Multi-echelon inventory allocation of multi-indenture spare parts considering maintenance ratio

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Abstract. The existing multi-indenture spare parts maintenance has a certain maintenance proportion, a multi-echelon inventory allocation method with maintenance ratio for multi-indenture spare parts was proposed. Multi-echelon inventory allocation model for multi-indenture spare parts was established, which integrates maintenance rate into METRIC (Multi-echelon technique for recoverable item control) model, and then the marginal analysis method was applied to solve this model. First, the theory of multi-echelon inventory allocation model with maintenance ratio for multi-indenture spare parts was elaborated, and the mathematical model was established with the spare parts inventory cost as the optimization objective, the fleet availability as the constraints. Then, the procedure of solving the model by using marginal analysis method is discussed. Finally, taking an aircraft multi-indenture LRU (Line repairable units) as the research object, multi-echelon inventory allocation was investigated by considering maintenance ratio. The feasibility and effectiveness of the developed method are verified by an example analysis.

1. Introduction

Spare parts plays an important role in maintaining the reliability and Airworthiness of aircraft. The composition of spare parts of civil aircraft is complex. Aircraft is composed of many systems and the system is a combination of LRU, LRU also contains many SRU. Therefore, the failure of an aircraft system is often the result of the interrelated failure between LRU and SRU.

Many scholars have studied the allocation of maintenance resources considering the maintenance ratio. In order to realize the allocation of civil aircraft maintenance resources, Boeing and Airbus Company takes the maintenance ratio into account and evaluates the demand for repairable parts [1]. In METRIC theory, maintenance ration is also introduced into multi-echelon spare parts inventory allocation. For example, Patriarca and other researchers use METRIC theory to study the maintenance resource allocation of civil aircraft [2]; De Smidt-Destombes explore the application of METRIC theory in multi-echelon spare parts inventory allocation of redundant systems [3]; Basten realize multi-echelon spare parts inventory allocation under LORA combined with METRIC theory [4]; Xue Tao introduced scrap flow into spare parts inventory allocation model, established a multi-echelon and multi-indenture spare parts inventory allocation model [5]; Li Yongkai introduced scrap into VARI-METRIC model complete the multi-echelon inventory allocation of repairable parts [6]; Feng Yunwen studied the multi-echelon inventory allocation of civil aircraft spare parts under the premise of considering the maintenance ratio [7-9].
Although the above research introduces maintenance ration into spare parts inventory allocation model, but there is no further study on the allocation scheme under multi-echelon and multi-indenture repair mode. Therefore, this paper introduces the factor of maintenance ratio into spare parts inventory allocation model, and modeling the spare parts inventory allocation under different repair modes in initial state of civil aircraft. The model is solved by the marginal analysis method, and then the spare parts of multi-echelon allocation mode considering maintenance ratio is studied.

2. Model description and assumptions
In general, the maintenance of civil aircraft mainly adopts two-level repair mode, including the Local Warehouse (LW) and the Central Department (CD), in which the terminal is the LW maintenance site of spare parts. And the CD is located behind the terminal, which is a repair center with higher repair capability. The schematic diagram of aircraft maintenance location and spare parts composition is shown in figure 1.

![Schematic diagram of aircraft maintenance location and spare parts composition](image)

**Figure 1.** Spare parts inventory and composition chart.

![Flow chart of spare parts maintenance and supply](image)

**Figure 2.** Flow chart of spare parts maintenance and supply.

When the aircraft's spare parts fail, the parts will be disassembled. If the parts are LRU, it will be sent to the LW for repair, and a spare part will be called from the LW for installation. Because of the limited maintenance capacity of the LW, if the LRU can’t complete repair in the LW, it will be sent to the CD for repair. If the failure part is SRU, send it to the CD for repair, and call a spare part from the
CD. If it can’t be repaired, choose scrap and purchase a new spare part. The repair process of spare parts is shown in figure 2.

In order to simplify the modeling process, there are some reasonable assumptions for multi-echelon inventory allocation method:

(1) The failure of LRU and SRU follows the Poisson Process, and different fault units are independent of each other in repair quantity.

(2) Each LRU does not belong to each other and SRU is independent of each other.

(3) The invalid parts can be repaired perfectly.

(4) When repaired at the LW and CD, the faulty parts are served according to the first-come-first-served principle.

(5) There is no lateral supply.

3. Establishment of model

3.1. Annual demand for spare parts

The average annual demand of LRU is:

\[ m_i = \frac{FH \times N \times QPA}{MTBUR} \tag{1} \]

where: \( i = 1, 2, 3 \ldots, k \) is the number of LRU, FH is the Annual Flight Time of Aircraft, N is the Fleet size, QPA is the installation Number of LRU in Single Aircraft, MTBUR is the mean time between unscheduled removal.

(1) Average annual demand of LRU in LW

Calculating the annual average demand of LRU for each LW based on the distance between terminals:

\[ m_{0j} = \frac{d_j}{\sum_{j=1}^{n} d_j} m \tag{2} \]

(2) Average annual demand of SRU in LW

The average annual demand of SRU is the average annual demand of LRU multiplied the maintenance ration of LRU in the LW, and then multiplied the SRU demand probability generated by LRU repair:

\[ m_{ij} = m_{0j} \lambda_{0j}^{\text{line}} \lambda_{ij}^{\text{cause}} \tag{3} \]

where, \( m_j \) is the annual average demand of \( i \)-th SRU at terminal \( j \), \( \lambda_{0j}^{\text{line}} \) is the LRU maintenance ration of terminal \( j \), \( \lambda_{ij}^{\text{cause}} \) is the probability of \( i \)-th SRU fault caused by terminal \( j \)'s LRU fault.

(3) Average annual demand of LRU in CD

Equal to the sum of the LRU requirements for CD applications that occur at each terminal.

\[ m_{00} = \lambda_{00}^{\text{shop}} \sum_{j=1}^{l} m_{0j}(1-\lambda_{0j}^{\text{line}}) \tag{4} \]

(4) Average annual demand of SRU in CD

Equal to the sum of all LW requirements, and plus the number of SRU required for CD repair.

\[ m_{i0} = \lambda_{i0}^{\text{shop}} (\sum_{j=1}^{l} m_{ij}(1-\lambda_{ij}^{\text{line}}) + m_{00} \lambda_{i0}^{\text{cause}}) \tag{5} \]

3.2. Expected demand for spare parts

The expected shortage of spare parts is as follows:

\[ EBO(s) = \sum_{x=s+1}^{\infty} \left( x-e^{-s} \right) \frac{\omega^x e^{-\omega}}{x!} \tag{6} \]

Expected demand of SRU in CD is:

\[ \mu_{i0} = m_{i0} \text{MSPT}_{i0} \]
\[ \mu_{i0} = m_{i0} \text{PT}_{i0} \tag{7} \]

where, MSPT\(_{i0}\) is the average repair time of \( i \)-th SRU CD, PT\(_{i0}\) is the purchase time of \( i \)-th SRU.

The expected demand of LRU in CD consists of two parts: the repairing spare parts and the spare parts that are delayed due to the shortage of the upper level.
\[ \mu_{00} = m_{00} MSPT_{00} + \sum_{i=1}^{l} f_{i0} EBO(s_{i0}|m_{i0} MSPT_{i0}) \] (8)

where
\[ f_{i0} = \frac{\mu_{00} - c_{i0}}{m_{i0}} \] (9)

The expected demand of LRU in LW consists of the repaired materials, the materials in transit, the materials that are delayed due to the shortage of the upper level and the material that are delayed in repairing due to the shortage of the next level.
\[ \mu_{0j} = m_{0j} \left(1 - \lambda_{0j}^{\text{line}}\right) \tau_{0j} + m_{0j} \lambda_{0j}^{\text{line}} MSPT_{0j} \]
\[ + f_{0j} EBO(s_{00}|E[X_{00}], Var[X_{00}]) + \sum_{i=1}^{l} EBO(s_{ij}|E[X_{ij}], Var[X_{ij}]) \] (10)

where
\[ f_{0j} = \frac{m_{0j} \left(1 - \lambda_{0j}^{\text{line}}\right)}{m_{00}} \] (11)
\[ Var[X_{00}] = m_{00} MSPT_{00} + \sum_{i=1}^{l} f_{i0} (1 - f_{i0}) EBO(s_{i0}|m_{i0} MSPT_{i0}) \]
\[ + \sum_{i=1}^{l} f_{i0}^2 VBO(s_{i0}|m_{i0} MSPT_{i0}) \] (12)
\[ Var[X_{ij}] = m_{ij} \left(1 - \lambda_{ij}^{\text{line}}\right) \tau_{ij} + \lambda_{ij}^{\text{line}} MSPT_{ij} \]
\[ + f_{ij} (1 - f_{ij}) EBO(s_{ij}|m_{ij} MSPT_{ij}) + f_{ij}^2 VBO(s_{ij}|m_{ij} MSPT_{ij}) \] (13)

Var[X_{00}] is the variance of the base repair LRU, Var[X_{ij}] is the variance of \( i-th \) SRU that the terminal \( j \) is repairing or replenishing.

3.3. Optimization model
In this paper, spare parts inventory cost as the optimization objective, the fleet availability as the constraints, a multi-echelon inventory allocation method is put forward:
\[ \begin{align*}
\min C & \\
\text{s.t. } A & > A_{\min}
\end{align*} \] (14)

where
\[ A = \prod_{i=1}^{k} \left(1 - \frac{EBO(s_{i})}{N_{i} X_{PA_{i}}}\right) \] (15)

\( C \) is the spare parts inventory cost, \( A \) is the fleet availability, \( A_{\min} \) is the minimum fleet availability requirements.

4. Solving process based on marginal analysis
The multi-echelon inventory allocation model solved by the method of marginal analysis. The solution steps are as follows.

Step 1 Determine the spare parts included in the research object, calculate the average annual demand of the CD, the average annual demand of the LW and the corresponding scrap volume.

Step 2 Calculate the expected demand, the expected shortage, and the fleet availability of CD and LW.

Step 3 Calculate the marginal increment of spare parts, calculate the maximum value of \( \Delta i = [EBO(s_{i}) - EBO(s_{i} + 1)]/C_{i} \), Increase the unit number of spares parts according to the maximum marginal increment.

Step 4 Calculate the total investment cost \( C \), if \( A > A_{\min} \), go to step 3; otherwise, end of the calculation.

5. Numerical example
A typical numerical example is provided to illuminate the application of our model. The fleet operation information is Fleet size \( N=10 \), Annual flight hours \( FH=3300h \). There are 3 terminals and one Central Department, and the distance between terminals is shown in figure 3. The maintenance ratio information and specific parameters of each spare part are shown in table 1 and table 2.
**Figure 3.** Spare parts maintenance structural diagram.

**Table 1.** Spare parts parameters.

| Spare Parts | MTBUR/h | SR | QPA | C/$ | $\lambda_{ij}$ | $\lambda_{0j}$ | $\lambda_{i0}$ | $\lambda_{ij}^{\text{shop}}$ | $\lambda_{i0}^{\text{shop}}$ | $\lambda_{ij}^{\text{cause}}$ |
|-------------|---------|----|-----|-----|---------------|--------------|--------------|----------------|----------------|------------------|
| LRU1        | 60000   | 0.3| 2   |     | 0.6           | 0.83         | 0.2          | 0.9            | 0.15           |
| LRU2        | 25000   | 0.2| 12  |     | 0.55         | 0.75         | 0.2          | 0.9            | 0.15           |
| LRU_SRU1    | 41500   | 0.1| 6   |     | 0.6          | 0.75         | 0.2          | 0.9            | 0.15           |
| SRU1.1      | 50000   | 0.2| 2   |     | 0.4          | 0.8          | 0.15         | 0.8            | 0.1            |
| SRU1.2      | 60000   | 1  | 2   |     | 0.6          | 0.2          | 0.7          | 0.15           |
| SRU1.3      | 50000   | 1  | 2   |     | 0.5          | 0.25         | 0.9          | 0.15           |
| LRU_SRU2    | 35000   | 0.5| 4   |     | 0.6          | 0.2          | 0.9          | 0.15           |
| SRU2.1      | 50000   | 1  | 2   |     | 0.6          | 0.75         | 0.2          | 0.75           | 0.18           |
| SRU2.2      | 50000   | 1  | 2   |     | 0.6          | 0.3          | 0.8          | 0.1            |
| SRU2.3      | 50000   | 1  | 2   |     | 0.6          | 0.2          | 0.85         | 0.15           |
| SRU2.4      | 33333.3 | 1  | 6   |     | 0.6          | 0.2          | 0.9          | 0.18           |
| LRU_SRU3    | 40000   | 0.5| 4   |     | 0.6          | 0.2          | 0.9          | 0.15           |
| SRU3.1      | 46666.67| 1  | 1   |     | 0.6          | 0.15         | 0.8          | 0.17           |
| SRU3.2      | 13333.33| 1  | 8   |     | 0.6          | 0.2          | 0.85         | 0.2            |
| SRU3.3      | 210000  | 1  | 6   |     | 0.6          | 0.3          | 0.75         | 0.15           |
| LRU3        | 15343.07| 0.1| 4   |     | 0.6          | 0.2          | 0.9          | 0.15           |

**Table 2.** Spare parts parameters.

| LRU/SRU | MSPT$_{i0}$ | PT$_{i0}$ | MSPT$_{00}$ | PT$_{00}$ | MSPT$_{ij}$ | $\tau_{ij}$ | $\tau_{0j}$ | MSPT$_{0j}$ |
|---------|-------------|-----------|-------------|-----------|-------------|------------|------------|-------------|
| LRU1    | 6           | 10        | 6           | 11        | 8           | 7          | 7          | 5           |

Based on the above data, according to formulas (1) to (5), the annual average demand and scrap amount can be obtained, and then the expected demand, expected shortage, and fleet availability are calculated according to formulas (6) to (13). Then, by using this data to establish an inventory allocation model. The inventory allocation and costs at the CD and LW are shown in table 3.

As can be seen from table 2, when the spare parts inventory quantity is 52, the constraint is met, and the fleet availability is 0.9711, and the total cost is 375,989 US dollars.

In order to verify the feasibility and effectiveness of this model, based on the spare parts parameters in table 1 and fleet operation information, the traditional METRIC model is solved by marginal analysis method without considering the influence of maintenance ratio. The relationship between fleet availability and spare parts costs is shown in figure 4.
Table 3. Spare parts inventory allocation.

|        | CD | LW1 | LW2 | LW3 |
|--------|----|-----|-----|-----|
| LRU1   | 0  | 0   | 0   | 0   |
| LRU2   | 3  | 1   | 1   | 1   |
| LRU_SRU1 | 3 | 1   | 0   | 1   |
| SRU1.1 | 0  | 0   | 1   | 1   |
| SRU1.2 | 2  | 0   | 1   | 1   |
| SRU1.3 | 2  | 0   | 1   | 1   |
| LRU_SRU2 | 2 | 0   | 0   | 0   |
| SRU2.1 | 1  | 0   | 1   | 1   |
| SRU2.2 | 2  | 0   | 0   | 1   |
| SRU2.3 | 1  | 0   | 1   | 1   |
| SRU2.4 | 2  | 0   | 1   | 1   |
| LRU_SRU3 | 2 | 0   | 0   | 0   |
| SRU3.1 | 0  | 0   | 1   | 1   |
| SRU3.2 | 2  | 0   | 1   | 1   |
| SRU3.3 | 1  | 0   | 0   | 1   |
| LRU3   | 3  | 1   | 1   | 1   |
| Costs/$ | 225725 | 37817 | 46717 | 65730 |

Figure 4. Costs and Fleet Availability Relationship Curve.

From figure 4, it can be seen that the fleet availability gradually reaches the minimum availability requirement with the increase of costs, but the growth rate of fleet availability gradually decreases. And, after the fleet availability reaching 0.90, the increase in fleet availability will cost more money. Therefore, the minimum fleet availability should adjust according to the budget to suit the airlines.

And it shows the spare parts inventory allocation considering maintenance ratio has a slightly lower fleet availability than that without considering maintenance ratio, but the spare parts cost is higher than the spare parts cost without considering maintenance ratio. The main reason for this result is that the multi-echelon inventory allocation without considering maintenance ratio is considered to have unlimited repair capability and no scrap. So, compared with the multi-echelon inventory allocation considering maintenance ratio, the allocation quantity of multi-echelon inventory allocation is smaller and the spare parts costs are lower. And the consideration factor is too idealized, the allocation result did not conform to the actual engineering. Therefore, the multi-echelon inventory allocation model considering maintenance ratio is much more fit the engineering reality.

6. Summary

In this paper, the engineering practices factors of maintenance ratio are introduced into spare parts allocation calculation, and a multi-echelon inventory allocation model of spare parts considering
maintenance ratio is constructed, which is solved by the marginal analysis. Also, the spare parts of a civil aircraft are used to calculate the multi-echelon inventory allocation, and the feasibility and validity of the proposed method are verified by comparing the multi-echelon inventory allocation without considering the maintenance ratio.

The model considers the maintenance proportion, the membership relationship of spare parts and multi-echelon inventory in actual engineering. It is more suitable for practical application, and it can optimize the configuration of complex spare parts for aircraft and reduce the operating cost of airlines.

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