Simulation of Ammunition Consumption Conversion Factor Considering Damage Degree

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Abstract. In the current measurement of ammunition consumption, deviations are caused by insufficient consideration of the degree of damage. In response to this problem, the issue of measuring ammunition consumption is studied. Considering the damage mechanism, target damage characteristics, ammunition-target assignment and other factors, the equivalent radius model of target damage is established based on the requirements of damage degree. Based on the rule of ammunition dispersion after weapon launch, a simulation method for ammunition consumption conversion factor about point target was established. Based on the most favorable distribution model, a simulation algorithm for calculating the ammunition consumption of area target is presented, and an example is given which presents the algorithm is scientific and rational.

Keywords: Equipment Support, Ammunition Consumption Forecast, Damage Degree, Ammunition Dispersion, Conversion Factor

1 Introduction
The prediction of ammunition consumption is the basis of ammunition support and the key link for effective implementation of ammunition precision support[1]. The damage degree is a specific measure of the ammunition effectiveness, a major indicator of whether combat effectiveness can be achieved, and a specific indicator of whether the combat command department will continue to combat. When using a suppressed weapon to hit a surface target, the amount of ammunition required is large. In the phase of ammunition demand prediction, the requirement of accuracy is high. Therefore, it is an important issue for the current demand for ammunition. When the ammunition under standard conditions strikes the standard target, the ammunition consumption under the standard damage level is the standard amount[2-5]. The ammunition consumption conversion factor is equal to the ammunition consumption in other different cases divided by the standard amount. The representation method is widely used because of its convenient calculation, but it is also subject to scientific constraints on the calculation method of the ammunition consumption conversion factor under different conditions.
In terms of macroscopic exploration, ammunition demand prediction focused on the overall impact of ammunition consumption under different influencing factors. In microscopic exploration, it mainly analyzed the matching optimization of different ammunition and targets. In this case, the ammunition demand was analyzed by analyzing the damage mechanism. In summary, the current research does not consider the degree of ammunition damage, but the ammunition consumption is closely related to the degree of damage. This paper considers the requirements of different damage levels for ammunition consumption, and analyzes the ammunition consumption by the kill radius of the main damage effect[6-8]. The uncertainty of the impact of ammunition dispersion on damage is analyzed, which has certain significance for accurately analyzing the consumption of ammunition.

2 Ammunition Consumption Prediction

2.1 Thinking about the Related Problems of Ammunition Consumption Prediction

Different ammunition damage requirements have a great impact on the number of ammunition consumption. In the actual combat process, it can be considered that the ammunition is launched to the target area one by one, and the damage degree to the target is gradually deepened. The ammunition consumption is mainly affected by the ability of ammunition to damage, the ability of target to resist damage and the ammunition dispersion. (1)Target damage mechanism. The damage capability of ammunition includes fragmentation, shock wave, shock vibration, temperature and pressure, etc., which have different damage effects for different targets. Meanwhile different targets have different tolerances for different damage effects, resulting in different levels of damage. The damage degree of target caused by the ammunition depends mainly on the most serious damage mechanism. (2)Ammunition dispersion rule. The damage degree of target is directly related to the distance between the ammunition and the target without the shelter. The distance is affected by Ammunition dispersion rule.

2.2 Model Hypothesis

Model hypothesis as follows: (1) The damage degree of target is only affected by the distance between ammunition and target. That is to say, the damage mechanism of ammunition is isotropic, and the target has no shelter. (2) The impact point shown in Fig. 1 falls within the ellipse and obeys to the two-dimensional Gaussian distribution.

![Fig.1 Ammunition dispersion rule and distribution center](image)

2.3 Procedure

An estimation model of ammunition hit probability based on equivalent radius damage is established. According to the situation of the warhead, it can be considered that the damage effect of the ammunition on the target is related to the distance between the ammunition and the target. Assume that the effective radius of ammunition killing is. If the distance between the ammunition bombing point and the target is less than, it is considered to be killing, otherwise it is considered that there is no killing effect. In order to achieve a given killing effect, the number of ammunition, whose distance between the explosion point and the target is less than, needs to achieve. On this basis, the relationship between ammunition scattering and conversion factor is studied.

If the deviation of shot is not considered, the ammunition is evenly distributed in the scattered ellipse, as shown in Fig. 2.
the coincident effect explosion case, that in area ammunition, suppose.

(2) When falls, it is an effective killing ammunition, then the probability of an ammunition falling into the killing radius is:

\[ P = \frac{S_1}{S_2} \]  \hspace{1cm} (1)

\( S_1 \) is the area of coincident portion between the ellipse and the circle with the radius \( R \). \( S_2 \) is the area of the ellipse.

Here are a few special cases to discuss.

(1) When \( R \geq B_d \). If the ammunition warhead has a relatively large killing effect, let \( R \geq B_d \), as shown in Figure 2(a). All ammunition will fall within the effective kill radius. The number of ammunition that falls within the effective kill radius reaches \( A \), and the given damage effect is achieved. In this case, the amount of ammunition consumed is \( A' \).

\[ \text{Fig. 2 Relationship between ammunition dispersion and kill radius} \]

(2) When \( B_d \geq R \geq B_f \). The ammunition distribution is \( B_d \geq R \geq B_f \), as shown in Fig.3, indicating that the vast majority of ammunition falls within the effective kill radius. The distance between the explosion point and the target is less than \( R \), and this case occurs \( A \) times, then the given damage effect is achieved. The actual amount of ammunition consumed is \( A' = \frac{A}{P} = \frac{A S_2}{S_1} \).

\[ \text{Fig. 3 Area solution diagram} \]

Suppose the intersection of the circle and the ellipse is \( A \), and its coordinate is \((x_0, z_0)\). as

\[
\begin{align*}
\left\{ \begin{array}{l}
 x_0^2 + z_0^2 = R^2 \\
 \frac{x_0^2}{B_d^2} + \frac{z_0^2}{B_f^2} = 1,
\end{array} \right.
\end{align*}
\]

and assume \( \Theta = \angle AOB_f \), \( \Theta = \arctan \left( \frac{x_0}{z_0} \right) \). \( S_0 \) is the area of coincident portion between the ellipse and graph \( AOB_f \). The integral method is used to solve the area \( S_0 \). As can be seen from Fig.3, \( S_0 \) can be obtained by subtracting the area of the graphic \( ADB_f \) by the area of the sector AOD.
The area of sector AOD is described as 
\[ S_{\text{sector AOD}} = \frac{\Theta}{2\pi} \pi R^2 = \frac{\Theta}{2} R^2, \]
and the area of graphic \( ADB_f \) is described as 
\[ S_{\text{ADB}_f} = \int_{x_1}^{x_2} \left( \sqrt{R^2 - x^2} - B_f \sqrt{1 - \frac{x^2}{B_f^2}} \right) dx. \]
Then, \( S_0 = S_{\text{sector AOD}} - S_{\text{ADB}_f}, \) the area of sector AOC is 
\[ S_0 = \frac{\pi}{2} - \frac{\Theta}{2\pi} \pi R^2 = \left( \frac{\pi}{4} - \frac{\Theta}{2} \right) \Theta R^2. \]

Thereby, the effective killing area \( S_i \) in this case can be obtained by 
\[ S_i = 4 \left( S_0 + S_0 \right), \]
and the area of ammunition dispersion is 
\[ S_2 = \pi B_f B_f, \] then
\[ P_i = \frac{4 \left( S_0 + S_0 \right)}{\pi B_f B_f} \tag{2} \]

(3) When \( R \leq B_f \)

This situation is shown in Fig.2(c). For the suppressed weapon, its range is long and its distribution is relatively large. The distance between the explosion point and the target is less than \( R \), and this phenomenon occurs \( A \) times. If striking \( A \) times to achieve a given damage effect, the actual amount of ammunition consumed is \( A' = \frac{A S_2}{S_1}. \) The effective killing area \( S_i = \pi R^2, \) with equation (2),
\[ P_i = \frac{R^2}{B_f B_f} \tag{3} \]

3 Simulation of Ammunition Consumption Conversion Factor

3.1 Analysis Method for Ammunition Consumption Conversion Factor about Point Target

Through the analysis in the previous section, the hitting probability of ammunition can be obtained. When calculating the consumption, it is necessary to consider the number of shots falling within the circle whose distance from the target is \( R \). The ammunition consumption prediction is the ratio of the number \( A \) of ammunition that achieves a given damage effect to the hitting probability[9, 10].

It can be considered that the probability of falling within the valid range is subject to the binomial distribution, denoted by \( P \).
\[ P \{ X = k \} = p^k q^{1-k}, k = 0,1 \tag{4} \]

Considering the three situations in which the ammunition dispersion and the kill radius may occur, combined with the single-shot hit probability, the expected ammunition consumption \( A' \) can be obtained. \( A' = \frac{A}{P} = \frac{A S_2}{C_s p^1 q^{1-k}} \).

Due to the different distribution of ammunition, the amount of ammunition consumed is different when the number of shells falling into the effective area is the same. the conversion factor \( P_i \) can be obtained.
\[ P_i = \frac{n_i}{n_b} \tag{5} \]

By simplifying other conditions, the conversion factor can only be corrected from the perspective of scattering, and the relationship can be determined more accurately. The conversion factor is a multiple relationship, so the algorithm is feasible.
3.2 Simulation Method for Ammunition Consumption Conversion Factor about Area Target

Estimation of area target ammunition consumption is different from point target, mainly reflected in the following aspects.

(1) The effect of damage is different. Area targets often do not require complete destruction like point targets, but require a certain damage rate. (2) The area target zone generally exceeds $B_T$, even if there is dispersion bias, it may still be within the target area. (3) In some cases, the purpose of firing the area target is to suppress, not to destroy the target.

For area targets, the impact of ammunition dispersion on the conversion factor of ammunition consumption is mainly reflected in the following aspects. (1) The increased dispersion has increased the probability of falling in non-target areas. (2) After the dispersion becomes larger, the uniformity of the falling point of the ammunition is deteriorated, which affects the ammunition covering effect.

Because the establishment of its mathematical model is relatively complex, this paper uses simulation methods to conduct research. To establish a simulation model, make the following assumptions. (1) Suppose the surface target is a rectangle with side lengths $a$ and $b$. (2) The area within the target area is segmented by a circle with a radius of damage radius $R$. (3) Use the center of the circle as the aiming point, determine the ammunition drop point, and launch the same amount of ammunition as the number of round. (4) On the condition that at least a piece of ammunition fell into each circle, it is considered that the target in the circle has been damaged, which serves as the basis for the statistical effect of damage. (5) According to the deadline for reaching the damage requirements, the number of ammunition launched is counted. (6) Analyze the amount of ammunition consumed in the case of different dispersions and give the conversion factor.

The simulation process of ammunition consumption when shooting area target with a given hit probability is as follows: (1) Set the initial conditions. Set the number of simulations to $L$ times, let initial value of simulation times $l = 0$, and total number of shots $h = 0$. (2) Start single simulation. Let $l = l + 1$. (3) Set initial condition of single simulation. According to the original question, determine the radius $R$ of the damage circle, and the number $N$ of damage rounds required to cover the target. Initially, the number of damaged rounds $n = 0$. Let $a(i) = 0, i = 1, 2, \cdots, N$. $a(i) = 0$ indicates that the target within the NO.$i$ covered circle has not been damaged. $a(i) = 1$ indicates that the target within the NO.$i$ covered circle has been damaged. Initial used ammunition $m = 0$. Set the cutoff condition to $n/N \geq \tau_0$. That is, when the number of damaged rounds exceeds the specified ratio $\tau_0$, it is considered that the damage request is reached and the shooting is no longer continued. (4) One cluster shot. Let $m = m + N$; Let $k = 0$. (5) Judging whether the NO.$k < N$ covered circle is damaged or not. Let $k < N$; In conjunction with the dispersion of ammunition, sampling is performed. If the drop falls within the covered circle, let $k < N$. (6) If $k < N$, turn to (5); otherwise, jump to (7). (7) Calculate the number $n$ of covered circles which have been damaged. If $n/N < \tau_0$, turn to (4); otherwise, let $h = h + m$, and jump to (3). (8) If $l < L$, turn to (2); otherwise, jump to (9). (9) Calculate $h/L$. This is the expected value of ammunition consumption in this case.

Using the process to simulate the expected value of ammunition consumption in the standard case and the expected value of ammunition consumption in the plateau mountain combat situation, the conversion factor can be obtained.

$$p_m = \frac{h_E}{h_S} \tag{6}$$

Among: $p_m$ is the ammunition consumption conversion factor for the area target; $h_E$ is the expected value of ammunition consumption under plateau mountain conditions; $h_S$ is the expected value of ammunition consumption under standard conditions.
4 Example Analysis
Assume that the enemy target is a brigade command post, which covers a rectangular area 150 meters long and 200 meters wide. It is 14 kilometers away from the enemy target. Using a high explosive bomb, whose damage radius is 25 meters. In the standard case, the ammunition dispersion tolerance is $B_{SS} = 40m$, $B_{SS} = 60m$. In a plateau, ammunition is scattered as $B_{SS} = 44m$, $B_{SS} = 59m$. The degree of damage required is $r_0 = 0.9$. Using above procedure, simulate 10,000 times and get the expected number of shots.

Calculate the number of coverage circles to 68, and apply equation (20), then get $P_x = 0.260417$ and $P_x = 0.240755$. On this basis, the MATLAB is used for programming calculation. Execute this procedure to obtain the expected ammunition consumption under standard conditions: $h_s = 402.8736$. Expected ammunition consumption in a plateau mountain environment: $h_e = 438.7008$. Obtaining the conversion factor under this condition: $P_m = \frac{h_e}{h_s} = 1.089$.

It can be seen from the above calculation process that the predicted ammunition consumption correction under this condition is obtained by multiplying the ammunition consumption under standard conditions by the conversion factor.

5 Conclusion
Through research, the following basic conclusions can be obtained:

(1) Different degrees of damage have been described by equivalent circles of different sizes. In turn, the ammunition consumption forecast has been simplified by the equivalent circle.

(2) The ammunition dispersion rule has been described by equivalent ellipses. Then the ammunition consumption of point target has been simulated.

(3) According to the ammunition distribution rule under the most favorable distribution, the ammunition consumption of area target has been simulated.

(4) If the damage capability is related to both the distance and the incidence angle of ammunition, a similar approach can be used.

References
[1] Li Wen-sheng, et al. troops ammunition service, Beijing: National Defense Industry Press, 2004.
[2] Dwayne M. Butler, Anthony Atler, Stephen M. Worman. Identifying Efficiencies in the Supply Chain for Training Ammunition [R]. 2016.
[3] AMES M.L. Army Ammunition Management Information System Challenges [R].2010.
[4] CHERYL D.MONROE. Supplying the Forces While Rightsizing Ammunition Storage Activities [R].2012.
[5] Robert Slak. A Study of Slovenian Armed Forces Ammunition Forecasting Methodology [D].Fort Leavenworth, Kansas, 2012.
[6] Katie Dang. Getting bombs on target: NOLSC Ammunition supply chain management team [R].2010.
[7] Joint Technical Coordinating Group for Ammunitions Effectiveness (JTCG/ME) Publications. 2006.
[8] Akst, George. Marine Corps War Reserve Ammunitions Requirement (WRMR) Model [R]. 2007.
[9] Wayne Zandbergen, William Inserra. Marine Corps Studies Program Support Final Report: USMC Ground Ammunition Requirements Study(M00264-06-D-0003) [R]. 2008.
[10] Jeffrey Brooks. Managing Ammunition to Better Address Warfighter Requirements Now and in the Future. [R]. 2008.