126 GeV Higgs and ATLAS bound on the lightest graviton mass in Randall-Sundrum model

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Abstract

In the search for extra dimension through dilepton events in 7-TeV proton-proton collision, the ATLAS detector at LHC has set stringent lower bound on the mass of the Randall-Sundrum (RS) lightest graviton Kaluza-Klein (KK) mode. Considering that the Randall-Sundrum model undertakes to resolve the well-known gauge hierarchy/fine tuning problem to restrict the Higgs mass within the estimated $\sim 126$ GeV against large radiative correction up to the cut-off of the model, we explore the allowed parameter space within which the RS model can be trusted. We show that the consistency of the model with ATLAS results constrains the cut-off of the theory which is at least two orders lower than the Planck/Quantum gravity scale implying the possible existence of a new Physics at this lower scale.

Introduction

One of the main goals of the experiments in the Large Hadron Collider (LHC), is the search for physics beyond standard model (BSM). The motivations for new physics beyond standard model stems from the large hierarchy of mass scales between the Planck and the TeV scales which results into the well-known fine tuning problem in connection with the mass of the Higgs boson, the only scalar particle in the standard model. It has been shown that due to large radiative corrections the Higgs mass can not be confined within TeV scale (the current estimated value $\sim 126$ GeV), unless some unnatural tuning is done order by order in the perturbation theory. Among several proposals to address this problem, the models with extra spatial dimensions draw lot of attention. In particular the warped geometry model proposed by Randall and Sundrum [1] has drawn special attention for the following reasons: (1) It resolves the hierarchy

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problem without introducing any other intermediate scale in the theory, (2) The modulus of the extra dimensional model can be stabilized [2], and (3) A warped solution, though not exactly same as RS model, can be found from string theory which as a fundamental theory predicts inevitable existence of extra dimensions [3]. Due to these unique features, experiments in LHC are designed to explore the signature of these warped extra dimensions through the dileptonic decay of Kaluza-Klein graviton KK modes present in these models.

In this work we propose to study some theoretical constraints of RS model and look for their consistency with the result obtained in LHC so far. We begin with a brief description of RS model.

RS Model (brief description)

Randall-Sundrum scenario [1] which is defined on a 5-dimensional anti de-Sitter space-time with one spatial direction orbifolded on $S^1/Z_2$ has the following features:

- Two flat 3-branes namely hidden/Planck brane and visible/standard model brane are located at the two orbifold fixed points. The brane tension of the standard model/visible brane is negative.
- 5-dimensional Planck scale is nearly equal to 4-dimensional Planck scale.
- Without introducing any extra scale, other than the Planck scale, in the theory one can choose the brane separation modulus $r_c$ to have a value $\sim M_{Pl}^{-1}$ such that the desired warping can be obtained between the two branes from Planck scale to TeV scale.
- The modulus can be stabilized to the above chosen value by introducing scalar in the bulk [2] without any further fine tuning.

We first briefly outline the RS model below:

The RS model is characterized by the non-factorisable background metric,

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2$$

with $\eta_{\mu\nu} = (-, +, +, +, +)$ and $\sigma = kr_c|\phi|$. $r_c$ is the compactification radius for the extra dimension and $k$ is of the order of 4-dimensional Planck scale $M_{Pl}$ and relates the 5D Planck scale $M$ to the cosmological constant $\Lambda$. The extra dimensional coordinate is denoted by $\phi$ and ranges from $-\pi$ to $+\pi$ following a $S^1/Z_2$ orbifolding. Two 3-branes are located at the orbifold fixed points $\phi = (0, \pi)$. The standard model fields are residing on the visible brane and only gravity can propagate in the bulk. Solving five dimensional Einstein’s equation and using orbifolded boundary condition, the warped solution for the metric turns out to be, $\sigma(\phi) = kr_c|\phi|$, Where $k = \sqrt{\frac{\Lambda}{24M^3}}$. The visible and Planck brane tensions are, $V_{hid} = -V_{vis} = 24M^3k^2$. All the dimensional parameters described above are related to the reduced 4-dimensional Planck scale $M_{Pl}$ as,

$$\overline{M}_{Pl}^2 = \frac{M_{Pl}^3}{k}(1 - e^{-2kr_c\pi})$$

(2)
For $kr_c \approx 12$, the exponential factor present in the background metric, which is often called warp factor, produces a huge suppression on the Planck scale mass parameters and reduced those parameters to TeV scale on the visible brane. Thus a scalar mass say mass of Higgs is given as,

$$m_H = m_0 e^{-kr_c \pi}$$  \hspace{1cm} (3)

Here, $m_H$ is Higgs mass parameter on the visible brane and $m_0$ is the the cut off scale of the theory, above which new physics beyond standard model is expected to appear. A natural choice for this would be Planck or quantum gravity scale beyond which standard model will not be valid.

**Theoretical restriction on $\epsilon = k/M_{Pl}$**

As argued in [1] we assumed that $k < M$ with $M \sim M_{Pl}$. This requirement emerges from the fact that $k$, which measures the bulk curvature must be smaller than the Planck scale so that the classical solutions for the bulk metric given by RS model is a valid one. Alternatively from the viewpoint of string theory it was argued in [4] that using the expression for D-3 brane tensions and the string scale which in turn is related to $M_{Pl}$ through Yang-Mills gauge coupling, the favoured range of $k/M_{Pl}$ is $0.01 \leq k/M_{Pl} \leq 1$.

It has been shown [4], that in such models the gravity which propagates in the bulk can be expanded into KK modes as,

$$h_{\alpha\beta}(x, \phi) = \sum_{n=0}^{\infty} h_{\alpha\beta}^{(n)}(x) \frac{\chi^n(\phi)}{\sqrt{r_c}}$$  \hspace{1cm} (4)

Where $h_{\alpha\beta}^{(n)}(x)$ are the KK modes of the graviton on the flat 3-brane and $\chi^n(\phi)$ is the wave function for the graviton. In [4] it has been shown that the solution for the graviton wave function is of the form,

$$\chi^n(\phi) = \frac{e^{2\sigma(\phi)}}{N_n} [J_2(z_n) + \alpha_n Y_2(z_n)]$$  \hspace{1cm} (5)

Where $J_2$ and $Y_2$ are the Bessel functions of order 2 and $z_n(\phi) = \frac{m_n e^{\sigma(\phi)}}{k e^{kr_c \pi}}$, $\alpha_n$ are constants and $N_n$ is the normalization factor for the wave function.

If we now choose a parameter $x_n = z_n(\pi)$ and consider the limit $m_n/k \ll 1, e^{kr_c \pi} \gg 1$ then applying the continuity condition that the wave function should be continuous at $\phi = 0$ and $\phi = \pi$ (i.e at the two orbifold fixed points), it produces $J_1(x_n) = 0$ and $\alpha_n \ll 1$. Hence the term with $Y_2(z_n)$ can be ignored in comparison to $J_2(z_n)$. The expression $J_1(x_n) = 0$ implies that $x_n$ are the roots of the Bessel function of the order of 1 and if we plot the $J_1(x_n)$ vs. $x_n$, we get the first few values of $x_n$ as follows, $x_1 = 3.83$, $x_2 = 7.02$, $x_3 = 10.17$ etc.

As it has been defined earlier that, $x_n = z_n(\pi) = \frac{m_n}{k} e^{kr_c \pi}$, we can find the KK mass tower of graviton from this expression,

$$m_n = x_n k e^{-kr_c \pi}$$  \hspace{1cm} (6)

We mentioned earlier that to study the phenomenology of this model the most important parameter is $\epsilon = k/M_{Pl}$. Hence let us focus on the first KK mass
mode of graviton (i.e \( m_1 \)) and slightly reparametrize the expression for \( m_1 \), from eq. (6).

\[
m_1 = x_1 k e^{-k r c \pi} \tag{7}
\]

From eq. (3) and using \( \epsilon = k / M_{Pl} \),

\[
m_1 = x_1 \epsilon \frac{m_H}{m_0} M_{Pl} \tag{8}
\]

Now from eq. (2), the exponential term is very small in comparison to 1. Hence we obtain,

\[
M_{Pl}^2 = \frac{M_3^3}{k}. \quad \text{In eq. (8), we use } M_{Pl}^2 = \frac{M_3^3}{k} \text{ and take the cut off scale as, } m_0 = \alpha M, \text{ where } M \text{ is the 5-dimensional Planck scale and } \alpha \text{ is any constant parameter. } \alpha = 1 \text{ implies that the cut-off scale is the quantum gravity scale } M_{Pl}, \text{ while } \alpha < 1 \text{ indicates the appearance of new physics below Planck scale. Hence we obtain,}
\]

\[
m_1 = x_1 e^{2/3} \frac{m_H}{\alpha} \tag{9}
\]

If we consider \( \alpha \) to be 1 i.e the cut-off scale is the 5-dimensional Planck scale \( M \) then values of \( m_1 \) for different values of \( \epsilon \) varying from 0.01 – 0.1 are shown in table (1) and plotted in figure (1).

| \( \epsilon = k / M_{Pl} \) | \( m_1 = x_1 e^{2/3} m_H (\text{GeV}) \) |
|-----------------------------|---------------------------------|
| 0.01                        | 22.39                           |
| 0.03                        | 46.59                           |
| 0.05                        | 65.49                           |
| 0.07                        | 81.96                           |
| 0.09                        | 96.91                           |
| 0.1                         | 103.96                          |

Table 1: Theoretical values of first KK mass mode of graviton From RS model when \( \alpha = 1, x_1 = 3.83 \) and \( m_H = 126.0 \text{GeV} \)

Figure 1: figure of \( \epsilon \) vs. \( m_1 \) where \( m_H = 126 \text{GeV} \) and \( m_1 \) is varying from 20 GeV to 110 GeV

The Experimental lower bound for the mass of the first KK mode of graviton for different values of \( \epsilon \) as reported by the ATLAS Collaboration [5] is shown in the figure (2). Some of the values of \( m_1 \) for different \( \epsilon \) are shown in table (2).
\[ \epsilon = k/M_{Pl} \]

| \( \epsilon \) | \( m_1 \) (TeV) |
|--------|---------|
| 0.01   | 1.01    |
| 0.03   | 1.48    |
| 0.05   | 1.88    |
| 0.07   | 2.04    |
| 0.09   | 2.17    |
| 0.1    | 2.22    |

Table 2: The mass table from the results of ATLAS detector of LHC

Figure 2: Graph of \( \epsilon \) vs. \( m_1 \) as reported by ATLAS Collaboration. RegionI has been ruled out which sets lower bound for the first graviton KK mode for different values of \( \epsilon \)

The tables as well as the plot clearly indicate that for the entire range of \( 0.01 < \epsilon < 1 \), the theoretical prediction for the first KK mode of graviton falls well within region-I of the ATLAS plot which has been ruled out in the search of graviton KK mode resonance in dilepton events. This implies a serious conflict between graviton KK modes as predicted in RS model and the result reported by ATLAS Collaboration.

For a possible resolution to this problem we calculate the threshold values of the parameter \( \alpha \) from the expression, \( \alpha = x_1 \epsilon^{2/3} m_H/m_1 \) and use the values of lower bound of \( m_1 \) for different \( \epsilon \) as reported by ATLAS data. These values are shown in table (3).

| \( \epsilon \) | \( m_1 \) from ATLAS (TeV) | values of \( \alpha \) |
|--------|-----------------|----------------|
| 0.01   | 1.01            | \( 2.2 \times 10^{-2} \) |
| 0.03   | 1.48            | \( 3.1 \times 10^{-2} \) |
| 0.05   | 1.88            | \( 3.4 \times 10^{-2} \) |
| 0.07   | 2.04            | \( 4.0 \times 10^{-2} \) |
| 0.09   | 2.17            | \( 4.4 \times 10^{-2} \) |
| 0.1    | 2.22            | \( 4.6 \times 10^{-2} \) |

Table 3: The values of \( \alpha \) for each \( \epsilon \) and corresponding mass of graviton from ATLAS data where \( x_1 = 3.83 \) and \( m_H = 126 \text{ GeV} \)
This indicates that the cut-off of the RS model namely $m_0$, must be at least two order lower then the Planck scale indicating the existence of new physics at a scale of the order of $10^{17}$ GeV.

**Conclusion**

Present estimation of the lower bound on the lightest RS graviton KK mode masses from dilepton events as reported by ATLAS Collaboration is in conflict with the requirement of resolving the gauge hierarchy /fine tuning problem unless the cut-off of the standard model is at least two order lower than the Planck/Quantum gravity scale. This indicates the possible appearance of a new physics at $\sim 10^{17}$ GeV.

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