Bimorphic Floquet topological insulators

Georgios G. Pyrialakos, Julius Beck, Matthias Heinrich, Lukas J. Maczewsky, Nikolaos V. Kantartzis, Mercedes Khajavikhan, Alexander Szameit and Demetrios N. Christodoulides

Topological theories have established a unique set of rules that govern the transport properties in a wide variety of wave-mechanical settings. In a marked departure from the established approaches that induce Floquet topological phases by specifically tailored discrete coupling protocols or helical lattice motions, we introduce a class of bimorphic Floquet topological insulators that leverage connectional chains with periodically modulated on-site potentials to reveal rich topological features in the system. In exploring a ‘chain-driven’ generalization of the archetypical Floquet honeycomb lattice, we identify a rich phase structure that can host multiple non-trivial topological phases associated simultaneously with both Chern-type and anomalous chiral states. Experiments carried out in photonic waveguide lattices reveal a strongly confined helical edge state that, owing to its origin in bulk flat bands, can be set into motion in a topologically protected fashion, or halted at will, without compromising its adherence to individual lattice sites.

Floquet engineering provides a powerful tool for shaping the topological structure of fermionic and bosonic settings alike. At its core, it relies on the fact that a proper periodic modulation can induce chirality to arrangements of non-interacting particles, thereby giving rise to a wide range of topological insulators (TIs). Under these conditions, symmetry-protected states emerge along the boundaries of a modulated lattice, allowing for scatter-free transport on edges or interfaces with static (topologically trivial) domains. As such, helical transport constitutes a hallmark signature of complex topological order, and its presence in Floquet-driven systems highlights how periodic modulations can systematically extend the original classification of topological phases beyond static systems with spin–orbit coupling or magnetic order. Along these lines, different topological invariants such as the winding number are required to describe the topological nature of gaps with vanishing Chern numbers (refs. 21–23). These new degrees of freedom have enabled the experimental realization of a wide variety of topological systems in photonic lattices, ranging from anomalous Floquet TIs to systems exhibiting Weyl point dynamics, Anderson TIs, TIs in synthetic dimensions, photonic Z2 TIs exhibiting fermionic time-reversal symmetry and topological lasers. Quite recently, anomalous driving protocols have enabled the observation of solitons in topological bandgaps, the creation of nonlinearity-induced TIs and investigations into the nonlinear dynamics of higher-order topological insulators.

Historically, photonic waveguide Floquet insulators can be divided into two distinct categories according to the character of the modulation involved. The first class is based on diatomic lattices, whereby a topological regime may be brought about by helical trajectories of the individual waveguide sites, introducing an effective chiral gauge field and, in turn, a non-trivial Chern insulating phase to the system. On the other hand, more complex Floquet phases can be synthesized by step-wise coupling protocols that periodically allow for the selective interaction between neighboring sites (along different lattice vectors) by varying their corresponding separations (Fig. 1c). Approaching the synthesis of topological phases from two opposite directions, namely discrete coupling steps as opposed to the effective-medium strategy of rapidly spiralling waveguides, these two regimes would appear to be mutually exclusive. In this work, we introduce a third paradigm that embodies the distinctive characteristics of both aforementioned classes while overcoming their most pressing physical constraints. To this end, we introduce interstitial elements between the sites of a static periodic lattice that act as Floquet drivers for the system (Fig. 1d). In doing so, these sites allow us to leverage the periodic modulation of their on-site potentials in order to synthesize hybrid topological systems whose band structure simultaneously hosts both conventional Chern insulator bands and anomalous topological bands, and is even capable of supporting topological phases with higher ranked invariants, in lattices with z-invariant site positions.

Floquet potentials in a chained honeycomb lattice. To exemplify our approach, let us consider a honeycomb lattice of weakly coupled elements, in which the three nearest-neighbour couplings vary independently in a cyclic fashion along the time (or propagation) coordinate. Despite a vanishing Chern number, such arrangements are known to be capable of supporting different insulating and non-trivial Floquet topological phases. The topological properties of this anomalous system are characterized by a three-dimensional winding number \( \mathcal{W} \), which considers the temporal evolution of the system and counts the edge states that cross a particular bandgap. The Chern number of any band can be associated with the difference between the adjacent gaps’ winding numbers, \( \mathcal{C} = \mathcal{W}_{\text{above}} - \mathcal{W}_{\text{below}} \) (refs. 21,22). A spatially fixed implementation of a non-trivial Floquet phase is forbidden by the symmetry rules of a diatomic lattice. In other words, no conceivable modulation of their two on-site potentials is capable of imbuing the Hamiltonian with helicity. In addition, any imbalance between the potentials inadvertently breaks the sublattice symmetry, and...
in turn introduces a trivial bandgap that prevents the topological transition.

To overcome these limitations, we introduce interstitial elements between the main sites of the potential lattice, one in each coupling path between adjacent sites of the two sublattices. Figure 2 illustrates how the band structure of the honeycomb system (Fig. 2a) is altered in the presence of these interstitial sites (Fig. 2b). Two copies of the honeycomb spectrum (Fig. 2a), each of which features a pair of Dirac cones in the first Brillouin zone, coexist symmetrically around multiple degenerate flat bands located at zero energy. The latter are composed of compact localized states that reside exclusively within the chain elements, locked by the destructive interference of light at the main sites. This particular modal structure is supported by the presence of additional symmetries in a system that hosts an odd total number of fundamental eigenmodes \(^{33,34}\), here originating from the \(5 \times 5\) bulk Hamiltonian

\[
H(\mathbf{k}) = \frac{3}{2} \left( \beta_j(t) \psi_j^\dagger \psi_j + \beta_j(t) \psi_j^\dagger \psi_j + \text{h.c.} \right)
\]

where \(\mathbf{k}\) is the Bloch vector momentum, \(t\) is the time variable, \(\psi^\dagger\) and \(\psi\) are the creation and annihilation operators on the lattice sites and \(\delta_A\) and \(\delta_B\) are lattice vector displacements between the \(j\)th chain element and the main site of sublattices A and B, respectively. Here, h.c. stands for the Hermitian conjugate. Out of the fifteen independent terms of the Hermitian Hamiltonian, only nine remain in equation (1), six involving nearest-neighbour interactions (with couplings \(c_1, c_2\), and \(c_3\)) and three involving on-site potential shifts exclusively at the interstitial sites (\(\beta_1, \beta_2, \beta_3 \neq 0; \beta_A, \beta_B = 0\)). Each of these terms can be addressed independently without perturbing the balance between sites A and B, thus fully preserving sublattice symmetry in the unit cell. In a continuous representation of the system (that is, the Schrödinger equation with a continuous refractive index profile), the time variation of both \(c\) and \(\beta\) can be realized via modification of the potential contrast at the interstitial sites—"in other words, it is specifically these sites that can be leveraged to bring about a topological phase transition.

We consider how the system responds under sinusoidal modulations that are chirally phase-shifted by \(2\pi/3\) between the three interstitial sites surrounding each primary site. In this case, the potential at the site between elements A and B of the \(j\)th chain is given by

\[
V(t) = 1 + \sin(2\pi T + 2\pi j/3), \quad j \in \{1,2,3\}
\]

where \(T\) denotes the Floquet period. Note that, by using an effective \(2 \times 2\) tight-binding representation of the system (Supplementary Section 1), this is equivalent to a continuous helical rotation of the principal direction of maximal coupling, despite the spatially static arrangement of the lattice sites. The topological changes due to the periodic drive can be traced by means of the unitary \(k\)-space evolution operator

\[
U(\mathbf{k}, t) = \mathcal{T} \exp \left( -i \int_0^t \text{d}t' H(\mathbf{k}, t') \right)
\]

where \(\mathcal{T}\) denotes time-ordering and \(H\) corresponds to the time-dependent Hamiltonian integrated over time \(t'\). In this respect, we decompose \(U = U_0 U_d\) into a product of a quasi-static term \(U_0 = \exp(-iH_{0d}t)\) with \(H_{0d} = i/T \log(U(\mathbf{k}, T))\) and a dynamic term \(U_d\) that accounts for the periodic micro-motion of the mode during the period of the drive with \(U_d(\mathbf{k}, T) = \mathbb{I}\) the identity operator\(^{35}\). Note that the effective Hamiltonian conforms to the traditional ten-fold way classification\(^{15}\), which, in this two-dimensional setting, is characterized by a \(Z\) topological invariant. As depicted in the example of Fig. 2c, for a Floquet period of \(T = 2\pi/7\), the quasi-energy
Observation of topological compact localized states. To experimentally probe the propagation dynamics of the different topological states in chain-driven lattices, we employ femtosecond laser direct-written photonic lattices as a platform for their implementation (Methods). The evolution of light in such systems is governed by a Schrödinger equation in which the propagation coordinate $z$ represents time and the refractive index profiles of the individual waveguides act as interacting potential wells. In this context, the effective refractive index $\Delta n_{\text{eff}}$ of each waveguide provides direct control over the on-site potential and can be seamlessly tuned by modulating the inscription velocity along the propagation coordinate $z$. In turn, evanescent coupling between adjacent waveguides instantiates the required hopping terms. Having confirmed numerically that the desired characteristics of the 5 × 5 tight-binding model can be faithfully reproduced (Supplementary Section II) within the experimentally accessible parameter range of our platform, we fabricated triangular chain-driven lattices composed of 42 unit cells (Fig. 3a). Despite its decidedly bristly appearance, the lattice is in fact terminated by the chained generalization of three zigzag-type edges, since the outermost waveguides represent interstitial sites. In a first set of experiments, we targeted the dispersive Chern states in the vicinity of the Dirac points by synthesizing a spectrally narrow wave packet of an appropriate wave vector via a tripartite excitation pattern with alternating phases injected into three consecutive primary sites along the vertical edge of the system. A series of measurements for different initial positions (Fig. 3c) clearly shows a systematic counterclockwise transport that is confirmed by extended-range numerical simulations (Fig. 3d) and allows light to circumnavigate the corners of the waveguide array. By contrast, when the alternating phase is removed from the excitation pattern, strong bulk diffraction was observed (Extended Data Fig. 1), highlighting the absence of the waveguide array. By contrast, when the alternating phase is removed from the excitation pattern, strong bulk diffraction was observed (Extended Data Fig. 1), highlighting the absence of the waveguide array. By contrast, when the alternating phase is removed from the excitation pattern, strong bulk diffraction was observed (Extended Data Fig. 1), highlighting the absence of the waveguide array. By contrast, when the alternating phase is removed from the excitation pattern, strong bulk diffraction was observed (Extended Data Fig. 1), highlighting the absence of the waveguide array. By contrast, when the alternating phase is removed from the excitation pattern, strong bulk diffraction was observed (Extended Data Fig. 1), highlighting the absence.
of the Chern state in the centre of the Brillouin zone (at Bloch momenta \( k_x = 0 \)).

Owing to their flat-band origins and Brillouin-zone-spanning nature (Fig. 4a), the anomalous topological edge states of the chain-driven lattice can be readily populated by injecting light into individual non-driven interstitial sites. In a second set of experiments, we therefore traced the propagation of such single-site excitations along the edge and around two corners of the system (Fig. 4b), and observed herhel topological transport in a counter-clockwise direction that, in contrast to the dispersive Chern channel, maintains the narrow width of the edge wave packet. This behaviour is also confirmed by numerical extended-range propagation simulations (Fig. 4c). Single-site excitations of bulk interstitial sites instead remain localized at their initial positions (Fig. 4d,e) as dictated by the quasi-static part \( U_0 \) of the evolution operator, despite the fact that the dynamic part \( U_1 \) intermittently allows light to enter the neighbouring sites during each Floquet cycle. Comparing these results to the discrete diffraction in a reference lattice of identical geometry implemented without the periodic modulation, we find that light injected into the interstitial sites on the edge as well as within the bulk of the non-driven lattice remains localized by virtue of the flat-band states residing there (Fig. 4f–i). This complementary behaviour opens up the possibility of imprinting arbitrary excitation patterns in the edge channels of chain-driven Floquet TIs. The fact that the Floquet drive can be temporarily frozen and resumed at will without any changes to the lattice geometry provides exceptional control over their topological transport dynamics: once synthesized, compact wave packets can propagate along the edge without being subjected to dispersive broadening, while the unique properties of the flat band allow them to be freely shifted between travelling and localized states without ever rendering them vulnerable to bulk diffraction (Supplementary Video 1).

Discussion

Notably, the chained lattice exhibits a number of higher-order topological phases that may occur for alternate periods of the drive. Changing the period \( T \) of the modulation, in relation to the scale given by the inverse coupling in the system, allows adjacent unit cells of the \( 2\pi/T \)-periodic quasi-energy spectrum to overlap in a topologically non-trivial fashion. In the conventional coupling-modulated honeycomb system, a critical period \( T_c = \pi/3 \) separates the Chern phase \( (C = \pm 1 \text{ and } W = 0) \) for \( T > T_c \) and the anomalous phase \( (C = 0 \text{ and } W = 1) \) for \( T < T_c \), as illustrated in Extended Data Fig. 2. By contrast, the upper- and lowermost bands of the chain-driven lattice exhibit \( |C| > 1 \) for any finite value of \( T \), allowing for the Chern-type and anomalous regimes to naturally coexist and interact in new and interesting ways: as \( T \) is increased above a new critical point \( T_{c'} = \pi > T_C \), the bands closer to zero energy enter a higher-order Chern phase \( (C = \pm 2) \) as additional pairs of topological edge states emerge in their neighbouring gaps (Extended Data Fig. 3 and Supplementary Fig. 5).

In this work, we proposed and experimentally demonstrated a bimorphic class of Floquet TIs based on periodic modulations of certain on-site potentials. We showed how a ‘chained’ honeycomb lattice, in which the exchange of population between primary sites is mediated by interstitial sites, can be endowed with rich topological features without resorting to magnetic interactions, helical lattice motion’ or complex coupling protocols. Beyond providing a complementary route towards inducing topology, our ‘chain-driven’ systems synergistically combine the characteristic features of conventional and anomalous Floquet TIs, allowing for the simultaneous existence of Chern-type chiral states and transport without dispersion in quasi-localized wave packets. These anomalous modes bifurcate from the flat band of the static system and, as such, can be readily converted into their likewise topologically protected,
compactly localized counterparts. While our findings are general and can be readily adapted to any topological platform that offers the means to dynamically control the on-site potential, such as cold atoms\(^{37}\), electronic circuits\(^{38}\) or even mechanics\(^{39}\), the capability to affect topological phase transitions without changes to the lattice geometry is of particular importance in the context of topological photonics, where curved waveguide trajectories inevitably entail additional losses. Moreover, it paves the way towards sophisticated designs involving several coexisting modulation periods and even regions with opposite helicity that can seamlessly interact without associated local coupling defects that would be inevitable in systems with modulated waveguide trajectories. Along these lines, we envision a new generation of low-loss robust photonic circuitry in which optically encoded packets of information can be transported, steered and even reshuffled without compromising their topological protection at any point.

**Online content**

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41563-022-01238-w.

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![Fig. 4 | Anomalous edge-state propagation and compact localized bulk states.](Image)
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Methods

**Experimental configuration.** The photonic structures used in our experiments are inscribed by focusing ultrashort laser pulses from a frequency-doubled fibre amplifier system (Coherent Monaco, wavelength 517 nm, repetition rate 333 kHz, pulse duration 270 fs) into the volume of a fused silica sample (Corning 7980, dimensions 1 mm × 20 mm × 150 mm, bulk refractive index $n_b = 1.457$ at 633 nm), inducing permanent refractive index changes along arbitrary three-dimensional trajectories as defined by the motion of a precision translation system (Aerotech ALS130). Due to the focusing conditions, these waveguides exhibit slightly elliptical mode fields with a typical refractive index contrast of up to $\Delta n = 2 \times 10^{-3}$. The selective modulation of the interstitial sites' index in a range of $\pm 10\%$ around this value was achieved by an appropriate modulation of the inscription speed between 92 and 156 mm min$^{-1}$. The ideal profile of the sinusoidal index modulation was approximated by twelve constant-index segments per Floquet period. Waveguide fluorescence imaging$^{[8]}$ shows that even for such a relatively coarse discretization, the modulated channels exhibit excess losses of only $0.096 \pm 0.010$ dB cm$^{-1}$ relative to the static waveguides (compare with Extended Data Fig. 4). With half of the lattice sites being modulated, the mean excess losses of the bimorphic Floquet TI are 0.048 $\pm 0.005$ dB cm$^{-1}$, substantially below the 1.7 dB cm$^{-1}$ of bending losses reported for conventional Floquet TIs based on helically modulated waveguides$^{[4]}$. The topological propagation dynamics were probed with coherent light from a tuneable supercontinuum source (NKT SuperK Extreme), allowing us to compensate for the micro-motion of the wave packets within the Floquet period and faithfully capture the dynamics according to the quasi-static evolution operator by varying the excitation wavelength between 570 nm and 633 nm. The appropriate intensity and phase distributions for the desired excitation conditions were synthesized with a spatial light modulator (Hamamatsu LCOS-SLM).

**Numerical simulations.** The numerical results are obtained by solving the paraxial Schrödinger equation as an eigenvalue problem (for computations of the band structure, via the finite-difference method) and as a propagation problem (for computations of the field dynamics, via the beam-propagation method). In this context, the ribbon band diagrams of Figs. 2 and 3 provide the necessary validation against a variety of defects was verified by beam-propagation method simulations (Supplementary Fig. 6).

To efficiently reduce the computational burden of the time-dependent eigenvalue problem, we decompose the solution space into a sinusoidal basis along the propagation axis. This approach relies on the expectation that the time-periodic part of the eigenmode solution will be related to the driving protocol used for the time-dependent modulation of the refractive index. This strategy leads into a solution space that is numerically large in the transverse plane (discretized by finite differences) but highly reduced along the z dimension. The size of the matrix generated through this process is optimally minimized so that it remains within reach of common eigenvalue decomposing techniques. More information is in Supplementary Section II.

**Reporting Summary.** Further information on research design is available in the Nature Research Reporting Summary linked to this article.

**Data availability**

The experimental data that support the findings of this study are available from M.H. upon reasonable request (matthias.heinrich@uni-rostock.de).

**Code availability**

The MATLAB codes corresponding to the beam-propagation method and band structure algorithms are available from G.G.P. upon reasonable request (pyrialak@knights.ucf.edu).

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**Author contributions**

G.G.P. initiated the idea, formulated the index-modulated lattice and performed the theoretical calculations and simulations. J.B. developed the experimental implementation, fabricated the samples and conducted the measurements. J.B., L.J.M. and M.H. evaluated the measurements and interpreted the data. M.K., N.V.K., A.S. and D.N.C. supervised the efforts of their respective groups. All authors discussed the results and cowrote the manuscript.

**Competing interests**

The authors declare no competing interests.

**Additional information**

Extended data are available for this paper at https://doi.org/10.1038/s41563-022-01238-w. The online version contains supplementary material available at https://doi.org/10.1038/s41563-022-01238-w.

**Correspondence and requests for materials** should be addressed to Demetrios N. Christodoulides.

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**Extended Data Fig. 1 | Comparison of staggered and flat-phased broad excitations.** (a-d): Staggered excitations of the primary waveguides of edge unit cells successfully populate the topological Chern mode near the edge of the Brillouin zone. (e-h) Absent the appropriate phase modulation, the injected wave packets instead represent a superposition of the bulk bands near the center of the Brillouin zone. As a result, the light diffracts freely across the entire lattice instead of being captured in the helical Chern channel.
Extended Data Fig. 2 | Honeycomb helical FTI. A driven honeycomb lattice with sinusoidally modulated time-periodic coupling terms exhibits a secondary topological phase in response to an increase of the driving period. At the critical driving period $T_c = \pi/3$ the gap at the Floquet zone collapses and reopens with a topologically non-trivial winding number. This corresponds to an anomalous phase with a trivial Chern number, signifying a topological phase transition.
Extended Data Fig. 3 | Chained honeycomb FTI. Above its critical modulation period \( T' = \pi \), the chain-driven honeycomb lattice enters a secondary topological phase characterized by the band diagram shown on the right. In this configuration, the bands manifest a non-trivial topological structure characterized by higher order Chern invariants \( \mathcal{C} = 2 \), and, in turn, the Chern gaps host an increased number of unidirectional edge states. Details on the modal amplitudes and propagation dynamics of the Chern states in the \( \mathcal{C} = 2 \) phase are provided in Supplementary Fig. 5.
Extended Data Fig. 4 | Losses in index-modulated waveguides. Waveguide fluorescence characterization confirms that even coarsely discretized index modulation (twelve constant-index segments approximating the ideal cosine Floquet cycle) only introduces excess losses of \((0.096 \pm 0.010)\text{dB/cm}\) relative to a straight constant-index waveguide. With half of the lattice sites being modulated, the mean excess losses of the bimorphic FTI are \((0.048 \pm 0.005)\text{dB/cm}\), substantially below the value of 1.7 dB/cm reported for bending losses in conventional FTIs based on helically modulated waveguides\(^5\). Note that the apparent oscillations in the normalized intensity of the modulated waveguide are due to the different concentration of color centers formed at different writing speeds, resulting in a modulation of the fluorescence efficiency between the individual segments of each modulation period. Due to the large number of periods, this oscillation does not notably impact the measurement of the loss coefficient \(\gamma\). The fluorescence itself is a feature of femtosecond laser-written waveguides in fused silica, and does not pose a substantial source of propagation losses\(^5\).
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Software and code

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Data collection

The experimental data (output intensity distributions) shown in this work were collected with a resolution of 12bit via standard LabView image acquisition routines.

Data analysis

Using Matlab, the recorded images were normalized to their respective peak values and subsequently exported as PNG files (8bit color map).

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All manuscripts must include a data availability statement. This statement should provide the following information, where applicable:
- Accession codes, unique identifiers, or web links for publicly available datasets
- A description of any restrictions on data availability
- For clinical datasets or third party data, please ensure that the statement adheres to our policy

The experimental data used to generate the figures in main manuscript and supplementary are available from the authors upon reasonable request.
Field-specific reporting

Please select the one below that is the best fit for your research. If you are not sure, read the appropriate sections before making your selection.

- Life sciences
- Behavioural & social sciences
- Ecological, evolutionary & environmental sciences

For a reference copy of the document with all sections, see nature.com/documents/nr-reporting-summary-flat.pdf

Life sciences study design

All studies must disclose on these points even when the disclosure is negative.

Sample size: Describe how sample size was determined, detailing any statistical methods used to predetermine sample size or if no sample-size calculation was performed, describe how sample sizes were chosen and provide a rationale for why these sample sizes are sufficient.

Data exclusions: Describe any data exclusions. If no data were excluded from the analyses, state so. If data were excluded, describe the exclusions and the rationale behind them, indicating whether exclusion criteria were pre-established.

Replication: Describe the measures taken to verify the reproducibility of the experimental findings. If all attempts at replication were successful, confirm this. If there are any findings that were not replicated or cannot be reproduced, note this and describe why.

Randomization: Describe how samples/organisms/participants were allocated into experimental groups. If allocation was not random, describe how covariates were controlled or if this is not relevant to your study, explain why.

Blinding: Describe whether the investigators were blinded to group allocation during data collection and/or analysis. If blinding was not possible, describe why or explain why blinding was not relevant to your study.

Behavioural & social sciences study design

All studies must disclose on these points even when the disclosure is negative.

Study description: Briefly describe the study type including whether data are quantitative, qualitative, or mixed-methods (e.g. qualitative cross-sectional, quantitative experimental, mixed-methods case study).

Research sample: State the research sample (e.g. Harvard university undergraduates, villagers in rural India) and provide relevant demographic information (e.g. age, sex) and indicate whether the sample is representative. Provide a rationale for the study sample chosen. For studies involving existing datasets, please describe the dataset and source.

Sampling strategy: Describe the sampling procedure (e.g. random, snowball, stratified, convenience). Describe the statistical methods that were used to predetermine sample size or if no sample-size calculation was performed, describe how sample sizes were chosen and provide a rationale for why these sample sizes are sufficient. For qualitative data, please indicate whether data saturation was considered, and what criteria were used to decide that no further sampling was needed.

Data collection: Provide details about the data collection procedure, including the instruments or devices used to record the data (e.g. pen and paper, computer, eye tracker, video or audio equipment) whether anyone was present besides the participant(s) and the researcher, and whether the researcher was blind to experimental condition and/or the study hypothesis during data collection.

Timing: Indicate the start and stop dates of data collection. If there is a gap between collection periods, state the dates for each sample cohort.

Data exclusions: If no data were excluded from the analyses, state so. If data were excluded, provide the exact number of exclusions and the rationale behind them, indicating whether exclusion criteria were pre-established.

Non-participation: State how many participants dropped out/declined participation and the reason(s) given or provide response rate or state that no participants dropped out/declined participation.

Randomization: If participants were not allocated into experimental groups, state so. Or describe how participants were allocated to groups, and if allocation was not random, describe how covariates were controlled.

Ecological, evolutionary & environmental sciences study design

All studies must disclose on these points even when the disclosure is negative.

Study description: Briefly describe the study. For quantitative data include treatment factors and interactions, design structure (e.g. factorial, nested, hierarchical), nature and number of experimental units and replicates.

Research sample: Describe the research sample (e.g. a group of tagged Passer domesticus, all Stenocereus thurberi within Organ Pipe Cactus National
Research sample

Monument), and provide a rationale for the sample choice. When relevant, describe the organism taxa, source, sex, age range and any manipulations. State what population the sample is meant to represent when applicable. For studies involving existing datasets, describe the data and its source.

Sampling strategy

Note the sampling procedure. Describe the statistical methods that were used to predetermine sample size OR if no sample-size calculation was performed, describe how sample sizes were chosen and provide a rationale for why these sample sizes are sufficient.

Data collection

Describe the data collection procedure, including who recorded the data and how.

Timing and spatial scale

Indicate the start and stop dates of data collection, noting the frequency and periodicity of sampling and providing a rationale for these choices. If there is a gap between collection periods, state the dates for each sample cohort. Specify the spatial scale from which the data are taken.

Data exclusions

If no data were excluded from the analyses, state so OR if data were excluded, describe the exclusions and the rationale behind them, indicating whether exclusion criteria were pre-established.

Reproducibility

Describe the measures taken to verify the reproducibility of experimental findings. For each experiment, note whether any attempts to repeat the experiment failed OR state that all attempts to repeat the experiment were successful.

Randomization

Describe how samples/organisms/participants were allocated into groups. If allocation was not random, describe how covariates were controlled. If this is not relevant to your study, explain why.

Blinding

Describe the extent of blinding used during data acquisition and analysis. If blinding was not possible, describe why OR explain why blinding was not relevant to your study.

Did the study involve field work?  

☐ Yes  ☒ No

Reporting for specific materials, systems and methods

We require information from authors about some types of materials, experimental systems and methods used in many studies. Here, indicate whether each material, system or method listed is relevant to your study. If you are not sure if a list item applies to your research, read the appropriate section before selecting a response.

Materials & experimental systems

n/a Involved in the study

☒ Antibodies
☒ Eukaryotic cell lines
☒ Palaeontology and archaeology
☒ Animals and other organisms
☒ Human research participants
☒ Clinical data
☒ Dual use research of concern

Methods

n/a Involved in the study

☒ ChiP-seq
☒ Flow cytometry
☒ MRI-based neuroimaging