PAULI–VILLARS REGULARIZATION IN A DISCRETE LIGHT-CONE MODEL

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Pauli–Villars regularization is successfully applied to nonperturbative calculations in a (3 + 1)-dimensional light-cone model. Numerical results obtained with discretized light-cone quantization compare favorably with the analytic solution.

1 Introduction

One of the challenges of using light-cone methods to solve nonperturbative problems in field theories such as quantum chromodynamics (QCD) is to develop a nonperturbative regularization and renormalization scheme. Here we discuss a new approach based on generalized Pauli–Villars regularization and discretized light-cone quantization (DLCQ). The DLCQ Fock basis is expanded to include momentum states of Pauli–Villars particles which provide the cancelations needed to regulate ultraviolet divergences. The determination of the number of Pauli–Villars particles and of their coupling strengths is done in perturbation theory, where loop integrals are rendered absolutely convergent.

We test this approach on a simple model designed to have an analytic solution for the lowest massive state. The model is related to an equal-time model of Greenberg and Schweber, which has also been translated to the light cone by Glazek and Perry.

2 Light-Cone Coordinates

We define light-cone coordinates and momentum components by

\[ x^\pm = t \pm z, \quad x_\perp = (x, y), \]

(1)
and
\[ p^+ = E \pm p_z, \quad p_\perp = (p_x, p_y), \quad p \equiv (p^+, p_\perp). \]  
\( \text{(2)} \)

The dot product is written
\[ p \cdot x = \frac{1}{2} (p^+ x^- + p^- x^+) - p_\perp \cdot x_\perp. \]  
\( \text{(3)} \)

The time variable is taken to be \( x^+ \). The conjugate variable \( p^- \) is the light-cone energy, and the associated operator \( \mathcal{P}^- \) determines the time evolution of the system. The mass-squared operator, frequently called the light-cone Hamiltonian, is
\[ H_{\text{LC}} = \mathcal{P}^+ \mathcal{P}^- - p_\perp^2, \]  
\( \text{(4)} \)

where \( \mathcal{P}^+ \) and \( \mathcal{P}_\perp \) are momentum operators conjugate to \( x^- \) and \( x_\perp \). In these coordinates, the fundamental eigenvalue problem is
\[ H_{\text{LC}} \Psi = M^2 \Psi, \quad \mathcal{P} \Psi = \mathcal{P} \Psi, \]  
\( \text{(5)} \)

where \( M \) is the mass of the state \( \Psi \).

The use of light-cone coordinates has several advantages. The generators of the Poincaré algebra have the largest possible nondynamical subset; in particular, boosts are kinematical. The perturbative vacuum is the physical vacuum, because \( p^+ = \sqrt{p^2 + m^2} + p_z > 0 \), and there is then no need to compute the vacuum state. Fock-state expansions are well defined with no disconnected vacuum pieces.

For field theories quantized in light-cone coordinates one does need to be careful about the use of Pauli–Villars regulators. The number of Pauli–Villars particles used in ordinary Feynman perturbation theory may not suffice. One concrete example can be found in Yukawa theory. To regulate the one-loop self-energy three heavy bosons are required, rather than the usual one. The reason that one is not sufficient is that symmetric integration techniques cannot be employed; the use of such techniques in equal-time quantization amounts to a regularization prescription which supplements the Pauli–Villars regularization.

To see the need for three heavy bosons explicitly, consider the self-energy integral
\[ I(\mu^2, M^2) \equiv -\frac{1}{\mu^2} \int \frac{dl^+ dl_\perp}{l^+(q^+ - l^+)^2} \times \frac{(q^+)^2 l_\perp^2 + (2q^+ - l^+)^2 M^2}{M^2 - D} \theta(\Lambda^2 - D), \]  
\( \text{(6)} \)

\(^{b}\)For theories with complicated vacuum structure in equal time formulations this simplicity of the vacuum is offset by a more complicated operator structure.
where $\mu$ is the ordinary boson mass, $M$ is the fermion mass, and $D$ is the invariant mass of the intermediate state

$$D = \frac{\mu^2 + l^2}{l^+/q^+} + \frac{M^2 + l^2}{(q^+ - l^+)/q^+}.$$  \hspace{1cm} (7)

The integral has been made finite by an invariant-mass cutoff $\Lambda^2$, and can be done exactly; however, an expansion in powers of the fermion mass is much more instructive. We find

$$I(\mu^2, M^2) \simeq \frac{\pi}{\mu^2} \left[ \left( \frac{\Lambda^2}{2} - \mu^2 \ln \Lambda^2 + \mu^2 \ln \mu^2 - \frac{\mu^4}{2\Lambda^2} \right) + M^2 \left( 3 \ln \Lambda^2 - 3 \ln \mu^2 - \frac{9}{2} + \frac{5\mu^2}{\Lambda^2} \right) + M^4 \left( \frac{2}{\mu^2} \ln(M^2/\mu^2) + \frac{1}{3\mu^2} - \frac{1}{2\Lambda^2} \right) \right].$$ \hspace{1cm} (8)

The leading term violates the chiral symmetry of the original theory; to remove this term from the infinite-cutoff limit requires three Pauli–Villars bosons with different masses $\mu_i$. The subtracted integral is

$$I_{\text{sub}}(\mu^2, M^2, \mu_i^2) = I(\mu^2, M^2) + \sum_{i=1}^{3} C_i I(\mu_i^2, M^2),$$ \hspace{1cm} (9)

and the $C_i$ are chosen to satisfy

$$1 + \sum_{i=1}^{3} C_i = 0, \quad \mu^2 + \sum_{i=1}^{3} C_i \mu_i^2 = 0, \quad \sum_{i=1}^{3} C_i \mu_i^2 \ln(\mu_i^2/\mu^2) = 0.$$ \hspace{1cm} (10)

Given this perturbative analysis of the regularization, the conjecture is that fermion self-energies in a nonperturbative calculation will be regularized by the same Pauli–Villars bosons with the same coupling strengths $C_i$.

### 3 A Soluble Model

To test this idea, we have constructed a model remotely related to Yukawa theory but which has the advantage of being analytically soluble. This type of model was investigated in equal-time quantization by Greenberg and Schweber and on the light cone by Glazek and Perry. It can be obtained from the

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\textsuperscript{3}To completely regularize Yukawa theory will require a $\phi^4$ term and perhaps a Pauli–Villars fermion.
Yukawa Hamiltonian\textsuperscript{[3]} by a number of severe modifications. The momentum dependence in the fermion kinetic energy becomes \((M_0^2 + M_0'p^+)/P^+\), where \(p^+\) and \(P^+\) are the longitudinal momenta of the fermion and system, respectively.\textsuperscript{[3]} Only the no-flip three-point vertex is kept and then only in a modified form where the longitudinal momentum dependence is simplified. The structure of the chosen interactions results in states populated by a fixed number of fermions and a cloud of bosons. We study only the lowest state with one fermion. One Pauli–Villars field is found to be sufficient in this case.

The resulting light-cone Hamiltonian \(H_{\text{eff}}^{\text{LC}} = P^+ P_{\text{eff}}\) is given by

\[
H_{\text{eff}}^{\text{LC}} = \int \frac{dp^+ d^2p_{\perp}}{16\pi^3 p^+} (M_0^2 + M_0'p^+) \sum_\sigma \bar{b}_{\sigma} p_{\sigma} b_\sigma \\
+ P^+ \int \frac{dq^+ d^2q_{\perp}}{16\pi^3 q^+} \left[ \frac{\mu^2 + q_{\perp}^2}{q^+} a_{\sigma}^\dagger a_{\sigma} + \frac{\mu^2 + q_{\perp}^2}{q^+} a_{1\sigma}^\dagger a_{1\sigma} \right] \\
+ g \int \frac{dp_{\perp 1}^+ d^2p_{\perp 2}}{\sqrt{16\pi^3 p_{\perp 1}^+}} \int \frac{dp_{\perp 2}^+ d^2p_{\perp 1}}{\sqrt{16\pi^3 p_{\perp 2}^+}} \int \frac{dq_{\perp 1}^+ d^2q_{\perp 2}}{16\pi^3 q^+} \sum_\sigma \bar{b}_{2\sigma} b_\sigma \\
\times \left[ \left( \frac{p_{\perp 1}^+}{p_{\perp 2}^+} \right)^\gamma \bar{a}_{\sigma}^\dagger p_{\sigma} - p_{\sigma} + q + \left( \frac{p_{\perp 2}^+}{p_{\perp 1}^+} \right)^\gamma \bar{a}_{1\sigma}^\dagger (p_{1\sigma} - p_{2\sigma} - q) \\
+ i \left( \frac{p_{\perp 1}^+}{p_{\perp 2}^+} \right)^\gamma \bar{a}_{1\sigma}^\dagger p_{\sigma} - p_{\sigma} + q + i \left( \frac{p_{\perp 2}^+}{p_{\perp 1}^+} \right)^\gamma \bar{a}_{\sigma}^\dagger (p_{1\sigma} - p_{2\sigma} - q) \right],
\]

where

\[
[a_{q}, a_{q'}^\dagger] = 16\pi^3 q^+ \delta(q - q'), \quad \{b_{\sigma q}, b_{\sigma' q'}^\dagger\} = 16\pi^3 p^+ \delta(p - p') \delta_{\sigma\sigma'}. \tag{12}
\]

The state vector is written as a Fock-state expansion

\[
\Phi_{\sigma} = \sqrt{16\pi^3 P^+} \sum_{n,n_1} \int \frac{dp^+ d^2p_{\perp}}{\sqrt{16\pi^3 p^+}} \prod_{i=1}^{n} \frac{dq_{i}^+ d^2q_{i\perp}}{\sqrt{16\pi^3 q_{i}^+}} \prod_{j=1}^{n_1} \frac{dr_{j}^+ d^2r_{j\perp}}{\sqrt{16\pi^3 r_{j}^+}} \\
\times \delta(P - p - \sum_{i} q_{i} - \sum_{j} r_{j}) \phi^{(n,n_1)}(q_{i}, r_{j}; P) \\
\times \frac{1}{\sqrt{n!n_1!}} \bar{b}_{\sigma}^\dagger \prod_{i} a_{\sigma}^\dagger \prod_{j} a_{1\sigma}^\dagger |0\rangle, \tag{13}
\]

\textsuperscript{d}This term has a structure similar to that of the self-induced inertia term shown in Eq. (C.2) of Ref.\textsuperscript{[10]}

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with \( n \) the number of ordinary bosons and \( n_1 \) the number of Pauli–Villars bosons. The normalization of the state is chosen to be

\[
\Phi_{\phi}^n \cdot \Phi_{\sigma} = 16\pi^3 P^+ \delta (P' - P).
\]

(14)

For the boson amplitudes this implies

\[
1 = \sum_{n_1} \prod_i \int dq_i^+ d^2 q_{i1} \prod_j \int dr_j^+ d^2 r_{j1} \times \left| \phi^{n_1} (q_1, P - \sum_i q_i - \sum_j r_j) \right|^2.
\]

(15)

A solution to the mass eigenvalue problem

\[
H_{L\Phi_{\sigma}}^\phi \Phi_{\sigma} = M^2 \Phi_{\sigma}
\]

must satisfy the following coupled set of integral equations:

\[
\left[ M^2 - M_0^2 - M_0^P - \sum_i \left( \frac{\mu^2 + q_{i1}^2}{y_i} - \sum_j \frac{\mu^2 + r_{j1}^2}{z_j} \right) \right] \phi^{(n_1)} (q_1, r_1, P)
\]

\[
= g \left\{ \sqrt{n + 1} \int dq_1^+ d^2 q_{11} \left( \frac{p^+ - q^+}{p^+} \right)^\gamma \phi^{(n+1, n_1)} (q_1, q_1, r_1, P - q_1)
\]

\[
+ \frac{1}{\sqrt{n}} \sum_i \left( \frac{p^+}{\sqrt{16\pi^3 q_i^+}} \right)^\gamma \times \phi^{(n-1, n_1)} (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_1, r_1, P + q_1)
\]

\[
+ i \sqrt{n + 1} \int dr_1^+ d^2 r_{11} \left( \frac{p^+ - r^+}{r^+} \right)^\gamma \phi^{(n+1, n_1)} (q_1, r_1, P - r_1)
\]

\[
+ \frac{i}{\sqrt{n_1}} \sum_j \left( \frac{p^+}{\sqrt{16\pi^3 r_j^+}} \right)^\gamma \times \phi^{(n_1, n_1-1)} (q_1, r_1, \ldots, r_{j-1}, r_{j+1}, \ldots, r_n, P + r_j) \right\}.
\]

(17)

The solution is

\[
\phi^{(n, n_1)} = \sqrt{Z (-g)^n (-ig)^{n_1} } \left( \frac{p^+}{p^+} \right)^\gamma \prod_i \frac{y_i}{\sqrt{16\pi^3 q_i^+ (\mu^2 + q_i^2)}}
\]
\[
\times \prod_j \frac{z_j}{\sqrt{16\pi^3 r_j^+ (\mu_1^2 + r_{\perp j}^2)}}.
\]

provided that \(M_0 = M\) and

\[
M'_0 = \frac{g^2/P^+ \ln \mu_1/\mu}{16\pi^2 \gamma + 1/2}.
\]

Given the structure of the individual amplitudes, a natural choice for the value of the interaction parameter \(\gamma\) is seen to be \(1/2\), because each amplitude is then proportional to the square root of the product of all longitudinal momenta. With this value of \(1/2\), the wave function normalization can be reduced to a rapidly converging infinite sum

\[
\frac{1}{Z} = \sum_{n, n_1} \frac{1}{(2n + 2n_1 + 1)!n_1!} \frac{(g/\mu)^{2n}(g/\mu_1)^{2n_1}}{(16\pi^2)^{n+n_1}},
\]

To fix the coupling \(g\) we set the value of the expectation value \(\langle \phi^2(0) \rangle \equiv \Phi^\dagger \phi^2(0) \Phi\). For the analytic solution (with \(\gamma = 1/2\)) it reduces to

\[
\langle \phi^2(0) \rangle = \sum_{n, n_1} \frac{2Zn}{(2n + 2n_1)!n_1!} \frac{(g/\mu)^{2n}(g/\mu_1)^{2n_1}}{(16\pi^2)^{n+n_1}}.
\]

From a numerical solution the quantity can be computed fairly efficiently in a sum similar to the normalization sum

\[
\langle \phi^2(0) \rangle = \sum_{n=1, n_1=0}^n \prod_i \int dq_i^+ d^2q_i, \prod_j \int dr_j^+ d^2r_j,
\]

\[
\times \left( \sum_{k=1}^n \frac{2}{q_k^+ / P^+} \right) \left| \phi^{(n,n_1)}(q_i, r_j; P - \sum_i q_i - \sum_j r_j) \right|^2,
\]

with the integrals computed from discrete approximations.

As a prediction of the model, we compute a distribution function for the physical bosons

\[
f_B(y) \equiv \sum_{n, n_1} \prod_i \int dq_i^+ d^2q_i \prod_j \int dr_j^+ d^2r_j \sum_{i=1}^n \delta(y - q_i^+ / P^+ + \sum_i q_i - \sum_j r_j) \left| \phi^{(n,n_1)}(q_i, r_j; P - \sum_i q_i - \sum_j r_j) \right|^2.
\]
and the average multiplicity
\[
\langle n_B \rangle = \int_0^1 f_B(y) dy.
\] (24)

For the analytic solution we obtain
\[
f_B(y) = \sum_{n,n_1} \frac{Z_n y (1-y)^{(2n+2n_1-1)} (g/\mu)^{2n} (g/\mu_1)^{2n_1}}{(2n + 2n_1 - 1)! n_1!} (16\pi^2)^{n+n_1}.
\] (25)

and
\[
\langle n_B \rangle = \sum_{n,n_1} \frac{Z_n}{(2n + 2n_1 + 1)! n_1!} (g/\mu)^{2n} (g/\mu_1)^{2n_1} (16\pi^2)^{n+n_1}.
\] (26)

Another prediction is the slope of the form factor for the dressed fermion. An expression can be constructed from the formalism developed by Brodsky and Drell. For the analytic solution of the model we obtain
\[
F'(0) = \sum_{n,n_1} \frac{Z_n (n/\mu^2 + n_1/\mu_1^2)}{(2n + 2n_1 + 3)! n_1!} (g/\mu)^{2n} (g/\mu_1)^{2n_1} (16\pi^2)^{n+n_1}.
\] (27)

Numerically, one can estimate \( F'(0) \) from
\[
\tilde{F}'(0) = - \sum_{n,n_1} \prod_i \int dq^+_i d^2q_{\perp i} \prod_j \int dr^+_j d^2r_{\perp j} \]
\[
\times \left[ \sum_i \left| \frac{y_i}{2} \nabla_{\perp i} \phi^{(n,n_1)}(q_i,\mathbf{L} - \sum_i q_i - \sum_j r_j) \right|^2 \right. \]
\[
+ \sum_j \left| \frac{z_j}{2} \nabla_{\perp j} \phi^{(n,n_1)}(q_j,\mathbf{L} - \sum_i q_i - \sum_j r_j) \right|^2 \right],
\] (28)

which differs from \( F'(0) \) by surface terms which vanish as \( \Lambda \to \infty \).

4 Numerical methods and results

We solve the model numerically by using DLCQ. We impose periodic boundary conditions for bosons and antiperiodic conditions for fermions in a light-cone box \(-L < x^- < L, -L_{\perp} < x, y < L_{\perp}\). This introduces a discrete grid:
\[
p^+ \to \frac{\pi}{L} n, \quad p_{\perp} \to \left( \frac{\pi}{L_{\perp}} n_x, \frac{\pi}{L_{\perp}} n_y \right).
\] (29)
Integrals are replaced by discrete sums

\[
\int dp^+ \int d^2 p_\perp f(p^+, p_\perp) \simeq \frac{2\pi}{L} \left( \frac{\pi}{L_\perp} \right)^2 \sum_n \sum_{n_x, n_y = -N_\perp}^{N_\perp} f(n\pi/L, n\pi_\perp/L_\perp).
\]

(30)

The limit \( L \to \infty \) can be exchanged for a limit in terms of the integer resolution

\[
K \equiv \frac{L}{\pi} p^+.
\]

(31)

Longitudinal momentum fractions are given by \( x = p^+/P^+ \to n/K \), with \( n \) odd for fermions and even for bosons. The light-cone Hamiltonian \( H_{LC} \) is independent of \( L \), but does depend on \( K \).

Because all \( n \) are positive, DLCQ automatically limits the number of particles to no more than \( \sim K/2 \). The integers \( n_x \) and \( n_y \) range between limits associated with some maximum integer \( N_\perp \) fixed by the invariant-mass cutoff

\[
\frac{m_i^2 + p^2_\perp}{x_i} \leq \Lambda^2
\]

(32)

imposed for each constituent. The eigenvalue problem (17) is then converted to a finite matrix problem. Typical basis sizes are given in Table 1. The Hamiltonian matrix is quite sparse. The lowest eigenvalue of the matrix is extracted with use of the Lanczos algorithm\(^{12}\) for complex symmetric matrices.

Some of the results obtained are listed in Table 2 and displayed in Figs. 1 and 2. Figure 1 shows that the results are quite insensitive to numerical resolution, while Fig. 2 illustrates how the analytic solution is approached as the cutoff is increased. These results demonstrate that the DLCQ approximation yields a good representation of the solution to the model.

5 Summary

We have developed and used a simple soluble model to test the feasibility of Pauli–Villars regularization in DLCQ. The number of Pauli–Villars Fock states required is not prohibitive and good results are obtained. From this success we can work toward approximation of Yukawa theory itself by adding fermion dynamics and gradually reinstating the full complexity of the interactions. We can also consider application to other theories, perhaps including QCD.

\(^e\)The imaginary couplings of the Pauli–Villars particles make the Hamiltonian matrix complex symmetric.
Table 1: Basis sizes for DLCQ calculations in the soluble model with parameters $M^2 = \mu^2$, $\mu_1^2 = 10\mu^2$, and $\Lambda^2 = 50\mu^2$. The numbers of physical states are in parentheses.

| $N_\perp$ | 7   | 9   | 11  | 13  | 15  | 17  |
|-----------|-----|-----|-----|-----|-----|-----|
| 1         | 18  | 38  | 36  | 65  | 110 | 185 |
|           | (7) | (12) | (19) | (30) | (45) | (67) |
| 2         | 218 | 265 | 590 | 1120 | 822 | 1410 |
|           | (127) | (119) | (343) | (754) | (453) | (626) |
| 3         | 958 | 1408 | 4460 | 17031 | 110254 | 32866 |
|           | (367) | (736) | (2671) | (9230) | (13213) | (13531) |
| 4         | 3714 | 9259 | 49394 | 50966 | 110254 | 32866 |
|           | (1399) | (5913) | (32363) | (32124) | (55319) | (172247) |
| 5         | 13702 | 54100 | 95176 | 386140 | 1553576 | 5699 |
|           | (5699) | (28065) | (66371) | (232400) | (1038070) |  |
| 6         | 35666 | 126748 | 536758 | 2907158 | (12991) | (276299) |
|           | (12991) | (69245) | (391511) | (2107688) | (1008539) |  |
| 7         | 79794 | 519325 | 1317392 | (61947) | (687394) |  |
| 8         | 172118 | 1165832 | (1008539) |  |

Table 2: Numerical parameter values and results from solving the model eigenvalue problem. The physical parameter values were $M^2 = \mu^2$ for the fermion mass, $\mu_1^2 = 10\mu^2$ for the Pauli–Villars mass, and $\langle \phi^2(0) \rangle = 1$ to fix the coupling $g$.

| $(\Lambda/\mu)^2$ | $K$ | $N_\perp$ | $\mu L_\perp/\pi$ | $(M_0/\mu)^2$ | $g/\mu$ | $\langle n_B \rangle$ | $100\mu^2 F'(0)$ |
|------------------|-----|-----------|-----------------|-------------|---------|----------------|----------------|
| 50               | 11  | 4         | 0.8165          | 0.8547      | 13.293  | 0.177           | -0.751          |
| 50               | 13  | 4         | 0.8165          | 0.8518      | 13.230  | 0.172           | -1.015          |
| 50               | 15  | 4         | 0.8165          | 0.8408      | 13.556  | 0.178           | -0.715          |
| 50               | 17  | 4         | 0.8165          | 0.8289      | 13.392  | 0.180           | -0.565          |
| 50               | 9   | 5         | 1.2062          | 0.8601      | 14.023  | 0.179           | -0.547          |
| 50               | 9   | 6         | 1.2247          | 0.8377      | 14.323  | 0.179           | -0.582          |
| 50               | 9   | 7         | 1.4289          | 0.8302      | 14.386  | 0.179           | -0.658          |
| 100              | 9   | 5         | 1.2062          | 0.8601      | 14.023  | 0.179           | -0.547          |
| 200              | 9   | 5         | 0.7143          | 1.0520      | 12.565  | 0.174           | -0.239          |
| $\infty$         | analytic | 1.0000 | 13.148          | 0.160      | -0.786          |  |
Figure 1: The boson distribution function $f_B$ at various numerical resolutions, with $\langle \phi^2(0) \rangle = 1$ and $\Lambda^2 = 50 \mu^2$. The solid line is the analytic result at infinite $\Lambda^2$. 
Figure 2: The boson distribution function $f_B$ for different cutoff values, with $\langle \phi^2(0) \rangle = 1$ and numerical resolution set at $K = 9$ and $N_\perp = 5$. The solid line is the analytic result at $\Lambda^2 = \infty$. 

\[ \begin{align*} 
\Lambda^2 &= 50 \mu^2 \\
\Lambda^2 &= 100 \mu^2 \\
\Lambda^2 &= 200 \mu^2 
\end{align*} \]
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