Development of the crack-line-update method for two-dimensional piercing simulations

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Abstract. We have developed the crack-line-update method for two-dimensional piercing simulations by a finite element method. This method is combined with a remeshing process involving an outer line and meshing. By using the crack-line-update method, crack lines, which are derived from a stress and strain analysis of the high-damage regions, are added to the outer line in every remeshing step. In this paper, we compare the simulations between the mesh deletion and crack-line-update methods. The crack-line-update method successfully simulated the arresting of cracks during the piercing process, whereas the mesh deletion method could not express such fracture behavior.

1. Introduction

Piercing is the most common method for making holes on metal sheets, and high residual stresses and work-hardening on pierced surfaces remarkably deteriorate their formability[1] and fatigue strength[2]. Delayed cracking caused by hydrogen embrittlement on the pierced surface of ultrahigh strength steel sheets is also an issue [3].

Numerical piercing simulations enable virtual piercing through trial-and-error in a variety of tool conditions. It can be used to accelerate tool development and suppress the abovementioned defects at low cost. Therefore, many researchers and engineers have developed accurate numerical piercing simulations. However, some problems still exist for the accurate simulation of state values on pierced surfaces.

An insufficient fracture expression in a finite element (FE) simulation is one of these problems. The conventional mesh deletion method, where almost zero stiffness is imposed on the elements, cannot simulate a realistic fracture surface during a piercing process as cracks are assumed to be included in deleted meshes. This assumption prevents the prediction of state values on the fractured surface. Node separation can address this problem; however, it requires many manual operations that involve much human power. Further, the simulation results with node separation are affected by the mesh shapes and arrangements.

Considering the above background, we developed the crack-line-update method for two-dimensional FE simulations of piercings. This method is combined with a remeshing process involving an outer line and meshing. By using the crack-line-update method, crack lines, which are derived from stress and strain analyses of high-damage regions, are added to the outer line in every
remeshing step. This method does not require manual operations and the meshes do not affect the crack-line derivation.

This paper shows the details of the algorithm for the derivation of a crack-line, and demonstrates the proposed method. The crack-line-update method successfully simulated the arresting of cracks during the piercing process, whereas the mesh deletion method could not express such fracture behavior.

2. The crack-line-update method

2.1. The algorithm for crack-line addition

A crack-line is introduced in such a way that it splits a set of elements in which the damage values for fracture analysis exceed the fracture criterion. Almost all of the fracture criteria are available to classify a fracture region. Figure 1 shows a schematic of the crack-line-update method.

A crack-line is assumed to be straight and begins at a node that has the maximum damage value on the surface of a fracture region. The nodal damage value is calculated by averaging the damage values of the elements where the node belongs. The direction of a crack-line is derived from the average value of stresses over all the elements belonging to the fracture region. We are now considering two types of derivations, which are explained in Sect. 2.2. The crack-line ends at the crosspoint at the edges of the fracture region. Doubling the nodes at the start point makes a crack-line a newly added outline.

The above process is repeated at every remeshing step, which involves outline extraction from old meshes and meshing on newly extracted shapes. The crack-line is added to the newly extracted shape. This method might seem similar to the node separation method; however, they are completely different. The direction and end of a crack-line is independent of the nodes of the old meshes.

To simplify the cracking behavior for the stable simulations, additional processes are included in the present version of the crack-line-update method. First, the crack-line added for the case of a fracture region including a surface, i.e., internal fractures, is neglected. Secondly, a crack-line is not added in case a crack does not lie on a fracture region. Such a case is possible for certain crack-line directions and starting positions.

2.2. Crack-line direction

Two types of derivations, the directions of maximum principal stress and shearing bands, are applied using the average stress over all the elements that belong to a fracture region.
In the derivation of the direction of the maximum principal stress, a crack-line direction is defined as that being perpendicular to the direction of maximum principal stress. This is based on the brittle fracture mechanism that is commonly known.

The other derivation is based on plastic strain localization, which is observed as the onset of a shear band. To analyze the shear band direction, this study adopted a Stören and Rice type bifurcation criterion[4] with the flow law proposed by Ito and Goya[5]. To explain the Stören and Rice bifurcation criterion, we define a homogeneous continuum, $\Omega$, in a critical mechanical equilibrium, wherein a shear band, $\partial \Omega$, has been formed as shown in Fig.2. The velocity gradient tensor in the shear band, $L^\alpha$, is expressed by the tensor out of the shear band, $L^\alpha$, and the normal vector of the shear band, $n$, as follows:

$$L^\alpha = L^\alpha + g \otimes n ,$$  \hspace{1cm} (1)$$

where $g$ denotes the jump in the velocity gradient, and is non-zero only within the shear band. The following time derivations of the first Piola–Kirchhoff stress tensors $\dot{\sigma}^\alpha$, $\partial \sigma^\alpha$ in and out of the shear band are derived from Eq.(1).

$$\dot{\sigma}^\alpha = A^\alpha \cdot \dot{L}^\alpha ,$$  \hspace{1cm} (2)$$

$$\partial \sigma^\alpha = A^\alpha \cdot \partial L^\alpha = A^\alpha \left( L^\alpha + g \otimes n \right) ,$$  \hspace{1cm} (3)$$

where

$$A^\alpha_{\beta\beta} = D^\alpha_{\beta\beta} + \frac{1}{2} \left( \sigma^\alpha_{\beta\beta} \delta^\alpha_{\beta\beta} - \sigma^\alpha_{\beta\beta} \delta^\alpha_{\beta\beta} - \sigma^\alpha_{\beta\beta} \delta^\alpha_{\beta\beta} - \sigma^\alpha_{\beta\beta} \delta^\alpha_{\beta\beta} \right) .$$  \hspace{1cm} (4)$$

In Eq.(4), $D$ is the tangential stiffness tensor for the Jauman stress rate; $\sigma$ is the Cauchy stress tensor; and $\delta$ is the Kronecker-Delta. Assuming a homogeneous stress and strain distribution before shear band onset, i.e., $A^\alpha = A^\alpha \equiv A$, the difference between $\dot{\sigma}^\alpha$ and $\partial \sigma^\alpha$, defined as $\Delta \sigma$, is expressed as:

$$\Delta \sigma = A \cdot (g \otimes n) .$$  \hspace{1cm} (5)$$

Using Eq.(5), the stress continuity through the shear band interface yields the following:

$$\Delta \sigma \cdot n = A \cdot (g \otimes n) \cdot n = 0 ,$$  \hspace{1cm} (6)$$

which can be written in the componential form as

$$(A_{\alpha\beta} n_{\alpha} n_{\beta}) g_{\delta} = 0 .$$  \hspace{1cm} (7)$$

Defining the tensor $Q$ such that $Q_{\alpha\beta} = A_{\alpha\beta} n_{\alpha} n_{\beta}$ ($Q$ is called acoustic tensor), Eq.(7) transforms to Eq.(8):

$$Q \cdot g = 0 .$$  \hspace{1cm} (8)$$

Here, $g$ does not have a unique solution in Eq.(8) when $Q$ is singular. Considering the definition of $g$, shear band onset is observed when $Q$ becomes singular.

For $D$, we use the appropriate Ito and Goya’s tangential stiffness tensor for a linear comparison of a solid, as shown in Eq.(9):

$$D = \frac{S}{b} \left\{ aE_{\perp} - T \left( s \otimes s \right) \right\} + \left( K - \frac{1}{3} a \right) \left( E_{\perp} \otimes E_{\perp} \right) ,$$  \hspace{1cm} (9)$$

where

Fig.2 Schematic of a shear band

Ω (Continuum zone)
\[ S^c \equiv \frac{S}{2G}, \quad a \equiv 1 + S^c - S^c T \| s \|^2, \quad b \equiv \left( 1 + S^c \right) a, \quad S^c = \frac{2H^c}{3K_c}, \quad T \equiv \frac{3(1 - K_c)}{2\sigma^3}; \]

G is the shear modulus, \( H^c \) the work hardening ratio, \( s \) the deviatoric stress tensor, and \( K_c \) a material parameter.

Based on the above bifurcation criterion, we propose a crack-line direction to \( n \) that minimizes the determinant of \( Q \).

3. Demonstration of two-dimensional piercing simulations

3.1. Boundary Conditions

A schematic of the boundary conditions is shown in Fig.3. A common piercing process was considered for this demonstration.

The tools and work (specimen) were modelled for the two-dimensional axisymmetric condition. The piercing punch diameter was set to 10 mm, and clearance (i.e., punch radius minus die radius) to 0.4 mm. The edges of the tools were rounded to 0.066 mm radius of curvature for numerical stability. The holder, whose displacements are fixed, constrains the bending of the specimen. The horizontal displacement of the specimen edge was fixed.

![Fig.3 Boundary conditions for FE analysis](image)

3.2. Solver condition

A commercial static explicit FE code TP-MFORM 2D produced by Trial Park Co Ltd. was used for this study. In this solver, the errors induced by the tangential approximation of the deformation process are controlled by limiting the increments in the steps of all the sources of non-linearities using the so-called \( r_{\text{min}} \) strategy[6].

Quadrant linear elements with selective reduced integration were used. The minimum mesh size around the tool edges was approximately 0.1×0.1 mm. The meshes for this simulation were automatically updated when the minimum mesh aspect ratio decreased to 0.9 so that severe mesh distortion did not stop the simulation by producing a singularity.

FE simulations using the crack-line-update and mesh deletion methods were performed.

3.3. Material properties

The plastic properties were described using the von Mises yield criterion and an associated flow rule. The following Swift hardening function was applied to the hardening law:

\[ \sigma_p = 843 \left( \varepsilon_p + 0.00465 \right)^{0.05}, \quad (10) \]

where \( \sigma_p \) and \( \varepsilon_p \) denote the yield stress and equivalent plastic strain respectively.

Cockcroft and Latham’s ductile fracture criterion[7] was used for the classification of fracture regions. The criterion is expressed as follows.
\[ \int_0^\tau \frac{\sigma_{\text{max}}}{\bar{\sigma}} d\varepsilon_p = C. \tag{11} \]

where \( \bar{\sigma} \) and \( \sigma_{\text{max}} \) are the equivalent and maximum principal stresses respectively. In the demonstration, \( C \) is set to 1.0, and the integral on the left of Eq. (11) is defined as a damage value.

### 3.4. Results and discussion

To begin with, the simulation of the crack-line-update method could not be completed because it was stopped by the self-contact problem. The self-contact is already developed for FE simulation, but our FE solver have not still included it. Therefore, this section describes a comparison of the simulation halfway for both the mesh deletion and crack-line-update methods.

Figure 4a shows the result in the case of mesh deletion. The elements estimated as fracture are deleted, and the propagation speed is comparably higher than that for the crack-line-update method. Indeed, the crack-lines shown in Figs. 4b and 4c did not overtake the punch bottom even as the fracture region spread through the punch edge (see Fig. 4a). The crack perpendicular to the maximum stress was elongated in the horizontal direction, and opened up without any further propagation (Fig. 4b). On the other hand, the crack in the shear band onset reveals slant propagation (Fig. 4c). Figure 4c shows the crack closing behavior, wherein the crack surface is thrusting onto the other surface.

In practical piercing, a crack is frequently arrested around the punch (or die) edge. Thus, the crack-line-update method is considered to successfully simulate such crack arresting behavior. In the case of the perpendicular to the maximum principal stress, the crack-line deviated from the thickness direction of the specimen; thus, the separation of the specimen will be substantially delayed compared to the mesh deletion case. For the case of crack-line-update using the shear-band direction, the result will be intermediate between that of the mesh deletion and crack-line-update method based on the maximum principal stress. The deviation in the crack-line is not very large. The closure of the crack probably suppresses the elongation of the crack-line. As future work, we must examine the validity of two derivations for the crack-line direction.

### 4. Summary

This study developed the crack-line-update method for two-dimensional piercing simulations using a finite element method. The crack-line-update method is combined with a remeshing process that involves an outer line and meshing. By using the crack-line-update method, crack lines, which are derived from a stress and strain analysis of the high-damage regions, are added to the outer line in every remeshing step.

The crack-line directions were derived in two ways: one is the perpendicular to the maximum principal stress direction, and the other is the shear-band direction based on the bifurcation criterion. For bifurcation analysis, the study applied Ito and Goya’s tangential stiffness.

Although the demonstrated simulations stopped due to the self-contact problem, they suggest the effectiveness of the crack-line-update method, which successfully simulated the arresting of a practical crack, whereas the mesh deletion method could not express such fracture behavior. Two types of crack-line directions resulted in contrasting differences in crack opening and closure. We plan to validate these two behaviors in our future work.

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(a) Conventional element stiffness reduction

(b) The crack-line update method using the maximum principal stress direction

(c) The crack-line update method using the bifurcation direction

Fig. 4 Results of the numerical demonstration

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