Renormalon Variety in Deep Inelastic Scattering.

R. Akhoury and V.I. Zakharov

The Randall Laboratory of Physics
University of Michigan
Ann Arbor, MI 48109-1120

Abstract

We discuss the renormalon-based approach to power corrections in non-singlet deep inelastic scattering structure functions and compare it with the general operator product expansion. The renormalon technique and its variations relate the power corrections directly to infrared-sensitive parameters such as the position of the Landau pole $\Lambda_{QCD}$ or the infinitesimal gluon mass $\lambda$. In terms of the standard OPE these techniques unify evaluations of the coefficient functions and of matrix elements. We argue that in case of deep inelastic scattering there is a proliferation of competing infrared sensitive parameters. In particular we consider the gluon and quark masses, virtuality of quarks and $\Lambda_{QCD}$ as possible infrared cut offs and compare the emerging results. In the standard renormalon technique where $\Lambda_{QCD}$ is the infrared parameter, the argument of the running coupling is crucial to obtain the correct $x$ dependance of the structure functions. Finally we discuss the limitations of the use of the renormalon based methods for determining of the $x$ dependance of the power corrections.
1 Introduction

Generically, renormalons allow for a parametrization of infrared sensitive contributions to hard processes in QCD within the framework of perturbation theory (for the basic ideas see [1, 2, 3] and references therein). If one considers only infrared safe observables then the infrared sensitive contributions are power suppressed. Moreover, in many cases it does not matter which particular infrared parameter is used in calculations. It can be, for example, the position of the Landau pole in the running coupling, $\Lambda_{QCD}$, or (at the one-loop level) a fictitious gluon mass $\lambda$. Since renormalons are a pure perturbative construct the answer is obtained directly in terms of the infrared parameters chosen. If, on the other hand, the treatment of the same process is possible within an operator product expansion the evaluation of the power corrections is a two-stage procedure. First, one calculates perturbatively a coefficient function in front of operators. The matrix elements of these operators provide then a measure of the infrared contribution, both perturbative and non-perturbative. Thus, renormalons fix the matrix elements perturbatively. Finally, one adjusts the overall scale of the matrix element to allow for its non-perturbative enhancement. This tacit assumption is behind almost all the applications of renormalons [4].

The best known example of this kind is the gluon condensate $\langle 0 | \alpha_s(G^a_{\mu\nu})^2 | 0 \rangle$. It can be treated non-perturbatively [4]. On the other hand if one introduces a finite gluon mass $\lambda$ then [3]:

$$\langle 0 | \alpha_s(G^a_{\mu\nu})^2 | 0 \rangle_{\lambda \neq 0} = - \frac{3\alpha_s}{\pi^2} \lambda^4 \ln \lambda^2$$  \hspace{1cm} (1)

where we keep only the term nonanalytic in $\lambda^2$ since only such terms can be consistently attributed to the infrared region. Instead of introducing $\lambda \neq 0$ one can evaluate the gluon condensate associated with a renormalon chain [1, 2]:

$$\langle 0 | \alpha_s(G^a_{\mu\nu})^2 | 0 \rangle_{\text{ren}} = p.v. \int \frac{3d^4k}{\pi^2} \alpha_s(k^2) = \text{const} \cdot \Lambda_{QCD}^4$$  \hspace{1cm} (2)

where $\alpha_s(k^2)$ is the running coupling corresponding to a single-term $\beta$-function and the
integration over the pole in $\alpha_s(k^2)$ has been defined as the principal value (p.v.). In this example the use of renormalons, i.e. of Eqs. (1), (2), does not enhance the predictive power of the theory. Indeed, Eqs. (1), (2) cannot be taken literally and one reserves for an unknown rescaling of the matrix element to fit the data. What renormalons do achieve is a short cut of the procedure since, say, terms of order $\lambda^4 \ln \lambda^2$ can be evaluated directly for physical quantities without invoking the operator product expansion as an intermediate step. This may be a decisive advantage of renormalons in cases where there is no OPE available.

Thus at first sight the alternative is that either there is an OPE and then the renormalons are a particular model for the matrix elements involved or there is no OPE and then the renormalons could be a substitution for it. This logic appears to be defied by the example of deep inelastic scattering. Indeed, the power corrections can be treated either via the OPE \[7, 8, 9\] and or via renormalons \[10, 11, 12, 13\]. While the OPE reserves for at least one unknown matrix element for each moment, renormalons are claimed to fix the whole $x$ dependence of the $1/Q^2$ correction \[11, 12, 13\]. Moreover the same approach can be used to study the power corrections to fragmentation functions \[14\].

Motivated by these very interesting observations we look, in this paper \[15\], into the anatomy of the infrared sensitive contributions in the renormalon calculus as applied to DIS. We find that already at the one-loop level there are a variety of possible infrared procedures which do not obviously give the same answer. For example, as discussed in more detail later in the paper, one possible strategy underlying the applications of renormalons is to reduce the matrix elements governing higher twist contributions to those of the leading twist. Typically, the reduction produces a factor $f$ proportional to:

$$f(\epsilon^2, m^2, \lambda^2) \sim \int_0^1 dy X(y) \ln(X(y))$$

where

$$X(y) = \frac{\epsilon^2}{Q^2} y (y - 1) + \frac{m^2}{Q^2} y^2 + \frac{\lambda^2}{Q^2} (1 - y)$$

$\lambda, m$ are the gluon and quark masses respectively and $\epsilon$ is the virtuality of the quark, $p^2 - m^2 \equiv$
\( \epsilon^2 \). This proliferation of the infrared sensitive parameters, that is \( \lambda, m, \epsilon \), is due to the fact that there are both soft gluon and soft quark lines. On the other hand the classical applications of renormalons assume that there is only a single gluon line made soft through an insertion of a renormalon chain, while the other lines are hard. Thus in case of DIS, the renormalon-related parameter \( \lambda \) determines the result only upon forcing the other parameters \( (m, \epsilon) \) to vanish - \( m^2, \epsilon^2 \ll \lambda^2 \), which is not necessarily natural. One could of course assume, for example, that to the contrary, \( m^2 \gg \lambda^2 \). In addition to \( \lambda \) and \( m \), one could also explore the infrared sensitivity by the standard renormalon chain technique in which the power corrections are proportional to \( \Lambda_{QCD}^2/Q^2 \), etc. In sections 2, 3 and 4 we consider in more detail the \( x \) dependence of the non-singlet structure functions with \( \lambda, \Lambda_{QCD} \) and \( m \), respectively, as dominating infrared parameters. Section 2 essentially reproduces the results of [11, 12, 13]. In section 3 we find that not only is the predicted \( x \) dependence a function of what the argument of the running is, but in addition, two arbitrary scales typified by two different integrals multiplying \( \Lambda_{QCD}^2/Q^2 \) are introduced, in general. This difference in fact arises because some contributions come from Feynman integrals which are collinear divergent whereas others from ones that are not. This is to be contrasted with the case discussed in section 2 where there is just one unknown parameter \( \lambda \) and is an indication of an inherent infrared instability. As for the dependence on the virtuality \( \epsilon \) it also indicates that in fact we are dealing with an infrared unsafe quantity. This would, in general, become manifest in the two-loop approximation for the power corrections. To see this one does not need to evaluate the second loop explicitly but it is enough to note that the anomalous dimensions of the operators governing the leading and higher twist contributions are different. This is discussed in section 4. In section 5 we present our conclusions.
2 Gluon Mass as an Infrared Cut-Off.

As is mentioned in the introduction there exist a variety of choices of infrared sensitive parameters. Thus far the dispersive approach to the running coupling [11], and the renormalon chain in the large $N_f$ limit [12] have been tried, with identical conclusions for the $x$ dependence of the DIS structure functions wherever there is overlap. To these we will add the cases of $\lambda, m \neq 0$ and of the Landau pole contribution in the running coupling. These cases are considered in this and subsequent sections.

To introduce notations, we consider the partonic structure tensor defined as

$$W_{\mu\nu}(p, q) = \frac{1}{8\pi} \sum M_\mu M_\nu^*$$

where $M_\mu$ is the amplitude for $\gamma^* + q \rightarrow q' + g + ...$ where $q$ is the parton (quark for our purposes) of momentum $p$. An average over the initial quark spins is understood and the standard decomposition of the $W_{\mu\nu}$ is used:

$$W_{\mu\nu} = \frac{1}{2} \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_L(z, Q^2) +$$

$$(p_\mu p_\nu - \frac{p \cdot q}{q^2} (p_\mu q_\nu + p_\nu q_\mu) + g_{\mu\nu} \frac{(p \cdot q)^2}{q^2}) \frac{F_2(z, Q^2)}{p \cdot q}$$

where $z = Q^2/2p \cdot q$, $q^2 = -Q^2$ and $p^2 = 0$. Note that we put the mass of the quark exactly zero while keeping a finite gluon mass so as to ensure the role of $\lambda \neq 0$ as a unique infrared cut-off. For the structure function $F_L$ only the diagram with emission of a real gluon contributes to lowest order in $\alpha_s$. For single gluon emission we have:

$$W_{\mu\nu} = \frac{C_F \alpha_s}{2} \int \frac{d^4k}{(2\pi)^4} 2\delta_+((k + q)^2 - \lambda^2) \frac{d_{\rho\sigma}(p - k)}{k^4} Tr \left[ p_{\rho} k^\mu \gamma_{\mu} (k + q)_{\nu} k_{\nu} \gamma_{\nu} \right]$$

where in the Feynman gauge $d_{\rho\sigma} = -g_{\rho\sigma}$.

To perform the integral we use a Sudakov parametrization:

$$k^\mu = \kappa p^\mu + \frac{k^2 + k_\perp^2}{2\kappa} n^\mu + k_\perp^\mu$$

5
where $n^2 = 0, p^2 = 0, n \cdot k_\perp = 0, n \cdot p = 1, p \cdot k_\perp = 0$ and we may choose

$$n^\mu = \frac{q^\mu + z p^\mu}{p \cdot q}.$$  \hspace{1cm} (7)

Moreover, for these variables:

$$\int d^4 k = \frac{\pi}{2} \int \frac{d\kappa}{\kappa} d^2 k d^2 k_{\perp}.$$  \hspace{1cm} (8)

For detecting the non-analytical terms the limits of the $k^2$ integration are also important:

$$\frac{\lambda^2 z}{1 - z} \leq k^2 \leq \frac{Q^2}{z} - \lambda^2.$$  \hspace{1cm} (9)

It is straightforward to find for $F_L$:

$$F_L = 2 C_F \frac{\alpha_s}{4\pi} \frac{4 z^2}{Q^2} \int \frac{dk^2}{k^4} (k^2 - \lambda^2)^2 \left(1 + \frac{k^2 - \lambda^2}{Q^2 z}\right),$$  \hspace{1cm} (10)

with the limits of integration as indicated in (9). From (10) we immediately get for the term independent of $\lambda$ and representing therefore the leading twist:

$$F_L = C_F \frac{\alpha_s}{4\pi} 4 z,$$  \hspace{1cm} (11)

which is a well known result. Furthermore for the terms $\lambda^2 \ln \lambda^2$ and $\lambda^4 \ln \lambda^2$ we get:

$$(F_L)_{\lambda^2} = C_F \frac{\alpha_s}{2\pi} \frac{4 z^2}{Q^2} \cdot 2 \lambda^2 \ln \lambda^2$$  \hspace{1cm} (12)

and

$$(F_L)_{\lambda^4} = C_F \frac{\alpha_s}{2\pi} \frac{4 z^3}{Q^4} (-3) \lambda^4 \ln \lambda^2,$$  \hspace{1cm} (13)

respectively. It is worth emphasizing again that equations (12,13) are understood in the sense that only non-analytic terms in $\lambda^2$ are kept.

The results (12) and (13) do reproduce the predictions based on the dispersive approach to the running coupling [11]. This coincidence of the results comes as no surprise since the dispersive formulation is, in fact, aimed at improving the high-energy behaviour of the theory with massive gluons. However, the change in this high-energy behaviour obviously does not affect the infrared sensitive pieces which are non-analytic in $\lambda^2$. 

6
3 Landau-Pole Parametrization.

As the next way to parametrize the infrared sensitive contributions we examine in this section the contribution of the Landau pole in the running coupling, in which case the power corrections are proportional to \((\Lambda_{QCD}^2/Q^2)^n\). For definiteness we again consider first the longitudinal structure function. Since now the infrared sensitive parameter will be \(\Lambda_{QCD}\) we can put \(\lambda = 0\). Moreover, we now account for the running of the effective coupling while still using the kinematics of one-gluon emission. The crucial issue then is what is the argument of the running coupling. We perform first the integration assuming that it is \(k_\perp^2\) that determines the running, i.e, \(\alpha_s(k_\perp^2)\). We shall see however that to reproduce the results of the previous sections one should assume that the true argument of the effective coupling is \(k_\perp^2/(1-z)\).

To depict the contribution to \(F_L\) of the Landau pole we first integrate over \(\kappa\) and \(k^2\) (see Eqs. (6), (8)). Thus we start with

\[
F_L = 2C_F \frac{4z^2}{Q^2} \int \frac{\alpha_s(k_\perp^2)}{4\pi} dk_\perp^2 d\kappa \frac{dk^2}{k^4} \delta \left( \frac{k_\perp^2}{k} + \frac{k^2(1-z)}{k} \right) \cdot (14)
\]

Then the integration over \(k^2\) is trivial while the integration over \(\kappa\) can be expressed in terms of \(\kappa_\pm\):

\[
2\kappa_\pm = 1 + z \pm \sqrt{(1-z)^2 - 4(k_\perp^2/Q^2)z(1-z)}.
\]

In this way we come to the following integral over \(k_\perp^2\):

\[
F_L = 2C_F \frac{4z^2}{Q^2} \int_0^{Q^2(1-z)/4z} \frac{\alpha_s(k_\perp^2)}{4\pi} \frac{dk_\perp^2}{\sqrt{(1-z)^2 - 4(k_\perp^2/Q^2)z(1-z)}}.
\]

In order to pick up the renormalon contributions we substitute the following representation in Eq. (16):

\[
\alpha_s(k_\perp^2) = \int_0^\infty d\sigma \left( \frac{k_\perp^2}{\Lambda_{QCD}^2} \right)^{-\sigma \beta_1}.
\]

\[
(17)
\]
where $\beta_1$ is the first coefficient in the $\beta$–function. Furthermore, to perform the integration over $k_{\perp}$ it is convenient to introduce a new variable, $y = 4z(1 - z)^{-1}k_{\perp}^2Q^{-2}$ which allows us to disentangle the $z$ and $k_{\perp}$ dependences:

$$F_L = \frac{2C_F}{4\pi} \cdot \frac{4z^2}{1 - z} \int_0^\infty d\sigma \left( \frac{\Lambda_{QCD}^2}{Q^2} \right)^{\sigma\beta_1} \left( \frac{1 - z}{4z} \right)^{1 - \sigma\beta_1} \int dy \ y^{-\sigma\beta_1}(1 - y)^{-1/2}. \quad (18)$$

Finally we get for $F_L$ the following expression

$$F_L = \frac{2C_F}{4\pi} \cdot \frac{4z^2}{1 - z} \int_0^\infty d\sigma \left( \frac{\Lambda_{QCD}^2}{Q^2} \right)^{\sigma\beta_1} \left( \frac{1 - z}{4z} \right)^{1 - \sigma\beta_1} \frac{\Gamma(1 - \sigma\beta_1)\Gamma(1/2)}{\Gamma(3/2 - \sigma\beta_1)} \quad (19)$$

which exhibits the infrared renormalons corresponding to the poles of $\Gamma(1 - \sigma\beta_1)$. Since $(\Lambda_{QCD}^2/Q^2)^{\sigma\beta_1} \sim exp(-\sigma/\alpha_s(Q^2/\Lambda_{QCD}^2))$ the corresponding power ambiguities are proportional to $(\Lambda_{QCD}^2/Q^2), (\Lambda_{QCD}^2/Q^2)^2...$

In order to extract the power behaviour one must define the integral over $\sigma$ using some prescription. We next outline how this integral may be defined via, say, a principal value prescription for the poles at $\sigma = n/\beta_1$, $n = 1, 2, ...$. First we note that small values of $\sigma$ correspond to large values of $k_{\perp}$ and hence for this we get the usual renormalization group improved perturbative answer. Keeping this in mind we may divide the integration region thus:

$$\sigma \in [0, \infty] = \left[0, \frac{s}{\beta_1}\right] + \sum_{n \geq 1} \left[\frac{n - s}{\beta_1}, \frac{n + s}{\beta_1}\right], \quad (20)$$

where, $0 < s < 1$, and a typical choice could be $s = 1/2$. The first region gives the perturbative answer, as we have checked, and the rest give the power corrections in the principal value prescription. To make this explicit, consider the contribution from the pole at $\sigma = n/\beta_1$ to the integral (see Eq. (19)):

$$\int_0^\infty d\sigma \left( \frac{4z\Lambda_{QCD}^2}{(1 - z)Q^2} \right)^{\sigma\beta_1} \frac{\Gamma(1 - \sigma\beta_1)\Gamma(1/2)}{\Gamma(3/2 - \sigma\beta_1)} \quad (21)$$

which is defined by:

$$\int_{\frac{n}{\beta_1}}^{\frac{n+1}{\beta_1}} d\sigma \left( \frac{4z\Lambda_{QCD}^2}{(1 - z)Q^2} \right)^{\sigma\beta_1} \frac{\Gamma(1 - \sigma\beta_1)\Gamma(1/2)}{\Gamma(3/2 - \sigma\beta_1)} \quad (22)$$
This can be written after some algebra as:

\[
\left( \frac{4z \Lambda_{QCD}^2}{(1-z)Q^2} \right)^n \bar{I}_n^{pv}
\]

where, the integral \( \bar{I}_n^{pv} \) is defined by:

\[
\bar{I}_n^{pv} = (-1)^n \frac{\pi^{3/2}}{\beta_1} \int_0^s \frac{d\sigma}{\sin \pi \sigma} \left[ \frac{(4z \Lambda_{QCD}^2)^\sigma (n+\sigma)}{\Gamma(1+n+\sigma)\Gamma(3/2-n-\sigma)} - \frac{(4z \Lambda_{QCD}^2)^{-\sigma} (n-\sigma)}{\Gamma(1+n-\sigma)\Gamma(3/2-n+\sigma)} \right]
\]

From this, for the first two leading power corrections we finally get:

\[
(F_L)_{1/Q^2} = \frac{C_F}{2\pi} \frac{4z^2 \Lambda_{QCD}^2}{1-z} \bar{I}_1^{pv},
\]

\[
(F_L)_{1/Q^4} = \frac{C_F}{2\pi} \frac{16z^3 \Lambda_{QCD}^4}{(1-z)^2} \bar{I}_2^{pv}.
\]

We would like to emphasize that we could have chosen some other prescription for defining the integral over \( \sigma \) and hence the overall scale of the power corrections is arbitrary. An important aspect of Eq. (25) is that the \( z \)-dependence of the power corrections differs from the corresponding \( z \)-dependence of the power corrections due to \( \lambda \neq 0 \) at the one-loop level (see Eqs. (12), (13)). Thus, the predictions for the \( z \)-dependence of the power corrections are sensitive to the exact argument of the running coupling.

Let us focus first on the question concerning the most sensitive point of the derivation. Imagine that the true argument of the running coupling is in fact \( k_\perp^2/(1-z) \), i.e.:

\[
\alpha_s(k_\perp^2) \rightarrow \alpha_s\left( \frac{k_\perp^2}{1-z} \right).
\]

This would correspond to a change in \( \Lambda_{QCD}^2 \) in the Eq. (17):

\[
\Lambda_{QCD}^2 \rightarrow (1-z) \cdot \Lambda_{QCD}^2
\]

which clearly would bring Eq.(23) in line with the predictions based on \( \lambda^2 \neq 0 \), see Eqs. (12), (13). The corresponding principal value integral will be denoted by \( I_n^{pv} \) and for future
reference it is given by:

\[ I_n^p = (-1)^n \frac{\pi^{3/2}}{\beta_1} \int_0^s \frac{d\sigma}{\sin \pi \sigma} \left[ \frac{(4z\Lambda_{QCD}^2)^\sigma (n + \sigma)}{\Gamma(1 + n + \sigma) \Gamma(3/2 - n - \sigma)} - \frac{(4z\Lambda_{QCD}^2)^{-\sigma} (n - \sigma)}{\Gamma(1 + n - \sigma) \Gamma(3/2 - n + \sigma)} \right] \]

(28)

Since \( F_L \) is not logarithmically enhanced in the leading order, one may think that there is no reason, a priori, to believe that the coupling in Eq. (16) should run like \( \alpha_s(k_{\perp}^2) \). However, since it is well known [16] that for the structure function \( F_2 \), perturbation theory up to the two loop order for the leading contributions which are logarithmically enhanced as \( z \to 1 \) is consistent with the identification \( \alpha_s(k_{\perp}^2) \), let us consider the \( 1/Q^2 \) corrections to it. Here we will find not only the pattern indicated above but in addition a different kind of arbitrariness related to the fact that the structure function \( F_2 \) receives contributions which are collinear divergent.

Using dimensional regularization to isolate the collinear divergences we readily obtain the following expression for the real contribution to \( F_2 \):

\[ F_2^{real} = 2C_F \frac{\alpha_s}{2\pi} \int_0^\infty \frac{Q^2(1-z)}{4z} \frac{d k_{\perp}^2}{k_{\perp}^2} \frac{k_{\perp}^{-2\epsilon}}{k_{\perp}^2} \frac{1}{\sqrt{(1-z)^2 - 4(k_{\perp}^2/Q^2)z(1-z)}} \]

\[ - C_F \frac{\alpha_s}{2\pi} (1 + z) \int_0^\infty \frac{Q^2(1-z)}{4z} \frac{d k_{\perp}^2}{k_{\perp}^2} \frac{k_{\perp}^{-2\epsilon}}{k_{\perp}^2} \frac{1}{\sqrt{(1-z)^2 - 4(k_{\perp}^2/Q^2)z(1-z)}} \]

\[ - C_F \frac{\alpha_s}{2\pi} \frac{3z}{1 - z} \frac{1}{Q^2} \int_0^\infty \frac{Q^2(1-z)}{4z} \frac{d k_{\perp}^2}{k_{\perp}^2} \frac{k_{\perp}^{-2\epsilon}}{k_{\perp}^2} \frac{1}{\sqrt{(1-z)^2 - 4(k_{\perp}^2/Q^2)z(1-z)}} \]

\[ + C_F \frac{\alpha_s}{2\pi} \frac{6z + 4z^2}{Q^2} \int_0^\infty \frac{Q^2(1-z)}{4z} \frac{d k_{\perp}^2}{k_{\perp}^2} \frac{k_{\perp}^{-2\epsilon}}{k_{\perp}^2} \frac{1}{\sqrt{(1-z)^2 - 4(k_{\perp}^2/Q^2)z(1-z)}}. \]

(29)

There are of course the virtual contributions which in particular transform factors like \( 1/(1-z) \) into "+ " distributions, however, we omit them for the moment as they are not crucial to the argument. It is easy to check that integration over \( k_{\perp} \) reproduces the well known result for this structure function to leading twist. Note that only the integral multiplying the first two terms produces a collinear divergence proportional to \( 1/\epsilon \) ( \( \epsilon \) is related to the number of
space time dimensions \((D)\) by \(D = 4 - 2\epsilon\) whereas the second type of integral does not.

In order to determine the \(\Lambda_{QCD}^2/Q^2\) contributions from the above using the renormalon method, let us first consider the result when we take \(\alpha_s\) to be \(\alpha_s(k_1^2/(1-z))\) for all the terms on the right hand side of Eq.(29). Proceeding as for the case of \(F_L\) we find:

\[
F_{2}^{\text{real}} = \frac{C_F}{2\pi} \left[ \left( \frac{2}{1-z} - (1+z) \right) \left( \frac{Q^2}{4} \frac{1-z}{z} \right)^{-2\epsilon} J + \left( -\frac{3}{4} \frac{1}{(1-z)} + \frac{3}{2} z \right) I \right],
\]

(30)

where \(J\) and \(I\) are two different types of integrals given by:

\[
J = \int_0^\infty d\sigma \left( \frac{4z\Lambda_{QCD}^2}{Q^2} \right)^{\sigma\beta_1} \frac{\Gamma(-\sigma\beta_1 - \epsilon)\Gamma(1/2)}{\Gamma(1/2 - \sigma\beta_1 - \epsilon)}
\]

(31)

\[
I = \int_0^\infty d\sigma \left( \frac{4z\Lambda_{QCD}^2}{Q^2} \right)^{\sigma\beta_1} \frac{\Gamma(1 - \sigma\beta_1)\Gamma(1/2)}{\Gamma(3/2 - \sigma\beta_1)}.
\]

(32)

Notice the collinear divergence in \(J\) for \(\sigma \sim 0\) which was identified with the perturbative region. One may imagine defining \(J\) and \(I\) by two different prescriptions for obtaining the power corrections (for which the ratios \(J/I\) would be different) and in this way we see that two arbitrary scales would appear in the predictions. Thus if \(J_n\) and \(I_n\) denote the appropriately defined integrals in the integration region over \(\sigma\) which produces the power correction \((1/Q^2)^n\), then:

\[
\left[ F_{2}^{\text{real}} \right]_{1/Q^2} = \frac{C_F}{2\pi} \left[ \left( \frac{2}{1-z} - (1+z) \right) 4zJ_1 + \left( -\frac{3}{4} \frac{1}{(1-z)} + \frac{3}{2} z \right) 4zI_1 \right] \frac{\Lambda_{QCD}^2}{Q^2}.
\]

(33)

In the principal value prescription, for example, \(J_n^{pv}\) is different from \(J_n^{pv}\) (see Eq.(28)) and is given by:

\[
J_n^{pv} = (-1)^n \pi^{3/2} \frac{1}{\beta_1} \int_0^\infty d\sigma \frac{\pi^{3/2}}{\sin\pi\sigma} \left[ \frac{(4z\Lambda_{QCD}^2)^{\sigma(n-1/2 + \sigma)}}{\Gamma(1+n+\sigma)\Gamma(3/2 - n - \sigma)} - \frac{(4z\Lambda_{QCD}^2)^{-\sigma(n-1/2 - \sigma)}}{\Gamma(1+n-\sigma)\Gamma(3/2 - n + \sigma)} \right]
\]

(34)

If we would have used the running \(\alpha_s(k_1^2)\) instead of \(\alpha_s(k_1^2/(1-z))\) then in particular, we would have an extra factor of \(1 - z\) in the denominators of Eq. (33). The \(z\) dependance of the higher twist contributions as given in Eq. (33) is of a similar type as that obtained from
the method of non zero gluon mass [13], however in this case we find that the expression for the power corrections involves two arbitrary scales $I(\Lambda_{QCD}^2/Q^2)$ and $J(\Lambda_{QCD}^2/Q^2)$ rather than a single gluon mass parameter $\lambda$. The predictions for the $z$ dependance will be different depending on which prescription we choose. The origin of these two different functions is that $J$ arises from integrals which are collinear divergent and $I$ from those that do not have this divergence. It is therefore not surprising to expect different functions appearing in this manner. This situation is not improved if we let the coupling in the various terms of Eq.(29) run in a different manner depending on whether they come from collinear or soft regions as $z \to 1$.

There are two aspects of the above discussion that we would like to emphasize. (1) The predictions for the observable $z$ dependence of the power corrections depend crucially on the argument of the running coupling. For the case of $F_L$, if the coupling runs as $\alpha_s(k^2_\perp)$ then the results disagree with the case $\lambda^2 \neq 0$ while the running as $\alpha_s(k^2_\perp / 1-z)$ reproduces the results of the previous section (see also [11]). The argument of the running coupling can be clarified in perturbation theory by two-loop calculations and has been studied for say the structure function $F_2$ in a number of papers as mentioned above [16]. (2) More importantly, we see that for the structure function $F_2$ the renormalon technique necessitates the introduction of two unknown scales (associated respectively with $I$ and $J$) rather than just one in the case of $\lambda^2 \neq 0$. Thus keeping in mind the transition to the non-perturbative case we should reserve for at least two independant rescaling functions. In this respect the picture is different from that in section 2. We will see a furthur indication of this in the next sections.

4 Quark Mass as an Infrared Parameter.

In an attempt to further quantify the infrared sensitivity in deep inelastic scattering phenomena, in this section, we consider the quark mass as an infrared parameter substituting say the gluon mass of section 2. It is convenient to discuss the role of the quark mass for the
moments of the structure functions, i.e. in the language of OPE. Moreover in this language, as discussed at the end of this section, it is most convenient to find the possible sources of infrared instability. In order to make the connection to earlier work [10] which also exploited the OPE more transparent we choose to consider in this section, the moments of the antisymmetric structure function $F_3$.

The forward Compton amplitude has the standard operator product expansion:

$$T_{\mu\nu} = i \int d^4x \exp(iqx) T(j_\mu(x) j^\dagger_\nu(0)) = \sum_{i,n} \left( \frac{2}{-q^2} \right)^n \cdot \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) q_{\mu_1} q_{\mu_2} C_i^n - (35)$$

where $C_i$ are the coefficient functions and the $\theta$ are the operators. We will be interested in the $1/Q^2$ correction keeping the leading twist contribution as well. The calculation of section 2 which reproduces the results of Ref. [11] corresponds to a two-step procedure in evaluating the matrix elements of the operators $\theta$ [12]. Namely, one first reduces the operators of higher twist to those of the leading twist and then expresses the latter in terms of phenomenological structure functions in a standard way.

We will explain this procedure in detail on the example of the lowest moment of $F_3$. The relevant operator is [7, 8]:

$$T^{A}_{\mu\nu} = 2i \frac{q^2}{q^2} \epsilon^{\mu\nu\alpha\beta} q_\alpha \left( L_\beta + \frac{4}{q^2} \theta_\beta \right)$$

where

$$L_\beta = \bar{q} \gamma_\beta q,$$

$$\theta_\alpha = g_s \bar{q} \tilde{G}^{a}_{\alpha\beta} t^a \gamma_\beta \gamma_5 q,$$

$$\tilde{G}^{a}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\gamma\delta} G^{a}_{\gamma\delta}. (39)$$

Now, we assume that the matrix element of the operator $\theta_\beta$ containing the gluon field can be reduced to that of $L_\beta$ by using perturbation theory: i.e, we simply evaluate the contributions
from the diagrams of Fig.1 with quark external states. (The open circles represent the insertion of the higher twist operator). In this way we get,

$$\langle p|T_A^{\mu\nu}|p\rangle \approx \frac{2i}{q^2}\epsilon_{\mu\nu\alpha\beta}q_\alpha q_\beta \left( \langle p|L_\beta|p\rangle - f(\epsilon^2, m^2, \lambda^2) \frac{C_F}{2\pi} \frac{4\alpha_s}{3} \langle p|L_\beta|p\rangle \right)$$  \hspace{1cm} (40)

where the factor \( f(\epsilon^2, m^2, \lambda^2) \) already mentioned in the Introduction is:

$$f(\epsilon^2, m^2, \lambda^2) = \frac{2}{q^2} \int_0^1 dx M^2 \ln(M^2), \quad M^2 = -\epsilon^2 x(1-x) + m^2 x^2 + \lambda^2 (1-x).$$  \hspace{1cm} (41)

The evaluation of the \( 1/Q^2 \) correction considered in section 2 \[11, 12, 13\] corresponds to \( f(0, 0, \lambda^2) = -2(\lambda^2/Q^2)\ln\lambda^2 \).

Eq. (41) demonstrates an origin of the renormalon ambiguities in DIS. From the point of view of the general OPE (see Eq. (36)) there is no reason whatsoever to treat the operators of the quark and gluon fields in an asymmetric way. The assumption that one can first integrate out the gluon line perturbatively and use a non-perturbative input on the matrix element of quark fields is arbitrary. This can be contrasted with the basic idea of renormalons\[1, 2\] when only one gluon line is made soft by means of an insertion of a large number of vacuum bubbles. Now we have both gluon and quark lines soft. It is only in this way that one can get \( 1/Q^2 \) corrections. Thus the estimates of the \( 1/Q^2 \) corrections in DIS can be understood only within a more general framework when one introduces infrared sensitive parameters in various ways. Then the quark mass is no worse a parameter than the gluon mass and one can look for \( m^2 \ln m^2 \) terms instead of \( \lambda^2 \ln \lambda^2 \) terms. However for the case of the quark mass one must be careful to take into account the purely kinematic higher twist contributions a la Nachtman\[18\]. To illustrate this point let us consider the second moment of the structure function \( F_3 \).

The relevant operator now is \[7, 8\]:

$$T_A^{\mu\nu} = \frac{4i}{q^2}\epsilon_{\mu\nu\alpha\beta}q_\alpha q_\beta (L_\beta - \frac{3}{32q^2} A_{\beta\gamma} + \frac{7}{16q^2} B_{\beta\gamma})$$  \hspace{1cm} (42)

where, the leading twist operator is also included and symmetrization and trace subtractions
are not explicitly shown. Further,

\[ L_{\beta\gamma} = i\bar{q}\gamma_{\beta}D_{\gamma}q \]  
\[ A_{\beta\gamma} = -2g\bar{q}D_{\alpha}G_{\alpha\gamma}^{a}t_{5}^{\alpha}\gamma_{\beta}q \]  
\[ B_{\beta\gamma} = g\bar{q}\{\tilde{G}_{\beta\alpha}^{a}, iD_{\gamma}\} + t_{5}^{\alpha}\gamma_{\alpha}\gamma_{\beta}q \]  

Just as before, the matrix elements of the operators containing the gluon field can be reduced to that of \( L_{\beta\gamma} \) by evaluating the contributions from Fig.1. The results for the two operators are:

\[ \langle A_{\beta\gamma} \rangle = \frac{C_{F}}{2\pi}\alpha_{s}\frac{4e_{\mu\nu\rho\delta}q_{\rho}q_{\delta}}{q^{4}} \left( \frac{f_{1}^{a}(p^{2}, m^{2}, \lambda^{2}) + f_{2}^{a}(p^{2}, m^{2}, \lambda^{2})}{q^{2}} \right) \langle L_{\beta\gamma} \rangle \]  
\[ \langle B_{\beta\gamma} \rangle = \frac{C_{F}}{2\pi}\alpha_{s}\frac{f_{b}(p^{2}, m^{2}, \lambda^{2})}{q^{2}} \langle L_{\beta\gamma} \rangle \]  

where,

\[ f_{1}^{a}(p^{2}, m^{2}, \lambda^{2}) = 9 \int_{0}^{1} dx(1 - x)M^{2}lnM^{2} \]  
\[ f_{2}^{a}(p^{2}, m^{2}, \lambda^{2}) = -3 \int_{0}^{1} dx(-p^{2}x(1 - x) + m^{2}x)lnM^{2} \]  
\[ f_{b}(p^{2}, m^{2}, \lambda^{2}) = -28 \int_{0}^{1} dxM^{2}lnM^{2} \]  

It is easy to verify that for \( p^{2} = m^{2} = 0 \) the results of ref.\[13\] are reproduced. For finite quark masses, we see from Eq.\[(46)\] and Eq.\[(49)\], that it appears in a different way than the gluon mass through the function \( f_{2}^{a} \). The origin of this function however is easily understood as a consequence of a kinematic effect due to finite quark masses. To see this we first note that in general the coefficient function \( C(q) \) will depend on the running quark mass,

\[ m(Q) = m(Q_{0}) + m(Q_{0})\gamma_{m}ln\frac{Q}{Q_{0}} \]  

where, \( \gamma_{m} \) is the mass anomalous dimension which at the one loop order is:

\[ \gamma_{m} = -3\frac{C_{F}}{2\pi}\alpha_{s} \]
The coefficient function itself may be expanded:

\[
C(q) = C^0(q,m) + \gamma_m \ln \frac{Q}{m} m \frac{\partial}{\partial m} C^0 + \ldots
\]  

(53)

where the ellipses denote other higher order terms. The mass dependence of the coefficient functions can be inferred to any order in \( m/Q \) by considering the structure functions as a function of the Nachtman variable \( \xi \): [18, 19]

\[
\xi = x [1/2 + (1/4 + m^2/Q^2)^{1/2}]
\]

(54)

Using this it is straightforward to reproduce the contribution \( f_2^q \). Thus we see that when the purely kinematic effects due to the quark masses are taken into account following a procedure analogous to that of Ref. [18], we get similar predictions for the infrared sensitivity keeping \( m \) as an infrared parameter or \( \lambda^2 \neq 0 \), at least up to the second moment of \( F_3 \).

The formulation discussed in this section is the appropriate language for the inclusion of the anomalous dimensions of the operators governing the power corrections. A discussion of this, as we see next, brings into question the infrared safety of the relations like those in Eqs. (12), (13) and (40).

Let us go back to the case of the first moment of \( F_3 \). The leading twist contribution and the first power corrections are governed then by the operators \( L_\beta \) and \( \theta_\beta \), respectively (see Eqs. (37, 38)). The anomalous dimensions of the operator \( L_\beta \) is obviously vanishing while that of the operator \( \theta_\beta \) is [7]:

\[
\gamma_\theta = -\frac{\alpha_s}{4\pi} \left( \frac{32}{9} \right).
\]

(55)

Since the anomalous dimensions of \( L_\beta \), and \( \theta_\beta \) are different, Eq. (40) can at best be true only for a particular choice of \( Q^2 \). In other words, the relations like Eqs. (40) and in general, (12), (13) and are not infrared safe. If one accounts first for a splitting of the quark into a collinear quark and gluon and then for the emission of a soft gluon, the corresponding contribution to the power correction is not suppressed by \( \alpha_s(Q^2) \). This is the meaning of a nonvanishing anomalous dimension of \( \theta_\beta \). Note that in this respect the properties of
the power like corrections differ from the properties of the leading twist contributions. In
the latter case there are no new collinear divergences in higher loops if one considers the
non-singlet DIS structure functions. This in turn means that the power corrections are
governed by an independent structure function and its reduction to the leading twist one
cannot be justified in any known approximation (for an earlier discussion, see [9]). As
mentioned in section 3, an indication of this is already present in the renormalon chain
method of obtaining the power corrections where these were found to involve two different
scales through the integrals $I$ and $J$.

5 Conclusions.

The picture of predictions for the power-like corrections to deep inelastic scattering is remark-
ably rich. From a purely phenomenological viewpoint, one deals not with a few numbers
fixed by renormalons as is usually the case but rather with a few functions which are $x$-
dependences of the power corrections to various structure functions [11, 12, 13]. This makes
comparison with the experimental data much more challenging. On the theoretical side,
as is emphasized in this paper, there is an unusual variety of possible choices of infrared
sensitive parameters. We considered in detail the consequences of choosing the gluon and
quark masses as well as the position of the Landau pole in the running coupling as infrared
parameters. To summarize the results, let us mention some of the conclusions made:

(1) Keeping the gluon mass $\lambda \neq 0$ and defining the infrared sensitive contributions as
those non-analytic in $\lambda^2$, results exactly in the same predictions as the dispersive approach
to the coupling [11] and the renormalon chain in the large $N_f$ limit [12]. The choice between
these techniques is purely a matter of convenience and the case of $\lambda \neq 0$ looks most simple
from the computational point of view.

(2) Parametrizing the power correction in terms of $\Lambda_{QCD}$ brings out two interesting
features. The predictions based on $\lambda \neq 0$ are reproduced if one assumes that the coupling
runs as \( \alpha_s(k^2/(1-z)) \). On the other hand, only by letting the coupling run as \( \alpha_s(k^2) \) do we get consistency with explicit two-loop calculations in case of the structure function \( F_2 \). This latter choice results in a different pattern of the power corrections to \( F_2 \) and to \( F_L \) than the case \( \lambda \neq 0 \). The power corrections to \( F_L(x, Q^2) \) are more important phenomenologically since \( F_L(x, Q^2) \) is proportional to \( \alpha_s(Q^2) \) in the leading-twist approximation. In fact here at the two-loop level, one would encounter diagrams of the type shown in Fig. 2 which can be shown to have power corrections of the type \( \frac{\lambda^2}{Q^2(1-z)} \) at the partonic level. It is not clear that such contributions can be neglected even though they arise at a higher perturbative order.

We also found that because of the collinear divergence, not one but two unknown scales are, in general, introduced in the renormalon chain method. One which is associated with \( I_1 \cdot \frac{\Lambda_{QCD}^2}{Q^2} \) and the other with \( J_1 \cdot \frac{\Lambda_{QCD}^2}{Q^2} \). This again is different from the case \( \lambda \neq 0 \) and is indicative of an inherent infrared instability.

(3) The use of the quark mass to identify the infrared sensitive contribution results, generally speaking, in a different pattern of power-like corrections. The use of the hypothesis on non-perturbative enhancement of the infrared sensitive contributions [4] is crucial at this point. Namely one splits \( m^2 \ln m \) terms into two pieces. One piece is a power-like corrections due to a finite mass of the target— the quark in our case. Such contributions may be treated in a manner similar to that in [5]. The other piece is to be treated as a signal for an infrared sensitive contribution associated with low \( k^2_\perp \). Allowing for a non-perturbative enhancement of this piece brings the predictions in line with the case \( \lambda \neq 0, m = 0 \).

(4) From the point of view of the OPE approach, the calculations of sections 2, and 3 correspond to an asymmetrical treatment of the effects due to the soft gluon and quark lines. One integrates over the gluon line perturbatively while the quark distribution (as manifested in the structure functions) are borrowed from experiment which implies the inclusion of both perturbative and non-perturbative infrared effects. The dependence on the quark virtuality inherent to the reduction factor [11] signals that the procedure cannot in fact be substantiated theoretically. However, the virtuality of the quark is not a convenient
parameter because it is manifestly not gauge invariant. To circumvent this we discussed the problem in the framework of the anomalous dimension of the operators governing the power corrections. Since these are different for the leading twist and the higher twist operators, we concluded that in general we are dealing with new infrared unsafe quantities at the level of the power corrections which cannot be cured by just introducing the same structure function as for the leading twist. Hence independant structure functions are required for the power corrections. Thus, experimental confirmation of the one-loop results given in [12, 13] and reviewed in section 2 of the paper would in fact bring about a puzzle because of a successful perturbative evaluation of an infrared unsafe quantity. One would then be confronted with the challenge of formulating the approximation involved in more precise terms.

6 Acknowledgements

We would like to thank S. Catani for an interesting discussion. We would also like to thank Yu. Dokshitzer and G. Marchesini for communications concerning Ref. [11]. This work was supported in part by the US Department of Energy.

References

[1] A.H. Mueller, Nucl. Phys. B250, 327 (1985).
[2] V.I. Zakharov, Nucl. Phys. B385, 452 (1992).
[3] R. Akhoury and V.I. Zakharov, hep-ph/9610492, to appear in the Proceedings of QCD '96, Montpellier.
[4] R. Akhoury and V. I. Zakharov, Phys. Lett. B357 (1995) 646; Nucl. Phys. B465, 295 (1996).
[5] M. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. B147 (1978) 385.
[6] V.P. Spiridonov and K.G. Chetyrkin, *Sov. J. Nucl. Phys.* **47** (1988) 522.

[7] E.V. Shuryak and A.I. Vainshtein, *Nucl. Phys.* **B199** (1982) 451.

[8] R.L. Jaffe and M. Soldate, *Phys. Rev.* **D26** (1982) 49.

[9] R. K. Ellis, W. Furmanski and R. Petronzio, *Nucl. Phys.* **B207** (1982) 1; *Nucl. Phys.* **B212** (1983) 29.

[10] A.H. Mueller, *Phys. Lett.* **B308** (1993) 355; X. Ji, *Nucl. Phys.* **B448** (1995) 51; V. M. Braun, hep-ph/9505317.

[11] Yu. L. Dokshitzer, G. Marchesini, and B.R. Webber, *Nucl. Phys.* **B469** (1996) 93.

[12] E. Stein, M. Meyer-Hermann, L. Mankiewicz, and A. Schäfer, *Phys. Lett.* **B376** (1996) 177; M. Meyer-Hermann, M. Maul, L. Mankiewicz, E. Stein, and A. Schäfer, hep-ph/9605223; M. Maul, E. Stein, A. Schäfer, and L. Mankiewicz, hep-ph/9612300.

[13] M. Dasgupta and B.R. Webber, *Phys. Lett.* **B382**, 273 (1996).

[14] M. Dasgupta and B.R. Webber, hep-ph/9608394; M. Beneke, V. M. Braun and L. Magnea, hep-ph/9609266.

[15] The results of this paper were briefly summarized in Ref. [3].

[16] J. Kodaira and L. Trentadue, *Phys. Lett.* **B112**, 66 (1982); G. Sterman, *Nucl. Phys.* **B281**, 110 (1987); S. Catani and L. Trentadue, *Nucl. Phys.* **B327**, 323 (1989).

[17] B. R. Webber, *Phys. Lett.* **B339**, 148 (1994).

[18] O. Nachtman, *Nucl. Phys.* **B63** (1973) 237.

[19] H. Georgi and H. D. Politzer, *Phys. Rev.* **D14** (1976) 1829.
Fig. 1

Fig. 2