Research Article

Thermal Radiation Effect on Unsteady Magneto-Convective Heat-Mass Transport Passing in a Vertical Permeable Sheet with Chemical Reaction

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The unsteady magneto-convective heat-mass transport passing in a vertical porous sheet with the thermal radiation and the chemical reaction effects has been examined numerically. The governing PDEs have been transferred into ODEs by applying the local similarity transformation. The nondimensional governing equations including the boundary conditions are solved by applying the superposition method with the help of the “MATLAB ODE45” software numerically. The influence of emerging nondimensional numbers/parameters, for example, the Prandtl number (Pr), thermal radiation parameter (R), Schmidt number (Sc), and chemical reaction parameter (Kr), on fluid velocity, concentration, and thermal radiation within the boundary layer has been examined. The outcomes indicate that enhancing values of the Soret and Dufour numbers reduce the thermal boundary layer thickness. Uplifting values of the thermal radiation (0.5-3.5) enhance the local skin friction coefficient and mass transfer rate by approximately 15% and 78% but decrease the heat transfer rate by 47%. The local skin friction coefficient enhances about 21%, and the mass transfer rate reduces about 64% due to an increase in the chemical reaction parameter (0.5-2.0). Finally, we compared our numerical results with previously published literature and observed them to have a good agreement.

1. Introduction

The MHD (hydromagnetic) free convective and heat transfer flow problems in a permeable medium play an important role in various industrial and scientific processes, for example, problems of boundary layer flow control, plasma studies, thermo nuclear fusion, furnace design, metallurgy, mineral and petroleum engineering, geothermal energy extraction, chemical engineering, and solar power technology. These types of problems act on the different engineering devices applying electrically conducting fluids, for example, MHD generators, plasma jet engines, MHD accelerators, MHD pumps, nuclear reactors, and MHD flow meters. The free magneto-convective heat-mass transport passing a permeable medium restricted by a vertical permeable sheet with constant heat flux has been examined by Raptis and Kafoussias [1]. The influences of natural convection and mass transport on the oscillatory flow through a moving vertical isothermal sheet under constant heat sources and suction effects have been explained by Raptis [2]. Sattar [3] discussed the impact of variable suction and Hall current effects on unsteady free magneto-convective heat-mass transport passing a permeable medium near a vertical permeable sheet with constant heat flux. The non-Darcy mixed convective flow along a vertical wall in a saturated permeable medium has been analyzed by Lai and Kulacki [4]. Many researchers such as Eckert and Drake [5], Pop and Ingham [6], Nield and Bejan [7], Gebhart et al. [8], and Incropera et al. [9] had well documented the comprehensive studies of free convective boundary layer flow over different geometrical bodies.
with heat and mass transfer in nonpermeable media. Hydro-
magnetic (MHD) manages the heat and force exchanged by
the surface in boundary layer flow problems. Srinivas and
Muthuraj [10] debilitated the homotopy analysis technique
to get an approximate solution for the hydromagnetic vis-
cous incompressible fluid flow under the permeability and
thermal radiation effects. The MHD viscoelastic fluid char-
acteristic passing a wall has been investigated by Rafa-
tari and Vajravelu [11]. Si et al. [12] explored the heat transfer
for micropolar fluid embedded in a porous medium. Con-
vective flows with simultaneous mass and heat transfer with
the effect of the chemical reaction and a magnetic field rise
in numerous transport processes both artificially and natu-
really in various engineering and science applications. This
concept plays a significant part in the chemical industry,
cooling and power drying industry, cooling of nuclear re-
tors, chemical vapor deposition on surfaces, petroleum
industries, etc. Free convective flow happens frequently in
nature. It happens owing to concentration distinctions and
owing to temperature differences or the combination of
these two, such that there exists differences in water mass
and in atmospheric flows, and therefore, the flow is affected
by such mass distinction. Nield and Bejan [7] explained the
flow past and through permeable media in detail. Hiremath
and Patil [13] discussed the influence of free or natural con-
vective flows on the oscillatory flow passing in a permeable
medium. At constant temperature, the free convection cur-
rents are bounded by a vertical plane surface. Sharma et al.
[14] explained the fluctuating mass and heat transfer on
three-dimensional flow past a permeable medium with the
variable permeability effect. Howell et al. [15] explained that
when technological methods take space, then the thermal
radiation heat transfer has become so significant at higher
temperatures. The impact of thermal radiation heat transfer
cannot be ignored. MHD flow, mass, and heat transfer
become more momentous in industrial areas with thermal
radiation effects. Various methods in science and engineer-
ing sides happen at high temperatures. The concept of heat
transfer of the thermal radiation becomes very notable for
the model of the relevant instruments. The final product
quality is dependent on the heat-controlling factors to a
great extent. The idea of radiative heat transfer in the process
may be conducted to the wished-for product with sought
qualities. The impact of thermal radiation on unsteady
magneto-convective heat-mass transport passing in a per-
meable space has been analyzed by Olanrewaju [16]. In
many practical applications in the presence of two forms of
chemical effects such as heterogeneous and homogeneous,
the heterogeneous and homogeneous reaction mass transfer
takes space by diffusive operations. These reactions include
the species’ molecular diffusion. A heterogeneous reaction
takes space within the phase boundary or in a limited area.
But a homogeneous reaction is similar to an internal source
of heat generation. It happens uniformly throughout a given
phase. The concentration is directly proportional to the rate
of reaction in the chemical reaction with the first order. The
diffusive species may be generated or absorbed due to vari-
ous types of chemical reactions in the presence of the ambi-
ent fluid. It may be greatly influenced by the quality and
properties of completed products. The impact of chemical
reaction and magnetic field on unsteady natural convection
fluid flow passing in a vertical porous sheet under the
diffusion-thermo and thermal-diffusion impacts has been
explained by Reddy et al. [17]. Further, Raju [18] has
observed the impact of the transverse magnetic field on an
unsteady free convection flow past a vertical sheet. He also
explained the numerical outcomes for the impacts of
thermal-diffusion and diffusion-thermo on their system with
heat sources and thermal radiation effects. Sharma and
Bhaskar [19] investigated the impacts of thermal radiation
and chemical reaction on the three-dimensional MHD incompressible and viscous flow. They also considered the
Dufour and Soret impacts on their system. The impact of
the magnetic field and thermal radiation on a transient natu-
ral convective nanofluid that streams along with a vertical
sheet has been analyzed by Kumar et al. [20]. Daniel et al.
[21] explained the combined viscous dissipation effects,
Joule heating, and thermal radiation on the steady two-
dimensional electrical MHD boundary layer nanofluid flow
over a porous linear stretching sheet. Also, they have used
the Keller box method for solving the coupled ODEs.
With the combination of mass and heat transfer proce-
dures, the flow is run by density differences created by con-
centration gradient, temperature gradient, and material
composition simultaneously. The Dufour (diffusion-thermo)
effect is the concentration differences which create the
energy flux. The Soret (thermal-diffusion) effect is the tem-
perature gradient which creates the mass flux. The Soret
influence, for example, has been used in mixture and for iso-
tope separation between gases with medium molecular
weight and gases with very light molecular weight. The
Dufour and Soret influences are faced in numerous practical
applications, for example, in the fields of chemical engineer-
ing and geosciences. The effects of the Dufour number and
Soret number on the hydromagnetic mixed convection-
radiation interaction along a porous surface submerged in
a permeable medium have been studied by Chamkha and
Ben-Nakhi [22]. Alam and Rahman [23] discussed the influ-
ence of Dufour and Soret effects on steady hydromagnetic
free convective heat and mass transfer flow through a verti-
cal permeable sheet embedded in a permeable medium.
Alam et al. [24] explained the Soret and Dufour effects on
unsteady free convective and mass flow through an impul-
sively started infinite vertical porous flat sheet in a porous
medium. They also used in their simulation the transversely
applied magnetic field effect. The impacts of Hall currents,
thermal radiation, and Dufour and Soret on MHD flow by
mixed convective heat flow over a vertical surface in perme-
able media have been investigated by Shateyi et al. [25]. They
have found the numerical solutions of this problem by using
MATLAB routine bvp4c. Hasanuzzaman et al. [26]
explained the effect of transpiration on unsteady free con-
vective and heat transfer flow around a vertical slender body.
They have also used the shooting technique for solving the
ODEs with the help of "MATLAB ODE45" software. Their
simulation is almost the same as our simulation. Hasanuzza-
man et al. [27] investigated the unsteady free magneto-
convective heat-mass transport passing in an infinite vertical
permeable sheet in the presence of Dufour and thermal diffusion effects.

The main result of this current research is to suppose the above problems passing a vertical permeable sheet taking into account the chemical reaction and thermal radiation effects. The main novelty of this study is to compare our results with a published paper. Another novelty of this paper is also enhanced by assuming the chemical reaction and thermal radiation under the Runge-Kutta-Merson integration technique which is not explained yet. Computations have been performed for a vast range of the dimensionless numbers/parameters; for example, thermal radiation parameter, Soret number, suction parameter, chemical reaction parameter, Prandtl number, Dufour number, Schmidt number, and magnetic number on temperature, concentration, and velocity profiles are discussed graphically and analyzed. Besides, the properties of the heat and mass transfer and skin friction coefficient have been explained in the tabular forms.

2. Governing Equations

The two-dimensional unsteady free magneto-convective heat-mass transport of an electrically conducting viscous fluid passing in a vertical permeable sheet at \( y = 0 \) is considered. The direction of an upward sheet is the \( x \)-axis. The \( y \)-axis is the normal of the plane sheet in the fluid. When a uniform magnetic field \( B \) is imposed on the sheet, then the permeable sheet is supposed to be electrically nonconducting along the \( y \)-axis as shown in Figure 1.

We presume that the induced magnetic field is insignificant for a very small flow magnetic Reynolds number compared with one of the research projects [28]. Then, the lines of the magnetic force are permanent relative to the fluid such that \( B = (0, B_0, 0) \). The current density is \( J = (J_x, J_y, J_z) \), and the continuity equation of charge is \( \nabla \cdot J = 0 \) which implies that \( J_y = \text{constant} \). The propagation direction is presumed only along the \( y \)-axis. This propagation direction does not have any change along the \( y \)-axis. So the differentiation of \( J_y \) with respect to \( y \) must be zero such as \( \partial J_y / \partial y = 0 \). Since this constant of integration is zero when the sheet is electrically nonconducting, \( J_y = 0 \) at the sheet and it is zero everywhere.

The one-dimensional problem under the Boussinesq approximation and the above assumptions may be used in the below form [24]:

\[
\begin{align*}
\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} & = \nu \frac{\partial^2 u}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} + K'(C - C_{\infty}) - \frac{\sigma' B_0^2 u}{\rho} .
\end{align*}
\]

where \( u \) and \( v \) are the components of the velocity in the \( x \) and \( y \) directions, respectively. The fluid density is \( \rho \), \( \nu \) is the kinematic viscosity, the coefficient of concentration expansion is \( \beta' \), the coefficient of thermal expansion is \( \beta \), the fluid temperature is \( T \), the wall temperature is \( T_w \), the fluid temperature in the free stream is \( T_{\infty} \), the component of radiative heat flux is \( q_r \), \( C \) is the fluid concentration, the wall concentration is \( C_w \), the free stream concentration is \( C_{\infty} \), the thermal conductivity of the sheet is \( k \), the concentration susceptibility is \( C_s \), the specific heat at constant pressure is \( C_p \), the mean temperature of the fluid is \( T_m \), \( k_T \) is the thermal diffusion ratio, the mass diffusivity coefficient is \( D_m \), \( g \) is the gravitational acceleration, and the chemical reaction rate of species concentration is \( K' \).

Upon using a similarity parameter \( \sigma \),

\[
\sigma = \sigma(t),
\]

where the time-dependent length scale is \( \sigma \). The solution of equation (1) is supposed in terms of this length scale given by

\[
v = -v_0 \frac{u}{\sigma} .
\]

Here, the dimensionless normal velocity at the sheet is \( v_0 \). \( v_0 < 0 \) represents blowing, and \( v_0 > 0 \) represents suction.
According to the Rosseland approximation [29], the radiative heat flux $q_r$ is given by

$$ q_r = -\frac{4\sigma^* \partial T^4}{3K^*} \frac{\partial y}{\partial y}, $$

(9)

where the constant of Stefan-Boltzmann is $\sigma^*$ and the coefficient of the mean absorption is $K^*$.

We consider from Raptis [30] that the difference between the fluid temperature and the free stream temperature is small enough.

Expanding in a Taylor series $T^4$ about $T_0$ and ignoring higher-order terms, we have

$$ T^4 \equiv 4T_0^4 T - 3T_0^4. $$

(10)

Now, the below similarity variables can be applied:

$$ \eta = \frac{y}{\sigma}, f(\eta) = \frac{u}{U_0}, \theta(\eta) = \frac{\theta - T_0}{T_w - T_0}, \phi(\eta) = \frac{C - C_0}{C_w - C_0}. $$

(11)

Applying equations (7)–(11), equations (1)–(4) are converted into the nondimensional coupled ODEs as follows:

$$ f'' + 2\xi f' + G_r \theta + G_m \phi - M f = 0, $$

(12)

$$ \theta'' + \frac{Pr}{1 + R} \left( 2\xi \theta' + Df \phi'' \right) = 0, $$

(13)

$$ \phi'' + 2Sc \phi' + ScSo \theta'' + K_r \phi = 0. $$

(14)

The converted boundary conditions are given by

$$ f = 1, \theta = 1, \phi = 1 \text{ at } \eta = 0, $$

(15)

$$ f = 0, \theta = 0, \phi = 0 \text{ at } \eta \to \infty, $$

(16)

where the local Grashof number is $G_r = g\beta(T_w - T_0) \alpha^2 / U_0 \nu$, the magnetic force parameter is $M = \alpha' B_0^2 \sigma^2 / \nu$, the Prandtl number is $Pr = \nu C_p / k$, the local modified Grashof number, the Dufour number is $D_f = D_m k_f (T_w - T_0) / C_m C_p \nu (T_w - T_0)$, the Soret number is $S_0 = D_m k_f (T_w - T_0) / \nu T_m (C_w - C_0)$, the Schmidt number is $Sc = \nu / D_m$, the thermal radiation parameter is $R = 16 \sigma^* T_0^3 / K^* K$, the chemical reaction parameter is $K_r = K' \sigma^2 / \nu$, and $\xi = \eta + (\nu \xi / 2)$.

The flow parameters are the skin-friction coefficient ($\tau$), the Nusselt number ($Nu$), and the local Sherwood number (Sh) which are defined as

$$ \tau \propto f', \quad Nu \propto -\theta', \quad Sh \propto -\phi'. $$

(17)

### 3. Numerical Solution

By applying the superposition method [31], the solutions of equations (12)–(14) with the boundary conditions (15)–(16) are obtained. The boundary value problems are converted into the initial value problem by using the superposition technique. This initial value problem may easily be integrated by using an initial value solver. So to convert equations (12)–(14) to an initial value problem, the functions $f(\eta)$, $\theta(\eta)$, and $\phi(\eta)$ are, respectively, decomposed to

$$ f(\eta) = f_1(\eta) + \mu f_2(\eta) + \lambda f_3(\eta) + \delta f_4(\eta), $$

(18)

$$ \theta(\eta) = \theta_1(\eta) + \mu \theta_2(\eta) + \lambda \theta_3(\eta) + \delta \theta_4(\eta), $$

(19)

$$ \phi(\eta) = \phi_1(\eta) + \mu \phi_2(\eta) + \lambda \phi_3(\eta) + \delta \phi_4(\eta), $$

(20)

where $\delta$, $\mu$, and $\lambda$ are arbitrary constants. Now, put the equations (18)–(20) in equations (12)–(14) and then separate the various coefficients to zero. Finally, we get the differential equations which are given by

$$ f_1'' + 2\xi f_1' - Mf_1 + G_r \theta_1 + G_m \phi_1 = 0, $$

$$ f_2'' + 2\xi f_2' - Mf_2 + G_r \theta_2 + G_m \phi_2 = 0, $$

$$ f_3'' + 2\xi f_3' - Mf_3 + G_r \theta_3 + G_m \phi_3 = 0, $$

$$ f_4'' + 2\xi f_4' - Mf_4 + G_r \theta_4 + G_m \phi_4 = 0, $$

$$ \theta_1'' + \frac{Pr}{1 + R} \left( 2\xi \theta_1' + Df \phi_1'' \right) = 0, $$

$$ \theta_2'' + \frac{Pr}{1 + R} \left( 2\xi \theta_2' + Df \phi_2'' \right) = 0, $$

$$ \theta_3'' + \frac{Pr}{1 + R} \left( 2\xi \theta_3' + Df \phi_3'' \right) = 0, $$

$$ \theta_4'' + \frac{Pr}{1 + R} \left( 2\xi \theta_4' + Df \phi_4'' \right) = 0, $$

$$ \phi_1'' + 2Sc \phi_1' + ScSo \theta_1'' + K_r \phi_1 = 0, $$

$$ \phi_2'' + 2Sc \phi_2' + ScSo \theta_2'' + K_r \phi_2 = 0, $$

$$ \phi_3'' + 2Sc \phi_3' + ScSo \theta_3'' + K_r \phi_3 = 0, $$

$$ \phi_4'' + 2Sc \phi_4' + ScSo \theta_4'' + K_r \phi_4 = 0, $$

(21)

The initial values of the decomposed functions $f_1(\eta), f_2(\eta), f_3(\eta), f_4(\eta), \cdots$ are now got passing in the boundary conditions (15) and (16) as

$$ f_1(\eta) = 1.0, f_2(\eta) = 0, f_3(\eta) = 0, f_4(\eta) = 0, $$

$$ \theta_1(\eta) = 1.0, \theta_2(\eta) = 0, \theta_3(\eta) = 0, \theta_4(\eta) = 0, $$

$$ \phi_1(\eta) = 1.0, \phi_2(\eta) = 0, \phi_3(\eta) = 0, \phi_4(\eta) = 0. $$

(22)

Also, as $\eta \to \infty$, using the boundary conditions (15) and (16) in (16)–(18), we obtain
\[ \mu = f_1(\theta_1 \phi_1 - \phi_1 \theta_1) + \theta_1(f_2 \phi_1 - f_1 \phi_1) + \phi_1(f_3 \theta_1 - f_3 \phi_1), \]
\[ \lambda = f_1(\theta_2 \phi_2 - \phi_2 \theta_2) + \theta_1(f_2 \phi_2 - f_1 \phi_2) + \phi_1(f_2 \theta_1 - f_2 \phi_1), \]
\[ \delta = f_1(\theta_3 \phi_3 - \phi_3 \theta_3) + \theta_1(f_3 \phi_3 - f_3 \phi_1) + \phi_1(f_3 \theta_1 - f_3 \phi_1). \] (23)

In (16)–(18), all the functional values are obtained as

\[ \frac{\partial f(\eta)}{\partial \eta} = \frac{\partial f_1(\eta)}{\partial \eta} + \mu \frac{\partial f_2(\eta)}{\partial \eta} + \lambda \frac{\partial f_3(\eta)}{\partial \eta} + \delta \frac{\partial f_4(\eta)}{\partial \eta}, \]
\[ \frac{\partial \theta(\eta)}{\partial \eta} = \frac{\partial \theta_1(\eta)}{\partial \eta} + \mu \frac{\partial \theta_2(\eta)}{\partial \eta} + \lambda \frac{\partial \theta_3(\eta)}{\partial \eta} + \delta \frac{\partial \theta_4(\eta)}{\partial \eta}, \]
\[ \frac{\partial \phi(\eta)}{\partial \eta} = \frac{\partial \phi_1(\eta)}{\partial \eta} + \mu \frac{\partial \phi_2(\eta)}{\partial \eta} + \lambda \frac{\partial \phi_3(\eta)}{\partial \eta} + \delta \frac{\partial \phi_4(\eta)}{\partial \eta}. \] (24)

The missing slopes are given by

\[ \frac{\partial f(0)}{\partial \eta}, \frac{\partial \theta(0)}{\partial \eta}, \frac{\partial \phi(0)}{\partial \eta}. \] (25)

Now, assuming the values of the missing slopes:

\[ \frac{\partial f(0)}{\partial \eta} = \mu, \quad \frac{\partial \theta(0)}{\partial \eta} = \lambda, \quad \frac{\partial \phi(0)}{\partial \eta} = \delta, \] (26)

the initial conditions for the missing slopes of the decomposed functions are observed easily. Integrate equations (12)–(14) by using an initial value solver to get the converged solutions which are focused graphically in Figures 2–15 by applying the Runge-Kutta-Merson integration scheme. Now, Sh, Nu, and \( \tau \) are, respectively, denoted as the Sherwood number, the Nusselt number, and the local skin friction coefficient which are proportionate to \( -\partial \phi(0)/\partial \eta \), \( -\partial \theta(0)/\partial \eta \), and \( \partial f(0)/\partial \eta \), respectively.

4. Results and Discussions

The impact of thermal radiation on unsteady magneto-convective heat-mass transport passing in a vertical permeable sheet under the chemical reaction effect has been analyzed numerically in this research. By applying the superposition technique, we have solved the set ODEs (10)–(12) with the boundary conditions (15) and (16) numerically. Also, we have used the “MATLAB ODE45” software. The impacts of the suction parameter \( \nu_0 \), the Dufour number \( (DF) \), the magnetic force parameter \( (M) \), the Soret number \( (So) \), the radiation parameter \( (R) \), the Schmidt number \( (Sc) \), the chemical reaction parameter \( (K_r) \), and the Prandtl number \( (Pr) \) on temperature, concentration, and velocity distributions are displayed in Figures 2–15. The values 7.0, 1.0, and 0.71 are supposed for \( Pr \) (1.0 and 7.0 for water at 17\(^\circ\)C and 0.71 for air at 20\(^\circ\)C). The values 0.75, 0.60, and 0.22 are also supposed for \( Sc \) (0.60 for vapor water, 0.22 for hydrogen, and 0.75 for oxygen). The remaining nondimensional parameter/number values are however taken arbitrarily.

4.1. Velocity Distributions for Various Values of Numbers/Parameters. Figure 2 depicts the velocity distribution for...
various values of the magnetic force parameter \( (M) \). From Figure 2, it is found that the fluid velocity reduces for rising values of magnetic parameter \( (M) \). With increasing values of the magnetic force parameter, a resistive kind of force, for example, a drag force, is generated. This resistive type of force is called Lorentz force. This is due to the fact that the magnetic force parameter produces a resistive kind of force (Lorentz force) which causes reduction in the fluid velocity.

Figure 3 represents the impact of the suction parameter \( (v_0) \) on the velocity profile. When \( v_0 > 0 \), then the whole system fluid suction event has happened. It can be concluded from Figure 3 that for the case of suction \( (v_0 > 0) \), the mass of the fluid in the computational domain decreases. For this reason, the frictional force reduces. So the velocity of fluid
reduces for increasing values of the suction. This is because suction stabilizes the boundary layer growth. The velocity distribution is observed to enhance and reaches the highest value in a region close to the leading edge of the sheet and then reduces to zero gradually. The Prandtl number \( \frac{\rho \nu \kappa}{\rho C_p} \) is proportional to the kinematic viscosity. From graph 4, it is noticed that when the Prandtl number enhances, then the fluid kinematic viscosity increases.

Table 1: Local skin friction coefficient and heat and mass transfer rates for various values of the magnetic force parameter \( (M) \).

| \( M \) | \( f'(0) \) | \( -\theta'(0) \) | \( -\phi'(0) \) |
|---|---|---|---|
| 0.5 | 10.566110056450 | 0.9329905238265 | 0.2120305487463 |
| 1.5 | 8.5790319819634 | 0.9329905238265 | 0.2120305487463 |
| 3.0 | 6.5412588977310 | 0.9329905238265 | 0.2120305487463 |
| 4.0 | 5.5464274696343 | 0.9329905238265 | 0.2120305487463 |

Table 2: Local skin friction coefficient and heat and mass transfer rates for different values of the suction parameter \( (v_0) \).

| \( v_0 \) | \( f'(0) \) | \( -\theta'(0) \) | \( -\phi'(0) \) |
|---|---|---|---|
| 0.0 | 10.6263717252703 | 0.776316119591959 | 0.2069420242735 |
| 0.5 | 10.566110056450 | 0.9329905238265 | 0.2120305487463 |
| 1.5 | 9.87472905951721 | 1.27771556584173 | 0.21670531323914 |
| 2.5 | 8.656095988809097 | 1.654923593722190 | 0.21710157759460 |
| 3.5 | 7.1517444437728 | 2.055580811460300 | 0.21982144380065 |
It is found that with the increasing values of the suction parameter, the fluid temperature reduces. This is due to the fact that the suction parameter decelerates fluid particles through the permeable wall decreasing the growth of the fluid thermal boundary layers. The Prandtl number ($Pr$) is inversely proportional to the thermal conductivity. The temperature distribution in Figure 9 is found to reduce temperature for uplifting values of $Pr$ (the thermal conductivity decreases). Physically, the lower thermal conductivity has relatively a higher Prandtl number. It reduces heat conduction and so temperature decreases. Hence, the rate of heat transfer enhances for uplifting values of $Pr$ so that the temperature profile lessens. Figure 10 shows the impact of thermal radiation parameter ($R$) on the temperature profile. From Figure 10, it is noticed that the temperature gradient at the surface decreases for rising values of the thermal radiation parameter. At the surface, the heat transfer rate decreases for increasing $R$. The thermal radiation parameter is accountable for the thermal boundary thickening. The fluid loses the heat energy from the flow region, and for this reason, the system cools. This is due to the fact that the Ros- seland approximation increases the temperature. Figure 11 depicts the effect of diffuso-thermal parameter, i.e., the Dufour number, $Df = Dm kT (Cw - Cs)/(Cs Cp(Tw - T∞))$, on the temperature profile. The Dufour effect mentions heat flux produced by a solutal (concentration) gradient. The temperature distribution accentuates for increasing the diffusion-thermo parameter ($Df$) as shown in Figure 11. The temperature distribution in the absence of the Dufour effect is smaller in comparison to that in the presence of the Dufour number effect. The thickness of the thermal boundary layer accelerates considerably in the presence of strong Dufour effects.

4.3. Concentration Distribution for Various Values of Numbers/Parameters. Figure 12 depicts the impact of the suction parameter ($v_o$) on the concentration distribution. Sucking decelerated fluid particles past the permeable wall decreases the concentration boundary layer growth. Figure 13 represents the effect of the various values of the Schmidt number ($Sc$) on the concentration distribution. The molecular (species) diffusivity is inversely proportional to the Schmidt number. When $Sc > 1$, then the rate of momentum diffusion exceeds the rate of species diffusion. But it was the opposite behavior for $Sc < 1$. The concentration (species) and momentum layers will have the same diffusivity rates and thickness for $Sc = 1$. The concentration distribution in Figure 13 is found to reduce the concentration for uplifting values of $Sc$. The associated depletion in mass diffusivity outcomes in a small forceful mass transfer decreases concentration levels and also reduces the thickness of the concentration boundary layer. This is because mass

Table 3: Local skin friction coefficient and heat and mass transfer rates for different values of the Prandtl number ($Pr$).

| $Pr$  | $f'(0)$ | $-\theta'(0)$ | $-\psi'(0)$ |
|-------|---------|---------------|-------------|
| 0.71  | 10.5661100056450 | 0.932990523828265 | 0.212030548746318 |
| 1.0   | 10.1474550375773 | 1.14348338490675 | 0.126571701930447 |
| 7.0   | 8.46412529013253 | 4.09185943257223 | -1.13908665092652 |

Table 4: Local skin friction coefficient and heat and mass transfer rates for different values of the radiation parameter ($R$).

| $R$   | $f'(0)$ | $-\theta'(0)$ | $-\psi'(0)$ |
|-------|---------|---------------|-------------|
| 0.5   | 10.5661100056450 | 0.932990523828265 | 0.212030548746318 |
| 1.5   | 11.2599969462999 | 0.694552002634953 | 0.305717248240946 |
| 2.5   | 11.7605822700024 | 0.574503600647324 | 0.350935901031615 |
| 3.5   | 12.1549714597407 | 0.49963529878373 | 0.378197640492878 |
The Soret number is defined by $So = \frac{D_m k_r (T_w - T_\infty)}{\nu T_m (C_w - C_\infty)}$; i.e., the Soret number is inversely proportional to the mass diffusivity coefficient ($D_m$). The Soret effect rises where large heavy molecules and small light molecules separate under a temperature gradient. The Soret number enhancement means that the mass diffusivity coefficient increases. For this reason, the concentration distribution increases significantly as the Soret number increases. This outcome is an important enhancement in concentration boundary layer thickness. The impact of the chemical reaction parameter ($K_r$) on the concentration profile is displayed in Figure 15. Rising values of the chemical reaction parameter increase the concentration of the fluid.

4.4. Heat and Mass Transfer Rates and Local Skin Friction. The authors have investigated not only the velocity, temperature, and concentration fields but also the values of the local skin friction coefficient and heat and mass transfer rates. The authors gave the local skin friction coefficient and heat and mass transfer rates in Tables 1–7. Tables 1–7 represent the impact of different values of the mentioned parameters/numbers on the values of heat and mass transfer rates and local skin friction coefficient. From Tables 1–7, it is found that the skin friction reduces for uplifting values of the magnetic force parameter, suction parameter, Prandtl number, and Schmidt number but enhances for enhancing values of the thermal radiation parameter, Soret number, and chemical reaction parameter.

The heat transfer rate enhances for uplifting values of the suction parameter and Prandtl number, but a reverse trend is found for the thermal radiation parameter. Also, the mass transfer rate enhances for rising values of the suction parameter and thermal radiation parameter but decreases for the Prandtl number, Soret number, Schmidt number, and chemical reaction parameter.

4.5. Comparison. The present research results have been compared with Alam et al. [24]. The comparison of the local Sherwood number and the local Nusselt number has been shown in Tables 8 and 9, respectively. From these tables, it is found that the comparisons of the present numerical

### Table 5: Local skin friction coefficient and heat and mass transfer rates for different values of the Soret number (So).

| So  | $f'(0)$ | $-\theta'(0)$ | $-\psi'(0)$ |
|-----|---------|---------------|-------------|
| 1.0 | 9.9357 | 0.9329990523828265 | 0.359608012618135 |
| 2.0 | 10.5611 | 0.9329990523828265 | 0.212030548746318 |
| 3.0 | 11.1964 | 0.9329990523828265 | 0.064453083187319 |

### Table 6: Local skin friction coefficient and heat and mass transfer rates for different values of the Schmidt number (Sc).

| Sc  | $f'(0)$ | $-\theta'(0)$ | $-\psi'(0)$ |
|-----|---------|---------------|-------------|
| 0.22 | 10.5611 | 0.932990523828265 | 0.212030548746318 |
| 0.50 | 9.61314175800702 | 0.932990523828265 | 0.2215338418678 |
| 1.00 | 9.0343 | 0.932990523828265 | 0.18186779242937 |

### Table 7: Local skin friction coefficient and heat and mass transfer rates for different values of the chemical reaction ($K_r$).

| $K_r$ | $f'(0)$ | $-\theta'(0)$ | $-\psi'(0)$ |
|------|---------|---------------|-------------|
| 0.5  | 10.5611 | 0.9329990523828265 | 0.212030548746318 |
| 1.0  | 11.1649 | 0.932990523828265 | 0.07583645608230 |
| 1.5  | 11.8324799143874 | 0.932990523828265 | -0.089346500221726 |
| 2.0  | 12.8169989130888 | 0.932990523828265 | -0.300468126650875 |

### Table 8: Comparison of the local Sherwood number (Sh) for different values of So and Df when $R = 0$ and $K_r = 0$ [24].

| So   | Df | Alam et al. [24] | Present results | Persistence of error |
|------|----|-----------------|----------------|---------------------|
| 1.0  | 0.06 | 0.315615 | 0.31607639 | 0.046139 |
| 0.5  | 0.12 | 0.468128 | 0.46965549 | 0.152749 |
| 0.4  | 0.15 | 0.496002 | 0.49747809 | 0.147609 |
| 0.2  | 0.30 | 0.549515 | 0.54972859 | 0.021359 |
| 0.1  | 0.60 | 0.575236 | 0.57553013 | 0.029413 |

### Table 9: Comparison of the local Nusselt number (Nu) for different values of So and Df when $R = 0$ and $K_r = 0$ [24].

| So   | Df | Alam et al. [24] | Present results | Persistence of error |
|------|----|-----------------|----------------|---------------------|
| 1.0  | 0.06 | 1.652241 | 1.65241042 | 0.016942 |
| 0.5  | 0.12 | 1.541984 | 1.54403130 | 0.204730 |
| 0.4  | 0.15 | 1.517881 | 1.51849539 | 0.061439 |
| 0.2  | 0.30 | 1.450355 | 1.45560581 | 0.525081 |
| 0.1  | 0.60 | 1.364651 | 1.36587384 | 0.131284 |
outcomes show a good agreement with previously published outcomes under the special cases. These comparisons ensure the accuracy and validity of the present research work.

5. Conclusions

The unsteady free magneto-convective heat-mass transport passing in a vertical permeable sheet has been analyzed numerically under the thermal radiation and the chemical reaction effects. From the above numerical simulations, the below conclusions can be illustrated:

(i) The local skin friction enhances about 15%, 13%, 28%, 56%, and 21% due to increasing thermal radiation parameter (0.5-3.5), Soret number (1.0-3.0), local Grashof number (5.0-10.0), modified Grashof number (5.0-10.0), and the chemical reaction parameter (0.5-1.0), respectively. Besides, rising values of the magnetic force number (0.5-4.0), suction parameter (0.5-1.5), Prandtl number (0.71-1.0), and Schmidt number (0.22-1.0) reduce the local skin friction by 48%, 7%, 20%, and 15%, respectively

(ii) Uplifting values of the suction parameter (0.5-1.5) and Prandtl number (0.71-1.0) enhance the heat transfer rate by 64% and 23%, respectively, but decrease about 47% for enhancing the thermal radiation parameter (0.5-3.5)

(iii) The mass transfer rate enhances about 5% due to enhancing the suction parameter (0.0-1.5). Besides, increasing the Soret number (1.0-3.0), Schmidt number (0.22-1.0), and chemical reaction parameter (0.5-2.0) decreases the mass transfer rate by 82%, 14%, and 64%, respectively

The result of this paper can be helpful for suspensions, production of paper, plasma studies, thermo nuclear fusion, furnace design, metallurgy, mineral and petroleum engineering, geothermal energy extraction, chemical engineering, solar power technology, etc.

Nomenclature

| Symbol | Definition |
|--------|------------|
| $u$    | Velocity component along the x-axis |
| MHD   | Hydromagnetic |
| $J$   | Density of current |
| $T_w$ | Wall temperature |
| $C$   | Fluid concentration |
| $C_{\infty}$ | Free stream concentration |
| $U_0(t)$ | Uniform surface velocity |
| $g$   | Acceleration due to gravity |
| $\beta$ | Volumetric expansion coefficient with temperature |
| $k$   | Thermal conductivity |
| $C_p$ | Specific heat at constant pressure |
| $k_f$ | Thermal diffusion ratio |
| $\sigma$ | Similarity parameter |
| $G_r$ | Local Grashof number |
| $M$   | Magnetic parameter |
| $D_f$ | Dafour number |
| $Sc$  | Schmidt number |
| $\tau$ | Local skin friction coefficient |
| $Sh$  | Sherwood number |
| $\theta$ | Dimensionless temperature |
| $q_r$ | Radiative heat flux |
| $R$   | Thermal radiation parameter |
| $v$   | Velocity component along the y-axis |
| $B$   | Uniform magnetic field |
| $T$   | Temperature of fluid |
| $T_{\infty}$ | Free stream temperature |
| $C_w$ | Wall concentration |

Data Availability

The datasets generated and/or analyzed during the current study are not publicly available due to the fact that we will use these data when extending our further research but are available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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