Baryons on the lattice

G.S. Bali

Dept. of Physics & Astronomy, The University of Glasgow, Glasgow G12 8QQ, UK

I comment on progress of lattice QCD techniques and calculations. Recent results on pentaquark masses as well as of the spectrum of excited baryons are summarized and interpreted. The present state of calculations of quantities related to the nucleon structure and of electromagnetic transition form factors is surveyed.

1. INTRODUCTION

In the past two years several narrow hadronic resonances have been discovered: new bottomonium and charmonium states, the $B_c$ and at least two new $D_s^*$ mesons. It is more than twenty years ago that new states with widths $< 10$ MeV have been seen last, in the $\Upsilon$ system. This is an exciting situation as only a rather small number of discovered hadrons, including most notably quarkonia, the hydrogen of QCD, are narrow. What makes this new era of hadron spectroscopy even more interesting is that many of these previously overlooked states appear to require more QCD than is allowed within simple quark-model $q\bar{q}$ mesons or $qqq$ baryons: at least some of the new states constitute the anti-thesis to the “hydrogen of QCD". If confirmed in a high statistics experiment, the $\Theta^+$ pentaquark baryon and possibly other exotic baryons will add even more to this excitement.

Lattice QCD is ideally positioned to compute the spectrum of reasonably stable hadrons as well as of non-perturbative properties relating to their internal structure. The above described experimental discoveries were paralleled by significant advances in lattice methods, enabling computer simulations of QCD to become a precision predictive tool. In many areas there are also lessons to be learned from combining lattice studies with effective field theory (EFT) methods and/or model assumptions.

We have also witnessed experimental progress in the study of the spectrum of excited baryons and of electromagnetic transition form factors. Generalised parton distributions (GPDs) of the proton are now being studied intensively. As a stable particle the nucleon lends itself to lattice studies. In the case of spin-independent structure functions it will be hard for lattice simulations to compete with the experimental precision. However, in addition to the theoretical satisfaction of verifying experimental measurements, this provides an ideal test ground for the methods and approximations employed in lattice studies. Once spin and transversity are included into the description of the nucleon, the experimental situation is far less clean and here there is real potential for lattice prediction rather than postdiction.

I will describe the present state of the field, interpret recent pentaquark and baryon mass calculations and briefly survey studies on the form and structure of baryons.
2. THE LATTICE: WHERE ARE WE? WHERE DO WE GO?

QCD can be regularized by introducing a space-time lattice cut-off $a$. The QCD coupling and $n_f$ quark masses whose values are not predicted by QCD should then be matched to reproduce $n_f+1$ experimental measurements of hadronic properties, for instance hadron masses. Everything else is a prediction and in this sense lattice QCD is a first principles approach. The confinement of colour implies that finite size effects are usually tiny, as long as the spatial box extent $L a \gg m^{-1}_\pi$. We are fortunate to find that lattice spacings $a^{-1} = 1 - 4$ GeV are sufficiently small to allow for controlled continuum limit extrapolations, $a \rightarrow 0$. This means that $L \ll 100$ is sufficient, which makes QCD tractable on computers.

On a lattice with $V = L^3 T$ points, the lattice Dirac operator is a huge matrix of dimension $12V$. The inversion of this operator represents the major computational task of lattice QCD and this makes simulations incorporating sea quarks expensive. The algorithmic cost explodes with small $\pi$ masses, $\propto 1/(m_\pi a)^3$. A smaller $m_\pi$ also requires a larger spatial lattice volume and the scaling behaviour, keeping $m_\pi L a$ fixed, is even worse: $\propto 1/(m_\pi a)^7$. This is the main reason why many simulations are performed in the quenched approximation including only valence quarks and neglecting the polarization of the QCD vacuum due to sea quarks. While this approximation violates unitarity and does not even qualify as a quantum field theory, light hadron masses seem to agree within 10% with experiment [1], indicating that the main effect of quark loops can be absorbed into redefinitions of the bare parameters of the theory. The quenched approximation however goes terribly wrong at least in the scalar and pseudoscalar flavour-singlet sectors.

In Figure 1 we display recent $n_f = 2$ QCD results obtained by the LHP and SESAM Collaborations [2] on the quark contribution $\Delta \Sigma$ to the proton spin, in the $\overline{MS}$ scheme at a scale $\mu = 2$ GeV. The normalization is such that $\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + J_g$, where $L_q$ is the contribution from the quark angular momentum and $J_g$ from the gluons. Similar results have been obtained by the QCDSF Collaboration [3]. In these simulations, $m_\pi > 550$ MeV. Obviously, for infinite quark masses we expect $\Delta \Sigma = 1$. It is therefore not
surprising that the experimental value is overestimated and it is clear that smaller quark masses are absolutely essential to allow for a meaningful chiral extrapolation. Fortunately, with the advent of new Fermion formulations \[5\] that respect an exact lattice chiral symmetry, a reduction of the quark mass towards values \(m_\pi \approx 180\text{ MeV}\) has become possible \[6\], albeit so far only in the quenched approximation.

In many lattice calculations it is sufficient to calculate quark propagators that originate from a fixed source point. In these cases only one column of the inverse Dirac matrix needs to be calculated, naively reducing the effort by a factor \(V\). In some cases diagrams with disconnected quark lines are needed. Examples are the physics of flavour singlet mesons, strong decays as well as parton distributions. In the latter case the complication can be avoided by assuming \(SU(2)\) isospin symmetry and only calculating differences between \(u\) and \(d\) quark distributions. Disconnected quark lines require all-to-all propagators and hence a complete inversion of the Dirac matrix appears necessary. This turns out to be prohibitively expensive in terms of memory and computer time. Fortunately, sophisticated noise reduced stochastic estimator techniques have been developed over the past few years and as a result tremendous progress was achieved. One such benchmark is the QCD string breaking problem, \(Q(r)\overline{Q}(0) \leftrightarrow \overline{B}(r)B(0)\), where \(B = \overline{Q}q\) and \(Q\) is a static quark. This represents one of the cleanest examples of a strong decay. Within both, the transition matrix element as well as the \(BB\) state all-to-all propagators are required. In Figure 2 we display the result of a recent SESAM Collaboration study \[3\] with \(n_f = 2\), \(m_q \approx m_s\) and \(a \approx 0.085\text{ fm}\). An extrapolation to physical light quark masses yields a string breaking distance \(r_c \approx 1.16\text{ fm}\). The gap between the two states in the string breaking region is \(\Delta E \approx 50\text{ MeV}\) and we are able to resolve this with a resolution of 10 standard deviations!

In conclusion, all the long standing “killers” \(m_q \ll m_s\), \(n_f > 0\) and all-to-all propagators have been successfully tackled. However, we are still a few years away from overcoming combinations of two of these simultaneously and possibly up to a decade separates us from precision simulations of flavour singlet diagrams with realistically light sea quarks. Not all these ingredients are always required at the same time. While ever bigger computers are an absolute necessity, most of the recent progress would have been impossible without novel methods. The gain factor from faster computers was almost 5,000 over the past 15 years. The factor from theoretical and algorithmic advances is harder to quantify.

3. PENT AQURK S

QCD goes beyond the quark model and hence hadronic states that do not fit into a naive quark model of \(q\overline{q}\) mesons and \(qqq\) baryons are of particular interest. Many of the observed hadrons will contain considerable higher Fock state components. Obviously, quantum numbers that are incomprehensible with a quark model meson or baryon interpretation provide us with the most clean-cut distinction. Such examples do exist in the Review of Particle Properties, namely the \(J^{PC} = 1^{-+}\) mesons \(\pi_1(1400)\) and \(\pi_1(1600)\). The minimal configuration required to obtain a vector state with positive charge either consists of two quarks and two antiquarks (tetraquark/molecule) or of quark, antiquark and a gluonic excitation (hybrid meson). These resonances are rather broad with widths \(\Gamma \approx 300\text{ MeV}\). However, the ratio \(\Gamma/m\) is very much the same as for the established \(\rho(770)\) vector meson.

Another clear indication of an \(n_{\text{quark}} > 3\) nature would for instance be a baryonic state
with strangeness $S = +1$. The minimal quark configuration in this case consists of five quarks (pentaquark): $uudd\bar{s}$. Over the past two years several experiments have presented evidence of a very narrow $\Theta^+(1530)$ resonance [7], with decay $\Theta^+ \to pK^0$ and $\Theta^+ \to nK^+$. The parity has not yet been established. However the mass is about 100 MeV above the $KN$ threshold and for $J^P = \frac{1}{2}^-$ an $S$-wave decay is possible, which might be difficult to reconcile with a width $\Gamma \ll 10$ MeV. For $\frac{1}{2}^+$ a $P$-wave is required, still a bit puzzling but less so. As the main decay channel does not require quark-antiquark pair creation one might hope to gain some insight from quenched lattice simulations and several attempts have been made [8,9,10,11,12,13].

Two groups [9,10] also investigated charmed pentaquarks and two studies [5,11] incorporated the $I = 1$ sector, in addition to $I = 0$. Two groups [10,11] employed chiral overlap Fermions while the others used conventional Wilson-type lattice quarks. Only the Kentucky group [11] went down to $m_\pi \approx 180$ MeV while all other $\pi$ masses were larger than 400 MeV. The Budapest-Wuppertal group [5] varied the lattice spacing and attempted a continuum-limit extrapolation. In all studies the negative parity mass came out to be lighter than the positive parity one, which is expected in the heavy quark limit.

There are two crucial questions to be asked: what happens when realistically light quark masses are approached? Do we see resonant or scattering states? Resolving a resonance sitting on top of a tower of $KN$ scattering states with different relative momenta appears rather hopeless at first. However, there are two discovery tools available: variation of the lattice volume and of the creation operator. By varying the volume (and the boundary conditions [10]) one will change the spectrum of $KN$ scattering states as well as the coupling of a given operator to $KN$ (the spectral weight).

For the $\frac{1}{2}^+$ state which can only decay into a $P$-wave, the mass of the scattering state will depend on the lattice size since the smallest possible non-vanishing lattice momentum is $\pi/(aL)$. For $\frac{1}{2}^-$ the volume dependence of the lowest scattering state mass will be weak, however, the scaling of the spectral weight with the volume provides us with additional information.

It turns out that the situation on the lattice is at least as ambiguous as the one encountered in experiment. To demonstrate this we display some Kentucky-Washington
results \[11\] in Figure 3. It appears that the \(\frac{1}{2}^-\) state dominantly couples to an S-wave \(KN\). The \(\frac{1}{2}^+\) displays the qualitative volume dependence of a P-wave, however, it does not share its non-interacting mass. It is most likely a P-wave scattering state. This interpretation is supported by the observed volume dependence of the spectral weight. At very small \(m_\pi\) the situation becomes further complicated by the fact that there is no axial anomaly in the quenched approximation. Hence the flavour singlet \(\eta'\) is degenerate with the \(\pi\). As this contribution comes in with a negative spectral weight, it is sometimes labelled a “ghost”. The \(\frac{1}{2}^+\) state can contain such a \(KN\eta'\) S-wave (dashed lines).

The \(\frac{1}{2}^-\) state becomes indistinguishable from a \(KN\) S-wave as the quark mass is reduced. This does not conclusively exclude the possible existence of a nearby resonant state which might only couple very weakly to the creation operator used in this particular study. In order to draw more definite conclusions a variation of the creation operator as well as of the volume appears necessary, which is a very ambitious project \[14\]. If the pentaquark really was such a narrow resonance as some experiments suggest then maybe a lattice operator can be constructed that has a large overlap with this state but only a very small coupling to \(KN\).

Lattice studies of diquark interactions in a simplified, more controlled environment represent an alternative strategy to the brute force simulation of unstable states. A baryon with one static and two light quarks constitutes one such arena. One can of course also investigate multiquark interactions in the nonrelativistic limit of infinitely heavy quark masses. Such tetra- and penta-quark potentials have been studied recently by two groups \[13,15,16\] and the results should provide model builders with some insight. However, it is not clear how to relate these findings to the light quark limit in which chiral symmetry appears to play a bigger rôle than instantaneous confining forces.

There exist quite a few narrow resonances very close to strong decay thresholds like the \(\Lambda(1405)\), the recently discovered \(X(3872)\) charmonium state and the \(a_0/f_0(980)\) system. It is very conceivable that such states contain a sizable multiquark component. The question then arises if these are would-be quark model states or if these are true molecules/multiquark-states, that appear in addition to the quark model spectrum. A fantastic arena to address this was provided by the recently discovered (probably scalar) \(D^*_s(2317)\) and (probably axialvector) \(D^*_s(2457)\) states. First lattice studies \[17,18\] have been performed, with somewhat contradictory interpretations of very compatible results. One might hope that a similar lattice effort will also be dedicated on the comparatively cleaner and easier question of tetraquarks as has been on pentaquarks.

4. EXCITED BARYONS

The spectrum of baryons has attracted renewed experimental and theoretical interest in recent years. There is the question if the states can be understood in terms of quark models and if so by what sort of interaction and assumptions. Do gluonic excitations or pentaquark components play a rôle for instance in the Roper resonance? What can we learn about quark-quark interactions within bound states? Quark model predictions are somewhat obscured as the corresponding decay widths set a limit on the precision that can be expected for the resulting masses. Are missing states really “missing” or are they just obliterated due to the presence of many very broad, overlapping resonances? Strongly
decaying hadrons also pose problems in lattice simulations. At present almost all calculations of baryonic resonances have been performed within the quenched approximation in which these are stable and hence the problem is circumvented.

This limitation can also be viewed as a virtue since most models suffer from an omission of quark pair creation effects too. Comparison with similar lattice results then allows to establish the validity range and applicability of a particular phenomenological model. One strength of lattice methods is that simulations are not limited to the quark mass parameters found in nature. Investigating the quark mass dependence of results is a powerful tool. In the limit of large quark masses one would expect to make contact with non-relativistic quark models while as $m_\pi \to 0$ overlap with chiral perturbation theory ($\chi$PT) predictions should be verifiable.

Based on the assumptions that QCD bound states are mesons and baryons, that there is a mass gap and spontaneous chiral symmetry breaking at zero quark mass, an effective low energy chiral effective field theory ($\chi$EFT) can be derived in the spirit of the Born-Oppenheimer approximation. This will, to leading order, describe interactions between the (fast moving) massless Nambu-Goldstone pions and other hadrons. In nature quarks and thus pions are not massless and the leading mass corrections are formally of order $m_\pi/\Lambda_{\chi SB}$ where $\Lambda_{\chi SB} \approx 4\pi F_\pi > 1$ GeV. The number of terms explodes at higher orders and predictive power is eventually lost, unless $m_\pi$ is sufficiently small to allow for an early truncation.

Lattice simulations with sea quarks have so far been limited to unrealistically heavy pions, heavier than about 400 MeV. Only recently masses as low as 180 MeV have become possible \cite{6}. To allow for a controlled extrapolation of lattice results to the physical region it is mandatory to establish an overlap between simulation data and $\chi$PT expectations. In general the size of this window will depend on the observable in question. Chiral lattice Fermion actions will make such a comparison with $\chi$PT cleaner. Only in this case an exact version of chiral symmetry can be formulated at finite lattice spacing. With other Fermion discretizations, strictly speaking, a comparison should only be attempted after extrapolating lattice results to the continuum limit. While even 400 MeV $\ll \Lambda_{\chi SB}$ (modulo the ambiguity of “$\ll$” vs. “$<$”) such a pion is still doomed to “see” some of the internal structure of the proton, with an inverse charge radius of about 250 MeV. Hence it is doubtful that the quark and gluon nature of QCD can completely be ignored with such a “hard” pion probe.

Naively, hadron masses are a polynomial in the quark mass, $m_q \propto m_\pi^2$. However, pion loops give rise to a non-analytic functional dependence on the quark mass. For instance the nucleon mass is given by

$$m_N(m_\pi) = m_N(0) + a_2 m_\pi^2 + a_3 m_\pi^3 + \left[ e_1'(\lambda) + a_4 + a_4' \ln \frac{m_\pi}{\lambda} \right] m_\pi^4 + a_5 m_\pi^5 + \cdots, \quad (1)$$

with a renormalization scale $\lambda$. The coefficients of the non-analytic terms can be related to phenomenological low energy constants. For instance $a_3 = -3 g_A^2/(32\pi F_\pi^2)$. Apart from such constants, $a_4$ and $a_4'$ contain terms $\propto m_N^{-1}$ and $a_4'$ a contribution $\propto a_2$. In the quenched approximation, the leading non-analytic term is not proportional to $m_\pi^3$ but to $m_\pi^2 \ln(m_\pi/\lambda)$, due to the $\eta'$ becoming an additional Goldstone pion.

In Figure 4 we show a comparison \cite{19,20} between $n_f = 2$ lattice data of the nucleon mass and the relativistic $\chi$PT expectation Eq. (1). The lattice results were obtained
Figure 4. Chiral extrapolation of the nucleon mass [19,20].

by the CP-PACS and JLQCD Collaborations [21] as well as by the UKQCD and QCDSF Collaborations [22], with (non-chiral) Wilson-type Fermions, on relatively fine lattices, $a < 0.15$ fm and large volumes, $aLm_\pi > 5$. The low energy parameters were fixed to phenomenological values and the fit comprises only of $m_N(0), c_2$ and $e_1(1$ GeV). The quantitative agreement between curve and data for $m_\pi > 600$ MeV is accidental [19]. A naive polynomial fit to the simulation data results in a nucleon mass much larger than the experimental value: lower order $\chi$PT only becomes applicable at smaller quark masses.

EFTs are based on the separation of scales. If the $\chi$EFT however is regulated in dimensional regularization then $\pi$ loop-integrals can receive significant contributions from momenta $q > \Lambda_{\chi SB}$. To enhance the convergence the Adelaide group [23] suggested a "finite range regularization" approach which amounts to introducing a momentum cut-off which is then varied to achieve "model independence". A hard cut-off can also be provided by lattice regularization of the chiral expansion [24,25]. Needless to say that all cut-off and scheme dependence will disappear at sufficiently high orders in the $p$-expansion.

During the past four years we witnessed many lattice publications on the spectrum of excited baryons [26]. While the extraction of a ground state mass is relatively straightforward, radial excitations either require high statistics and some confidence in the fitting procedures or the design of sophisticated, non-local creation operators [27]. All but one study [28] have been performed in the quenched approximation. Other strategies were implementations of improved Wilson-type actions like the $D_{234}$ action by Lee et al. [30] or the FLIC action by Melnitchouk et al. [31] as well as a recent study with Wilson-clover Fermions [32]. The BGR Collaboration employed chirally improved Fermions [27], the Riken-BNL group used chiral domain wall Fermions [33] and the Taiwan [34] as well as the Kentucky-Washington groups [6] made use of overlap Fermions.

Early articles shared the observation of the positive parity state being much heavier than the Roper $N'(1440)$ resonance while the negative parity ground state was compatible with
the orbital excitation, $N^*(1535)$. One explanation would be that the resonance observed in nature might have little overlap with the dominantly $qqq$ state created on the lattice; pentaquark or gluon components might be necessary. Alternatively, maybe one should not take the exact position of a resonance with a width of $O(200\text{ MeV})$ overly seriously.

Figure 5. The $N$, $N'$ and $N^*$ masses for overlap Fermions \cite{6} at $a \approx 1\text{ GeV}$.

Figure 6. Chirally improved Fermions \cite{27}: $0.06 < m_\pi^2/\text{GeV}^2 < 0.65$, $a^{-1} \approx 1.33\text{ GeV}$.

New results obtained by the Kentucky-Washington group \cite{6} and the BGR Collaboration \cite{27} at lighter pion masses, $m_\pi > 180\text{ MeV}$ and $m_\pi > 250\text{ MeV}$, respectively, are however compatible with experiment. We display these in Figures 5 and 6. Rather unsurprisingly the creation operator requires a node in its spatial wave function to produce a significant overlap with the radial excitation. This complication was circumvented in Ref. \cite{6} by a sophisticated fitting procedure while in Ref. \cite{27} such an adequate operator has been constructed, using a variational principle. Making contact with the light quark regime seems hopeless for $m_\pi > 500\text{ MeV}$. However, the authors of two recent studies \cite{32,34} manage to extrapolate their results to the experimental values from such pion masses as well. Note that the BGR Collaboration only sees a clear signal of the radial excitation for $m_\pi > 400\text{ MeV}$ while the Bayesian fitting procedure of the Kentucky group yields results at any quark mass. The Taiwan group \cite{34} in addition predicts the spectrum of doubly charmed baryons, with findings roughly compatible with earlier studies \cite{35,36} as well as with the SELEX candidate(s).

One might hope that in the near future the $N'$ state can cleanly be disentangled from a $P$-wave $\eta'\bar{N}$ scattering state or, at very small masses, from $S$-wave $\pi\pi N/\eta/\eta'\bar{N}$ states. By studying the volume dependence and spectral weights the Kentucky group has taken steps in this direction. Most of the lattice studies include additional baryonic resonances where similar problems need to be addressed. In conclusion, we are close to an understanding of the transition between the heavy and light quark limits in the quenched approximation, an information invaluable to model builders.
5. FORM AND STRUCTURE

Quite a few results on moments of GPDs, most notably from the QCDSF [37], SESAM and LHP Collaborations [38], exist. A nice review of the state-of-the-art concerning spin-independent parton distributions and the axial charge can be found in Ref. [39]. The main problem here is an overestimation by about 60% of \( \langle x \rangle_{u-d} \) if linearly extrapolated in \( m_\pi^2 \), relative to experiment. It is not clear whether this difference will reduce as the quark mass is decreased or if this has to do with the non-chiral Fermion formulation used. This issue will be clarified in the near future [40].

There has also been progress in resolving the momentum dependence of electromagnetic \( \gamma^* N \to \Delta \) transition form factors in a quenched study [41], in the region \( 0.1 \text{ GeV}^2 < Q^2 < 1.4 \text{ GeV}^2 \). The magnetic dipole form factor is significantly overestimated at large \( Q^2 \), due to the unrealistically small charge radii of \( \Delta \) and \( N \). One might hope that such effects cancel in part from form factor ratios. \( R_{EM} = G_{E2}/G_{M1} \) is fairly constant at -0.02(1), once extrapolated to the chiral limit. In contrast, \( R_{CM} = G_{C2}/G_{M1} \) decreases monotonously from -0.01(1) at 0.1 GeV\(^2\) down to -0.09(3) at \( Q^2 > 1 \text{ GeV}^2 \). This behaviour is in good agreement with \( Q^2 > 0.4 \text{ GeV}^2 \) CLAS data while it is hard to reconcile with the OOPS point \( R_{SM} = [-6.1 ± 0.2 ± 0.5] \% \) at \( Q^2 ≈ 0.13 \text{ GeV}^2 \).

6. SUMMARY

Many lattice studies now include sea quarks. Within the quenched approximation, light quark masses close to the physical limit have been realised and the lattice provides a powerful tool for exploring the validity range of chiral expansions. Lattice pentaquark studies still yield ambiguous results. A systematic study using a large set of creation operators, of lattice volumes, spacings and quark masses is possible but ambitious. Latest lattice data suggest that the mass of the Roper resonance can be reproduced in the quenched approximation, thus indicating a non-exotic leading Fock component. A lot of progress has been made in understanding the structure of the nucleon and there is a strong push towards reducing the lattice quark masses, closer to the physical limit.

ACKNOWLEDGMENTS

I thank Thomas Hemmert for useful comments. This work is part of the EC Hadron Physics I3 Contract No. RII3-CT-2004-506078. GB is supported by a PPARC Advanced Fellowship (grant PPA/A/S/2000/00271) as well as by PPARC grant PPA/G/0/2002/0463.

REFERENCES

1. C. Gattringer et al. [BGR Collab.], Nucl. Phys. B 677 (2004) 3.
2. J. W. Negele et al., Nucl. Phys. Proc. Suppl. 128 (2004) 170.
3. G. S. Bali, T. Düssel, T. Lippert, H. Neff, Z. Prkacin and K. Schilling [SESAM Collab.], arXiv:hep-lat/0409137 and in preparation.
4. M. Göckeler et al. [QCDSF Collab.], Phys. Rev. Lett. 92 (2004) 042002.
5. H. Neuberger, Phys. Lett. B 417 (1998) 141 and references therein.
6. N. Mathur et al., Phys. Lett. B 605 (2005) 137.
7. See e.g. T. Nakano, these proceedings; M.V. Polyakov, these proceedings.
8. F. Csikor, Z. Fodor, S. D. Katz and T. G. Kovacs, JHEP 0311 (2003) 070.
9. S. Sasaki, Phys. Rev. Lett. 93 (2004) 152001.
10. T. W. Chiu and T. H. Hsieh, arXiv:hep-ph/0403020 and arXiv:hep-ph/0501227
11. N. Mathur et al., Phys. Rev. D 70 (2004) 074508.
12. N. Ishii et al., Phys. Rev. D 71 (2005) 034001; arXiv:hep-lat/0501022
13. C. Alexandrou, G. Koutsou and A. Tsapalis, arXiv:hep-lat/0409065
14. G. T. Fleming, arXiv:hep-lat/0501011
15. C. Alexandrou and G. Koutsou, Phys. Rev. D 71 (2005) 014504.
16. F. Okiharu, H. Suganuma and T. T. Takahashi, arXiv:hep-lat/0407001
17. G. S. Bali, Phys. Rev. D 68 (2003) 071501.
18. A. Dougall et al. [UKQCD Collab.], Phys. Lett. B 569 (2003) 41.
19. M. Procuta, T. Hemmert and W. Weise, Phys. Rev. D 69 (2004) 034505.
20. M. Göckeler, arXiv:hep-lat/0412013.
21. A. Ali Khan et al. [CP-PACS Collab.], Phys. Rev. D 65 (2002) 054505; erratum ibid D 67 (2003) 059901; S. Aoki et al. [JLQCD Collab.], Phys. Rev. D 68 (2003) 054502.
22. C. R. Allton et al. [UKQCD Collab.], Phys. Rev. D 65 (2002) 054502; A. Ali Khan et al. [QCDSF and UKQCD Collabs.], Nucl. Phys. B 689 (2004) 175.
23. D. B. Leinweber, these proceedings; R. D. Young et al., Phys. Rev. D 66 (2002) 094507; D. B. Leinweber et al., arXiv:hep-lat/0501028
24. I. A. Shushpanov and A. V. Smilga, Phys. Rev. D 59 (1999) 054013.
25. R. Lewis and P. P. Ouimet, Phys. Rev. D 64 (2001) 034005; B. Borasoy and R. Lewis, Phys. Rev. D 71 (2005) 014033.
26. See e.g. D. B. Leinweber et al., arXiv:nucl-th/0406032
27. T. Burch et al. [BGR Collab.], Phys. Rev. D 70 (2004) 054502. D. Brommel et al., Phys. Rev. D 69 (2004) 094513. T. Burch et al., arXiv:nucl-th/0501025
28. C. Maynard and D. Richards [UKQCD], Nucl. Phys. Proc. Suppl. 119 (2003) 287.
29. M. Göckeler et al. [LHP-UKQCD-QCDSF Collabs.], Phys. Lett. B 532 (2002) 63.
30. F. X. Lee et al., Nucl. Phys. Proc. Suppl. 106 (2002) 248.
31. W. Melnitchouk et al., Phys. Rev. D 67 (2003) 114506; J. M. Zanotti et al. [CSSM Lattice Collab.], Phys. Rev. D 68 (2003) 054506.
32. D. Guadagnoli et al., Phys. Lett. B 604 (2004) 74 and these proceedings.
33. S. Sasaki, T. Blum and S. Ohta, Phys. Rev. D 65 (2002) 074503; S. Sasaki, Prog. Theor. Phys. Suppl. 151 (2003) 143.
34. T. W. Chiu and T. H. Hsieh, these proceedings: arXiv:hep-lat/0501021 T. W. Chiu and T. H. Hsieh, arXiv:hep-lat/0406016
35. R. Lewis et al. Phys. Rev. D 64 (2001) 094509 and D 66 (2002) 014502.
36. J. Flynn, F. Mescia and A. Tariq [UKQCD Collab.], JHEP 0307 (2003) 066.
37. M. Göckeler et al. [QCDSF Collab.], Phys. Rev. Lett. 92 (2004) 042002; arXiv:hep-lat/0409162 arXiv:hep-lat/0501029
38. P. Hägler et al. [LHP and SESAM Collabs.], Phys. Rev. D 68 (2003) 034505; Phys. Rev. Lett. 93 (2004) 112001; arXiv:hep-ph/0410017
39. W. Schroers, these proceedings: arXiv:hep-ph/0501156
40. M. Gürtler et al. arXiv:hep-lat/0409164 R. Horsley, arXiv:hep-lat/0412007
41. C. Alexandrou et al., Phys. Rev. Lett. 94 (2005) 021601.