CHIRAL SYMMETRY IN NUCLEAR PHYSICS

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How chiral symmetry – which is a basic ingredient of quantum chromodynamics (QCD) for light-quark hadrons – enters and plays an eminent role in nuclear physics is discussed. This is done in two steps. In the first step, I introduce the notion of the Cheshire Cat Principle which describes how a strong-interaction phenomenon can be described in various different languages. In the second step, I treat two cases of widely different density regimes. The first case deals with formulating nuclear physics of dilute nuclear systems with a focus on both bound and scattering processes in two-nucleon systems in terms of effective field theories (EFT), e.g., in the framework of chiral perturbation theory. The second case requires formulating effective field theories for dense hadronic matter such as heavy nuclei and nuclear matter under normal as well as extreme conditions expected to be encountered in compact stars and in heavy-ion collisions at relativistic energy. The first case is rigorously formulated but the second relies on several assumptions that appear to be reasonable but need ultimately to be justified. The Cheshire Cat Principle is proposed to play an essential role in lending support to the latter development.

1 Introduction

Chiral symmetry, now known to be associated with the fact that the up and down quarks are very light, has been playing an important role in nuclear physics since a long time, since even before the advent of QCD. In this lecture, I will describe some of the more recent developments in this area in which I have participated.

One of the most notable results obtained in nuclear physics that are consequences of chiral symmetry is the exchange current contribution to nuclear responses to electroweak currents. Although exchange currents are clearly es-

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established by theory and experiment, the role of chiral symmetry – specifically in terms of the pion – is not well appreciated in nuclear physics community. I believe that this issue marks the turning point from the “classical” nuclear physics to the next generation of hadronic physics probing hadronic matter under extreme density and/or temperature conditions.

It was pointed out in 1971 by Chemtob and Rho that the meson-exchange current, a long-standing problem in nuclear physics, could be efficiently organized in such a way that the main contribution be calculated by low-energy theorems based on chiral symmetry. Riska and Brown, by exploiting the Chemtob-Rho organization, succeeded to explain quantitatively the radiative \( np \) capture process \( n + p \rightarrow d + \gamma \). In 1978, Kubodera, Delorme and Rho proposed what is now called the “chiral filter hypothesis” which states that whenever soft-pion exchanges are un-suppressed by kinematics or selection rules, they should provide the dominant model-independent contribution with the remaining corrections markedly suppressed while whenever they are suppressed for the same reason, then short-distance effects can and do enter at the same level as higher order corrections, rendering systematic calculations difficult if not unfeasible. The predictions that followed from this hypothesis were that the soft-pion effects should be prominent in (1) nuclear electromagnetic M1 transitions and (2) weak axial-charge transitions. Both these predictions were unambiguously confirmed by experiments. The former was confirmed in the large momentum transfer electrodisintegration of the deuteron \( e + d \rightarrow n + p + e \) performed in 1980’s and the latter in a series of experiments and analyses of Warburton in early 1990’s. These predictions which were made without direct recourse to low-energy effective field theories of QCD were more recently given a justification in terms of chiral perturbation theory in 1991 and more rigorously treated in the context of effective field theories using a cut-off regularization as described below.

What I believe to be the most crucial aspect in understanding nuclear phenomena in the context of QCD is that a low-energy process can be described very accurately in terms of variables that represent physical (color singlet) objects, that is, the hadron degrees of freedom that are measured in the laboratory although the appropriate QCD variables are quarks and gluons. The way this can be understood from the point of view of QCD is in terms of what is now called “Cheshire Cat Principle” (CCP) which in a nutshell can be

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In the context of chiral perturbation theory, it is natural that the pion-exchange currents dominate whenever they are unsuppressed over short-ranged terms although in the potential both the pion-exchange and contact interactions without derivatives figure on the same footing. The chiral filter phenomena in the currents can be understood by a simple chiral counting as shown in this paper.
summarized as an (albeit approximate) equivalence between the description in terms of hadronic variables and that in terms of QCD variables for processes that take place in long-wavelength kinematic regimes. In certain cases, the CCP can be exact. For instance, in two space-time dimensions, the bosonization technique allows one to precisely describe fermionic excitations in terms of bosonic and vice versa and in the presence of supersymmetry, there can be exact “duality,” a concept which is becoming very popular in particle physics community. In nonsupersymmetric and dimension-four world, there is no known transformation that can give an exact CCP but there is no proof that such a transformation does not exist. I will show a recent case where in QCD at four dimensions a fairly good CCP can be seen in nucleon structure. Another intriguing case, though somewhat different in nature, is discussed in [11].

At very low energy and low density, nuclear physics can be accurately described in terms of low-mass hadronic variables with a cutoff set at the chiral scale given by the vector meson mass, $\sim 1$ GeV. I will illustrate this in terms of effective field theories that involve only the nucleon fields and pionic fields. In fact we will see that even the pion can be integrated out by setting an effective cutoff at the pion mass and yet a quantitative agreement can be obtained in the nucleon-only theory. I will focus on two-nucleon systems which can be described in terms of a Lagrangian whose parameters are defined in matter-free space.

I will next jump over the intermediate density regime to one for which no systematic chiral perturbation theory approach is available. This is the case with nuclear matter where new scales enter due to the presence of a Fermi sea. For this case, two effective field theories will figure; one, chiral Lagrangian field theory in a dense background and the other, Fermi-liquid fixed-point theory. These two by themselves are superb effective field theories. I will argue that in going to dense hadronic matter beyond nuclear matter density, the two effective field theories could be married once Brown-Rho scaling is introduced. The arguments used here are more conjectural than rigorous but I will show a number of evidence that the marriage is a successful one.

2 Cheshire Cat Principle (CCP)

When the large $N_C$ property of QCD which implies the skyrmion picture for baryons and the bag structure of QCD with asymptotic freedom are properly combined, we have the chiral bag. When the topological structure of the skyrmion is incorporated into the chiral bag, we obtain the (approximate)
Cheshire Cat structure. It has been shown\cite{2}\cite{13} that at least at low energies, practically all physical observables, with few exceptions, obey the Cheshire Cat Principle, in some cases exactly (such as the baryon charge thanks to topological invariance) and some approximately (those properties not connected to topology). The reason why the Cheshire Cat Principle is operative is probably connected to the existence of something deep in the form of a web of dualities in gauge theories.

Among the few cases that appeared to deviate from the Cheshire Cat Principle, a most recent and intriguing one is the flavor singlet axial charge (FSAC) of the proton, sometimes mistakenly identified as “proton spin.” A straightforward calculation of the FSAC in the chiral bag would indicate that the FSAC is small at small bag radius and grows to order unity at large bag radius which would mean that the Cheshire Cat is grossly violated. The purpose of this part of the lecture is to show that if certain vacuum properties of the bag related to anomalies are properly taken into account, the CC can be recovered\cite{14}. Here breaking of chiral symmetry (i.e., chiral $U_A(1)$) due to quantum effects will be involved.

2.1 Chiral Bag

The most convenient formalism to discuss the CCP is the chiral bag model (CBM) which is a model that interpolates between the “bagged” quark description known as MIT bag and the topological model, skyrmion, obtainable in the large $N_C$ limit of QCD. This model consists of the bag of radius $R$ within which the QCD degrees of freedom, quarks and gluons, live surrounded by the cloud of pseudo-Goldstone bosons (pions, kaons etc.) and, if needed for short-distance physics, massive mesons with communication between the inside and the outside mediated by suitable boundary conditions. The power of this model is that it can combine both long-distance (outside) and short-distance (inside) physics controlled by the boundary terms. Thus it can describe long-wavelength processes as well as short-wavelength processes within a single model which is difficult for the skyrmion or the MIT bag separately. For convenience, one usually uses – as we shall do for this lecture – a spherical geometry for the bag with a sharp boundary but in principle one could use an arbitrarily shaped bag with a fuzzy surface. Because of the leakage of various charges across the boundary, it should not really matter what shape and how sharp the boundary should be in. The CCP in (octet or triplet) flavor space has been extensively discussed, e.g., in\cite{3}, so I shall not go into that. Let me just focus on the flavor singlet $U_A(1)$ channel which is of particular interest because of the chiral anomaly associated with the massive $\eta'$ degree of freedom. The
model we consider has the $\eta'$ field (which we shall simply call $\eta$) outside\(^3\). This $\eta$ field couples on the surface to the quarks living inside. The coupling condition follows from the equation of motion for the quark field and takes the form

$$in^\mu \gamma_\mu \psi = e^i R \gamma_5 \psi(R)$$

where $f$ is a mass-dimension-1 constant related to the $\eta$ decay constant and $n^\mu$ is a unit normal to the surface. This condition assures at the classical level that the baryon current is conserved\(^4\).

When the $U_A(1)$ field is present, it can also couple to the gluons living inside. That gives a boundary condition that involves the $\eta$ field and the gluon field. To deduce this condition, consider the normal components of the FSAC on both sides of the boundary. Coming from outside, one has

$$fn^\mu \partial_\mu \eta(R).$$

This is well-defined at the surface since it is a nice local operator. Now coming from inside, we have

$$\frac{1}{2}n^\mu \bar{\psi} \gamma_\mu \gamma_5 \psi(R).$$

This however is not well-defined since the bilinear in fermion fields at the same space-time point is singular. Regularizing it by point-splitting a la Schwinger reveals that the regularized form of (3) has an additional term involving the Chern-Simons current $K^\mu = \epsilon^{\mu\nu\alpha\beta} (G^a_\nu G^a_{\alpha\beta} - \frac{2}{3} f^{abc} g G^a_\nu G^b_\alpha G^c_{\beta})$ given in terms of the color gauge field $G^a_\mu$,

$$\frac{1}{2} \hat{n} \cdot (\bar{\psi} \gamma_5 \psi)_{\text{NO}} + \frac{N_F g^2}{16\pi^2} \hat{n} \cdot K$$

where the subscript “NO” stands for normal ordering. Now the Chern-Simons current on the surface is not gauge-invariant, so (4) is not gauge-invariant. This violation of gauge invariance, which is not acceptable, can be understood as "color anomaly." The color anomaly arises as a consequence of the celebrated chiral anomaly associated with the $U_A(1)$ symmetry which is broken due to

\(^3\)The pion field outside plays an important role in inducing the leakage of the baryon charge. This phenomenon is very well understood and will be taken into account in the result although we will not discuss it here.

\(^4\)We know however that the hedgehog pions induce a violation of the baryon current but the $\eta$ field does not participate in this violation since there is no topological component associated with this pseudoscalar meson.
quantum fluctuations in the vacuum. The flavor singlet axial current inside has the anomaly and hence the color leaks out as a consequence. To prevent the leakage of the color, the simplest (though perhaps not the only) way is to put a boundary condition that “soaks up” this leaking color charge. It has been shown that the following surface counter term does the job:

\[ \mathcal{L}_{CT} = i \frac{g^2}{32\pi^2} \oint_\Sigma d\beta K^{\mu} n_\mu (\text{Tr} U^\dagger \text{Tr} U - \text{Tr} U) \]  

(5)

where \( N_F \) is the number of flavors (here taken to be =3), \( \beta \) is a point on a surface \( \Sigma \), \( n^\mu \) is the outward normal to the bag surface, \( U \) is the \( U(N_F) \) matrix-valued field written as \( U = e^{i\pi f} e^{i\eta f} \). Note that (5) manifestly breaks color gauge invariance (both large and small, the latter due to the bag), so the action of the chiral bag model with this term is not gauge invariant at the classical level but as shown in [4], when quantum fluctuations are calculated, there appears an induced anomaly term on the surface which exactly cancels this term. Thus gauge invariance is restored at the quantum level. Presence of the boundary term (5) implies that the color electric and magnetic fields must satisfy boundary conditions that have coupling to the \( \eta \) field,

\[ \hat{n} \cdot \vec{E}^a = -\frac{N_F g^2}{8\pi f} \hat{n} \cdot \vec{B}^a \eta, \]  

(6)

\[ \hat{n} \times \vec{B}^a = \frac{N_F g^2}{8\pi f} \hat{n} \times \vec{E}^a \eta. \]  

(7)

These together with the “matter” boundary conditions (1) and the axial current continuity

\[ \frac{1}{2} \hat{n} \cdot (\bar{\psi} \gamma_5 \psi)_{NO}(R) = f n^\mu \partial_\mu \eta(R) \]  

(8)

define the surface boundary conditions of our CBM to be satisfied by the usual equations of motion of the quarks and gluons inside the bag and of the massive \( \eta \) field outside of the bag.

### 2.2 Flavor-Singlet Axial Charge (FSAC)

We now study the flavor singlet axial charge of the proton. We define the axial current as

\[ A^\mu = A_B^\mu \Theta_B + A_M^\mu \Theta_M. \]  

(9)
Since we will be dealing only with the flavor-singlet axial current, we will omit the flavor index in the current. We shall use the short-hand notations \( \Theta_B = \theta(R - r) \) and \( \Theta_M = \theta(r - R) \) with \( R \) being the radius of the bag. We demand that the \( U_A(1) \) anomaly be given in this model by

\[
\partial_\mu A_\mu = \frac{\alpha_s N_F}{2\pi} \sum_a \vec{E}_a \cdot \vec{B}_a \Theta_B + f m^2_\eta \eta \Theta_M. \tag{10}
\]

Our task is to construct the FSAC in the chiral bag model that is gauge-invariant and consistent with this anomaly equation. Our basic assumption is that in the nonperturbative sector outside of the bag, the only relevant \( U_A(1) \) degree of freedom is the massive \( \eta' \) field. This assumption allows us to write

\[
A^\mu_B = A^\mu_\eta = f \partial_\mu \eta \tag{11}
\]

with the divergence

\[
\partial_\mu A^\mu_\eta = f m^2_\eta \eta. \tag{12}
\]

Now the question is: what is the gauge-invariant and regularized \( A^\mu_B \) such that the anomaly (10) is satisfied? To address this question, we rewrite the current (11) as

\[
A^\mu = A^\mu_{BQ} + A^\mu_{BG} + A^\mu_\eta \tag{13}
\]

such that

\[
\partial_\mu (A^\mu_{BQ} + A^\mu_\eta) = f m^2_\eta \eta \Theta_M, \tag{14}
\]

\[
\partial_\mu A^\mu_{BG} = \frac{\alpha_s N_F}{2\pi} \sum_a \vec{E}_a \cdot \vec{B}_a \Theta_B. \tag{15}
\]

The subindices \( Q \) and \( G \) imply that these currents are written in terms of quark and gluon fields respectively. In writing (14), we have ignored the up and down quark masses.

The matrix element of the “matter” currents satisfying (14) is quite simple. The quark current inside the bag is

\[
A^\mu_{BQ} = \bar{\Psi} \gamma^\mu \gamma_5 \Psi \tag{16}
\]

where \( \Psi \) should be understood to be the bagged quark field. The proton matrix element

\[
a^0_{BQ} \equiv \langle p | \int_B d^3r \bar{\Psi} \gamma_3 \gamma_5 \Psi | p \rangle \tag{17}
\]
Figure 1: Various contributions to the flavor singlet axial current of the proton as a function of bag radius and comparison with the experiment: (a) “matter” (quark plus $\eta$) contribution ($a_{BQ}^0 + a_{\eta}^0$), (b) the contribution of the static gluons due to quark source ($a_{G,\text{stat}}^0$), (c) the gluon vacuum contribution ($a_{G,\text{vac}}^0$), and (d) their sum ($a_{\text{total}}^0$). The shaded area corresponds to the range implied by experiments.

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can be computed readily taking into account of the leakage of the baryon charge due to the hedgehog pion we know how to compute. As for the $\eta$ contribution, one sees that the boundary condition (8) relates it directly to the quark contribution. A simple calculation gives

$$a_{\eta}^0 = \frac{1 + y_{\eta}}{2(1 + y_{\eta}) + y_{\eta}^2} \langle p | \int_B d^3 r \bar{\Psi} \gamma_3 \gamma_5 \Psi | p \rangle$$  \hspace{1cm} (18)$$

with $y_{\eta} = m_{\eta} R$. The first crucial observation at this point is that the matter contribution, the sum of (17) and (18), will vanish as the radius is shrunk to zero and will increase monotonically as the bag is increased. This can be seen in Fig.1. This feature by itself would show a violent breakdown of the CCP.

We now turn to the gluon contribution which is a lot more difficult to calculate accurately. Only an approximate calculation exists at the moment but I believe it is qualitatively correct.

To see what is involved, we first write down the relevant matrix element
which follows from the relation (13),

\[ a_{G}^{0} = \langle p | - \frac{N_{F}G_{a}}{\pi} \int_{B} d^{3}r \epsilon_{3} \vec{E}^{a} \cdot \vec{B}^{a} | p \rangle. \] (19)

It is not obvious but this matrix element can be shown to be gauge-invariant. Since we are dealing with a cavity system and the color anomaly which requires coupling to the \( \eta \) field in the gluon boundary conditions (6) and (7), we have a double Casimir problem here. Only when the Casimir contributions are properly taken into account can we recover a fully consistent gauge-invariant result. Doing this calculation in full generality is not going to be an easy matter, so we shall proceed less rigorously.

The gluon contribution can be roughly divided into two parts: One is the (presumably dominant) effect coming from the quark and \( \eta \) sources and the other is the contribution coming from a vacuum change due to the \( \eta \) coupling. The first involves the valence quarks and static gluon fields. The calculation for this was already made in previous papers (references are given in 14) and only a slight extension including the \( \eta \) field in the boundary condition yields the desired result. The result shown in Fig.1 as \( a_{G,\text{stat}}^{0} \) provides us with the second important information and that is that the dominant gluon contribution comes with a negative sign, so canceling nearly completely the matter contribution. It seems reasonable as a first approximation to conclude that \( a_{BQ}^{0} + a_{\eta}^{0} + a_{G,\text{stat}}^{0} \approx 0 \). What is left must therefore be the Casimir contribution.

The simplest way to compute the Casimir effect is to assume CCP and check that the assumption is verified a posteriori. In 15, it is shown that the CCP allows us to write – via the color boundary conditions (8) and (9) – the following local operator

\[ E^{a} \cdot B^{a}(x) \approx - \frac{N_{F}g^{2}}{8\pi^{2}} \eta(R) \frac{1}{2} G^{2}(x). \] (20)

Here only the term up to the first order in \( \eta \) is retained in the right-hand side. Applied to the CBM, the couplings are to be understood as the average bag couplings and the gluon fields are to be expressed in the cavity vacuum through a mode expansion. In fact, by comparing the expression for the \( \eta' \) mass derived in 13 using Eq.(21) with that obtained by Novikov et al [16] in a QCD sum-rule method, we note that the matrix element of the \( G^{2} \) in (20) should be evaluated in the absence of light quarks. This means, in the bag model, the cavity vacuum. The corresponding Casimir calculation involves a mode sum regularized, say, by the heat-kernel method. There is some subtlety involved

\[ ^{#3}\text{Such a calculation is in progress at the time of this writing.} \]
in this calculation but we will not go into details here (see for details). The results are summarized in Fig.1 compared with the presently accepted range of the experimental values obtained in deep inelastic lepton scattering from the nucleon. We see that the CCP is recovered via the Casimir effect.

To summarize, the CCP holds fairly well in all low-energy properties of baryon dynamics. More tantalizingly, there is a hint for a one-to-one mapping between quark-gluon matter at high density and hadron matter at low density. We will later argue that the CCP holds in normal nuclear matter as well as in high density matter before the phase transition.

3 Effective Field Theories (EFT) For Nuclei

3.1 The Essence of EFT

The idea of effective quantum field theory is extremely simple. At low energy where non-perturbative effects of QCD dominate, the relevant degrees of freedom are not quarks and gluons but hadrons. We do not have to know how the quark-gluon variables get transformed into hadron variables. We may simply assume the CCP described above as the mechanism for the transformation. Let the relevant hadronic degrees of freedom be represented by the generic field $\Phi$. Separate the field into two parts; the low-energy part $\Phi_L$ in which we are interested and the high-energy part $\Phi_H$ which does not interest us directly. We delineate the two parts at the momentum scale characterized by a cutoff, say, $\Lambda_1$. In the generating functional $Z$ that we want to compute, integrate out the $\Phi_H$ field and define an effective action $S^{eff}_{\Lambda_1}[\Phi_L] = \frac{1}{i} \ln \left( \int [d\Phi_H] e^{iS[\Phi_L, \Phi_H]} \right)$. Then what we need to compute is

$$Z = \int_{\Lambda_1} [d\Phi_L] e^{iS^{eff}_{\Lambda_1}[\Phi_L]}.$$  \hspace{1cm} (21)

Given the right degrees of freedom effective below the cutoff $\Lambda_1$, we expand the effective action as $S^{eff}_{\Lambda_1}[\Phi_L] = \sum_i C_i Q_i[\Phi_L]$, where $Q_i[\Phi_L]$ are local field operators allowed by the symmetries of the problem, and $C_i$’s are constants, which are required to satisfy the “naturalness condition”. The $Q_i[\Phi_L]$’s are ordered in such a manner that, for low-energy processes, the importance of the $Q_i[\Phi_L]$ term diminishes as $i$ increases. The contribution of the $\Phi_H$ degrees of freedom integrated out from the action is not simply discarded but instead gets lodged in the coefficients $C_i$ as well as in higher-dimensional operators. The strategy of effective field theories is to truncate the series at a manageable finite order, and obtain desired accuracy to capture the essence of the physics involved. The separation of $\Phi$ into $\Phi_L$ and $\Phi_H$ involves two steps: The first
is to “decimate” the degrees of freedom, that is, to choose the fields $\Phi_L$ relevant for the physics to be incorporated explicitly into the Lagrangian, and the second is a regularization procedure which regulates the high-momentum contribution. Let $m$ be the lightest mass of the degrees of freedom that are integrated out. Clearly the most physically transparent regularization is to introduce a momentum cutoff with $\Lambda_1 \sim m$. A different cutoff introduces different coefficients $C_i$ for the given set of fields. The idea is then to pick a value of the cutoff that is low enough to avoid unnecessary complications but high enough to avoid throwing away relevant physics.

This defines an effective field theory. The rest is just prescription and work. Technical details may differ, some more convenient and more elegant than others, but when done well, they should all give equally good results.

In the full (untruncated) theory, the location of the cutoff is entirely arbitrary, so we could have chosen any values of the cutoff other than $\Lambda_1$ without changing physics. The cut-off dependence of $Q_i[\Phi_L]$ is compensated by the corresponding changes in $C_i$ through the renormalization group equation. In such a theory, which is rarely available in four dimensions, physical observables should be strictly independent of where the cutoff is set. This of course will not be the case when the series is truncated. However, for a suitably truncated effective theory to be predictive, the observables should be more or less insensitive to where the cutoff is put. Should there be strong cutoff dependence, it would signal that there is something amiss in the scheme. For instance, it could be a signal for the presence of new degrees of freedom or of “new physics” that must be incorporated and/or for the necessity of including higher-order terms.

An important aspect of effective field theory is that the result should not depend upon what sort of regularization one uses. In principle, therefore, one could equally well use the dimensional regularization or the cut-off regularization or something else as long as one makes sure that all relevant symmetries are properly implemented. Some of these issues in the context of the dimensional regularization and its variants were addressed recently in [20]. Here we will use the cut-off regularization for the reason that the procedure is simple and physically transparent when limited to the order we shall adopt, namely, to the next-to-leading order (NLO). For some of the problems treated here such as neutron-proton $(np)$ scattering, one could also use in a predictive manner a modified dimensional regularization in which certain power divergences are removed (i.e., PDS scheme).
3.2 An EFT At Work

We shall apply the above strategy to two-nucleon systems at very low energy. Suppose we look at processes that involve momentum $p \ll m_\pi \approx 140$ MeV. We can approach the problem in two ways. Since the momentum involved is much less than the pion mass, we can simply integrate out the pion and treat the nucleon field only. This means that the physics of the pion will appear in the coefficients of the terms we write down in the Lagrangian that we wish to solve. Next we will restore the pion and put it explicitly into the calculation. The coefficients of the terms in the Lagrangian will now be different from the case when the pion was integrated out. We will compare the two cases and learn how “new physics” in the form of the pion emerges.

As mentioned, if done correctly, it should not matter which regularization one uses. Here we shall use the cut-off regularization which is now known to be the simplest and trouble-free regularization in the market. There have been lots of papers written in connection with the $np$ scattering at low energy discussing the merits and demerits of various regularization schemes. Readers interested in what the hot debates are all about could find them in [25]. The issue is quite interesting theoretically but at the end of the day, it is fair to say that using the cut-off regularization is still the physically most transparent (although perhaps not the most elegant) way to do the calculation. I won’t go into this matter here which is in some sense academic. I could also spend all my lecture time discussing scattering only but I won’t do this either. Instead I will treat all two-nucleon problems at the same time, including $np$ and $pp$ channels, bound states, scattering states as well as electroweak transitions involving the deuteron and unbound states. Work along this line with a cutoff regularization was done previously by Ordóñez et al [26].

• Counting rule

Since we are going to use the cut-off regularization, Weinberg’s original chiral counting rule is perfectly fine. This will deviate from the PDS scheme in treating the pion when pions figure in the theory. As I shall mention later, Weinberg’s counting scheme has a predictive power in the form of chiral filter which the PDS scheme lacks. More on this later.

Start with the case where the pion is present. Let $Q$ be the typical momentum scale of the process or the pion mass, which is regarded as “small” compared to the chiral scale $\Lambda_\chi \sim 1$ GeV. The counting rule given by Weinberg is that, a Feynman graph comprised of $A$ nucleons, $N_E$ external (electroweak)
fields, \( L \) irreducible loops and \( C \)-separated pieces is order of \( \mathcal{O}(Q^\nu) \) with

\[
\text{ChPT: } \nu = 2L + 2(C-1) + 2 - (A + N_E) + \sum_i \bar{\nu}_i, \tag{22}
\]

\[
\bar{\nu}_i \equiv d_i + \frac{n_i}{2} + e_i - 2 \tag{23}
\]

where we have characterized each vertex \( i \) by \( d_i \) the number of derivatives and/or the power of the pion mass, \( n_i \) that of nucleon fields and \( e_i \) that of external (electroweak) fields. The quantities \( \bar{\nu}_i \) and \( C \) are defined so that \( \bar{\nu}_i \geq 0 \) even in the presence of external fields, \( \mathbf{S} \) and \( (C-1) = 0 \) for connected diagrams.

Now if the pion is integrated out, there are no irreducible loops and the corresponding counting rule can be obtained simply by putting the number of loops \( L \) equal to zero,

\[
\text{EFT: } \nu = 2(C-1) + 2 - (A + N_E) + \sum_i \bar{\nu}_i \tag{24}
\]

with the same definition for \( \bar{\nu}_i \) given in eq.(23). But the meaning of \( d_i \) in eq.(23) is changed: Since the pion field is integrated out, the \( d_i \) stands here only for the number of derivatives, and not the power of the pion mass. Then, up to the next-to-leading order (NLO), the resulting irreducible vertex or potential consists of contact interactions and two spatial derivatives thereof.

Electromagnetic interactions have different counting rules and should be treated separately. Since the Coulomb interaction (between protons) is given as \( \frac{\alpha q^2}{r^2} \) (where \( \alpha \approx 1/137 \) is the fine-structure constant and \( q \) is the momentum transferred), it is of relevance in an extremely small momentum region while it becomes irrelevant in other regions due to the smallness of \( \alpha \). Since we will be interested also in the proton fusion at threshold, we will explicitly include the Coulomb potential in the \( pp \) sector.

- **Putting the cutoff**

To the order we consider here for the irreducible graphs, that is, to NLO, all we need is to put the cutoff on the irreducible vertex (call potential from now on). Given a potential in the momentum space \( V \) which we are to iterate to infinite order in the \textit{reducible} channel to get the bound or quasi-bound state, the regularized potential in coordinate space is taken as

\[
V(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot r} S_A(q^2) V(q), \tag{25}
\]
where $S_\Lambda(q^2)$ is the regulator with a cutoff $\Lambda$. For our purpose it is convenient to take the Gaussian regulator

$$S_\Lambda(q^2) = \exp\left(-\frac{q^2}{2\Lambda^2}\right).$$  \tag{26}$$

Summing the infinite diagrams with the potential is equivalent to solving Schrödinger equation with the potential $V(r)$. Since in our scheme we should not let $\Lambda$ become too large, the Schrödinger equation is well-defined.

• **Potential without the pion**

  In the absence of the pion, the potential to NLO is simply

$$V_{\text{no}\rightarrow\pi}(q) = \frac{4\pi}{M} \left[ C_0 + (C_2\delta^{ij} + D_2\sigma^{ij})q^i q^j \right] + Z_1 Z_2 \frac{\alpha}{q^2}$$  \tag{27}$$

with

$$\sigma^{ij} = \frac{3}{\sqrt{8}} \left( \frac{\sigma_1^i \sigma_2^j + \sigma_1^j \sigma_2^i}{2} - \frac{\delta^{ij}}{3} \sigma_1 \cdot \sigma_2 \right),$$  \tag{28}$$

where $q$ is the momentum transferred, $M \simeq 940$ MeV the nucleon mass, and $Z_1 Z_2 = 1$ for the $pp$ channel and zero otherwise. For the proton fusion process, we will also consider $O(\alpha^2)$ corrections, i.e., the vacuum polarization (VP) potential and the two-photon-exchange (C2) potential; as in 28, these will be treated perturbatively. The parameters $C$’s and $D_2$ are defined for each channel. The $D_2$ term, however, is effective only for the spin-triplet channel. Thus, there are two parameters ($C_0$ and $C_2$) for each of the $pp$ and $np$ $^1S_0$ channels, and three ($C_0$, $C_2$ and $D_2$) for the $^3S_1$ channel. These parameters will be determined from scattering and bound-state experimental data.

• **Potential with the pion**

  To the order considered, the contribution of the pion is simple and parameter-free. There are no loops in the irreducible channel, so all we need is the potential

$$V(q) = -\tau_1 \cdot \tau_2 \frac{g_A^2}{4f_\pi^2} \frac{\sigma_1 \cdot q \sigma_2 \cdot q}{q^2 + m_\pi^2} + V_{\text{no}\rightarrow\pi}(q)$$  \tag{29}$$

where $g_A \simeq 1.26$ is the axial-vector coupling constant and $f_\pi \simeq 93$ MeV is the pion decay constant.
• Renormalization

Since the cutoff is finite, there are no divergences, so for a given $\Lambda$, the renormalization amounts to determining the constants that appear in the potential. All the constants are fixed in our procedure by the scattering lengths and effective ranges for $np$ and $pp$ in $^1S_0$ channel and the binding energy of the deuteron ($B_d$), the deuteron $D/S$ ratio ($\eta_d$) and the wave function normalization factor ($A_s$), all of which are given very accurately by experiments. For the $pp$ channel, one has to take into account the Coulomb and other electromagnetic effects. This can be done in various ways with little ambiguity\textsuperscript{29}.

• Observables

The following observables will be predicted and compared with experiments.

1. Phase shifts for $np$ $^1S_0$ channel.
2. The charge radius $r_d$, quadrupole moment $Q_d$, D-state probability $p_D$ and magnetic moment $\mu_d$ of the deuteron.
3. The radiative $np$ capture $n + p \rightarrow d + \gamma$.
4. The solar fusion process $p + p \rightarrow d + e^+ + \nu_e$.

3.3 The Power of EFT: Comparison with Experiments

We will now show that the theory works impressively. We will also see the power of the chiral filter in our scheme. The deuteron properties and electroweak matrix elements will be treated with and without the pion. The scattering phase shifts will be shown later.

• Results without the pion

Without the pion, we expect that the cutoff should be around the pion mass $\sim 140$ MeV. The results are listed in Table 1. They are remarkably stable against the cutoff as long as it is near or above the pion mass. The agreement with experiment is also quite impressive.

• Results with the pion

With the pion figuring explicitly, we expect the cutoff to be at the next mass scale $\gtrsim 2m_\pi \sim 300$ MeV. The results are given in Table 2. The results are even more spectacular.
Table 1: The next-to-leading order (NLO) results without the pion field. The static properties of the deuteron, the M1 and GT amplitudes are listed for various choices of the cutoff $\Lambda$. The low-energy input parameters are the scattering lengths and effective ranges for the $np$ and $pp$ $^1S_0$ channels, and $B_d$, $A_s$ and $\eta_d$ for the deuteron channel.

| $\Lambda$ (MeV) | 140 | 150 | 175 | 200 | 225 | 250 | Exp. $v_{1p}$|
|-----------------|-----|-----|-----|-----|-----|-----|-------------|
| $r_d$ (fm)      | 1.973 | 1.972 | 1.974 | 1.978 | 1.983 | 1.987 | 1.966(7) | 1.967 |
| $Q_d$ (fm$^2$)  | 0.259 | 0.268 | 0.287 | 0.302 | 0.312 | 0.319 | 0.286 | 0.270 |
| $P_D$ (%)       | 2.32 | 2.83 | 4.34 | 6.14 | 8.09 | 9.90 | 5.76 |
| $\mu_d$        | 0.867 | 0.864 | 0.855 | 0.845 | 0.834 | 0.823 | 0.8574 | 0.847 |
| $M_{M1}$ (fm)   | 3.995 | 3.989 | 3.973 | 3.955 | 3.936 | 3.918 | 3.979 |
| $M_{GT}$ (fm)   | 4.887 | 4.881 | 4.864 | 4.846 | 4.827 | 4.810 | 4.859 |

Table 2: The NLO results with pion field. See the caption of Table 1.

| $\Lambda$ (MeV) | 200 | 250 | 300 | 350 | 400 | 500 | Exp. $v_{1p}$|
|-----------------|-----|-----|-----|-----|-----|-----|-------------|
| $r_d$ (fm)      | 1.963 | 1.965 | 1.966 | 1.967 | 1.968 | 1.968 | 1.966(7) | 1.967 |
| $Q_d$ (fm$^2$)  | 0.261 | 0.268 | 0.272 | 0.273 | 0.274 | 0.274 | 0.286 | 0.270 |
| $P_D$ (%)       | 3.16 | 4.11 | 4.77 | 5.16 | 5.35 | 5.39 | 5.76 |
| $\mu_d$        | 0.862 | 0.856 | 0.853 | 0.850 | 0.849 | 0.849 | 0.857 | 0.847 |
| $M_{M1}$ (fm)   | 3.987 | 3.976 | 3.968 | 3.963 | 3.958 | 3.952 | 3.979 |
| $M_{GT}$ (fm)   | 4.884 | 4.874 | 4.867 | 4.862 | 4.859 | 4.854 | 4.859 |
• **Meson-exchange current corrections**

In order to complete the calculations for the \( np \) capture and the proton fusion process, the exchange-current corrections should be calculated.

For the M1 transition for the \( np \) capture, the leading correction to the dominant single-particle matrix element comes at the next order from one-soft-pion exchange. This is in accordance with the chiral filter argument. A detailed benchmark calculation in ChPT that includes loop corrections (NNLO relative to the leading M1 matrix element) finds 9.29% correction to the cross section for the radiative \( np \) capture getting \( \sigma^\text{th} = 334 \pm 3 \text{ mb} \) in agreement with the experiment \( \sigma^\text{exp} = 334 \pm 0.5 \text{ mb} \), the 1% uncertainty in the theoretical result being due to short-distance physics incompletely accounted for in chiral perturbation theory.

The meson-exchange correction to the Gamow-Teller matrix is a different story since it is not protected by the chiral filter. In order to assure an accuracy, one would have to go to \( \mathcal{O}(Q^n) \) with \( n \geq 3 \). So far, calculations have been done up to \( \mathcal{O}(Q^3) \) but there is no reason to believe that terms of order with \( n \geq 4 \) are negligible compared with what has been calculated. The predicted \( S \) factor for the solar burning process comes out to be \( S^\text{ChPT} = 4.05 \times 10^{-25} \text{ MeV} - \text{barn} \) with a possible uncertainty of \( \sim 2\% \) due to higher-order corrections in the meson-exchange currents that have not been evaluated yet. There is no direct check of this prediction. It is however quite close to what has been obtained by the usual potential model calculation (with realistic potentials), \( S^\text{pot} = 4.00 \times 10^{-25}(1 \pm 0.007 \pm 0.020 - 0.011) \text{ MeV} - \text{barn} \). As is well known now, this leads to the celebrated solar neutrino problem within the Standard Model. The neutrino mass resolution to the problem is a hot topic lying in a different domain of physics.

• **\( np^1 S_0 \) phase shifts**

Because of the large scattering length involved in this channel, naive dimensionally regularized EFTs do not work in a natural way. Much “sweat and blood” have been shed on the issue of how to rescue the dimensional regularization scheme from demise. The final rescue was found in the PDS scheme with a slight modification of the counting rule together with the demotion of the pion to a sub-dominant role. All this technical problem is avoided in a straightforward way in the cut-off regularization scheme. It is now generally accepted that the PDS scheme is equivalent to the cut-off scheme if the chiral counting is done properly to the same order.

The calculation with a cutoff to NLO in the irreducible vertex (or poten-
tial) without any additional fiddling is fully consistent. Figure shows that the phase shifts come out very well, fairly independently of the cutoff, up to $p \sim 100$ MeV if the pion is present and to $p \sim 70$ MeV if the pion is absent. Again the presence of the pion as an explicit degree of freedom markedly improves the result.

Figure 2: The $np \, ^1S_0$ phase shift (degrees) vs. the center-of-mass (CM) momentum $p$. We show the predicted results both without (left) and with (right) the pion field. For each case, our NLO results are given for two extreme values of the cutoff: The solid curve represents the lower limit, $\Lambda = 100$ MeV (without the pion) and 200 MeV (with the pion), and the dotted curve the upper limit, $\Lambda = 300$ MeV (without the pion) and 500 MeV (with the pion). The “experimental points” (obtained from the Nijmegen multi-energy analysis) are given by the solid circles. Our theory with $\Lambda = 150$ MeV (without the pion) and $\Lambda = 250$ (with the pion) are shown by the dashed line to show that the theory is in almost perfect agreement with the data up to $p \lesssim 200$ MeV.

The goodness of EFT: Insensitivity to $\Lambda$

Although there is little dependence on the value of $\Lambda$, the results are not entirely independent of it. This is expected since the series is truncated in the way the cutoff procedure goes. The reason is simply that the (chiral) counting in the scheme with the cutoff is not fully consistent, with the error residing in higher orders. That is, a genuine $\Lambda$ independence requires including certain

Furthermore it is so simple that it makes one wonder what all this fuss about the PDS and other schemes is about.
terms of the next order and we are not including them here, so they represent
the error committed. That the results are quite insensitive to the cutoff value
indicates however that the error committed is small enough.

![Graph](image)

Figure 3: $\mathcal{E}(M1)$ (upper) and $\mathcal{E}(GT)$ (lower) vs. the cutoff $\Lambda$. The solid curves
represent the NLO results with pions and the dotted curves without pions.

Let us quantitatively examine whether and how our theory is a truly effective
effective theory. We do this using the electroweak processes. This test has
not yet been made by other workers in the field. For this we should demand
not only that the theory agree with experiments but also that the result not
depend sensitively on the cutoff. We test the latter by looking at the following
quantities:

\[
\mathcal{E}(M1) \equiv \frac{M^{\text{th}}_{M1} - M^{18}_{M1}}{M^{18}_{M1}}, \quad \mathcal{E}(GT) \equiv \frac{M^{\text{th}}_{GT} - M^{18}_{GT}}{M^{18}_{GT}},
\]

(30)

where $M^{\text{th}}_{M1}$ and $M^{18}_{M1}$ denote, respectively, the M1 transition
matrix element of our NLO calculation and that of the Argonne $v_{18}$ potential, \cite{9} and similarly
for $\mathcal{E}(GT)$. Here we are taking the Argonne potential as "experiment" since
it is fitted to experiments. Since these quantities are not the entire story that
can be directly compared with experiments (due to the exchange currents),
this is the best we can do to NLO. We see in Fig. 3 that our criterion is very
well satisfied. This is the more so when the pion is incorporated. We see in a
quantitative way how accounting for “relevant degrees of freedom” in effective theories improves the result as well as the consistency of the theory.

- The chiral filter

I mentioned above that the advantage over other consistent schemes (such as PDS) of the Weinberg counting combined with the cut-off regularization we adopted is that the chiral filter mechanism allows one to make certain predictions, not postdictions that are not easily accessible to other schemes. The soft pion plays a particularly prominent role in nuclear processes and this predominance is visible in the scheme. Thus the importance of the soft-pion exchange in nuclear isovector M1 transitions and in nuclear axial-charge transitions was predicted before experiments came to confirm it. A further forecast we can make is that when the pion-exchange is suppressed for whatever reason, chiral perturbation expansion will lose much, if not all, of its predictive power and one would be forced to adopt a different strategy than the usual one. Indeed this is exemplified by the inability of low-order chiral perturbation calculations to explain the experimental cross section for the \( p + p \rightarrow p + p + \pi^0 \) process. In other schemes, the particular role of pions could be hidden, though not absent, and one would have difficulty in “seeing” it and hence in making predictions that hinge on it.

As a specific example, let me take the radiative \( np \) capture. The precise agreement between the chiral perturbative calculation and the experiment for the capture cross section obtained in \(^23\) is the bench-mark in nuclear physics and the result was understood due to the clear identification of the important role of the pion exchange. In the recent calculation of the same process in the PDS scheme \(^31\), the authors emphasize the fact that the pion-exchange graph in their scheme is only one of several NLO terms in the expansion, is scale- and scheme-dependent and hence has no particular significance. Since the experimental cross section is used to fix one unknown NLO dimension-6 counter term, this calculation is not a prediction. Indeed the counter term so extracted turns out to be as large as the terms explicitly calculated, and there is nothing which says that higher order (nonanalytic) terms and counter terms will be negligible in the scheme.

I should hasten to say that there is nothing basically wrong in this PDS calculation. It is just that it does not seem as predictive. My personal opinion on this matter is this. The PDS scheme is perhaps more consistent in the spirit of EFT than the scheme of ref\(^2\). However by strict adherence to the consistency of the counting, it misses an important piece of physics, namely, the short-range correlation that is present in nuclear wave functions.
So what is this short-range correlation (SRC)? As far as I can see, this is not understood within the framework of EFT. It is an effect of short-distance physics that is “integrated out” from the EFT, so in principle if the expansion of the EFT makes sense, the SRC effect presumably is lodged in the parameters of the EFT. In this scheme, then, the question is: How far does one have to go in the expansion to capture the essence of SRC physics? In the nuclear physics community, the SRC appears in the wave function as a sort of “black box” determined by looking at experiments. In [23], the SRC so invoked is found to suppress all contact terms in the current [7]. The result shows that this is consistent with the chiral filter, the precise agreement providing a support for this reasoning: The error incurred is found to be negligible. It seems possible to justify this SRC mechanism by resorting, along the line developed by Lepage [21], to the OPE in the wave function to factorize the short-distance (asymptotically free) part of the QCD interaction inaccessible to chiral perturbation theory. By delegating the short-distance physics to the wave function as we did, we “enhance” the terms which are allowed by chiral filter and “suppress” those which are not. This is why there is no predictive power for our ChPT when soft-pion terms are suppressed. On the other hand, the PDS scheme may allow a “systematic calculation,” but there is no reason to expect that it would work well.

4 Effective Field Theories For Dense Matter

The strategy found to be so successful at low energy for two-nucleon systems is being extended to three-body systems [32], but it would require a quantum jump to get to nuclear matter and denser matter. This means that there cannot be a “straightforward” application of chiral perturbation theory in the immediate future. All we can do for the moment is to resort to some educated guesses and check simplifying assumptions we make against nature to see whether our guesses pass the test. We eliminate wrong guesses and adopt the right ones by trial and error [3]. I will describe in the rest of this lecture how we move forward in this direction and meet with some success. This part is based on a series of papers written recently [33, 34, 35, 36].

The basic idea is to combine two superbly effective field theories available

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#7 Note that contact terms in the irreducible diagrams contributing to the potential are not suppressed by the same mechanism as explained in [8]. This is the modern understanding of the chiral filter conjecture.

#8 My point is that if we sit and worry about the enormous complications brought in by many-body dynamics under extreme conditions, we will get nowhere. The analogy would be a centipede who sits and worries how it is going to march forward with so many legs potentially in the way.
in the market via a simple scaling assumption which is known in the literature as “BR scaling.” The two EFTs that I will invoke are chiral Lagrangian field theory we now understand well and Landau Fermi liquid theory for dense matter with a Fermi sea which can be formulated as an effective field theory. We know how the first works in dilute systems and how the latter works in strongly correlated fermionic systems. Our aim is to bridge the two in a way that can be applied to dense hadronic matter.

4.1 Nuclear Matter as a Chiral Fermi Liquid

The first question we have to address is: What is nuclear matter in chiral Lagrangian field theory? One might address this question in terms of a skyrmion with a large winding number $A$ (see for references) but so far very little has been learned from this. It seems more profitable to start with a chiral Lagrangian with nucleons and mesons incorporated, calculate the effective action in a suitable chiral expansion and look for a (nontopological) soliton with a given baryon number $A$. Such a solitonic solution would then describe the ground state of a system with baryon number $A$ which could be closely resembling a heavy nucleus in its ground state. Lynn has argued, using a somewhat oversimplified model, that such a system is a chiral liquid that gives a realistic description of a droplet of nuclear matter. Although there is no convincing derivation of the chiral liquid solution, the reasoning is very plausible and I am going to assume that this conjecture is correct. The next crucial step is to identify this chiral liquid structure with the mean field solution of a phenomenologically successful effective chiral action such as that written by Furnstahl et al. in terms of an expansion based on “naturalness condition.” The effective chiral Lagrangian theory of Furnstahl et al for nuclear matter is a generalization of Walecka model, so we arrive at the first conclusion that the chiral liquid structure can be identified with the mean field solution of a Walecka-type model.

4.2 Effective Chiral Lagrangian with BR Scaling

I will next argue that when BR scaling is implemented into an effective chiral Lagrangian, the mean-field theory of this Lagrangian corresponds to Walecka mean field theory extended a la Furnstahl et al.

- BR scaling

It is highly likely that when baryon matter is present, the vacuum itself on which excitations take place is modified. This is so since the quantity
that characterizes the vacuum, the quark condensate $\langle \bar{q}q \rangle$, must change in
the presence of matter as indicated by such models as Nambu-Jona-Lasinio
model (see 4). We may consider writing an effective Lagrangian in a space that
contains a matter of density $\rho$ directly in terms of the parameters defined in
that space. The parameters then must change as the density changes. One
can think of the parameters depending on some field variables that change
according to the change of the condensate and hence indirectly according to
density. We will then assume that the effective Lagrangian defined in that
space characterized by a particular field value (or density) be dictated by the
symmetries of QCD. In 3, assuming the relevant symmetries to be the chiral
symmetry and the scale symmetry of QCD phrased in the form of large $N_c$
theory, i.e., skyrmion, the mass parameters and the (pion decay) coupling
constant of the effective theory are found to scale as

$$\frac{M_N^*}{m_N} \approx \frac{m_\omega^*}{m_\omega} \approx \frac{m_\rho^*}{m_\rho} \approx \frac{m_\phi^*}{m_\phi} \approx \frac{f_\pi^*}{f_\pi} \equiv \Phi. \quad (31)$$

Here the $\star$ represents in-medium quantity, $M_N^*$ a scaling nucleon mass unaf-
fected by the pion cloud (the “Landau effective mass” of the nucleon $m_N^*$ will
be defined later), $\omega$ and $\rho$ are respectively the isoscalar vector meson and the
isovector vector meson degrees of freedom and $\phi$ an isoscalar scalar meson field
which is a chiral singlet effective in nuclear matter at a mass $\sim 500$ MeV.

• Chiral symmetry and Walecka model

It has often been asked why Walecka model 39 works. In fact, Cohen et
al 40 gave a QCD sum rule justification of the large vector and scalar mean
fields found in the Walecka-type models. Contrary to what some people have
claimed, the success of the model is not due to any of the following: (1) renor-
malizability, (2) relativity, (3) accuracy of the mean field at large density.

The theory is an effective theory and not a fundamental theory, so neither
does renormalizability have any raison d'être nor should it be valid at asymp-
totic density at which the model – unless fundamentally changed – should be
irrelevant. Since heavy-baryon (nonrelativistic) chiral perturbation treatment
(as given below) successfully reproduces the result of Walecka-type models,
relativity is clearly not an essential ingredient for the success either.

So why does Walecka model work?

I suggest the reason is that it is consistent with chiral symmetry, as explained in 41,42. To see this, suppose we write an effective Lagrangian in
heavy-fermion formalism with all except nucleon fields integrated out. The
Lagrangian will consist of the usual nucleon bilinear term and multi-Fermion
interactions, \((\bar{N} \cdots N)^n\) with \(n \geq 2\) with various covariants and derivatives appearing so that chiral symmetry is preserved. In particular the four-Fermi interactions of the form

\[
\bar{N} \Gamma^\alpha N \bar{N} \Lambda_\alpha N
\]

(32)

where \(\Gamma^\alpha = \Lambda^\alpha = 1\) and \(\Gamma^\alpha = \Lambda^\alpha = \gamma^\alpha\) have respectively scalar meson (“\(\sigma\)”)) and isoscalar vector meson (“\(\omega\)”)) quantum numbers which would, when suitably treated in mean field, give rise to the same effects as the \(\sigma\) and \(\omega\) fields in a Walecka-type theory. It is clear from this argument that the scalar is not the sigma of the linear sigma model. It is a chiral singlet and hence safe from the disaster in nuclear matter that the \(\sigma\) of the linear sigma model produces.

The most reasonable way of interpreting the scalar is that it is a “quarkonium” component of the dilaton that figures in the trace anomaly. This is therefore a chiral singlet. In free space, it must be massive \(\sim 700\) MeV but go down to \(\sim 500\) MeV at nuclear matter density. This scalar must ultimately join the pions at the chiral phase transition to make up the quartet of the \(O(4)\) chiral symmetry. This could occur via Weinberg’s mended symmetry. Thus as density increases toward the chiral symmetry restoration point, the mass of the scalar must drop. BR scaling prescribes how the mass could drop.

The vector meson \(\omega\) is also a chiral singlet. According to BR scaling, its mass must drop as a function of density. How does this happen? The way to see that the mass dropping is “universal” as in (31) is that it is essentially a scalar tadpole acting on all hadrons in the way that the Nambu-Jona-Lasinio model would describe, namely through the dynamically generated mass of a constituent quark making up the hadrons. It is the higher dimension multi-Fermi operators present in the effective Lagrangian that would generate the tadpole mechanism.

• **The chiral liquid as Landau Fermi liquid**

If we assume that we have a chiral liquid defined with the Fermi momentum \(k_F\), then excitations around the liquid ground state can be treated again as an effective field theory as described beautifully in [18]. This will give essentially Landau Fermi liquid theory, with two fixed points, one the effective mass of the nucleon \(m_N^\star\) which we will identify as Landau effective mass, and the other, Fermi liquid interactions \(\mathcal{F}\) between quasiparticles.

The next important argument we need can be borrowed from Matsui [2] who showed that Walecka theory for nuclear matter can be mapped to Landau Fermi liquid theory with the vector-meson-mediated interaction related to the Landau parameter \(F_1\). Since this Landau parameter governs also the Landau
effective mass of the nucleon, the scaling of the nucleon mass must be related
mainly to the vector degree of freedom. With BR scaling this means that
the scaling of all hadrons must be governed by the Fermi-liquid fixed-point
parameter $F_1$.

Our conclusion then is: chiral Lagrangian with BR scaling, via its identifi-
cation with Walecka mean field theory, can be mapped to Landau Fermi liquid
fixed point theory. For details of the reasoning, I refer to [33, 34, 35, 36].

4.3 Predictions at Normal Nuclear Matter Density

Here we shall establish a crucial relation between BR scaling and the Land-
au parameter $F_1$ and give some predictions based on the relation.

- **Nuclear matter**

  We learned from the chain of arguments developed above that in order
to describe nuclear matter, all we need to do is to write the simplest form
of Walecka model with masses and coupling constants affixed with a star and let
the masses scale according to (31). The Lagrangian is

$$L = \bar{N} [\gamma_{\mu} (i \partial^\mu - g_v^\star (\rho) \omega^\mu) - M^\star (\rho) + h^\star \phi] N$$

$$+ \frac{1}{2} [ (\partial \phi)^2 - m_{\omega}^2 (\rho) \phi^2 ] - \frac{1}{4} F_{\omega}^2 + \frac{1}{2} m_{\phi}^2 (\rho) \phi^2$$

where $N$ is the nucleon field, $\omega_{\mu}$ the isoscalar vector field and $\phi$ an isoscalar
scalar field. Pseudoscalar fields can be suitably incorporated as done later but
they do not figure in the symmetric nuclear matter that we are concerned with
here. The scaling behavior of the constants $g_v$ and $h$ is left arbitrary. It was
shown in [31] that in the mean field, this Lagrangian with the BR scaling (31),
a suitable scaling of the constant $g_v$ and no scaling of $h$ gives a surprisingly
good description of the ground state with a compression modulus well within
the accepted value $200 \sim 300$ MeV. The result is given in Fig.4. The scaling
$\Phi(\rho_0) = 0.78$ needed here is forced on us, as described below, by QCD sum-rule
calculations and orbital gyromagnetic ratios in heavy nuclei. Contrary to the
worry expressed by some people, when properly interpreted, the density depen-
dence of the parameters of the Lagrangian does not spoil any thermodynamic
consistency of the theory [4].

\#9 Note that this differs from the usual Walecka model where the nucleon mass shift comes
from the VEV of the $\phi$ field.
Figure 4: $E/A - M$ vs. $\rho$ for this model (S3) compared with the elaborate model (labeled as FTS1) of Furnstahl et al.\cite{38}. $B1$ and $B3$ include higher polynomial terms in the Lagrangian\cite{33} that represent fluctuations beyond the mean field.

- BR scaling $\Phi$ and Landau parameter $F_1$

From Galilean invariance (or Lorentz invariance) follows the Landau effective mass of the nucleon

$$\frac{m_N^*}{m_N} = 1 + F_1/3 = \left(1 - \tilde{F}_1/3\right)^{-1} \quad (34)$$

where $\tilde{F} = (m_N/m_N^*)F$. On the other hand, the chiral Lagrangian with BR scaling gives in tree order

$$\frac{m_N^*}{m_N} = \Phi(1 + F_1^\pi/3) = \left(\Phi^{-1} - \tilde{F}_1^\pi/3\right)^{-1} \quad (35)$$

where $F_1^\pi$ is the contribution from the pion to the Landau $F_1$ parameter. Now comparing $(34)$ and $(35)$, we see that

$$\tilde{F}_1 - \tilde{F}_1^\pi = 3(1 - \Phi^{-1}). \quad (36)$$

It seems reasonable to assume that the contribution to $F_1$ from the $\omega$ channel saturates the rest. Then we have

$$\tilde{F}_1^\omega = 3(1 - \Phi^{-1}). \quad (37)$$
This is a crucial relation between BR scaling and the $\omega$ degree of freedom. It shows that the nucleon effective mass is primarily governed by the $\omega$ degree of freedom as mentioned above.

Calculation of the response of a nucleon sitting on top of the Fermi sea to electromagnetic current gives the gyromagnetic ratio that agrees precisely with the Landau-Migdal formula:

\[ g_l = \frac{1 + \tau_3}{2} + \delta g_l \]  

with

\[ \delta g_l = \frac{1}{6} (\tilde{F}_1' - \tilde{F}_1) \tau_3 \]  

where $F_1'$ is the isovector counterpart to $F_1$. The chiral Lagrangian with BR scaling gives in tree order

\[ \delta g_l = \frac{4}{9} [\Phi^{-1} - 1 - \tilde{F}_1^\tau] \tau_3. \]  

Since the $F_1^\tau$ is completely known by chiral symmetry, given a $\delta g_l$ from experiment, one can determine $\Phi$ at nuclear matter density. We can turn the problem around and calculate $\delta g_l$, given $\Phi(\rho_0)$. We shall do the latter by noting that $\Phi(\rho_0)$ can be calculated from Gell-Mann-Oakes-Renner formula for the pion in medium or from QCD sum-rule calculations of the vector meson mass in medium and using BR scaling. The two ways give the same answer. Take the QCD sum-rule value:

\[ \Phi(\rho_0) = 0.78 \pm 0.08. \]  

With this value and the known $F_1^\tau$ at $\rho = \rho_0$ (for which the Landau mass comes out to be $m_N^*(\rho_0)/m_N \approx 0.70$), we get

\[ \delta g_l(\rho_0) = 0.227\tau_3. \]  

This agrees precisely with the experimental value obtained for the proton from giant dipole resonances in heavy nuclei

\[ \delta g_l^{(exp)} = 0.23 \pm 0.03. \]

- **Fluctuations around nuclear matter ground state**

  Pseudo-Goldstone octet bosons fields can be readily restored in the Lagrangian in a way consistent with chiral symmetry. Their fluctuations
will describe their properties in the medium defined by the scaling parameter $\Phi$. For example, kaonic fluctuations would have the form

$$L^{eff} = \frac{-6i}{8f_\pi^2} \overline{K} \partial_i K N \dagger N + \frac{\Sigma_{KN}}{f_\pi^2} \overline{K} K$$

where $\Sigma_{KN}$ is the $KN$ sigma term. In tree order (i.e., mean field), $\overline{NN} \approx N \dagger N \approx \rho$. The potential felt by the kaon in the background of nuclear matter is given by

$$V_{K^\pm} = \pm \frac{3}{8f_\pi^2} \rho_s$$

and

$$S_{K^\pm} = -\frac{\Sigma_{KN}}{f_\pi^2} \rho_s$$

where $\rho = \langle N \dagger N \rangle \approx \rho_s = \langle \overline{NN} \rangle$. At nuclear matter density, we can identify these results as one-third of the corresponding potentials for nucleons, so we can write

$$V_{K^\pm} \approx \pm \frac{1}{3} V_N$$

and

$$S_{K^\pm} \approx \frac{1}{3} S_N.$$ 

Phenomenology in Walecka-type mean-field theory gives $(S_N - V_N) \lesssim -600 \text{ MeV}$ for $\rho = \rho_0$. This leads to the prediction that at nuclear matter density

$$S_{K^-} + V_{K^-} \lesssim -200 \text{ MeV}.$$ 

This is consistent with the result coming from the analysis in K-mesic atoms $(S_{K^-} + V_{K^-})_{\text{atom}}^{K^-} \sim -200 \text{ MeV}$. This order of the mass shift in the kaon mass is also seen in heavy-ion experiments and found to lead to an interesting consequence on the formation of neutron stars and light-mass black holes. This issue is discussed by Li, Lee and Brown.

**Strongly enhanced axial-charge transitions in heavy nuclei**

As mentioned at the beginning, Warburton’s experiments and analyses on weak axial-charge transitions confirmed one of the chiral filter predictions. There was also a surprise there which excited quite a few theorists. Warburton
found that not only was there a strong mesonic contribution to the axial-charge matrix element but even more interestingly, the enhancement over the single-particle matrix element in heavy nuclei (such as lead) was 100%. It turns out that this strong enhancement in dense nuclei can be simply explained by BR scaling as proposed by Kubodera and Rho [49].

Consider, following Warburton, the quantity $\epsilon_{MEC}$ which is the ratio of the measured axial-charge matrix element over the theoretical single-particle matrix element. Warburton’s result for the lead region was that

$$ \epsilon_{MEC}^{exp} \approx 1.9 \sim 2.0. \quad (50) $$

Now using our chiral Lagrangian with BR scaling in the tree order, we get the very simple formula [36]

$$ \epsilon_{MEC}^{th} \approx \Phi^{-1}(1 + R) \quad (51) $$

where $R$ is the ratio of the (soft-pion dominated) meson-exchange current matrix element over the single-particle matrix element which for $\rho = \rho_0$ comes out to be $0.56 \sim 0.61$ (this is a quite reliable range protected by the chiral filter mechanism [48]). For the lead region in question, we may take $\rho \approx \rho_0$. We find for the range involved for $R$,

$$ \epsilon_{MEC}^{th}(\rho_0) = 2.0 \sim 2.1. \quad (52) $$

In my opinion, this is as strong a confirmation of the notion of BR scaling as the orbital gyromagnetic ratio discussed above.

4.4 Predictions For Higher Density $\rho > \rho_0$

We have so far seen that used at the mean field level, the chiral effective Lagrangian with BR scaling can reasonably well describe nuclear properties at nuclear matter density. The question is: Can one extend the treatment to higher densities at which various phase changes such as kaon condensation, color superconductivity, chiral restoration etc. could take place? In [48], it was argued that kaon condensation could take place at $\rho \lesssim 3 \rho_0$ within the framework. What about the others? For this, one must go beyond the mean field with the effective Lagrangian but so far there is no systematic formulation of going higher order with the Lagrangian with BR scaling.

The mean field arguments are expected to work provided quasiparticle notions apply to the hadrons relevant to the process. For instance, BR scaling has been successfully applied to the CERES dilepton processes [49] which involve densities of $2 \sim 3 \rho_0$ and temperatures of $\gtrsim 100$ MeV. Here the principal
mechanism that is invoked is the mass shift of the \( \rho \) meson with the width playing a secondary role. The reasoning here is that when one fluctuates around the background at a given density with scaling masses and other parameters, most of the strong correlations are included in those effective parameters and the residual interactions on top of the given background are weak so that a weakly-interacting local field description is valid. Thus it may be that viewed from BR scaling, widths do not destroy the quasiparticle picture.

On the other hand, if one were to build correlations starting from the matter-free background, one is in a strong-coupling regime and it seems inevitable that interactions broaden the width of the hadrons involved, in particular, of the \( \rho \) meson. It is found that the medium-broadened width of the vector meson can also explain reasonably well the CERES data\textsuperscript{52}.

Is there any connection between the mass-shifted vector meson defined at a density \( \rho \neq 0 \) and the broadened-width vector meson built on the \( \rho = 0 \) background? My conjecture is that there is a dual description of the same process in terms of different languages, similar to the CCP discussed in the first lecture. Here the languages dual to each other are the “partonic” picture of ref.\textsuperscript{51} (with BR-scaling hadrons) and the “hadronic” picture of ref.\textsuperscript{52} (with many-body interactions). This is somewhat like the quark-hadron duality one sees in heavy-light meson decay processes\textsuperscript{53}. How this dual structure can work in the case of the CERES dilepton process is explained in a recent paper co-authored by the two schools – BR scaling and dynamical width\textsuperscript{54}.

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