Experience in DNS of turbulent free convection in air along an isothermal vertical plate

E M Smirnov¹, A M Levchenya¹, N G Ivanov¹, A A Smirnovsky¹ and P E Smirnov²

¹Peter the Great St.-Petersburg Polytechnic University, St.-Petersburg, Russia
²ANSYS Germany GmbH, Otterfing, Germany

E-mail: smirnov_em@spbstu.ru

Abstract. The paper presents results of direct numerical simulation of the turbulent free convection boundary layer developing along a vertical heated plate up to the local Grashof number, $Gr$, of $8 \times 10^{10}$ (Pr=0.7). A span-distributed macro-roughness of a selected form is used for enhancing transition to turbulence, so that the $Gr$–value corresponding to the end point of transition would be close to that reported for the case of natural transition. The results obtained for the first- and the second-order turbulent flow and heat transfer statistics in the developed turbulent regime are in a good accordance with the experimental data reported by Tsuji and Nagano (1988).

1. Introduction

The turbulent free convection boundary layer (FCBL) in air along a vertical heated plate was a subject of numerous experimental, analytical and computational studies. Nevertheless, this case still attracts a great attention of researchers. To a large extent, it is caused by many specific difficulties and uncertainties arising at measurements of these kind of low-speed non-isothermal flows [1], as well as by known limitations of the RANS models as applied for prediction of buoyancy-controlled flows. Problems of heat transfer augmentation in the turbulent FCBL represent another area of great interest.

For the last decade, several studies of the transitional and turbulent vertical-plate FCBL have been carried out based on the DNS method [2-5] and with the LES approach [6-7]. Results of time-developing DNS was reported in [2,3]. For space-developing DNS or LES, different approaches were used for enhancing transition to turbulence. In [4], random perturbations were introduced into the upstream boundary layer to model a “natural transition”. A span-distributed macro-roughness for triggering transition was used in [5]. The LES study presented in [6] was performed with no special means, and the transition position was “controlled” by the domain size and the grid. In the most recent LES [7], velocity oscillations of natural frequency were introduced on the wall to mimic the initial stage of transition process. The present DNS study relies on the triggering transition method suggested in [5] and is focused on the refined prediction of the first- and the second-order turbulent flow statistics.
2. Mathematical model and numerical aspects

2.1. Description of the case

It is assumed that the free convection boundary layer of air (Pr=0.7) is developing along an isothermally heated vertical flat plate (plane ABCD in Figure 1a). Macro-roughness in the form of a periodic row of rectangular obstacles triggering the laminar-to-turbulent transition is installed on the plate surface at a stream-wise position (x=x₀) that falls into the unstable laminar flow region.

The thermal boundary conditions correspond to those adopted in the experimental study of air convection by Tsuji and Nagano [1]: the plate surface temperature, Tₑ, is set to 60°C, and the ambient fluid temperature, Tₐ, is 16°C. The local Grashof number is defined traditionally, as Grₓ= gβΔTₓ/ν², where ΔT=Tₑ-Tₐ, g is the gravity acceleration, ν is the fluid kinematic viscosity.

The vertical size of the plate is set to L=AB=DC=2.5 m. The maximum Grashof number, evaluated for x=L, is 8.2×10¹⁰. The computational domain span is 0.48 m, and the periodicity boundary conditions are prescribed on planes ABFE and DCGH. Segment AD corresponds to the upstream edge of the plate (x=0). The slip adiabatic wall condition is imposed on the lower plane (plane ADHE in Figure 1a). The boundary counterpart to the heated plate (plane EFGH) is placed at a distance of 0.40 m. At this boundary, being in fact the inlet one, zero value of the reduced total pressure is prescribed, with normal-to-boundary inflow, and the fluid temperature is set to Tₑ. The major part of the upper boundary (BCGF) is considered as an outlet, where zero value of the reduced static pressure is prescribed (more details are given in Section 2.2). The reference pressure is set as 101325 Pa.

Eight rectangular adiabatic obstacles, each of lₓ×lᵧ×lₜ=8×8×30 mm in size, are positioned symmetrically with respect to the confining planes ABFE and DCGH. The span-wise distance between the obstacles is 30 mm. The upstream faces of the obstacles are placed at x=x₀=0.496 m. In the simulated case, Grₓ,0=Grₓ(x=x₀)=6.0×10⁸ that is about two times lower than the experimental values reported in the literature for the start of the natural laminar-to-turbulent transition. According to the known self-similar solution for the laminar free convection layer, the whole thickness, δₓ,0 of the layer evaluated at x=x₀ is about 20 mm that exceeds two and a half times the obstacle height lₓ=8 mm.

The simulation is performed based on the 3D incompressible-fluid Navier–Stokes equations written with the Boussinesq approximation. Physical properties of air are evaluated at the mean temperature Tₓ=(Tₑ+Tₐ)/2, except that the thermal expansion coefficient β is evaluated at T=Tₑ.

![Figure 1. (a) Computational domain scheme and coordinate system, (b) an instant pattern of Q-criterion iso-surfaces colored with velocity values, (c, d) maps of the time-averaged shear stress and heat flux on the plate surface.](image-url)
2.2. Numerical aspects

The computational grid used for simulation consisted of about 105 million of hexahedral cells. The grid points were clustered near the plate. Downstream of the obstacles, the averaged value of the normalized distance from the center of the first computational cell to the wall, $Y^+$, was about 0.3. Normalized cell sizes in the $x$- and $z$- directions were $\Delta x^+ \approx \Delta z^+ \approx 6$.

Calculations were performed with ANSYS Fluent in version 17.0 using the NITA solver (Non-Iterative Time-Advancing) with the Fractional-Step option. The second-order central scheme was applied for evaluation of convective fluxes, both for the momentum and energy equations. The time advancing was carried out with a step of 0.002 seconds. Samples of duration of about 60 seconds were calculated for getting statistics after a transient period. As for the upper boundary (plane BCGF in Figure 1a), a special technique was applied that provided automatic prescribing of the slip-wall condition on those parts of the boundary, where a trend to inflow could occur (far away from the plate).

3. Results

Figure 1b presents a plot of the Q-criterion iso-surfaces (coloured with the stream-wise velocity). It gives evidence that a large portion of the plate is covered by the turbulent boundary layer. The maps of the time-averaged shear stress, $\tau_w$, and the heat transfer rate, $q_w$, on the plate surface (Figure 1c,d) show that traces of the disturbing action of the introduced macro-roughness almost disappear to the second half of the plate height. Consequently, it allows one to assume that here the turbulent free-convection boundary layer is approaching its natural state.

Figure 2a shows the skin friction coefficient versus $Gr_x$. Hereinafter, the computed data presented were obtained by applying additionally a procedure of span-averaging. The skin friction coefficient is defined as $C_f = 2 \tau_w/\rho v_b^2$, where $\rho$ is the fluid density and $v_b$ is the buoyancy velocity defined as $v_b = (g\beta \Delta T \nu)^{1/3}$. At $Gr_x > 10^{10}$, the computed $C_f$-values are in a good agreement with the correlation established in [1] for the turbulent flow regime. Comparison of the computed local Nusselt numbers with the measurement data [1] is given in Figure 2b. Here one can see that the predicted position of a rapid increase in $Nu_x$ from the “laminar” to the “turbulent” values is slightly shifted to higher values of $Gr_x$, as compared with the experiments [1], where the environment was responsible for the transition to turbulence. After the end point of the predicted transition interval, the computed $Nu_x$-values are in a good agreement with the corresponding experimental correlation.

Figure 3 shows a comparison of the computed profiles of the mean vertical velocity and temperature with experimental data. Among the data given in [1] (available also in the ERCOFTAC Classic Database), the case of $Gr_x = 3.62 \times 10^{10}$ was chosen. This case corresponds to the measurement position of $x=1.9$ m. The computed profiles were extracted from the DNS solution at the same
position. The traditional wall scales, \( u^* = \sqrt{\tau_w/\rho} \) and \( t^* = q_w/(\rho C_p u^*) \), are used for normalising the variables, so that \( y^* = y u^*/\nu \), \( U^* = U/u^* \), and \( T^* = (T_w - T)/t^* \). Generally, a very good agreement between the computed and measurement results is observed.

Normalized profiles of the RMS values of the velocity and temperature fluctuations (denoted as \( u, v, w, t \)) and the correlations of the velocity and temperature fluctuations are shown in Figure 4. Unfortunately, in case of \( Gr_x = 3.62 \times 10^{10} \), the measurement data set does not include some profiles (a complete set is given in [8] for \( Gr_x = 8.99 \times 10^{10} \), but this \( Gr_x \)-value is beyond the present DNS). When comparing the available profiles for the RMS of the stream-wise velocity and temperature fluctuations (Figure 4a, b), one can conclude that the DNS predicts somewhat higher intensity of fluctuations, but distinctions are far to be dramatic. The DNS predicts a non-monotonic behaviour of the correlation \( \langle ut \rangle \) in the near wall region (Figure 4c). For the measurements, this feature was detected only at considerably higher values of the Grashof number [1,8].

![Figure 3](image1.png)

**Figure 3.** Comparison of the computed profiles of the mean (a) velocity and (b) temperature with experimental data for the free convection boundary layer at \( Gr_x = 3.62 \times 10^{10} \).

![Figure 4](image2.png)

**Figure 4.** Turbulence characteristics in the free convection boundary layer at \( Gr_x = 3.62 \times 10^{10} \).
Conclusions
A valuable experience has been gained from DNS of the spatially developing vertical-plate turbulent free convection boundary layer with triggering of transition by a macro-roughness. The results obtained for the developed turbulent regime are generally in a good accordance with the recognized experimental data [1]. The computational model developed can be used for performing high fidelity studies of heat transfer augmentation in the transitional and turbulent free convection boundary layers.

Acknowledgments
The study is supported by the Russian Science Foundation under grant no. 18-19-00082.

References
[1] Tsuji T and Nagano Y 1988 *Int. J. Heat Mass Transfer* 31 1723–34
[2] Abedin M Z, Tsuji T and Hattori Y 2009 *Int. J. Heat Mass Transfer* 52 4525
[3] Abramov A G, Smirnov E M and Goryachev V D 2014 *Fluid Dynamics Research* 46 041408
[4] Zhao Y, Lei C, Patterson J C 2016 *Int. Communication Heat Mass Transfer* 76 366–75
[5] Smirnov E M, Abramov A G, Smirnovsky A A and Smirnov P E 2018 *Journal of Physics: Conference Series* 1128 012090
[6] Nakao K, Hattori Y and Suto H. 2017 *International Journal of Heat and Fluid Flow* 63 128–38
[7] Ortiz A V and Koloszar L 2020 *Computers and Fluids* 199 104417
[8] Tsuji T and Nagano Y 1988 *Int. J. Heat Mass Transfer* 31 2101–11