hermiticity of $\hat{\rho}$ (but not $\rho$) and it would influence the behavior of the neutrino oscillation. For simplicity, later on we will write this term just as $K \hat{\rho} \tilde{K}'$ and the free parameters are absorbed in $K$ and $\tilde{K}'$.

(iii) $\tilde{H} = H + L$ is a $2 \times 2$ hermitian matrix, so it can be decomposed as

$$\tilde{H} = a_0 \sigma_0 + a_1 \sigma_1 + a_2 \sigma_2 + a_3 \sigma_3. \tag{14}$$

The first term in this expression is trivial, so we will neglect it in later calculations, and furthermore from the explicit form of $\tilde{H}$ given in eqs.(2) and (13), take $a_2 = 0$ in later calculations.
Due to energy conservation, \([K, \hat{H}] = 0\) must be satisfied, a general form of \(K\) should be
\[
\alpha (a_1 \sigma_1 + a_3 \sigma_3),
\]
where \(\alpha\) is an arbitrary parameter. \(\tilde{K}'\), being a hermitian \(2 \times 2\) matrix, can be written as
\[
\tilde{K}' = \lambda_1 \sigma_1 + \lambda_2 \sigma_2 + \lambda_3 \sigma_3,
\]
where a possible but trivial term, \(\lambda_0 \sigma_0\), has been dropped out.

Just for illustration and convenience for later discussions, following ref. [9], we choose the simplest form for \(\tilde{K}'\) as \(\lambda \sigma_3\) (i.e. \(\lambda_1 = \lambda_2 = 0\) and \(\lambda_3 = \lambda\)). Thus the extension term becomes
\[
K \hat{\rho} \tilde{K}' = (a_1 \sigma_1 + a_3 \sigma_3) \hat{\rho} \lambda \sigma_3, \tag{15}
\]
where the arbitrary parameter \(\alpha\) is absorbed into \(\lambda\). So far, we cannot derive this term from a well-established theory yet, so that we leave it aside as an open question.

As usually adopted method [6, 7, 9], we write \(\hat{\rho}\) in a four-vector form. In fact, this step is not trivial at all because of the QM extension term, (below we will give more discussions on this point).

One can decompose \(\hat{\rho}\) as
\[
\hat{\rho} = \hat{\rho}_0 \sigma_0 + \hat{\rho}_1 \sigma_1 + \hat{\rho}_2 \sigma_2 + \hat{\rho}_3 \sigma_3.
\]
Thus
\[
\frac{d}{dt} \hat{\rho} = \hat{H} \hat{\rho} + K \hat{\rho} \tilde{K}'
\]
would become a form
\[
i \frac{d}{dt} \hat{\rho}_a = (\hat{H}^{qm}_{ab} + \delta H_{ab}) \hat{\rho}_b, \quad a, b = 0, 1, 2, 3, \tag{16}
\]
where both \(\hat{H}^{qm}\) and \(\delta H\) are \(4 \times 4\) matrices which correspond to \(\hat{H} = H + L\) and \(K \hat{\rho} \tilde{K}'\) respectively.

Thus we have
\[
i \frac{d}{dt} \begin{pmatrix}
\hat{\rho}_0 \\
\hat{\rho}_1 \\
\hat{\rho}_2 \\
\hat{\rho}_3
\end{pmatrix} = \begin{pmatrix}
a_0 + \lambda a_3 & a_1 & i \lambda a_1 & a_3 \\
a_1 & a_0 - \lambda a_3 & -i a_3 & \lambda a_1 \\
-i \lambda a_1 & i a_3 & a_0 - \lambda a_3 & -i a_1 \\
a_3 & \lambda a_1 & i a_1 & a_0 + \lambda a_3
\end{pmatrix}
\begin{pmatrix}
\hat{\rho}_0 \\
\hat{\rho}_1 \\
\hat{\rho}_2 \\
\hat{\rho}_3
\end{pmatrix} \equiv H_{(4 \times 4)} \hat{\rho}^V. \tag{17}
\]
It is easy to check that in the \((4 \times 4)\) form, \(H_{(4 \times 4)} = (\hat{H}^{qm}_{ab} + \delta H_{ab})_{4 \times 4}\) still retains hermiticity.

(iv) Solution of eq.(17) can be obtained by diagonalizing the \(4 \times 4\) matrix. Because it is a hermitian matrix, so it can be diagonalized via a unitary matrix \(V\), as \(diag(H_{(4 \times 4)}) = VH_{(4 \times 4)}V^\dagger\).
Thus
\[ \hat{\rho}_a(t) = \sum_{b,c} V_{ab} V_{bc} \hat{\rho}_c(t=0) e^{-i\beta_b t}, \] (18)
where \( \beta_b \) are eigenvalues of \( H_{4\times4} \) in eq.(17). Since at the moment \( t=0 \), the system is at a pure state, \( \hat{\rho}(t=0) = \rho(t=0) \). Without losing generality we assume a pure state at the initial state, thus
\[ \rho(t=0) = \hat{\rho}(t=0) = \left| \nu_e \right\rangle \left\langle \nu_e \right| = \begin{pmatrix} \cos^2 \theta_v & -\sin \theta_v \cos \theta_v \\ -\sin \theta_v \cos \theta_v & \sin^2 \theta_v \end{pmatrix}. \] (19)

Converting the initial condition in \( 2 \times 2 \) matrix form into a four-vector form and substituting those \( \hat{\rho}_c(t=0) \) into the solution (18), we obtain \( \hat{\rho}_a(t) \) and \( \rho_e(t) \equiv \hat{\rho}_e(t) \hat{\rho}_e^\dagger(t) \) in \( (2 \times 2) \) matrix form.

In fact, a complete form of \( K \hat{\rho} K' \) which replaces the simple one in eq.(15), does not change the whole physical picture. Our full derivation shows that to obtain the formulation with all the \( \lambda_i \) (i=1,2,3) existing, one only needs to re-write \( \lambda \) of eq.(15) in a complicated combination of \( \lambda_i \) whereas the general form remains unchanged. So a solution derived from the simple expression does not lose generality. Here we omit the details to save space.

(v) The oscillation rate is defined as
\[ P_{\nu_e \rightarrow \nu_\mu}(t) \equiv | <\nu_\mu|\nu_e(t) > |^2 = Tr(\rho_\mu^\dagger \rho_e(t)). \] (20)

Thus we have the final expression of \( P \) as
\[ P(l) = \frac{1}{2} \sin^2 2\theta_v (\frac{l m}{l v})^2 [1 - \cos(2\frac{\pi l}{l m}) \cos(2\frac{\pi \lambda l}{l m}) + \cos 2\theta_v \sin(2\frac{\pi l}{l m}) \sin(2\frac{\pi \lambda l}{l m})], \] (21)
where \( l \) is the distance from the production site of \( \nu_e \) to the detection point. Note that our \( \lambda \) is dimensionless and it is slightly different from Reznik’s notation where his \( \lambda \) has a dimension of energy. It is a modified expression of the neutrino oscillation in medium and simply an extension from that given by Mikheev, Smirnov and Wolfenstein.

Note that we have checked that the extension of QM which we are considering, is of unitarity. Because of conservation of possibilities the constraint
\[ P_{\nu_e \rightarrow \nu_\mu}(t) + P_{\nu_e \rightarrow \nu_e}(t) = 1 \]
is satisfied exactly.

As \( \lambda = 0 \), which implies the extra term \( K \hat{\rho} K' \) disappears, the expression reduces back to the form given in ref.[3] and [4]. For non-zero \( \lambda \) values, the oscillation is not simply harmonic, but in
a modified mode. In general the oscillation period is shortened, so for the solar neutrino case the result favors a larger reduction rate of the electron neutrinos produced in the Sun.

(vi) To compare various possibilities, let us consider another type of environmental influence on the neutrino oscillation due to the ‘micro black holes everywhere’ although the authors of ref. [6] originally proposed it for the quantum system $K^0 \rightarrow \bar{K}^0$ mixing. According to the mechanism given in ref. [6], the oscillation rate may be obtained:

$$P(t) = |<\nu_\mu |\nu_e(t)>|^2 = \frac{1}{2} \sin^2 2\theta_v (1 - e^{-\frac{i\alpha + \gamma}{2} t} \cos \frac{2\pi t}{l_v}),$$

(22)

where an approximation

$$\Delta/k \gg |\alpha|, |\beta|, |\gamma| \quad (or \ \alpha \sim \beta \gg |\beta|)$$

is required.

That is a modified vacuum oscillation and the damping factor indicates an energy loss at the evolution process. It is understood that as the neutrinos propagate in the physical vacuum full with micro black holes, their evolution behavior are affected by the black holes. Simultaneously the quantum system turns from a pure state into a mixed state and this trend is non-reversible since the black holes never release anything out (here we do not refer to the quantum tunneling effect of black holes). The new behaviors of the system evolution will be discussed in our coming work [13].

**Discussions and implications**

(i) It is definite that the solar environment can significantly influence the neutrino oscillation. Mikheev, Smirnov and Wolfenstein’s work accounted such effect by adding an extra hamiltonian to the system. Here in a more general framework, as Reznik suggested, there is a natural way to include the influence from environment, especially the system may be oscillating between pure and mixed states. Namely, the environment interacts with the quantum system in an unknown and complicated way. Such an interaction is not only depicted by a hermitian Hamiltonian, but also by an additional term, which violates the hermiticity of $\hat{\rho}$. It makes the system evolve between pure and mixed states. The MSW’s Hamiltonian only results in a hermitian part of $\hat{\rho}$. It is also noted that the interaction of the neutrino system with the magnetic field of the Sun [10] can also induces an external environmental effect but it may also be absorbed into $L$. Only the extension part can cause the system to alternate between pure and
mixed states. Here the situation is different from that of the ‘micro black holes’. There are interactions between the neutrino system and the environment, although we cannot theoretically derive them precisely at present. In general the energy of the whole system is conserved, even though it evolves from a pure state into a mixed state and vice versa. From the analytic expression, we also observe that the oscillation behavior of the neutrino system with the QM extension term is quite different from that without it. In the regular QM region where $K\hat{\rho}K'$ is absent (i.e. $\lambda = 0$), the oscillation rate $|\langle \nu_\mu | \nu_e(t) > |^2$ can vary between 0 and $\sin^2(2\theta_m)$ (or $\sin^2(2\theta_v)$ for the vacuum case) on the contrary, due to the presence of the QM extension term, $|\langle \nu_\mu | \nu_e(t) > |^2$ can never be zero except at the moment $t=0$.

For the solar neutrino problem since the initial $\nu_e$ is produced all over the Sun and the distribution of the chemical elements in the Sun varies, one must integrate the transition probability over the whole production region and energy spectrum of neutrinos. It is

$$\frac{1}{6} \int_E \int_{-L}^{+L} P(L-l) \eta(L-l, E) dl dE,$$

where the origin is chosen at the center of the Sun, and the time, which the neutrino takes for traveling from $l$ to the surface of the Sun, is $t = l/v \sim l/c$ and $v$ is the speed of the neutrino, which is close to that of light $c$, as long as neutrino masses are supposed to be very tiny. The factor $1/6$ accounts for an average probability in the propagation direction to the Earth. If the distribution of the neutrino production $\eta(L-l, E)$ does not vary drastically, it can be approximately treated as a constant, the integration over sites can be carried out analytically.

To solve the solar neutrino problem with MSW solution as suggested by many literatures, $\Delta \equiv m_1^2 - m_2^2$ is comparatively large. We have $L \gg l_m$, i.e. the solar size is much longer than the oscillation length. In the case the extra contribution in eq.(21) would disappear as well as the oscillation term in eq.(4) which is proportional to $\frac{2l_m}{2\pi L} \cos(2\pi L/l_m)$, unless $\lambda$ is close to unity. If $\lambda \sim 1$, the modified expression (21) can be averaged as

$$\frac{1}{L} \int_{-L}^{+L} P(L-l) dl = \frac{1}{2} \sin^2 \theta_v \left( \frac{l_m}{l_v} \right)^2 \cos^2 \theta_v. \quad (23)$$

Note that, as long as $\lambda \neq 1$ and $l_m \ll L$, the extension term $K\hat{\rho}K'$ does not change the physics obtained in the regular framework without such a term, whereas $\lambda \to 1$, an extra factor $\cos^2 \theta_v$
would be attached to the original expression that is observable. If $\theta_e$ is small, this effect can also be neglected.

In fact, the medium is not uniform, i.e. $\eta(L - l, E)$ is not a constant over $l$ and $\lambda$ may be environment dependent, there may be some effects which can influence the data fitting of $\theta_m$, but to take into accounts of all the detail, one needs better understanding of the solar model and the QM extension effect.

It is also interesting to note that if $\Delta \equiv m_1^2 - m_2^2$ is very small, for instance, which is close to the expected value for the vacuum oscillation solution, i.e. the oscillation in medium cannot complete in one cycle in the solar neutrino case, the effects of the extension term, no matter from the micro black holes or the dense environment, increases the transition probability from $\nu_e$ to $\nu_\mu$.

(ii) The interesting QM extension effect is possibly observable in other neutrino oscillation experiments. The most likely possibility is in the planned long baseline accelerator experiments, KEK-SuperKamiokande(250Km), CERN-Gran Sasso(730Km) and Fermilab-Soudan2(730Km)\[12]. The average energies of $\mu$ neutrino beams are approximately 1 GeV, 6 GeV and 10 GeV respectively. In all these long baseline experiments neutrino oscillation is expected to be seen if the neutrino mass square difference is not much smaller than $10^{-3}\text{ eV}^2$ and the mixing angle is not too small. With the new QM extension effect being taken into account, we need to consider two cases. If $\lambda$ is not much smaller than one, then the new effect may modify neutrino oscillation significantly. And if $\lambda$ is much larger than one, then the oscillation is observable for even smaller neutrino mass square difference $10^{-3}\lambda^{-1}\text{ eV}^2$. If the distance of the experiment is chosen as $10^4\text{Km}$, that is about the diameter of the Earth, the energy of the neutrino beam is about 1GeV, for neutrino mass square difference as small as $10^{-5}\text{eV}^2$, the oscillation is expected to be seen. So long as $\lambda$ is not too smaller than one, then the QM extension effect is also to be observed for the mass squared difference not too smaller than $10^{-5}\text{ eV}^2$.

(iii) Ellis et al. \[3] and Reznik \[9] introduced two different mechanisms which both can induce transition (or oscillation) of quantum systems from a pure state to a mixed one. Ellis et al. added the extra $\delta H_{(4 \times 4)}$ of $(4 \times 4)$ form in the evolution equation for $\rho^V$ of the four-vector form and this new term is not hermitian. When one tries to turn the equation back to a $(2 \times 2)$ form, it is
found that there is no way to make a Schrödinger-type equation for the $(2 \times 2)$ $\rho$-matrix such as

$$i(d/dt)\rho = H_{(2\times2)}\rho.$$  

Reznik’s formulation, instead, gives an equation for $\dot{\rho}$ of $2 \times 2$ form where he introduced the extension term $K\dot{\rho}K'$ and the equation also cannot be converted into an equation for $\rho$, even in the four-vector form, but there $H_{(4\times4)}$ is a hermitian matrix. Generally the resultant $(2 \times 2)\dot{\rho}$ is not hermitian, while $\rho$ is. One can notice that the differences of Ellis et al.’s mechanism from Reznik’s explicitly.

Most generally, if $\rho$ (or $\dot{\rho}$) is in a four vector form, the generalized Schrödinger-type equation reads

$$i\frac{d}{dt}\rho^V = H_{(4\times4)}\rho^V,$$ (or for $\dot{\rho}$),

it can be converted into a $(2 \times 2)$ matrix equation where $\rho$ (or $\dot{\rho}$) is a $2 \times 2$ matrix as

$$i\frac{d}{dt}\rho = L\rho + \rho R + L\rho K + \Delta\rho,$$ (or for $\dot{\rho}$),

where $L, R, K, \Delta$ are operators and can be decomposed as

$$O = O_0\sigma_0 + O_i\sigma_i \quad (O = L, R, K, \Delta; \quad i = 1, 2, 3),$$

the coefficients $O_\alpha (\alpha = 0, 1, 2, 3)$ would be complex as the $L\rho(\dot{\rho})K$ term exists. Writing out $H_{(4\times4)}$, the real part of $O_\alpha$ correspond to the hermitian components of $H_{(4\times4)}$ while the imaginary parts to the anti-hermitian ones.

Because of the existence of the anti-hermitian components, the energy of the system is not conserved as in Ellis et al.’s $H_{(4\times4)}$. In our formulation, the $H_{(4\times4)}$ is hermitian, so the energy of the system is conserved. Both the extra term $K\rho K'$ and $K\dot{\rho}K'$ demand $(d/dt)Tr(\rho^2) \neq 0$, i.e. a system evolves from a pure state into a mixed state.

(iv) We investigate the general properties of a quantum system which evolves from a pure state into a mixed one due to either the micro black holes in the background or certain environment in the case of neutrino oscillation. With the generalized Schrödinger equation, we have derived an expression for $\dot{\rho}$ which is non-hermitian and the density matrix $\rho = \dot{\rho}\dot{\rho}^\dagger$. In this result, $\dot{\rho} \neq \dot{\rho}^\dagger$, naturally, one can decide that $\dot{\rho}^\dagger\dot{\rho}$ describes the environment. In the case, $(d/dt)Tr\rho^2 \neq 0$, thus the system evolves between pure and mixed states.
Thus we have derived the $\nu_e \rightarrow \nu_\mu$ transition rate and find that the newly obtained expression is a modification of the famous Mikheev-Smirnov-Wolfenstein’s formulation.

In the modified expression, $\sin^2(2\theta_v)(\frac{l_{tm}}{\lambda})^2 = \sin^2(2\theta_m)$ keeps unchanged as in the MSW theory, but the oscillation form $(1 - \cos(\frac{2\pi t}{l_m}))$ is modified into a more complicated one as $(1 - \cos(\frac{2\pi t}{l_m}) \cos(\frac{2\pi \lambda}{l_m}) + \cos(2\theta_v) \sin(\frac{2\pi t}{l_m}) \sin(\frac{2\pi \lambda}{l_m}))$. It obviously favors $\nu_e \rightarrow \nu_\mu$.

(v) The new extra term(s) either from micro black holes or from a matter environment should be derived from certain theories in principle, whereas so far no one has a reasonable way to derive it(them) out theoretically. However as pointed out in the paper, the parameters, such as $\lambda$ for instance, can be phenomenologically determined by fitting data if they fall into a suitable region. The environmental influence may be observable in the solar neutrino and/or the planned long-baseline experiments through neutrino oscillations. Therefore we conclude that the environmental influence may be experimentally accessible and certain constraints on the ‘extra’ terms may be set from the solar neutrino observation and the planned long-baseline neutrino oscillation experiments in the foreseeable future.

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