Yang-Mills streamlines and semi-classical confinement

Kurt Langfeld* and Ernst-Michael Ilgenfritz†

*School of Computing & Mathematics, University of Plymouth, Plymouth PL4 8AA, UK
†Institut für Physik, Humboldt-Universität zu Berlin, D-12489 Berlin, Germany

Abstract. Semi-classical configurations in Yang-Mills theory have been derived from lattice Monte Carlo configurations using a recently proposed constrained cooling technique which is designed to preserve every Polyakov line (at any point in space-time in any direction). Consequently, confinement was found sustained by the ensemble of semi-classical configurations. The existence of gluonic and fermionic near-to-zero modes was demonstrated as a precondition for a possible semi-classical expansion around the cooled configurations as well as providing the gapless spectrum of the Dirac operator necessary for chiral symmetry breaking. The cluster structure of topological charge of the semi-classical streamline configurations was analysed and shown to support the axial anomaly of the right size, although the structure differs from the instanton gas or liquid. Here, we present further details on the space-time structure and the time evolution of the streamline configurations.

Keywords: semiclassical, Yang-Mills theory, confinement, chiral symmetry breaking, axial anomaly

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Introduction: The lattice approach has been very successful to detail the low energy properties of QCD as encoded in the hadron spectrum and hadron structure. The success of the vortex and monopole picture [1] feeds the hope that many features of the vacuum structure, such as quark (de-) confinement, spontaneous chiral symmetry breaking (and restoration) and the (explicit) axial anomaly, can be understood as coming from a common source. A qualitative explanation of these phenomena would be helpful for the understanding of the phase structure of QCD under extreme conditions where direct simulations are prevented by the notorious sign-problem.

In the late seventies the hope was stirred that the instanton solutions [2] could be the building blocks to construct a semi-classical model to explain all these outstanding features [3, 4]. One of the aims was to find an effective description in terms of an enormously reduced number of degrees of freedom and a appropriate probability measure for them [5]. A characteristic aspect of this model was that, in absence of quantum fluctuations, action would be nearly and topological charge would be strictly quantized. The original hope was not fulfilled as far as the model failed to explain confinement.

Analyzing lattice fields, one has understood in the mean time (with filtering techniques based on certain scalar and fermionic operators [6, 7]) that at a certain momentum scale of resolution the topological density seems to exist in the form of instanton-like structures. A careful tuning of cooling or smearing applied to the gauge field can reproduce this structure. Unfortunately, it turned out, using suitably stopped standard cooling, that the reconstructed apparent instanton configurations can explain the spontaneous breakdown of chiral symmetry but likewise fail to confine quarks [8, 9].

The most popular picture both of confinement and chiral symmetry breaking rests on vortices (for a review see [1]) defined by the maximal centre gauge [10]. Although networks of thin, percolating centre vortices are infinite action configurations, their properties nicely scale with the lattice regulator granting the vortex configurations a meaningful interpretation in the continuum limit [11]. It was pointed out in [12, 13] that the balance between vortex entropy and vortex action must be intrinsically fine-tuned in Yang-Mills theory to explain these scaling properties. A natural explanation for this fine-tuning was firstly offered by one of us in [14]: the vortices are the singular image, in a certain gauge, of otherwise smooth semi-classical configurations which confine and which, this time, are probably not built out of instantons. In our recent paper [15], we have expanded the cooling techniques and did find ensembles of semi-classical configurations which confine quarks. We here summarise the ideas and main results of this approach and present new result on the space-time structure of these configurations.

Constrained cooling: Recently, we supplemented cooling of lattice configurations with certain constraints expressing conditions valid in the infrared [14, 15]. This should allow us to let cooling run non-supervised, providing ensembles of genuinely semi-classical configurations which conserve the property of confinement. Originally, the name “streamline” referred to instanton anti-instanton pairs living in concurrence with the empty perturbative vacuum [16]. These pairs would annihilate under cooling, if the constraints were not in place.

The idea in [15] was to reduce the action of $SU(2)$ lattice configurations with a huge number of constraints to preserve each Polyakov loop on the lattice. It might be
FIGURE 1. (Euclidean) time evolution of the topological charge density of the two biggest clusters.
surprising that these conditions, implemented by a Lagrange multiplier for each link, do not prevent the lattice field to develop into a smooth field free of UV fluctuations. Our numerical experiments have shown that the ensembles obtained via constrained cooling are smooth low-action configurations which can be easily studied by means of the gluonic action density, the topological charge density and the modes of gluonic and fermionic operators.

**Summary of main results:** It turned out that these configurations are suitable background fields for a semiclassical evaluation of the partition function: an investigation of the gauge invariant spectrum of the Yang-Mills Hessian [17] shows a clear separation between close-to-zero modes, which encode the collective degrees of freedom of the semi-classical background, and the bulk of gluonic modes, which merely counts towards its quantum weight (entropy).

The topological charge of the streamline configurations can be determined by simple means using the field-theoretic definition using the field-strength tensor. It turns out that the topological susceptibility is protected by the constraints yielding the value known for $SU(2)$ gauge theory [18].

In order to trace out the spontaneous breakdown of chiral symmetry, we calculated the near-zero eigenspectrum of the staggered quark operator using backgrounds fields obtained by constrained cooling. Although we have worked with staggered fermions for which the index theorem, the close relationship between zero modes and topological charge is spoiled, the presence of a band of close-to-zero modes proves that the chiral symmetry is spontaneously broken. Here, the mechanism for spontaneous symmetry breaking seems to be very much the same as for instanton liquids: while a particular instanton gives rise to fermionic zero modes, a liquid of instantons naturally induces a spectral density and hence spontaneous symmetry breaking via the Banks-Casher relation. We stress that, while the near-to-zero spectral properties of the streamline configurations are non-trivial, the spectral properties of configurations emerging from standard cooling are largely different: For the latter, the Creutz ratios probing confinement, the topological susceptibility and the near-to-zero modes of the quark operator are fading away with an increasing number of standard cooling iterations.

The space-time texture of the topological charge gives the possibility to confront the result of constrained cooling with any of the popular models. A lattice field, originally prepared at $\beta = 2.5$ with Wilson action, underwent constrained cooling. Table 1 lists its 10 biggest clusters sorted according to the modulus of the topological charge $Q_{\text{lump}}$ of the cluster. $N_{\text{lump}}$ denotes the number of space time points which belong to a particular cluster, and $A_{\text{lump}}$ denotes the action of the cluster. The total topological charge of the configuration of $Q \approx 4$ is contributed roughly from two large clusters with charge 15.7 and $-11.2$. Fig. 1 shows the positive (red) and negative (blue) charge densities of just these two clusters. All 24 time slices of a $24^4$ lattice are seen in consecutive order. We refer to [15] for details on the cluster search.

By contrast to a liquid of instantons, we found that individual clusters percolate and can have any non-integer topological charge and that only the sum of all cluster contributions is quantised (as it must be). One can see how parts of the two clusters appear and disappear in the successive time-slices.

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**Table 1.** Data of the visualised configuration.

| num | $N_{\text{lump}}$ | $Q_{\text{lump}}$ | $A_{\text{lump}}$ |
|-----|------------------|------------------|------------------|
| 1   | 171726           | 0.1573E+02       | 0.3491E+02       |
| 2   | 158554           | -0.1125E+02      | 0.2904E+02       |
| 3   | 4                | 0.1919E-03       | 0.1177E-02       |
| 4   | 7                | 0.1777E-03       | 0.1315E-02       |
| 5   | 7                | -0.1727E-03      | 0.1839E-02       |
| 6   | 3                | 0.1726E-03       | 0.8142E-03       |
| 7   | 6                | 0.1385E-03       | 0.1192E-02       |
| 8   | 4                | 0.1378E-03       | 0.7320E-03       |
| 9   | 5                | -0.1354E-03      | 0.7577E-03       |