Unitarization of Gluon Exchange Amplitudes
and Rapidity Gaps at the Tevatron

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Abstract

Rapidity gaps between two hard jets at the Tevatron have been interpreted as being due to the exchange of two gluons which are in an overall color-singlet state. We show that this simple picture involves unitarity violating amplitudes. Unitarizing the gluon exchange amplitude leads to qualitatively different predictions for the fraction of $t$-channel color singlet exchange events in forward $qq$, $qg$ or $gg$ scattering, which better fit Tevatron data.
I. INTRODUCTION

Over the past few years rapidity gaps, i.e. pseudorapidity regions without hadronic activity, have been observed in hadronic collisions at both the HERA ep collider [1] and in p\bar{p} collisions at the Fermilab Tevatron [2–4]. Such rapidity gaps are widely attributed to the exchange of color singlet quanta between incident partons [6–9], the exchange of two gluons in a color singlet state being the simplest such model [6]. At the Tevatron, a fraction \( f_{\text{gap}} \approx 1\% \) of all dijet events with jet transverse energies \( E_{Tj} \gtrsim 20 \text{ GeV} \) and jet separations of more than three units of pseudorapidity exhibit rapidity gaps between the jets. This observation is particularly striking since it demonstrates that color singlet exchange effects in QCD events are relevant at momentum transfers of order 1,000 GeV\(^2\), raising the hope that perturbative methods can be used for quantitative descriptions.

A gap fraction of order one percent was in fact predicted by Bjorken [7], in terms of a fraction \( f_s \approx 0.15 \) of dijet events which are due to \( t\)-channel color-singlet exchange and a survival probability \( P_S \) of rapidity gaps of order 10\% [7,10],

\[
  f_{\text{gap}} = f_s \ P_S .
\]

(1)

Here the survival probability estimates the fraction of hard dijet events without an underlying event, i.e. without soft interactions between the other partons in the scattering hadrons. Such multiple interactions would fill the rapidity gap produced in the hard scattering process. For \( Q\bar{q} \) elastic scattering, Bjorken estimated the color-singlet fraction \( f_s \) in terms of the imaginary part of the two-gluon \( t\)-channel exchange amplitude, which is known to dominate the forward scattering amplitude for \( t\)-channel color-singlet exchange. In impact parameter space, at impact parameters small compared to \( R = \mathcal{O}(1/\Lambda) \), the result is

\[
  f_s^{\text{impact}} = \frac{2}{9} \left[ \frac{1}{2} \alpha_s \left( \frac{1}{b^2} \right) \frac{\log R^2}{b^2} \right]^2 = \frac{1}{18} \left[ \frac{12\pi}{(33 - 2n_f) \log \frac{1}{b^2} \log R^2} \right]^2 \approx \frac{1}{2} \left[ \frac{4\pi}{33 - 2n_f} \right]^2 = 0.15 .
\]

(2)

Here \( 2/9 \) is the relative color factor of the two-gluon color-singlet to the one-gluon color-octet exchange cross section and \( R \) is an infrared cutoff parameter which regularizes the
two-gluon loop-integral. This model for the calculation of the color singlet fraction $f_s$, with the two gluon-exchange amplitude replaced by its imaginary part, will be called the two-gluon exchange model in the following.

In this model, the color singlet fraction grows with the color charge of the scattered partons. For $qq$ and $gg$ elastic scattering $f_s$ would be larger by factors $9/4$ and $(9/4)^2$, respectively [9]. This results in a substantial decrease of the observable gap fraction as the contribution from gluon induced dijet events is reduced, e.g. by increasing the average transverse momentum of the observed jets and thereby the Feynman-$x$ values of the incident partons. Such measurements have recently been reported by both the CDF [5] and the D0 [4] Collaborations, and no such effect is observed. In fact, the D0 data are compatible with a slight increase of the gap fraction with increasing jet $E_T$, casting doubt on the validity of the two-gluon exchange model [12].

In this paper we reconsider the basic ideas behind the two-gluon exchange model. We demonstrate its limitations and show that, even when starting from this perturbative picture of rapidity gap formation, the determination of the color singlet exchange fraction $f_s$ is essentially nonperturbative. We start from a basic feature of the two-gluon exchange model: unitarity fixes the imaginary part of the $t$-channel two-gluon exchange amplitude in terms of the Born amplitude and this imaginary part dominates $t$-channel color singlet exchange [7]. Rewriting this relationship in terms of phase shifts, the one- and two-gluon exchange amplitudes are found to be too large to be compatible with unitarity. Phase shift unitarization leads to a more realistic description, in which the total differential cross section remains unchanged compared to the Born result, but with $t$-channel color singlet exchange fractions which differ substantially from the expectations of the two-gluon exchange model. These features are demonstrated analytically for fixed values of the strong coupling constant, $\alpha_s$, in Section II. In Section III we then perform a numerical analysis for running $\alpha_s$, showing that the key properties of the fixed-$\alpha_s$ results remain unchanged.

The predicted color singlet fractions are found to very strongly depend on the regularization of gluon exchange at small momentum transfer, however, and thus cannot be reliably
calculated within perturbation theory. Within our unitarized model the non-perturbative
effects can be summarized in terms of two parameters, the survival probability of gaps,
$P_s$, and a universal Coulomb phase shift, $\psi_0$. Implications for the formation of gaps at
the Tevatron are analyzed in Section V. In particular we calculate how the gap fraction
between two hard jets varies with jet transverse energies and jet pseudorapidity separation
and then compare predicted fractions with Tevatron data [4,5]. Our conclusions are given
in Section V.

II. ELASTIC SCATTERING AMPLITUDE AND UNITARIZATION

Consider the elastic scattering of two arbitrary partons, $p$ and $P$,

$$p(i_1) + P(j_1) \rightarrow p(i_2) + P(j_2),$$

at momentum transfer $Q^2 = -t$. Here $i_1, \ldots, j_2$ denote the colors of the initial and final
state partons. The cross section and the partial wave amplitudes are completely dominated
by the forward region, $Q^2 \ll s$, where the Rutherford scattering amplitude,

$$\mathcal{M} = -8\pi \alpha_s \frac{s}{t} T^a \otimes T'^a = 8\pi \alpha_s \frac{s}{Q^2} F_c = \mathcal{M}_0 F_c,$$

provides an excellent approximation. Note that helicity is conserved in forward scattering,
hence spin need not be considered in the following. The only process dependence arises from
the color factor $F_c = T^a \otimes T'^a$.

A. Diagonalization in Impact Parameter and Color Space

In order to study unitarity constraints, we need to diagonalize the amplitude in both
momentum/coordinate space and in color space. The first step is most easily achieved by
transforming to impact parameter space,

$$T(b) = \int \frac{d^2q}{(2\pi)^2} \mathcal{M}(q)e^{-iq\cdot b}.$$
Neglecting multi-parton production processes, i.e. inelastic channels, unitarity of the S-matrix implies the relation

$$\mathcal{I}m\ T(b) = \frac{1}{4s}|T(b)|^2 ,$$  

(6)

for the full $2 \rightarrow 2$ scattering amplitude $T(b)$. Eq. (6) represents a matrix relation in color space. More fully it can be written as

$$\mathcal{I}m\ T(b)_{i_{2}j_{2},i_{1}j_{1}} = \frac{1}{4s} \sum_{i,j} T(b)_{i_{2}j_{2},ij} T^\dagger(b)_{ij,i_{1}j_{1}} ,$$  

(7)

where the sum runs over the dimension of the color space, $d_{C} = 9$ for $Qq$ and $Q\bar{q}$ scattering and $d_{C} = 24$ (64) for $qg$ ($gg$) elastic scattering.

Since the color factors can be written as hermitian matrices, the right-hand side of Eq. (7) represents a simple matrix product of the color matrices. This product is easily diagonalized by decomposing the color factors $F_{c}$ into a linear combination of projection operators onto the irreducible color representations which are accessible in the s-channel,

$$F_{c} = (F_{c})_{i_{2}j_{2},i_{1}j_{1}} = \sum_{k} f_{k} (P_{k})_{i_{2}j_{2},i_{1}j_{1}} = \sum_{k} f_{k} P_{k} .$$  

(8)

For the case of quark-antiquark elastic scattering, for example, with color decomposition $3 \otimes \bar{3} = 1 \oplus 8$, the color factor can be written in terms of Gell-Mann matrices as

$$(F_{c})_{i_{2}j_{2},i_{1}j_{1}} = \left(\frac{\lambda^{a}}{2}\right)_{i_{2}i_{1}} \left(\frac{\lambda^{a}}{2}\right)_{j_{1}j_{2}} = \frac{4}{9} \delta_{j_{1}i_{1}} \delta_{i_{2}j_{2}} - \frac{1}{3} \left(\frac{\lambda^{a}}{2}\right)_{j_{1}i_{1}} \left(\frac{\lambda^{a}}{2}\right)_{i_{2}j_{2}} = \frac{4}{3} P_{1} - \frac{1}{6} P_{8} .$$  

(9)

For all cases, $Q\bar{q}$, $Qq$, $gg$ and $gg$ elastic scattering, the decomposition into s-channel projectors is summarized in Table I. This color decomposition, combined with the transformation to impact parameter space, diagonalizes the unitarity relation for elastic scattering amplitudes.

**B. Phase Shift Analysis at Leading Order in $\alpha_{s}$**

Expanding the full $2 \rightarrow 2$ amplitude $T(b)$ into $s$-channel projectors, $T(b) = \sum_{k} T_{k}(b)\ P_{k}$, the individual coefficients $T_{k}$ are seen to satisfy the unitarity relation (6). The full scattering...
TABLE I. Representations and color operators for QCD elastic scattering. The indices of the projection operators \( P_k \) in the last column represent the dimensionalities of the irreducible color representations in the \( s \)-channel. Results for the \( 8 \otimes 8 \) decomposition are taken from Ref. [13].

| process | product representation and decomposition | color operator \( F_c \) |
|---------|----------------------------------------|--------------------------|
| \( Qq \) (\( Qq \)) | \( 3 \otimes 3 \) \( \overline{3} \oplus 6 \) | \(-\frac{2}{3}P_3 + \frac{1}{3}P_6 \) |
| \( Q\overline{q} \) | \( 3 \otimes \overline{3} \) \( 1 \oplus 8 \) | \( \frac{4}{3}P_1 - \frac{1}{6}P_8 \) |
| \( gg \) (\( g\overline{g} \)) | \( 8 \otimes 3 \) \( 3 \oplus \overline{6} \oplus 15 \) | \( \frac{3}{2}P_3 + \frac{1}{2}P_6 - \frac{1}{2}P_{15} \) |
| \( gg \) | \( 8 \otimes 8 \) \( 1 \oplus 8^S \oplus 8^A \oplus 10 \oplus \overline{10} \oplus 27 \) | \( 3P_1 + \frac{3}{2}P_8^S + \frac{3}{2}P_8^A - P_{27} \) |

Within perturbation theory, the individual phase shifts \( \delta_k(b) \) can be expanded in a power series in \( \alpha_s \). The lowest order term is fixed by the Fourier transform of the Born amplitude \( \mathcal{M}_0 \), which, however, diverges at small \( |q| \) and needs to be regularized. This is most easily done by an infrared cutoff, \( |q| > 1/R \), of the integral \( \mathcal{M}_0 \). One can interpret this infrared cutoff as a consequence of confinement; the color singlet nature of hadrons at scales larger than \( \approx 1 \) fm does not allow long wave-length gluons to couple and, hence, soft gluon exchange must be suppressed. The cutoff \( R \) is related to the size of the hadronic wave-function \( [14] \) and can be considered as a nonperturbative parameter in the following.

With the cutoff \( |q| > 1/R \), the Fourier transform of the Rutherford amplitude \( \mathcal{M}_0 \) to
impact parameter space is given by

\[ T_0(b) = \frac{4s}{2} \left( \log \frac{R^2}{b^2} + 2(\log 2 - \gamma) \right) F_c \equiv 4s \delta_0(b) F_c. \] (11)

Here \( \gamma = 0.577215 \ldots \) is Euler’s constant, and terms of order \( b^2/R^2 \) are neglected. Comparison with (10) yields

\[ \delta_k(b) = f_k \delta_0(b) + O(\alpha_s^2), \] (12)

where the \( f_k \) are taken from Table I. Keeping the lowest order term in (12) only, the transformation back to momentum space can be performed analytically for the full amplitude in (10), with the result

\[ \mathcal{M}(q) = 8\pi\alpha_s \frac{s}{q^2} \sum_k f_k P_k \exp \left( i\alpha_s f_k \log R^2 q^2 + O((\alpha_s f_k)^3) \right). \] (13)

As in the analogous QED case [15], the coefficient \( \mathcal{M}_k \) of each projector \( P_k \) is just the Born amplitude, multiplied by an infrared divergent phase factor \( i.e. \)

\[ \mathcal{M} = \sum_k \mathcal{M}_k P_k = \sum_k f_k \mathcal{M}_0 e^{i f_k \psi} P_k, \] (14)

with

\[ \psi = \alpha_s \log R^2 q^2 + O(\alpha_s^3). \] (15)

This general structure has important consequences for the total differential cross section, summed over all colors, and for the \( t \)-channel color singlet exchange rate. From (14) the color summed amplitude squared is given by

\[ \sum_{\text{colors}} |\mathcal{M}|^2 = \sum_k |\mathcal{M}_k|^2 d_k \] (16)

where \( d_k \) is the dimensionality of the \( k \)th irreducible color representation in the \( s \)-channel (see Table I). Since each \( \mathcal{M}_k \) equals its tree level value \( f_k \mathcal{M}_0 \), up to a phase, the total differential cross section remains unchanged by our phase shift unitarization.

*This is not necessarily true for other unitarization prescriptions. For example, for a “K-matrix
In order to understand the rapidity gap rate in hadronic collisions, we need the fraction of dijet events which are produced without color exchange in the $t$-channel. The color factor describing this situation is given by

$$F_s = \delta_{i_2i_1}\delta_{j_2j_1} = I,$$

which is just the unit operator, as far as the decomposition into $s$-channel projectors is concerned. For any given $2 \to 2$ process we define the $t$-channel color singlet exchange amplitude, $M_s$, as the coefficient of $F_s$. The exchange of color octet quanta in the $t$-channel or of yet higher color representations will be orthogonal to this term, i.e. no interference terms arise once the squared amplitude is summed over all colors, and this makes the definition of $M_s$ unique. Decomposing the full amplitude as $M = \sum_k M_k P_k = M_s F_s + \ldots$, the color singlet fraction, $f_s$ of Eq. (2), for any particular $2 \to 2$ process, is then given by

$$f_s = \frac{\sum\text{colors} |M_s I|^2}{\sum\text{colors} |M|^2} = \frac{\sum_k |M_k d_k|^2}{d_C \sum_k |M_k|^2 d_k},$$

where $d_C = \sum_k d_k$ is the dimensionality of the full color space.

Let us apply this expression to the full, unitarized amplitude of Eq. (14). The lowest order Rutherford amplitude $M_0$ cancels in the ratio of Eq. (18), which hence can be evaluated in terms of the coefficients $f_k$ and the dimensionalities $d_k$ of Table I. We find

$$f_s(Qq \to Qq) = \frac{8}{9} \sin^2 \frac{\psi}{2},$$

$$f_s(Q\bar{q} \to Q\bar{q}) = \frac{32}{81} \sin^2 \frac{3\psi}{4},$$

unitarization”, with $T_k(b) = 4s f_k T_0(b)/(4s - if_k T_0(b))$, we numerically find substantially reduced unitarized cross sections. For $gg$ scattering, $d\sigma(gg \to gg)/d\cos \theta$ can be a factor 5 lower than the Born result, even for large scattering angles. Such a unitarization procedure would be completely unacceptable phenomenologically.
\[ f_s(qg \rightarrow qg) = \frac{1}{8} \sin^2 \psi + \frac{15}{32} \sin^2 \psi , \]
\[ f_s(gg \rightarrow gg) = \frac{9}{16} \sin^2 \frac{5\psi}{4} - \frac{1}{16} \sin^2 \frac{3\psi}{4} + \frac{9}{128} \sin^2 2\psi . \]

These color singlet fractions are plotted as a function of \( \psi \) in Fig. 1.

![Graph showing the fraction of t-channel color singlet exchange events in Qq, Q\bar{q}, qg and gg elastic scattering as a function of the universal phase \( \psi \). See text for details.](image)

**FIG. 1.** Fraction of \( t \)-channel color singlet exchange events in \( Qq, Q\bar{q}, qg \) and \( gg \) elastic scattering as a function of the universal phase \( \psi \). See text for details.

From (15) we see that, formally, the universal phase \( \psi \) is of order \( \alpha_s \) and thus the color singlet fractions are of \( O(\alpha_s^2) \), which agrees with Bjorken’s result of Eq. (2). Expanding (20) to lowest order, one obtains

\[ f_s(Q\bar{q} \rightarrow Q\bar{q}) \approx \frac{2}{9} \psi^2 = \frac{2}{9} |\alpha_s \log R^2 q^2|^2 , \]

which appears to be four times larger than the result given in (2). This conundrum is resolved by observing that (2) represents a cross section ratio in impact parameter space while (15-22) are the color singlet fractions in momentum space. Indeed, in impact parameter space and to leading order in \( \alpha_s \), Eqs. (10-12) imply
which agrees with Eq. (2).† The factor four difference between the color singlet fractions in momentum and impact parameter space can be traced to a binomial factor 2 in the Fourier transform to momentum space of \( \log^2 \frac{R^2}{b^2} = \log(qR)^2 - \log(bq)^2 \) compared to the transform of \( \log \frac{R^2}{b^2} = \log(qR)^2 - \log(bq)^2 \): the relevant term is the one linear in \( \log(bq)^2 \), which is enhanced by a factor 2 in the first case. Since all experiments are analyzed in momentum space, the predicted color singlet fractions in the two-gluon exchange model need to be multiplied by a factor 4 compared to the results derived from (3). For gluon-gluon scattering in particular, this would lead to a color singlet fraction of \( f_s(gg) = 0.15 \times 4 \times (9/4)^2 \approx 3 \), which obviously cannot hold.

The reason for this problem is the fact that, with the same arguments as used in Eq. (2), the phase appearing in Eqs. (19-22) is approximately given by \( \psi \approx 12\pi/(33 - 2n_f) = 1.64 \) which is too large to make use of a small angle expansion. Instead, (22) predicts \( f_s(gg \rightarrow gg) = 0.39 \), which is still a surprisingly large color singlet fraction but almost an order of magnitude smaller than the two-gluon exchange result.

Indeed, a resummation of higher order terms is required by unitarity. In the two-gluon exchange model, in impact parameter space, the amplitude for a particular color representation in the s-channel is given by

\[
\frac{1}{4s} T_k(b) = \delta_0(b)f_k + i(\delta_0(b)f_k)^2
\]

with \( \delta_0(b) \approx 6\pi/(33 - 2n_f) = 0.82 \), for small impact parameters. Here box corrections to the real part are neglected. The unitarity relation (5), on the other hand, implies

\[
|\mathcal{R}e\left( \frac{1}{4s} T_k(b) \right)| < 0.5 ,
\]

\footnote{For \( Q\bar{q} \rightarrow Q\bar{q} \) scattering and in impact parameter space we thus agree with Ref. [7] while the additional factor four in momentum space was missed in Ref. [3].}
a condition which is violated for all color channels with $|f_k| \geq 2/3$ in Table I. Since the two-gluon exchange model violates unitarity, we study implications of the unitarized extension of this model in the following.

So far we have estimated the value of the universal phase $\psi$ by replacing $\alpha_s$ by its running value $\alpha_s(Q^2)$ in (13),

$$\psi(Q^2) = \alpha_s(Q^2) \log R^2 Q^2 = \frac{4\pi}{11 - \frac{2}{3}n_f} \left(1 + \frac{\log R^2 \Lambda^2}{\log Q^2 / \Lambda^2}\right) = \frac{4\pi}{\beta_0} + \alpha_s(Q^2) \log R^2 \Lambda^2,$$

and then using the asymptotic expression for $\log Q^2 \to \infty$, i.e. neglecting the $O(\alpha_s)$ term in (27). This corresponds to setting the cutoff parameter $R = 1/\Lambda$. In a more complete calculation $R$ describes the transverse length scale at which color screening, due to other partons in the proton, sets in, thus suppressing the effective gluon coupling. In effect, $\kappa \equiv R\Lambda \approx 1$ is a non-perturbative parameter for which we only have a rough guess, and which may be uncertain to at least a factor three, if not an order of magnitude. With $4\pi/\beta_0 = 0.52\pi = 1.64$, and $\alpha_s(Q^2) \approx 0.14$ at momentum transfers relevant for the Tevatron rapidity gap data, a variation of $\kappa$ by a factor 10 leads to changes in $\psi$ by $30\%$ or more. A change of this order, in particular an increase of $\psi$, can drastically change the predicted color singlet fractions for individual processes, as is obvious from Fig. I.

### III. RUNNING COUPLING EFFECTS

An increase of $\psi$ is, in fact, to be expected when including running coupling effects in the determination of the tree level phase shift $\delta_0(\mathbf{b})$ in Eq. (11). A running coupling increases the average size of the Born amplitude (11) in the Fourier transform to impact parameter space, which leads to larger values of $\delta_0(\mathbf{b})$ in (11) and this translates into a larger phase $\psi$ in (13).

We have analyzed this question quantitatively by determining the partial wave phase shifts which correspond to the Born amplitude, with running $\alpha_s(Q^2)$, and then unitarizing the partial wave amplitudes as in (10). In terms of the Born amplitude $\mathcal{M}_0(Q^2)$ the phase shifts for fixed angular momentum $J$ are, to lowest order, given by
\[ \delta_j = \frac{1}{32\pi} \int d\cos \theta \, P_j(\cos \theta) \, M_0 \left( Q^2 = \frac{s}{2}(1 - \cos \theta) \right). \quad (28) \]

This integral is singular at \( Q^2 = 0 \), via the \( 1/Q^2 \) pole of the Rutherford amplitude, and at \( Q^2 = \Lambda^2 \), via the Landau pole of \( \alpha_s(Q^2) \). Both singularities need to be regularized, for which we introduce two (independent) mass parameters, \( M \) and \( M_\alpha \). We thus replace the Born amplitude by

\[ \mathcal{M}_0(Q^2) = 8\pi \, \alpha_s(Q^2, M^2_\alpha) \, \frac{s}{Q^2 + M^2} = \frac{32\pi^2}{\beta_0} \, \frac{1}{\log \frac{Q^2 + M^2_\alpha}{\Lambda^2}} \, \frac{s}{Q^2 + M^2}. \quad (29) \]

We do not expect this amplitude to correctly describe the actual QCD matrix elements at low \( Q^2 \). By varying the infrared cutoff parameters \( M \) and \( M_\alpha \) we rather explore the importance of the small \( Q^2 \) region and, thus, the importance of non-perturbative effects which we are unable to calculate. The resulting unitarized amplitudes are now given by

\[ \mathcal{M}_k(\theta) = 8\pi \sum_{J=0}^{\infty} (2J + 1) P_J(\cos \theta) T_{k,J} \quad (30) \]

with

\[ i T_{k,J} = \exp \left( i \frac{4\pi}{\beta_0} f_k \int_{-1}^{1} dx P_J(x) \frac{1}{(z - x) \log \frac{s}{2\Lambda^2}(z_\alpha - x)} \right) - 1. \quad (31) \]

Here the \( P_J(x) \) are Legendre polynomials, and \( z = 1 + 2M^2/s \) and \( z_\alpha = 1 + 2M^2_\alpha/s \) contain the two regularization parameters.

The integration in (31) and the partial wave sum in (30) are performed numerically. Results are shown in Fig. 2, where possible variations of the color singlet fraction with the regularization parameters \( M \) and \( M_\alpha \) are explored. A very strong variation of \( f_s \) is found for individual scattering processes. But, as we shall demonstrate below, these variations tend to be averaged out to a large extent when summing over the various partonic subprocesses which contribute to actual dijet data.

The parameter dependence of \( f_s \) for running strong coupling constant in Fig. 4 is reminiscent of the variation of \( f_s \) with the universal phase \( \psi \), which is shown in Fig. 1 for our analytical results (13–22). The similarities between the running coupling partial wave expansion (RPWE) and the impact parameter space (IP) results go far deeper, in fact 16.
FIG. 2. Dependence of the $t$-channel color singlet-exchange fraction on the regularization parameters (a) $M$ and (b) $M_\alpha$ (see Eq. (29)). Results are shown for fixed momentum transfer, $Q = 30$ GeV, and fixed forward scattering angle.

Similar to the analytical IP results, the RPWE calculation leads to a total differential cross section, summed over all colors, which agrees with the Born result to few percent accuracy. In addition, the color singlet fractions at fixed $Q^2$ are found to be independent of scattering angle or parton center of mass energy. Indeed, with an accuracy of a few percent, the numerical results of the RPWE calculation can be parameterized as in \[ \psi(Q^2, M, M_\alpha) \] which is universal for all subprocesses. As is obvious from Fig. 2 the dependence of $\psi$ on the regularization parameters is quite strong. Its $Q^2$-dependence, on the other hand, is logarithmic only and the ansatz

\[ \psi(Q^2) = \psi_0 + \psi_1 \log \frac{Q}{Q_0} + \psi_2 \log^2 \frac{Q}{Q_0} \] \hspace{1cm} (32)

provides an excellent parameterization of the RPWE results. Representative values for the coefficients $\psi_i$ are given in Table I. Only $\psi_0$ is found to depend appreciably on the regularization parameters while $\psi_1 = 0.256$ and $\psi_2 = -0.019$ (for $Q_0 = 50$ GeV) are constant within the numerical uncertainty. The variation of $\psi(Q)$ with momentum transfer is modest, with $\Delta \psi(Q) \approx \pm 0.2$ in the interval $20$ GeV $< Q < 100$ GeV. In this region, which is of interest for the Tevatron, the RPWE values for $\psi(Q^2)$ in general are substantially larger
TABLE II. Dependence of the universal phase $\psi(Q^2)$ on the regularization parameters $M$ and $M_\alpha$ (see Eq. (29)). The $\psi_i$ are the coefficients of the expansion in (32) for $Q_0 = 50$ GeV.

| $M$ [GeV] | $M_\alpha$ [GeV] | $\psi_0$ | $\psi_1$ | $\psi_2$ |
|-----------|------------------|---------|---------|---------|
| 0.01      | 0.2              | 8.87    | 0.254   | -0.019  |
| 0.01      | 1.0              | 4.63    | 0.254   | -0.020  |
| 0.01      | 5.0              | 3.25    | 0.257   | -0.017  |
| 0.10      | 0.2              | 4.46    | 0.255   | -0.017  |
| 0.10      | 1.0              | 3.07    | 0.255   | -0.018  |
| 0.10      | 5.0              | 2.31    | 0.257   | -0.020  |
| 1.00      | 0.2              | 1.71    | 0.255   | -0.020  |
| 1.00      | 1.0              | 1.59    | 0.256   | -0.020  |
| 1.00      | 5.0              | 1.38    | 0.257   | -0.021  |

than for the analytic impact parameter space calculation, thus confirming the qualitative arguments made earlier.

Beyond demonstrating the relation of the numerical RPWE results to the analytical expressions derived in impact parameter space, the existence of the simple parameterization in (32) is very important in order to compare our calculations to experimental data. The numerical integrations which need to be done in (31) are too slow to be performed for individual phase space points in a Monte Carlo calculation of dijet cross sections. With the above observations this is not necessary, however, since instead we can use the analytical results of (19-22) together with the parameterization of Eq. (32).
IV. COMPARISON WITH TEVATRON DATA

Both the D0 [4] and the CDF [5] collaborations at the Tevatron have analyzed the fraction of dijet events with rapidity gaps, as a function of both the transverse energy, $E_T$, and the pseudorapidity separation, $\Delta \eta = |\eta_{j1} - \eta_{j2}|$, of the two jets. As these phase space variables change, the composition of dijet events varies, from mostly gluon initiated processes at small $E_T$ and $\Delta \eta$ (and, hence, small Feynman-x, $x_F$) to $Q\bar{q}$ scattering at large values. A dependence of the gap fraction on the color structure of the scattering partons would thus be reflected in a variation with $E_T$ and $\Delta \eta$.

Bjorken’s two gluon exchange model, which is equivalent to the small $\psi$ region in our analysis, predicts a larger fraction of color singlet exchange events for gluon initiated processes [7,9] (see Fig. 1 for $\psi \lesssim 0.3\pi$). The gap fraction should thus decrease with increasing $E_T$ or $\Delta \eta$. The opposite behavior is expected in statistical models of color rearrangement [11,12]. Here the eight color degrees of freedom for gluons, as compared to three for quarks, make it less likely for gluon initiated processes that $t$-channel color singlet exchange is achieved by random color rearrangement. This would lead to a smaller gap fraction at small $x_F$ and therefore small $E_T$ or $\Delta \eta$.

In the unitarized RPWE framework, the dependence on the regularization parameters is sufficiently strong to encompass both scenarios. This is demonstrated in Figs. 3 and 4, where the results of the running coupling analysis for three choices of the regularization parameters are compared with Tevatron data, taken at $\sqrt{s} = 1800$ GeV. The data correspond to dijet events with two opposite hemisphere jets of $E_T > 20$ GeV, $|\eta_j| > 1.8$ (CDF) or $E_T > 30$ GeV, $|\eta_j| > 1.7$ (D0). D0 data are taken from Ref. [4] and show the fraction of dijet events with rapidity gaps. CDF [8] shows the ratio of gap fractions in individual $\Delta \eta$ and $E_T$ bins to the overall gap fraction in the acceptance region. For comparing our calculation with the data we fix the survival probability $P_s$ in Eq. (1) to reproduce the overall gap fraction in the acceptance region, which was measured as $f_{gap} = (0.85 \pm 0.06 \pm 0.07)\%$ for the D0 sample and $f_{gap} = (1.13 \pm 0.12 \pm 0.11)\%$ for the CDF sample. Required survival probabilities strongly
depend on regularization parameters and vary between 1.9% and 5.5%, which is on the low side of previous estimates [7,10]. For a given choice of regularization parameters, predictions for the $E_T$ or $\Delta \eta$ dependence of the gap fraction are quite similar for the CDF and D0 cuts. To the extent that the two data sets are consistent within errors, it is not yet possible to discriminate between different choices of regularization parameters, i.e. to obtain sensitivity to the non-perturbative dynamics.

D0 data somewhat favor color singlet fractions which grow with $x_F$ and which are more in line with expectations from color evaporation models [11,12]. Note that our unitarized gluon exchange model, with $M = 0.2$ GeV and $M_\alpha = 0.5$ GeV is able to describe this trend, even though it is an extension of the two-gluon exchange model. CDF data slightly prefer a gap fraction which decreases with increasing $\Delta \eta$ and, hence, with larger $x_F$. The unitarized gluon exchange model, with $M = 0.1$ GeV and $M_\alpha = 0.2$ GeV describes such a situation. Comparison with Fig. 2 shows that this set of parameters predicts a much smaller gap fraction for $Q\bar{q}$ scattering than for gluon initiated processes, which is qualitatively similar to the two-gluon exchange model [7,9]. Indeed, the shape of the gap fraction for the two gluon exchange model is very similar to the long-dashed curves in Fig. 4.

Clearly, the data are not yet precise enough to unambiguously distinguish between these different scenarios. On the theoretical side, the variation of the RPWE predictions with model parameters highlights the limitations of a perturbative approach to the color singlet exchange probability. Taking the phase $\psi_0$ and the survival probability $P_s$ as free parameters, the unitarized two-gluon exchange model is clearly capable of fitting the present Tevatron data, however.

V. DISCUSSION AND CONCLUSIONS

The formation of rapidity gaps in hadronic scattering events is a common occurrence, and its ubiquity asks for a theoretical explanation within QCD. The formation of gaps between two hard jets at the Tevatron is particularly intriguing and is commonly being explained in
FIG. 3. Dependence of rapidity gap fraction as a function of (a) $\Delta \eta$ and (b) jet $E_T$ for three choices of regularization parameters which are given in units of GeV. Symbols with error bars correspond to the D0 data as given in Ref. [4]. Also shown are the survival probabilities needed to reproduce the overall rapidity gap fraction of 0.85% as measured by D0.

terms of color singlet exchange in the $t$-channel, be it via an effective color singlet object like the “Pomeron” or via a statistical color rearrangement, in terms of multiple soft gluon exchange.

“Pomeron” exchange models build on the observation that color singlet exchange in the $t$-channel can be achieved in QCD via the exchange of two gluons, with compensating colors [5]. When trying to build a quantitative model for the formation of rapidity gaps [7], one encounters infrared divergences in the color singlet hard scattering amplitude, which in a full treatment would be regularized by the finite size and the color singlet nature of physical hadrons [14]. In turn, this indicates that non-perturbative information may be indispensable for a quantitative understanding of the hard color singlet exchange process.

We have analyzed this question within a particular model, based on the unitarization of single gluon exchange in the $t$-channel. The Low-Nussinov model [8] corresponds to a truncation of the unitarization at order $\alpha_s^2$. We find that, for any reasonable range of regularization parameters, the two-gluon exchange approximation violates partial wave unitarity,
FIG. 4. Dependence of rapidity gap fraction as a function of a) $\Delta \eta$ and b) $E_T$. Shown is the gap fraction in a given bin, normalized to the inclusive gap fraction of 1.13% as measured by CDF. CDF data are shown with error bars together with predictions of the unitarized RPWE calculation, for the three choices of regularization parameters as in Fig. 3.

and thus a fully unitarized amplitude is needed for phenomenological applications.

The unitarization of hard elastic quark and gluon scattering is not unique, of course, but any acceptable method must preserve the successful description of hard dijet events by perturbative QCD. The phase shift approach used here fulfills this requirement: the unitarization does not change the Born-level predictions for the color averaged differential cross sections. As a corollary, the color-inclusive dijet cross section is independent of the regularization parameters which need to be introduced for the full phase shift analysis.

The situation is entirely different when considering the $t$-channel color singlet exchange component which is introduced by the exchange of two or more gluons or by unitarization. The $t$-channel color singlet exchange fraction, $f_s$, is strongly affected by the full unitarization and deviates from the expectations of the two-gluon exchange approximation, changing even the qualitative predictions of the Low-Nussinov model. These strong unitarization effects are reflected by a strong dependence on the precise regularization procedure. This cutoff dependence, which parameterizes non-perturbative effects, does not allow to make...
quantitative predictions for the color singlet exchange fractions in particular partonic subprocesses. These limitations, which have been demonstrated here for the two-gluon exchange approximation to the Pomeron, may be generic to Pomeron exchange models, and should be analyzed more generally.

In spite of these limitations, we find some intriguing features of the unitarized gluon exchange amplitudes. For all partonic subprocesses and for all regularization parameters, the color singlet exchange fractions can be described in terms of a single universal phase, $\psi(Q^2)$, which absorbs all non-perturbative effects. This suggests a unified phenomenological description of the rapidity gap data, via the gap survival probability $P_s$ and the phase $\psi(Q^2)$. Such an analysis goes beyond the transverse momentum and pseudorapidity dependence of rapidity gap fractions which have just become available, and should best be performed directly by the experimental collaborations.

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