The Measurement of Standing Wave Patterns by using High-speed Digital Image Correlation

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Abstract Paper deals with the measurement of standing wave patterns of a square plate by using Digital Image Correlation method that allows perform measurements of 3D displacements and strains. The experiment described in the paper had two phases. In the first phase, the natural frequencies of the plate were determined. The corresponding mode shapes of vibration were measured in the second phase. For the purpose of capturing a deformation of vibrating surface the correlation system Q-450 with two high-speed cameras had to be used for measurement.

Keywords: Chladni patterns, mode shapes of vibration, digital image correlation

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1. Introduction

All objects have tendency to vibrate naturally at certain frequencies. These frequencies are known as the natural frequencies or eigen-frequencies of a structure. If the amplitudes of the vibrations are large enough and if natural frequency is within the human frequency range, then the vibrating object will produce sound waves that are audible. Each of the natural frequencies at which an object vibrates is associated with a standing wave pattern. When an object is forced into resonance vibrations at one of its natural frequencies, it vibrates in such a manner that the vibration causes the sand to move and concentrate along the nodal lines where the surface is still.

In 1787, a German physicist Ernst Chladni described a technique that allows show the mode shapes of vibration of a solid surface. Chladni repeated the pioneering experiments of Robert Hooke who, in 1680, had observed the different patterns associated with the vibrations of glass plates. Hooke ran a violin bow along the edge of a plate covered with flour and saw how these patterns emerge. Chladni's technique consisted of drawing a bow over a piece of metal whose surface was lightly covered with sand. The plate was bowed until it reached resonance, when the vibration causes the sand to move and concentrate along the nodal lines where the surface is still. The patterns formed by these lines are what are now called Chladni patterns or Chladni figures. Variations of this technique can be still used e.g. in the design and construction of acoustic instruments [2].

Natural frequencies and mode shapes of vibration are the key structural properties that together with a damping describe a dynamic behavior of a structure. These parameters are uniformly referred to as modal parameters and depend on the geometry, material properties and boundary conditions of the system. The process of determining (or estimating) the modal parameters is called Experimental Modal Analysis (EMA). This method is based on the investigation of the relationship between the excitation and the response of a structure. Its aim is a decomposition of vibration into modal contributions so-called modes, each of which is characterized by its modal parameters [3]. Modal analysis is the youngest field of dynamics whose beginnings date back to the forties of the last century. The invention of the fast Fourier Transformation (FFT) algorithm by Cooly and Tukey in 1965 finally paved the way for rapid and prevalent application of experimental modal analysis. Over the last fifty years, the different measurement techniques and the
estimation algorithms have been developed and are being used successfully [4].

Accelerometers are most often used to measurement of vibration responses. They are very popular due to easy application and technical parameters. Accelerometers have wide dynamic and frequency range and relatively low weight. On the other hand, one accelerometer can measure a response at only one point. In addition, transducer applied on a structure influences the structural properties of an object at the place of its application. Progress achieved in computer technology and optics led to an improvement of unconventional optical methods that allow perform full-field contactless measurements with high spatial and time resolution. Thanks to this, it is possible to obtain response from any point of object without affecting the structure. The most famous optical techniques used in vibration analysis are Laser Doppler Vibro-Scanners (LDVS) and systems working on principle of Digital Image Correlation (DIC) [5].

2. Digital Image Correlation

The digital image correlation method is a modern contactless optic method, which uses digital image registration technique to precise measurement of plane and spatial deformations. Thanks to cameras with a high resolution, correlation system is able to observe a wide range of an object’s surface points with variable contrast. It allows visualize measured quantities in the whole observed area. In the case, when the high-speed digital cameras are used, the correlation system can be applied to solve various dynamic problems in mechanics. The flexible area of measurement (mm$^2$ to m$^2$), material and geometry independence and spatial visualization of measured parameters are the characteristic features of DIC method resulting from its concept. This is the reason why digital imagine correlation has been successful applied for many areas of science and survey. It has been applied for testing of components in engineering and microelectronic, determination of material properties, analyzing of vibration and modal parameters, FEA validation and so on.

With knowledge of the imaging parameters for each camera and the orientations of the cameras with respect to each other, the position of each object point in three dimensions is calculated. In order to evaluate surface displacements and strains on the object surface, a series of measurements is taken, while the specimen surface is moved due to a loading. The correlation algorithm tracks the observed gray value patterns for each camera and transforms corresponding facet positions in both cameras into 3D coordinates for each step, resulting in a track of each surface facet in 3D space. As the surface deformation is measured pointwise, displacements of individual surface points and subsequently surface strains can be evaluated.

3. Experimental measurement

The aim of an experiment was to capture standing wave patterns by using 3D DIC correlation system Q-450 produced by Dantec Dynamics company. This system uses two high-speed digital cameras Speed Sense Phantom with 1.3 Mpx CCD sensors. When the full resolution of sensor is used, the maximal acquisition rate of cameras is 3340 fps.

The object of measurement was a plate of square shape with dimensions 250 x 250 mm. The plate was made of poly methyl methacrylate sheet (PMMA) of thickness of approx. 2.0 mm. In the middle, the plate was attached to the drive rod of electromagnetic exciter. The exciter was connected to amplifier that amplifies the harmonic signal from signal generator.

3.1. Determination of Natural Frequencies

The first step of the experiment was to determine natural frequencies of the plate. For that purpose, a periodic swept sine signal was used as the excitation signal with a frequency varying from 15 Hz to 750 Hz linearly during the 60 seconds. The vibration responses of the plate were measured three times, at three different points, by one-point laser Doppler vibrometer PDV-100. The experimental setup is shown in Figure 3.
The approximate values of natural frequencies were determined in time domain. If a frequency of excitation signal corresponds to arbitrary natural frequency of the plate, a resonance occurs and responses of the plate rise significantly (i.e. amplitudes of vibration are maximal). There is a relatively simple to identify these moments in the time course of the measured response. This is the way how to obtain a rough estimate of natural frequencies because there is a difficult to distinguish the frequencies of the coupled and closed modes of the plate. Figure 4 shows the velocity response measured at one of analyzed points.

![Figure 4. Time course of measured response of the plate](image)

The values of natural frequencies of the plate that were determined by the mentioned way are listed in Table 1.

| f [Hz] | 29 | 96 | 145 | 172 | 270 | 378 | 520 | 623 | 730 |
|-------|----|----|-----|-----|-----|-----|-----|-----|-----|

3.2. Measurement of Standing Wave Patterns

To capture standing wave patterns, the harmonic sine signal was fed to the input of the exciter. The plate was excited by its natural frequencies from Table 1, individually. The cameras recorded an oscillating surface of the plate with sampling frequency of 2000 fps. Figure 5 shows the configuration of the correlation system Q-450 during the measurement.

![Figure 5. Experimental configuration of system Q-450 during the measurement of standing wave patterns of the plate](image)

Of course, before the measurement, there was necessary to create a stochastic speckle pattern on the top side of the plate. We used a self-adhesive vinyl foil with pre-printed pattern. The view from the camera 1 and camera 2 is shown in Figure 6.

![Figure 6. View from the cameras to a measured surface of the plate](image)

Considering that the resultant relative displacements are being determined in a correlation process when digital images recorded in the individual time steps are being compared to each other, the images capturing the stationary and undeformed plate had to be used as reference.
In order to be possible to visualize the standing wave patterns (similarly as in the case of Chladni patterns), the resultant deformation of the plate had to be plotted in the form of absolute displacements. In addition, there was used a function that removes the movements of rigid body. The individual standing wave patterns of the plate are given in Table 2.

| Table 2. Standing wave patterns of the analyzed plate |
|-----------------------------------------------------|
| ![Patterns](image1.png) |
| ![Patterns](image2.png) |
| ![Patterns](image3.png) |
| ![Patterns](image4.png) |
| ![Patterns](image5.png) |
| ![Patterns](image6.png) |

3.3. Chladni Patterns

For the purpose of verification, we performed also modified Chladni’s experiment. Experimental procedure was similar as in the previous measurement, i.e. the plate was excited in successive steps by harmonic signal with the frequency corresponding to natural frequencies of the plate. In this case, the plate was covered by salt that moved to nodal place of the vibrating plate. Some of the Chladni patterns obtained by this way are given in Table 3.

| Table 3. Chosen Chladni patterns of the analyzed plate |
|-------------------------------------------------------|
| ![Patterns](image7.png) |
| ![Patterns](image8.png) |
| ![Patterns](image9.png) |
| ![Patterns](image10.png) |

4. Conclusions

The paper presents the methodology of measurement the standing wave patterns by using digital image correlation method. The accuracy of measurement depends on several factors. The one of them is the knowledge of the accurate values of the natural frequencies. The best way to obtain these values is experimental modal analysis. The second factor is quality of the excitation. If the intensity of excitation is too low, some modes will not be adequately excited. This is seen in the case of modes with frequency 520 Hz and 730 Hz. The last factor is the sampling frequency of cameras. The higher time resolution will be the more accurate mode shapes of vibration can be captured.

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