PHYSICS

Manipulating Hubbard-type Coulomb blockade effect of metallic wires embedded in an insulator

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ABSTRACT

Correlated states have emerged in low-dimensional systems owing to enhanced Coulomb interactions. Elucidating these states requires atomic-scale characterization and delicate control capabilities. Herein, spectroscopic imaging-scanning tunneling microscopy was employed to investigate the correlated states residing in 1D electrons of the monolayer and bilayer MoSe₂ mirror twin boundary (MTB). The Coulomb energies, determined by the wire length, drive the MTB into two types of ground states with distinct respective out-of-phase and in-phase charge orders. The two ground states can be reversibly converted through a metastable zero-energy state with in situ voltage pulses, which tune the electron filling of the MTB via a polaronic process, substantiated by first-principles calculations. Our Hubbard model calculation with an exact diagonalization method reveals the ground states as correlated insulators from an on-site U-originated Coulomb interaction, dubbed the Hubbard-type Coulomb blockade effect. Our study lays a foundation for understanding and tailoring correlated physics in complex systems.

Keywords: STM, one-dimensional, Coulomb effect, electron correlation, Hubbard

INTRODUCTION

In reduced dimensions, the screening effect is suppressed, resulting in enhanced Coulomb interactions. Owing to their structural simplicity, low-dimensional systems provide model platforms for studying correlated physics [1–5]. In particular, 1D electrons are endowed with divergent electron susceptibility owing to their perfect Fermi surface nesting, making them susceptible to interactions [6,7]. These are exemplified by the Peierls-type charge density wave (CDW) originating from the periodic lattice distortion induced by electron–phonon interactions [8–10]. They are also exemplified by the fractionalized spin-charge separation in the Tomonaga Luttinger liquid (TLL) as a result of electron–electron interactions [11,12]. Thus, interesting 1D electrons serve as paradigmatic systems for the investigation of emergent correlated states [13].

Actual 1D systems are subject to quantum fluctuations that profoundly influence correlated states [14–16]. Inter-chain coupling can suppress such an influence; however, it also introduces complications to the system. Recent studies have identified a new 1D metallic system residing in the mirror twin boundary (MTB) of transition metal dichalcogenides [17,18]. This system is devoid of inter-chain coupling and has negligible quantum fluctuations because of its embedment inside the bulk insulating matrix [16,18]. These properties render it an ideal candidate for studying correlated physics in 1D systems.

In correlated phases, the electron-filling adjustment is not only decisive for pinning down the nature of the ground states, but also engages with Coulomb interactions for tuning phase transitions. It is desirable to investigate the evolution of the correlated phases with the in situ response of electron filling. Conventional methods of carrier doping...
inevitably introduce disorders into the system. In low-dimensional systems, electric gating acts as another means of tuning electron filling. Recent achievements in such phase transitions between Mott-like insulators, superconductors and topological orders have been realized in twisted magic-angle graphene superlattices \cite{19–21}. However, the electric-gating method requires sophisticated procedures for device fabrication.

In this study, we characterize the correlated states in 1D MTB wires and demonstrate their \textit{in situ} manipulation using spectroscopic imaging-scanning tunneling microscopy (SI-STM) at 4 K. MTBs are driven into two types of correlated insulating states arising from the dubbed Hubbard-type Coulomb blockade (CB) effect. The two ground states can be switched controllably with voltage pulses, which tune the electron filling of the MTB. Our study clarifies the nature of the correlated ground states and demonstrates the feasibility of local control.

RESULTS AND DISCUSSION

Figure 1a shows the topography of the MTBs, which appears as bright straight-line defects embedded in the monolayer and bilayer MoSe\textsubscript{2} films on the graphene-covered SiC substrate. Their crystal structure (Fig. 1b, inset) conforms to our atomic-resolution imaging (Note 1 in the Supplementary Material). The line spectra along the wire (Fig. 1c) exhibit discrete levels (Fig. 1d), which are quantum well states (QWSs) due to electron confinement along the finite MTB (Note 2 in the Supplementary Material). The spectrum of the MTB (Fig. 1b) features an abrupt conductance onset peak at 0.27 eV, a series of peaks related to the QWSs and a spectral dip at $-0.65$ eV. These spectroscopic features are effectively captured by density functional theory (DFT) calculations with an MTB Mo–Mo bond of 2.66 Å. After structural relaxation, the calculated MTB exhibits a stable 1 × 1 structure without any signature of spontaneous distortion, excluding the possibility of the CDW state from the Peierls mechanism. The calculated MTB lattice constant (0.33 nm) agrees with the measured value (0.34 nm).

The low-energy states around the Fermi level $E_F$ (Fig. 2a and b) exhibit full gaps and strong peaks at the gap edges (Fig. 2a and b, red arrows), with evidently higher intensities than the QWSs. MTBs of different lengths all exhibit satellite peaks of 14 ± 1 meV spacing, whose intensities follow those of the QWSs and the low-energy peaks (Fig. 2a and b, blue arrows), and have thus been ascribed as phonon peaks previously \cite{18}, instead of Coulomb staircases. Interestingly, two types of MTBs are observed. For the Type-1 MTB [Fig. 2(a)], the conductance modulation of its two low-energy peaks is out of phase. The out-of-phase relationship gradually diminishes at the wire ends. Its spectral gap size, $E_{\Delta 1}$, is uniform throughout the wire and its magnitude is much larger than the energy spacing between the QWSs, $\Delta E_{\text{QWS}}$. Type-2 MTB (Fig. 2b) has a similar gap, but with substantial differences. First, the gap size $E_{\Delta 1}$ is close to $\Delta E_{\text{QWS}}$. Second, the spatial modulation of its low-energy peaks is strictly in phase. Third, the two second low-energy peaks (Fig. 2b, purple arrows) are even more enhanced. We define the energy separation between the first occupied peak and the second unoccupied peak near $E_F$ as $E_{\Delta 2}$. Hereinafter, Type-1 and Type-2 MTBs are equivalently designated with their phase relations as the out-of-phase and in-phase states, respectively.

Statistics for $>80$ MTBs indicate that the ratio between Type-1 and Type-2 wires is $\sim$4:1. The more abundant Type-1 MTB is possibly related to its larger gap around $E_F$, which decreases the system energy more effectively. Moreover, the gap sizes of both low-energy states exhibit a pertinent inversely proportional relationship with the wire length, $L$ (Fig. 2c and d) \cite{22,23}. A similar relationship exists between $\Delta E_{\text{QWS}}$ and $L$, which prompts us to evaluate their relations. For the Type-1 MTBs with various lengths, their ratios of $E_{\Delta 1}/\Delta E_{\text{QWS}}$ are similar, with statistics of 2.25 ± 0.14 (standard
(c) and (d) Statistics showing $E_g$ of the (a) out-of-phase and (b) in-phase MTB.

The different states of the MTBs have the same overall electronic structure (Fig. 3d). $E_F$ of the MTB is tuned by pulses, demonstrating that it accommodates additional charges, as is evident from the energy shift of the characteristic band onset (Fig. 3d, inset). Because the residual charge in the MTB is changed by the pulses, the center of the low-energy gap changes correspondingly. The additional charge is not trapped by the localized defects below the graphene [25], which would otherwise cause electronic inhomogeneity along the MTB. To understand the charge stabilization mechanism, DFT calculations were performed, with one additional electron added per six MTB units. This results in a uniform increase in $E_F$ by 81 meV (Fig. 3c). Concomitantly, the MTB Se–Se (Mo–Mo) bond length slightly increases by 0.024 pm (0.0073 pm) (Fig. 3f) and the added charges are distributed mainly on the Mo atoms of the MTB (Fig. 3g) as well as on the edges of the simulation structure (Note 4 in the Supplementary Material). Hence, the collective structural relaxation of the MTB bonds stabilizes the injected charge through a polaronic process [26].

Hereinafter, we elucidate the ground states of the MTBs. The Peierls-type CDW state suggested in [18] could be excluded because CDW should be anti-phase, contradicting our in-phase state, and the CDW gap should be invariant with the wire length and electron filling, which is distinct from Figs 2c and d and 3a–c. Notably, in the 2D case, in the two in-phase levels could be essentially from the same QWS, which is gapped by spin degeneracy lifting via a Mott-like insulating mechanism, as in [19]; (iii) the Coulomb interaction should also exist in Type-1 MTBs, which evokes an out-of-phase gap. These implications can be examined by adjusting the electron filling, for which we expect an effect of the Coulomb interaction. By applying tip pulses of 1.9 V, all discrete levels of an out-of-phase MTB (Fig. 3a) rigidly shift toward higher energy, suggesting that the pulses change the residual charge in the MTB. The out-of-phase gap disappears immediately when the upper gap edge level partially overlaps with $E_F$ (Fig. 3b). In this new state, dubbed the zero-energy state, all the discrete levels become equidistant in energy with the adjacent level-node differing by one. The zero-energy state is irrelevant to the topological end state [24] because it is distributed throughout the MTB. Further pulsing drives the MTB into an in-phase state (Fig. 3c). The zero-energy state is unstable and can spontaneously transform into an in-phase or out-of-phase state during the spectroscopic measurements. Such phase transitions are fully reversible with opposite polarity pulses of $-1.9$ V (Note 3 in the Supplementary Material).

Figure 2. Two types of MTB. (a) and (b) Left: 2D conductance plot ($V_g = 200$ mV, $I = 100$ pA, $V_{mod} = 1.414$ mV (rms)) of the (a) out-of-phase and (b) in-phase MTB. Right: Point spectra extracted along two conductance modulation extrema from the red and black lines. (c) and (d) Statistics showing $E_\phi$ and $E_{\phi}^f$ as a function of $L$. The blue curves are fitted to the data on graphene (GR), yielding $E_\phi = \frac{9.44 \pm 0.006}{L}$ (nm) and $E_{\phi}^f = \frac{9.33 \pm 0.006}{L}$ (nm). (e) and (f) Statistics showing the ratios of $E_\phi/\Delta E_{QWS}$ and $E_{\phi}^f/\Delta E_{QWS}$, respectively, whose statistical values on the graphene substrate are marked with blue lines. In (c)–(f), the open (solid) dots mark the corresponding values measured on monolayer (ML) [double-layer (DL)] MTBs, with the black (red) color representing the data on the graphene (HOPG) substrate.
contrast to the 1D case, the anti-phase relation of the CDW state could be violated in specific systems by other factors, such as the special band structure [27] or strong lattice distortions [28]. The MTB can be considered an elongated quantum dot whose capacitance is proportional to $L$. This may generate a classical CB gap that is sensitive to electron filling [29]. However, this is unlikely. First, the capacitance of a monolayer MTB of a given length relative to the substrate should be approximately twice that of a bilayer MTB, which contradicts Fig. 2c and d. Second, the capacitance of the MTBs on HOPG should differ from that on graphene, but their spectral gaps exhibit the same length dependence (Fig. 2c–f). Furthermore, the spectral gap sizes are invariant upon changing the tip–MTB separations, whereas the capacitance varies (Note 5 in the Supplementary Material). Third, tip-pulsing a half-covered MTB and two contacting MTBs can only switch the ground state of the partial MTB while leaving the other half intact (Note 5 in the Supplementary Material). The long-range Coulomb interactions would mutually affect the ground states of the two contacting MTBs. This suggests that the Coulomb interaction is essentially local and drives the MTB into correlated insulating states.

We discuss the nature of the different correlated states of the MTB. In the case of the zero-energy state, the tail of the zero-energy discrete levels overlaps the Fermi level of the substrate (see Supplementary Fig. S9), which allows the charge density to fluctuate between the MTB and substrate without an energy cost. The charge fluctuation can screen the Coulomb interaction, rendering the intrinsic QWS energies detected. It is observed that, at the zero-energy state, the center of the zero-energy discrete level is always either slightly above or below $E_F$ (Supplementary Fig. S9). With further pulsing, the center of the discrete level shifts right at $E_F$ and the intra-level Coulomb repulsion opens the correlated spin–split gap, i.e. the in-phase state. Because the spin–split gap comes from the same discrete level, its spatial charge order is maintained in phase. For scenarios in which $E_F$ is located between the two
QWS levels, i.e. the out-of-phase state, the inter-level Coulomb interaction contributes an additional gap opening to the otherwise equidistantly spaced QWS energies. As the two discrete levels around $E_g$ are two QWSs with adjacent wave vectors, their real-space charge-density oscillations are out of phase in the middle of the MTB, whose phase difference gradually diminishes when approaching the wire ends.

To quantitatively depict such an ascription, we consider a 1D Hubbard model [30] in a finite chain with $N$ atoms to describe the MTB, which is determined as follows:

$$H = -\sum_{i,\sigma} [t(c_{i,\sigma}^\dagger c_{i+1,\sigma} + c_{i+1,\sigma}^\dagger c_{i,\sigma}) - \mu n_{i,\sigma}] + \frac{U}{2} \sum_i (n_i)^2,$$  \hspace{1cm} (1)

where the notations are standards. Confinement of the first term in Equation (1) on the finite chain results in discrete QWSs, and their eigen functions can be labeled by wave vector $k$.

First, we perform a mean-field analysis, where the Fock term in the Hubbard interaction can be neglected [31], and the effective Coulomb interaction is as follows:

$$H_U = \sum_k u_1 n_{k,\uparrow} n_{k,\downarrow} + \frac{1}{2} \sum_{k\neq k'} u_2 n_{k} n_{k'},$$ \hspace{1cm} (2)

where $u_1 = \frac{U}{2\pi}$ and $u_2 = \frac{U}{\pi}$. At low energies, only the Coulomb interaction between the two relevant one-electron states, namely the highest occupied level $k_0$ and an unoccupied level $k$, hosting the tunneling electron, must be considered. In the case of the doubly occupied $k_0$ state, corresponding to the Type-1 MTB, electron tunneling into any $k_0$ state experiences the Coulomb interaction of two electrons, leading to a spectral gap $E_{g0} = \Delta E_{QWS} + 2u_2$ (Fig. 4a). Similarly, when the $k_0$ state is singly occupied, corresponding to the Type-2 MTB, electron tunneling into any $k_0$ state only experiences the Coulomb interaction of one electron whose strength depends on the specific $k_0$ state. Then, the spectral gap would be $E_{g1} = u_1$ for tunneling into the lowest unoccupied level ($k = k_0$) and $E_{g2} = \Delta E_{QWS} + u_2$ for tunneling into the second lowest unoccupied level ($k \neq k_0$) (Fig. 4c). In contrast, if the $k_0$ state is at $E_F$, charge fluctuations effectively screen the Coulomb interaction, removing the correlated gap (Fig. 4b). Such modeling analysis can determine $U$ using experimentally extracted parameters (Note 7 in the Supplementary Material). For the in-phase state, a comparison of the experimental and theoretical $E_{g1}$ yields $U = 1.9 \pm 0.1$ eV. A similar comparison of the out-of-phase state determines a similar $U = 1.4 \pm 0.2$ eV. The theory also yields $\frac{E_{g1}}{E_{g2}} = 1.43$ and $\frac{E_{g0}}{\Delta E_{QWS}} = 2.22$, conforming to the measured respective values of $1.68 \pm 0.07$ and $2.25 \pm 0.14$.

The picture of the in-phase and out-of-phase states can be numerically verified beyond the mean-field level. In Fig. 4d–f, the intensity plot of the local density of states (LDOS) of Equation (1) in a 12-site chain with an open-boundary condition obtained using the exact diagonalization method is shown. Figure 4d presents the result of the $U = 0$ case, exhibiting discrete QWSs with different numbers of nodes. Owing to the finite-size effect, the gap between different QWSs varies, and that relevant to Fig. 4e and f is $\Delta E_{QWS} \sim 0.43t$. In Fig. 4e and f, a strong Hubbard interaction ($U = 8.0t$) is turned on for even-filled (eight electrons) and odd-filled (seven electrons) systems, respectively. When the system is even-filled (Fig. 4e), the node number of the LDOS across $E_F$ changes by one, and the spectral gap is $E_{g0} \sim 0.45t$, which is slightly greater than $\Delta E_{QWS}$. In contrast, when the system is odd-filled (Fig. 4f), the node number of the LDOS remains the same, with the first spectral gap $E_{g1} \sim 0.34t$ and the second $E_{g2} \sim 0.47t$. This numerical result reproduces the observed in-phase state (corresponding to Fig. 4f) and out-of-phase state (corresponding to Fig. 4e) well, except for the different gap ratios $\frac{E_{g2}}{E_{g1}} \sim 1.38$ and $\frac{E_{g0}}{\Delta E_{QWS}} \sim 1.05$. Such deviations could be attributed to finite-size effects, as the realized systems are much larger in the experiments. However, it should be noted that the node change feature across $E_F$ is robust against the system size, electron numbers and even the Hubbard interaction strength, as long as the electron number parity is fixed (Note 8 in the Supplementary Material).

Moreover, as indicated by the colors of the balls in Fig. 4d–f, we observe that in the out-of-phase state, the two energy levels near $E_F$ always contain contributions from both spin-up and spin-down electrons, whereas in the in-phase state, the two energy levels near $E_F$ with the same number of nodes always come from electrons with opposite spins, which is consistent with the mean-field picture.

As expressed in Equation (2), the electron confinement of the wave functions renders the effective Coulomb interaction normalized by the chain site $N$, which becomes the on-site $U$ at the single-site limit. The effective Coulomb energy is analogous to the charging energy $e^2/(2C)$ in the classical CB effect, where the capacitance $C$ is proportional to the wire length. Thus, its dictation on the correlated insulating state can be considered a Hubbard-type CB effect. The on-site $U$ suggests TLL behavior in the MTB [22,23], as observed from the Fourier transformation of a conductance plot along the MTB,
Figure 4. Results of Hubbard model on a finite chain. (a)–(c) Schematics showing an energy-level diagram at the mean-field level, i.e. (a) out-of-phase state, (b) zero-energy state and (c) in-phase state, respectively. Each level is marked with its wave vector and energy level is highlighted in red. (d)–(f) Intensity plot of the LDOS on a 12-site chain with an open-boundary condition obtained by exact diagonalization method with (d) $U = 0$, eight electrons, (e) $U = 8.0t$, eight electrons and (f) $U = 8.0t$, seven electrons. The Type-1 and Type-2 MTBs have an even and an odd number of electrons, respectively. The spin-up (spin-down) electrons are depicted with red (blue) balls. The solid (hollow) balls represent electrons residing in occupied (unoccupied) levels.

**CONCLUSION**

In conclusion, we discovered a Hubbard-type CB effect, which could establish an alternative platform for emulating the Hubbard model in artificial quantum systems, as recently demonstrated with bosons [32] or fermions [33]. The Hubbard-type CB effect does not explicitly require inputs from the specific system of MoSe$_2$ MTB, which is beyond the scope of clarifying the controversies over its ground state. Such an effect, in contrast to the classical CB effect due to the static Coulomb repulsion energy of electrons, features a quantum many-body effect due to the strong interactions between electrons. Our strategy of in situ electron-filling adjustment allows the addressing and manipulation of individual nano-objects with atomic-scale precision, which may foster the study of correlation physics.

**MATERIALS AND METHODS**

The experiments were conducted using custom-made Unisoku cryogenic STM [34] at 4 K. An electrochemically etched W wire was used as the STM tip, which was cleaned on an Ag(111) surface prior to performing the measurements. Tunneling spectra were obtained using a lock-in technique with a modulation voltage of 983 Hz. Monolayer and bilayer MoSe$_2$ films were grown on graphene-covered SiC(0001) substrates or HOPG substrates with molecular beam epitaxy by co-depositing high-purity Se (purity 99.999%) and Mo (purity 99.95%) atoms with a flux ratio of ~10:1. The substrate temperature was maintained at ~530 K during the growth, with subsequent annealing at 870 K for 10 mins. The detailed preparation procedures for the graphene substrate are described in [5]. The HOPG substrate was cleaved ex situ and further degassed in a vacuum chamber at ~1170 K for 0.5–2 h before MoSe$_2$ growth.

DFT calculations were performed using the Vienna ab initio simulation package (VASP) [35] with the Perdew–Burke–Ernzerhof (PBE) exchange-correlation functional [36] and projector-augmented wave pseudo-potential. A plane-wave basis set with an energy cut-off of 300 eV was applied. We used periodic boundary conditions along the MTB and added a vacuum layer of 10 Å on each side of the structure perpendicular to the MTB. In addition, a vacuum layer of 15 Å was placed normal to the basal plane, which was sufficiently large to eliminate artificial interactions among the periodic images. The structure model contained 34 Se atoms and 16 Mo atoms with a lattice constant of 3.31 Å along the MTB. Geometry optimization was performed until the residual force on each atom was <0.02 eV/Å. We used a 1 × 18 × 1 k-mesh for structure relaxation and electronic structure calculations, and 51 k-point sampling along the MTB for the band structure calculation.

**SUPPLEMENTARY DATA**

Supplementary data are available at NSR online.

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AUTHOR CONTRIBUTIONS

X.Y., J.J.X., Z.H.L. and K.F. carried out the experiments with the help of W.H.Z., Z.M.Z., L.Q., Z.H.Z. and Y.L. Z.L.G. and J.X.L. did the exact diagonalization method calculation. H.W. and S.M. did the DFT calculations. N.N. constructed the theoretical modeling with discussions from H.W.L. Y.S.F., Z.L.G., J.X.L. and N.N. wrote the manuscript with contributions from H.W. All authors commented on the manuscript.

Conflict of interest statement. None declared.

REFERENCES

1. Patrik F. Lecture Notes on Electron Correlation and Magnetism. Singapore: World Scientific, 1999.
2. Adler F, Rachel S and Laubach M et al. Correlation-driven charge order in a frustrated two-dimensional atom lattice. Phys Rev Lett 2019; 123: 086401.
3. Ugeda MM, Bradley AJ and Zhang Y et al. Characterization of collective ground states in single-layer NbSe2. Nat Phys 2018; 12: 92–7.
4. Xi X, Zhao L and Wang Z et al. Strongly enhanced charge-density-wave order in monolayer NbSe2. Nat Nanotechnol 2015; 10: 765–9.
5. Peng L, Zhao J and Cai M et al. Mott phase in a van der Waals transition-metal halide at single-layer limit. Phys Rev Res 2020; 2: 023264.
6. Thierry G. Quantum Physics in One Dimension. Oxford: Clarendon Press, 2004.
7. Gröner G. Density Waves in Solids. Boca Raton: CRC Press, 2018.
8. Monceau P. Electronic crystals: an experimental overview. Adv Phys 2012; 61: 325–581.
9. Cheon S, Kim TH and Lee SH et al. Chiral solitons in a coupled double Peierls chain. Science 2015; 350: 182–5.
10. Zeng CG, Kent P and Kim TH et al. Charge-order fluctuations in one-dimensional silicides. Nat Mater 2008; 7: 539–42.
11. Blumenstein C, Schäfer J and Mietke S et al. Atomically controlled quantum chains hosting a Tomonaga–Luttinger liquid. Nat Phys 2011; 7: 776–80.
12. Stühler R, Reis F and Müller T et al. Tomonaga–Luttinger liquid in the edge channels of a quantum spin Hall insulator. Nat Phys 2020; 16: 47–51.
13. Chen C, Jiang K and Zhang Y et al. Atomic line defects and zero-energy end states in monolayer FeTeSe high-temperature superconductors. Nat Phys 2020; 16: 536–40.
14. Mermin ND and Wagner H. Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic Heisenberg models. Phys Rev Lett 1966; 17: 1133–6.
15. Lee PA, Rice TM and Anderson PW. Fluctuating effects at a Peierls transition. Phys Rev Lett 1973; 31: 462–5.
16. Yang X, Xian J and Li G et al. Possible phason-polaron on purely one-dimensional charge order of Mo2Se4 nanowires. Phys Rev X 2020; 10: 031061.
17. Liu HJ, Jiao L and Yang F et al. Dense network of one-dimensional midgap metallic modes in monolayer MoSe2 and their spatial undulations. Phys Rev Lett 2014; 113: 066105.
18. Banja S, Wickenburg S and Liu ZF et al. Charge density wave order in 1D mirror twin boundaries of single-layer MoSe2. Nat Phys 2016; 12: 751–6.
19. Cao Y, Fatemi V and Demir A et al. Correlated insulator behaviour at half-filing in magic-angle graphene superlattices. Nature 2018; 556: 80–4.
20. Cao Y, Fatemi V and Fang S et al. Unconventional superconductivity in magic-angle graphene superlattices. Nature 2018; 556: 43–50.
21. Chen GR, Sharpe AL and Fox EJ et al. Tunable correlated Chern insulator and ferromagnetism in a moiré superlattice. Nature 2020; 579: 56–61.
22. Jolie W, Murray C and Weiss PS et al. Tomonaga-Luttinger liquid in a box: electrons confined within MoSe2 mirror-twin boundaries. Phys Rev X 2019; 9: 011055.
23. Xia YP, Zhang JQ and Jin YJ et al. Charge density modulation and the Luttinger liquid state in MoSe2 mirror twin boundaries. ACS Nano 2020; 14: 10716–22.
24. Zhang X, Gu QQ and Sun HG et al. Eightfold fermionic excitation in a charge density wave compound. Phys Rev B 2020; 102: 035125.
25. Lee J, Wong D and Velasco J et al. Imaging electrostatically confined Dirac fermions in graphene quantum dots. Nat Phys 2016; 12: 1032–6.
26. Patera LL, Queck F and Scheuerer P et al. Mapping orbital changes upon electron transfer with tunnelling microscopy on insulators. Nature 2019; 566: 245–8.
27. Sacks W, Rotichov D and Klein J. Voltage-dependent STM image of a charge density wave. Phys Rev B 1998; 57: 13118–31.
28. Dai JX, Calleja E and Alldredge J et al. Microscopic evidence for strong periodic lattice distortion in two-dimensional charge-density-wave systems. Phys Rev B 2014; 89: 165140.
29. Hansson N, Nalwoga B and Petta JR et al. Spin dynamics in few-electron quantum dots. Rev Mod Phys 2007; 79: 1217–65.
30. Lof RW, van Veenendaal MA and Koonmans B et al. Band gap, excitons, and Coulomb interaction in solid C60. Phys Rev Lett 1992; 68: 3924–7.
31. Tarucha S, Austing DG and Tokura Y et al. Direct Coulomb and exchange interaction in artificial atoms. Phys Rev Lett 2000; 84: 2485–8.
32. Baier S, Mark MJ and Petter D et al. Extended Bose-Hubbard models with ultracold magnetic atoms. Science 2016; 352: 201–5.
33. Hensgens T, Fujita T and Janssen L et al. Quantum simulation of a Fermi–Hubbard model using a semiconductor quantum dot array. Nature 2017; 548: 70–3.
34. Peng L, Yuan Y and Li G et al. Observation of topological states residing at step edges of WTe2. Nat Commun 2017; 8: 659.
35. Kresse G and Furthmüller J. Efficient iterative schemes for ab initio total-energy calculations using a plane-wave basis set. Phys Rev B 1996; 54: 11169–86.
36. Perdew JP, Burke K and Ernzerhof M. Generalized gradient approximation made simple. Phys Rev Lett 1996; 77: 3865–8.