On Controller Tuning with Time-Varying Bayesian Optimization

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Abstract—Changing conditions or environments can cause system dynamics to vary over time. To ensure optimal control performance, controllers should adapt to these changes. When the underlying cause and time of change is unknown, we need to rely on online data for this adaptation. In this paper, we will use time-varying Bayesian optimization (TVBO) to tune controllers online in changing environments using appropriate prior knowledge on the control objective and its changes. Two properties are characteristic of many online controller tuning problems: First, they exhibit incremental and lasting changes in the objective due to changes to the system dynamics, e.g., through wear and tear. Second, the optimization problem is convex in the tuning parameters. Current TVBO methods do not explicitly account for these properties, resulting in poor tuning performance and many unstable controllers through over-exploration of the parameter space. We propose a novel TVBO forgetting strategy using Uncertainty-Injection (UI), which incorporates the assumption of incremental and lasting changes. The control objective is modeled as a spatio-temporal Gaussian process (GP) with UI through a Wiener process in the temporal domain. Further, we explicitly model the convexity assumptions in the spatial dimension through GP models with linear inequality constraints. In numerical experiments, we show that our model outperforms the state-of-the-art method in TVBO, exhibiting reduced regret and fewer unstable parameter configurations.

I. INTRODUCTION

Automated controller tuning aims to find optimal controllers not only from model knowledge but also include data generated from the plant in the search. An emerging method for controller tuning is Bayesian optimization (BO) [1]. It is well-suited for this task as it can incorporate a broad spectrum of prior knowledge about the control objective in a probabilistic model. Most work on controller tuning with BO has focused on tuning problems where the dynamics governing the system do not change over time [2–8]. However, in practice, control performance might decay when the underlying system changes, e.g., due to wear and tear or payload differences. Herein we consider changes that are unknown a priori so that tuning must be online and therefore sample efficient.

In general, the time-varying optimization problem with arbitrary temporal changes is infeasible, and we need some regularity assumptions to derive meaningful algorithms. In time-varying Bayesian optimization (TVBO) [9] these are formulated as a prior distribution that defines correlations of the objective over time. This prior knowledge restricts the temporal change of the objective function and thus guides the optimization algorithm in the exploration-exploitation trade-off when learning about changes. Especially in online controller tuning, we want to include as much prior information as possible to avoid over-exploration and increase sample efficiency, performance, and safety.

In this work, we distinguish between two different kinds of time variations: variations in the objective function (modeled as a sample from a stochastic process) and variations of the process itself. For clarity, we reserve the word stationary process to qualify the second kind of time variations. Therefore, a time-varying objective function can either result from a stationary or a non-stationary process (see Def. 4). Current TVBO methods exclusively focus on stationary processes, which exhibit a catastrophic forgetting behavior over time as the posterior prediction falls back to the prior (B2P) over time (red). In contrast, we propose modeling the control objective as a non-stationary spatio-temporal process via uncertainty injection (UI), better capturing engineering knowledge and intuition. The model can remember relevant past information in the form of the posterior mean (blue). As new data is added ($t_1 + 1$, bottom right), this information can be leveraged by a TVBO algorithm as it does not need to re-explore the whole domain.

Fig. 1: Intuition behind the proposed forgetting strategy (blue) in comparison to the state-of-the-art in TVBO (red). At $t_0$, recent data of the control objective is available (top). Over time forgetting occurs ($t_1$, bottom left) and uncertainty increases if no new data is added. Existing approaches assume a stationary spatio-temporal process whose posterior falls back to the prior (B2P) over time (red). In contrast, we propose modeling the control objective as a non-stationary spatio-temporal process via uncertainty injection (UI), better capturing engineering knowledge and intuition. The model can remember relevant past information in the form of the posterior mean (blue). As new data is added ($t_1 + 1$, bottom right), this information can be leveraged by a TVBO algorithm as it does not need to re-explore the whole domain.
tal and lasting changes such as wear and tear. For example, friction only tends to increase over time and is usually not a mean-reverting process. In addition, mean-reversion leads to undesirable exploration behavior when combined with a constant prior mean, another prevalent assumption in BO. In that case, TVBO has to explore the whole domain over and over again. Instead of B2P forgetting, we propose a novel TVBO forgetting strategy using Uncertainty-Injection (UI), which incorporates the assumption about incremental and lasting changes in the control objective. Specifically, UI results in a non-mean reverting process which is beneficial, for example, when the dynamics and control performance changes over time due to increased friction. We also directly incorporate prior knowledge of the objective’s convexity as shape constraints for efficient exploration to further guide the optimization.

**Contribution:** In summary, we propose a novel model for TVBO that encodes relevant assumptions for online controller tuning, a non-stationary time-varying optimization problem. In particular, we propose to use UI instead of B2P (see Fig. 1). With UI in TVBO, the model remembers the posterior mean over time while allowing for a wide variety of changes in the objective, such as drifts and jumps, without over-exploration. This better captures our prior knowledge about temporal changes in control systems. Additionally, accounting for many convex control objectives, we explicitly incorporate a priori knowledge about convexity into the TVBO algorithm. The convexity assumption allows global extrapolation from local queries. The main contributions are:

- **UI in TVBO (UI-TVBO):** A surrogate model for TVBO modeling non-stationary temporal change in the objective functions using a Wiener process.
- **We use linearly inequality constrained Gaussian processes (GPs) [10] to model convex time-varying objective functions for TVBO.**

We show, based on an linear quadratic regulator (LQR) problem under non-stationary changes, that UI-TVBO needs to explore less and, thereby, yields improved regret compared to the state-of-the-art. Additionally, including the prior knowledge of convexity further reduces the regret and prevents unstable parameter configurations.

**II. Problem Formulation**

We seek to sequentially optimize controller parameters $\theta_t$ of an unknown time-varying objective function $f_t : \mathcal{X} \times \mathcal{T} \mapsto \mathbb{R}$. Thus, the optimization problem can be written as

$$\theta^*_t = \arg \min_{\theta \in \mathcal{X}} f_t(\theta)$$

at the discrete time step $t \in \mathcal{T} = \{1, 2, \ldots\}$ within a feasible set $\theta \in \mathcal{X} \subset \mathbb{R}^D$. At each time step, an algorithm can query the objective function based on past information at a chosen location $\theta_t$ and obtains a noisy observation of the form $y_t = f_t(\theta_t) + w_t$ with zero mean i.i.d. Gaussian noise $w_t \sim \mathcal{N}(0, \sigma_w^2)$ where $\sigma_w^2$ is the noise variance. The performance of the tuning algorithm is defined in terms of regret measuring the difference between optimal and chosen parameters as formalized in the following definition.

**Definition 1 (Regret (cf. [9])).** Let $\theta^*_t$ be the optimum of the time-varying function $f_t(\theta)$ as $\theta^*_t = \arg \min_{\theta \in \mathcal{X}} f_t(\theta)$ at time step $t$ and suppose $\theta_t$ is queried at time step $t$. Then the regret after $T$ time steps is $R_T := \sum_{t=1}^{T} (f_t(\theta_t) - f_t(\theta^*_t))$.

Since solving (1) for arbitrary changes in $f_t$ is not feasible, we must formulate some assumptions. We assume that the objective function can be modeled as a GP [11]. Where applicable, we assume spatial convexity of the objective function modeled as an inequality constraint GP [10].

**Assumption 1 (Smoothness and temporal change).** The objective function $f_t$ is a sample path from a known GP prior $\phi \sim \mathcal{GP}(\mu_0, k)$ with a separable spatio-temporal kernel $k : \mathcal{X} \times \mathcal{T} \mapsto \mathbb{R}$ as

$$k((\theta, t), (\theta', t')) = k_S(\theta, \theta')k_T(t, t').$$

Moreover, the mean function $\mu_0$ is at least twice differentiable, and the covariance function $k$ is at least four times differentiable, both w.r.t. $\theta$.

**Assumption 2 (Convexity).** Every realized sample $f_t$ from $\phi$ is at least twice differentiable with respect to $\theta$ and the Hessian $\nabla^2 f_t$ is semi positive-definite $\forall t \in \mathcal{T}$ and $\forall \theta \in \mathcal{X}$.

Many relevant controller tuning problems can be formulated as convex optimization problems (e.g., the LQR problem) making Assumption 2 especially suitable in the context of control systems. For further clarity, we distinguish between two types of time-varying optimization problems, depending on the stationarity of the process.

**Definition 2.** For a stationary spatio-temporal process it holds that for all $\theta \in \mathcal{X}$, $\tau, t_1, \ldots, t_n \in \mathcal{T}$, and $n \in \mathbb{N}$ $P(\phi_{t_1}(\theta), \ldots, \phi_{t_n}(\theta)) = P(\phi_{t_1+\tau}(\theta), \ldots, \phi_{t_n+\tau}(\theta))$.

**Definition 3.** Stationary time-varying optimization (STVO) problems are optimization problems of the form (1) under Assumption 1 where $\phi$ is a stationary spatio-temporal process as in Def. 2.

**Definition 4.** Non-stationary time-varying optimization (NSTVO) problems are optimization problems of the form (1) under Assumption 1 where $\phi$ is a non-stationary spatio-temporal process.

To the best of our knowledge, prior work in TVBO is based on [9] and, therefore, considers problems of the form in Def. 3. Instead, the approach presented herein explicitly models NSTVO problems (Def. 4), as these often occur in physical systems (e.g., due to wear and tear or load changes).

**III. Related Work**

We propose a spatio-temporal model for online controller tuning using TVBO and utilize prior knowledge about the shape of the objective function to guide the optimization. We discuss prior work on TVBO, BO for controller tuning, and shape constraints on the surrogate model in BO. We
also briefly touch on probabilistic models for time-varying functions. We omit work on time-varying convex optimization that is not in the BO setting and refer the reader to Simonetto et al. [12] for a recent overview.

**Time-Varying Bandit Optimization:** Minimizing regret over finite actions in a time-varying environment is studied in dynamic multi-armed bandits (MABs). Here, both forgetting strategies – UI [13, 14] as well as B2P [15] – have been considered. GP-based TVBO with a spatio-temporal process can be considered as a special case of contextual BO [16] but it was first explicitly discussed by Bogunovic et al. [9]. The authors introduce a stationary spatio-temporal process as well as a time-varying UCB variant with regret bounds to solve STVO problems. The time-varying UCB variant increases the UCB exploration parameter over time, which leads to undesirable high exploration in controller tuning problems and is difficult to tune in practice. The work of Bogunovic et al. [9] has been adapted to different applications in low-dimensional settings with up to four parameters, such as controller learning [17], safe adaptive control [18], and online hyperparameter optimization [19]. In contrast to the algorithms based on Bogunovic et al. [9], we use the standard UCB algorithm for TVBO without increasing exploration over time. The main difference between UI-TVBO and the methods based on Bogunovic et al. [9] is that UI-TVBO intends to solve NSTVO problems such as controller tuning.

**BO for Controller Tuning:** BO has emerged as a powerful and popular method to tune controllers given only limited system knowledge (e.g., [2–8]). However, controller tuning with changing dynamics is significantly less explored in literature. König et al. [18] considers an adaptive control problem for safe model-free adaptive control where they adopt the setting of Bogunovic et al. [9].

**Shape Constraints in BO:** The main benefit of incorporating prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function. A prior knowledge in BO is an increase in sample efficiency as it reduces the hypothesis space of the objective function.

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**Probabilistic Models for Time-Varying Functions:** Modeling temporal and spatio-temporal processes with a GP is a topic of interest in regression and Bayesian filtering [23–25]. Analogous to their approaches, we utilize a separable spatio-temporal kernel. Van Vaerenbergh et al. [23] further discussed the remembering-forgetting trade-off when modeling a time-varying function and introduced the terms B2P and UI. While B2P has been the implicit forgetting strategy in prior work on TVBO, we argue that UI is a more suitable modeling approach for many practical problems such as online controller tuning.

**IV. METHODS**

Motivated by the online controller tuning problem, we present a non-stationary surrogate model for TVBO based on UI, UI-TVBO. Furthermore, we present an approach to incorporate Assumption 2 into TVBO. The TVBO algorithm is shown in Algorithm 1.

**Algorithm 1 TVBO algorithm.**

1. **Input:** $GP(0, k), \mathcal{X} \subset \mathbb{R}^D, \mathcal{D}_N = \{(t_j, \theta_j, y_j)\}_{j=0}^N$
2. for $t = N + 1, N+2, \ldots$ do
3. \hspace{1em} Train GP model with $\mathcal{D}_t$
4. \hspace{1em} Optimize $\theta_{t+1} = \arg\min_{\theta \in \mathcal{X}_{t+1}} \alpha(\theta, t+1|D)$
5. \hspace{1em} Query objective function $y_{t+1} = f_{t+1}(\theta_{t+1}) + w$
6. \hspace{1em} $\mathcal{D}_{t+1} = \mathcal{D}_t \cup \{(y_{t+1}, \theta_{t+1}, t+1)\}$
with the model of a Wiener process. The posterior covariance for \( r = 0 \), where \( k_{S,se} = \sigma_{k}^{2} \), is given by

\[
\text{cov}[(\theta, t), (\theta', t')] = \sigma_k^2 k_{T,wp}(t', t') - \sigma_k^2 k_{T,wp}(t', t) \\
= \sigma_k^2 k_{T,wp}(t', t') - \sigma_k^2 k_{T,wp}(t, t') \\
= \sigma_k^2 \sigma_w^2 (t' - t). \tag{6}
\]

We thus re-parametrize the increase in variance as \( \sigma_w^2 \) with \( \sigma_w^2 = \sigma_k^2 \sigma_w^2 \) for UI-TBBO. Note, that, as desired, for \( \sigma_w^2 \to 0 \) UI-TBBO converges to the time-invariant model as \( \lim_{\sigma_w^2 \to 0} k_{T,wp} = 1 \) with the proposed parametrization. We formalize the assumptions about the temporal changes when using a Wiener process in Assumption 3.

**Assumption 3** (Temporal Wiener process), \( \phi_t(\theta) \) follows a Wiener process with known variance \( \sigma_w^2 \) at all \( \theta \). Well known results mean that \( \delta(\theta) = \phi_{t+1}(\theta) - \phi_t(\theta) \) at point \( \theta \) is an i.i.d. normal distribution \( \delta(\theta) \sim N(0, \sigma_w^2) \).

B. Modeling Convex Time-Varying Objective Functions

In online controller tuning, it is desirable to limit the exploratory behavior of the algorithm as it directly influences the performance. This can be achieved by embedding prior knowledge about the shape of the objective function into TVBO. Since many control objectives are convex, we formulate a surrogate model that approximates Assumption 2. When choosing the controller parameters for \( t + 1 \), we have a current estimate of the optimum \( \hat{\theta}_t^* \) and want to find a good \( \hat{\theta}_{t+1}^* \). For this, we optimize the BO acquisition function operating on the posterior of \( \phi_{t+1}(\theta) \). Since, by Assumption 2, the objective is convex, we modify the posterior using inequality constraints on the second derivative for each dimension using the method proposed by Agrell [10]. For more than one dimension, this is only an approximation of Assumption 2 as it enforces convexity only in coordinate directions but does not enforce the Hessian to be positive semi-definite.

The inequality constraints are enforced on a finite set of virtual observation points (VOPs). Given a sufficient amount of VOPs combined with the smooth SE kernel, we can approximate convexity in the feasible set \( X \). In order to sample from the constrained posterior (cf. [10, Algorithm 3]), we use the sampling method introduced by Botev [26] but switch for computational reasons to a Gibbs sampling algorithm [21] if the distribution is high-dimensional or the covariance matrix close to singular.

Since the acquisition function is optimized only at time step \( t + 1 \) of TVBO, it is sufficient to place VOPs in \( X \) only at \( t + 1 \). Ideally, the VOPs are placed densely within \( X \). However, sampling becomes computationally expensive for large numbers of VOPs [10]. This limits constraining the posterior to only low-dimensional problems. However, it is still suitable for PID-controller and low-dimensional LQR tuning. We will consider the latter in the numerical results.

The method of Agrell [10] requires the derivatives of the kernel w.r.t. to \( \theta \). We obtain them by leveraging the separable spatio-temporal kernel in (2). Since the temporal kernel is independent of \( \theta \), we can write \( \frac{\partial}{\partial \theta} k = (\frac{\partial}{\partial \theta} k_S) k_T \). Calculating the necessary Gram matrices is then straightforward [10, Section 4.2]. This concept of constraining the posterior in a time-varying context is visualized in Fig. 2. For large domains, we avoid computational bottlenecks by enforcing local convexity. We place equidistant VOPs in a subset of the feasible set \( X_{t+1} \subseteq X \) around the current best estimate \( \hat{\theta}_t^* \). The acquisition function is then optimized in \( X_{t+1} \).

**Remark.** If \( \theta_t^* \) is in \( \hat{X_t} \), the regret is unaffected by the local optimization. This is usually true for gradually changing processes. Nevertheless, depending on the rate of change \( \sigma_w^2 \) this might not hold. In our empirical evaluation, we observed that even if the optimizer jumps out of \( \hat{X_t} \), the algorithm will recover and find a \( \hat{X_t} \) that contains the optimum.

The method presented in this section is independent of the temporal process of \( \phi \) and can be used for all modeling approaches in TVBO. Hence, we will compare constrained variations for both B2P (C-TV-GP-UCB) and UI (C-TV-TBBO) in the numerical results.

V. Numerical Results

To evaluate our modeling approach, we benchmark it against the state-of-the-art model TV-GP-UCB with a forgetting factor of \( \varepsilon \in (0,1) \) [9] on a LQR tuning problem. The LQR problem is a classic problem in optimal control theory controlling a linear dynamical system by minimizing a quadratic cost function. It satisfies Assumption 2 making it suitable to test our methods. In addition to TV-GP-UCB, we compare our method against the initially optimal state-feedback control as a baseline to investigate whether tuning actually improves performance.
For the system at hand, we consider a cart-pole system linearized around the upper equilibrium point. The linear state space model is the following:

$$\dot{x} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -\frac{1}{T_c} & 0 & 0 \\
0 & \frac{1}{2J_a} & \frac{1}{2J_a} & -\frac{\tau_p(t)}{J_a} \\
0 & \frac{1}{2m} & \frac{1}{2m} & 0 & 1
\end{bmatrix} x + \begin{bmatrix}
0 \\
0 \\
\frac{K_u}{T} \\
-\frac{1}{2}\frac{m}{J_a} K_u \frac{1}{T}
\end{bmatrix} u \tag{7}$$

with $x = [x, \dot{x}, \varphi, \dot{\varphi}]$, where $x$ is the cart position and $\varphi$ the pole angle. Furthermore, $m_p = 0.0804$ kg, $l = 0.147$ m, $J_d = 0.5813 \cdot 10^{-3}$ kg m$^2$, $\tau_p(t)$ are the mass, length, moment of inertia, and time varying-friction of the pole, respectively. The input $u$ is the desired cart velocity and we choose $T_1 = 1$ s and $K_u = 1$. To induce a non-stationary smooth change in the dynamics model, the initial friction $\tau_p^0 := \tau_p(0) = 2.2 \cdot 10^{-3}$ N m s in the bearing is increased at $t_1 = 50$ until $t_2 = 100$ as

$$\tau_p(t) = \begin{cases}
\tau_p^0, & t < t_1 \\
\tau_p^0 + \frac{\tau_p^0}{2} (1 - \cos \left( \frac{\pi}{2} (t - t_1) \right)), & t_1 \leq t \leq t_2 \\
3\tau_p^0 + \frac{\tau_p^0}{2} \sin \left( -\frac{\pi}{t_2} \right), & t > t_2
\end{cases} \tag{8}$$

A lasting increase in friction is a realistic real-world scenario caused by, e.g., wear and tear.

We choose the objective function to be the approximate LQR cost over a simulation horizon $M$ as

$$\hat{J}_t = \frac{1}{M} \sum_{m=0}^{M-1} x_m^T Q x_m + u_m^T R u_m \tag{9}$$

with the positive definite weight matrices $Q \in \mathbb{R}^{4 \times 4}$ and $R \in \mathbb{R}$, and where $m$ is a discrete simulation time step (not a TVBO time step). Due to the change in friction over time, the cost function in (9) is time-varying.

The state-feedback controller at time step $t$ is defined as $u_m = -K_t^T x_m$. We directly optimize the controller gains $\theta_t$ as $\theta_t := K_t = [\theta_0, \theta_1, \theta_2, \theta_3]$. At each time step $t$ of TVBO, we perform a closed-loop simulation given the chosen parameters of the controller over $M = 1000$ discrete time steps with a sampling time of 0.02s. The accumulated cost is then calculated using (9) with $Q = 10 \cdot I$ and $R = 1$, and the result is provided as noisy observation (due to process noise) to the TVBO algorithm. Given the linear state space model, it is possible to calculate the optimal state-feedback controller at each time step $K_t^*$ using the algebraic Riccati equation if the system dynamics were known.

We use the standard TVBO algorithm shown in Algorithm 1 and choose LCB as the acquisition function as $\alpha(\theta, t + 1|D) = \mu_{t+1}(\theta) - \sqrt{\beta_{t+1}} \sigma_{t+1}(\theta)$ with an exploration-exploitation factor of $\beta_{t+1} = 2$. By setting $\beta_{t+1}$ to a constant, we omit its poly-logarithmic increase as in Bogunovic et al. [9]. Especially in controller tuning, it is undesirable to increase explorative behavior over time due to the increased risk of the controller becoming unstable. For convex function it is also unnecessary. We further set the forgetting factors to $\tilde{\sigma}_w^2 = \varepsilon = 0.03$, which showed good empirical performance for all approaches in preliminary experiments. The algorithm is implemented using BoTorch [27] and we use GPYtorch [28] for the GP surrogate model.

For the 2D problem we set $\theta_{t,0}$ and $\theta_{t,1}$ to their optimal values (i.e., the first two entries of $K_t^*$). At the start, we are given an initial set of $N$ parameters $\mathcal{D}_N = \{(t_j, \theta_{j,0}, \theta_{j,1})\}_{j=0}^N$ where $y_j$ and $t_j$ are the corresponding observations and time stamps. For both the 2D LQR problem and 4D LQR problem, the initial set is composed of $N = 30$ stable controller parametrizations, and the performance is normalized to a mean of zero and unit variance. The feasible set $\mathcal{X}$ is chosen as a hyperbox, also allowing unstable controller configurations. We determine unstable controllers heuristically via a cost threshold. In order to not deteriorate the hyperparameter optimization of the GP, the observations for unstable controller configurations $\hat{y}_i$ are based on the current posterior as $\hat{y}_i(\theta_i) = \mu_i(\theta_i) + 3\sigma_i(\theta_i)$. Intuitively, this means the cost for an unstable controller is set “as high as possible” given the

\footnote{Code and parameters published at https://github.com/brunzema/uitvbo.}
likelihood function of the current posterior. To guide the hyperparameter optimization, we use Gamma priors on the length scales of the spatial kernel.

Table I shows the results of 25 different runs of both problems each with a time horizon of $T = 300$. For calculating the regret (Def. 1), unstable iterations are skipped since $f_i(\theta_3^*) - f_i(\theta_3^*)$ is not well defined for the corresponding iteration. Note that this favors the approach with more unstable controllers, which in all cases is TV-GP-UCB. In both the 2D and 4D LQR problem, our approach UI-TVBO significantly outperforms the state-of-the-art in terms of regret, yields a smaller variance and drastically reduces the number of unstable controllers. Furthermore, UI-TVBO shows smaller regret than the initial optimal controller $K_0^*$ in the 2D problem, whereas TV-GP-UCB does not. Due to the large sampling space, both methods cannot outperform $K_0^*$ in 4D. Consequently, no adaptive tuning would have resulted in a lower overall cost. If we tighten the bounds on the feasible set to consider only stable parameter configurations (bottom, Table I), UI-TVBO again outperforms $K_0^*$ in 4D. Including the convexity constraints prevents unstable configurations and further decreases the regret and its variance. This suggests that prior knowledge about the convexity of the problem can be enough to make controller tuning safe in some instances.

To further compare the behavior of the models, Fig. 3 shows the normalized mean regret (thick lines) as well as the individual runs (thin lines) over time. Following the non-stationary change, TV-GP-UCB further increases in normalized regret and individual runs display a sawtooth-like behavior caused by excessive exploration of B2P forgetting. In contrast, UI-TVBO (empirically) displays the desirable asymptotically decreasing regret [9, 16].

Fig. 4 shows the query “trajectory” of the parameter $\theta_3^*$ of one run over time. We observe that after the non-stationary change, TV-GP-UCB starts to explore aggressively at the domain boundaries, resulting in several unstable controllers. While UI-TVBO still yields two unstable controllers, the required exploration is significantly reduced as the model was able to adapt to the change in dynamics. If the convexity assumption is included (green), the model can extrapolate within the bound from local queries (cf. Fig. 2). Hence, the sampling radius around the optimum is reduced, resulting in a superior tuning performance.

VI. CONCLUSION

We proposed a modeling approach to TVBO for controller tuning, which utilizes the concept of UI to cope with non-stationary optimization problems. Using an LQR tuning problem of an inverted pendulum with time-varying dynamics, we demonstrated that our modeling approach UI-TVBO outperforms the state-of-the-art in terms of regret while yielding substantially fewer unstable parameter configurations.

Furthermore, we successfully merged TVBO with shape constraints to further improve performance. While the results look very promising, scaling shape constraints (such as convexity) for GPs to higher dimensions remains an open and important research question. Improvements therein would greatly benefit the applicability of BO and TVBO in controller tuning for high dimensional systems. Extending the presented approach using ideas from hard shape constraints as by Aubin-Frankowski and Szabo [29] can be a promising direction for future research.

UI-TVBO can model non-stationary changes. However, it cannot predict changes or learn them from data. For future work, we plan to model drift which would allow us to predict the change in the optimizer, which is also discussed as a relevant research direction by Simonetto et al. [12].

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