Investment strategy based on a company growth model

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Summary. We first estimate the average growth of a company’s annual income and its variance by using both real company data and a numerical model which we already introduced a couple of years ago. Investment strategies expecting for income growth is evaluated based on the numerical model. Our numerical simulation suggests the possibility that an investment strategy focusing on the medium-sized companies gives the best asset growth with relatively low risk.

Key words. Investment strategy, Income, Wealth.

1 Introduction

Studies on wealth distribution can be traced back more than 100 years. It is widely known that Pareto published the first report on the personal income distribution in 1897 [V. Pareto]. He showed that the probability density distribution of income follows a power law distribution in the high-income range. He claimed that a kind of social structure can be characterized by the power exponent because smaller value implies that the tendency of monopoly is stronger.

By analyzing a huge database of company incomes, our group has been investigating statistical properties of time evolution of company’s income [T. Mizuno\textsuperscript{(1)}, T. Mizuno\textsuperscript{(2)}]. In those papers, we showed that prediction of the average income in the future is possible although the income of each company has violent fluctuations. A country needs to implement many policies of company’s income because company’s
income is closely related to the tax. In the present paper we estimate the average growth rate of a company by using a numerical model that is derived from the real data and discuss the performance of investment strategy for companies.

2 An empirical model for company’s income growth

We have already introduced an empirical model describing the stochastic dynamics of company’s income growth by the following equation [T. Mizuno(1), T. Mizuno(2)],

\[ I_k(t + 1) = \alpha(t) \cdot b(t) \frac{\sigma(I)}{\sigma_0} I_k(t) + f(t) \]  

(1)

where \( I_k(t) \) is the income of company \( k \) at the \( t \)-th year, \( b(t) \) and \( f(t) \) are random noises. The factor \( \alpha(t) \) is a stochastic variable taking either \( P \) or \(-1 \). The probability densities of these functions can be estimated from the company data. The power exponent \( \sigma(I)/\sigma_0 \) characterizes the size dependence of company growth statistics. This model is simple but is confirmed to reproduce known empirical laws of company’s income.

3 The growth rate of company’s income

For the purpose of discussing the investment strategy for income growth of American companies, we apply Eq.(1) with estimation the coefficient \( \alpha(t) \), \( b(t) \), \( f(t) \), \( \sigma(I)/\sigma_0 \) from a data set of income of American companies from 1989 to 1995 [T. Mizuno(2)][Moody’s company data]. In order to find out an investment strategy we observe how a company of initial size \( I(t = 0) \) grows in the next 5 years. We calculate the averaged growth rate of the cumulative income, that is defined by the ratio of the cumulative income normalized by the initial income, \( \Sigma I(t = 5)/I(0) \), where \( \Sigma I(t = 5) = I(t = 1) + \cdots + I(t = 5) \). In Fig.1 the numerical values of \( \Sigma I(t = 5)/I(0) \) estimated by Eq.(1) are compared with the values estimated directly from the real data of American companies (). From this figure we notice that the expectation of the cumulative income \( \Sigma I(t = 5) \) is proportional to the initial income \( I(0) \) for the companies whose initial income is larger than one million dollars/year. For the companies with initial incomes smaller than one million dollars/year the expectation value of cumulative income is nearly independent of the initial value of income. We can also estimate the income growth of a long term by using the numerical model. We show the case of \( \Sigma I(t = 15) \) by a line of USA,\( S(t = 15) \) in Fig.1. Companies with initial incomes smaller than two million dollars/year also have the large average value of \( \Sigma I(t = 15)/I(0) \). These results imply that investing such smaller income companies will produce high growth on average.

Although a small company’s growth rate is large on average, its variance is also large. In Fig.2 we plot the probability densities of growth rates for the term of 5 years \( (\Sigma I(t = 5)) \) for three different initial income ranges. The probability density tails are very large for smaller companies as expected. In order to clarify the dependence on the initial income \( I(0) \) for this variance, we show the standard deviation of the
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Fig. 1 Averaged growth rate as a function of the initial income. Plots of USA are real data for 5 years. Plots of USA_S are results estimated by the numerical model.

Fig. 2 The probability density of growth rate for the term of 5 years ($\Sigma I(t = 5)$).

Fig. 3 The standard deviation of the growth rate.
probability density distribution of $\Sigma I(t=5)/I(0)$ in Fig.3. For companies with initial incomes smaller than one million dollars/year the standard deviation decreases rapidly in inverse proportion to initial income. For companies with initial incomes larger than one million dollars/year the standard deviation decreases slowly depending on initial income. Therefore, the risk caused by investment is small for large income companies.

4 The efficient investment strategy

The relation between profits and risks is important when considering an efficient investment strategy. For the case of initial income $I(0) = 10 \times 1000$ dollars) the distribution in Fig.2 is clearly asymmetric, namely, the average of return is positive. But there is a large possibility of taking very large loss because the fluctuations are so large. Namely, this case corresponds to the high-risk-high-return strategy, not a reasonable investment. In order to discuss a rational investment strategy with relatively low risk, we observe the relation between the average of $\Sigma I(t=5)/I(0)$ and the standard deviation. We define the investment efficiency $E(c, I(0))$ by the following equation,

$$E(c, I(0)) = \left\langle \sum I(5) / I(0) \right\rangle - c \cdot \sigma \left( \sum I(5) / I(0) \right), \quad (2)$$

where, $c$ is a constant showing the balances between profits and risks. We assume $c = 1/5$ $1/9$ although this balance changes with contents of each investment. We show the relation between the investment efficiency $E$ and the initial income $I(0)$ in the case of $c = 1/8$ in Fig.4. In this case the best reasonable investment targets are the companies of size around the peak point of the investment efficiency $E$, that is, the companies with income $1,000,000$-$10,000,000$ dollars/year for investment period of 5 years.
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5 Discussion

We discussed the expected profits and risks for various investment strategies by using the simple model of company income and showed that investment to the medium-sized companies is efficient with relatively low risk. Although we showed results only for companies in USA, similar results hold also for other countries such as Japan and England.

Turning our eyes to politics, Japanese government and banks tends to support large companies, however, judging from our results such investments can be categorized as too much low-risk-low-return strategy. Financial support to medium-sized companies will contribute to recovery of economical growth.

The correlation between the income and other indices (asset, etc.) [H. Takayasu] and the dynamics of other indices [M. H. R. Stanley] have been reported. We are going to use other indices of wealth in addition to income for the investment strategy in the future works. We expect that the investment strategy of high accuracy can be obtained by using these results.

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