Experimental study of temperature gradient on solid dissolution process exposed to transverse rotating magnetic field

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Abstract. The main purpose of this work is to study the effect of transverse rotating magnetic field (TRMF) on the dissolution process of rock-salt sample. Moreover, the experimental study of the influence of the temperature gradient between the surface of sample and the solvent temperature on this process is presented in this paper. The results of investigations are worked out by means of the novel type dimensionless equations including standard and magnetic numbers. The obtained results are compared with previous data given in literature.

1. Introduction
Transfer of the solute into the main body of the fluid occurs in the three ways, dependent upon the conditions. For an infinite stagnant fluid, transfer will be by the molecular diffusion augmented by the gradients of temperature and pressure. The natural convection currents are set up owing to the difference in density between the pure solvent and the solution. This difference in inducted flow helps to carry solute away from the interface. The third mode of transport is depended on the external effects. In this way, the forced convection closely resembles natural convection expect that the liquid flow is involved by using the external force.

One of the key aspects in the dynamic behaviour of the mass-transfer processes is the role of hydrodynamics. On a macroscopic scale, the improvement of hydrodynamic conditions can be achieved by using various techniques of mixing, vibration, rotation, pulsation and oscillation in addition to other techniques like the use of fluidization, turbulence promotes or magnetic and electric fields etc. The transverse rotating magnetic field (TRMF) is a versatile option for enhancing several physical and chemical processes. Studies over the recent decades were focused on application of magnetic field (MF) in different areas of engineering processes [1,2]. Static, rotating or alternating MFs might be used to augment the process intensity instead of mechanically mixing. The practical applications of TRMF are presented in the relevant literature [3-9].

It should be noticed that the temperature gradient induces buoyancy-driven convective flow in the fluid. This temperature gradient has a significant practical interest to the mass transfer process. It is reported that the difference between the surface temperature of solid sample and the liquid temperature has strong influence on the dissolution process [3,4]. The main objective of the present study is to investigate the solid dissolution process that is induced under the action of TRMF and the gradient temperature between solid surface and liquid. According to the information available in technical
literature, the usage of TRMF and gradient temperature is not theoretical and practical analyzed. The obtained experimental data are generalized by using the empirical dimensionless correlations.

2. **Theoretical background**

2.1. **Influence of transverse rotating magnetic field on solid dissolution process**

Under convective conditions a relationship for the mass-transfer similar to the relationships obtained for heat-transfer may be expected of the form

\[ Sh = f(Re, Sc) \]  

(1)

The two principle dimensionless groups of relevance to mass-transfer are Sherwood and Schmidt numbers. The Sherwood number can be viewed as describing the ratio of convective to diffusive transport, and finds its counterpart in heat transfer in the form of the Nusselt number. The Schmidt number is a ratio of physical parameters pertinent to the system. This dimensionless group corresponds to the Prandtl number used in heat-transfer. Moreover, this number provides a measure of the relative effectiveness of momentum and mass transport by diffusion. Added to these two groups is the Reynolds number, which represents the ratio of convective-to-viscous momentum transport. This number determines the existence of laminar or turbulent conditions of fluid flow. For small values of the Reynolds number, viscous forces are sufficiently large relative to inertia forces. But, with increasing the Reynolds number, viscous effects become progressively less important relative to inertia effects.

Equation (1) to be of practical use, it must be rendered quantitative. This may be done by assuming that the functional relation is in the following form

\[ Sh = a_i Re^{b_i} Sc^{c_i} \]  

(2)

The mass transfer process under action of transverse rotating magnetic field may be described by using the following relation [1]

\[ Sh = f(Ta_m, Sc) \]  

(3)

Basing on the considerations given above, the correlations of mass transfer process under the transverse rotating magnetic field action have the general form

\[ Sh = a_2 Ta_m^{b_2} Sc^{c_2} \]  

(4)

2.2. **Influence of temperature gradient on solid dissolution process**

As mentioned above, the temperature gradient has strong influence on the solid dissolution process. The heat transfer from the sample to the ambient fluid may be modeled by means of the well-known Nusselt type equation

\[ Nu = f(Re, Pr) \]  

(5)

It should be assumed that the relationship for the heat transport under the transverse rotating magnetic field conditions can be characterized in the following general form

\[ Nu = f(Ta_m, Pr) \]  

(6)

The above Eq.(6) may be practically expressed as follows

\[ Nu = a_3 Ta_m^{b_3} Pr^{c_3} \]  

(7)

2.3. **Description of solid dissolution process under the action of transverse rotating magnetic field**
Taking into consideration the above equations (Eq.(4) and Eq.(7)), the solid dissolution process under the action of TRMF and the gradient temperature between the solid surface and the liquid may be described by means of the following equations system

\[
\begin{align*}
\text{Sh} &= a_2 \text{Ta}_m^{b_3} \text{Sc}^{c_3} \\
\text{Nu} &= a_3 \text{Ta}_m^{b_4} \text{Pr}^{c_3}
\end{align*}
\]

The ratio of Sherwood and Nusselt numbers is given by

\[
\frac{\text{Sh}}{\text{Nu}} = a_4 \text{Ta}_m^{b_5} \left( \frac{\text{Sc}}{\text{Pr}} \right)^{c_4}
\]

where

\[
\begin{align*}
a_4 &= \frac{a_3}{a_2} \quad \text{and} \quad a_2 \neq a_3 \\
b_4 &= b_2 - b_3 \quad \text{and} \quad b_2 \neq b_3 \\
c_4 &= 0.33 \quad \text{and} \quad c_2 = c_3
\end{align*}
\]

The exponent upon of the Schmidt number is to be 0.33 [11] as there is some theoretical and experimental evidence for this value [12], although reported values vary from 0.56 to 1.13 [13,14], the ratio of the Schmidt and Prandtl numbers is called as the dimensionless Lewis number (the ratio of thermal diffusivity to mass diffusivity)

\[
\text{Le} = \left( \frac{\text{Sc}}{\text{Pr}} \right) \Rightarrow \text{Le} = \left( \frac{\nu}{D_i} \right) \left( \frac{a}{\nu} \right) \Rightarrow \text{Le} = \frac{a}{D_i}
\]

Therefore the ratio of Sherwood and Nusselt numbers is defined as follows

\[
\frac{\text{Sh}}{\text{Nu}} = a_4 \text{Ta}_m^{b_5} \text{Le}^{0.33}
\]

The Sherwood number and the magnetic Taylor number may be defined as follows [1]

\[
\text{Sh} = \left( \frac{(\beta_i)_v}{\rho D_i} \right)^2 \Rightarrow \text{Sh} = \frac{\left[ \beta_i \right]_{\text{avg}} \left[ F_m \right]_{\text{avg}}}{V_i} D_i^2
\]

\[
\text{Ta}_m = \text{Ha}^2 \text{Re}_m \Rightarrow \text{Ta}_m = \left( \frac{\sigma B^2 l^2}{\nu \rho} \right) \left( \frac{w_o D}{\nu} \right) \Rightarrow \text{Ta}_m = \left( \frac{\sigma \left[ B_{\text{TRMF}} \right]_{\text{avg}}}{V_i \rho_i} \right) \left( \frac{D^2}{D_i} \right) \left( \frac{\omega_{\text{TRMF}} D^2}{V_i} \right)
\]

Moreover, the Nusselt number is given by the relation

\[
\text{Nu} = \frac{\alpha_i d_s}{\lambda_i} \Rightarrow \text{Nu} = \left[ \alpha_i \right]_{\text{avg}} D_i
\]
The volumetric mass transfer coefficient \( \left( \beta_i \right)_v \) may be described by relation [1]

\[
\left( \beta_i \right)_v = \frac{\beta_i dF_m}{dV} \Rightarrow \left[ \left( \beta_i \right)_v \right]_{avg} = \frac{\left[ \beta_i \right]_{avg} \left[ F_m \right]_{avg}}{V_i}
\]

(16)

The heat transfer from the sample to liquid may by modelled by the following relationship

\[
Q_i = Q_i \Rightarrow \alpha_i F_m \Delta T_i = m_i c_{p_i} \Delta T_2
\]

(17)

The averaged heat transfer coefficient is given as follows

\[
\left[ \alpha_i \right]_{avg} = \frac{m_i \left[ c_{p_i} \right]_{avg} \left( \left[ T_i \right]_{t_2} - \left[ T_i \right]_{t_1} \right)}{\left[ F_m \right]_{avg} \left( \left[ T_i \right]_{t_3} - \left[ T_i \right]_{t_4} \right)}
\]

(18)

3. Experimental details

A schematic of the experimental apparatus is presented in figure 1. This setup may be divided into: a generator of the rotating electromagnetic field (1), a glass container (2) with the conductivity samples (3-4), an electric control box (5) and an inverter (6) connected with multifunctional electronic switch (8) and a personal computer (7) loaded with special software. This software made possible the electromagnetic field rotation control, recording working parameters of the generator and various state parameters. The more detailed description of this experimental set up is given in the relevant paper (see Ref. [1]).

For the experimental measurements, MF is generated by coils located axially around of the cylindrical container. As mentioned above, this field is rotated around the container with the constant angular frequency, \( \omega_{TRMF} \). The TRMF strength is determined by measuring a magnetic induction. The values of the magnetic induction at different points inside the glass container are detected by using a Hall sample connected to the personal computer. The typical example of the dependence between the spatial distributions of magnetic induction and the various values of the alternating current frequency for the cross-section of container is given in Ref. [15]. The obtained results in this paper suggest that the averaged values of magnetic induction may be analytically described by the following relation

\[
\left[ B_{TRMF} \right]_{avg} = 14.05 \left[ 1 - \exp\left( -0.05 f_{TRMF} \right) \right]
\]

(19)

The experimental procedure is described in the relevant literature (see Ref. [1]). In the case of these experimental investigations the gradient temperature between the surface and liquid was caused by using the cartridge heater (power ~1200W). This tubular device was inserted into drilled holes of rock-salt sample for heating. Moreover, the heating set-up was contained the temperature controller and sensors. The sensors for the temperature control were placed between the working surface of the sample and the heater. These sensors were also located on the surface of the solid sample. The sketch of rock-salt sample with the heating set-up is graphically presented in figure 2. The sample was kept at a constant temperature (65°C, 70°C or 80°C). The heat transfer from the sample to ambient fluid was realized for the various temperatures (20 °C, 40 °C and 60°C). The system of temperature sensors was used to control the temperature of the water during the solid dissolution process.
4. Results and discussion

The effect of dissolution process under the action of TRF can be described by using the variable $\text{ShSc}^{-0.33}$ proportional to the term $a \left(\text{Ta}_m\right)^b$. The experimental results obtained in this work are graphically illustrated in $\log\left(\text{ShSc}^{-0.33}\right)$ versus $\log\left(a \left(\text{Ta}_m\right)^b\right)$ in figure 3. Moreover, the influence of the temperature gradient between the surface temperature of solid and the liquid temperature on the mass transfer coefficient is presented in this figure. In order to establish the effect of all important parameters on the dissolution process in the analyzed set-up, we propose the following relationship to work out the experimental database

$$\frac{\text{Sh}}{\text{Sc}^{-0.33}} = a \left(\text{Ta}_m\right)^b$$

(20)

The presented results in figure 3 suggest that these points may be described by a unique monotonic function. The constants and exponents are computed by employing the Matlab software and the principle of least squares and the proposed relationships are collected in table 1.
Fig. 3. The graphical presentation of mass transfer data under the action of TRMF: a) $T_s = \text{var}; T_i = 20^\circ C$, b) $T_s = \text{var}; T_i = 40^\circ C$ and c) $T_s = \text{var}; T_i = 60^\circ C$

Table 1. The developed relationships for the obtained experimental data

| Temperature of surface of salt-rock sample | Temperature of liquid |
|------------------------------------------|-----------------------|
|                                          | 20°C                  | 40°C                  | 60°C                  |
| 65°C                                     | $\frac{Sh}{Sc^{0.33}} = 80.36 \left(T_a\right)^{0.148}$ | $\frac{Sh}{Sc^{0.33}} = 79.71 \left(T_a\right)^{0.08}$ | $\frac{Sh}{Sc^{0.33}} = 51.95 \left(T_a\right)^{0.06}$ |
| 70°C                                     | $\frac{Sh}{Sc^{0.33}} = 85.54 \left(T_a\right)^{0.132}$ | $\frac{Sh}{Sc^{0.33}} = 84.13 \left(T_a\right)^{0.05}$ | $\frac{Sh}{Sc^{0.33}} = 67.52 \left(T_a\right)^{0.04}$ |
| 80°C                                     | $\frac{Sh}{Sc^{0.33}} = 92.87 \left(T_a\right)^{0.09}$ | $\frac{Sh}{Sc^{0.33}} = 92.85 \left(T_a\right)^{0.04}$ | $\frac{Sh}{Sc^{0.33}} = 73.71 \left(T_a\right)^{0.03}$ |

As can be clearly seen (see Figure 3) mass transfer rates expressed as $\left(\frac{Sh}{Sc^{-0.33}}\right)$ increase with increasing the values of magnetic Taylor number. It is found that as the intensity of magnetic field increases, the velocity of liquid inside the cylindrical container increases. It may be concluded that the TRMF strongly influenced on the mass transfer process. It should be noticed that this process may be improved by means of the gradient temperature between the surface of rock-salt sample and the liquid.
Figure 3 shows that Sherwood number increases with the increasing difference between the temperature of rock-salt surface and the liquid temperature. It is clear that the effect of TRMF on the dissolution process is also depended on the temperature gradient.

The enhancement due to heat transfer process is modeled in terms given in Eq.(20). The graphical presentation of the calculated experimental points is presented in figure 4.

![Graphical presentation of mass and heat transfer data at transverse rotating magnetic field](image)

5. Conclusion

The novel approach to the mixing process presented and based on the application of transverse rotating magnetic field to produce better hydrodynamic conditions in the case of the mass-transfer process. From practical point of view, the dissolution process of solid body is involved by using the turbulently agitated systems. In previous publications are not available data describing the mass-transfer operations of the dissolution process under the transverse rotating magnetic field conditions and the temperature gradient. Therefore, the experimental investigations have been conducted to explain the influence of this kind of magnetic field on the mass-transfer enhancement. Moreover, the influence of the transverse rotating magnetic field and the temperature gradient on this process is described using the non-dimensional parameters formulated on the base of fluid mechanics equations.

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Nomenclature

| Symbol | Definition                  | Unit            |
|--------|-----------------------------|-----------------|
| $B$    | magnetic induction          | kg · A$^{-1}$ · s$^{-2}$ |
| $c_p$  | specific heat capacity of liquid | J · kg$^{-1}$ · deg$^{-1}$ |
| $d_s$  | sample diameter             | m               |
| $D$    | diameter of container       | m               |
| $D_i$  | diffusion coefficient       | m$^2$ · s$^{-1}$ |
$f_{\text{TRMF}}$ frequency of electrical current (equal to frequency of TRMF) \( s^{-1} \)

$F_m$ cylindrical surface of dissoluble sample \( m^2 \)

$m_l$ mass of liquid \( kg \)

$Q_l$ heat flow from liquid \( W \)

$Q_s$ heat flow from sample \( W \)

$T$ temperature \( deg \)

$[T]_{t_1}$ temperature of liquid at moment $t_1$ \( deg \)

$[T]_{t_2}$ temperature of liquid at moment $t_2$ (after time of dissolution process) \( deg \)

$T_s$ temperature of sample \( deg \)

$V$ volume of liquid \( m^3 \)

$w$ velocity \( m \cdot s^{-1} \)

\textbf{Greek letters}

$\alpha_s$ heat transfer coefficient \( W \cdot m^{-2} \cdot deg^{-1} \)

$\beta_i$ mass transfer coefficient \( kg \cdot m^{-2} \cdot s^{-1} \)

$\lambda$ thermal conductivity of liquid \( W \cdot m^{-1} \cdot deg^{-1} \)

$\nu$ kinematic viscosity \( m^2 \cdot s^{-1} \)

$\rho$ density \( kg \cdot m^{-3} \)

$\sigma_e$ electrical conductivity \( A^2 \cdot s^3 \cdot kg^{-1} \cdot m^{-3} \)

$\omega_{\text{TRMF}}$ angular velocity of transverse rotating magnetic field \( rad \cdot s^{-1} \)

\textbf{Subscripts}

avg averaged value

$l$ liquid

$s$ sample

$V$ volumetric

\textbf{Abbreviation}

TRMF transverse rotating magnetic field

\textbf{References}

[1] Rakoczy R, Masiuk S 2010 \textit{AIChE Journal} \textbf{56} 1416

[2] Rakoczy R 2010 \textit{Chemical Engineering and Processing: Process Intensification} \textbf{49} 42

[3] Volz M, Mazuruk K 1999 \textit{International Journal of Heat and Mass Transfer} \textbf{42} 1037

[4] Melle S, Calderon O, Fuller G, Rubio M 2002 \textit{Journal of Colloid and Interface Science} \textbf{247} 200

[5] Walker J, Volz M, Mazuruk K 2004 \textit{International Journal of Heat and Mass Transfer} \textbf{47} 1877

[6] Nikrityuk P, Eckert K, Grundmann R 2006 \textit{International Journal of Heat and Mass Transfer} \textbf{49} 1501

[7] Yang M, Ma N, Bliss D, Bryant G 2007 \textit{Chemical Engineering Science} \textbf{23} 707

[8] Fraňa K, Stiller J, Grundmann R 2006 \textit{Magnetohydrodynamics} \textbf{42} 187

[9] Spitzer K 1999 \textit{Crystal Growth and Characterization of Materials} \textbf{38} 39

[10] Rakoczy R 2011 Theoretical and experimental analysis of the influence of the rotating magnetic field on the selected unit operations and processes of chemical engineering (West Pomeranian University of Technology Publishing House, Szczecin, Poland in polish).

[11] Noordij P, Rotte J 1967 \textit{Chemical Engineering Science} \textbf{22} 1475

[12] Sugano Y, Rutkowski D 1968 \textit{Chemical Engineering Science} \textbf{23} 707

[13] Lemlich R, Levy M 1961 \textit{AIChE Journal} \textbf{7} 240

[14] Rakoczy R, Masiuk S 2009 \textit{Chemical Engineering and Processing: Process Intensification} \textbf{48} 1229