Co-optimization of the shape, orientation and layout of offshore wind farms

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Abstract. Previous studies on wind farm design optimization usually assume that the wind farm boundary is fixed. However, due to the uneven distribution of energy for different wind directions, the wind farm boundary, which determines the possible shape and orientation of the wind farm, can have large influences on the wind farm’s energy production. Towards an overall design optimization, the shape and orientation of wind farm should also be considered as part of the design variables. To tackle this problem, we formulate a co-optimization problem of the wind farm’s shape, orientation and layout to maximize AEP under several realistic constraints. As an initial effort, we limit the wind farm boundary to parallelogram shapes. A two-stage optimization method is then proposed to solve the co-optimization problem. The case study on the Horns Rev 1 offshore wind farm demonstrates the importance of choosing proper shape and orientation and the effectiveness of the proposed optimization method.

1. Introduction
The fast development of offshore wind energy in recent years has resulted in the proliferation of large offshore wind farms. Planning and designing these wind farms thus becomes an important yet challenging task, as many decisions need to be made and various engineering problems need to be solved under multi-disciplinary considerations [1, 2]. As large and complex projects that involve many stakeholders and consist of different planning, design and engineering tasks, offshore wind farm projects often face many types of risks and uncertainties in the planning phase alone [3]. To ensure a successful offshore wind farm project, a good planning and design of the wind farm needs to be first conducted before the actual wind farm construction starts.

The planning and design of an offshore wind farm can be divided into two phases: the planning phase in which the marine spatial planning is finished and the design phase in which the detailed wind farm design is worked out. In the planning phase, the main job is to select the site of the offshore wind farm, specify the capacity limit and designate a given area which allows installation of wind turbines. This planning of course has to be done thorough and well by respecting the vulnerability of the marine environment and constraints and requirements posed by all sea uses [4]. In Denmark, which is a pioneering country in offshore wind development, this task is done by a specially established spatial planning committee for offshore wind, which is led by the Danish Energy Agency (DEA) and consists of government authorities responsible for the natural environment, safety at sea and navigation, offshore resources extraction, visual interests and grid transmission conditions [5]. The committee finds appropriate sites for offshore wind farms by taking into account various aspects of the offshore wind development, such as
environmental impacts, other sea uses, grid capacity, wind resource and water depth. Then preliminary site investigations are carried out by the authorities to specify the capacity limit and designated area well before the tendering process begins. Finally, the authority (DEA in the case of Denmark) announces a tender for an offshore wind farm project with a specific total capacity within a designated geographical area [5].

After the planning phase is done, and a developer has won the tender for an offshore wind farm, the design phase starts. In this phase, the overall design of the wind farm needs to be specified, which include the type of turbines, the number of turbines and the location of turbines [6], the electrical system [7] and the foundations [8]. Among various design problems, Wind farm layout optimization is the basic one that has crucial importance for successful offshore wind farm development [9]. Normally, this problem is formulated as determining the locations of wind turbines inside a wind farm to maximize/minimize an objective function, such as the annual energy production (AEP) [10, 11], and levelized cost of energy (LCOE) [6, 12]. Few studies considered design variables beyond the turbine locations, such as hub heights [13], and turbine types and number of turbines [6].

The procedure in the planning phase can be termed as macro-siting, while the procedure in the design phase can be called as micro-siting [14]. Usually these two stages are carried out separately, similar to the way offshore wind farms are developed in Denmark.

Most of the studies in the literature focus on the micro-siting problem, i.e., wind farm layout optimization. In these studies, the designated area with a specific boundary is taken as a given and usually treated as a constraint. This boundary is usually specified in the planning phase by government authorities under various considerations such as wind resource, water depth, environmental impacts and other sea uses [5]. Although expertise within the technical fields of wind power as well as turbine, foundation and grid technologies is involved when specifying the boundary, the detailed layout or design of the wind farm to be constructed is usually not considered [5]. Nevertheless, this boundary will determine the possible shape and orientation of wind farm. Due to the uneven distribution of wind coming from different wind directions, this boundary can influence the performance of the designed wind farm. Therefore, an overall optimization of a wind farm design needs to consider its shape and orientation. However, this has been largely neglected in the literature as most of the studies focus on the design phase. One exception is [15], in which a sensitivity analysis of the performance of a wind farm to the aspect and orientation of its rectangle boundary was conducted.

In this study we consider the co-optimization of shape, orientation and layout of offshore wind farms for the first time. The objective of this study is twofold: firstly, to demonstrate the importance of selecting proper shape and orientation of wind farm boundary; and secondly, to develop a co-optimization method for optimizing the shape, orientation and layout of offshore wind farms. A lot of realistic aspects important for real offshore wind farms, e.g., water depth, exclusive zones and detailed cost of balance of plant, are not considered in this study, which makes the nature of this study preliminary. Nevertheless, in this study we intend to highlight the importance of considering the shape, orientation and layout of offshore wind farms together and show the potential benefit of considering layout optimization in the planning phase, especially when designating the wind farm area.

2. Problem formulation

This section presents the formulation of the co-optimization problem for offshore wind farms, which aims to optimize the shape, orientation and layout simultaneously.

2.1. Shape, orientation and layout

The majority of studies on wind farm design assume that the boundary of a wind farm is given, i.e., with a fixed shape and orientation, and the task of wind farm design optimization is to find
the best layout for wind turbines in this given boundary. In practice, this boundary is usually
determined by the energy or planning agency of the government, which often happens in the
planning phase and well before the final determination of the wind farm layout. This, of course
could potentially lead to sub-optimal solutions for a given wind farm site. For example, the
designated areas for two large offshore wind farm in Denmark were determined long before the
tendering process begins and presented in a public report that explains the tenders and invites
interested companies for dialogues [16]. The shapes of the designated areas are shown in Fig. 1.

![Designated areas for two Danish offshore wind farms](image1)

(a) Horns Rev 3  
(b) Kriegers Flak

Figure 1: Designated areas for two Danish offshore wind farms (adapted from [16])

Usually the layouts selected by the wind farm developers respect the boundaries defined by
the designated areas. Examining the layouts of existing wind farms, we can find a large variety
of boundaries, each with a different shape and orientation. Five of such wind farms are shown
in Fig. 2.

![Layouts of five offshore wind farms](image2)

Figure 2: Layouts of five offshore wind farms (adapted from [17])

In order to focus mainly on the effects of wind farm shape, orientation and layout, we neglect
some of the limiting factors that may be important in reality, such as water depth, sailing routes
and preserved zones in this study. Thus, for a wind farm occupying a given size of area, the number of potential shapes and orientations it can take is infinitely large. To make the problem of selecting proper shape and orientation solvable, we have to make some assumptions to limit the design space. Inspired by the shape of the Horns Rev 1 wind farm shown in Fig. 2, we consider a special type of wind farm shape, parallelogram.

Fig. 3 shows such a parallelogram wind farm boundary. Clearly its shape is defined by the lengths of its two edges and the angle between these two edges, which are denoted as $L_1$, $L_2$ and $\theta$, and its orientation is determined by the angle ($\alpha$) between one edge with the $x$ direction.

Assuming the area of the parallelogram is fixed as $S$, we can use three design variables, $L_1$, $\theta$ and $\alpha$ to represent the shape and orientation of a given wind farm boundary, while the remaining variable $L_2$ can be derived as:

$$L_2 = \frac{S}{L_1 \sin \theta} \tag{1}$$

For a wind farm composed of $N_{wt}$ wind turbines, its layout can be defined by their $x$ and $y$ coordinates that can be denoted as:

$$X = [x_1, x_2, ..., x_{N_{wt}}], Y = [y_1, y_2, ..., y_{N_{wt}}] \tag{2}$$

Thus, there are $2N_{wt} + 3$ independent design variables for defining the shape, orientation and layout of a wind farm occupying a given size of area in a parallelogram shape boundary. These design variables are $L_1$, $\theta$, $\alpha$, $X$ and $Y$.

### 2.2. Objective function

The optimization objective in this study is taken as maximizing the wind farm’s annual energy production (AEP):

$$\max_{L_1, \theta, \alpha} \max_{X, Y} \text{AEP}(X, Y | L_1, \theta, \alpha) \tag{3}$$

Note that the above equation means that the co-optimization problem can be divided into tasks in two stages: first to select the shape and orientation of the parallelogram boundary defined by ($L_1$, $\theta$, $\alpha$), and then to choose the optimal layout ($X$, $Y$) inside this boundary for maximizing the AEP. Details of how to solve the optimization problem in two stages are described in Section 3.

To calculate the AEP, the Jensen wake model [18] is used to compute the wake deficits. For the sake of brevity, details of how to calculate the AEP of a given wind farm using the Jensen wake model are not described here but referred to [11].
2.3. Constraints

Four kinds of constraints are considered in this study. The first kind is on the ratio of the lengths of the two edges to avoid the possibility of choosing an extremely long and narrow parallelogram:

\[
\frac{1}{R} \leq \frac{L_1}{L_2} \leq R
\]  

(4)

where \( R \) is the maximal allowed ratio.

The second kind of constraints are the minimal distance requirements between any two turbines, governed by:

\[
\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq d_{\text{min}}, \text{ for } i, j = 1, 2, \ldots, N_{\text{wt}} \text{ and } i \neq j
\]  

(5)

where \( d_{\text{min}} \) is the required minimal distance.

The third kind of constraints require all turbines to be located inside the parallelogram boundary:

\[
[x_i, y_i] \in B(L_1, \theta, \alpha), \text{ for } i = 1, 2, \ldots, N_{\text{wt}}
\]  

(6)

where \( B(L_1, \theta, \alpha) \) represents the domain defined by the parallelogram boundary.

The last kind of constraint limits the total length of the internal cable network \( CL(X, Y) \) to be less than a given threshold value \( CL_{\text{max}} \), as:

\[
CL(X, Y) \leq CL_{\text{max}}
\]  

(7)

Note that in this study \( CL(X, Y) \) is the total length of the optimal internal cable network, which is assumed to be the minimal spanning tree that connects all turbines. This network topology can be found and its total length can be calculated by Prim’s algorithm [19], a widely used greedy algorithm for minimal spanning tree problems. Details of how to calculate \( CL(X, Y) \) are referred to [20]. It is also worth to note that optimal cable network design in reality is more complex than the simple formulation used here, as different types of cable will be needed for different connections depending on the maximal power flows and also the location of substation needs to be considered. Nevertheless, we adopt this simple formulation that depends only on the total length, as electrical system optimization is not the focus of the current study.

3. Optimization method

A two-stage optimization strategy is proposed to solve the co-optimization problem defined in the previous section.

In the first stage, a heuristic algorithm is used to select good shapes and orientations (S&Os) of the wind farm. This is done by discretizing the valid ranges of \( L_1, \theta \) and \( \alpha \) and iterating over the possible combinations of these design variables. For each considered combination that satisfies the constraints defined by Eqs. (1) and (4), a grid-like layout is generated by filling \( N_{\text{wt}} \) turbines inside the parallelogram, with \( N_1 \) columns and \( N_2 \) rows along the two directions of the parallelogram’s two edges. \( N_1 \) and \( N_2 \) are determined by requiring the spacing values along two directions, i.e., \( L_1/N_1 \) and \( L_2/N_2 \), be as close as possible. The resulting layout is then checked against the constraints defined in Eq. (5). If the layout is feasible, its AEP is evaluated and the results are appended to a list of found results. From the found results, several top performing S&Os and their corresponding grid-like layouts can be selected. The process of the first stage optimization is depicted in the flow chart in Fig. 4.

As one could notice from the flowchart, the constraint on cable length, as given by Eq. (7), is not considered. This is to accommodate the fact that the grid-like layout generated in this stage can also violate the constraint on cable length. Thus, in order to explore a larger part of the design space of shape and orientation, this constraint is first neglected in this stage.
Figure 4: First stage optimization to select good shapes and orientations.

After the proper S&Os have been selected in the first stage, the second stage optimization tries to find the optimal layout for each selected S&O, starting from the corresponding grid-like layout found in the first stage optimization.

Note that since the constraint on cable length has been neglected in the first stage, the starting grid-like layout for a selected S&O can violate this constraint. To remedy this problem, the original objective function is extended by adding a penalty function term based on the cable length constraint. The extended objective function is governed by:

$$\text{max} \left[ \text{AEP}(X, Y) + w \text{max} (\text{CL}(X, Y) - \text{CL}_{\text{max}}) \right] \quad (8)$$

where $w$ is a penalty coefficient, usually taken as a large number.

For each of the selected S&O (defined by $L_k^i$, $\theta^k$, and $\alpha^k$), the second stage optimization problem becomes a conventional layout optimization problem, with objective function defined by Eq. (8) and constraints defined by Eqs. (5, 6). To solve this problem, the Random Search (RS) algorithm [11] is employed. This algorithm tries to improve the layout by iteratively moving...
a randomly chosen turbine to a random position inside the boundary while respecting all the constraints. Details of this algorithm can be found in [11].

After the second stage optimization, several optimized S&Os and layouts are obtained, which could then be further analyzed and selected by the wind farm developer, maybe by taking other design requirements and factors into consideration. Nevertheless, how to make the final decision is beyond the scope of this study, as we mainly aim to demonstrate the importance of choosing the proper shape and orientation and to propose a method to co-optimize the shape, orientation and layout of offshore wind farm.

4. Case study
To demonstrate the effectiveness of the co-optimization method, Horns Rev 1 offshore wind farm is studied here. This wind farm is located in Denmark and composed of 80 wind turbines, each with a rated power of 2 MW. Its layout can be seen in Fig. 2. In previous studies, such as in [11, 20], the layout optimization of this wind farm has been studied, assuming a fixed shape and orientation of wind farm boundary. The wind condition at this site can be described by a joint distribution of wind speed and wind direction [21], as shown in Fig. 5.

Figure 5: Joint distribution of wind speed and wind direction at the Horns Rev 1 site (adapted from [21]).

Here we assume the shape and orientation of the wind farm’s boundary can be changed. Note this boundary is required to be a parallelogram shape with the same size of area of the original wind farm. The co-optimization problem can then be solved by the two-stage optimization method proposed in this study. We solve this problem with the parameters as summarized in Table 1. Note that the area size $S$ and the cable length $CL_{max}$ are calculated from the original layout of the Horns Rev 1 wind farm.
Table 1: Parameters used in the case study.

| Parameter | Unit | Value | Meaning |
|-----------|------|-------|---------|
| $N_{wt}$  | –    | 80    | number of turbines |
| $D$       | m    | 80    | rotor diameter |
| $R$       | –    | 5     | maximal ratio between two edges |
| $S$       | km$^2$ | 19.61 | area size occupied by the wind farm boundary |
| $d_{min}$ | m    | 320   | minimal distance between two turbines |
| $w$       | –    | 1e5   | penalty coefficient for cable length constraint |
| $CL_{max}$| km   | 44.23 | maximal total cable length |

The co-optimization results, for the original and three selected S&Os, are shown in Fig. 6.

Figure 6: Grid-like initial layout and optimized layout for (a) original shape and orientation (S&O), (b) S&O 1, (c) S&O 2 and (d) S&O 3.
The performance of the optimized wind farm, including the grid-like layouts obtained in the first stage optimization and the final optimized layouts, are shown in Table 2. Note that in this table, all relative improvements (Imp) are calculated with regards to the original Horns Rev 1 wind farm.

Table 2: Performance of the grid-like and optimized layouts for different shape and orientations.

| Case     | Grid-like layout | Optimized layout | Optimized layout |
|----------|------------------|------------------|------------------|
|          | AEP [GW h]  | Imp [%] | AEP [GW h]  | Imp [%] | CL [km] | Imp [%] |
| Original | 702.44      | 0       | 704.19      | 0.36    | 38.96   | 11.91   |
| S&O 1    | 714.59      | 1.73    | 717.98      | 2.16    | 39.19   | 11.39   |
| S&O 2    | 714.05      | 1.65    | 718.09      | 2.12    | 38.46   | 13.05   |
| S&O 3    | 711.79      | 1.33    | 716.16      | 1.90    | 35.79   | 19.08   |

The results of co-optimization clearly show that different shapes and orientations can lead to quite different performance on AEP. If we stick to the shape and orientation of the original wind farm, layout optimization can manage to increase the AEP only by a small amount, i.e., 0.36%. On the other hand, if we allow a different shape and orientation, more substantial improvement (larger than 2%) can be obtained. Even if we only allow grid-like layout, a wind farm with 1.73% higher AEP can be found, when choosing S&O 2. Examining the total length of the internal cables, the optimized wind farms also significantly improves, with reductions of more than 10%. Since the occupied area is required to be the same size for different S&Os, we can conclude that the wind farms obtained by simultaneously optimizing the shape, orientation and layout are superior than the original wind farm, at least according to the problem formulation adopted in this study.

For each S&O, the results described above is obtained by applying the co-optimization method with the maximal number of evaluations set as 20000 for the RS algorithm used in the second stage optimization. The evolutionary histories of the four second stage optimization runs are shown in Fig. 7.

![Figure 7: Evolutionary histories of second stage optimization runs for different S&Os.](image-url)
From these histories, one can clearly see the importance of a good initial layout for achieving a higher AEP. In this study, this good initial layout is the grid-like layout determined by the S&O, which is selected by the first stage optimization. It demonstrates the crucial role of a wind farm’s shape and orientation.

5. Conclusions
The shape and orientation of an offshore wind farm can have a large influence on its AEP performance. Following the current common practice of separating planning and design phases, most studies on wind farm design optimization assumes the shape and orientation of the wind farm boundary as given. In this study, we propose to also include the shape and orientation in the design variables. Out of the infinite possibilities of wind farm boundary shapes, we choose to consider the parallelograms only, mainly to limit the complexity of problem formulation and also partly inspired by a real offshore wind farm, Horns Rev 1. A complete formulation of the co-optimization problem is first developed, considering realistic constraints on the shape, mutual distances between turbines and total length of internal cables. A two-stage optimization algorithm is then proposed to solve the co-optimization problem by: first selecting several good shapes and orientations with a newly developed heuristics; then optimizing the wind farm layouts using the Random Search algorithm. The case study using the Horns Rev 1 wind farm demonstrates the crucial importance of the shape and orientation and the effectiveness of the proposed co-optimization method. The findings shown in this study also suggest the potential benefits of considering layout optimization in the planning phase of offshore wind farms, i.e., taking layout optimization into consideration when designating the wind farm areas.

Future studies will consider objective functions beyond AEP, investigate wind farm boundary shapes other than parallelograms and investigate the co-optimization problem in a more realistic setting with considerations of water depths, exclusive zones and other limiting factors.

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