Observation of partial and infinite-temperature thermalization induced by continuous monitoring on a quantum hardware

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On a quantum superconducting processor we experimentally observe partial and infinite-temperature thermalization induced by a sequence of repeated quantum projective measurements, interspersed by a unitary (Hamiltonian) evolution. Specifically, on a qubit and two-qubit systems, we test the state convergence of a monitored quantum system in the limit of a large number of quantum measurements, depending on the non-commutativity of the Hamiltonian and the measurement observable. In most cases, where the Hamiltonian and observable do not commute, the convergence is uniform towards the infinite-temperature state. Conversely, whenever the two operators have one or more eigenvectors in common in their spectral decomposition, the state of the monitored system converges differently in the subspaces spanned by the measurement observable eigenstates. As a result, we experimentally show that the convergence does not tend to a completely mixed (infinite-temperature) state, but to a block-diagonal state in the observable basis, with a finite effective temperature in each measurement subspace. Finally, we quantify the effects of the quantum hardware noise on the experimental data by modelling them by means of depolarizing quantum channels.

I. INTRODUCTION

In a quantum dynamical system, the application of a measurement necessarily entails a back-action on the system dynamics. Specifically, if one takes into account the von-Neumann postulate of quantum measurement (that for all practical purposes is just an idealization), after the measurement of a quantum observable, the quantum system instantaneously collapses onto an eigenstate of the observable. Such a mechanism is the well-known projective measurement

Quantum monitoring emphasizes the quantum measurement postulate since it prescribes the application over time of a sequence of projective measurements in single realizations of quantum system dynamics. In correspondence of each projective measurement, the quantum system collapses and re-starts its evolution from one of the quantum observable eigenstates with a given probability. This entails the onset of an ensemble of quantum system trajectories; the dimension of such an ensemble grows as \( d^n \) with \( d \) dimension of the quantum system and \( n \) number of projective measurements applied in the single dynamical process.

In the last decade, several research groups in the quantum thermodynamics community have studied the property and effects of fluctuations due to measurement back-action\(^4\) in a sequence of quantum measurements from the point of view of thermodynamic quantities (work, heat, entropy) evaluated at least over two times\(^5\). For more details, we refer the reader to a couple of recent reviews on the relation between quantum thermodynamics and continuous quantum monitoring\(^24\)–corresponding to the discrete case– from the other hand.

Another research line around quantum monitoring is the investigation of the behavior of the monitored system, both in the asymptotic limit of a large number of measurements\(^22,23\) and in the classical limit of the quantum system’s size approaching macroscopicity\(^23\). The tendency of the monitored quantum system to converge towards a steady state has been analyzed also in the more general case in which an additional source of noise affects the evolution between consecutive measurements\(^24–29\). Specifically, in agreement with Refs.\(^30,31\), it has been determined that for any quantum observable the probability distribution of its outcomes, in the single realization of the system evolution, has an exact large-deviation form with an exponentially decaying profile in the number of measurements. This has allowed the derivation of the most probable distribution of the observable outcomes\(^25\).

Furthermore, in this framework, it is worth mentioning also studies on quantum monitoring in the condensed matter context. It has been shown, indeed, that in quantum many-body systems the quantum measurement back-action in a sequence of measurements locally acting on the system is responsible for spontaneous symmetry breaking\(^32\), as well as for entangling-disentangling trans-
heat-engine fluctuation relation

ators have been realized

ments

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ordered states

XXZ spin chain

of Kardar-Parisi-Zhang scaling in the dynamics of an

XXZ spin chain, and the observation of Leggett-Garg’s inequalities violations, investigations of the irreversibility and implications of the third law of thermodynamics, the inspection of dissipative collective effects on noisy quantum computers, experimental validation of the Kibble-Zurek mechanism, evidence of Kardar-Parisi-Zhang scaling in the dynamics of an XXZ spin chain, and the realization of topological ordered states and topological edge states. Finally, in the context of quantum thermodynamics, several experiments have been performed on quantum hardware to benchmark quantum fluctuation relations and test their robustness against intermediate projective measurements. Moreover, quantum heat engines and refrigerators have been realized, then verifying the so-called heat-engine fluctuation relation and implementing the concept of quantum-measurement-cooling. All such thermodynamics experiments, apart from their interest from a fundamental standpoint, may also be useful to characterize the energetic cost of quantum computation, and investigate the thermodynamics of quantum annealers. These tasks are extremely important, since understanding and controlling the energy exchanges and dissipation of quantum hardware is necessary for their improvement.

We thus wonder: may quantum monitoring be effectively applied on a (commercial) quantum device? May such devices be exploited to investigate the open research issues under quantum monitoring, especially in the limit of a large number of measurements? In this paper, on IBM Quantum hardware, encoding single-qubit and two-qubit systems, we experimentally observe the phenomena of partial and infinite-temperature thermalization, recently found in, in the subspaces spanned by the measurement quantum observable. Infinite-temperature thermalization (ITT) denotes the tendency of a monitored quantum system to end up towards the completely mixed state in the limit of a large (ideally infinite) number of measurements. Such behavior complies with the fact that the dynamics originating by a sequence of projective measurements interspersed with a unitary evolution is described by a unitary quantum map. However, as determined in Ref., ITT occurs under some conditions: if (i) the Hamiltonian \( H \) of the system and the measurement observable \( \mathcal{O} \) do not have eigenvectors in common in their spectral decomposition, and (ii) the size of the monitored quantum system is finite. Conversely, in case \( H \) and \( \mathcal{O} \) share a common nontrivial subspace (it is indeed enough that they are commuting, or even nearly commuting, operators), the so-called partial thermalization of the system’s state is recovered in the limit of \( n \to \infty \). Partial thermalization manifests itself in the fact that the Hamiltonian \( H \), once expressed in the basis decomposing the measurement observable \( \mathcal{O} \), is a block matrix and, at the same time, the largest eigenvalue of the matrix containing the transition probabilities to jump from an eigenstate of \( \mathcal{O} \) to another in single trajectories is degenerate. As a result, the monitored quantum system converges to a block-diagonal density operator in the quantum observable basis that is characterized by a finite effective temperature in each measurement subspace. Also, the period \( \tau \) between measurements plays a role. First of all, as long as \( n\tau \ll 1 \) and the monitored system is initialized in an eigenstate of the measurement observable, the system dynamics is ‘frozen’ in the initial condition in agreement with the so-called quantum Zeno effect. Secondly, one can observe resonance-like behaviors when the energy gap of the monitored system is commensurable with the inverse of \( \tau \). This entails that, for fixed values of \( \tau \), the system jumps from an eigenstate of \( \mathcal{O} \) to another as a function of \( n \), integer number. Such resonance-like behavior is delectable in the expectation value of \( \mathcal{O} \). Finally, due to the hard constraints imposed on the number of measurements by the noise in the quantum hardware, we model its (dynamical) effect on the measured statistics as a function of \( n \). Specifically, we determine that, for single-qubit and two-qubit systems, ITT and partial thermalization are disturbed by a noise process modeled by a depolarizing quantum channel, which might have originated from using controlled-NOT gates. Despite the presence of noise hindering us to apply an arbitrarily large number of quantum measurements in a single trajectory, our experiments may help to understand the interplay of unitary dynamics interspersed by projective measurements in contact with an environment leading to decoherence.

II. QUANTUM MONITORING

Let us briefly recall the main theoretical results for the protocol we are going to take into account. To this extent, we consider a quantum system that is defined in a \( N \)-dimensional Hilbert space and evolves with a time-independent Hamiltonian \( H \). At time \( t = 0 \), the system is initialized in the pure state \( \rho_0 \equiv |\Psi_0\rangle\langle \Psi_0| \) (in Ref., \( |\Psi_0\rangle \) is an eigenstate of the Hamiltonian, obtained by performing a first projective measurement of the energy). At times \( t_n \equiv n\tau \), with \( n \) integer number, the system is subjected to a sequence of projective measurements of a generic observable \( \mathcal{O} = \sum_k \mathcal{O}_k \). In general, \( [H, \mathcal{O}] \neq 0 \). Between two consecutive measurements, the monitored
system follows the unitary evolution $U \equiv \exp(-iHt/\hbar)$, where $\hbar$ is the reduced Planck constant that is set to 1 from now on. Then, after each projective measurement
\[ M(\rho) \equiv \frac{\Pi_k \rho \Pi_k}{\rho_k}, \]
the system collapses in one of the eigenstates $|\alpha_k\rangle$, of $O$ with probability $P_{|\alpha_k\rangle} = \text{Tr}[\rho \Pi_k]$. The protocol is sketched in Fig. 1. Notice that, if $[H,O] \neq 0$, $\{|\alpha_k\rangle\}$ are not eigenstates decomposing $H$. This means that, in a sequence of projective measurements of $O$ interspersed by the unitary evolution $U$, the system dynamics
\[ \rho_n = \sum_{k_1, \ldots, k_t} \Pi_{k_1} U \ldots \Pi_{k_t} U \rho_0 U^\dagger \Pi_{k_t} \ldots U^\dagger \Pi_{k_1} \rho_n \]
originate, in a not trivial way, a bunch of quantum trajectories as well as a multi-time statistics for the outcomes of $O$. In particular, the transition probability between two eigenstates of $O$ at two consecutive times is given by the matrix element
\[ L_{k,k'} \equiv |\langle \alpha_{k'} | U(\tau) | \alpha_k \rangle|^2. \]
As a consequence, the state of the system at time $t = n \tau$ is equal to the statistical mixture
\[ \rho_n = \sum_k P^n_{|\alpha_k\rangle} |\alpha_k\rangle \langle \alpha_k|, \]
where the probabilities $P^n_{|\alpha_k\rangle}$ are given by
\[ P^n_{|\alpha_k\rangle} = \sum_{k'} (L^n)_{k,k'} P^0_{|\alpha_{k'}\rangle} \]
with $P^0_{|\alpha_k\rangle} = |\langle \alpha_k | \Psi_0 \rangle|^2$.

The occurrence of the observable outcomes (at discrete times), in the single trajectory, is effectively described by a Markov process. The condition $\text{Tr}[\rho_n] = 1$, ensuring the normalization of the probabilities, is preserved at any time by the fact that matrix $L$ is stochastic, namely
\[ \sum_{k'} L_{k,k'} = 1 \]
for all $k' = 1, \ldots, N$. The behavior of the system dynamics at large times ($n$ large) can be inferred by looking at the spectrum of $L$, as proved in Ref.\textsuperscript{9}. As $L(\tau)$ is symmetric and stochastic, it follows that the spectrum $\{\lambda_k\}$ of $L$ is real and such that $-1 \leq \lambda_k \leq 1 \forall k$, while the largest eigenvalue $\lambda_0 \equiv 1$ is. In the $\{\alpha_k\}$ basis, it corresponds to the eigenvector
\[ |v_0\rangle = \frac{1}{\sqrt{N}} (1, \ldots, 1)^T. \]
As shown in Ref.\textsuperscript{23}, if $H$ is irreducible in the basis $\{|\alpha_k\rangle\}$, the Perron-Frobenius theorem guarantees that $\lambda_0$ is non-degenerate. Thus, for large $n$,
\[ L^n \rightarrow |v_0\rangle \langle v_0|. \]

III. SINGLE-QUBIT

To experimentally explore the aforementioned dynamics, we begin by considering a single-qubit $q_0$. The Hilbert space of the corresponding two-level system is described by the two eigenstates of $\sigma^z = |0\rangle \langle 0| - |1\rangle \langle 1|$, commonly known as the computational basis $|k\rangle$ with $k = 0, 1$. The starting point of our protocol is the state $|\Psi_0\rangle = |0\rangle$, which undergoes a cyclic dynamics given by an evolution governed by the following Hamiltonian:
\[ H_{q_0} = \frac{1}{2} \sigma^z, \]
up to a time $\tau$ followed by a projective measurement in the $\sigma^z$ basis.

We are going to monitor the ‘magnetization’ along $z$ of the qubit that is given by the imbalance of its populations and reads as
\[ \langle \sigma^z \rangle (n\tau) = P^n_{|0\rangle} - P^n_{|1\rangle}. \]

While in this case $P^0_{|0\rangle} = 1$ and $P^0_{|1\rangle} = 0$ ($|\Psi_0\rangle = |0\rangle$ indeed), in order to compute the probabilities $P^n_{|k\rangle}$ ($k = 1, 2$) at later times we need the transition matrix $L$ defined in Eq. (3). In the single-qubit case, the transition matrix explicitly reads as
\[ L = \begin{pmatrix} \cos^2 \left(\frac{\lambda}{2}\right) & \sin^2 \left(\frac{\lambda}{2}\right) \\ \sin^2 \left(\frac{\lambda}{2}\right) & \cos^2 \left(\frac{\lambda}{2}\right) \end{pmatrix}. \]

Then, the spectral decomposition of $L(\tau)$ allows us to compute
\[ L^n = \lambda^n_0 |v_0\rangle \langle v_0| + \lambda^n_1 |v_1\rangle \langle v_1|, \]
compute the ‘magnetization’ along $z$ of the qubit

$$\langle \sigma^z_{q_0} \rangle (n \tau) = (\cos \tau)^n$$

(14)

that, in most cases, relaxes to the infinite-temperature thermal expectation value, namely $\langle \sigma^z \rangle = 0$. However, as already noticed, exceptions to this general behavior do arise for some particular choices of $\tau$, i.e., $\tau = p \pi$ with $p \in \mathbb{Z}$, corresponding to resonance-like behaviors. This is clear from Figure 2(a), where the experimental data for $\langle \sigma^z_{q_0} \rangle$—obtained from the protocol implementation on a real quantum processor—are shown. ITT is achieved for most of evolution times $\tau$ except for a set of points near the fine-tuned cases $\tau = 0, \pi$. On top of this behavior, the resonance can be clearly identified. When $\tau = 0$ then the system is frozen in the initial state $|\Psi_0\rangle = |0\rangle$ with $\langle \sigma^z_{q_0} \rangle = 1$, see Fig. 2(b) red circles. Close to this value of $\tau$, the relaxation timescale is $n \sim O(\tau^{-2})$ so that the dynamics of the monitored system is frozen within it. This is nothing but a manifestation of the quantum Zeno effect that prevents thermalization, as well as any other dynamics, as a consequence of the frequent monitoring and resulting collapse on the initial state. Moreover, whenever $\tau = \pi$, we find a second resonance effect, as $\langle \sigma^z_{q_0} \rangle = (-1)^n$ (see Eq. (14)). The latter stems from taking, at $\tau = \pi$, the energy gap of the qubit commensurable with the inverse of the period, as shown in Fig. 2(b) blue crosses. In other words, in this case, the unitary evolution simply brings the system back and forth between the eigenstates of the measured observable, corresponding to $\langle \sigma^z_{q_0} \rangle = \pm 1$, respectively. In this case, the thermalization timescale goes as $n = O(\tau^{-2})$ as well. Interestingly, this mechanism is analogous to the well-known phenomenon of period doubling of Floquet driven systems near to resonances, extensively studied in disordered, two-dimensional, and long-range settings.\(^{39,48,70-81}\) Such effects are well highlighted, albeit in a different way, also by Fig. 2(c) where we fix the value of $n$ (in the figure, $n = 1, 2, 8, 9, 30, 31$) and $\langle \sigma^z_{q_0} \rangle$ is plotted as a function of $\tau \in [0, \pi]$. In the subinterval $\tau \in [\pi/2, \pi]$, we observe that the monitored system (starting from $|\Psi_0\rangle = |0\rangle$) tends to flip towards the excited state $|1\rangle$ or to stay in the ground $|0\rangle$, depending on the fact that $n$—integer number—is even ($\langle \sigma^z_{q_0} \rangle > 0$) or odd ($\langle \sigma^z_{q_0} \rangle < 0$). Moreover, these behaviors occur in a more discontinuous way the greater the number of projective measurements. A discontinuity is found in the limit of $n \to \infty$, whereby in such a limit resonances are events of measure zero (i.e., occurring with zero probability almost surely).

IV. PARTIAL THERMALIZATION OF TWO QUBITS

We consider a composite system of two non-interacting qubits $q_0, q_1$ whose dynamics is described by the Hamiltonian

$$H = \frac{1}{2} \left( \sigma^x_{q_0} + \sigma^x_{q_1} \right)$$

(15)
and, at time $t = 0^-$, starts from the product state $\rho_0 = |00\rangle\langle 00|$. Our protocol is based on applying a sequence of evolutions intertwined by projective measurements of an observable whose eigenbasis (spectral decomposition) contains entangled states with respect to the computational basis. In this regard, the change of basis from a generic two-qubit basis $\{|\phi_k\rangle\}$, with $k = 0, 1, 2, 3$, to the computational basis $\{|k\rangle\}$, with $k = 00, 01, 10, 11$, is given by the following relation:

$$
\begin{pmatrix}
|00\rangle \\
|01\rangle \\
|10\rangle \\
|11\rangle
\end{pmatrix} = V
\begin{pmatrix}
|\phi_0\rangle \\
|\phi_1\rangle \\
|\phi_2\rangle \\
|\phi_3\rangle
\end{pmatrix},
$$

where $V$ is the matrix for the change of basis. We notice that, while the evolution acts locally and independently on the two qubits (due to our choice of the Hamiltonian $H$), the projective measurements may create entanglement as they allow for an effective exchange of information. However, an equivalent protocol could be achieved (with the same circuit complexity) by choosing an entangling dynamics and taking $\sigma_z^k$ as local observables whose spectral decomposition is the computational basis of the single-qubit. In fact, as a by-product of our protocol, we have determined that the same statistics of measurement outcomes can be recovered in the following two ways. (i) By letting evolve a quantum system with local time-independent Hamiltonian and then (repeatedly) measuring it on a generic (entangling) basis. Or (ii) by implementing dynamics under a generic entangling Hamiltonian and measuring the system in the local computational basis. Furthermore, we are also going to take the opportunity to show the way of realizing quantum measurements in a generic basis, decomposing a given quantum observable, on quantum hardware.

We stress that, on the IBM Quantum hardware, the only native measurement allowed is over the computational basis corresponding to the projectors $\Pi_k = |k\rangle\langle k|$. Thus, in order to perform measurements over the eigenvectors of an entangled basis

$$
\pi_k = |\phi_k\rangle\langle \phi_k|,
$$

first we have to apply the change of matrix $V$ that maps the entangled basis $\{|\phi_k\rangle\}$ into the $\sigma_z^k$ eigenbasis. Then, we need to measure along $\sigma_z^k$ and, finally, we have to perform the inverse unitary gate $V^\dagger$ that is implicitly defined by the relation

$$
\pi_k = V^\dagger \Pi_k V.
$$

In the following, our analysis will focus on two specific measurement bases, the single-triplet and the Bell bases, in which the conditions allowing partial thermalization are discussed.

### A. Singlet-Triplet basis

The change-of-basis matrix $V$ that maps the singlet-triplet basis into the computational basis is given by

$$
\begin{pmatrix}
|00\rangle \\
|01\rangle \\
|10\rangle \\
|11\rangle
\end{pmatrix} = V
\begin{pmatrix}
|\phi_0\rangle \\
|\phi_1\rangle \\
|\phi_2\rangle \\
|\phi_3\rangle
\end{pmatrix},
$$

where the states

$$
|\psi_0\rangle = |00\rangle, \\ |\psi_1\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle), \\ |\psi_2\rangle = |11\rangle,
$$

constitute the triplet state while

$$
|\psi_3\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)
$$

is the singlet state. The latter is anti-symmetric under permutation $P : q_0 \leftrightarrow q_1$ of the two qubits, meanwhile the other three states are symmetric under such a transformation.

Since the Hamiltonian (15) is invariant under $P$ and the initial state has a zero overlap with the subspace spanned by $|\psi_2\rangle$ (i.e., the anti-symmetric manifold), we therefore expect a partial thermalization in the symmetric manifold of the Hilbert space. In fact, in the absence of noise, the singlet state $|\psi_3\rangle$ is a dark state, and the Hamiltonian in the measurement basis reads as

$$
V^\dagger HV = \frac{1}{\sqrt{2}}
\begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{pmatrix}.
$$

Then, we may write the transition matrix $L(\tau)$ as

$$
L(\tau) = \begin{pmatrix}
\cos^2 \frac{\tau}{2} & \frac{1}{2} \sin^2 \tau & 0 & \frac{1}{2} \sin^2 \frac{\tau}{2} \\
\frac{1}{2} \sin^2 \frac{\tau}{2} & \cos^2 \tau & 0 & \frac{1}{2} \sin^2 \frac{\tau}{2} \\
0 & 0 & 0 & 1 \\
\frac{1}{2} \sin^2 \frac{\tau}{2} & \frac{1}{2} \sin^2 \tau & 0 & \cos^2 \frac{\tau}{2}
\end{pmatrix},
$$

and we have $P_{|\psi_3\rangle} = \delta_{k,0}$ due to the fact the initial state $|\psi_0\rangle$ is $|00\rangle$.

Let us now show $L^n(\tau)$, which is provided by the following expression:

$$
L^n(\tau) = \sum_{j=0}^{3} \lambda_j^n |v_j\rangle\langle v_j|
$$

where

$$
\lambda_0 = 1, \\ |v_0\rangle = \frac{1}{\sqrt{3}} (1 \ 1 \ 0 \ 1)^T, \\ \lambda_1 = 1, \\ |v_1\rangle = (0 \ 0 \ 1 \ 0)^T, \\ \lambda_2 = \cos \tau, \\ |v_2\rangle = \frac{1}{\sqrt{2}} (1 \ 0 \ 0 \ -1)^T, \\ \lambda_3 = \frac{1}{4} (1 + 3 \cos \tau), \\ |v_3\rangle = \frac{1}{\sqrt{6}} (1 \ -2 \ 0 \ 1)^T.
$$
Thus, the probabilities \( P_n^{\psi_k} = \text{Tr} [\rho_n |\psi_k\rangle \langle \psi_k|] \) are

\[
P_n^{\psi_0} = \frac{1}{6} [3 \cos^n(\tau) + 2^{-2n}(3 \cos(2\tau) + 1)^n + 2],
\]

\[
P_n^{\psi_1} = \frac{1}{3} \left[ 1 - 2^{-2n}(3 \cos(2\tau) + 1)^n \right],
\]

\[
P_n^{\psi_2} = 0,
\]

\[
P_n^{\psi_3} = \frac{1}{6} \left[ -3 \cos^n(\tau) + 2^{-2n}(3 \cos(2\tau) + 1)^n + 2 \right],
\]

which in the limit of \( n \to \infty \) read

\[
P_{\infty}^{\psi_0} = \begin{cases} \frac{1}{3} & \text{if } \tau \neq p\pi, \ p \in \mathbb{Z}, \\ 1 & \text{if } \tau = 2p\pi, \ p \in \mathbb{Z}, \\ \frac{1}{2} ((-1)^n + 1) & \text{if } \tau = (2p+1)\pi, \ p \in \mathbb{Z}, \end{cases}
\]

\[
P_{\infty}^{\psi_1} = \begin{cases} \frac{1}{3} & \text{if } \tau \neq p\pi, \ p \in \mathbb{Z}, \\ 0 & \text{if } \tau = p\pi, \ p \in \mathbb{Z}, \end{cases}
\]

\[
P_{\infty}^{\psi_2} = 0,
\]

\[
P_{\infty}^{\psi_3} = \begin{cases} \frac{1}{3} & \text{if } \tau \neq p\pi, \ p \in \mathbb{Z}, \\ 0 & \text{if } \tau = 2p\pi, \ p \in \mathbb{Z}, \\ \frac{1}{2} ((-1)^n + 1) & \text{if } \tau = (2p+1)\pi, \ p \in \mathbb{Z}. \end{cases}
\]  

The theoretical values of the probabilities in Eqs. (26), plotted in Fig. 8(a), have to be compared with the ones obtained experimentally, which are shown in Fig. 3. Since the initial density operator \( \rho_0 = |00\rangle \langle 00| \) is also an eigenprojector of the measurement basis, we are able to observe the quantum Zeno effect for \( \tau = 0 \), as in the previous paragraph. For small \( n \) and far from resonances, the state of the system quickly relaxes towards an equally populated mixed state in the triplet subspace \( \{|\psi_0\rangle, |\psi_1\rangle, |\psi_3\rangle\} \). This corresponds to a partial thermalization of the monitored quantum system, since the subspaces of the singlet and triplet states are not mixed by the dynamics. In Fig. 4 we plot the elements of the initial theoretical density operator and the ones obtained after \( n = 30 \) projective measurements of our protocol. For large \( n \), the non-trivial structure of the density operator in the computational basis takes into account the fact that continuous monitoring protects the symmetric manifold of the Hilbert space by maintaining the system within it over time. However, in our experiments, by increasing the number of measurements, the noise brings the system to the infinite-temperature state, thus restoring the ITT regime. We remind the reader to Sec. V for a detailed analysis of the noise affecting the quantum hardware that we employed.

## Bell basis

The second measurement basis that we have tested is the Bell basis. The change-of-basis matrix \( V \) from the Bell states to the computational basis is

\[
\begin{pmatrix}
|00\rangle \\
|01\rangle \\
|11\rangle
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
1 & 0 & 1
\end{pmatrix} \begin{pmatrix}
|\beta_0\rangle \\
|\beta_1\rangle \\
|\beta_2\rangle
\end{pmatrix},
\]

from which we have

\[
|\beta_0\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle), \quad |\beta_1\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle), \quad |\beta_2\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle), \quad |\beta_3\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle).
\]
Figure 5. Experimental values of the probabilities to measure two rotating qubits, subjected to continuous monitoring, in the subspaces of the system Hilbert space spanned by the Bell basis $\{|\beta_k\rangle\}$, with $k = 0, 1, 2, 3$.

Then, following the same procedure of the previous paragraph, the Hamiltonian in the measurement basis is

$$V^\dagger HV = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (30)$$

This means that in this case the initial state $|00\rangle$ is no longer an eigenstate of the measurement basis, so that

$$P^0_{|\beta_k\rangle} = \frac{1}{2} (\delta_{k,0} + \delta_{k,2}). \quad (31)$$

Moreover, the transition matrix $L(\tau)$ is given here by

$$L(\tau) = \begin{pmatrix} \cos^2(\tau) & \sin^2(\tau) & 0 & 0 \\ \sin^2(\tau) & \cos^2(\tau) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (32)$$

with the result that

$$L^n(\tau) = \sum_{j=0}^3 \lambda_j^n |v_j\rangle\langle v_j| \quad (33)$$

where

\begin{align*}
\lambda_0 &= 1, & |v_0\rangle &= (0 \ 0 \ 0 \ 1)^T, \\
\lambda_1 &= 1, & |v_1\rangle &= (0 \ 0 \ 1 \ 0)^T, \\
\lambda_2 &= 1, & |v_2\rangle &= \frac{1}{\sqrt{2}} (1 \ 1 \ 0 \ 0)^T, \\
\lambda_3 &= \cos 2\tau, & |v_3\rangle &= (1 \ -1 \ 0 \ 0)^T. 
\end{align*} \quad (34) \quad (35)

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Moreover, the transition matrix $L(\tau)$ is given here by

$$L(\tau) = \begin{pmatrix} \cos^2(\tau) & \sin^2(\tau) & 0 & 0 \\ \sin^2(\tau) & \cos^2(\tau) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (32)$$

with the result that

$$L^n(\tau) = \sum_{j=0}^3 \lambda_j^n |v_j\rangle\langle v_j| \quad (33)$$

where

\begin{align*}
\lambda_0 &= 1, & |v_0\rangle &= (0 \ 0 \ 0 \ 1)^T, \\
\lambda_1 &= 1, & |v_1\rangle &= (0 \ 0 \ 1 \ 0)^T, \\
\lambda_2 &= 1, & |v_2\rangle &= \frac{1}{\sqrt{2}} (1 \ 1 \ 0 \ 0)^T, \\
\lambda_3 &= \cos 2\tau, & |v_3\rangle &= (1 \ -1 \ 0 \ 0)^T. \quad (34) \quad (35)
\end{align*}

It follows that $P^0_{|\beta_k\rangle} = \text{Tr} [\rho_n |\beta_k\rangle\langle \beta_k|]$ are

$$P^0_{|\beta_k\rangle} = \frac{1}{4} (1 + \cos^n 2\tau), \quad P^0_{|\beta_k\rangle} = \frac{1}{4} (1 - \cos^n 2\tau), \quad (36)$$

which in the limit of $n \to \infty$ read

\begin{align*}
P^\infty_{|\beta_k\rangle} &= \begin{cases} 
\frac{1}{4} & \text{if } \tau \neq \frac{p \pi}{2}, \ p \in \mathbb{Z}, \\
\frac{1}{2} & \text{if } \tau = \frac{p \pi}{2}, \ p \in \mathbb{Z}, \\
\frac{1}{4} (1 + (-1)^n) & \text{if } \tau = \frac{p \pi + \frac{p \pi}{2}}, \ p \in \mathbb{Z},
\end{cases} \\
P^\infty_{|\beta_k\rangle} &= \begin{cases} 
\frac{1}{4} & \text{if } \tau \neq \frac{p \pi}{2}, \ p \in \mathbb{Z}, \\
0 & \text{if } \tau = \frac{p \pi}{2}, \ p \in \mathbb{Z}, \\
\frac{1}{4} (1 + (-1)^{n+1}) & \text{if } \tau = \frac{p \pi + \frac{p \pi}{2}}, \ p \in \mathbb{Z},
\end{cases} \\
P^\infty_{|\beta_k\rangle} &= \frac{1}{2}, \\
P^\infty_{|\beta_k\rangle} &= 0. \quad (37)
\end{align*}

The theoretical populations of Eqs. (36), plotted in Fig. 8(c), have to be compared with the ones obtained experimentally, shown in Fig. 5. In this case, the initial state of the dynamics, $|00\rangle = \frac{1}{\sqrt{2}} (|\beta_0\rangle + |\beta_2\rangle)$, is a superposition of two of the states of the measurement basis. Once again, at short time scales, partial thermalization is observed. In fact, $P^0_{|\beta_2\rangle} \approx 0$, $P^0_{|\beta_2\rangle} \approx 1/2$ $\forall n$ and only the complementary subspace is mixed by the dynamics. Moreover, there is less noise in comparison with the case-study in the previous paragraph. This could be due to the fact that a smaller number of CNOTs is needed to transpile the continuous measurement protocol into the native gates of the used quantum hardware. As in Fig. 4, we plot in Fig. 6 the elements of the initial theoretical
density operator and the ones obtained after \( n = 30 \) projective measurements of our protocol. Once again, for large \( n \), the non-trivial structure of the density operator in the computational basis allows for the protection of the symmetric manifold of the Hilbert space by maintaining the system within it over time.

V. EFFECTS OF THE QUANTUM HARDWARE NOISE

In our experiments, the simplest model able to describe the noise on the hardware is the global depolarizing channel

\[
\mathcal{E}[\rho] = (1 - \gamma)\rho + \gamma \frac{I}{2^N}, \tag{38}
\]

where \( N \) is the number of qubits and \( \gamma \), the strength of the noise, is the probability of inducing a completely mixed state. As previously stated, we have evidence that this effect is mostly generated by the CNOTs implemented on the hardware.\(^{82,83}\) We stress that our model aims to describe all the noise contributions with only one free parameter. We can thus write the quantum dynamical map that evolves the density operator over one cycle of the protocol as

\[
\rho_{n+1} = (1 - \gamma) \sum_k \pi_k U \rho_n U^\dagger \pi_k + \gamma \frac{I}{2^N}. \tag{39}
\]

Thus, since both the quantum map and the depolarizing channel are unital, they commute. This entails that

\[
\rho_n = \mathcal{E}[\rho_n^{\text{noisless}}] = (1 - \gamma)^n \rho_n^{\text{noisless}} + (1 - (1 - \gamma)^n) \frac{I}{2^N}, \tag{40}
\]

so that

\[
P_{\psi_n}^{\text{noisy}} = (1 - \gamma)^n P_{\psi_n}^{\text{noisless}} + (1 - (1 - \gamma)^n) \frac{I}{2^N}. \tag{41}
\]

As stated previously the quantum dynamical map is unital, thus the identity is a fixed point of the dynamics. This means that the repeated application of the map will bring the system towards the completely mixed state with a timescale

\[
n_{\text{noise}} \propto \frac{1}{\ln(1 - \gamma)}. \tag{42}
\]

For large \( n \), the noise hinders partial thermalization and entails the ITT in the whole Hilbert space. Therefore, partial thermalization can be observed only for a time window from the relaxation time of the Markov chain modelling the dynamics of \( P_{\psi_k}^{\text{noisy}} \) up to the depolarizing time.

The value of the parameter \( \gamma \) is estimated from the experimental data that are shown in Figs. 3 and 5. For such a purpose, first of all, we compute the time-average of the measured probabilities, i.e.,

\[
\frac{P_n}{\langle \phi_k \rangle} = \frac{1}{\pi} \int_0^\pi \mathrm{d}\tau \mathrm{Tr}[\rho_n |\phi_k\rangle \langle \phi_k|]; \tag{43}
\]

this allows us to reduce the impact of the errors coming from the local rotations along \( \pi \) that tends to be canceled by averaging. Then, we fit the averaged probabilities (43) with the results of the theoretical simulations for the noisy dynamics that are obtained by evolving \( \rho_0 \) under the quantum map in Eq. (39). The results coming from the numerical simulations of our theoretical model for the noisy protocol are shown in Fig. 8(b)(d), with the estimated \( \gamma \) fitted over the experimental data of Figs. 3 and 5, respectively. A very good agreement is found when comparing the result of noisy simulations Fig. 8(b)(d) with the experimental data Figs. 3 and 5, especially for \( n \in [1,15] \). This provides us evidence that, despite its simplicity, the noise model introduced in Eq. (39) is able to capture the main features of the real noise affecting the quantum hardware.

In particular, we have found that in the case-study with the single-triplet measurement basis the strength of the noise \( \gamma \approx 0.012 \), meaning that after \( n_{\text{noise}} \approx 8 \) the effect of partial thermalization is hidden by the noise. In fact, by inspecting Fig. 7(a), all the time-averaged probabilities are equal for \( n \approx 24 \), which entails that the system is in a completely mixed state. On the other hand, the continuous monitoring through a Bell measurement basis is less noisy. The strength of the noise, indeed, is estimated to \( \gamma \approx 0.033 \), which means that \( n_{\text{noise}} \approx 30 \). This is reflected in Fig. 7(b) where the time-averaged
probabilities for $n = 24$ show that the state of the monitored quantum system is not yet completely thermalized to infinite temperature within the whole Hilbert space.

As a final remark, we stress that only one free-parameter is needed to theoretically reproduce the experimental data. This suggests that recently introduced mitigation techniques, such as the so-called zero noise extrapolation\cite{84,85,86}, can actually provide a good solution to extract meaningful results from NISQ era intermediate scale quantum devices, with a noise strength that is still far from the fault-tolerant error correction threshold\cite{87} as the one provided by IBMQ.

VI. CONCLUSIONS

In this paper, we have experimentally observed, to our knowledge for the first time, the phenomena of partial and infinite-temperature thermalization on quantum hardware. Specifically, by resorting to IBM quantum machines, ibm_lagos an IBM Quantum Falcon r5.11H processor, we have implemented a sequence of projective measurements, between unitary evolutions, on single-qubit and two-qubit systems. Then, we analyzed the behavior of the monitored system as a function of the number of projections (in our experiments, $n \in [1, 32]$ and $n \in [1, 24]$ for the single-qubit and two-qubit systems respectively). As predicted in the theoretical paper\cite{23}, there exist parameter ranges such that partial (infinite-temperature) thermalization occurs depending on the (non)commutativity of $H$, system Hamiltonian, and $O$, measurement observable. Moreover, we have found that on a quantum hardware the noise is the main obstacle to the long-time stability of partially thermalized states. Yet, we were able to detect them (i.e., partial thermalization) at intermediate time scales that were still resilient to the detrimental effect of the noise.

Our findings are not exclusively aimed at testing
thermalization induced by a sequence of projective measurements on an experimental quantum platform, but they are also expected to have some technological applications for quantum state preparation, provided the system Hamiltonian and the measurement observable. Indeed, we have here demonstrated that continuous monitoring may make the system converge to unconventional quantum states (neither thermal nor ground states) that are described by a finite effective temperature in each subspace defined by the measurement observable. Hence, these states are not just diagonal in the observable basis but they are block-diagonal, meaning that some coherence terms in such basis are preserved asymptotically in the large $n$ limit.

Finally, as an outlook, it would be worth extending our analysis to the continuous monitoring of quantum many-body systems via a sequence of quantum measurements of local observables, say single spin measurements. In this way, connections with numerical results of the recent literature on measurement-induced phase transitions might be determined, and analytical derivations could be accordingly provided.

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CONFLICT OF INTERESTS

The authors have no conflicts to disclose.

DATA AVAILABILITY

The data that supports the findings of this study are available from the authors upon reasonable request.

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