The ion acoustic instability of the rotating cylindrical helicon discharge plasma

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(Dated: 8 February 2022)

The kinetic theory for the cylindrical plasma, produced by the cylindrically symmetric (azimuthal mode number \( m = 0 \)) helicon wave, is developed with accounting for the cylindrical geometry and the radial inhomogeneity of the helicon wave and plasma. This theory reveals macroscale effect of the azimuthal steady rotation of electrons with a radially inhomogeneous angular velocity, caused by radial inhomogeneity of the helicon electric field. It is found that this sheared rotation as well as the electron density and temperature inhomogeneity are responsible for the development of the high frequency ion acoustic instability of the inhomogeneous cylindrical plasma. This instability is spatially localized in the region of strong gradients of the helicon wave electric field and of the plasma density, where it is more stronger than the parametric instabilities driven by the oscillating motion of the electrons relative to ions in the helicon wave.

PACS numbers: 52.35.Ra, 52.35.Kt

I. INTRODUCTION

The helicon plasma sources attract great interest in plasma community and have various applications due to the remarkably strong absorption of helicon waves in plasmas and anomalously strong electron heating. The helicon plasma is generated by using the helicon wave, which is the whistler wave in the bounded plasma. The frequency of helicon/whistler wave lies between ion and electron cyclotron frequencies. The linear theory of the helicon wave predicts that this wave has phase velocity much above the electron thermal velocity and, therefore, the absorption of the helicon wave in the collisionless plasma by electrons due to the electron Landau damping is a negligibly weak effect. That is why the experimentally observed unusually high absorption rate of the helicon wave, that testified about strong interaction of the helicon wave with electrons, was unpredictable. Although a large number of studies have been curried, the mystery of why helicon discharges are so efficient is still unresolved.

The anomalous absorption of helicons and plasma heating was observed a very long time ago in the first experimental studies of the basic plasma physics processes. It was claimed in these papers that the anomalous absorption of a large amplitude whistler wave was caused by the development of the current driven ion acoustic instability or by the development of the resonant decay instability. The Boswell’s experiments gave impetus to the active theoretical investigations of the plasma instabilities driven by the helicon wave. It was found that the development of the parametric instabilities in the helicon wave, that originated from the oscillatory motion of electrons relative to ions in the pumping helicon field, may be the cause of the anomalous absorption of the helicon wave and of the anomalous heating of electrons, resulted from the interaction of electrons with ion acoustic turbulence. Since then, the plasma turbulence in helicon plasma was investigated in several experiments, in which the detected short scale fluctuations were identified as the ion acoustic waves and was found that the level of these fluctuations increases with RF power.

The developed theory of the parametric instabilities of the helicon plasma is grounded on the approximation of the spatially uniform helicon wave. However, the helicon wave field in cylindrical helicon sources is as a rule spatially inhomogeneous with radial inhomogeneity length comparable with, or less than, the radius of plasma cylinder. The effect of the cylindrical geometry of the plasma and of the helicon wave field, and the effect of the spatial inhomogeneity of the helicon wave on the parametric microturbulence is usually ignored assuming that the approximation of the uniform helicon field is sufficient for the proper description of the parametric instabilities the wavelengths of which are much less than the radial inhomogeneity scale length of the helicon wave in plasma cylinder. It was found in Refs.¹⁰,¹¹, however, that the electromagnetic field inhomogeneity in the inductive plasma sources may be the powerful source of the instabilities development. It was derived that the accelerated motion of electrons relative to ions under the action of the ponderomotive force, formed in the skin layer of the inductively coupled plasma, is the much more
stronger source of the instabilities development than the quiver motion of the electron in the electromagnetic field. The spatial structure of the helicon wave in the helicon sources depends on the antenna design, on the magnitude of the confined magnetic field, on the input RF power and RF frequency, on the magnitude and radial profile of the electron density, etc. The focus of this paper is the two-scale kinetic theory of the microscale instabilities of the helicon plasma driven by the macroscale plasma flow, formed by the radially inhomogeneous helicon wave in the radially inhomogeneous plasma. In Sec. III using the Vlasov-Poisson model we develop the theory of the stability of the cylindrical plasma in the field of the azimuthally symmetric \((m = 0)\) radially inhomogeneous helicon wave. In high frequency helicon wave field, electrons experience the oscillating motion relative to the practically unmovable ions. This motion is known as a source of the development of the parametric instabilities investigated usually employing the approximation of the uniform helicon wave. The developed in this paper theory reveals new macroscale effect of the azimuthal steady rotation of electrons with radially inhomogeneous angular velocity. This effect was detected experimentally in the helicon plasma source, that uses azimuthally symmetric antenna, long time ago, but was not explained yet. We found that this rotation is caused by the radial inhomogeneity of the cylindrical helicon wave and is spatially localized in the region of strong gradients in the helicon wave. The main result of this section is the derived basic integral equation for the electrostatic potential, which governs the microinstabilities of the radially inhomogeneous cylindrical plasma driven by the azimuthally symmetric inhomogeneous helicon wave. The solution of this integral equation for the short scale high frequency ion acoustic instability is presented in Sec. III. The Conclusions are given in Sec. IV.

II. BASIC EQUATIONS

We consider an axially symmetric plasma in a uniform axial magnetic field \(B_0\), directed along \(z\) axes, and in the electric \(E_1 = E_{1r}(r, z, t) + E_{1\varphi}(r, z, t)\), and magnetic \(B_{1z}(r, z, t)\) fields of the azimuthally symmetric helicon wave, excited by the loop antenna located on the boundary \(r = r_0\) of the cylindrical chamber. The helicon wave field is given by the relations

\[
E_{1r}(r, z, t) = E_{1r}(r) \sin (k_0 z - \omega_0 t), \\
E_{1\varphi}(r, z, t) = E_{1\varphi}(r) \cos (k_0 z - \omega_0 t), \\
B_{1z}(r, z, t) = B_{1z}(r) \sin (k_0 z - \omega_0 t),
\]

where \(\omega_0\) is the frequency of the helicon wave, \(k_0\) is the wavenumber component along magnetic field \(B_0\). In such a fields, a plasma in equilibrium has an azimuthally symmetric radially inhomogeneous density profile. The radial profiles of the electric and magnetic fields depends on the plasma density profiles and differ from the fields

\[
E_{1r}(r) = E_{1r} \tilde{J}_1(k_{0\perp} r), \\
E_{1\varphi}(r) = E_{1\varphi} \tilde{J}_1(k_{0\perp} r), \\
B_{1z}(r) = B_{1z} \tilde{J}_0(k_{0\perp} r),
\]

where \(k_{0\perp} = \omega_0^2 \omega_{ce} / (\omega_{ce} k_0 c^2)\), \(J_{0,1}(k_{0\perp} r)\) are the Bessel functions, known for the helicon plasmas with a uniform density. In the cylindrical coordinates \(r, \varphi, z\) for the electron position and \(v_{\perp}, \phi, v_z\) for the electron velocity, the Vlasov equation for electrons has a form

\[
\frac{\partial F_e}{\partial t} + v_{\perp} \cos \phi \frac{\partial F_e}{\partial r} + \frac{v_{\perp}}{r} \sin \phi \frac{\partial F_e}{\partial \varphi} + v_z \frac{\partial F_e}{\partial z} - \frac{e}{m_e} \left( \sin \phi E_{\varphi} + \cos \phi E_r \right) \frac{\partial F_e}{\partial v_{\perp}} - \left[ \omega_{ce} + \frac{v_{\perp}}{r} \sin \phi \right] \frac{m_e v_{\perp}}{e} + \frac{e B_{1z}(r, z, t)}{m_e c} \right] \\
\sin (\phi E_r - \cos \phi E_{\varphi}) \frac{\partial F_e}{\partial \phi} + \frac{e}{m_e} E_r \frac{\partial F_e}{\partial v_z} = 0,
\]

where \(\omega_{ce} = e B_0 / m_e c\) is the electron cyclotron frequency. In this equation, electric field \(E(r, \varphi, z, t) = E_r(r, \varphi, z, t) + E_{\varphi}(r, \varphi, z, t) + E_z(r, \varphi, z, t)\) is

\[
E (r, \varphi, z, t) = E_1 (r, z, t) + \tilde{E} (r, \varphi, z, t),
\]

where

\[
\tilde{E}(r, \varphi, z, t) = -\nabla \Phi (r, \varphi, z, t)
\]

is the electrostatic electric field of the plasma response on the helicon wave. This self consistent electric field is determined by the Poisson equation,

\[
- \Delta \Phi (r, \varphi, z, t) = 4\pi \sum_{\alpha = e, i} e_{\alpha} \int f_\alpha (v, r, \varphi, z, t) dv,
\]

in which \(f_\alpha\) is the fluctuating part of the distribution function \(F_\alpha\), \(f_\alpha = F_\alpha - F_{\alpha0}\), where \(F_{\alpha0}\) is the equilibrium distribution function. The Vlasov equation (3) for electron and ion components, and the Poisson equation (6) compose the basic system of equations of our studies.

III. THE SOLUTION OF THE VLASOV EQUATION FOR THE CYLINDRICAL PLASMA IN THE FIELD OF THE AZIMUTHALLY SYMMETRIC HELICON WAVE

The Vlasov equation for the electron distribution function \(F_e (v_{\perp}, \phi, v_z, r, \varphi, z, t)\) of the helicon sources contains two different spatial scales: the macroscale of the radial inhomogeneity of the helicon wave, which is commensurable with radial scale of the plasma density inhomogeneity, and the microscale commensurable with the thermal Larmor radius of electrons. In this section, we derive the two-scale solution of Eq. (3) by the transformation of...
the \( r, \varphi, v_L, \phi \) coordinates to the cylindrical guiding center coordinates \( R_e, \psi, \rho_e, \delta \) determined by the relations:

\[
R_e^2 = \frac{1}{\omega_{ce}^2} \left( v_{L1}^2 + 2v_{L1}r_{ce} \sin \phi + r_{ce}^2 \right), \quad E_{1\varphi} = \arcsin \frac{v_1}{\omega_{ce}^2} \left( 1 + v_{L1}^2 (v_{L1}^2 + 2v_{L1}r_{ce} \sin \phi) \right)^{1/2} \, .
\]

(7)

The geometric interpretation of the cylindrical guiding center coordinates for an electron is presented in Fig. 1. In coordinates \( R_e, \psi, \rho_e, \delta, z, t \), Eq. (3) transforms to the following equation for \( F_e (R_e, \psi, \rho_e, \delta, z, t) \):

\[
\frac{\partial F_e}{\partial t} + v_z \frac{\partial F_e}{\partial z} + \frac{c}{B_0} \left( E_{1\varphi} - \omega_{ce} \right) \frac{\partial F_e}{\partial \phi} + \frac{c}{B_0} E_{1\varphi} \cos \delta + E_{1\varphi} \sin \delta \frac{\partial F_e}{\partial \rho_e} - \frac{c}{B_0} \frac{1}{\omega_{ce}} \left( E_{1\varphi} \sin \delta - E_{1\varphi} \cos \delta \right) \frac{\partial F_e}{\partial \psi} + \frac{c}{B_0} \frac{1}{\omega_{ce}} \frac{\partial \Phi}{\partial \psi} - \frac{c}{B_0} \frac{1}{\omega_{ce}} \frac{\partial \Phi}{\partial \psi} - \frac{c}{B_0} \frac{1}{\omega_{ce}} \frac{\partial \Phi}{\partial \rho_e} - \frac{c}{B_0} \frac{1}{\omega_{ce}} \frac{\partial \Phi}{\partial \rho_e} - \frac{c}{B_0} \frac{1}{\omega_{ce}} \frac{\partial \Phi}{\partial \rho_e} = 0.
\]

(12)

The Vlasov equation (12) with \( \Phi = 0 \) is the equation for the equilibrium electron distribution function \( F_{e0} \). Consider now the system of equations for the characteristics of equation for \( F_{e0} \):

\[
dt = \frac{dR_e}{B_0 E_{1\varphi}} = \frac{d\psi}{E_{1\varphi}} (E_{1\varphi} \sin \delta - E_{1\varphi} \cos \delta) + \frac{d\rho_e}{E_{1\varphi} (E_{1\varphi} \sin \delta - E_{1\varphi} \cos \delta)} = \frac{dz}{v_z}. \]

(13)

With approximations \( E_{1\varphi} (r) \approx E_{1r} (R_e) \) and \( E_{1\varphi} (r) \approx E_{1\varphi} (R_e) \), which follows from the relation

\[
r = \left( R_e^2 - 2\rho_e R_e \sin \delta + \rho_e^2 \right)^{1/2} \approx R_e \]

(14)

in the limit \( \rho_e \ll R_e \), the system of equations for the guiding center coordinates \( R_e, \psi \):

\[
dt = \frac{dR_e}{E_{1\varphi} (R_e) \cos (\omega_0 t - k_{0z} z_1 + v_z t)} = \frac{d\rho_e}{E_{1\varphi} (R_e) \sin (\omega_0 t - k_{0z} z_1 + v_z t)},
\]

(15)

where \( z_1 = z - v_z t \) is the integral of system (13), becomes separate from the system of equation for the coordinates \( \rho_e \) and \( \delta \) of the Larmor motion.

Now we consider the approximate solution to the nonlinear equation for \( R_e \):

\[
\frac{dR_e}{dt} = E_{1\varphi} (R_e) \cos (\omega_0 t - k_{0z} z_1),
\]

(16)

in which the term \( k_{0z} v_z t \) is omitted, because for the helicon wave \( \omega_0 \gg k_{0z} v_e t \), where \( v_e \) is the electron thermal velocity. That solution is derived in the form

\[
R_e (t) \approx R_{e1} + \frac{c}{B_0 \omega_0} E_{1\varphi} (R_{e1}) \sin (\omega_0 t - k_{0z} z_1)
\]

(17)

which is the power series expansion in \( |\xi / R_{e1}| \ll 1 \), where \( \xi = \frac{c}{B_0 \omega_0} E_{1\varphi} (R_{e1}) \) is the amplitude of the displacement of an electron along the coordinate \( R_e \) and \( R_{e1} \) is the integral of Eq. (10).

The approximate solution to the equation for the angle \( \psi \):

\[
\frac{d\psi}{dt} = \frac{c}{B_0 R_e} E_{1r} (R_e) \sin (\omega_0 t - k_{0z} z_1)
\]

(18)
It follows from (19) that the secular growth of \( \psi \) becomes dominant at time \( t \), with the first and second order of \( |\xi|/R_{e1} |^2 \) has a form

\[
\psi \approx \psi_1 - \frac{c}{B_0 R_{e1} \omega_0} E_{1r}(R_{e1}) \cos (\omega_0 t - k_{z0} z_1)
- \frac{c^2}{2 B_0^2 R_{e1}^2 \omega_0} E_{1\varphi}(R_{e1}) \left( E_{1r}(R_{e1}) - R_{e1} \frac{\partial E_{1r}(R_{e1})}{\partial R_{e1}} \right)
\times \left[ t - \frac{1}{2 \omega_0} \sin 2 (\omega_0 t - k_{z0} z_1) \right],
\]

(19)

It contains the terms oscillating on frequencies \( \omega_0 \) and \( 2 \omega_0 \), and the term corresponding to the rotation of the guiding center coordinate with stationary radially inhomogeneous angular velocity \( \Omega_c (R_{e1}) \),

\[
\Omega_c (R_{e1}) = \frac{c^2 E_{1\varphi}(R_{e1})}{2 B_0^2 R_{e1}^2 \omega_0}
\times \left[ E_{1r}(R_{e1}) - R_{e1} \frac{\partial E_{1r}(R_{e1})}{\partial R_{e1}} \right].
\]

(20)

It follows from (19) that the secular growth of \( \psi \) becomes dominant at time \( t \), at which

\[
\omega_0 t > \frac{2 B_0 R_{e1} \omega_0}{c E_{1\varphi}} > 1.
\]

(21)

Equation (21) determines the condition under which the theory of the parametric instabilities, based on the approximation of the uniform helicon wave, becomes worse. In what follows, we assume that condition (21) is valid for the selected plasma and helicon wave parameters and the sources of the instabilities development will be different from the oscillating motion of electrons relative to ions. For time \( t \), for which condition (21) is valid, the approximation

\[
\psi = \psi_1 - \Omega_c (R_{e1}) t
\]

(22)

may be used. For \( B_0 = 100 G, \omega_0 = 10^7 c^{-1}, R_{e1} = 2.5 \) cm, \( E_{1\varphi} = 5 V/cm \), condition (21) is satisfied for \( \omega_0 t > 10, i.e. at time t > 10^{-6} c. \)

Because \( \omega_{ce} \) is much larger than any other term in the equation for \( d\delta/dt \) of system (13), the solution for \( \delta_1 \) with a great accuracy is

\[
\delta = \delta_1 - \omega_{ce} t.
\]

(23)

The solution of the equation for the radius \( \rho_e \) of the electron Larmor orbit,

\[
\frac{d\rho_e}{dt} = \frac{c}{B_0} \left[ E_{1r}(R_e) \sin (k_{z0} z_1 - \omega_0 t) \cos (\delta_1 - \omega_{ce} t)
+ E_{1\varphi}(R_e) \cos (\omega_0 t - k_{z0} z_1) \sin (\delta_1 - \omega_{ce} t) \right],
\]

(24)

where \( \delta_1 \) is given by Eq. (23) is

\[
\rho_e = \rho_{e1} - \frac{c}{B_0 \omega_{ce}} \left[ E_{1r}(R_{e1}) \sin (\omega_0 t - k_{z0} z_1)
\times \sin (\omega_{ce} t - \delta_1)
- E_{1\varphi}(R_{e1}) \cos (\omega_0 t - k_{z0} z_1) \cos (\omega_{ce} t - \delta_1) \right].
\]

(25)

with accuracy to terms of the order of \( O \left( \frac{\omega_0}{\omega_{ce}} \approx 1 \right). \) It is easy to check that the Vlasov equation for the equilibrium electron distribution function \( F_{e0} \) in variables \( R_{e1}, \rho_e, \psi_1, \rho_{e1}, \delta_1, v_z, z_1 \) determined by the solutions (17), (19), (24), (25), reduces to the equation

\[
\frac{\partial F_{e0}}{\partial t} = 0,
\]

and, therefore, \( F_{e0} = F_{e0} (R_{e1}, \psi_1, \rho_{e1}, \delta_1, v_z, z_1) \), and does not depend on time variable.

The equation for the perturbation \( f_{e} (R_{e1}, \psi_1, \rho_{e1}, \delta_1, v_z, z_1, t) \) becomes

\[
\frac{\partial}{\partial t} f_{e} (R_{e1}, \psi_1, \rho_{e1}, \delta_1, v_z, z_1, t)
= \frac{c}{B_0 R_{e1}} \left( \frac{\partial \Phi}{\partial \psi} - \frac{\partial \Phi}{\partial \psi_1} \right) \frac{\partial F_{e0}}{\partial R_{e1} (R_{e1}, \rho_{e1}, v_z, z_1)}
\times \left( \frac{\partial}{\partial \rho_{e1}} + \frac{e}{m_e} \frac{\partial \Phi}{\partial v_{ez}} \right).
\]

(26)

In this equation, potential \( \Phi (R_{e1}, \psi_1, \rho_{e1}, \delta_1, v_z, z_1, t) \) is presented in the form of the Fourier-Bessel transformation,

\[
\Phi (R_{e1}, \psi_1, \rho_{e1}, \delta_1, v_z, z_1, t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int dk_{\perp} dk_z d\theta d\omega \Phi (k_{\perp}, \theta, k_z, \omega)
\times J_n(k_z \rho_{e1}) J_{n+m} (k_{z0} R_{e1})
\times \exp \left[ -i n (\delta_1 - \omega_{ce} t) - i m (\theta - \psi_1 + \Omega_c (R_{e1}) t) \right]
+i (m + n) \frac{\pi}{2} - i (\omega - k_z v_z) t + ik_z z_1,
\]

(27)
It was derived from the Fourier-Bessel transform

\[
\Phi(r, \varphi, z, t) = \int \Phi(k, \omega) e^{-i\omega t} d\omega \\
\times e^{ik_{L}r \cos(\theta-\varphi)} k_{L}dk_{L}d\theta e^{ik_{z}z}dk_{z},
\] (28)

in which the identity

\[
k_{L}r \cos(\theta-\varphi) = k_{L}R_{e} \cos(\theta-\psi) + k_{L}e \sin(\theta-\psi-\delta)
\] (29)

and Eqs. (22) and (23) were employed.

IV. THE ION ACOUSTIC INSTABILITY OF THE CYLINDRICAL PLASMA IN THE FIELD OF THE AZIMUTHALLY SYMMETRIC HELICON WAVE

By using the solution for \(f_{e}\) of Eq. (23) in the Poisson equation (6), we derive the Fourier transformation of the Poisson equation,

\[
k^{2}\Phi(k, \omega) = 4\pi e_{i}n_{i}(k, \omega) + 4\pi e_{n}e(k, \omega),
\] (30)

where

\[
n_{e}(k, \omega) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-i\omega t} n_{em}(k, z, \omega)
\]

\[
= -\omega_{ce}^{2} e_{me}^{2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int dR_{e1} R_{e1} \int dv_{z}
\times \int d\rho_{e1}\rho_{e1} \int dk_{1}k_{1}\Phi_{m}(k_{1}, z, \omega) J_{n}(k_{1}\rho_{e1})
\times J_{n}(k_{1}\rho_{e1}) J_{n+m}(k_{1}R_{e1}) J_{n+m}(k_{1}R_{e1})
\times \left[\frac{\omega - n\omega_{ce} + m\Omega_{e}(R_{e1}) - k_{e}v_{z}}{\omega_{ce}R_{e1}} \frac{1}{\rho_{e1}} \frac{\partial F_{e0}}{\partial \rho_{e1}} + k_{z} \frac{\partial F_{e0}}{\partial v_{z}}\right].
\] (31)

In Eq. (31), the \(m\)-th Fourier harmonic \(\Phi_{m}(k_{1}, z, \omega)\) of the potential \(\Phi\) is determined by the relation

\[
\Phi_{m}(k_{1}, z, \omega) = \frac{1}{2\pi} \int d\theta_{1} \Phi(k_{1}, \theta_{1}, z, \omega) e^{im\theta_{1}}.(32)
\]

The equation for \(\Phi_{m}(k_{1}, z, \omega)\) follows from (30)-(32),

\[
\Phi_{m}(k_{1}, z, \omega) \left(1 - \frac{\omega_{pe}^{2}}{\omega^{2}}\right)
+ 8\pi^{2} e_{me}^{2} \omega_{ce}^{2} \sum_{n=-\infty}^{\infty} \int dR_{e1} R_{e1} \int dv_{z}
\times \int d\rho_{e1}\rho_{e1} \int dk_{1}k_{1}\Phi_{m}(k_{1}, z, \omega) J_{n}(k_{1}\rho_{e1})
\times J_{n}(k_{1}\rho_{e1}) J_{n+m}(k_{1}R_{e1}) J_{n+m}(k_{1}R_{e1})
\times \left[\frac{\omega - n\omega_{ce} + m\Omega_{e}(R_{e1}) - k_{e}v_{z}}{\omega_{ce}R_{e1}} \frac{1}{\rho_{e1}} \frac{\partial F_{e0}}{\partial \rho_{e1}} + k_{z} \frac{\partial F_{e0}}{\partial v_{z}}\right] = 0.(33)
\]

In Eq. (33), the approximation of the unmoving ions in the helicon wave was used, which is applicable for the treating of the instabilities with the growth rate \(\gamma(k) \gg \omega_{ce}/2\pi\) and \(k_{e}\rho_{e} \gg 1\). Here, we derive the solution to Eq. (33) in the short wavelength limit

\[
k_{L}R_{e1} \sim m \gg 1,
\] (34)

analysed in Ref.12 in the studies of the drift turbulence of the azimuthally symmetric radially nonuniform plasma and in Ref.13 in the studies of the shear flow driven ion cyclotron and ion acoustic instabilities of the cylindrical inhomogeneous plasma. It follows from Refs.17,18 that the solution to Eq. (33) under condition (34), is

\[
\Phi_{m}(k_{L}, z, \omega) = \Phi_{m}(k_{L}, z, \omega_{m}(k_{L}, z))
\] (35)

for \(\omega = \omega_{m}(k_{L}, z)\) and \(\Phi_{m}(k_{L}, z, \omega) = 0\) for \(\omega \neq \omega_{m}(k_{L}, z)\), where \(\omega_{m}(k_{L}, z)\) is the solution to the equation

\[
\varepsilon_{m}(k_{L}, z, \omega) = 1 - \frac{\omega^{2}}{\omega_{pe}^{2}}
+ \frac{8\pi^{2} e_{me}^{2} \omega_{ce}^{2}}{k^{2}m_{e}} \sum_{n=-\infty}^{\infty} \int d\rho_{e1}\rho_{e1} \int dv_{z}
\times \left[\frac{J_{2}(k_{1}\rho_{e1})}{(\omega - n\omega_{ce} + m\Omega_{e}(R_{e1}) - k_{e}v_{z}) \omega_{ce}R_{e1}} \frac{1}{\rho_{e1}} \frac{\partial F_{e0}}{\partial \rho_{e1}}
+ \frac{n}{\omega_{ce}} \frac{1}{\rho_{e1}} \frac{\partial F_{e0}}{\partial \rho_{e1}} + k_{z} \frac{\partial F_{e0}}{\partial v_{z}}\right] \frac{R_{e1} = R_{e0} = m_{e}^{1/2}}{R_{1}} = 0.
\] (36)

The cylindrical plasma excited by the cylindrically symmetric helicon wave has an azimuthally symmetric radially inhomogeneous density profile. For the Maxwellian distribution \(F_{e0}(\rho_{e}, v_{z}, R_{e0})\),

\[
F_{e0}(\rho_{e}, v_{z}, R_{e0}) = \frac{n_{e0}(R_{e0})}{(2\pi)^{3/2} v_{T e}^{3}}
\times \frac{1}{\rho_{T e}^{2} - v_{z}^{2}} \frac{1}{v_{T e}^{3}},
\] (37)

where \(\rho_{T e} = v_{T e}(R_{e0})/\omega_{ce}\), and \(v_{T e}(R_{e0}) = T_{e}(R_{e0})/m_{e}\), we derive from Eq. (36) the following dispersion equation for the low frequency perturbations with \(\omega \ll \omega_{ce}\):

\[
\varepsilon_{m}(k_{L}, z, \omega) = 1 - \frac{\omega^{2}}{\omega_{pe}^{2}} + \frac{1}{k^{2}\lambda_{D e}^{2}}
\times \left[1 + i \frac{\pi}{2} \frac{\omega - m\Omega_{e}(R_{e0})}{k_{z}v_{T e}} W(z e) A_{e0}\right] = 0.
\] (38)

In Eq. (38), \(W(z e) = e^{-z_{e}^{2}} (1 + (2i/\sqrt{\pi}) \int e^{t^{2}} dt)\) is the Faddeeva function with argument \(z_{e} = (\omega - m\Omega_{e}(R_{e0}))/\sqrt{2k_{z}v_{T e}}\), \(A_{e0} = I_{0}(k_{L}^{2}r_{e}^{2}) e^{-k_{z}^{2}v_{T e}^{2}}\), and \(I_{0}\) is the modified Bessel function of the first kind and order 0. The frequency \(\Omega_{e}(R_{e0})\) is determined as

\[
\Omega_{e}(R_{e0}) = \Omega_{e}(R_{e0}) + \omega_{de}.
\] (39)
This frequency reveals the coupled effect of the plasma inhomogeneity, determined by the local electron diamagnetic drift frequency $\omega_{de}^{*}(R_{e0})$, 

$$\omega_{de}^{*}(R_{e0}) = \omega_{de}^2 \frac{\partial \ln n_{e0}(R_{e0})}{R_{e0}\partial R_{e0}} \left(1 - \frac{1}{2} \frac{\eta_{e}}{n_{e0}}\right), \quad (40)$$

where $\eta_{e} = \partial \ln T_e/\partial \ln n_{e0}$, of the radially inhomogeneous plasma with a cylindrical geometry, and the effect of the spatial inhomogeneity of the helicon wave field, determined by the frequency $\Omega_{ce}(R_{e0})$. It follows from Eqs. (20) and (40), that

$$\frac{\omega_{de}^{*}}{\Omega_{ce}(R_{e0})} \sim \frac{v_{Tc}^{2} \omega_{0}}{\omega_{ce}^{2} \omega_{0}}. \quad (41)$$

where

$$v_{Tc}^{2} = \frac{e^{2} E_{1e} E_{1v}}{B_{0}^{2}}. \quad (42)$$

Equation (41) reveals that the effect of the plasma inhomogeneity is dominant in $\Omega_{ce}(R_{e0})$ when

$$v_{Tc} > \tilde{v}_{e} \left[\frac{\omega_{0}}{\omega_{ce}}\right]^{1/2}. \quad (43)$$

The opposite case of $|\Omega_{ce}(R_{e0})| > |\omega_{de}^{*}|$ occurs when

$$\tilde{v}_{e} > v_{Tc} \left[\frac{\omega_{ce}}{\omega_{0}}\right]^{1/2}, \quad (44)$$
i.e. in the case of strong RF input power and a weak magnetic field.

The solution to Eq. (38) for the adiabatic electrons ($|z_e| < 1$) is $\omega(k) = \omega_s + \delta \omega(k)$, where $\omega_s(k)$ is the frequency of the ion acoustic wave, $\omega_{de}^{*}(k) = k^{2} v_{s}^{2} \left(1 + k^{2} \lambda_{De}^{2}\right)^{-1}$, $v_{s} = (T_{e}/m_{i})^{1/2}$ is the ion acoustic velocity, and $\delta \omega(k)$ with an accuracy to terms on the order of $(\delta \omega(k)/\omega_{s})^{2} \ll 1$ is

$$\delta \omega(k) = -i \frac{\sqrt{\pi}}{2} \omega_{s} z_{e0} \left(1 + k^{2} \lambda_{De}^{2}\right) W(z_{e0}) A_{e0} \quad (45)$$

with $z_{e0} = \left[\omega_{s}(k) - m_{i} \tilde{\Omega}(R_{e0})\right]/\sqrt{2} k_{z} v_{Tc}$, where $|z_{e0}| < 1$ when $k_{z}/k > \sqrt{m_{e}/m_{i}}$. The ion acoustic instability develops when

$$m_{i} \tilde{\Omega}(R_{e0}) > k v_{s}, \quad (46)$$

with the growth rate $\gamma_{s}(k) = \text{Im} \delta \omega(k)$ equal to

$$\gamma_{s}(k) = -\text{Im} \delta \omega(k) \approx \frac{\sqrt{\pi}}{2} \frac{\omega_{s}(k) z_{e0}}{\left(1 + k^{2} \lambda_{De}^{2}\right)} e^{-z_{e0}} A_{e0}. \quad (47)$$

Note, that because $R_{e0} = m_{i}/k_{z}$, condition (46) may be presented in the form $R_{e0} \tilde{\Omega}(R_{e0}) > v_{s}$ i.e. the “azimuthal electron current velocity” should be larger than the ion acoustic velocity. The maximum growth rate (47) attains for $z_{e0} = -1/\sqrt{2}$ and for $k_{z} \rho_{e} \gg 1$ it is equal to

$$\gamma_{max}(k) \approx 0.054 \frac{(\omega_{ce} \omega_{i0})^{1/2}}{(1 + k^{2} \lambda_{De}^{2})^{3/2}}. \quad (48)$$

It follows from Eqs. (45), (46), and (47) that by the transformations

$$\frac{m}{R_{e0}} \rightarrow k_{y}, \quad m \omega_{de}^{*} \rightarrow k_{y} v_{de}, \quad m \tilde{\Omega} \rightarrow k_{y} V_{0 \perp}, \quad (49)$$

Eq. (45) and its solutions (46), (47) becomes equal to the local dispersion equation for the slab model of the inhomogeneous plasma with electron current flowing perpendicularly to a magnetic field and to its solution for the ion acoustic current driven instability, respectively[20]. By using this similarity of the considered microscale ion acoustic instability in the cylindrical and in the slab plasma geometries, we can employ the estimates for the energy density $W_{E} = (4\pi)^{-1} \int dk k^{2} \phi^{2}(k)$ of the electric field

$$W_{E} \sim 5 \cdot 10^{-4} \frac{\omega_{ce} T_{e}}{\omega_{pe} T_{i}}, \quad (k\lambda_{De} \sim 1, \gamma = \gamma_{max}) \quad (50)$$

at the saturation stage of the instability, resulted from the induced scattering of the ion acoustic wave by the unmagnetized ions[21]. The interaction of the magnetised electrons with ion acoustic turbulence under condition of the Cherenkov resonance results in the growth of the electron temperature $T_{e\parallel}$ along the magnetic field determined by the equation

$$n_{e} \frac{dT_{e\parallel}}{dt} \sim \frac{R_{e0} \tilde{\Omega}(R_{e0})}{v_{s}} \left[\omega_{ei} \omega_{ce}\right]^{1/2} W_{E} \sim \nu_{eff} \frac{k_{z}^{2} \omega_{pe}}{\omega_{ce}} W_{0}(R_{e0}), \quad (51)$$

where $W_{E}$ is determined by Eq. (50), $W_{0}(R_{e0})$ is the energy density of the helicon wave at radius $R_{e0}$ and $\nu_{eff}$ is the effective collision frequency of the electrons with electric field of the ion acoustic turbulence.

V. CONCLUSIONS

In this paper, we present the theory of the microin-stabilities of the cylindrical plasma excited by the cylindrically symmetric helicon wave with accounting for the cylindrical geometry and the radial inhomogeneities of the helicon wave and of a plasma. This theory reveals new macroscopic effect of the azimuthal steady rotation of electrons with a radially inhomogeneous angular velocity, caused by the radial inhomogeneity of the helicon wave. It is found, that this effect is responsible for the development of the ion acoustic instability driven by the azimuthal electron current and plasma inhomogeneity at radius $R_{e0}$ where condition (48) holds. In the vicinity of $R_{e0}$ this effect dominates over the effect of the oscillating motion of the electrons relative to ions, which is...
basic in the theory of the parametric instabilities studied in Refs. 6, 7 with approximation of the spatially uniform helicon pumping wave.

ACKNOWLEDGMENTS

This work was supported by National R&D Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (Grant No. NRF-2017R1A2B2011106) and BK21 FOUR, the Creative Human Resource Education and Research Programs for ICT Convergence in the 4th Industrial Revolution, and by the National Research Foundation of Ukraine (Grant No.2020.02/0294).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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