A Note on the Collision of Reissner-Nordström Gravitational Shock Waves in AdS

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Abstract

We study the collision of two Reissner-Nordström gravitational shock waves in AdS and show that the charge completely prevents the formation of marginally trapped surfaces of the Penrose type with topology $S^{D-2}$, independently of the energy and the value of the impact parameter. In the case of head-on collisions, a numerical analysis shows that no trapped surfaces with topology $S^1 \times S^{D-3}$ form either.
Introduction. The study of collision of gravitational waves has been a very active area of research in the field of general relativity (see [1] for a summary of results). The interest in the subject has been renewed in the context of the AdS/CFT correspondence [2]. The underlying reason is the expectation that colliding waves in AdS spacetime could provide a reliable gravitational dual of the high-energy collision of two “nuclei”, modeled by energy lumps in the holographic strongly coupled gauge theory [3, 4]. Although these energy lumps do not reproduce all the properties of heavy ions, these theoretical experiments can be used as models where collective gauge theory effects, relevant for realistic heavy ion collisions, are tractable. An expected outcome of the collisions of two gravitational waves is the formation of a black hole. Holographically, black hole formation can be interpreted as the thermalization of the gauge theory plasma resulting from the collision of the lumps.

In spite of the recent progress in the numerical study of the problem of the collision of two gravitational waves [5], a full understanding of the highly nonlinear dynamics dominating the physics in the interaction region is still missing. An alternative to a full numerical simulation of the collision process (in the spirit of numerical general relativity) is to look for the formation of a trapped surface that would signal the eventual presence of an event horizon [3]. Although in many cases this also involves numerical analysis, the approach is technically simpler. The idea [6] consists in looking for marginally trapped surfaces with topology $S^{D-2}$, lying on the hypersurface \( \{u = 0, v < 0\} \cup \{u < 0, v = 0\} \), which does not involve solving for the geometry in the interaction region. The information about the collision of the waves is nevertheless encoded in the nontrivial matching condition at \( u = v = 0 \). In the following, we will refer to these marginally trapped surfaces as of Penrose type.

Varying the parameters of the collision, it is possible to find thresholds for the formation of Penrose trapped surfaces. Holographically, this can be seen as a threshold for the onset of the plasma thermalization after the collision takes place. This is what happens, for example, for large enough impact parameter [7] [8], or when the sizes of the two colliding lumps are very different [8]. A more intriguing threshold was found in [9] associated with the transverse size of the gravitational source of the shock wave in AdS, which however does not induce a change in the holographic energy-momentum tensor.

Trapped surfaces with topology $S^{D-3}$ in the collision of RN-AdS shock waves. It would be desirable to extend the analysis of the formation of marginally trapped surfaces of
Penrose type to more general types of incoming waves, in particular those obtained taking an
infinite boost limit \cite{10} of a $D$-dimensional Reissner-Nordström (RN) solution asymptotically
anti-de Sitter (AdS)

$$ds^2 = -\left[1 - \frac{2G_NM}{r^{D-3}} + \frac{G_NV^2}{r^{2(D-3)}} + \frac{r^2}{L^2}\right]dt^2$$
$$+ \left[1 - \frac{2G_NM}{r^{D-3}} + \frac{G_NV^2}{r^{2(D-3)}} + \frac{r^2}{L^2}\right]^{-1}dr^2 + r^2d\Omega_{D-2},$$

(1)

where $L$ is the radius of AdS, and $M$ and $Q$ are given in terms of the mass $m$ and electric
charge $q$ by

$$M = \frac{8\pi m}{(D-2)\Omega_{D-2}}, \quad Q^2 = \frac{8\pi q^2}{(D-2)(D-3)},$$

(2)

with $\Omega_N$ the volume of the $N$-dimensional sphere $S^N$. Performing a boost with Lorentz parameter $\gamma$, the shock wave geometry is obtained in the limit $\gamma \to \infty$ while keeping

$$\mu \equiv \gamma M, \quad e^2 \equiv \gamma Q^2$$

(3)

fixed \cite{11, 12, 13}. The metric takes the form (in Poincaré coordinates)

$$ds^2 = \frac{L^2}{z^2} \left[-dudv + dz^2 + d\vec{x}_T^2 + \frac{z}{L}\Phi(q)_{RN}\delta(u)du^2\right].$$

(4)

The metric function $\Phi(q)_{RN}$ depends on the AdS chordal coordinate

$$q = (z - L)^2 + \vec{x}_T^2$$

(5)

and can be expressed as

$$\Phi(q)_{RN} = \Phi(q)_D - \frac{e^2}{2\mu}\Phi(q)_{2D-3},$$

(6)

where $\Phi(q)_D$ is the profile of the Aichelburg-Sexl shock wave in AdS$_D$ \cite{15, 3}. Therefore we have

$$\Phi(q)_{RN} = L^{2^{3-D}\sqrt{\pi}} G_N\left(\frac{D}{2}\right) \frac{G_NM}{L^{D-3}} q^{2-D} 2F_1 \left(D - 2, \frac{D}{2} ; D ; -\frac{1}{q}\right)$$

$$- L^{2^{5-2D}\sqrt{\pi}} G_N e \left(\frac{2D-3}{2}\right) q^{5-2D} 2F_1 \left(2D - 5, \frac{2D-3}{2} ; 2D - 3 ; -\frac{1}{q}\right).$$

(7)
To avoid confusion, we recall that the energy parameter $\mu = \gamma M$ used here and in [13] is related to the parameter $E = \gamma m$ of Ref. [3] by the rescaling $E = \frac{D-2}{8\pi} \Omega_{D-2} \mu$.

We analyze the problem of collision of two shock waves. Generically, in the region outside the interaction wedge, the metric reads

$$
\begin{align*}
&ds^2 = \frac{L^2}{z^2} \left[ -du dv + dz^2 + dx_T^2 + \frac{z}{L} \Phi_-(z, x_T) \delta(u) du^2 + \frac{z}{L} \Phi_+(z, x_T) \delta(v) dv^2 \right].
\end{align*}
$$

(8)

It is convenient to change coordinates $(u, v, z, x_T) \Rightarrow (U, V, Z, x_T)$ to remove the distributional terms in the metric of the incoming waves (see the Appendix of Ref. [8]). Following Penrose [6], we look for surfaces $S$ with topology $S^{D-2}$ contained in the past ligh-cone $\{ U \leq 0, V = 0 \} \cup \{ U = 0, V \leq 0 \}$. This is parametrized in terms of two nonnegative functions $\psi^\pm(Z, x_T)$ as

$$
S^+ = \left\{ U = -\psi^+(Z, x_T) \right\}, \quad S^- = \left\{ U = 0 \quad V = -\psi^-(Z, x_T) \right\},
$$

(9)

where $S = S^+ \cup S^-$. Defining

$$
\Psi^\pm(Z, x_T) = \frac{Z}{L} \psi^\pm(Z, x_T),
$$

(10)

the condition that the expansion of the congruence of null geodesics normal to $S$ vanishes reads

$$
\left( \Box_{\mathbb{H}^{D-2}} - \frac{D-2}{L^2} \right) (\Phi^\pm - \Psi^\pm) = 0,
$$

(11)

where $\Phi^\pm$ are the profiles of the two incoming waves.

In the case of head-on collisions, we can exploit the $O(D-2)$ invariance of the system and assume that $\Psi^\pm(Z, x_T) = \Psi^\pm(q)$ only depends on the chordal coordinate [5]. In terms of it, the metric of the hyperbolic transverse space takes the form

$$
\begin{align*}
&ds^2_{\mathbb{H}^{D-2}} = L^2 \left[ \frac{dq^2}{q(q+1)} + 4q(q+1)d\Omega^2_{D-3} \right],
\end{align*}
$$

(12)

and the trapped surface equation (11) is given by

$$
\begin{align*}
&\left[ q(q+1) \frac{d^2}{dq^2} + \frac{D-2}{2} (1+2q) \frac{d}{dq} - (D-2) \right] \left[ \Phi^\pm(q) - \Psi^\pm(q) \right] = 0.
\end{align*}
$$

(13)

Since $S$ has two branches, we have to make sure that the null geodesics are continuous across the $(D-3)$-dimensional surface $C = S^+ \cap S^-$ defined by $\Psi^-(q) = 0$. This is implemented by the boundary condition

$$
\begin{align*}
g^{ab} \partial_a \Psi^\pm \partial_b \Psi^\pm \bigg|_C = 4,
\end{align*}
$$

(14)
where $g_{ab}$ is the metric on $\mathbb{H}_{D-2}$. Using the chordal coordinate, the surface $C$ is parametrized by $q = q_C$, and the previous condition can be written as
\[
\Psi'(q_C)^2 = \frac{4L^2}{q_C(q_C + 1)} \quad \Rightarrow \quad \Psi'(q_C) = -\frac{2L}{\sqrt{q_C(q_C + 1)}}. \tag{15}
\]

The sign is fixed by the requirement that the trapped surface has topology $S^{D-2}$, which means that $\Psi_\pm(q) > 0$ for $0 \leq q < q_C$. The trapped surface is determined by the solution to the differential equation $\Psi$ subjected to the two boundary conditions
\[
\begin{align*}
\Psi(q_C) &= 0 \\
\Psi'(q_C) &= -\frac{2L}{\sqrt{q_C(q_C + 1)}}
\end{align*} \tag{16}
\]

For simplicity, let us assume collision between two identical shock waves, so we have $\Psi_+(q) = \Psi_-(q) \equiv \Psi(q)$ and $\Phi_+(q) = \Phi_-(q) \equiv \Phi(q)_{\text{RN}}$. The general solution to the differential equation $\Psi$ can be written
\[
\Psi(q) = \Phi(q)_{\text{RN}} + C_1\Phi_1(q) + C_2\Phi_2(q), \tag{17}
\]
where $C_{1,2}$ are two integration constants and $\Phi_{1,2}(q)$ are the two independent solutions to the homogeneous differential equation
\[
\begin{align*}
\Phi_1(q) &= 1 + 2q \\
\Phi_2(q) &= q^{2-D} F_1 \left(D - 2, \frac{D}{2}; D; \frac{-1}{q} \right).
\end{align*} \tag{18}
\]

If the trapped surface has topology $S^{D-2}$, the function $\Psi(q)$ has to be regular for $0 \leq q \leq q_C$ and this forces $C_2 = 0$. Imposing besides $\Phi_1(q_C) = 0$ determines the remaining constant to be
\[
C_1 = -\frac{1}{1 + 2q_C}\Phi(q_C)_{\text{RN}}. \tag{19}
\]

The value of $q_C$ is now determined by the second condition $\Phi$ Plugging
\[
\Psi(q) = \Phi(q)_{\text{RN}} - \frac{1 + 2q}{1 + 2q_C}\Phi(q_C)_{\text{RN}}, \tag{20}
\]
into the second equation in 16, we find the algebraic equation
\[
\Phi'(q_C)_{\text{RN}} - \frac{2}{1 + 2q_C}\Phi(q_C)_{\text{RN}} + \frac{2L}{\sqrt{q_C(q_C + 1)}} = 0. \tag{21}
\]
Figure 1: Plot of the function $\Psi(q)$ for $D = 4$ (left panel) and $D = 5$ (right panel). In all cases the energy of the head-on collision is $G_N\mu/L^{D-3} = 1$ and the charge parameter (from top to bottom) $\sqrt{G_N e/L^{D-3}} = 0.5, 0.75$ and 1.0.

The problem of the existence of trapped surfaces of the Penrose type in the collision of two RN-AdS shock waves has been studied in [13] (see also [14]), as well as in a first version of this paper. The analysis presented there, however, has a fundamental flaw consisting in that the function $\Psi(q)$ has a second zero below $q_c$ and therefore the surface defined by it does not have topology $S^{D-2}$ as assumed. The origin of this problem lies in the fact that the charge-dependent term in $\Phi_{RN}(q)$ shown in Eq. (7) tends to minus infinity as $q \to 0^+$. Indeed, whereas in this limit the $\mu$-dependent term diverges as $q^{4-D}$ for $D > 4$ and $-\log q$ for $D = 4$, the $e$-dependent term diverges as $-q^{7-D}$ for $D \geq 4$. This second term dominates near $q = 0$ for any $e > 0$ (and $D \geq 4$), so $\Psi(q)$ goes to minus infinity as $q \to 0^+$ [see Eq. (20)]. This behavior is explicitly shown in Fig. 1, where the data for the function $\Psi(q)$ is plotted for $D = 4$ and $D = 5$ and various values of the charge.

On physical grounds it is to expect that this problem remains when considering a non-vanishing impact parameter for the collision. To check this requires solving numerically the Laplace-type equation (11) with the appropriate boundary conditions [16, 7, 8]. We work in radial coordinates $r = 2L\sqrt{q(q+1)}$, where the metric of the transverse hyperbolic space takes

\footnotesize
5We thank an anonymous referee for pointing out this basic problem that we overlooked in the previous version of this paper.
the form
\[ ds^2_{\mathbb{H}_{D-2}} = \frac{dr^2}{1 + \frac{r^2}{L^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\Omega^2_{D-4}. \] (22)

The system is \(O(D-3)\)-invariant and the Penrose trapped surface is parametrized by \(\Psi_\pm(r, \theta)\). To solve for these functions, it is convenient to define \(H_\pm(r, \theta) = \Phi_\pm(r, \theta)_{RN} - \Psi_\pm(r, \theta)\), where \(\Phi_\pm(r, \theta)_{RN}\) are the profiles of the incoming RN-AdS shock waves whose sources are located respectively at \(r_\pm = \frac{b}{2}, \theta_+ = 0, \theta_- = \pi, \vartheta_\pm = 0\), with \(\vartheta_\pm\) the angular coordinates on \(S^{D-4}\).

\(H_\pm(r, \theta)\) satisfies the equation
\[
\left[ \left(1 + \frac{r^2}{L^2} \right) \partial_r^2 + \frac{(D-3)\Lambda^2 + (D-2)r^2}{rL^2} \partial_r + \frac{1}{r^2} \partial_\theta^2 + \frac{D-4}{r^2 \tan \theta} \partial_\theta - \frac{D-2}{L^2} \right] H_\pm(r, \theta) = 0. \] (23)

This has to be solved in a domain \(0 \leq \theta \leq \pi, 0 \leq r(\theta) \leq LG(\theta)\). As a consequence of the symmetry of the problem, the functions \(\Phi_\pm(r, \theta)_{RN}\) and \(\Psi_\pm(r, \theta)\) satisfy
\[
\Psi_\pm(r, \theta) = \Psi_\mp(r, \pi - \theta), \quad \Phi_\pm(r, \theta)_{RN} = \Phi_\mp(r, \pi - \theta)_{RN}. \] (24)

This implies Neumann boundary conditions for the function \(H_\pm(r, \theta)\) on the lower boundary, \(\partial_\theta H_\pm(r, 0) = 0 = \partial_\theta H_\pm(r, \pi)\).

The function \(G(\theta)\) determining the shape of the candidate trapped surface also has the symmetry \(G(\theta) = G(\pi - \theta)\). It has to be chosen such that the following conditions are satisfied
\[
H_\pm(r, \theta) \bigg|_{r=LG(\theta)} = \Phi_\pm(LG(\theta), \theta)_{RN},
\]
\[
\left[ \left(1 + \frac{r^2}{L^2} \right) (\partial_r \Psi_+)(\partial_r \Psi_-) + \frac{1}{r^2} (\partial_\theta \Psi_+)(\partial_\theta \Psi_-) \right] \bigg|_{r=LG(\theta)} = 4. \] (25)

We solve this boundary problem for the collision of two identical shocks using the method devised in [16]: we solve Eq. (23) numerically in a 50(angular) \(\times\) 100(radial) grid and find \(G(\theta)\) through a trial-and-correction loop. In the following we summarize our results for the collision of two waves of the same energy. The details of the implementation of the method can be found in [8], where it was applied to the off-center collision of two Aichelburg-Sexl-AdS shock waves in various dimensions.

The numerical data obtained shows that \(\Psi(r, \theta)\) not only vanishes at the boundary \(r = LG(\theta)\), but also in the interior. In Fig. 2 we have plotted the radial profile of the function \(\Psi(r, \theta_0)\) at \(\theta_0 = 0\), both for \(D = 4\) (left panel) and \(D = 5\) (right panel). The conclusion is that there are no trapped surfaces of the Penrose type with topology \(S^{D-2}\) produced in the collision of two RN-AdS shock waves, both with or without impact parameter.
Figure 2: Plot of the section $\theta = 0$ of the function $\Psi(q, \theta)$ in four (left panel) and five dimensions (right panel) with nonvanishing impact parameter $b/L = 0.3$. Again, the energy of the collision is $G_N \mu / L^{D-3} = 1$ and the charge parameter (from top to bottom) $\sqrt{G_N e / L^{D-3}} = 0.5, 0.75$ and 1.0.

**Changing the topology.** Given the results above, it seems natural to look for marginally trapped surfaces with topology $S^1 \times S^{D-3}$. Mathematically, this means that we allow for the possibility of $\Psi(q)$ vanishing at two points that we denote by $q_{\text{in}}$ and $q_{\text{out}}$, and such that

$$\Psi_{\pm}(q) > 0 \quad \text{for} \quad q_{\text{in}} < q < q_{\text{out}}.$$  \hfill (26)

The surface $C$ has therefore two components $C_{\text{in}}$ and $C_{\text{out}}$ defined respectively by $q = q_{\text{in}}$ and $q = q_{\text{out}}$. Let us analyze now the case of a symmetric head-on collision. The condition for the continuity of the congruence of normal null geodesics across $C$ now translates into the couple of equations

$$\Psi'(q_{\text{out}})^2 = \frac{4L^2}{q_{\text{out}}(q_{\text{out}} + 1)} \quad \Rightarrow \quad \Psi'(q_{\text{out}}) = -\frac{2L}{\sqrt{q_{\text{out}}(q_{\text{out}} + 1)}},$$

$$\Psi'(q_{\text{in}})^2 = \frac{4L^2}{q_{\text{in}}(q_{\text{in}} + 1)} \quad \Rightarrow \quad \Psi'(q_{\text{in}}) = \frac{2L}{\sqrt{q_{\text{in}}(q_{\text{in}} + 1)}}.$$  \hfill (27)
The choice of signs is fixed by Eq. (26). Hence, we have to solve the differential equation (13) with the boundary conditions

\begin{align}
\Psi(q_{\text{out}}) &= 0, \quad (28) \\
\Psi(q_{\text{in}}) &= 0, \quad (29) \\
\Psi'(q_{\text{out}}) &= -\frac{2L}{\sqrt{q_{\text{out}}(q_{\text{out}}+1)}}, \quad (30) \\
\Psi'(q_{\text{in}}) &= \frac{2L}{\sqrt{q_{\text{in}}(q_{\text{in}}+1)}}. \quad (31)
\end{align}

In the case of the solutions found in [13], it is important to notice that Eq. (31) is not satisfied at the internal zero of \( \Psi(q) \). Hence, to have a chance of finding trapped surfaces of nontrivial topology we have to be more general. An important change introduced by the \( S^1 \times S^{D-3} \) topology of the trapped surface occurs in the form of the general solution to Eq. (13). Now the region of interest excludes both \( q = 0 \) and \( q = \infty \), so there is no reason to set \( C_2 = 0 \) in (17) as we did when assuming that the trapped surface had topology \( S^{D-2} \). In fact, \( C_1 \) and \( C_2 \) are determined by Eqs. (28) and (29)

\begin{align}
C_1 \Phi_1(q_{\text{out}}) + C_2 \Phi_2(q_{\text{out}}) &= -\Phi(q_{\text{out}})_{RN}, \\
C_1 \Phi_1(q_{\text{in}}) + C_2 \Phi_2(q_{\text{in}}) &= -\Phi(q_{\text{in}})_{RN}. \quad (32)
\end{align}

Once these constants are solved in terms of \( q_{\text{in}} \) and \( q_{\text{out}} \), we impose the conditions (30) and (31). This provides two algebraic equations that, in principle, are enough to determine the values of the internal and external radii of the trapped surface.

It is interesting to notice that the resulting function \( \Psi(q) \) is independent of the mass parameter \( \mu \). The right-hand side of both equations in (32) is the sum of two terms, respectively of order \( \mu^0 \) and \( \mu \). We write \( C_{1,2} = C_{1,2}^{(0)} + \mu C_{1,2}^{(1)} \), whereas Eq. (6) can be recast as

\[ \Phi(q)_{RN} = A_D \mu \Phi_2(q) + \Phi(q) \quad \text{with} \quad A_D = L^{2D-3} \frac{\sqrt{\pi} \Gamma \left( \frac{D}{2} \right)}{\Gamma \left( \frac{D+1}{2} \right)} \left( \frac{G_N}{L^{D-3}} \right). \quad (33) \]

Solving the system (32) order by order in \( \mu \) gives \( C_1^{(1)} = 0 \), \( C_2^{(1)} = -A_D \). From Eq. (17) we find that the \( \mu \)-dependent part in \( \Psi(q) \) cancels.

A numerical analysis of the system of equations (30)-(31), with the values of the constants found from (32), renders no nonvanishing solutions for \( q_{\text{in}} \) and \( q_{\text{out}} \). This implies that there are no trapped surface of topology \( S^1 \times S^{D-3} \) formed as the result of the head-on collision of two
Figure 3: Plot of the implicit equations (30) and (31) for two values of the charge in $D = 5$. The two curves approach each other close to the origin but do not cross. The dashed line represents the diagonal $q_{in} = q_{out}$, and shows that both curves lie in the “physical” region $q_{out} > q_{in}$.

RN-AdS shock waves. This is pictorially illustrated in Fig. 3 where the two curves defined by Eqs. (30) and (31) do not cross each other outside the origin.

**Closing remarks**  The study of the problem of collisions of two RN-AdS shock waves presented here shows that the charge parameter $e$ completely prevents the formation of marginally trapped surfaces in the region $\{u = 0, v \leq 0\} \cup \{u \leq 0, v = 0\}$ with topology $S^{D-2}$, independently of the value of the impact parameter. In the case of head-on collisions, we have not found any trapped surface with topology $S^1 \times S^{D-3}$ in the same region either.

These results do not preclude the existence of trapped surfaces in other regions of the spacetime formed as the result of the collision. In [17, 12] the problem of the collision of both Aichelburg-Sexl and RN shock waves in flat space was analyzed, looking for trapped surfaces lying in the future light-cone $\{u = 0, v \geq 0\} \cup \{u \geq 0, v = 0\}$. In the case of AdS, the analysis needed to find these trapped surfaces is more involved than the one required to find the Penrose trapped surface, since the change of coordinates needed to eliminate the distributional terms in the metric is nontrivial in the “future ligth-cone” region. As a consequence, the equations determining the trapped surface are much more complicated and have to be solved numerically, even in the case of head-on collisions. This problem is under current investigation.
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