Primordial black holes, dark matter and hot-spot electroweak baryogenesis at the quark-hadron epoch

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Abstract

We propose a scenario in which the quantum fluctuations of a light stochastic spectator field during inflation, for instance the QCD axion, generate large curvature fluctuations in the radiation era. This inevitably leads to the existence of Hubble patches in which primordial black holes (PBHs) form at the quark-hadron epoch with a density comparable to the dark matter. The PBHs naturally have an extended mass function, with a density peaking at the Chandrasekhar mass, and extending to the mass required to explain the black hole coalescences observed by LIGO/Virgo, without violating current astrophysical constraints. At the quark-hadron epoch, the entropy production and temperature increase in PBH-forming regions provide the ingredients for a novel scenario of hot spot electroweak baryogenesis. Baryons are produced very efficiently around these collapsing pockets and then quickly diffuse to the entire Universe. This explains why the PBH collapse fraction at the QCD epoch is of order the observed baryon-to-photon ratio and why baryons and dark matter have comparable densities. If the stochastic spectator field is the QCD axion, this also explains the strong CP problem. Parameter fine-tunings are replaced by a single anthropic selection argument involving the stochasticity of the spectator field during inflation and the requirement that galaxies can form. Finally, we identify several observational predictions of our scenario that should be testable within the next few years.

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I. INTRODUCTION

Primordial black holes (PBHs) have been a focus of great interest for nearly 50 years \[1–5\]. One reason for this is that only PBHs could be small enough for Hawking radiation to be important \[4\], those smaller than about $10^{15}\text{g}$ having evaporated by now with many interesting cosmological consequences \[6\]. Recently, however, attention has shifted to PBHs larger than $10^{15}\text{g}$, which have not yet evaporated. This is because of the possibility that they provide the dark matter, an idea that goes back to the earliest days of PBH research \[3, 5, 7\] but which has been the focus of intensive recent work \[8–12\]. Since PBHs formed in the radiation-dominated era, they are not generated from baryons and therefore circumvent the well-known big bang nucleosynthesis (BBN) constraint that baryons can have at most 5% of the critical density \[13\], which is well below the 25% associated with the dark matter.

There are various reasons for invoking PBH dark matter: (1) the failure to find alternative dark matter candidates; (2) the fact that black holes definitely exist, so that one does not need to invoke new physics; (3) tentative observational evidence from a variety of lensing, dynamical and gravitational wave effects. Even if non-evaporating PBHs do not provide all the dark matter, they could still have interesting cosmological and astrophysical effects. For example, they have been invoked to explain the seeding of the supermassive black holes in galactic nuclei \[14–17\], the generation of large-scale structure through Poisson fluctuations \[18, 19\], the minimum radius and the large mass-to-light ratios of ultra-faint dwarf galaxies \[12\], and the generation of correlations between the soft X-ray and infrared backgrounds \[22, 23\]. It has also been proposed \[24, 25, 30\] that PBHs could explain the detection of gravitational waves from binary black hole mergers seen by LIGO/Virgo \[26–29\]; this may only require a small fraction of the dark matter to be in such PBHs \[30–32\] but the possibility of them being all the dark matter is not excluded \[33\].

Even if PBHs play none of these roles, all these effects can be used to place interesting constraints on the number of PBHs and this in turn constrains the cosmological models which generate them. The constraints are most usefully expressed as limits on the fraction $f(M)$ of the dark matter in PBHs of mass $M$ and – as reviewed in Ref. \[8\] – there are only a few mass windows where they could provide all the dark matter ($f = 1$). The most interesting is the IMBH range $(1 – 100\,M_{\odot})$, since this would also have implications for the LIGO/Virgo events. The other windows are the lunar-mass range $(10^{20} – 10^{24}\text{g})$, the
asteroid-mass range \((10^{16} - 10^{17} \text{g})\) and Planck mass relics of evaporation \((10^{-5} \text{g})\). Recently it has been argued that the PBH dark matter proposal may require an extended PBH mass function \([8, 16]\) and this is certainly the most natural situation. Depending on the constraint, this may either help or hinder the PBH dark matter proposal \([11, 12, 34, 35]\).

One criticism of the PBH scenario is that it requires fine-tuning of the initial collapse fraction. If we assume the PBHs all have mass of about \(M\) (i.e. a spread of masses \(\Delta M \sim M\)) and form at time \(t\) in a radiation-dominated early Universe, then the collapse fraction at formation \(\beta(M)\) is related to the current dark matter fraction \(f(M)\) by
\[
\beta(M) \sim f(M)(t/t_{\text{eq}})^{1/2} \sim 10^{-9} f(M)(M/M_\odot)^{1/2}
\]
where \(t_{\text{eq}}\) is time of matter-radiation equality and we assume the PBHs have of order the horizon mass, \(M \sim 10^5 (t/s)/M_\odot\) at formation \([36]\). Therefore \(\beta(M)\) is very small even if \(f(M) = 1\). There are many PBH formation mechanisms \([37]\), including inflation \([7, 38–41]\), curvaton \([42, 43]\), an early matter-dominated era \([44, 45]\), bubble collisions \([46, 47]\) and collapse of cosmic strings \([48, 49]\) or domain walls \([50, 51]\). In most of them the collapse fraction is very sensitive to some cosmological parameter, so perhaps the most attractive scenario is the one that can most naturally explain the fine-tunings.

In this paper, we propose a specific scenario in which the PBHs provide the dark matter (DM) and form at the QCD epoch. This idea has a long history and was originally based on the idea that there may have been a 1st order phase transition at the QCD epoch \([52]\). This is no longer plausible but - for given spectrum of primordial fluctuations - one might expect PBH formation to be enhanced at this time because of the slight softening of the equation of state expected \([53–55]\), a possibility that was recently revived by several authors \([56–59]\). For a while the possibility that the dark matter could be PBHs formed at the QCD epoch seemed to be supported by microlensing observations. More extensive data then appeared to exclude this possibility \([60, 61]\). However, it has recently been claimed that microlensing constraints are less stringent - and may even allow 100\% of the DM - when one takes into account more realistic DM profiles \([35, 62]\), uncertainties in the detection efficiency \([63, 64]\) or the possibility that the PBHs are grouped into clusters \([65]\).

Our own proposal combines two new ideas: a PBH formation scenario involving an inflationary spectator field and an efficient phase of *hot-spot* electroweak baryogenesis induced by the out-of-equilibrium collapse to form PBHs. In some regions of the Universe, a light spectator field, such as the QCD axion, can populate the slow-roll region of its potential,
due to quantum stochastic fluctuations during inflation. These regions undergo an extra expansion when the field dominates the density of the Universe, well after the end of inflation, which generates super-horizon curvature fluctuations. These fluctuations collapse at the QCD epoch to form PBHs and, at the same locations, generate the baryons, which quickly diffuse to the entire Universe, leading to the observed matter-antimatter asymmetry before big-bang nucleosynthesis. Our model therefore uses the power of gravity to provide the local out-of-equilibrium and kinetic energy conditions for very efficient electroweak baryogenesis at the quark-hadron epoch. This naturally explains why the dark matter and visible stars have comparable masses, both of these being close to the Chandrasekhar limit, and comparable densities. It also explains why the collapse fraction $\beta$ at the QCD epoch is of order the cosmological baryon-to-photon ratio $\eta$ and why the baryons and PBHs have similar densities.

Although our model explains why the PBH collapse fraction is of order the baryon-to-photon ratio, it does not explain why these quantities have their actual observed values ($\beta \sim \eta \sim 10^{-9}$). However, it is well known that various constraints on the value of $\eta$ are required in order that galaxies can arise [71]. These constraints can be regarded as “anthropic” selection effects, in which case our model implies that $\beta$ is also anthropically constrained. Although anthropic arguments are unpopular in some quarters, in recent years they have become more fashionable because the multiverse hypothesis allows them to be interpreted as a selection effect [72–74]. Several other authors have also recently considered anthropic aspects of PBH formation [75, 76]. In our scenario, the stochasticity of the spectator field during inflation naturally allows to invoke such an anthropic selection. The most important constraint concerns the closeness of the times of matter-radiation equality and photon decoupling. We argue that only Hubble patches where $\beta \sim \eta \sim 10^{-9}$ lead to the formation of galaxies. In others, either the diffusion damping scale is enhanced and galaxy formation is suppressed, or sub-galactic structure formation is boosted in such a way that PBH should quickly accrete most of the baryonic matter, making galaxy formation difficult.

The plan of this paper is as follows. In Section 2 we discuss the mass and collapse fraction of PBHs forming at the QCD epoch, explaining why these are close to the Chandrasekhar mass and baryon-to-photon ratio, respectively. In Section 3 we describe our favored PBH formation scenario, this involving a spectator field which drives a second inflationary phase in some regions of the Universe; we analyse the form of the curvature fluctuations required
to generate the PBHs and the expected PBH mass function. In Section 4 we argue that PBH production at the QCD epoch could explain the observed baryon asymmetry. In Section 5 we discuss the present constraints on PBH from the LIGO observations and show that our predicted PBH mass function is consistent with these. In Section 6 we discuss the anthropic constraints on the collapse fraction. In Section 7 we draw some general conclusions.

II. FINE-TUNING OF PBH MASS AND COLLAPSE FRACTION

PBHs could have formed at any time in the early Universe, so their initial mass could take any value from the Planck mass up to millions of solar masses. Invoking PBHs formed at the QCD epoch as the dark matter is attractive for three reasons:

1. the quark-hadron transition may naturally enhance gravitational collapse;

2. this explains why the dark objects and visible stars both have masses comparable to the Chandrasekhar mass;

3. the PBH collapse fraction required to provide the dark matter is the baryon-to-photon ratio and this arises naturally if the PBHs are responsible for baryogenesis.

Point (1) has been emphasized by previous authors, most recently Byrnes et al [56]. As regards (2), one expects a spectrum of masses, spanning the range $0.01 - 100 M_{\odot}$. We are mainly interested in the mass where most of the density resides, but an extended mass function has important implications for LIGO observations since one expects the gravitational wave signal from binary PBH coalescences to peak at a larger value of $M$ than the density. In this section, we first focus on point (2) and emphasize the link between the mass of the Hubble horizon at the QCD epoch and the Chandrasekhar mass. We then focus on point (3) but leave a detailed discussion of the scenario which explains the coincidence between the dark matter fraction and the baryon-to-photon ratio until the following two sections, The key point is that the baryon asymmetry generated locally (i.e. around each black hole) is $\mathcal{O}(1)$ but this was reduced to $\beta$ after the diffusion of baryons to the rest of the universe. This model could also naturally explains another apparent fine-tuning – why the PBH density is just a factor of 6 above the baryonic density.
A. Chandrasekhar and QCD epoch horizon mass coincidence

The Chandrasekhar limit is the maximum mass of a white dwarf, this representing a balance between gravity and the electron degeneracy pressure. It can be shown to be

\[ M_{\text{Ch}} = \frac{\omega}{\mu^2} \left( \frac{3\pi}{4} \right)^{1/2} \frac{m_p^3}{m_p^2} \approx 5.6 \mu^{-2} M_\odot, \]  

(2.1)

where \( M_P \) is the Planck mass, \( m_p \) is the proton mass, \( \omega = 2.018 \) is a constant that appears in the solution of the Oppenheimer-Volkov (Lane-Emden) equation and \( \mu \) is the number of electrons per nuclei (1 for hydrogen, 2 for helium). Stars more massive than \( M_{\text{Ch}} \) cannot avoid gravitational collapse to a neutron star and ones somewhat larger than this collapse to a black hole. Therefore, this is a lower limit on the mass of a black hole arising from stellar evolution. One can also show that all hydrogen-burning main-sequence stars have a mass in the range 0.1 to 10 times \( M_{\text{Ch}} \) \( \text{[71]} \). The lower limit comes from the nuclear ignition condition and the upper limit from the instability associated with radiation-pressure-dominated stars.

In terms of fundamental units, one has \( M_{\text{Ch}} \sim \alpha_G^{-3/2} m_p \), where

\[ \alpha_G \equiv Gm_p^2/(\hbar c) = m_p^2/M_P^2 \sim 10^{-38} \]  

(2.2)

is the gravitational fine structure constant, so all stars have a mass within an order of magnitude of this.

Let us now consider the mass of a black hole which forms from the gravitational collapse of a large curvature perturbation during the radiation era of the early universe. In this case, some fraction \( \gamma \) of the relativistic gas within the particle horizon collapses to form a PBH. Although the precise fraction is uncertain, the standard assumption is that \( \gamma \approx 0.2 \) \( \text{[6]} \). During the radiation era, the density is \( \rho_r \approx 3/(32\pi G t^2) \) and the Hubble horizon (also particle horizon) size is \( d_H \approx 2ct \), so the mass of a PBH forming at time \( t \) is

\[ M = \frac{4\pi}{3} \gamma \rho_r d_H^3 = \frac{\gamma c^3 t}{G}. \]  

(2.3)

We can express this in terms of the temperature, using the relation

\[ k_B T \approx (32 g_* \pi^3 G t^2 / 45)^{-1/4} \sim \alpha_G^{-1/4} g_*^{-1/4} m_p (t/t_p)^{-1/2}. \]  

(2.4)

where \( k_B \) is the Boltzmann constant, \( g_* \) is the number of relativistic degrees of freedom and \( t_p = \hbar/(m_p c^2) \sim 10^{-23} \) s is the proton timescale. At the QCD epoch this gives

\[ M = \frac{\gamma \xi^2}{g_*^2} \left( \frac{45}{16\pi^3} \right)^{1/2} \frac{M_P^3}{m_p^2} = \left[ \frac{\gamma \xi^2 \mu^2}{g_*^{1/2} \pi^2 \omega} \right] M_{\text{Ch}} \approx 1.0 \left( \frac{\gamma}{0.2} \right) \left( \frac{g_*}{10} \right)^{-1/2} \left( \frac{\xi}{5} \right)^2 M_\odot. \]  

(2.5)
where $g_\ast$ is normalised appropriately and $\xi = m_p/(k_B T) \approx 5$ is the ratio of the proton mass to the QCD transition temperature. The middle expression shows that PBHs naturally have around the Chandrasekhar mass if they form at the QCD epoch, the factor in square brackets being close to 1 for the relevant parameter choices.

We stress that there are several important differences between a star and a PBH forming at the QCD epoch, despite their similar masses. A region collapsing to a PBH has around the Hubble horizon size, $d_H \sim (M/M_\odot)$ km, at maximum expansion and does not collapse much before forming an event horizon, whereas stars have radii of order $10^6$ km and collapse by a factor of $10^6$. Consequently the final spin of stellar black holes is expected to be large, due to conservation of angular momentum, while that of PBH should be negligible \cite{77,78}.

**B. PBH collapse fraction and baryon-to-photon ratio coincidence**

We denote the total fraction of the dark matter in PBHs (allowing for an extended mass function) by $f_{\text{tot}}$ and the ratio of the dark matter and baryonic densities by $\chi$. The most recent Planck measurements \cite{114} give the cold dark matter and baryon density parameters $\Omega_c h^2 = 0.12$ and $\Omega_b h^2 = 0.022$, so $\chi \simeq 5.5$. The ratio of the PBH density to the baryonic density is then $\Omega_{\text{PBH}}/\Omega_b = f_{\text{tot}} \chi$ and this is constant after PBH formation, since they both scale as $a^{-3}$ if we neglect PBH accretion.

The CMB density scales as $a^{-4}$, so the ratio of the PBH density to the CMB density scales as $a \propto t^{1/2}$ in the radiation era. The PBH collapse fraction is determined by evaluating this ratio at the PBH formation time, which we denote as $t_{\text{form}}$. To determine this, we need the relationship between the temperature and time. Before matter-radiation equality, this is given by Eq. (2.4). If the collapse fraction at the PBH formation time is $\beta$, then Eq. (2.4) implies that at any subsequent epoch within the radiation era

$$\frac{\rho_{\text{PBH}}}{\rho_\gamma} = f_{\text{tot}} \chi \frac{\rho_b}{\rho_\gamma} \sim f_{\text{tot}} \chi \frac{n_b m_p}{n_\gamma k_B T} = f_{\text{tot}} \chi \frac{\eta m_p}{k_B T} \sim f_{\text{tot}} \chi \eta g_\ast^{1/4} \alpha_G^{1/4} \left( \frac{t}{t_p} \right)^{1/2}, \quad (2.6)$$

where we have used Eq. (2.4) and

$$\eta \equiv n_b/n_\gamma = 2.8 \times 10^{-8} \Omega_b h^2 = 6.1 \times 10^{-10} \quad (2.7)$$

denotes the ratio of the baryon and photon number densities. Since the ratio of the PBH and photon densities can also be written as $\beta(t/t_{\text{form}})^{1/2}$, this implies

$$\beta \sim f_{\text{tot}} \chi \eta g_\ast^{1/4} \alpha_G^{1/4} (t_{\text{form}}/t_p)^{1/2}. \quad (2.8)$$

8
We can also express this in terms of the mass of the PBH, Eq. (2.3) implying that this is

\[ M \sim \gamma \alpha_G^{-1} m_p (t_{\text{form}}/t_p). \]  

(2.9)

Eqn (2.8) then gives

\[ \beta \sim f_{\text{tot}}^{\gamma g_*^{1/4} \gamma^{-1/2} \eta \alpha_G^{3/4} (M/m_p)^{1/2}} \sim f_{\text{tot}}^{\gamma^{-1/2} \eta g_*^{1/4} (M/M_{\odot})^{1/2}}, \]  

(2.10)

where we have used \( M_{\odot} \sim \alpha_G^{-3/2} m_p \) at the last step.

The collapse fraction required for PBHs to provide the dark matter takes a very simple form if they are produced at the QCD transition. This is because, from Eqs. (2.10) and (2.1), the collapse fraction is just

\[ \beta \sim f_{\text{tot}}^{\gamma g_*^{1/4} \gamma^{-1/2} \eta} \sim 10^{-9}, \]  

(2.11)

where we assume \( f_{\text{tot}}^{\approx 1}, \chi \approx 6, g_* \approx 10 \) and \( \gamma \approx 0.2 \) at the last step. Since the photon energy at the QCD epoch is comparable to the proton mass, one necessarily has \( \rho_b/\rho_\gamma \sim \eta \) then. Therefore one also has \( \beta \sim \eta \) if the PBH and baryon densities are comparable, so no fine-tuning of the collapse fraction is required if there is some natural way in which the value of \( \eta \) is associated with the value of \( \beta \). One might envisage three possibilities.

- The photons may have been generated by the PBHs (e.g. via accretion) in such a way that the photon-to-baryon ratio \( S \equiv \eta^{-1} \) is of order \( \beta^{-1} \). This is not impossible since observations only require the Universe to be radiation-dominated at BBN and this just implies \( S > 10^4 \) [see Sec. VI].

- The collapse fraction \( \beta \) may have been determined by \( \eta \) in some way. For example, since most antiprotons annihilate just before the QCD phase transition, leaving \( 1/\eta \) photons for each surviving proton, one just needs the surviving protons (or at least 80% of them) rather than the photons to go into the PBHs.

- The baryon asymmetry may have been generated by PBHs, so that \( \eta \) is naturally driven to \( \beta \) for PBH formation at the QCD transition. In this paper, we propose a scenario in which large curvature fluctuations provide efficient baryogenesis in the regions that collapse to PBHs. This leads to \( \eta \gg 1 \) locally but \( \eta \sim \beta \) after the plasma is homogenized.

Since we have not found any convincing scenario in which the first two possibilities arise naturally, we focus only on the third possibility in the rest of this paper.
III. FINE-TUNING OF CURVATURE FLUCTUATIONS

Forming PBHs with the dark matter abundance requires curvature fluctuations larger than those expected for slow-roll inflation in most single field scenarios \[80, 81\]. Invoking the softening of the equation of state during the QCD transition boosts the formation of stellar-mass PBHs but does not alleviate the need for large curvature fluctuations. However, if the PBHs form from inhomogeneities, \( \beta \) is exponentially sensitive to the amplitude of the power spectrum of curvature fluctuations, so there is still an important fine-tuning issue to resolve. In this section, we propose a new PBH formation mechanism, involving a light stochastic spectator field, whose quantum fluctuations during inflation provide the key ingredient to resolving the fine-tuning. Instead of being artificially fine-tuned, the mean value of this field within our observable Universe is explained by anthropic selection. We then propose identifying this scalar field with the QCD axion. Another key feature of our scenario is that it provides a novel mechanism for electroweak baryogenesis at the QCD epoch, which naturally produces a baryon-to-photon ratio of order the PBH collapse fraction.

A. Basic idea

The basic idea is that quantum stochastic fluctuations in the spectator field during inflation lead it to acquire different mean values in different Hubble patches today. There are a huge number of these patches, so there necessarily exist some (e.g. the one corresponding to our Universe) in which the spectator field on leaving the horizon has the value required for subsequent quantum fluctuations to induce large curvature fluctuations over different horizon-sized regions at PBH formation. More precisely, the spectator field within these regions remains frozen during the radiation era until its potential energy starts to dominate the density of the Universe, well after inflation. At this point the field triggers a second inflationary phase (for at most a few e-folds) within these regions, whereas in the rest of the patch it quickly rolls down its potential without inflating. This extra expansion generates local non-linear curvature fluctuations, which later re-enter the horizon and collapse to form PBHs. However, in the rest of the Universe the curvature fluctuations are statistically Gaussian and behave as expected in standard slow-roll inflation, unaffected by the spectator field.
B. The stochastic spectator during inflation

We define three characteristics wave-numbers: the scale of the observable Universe ($k_{H_0} \simeq 2.3 \times 10^{-4} \text{ Mpc}^{-1}$), the CMB pivot scale ($k_*= 0.05 \text{ Mpc}^{-1}$) and the PBH/QCD scale ($k_{QCD} \simeq 10^6 \text{ Mpc}^{-1}$). There are about 22 e-folds of inflation between the observable Universe and the PBHs exiting the horizon and about 17 e-folds between the CMB pivot and PBH scales doing so. During inflation we assume that the Hubble rate can be reconstructed from a truncated hierarchy of Hubble-flow slow-roll parameters,

$$
\epsilon_1 \equiv -\frac{d(\ln H)}{dN}, \quad \epsilon_2 \equiv \frac{d(\ln \epsilon_1)}{dN}, \quad \epsilon_3 \equiv \frac{d(\ln |\epsilon_2|)}{dN},
$$

(3.1)

where $N$ is the number of e-folds since horizon exit of the CMB pivot scale. Then one has

$$
\epsilon_2(N) = \epsilon_{2*} \exp \left( \int_0^N \epsilon_3(N') dN' \right),
$$

(3.2)

$$
\epsilon_1(N) = \epsilon_{1*} \exp \left( \int_0^N \epsilon_2(N') dN' \right),
$$

(3.3)

$$
H(N) = H_* \exp \left( -\int_0^N \epsilon_1(N') dN' \right).
$$

(3.4)

Assuming slow-roll inflation, the scalar power spectrum amplitude $A_s$ and the spectral index $n_s$ measured by Planck [107] are given to first order in the Hubble-flow parameters by

$$
A_s = 2.1 \times 10^{-9} \simeq \frac{H_*^2}{8\pi \epsilon_{1*} M_P^2}, \quad n_s = 0.9649 \pm 0.0042 \simeq 1 - 2\epsilon_{1*} - \epsilon_{2*},
$$

(3.5)

where $M_P$ is the reduced Planck mass.

Inflation is driven by a scalar field slowly rolling down its potential. Three effective benchmark models are considered, all agreeing with the amplitude and spectral index measurements:

- Model 1: $\epsilon_{1*} = 0.01, \epsilon_{2*} = \epsilon_{3*} = 2\epsilon_{1*} = 0.02, H_* = 2.3 \times 10^{-5} M_P$, as expected for a quadratic potential $V(\phi) \propto \phi^2$.
- Model 2: $\epsilon_{1*} = 0.005, \epsilon_{2*} = \epsilon_{3*} = 4\epsilon_{1*} = 0.02, H_* = 1.6 \times 10^{-5} M_P$, as expected for a linear potential $V(\phi) \propto \phi$.
- Model 3: $\epsilon_{1*} \lesssim 10^{-3}, \epsilon_{2*} = 0.04, \epsilon_{3*} = 0, H_* \lesssim 7.3 \times 10^{-6} M_P$, as expected for small-field or plateau-like potentials.
As explained later, if the spectator field dominates the density of the Universe much above the GeV scale, a significant decrease of $H(N)$ during inflation is needed to avoid an over-production of light PBHs (i.e. $\epsilon_1 \gtrsim 10^{-3}$), which excludes Model 3 but also predicts a detectable tensor-to-scalar ratio, $r \approx 16\epsilon_{1*} \gtrsim 2 \times 10^{-2}$. Model 1 is disfavored by the current limits on this ratio, so Model 2 is preferred in this case and the others should be regarded as two extreme possibilities. However, the requirement on the Hubble rate variation during inflation is relaxed if the spectator field dominates at the GeV scale or below, as in the case of the QCD axion, and Model 3 then leads to a more generic PBH mass distribution that is only marginally impacted by the exact shape of the inflationary potential. This extends our scenario to any inflationary model. Our approach is relatively simplistic compared to numerical computation of inflationary predictions but it is precise enough to understand the basic physical principles behind our mechanism of PBH formation.

We assume that there exists a light spectator field, $\psi$, with a potential $V(\psi)$ whose shape is discussed later, having a mass $m_\psi \ll H_{\text{inf}}$ during inflation. In a coarse-grained model, its quantum fluctuations during one e-fold of inflation are $\Delta \psi_{\text{stoch}} \sim H/2\pi$ within each Hubble volume. If the field is light enough not to reach adiabatic equilibrium during inflation, then the variance of the fluctuations $\delta \psi$ monotonically increases like

$$\langle \delta \psi^2 \rangle \approx \int_0^N \frac{H(N')^2}{4\pi^2} dN'.$$  

(3.6)

The evolution of $\langle \delta \psi^2 \rangle$ for our three benchmark inflation models is represented in Figure 1. If $\epsilon_1$ were constant, one would have

$$\langle \delta \psi^2 \rangle \approx \frac{H_{\text{CMB}}^2}{8\pi^2 \epsilon_1} \left(1 - e^{-2\epsilon_1 N}\right),$$  

(3.7)

which grows linearly with the number of e-folds before reaching a plateau when $N \gtrsim 1/(2\epsilon_1)$. In a realistic scenario $\epsilon_1$ varies during inflation, but qualitatively the spectator field variance follows a similar behavior. The stochastic dynamics of the spectator field during inflation is described by the Fokker-Planck equation and admits a Gaussian solution for the probability density distribution $[85]$:

$$P(\psi, N) = \frac{1}{\sqrt{2\pi \langle \delta \psi^2 \rangle}} \exp\left[ -\frac{(\psi - \langle \psi \rangle)^2}{2\langle \delta \psi^2 \rangle}\right],$$  

(3.8)

where $\langle \psi \rangle$ is the mean value within the patch corresponding to our Universe. One infers that the probability density of having a local field variation $\Delta \psi(N, x) \equiv \psi(N, x) - \psi(N - 1, x)$
during one expansion e-fold is

\[ P(\Delta \psi, N) = \frac{1}{\sqrt{2\pi(H(N)^2/4\pi^2)}} \exp \left[ -\frac{\Delta \psi^2}{2(H(N)^2/4\pi^2)} \right]. \]  \hspace{1cm} (3.9)

We will use this expression in the following section to compute the PBH abundance today.

C. The stochastic spectator after inflation

After inflation, the spectator field remains constant on super-Hubble scales as long as \( m \ll H \) and \( \rho \gg \rho_\psi \simeq V(\psi) \). In the different patches of the multiverse, \( \langle \psi \rangle \) can take up to super-Planckian values if inflation lasts for a sufficient number of e-folds. In patches where \( \langle \psi \rangle \) is close enough to field values for which the potential is sufficiently flat to induce slow-roll (e.g. \( \psi_{\text{cr}} \simeq \sqrt{2} M_P \) for a quadratic potential \( V = m_\psi^2 \psi^2 \)), at some time during the radiation era the spectator field starts to dominate the density of the Universe and eventually induces in some regions a second (short) inflationary phase, generating large curvature fluctuations and leading to PBH formation. In other regions, the field quickly rolls down the potential without inflating. In other patches where \( \langle \psi \rangle \) is far from the slow-roll region, the stochastic field fluctuations are not able to produce a second inflationary phase in any region, so there is no PBH formation. We focus here on the patches that have exactly the required value of \( \langle \psi \rangle \) for the subsequent field fluctuations to produce PBHs with an abundance compatible with the dark matter.

We distinguish two possible behaviors, depending on the size of the slow-roll region able to generate \( \mathcal{O}(1) \) curvature fluctuations in the spectator field potential, \( \Delta \psi^{sr} \), compared to the characteristic size of the quantum fluctuations during inflation, \( \Delta \psi^{\text{stoch}} \).

*Case 1: \( \Delta \psi^{sr} \gg \Delta \psi^{\text{stoch}} \).* From Eq. (2.21) of Ref. [85], the probability that the stochastic fluctuations lead the spectator field to acquire a local value \( \psi > \psi_{\text{cr}} \), the critical field value above which the slow-roll conditions are satisfied, is given by

\[ P_1 = \int_{\psi > \psi_{\text{cr}}} P(\psi, N)d\psi = \frac{1}{2} \text{erfc} \left[ \frac{\psi_{\text{cr}} - \langle \psi \rangle}{\sqrt{2\langle \delta \psi^2 \rangle}} \right]. \]  \hspace{1cm} (3.10)

This provides a first condition for these regions to undergo an extra inflationary phase with \( N_{\text{extra}} \sim \mathcal{O}(1) \). One can recognize a similar behavior when computing \( \beta \) for Gaussian curvature fluctuations with a variance \( \sigma \), viz. \( \beta = \text{erfc}(\zeta_{\text{tr}}/\sqrt{2\sigma^2}) \). If \( P_1 \) were the only condition for PBH formation, since \( \langle \delta \psi^2 \rangle \) is a growing function of time during inflation, the
FIG. 1: Evolution of the spectator field variance as a function of the number of e-folds since the current Hubble scale, $k_{H_0} = 2.3 \times 10^{-4}\text{Mpc}^{-1}$, exited the horizon for Models 1 (dashed red), 2 (solid blue) and 3 (dotted yellow). The vertical dotted lines represent the e-fold numbers corresponding to PBH masses of 0.01$M_\odot$, $1M_\odot$ and $100M_\odot$ (left to right).

FIG. 2: Collapse fraction of PBHs at formation, assuming a curvature threshold $\zeta_{th} = 1.02$, as a function of the number of e-folds since the current Hubble scale, $k_{H_0} = 2.3 \times 10^{-4}\text{Mpc}^{-1}$, exited the horizon for Models 1 (dashed red), 2 (solid blue) and 3 (dotted yellow), with $\epsilon_\psi = 0.5H_\star^2/m_{pl}^2$ and $\psi_{cr} - \langle \psi \rangle = 10^{-4}H_\star^2$ (thin lines) or $\psi_{cr} - \langle \psi \rangle = 0.5H_\star^2$ (thick lines). The vertical dotted lines represent the e-fold numbers for PBH masses of 0.01$M_\odot$, $1M_\odot$ and $100M_\odot$ (left to right).
model would generally lead to an overproduction of light PBHs. But PBH formation occurs only if a second condition is satisfied: \( \Delta \psi > \Delta \psi_{\text{tr}} \) where \( \Delta \psi_{\text{tr}} \) is the threshold fluctuation required to induce an extra e-folding \( \delta N = \zeta_{\text{tr}} \). Indeed, in our coarse-grained picture, only these regions will experience a curvature fluctuation (defined as the local curvature minus the mean curvature in the surrounding superhorizon region) leading to gravitational collapse when it re-enters inside the horizon. This second condition has probability

\[
P_2 = \int_{\Delta \psi > \Delta \psi_{\text{tr}}} P(\Delta \psi, N) d\Delta \psi = \frac{1}{2} \text{erfc} \left[ \frac{\Delta \psi_{\text{tr}}}{\sqrt{2}H(N)/(2\pi)} \right].
\]

In this simplified pictured, PBH formation occurs within one e-fold of expansion with probability

\[
P_{\text{PBH}} = \frac{d\beta(t)}{d \ln M} = P_1(N_t) \times P_2(N_t) = \frac{1}{4} \text{erfc} \left[ \frac{\psi_{\text{tr}} - \langle \psi \rangle}{\sqrt{2} (\delta \psi^2)} \right] \text{erfc} \left[ \frac{\Delta \psi_{\text{tr}}}{\sqrt{2}H(N)/(2\pi)} \right],
\]

where \( N_t \) denotes the number of e-folds when the scale associated with PBHs of mass \( M \) exits the Hubble horizon. The classes of plateau-like and large-field potentials correspond to this case.

**Case 2:** \( \Delta \psi_{\text{sr}} \ll \Delta \psi_{\text{stoch}} \). If the slow-roll region of the spectator field potential is tiny compared to its quantum fluctuations during inflation, as is the case for the QCD axion discussed later, the probability of PBH formation is related to the probability that the field ends up in the slow-roll region, producing \( \mathcal{O}(1) \) curvature fluctuation. This assumes that the field distribution one e-fold earlier is given by Eq. (3.8) with \( N \rightarrow N - 1 \). If one denotes by \( \psi_{\min} \) and \( \psi_{\max} \) the minimum and maximum field values in this region (so that \( \psi_{\max} = -\psi_{\min} = \Delta \psi_{\text{sr}} \) for a symmetric potential), one obtains

\[
P_{\text{PBH}} = \int d\psi P(\psi, N - 1) \times \frac{1}{2} \left[ \text{erf} \left( \frac{\psi_{\max} - \psi}{\sqrt{2} (H_N^2/(4\pi^2))} \right) + \text{erf} \left( \frac{\psi_{\min} + \psi}{\sqrt{2} (H_N^2/(4\pi^2))} \right) \right].
\]

For a symmetric potential, in the limit \( \Delta \psi_{\text{sr}} \ll \Delta \psi_{\text{stoch}} \), this gives

\[
P_{\text{PBH}} = \int d\psi P(\psi, N - 1) \sqrt{\frac{2}{\pi}} \frac{\Delta \psi_{\text{sr}}}{H_N^2/(2\pi)} \exp \left[ -\frac{\psi^2}{2H_N^2/(4\pi^2)} \right].
\]

After integrating over the field distribution, one obtains

\[
P_{\text{PBH}} = \sqrt{\frac{2}{\pi}} \frac{\Delta \psi_{\text{sr}}}{\sqrt{H_N^2/(4\pi^2) + \langle \delta \psi^2 \rangle_{N-1}}} \exp \left[ -\frac{\langle \psi \rangle^2}{2(H_N^2/(4\pi^2) + \langle \delta \psi^2 \rangle_{N-1})} \right],
\]

which can be suppressed to any low value in patches where \( \langle \psi \rangle < \sqrt{\langle \delta \psi^2 \rangle} \). This mechanism allows PBH formation with \( P_{\text{PBH}} \sim 10^{-9} \) for small field, double-well or axionic potentials. An
important difference with the previous case is that when \( \langle \psi \rangle \ll \sqrt{H_{N}^{2}/4\pi^{2} + \langle \delta \psi^{2} \rangle_{N-1}} \), the PBH probability \( P_{PBH} \) becomes inversely proportional to \( \sqrt{H_{N}^{2}/4\pi^{2} + \langle \delta \psi^{2} \rangle_{N-1}} \), instead of involving an erfc function. As expected, it is roughly determined by the ratio of the width of the slow-roll region to the range explored by the field fluctuations. If \( H(N) \) is not drastically reduced during inflation, this mechanism would overproduce light PBHs. Nevertheless, as in the first case, if stochastic spectator domination does not occur too much before PBH formation, as expected if the field is identified with the QCD axion, this naturally introduces a cut-off at small masses, so the model is viable.

D. Short second phase of inflation

Equations (3.12) and (3.15) give the probability that a region in our Universe will collapse to form a PBH when it re-enters the horizon during the radiation era. A large curvature fluctuation is generated by the short extra expansion induced when the spectator field slowly rolls towards the bottom of its potential. One can use the stochastic \( \delta N \) formalism to link the local curvature fluctuation to this extra expansion, \( \zeta(x) \approx \delta N(x) \), this itself being due to a spectator field fluctuation \( |\Delta \psi| \) during inflation, which remains frozen until the field density dominates. For simplicity, we neglect the impact of the radiation density during the extra expansion and assume that the spectator field evolution respects the slow-roll conditions until \( \psi \) reaches the critical value \( \psi_{cr} \) where the slow-roll parameter \( \epsilon_{\psi} \equiv M_{P}^{2}(V'/V)^{2} \approx 1 \). If \( \psi_{m} \equiv \max(\min)(\psi_{cr}, \psi - \Delta \psi) \), the maximum (minimum) between the slow-roll region and the field value on immediate super-bubble scales, one obtains for an increasing (decreasing) potential

\[
\zeta(x) = \Delta N(x) = \frac{1}{M_{P}^{2}} \int_{\psi_{m}}^{\psi} \frac{V(\chi)}{V'(\chi)} d\chi.
\]

Determining the distribution of curvature fluctuations more accurately would require numerically implementing the stochastic \( \delta N \) formalism and solving the exact field and expansion dynamics for a large number of field trajectories and a given potential. This is left for a future work.

One can get an approximation for \( \zeta(x) \) by assuming that \( \epsilon_{\psi} \) remains constant and that the extra inflation ends abruptly when the field reaches the value \( \psi_{cr} \), such that

\[
\zeta(x) \sim \frac{\min(|\psi - \psi_{cr}|, |\Delta \psi|)}{M_{P} \sqrt{\epsilon_{\psi}(\psi)}}.
\]
Since $|\Delta \psi| \sim H_{\text{inf}}$, the condition for the curvature fluctuation to exceed the threshold for PBH formation is $\epsilon_\psi \sim H_{\text{inf}}^2/M_P^2$, so the spectator field potential must be very flat. Note that slow-roll is violated at $\psi_{\text{cr}} \approx \sqrt{2} M_P$ for the simplest quadratic or quartic potentials, so the stochastic field fluctuations, $\Delta \psi \sim H_{\text{inf}} \lesssim 10^{-5} M_P$, are unable to drive the field in the flat region of the potential where $\epsilon_\psi$ would be low enough to induce a large curvature fluctuation. Instead, one needs plateau-like or small-field (e.g. double-well axionic) potentials in this context. Note also that, since the stochastic field quantum fluctuations are Gaussian, curvature fluctuations much larger than $O(1)$ (i.e. deriving from an extra inflationary phase lasting more than a few e-folds) are exponentially suppressed. In the next section, a more concrete and refined calculation is performed for the particularly interesting case in which the stochastic spectator is identified with the QCD axion.

E. The case of the QCD axion

A natural candidate for the light spectator field during inflation is the QCD axion. Its existence is well-motivated theoretically, since it provides a robust solution to the strong CP problem. The standard model is augmented with an extra pseudo-Goldstone boson, that naturally relaxes the CP violation parameter to zero. We assume that the associated Peccei-Quinn symmetry is spontaneously broken prior to inflation. As discussed in the next section, the QCD axion not only naturally generates the large curvature fluctuations required to produce enough PBHs to explain the dark matter, but it also has all the ingredients required for cold baryongenesis at the time and place of PBH formation without fine-tuning.

The axion potential at temperature $T \lesssim \text{GeV}$ is given by

$$V(a) = m_a^{\text{eff}}(T)^2 f_a^2 \left[1 + \cos \left(\frac{a}{f_a}\right)\right],$$

(3.18)

with an effective axion mass

$$m_a^{\text{eff}}(T) \simeq \begin{cases} m_a^2 \left(\frac{T}{T_c}\right)^{-\alpha} & \text{if } T \gtrsim T_c \approx 100 \text{ MeV} \\ m_a^2 & \text{if } T \lesssim T_c \approx 100 \text{ MeV} \end{cases},$$

(3.19)

where $a$ is the axion field, $m_a$ is the zero temperature axion mass, and $f_a$ is the axion scale that fixes its mass and couplings. As a benchmark, we assume $T_c = 100 \text{ MeV}$ and a power-law index $\alpha = 7$. Possibly $f_a$ could be as high as the GUT scale.
As long as its fluctuations are super-horizon-scale, the axion field obeys the Klein Gordon equation, is frozen by the Hubble damping term and lies very close to the top the potential (i.e. \( a \ll f_a \)) in the patch corresponding to our Universe. At a temperature of about 1 GeV, it quickly rolls down the potential without inflating in most regions of the Universe.

In our scenario, in a small fraction of the regions of the Universe, the axion field lies in the tiny slow-roll region of the potential. In these regions, the axion remains frozen and starts to dominate the density when \( \rho_a \simeq m^2 f_a^2 \gtrsim \rho_r \), which happens at \( \rho_r^{1/4} \sim 100 \) MeV for our benchmark model, just after the QCD transition. This induces in some regions a short extra inflation phase, which produces a large local curvature fluctuation that collapses when it becomes sub-Hubble (immediately after their generation for the QCD axion). Note that the transition to axion domination is relatively sudden due to the strong temperature dependence of the axion mass. The evolution of the axion potential and the radiation density as a function of the temperature is represented in Fig. 3.

Assuming that the slow-roll conditions are satisfied, the end of this short extra-inflation era occurs when \( \epsilon_a = 1 \) and the axion field has a value \[112\]

\[ a_{\text{end}} = f_a \arccos \left( \frac{1 - 2 f_a^2 / M_P^2}{1 + 2 f_a^2 / M_P^2} \right) \simeq f_a \arccos \left( 1 - 4 f_a^2 / M_P^2 \right) \simeq f_a \sqrt{8 \left( \frac{f_a}{M_P} \right)} \ll f_a . \] (3.20)

The last approximations are valid when \( f_a \ll M_P \), which applies at the GUT scale and below. The number of e-folds generated between \( a \) and \( a_{\text{end}} \) is

\[ \Delta N = \frac{f_a^2}{M_P^2} \ln \left[ \frac{1 - \cos(a_{\text{end}}/f_a)}{1 - \cos(a/f_a)} \right] \simeq \frac{f_a^2}{M_P^2} \ln \left( \frac{a_{\text{end}}^2}{a^2} \right) . \] (3.21)

This relation can be inverted and the value of \( a_{\text{end}} \) inserted to give

\[ a = f_a \arccos \left[ 1 - \frac{4 f_a^2}{M_P^2 + 2 f_a^2} \exp \left( - \frac{M_P^2}{f_a^2} \Delta N \right) \right] . \] (3.22)

One needs \( \Delta N = \zeta_{\text{th}} = 1.02 \), so the field range of interest is exponentially suppressed for \( f_a \ll M_P \), so PBH formation is very unlikely for \( H_* \gtrsim 10^{-6} M_P \). But interestingly, the stochastic quantum fluctuations of the axion field, \( a \sim H_{\text{inf}} \), for our three models can produce \( \mathcal{O}(1) \) curvature fluctuations \( [\Delta N \sim \mathcal{O}(1)] \) with a probability \( P_{\text{PBH}} \sim 10^{-9} \) if \( f_a \simeq 0.1 M_P \), i.e. if the Peccei-Quinn symmetry is broken at about the GUT scale. But the exact value of \( f_a \) does not need to be fine-tuned, thanks to the stochasticity of \( \langle a \rangle \). Typically, a slight shift in \( \langle a \rangle \) would enhance or suppress the resulting PBH abundance, allowing it to be tuned to the dark matter density as a selection effect. Nevertheless, a much larger or lower value of
$f_a$ would lead to an exponential suppression or enhancement of large curvature fluctuations, respectively, thereby overproducing or underproducing PBHs.

Note that $\langle a\rangle$ is initially very close to the top of the potential in our Universe, and the axion density is negligible compared to the dark matter density today, so that a value of $f_a$ at the GUT scale is compatible with the current constraints on isocurvature modes and black holes superradiance. In order to solve the strong CP problem, the axion mass in our scenario must be about $m_a \simeq 6\mu\text{eV}(10^{12}\text{GeV}/f_a) \sim 10^{-11}\text{eV}$.

**F. PBH mass distribution**

A black hole is formed when the curvature fluctuation exceeds some threshold $\zeta_{tr}$. The exact value of $\zeta_{tr}$ depends on the equation of state $w = P/\rho$ and has been computed in the spherically symmetric situation analytically by Harada et al. \[108\] and using numerical relativity by Musco and Miller \[109\]. As already mentioned, PBHs in the stellar-mass range form during the QCD cross-over because the sound-speed reduction lowers the curvature

![Graph](https://via.placeholder.com/150.png?text=Graph)
threshold\cite{thresh,thresh2}. Recently, Byrnes et al.\cite{PBHmass} have computed the expected PBH mass function more accurately, based on the latest results of lattice QCD simulations. We adopt their methodology here. If the entire Hubble volume at re-entry collapses to form a PBH, one typically gets a peak in the PBH density at around $2-3M_\odot$. More realistically, the size is fixed by the mass inside the fluctuation at turn-around and both analytic considerations and numerical simulations show that the final PBH mass is only one fifth of the Hubble mass ($\gamma \simeq 0.2$), so the peak is more likely around $0.5M_\odot$.

For Case 1, the PBH mass functions obtained in our three models are shown in Fig. 4, on the assumption that the PBHs provide all the dark matter (i.e. $f_{DM} = 1$). In order to avoid an overproduction of light PBHs, it is essential that either the Hubble rate varies sufficiently during inflation, which is why we require $\epsilon_1 \gtrsim 10^{-3}$, or that the spectator field dominates the energy density of the Universe below the TeV scale. A second (lower) peak is expected within the range $10^{-23} - 30M_\odot$ and this could explain the LIGO/Virgo black holes. For Case 2, with field domination above the GeV scale, the mass function is identical to that in Case 1 of Model 3. However, for the QCD axion scenario, the field dominates at the quark hadron epoch, which induces a cut-off in the mass function, depending on the temperature dependence of the potential. This leads to the PBH mass function represented in Fig. 5. As discussed in Section V, such a mass distribution passes all the current astrophysical and cosmological constraints, at least when one accounts for the astrophysical uncertainties associated with microlensing and supernova lensing observations. Note that - for the QCD axion scenario - the cut-off scale prevents PBH formation below about a solar mass, which could also explain the lack of microlensing events from sub-solar objects.

IV. FINE-TUNING OF THE BARYON ASYMMETRY OF THE UNIVERSE

In this section we argue that the observed baryon asymmetry may have been generated by PBHs in such a way that $\eta$ is naturally driven to $\beta$ for PBH formation at the QCD transition. The large curvature fluctuations, and subsequent gravitational collapse to PBHs upon horizon re-entry, would have provided the out-of-equilibrium condition required for efficient baryogenesis only within those regions, leading to $\eta_{\text{local}} \gtrsim 1$. The plasma would have then homogenized before nucleosynthesis, distributing those baryons to the rest of the universe and naturally explaining why $\eta \sim \beta$. 
FIG. 4: Top: Collapse fraction of PBHs at formation, assuming the equation of state parameter during the QCD cross-over given in Ref. [56], for the three inflation models and an efficiency factor $\gamma = 0.2$. [Here $\epsilon_\psi$ is chosen so that the PBHs provide all the dark matter and the spectator field potential belongs to Case 1.] Bottom: corresponding PBH mass function, $f_{DM} \equiv d\beta/d\ln M$, the vertical lines representing the mass of PBHs formed at different temperatures. The mass function expected in the Byrnes et al. model [56] with a nearly scale-invariant spectrum of curvature fluctuations with spectral index $n_s = 0.96$ [and $\gamma = 0.2$] is also shown.
FIG. 5: Possible PBH mass function in the QCD axion scenario [or for potentials belonging to Case 2], for the three considered inflation models and, in the case of Model 2, for different values of the energy cut-off scale, 200 MeV, 140 MeV and 80 MeV (from left to right). The equation of state during the QCD cross-over given in Ref. 56, and the PBH efficiency factor is \( \gamma = 0.2 \).

A. Entropy production at PBH formation

The collapse of a large curvature fluctuation to PBH at horizon reentry is an extremely violent and far-from-equilibrium process. It is also responsible for a huge production of gravitational entropy, which we now evaluate. The entropy of the gas of relativistic particles within the horizon in the early universe can be written as \((k_B = 1)\)

\[
S_{\text{gas}} = \frac{2\pi^2}{45} g_* S(T) T^3 V_H,
\]

where \(V_H = \frac{32\pi}{(3 c^3 t^3)}\) is the horizon volume in the radiation era. On the other hand, the gravitational entropy of a PBH formed from the gravitational collapse of this volume is

\[
S_{\text{PBH}} = \frac{c^3 A}{4\hbar G} = \frac{4\pi G}{\hbar c} M^2 \simeq 4\pi \gamma^2 \left( \frac{t}{t_p} \right)^2,
\]

where \(A\) is the black hole area and the PBH mass is given by Eq. (2.3). The ratio between these two quantities depends on the time of PBH formation,

\[
\frac{S_{\text{PBH}}}{S_{\text{gas}}} = \gamma^2 \left( \frac{810}{32\pi} \right)^{1/2} \frac{g_*^{1/2}(T) M_P}{g_* S(T) T} \simeq 0.9 \times 10^{20} \gamma^2 \frac{200 \text{ MeV}}{T},
\]
so a PBH formed at the QCD epoch is responsible for an entropy increase which is huge compared with the entropy of the particles themselves. Even though this provides an upper bound on the available entropy increase, the process described below requires significantly less entropy production and does not require all the gravitational degrees of freedom of the PBH.

**B. Energy production at PBH formation**

Since the gravitational collapse is not 100% efficient, a large fraction of the mass within the horizon is expelled away from the PBH by conservation of linear momentum. The expelled particles are accelerated and acquire a kinetic energy equivalent to the difference in potential energy before and after the collapse. Since the radius of the PBH at formation is a factor $\gamma$ smaller than the horizon size, the total amount of kinetic energy released by the collapse into the remaining gas of relativistic particles is

$$K \approx \left(1 - \frac{1}{\gamma}\right) \frac{c^3}{G}.$$  \hspace{1cm} (4.4)

This is equivalent to several solar masses of energy at the QCD epoch. The smaller the value of $\gamma$, the more the energy at our disposal.

Now we can estimate the kinetic energy per particle generated by the collapse. The kinetic energy will be distributed equally among all degrees of freedom. If we concentrate on the most massive particles, the newly created protons, the fraction of energy transferred into protons and antiprotons at $T \sim 200$ MeV (taking into account the degrees of freedom of photons, neutrinos, electrons, muons and pions) is

$$2 \times 2/(2 + 3 \times 2 + 2 \times 2 + 2 \times 2 + 3) = 4/19 \approx 1/5,$$

while their number density is that of a non-relativistic particle (for $70 < T < 200$ MeV, after the QCD transition and before proton freeze-out, as we are assuming),

$$n_p(x) = 2 \left(\frac{m_p T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_p}{T}\right) = 1.59 \times 10^{40} x^{-3/2} e^{-x} \text{ cm}^{-3},$$  \hspace{1cm} (4.5)

with $x = m_p/T$. Therefore, the kinetic energy per nucleon that each proton and antiproton within the horizon has acquired from the collapse of the PBH is

$$K/N_p \sim 140 \ x^{-5/2} \ e^x \text{ GeV},$$  \hspace{1cm} (4.6)
where we have assumed $\gamma \simeq 0.2$. From Eq. (4.4), the fraction is $(1-\gamma)/\gamma$ for lower efficiencies. The density of the relativistic plasma surrounding the collapsed horizon is huge at that time,

$$n_{\text{gas}} = 1.64 \times 10^{41} \text{ cm}^{-3}. \quad (4.7)$$

These relativistic particles constitute the target onto which protons collide, accelerated by the violent collapse of the PBH, in a similar way to that of present accelerators although at immensely larger densities. For example, at $T = 120$ MeV, immediately after the QCD quark-hadron transition, the density of protons is $3 \times 10^{35}$ cm$^{-3}$ and the kinetic energy per proton is 2 TeV, smashing into a wall of $3.4 \times 10^{38}$ particles per cm$^3$. At those energies and densities, there is copious production of W bosons within a Hubble time of 40 $\mu$s, via cross-sections of order the microbarn.

C. Hot spot electroweak baryogenesis at the QCD epoch

PBH formation via the gravitational collapse of a dense relativistic gas of particles in the early Universe is an extremely violent process, with the kinetic energies of protons and antiprotons being well above the plasma temperature producing a high density gas of gauge bosons. Although this process occurs while the rest of the universe is well below the electroweak scale, the plasma is extremely hot locally and the rate of events is significantly enhanced with respect to the surrounding thermal state. The horizon that collapses is a hot fireball where high energy sphaleron transitions can take place very far from equilibrium, similar to the conditions achieved with heavy ion collisions in high energy colliders but at much larger energies and densities. (A somewhat related scenario was proposed in Ref. [86] in the context of low-scale reheating after inflation.) We will show that these conditions are enough to generate the observed Baryon Asymmetry of the Universe (BAU).

According to Sakharov [87], baryogenesis requires three ingredients: (1) baryon number violation; (2) C and CP violation; and (3) out-of-equilibrium conditions to avoid any acquired asymmetry being washed out. CP violation in the Standard Model (SM) is realized in the hadronic sector via the complex phases of the CKM matrix

$$V_{\text{CKM}} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}. \quad (4.8)$$
The amount of CP violation is proportional to the Jarlskog determinant and given by

$$\delta_{\text{CP}}(T) = \frac{J}{T^{12}} \simeq \left(\frac{20.4 \text{ GeV}}{T}\right)^{12} K,$$

with

$$J = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_d^2)(m_d^2 - m_d^2) \cdot K$$

$$K = \text{Im} V_{ij} V_{ji}^* = s_1^2 s_2 c_1 c_2 c_3 \sin \delta = (3.06 \pm 0.2) \times 10^{-5}.$$  

Since $m_t = 172$ GeV, $m_b = 4.5$ GeV, $m_c = 1.27$ GeV and $m_s = 0.96$ GeV, we find $J/K = (20.4 \text{ GeV})^{12}$, as indicated in Eq. (4.9), so $\delta_{\text{CP}}$ is extremely temperature-sensitive.

At the classical level, the baryon and lepton symmetries are accidentally conserved in the SM. However, the chiral anomaly implies that the currents are not conserved at the quantum level:

$$\partial_\mu j_\mu^B = \partial_\mu j_\mu^L = 3 \alpha_W \pi F_{\mu\nu} \tilde{F}^{\mu\nu} \implies \Delta B = \Delta L = 3 \Delta N_{\text{CS}},$$

where the Chern-Simmons number $N_{\text{CS}}$ characterizes the different electroweak (EW) vacua and corresponds to the Higgs windings around its potential. Each winding generates a three unit baryon number jump.

The sphaleron rate $\Gamma_{\text{sph}}$ describes the rate per unit time and volume at which long-wavelength configurations wrap around the SM false vacuum and make transitions from one Chern-Simmons number to the next. This induces the baryon number violation given by Eq. (4.12). The rate depends very strongly on temperature:

$$\Gamma_{\text{sph}}(T) \sim \begin{cases} \alpha_W^4 T^4, & T > 200 \text{GeV}, \\ \text{const.} \left(\frac{E_{\text{sph}}}{T}\right)^3 m_W^4(T) e^{-\frac{E_{\text{sph}}}{T}}, & T < 200 \text{GeV}, \end{cases}$$

with $E_{\text{sph}} \simeq 2 m_W / \alpha_W$, $m_W^4(T) = \pi \alpha_W (v^2(T) + T^2)$ with $v(T) = v (1 - T^2/12v^2)$ and $v = 245$ GeV being the Higgs vacuum expectation value at zero temperature.

CP violation enters the dynamics through an effective operator:

$$\mathcal{O} = \frac{3 \alpha_W}{8 \pi} \theta F_{\mu\nu} \tilde{F}^{\mu\nu},$$

which induces an effective chemical potential for baryon production,

$$\mu_{\text{eff}} = \delta_{\text{CP}}(T) \frac{d\theta}{dt},$$
with $\Delta\theta \sim \pi$ for each jump in $N_{\text{CS}}$ and the CP-violation dimensionless parameter given by Eq. (4.9). The out-of-equilibrium evolution of the baryon number density $n_b$ can be described by an approximate Boltzmann equation,

$$\frac{dn_B}{dt} + \Gamma_B n_B = \Gamma_{\text{sph}} \frac{\mu_{\text{eff}}}{T_{\text{eff}}}.$$  (4.16)

Here $\Gamma_B = \frac{39}{2} \Gamma_{\text{sph}}(T_{\text{th}})/T_{\text{eff}}^3$ would be responsible for erasing the baryon density after baryon production, but since the plasma surrounding the PBH collapse has a significantly lower temperature, the sphaleron transitions are quenched immediately. As long as the right-hand side of Eq. (4.16) is large, the approximate solution for $T_{\text{eff}} \gg T_{\text{th}}$ is

$$n_b = \int dt \Gamma_{\text{sph}}(t) \frac{\mu_{\text{eff}}}{T_{\text{eff}}} \simeq \Gamma_{\text{sph}}(T_{\text{eff}}) \frac{\delta_{\text{CP}}}{T_{\text{eff}}} \Delta \theta.$$  (4.17)

This assumes that the sphaleron rate is dominant during the hot phase of the fireball expansion after the PBH collapse, at $T_{\text{eff}} \lesssim 500$ GeV, while the CP violation is produced in the diffusion of those quarks and leptons through the surrounding thermal plasma towards a temperature around $T_{\text{th}} = 70$ MeV. The entropy density at the end of this process is

$$s = \frac{2\pi^2}{45} g_s S(T_{\text{th}}) T_{\text{th}}^3$$  (4.18)

with $T_{\text{th}} \sim 70$ MeV and $g_s S(T_{\text{th}}) = 10.75$. Therefore

$$\eta = \frac{n_B}{n_{\gamma}} \simeq \frac{7n_B}{s} \simeq \frac{315}{2\pi^2 g_s S} \frac{\Gamma_{\text{sph}}(T_{\text{eff}})}{T_{\text{eff}} T_{\text{th}}^3} \delta_{\text{CP}}(T_{\text{th}}) \Delta \theta,$$  (4.19)

which is very large for $T_{\text{eff}} \sim 500$ GeV and $\Delta \theta \sim 1$. Thus it is possible to generate a large BAU at the QCD scale, $\eta_{\text{local}} \gg 1$ using only SM ingredients. This means that the regions around the pockets that are collapsing to form PBHs are saturated with baryons, so that the local photon number can be highly suppressed. There is no need for any tuning, as long as the effective and thermal temperatures are low enough to induce $\eta_{\text{local}} \gtrsim 1$ locally.

The final value of the BAU, $\eta \sim 10^{-9}$, arises from the fact that the BAU is produced initially only in the hot spots around horizon domains that have collapsed to PBHs and then radiated away to the rest of the universe. If the BAU occurs at the QCD scale (i.e. $t_{\text{QCD}} \sim 10^{-5}$ s) and a fraction $\beta \sim 10^{-9}$ of all horizon volumes become PBHs, then the typical distance between PBH domains where the BAU has been generated is

$$d \sim \beta^{-1/3} d_H(t_{\text{QCD}}) \sim 10^{-2} \text{ light-seconds}.$$  

Moving at the speed of sound, $c_s \simeq c/\sqrt{3}$, baryons diffuse until they uniformly distribute the localized BAU to the rest of the Universe well before BBN ($t_{\text{BBN}} \sim 1 - 180$ s). This explains the origin of the relation $\eta \sim \beta$.  

26
The model also explains why the baryonic and dark matter densities are comparable. In fact, it may even predict the precise value, \( \chi \simeq 5.5 \simeq 30\gamma \). The reason is that only PBHs formed below the 100 MeV scale will acquire enough kinetic energy to induce efficient baryogenesis \( (\eta \gtrsim 1) \). For the PBH mass spectrum induced by the sound-speed reduction during the QCD transition, only a few percent of the PBH density is formed at a temperature below 100 MeV; this is consistent with the required fraciton \( \sim 1/30 \).

V. ASTROPHYSICAL CONSTRAINTS AND GRAVITATIONAL WAVES

The question of whether solar-mass PBHs can explain all or only a fraction of the DM is still open and actively debated. In this section we discuss this important issue in the context of our PBH formation scenario. Motivated by the unexpectedly large masses and low effective spins of the black holes detected by LIGO/Virgo, the constraints on PBHs in the mass range \( 1 - 100M_\odot \) have recently been improved, e.g. from the dynamical heating of ultra-faint dwarf galaxies (UFDGs) and their star clusters \([88, 91]\), from anisotropies in the cosmic microwave background \([20, 21, 92]\), from the non-observation of X-ray and radio sources towards the Galactic center \([93, 94]\) or in the interstellar medium \([95]\). These constraints complement previous ones from the non-disruption of wide stellar binaries in the galactic halo \([96, 97]\).

If the PBHs have a monochromatic mass distribution, then they could account for no more than 10% of the DM in the mass range \( 1 - 100M_\odot \). Recently these constraints have been re-analyzed for an extended mass function \([8, 11]\) - in particular, for the lognormal mass distribution expected in many inflationary scenarios \([35, 98]\) - and the most stringent ones then come from the stability of compact UFDGs and the central star cluster in Eridanus II \([89, 90]\). These can be more or less stringent, depending on whether or not there is a central intermediate-mass black hole, but they exclude PBHs heavier than \( 10M_\odot \) from providing most of the DM. Also, all these constraints depend on questionable astrophysical assumptions, such as the galactic halo profile, the efficiency of PBH accretion, the PBH halo mass function etc.

Our scenario predicts that all the DM comprises PBHs but with the wide mass distribution shown in Fig. 4. This peaks around \( 0.5M_\odot \) and about 90% of the DM is in PBHs between 0.1 and \( 2M_\odot \). Below \( 0.1M_\odot \) and above \( 10M_\odot \), PBHs contribute no more than a
few percent of the DM, so one evades all the above constraints. The LIGO/Virgo black
holes can also be explained as coming from the tail of the PBH distribution, as suggested
in Ref. [12]. The merging rates required are compatible with PBHs forming binaries by
capture in haloes at late times if PBHs provide all the DM [24]. The merging rate of PBH
binaries formed before matter-radiation equality does not allow more than 1% of the DM for
a monochromatic mass function [30] but the rate is suppressed for a wide-mass distribution
because of binary disruption by surrounding PBHs [33]. In our model, the merging rates
of PBHs heavier than $10M_\odot$ is additionally suppressed, since they contribute only a few
percent of the DM.

The abundance of sub-solar PBH is also constrained by the LIGO/Virgo limits on the
merging rates of sub-solar equal-mass binaries [100, 101] but again this applies only for a
monochromatic distribution. For a wide distribution, PBH mergers will only rarely have
equal masses, so the rate will be spread out in the progenitor mass space and could be also
impacted by early disruptions.

In our model, the peak is at around a solar mass, which is in the range probed only by
star and supernovae microlensing constraints. However, these constraints are less stringent
- and may even allow 100% of the DM - if one uses more realistic DM profiles [35, 62] or if
the PBHs are clustered [65]. The supernova constraints [102] are particularly controversial
since the current analysis does not include some astrophysical uncertainties [103].

Any conclusion regarding the contribution of PBHs to the DM depends crucially on
the reliability of all these constraints. Taking into account the current uncertainties, it is
still possible that all the DM comprises PBHs with an extended mass function of the kind
predicted in our scenario. Upcoming microlensing and supernovae surveys can clearly probe
our model - microlensing events in M31 [104, 105] are relevant here and new results from
the OGLE survey are expected very soon - as are searches for sub-solar-mass black holes
with gravitational wave experiments.

VI. FINE-TUNING OF DARK MATTER AND BARYON ABUNDANCES

The scenario proposed in this paper explains why the PBH collapse fraction is of order
the baryon-to-photon ratio and why the PBH density should be of the order of the baryon
density. It does not explain the actual values of these quantities ($\beta \sim \eta \sim 10^{-9}$) but it does
allow $\beta$ and $\eta$ to take the observed values in a large number of Hubble patches as a result of the stochasticity of the spectator field during inflation. We can then invoke an anthropic selection argument because - independent of the nature of the dark matter - there are various constraints on the value of $\eta$ required in order that galaxies can form [71]. These constraints are sometimes described as “anthropic” but in the present case they are really no more than a selection effect. In any case, any constraint on $\eta$ implies an equivalent constraint on $\beta$ in our model, so in this sense there are anthropic aspects to the PBH collapse fraction.

The most important constraint concerns the closeness of the times of matter-radiation equality ($t_{eq}$) and photon decoupling ($t_{dec}$). This corresponds to $\eta$ being roughly $\alpha^4$, where $\alpha \equiv e^2/(\hbar c) \sim 10^{-2}$ is the electric fine structure constant. A larger dark matter density would increase the gap between $t_{eq}$ and $t_{dec}$. We now discuss this argument in more detail, expanding on the analysis originally given in Ref. [71]. The energy density and number density for black-body photons are

$$\rho_\gamma = a_S T^4, \quad n_\gamma = \frac{30\zeta(3) a_S T^3}{\pi^4 k_B} = 3.7 a_S T^3 / k_B \Rightarrow \rho_\gamma = 0.27 n_\gamma k_B T,$$

(6.1)

where $a_S$ is the black-body Stephan-Boltzmann constant. The photon entropy density and entropy per baryon are respectively

$$s = 0.37 n_\gamma, \quad \sigma = \frac{4 a_S T^3}{3 n_b k_B} = 0.37 \eta^{-1},$$

(6.2)

with the baryon to photon ration given by Eq. (2.7). The photon temperature evolves as

$$k_B T(z) = k_B T_0 (1 + z) = 2.5 \times 10^{-13} m_p (1 + z),$$

(6.3)

so the redshift, density and temperature at matter-radiation equality are related by

$$\rho_m = \left(\frac{1 + z_{eq}}{1 + z} \right) \rho_T = \frac{T_{eq}}{T} \rho_t,$$

(6.4)

where $\rho_m$ is the total matter density (including any dark component). If the ratio of the dark matter density (including any PBHs) to the baryon density is denoted by $\chi$, then the total matter density is $1 + \chi$ times the baryon density, so at matter-radiation equality we have

$$\rho_m = (1 + \chi) n_b m_p = \rho_t = 0.27 n_\gamma T_{eq},$$

(6.5)

which implies

$$T_{eq} \approx 3.7(1 + \chi) m_p \approx 2 \times 10^4 (1 + \chi) K,$$

(6.6)
using Eq. (2.7). The Saha equation implies that the temperature at decoupling is less than the H-ionization energy by a factor of one hundred. Since the ionization energy is $\alpha^2 m_e c^2$, where $m_e$ is the electron mass, this gives

$$T_{\text{dec}} \sim 0.01 \alpha^2 m_e \sim 0.1 \alpha^4 m_p \sim 10^3 \text{K},$$

(6.7)

where we have used the relation $m_e/m_p \approx 10 \alpha^2$ (required for chemistry). From Eq. (6.6) the coincidence $T_{\text{eq}} \sim T_{\text{dec}}$ corresponds to the condition

$$\eta \sim 0.5 (1 + \chi)^{-1} \alpha^4 \sim 5 \times 10^{-9} (1 + \chi)^{-1}.$$  

(6.8)

Alternatively, from Eq. (6.6), the time of matter-radiation equality is

$$t_{\text{eq}} \sim (1 + \chi)^{-2} \eta^{-2} \alpha_G^{-1/2} t_p \sim 10^{12} (1 + \chi)^{-2} \text{s}.$$  

(6.9)

After $t_{\text{eq}}$, eqn (2.4) is replaced by

$$T \sim T_{\text{eq}} (t/t_{\text{eq}})^{-2/3} \sim (1 + \chi)^{-1/3} \eta^{-1/3} \alpha_G^{-1/3} m_p (t_p/t)^{2/3},$$

(6.10)

so from Eq. (6.7) the time of decoupling is

$$t_{\text{dec}} \sim 10 (1 + \chi)^{-1/2} \eta^{-1/2} \alpha_G^{-1/2} \alpha^{-6} t_p \sim 10^{13} (1 + \chi)^{-1/2} \text{s}.$$  

(6.11)

The coincidence $t_{\text{eq}} \sim t_{\text{dec}}$ therefore corresponds to the condition (6.8), as expected. It is unexplained by standard physics.

In our scenario, $\chi$ is naturally of order one because both the baryons and PBHs originate from the same collapsing curvature fluctuations. Given the observed value of $\chi$, the epoch of matter-radiation equality and decoupling are further apart for $\eta \sim \beta \ll 10^{-9}$. Equality will take place after decoupling, increasing the diffusion damping scale and the Silk mass much beyond the size and mass of galaxy clusters, so this would erase fluctuations and suppress the formation of such structures. In the opposite case, $\eta \sim \beta \gg 10^{-9}$, the diffusion damping scale is reduced, which would boost the formation of structures of the size of dwarf galaxies and below. However PBHs would accrete most of the baryonic matter in less than one billion years, and the Universe would be composed only of PBH. This argument was recently used as an attempt to explain the large mass-to-light ratio of ultra-faint dwarf galaxies [12].

30
There is another anthropic constraint on the value of $\eta$. By the Dicke anthropic argument \[106\], the current age of the Universe must be of order the main-sequence time of a star, which implies

$$t_0 \sim 100 \alpha_G^{-1} t_p \sim 10^{17} \text{s}.$$ \hfill (6.12)

For life to arise, we require this time to exceed both $t_{\text{eq}}$ and $t_{\text{dec}}$, which implies

$$\eta > \max[(1 + \chi)^{-1} \alpha_G^{1/4}, 100 (1 + \chi)^{-1} \alpha_G \alpha^{-12}],$$ \hfill (6.13)

where the first term is the larger and of order $10^{-10}$. For the Universe to be radiation-dominated at BBN, we require

$$\eta < \alpha_G^{-1/4} (t_p/t_{\text{NS}})^{1/2} \sim 10^{-4}.$$ \hfill (6.14)

where $t_{\text{NS}}$ is the time of cosmological nucleosynthesis. All the above arguments are equivalent to the ones in Ref. \[71\], apart from the presence of the $1 + \chi$ term. In this context, we stress that it is always the combination $(1 + \chi) \eta$ which appears, indicating that it is the entropy per dark matter particle rather than entropy per baryon which is relevant.

VII. CONCLUSION

The early universe can be used as a probe of fundamental physics at much higher energies than those explored in particle accelerators today. We can extrapolate the fundamental interactions of the Standard Model of particle physics to the dense and hot early universe and see whether there are the necessary conditions for the BAU to develop. In the scenario that we have here proposed, the BAU is generated at the violent process of PBH formation during the quark-hadron transition, triggered by the sudden drop in the radiation pressure, in the presence of large amplitude curvature fluctuations. Baryon number violation is driven by out-of-equilibrium sphaleron processes that are immediately quenched by the surrounding plasma in the expanding universe, preventing baryon wash-out, while the only CP violation needed is that of the CKM phases of the standard model. Moreover, the same small fraction of domains that act as hot spots for the efficient production of baryons is responsible for the present low value of the BAU and the dominance of PBH over other forms of matter, while at the same time explaining why baryons and dark matter have similar densities today.
It is interesting how this scenario resolves two of the more acute problems of cosmology, the origin of the baryon asymmetry and the nature of dark matter, in one go. Rather than relying on new particle physics interactions at high energy to generate the baryon asymmetry simultaneously on all locations in the universe, this scenario suggests it occurs only locally, on just a few rare domains, during the violent gravitational collapse associated with the formation of primordial black holes, and is later radiated (diffused) to the rest of the universe. The connection between the rareness of those domains, responsible for a late matter domination (thus leaving enough time for the subsequent stellar evolution and structure formation), and the low baryon-to-photon ratio is a completely new way of approaching the problem. Dark matter (in the form of PBH) and baryons are then linked together, explaining their order-one relative ratio. If LIGO-Virgo interferometers map out in the next few years the mass distribution of coalescing black holes and turns out to be like that of Fig. 5, then we will conclude that the QCD epoch played a crucial role in the evolution of the universe, generating at the same time the matter-antimatter asymmetry and the dark matter, setting the stage for subsequent primordial nucleosynthesis, stellar evolution and structure formation.

Without any parameter fine-tuning, our scenario explains why PBHs should have an abundance comparable to the dark matter in our Universe, why the dark matter density is comparable to the baryon density, and why the baryon-to-photon ratio is of order $10^{-9}$. The different fine-tunings are replaced by a single anthropic selection argument associated with the formation of galaxies. The quantum fluctuations of a light stochastic spectator field during inflation, which are the basis of our anthropic argument, provide the rare super-horizon curvature fluctuations that are produced during a short transient phase well after inflation, when the field dominates the density of the Universe. These fluctuations collapse into solar-mass PBHs at the quark-hadron epoch, also producing all the conditions required to generate a strong baryon asymmetry through electroweak baryogenesis, which then quickly diffuses to the entire Universe. An important feature of this scenario is that such curvature fluctuations are only local, and thus highly non-Gaussian, whereas the curvature fluctuations in the rest of the Universe remain Gaussian and follow the predictions of standard slow-roll inflation. This avoids the need for an enhancement in the primordial power spectrum on some scale, which has long been considered unnatural and indeed one of the principal argument against PBHs. Note that the existence of any light spectator field
during inflation, as long as its density exceeds the QCD scale, inevitably leads to Hubble patches in which PBHs and baryons are formed with the observed relative abundances.

Several possible observable predictions of our model have been considered. If the spectator field potential is of the plateau-type with $V^{1/4}$ being above the TeV scale, some variation of the Hubble rate during inflation is needed to avoid an overproduction of light PBHs. This generates a tensor-to-scalar ratio $r \gtrsim 0.08$, which would be in tension with current CMB observations and easily detectable with upcoming ones. If $100 \text{ MeV} \lesssim V^{1/4} \lesssim 1 \text{ TeV}$, this condition is relaxed and - for an inflation scale $H_{\text{inf}} \lesssim 10^{-6} \bar{M}_P$ - the generic PBH mass function is different from that expected for a nearly scale-invariant power-spectrum enhancement (see Fig. 4). Another notable difference is the existence a low-mass cut-off that depends on the energy scale at which the field starts to dominate the density of the Universe. Stellar or quasar microlensing searches and sub-solar PBH searches with gravitational wave interferometers are thus ideal for testing and distinguishing between the different scenarios. If the potential is of the double-well type, the probability of PBH formation is also modified, thereby changing the mass function, as indicated in Fig. 5.

Finally, we propose that the QCD axion plays the role of the spectator field. In this case, the strong temperature dependence of the potential induces a cut-off below the solar-mass scale, which could explain the lack of microlensing events from sub-solar compact objects. The high-mass tail of the PBH distribution is naturally suppressed because of the equation of state evolution through the QCD transition and this could explain the merger rates, spins and masses of the LIGO/Virgo black holes.

We have addressed various fine-tuning issues in this paper and in concluding we summarize the connection between them:

* The similarity of the dark matter (PBH) and baryon densities today ($\chi \approx 6$) is unexplained in most models of PBH formation. This ratio is constant after baryogenesis and PBH formation but its actual value is unspecified and could either be very large or very small. In our model, we expect $\chi \sim 1$ because the local baryon asymmetry generated around each PBH is $\mathcal{O}(1)$ and we may even be able to predict its rough order of magnitude.

* The usual criticism of the PBH dark matter proposal is that it requires fine-tuning of the PBH collapse fraction $\beta$. This needs to be tiny but not too tiny. Given the sensitivity of $\beta$ to the amplitude of the fluctuations, one would expect the current PBH density to be either negligible or huge, leaving too few baryons to make galaxies. However, in our proposal
the collapse fraction is necessarily of order $\eta$ because the baryon asymmetry is generated by the PBHs. It is $O(1)$ locally around each domain that collapses to PBH, but reduced precisely by the factor $\beta$ after the asymmetry has diffused throughout the Universe.

As discussed in Section VI, there are long-standing tunings involving the baryon-to-photon ratio $\eta \sim 10^{-9}$. For example, it needs to be more than $\alpha^{-1/4}_G \sim 10^{-10}$ to ensure the lifetime of stars exceeds the time of matter-radiation equality but less than $\alpha^{-1/4}_G (t_\text{p}/t_{\text{NS}})^{1/2} \sim 10^{-4}$ to avoid all the Universe going into helium at cosmological synthesis. The comparability of the times of decoupling and matter-radiation equality requires the condition $\eta \sim \alpha^4$, although the precise form of this relation depends on the amount of dark matter. These conditions might be interpreted anthropically.

We have not addressed the other well-known fine-tuning problem: the comparability of the dark energy and dark matter densities ($\Omega_{\text{DE}}/\Omega_{\text{CDM}} \approx 3$), this only applying at a particular epoch. This has recently been addressed by Tzikas et al. [113] by invoking a cosmological model in which the number of effective spatial dimensions is reduced from three to one at early times. However, this proposal is not compatible with our own since the QCD epoch is long after the $1 + 1$ phase.

In the present paper, we have presented a broad outline of our scenario and a more quantitative approach is required to derive accurate observational predictions. Possible refinements would include a more accurate description of the stochastic dynamics of the spectator field during inflation, the computation of the exact spectator field dynamics after inflation with the $\delta N$ stochastic formalism, the details of the black hole collapse to extract a more accurate value of the efficiency factor $\gamma$, and a more accurate derivation of the dark matter to baryon ratio $\chi$ in our model of hot-spot electroweak baryogenesis. We should also consider in more details some concrete realizations of our scenario, when the spectator field is embedded in a high-energy physics framework.

There could also be interesting observational consequences related to the fact that there could exist extremely rare but large curvature fluctuations on cosmological scales. For example, these could help explain the cold spot observed in the CMB, or the existence of large underdense voids. These rare but non-linear cosmic inhomogeneities might even be used to mimic the dark energy. Our scenario could thus provide a new framework to explain the same order of magnitude of the dark energy and dark matter densities today.
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