Redundancy-Free UAV Sensor Fault Isolation And Recovery

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Abstract—The sensory system of unmanned aerial vehicles (UAVs) plays an important role in flight safety. Thus, any sensor fault/failure can have potentially catastrophic effects on vehicle control. The recent advance in adversarial studies demonstrates successful sensing fault generation by targeting the physical vulnerabilities of the sensors. It poses new security challenges for sensor fault detection and isolation (FDI) and fault recovery (FR) research because the conventional redundancy-based fault-tolerant design is not effective against such faults. To address these challenges, we present a redundancy-free method for UAV sensor FDI and FR. In the FDI design, we used a basic state estimator for a rough early warning of faults. We then refine the design by considering the unmeasurable actuator state and modeling uncertainties. Under such novel strategies, the proposed method achieves fine-grained fault isolation. Based on this method, we further designed a redundancy-free FR method by using complementary sensor estimations. In particular, position and attitude feedback can provide backup feedback for each other through geometric correlation. The effectiveness of our approach is validated through simulation of several challenging sensor failure scenarios. The recovery performance is experimentally demonstrated by a challenging flight tasks-restoring control after completely losing attitude feedback. With the protection of FDI and FR, flight safety is ensured. This UAV security enhancement method is promising to be generalized for other types of vehicles and can serve as a compensation to other fault-tolerant methodologies.

I. Introduction

Cyber-Physical Systems (CPSs) hold a great promise in bridging the gap between the virtual cyber domain and the actual physical world. As one of the most pervasive CPSs, unmanned aerial vehicles (UAVs) have drawn a surge of interest and usage in many applications, such as search and rescue, surveying and inspection, precision agriculture, aerial photography, and private hobbies [1]. The key to achieving such extensive applicability of UAVs is their sophisticated suite of onboard sensors, which relay essential information for flight guidance and further allow for advanced system control, from simple attitude stabilization to high-level path-planning and navigation. To this end, perceiving the physical world plays an important role in UAV system safety and stability. Any sensor error may result in catastrophic system failure, causing the vehicle to lose control or even crash [2], [3]. From a security standpoint, this reliance on sensors provides a clear potential vulnerability to targeted attacks in both the cyber and physical domains [4]–[10].

To date, the ever-improving reliability design of UAVs significantly reduces the impact of the regular sensor errors on UAV function [2]. Besides, the targeted adversarial attack methodologies are evolving rapidly. Such attacks are not limited to traditional hacking in the cyber domain [3]–[6], but are progressively expanding to involve physical vulnerabilities of the system, which makes the attack more effective and stealthy [8]–[10]. Actually, a number of the state-of-the-art sensor attacks consider both cyber and physical properties of the system. These cross-layer attacks have exemplified to be effective against many critical sensors of UAVs, which is hard to resist. As presented in [9], Son et al. have disrupted the operation of drones successfully by using intentional acoustic noise to generate failure-inducing resonance in the inertial measurement unit (IMU) that provides the attitude feedback. Trippel et al. have manipulated the acceleration measurements with a similar approach and have demonstrated a potential to take over the control and misguided the vehicle completely [10]. These sensor faults are very difficult to diagnosis while can currently overcome the well-designed fault-tolerant modes, including redundant system and robust observer [3], [11]. Accordingly, security researchers frequently emphasize the importance of sensor safety and have conducted many related studies to address these safety issues.

Available studies of UAV sensor security can be divided into two main groups: those focusing on fault detection and isolation (FDI), and those investigating fault recovery (FR). Addressing the former group, FDI determines whether the sensor faults and the malicious attacks can be accurately...
detected and isolated to avoid further damage. Several fruitful sensor FDI designs that are based on different theoretical foundations [12], [13], including knowledge-based interpretation [14], signal processing [15], and model-based estimation [11], demonstrate tremendous progress in the development of FDI methods. Nevertheless, the interest of past research is to detect anomalies in flight states, which are not equivalent to physical system failures. For UAVs, flight states are generated by sensor fusion [16]. Thus, a flawed sensor may induce multiple flight state anomalies simultaneously, which is more threatening for flight safety and complicates pinpointing the failure source. Therefore, for FDI implementation, there remain some open challenges [12], [13]. First, a vehicle can yield a similar abnormal action to two completely different types of failures, e.g., sensor or actuator failure. Without considering the inherent coupling of the system dynamics, it is hard to dig out the source of such a fault. Second, when multiple flight states malfunction simultaneously, the strap-down structure of the sensory system on UAVs make it challenging to identify the exact failure sensor. The high-level sensor abnormalities can affect low-level sensing in cascade for UAVs, e.g., attitude twitching can be subject to the frequent loss of position feedback. This fact mislead the FDI system to carry out incorrect isolation. Third, sensor FDI requires some level of adaptability that can tolerate physical parameter uncertainties to accommodate the modeling error. Thus, developing a FDI method that can solve the above shortcomings would be very meaningful for fault tolerant and improve the overall security of UAV systems.

A complete UAV sensor safety design must contain both an FDI unit as well as a fault recovery function to tolerate the detected flaws. When faults are identified and isolated by the FDI, the recovery logic should then be able to maintain the flight state with as much stability as possible using the remaining incomplete sensory system. In particular, when the faulty sensor has been isolated, the absent state feedback must be compensated to ensure the controllability of the vehicle. In order to recover the sensor readings, installation of a backup sensory system is the most widely used approach [3], [13], [17], [18]. Through a simple comparison and replacement, this hardware redundancy is simple and effective against the traditional software-based sensor faults and attacks, such as numeric error, trojans and data spoofing. Though redundancy has been demonstrated as an adequate defense strategy in many cases, this approach is not sufficient when the UAV encounters some well-designed adversary attacks that concern both cyber-physical properties of the targeted sensors. This is because the redundant sensors are provided with the same physical vulnerabilities as the original ones. For instance, a redundant attitude sensor would be incapable of nullifying the effects of resonating the inertial sensors via external excitation [9], [10]; both original and redundant sensors would fail. For position feedback, redundancy cannot troubleshoot artificially disturbed or blocked satellite signals [19]. As alternatives to a redundant approach, researchers have proposed to use atypical sensing approaches, such as equipping coplanar [20] and multiple-antenna GPS for attitude control [21], employing accelerations integral [22], [23] and landmark calibration [24] for position reckoning, and relying on vision sensors for both attitude and position compensation [25], [26]. Although these methods do address the drawbacks of identical hardware redundancy, they significantly increase the complexity of the system. Additional system complexity would likely result in greater vulnerability to attack and possess more natural sensor error, which weakens the reliability of the entire system [2]. Moreover, the additional size, weight, power and computational load necessary is not compatible with some UAVs that have very limited physical and computational resources while requiring simultaneous high performance and security demands.

In this work, we propose a generic fault-tolerant method for the UAV sensory system. The proposed method consists of a fine-grained sensor FDI architecture and a sensor complementary FR in parallel. Instead of adding auxiliary sensors, the proposed method is intended to minimize the modification of the original system by only using the original sensor arrays: the classic combination of a position sensor (e.g., GPS, LiDAR) and an attitude sensor (e.g., IMU). The FDI unit design is based on smooth variable structure filter (SVSF), which is a sliding-mode-based state estimator with a prediction-correction workflow that was first presented in 2007 [27]. In order to determine the shortcomings of the FDI as mentioned above, we expand the traditional observing state variables to govern the unmeasurable terms, e.g., actuator states. A reduced order observer is then constructed to differentiate the faulty components explicitly, able to distinguishing different faults with similar outcomes. As presented in [27], we employ the chattering information of physical parameter estimation to assist the early warning generation if the fault appears. The chattering signals associated with the state estimator to pinpoint the compromised sensor and achieve accurate fault isolation. For the purpose of recovery, we present a geometrical approach that make full use of the original UAVs’ sensors to achieve a mutual protection. When the inertial sensors is flawed, the position information can be used to derive the attitude states for stabilization. Conversely, if the position feedback is blocked, the current relative position can be estimated using the corresponding inertial measurements. This method assumes that all of the onboard sensors are not defective at the same time. Supporting this assumption, we offer that it is hard to compromise multiple sensors simultaneously because they typically measures different physical terms, and possess distinct working principle, communication methods, and signal bandwidths. The novelty and effectiveness of the proposed method has been demonstrated by both simulation and experimental test through some challenging FDI/FR tasks.

II. TEST PLATFORM DESCRIPTION

As shown in Fig.2 in this work, the test platform is an ultra-compact quad-copter aerial vehicle: Crazepony MINI. It weighs 35g and has a 140mm wheelbase. The propeller length is 75mm. The vehicle frame is designed as a circuit
body, which is embedded four motor drivers, a STM32 microcontroller with the ARM-CortexM4 core, a MPU-6050 IMU that consists of a 3-axis gyroscope and accelerometer, and an extended compass HMC5883L. The usage of the compass is irreplaceable, since, the yaw angle is observable only in presence of horizontal accelerations in strapdown inertial navigation systems [28]. Therefore, without magnetic field compensation, the yaw angle error is not bounded in hover flight, which causes an erroneous propagation of sensor fusion result, leading to a deterioration in accuracy, or even complete divergence. The position feedback is generated by an external motion capture system-VICON. The entire setup constructs a typical UAV sensory system, wherein the position and attitude feedback come from different sensors, with different communication channels, and different signal frequencies corresponding to the relevant system bandwidth. These detailed discrepancies are shown in Table I.

The coordinates definition is shown in Fig. 2. The vehicle is modeled as the standard rigid body dynamics as shown

\[
P = V, \quad m\dot{P} = Rf + mg, \\
\dot{R} = R[\omega], \quad J \omega = \tau - \omega \times J \omega.
\]  

(1)

where \( P = [x, y, z]^T \) is the position vector of the vehicle in the inertial frame \( XYZ \) which is defined by North-East-Up (NEU); \( V = [\dot{x}, \dot{y}, \dot{z}]^T \) is the velocity vector of the vehicle in the inertial frame; \( m \) is the total mass; \( g = [0, 0, -g]^T \) is the gravity acceleration vector, \( g = 9.8m/s^2 \); \( R \) is the rotation matrix; \([\omega]\) denotes the skew-symmetric matrix mapping from vector dot product to vector cross product; \( J \) is the inertia matrix of the vehicle; \( f \) represents the vector in the body frame \( x^b, y^b, z^b \), e.g., the thrust vector \( f = [0, 0, f_z]^T \), the vehicle angular velocity \( \omega = [p, q, r]^T \), and the 3-axis torque vector \( \tau = [\tau_x, \tau_y, \tau_z]^T \).

The motor can be approximated by a first order system

\[
\dot{\Omega} = -\alpha \Omega + K_m u,
\]  

(2)

where \( \Omega \) is the motor speed, \( \alpha \) is the pole location, \( K_m \) is the lumped motor drive gain, and \( u \) is the motor input voltage.

With the motor speed, the lift and drag force of each rotor can be simplified as

\[
F_{Li} = C_L \Omega_i^2, \quad F_{Di} = C_D \Omega_i^2,
\]

where \( i \) is the motor index, \( C_L \) and \( C_D \) are the aerodynamic lift and drag coefficient of the rotor, respectively.

### Table I

| State feedback | Sensor                  | Communication | Frequencies |
|----------------|-------------------------|---------------|-------------|
| \( x, y, z \)  | IMU (Accelerometer)     | I2C           | 1000Hz      |
| \( p, q, r \)  | IMU (Gyroscope)         | I2C           | 1000Hz      |
| \( \phi, \theta \) | IMU (Sensor fusion)    | SPI           | 400Hz       |
| \( \psi \)     | Compass (Sensor fusion) | SPI           | 100Hz       |
| \( x, y, z \)  | VICON                   | Serial        | 50Hz        |

III. Sensor Fault Detection And Isolation

As introduced in Section IV, the relatively fixed sensor types in the UAV system provide the clear targets for adversaries to generate system fault and hijack the control. In order to enhance the overall security of UAVs, we present a novel sensor FDI architecture for fine-grained fault isolation that initiates the corresponding FR mode consequently. Unlike the previous FDI studies, the proposed method solved some open challenges without any redundant hardwares. According to our design, the unmeasurable actuation states has been considered as a part of the vehicle dynamics, so the sensor attack and actuator attack can easily be distinguished. Furthermore, by monitoring the changes in the physical parameters of the vehicle, the proposed method can also deal with more complex faults and vandalism, especially when the key sensors are completely failed or multiple attacks are activated simultaneously.

In this section, the thorough algorithm structure and formulations of the SVSF-based FDI method are introduced. We present the novel designs in our FDI logic in detail, which include actuator fault distinction and model uncertainties observation. Through the adoptions of these design tricks, the proposed FDI method achieves accurate fault isolation in some challenging faulty scenarios as shown in Section V.

A. State Estimation and Basic Fault Detection

The feasibility of using SVSF for UAV parameter and state estimation has been demonstrated in several previous studies [18], [29]. The results of these works exemplify the robustness of this method on system estimation under the bounded system uncertainties and disturbances.
The typical SVSF state estimator consists of two stages: prediction and correction. In one iteration, a model-based prediction function \( \hat{F} \) generates a priori state estimation \( \hat{x}_{k|k-1} \), then a discrete corrective action is taken by adding a corrective gain \( K_k \). The corrective gain is not only used to guarantee the stability of the estimator but also rectify the bounded estimation error robustly. Subsequently, the updated posteriori estimation \( \hat{x}_{k|k} \) and state measurement \( z_{k|k} \) carry out the next iteration.

In order to implement SVSF properly, we rewrite the nonlinear UAV model to a discrete form

\[
\begin{align*}
x_{k+1} &= \hat{F}(x_k, u_k, \tilde{d}_k), \\
z_{k+1} &= Hx_k + \tilde{n}_k + f, 
\end{align*}
\]

where the vehicle states is given by \( x = [P, V, \Theta, \omega_b]^T \), wherein \( \Theta = [\phi, \theta, \psi]^T \) represents vehicle’s 3-axis Euler angle; \( k \) is the time step; \( u \) is the input voltages of the four motors; \( \tilde{d} \) is the external disturbance; \( z \) is the state measurement; \( H = I_{12} \) where \( I_{12} \) is a twelfth-order identity matrix; \( \tilde{n} \) is the measurement noise; \( f \) represents the sensor fault.

Following the above discrete model, a prediction function \( \hat{F} \) can be defined accordingly. The detailed formulation of the state variables is given in Appendix A.

With \( \hat{F} \), the prediction stage can be expressed by

\[
\begin{align*}
\hat{x}_{k|k-1} &= \hat{F}(\hat{x}_{k-1|k-1}, u_{k-1}), \\
e_{k|k-1} &= z_k - \hat{H}\hat{x}_{k|k-1},
\end{align*}
\]

where \( \dot{\cdot} \) denotes the corresponding estimation variable. The estimation result is defined by \( \hat{z}_k = \hat{H}\hat{x}_k \).

The correction stage is formulated as

\[
\begin{align*}
K_k &= \hat{H}^{-1}_k (e_{k|k-1} + \gamma |e_{k|k-1}| \circ \text{sgn}(e_{k|k-1})), \\
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k e_{k|k}, \\
e_{k|k} &= z_k - \hat{H}\hat{x}_{k|k}.
\end{align*}
\]

Assuming \( \tilde{d} \) and \( \tilde{n} \) are bounded and \( f = 0 \), the stability proof of the estimator is similar to [27] except for the different state variables.

If the vehicle is subject to sensor failure, i.e., \( f \neq 0 \), the unknown faulty information would be involved in the estimator calculation. To detect it, we implement a widely used sensor FDI procedure-residual check: examining the discrepancy between estimation and actual sensor readings.

From the system model, since \( x \rightarrow z \) is a one-to-one mapping, the estimation result is dedicated to each certain state. Therefore, the residual vector \( r \) can be generated from the correction stage which is defined by

\[
r = k_r \sum_{i=k}^{k+T} (|z_i - \hat{x}_i|),
\]

where \( k_r \) are weighting coefficients, which are determined by the different sensitivities of the sensors as well as the corresponding sensor calibration results; \( T \) is the window size of the checking process, which is used to minimize the false positives.

If the residual rises beyond a certain threshold, a fault is supposed to be present and will be reported to begin the execution of isolation logic.

### B. Novel Designs For Fine-grained FDI

Accurately judging the flawed sensor is critical to achieving correct isolation. More the most part, the abnormality of the state parameters relies solely on the interpretation of the residuals, which just exposes the presence of the fault yet cannot pinpoint its hardware source explicitly. For UAVs, this happens for two main reasons. First, sensing and actuation of the UAVs are inherently coupled in the system dynamics, which results in difficulties of differentiating wether the failure is from sensor or actuator. Second, some of the state parameters cannot be measured directly. Instead, they are updated through a sensor fusion algorithm that combines multiple sensors’ feedback. This synthesis of sensor feedback weakens the impact of any single sensor and mitigates the impact of the faults to some degree. These general limitations motivate us to improve the current design to address these issues. In this subsection, following the existing SVSF-based estimator, we mine more potential features for sensor FDI. With such findings, we propose a novel FDI architecture that solved the mentioned open challenges, e.g., decoupling the sensor and actuator faults, diagnosing multiple faults and accurately isolating the source of faulty elements.

1) Differentiating Sensor/Actuator Faults: As UAV sensors, the actuation system of the UAVs is also vulnerable and can be easily compromised via similar cyber and physical domains strategies. Actuator failures not only affect the normal flight, due to the implicit dynamics from the actual system input \( \Omega \) to the state \( x \) (untrusted \( u \) in between), they also introduce the need of FDI design to distinguish the exact sensing and actuation faults [18]. The existing estimated states are not sufficient to address this problem. To adjust, the motor states should be added in the overall vehicle model. Therefore, we expand the current state space to

\[
x_e = \begin{bmatrix} x \\ x_m \end{bmatrix} = \begin{bmatrix} x \\ [\Omega_1, \Omega_2, \Omega_3, \Omega_4]^T \end{bmatrix}
\]

where \( x_m \) represents the vector of the motor speed.

For the UAVs, without the respective sensors, e.g., Hall sensor or encoder, the motor speed is unmeasurable. Therefore, the expansion of the state space causes the system to have fewer measurements than current states. To extract the motor speed information, a reduced order observer is constructed as follows.

The original system can be partitioned as

\[
\begin{align*}
x_{e_{k+1}} &= \begin{bmatrix} x_{k+1} \\ x_{m_{k+1}} \end{bmatrix} = \begin{bmatrix} \hat{F}(x_k, x_m, \tilde{d}_k) \\ T_m(u_k) \end{bmatrix}, \\
z_{k+1} &= \begin{bmatrix} z_{k+1} \\ z_{m_{k+1}} \end{bmatrix} = \begin{bmatrix} H \\ 0 \end{bmatrix} x_e + \tilde{n}_k + f,
\end{align*}
\]

where \( \Theta \) is a 4x12 zero matrix.

The prediction function \( \hat{F} \) is changed accordingly,


\[ \dot{x}_{c,k|k-1} = \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{m}_{m|k-1} \end{bmatrix} = \begin{bmatrix} \hat{f}^{\prime}(\hat{x}_{k|k-1}, \hat{m}_{m|k-1}) \\ \hat{f}_{m}(\hat{x}_{k|k-1}, \hat{m}_{m|k-1}) \end{bmatrix} \]  

(10)

With the same corrective percedure as presented in equation (6), the motor speed can be observed by

\[ \dot{x}_{m} = A^{-1} \dot{M}(\hat{x}_{k}, \hat{x}_{k-1}), \]  

(11)

where \( A \) is the force mapping as shown in equation (3),

\[ \dot{M}(\hat{x}_{k}, \hat{x}_{k-1}) = \frac{m(\hat{x}_{k} - \hat{x}_{k-1} + gdt)/(C\theta_{k-1}S\phi_{k-1})}{dt} \]

(12)

where \( C \cdot S \cdot \) denote \( \cos(\cdot) \) and \( \sin(\cdot) \) respectively for compact notation; \( J_x, J_y, J_z \) are the moment of inertias and mass of the vehicle; \( J_1 = J_x - J_z, J_2 = J_y - J_z, J_3 = J_x - J_y; \) \( dt \) is the sampling time.

Consequently, the actuator fault residual is defied by

\[ r_m = \sum_{i=k}^{k+T} (x_{m,i} - \hat{x}_{m,i}). \]  

(13)

For FDI, if both \( r_m \) and \( r \) diverge, it is an actuator failure. Otherwise, only \( r \) should drift, indicating a sensor failure.

2) A Cascade FDI Architecture-Residuals Associated With Physical Parameter Changes: The proposed state estimator can also be applied to adapt to the physical model variations. During the real flight, the physical parameters of the vehicle (mass and inertia) will not usually change significantly. By utilizing this certainty, if a sensor failure appears, we can track the varying model parameters in reverse and trace the source of the variations to the specific faulty components.

For our estimator, due to the zero width smoothing layer in \( K_k \), the modeling error is referred to as a chattering signal.

The a priori chattering of \( Ch = [Ch_x, Ch_y, Ch_q, Ch_r]^T \) is defined as

\[ Ch_{k|k-1} = dt \begin{bmatrix} \Delta \frac{C_{\phi_{k-1}}}{C_{\phi_{k-1}}} (C\psi_{k-1} C\phi_{k-1}) \hat{f}_z \\ \Delta (C_{\psi_{k-1}}) \hat{f}_x \\ \Delta (C_{\psi_{k-1}}) \hat{f}_y \\ \Delta (C_{\psi_{k-1}}) \hat{f}_z \end{bmatrix} \]

where \( \hat{f}_x, \hat{f}_y, \hat{f}_z \) is the estimated unscaled control force and torques.

In normal flights, the chattering is bounded and can be used to refined model parameters through a short time on-line calibration. The calibrated model contains a set of meaningful physical parameters that relevant to the inception of the fault in the UAV system. For example, by using CH\(_x\), the uncertainty of the vehicle mass \( \Delta_m \) can be estimated by

\[ \Delta_m = \sum_{i=1}^{l} \left( \frac{Ch_{m|k-1}}{dtC_i C\phi_{k-1} C\phi_{k-1}} \right) \hat{f}_z \]  

(14)

where \( l \) is the step size of the samples.

The uncertainties of vehicle body inertia can be calculated with the similar approach [18].

The sensor fault causes additional force and torque to be exerted, yielding divergent values in the corresponding physical parameters, which can in turn be employed to pinpoint the compromised sensor. For example, in a real flight, although the compromised accelerometer and GPS may generate the similar behavior such as undesired ascending or descending, the accelerometer failure will definitely induce \( CH_z \) to diverge while the spoofed GPS will not. Therefore, residuals checking and model uncertainties interpretation constitute a cascade structure as shown in Algorithm[1] which is capable of achieving fine-grained faulty element isolation.

**Algorithm 1 Fine-grained UAV Sensor FDI**

1. \( r \) residue of the vehicle state
2. \( r_m \) residue of the actuator state
3. \( A \) vehicle physical parameter uncertainties
4. \( \Delta \) sensor faults
5. procedure FDI(r, \( r_m \), \( \Delta \))
6. \( t \) + ;
7. if \( r > \) threshold then \( \Rightarrow \) fault early warning
8. if \( r_m > \) threshold then \( \Rightarrow \) check rotor status
9. actuator_fault_alert();
10. else
11. if \( \Delta_m > \) threshold then \( \Rightarrow \) accelerometer_isolate();
12. else if \( \Delta_f > \) threshold then \( \Rightarrow \) gyroscope_isolate();
13. else
14. position_sensor_isolate();
15. end if
16. end if
17. window_size
18. \( t \) ← 0
19. \( r \) ← 0
20. \( r_m \) ← 0
21. end if
22. end procedure

IV. FLIGHT STABILITY RECOVERY

Most of the current methods for sensor fault recovery are based on model-based estimation or sensor redundancy, which are no longer sufficient to tolerant the advanced hybrid attack (crossing cyber-physical layer) that deployed based on the inherent physical vulnerabilities of sensors. As introduced in Section[1], adversaries can utilize such vulnerabilities to generate an unpredictable fault by using either direct contact or non-contact ways. If successful, the vehicle fully loses sensor readings from the original and redundant systems because the same type of sensors have the same physical properties, causing the recovery function to fail. For example: if inertia sensors (IMU) is compromised by the method presented in [9], the vehicle would miss all of the inertia related feedback that contains 3-axis acceleration, 3-axis angular speed, and roll, pitch attitude; similarly, if missing position information [19], the vehicle can hold the attitude, but there is no guarantee that it is flying on the correct track.

Fortunately, the vulnerabilities of UAVs onboard sensors are very different: inertial components are sensitive to vibrations; magnetic components cannot be resistant to magnetic...
field distortion; position sensing is usually derived from external references (satellites, landmarks or cameras), focusing on communication quality. Because of these differences, for adversaries, it is almost impossible to develop an all-in-one attack. This fact inspired us to use the trusted sensor to complement and accommodate the faults of the others. This complementary approach does not require hardware retrofit, which fits those small-size vehicles that have stringent size, weight, power, and computational load constraints.

In this section, we present a sensor complementary method for fault recovery without hardware redundancy. In particular, due to the geometric correlations of the vehicle dynamics, position and attitude feedback can be used to compensate each other. During the recovery process, with the fine-grained FDI, the compromised sensor reading will be totally rejected, altering to use the compensations from other trusted sensor. We present the inertia sensor failure case first, i.e., using position information to derive an alternative attitude for flight control, which is challenging without accessorial sensors. Then, we discuss the scenario in which the vehicle loses position feedback. Similar to the former case, we utilize inertia measurement to compensate position drift. The successful recovery cases as shown in Section V validate the performance of the proposed FR method.

A. Inertia Sensor Isolated

For the UAV system, if the inertia sensor is isolated, except for the absolute yaw angle measured by compass, all of the body frame information becomes unreachable. This is a very severe issue as the vehicle is certain to lose stability and drop soon after entirely losing attitude feedback control. Therefore, the corresponding failure recovery is essential to ensure the flight safety. However, with the very limited remaining feedback, \((x, y, z, \psi)\), it is challenging to restore the current attitude. In order to fix the incomplete attitude information, we present a geometric approach that associates attitude with position data.

The essence of UAV attitude generation is the direction change of the thrust vector. For our test platform, by ignoring the relative small body damping, the thrust vector \(\mathbf{T}\) equals to the resultant vector of \(\mathbf{g}\) and \(\mathbf{P}\). For our test platform, since the thrust vector is always perpendicular to the rotor-disk plane, it determines the roll and pitch angle specifically. Based on this phenomenon, we can use the following geometrical approach for attitude recovery.

From equation, the thrust vector in the inertial frame is \(\mathbf{T} = m\mathbf{P} - mg\). It is normalized as \(\mathbf{t} = \frac{\mathbf{T}}{|\mathbf{T}|} = [t_1, t_2, t_3]^T\) for later calculation.

The compass data represents the heading direction of the vehicle, which defines the first coordinate transformation \(\mathbf{R}_1\) as shown in Fig. 3:

\[
\mathbf{R}_1 = \begin{bmatrix}
C\psi & S\psi & 0 \\
-S\psi & C\psi & 0 \\
0 & 0 & 1
\end{bmatrix},
\]

where \(\psi \in (0, 2\pi)\).

Through \(\mathbf{R}_1\) rotation, the original north-east-up coordinate \(XYZ\) is transferred to \(X_1Y_1Z_1\).

With the auxiliary coordinate \(X_1Y_1Z_1\), the vehicle tilting angles along the pitch and roll direction are determined by \(\mathbf{R}_2\) and \(\mathbf{R}_3\), respectively:

\[
\mathbf{R}_2 = \begin{bmatrix}
C\dot{\theta} & 0 & S\dot{\theta} \\
0 & 1 & 0 \\
-S\dot{\theta} & 0 & C\dot{\theta}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
Z_1 \cdot Z_2 & 0 & Y_1 \cdot (Z_1 \times Z_2) \\
0 & 1 & 0 \\
-Y_1 \cdot (Z_1 \times Z_2) & 0 & Z_1 \cdot Z_2
\end{bmatrix}
\]

\[
\mathbf{R}_3 = \begin{bmatrix}
1 & 0 & 0 \\
0 & C\dot{\phi} & -S\dot{\phi} \\
0 & S\dot{\phi} & C\dot{\phi}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1 & 0 & 0 \\
0 & Z_2 \cdot t & -X_2 \cdot (Z_2 \times t) \\
0 & X_2 \cdot (Z_2 \times t) & Z_2 \cdot t
\end{bmatrix}
\]

Through rotation \(\mathbf{R}_2\) and \(\mathbf{R}_3\), the coordinate transformation follows the sequence of \(X_1Y_1Z_1 \rightarrow X_2Y_2Z_2 \rightarrow X_3Y_3Z_3\), wherein \(X_1Y_1Z_1\) is the current body frame coordinate. The corresponding \(\dot{\theta}\) and \(\dot{\phi}\) in \(\mathbf{R}_2\) and \(\mathbf{R}_3\) are complementary attitude feedback which is generated based on yaw angle \(\psi\) and the thrust vector \(\mathbf{t}\) in the inertial frame. In normal flight conditions, both \(\dot{\psi}, \dot{\phi} \in (-\frac{\pi}{2}, \frac{\pi}{2})\).

With \(\psi\) and \(\mathbf{t}\), we know \(X_1 = [C\psi, S\psi, 0]^T\) and \(\mathbf{t} = [t_1, t_2, t_3]^T\). The unknown terms in \(\mathbf{R}_2\) and \(\mathbf{R}_3\) matrices can be derived by...
raw measurements are modeled by information, we continually use the normal flight data to in unintended positional shift. gration increases the cumulative sensor error and degrades a vast error relative to the actual position, because the inte-
data. This intuitive method is referred as Dead-Reckoning by performing a double integral operation on acceleration ease theoretically, i.e., traveling distances can be acquired does not affect flight stability. Moreover, the inertia sensor fault of the position feedback is much 'safer' because it B. Position Feedback Isolated control, the control gains must be fine-tuned to reduce the feedbackfrequency of the recovered attitude is insufficient for of the physical system needs to be considered when this method is applied to small-size, light-weight platforms. If the feedback frequency of the recovered attitude is insufficient for control, the control gains must be fine-tuned to reduce the closed-loop sensitivity or using a digital control law.

B. Position Feedback Isolated

For the UAVs, compared to the inertia sensor failure, a fault of the position feedback is much 'safer' because it does not affect flight stability. Moreover, the inertia sensor can be used to obtain the position feedback with relative ease theoretically, i.e., traveling distances can be acquired by performing a double integral operation on acceleration data. This intuitive method is referred as Dead-Reckoning [30]. However, the result of Dead-Reckoning always shows a vast error relative to the actual position, because the integration increases the cumulative sensor error and degrades the accuracy of the position estimate significantly, resulting in unintended positional shift.

In order to alleviate the position drift without the aiding information, we continually use the normal flight data to calibrate the inertial sensor. With the aligned sensors, the raw measurements are modeled by

\[ \begin{align*}
    \mathbf{C}_d &= \mathbf{Z}_1 \cdot \mathbf{Z}_2 = \frac{t \times \mathbf{Y}_1}{|t \times \mathbf{Y}_1|} = \frac{t_3}{\lambda(t, \psi)} \\
    \mathbf{S}_d &= \mathbf{Y}_1 \cdot (\mathbf{Z}_1 \times \mathbf{Z}_2) = \frac{t \times \mathbf{X}_1}{|\mathbf{Y}_1 \times t|} = \frac{t_1 \mathbf{C}_\psi + t_2 \mathbf{S}_\psi}{\lambda(t, \psi)}, \quad (17)
    \mathbf{C}_\hat{d} &= \mathbf{Z}_2 \cdot \mathbf{t} = \lambda(t, \psi) \\
    \mathbf{S}_\hat{d} &= -\mathbf{X}_2 \cdot (\mathbf{Z}_2 \times \mathbf{t}) = \mathbf{Y}_2 \cdot \mathbf{t} = -t_1 \mathbf{S}_\psi + t_2 \mathbf{C}_\psi
\end{align*} \]

where \( \lambda(t, \psi) = \sqrt{t_1^2 + (t_1 \mathbf{C}_\psi + t_2 \mathbf{S}_\psi)^2} \).

The detailed calculation is given in Appendix B.

From equation (17), the complimentary feedback of roll and pitch attitude can be determined mathematically using inverse trigonometric functions. For implementation, we calculated both arccos(·) and arcsin(·) for cross-validation to avoid the inconsistent direction of rotation.

By taking advantage of the phenomenon that the thrust orientation always follows the inclination of the vehicle body, the proposed method above is able to recover attitude information with limited sensor readings. It enhances flight safety significantly in inertia sensor failure condition. Furthermore, the proposed method can also be generalized to any sensor that can generate the thrust vector, such as vision sensors, infrared sensors, acoustic sensors, and laser sensors. Meanwhile, this complementary strategy can also be used for roughly detecting sensor failure through a comparison with raw sensor readings. For implementation, due to the relatively low update frequency of position feedback, the bandwidth of the physical system needs to be considered when this method is applied to small-size, light-weight platforms. If the feedback frequency of the recovered attitude is insufficient for control, the control gains must be fine-tuned to reduce the closed-loop sensitivity or using a digital control law.

V. Simulation And Experimental Result

For validation purpose, we deployed our method onto a customized simulation tool (programmed in C++ with Python binding) based on DART physics engine, which is used to solve vehicle dynamics. The physical parameters of the vehicle that applied in the simulation were obtained by conducting system identification on the real platform. Among several simulation validations, we picked attitude recovery for the experimental demonstration because this kind of failure is very challenging for most of the current fault-tolerant schemes of UAVs [9].

A. Simulation Validations

1) Differentiating Sensor And Actuator Faults: This test aims to demonstrate the importance and method of distinguishing sensor and actuator faults. The quadcopter used has been compromised with two different faults-actuator fault and inertia sensor fault. These two faults target completely separate UAV components but produce comparable residual trends and flight paths as shown in Fig.4. To launch the sensor fault, we input noisy sensor readings and simulate the attitude feedback of the vehicle incorrectly; for the actuator fault, we similarly compromised the motor speed to provide uncontrollable thrust. As a result, the control effort could not be corrected by the original control software. During the flight, the quadcopter is initially set to maintain a constant hovering position. Once a fault was triggered, the component failure immediately affected the control performance of the vehicle. Without measuring the motor speed, differentiating actuator and sensor faults become a challenge. However, the unknown thrust and corresponding motor speed can be estimated using the trusted state measurement and the vehicle stability can be maintained by fault-tolerant control accordingly. To validate the estimation results, we show the real speed of the attacked motor (unit in rpm-revolutions per minute) as a reference.
Although the residual values and flight paths resulting from the different faults are similar, the two faults can be differentiated with relative ease using the $r_m$ values.

2) Fine-grained Fault Isolation: In the second test, the position and gyroscope sensors were failed individually to show the effectiveness of using the chattering signal to pinpoint the sensor faults with similar effects on UAV behavior. In this case, the hacked states are the x-position and the corresponding direction of angular speed in the body frame, which fails the two separate sensors while producing very similar control results.

As evident in Fig. 5, the flight paths, attitudes, and residuals of the vehicle are very close to each type of fault. As an underactuated, highly sensitive system, the coupled dynamics and the highly irregular nature caused the UAVs’ irregular flight performance resulting from the sensor faults. Thus, there is not a consistent method of explicitly determining which sensor has failed using the state residuals or from sensor readings.

Chattering signals, rendered by physical model variations, however, offer a clear solution to address this problem since the basic physical parameters (mass and inertias) usually does not change during the flight. As Fig. 5 shown, because the gyroscope sensor failure causes the body inertia estimation to diverge, its chattering signal appears much more sensitive in the case of a fault than the position sensor is. Consequently, the significantly larger fluctuations in chattering can reliably be associated with a gyroscope sensor failure.

3) Recovery Logic: The last validation test assesses the effectiveness of our proposed method of attitude/position feedback recovery and re-stabilizing the vehicle. When the attitude feedback is completely lost, the vehicle ordinarily drops immediately without any redundancy sensing as shown in Fig. 6 (a). Utilizing the position sensor to restore approx-
imate attitude information, the UAV was able to maintain its desired attitude and stay within a bounded area (a 0.2m circular around area for 16 seconds in this test). While the vehicle tracking is not ideal due to the low feedback frequency inherent with this method, it does safely maintain stability as demonstrated in Fig 6(b).

For restoring relative position information from attitude feedback, implementing drift compensation shows obvious lessened positional drift comparing to the direct Dead-Reckoning as shown in Fig 6(c). When the UAV was assigned a default point to point flight from (0,0) to (7.5), the Dead-Reckoning method allowed the vehicle to drift more than twice as much as the proposed method, although both methods were able to maintain attitude stability over the course of the flight.

B. Experimental Results

This experiment aims to counter the severe oscillated IMU measurements, which is a very threatening sensor fault for UAVs as presented in [9] since the traditional fault-tolerant strategies becomes ineffective though such methods have shown high reliability in many other tasks. For demonstration, we follow the similar sensor failure pattern as presented in [9] by maliciously aliasing a high-low-frequency signal on the IMU sensor via trojan input to mimic gyro’s resonating output which is shown in Fig 7. The abnormal oscillating of the gyro readings need to be diagnosed and isolated immediately while the position feedback can provide pseudo attitude to complement the flawed feedback and stabilize the vehicle. During the experiment, we first triggered the fault without a fault-tolerant scheme protection: the vehicle attitude drops immediately as shown in Fig 7. We then utilized the proposed system FDI and FR and the UAV demonstrated stable hovering flight with the faulty sensor. From the experiment result as shown in Fig 8, it can maintain its stabilities for a longterm running. Compare to the corresponding simulation case, due to the larger numerical error and limited computation resources of the onboard implementation, the position drift in experimantal test is larger but it is still bounded.

VI. Conclusion

In this work, a redundancy-free UAV sensor fault-tolerant method was presented, which included two main parts: FDI and FR. At all times, a state-estimator-based FDI was employed which took into account the unmeasurable actuator state and modeling uncertainty. With this functions, if a fault was taking place, it can rapidly determine the fault and perform fine-grained fault isolation. To recover from the sensor faults, we proposed a geometrical method. The
Fig. 8. Left side figure presents a stable flight with the inertia sensor isolated condition, using complementary attitude for control from position feedback. The corresponding time sequences is composed and shown on the right side one.

attitude and position sensors used to vehicle control would simultaneously provide their intended feedback information as well as estimated feedback to supplement the isolated faulty sensor. The proposed UAV sensor fault-tolerant schemes have been validated through both simulation and experimental tests using a small-sized quadcopter platform. The result shows that this new fault-tolerant design allows the vehicle to recover in case of both cyber and physical domain attacks and differentiate faults with similar effects of the system flight. Given the demonstrated success of the FDI and FR design capabilities, the proposed method holds great promise in being portable to other types of autonomous vehicles (e.g., a fixed-wing vehicle or a flapping-wing vehicle) and compliment other fault-tolerant methodologies, as shown in our ongoing work.

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A. Detailed Discrete Model of The Test Platform

\[ x_k = x_{k-1} + dt \dot{x}_{k-1}, \]
\[ y_k = y_{k-1} + dt \dot{y}_{k-1}, \]
\[ z_k = z_{k-1} + dt \dot{z}_{k-1}, \]
\[ \dot{x}_k = \dot{x}_{k-1} + \frac{dt}{m} \left( C \phi_{k-1} S \theta_{k-1} C \phi_{k-1} + S \phi_{k-1} S \phi_{k-1} \right), \]
\[ \dot{y}_k = \dot{y}_{k-1} + \frac{dt}{m} \left( S \phi_{k-1} S \theta_{k-1} C \phi_{k-1} - C \phi_{k-1} S \phi_{k-1} \right), \]
\[ \dot{z}_k = \dot{z}_{k-1} + \frac{dt}{m} \left( C \phi_{k-1} C \phi_{k-1} \right), \]
\[ C_L (\Omega_{k-1}^2 + \Omega_{k-1}^2 + \Omega_{k-1}^2 + \Omega_{k-1}^2) - g dt, \]
\[ \phi_k = \phi_{k-1} + dt (C \phi_{k-1} q_{k-1} - S \phi_{k-1} r_{k-1}), \]
\[ \theta_k = \theta_{k-1} + dt (C \phi_{k-1} q_{k-1} - S \phi_{k-1} r_{k-1}), \]
\[ \psi_k = \psi_{k-1} + dt (S \phi_{k-1} \theta_{k-1} q_{k-1} - C \phi_{k-1} r_{k-1}), \]
\[ p_k = p_{k-1} + dt \left( \frac{J_y - J_z}{J_y} \right) q_{k-1} r_{k-1} + \]
\[ dt \frac{C_L}{J_x} \left( -d \Omega_{k-1}^2 + d \Omega_{k-1}^2 + d \Omega_{k-1}^2 - d \Omega_{k-1}^2 \right), \]
\[ q_k = q_{k-1} + dt \left( \frac{J_y - J_z}{J_z} \right) p_{k-1} r_{k-1} + \]
\[ dt \frac{C_L}{J_x} \left( -d \Omega_{k-1}^2 + d \Omega_{k-1}^2 + d \Omega_{k-1}^2 - d \Omega_{k-1}^2 \right), \]
\[ r_k = r_{k-1} + dt \left( \frac{J_y - J_z}{J_z} \right) p_{k-1} q_{k-1} + \]
\[ dt \frac{C_L}{J_x} \left( -d \Omega_{k-1}^2 + d \Omega_{k-1}^2 + d \Omega_{k-1}^2 - d \Omega_{k-1}^2 \right). \]

B. Derivation of The Complementary Roll And Pitch

Remark:
\[ a \times b = -b \times a, \]
\[ a \cdot (b \times c) = b \cdot (c \times a) = c \cdot (a \times b), \]
\[ (a \times b) \cdot (c \times d) = (a \cdot c) \cdot (b \cdot d) - (a \cdot d) \cdot (b \cdot c), \]
\[ a \times (b \times c) = b \cdot (a \cdot c) - c \cdot (a \cdot b). \]

Given: \( X_1, Z_1, t \); and \( X_2 \) is perpendicular to \( X_1 \) and \( t \), some useful preliminary results can be calculated:

\[ Y_1 = Z_1 \times X_1 = (-S\psi, C\psi, 0); \]
\[ Y_1 \times t = t \times (X_1 \times Z_1) = X_1 \cdot (t \times Z_1) - Z_1 \cdot (t \times X_1); \]
\[ X_2 = \frac{Y_1 \times t}{|Y_1 \times t|}; \]
\[ Z_2 = X_2 \times Y_1. \]

Therefore,

(1) \[ Z_1 \cdot Z_2 = Z_1 \cdot (X_2 \times Y_1) = \frac{(Y_1 \times t) \cdot (Y_1 \times Z_1)}{|Y_1 \times t|} \]
\[ = \frac{t \cdot Z_1}{|Y_1 \times t|} = \frac{t_3 \lambda}{\lambda(t, \psi)}. \]

(2) \[ Y_1 \cdot (Z_1 \times Z_2) = Z_2 \cdot (Y_1 \times Z_1) = (X_2 \times Y_1) \cdot (Y_1 \times Z_1) \]
\[ = (X_2 \times Y_1) (Y_1 \cdot Z_1) - (X_2 \cdot Z_1) \cdot (Y_1 \cdot Y_1) = - (X_2 \cdot Z_1) \]
\[ = \frac{Y_1 \times t}{|Y_1 \times t|} \cdot Z_1 = \frac{t_1 C\psi + t_2 S\psi}{\lambda(t, \psi)}. \]

(3) \[ Z_2 \cdot t = t \cdot (X_2 \times Y_1) = X_2 \cdot (Y_1 \times t) \]
\[ = X_2 \cdot X_2 \cdot |Y_1 \times t| = \lambda(t, \psi). \]

(4) \[ (X_2 \cdot Z_2 \times t) = (Z_2 \cdot t \times X_2) = (X_2 \times Y_1) \cdot (t \times X_2) \]
\[ = -Y_1 \cdot t = S\psi t_1 - C\psi t_2. \]