Compressed Channel Estimation for IRS-Assisted Millimeter Wave OFDM Systems: A Low-Rank Tensor Decomposition-Based Approach

Xi Zheng, Peilan Wang, and Jun Fang, Senior Member and Hongbin Li, Fellow, IEEE

Abstract—We consider the problem of downlink channel estimation for intelligent reflecting surface (IRS)-assisted millimeter Wave (mmWave) orthogonal frequency division multiplexing (OFDM) systems. By exploiting the inherent sparse scattering characteristics of mmWave channels, we show that the received signals can be expressed as a low-rank third-order tensor that admits a tensor rank decomposition, also known as canonical polyadic decomposition (CPD). A structured CPD-based method is then developed to estimate the channel parameters. Our analysis reveals that the training overhead required by our proposed method is as low as $O(U^2)$, where $U$ denotes the sparsity of the cascade channel. Simulation results are provided to illustrate the efficiency of the proposed method.

Index terms—Intelligent reflecting surface, millimeter wave communications, channel estimation.

I. INTRODUCTION

IRS has emerged as a promising solution to address the blockage issue and extend the coverage for mmWave communications. Nevertheless, due to the passive nature of reflecting elements and the large size of the channel matrix resulting from massive units at the IRS, channel estimation for IRS-assisted mmWave systems is very challenging. To reduce the training overhead, some previous studies exploited the inherent sparsity of mmWave channels and developed compressed sensing-based methods to estimate the cascade BS-IRS-user channel $[1]–[3]$. These works are mainly concerned with the estimation of narrowband channels. MmWave systems, however, are very likely to operate on wideband channels with frequency selectivity. As for wideband channels, the work $[4]$ considered channel estimation for IRS-assisted mmWave OFDM systems, where a distributed orthogonal matching pursuit (OMP) algorithm was proposed by utilizing the common angular-domain sparsity shared by different subcarriers. The work $[4]$, however, assumes that the BS-IRS channel is LOS-dominated and known $a$ priori.

Recently, some tensor decomposition-based methods, e.g. $[5]–[7]$, were proposed for IRS-assisted systems by exploiting the intrinsic multi-dimensional structure of the received signals. These works, however, did not utilize the sparse scattering characteristic of the mmWave channel. In their formulation, the CP rank of the constructed tensor is equal to the number of reflecting elements at the IRS. As a consequence, these methods require a training overhead proportional to the number of reflecting elements, which is usually large in practice.

In this paper, we develop a new tensor-decomposition channel estimation method for IRS-assisted mmWave OFDM systems. Different from $[5]–[7]$, our work formulates the received signal as a low-rank third-order tensor by exploiting the inherent sparse structure of the cascade channel. The CP rank of the constructed tensor is equal to the sparsity of the cascade channel. This low-rank structure enables to obtain a reliable estimate of the cascade channel using only a very small amount of training overhead. Another challenge of our problem lies in that, due to the nature of the cascade channel, one of the factor matrices of the tensor has redundant columns. As a result, the Kruskal’s condition, which is essential to the uniqueness of the CPD, does not hold and existing CPD-based methods, e.g. $[8]$, cannot be applied. To address this difficulty, in our work, the Vandermonde structure of the factor matrix is invoked to develop a structured CPD method for channel estimation.

II. SYSTEM MODEL

Consider an IRS-assisted mmWave OFDM system, where an IRS is deployed to assist data transmission from the BS to an omnidirectional-antenna user. For simplicity, we assume that the direct link between the BS and the user is blocked due to poor propagation conditions. The total number of OFDM tones (subcarriers) is $P_0$, among which $P$, say $\{1, 2, \cdots, P\}$, subcarriers are selected for training. The BS is equipped with a uniform linear array (ULA) with $N$ antennas and $R$ radio frequency (RF) chains, where $R \ll N$. The IRS is a uniform planar array (UPA) with $M = M_x \times M_y$ passive reflecting elements. Each element can independently reflect the incident signal with a reconfigurable phase shift. Denote $\Phi \triangleq \text{diag}(\mathbf{v}) \triangleq \text{diag}(e^{j\gamma_1}, e^{j\gamma_2}, \cdots, e^{j\gamma_M})$ as the phase-shift matrix, where $\mathbf{v} \in \mathbb{C}^M$ is the phase-shift vector, $\gamma_i \in [0, 2\pi]$ denotes the phase shift coefficient associated with the $i$th passive reflecting element.
In this paper, we adopt a geometric wideband mmWave channel model \[9\] to characterize the channel between the BS (IRS) and the IRS (user). Specifically, the BS-IRS channel in the delay domain can be expressed as
\[
G_r(t) = \sum_{l=1}^L \alpha_l a_{IRS}(\theta_{a,l}, \varphi_{e,l}) a_{BS}^T(\phi_l) \delta(t - \tau_l) \tag{1}
\]
where \(L\) is the total number of paths between the BS and the IRS, \(\alpha_l\) is the complex gain associated with the \(l\)th path, \(\phi_l\) represents the angle of departure (AoD), \(\{\theta_{a,l}, \varphi_{e,l}\}\) denote the azimuth and elevation angles of departure, and \(\delta(t - \tau_l)\) denotes the time delay.

Similarly, the IRS-user channel in the delay domain is modeled as
\[
r_r(t) = \sum_{l=1}^{L_r} g_l a_{IRS}(\chi_{a,l}, \varphi_{e,l}) \delta(t - \kappa_l) \tag{2}
\]
where \(L_r\) is the number of paths between the IRS and the user, \(g_l\) denotes the associated complex path gain, \(\{\chi_{a,l}, \varphi_{e,l}\}\) denote the azimuth and elevation angles of arrival, and \(\kappa_l\) is the time delay.

Accordingly, the frequency-domain BS-IRS and IRS-user channel matrices associated with the \(p\)th subcarrier can be respectively written as
\[
G_p = \sum_{l=1}^L \alpha_l e^{-j2\pi f_s \tau_l} a_{IRS}(\theta_{a,l}, \varphi_{e,l}) a_{BS}^T(\phi_l) \tag{3}
\]
\[
r_p = \sum_{l=1}^{L_r} g_l e^{-j2\pi f_s \kappa_l} a_{IRS}(\chi_{a,l}, \varphi_{e,l}) \tag{4}
\]
where \(f_s = 1/T_s\) is the sample frequency.

To facilitate the algorithmic development, we consider a framed-based downlink training protocol. For each subcarrier, the BS employs \(T\) different beamforming vectors at \(T\) consecutive time frames. Each time frame is divided into \(Q\) time slots. At the \(q\)th time slot, the IRS uses an individual phase-shift matrix, denoted as \(\Phi_q\), to reflect the incident signal. The beamforming vector associated with the \(p\)th subcarrier at the \(q\)th time frame can be expressed as \(x_p(t) = F_{RF}(t) f_{BB,p}(t)s_p(t)\), where \(s_p(t)\) denotes the \(p\)th subcarrier’s pilot symbol, \(f_{BB,p}(t)\) is the digital precoding vector for the \(p\)th subcarrier, and \(F_{RF}(t)\) is a RF precoder common to all subcarriers. For simplicity, we assume that \(f_{BB,p}(t) = f_{BB}(t)\) and \(s_p(t) = 1, \forall p\), in which case we have
\[
x_p(t) = f(t) \triangleq F_{RF}(t) f_{BB}(t), \forall p \tag{5}
\]
The transmitted signal arrives at the user via propagating through the BS-IRS-user channel. At the \(t\)th time frame, the received signal associated with the \(p\)th subcarrier at the \(q\)th time slot can thus be written as
\[
y_{p,q}(t) = r_p^T \Phi_q G_p f(t) + n_{p,q}(t) = v_q^T H_p f(t) + n_{p,q}(t) \tag{6}
\]
where \(\Phi_q = diag(v_q)\), \(H_p \triangleq diag(r_p)G_p\) denotes the cascade BS-IRS-user channel associated with the \(p\)th subcarrier, and \(n_{p,q}(t)\) denotes the additive Gaussian noise.

Substituting (3)–(4) into (6), we arrive at
\[
H_p = \sum_{m=1}^{L_r} \sum_{n=1}^L \beta_{m,n} e^{-j2\pi f_s \tau_n} a_{IRS}(\chi_{a,m} + \varphi_{e,n}, \chi_{e,m} + \varphi_{a,n}) a_{BS}^T(\phi_n) \tag{7}
\]
where the mapping process \(\{a\}\) is defined as
\[
\{a\} = \sum_{u=1}^{L_r} \beta_{u} e^{-j2\pi f_s \tau_u} a_{IRS}(\omega_{a,u}, \omega_{e,u}) a_{BS}^T(\phi_u) \tag{8}
\]

Our objective is to estimate the cascade channel matrices \{\(H_p\)\} from the received measurements \{\(y_{p,q}(t)\)\}. Note that in the data transmission stage, the knowledge of \{\(H_p\)\} suffices for joint active and passive beamforming, i.e. optimizing \(v\) and \(\{f_p = F_{RF} f_{BB}(t)\}_p\) to maximize the spectral efficiency.

### III. Proposed CPD-Based Method

#### A. Low-Rank Tensor Representation

Substituting (7) into (9), we obtain
\[
y_{p,q}(t) = \sum_{u=1}^{L_r} \beta_{u} e^{-j2\pi f_s \tau_u} v_q^T a_{IRS}(\omega_{a,u}, \omega_{e,u}) a_{BS}^T(\phi_u) f(t) + n_{p,q}(t) \tag{9}
\]
Define \(y_q(t) \triangleq [y_{p,1}(t) \cdots y_{p,Q}(t)]^T \in \mathbb{C}^Q\). The received signal at the \(q\)th time frame can be written as
\[
y_p(t) = \sum_{u=1}^{L_r} \beta_{u} e^{-j2\pi f_s \tau_u} V^T a_{IRS}(\omega_{a,u}, \omega_{e,u}) a_{BS}^T(\phi_u) f(t) + n_p(t) \tag{10}
\]
where
\[
V \triangleq [v_1 \cdots v_Q] \in \mathbb{C}^{M \times Q} \tag{11}
\]
\[
n_p(t) \triangleq [n_{p,1}(t) \cdots n_{p,Q}(t)]^T \in \mathbb{C}^Q \tag{12}
\]

After receiving signals across all \(T\) time frames, the received signal associated with the \(p\)th subcarrier can be further expressed as a matrix
\[
Y_p = \sum_{u=1}^{L_r} \beta_{u} e^{-j2\pi f_s \tau_u} \tilde{a}_{IRS}(\omega_{a,u}, \omega_{e,u}) a_{BS}^T(\phi_u) + N_p \tag{13}
\]
where
\[ Y_p \triangleq [y_p(1) \; \cdots \; y_p(T)] \in \mathbb{C}^{Q \times T} \]
\[ \tilde{a}_{\text{IRS}}(\omega_{a,u}, \omega_{c,u}) \triangleq V^T a_{\text{IRS}}(\omega_{a,u}, \omega_{c,u}) \in \mathbb{C}^{Q} \]
\[ a_{\text{BS}}(\phi_u) \triangleq F^T a_{\text{BS}}(\phi_u) \in \mathbb{C}^{T} \]
\[ F \triangleq [f(1) \; \cdots \; f(T)] \in \mathbb{C}^{N \times T} \]
\[ N_p \triangleq [n_p(1) \; \cdots \; n_p(T)] \in \mathbb{C}^{Q \times T} \quad (14) \]

As signals from multiple subcarriers are available at the receiver, the received signal can be expressed as a third-order tensor \( \mathcal{Y} \in \mathbb{C}^{Q \times T \times P} \). It can be readily verified that the tensor \( \mathcal{Y} \) admits a CPD form as
\[ \mathcal{Y} = \sum_{u=1}^{U} \tilde{a}_{\text{IRS}}(\omega_{a,u}, \omega_{c,u}) \circ (\beta_u a_{\text{BS}}(\phi_u)) \circ g(t_u) + \mathcal{N} \quad (15) \]

where \( U \triangleq LL_z, \mathcal{N} \in \mathbb{C}^{Q \times T \times P} \) is the tensor representation of the observation noise, and
\[ g(t_u) \triangleq [e^{j2\pi \frac{t_u}{L_z} 1} \; \cdots \; e^{j2\pi \frac{t_u}{L_z} P_n}]^T \in \mathbb{C}^P \quad (16) \]

Define
\[ A \triangleq [\tilde{a}_{\text{IRS}}(\omega_{a,1}, \omega_{c,1}) \; \cdots \; \tilde{a}_{\text{IRS}}(\omega_{a,U}, \omega_{c,U})] \in \mathbb{C}^{Q \times U} \quad (17) \]
\[ B \triangleq [\beta_1 a_{\text{BS}}(\phi_1) \; \cdots \; \beta_U a_{\text{BS}}(\phi_U)] \in \mathbb{C}^{T \times U} \quad (18) \]
\[ C \triangleq [g(t_1) \; \cdots \; g(t_U)] \in \mathbb{C}^{P \times U} \quad (19) \]

Here \( \{A, B, C\} \) are the factor matrices of the tensor \( \mathcal{Y} \). We see that the channel parameters \( \{\omega_{a,u}, \omega_{c,u}, \phi_u, t_u, \beta_u\}_{u=1}^U \) can be readily estimated from the factor matrices. Inspired by this observation, we first estimate the three factor matrices from the tensor \( \mathcal{Y} \), and then estimate the associated channel parameters based on the estimated factor matrices.

### B. Uniqueness Condition

A well-known sufficient condition for the uniqueness of the CP decomposition is given in [10] and summarized as

**Theorem 1.** Let \( \chi \in \mathbb{C}^{I \times J \times K} \) be a third-order tensor decomposed of three factor matrices \( A(1) \in \mathbb{C}^{I \times R}, A(2) \in \mathbb{C}^{J \times R} \) and \( A(3) \in \mathbb{C}^{K \times R} \), if the condition

\[ k_A(1) + k_A(2) + k_A(3) \geq 2R + 2 \quad (20) \]

is satisfied, then the CPD of \( \chi \) is unique up to scaling and permutation ambiguities. Here \( k_A \) denotes the \( k \)-rank of \( A \), which is defined as the largest value of \( k_A \) such that every subset of \( k_A \) columns of \( A \) is linearly independent.

Clearly, the above Kruskal’s condition (20) does not hold if \( k_A(1) = 1, 2 \), or 3, Unfortunately, in our problem, the factor matrix \( B \) has redundant columns when \( L_z \neq 1 \), in which case we have \( a_{\text{BS}}(\phi_u) = a_{\text{BS}}(\phi_u) \), for any \( u \in \{u : \text{mod}(u, L) = n\} \). Redundant columns indicate that \( k_B = 1 \). Hence the Kruskal’s condition (20) cannot be satisfied.

To address this difficulty, note that the factor matrix \( C \) (cf. (19)) is a Vandermonde matrix. Previous studies show that, even if the Kruskal’s condition does not hold valid, the CPD is still unique when one of its factor matrices has a Vandermonde structure. The uniqueness result was summarized as follows.

**Theorem 2.** Let \( \chi \in \mathbb{C}^{I \times J \times K} \) be a third-order tensor decomposed of three factor matrices \( A(1) \in \mathbb{C}^{I \times R}, A(2) \in \mathbb{C}^{J \times R} \) and \( A(3) \in \mathbb{C}^{K \times R} \), where \( A(3) \) is a Vandermonde matrix with distinct generators. If the condition

\[
\begin{align*}
\text{rank}(A(3) \otimes A(2)) &= R \\
\text{rank}(A(1)) &= R
\end{align*}
\]

is satisfied, then the CPD is unique, where \( A \) denotes a submatrix of \( A \) that is obtained by removing the bottom row of \( A \), and \( \otimes \) denotes the Khatri-Rao product.

**Proof.** See [11].

From Theorem 2, we know that if

\[
\begin{align*}
\text{rank}(C \otimes B) &= U \\
\text{rank}(A) &= U
\end{align*}
\]

is satisfied and \( C \) is a Vandermonde matrix with distinct generators, then the CP decomposition of \( \chi \) is unique.

We first examine the rank of \( (C \otimes B) \). Note that the factor matrix \( C \) is a Vandermonde matrix with distinct generators, as we generally have \( t_i \neq t_j, \forall t_i \neq t_j \). Thus the matrix \( (C \otimes B) \) has full column rank even if \( B \) has linearly dependent columns, provided that \((P - 1)T \geq U \) [11]. On the other hand, recall that the factor matrix \( A \) has a form as

\[ A = V^T [a_{\text{IRS}}(\omega_{a,1}, \omega_{c,1}) \; \cdots \; a_{\text{IRS}}(\omega_{a,U}, \omega_{c,U})] = V^T A_{\text{IRS}} \quad (23) \]

Note that \( A_{\text{IRS}} \) is a matrix consisting of a set of steering vectors characterized by different angular parameters. When entries of \( V \) are chosen uniformly from a unit circle, it is shown in [8] that the k-rank of \( A \) is equal to \( \min(Q, U) \). When \( Q \geq U \), we have \( \text{rank}(A) = U \).

In summary, conditions (22) are generally satisfied when \( \min((P - 1)T, Q) \geq U \). Since the total number of measurements required for our method is \( PTQ \), conditions (22) imply that our proposed method has a sample complexity of \( O(U^2) \), which only depends on the sparsity of the cascade channel.

### C. CP Decomposition

We introduce the method [11], [12] to recover the factor matrices of \( \mathcal{Y} \) by exploiting the Vandermonde structure inherent in the factor matrix. Consider the mode-1 unfolding of the received tensor \( \mathcal{Y} \):

\[ Y_{(1)}^T = (C \otimes B) A^T + N_{(1)}^T \quad (24) \]

and perform the truncated singular value decomposition (SVD)

\[ Y_{(1)}^T = U \Sigma V^H \in \mathbb{C}^{T \times P \times Q}, \text{ where } U \in \mathbb{C}^{T \times P}, \Sigma \in \mathbb{C}^{P \times U}, \text{ and } V \in \mathbb{C}^{Q \times U}. \]

Here \( U \) can be estimated via a minimum description length (MDL) criterion [13].

Ignoring the noise, from (22), we know that there exists a nonsingular matrix \( M \in \mathbb{C}^{U \times T} \) such that

\[ U M = C \otimes B \quad (25) \]
The above equation implies that
\[ U_1 M = \mathcal{C} \odot B \]  
\[ U_2 M = \overline{\mathcal{C}} \odot B \]

where \( \overline{A} \) denotes a submatrix of \( A \) obtained by removing the top row of \( A \), and
\[ U_1 = U(1 : (P - 1)T, :) \in \mathbb{C}^{(P-1)T \times U} \]
\[ U_2 = U(T + 1 : PT, :) \in \mathbb{C}^{(P-1)T \times U} \]

On the other hand, by utilizing the Vandermonde structure of \( C \), we have
\[ (\mathcal{C} \odot B) Z = \overline{\mathcal{C}} \odot B \]

where \( Z \triangleq \text{diag}(z_1, \ldots, z_U) \), and \( z_u \triangleq e^{-j2\pi \frac{\ell_u}{M}} \) is the generator of the factor matrix \( C \). Combining (26)–(30), we arrive at
\[ U_2 M = U_1 M Z \]

Since \( \mathcal{C} \odot B \) is full column rank, both \( U_1 \) and \( U_2 \) are full column rank. Therefore, the generators \( \{\hat{z}_u\}_{u=1}^U \) and \( \hat{M} \) can be obtained from the eigenvalue decomposition (EVD) of \( U_1^H U_2 = \hat{M} \hat{Z} \hat{M}^{-1} \). Each column of the factor matrix \( C \) can be estimated as
\[ \hat{c}_u = [\hat{z}_u^0 \hat{z}_u^1 \ldots \hat{z}_u^P]^T \]

According to (25), the column of the factor matrix \( B \) can be estimated as
\[ \hat{b}_u \triangleq (\hat{c}_u^H \otimes I_T) U \hat{M}(:, u) \]

Finally, given \( \hat{B} \) and \( \hat{C} \), the factor matrix \( A \) can be given as
\[ \hat{A} = Y_{(1)} \left( (\hat{C} \odot \hat{B})^T \right)^\dagger \]

D. Channel Estimation

After obtaining \( \hat{A}, \hat{B} \) and \( \hat{C} \), we now proceed to estimate the channel parameters \( \{\hat{\omega}_{a,u}, \hat{\omega}_{e,u}, \hat{\phi}_u, \hat{\beta}_u\}_{u=1}^U \). From the above discussion, we know that the estimated \( \{\hat{A}, \hat{B}, \hat{C}\} \) and the true factor matrices \( \{A, B, C\} \) are related as
\[ \hat{A} = A \Psi_1 \Gamma + E_1 \]
\[ \hat{B} = B \Psi_2 \Gamma + E_2 \]
\[ \hat{C} = C \Gamma + E_3 \]

where \( \{\Psi_1, \Psi_2\} \) are nonsingular diagonal matrices which satisfy \( \Psi_1 \Psi_2 = I_U \), and \( \{E_1, E_2, E_3\} \) are estimation errors. \( \Gamma \) is an unknown permutation matrix. This permutation matrix \( \Gamma \) is common to all factor matrices, and thus can be ignored.

From an estimated generator \( \{\hat{z}_u\} \), the delay parameter \( i_u \) can be estimated as
\[ i_u \triangleq -\frac{P_0}{2\pi f_s} \text{arg}(\hat{z}_u) \]

where \text{arg}(\hat{z}_u) denotes the argument of the complex number \( \hat{z}_u \). Recall that each column of the factor matrix \( A \) is characterized by angle parameters \( \{\omega_{a,u}, \omega_{e,u}\} \). Therefore these two angle parameters can be estimated through a correlation-based estimator:
\[ \hat{\omega}_{a,u}, \hat{\omega}_{e,u} = \arg \max_{\omega_{a,u}, \omega_{e,u}} \left| \hat{a}_{1u}^H \hat{a}_{\text{IRS}}(\omega_{a,u}, \omega_{e,u}) \right| \]

where \( \hat{a}_u \) denotes the \( u \)th column of \( \hat{A} \). Similarly, the AoD associated with the BS, can be estimated as
\[ \hat{\phi}_u = \arg \max_{\phi_u} \frac{\|\hat{b}_u^H \hat{a}_{\text{BS}}(\phi_u)\|}{\|\hat{b}_u\| \|\hat{a}_{\text{BS}}(\phi_u)\|} \]

where \( \hat{b}_u \) denotes the \( u \)th column of \( \hat{B} \).

Next, we try to recover the composite path loss gains \( \{\hat{\beta}_u\} \). After obtaining \( \{\hat{\omega}_{a,u}, \hat{\omega}_{e,u}\} \), the factor matrix \( A \) can be accordingly estimated as
\[ \hat{A} = [\hat{a}_{\text{IRS}}(\hat{\omega}_1, \hat{\omega}_1) \ldots \hat{a}_{\text{IRS}}(\hat{\omega}_u, \hat{\omega}_e)] \]

Ignoring the estimation errors, \( \hat{A} \) and \( \hat{A} \) are related as \( \hat{A} = A \Psi_1 \). Hence the nonsingular diagonal matrix \( \Psi_1 \) can be estimated as \( \Psi_1 = \hat{A} \hat{A} \). Since \( \Psi_1 \Psi_2 = I_U \), \( \Psi_2 \) can be obtained as \( \Psi_2 = \Psi_1^{-1} \).

On the other hand, after obtaining \( \hat{\phi}_u \), we can construct a new matrix
\[ \hat{B} = [\hat{a}_{\text{BS}}(\hat{\phi}_1) \ldots \hat{a}_{\text{BS}}(\hat{\phi}_U)] \]

Ideally we should have \( B = \hat{G} \hat{B} \), where \( \hat{G} \triangleq \text{diag}(\hat{\beta}_1, \ldots, \hat{\beta}_U) \). Moreover, ignoring estimation errors, we should have \( \hat{B} = B \Psi_2 \). Therefore \( G \) can be estimated as
\[ \hat{G} = \hat{B}^H \hat{B} \Psi_2^{-1} \]

Finally, the cascade channels \( \{H_p\} \) can be estimated after those parameters \( \{\hat{\omega}_{a,u}, \hat{\omega}_{e,u}, \hat{\phi}_u, i_u, \hat{\beta}_u\}_{u=1}^U \) are obtained.

IV. Simulation Results

In this section, we present simulation results to evaluate the performance of the proposed structured CPD-based (SCPD) method. We assume that the BS employs a ULA with \( N = 64 \) antennas and \( R = 1 \) RF chain, the IRS is equipped with \( M = 16 \times 16 \) passive reflecting elements. In our simulations, the angular parameters \( \{\theta_{a,l}, \theta_{e,l}\}_{l=1}^L \), \( \{\phi_i\}_{i=1}^L \), and \( \{\chi_a, \chi_e\}_{l=1}^L \) are randomly generated from \([0, 2\pi]\), where we set \( L = 2 \).
and $L_r = 2$. The delay spreads $\{\tau_l\}_{l=1}^{L_r}, \{\kappa_l\}_{l=1}^{L_r}$ are drawn from a uniform distribution $U(0, 100\text{ns})$. The complex gains $\{\alpha_l\}_{l=1}^{L_r}, \{\eta_l\}_{l=1}^{L_r}$ follow a circularly symmetric Gaussian distribution $\mathcal{C}\mathcal{N}(0, (c/4\pi D_2 f_c)^2)$, $\mathcal{C}\mathcal{N}(0, (c/4\pi D_2 f_c)^2)$, where $c$ is the speed of light, $D_1$ is the distance from the BS to the IRS, $D_2$, $\eta_l$ denotes the length of the $l$th path from the IRS to the user, and $f_c$ is the carrier frequency. We set $D_1 = 30m$ and $f_c = 28\text{GHz}$ in our experiments. The total number of subcarriers is set to $P_0 = 128$, among which $P$ subcarriers are used for training. The sampling rate is set to $f_s = 0.32\text{GHz}$. The signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = \frac{\|\mathbf{Y} - \mathbf{N}\|^2_F}{\|\mathbf{N}\|^2_F}$$  \hspace{1cm} (41)

We firstly examine the estimation accuracy of the channel parameters $\{\omega_{a,u}, \omega_{a,c}, \phi_u, \zeta_u, \beta_u\}_{u=1}^{U}$. The CRB results are also included to provide a benchmark for evaluating the performance of our proposed method. Note that our estimation problem has a form similar to that of [8]. Therefore its CRB can be derived by following the derivations developed in [8]. In Fig. 2 we depict the mean square errors (MSEs) of our proposed method versus the SNR, where we set $P = 16$, $T = 16$, $Q = 16$. From Fig. 2 we see that our proposed method can obtain accurate estimates of the angular parameters and the time delays. Its estimation errors are close to the theoretical lower bound. The estimates of the composite path gains are not as close to the CRB as other parameters, probably because the composite path gains are not directly estimated from the factor matrices.

Next, we report the overall channel estimation performance. By transforming the channel estimation problem into a MMV compressed sensing problem, the simultaneous-OMP method (SOMP) [14] can also be used to estimate the channel. For the SOMP, two different grids are employed to discretize the continuous parameter space: the first grid discretizes the AoA-AoD-time delay space into $(32 \times 32) \times 128 \times 64$ points, and the second grid discretizes the AoA-AoD-time delay space into $(64 \times 64) \times 256 \times 128$ points. In Fig. 3 we plot the estimation performance of respective methods as a function of the SNR and the number of time frames $T$. The performance is evaluated via the normalized mean squared error (NMSE) of the cascaded channel, which is defined as

$$\sum_{p=1}^{P} \frac{\|\hat{H}_p - H_p\|_F^2}{\|H_p\|_F^2} / \sum_{p=1}^{P} \|H_p\|_F^2.$$  \hspace{1cm} (42)

From these results, we see that the proposed method presents a substantial performance improvement over the SOMP method. In addition, we observe that our proposed method can provide reliable channel estimation when $P = Q = 8$ and $T = 4$, which corresponds to a total number of 256 measurements for training. This result indicates that the proposed method can achieve a substantial training overhead reduction.

V. Conclusion

In this paper, we developed a CPD-based channel estimation method for IRS-assisted mmWave OFDM systems. The proposed method exploits the inherent low-rank structure of the cascade channels and the inherent Vandermonde structure of the factor matrix. Our analysis shows that the proposed method only requires a modest amount of training overhead to extract the channel parameters. Simulation results were provided to illustrate the efficiency of the proposed method.

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