\( \bar{K} \)-induced formation of the \( f_2(1270) \) and \( f'_2(1525) \) resonances on proton targets

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We investigate the production of \( f_2(1270) \) and \( f'_2(1525) \) mesons in the \( K^{-} p \rightarrow \Lambda f_2(1270) \), \( K^{-} p \rightarrow \Lambda f'_2(1525) \) and \( K^{-} p \rightarrow K^{+} K^{-} \Lambda \) reactions within an effective Lagrangian approach. For \( K^{-} p \rightarrow f_2 \Lambda \) reaction, by considering the contributions from the \( t \)-channel \( K^{+} \) exchange and \( u \)-channel nucleon pole, we get a fairly good description of the experimental measurements about the total and differential cross sections. Based on the studies of the \( K^{-} p \rightarrow f_2 \Lambda \) reaction, we investigate \( K^{-} p \rightarrow K^{+} K^{-} \Lambda \) reaction including the contributions from the \( f_2(1270) \) and \( f'_2(1525) \) mesons decaying into \( K^{+} K^{-} \) pair. The total cross sections and invariant mass distribution of the \( Kp \rightarrow K^{+} K^{-} \Lambda \) reaction are predicted. The results can be tested in future experiments and therefore offer new clues on the nature of these tensor states.

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I. INTRODUCTION

Since the charmonium-like resonance \( XYZ \) and the new baryonic \( P_c \) states are observed, a lot of experiments spring up to study the exotic states from Belle, BaBar, BESIII, LHCb, CDF, D0 and other collaborations. And people believe that the traditional convention, the meson is made up of quark and antiquark as well as baryon is made up of three quarks, is broken. Furthermore, it is not surprising to find out that many low-lying states, even those long believed to be conventional \( q\bar{q} \) (or \( qq\bar{q} \)) states, may have large components of other nature. This observation has attracted a lot of attention from the theoretical side. Various explanations of these states have been proposed, such as molecules, multi-quark states, kinematic effects, or mixtures of components of different nature. Nevertheless, up to now none of them has been accepted unanimously.

Indeed, it has been shown that many of the low-lying mesonic states can be understood not only as \( q\bar{q} \) states but also as meson-meson molecules, dynamically generated in the so-called unitary approaches. For example, \( f_2(1270), f'_2(1525) \) and \( K_2^{*}(1430) \) are listed in the Particle Data Group as the lightest tensor states and correspond to the \( 2^{++} \) ground-state nonet. On the other hand, these states have been studied tensor mesons in the unitary approach based on the hidden-gauge Lagrangians. It was found that the \( f_2(1270), f'_2(1525) \), and \( K_2^{*}(1430) \) are dynamically generated from vector-meson-vector meson interactions. Within this picture, the branching ratios into pseudoscalar-pseudoscalar and vector-vector final states are all consistent with data. In Ref. [4], the two-photon decay widths of the \( f_0(1370) \) and \( f_2(1270) \) have been calculated and found to agree with data. Furthermore, the ratios of the \( J/\psi \) decay rates into a vector meson (\( \phi, \omega, \text{or} K^{*} \)) and one of the tensor states \( f_2(1270), f'_2(1525), \) and \( K_2^{*}(1430) \)] have been calculated, and the agreement with data is found to be quite reasonable [5]. Following the same approach, in Ref. [6] it is shown that the ratio of the \( J/\psi \) decay rates into \( \gamma f_2(1270) \) and \( \gamma f'_2(1525) \) also agrees with data. The agreement with experimental data turns out to be quite good in general, providing support to the underlying assumption that these states contain large meson-meson components.

Assuming that the \( f_2(1270) \) resonance is a \( \rho - \rho \) molecular state, it was found in Ref. [7] that the differential cross section as well as the \( t \) dependence of the \( \gamma p \rightarrow p f_2(1270) \) reaction are in good agreement with the experimental results [8] and provide support for the molecular picture of the \( f_2(1270) \) resonance in the first baryonic reaction. In a recent work [9], taking the \( f'_2(1525) \) resonances are dynamically generated states from vector-meson-vector-meson interactions in the \( s \)-wave with spin \( S = 2 \), the \( \gamma p \rightarrow f'_2(1525) \) reaction has been studied.

In addition to the photoproduction processes, the \( \bar{K} N \) scattering can also provide important information on tensor mesons. Hence, we study the \( Kp \rightarrow \Lambda f_2 \) \( (f_2 \equiv f_2(1270), f'_2(1525)) \) reaction within an effective Lagrangian approach in this work. The contributions from the Born terms including \( t \)-channel \( K^{+} \) exchange and \( u \)-channel nucleon pole are considered. Furthermore, for the low energy of the \( K^{-} p \rightarrow K^{+} K^{-} \Lambda \) reaction, we pay especially attention on the role of the \( f_2(1270) \) and \( f'_2(1525) \) mesons.

This paper is organized as follows. After the introduction, we present the calculation of the production of \( f_2(1270), f'_2(1525) \) via \( \bar{K} \)-induced formation on proton targets. In Sec. III, the contribution to the \( K^{+} K^{-} \Lambda \) final state is discussed, and the corresponding total cross sections and invariant mass distribution of these reactions are given. This work ends with the conclusion.

II. THE \( K^{-} p \rightarrow f_2(1270) \Lambda, f'_2(1525) \Lambda \) REACTION

We choose the production process \( K^{-} p \rightarrow f_2(1270) \Lambda, f'_2(1525) \Lambda \) which can couple to \( K^{+} K^{-} \) [2]. First, we mainly concentrate on the production probability of \( f_2 \) in the \( K^{-} p \rightarrow f_2 \Lambda \) process, where
The basic tree level Feynman diagrams for the $K^- p \rightarrow f_2 \Lambda$ reaction are shown in Fig. 1, where the contributions from the $t$-channel $K^+$ exchange (a), $u$-channel nucleon pole (b), we also show the definition of the kinematical $(p_1, p_2, p_3, q, p_5)$ that we use in the present calculation. In addition, we use $k_1 = p_1 - q$, and $k_5 = p_1 - p_5$.

$t$–channel $K^+$ exchange process [Fig. 1(a)], and the nucleon pole [Fig. 1(b)] are taken into account. While the $s$-channel processes are neglected since the information of those processes is scarce and we expect these contributions to be small.

The obtained total cross sections for $K^- p \rightarrow f_2(1270)\Lambda$ with the typical cutoff $\Lambda_K = 0.51 - 0.53$ GeV. The experimental data are taken from Ref. [11](circle), Ref. [11](star) and Ref. [12](square)

The coupling constant $g_{Kp\Lambda}$ can be determined by flavor $SU(3)$ symmetry relations, which give $g_{Kp\Lambda} = 13.24$ [13]. While the value of the coupling constant $g_{f_2K^+K^-}$ can be determined from the partial decay width of $f_2 \rightarrow K^+ K^-$, which can be obtained from Eq. (2),

$$
\Gamma_{f_2 \rightarrow K^+ K^-} = \frac{m_{f_2}}{480\pi} g_{f_2K^+K^-}^2 (1 - \frac{4m_K^2}{m_{f_2}^2})^{5/2},
$$

FIG. 2: Feynman diagrams for $K^- p \rightarrow f_2(1270)\Lambda, f_2(1525)\Lambda$ reaction. The contributions from $t$-channel $K^+$ exchange (a), $u$-channel nucleon pole (b). we also show the definition of the kinematical $(p_1, p_2, p_3, q, p_5)$ that we use in the present calculation. In addition, we use $k_1 = p_1 - q$, and $k_5 = p_1 - p_5$. 

FIG. 3: The obtained total cross section for $K^- p \rightarrow f_2(1525)\Lambda$ with the typical cutoff $\Lambda_K = 0.51 - 0.53$ GeV. The experimental data are taken from Ref. [11](circle), Ref. [11](star) and Ref. [12](square)
determine the coupling constant for $K$ from its decay into two pions, which can then be used to determine the universal coupling constant of the $K\Lambda$ reaction.

By using the tensor meson dominance (TMD) [17], one can determine the universal coupling constant of the $f_2$ meson from its decay into two pions, which can then be used to determine the coupling constant for $f_2 \to NN$ reaction (See Table I).

| Resonance  | channel | $g_{f_2KK}$ | $G_{f_2NN}$ | $F_{f_2NN}$ |
|------------|---------|-------------|-------------|-------------|
| $f_2(1270)$ | $K\bar{K}$ | 9.96        | --          | --          |
|            | $\bar{N}\bar{N}$ | --          | 2.19        | 0           |
| $f_2(1525)$ | $K\bar{K}$ | 15.78       | --          | --          |
|            | $\bar{N}\bar{N}$ | --          | 0           | 0           |

where $m_{f_2}$ and $m_K$ are mass of $f_2$ and kaon, respectively.

With mass ($m_{f_2} = 1.275$ GeV, $1.525$ GeV, $m_K = 0.494$ GeV), total decay width ($\Gamma_{f_2(1270)} = 0.185$ GeV, $\Gamma_{f_2(1525)} = 0.073$ GeV), and decay branching ratio of $f_2 \to K^+K^-$ [Br($f_2(1270) \to K^+K^-) = 0.046$, Br($f_2(1525) \to K^+K^-) = 0.887$], from Eq. (4), we obtain the coupling constant are shown in Tab. I.

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Ref. [18] for details. And are shown in Table I.

In evaluating the scattering amplitudes of the $K^-p \to f_2\Lambda$ reaction, we need to include the form factors because the hadrons are not pointlike particles. For the t-channel $K^+$ meson exchange, we adopt here a common scheme used in many previous works [19]

$$F_K(k_t^2) = \frac{\Lambda^2 - m_k^2}{\Lambda^2 - k_t^2},$$

where $\Lambda_K$ is the cutoff parameter.

For the $s$-channel and $u$-channel processes, we adopt a form factor [19],

$$\mathcal{F}_B(q_{ex}^2, M_{ex}) = \frac{\Lambda_B^4}{\Lambda^4 + (q_{ex}^2 - M_{ex}^2)^2},$$

where $q_{ex}$ and $M_{ex}$ are the four-momentum and the mass of the exchanged hadron, respectively. The cut-off parameter is constrained between 0.8 and 1.5 GeV for all channels. For simplicity, we take $\Lambda_B = 0.98$ GeV [19] for the $u$-channel nucleon pole, and the $s$-channel resonance exchanges.

With the above preparation, we finally obtain the amplitude of the $K^-(p_1)p(p_2) \to f_2(q)\Lambda(p_3)$ process,

$$T_f^{j_3} = \frac{-i}{2m_{f_2}} \frac{g_{KpN}g_{f_2K^+}}{M_K + M_\Lambda} \bar{u}(p_5, s_5)\gamma_5u(p_2, s_2) \frac{1}{k_t^2 - m_k^2} F_k(k_t^2)$$

$$\times T_f^{\gamma\gamma}(q, \lambda)[-k_t^2 - p_{1\mu}p_{1\nu} + 2k_{t\mu}p_{1\nu}],$$

(7)

$$T_f^{s_3} = \frac{-i}{2M_N(M_N + M_\Lambda)} \bar{u}(p_5, s_5)\gamma_1p_1\frac{k_5 + M_N}{k_5^2 - M_N^2}$$

$$\times (\gamma_{\mu}p_{2\nu} + \gamma_{\nu}p_{2\mu}) T_f^{\gamma\gamma}(q, \lambda)u(p_2, s_2)\mathcal{F}_N(k_t^2, M_N^2),$$

(8)

where $s_3$, $p_5$ and $s_5$, $p_2$ denote the spin polarization variables and the four-momenta of the outgoing $\Lambda$ and the initial proton, respectively, while $q, \lambda$ are the four-momenta and spin polarization variables of the $f_2$ meson. The $\bar{u}(p_5, s_5)$ and $u(p_2, s_2)$
are the Dirac spinors for the \( \Lambda \) and proton, respectively, while the \( T_{\mu\nu}^{ff}(q, \lambda) \) is the polarization tensor of the \( f_2 \).

By defining \( t = (p_1 - q)^2 = k_1^2 \), \( s = (p_1 + p_2)^2 \), the corresponding unpolarized differential cross section reads as

\[
\frac{d\sigma}{d t} = \frac{M_N M_\Lambda}{16\pi s} \frac{1}{|p_{1\text{cm}}|^2} \left( \frac{1}{2} \sum_{s_1, s_2} |T_{\mu\nu}^{ff}|^2 \right). \tag{9}
\]

The total cross section can be obtained by integrating over the range of \( |t| \).

The sum over polarizations, in Eq. (4), can be easily done thanks to

\[
\sum_{f} T_{\mu\nu}(q, \lambda) T_{\mu\nu}^*(q, \lambda) = P_{\text{prop}}
\]

\[
= \frac{1}{2} (\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta} + \bar{g}_{\mu\alpha}\bar{g}_{\nu\beta}) - \frac{1}{3} \bar{g}_{\mu\nu}\bar{g}_{\alpha\beta}
\tag{10}
\]

for the tensor \( f_2 \) meson, where \( \bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{q^\mu q^\nu}{m_f^2} \).

As shown in Figs. 2 and 3, we present the variation of the differential cross section for \( f_2(1270) \) and \( f_2'(1525) \). The obtained differential cross section for \( K^- p \to f_2(1270)\Lambda \) and \( K^- p \to f_2'(1525)\Lambda \) with the typical cutoff \( \Lambda_K \) are also analyzed in Fig 4 and Fig 5, respectively. The experimental data denoted by the squares are taken from Ref. [11]. We see that our theoretical results, which is obtained including the contributions from the \( t \)-channel \( K^+ \) exchange and nucleon pole, can give a reasonable description of the experimental data.

III. THE \( K^- p \to f_2(1270)\Lambda, f_2'(1525)\Lambda \to K^+ K^- \Lambda \) REACTION

Turning now to the \( K^- p \to f_2\Lambda \to K^+ K^- \Lambda \) reaction, described in Fig 6, the scattering amplitude \( M_{\text{prop}}(K^- p \to K^+ K^- \Lambda) \) is

\[
M_{\text{prop}}^{f_2}(K^- p \to K^+ K^- \Lambda) = i \left( \frac{g_{f_2 K^- K^+ \Lambda}}{2m_f} \right) \frac{g_{K^- p \Lambda}}{M_N + M_\Lambda} \left\{ \begin{array}{c}
p_1^a (p_3 - p_4)^b (p_3 - p_4)^c p_3^d G_{\text{prop}}(q) \\
[k_2^c (p_1 - k_2)^d - (p_1 - k_2)^c p_1^d] \frac{1}{k_1^2 - m_k^2}
\end{array} \right. \nonumber
\]  

\[
\times \bar{u}(p_5, s_5) k_1^2 \gamma_\mu (p_2, s_2) F_K(k_1^2) F_f(q^2, M_f) \tag{11}
\]

\[
M_{\text{prop}}^{f_2'}(K^- p \to K^+ K^- \Lambda) = -i \left( \frac{g_{f_2' K^- K^+ \Lambda}}{2m_f} \right) \frac{g_{K^- p \Lambda}}{M_N + M_\Lambda} \left\{ \begin{array}{c}
p_1^a (p_3 - p_4)^b (p_3 - p_4)^c p_3^d G_{\text{prop}}(q) \\
[k_2^c (p_1 - k_2)^d - (p_1 - k_2)^c p_1^d] \frac{1}{k_1^2 - m_k^2}
\end{array} \right. \nonumber
\]  

\[
\times \bar{u}(p_5, s_5) \gamma_\mu (k_1^2 + M_k) (\gamma_\alpha p_{2\alpha} + \gamma_\beta p_{2\beta})
\] 

\[
\times \bar{u}(p_2, s_2) F_K(k_1^2) F_f(q^2, M_f). \tag{12}
\]

where \( G_{\text{prop}}^T \) is the propagator of the tensor meson \( f_2 \) and can read

\[
G_{\text{prop}}^T = \frac{P_{\text{prop}}}{q^2 - m_{f_2}^2 + i m_{f_2} \Gamma_{f_2}}. \tag{13}
\]

FIG. 7: (color online). The cross section for \( K^- p \to f_2(1270)\Lambda, f_2'(1525)\Lambda \to K^+ K^- \Lambda \) reaction.

Furthermore, the corresponding \( K^+ K^- \) invariant mass spectrum for the \( K^- p \to K^+ K^- \Lambda \) reaction with \( \Lambda_K = 0.52 \) GeV at beam momentum \( P_{K^-} = 3.9 \) and 4.6 GeV are calculated and shown in Fig 8. The dashed lines are pure phase space distributions, while, the solid lines are full results from our model.
The $K^- K^+$ invariant mass spectrum for the $K^- p \rightarrow K^+ K^- \Lambda$ reaction at beam momentum $P_{K^-} = 3.9$ and $4.6$ GeV. The dashed lines are pure phase space distributions, while, the solid lines are full results from our model.
From Fig 8 we can see that there is a clear peak in the $K^-K^+$ invariant mass distribution, which is produced by including the contribution from the $f_2(1270)$ and $f_2'(1525)$, respectively.

IV. SUMMARY

In this work, we perform a calculation of the $f_2[= f_2(1270)$ $f_2'(1525)]$ tensor meson production in the $K^-p \rightarrow f_2\Lambda$ and $K^-p \rightarrow K^-K^+\Lambda$ reaction within the effective Lagrangian method. For the $K^-p \rightarrow f_2\Lambda$ reaction, by considering the contributions from the t-channel $K^*$ exchange and u-channel nucleon pole, we get a fairly good description of the experimental total cross section data. Our model shown the differential cross section $d\sigma/dt$ as function as $-t$, and get a good description of the experimental differential cross section data.

Basing on our results of $K^-p \rightarrow f_2\Lambda$ reaction, we have studied the $K^-p \rightarrow K^-K^+\Lambda$ reaction. In this case, we have considered the contributions from only the $f_2(1270)$ and $f_2'(1525)$ mesons. The invariant mass distribution for the Dalitz process $K^-p \rightarrow K^-K^+\Lambda$ shows an obvious peak at $M_{K^-K^+}\approx 1.63$ GeV and $M_{K^-}\approx 2.33$ GeV, respectively, which can be checked by further experiment.

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