Dipole anisotropy in gravitational wave source distribution

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Abstract. Our local motion with respect to the cosmic frame of rest is believed to be dominantly responsible for the observed dipole anisotropy in the Cosmic Microwave Background Radiation (CMBR). We study the effect of this motion on the sky distribution of gravitational wave (GW) sources. We determine the resulting dipole anisotropy in GW source number counts, mass weighted number counts, which we refer to as mass intensity, and mean mass per source. The mass $M$ dependence of the number density $n(M)$ distribution of BBH is taken directly from the data. We also test the anisotropy in the observable mean mass per source along the direction of the CMB dipole. The current data sample is relatively small and consistent with isotropy. The number of sources required for this test is likely to become available in near future.

Keywords: gravitational wave detectors, gravitational waves / experiments, gravitational waves / sources, gravitational waves / theory

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1 Introduction

The standard $\Lambda$CDM model of modern cosmology is based on the assumption of isotropy and homogeneity of the Universe at large distance scales, known as the Cosmological Principle. A testable consequence of this would be an isotropic distribution of galaxies at large distance scales. There are many indications that the Universe may be isotropic at scales $\gtrsim 100$ Mpc [1–9]. The most used source for the measurement of the isotropy of the Universe is the Cosmic Microwave Background Radiation (CMBR). The analysis of CMBR temperature and polarisation fluctuations using Planck observations show small-scale statistical deviations which are consistent with the cosmological principle. The largest temperature anisotropy in the CMBR is the dipole which is interpreted in terms of our local motion with respect to the CMB rest frame and has been measured in refs. [2, 8–12]. Considering the dipole as being solely caused by our motion, the velocity is found to be $(369.82 \pm 0.11 \text{ Km s}^{-1})$ in the direction, $l = 264.021^\circ \pm 0.011^\circ$, $b = 48.253 \pm 0.0005^\circ$ in galactic coordinates [8, 10, 11]. In J2000 equatorial coordinates, the direction parameters are $RA = 167.9^\circ$, $DEC = -6.93^\circ$.

Our local motion is also expected to generate a dipole anisotropy in the large scale structure [13–15]. Such an anisotropy has been observed in the distribution of radio sources [16–21] and in the diffuse X-ray background [22]. These studies find reasonable agreement with the direction of CMBR dipole. However, surprisingly the dipole amplitude in radio observations is found to be much larger than predicted [18–21]. A strong signal of deviation from the CMB prediction is also seen in the infrared observations [23]. A recent study attributes these deviations, along with the observed Hubble tension, to a large scale inhomogeneity in the Universe, arising due to cosmic super-horizon perturbation modes [24]. While these inferences may be speculative, the observation of gravitational waves provides an independent probe to study the isotropy of the Universe and test these claims.

The detection of gravitational waves by Advanced LIGO [25] and Advanced Virgo [26] detectors revealed the existence of a detectable population of coalescing stellar-mass binary black holes (BBHs). Studies of gravitational wave signals from resolved, well-localized BBH mergers present another tool to probe the anisotropy of the Universe. Advanced LIGO and Advanced Virgo released the details of more than 50 compact binary merger events over the observations of the first run (O1), second run (O2) and half of the third run (O3a). These observations are grouped in the gravitational-wave transient catalog GWTC-1 [27], GWTC-2 [28] and GWTC-2.1 [29]. With the completion of the second part of the third observing run (O3b) in collaboration with LIGO, Virgo, and KAGRA (LVK), the total number of
gravitational-wave merger events that includes BBHs, neutron star-black holes (NSBHs) and binary neutron stars (BNS) has been increased to 90. GWTC-3 is the cumulative catalog describing all the gravitational-wave transients found in observing runs up to the end of the third observing run (O3) of LIGO-Virgo detectors [30]. These BBH detections enable determining the astrophysical properties of the BBH population, such as mass and spin distribution of BH populations, the merger rate of such systems and, their evolution across cosmic time [31–33]. Compact binary sources of gravitational wave transient catalog are also used to get some information about the overall sky distribution of events [34, 35]. The sky positions of the GW events are localized by performing coherent analyses of multiple detectors’ data using full Bayesian parameter estimation method [36–38] or rapid Bayesian reconstruction method [39]. During O1 and large part of O2 only the two LIGO detectors (HL) network was operational, causing sky localization areas of the sources to be broader in the range of hundreds to thousands square degrees [27, 40]. The addition of Advanced Virgo to the gravitational-wave detectors network during O2 and O3 has yielded many well-localized sources observed by all three detectors of the Advanced LIGO, Virgo (HLV) network with sky areas lesser than 100 square degrees. Such gravitational-wave sources with substantially small areas can be utilitized to study the isotropy of the Universe [34, 35, 41–43]. The sky localization areas of sources will improve further during the O4 observation of the Advanced LIGO, Virgo, and KAGRA (HLVK) network [44]. With a future five detectors network including the third LIGO detector in India, a significant fraction of gravitational-wave sources will be localized to a few square degrees [45].

In this paper, we determine the dipole anisotropy in the extracted black hole mass distribution arising due to our local motion. We also use LIGO/Virgo data [46] to explore the dipole anisotropy in the distribution of BBH mergers. Such an analysis requires the distribution of number counts per unit mass, which we extract directly from data. We assume a functional form for the distribution law for the total mass of BBH and fit the parameters of the distribution using the data. We determine the expected dipole anisotropy in this mass distribution of BBH caused by our local motion. We also determine the dipole anisotropy in mass intensity and mass per source. Next, we extract sky locations of sources from available posterior distributions of BBH merger events and test the anisotropy in distribution of sources with respect to the CMB direction.

The manuscript is structured as follows. In section 2, we outline the effect of local motion on the observed GW source mass distribution, assuming the power law distribution. We present a generalized mass distribution in section 3 and obtain the fit parameters from the GW data for the various cuts on BBH total mass. We also determine the expected dipole anisotropy in GW sources. In section 4, we investigate the anisotropy in the observed BBH event and then we conclude our results in section 5.

2 The effect of local motion on GW source distribution

We start with the assumption that the GW sources are distributed isotropically in the cosmic frame of rest. Our local motion with respect to this frame will introduce two effects, namely, Doppler boosting and wave aberration. The combined effect is expected to change the observed source count and mass intensity (mass weighted number counts) of GW sources in the sky as a function of the sky position. The effect is similar to that in the case of radio sources [13] except that in the present case we need to use the mass distribution of sources instead of the brightness distribution in the case of radio sources. We essentially
determine the mass distribution of sources, assumed to be isotropic in the cosmic frame of rest, directly from data. The observations are made by imposing a lower limit on the mass. Due to Doppler boosting, the observed source mass gets a direction dependent contribution and furthermore, aberration effect leads to a direction dependent change in the solid angle of observation. These effects together lead to a dipole anisotropy in the number counts as well as in mass intensity, as we explain below.

Consider a binary system of component masses $m_1$ and $m_2$ in the comoving frame. The total mass $M$, chirp mass $\mathcal{M}_c$ and symmetric mass ratio $\eta$ are defined as $M = m_1 + m_2$, $\mathcal{M}_c = M\eta^{3/5}$ and $\eta = m_1m_2/M^2$ respectively. From the observed GW waveform, we can measure the Chirp mass of binary [47],

$$\mathcal{M}_c = \frac{c^3}{G} \left[ \frac{5}{96} \pi^{-8/3} \nu_{\text{obs}}^{-11/3} \nu_{\text{obs}}^{-1} \right]^{3/5}. \quad (2.1)$$

Here, $\mathcal{M}_c$ is the chirp mass observed in the moving local frame and $\nu_{\text{obs}}$ is the GW frequency in the local frame.

We next determine the anisotropy in the distribution of $\mathcal{M}_c$ generated due to our motion with respect to the cosmic frame of rest. The chirp mass is affected by the relative motion of the detector with respect to the source. However, this effect will maintain the isotropy of this mass distribution in the cosmic frame of rest and the anisotropy will be generated purely due to our motion relative to this frame. Furthermore, the chirp mass $\mathcal{M}_c$ and the total mass $M$ are related by $\mathcal{M}_c = M\eta^{3/5}$, where $\eta$ depends on the mass ratio $m_1/m_2$. Assuming that the mass ratio is distributed isotropically in the cosmic frame of rest, our analysis is also directly applicable to the total mass $M$.

Due to the Doppler effect, the shift in frequency of GW is given by [13]

$$\nu_{\text{obs}} = \nu_{\text{rest}}\delta \quad (2.2)$$

where, $\delta \approx (1 + \frac{v}{c}\cos(\theta))$ is the Doppler factor, $v$ is the velocity of observer making an angle $\theta$ with the source. As the observed frequency is Doppler shifted, the mass in the rest frame and the moving frame is related by

$$\left(\nu_{\text{obs}}^{-11/3} \nu_{\text{obs}}^{-1}\right)^{3/5} = \frac{1}{\delta} \left(\nu_{\text{rest}}^{-11/3} \nu_{\text{rest}}^{-1}\right)^{3/5}$$

This leads to,

$$\mathcal{M}_{\text{obs}} = \frac{1}{\delta} \mathcal{M}_{\text{rest}}, \quad (2.3)$$

where $\mathcal{M}_{\text{obs}}$ and $\mathcal{M}_{\text{rest}}$ are chirp masses in the local frame and the cosmic rest frame, respectively. Furthermore, the aberration effect changes the solid angle in the direction of motion, i.e., $d\Omega_{\text{obs}} = d\Omega_{\text{rest}}\delta^{-2}$.

Let us first assume that the number count distribution has a power law BBH total mass $M$ dependence in the cosmic frame of rest, i.e.,

$$\frac{dN}{d\Omega}(M > M_{\text{min}}) = kM^{-\alpha}. \quad (2.4)$$

$M$ will also follow the relation, eq. (2.3), between rest frame and moving frame. Let us denote the differential number count per unit solid angle per unit mass by $n(\theta, \phi, M)$, where $(\theta, \phi)$
are polar angles corresponding to the direction of observation or sky location of the source. Assuming isotropy, we have, in the rest frame

$$n_{\text{rest}}(\theta, \phi, M_{\text{rest}}) \equiv \frac{d^2 N_{\text{rest}}}{d\Omega_{\text{rest}} dM_{\text{rest}}} = k\alpha M^{-\alpha-1}_{\text{rest}}. \quad (2.5)$$

Let $d^2 N_{\text{obs}}$ be the number of sources in bin $d\Omega_{\text{obs}} dM_{\text{obs}}$ and $d^2 N_{\text{rest}}$ be the number of sources in the corresponding bin in the rest frame. We have

$$d^2 N_{\text{obs}} = d^2 N_{\text{rest}} = n_{\text{rest}} d\Omega_{\text{rest}} dM_{\text{rest}} = k\alpha M_{\text{rest}}^{-\alpha-1} \delta^2 d\Omega_{\text{obs}} dM_{\text{rest}}. \quad (2.6)$$

Using the relation for $M_{\text{rest}}$, we obtain,

$$d^2 N_{\text{obs}} = k\alpha \delta^{2-\alpha} M_{\text{obs}}^{-\alpha-1} d\Omega_{\text{obs}} dM_{\text{obs}}. \quad (2.7)$$

Integrating over $M$ from $M_{\text{min}}$ to $\infty$, we get

$$\frac{dN_{\text{obs}}}{d\Omega_{\text{obs}}} = k(M_{\text{min}})^{-\alpha} \delta^{2-\alpha} = \frac{dN_{\text{rest}}}{d\Omega_{\text{rest}}} \delta^{2-\alpha}. \quad (2.8)$$

Hence, the Doppler boosting and aberration, at leading order, will produce a dipole anisotropy in GW source count, given by

$$\vec{D} N(v) = [2 - \alpha](\vec{v}/c). \quad (2.9)$$

It is clear that the effect depends on the mass distribution function due to the dependence on the parameter $\alpha$. The term $2\vec{v}/c$ in eq. (2.9) is due to the aberration. The contribution due to the Doppler effect is directly proportional to $\alpha$ and hence is strongest for steeper distribution functions. However overall this cancels the effect due to aberration and for typical values of $\alpha$, the dipole reduces with an increase in $\alpha$.

### 3 Generalized mass distribution

Let us generalize the distribution of total mass $M$ of the binary which takes into account the deviation from a pure power law behavior. We assume the following functional form of $n(\theta, \phi, M_{\text{rest}})$

$$n(\theta, \phi, M_{\text{rest}}) \equiv \frac{d^2 N_{\text{rest}}}{d\Omega_{\text{rest}} dM_{\text{rest}}} = kM_{\text{rest}}^{-1-f(M_{\text{rest}})}, \quad (3.1)$$

where,

$$f(M_{\text{rest}}) = \beta \exp \left[ -\frac{(M_{\text{rest}} - \mu)^2}{2\sigma^2} \right].$$

We fit this functional form to the binary mass data, as shown in figure 1. We take the values of binary masses directly from the GWTC event portal [48] of the Gravitational Wave Open Science Center (GWOSC) [46]. Here we have used the total masses of binary in the detector frame. We see from the figure that the generalized distribution function provides a good fit to the data. This justifies our use of the distribution function given in eq. (3.1). We find that our functional form of mass distribution is similar to other mass models [49–51] and provides as good a fit as other distribution functions used in the literature.
Figure 1. The fit to the distribution of number counts per unit mass, dN/dM, for the functional form given in eq. (3.1), over the mass range $M > 3M_\odot$. The fit parameters are $\beta = -1.072$, $\mu = 51.695M_\odot$, $\sigma = -147.71M_\odot$.

Following the steps described in section 2, we obtain the generalized form of eq. (2.7), at leading order in $(v/c) \cos \theta$,

$$d^2N_{\text{obs}} = kM^{-1- f(M_{\text{obs}})} \left[ 1 - M_{\text{obs}} \frac{v}{c} \cos \theta \frac{df(M_{\text{obs}})}{dM} \ln (M_{\text{obs}}) \right] \delta^2 - f(M_{\text{obs}}) d\Omega_{\text{obs}} dM_{\text{obs}}. \quad (3.2)$$

Integrating $M_{\text{obs}}$ from $M_{\text{min}}$ to $M_{\text{max}}$, we get

$$\frac{dN_{\text{obs}}}{d\Omega_{\text{obs}}} = k \left( I_1 + (2I_1 + I_2) \frac{v}{c} \cos \theta \right)$$

$$= \frac{dN_{\text{rest}}}{d\Omega_{\text{rest}}} \left( 1 + a \frac{v}{c} \cos \theta \right), \quad (3.3)$$

where,

$$\frac{dN_{\text{rest}}}{d\Omega_{\text{rest}}} = kI_1 \quad (3.4)$$

$$a = 2 + \frac{I_2}{I_1} \quad (3.5)$$

$$I_1 = \int_{M_{\text{min}}}^{M_{\text{max}}} M^{-1- f(M_{\text{obs}})} dM_{\text{obs}} \quad (3.6)$$

$$I_2 = - \int_{M_{\text{min}}}^{M_{\text{max}}} [f(M_{\text{obs}}) + M_{\text{obs}} f'(M_{\text{obs}}) \ln (M_{\text{obs}}) M^{-1- f(M_{\text{obs}})} dM_{\text{obs}}. \quad (3.7)$$

If the sources are distributed isotropically in the cosmic rest frame, the observed amplitude of Dipole anisotropy in source counts will be

$$|D_N| = a \frac{v}{c} \cos \theta. \quad (3.8)$$

Integrating $I_1$ and $I_2$ from $M_{\text{min}} = 3M_\odot$ to $M_{\text{max}} = 400M_\odot$ and using the fit parameters, we obtain

$$I_1 = 179.3, \quad I_2 = -1.56, \quad a = 1.991 \quad (3.9)$$

Taking the local velocity to be, $v=370$ Km/s, we expect that the amplitude of dipole anisotropy will be $|D_N| \approx 2.46 \times 10^{-3}$ for $M > 3M_\odot$. 

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We next generalize this calculation to mass intensity, $M_I$. This is defined as the number counts weighted by mass, i.e.,

$$d^2M_I = Md^2N.$$  \hfill (3.10)

We have,

$$d^2M_{I,obs} = M_{obs}d^2N_{obs} = M_{obs}n_{rest}d\Omega_{rest}dM_{rest}.$$  \hfill (3.11)

Using eq. (3.2) and integrating over $M_{obs}$ from $M_{min}$ to $M_{max}$, we obtain

$$\frac{dM_{I,obs}}{d\Omega_{obs}} = \frac{dM_{I,rest}}{d\Omega_{rest}} \left(1 + b\frac{v}{c}\cos\theta\right),$$  \hfill (3.12)

where,

$$\frac{dM_{I,rest}}{d\Omega_{rest}} = kI_3$$  \hfill (3.13)

$$b = 2 + \frac{I_4}{I_3}$$  \hfill (3.14)

$$I_3 = \int_{M_{min}}^{M_{max}} M_{obs}^{-f(M_{obs})}dM_{obs}$$  \hfill (3.15)

$$I_4 = -\int_{M_{min}}^{M_{max}} [f(M_{obs}) + M_{obs}f'(M_{obs})\ln M_{obs}] M_{obs}^{-f(M_{obs})}dM_{obs}$$  \hfill (3.16)

Therefore, the observed dipole anisotropy in mass intensity is given by

$$D_M = b\frac{v}{c}\cos\theta.$$  \hfill (3.17)

We also consider another observable, mass per source in angular bin of size $d\Omega$, defined as,

$$\frac{dm}{dN} = \frac{(dM_I/d\Omega)}{(dN/d\Omega)}.$$  \hfill (3.18)

Using eq. (3.3) and eq. (3.12), the observed value of mass per source, at leading order in $(v/c)\cos(\theta)$, is given by

$$\frac{dm_{obs}}{dN_{obs}} = \frac{dm_{rest}}{dN_{rest}} \left(1 - d\frac{v}{c}\cos\theta\right),$$  \hfill (3.19)

where $d = (a - b)$. Hence, the observed dipole anisotropy in mass per source will be

$$D_m = -d\frac{v}{c}\cos\theta.$$  \hfill (3.20)

It is quite interesting that the dipole in this parameter is opposite to that in number counts. Hence it may be very useful to test the kinematic origin of the dipole.

The fit parameters corresponding to eq. (3.1) and values of $a$, $b$ and $d$ are given in table 1 for different cuts on the mass of binary. We point out that the parameter $d$ is also quite large. This is in contrast to a pure power law, in which case we do not expect any dipole anisotropy in mass per source in analogy to brightness per source in the case of radio data [21]. In comparing with the results of matter dipole [13] we find that the signal in number counts is reduced by a factor of 2. We use the procedure used in [13] to estimate the number of events required for three-sigma detection of the dipole. The dipole amplitude in
Table 1. The fit parameters corresponding to eq. (3.1) for various cuts on mass and values of $a$, $b$ and $d$ for each set of parameters are given.

| $M(M_\odot) >$ | 3  | 10  | 20  | 30  |
|----------------|----|-----|-----|-----|
| $\beta$       | -1.072 | -1.074 | -1.106 | -1.082 |
| $\mu$         | 51.695 | 60.531 | 27.771 | 87.903 |
| $\sigma$      | -147.71 | -134.69 | -167.52 | -78.874 |
| $a$           | 1.991 | 1.95 | 1.846 | 1.879 |
| $b$           | 1.039 | 1.028 | 1.01 | 0.991 |
| $d$           | 0.952 | 0.922 | 0.836 | 0.887 |

The number counts is expected to be $2.46 \times 10^{-3}$. The number of sources required within about a steradian of the dipole axis is given by

$$3\sqrt{2/n} \approx 4.92 \times 10^{-3},$$

(3.21)

Hence for a three-sigma detection, we would require approximately $7 \times 10^5$ of GW sources. This may be within reach of future next-generation detectors, such as Einstein Telescope and Cosmic Explorer [52–54]. These detectors might observe $\sim 10^5$ GWs per year coming from BBHs and BNSs mergers with signal to noise ratio threshold of more than 9, as discussed in [41]. These number of GWs events are estimated for a redshift shell of $z = 2$. Therefore, it may take $\sim 7$ years of observation to detect the $3\sigma$ level dipole anisotropy in source number counts. In any case, given that this effect will also be observed in galaxy surveys, even a one-sigma detection, requiring an order of magnitude fewer sources, would be very useful to establish consistency. We next determine the number of sources needed to observe the dipole in mass per source. This is dependent on the standard deviation in the mass distribution. Given that the velocity is about 370 Km/sec and $d = 0.95$ we expect the dipole amplitude to be approximately $10^{-3}$. We find that a three-sigma detection in this case requires about 4 million sources. It may take $\sim 40$ years of observation to detect the $3\sigma$ level dipole anisotropy in mass per source.

4 Data analysis

The currently available GW data is rather limited and we do not expect to obtain a signal of local velocity in this data. The resulting dipole anisotropy signal is of similar strength as in radio sources and hence we require roughly $8 \times 10^5$ sources to extract a signal at 3 sigma significance [13]. However, the required signal is within the reach of gravitational wave detectors in near future. In order to illustrate the procedure we extract the dipole in the mass per source. We use this observable rather than number counts since it is not affected by several biases which may be present in such a small sample. For example, the number counts would be strongly affected if the sky is not uniformly sampled or the detector efficiency depends on the direction of observation. These will not significantly affect the observable mass per source.

The posterior samples containing information about the parameters of events were taken from the LIGO/Virgo data released via the Gravitational-Wave Open Science Center (GWOSC) [46]. We extracted the mass and location of each source using the given probability distribution. For few sources, the probability distribution of their sky location (RA,
Figure 2. Histogram of RA of the skymap of the event GW191219_163120 distributed in a bimodal fashion and the fitted two-component mixture model (eq. (4.1)) on RA samples with parameters shown in the figure.

DEC) are uni-modal and are completely in either the forward or backward hemisphere with respect to the CMB dipole direction \((RA = 167.9^o, DEC = -6.93^o)\). Other sources have multi-modal probability distribution of sky location and spread across both hemispheres. As CMB dipole is near to the Celestial equator, so we assume that the position of any event with respect to the CMB dipole direction will only be decided by the angle RA of that event. To estimate the probability of occurrence of these sources in any one of the hemispheres we use the idea of finite mixture modeling for the probability distribution of RA.

Let \(f(x)\) be the probability distribution of RA, we can write it as

\[
f(x) = pf_1(x) + (1-p)f_2(x)
\]

where we have taken \(f_1\) and \(f_2\) both Gaussian and there would be 5 parameters to estimate from the data, \(p, \sigma_1, \sigma_2, \mu_1,\) and \(\mu_2,\) defined as mixture proportion, the standard deviations and means respectively for the distributions \(f_1\) and \(f_2.\) One such fitting is shown in figure 2 for the event GW191219_163120. It may be possible to improve the fit using a different functional form. This may improve the extracted position coordinates of the sources and hence lead to a better determination of the dipole. However, the dominant source of error is the intrinsic fluctuations in mass values and the error in mass values or positions is comparatively negligible. For example, the error in the mean value of mass per source in any region of the sky is dominated by the spread in mass values rather than the error in individual masses. Similarly, the position errors are relatively small and negligible after averaging over large number of sources. Hence, we find that our analysis is sufficiently reliable and postpone further refinements to future research.

From figure 2, we can say that probability of the event at \(RA_1 = 54\) is 0.55 and the probability of the event at \(RA_2 = 221\) is 0.45. To get an estimate of the declination angle (DEC) of the event at \(RA_1\) and \(RA_2,\) we look into the skymap of the given event and
approximated the corresponding angles as $DEC_1 = -35$ and $DEC_2 = 35$ directly from the given skymap, as shown in figure 3.

We have done the same analysis for all the events and determined the angles RA, DEC and corresponding probabilities of each event. The resulting skymap of all events weighted with their probability is shown in figure 4. Here colorbar represents the probability of the event.

We determine the cosine of the angle ($\cos \theta$) between the unit vector along the CMB dipole and the unit vector along the direction of each source. In the forward hemisphere with respect to CMB dipole, $\cos \theta$ will be positive and in the backward hemisphere, $\cos \theta$ will be negative. We use the masses of events in the detector frame. The results are shown in table 2.

In order to determine if the sources are distributed isotropically we determine the observable mean value of the mass per source in each hemisphere. We define mean mass per
source $m$ as:

$$m = \frac{\sum_{i=1}^{N} M_i p_i}{\sum_{i=1}^{N} p_i}. \quad (4.2)$$

where $M_i$ is the observed mass of the source and $p_i$ the probability that the source lies in a particular hemisphere. From data we find that the mean mass per source in forward hemisphere is $86.19^{+6.44}_{-2.35} M_\odot$ whereas in backward hemisphere it is $84.63^{+3.04}_{-3.0} M_\odot$. We find that the difference between the two is not statistically significant and hence the data is consistent with isotropy.

We perform a more detailed analysis by determining the best fit value of the dipole amplitude. We test the following model for the mass distribution on sky,

$$M_{th}(\theta, \phi) = M_0 + \delta M \cos \theta \quad (4.3)$$

where $\theta$ is the polar angle in a frame with $z$-axis pointing along the CMB dipole axis and $\phi$ being the corresponding azimuthal angle. Here $M_0$ is the monopole term and $\delta M$ is the dipole amplitude. We point out that if we do a direct $\chi^2$ fit to data using the error values in the masses, the fit keeping only the monopole and dipole terms is not found to be very good due to the large dispersion in the mass values. The fit is expected to improve as we collect larger data samples. Alternatively, we may need to include higher multipoles which can be done using the formalism developed in [55]. We determine the two parameters $M_0$ and $\delta M$ by minimizing the weighted least square difference. This is defined as

$$\Delta = \frac{\sum_{i=1}^{N} p_i (M_i - M_0 - \delta M \cos \theta)^2}{\sum_{i=1}^{N} p_i}. \quad (4.4)$$

The best-fit value of parameters is found to be: $M_0 = 85.3 M_\odot$ and $\delta M = 6.2 M_\odot$. We point out that the sample has dispersion which is much larger than the error bars on individual masses. Hence we ignore the errors in masses while determining the best-fit values. We determine the statistical significance of the dipole by simulations. We generate 100 random samples by randomly permuting the mass values among different sources while keeping their angular positions fixed. This method is useful since it maintains the observed sky distribution of sources and also preserves the observed mass distribution. This is expected to be more reliable in comparison to an alternate procedure in which the random samples are generated by sampling an isotropic mass distribution. We determine the best-fit values of each of these samples and determine the probability or p-value that a random isotropic sample can generate the dipole with an amplitude equal to or larger than that seen in the data. The result is shown in figure 5.

We find that the p-value of our statistical test is 0.3. This shows that the dipole seen in data is not significant. Hence we conclude that the data is consistent with isotropy.

5 Conclusions

The search for dipole anisotropy in large scale structures is turning out to be very interesting due to results obtained by radio [18–21] and infrared surveys [23]. Both show a significant
Figure 5. Distribution of dipole amplitude obtained from random samples. The blue line indicates the amplitude obtained from the real data ($\delta M/M_0 = 0.072$).

deviation from the amplitude expected on the basis of the CMB dipole while the direction is found to be in reasonable agreement. Such a trend has raised the possibility that the Universe might be intrinsically anisotropic and has led to considerable theoretical effort [56]. The GW observations open up a new and independent domain to test this phenomenon. Here we have determined the signal of dipole anisotropy in the GW observations predicted due to our velocity with respect to the cosmic frame of rest. We considered three different observables, the number counts, the number counts weighted by mass (also called mass intensity), and the mean mass per source. We assume a functional form for the mass dependence of the number count distribution and extract the parameters of this function directly from data. We find that all three observables acquire a significant dipole anisotropy due to our local motion, which can be tested reliably once a sufficiently large sample of GW becomes available. We also note that the peculiar velocities of the galaxies and the binaries within them might affect the dipole detection probability by widening the distribution of inferred masses. Hence, the required number of detections may be larger than estimated for obtaining a 3$\sigma$ signal. It may be useful to consider the contribution of such environments in a future study to assess their contribution.

In order to illustrate our procedure, we have also analyzed the available data in order to determine whether the mean mass per source shows any deviation in the two opposite hemispheres in the direction of the CMB dipole. We find that the sample is consistent with isotropy. The data sample is currently very small and considerably more data is required to determine the signal of local motion. The signal is within the reach of future gravitational wave observatories and we look forward to a more detailed study of this phenomenon with future data.
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| S.No | Event               | M_{det}(M_\odot) | RA(degree) | DEC(degree) | p   | \cos \theta |
|------|---------------------|------------------|------------|-------------|-----|--------------|
| 1    | GW150914_095045     | 70.92_{-3.37}^{+4.57} | 141        | -67         | 1   | 0.45697237   |
| 2    | GW151012_095443     | 46.4_{-4.5}^{+4.4}  | 59.3       | 41          | 0.51| -0.31812088  |
| 3    | GW151012_095443     | 46.4_{-4.5}^{+4.4}  | 244        | -40         | 0.49| 0.26023751   |
| 4    | GW151226_033853     | 23.7_{-1.4}^{+2.3}  | 53         | 45          | 0.4 | -0.380593    |
| 5    | GW151226_033853     | 23.7_{-1.4}^{+2.3}  | 208        | -38         | 0.6 | 0.6726463    |
| 6    | GW170104_101158     | 60.4_{-4}^{+1}      | 345        | -22         | 0.1 | -0.87403264  |
| 7    | GW170104_101158     | 60.4_{-4}^{+1}      | 130        | 45          | 0.9 | 0.46857325   |
| 8    | GW170608_020116     | 19.8_{-0.33}^{+0.28} | 122        | 34          | 1   | 0.5052529    |
| 9    | GW170729_185629     | 123_{-21}^{+21}     | 302        | -69         | 0.9 | -0.13492819  |
| 10   | GW170729_185629     | 123_{-21}^{+21}     | 156        | 50          | 0.1 | 0.53195002   |
| 11   | GW170809_082821     | 70.6_{-4.5}^{+6.0}  | 15         | -28         | 1   | -0.72362404  |
| 12   | GW170814_103043     | 68.8_{-3}^{+3}      | 47         | -45         | 1   | -0.27515847  |
| 13   | GW170817             | 2.75_{-0.016}^{+0.016} | 197       | -23         | 1   | 0.84557982   |
| 14   | GW170818_022509     | 75.76_{-6}^{+6}     | 341        | 22          | 1   | -0.95894268  |
| 15   | GW170823_131358     | 90.8_{-9}^{+11}     | 54         | 30          | 0.37| -0.40862792  |
| 16   | GW170823_131358     | 90.8_{-9}^{+11}     | 252        | -35         | 0.63| 0.15279336   |
| 17   | GW190403_051519     | 236.18_{-54}^{+40} | 334        | -50         | 0.25| -0.52697769  |
| 18   | GW190403_051519     | 236.18_{-54}^{+40} | 191        | 35          | 0.75| 0.67876321   |
| 19   | GW190408_181802     | 55.6_{-3.83}^{+3.42} | 349        | 53          | 1   | -0.69366892  |
| 20   | GW190412             | 42.15_{-4.51}^{+3.6} | 218        | 35          | 1   | 0.45240024   |
| 21   | GW190413_052954     | 90.84_{-13}^{+15}   | 66.3       | -44         | 0.76| -0.05977149  |
| 22   | GW190413_052954     | 90.84_{-13}^{+15}   | 192        | 75          | 0.24| 0.11798748   |
| 23   | GW190413_134308     | 135_{-17.75}^{+17.73} | 154       | -30         | 1   | 0.8948518    |
| 24   | GW190421_213856     | 108.9_{-12.44}^{+15.31} | 200       | -47         | 1   | 0.66175767   |
|     | GW190426_190642   | 312.16^{+61}_{-44} | 57     | -13 | 0.8 | -0.31791357 |
|-----|------------------|---------------------|--------|-----|-----|------------|
| 25  | GW190426_190642   | 312.16^{+61}_{-44} | 254    | 23  | 0.2 | 0.01500671 |
| 26  | GW190503_185404   | 91.63^{+11.26}_{-12.13} | 95     | -50 | 1   | 0.280053   |
| 27  | GW190512_180714   | 45.31^{+2.88}_{-2.8} | 250    | -27 | 1   | 0.17634627 |
| 28  | GW190513_205428   | 73.6^{+12.57}_{-6.72} | 50     | 42  | 0.9 | -0.42593418|
| 29  | GW190517_055101   | 85.4^{+7}_{-7.31}   | 233    | -45 | 1   | 0.38085937 |
| 30  | GW190519_153544   | 154.8^{+18}_{-18}   | 353    | 45  | 0.7 | -0.78447906|
| 31  | GW190519_153544   | 154.8^{+18}_{-18}   | 183    | -16 | 0.3 | 0.9544912  |
| 32  | GW190521_030229   | 243.5^{+58}_{-36}   | 352    | -40 | 0.48| -0.68094522|
| 33  | GW190521_030229   | 243.5^{+58}_{-36}   | 192    | 30  | 0.52| 0.72443384 |
| 34  | GW190527_092055   | 84.43^{+5.92}_{-10.49}| 301    | -30 | 0.64| -0.52708111|
| 35  | GW190527_092055   | 84.43^{+5.92}_{-10.49}| 42     | -30 | 0.36| -0.44377511|
| 36  | GW190602_175927   | 171.9^{+22.64}_{-20.58}| 71     | -35 | 0.6 | -0.02845585|
| 37  | GW190602_175927   | 171.9^{+22.64}_{-20.58}| 81     | -35 | 0.4 | 0.11318094 |
| 38  | GW190620_030421   | 140.5^{+20}_{-22}   | 40     | -18 | 0.25| -0.54266866|
| 39  | GW190620_030421   | 140.5^{+20}_{-22}   | 252    | 30  | 0.75| 0.02804227 |
| 40  | GW190630_185205   | 69.7^{+2}_{-4}      | 340    | -20 | 0.79| -0.88270748|
| 41  | GW190630_185205   | 69.7^{+2}_{-4}      | 166    | 15  | 0.21| 0.92711366 |
| 42  | GW190701_203306   | 129.6^{+16.61}_{-14.75}| 40     | -7  | 1   | -0.59054774|
| 43  | GW190706_222641   | 180.3^{+22.4}_{-27.8}| 336    | -40 | 0.08| -0.6654857 |
| 44  | GW190706_222641   | 180.3^{+22.4}_{-27.8}| 148    | 27  | 0.92| 0.77690516 |
| 45  | GW190707_093326   | 23.36^{+1.54}_{-0.69} | 310    | 50  | 0.43| -0.59593266|
| 46  | GW190707_093326   | 23.36^{+1.54}_{-0.69} | 150    | -30 | 0.57| 0.87841258 |
| 47  | GW190708_232457   | 37.12^{+2.83}_{-1.67} | 350    | -30 | 0.34| -0.79879279|
| 48  | GW190708_232457   | 37.12^{+2.83}_{-1.67} | 170    | 15  | 0.66| 0.92699865 |
| 49  | GW190719_215514   | 91.43^{+7.4}_{-15}  | 340    | -30 | 0.4  | 0.79121117 |
| 50  | GW190719_215514   | 91.43^{+7.4}_{-15}  | 150    | 30  | 0.6  | 0.75775595 |
| 51  | GW190720_000836   | 25.3^{+1.28}_{-1.52} | 300    | 36  | 1   | -0.6093442 |
| 52  | GW190725_174728   | 21.83^{+0.31}_{-0.29} | 284    | -10 | 0.8 | -0.40913847|
| 53  | GW190725_174728   | 21.83^{+0.31}_{-0.29} | 81     | 20  | 0.2 | 0.00917921 |
| 54  | GW190727_060333   | 104.6^{+13}_{-11}   | 352    | 50  | 0.46| -0.72888692|
| 55  | GW190727_060333   | 104.6^{+13}_{-11}   | 142    | -70 | 0.54| 0.4187993  |
| 56  | GW190728_064510   | 24.5^{+0.73}_{-0.73} | 315    | 10  | 0.7  | -0.84177514|
| 57  | GW190728_064510   | 24.5^{+0.73}_{-0.73} | 94     | -60 | 0.3  | 0.24213604 |
| 58  | GW190731_140936   | 109.4^{+14.76}_{-14.32} | 66     | -70 | 0.36| 0.0433694 |
| 59  | GW190731_140936   | 109.4^{+14.76}_{-14.32} | 188    | -40 | 0.64| 0.79168889 |
| 60  | GW190803_022701   | 100^{+14.14}_{-11.97} | 94     | 32  | 1   | 0.16951977 |
| Event ID        | Event ID        | Sensitivity | Flare | Duration | Time | Distance | Probability | Confidence | Distance Error | Confidence Error |
|-----------------|-----------------|-------------|-------|----------|------|----------|-------------|------------|----------------|-----------------|
| GW190805_211137| GW190805_211137| 147.36 ± 2.14 | 350   | -20      | 0.9  | -0.89093402 |
| GW190814         |                 | 27.21 ± 1.32  | 13    | -25      | 1    | -0.7637364  |
| GW190828_063405  |                 | 78.57 ± 6.5   | 327   | 30       | 0.2  | -0.86346248 |
| GW191129_134029  |                 | 94.93 ± 13    | 134   | -30      | 0.53 | -0.77388862 |
| GW191109_010717  |                 | 23.26 ± 10.45 | 319   | 55       | 1    | -0.59731342 |
| GW191113_071753  |                 | 43.36 ± 13.45 | 55    | 15       | 0.4  | -0.40434715 |
| GW191113_071753  |                 | 43.36 ± 13.45 | 228   | -40      | 0.6  | 0.45665057  |
| GW191126_115259  |                 | 62.76 ± 19.25 | 323   | 32       | 0.46 | -0.61220318 |
| GW191126_115259  |                 | 62.76 ± 19.25 | 284   | 15       | 0.32 | -0.45307229 |
| GW191124_110529  |                 | 62.76 ± 19.25 | 132   | 15       | 0.68 | 0.74549564  |
| GW191215_223052  |                 | 58.41 ± 8.36  | 326   | 20       | 0.47 | -0.90677818 |
| GW191215_223052  |                 | 58.41 ± 8.36  | 116   | -45      | 0.53 | 0.51843982  |
| GW191216_213338  |                 | 21.22 ± 0.66  | 320   | 32       | 1    | -0.80793589 |
| GW191219_163120  |                 | 36.03 ± 2.61  | 54    | -35      | 0.55 | -0.2602422  |
| GW191219_163120  |                 | 36.03 ± 2.61  | 221   | 35       | 0.45 | 0.41903645  |
| GW191222_033537  |                 | 119.21 ± 15.78| 50    | 30       | 0.1  | -0.46260687 |
| GW191222_033537  |                 | 119.21 ± 15.78| 209   | -40      | 0.9  | 0.65602023  |
Table 2. The masses of GW events in detector frame, extracted location of GW sources, probability(p) of occurrence of event in forward hemisphere (cos $\theta > 0$) or in backward hemisphere (cos $\theta < 0$), and direction with respect to CMB dipole.
References

[1] COBE collaboration, Structure in the COBE differential microwave radiometer first year maps, *Astrophys. J. Lett.* **396** (1992) L1 [SPIRE].

[2] D.J. Fixsen et al., The cosmic microwave background spectrum from the full COBE FIRAS data set, *Astrophys. J.* **473** (1996) 576 [astro-ph/9605054] [SPIRE].

[3] K.K.S. Wu, O. Lahav and M.J. Rees, The large-scale smoothness of the universe, *Nature* **397** (1999) 225 [astro-ph/9804062] [SPIRE].

[4] C. Blake and J. Wall, Detection of the velocity dipole in the radio galaxies of the NRAO VLA sky survey, *Nature* **416** (2002) 150 [astro-ph/0203385] [SPIRE].

[5] C. Marinoni, J. Bel and A. Buzzi, The scale of cosmic isotropy, *JCAP* **10** (2012) 036 [arXiv:1205.3309] [SPIRE].

[6] C.A. Meegan et al., Spatial distribution of gamma-ray bursts observed by BATSE, *Nature* **355** (1992) 143 [SPIRE].

[7] C. Scharf et al., Evidence for X-ray emission from a large scale filament of galaxies?, *Astrophys. J. Lett.* **528** (2000) L73 [astro-ph/9911277] [SPIRE].

[8] PLANCK collaboration, Planck 2018 results. I. Overview and the cosmological legacy of Planck, *Astron. Astrophys.* **641** (2020) A1 [arXiv:1807.06205] [SPIRE].

[9] PLANCK collaboration, Planck 2013 results. XXVII. Doppler boosting of the CMB: eppur si muove, *Astron. Astrophys.* **571** (2014) A27 [arXiv:1303.5087] [SPIRE].

[10] A. Kogut et al., Dipole anisotropy in the COBE DMR first year sky maps, *Astrophys. J.* **419** (1993) 1 [astro-ph/9312056] [SPIRE].

[11] WMAP collaboration, Five-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: data processing, sky maps, and basic results, *Astrophys. J. Suppl.* **180** (2009) 225 [arXiv:0803.0732] [SPIRE].

[12] PLANCK collaboration, Planck 2018 results. VII. Isotropy and statistics of the CMB, *Astron. Astrophys.* **641** (2020) A7 [arXiv:1906.02552] [SPIRE].

[13] G.F.R. Ellis and J.E. Baldwin, On the expected anisotropy of radio source counts, *Mon. Not. Roy. Astron. Soc.* **206** (1984) 377.

[14] A. Baleiśis, O. Lahav, A.J. Loan and J.V. Wall, Searching for large scale structure in deep radio surveys, *Mon. Not. Roy. Astron. Soc.* **297** (1998) 545 [astro-ph/9709205] [SPIRE].

[15] J.J. Condon et al., The NRAO VLA sky survey, *Astron. J.* **115** (1998) 1693 [SPIRE].

[16] C. Blake and J. Wall, Detection of the velocity dipole in the radio galaxies of the NRAO VLA sky survey, *Nature* **416** (2002) 150 [astro-ph/0203385] [SPIRE].

[17] F. Crawford, Detecting the cosmic dipole anisotropy in large-scale radio surveys, *Astrophys. J.* **692** (2009) 887 [arXiv:0810.4520] [SPIRE].

[18] A.K. Singal, Large peculiar motion of the solar system from the dipole anisotropy in sky brightness due to distant radio sources, *Astrophys. J. Lett.* **742** (2011) L23 [arXiv:1110.6260] [SPIRE].

[19] C. Gibelyou and D. Huterer, Dipoles in the sky, *Mon. Not. Roy. Astron. Soc.* **427** (2012) 1994 [arXiv:1205.6476] [SPIRE].

[20] M. Rubart and D.J. Schwarz, Cosmic radio dipole from NVSS and WENSS, *Astron. Astrophys.* **555** (2013) A117 [arXiv:1301.5559] [SPIRE].

[21] P. Tiwari et al., Dipole anisotropy in sky brightness and source count distribution in radio NVSS data, *Astropart. Phys.* **61** (2014) 1 [arXiv:1307.1947] [SPIRE].
[22] S.P. Boughn, R.G. Crittenden and G.P. Koehrsen, The large scale structure of the X-ray background and its cosmological implications, Astrophys. J. 580 (2002) 672 [astro-ph/0208153] [inSPIRE].

[23] N.J. Secrest et al., A test of the cosmological principle with quasars, Astrophys. J. Lett. 908 (2021) L51 [arXiv:2009.14826] [inSPIRE].

[24] P. Tiwari, R. Kothari and P. Jain, Superhorizon perturbations: a possible explanation of the Hubble-Lemaître tension and the large-scale anisotropy of the universe, Astrophys. J. Lett. 924 (2022) L36 [arXiv:2111.02685] [inSPIRE].

[25] LIGO Scientific collaboration, Advanced LIGO, Class. Quant. Grav. 32 (2015) 074001 [arXiv:1411.4547] [inSPIRE].

[26] VIRGO collaboration, Advanced Virgo: a second-generation interferometric gravitational wave detector, Class. Quant. Grav. 32 (2015) 024001 [arXiv:1408.3978] [inSPIRE].

[27] LIGO Scientific and Virgo collaborations, GWTC-1: a gravitational-wave transient catalog of compact binary mergers observed by LIGO and Virgo during the first and second observing runs, Phys. Rev. X 9 (2019) 031040 [arXiv:1811.12907] [inSPIRE].

[28] LIGO Scientific and Virgo collaborations, GWTC-2: compact binary coalescences observed by LIGO and Virgo during the first half of the third observing run, Phys. Rev. X 11 (2021) 021053 [arXiv:2010.14527] [inSPIRE].

[29] LIGO Scientific and Virgo collaborations, GWTC-2.1: deep extended catalog of compact binary coalescences observed by LIGO and Virgo during the first half of the third observing run, arXiv:2108.01045 [inSPIRE].

[30] LIGO Scientific et al. collaborations, GWTC-3: compact binary coalescences observed by LIGO and Virgo during the second part of the third observing run, arXiv:2111.03606 [inSPIRE].

[31] LIGO Scientific and Virgo collaborations, Binary black hole population properties inferred from the first and second observing runs of advanced LIGO and advanced Virgo, Astrophys. J. Lett. 882 (2019) L24 [arXiv:1811.12940] [inSPIRE].

[32] LIGO Scientific and Virgo collaborations, Population properties of compact objects from the second LIGO-Virgo gravitational-wave transient catalog, Astrophys. J. Lett. 913 (2021) L7 [arXiv:2010.14533] [inSPIRE].

[33] KAGRA et al. collaborations, Population of merging compact binaries inferred using gravitational waves through GWTC-3, Phys. Rev. X 13 (2023) 011048 [arXiv:2111.03634] [inSPIRE].

[34] E. Payne, S. Banagiri, P. Lasky and E. Thrane, Searching for anisotropy in the distribution of binary black hole mergers, Phys. Rev. D 102 (2020) 102004 [arXiv:2006.11957] [inSPIRE].

[35] R. Stiskalek, J. Veitch and C. Messenger, Are stellar-mass binary black hole mergers isotropically distributed?, Mon. Not. Roy. Astron. Soc. 501 (2021) 970 [arXiv:2003.02919] [inSPIRE].

[36] J. Veitch et al., Parameter estimation for compact binaries with ground-based gravitational-wave observations using the LALInference software library, Phys. Rev. D 91 (2015) 042003 [arXiv:1409.7215] [inSPIRE].

[37] G. Ashton et al., BILBY: a user-friendly Bayesian inference library for gravitational-wave astronomy, Astrophys. J. Suppl. 241 (2019) 27 [arXiv:1811.02042] [inSPIRE].

[38] I.M. Romero-Shaw et al., Bayesian inference for compact binary coalescences with BILBY: validation and application to the first LIGO-Virgo gravitational-wave transient catalogue, Mon. Not. Roy. Astron. Soc. 499 (2020) 3295 [arXiv:2006.00714] [inSPIRE].
[39] L.P. Singer and L.R. Price, *Rapid Bayesian position reconstruction for gravitational-wave transients*, Phys. Rev. D 93 (2016) 024013 [arXiv:1508.03634] [inSPIRE].

[40] LIGO Scientific and Virgo collaborations, *Binary black hole mergers in the first advanced LIGO observing run*, Phys. Rev. X 6 (2016) 041015 [Erratum ibid. 8 (2018) 039903] [arXiv:1606.04856] [inSPIRE].

[41] S. Mastrogiavanni, C. Bonvin, G. Cusin and S. Foffa, *Detection and estimation of the cosmic dipole with the Einstein telescope and cosmic explorer*, Mon. Not. Roy. Astron. Soc. 521 (2023) 984 [arXiv:2209.11658] [inSPIRE].

[42] R. Essick et al., *Anisotropy measurement with gravitational wave observations*, Phys. Rev. D 107 (2023) 043016 [arXiv:2207.05792] [inSPIRE].

[43] M. Cavaglia and A. Modi, *Two-dimensional correlation function of binary black hole coalescences*, Universe 6 (2020) 93 [arXiv:2005.06004] [inSPIRE].

[44] KAGRA et al. collaborations, *Prospects for observing and localizing gravitational-wave transients with advanced LIGO, advanced Virgo and KAGRA*, Living Rev. Rel. 21 (2018) 3 [arXiv:1304.0670] [inSPIRE].

[45] M. Saleem et al., *The science case for LIGO-India*, Class. Quant. Grav. 39 (2022) 025004 [arXiv:2105.01716] [inSPIRE].

[46] LIGO Scientific and Virgo collaborations, *Open data from the first and second observing runs of advanced LIGO and advanced Virgo*, SoftwareX 13 (2021) 100658 [arXiv:1912.11716] [inSPIRE].

[47] LIGO Scientific and Virgo collaborations, *Observation of gravitational waves from a binary black hole merger*, Phys. Rev. Lett. 116 (2016) 061102 [arXiv:1602.03837] [inSPIRE].

[48] LIGO Scientific et al. collaborations, *GWOSC event portal snapshots*, Zenodo, October 2022 [DOI:10.5281/ZENODO.7249086].

[49] C. Talbot and E. Thrane, *Measuring the binary black hole mass spectrum with an astrophysically motivated parameterization*, Astrophys. J. 856 (2018) 173 [arXiv:1801.02699] [inSPIRE].

[50] N. Farrow, X.-J. Zhu and E. Thrane, *The mass distribution of galactic double neutron stars*, Astrophys. J. 876 (2019) 18 [arXiv:1902.03300] [inSPIRE].

[51] M. Fishbach, R. Essick and D.E. Holz, *Does matter matter? Using the mass distribution to distinguish neutron stars and black holes*, Astrophys. J. Lett. 899 (2020) L8 [arXiv:2006.13178] [inSPIRE].

[52] D. Reitze et al., *Cosmic explorer: the U.S. contribution to gravitational-wave astronomy beyond LIGO*, Bull. Am. Astron. Soc. 51 (2019) 035 [arXiv:1907.04833] [inSPIRE].

[53] M. Punturo et al., *The Einstein telescope: a third-generation gravitational wave observatory*, Class. Quant. Grav. 27 (2010) 194002 [inSPIRE].

[54] S. Dwyer et al., *Gravitational wave detector with cosmological reach*, Phys. Rev. D 91 (2015) 082001 [arXiv:1410.0612] [inSPIRE].

[55] R. Kothari et al., *A study of dipolar signal in distant quasars with various observables*, arXiv:2208.14397 [inSPIRE].

[56] L. Perivolaropoulos and F. Skara, *Challenges for ΛCDM: an update*, New Astron. Rev. 95 (2022) 101659 [arXiv:2105.05208] [inSPIRE].