Matter-gravity entanglement entropy and the information loss puzzle

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Abstract

Since Hawking’s 1974 discovery, we expect a black hole formed by collapse will emit radiation and eventually disappear. One aspect of the information loss puzzle is the challenge to define an objective notion of physical entropy which, in accordance with the Second Law, increases for this, and other, closed unitarily evolving systems. We question whether coarse-grained entropy can meet this challenge but argue that matter-gravity entanglement entropy can. We argue that due to (usually neglected) photon-graviton interactions, if the evaporation is slowed down by putting the black hole in a slightly permeable box, the radiation remaining after a large black hole has evaporated will (be pure and) mainly consist of roughly equal numbers of photons and gravitons entangled with one another — with a (matter-gravity entanglement) entropy greater than that of the freshly formed black hole. We also give arguments that, in the absence of such a box, the final state would be similar. If it is so, this would seem to improve the prospects for the resolution of the firewall puzzle since late emitted photons/gravitons would not be needed to purify early emitted photons/gravitons; instead they would purify one other.

After the (theoretical) discovery of black holes [1] and prior to Hawking’s (theoretical) discovery [2] that black holes emit thermal radiation, it had seemed that it would be possible to violate the second law of thermodynamics by simply depositing a package of entropic matter into a black hole. Bekenstein had pointed out [3] that the second law could be saved if a black hole itself had an entropy and argued that this entropy should be proportional to its area. Hawking’s discovery showed that this was indeed the case and (in units with $G = h = c = k = 1$) fixed the constant of proportionality to be 1/4. Thus an important puzzle was solved. However Hawking’s discovery also led us to question whether we had ever actually properly understood the second law. One version of the second law is the statement that the entropy of a closed system increases monotonically with time. And, in a quantum theoretic description based on Hilbert spaces and time-evolving density operators, there is a natural quantity [4] with the properties one expects of an entropy, namely the von Neumann entropy, $S^{vN}$ (sometimes called the ‘fine-grained’ entropy) defined, for any density operator, $\rho$, to be $-\text{tr}\rho \log \rho$. And yet, one must face the fact that the following three assumptions

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are in contradiction with one another because von Neumann entropy is a unitary invariant. Let us call this the second law puzzle.

Prior to Hawking’s discovery, this had seemed not necessarily to be a problem because one could take the view that the physical entropy of a closed system is not its von Neumann entropy but rather some sort of coarse-grained entropy \[5\]. But, notoriously, coarse-graining is a subjective procedure \[5, 6\] (it depends on judgments about what states we, as physicists, can distinguish\[1\] and thereby also necessarily partly vague and yet there doesn’t seem to be anything subjective or vague about (one quarter of) an area\[2\].

Thus the second law puzzle acquires a new potency when the closed system in question involves a black hole; in particular, for what we shall call here the (gedanken) black hole collapse system (see Figure 1) which consists initially of a compact star in an otherwise flat empty universe which collapses to a black hole which subsequently Hawking radiates and eventually disappears, leaving only Hawking radiation streaming away from where the black hole had been. (For simplicity we shall assume the black hole is spherical and uncharged.) We shall call the special case of the second law puzzle in the case of such a black-hole collapse system the black hole information loss puzzle. This is not always how the puzzle is stated, and, from some other perspectives, it might seem only to capture part of the information loss puzzle. But, at least some of the other aspects of the puzzle as some others perceive it seem to arise from a wish for a semiclassical understanding of the black hole collapse system and I want to suggest that, at the level of full quantum gravity, it is the appropriate statement.

The first statement of an information loss puzzle was by Hawking himself \[7\] who seems to have assumed that physical entropy is von Neumann entropy, thus implicitly accepting the truth of Point (3). Thus the puzzle, as Hawking perceived it, amounted to what we might call the unitarity puzzle: (In the presence of (3)) (1) and (2) above cannot both hold. His proposed resolution was to accept the truth of (2) and deny the truth of (1) – i.e. to posit that unitarity doesn’t hold in quantum gravity. Instead he proposed a generalized form of dynamics (or rather of scattering) in terms of a ‘superscattering operator’ acting on density operators. In later work \[8, 9\], partly influenced by progress in string theory, he changed his mind and argued that unitarity does, after all, hold. However he appears not to have addressed the questions of how physical entropy should then be defined and how the second law (i.e. entropy increase) can nevertheless hold.

A paper which has influenced much recent work on the information loss puzzle is that of Page \[10\]. Page considers a semiclassical description of a black hole collapse system for a

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1 A particular form of quantum coarse-graining – which is often applied to states which are energy eigenstates or close to energy eigenstates – is to define the entropy of a (pure) state with (approximate) energy \(E\) to be the logarithm of the number of energy eigenstates with energy in a small range about \(E\) (or of the number with energy less than \(E\)) or, in the case of a highly degenerate energy level, just the number of energy eigenstates in that level. This is often called the method of computing entropy by ‘counting states’. This is subjective in that it relies on the judgment that we can’t distinguish the given state from other states in that interval, or other states of the same energy (and in the case of ‘energy less than \(E\)’, the additional states don’t matter because there aren’t very many of them). Another criticism of this notion of coarse-grained entropy is that, if the Hamiltonian of which these are energy eigenstates is the Hamiltonian of a theory of everything, then, taken literally, the definition would entail that any system with some given energy \(E\) – be it a black hole, a neutron star or, even, say, a cat – would be assigned a coarse grained entropy equal to that of a black hole with the same energy!

2 For how many physicists reacted to this in the years after 1974, see for example this quote from the 1979 article \[6\] by Penrose: “Now recall that in the Bekenstein-Hawking formula, an entropy measure is directly put equal to a precise feature of spacetime geometry, namely the surface area of a black hole. Is it that this geometry is now subjective with the implication that all spacetime geometry (and therefore all physics) must in some measure be subjective? Or has the entropy, for a black hole, become objective? If the latter, then may not entropy also become objective in less extreme gravitational situations . . .”
large black hole and, after the formation of the black hole, considers this to consist of two interacting quantum systems, the ‘black hole’ and ‘the radiation’, spatially localized with a fuzzy boundary in between them at around a few Schwarzschild radii. (So what Page regards as the ‘black hole’ includes [the part of the radiation field in] a small region outside the horizon.) He assumes that a freshly formed black hole may be considered to be in a pure state (see below) and studies the evolution of the total state consisting initially of such a pure black hole state together with a pure initial state of the radiation field. (As an alternative to this, one might have considered an initial total pure state before the stellar collapse to a black hole.) He then argues that for such a state to evolve into a pure state at late times (after the full evaporation of the black hole) so as to be consistent with unitarity, the late emitted photons need to purify the early emitted photons. Recently there has been much work which claims to show that this indeed happens, as reviewed, for example, by Almheiri et al. and in which a number of new types of argument are marshalled which are claimed to show that the von Neumann entropy of the radiation for the Page initial state indeed goes to zero (as required for unitarity if the black hole disappears) at late times.

One might well find it strange that the freshly formed black hole can, according to Page (and also according to the more recent work cited in which follows Page) be described as being in a pure state given that it will have a large physical entropy (namely a quarter of the area of its horizon). But, surprisingly in view of what we wrote above, Page and some subsequent authors view the physical entropy of the black hole (and also of the radiation) as given by a coarse-graining procedure. As far as the second-law aspect of the information loss puzzle is concerned, the recent review tells us that “coarse-grained entropy obeys the second law of thermodynamics” and refers the reader to earlier work by Wall for arguments for the ‘generalized second law’ by which they (and Wall and others) mean the statement that $S_{\text{blackhole}} + S_{\text{radiation}}$ grows monotonically in time when $S_{\text{blackhole}}$ is identified with $1/4$ of the horizon area and $S_{\text{radiation}}$ is defined to be a certain coarse-grained entropy for the Hawking radiation.

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3 More precisely he assumes the only massless particles in nature are photons and gravitons and that one starts with a (star that collapses to a) black hole which is sufficiently large that most of the Hawking radiation that gets produced will consist mainly of photons and gravitons.
Let us also mention here that there is a well-known difficulty with the approach of Page and of the authors reviewed in [11] which is that, by monogamy [13], the late emitted particles cannot simultaneously purify the early emitted particles and also be entangled (as one might expect them to be on a semiclassical understanding) with "quanta" which fall through the horizon. We shall call this the firewall puzzle in view of a much discussed paper [11] which emphasizes this difficulty and points out that one way out of it would be to posit the existence of a 'firewall' preventing the latter entanglement. (See also the earlier papers [15, 16] where statements of essentially the same puzzle appeared.) It seems [11] to be unclear whether or to what extent the work reviewed in [11] may be considered to fully resolve this firewall puzzle.

Starting in 1998 [17, 18] I have taken a different path in attempting to resolve the information loss puzzle and the other puzzles about quantum black holes. I took seriously the suggestion (see Footnote 2) that consideration of the Hawking effect (and especially of the fact that there is apparently nothing subjective about the Hawking area formula for black hole entropy) may lead us to a new understanding of the origin of thermodynamics, and in particular of the notion of entropy, in which quantum gravity plays a fundamental role. Also, it seemed to me that it is not a puzzle that can be understood in a semi-classical framework, but must be regarded as a puzzle about full (albeit low energy – see below) quantum gravity.

Of particular relevance to my train of thought was yet another puzzle about quantum black holes, which, as far as I can see, is not addressed in the approach described in [10, 11]. Namely the thermal atmosphere puzzle. This concerns not a black hole collapse system but rather the equilibrium state that we assume [19] to be possible if one confines a (spherical, uncharged) black hole, together with its Hawking radiation, to a box. See Figure 2. We shall call this the (closed) black hole equilibrium system. The puzzle is that there are arguments [20, 21] that the entropy of such a black hole equilibrium may be identified with the entropy of the gravitational field of the black hole and there are also arguments [20, 21] that it may be identified with the entropy of the thermal atmosphere (which consists mostly of matter, albeit a small part of it will consist of gravitons – see the last paragraphs of this essay for a more detailed discussion on this point). And if we were to add those two entropies we would get the wrong answer. (Twice the correct answer.)

It struck me that this situation is reminiscent of the fact (familiar from quantum information theory) that, if one has a (pure) vector state, Ψ, on a bipartite system – described by a total Hilbert space, Ψ, that arises as the tensor product, $\mathcal{H}_A \otimes \mathcal{H}_B$, of an ‘A’ Hilbert space and a ‘B’ Hilbert space, then the partial trace, $\rho_A = \text{Tr}_{\mathcal{H}_B}(\Psi \langle \Psi \rangle)$, of $\langle \Psi | \Psi \rangle$ over $\mathcal{H}_B$ (also known as the reduced state of the ‘A’ system) is, in general, an impure density operator. (Similarly with $A \leftrightarrow B$.) When it is impure, one says that the total state, Ψ, is entangled and one defines its A-B entanglement entropy to be the von Neumann entropy, $S^\text{vN}(\rho_A) = -\text{tr}(\rho_A \log \rho_A)$ of $\rho_A$ – which, it turns out, is necessarily equal to the von-Neumann entropy, $S^\text{vN}(\rho_B)$ of $\rho_B$. One also says that System A purifies System B (and vice-versa).

We refer to [22, 23], where we argue that the best we can do in terms of a classical spacetime picture of the black hole equilibrium system is to picture it as just the exterior Schwarzschild spacetime but with a non-classically describable edge region, a few Planck lengths thick, around where, classically, the horizon would be. (This is reminiscent of the firewall of [14, 15, 16] discussed above except the claim here is that it arises as a special feature of enclosed horizons.) So Figure 2 here is misleading and we should really think of the gravitational field of the black hole as residing in that edge region outside of/around the horizon. Let us also mention here, as an aside, that a similar picture is also expected to apply to an AdS black hole and it suggests (see [22, 23] and see also [23]) that AdS/CFT is not a bijection between the boundary CFT and the full quantum gravity theory in the bulk, but just between the boundary CFT and the matter sector of the bulk, with the gravity sector purifying the matter sector. It remains, however, an unsolved problem to identify what the matter-gravity split corresponds to in the relevant string theory.

Note that, unlike in classical statistical physics, it is perfectly possible therefore to have a pure state on a total system consisting of two subsystems (A and B) each of which, considered separately, are in mixed states.
The obvious connection to try to make is to identify the von Neumann entropy of $\rho_A$ in the above story with ‘the entropy of the gravitational field of the black hole’ and ‘the von Neumann entropy of $\rho_B$’ with ‘the entropy of the thermal atmosphere’ and the A-B entanglement entropy with the entropy of the entire black hole equilibrium system. Thus it would seem that the thermal atmosphere puzzle would be resolved if we assume the black hole equilibrium system to be correctly described by a pure total state, $\Psi$, of quantum gravity entangled between the gravitational field of the black hole and its atmosphere in such a way that their reduced states are both approximately Gibbs states at the Hawking temperature. And if we identify the physical entropy of the entire closed system with the total state’s entanglement entropy between the gravitational field of the black hole and its thermal atmosphere. (We remark that, previously, the black hole equilibrium system has been modelled by a total Gibbs state at the Hawking temperature or [as in Hawking’s discussion of black hole thermodynamics] in terms of a total microcanonical ensemble.) As to evidence that the matter-gravity entanglement entropy takes the Hawking value, one can make a semiqualitative argument based on string theory ideas that it goes like the square of the black hole mass (and hence like its area) but, along with other string theory approaches to the entropy of Schwarzschild (i.e. spherical, uncharged) black holes, cannot obtain the factor of $1/4$. 

And in consequence the von Neumann entropy is very far from being additive. Also, in view of this fact, this way of obtaining a mixed state from a pure state cannot be regarded as a sort of ‘coarse-graining’. (It is unfortunate that in my first account of this matter gravity entanglement hypothesis, I attempted to explain it in the language of microstates and macrostates and coarse-graining and this aspect of that paper was, in hindsight, unclear and misleading. This was corrected in my subsequent papers.)

It should be mentioned that, aside from our reasons to reject a description in terms of a total Gibbs state in favour of a description in terms of a total pure state, it is anyway problematic to regard a quantum black hole, in the absence of a cosmological constant, as being in a total Gibbs state at the Hawking temperature because it would have a negative specific heat (although this difficulty is absent for Schwarzschild AdS black holes). Note anyway that one can infer from the work in that the sense in which our reduced states of matter and gravity are expected to be “approximately thermal” also avoids that difficulty.

This argument (see or for a review) is a modification of the Horowitz-Polchinski semi-qualitative entropy calculation of the entropy of a Schwarzschild black hole, which, in turn, is based on Susskind’s picture of the weak string coupling limit (where the string length scale is adjusted to keep Newton’s constant of gravitation constant) of a such a black hole as being a long string in Minkowski space. That argument is subject to the criticisms we gave of ‘counting states’ in Footnote of this matter gravity entanglement hypothesis. Instead of looking at the weak string coupling limit of a bare black hole, we look at the weak string coupling limit of the entire black hole equilibrium system (which we take to be a random pure state which arises as a superposition of energy eigenstates with energy in a narrow band around the black hole mass) involving a long string surrounded by a stringy atmosphere of small strings in a box. We then argue that the matter-gravity entanglement entropy is approximately given by the entanglement entropy of the long string with its stringy atmosphere and, tweaking the procedure in, equate this with the entanglement entropy of our black hole equilibrium system when the string length is comparable to the black hole mass. We find that that this will go like the square of the black hole mass (while the argument is not subject to the criticisms of Footnote of ‘counting of states’). We also hope that e.g. the original Strominger-Vafa computation of the black hole entropy of (near) extremal black
We want to go further though and to take this as a clue for the definition of physical entropy for an arbitrary closed system, be it our black hole equilibrium system, be it the black hole collapse system, be it the universe! And we want the notion of physical entropy to make sense, in the case of the black hole collapse system, not only during the period of time when there’s a black hole present (and emitting radiation) but also before the star collapses and after the black hole has completely evaporated – and in the case of the universe, at early times soon after the big bang as well as later. Bearing in mind that in the case of the black hole equilibrium system, we can roughly equate ‘the thermal atmosphere’ with ‘the matter atmosphere’ ([near the horizon] gravitons are expected to only be a small part the thermal atmosphere – we will return to this point later) this suggests that we take our definition for the physical entropy of a closed system to be its matter-gravity entanglement entropy. This in turn suggests that we will need as our fundamental framework a low-energy version of quantum gravity (where low energy means below something like the Planck energy, $10^{19}$ GeV) for presumably ‘matter’ and ‘gravity’ are not fundamental notions in exact quantum gravity but emerge at such ‘low’ energies. In consequence, we expect that, on the approach to understanding thermodynamics entailed by this proposed definition for physical entropy, thermodynamics is itself an emergent theory that only makes sense at energies below the Planck energy – which only starts to make sense a few Planck times after the big bang when presumably a sort of phase transition occurs and the distinction between matter and gravity starts to emerge.

On the assumption that low energy quantum gravity is a quantum theory of a conservative type with a total Hilbert space that arises as the tensor product of a gravity Hilbert space and a matter Hilbert space and with a time-evolution, $U(t) = e^{iHt}$ which is unitary, we then, arguably have a resolution to the information loss puzzle (and, in the case of the universe, of the second law puzzle) since we now have a notion of physical entropy – namely matter-gravity entanglement entropy – which is not subjective and not a unitary invariant. If we make the

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8 We also have an extension of this hypothesis to include open systems, which also, in a sense, offers a completion of the well-known ‘environmental decoherence’ paradigm. For this, we refer to [34, Endnote (xii)] or [35] or [36].

9 Likewise, our notions of ‘entropy’ and of ‘thermodynamics’ necessarily will not apply to a fictitious universe made out of pure gravity (even though such a universe could contain black holes). This should not worry us though since, as far as I understand, it is believed that a quantum theory of the pure gravitational field cannot be constructed in a mathematically meaningful way and anyway, we only are interested in having a theory which applies to our universe where, as it happens, there is both gravity and matter.

10 It might be objected that the usual constraints of canonical quantum gravity prevent the quantum gravity Hilbert space from arising as a tensor product in this way just as, e.g., in quantum electrodynamics in Coulomb gauge, because of the Gauss law constraint, the longitudinal modes of the electromagnetic field are made out of charged matter operators. However, I have recently shown (see [37] and a forthcoming briefer account [38]) that a (unitarily) equivalent description of QED is possible in which Gauss’s law holds as an operator equation which picks out a physical subspace of a total Hilbert space which does arise as a tensor product of a charged matter Hilbert space and an electromagnetic field Hilbert space – albeit all elements of that physical subspace (including the vacuum) have a certain degree of entanglement between charged matter and longitudinal photons. Work is in progress on generalising this construction to (first, linearized) quantum gravity.

11 From what we know about the nature of ‘time’ in general, or even special, relativity, to talk about ‘time-evolution’ in quantum gravity in such a naive way raises difficult questions to which, we must admit, in the case of our black hole collapse system, we are presently unable to provide a clear answer except in a hand-waving sense or in the limit of weak string coupling and large string length – see Footnote 7 and [27, 28, 29, 30]. Let us note though that in the case of our black hole equilibrium system, there is an obvious notion of time with respect to which the system is static. Also, in the case of the universe, it is natural to identify ‘time’ here with cosmological time.
additional assumption that the total Hamiltonian, $H_{\text{total}}$, arises as the sum

$$H_{\text{total}} = H_{\text{gravity}} + H_{\text{matter}} + H_{\text{interaction}},$$

of a gravity Hamiltonian, a matter Hamiltonian and an interaction term, and if we further assume that the initial state (in the case of the universe, a few Planck times after the big bang) is unentangled between matter and gravity (or rather – cf. Footnote 10 has a low degree of matter-gravity entanglement) then it is reasonable to expect that the degree of entanglement, and therefore the physical entropy as we define it, will increase because of the interaction. (If our systems had a finite number of degrees of freedom, one might anticipate Poincaré recurrence times when the entropy returned to a low value, but if the number of degrees of freedom is large [or infinite] that might well not happen for a very long time [or ever]. Thus not only do our hypotheses seem to offer a resolution to the second law puzzle, but they also offer a plausible mechanism for why the second law holds – i.e. why the physical entropy of a closed system increases with time. We shall call this our general argument for entropy increase.

While there may have been “no need to have questioned unitarity in the first place” and while we now have a general argument for entropy increase (for systems that evolve unitarily) one of course still wants to understand, in the case of the black hole collapse system, how the entropy increase of the second law can be understood in terms of the details of the collapse and evaporation process (while unitarity is maintained). Here we remark that, on our matter-gravity entanglement hypothesis, it seems that any configuration of ordinary matter will have a nonzero entropy, albeit this is expected to be small unless the system is highly relativistic and/or involves strong gravitational fields. (See [18, 34] where we also give some evidence that the entropy will be lower e.g. for a gas and higher when matter is in a more condensed state.) Thus our collapsing star will already have a small nonzero entropy and presumably this will increase rapidly to the Hawking value as the black hole forms, presumably because of a large degree of matter-gravity entanglement happening around where, in a classical description, a horizon is supposed to form. But also, if the second law is to hold, then it must be that, after the full evaporation of the black hole, the state of Hawking radiation that remains, streaming outwards from the centre where the black hole had been, has a physical entropy even greater than the entropy that the black hole had when it was freshly formed (i.e. as we know from Hawking, one quarter of the area of its horizon). How can that be, if we define physical entropy to be matter-gravity entanglement entropy? Well, if the initial black hole was large enough and assuming that the only massless particles in nature are photons and gravitons (see Footnote 3) then (cf. [10]) the late time Hawking radiation will mainly consist just of photons and gravitons. For this to have the necessary big entropy as we define it (and for the total state to remain pure!) it presumably must be that the photons and gravitons are highly entangled with one another (while in a total pure state).

Now there immediately appears a difficulty. According to the 1976 Hawking-effect calculations of Page [41] because of their higher spin, fewer gravitons will be emitted than photons. Moreover, even if one purifies as many photons as one can with the fewer gravitons predicted to be emitted, the quantitative results of [41] tell us that the matter-gravity entanglement entropy falls short of the entropy of the freshly formed black hole. And there is no mechanism for any such purification in those 1976 calculations.

However there seem to be a couple of loopholes. Those calculations in [41], as indeed the original black hole evaporation calculation [2] of Hawking are done in the framework of quantum field theory in curved spacetime and it may well be that, at least after a black hole has been radiating for some time, the quantum fluctuations around the horizon build up and render that approximation to quantum gravity (and also semiclassical gravity where one incorporates
a backreaction of the expectation value of the stress energy tensor) inapplicable.\footnote{\textsuperscript{12}}

Secondly, \cite{11} neglects photon-graviton interactions. Let us temporarily leave aside our black hole evaporation system and look in some detail at states of our black hole equilibrium system. We model these as random pure total states which have (or rather arise as superpositions of energy eigenstates with energies in a narrow band around) some energy, $E$, in a spherical gedanken box of volume $V$. Although we are dealing with randomly chosen pure total states and Hawking \cite{19} studied a microcanonical ensemble, we expect that the broad features of the thermodynamical behaviour that is predicted on our assumptions will be qualitatively similar to those described in \cite{19, Page 195}. (In the language of \cite{27, 28} we are replacing the ‘traditional’ by the ‘modern’ explanation of the origin of thermodynamic behaviour.) If $V$ is sufficiently large, we expect that the box would contain just radiation and no black hole and for that radiation to mainly consist just of massless particles – i.e. photons and gravitons. But now we would expect the total state to involve equal numbers of photons and gravitons and moreover (on our assumption of a total pure state) for that state to be highly entangled between the photons and gravitons. After all, all that seems to matter here is that we have a total pure state of two species of massless boson, each with two helicity states and weakly coupled to one another. (It might help to notice that were such a pure total state not be entangled, in view of the interaction term, $H_I$, in the total Hamiltonian, it could of course not be an energy eigenstate!)

Next consider a suitably smaller volume $V$ (but not so small as for its radius to be within the Schwarzschild radius for energy $E$) then (cf. again \cite{19, Page 195}) we would expect an energy eigenstate with energy $E$ chosen at random to very probably consist of a black hole (let us assume, for simplicity, located centrally) surrounded by an atmosphere. Further, we would expect this atmosphere to roughly consist of an inner region, close to the black hole’s horizon, where, because of the blueshift, all matter degrees of freedom are effectively massless and (cf. ’t Hooft’s brick wall discussion \cite{39, 40}) for this region to account for most of the (matter-gravity entanglement) entropy, and an outer region consisting just of massless particles – i.e. just photons and gravitons. And, similarly to in the case where a black hole is absent, we would expect those outer photons and gravitons to be in roughly equal numbers and highly entangled with one another, albeit their contribution to the overall (matter-gravity entanglement) entropy will be small compared to that of the inner region. (It seems reasonable to expect that the entropy of the inner and outer regions will roughly add up to the total entropy.) In particular, those outer photons will (consistently with monogamy \cite{13}) not be strongly entangled with the gravitational field of the inner region.\footnote{\textsuperscript{13}} Now, suppose we were to make a small hole in the box in the latter situation – or rather, to preserve spherical symmetry, suppose we were to make the box slightly permeable. Presumably what would come out would be radiation consisting of photons and gravitons – in roughly equal numbers and highly entangled with one another, while the escaped photons and graviton would be replenished (at the expense of the inner region) as the system inside the box strives to return to equilibrium. And so on.

\textsuperscript{12}We remark here that Almheiri et al. \cite{11} rule that out, writing “\textit{When the black hole reaches the final stages of evaporation, its size becomes comparable to the Planck length and we can no longer trust the semiclassical gravity description.}” and go on to indicate that they advocate trusting semiclassical gravity until those final stages. However we are unaware of any strong arguments that semiclassical gravity can be trusted for so long.

\textsuperscript{13}It is tempting to identify the combined gravitational and matter degrees of freedom of the inner region with “the black hole”. See also Footnote\cite{4}. Note that we don’t expect the inner region to be describable in terms of weakly coupled quantum fields albeit (cf. Footnote\cite{7}) it may be understandable as a weakly coupled system in a weak string-coupling limit, while, on the other hand, in the outer region, the photons and gravitons would be describable as weakly coupled in low-energy perturbative quantum gravity. Note also that, in the inner region, the gravitational field is expected to perform the feat of simultaneously purifying \textit{all} of the matter degrees of freedom. Again this seems not to be understandable in a quantum field description but may be understandable in string theory – see also Footnotes\cite{4} and \cite{7}.
So, if we start with a black hole in a spherical box in equilibrium and then permit it to evaporate slowly by making the box slightly permeable, it seems reasonable to expect that what we will be left with after it has fully evaporated, is radiation consisting of photons and gravitons – in roughly equal numbers and highly entangled with one another.

It seems plausible that the same will be true if the initial state inside the slightly permeable, box is not an equilibrium state but the state of a black hole freshly formed by collapse since we would anyway expect that such a system would strive to surround itself with an atmosphere by Hawking radiating.

Returning to the standard black hole collapse system, where Hawking evaporation takes place without being slowed down by enclosing the black hole in a slightly permeable box, I am unable to give as compelling an argument as for the case when the black hole is enclosed by such a box. However it seems possible that, at the same low-energy quantum gravity level of description and again due to photon-graviton interactions (which are neglected in a quantum field theory in curved spacetime treatment) the Hawking radiation would still resemble, at least after some time, the state of entangled photons and gravitons argued for above when a slightly permeable box surrounds the black hole. One possible mechanism for this would be that, because of their interaction, radiated photons would dress themselves with clouds of soft gravitons and vice versa. (An interesting recent paper on the mathematics of such clouds for the scattering of massless particles is [42].)

If it is so, then, plausibly, after the black hole had fully evaporated, the entropy (as we define it) of the entangled state of photons and gravitons that remains would exceed that of the freshly formed black hole.

Amongst the many questions that remain to be understood are the question of what happens for an initially non-large black hole, or after an initially large black hole has reduced in size, when Hawking radiation of massive particles becomes important. It also remains to understand whether/how such a result could be reconciled with the results reported in [11]. But in any case, we reiterate that something like the above scenario must presumably hold according to our general argument for entropy increase which, as we argued above, predicts that the total state at late times (will be pure and) will have a (matter-gravity entanglement) entropy higher than that of the freshly formed black hole.

The prospects would also seem to be improved for resolving the firewall puzzle. For, after all, if the photons and gravitons of Hawking radiation are emitted in roughly equal numbers and highly entangled with one another, then the late emitted photons and gravitons will not be needed to purify the early emitted photons and gravitons; they will, instead, purify one other.

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