Happy recollections of five decades in physics

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These two days of discussions on physics with special attention to my scientific quests gives me great gratification. So many of my friends who have distinguished themselves as physicists have put their minds to present such excellent expositions. The contributions that I have made over the last half century, when handled by these experts, show their interconnection through the paths chosen by them, and also seem to show the interconnection between these ideas.

1. The Vector-Axial Vector Theory of Weak Interaction

When I joined the University of Rochester as a graduate student, I had to undergo a written qualifying examination, and I had to review various subjects like quantum mechanics, nuclear physics, electromagnetic theory, and classical mechanics. The study of nuclear physics gave me a chance to review the evidence for the relativistic form of the beta decay interaction. It was clear that the data was inconsistent, but the popular vote seemed to be for an S-T interaction. But this gave no chance for combining the muon decay with the beta decay. After the Ph.D. qualifying examination, Robert Marshak asked me to look into the problem of beta decay and muon decay interaction. The decay of the charged pion could be only axial vector or pseudoscalar interaction. So to make possible a Universal Fermi interaction, the beta decay had to contain A (axial vector); but if that was so, the companion interaction should be V (Vector). It then has a great formal similarity with the electromagnetic interaction, which was pure vector. So my conclusion was that in view of the inconsistency of the data, on angular correlation and decay rates, there should be some experiments that were wrong. I wanted to present this result at the Rochester Conference on High Energy Physics in the spring of 1957; it would have resolved the problem of weak interaction. But I was not given the chance. By the time I got to present it, I had talked to Murray Gell-Mann about what I wanted to present at the Rochester Conference in Spring; finally my manuscript was presented by Robert Marshak at a conference in Italy. While this was going on, Richard Feynman and Murray Gell-Mann wrote a paper with the assertion that the beta interaction was V-A, but with no analysis of experiments. This was fifty years ago, but there are still people who call it the Feyman-Gell Mann theory. There are even textbook writing “experts” who are innocent of this knowledge. (On one occasion when I went to McGill, I wanted to tell the sorry to the author of a standard book on Nuclear Physics. Even though he and I were present at a dinner, he bolted his food and made himself scarce.)

The vectorial nature of the leptonic interaction current suggests similarity between electromagnetism and weak interactions. It was suggestive to later investigators that gauge couplings may be involved. Once this beta interaction was understood, the other weak interactions became understood one by one. The most remarkable development was the electro-weak unified theory of Weinberg and Salam.
In his opening on the V-A-theory, Steven Weinberg pointed out that the discovery of V-A gave the key to future developments of gauge theories of electron weak unification. When we studied the symmetries of the V-A interaction and the isotopic spin selection rules and considered the SU(2) characterization generated by the weak electron-neutrino currents, little could one imagine that the theory would suggest the gauge theory of electro-weak interactions. With the participation of Susumo Okubo, it gave us an opportunity to find the solution of the singular integral equation for decay. Ashok Das and Sandip Pakvasa surveyed the present status of the theory of weak interactions and the “coming of age” of neutrinos.

2. Symmetries
Symmetries were the tool for understanding much of atomic and nuclear physics. In atomic physics, the interactions were well known, but for complex spectra one had to emphasize symmetry aspects. In nuclear physics symmetries provided the vehicle for the computations. But when it comes to particle physics and field theory, symmetries gave us mass and cross section sum rules. For example, Heitler showed that charge independence gives the sum rule for pion-nucleon scattering:

\[
\sigma(p^+ \rightarrow p^0) = \sigma(p^- \rightarrow n^0) = \sigma(p^0 \rightarrow n^0)
\]

On the other hand, Lie algebras were used to study the mass and electromagnetic properties of observed particles. Marshak, Okubo, and Sudarshan found a charge independence relation between the magnetic moments of the three \(\Sigma\) hyperons. Okubo used the seminal method of the Wigner-Eckart theorems to discover the mass formula for SU(3) multiplets. Macfarlane and Sudarshan found results for transition magnetic moments of \(\Sigma^0\) and their transition masses in relation to the magnetic moments and mass differences within the multiplet. In the meantime, Levinson, Lipkin, and Meshkov, and our group in Rochester were leapfrogging through the question of combining the internal symmetries with relativistic invariance raised by McGlinn followed up by Han, Sudarshan, and finally by O’Raifeartaigh.

Symmetry is not only in quantum particle physics, but also in Classical Mechanics. Currie, Jordan, Mukunda, and Sudarshan studied the consequences for classical dynamics, and one result was the No Interaction theorem for relativistic classical Hamiltonian dynamics. Later, in an extended collaboration led by Marmo and Wang on nonholonomic constraints we found ways of going beyond the no-interaction limitation. The reviews of Sydney Meshkov, Moo-Young Han, N. Mukunda, and Thomas Jordan commented on these topics authoritatively, and brought out the vitality of symmetry ideas. It was a major milestone when Susumu Okubo discovered the broken SU(3) mass formula. When SU(3) was introduced into particle physics, the formalisms got new applications.

3. Spin and Statistics
From the works of Stoner in magnetism, followed up by Pauli in atomic spectra, one arrived at the basic principle of atomic structure, namely the Pauli principle. At about the same time, Bose had shown that photons obeyed a different statistics. Many people consider that Pauli proved the spin-statistics theorem for relativistic quantum field theory, but the main consequences are in the non-relativistic domain of atomic structure and the specific heat of dielectrics; so a non-relativistic and a more complete proof was necessary.

For more than forty years I have held that we must have a simple and direct derivation of the spin-statistics connection, not restricted to relativistic quantum theory. Some time ago, Neuenschwander also raised the same question.

My attempts to do this, beginning in 1968, have been elegantly summarized by Shaji. Luis Boya showed how to extend it to even theories with higher space dimensions, David Finkelstein related it to super-conductivity, and Ian Duck reviewed the whole domain in his spontaneous poetry. Tom Imbo and I explored the spin-statistics connection on manifolds with nontrivial
topology of the configuration space. Unfortunately, a health emergency prevented him from being present here.

4. Quantum Coherence Optics

I am again reminded of my youth spent in Rochester. I trace my love of optics to Professor M. A. Thangaraj, who taught me optics in my undergraduate class in India; but the development in coherence optics was annotated to me by Emil Wolf and Leonard Mandel. While there were statements made about “Quantum Coherence” that cannot be spanned by “Classical Optics,” only classical coherence optics embellished with quantum labels was treated in “Quantum Optics” by other authors in which only classical distribution functions were postulated. In 1963 I gave the first and only formulation of optical coherence in quantum field theory, first in a short paper in Physical Review Letters and later in an extended paper (of about a hundred pages) in the Journal of Mathematical and Physical Sciences. Chandra Lal Mehta, who was at Rochester, in the early sixties, contributed significantly to the development of “quantum” optics. He has reviewed for us the theory and its history. The quantum aspect of coherence leads to anti-bunching and negative intensity correlations, and this was demonstrated experimentally by the work of Jeff Kimble and Leonard Mandel. He also did remarkable experiments in “Squeezed Light,” which is an extreme case of quantum optics. Kimble’s “Assessment” gives us a firsthand account of nonclassical light and its experimental confirmation. In his presentation, Rajiah Simon traces the relevance of nonclassical light in Information Science. He also puts on record the history of the discovery of the Diagonal (Coherent) Representation. Simon’s analysis of false claims should convince any intelligent scientist. It is indeed a treat to hear the field being reviewed by four people who were pioneers in quantum optics and who continue to enrich the subject. Little did I realize when I wrote down the Diagonal Representation, that it would have such ramifications, or that false claims would be made to confuse the physicists who have not done their homework.

5. Quantum Zeno Effect

In classical physics we have the physics of discrete particles as well as those of waves. For these waves the “intensity” or “density” or any physical property is proportional to the square of the amplitude. But it is the amplitude that obeys equations of motion and in particular, the change from one wave amplitude to another is linear in time. Hence transition probabilities should be quadratic in time. On the other hand, in the decay of radioactive nuclei the rate is constant, and the probability grows linearly with time. How come we can get a constant rate (and linear growth of probability)?

There are two ways of obtaining this. One is best illustrated by Dirac’s calculation of the rate of decay of an excited atom. Here we take into account a large continuum of energy levels of the final states; the integrated probability of transition is well approximated (for times not too short) by an expression linear in time. This is generally the accepted method to deal with an unstable system. Another method is to deduce a Markovian (probabilistic) evolution by resetting the phases at a very short time and this leads to the Kossakowski semigroup. In either version, we get a constant rate (and the radioactive law).

But what about the exact time dependence for very short times when the transition rate is proportional to the square of the time? If we were to consider this as a measurement and reset the system at this time interval, we could proceed to determine at a later time whether the system has decayed or not. So the survival probability of decay over a time t is given by and is the limit of $P(t) = (P(t/N))^N$. For large N, the survival amplitude of the original state approaches unity, and hence the decay is inhibited. This is the Quantum Zeno effect of Misra and Sudarshan. Prashant Valanju was the first to demonstrate this effect by an analysis of the experimental data on hadron-nucleus collisions for his doctoral thesis.
The presentations by Marmo, Pascazio, and Chiu discussed various aspects of Quantum Zeno Effect. Clearly this effect provides a mechanism for stabilizing qubits and controlling decoherence.

Giuseppe Marmo and Saverio Pascazio have put the theory in a proper and rigorous mathematical framework including the Trotter formula, and showed its manifold manifestations\textsuperscript{19}. Charles Chiu reviewed this effect with particular attention to excited atom decay and nucleon-nucleus collisions\textsuperscript{20} and exhibited the details of the true evolution of a metastable state\textsuperscript{21}. He also connected it with Aharonov-Vardi\textsuperscript{n}man’s\textsuperscript{22} work on the technique of realizing a simple Feynman path experimentally. Wayne Itano critically reviewed the work of his group on the experimental verification of QZE. Their experimental\textsuperscript{23} results agree with the work of Raizen and collaborators\textsuperscript{24} in Austin and yield sufficient experimental verification of the Quantum Zeno Effect.

6. Tachyons
The story of superluminal motion is an interesting one. In 1959 summer I wrote a brief paper on particles traveling faster than light and sent it to Physical Review. It promptly came back with a referee report which said it was “all wrong,” and the Editor regretted that in view of this report, they cannot publish it. I requested a second referee whose report said it was quite correct, but all the results were “well known,” and so publication was not recommended. I requested a third referee, since the first one did not say what was wrong and the second referee said “it all well known,” but did not indicate where. The third referee was the most gifted. He said, “I have read the manuscript, and the two referee reports. I agree with both of them.” I requested that the Editor read the paper and the confusing and contradictory reports. As was to be expected, I did not even receive the courtesy of a reply. But when someone else wrote a paper several years later (which was clearly inconsistent), the same Editor published it\textsuperscript{25} (and did not send it to me refereeing)! Eventually Oleka-Myron Bilaniuk and I rewrote the article and got it published in the American Journal of Physics. It generated a lot of correspondence and discussions. The revolutionary idea came to many people’s attention due to the excellent article in “Discovery” by David Freedman\textsuperscript{26}. Erasmo Recami has been the most eloquent exponent of superluminal motion. He showed that many interesting consequences result from superluminal motion. He has reviewed the experimental searches so far, and gave an exposition of the visual effects of such motion, which are quite characteristic. Oleka-Myron Bilaniuk gave his simple but elegant exposition of this unusual physical system. Samir Bose addressed the question whether special relativity theory forbids faster than light particles and its possible relevance in general relativistic Cosmology.

7. Open Systems and Dynamical Maps, Irreversibility
In the course of being a research guide, I have learned new things from my younger colleagues. I feel at ease not only with Fiber Bundles, but also with manifolds and homotopy. But I feel uneasy when people talk about information and entropy, not because others do not make sense, but my feeling that information and order are encoded and we do not have the decoder. The next session involved one of the most challenging research areas, beginning with the question of how can we get irreversibility from reversible equations of motion\textsuperscript{28}. After much study and critical examination of the various trendsetters, I am still of the belief that the question is open. For a many particle system the trajectories are quite complicated, and resemble the strands of cooked spaghetti\textsuperscript{29}. (See Fig. 1.) There seems to be no pattern and no predictability about the various trajectories. But if we insert a fork and pull out a few strands, they are almost parallel near the fork (Fig. 2). One also recognizes that in both directions the strands end up in complicated patterns. Seeing regularity at the present time does not say that it was regular at either end. If the trajectories belong to a confined space, it is likely that approximately the same regularity
reappears (Fig. 3). The reason that regular strands get tangled up is not cussedness of the universe, it is our arbitrary imposition of a particular order that is described by the dynamics. We are also aware of arbitrary choice of ‘order’ when we consider the motion of a quantum particle in a potential. We identify the “in” solution (Fig. 5) with ‘order’ in the far past, but disarray in the future. But there is an equivalent set of ‘out’ states with order in the far future but disarray in the far past, and even at the present (Fig. 6).

Classical stochastic processes involve a non-negative probability distribution obeying a linear equation of motion. This is a Markov process. The stochastic propagation in time is then given by the solution to a partial differential equation akin to a non-negative stochastic matrix (or structure kernel). It is interesting that for a discrete probability vector stochastic evolution is more natural than any continuous evolution in time.

What about Quantum Mechanics? In this case, probabilities are bilinear in the amplitude. So we must work only with these quantities. But these are precisely the elements of the density matrix $\rho$. Only non-negative, unit trace density matrices are acceptable; and the quantum stochastic process must map these matrices into other density matrices. The quantum density matrices constitute a convex set

$$\rho = \cos^2(\theta) \rho_1 + \sin^2(\theta) \rho_2$$

is an allowed density matrix for all values of $\theta$. Since the stochastic super matrices takes them into the convex set, stochastic matrices constitute a convex set.

The implication of this requirement on the stochastic matrices is seen best by constructing a super matrix $B$, by defining

$$B_{\alpha^r,\alpha^s} = A_{\alpha^r\alpha^s}$$

For the finite dimensional case the eigenvectors $v(\alpha)$ of $B$ give

$$B_{\alpha^r,\alpha^s} = \sum_{\alpha} \mu(\alpha) v_{\alpha^r}(\alpha) v_{\alpha^s}^*(\alpha)$$
for the case all $\mu(\alpha)>0$, we could consider the stochastic map in the form

$$B_{ss'} = \sum_{\alpha} C_{ss'}(\alpha) C_{s's'}^*(\alpha)$$

These cases with all $\mu(\alpha) > 0$ are called “completely positive” maps.

Every such map can be obtained by embedding $\rho$ in a larger super density matrix $R$, by writing

$$R = \sum \rho_{nn} X_{n'n'}$$

and then evolving $R$ by a unitary super matrix $U$ where

$$U_{n'n'}^{11} = C_{n'n'}(\alpha)$$

and taking the trace over $\alpha$.

This theory of dynamical maps has been extended, refined (and plagiarized). It has also been applied to quantum computing and quantum tomography. The maps provide a method to derive the Kossakowski stochastic semigroups. Asorey’s masterly presentation discussed these matters and some newer developments. Hegerfeldt discussed quantum jumps in quantum optics. Chisolm gave an exposition of the treatment of decoherence. This work on stochastic mapping is being pursued with Chelikowsky, Shaji, Jordan, Rodriguez, Asorey, Kossakowski, and Marmo.

It was a gratifying experience to have so many of my former collaborators and former students review these developments spanned by five decades.

8. Causality versus Reversibility

Perhaps our ideas of causality, the requirement that cause precedes effect is dependent on this concept of order. A collimated beam of particles gets scattered by a potential. But what then of the ‘out’ states? A potential seems to be able to take a diffuse beam and make them collimated. Does it jibe with our idea of cause and effect?

As mentioned earlier, we take a bunch of (cooked) spaghetti strands, pick them up and put them on a separate plate, you may find that somewhere along their length more or less the same regularity may appear. This kind of recurrence is explored on general grounds (Fig. 4).

It is interesting to recognize that when tachyons are involved emission and absorption could change depending on the focus of the observer. This is an aspect of the Reinterpretation Principle that Recami and Bilaniuk discussed.

9. Reincarnation of Scientific Discoveries

In Indian tradition, such recurrences are part of the cosmology, except that it would then be called “reincarnation.”

But to my dismay, this happens to scientific discoveries also. When Jagadis Chandra Bose designed and demonstrated long distance propagation of radio waves in England, it was rediscovered by Giulio Marconi, and Marconi was awarded the Nobel Prize and the credit for the discovery, even though Marconi did it after Bose and Aryabhata’s result for the power series for arctangent is assigned the name Gregory’s series. In my own work, my work on quantum stochastic processes was rediscovered (with Greek and Latin characters interchanged) some seven years later. Andrzej Kossakowski discovered the generic semigroup structure, but many people ascribe it to only Lindblad, perhaps because he did it later.

Some of these developments must have been clear to you from live presentations by Manuel Asorey. The presentations of Gerhard Hegerfeldt and Eric Chisolm were so clear that I need not elaborate on their clear expositions.

10. In Gratitude and Appreciation: Beyond Seven Quests

I am grateful to Rodger Walser, Mary Ann Rankin, Juan Sanchez, and John Markert, as well as to Austin Gleeson, Swadesh Mahajan, and V. V. Raman for their kind words of praise and encouragement. I need also to thank Alaka Valanju, Gopalakrishnan Bhamathi, Prashant Valanju and Rodger Walser for their tireless efforts in bringing about this Seven Quests Symposium.
These seven quests by no means exhaust the scientific topics that I am interested in. Stanley Deser reminds me of our work with Gilbert on 3- and 4-point function in quantum field theory. Similarly the work on axiomatic field theory, quantum mechanics in dual spaces and work on inconsistency of charged spin 3/2 fields and topics like Fiber Bundles in Classical mechanics, the structure of the Dirac brackets, relativistic interactions using covariant constraints, theory of rays in quantum optics and radiative transfer, with the scattering of charged particles in nuclear emulsion, multiple meson production, the theory of weak measurements and measurement theory, and several other topics are not covered by the seven quests. But the essential point to be brought out is that all these are inter-related. For example, the study of the convex set of statistical states led to the discovery of the incompleteness of the Wightman axioms.

Even more important is the gratifying recognition that most of the young scientists who studied with me for their doctoral work have gone on to be excellent and productive scientists.

Substances and process are but two aspects of reality; and in quantum theory we see this identity even more directly. This wisdom should translate into an understanding of the process of measurements, and ultimately, to unite scientific theory and experiment with aspects of human consciousness. This is the ultimate frontier.

Recently I came across my scientific genealogy (published in these proceedings). It is indeed gratifying to see Hans Bethe, Arnold Sommerfield, Felix Klein, Carl Friedrich Gauss, as well as Joseph Fourier, Simeon Poisson, Joseph Lagrange, and Leonard Euler along the chain. These intellectual giants continue to influence scientists, some of them through me!

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