

Actions for twisted spectral triple  
and the transition from the Euclidean to the Lorentzian

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Abstract

This is a review of recent results regarding the application of Connes’ noncommutative geometry to  
the Standard Model, and beyond. By twisting (in the sense of Connes-Moscovici) the spectral triple of  
the Standard Model, one does not only get an extra scalar field which stabilises the electroweak vacuum,  
but also an unexpected 1-form field. By computing the fermionic action, we show how this field induces  
a transition from the Euclidean to the Lorentzian signature. Hints on a twisted version of the spectral  
action are also briefly mentioned.

1 Introduction

Noncommutative geometry “a la Connes” [10] allows to obtain the Lagrangian of the Standard Model of elementary particles - including the Higgs sector - minimally coupled with Einstein-Hilbert action (in Euclidean signature) from geometrical principles. In addition, it offers some guidelines to go beyond the Standard Model by playing with the mathematical rules of the game (for a recent review, see [9]).

Early attempts “beyond the SM” were considering new fermions (see e.g. [28] and other papers of the same author). One may also relax one of the axioms of noncommutative geometry, the first-order condition discussed below [8]; or modify another axiom regarding the real structure (also discussed below) [3, 4]. Other proposals are based on some Clifford bundle structure [16], or non-associativity [2]. Here we focus on a model consisting in twisting the original noncommutative geometry [19, 23, 22].

From the examples listed above, all but the first are minimal extensions of the Standard Model: they allow to produce the kind of extra scalar field $\sigma$ suggested by particle physicists to stabilise the electroweak vacuum (which also permits to make the calculation of the Higgs mass in noncommutative geometry compatible with its experimental value), without adding new fermions.

By using twisted noncommutative geometry, one gets in addition an supplementary piece, namely a 1-form field, which surprisingly turns out to be related to the transition from Euclidean to Lorentzian signature.

We give an overview of these results below, beginning in §2 with a recalling on the Standard Model in noncommutative geometry. Then we summarise in §3 how to apply a twist to the geometry, and why this is related to a transition from the Euclidean to the Lorentzian. In §4 we show how this transition is actually realised at the level of the fermionic action. We also stress some projects regarding the spectral (i.e. bosonic) action.
2 The Standard Model in noncommutative geometry

2.1 Spectral triple

Definition 2.1. [12] A spectral triple consists of an involutive algebra $A$ acting on a Hilbert space $\mathcal{H}$, together with selfadjoint operator $D$ on $\mathcal{H}$ such that the commutator $[D, a]$ is bounded for any $a \in A$. It is graded if, in addition, there exists a selfadjoint operator $\Gamma$ which squares to $\mathbb{I}$, and such that

$$\{\Gamma, D\} = 0, \quad [\Gamma, a] = 0 \quad \forall a \in A. \quad (2.1)$$

Any closed Riemannian (spin) manifold $M$ defines a spectral triple

$$C^\infty(M), \quad L^2(M, S), \quad \mathcal{F} = -i \gamma^\mu \nabla_\mu \quad (2.2)$$

where $C^\infty(M)$ is the algebra of smooth functions on $M$, acting by multiplication on the Hilbert space $L^2(M, S)$ of square integrable spinors, and $\mathcal{F}$ is the Dirac operator, with $\nabla_\mu = \partial_\mu + \omega_\mu$ the covariant derivative associated with the spin connection $\omega_\mu$. For an even dimensional manifold, the spectral triple is graded with grading the product of the Dirac matrices, that is $\gamma^5$ for a four dimensional manifold.

A supplementary structure that plays an important role in the construction of physical models is the real structure [11]. The latter consists of an antilinear operator $J$ such that

$$J^2 = \mathbb{I}, \quad JD = \epsilon' DJ, \quad J\Gamma = \epsilon'' \Gamma J \quad (2.3)$$

where $\epsilon, \epsilon', \epsilon'' = \pm 1$ define the so-called $KO$-dimension $k \in [0, 7]$ of the spectral triple. In addition, the operator $J$ implements a map $a \rightarrow a^\circ := Ja^* J^{-1}$ from $A$ to the opposite algebra $A^\circ$ (the same object of $A$ as a vector space, but with opposite product: $a^\circ b^\circ = (ba)^\circ$). This allows to define a right action of $A$ on $\mathcal{H}$,

$$\psi a := a^\circ \psi, \quad (2.4)$$

which is asked to commute with the left action (the order zero condition)

$$[a, Jb^* J^{-1}] = 0 \quad \forall a, b \in A. \quad (2.5)$$

For the spectral triple (2.8), the real structure is $J = i \gamma^0 \gamma^2 cc$ where $cc$ denotes the complex conjugation. This coincides with the charge conjugation operator of quantum field theory.

Finally, one requires that the following first order condition holds

$$[[D, a], Jb^* J^{-1}] = 0 \quad \forall a, b \in A \quad (2.6)$$

which is an algebraic formulation of $D$ being a first-order differential operator.

With other extra-conditions, spectral triples provide a spectral characterization of manifolds [13]. Namely, any closed Riemannian (spin) manifold defines a spectral triple (2.8); conversely, given a spectral triple $(A, \mathcal{H}, D)$ with $A$ commutative and unital, then there exists a Riemannian manifold $M$ such that $A = C^\infty(M)$.

This motivates the definition of a noncommutative geometry as a spectral triple $(A, \mathcal{H}, D)$ where $A$ is non-necessarily commutative.

2.2 Gauge theory

A gauge theory (on a four dimensional manifold $M$) is described by [12, 14] the product, in the sense of spectral triple, of a manifold (2.2) by a finite-dimensional spectral triple

$$A_F, \mathcal{H}_F, D_F \quad (2.7)$$

that encodes the gauge degrees of freedom. The product triple is

$$A = C^\infty(M) \otimes A_F, \quad \mathcal{H} = L^2(M, S) \otimes \mathcal{H}_F, \quad D = \mathcal{F} \otimes \mathbb{I}_F + \gamma^5 \otimes D_F \quad (2.8)$$
where $\mathbb{I}_F$ is the identity operator on $\mathcal{H}_F$.

The connection 1-forms, generalised to the noncommutative setting, are elements of

$$\Omega^1_D(A) := \{\Sigma_i a_i [D, b_i^*] \}, \quad a_i, b_i \in A,$$

and the associated covariant Dirac operator is

$$D_A = D + A + JAJ^{-1} \quad \text{with} \ A \in \Omega^1_D(A).$$

A gauge transformation is implemented by the conjugate action of

$$\text{Ad}(u) : \psi \to u\psi u^* = u(u^*)^\circ \psi = uJuJ^{-1}\psi,$$

for $u$ is a unitary element of $A$. Namely, a gauge transformation maps the covariant operator $D_A$ to

$$\text{Ad}(u) D_A \text{Ad}(u)^{-1} = D_{A^u}$$

where $A^u$ is the gauge transformed of the potential $A$, given by

$$A^u := u[D, u^*] + uAu^*.$$  

### 2.3 The Standard Model

The finite dimensional spectral triple that describes the Standard Model is [7]

$$\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}), \quad \mathcal{H}_F = \mathbb{C}^{32=2 \times 2 \times 8}, \quad D = D_0 + D_R$$

where $\mathbb{H}$ is the algebra of quaternions. The algebra is such that its group of unitary element gives back the gauge group of the standard model, and 32 is the number of fermions per generation (6 coloured quarks and two leptons, that exists in two chiralities, together with their antiparticles). The operator $D$ is a $32 \times 32$ matrix, which divides into a block diagonal part $D_0$ which contains the Yukawa couplings of the electron, the quarks up and down, and the (Dirac) mass of the electronic neutrino; and an off-diagonal part $D_R$ which contains only one non-zero entry $k_R$ (Majorana mass of the electronic neutrino). The structure is then repeated for the other two generations of fermions.

A generalised 1-form (2.9) then divides into two pieces,

$$A = \gamma^5 \otimes H - i \sum \mu \gamma^\mu \otimes A_\mu,$$

where $H$ is a scalar field on $\mathcal{M}$ with value in $\mathcal{A}_F$, that identifies with the Higgs field, while $A_\mu$ is a 1-form field with value in $\text{Lie}(U(\mathcal{A}_F))$, whose components give the gauge fields of the standard model.

The fermionic action of the Standard Model is retrieved as

$$S^f(D_A) = \mathfrak{A}_{D_A}(\xi, \bar{\xi})$$

with $\xi$ the Grassman variables associated to a $+1$ eigenvector of the grading operator, and

$$\mathfrak{A}_{D_A}(\xi, \xi') = \langle J\xi, D_A\xi' \rangle.$$  

is the bilinear form defined by the covariant Dirac operator and the real structure. The asymptotic expansion $\Lambda \to \infty$ of the spectral action

$$\text{Tr} f \left( \frac{D^2_A}{\Lambda^2} \right)$$

($f$ being a smooth approximation of the characteristic function of $[0,1]$) yields the bosonic Lagrangian of the standard model coupled with the Einstein-Hilbert action in Euclidean signature.
The spectral action provides initial conditions at a putative unification scale. Physical predictions are obtained by running down the parameters of the theory under the renormalisation group equation. Assuming there is no new physics between the unification scale and our scale, one finds a mass of the Higgs boson around $170 \text{ GeV}$, which is not in agreement with the experimental value $m_H = 125, 1 \text{ GeV}$.

But it was well known in particle physics that for a Higgs boson with mass $m_H \leq 130 \text{ GeV}$, the quartic coupling of the Higgs field becomes negative at high energy, meaning the electroweak vacuum is meta-stable rather than stable [18, 5]. Such instability can be cured by a new scalar field $\sigma$, that couples to the Higgs in a suitable way [24].

In the spectral triple of the Standard Model, such a field $\sigma$ is obtained by turning into a field the neutrino Majorana mass $k_R$ which appears in the off-diagonal part $D_R$ of the finite dimensional Dirac operator $D_F$ [6]:

$$k_R \to k_R \sigma.$$ 

In addition, by altering the running of the parameters under the equations of the group of renormalisation, $\sigma$ makes the computation of $m_H$ compatible with $125 \text{ GeV}$.

The point is that the field $\sigma$ cannot be obtained on the same footing as the Higgs, that is as a component of a generalised 1-form (2.9), for

$$[\gamma^5 \otimes D_R, a] = 0 \quad \forall a, b \in A = C^\infty(\mathcal{M}) \otimes \mathcal{A}_F.$$ 

This motivates to modify the spectral triple of the Standard Model. Several ways have been explored, some of them listed in the introduction. Here we follow the path consisting in twisting the spectral triple. This is a way to implement on a solid mathematical ground the idea of grand-symmetry, first introduced in [21].

### 3 Twisted spectral triples and Lorentz signature

#### 3.1 Minimal twist of the Standard Model

Given a triple $(\mathcal{A}, \mathcal{H}, D)$, instead of asking the commutators $[D, a]$ to be bounded, one asks the boundedness of the twisted commutators

$$[D, a]_\rho := Da - \rho(a)D \quad \text{for some fixed } \rho \in \text{Aut}(\mathcal{A}). \quad (3.19)$$

Such a variation of the original definition of the spectral triples were introduced in [15] with purely mathematical motivations. Later it was realised that twisted spectral triples provide a way to generate the field $\sigma$ required to stabilise the electroweak vacuum and fit the calculation of the Higgs mass [23].

The idea is to introduce a twisting automorphism $\rho$ in the spectral triple of the Standard Model, with minimal changes. In particular, we keep the Hilbert space and the Dirac operator unchanged, since they encode the fermionic content of the theory, and there are so far no indications of new fermions beyond those of the Standard Model. These requirements make necessary to double the algebra [25]. Namely, one considers the triple

$$\mathcal{A} = (C^\infty(\mathcal{M}) \otimes \mathcal{A}_F) \otimes \mathbb{C}^2, \quad \mathcal{H} = L^2(\mathcal{M}, S) \otimes \mathcal{H}_F, \quad D = \partial \otimes 1_{32} + \gamma^5 \otimes D_F$$

with automorphism $\rho$ the flip

$$\rho((f, g) \otimes m) = (g, f) \otimes m \quad f, g \in C^\infty(\mathcal{M}), m \in \mathcal{A}_F.$$ 

Instead of (2.9), one considers a twisted generalised form

$$A_\rho = \Sigma_i a_i [D, b_i^\rho], \quad a_i, b_i \in \mathcal{A}, \quad (3.21)$$

and the associated twisted-covariant Dirac operator

$$D_{A_\rho} := D + A_\rho + J A_\rho J^{-1}. \quad (3.22)$$
One generates in this way the extra-scalar field $\sigma$, since the twisted commutator $[\gamma^5 \otimes D_R, a]_\rho$ is no longer zero, and yields precisely the kind of scalar field $\sigma$ discussed above.

But there is also an unexpected guest, namely a 1-form field $f_\mu dx^\mu$, coming from the twisted commutator of the free part $\phi \otimes \mathbb{I}_F$.

### 3.2 Lorentzian inner product from twist

A gauge transformation for a twisted spectral triple is given by the twisted conjugate action of the operator $U = \text{Ad}(u)$ in (2.11), that is

$$\rho(U) D_{A_\rho} U^{-1} = D_{A_{\rho}^u}$$

where $\rho(U) = \rho(u) J \rho(u) J^{-1}$ with $u$ a unitary of $\mathcal{A}$ and [26]

$$A_{\rho}^u := \rho(u) [D, u^*]_\rho + \rho(u) A u^*.$$  (3.24)

This is a twisted version of the law of transformation of the gauge potential (2.13).

The main difference with the usual gauge transformation (2.12) is that the latter preserves the selfadjointness of the operator $D_{A_\rho}$, whereas (3.23) does not preserve the selfadjointness of $D_{A_{\rho}^u}$. However, it preserves the adjointness with respect to the inner product induced by the twist, which is defined as follows.

**Definition 3.1.** Let $\rho$ be an automorphism of the algebra $B(\mathcal{H})$ of bounded operators on an Hilbert space $\mathcal{H}$. A $\rho$-twisted inner product $\langle \cdot, \cdot \rangle_\rho$ is an inner product on $\mathcal{H}$ such that

$$\langle \psi, O \phi \rangle_\rho = \langle \rho(O)^\dagger \psi, \phi \rangle_\rho \quad \forall O \in B(\mathcal{H}), \ \psi, \phi \in \mathcal{H},$$

where $^\dagger$ is the adjoint with respect to the initial inner product. We denote the $\rho$-adjoint of $O$ as $O^\rho := \rho(O)^\dagger$.

If $\rho$ an inner automorphism of $B(\mathcal{H})$, that is $\rho(O) = R O R^\dagger$ for a unitary operator $R$ on $\mathcal{H}$, then a natural $\rho$-product is

$$\langle \psi, \Phi \rangle_\rho = \langle \psi, R \Phi \rangle.$$  

In the twisted spectral triple of the Standard Model, the flip $\rho$ is an inner automorphism of $B(L^2(\mathcal{M}, S))$, with $R = \gamma^0$. Thus the $\rho$-twisted inner product is nothing but the Krein product for the space of spinors on a Lorentzian manifold. Furthermore, extending $\rho$ to the whole of $B(L^2(\mathcal{M}, S))$, one finds

$$\rho(\gamma^0) = \gamma^0, \quad \rho(\gamma^j) = -\gamma^j \quad \text{for} \quad j = 1, 2, 3.$$  

The flip $\rho$ is the in some sense the square of the Wick rotation [17]

$$W(\gamma^0) = \gamma^0, \quad W(\gamma^j) = i \gamma^j.$$  

that is $\rho = W^2$.

This suggests that the twisting procedure has something to do with a transition from the Riemannian to the Lorentzian signatures. This is confirmed by studying the fermionic action for a twisted spectral triple.

### 4 Actions for twisted spectral triples

#### 4.1 Twisted fermionic action

The fermionic action $S_\phi^f$ for a twisted spectral triple is defined [20] substituting the inner product in (2.17) with the twisted $\rho$-product, and considering the twisted covariant operator $D_{A_{\rho}^u}$ instead of $D_{A_\rho}$. Also, one does not restrict to an eigenspace of the grading operator, but consider instead an eigenvector of the unitary $R$ that implements the twist. This is required to guarantee that the fermionic action is antisymmetric as a bilinear form, allowing thus the switch to Grassmann variables.
This has important consequences, most easily seen in the simplest example of the minimal twist of a manifold (of even dimension $2m$):

$$A = C^\infty(M) \otimes \mathbb{C}^2, \quad \mathcal{H} = L^2(M, S), \quad D = \partial; \quad \rho$$

where the representation $\pi$ of $A$ on $\mathcal{H}$ is

$$\pi(f, g) = \begin{pmatrix} f\mathbb{1}_{2^{m-1}} & 0 \\ 0 & g\mathbb{1}_{2^{m-1}} \end{pmatrix},$$

and $\rho$ is the flip

$$\rho(f, g) = (g, f) \quad \forall (f, g) \in A \simeq C^\infty(M) \oplus C^\infty(M).$$

(4.25)

A twisted generalised 1-form is parametrised by a 1-form field $f_\mu$ (there is no scalar field $\sigma$). The twisted fermionic action, in dimension 4 has been computed in [27]. One finds

$$S^f(\partial_\rho) = 2 \int_M d\mu \tilde{\zeta}^\dagger \sigma_2 (i f_0 \mathbb{1}_2 - \sum_{j=1}^3 \sigma_j \partial_j) \tilde{\zeta} \quad \text{where} \quad \zeta = \begin{pmatrix} \xi \\ \zeta \end{pmatrix} \in \mathcal{H}_R,$$

(4.26)

with $\mathcal{H}_R$ the +1 eigenspace of the unitary $R$. The striking point is the disappearance of the derivative $\partial_0$, substituted with the component $f_0$ of the twisted fluctuation. It reminds the Weyl Lagrangian $\bar{\psi} \overleftarrow{\partial_\mu} \sigma_\mu \psi$ where $\overleftarrow{\sigma}_\mu := \{\mathbb{1}_2, -\Sigma_{j=1}^3 \sigma_j\}$. Actually, it is tempting to identify $\tilde{\zeta}$ with $\psi$, then to assume

$$\partial_0 \psi = i f_0 \tilde{\zeta},$$

that is

$$\tilde{\zeta}(x_0, x_j) = \psi(x_0, x_j) = e^{it f_0} \psi(x_j).$$

But then $\tilde{\zeta}^\dagger \sigma^2$ has no reason to identify with $i \bar{\psi} \overleftarrow{\partial_\mu} \psi$. In other terms, there are not enough degrees of freedom to identify (4.26) with the Weyl Lagrangian.

This is cured by considering a double manifold, that is

$$A = (C^\infty(M) \otimes \mathbb{C}^2) \otimes \mathbb{C}^2, \quad \mathcal{H} = L^2(M, S) \otimes \mathbb{C}^2, \quad D = \partial \otimes \mathbb{1}_2$$

(4.27)

with representation

$$\pi(a = (f, g), a' = (f', g')) = \begin{pmatrix} f\mathbb{1}_2 & 0 & 0 & 0 \\ 0 & f'\mathbb{1}_2 & 0 & 0 \\ 0 & 0 & g\mathbb{1}_2 & 0 \\ 0 & 0 & 0 & g'\mathbb{1}_2 \end{pmatrix}. \quad (4.28)$$

The minimal twist is then given by $A \otimes \mathbb{C}^2$ acting on the same Hilbert space, with the same Dirac operator, and the automorphism is the flip. Then the twisted fermionic action gives back the Weyl equation in Lorentzian signature.

A similar result holds for the minimal twist of the spectral triple of electrodynamics [29] in Euclidean signature. The twisted fermionic action yields the Dirac equation in Lorentzian signature (and in the temporal gauge of Weyl).

As a conclusion, the component $f_0$ of the 1-form field that parametrises a general twisted 1-form (3.21) gets interpreted as the energy of a plane wave solution of the Weyl/Dirac equation in Lorentzian signature, even though one started with a Riemannian manifold. After a Lorentz transformation, the other components $f_i$, $i = 1, 2, 3$ get interpreted as spatial momenta (see [27] for details).
4.2 Spectral action for twisted spectral triple

To adapt the spectral action (2.18) to the twisted case, there are several options that are still “work in progress”. First of all, since the selfadjointness of the twisted covariant Dirac operator $D_{A_\rho}$ is not preserved by a twisted gauge transformation (3.23), $(D_{A_\rho})^2$ may not remain positive (nor selfadjoint, not even normal), so that there is no guaranty to make sense of $f(D_{A_\rho})$ thanks to the spectral theorem. This difficulty can be overcome by considering $(D_{A_\rho})^\dagger D_{A_\rho}$ instead of $(D_{A_\rho})^2$, as was done in [23]. As noticed in [20], under a twisted gauge transformation $(D_{A_\rho})^\dagger D_{A_\rho}$ is mapped to $U D_{A_\rho}^\dagger D_{A_\rho}^\dagger U^\dagger$, which has the same trace as $D_{A_\rho} D_{A_\rho}^\dagger$. Hence the action

$$\text{Tr}f\left(\frac{(D_{A_\rho})^\dagger D_{A_\rho}}{\Lambda^2}\right)$$

is well defined and gauge invariant.

Alternatively, one may use the twisted $\rho$-product of definition 2, and consider the trace of $(D_{A_\rho})^\dagger D_{A_\rho}$. Although the gauge invariance is not obvious and, for the same reasons explained above, the cut-off by $f(\Lambda)$ is not guaranteed by the spectral theorem, it is intriguing that in the case of the minimal twist of a Riemannian manifold, one has [1]

$$D^+ D = -\partial^2_0 + \sum_j (\partial^2_j)^2 + 2i\gamma^0 \gamma^j \partial_j \partial_j,$$

that is the sum of the squared of the free Dirac operator on Minkowski space $\phi_M$ with a correction term with vanishing trace. This tends to confirm the idea that a transition from the Euclidean to the Lorentzian does occur at the level of the action, if this is not at the level of the $\gamma$ matrices. To deal with the cut-off, one should use some technics of algebraic quantum field theory. In particular, one may select positive frequencies using another state than the trace truncated by the energy cut-off $\Lambda$. This will be investigated in some future works.

A third option is to consider from the start a $\rho$-adjoint Dirac operator, for example the free Dirac operator in Minkowski space $\phi_M$. Then [20] $\phi_M^\dagger \phi_M$ is (up to a sign), the Laplacian in Euclidean signature. The transition is then from the Lorentzian to the Riemannian. A similar calculation with a $\rho$-adjoint twisted covariant Dirac operator yields results similar to those of [23]. This will be explained in some future work.

More generally, this last example questions the definition of twisted spectral triple: would it make sense to impose the Dirac operator to be $\rho$-adjoint with respect to the twisted product rather than imposing the selfadjointness with respect to the initial Hilbert product ?

References

[1] In preparation. 2020.

[2] L. Boyle and S. Farnsworth. Noncommutative geometry, non-associative geometry and the standard model of particle physics. New J. Phys., 16(123027), 2014.

[3] T. Brzezinski, N. Ciccoli, L. Dabrowski, and A. Sitarz. Twisted reality condition for Dirac operators. Math. Phys. Anal. Geo., 19(3:16), 2016.

[4] T. Brzezinski, L. Dabrowski, and A. Sitarz. On twisted reality conditions. Lett. Math. Phys., 109(3):643–659, 04 2018.

[5] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, and A. Salvio. Investigating the near-criticality of the Higgs boson. JHEP, 89, 2013.

[6] A. H. Chamseddine and A. Connes. Resilience of the spectral standard model. JHEP, 09:104, 2012.

[7] A. H. Chamseddine, A. Connes, and M. Marcolli. Gravity and the standard model with neutrino mixing. Adv. Theor. Math. Phys., 11:991–1089, 2007.
[8] A. H. Chamseddine, A. Connes, and W. van Suijlekom. Beyond the spectral standard model: emergence of Pati-Salam unification. JHEP, 11:132, 2013.

[9] A. H. Chamseddine and W. D. van Suijlekom. A survey of spectral models of gravity coupled to matter. In: Chamseddine A., Consani C., Higson N., Khalkhali M., Moscovici H., Yu G. (eds) Advances in Noncommutative Geometry. Springer, Cham, pages 1–51, 2019.

[10] A. Connes. Noncommutative Geometry. Academic Press, 1994.

[11] A. Connes. Noncommutative geometry and reality. J. Math. Phys., 36:6194–6231, 1995.

[12] A. Connes. Gravity coupled with matter and the foundations of noncommutative geometry. Commun. Math. Phys., 182:155–176, 1996.

[13] A. Connes. On the spectral characterization of manifolds. J. Noncomm. Geom., 7(1):1–82, 2013.

[14] A. Connes and M. Marcolli. Noncommutative geometry, quantum fields and motives. AMS, 2008.

[15] A. Connes and H. Moscovici. Type III and spectral triples. Traces in number theory, geometry and quantum fields, Aspects Math. Friedt. Vieweg, Wiesbaden, E38:57–71, 2008.

[16] L. Dabrowski, F. D’Andrea, and A. Sitarz. The standard model in noncommutative geometry: fundamental fermions as internal forms. Lett. Math. Phys., 108:1323–1340, 03 2018.

[17] F. D’Andrea, M. A. Kurkov, and F. Lizzi. Wick rotation and fermion doubling in noncommutative geometry. Phys. Rev. D, 94(2):025030, 2016.

[18] G. Degrassi, S. D. Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, and A. Strumia. Higgs mass and vacuum stability in the standard model at nlo. JHEP, 08:098, 2012.

[19] A. Devastato. Noncommutative geometry, grand symmetry and twisted spectral triple. J. Phys.: Conf. Ser., 634:012008, 2015.

[20] A. Devastato, S. Farnsworth, F. Lizzi, and P. Martinetti. Lorentz signature and twisted spectral triples. JHEP, 03(089), 2018.

[21] A. Devastato, F. Lizzi, and P. Martinetti. Grand Symmetry, Spectral Action and the Higgs mass. JHEP, 01:042, 2014.

[22] A. Devastato, F. Lizzi, and P. Martinetti. Higgs mass in noncommutative geometry. Fortschritte der Physik, 62(9-10):863–868, 2014.

[23] A. Devastato and P. Martinetti. Twisted spectral triple for the standard and spontaneous breaking of the grand symmetry. Math. Phys. Anal. Geo., 20(2):43, 2017.

[24] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, H. M. Lee, and A. Strumia. Stabilization of the electroweak vacuum by a scalar threshold effect. JHEP, 06(031), 2012.

[25] G. Landi and P. Martinetti. On twisting real spectral triples by algebra automorphisms. Lett. Math. Phys., 106:1499–1530, 2016.

[26] G. Landi and P. Martinetti. Gauge transformations for twisted spectral triples. Lett. Math. Phys., pages 2589–2626, 2018.

[27] P. Martinetti and D. Singh. Lorentzian fermionic action by twisting euclidean spectral triples. Preprint: arXiv 1907.02485, 2019.

[28] C. A. Stephan. New scalar fields in noncommutative geometry. Phys. Rev. D, 79:065013, 2009.

[29] K. van den Dungen and W. D. van Suijlekom. Electrodynamics from noncommutative geometry. J. Noncommut. Geom., 7:433–456, 2013.