Superconductivity Induced by the Proximity Effect of Singlet Resonating Valence Bond Order in High-$T_c$ Superconductors

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Effects of inhomogeneous doping on the high-$T_c$ cuprate superconductors are studied within the framework of the $t$-$J$ model. Especially, the boundary between two non-superconducting regions with doping rates much higher and lower than the optimal one is examined. It is found that the critical temperature of the optimally doped sample and the critical current density can be the same order as that in the superconducting state below the critical temperature. We also point out an experimental possibility to observe this phenomenon.

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It has been recognized that there are two important ingredients for the superconductivity in high-$T_c$ cuprates, which are the spin correlation, leading to the spin gap, and the doped charge carriers with a sufficiently high density. In the slave-boson mean-field theory of the $t$-$J$ model, the former is realized as the singlet resonating valence bond (s-RVB) order of spinons and the latter the Bose condensation of holons. In this framework, it is assumed that the spinons and the holons are well-defined low-lying excitations, and the spin-charge separation is a good picture. Although there still remains some controversy on this point, it has been clarified that some physical properties of the high-$T_c$ cuprates can be well described by the $t$-$J$ model, for example, the transport properties and the magnetic properties. In this Letter, we take the $t$-$J$ model to be our starting point.

We consider the situation where the doping rate is spatially varying. We especially concentrate on the case where two non-superconducting regions, one is too overdoped and the other is too under-doped, are in contact with each other within the same CuO$_2$ plane. We expect that the s-RVB order and the holon condensate can exist simultaneously at the boundary, thus giving rise to superconductivity. Since the s-RVB order and the holon condensation can occur at temperatures higher than the critical temperature of the optimally doped sample, $T_{\text{opt}}$, this “boundary superconductivity” may also occur above $T_{\text{opt}}$.

Our idea is similar with the “spin-gap proximity effect” previously introduced by Emery, Kivelson and Zacher [1] based on a somewhat different model for high $T_c$ superconductors, discussing the stripe order and its significance for the superconductivity. Recently, Kivelson has proposed a possibility to raise critical temperature using spin-gap proximity effect [2]. He proposes making a stack of spin-gap material and hole-rich material in a layered structure. In contrast to this, we consider the situation where the spin-gap and hole-rich regions exist within a CuO$_2$ plane. This kind of situation has not been studied within the framework of the $t$-$J$ model and we consider this is of interest from both theoretical and experimental points of view.

First we introduce the $t$-$J$ model which describes the correlated electrons in each CuO$_2$ plane. The Hamiltonian is given by

\[
\mathcal{H} = -t \sum_{\langle i,j \rangle \sigma} (\hat{f}_{i\sigma} \hat{f}_{j\sigma} + h.c.) + J \sum_{\langle i,j \rangle} \hat{S}_i \cdot \hat{S}_j - \sum_i \lambda_i (\sum_\sigma \hat{f}_{i\sigma} \hat{f}_{i\sigma} + \hat{b}_{i}^\dagger \hat{b}_{i} - 1), \tag{1}
\]

where $i$ and $j$ denote lattice points in the CuO$_2$ plane, $\langle i,j \rangle$ means the nearest neighbors, $\hat{f}_{i\sigma}$ ($\sigma = \uparrow, \downarrow$) and $\hat{b}_{i}$ are the annihilation operators of the spinon and the holon, respectively, $\hat{S}_i$ denotes $\frac{1}{2} \sum_\alpha \hat{\sigma}_{i\alpha} \hat{\sigma}_{i\beta}$ with $\hat{\sigma}_{i\alpha}$ being Pauli matrices, and $\lambda_i$ is the Lagrange multiplier for the constraint, $\sum_\sigma \hat{f}_{i\sigma} \hat{f}_{i\sigma} + \hat{b}_{i}^\dagger \hat{b}_{i} = 1$. In this Letter the lattice spacing is taken to be unity. In addition to Eq. (1), we later introduce the coupling to the electromagnetic field.

The Eq. (1) is treated by the so-called slave-boson mean-field theory. The local constraint represented by $\lambda_i$ is replaced by the global one, which is described by the constant chemical potential for spinons $\mu_F$ and that for holons $\mu_B$. We define the statistical averages, $\langle i,j \rangle$ if $\langle i,j \rangle$ means the nearest neighbors, $\hat{f}_{i\sigma}$ ($\sigma = \uparrow, \downarrow$) and $\hat{b}_{i}$ are the annihilation operators of the spinon and the holon, respectively, $\hat{S}_i$ denotes $\frac{1}{2} \sum_\alpha \hat{\sigma}_{i\alpha} \hat{\sigma}_{i\beta}$ with $\hat{\sigma}_{i\alpha}$ being Pauli matrices, and $\lambda_i$ is the Lagrange multiplier for the constraint, $\sum_\sigma \hat{f}_{i\sigma} \hat{f}_{i\sigma} + \hat{b}_{i}^\dagger \hat{b}_{i} = 1$. In this Letter the lattice spacing is taken to be unity. In addition to Eq. (1), we later introduce the coupling to the electromagnetic field.

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and \( \eta \) to be constant. Including also the scalar and vector potential, \( \varphi \) and \( \vec{A} \), and the fictitious gauge fields, \( a_0 \) and \( \vec{a} \), which represent the fluctuation around the mean-field solution \([3]\), we obtain the following Lagrangian,

\[
\mathcal{L} = \sum_{i\sigma} f_{i\sigma}^\dagger \left\{ \hbar \partial_x - \mu_F + i a_0(\vec{r}_i) \right\} f_{i\sigma} \\
+ \sum_i b_i^\dagger \left\{ \hbar \partial_x - \mu_B + i (e \varphi(\vec{r}_i) + a_0(\vec{r}_i)) \right\} b_i \\
+ i \varphi(\vec{r}_i) \rho(\vec{r}_i) \\
- \left( \tau \eta + \frac{3}{2} J \xi \right) \sum_{ij\sigma} \left\{ e^{i\theta_{ji}} f_{i\sigma}^\dagger f_{j\sigma} + \text{h.c.} \right\} \\
- \xi \sum_{ij} \left\{ e^{i\theta_{ji}} b_i^\dagger b_j + \text{h.c.} \right\} \\
- \sum_i \left[ \Delta^2 \chi_{ij} + \Delta^2 \chi_{ji} + \Delta \chi_{ij} \chi_{ji} + \text{h.c.} \right] - \frac{8}{3J} \left( |\Delta^2 \chi_{ij}|^2 + |\Delta \chi_{ij}|^2 \right),
\]

where \( \chi_{ij} = f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \) and

\[
\theta_{ji}^\beta = \frac{1}{2} \int_{\vec{r}_i} \vec{a} \cdot d\vec{r}, \quad \theta_{ji}^\gamma = \frac{1}{2} \int_{\vec{r}_j} \left( \vec{a} + \frac{e}{c} \vec{A} \right) \cdot d\vec{r}.
\]

The chemical potentials \( \mu_F \) and \( \mu_B \) are determined from

\[
\langle f_{i\sigma}^\dagger f_{i\sigma} \rangle = 1 - \tilde{\delta}, \quad \langle b_i^\dagger b_i \rangle = \tilde{\delta},
\]

where \( \tilde{\delta} \) is the average holon density in the whole system. The background charge density, originating from doping, is denoted by \( \rho(\vec{r}_i) \), whose spatial average equals \( -e\tilde{\delta} \). In this Letter \( a_0 \) and \( \vec{a} \) are treated perturbatively on the same line with Refs. \([3] \) and \([4] \).

First we study how the distribution of the holons, namely \( \delta(\vec{r}_i) \), is determined under an inhomogeneous doping, which is described by \( \rho(\vec{r}_i) \). For this purpose, we neglect the terms including \( \Delta_{ij} \) and \( \vec{a} \) and \( \vec{A} \). Then the spinon and the holon degrees of freedom can be integrated out, giving rise to terms, \( \pi_F a_0^2 + \pi_B (a_0 + e \varphi)^2 \), where \( \pi_F \) and \( \pi_B \), respectively, are the polarization functions of spinons and holons. Now the magnitudes of \( \pi_F \) and \( \pi_B \) determine which of the spinon and the holon couples to \( \varphi \). In this Letter we assume \( \pi_F \gg \pi_B \), which may be valid in lower doping region. Then \( a_0 \) is almost fixed to zero and \( \varphi \) couples to the holons. Now \( \delta \) and \( \varphi \) are determined from \( \rho \) in a self-consistent way.

We especially consider the following situation. The CuO\(_2\) planes, described by the above Lagrangian, are laid parallel to \( x-y \) plane and stacked in \( z \)-direction. The region \( x < 0 \) (\( x > 0 \)) is filled with a s-RVB ordered (hole-rich) material with a low (high) enough doping rate. Both of the regions are non-superconducting. The system is uniform in \( y \)- and \( z \)-direction. We assume that the doping rate, which is described by \( \rho(\vec{r}_i) \), changes at \( x = 0 \) abruptly. In order to preserve the charge neutrality in the bulk, namely at \( x \to \pm \infty \), the statistical average \( \langle \varphi(\vec{r}) \rangle \) must take constant imaginary values, which differ between \( x \to \infty \) and \( x \to -\infty \). (Note that the chemical potential \( \mu_B \) is a constant in the whole system.) Near the boundary \( \langle \varphi(\vec{r}) \rangle \) changes smoothly connecting these two bulk limits. As in the case of semiconductor junctions \([5] \), the smooth connection is enabled by the polarization charge appearing at the boundary. The depth of the charged region is given by the screening length of the holons. This length is extremely long (short) for \( x < 0 \) (\( x > 0 \)) and there still remains a sharp drop of the holon density at the boundary. Therefore, in this Letter, we approximate \( \delta \) by a step function (see Fig. \([1] \)).

Next we study the s-RVB order by introducing the Ginzburg-Landau (GL) free energy of the order parameter \( \Delta_d = \frac{1}{2} (\Delta_x - \Delta_y) \). Here we assume that the s-RVB order has \( d \)-wave symmetry. The GL free energy is obtained by the perturbative expansion of the free energy with respect to \( \Delta_d \) \([6] \). We introduce the half band width \( D = 4(\tau \eta + 3J \xi/8) \) and the strength of attractive interaction \( V = 3J/8 \). The dispersion of spinons, \( (D/2)(\cos k_x + \cos k_y) \), is approximated by a parabolic one, \( D(-1 + |k|^2/4) \), and then the density of states at Fermi energy becomes \( N(0) = 1/(\pi D) \). At a temperature \( T < D \), the solution for \( \sum_{ij} f_{i\sigma}^\dagger f_{i\sigma} = 1 - \delta \) is approximately given by \( \mu_F = -\delta/N(0) \) and the critical temperature \( T_d \) becomes \( T_d \approx T^* \exp[-1/(2\pi N(0)(1 - \delta)^2)] \) with \( T^* = 2D e^{\gamma}/\pi \) where \( \gamma \) is the Euler’s constant. This expression is valid in the “weak coupling regime”, \( V N(0) \ll 1 \). The GL free energy density of \( \Delta_d \) is given by

\[
F_{\text{s-RVB}} = F_0 + \alpha_d |\Delta_d|^2 + \beta |\Delta_d|^4 + \gamma_d |\bar{\Pi}| |\Delta_d|^2,
\]

where \( \bar{\Pi} = -i \nabla + (2/h) \vec{a} + (2\pi/\phi_0) \vec{A} \) with \( \phi_0 = hc/(2e) \). The GL coefficients are given by

\[
\alpha_d = \frac{2N(0)(1 - \delta)^2}{T_d} \equiv \frac{c_1}{T_d},
\]

\[
\beta = \frac{21(3)}{2\pi^2 T^2} \equiv \frac{c_2}{T^2},
\]

\[
\gamma_d = \frac{7(3)}{16\pi^2 T^2} \equiv \frac{c_3}{T^2}.
\]
where $\zeta(x)$ is the zeta function and $c_1$ and $c_2$ are constants. The doping dependence of the GL coefficients comes from $T_d$, $\alpha_d$ and $\beta$. Since the largest dependence appears from $T_d$, we neglect the $(1-\delta)^n$-factors in $\alpha_d$ and $\beta$ in the following. Then the doping rate affects $T_d$ only.

Here we consider the situation where the transition temperatures in the regions $x < 0$ and $x > 0$, denoted by $T_d$ and $T_d'$, respectively, satisfies the condition, $T_d > T > T_d'$. It is also assumed that holons are condensed in the region $x > 0$. Note that the quantity with (without) dash ($) is defined in $x > 0$ ($x < 0$). We first determine the spatial variation of $\Delta_d$, disregarding $\vec{a}$ and $\vec{A}$. Here $\Delta_d$ is assumed to be real. The expectation value of $\Delta_d$ is $\sqrt{|\alpha_d|/(2\beta)} = \Delta_d$ at $x \to -\infty$ and 0 at $x \to \infty$. In $x < 0$ we introduce a new variable $\delta \Delta_d = \Delta_d - \Delta_d$. The spatial variation of $\Delta_d$ is then governed by the following equations,

$$2\alpha_d \delta \Delta_d(x) - g_d \partial_x^2 \delta \Delta_d(x) = 0 \quad (x < 0),$$

$$\alpha' \delta \Delta_d(x) - g_d \partial_x^2 \delta \Delta_d(x) = 0 \quad (x > 0). \tag{9}$$

We can easily see that $\Delta_d$ behaves as $\Delta_d + g_1 \exp(x/\xi_F)$ for $x < 0$ and $g_2 \exp(-x/\xi_F')$ for $x > 0$, where $\xi_F = \sqrt{\gamma_d/2|\alpha_d|}$, $\xi_F' = \sqrt{\gamma_d/\alpha_d'}$, and $g_1$ and $g_2$ are constants.

The solution in the whole region is obtained by connecting these two solutions at $x = 0$ with boundary condition,

$$\partial_x \Delta_d(-0) = \partial_x \Delta_d(+0), \quad \Delta_d(-0) = \Delta_d(+0), \tag{10}$$

(see Fig. 1). This condition is in contrast with that for the proximity effect in ordinary superconductor-normal metal junction, where the change of the BCS interaction causes a discontinuity of the gap function $\Delta_d$. (Note that the quantity which is continuous at the boundary is not the gap function but the anomalous amplitude.)

The constants $g_1$ and $g_2$ are obtained as

$$g_1 = \frac{\xi_F}{\xi_F + \xi_F'} \Delta_d, \quad g_2 = \frac{\xi_F'}{\xi_F + \xi_F'} \Delta_d. \tag{11}$$

Note that non-zero $\Delta_d$ in $x > 0$ is due to the proximity effect of s-RVB order and $\xi_F'$ gives the proximity length. The temperature dependence of the coherence lengths at $T_d' \ll T < T_d$ are as follows:

$$\xi_F \simeq \sqrt{\frac{c_2}{2} D T_d \frac{1}{\sqrt{1 - T/T_d}}}, \quad \xi_F' = \frac{\sqrt{c_2}}{\sqrt{\ln T/T_d}} \frac{D}{T} \tag{12}$$

It is interesting to see that $\xi_F'$ is, except for the logarithmic factor, analogous to the ordinary proximity length in the clean limit $\propto \hbar v_F/T$ where $v_F$ is the Fermi velocity. If $T$ is close to $T_d$, $\xi_F$ becomes much larger than $\xi_F'$ and the amplitude of $\Delta_d$ in the proximity region, i.e., $g_2$, becomes much smaller than $\Delta_d$.

However if $T$ is not too close to $T_d$, $\xi_F$ and $\xi_F'$ can be the same order and $g_2$ becomes comparable to $\Delta_d$. Noting that the holons are already condensed in the proximity region, this result means the induced superconductivity in the boundary area.

The strength of the induced superconductivity becomes more apparent by estimating the supercurrent which can flow parallel to the boundary. In this region the gauge field $\vec{a}$ is massive because of the holon condensation, and $\vec{a}$ in Eq. (3) can be set to zero. Then the response to the actual vector potential $\vec{A}(\vec{r})$ is dominated by the spinons. Here we take the vector potential to be constant and parallel to $y$-direction. The total current in $y$-direction is given by

$$J = 2c\gamma d A_y \int_0^\infty \Delta_d^2(x) \, dx$$

$$= c\gamma d A_y \left\{ \frac{\xi_F(\Delta_d)}{\xi_F'(\Delta_d) + \xi_F^2} \right\} \Delta_d^2, \tag{12}$$

where the $A_y$-dependent proximity length $\xi_F'(\Delta_d) = \xi_F'/\sqrt{1 + p' A_y^2}$ decreases with increasing $A_y$, with $p'$ being $(2\pi \xi_F^2/\phi_0)^2$. The maximum of $J$ as a function of $A_y$ is given approximately by

$$J_{\text{max}} = 2c \left( \frac{2\pi}{\phi_0} \right)^2 \gamma_d \frac{1}{3\sqrt{3p'}} \frac{\xi_F^3}{\xi_F^2} \Delta_d^2, \tag{13}$$

For clarity, here we introduce the critical current density $j_c$ in the bulk superconducting phase. This quantity (in overdoped region) is calculated from Eq. (12) by assuming that holons are condensed and $\vec{a}$ is massive in the whole region. We obtain

$$j_c = 2c \left( \frac{2\pi}{\phi_0} \right)^2 \gamma_d \frac{1}{3\sqrt{3p'}} \Delta_d^2, \tag{14}$$

where $p = 2(2\pi \xi_F^2/\phi_0)^2$. It is clear that $J_{\text{max}} = \sqrt{2j_c \xi_F^2}/\xi_F$. If $\xi_F' \simeq \xi_F$, the critical current density in the proximity region, defined by $J_{\text{max}}/\xi_F'$, can be the same order as $j_c$.

Finally we point out some experimental set-ups to observe the effect predicted in this Letter. The simplest way may be to utilize the field effect transistor (FET) recently developed by Shon et al. [15, 16]. We propose two types of experiments, which we call “half-gate” experiment and “side-gate” experiment. The experimental set-ups are depicted in Fig. 2. In the former case (Fig. 2(a)) the gate electrode is attached parallel to the CuO$_2$ plane, but it covers only half of it. As a result, a boundary between the s-RVB ordered region (uncovered by the gate) and the hole-rich region (covered by the gate) is formed under the edge of the gate. (Here we are assuming that the gate induces doping to the undoped compound.) However, in order to observe the effect studied in this Letter, the insulating layer separating the gate and the sample must be smaller than the s-RVB coherence length (approximately the superconducting coherence length below $T_c \approx$ several nm.), which may be hardly realized.

The other configuration is the “side-gate” as depicted in Fig. 2(b). In this case the gate is applied to the CuO$_2$ plane from the side and by applying a voltage a hole-rich layer is formed under the gate. This situation is a
FIG. 2: Experimental set-ups using field effect transistor: (a) half-gate configuration and (b) side-gate configuration. The \( \text{Cu}_2\text{O}_2 \) planes are shown in an exaggerated scale as compared to the electrodes.

little different from one considered in this Letter where the hole-rich region is infinitely wide. However, the same mechanism studied by us may also work. In this case, the thickness of the superconducting region is limited not by the s-RVB proximity length but by the thickness of the hole-rich region. This configuration can be realized more easily.

There may be other ways. One might introduce the spatial variation of doping chemically, for example, by using the epitaxial growth [20]. If this can be realized, it may fit the situation studied in this Letter best.

In either case, the width of the superconducting region is very narrow and the fluctuation is essential[21]. Therefore it is not easy to observe good superconducting behavior. However, we expect that whether the superconducting correlation exists or not is an experimentally detectable fact.

In summary, we have studied the effect of inhomogeneous doping in the high-\( T_c \) superconductors. We have shown, based on the \( t-J \) model, a possibility of superconductivity at the boundary between the s-RVB ordered and the hole-rich regions, caused by the proximity effect of the s-RVB order. We have also suggested experimental set-ups to observe this effect. Since our proposal is based on the spin-charge separation picture, these experiments may also shed a renewed light on the mechanism of high-\( T_c \) superconductivity. On the other hand, from a theoretical point of view, the extension of the present study to more general inhomogeneous cases is also important in understanding recently found inhomogeneous phenomena in cuprates [22, 23, 24].

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[1] P. W. Anderson, Science 235, 1196 (1987).
[2] G. Baskaran, Z. Zou and P. W. Anderson, Solid State Commun. 63, 973 (1987).
[3] P. W. Anderson and Z. Zou, Phys. Rev. Lett. 60, 132 (1988).
[4] Y. Isawa, S. Maekawa and H. Ebisawa, Physica B 148, 391 (1987).
[5] P. W. Anderson, G. Baskaran, Z. Zou and T. Hsu, Phys. Rev. Lett. 58, 2790 (1987).
[6] Y. Suzumura, Y. Hasegawa and H. Fukuyama, J. Phys. Soc. Jpn. 57, 2768 (1988).
[7] G. Kotliar and J. Liu, Phys. Rev. B38, 5142 (1988).
[8] N. Nagaosa and P. A. Lee, Phys. Rev. Lett. 64, 2450 (1990); Phys. Rev. B46, 5621 (1992).
[9] T. Tanamoto, K. Kuboki and H. Fukuyama, J. Phys. Soc. Jpn. 60, 3072 (1991).
[10] T. Tanamoto, H. Kohno and H. Fukuyama, J. Phys. Soc. Jpn. 62, 717 (1993); ibid. 62, 2793 (1993).
[11] V. J. Emery, S. A. Kivelson and O. Zacher, Phys. Rev. B56, 6120 (1997); ibid. 59, 15641 (1999).
[12] S. A. Kivelson, cond-mat/0109151.
[13] This is not exact since now we consider the inhomogeneous doping. However, rather weak doping dependence of \( \xi \) and \( \eta \) within the mean-field theory [1, 2] may justify this approximation.
[14] N. Nagaosa and P. A. Lee, Phys. Rev. B45, 966 (1992).
[15] S. Z. Sze, Physics of Semiconductor Devices, (John Wiley and Sons, New York, 1981).
[16] D. L. Feder and C. Kallin, Phys. Rev. B55, 559 (1997).
[17] P. G. de Gennes, Superconductivity of Metals and Alloys, (W. A. Benjamin, New York, 1966).
[18] J. H. Shion, Ch. Kloc and B. Batlogg, Nature 406, 702 (2000); ibid. 408, 549 (2000).
[19] J. H. Shion et al., Nature 413, 434 (2001).
[20] T. Terashima et al., Jpn J. Appl. Phys. 28, L987 (1989).
[21] A. Kumagai, M. Hayashi and H. Ebisawa, J. Phys. Soc. Jpn. 70, 509 (2001).
[22] I. Iguchi, T. Yamaguchi and A. Sugimoto, Nature 412, 420 (2001).
[23] X. A. Xu, N. P. Ong, Y. Wang, T. Kakeshita and S. Uchida, Nature 406, 486 (2000).
[24] K. M. Lang, V. Madhaven, J. E. Hoffman, E. W. Hudson, H. Eisaki, S. Uchida and J. C. Davis, Nature 415, 412 (2002).