Large radii and string unification

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Abstract

We study strong coupling effects in four-dimensional heterotic string models where supersymmetry is spontaneously broken with large internal dimensions, consistently with perturbative unification of gauge couplings. These effects give rise to thresholds associated to the dual theories: type I superstring or M-theory. In the case of one large dimension, we find that these thresholds appear close to the field-theoretical unification scale $\sim 10^{16}$ GeV, offering an appealing scenario for unification of gravitational and gauge interactions. We also identify the inverse size of the eleventh dimension of M-theory with the energy at which four-fermion effective operators become important.

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Large internal dimensions in string theories have been studied in connection with perturbative breaking of supersymmetry \[1\]–\[7\]. Their inverse size is proportional to the scale of supersymmetry breaking which is expected to be of the order of the electroweak scale. The existence of such large dimensions is consistent with perturbative unification in a class of four-dimensional models which include some orbifold compactifications of the heterotic superstring \[3\]. Present experimental limits have been obtained from an analysis of effective four-fermion operators, yielding \( R^{-1} \gtrsim 200 \text{ GeV} \) or 1 TeV in the case of one or two large dimensions, respectively \[5\]. The main experimental signature of these models is the direct production of Kaluza-Klein excitations for gauge bosons which can be detected at future colliders \[6\].

The presence of large internal dimensions implies that the ten-dimensional heterotic string is strongly coupled \[8\]. In spite of this, in the class of models mentioned above, the radiative corrections to the four-dimensional couplings remain small \[3\]. This happens for instance when the corresponding Kaluza-Klein excitations are organized in multiplets of \( N = 4 \) supersymmetry (possibly spontaneously broken to \( N = 2 \) \[8\]) leading to cancellations among particles of different spins. However, the fact that the ten-dimensional coupling is strong raises the question of possible large corrections to other quantities of the four-dimensional effective field theory, such as non-renormalizable operators \[5, 10\]. Although this problem is difficult to handle using perturbative methods, it can be studied using recent results on string dualities.

There is a growing evidence that the strongly coupled heterotic string in ten dimensions is equivalent to the weakly coupled type I superstring \[11\] or to the eleven-dimensional M-theory \[12\]. The corresponding duality relations imply the existence of different thresholds associated to these dual theories where the effective theory changes regime. These thresholds may also appear as energy scales at which non-renormalizable operators become important \[10\]. An analysis of type I superstring and M-theory thresholds in models
with six large internal dimensions reveals that these thresholds appear much below the compactification scale $R^{-1}$, implying for the latter a lower bound $\sim 4 \times 10^7$ GeV.$^\text{[10]}$.

In this letter we study strong coupling effects in the class of models of refs. $^\text{[3]}$–$^\text{[5]}$ with anisotropic compactification space and where supersymmetry breaking is induced by the large internal dimension(s). We find that the threshold of dual theories appear now much above the compactification scale. Moreover, in the case of one large dimension at the TeV range, these thresholds are close to the experimentally inferred unification scale $\sim 10^{16}$ GeV, while the inverse size of the eleventh dimension of M-theory is at an intermediate scale $\sim 10^{13}$ GeV. This offers an alternative economical scenario for unification of gravitational and gauge interactions in the context of open strings or M-theory.$^\text{[1]}$. In fact both the unification and supersymmetry breaking scales, along with the electroweak one, can in principle be determined by a single dynamical calculation in the low energy theory.$^\text{[4]}$.

For type I strings, we establish the precise relation between the open string scale and the unification mass in the low energy theory. We also analyze the role that non-renormalizable operators play concerning the different thresholds. We find that, while the dimension eight operators $F_{\mu \nu}^4$ lead to the threshold of type I superstrings, the dimension six four-fermion operators reproduce the threshold of the eleventh dimension of M-theory, providing additional evidence for heterotic – M-theory duality.

### Type I superstring threshold

In the heterotic string, the ten-dimensional string coupling $\lambda_H$ and the string scale $M_H \equiv \alpha_H^{-1/2}$ are expressed in terms of four-dimensional parameters as:

$$\lambda_H = 2(\alpha_G V)^{1/2} M_H^3, \quad M_H = \left(\frac{\alpha_G}{8}\right)^{1/2} M_P,$$  

$^1$For a different approach see ref. $^\text{[13]}$. 

$^\text{[10]}$
where \((2\pi)^6V\) is the volume of the six-dimensional internal manifold, \(\alpha_G\) is the gauge coupling at the unification scale and \(M_P = G_N^{-1/2}\) is the Planck mass. Using the experimental values \(M_P = 1.2 \times 10^{19}\) GeV and \(\alpha_G \sim 1/25\) (assuming minimal supersymmetric unification), one finds \(M_H \sim 10^{18}\) GeV while \(\lambda_H\) grows to huge values as the internal volume gets large.

In ten dimensions, the heterotic \(SO(32)\) and type I strings are related by duality as \([11]\):

\[
\lambda_I = \frac{1}{\lambda_H} \quad M_I = M_H \lambda_H^{-1/2} .
\]

This implies that in the limit of large volume, the dual type I superstring is weakly coupled \((\lambda_I \ll 1)\) and its scale \(M_I \ll M_H\). In terms of four dimensional parameters,

\[
M_I = \left(\frac{\sqrt{2}}{\alpha_G M_P}\right)^{1/2} V^{-1/4} .
\]

When the internal manifold is large and isotropic, \(V = R^6\) and the type I threshold \(M_I \sim (\alpha_G M_P)^{-1/2} R^{-3/2}\) is much below the compactification scale \(R^{-1}\) \([10]\). In this case, it might be possible to lower the open string scale down to the TeV region, which would be the threshold of a genuine four-dimensional string \([14]\). All phenomenology should then be reexamined. The open string scale \(M_I\) appears also as the threshold at which the dimension eight effective operators \(F_{\mu\nu}^4\) become important \([10]\). On the heterotic side these operators receive contributions at the one loop level, while on the type I side they arise at the tree level \([15]\).

Equation (3) shows that the situation is reversed for anisotropic internal manifolds with less than four large dimensions. The open string scale becomes now larger than the compactification scale. In particular, for the class of models where supersymmetry breaking is tied to the size of just one large dimension, \(V \sim R\) and

\[
M_I = \left(\frac{\sqrt{2}}{\alpha_G}\right)^{1/2} R^{-5/4} M_P^{3/4} ,
\]
where $r$ is the size of the five “small” internal radii in units of $M_P$. For $R^{-1} = 1$ TeV and $r = \mathcal{O}(1)$, one obtains $M_I \sim 7 \times 10^{15}$ GeV which is very close to the gauge coupling unification scale. In this way, when going up in energies, the physical picture is the following. Between the TeV and the unification scale the effective theory can be studied using perturbation theory in the heterotic string. It behaves as five dimensional but with peculiarities related to the orbifold character of the compactification and the mechanism of supersymmetry breaking [3]–[5]. In particular, chiral states (quarks and leptons) do not have Kaluza-Klein excitations, while the couplings run with the energy as in four dimensions. At the unification scale, the theory becomes a genuine type I string weakly coupled.

In order to make precise the relation of the open string scale $M_I$ with the unification mass, one has to take into account the string threshold corrections to gauge couplings which can be computed on the type I side for any particular model. A direct one loop computation in the string theory gives [16]:

$$
\frac{4\pi}{\alpha_i} = \frac{4\pi}{\alpha_G} + \int \frac{dt}{t} B_i(t),
$$

where $i$ labels the gauge group factor and the integrand $B_i$ depends on the specific model. In the ultraviolet limit $t \to 0$, $B_i \sim 1/t + \mathcal{O}(e^{-1/t})$ and the integral appears to have a quadratic divergence. This is the only short-distance divergence which can be present in string theory and it is associated to tadpoles of massless particles. The tadpole cancellation implies a particular regularization of the different open string diagrams which makes the result ultraviolet finite [16]. It consists to cutoff the contributions from the annulus at $t = 1/\Lambda^2$ and from the Möbius strip at $t = 1/4\Lambda^2$. This leads to the prescription:

$$
\int_0^1 \frac{dt}{t} B_i \equiv \lim_{\lambda \to \infty} \left\{ \frac{4}{3} \int_{1/\lambda^2} B_i - \frac{1}{3} \int_{1/4\lambda^2} B_i \right\}.
$$

The integral (5) still has a logarithmic infrared divergence, since as $t \to \infty$, $B_i$ goes to a constant $b_i$. This is a physical divergence which reproduces the correct low-energy
running of the gauge couplings \( \alpha_i \) with beta function coefficients \( b_i \). It can be regularized by introducing an infrared cutoff at \( t = 1/\alpha_i' \mu^2 \). To compare the string expression with the field theoretical couplings in a particular renormalization scheme, we have to add in the r.h.s. of eq. (5) an appropriate constant term \([7]\). In the \( \overline{\text{DR}} \) renormalization scheme, or equivalently in the Pauli-Villars (PV) scheme \([8]\), the constant is:

\[
\lim_{\mu \to 0} \left\{ b_i \ln \frac{\Lambda^2_{\text{PV}}}{\mu^2} - b_i \int_0^{1/\alpha_i' \mu^2} \frac{dt}{t} \left( 1 - e^{-\pi \alpha_i' \Lambda^2_{\text{PV}} t} \right) \right\} = -b_i \ln (\pi e^\gamma),
\]

where \( \gamma \approx 0.6 \) is the Euler’s constant. By adding the constant \([8]\) in eq. (5) one finds:

\[
\frac{4\pi}{\alpha_i} = \frac{4\pi}{\alpha_G} + b_i \ln \frac{M^2_{\text{I}}}{\mu^2} + \Delta_i^I,
\]

where the type I string threshold corrections \( \Delta_i^I \) in the \( \overline{\text{DR}} \) scheme are given by:

\[
\Delta_i^I = \lim_{\varepsilon \to 0} \left\{ \int_0^{1/\varepsilon} \frac{dt}{t} B_i(t) + b_i \ln \varepsilon \right\} - b_i \ln (\pi e^\gamma).
\]

Notice that the threshold corrections in the heterotic string have a similar expression as an integral over the complex modular parameter \( \tau \) of the world-sheet torus. The main difference is that the ultraviolet divergence is now regularized by the restriction to the fundamental domain \( \Gamma \) of the modular group. The limit \( \varepsilon \to 0 \) in eq. (9) can then be taken easily by subtracting and adding \( b_i \) to the integrand \( B_i \):

\[
\Delta_i^H = \int_{\Gamma} \frac{d^2 \tau}{\text{Im} \tau} (B_i(\tau, \bar{\tau}) - b_i) + b_i \lim_{\varepsilon \to 0} \left\{ \int_{\Gamma} \frac{d^2 \tau}{\text{Im} \tau} \ln \varepsilon \right\} - b_i \ln (\pi e^\gamma)
\]

\[
= \int_{\Gamma} \frac{d^2 \tau}{\text{Im} \tau} (B_i(\tau, \bar{\tau}) - b_i) + b_i \ln \frac{2e^{1-\gamma}}{\pi \sqrt{27}}.
\]

The last constant can be absorbed in the definition of the string unification scale (see eq. (8)), while the remaining integral defines the threshold corrections \([7]\).

Such a procedure cannot be adopted in the open string case, since it gives rise to artificial logarithmic ultraviolet divergences which cannot be regularized by the prescription
However, eq. (9) provides a well defined way for computing threshold corrections in type I string models. For the class of models we consider in this work, threshold effects depend mainly on the size of the “small” dimensions $r$ and can provide model dependent corrections to the unification scale $M_I$.

**M-theory threshold**

The strong coupling limit of the heterotic $E_8 \times E'_8$ superstring in ten dimensions is believed to be described by the eleven-dimensional M-theory compactified on the semi-circle $S^1/\mathbb{Z}_2$ of radius $\rho$ \cite{12}. The relations between the eleven- and ten-dimensional parameters are:

$$M_{11} = M_H \left(\frac{\sqrt{2}}{\lambda_H}\right)^{1/3} \quad \rho^{-1} = \frac{1}{\sqrt{2\lambda_H}} M_H ,$$

where we have defined the eleven-dimensional scale $M_{11} = 2\pi(4\pi \kappa^2)^{-1/9}$ \cite{12}. When the ten-dimensional heterotic coupling is large ($\lambda_H \gg 1$), the radius of the semi-circle is large and M-theory is weakly coupled on the world-volume.

Using eq. (11), one can express $M_{11}$ and $\rho$ in terms of the four-dimensional parameters:

$$M_{11} = (2\alpha_G V)^{-1/6} \quad \rho^{-1} = \left(\frac{2}{\alpha_G}\right)^{3/2} M_P^{-2} V^{-1/2} .$$

It follows that the M-theory threshold $M_{11}$ is always above (or of the order of) the compactification scale $R^{-1}$. Furthermore, in the case of one large dimension, $V \sim R$ and the scale of the eleventh dimension $\rho^{-1}$ is also larger than $R^{-1}$:

$$M_{11} = (2\alpha_G R)^{-1/6} r^{-5/6} M_P^{5/6} \quad \rho^{-1} = \left(\frac{2}{\alpha_G}\right)^{3/2} r^{-5/2} R^{-1/2} M_P^{1/2} .$$

For $R^{-1} = 1$ TeV and $r = \mathcal{O}(1)$, one obtains $M_{11} \sim 3 \times 10^{16}$ GeV which essentially coincides
with the gauge coupling unification scale\footnote{A precise connection between $M_{11}$ and the field theoretical unification mass needs a genuine calculation in the M-theory which is not available at present. Here, we assume that the unification mass is reasonably approximated by $M_{11}$, as it was the case for the type I threshold.}. Moreover, the eleventh dimension threshold is at the intermediate scale $\rho^{-1} \sim 4 \times 10^{13}$ GeV. Thus, in the region between the TeV and the intermediate scale, the effective five dimensional theory has a perturbative heterotic string description (as in the case of type I strings below $M_I$). Above the intermediate scale, strong coupling effects are relevant and the eleventh dimension of M-theory opens up. Finally, at the unification scale, gravity becomes important through M-theory interactions.

The situation is reversed if there are more than two large dimensions in the internal volume $V$. The threshold of the eleventh dimension $\rho^{-1}$ is now below the compactification scale $R^{-1}$ \footnote{A precise connection between $M_{11}$ and the field theoretical unification mass needs a genuine calculation in the M-theory which is not available at present. Here, we assume that the unification mass is reasonably approximated by $M_{11}$, as it was the case for the type I threshold.}. In this case, as going up in energies, the theory would first become five-dimensional at the scale $\rho^{-1}$, while the other large dimensions will open up at a higher scale $R^{-1} < M_{11}$. Moreover, in all these regions the theory does not have a perturbative string description. For the case of two large dimensions $R^{-1} \sim 10^{-2} \rho^{-1}$, while for the case of six $R^{-1} \sim M_{11}$.

One may ask the question whether the scale of the eleventh dimension $\rho^{-1}$ can appear on the heterotic side as a threshold at which some non-renormalizable effective operators become important, in a similar way as the open string threshold $M_I$ was determined from an analysis of the dimension eight operators $F_{\mu\nu}^4$. In the following we will argue that the dimension six four-fermion operators are the relevant ones.

Let us consider indeed the effective interaction of four chiral fermions corresponding to twisted states in orbifold models with $2 \leq d \leq 6$ large internal dimensions of common size $R$. At energies below $R^{-1}$ the (tree-level) result can be obtained directly in the effective field theory by summing over all Kaluza-Klein excitations exchanged between the two fermion lines \footnote{A precise connection between $M_{11}$ and the field theoretical unification mass needs a genuine calculation in the M-theory which is not available at present. Here, we assume that the unification mass is reasonably approximated by $M_{11}$, as it was the case for the type I threshold.}. In the case where all fermions arise at the same fixed point of the orbifold,
the coupling of two twisted states with one excited (untwisted) mode, labeled by the $d$-dimensional vector $\vec{n}$, is \cite{20}:

$$g_{\vec{n}} = g \delta^{-\vec{n}^2 \alpha_H'/2R^2},$$  \hspace{1cm} (14)

where $g$ is the four-dimensional string coupling and the value of the constant $\delta \geq 1$ depends on the orbifold. The strength $\xi^2$ of the corresponding effective operator can be written as:

$$\xi^2 = \alpha_G R^2 \sum_{\{\vec{n}\} \neq 0} \frac{\delta^{-\vec{n}^2 \alpha_H'/2R^2}}{\vec{n}^2} \sim \alpha_G R^d \alpha_H'^{(2-d)/2},$$  \hspace{1cm} (15)

in the large $R$ limit. In terms of four-dimensional parameters, using eq. (1) and the relation $V \sim R^d \alpha_H'^{(6-d)/2}$, one finds:

$$\xi^2 \sim \alpha_G^3 M_P^4 V .$$  \hspace{1cm} (16)

The scale $\xi^{-1}$ defines the energy threshold at which the dimension six four-fermion operators become important. Experimental bounds on $\xi^{-1}$ are obtained by identifying it with the scale of compositeness and they yield typically $\xi^{-1} \gtrsim 1$ TeV \cite{3}.

Note that the expression (14) for $\xi^{-1}$ is identical to the scale of the M-theory eleventh dimension (12). We believe this is not an accident but a consequence of heterotic – M-theory duality. In fact, as we discussed above, on the M-theory side the lowest threshold is $\rho^{-1}$ and to first approximation the four-fermion operator receives contributions only from the exchange of the Kaluza-Klein modes associated to this single extra dimension. A similar computation as in eq. (15) for $d = 1$, $R = \rho$ and $g_{\vec{n}} = \mathcal{O}(1)$, then gives $\xi \sim \rho$.

To make the above argument, it is crucial that the heterotic string is compactified on an orbifold with twisted sectors, implying on the M-theory side that the internal seven-dimensional space is not a product of the orbifold with the semi-circle and $\mathbb{Z}_2$ has a non-trivial action. Otherwise, if the heterotic string were compactified on a smooth manifold $M_6$ (or on an orbifold without fixed points), its dual model would be, by an adiabatic argument, M-theory compactified on $M_6 \times S^1/\mathbb{Z}_2$ \cite{21, 13}. In this case, ordinary matter originated
from $E_8 \times E'_8$ arises at the two fixed points of the semi-circle and has only gravitational interactions with Kaluza-Klein states associated to the eleventh dimension \[^{[10]}\]. Therefore, the above four-fermion operator could not be used to extract information on the scale $\rho^{-1}$.

Finally, it would be interesting to consider, in the context of M-theory, the possibility of breaking spontaneously $N = 1$ supersymmetry using the radius of the eleventh dimension by a mechanism analogous to the Scherk-Schwarz compactification \[^{[22]}\]. Here, there are two possibilities:

- In the case where M-theory is compactified on a seven-dimensional space which does not contain the semi-circle as a product factor, the scale of supersymmetry breaking in the observable sector ($m_{\text{susy}}$) would be generically proportional to $\rho^{-1} \sim 1$ TeV. From eq. (12), the $d$ additional large dimensions would then open up at an intermediate scale varying between $10^6$ and $10^{13}$ GeV corresponding to $d = 3$ and $d = 6$, respectively.

- In the case where the internal manifold of M-theory is $M_6 \times S^1/Z_2$, supersymmetry is broken only in the gravitational sector (at the lowest order) and will be communicated to the observable world by gravitational interactions, yielding $m_{\text{susy}} \sim \rho^{-2}/M_P$. As a result, the threshold of the eleventh dimension $\rho^{-1}$ should be at an intermediate scale $\sim 10^{12}$ GeV. Interestingly enough, for $d = 6$ the inverse size of the six-dimensional internal manifold $M_6$ is now of the order of the gauge coupling unification mass $\sim 10^{16}$ GeV. It is suggestive that this situation could describe ordinary gaugino condensation in the dual strongly coupled heterotic string \[^{[23]}\].
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