On asymptotic behaviour of galactic rotation curves in superfluid vacuum theory

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According to the Newton’s theory of gravity, rotation curves, i.e., rotation velocities as functions of distances from a gravitating center, of free-falling non-relativistic test particles must have a Keplerian form, which means that their velocity must be inversely proportional to the square root of distance. However, numerous astronomical observations, of both stars and luminous gas in galaxies, show significant deviation from this behavior. For most of galaxies, the data shows rotation curves are flat (FRC), i.e., the tangent lines of rotation curves are approximately horizontal, for a wide range of distances from a center [1].

There are currently two main approaches to address this problem. One approach is popular among particle physicists working in the Standard Model (SM) theory of strong and electroweak interactions and its extensions. This approach assumes that some kind of corpuscular matter exists, referred to as dark matter (DM), which interacts with known SM particles via gravity, but otherwise very weakly, if at all. Such matter seems to be truly elusive: its particles must be abundant in galaxies, including our Milky Way; yet they do not, to the best of our knowledge, affect Earth’s particle experiments.

The other approach assumes that DM-attributed phenomena are not caused by any unknown corpuscular matter; but that gravity alone alters its behaviour from being Newtonian-like at length scales which are very different from our habitual scale. This trivially resolves the question of the absence of SM-DM interactions in Earth’s experimental conditions. The scale dependence of gravity also explains the significantly non-Keplerian behaviour of galactic rotation curves, while orbits of planets and satellites remain Keplerian to a high degree of precision. This approach is also very popular: in fact, it is hard to find a theory of modified gravity which does not explain the non-Keplerian phenomenon of galactic rotation curves. However, this approach is not without its drawbacks. First, because the number of mutually exclusive gravity theories is currently so large, this approach’s status quo lacks universality and unification. More importantly, most theories of modified gravity (that we know of) require a different set of parameters for each galaxy; which raises the question of why every galaxy, or even every isolated supermassive body, should “have” its own theory of gravity.

Therefore, some criteria for a convincing theory of astronomical scale phenomena are necessary; in addition to those of bona fide unification, universality across all galaxies, and minimal free parameters. One requirement is that a candidate theory should predict hitherto unknown phenomena at a galactic scale, or even beyond, which can be empirically verified. Another expectation is that it should describe phenomena, which are attributed not only to dark matter, but also to dark energy (DE). These would include the accelerating expansion of the Universe, Hubble law and the notable discrepancy between outcomes of the different measurement methods of the Hubble constant; those based on cosmic microwave background radiation (suggesting a value $67.4 \pm 0.5 \text{ km/s/Mpc}$ for a Hubble constant); versus other methods, such as distance ladder measurements using Cepheids ($74 \pm 1 \text{ km/s/Mpc}$) and red giants ($70 \pm 2 \text{ km/s/Mpc}$), and geometric distance measurements using megamasers ($74 \pm 3 \text{ km/s/Mpc}$).

There is also growing consensus that a convincing explanation of DM- and DE-attributed phenomena cannot be based on a stand-alone model, but must be a part of a fundamental theory, involving other fundamental interactions known to date. Such a theory should definitely operate at an utterly quantum level, which implies re-
formulating the concept of gravity as a quantum phenomenon, commonly referred to as quantum gravity.

One of the most viable candidates for such a theory of physical vacuum and quantum gravity is superfluid vacuum theory (SVT), a post-relativistic approach to high-energy physics and gravity. This is essentially a framework, which evolved from Dirac’s idea of non-removable quantum matter filling an otherwise empty three-dimensional space \( \mathbb{R}^3 \); a pedagogical introduction can be found in monographs \([3, 4]\). As a matter of fact, superfluid vacuum theory possesses features of both corpuscular DM and modified-gravity approaches: it is not only an essentially quantum many-particle theory in its foundations, but it also induces and includes relativistic gravity and spacetime in the “phononic” or linear low-momentum limit of dispersion relations for excitations, which are thus observed as relativistic particles.

Within the framework of the superfluid vacuum paradigm, various theories currently co-exist, which share the main idea, but which offer different views of the dynamics and structure of the physical vacuum. One of these theories is logarithmic superfluid vacuum theory, whose foundations can be found in \([5, 6]\). To the best of our knowledge, this model is not only free from the above-mentioned limitations, but also explains the known DM and DE-attributed phenomena, and predicts a number of new effects, including vacuum Cherenkov radiation, deformed dispersion relations, mass generation and superfluid stars \([7, 12]\). More relevant to the purposes of this paper, works \([6, 11]\) describe a number of astronomical-scale effects predicted by logarithmic SVT; including various expansion mechanisms and non-Keplerian (flat followed by non-flat) behaviour of galactic rotation curves. The latter will be our main focus here.

Logarithmic superfluid vacuum theory assumes that physical vacuum is quantum Bose liquid described by a condensate wavefunction obeying a nonlinear wave equation with logarithmic nonlinearity, whose parameters are: the inertial mass \( m \) of fundamental background condensate’s particles, the critical density \( \bar{\rho} \), and the real-valued constant parameters \( b_0 \) and \( q \) of the nonlinear coupling. A vast amount of mathematical literature dealing with such equations exists, to mention just very recent examples \([13, 12]\). In the general theory of superfluids, logarithmic nonlinearity describes many-body effects – in fact, it can be used as a leading-order or robust approximation for a large class of strongly interacting liquids in which the characteristic kinetic energies of particles are less than the inter-particle potentials \([19]\). Logarithmic fluid models are known to work very well for laboratory superfluids, such as the helium II phase, where they have resolved a number of long-standing problems \([21, 22]\) and predicted new effects \([9]\).

When applying them to the theory of physical vacuum, one can show that this background superfluid induces a four-dimensional spacetime, while photon-like excitations are somewhat analogous to acoustic waves in laboratory superfluids, traveling at the speed \( c_b = \sqrt{b_0/\bar{m}} \) in the approximation of a homogeneous isothermal liquid \([6, 11]\). In the “phononic” (low-momentum) limit, \( c_b \to c(0) \approx c \) where \( c = 2.998 \times 10^5 \) km/s is called, for historical reasons, the speed of light in vacuum.

For our purposes here, we assume spherical symmetry, and omit terms which decay faster than an inverse distance at spatial infinity, in which case the induced metric can be written in static coordinates in the form \([11]\):

\[
ds^2 = -K^2 c(0)^2 dt^2 + \frac{1}{K^2} dr^2 + r^2 d\sigma^2, \tag{1}
\]

where \( d\sigma^2 = d\theta^2 + \sin^2\theta d\varphi^2 \) is the line element of a unit two-sphere, \( \delta_0 = 2(a_1b_0 + a_2q/\ell^2)/mc^2(0) \), \( \ell = (m/\bar{\rho})^{1/3}, r_H = 2a_1q/mc^2(0) \), \( \beta = \sqrt{(2\chi b_0/mc^2(0))/R_{2\Phi}} \), \( R_{2\Phi} = mc^2(0)/2a_1b_0, R_{4\Phi} = \ell / (mc^2(0)/2a_2b_0) \).

One can see that the induced spacetime metric depends on a number of parameters, which can be divided into two groups. The first consists of parameters of the model’s Hamiltonian: fundamental mass scale \( m \), fundamental density scale \( \bar{\rho} \), and parameters of the nonlinear coupling, \( b_0 \) and \( q \). However, only two of these parameters, \( m \) and \( \bar{\rho} \), are \textit{ab initio} fixed; whereas the other two, \( b_0 \) and \( q \), can vary depending on the environment, because the nonlinear coupling is a linear function of quantum temperature. The latter is defined as a thermodynamic conjugate of quantum information entropy, sometimes dubbed as the Everett-Hirschman entropy, and is conjectured to be linearly related to conventional (thermal) temperature \([20]\).

The second group are parameters \( \alpha \)'s and \( \chi \) of a trial wavefunction of the state \( |\Psi_{vac}\rangle \) our superfluid is in, which is defined as a solution of a logarithmic quantum wave equation, further details can be found in section 3 of \([11]\). We expect that this state is stable, or at least metastable, with a sufficiently large lifetime; therefore it is thus natural to assume that it is a ground state, or close to, stationary and rotationally invariant. Because such a wavefunction is obviously affected by surrounding matter, parameters \( \alpha \)'s and \( \chi \) are also environment-dependent.

When it comes to galactic scale effects, this means that only parameters \( m \) and \( \bar{\rho} \) are galaxy-independent; others would generally vary from galaxy to galaxy, because every galaxy has its own quantum temperature and localized vacuum wavefunction; the latter to be regarded as a perturbation of the vacuum wavefunction for a larger superset configuration of gravitating matter. This resolves the above-mentioned issue of the modified gravity approach.

Furthermore, different terms of the metric \([11]\) come into play at different length scales. The Schwarzschild term, \(-r_H/r\), should dominate in the inner regions of galaxies, where it induces the Keplerian-type orbits. On the contrary, the quadratic (de Sitter) term dominates at the largest length scale. It is probably negligible at
stellar disc \[27, 28\], then we obtain take into account the combined contribution of gas and attraction, at the distance \(\ell\). Finally, the linear term, \(r/R_{\text{GR}}\), becomes significant at extragalactic distances, but probably at the outskirts of galaxies as well, depending on a value of \(R_{\text{GR}}\).

These features seem plausible for describing DM- and DE-attributed phenomena \[22\]. It is also worth noting that while the linear and quadratic terms were already known to occur in some theories of modified gravity \[24–26\], the logarithmic term’s occurrence is, to the best of our knowledge, unique to the logarithmic superfluid vacuum theory.

The non-relativistic rotational velocity curves can be estimated using a simple formula \(v^2(R) = \frac{1}{2} \frac{d}{dr}(c_{0}^2 K^2)\), where \(R\) is the orbit’s radius. If we take into account the combined contribution of gas and stellar disc \[27, 28\], then we obtain

\[
v = \sqrt{v_N^2 + v_X^2 + \frac{a_1 b_0}{m \ell} R - \frac{2a_2 b_0}{m \ell^2} R^2},
\]

where

\[
v_N^2 = \frac{4}{3} v_{\text{HI}}^2 + \frac{4}{3} v_\star^2 + \frac{GM_*}{2h_R^3} R^2 B \left( \frac{R}{2h_R} \right),
\]

\[
v_X^2 = \frac{1}{2} R \frac{d}{dr} \left[ c_{0}^2 (\beta^2 \ln \left( \frac{R}{\ell} \right) \right] = \chi b_0 \frac{\ell}{m},
\]

where \(M_*\) and \(h_R\) are, respectively, the total gravitational mass and surface brightness’ scale length of the stellar disc, \(B(x) = I_0(x) K_0(x) - I_1(x) K_1(x)\), \(I_n(x)\) and \(K_n(x)\) are modified Bessel functions of the first and second kind, respectively, and the mass ratio between helium and neutral hydrogen (HI) is assumed to be 1/3. Notice that the contribution \(1\) is independent of \(R\) which proves our earlier statement about the relation between the FRC phenomenon and logarithmic term in the induced metric. Thus, a rotation curve would be asymptotically flat if \(a_1 = a_2 = 0\) identically, while the Keplerian term \(v_N^2\) rapidly decreases as \(R\) grows, thus making the term \(1\) to predominate in Eq. \(2\) at large \(R\). However, if either or both of \(a_1\) and \(a_2\) are not zero then we have what we call the asymptotically non-flat behaviour of a rotation curve.

Furthermore, let us consider the asymptotic behaviour of orbital velocity in the utmost outer regions of galaxies, but not much beyond; therefore we can neglect the de Sitter term. Our goal here is preliminary estimates, therefore we will not consider gas and disc contributions, for the sake of simplicity and minimal assumptions for galactic luminous matter. Fortunately, because we are considering distances so far away from the galactic center, almost at the galaxy’s border, luminous matter contributions should not be of a leading order of magnitude; although they are still significant \[27\].

From Eq. \(2\) we thus obtain a predicted asymptotic value of this velocity

\[
v \approx v_{\text{out}} = v_X \sqrt{1 + \frac{R}{L_X}},
\]

where a value \(L_X = \chi \ell / a_1\) thus defines the characteristic length scale of the crossover between flat and linear regimes of rotation curves. Because \(\ell\) is positive-definite, the sign of \(L_X\) equals that of the product of parameters \(a_1\) and \(\chi\) of a galaxy’s vacuum wavefunction.

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**TABLE I:** Results of the linear two-parameter best fit to the velocity-squared rotation curves of sample galaxies.

|       | \(R_{\text{min}}\), kpc | \(R_{\text{max}}\), kpc | \(L_X\), kpc | \(v_{\text{min}}\), km/s | \(v_{\text{max}}\), km/s | \(v_X\), km/s |
|-------|-----------------|-----------------|----------|-----------------|-----------------|-------|
| DDO 154 | 10.96           | 13.78           | 13.47    | 43.62           | 46.97           | 43.81 |
| NGC 2403 | 12.73           | 15.52           | 13.95    | 132.3           | 136.2           | 133.0 |
| NGC 2841 | 34.69           | 44.14           | 389.0    | 251.4           | 264.5           | 260.7 |
| NGC 2903 | 22.66           | 28.51           | -91.04   | 178.5           | 182.5           | 181.0 |
| NGC 3198 | 34.15           | 43.18           | 173.0    | 145.6           | 150.3           | 146.4 |
| NGC 3521 | 27.24           | 34.25           | 2439     | 187.0           | 194.0           | 188.1 |
| NGC 5055 | 24.98           | 31.59           | -41.66   | 175.2           | 183.8           | 189.4 |
| NGC 7331 | 29.92           | 39.55           | 159.9    | 221.2           | 238.3           | 233.1 |
To analyze experimental data, it is convenient to square both sides of Eq. (5) and perform the least-square fitting of a linear function of radial distance, $v^2 \rightarrow c_1 R + c_0$. Sample disc galaxies from the THINGS database are listed in Table I for each of those we took the ten furthest orbits data points known to the database. From each galaxy's data set we derived two subsets: the main subset, with the smallest-$R$ twenty per cent of points dropped, and an auxiliary subset, with the highest-$R$ twenty per cent of points omitted.
An auxiliary subset is expected to estimate parameters of the flat regime, such as $v_\chi$. This velocity is determined by evaluating the best-fitting line of the subset at the smallest-$R$ point of the subset, $R_{\text{min}}$. The data point which corresponds to $R_{\text{min}}$ is denoted as $v_{\text{min}}$.

The main subset is used to estimate possible deviations from the flat regime, as predicted by Eq. (5). The data point corresponding to the largest-$R$ point of the subset, $R_{\text{max}}$, is denoted as $v_{\text{max}}$. The value $L_\chi$ is determined as the ratio of the corresponding $v^2_\chi$ and a linear coefficient of the best-fitting line of the subset.

The outcome of the fitting procedure is given in Table I and Fig. 1. Substantial inclinations of solid lines can clearly be seen in the figure. This indicates the presence of a crossover transition from flat to linear regimes of rotation with increasing $R$, as predicted by Eq. (5): because tangent lines of best fits become no longer exactly horizontal for the furthest data points, then rotation curves become non-flat as one approaches galactic boundaries.

For a number of galaxies, NGC 2841, NGC 3198, NGC 7331 and especially NGC 3521, the computed characteristic linear-regime length value $|L_\chi|$ turns out to be much larger than $R_{\text{max}}$, by one, or even two orders of magnitude. This is because the majority of data points still belong to the flat regime; while the full crossover to the post-flat regime occurs beyond the chosen data range. This is still compatible with the theory; because the linear term in the metric is expected to predominate at extragalactic length scales, while the logarithmic term is most significant in the outer regions of galaxies.

These preliminary estimates are sufficient for the purposes of the current paper, while the detailed fitting of rotation curves, which obviously require the combined contribution from the gas and stellar disc been taken into account, will remain as the subject of future studies.

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[1] V. C. Rubin, W. K. Ford Jr., and N. Thonnard, Astrophys. J. 238, 471 (1980).
[2] P. A. M. Dirac, Nature 168, 906 (1951).
[3] G. E. Volovik, The Universe in a Helium Droplet (Oxford University Press, Oxford, 2009).
[4] K. Huang, A Superfluid Universe (World Scientific, Hackensack, 2016).
[5] K. G. Zloshchastiev, Grav. Cosmol. 16, 288 (2010).
[6] K. G. Zloshchastiev, Acta Phys. Polon. 42, 261 (2011).
[7] K. G. Zloshchastiev, Phys. Lett. A 375, 2305 (2011).
[8] V. Dzhunushaliev and K. G. Zloshchastiev, Central Eur. J. Phys. 11, 325 (2013). [arXiv:1204.6380].
[9] K. G. Zloshchastiev, Int. J. Mod. Phys. B 33, 1950184 (2019).
[10] K. G. Zloshchastiev, Int. J. Mod. Phys. A 35, 2040032 (2020).
[11] K. G. Zloshchastiev, Universe 6, 180 (2020).
[12] K. G. Zloshchastiev, Low Temp. Phys. 47, 89 (2021).
[13] L. Zhang and W. Hou, Appl. Math. Lett. 102, 106149 (2020).
[14] C. O. Alves and C. Ji, Discret. Contin. Dyn. Syst. 40, 2671 (2020).
[15] J. Shertzer and T. C. Scott, J. Phys. Commun. 4, 065004 (2020).
[16] T. Boudjeriou, J. Elliptic Parabol. Equ. 6, 773 (2020).
[17] F. C. E. Lima and C. A. S. Almeida, Europhys. Lett. (EPL) 131, 31003 (2020).
[18] Z. Zhou and Z. Yan, Phys. Lett. A 387, 127010 (2020).
[19] Q. Yang and C. Bai, AIMS Math. 6, 886 (2021).
[20] K. G. Zloshchastiev, Z. Naturforsch. A 73, 619 (2018).
[21] K. G. Zloshchastiev, Eur. Phys. J. B 85, 273 (2012).
[22] T. C. Scott and K. G. Zloshchastiev, Low Temp. Phys. 45, 1231 (2019).
[23] K. Pardo and D. N. Spergel, Phys. Rev. Lett. 125, 211101 (2020).
[24] P. D. Mannheim and D. Kazanas, Astrophys. J. 342, 635 (1989).
[25] D. Grumiller, Phys. Rev. Lett. 105, 211303 (2010). Erratum: Phys. Rev. Lett. 106, 039901 (2011).
[26] H.-N. Lin, M.-H. Li, X. Li, and Z. Chang, Mon. Not. R. Astron. Soc. 430, 450 (2013).
[27] A. Toomre, Astrophys. J. 138, 385 (1963).
[28] S. Casertano, Mon. Not. R. Astron. Soc. 203, 735 (1983).