Gravitational wave constraints on Dirac–Born–Infeld inflation

James E Lidsey and Ian Huston
Astronomy Unit, School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, UK
E-mail: J.E.Lidsey@qmul.ac.uk and I.Huston@qmul.ac.uk

Received 10 May 2007
Accepted 11 June 2007
Published 2 July 2007

Abstract. An upper bound on the amplitude of the primordial gravitational wave spectrum generated during ultra-violet Dirac–Born–Infeld (DBI) inflation is derived. The bound is insensitive to the form of the inflaton potential and the warp factor of the compactified dimensions and can be expressed entirely in terms of observational parameters once the volume of the five-dimensional sub-manifold of the throat has been specified. For standard type-IIB compactification schemes, the bound predicts undetectably small tensor perturbations with a tensor–scalar ratio \( r < 10^{-7} \). This is incompatible with a corresponding lower limit of \( r > 0.1(1 - n_s) \), which applies to any model that generates a red spectral index \( n_s < 1 \) and a potentially detectable non-Gaussianity in the curvature perturbation. Possible ways of evading these bounds in more general DBI-type scenarios are discussed and a multiple-brane model is investigated as a specific example.

Keywords: inflation, physics of the early universe

ArXiv ePrint: 0705.0240
1. Introduction

The inflationary scenario provides the theoretical framework for the early history of the universe. It has now been successfully tested by observations, including the third-year data from the Wilkinson Microwave Anisotropy Probe (WMAP3) [1]. Despite this success, however, the high-energy physics that resulted in a phase of accelerated expansion is still not well understood. String/M-theory unifies the fundamental interactions including gravity and it is natural and important to develop inflationary models within string theory and to confront them with cosmological observations.

One class of string-theoretic models that has received considerable attention is D-brane inflation [2]–[26]. (For recent reviews, see, e.g., [27,28].) Of particular interest is the Dirac–Born–Infeld (DBI) inflationary scenario [7,12]. This is based on the compactification of type-IIB string theory on a Calabi–Yau (CY) three-fold, where the form-field fluxes generate locally warped regions known as ‘throats’. The propagation of a D3-brane in such a region can drive inflation, where the inflaton field is identified with the radial position of the brane along the throat. Since this is an open string mode, the field equation for the inflaton is determined by a DBI action.

The purpose of the present work is to explore the observational consequences of DBI inflation. In general, primordial gravitational wave fluctuations and non-Gaussian statistics in the curvature perturbation provide two powerful discriminants of inflationary scenarios. The nature of the DBI action is such that the sound speed of fluctuations in the inflaton can be much less than the speed of light. This induces a large and potentially detectable non-Gaussian signal in the density perturbations [7,12,29,30].

The gravitational wave background generated in DBI inflation was recently investigated by Baumann and McAllister (BM) [31]. By exploiting a relationship due originally to Lyth [32], these authors derived a field-theoretic upper limit to the tensor amplitude and concluded that rather stringent conditions would need to be satisfied for these perturbations to be detectable. Moreover, the special case of DBI inflation driven...
by a quadratic potential is incompatible with the WMAP3 data when this constraint is imposed [33].

Our aim is to derive observational constraints on DBI inflation that are insensitive to the details of the throat geometry and the inflaton potential. In general, there are two realizations of the scenario, which are referred to as the ultra-violet (UV) and infra-red (IR) versions, respectively. These are characterized by whether the brane is moving towards or away from the tip of the throat. We focus initially on the UV scenario and derive an upper bound on the gravitational wave amplitude in terms of observable parameters. This limit arises by considering the variation of the inflaton field during the era when observable scales cross the Hubble radius, and we find in general that the tensor–scalar ratio must satisfy

$$r \lesssim 10^{-7}.$$  

This is below the projected sensitivity of future Cosmic Microwave Background (CMB) polarization experiments [34,35]. On the other hand, the WMAP3 data favour a red perturbation spectrum with $n_s < 1$ when the tensor modes are negligible and the scalar spectral index is effectively constant [1]. For models which generate such a spectrum, we identify a corresponding lower limit on the tensor modes such that $r \gtrsim 0.1(1 - n_s)$. This is incompatible with the upper bound on $r$ when $1 - n_s \simeq 0.05$, as inferred by the observations.

Hence, a reconciliation between theory and observation requires either a relaxation of the upper limit on $r$ or a blue spectral index ($n_s > 1$). The DBI scenario would need to be generalized in a suitable way for the upper bound on $r$ to be weakened. We identify necessary conditions that a generalized action must satisfy for the Baumann–McAllister constraint to be relaxed. We then show that such conditions can be realized in a recently proposed IR version of DBI inflation driven by multiple coincident branes [36].

The paper is organized as follows. We briefly review DBI inflation in section 2. In sections 3 and 4, we derive the respective limits on the amplitude of the tensor perturbations. We determine how the BM bound may be relaxed in generalized DBI models in section 5 and discuss multi-brane IR scenarios in section 6. Finally, we conclude in section 7. Units are chosen such that $M_P \equiv (8\pi G)^{-1/2} = 2.4 \times 10^{18}$ GeV defines the reduced Planck mass and $c = \hbar = 1$.

2. DBI inflation

Flux compactification of type-IIB string theory to four dimensions results in a warped geometry, where the six-dimensional CY manifold contains one or more throats (see [37,38] for reviews). The metric inside a throat has the generic form

$$ds_{10}^2 = h^2(\rho) ds_4^2 + h^{-2}(\rho) \left( d\rho^2 + \rho^2 ds_{X_5}^2 \right),$$

where the warp factor $h(\rho)$ is a function of the radial coordinate $\rho$ along the throat and $X_5$ is a Sasaki–Einstein five-manifold. In many cases, the ten-dimensional metric (1) can be approximated locally by the geometry $\text{AdS}_5 \times X_5$, where the warp factor is given by $h = \rho/L$ and the radius of curvature of the $\text{AdS}_5$ space is defined by

$$L^4 \equiv \frac{4\pi^4 g_s N}{\text{Vol} (X_5) m_s^4},$$

such that $\text{Vol}(X_5)$ is the dimensionless volume of $X_5$ with unit radius, $N$ is the D3 charge of the throat, $g_s$ is the string coupling and $m_s$ is the string mass scale. In the Klebanov–Strassler (KS) background [39], the throat is a warped deformed conifold and corresponds...
to a cone over the manifold $X_5 = T^{1,1} = SU(2) \otimes SU(2)/U(1)$ in the UV limit ($\rho \to \infty$). This has a volume $\text{Vol}(T^{1,1}) = 16\pi^3/27$ and topology $S^2 \times S^3$, where the $S^2$ is fibred over the $S^3$. The conical singularity at the tip of the throat is smoothed out by an $S^3$ ‘cap’ due to the wrapping of the fluxes along the cycles of the conifold [39, 40] and the warp factor asymptotes to a constant value in this region.

In general, the low-energy world-volume dynamics of a probe D3-brane in a warped throat is determined by an effective, four-dimensional DBI action. The inflaton field is related to the radial position of the brane by $\phi \equiv \sqrt{T_3} \rho$, where $T_3 = m_s^4/[(2\pi)^3 g_s]$ is the brane tension. The action is then given by

$$S = \int d^4x \sqrt{|g|} \left[ \frac{M_p^2}{2} R + P(\phi, X) \right] \quad (3)$$

$$P(\phi, X) = -T(\phi) \sqrt{1 - 2T^{-1}(\phi)X} + T(\phi) - V(\phi), \quad (4)$$

where $R$ is the Ricci curvature scalar, $X \equiv -\frac{1}{2} g_{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$ is the kinetic energy of the inflaton, $V(\phi)$ denotes the field’s interaction potential and $T(\phi) = T_3 h^4(\phi)$ defines the warped brane tension. We refer to $P(\phi, X)$ as the ‘kinetic function’ for the inflaton.

We consider a spatially flat and isotropic cosmology sourced by a homogeneous scalar field. In this case, the Friedmann equations for a monotonically varying inflaton can be expressed in the form [7]

$$3M_p^2 H^2(\phi) = V(\phi) - T(\phi) \left[ 1 - \sqrt{1 + 4M_p^4 T^{-1} H'^2} \right] \quad (5)$$

$$\dot{\phi} = -\frac{2M_p^2 H'}{\sqrt{1 + 4M_p^4 T^{-1} H'^2}}, \quad (6)$$

where $H = H(\phi)$ represents the Hubble parameter as a function of the field and a prime denotes $d/d\phi$.

An important consequence of the non-trivial kinetic structure of the DBI action is that the sound speed of fluctuations in the inflaton differs from unity:

$$c_s = \frac{1}{P_{,X}} = \sqrt{1 - 2T^{-1} X}, \quad (7)$$

where a subscripted comma denotes partial differentiation. Furthermore, the condition that the sound speed be real imposes an upper bound on the kinetic energy of the inflaton, $\dot{\phi}^2 < T(\phi)$, which is independent of the steepness of the potential. The motion of the brane is said to be ‘relativistic’ when this bound is close to saturation.

We now define the epoch that is directly accessible to cosmological observations as ‘observable inflation’. Depending on the reheating temperature, this occurred some 30 to 60 e-foldings before the end of inflation. The inflationary dynamics during this phase can be quantified in terms of three parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{XP_{,X}}{M_p^2 H^2} = \frac{2M_p^2}{\gamma} \left( \frac{H'}{H} \right)^2 \quad (8)$$

$$\eta \equiv \frac{2M_p^2 H''}{\gamma \dot{H}} \quad (9)$$

$$s \equiv \frac{c_s}{\dot{c_s}/H} = \frac{2M_p^2}{\gamma} \frac{H'}{H \gamma} \quad (10)$$
where $\gamma \equiv 1/c_s$. We will assume that the ‘slow-roll’ conditions $\{\epsilon, |\eta|, |s|\} \ll 1$ applied during observable inflation. In this regime, the amplitudes and spectral indices of the two-point functions for the scalar and tensor perturbations are given by [41]

$$P_S^2 = \frac{1}{8\pi^2 m_p^2 c_s \epsilon} \frac{H^2}{4\pi^2 \dot{\phi}^2}$$

$$P_T^2 = \frac{2}{\pi^2 m_p^4} \frac{H^2}{H^4}$$

$$1 - n_s = 4\epsilon - 2\eta + 2s$$

$$n_t = -2\epsilon$$

respectively, where the quantities on the right-hand sides are evaluated when the scale with comoving wavenumber $k = aH\gamma$ crossed the Hubble radius during inflation. The tensor–scalar ratio, $r \equiv P_T^2/P_S^2$, is directly related to the tensor spectral index by [41]

$$r = -8c_s n_t = 16c_s \epsilon.$$  

Hence, a sound speed different to unity leads to a violation of the standard inflationary consistency equation, which might be detectable in the foreseeable future [42,43]. Recent observations of the CMB indicate that $P_S^2 = 2.5 \times 10^{-9}$ and $r < 0.55$ [1]. The best-fit value for a constant spectral index is $n_s = 0.987^{+0.019}_{-0.017}$ if $r \neq 0$ is assumed as a prior. This is strengthened to $n_s = 0.948^{+0.015}_{-0.018}$ for a prior of $r = 0$ [1].

A further important consequence of a small sound speed is that departures from purely Gaussian statistics may be large [7,12,29,30]. It is conventional to quantify such deviations by expressing the non-Gaussian curvature perturbation $\mathcal{R}$ in the form $\mathcal{R} = \mathcal{R}_G + f_{NL}(\mathcal{R}_G^2 - \langle \mathcal{R}_G^2 \rangle)$, where $\mathcal{R}_G$ represents the Gaussian contribution, the quadratic piece is a convolution and $f_{NL}$ defines the ‘non-linearity’ parameter [44]. In general, this parameter is a function of the three momenta which form a triangle in Fourier space. However, in the limit where these momenta have equal magnitude, the non-linearity parameter is given to leading order by [30,43]

$$f_{NL} = \frac{1}{3} \left( \frac{1}{c_s^2} - 1 \right).$$

Currently, WMAP3 constrains this parameter to be $|f_{NL}| \lesssim 300$ [1,45].

Finally, equations (7), (11) and (16) may be combined to constrain the warped brane tension during observable inflation:

$$\frac{T(\phi_*)}{m_P^2} = \frac{\pi^2}{16} r^2 P_S^2 \left( 1 + \frac{1}{3f_{NL}} \right),$$

where a subscript ‘*’ denotes that the quantity is to be evaluated at that epoch.

In the following, we first consider the UV version of DBI inflation, where the brane moves relativistically towards the tip of the throat. We will assume implicitly that the sound speed is sufficiently small to generate a non-Gaussianity in excess of $f_{NL} \gtrsim 5$, since this is the projected sensitivity of the Planck satellite [46]. Furthermore, we consider an arbitrary warp factor and inflaton potential, subject only to the condition that a sufficiently long phase of quasi-exponential expansion can be maintained to solve the horizon problem of the hot big bang model.
3. An upper bound on the primordial gravitational waves

Recently, Baumann and McAllister (BM) [31] derived a field-theoretic upper bound on the tensor–scalar ratio by noting that the four-dimensional Planck mass is related to the volume of the compactified CY manifold, \( V_6 \), such that \( M_P^2 = \frac{V_6}{\kappa_{10}^2} \) (w.e.r.e \( \kappa_{10}^2 \equiv \frac{1}{2} (2\pi)^7 g_s^2 m_s^{-8} \)). In general, the compactified volume is comprised of bulk and throat contributions, \( V_6 = V_{6,\text{bulk}} + V_{6,\text{throat}} \), where the latter is given by

\[
V_{6,\text{throat}} = \text{Vol}(X_5) \int_0^{\rho_{UV}} d\rho \frac{\rho^5}{h^3(\rho)},
\]  

and \( \rho_{UV} \) denotes the radial coordinate at the edge of the throat (defined as the region where \( h(\rho_{UV}) \) is of order unity). If one assumes that the bulk volume is non-negligible relative to that of the throat (i.e. \( V_{6,\text{throat}} < V_6 \)), it follows that \( M_P^2 > V_{6,\text{throat}} \kappa_{10}^2 \). For a warped AdS_5 \times X_5 geometry, this leads to an upper limit on the total variation of the inflaton field in the throat region:

\[
\frac{\phi_{UV}}{M_P} < \frac{2}{\sqrt{N}}.
\]

Condition (19) may be converted into a corresponding limit on the tensor–scalar ratio by noting from the definition (8) that \( \dot{\phi}^2/M_P^2 = 2\epsilon H^2/P_X \). This implies that the variation of the inflaton field is given by the Lyth bound [31, 32]

\[
\frac{1}{M_P^2} \left( \frac{d\phi}{dN} \right)^2 = \frac{r}{8},
\]

where \( N \equiv \int dt H \). Since \( \phi_* < \phi_{UV} \), this results in an upper bound on the observable tensor–scalar ratio [31]:

\[
r_* < \frac{32}{N(N_{\text{eff}})^2},
\]

where \( N_{\text{eff}} \equiv \int_0^{N_{\text{end}}} dN (r/r_*)^{1/2} \) is a model-dependent parameter that quantifies how \( r \) varies during the final stages of inflation.

Typically, one expects \( 30 \lesssim N_{\text{eff}} \lesssim 60 \), although smaller values may be possible if the slow-roll conditions are violated after observable scales have crossed the horizon. Furthermore, \( N \gg 1 \) is necessary for backreaction effects to be negligible. Hence, the constraint (21) imposes a strong restriction on DBI inflationary models. On the other hand, the numerical value of \( N_{\text{eff}} \) is uncertain. Our aim here is to focus on the range of values covered by the inflaton field during the observable stages of inflation. This will result in a constraint on the tensor modes that can be expressed in terms of observable parameters.

To proceed, we denote the change in the value of the inflaton field over observable scales by \( \Delta \phi_* = \sqrt{T_3} \Delta \rho_* \). Since the brane moves towards the tip of the throat in UV DBI inflation, it follows that \( \rho_* > \rho_{\text{end}} > 0 \), which implies that

\[
\rho_* > |\Delta \rho_*|.
\]

This change in the inflaton value will correspond to a fraction of the throat volume, \( |\Delta V_{6,*} | < V_{6,\text{throat}} \lesssim V_6 \), where equality in the second limit arises if the bulk volume is
negligible. Hence, $|\Delta \phi_s|$ is bounded such that

$$
\left( \frac{\Delta \phi}{M_P} \right)_s^2 < \frac{3 \kappa_{10}^2 (\Delta \rho_s)^2}{|\Delta V_{6,s}|}.
$$

(23)

The observations of the CMB that directly constrain the primordial tensor perturbations only cover multipole values in the range $2 \leq l \lesssim 100$. This is equivalent to no more than $\Delta N_s \simeq 4$ e-foldings of inflationary expansion and, in general, corresponds to a narrow range of inflaton values. To a first approximation, therefore, the fraction of the throat volume (18) that is accessible to cosmological observation can be estimated to be

$$
|\Delta V_{6,s}| \simeq \text{Vol}(X_5) \frac{|\Delta \rho_s| \rho_s^2}{h_s^4}.
$$

(24)

Combining the inequality (22) with equation (24) then implies that

$$
|\Delta V_{6,s}| > \text{Vol}(X_5) \frac{(\Delta \rho_s)^6}{h_s^4},
$$

(25)

and substituting the CMB normalization (17) and condition (25) into the bound (23) yields the upper limit

$$
\left( \frac{\Delta \phi}{M_P} \right)_s^6 < \frac{\pi^3}{16 \text{Vol}(X_5)} r_s^2 P_s^2 \left( 1 + \frac{1}{3 f_{NL}} \right).
$$

(26)

Hence, employing the Lyth bound in the form $(\Delta \phi_s/M_P)^2 \simeq r(\Delta N_s)^2/8$ results in a very general upper limit on the tensor–scalar ratio:

$$
r_s < \frac{32 \pi^3}{(\Delta N_s)^6 \text{Vol}(X_5)} P_s^2 \left( 1 + \frac{1}{3 f_{NL}} \right).
$$

(27)

Condition (27) is only weakly dependent on the level of non-Gaussianity when $f_{NL} > 5$ and we may therefore neglect the factor involving this parameter. Substituting the WMAP3 normalization $P_s^2 = 2.5 \times 10^{-9}$ then implies that

$$
r_s < \frac{2.5 \times 10^{-6}}{(\Delta N_s)^6 \text{Vol}(X_5)}.
$$

(28)

Furthermore, the most optimistic estimate for the minimum number of e-foldings that could be probed by observation is $\Delta N_s \simeq 1$, whereas a generic compactification arises when the volume of the Einstein five-manifold is $\text{Vol}(X_5) \simeq \mathcal{O}(\pi^3)$ [39]. This yields a model-independent upper bound on the tensor–scalar ratio for standard UV DBI inflation:

$$
r_s < 10^{-7}.
$$

(29)

This is significantly below the sensitivity of future CMB polarization experiments, which will measure $r \gtrsim 10^{-4}$ [34, 35]. If CMB observations are able to span the full range of e-foldings such that $\Delta N_s \simeq 4$, this constraint is strengthened to $r_s \lesssim 2 \times 10^{-11}$.

Before concluding this section, we should remark that the estimate (24) was derived under the assumption that the integrand in equation (18) is constant. This inevitably introduces errors into the bound (27). However, the two limiting cases of interest
Gravitational wave constraints on DBI inflation in KS-type geometries arise when the warp factor scales either as $h \propto \rho$ or as $h \approx \text{constant}$ [39, 40]. In both cases the integral (18) can be performed analytically. Indeed, if we specify $h \propto \rho^\alpha$ for some constant $\alpha$, evaluate the integral between $\rho_*$ to $\rho_* + \Delta \rho_*$, and expand to second order in a Taylor series, we find that

$$\Delta V_{6,*} \simeq \text{Vol}(X_5) \frac{\rho_*^5}{h^4(\rho_*)} (\Delta \rho_*) \left[ 1 + \frac{(5 - 4\alpha)(\Delta \rho_*)}{2 \rho_*} \right].$$

(30)

This implies that the error in equation (24) is no greater than about $3(\Delta \rho_*/\rho_*)$ if $0 \leq \alpha \leq 1$. More generally, it follows that a similar error will arise for any warp factor $h \propto \rho^{\alpha(\rho)}$, where the function $\alpha(\rho)$ satisfies $0 \leq \alpha(\rho) \leq 1$ over observable scales. We conclude, therefore, that equation (24) provides a sufficiently good estimate of the volume element for a generic warp factor.

4. A lower bound on the primordial gravitational waves

The analysis of the previous section indicates that standard versions of UV DBI inflation generate a tensor spectrum that is unobservably small. Hence, $r = 0$ can be assumed as a prior when discussing the WMAP3 data. However, in this case the data exclude a scale-invariant density spectrum at the 3σ level when the running in the spectral index, $\alpha_s \equiv \frac{d \ln n_s}{d \ln k}$, is negligible [1]. Furthermore, a blue spectral index is only marginally consistent with the data when $r \neq 0$ and $\alpha_s = 0$. (The inferred upper limit is $n_s < 1.006$.) Although the results from WMAP3 do allow for a blue spectrum if there is significant negative running in the spectral index, we will focus in this section on models that generate a red spectral index $n_s < 1$, since these seem to be preferred by the current data.

In general, the spectral index may be related to the tensor–scalar ratio. After differentiating equation (7) with respect to cosmic time, and employing equations (6) and (13), we find that

$$1 - n_s = 4\epsilon + \frac{2s}{1 - \gamma^2} \mp \frac{2M_P^2 T'|H'|}{\gamma TH},$$

(31)

where the minus (plus) sign corresponds to a brane moving down (up) the warped throat. The second term in equation (31) can be converted into observable parameters by defining the ‘tilt’ of the non-linearity parameter, $n_{NL} \equiv d \ln f_{NL}/d \ln k$ [15]. This implies that $s = -3f_{NL}n_{NL}/[2(1 + 3f_{NL})]$ and substitution of equations (13)–(16) into equation (31) then yields

$$1 - n_s = \frac{r}{4} \left[ \sqrt{1 + 3f_{NL}} + \frac{n_{NL}}{1 + 3f_{NL}} \mp \sqrt{\frac{r}{8}} \left( \frac{T'}{T} M_P \right)_* \right].$$

(32)

In [43], brane inflation near the tip of a KS-type throat was considered, where the warped brane tension asymptotes to a constant value. In this regime, equation (32) reduces to the condition $r \simeq 2.3(1 - n_s)/\sqrt{f_{NL}}$ when $f_{NL}$ is sufficiently large to be detectable by Planck, i.e., $|f_{NL}| > 5$. It then follows from the WMAP3 best-fit value $n_s \simeq 0.987$ and upper limit $|f_{NL}| < 300$ [45] that the gravitational wave amplitude is

\[1\] As we shall see in the following section, even an order of magnitude error will make little difference to our final conclusions.
bounded both from above and below such that \(0.001 \lesssim r \lesssim 0.01\). These bounds follow from current WMAP3 limits on the spectral index and the non-linearity parameter, but do not take into account the field-theoretic upper bound that must be imposed on the variation of the inflaton field during inflation.

More generally, in UV DBI inflation where the brane moves towards the tip of the throat, it is reasonable to assume that the warp factor decreases monotonically with the radial coordinate over the observable range of inflaton values, i.e., that \(dh/d\rho \geq 0\). This condition is satisfied for AdS\(_5 \times X_5\) compactifications and KS-type solutions. Consequently, the third term in equation (32) will be semi-negative definite, which implies that

\[
\frac{r}{4} \sqrt{1 + 3 f_{\text{NL}}} + \frac{n_{\text{NL}}}{1 + 3 f_{\text{NL}}} > 1 - n_s. \tag{33}
\]

Condition (33) is a consistency relation on UV DBI inflation in terms of observable parameters and it may be combined with the upper bound (27) to confront the scenario with observations. Firstly, let us assume that the tensor–scalar ratio is negligible. The WMAP3 data imply that \(1 - n_s > 0.037\), and this is only compatible with condition (33) if

\[
3f_{\text{NL}} < 2(1 - n_s), \tag{34}
\]

However, when \(f_{\text{NL}} \gg 1\), this would violate the slow-roll conditions that must be satisfied for a consistent derivation of the perturbation spectra (11). For example, the conservative bound \(|s| < 0.1\) with \(1 - n_s \simeq 0.05\) is violated if \(f_{\text{NL}} > \mathcal{O}(5)\).

In view of this, let us consider the case where the tensor perturbations are non-negligible. The magnitude of the second term in condition (33) is suppressed by a factor of \(f_{\text{NL}}^{3/2} \gg 1\) relative to the first. This is expected to be a significant effect in DBI inflation. Consequently, by saturating the WMAP3 limit \(|f_{\text{NL}}| < 300\) [45], we arrive at a lower bound on the tensor–scalar ratio which applies to any model for which the ratio \(n_{\text{NL}}/f_{\text{NL}}\) is negligible:

\[
r_s > \frac{4(1 - n_s)}{\sqrt{3f_{\text{NL}}} > 1 - n_s} \tag{35}
\]

This requires \(r > 0.002\) for the WMAP3 best-fit value \(1 - n_s \simeq 0.013\), which is incompatible with the upper limit (29).

In general, therefore, it is difficult to simultaneously satisfy the bounds on \(r\) with the WMAP3 data in standard UV DBI inflation. There is a small observational window where a blue spectrum is consistent with the data, in which case the lower limit (35) does not apply. However, if the tensor modes are negligible, as implied by the inequality (29), the data strongly favour a red spectral index with \(n_s < 0.963\), and this violates the condition (35). A significant detection of a red spectral index requires either a violation of the slow-roll conditions or a sufficiently small value for the volume of \(X_5\). In particular, combining the limits (28) and (35) results in the condition

\[
\text{Vol}(X_5) < \frac{2 \times 10^{-5}}{(1 - n_s)(\Delta N_\ast)^6}, \tag{36}
\]

and we find that \(\text{Vol}(X_5) \lesssim 10^{-7}\) for typical values \(1 - n_s \simeq 0.05\) and \(\Delta N_\ast \simeq 4\). This is comparable to the limit on the volume derived for the special case of a quadratic inflaton
potential [31]. As noted in [31, 33], condition (36) may be achieved if $X_5$ corresponds to a $Y^{p,q}$ space, which has arbitrarily small volume in the limit $q = 1$ and $p \to \infty$ [47]. Small volumes could also be realized by orbifolding the $S^2$ symmetry of a KS-type throat.

On the other hand, our upper limit (28) on the gravitational waves follows as a consequence of assuming the constraint (22). This could be violated in IR versions of the scenario, where observable scales crossed the Hubble radius when the brane was near the tip of the throat and $\phi \ll M_P$ [13, 15]. Nonetheless, we emphasize that the upper bound (28) on the tensor modes will also apply to any IR DBI model for which $|\Delta \phi_*| < \phi_*$. In view of the above discussion, therefore, we will proceed in the following section to discuss a framework for generalizing the DBI scenario so that the constraints on the tensor modes can be satisfied.

5. Relaxing the Baumann–McAllister bound

In this section, we take a phenomenological approach and consider a general kinetic function of the form

$$ P = -f_1(\phi) \sqrt{1 - f_2(\phi) X} - f_3(\phi), \quad (37) $$

where $f_i(\phi)$ are unspecified functions of the inflaton field. A direct comparison with equation (4) indicates that the standard DBI action exhibits two important properties. The first is the condition that $f_1 f_2 = 2$. This implies that $c_s P_{,X} = 1$ and greatly simplifies the form of equation (20). The second property is that the warp factor uniquely determines the kinetic structure of the action, i.e., $h^4 \propto f_1 \propto f_2^{-1}$.

Equation (37) can be transformed into a similar form to that of equation (4) through the field redefinition

$$ \varphi \equiv \int d\phi \sqrt{\frac{f_1 f_2}{2}}. \quad (38) $$

This implies that the sound speed of fluctuations in the inflaton is given by

$$ c_s = \sqrt{1 - f_2^2 X} = \frac{f_1 f_2}{2} \frac{1}{P_{,X}}, \quad (39) $$

and the scalar perturbation amplitude by

$$ P_{,S}^2 = \frac{1}{2\pi^2 f_1 f_2 \phi^2} \frac{H^4}{f_3}. \quad (40) $$

However, the consistency equation (15) and non-Gaussianity constraint (16) remain unaltered for this more general class of models [43]. It follows, therefore, that the CMB normalization condition (17) generalizes to a constraint on the value of $f_1(\phi_*)$:

$$ \left( \frac{f_1}{M_P^2} \right)_* \approx \frac{\pi^2}{16} 2 P_{,S}^2 \left( 1 + \frac{1}{3 f_{NL}} \right). \quad (41) $$

It is assumed implicitly that the functions have a suitable form for generating a successful phase of inflation.
Finally, the expression for the scalar spectral index follows most directly by generalizing equation (32) via the correspondence (38). It is straightforward to show that

\[ 1 - n_s = \frac{r}{4} \sqrt{1 + 3f_{NL}} + \frac{n_{NL}}{1 + 3f_{NL}} \mp \sqrt{\frac{r}{4f_1f_2}} \left( \frac{f_1'}{f_1} M_P \right)^* . \tag{42} \]

Equation (38) also allows us to deduce that the infinitesimal variation of the effective field \( \varphi \) is given by the Lyth bound, \( (\Delta \varphi / M_P)^2 \simeq r(\Delta N)^2 / 8 \). This implies that the variation of the inflaton field during inflation is

\[ \int_0^{\phi_*} \frac{d\phi}{M_P} \sqrt{ \frac{f_1f_2}{2} } = \sqrt{\frac{r_*}{8N_{\text{eff}}}} . \tag{43} \]

If we now restrict our attention to the observable stage of inflation, and further assume that the variation of \( f_1f_2 \) is negligible during that epoch, we find that

\[ \left( \frac{\Delta \phi}{M_P} \right)^2 \simeq \frac{(\Delta N)^2}{4f_1f_2} r_* . \tag{44} \]

On the other hand, the BM bound restricts the maximal variation of the scalar field \( \phi \) in the throat region. This will be determined by expression (19) for generic warped geometries that are asymptotically AdS\(_5 \times X_5 \) away from the tip of the throat. Moreover, if observable scales leave the horizon while the brane is inside the throat, the change in the field value must satisfy \( |\Delta \phi| < \phi_{UV} \). Hence, it follows from equation (44) that\(^3\)

\[ r_* < \frac{16}{N(\Delta N)^2} f_1f_2 . \tag{45} \]

We will refer to condition (45) as the generalized BM bound. Combining expressions (41) and (45) then results in a necessary condition on the value of \( f_2(\phi_*) \) for the generalized BM bound to be satisfied:

\[ \frac{f_2(\phi_*) M_P^4}{N} > \frac{(\Delta N)^2}{\pi^2} \frac{1}{rP_S^2} . \tag{46} \]

This condition can also be interpreted as a lower limit on \( r \) for a given value of \( f_2(\phi_*) \).

In UV models, identical arguments that led to the lower limit (35) on the tensor–scalar ratio will also apply in this more general context if, as expected, \( f_1(\phi) \) is a monotonically increasing function. A necessary condition for the lower and upper limits (35) and (45) to be compatible, therefore, is that

\[ f_1f_2 > \frac{N(\Delta N)^2(1 - n_s)}{4\sqrt{3f_{NL}}} . \tag{47} \]

In IR scenarios, however, the positive sign will apply in the last term of the right-hand side of equation (42). Hence, assuming \( f_1' > 0 \) and neglecting the term proportional to \( n_{NL}/f_{NL} \) yields an upper limit on the tensor–scalar ratio:

\[ r_* < \frac{4(1 - n_s)}{\sqrt{3f_{NL}}} . \tag{48} \]

\(^3\) We are being conservative by restricting our discussion to the observable phase of inflation. More generally, if \( f_1f_2 \) remains nearly constant over the last \( N \) e-foldings of inflation, we may substitute \( \Delta N \rightarrow N_{\text{eff}} \), where \( N_{\text{eff}} \) may be as large as 60. Thus, the generalized bound (45) should be regarded as a necessary (but not sufficient) condition to be satisfied by the tensor modes.
Combining conditions (46) and (48) therefore leads to a constraint on \( f_2(\phi_*) \) for the generalized BM bound to be satisfied in IR inflation:

\[
\frac{f_2 M_P^4}{N} > \frac{\sqrt{3} (\Delta N_*)^2}{4 \pi^2} \sqrt{f_{\text{NL}}} \frac{\sqrt{f_{\text{NL}}}}{1 - n_s} P_S^2
\]  

(49)

To summarize this section, expression (45) implies that the BM bound (21) could be relaxed if \( f_1 f_2 \gg 1 \) on observable scales. It is therefore important to develop string-inspired models where this condition arises naturally. This will be the focus of the next section.

6. IR inflation and multiple branes

Recently, an interesting version of IR DBI inflation was proposed by Thomas and Ward [36]. In this model, the flux annihilation process generates \( n \) coincident branes that are initially located at the bottom of a throat region. The dynamics of this configuration is determined by the non-Abelian world-volume theory [48, 49]. This theory exhibits extra stringy degrees of freedom which arise due to the fuzzy nature of the geometry. For the case where a fuzzy two-sphere is embedded in a three-cycle in the \( X_5 \) manifold, the kinetic structure of the action is given in the large-\( n \) limit by [36]

\[
P = -n T_3 h^4(\phi) W(\phi) \sqrt{1 - 2 T_3^{-1} h^{-4}(\phi) X - h^4(\phi) + V(\phi)},
\]  

(50)

where

\[
W(\phi) \equiv \sqrt{1 + C^{-1} h^{-4}(\phi) \phi^4}
\]  

(51)

defines the so-called ‘fuzzy’ potential, \( C = \pi^2 \hat{C} T_3^2 / m_4^4 \) is a model-dependent constant and \( \hat{C} \simeq n^2 \) is the quadratic Casimir of the \( n \)-dimensional representation of \( SU(2) \). Comparison with equation (37) implies that \( f_1 f_2 = 2 n W \) and \( f_2 = 2 / (T_3 h^4) \). Hence, the new features of this model relative to the standard single-brane scenario are parametrized in terms of the fuzzy potential \( W(\phi) \). This configuration is conjectured to be dual to a D5-brane which is wrapped around a two-cycle of the throat [50]–[52].

The regime \( W \gg 1 \) is of interest for relaxing the gravitational wave constraints\(^4\). The generalized BM bound (46) may now be expressed as a limit on the value of the warp factor \( h(\phi_*) \) on CMB scales:

\[
\frac{N T_3 h_*^4}{M_P^4} < \frac{8 \pi^2 (1 - n_s) P_S^2}{\sqrt{3} f_{\text{NL}}(\Delta N_*)^2}
\]  

(52)

We now consider whether this limit can be satisfied for reasonable choices of parameters when the warped compactification corresponds to an AdS\(_5\) or KS throat, respectively. Recall that the warp factor for the AdS\(_5\) throat is given by \( h = \phi / (\sqrt{T_3} L) \). Condition (52) therefore reduces to a constraint on the value of the inflation during observable inflation:

\[
\frac{\phi_*^4}{M_P^4} < \frac{8 \pi^2 (1 - n_s) P_S^2 \lambda}{\sqrt{3} f_{\text{NL}}(\Delta N_*)^2 N},
\]  

(53)

\(^4\) Note that the case \( n \gg 1 \) and \( W \sim 1 \) will not significantly relax the BM bound, since we require \( n \ll N \) for backreaction effects to be negligible.
where $\lambda \equiv \pi N/[2\text{Vol}(X_3)]$. However, non-perturbative string effects are expected to become important below a cutoff scale, $\phi_{\text{cut}} = h_{\text{cut}} \lambda^{1/4} m_s$, where $h_{\text{cut}}$ is the value of the warp factor at that scale. For consistency, therefore, one requires $\phi_s > \phi_{\text{cut}}$, which implies an upper limit on the D3-brane charge:

$$N < \frac{8\pi^2 (1 - n_s) P_s^2}{\sqrt{3} f_{\text{NL}} (\Delta N_s)^2} \left( \frac{M_p}{h_{\text{cut}} m_s} \right)^4.$$  \hspace{1cm} (54)

Assuming the typical values $m_s \sim 10^{-2} M_p$, $\Delta N_s \simeq 4$ and $h_{\text{cut}} \sim 10^{-2}$ implies $N < 7 \times 10^7 (1 - n_s) f_{\text{NL}}^{-1/2} < 2 \times 10^6$, where the latter inequality follows for $1 - n_s < 0.05$ and $f_{\text{NL}} > 5$.

For an AdS_5 throat, the fuzzy potential is a constant and the condition that $W \gg 1$ becomes

$$\hat{C} \ll \frac{4\pi^2 g_s N}{\text{Vol}(X_3)}.$$  \hspace{1cm} (55)

Hence, combining inequalities (54) and (55) implies that

$$\hat{C} \ll \frac{32\pi^4 (1 - n_s) P_s^2}{\sqrt{3} f_{\text{NL}} (\Delta N_s)^2} \frac{g_s}{\text{Vol}(X_3)} \left( \frac{M_p}{h_{\text{cut}} m_s} \right)^4,$$  \hspace{1cm} (56)

and specifying $g_s \sim 10^{-2}$ and $\text{Vol}(X_3) \simeq \pi^3$ then yields the limit $\hat{C} \ll 10^6 (1 - n_s) f_{\text{NL}}^{-1/2} \leq 2 \times 10^4$, or equivalently, $n \ll 150$. Furthermore, since $f_1 f_2 \simeq \text{constant}$, the inequality (45) may be strengthened by a factor of $(N_{\text{eff}}/\Delta N_s)^2$ by substituting $\Delta N_s \rightarrow N_{\text{eff}}$. This ratio could be as high as $(60/4)^2 \approx 200$, which would rule out this particular model.

Since the branes are initially located at the tip of the throat, another case of interest is the IR limit of the KS geometry, where the warp factor asymptotes to a constant value [53]:

$$h_{\text{tip}} = \exp \left( -\frac{2\pi K}{3 M g_s} \right),$$  \hspace{1cm} (57)

and $K, M \in \mathbb{Z}^+$ are the units of flux associated with the NS–NS and R–R three-forms, respectively, such that $N = M K$. In this case, the generalized BM bound (52) becomes

$$\frac{8\pi K}{3 M g_s} - \ln N > 4 \ln \left( \frac{m_s}{g_s^{1/4} M_p} \right) - \ln \left( \frac{64\pi^5 (1 - n_s) P_s^2}{\sqrt{3} f_{\text{NL}} (\Delta N_s)^2} \right).$$  \hspace{1cm} (58)

The radius of the three-sphere at the tip of the KS throat is of the order $(g_s M)^{1/2}$ in string units and this must be large (and at the very least should exceed unity) for the supergravity approximation to be reliable. Substituting this requirement into expression (58) results in a necessary (but not sufficient) condition on the D3-brane charge for the generalized BM bound to be satisfied:

$$\frac{1}{N} \exp \left( \frac{8\pi g_s N}{3} \right) > \sqrt{3} f_{\text{NL}} (\Delta N_s)^2 \frac{1}{64\pi^5 (1 - n_s) P_s^2 g_s} \left( \frac{m_s}{M_p} \right)^4.$$  \hspace{1cm} (59)

Recalling that a necessary condition for the backreaction of the branes to be negligible is $N \gg n \gg 1$ implies that the exponential term in (59) will dominate unless the string
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coupling constant is extremely small. Hence, for the parameter estimations quoted above, we deduce the lower limit

\[ N - 12 \ln N > -6.8 + 12 \ln \left( \frac{\sqrt{N L}}{1 - n_s} \right), \]  

which becomes \( N \gtrsim 10^2 \) for \( 1 - n_s \simeq 0.05 \) and \( f_{\text{NL}} > 5 \).

In general, however, the \( K \) and \( M \) units of flux are not independent. F-theory compactification on Calabi–Yau four-folds provides a geometric way of parameterizing type-IIB string compactifications [53]–[58]. Global tadpole cancellation constrains the topology of the four-fold and this restricts the brane and flux configurations. When the KS system is embedded into F-theory, the constraint is given by [53]

\[ \frac{\chi}{24} = n + MK, \]  

(61)

where \( \chi \) is the Euler characteristic of the four-fold. Hence, \( N = MK < \chi/24 \), and together with condition (60), this implies that

\[ \chi > 2400, \]  

(62)

for \( N > 10^2 \). It is known that the Euler number for four-folds corresponding to hypersurfaces in weighted projective spaces can be as high as \( \chi \leq 1820448 \) [58], so there are many compactifications that could in principle satisfy the generalized BM bound. On the other hand, the above limit on the Euler characteristic does allow us to gain some insight into the topology of the extra dimensions, since compactifications which result in a small Euler characteristic would be incompatible with the generalized BM bound.

7. Conclusion

In this paper, we have derived an upper limit to the amplitude of the primordial gravitational wave spectrum generated during UV DBI inflation. We considered the maximal inflaton field variation that can occur during the observable stages of inflation and assumed only that the brane was inside the throat during that epoch. The bound (28) is valid for an arbitrary inflaton potential and warp factor (modulo some weak caveats) and can be expressed entirely in terms of observable parameters once the volume of the five-dimensional sub-manifold of the throat has been specified. The inferred upper limit on \( r \) is surprisingly strong. We find that the standard UV scenario predicts tensor perturbations that are undetectably small, at a level \( r^* \lesssim 10^{-7} \).

The current WMAP3 data favour models that generate a red spectral index \( n_s < 1 \) when both the gravitational waves and running in the scalar spectral index are negligible. For UV versions of the scenario, we have identified a corresponding lower limit on \( r \) which applies in this region of parameter space, \( r^* \gtrsim 0.1(1 - n_s) \). It is clear that the standard model cannot satisfy both the upper and lower bounds on the tensor modes for the observationally favoured value \( 1 - n_s \simeq 0.05 \).

The generality of our analysis implies that modifying either the inflaton potential or the form of the warp factor is unlikely to resolve this discrepancy. On the other hand, there are a number of possible ways of reconciling theory with observation. In general, either the upper or lower limit on \( r \) needs to be relaxed. Weakening the latter would require a violation of the slow-roll conditions or a blue spectral index. A value of \( n_s > 1 \) is
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compatible with WMAP3 if the running of the spectral index is sufficiently negative, but is only marginally consistent if just the tensor modes are non-negligible. The upper limit on $r$ can be weakened by reducing the volume of $X_5$ or by generalizing the DBI action. Furthermore, it need not necessarily apply in IR versions of the scenario, although the BM bound will still hold in such cases.

We considered a generalized version of the DBI action and identified a necessary condition on the form of such an action for the BM bound to be relaxed. As a concrete example, we investigated a version of IR inflation that is driven by multiple coincident branes and found that the bounds on the tensor–scalar ratio can indeed be made compatible if the brane charges satisfy appropriate conditions.

The upper bound on $r$ that we derived in section 3 arises because the warp factor in standard DBI models completely specifies the kinetic energy of the inflaton field. Deriving a corresponding bound for the generalized model (37) would be more involved, since the CMB normalization (41) only directly constrains the function $f_1(\phi)$ and this may not necessarily depend on the warp factor. Nonetheless, the constraints (23) and (25) can be combined with equation (44) to derive a limit on the tensor–scalar ratio in terms of the warp factor and the kinetic function (37) and we find that

$$r_\ast < \frac{2}{\pi^{2/3}(\Delta N_s)^2} \left(\frac{m_s}{M_P}\right)^{4/3} \frac{h_s^{4/3}(f_1 f_2)_s}{g_s \text{Vol}(X_5)}.$$  (63)

For a specific model where the warp factor and the functions $f_i(\phi)$ are determined by particle physics considerations, condition (63) may be interpreted as a bound that relates the tensor modes directly to the value of the inflaton field during observable inflation. This constraint provides a consistency check that any given model must satisfy irrespective of the form of the inflaton potential.

In conclusion, therefore, primordial gravitational wave constraints combined with cosmological observations of the density perturbation spectrum act as a powerful discriminant of DBI inflationary models. They also serve as an important observational guide for identifying viable generalizations of the scenario.

Acknowledgments

IH is supported jointly by a Queen Mary studentship and the Science and Technology Facilities Council (STFC). We thank S Thomas and J Ward for helpful discussions.

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