Recent Progress on the BNL Muon ($g - 2$) Experiment

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B. Lee Roberts1, H.N. Brown2, G. Bunce2, R.M. Carey1, P. Cushman9, G.T. Danby2, P.T. Debevec7, M. Deile11, H. Deng11, W. Deninger7, S.K. Dhawan11, V.P. Druzhinin3, L. Duong9, E. Efstathiadis1, F.J.M. Farley11, G.V. Fedotovich3, S. Giron9, F. Gray7, D. Grigoriev3, M. Grosse-Perdekamp11, A. Grossmann6, M.F. Hare1, D.W. Hertzog7, V.W. Hughes11, M. Iwasaki10, K. Jungmann6, D. Kawall11, M. Kawamura10, B.I. Khazin3, J. Kindem9, F. Krienen1, I. Kronkvist9, R. Larsen2, Y.Y. Lee2, I. Logashenko1, R. McNabb9, W. Meng2, J. Mi2, J.P. Miller1, W.M. Morse2, D. Nikas2, C.J.G. Onderwater7, Y. Orlov4, C.S. Özben2, J.M. Paley1, C. Polly7, J. Pretz11, R. Prigl2, G. zu Putlitz6, S.I. Redin11, O. Rind11, N. Ryskulov3, S. Sedykh7, Y.K. Semertzidis2, Yu.M. Shatunov3, E.P. Sichtermann11, E. Solodov3, M. Sossong7, A. Steinmetz11, L.R. Sulak1, C. Timmermans9, A. Trofimov1, D. Urner7, P. von Walter6, D. Warburton2, D. Winn5, A. Yamamoto8, D. Zimmerman9,

1Department of Physics, Boston University, Boston, MA 02215, USA, 2Brookhaven National Laboratory, Upton, NY 11973, USA, 3Budker Institute of Nuclear Physics, Novosibirsk, Russia, 4Newman Laboratory, Cornell University, Ithaca, NY 14853, USA, 5Fairfield University, Fairfield, CT 06430, USA, 6Physikalisches Institut der Universit"at Heidelberg, 69120 Heidelberg, Germany, 7University of Illinois at Urbana-Champaign, IL 61801, USA, 8KEK, Tsukuba, Ibaraki 305-0801, Japan, 9University of Minnesota, Minneapolis, MN 55455, USA, 10Tokyo Institute of Technology, Tokyo, Japan, 11Yale University, New Haven, CT 06520, USA

Abstract

The status of the muon ($g - 2$) experiment at the Brookhaven AGS is reviewed. An accuracy of 1.3 ppm on the $\mu^+$ anomalous magnetic moment has been achieved and published. This result differs with the standard model prediction by about 2.5 standard deviations. A data sample with approximately seven times as much data is being analyzed, with a result expected in early 2001.

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1 Introduction

Measurements of the $g$-factors of elementary particles have been intimately tied to the development of our understanding of subatomic physics. The proportionality constant, $g$, relates the spin and magnetic moment of an elementary particle through the relationship

$$\bar{\mu}_s = g \left( \frac{e}{2m} \right) \vec{S},$$

where

$$a = \frac{(g - 2)}{2}$$

is the anomalous part of the magnetic moment. In the Dirac theory, $g = 2$. The muon anomalous moment is dominated by the lowest order radiative correction $\alpha^2 \pi$, which was first calculated in 1948 by Schwinger.\[1\] QED calculations of the electron and muon anomalies have been carried out to eight-order (with an estimate of tenth-order) by Kinoshita and others.\[2\]

$$\gamma \mu (a) \gamma \mu \gamma (b)$$

Figure 1: The Feynman graphs for (a) $g = 2$; and (b) the lowest order radiative correction (Schwinger term).

The electron anomalous moment is now measured to an experimental accuracy of a few parts per billion, and is well described by QED calculations.\[2\] To the level of measurement, only photons and electrons contribute, and $a \simeq 1 \times 10^{-3}$. There is no evidence to date, either from $g$-factor measurements or $e^+e^-$ scattering, to indicate that the electron has any internal structure.

While the $g$-factor of the electron has provided a testing ground for QED, the anomalous magnetic moment of the muon has provided an even richer source of information, since the contribution of heavy virtual particles to the anomaly scales as the mass of the lepton squared. In a series of three elegant experiments at CERN,\[3\] virtual muons and hadrons have been shown to contribute at measurable levels.

The CERN experiments measured $\mu = [0.001 165 9230(85)] (e \hbar / 2m_{\mu})$, a precision of $\pm 7.3$ parts per million (ppm) for $a_{\mu}$. This result tested QED to a high level, and showed for the first time the contribution of virtual hadrons to the magnetic moment of a lepton, the sensitivity was not sufficient to observe the predicted electroweak contribution of 1.3 ppm from virtual $W^\pm$ and $Z^0$ gauge bosons. The goal of our experiment is an overall accuracy of $\pm 0.35$ ppm which allows sufficient sensitivity to measure the electroweak contribution, as well as to search for physics beyond the standard model.

The standard model theory has been reviewed at this conference by Prades.\[6\] The theoretical value of $a_{\mu}$ consists of contributions from QED, virtual hadrons, and virtual electroweak gauge bosons.\[7\] The theoretical uncertainty is dominated by the hadronic contribution, which must be calculated using data from $e^+e^- \rightarrow$ hadrons.
along with a dispersion relation, or in the case of the hadronic light-by-light term, from a model calculation. The most important diagrams are shown in Fig. 2. The largest contribution (and the largest uncertainty) comes from the hadronic vacuum polarization diagram (Fig. 2(a)).

Figure 2: Hadronic contributions to the muon anomalous moment. In these diagrams, $H$ refers to a loop with hadrons (quarks). (e) shows the hadronic light-by-light contribution discussed by Prades at this meeting.

The one-loop electroweak contributions to $a_\mu$ have been available for some time, and now higher order calculations which include both fermionic and bosonic two-loop terms are available, with the next order leading logs also evaluated.

One of the main motivations for our measurement was to confront the standard model, and to search for possible contributions from non-standard model physics such as supersymmetry, muon or $W$-gauge-boson substructure. Theoretical interest in possible non-standard model contributions to the muon $(g - 2)$ value has risen substantially in the past five years, and a great deal has been written about possible contributions to the muon $(g - 2)$ value from non-standard model physics.

Just as proton substructure produces a $g$-value which is not equal to two, muon (or $W$) substructure would also contribute to the anomalous moment, the critical issue being the scale of the substructure. A standard model value for $(g - 2)$ at the 0.35 ppm level would restrict the substructure scale to around 5 TeV. If leptoquarks exist, they too could contribute to the non-standard model value of $(g - 2)$. The muon $(g - 2)$ obtains its sensitivity to $W$ substructure and anomalous gauge couplings through the $WW\gamma$ triple gauge vertex in the single loop $W$ contribution.

Figure 3: The lowest order supersymmetric contributions to $(g - 2)$.

Supersymmetry has become a serious candidate for physics beyond the standard model. There is a large sensitivity to almost any supersymmetric model with large $\tan \beta$. The SUSY contribution is shown in Fig. 3. In the case of large $\tan \beta$, the chargino ($\chi^-$) diagram dominates the contribution to $(g - 2)$ from SUSY, and is given
by \[\text{(7)}\]

\[a_\mu(\text{SUSY}) \simeq \frac{\alpha}{8\pi \sin^2 \theta_W} \frac{m_\mu^2}{\tilde{m}^2} \tan \beta \simeq 140 \times 10^{-11} \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2 \tan \beta, \tag{2}\]

where \(\tilde{m}\) is the largest mass in the loop. The goal of E821 is to reach a precision of \(\pm 40 \times 10^{-11} \ (\pm 0.35 \text{ ppm})\), so the factor of 140 in Eq. 2 corresponds to 1.2 ppm in \(a_\mu\).

For \(\tilde{m} = 750 \text{ GeV} \) and \(\tan \beta = 40\), \(a_\mu(\text{SUSY}) = 100 \times 10^{-11}\), a contribution which is 2.5 times larger than the sensitivity we hope to achieve. For \(\tilde{m} = 500 \text{ GeV}\), the effect is \(224 \times 10^{-11}\) or 5.6 ppm.

## 2 The Experimental Technique

The experimental technique is a refinement of the technique used in the third CERN experiment, \[\text{(8)}\] with the addition of direct injection of a muon beam into the storage ring. A superconducting magnetic storage ring with a vertical magnetic field of 1.45 T, central orbit radius of 711.2 cm, and central momentum of 3.1 GeV stores a bunch of muons provided by the AGS. Vertical focusing is provided by electrostatic quadrupoles which are placed in the ring with four-fold symmetry. The injected beam is kicked onto a stable orbit by a fast kicker system which uses a current distribution to provide the \(\sim 0.1 \text{ Tm}\) kick needed to store the beam. The residual magnetic field from the fast kicker has been measured using the Faraday effect, and it was found to contribute less than 0.1 ppm to the integrated \(Bdl\) seen by the muons for times greater than 20 \(\mu\)s after injection.

A charged particle moving transverse to a uniform magnetic field will go in a circle with the orbital cyclotron frequency

\[\omega_c = \frac{(eB)}{(m\gamma)}. \tag{3}\]

The spin precession frequency in this same magnetic field is given by

\[\omega_s = \frac{geB}{2m} + (1 - \gamma) \frac{eB}{m\gamma}. \tag{4}\]

Thus the spin vector of a charged particle moving transverse to a uniform magnetic field will precess relative to the momentum vector with a frequency \(\omega_a\), which is given by the difference between the orbital cyclotron frequency \(\omega_c\) and the spin precession frequency \(\omega_s\). This frequency is

\[\omega_a = \omega_s - \omega_c = \frac{e}{m} a_\mu B, \tag{5}\]

where \(\omega_a\) is directly proportional to the anomalous moment and is independent of the particle’s momentum.
Vertical focusing must be provided to keep the muon beam stored, which can be accomplished with magnetic multipoles, or with an electrostatic quadrupole field. However, if magnetic multipoles are used, it is difficult to determine the average $B$ field to the accuracy needed for a precision measurement of $a_\mu$. In a region in which both magnetic and electric fields are present, the relativistic formula for the precession is given by

$$\vec{\omega}_a = \frac{d\Theta_R}{dt} = -\frac{e}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right],$$  

(6)

where $\Theta_R$ is the angle between the muon spin direction in its rest frame and the muon velocity direction in the laboratory frame. The other quantities refer to the laboratory frame. If the muon beam has the "magic" value of $\gamma = 29.3$, then the coefficient of the $\vec{\beta} \times \vec{E}$ term is zero, and the electric field does not cause spin precession. Thus the precession of the spin relative to the momentum is determined entirely by the magnetic field, and one can use electrostatic quadrupoles for vertical focusing. Because the muon’s lifetime is relatively long, and because muons are produced fully-polarized along their direction of motion in pion decay at rest, it is possible to produce a beam of polarized muons. With our kicker we store $\sim 10^4$ per fill of the storage ring.

In the three-body decay $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$, the highest energy positrons are preferentially emitted parallel to the muon spin direction in the muon rest frame. When emitted parallel to the muon momentum, the highest energy electrons in the muon rest frame are Lorentz boosted to become the highest energy electrons in the lab frame. Therefore, the number of high energy electrons is a maximum when the muon spin is parallel to the momentum, and a minimum when it is anti-parallel, thus making it possible to measure the spin (or anomalous) precession frequency by counting high energy electrons as a function of time. This time spectrum will show the muon lifetime modulated by the spin precession frequency. In a perfect experiment, the positron time spectrum would be given by

$$N(t) = N_0 e^{-t/\gamma\tau} (1 + A \cos (\omega_\mu t + \phi)).$$  

(7)

The real experiment has pulse pileup, muons lost from the storage ring other than by decay, and because the detector acceptance depends on the radial position of the muon decay, the coherent motion of the beam in the storage ring modulates the time spectrum with the coherent betatron frequency. In the four independent analyses of the 1999 data set, pileup was either fit directly, or subtracted from the data set. A picture of the pileup subtracted data and a ten parameter fit to it is given in Fig. 4 below.

Equation (6) gives the principal elements needed to obtain a value for the muon anomaly. One needs to measure the muon frequency $\omega_\mu$ as well as the magnetic field weighted over the muon distribution. The field is measured with NMR techniques, in a four-step procedure. During data collection, the field is monitored with 377 fixed
Figure 4: The positron time spectrum from the 1999 data set with an energy threshold of 2.0 GeV. The data and fit to a 10 parameter function are shown, where $\chi^2/\nu$ for the fit is 3818/3799. There are $0.95 \times 10^9$ events in the histogram.

NMR probes which are outside of the beam vacuum chamber. Every few days an NMR mapping trolley is used to map the field inside of the storage region, which calibrates the fixed probes. Before and after the running period, the NMR probes in the trolley are cross calibrated with a calibration probe located at one point in azimuth, which plunges into the storage region to measure the field at the position of each of the 17 trolley probes. This plunging probe, is then calibrated with a special probe which has a spherical water sample, thus giving us the spin rotation frequency of a free proton, $\omega_p$, in our magnetic field. We compute the ratio $R = \omega_a/\omega_p$, and the anomaly is given by $a_\mu = R/(\lambda - R)$. The constant $\lambda = \mu_\mu/\mu_p$, the ratio of the magnetic moments of the muon and proton, is known independently from other experiments.

The analysis was performed “blind” meaning that arbitrary offsets were put on the two frequencies $\omega_a$ and $\omega_p$ during the analysis, so that it was impossible to determine the value of without knowing these two offsets. Two independent analyses of $\omega_p$ and four of $\omega_a$ were performed. Only after the separate analyses of these frequencies were consistent and well studied, were the offsets revealed, and the value of $a_\mu$ computed.

The value obtained was $a_{\mu^+} = 11 659 202(14)(6) \times 10^{-10}$ (1.3 ppm). The standard model value used for comparison was $a_{\mu} = 11 659 159.6(6.7) \times 10^{-10}$ (0.66 ppm). The weighted averaged of the experimental values gives a difference from the standard model of $a_\mu(exp) - a_\mu(th) = +43(16) \times 10^{-10}$. The individual measurements are shown graphically in Fig. 5.

While this difference with the standard model is quite interesting, it is far from definitive. As discussed at this conference by Prades, the theoretical value is undergoing detailed study by a number of people. As additional data from Novosibirsk,
Daφne and BES become available, along with the full $\tau$-decay data set from LEP and CLEO, our knowledge of the theoretical hadronic contribution will improve further.

The experimental value is also a work in progress. We collected about 4 billion positrons in our 2000 run, and about 3 billion electrons in our 2001 run. Thus we expect our statistical error to improve by about $\sqrt{7}$. We are constantly improving our understanding of the systematic errors, and believe that the final total systematic error should be about 0.3 ppm. Recently, scientific approval for an additional data collection period was given by the Laboratory, but funding will have to be found if we are to collect these additional data. If we obtain the additional data, we expect to reach a statistical error of about 0.33 ppm.

Much progress has been made since our collaboration began almost two decades ago. We have solved many interesting technical issues, and have obtained an answer at the part per million level. This new result presented us with a surprise which has been received with widespread interest. Our collaboration is working hard to finish the analysis of the additional data sets in order to clarify whether this potential signal for new physics will remain at the end of the day. Stay tuned.

References

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