Gaussian and non-Gaussian distributed random analytical and entire functions

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Abstract

We investigate the complex Gaussian as well as non-Gaussian distributed random analytical and entire functions (complex entire random field) and calculate their domain of definiteness (radius of convergence) as well as some important characteristics: order and type.

As a consequence we deduce that all the mentioned characteristics, under very natural conditions, are deterministic (non-random) with probability one and we calculate them. Moreover we exhibit some examples to show the exactness of the obtained results.

Keywords: Gaussian and non-Gaussian distributed random variables, entire functions, Taylor (power) series, radius of convergence, tail function, centered variables and random functions, Lemma of Borel-Cantelli, complex variables and functions, order and type of an entire function, distribution of zeros.

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1 Introduction.

Define the following random entire Gaussian distributed centered complex function of the form (power, Taylor series)

\[ f(z) = f(z, \omega) = f[\{\xi_k\}](z) \overset{def}{=} \sum_{k=0}^{\infty} \xi_k z^k. \]  (1.1)

Here \( z \) is a complex variable, \( \{\xi_k\}, k = 0, 1, 2, \ldots \), are centered (mean zero) Gaussian distributed complex random variables (r.v.), defined on some sufficiently rich probability space \( \Omega = (\{\omega\}, B, P) \):

\[ \xi_k = \eta_k + i \zeta_k, \quad i^2 = -1, \]
not necessarily independent and, in general case, the variables \( \eta_k, \zeta_k \), are also dependent. Here the random variables \( \eta_k, \zeta_k \) are real valued.

In Section 2 we analyze the Gaussian case while in Section 3 we consider the non-Gaussian case.

Denote also

\[
\beta_k^2 = \text{Var} \eta_k^2 = \mathbb{E} \eta_k^2, \quad \gamma_k^2 = \text{Var} \zeta_k^2 = \mathbb{E} \zeta_k^2,
\]

(1.2)

\[
\sigma_k^2 = \beta_k^2 + \gamma_k^2.
\]

(1.3)

In this short report we study the following characteristics of the introduced function: radius of convergence, order and type, etc.

Some properties of these random entire functions are studied in many works, see e.g. [3, 6, 8, 11, 12, 16, 17]. Throughout this paper the letters \( C, C_j(\cdot) \) will denote, as ordinary, various non-essential positive finite constants which may differ from one formula to the next even within a single string of estimates and which does not depend on the variables. We make no attempts to obtain the best values for these constants.

2 Main results. Gaussian entire random function.

Radius of convergence. Gaussian case.

**Theorem 2.1.** The radius \( R \) of convergence of the Taylor series (1.1) is non-random a.e. and may be calculated by the formula

\[
R = \frac{1}{\lim_{n \to \infty} n \sqrt{\sigma_n}}.
\]

(2.1)

The proof of Theorem 2.1 is a consequence of the following auxiliary important lemma.

**Lemma 2.2.** Let \( \{\xi_n\}, n = 1, 2, \ldots \) be a sequence of centered Gaussian distributed random variables. We state that, with probability one,

\[
\lim_{n \to \infty} \sqrt[\text{radical.alt1}]{\frac{\xi_n}{\text{divides.alt0} \sigma_n}} = \lim_{n \to \infty} \sqrt{\frac{\xi_n}{\text{divides.alt0} \sigma_n}}.
\]

(2.2)

**Proof.**

*Upper estimate.*

We follow the arguments in [3, Lemma 7]. Suppose, without loss of generality, that

\[
\lim_{n \to \infty} \sqrt{\sigma_n} = 1.
\]

Let us consider at first the case when \( \xi_n \) are numerical valued variables. Let \( \epsilon > 0 \) be given. There exists an integer number \( n_0 = n_0(\epsilon) \) for which
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\[ \sigma_n < (1 + \epsilon)^n, \quad n \geq n_0. \]

The probability \( P_n(\epsilon) := P\left( \sqrt[n]{|\xi_n|} > (1 + \epsilon)^2 \right) \) allows the following estimate

\[ P_n(\epsilon) \leq 2 \exp\left( -0.5(1 + \epsilon)^n \right). \quad (2.3) \]

Since \( \forall \epsilon > 0 \Rightarrow \sum_{n=1}^{\infty} P_n(\epsilon) < \infty, \)
we deduce, by virtue of the lemma of Borel - Cantelli, that

\[ \lim_{n \to \infty} \bar{\lim}_{n \to \infty} \sqrt[n]{|\xi_n|} \leq 1 \text{ a.e.} \]

**Lower estimate.**

One can suppose, without loss of generality, \( \sigma_n = 1, \quad n \geq 1. \) Let again \( \epsilon = \text{const} \in (0, 1). \)
Consider the probability

\[ P^{(n)}(\epsilon) \overset{\text{def}}{=} P\left( \sqrt[n]{|\xi_n|} \leq 1 - \epsilon \right). \]

We have

\[ P^{(n)}(\epsilon) = P\left( |\xi_n| \leq (1 - \epsilon)^n \right) \leq C (1 - \epsilon)^n, \]
so that, as before,

\[ \forall \epsilon \in (0, 1) \Rightarrow \sum_{n=1}^{\infty} P^{(n)}(\epsilon) < \infty. \]

Therefore,

\[ \lim_{n \to \infty} \bar{\lim}_{n \to \infty} \sqrt[n]{|\xi_n|} \geq 1 \text{ a.e.} \]

Let us return to the general case, i.e. the complex case (1.2), (1.3). As above, \( \xi_n = \eta_n + i \zeta_n, \quad i^2 = -1, \) so that the value \( \|\xi_n\|^2 \) may be represented in the form

\[ \|\xi_n\|^2 = \beta_n^2 \eta_n^2 + \gamma_n^2 \zeta_n^2, \]
where \( \text{Law}\{\eta_n\} = \text{Law}\{\zeta_n\} = N(0, 1), \) and one can suppose, without loss of generality,

\[ \beta_n^2 + \gamma_n^2 = \sigma_n^2 = 1. \]

Denote, as usually, by \( \|\tau\|_p \) the ordinary Lebesgue - Riesz \( L_p \) norm of the random variable \( \tau : \)

\[ \|\tau\|_p \overset{\text{def}}{=} \left[ \mathbf{E}|\tau|^p \right]^{1/p}, \quad p \geq 1. \]

It is well known that if \( \text{Law}(\tau) = N(0, 1), \) then

\[ \|\tau\|_p \asymp C_1 p^{1/2}, \quad p \geq 1, \]
therefore

$$||\tau^2||_p \leq C_2 \ p, \ p \geq 1.$$  

We apply the triangle inequality for the Lebesgue - Riesz norm

$$||\xi_n||^2 \leq C_3 \ p,$$

or equally

$$||\xi_n||_p \leq C_4 \ \sqrt{p}, \ p \geq 1.$$  

Hence, the r.v. - s \ \{\tau_n\}, \ n = 1, 2, \ldots \ are uniformly subgaussian:

$$\sup\limits_n P(|\xi_n| > u) \leq 2 \exp(-C_5 u^2), \ u \geq 0.$$  

It follows immediately as before, from the last relation, that with probability one

$$\lim\limits_{n \to \infty} \sqrt[n]{|\xi_n|} \leq 1.$$  

The inverse relation

$$\lim\limits_{n \to \infty} \sqrt[n]{|\xi_n|} \geq 1$$

may be proved quite analogously to that in the one - dimensional case.

Lemma 2.2 and consequently Theorem 2.1 are proved.

As a slight consequence, if

$$\lim\limits_{n \to \infty} \sqrt[n]{\sigma_n} = 0, \ (2.4)$$

then \ \( R = \infty. \)

**Order and type for the Gaussian entire functions.**

Suppose in this subsection that the condition \ (2.4) \ is satisfied; then the random function \( f = f(z) \) \ from the definition of the Taylor (power) series \ (1.1) \ is entire on the whole complex plane with probability one: \( R = \infty. \)

Recall the following classical definitions from the theory of an analytical complex functions, see e.g. [10, chapter 1]. Let the complex valued entire function \( g = g(z) \) \ be defined (and be analytical) on the whole complex plane. Denote

$$M_g(r) := \sup\limits_{z:|z|=r} |g(z)| = \sup\limits_{z:|z|=r} |g(z)|.$$

The value

$$\rho[g] \overset{def}{=} \lim\limits_{r \to \infty} \left\{ \frac{\ln \ln M_g(r)}{\ln r} \right\}$$
is said to be Order of the function $g$. The Type $\beta[g]$ of these function $g$ is defined by an equality

$$\beta[g] \overset{\text{def}}{=} \lim_{r \to \infty} \left\{ \ln M_g(r) \over r^{\rho(g)} \right\}.$$ 

Let us return to the source random complex function $f = f(z)$ in (1.1).

**Theorem 2.3.** Both the characteristics for the random Gaussian function $f = f(z)$, order and type, are a.e. non-random and may be calculated correspondingly by the relations

$$\rho[f] = \lim_{n \to \infty} \left\{ {n \ln n \over \ln |\xi_n|} \right\}, \quad (2.5)$$

$$\beta[f] = \lim_{n \to \infty} \left\{ n^{1/\rho[f]} \sqrt{\xi_n} \right\}. \quad (2.6)$$

**Proof.** The equality (2.6) and in particular, the non-randomness of the value $\beta[f]$, can be ground quite alike ones in the Theorem 2.1 and Lemma 2.2.

Further, it is known, see e.g. [10, chapter 1, pp. 5-15], that for the entire function of the form

$$g(z) = \sum_{k=0}^{\infty} c_k z^k$$

there holds

$$\rho[g] = \lim_{n \to \infty} \left\{ {n \ln n \over \ln |c_n|} \right\}, \quad (2.7)$$

$$\beta[g] = \lim_{n \to \infty} \left\{ n^{1/\rho[f]} \sqrt{|c_n|} \right\}. \quad (2.8)$$

Therefore

$$\rho[f] = \lim_{n \to \infty} \left\{ {n \ln n \over \ln |\xi_n|} \right\}, \quad (2.9)$$

$$\beta[f] = \lim_{n \to \infty} \left\{ n^{1/\rho[f]} \sqrt{\xi_n} \right\}. \quad (2.10)$$

It is no hard to ground alike to the proof of Lemma 2.2 that both the last expressions in the right - hand sides of equations (2.9) and (2.10) are actually non-random and coincide, correspondingly, with the ones in Theorem 2.1.

We will ground in the next section a more general case, where the r.v. $\{\xi_n\}$ are not necessarily Gaussian.
3 Non-Gaussian case.

Let us consider now the series of the form
\[ h(z) = h(z, \omega) = h(\{\eta_k\})(z) \overset{\text{def}}{=} \sum_{k=0}^{\infty} \eta_k z^k. \]  
(3.1)

Here \( \{\eta_k\}, \ k = 0, 1, 2, \ldots \), are random variables, not necessarily independent, Gaussian or centered.

Introduce the following tail functions
\[ T_k(u) := \mathbb{P}(|\eta_k| > u), \quad S_k(u) := \mathbb{P}(|\eta_k| \leq u) = 1 - T_k(u), \quad u \geq 0. \]  
(3.2)

\textbf{Theorem 3.1.} (Upper bound). Suppose that there exists a positive sequence of deterministic numbers \( \sigma_k \) such that the uniform tail function
\[ T(u) \overset{\text{def}}{=} \sup_k \mathbb{P}(|\eta_k|/\sigma_k > u), \quad u \geq 1 \]  
(3.3)
satisfies the following condition
\[ \forall Q > 1 \implies \sum_{k=1}^{\infty} T(Q^k) < \infty. \]  
(3.4)

Then, with probability one,
\[ \lim_{n \to \infty} \sqrt{n} |\eta_n| \leq \lim_{n \to \infty} \sqrt{\sigma_n}. \]  
(3.5)

More generally assume that, for all the values \( Q > 1 \),
\[ \sum_{k=1}^{\infty} \mathbb{P}(|\eta_k|/\sigma_k > Q^k) < \infty. \]  
(3.6)

Then the relation (3.5) again holds true by virtue of the well-known Lemma of Borel - Cantelli.

\textbf{Remark 3.1.} The variables \( \sigma_k \) in (3.3) may be chosen, for instance, as follows. Let \( \| \cdot \| \) be some rearrangement invariant norm defined on spaces of random variables, for example, Lebesgue-Riesz, Orlicz or Grand Lebesgue Space one, see [5, 4, 9]. One can put
\[ \sigma_k := \| \eta_k \|, \quad k = 1, 2, \ldots, \]  
if, of course, \( \sigma_k \in (0, \infty) \).

\textbf{Corollary 3.1.} Suppose that there exists a finite constant \( C < \infty \) and a non-negative r.v. \( \nu \) for which
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\[ \sup_{k=0,1,\ldots} P \left( \frac{|\eta_k|}{\sigma_k} > y \right) \leq C P(\nu > y), \quad y \geq 1. \]

Then the condition (3.4) is quite equivalent to the following one

\[ E \ln(e + \nu) < \infty. \quad (3.7) \]

(See [13]).

Note that the condition (3.7) is very weak; it is satisfied if, for example,

\[ \exists b = \text{const} > 0 : P(\nu > z) \leq C z^{-b}, \quad z \geq 3. \quad (3.8) \]

Let us ground an opposite estimate for the radius of convergence. We find quite analogously that if

\[ \forall q \in (0,1) \Rightarrow \sum_{k=1}^{\infty} P \left( \frac{|\eta_k|}{\sigma_k} < q^k \right) < \infty, \quad (3.9) \]

then

\[ \lim_{n \to \infty} \sqrt[n]{|\eta_n|} \geq \lim_{n \to \infty} \sqrt[n]{\sigma_n}. \quad (3.10) \]

To summarize:

**Theorem 3.2.** Suppose that both the relations (3.6) and (3.9) are satisfied. Then the radius \( R \) of convergence for the power (Taylor) series (3.1) is deterministic almost surely and may be calculated by the formula

\[ R = \frac{1}{\lim_{n \to \infty} \sqrt[n]{\sigma_n}}. \quad (3.11) \]

**Corollary 3.2.** Suppose that there exists a non-negative r.v. \( \tau \) for which \( \tau \geq 4 \) and, for some finite constant \( C < \infty \), holds

\[ \sup_{k=0,1,\ldots} P \left( \frac{\sigma_k}{|\eta_k|} > y \right) \leq C P(\tau > y), \quad y \geq 1. \]

Then the condition (3.9) is quite equivalent to the following one

\[ E \ln(e + \tau) < \infty. \quad (3.12) \]
Example 3.1. Evidently, if the r. v.-s \( \{ \eta_k \} \) are independent, then the radius of convergence of the series (3.1) is non-random.

Let now the sequence of coefficients \( \{ \eta_k \} \) in the (3.1) be such that

\[
P(\forall k = 0, 1, \ldots \Rightarrow \eta_k = 1) = \frac{1}{2},
\]

\[
P(\forall k = 0, 1, \ldots \Rightarrow \eta_k = 2^{-k}) = \frac{1}{2}.
\]

On the other words, the infinite-dimensional vector of coefficients \( \eta = \{ \eta_k \}, k = 0, 1, 2, \ldots \), has the following distribution

\[
P(\eta = (1, 1, \ldots, 1, \ldots)) = 1/2,
\]

\[
P(\eta = (1, 1/2, 1/4, \ldots, 1/2^k, \ldots)) = 1/2.
\]

Then the radius of convergence \( R \) of the series (3.1) is a random variable such that

\[
P(R = 1) = P(R = 2) = 1/2.
\]

Example 3.2. Let \( \sigma_k = 1 \) and \( \tau \) be a positive random variable such that \( \tau \geq e \) and

\[
P(\tau \geq y) = \frac{1}{\sqrt{\ln y}}, \quad z \geq e.
\]

Let us consider the following power series

\[g_\tau(z) := \sum_{k=0}^{\infty} \tau \cdot z^k,\]

i.e. here \( \eta_k = \tau, k = 0, 1, \ldots \). The radius of convergence is equal to one, despite that

\[
\forall Q > 1 \Rightarrow \sum_{k=0}^{\infty} P(\tau > Q^k) = \infty.
\]

Let us ground the relations (2.5) and (2.6) of the Theorem 2.3 in the general case, i.e. non-Gaussian case.

Denote as before

\[
\rho \overset{\text{def}}{=} \lim_{n \to \infty} \frac{n \ln n}{\ln \sigma_n}.
\]

Question: under which conditions the following relation

\[
\lim_{n \to \infty} \frac{n \ln n}{\ln |\eta_n|} = \rho
\]  (3.13)
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Holds with probability one?

Namely one can assume, without loss of generality, \( \rho \in (0, \infty) \). Let \( \epsilon \in (0, \min(1, \rho/2)) \) be an arbitrary constant. We have, for all the sufficiently greatest values \( n \geq n_0 = n_0(\epsilon) \),

\[
\frac{n \ln n}{|\ln \sigma_n|} \leq \rho(1 + \epsilon).
\]

One can suppose \( \sigma_n \in (0, 1) \); therefore

\[
0 < \sigma_n \leq \frac{n^{-n \ln n}}{\rho^n (1 + \epsilon)^n} = n^{-n / \rho(1 + \epsilon)}.
\]

We deduce, from the direct definition of the variables \( \sigma_n \),

\[
P\left( \frac{k \ln k}{|\ln |\eta_k||} < \rho(1 + 2\epsilon) \right) = P\left( \frac{|\eta_k|}{\sigma_k} > k^{k(1+\epsilon)(2+\epsilon)} \right) \leq T_k \left( k^{k(1+\epsilon)(2+\epsilon)} \right), \quad k \geq 1.
\]

We conclude again as a consequence, by virtue of lemma of Borel-Cantelli, the following result.

**Proposition 3.3.** If

\[
\forall \delta > 0 \Rightarrow \sum_{k=1}^{\infty} T_k \left( k^{\delta k} \right) < \infty,
\]

then, with probability one,

\[
\lim_{n \to \infty} \left\{ \frac{n \ln n}{|\ln |\eta_n||} \right\} \leq \rho.
\]

**Corollary 3.3.** Suppose that there exists a random variable \( \nu \), \( \nu \geq 4 \), and a constant \( C < \infty \) for which

\[
\sup_{k \geq 1} P(|\eta_k| > z) \leq C \times P(\nu > z), \quad z \geq 4.
\]

The condition (3.15) is satisfied if, for instance,

\[
\forall \delta > 0 \Rightarrow \sum_{k=4}^{\infty} P \left( \nu \geq k^{\delta k} \right) < \infty.
\]

The last relation may be transformed in turn as follows:

\[
E \left\{ \frac{\ln \nu}{\ln \ln \nu} \right\} < \infty.
\]

Analogously can be justified the following fact.
Proposition 3.4. (Lower bound)

Suppose that

\[ \forall \delta > 0 \Rightarrow \sum_{k=1}^{\infty} P \left( \frac{\sigma_k}{|\eta_k|} > k^\delta \right) < \infty. \]  \tag{3.17}

Then, with probability one,

\[ \lim_{n \to \infty} \left\{ \frac{n \ln n}{|\ln |\eta_n||} \right\} \geq \rho. \]  \tag{3.18}

Proof. Indeed, suppose for simplicity

\[ \frac{n \ln n}{|\ln |\sigma_n||} = \rho = \text{const} \in (0, \infty), \quad n \geq 4, \]

then

\[ \sigma_n = n^{-n/\rho}. \]

Let as above \( \epsilon \) be an arbitrary number from the (open) interval \( (0, \min(1, \rho/2)) \). Consider a sequence of events

\[ E_k(\epsilon) = \left\{ \frac{k \ln k}{|\ln |\eta_k||} \leq \rho - \epsilon \right\}. \]

We have

\[ P(E_k(\epsilon)) = P \left( \frac{\eta_k}{\sigma_k} < k^{-k/\rho(\rho-\epsilon)} \right) = P \left( \frac{\sigma_k}{\eta_k} > k^{k/\rho(\rho-\epsilon)} \right). \]

It follows immediately, from the condition (3.17), that

\[ \forall \epsilon \in (1, \min(1, \rho/2)) \Rightarrow \sum_{k=4}^{\infty} P(E_k(\epsilon)) < \infty. \]

It remains to mention once again the lemma of Borel - Cantelli. \( \square \)

Remark 3.2. As above, the condition (3.17) is satisfied in turn if there exists a random variable \( \zeta, \zeta \geq 4 \), for which

\[ \sup_{k \geq 4} P \left( \frac{\sigma_k}{|\eta_k|} < z \right) \leq C \cdot P(\zeta > z), \quad z \geq 4, \]

and such that

\[ E \left\{ \frac{\ln \zeta}{\ln \ln \zeta} \right\} < \infty. \]

Remark 3.3. Of course, if all the conditions of the Propositions 3.1 and 3.2 are satisfied, then with probability one

\[ \lim_{n \to \infty} \left\{ \frac{n \ln n}{|\ln |\eta_n||} \right\} = \rho. \]  \tag{3.19}
Remark 3.4. The mentioned problem was considered in many works, e.g. [1] [2] [14] [15], etc. As a rule, it was investigated only the case when the coefficients \( \{ \eta_k \} \) are independent or at last pairwise independent.

4 Concluding remarks.

A. Note that the classical methods offered by V.M. Zolotarev in [18] and, after by W. Höeffding in [7], bring more complicated computations.

B. The asymptotical behavior of the coefficients and, as a consequence, the asymptotical behavior of the function itself, allows to calculate the asymptotical distribution of its zeros, see e.g. the classical monograph on B.Ya. Levin [10]; see also [8] [12] [16] [17].

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