Non-spherical nucleon clusters in the mantle of a neutron star: CLDM based on Skyrme-type forces

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Abstract. Neutron stars are superdense compact astrophysical objects. The central region of the neutron star (the core) consists of locally homogeneous nuclear matter, while in the outer region (the crust) nucleons are clustered. In the outer crust these nuclear clusters represent neutron-rich atomic nuclei and all nucleons are bound within them. Whereas in the inner crust some neutrons are unbound, but nuclear clusters still keep generally spherical shape. Here we consider the region between the crust and the core of the star, so-called mantle, where non-spherical nuclear clusters may exist. We apply compressible liquid drop model to calculate the energy density for several shape types of nuclear clusters. It allows us to identify the most energetically favorable configuration as function of baryon number density. Employing four Skyrme-type forces (SLy4 and BSk24, BSk25, BSk26), which are widely used in the neutron star physics, we faced with strong model dependence of the ground state composition. In particular, in agreement with previous works within liquid drop model, mantle is absent for SLy4 (nuclear spheres directly transit into homogeneous nuclear matter; exotic nuclear shapes do not appear).

1. Introduction

The matter of the outer crust of the neutron star (corresponds to density $\rho < \rho_{\text{drip}}$, where $\rho_{\text{drip}} \approx 4.3 \times 10^{11}$ g cm$^{-3}$ is the neutron drip density \cite{1, 2}) is composed of neutron rich nuclei up to the drip line. Except outermost layers, these nuclei are subjected to beta-decays in the terrestrial conditions. However in the crust they are surrounded by degenerate electrons with high chemical potential and beta-decay is blocked by the Pauli principle. In the inner crust ($\rho > \rho_{\text{drip}}$) part of the neutrons are not localized within the nuclei (nuclear clusters) and occupy continuous part of the energy spectrum \cite{1, 2}. Although the nuclear clusters in the inner crust are highly neutron rich, they can be considered spherically symmetric with reasonable accuracy. However, at even higher density (in the deeper layer of the neutron star) more exotic strongly non-spherical (two or one dimensional) configurations of clusters are predicted to appear \cite{1, 2}. This layer usually called mantle and precedes the transition to the neutron star core, where nucleons are not clusterized. The nuclear clusters in the mantle are often called “pasta phases” due to their pasta-like structure.

For the first time the mantle was considered in \cite{3}. According to this seminal work the geometry of nucleon clusters changes with the density increase in the following order: spherical clusters — cylindrical clusters (two-dimensional, so-called “spaghetti”), plate-like nuclei (one-
dimensional, “lasagna”) — neutron cylinders surrounded by nuclear matter (two-dimensional, “antispaghetti” or “bucatini” phase) – neutron spheres surrounded by nuclear matter (three-dimensional, “Swiss cheese” phase). Two latter phases are often refereed as inverted ones. For recent studies in the nuclear pasta field see, e.g., [4, 5, 6, 7, 8, 9] and references therein.

Here we analyze the mantle within the compressible liquid drop model (CLDM), which allows us to write down the energy density of the clustered nucleonic matter in algebraic form (1). As a result, the problem of energy minimization is reduced to a system of algebraic equations [1], which express the equilibrium conditions naturally [2, 10]. It significantly speeds up the calculations in comparison to more detailed methods, such as, extended Thomas-Fermi (ETF) approach based [11] (see, e.g., [7, 12] for application of this approach to the neutron star mantle).

We apply four sets of parameters of CLDM model, corresponding to four Skyrme-type forces, commonly applied in the neutron star physics. Namely, the first one – SLy4 – was developed by Lyon group [13], and chosen here as originally constructed for the description of neutron-rich nuclei. It reproduces the properties of doubly magic nuclei and is consistent with the UV14+VII model for pure neutron matter, which was developed in [14]. The SLy4 was further improved by Brussels group, leading to a wide set of functionals traditionally named BSk. Among them we select BSk24, BSk25, BSk26 which was fitted to reproduce the masses of the Atomic Mass Evaluation 2012 (AME2012) [15] and several realistic models of the strong interaction in homogeneous neutron matter, but differ by the symmetry energy coefficients (see [16] for details).

2. Compressible liquid drop model

Within the CLDM model the nucleon system is modeled as composed of two regions, filled by homogeneous phases and separated by sharp interface. The energy density of homogeneous phases is calculated in the same way as for bulk matter, while the thermodynamic properties of the interface can be calculated within ETF approach (e.g., [17, 18, 19]). For convenience, the phase boundary is chosen in such a way that only neutrons are adsorbed on the surface [1, 17]; moreover it simplifies the Coulomb energy calculation.

For non-inverted phases, the denser part is the nucleon cluster consisting of protons and neutrons, whereas the lower-density region is presented by degenerate neutrons, which surrounds the cluster. The total energy density in the CLDM can be written as:

\[
\epsilon = u \epsilon^{\text{bulk}}(n_{ni}, n_{pi}) + (1 - u) \epsilon^{\text{bulk}}(n_{no}, 0) + \epsilon_e(n_e) + \epsilon_s(n_s, u, r_p) + \epsilon_C(n_{pi}, r_p, u),
\]  

(1)

where \(\epsilon^{\text{bulk}}(n_{ni}, n_{pi})\) and \(\epsilon^{\text{bulk}}(n_{no}, 0)\) are contributions from the bulk energy densities of the nucleons (as usual, they describe only nuclear interactions and neglect the Coulomb interaction of protons), analytical expressions for them were taken from [13, 16, 20, 21]. The bulk energy densities include the nucleon rest mass; differences in the masses of the proton and the neutron are taken into account. \(\epsilon_e(n_e)\) is the energy density of an ideal degenerate relativistic electron gas (e.g., [1, 2]), \(\epsilon_C(n_{pi}, r_p, u)\) is the Coulomb energy density (see below), finally \(\epsilon_s(n_s, u, r_p)\) is the surface energy density.

According to Eq. (1), the energy density depends on the following six parameters: \(n_{ni}, n_{pi}, n_{no}\) — the number densities of the neutrons and protons in the cluster and unbound neutrons, respectively (we checked that for the considered Skyrme-based models, protons do not appear outside the cluster), \(r_p\) is the proton radius of the cluster, \(u\) is the fraction of volume filled by cluster, and \(n_s\) is the surface density of adsorbed neutrons. The number density of electrons \(n_e\) is given by the electrical neutrality condition \(n_e = n_{pi}u\).

For inverted configurations (antispaghetti and Swiss cheese phases) \(r_p\) has a bit different meaning and describes the linear size of the volume filled by unbound neutrons and \(u\) is the ratio of the remaining volume to the entire volume of the cell. Then Eq. (1) is converted to the
expression:
\[
\epsilon = u\epsilon_{\text{bulk}}(n_{ni}, n_{pi}) + (1 - u)\epsilon_{\text{bulk}}(n_{no}, 0) + \epsilon_e(n_e) + \epsilon_s(\nu_s, 1 - u, r_p) + \epsilon_C(n_{pi}, r_p, 1 - u),
\]
which still depends on the same six parameters.

The Coulomb energy density \(\epsilon_C(n_{pi}, r_p, u)\) is calculated for electrically neutral Wigner-Seitz cell which is approximated by a cell with high rotational symmetry (namely, spherical, cylindrical, and planar cells for 3D, 2D, and 1D phases, respectively); the nuclear cluster is assumed to be located in the center of the cell. For solid state physics similar approach was suggested in [22] and often applied for neutrons star crust and mantle, being refereed as Wigner-Seits approximation.

The surface energy density, surface tension and number density of neutrons, adsorbed on the surface are calculated for plane boundary according to the formalism of [17, 18]; the results are described in thermodynamically consistent manner, using the approach of [10]. For the SLy4 and BSk24 models results are taken from [19], for BSk25 and BSk26 models similar calculations were performed in this work. It guarantees self-consistency of the model, since the surface terms are calculated using the same nuclear interactions for the whole range of possible two-phase equilibrium conditions without usage of phenomenological parameterizations like it was applied in [23] and many other papers in the field. These phenomenological parameterizations give rather accurate description of the surface tension at not too low proton fraction, however they becomes qualitatively inaccurate if proton fraction becomes very small. Namely, they predict surface tension to vanish only at vanishing proton fraction, in spite that accurate consideration of the two phase boundary for considered Skyrme-based models indicates that the surface tension vanishes at low, but finite (\(\sim 1 - 2\%\)) critical proton fraction, where the only solution for two phase equilibrium condition is equal composition in both phases (i.e., there are no surface which separates the phases because both phases becomes the same, see, e.g., [18, 19]).

We neglect curvature corrections because the approach suggested in [17, 18] seems to violate general thermodynamic relations for these corrections (the curvature correction for surface tension is fully determined by geometrical component, but curvature correction to the surface energy density includes also dynamical contribution, see [17, 18] for details). We plan to develop thermodynamically consistent approach for the curvature corrections and include them in subsequent work.

We apply zero-temperature approximation and minimize the energy density, given by equation (1) (or equation (2) for inverted phases), at constant baryon number density \(n_b\) to find CLDM parameters, which describe the ground state of the system. The minimization is performed for each of the five different shapes of clusters (spheres, spaghetti, lasagna, antispaghetti and Swiss cheese phases); the energy density of homogeneous nuclear matter is also calculated. As a next step, we compare energy density for these shapes and homogeneous nuclear matter to determine the most energetically favorable configuration for fixed \(n_b\). Being interested in the density range typical for the mantle of neutron stars, we determine the sequence of the phases in the mantle or conclude that the mantle should be absent for the given model, i.e. spherical nuclei are the most favorable up to the crust-core interface.

3. Results and conclusions

The energy density for considered set of cluster shapes is shown in Figure 1 for four parameterizations of our CLDM model, based on non-relativistic Skyrme-type effective interactions SLy4, BSk24, BSk25 and BSk26. To improve visibility, the energy density for spherically symmetric clusters are subtracted.

For SLy4 model spherical nuclei remain energetically favorable up to the transition (at \(\approx 0.073 \text{ fm}^{-3}\)) to the neutron star core (homogeneous nuclear matter). This result is consistent with [4]. Nevertheless, there are differences in the transition density (0.077 \text{ fm}^{-3} reported in [4]).
Figure 1. Difference between energy density of the system for a given shape of nuclear cluster and spherical nuclear clusters as a function of the number density of nucleons. Panels (a), (b), (c), and (d) are for SLy4, BSk24, BSk25, and BSk26, respectively.

This discrepancy is likely associated to the fact that authors of [4] include curvature corrections to the surface tension into their calculations. As noted in previous section, the curvature corrections are neglected here due to doubts in the thermodynamic consistency. SLy4 was also considered in [12] by minimization of thermodynamic potential $Ω$ within ETF approach. This work suggests that spaghetti and lasagna should be energetically favorable at $0.061 - 0.073 \text{ fm}^{-3}$ and $0.073 - 0.081 \text{ fm}^{-3}$, respectively.

For BSk24 our calculations predict spherical nuclei to be energetically favorable up to density $\sim 0.07 \text{ fm}^{-3}$, after which the transition to the cylindrical clusters occurs. Subsequent density increase leads to transition into homogeneous nuclear matter at $n_b \sim 0.075 \text{ fm}^{-3}$. These results qualitatively agree with [7], based on ETF approach. However, quantitative difference is present: [7] predict that the “spaghetti” phase should become favorable already at $n_b \sim 0.05 \text{ fm}^{-3}$.

Meanwhile for BSk25, our model predicts phase transition between spherical and cylindrical clusters to occur at $n_b \approx 0.063 \text{ fm}^{-3}$. At $n_b \sim 0.077 \text{ fm}^{-3}$ “lasagna” becomes energetically favorable and, finally, at $n_b \sim 0.08 \text{ fm}^{-3}$ the mantle ends and uniform nuclear matter becomes the ground state. It should be mentioned, that BSk25 demonstrates some abnormal behavior and fails to satisfy detailed theoretical calculations at the subnuclear densities (see [16] for details).

Finally, for BSk26 the lasagna phase is absent in mantle, which is composed of cylindrical clusters, being located at $0.075 \text{ fm}^{-3} \lesssim n_b \lesssim 0.078 \text{ fm}^{-3}$. We are not aware for previous calculations of the mantle structure for BSk25 and BSk26 models.

It is worth noting that the inverted phases (antispaghetti and Swiss cheese) are not energetically favorable and predicted to be absent in neutron stars, if they are considered within...
Sly4, BSk24, BSk25, and BSk26 models. However, the differences of the energy density for different configurations of clusters are very small (several tens of keV per nucleon, see figure 1) and are comparable to the characteristic uncertainties in the description of nuclear matter (even the experimentally available set of masses of terrestrial nuclei is reproduced by theoretical models at the same level of accuracy, see, e.g., [16]). It suggests that the predictions of the neutron star mantle structure are model dependent.

We plan to perform calculations including the curvature corrections to the surface properties of nuclei within a thermodynamically consistent approach. We expect that it would allow us to make results of CLDM model closer to ETF calculations.

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