Transverse-to-longitudinal emittance-exchange in optical wavelength

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Abstract

Emittance exchange is a promising technique for next-generation accelerator-based applications. A novel technique is proposed in this paper to exchange emittance of the electron beam between transverse and longitudinal planes in optical wavelength. The emittance exchange configuration consists of a dual-tilted-laser modulator sandwiched by two identical doglegs. Analytical and simulation results demonstrate that the emittance exchange for the electrons at the zero-crossing phase of each laser cycle can be easily achieved with this technique. The proposed technique is quite promising for improving the performance of compact high-gain free-electron laser facilities. Meanwhile, it holds the feasibility to flexibly tailor a beam’s longitudinal shape in the optical scale.

1. Introduction

Next-generation accelerator-based applications (e.g., linear colliders, storage rings or free-electron lasers) demand precise manipulations of the six-dimensional (6D) phase space of relativistic electron bunches. Emittance exchange (EEX) technique is one of these promising techniques which rearranges a beam’s distribution, or more specifically, exchanges the projected emittance between different planes in 6D phase space. Of particular interest is the transverse-to-longitudinal EEX that exchanges the emittance of one transverse plane with the longitudinal plane.

X-ray free-electron laser (FEL) facilities, such as free-electron laser in Hamburg (FLASH) (Ackermann et al (2007)), the linac coherent light source (LCLS) (Emma et al (2010)), the spring-8 Angstrom compact free-electron laser (SACLA) (Ishikawa et al (2012)), the Trieste FERMI (Allaria et al (2012)), the Swiss x-ray free-electron laser (SwissFEL) (Milne et al (2017)) and the Pohang Accelerator Laboratory x-ray free-electron laser (PAL-XFEL) (Kang et al (2017)) are now under operation worldwide. These facilities are always so large and expensive that only a few countries can afford them. The main challenge for down-size and improving the performance of an x-ray FEL remains the achievement of bright electron beams in the transverse plane. Limited by its initial value at the cathode of the thermionic or photocathode guns, the normalized transverse emittance of an electron beam usually ranges from 0.5 to 1 mm-mrad.

Transverse-to-longitudinal EEX was thereby introduced to obtain a smaller transverse emittance for enhancing the FEL gain (Cornacchia and Emma (2002), Emma et al (2006)). However, with a typical configuration of two identical doglegs and a radio-frequency (RF) deflecting cavity in the middle, the EEX technique exchanges the emittances within the whole bunch, which indicates it will only be effective for a low charge beam with ultra-short bunch length at the gun exit.

Subsequent developments of theoretical explorations and experimental investigations on EEX (Kim and Sessler (2006), Xiang (2010), Xiang and Chao (2011), Ruan et al (2011)) expanded its application. The EEX scheme is now beyond its original application of exchanging emittances but also used to tailor a beam’s longitudinal phase space by shaping its initial transverse distribution to meet various requirements.
Figure 1. Schematic layout of the beamline for the proposed scheme.

(Sun et al (2010), Jiang et al (2011), Graves et al (2012), Ha et al (2017), Gao et al (2018)). For instance, EEX has been used to generate a subpicosecond bunch train by modulating its initial transverse distribution with a multi-slit mask (Sun et al (2010)). The duration and separation of the bunch train can be adjusted by customizing the mask. However, the demand in a shorter duration of the bunch train requires a mask with more precise structure. This brings realistic problems.

The difficulties encountered in FEL and bunch shaping have prompted us to find a new EEX solution. Taking benefit of a multidimensional laser-electron manipulation method (Wang et al (2019)), we propose a novel technique in this paper to achieve periodic transverse-to-longitudinal EEX in the scale of optical laser wavelength. With a dual-tilted-laser modulator sandwiched by two identical doglegs, this technique could easily exchange the emittance for the electrons at the zero-crossing phase of each laser cycle. The proposed technique is quite promising for improving the performance of a compact high-gain FEL facility. It can also be used to flexibly tailor a beam’s longitudinal shape in the optical scale.

2. Schematic layout

Figure 1 illustrates the schematic layout of the proposed scheme, where one can find the beamline of two identical doglegs and a dual-tilted-laser modulator. Dual-tilted-laser modulator is a modulator where the interaction of two tilted lasers (i.e., ‘laser 1’ and ‘laser 2’ in figure 1) and the electron beam occurs. Cartesian coordinates are adopted here, where $x$, $y$ and $z$ represent the horizontal, vertical and longitudinal coordinates, respectively. It is assumed that the two seed lasers are incident in the $z–y$ plane and the electric fields of them are both polarized in the $x$ direction. Besides, the two seed lasers have opposite incident angles and a delayed phase of $\pi$ between them. When the electron beam co-propagates with the seed lasers through the modulator, they interact with each other.

For simplicity, we first consider the interaction of one obliquely incident laser with the electron beam in the modulator. The laser incident angle is $\theta$. The electron beam would get energy and angular modulations, simultaneously (Feng et al (2015), Wang et al (2019))

$$\Delta\delta = \Lambda \sin(k_z z + \theta k_y),$$

$$\Delta y' = \Lambda \theta \sin(k_z z + \theta k_y),$$

where $\delta = (\gamma - \gamma_0)/\gamma_0$ is the dimensionless energy deviation with respect to the reference particle, $\gamma$ is the relativistic factor, $y' = dy/dz$ is the vertical divergence, $k_z = 2\pi/\lambda_0$ is the wave number of the seed laser, $\lambda_0$ is the wavelength of the seed laser and $\Lambda = \Delta\gamma/\gamma_0$ is the laser-induced energy modulation amplitude. By injecting another seed laser with an incident angle of $-\theta$ and a phase shift of $\pi$ into the modulator, the modulations become

$$\Delta\delta = 2\Lambda \cos(k_z z) \sin(\theta k_y),$$

$$\Delta y' = 2\Lambda \theta \sin(k_z z) \cos(\theta k_y).$$

When the vertical beam size is relatively small (i.e., $\sigma_y \ll 1/(\theta k_z)$), equations (2a) and (2b) reduce to

$$\Delta\delta = 2k_z \Lambda \theta y \cos(k_z z),$$

$$\Delta y' = 2\Lambda \theta \sin(k_z z).$$

Considering electrons within one seed laser wavelength range (e.g., $-\lambda_0/2 \leq z < \lambda_0/2$), the particles around zero point get almost linear energy and angular chirps $\delta \approx hy$ and $\Delta y' \approx hz$, respectively. Hereafter we will call these particles gold particles, which occupy about 1/8 of the seed laser wavelength in each laser period. Let the coordinate
vector be \((y, y', z, \delta)\). Under thin lens approximation, the corresponding transport matrix of the gold particles for the modulation section can be written as

\[
R_M = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & h & 0 \\
0 & 0 & 1 & 0 \\
h & 0 & 0 & 1
\end{pmatrix}.
\] (4)

This matrix shows a perfect equivalence to the matrix of a RF deflecting cavity, superficially. However, the application scopes of these two matrices are different. The matrix of a RF deflecting cavity normally applies to the whole electron bunch. While in our scheme, the electron beam is periodically modulated at the laser wavelength. The matrix is only applicable to the gold particles with a period of the laser wavelength. Apparently, the dual-tilted-laser modulator enables a finer and smaller-scale electron beam manipulation.

Two identical doglegs are used in our scheme. Under small-angle approximation, the transport matrix of a dogleg is

\[
R_D = \begin{pmatrix}
1 & L & 0 & \eta \\
0 & 1 & 0 & 0 \\
0 & \eta & 1 & \xi \\
0 & 0 & 0 & 1
\end{pmatrix},
\] (5)

where \(L\) and \(\eta\) are the length and dispersion of the dogleg, respectively. These parameters satisfy the relationship \(L\xi = \eta^2\). By setting the dispersion strength \(\eta = -\frac{1}{\pi}\), the transport matrix for the whole beam line is

\[
R = R_D \cdot R_M \cdot R_D = \begin{pmatrix}
0 & 0 & hL & 0 \\
0 & 0 & h & h\xi \\
h\xi & 0 & 0 & 0 \\
h & hL & 0 & 0
\end{pmatrix}.
\] (6)

The elements of the \(2 \times 2\) matrix in the upper left and lower right corners all equal to 0 indicate that complete EEX is achieved (Cornacchia and Emma (2002), Emma et al (2006)). Since the gold particles occupy about 1/8 of the laser wavelength in each period, their longitudinal emittance is usually much smaller than the transverse emittance. Through the transverse-to-longitudinal EEX, the electron beam with a smaller transverse emittance and larger longitudinal emittance could be obtained.

3. Physical mechanism

To better illustrate the feasibility of the proposed scheme, we reveal the physical mechanism behind the proposed technique in this section. The following simulations use an electron beam with energy of 2 GeV, peak current of 3000 A, normalized emittance of 1 mm mrad and relative energy spread of \(5 \times 10^{-5}\). The beta function at the entrance of the first dogleg is assumed to be \(\beta_y = 12\) m. Then the beam size is \(\sigma_y = 55\) \(\mu\)m. The dogleg section has a length of 10 m, dispersion of \(\eta = 9.37\) cm and momentum compaction of \(\xi = 0.88\) mm.

The modulation process was simulated with a three-dimensional (3D) algorithm, which is based on the fundamentals of electrodynamics when considering the appearance of the electromagnetic field of two tilted laser pulses with opposite incident angles and the magnetic field of the modulator (Wang et al (2019)). The modulator is composed of a two-period undulator magnet with 20 cm period. The two laser pulses are assumed round and much longer than the electron beam with a central wavelength of 10.6 \(\mu\)m, peak power of 26 GW, incident angles of 3 mrad and -3 mrad, respectively. To fully cover the electron beam, the laser waist is set to be 1.2 mm. From these parameters, we can conclude that the initial longitudinal emittance for the gold particles is 0.075 mm mrad. We would also know that the vertical beam size \(\sigma_y\) is approximately one-tenth of \(1/(\theta k_s)\). When the electron beam is wiggling in the modulator, the laser–electron interaction will introduce energy and angular modulations on the electron beam simultaneously. The laser-induced vertical divergence and energy deviation are illustrated in figure 2. One can see that the divergence and energy are both periodically modulated and the longitudinal distance between the adjacent modulation peaks is exactly the wavelength of the seed laser. Besides, the energy is approximately linearly modulated in the vertical direction, while the angular modulation does not substantially change with the \(y\) coordinate. Moreover, the coefficient ratio of the induced angle modulation and the energy modulation is equal to \(k_y \lambda\), which is about 160 in this case. These simulation results are consistent with the theoretical predictions in equations (3a) and (3b).

After the electron beam passing through the second dogleg, the EEX process is accomplished. The longitudinal emittance for the gold particles is completely exchanged with the initial vertical emittance and
Figure 2. Distributions of the induced energy deviation (a) and vertical divergence (b) after the laser-electron interaction with a 3D simulation algorithm. The gold particles are distributed between the magenta dashed lines.

Figure 3. The initial longitudinal (a) and vertical (c) phase spaces versus the final phase spaces (b), (d) of the electron beam within one lasing cycle. 'Overlap sep' in (c) and (d) is an introduced linear coordinate to separate the overlap of each color region.

now becomes 1 mm mrad. The vertical emittance now becomes 0.16 mm mrad. The incomplete exchange comes from nonlinearity of sinusoidal modulation. Figure 3 demonstrates the initial and final distributions of the electron beam in the longitudinal and vertical phase spaces, respectively. Particles in one laser cycle are divided into eight regions with different colors. The gold particles are marked with the red dots. As can be seen from figures 3(a) and (b), after the electron beam passes through the proposed beamline, the bunch length of the gold particles is slightly increased while the energy spread is increased by an order of magnitude. Figures 3(c) and (d) show the projected phase space of the particles in each section. Since projected phase space is a direct map of emittance, these figures indicate that, after the exchanger, the gold particles’ vertical emittance is apparently reduced. As for the particles in the nonlinear regions, their longitudinal and vertical emittances all increase with the distance to the center region.

Figure 4 shows the longitudinal current distributions within one laser wavelength. A current spike with FWHM of 5.2 fs appears in the center region, which takes about one-seventh of the laser wavelength. The
Figure 4. Longitudinal current distribution within one laser wavelength. The black dot-dashed line is the density distribution. Solid lines of other colors are statistics for each corresponding color slice.

Figure 5. Correlations of the normalized vertical emittance change ($\Delta \epsilon_y / \epsilon_y$) to seed laser power ($P_s$) and laser position shift ($\Delta y$).

peak current now reads 11 kA, in which the gold particles contribute about 3150 A. Particles in the adjacent nonlinear regions also contribute to the current spike as they are flocked into the center area.

It is found that the optimum wavelength of the incident laser is around 10 μm for this case. For a shorter wavelength, the linear part of the laser field is too small, which causes non-gold particles to easily mix with gold particles, thereby increasing the emittance. For a longer wavelength, the longitudinal emittance will become very large, which will also result in a growth of transverse emittance after the proposed EEX. It is worth pointing out that the optimum wavelength is determined by the parameters of the electron beam.

It should be emphasized, however, that power jitter and transverse position jitter of the seed laser may have impacts on the proposed EEX scheme. Here a demonstration of the correlations of the normalized vertical emittance change ($\Delta \epsilon_y / \epsilon_y$) to seed laser power ($P_s$) and the laser position shift ($\Delta y$) are shown in figure 5. Only gold particles are considered here. One can see that for a 10% emittance growth, the tolerance of laser power is about 5.8% and tolerance of position jitter is up to 83% of the electron beam size in the modulator. These requirements are trivial and it reveals the stability of the proposed scheme.

4. Applications of the proposed technique

The proposed scheme could achieve EEX in optical wavelength, which may open up new opportunities for advanced beam manipulations and diagnostics. In this section, the proposed EEX is applied to the x-ray
FEL and bunch shaping, respectively, to illustrate the practicality of this technique. The parameters used here are the same as those in the previous content.

4.1. A compact x-ray FEL
The performance of a high gain FEL depends critically on the Pierce parameter \( \rho \) (Bonifacio et al (1984)):

\[
\rho = \frac{1}{4\gamma} \left[ \frac{I \lambda_s^2 K^2 [J]_1^2}{I_a \pi^2 \sigma_x \sigma_y} \right],
\]

where \( \gamma \) is the relativistic factor, \( I \) is the beam current, \( I_a = 17 \) kA, \( [J] \) is the Bessel function coupling factor associated with the planar undulator, \( \sigma_x \) and \( \sigma_y \) are the rms horizontal and vertical electron beam sizes in the undulator, respectively. FEL saturation power is approximately \( \rho P_{\text{beam}} \), where \( P_{\text{beam}} \) is the electron beam power. The saturation length is denoted as

\[
L_g = (1 + \varsigma)L_{g0},
\]

where \( L_{g0} = \lambda_s/(4\pi \sqrt{3} \rho) \), \( \varsigma \) is a gain degradation parameter (Yu et al (1990), Xie (1995)). The normalized transverse emittance of the electron beam has to satisfy the condition in order to obtain a relatively short gain length (Hemsing et al (2014))

\[
\epsilon_n < \beta \lambda_s \gamma/(4\pi L_{g0}),
\]

where \( \beta \) is the beta function in the undulator, \( \lambda_s \) is the radiation wavelength. In some scenarios, when short-wavelength radiation is generated by a medium-energy electron beam with relatively low emittance, equation (9) is not satisfied. This is where using the proposed EEX could effectively reduce the transverse emittance in order to reduce the saturation length and increase the saturation power as well.

Figure 6 depicts the superiority of the proposed technique. The comparisons are made between self-amplified spontaneous emission (SASE) (Kondratenko and Saldin (1980), Bonifacio et al (1984)) and the proposed scheme. One can see from figures 6(a) and (b) that the intensity core in vertical phase space of the case with the proposed technique is much higher than the case without it. This high-intensity core with small emittance would dominate the amplification process of FEL radiation (Röhrs et al (2009)).

After the exchanger, the electron beam is injected into the radiator with period length of 2 cm, peak magnet strength of 0.6 T and undulator parameter \( K \) of 1. The average beta function is 6.9 m. Under this radiator setup, the estimated saturation length and power of SASE are about 26 m and 4.1 GW, respectively. One can also get the saturation length and power of SASE with only gold particles, which are 18 m and 6.5 GW, respectively. Considering that there is no obvious boundary between gold particles and non-gold particles, particles in the nonlinear regions with small emittance will also help the radiation amplification processes, thus obtaining a higher radiation power and a shorter saturation length.

Figure 6(c) presents the steady-state simulation results of the radiation processes with the help of GENESIS (Reiche (1999)). Steady-state simulations could give brief verification of the 3D analysis of FEL’s
performance. Simulation results show that, for SASE FEL, the radiation is saturated at approximately 32 m with peak power of about 2.6 GW. The performance of the SASE FEL in the simulation is worse than 3D analysis, which may come from non-perfect betamatch of the electron beam. This shall not affect the relative comparisons of the two schemes. With the proposed technique, 8.4 GW saturation power can be achieved with a 10 m-long undulator. These results show that the proposed technique could effectively increase the saturation power and significantly reduce the required undulator length to reach FEL saturation.

Figure 7. Time dependent simulations: comparisons of SASE (blue dotted line) and proposed scheme (red solid line) of (a) peak power and (b) the temporal FEL profile at $z = 40$ m.

Time dependent simulations are shown in figure 7 to reveal the peak power evolution processes and the temporal FEL profiles. Electron beam in one seed laser wavelength is considered here. The peak power of the proposed scheme could be over 34 GW at 22 m. The pulse duration is about 3.3 fs (FWHM).

4.2. Longitudinal bunch shaping

The proposed technique also holds the ability to be used for longitudinal bunch shaping with the help of a multi-slit mask, similar as the one that has been done with the traditional EEX (Sun et al (2010)), but in optical wavelength scale here. We take the same parameters to simulate this case. Simulation results are briefly illustrated in figure 8. The width and spacing of the slit are assumed to be 22 $\mu$m and 28 $\mu$m, respectively. The electron beam firstly traverses the mask and the transverse phase space becomes like figure 8(a). After passing through the proposed EEX beamline, the electron beam now has a strip structure in the longitudinal phase space, as can be seen from figure 8(b). The current profile in figure 8(b) shows the

Figure 8. Longitudinal bunch shaping with a multi-slit mask. (a) The transverse bunch distribution after the mask. The longitudinal beam distribution (b) and current profile (c) after the proposed beamline.
mask’s influence more intuitively, where several spikes superimpose on the originally smooth current profile. Further research indicates that the degree of aggregation of these spikes can be easily adjusted by the laser-induced modulation amplitude without changing the structure of the slit.

5. Summary and discussion

In conclusion, an easy-to-implement technique for exchanging transverse-to-longitudinal emittance in the optical wavelength is proposed in this paper. Taking advantage of the laser–electron manipulation method, the proposed technique could effectively achieve EEX for the electrons at zero-crossing phase of each lasing cycle. Analytical derivations and simulations demonstrate the validity of this technique. This technique could significantly reduce the required undulator length to reach FEL saturation, and meanwhile, increase the saturation power. Besides, applying this technique to bunch shaping, one can obtain a periodic and multi-spiked current profile in one laser wavelength.

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References

Ackermann W et al 2007 Nat. Photon. 1 336
Allaria E et al 2012 Nat. Photon. 6 699–704
Bonifacio R, Pellegrini C and Narducci L M 1984 Opt. Commun. 50 373–8
Cornacchia M and Emma P 2002 Phys. Rev. Spec. Top. Accel. Beams 5 084001
Emma P, Huang Z, Kim K J and Piot P 2006 Phys. Rev. Spec. Top. Accel. Beams 9 100702
Emma P et al 2010 Nat. Photon. 4 441
Feng C, Xiang D, Deng H, Huang D, Wang D and Zhao Z 2015 Opt. Express 23 14993–5002
Gao Q et al 2018 Phys. Rev. Lett. 120 114801
Graves W, Kärtner F, Moncton D and Piot P 2012 Phys. Rev. Lett. 108 263904
Ha G et al 2017 Phys. Rev. Lett. 118 104801
Hemsing E, Stupakov G, Xiang D and Zholents A 2014 Rev. Mod. Phys. 86 897
Ishikawa T et al 2012 Nat. Photon. 6 510
Jiang B, Power J, Lindberg R, Liu W and Gai W 2011 Phys. Rev. Lett. 106 114801
Kang H S et al 2017 Nat. Photon. 11 708–13
Kim K J and Sessler A 2006 AIP Conf. Proc. 821 115–38
Kondratenko A M and Saldin E L 1980 Part. Accel. 10 207–16
Milne C J et al 2017 Appl. Sci. 7 720
Reiche S 1999 Nucl. Instrum. Meth. A 429 243–8
Röhrs M, Gerth C, Schlarb H, Schmidt B and Schmüser P 2009 Phys. Rev. Spec. Top. Accel. Beams 12 050704
Ruan J et al 2011 Phys. Rev. Lett. 106 244801
Sun Y et al 2010 Phys. Rev. Lett. 105 234801
Wang X, Feng C, Tai C Y, Zeng L and Zhao Z 2019 Phys. Rev. Accel. Beams 22 070701
Xiang D 2010 Phys. Rev. Spec. Top. Accel. Beams 13 010701
Xiang D and Chao A 2011 Phys. Rev. Spec. Top. Accel. Beams 14 114001
Xie M 1995 Proc. Part. Accel. Conf. (IEEE) 1 183–5
Yu I H, Krinsky S and Gluckstern R 1990 Phys. Rev. Lett. 64 3011