Conditions for a Monotonic Channel Capacity

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Abstract—Motivated by results in optical communications, where the performance can degrade dramatically if the transmit power is sufficiently increased, the channel capacity is characterized for various kinds of memoryless vector channels. It is proved that for all point-to-point channels, the channel capacity is a nondecreasing function of power. As a consequence, maximizing the mutual information over all input distributions with a certain power is for such channels equivalent to maximizing it over the larger set of input distributions with upperbounded power. For interference channels such as optical wavelength-division multiplexing systems, the primary channel capacity is always nondecreasing with power if all interferers transmit with identical distributions as the primary user. Also, if all input distributions in an interference channel are optimized jointly, then the achievable sum-rate capacity is again nondecreasing. The results generalize to the channel capacity as a function of a wide class of costs, not only power.

I. INTRODUCTION

IN THE MOST cited paper in the history of information theory [1], Shannon in 1948 proved that with adequate coding, reliable communication is possible over a noisy channel, as long as the rate does not exceed a certain threshold, called the channel capacity. He provided a mathematical expression for the channel capacity of any point-to-point channel, based on its statistical properties. The expression is given as the supremum over all possible input distributions of a quantity later called the mutual information [2], [3]. The channel capacity is often studied as a function of a cost, such as the transmit power. More specifically, the capacity–cost function is defined as the supremum of the mutual information over all input distributions whose cost is either equal to a given constant or upperbounded by a constant—the convention differs between disciplines. We will return to the distinction between the two definitions at the end of this section.

For the additive white Gaussian noise (AWGN) channel, the channel capacity is known exactly [1 Sec. 24], [4 Ch. 9]. In recent years, the problem of calculating or estimating the channel capacity of more complicated channels has received a lot of attention (see surveys in [5]–[8]). Due to the absence of exact analytical solutions and the computational intractability of optimizing over all possible input distributions, most investigations of the channel capacity of non-AWGN channels rely on bounding techniques and asymptotic analysis.

The main motivation for this paper comes from the type of nonlinear distortion encountered in fiber-optical communications. In contrast to linear channels, an optical fiber has the peculiar property that communications becomes virtually impossible if the signal amplitude is too high [9]–[15], [16]. This phenomenon is well-known from experiments and simulations, and can also be explained theoretically. The lightwave propagation in an optical fiber is governed by a nonlinear differential equation, the nonlinear Schrödinger equation or, if polarization effects are considered, the Manakov equation [17], [18], [19]. These equations include a nonlinear distortion term, whose amplitude is proportional to the cubed signal amplitude. At high enough signal amplitudes, this nonlinear distortion dominates the other terms in the differential equation, effectively dawning the signal.

Similarly, one might expect that the nonlinear distortion would force the mutual information and channel capacity down to zero at sufficiently high power, and in the past two decades, many results have been published in optical communications to support this conjecture [9]–[8], [20]–[37]. Already in 1993, Splett et al. modeled the interference from four-wave mixing in a wavelength-division multiplexing (WDM) system as an AWGN component, under some conditions on the noise and dispersion, in what might have been the first study ever of the channel capacity of a nonlinear optical link [38]. The variance of this AWGN depends nonlinearly on the transmit power, which is assumed equal on all wavelengths. Similar nonlinear channel models have been rediscovered, modified, and further analyzed in [11]–[13], [21], [25], [36], [39]. Due to the signal-dependent noise, their channel capacities are not monotonic: As the transmit power (or signal-to-noise ratio) increases, the channel capacity increases towards a peak and then decreases again as the power is further increased. Other channel models with signal-dependent AWGN were presented in [35], [37] and have similar nonmonotonic channel capacities. An essential assumption, explicit or implicit, in the derivation of these AWGN-based models is that the transmitted signal consists of independent, identically distributed symbols, but not in the presence of error-correction coding, since coding introduces correlation between symbols. Using a model derived under certain conditions on the transmitted signal is particularly risky in channel capacity calculations, since the channel capacity is by definition the maximum achievable rate using any transmission scheme—including those for which the constrained model is not valid.

A continuous-time channel model for cross-phase modulation (XPM) was presented by Mitra and Stark [20]. Although no discrete-time XPM model was obtained, they showed that the channel capacity of the XPM channel model is lowerbounded by the capacity of a signal-dependent AWGN channel, and that this lower bound is nonmonotonic. They further conjectured that the true channel capacity would have a similar nonmonotonic behavior as its lower bound. Many
variants of the Mitra–Stark lower bound have been presented in recent years, often along with the conjecture that the true channel capacity is also nonmonotonic [23, 25, 28, 33, 40]. This conjecture was disproved in the zero-dispersion case by Turitsyn et al. [41], who showed that the lower bound based on the AWGN channel [22] is very far from the true channel capacity, and that the channel capacity in fact grows logarithmically with power under certain conditions.

Another type of lower bound on channel capacity is obtained by fixing the input distribution and calculating the mutual information [7, 27, 42, 34, 42, 43] or by optimizing the mutual information over a subset of all possible input distributions [7, 50, 61, 34, 55]. All these lower bounds consistently show a nonmonotonic behavior, decreasing towards zero after a peak at a finite power, and the conjecture that the channel capacity would have a similar nonmonotonic behavior as its lower bounds is often repeated.

We believe that the results cited above, while mathematically correct, do not fully exploit the potential of capacity-achieving coding over nonlinear optical channels. Whereas previous studies have shown that for many cost-dependent channel models, the capacity is a nonmonotonic function of transmit power, we prove in this work mathematically that for any cost-independent channel model, the capacity is monotonic (nondecreasing but not necessarily strictly increasing). Thus, these two classes of models behave entirely differently in the nonlinear regime. The results are also extended to a wide class of cost functions and to three specific multiuser scenarios.

The presented theory holds regardless of whether the capacity–cost function is defined by maximizing over all input distributions with exactly the given cost or with an upperbounded cost. The proofs are developed assuming the former definition, and they are all trivial for the latter. An interesting consequence of the nondecreasing channel capacity is that the two definitions of the capacity–cost function are fully equivalent.

II. CHANNEL CAPACITY AND COST

Let \( X \) and \( Y \) be real, \( n \)-dimensional vectors, representing the input and output, resp., of a discrete-time memoryless communication channel. Their respective domains, or alphabets, are denoted by \( \mathcal{X} \subseteq \mathbb{R}^n \) and \( \mathcal{Y} \subseteq \mathbb{R}^n \). The joint distribution \( f_{X,Y}(x,y) \) for \( x \in \mathcal{X} \) and \( y \in \mathcal{Y} \) can be factorized as \( f_{X,Y}(x,y) = f_X(x)f_{Y|X}(y|x) \), where \( f_X \) represents the input distribution (which is in practice determined by the modulation format) and \( f_{Y|X} \) represents the channel. We denote the mutual information between \( X \) and \( Y \) with \( I(X;Y) \), while \( I(X;Y|Z) \) denotes a conditional mutual information. The entropy and conditional entropy are denoted by \( H(X) \) and \( H(X|Z) \), resp., and the differential entropy and conditional differential entropy are denoted by \( h(X) \) and \( h(X|Z) \), resp.

Using error-correction coding, codewords of input symbols \( x \in \mathcal{X} \) are selected from a codebook. The rate of transmission, in bits per second, is the logarithm of the codebook size divided by the codeword length. The codewords can be transmitted with arbitrarily small error probability if the codewords are sufficiently long and the rate is sufficiently small. Such a rate is called an achievable rate, and the supremum of all achievable rates, over all possible codes and block lengths, is defined as the operational channel capacity.

Shannon’s channel coding theorem [1] states that the operational channel capacity is equal to the information channel capacity, which is defined as the supremum of the mutual information \( I(X;Y) \) between the channel input and output, where the supremum is taken over all input distributions \( f_X \). The capacity-achieving distribution may be continuous or discrete [44, 45].

In this work, the channel capacity is characterized as a function of some kind of cost. Closely following the definitions in [45], [46, Ch. 2], we define the cost function \( b(x) \) as a deterministic, real, nonnegative function of an input symbol \( x \in \mathcal{X} \). The codeword cost is defined as the sum of the cost function of all symbols in the codeword divided by the codeword length. The most common cost function, and the only one that will be exemplified in this paper, is the transmit power, which is obtained by setting \( b(x) = \|x\|^2 \).

The capacity–cost function can be defined in two, subtly different, ways, depending on whether the cost of every codeword is upperbounded by a cost \( \beta \) or exactly \( \beta \). The channel coding theorem extends straightforwardly to both these cases. In the first case, which is most common in classical information theory, the maximum achievable rate using a codebook in which all codewords have at most a given cost \( \beta \) is [4 Ch. 9], [46 Sec. 7.3], [47 Sec. 3.3]

\[
\tilde{C}(\beta) \triangleq \sup_{f_X \in \Omega(\beta)} I(X;Y),
\]

where \( \Omega(\beta) \) is the set of all distributions \( f_X \) over \( \mathcal{X} \) such that \( \mathbb{E}[b(X)] \leq \beta \). Analogously, the maximum achievable rate using a codebook in which all codewords have exactly the same cost \( \beta \) can be shown to be

\[
C(\beta) \triangleq \sup_{f_X \in \Omega(\beta)} I(X;Y),
\]

where \( \Omega(\beta) \) is the set of all distributions \( f_X \) over \( \mathcal{X} \) such that \( \mathbb{E}[b(X)] = \beta \). This case is prevalent in optical information theory [20, 25, 30, 26 eq. (11.5)] and also sometimes considered in wireless communications [48].

The second definition of the capacity–cost function is considered in this paper, i.e., every codeword has the same cost. This is partly because the work was inspired by capacity results in optical communications, where this is the conventional definition, and partly because the fundamental question considered in this paper, about the monotonic behavior of channel capacity, is trivial in terms of \( \tilde{C}(\beta) \) [29, 49 Ch. 2]. That \( \tilde{C}(\beta) \) is nondecreasing for all channels follows from [1] and the fact that \( \Omega(\beta) \subseteq \Omega(\beta') \) for all \( \beta \geq \beta' \). However, the two definitions are in fact equivalent for point-to-point channels, as a consequence of Corollary [3] in the next section.

\[\text{1} \text{With a slight abuse of notation, we also include distributions that have no pdf [4 Sec. 8.5].}\]
III. POINT-TO-POINT CHANNELS

In this section, we are concerned with a discrete-time, memoryless vector channel between a single transmitter and a single receiver, formally defined as follows.

**Definition 1:** A point-to-point channel is a memoryless relationship $f_{Y|X}$ between vectors $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$, which is independent of the codeword cost.

Such a relationship can represent a continuous-time band-limited channel by sampling the transmitted and received waveforms at the Nyquist rate [11, Sec. 23], and it can represent channels with an arbitrarily long (finite) memory by choosing the dimension $n$ much larger than the channel memory [46, Sec. 4.6]. The dimensions may also, in addition to time, represent frequency (wavelength), space, polarization, lightwave modes, or all of these. Hence, the theory applies to a wide variety of channels in different applications.

The capacity is commonly studied as a function of the transmit power, which is obtained by setting $b(x) = \|x\|^2$ for all $x \in \mathcal{X} = \mathbb{R}^n$. The results in this paper hold not only for transmit power but also more generally for any unbounded cost function, according to the following definition.

**Definition 2:** An unbounded cost function $b(x)$ over a domain $\mathcal{X}$ is a real, nonnegative function such that for any given $b_0 \geq 0$, there exists a vector $x \in \mathcal{X}$ for which $b(x) = b_0$.

The main result for point-to-point channels is the following theorem, which implies that the channel capacity will either increase indefinitely or converge to a finite value as the cost increases, depending on the channel. However, it cannot have a peak for any channel or any cost. Despite its simple nature, it has to our knowledge not been stated before.

**Theorem 1 (Law of Monotonic Channel Capacity):** Let $f_{Y|X}(y|x)$ be a point-to-point channel defined on $x \in \mathcal{X}$ and $y \in \mathcal{Y}$. Let $b(x)$ be an unbounded cost function on $\mathcal{X}$. Then $C(\beta)$ is a nondecreasing function of $\beta$.

**Proof:** We will show that for any given pair of costs $\beta \geq \beta' \geq 0$, $C(\beta) \geq C(\beta')$. Let, for any $0 < \epsilon \leq 1$,

$$\beta'' \triangleq \beta' + \frac{\beta - \beta'}{\epsilon}. \quad (3)$$

We define a time-sharing random symbol $X \in \mathcal{X}$ given an auxiliary binary random variable $Q$ such that

$$X \triangleq \begin{cases} X', & Q = 0, \\ X'', & Q = 1. \end{cases} \quad (4)$$

where $\Pr\{Q = 1\} = \epsilon$ and the distributions of $X' \in \mathcal{X}$ and $X'' \in \mathcal{X}$ satisfy $\mathbb{E}[b(X')] = \beta'$ and $\mathbb{E}[b(X'')] = \beta''$, resp. Such distributions exist, by assumption, for any costs $\beta', \beta'' \geq 0$. Thus

$$\mathbb{E}[b(X)] = (1 - \epsilon)\mathbb{E}[b(X')] + \epsilon\mathbb{E}[b(X'')]$$

$$= (1 - \epsilon)\beta' + \epsilon\beta''$$

$$= \beta. \quad (5)$$

Because $Q \to X \to Y$ is a Markov chain, the mutual information can be bounded as

$$I(X;Y) \geq I(X;Y|Q) = (1 - \epsilon)I(X;Y|Q = 0) + \epsilon I(X;Y|Q = 1)$$

$$\geq (1 - \epsilon)I(X;Y|Q = 0) = (1 - \epsilon)I(X';Y'), \quad (6)$$

where the first inequality follows from [4] eq. (2.122)] and $Y'$ is defined as the channel output when the input is $X'$. This inequality holds for any $0 < \epsilon \leq 1$ and any distributions $f_{X'} \in \Omega(\beta')$ and $f_{X''} \in \Omega(\beta'')$. Choosing $f_{X'}$ as a capacity-achieving distribution at cost $\beta'$, the right-hand side of (6) becomes $(1 - \epsilon)C(\beta')$. Thus,

$$C(\beta) \geq I(X;Y) \geq (1 - \epsilon)C(\beta'). \quad (7)$$

If now $C(\beta) < C(\beta')$, then (6) would yield a contradiction in the range $0 < \epsilon < 1 - C(\beta')/C(\beta')$. Hence, $C(\beta) \geq C(\beta')$. 

Intuitively, an input distribution with a nondecreasing mutual information for a given channel can be constructed by combining two parts, a high-probability part at a moderate cost, which does not vary much as the overall average cost is changed, and a low-probability part, a “satellite,” which absorbs the whole increase in average cost by moving away from the other part [14], [15]. As $\epsilon \to 0$, the input distribution $f_X$ becomes more and more like $f_{X'}$, while the cost remains at $\beta$ because the lower cost $\beta' < \beta$ is balanced by another cost $\beta'' \gg \beta$.

The theorem can also be proved in terms of the operational capacity, as outlined in the following. Consider, for a given channel, a capacity-achieving code at cost $\beta'$. From (2), all codewords have the same cost $\beta'$. We construct a new code by appending a single symbol to each codeword, such that the new codewords have cost $\beta > \beta'$. This last symbol can be ignored by the receiver and does not influence the error probability. If the codeword length is large enough, the rate loss from the extra symbol is negligible. This shows that $C(\beta')$ is an achievable rate at cost $\beta$, and thus, $C(\beta) \geq C(\beta')$.

By writing the channel capacity (1) as $C(\beta) = \sup_{\beta' \leq \beta} C(\beta')$, which by Theorem 1 is equal to $C(\beta)$, the following corollary is obtained.

**Corollary 2:** Under the conditions of Theorem 1, $\hat{C}(\beta) = C(\beta)$ for all $\beta \geq 0$.

This means that the two definitions (1) and (2) are equivalent: The cost-limited channel capacity $C(\beta)$ is achieved by an input distribution $f_X$ for which the cost equals the maximum allowed value $\beta$. A practical interpretation is that when designing a capacity-achieving code for a nonlinear channel, it suffices to consider only codes for which all codewords have the same cost $\beta$.

As stated in Definition 1 the channel distribution $f_{Y|X}$ considered in Theorem 1 does not change with the cost $\beta$. In other words, the channel remains the same regardless of which codebook is used. This is a standard assumption in information theory [46, Sec. 4.2], and it is not considered to restrict generality. On the other hand, this assumption has
often been relaxed in optical communications. The information capacity \( \beta \) of a cost-dependent channel model may decrease to zero at sufficiently high cost (power) \([6], [7], [20], [24], [25], [31]–[33], [35], [38]\), thus contradicting Theorem \(1\). We believe that cost-dependent channel models should be avoided in this context, because information-theoretic tools are lacking to handle and interpret them. It is not even clear whether the optimization problem \( \sup_{f_X} I(X;Y) \) has any operational meaning in terms of maximum achievable rates for point-to-point channels. Shannon’s channel coding theorem, in its standard memoryless form, assumes that the channel operates on each symbol \( X \) independently, which is not the case if \( f_{Y|X} \) depends on \( \beta \). Furthermore, such models are questionable from a physical viewpoint, as they imply an infinite channel memory \([14]\).

IV. INTERFERENCE CHANNELS

We consider a discrete-time, memoryless interference channel with \( k \) users, each with the purpose of transmitting a message to a transmitter to a receiver \([47]\), for example an optical WDM system. The input and output are denoted by \( X_i \) and \( Y_i \), resp., for \( i = 1, \ldots, k \). The \( i \)th user attempts to recover \( X_i \) based on \( Y_i \), without knowledge of \( Y_j \) for \( j \neq i \). The statistics of the received vectors is given by the conditional distribution \( f_{Y_1} \ldots f_{Y_k} x_1 \ldots x_k \), which does not change with the cost. Independent data is transmitted by each user, and the joint input distribution \( f_{X_1} \ldots f_{X_k} \) is therefore equal to the product of the marginal distributions \( f_{X_1} \cdot \cdots \cdot f_{X_k} \). All input distributions \( f_{X_i} \) are known to all users. From the viewpoint of user \( i \), all interfering input symbols \( X_j \) for \( j \neq i \) are assumed to be independent between channels uses. This assumption, which is conventional in optical communications, is valid if the codebook of user \( j \) is not known to user \( i \) or if user \( j \) transmits uncoded data.

Three scenarios, or behavioral models \([51]\), are considered in the following subsections. Two of them correspond to selfish optimization by individual users, which is presently the dominant optimization approach in optical information theory. The aim is to determine the maximum achievable rate of the channel, assuming that the interference caused by the other users can be included in the channel model. Since the conditional distribution \( f_{Y_1|X_1} \) in this case depends on the distributions \( f_{X_2}, \ldots, f_{X_k} \), but not on \( f_{X_1} \), Theorem \(1\) applies and the capacity–cost function for the primary user is nondecreasing.

A. Fixed Interference Distributions

Suppose that the distributions \( f_{X_2}, \ldots, f_{X_k} \) are fixed and do not change even if \( f_{X_1} \) would change. From the viewpoint of the primary user, the interference caused by the other users can be included in the channel model. Since the conditional distribution \( f_{Y_1|X_1} \) in this case depends on the distributions \( f_{X_2}, \ldots, f_{X_k} \), but not on \( f_{X_1} \), Theorem \(1\) applies and the capacity–cost function for the primary user is nondecreasing.

B. Adaptive Interference Distributions

In this section, we consider the scenario where all users apply the same input distribution, or linearly rescaled versions thereof. If the primary user’s distribution is \( f_{X_1} \), then the other distributions are

\[ f_{X_i}(x) = \alpha_i^n f_{X_1}(\alpha_i x), \quad i = 2, \ldots, k \]  

(14)

for some given constants \( \alpha_2, \ldots, \alpha_k \). An important special case is \( \alpha_2 = \cdots = \alpha_k = 1 \), which makes all distributions \( f_{X_2}, f_{X_3}, \ldots, f_{X_k} \) identical.

The primary channel is described by

\[ f_{Y_1|X_1}(y_1|x_1) = \mathbb{E}[f_{Y_1|X_1, X_2, \ldots, X_k}(y_1|x_1, X_2, \ldots, X_k)], \]  

(15)

where the expectation is taken over all interferers \( X_2, \ldots, X_k \). This expression changes with \( f_{X_1} \), because \( f_{X_2}, f_{X_3}, \ldots, f_{X_k} \) change with \( f_{X_1} \). Hence, the channel is cost-dependent and Theorem \(1\) does not apply. It cannot nevertheless be proven that the channel capacity is monotonic.

**Theorem 4:** If the \( k \) input distributions are related as in \(14\) for some given constants \( \alpha_2, \ldots, \alpha_k \), then the capacity–cost function of the primary channel is nondecreasing, for any interference channel \( f_{Y_1|X_1, X_2, \ldots, X_k} \).

**Proof:** First, we verify that the channel coding theorem holds in the considered scenario. For any fixed primary user distribution \( f_{X_1} \), the interferers’ distributions \( f_{X_2}, \ldots, f_{X_k} \) are also fixed via \(14\), and the primary transmitter–receiver pair is characterized by \(15\). By the point-to-point channel coding theorem \([11] Sec. 23\), \([46] Ch. 7\), \([47] Ch. 3\), the mutual information \( I(X_1;Y_1) \) of this joint distribution corresponds to an achievable rate of the primary channel. Hence, all rates below the primary channel capacity

\[ C(\beta) \triangleq \sup_{f_{X_1} \in \Omega(\beta)} I(X_1;Y_1) \]  

(16)

are achievable and the channel coding theorem holds.

To prove that \( C(\beta) \) is nondecreasing, let \( f_{X_1} \in \Omega(\beta') \) be a capacity-achieving distribution\(^3\) at some cost \( \beta' \geq 0 \). We will show that \( C(\beta) \geq C(\beta') \) for any \( \beta \geq \beta' \).

\(^3\)Or, more precisely, a capacity-approaching sequence of distributions.
For any given $\beta \geq \beta'$ and $0 < \epsilon \leq 1$, let

$$\beta'' \triangleq \beta' + \frac{\beta - \beta'}{\epsilon}$$

(17)

and let $f_{X''_i}$ be any distribution over $\mathcal{X}$ with $E[b(X''_i)] = \beta''$. We now define a time-sharing random vector $X_1$ given an auxiliary binary random variable $Q_i$ such that

$$X_1 \triangleq \begin{cases} X'_i, & Q_1 = 0, \\ X''_i, & Q_1 = 1, \end{cases}$$

(18)

where $\Pr\{Q_1 = 1\} = \epsilon$. This vector satisfies

$$E[b(X_1)] = (1 - \epsilon)E[b(X'_i)] + \epsilon E[b(X''_i)] = (1 - \epsilon)\beta' + \epsilon\beta'' = \beta.$$  

(19)

As illustrated in Fig. 1, the interference can be generated by an analogous time-sharing method, using the auxiliary variables $Q_2, \ldots, Q_k$. These variables have the same distribution as $Q_1$ and are independent of each other and also of $Q_1$. They control the interferers $X_2, \ldots, X_k$ such that $X_i = X'_i$ if $Q_i = 0$ and $X_i = X''_i$ if $Q_i = 1$, where

$$f_{X'_i}(x) = \alpha^n f_{X'_i}(\alpha; x),$$

(20)

$$f_{X''_i}(x) = \alpha^n f_{X''_i}(\alpha; x)$$

(21)

for $i = 2, \ldots, k$. Obviously, the time-sharing symbol $X_1$ has the desired distribution \[14\].

The mutual information of the primary channel can be bounded as

$$I(X_1; Y_1) \leq I(X_1; Y_1 | Q_1)$$

$$\geq I(X_1; Y_1 | Q_1, Q_2, \ldots, Q_k)$$

$$- H(Q_2, \ldots, Q_k),$$

(22)

where (22) holds because $Q_1 \rightarrow X_1 \rightarrow Y_1$ is a Markov chain and (23) follows by setting $Z = [Q_2, \ldots, Q_k]$ in Lemma 3. The first term of the right-hand side of (22) can be bounded as

$$I(X_1; Y_1 | Q_1, Q_2, \ldots, Q_k)$$

$$= \sum_{(q_1, \ldots, q_k) \in \{0, 1\}^k} \Pr\{Q_1 = q_1, \ldots, Q_k = q_k\} \cdot I(X_1; Y_1 | Q_1 = q_1, \ldots, Q_k = q_k)$$

$$\geq \Pr\{Q_1 = \cdots = Q_k = 0\} \cdot I(X_1; Y_1 | Q_1 = \cdots = Q_k = 0)$$

$$= (1 - \epsilon)h I(X'_1; Y'_1)$$

$$= (1 - \epsilon)h C(\beta').$$

(24)

The second term of the right-hand side of (22) is

$$H(Q_2, \ldots, Q_k) = \sum_{i=2}^k H(Q_i)$$

$$= (k - 1)H_2(\epsilon),$$

(25)

where $H_2(u) \triangleq -u \log_2 u - (1 - u) \log_2(1 - u)$. Combining (16), (23), (24), and (25) yields

$$C(\beta) = \sup_{f_{X_1} \in \Omega(\beta)} I(X_1; Y_1)$$

$$\geq \sup_{0 < \epsilon \leq 1} \left[ (1 - \epsilon)h C(\beta') - (k - 1)H_2(\epsilon) \right]$$

$$= \lim_{\epsilon \to 0} \left[ (1 - \epsilon)h C(\beta') - (k - 1)H_2(\epsilon) \right]$$

$$= C(\beta'),$$

(26)

which completes the proof. \[ \square \]

Intuitively, the proof relies on constructing a “satellite distribution” \[14\] for $X_1$, where the “satellite,” denoted by $X''_i$ in (18), carries a much higher cost than $X'_i$ and occurs with lower probability.

### C. Joint Optimization

In the third and last scenario, we assume that the system includes a mechanism to optimize the transmission schemes of all users jointly, for example via a central network controller. As in the previous two scenarios, the transmitters and receivers are still operated separately, in the sense that the transmitters and receivers do not exchange information about their respective signals\[4\].

Let $R_i$ be an achievable rate for the transmitter–receiver pair $i = 1, \ldots, k$ and let $R \triangleq (R_1, \ldots, R_k)$ be a vector of rates that can be simultaneously achieved over the interference channel, with arbitrarily small error probability. The capacity region $C(\beta)$, where $\beta \triangleq (\beta_1, \ldots, \beta_k)$, is defined as the closure of the set of all achievable rate vectors $R$ when every codeword used by user $i = 1, \ldots, k$ has the exact cost $\beta_i$ \[47\] Sec. 4.1, 6.1]. While no analytical expression is known for the capacity region of general interference channels \[47\] Ch. 6], we can still derive fundamental properties by using the time-sharing principle.

\[4\]If data instead is jointly encoded over all transmitted signals $X_1, \ldots, X_k$ and jointly decoded based on all received signals $Y_1, \ldots, Y_k$, then the channel is equivalent to a high-dimensional point-to-point channel and Theorem 1 applies.
Theorem 5: Let \( \beta = (\beta_1, \ldots, \beta_k) \) and \( \beta' = (\beta'_1, \ldots, \beta'_k) \) be two cost vectors such that \( \beta_i \geq \beta'_i \geq 0 \) for \( i = 1, \ldots, k \). Then their capacity regions satisfy \( \mathcal{C}(\beta) \supseteq \mathcal{C}(\beta') \).

Proof: Let, for any \( 0 < \epsilon \leq 1 \),
\[
\beta'' = \beta' + \frac{\beta - \beta'}{\epsilon}.
\] (27)

Let \( \mathbf{R}' \) and \( \mathbf{R}'' \) be achievable rate vectors at costs \( \beta' \) and \( \beta'' \), resp. By time sharing \([4, \text{Sec. 15.3.3}],[47, \text{Sec. 4.4}]\), the rate
\[
(1 - \epsilon)\mathbf{R}' + \epsilon\mathbf{R}'' \geq (1 - \epsilon)\mathbf{R}'
\] (28)
is achievable at cost
\[
(1 - \epsilon)\beta' + \epsilon\beta'' = \beta.
\] (29)

The capacity region \( \mathcal{C}(\beta) \) thus includes all rate vectors of the form \( (1 - \epsilon)\mathbf{R}' \), where \( \mathbf{R}' \) is achievable at cost \( \beta' \) and \( \epsilon \) is an arbitrarily small positive number. Since the capacity region by definition is the closure of all achievable rate vectors \([47, \text{Sec. 4.1, 6.1}]\), \( \mathcal{C}(\beta) \) also includes \( \lim_{\epsilon \to 0} (1 - \epsilon)\mathbf{R}' = \mathbf{R}' \). In conclusion, \( \mathbf{R}' \in \mathcal{C}(\beta) \) for all \( \mathbf{R}' \in \mathcal{C}(\beta') \), which implies \( \mathcal{C}(\beta) \supseteq \mathcal{C}(\beta') \). \( \square \)

The capacity region is a \( k \)-dimensional object, and it varies as a function of the \( k \)-dimensional vector \( \beta \). The following two corollaries exemplify how linear combinations of the achievable rates change when the cost is varied linearly.

Corollary 6: If the cost is varied along a line as
\[
\beta = \beta_0 + \mu\Delta,
\] (30)
where all components of \( \beta_0 \) and \( \Delta \) are nonnegative, then all achievable rates \( R_1, \ldots, R_k \) are nondecreasing functions of \( \mu \geq 0 \), and the achievable sum rate \( R_1 + \cdots + R_k \) is also a nondecreasing function of \( \mu \geq 0 \).

Corollary 7: If all transmitters obey the same cost constraint \( \beta_1 = \cdots = \beta_k = \beta \), then all achievable rates \( R_1, \ldots, R_k \) are nondecreasing functions of \( \beta \).

V. Numerical Example

In this section, examples are given for mutual information and channel capacity as functions of the transmit power, for a simple nonlinear channel. The studied channel is chosen for its simplicity, not for its resemblance to any particular physical system, because evaluating the channel capacity is numerically possible only for very low-dimensional, memoryless channels, which unfortunately excludes more realistic channel models.

A. A Nonlinear Channel

We consider a very simple channel with nonlinear distortion and additive noise, represented as
\[
Y = a(X) + Z,
\] (31)
where \( X \) and \( Y \) are the real, scalar input and output of the channel, resp., \( a(\cdot) \) is a given deterministic function and \( Z \) is white Gaussian noise with zero mean and variance \( \sigma_Z^2 \). For a given channel input \( x \), the channel output is represented by the conditional probability density function (pdf)
\[
f_{Y|X}(y|x) = \frac{1}{\sigma_Z} f_G \left( \frac{y - a(x)}{\sigma_Z} \right),
\] (32)
where \( f_G(x) = (1/\sqrt{2\pi}) \exp(-x^2/2) \) is the zero-mean, unit-variance Gaussian pdf. Since \( f_{Y|X}(y|x) \) is Gaussian for any \( x \), the conditional entropy is \([1, \text{Sec. 20}],[47, \text{Sec. 8.1}]\)
\[
h(Y|X) = \frac{1}{2} \log_2 2\pi e \sigma_Z^2.
\] (33)

For a given input distribution \( f_X \), the output distribution \( f_Y \) is obtained by marginalizing the joint distribution \( f_{X,Y}(x,y) = f_X(x)f_{Y|X}(y|x) \), and the mutual information is calculated as
\[I(X;Y) = h(Y) - h(Y|X).\]

In this example, we select \( a(x) \) in (31) as a smooth clipping function
\[
a(x) = a_{\max} \tanh \left( \frac{x}{a_{\max}} \right),
\] (34)
where \( a_{\max} > 0 \) sets an upper bound on the output. If the instantaneous channel input \( X \) has a sufficiently high magnitude compared with \( a_{\max} \), the channel is essentially binary. For \( X \) close to zero, on the other hand, the channel approaches a linear AWGN channel.

The channel parameters are \( a_{\max} = 10 \) and \( \sigma_Z = 1 \) throughout this section. The function \( a(x) \) in (34), which represents the nonlinear part of the channel (31), is shown in Fig. 2.

B. Mutual Information

The mutual information \( I(X;Y) \) is evaluated by numerical integration, as a function of the average transmit power \( p = \mathbb{E}[X^2] \). No optimization over input distributions is carried out. The input distribution \( f_X(x) \) is constructed from a given unit-power distribution \( g(x) \), rescaled to the desired power \( p \) as \( f_X(x) = \alpha g(\alpha x) \), where \( \alpha = 1/\sqrt{p} \). The results are presented in Fig. 3 for three continuous input pdfs \( f_X(x) \): zero-mean Gaussian, zero-mean uniform, and single-sided exponential, defined as, respectively,
\[
f_{X_1}(x) = \frac{1}{\sqrt{p}} f_G \left( \frac{x}{\sqrt{p}} \right).
\] (35)
\[
f_{X_2}(x) = \begin{cases} 
\frac{1}{2\sqrt{3p}}, & -\sqrt{3p} \leq x \leq \sqrt{3p}, \\
0, & \text{elsewhere},
\end{cases}
\] (36)
\[
f_{X_3}(x) = \begin{cases} 
\sqrt{\frac{2}{p}} e^{-x\sqrt{2/p}}, & x \geq 0, \\
0, & x < 0.
\end{cases}
\] (37)

At asymptotically low power \( p \), the channel is effectively an AWGN channel. In this case, the mutual information is governed by the mean value of the input distribution, according to
even if the probability mass at $m$ is achieved by zero-mean distributions. The discrete distributions have uniform probabilities and equal spacing. The AWGN channel capacity is included for reference (dotted).

Fig. 3. Mutual information for the nonlinear channel in (31) with $\sigma_{Z} = 1$ and $\sigma_{Z} = 1$ with various continuous (solid) and discrete (dashed) input distributions. The discrete distributions have uniform probabilities and equal spacing. The AWGN channel capacity is included for reference (dotted).

All zero-mean input distributions achieve approximately the same mutual information, which approaches the AWGN channel capacity. The asymptotic mutual information for the exponential distribution, whose mean is $\sqrt{p}/2$, is half that achieved by zero-mean distributions.

The mutual information curves for all three input pdfs reach a peak around $p = 100$, when a large portion of the input samples still fall in the linear regime of the channel. When the transmit power $p$ is further increased, the mutual information decreases towards a value slightly less than 1 asymptotically for the zero-mean input pdfs and 0 for the exponential input. The asymptotes are explained by the fact that at high enough power, almost all input samples fall in the nonlinear regime, where the channel behaves as a 1-bit noisy quantizer.

Similar results for various discrete input distributions are also included in Fig. 3. The studied one-dimensional constellations are on-off keying (OOK), binary phase-shift keying (BPSK), and $m$-ary pulse amplitude modulation ($m$-PAM). The constellation points are equally spaced and the input samples $X$ are chosen uniformly from these constellations. The mutual information for $m$-PAM constellations with $m \geq 4$ exhibits the same kind of peak as the continuous distributions in Fig. 3; indeed, a uniform distribution over equally spaced $m$-PAM approaches the continuous uniform distribution as $m \to \infty$.

Similarly to the continuous case, the mutual information for zero-mean discrete input distributions approach the AWGN channel capacity as $p \to 0$. Half this channel capacity is achieved by the OOK input, which has the same mean value $\sqrt{p}/2$ as the exponential input above. The asymptotics when $p \to \infty$ depends on whether $m$ is even or odd. For any even $m$, the channel again acts like a 1-bit quantizer and the asymptotic mutual information is slightly less than 1. For odd $m$, however, here exemplified by 3-PAM, there is a nonzero probability mass at $X = 0$, which means that the possible outputs are not only $Y = \pm a_{\text{max}} + Z$ but also $Y = 0 + Z$. Hence, the channel asymptotically approaches a ternary-output noisy channel, whose mutual information is upperbounded by $\log_{2} 3 = 1.58$.

To summarize, this particular channel has the property that the mutual information for any input distribution approaches a limit as $p \to \infty$, and this limit is upperbounded by $\log_{2} 3$. It might seem tempting to conclude that the channel capacity, which is the supremum of all mutual information curves, would behave similarly. However, as we shall see in the next section, this conclusion is not correct, because the limit of a supremum is in general not equal to the supremum of a limit. Specifically, the asymptotical channel capacity is $\lim_{p \to \infty} C(p) = \lim_{p \to \infty} \sup_{\sigma_{Z}} I(X;Y)$, which is not equal to $\sup_{\sigma_{Z}} \lim_{p \to \infty} I(X;Y) \leq \log_{2} 3$.

C. Channel Capacity

The standard method to calculate the channel capacity of a discrete memoryless channel is by the Arimoto–Blahut algorithm [53], [54], [4, Sec. 10.8], [55, Ch. 9]. It has been extended to continuous-input, continuous-output channels in [56] and furthermore to cost-constrained inputs in [57]. The idea in [57] is to represent distributions by lists of samples, so-called particles. A particle-based input distribution has the form

$$f_X(x) = \sum_{i=1}^{s} w_i \delta(x - c_i), \quad (38)$$

where $\delta(\cdot)$ is the Dirac delta function, $s$ is the number of particles, $c = (c_1, \ldots, c_s)$ are the particles, and $w = (w_1, \ldots, w_s)$ are the probabilities, or weights, associated with each particle. If $s$ is large enough, any distribution can be represented in the form (38) with arbitrarily small error. With this representation, the yield $h(Y)$, and thereby $I(X;Y)$, by numerical integration.

Since $h(Y|X)$ is constant, the capacity is obtained by maximizing $h(Y)$ subject to constraints on the total probability and power. This problem is in general nonconvex. In [57], the optimization is done by alternating optimization [55, Sec. 9.1], first finding $w$ for a given $c$ using the Arimoto–Blahut algorithm and then finding $c$ for a given $w$ using a gradient search, and so on. Here, we apply gradient search techniques for both steps. The objective is to maximize the Lagrangian function

$$L(c, w, \lambda_1, \lambda_2) \equiv h(Y) + \lambda_1 \left( \sum_{i=1}^{s} w_i - 1 \right) + \lambda_2 \left( \sum_{i=1}^{s} w_i c_i^2 - p \right), \quad (40)$$

where the Lagrange multipliers $\lambda_1$ and $\lambda_2$ are determined to maintain the constraints $\sum w_i = 1$ and $\sum w_i c_i^2 = p$ during the optimization process. The gradients of $L$ with respect to $c$ and $w$ are calculated, and a steepest descent algorithm (or more accurately, “steepest ascent”) is applied to maximize $L$. 


In each iteration, a step is taken in the direction of either of the two gradients. The step size is determined using the golden section method. Several initial values \( c, w \) were tried, and \( s \) was increased until convergence. In all cases, \( s = 16 \) turned out to be a sufficient number of particles. The topography of \( L \) as a function of \( c \) and \( w \) turned out to include vast flat fields, where a small step has little influence on \( L \). This made the optimization numerically challenging. No suboptimal local maxima were found for the studied channel and constraints, although for nonlinear channels in general, the mutual information as a function of the input distribution may have multiple maxima.

This channel capacity, numerically obtained by the above method, is shown in Fig. 4 for the studied channel (thick solid curve). As promised by the law of monotonic channel capacity (Theorem 1), the curve differs from most mutual information curves by not having a peak at any \( p \). The channel capacity follows the mutual information of the Gaussian distribution closely until around \( p = 100 \). However, while the Gaussian case attains its maximum mutual information \( I(X; Y) = 2.44 \) bits/symbol at \( p = 130 \) and then begins to decrease, the channel capacity continues to increase towards its asymptote \( \lim_{p \to \infty} C(p) = 2.54 \) bits/symbol. The fact that the capacity curve rises somewhat over the peak and not only flattens out is encouraging for future work on capacity-achieving coding for more realistic nonlinear channel.

This asymptotical channel capacity can be explained as follows. Define the random variable \( A \triangleq a(X) \). Since \( a(\cdot) \) is a continuous, strictly increasing function, there is a one-to-one mapping between \( X \in (-\infty, \infty) \) and \( A \in (-a_{\text{max}}, a_{\text{max}}) \). Thus \( I(X; Y) = I(A; Y) \), where \( Y = A + Z \). This represents a standard discrete-time AWGN channel whose input \( A \) is subject to a peak power constraint. The capacity of a peak-power-constrained AWGN channel was bounded already in [1] Sec. 25] and computed numerically in [44], where it was also shown that the capacity-achieving distribution is discrete. The asymptote in Fig. 4 which is 2.54 bits/symbol or, equivalently, 1.76 nats/symbol, agrees perfectly with the amplitude-constrained capacity in [44] Fig. 2 for \( a_{\text{max}}/\sigma_Z = 10 \).

Some almost capacity-achieving input distributions are shown in Fig. 5 numerically optimized as described above. For \( p = 10 \), the optimized discrete input distribution is essentially a nonuniformly sampled Gaussian pdf, and the obtained channel capacity, 1.61, has the same value as the mutual information of a continuous Gaussian pdf, shown in Fig. 3. For \( p = 100 \) and 1000, the distribution is more uniform in the range where the channel behaves more or less linearly, which for this channel is approximately at \( -a_{\text{max}}/2 < x < a_{\text{max}}/2 \), with some high-power outliers in the nonlinear range \( |x| > a_{\text{max}} \). In all cases, increasing the number of particles \( s \) from what is shown in Fig. 5 does not increase the mutual information significantly, from which we infer that these discrete input distributions perform practically as well as the best discrete or continuous input distributions for this channel.

Although the capacity-achieving distributions would look quite different for other types of nonlinear channels, a general observation can be made from Fig. 5. Even at high average power, the input should consist of samples with moderate power, for which the channel is good, most of the time. The high average power is achieved by a single particle having a very large power; thus, the capacity-achieving distribution is a satellite distribution [14]. This single particle, or satellite, corresponds to \( X'' \) and \( X'' \) in the proofs of Theorems 1 and 2, resp., which as \( \epsilon \to 0 \) have high cost (power) and low probability.

VI. SUMMARY AND CONCLUSIONS

It was proved that the channel capacity is a nondecreasing function of a cost (such as transmit power) in the following cases.

- Point-to-point memoryless vector channels \( f_{Y|X} \) that do not change with the input distribution \( f_X \).
- Interference channels where all users, except the one of interest, transmit data from fixed input distributions.
- Interference channels where all users transmit data from the same (optimized) distribution.
- Interference channels where the distributions of all users are optimized jointly.

The mutual information may be decreasing with cost in all these cases, but not the channel capacity in Shannon’s sense. In contrast, there are numerous examples in the literature where the channel capacity has a peak at a certain cost, after which it decreases towards zero [44–48, 20–38]. These examples all pertain to one of the following cases:

- Point-to-point channels that change depending on the transmitter settings, typically as a function of the transmit power [59].
Interference channels where the transmission scheme of one user (the one of interest) is optimized while the other users satisfy the same power constraint by pure amplification \[21\].

A practical interpretation is that when designing codes for nonlinear channels under the constraint of a maximum average power, it suffices to consider codes in which all codewords satisfy the power constraint with equality. This is in contrast to previous works in optical communications, which often assumed the existence of an optimal (finite) power. Further research is needed to show whether the new approach is just a way to achieve the same rates as before at a higher power, or if it may lead to significantly increased achievable rates.

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