Numerical Prediction of Sand Erosion in Elbows

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Abstract. In the present article, turbulent flow of crude oil and sand particles in piping system with a 90° elbow is analysed using computational fluid dynamics simulations. The flow of the crude oil is modelled as continuous fluid which is governed by the continuity and Reynolds-averaged Navier–Stokes equations. The Discrete phase model is used for sand particle flow. The governing parameters considered in the present study are: the inlet velocity, spherical sand particle size and sand mass flow rate. The maximum erosion rate is found on the outer surface of the elbow where the sand and fluid impinging the surface before changing the flow direction. It is found that the erosion rate is increasing with the increase of the inlet velocity of the crude oil. The numerical results show that the large sand particles with constant sand flow rate causing less erosion rate than those of smaller diameter. The results show also that the increase of the sand mass flow rate leads to the increase of the erosion rate.

1. Introduction

In the oil and gas industry sand is a source of several flow assurance problems. If sand erosion is not monitored, predicted and controlled properly, entire production processes can be impeded and even shut down for long periods of time. Sand erosion is a mechanical process where it is occurring when a sand particle repeatedly impinges on the inner surface of the pipe system gradually removed its material. In order to protect the pipeline and equipment from potential hazards or to design the pipeline and equipment, it is necessary to understand the wearing of piping system caused by sand impingements. The estimation of sand erosion in a multiphase flow is a complicated process, depends on many parameters, such as: sand properties and particle tracks, fluid flow, surface conditioning and multi-phase effects among others.

Calculating impact speed is one of the challenging parts of erosion prediction. Low particle velocity has low particle kinetic energy. Hence, low impact to the pipe wall create minor erosion. Higher particle velocity has higher kinetic energy resulting in higher material erosion rate of the pipes. Okonkwo et al. [1] studied various combinations of impact angles and particle speeds and demonstrated that the predominant erosion mechanism happening under lower impact angles and higher speeds.

The literature review shows a vast use of computational fluid dynamics (CFD) techniques in predicting sand erosion in the oil and gas applications. CFD modeling can predict the detailed information on the location and the rate of erosion. The results of the CFD modeling have the potential to optimize the design prior to fabrication and testing. The literature shows many erosion models using either single phase or multi-phase modeling. The simple and easy to employ sand erosion model is using the Discrete Phase Model (DPM), which applicable for sand volume fractions less than 10 % [2].
Zhang et al. [3] used CFD methodology to study the erosion wear of fracturing pipeline under the action of multiphase flow in oil & gas industry and found that the significant factors influencing the erosion rate is the flow velocity of fluid and the impact angle of the solid particles. Karimi et al. [4] utilized CFD to calculate erosion caused by fine particles in different geometries. The geometries include submerged jet impingement and elbows. Simulation results are compared with experimental data available in the literature for elbows and conducted during this investigation for fine particles submerged in an impinging slurry jet. Droubi et al. [5] analysed numerically the sand erosion in piping system with 90° bend. The Erosion rate was found to decrease with the increase of either the pipe diameter or the bend radii. The capability of CFD for multiphase flow simulation using Volume of Fluid (VOF) method has been tested by Zahedi et al. [6] for air-water flow with emphasis on erosion prediction. They concluded that a larger experimental database is necessary to study the behaviour and relation between superficial gas and liquid velocities with phase velocities in a bend and their effects on particle impact speeds and angles. Huey and Saeid [7] considered numerical simulations to study the performance analysis and sand erosion in a centrifugal pump. The CFD simulation results show that the erosion rate increases with increasing rotational speed and particle size. Erosion mostly occurred at the leading edge of the blade near the shroud of the impeller at different particle diameters and impeller rotational speed. Recently, Saeid [8] considered the discrete phase model to numerically predict the sand erosion in a check valve used in oil production system. The literature review show that most of the CFD-based erosion models focus on the pipe bend with gas–solid flow or water–sand flows. Therefore, the aims of the current study are to simulate the crude oil flow in a pipe elbow. The study will include identifying the governing parameters effecting the sand erosion in the piping system in order to minimize the damage in the system. The piping system used in the present investigations is shown in Figure 1. The diameter of the piping system is \( D = 100 \) mm and the bend radius \( R = 1.5D \). The inlet pipe length is extended to be 30\( D \) in order to eliminate the effects of the boundary layers entrance region and ensure the fully developed flow at the elbow. Similarly, the outlet pipe is extended to be 15\( D \) to eliminate the effects of the reversed flow to the solution domain.

![Figure 1. Flow geometry (Not to scale).](image)

2. Mathematical Model
The crude oil is treated as continuous phase and the governing equations are based on the mass and momentum balance. The sand particles are treated as the dispersed phase, which can be modelled using the Discrete Phase Model (DPM). The flow of the continuous phase (crude oil) is selected in the turbulent regime. The continuity and the Reynolds-averaged Navier-Stokes (RANS) equations are:

\[
\frac{\partial}{\partial x_i} (\rho u_i) = 0 \tag{1}
\]

\[
\frac{\partial}{\partial x_j} (\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i} \left\{ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_l}{\partial x_l} \right) \right\} + \frac{\partial}{\partial x_j} (\rho \bar{u_i} \bar{u_j}) + S \tag{2}
\]
where \(u_i\) and \(\bar{u}_i\) are the average and fluctuating components of the velocity vector and the overbar represents the average value of the fluid velocity. \(S\) is the momentum source term. The Boussinesq approximation is used to relate the Reynolds stresses to the average velocity gradients as:

\[
-\rho \bar{u}_i \bar{u}_j = \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \left( \rho k + \mu_t \frac{\partial u_k}{\partial x_k} \right) \delta_{ij}
\]  

(3)

where the eddy viscosity \(\mu_t\) calculation depends on the turbulence model and \(k\) is the turbulent kinetic energy of the flow. In the present study, two turbulence models are used. They are the realizable \(k - \varepsilon\) with scalable wall functions and the SST \(k - \omega\) with near surface wall corrections model [2].

The discrete phase model is employed, in which the particle trajectories are calculated through the solution domain. The calculation of trajectories is done by solving the force balance for a single particle using Newton’s second law as follows [2]:

\[
\frac{du_{p_i}}{dt} = F_D(u_i - u_{p_i}) + \frac{g(\rho_p - \rho)}{\rho_p} + F_i
\]  

(4)

where \(u_{p_i}\) is the particle velocity vector in the \(i\) direction and \(\rho_p\) is the density of the particle. The buoyancy force term is neglected in the present study as the sand particles and crude oil have insignificant difference in densities. \(F_i\) is an additional acceleration of the fluid surrounding the particle, and \(F_D\) is the drag force/ particle mass and \(F_i\) is the force needed to accelerate/decelerate the fluid around the particle. These terms are defined by [2]:

\[
F_D = \frac{18\mu}{\rho_p d_p^2} \frac{C_D Re_d}{24} \quad \text{and} \quad F_i = \frac{1}{2} \frac{\rho}{\rho_p} \frac{d(u_i - u_{p_i})}{dt}
\]  

(5)

where \(\mu\) is the dynamic viscosity of the continuous phase (fluid), \(dp\) is the diameter of the particle, and \(Re_d\) is the Reynolds number, which is given by: \(Re_d = \rho d_p |u_{p_i} - u_i| / \mu\). The drag coefficient \(C_D\) in equation (5) for smooth spherical particles applicable for wide ranges of \(Re\) given by [2]:

\[
C_D = a_1 + \frac{a_2}{Re_d} + \frac{a_3}{Re_d^2}
\]  

(6)

The default values of the model constants \((a_1, a_2\) and \(a_3\)) are used in the present investigations [2]. The rate of erosion is defined as the velocity of the mass removal of material from a surface per unit area (kg/m²s). The aim of the current study is to monitor the particle erosion rate at wall boundaries, which is given in the following correlation [2]:

\[
R_{erosion} = \sum_{p=1}^{N_p} \frac{\dot{m}_p C(d_p) f(\alpha) \nu^{b(\nu)}}{A_{face}}
\]  

(7)

where \(\dot{m}_p\) = mass flow rate of a particle (kg/s), \(C(d_p)\) = particle diameter function, \(f(\alpha)\) = impact angle function, \(\nu =\) relative solid particle velocity, \(b(\nu) =\) function of particle velocity and \(A_{face}\) = area of the mesh face at the surface. The default values of \(C(d_p) = 1.8 \times 10^{-9}, f(\alpha) = 1,\) and \(b(\nu) = 0\) are used in the present study.

3. Numerical Solution Method

The geometry was created in ANSYS 18.1 [2]. The geometry is then meshed by dividing it into smaller tetrahedral cells. Five inflation layers are generated near walls and the cells are clustered near the elbow area as shown in Figure 2. The domain is divided into 398421 tetrahedral elements after performing the suitable mesh sensitivity analyses. The mesh is then transferred to the solver, which is FLUENT software [2]. Medium density crude oil is used as the working fluid and it is considered as incompressible Newtonian fluid with density \(\rho = 850\) kg/m³ and dynamic viscosity \(\mu = 0.009\) kg/m/s [9]. The sand material is considered as inert anthracite spherical particles with uniform diameter and density of 1550 kg/m³ injected from the inlet face with different mass flow rates. The no-slip boundary
condition is selected at all the solid surfaces of the system. The boundary condition at the inlet is selected to be constant velocity inlet in the range 1 m/s to 100 m/s and the pressure at the outlet is set to atmospheric (zero gage). The boundary conditions for particles are set as reflected by the walls and escape through the inlet and outlet boundaries. The inlet sand particles velocity is set to zero for all cases. The numerical solution of the governing equations is based on the second order upwind scheme. SIMPLE algorithm [2] is employed to solve the discretized equations. This is an iterative solution and needs under relaxation to control the changes in the values during iteration. The values of the under relaxation factors of 0.3, 0.7 and 0.8 for pressure, momentum and turbulence model equations respectively are used. The under relaxation factors of 0.5 is used for the interphase exchange of momentum during the calculation. The maximum residual in continuity equation and all variables were lower than $10^{-3}$ at the end of the iterations. The numerical solution of the Discrete Phase Model is computed by integrating equation (4) using the implicit Euler method.

### 4. Results and Discussion

The velocity profile of crude oil flow in the inlet pipe (at 0.2 m from the elbow) with inlet velocity of 30 m/s is used for validation. The power-law velocity profile express as [10]:

$$\frac{u}{u_{\text{max}}} = (1 - \frac{r}{R})^{1/n}$$  \hspace{1cm} (8)

where $r$ is the radial distance from the centreline of the pipe and $R$ is the total radius of the pipe. The results presented in Figure 3 show that prediction using the realizable $k - \varepsilon$ with scalable wall functions is almost similar to that obtained using SST $k - \omega$ model. However, the solution is converged faster when the realizable $k - \varepsilon$ with scalable wall functions is used. Therefore the realizable $k - \varepsilon$ with scalable wall functions it is used for results generation in the present study. Different mesh sizes are used for grid independent study and the accuracy of the results is verified by checking the velocity profiles and mass balance of the flow using the realizable $k - \varepsilon$ with scalable wall functions. The total number of cells (control volumes) in the fine mesh is 581 921 cell. The moderated mesh contains 398 421 cell and it is 252 384 cell for the coarse mesh for the same geometry. The results presented in Figure 3 for the velocity profile in the same location show negligible difference using moderated and fine mesh sizes with same trend as the power law profile. Therefore, the moderated size of the mesh is used to generate the results in the present study. The global mass balance is monitored and satisfied in the converged solution within ± 0.01% in all cases.

![Figure 2. Mesh geometry (moderate mesh)](image)

![Figure 3. Velocity profiles with inlet velocity 30 m/s.](image)

To understand the flow structure and the particles-fluid interaction and its effect on the surface erosion, the governing equations are solved and the erosion rate is calculated for different conditions. Figure 4 shows the crude oil and particle flow details near the elbow at the med-section where the flow direction changed suddenly with inlet velocity of 30 m/s. The spherical sand particles are considered with a diameter of 1 μm and sand flow rate of $10^{-20}$ kg/s. The fluid pressure at the elbow is maximum...
at the outer surface of the elbow where the fluid imping the elbow and on the opposite side the pressure is minimum due to the fluid acceleration as shown in Figure 4(a). The discrete phase model results show the particle tracking with a velocity almost similar to the fluid velocity as shown in Figures 4(a) and 4(b). The maximum erosion rate happening on the outer surface of the elbow where the sand and fluid impinging the wall as shown in Figure 4(c). After impinging the outer surface of the elbow, the sand particles are directed to the inner surface of the outlet pipe and create another region of high erosion rate as shown in Figures 4(b) and 4(c).

Figure 4. Fluid and particle flow details near the elbow with inlet velocity of 30 m/s.

The simulation results show that the inlet velocity of the crude oil, the size of the sand particles and its mass flow rate are important parameters on the rate of sand erosion in the system. In practice, the sand production rate during the crude oil exaction is a complex process with different sand shape and size especially in the new oil wells. To simplify the problem, it is assumed that the sand particles are of spherical shape with a diameter in the range of 1 μm to 50 μm and flow rate in the range of $1 \times 10^{-20}$ kg/s to $3 \times 10^{-20}$ kg/s with crude oil inlet velocity in the range 1 m/s to 100 m/s. Figure 5(a) show that the erosion rate are increasing with the increase of the inlet fluid velocity while keeping the inlet sand particles velocity at zero. The sand particles are carried by the fluid and therefore will have more kinetic energy with high inlet velocity of the fluid which leads to increase the erosion rate. The location of the maximum erosion rate is always same as that presented in Figure 4(c).

Figure 5. Variation of the area-weighted average erosion rate with inlet velocity and sand diameter.
It is observed that the large sand particles with constant sand flow rate causing less erosion rate than those of smaller diameter as shown in Figure 5 (b). This is due to less number of large size particles in the same sand flow rate compare to the smaller particles. The fluid-sand flow interaction leads to different particle impact angles and velocities at different particle diameter. The particles with small diameter can be carried out by the fluid and impact on the elbow surfaces. However the flow of the big size particles need more fluid kinetic energy and will have less impact force on the elbow surfaces and therefore less erosion rate. The results presented in Figure 5 also show the increase of the erosion rate with the increase of the sand mass flow rate. This is due to the increase of the number of impacts of sand particles on the surface of the piping system.

5. Conclusions
Parametric study is carried out to investigate the effect of the governing parameters on the sand erosion in a crude oil flow through a 90° pipe elbow. Numerical simulations were performed for different crude oil inlet velocity, spherical sand diameter and sand flow rate. The simulations in the present study were performed by solving the fluid dynamics field followed by particle tracking and finally erosion rate calculation. The maximum erosion rate is calculated on the outer surface of the elbow where the sand and fluid coming from the inlet pipe impinging the surface facing the flow before changing the direction. After that the sand particles are directed to the inner surface of the outlet pipe which create another region of high erosion rate. It is found that the maximum erosion and the area-weighted average erosion rate are increasing with the increase of the inlet velocity of the crude oil while keeping the inlet sand particles velocity at zero due to fluid-sand interaction. The numerical results show the large sand particles with constant sand flow rate causing less erosion rate than those of smaller diameter. This is because the flow of the big size particles need high fluid kinetic energy and therefore have less impact and less erosion rate compared with small size particles. Due to the increase of the number of impacts on the surface of the piping system the more sand flow rate generates more erosion in the system. Finally the current findings suggest to lower production velocity and avoid sudden change in the flow direction if possible to reduce the sand erosion.

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