Vortices in gauge models at finite density with vector condensates

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There exists a class of gauge models incorporating a finite density of matter in which the Higgs mechanism is provided by condensates of gauge (or gauge plus scalar) fields, i.e., there are vector condensates in this case. We describe vortex solutions in the simplest model in this class, the gauged $SU(2) \times U(1)$ model with the chemical potential for hypercharge $Y$, in which the gauge symmetry is completely broken. It is shown that there are three types of topologically stable vortices in the model, connected either with photon field or hypercharge gauge field, or with both of them. Explicit vortex solutions are numerically found and their energy per unit length are calculated. The relevance of these solutions for the gluonic phase in the dense two-flavor QCD is discussed.

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I. INTRODUCTION

Vortices are very special soliton-like excitations. In condensed matter physics, vortex solutions were discovered by Abrikosov 1. They play a crucial role in the dynamics of type II superconductors. In relativistic quantum field theory, vortices were first considered by Nielsen and Olesen 2. They can be important in cosmology, astroparticle physics, and for the confinement dynamics in QCD-like theories (for reviews, see Ref. 3). Vortex and vortex-like solutions in relativistic models at finite density of quark matter were studied in Ref. 4.

The existence of vortex solutions is usually connected with the Higgs mechanism in abelian $U(1)$ gauge dynamics. The major degrees of freedom are a complex scalar Higgs field and a $U(1)$ gauge field. In this paper, we will describe vortex solutions in a somewhat different class of gauge models incorporating a finite density of matter. The Higgs mechanism in these models is provided by condensates of gauge (or gauge plus scalar) fields, i.e., there are vector condensates in this case. This class in particular includes the $SU(2)_L \times U(1) \sigma$ model, (nonrenormalizable) models including massive vector bosons with a chemical potential for electric charge, and the gauged linear $SU(2)_L \times U(1)_Y \sigma$-model (without fermions) with a chemical potential for hypercharge.

It is also noticeable that a recently revealed gluonic phase in neutral two-flavor quark matter also relates to this class. In the gluonic phase, vector condensates of gluons cure a chromomagnetic instability in the two-flavor superconducting (2SC) solution and lead to spontaneous breakdown of the $SU(2)_c \times U(1)_{em} \times SO(3)_{rot}$ symmetry down to $SO(2)_{rot}$. Here $SU(2)_c$ and $U(1)_{em}$ are the color and electromagnetic gauge symmetries in the 2SC medium, and $SO(3)_{rot}$ is the rotational group (recall that in the 2SC solution the color $SU(3)_c$ is broken down to $SU(2)_c$). In other words, the gluonic phase describes an anisotropic medium in which the color and electric superconductivities coexist. Because it is naturally to expect that cold quark matter may exist in the core of compact stars (for reviews, see Ref. 14), it would be of great interest to describe the dynamics of this phase in more detail, in particular, to clarify whether vortex excitations exist there.

Since the dynamics of the gluonic phase is very rich and complicated, as a first step, it would be appropriate to use a

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1 Ungauged linear $SU(2)_L \times U(1)_Y \sigma$-model with a chemical potential for hypercharge is a toy model for the description of the dynamics of the kaon condensate in high density QCD. In particular, it realizes the phenomenon with abnormal number of Nambu-Goldstone (NG) bosons, when spontaneous breakdown of continuous symmetries leads to a lesser number of NG bosons than that required by the Goldstone theorem (for a recent discussion of this model, see Ref. 11).
simpler model relating to the same universality class, i.e., with the same sample of the symmetry breaking. Fortunately, such a model exists: It is the gauged \( \sigma \)-model with the chemical potential for the hypercharge introduced and studied in Ref. \[8\]. Let us describe it in more detail. Its Lagrangian density reads (we use the metric \( g^{\mu \nu} = \text{diag}(1, -1, -1, -1) \)):

\[
\mathcal{L} = -\frac{1}{4} F^{(a)}_{\mu \nu} F^{\mu \nu(a)} - \frac{1}{4} F^{(Y)}_{\mu \nu} F^{\mu \nu(Y)} + \left[ (D_{\mu} - i \mu \delta_{\mu 0}) \Phi \right]^\dagger (D^{\nu} - i \mu \delta^{\nu 0}) \Phi - m^2 \Phi \dagger \Phi - \lambda (\Phi \dagger \Phi)^2,
\]

(1)

where the covariant derivative \( D_{\mu} = \partial_{\mu} - ig A_{\mu} - (ig'/2) B_{\mu}, \Phi \) is a complex doublet field \( \Phi^T = (\varphi^+, \varphi_0) \), and the chemical potential \( \mu \) is provided by external conditions (to be specific, we take \( \mu > 0 \)). Here \( A_{\mu} = A^a_{\mu} \tau^a/2 \) are \( \text{SU}(2)_L \) gauge fields (\( \tau^a \) are three Pauli matrices) and the field strength \( F^{(a)}_{\mu \nu} = \partial_{\mu} A^{(a)}_{\nu} - \partial_{\nu} A^{(a)}_{\mu} + g \epsilon^{abc} A^{(b)}_{\mu} A^{(c)}_{\nu} \). \( B_{\mu} \) is a \( U_Y(1) \) gauge field with the field strength \( F^{(Y)}_{\mu \nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} \). The hypercharge of the doublet \( \Phi \) equals +1. This model has the same structure as the electroweak theory without fermions and with chemical potential for hypercharge \( Y \). Note that the terms with the chemical potential are \( \text{SU}(2)_L \times U(1)_Y \) (and not \( \text{SU}(2)_L \times \text{SU}(2)_R \)) symmetric. This follows from the fact that the hypercharge generator \( Y \) is \( Y = 2 I_R^3 \) where \( I_R^3 \) is the third component of the right handed isospin generator. Henceforth we will omit the subscripts \( L \) and \( R \), allowing various interpretations of the \( \text{SU}(2)_L \).

Because the chemical potential explicitly breaks the Lorentz symmetry, the symmetry of the model is \( \text{SU}(2)_L \times U(1)_Y \times \text{SO}(3)_{\text{rot}} \). As was shown in Ref. \[8\], for sufficiently large values of the chemical potential \( \mu \), the condensates of both the scalar doublet \( \Phi \) and the gauge field \( A_{\mu} \) occur. These condensates break the symmetry \( \text{SU}(2)_L \times U(1)_Y \times \text{SO}(3)_{\text{rot}} \) down to \( \text{SO}(2)_{\text{rot}} \). This is the same symmetry breaking pattern as in the gluonic phase of dense QCD \[12\], while the gauged \( \sigma \)-model is much simpler: It is renormalizable and for small coupling constants \( g, g' \) and \( \lambda \), the tree approximation is reliable there.

As will be shown below, topologically stable vortex solutions exist in this model indeed. Their structure is much richer and more complicated than that in the Abelian gauge model. In particular, there are different types of vortices connected either with photon field or hypercharge gauge field \( B_{\mu} \), or with both of them.

The paper is organized as follows. In Section \[III\] a general analysis of vortex solutions in this model is realized. We discuss a topological stability of these solutions and introduce three types of vortices: Magnetic vortices, hypermagnetic vortices, and hybrid ones. They are connected with photon field, hypercharge field \( B_{\mu} \), and both of them, respectively. In Section \[III\] an explicit solution for magnetic vortices is obtained. In Sections \[IV\] and \[V\] explicit solutions for hypermagnetic vortices and hybrid ones, respectively, are derived. In Section \[VI\] we summarize the main results of the paper.

## II. VORTEX SOLUTIONS: GENERAL CONSIDERATION

The equations of motion in the gauged \( \sigma \)-model at finite density follow from Lagrangian \[1\]:

\[
-(D_{\nu} - i \mu \delta_{\nu 0})(D^{\nu} - i \mu \delta^{\nu 0}) \Phi - m^2 \Phi \dagger \Phi - 2 \lambda (\Phi \dagger \Phi) \Phi = 0,
\]

(2)

\[
\partial^\mu F^{(a)}_{\mu \nu} + ig \left[ \Phi \dagger \frac{\tau^a}{2} \left( \partial_{\nu} - i \frac{g'}{2} B_{\nu} \right) \Phi - \left( \partial_{\nu} - i \frac{g'}{2} B_{\nu} \right) \Phi \right] \dagger \frac{\tau^a}{2} + g \epsilon^{abc} A^{(b)}_{\mu} F^{(c)}_{\mu \nu} + \frac{g^2}{2} A^{(a)}_{\mu} \Phi \dagger \Phi + 2g \mu \delta_{\nu 0} \Phi \dagger \frac{\tau^a}{2} \Phi = 0,
\]

(3)

\[
\partial^\mu F^{(Y)}_{\mu \nu} + ig' \left( \Phi \dagger \frac{\tau^a}{2} \left( \partial_{\nu} - i \mu \delta_{\nu 0} \right) \Phi - \left( \partial_{\nu} - i \mu \delta_{\nu 0} \right) \Phi \right) \dagger \frac{\tau^a}{2} = 0.
\]

(4)

As was shown in Ref. \[8\], for sufficiently large values of the chemical potential \( \mu \), the vacuum solution of \[2\] and \[3\] is given by

\[
W^{(-)} = (W^{(+)} *) = C = \sqrt{\frac{\mu v_0}{\sqrt{2}g} - \frac{v_0^2}{4}} \quad A^{(3)} = \frac{v_0}{\sqrt{2}}, \quad \Phi^T = (0, v_0),
\]

(5)
\[ \nu_0 = \frac{\sqrt{(g^2 + 64\lambda)\mu^2 - 8(8\lambda - g^2)m^2} - 3g\mu}{2(8\lambda - g^2)}, \]

\[ W_\mu^{(\mp)} = \frac{1}{\sqrt{2}}(A_\mu^{(1)} \pm iA_\mu^{(2)}), \Phi^T = (\varphi^+, \varphi_0) \] and all other fields are equal to zero. \(^2\) It is clear that this solution implies that the initial symmetry \( SU(2) \times U(1)_Y \times SO(3)_{rot} \) is spontaneously broken down to \( SO(2)_{rot} \). In particular, the electromagnetic \( U(1)_{em} \), with electric charge \( Q_{em} = I_3 + Y/2 \), is spontaneously broken by the condensate of \( W \) bosons, i.e., electric superconductivity takes place in this medium.

Note that because the \( U(1)_Y \) symmetry is local, for nonzero chemical potential \( \mu \) one should introduce a source term \( B_0J_0 \) in Lagrangian density \( \Phi \) in order to make the system neutral with respect to hypercharge \( Y \). This is necessary since otherwise in such a system thermodynamic equilibrium could not be established. The value of the background hypercharge density \( J_0 \) (representing very heavy particles) is determined from the requirement that \( B_0 = 0 \) is a solution of the equation of motion for \( B_0 \) (Gauss’s law) \(^2\). There exists an alternative description of this dynamics in which a background hypercharge density \( J_0 \) is considered as a free parameter and \( \mu \) is taken to be zero. Then Gauss’s law will define the vacuum expectation value \( \langle B_0 \rangle \). It is not difficult to check that these two approaches are equivalent if the chemical potential \( \mu \) in the first approach is taken to be equal to the value \( \nu_0 \langle B_0 \rangle \) from the second one. In this paper, following Refs. \(^8\) \(^15\), we use the first approach.

As was shown in Ref. \(^8\), in accordance with the sample of spontaneous breakdown of the global symmetry, \( SO(3)_{rot} \to SO(2)_{rot} \), there are two gapless NG bosons in this model. The other excitations are massive (the Higgs mechanism). Because electric superconductivity is realized in the model, it is naturally to expect that vortices may exist there. As will be shown in this paper, this is indeed the case.

Vortices are topologically nontrivial configurations with fields approaching their vacuum values at spatial infinity \(^R\). Mathematically, vortices are topologically stable whenever the first homotopy group \( \pi_1(G_{int}/H_{int}) \) of the vacuum manifold \( G_{int}/H_{int} \) is nontrivial (here \( G_{int} \) is an internal symmetry group of the action in a model and its subgroup \( H_{int} \) is a symmetry group of the vacuum.) We emphasize that this criterion of the topological stability is sufficient but not necessary. In particular, as will be shown below, magnetic vortices, the most interesting vortex solutions in the present model, are topologically stable due to another criterion.

According to Eq. \(^R\), two complex fields \( W_z^{(-)} \) and \( \varphi_0 \) have nonzero vacuum expectation values in the ground state of the gauged \( \sigma \)-model at finite density. Therefore a priori there can be different types of topologically stable vortex solutions: Both the field \( W_z^{(-)} \) and the field \( \varphi_0 \) can wind around the spatial circle at infinity. I.e., in the cylindrical coordinates \( (t, \rho, \phi, z) \), as \( \rho \to \infty \), the asymptotics of these fields take the form
\[ W_z^{(-)} \to C e^{il\phi}, \varphi_0 \to \nu_0 e^{in\phi}, \]

where \( l \) and \( n \) are integer. Let us turn to the analysis of a topological stability of solutions with these asymptotics.

In this model, the internal group \( G_{int} \) is \( SU(2) \times U(1)_Y \) and the symmetry group \( H_{int} \) of vacuum \(^X\) is trivial. Therefore the first homotopy group of the vacuum manifold in the model is
\[ \pi_1(G_{int}/H_{int}) = \pi_1(SU(2) \times U(1)_Y) = \pi_1(U(2)) + \pi_1(U(1)_Y) = \pi_1(U(1)_Y) = Z, \] where \( Z \) is the group of integer numbers (here we use the fact that since the \( SU(2) \) manifold is simply connected, \( \pi_1(SU(2)) = 0 \)). This seems to suggest that there exist only topologically stable vortex solutions connected with the hypercharge \( U(1)_Y \). However, as will be shown below, the situation in this model is more sophisticated and there exist also other topologically stable vortices. The point is that while the scalar field \( \Phi \) is assigned to the fundamental representation of the \( SU(2) \) group, the gauge field \( A_\mu^{(a)} \) is assigned to the adjoint one. Because of that, \( SU(2) \) gauge transformations for \( A_\mu^{(a)} \) reduce to \( SO(3) \) ones. It is important that, unlike \( SU(2) \), the manifold of \( SO(3) \) is not simply connected and \( \pi_1(SO(3)) = Z_2 \) with \( Z_2 = 0, 1 \). As will become clear in a moment, this point is quite important in the analysis of the topological stability of the solutions.

\(^2\) Here “sufficiently large values of \( \mu \)” means the following: When \( m^2 > 0 \), \( \mu \) should be larger than the critical value \( \mu_{cr} = m \), and for \( m^2 < 0 \), \( \mu \) should be larger than \( \mu_{cr} = g|m|/2\sqrt{\lambda} \) (the critical value \( g|m|/2\sqrt{\lambda} \) coincides with the mass of \( W \) boson in the vacuum theory with \( \mu = 0 \) and \( m^2 < 0 \)).
We begin by considering the conventional case when $l = 0$ in asymptotics (7): 

$$W_z^{(-)} \rightarrow C, \quad \varphi_0 \rightarrow v_0 e^{i n \phi}. \quad (8)$$

This configuration can be obtained from the vacuum configuration $[5]$ by using the $U(1)_Y$ gauge transformation $e^{i n \phi Y}$. It is clear that in this case the solutions with different $n$ are assigned to different topological classes described by the homotopy group $\pi_1(U(1)_Y) = Z$. Therefore, if such vortex solutions exist, they are topologically stable. Due to the evident reasons, we will call them hypermagnetic vortices. They possess hypermagnetic fluxes with a (hypermagnetic) winding number $H = n$.

Let us now turn to a more interesting case of asymptotics (7) with $n = 0$: 

$$W_z^{(-)} \rightarrow C e^{i l \phi}, \quad \varphi_0 \rightarrow v_0. \quad (9)$$

It is a rather unusual situation: The phase $l \phi$ is now related to a vector field and not to a scalar one. It is clear that configuration (9) can be obtained from vacuum configuration $[5]$ by using the gauge transformation 

$$e^{-i l \phi Q_{em}} = e^{-i l \phi Y^3/2} e^{-i l \phi Y/2}, \quad (10)$$

where electric charge $Q_{em} = I_3 + \frac{Y}{2}$ is equal to $-1$ for the field $W_z^{(-)}$ and zero for the scalar field $\varphi_0$.

Because for the vector fields the $SU(2)$ transformations are reduced to the $SO(3)$ ones, it is important to distinguish two cases: with $l$ being even and $l$ being odd. In the first case, with $l = 2k$, as the angle $\phi$ running from $0$ to $2\pi$, the gauge transformation (10) defines closed loops both on the manifold of $SU(2)$ and that of $U(1)_Y$. Then, because $\pi_1(SU(2) \times U(1)_Y) = \pi_1(U(1)_Y) = Z$, we conclude that these solutions are topologically equivalent to the hypermagnetic vortex solutions with the winding number $H = -k$ considered above.

The case with odd $l = 2k + 1$ is more sophisticated. In this case, as $\phi$ changing from $0$ to $2\pi$, the gauge transformation (10) does not define closed loops neither on the $SU(2)$ manifold nor on the $U(1)_Y$ one. Therefore in this case one cannot use the homotopic group $\pi_1(SU(2) \times U(1)_Y)$ for studying the topological stability of these solutions. Let us show that for this purpose one can use the homotopic group $\pi_1(SO(3))$ instead. The reasons are the following:

Because the hypercharge of the gauge field $A_{\mu}^{(a)}$ is zero, the action of the gauge transformation (10) on it is given by the operator $e^{-i(2k+1)\phi I_3^{adj}/2}$, where $I_3^{adj}$ is a $SU(2)$ generator in the adjoint representation. As the angle $\phi$ changing from $0$ to $2\pi$, this transformation does define a closed loop on the $SO(3)$ manifold, which coincides with the vacuum manifold of $A_{\mu}^{(a)}$. Therefore, one can indeed use the homotopic group $\pi_1(SO(3)) = Z_2$ for the description of the topological stability of these solutions.

Now, because odd values of $l$ correspond to the element $+1$ of $\pi_1(SO(3)) = Z_2$, we conclude that this loop cannot be unwound and, therefore, these vortex solutions are topologically stable. Due to the form of gauge transformation (10), we will call them magnetic vortices. They possess magnetic fluxes with a (magnetic) winding number $M = -(2k + 1)$. The minus sign here corresponds to a negative electric charge of $W^-$ bosons. A detailed study of the topological structure of the set of these solutions is beyond the scope of this paper. Here we will only argue that the solutions with different values of $k$ are topologically inequivalent. The argument is based on gauge transformation (10). It implies that although for odd $l = 2k + 1$ the path defined by this transformation on the $U(1)_Y$ manifold is not closed, it contains $k$ windings. Therefore the solutions with different $k$ should be assigned to different topological classes, i.e., they cannot be transformed from one into another by using a continuous deformation.

The last case is that with the “hybrid” asymptotics

$$W_z^{(-)} \rightarrow C e^{i l \phi}, \quad \varphi_0 \rightarrow v_0 e^{i n \phi} \quad (11)$$

with both $l$ and $n$ being nonzero. In this case the gauge transformation (10) is replaced by 

$$e^{-i l \phi Q_{em}} e^{i n \phi Y} = e^{-i l \phi Y^3/2} e^{i(2n-l)\phi Y/2}. \quad (12)$$

---

3 These two manifolds coincide because vacuum configuration $[5]$ includes two linearly independent isovectors, $A_{\mu}^{(1)}$, and $A_{\mu}^{(3)}$, and there is no subgroup in $SO(3)$ under which two linearly independent isovectors are invariant.
Using the same arguments as above, we conclude the following. For even $l = 2k$, these solutions are topologically equivalent to the hypermagnetic vortices with fluxes corresponding to the winding number $H = n - k$. As to odd $l = 2k + 1$, they yield new topological classes of solutions. These solutions carry both magnetic and hypermagnetic fluxes with winding numbers $(M, H)$, with $M$ taking odd integers $-l = -(2k + 1)$ and $H = n$. It will be shown in Sec. IV that the hybrid vortices can be considered as composites of the magnetic and hypermagnetic ones.

As to the topological structure of the solutions for hybrid vortices with odd $l = 2k + 1$, a comparison of Eq. (12) with Eq. (10) suggests the following generalization of the conclusion that was made above for the case of the magnetic vortices: The hybrid vortices with different $n - k$ are topologically inequivalent.

This concludes the analysis of the topological stability of vortex solutions. As will be shown below, these three types of vortex solutions exist in the present model indeed.

### III. MAGNETIC VORTICES

In this section, we will analyze the solutions for the magnetic vortices having asymptotics (9) with $l$ being odd. To be concrete, we will take $l = -1$. In accordance with the discussion in the previous section, one should expect that the magnetic winding number $M$ equals $+1$ in this case.

Vortex solutions are static $z$-independent configurations. We will consider the following ansatz for them in the cylindrical coordinates $(t, \rho, \phi, z)$ (the reasons for choosing this ansatz will be discussed in more detail below):

$$W_z(t, \rho, \phi, z) = \frac{W(t)e^{-\phi}}{\sqrt{2}}, \quad A_4^\prime(t, \rho, \phi, z) = A_4^\prime(t, \rho, \phi, z), \quad A_\phi(t, \rho, \phi, z) = A_\phi(t, \rho, \phi, z),$$

$$Z_\phi(t, \rho, \phi, z) = Z_\phi(t, \rho, \phi, z), \quad \varphi_0(t, \rho, \phi, z) = \varphi_0(t, \rho, \phi, z) = \varphi_0(t, \rho, \phi, z),$$

where $A_\phi$ and $Z_\phi$ are angular components of the electromagnetic field $A_\mu = \frac{g' A_4^\prime + g B_\phi}{\sqrt{g'^2 + g_\phi^2}}$ and the Z-boson field $Z_\mu = \frac{g A_4^\prime - g B_\phi}{\sqrt{g'^2 + g_\phi^2}}$ (all other components of the fields are equal to zero). 4

Using equations of motion (2)-(11), one can check that ansatz (13) is consistent with them and they reduce to the following system of five coupled nonlinear ordinary differential equations:

$$W'' + \frac{W'}{\rho} - \frac{W}{\rho^2} + \frac{2g(g' A_\phi + g Z_\phi) W}{\sqrt{g'^2 + g_\phi^2}} + g^2 \left( \frac{A_4^\prime}{g'^2 + g_{\phi}^2} \right) W - \frac{g^2}{2} \rho W v^2 = 0,$$

$$\frac{(A_4^\prime)^\prime}{\rho} + \frac{(A_4^\prime)^\prime}{\rho} - g^2 W^2 A_4^\prime - \frac{g^2}{2} A_4^\prime v^2 + g \mu v^2 = 0,$$

$$A_\phi'' + \frac{A_\phi'}{\rho} - \frac{A_\phi'}{\rho^2} + \frac{gg' W^2}{\sqrt{g'^2 + g_\phi^2}} \left( \frac{1}{\rho} - \frac{g(g' A_\phi + g Z_\phi)}{\sqrt{g'^2 + g_\phi^2}} \right) = 0,$$

$$Z_\phi'' + \frac{Z_\phi'}{\rho} - \frac{Z_\phi'}{\rho^2} - \frac{g^2 + g_\phi^2}{2} Z_\phi v^2 + \frac{g^2 W^2}{\sqrt{g'^2 + g_\phi^2}} \left( \frac{1}{\rho} - \frac{g(g' A_\phi + g Z_\phi)}{\sqrt{g'^2 + g_\phi^2}} \right) = 0,$$

$$v'' + \frac{v'}{\rho} - \frac{g^2 + g_\phi^2}{4} Z_\phi^2 v - \frac{g^2}{4} W^2 v + \left( \frac{g A_4^\prime}{2} - \mu \right)^2 v - m^2 v - 2\lambda v^3 = 0.$$

4 Recall that in the cylindrical coordinates, a vector field $\vec{A} \equiv (A_x, A_y, A_z)$ is decomposed as $\vec{A} = A_\rho \hat{\rho} + A_\phi \hat{\phi} + A_z \hat{z}$, where $A_\rho = A_x \cos \phi + A_y \sin \phi$, $A_\phi = -A_x \sin \phi + A_y \cos \phi$. 5
Note that although the phase in the exponent \( e^{-i\phi} \) in \( W_t^{(-)} \) in Eq. (12) is of the electromagnetic \( U(1)_{em} \) origin, this ansatz contains in addition also \( Z_\phi \) field. This is because the \( W \) condensate mixes the gauge fields \( A_\mu \) and \( Z_\mu \) (see Eqs. (13) and (17)).

It is instructive to compare ansatz (13) and equations of motion (14)- (18) with those in the Abelian Higgs model. Obviously, \( W_t^{(-)} \) plays the role of the Higgs field. The fields \( A_\phi \) and \( Z_\phi \) are analogous to the Abelian gauge field. The fields \( A_\mu^{(3)} \) and \( v \) do not have analogs in the Abelian Higgs model. Of course, they are present in ansatz (13) in order to ensure the correct vacuum asymptotics at infinity. Thus, the fields \( A_\mu^{(3)} \) and \( v \) have a somewhat different status in the vortex ansatz comparing to the fields \( W_t^{(-)} \), \( A_\phi \), and \( Z_\phi \). We will discuss this point in more detail below, when we consider the ultraviolet boundary conditions for these fields.

The infrared boundary conditions at \( \rho \to \infty \) follow from the requirement of finiteness of the energy per unit length of a vortex, which implies that fields tend to their vacuum values:

\[
W(\rho) \to \sqrt{\frac{2\mu v_0}{g}} - \frac{v_0^2}{2}, \quad A_\mu^{(3)}(\rho) \to \frac{v_0}{\sqrt{2}}, \quad v(\rho) \to v_0, \quad A_\phi(\rho) \to 0, \quad Z_\phi(\rho) \to 0. \tag{19}
\]

As to the ultraviolet boundary conditions at \( \rho = 0 \), because \( \rho = 0 \) is a regular singular point of the system of differential equations (14)-(18), these boundary conditions follow from the equations themselves if one utilizes the condition of regularity of the solution at this point:

\[
W(0) = A_\phi(0) = Z_\phi(0) = 0, \quad (A_\mu^{(3)})'(0) = v'(0) = 0. \tag{20}
\]

The ultraviolet boundary conditions (20) for the fields \( W \) and \( A_\phi, Z_\phi \) are like those for a Higgs field and a gauge field, respectively, in the Abelian Higgs model: they guarantee that these fields are single valued at \( \rho = 0 \). Unlike them, the fields \( A_\mu^{(3)} \) and \( v \) have Neumann boundary conditions which imply that there is no influx of momentum for these fields at the boundary \( \rho = 0 \).

We solved numerically Eqs. (14)-(18) with boundary conditions (19)-(21) by using MATLAB. To be concrete, we considered the case with \( m^2 > 0 \) and chose the parameters \( \mu \) and \( g v_0/\sqrt{2} \) as in Ref. [8]: \( \mu/m = 1.1 \) and \( g v_0/\sqrt{2m} = 0.1 \). For the values of the coupling constants \( g \) and \( g' \) we used those from the electroweak theory: \( g = 0.65 \) and \( g' = 0.35 \). Then, using Eq. (17), one gets the value of the coupling constant \( \lambda: \lambda = 0.53 \). Therefore we work in the weak coupling regime when the tree approximation is reliable.

The solution is shown in Fig. 1, where dimensionless fields \( W/m, A_\mu^{(3)}/m, A_\phi/m, Z_\phi/m, v/m \) are plotted against \( r = \rho m \). The boundary conditions (19) were set at \( r_{max} = 80 \) and it was checked that starting from \( r_{max} = 40 \) the form of the solution practically does not depend on the choice of \( r_{max} \).

Substituting ansatz (13) into Eq. (11), we calculate the effective potential \( (V = -\mathcal{L}) \):

\[
V = \frac{(W')^2}{2} + \frac{W^2}{2} \left( \frac{1}{\rho} - \frac{g(g' A_\phi + gZ_\phi)}{\sqrt{g^2 + g'^2}} \right)^2 - \frac{(A_\mu^{(3)})^2}{2} - \frac{g^2(A_\mu^{(3)})^2 W^2}{2} + \frac{1}{2} \left( A_\mu^{(3)} + \frac{A_\phi}{\rho} \right)^2 + \frac{1}{2} \left( Z_\phi + \frac{Z_\phi}{\rho} \right)^2 - \left( \frac{gA_\mu^{(3)}}{2} - \mu \right)^2 v^2 + \frac{g^2}{4} Z_\phi^2 v^2 + m^2 v^2 + \lambda v^4 + \frac{g^2}{4} W^2 v^2. \tag{22}
\]

Subtracting the energy density of the vacuum solution, we find that the energy of the magnetic vortex per unit length is equal to 1.27 in units of \( m^2 \).

Note that the \( SU(2) \times U(1)_Y \) invariant definition of the electromagnetic field strength \( F^{(em)}_{\mu\nu} \) is

\[
F^{(em)}_{\mu\nu} = \frac{g F^{(Y)}_{\mu\nu}}{\sqrt{g^2 + g'^2}} - \frac{2g'}{\sqrt{g^2 + g'^2}} \frac{\Phi^\dagger F^{(em)}_{\mu\nu} \Phi}{\Phi^\dagger \Phi}. \tag{23}
\]
FIG. 1: Dimensionless fields $\frac{W}{m}$, $\frac{A^{(3)}_i}{m}$, $\frac{A_i}{m}$, $\frac{Z_\phi}{m}$, and $\frac{v}{m}$ as functions of $r = \rho m$ for the magnetic vortex solution.

For the vortex solution with ansatz (13), it yields the conventional relation for the magnetic field $\vec{H}$:

$$\vec{H} = \text{curl} \ A.$$  \hspace{1cm} (24)

It is well known that the magnetic flux of vortices is quantized and related to the winding number, which is a topological invariant. It is easy to check that this property takes place in the present case. Indeed, as follows from Eqs. (16) and (17), while the field $A_\phi$ has the asymptotics $\frac{1}{e\rho}$ as $\rho \to \infty$ (with $e = \frac{\sqrt{g''}}{\sqrt{g'^2 + g''^2}}$), the field $Z_\phi$ rapidly decreases in this limit. Therefore we find that for the magnetic vortex only the flux related to the electromagnetic field $A_\mu$ is nonzero. Then, using Eq. (24), we get:

$$ F_{em}^{(M)} = \int dxdy H_z = \oint d\vec{x} \cdot A = \int d\phi \, \rho A_\phi |_{\rho = \infty} = \frac{2\pi}{e}. $$ \hspace{1cm} (25)

I.e., as was expected, this vortex solution corresponds to the winding number +1.

\section{IV. HYPERMAGNETIC VORTICES}

A hypermagnetic vortex possesses asymptotics [8]. To be concrete, we will take $n = -1$. As follows from the discussion in Sec. II, the hypermagnetic winding number $H$ should be now equal to -1.

In this case, we will use the following ansatz:

$$ W_z^{(-)}(t, \rho, \phi, z) = \frac{W(\rho)}{\sqrt{2}}, \quad A^{(3)}_{i}(t, \rho, \phi, z) = A^{(3)}_{i}(\rho), \quad A_{\phi}(t, \rho, \phi, z) = A_{\phi}(\rho), \quad Z_{\phi}(t, \rho, \phi, z) = Z_{\phi}(\rho), \quad \phi_0(t, \rho, \phi, z) = v(\rho)e^{-i\phi}. $$ \hspace{1cm} (26)

For this ansatz, the equations of motion (2)-(4) yield

$$ W'' + \frac{W'}{\rho} + g^2 \left( (A^{(3)}_{i})^2 - \frac{(g'A_{\phi} + gZ_{\phi})^2}{g'^2 + g''^2} \right) W - \frac{g^2}{2} Wv^2 = 0, $$ \hspace{1cm} (27)
\[(A_i^{(3)})'' + \frac{(A_i^{(3)})'}{\rho} - g^2 W^2 A_i^{(3)} - \frac{g^2}{2} A_t^{(3)} v^2 + g\mu v^2 = 0, \quad (28)\]

\[A''_t + \frac{A'_t}{\rho} - \frac{A_t}{\rho^2} - \frac{g^2 g' (g' A_t + g Z_\phi)}{g^2 + g'} W^2 = 0, \quad (29)\]

\[Z''_\phi + \frac{Z'_\phi}{\rho} - \frac{Z_\phi}{\rho^2} + \frac{\sqrt{g^2 + g'^2}}{2} \left( \frac{2}{\rho} - \sqrt{g^2 + g'^2} Z_\phi \right) v^2 - \frac{g^3 (g' A_t + g Z_\phi)}{g^2 + g'} W^2 = 0, \quad (30)\]

\[v'' + \frac{v'}{\rho} - \frac{1}{4} \left( \frac{2}{\rho} - \sqrt{g^2 + g'^2} Z_\phi \right)^2 v - \frac{g^2}{4} W^2 v + \left( \frac{g A_t^{(3)}}{2} - \mu \right)^2 v - m^2 v - 2\lambda v^3 = 0. \quad (31)\]

The infrared boundary conditions for the hypermagnetic vortex ansatz at \(\rho \to \infty\) are the same as for the magnetic vortex and given in Eq. (19). The ultraviolet boundary conditions at \(\rho = 0\) are

\[v(0) = A_t(0) = Z_\phi(0) = 0, \quad (32)\]

\[(A_i^{(3)})'(0) = W'(0) = 0. \quad (33)\]

Comparing these boundary conditions with those in Eqs. (20) and (21) for the magnetic vortex, one can see that the ultraviolet boundary conditions for \(W\) and \(v\) are now interchanged. Of course, this point is related to the fact that the phase in the scalar field \(\phi_0\) (and not vector field \(W\)) is relevant for the hypermagnetic U(1)_Y vortex. The numerical solution of Eqs. (27)-(31) with boundary conditions (19), (32), and (33) is shown in Fig. 2, where we use the same set of parameters as for the magnetic vortex.

The effective potential for ansatz (20) is

\[V = \frac{(W')^2}{2} + \frac{g^2 (g' A_t + g Z_\phi)^2}{g^2 + g'^2} W^2 - \frac{(A_i^{(3)})'^2}{2} \]

\[-\frac{g^2 (A_t^{(3)})^2 W^2}{2} + \frac{1}{2} \left( A'_t + \frac{A_t}{\rho} \right)^2 + \frac{1}{2} \left( Z'_\phi + \frac{Z_\phi}{\rho} \right)^2 - \left( \frac{g A_t^{(3)}}{2} - \mu \right)^2 v^2 \]

\[+ (v')^2 + \frac{1}{4} \left( \frac{2}{\rho} - \sqrt{g^2 + g'^2} Z_\phi \right)^2 v^2 + m^2 v^2 + \lambda v^4 + \frac{g^2}{4} W^2 v^2. \quad (34)\]

Subtracting the energy density of the vacuum solution, we find that the energy density of the hypermagnetic vortex per unit length in units of \(m^2\) is equal to 0.79.

It follows from Eqs. (24) and (30) that \(A_t\) and \(Z_\phi\) have the asymptotics \(-\frac{2g}{\rho' \sqrt{g^2 + g'^2}}\) and \(\frac{2}{\rho' \sqrt{g^2 + g'^2}}\) as \(\rho \to \infty\). Therefore, the fluxes of both fields \(A_t\) and \(Z_\phi\) are nonzero

\[F_{em}^{(H)} = -\frac{4\pi g}{g' \sqrt{g^2 + g'^2}}, \quad (35)\]

\[F_{Z}^{(H)} = \frac{4\pi}{\sqrt{g^2 + g'^2}}. \quad (36)\]
It is not difficult to see that these two fluxes correspond to the hypermagnetic flux of the $U(1)_Y$ gauge field $B_\mu$ (the hypermagnetic flux of the $A_{\mu}^{(3)}$ field is zero):

$$F_Y^{(H)} = -\frac{4\pi}{g'}.$$  
(37)

Note that because the coupling of the Higgs field with the field $B_\mu$ in the covariant derivative $D_\mu = \partial_\mu - igA_\mu - \left(\frac{ig'}{2}\right)B_\mu$ is expressed through $g'/2$ rather than $g'$, this flux corresponds to the winding number -1, as was expected.

V. HYBRID VORTICES

The hybrid vortex solutions carry both the magnetic and hypermagnetic fluxes with winding numbers $(M, H)$. Their asymptotics are given in Eq. (11). To be concrete, we will consider the solutions with $l = -1, n = \mp 1$ corresponding to $(M, H) = (1, \mp 1)$, i.e., we consider hybrid vortices with the equal and opposite phases of the $W_{\pm}^{(-)}$ and $\varphi_0$ fields.

Clearly, the ansatz for these vortices should have the form

$$W_{\pm}^{(-)}(t, \rho, \phi, z) = \frac{W(\rho)e^{-i\phi}}{\sqrt{2}}, \quad A_{\pm}^{(3)}(t, \rho, \phi, z) = A_{\pm}^{(3)}(\rho), \quad A_{\phi}(t, \rho, \phi, z) = A_{\phi}(\rho),$$
$$Z_{\phi}(t, \rho, \phi, z) = Z_{\phi}(\rho), \quad \varphi_0(t, \rho, \phi, z) = v(\rho)e^{-i\sigma\phi},$$  
(38)

where $\sigma = \pm$.

The equations of motion for these vortices are sort of a combination of the equations of motion for the magnetic and hypermagnetic ones:

$$W'' + \frac{W'}{\rho} - \frac{W}{\rho^2} + \frac{2g(g' A_{\phi} + gZ_{\phi})}{\sqrt{g^2 + g'^2}} W + g^2 \left( (A_{\pm}^{(3)})^2 - \frac{(g' A_{\phi} + gZ_{\phi})^2}{g^2 + g'^2} \right) W - \frac{g^2}{2} Wv^2 = 0,$$  
(39)
\[
(A_t^{(3)})'' + \frac{A_t^{(3)\prime}}{\rho} - \frac{g^2 W^2 A_t^{(3)}}{2} - \frac{g^2}{2} A_t^{(3)} v^2 + g \mu v^2 = 0 ,
\]

\[
A_\phi'' + \frac{A_\phi'}{\rho^2} - \frac{A_\phi}{\rho^2} + \frac{g g' W^2}{\sqrt{g^2 + g'^2}} \left( \frac{1}{\rho} - \frac{g(g' A_\phi + g Z_\phi)}{\sqrt{g^2 + g'^2}} \right) = 0 ,
\]

\[
Z_\phi'' + \frac{Z_\phi'}{\rho^2} + \frac{Z_\phi}{\rho^2} + \frac{2 \sigma}{\rho} - \sqrt{g^2 + g'^2} Z_\phi v^2 + \frac{g^2 W^2}{\sqrt{g^2 + g'^2}} \left( \frac{1}{\rho} - \frac{g(g' A_\phi + g Z_\phi)}{\sqrt{g^2 + g'^2}} \right) = 0 ,
\]

\[
v'' + \frac{v'}{\rho} - \frac{\left( \frac{2 \sigma}{\rho} - \sqrt{g^2 + g'^2} Z_\phi \right)^2 v}{4} - \frac{g^2}{4} W^2 v + \left( \frac{g A_t^{(3)}}{2} - \mu \right)^2 v - m^2 v - 2 \lambda v^3 = 0 .
\]

The infrared boundary conditions at infinity remain the same as before. Requiring the regularity of solutions at \( \rho = 0 \) leads to the following ultraviolet boundary conditions:

\[
W(0) = v(0) = A_\phi(0) = Z_\phi(0) = 0 ,
\]

\[
(A_t^{(3)})'(0) = 0 .
\]

Note that in contrast to the cases of the magnetic and hypermagnetic vortices, now both functions \( W(\rho) \) and \( v(\rho) \) have the Dirichlet boundary conditions at \( \rho = 0 \). The reason of this is clear: Because both \( W_2^{-}(\rho) \) and \( \phi_0 \) have nontrivial phases for hybrid vortices [53], the regularity of solutions at \( \rho = 0 \) can be ensured only if the moduli of these fields vanish at this point.

The numerical solutions of Eqs. (39)–(43) with boundary conditions (19), (41), and (45) are shown in Fig. 3 and Fig. 4. For these solutions, the same set of parameters was used as in the previous cases.

The effective potential for ansatz (43) is equal to

\[
V = \frac{(W')^2}{2} + \frac{W^2}{2} \left( \frac{1}{\rho} - \frac{g(g' A_\phi + g Z_\phi)}{\sqrt{g^2 + g'^2}} \right)^2 - \frac{(A_t^{(3)\prime})^2}{2}
\]

\[
- \frac{g^2 (A_t^{(3)})^2 W^2}{2} + \frac{1}{2} \left( A_\phi + \frac{A_\phi}{\rho} \right)^2 + \frac{1}{2} \left( Z_\phi + \frac{Z_\phi}{\rho} \right)^2 - \left( \frac{g A_t^{(3)}}{2} - \mu \right)^2 v^2
\]

\[
+ (v')^2 + \frac{1}{4} \left( \frac{2 \sigma}{\rho} - \sqrt{g^2 + g'^2} Z_\phi \right)^2 v^2 + m^2 v^2 + \lambda v^4 + \frac{g^2}{4} W^2 v^2 .
\]\n
Subtracting the energy density of the vacuum solution, we find that in units of \( m^2 \), the energy per unit length of hybrid vortices with the equal and opposite phases are equal to 1.49 and 2.3, respectively.

It follows from Eqs. (41) and (42) that \( A_\phi \) and \( Z_\phi \) have the asymptotics \( \frac{g^2 (1 - 2 \sigma) + g'^2}{\rho g g' \sqrt{g^2 + g'^2}} \) and \( \frac{2 \sigma}{\rho \sqrt{g^2 + g'^2}} \) as \( \rho \to \infty \). Therefore, the fluxes of hybrid vortices are equal to

\[
\mathcal{F}^{(MH)}_{em} = \frac{2 \pi g [g^2 (1 - 2 \sigma) + g'^2]}{g g' \sqrt{g^2 + g'^2}} ,
\]

\[
\mathcal{F}^{(MH)}_Z = \frac{4 \pi \sigma}{\sqrt{g^2 + g'^2}} .
\]
FIG. 3: Dimensionless fields $W(r)/m$, $A^{(3)}(r)/m$, $A_\phi(r)/m$, $Z_\phi(r)/m$, and $v(r)/m$ as functions of $r = \rho m$ for the hybrid vortex solution with equal phases.

FIG. 4: Dimensionless fields $W(r)/m$, $A^{(3)}(r)/m$, $A_\phi(r)/m$, $Z_\phi(r)/m$, and $v(r)/m$ as functions of $r = \rho m$ for the hybrid vortex solution with opposite phases.
These expressions can be rewritten in the following transparent way:

\[ \mathcal{F}^{(MH)}_{em} = 1 \cdot \mathcal{F}^{(M)}_{em} + \sigma \cdot \mathcal{F}^{(H)}_{em}, \]  

where \( \mathcal{F}^{(M)}_{em,Z} \) and \( \mathcal{F}^{(H)}_{em,Z} \) are the fluxes of the magnetic and hypermagnetic vortices given by Eqs. (25) and (35), (36), respectively. This suggests that the hybrid vortices can be considered as composites of the magnetic and hypermagnetic ones. Note that the sum of the energies per unit length for the magnetic vortex and the hypermagnetic one is equal to 1.27 + 0.79 = 2.06. On the other hand, because the energies for the hybrid vortices with \( \sigma = + \) and \( \sigma = - \) are equal to 1.49 and 2.3, respectively, they satisfy the inequality 1.49 < 2.06 < 2.3. This inequality implies that while the hybrid vortex with equal phases is stable, the hybrid vortex with opposite phases is not.

VI. CONCLUSION

The set of vortex solutions we obtained in the gauged \( SU(2) \times U(1)_Y \) \( \sigma \)-model with the chemical potential for hypercharge is quite rich. In particular, there are different types of vortices connected either with photon field or hypercharge gauge field, or with both of them. The richness of this set is provided by the structure of the ground state [5], which includes vector condensates. This ground state describes an anisotropic medium with electric superconductivity.

It is quite noticeable that the sample of symmetry breaking in this model is the same as in the gluonic phase in neutral two-flavor QCD [12], while the present model is much simpler. The point is that because of a nonzero baryon density in the gluonic phase, its dynamics in the hard-dense-loop approximation is mostly provided by quark loops. And since most of the initial symmetries (including rotational \( SO(3)_{rot} \) in this phase are spontaneously broken, the calculation of its effective action is quite involved. This makes the study of vortex solutions in the gluonic phase to be quite a difficult problem. On the other hand, vortices could be very important for understanding the dynamics of quark matter in compact stars. The existence of such solutions in a related but much simpler model is encouraging.

It is instructive to compare the present vortex solutions with those in such complex condensed matter systems as the \( B \) phase in \( ^3\)He [16] and high-\( T_c \) superconductors [17]. Unlike the conventional Abrikosov-Nielsen-Olesen vortices [1, 2], the initial symmetries are not completely restored in the core of vortices in those systems. In \( ^3\)He, vortices with an A-phase core or with asymmetric core are realized. In high-\( T_c \) superconductors, there is an antiferromagnetic condensate in the vortex core. A similar situation takes place for the magnetic and hypermagnetic vortices in the present model. In the core of a magnetic vortex, while the order parameter \( W_2^{(-)} \) is zero, the order parameter \( \varphi_0 \) is not (see Fig. 1). Therefore, in its core only the \( U(1)_{em} \) symmetry is restored. In the core of a hypermagnetic vortex, we found that while \( \varphi_0 = 0 \), the order parameter \( W_2^{(-)} \) is not zero (see Fig. 2). Therefore, only the \( U(1)_Y \) symmetry is restored in the core in this case. On the other hand, the whole abelian \( U(1)_{em} \times U(1)_Y \) gauge symmetry is restored in the core of a hybrid vortex (see Figs. 3 and 4).

Recently, there has been a considerable interest in systems with coexisting order parameters (such as high-\( T_c \) superconductors) in condensed matter [17]. Generating vector condensates is a very natural way of creating such systems (for example, in the present model, electric superconductivity coexists with spontaneous rotational symmetry breaking). This possibility deserves further study.

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