To the question of a common field theory

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Abstract. The paper suggests an alternative approach devoted to the derivation of the equations of the theory of gravitation, the electromagnetic field and other fields, taking into account their interaction with each other. The theory is based on the calculation of the potential and kinetic energy of the surface of a five-dimensional hypermembrane, where the vector of its displacement $\xi$, associated with fields of purely physical nature and it plays the role of the fifth dimension.

1. Basic functional of the theory

In this paper we will discuss one alternative approach devoted to the theory of generalized physical fields, the interaction between which, thanks to the calculation method proposed below, can be calculated exactly with a single parameter of the theory, denoted by the letter $L_\alpha$ and having a dimension of the length. We calculate this parameter precisely, starting from the general properties of the theory of dimensions. Note that the number of articles devoted to the General Theory of Relativity (abbreviated GTR) and the “M-theory” has a huge amount of the work. That's why we will not cite here any original works for the simple reason that simply not to offend those on whom we did not refer, and who either did not notice, or simply missed in the sea of publications on this big topic. In this connection we confine ourselves to a number of classical monographs and we will dwell in detail on our understanding of the question concerning the formal mathematical side of the possible combination of interactions between the currently known physical fields. We also emphasize that the theory presented below is not of a quantum but of a classical nature. It is worthwhile to note in this connection that the transition to the quantum limit is not difficult, since the algorithm for quantizing the amplitudes of interacting fields developed to date is rather well developed. Firstly we begin with some general provisions of GTR. The fundamental functional of the Hilbert-Einstein theory is a classical action and it has the form \[ (1) \]

\[
S\left( g^{\alpha} \right) = \frac{c^4}{8\pi G} \int R\sqrt{-g} d\Omega,
\]

where $d\Omega = dx^0 dx^1 dx^2 dx^3, dx^0 = cdt$ - the four-dimensional volume in which $dt$ - own time interval and the contravariant components $dx^1, dx^2, dx^3$ - contravariant coordinates, $c$ - is the velocity of the light in a vacuum, $G$ - constant of gravitation, $R$ - curvature of the space (it will be told just below), which is the convolution of the Riemann tensor, i.e. $R = R^\mu = R^\mu_{\alpha\beta\gamma}$, where fourth-rank tensor $R^\mu_{\alpha\beta\gamma}$ - Riemann tensor, $-g$ - determinant of the covariant metric tensor $g_{\alpha\beta}$. And at last the parameter $\alpha$ in (1) is the coefficient which is a certain adjustable parameter that ensures a correct transition in the nonrelativistic approximation to the Newton potential, following from the Poisson equation. In [1] it is assumed equal to 16.
In calculating the variation of expression (1) with respect to the components of the contravariant metric tensor $g^{ik}$ the remarkable theorem of Hilbert comes to the aid that the variation of the covariant Ricci tensor $R_{ik}$, exactly is 

$$\delta R_{ik} = \frac{\partial R_{ik}}{\partial g^{mn}} \delta g_{mn} = 0.$$ 

This immediately leads to the Hilbert-Einstein equation in the absence of the matter: $R_{ik} - \frac{1}{2} R g_{ik} = 0$. In the event that matter is taken into account, an energy-momentum tensor $T_{ik}$ with a multiplier appears on the right $\frac{\alpha \pi G}{4c^4}$. As it was noted in [3] that the basic equation of the general relativity theory must always be solved only in the linear approximation with respect to small corrections to the pseudo-Euclidean metric

$$g_{0i} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}.$$ 

We note here that this assertion follows from a comparison of the scales of the left and right sides of the basic equation GTR $R_{ik} - \frac{1}{2} R g_{ik} = \frac{k \pi G}{4c^4} T_{ik}$. At the same time, it evidently follows from this that the characteristic scale of the change of the coordinates in order of magnitude should correspond to the distances $\delta x \sim \frac{c}{\sqrt{\rho G}} \sim 10^{13}$ cm (approximately one hundred million kilometers). It is clear that if $\delta x \ll \frac{c}{\sqrt{\rho G}}$, then we have in the right to not take matter into account at all, and solve the equation of general relativity without the right-hand side of. The latest, however, will relate only to geometry, but not to physics. The GRT equation acquires the physical meaning only in the case when the solution is sought in the form of a small additive correction to the pseudo-Euclidean metric (which was noted in [3]). This fact turned out to be the key to the theory, which will be discussed now. Its main idea is as follows.

Let us imagine a formal dimensional $n + 1$ manifold surrounded by $n$ dimensional hypersurface that makes small oscillations with respect to some fixed equilibrium position with some purely conventional name of the type “Beginning of the world” or “global equilibrium”, but for which the main characteristic feature is the condition that the scalar curvature Riemann $R = 0$. Such a rather pathetic name, however, quite adequately characterizes the essence of the described. Indeed, if there exist small oscillations of the hypersurface of four-dimensional space-time, then they can be generated only by the physical fields known now. It is the electromagnetic (potential $A^i = (i \psi, A)$, where $i$ - imaginary unit, A - the magnetic field potential, $\psi$ - the electric field potential (potential $A^i = (i \psi, A)$, the gravitational (potential $\phi$)), strong (potential $\phi_s$), electroweak and, which has some potential $\phi_D$ while only a purely hypothetical “dark matter”. This means that as soon as at least one of these fields appears, the process of oscillations of the hypersurface with the “simultaneous” appearance of one of the set of Universes of our World begins automatically. Such a picture will not contradict the theory of the “Big Bang” in according to which, at the time (if one can speak about it) bubble bursts, all known and unknown physical fields appear simultaneously, leading (the proof of this, see below) to fluctuations of the space-time Continuum, which is its inalienable property. From a formal point of view, it means that $R \neq 0$. The transition from a flat world $R = 0$ to ours $R \neq 0$ means only that, firstly, there is a kind of “geometric phase transition” and secondly, as a consequence, that a small value $\delta R$ should be proportional to the deviations of the hypersurface from the position of
“global equilibrium”. At the same time, it becomes perfectly clear that such hypersurfaces surrounding any material objects can be as many as you like. The magnitude of the deviation, which we will denote as a vector by a letter $\xi$ for different physical fields has a different scale of magnitude. For example, for a gravitational potential, it will be much larger than for a nuclear one. From simple analogies with vibrations of a one-dimensional elastic string in a two-dimensional (time-coordinate) case for its potential energy, one can write that $U_i = B_i \int \frac{dl}{R^2}$, where $B_i$ - a certain constant that provides the correct dimensionality of energy, which is a rigidity, $dl$ - is the element of a string length, $R$ - its radius of curvature at a given point. By introducing a small displacement of the string $\xi = \xi(x,t)$, on a three-dimensional manifold $\xi - x - t$, the element of length on the pseudo-plane $x - t$ can be represented as with a metric tensor $\hat{g}_{ab} = \left( \begin{array}{cc} a^2 - \xi^2 & i \xi_i \xi_i \\ i \xi_i \xi_i & -(1 + \xi_i^2) \end{array} \right)$, where $a$ - is the velocity of the longitudinal vibrations of the string, $\xi_i = \frac{\partial \xi}{\partial x}$, $\xi = \frac{\partial \xi}{\partial t}$ ($c = 1$). In doing so, we took advantage of the presentation of $dx^i = adt, dx^2 = idx$. As we can see, the determinant of this transformation is $g = -a^2 \left(1 + \xi_i^2 - \frac{\xi_i^2}{a^2}\right)$. Therefore, the total potential energy can be represented as a double integral:

$$U_i = -\frac{B_i a}{L} \int \frac{-g dx dt}{R^2},$$

(2)

where $d\Sigma_3 = a\sqrt{1 + \xi_i^2 - \frac{\xi^2}{a^2}} dx dt$ - pseudo-plane element of the hypersurface, $L$ - the parameter having the dimension of the length. In this example this is the length of the string. From the extremum of the functional (2) with respect to the variable $\xi$, i.e. $\delta U_i(\xi) = 0$, in the case of small oscillations, when $|\xi|, |\dot{\xi}| << 1$, we are getting the well-known equation $\frac{\partial^2 \xi}{\partial t^2} = a^2 \frac{\partial^2 \xi}{\partial x^2}$ (see, for example, [4]). Its solution defines a family of extremals that provide a minimum of potential energy (2). In this example, the metric describes a two-dimensional problem on a pseudo-plane, but for a one-dimensional line. The generalization (2) to the $n$-dimensional case is obvious. In the framework of the problem we are considering and we must write down the potential energy of the five-dimensional hypermembrane in the following form:

$$U_3 = B_3 \int R d\Sigma_4,$$

(3)

where $B_3$ - is the same of the constant value as in the formula (2), but of a different dimension, the element of the hypersurface $d\Sigma = cdx dy dz dt \sqrt{1 + \xi_i^2 + \xi_i^2 + \xi_i^2 - \frac{\xi^2}{c^2}}$, but here the value $R$ is the scalar curvature of Riemann. We note the fundamental difference between expression (3) for the potential energy from the Hilbert-Einstein action (1). When constructing the action of $S$ functional in (1), the energy density was multiplied by $dx^i = cdt$. As a result, the Hilbert-Einstein functional (abbreviated H-E) was obtained, the variation of which leads to the basic equation of GRT in the absence of the matter [1-2]. $\left( R_k^l - \frac{1}{2} R \delta_k^l \right) = 0$. As can be seen from (3), at our disposal there is already ready potential energy describing the displacement of points lying on the hypersurface of the space-time continuum, but not action. This means that in the five-dimensional case the membrane will be a four-dimensional...
hypersurface with an area element of \( d\Sigma_4 = cdx dy dz dt \sqrt{1 + \xi_i^2 + \xi_j^2 + \xi_k^2 - \frac{\xi^2}{c^2}} \). In this case, small displacements of this hypersurface will obey the usual d'Alembert equation \( \Box \xi = 0 \), where the d'Alembert operator is \( \Box = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta \), \( c \) - the velocity on the three-dimensional hypersurface of the four-dimensional manifold, where \( \xi^4 \) is the fourth dimension. Formally, we are talking about a five-dimensional continuum, in which time plays the role of the fifth dimension. The reasoning above leads us directly to the formulation of our problem, automatically indicating the course of its solution. Indeed if we assume that the solution of the problem should be sought in a five-dimensional (at least) space in which the formally introduced fourth vector dimension must have a purely physical nature (see below), then the role of the fifth dimension will play time. We note here that the approach under consideration has no points of intersection with the 11-dimensional “M-theory”, but has a purely alternative classical character. Thus, the potential energy of oscillations of the four-dimensional hypermembrane in the five-dimensional continuum can be easily written in the obvious form:

\[
U_4 = B_4 \int_{\Sigma_0} R d\Sigma_4,
\]

where \( d\Sigma_0 = \sqrt{-g} dx^1 dx^2 dx^3 dx^0 \) - is the element of its hypersurface in the case of the pseudo-Euclidean metric. The element of the metric on it is representable in the following general form

\[
d\xi^2 = c^2 dt^2 - \frac{(dx^1)^2}{k^2} - \frac{(dx^2)^2}{k^2} - \frac{(dx^3)^2}{k^2} - d\xi_5^2,
\]

where the introduced coefficient \( k \) allows to automatically take into account the transition from macroscopic dimensions to smaller ones and is simply a scale factor, \( \xi_5 \) - the vector characterizing a purely physical fifth dimension (see above text). It is a consideration of the influence of various physical fields under the action of which and quite definite oscillations of the hypermembrane occur relative to the position of the “Global Equilibrium”. We define it as a vector with components \( \xi = (\xi_1, \xi_2, \xi_3, \xi_5) \), which will be denoted below as \( \xi^a \) with a Greek index of \( \alpha \), running four values, exactly \( \alpha = 1, 2, 3, 4 \) as well as the upper Latin index \( \xi_i \), where up index \( i = 1, 2, 3, 4 \). This four-dimensional displacement vector \( \xi_i^a \), is proportional to \( A' = (\psi, -i A) \) - (A - the potential of the electromagnetic field, \( \psi \) - magnetic field potential, electric). As for the proportionality factor, which provides the correct dimensionality for the displacement, it will be strictly calculated below. The displacement \( \xi_z = L_0 \frac{\varphi}{c^2} \), where \( L_0 \) - is the only parameter of the problem, having the dimension of the length, and \( \varphi \) - is the potential of the gravitational field. As to the displacements \( \xi_1, \xi_4 \), it is possible, for example, to mean the potential for strong interaction and the potential of the “Dark matter”. At least their physical meaning will be completely transparent after the derivation of the basic equation for \( \xi_5 \). It is worth emphasizing once again that the hypersurface under consideration is an inalienable property of any space-time continuum, and it is always present. Its oscillations are generated by all existing physical fields: gravitational, electromagnetic, nuclear, “Dark matter” (if it exists), etc. Between these fields there are interactions that can be calculated exactly within the framework of the proposed model. They will appear purely formally in terms proportional to the even powers of the displacement vector of the hypersurface \( \xi_5^{2n} \) for \( n \geq 2 \), and are found from the product of Riemann’s scalar curvature \( R \) to the Jacobian \( \sqrt{-g} \), where \( (-g) \) - the determinant of the covariant metric tensor given on this hypersurface. Moreover, the metric on this hypersurface is non-Euclidean (formulas (5) and (6)). The representation of the displacement vector \( \xi^a \)
of the points of the surface of a physical hypermembrane in the form \( \zeta_i = (\zeta_{i1}, \zeta_{i2}, \zeta_{i3}, \zeta_{i4}) \) is only a hypothesis. However, this assumption allows us to enter into the definition of \( \zeta_i \) all the basic physical parameters that take into account their interaction with the gravitational field. Let us note, by the way, that this will lead to all the main conclusions of the general theory of relativity and, in particular, to the gravitational waves and the Newton potential.

In addition, we arrive at the correct formulation of the theory of the electromagnetic field, which includes taking into account the gravitational field and the remaining fields (see below). Taking into account that \( \zeta_i = (\zeta_{i1}, \zeta_{i2}, \zeta_{i3}, \zeta_{i4}) \) we obtain

\[
\begin{align*}
\begin{cases}
\frac{dx^i}{c} = c^4 \left( 1 - \frac{\beta^2}{c^2} \right) dt^2 - dx^2 \left( 1 + \frac{\beta^2}{c^2} \right) - dy^2 \left( 1 + \frac{\beta^2}{c^2} \right) - dz^2 \left( 1 + \frac{\beta^2}{c^2} \right) \\
-2dxdy = 2dxdz = 2dxdy + 2ydydz = 2ydydz - 2xdxdy + 2zdzd\zeta^2 - 2ydydz = 2zdzd\zeta^2,
\end{cases}
\end{align*}
\]

where \( i \) is an imaginary unit, \( dx = \frac{dx^i}{k}, \ dy = \frac{dx^j}{k}, \ dz = \frac{dx^k}{k} \) - the introduced abbreviated notations.

We note that we do not introduce here the covariant and contravariant metric tensors for a purely pseudo-Euclidean metric as for example in [1], but also use very convenient imaginary coordinates due to a simple rule \( dx' = \left( dx^0, i \frac{dx^1}{k}, i \frac{dx^2}{k}, i \frac{dx^3}{k} \right) \), where \( dx^0 = cdt \). Within the framework of expression (5) it is simply pointless to speak of a pseudo-Euclidean metric, since the covariant and contravariant metric tensors on this hypersurface are \( 4 \times 4 \) matrices. Indeed

\[
\begin{align*}
\hat{g}_{ik} &= \begin{pmatrix}
1 - \frac{\beta^2}{c^2} & -i \frac{\beta^2}{c^2} & -i \frac{\beta^2}{c^2} & -i \frac{\beta^2}{c^2} \\
-i \frac{\beta^2}{c^2} & 1 + \frac{\beta^2}{c^2} & -i \frac{\beta^2}{c^2} & -i \frac{\beta^2}{c^2} \\
i \frac{\beta^2}{c^2} & -i \frac{\beta^2}{c^2} & 1 + \frac{\beta^2}{c^2} & -i \frac{\beta^2}{c^2} \\
i \frac{\beta^2}{c^2} & -i \frac{\beta^2}{c^2} & -i \frac{\beta^2}{c^2} & 1 + \frac{\beta^2}{c^2}
\end{pmatrix}.
\end{align*}
\]

The determinant of this covariant tensor is given by a simple formula

\[
-g = 1 + \beta^2 + \beta^2 + \beta^2 - \frac{\beta^2}{c^2}.
\]

As can be seen this expression reduces to the formulas (2) and (3) with decreasing dimensionality, as it should be. Hence the sought potential energy of a hypermembrane will be

\[
U_4 = \frac{c^4}{a\pi G L_4} \int_{V_0} R \sqrt{1 + \frac{\beta^2}{c^2} + \frac{\beta^2}{c^2} + \beta^2} \, d\omega = dx dy dz d\omega,
\]

where \( d\omega = dx dy dz dx^0 \) - a hypersurface element and \( dx^0 = cdt \). To calculate the variation of the functional (6) with respect to the vector displacement function of the surface of the hypermembrane \( \zeta(\mathbf{r}, x^0) \), we must first calculate the integrand in (6) in terms of the displacement function \( \zeta(\mathbf{r}, x^0) \). To do this, proceed as follows. We shall not search separately for the quantities entering into (6), but we
immediately use the definition of the scalar curvature of the Riemann tensor, which we write in the usual form as \( R = R^i_{i} = R^i_{jkl} \), where \( R^i_{jkl} = \frac{\partial \Gamma^i_{jk}}{\partial x^l} - \frac{\partial \Gamma^i_{jl}}{\partial x^k} + \Gamma^i_{lm} \Gamma^m_{jk} - \Gamma^i_{jm} \Gamma^m_{kl} \). Hence, the scalar is
\[
R = R^i_{i} = \frac{\partial \Gamma^i_{jk}}{\partial x^l} - \frac{\partial \Gamma^i_{jl}}{\partial x^k} + \Gamma^i_{lm} \Gamma^m_{jk} - \Gamma^i_{jm} \Gamma^m_{kl}.
\]
Since the Cristoffel’s symbol is \( \Gamma^i_{jk} = \frac{1}{2} g^{il} \left( \frac{\partial g_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l} \right) \), then
\[
\Gamma^i_{jk} = \frac{1}{2} g^{il} \left( \frac{\partial g_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l} \right), \quad \text{and} \quad \Gamma^i_{ii} = \frac{1}{2} \nabla_i g.
\]
In the first approximation, we are only interested in terms quadratic in bias \( \tilde{\xi} \) in the potential energy (6). The next approximation as is easy to see will be of order \( \tilde{\xi}^4 \), and allows us to take into account the various anharmonisms associated with the interaction of the fields, as we discussed above. To calculate the quadratic terms, we will need separate Cristoffel’s symbols. Indeed for example
\[
\Gamma^0_{00} = \frac{1}{2} g^{00} \left( \frac{\partial g_{00}}{\partial x^0} \right) = -\frac{1}{g} \left( \xi_x^2 + \xi_y^2 + \xi_z^2 \right),
\]
where we can assume that the determinant of the metric tensor is approximately equal to \(-1\), because the magnitude of the displacement \( \tilde{\xi} \) is small in comparison with the length parameter \( \sqrt{\rho G} \), where \( \rho \) - is the density of the matter. Note that we assume here and further the velocity of light is unity (\( c = 1 \)). As a result, we find
\[
\Gamma^1_{11} \approx \tilde{\xi}_x^2 \quad \Gamma^2_{22} = -\tilde{\xi}_y^2 \quad \Gamma^3_{33} \approx -\tilde{\xi}_z^2.
\]
Similarly, all other components of the Cristoffel’s symbol are calculated. For example, \( \Gamma^1_{00} = -\tilde{\xi}_x^2 \), \( \Gamma^1_{11} = i \tilde{\xi}_x \tilde{\xi}_y \), \( \Gamma^1_{22} = i \tilde{\xi}_x \tilde{\xi}_y \). As a result we have that \( \Gamma^i_{ij} = i \tilde{\xi}_x \tilde{\xi}_y \), \( \Gamma^i_{ik} = i \tilde{\xi}_x \tilde{\xi}_z \), \( \Gamma^i_{ik} = i \tilde{\xi}_y \tilde{\xi}_z \), where \( \Box \) - is the D’Alembert operator. Therefore, in the approximation that is quadratic in the bias \( \tilde{\xi} = (\tilde{\xi}_x, \tilde{\xi}_y, \tilde{\xi}_z) \), we find the following expression
\[
R \approx \frac{\partial \Gamma^i_{ij}}{\partial x^j} \frac{\partial \Gamma^i_{ij}}{\partial x^j} = \frac{\partial}{\partial t} \left( \tilde{\xi}_x \Box \tilde{\xi}_x - \tilde{\xi}_y \Box \tilde{\xi}_y - \tilde{\xi}_z \Box \tilde{\xi}_z \right) - \frac{\partial}{\partial x^i} \left( \tilde{\xi}_i \Box \tilde{\xi}_i - \tilde{\xi}_i \Box \tilde{\xi}_i - \tilde{\xi}_i \Box \tilde{\xi}_i \right) - \frac{1}{2} \Box g.
\]
Hence,
\[
\Box g = -2 \left( \tilde{\xi}_x \Box \tilde{\xi}_x + \tilde{\xi}_y \Box \tilde{\xi}_y + \tilde{\xi}_z \Box \tilde{\xi}_z \right) - \frac{1}{2} \left( \tilde{\xi}_x \Box \tilde{\xi}_x + \tilde{\xi}_y \Box \tilde{\xi}_y + \tilde{\xi}_z \Box \tilde{\xi}_z \right).
\]
Therefore, from (7), we have
\[
R \approx 2 \left( \Box \tilde{\xi} \right)^2 + \nabla \tilde{\xi}_x \cdot \nabla \tilde{\xi}_x + \nabla \tilde{\xi}_y \cdot \nabla \tilde{\xi}_y + \nabla \tilde{\xi}_z \cdot \nabla \tilde{\xi}_z + \nabla \tilde{\xi}_x \cdot \nabla \tilde{\xi}_y + \nabla \tilde{\xi}_y \cdot \nabla \tilde{\xi}_z + \nabla \tilde{\xi}_z \cdot \nabla \tilde{\xi}_x + \nabla \tilde{\xi}_x \cdot \nabla \tilde{\xi}_z - \nabla \tilde{\xi}_y \cdot \nabla \tilde{\xi}_y - \nabla \tilde{\xi}_z \cdot \nabla \tilde{\xi}_z - \nabla \tilde{\xi}_x \cdot \nabla \tilde{\xi}_x - \nabla \tilde{\xi}_y \cdot \nabla \tilde{\xi}_y - \nabla \tilde{\xi}_z \cdot \nabla \tilde{\xi}_z.
\]
And, consequently, the potential energy of a hypermembrane curved by physical fields can be written in the form of the following quadratic functional:
\[
U_4 \approx \frac{c^4}{8 \pi G \rho} \int \left( \Box \tilde{\xi} \right)^2 + \nabla \tilde{\xi}_x \cdot \nabla \tilde{\xi}_x + \nabla \tilde{\xi}_y \cdot \nabla \tilde{\xi}_y + \nabla \tilde{\xi}_z \cdot \nabla \tilde{\xi}_z + \nabla \tilde{\xi}_x \cdot \nabla \tilde{\xi}_y + \nabla \tilde{\xi}_y \cdot \nabla \tilde{\xi}_z + \nabla \tilde{\xi}_z \cdot \nabla \tilde{\xi}_x + \nabla \tilde{\xi}_x \cdot \nabla \tilde{\xi}_z - \nabla \tilde{\xi}_y \cdot \nabla \tilde{\xi}_y - \nabla \tilde{\xi}_z \cdot \nabla \tilde{\xi}_z - \nabla \tilde{\xi}_x \cdot \nabla \tilde{\xi}_x - \nabla \tilde{\xi}_y \cdot \nabla \tilde{\xi}_y - \nabla \tilde{\xi}_z \cdot \nabla \tilde{\xi}_z \right) d \omega.
\]
Recall that the “nabla” operator involved here means differentiation only with respect to the spatial coordinates, that is \( \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \). Varying expression (8) by displacements \( \zeta^a \) we easy obtain

\[
\delta U_4 \approx \frac{2 \epsilon^4}{\alpha \pi G L_\alpha} \int_{\Sigma_\alpha} \Box^2 \zeta^a \delta \zeta^a d\omega.
\]  

To take into account the kinetic energy of the surface of the oscillating hypermembrane, let us write down its relativistic invariant expression, introducing the displacement velocity \( \frac{\partial \zeta^a}{\partial t} \) in the following form:

\[
E_k = \frac{\epsilon^2}{2 L_\alpha} \int_{\Sigma_\alpha} \rho_{\alpha \beta} \frac{\partial \zeta^a}{\partial t} \frac{\partial \zeta^b}{\partial x^\beta} d\omega,
\]

where \( \rho_{\alpha \beta} \) is the matrix of the density of the corresponding matter and as a rule should have a diagonal form \( \rho_{\alpha \beta} = \rho_{\alpha} \delta_{\alpha \beta} \), where \( \delta_{\alpha \beta} \) - the Kronecker’s symbol. In accordance with this the general formula (10) can be rewritten as:

\[
E_k = \frac{\epsilon^2}{2 L_\alpha} \int_{\Sigma_\alpha} \rho_{\alpha} \frac{\partial \zeta^a}{\partial t} \frac{\partial \zeta^a}{\partial x^\alpha} d\omega,
\]

where \( dx^\alpha = (c dt, idx, idy, idz) \). Varying expression (11) by \( \tilde{\zeta} \), after elementary integration by parts we obtain

\[
\delta E_k = \frac{\epsilon^2}{2 L_\alpha} \int_{\Sigma_\alpha} \rho_{\alpha} \frac{\partial \zeta^a}{\partial t} \frac{\partial \zeta^a}{\partial x^\alpha} d\omega = \frac{\epsilon^2}{L_\alpha} \int_{\Sigma_\alpha} \rho_{\alpha} \frac{\partial^2 \zeta^a}{\partial x^\alpha} d\omega = -\frac{\epsilon^2}{L_\alpha} \int_{\Sigma_\alpha} \rho_{\alpha} \frac{\partial \zeta^a}{\partial t} \Box \zeta^a d\omega.
\]

The sum of the expressions (9) and (12) must be zero, i.e

\[
\frac{2 \epsilon^4}{\alpha \pi G L_\alpha} \int_{\Sigma_\alpha} \Box^2 \zeta^a \delta \zeta^a d\omega - \frac{\epsilon^2}{L_\alpha} \int_{\Sigma_\alpha} \rho_{\alpha} \delta \zeta^a \Box \zeta^a d\omega = 0.
\]

Therefore, because the variation of \( \delta \zeta^a \) are the arbitrariness we are coming to the desired equation for the displacement function

\[
\Box^2 \zeta^a = \kappa \zeta^a \zeta^a,
\]

where parameter \( \kappa = \frac{\alpha \pi G \rho_{\alpha}}{2 \epsilon^2} \). On the basis of equation (13), we can make the following rather important statement. The above displacement \( \zeta \) is an inherent property of any matter and thanks to this parameter, we have at our disposal the distribution of all fields in the space-time continuum, which can be mutually generated by any other physical fields (electromagnetic, strong, gravitational, dark matter, etc.). It should be noted that H-E equations do not have such an opportunity, since they contain a completely different idea, namely, that the metric properties of the space-time continuum change due to the presence of matter. This rather fundamental difference allows us, with the help of the proposed model to introduce, for example, the interaction of a gravitational field with a nuclear field, to assert that it leads to the formation of an electromagnetic field. Similarly the EM field can interact with the nuclear potential \( \phi = \phi(A, \psi_\alpha) \), as a result of which a gravitational field with a potential is generated. It is worth emphasizing that the marked effects will appear only in the nonlinear approximation with respect to the displacement of the surface \( \zeta \) and therefore with very weak interaction, which is in general quite unsurprising.
2. The basic equations

Indeed, for the displacement $\vec{\zeta}$ in the absence of nonlinear terms in the metric of the hypersurface, and in the absence of currents (we shall take them into account below), we have equation (13), which, when taking into account the nonlinear terms $\vec{\zeta}$ must be of a much more complicated form:

$$\Box^2 \zeta^a - \kappa_a^2 \zeta^a = B_{ijklm}^a \frac{\partial}{\partial x^i} \left( \zeta^j_{,k} \zeta^l_{,m} \right),$$

(14)

where the down suffix members of $\vec{\zeta}$ mean the partial derivatives with respect to the corresponding arguments, i.e. $\zeta^a_i = \frac{\partial \zeta^a}{\partial x^i}$, $\zeta^a_{,i} = \frac{\partial^2 \zeta^a}{\partial x^i \partial x^j}$. A dimensionless tensor of sixth rank $B_{ijklm}^a$ can be easily calculated from the general definitions for $g_{ik}$ and $g^{ik}$. We also note that the length parameter $L_G$ introduced above is also included in the definition of the tensor $B_{ijklm}^a$. As we can see the function itself $\vec{\zeta}$ is not explicitly included in the definition of the metric. Its definition includes only derivatives of displacement $\vec{\zeta}$. Therefore in order to find them we differentiate equation (14) with respect to $\Box$. Then, $\Box^2 \zeta^a = \kappa_a^2 \Box \zeta^a$. The solution of this equation, like equations (14), is completely identical and in a spherically symmetric case must have purely radial dependences. In order to verify this, we return to equation (14), and find its solution. To do this, it is convenient to enter a new function $u^a = - \Box \zeta^a$. It follows from (14) that $\Box u^a - \kappa_a^2 u^a = 0$, where $\kappa_a^2 = \frac{\alpha \kappa^2 G p_a}{2c^2}$. In the simplest variant we assume that the solution of the equation can be sought in the form of an expansion into the Fourier integral with respect to the time argument, i.e. $u^a (r,t) = \frac{1}{2\pi} \int_0^\infty \zeta^a (r) e^{i\omega t} d\omega$, where $u^a (r)$ is a Fourier–image of the function $u^a$. As a result the equation for the Fourier transformations is $\Delta u^a + \alpha_a^2 u^a = 0$, where $\alpha_a = \sqrt{\kappa_a^2 + \frac{\omega^2}{c^2}}$ is the parameter. After all the simple mathematical actions the final solution can be written in the following form

$$\zeta^a (r,t) = \frac{1}{4\pi r^2} \int_0^\infty \frac{\cos \frac{\omega r}{c} \omega^2 e^{-\omega t}}{\omega} \left[ \frac{C_{1a}^a \left( e^{i(a+a_r)R} - 1 \right) + C_{2a}^a \left( e^{-i(a+a_r)R} - 1 \right)}{a_a + \omega} - \frac{C_{1a}^a \left( e^{i(a-a_r)R} - 1 \right) + C_{2a}^a \left( e^{-i(a-a_r)R} - 1 \right)}{a_a - \omega} \right] d\omega.$$

If we assume that the frequency of oscillations is an monochromatic, i.e. $\zeta^a_{\omega_a} (r) = 2\pi \delta (\omega - \omega_0)$, we get

$$\zeta^a (r,t) = \frac{\cos \frac{\omega r}{c}}{2r \omega_0} \left[ \frac{C_{1a}^a \left( e^{i(a+a_r)R} - 1 \right) + C_{2a}^a \left( e^{-i(a+a_r)R} - 1 \right)}{a_a + \omega} - \frac{C_{1a}^a \left( e^{i(a-a_r)R} - 1 \right) + C_{2a}^a \left( e^{-i(a-a_r)R} - 1 \right)}{a_a - \omega} \right] e^{-\omega t},$$

where $\omega = \omega_0 = \text{const}$.

As can be seen in the stationary case for $\omega = 0$, we find

$$\zeta^a (r) = \frac{2R}{r \kappa} \cdot \frac{C_a^a e^{i\omega R} - C_a^a e^{-i\omega R}}{2i}.$$ 

Considering, that $C_1^a = C_2^a = -C_a^a$, we will find

$$\zeta^a (r) = C_a \frac{2R \sin \kappa R}{r \kappa}.$$
From the definition of the displacement vector of the hypermembrane \( \xi = (\xi_1, \xi_2, \xi_3, \xi_4) \) and because of the smallness of \( \kappa \), we obtain classical solutions for the gravitational static field in the form \( \xi_2 = L_\sigma \phi = C \frac{2R}{r} \). From the condition of “cross-linking” at the boundary of \( -\nabla \phi \rvert_{rr} = g = G \frac{mR}{r^2} \), where \( g \) - is the acceleration of free fall, and \( m \) - is the mass of the body, as it should be, we come to Newton’s law \( \phi = \frac{mG}{r} \). However the most physical solution can be found after introducing the interaction of “currents” leading to inhomogeneous equations, in contrast to the homogeneous (14) (see below). Thanks to the smallness of the product \( \kappa R \), one can also obtain the potential of the electric field, which appears in the definition \( \xi = (\xi_1, \xi_2, \xi_3, \xi_4) \) in the form of a vector \( \xi_i = L_\sigma (\gamma r,-\gamma A) \), where \( A \) - is the vector potential, \( \gamma r \) - is the electric potential (about parameter \( \gamma \) we’ll talk a little lower). So we have \( \gamma r = \frac{q}{r} \), where \( q \) - is a charge. We also note that, from the above general solution, we obtain delayed potentials of the electromagnetic field, as well as other consequences of the theory of the electromagnetic field. As a result \( \xi_i = D_\alpha \), where \( D_\alpha \) - some permanents. By virtue of this solution, the covariant metric tensor is very easy to write in explicit form. Its determinant is \( g = \left(1 + \frac{b_\alpha^2}{k^2 r^2}\right) \), where \( b_\alpha^2 = \left(D_\alpha^2\right)^2 + \left(D_\alpha^2\right)^2 - \left(D_\alpha^2\right)^2 \). As can be seen from the solution (14), all known, including purely hypothetical fields that is the gravitational, electromagnetic, nuclear and “Dark matter” should influence the metric properties of the space-time continuum. As for the \( D_\alpha \) they enter the solution for the time being as phenomenological constants. If we assume that \( D_\alpha^2 = D_\beta^2 = 0 \), we arrive to a metric \( g_{00} = 1 - \frac{D_0^2}{k^2 r^2}, \ g_{11} = -1 - \frac{D_1^2}{k^2 r^2} \), \( g_{10} = g_{01} = -\frac{D_1 D_0}{k^2 r^2} \), where \( D_0^2 = D_0^2 D_0^2, \ D_0 D_1 = D_0^2 D_1^2, \ D_1^2 = D_1^2 D_0^2 \), quite strongly resembling the decisions of Schwarzschild and Kerr. In this case the metric tensor will have the following form:

\[
\hat{g}_{\alpha \beta} = \begin{pmatrix}
1 - \frac{D_0^2}{k^2 r^2} & -\frac{D_1 D_0}{k^2 r^2} & 0 & 0 \\
-\frac{D_1 D_0}{k^2 r^2} & 1 - \frac{D_1^2}{k^2 r^2} & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}.
\]

(15)

Here it is worth noting that although the dependence of the components of the metric tensor in the Schwarzschild solution is \( \frac{1}{r} \), and not \( \frac{1}{r^2} \), as in (15), this is by and large the same thing since the correctness of the solution (15) is verified, as well as the Schwarzschild solution to the present moment time, unfortunately, impossible. Moreover in this connection underline that the decision (15) describing the metric on the hypersurface but not the volume one as in Schwarzschild’s decision. In addition, we’d like note that in the solution (15) was obtained, the off-diagonal components of the metric tensor \( g_{01} = g_{10} \) turned out to be non-zero. Perhaps some suspicion that in Schwarzschild they were simply lost are creeping in. In order for formula (15) not to look purely formal, we need to take
into account in the general equation (14) the currents on the surface of the hypermembrane. For this it is convenient to introduce the following quantities:

\[ \varepsilon_f \int f^a \nabla \xi^a d\omega, \quad f^a = \frac{e^2}{L_G} \left( \frac{\rho_q q u_{i}}{\omega L c^2} - \rho_{\gamma} - \rho_{\pi} - \rho_{D} \right) . \]  

(16)

where \( \varepsilon_f \) - is the energy associated with the displacement of the surface by the external “currents”, \( f \) - is the density of the volume force, all of \( \rho \) - are the corresponding to the density (charge, matter, nuclear matter, dark matter). \( \xi^a \) - displacement of the surface of the hypermembrane, which is chosen in the only possible form, preserving the correct dimensionality for the energy \( \varepsilon_f \). The dimensionless four-velocity

\[ u' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left( 1, \frac{v}{c} \right) \]

and the contravariant four-potential of the EM field is \( A' = (\psi, -i A) \). When the energy (16) is varied by the displacement \( \xi^a \) after double integration by parts and using the Stokes formula for transforming the integral over the hypersurface into an integral over the hypercontour, in which the variation \( \delta \xi^a = 0 \), and then substituting the result in the right-hand side of equation (14) after an elementary transformation equation with allowance for nonlinear terms:

\[ \square^2 \xi^a - \kappa^2 \square \xi^a = -L_G^2 \frac{4\pi}{c^4} \square f^a + B_{\text{dilat}} \frac{\partial}{\partial \xi^a} \left( z^b z^c z^d z^e \right). \]  

(17)

Let us analyze equation (17) in the case when there are no interactions that is a dimensionless tensor \( \hat{B} = 0 \).

3. Parameter \( L_G \)

In fact, choosing as the first example the electromagnetic potential, that is, assuming that the displacement due to it is

\[ \epsilon_{qi} = -L_G^2 \frac{A'_i}{e} , \]

we are finding from equation (17)

\[ \square^2 A' - \kappa^2 A' = -L_G^2 \frac{4\pi G}{c^4} \square f^a . \]

Substituting here from (16) \( f^a \) and “removing” one D’Alembert operator we get:

\[ \square A' - \kappa^2 A' = 4\pi \rho_q u' \frac{e^2 G}{L_G^2 c^4} . \]  

(18)

In order that in the limiting case of the absence of gravity that is neglecting the second term on the left-hand side of the eq. (18) it becomes the usual equation of the theory of electromagnetic potential we should do this

\[ L_G = \frac{e}{c^2} \sqrt{G} . \]  

(19)

Then instead of (18) we obtain a generalized equation of the EM potential theory with allowance for the influence of gravitational forces:

\[ \Delta A' - \frac{1}{c^2} \frac{\partial^2 A'}{\partial t^2} + \kappa^2 A' = -4\pi \rho_q u'. \]  

(20)
Substituting the numerical values of the constants in (19) leads to the following: \( L_0 \sim 10^{-34} \text{ cm} \). This negligible size, nevertheless, will play an important role if we need to take into account the nonlinear interactions between the fields (see below) that enter the equation (17) in the form of tensor \( \hat{B} \). As we see from equation (17) in the absence of the matter and nonlinear interactions, when \( \rho_a = 0 \), \( \kappa = 0 \) and \( \hat{B} \), the solution should be sought in the form \( \xi (r,t) = \xi_0 e^{ikr-i\omega t} \), where \( \xi_0 \) - the amplitude of the oscillations. As a result, we immediately obtain the dispersion equation \( (k^2c^2-\omega^2)=0 \), from which the dispersion laws of the gravitational and electromagnetic waves automatically follow in the usual form \( \omega = ck \). If we return to eq. (17) and put \( \xi_0 = L_0 \frac{\phi}{c^2} \), we immediately obtain from it
\[
\frac{L_0}{c^2} \Box \phi - \kappa^2 \frac{L_0}{c^2} \phi = L_0 \frac{\alpha \pi G \rho c^3}{c^4 L_0} \Box \rho. \quad \text{“Removing” here the D’Alembert operator and, making simple abbreviations we are find up to the fundamental solution} \quad \frac{C}{r}, \quad \text{i.e.} \quad \Box \phi - \kappa^2 \phi = \alpha \pi G \rho + \frac{C}{r}. \quad \text{Setting} \quad C = 0, \quad \text{in the stationary case, we obtain the Helmholtz equation for the gravitational potential, in which the constant} \quad \alpha \quad \text{should be set equal to} \quad 4 \quad \text{and hence}
\[
\Delta \phi + \kappa^2 \phi = -4\pi G \rho. \tag{21}
\]
The solution of equation (21) is
\[
\phi(r) = G \int \frac{\rho(r') \cos \kappa |r-r'|}{|r-r'|} dV'. \tag{22}
\]
Assuming the density is homogeneous, the integral is easily calculated in a spherical coordinate system centered at its center. As a result we have the following one
\[
\phi(r) = \frac{2\pi G \rho}{r} \int_0^\pi \cos \kappa \sqrt{r'^2 - 2rr'\cos \theta + r'^2} \left( \sqrt{r'^2 - 2rr'\cos \theta + r'^2} \right). 
\]
The internal integral along the azimuthal angle can be taken trivially and, as a result, the solution for the external problem will be:
\[
\phi'(r) = \frac{2\pi G \rho}{rk^3} \int_0^\theta \sin \kappa (r + r') - \sin \kappa |r-r'| \sqrt{r'^2 - 2rr'\cos \theta + r'^2} \frac{4\pi G \rho \cos \kappa r'}{kr} r' \sin kr' dr'. 
\]
\[
= \frac{4\pi G \rho \cos kr}{rk^3} (\sin \kappa R - \kappa R \cos \kappa R). 
\]
Consequently \( \phi'(r) = \frac{4\pi G \rho \cos kr}{rk^3} (\sin \kappa R - \kappa R \cos \kappa R) \). Since \( \kappa R \ll 1 \) then immediately follows Newton’s law with an extremely small correction:
\[
\phi'(r) = \frac{MG}{r} \left( 1 - \frac{(kr)^2}{2} - \frac{(kR)^2}{10} \right), \tag{24}
\]
where \( M \) - is the mass of a spherical object. We recall that this solution was obtained from the solution of the external problem for the Helmholtz equation. It is easy to verify that for the inner problem, starting from the general solution given above, after simple computations, we obtain:
\[
\phi^-(r) = \frac{2\pi G \rho}{R^2} \left( R^2 - \frac{(kr)^2}{3} - \frac{(kR)^2}{6} - \frac{k^2R^4}{4} + \frac{(kr)^2}{60} \right). \tag{25}
\]
As can be seen from the solutions (24) and (25), they are continuously “sewn” on the boundary of the body when \( r = R \), that is \( \phi'(r) \bigg|_{r=R} = \phi^-(r) \bigg|_{r=R}. \)
4. Conclusion
To summarize, we note:
1. The functional that was used to derive the basic equation (17) is the total energy of the hypermembrane in the space-time continuum. The sum of the potential energy and work produced by all external non-conservative forces, which differs significantly from the Hilbert-Einstein method, where the approach of classical mechanics, connected with the calculation of the variation of the action functional.
2. All the physics of the problem being solved lies in the fourth spatial coordinate having vector character $\xi$, all components of which are determined by the potentials of the physical fields.
3. The proposed approach makes it possible to find the change in the metric under the influence of different types of fields, and to calculate the interaction between fields of different physical nature, which is important in the microscopic study and evaluation of quantum contributions to various gravitational effects.

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