Jamming transition in a driven lattice gas

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Abstract – We study a two-lane driven lattice gas model with oppositely directed species of particles moving on two periodic lanes with correlated lane switching processes. While the overall density of individual species of particles is conserved in this system, the particles are allowed to switch lanes with finite probability only when oppositely directed species meet on the same lane. This system exhibits a unique behavior, wherein phase transition is observed between a homogeneous absorbing phase, characterized by complete segregation of oppositely directed particles between the two lanes, and a jammed phase. The transition is accompanied by a finite drop of current in the lattice, emergence of a cluster comprised of both species of particles in the jammed phase, and is determined by the interplay of the relative rates of translation of particles on the same lane and their lane switching rates. These findings may have interesting implications for understanding the phenomenon of jamming in microtubule filaments observed in the context of axonal transport.

Unlike one-dimensional (1D) systems in thermal equilibrium, 1D and quasi-1D driven diffusive systems have a stationary-state behavior, characterized by macroscopic currents. These systems can exhibit spontaneous symmetry breaking, phase separation and condensation, resulting in very rich and complex phase diagrams which is in contrast to 1D equilibrium lattice gas models [1–10]. One of the motivations for studying such systems is their amenability in providing a framework for studying a varied class of driven biological processes, such as transport on biofilaments [11–13], growth of fungal filaments [14,15], transport across biomembranes [16] among others.

In this letter, we study a periodic two-lane driven lattice gas system with oppositely directed species and conserved overall density of individual species [17]. This system incorporates bidirectionality and correlated lane switching processes, wherein oppositely directed species can switch lanes with a finite probability only when they encounter each other and not otherwise. Such correlated lane switching mechanism fundamentally alters the steady-state properties. We find that the system exhibits a unique behavior, wherein a phase transition is observed between a jammed clustered phase and a homogeneous absorbing phase, characterized by complete segregation of oppositely directed particles between the two lanes. The jammed phase in each lane is characterized by a large cluster comprised of both species of particles and no vacancies, which is surrounded by a region of single-species fluid phase in rest of the lane. This phase transition is distinct from phase transitions observed for other multi-species driven lattice gas models with conserved particle densities, such as the ones discussed in [6], where a transition between a disordered fluid phase and a phase with strong phase separation with three pure macroscopic domains of the individual species and vacancies, respectively, is seen. It is also different from the transition seen between a two-species homogeneous phase to a condensate phase [9]. While transition from a jammed state to free flowing state of particles has been observed for driven systems which exchange particles with environment [18,19], both the mechanism and the nature of the steady state is very different for our case, owing to the constraint of particle number conservation. Further, the behavior of this system is in contrast to other two-lane models [20–22], and to a closely related periodic two-lane model with conserved particle number and uncorrelated lane switching mechanism [23], where the steady state is characterized by large but finite-size clusters and no phase transition is observed in the thermodynamic limit of $N \to \infty$ [8,23].

From a biological standpoint, bidirectional transport of cellular cargoes on multiple parallel filaments have been observed, in the context of axonal transport in neurons [24].
along with filament switching of the motors between neighbouring microtubule (MT) filaments [25]. In a varied class of motor-MT systems including neurons in brain cells, a jamming phenomenon has been observed and it has been attributed to the changes in motor properties such as their processivity and (un)binding rates from the filament [26,27]. In this context, this simple model that we study may be useful in providing some insight into the physical mechanism responsible for jamming caused by changes in lane switching and translocation rate of motors on filaments.

The model that we study is comprised of two periodic lattices of length \( L \) and \( N \) sites with a lattice spacing \( \epsilon = L/N \). Each lattice site can either be empty or it can be occupied by a (+) particle or a (−) particle. In each lattice, a (+) particle hops to the right with a rate \( \alpha \) if the adjacent site to the right is vacant. Similarly, a (−) particle hops to the left with the same rate \( \alpha \) if the adjacent site to the left is unoccupied. For a (+) particle on a lattice site \( i \), if the neighbouring site to the right on the same lattice is occupied by a (−) particle, then two different processes can take place; either the (+) particle hops to the neighbouring site at \( i + 1 \) on the same lattice while the neighbouring (−) particle hops to the site \( i \) with a rate \( \beta \), or the (+) particle in lattice 1 switches with a rate \( \gamma_1(\gamma_2) \) to the corresponding site \( i \) on the other lattice if that site is vacant. Similarly for (−) particles if the neighbouring site to the left is occupied with a (+) particle, then the (−) particle hops to the neighbouring site at \( i - 1 \) on the same lattice while the neighbouring (+) particle hops to the site \( i \) with rate \( \beta \), or the (−) particle switches to the other lattice with a rate \( \mu_1(\mu_2) \) if the site \( i \) of the opposite lane is vacant. We study the system for which the total conserved densities of (+) and (−) are equal, so that \( \rho_+ = \rho_- = \rho_0 \), where \( \rho_+ \) and \( \rho_- \) are the conserved total density of (+) and (−) particles, respectively. We choose \( \gamma_1 = \mu_1 = \gamma \) and \( \gamma_2 = \mu_2 = \mu \) with \( \mu > \gamma \) and set \( \alpha = 1 \), expressing the other rates in terms of it. We study the system using a combination of Mean-Field (MF) analysis and Monte Carlo (MC) simulations\(^1\). The corresponding evolution equations for the mean site occupation density for the (+) and (−) particles in each lane can be expressed in terms of the loss and gain terms arising out of translation and lane switching processes. These loss and gain terms can be written as combinations of various correlators of site occupation number of each species [17]. MF approximation amounts to factorizing all the expectation values of two-point correlators arising out of the different combinations of the site occupation numbers of the two species. For going over to a continuum description, we normalize the total length of the lattice \( L \) to 1 and let \( N \to \infty \) so that \( \epsilon \to 0 \) in the thermodynamic limit [13,17]. We denote the mean densities as a function of the relative position \( x \) along the lanes as \( p_1(x) \), \( p_2(x) \), \( n_1(x) \) and \( n_2(x) \), corresponding to the density of (+) in lane 1, (+) in lane 2, (−) in lane 1, and (−) in lane 2, respectively. The MF evolution equations then read as [17]

\[
\partial_t p_1 = \mu p_2 n_2 (1 - p_1 - n_1) - \gamma p_1 n_1 (1 - p_2 - n_2) + \epsilon [\mu p_2 (1 - p_1 - n_1) \partial_x n_2 - \gamma p_1 (1 - p_2 - n_2) \partial_x n_1] - \epsilon \partial_x [\alpha p_1 (1 - p_1 - n_1) + \beta p_1 n_1] + O(\epsilon^2),
\]

\[
\partial_t p_2 = \gamma p_1 n_1 (1 - p_2 - n_2) - \mu p_2 n_2 (1 - p_1 - n_1) + \epsilon [\gamma p_1 (1 - p_2 - n_2) \partial_x n_1 - \mu p_2 (1 - p_1 - n_1) \partial_x n_2] - \epsilon \partial_x [\alpha p_2 (1 - p_2 - n_2) + \beta p_2 n_2] + O(\epsilon^2),
\]

\[
\partial_t n_1 = \gamma p_2 n_2 (1 - p_1 - n_1) - \mu p_1 n_1 (1 - p_2 - n_2) + \epsilon [\mu n_1 (1 - p_2 - n_2) \partial_x p_1 - \gamma n_2 (1 - p_1 - n_1) \partial_x p_2] + \epsilon \partial_x [\alpha n_1 (1 - p_1 - n_1) + \beta p_1 n_1] + O(\epsilon^2),
\]

\[
\partial_t n_2 = \mu p_1 n_1 (1 - p_2 - n_2) - \gamma p_2 n_2 (1 - p_1 - n_1) + \epsilon [\gamma n_2 (1 - p_1 - n_1) \partial_x p_2 - \mu n_1 (1 - p_2 - n_2) \partial_x p_1] + \epsilon \partial_x [\alpha n_2 (1 - p_2 - n_2) + \beta p_2 n_2] + O(\epsilon^2),
\]

where we have displayed terms up to the first power of \( \epsilon \).

For \( \mu > \gamma \), the homogeneous MF steady-state solution for the density is \( p_1 = 2p_0, p_2 = 0, n_1 = 0 \) and \( n_2 = 2p_0 \). This MF solution corresponds to segregation of oppositely directed particles between the two lane. Numerical simulation in this phase confirms that the system phase segregates with all the (+) particles occupying lane 1, while all the (−) particles occupy lane 2. The corresponding currents are \( J_1^+ = 2p_0 (1 - 2p_0) \), \( J_1^- = 0 \), \( J_2^+ = 0 \), and \( J_2^- = -2p_0 (1 - 2p_0) \) [17]. This steady state is an absorbing state because once the system gets into this configuration, there is no particle exchange between the lanes, and no microscopic site dynamics can take it out of this state. We also define an order parameter \( \phi \) which is the ratio of the density of (−) particles in lane 1 and the fixed total density of particles \( \rho_T = 2p_0 \). We also look at the relative cluster size \( \Delta \), defined as the ratio of the cluster size in lane 1 and the length of the lattice \( L \). In the absorbing phase, \( \phi = 0 \) while the relative cluster size is of \( O(0) \).

Apart from the absorbing phase, numerical simulations for this model also exhibit a jammed phase for a certain range of parameters (fig. 1), where we observe formation of a single cluster in each of the lanes. This cluster is comprised of both (+) and (−) with no vacancies, with a density of 1/2 for both the species (fig. 2). In the rest of the region outside the cluster, there is a homogeneous distribution of (+) and vacancies with absence of (−) in lane 1, and homogeneous distribution of (−) and vacancies with absence of (+) in lane 2. However this apparent phase separation and formation of a single condensate in each lane, that we observe in numerical MC simulations (done for system sizes up to \( 10^5 \)) may not hold true in general in the thermodynamic limit of \( N \to \infty \). This will

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\(^1\)In MC simulations, we wait for an initial transient \( \geq 20000N \) swaps. Averaging is done typically over \( 10^3–10^4 \) time swaps with a period \( \geq 20N \).
phase with equal to 1/2 so that the cluster is in a maximal current

MC simulation is done for $J$ outside the cluster in magnitude with opposite sign inside the cluster, while (corresponding to fig. 1). Parameter values used for MC simulation for different system sizes $N$ overlap with each other and with the MF prediction (solid lines), in the jammed phase. MC simulations are done for parameter values $K = 1.2$, $\gamma = 0.4$, $\rho_o = 0.1$.

indeed be the case if the distribution of the cluster sizes is such that the mean cluster size is of the same order or larger than the system sizes accessed by simulations [1,8]. We differ the discussion about this issue towards the end. Instead we focus on the jammed steady state of the finite-size systems that we can access through simulations. In order to do a MF analysis of this phase we assume that for the single cluster, the densities of (+) and (−) are equal to 1/2 so that the cluster is in a maximal current phase with $J_1^+ = J_2^+ = \frac{\beta}{2}$. Outside the cluster $J_1^+ = J_2^+ = \frac{\gamma}{2}$ and $J_2^- = 0$. The currents for (−) are exactly the same in magnitude with opposite sign inside the cluster, while outside the cluster $J_2^- = -\frac{\beta}{2}$ and $J_1^- = 0$. The overall total current of (+) and (−), obtained by adding the current in lanes 1 and 2, remains constant both inside and outside the cluster (fig. 2). The clusters in both the lanes are co-localized. The densities outside the cluster obtained from MC simulation match well with the MF expression for density $\rho_1 = 1/2(1 - \sqrt{1 - 2\beta})$, which can be obtained by applying the current continuity condition inside and outside the cluster in each lane. The MF expression for the relative cluster size $\Delta$ is obtained by equating the total conserved density of the particles to the individual expression of densities inside and outside the cluster,

$$\Delta = \frac{4\rho_o - 1 + \sqrt{1 - 2\beta}}{1 + \sqrt{1 - 2\beta}}. \quad (5)$$

This matches well with the MC simulation results (fig. 3). $\Delta$ takes a value of $2\rho_o$ for $\beta \to 0$. The MF expression for the order parameter is $\phi = \frac{\Delta}{4\rho_o}$ and in the limit of $\beta \to 0$ it assumes a value of 1/2.

The entire phase diagram can be specified in terms of the lattice hopping rate $\beta$, the lane switching rate $\gamma$, the switching rate constant $K \equiv \frac{\beta}{\gamma}$ and the fixed density $\rho_o$. We obtain the phase diagram using MC simulations. In fig. 4 we show the phase diagram in the $\beta$-$\gamma$ plane for a fixed value of $K$ and $\rho_o$. Close to the phase transition boundary in the $\beta$-$\gamma$ phase plane, the system is sensitive to the initial starting configuration. For instance, for the same value of $\gamma$, $\beta$ and $K$ but different initial conditions, the final steady state obtained by MC simulations can be either a jammed state or an absorbing state, if the $\gamma$ and $\beta$ values lie in the vicinity of the phase boundary. The phase boundary for an initial starting condition of equal density of (+) and (−) in both lanes is shifted when compared to an initial condition where the initial density of (−) in lane 1 is 20% of the fixed density $\rho_o$ (fig. 4). Thus, the phase boundary appears as a narrow band of region in the phase plane rather than a sharp line, indicating that self-averaging does not occur in the vicinity of the transition boundary. However, for the condition of same
specified initial density in each lane, the phase transition point \( \beta_c \) remains almost unchanged for different system sizes (Fig. 4, inset). One way of qualitatively understanding this dependence on the initial configuration is as follows: The initial configuration is characterized in terms of the initial occupation number of (+) and (−) in each lane. The jammed phase and the absorbing phase corresponds to the fixed points of the system, which can be stable or unstable depending on the parameter values of \( \gamma, \beta \) and \( K \). Changing these parameter values alters the zones of attraction about these fixed points. Starting from a given initial configuration, the system navigates in the configuration space to reach the stable fixed point. However, if the system hits the configuration corresponding to the absorbing state, it remains trapped in it and the flows in the configuration space is halted. Thus, the flow of the system configuration to the eventual steady-state phase would be influenced by the initial configuration, especially in the vicinity of the phase transition boundary where the gradient of the flows to the two different fixed points becomes comparable, i.e., if the initial configuration of the system is such that it is very close to the absorbing phase configuration, then stochastically the system is more likely to hit the absorbing state than a starting configuration which is far in terms of these occupation numbers.

However, deep inside a particular phase (beyond the region of the narrow band), the final steady state becomes independent of the initial configuration and it is uniquely determined in terms of the density, current, and the order parameter \( \phi \) corresponding to that particular phase, implying that the effects of the initial configurations are diminished compared to the overall relative flows which determines the steady state. To illustrate this point, we define the kink number \( N_k \), which corresponds to the total number of times a (+) is followed by a (−) in the same lane. Figure 5 shows the temporal evolution of the relative kink number \( N_k/N \) for different initial conditions, where the final steady state corresponds to the same jammed phase. In fact, for the absorbing state, \( N_k \) is zero while \( N_k/N \) is a finite value, whose average value is independent of the system size. Further, we have checked that the relative fluctuations of the relative kink number decreases with the system size, indicating that the system gets kinetically trapped in the jammed state even in the thermodynamic limit of \( N \to \infty \).

In order to understand what determines the phase boundary between the jammed and the absorbing phase, we look at the temporal behavior of the system in the vicinity of the absorbing state. In particular, we perform a linear stability analysis of the MF steady-state fixed point corresponding to the absorbing steady state. For \( \mu > \gamma \), the homogeneous MF steady-state solution for the density is \( p_1 = 2 \rho_o, p_2 = 0, n_1 = 0 \) and \( n_2 = 2 \rho_o \), which is obtained from eqs. (1)–(4). For performing a linear stability analysis about the homogeneous MF steady state, we have to take into account the terms in eqs. (1)–(4) which are \( O(\epsilon^2) \). Following the usual procedure of retaining terms up to linear order in fluctuations of the variables, \( p_1, p_2, n_1 \) and \( n_2 \) and taking spatial Fourier transforms of the fluctuations, \( \delta p_1, \delta p_2, \delta n_1 \) and \( \delta n_2 \), we obtain the corresponding eigenvalues, which determines the stability of the MF homogeneous phase. The corresponding eigenvalues are

\[
\lambda_{1,2} = -i\epsilon q M - \left[ \mu A + \frac{1}{2} \epsilon^2 q^2 M + \gamma A \left( 1 + \frac{1}{4} \epsilon^2 q^4 \right) \right],
\]

\[
\lambda_{3,4} = \pm i\epsilon \alpha (1 - 2 \rho_o) - \frac{1}{2} \epsilon^2 q^2 \alpha.
\]

Here, \( M = \alpha (1 - 2 \rho_o) + 2 \rho_o \beta \) and \( A = 2 \rho_o (1 - 2 \rho_o) \).
Fig. 6: (Colour on-line) (a) MF linear stability line as a function of $K$: increasing $K$ tends to increase the region of the homogeneous absorbing phase. Here $\rho_o = 0.1$. (b) Effect of variation of $\rho_o$: (i) $\rho_o = 0.1$, (ii) $\rho_o = 0.2$, (iii) $\rho_o = 0.4$. Here $K = 1.2$. The inset shows the marginally stable mode (solid line) corresponding to the circle on the stability line in fig. 6(b), and the effect on the mode stability by variation of $\gamma$. Here $\beta = 0.2$.

Of the four eigenvalues, only for $\lambda_1$ the real part of the eigenvalue can be positive depending on the values of the parameters and wave number $q$. The other three eigenvalues always correspond to stable modes of fluctuations. Figure 6 (inset) shows the variation for $\lambda_1$ as a function of the wave number $q$. In fact the long-wavelength limit ($q = 0$) fluctuation is always stable as $\lambda_1(q = 0) = (-\mu + \gamma)A$ is always negative for $\mu > \gamma$. For a certain critical value of $\gamma$, $\lambda_1$ becomes positive. However, the maximum value of the wave number $q_m$, is limited by the lattice spacing, so that $q_m = \frac{2\pi N}{L}$, corresponding to a fluctuation at the scale of one lattice spacing. Now the expression for the stability line can be obtained by substituting the expression $q_m$ in eq. (6) and equating it to zero. Setting $\alpha$ to 1, we obtain the equation for MF linear stability line,

$$\gamma = 2\pi^2 \frac{1}{2\rho_o} + \beta \frac{1 - 2\rho_o}{1 + 4\pi^2} - K.$$  

Figures 6(a) and (b) show the variation of the position of the MF stability boundary with $\rho_o$ and $K$, respectively. Comparing the MF stability line of fig. 6(a) (solid lines), with the numerical phase boundary in fig. 4, we can see that it does not agree with the numerical simulation result. Since the MF stability line is determined by the instabilities of large wave number fluctuations, it is only expected that, at the scale of lattice spacing, the correlations of fluctuations between the neighbouring lattice sites cannot be neglected, leading to inaccuracies in the MF analysis.

Finally we look at the issue of formation of a single condensate in the jammed phase in the thermodynamic limit for our system. For some driven systems, an apparent condensation phenomenon was observed in numerical simulations for finite-size systems [6,23], while subsequently it was shown that phase separation did not exist in the thermodynamic limit of $N \to \infty$ [8]. For condensation to occur, the cluster size has to diverge with the system size. However, studying systems of finite-size lattices, it is difficult to distinguish between the case of the cluster size which diverges with the system size, and large but finite-size clusters. This can happen if the distribution of the cluster sizes is such that the mean cluster size is of the same order or larger than the system sizes [1,8]. For many models which carry a non-vanishing current in the thermodynamic limit, the current in a finite cluster of size $n$ takes an asymptotic form $J_n \sim J_{\infty} (1 + b/n^\alpha)$ to leading order in $1/n$ [8,9]. Using a correspondence between the asymptotic form of the current in the cluster for zero-range process (ZRP) and such models, a criterion has been proposed which specifies that phase separation leading to the formation of a single condensate can occur for either $\sigma < 1$ and $b > 0$ or for $\sigma = 1$ and $b > 2$ [8]. This conjecture has been applied for a two-lane model [23], by performing a MC simulation for the open two-lane system of size $n$, without vacancies and with equal rate of particle entry and exit at the boundaries. It has been used to determine the finite-size corrections to current $\Delta_n = (J_n - J_{\infty})/J_{\infty}$ and extract the corresponding values of $\sigma$ and $b$ [8]. However, there are two issues that we wish to highlight when we apply this criterion for our case: i) The region adjoining the cluster is comprised of a fluid phase of $(+)$ alone, while for lane 2 this region is solely a fluid phase of $(\bar{\cdot})$ (fig. 2). Thus, it is a priori not clear whether the simulation for the open system should be performed with equal entry and exit rates of the particles at the boundaries. We perform the MC simulations for both cases, e.g., with equal boundary rates of entry and exit for each particles in both the lanes, and with no entry of $(\bar{\cdot})$ particles in lane 1 and no entry of $(\cdot)$ particles in lane 2. ii) In the MC simulations, we find that the root mean square (RMS) fluctuations of the measured current $\delta J_n$ are such that $\delta J_n > \Delta_n$ (see footnote 2). This implies that the estimates of $b$ and $\sigma$

\[\delta J_n \approx 0.005 \text{ while } \Delta_n \approx 0.0002 \] for $N = 10^3$,

\[\delta J_n \approx 0.0015 \text{ while } \Delta_n \approx 0.0003.\]
obtained from fitting the data would be rather unreliable. In fig. 7 we show that the straight line fit (with $\sigma = 1$) for the data points obtained for unequal entry rates of particles in two lanes corresponds to $b = 2.86$. For equal entry and exit rates, we find that $b \approx 3.03$. Thus, both of these data sets suggest *condensation*. However, owing to the limitations of high $\sigma$, we find that $n$ values of the fluctuations of the current when compared with being able to resolve the nature of the jammed phase is quite unique. The reason for not being able to resolve the nature of the jammed phase is due to the limitations placed by the relatively high values of the fluctuations of the current when compared with the finite-size correction to current $\Delta n$. The MF theory within a particular phase is able to accurately predict the steady-state profile, although the transition itself is not well described by a MF analysis. Suitable modifications of this model may provide some insight into understanding jamming on MT filaments observed in the context of axonal transport.

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