Abstract

We study the journey planning problem for multimodal networks consisting of public transit and a non-schedule-based transfer mode (e.g., walking, bicycle, e-scooter). So far, all efficient algorithms for this problem either restrict usage of the transfer mode or Pareto-optimize only two criteria: arrival time and the number of used public transit trips. However, we show that both limitations must be lifted in order to obtain high-quality solutions. In particular, the time spent using the (unrestricted) transfer mode must be optimized as a third criterion. We present McTB, the first algorithm that optimizes three criteria efficiently by avoiding costly data structures for maintaining Pareto sets. To enable unlimited transfers, we combine it with a three-criteria extension of the ULTRA \cite{3,16} preprocessing technique. Furthermore, since full Pareto sets become impractically large for more than two criteria, we adapt an approach by Dellinger et al. \cite{5} to restrict the Pareto set in a methodical manner. Extensive experiments on real-world data show that our algorithms are fast enough for interactive queries even on large country-sized networks. Compared to the state of the art for multicriteria multimodal journey planning, MCR \cite{6}, we achieve a speedup of up to 80.

1 Introduction

With the emergence of new, flexible transport options such as rental bikes and e-scooters, the ability to plan multimodal journeys that combine different transportation modes is becoming more important than ever. In this work, we consider bimodal transportation networks, consisting of public transit and a transfer graph which represents a secondary, non-schedule-based transfer mode (e.g., walking, bicycle, e-scooter). While many efficient journey planning algorithms have been developed for both road and public transit networks \cite{2}, solving the combined multimodal problem is more challenging. Fully multimodal algorithms such as MCR \cite{6} are slow because they explore the transfer graph with costly Dijkstra searches. Some public transit algorithms support walking as a transfer mode \cite{7,8,18,10,9}, but only in a limited capacity, for example by requiring the transfer graph to be transitively closed. Unfortunately, such restrictions lead to much longer travel times for many queries and therefore significantly worsen the solution quality \cite{17,14}. A recently proposed approach for lifting these restrictions is ULTRA \cite{3,16}. It enables any public transit algorithm to support unlimited transfers by precomputing a small set of transfer shortcuts.

Like many public transit algorithms, ULTRA supports Pareto optimization of two criteria: arrival time and the number of used public transit trips. Road-based algorithms such as RAPTOR \cite{7,8} and Trip-Based Routing (TB) \cite{18} can handle the second criterion efficiently by avoiding explicit representation of Pareto sets. By contrast, algorithms for more than two criteria (e.g., McRAPTOR \cite{7,8}) require expensive dynamic data structures to maintain the Pareto sets and are therefore too slow for interactive applications.

For public transit, it is commonly accepted that optimizing only the arrival time is not enough to obtain high-quality solutions. The number of trips measures the discomfort associated with using public transit: Since changing vehicles is cumbersome, many passengers will accept a slightly later arrival time to save an additional trip. With an unlimited transfer mode, we argue that it becomes necessary to also minimize the transfer time, i.e., the time that is spent using the transfer mode\footnote{Note that our definition of transfer time excludes the waiting time that may occur when transferring between two vehicles.}. This is not as important if transfers are restricted, since most journeys will have a low transfer time anyway. With unlimited transfers, however, journeys that are optimal according to the other two criteria may require an excessive amount of transfer time. As shown by our experiments in Section \ref{sec:experiments}, there are often alternatives which are only slightly worse in the other two criteria but require less transfer time. Such alternatives will be missed when optimizing only two criteria, even with restricted transfers. Computing satisfactory solutions in a multimodal context therefore requires a third criterion that measures the discomfort associated with the transfer mode.

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Our Contribution. To enable efficient three-criteria optimization, we introduce McTB (Section 4), the first algorithm which can optimize a third criterion (e.g., transfer time) without explicit representation of Pareto sets. Our algorithm is based on TB, which operates on stop events, i.e., visits of public transit vehicles at stops. Since each stop event is associated with a particular arrival time, this removes the need to maintain arrival times explicitly. McTB builds on this idea by tracking the currently best value for the third criterion at each stop event. Combined with a round-based exploration of the network, this makes the third criterion the only one whose value must be tracked explicitly. Consequently, McTB does not require costly dynamic data structures in order to maintain Pareto sets, except for a single set of solutions at the target vertex.

TB requires a preprocessing phase which precomputes transfers between stop events. As shown in [10], this preprocessing phase can be replaced with a modified version of ULTRA, thereby enabling unlimited transfers. To apply this to McTB, we develop McULTRA, a three-criteria extension of ULTRA (Section 3). Analogously to ULTRA, McULTRA can be combined with any three-criteria public transit algorithm which normally requires transitively closed transfers.

For three or more criteria, the number of Pareto-optimal journeys becomes impractically large [6][11][5]. Besides causing high query times, this makes it difficult for users to choose between many similar alternatives. Several techniques have been proposed for filtering the Pareto set [6][1]. Restricted Pareto sets [5] are of particular interest because they can be computed much faster than full Pareto sets, but their definition is independent of the algorithm used to compute them. Delling et al. [5] introduced BM-RAPTOR, a (Mc)RAPTOR-based algorithm which computes restricted Pareto sets in a network with limited transfers. In Section 3, we show that only minor changes are necessary to make BM-RAPTOR utilize McULTRA shortcuts, thus enabling fast computation of restricted Pareto sets in a multimodal network. To achieve even faster query times, we re-engineer the pruning scheme of BM-RAPTOR to support (Mc)TB as the underlying query algorithm.

Section 2 presents an experimental evaluation on real-world multimodal networks. We show that the number of required McULTRA shortcuts remains low when adding transfer time as a third criterion. Combining McULTRA and McTB yields a speedup of 6–8 compared to MCR, the fastest previously known multimodal algorithm for three criteria. ULTRA-BM-TB, our new algorithm for restricted Pareto sets, achieves interactive query times even on large networks, yielding a speedup of 30–80 compared to MCR.

2 Preliminaries

Following the notation in [8] and [10], a public transit network is a 4-tuple \((S, T, R, G)\) consisting of a set of stops \(S\), a set of trips \(T\), a set of routes \(R\) and a directed, weighted transfer graph \(G = (V, E)\). Each stop \(v \in S\) represents a location in the network where passengers can enter or exit a vehicle. A trip \(T = (e_0, \ldots, e_k) \in T\) is a sequence of stop events performed by a single vehicle. Each stop event \(e_i\) represents a visit of the vehicle at a stop \(v(e_i) \in S\) with arrival time \(\tau_{arr}(e_i)\) and departure time \(\tau_{dep}(e_i)\). If a departure buffer time must be observed before entering \(T\) via \(e_i\), this can be handled implicitly by reducing \(\tau_{dep}(e_i)\) accordingly [10]. The \(i\)-th stop event of a trip \(T\) is denoted by \(T[i]\) and the number of stop events in the trip by \(|T|\). The routes \(R\) represent a partition of the trips such that all trips of a route share a single set of solutions at the target vertex.

Given a source vertex \(s \in V\) and a target vertex \(t \in V\), an \(s-t\)-journey represents the movement of a passenger from \(s\) to \(t\) through the network. It consists of an alternating sequence of trip legs (i.e., subsequences of trips) and transfers (i.e., paths in the transfer graph, which may be empty). The transfer between \(s\) and the first trip leg is called the initial transfer, whereas the final transfer connects the final trip leg to \(t\). The remaining transfers, which connect two trip legs, are called intermediate transfers.

Problem Statement. An \(s-t\)-journey \(J\) is evaluated according to the three criteria arrival time \(\tau_{arr}(J)\) at \(t\), number of used trips \(|J|\) (i.e., the number of trip legs), and transfer time \(\tau(J)\) (i.e., the total time spent traversing the transfer graph). A journey \(J\) weakly dominates another journey \(J'\) if \(J\) is not worse than \(J'\) according to any of the three criteria. Moreover, \(J\) strongly dominates \(J'\) if \(J\) weakly dominates \(J'\) and \(J\) is strictly better than \(J'\) according to at least one criterion.

Given source and target vertices \(s, t \in V\) and a departure time \(\tau_{dep}\), the objective is to compute a (full or restricted) Pareto set of \(s-t\)-journeys that depart no earlier than \(\tau_{dep}\). A (full) Pareto set \(J\) is a set of minimal size such that every valid journey is weakly dominated by a journey in the set. An anchor Pareto set \(J_A\) is a Pareto set considering only the two criteria
arrival time and number of trips. Each journey \( J \) in the full Pareto set has a corresponding anchor journey \( A(J) \), which is the journey in \( J_A \) with the highest number of trips not greater than \( |J| \). Given a trip slack \( \sigma_{tr} \geq 1 \) and an arrival slack \( \sigma_{arr} \geq 1 \), the restricted Pareto set \( \mathcal{J}_R \) is defined as

\[
\mathcal{J}_R := \{ J \in \mathcal{J} \mid |J| \leq |A(J)| \cdot \sigma_{tr} \text{ and } \tau_{arr}(J) - \tau_{dep} \leq (\tau_{arr}(A(J)) - \tau_{dep}) \cdot \sigma_{arr} \}.
\]

The restricted Pareto set contains every journey from the full Pareto set whose arrival time and number of trips do not exceed their respective slack compared to its anchor journey. In \cite{bag}, the restricted Pareto set was defined using absolute slack values (e.g., \( \sigma_{arr} = 60 \text{ min} \)). We argue that it is more natural to use relative slacks, which depend on the length of the anchor journey. Passengers are more willing to take long detours if the journey is already long, whereas a 60-minute detour on a 15-minute journey is not attractive. All presented algorithms for computing restricted Pareto sets can be easily modified to support absolute slacks instead.

**Algorithms.** We give a brief overview of the algorithms we build on. RAPTOR \cite{bag, bat} is a round-based algorithm which optimizes arrival time and number of trips in a network with transitively closed transfers. Round \( i \) finds journeys with exactly \( i \) trips by performing two steps: First, all routes visiting stops whose earliest arrival time was improved in round \( i-1 \) are collected and scanned. Afterwards, edges in the transfer graph are relaxed. McRAPTOR extends RAPTOR for an arbitrary number of additional criteria. For each stop and round, it maintains a bag of Pareto-optimal labels instead of a single earliest arrival time.

Bounded McRAPTOR (BM-RAPTOR) \cite{bag} is an extension of RAPTOR which computes restricted Pareto sets. Three variants of BM-RAPTOR were proposed in \cite{bag}: we focus on the fastest variant (Tight-BMRAP). It performs three phases: the forward pruning search is a two-criteria RAPTOR query which computes the earliest arrival time \( \tau_{arr}(v,i) \) per stop \( v \) and round \( i \), and thereby the anchor Pareto set \( J_A \). Then, a backward pruning search is run for each anchor journey, in order of most used trips to fewest. Collectively, these compute a latest departure time \( \tau_{dep}(v,i) \) for each stop \( v \) and round \( i \). This is the latest departure time at \( v \) such that a can be reached with \( i \) remaining trips without exceeding the trip or arrival slack. The backward pruning search for an anchor journey \( J \in J_A \) is a reverse RAPTOR query which starts at \( t \) with the arrival time \( \tau_{dep}(J) + (\tau_{arr}(J) - \tau_{dep}) \cdot \sigma_{arr} \). Let \( K \) denote the maximum number of trips of any journey in \( J_A \). The search is run for \( n = \min(|J| - 1, |J| \cdot \sigma_{tr}) \) rounds, where \( J' \) is the journey in \( J_A \) with the next higher number of trips, or \( n = K \cdot \sigma_{tr} \) rounds if \( |J| = K \). When the backward search finds a departure at a stop \( v \) in round \( i \) with departure time \( \tau \), it is discarded if \( \tau < \tau_{arr}(v,n-i) \). Otherwise, it is written to \( \tau_{dep}(v,K \cdot \sigma_{tr} - (n-i)) \). The latest departure times \( \tau_{dep} \) are not reinitialized between backward searches. Finally, the main search is a McRAPTOR query from \( s \) to \( t \). If it arrives at a stop \( v \) in round \( i \) with arrival time \( \tau \), the arrival is discarded if \( \tau > \tau_{dep}(v,K \cdot \sigma_{tr} - i) \).

A faster alternative to RAPTOR is Trip-Based Routing (TB) \cite{bag}, which requires precomputed transfers between stop events. Instead of storing arrival times at stops, it maintains a reached index \( v(T) \) for each trip \( T \), which is the index of the first stop along \( T \) that has been reached via some trip \( T' \preceq T \). Like RAPTOR, TB operates in rounds, where each round scans segments of trips which were newly reached in the previous round. For each stop event along a scanned trip segment, outgoing transfers to other stop events and to the target vertex are relaxed.

ULTRA \cite{bag, bag} is a preprocessing technique which allows any public transit algorithm that requires a transitively closed transfer graph to handle unlimited transfers instead. To this end, it precalculates a set of transfer shortcuts representing all intermediate transfers required to find a two-criteria Pareto set. There are two variants of ULTRA: The stop-to-stop variant computes shortcuts between stops. These are sufficient for most algorithms, including RAPTOR. The event-to-event variant computes shortcuts between stop events, enabling integration with TB \cite{bag}. Both variants work by enumerating journeys with at most two trips. Journeys that consist of two trips connected by an intermediate transfer are called candidates, while all other journeys are called witnesses. If a candidate is not dominated by any witness, a shortcut representing its intermediate transfer is added. An ULTRA query uses Bucket-CH \cite{bag, bag, bag} to explore initial and final transfers. Afterwards, a public transit algorithm of choice is run, using the transfer shortcuts as the transfer graph.

**3 McULTRA Shortcut Computation.** To enable unlimited transfers while optimizing a third criterion, we propose McULTRA, an adaptation of ULTRA. As with original ULTRA, we present two variants: The stop-to-stop variant can be combined with any algorithm that requires transitively closed transfers, using the same query framework as ULTRA. The event-to-event variant is intended for combination with McTB, which we present in Section 4. Since the shortcut computation is mostly identical for both variants, we mainly describe the event-to-event variant and mention differences where appropriate.
The basic outline of the shortcut computation remains unchanged: For every stop \( s \in S \), the algorithm collects all departure times of trips at \( s \) and then runs an iteration of McRAPTOR for each departure time in descending order. The McRAPTOR iterations are restricted to two rounds (each consisting of route scans followed by transfer relaxations), and vertex bags are not cleared between them. During the final transfer relaxation phase of each iteration, labels representing undominated candidates are extracted and shortcuts are inserted for them. Adding a third criterion requires the use of McRAPTOR instead of RAPTOR, and therefore vertex bags instead of mere arrival times. In the following, we adapt the optimizations included in the original ULTRA shortcut computation and describe additional optimizations for the three-criteria setting. Several optimizations rely on the following simple observation:

**Lemma 3.1.** For a fixed number of trips \( i \) and a fixed stop event \( \epsilon \), the three-criteria Pareto set contains at most one journey with \( i \) trips that uses \( \epsilon \) as the final stop event.

**Proof.** All journeys with \( i \) trips and final stop event \( \epsilon \) are equivalent according to arrival time and number of trips. The best journey according to the third criterion therefore dominates the others.

**Route Scans.** When scanning a route, RAPTOR maintains a single active trip along the route, which is the earliest trip that can be entered so far. In McRAPTOR, the active trip is replaced with a route bag containing all Pareto-optimal labels, each of which has its own active trip. No two labels can have the same active trip: When exiting at any stop along the route, they will share the same final stop event, and by Lemma 3.1 the label that is better in the third criterion will dominate the other one.

Bags are always required in the second route scanning phase of each McRAPTOR iteration. For the first route scan, the simpler RAPTOR variant with a single active trip is sufficient as long as the third criterion only affects the transfer graph (which is the case for transfer time). In this case, because candidates do not have an initial transfer, all candidates labels will have value 0 in the third criterion during the first route scan. Therefore, the candidate with the earliest arrival time dominates all others. While the same is not true for witnesses, it is not necessary to find all witnesses since failing to find one may only lead to superfluous shortcuts.

**Dijkstra Searches.** The single-criterion Dijkstra searches of ULTRA are replaced with a standard multi-criteria Dijkstra. Given an arrival weight \( w_{arr} > 0 \) and a transfer weight \( w_t > 0 \) (assuming the third criterion is transfer time), a label with arrival time \( \tau_{arr} \) and transfer time \( \tau_t \) has a key of \((w_{arr} \cdot \tau_{arr} + w_t \cdot \tau_t)/(w_{arr} + w_t)\) in the priority queue. It is also possible to assign a weight of 0 to one of the two criteria as long as it is used as a tiebreaker in the case of equality. This ensures that the order in which labels are settled does not conflict with Pareto dominance, i.e., if label \( \ell_1 \) is settled before label \( \ell_2 \), \( \ell_2 \) may not dominate \( \ell_1 \). The stopping criteria for the Dijkstra searches used by the original ULTRA shortcut computation can be carried over to McULTRA: The final Dijkstra search of each iteration is stopped once all candidates have been extracted from the queue. The stopping criterion for the intermediate Dijkstra search is based on the intermediate witness limit \( \tau_1 \). If \( \tau_2 \) is the key of the last candidate extracted from the queue, the search is stopped once the key of the queue head exceeds \( \tau_2 + \tau_1 \).

**Parent Pointers.** Two-criteria ULTRA uses per-stop parent pointers to extract shortcuts and to distinguish between candidates and witnesses. This is no longer sufficient for three criteria since a stop may now have multiple candidate labels which represent different shortcuts. By Lemma 3.1, each stop event may have at most one candidate. Thus, McULTRA maintains two pointers \( \text{parent}_1[\epsilon] \) and \( \text{parent}_2[\epsilon] \) per stop event \( \epsilon \), where \( \text{parent}_k[\epsilon] \) is the parent stop event for reaching \( \epsilon \) with \( k \) trips. Each candidate label includes a pointer to the last stop event where a trip was entered or exited. To distinguish them from candidates, witness labels set this pointer to \( \perp \). When a candidate journey with final stop event \( \epsilon_1 \) is found, the corresponding shortcut can be extracted as \((\text{parent}_1[\text{parent}_2[\epsilon_1]], \text{parent}_2[\epsilon_1])\).

A crucial optimization of ULTRA is that once a shortcut is added to the result, all remaining candidates which represent the same shortcut are turned into witnesses. To achieve this efficiently, each stop event \( \epsilon \) maintains a list of child stop events:

\[
\text{children}_2[\epsilon] := \{ \epsilon_1 \mid \text{parent}_2[\epsilon_1] = \epsilon \}
\]

Once a shortcut \((\epsilon_0, \epsilon_d)\) is inserted, every child \( \epsilon_t \in \text{children}_2[\epsilon_0] \) is turned into a witness by setting \( \text{parent}_2[\epsilon_t] = \perp \). Consequently, a label that points to a stop event \( \epsilon_t \neq \perp \) is only considered a candidate if \( \text{parent}_2[\epsilon_1] \neq \perp \). After the final Dijkstra search is stopped, there may be former candidate labels left in the queue which point to a valid final stop event \( \epsilon_t \) but for which \( \text{parent}_2[\epsilon_1] = \perp \). These labels must be retained as witnesses for later McRAPTOR iterations, but their stop event pointers are set to \( \perp \). Otherwise, they might be misidentified as candidates if a later iteration sets \( \text{parent}_2[\epsilon_1] \) to a valid stop event again.

**Final Transfer Pruning.** The event-to-event variant of original ULTRA uses a weaker pruning rule
for candidates than the stop-to-stop variant: A candidate is discarded if it is strongly dominated by a witness or weakly dominated by a candidate, but not if it is only weakly dominated by a witness. This difference is carried over to McULTRA. Preliminary experiments showed that the final Dijkstra search takes much longer in the event-to-event variant than in the stop-to-stop variant, which is not the case for original ULTRA. The reason for this is that the stopping criterion is applied later due to undominated candidate labels with a very high key. These labels are only weakly dominated by equivalent labels which were candidates in previous iterations. To remedy this, we distinguish between proper candidates, which are not dominated by any journey, and improper candidates, which are weakly dominated by a witness. We make the stopping criterion of the final Dijkstra search stricter by introducing a final witness limit $\bar{\tau}_2$. Let $\tau_c$ be the key of the last proper candidate extracted from the queue. The search is stopped once the key of the queue head exceeds $\tau_c + \bar{\tau}_2$. Afterwards, all remaining improper candidates are removed from the queue and shortcuts are inserted for them.

4. McTB Query Algorithm

As a faster alternative to McRAPTOR, we propose McTB, a new three-criteria query algorithm based on TB. The original preprocessing phase of TB computes event-to-event transfers based on a limited transfer graph. Since we focus on a multimodal scenario, we use a set $\mathcal{E}^t$ of event-to-event McULTRA shortcuts instead and obtain a multimodal algorithm, ULTRA-McTB. Pseudocode for the trip enqueuing and scanning procedures of McTB is given in Algorithm 1. For ease of exposition, we assume that the third criterion optimized by McTB is transfer time. However, McTB supports any third criterion whose values can be totally ordered.

McTB replaces the indices $r(\cdot)$ used by TB with a transfer time label $\tau_t(T, i)$ for each stop event $T[i]$ of $T$, which represents the minimum transfer time needed to reach $T[i]$ (or $\infty$ if $T[i]$ is not reachable). By Lemma 3.1, the journey represented by $\tau_t(T, i)$ is not dominated by any other journeys ending at $T[i]$ found so far. Two invariants are upheld for $\tau_t(T, i)$: For each $T' \succ T$, $\tau_t(T', i) \leq \tau_t(T, i)$ holds since a passenger can reach $T'[i]$ by traveling to $T[i]$ and waiting. Similarly, remaining seated in a trip does not increase the transfer time, so $\tau_t(T, j) \leq \tau_t(T, i)$ holds for each $i < j < |T|$. 

Initial Transfer Evaluation. Like every ULTRA query, ULTRA-McTB explores initial and final transfers with a Bucket-CH search. This yields the minimal arrival time $\tau_{arr}(s, u)$ for each stop $u$ reachable via an initial transfer, and the minimal target transfer time $\tau_t(v, t)$ for each stop $v$ from which $t$ is reachable via a final transfer. Afterwards, the algorithm identifies trips that can be entered via an initial transfer. This is done as in the original TB query: For each stop $v$ reachable via an initial transfer and each route visiting $v$, the earliest reachable trip at $v$ is found via binary search and collected for the first trip scanning phase via the $\text{Enqueue}$ procedure. Note that two-criteria ULTRA-TB replaces this step with RAPTOR-like route scans, exploiting the fact that the earliest reachable trip of a route never increases along its stop sequence. However, once a third criterion is introduced, it is no longer sufficient to consider the earliest reachable trip of the route, since entering an earlier trip at a later stop may increase the transfer time.

Trip Enqueuing. Like the original TB query, the McTB query works in rounds, where round $n$ scans trip segments that were collected in queue $Q_n$ during round $n - 1$. A trip segment $(T, j, k, \tau)$ represents the subsequence of $T$ from $T[j]$ to $T[k]$. Here, the trip segment also stores the transfer time $\tau$ with which it was reached. Trip segments are inserted into the queue by the $\text{Enqueue}$ procedure (lines 1–2). When a trip $T$ is entered with transfer time $\tau$ at index $i$, the first index where it may be exited is $j := i + 1$. If $\tau(T, j) \leq \tau$ (line 2), $T$ does not improve the transfer time at $T[j]$ or any later stop event, so the trip segment is discarded. Otherwise, the end of the trip segment is set to the last index $k$ for which $\tau(T, k) > \tau$ holds (line 3). Finally, the trip segment is added to the queue (line 4) and the transfer time labels are updated to $\tau$, satisfying the two invariants (lines 5–7).

Trip Scanning. To prevent redundant portions of $x$ that may have been enqueued. In this case, scanning the portion of $T$ between $j'$ and $k$ is redundant because it overlaps with $x'$. The pruning step identifies redundant portions of $x$ and adjusts the end index $k$ accordingly. If such an $x'$ exists, either $T' \prec T$ or $\tau' \prec \tau$ must hold, since otherwise the check in line 2 would have discarded $x'$. If $\tau' \prec \tau$, then enqueuing $x'$ ensures $\tau(T', j') < \tau$, which is checked in line 11. If $T' \prec T$, the trip $\text{pred}(T)$ which immediately precedes $T$ in the route must have $\tau_t(\text{pred}(T), j') \leq \tau_t(T', j') \leq \tau'$ and $\tau' \leq \tau$. This is checked in line 12.

Following the pruning step, final transfers are evaluated for all stops along the trip segment $x = (T, j, k, \tau)$. Line 13 adds new journeys to the set $\mathcal{J}$ of Pareto-optimal journeys at $t$, which is the only dynamic data structure maintained by the algorithm. Outgoing trans-
Algorithm 1: Trip enqueuing and scanning procedures of the McTB query.

1. Procedure Enqueue $(T, j, Q, \tau)$
   2. if $\pi(T, j) \leq \tau$ then return
   3. $k \leftarrow \max\{i \in \{j, \ldots, |T| - 1\} | \pi(T, i) > \tau\}$
   4. $Q \leftarrow Q \cup \{(T, j, k, \tau)\}$
   5. for each $T' \geq T$ do
      6. for $i$ from $j$ to $|T| - 1$ do
         7. $\pi(T', i) \leftarrow \min(\tau, \pi(T', i))$
   8. Procedure Scan($Q_n$)
   9. for each $(T, j, k, \tau) \in Q_n$ do
      10. for $i$ from $j + 1$ to $k$ do
         11. if $\pi(T, i) < \tau$ then $k \leftarrow i - 1$
         12. else if $\pi(T, i) = \tau$ and $\text{pred}(T) \neq \bot$ and $\pi(\text{pred}(T), i) \leq \tau$ then
            13. $k \leftarrow i - 1$
      14. for each $(T, j, k, \tau) \in Q_n$ do
      15. $J \leftarrow \text{Pareto set of } J \cup \{(\tau_{\text{arr}}(T[i]) + \pi(t(T[i]), t), n, \tau + \pi(t(T[i]), t))\}$
      16. for each $(T, j, k, \tau) \in Q_n$ do
         17. if $J$ dominates $(\tau_{\text{arr}}(T[j]), n + 1, \tau)$ then continue
      18. for $i$ from $j$ to $k$ do
         19. for each $e = (T[i], T'[i']) \in \mathcal{E}^*$ do
            20. Enqueue($T, i', 1, Q_{n+1}, \tau + \pi(e)$)

5 Bounded Query Algorithms

In this section, we introduce algorithms for computing restricted Pareto sets in a network with unlimited transfers. Note that simply combining BM-RAPTOR with McULTRA shortcuts is not sufficient. BM-RAPTOR requires that the forward and backward pruning searches find optimal arrivals or departures at each stop. However, since ULTRA-RAPTOR explores final transfers with a backward Bucket-CH search from $t$, optimal arrivals via a transfer at stops other than $t$ may not be found. We show that only small modifications are necessary to make ULTRA-RAPTOR support pruning searches. Furthermore, we introduce a new TB-based algorithm for computing restricted Pareto sets.

5.1 ULTRA-BM-RAPTOR For the pruning searches, we replace ULTRA-RAPTOR with a slightly modified algorithm, which we call pRAPTOR. Aside from using the three-criteria McULTRA shortcuts for the intermediate transfers, the only difference is the transfer relaxation phase. Normally, ULTRA-RAPTOR relaxes the outgoing shortcut edges of all stops whose arrival time was improved during the preceding route scanning phase. pRAPTOR additionally relaxes the edges of stops whose arrival time was improved during the previous transfer relaxation phase. This allows pRAPTOR to find journeys that use multiple shortcut edges in a row. However, each additional edge is counted as an additional trip.

Theorem 5.1 shows that pRAPTOR can be used to perform pruning searches. Combined with ULTRA-McRAPTOR for the main search, this yields ULTRA-BM-RAPTOR as a multimodal algorithm for computing restricted Pareto sets.

Theorem 5.1. Let $\tau_{\text{arr}}(v, i)$ denote the arrival time at $v$ found by pRAPTOR in round $i$. Consider a journey $J$ which is Pareto-optimal for three criteria. Given a stop $v$ visited by $J$, let $\tau_{\text{arr}}(J, v)$ denote the arrival time of $J$ at $v$. For $1 \leq i \leq |J|$, let $T_i$ be the $i$-th trip of $J$, $u_i$ the stop where $T_i$ is entered and $v_i$ the stop where $T_i$ is exited. Then $\tau_{\text{arr}}(u_i, i - 1) \leq \tau_{\text{arr}}(J, u_i)$ and $\tau_{\text{arr}}(v_i, i) \leq \tau_{\text{arr}}(J, v_i)$.

Proof. First we show that $\tau_{\text{arr}}(u_i, i - 1) \leq \tau_{\text{arr}}(J, u_i)$ implies $\tau_{\text{arr}}(v_i, i) \leq \tau_{\text{arr}}(J, v_i)$. If pRAPTOR arrives at $u_i$ no later than $J$, it will scan $T_i$ or an earlier trip of the same route during the route scanning phase of round $i$, and thereby reach $v_i$ with an arrival time of $\tau_{\text{arr}}(J, v_i)$ or better. For $i < |J|$, we show that this in turn implies $\tau_{\text{arr}}(u_{i+1}, i) \leq \tau_{\text{arr}}(J, u_{i+1})$. If the arrival via the route of $T_i$ in round $i$ improves the previous value of $\tau_{\text{arr}}(v_i, i)$, then the following transfer phase in round $i$ will relax the shortcut $(v_i, u_{i+1})$ and find a suitable arrival at $u_{i+1}$. Otherwise, $\tau_{\text{arr}}(v_i, j) \leq \tau_{\text{arr}}(J, v_i)$ must hold for some $j < i$. Then the transfer phase of round $j + 1 \leq i$ will relax $(v_i, u_{i+1})$ and arrive at $u_{i+1}$ in time. Since the base case $i = 1$ follows from the correctness of Bucket-CH and RAPTOR, the claim is proven by induction. \qed
5.2 ULTRA-BM-TB To achieve even faster query times, we introduce ULTRA-BM-TB, a TB-based algorithm for computing restricted Pareto sets. ULTRA-BM-TB follows the same query framework as BM-RAPTOR, but uses a variant of ULTRA-TB for the pruning searches and ULTRA-McTB for the main search. Switching to TB-based algorithms necessitates different data structures for pruning. The main search of BM-RAPTOR relies on earliest arrival times $\tau_{arr}(\cdot, i)$ and latest departure times $\tau_{dep}(\cdot, i)$ for each round $i$, which are computed by the pruning searches. A natural adaptation of these data structures for TB is to replace them with a forward reached index $\mathcal{F}(T, i)$ and a backward reached index $\mathcal{F}(T, i)$ for each trip $T$ and round $i$.

The forward reached index $\mathcal{F}(T, i)$ is the first stop index along $T$ that is reachable from $s$ with $i$ trips, while the backward reached index $\mathcal{F}(T, i)$ is the last stop index along $T$ from which $t$ is reachable with $i$ trips and without exceeding the trip or arrival slack. These reached indices can be used for pruning in the Enqueue procedure: When entering a trip $T$ at stop event $T[k]$ in round $i$, a backward pruning search running for $n$ rounds does not enqueue the trip if $k < \mathcal{F}(T, n - i)$. Likewise, the main search does not enqueue the trip if $k > \mathcal{F}(T, K \cdot \sigma_{tr} - i)$.

Computing Per-Round Reached Indices. The presented pruning scheme requires the pruning searches to output one reached index per stop and round, whereas the original TB query only maintains one reached index per stop. For the forward pruning search, this can be changed by simply initializing $\mathcal{F}(v, i)$ with $\mathcal{F}(v, i - 1)$ for each stop $v$ at the start of round $i$. The backward pruning searches require a different approach because they do not access the rounds of $\mathcal{F}(\cdot, \cdot)$ in order. Before the first backward search is started, the backward reached indices for all $K \cdot \sigma_{tr}$ rounds are initialized with $\infty$. Whenever a backward reached index $\mathcal{F}(T, i)$ is set to a value $k$, this value is propagated to the following rounds by setting $\mathcal{F}(T, j)$ to $\min(\mathcal{F}(T, j), k)$ for all $i < j \leq K \cdot \sigma_{tr}$.

Shortcut Augmentation. To establish a correct pruning scheme, the pruning searches must fulfill a condition analogous to that stated by Theorem 5.1. For this purpose, we introduce the set of augmented shortcuts $\mathcal{E}_{\text{aug}}$ for a given set $\mathcal{E}$ of event-to-event McULTRA shortcuts:

$$\mathcal{E}_{\text{aug}} = \{ (T_a[i], T_b[j]) | \exists T_c : (T_a[i], T_b[j]) \in \mathcal{E} \land \forall T_d \supseteq T_a \forall T_c \supset T_b : (T_c[i], T_b[j]) \notin \mathcal{E} \}$$

Figure 1 illustrates how shortcut augmentation works. For each shortcut $(T_a[i], T_b[j]) \in \mathcal{E}$ and each trip $T_a$ preceding $T_c$ in the same route, it ensures that a shortcut exists from $T_a[i]$ to either $T_b[j]$ or an earlier trip of the same route at $j$. Clearly, this can be achieved by adding the shortcut $(T_a[i], T_b[j])$. However, if a shortcut $(T_a[i], T_b[j])$ exists for $T_a \supseteq T_c$ and $T_b \supset T_c$, the shortcut $(T_a[i], T_b[j])$ is superfluous because there must already be a shortcut from $T_a[i]$ to $T_c[j]$ or an earlier trip at $j$.

Lemma 5.1 and Theorem 6.2 prove that using an ULTRA-TB query with per-round reached indices and augmented McULTRA shortcuts for the pruning searches yields a correct pruning scheme.

**Lemma 5.1.** For any $(T_a[i], T_b[j]) \in \mathcal{E}$ and all $T_a \supset T_c$, there exists a $T_f \supseteq T_b$ with $(T_a[i], T_f[j]) \in \mathcal{E}_{\text{aug}}$.

**Proof.** Consider a trip $T_a \supset T_c$. Assume that for every $T_f \supset T_b$, there is no shortcut $(T_a[i], T_f[j])$ in $\mathcal{E}_{\text{aug}}$. In particular, this means $(T_a[i], T_b[j]) \notin \mathcal{E}_{\text{aug}}$. Since $T_a \supseteq T_c$ and $(T_a[i], T_b[j]) \in \mathcal{E}$, it follows by definition of $\mathcal{E}_{\text{aug}}$ that there must be a $T_g \supseteq T_a$ and $T_b \supset T_c$ such that $(T_g[i], T_b[j]) \in \mathcal{E}$. Among all such $T_c$, consider the one with the earliest departure time. Then it follows by definition of $\mathcal{E}_{\text{aug}}$ that $(T_g[i], T_f[j]) \in \mathcal{E}_{\text{aug}}$ since there is no edge $(T_g[i], T_f[j])$ in $\mathcal{E}$ with $T_f \supseteq T_g$ and $T_b \supset T_c$. This contradicts our assumption.

**Theorem 5.2.** Consider a journey $J$ which is Pareto-optimal for three criteria. For $1 \leq i \leq |J|$, let $T_i$ be the $i$-th trip of $J$ and $T_i[k]$ the stop event where $J$ exits $T_i$. Then an ULTRA-TB query using augmented McULTRA shortcuts has a reached index $r(T_i) \leq k_i$ after round $i$.

**Proof.** By induction over $i$. The base case $i = 1$ follows from the correctness of Bucket-CH and TB. Assume the claim is true for $i - 1$. Since $r(T_{i-1}) \leq k_{i-1}$ after round $i - 1$, there must be a trip $T_{i-1}' \supset T_{i-1}$ that was exited at $T_{i-1}'[k_{i-1}]$ in a round $j < i$. We know that $(T_{i-1}[k_{i-1}], T_j[\ell_j]) \in \mathcal{E}$ for the stop event $T_j[\ell_j]$ where $J$ enters $T_j$. Then $(T_{i-1}'[k_{i-1}], T_j[\ell_j]) \in \mathcal{E}_{\text{aug}}$ for some $T_{i-1}' \supset T_i$ by Lemma 5.1. Thus, $r(T_{i-1}) \leq r(T_{i-1}') \leq \ell_j + 1 \leq k_i$ after round $i$. \(\square\)
In general, the transfer times $\tau$ with a turbo frequency of 3.4 GHz, 1024 GiB AMD Epyc Rome 7742 CPUs clocked at 2.25 GHz, computations were run on a machine with two 64-core −→

Experiments

All algorithms were implemented in C++17 compiled with GCC 9.3.1 and optimization flag -O3. Shortcut computations were run on a machine with two 64-core AMD Epyc Rome 7742 CPUs clocked at 2.25 GHz, with a turbo frequency of 3.4 GHz, 1024 GiB of DDR4-3200 RAM, and 256 MiB of L3 cache. All other experiments were conducted on a machine with two 8-core Intel Xeon Skylake SP Gold 6144 CPUs clocked at 3.5 GHz, with a turbo frequency of 4.2 GHz, 192 GiB of DDR4-2666 RAM, and 24.75 MiB of L3 cache. Source code for our algorithms is publicly available.

Table 1 gives an overview of the networks we use for evaluation. All three networks were previously used in [15]. The London and Switzerland networks were sourced from Transport for London and a publicly available GTFS feed, respectively. The Germany network was provided by Deutsche Bahn. Unlimited transfer graphs were extracted from OpenStreetMap. Unless otherwise noted, travel times were computed by assuming a constant speed of 4.5 km/h, representing walking. For comparison with McRAPTOR and BM-RAPTOR, transitively closed transfer graphs were generated using the methods reported in [17]. To aid reproducibility, we make the London and Switzerland networks publicly available. Unfortunately, we cannot provide the Germany network as it is proprietary.

Impact of Optimizing Transfer Time

To evaluate the impact of optimizing transfer time on the solution quality, we compared minimal transfer times in the two-criteria Pareto set to those in restricted three-criteria Pareto sets. We chose restricted Pareto sets instead of a full one in order to exclude undesirable journeys with a low transfer time but excessive costs in the other criteria. Figure 2 shows the results on Switzerland and London for different transfer speeds. For walking as the transfer mode, more than 25% of the transfer time

Optimizations. As described thus far, the pruning scheme causes unnecessary work during the backward searches: The initial transfer phase of a backward search collects all routes from which $t$ is reachable and scans them to find trip segments to enqueue. This is efficient for a normal ULTRA-TB query because $t$ is typically reachable from most stops. In a bounded query, however, most stops are not reachable from $s$ in time to produce an optimal journey when transferring from there to $t$. For such stops, the Enqueue operation will be called, but no trip segments will be enqueued because they will be pruned by the forward reached indices. To avoid unnecessary Enqueue calls at these stops, we replace the forward reached indices $\tau(v, i)$ with earliest arrival times at stops. As in the RAPTOR-based algorithm, the forward pruning search now computes an earliest arrival time $\overrightarrow{\tau}(v, i)$ per stop $v$ and round $i$.

A corollary of Theorem 5.2 is that $\overrightarrow{\tau}(v, i)$ is never worse than the arrival time of a Pareo-optimal journey at $v$ in round $i$ via a trip, allowing the backward search to use it for pruning. When entering a trip $T$ at stop event $T[k]$ in round $i$, a backward search running for $n$ rounds does not enqueue the trip segment if $\overrightarrow{\tau}(v(T[k]), n - i) > \tau_{dep}(T[k])$. To explore initial transfers, a backward search with $n$ rounds for a journey $J$ first collects all stops $v$ from which $t$ is reachable and for which $\overrightarrow{\tau}(n) + \tau_t(v, t) - \tau_{dep} \leq (\overrightarrow{\tau}(J) - \tau_{dep}) - \sigma_{arr}$. For each such stop $v$ and each route visiting $v$, the latest reachable trip is found via binary search and the Enqueue operation is called.

To compute $\overrightarrow{\tau}(v, i)$, the forward pruning search adds an extra operation during the trip scanning phase. When relaxing the outgoing transfers of a stop event $T[\ell]$ in round $i$, $\overrightarrow{\tau}(v(T[\ell]), i)$ is now set to the minimum of itself and $\overrightarrow{\tau}(T[\ell])$. When starting a new round $i$, the arrival time $\overrightarrow{\tau}(v, i)$ for each stop $v$ is initialized with $\overrightarrow{\tau}(v, i - 1)$.

A final optimization concerns the main search. Normally, the transfer times $\tau(v, i)$ maintained by McTB are cleared at the start of each query. In the context of ULTRA-BM-TB, where most of the search space is pruned, this is often more expensive than the main search itself. Hence, we mark each transfer time $\tau_t(v, i)$ with a timestamp. When $\tau_t(v, i)$ is accessed and its timestamp does not match that of the current query, the value is reset to $\infty$.

6 Experiments

Table 1: Sizes of the public transit networks, including the unrestricted transfer graphs as well as the transitively closed transfer graphs used for comparison with McRAPTOR and BM-RAPTOR.

|                | London | Switzerland | Germany |
|----------------|--------|-------------|---------|
| Stops          | 19 682 | 25 125      | 243 167 |
| Routes         | 1 955  | 13 786      | 230 225 |
| Trips          | 11 450 | 350 006     | 2 381 394 |
| Stop events    | 4 508 644 | 4 686 865 | 48 380 936 |
| Vertices       | 18 164 | 603 691     | 6 870 496 |
| Edges          | 575 364 | 1 853 260 | 21 367 044 |
| Trans. edges   | 3 212 206 | 2 639 402 | 22 571 280 |

†https://github.com/kit-algo/ULTRA-Transfer-Time
‡https://data.london.gov.uk
§https://gtfs.geops.ch/
¶https://download.geofabrik.de/
‖https://ii11www.iti.kit.edu/PublicTransitData/ULTRA/
Figure 2: Comparison of optimal transfer times in the two-criteria Pareto set $J_A$ versus a restricted three-criteria Pareto set $J_R$ on the Switzerland and London networks for different transfer speeds. 10000 random queries were run for each choice of slack values. We then calculated the transfer time savings as $\Delta_t = (\tau_A - \tau_R)/\tau_A$, where $\tau_R$ is the lowest transfer time in $J_R$ and $\tau_A$ the lowest transfer time in $J_A$. Shown is the percentage of queries where $\Delta_t$ exceeds the specified threshold.

can be saved for most queries, even with small slack values. In up to 50% of all queries, more than half of the transfer time can be saved. This confirms that adding transfer time as a third criterion often significantly improves the quality of the found journeys. The savings become even more pronounced as the transfer speed increases. For 10 km/h (e.g., slow cycling), the vast majority of queries allow at least a moderate improvement in transfer time. Often, improvements are possible even without allowing a trip slack. This underscores that optimizing the transfer time becomes especially crucial as the transfer speed increases. Fast transfer modes are frequently competitive with public transit in terms of travel time, but not necessarily in terms of comfort. If the transfer time is not considered, optimal journeys will often avoid trip legs altogether in favor of long transfers.

For walking as the transfer mode, both networks behave similarly. London has a slightly larger share of queries with savings above 25%, whereas very large savings are more frequent on the Switzerland network. The impact of the transfer speed is more drastic for London, where large savings are more common for a transfer speed of 10 km/h than on the Switzerland network. This can be explained by different network
Table 2: Shortcut computation results for ULTRA and McULTRA. Times are formatted as hh:mm:ss. For event-to-event McULTRA, $|\text{Et}_\text{aug}|$ and $|\text{Et}_\text{aug}^{-1}|$ are the number of augmented forward and backward shortcuts, respectively.

| Network | Variant | ULTRA | McULTRA |
|---------|---------|-------|---------|
|         | Time | # Shortcuts | Time | # Shortcuts | $|\text{Et}_\text{aug}|$ | $|\text{Et}_\text{aug}^{-1}|$ |
| London  | Stop  | 00:03:18 | 150738 | 00:14:11 | 136755 |
|         | Event | 00:03:57 | 17384810 | 00:28:35 | 18841658 | 51054754 | 58136150 |
| Switzerland | Stop | 00:01:51 | 135694 | 00:03:48 | 214826 |
|         | Event | 00:02:09 | 11662444 | 00:08:08 | 13761297 | 44417661 | 44643118 |
| Germany | Stop  | 02:42:59 | 2069438 | 04:19:50 | 2924040 |
|         | Event | 02:42:55 | 122952170 | 10:23:36 | 186748113 | 557609345 | 571099035 |

Table 3: Performance of full Pareto set algorithms, averaged over 10,000 random queries. RAPTOR query times are divided into phases: initialization, including clearing vertex bags and exploring initial/final transfers (Init.), collecting (Collect) and scanning (Scan) routes, and relaxing intermediate transfers (Relax). Also reported are the number of rounds (Rnd.) and the number of found journeys (Jrn.). McRAPTOR* only supports stop-to-stop queries with transitive transfers.

| Network | Algorithm | Full graph | Rnd. | Jrn. | Time [ms] | Init. | Collect | Scan | Relax | Total |
|---------|-----------|------------|------|------|----------|-------|---------|-------|-------|-------|
| London  | McRAPTOR* | o          | 12.6 | 21.7 | 21.7     | 16.1  | 2.5     | 92.8  | 157.3 | 268.6 |
|         | MCR       | •          | 14.6 | 30.5 | 37.5     | 4.5   | 4.5     | 147.6 | 269.5 | 459.2 |
|         | ULTRA-McRAPTOR | •          | 14.6 | 30.5 | 27.9     | 3.9   | 3.9     | 144.7 | 53.7  | 230.2 |
|         | ULTRA-McTB | •          | 14.8 | 30.5 | –        | –     | –       | –     | –     | 79.2  |
| Switzerland | McRAPTOR* | o          | 24.3 | 20.7 | 20.7     | 8.0   | 8.0     | 120.7 | 132.4 | 281.7 |
|         | MCR       | •          | 32.7 | 30.5 | 82.8     | 20.3  | 20.3    | 278.7 | 409.9 | 791.7 |
|         | ULTRA-McRAPTOR | •          | 32.7 | 30.5 | 56.0     | 18.3  | 18.3    | 272.9 | 66.8  | 414.0 |
|         | ULTRA-McTB | •          | 33.5 | 30.5 | –        | –     | –       | –     | –     | 130.2 |
| Germany | McRAPTOR* | o          | 31.0 | 38.3 | 423.8    | 335.4 | 335.4   | 4399.7 | 3406.9 | 8565.8 |
|         | MCR       | •          | 36.4 | 57.1 | 929.0    | 454.0 | 454.0   | 7192.3 | 15554.7 | 24131.1 |
|         | ULTRA-McRAPTOR | •          | 36.4 | 57.1 | 716.9    | 430.0 | 430.0   | 7366.4 | 1647.9 | 10160.6 |
|         | ULTRA-McTB | •          | 36.1 | 57.1 | –        | –     | –       | –     | –     | 3967.3 |

topologies: The London network consists of one large metropolitan area. Here, public transit is common but often features many intermediate stops, so the average travel speed is fairly low across long distances. If the transfer mode is fast enough, it becomes competitive in terms of travel time. By contrast, long-distance journeys in Switzerland often involve high-speed trains, where even fast transfer modes are not competitive.

**Shortcut Computation.** We now evaluate the McULTRA shortcut computation. Following [3], the transfer graphs were contracted up to an average vertex degree of 20 for Germany and 14 for the other networks. As mentioned in Section 3, the first route scanning phase of each McRAPTOR iteration may use simplified RAPTOR-like route scans, but this may lead to superfluous shortcuts. On the Switzerland network with both witness limits set to $\infty$, using McRAPTOR-like route scans yields 4% fewer shortcuts for the stop-to-stop variant and 1% for the event-to-event variant. However, this increases the shortcut computation time by 16% and 10%, respectively. Hence, RAPTOR-like route scans are used for all subsequent experiments. Similarly, the intermediate witness limit is always set to $\bar{\tau}_1 = 0$ since this reduces the computation time by 60% while only producing 2% more shortcuts.

Figure 3 shows the impact of the Dijkstra label key and the final witness limit $\bar{\tau}_2$ on the computation time and the number of shortcuts in the event-to-event variant. Especially when the arrival time is weighted heavily, imposing a final witness limit saves considerable computation time, at the cost of producing noticeably more shortcuts. The latter can be mitigated
Figure 3: Impact of key choice and final witness limit $\bar{\tau}_2$ on the shortcut computation time and the number of shortcuts of event-to-event McULTRA, measured on the Switzerland network. For a label with arrival time $\tau_{\text{arr}}$ and transfer time $\tau_t$, a key ratio of $w_{\text{arr}} : w_t$ indicates a key value of $(w_{\text{arr}} \cdot \tau_{\text{arr}} + w_t \cdot \tau_t)/(w_{\text{arr}} + w_t)$. A weight of 0 indicates that the criterion is only used as a tiebreaker.

Figure 4: Impact of transfer speed for the Switzerland network. Left: Ratio of shortcuts compared to a transfer speed of 4.5 km/h. Speed limits in the network were obeyed for the lines with filled circles and ignored for the lines with empty circles. Right: Query performance of MCR and ULTRA-based algorithms, averaged over 1000 random queries. Speed limits were obeyed. For the RAPTOR-based algorithms, query times are divided into route collecting/scanning, transfer relaxation, and remaining time.

by weighting the transfer time more heavily, but this increases the computation time for strict final witness limits. This is explained by the fact that many improper candidates are weakly dominated by witnesses found in previous McRAPTOR iterations, and therefore tend to have higher arrival times than proper candidates. Increasing the arrival time weight moves proper candidates towards the front of the queue, allowing the stopping criterion to be applied earlier. To strike a balance between computation time and number of shortcuts, we choose weights $w_{\text{arr}} = 1$ and $w_t = 0$ as well as a final witness limit $\bar{\tau}_2 = 60$ min for all subsequent experiments.
Table 4: Performance of bounded queries with $\sigma_{\text{arr}} = \sigma_{\text{tr}} = 1.25$, averaged over 10000 random queries. Query times are divided into forward and backward pruning searches and main search. Also reported are the number of rounds performed by the main search (Rnd.) and the number of found journeys (Jrn.). Performance of two-criteria algorithms is shown for comparison. (BM-)RAPTOR* only supports stop-to-stop queries with transitive transfers.

| Network | Algorithm      | Full graph | Rnd. | Jrn. | Time [ms] |
|---------|----------------|------------|------|------|-----------|
|         |                |            |      |      | Forward | Backward | Main | Total |
| London  | RAPTOR*        | ◯          | 7.8  | 2.5  | –       | –       | –    | 5.2   |
|         | ULTRA-RAPTOR   | ●          | 8.2  | 4.3  | –       | –       | –    | 6.3   |
|         | ULTRA-TB       | ●          | 7.1  | 4.3  | –       | –       | –    | 4.3   |
|         | BM-RAPTOR*     | ◯          | 5.3  | 8.0  | 23.2    | 5.2     | 16.4 | 44.7  |
|         | ULTRA-BM-RAPTOR| ●          | 4.6  | 12.5 | 10.7    | 3.0     | 7.7  | 21.3  |
|         | ULTRA-BM-TB    | ●          | 4.6  | 12.5 | 10.0    | 1.8     | 3.8  | 15.5  |
|         | RAPTOR*        | ◯          | 8.3  | 2.5  | –       | –       | –    | 12.3  |
|         | ULTRA-RAPTOR   | ●          | 8.3  | 4.6  | –       | –       | –    | 12.8  |
|         | ULTRA-TB       | ●          | 7.5  | 4.6  | –       | –       | –    | 5.0   |
| Switzerland | BM-RAPTOR* | ◯          | 5.7  | 4.8  | 23.9    | 4.4     | 10.9 | 39.3  |
|         | ULTRA-BM-RAPTOR| ●          | 4.8  | 9.2  | 21.4    | 3.4     | 8.9  | 33.7  |
|         | ULTRA-BM-TB    | ●          | 4.8  | 9.2  | 12.1    | 1.4     | 2.5  | 15.9  |
|         | RAPTOR*        | ◯          | 10.2 | 2.8  | –       | –       | –    | 34.5  |
|         | ULTRA-RAPTOR   | ●          | 10.4 | 5.3  | –       | –       | –    | 365.2 |
|         | ULTRA-TB       | ●          | 9.7  | 5.3  | –       | –       | –    | 89.1  |
| Germany | BM-RAPTOR*     | ◯          | 7.0  | 7.9  | 465.6   | 51.8    | 150.0| 667.5 |
|         | ULTRA-BM-RAPTOR| ●          | 5.9  | 13.8 | 550.4   | 40.8    | 120.8| 711.9 |
|         | ULTRA-BM-TB    | ●          | 5.9  | 13.8 | 236.1   | 21.2    | 35.5 | 292.9 |

Table 2 shows overall results for the shortcut computation on all networks. Compared to two-criteria ULTRA, the number of shortcuts increases by less than a factor of 2. The shortcut computation takes 2–3 times longer in the stop-to-stop variant and 4–5 times longer in the event-to-event variant, with slightly higher values for the London network. By contrast, three-criteria MCR has a slowdown of about 20 compared to its two-criteria variant, MR-$\infty$, indicating that the shortcut computation scales much better for the additional criterion than MCR.

**Full Pareto Queries.** We evaluated two query algorithms for computing full Pareto sets: ULTRA-McRAPTOR (combining McRAPTOR with stop-to-stop McULTRA shortcuts) and ULTRA-McTB (combining McTB with event-to-event McULTRA shortcuts). Query times are reported in Table 3. Compared to MCR, the fastest previously known algorithm, ULTRA-McRAPTOR is about twice as fast and ULTRA-McTB achieves a speedup of 6–8. As in the two-criteria scenario, ULTRA-McRAPTOR achieves its speedup mostly by significantly reducing the transfer relaxation time, yielding similar query times to McRAPTOR on a transitivity closed transfer graph. The slight slowdown compared to transitive McRAPTOR is explained by the significantly higher number of found journeys, indicating that unlimited transfers are especially crucial in a multicriteria setting.

**Impact of Transfer Speed.** Figure 4 shows how McULTRA is affected by the speed of the transfer mode. For two-criteria ULTRA, Baum et al. [4] observed that the number of shortcuts declines once the transfer speed becomes competitive with public transit. With arrival time and number of trips as criteria, there is no reason to use public transit unless it saves travel time, so fewer shortcuts are produced for higher transfer speeds. This is no longer the case when adding transfer time as a criterion, since making a public transit detour can save transfer time. Consequently, the number of shortcuts increases for high transfer speeds. This effect is much stronger for the stop-to-stop variant than for the event-to-event variant, indicating that most of the additional shortcuts are only required at a few specific times during the day. All query algorithms become slower for higher transfer speeds as the search space increases. ULTRA-McRAPTOR is practical for transfer speeds up to 20 km/h (which is faster than the average travel speed via bicycle or e-scooter), but loses its advantage over
Table 5: Performance of bounded queries for different slack values on the Switzerland network, averaged over 10,000 random queries. Query times are divided into forward and backward pruning searches and main search. Also reported are the number of rounds performed by the main search (Rnd.) and the number of found journeys (Jrn.).

| Algorithm       | \(\sigma_{tr}\) | \(\sigma_{arr}\) | Full graph | Rnd. | Jrn. | Time [ms] |
|-----------------|------------------|-------------------|------------|------|------|-----------|
|                 |                  |                   |            |      |      | Forward   | Backward | Main   | Total  |
| BM-RAPTOR*      | 1.25             | 1.2               | ∘          | 5.7  | 4.5  | 22.9      | 3.4       | 8.3    | 34.5   |
|                 | 1.25             | 1.25              | ∘          | 5.7  | 4.8  | 23.9      | 4.4       | 10.9   | 39.3   |
|                 | 1.25             | 1.3               | ∘          | 5.7  | 5.1  | 24.7      | 5.3       | 13.2   | 43.2   |
|                 | 1.25             | 1.5               | ∘          | 5.7  | 5.8  | 27.8      | 8.5       | 22.4   | 58.7   |
|                 | 1.5              | 1.2               | ∘          | 6.5  | 5.1  | 23.3      | 5.4       | 14.6   | 43.2   |
|                 | 1.5              | 1.25              | ∘          | 6.5  | 5.4  | 24.1      | 6.9       | 19.2   | 50.2   |
|                 | 1.5              | 1.3               | ∘          | 6.5  | 5.9  | 24.5      | 8.1       | 23.1   | 55.7   |
|                 | 1.5              | 1.5               | ∘          | 6.5  | 6.9  | 26.3      | 12.9      | 39.0   | 78.2   |
| \(\sigma_{tr}\) | 1.25             | 1.2               | •          | 4.8  | 8.7  | 21.0      | 2.9       | 7.5    | 31.4   |
|                 | 1.25             | 1.25              | •          | 4.8  | 9.2  | 21.4      | 3.4       | 8.9    | 33.7   |
|                 | 1.25             | 1.3               | •          | 4.8  | 9.6  | 22.2      | 4.0       | 10.2   | 36.3   |
|                 | 1.25             | 1.5               | •          | 4.8  | 10.6 | 24.5      | 6.5       | 15.9   | 46.8   |
| ULTRA-BM-RAPTOR | 1.5              | 1.2               | •          | 5.6  | 9.3  | 20.7      | 3.4       | 8.9    | 33.0   |
|                 | 1.5              | 1.25              | •          | 5.6  | 10.0 | 21.4      | 4.2       | 10.7   | 36.2   |
|                 | 1.5              | 1.3               | •          | 5.6  | 10.5 | 22.3      | 4.8       | 12.4   | 39.5   |
|                 | 1.5              | 1.5               | •          | 5.6  | 11.9 | 24.3      | 7.7       | 20.2   | 52.3   |
|                 | 1.25             | 1.2               | •          | 4.8  | 8.7  | 11.3      | 1.1       | 1.7    | 14.1   |
|                 | 1.25             | 1.25              | •          | 4.8  | 9.2  | 12.1      | 1.4       | 2.5    | 15.9   |
|                 | 1.25             | 1.3               | •          | 4.8  | 9.6  | 12.9      | 1.7       | 3.3    | 17.9   |
|                 | 1.25             | 1.5               | •          | 4.8  | 10.6 | 16.5      | 3.3       | 7.4    | 26.7   |
| ULTRA-BM-TB     | 1.5              | 1.2               | •          | 5.7  | 9.3  | 11.7      | 1.3       | 2.4    | 15.5   |
|                 | 1.5              | 1.25              | •          | 5.7  | 10.0 | 12.5      | 1.7       | 3.5    | 17.8   |
|                 | 1.5              | 1.3               | •          | 5.7  | 10.5 | 13.3      | 2.1       | 4.8    | 20.3   |
|                 | 1.5              | 1.5               | •          | 5.7  | 11.9 | 15.7      | 4.0       | 10.6   | 30.4   |

MCR for transfer speeds above 30 km/h. By contrast, ULTRA-McTB maintains a speedup of 5–6 even for transfer speeds up to 40 km/h, due to the slower increase in the number of shortcuts.

**Bounded Queries.** We conclude by evaluating our bounded query algorithms. As shown in Table 2, shortcut augmentation increases the number of shortcuts by a factor of 3. The augmentation takes less than a minute even for Germany and is therefore negligible compared to the shortcut computation time. Query times for the bounded algorithms are shown in Table 4. Based on the results from Figure 2, we chose slack values \(\sigma_{arr} = \sigma_{tr} = 1.25\) as a good tradeoff between solution quality and query speed. The bounded algorithms are a factor of 2–4 slower than their two-criteria counterparts. This is consistent with the slowdown observed for BM-RAPTOR compared to RAPTOR on limited transfer graphs. Most of the running time is taken up by the forward pruning search, indicating that the pruning scheme is highly effective. Due to weaker target pruning and the algorithmic changes discussed in Section 5, the forward pruning search is slower than a two-criteria query, particularly for ULTRA-BM-TB. Nevertheless, ULTRA-BM-TB is up to 2.5 times faster than ULTRA-BM-RAPTOR. Compared to the ULTRA-based algorithms for full Pareto sets, the bounded variants achieve a speedup of around an order of magnitude. Compared to MCR, which was previously the fastest algorithm for this problem setting, ULTRA-BM-TB achieves a speedup of 30–80.
ately by the arrival slack, due to the weaker target pruning. The backward pruning searches and main search take significantly longer for higher slack values, and start to dominate the overall running time for $\sigma_{\text{tr}} = 1.5$ and $\sigma_{\text{arr}} = 1.5$. However, even for high slack values, the bounded algorithms remain much faster than their unbounded counterparts.

7 Conclusion

We showed that in order to improve the solution quality in a multimodal network with unlimited transfers, it is necessary to optimize transfer time as a third criterion. To achieve this, we developed McTB, a fast three-criteria algorithm that avoids costly dynamic data structures. To enable unlimited transfers, we proposed McULTRA, a three-criteria extension of ULTRA. Our shortcut computation algorithm runs in reasonable time and produces less than twice as many shortcuts as two-criteria ULTRA. The combination of McULTRA and McTB achieves a speedup of 6–8 over the state of the art and remains practical even for fast transfer modes. Finally, we developed RAPTOR- and TB-based algorithms for computing restricted Pareto sets in a multimodal network by using ULTRA shortcuts. The latter is up to 80 times faster than MCR, the fastest previously known algorithm for three-criteria multimodal queries.

Future work could involve adapting ULTRA for additional criteria, such as fare or occupancy rate. It is unclear whether an efficient TB-based algorithm can be designed for more than three criteria. However, the results reported in [5] suggest that BM-RAPTOR scales well for additional criteria since the pruning searches take up the majority of the running time. Therefore, combining TB-based pruning searches with a McRAPTOR-based main search may be a promising approach for designing an efficient bounded query algorithm.

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