This article summarizes the proposal to address the cosmological constant problem within the framework of supersymmetric large extra dimensions (SLED). The proposed mechanism is described, emphasizing the relaxation mechanism which ensures that low-energy particles like the electron do not contribute too large a vacuum energy. This is followed by a preliminary discussion of outstanding issues and observational consequences.

1 Introduction

Current measurements indicate that the universal expansion is currently accelerating in a way which is consistent with the existence of a small but nonzero cosmological vacuum energy density which is of order $\rho = v^4$, with $v \sim 3 \times 10^{-3}$ eV in units for which $\hbar = c = 1$. This discovery is a challenge because all known theories of microscopic physics appear to predict that a particle of mass $m$ contributes to $\rho$ an amount which is of order $\delta \rho \sim m^4$, and almost all known elementary particles have masses much larger than $10^{-3}$ GeV. This is particularly disturbing because it indicates a problem with our description of low energy physics — such as the physics of the electron (for which $m_e = 5 \times 10^5$ eV) — which we normally think we understand.

Supersymmetry is one of the few symmetries which may be helpful. This is because sufficiently many supersymmetries can explain forbid a cosmological constant in the theory at very short distances. Furthermore, even 4D $N = 1$ supersymmetry can also partially explain why the process of integrating out particles does not produce too large a vacuum energy, since supersymmetry enforces a cancellation between the contributions of bosons and fermions. Unfortunately, this cancellation is only partial if supersymmetry is spontaneously broken, leaving a residual value which must vanish either as $m_{sb}^2 M^2$ or $m_{sb}^4$, where $M$ is the largest mass scale in the problem and $m_{sb}$ is the supersymmetry-breaking scale. Unfortunately, the absence of observed superpartners for particles like the electron already implies that $m_{sb}$ must be at least as large
as hundreds of GeV.

1.1 Supersymmetric Large Extra Dimensions (SLED)

In the SLED proposal\textsuperscript{1} it is supposed that progress can be made in understanding the observed size of the vacuum energy within the framework of supersymmetric large extra dimensions. In this picture the world has two unwarped extra dimensions whose radii satisfy \(1/r \sim v \sim 10^{-2} \) eV — in which case \( r \sim 1/v \sim 0.01 \) mm — such as has been proposed in the Large Extra Dimensional (LED) scenario.\textsuperscript{2} Dimensions this large can have escaped observational detection provided all of the known particles (except the graviton) are trapped on a 4-dimensional surface (3-brane) which is localized within the two extra dimensions. Dimensions this large have previously been motivated to understand the hierarchy problem, since they predict that the effective 4D Planck scale, \( M_p \), is related to the underlying scale of 6D gravity, \( M \), by the relation \( M_p^2 = M^2 r \). If \( 1/r \sim 0.01 \) eV then the correct size, \( M_p \sim 10^{19} \) GeV, is obtained for \( M \) near the weak scale: \( M \sim 10 \) TeV.\textsuperscript{a}

The difference between the SLED and LED proposals is that the theory is also required to be supersymmetric, as might naturally be expected if it were to arise as a consequence of an underlying theory such as string theory. In this case the absence of supersymmetry in collider experiments can be understood if supersymmetry is strongly broken on our brane, perhaps at the scale \( M \sim 10 \) TeV. The main requirement of the SLED proposal is that the supersymmetry breaking scale in the 6D bulk is the natural size which would be inherited due to its gravitational strength couplings to the branes, that is \( m_{sb} \sim M^2/M_p \sim 1/r \sim v \sim 0.01 \) eV. In this case the observed vacuum energy density is close to the supersymmetry-breaking scale in the bulk.

2 The Cosmological Constant Problem Seen from 6 Dimensions

This section describes the contributions to the observed vacuum energy which can be expected within the SLED framework. In order to do so we first imagine integrating out all brane modes; and then integrating out all of the bulk physics to obtain the effective vacuum energy on scales much larger than the extra-dimensional size, \( r \).

2.1 Dynamical Relaxation and the Cancellation of Brane Tensions

One of the biggest problems for understanding the size of the vacuum energy has always been how to understand why well-understood particles like the electron do not contribute too large an amount. Since supersymmetry is badly broken on the brane within SLED, integrating out the known particles (like the electron) indeed does contribute a large vacuum energy density, which in total is naturally of order \( M^4 \). But this energy density is not a cosmological constant. Rather, it is a contribution to the tension of the brane — \( \delta T \sim M^4 \) — and so is an energy source which is localized within the extra dimensions at the position of the brane. We must ask how this energy source curves the extra dimensions, and then see what the implications are for the effective 4D cosmological constant which would be observed on distance scales larger than \( r \sim 0.1 \) mm.

It happens that the curvature of the extra dimensions due to this localized energy source can be computed using the classical equations of motion in the bulk, with the result that the geometry acquires a conical singularity at the position of the branes. This singularity contributes

\textsuperscript{a}We use here the Jordan frame, for which \( M_p \) is \( r \) dependent but the electroweak scale, \( M \), is not. It is in this frame that the Kaluza-Klein mass scale is \( m_{\text{KK}} \sim 1/r \). The constraint \( r < 0.01 \) mm comes from the requirement of not having excessive energy loss from astrophysical objects like supernovae.\textsuperscript{3}
a delta-function contribution to the two-dimensional curvature given by

\[ R_2 = -\frac{2}{e_2} \sum_i \left( \frac{T_i}{M^4} \right) \delta^2(y - y_i) + \ldots, \]

(1)

where \( e_2 \) is the volume element for the internal two dimensions, \( y_i \) denotes the position of the \( i \)'th brane and the ellipses denote contributions to \( R_2 \) which are smooth at the position of the brane.

The leading contribution to the effective 4D cosmological constant on long distance scales is obtained by integrating out the bulk gravitational degrees of freedom at the classical level. In this result an interesting cancellation occurs between the brane tensions and the classical bulk curvature given by eq. (1). The effective 4D cosmological constant obtained at this order is

\[ \rho_{\text{cl}} = \sum_i T_i + \int_M d^2y \ e_2 \left[ \frac{M^4}{2} R_2 + \ldots \right] = 0, \]

(2)

where the sum on \( i \) is over the various branes in the two extra dimensions and \( \ldots \) denotes all of the other terms besides the Einstein-Hilbert term in the supersymmetric bulk action. Interestingly the sum over brane tensions, \( T_i \), precisely cancels the contribution of the singular part of the curvature, eq. (1), to which they give rise and for supersymmetric theories the same kind of cancellation also occurs amongst the remaining terms in \( \rho_{\text{cl}} \) once these are evaluated using the smooth parts of the geometry and the other bulk fields obtained using the classical field equations. This cancellation has its roots in a classical scale invariance of the 6D supergravity action.

The above arguments have recently been verified in a very explicit way, through the construction of a very wide class of solutions to the classical field equations of 6D supergravity. The authors of this reference construct the most general 2-brane solutions which are (i) maximally symmetric in the 4 large dimensions, (ii) axially symmetric in the 2 extra dimensions, and (iii) for which the 6D dilaton — a 6D superpartner to the metric — and warp factor are nonsingular at the position of the two branes. The intrinsic geometry of the large 4 dimensions is flat for every single one of these solutions, as the above arguments would require. In this way we see very robustly how the classical bulk response can systematically cancel the contributions of any brane tensions to the effective 4D vacuum energy.

2.2 Bulk Loops

The above is a classical argument within the bulk, and so a central question asks for the size of quantum corrections. It is now argued that these can be acceptably small provided that the supersymmetry breaking scale in the bulk is \( m_{sb} \sim 1/r \sim v \), as argued earlier. As we have seen, a contribution of the form \( \delta \rho \sim m_{sb}^4 \) gives precisely the observed value when \( m_{sb} \) is this large.

Although \( m_{sb}^4 \) can arise for the vacuum energy in a supersymmetric theory, it is not automatic since supersymmetry just requires the result to vanish as \( m_{sb} \to 0 \). In particular, it does not preclude the appearance of contributions of order \( m_{sb}^2 M^2 \). Such terms also have a natural interpretation in 6 dimensions as arising as local curvature-squared contributions to the effective 6D action as high-energy modes having wavelength \( \lambda \sim 1/M \) are integrated out. Although such contributions can arise for some 6D supergravities, they need not for all supergravities.

Detailed calculations to determine the size of the quantum contributions are underway, and at present suggest that several conclusions may be drawn. In general there appear to be two reasons why such curvature-squared terms do not contribute for some 6D supergravities.

\[ ^{b} \]This last condition is equivalent to choosing the brane-dilaton coupling to preserve the classical scale invariance of the bulk supergravity action.\[ ^{8} \]
Their presence can be forbidden by the existence of additional supersymmetries at the TeV scale $M$, such as what would arise if the 6D theory were to arise as the low-energy limit of a higher-dimensional supergravity at these scales. Alternatively, even if they arise these local curvature-squared terms may vanish once evaluated at the classical solution, such as would occur for flat toroidal solutions. Since the 6D classical field equations can tie the value, $\phi$, of the 6D dilaton to the size of the extra dimensions by $e^{\phi} \sim 1/r^2$, it can also happen that it is sufficient to have the dangerous $m_{sb}^2 M^2$ terms vanish only for a few orders in the loop expansion (such as one loop) in order to obtain a satisfactorily small vacuum energy.8

Indirect evidence that ultraviolet extensions of 6D supergravity exist for which the loop corrections are of order $\rho \sim m_{sb}^4$ comes from direct one-loop string-based calculations of the vacuum energy. These calculations have been performed for toroidal compactifications with supersymmetry broken by boundary conditions and/or the presence of branes,9 and give a result which is of order $\rho \sim m_{sb}^4 \sim 1/r^4$.

2.3 Topological Constraints

The ability to identify broad classes of solutions in 6D supergravity7,6 allows one to see whether or not the classical cancellation of the vacuum energy has somehow been ensured through a hidden fine-tuning of some of the parameters of the theory. This kind of hidden fine-tuning has been been raised as an objection10 to the 5D models of ref.11. Examination of the general solutions shows that for all of them the brane tensions (and other parameters) indeed satisfy two constraints. These constraints simply express the topological statement that the Euler number of the internal geometry is the same as for a 2-sphere, and that the background Maxwell fields in the solutions have the Chern class appropriate to a magnetic monopole.7,6 These constraints should be regarded as being topological integrability conditions which must be satisfied in order for a solution to the classical field equations to exist, but for all choices for which solutions exist the intrinsic 4D geometry is flat.

As far as naturalness is concerned, the central point is whether these constraints remain satisfied as successive scales are integrated out from the high scale, $M \sim 10$ TeV, to the low scale $1/r \sim 10^{-2}$ eV. The topological constraints are expected to be stable against integrating out the scales between $M$ and $1/r$ precisely because they are topological. Being topological, they state that a particular combination of brane and bulk quantities is an integer and so cannot renormalize as short-distance modes are integrated out.6

The topological constraints do contain a danger, however, since they show is that for generic tensions the internal 2D geometry warps. This is dangerous from the present perspective because a vacuum energy of order $m_{sb}^4$ is only small enough if $m_{sb}^4 \sim 1/r \sim 0.01$ eV. But the value of $r$ is also fixed by the hierarchy problem, and typically only takes the numerical value $0.01$ eV in the unwarped case.6 This is because warping in general also contributes to the hierarchy — à la Randall and Sundrum12 — and so can allow the Kaluza Klein masses to be larger than in the unwarped case.

3 Observational Consequences

Although it is motivated by the cosmological constant problem, the SLED proposal has many rich experimental implications: for cosmology, for tests of gravity and for high-energy accelerators. Many of these implications are interesting in their own right, since they are very different from the usual paradigm of weak-scale supersymmetry breaking which leads to the Supersymmetric Standard Model. This section closes with a summary of some of these potential implications.13

- Late-Time Cosmology As might be expected for a relaxation mechanism, given that the extra dimensions are presently $r \sim 0.01$ mm in size the 6D SLED proposal implies the
existence of a very light scalar field whose mass at present is of order the present-day Hubble scale: \( m_\phi \sim H_0 \sim (M_p r^2)^{-1} \sim 10^{-33} \text{ eV} \). It is remarkable that such a small scalar mass is stable against radiative corrections provided that the corrections to the 4D vacuum energy are of order \( 1/r^4 \), as argued above. As a result the phenomenology of the Dark Energy naturally takes a ‘quintessence’ form, with an Albrecht-Skordis potential. Studies are underway to determine whether such cosmologies can satisfy observational constraints, such as the limits on the Dark Energy equation of state and limits on the evolution of Newton’s constant.

- **Early-Time Cosmology** LED models are known to be subject to strong constraints, such as due to the requirement that there not be too much population of bulk modes during nucleosynthesis. SLED models typically feel these bounds even more strongly due to the larger number of states which are available in the bulk. In particular it is not yet clear how weak-scale cosmology proceeds in these models, including issues such as the origin within SLED of Dark Matter.

- **Long-Range Tests of Gravity** The existence of such a light scalar implies the existence of a very long-range force which acts in competition with gravity. Strong limits exist on such forces which provide potentially severe constraints on 6D SLED models. A potential loophole which these models may use to thread these bounds relies on the time-dependence of the scalar couplings to ordinary matter.

- **Astrophysical Constraints** 6D LED models are known to be subject to strong constraints from supernovae and astrophysics, which constrain the rate with which such systems can drain their energy by radiation into the extra dimensions. This is the origin of the estimate that \( M \sim 10 \text{ TeV} \) (and hence \( r \sim 0.01 \text{ mm} \)).

- **Short-Range Tests of Gravity** LED and SLED models predict deviations to Newton’s Law at sub-millimetre distances due to the opening up there of the extra dimensions. However for SLED the successful description of the vacuum energy density fails if \( r \) becomes much smaller than 0.01 mm, and so there is a definite prediction of the scale at which new phenomena must become visible. The predicted range for deviations from Newton’s Law is of order \( r/(2\pi) \sim 1 \mu \text{m} \), and shorter distances cannot be entertained without giving up the explanation for the vacuum energy.

- **Accelerator Physics** Since gravitational physics must be at the TeV scale, there should be observable signals of extra-dimensional gravity at TeV-scale accelerators like the Large Hadron Collider just as for the non-supersymmetric LED picture. Although detailed modelling of the resulting signatures is in its infancy, the resulting missing-energy signals would look very different from what would be expected for supersymmetric extensions of the Standard Model.

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