Velocity distribution in a turbulent flow near a rough wall

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Abstract. Velocity distribution in the zone of developed wall turbulence, regardless of the conditions on the wall, is described by the well-known Prandtl logarithmic profile. In this distribution, the constant, that determines the level of velocity, is determined by the nature of the interaction of the flow with the wall and depends on the viscosity of the fluid, the dynamic velocity, and the parameters of the wall roughness. In extreme cases depending on the ratio between the thickness of the viscous sublayer and the size of the roughness the constant takes on a value that does not depend on viscosity, or leads to a ratio for a smooth wall. It is essential that this logarithmic profile is the result not only of the Prandtl theory, but can be derived from general considerations of the theory of dimensions, and also follows from the condition of local equilibrium of generation and dissipation of turbulent energy in the wall area. This allows us to consider the profile as a universal law of velocity distribution in the wall area of a turbulent flow. The profile approximation up to the maximum speed line with subsequent integration makes possible to obtain the resistance law for channels of simple shape. For channels of complex shape with rough walls, the universal profile can be used to formulate the boundary condition when applied to the calculation of turbulence models. This paper presents an empirical model for determining the constant of the universal logarithmic profile. The zone of roughness is described by a set of parameters and is considered as a porous structure with variable porosity.

1. Introduction
The velocity distribution in the zone of developed wall turbulence, regardless of the conditions on the wall, is described by the well-known logarithmic Prandtl profile [1]

\[ \frac{V}{V_*} = 2.5 \ln \frac{y}{C} , \]

(1)

, where \( y \) – distance from wall, \( V_* = \sqrt{\tau_{st}/\rho} \) – dynamic speed.

The constant in (1), which determines the level of velocity, is defined by the nature of the interaction of the turbulent flow with the wall and depends on the viscosity of the fluid, the dynamic velocity and the parameters determining the dimensions and shape of the roughness protrusions of the walls. It is essential that this logarithmic profile is the result not only of the Prandtl theory, but can be established from general considerations of dimensional theory, and also follows from the condition of
local equilibrium of generation and dissipation of turbulent energy in this area [2]. This allows us to consider the profile as a universal law of velocity distribution in the wall region of a turbulent flow.

In accordance with the production conditions, the profile (1) is valid in a sufficiently narrow wall zone (for a round pipe it is less than 0.15 $r_o$). However, for channels of simple geometry fitting the profile to the maximum speed line with subsequent integration makes it possible to obtain the law of resistance for the channel. In this way Prandtlm obtained the formula

$$ \frac{1}{\sqrt{\lambda}} = 2 \log(Re \sqrt{\lambda}) - 0.8 $$

which is considered to be the most accurate for calculating the resistance of round tubes with smooth walls in a wide range of Reynolds numbers.

For channels of complex geometry with rough walls, a universal profile for the known $C$ can be used to formulate the boundary condition when applied to the calculation of turbulence models. These circumstances, in our opinion, emphasize the relevance of the correct definition of the constant in (1). The value of the constant $C$ depends on the ratio between the thickness of the viscous sublayer

$$ \delta_v \approx \frac{5 V}{V_c} $$

and the height of the roughness projections, which is denoted by $\delta_r$. If the roughness protrusions are buried in a viscous sublayer $\delta_v < \delta_r$, the flow does not notice the roughness and the pipe appears as smooth. The hydraulic resistance depends only on the Reynolds number.

When excess of the dimensions of the viscous sublayer by the roughness protrusions is noticeable, the constant $C$ ceases to depend on the viscosity and is determined only by the dimensions of the roughness. The coefficient of pipe resistance in this case does not depend on $Re$ and is determined only by the relative dimensions of the roughness. This mode is referred to as the "full roughness" or "absolutely rough wall" mode.

Deviation from this pattern is observed only for artificial roughness of very large sizes [3]. With further investigation, such roughness is not considered.

2. General type of parameter $C$

As noted, the "smooth wall" mode is realized when the dimensions of the projections $\delta_v$ is smaller than thickness of the viscous sublayer. According to Nikuradze's measurements [4]

$$ C_v = 2.5 \ln \frac{V}{V_c} + 5.5 $$

and the profile (1) takes the form

$$ \frac{V}{V_c} = 2.5 \ln \frac{V}{V_c} + 5.5. $$

For a channel with a specific surface roughness, when Reynolds number increases, the thickness of the viscous sublayer decreases almost inversely $Re$ ($\delta_v = d \frac{5 \sqrt{\delta}}{Re \sqrt{\lambda_r}}$) and a transition to the regime of "complete manifestation of roughness" is realized. In this case $C = C_v$, the value of $C$ "does not depend on the viscosity of the liquid and is determined by a combination of dimensional and dimensionless quantities that determine the dimensions and geometry of the rough surface. The "lower" limit value of $C$ is the same for any wall roughness and is determined according to (2), "upper" $C_w$ is individual for each kind of roughness. Description of such transition, when the ratio between the height roughness
and the thickness of the viscous sublayer is changing, i.e. when the parameter \( \frac{\delta V_0}{v} \) is changing, is possible, if we introduce a universal transition function \( \varphi(\frac{\delta V_0}{v}) \) such that

\[
C = C_\varphi (1 - \varphi) + C_\varphi \varphi
\]

(3)

As \( \frac{\delta V_0}{v} \) increases, the value of \( \varphi \) changes from zero to one.

At the present time the most studied type of roughness is the sand roughness created by Nikuradze by applying a monolayer of grains of equal size with the help of glue on the inner surface of the pipe a monolayer of grains of equal \( \Delta \) [5]. According to his measurements, the regime of full manifestation of roughness occurs when \( \frac{\Delta V_0}{v} \geq 70 \) and

\[
C_\varphi = 8.48 - 2.5\ln \Delta.
\]

(4)

The coefficient of hydraulic resistance of a round pipe with sand roughness of the walls in the mode of full manifestation of roughness is found in the form

\[
\frac{1}{\sqrt{\lambda}} = 21\log \frac{d}{\Delta} + 1.17.
\]

(5)

The logarithmic profile (1) can be represented in another equivalent form [4,6]

\[
\frac{V_0}{V_s} = 2.5\ln \frac{y}{\delta_t} + \Phi,
\]

(6)

where \( \delta_t \) – height roughness. The constant \( \Phi \) in (6) is related to the constant \( C \) in (1) by the obvious relation

\[
\Phi = C + 2.5\ln \delta_t.
\]

(7)

The parameter \( \Phi \) in accordance with the definition (6) can be determined from the experimental data on the velocity distribution \( V_0(y) \) in the near-wall region of the flow. The results of the determination of \( \Phi \) for the sand roughness of the wall, obtained from the experimental measurements of Nikuradze [5] and presented in [4,6], are shown in figure 1.

In the same papers, as noted in [4], in order to increase the reliability of the data, the results of the determination of \( \Phi \) from the experimental data on the resistance of pipes with sand roughness of the walls (dark dots) are given. In the latter case, to determine \( \Phi \) the relation is used

\[
\sqrt{\frac{8}{\lambda}} = 2.5\ln \frac{\delta_t}{\Delta} - 3.75 + \Phi,
\]

(8)

which is obtained as a result of averaging (6) over the pipe cross section, taking into account the relationship between the dynamic and average velocities \( \frac{V_0}{V_\varphi} = \sqrt{\frac{\lambda}{8}} \).

The data obtained by different methods, in fact, are in agree, that, in our opinion, without disproving the reliability, rather indicates the admissibility of profile fit (6) when determining the resistance or, what is the same, admissibility of fitting (1) on the central part of the flow up to the axis of the pipe. For the parameter \( \Phi \) the identity (3) is identically true for the same transition function \( \varphi(\frac{\delta V_0}{v}) \)

\[
\Phi = \Phi_\varphi (1 - \varphi) + \Phi_\varphi \varphi,
\]

(9)
Where the limiting values $\Phi_r$ and $\Phi_m$ in accordance with (2), (4) and (7) are equal for the sand roughness

\[ \Phi_s = 2.5 \lg \frac{\Delta V_s}{V} + 5.5, \quad (10) \]

\[ \Phi_r = 8.48. \quad (11) \]

Limit values (10 and 11) are consistent with the data in Figure 1. The straight line in the left part of the figure corresponds to (10). Sandy roughness is "unique," the uniqueness lies in the fact that it is characterized by a single scale-the diameter of a grain of sand. Diameter of a grain of sand $\Delta$ determines the height of roughness $\delta = \Delta$, and characterizes the scale of the change in properties in transverse directions. This property of sand roughness - the uniqueness of the scale, allows us to use the data on $\Phi_s$, presented in [6], to determine the transition function. Comparing with (9), for $\varphi(\frac{\Delta V_s}{V})$ we have

\[ \varphi = \frac{\Phi - \Phi_s}{\Phi_r - \Phi_s}, \quad (12) \]

, where $\Phi_r$ and $\Phi_s$ are given (10) and (11).

The results obtained after the digitization of the curve and the calculations by (12) are shown in figure 1.

![Figure 1 - Dependence of the weight function $\varphi(\delta_s V_s / V)$ vs $\lg(\delta_s V_s / V)$](image)

The dependence approximating the curve in figure 1 has the form

\[ \varphi(\frac{\delta_s V_s}{V}) = -0.27 + 1.16 \cdot \lg \frac{\delta_s V_s}{V} - 0.26 \cdot \lg^2 \frac{\delta_s V_s}{V}. \quad (13) \]

With a known transition function $\varphi(\frac{\delta_s V_s}{V})$, the velocity distribution in the near-wall region of a developed turbulent flow is determined by a single parameter $\Phi_r$, videlicet

\[ \frac{V}{V_s} = 2.5 \ln \frac{y}{\delta_s} + \Phi_r (1 - \varphi) + \Phi_s \varphi \]

or taking into account that $\Phi_s$ it is the same for any roughness (10)
Thus, the problem of a complete description of the influence of roughness on the interaction of a turbulent flow with a wall has been reduced to the determination of a single parameter $\Phi_f$, which value depends only on the quantities that determine the size and geometry of the roughness.

3. Parameters determining the structure of wall roughness

To describe the individual geometric properties of roughness, which affect the flow patterns, it is necessary to introduce a set of dimensions and dimensionless quantities, while focusing on the characteristics of roughness introduced by state standards [7].

The main parameter is a parameter with length dimension, characterizing the altitude roughness size $\delta_f$, introduced already in determining the transition function $\phi$, which is used for comparison with the thickness of a viscous sublayer in describing the distribution of velocities $(\delta_f/V, / \nu)$ and with the characteristic size of the channel as a whole in determining its hydraulic resistance $(\delta_f / d)$. The value $\delta_f$ correlates with the parameter $\text{Ra}$ introduced by GOST [8].

As a characteristic latitudinal scale of roughness $\delta_f$, the "average pitch of irregularities in the $S_m$ profile" is to be used, determined by GOST—the length of the segment of the roughness divided by three points of intersection of the profilogram with a straight line at the level of half the height of the roughness (Figure 2). The points of intersection are distinguished on the profilogram by one projection and one cavity.

![Figure 2 - Scheme of the profile unevenness step](image)

For natural roughness, the latitudinal dimension, as a rule, considerably exceeds the height of the roughness [7].

The roughness zone can be considered as a porous structure, part of the volume of which is a solid component, and a part is filled with a liquid. The porous structure of the roughness is "simplified," in the sense that in each component there are no isolated inclusions of the other component. In other words, the components are simply connected. If we determine the porosity in the structure in the $y$-section from the wall, as the ratio of the area occupied by the liquid phase to the sum of the phase areas, the porosity in the rough structure will vary from one outside the roughness ridges (at $y > \delta_f$) to zero. The law of porosity variation along the thickness of the structure $\varepsilon(\frac{y}{\delta_f})$ is determined by the shape of the roughness protrusions and, therefore, is also a factor determining the interaction of the flow with the wall. The average value of the roughness porosity $\bar{\varepsilon}$, another quantitative parameter that determines the wall roughness, depends on the form of the function $\varepsilon(\frac{y}{\delta_f})$.

Thus, the minimal, as it seems to us, set of parameters characterizing the roughness includes: $\delta_f$—altitude parameter, determining the average height of the roughness protrusions; $\delta_f$—Latitude parameter equal to the profile unevenness step and defined by GOST, $\bar{\varepsilon}$—average porosity value of the roughness zone, or (and) the law of porosity variation along the height of the roughness.
4. Natural roughness

Roughness, formed naturally in the process of manufacturing and operating the surface. For simplicity, we assume that it is homogeneous; Properties do not change when moving along the surface and isotropic – the properties do not depend on the direction of flow past the surface.

Such roughness can be considered as a set of roughness protrusions of medium height \( \delta_{\perp} \), evenly distributed over the surface. The latitudinal parameter \( \delta_{\perp} \), characterizing the periodic property of the structure, is naturally identifiable with the average distance between the ridges of the roughness. При этом средняя площадь, приходящаяся на один выступ, будет пропорциональна \( \delta_{\perp}^2 \). Indeed, the densest arrangement of the protrusions with the distance \( \delta_{\perp} \) between the centers corresponds to a triangular package, while \( S_1 = \frac{\sqrt{3}}{2} \delta_{\perp}^2 = 0.721 \delta_{\perp}^2 \). With the most free square packing of projections with the distance between the centers \( \delta_{\perp} \), \( S_1 = \delta_{\perp}^2 \). For a uniformly chaotic package of projections with the same average distance between the protrusions \( \delta_{\perp} \), for an average area per one projection, let us take the average of the limiting values

\[
S_1 = 0.86 \delta_{\perp}^2. \tag{15}
\]

\( S_1 \) represents the area of an equivalent cell of a roughness structure containing one projection. Assuming for simplicity that the cell is cylindrical, we find for the radius \( r_0 = \left( \frac{S_1}{\pi} \right)^{0.5} = 0.523 \delta_{\perp} \).

The equivalent projection also has the form of a circle in cross section. At the base the radius \( r_0 \) of the protrusion, in the upper part (at \( y = \delta_{\perp} \)) zero, the height of the radius is determined by the porosity.

\[
r(y) = r_0 \left( 1 - \varepsilon \right)^{0.5}. \tag{16}
\]

If we take \( \varepsilon = \left( \frac{y}{\delta_{\perp}} \right)^n \), then for the average porosity of the structure of the roughness we find

\[
\varepsilon = \frac{1}{1 + n}, \quad \text{and}
\]

\[
n = \frac{1}{\varepsilon} - 1. \tag{17}
\]

Taking into account (16) and (17) for the equivalent projection, we finally have

\[
r(y) = 0.523 \delta_{\perp} \left( 1 - \left( \frac{y}{\delta_{\perp}} \right)^n \right)^{0.5}, \tag{18}
\]

where \( n \) is determined by the average porosity value \( \varepsilon \) (17).

The shape of the roughness protrusion at different values \( \delta_{\perp} \), \( \delta_{\parallel} \) and \( n \) is shown in Figure 3.
From the general analysis in point 1 it follows that for the case of natural roughness of the walls in the regime of complete roughness manifestation it should depend on $\delta_\perp$, $\delta_\parallel$, and related $n$ or $\varepsilon$. As $\Phi_r$ – dimensionless parameter, then

$$\Phi_r = \Phi_r(\frac{\delta_\perp}{\delta_\parallel}, \varepsilon). \quad (19)$$

The form of the function (19) can be determined by direct calculation of the surface flow formed by the regular arrangement of the roughness protrusions from Fig. 4 for various $\delta_\perp$, $\delta_\parallel$, and $\varepsilon$ at the regime of full manifestation of roughness, i.e. when $\frac{\delta_\perp V_\parallel}{V} > 70$.

The value $\Phi_r$ is determined either by the velocity distribution (6) or by the resistance of the round tube (8). Because methods for determining $\delta_\parallel$ and $\delta_\perp$ for roughness are known for use (19) for prognostic purposes, additional studies are needed to determine the average porosity of the structure of natural roughness.

5. Artificial roughness

The development of surfaces with artificial roughness and the study of their hydraulic characteristics was carried out with the aim of intensifying heat transfer with their help [9, 10]. For the most part, the artificial roughness is created by slicing, winding, rolling on the surface of projections of various shapes. The protuberances flow around the transverse flow of the liquid or at an angle that differs little from the direct one, determined by the ratio of the pitch and the cutting initiation (winding) and the diameter of the channel. Various types of artificial roughness are shown in figure 4.
For artificial roughness, the parameters introduced in clause 1 are most easily defined. The height dimension $\delta_\parallel$ is equal to the winding height, the latitudinal dimension $\delta_\perp$ is equal to the winding pitch, and the shape of the projections is uniquely determined by the height distribution of the porosity.

As an example, consider the roughness formed by rectangular projections of height "k" and width "b" located in step "s" (figure 4). For this roughness $\delta_\parallel = k$, $\delta_\perp = s$, and the porosity of the rough structure does not vary in height and is equal to $\varepsilon = 1 - \frac{b}{s}$. From the last relation follows $b = \delta_\perp (1 - \varepsilon)$. For the roughness under consideration, the paper [11] gives data for the parameter C + D, in our transcription of the value of $\Phi$ in the regime of full manifestation of roughness, i.e. $\Phi_r$, depending on the $\frac{s}{k} \equiv \frac{\delta_\perp}{\delta_\parallel}$ for different values of the ratio $\frac{k}{b} \equiv \frac{\delta_\parallel}{\delta_\perp} \frac{1}{1 - \varepsilon}$. Digitizing the graphic data of Figure 6 using the program GetData Graph Digitizer allowed to obtain an array $(\frac{\delta_\perp}{\delta_\parallel}, \varepsilon; \Phi_r)$ (See Table 1), a numerical description of the dependence (19).

![Figure 5](image)

**Figure 5** - Dependence of the parameter $\Phi_w$ on the relative step of the elements of roughness for values $\frac{k}{b}$ equal to 10 (1), 5 (2), 2 (3), 1 (4), 0.7 (5), 0.5 (6)

**Table 1** - Digitized data $\Phi_r$ on Figure 6 and the results of the calculation of $\Phi_w$ using formula (20)

| L  | 3,0    | 4,0    |
|----|--------|--------|
| $\varepsilon$ | 0,833  | 0,750  | 0,950  | 0,975  |
| $\Phi_r$   | 5,938  | 5,397  | 3,535  | 2,850  |
| $\Phi_w$   | 4,700  | 5,201  | 4,159  | 3,325  |

| L  | 5,0    | 6,0    |
|----|--------|--------|
| $\varepsilon$ | 0,714  | 0,917  | 0,967  |
| $\Phi_r$   | 5,348  | 5,606  | 3,328  | 2,830  |
| $\Phi_w$   | 5,304  | 5,507  | 4,118  | 3,007  |
In the graphical form, the dependence $\Phi_1(\frac{\delta}{\delta_t}, \varepsilon)$ is shown in Figure 6 (data is smoothened).

| L  | 7.0          | 8.0          |
|----|--------------|--------------|
| $\varepsilon$ | 0.714 0.796 0.857 0.929 0.971 0.986 0.750 0.821 0.875 0.938 0.975 0.988 |
| $\Phi_t$   | 4.493 3.805 3.449 3.114 2.688 2.364 3.882 3.519 3.275 3.037 2.661 2.358 |
| $\Phi_c$   | 4.916 4.236 3.726 3.130 2.773 2.654 4.425 3.829 3.383 2.862 2.549 2.445 |

In the graphical form, the dependence $\Phi_1(\frac{\delta}{\delta_t}, \varepsilon)$ is shown in Figure 6 (data is smoothened).

The dependence is close to linear. By the method of least squares with a standard deviation of 10% it was obtained

$$\Phi_t = 12.23 + 0.19 \cdot L - 8.34 \cdot \varphi.$$  \hspace{1cm} (20)

The deviation can be significantly reduced by using a nonlinear representation of the dependence, but it is advisable to do this using the original experimental data array. In this case, for the resistance of a pipe with a given roughness in the regime of absolute wall roughness, we obtain:

$$\sqrt{\frac{8}{L}} = 2.5 \ln \frac{\rho_t}{\lambda} - 3.75 + \Phi_w(\frac{\delta}{\delta_t}, \varepsilon),$$ \hspace{1cm} (21)

Where $\Phi_w(\frac{\delta}{\delta_t}, \varepsilon)$ is given by (20).

Equating (21) to the resistance of a round tube with a sand roughness $\Delta$ in the mode of full manifestation of the roughness

$$\sqrt{\frac{8}{\lambda}} = 2.5 \ln \frac{\rho_t}{\Delta} - 3.75 + 8.48,$$ \hspace{1cm} (22)

we find for our roughness with rectangular projections the value of the equivalent sand roughness

$$\Delta = \delta_t \exp(0.4 \cdot (8.48 - \Phi_t)).$$ \hspace{1cm} (23)
Knowledge of $\Phi_r$ allows us to describe the parameter $\Phi$ in an arbitrary case, since the transition function $\varphi(\frac{\delta V_i}{\nu})$ is assumed to be the same for any roughness, namely

$$\Phi = \Phi_s + (\Phi_i - \Phi_s) \varphi(\frac{\delta V_i}{\nu}),$$

(24)

according to (10) $\Phi_s = 2.5 \lg \frac{\delta V_i}{\nu} + 5.5$, $\varphi$ is given by (13). The coefficient of hydraulic resistance of a circular pipe in the general case as a function of the Reynolds number and roughness parameters is determined by the transcendental equation for $\Phi$ according to (24)

$$\sqrt{\frac{8}{\lambda}} = 2.5 \ln \frac{d}{2\delta_t} - 3.75 + \Phi_s(\frac{\delta V_i}{\nu}, \frac{\delta_t}{\delta_t}, \varepsilon),$$

(25)

where $\frac{\delta V_i}{\nu} = \frac{\delta_t}{d} \text{Re} \sqrt{\frac{2}{8}} \cdot \text{Re} = \frac{V_0 d}{\nu}$.

6. Conclusion

With the help of the introduced function of transition to the regime of complete manifestation of roughness (13), the velocity distribution in the zone of developed near-wall turbulence was obtained to within a scalar parameter $\Phi_r$ (14), the numerical value of which does not depend on the flow regime and fluid properties, but is determined only by the dimensions and geometry of the roughness zone. To determine $\Phi_r$, it is supposed using the values of high and latitude scales of roughness and porosity of rough structure provided by GOST.

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