Relaxing constraints on dark matter annihilation

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ABSTRACT

The relic abundance of thermal dark matter particles is generally assumed to be inversely proportional to their annihilation rate, which is therefore constrained by the present matter density, \( \langle \sigma_{\text{ann}} v \rangle \sim 10^{-26} \Omega_{\text{dm}}^{-1} \) cm\(^3\)·sec\(^{-1}\). Here we point out that much lower values of \( \langle \sigma_{\text{ann}} v \rangle \) are possible for heavy dark matter candidates \( (m_X \gtrsim 10 \text{ TeV}) \) that couple to other particle species through the electroweak force. With heavy dark matter particles present the early universe may evolve according to the following scenario. After an early entry into matter-dominated phase, dark matter particles form self-gravitating microhalos. Collisional interaction between dark matter particles and the surrounding radiation field eventually leads to microhalos gravothermal collapse and annihilation of most dark matter particles. For sufficiently heavy dark matter candidates \( (m_X \gtrsim 10 \text{ TeV}) \) the universe can return to radiation-dominated phase before the nucleosynthesis and thereafter follow the "standard" scenario.

Subject headings: cosmology: theory – early universe

1. Introduction

While there is a strong evidence for the existence of non-baryonic dark matter, little is known about its origins. In the currently popular scenario, dark matter particles and antiparticles start in thermal equilibrium with radiation. When temperature drops below the particle mass, \( m_X \), their comoving density begins an exponential fall until the annihilation time-scale, \( t_{\text{ann}} = (m_X \langle \sigma_{\text{ann}} v \rangle)^{-1} \), becomes larger than the age of the Universe. From this point the comoving density of dark matter particles is assumed frozen. For the relic abundance of dark matter particles to be consistent with the present value of \( \Omega_{\text{dm}} \) their annihilation rate at the time of freeze-out must equal (Kolb & Turner 1989)

\[
\langle \sigma_{\text{ann}} v \rangle \sim 3 \cdot 10^{-26} \text{ cm}^3\cdot\text{sec}^{-1}. \tag{1}
\]

Unless dark matter is of non-thermal origin (Chung, Kolb & Riotto 1998), lower values of \( \langle \sigma_{\text{ann}} v \rangle \) would overclose the universe and therefore can be excluded.

If dark matter particles interact through any known force, during the decoupling epoch, when \( T \sim m_X \), their annihilation cross-section can not exceed \( \sigma_{\text{ann}} \lesssim \alpha/T^2 \sim \alpha/m_X^2 \). Using this fact, Griest & Kamionkowski (1990) obtained from (1) an upper limit on the mass of dark matter particles

\[
m_X \lesssim 300 \text{ TeV}. \tag{2}
\]

However, in this paper we point out that dark matter candidates with higher mass and lower annihilation cross-section are not necessarily inconsistent with the present value of \( \Omega_{\text{dm}} \). Unlike the standard model, which assumes the dark matter comoving density freezes out soon after the temperature of the Universe drops below \( m_X \), we show that it is possible for it to drop again later, after dark matter particles form virialized halos.

2. Dark matter annihilation inside halos

Unlike the standard model, where the matter becomes dominant only around \( z \sim 10^4 \), in a Universe with low \( \langle \sigma_{\text{ann}} v \rangle \) and high \( m_X \) the initial transition to matter domination happens much earlier. Consequently the growth of density perturbations and the formation of the first gravita-
tionally bound halos would be likewise shifted to a much earlier epoch.

The collapse of dark matter particles into halos gives a strong boost to the dark matter annihilation rate. Whereas previously the time-scale for particle annihilation was increasing faster than the age of the Universe, freeze-out of the local density inside gravitationally bound halos causes the reversal of this trend. Once the Hubble time begins to catch up with the annihilation time-scale, the dark matter comoving density is no longer constant.

Subsequent evolution of the Universe depends on the nature of the dark matter particles. For purely collisionless dark matter it can be shown (see Appendix A) that the comoving density eventually approaches an asymptote, \( \rho_m(1 + z)^{-3} \propto t^{-1/4} \). This moderate rate of decline turns out to be insufficient for the Universe to return to radiation dominated phase before the nucleosynthesis. However, annihilation inside halos can be greatly sped up if dark matter particles interact with the surrounding radiation (made up mostly of relativistic quarks and leptons).

At present weakly interacting massive particles (WIMPs) are the most popular candidates for the role of dark matter. The collisional coupling between WIMPs and other species is usually assumed to be negligible. At our epoch this assumption is generally correct, as the Universe temperature is far below the mass scale of \( W^\pm \) and \( Z \) bosons which mediate the weak force. However, in the early universe, when \( T \gtrsim 300 \text{ GeV} \), the coupling becomes very efficient. At high temperatures WIMPs scattering cross-section rises to \( \sigma_{\text{elastic}} \sim \alpha^2 / T^2 \), making the ratio between the relaxation time and the age of the Universe

\[
\frac{\tau_{\text{rel}}}{t_{\text{Hubble}}} \sim \frac{m_X + T}{\alpha^2 m_{Pl}},
\]

where \( m_{Pl} \) is the Planck mass. For \( m_X \lesssim 10^{13} \text{ TeV} \) and \( 0.1 \lesssim T \lesssim 10^{13} \text{ TeV} \) this ratio drops below unity, implying that WIMPs kinetic energy is tightly coupled to the radiation.

Collisional coupling between dark matter and radiation has a two-fold effect on the formation of self-gravitating dark matter halos, which is expected to begin after the matter-radiation equality. On small scales, \( T_{\text{virial}} < T_\gamma \), the energy transfer from radiation to dark matter prevents the collapse of the low-mass halos. On large scales, \( T_{\text{virial}} \gtrsim T_\gamma \), the effect is the opposite. Since self-gravitating objects have negative heat capacity, the energy loss to the surrounding radiation field would lead dark matter particles to collapse deeper into the gravitational well, thereby raising the virial temperature. Thus the continuous energy loss to the background radiation results in a gravothermal collapse.\footnote{During the epoch of reionization, \( 6 \lesssim z \lesssim 20 \), inverse Compton scatterings between electrons and the CMB photons have a similar effect on baryonic halos. However, there the gravothermal collapse is generally avoided either by energy injection from stars and miniquasars or by hydrogen recombination, which depletes the free electron abundance.}

The time-scale for particle diffusion in self-gravitating objects, which in our case also sets the time-scale for gravothermal collapse of larger halos, is

\[
\tau_{\text{coll}} \sim \frac{(\sigma_{el}v)}{Gm_x},
\]

where \( \sigma_{el} \sim \alpha^2 / T^2 \) is the scattering cross-section for the weakly interacting particles. If, as generally predicted by the inflation models, the density perturbation spectrum is scale-invariant, then the velocity dispersion of dark matter particles in halos with a size above the damping scales should be of order \( v \sim 0.1c \) and the virial temperature \( T \sim 0.01m_X \). Thus in large halos the gravothermal collapse proceeds on a timescale

\[
\tau_{\text{coll}} \sim 10^3 \left( \frac{m_X}{1 \text{ TeV}} \right)^{-3} \text{ sec}. \quad (5)
\]

If halos consist equally of dark matter particles and anti-particles (i.e. there is no dark matter asymmetry) at some point during the gravothermal collapse nearly all of them may annihilate into radiation. Provided the mass of the particle is greater than \( \sim 10 \text{ TeV} \), halos collapse and the resulting conversion of dark matter to radiation would end before nucleosynthesis and thereafter the evolution of the Universe can proceed according to the “standard” scenario.

3. Discussion

It follows from our scenario that the existence of stable particles with high mass and low annihilation cross-section does not violate the constraints on the present matter density. If several species of
such particles existed in the early universe, each could have been eliminated in separate periods of matter domination followed by return to radiation domination.

To satisfy the constraints on abundance of the light elements the last reversal to radiation domination must have happened prior to the nucleosynthesis. Nevertheless it is possible for the transitional matter dominated phase to leave its imprint on the presently observed universe. Besides leaving a small remnant of heavy particles, which at present compose the dark matter, the transitional matter dominated phase might affect later universe by creating a population of primordial black holes and baryon asymmetry.

The violation of CP-symmetry in the early universe (Sakharov 1967) makes it natural to expect a small differential between the coupling strengths of particles and anti-particles. Inside collapsing halos the drag force produced on dark matter particles through elastic scatterings with the escaping radiation may be different from the force on dark matter anti-particles. As a result, spatial segregation of dark matter particles and anti-particles would be produced. Similarly, a differential drag by the infalling dark matter particles on the outflowing quarks and anti-quarks (or leptons and anti-leptons) would produce local over/underabundances of baryons and leptons. During halos gravothermal collapse the existence of such segregation would make it impossible for the entire halo to annihilate into radiation. Therefore the collapse of particles (or anti-particles) dominating at the halo center should lead to the formation of a black hole.

Since black holes do not conserve baryon numbers (Hawking 1974; Zeldovich 1976; Carr 1976), formation of a central black hole would convert a local particle asymmetry into a global one. Depending on their initial mass, these primordial black holes may have evaporated long before the present epoch, possibly further amplifying the matter-antimatter asymmetry in the process (Dolgov 1982). Alternatively, if their mass was sufficiently large, significant population of primordial black holes may still be found in our Universe.

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Appendix A: Annihilation of collisionless particles in virialized halos

We consider the evolution of matter density field, assuming the annihilation cross-section scales as \((\sigma_{\text{ann}} v) \propto v^{-x}\) at non-relativistic velocities. While it is common to assume \(x = 0\), many new particle candidates with \(x = 1\) have been proposed (e.g. Hisano et al. 2004; Profumo 2005; Lattanzi & Silk 2008). Here we shall assume that \(x\) can take any value in the physically plausible range \(0 \leq x < 2\).

After decoupling from radiation matter density drops to a point when the annihilation timescale, \(t_{\text{ann}} = (n_x \langle \sigma_{\text{ann}} v \rangle)^{-1}\), becomes longer than the Hubble time, so that the comoving density is frozen. As long as the density field remains nearly homogeneous with the density and velocity dispersion falling respectively as \(a^{-3}\) and \(a^{-2}\) with the increasing scale factor, \(a, t_{\text{ann}}/t_{\text{Hubble}}\) continues to rise. However, this trend is reversed after matter collapses into halos. Inside an isolated gravitationally bound halos both the local density and velocity dispersion remain roughly constant, freezing the growth of \(t_{\text{ann}}\) until it becomes comparable with \(t_{\text{Hubble}}\). At this point a halo would start losing mass at a significant rate via particle annihilation. The subsequent halo evolution of can be described as following.

The mass loss from particle annihilation and the resulting decrease of binding energy would cause the halo to expand, so that the annihilation timescale tracks the age of the halo. Because the dynamic time is much shorter than \(t_{\text{Hubble}}\), the halo always remains close to dynamical equilibrium. Since the gravitational binding energy is proportional to \(E_g \propto M^2/R\), where \(M\) and \(R\) are the total mass and the radius of the halo, changing the mass and radius by \(dM\) and \(dR\) respectively, changes \(E_g\) by

\[
dE_g = E_g \left(2 \frac{dM}{M} - \frac{dR}{R}\right). \tag{6}\]

The change of halo kinetic energy, \(E_k\), in the annihilation process depends on the velocity dependence of \(\sigma_{\text{ann}}\) and the particle velocity distribution. Assuming Maxwell-Boltzmann distribution, we find that the average kinetic energy of an an-
A nilhating particle is \((1 - x/2)(m_X \langle v^2 \rangle /2)\), where \(\sqrt{\langle v^2 \rangle}\) is the local velocity dispersion. Thus

\[
\frac{dE_k}{E_k} = \left(1 - \frac{x}{2}\right) \frac{dM}{M}. \tag{7}
\]

Combining the virial theorem, \(E_g = -2E_k\), with equations (6) and (7), we find that the radius of the halo increases with the falling mass as \(R \propto M^{-1+\frac{x}{2}}\). Further, by using the scalings \(\rho \propto M/R^3\), \(v \propto \sqrt{M/R}\) and \(\langle v^2 \rangle \propto t^{-1}\), we find that the total mass of the halo falls with time as \(M \propto t^\mu\), \(\mu = -4/(16+2x-x^2)\). For the entire range \(0 \leq x < 2\), \(\mu\) is confined to a narrow range \(-0.25 \leq x < -0.235\).

Modeling the Universe as an ensemble of isolated halos, we can follow the global evolution of matter and radiation density fields

\[
\frac{d^2a}{adt^2} = -\frac{4}{3} \pi G(\rho_m + 2\rho_\gamma), \tag{8}
\]
\[
\frac{d\rho_m}{dt} = -\left(3 \frac{da}{dt} - \frac{\mu}{t}\right) \rho_m, \tag{9}
\]
\[
\frac{d\rho_\gamma}{dt} = -4 \frac{da}{dt} \frac{\rho_m}{t}, \tag{10}
\]

where \(\rho_\gamma\) and \(\rho_m\) are respectively, the mean densities of radiation and matter and \(a\) is the expansion factor. Solving the equations, we find the asymptotic solutions

\[
a \propto t^{(2+\mu)/3}, \tag{11}
\]
\[
\rho_m = \frac{2 + 5\mu + 2\mu^2}{12\pi G t^2}, \quad \rho_\gamma = -\frac{(2 + \mu)\mu}{8\pi G t^2}, \tag{12}
\]
\[
\frac{\rho_m}{\rho_\gamma} = \frac{2(1 + 2\mu)}{3\mu}. \tag{13}
\]

For \(\mu \approx -1/4\) we get \(\Omega_m \approx 4/7\) and \(\Omega_\gamma \approx 3/7\).

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