CP violation in production of neutralinos in $e^+e^-$ collisions

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ABSTRACT: We propose T-odd and CP-odd odd asymmetries in order to analyze the impact of CP violating phases in neutralino production and subsequent leptonic two-body decays. We present numerical results of these asymmetries and of the cross sections for complex parameters $\mu$, $M_1$ and $A_\tau$, which may be present in the neutralino and stau sector of the Minimal Supersymmetric Standard Model if CP is violated. The asymmetries arise on tree level and thus could be large enough to be observed at a linear $e^+e^-$ collider in the $\sqrt{s} = 500$ GeV range with high luminosity. We discuss the feasibility for measuring the asymmetries by analyzing their statistical errors. Moreover we study the beam polarization dependence of the asymmetries and of the cross sections and show that they both can be enhanced considerably.

1. Introduction

The only source of CP violation in the Standard Model is given by one phase in the Kobayashi-Maskawa matrix. However, this phase alone cannot account for the observed baryon asymmetry of the universe \[\ddagger\], and further sources of CP violation have to be introduced. In the Minimal Supersymmetric Standard Model (MSSM) several supersymmetric (SUSY) breaking parameters and the higgsino mass parameter $\mu$ can be complex. The phases of the SUSY parameters are restricted by the experimental upper limits on the electric dipole moments (EDMs) \[\|\] of electron, neutron and of the $^{199}$Hg and $^{205}$Tl atoms. However, there can be strong cancellations between the different SUSY contributions to the EDMs, which can weaken the restrictions on the phases \[\|\]. Independently from the EDMs, an unambiguous determination of the values of the phases is necessary in order to

\[\ddagger\]Speaker.
clarify whether the SUSY phases are candidates for causing the baryon asymmetry of the universe.

The phases have also impact on the phenomenology of production and decay of SUSY particles, in particular at a future linear $e^+e^-$ collider \[5\], and give rise to an important class of CP and T-odd observables, which involve triple products \[6, 7, 8\]. They allow us to define various CP and T asymmetries which are sensitive to the different CP phases. On the one hand, these observables are naturally large because they are present at tree level. On the other hand, they also allow a determination of the sign of the phases, which could not be achieved if only CP-even observables would be studied.

In this talk, we study neutralino production (for recent studies with complex parameters and polarized beams see \[8, 9, 10\])

\[
e^+ + e^- \rightarrow \tilde{\chi}_i^0 + \tilde{\chi}_j^0 \tag{1.1}
\]

and the subsequent lepton two-body decay of one of the neutralinos

\[
\tilde{\chi}_i^0 \rightarrow \ell + \ell_1, \tag{1.2}
\]

and of the decay slepton

\[
\ell \rightarrow \tilde{\chi}_1^0 + \ell_2; \quad \ell_{1,2} = e, \mu, \tau. \tag{1.3}
\]

The triple product $T = (p_{e^-} \times p_{\ell_2}) \cdot p_{\ell_1}$ defines the T-odd asymmetry of the cross section $\sigma$ for the processes (1.1)-(1.3):

\[
\mathcal{A}_T = \frac{\sigma(T > 0) - \sigma(T < 0)}{\sigma(T > 0) + \sigma(T < 0)}. \tag{1.4}
\]

The dependence of $\mathcal{A}_T$ on $\varphi_{M_1}$ and $\varphi_{\mu}$, which are the phases of the complex gaugino mass parameter $M_1$ and the complex higgsino mass parameter $\mu$, respectively, was studied in \[11\], and for the neutralino three-body decays also in \[4, 7, 8, 12\].

In case the neutralino decays into a $\tau$-lepton, $\tilde{\chi}_i^0 \rightarrow \tilde{\tau}_k^\pm + \tau_k^\mp$, $k = 1, 2$, the transverse $\tau^-$ polarization $P_2$ and the transverse $\tau^+$ polarization $\bar{P}_2$, give rise to the CP-odd asymmetry

\[
\mathcal{A}_{CP} = \frac{1}{2}(P_2 - \bar{P}_2), \tag{1.5}
\]

which is also sensitive to the phases $\varphi_{A_\tau}$ of the complex trilinear scalar coupling parameter $A_\tau$ of the stau sector of the MSSM. Without measuring the transverse $\tau^\pm$-polarizations, a sensitivity to $\varphi_{A_\tau}$ is not obtained \[13\], and one would only be sensitive to CP violation in the production process (1.1) \[11\]. The components of the $\tau^-$ polarization vector are given by \[14, 15\]

\[
P_1 = \frac{\text{Tr}(\sigma_i)}{\text{Tr}(\sigma)}, \tag{1.6}
\]

with $\sigma$ the hermitean spin density matrix of the $\tau^-$ and $\sigma_i$ the Pauli matrices. The $\tau^-$ polarization $P = (P_1, P_2, P_3)$ is defined in a coordinate system in which $P_3$ is the longitudinal
polarization, and $P_1$ is the transverse polarization in the plane formed by $p_{e^-}$ and $p_\tau$. The T-odd component $P_2$ is the polarization perpendicular to $p_\tau$ and $p_{e^-}$ and is proportional to the triple product $s_\tau^2 \cdot (p_\tau \times p_{e^-})$ [15], where $s_\tau^2$ is the $\tau$ spin basis vector. The dependence of $A_{\text{CP}}$ on $\varphi_{M_1}$, $\varphi_{\mu}$ and $\varphi_{A_\tau}$ was studied in [15, 16]. Also the beam polarization dependence of $A_T$, $A_{\text{CP}}$ [17] and of the cross sections [17, 18] was studied.

\begin{figure}[ht]
\centering
\includegraphics[width=0.5\textwidth]{neutralino_production_decay.png}
\caption{Schematic picture of the neutralino production and decay process.}
\end{figure}

2. Numerical results

We present numerical results for $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ with the subsequent leptonic decay of $\tilde{\chi}_2^0$ for a linear collider with $\sqrt{s} = 500$ GeV and longitudinally polarized beams. For $A_T$, Eq. (1.4), we study the neutralino decay into the right selectron and right smuon, $\tilde{\chi}_2^0 \rightarrow \tilde{\ell}_R\ell_1$, $\ell = e, \mu$ and for $A_{\text{CP}}$, Eq. (1.5), that into the lightest scalar tau, $\tilde{\chi}_2^0 \rightarrow \tilde{\tau}_1\tau$. For a schematic picture of the production and decay process, see Fig. 1. We study the dependence of the asymmetries and the cross sections on the parameters $\mu = |\mu| e^{i\varphi_{\mu}}$, $M_1 = |M_1| e^{i\varphi_{M_1}}$, $A_\tau = |A_\tau| e^{i\varphi_{A_\tau}}$ and on the beam polarizations $P_{e^-}$ and $P_{e^+}$. We assume $|M_1| = 5/3M_2\tan^2\theta_W$ and use the renormalization group equations [19] for the selectron and smuon masses, $m_\tilde{\ell}_R^2 = m_0^2 + 0.23M_2^2 - m_Z^2 \cos 2\beta \sin^2\theta_W$ with $m_0 = 100$ GeV. The interaction Lagrangians and details on stau mixing can be found in [11].

For the calculation of the neutralino width and branching ratios we neglect three-body decays and include the following two-body decays

\begin{equation}
\tilde{\chi}_2^0 \rightarrow \tilde{\tau}_m \tau, \tilde{\ell}_R, \ell, \tilde{\chi}_1^0 Z, \tilde{\chi}_1^0 \tilde{W}^\pm, \tilde{\chi}_1^0 H_1^0, \ell = e, \mu, \quad m, n = 1, 2, (2.1)
\end{equation}

with $H_1^0$ being the lightest neutral Higgs boson. The Higgs parameter is chosen $m_A = 1000$ GeV. The decays $\tilde{\chi}_2^0 \rightarrow \tilde{\tau}_n^+ H_1^0$, with $H_1^\pm$ being the charged Higgs bosons, and the decays $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 H_2^{0, 2, 3}$, with $H_2^{0, 2, 3}$ being the neutral Higgs bosons, are thus ruled out in our scenarios. For the branching ratios of the sleptons we take $\text{BR}(\tilde{\ell}_R \rightarrow \tilde{\chi}_1^0 \ell) = 1$, for $\ell = e, \mu$. 

- 3 -
2.1 Asymmetry $\mathcal{A}_T$

In Fig. 3a, we show contour lines of the asymmetry $\mathcal{A}_T$ in the $|\mu|-M_2$ plane for $\varphi_{M_1} = 0.1\pi$ and $\varphi_\mu = 0$. It is remarkable that $\mathcal{A}_T$ can be close to 6%, even for the small value of $\varphi_{M_1} = 0.1\pi$ and for $\varphi_\mu = 0$. A small value of the phases, especially $|\varphi_\mu| \lesssim \pi/10$, is suggested by constraints on electron and neutron EDMs. In Fig. 3b we show the cross section $\sigma = \sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \times \text{BR}(\tilde{\chi}_2^0 \to \ell_R\ell_1) \times \text{BR}(\ell_R \to \tilde{\chi}_1^0\ell_2)$, with $\text{BR}(\tilde{\chi}_2^0 \to \ell_R \to \tilde{\chi}_1^0\ell_2) = 1$, in the $|\mu|-M_2$ plane for $\varphi_{M_1} = 0.1\pi$, $\varphi_\mu = 0$, taking $\tan \beta = 10$, $m_0 = 100$ GeV, $A_\tau = -250$ GeV, $\sqrt{\sigma} = 500$ GeV and $(P_{\ell^-}, P_{\ell^+}) = (0.8, -0.6)$. The area $A$ (B) is kinematically forbidden since $m_{\tilde{\chi}_1^0} + m_{\tilde{\chi}_2^0} > \sqrt{\sigma}$ ($m_{\tilde{\chi}_1^0} > m_{\tilde{\chi}_2^0}$). The gray area is excluded by $m_{\tilde{\chi}_1^0} < 104$ GeV.

In Fig. 3a we show the $\varphi_\mu-\varphi_{M_1}$ dependence of $\mathcal{A}_T$ for $|\mu| = 240$ GeV and $M_2 = 400$ GeV. The value of $\mathcal{A}_T$ depends stronger on $\varphi_{M_1}$ than on $\varphi_\mu$. It is remarkable, that maximal phases $\varphi_\mu, \varphi_{M_1} = \pm \pi/2$ do not lead to the highest values of $\mathcal{A}_T$.

The relative statistical error of $\mathcal{A}_T$ is given by $\delta \mathcal{A}_T = \Delta \mathcal{A}_T / |\mathcal{A}_T| = S / (|\mathcal{A}_T| \sqrt{N})$, with $S$ standard deviations and $N = L \cdot \sigma$ the number of events with $L$ the total integrated luminosity and $\sigma$ the total cross section. Assuming $\delta \mathcal{A}_T \approx 1$, it follows $S \approx |\mathcal{A}_T| \sqrt{N}$. For measuring $\mathcal{A}_T$, it is thus crucial to have large $\mathcal{A}_T$ and large $\sigma$, which can both be enhanced.
by using longitudinally polarized $e^+$ and $e^-$ beams. In Fig. 3 we show the contour lines of the standard deviations $S = |A_T| \sqrt{\mathcal{L} \cdot \sigma}$ for $\mathcal{L} = 500 \text{ fb}^{-1}$. In this scenario, $S$ can be enhanced for positive $P_{e^-}$ and negative $P_{e^+}$.

**Figure 3:** Contour lines of the asymmetry $A_T$ in the $\varphi_{\mu} - \varphi_{M_1}$ plane for $M_2 = 400 \text{ GeV}$ and $|\mu| = 240 \text{ GeV}$, taking $\tan \beta = 10$, $m_0 = 100 \text{ GeV}$, $\sqrt{s} = 500 \text{ GeV}$ and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$.

**Figure 4:** Contour lines of the standard deviations $S$ in the $P_{e^-} - P_{e^+}$ plane for $M_2 = 400 \text{ GeV}$, $|\mu| = 240 \text{ GeV}$, $\varphi_{M_1} = 0.1 \pi$ and $\varphi_{\mu} = 0$, taking $\tan \beta = 10$, $m_0 = 100 \text{ GeV}$, $\sqrt{s} = 500 \text{ GeV}$ and $\mathcal{L} = 500 \text{ fb}^{-1}$.

### 2.2 Asymmetry $A_{\text{CP}}$

In Fig. 3a we show the $\varphi_{\mu} - \varphi_{M_1}$ dependence of the $\tau$ polarization asymmetry $A_{\text{CP}}$, Eq. (1.5), for $\varphi_{\mu} = 0$, $|\mu| = 300 \text{ GeV}$ and $M_2 = 200 \text{ GeV}$. We have chosen a small value of $\tan \beta = 5$ and a large value of $|A_{\tau}| = 1 \text{ TeV}$ because $A_{\text{CP}}$ increases with increasing $|A_{\tau}| \gg |\mu| \tan \beta$ [15]. The cross section $\sigma = \sigma(e^+e^- \rightarrow \tilde{\chi}_1 \tilde{\chi}_2) \times \text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\tau}_1^+\tau^-)$ is shown in Fig. 3b. Also $\sigma$ is very sensitive to variations of the phases and varies between $30 \text{ fb}$ and $180 \text{ fb}$. The choice of $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$ enhances $\sigma$ and $A_{\text{CP}}$.

The feasibility for measuring $A_{\text{CP}}$ depends strongly on the sensitivity $S$ for measuring the $\tau$ polarization [21]. In [21] a sensitivity of $S = 0.35$ has been obtained. For an ideal detector and considering statistical errors only, the sensitivity for measuring $A_{\text{CP}}$ at 95% C.L. can be calculated to $S = \sqrt{2}/(|A_{\text{CP}}| \sqrt{\mathcal{N}})$ [15]. In Fig. 4 we show $S$ in the $P_{e^-} - P_{e^+}$ plane. A sensitivity of $S < 0.35$ can be obtained using polarized beams with negative $P_{e^-}$ and positive $P_{e^+}$.

In Fig. 7 we show for $M_2 = 400 \text{ GeV}$ and $|\mu| = 300 \text{ GeV}$ the contour lines of $A_{\text{CP}}$ in the $\varphi_{\mu} - \varphi_{M_1}$ plane. The asymmetry $A_{\text{CP}}$ is very sensitive to variations of the phases $\varphi_{M_1}$ and $\varphi_{\mu}$, and can reach values up to 65%.
\begin{align*}
\varphi_{M_1}/\pi & \quad A_{\text{CP}} \text{ in } \% \\
\varphi_{A_\tau}/\pi & \quad \sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\tau}_1^\pm) \text{ in fb}
\end{align*}

Figure 5: Contour lines of the asymmetry $A_{\text{CP}}$ (5b) and $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\tau}_1^\pm) \times \text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\tau}_1^\pm\ell^-)$ (5a) in the $\varphi_{A_\tau}$-$\varphi_{M_1}$ plane for $|A_\tau| = 1$ TeV, $M_2 = 200$ GeV, $|\mu| = 300$ GeV, taking $\tan\beta = 5$, $m_0 = 100$ GeV, $\sqrt{s} = 500$ GeV and $(P^-_e, P^+_e) = (-0.8, 0.6)$.

3. Summary and conclusion

We have proposed a T and a CP-odd asymmetry in $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\tau}_1^\pm$ and the subsequent leptonic two-body decay of $\tilde{\chi}_2^0$, in order to analyze CP violation caused by the phases $\varphi_{M_1}$, $\varphi_\mu$ and $\varphi_{A_\tau}$. In a numerical study for the decay $\tilde{\chi}_2^0 \rightarrow \bar{\ell}_R\ell_1$, $\bar{\ell}_R \rightarrow \tilde{\chi}_1^0\ell_2$ with $\ell_{1,2} = e, \mu$, we have shown that the asymmetry $A_T$ of the triple product $(\vec{p}_{e^-} \times \vec{p}_{\ell_2}) \cdot \vec{p}_{\ell_1}$, can reach values up to 6% even for small phases, e.g. $\varphi_{M_1} = 0.1\pi$ and $\varphi_\mu = 0$. The asymmetry $A_T$ and the cross section $\sigma$ can be enhanced by longitudinally polarized beams. For the neutralino decay $\tilde{\chi}_2^0 \rightarrow \tilde{\tau}_1^\pm\ell^\mp$, we have given numerical examples for the CP-odd $\tau$ polarization asymmetry $A_{\text{CP}}$, which is also sensitive to $\varphi_{A_\tau}$. The asymmetry can reach values up to 60%. Longitudinally polarized beams enhance $A_{\text{CP}}$ and the cross section, such that the sensitivity for measuring the $\tau$ polarization can be reduced significantly. In a numerical example, we have shown that a sensitivity of $S < 0.35$ can be achieved to measure a phase of $\varphi_{A_\tau} = 0.2\pi$. Depending on the MSSM scenario, the asymmetries should be accessible in future electron-positron linear collider experiments with longitudinally polarized beams in the 500 GeV c.m.s. energy range.

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\[ S = \sqrt{2} / (|A_{CP}| \sqrt{\mathcal{L}} \cdot \sigma) \]

**Figure 6:** Contour lines of the sensitivity \( S \) in the \( P_{e^-} - P_{e^+} \) plane for \( M_2 = 200 \text{ GeV} \), \( |\mu| = 300 \text{ GeV} \), \( \varphi_A = 0.2 \pi \), \( |A_r| = 1 \text{ TeV} \), \( \varphi_{\mu} = \varphi_{M_1} = 0 \), taking \( \tan \beta = 5 \), \( m_0 = 100 \text{ GeV} \), \( \sqrt{s} = 500 \text{ GeV} \) and \( \mathcal{L} = 500 \text{ fb}^{-1} \).

\[ A_{CP} \text{ in } \% \]

**Figure 7:** Contour lines of \( A_{CP} \) in the \( \varphi_{\mu} - \varphi_{M_1} \) plane for \( M_2 = 400 \text{ GeV} \), \( |\mu| = 300 \text{ GeV} \), \( \tan \beta = 5 \), \( m_0 = 100 \text{ GeV} \), \( \varphi_A = 0 \), \( |A_r| = 250 \text{ GeV} \), \( \sqrt{s} = 500 \text{ GeV} \) and \( (P_{e^-}, P_{e^+}) = (-0.8, 0.6) \).

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