Simultaneously Transmitting And Reflecting (STAR) RIS Assisted NOMA Systems

Jiajiao Zuo*, Yuanwei Liu†, Zhiguo Ding‡ and Lingyang Song§
* School of Internet of Things, Nanjing University of Posts and Telecommunications, Nanjing 210003, China.
† Jiangsu Key Laboratory of Broadband Wireless Communication and Internet of Things, Nanjing University of Posts and Telecommunications, Nanjing 210003, China.
‡ School of Electronic Engineering and Computer Science, Queen Mary University of London, London E1 4NS, U.K.
§ Department of Electronics, Peking University, Beijing 100871 China.
E-mail: zuojiakuo@njupt.edu.cn, yuanwei.liu@qmul.ac.uk, zhiguo.ding@manchester.ac.uk, lingyang.song@pku.edu.cn.

Abstract—In this paper, a novel simultaneously transmitting and reflecting reconfigurable intelligent surface (STAR-RIS) assisted non-orthogonal multiple access (NOMA) system is proposed, where the STAR-RIS can simultaneously transmit and reflect the incident signals. Our objective is to maximize the achievable sum rate by jointly optimizing the decoding order, power allocation coefficients, active beamforming, transmission and reflection beamformings. However, the formulated problem is non-convex with intricately coupled variables. To tackle this challenge, a suboptimal two-layer iterative algorithm is proposed. Specifically, in the inner-layer iteration, for a given decoding order, the power allocation coefficients, active beamforming, transmission and reflection beamformings are optimized alternatively. For the outer-layer iteration, the decoding order of NOMA users in each cluster is updated with the solutions obtained from the inner-layer iteration. Simulation results are provided to demonstrate that the proposed STAR-RIS-NOMA system outperforms conventional NOMA systems.

I. INTRODUCTION

Reconfigurable intelligent surfaces (RISs) are promising candidates for improving the performance of future sixth-generation (6G) wireless communication networks [1, 2]. By properly adjusting the amplitude and phase response of these elements, the propagation of the incident wireless signals can be reconfigured. However, RISs can only reflect the incident signals. To overcome this limitation, recently, a novel RIS, named simultaneous transmitting and reflecting RISs (STAR-RISs) [3] or intelligent omni-surface (IOS) [4], is proposed. Different from traditional reflection-only RISs, STAR-RISs can simultaneously transmit and reflect the incident signals, which leads to a full-space coverage. The transmitted and reflected signals can be reconfigured by a STAR-RIS element via its corresponding transmission and reflection coefficients, which introduces additional degree-of-freedoms (DoFs) to control the signal propagation [3]. Despite the above appealing advantages, to the best of our knowledge, research on STAR-RIS aided wireless communication systems is still in its infancy.

For the STAR-RISs in [5], the channel models for the near- and far-field regions were proposed and the closed-form expressions for channel gains of users receiving the transmission and reflection signal were derived. In [6], the fundamental coverage characterization of STAR-RIS assisted two-user communication networks was investigated. The sum coverage range maximization problems were formulated for both non-orthogonal multiple access (NOMA) and orthogonal multiple access (OMA) systems. The power consumption minimization problems for the STAR-RIS assisted unicast and multicast systems were studied in [7], where the active and passive beamformings were jointly optimized for different operating protocols of STAR-RIS. The IOS in an indoor multi-user downlink communication system was studied in [8], where a joint IOS analog beamforming and small base station digital beamforming optimization problem was formulated to maximize the sum-rate of the system. In [9], the optimal phase shifts of the IOS was analyzed and a branch-and-bound based algorithm was proposed to design the IOS phase shifts in a finite set.

On the other hand, as a promising technique for enhancing spectrum efficiency and supporting massive connectivity, NOMA has also received significant attention [10]. NOMA outperforms conventional OMA techniques by simultaneously sharing the communication resources between all users via the power or code domain [11]. Inspired by the aforementioned discussion, it is interesting to investigate the promising applications of the STAR-RIS technique in NOMA systems for further performance improvement.

In this paper, we propose a downlink STAR-RIS assisted NOMA (STAR-RIS-NOMA) communication system and formulate a joint optimization problem over the decoding order, power allocation coefficients, active beamforming, transmission and reflection beamforming.

II. SYSTEM MODEL AND PROBLEM FORMULATION

As shown in Fig. 1, we consider the downlink transmission in a STAR-RIS-NOMA communication system, where the direct base station (BS)-user links are blocked and the BS communicates with K single-antenna users with the aid of a STAR-RIS. Assume that the BS is equipped with N_T transmit antennas, while the STAR-RIS is equipped with M elements. The K users are grouped into C clusters. The number of users in cluster c is K_c, where \sum_{c=1}^{C} K_c = K. For expression convenience, let
Fig. 1: STAR-RIS-NOMA system

\[ u_p = \begin{bmatrix} \sqrt{\beta_p^1 e^{j\phi_p}}, \sqrt{\beta_p^2 e^{j\phi_p}}, \ldots, \sqrt{\beta_p^M e^{j\phi_M}} \end{bmatrix}^H \]

be the transmission \((p = t)\) or reflection \((p = r)\) beamforming vector, \(\Theta_p = \text{diag}(u_p)\) be the corresponding diagonal beamforming matrix. Moreover, the set of constraints to the transmission and reflection coefficients is denoted by:

\[
\mathbb{R}_{\beta,\theta} = \left\{ \begin{array}{l}
\left[ \beta_m^t, \beta_m^r, \theta_m^t, \theta_m^r \right] \mid \beta_m^t, \beta_m^r \in [0, 1]; \beta_m^t + \beta_m^r = 1; \\
\theta_m^t, \theta_m^r \in [0, 2\pi)
\end{array} \right\}.
\]

(1)

Denote by \(h_{c,n} = g_{c,n}^H \Theta_{c,n} F\) the combined channel of the BS-RIS-user link for user \(n\) in cluster \(c\), where \(F\) is the channel matrix from the BS to the RIS, \(g_{c,n}\) is the channel vector from the RIS to user \(n\), \(\Theta_{c,n}\) is the RIS coefficient diagonal matrix which is defined as

\[
\Theta_{c,n} = \begin{cases} 
\Theta_{t}, & \text{if user } n \text{ is in the transmission space (TS)}, \\
\Theta_{r}, & \text{if user } n \text{ is in the reflection space (RS)}.
\end{cases}
\]

(2)

Denote by \(w_c\) the beamforming vector for cluster \(c\), the received signal at the \(n\)-th user in cluster \(c\) is given by

\[
y_{c,n} = h_{c,n} w_c \left( \sqrt{\rho_{c,n} s_{c,n}} + \sqrt{\sum_{j=1,j\neq n}^{K_c} \rho_{c,j} s_{c,j}} \right) + \sum_{c=1,c\neq c}^{K_c} \sum_{i=1}^{K_c} h_{c,n} w_{c,i} \sqrt{\rho_{c,i}} s_{c,i} + z_{c,n},
\]

where \(\rho_{c,n}\) and \(s_{c,n}\) are the power allocation coefficient and the desired signal of user \(n\) in cluster \(c\). The power allocation coefficients satisfy \(\sum_{n=1}^{K_c} \rho_{c,n} = 1\). In addition, \(z_{c,n}\) is the complex circular i.i.d. additive Gaussian noise with \(z_{c,n} \in \mathcal{CN}(0, \sigma^2)\), where \(\sigma^2\) is the noise power.

Without loss of generality, denote by \(D_c(k)\) the user index that corresponds to the \(k\)-th decoded user in cluster \(c\). For any two users \(D_c(j)\) and \(D_c(k)\) with decoding order \(j > k\), after applying the successive interference cancellation (SIC) decoding procedure, the signal-to-interference-plus-noise ratio (SINR) for user \(D_c(j)\) to decode user \(D_c(k)\) is given by

\[
\text{SINR}_{D_c(j), D_c(k)} = \frac{\left| h_{c,D_c(j)} w_c \rho_{c,D_c(j)} \right|^2 e^{-j\phi_c}}{\left| h_{c,D_c(j)} w_c \right|^2 \rho_{c,D_c(k)} + I_{c,D_c(k)} + \sigma^2},
\]

(4)

where \(\rho_{c,D_c(k)} = \sum_{n=k+1}^{K_c} \rho_{c,D_c(n)}\) and \(I_{c,D_c(k)} = \sum_{c \neq c, j}^{C} \left| h_{c,D_c(j)} w_{c,i} \right|^2\). The corresponding achievable rate is

\[
R_{D_c(j), D_c(k)} = \log_2 \left( 1 + \text{SINR}_{D_c(j), D_c(k)} \right) \quad \text{and the achievable rate of user } D_c(k) \text{ is } R_{D_c(k)} = \log_2 \left( 1 + \text{SINR}_{D_c(k)} \right).
\]

Finally, the overall achievable sum rate can be written as

\[
R_{\text{sum}} = \sum_{c=1}^{C} \sum_{k=1}^{K_c} R_{D_c(k)}.
\]

To guarantee the SIC performed successfully, the condition \(R_{D_c(j), D_c(k)} \geq R_{D_c(k)}\) with \(j > k\) should be satisfied. As a result, there will be \(K_c \times (K_c - 1)\) SIC decoding rate conditions for cluster \(c\) with \(K_c\) users. It is worth noting that the SIC decoding rate conditions depend not only on the active beamforming vectors and power allocation coefficients, but also on the transmission and reflection beamforming vectors.

The joint optimization problem for the STAR-RIS-NOMA system is formulated as

\[
\max_{D_c, \rho_{c,D_c}, w_c, u_p} \sum_{c=1}^{C} \sum_{k=1}^{K_c} R_{D_c(k)}
\]

\[
s.t. \quad R_{D_c(k)} \geq R_{D_c(k)}^{\min},
\]

\[
P_{\text{max}} = \sum_{c \in C} \sum_{k=1}^{K_c} \rho_{c,D_c(k)},
\]

\[
D_c \in D,
\]

(5a)

(5b)

(5c)

(5d)

(5e)

(5f)

(5g)

where \(P_{\text{max}}\) is the total transmit power budget, \(D\) denotes the combination set of all possible decoding orders.

### III. SOLUTION OF THE PROBLEM

To solve problem (5), we first determine the decoding order for each cluster. Then, under the fixed decoding order, the joint optimization problem over power allocation coefficients, active beamforming, transmission and reflection beamforming are solved based on the alternating optimization.

#### A. Equivalent-Combined Channel Gain based Decoding Order

Since decoding order is an essential problem for the considered STAR-RIS-NOMA system, which should be determined before solving the optimization problem. For cluster \(c\) with \(K_c\) users, under given active beamforming vectors \(\{w_c\}\), transmission beamforming vector \(u_r\) and reflection beamforming vector \(u_r\), the optimal decoding order is defined as [12]

\[
\Gamma_{c,D_c(1)} \leq \Gamma_{c,D_c(2)} \leq \cdots \leq \Gamma_{c,D_c(K_c)},
\]

(6)

where \(\Gamma_{c,D_c(k)}\) is the equivalent-combined channel gain, which can be expressed as follows

\[
\Gamma_{c,D_c(k)} = \left| g_{c,D_c(k)}^H \Theta_{c,D_c(k)} F w_c \right|^2 / \left( \sum_{c=1,c\neq c}^{K_c} g_{c,D_c(k)}^H \Theta_{c,D_c(k)} F w_{c,i}^2 + \sigma^2 \right)^2.
\]

(7)

It is noted that the above decoding order is a function of the vectors of active, transmission and reflection beamforming. Due to

\[
\sum_{c=1,c\neq c}^{K_c} \sum_{i=1}^{K_c} \left| g_{c,D_c(k)}^H \Theta_{c,D_c(k)} F w_{c,i} \right|^2 \rho_{c,i} =
\]

\[
\sum_{c=1,c\neq c}^{K_c} \sum_{i=1}^{K_c} \left| g_{c,D_c(k)}^H \Theta_{c,D_c(k)} F w_{c,i} \right|^2 \rho_{c,i} =
\]
\[
\sum_{c=1}^{C} \sum_{i \neq c}^{K_c} \left| g_{c,i}^{H} \Theta_{c,i} \left( k \right) F \right|^2 w_{c,i}^2 \text{ with } \sum_{i=1}^{K_c} \rho_{c,i} = 1. \text{ Therefore, the power allocation coefficients have no impact on the decoding order. The detailed proof of this property can be found in [12].}
\]

**Lemma 1:** For any two users \( k \) and \( j \) belong to cluster \( c \), if the decoding order of the two users satisfies
\[
D^{-1}(j) > D^{-1}(k),
\]
where \( D^{-1}(\cdot) \) is the inverse of mapping function \( D_c(\cdot) \).

Then, under the optimal decoding order, the following SIC condition is guaranteed:
\[
R_{c,k}^j \geq R_{c,k}^k.
\]

**Proof:** Similar proof can be found in [12].

Lemma 1 indicates that the SIC constraints in (5c) can be removed under the optimal decoding order. Without loss of generality, let \( D_c(k) = k \). Then, problem (5) under the given decoding order can be rewritten as:
\[
\begin{align*}
\max_{\rho_{c,k},w_{c,k}} & \sum_{c=1}^{C} \sum_{k=1}^{K_c} \log_2 \left( 1 + \text{SINR}^k_{c,k} \right), \\
\text{s.t.} & \quad R_{c,k}^k \geq R_{c,k}^\text{min}, \\
& \quad \sum_{k=1}^{K_c} \rho_{c,k} = 1, \\
& \quad (5d), (5f), \\
\end{align*}
\]

In the following, we solve the reduced problem (10) instead of the original problem (5).

**B. Power Allocation Coefficients Optimization**

Since the inter-cluster interference has no relationship with \( \{\rho_{c,k}\} \), the optimization problem in (10) for the power allocation coefficients can be decomposed into \( C \) decoupled subproblems. Thus, the power allocation coefficients optimization problem for cluster \( c \) is simplified as:
\[
\begin{align*}
\max_{\rho_{c,k}} & \quad \sum_{k=1}^{K_c} R_{c,k}^k, \\
\text{s.t.} & \quad (10b), (10c), \\
\end{align*}
\]

Problem (11) has been solved by [13] and the optimal power allocation coefficients can be expressed as:
\[
\begin{align*}
\rho_{c,1}^* &= \frac{1}{1 + \frac{\rho_{c,2}}{2}} \\
\rho_{c,2}^* &= \frac{\rho_{c,2}}{1 + \frac{\rho_{c,2}}{2}} \\
& \vdots \\
\rho_{c,K_c-1}^* &= \frac{\rho_{c,K_c-1}}{1 + \frac{\rho_{c,K_c-1}}{2}} \\
\rho_{c,K_c}^* &= 1 - \sum_{i=1}^{K_c-1} \rho_{c,i}^*.
\end{align*}
\]

where \( r_{c,k}^\text{min} = 2R_{c,k}^\text{min} - 1 \).

**C. Active Beamforming Optimization**

Before solving the active beamforming optimization problem in (10), we introduce slack variables \( \{A_{c,k}, B_{c,k}\} \), where \( A_{c,k} \) and \( B_{c,k} \) are defined as
\[
\frac{1}{A_{c,k}} = |h_{c,k}w_{c,k}^2| \rho_{c,k},
\]

\[
B_{c,k} = |h_{c,k}w_{c,k}^2| \bar{p}_{c,k} + \sum_{c=1}^{C} \sum_{e \neq c} |h_{c,e}w_{c,e}^2| + \sigma^2.
\]

Thus, the active beamforming optimization problem in (10) can be equivalently reformulated as
\[
\max_{w_{c,k},A_{c,k},B_{c,k},R_{c,k}^k} \sum_{c=1}^{C} \sum_{k=1}^{K_c} R_{c,k}^k,
\]

\[
\text{s.t.} \quad \log_2 \left( 1 + \frac{1}{A_{c,k}B_{c,k}} \right) \geq R_{c,k}^k, \\
\frac{1}{A_{c,k}} \leq |h_{c,k}w_{c,k}^2| \rho_{c,k},
\]

\[
B_{c,k} \geq |h_{c,k}w_{c,k}^2| \bar{p}_{c,k} + \sum_{c=1}^{C} \sum_{e \neq c} |h_{c,e}w_{c,e}^2| + \sigma^2,
\]

\[
(5d), (10b),
\]

We further define \( H_{c,k} = h_{c,k}w_{c,k}^H \) and \( W_c = w_c w_c^H \), where \( W_c \succeq 0 \) and rank \( (W_c) = 1 \). Then, we have:
\[
|h_{c,k}w_{c,k}^2| = \text{Tr}(W_c H_{c,k}).
\]

Finally, problem (15) can be further expressed as:
\[
\max_{w_{c,k},A_{c,k},B_{c,k},R_{c,k}^k} \sum_{c=1}^{C} \sum_{k=1}^{K_c} R_{c,k}^k,
\]

\[
\text{s.t.} \quad \frac{1}{A_{c,k}} \leq \text{Tr}(W_c H_{c,k}) \rho_{c,k}, \\
B_{c,k} \geq \text{Tr}(W_c H_{c,k}) \bar{p}_{c,k} + \sum_{c=1}^{C} \text{Tr}(W_c H_{c,k}) + \sigma^2,
\]

\[
\sum_{c \in \mathcal{C}} \text{rank}(W_c) \leq P_{\text{max}},
\]

\[
W_c \succeq 0, \\
(10b), (15b).
\]

For the non-convex constraint (15b), according to the first-order Taylor expansion, we have the lower bound of
\[
\log_2 \left( 1 + \frac{1}{A_{c,k}B_{c,k}} \right),
\]

which is defined as:
\[
\bar{R}_{c,k}^0 = \log_2 \left( 1 + \frac{1}{A_{c,k}^{(\tau_1)}B_{c,k}^{(\tau_1)}} \right) - \log_2 \left( \frac{A_{c,k}^{(\tau_1)} - A_{c,k}^{(\tau_1)}}{A_{c,k}^{(\tau_1)}} \left( 1 + B_{c,k}^{(\tau_1)}B_{c,k}^{(\tau_1)} \right) \right) - \log_2 \left( \frac{B_{c,k}^{(\tau_1)} - B_{c,k}^{(\tau_1)}}{B_{c,k}^{(\tau_1)}} \right). 
\]

(17)
where the points \( A_{c,k}^{(\tau_1)} \) and \( B_{c,k}^{(\tau_1)} \) are the values of \( A_{c,k} \) and \( B_{c,k} \) in the \( \tau_1 \)-th iteration, respectively.

Now, the non-convex rank-one constraint (16e) is the remaining obstacle to solve problem (16). By relaxing constraint (16e), the considered problem becomes a convex semi-definite programming (SDP) problem, which is given as follows:

\[
\begin{align*}
\max_{\mathbf{W}_c, A_{c,k}, B_{c,k}, R_{c,k}} & \quad \sum_{c=1}^{C} \sum_{k=1}^{K_c} R_{c,k}, \\
\text{s.t.} & \quad \tilde{R}_{c,k} \geq R_{c,k}, \\
& \quad (10b), (16b), (16c), (16d), (16f).
\end{align*}
\]  

**Theorem 1:** The optimal \( \{ \mathbf{W}_c^* \} \) to problem (18) without the rank-one constraint (16e) always satisfy \( \text{rank} (\mathbf{W}_c) = 1 \).

**Proof:** Similar proof can be found in [14].

Theorem 1 represents the fact that we can obtain the optimal \( \{ \mathbf{W}_c^* \} \) of problem (16) by solving problem (18) without the rank-one constraint. Problem (18) is a standard convex SDP, which can be solved efficiently by CVX tool [15]. Algorithm 1 summarizes the proposed successive convex approximation (SCA) based algorithm to solve problem (15).

**Algorithm 1** Successive Convex Approximation (SCA) Based Algorithm for obtaining \( \{ \mathbf{W}_c^* \} \)

1: Initialize feasible points \( \{ A_{c,k}^{(0)} \}, \{ B_{c,k}^{(0)} \} \) and set the iteration index \( \tau_1 = 0 \).
2: repeat
3: \quad update \( \{ A_{c,k}^{(\tau_1 + 1)} \}, \{ B_{c,k}^{(\tau_1 + 1)} \} \) and \( \{ \mathbf{W}_c^{(\tau_1 + 1)} \} \) by solving problem (18);
4: \quad \tau_1 = \tau_1 + 1;
5: until the objective value of problem (18) converge.
6: Output: \( \mathbf{W}_c^* \)

**D. Transmission and Reflection Beamformings Optimization**

We denote \( \mathbf{H}_{c,k,n} = \text{diag} (\mathbf{H}_{c,k}^H) \mathbf{F}_{n} \) and \( \mathbf{U}_p = u_p^\mathsf{H} u_p \), where \( \mathbf{U}_p \succeq 0 \), \( \text{rank} (\mathbf{U}_p) = 1 \) and \( |\mathbf{U}_p|_{m,m} = \beta_p \), \( p \in \{ t, r \} \). Hence, we have:

\[
|\mathbf{g}_{c,k}^H \Theta_{c,k} \mathbf{F}_{w,c,k}|^2 = |\mathbf{v}_{c,k}^H \mathbf{H}_{c,k}|^2 = \text{Tr} (\mathbf{V}_{c,k}^H \mathbf{H}_{c,k}^H),
\]  

where \( \mathbf{H}_{c,k}^H = \mathbf{H}_{c,k} \) and the matrix(vector) \( \mathbf{V}_{c,k} (\mathbf{v}_{c,k}) \) is defined as:

\[
\mathbf{V}_{c,k} (\mathbf{v}_{c,k}) = \left\{ \begin{array}{ll}
\mathbf{U}_t (u_t), & \text{if user } k \text{ is in TS}, \\
\mathbf{U}_r (u_r), & \text{if user } k \text{ is in RS}.
\end{array} \right.
\]

Thus, the transmission and reflection beamforming optimization problem in (10) with fixed active beamforming vectors and power allocation coefficients can be formulated as

\[
\begin{align*}
\max_{\mathbf{U}_p, A_{c,k}, B_{c,k}, R_{c,k}} & \quad \sum_{c=1}^{C} \sum_{k=1}^{K_c} R_{c,k}, \\
\text{s.t.} & \quad \frac{1}{A_{c,k}} \leq \text{Tr} (\mathbf{V}_{c,k} \mathbf{H}_{c,k,c}) \beta_p, \\
& \quad B_{c,k} \geq \text{Tr} (\mathbf{V}_{c,k} \mathbf{H}_{c,k,c}) \beta_p + \sum_{c=1, c \neq c}^{C} \text{Tr} (\mathbf{V}_{c,k} \mathbf{H}_{c,k,c}) + \sigma^2, \\
& \quad \beta_p^t + \beta_p^r = 1, \\
& \quad |\mathbf{U}_p|_{m,m} = \beta_p, \\
& \quad \mathbf{U}_p \succeq 0, \quad \text{rank} (\mathbf{U}_p) = 1, \\
& \quad (10b), (15b).
\end{align*}
\]  

**Algorithm 2** is a standard convex SDP, which can be solved efficiently [15]. The details of the proposed sequential constant relaxation algorithm is presented in Algorithm 2.

**E. Proposed Two-layer Iterative Algorithm**

Based on the above discussions, we provide the details of the proposed two-layer iterative algorithm to solve the original problem (5) in Algorithm 3. The inner-layer iteration is guaranteed to converge, because the objective function’s value of problem (10) is monotonically non-decreasing after each iteration. For the outer-layer iteration, it is easy observed from step 13 to 17 that the achievable sum rate is monotonically non-decreasing after each iteration. Since both the inner- and outer-layer iterations converge, the proposed Algorithm 3 converges.

The complexity of Algorithm 1 to solve the active beamforming optimization problem (15) is \( O_1 \triangleq \mathcal{O} (\tau_1^{\max} \max (N_T, (2K + 1))^4 \sqrt{N_T} \log_2 \frac{1}{\epsilon_2}) \), where \( \tau_1^{\max} \) is the number of iterations for Algorithm 1 and \( \epsilon_1 \) is the solution accuracy. The complexity of Algorithm 2 to solve the transmission and reflection beamforming optimization problem (21) is \( O_2 \triangleq \mathcal{O} (\tau_2^{\max} \max (M, (2K))^4 \sqrt{M} \log_2 \frac{1}{\epsilon_2}) \), where \( \tau_2^{\max} \) is...
Algorithm 2 Sequential constraint relaxation Algorithm for obtaining \( \{U_p^*\} \)

1: Initialize feasible points \( U_p^{(0)} \), step size \( \Delta^{(0)} \), error tolerance \( \varepsilon \), set relaxation parameter \( \epsilon^{(\tau_2)} = 0 \) and the iteration index \( \tau_2 = 0 \).
2: repeat
3: Solve problem (24) to obtain \( U_p \);
4: if problem (24) is solvable
5: Update \( U_p^{(\tau_2+1)} = U_p^{(\tau_2)} \);
6: Update \( \Delta^{(\tau_2+1)} = \Delta^{(\tau_2)} \);
7: else
8: Update \( U_p^{(\tau_2+1)} = \sum_{k} U_p^{(\tau_2)} \);
9: update \( \Delta^{(\tau_2+1)} = \Delta^{(\tau_2)} \);
10: end
11: Update \( \tau_2 = \tau_2 + 1 \);
12: Update \( \epsilon^{(\tau_2+1)} \) via (23);
13: until \( |1 - \epsilon^{(\tau_2)}| \leq \varepsilon \) and the objective value of problem (24) converge.
14: Output: \( U_p^* \)

Algorithm 3 Two-Layer Iterative Algorithm

1: Initialize \( \{\rho_{c,k}^{(0)}\}, \{w_c^{(0)}\}, \{u_p^{(0)}\} \) and error tolerance \( \Delta \);
2: Set the iteration index \( \tau_0 = 0 \).
3: repeat
4: Calculate \( \{\Gamma_{c,D_0(k)}\} \) via (7);
5: Update \( \{D_{c}\} \) according to (6);
6: Calculate \( R_{\text{sum}} \left( \rho_{c,k}^{(\tau_0)}, w_c^{(\tau_0)}, u_p^{(\tau_0)} \right) \);
7: repeat
8: Update \( \{\rho_{c,k}\} \) according to (12);
9: Update \( \{w_c\} \) via Algorithm 1;
10: Update \( \{u_p\} \) via Algorithm 2;
11: until the objective value of problem (10) converges.
12: Record the obtained solutions \( \{\rho_{c,k}, \{w_c\}, \{u_p\}\} \);
13: calculate \( R_{\text{sum}}(\rho_{c,k}, w_c, u_p) \) if \( R_{\text{sum}}(\rho_{c,k}, w_c, u_p) \geq R_{\text{sum}}(\rho_{c,k}^{(\tau_0)}, w_c^{(\tau_0)}, u_p^{(\tau_0)}) \)
14: \( \rho_{c,k}^{(\tau_0+1)} = \rho_{c,k}^{(\tau_0)}, w_c^{(\tau_0+1)} = w_c^{(\tau_0)}, u_p^{(\tau_0+1)} = u_p^{(\tau_0)} \);
15: else
16: \( \rho_{c,k}^{(\tau_0+1)} = \rho_{c,k}^{(\tau_0)}, w_c^{(\tau_0+1)} = w_c^{(\tau_0)}, u_p^{(\tau_0+1)} = u_p^{(\tau_0)} \);
17: end if
18: Update \( \tau_0 = \tau_0 + 1 \);
19: until \( \frac{R_{\text{sum}}^{(\tau_0)} - R_{\text{sum}}^{(\tau_0-1)}}{R_{\text{sum}}^{(\tau_0-1)}} < \Delta \).
20: Output: the optimal \( \{\rho_{c,k}^{*}\}, \{w_c^{*}\}, \{u_p^{*}\} \);

THE IV. NUMERICAL RESULTS

In this section, numerical simulations are conducted to evaluate the performance of the proposed algorithm. Without loss of generality, we assume that there are three clusters in the STAR-RIS-NOMA system and each cluster contains three users. Specifically, cluster 1 is located in reflection space, and clusters 2 and 3 are located in the transmission space. Furthermore, we assume that the users in each cluster are randomly placed in their own circle. To model small-scale fading, we adopt Rician fading for all channels involved. The specific parameter value settings are summarized in Table I unless otherwise specified.

A. Impact of The Number of RIS Elements

To evaluate the performance of the proposed STAR-RIS-NOMA system, two benchmark schemes are considered, namely, traditional RIS-NOMA and traditional RIS-OMA systems. For the two traditional RIS assisted systems, to achieve the full-space coverage, one transmitting-only RIS and one reflecting-only RIS are employed and deployed adjacent to each other at the same location as the STAR-RIS. For a fair comparison, each traditional reflecting/transmitting-only RIS is assumed to have \( M/2 \) elements \([3, 7]\). For the RIS-OMA system, the BS serves all users through time division multiple access with the aid of the traditional reflecting/transmitting-only RISs. In addition, we employ the Gaussian randomization based SDP algorithm proposed in \([18]\) to solve the passive beamforming optimizations for the RIS-NOMA and RIS-OMA systems. Fig. 2 shows the achievable sum rate versus the number of RIS’s elements \( M \). It is first observed that the achievable sum rate achieved by the proposed STAR-RIS and traditional RIS based systems increases as \( M \) increases. Second, the proposed STAR-RIS-NOMA system always outperforms traditional RIS based systems, because the system can make full use of all the introduced DoFs to enhance the desired signal and mitigate the inter- and intra-cluster interference.

B. Impact of Decoding Order

In Fig. 3, we evaluate the impact of the decoding order on the achievable sum rate performance. Three decoding order determination methods are compared with our proposed method. The first benchmark scheme named Exhaust-Search method, finds the optimal decoding order via exhaustive search. The second benchmark scheme named Random method, selects the decoding order randomly. The third benchmark scheme named

![Fig. 2: The achievable sum rate versus the number of RIS elements, \( N_T = 4 \)](image-url)
Combined-Channel-Gain method, corresponds to employing combined channel gains, i.e., $\left\{g_{c,k}^H \Theta_{c,k} F_{w,c}\right\}$, to determine the decoding order.

From Fig. 3, it can be found that the proposed method can achieve performance close to that achieved by the Exhaust-Search method. Though some performance loss is incurred by the proposed method, the complexity of the proposed method is much lower than that of the Exhaust-Search method. In addition, our proposed method outperforms the Combined-Channel-Gain and Random methods. The reasons behind this can be explained as follows. For the multiple-cluster RIS-NOMA systems, the decoding order is determined not only by the users’ combined channel gain, but also by the interference channel gains from other clusters. Therefore, our proposed decoding order determination method is more reasonable.

![Fig. 3: The achievable sum rate versus the number of RIS elements, $N_T = 4, P_{\text{max}} = 35\text{dBm}$]

V. CONCLUSIONS

In this paper, we investigated a new joint optimization problem for STAR-RIS-NOMA system, where the decoding order, power allocation coefficients, active beamforming, transmission and reflection beamformings are jointly optimized to maximize the achievable sum rate. We propose a novel two-layer iterative algorithm to solve the formulated non-convex optimization problem. Simulation results validate the effectiveness of the proposed STAR-RSI-NOMA system. Besides, it is found that the proposed decoding order determination scheme can achieve near-optimal performance.

REFERENCES

[1] M. Renzo, A. Zappone, M. Debbah, M.-S. Alouini, C. Yuen, J. D. Rosny, and S. Tret’yakov, “Smart radio environments empowered by

reconfigurable intelligent surfaces: how it works, state of research, and the road ahead,” IEEE J. Sel. Areas Commun., vol. 38, pp. 2450–2525, Nov. 2020.

[2] Q. Wu and R. Zhang, “Towards smart and reconfigurable environment: intelligent reflecting surface aided wireless network,” IEEE Commun. Mag., vol. 58, pp. 106–112, Jan. 2020.

[3] Y. Liu, X. Mu, J. Xu, R. Schober, Y. Hao, H. V. Poor, and L. Hanzo, “STAR: Simultaneous transmission and reflection for 360° coverage by intelligent surfaces,” [Online]. Available:https://arxiv.org/abs/2103.09104, 2021.

[4] H. Zhang, S. Zeng, B. Di, Y. Tan, M. Renzo, M. Debbah, L. Song, Z. Han, and H. Poor, “Intelligent reflective-transmissive metasurfaces for full-dimensional communications: principles, technologies, and implementation,” [Online]. Available:https://arxiv.org/abs/2104.12313, 2021.

[5] J. Xu, Y. Liu, X. Mu, and O. Dobre, “SRAR-RISs: Simultaneous reflecting and refracting reconfigurable intelligent surfaces,” [Online]. Available:https://arxiv.org/abs/2101.09663, 2021.

[6] C. Wu, Y. Liu, X. Mu, X. Gu, and O. Dobre, “Coverage characterization of STAR-RIS networks: NOMA and OMA,” [Online]. Available:https://arxiv.org/abs/2104.10006, 2021.

[7] X. Mu, Y. Liu, L. Guo, J. Lin, and R. Schober, “Simultaneously transmitting and reflecting (STAR) RIS aided wireless communications,” [Online]. Available:https://arxiv.org/abs/2104.01421, 2021.

[8] S. Zhang, H. Zhang, B. Di, Y. Tan, M. Renzo, Z. Han, H. Poor, and L. Song, “Intelligent omni-surface: ubiquitous wireless transmission by reflective-transmissive metasurface,” [Online] Available:https://arxiv.org/abs/2011.00765, 2020.

[9] S. Zhang, H. Zhang, B. Di, Y. Tan, Z. Han, and L. Song, “Beyond intelligent reflecting surfaces: reflective-transmissive metasurface aided communications for full-dimensional coverage extension,” IEEE Trans. Veh. Technol., vol. 69, pp. 13 905–13 909, Nov. 2020.

[10] Z. Ding, Y. Liu, J. Choi, Q. Sun, M. Elkashlan, I. Chih-Lin, and H. V. Poor, “Application of non-orthogonal multiple access in LTE and 5G networks,” IEEE Commun. Mag., vol. 55, no. 2, pp. 185–191, Feb. 2017.

[11] Y. Liu, Z. Qin, M. Elkashlan, Z. Ding, A. Nallanathan, and L. Hanzo, “Non-orthogonal multiple access for 5G and beyond,” Proc. IEEE, vol. 105, no. 12, pp. 2347–2381, Dec. 2017.

[12] J. Cui, Z. Ding, P. Fan, and N. Al-Dhahir, “Unsupervised machine learning-based user clustering in millimeter-wave-NOMA systems,” IEEE Trans. Wireless Commun., vol. 17, no. 11, pp. 7425–7440, Nov. 2018.

[13] L. Zhu, J. Zhang, Z. Xiao, X. Cao, D. O. Wu, and X.-G. Xia, “Millimeter-wave NOMA with user grouping, power allocation and hybrid beamforming,” IEEE Trans. Wireless Commun., vol. 18, no. 11, pp. 5065–5079, Nov. 2019.

[14] X. Mu, Y. Liu, L. Guo, J. Lin, and N. Al-Dhahir, “Exploiting intelligent reflecting surfaces in NOMA networks: Joint beamforming optimization,” IEEE Trans. Wireless Commun., vol. 19, no. 10, pp. 6884–6898, Oct. 2020.

[15] M. Grant and S. Boyd, “CVX: Matlab software for disciplined convex programming, version 2.1,” http://cvxr.com/cvx, Mar. 2014.

[16] J. Zuo, Y. Liu, Z. Qin, and N. Al-Dhahir, “Resource allocation in intelligent reflecting surface assisted NOMA systems,” IEEE Trans. Commun., vol. 68, no. 11, pp. 7170–7183, Nov. 2020.

[17] X. Mu, Y. Liu, L. Guo, J. Lin, and R. Schober, “Joint deployment and multiple access design for intelligent reflecting surface assisted networks,” [Online]. Available:https://arxiv.org/abs/2101.09663, 2021.

[18] G. Yang, X. Xu, and Y.-C. Liang, “Intelligent reflecting surface assisted non-orthogonal multiple access,” 2020 IEEE Wireless Communications and Networking Conference (WCNC), pp. 1–6, 2020.

| Parameter | Value |
|-----------|-------|
| The locations of the BS and STAR-RIS | (0, 0, 20), (0, 30, 20) |
| The central coordinates of the three clusters’ circles | (0, 25, 0), (0, 35, 0), (5, 30, 0) |
| The radius of the three groups’ circles | 5 m, 5 m, 5 m |
| The path loss exponents of the BS-RIS and RIS-user links | 2.2, 2.2 |
| The path loss at 1 meter | 30 dB |
| The Rician factors of the BS-RIS and RIS-user links | 3 dB, 3 dB |
| The minimum QoS requirement for each user | 0.1 bits/s/Hz |
| Noise power | 90 dBm |

TABLE I: Simulation Parameters