RELATIVISTIC CONTRIBUTIONS TO THE PHOTON ASYMMETRY IN DEUTERON PHOTODISINTEGRATION

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Key-words: deuteron photodisintegration, polarization observables, Bethe-Salpeter.

Abstract

The cross section asymmetry $\Sigma$ in deuteron disintegration with linearly polarized light is considered in the framework of the Bethe-Salpeter formalism. Relativistic contributions to the asymmetry are investigated within the approximation, which takes into account the one-body part of the electromagnetic current operator and the dominant positive-energy partial amplitudes of the Bethe-Salpeter vertex function of the deuteron.

1 Introduction

Interest to the study of beam-polarization observables (namely, the polarization-difference cross sections and the cross-section asymmetry) in the exclusive deuteron photodisintegration is twofold. In the energy range from below pion threshold to above the $\Delta$ resonance, it is to find features characterizing sensitivity of these observables to the short-range behavior of the tensor force in models of the nucleon-nucleon (NN) interaction [1]. The study of the polarization observables also offers a testing ground for models of the $N\Delta$ interaction, as the polarization-dependent cross sections are sensitive to interference terms due to the meson-exchange as well as delta-isobar currents (Ref. [2, 3] and references therein). On the other hand, recent experiments on two-body photodisintegration of the deuteron at the TJNAF and the Yerevan Synchrotorn are devoted to measuring both the differential cross section and the asymmetry in the range, when the photon energy is commensurate with the rest mass of the nucleon. Instead of the reaction mechanism related to the conventional mesonic picture of the NN interaction, here one looks for the partonic phenomena such as the scaling of the cross sections and the hadronic helicity conservation [4, 5].

Relativistic contributions (RC) are expected to be significant in deuteron photodisintegration, especially when the photon momentum involved in the process, becomes comparable with the rest mass of the nucleon. So far, a through treatment of RC has been based on the Blankenbecler-Sugar reduction of the Bethe-Salpeter (BS) equation with spinor reduction carried out to the order of $O(1/m^2)$ for the charge and current densities [6]. In the nonrelativistic formalism relativistic corrections has been investigated in a number of papers, for example see Ref. [7] where it is shown that the inclusion of RC leads to an important reduction of the forward differential cross section. Inclusion of leading order RC, this is the spin-orbit term, in the photodisintegration channel results in large positive contribution reducing the value of the sum rule in absolute size by more than 30 percent!
In this talk we present some results concerning the investigation of the RC in the beam-polarization observables in deuteron photodisintegration. This is a continuation of the approach of Ref. [8] applied to the analysis of the differential cross section. A relativistic formulation is based on the Bethe-Salpeter (BS) equation for the description of the dynamics of 2N system and the gauge invariant procedure for the construction of the electromagnetic (EM) current.

2 Definition of observables

The kinematics of the process

\[ \vec{\gamma}(q) + D(K) \rightarrow P(k_p) + N(k_n) \]

for disintegration of the deuteron induced by a linearly polarized photon in the c.m. frame is organized as follows:

\[ K = (\sqrt{M^2 + K^2}, K) \quad \text{and} \quad q = (\omega, \omega) \]

is the four momenta of the deuteron and the real photon, respectively, with \( K + \omega = 0 \); \( k_p \) and \( k_n \) are the four momenta of the proton and neutron, \( k_{p,n}^2 = m^2 \). Further, it is convenient to introduce the total \( P = k_p + k_n \), \( P \equiv (\sqrt{s}, 0) \), and the relative asymptotic \( p = \frac{1}{2}(k_p - k_n) \), \( p = (0, k_p) \), four momenta.

The differential cross section for complete linear polarization of the photon is written in the form

\[ d\sigma(E_\gamma, \Theta, \Phi) = d\sigma_0(E_\gamma, \Theta)[1 + \Sigma(E_\gamma, \Theta) \cos(2\Phi)], \]

where \( d\sigma_0 \) is the differential cross section in the case of the unpolarized initial configuration, \( \Sigma \) is the cross section azimuthal asymmetry, \( \Theta \) is the angle between the momentum of the photon \( \omega \) and that of the outgoing proton \( k_p \), and \( \Phi \) is the angle between the polarization plane and the reaction plane. If the photon three momentum is taken to be in the +z direction \( \omega = (0, 0, \omega) \), the latter is defined by the polarization vector of the photon \( \epsilon_\lambda \) and \( \omega \) and the former — by vectors \( \omega \) and \( k_p \). In the transverse gauge for the radiation field, following the property \( \omega \cdot \epsilon_\lambda = 0 \), one may write for the photon helicities \( \lambda = \pm 1 \)

\[ \epsilon_\pm = \pm \frac{1}{\sqrt{2}}(0, 1, \pm i, 0). \]

The asymmetry \( \Sigma \) is the ratio of the difference to the sum of cross sections obtained with the photon’s electric vector parallel and perpendicular to the reaction plane, respectively

\[ \Sigma = \frac{(d\sigma_{||} - d\sigma_{\perp})}{(d\sigma_{||} + d\sigma_{\perp})} \]

Both the polarization-independent cross section \( d\sigma_0/d\Omega = \frac{1}{2}(d\sigma_{||}/d\Omega + d\sigma_{\perp}/d\Omega) \) and the polarization-difference cross sections \( d\Delta/d\Omega = \frac{1}{2}(d\sigma_{||}/d\Omega - d\sigma_{\perp}/d\Omega) \) are expressed in terms of the invariant reduced amplitude \( t_{Sm_s{}\lambda_m d}(\Theta) \), depending on the photon helicity \( \lambda \) and projections of the good total angular momentum of the deuteron \( m_d = \pm 1, 0 \) and of the total spin \( S = 0, 1 \) of np pair \( m_s \) on to quantization axis

\[ \frac{d\sigma_0}{d\Omega_p} = \frac{\alpha}{16\pi s} \frac{|k_p|}{\omega} \sum_{S_m S_\lambda m_d} |t_{S_m s{}\lambda_m d}|^2, \]

\[ \Sigma = - \frac{\sum_{S_m S_\lambda m_d} t_{S_m s{}\lambda_m d} t_{S_m s -\lambda_m d}^*}{\sum_{S_m S_\lambda m_d} |t_{S_m s{}\lambda_m d}|^2}, \]
where $\alpha = e^2/(4\pi)$ is the fine-structure constant, and taking the cross section as functions of the laboratory photon energy $E_\gamma$ and the c.m. angle $\Theta$, one relates the variables as

\[ s = M_d(M_d + 2E_\gamma), \quad |k_p| = \sqrt{\frac{4}{s} - m^2}, \quad \omega = \frac{M_d}{\sqrt{s}}E_\gamma. \]

\section{Few theory}

Notwithstanding that the nonrelativistic interpretation of deuteron photodisintegration is successful at low and intermediate energies, a systematic exposition of the covariant formalism at this energy range for the 2N system is still missing. Essential ingredients of the nonrelativistic formulation are the pure one-body EM plus meson-exchange currents as incorporated in the Siegert operators, explicit pair and pionic current, the spin-orbit and further relativistic corrections and delta-isobar configurations \[10\]. At present it is impossible to claim quantitative description of the process in question, using the fully relativistic analysis with the inclusion of the all relevant physics. In order to study RC here we apply the four-dimensional (4D) formalism of the BS equation. The reason for the application of the 4D-formalism is to preserve generality and to avoid some difficulties arising in considering EM process on few nucleon systems. It is known the relativistic effects can take different forms depending upon the organization of the dynamics and its forms \[11\]. And, if a quasipotential formalism is chosen with the same organization of the 2N interaction, equivalent results should be found in reduction of the exact BS matrix elements. The leading terms of such reduction should correspond to the nonrelativistic approximation, and the next order terms should produce $O(\frac{1}{m^2})$-corrections.

In the BS formalism the matrix elements associated with the physical observables in Eq. (5) are obtained via the recipe for the EM current operator construction of Refs. \[15, 14\], which is consistent with the form of the NN interaction kernel of the BS equation and gauge invariance. The dynamical model we assume in this work is the one-body plane-wave approximation described in Ref. \[8\]. In terms of the BS amplitudes for the deuteron $\psi_{Km_d}$ and the np pair $\chi_{ppSm_s}$, the c.m. transition amplitude is written in the form

\[ T_{Sm_s\lambda m_d}(\Theta, \Phi) = \frac{1}{4\pi^3} \int d^4k d^4l \bar{\chi}_{ppSm_s}(l) \epsilon_{\lambda} \cdot \Lambda(l, k; \mathbb{P}, \mathbb{K}) \psi_{Km_d}(k), \]

where $T_{Sm_s\lambda m_d}(\Theta, \Phi) = e^{i(\lambda + m_d)\Phi} t_{Sm_s\lambda m_d}(\Theta)$, $\Lambda$ denotes the Mandelstam vertex, which determines the EM interaction with 2N system in the framework of the BS formalism. Here all amplitudes are the $4 \times 4$ matrices in the spinor subspace. The four momentum conservation at the photon-deuteron vertex gives $\mathbb{P} = \mathbb{K} + q$. Owing to the fact that the EM interaction does not conserve the total isotopic spin, the invariant amplitude in Eq. (6) contains isovector ($\Delta T = 1$) and isoscalar ($\Delta T = 0$) transitions corresponding to a given spin state $|\mathbb{P}pSm_s\rangle$ of the np pair. The complete description of our formalism can be found in Ref. \[8\].

\section{Analysis of relativistic effects}

There are two kinds of the relativistic corrections to the nonrelativistic impulse approximation charge and current densities: (i) terms generated from the spinor reduction of the EM current operator, and (ii) terms generated from boosting of the initial deuteron wave
function from the rest frame to the c.m. frame of the reaction with the deuteron moving with a velocity $\frac{\mathbf{v}_d}{M_d}$ [6, 7]. In a more complete theoretical description, in addition to the one-body EM current, meson-exchange currents, giving rise to two-body operators, are included. Here the inclusion of energy transfer effects are important [12, 13].

Using the BS formalism one may develop, in principle, systematic studies of the relativistic effects in the EM interactions with 2N systems. Here one distinguishes several classes of the relativistic effects both of the kinematical and dynamical origin. First of all, the relativistic effects in the internal dynamics brought by the generalized relativistic wave function of the deuteron in its rest frame. These are effects of the negative-energy partial-states in the vertex function, the retardation (the relative-energy dependence), and the nonstatic single-particle propagators.

Second, one has to boost the wave function to a moving frame as

$$\psi_{\mathbf{K}d}(k) = \Lambda^{(1)}(\mathcal{L})\Lambda^{(2)}(\mathcal{L})\psi_{\mathbf{K}(0)d}(L^{-1}k),$$

where $\Lambda^{(i)}(\mathcal{L})$, $i = 1, 2$ is the booster to the c.m. frame and $\mathcal{L}$ is the Lorentz transformation matrix corresponding to $\mathbf{K}(0) = (M_d, 0)$. The correct treatment means that the spin booster $\Lambda^{(i)}(\mathcal{L}) \neq 1$ (the effect of the spin precession) and the effect of boosting on the internal space-time variables $L \neq 1$ (the Lorentz contraction).

*Figure 1. Some relativistic contributions to the cross section asymmetry. Solid line is the exact positive-energy one-body BS calculation, dot-dashed line is the static approximation to the exact result, dotted line is the static approximation with taking into account the booster $\Lambda$, and dashed line is the nonrelativistic impulse approximation. The transverse gauge is used for all the curves.*

Third, the correct treatment of the conserved EM current operator. A method of minimal substitution can be used for constructing the EM two-body current operator of two-nucleon system within the BS formalism [14, 15]. The relativistic two-body current operator is derived using the Ward-Takahashi identities for the one-body current operators and the
relativistic equation for the bound state. The result depends on the charge-exchange and nonlocal properties of the NN interactions. The matrix element in Eq. (7) is calculated in Figure 2.

**Figure 2.** Dependence of the cross-section asymmetry on the interaction kernel of the BS equation. Dashed curve is the positive-energy BS calculation with the one-boson-exchange vertex function, dot line is the same with Graz-II vertex function. Shaded area shows the effects of the retardation in the Graz-II vertex function.

The one-body plane wave approximation with the transverse gauge. The vertex function is the solution of the homogeneous BS equation with the Graz-II separable interaction [16]. The dominant positive-energy components of the vertex function of the deuteron are considered. In Fig. 4 we present results of the dependence of the asymmetry at $\Theta = 90^\circ$ on the laboratory photon energy $E_\gamma$. Here the curves concerning the number of approximations with respect to the exact positive-energy BS calculation (solid curve) are depicted. The static approximation (dot-dashed curve) amounts to neglecting the boost on the internal and spin variables of the BS amplitude for the deuteron. Such RC as the Lorentz contraction and spin precession are excluded. The effect due to the booster on the spin variables is shown by dotted line in Fig. 4.

We find that the asymmetry $\Sigma$ bears rather strong dependence on the interaction kernel used in solving the BS equation for the deuteron. Its is shown in Fig. 4. The solid and dash curves are the calculations with the positive-energy vertex functions of the deuteron, which are solutions of the BS equation with the one-boson-exchange and Graz-II interaction (strength of the D-state is equal to 5%) kernels, respectively. Since a realistic BS amplitude is computed using the Wick rotation [17], it can be taken in our analysis only at the relative energy equal to zero. The effect of the retardation (the relative energy dependence of the vertex function) in case of the separable interaction is highlighted in Fig. 4. The deviations between two curves are explained by, first, that the minima in the S-waves of the two vertex functions are shifted on approximately 100 MeV, and, second, D-wave of the realistic vertex function is much softer than that of the separable one. The latter is rather crucial for the
behavior of the asymmetry for the photon energies higher than 600 MeV.

In summary, we conclude that the relativistic effects could make a significant contribution to the polarization-beam observables in deuteron photodisintegration, though for their simple assessment one should encourage us to conduct a more complete analysis.

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