A Rejuvenated Universe Without Initial Singularity

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Abstract

Cosmological observations strongly suggest the presence of dark energy which comprises the majority of the current energy density of the universe. The equation of state relating the pressure and energy density of this dark energy, \( p = w \rho \), appears to have \( w \approx -1 \), with most analyses preferring \( w < -1 \) when these values are considered as part of the parameter space. If \( w < -1 \) is the future behavior of the dark energy, the scale factor, expansion rate, and energy density of the universe will diverge in a finite time as its apparent expansion age tends to zero. We hypothesize that \( w > -1 \) is restored at a late enough time, perhaps due to a phase transition of the dark energy, and show that this produces conditions observationally indistinguishable from a Hot Big Bang. This process of rejuvenation may have occurred in the past, making our universe much older than it appears and eliminating the Big Bang singularity.

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The expansion of the universe follows the Friedmann equations. The rate of change of the scale factor $a$ is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \quad (1)$$

where $H$ is the Hubble “constant”, $G$ is Newton’s constant, $\rho$ is the total energy density, and $k$ is the spatial curvature. The scale factor is arbitrary up to a multiplicative constant but the fractional rate of expansion $\dot{H}$ is observable, leading to Hubble's law of recession $v = H d$ and an apparent expansion age of the universe given by $t_{\text{expansion}} = H^{-1}$. The second Friedmann equation illustrates the impact of pressure upon the acceleration or deceleration of the expansion,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) = -\frac{4\pi G}{3}\rho(1 + 3w_{\text{eff}}). \quad (2)$$

Hence the strong observational evidence for accelerating expansion [1, 2] implies that the average equation of state of the radiation, matter, and dark energy in the universe has $w_{\text{eff}} < -1/3$. If no interchange is occurring between the matter and dark energy, they are separately conserved, and their energy densities are related to the scale factor by $\rho_m = \rho_{m,0}(a/a_0)^{-3}$ and $\rho_\phi = \rho_{\phi,0}(a/a_0)^{-3(1+w)}$.

Observations show that our universe is nearly flat ($k = 0$) and has dark energy with $w \simeq -1$ which dominates the current energy density [3, 4, 5, 6, 7, 8, 9, 10, 11]. Most analyses prefer $w < -1$, dubbed phantom energy [12], when these values are considered as part of the parameter space, although $w \geq -1$ is often assumed as a theoretical prior.

We can set the scale factor today $a_0 = 1$ and specialize (1) to analyze the past and future expansion rate of the universe in units of the current Hubble constant $H_0 \approx 70\text{km/s/Mpc}$ [13], resulting in a simple differential equation,

$$\dot{a} = (\Omega_{m,0} a^{-1} + \Omega_{\phi,0} a^{-1-3w})^{1/2}, \quad (3)$$

where WMAP determined $\Omega_{m,0} = 8\pi G \rho_{m,0}/(3H_0^2) = 0.26$ and $\Omega_{\phi,0} = 0.74$. Figure illustrates the behavior of the scale factor, energy density, and expansion age of the universe for the cases $w = -2/3, -1, -4/3$. In each of the models, our Universe appears to have begun in a Big Bang when the scale factor and expansion age were equal to zero and the energy density was infinite. This initial singularity occurs at a slightly different time in the three models, which reflects the amount by which the true age of the universe today in each model differs from the current expansion age of 14 billion years. Allowing $w < -1$ violates the
dominant energy condition of general relativity and allows the energy content of the dark energy to flow faster than the speed of light \[14\]. This dominant energy condition is indeed violated by some proposed models, although care must be taken to avoid “tachyonic” modes with negative squared effective mass that are unstable to growth \[12, 15, 16, 17, 18\]. A thorough investigation by \[19\] indicates that viable models with \( w < -1 \) can be constructed but require significant care. \( w < -1 \) is equivalent to an effective cosmological constant that increases with time, and it is possible that string physics \[20\] or other extra-dimensional effects could give the dark energy an effective \( w < -1 \) without creating instabilities. Moreover, the phenomenology described by \( w < -1 \) could instead be achieved with a pure cosmological constant and an increasing value of Newton’s gravitational constant \( G \) (see \[21\]). With the running of coupling constants intrinsic to M-theory and the recent barrage of models with large extra dimensions, it therefore seems premature to dismiss the idea that the dark energy has \( w_{eff} < -1 \) as unphysical.

The behavior of the \( w = -4/3 \) model is generic for all models with \( w < -1 \) even if \( w \) is not constant; the universe undergoes superinflation where the scale factor diverges within finite time as the energy density becomes infinite and the expansion age reaches zero. Let \( H_d \) be the expansion rate at a time \( t_d \) where matter has become irrelevant and dark energy dominates the energy density. Then if the equation of state is constant with \( w < -1 \), for \( t > t_d \) \[22\] has the solution

\[
a = a_d \left(1 - \frac{3}{2} (-1 - w) H_d (t - t_d) \right)^\frac{2}{3(-1-w)} = a_d \left( \frac{3}{2} (-1 - w) H_d (t_\infty - t) \right)^\frac{2}{3(-1-w)},
\]

where the time of scale factor divergence \( t_\infty \) is given by \[23\]

\[
t_\infty = t_d + \frac{2}{3 (-1 - w) H_d}.
\]

The conformal time is well behaved, approaching a finite positive value as \( t \to t_\infty \). For \( w < -1 \), the physical observables of energy density and expansion age at divergence are identical to those that exist at the initial moment of the Big Bang cosmology, with the notable exception that the energy density is in the form of dark energy whose behavior is closer to a cosmological constant than to matter or radiation. The moment of divergent scale factor represents a singularity, and observers in this superinflationary universe have event horizons that shrink to zero size at the moment the scale factor diverges because even nearby objects are receding faster than the speed of light. While \( H(t) \) is observable, determining
the true age of the universe requires a full knowledge of the historical equation of state of each component of matter, radiation, and dark energy. Age determinations from galaxy recession velocities (the expansion age), stellar evolution, and nuclear isotope abundances will differ from each other in a stage of dark energy domination as the expansion age differs greatly from the true age of the universe.

As the moment of divergence approaches, the energy density of the universe reaches scales at which physics beyond the standard model may alter the behavior. Figure 2 shows a toy model in which the dark energy undergoes a phase transition from $w = -4/3$ to $w = 1/3$. A final singularity is avoided, and a phase of decelerating expansion begins which looks just like the Big Bang but without an initial singularity. The singularity theorems of general relativity do not apply because they presume that the strong energy condition ($\rho + p > 0$) holds [14] but this is not applicable for $\rho > 0, w < -1$. Hence a runaway universe that undergoes this sort of phase transition ends up rejuvenated with a small expansion age and re-energized with a correspondingly large energy density.

To create a Hot Big Bang in which nucleosynthesis and the cosmic background radiation are consistent with observations, the phase transition must produce $w = 1/3$ at the end and then generate matter through thermal production. An intermediate step could be to produce a $w = 0$ (matter) component that decays into radiation ($w = 1/3$) as at the end of the standard inflationary scenario [24, 25, 26, 27, 28]. One possible mechanism to generate matter from the dark energy would be gravitational particle production into nonrelativistic particles of mass $M_X$, which occurs once $\dot{H}/H > M_X$ [29] and is therefore guaranteed to happen at some point in the approach to scale factor divergence regardless of the $M_X$ available. A radiation-dominated Hot Big Bang would then be produced by particle decay. However, it seems impossible to get enough of the energy density into matter by this mechanism to make $w_{eff} > -1$ because the gravitational particle production ends as $w_{eff} \to -1$ and $\dot{H}/H \to < M_X$. Decay of the matter ($w = 0$) into radiation ($w = 1/3$) could then make $w_{eff} > -1$ but could not achieve $w_{eff} > -1/3$ which is required to end the accelerating expansion and prevent a return to an epoch of dark energy domination. Another possible mechanism would be for the kinetic energy of a scalar phantom energy field to come to dominate its potential; since $w = (1/2\dot{\phi}^2 - V)/(1/2\dot{\phi}^2 + V)$, kinetic energy domination produces $w = 1$.

For a flat universe, the relationship between $H$ and $\rho$ is determined entirely by [11] so
rejuvenation leads to the precise physical observables which characterize the Big Bang. The runaway expansion of a $w < -1$ universe is guaranteed to reduce any curvature tremendously because of its $a^{-2}$ dependence, solving the flatness problem. The horizon problem of the Big Bang model is also resolved as regions initially under causal contact are expanded to very large, apparently superhorizon, size before $t = 0$. The presence of event horizons in the rejuvenated universe depends upon the relative length of decelerating and accelerating expansion epochs. The monopole problem is of interest as any massive remnants produced at the maximum energy density will remain today unless the equation of state of the universe remains near $w = -1$ for long enough to reduce their abundance significantly. The precise spectrum of density perturbations generated in a rejuvenated universe will depend upon the details of the dark energy and its phase transition, but a stage resembling the standard inflationary scenario is necessary to produce a nearly scale-invariant spectrum of density fluctuations consistent with observations of cosmological structure. There are interesting similarities between the rejuvenation scenario and the model of Pre-Big-Bang inflation based upon the scale factor duality of string theory [30, 31]. However, the PBB model has growing curvature and, for three spatial dimensions, has $w = -1/3$ at $t < 0$. Recent attempts to avoid a Big Bang singularity such as PBB or the Ekpyrotic Universe [32, 33] appeal to flat, static spacetime as an ideal set of initial conditions, although fine-tuning of initial curvature appears necessary. Rejuvenation instead uses the current observed state of our universe as initial conditions, with added assumptions about the equation of state of the dark energy and its future behavior.

This discussion of the rejuvenation scenario leads to three separate hypotheses, each of which makes predictions that can be tested against cosmological observations. The first is that rejuvenation will occur in the future of our universe; this requires $w < -1$ for a long enough period to increase the energy density to levels expected in the first second of the Hot Big Bang model. While this does not technically require $w < -1$ or $dw/dt < 0$ today, future rejuvenation will seem extremely improbable if neither of those conditions is met by the current dark energy. There are observational approaches using Type Ia supernovae, gravitational lensing, and the evolution of the number abundance of dark matter halos that should reveal if the present dark energy has $w < -1$ or $dw/dt < 0$. [34, 35, 36, 37] Unfortunately, neither of those conditions would guarantee a sufficiently long period of $w < -1$ to allow rejuvenation; proof of that would require a fundamental understanding of
the nature of the dark energy.

If the dark energy has a low sound speed, existing density inhomogeneities allow the runaway universe to provide a foundation for eternal (chaotic) inflation \[27, 28\] as regions with lower than average mass density today already have higher than average expansion rates and will undergo rejuvenation sooner than average. If the dark energy equation-of-state is constant throughout space, regions that are gravitationally bound in the current universe will never achieve dark energy domination as they have already decoupled from the universal expansion. Thus density fluctuations become destiny fluctuations, and the edge of bound regions becomes an event horizon at the moment of scale factor divergence if rejuvenation does not occur. If, on the other hand, the sound speed of the dark energy is the speed of light as is typical for scalar-field “quintessence” models, the dark energy density will roughly equilibrate (this precludes a universally constant value of \( w \)) and current gravitationally bound objects will be ripped apart - this has been dubbed “Cosmic Doomsday” by \[38\]. Note that current density fluctuations will prevent even this fate from being synchronized on scales of order the horizon size. Would black holes from our current universe survive rejuvenation? It appears so, but like any pre-inflationary relic their density will be diluted by the expansion to the point where observers in the post-rejuvenation phase would be quite unlikely to find them.

The second hypothesis is that rejuvenation generated the hot early conditions of our universe, avoiding a Big Bang singularity and making our universe much older than it appears. The apparent age of the universe determined from its expansion dynamics, content of matter and dark energy, and from nucleosynthetic ages will all be misleading in this case. Evidence that the fundamental physics of the dark energy allows a direct connection between the dark energy of a previous super-inflationary phase and the dark energy today would make our second hypothesis easier to believe, but this connection is not necessary for the hypothesis to be correct. Rejuvenation predicts that the amplitude of primordial density fluctuations should be increasing with decreasing physical scale i.e. the scalar spectral index \( n = d \log P(k)/d \log k > 1 \) over a range of physical scales that exited the horizon while the expansion was still accelerating. This provides an intriguing explanation for the anomalously low values of the quadrupole and octopole in the WMAP skymaps \[39\]. It appears possible for rejuvenation models to generate primordial power spectra that run from blue to red i.e. \( n > 1 \) to \( n < 1 \) over the range of scales probed by current observations.
of cosmological structure, as preferred by recent WMAP results \[40\]. These observable perturbations were generated about 50 e-foldings before the end of inflation or rejuvenation \[41\], although the number of e-folds can be smaller in the latter case. The standard slow-roll inflationary perturbation solution yields a formally infinite contribution to the primordial power spectrum at the scale corresponding to the horizon size when $w$ crosses through the value $-1$. Forming a viable cosmological model of rejuvenation will require careful attention to this problem, which could be resolved by ending slow-roll early in the phase transition. The detailed physics of cosmological perturbations generated during rejuvenation will be addressed in future work.

The third hypothesis is that we live in “Phoenix” universe which undergoes successive stages of rejuvenation, becoming larger and even closer to flat each time. This contrasts with all previous models of this type which have oscillated between Big Bang and Big Crunch, including the recently proposed Cyclic Universe \[42\]. Beyond requiring the first and second hypotheses to be true and thereby echoing their predictions, this third hypothesis might best be relegated to the realm of metaphysics, as the current universe likely offers few clues to the possible existence of multiple rejuvenation phases in the past or future. Nonetheless, a full physical understanding of the dark energy could illuminate an oscillatory nature, perhaps due to a scalar field endlessly rolling up and then falling back down its potential – a “Sisyphus” universe. Rejuvenation does not, however, allow for a periodic universe where the future actually produces the past. The monotonically increasing scale factor makes the universe much larger at a given approach to singularity than at the previous approach, with increased entropy and density inhomogeneities. The presence or absence of event horizons, which are anathema to M-theory, would depend on the relative rate of expansion during accelerating and decelerating periods back to the infinite past and out to the infinite future; this is an issue worth exploring in more detail for both rejuvenation and the Cyclic Universe models.

It should be noted that there is still no satisfactory explanation for the presence of dark energy today. One can postulate that the decay of the dark energy of the previous inflationary phase into matter and radiation was incomplete, but details await a physical theory that accounts for the dark energy and any phase transitions it may undergo. A particular challenge is to explain the time-asymmetry of the rejuvenation phase transition i.e. why $w$ remains greater than $-1$ as the universe cools to the temperature where $w < \ldots$
−1 occurred previously. Pending a well-defined physical theory for the dark energy, the rejuvenated universe scenario provides a plausible connection between the mystery of the present dark energy and the initial conditions for the inflationary phase of the “early” universe. It remains a significant observational challenge to determine whether our universe originated in a Big Bang singularity or whether phantom energy provided it with a fountain of youth.

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FIG. 1: The effects of dark energy with constant $w = -2/3$ (dashed), $w = -1$ (dotted), and $w = -4/3$ (solid). Vertical dotted line indicates the present era. Top panel shows the scale factor $a$ in units of the scale factor today versus time in units of the current expansion age $t_0 = H_0^{-1}$. The scale factor is growing exponentially for $w = -1$ but diverges when the universe reaches about 3.3 times its current expansion age for $w = -4/3$. The middle panel shows that energy density in units of the current energy density becomes constant for $w = -1$ but diverges if $w = -4/3$. The bottom panel shows that the expansion age $H^{-1}$ becomes constant for $w = -1$ but decreases to zero for $w = -4/3$. If $w = -4/3$, the observable quantities of energy density and expansion age return to their Big Bang values as the scale factor diverges.
FIG. 2: A toy model illustrating how a final singularity can be avoided by changing the equation of state of the dark energy before the scale factor diverges. The characteristic timescale for this transition is $t_{tr}$, which might correspond to a Planck time of $10^{-43}$ seconds. The top panel shows $w$ undergoing a transition from $-4/3$ (phantom energy) to $1/3$ (radiation) with the closest approach to a singularity at the moment when $w = -1$ marked as $t = 0$. The scale factor (second panel) expands throughout the entire transition but the acceleration of the expansion ceases. The energy density (third panel) reaches its maximum value $\rho_*$ at $t = 0$ and then begins to decrease. The apparent age of the universe, its expansion age (bottom panel), reaches a minimum of $t_*$ at $t = 0$. 