DALITZ ANALYSES: A TOOL FOR PHYSICS WITHIN
AND BEYOND THE STANDARD MODEL

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Abstract

Dalitz analyses are introduced as the method for studying hadronic decays. An accurate description of hadron final states is critical not only to an understanding of the strong coupling regime of QCD, but also to the precision extraction of CKM matrix elements. The relation of such final state interactions to scattering processes is discussed.

1 Motivation

This serves as the introduction to following talks on Dalitz plot analyses. I will first remind you of the motivation for such analyses and discuss some aspects of how to perform them and I leave it to others to describe detailed results.

We know that there is physics beyond the Standard Model but we do not yet know what this is. Insight is provided by precision measurements of the CKM matrix elements, and in particular the length of the sides and the angles of the CKM Unitarity triangle. Such measurements in heavy flavour decays probe the structure of the weak interaction at distance scales of 0.01fm. However, the detector centimetres away records mainly pions and kaons as the outcome of a typical process like $B \to D(\to K\pi\pi)K$ decay. Consequently, the very short distance interaction we wish to study is only seen through a fog of strong interactions, in which the quarks that are created exchange gluons, bind to form hadrons and then these hadrons interact for times a 100 times longer than the basic weak interaction. To uncover this basic interaction we need to understand the nature of these strong processes, or at least to model them very precisely. Indeed, the biggest uncertainties in determining the CKM angle $\phi_3$, or $\gamma$, from the difference of $D$ and $\bar{D}$ decays is due to our inability to model the final state interactions $[1, 2, 3]$. Dalitz plot analyses are the way to improve this.
What helps here is that the most common particles for $B$’s, $D$’s, and even $J/\psi$’s and $\phi$’s, to decay into are pions and kaons. These, being the lightest of all hadrons, have final state interactions that are common to all these processes. So each can teach us about the other and in turn heavy flavour decays are now the richest source of information about light quark dynamics.

![Dalitz plot example](image)

Figure 1: Example of a Dalitz plot, here for $D^0 \rightarrow K_s \pi^+ \pi^-$ decay.

As soon as one has more than two particles in the final state, the events are most readily pictured in a Dalitz plot, an example of which is shown in Fig. 1. This was invented by Richard Dalitz (known to everyone as Dick) more than 45 years ago [1]. Sadly Dick died earlier this year aged 81 and this talk is dedicated to him. For a three body decay, like $D \rightarrow K \pi \pi$, that we will concentrate on here, the data are plotted with the mass squared of the $K\pi$ and $\pi\pi$ systems on the $x, y$ axes, Fig. 1. Now the first thing to notice about such a plot is that the events are not uniformly distributed. The decay does not proceed by $D$ decaying to the three body system $K\pi\pi$ directly, rather there are a series of bands which show that the $D$ likes to emit a $K$ and then form a resonance like the $\rho$, which later decays into two pions, or the $D$ emits a $\pi$ forming a $K^*(892)$, which then later decays to $K\pi$. Thus to describe the basic matrix element for the decay we need to know how to represent the vector mesons like $\rho$ and $K^*$. These, being reasonably long lived, have a magnitude and phase change across the resonance well described by a simple Breit-Wigner formula, with a pole in the complex energy plane on the nearby unphysical sheet. However, this represents only a small fraction of the events in the Dalitz plot.
The simplest way for any state of heavy flavour to lose mass is to emit a $\pi$ or $K$ and form a scalar meson. Having $J^{PC} = 0^{++}$ this produces no change in angular momentum and so is almost cost free. Indeed, any system can lower its mass by forming a scalar with $I = 0$, since this has vacuum quantum numbers. This we can study in $\pi\pi$ scattering. There we see that the cross-section, Fig. 2, does not appear to have any structures looking like simple Breit-Wigners, rather it has a series of broad peaks with deep narrow dips between $\pi\pi$. The increase from threshold is related to the short-lived $\sigma$ and the dramatic dip at $KK$ threshold to the $f_0(980)$. Neither can be represented by a simple Breit-Wigner and certainly naively summing Breit-Wigners would violate the conservation of probability. Consequently, we will need a better way to treat these broad and overlapping structures throughout the $D$ and $D_s$ decay regions. Indeed data on these decays provide unique information about the strong coupling regime of QCD, which defines the Higgs sector of chiral symmetry breaking. Even if we do not care about this key aspect of the Standard Model, we nevertheless need an accurate way to parametrize its effects to determine the CKM matrix elements.

We have seen that much of 3-body decays proceeds as a two stage process. This is embodied in the isobar picture, Fig. 3. There one assumes that parent particle $P$ spits out a particle $c$, for instance, to produce a resonance $R$ that subsequently decays into $ab$, particle $c$ being regarded as a spectator as far as the final state is concerned. Such a picture immediately implies a modelling for the process $ab \rightarrow ab$, in which the same resonance $R$ appears. One must check that these descriptions are consistent. The resonance must have

Figure 2: Sketch of the modulus squared of the $I = J = 0 \pi\pi \rightarrow \pi\pi$ amplitude from Ref. 5. Resonances in the PDG tables [6] are indicated.
Figure 3: Decay of a parent particle $P$ to $abc$. $R$ is a resonance in the $ab$ channel, which is the basis of the isobar model of sequential decays: $P \rightarrow Rc$, then $R \rightarrow ab$, plus contributions $P \rightarrow R'a$, then $R' \rightarrow bc$ and $P \rightarrow R''b$ and $R'' \rightarrow ca$ with $c$, $a$ and $b$ as spectators, respectively.

the same mass and total width, as well as partial width to the $ab$ channel. This constraint is encoded in unitarity, which enforces the conservation of probability. Indeed, unitarity does not differentiate between the resonance and its background, but treats them both together. This is particularly important for short-lived states like the $\sigma$, where such a distinction is totally semantic. Thus unitarity requires that if the particle $c$ is a spectator then for instance the imaginary part of the amplitude for $P \rightarrow \pi\pi(c)$ is equal to the sum shown in Fig. 4, where one sums over all kinematically allowed hadronic intermediate states. This means that if we represent the scattering amplitude for these intermediate processes $ab \rightarrow h_n$ by the matrix $T$, then the amplitude for the decay $F$ is given by the vector equation

$$F(P \rightarrow ab(c)) = \sum_n \alpha_n T(h_n \rightarrow ab),$$

(1)

where the functions $\alpha_n$ represent the basic coupling of $P \rightarrow h_n(c)$ for each hadronic intermediate state $h_n$.

Figure 4: Unitarity for the $ab$ system in the decay $P \rightarrow (ab)c$, where $c$ is a spectator. The dashed lines denote the particles in the intermediate state are on mass shell.

When $c$ is a spectator these coupling functions are real. The hadronic amplitudes $T$ can be conveniently represented by a $K$-matrix form to ensure unitarity for the hadronic amplitudes (or by related forms I call the $L$-matrix if we are also to require the correct analyticity). At the lowest energies when $ab$ is the only accessible channel this implies Watson’s theorem [4], namely that the phase of each decay amplitude (with definite isospin and spin), must equal that for the corresponding hadronic scattering amplitude for $ab \rightarrow ab$. 

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2 Semi-leptonic decays

Now we can test this by considering the semileptonic decay $D^+ \rightarrow K^-\pi^+(\mu^+\nu_\mu)$, where clearly only the $K\pi$ system can have strong final state interactions. The dominant low energy signal is in the region of the $K^*(892)$, so FOCUS \cite{8} looked at the mass region from 800 MeV to 1 GeV in two bins. If the vector $K^*$ was all there was then the angular distribution of the final state hadrons in the $K\pi$ rest system would be proportional to $\cos^2 \theta$ and so forward-backward symmetric. However a marked asymmetry is found, Fig. 5. This means the $P$-wave $K\pi$ amplitude must interfere with some other wave and at low energies that is the $S$-wave. It is here that many have argued lies the scalar $\kappa$. What does experiment tell us?

Figure 5: Feynman graph of the semi-leptonic decay of a $D^+$. Asymmetry in $K\pi$ angular distribution in the $K^*(892)$ mass region from the FOCUS experiment \cite{8}.

Let us first look at $K\pi$ elastic scattering. This we learn about by studying high energy $K\pi$ production with $K$-beams at small momentum transfers, where the reaction is controlled by one pion exchange. Then one can extract the cross-section for $K\pi \rightarrow K\pi$ scattering. This was done 25 years ago by LASS \cite{9}. The cross-section shows clear $K^*(892)$ and $K^*_2(1430)$ peaks. By performing a partial wave analysis \cite{9}, one confirms that these have spin-1 and 2 respectively with the magnitude and phase of well-defined resonances. What we are interested in here is the $S$-wave underneath these. This shows a broad rise in magnitude and phase from 825 MeV (where data begin) with a peak at 1400 MeV and phase rising through 90°. This is characterised by the $K^*_0(1430)$ with a width of $\sim 300$ MeV. There is no hint of a low mass $\kappa$, which if it were simply describable by a Breit-Wigner, would have forced the phase to have already reached 90° by 800 MeV. What does $D$ semileptonic decay tell us? FOCUS \cite{8} found that their forward-backward asymmetry, Fig. 5, requires an $S$-wave phase of 45° in the $K^*(892)$ region, exactly as LASS has measured \cite{9}.

The statistics at FOCUS only allow a determination of the $K\pi$ $S$-wave phase in the $K^*(892)$ region and then only in very wide bins. However, such $D$ semileptonic decays with the event rates of CLEO-c and $B$-factories should provide an accurate determination of the low energy $K\pi$ phase-shifts, in the same way as $K_{e4}$ decays have done for $\pi\pi$ phases. These,
when combined with dispersion relations and three channel crossing symmetry, have resulted in a rather precise determination of the position of the $\sigma$-pole $^{10}$. If we are going to be able to determine whether and where a $\kappa$ exists, then we will need precision $K\pi$ information below the range accessed by LASS. Semileptonic $D$ decays are the theoretically unambiguous way to go.

3 Hadronic final states

While we wait for that, let us turn to 3-hadron decays and let us focus on the Cabibbo-favoured $D^+ \to K^-\pi^+\pi^+$ channel. Let us consider this with increasing levels of sophistication. First let us assume an isobar picture. The Dalitz plot is to be described by a sum of isobars in the three di-meson channels, Fig. 3. Since the $\pi\pi$ channel has $I = 2$, it has no known resonances and so is set to zero. The $K\pi$ channels are described simply by summing Breit-Wigners with parameters taken from the PDG tables $^{6}$, the $K^*(892)$, $K^*_1(1410)$, $K^*_2(1430)$ and of course the broad $K^*_0(1430)$. To this is added the 3-body interaction matrix element, which is presumed constant in both magnitude and phase across the Dalitz plot. The resultant fit to the E791 data $^{11}$ is very poor and 90% of the decay is ascribed to the direct 3-body term. Since the Dalitz plot displays distinct 2-body structures, no wonder the fit is poor. The next step is to add a Breit-Wigner for another scalar, they call the $\kappa$. It is just added. Its mass and width are determined by the fit $^{12}$ to be $M = 797$ MeV, $\Gamma = 410$ MeV. The fit dramatically improves. The direct 3-body fraction is down to a more believable 14%, and the $\kappa$ contributes 43%. However modelling the scalar channel by a sum of the $\kappa$ and $K^*_0(1430)$ Breit-Wigners is not only in violation of the conservation of probability, but implies a model for $K\pi$ elastic scattering in total disagreement with the LASS results $^{9}$. We clearly have to do better.

Rather than be tied to specific forms for the complicated $S$-wave amplitude, Brian Meadows $^{11,13}$ has tried something much more promising. With $P$ and $D$-wave $K\pi$ interactions given as before by sums of Breit-Wigners from the PDG tables $^{6}$, the $S$-wave is parametrized in terms of a magnitude and a phase in each bin across the Dalitz plot. Fitting gives the phases shown in Fig. 6. These are determined almost to threshold and, if they had greater precision and we applied the right tools for analytic continuation, might result in locating a $\kappa$ pole on the nearby unphysical sheet. So how do these phases compare with the $K\pi$ elastic $S$-wave?

In Fig. 6 the $K\pi$ phases from Brian Meadows’ E791 fit $^{13}$ and the phases for $K^-\pi^+ \to K^-\pi^+$ scattering from LASS $^{9}$ are shown. The latter are absolute. Those from $D$-decay are relative, relative to the $P$-wave fixed to be 90° at 892 MeV. Consequently, we are free to raise the E791 phase up to be zero at threshold. We then see better agreement, with a common rising trend with $K\pi$ mass. However, Fig. 6 shows these are clearly not the same. There are several possible reasons for this. $K^-\pi^+$ interactions involve both $I = 1/2$, 3/2 components. In elastic scattering the relative strength of these is fixed by Clebsch-Gordan coefficients, while in $D$-decay this is determined by dynamics. We may have the prejudice
that $I = 1/2$ is dominant, but that does not mean the $I = 3/2$ component is negligible. Thus if we assume Watson’s theorem, which holds in the elastic region effectively up to $K\eta'$ threshold, one can determine the relative amounts of $I = 1/2$ and $I = 3/2$ contributions, as in Ref. 14. However, what I want to present here is something a little different.

Figure 7: Unitarity for the $K\pi$ system in $D$-decay in the elastic region. The dashed lines denote the particles in the intermediate state are on mass shell.

So far we have considered the final state interaction of the $K$ and a $\pi$ where the second pion is merely a spectator. Let us now ask what happens if we try to include a subsequent interaction of the $K$ with this pion. There has for long been a body of work on such multiparticle interactions [15], particularly by Ascoli and collaborators [16] on 3 pion final states dating from the discovery of the $a_1$ and its possible structures. Such multiparticle final states have been investigated by Anisovich et al. [17] in $\bar{p}p$ annihilations at LEAR. Much more recently Caprini [18] has shown that one can deduce a unitarity relation with rescattering, implicit in these studies. This relation shows that for each partial wave amplitude the imaginary part of the $D \to K\pi\pi$ is a sum of contributions, Fig. 7.
Figure 8: The solid curve denotes a fit to the LASS results in Fig. 6 consistent with Chiral Perturbation Theory [19] for the $I = 1/2$ S-wave $K\pi$ phase compared with the S-wave $K\pi$ phase from the Meadows' analysis [11, 13] of the E791 results on $D \rightarrow K\pi\pi\pi$ decay. The upper line displays a preliminary calculation of how the elastic scattering phase incorporated in the first graph on the right hand side of the unitarity relation shown in Fig. 7 is modified by the rescattering corrections from the second (and third) graphs in Fig. 7. The curve closest to the E791 phases includes feedback.

If there was just the first term in Fig. 7 then Watson’s theorem would hold in the region of elastic unitarity. The second and third graphs in Fig. 7 give corrections. Since the Dalitz plot for $D^+ \rightarrow K^-\pi^+\pi^+$ is symmetric in the two $K\pi$ systems, and assuming the $\pi^+\pi^+$ amplitude to be negligible, we can then use the Meadows’ results to compute the rescattering corrections given by these graphs. There are some technicalities I won’t go into here, but the crude result gives the phase of the S-wave $K^-\pi^+$ interaction in $D$-decay to be the upper line in Fig. 8. The corrected phase is computed at each datum of the Meadows’ analysis and then these are simply connected by straight lines. The similar line a little bit lower (and closer to the data) involves including feedback to improve the phase variation across the Dalitz plot.

These preliminary results are encouraging. However, details still need to be checked before they can be considered definitive. Nevertheless, this method holds out the prospect of using high statistics data on hadronic $D$-decays from BaBar and Belle to determine the near threshold $K\pi$ phase-shift with precision, independently of summing higher orders in Chiral Perturbation Theory. So while there is no $\kappa(900)$ [23], there may be a scalar much closer to threshold deep in the complex plane. From the Roy equation analysis of $\pi\pi$ scattering we now know the position of the $\sigma$-pole rather precisely. All analyses should find the same result. The variation between treatments discussed in Refs. 20, 21 is unacceptable in the
era of precision physics. The application of the rescattering corrections to \( J/\psi \rightarrow \omega \pi \pi \) may well, with higher statistics accessible at BESIII, bring the mass and width from a simple Breit-Wigner treatment \[22\] in line with the true \( \sigma \)-pole position \[10\]. That is the challenge for BES.

A precise description of these hadronic final state interactions, especially those in scalar channels, is essential to reducing the uncertainty in the CKM triangle. This reduction is crucial to learning about physics both within and beyond the Standard Model. Dalitz analyses are at the heart of this programme, as others will describe.

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