Abstract

In the present paper, firstly the nature of zero-truncated Poisson distribution (ZTPD), zero-truncated Poisson-Lindley distribution (ZTPLD) and zero-truncated Poisson-Sujatha distribution (ZTPSD) have been discussed with graphs of their probability functions for different values of their parameter. Over-dispersion, equi-dispersion and under-dispersion of ZTPLD and ZTPSD have been discussed using index of dispersion. A simple and interesting method of finding moments of ZTPSD has been suggested and thus the first two moments about origin and the variance have been obtained. The estimation of parameter of ZTPD, ZTPLD and ZTPSD has been discussed using maximum likelihood estimation and method of moments.

The goodness of fit of ZTPD, ZTPLD, and ZTPSD using maximum likelihood estimate in zero-truncated data arising from demography, biological sciences and migration has been discussed and observed that in most data-sets relating to demography and biological sciences, ZTPSD gives better fit than ZTPD and ZTPLD.

Keywords: zero-truncated distribution, poisson-lindley distribution, poisson-sujatha distribution, moments, estimation of parameter, goodness of fit

Introduction

Zero-truncated distributions, in probability theory, are certain discrete distributions having support the set of positive integers. These distributions are applicable for the situations when the data to be modeled originate from a mechanism that generates data excluding zero-counts.

Let \( P(x; \theta) \) is the original distribution with support non negative positive integers. Then the zero-truncated version of \( P(x; \theta) \) with the support of positive integers is given by

\[
P(x; \theta) = \frac{P(x; \theta)}{1 - P(0; \theta)}, \quad x = 1, 2, 3, ...
\]  

(1.1)

The Poisson-Lindley distribution (PLD) having probability mass function (p.m.f.)

\[
P_0(x; \theta) = \frac{\theta^2 (x+\theta+2)}{(\theta+1)^{x+3}}, \quad x = 0, 1, 2, 3, ..., \theta > 0
\]  

(1.2)

has been introduced by Sankaran\(^1\) to model count data. Recently, Shanker & Hagos\(^2\) have done an extensive study on its applications to Biological Sciences and found that PLD provides a better fit than Poisson distribution to almost all biological science data. The PLD arises from the Poisson distribution when its parameter \( \lambda \) follows Lindley\(^3\) with probability density function (p.d.f.)

\[
g(\lambda; \theta) = \frac{\theta^3}{\theta+1} (1+\lambda)^{-\lambda}; \lambda > 0, \theta > 0
\]  

(1.3)

Detailed study of Lindley distribution (1.3) has been done by Ghitany et al.,\(^4\) and shown that (1.3) is a better model than exponential distribution for modeling some lifetime data. Recently, Shanker et al.,\(^5\) showed that (1.3) is not always a better model than the exponential distribution for modeling lifetime’s data. In fact, Shanker et al.,\(^6\) has done a very extensive and comparative study on modeling of lifetime data using exponential and Lindley distributions and discussed various lifetime data-sets to show the superiority of Lindley over exponential and that of exponential over Lindley distribution. The PLD has been extensively studied by Sankaran\(^1\) and Ghitany & Mutairi\(^5\) and its various properties have been discussed by them. The Lindley distribution and the PLD have been generalized by many researchers. Shanker & Mishra\(^7\) obtained a two parameter Poisson-Lindley distribution by compounding Poisson distribution with a two parameter Lindley distribution introduced by Shanker & Mishra.\(^8\) A quasi Poisson-Lindley distribution has been introduced by Shanker & Mishra by compounding Poisson distribution with a quasi Lindley distribution introduced by Shanker & Mishra.\(^9\) Shanker et al.,\(^10\) obtained a discrete two parameter Poisson-Lindley distribution by mixing Poisson distribution with a two parameter Lindley distribution for modeling waiting and survival times data introduced by Shanker et al.,\(^11\) Further, Shanker & Tekie\(^12\) obtained a new quasi Poisson-Lindley distribution by compounding Poisson distribution with a new quasi Lindley distribution introduced by Shanker & Amanuel.\(^13\)

Recently Shanker\(^14\) has obtained Poisson-Sujatha distribution (PSD) having p.m.f.

\[
P_0(x; \theta) = \frac{\theta^3}{\theta^2+2} \left(\theta^3+3\theta^2+4\right)^{-\theta-1}; \quad x = 0, 1, 2, 3, ..., \theta > 0
\]  

(1.4)

to model count data in different fields of knowledge. The PSD arises from the Poisson distribution when its parameter \( \lambda \) follows Shanker R\(^1\) with probability density function (p.d.f.)

\[
g(\lambda; \theta) = \frac{\theta^3}{\theta^2+2} (1+\lambda)^{-\lambda}; \lambda > 0, \theta > 0
\]  

(1.5)

Detailed discussion about its various properties, estimation of the parameter and applications for modeling lifetime data has been mentioned in Shanker\(^14\) and shown by Shanker\(^15\) that (1.5) is a better model than the exponential and Lindley\(^1\) distributions for modeling lifetime data. Shanker & Hagos\(^16,18\) has also obtained the size-biased and zero-truncated version of PSD and discussed their properties.
estimation of parameter and applications in different fields of knowledge.

In this paper, the nature of zero-truncated Poisson distribution (ZTPD), zero-truncated Poisson-Lindley distribution (ZTPLD) and zero-truncated Poisson-Sujatha distribution has been compared and studied using graphs for different values of their parameter. A simple method for obtaining the moments of ZTPSD has been suggested and the first two moments about origin and variance have been obtained. ZTPD, ZTPLD and ZTPSD have been fitted to a number of data-sets from demography and biological sciences to study their goodness of fit and superiority of one over the others.

Zero-truncated poisson, poisson-lindley and poisson-sujatha distributions

Zero-truncated poisson distribution (ZTPD)

Using (1.1) and the p.m.f. of Poisson distribution, the p.m.f. of zero-truncated Poisson distribution (ZTPD) given by

$$P_r(x;\theta) = \frac{\theta^x}{e^\theta - 1} \cdot \Theta x = 1, 2, 3, ..., \theta > 0 \quad (2.1.1)$$

was obtained independently by Plackett and David & Johnson to model count data excluding zero counts. An extension of a truncated Poisson distribution and estimation in a truncated Poisson distribution when zeros and some ones are missing has been discussed by Ghitany et al., to model count data for the missing zeros.

Shanker et al., have done extensive study on the comparison of ZTPD and ZTPLD with respect to their applications in data sets excluding zero-counts and showed that in demography and biological sciences ZTPD gives better fit than ZTPLD while in social sciences ZTPLD gives better fit than ZTPD.

Zero-truncated poisson-lindley distribution (ZTPLD)

Using (1.1) and (1.2), the p.m.f. of zero-truncated Poisson-Lindley distribution (ZTPLD) given by

$$P_r(x;\theta) = \frac{\theta^x}{\theta^2 + 3\theta x + (\theta + 1)^2} \cdot \Theta x = 1, 2, 3, ..., \theta > 0 \quad (2.2.1)$$

was obtained by Ghitany et al., to model count data for the missing zeros.

Shanker et al., have done extensive study on the comparison of ZTPD and ZTPLD with respect to their applications in data sets excluding zero-counts and showed that in demography and biological sciences ZTPD gives better fit than ZTPLD while in social sciences ZTPLD gives better fit than ZTPD.

Zero-Truncated Poisson-Sujatha Distribution (ZTPSD)

Using (1.1) and (1.4), the p.m.f. of zero-truncated Poisson-Sujatha distribution (ZTPSD) can be obtained as

$$P_r(x;\theta) = \frac{\theta^x}{\theta^4 + 4\theta^3 + 10\theta^2 + 7\theta + 2} \cdot \Theta x = 1, 2, 3, ..., \theta > 0 \quad (2.3.1)$$

The ZTPSD can also arise from the size-biased Poisson distribution (SBPD) with p.m.f.

$$g(x|\lambda) = \frac{\lambda^x}{(x-1)!} \cdot \Theta x = 1, 2, 3, ..., \lambda > 0 \quad (2.3.2)$$

When its parameter $\lambda$ follows a distribution having p.d.f.

$$h(\lambda;\theta) = \frac{\theta^3}{\theta^4 + 4\theta^3 + 10\theta^2 + 7\theta + 2} \left[ \left( \theta + 1 \right) \lambda^2 + \left( \theta + 3 \right) \lambda + \left( \theta^2 + 3\theta + 4 \right) \right] e^{-\theta \lambda} \quad (2.3.3)$$

The p.m.f. of ZTPSD is thus can be obtained as

$$P(x;\theta) = \int_0^{\infty} \frac{\theta^3}{\theta^4 + 4\theta^3 + 10\theta^2 + 7\theta + 2} \left[ \left( \theta + 1 \right) \lambda^2 + \left( \theta + 3 \right) \lambda + \left( \theta^2 + 3\theta + 4 \right) \right] e^{-\theta \lambda} d\lambda \quad (2.3.4)$$

Moments and related measures of ztpsd

The $r^{th}$ moment about origin of ZTPSD (2.3.1) can be obtained as

$$\mu_r = E \left[ X^r \right] = \frac{\theta^3}{\theta^4 + 4\theta^3 + 10\theta^2 + 7\theta + 2} \sum_{r=1}^{\infty} \frac{\lambda^r}{(x-1)!} \left[ \left( \theta + 1 \right) \lambda^2 + \left( \theta + 3 \right) \lambda + \left( \theta^2 + 3\theta + 4 \right) \right] e^{-\theta \lambda} d\lambda \quad (3.1)$$

Clearly the expression under the bracket in (3.1) is the $r^{th}$ moment about origin of the SBPD. Taking $r = 1$ in (3.1) and using the first moment about origin of the SBPD, the first moment about origin of the ZTPSD (1.3) can be obtained as

$$\mu_1 = \frac{\theta^3}{\theta^4 + 4\theta^3 + 10\theta^2 + 7\theta + 2} \left[ \left( \theta + 1 \right) \lambda^2 + \left( \theta + 3 \right) \lambda + \left( \theta^2 + 3\theta + 4 \right) \right] e^{-\theta \lambda} d\lambda \quad (3.2)$$

Citation: Shanker R, Fesshaye H. On zero-truncation of poisson, poisson-lindley and poisson-sujatha distributions and their applications. Biom Biostat Int J. 2016;3(4):125–135. DOI: 10.15406/bbij.2016.03.00072
Again taking $r=2$ in (3.1) and using the second moment about origin of the SBPD, the second moment about origin of the ZTPSD (2.3.1) can be obtained as

$$
\mu_2 = \frac{\theta^4}{\theta^2 + 4\theta^3 + 10\theta^2 + 7\theta + 2} d\lambda e^{-\theta \lambda} \lambda^2 + (\theta + 1) \lambda^2 + (\theta + 3) \lambda (3.3.3)
$$

Similarly, taking $r=3$ and $4$ in (3.1) and using the respective moments of SBPD, the third and the fourth moment about origin of ZTPSD can be obtained. The variance of ZTPSD (2.3.1) is thus obtained as

$$
\mu_2 = \frac{\theta^4 + 10\theta^3 + 58\theta^2 + 210\theta + 503\theta^3 + 760\theta^2 + 686\lambda^3 + 352\theta^2 + 96\theta + 12}{\theta^2 (\theta^4 + 4\theta^3 + 10\theta^2 + 7\theta + 2)}
$$

The index of dispersion of ZTPSD (2.3.1) is given by
It can be easily verified that the ZTPSD is over dispersed \( \mu<\theta^2 \), equi-dispersed \( \mu=\theta^2 \) and under dispersed \( \mu>\theta^2 \) for \( \theta=(\mu) \theta^* \approx 1.548328 \) respectively. It is to noted that ZTPLD is over dispersed \( \mu<\theta^2 \), equi-dispersed \( \mu=\theta^2 \) and under dispersed \( \mu>\theta^2 \) for \( \theta=(\mu) \theta^* \approx 1.258627 \) respectively.

### Estimation of the parameter

#### Estimation of parameter of ZTPD

Let \( x_1,x_2,\ldots,x_n \) be a random sample of size \( n \) from the ZTPD (2.1.1). The maximum likelihood estimate (MLE) and method of moment estimate (MOME) of \( \theta \) of ZTPD (2.1.1) is given by the solution of the following non linear equation

\[
e^\theta(\bar{x}-\theta)-\bar{x}=0,
\]

where \( \bar{x} \) is the sample mean.

#### Estimation of parameter of ZTPLD

**Maximum Likelihood Estimate (MLE):**

The maximum likelihood estimate \( \hat{\theta} \) of \( \theta \) of ZTPLD is the solution of the following non-linear equation

\[
\frac{2n}{\hat{\theta}^2} n(2\theta+3) \bar{x} - n \sum_{x=1}^{x} x f_x = 0
\]

Where \( \bar{x} \) is the sample mean. This non-linear equation can be solved by any numerical iteration methods such as Newton-Raphson, Bisection method, Regula-Falsi method etc.

**Method of Moment Estimate (MOME):**

Equating the population mean to the corresponding sample mean, the method of moment estimate (MOME) \( \hat{\theta} \) of \( \theta \) of ZTPSD is the solution of the following non-linear equation

\[
(1-\bar{x})\theta^3 + (3\bar{x}-4)\theta^2 + (\bar{x}-2)\theta - 2 = 0, \quad \bar{x} > 1,
\]

where \( \bar{x} \) is the sample mean.

### Applications and goodness of fit

In this section, an attempt has been made to test the suitability of ZTPD, ZTPLD and ZTPSD in describing the neonatal deaths as well as of infant and child deaths experienced by mothers. The data-sets considered here are the data of Sri Lanka and India. The data-sets of Meegama have been used as the data of Sri Lanka whereas the data from the survey conducted by Lal and the survey of Kadam, Patna, conducted in 1975 and referred to by Mishra have been used as the data of India. Further, ZTPD, ZTPLD, and ZTPSD have also been fitted to data on migration. It is obvious from the fitting of ZTPD, ZTPLD and ZTPSD that ZTPSD and ZTPLD are competitive distributions for modeling data from demography (Table A1-A8).

### Table A1

| Number of Neonatal Deaths | Observed number of Mothers | Expected Frequency |
|---------------------------|----------------------------|--------------------|
|                           | ZTPD                       | ZTPLD              | ZTPSD              |
| 1                         | 409                        | 399.7              | 408.1              | 408.2 |
| 2                         | 88                         | 102.3              | 89.4               | 89.2  |
| 3                         | 19                         |                     |                    |       |
| 4                         | 5                          |                     | 19.3               | 19.3  |
| 5                         | 1                          |                     |                    |       |
| Total                     | 522                        | 522.0              | 522.0              | 522.0 |

**ML Estimate**

| \( \hat{\theta} = 0.512047 \) |
| \( \hat{\theta} = 4.199697 \) |
| \( \hat{\theta} = 4.655303 \) |

| d.f. | P-value |
|------|---------|
| 1    | 0.0627  |
| 2    | 0.9301  |
| 2    | 0.945   |

**Citation:** Shanker R, Fesshaye H. On zero-truncation of poisson, poisson-lindley and poisson-sujatha distributions and their applications. *Biom Biostat Int J.* 2016;3(4):125–135. DOI: 10.15406/bbij.2016.03.00072
| Number of Neonatal Deaths | Observed number of Mothers | Expected Frequency |
|---------------------------|---------------------------|--------------------|
|                           |                           | ZTPD   | ZTPLD  | ZTPSD  |
| 1                         | 71                        | 66.5   | 72.3   | 72.1   |
| 2                         | 32                        | 35.1   | 28.4   | 28.6   |
| 3                         | 7                         |        |        |        |
| 4                         | 5                         |        | 10.9   | 10.9   |
| 5                         | 3                         |        |        |        |
| Total                     | 118                       | 118.0  | 118.0  | 118.0  |

ML Estimate
θ = 2.049609
θ = 0.696
θ = 2.274
θ = 2.215

| d.f. | P-value |
|------|---------|
| 1    | 0.4041  |
| 2    | 0.3208  |
| 2    | 0.3303  |

Table A3: The number of mothers of the urban area with at least two live births by the number of infant and child deaths

| Number of Infant and Child Deaths | Observed number of Mothers | Expected Frequency |
|----------------------------------|---------------------------|--------------------|
|                                  |                           | ZTPD   | ZTPLD  | ZTPSD  |
| 1                                | 176                       | 164.3  | 171.6  | 171.6  |
| 2                                | 44                        | 61.2   | 51.3   | 51.3   |
| 3                                | 16                        |        |        |        |
| 4                                | 6                         |        | 15     | 15.1   |
| 5                                | 2                         |        |        |        |
| Total                            | 244                       | 244.0  | 244.0  | 244.0  |

ML Estimate
θ = 0.744522
θ = 3.366836
θ = 1.882
θ = 1.869

| d.f. | P-value |
|------|---------|
| 1    | 0.0069  |
| 2    | 0.3902  |
| 2    | 0.3927  |

Table A4: The number of mothers of the rural area with at least two live births by the number of infant and child deaths

| Number of Infant and Child Deaths | Observed number of Mothers | Expected Frequency |
|----------------------------------|---------------------------|--------------------|
|                                  |                           | ZTPD   | ZTPLD  | ZTPSD  |
| 1                                | 745                       | 708.9  | 738.1  | 738.1  |
| 2                                | 212                       | 255.1  | 214.8  | 214.9  |
| 3                                | 50                        | 61.2   | 61.3   | 61.3   |

Citation: Shanker R, Fesshaye H. On zero-truncation of poisson, poisson-lindley and poisson-sujatha distributions and their applications. Biom Biostat Int J. 2016;3(4):125–135. DOI: 10.15406/bbij.2016.03.00072
### Table A5 The number of literate mothers with at least one live birth by the number of infant deaths

| Number of Infant Deaths | Observed number of Mothers | Expected Frequency |
|-------------------------|----------------------------|--------------------|
|                         |                            | ZTPD               |
| 1                       | 683                        | 659.0              |
| 2                       | 145                        | 177.4              |
| 3                       | 29                         | 34.6               |
| 4                       | 11                         | 34.6               |
| 5                       | 5                          | 873.0              |
| Total                   | 873                        | 873.0              |

ML Estimate

\[
\hat{\theta} = 0.538402 \quad \hat{\theta} = 4.00231 \quad \hat{\theta} = 4.462424
\]

\[
8.718 \quad 5.310 \quad 5.463
\]

d.f. 1 2 2

P-value 0.0031 0.0703 0.0651

### Table A6 The number of mothers of the completed fertility having experienced at least one child death

| Number of Child Deaths | Observed number of Mothers | Expected Frequency |
|------------------------|----------------------------|--------------------|
|                        |                            | ZTPD               |
| 1                      | 89                         | 76.8               |
| 2                      | 25                         | 39.9               |
| 3                      | 11                         | 32.3               |
| 4                      | 6                          | 32.5               |
| 5                      | 3                          | 12.2               |
| 6                      | 1                          | 12.2               |
| Total                  | 135                        | 135.0              |

Citation: Shanker R, Fesshaye H. On zero-truncation of poisson, poisson-lindley and poisson-sujatha distributions and their applications. Biom Biostat Int J. 2016;3(4):125–135. DOI: 10.15406/bbij.2016.03.00072
### Table A7: The number of mothers having at least one neonatal death

| Number of Neonatal Deaths | Observed number of Mothers | Expected Frequency |  |
|---------------------------|----------------------------|--------------------|---|
|                           |                            | ZTPD               | ZTPLD | ZTPSD |
| 1                         | 567                        | 545.8              | 561.4 | 561.5 |
| 2                         | 135                        | 162.5              | 139.7 | 139.5 |
| 3                         | 28                         | 32.3               | 34.2  | 34.2  |
| 4                         | 11                         |                    |       |       |
| 5                         | 5                          |                    |       |       |
| Total                     | 746                        | 746.0              | 746.0 | 746.0 |

ML Estimate:
- \( \hat{\theta} = 0.595415 \)
- \( \hat{\theta} = 3.625737 \)
- \( \hat{\theta} = 1.539511 \)

- d.f.: 2
- P-value: 0.1467

### Table A8: Number of households having at least one migrant according to the number of migrants, reported by Singh & Yadav

| Number of Migrants | Observed Frequency | Expected Frequency |  |
|--------------------|--------------------|--------------------|---|
|                    |                    | ZTPD               | ZTPLD | ZTPSD |
| 1                  | 375                | 354.0              | 379.0 | 378.3 |
| 2                  | 143                | 167.7              | 137.2 | 137.8 |
| 3                  | 49                 | 53.0               | 48.4  | 48.7  |
| 4                  | 17                 |                    |       |       |
| 5                  | 2                  |                    |       |       |
| 6                  | 2                  | 16.8               | 16.8  |       |
| 7                  | 1                  |                    |       |       |
| 8                  | 1                  |                    |       |       |
| Total              | 590                | 590.0              | 590.0 | 590.0 |

ML Estimate:
- \( \hat{\theta} = 0.947486 \)
- \( \hat{\theta} = 2.722929 \)

- d.f.: 2
- P-value: 0.8225

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Citation: Shanker R, Fesshaye H. On zero-truncation of poisson, poisson-lindley and poisson-sujatha distributions and their applications. Biom Biostat Int J. 2016;3(4):125–135. DOI: 10.15406/bbij.2016.03.00072
Biological sciences

In this section, an attempt has been made to test the goodness of fit of ZTPD, ZTPLD and ZTPSD on many data-sets relating to biological sciences and it is obvious from the fitting of these distributions that ZTPSD gives much closer fit than ZTPD and ZTPLD in almost all cases. Thus in biological sciences ZTPSD is a better model than ZTPD and ZTPSD to model zero-truncated count data (Table B1-B6).

Table B1 Number of European red mites on apple leaves, reported by Garman

| Number of European Red Mites | Observed Frequency | Expected Frequency |
|-----------------------------|--------------------|--------------------|
|                             | ZTPD               | ZTPLD              | ZTPSD              |
| 1                           | 38                 | 28.7               | 36.1               | 35.5               |
| 2                           | 17                 | 25.7               | 20.5               | 20.8               |
| 3                           | 10                 | 15.3               | 11.2               | 11.5               |
| 4                           | 9                  |                    |                    |                    |
| 5                           | 3                  |                    |                    |                    |
| 6                           | 2                  | 5.9                | 6.1                |                    |
| 7                           | 1                  |                    |                    |                    |
| 8                           | 0                  |                    |                    |                    |
| Total                       | 80                 | 80.0               | 80.0               | 80.0               |

ML Estimate

\[ \hat{\theta} = 1.791615 \]
\[ \hat{\theta} = 1.539511 \]

9.827 2.467 2.444

d.f. 2 3 3

P-value 0.0073 0.4854

Table B2 Number of yeast cell counts observed per mm square, reported by Student

| Number of Yeast Cells Counts per mm Square | Observed Frequency | Expected Frequency |
|-------------------------------------------|--------------------|--------------------|
|                                           | ZTPD               | ZTPLD              | ZTPSD              |
| 1                                         | 128                | 121.3              | 127.6              | 127.4              |
| 2                                         | 37                 | 49.2               | 40.9               | 41                 |
| 3                                         | 18                 |                    |                    |                    |
| 4                                         | 3                  |                    |                    |                    |
| 5                                         | 1                  |                    |                    |                    |
| 6                                         | 0                  |                    |                    |                    |
| Total                                     | 187                | 187.0              | 187.0              | 187.0              |

ML Estimate

\[ \hat{\theta} = 0.811276 \]
\[ \hat{\theta} = 3.115436 \]

5.228 1.034 1.013

d.f. 1 1 1

P-value 0.0222 0.3141

Citation: Shanker R, Fesshaye H. On zero-truncation of poisson, poisson-lindley and poisson-sujatha distributions and their applications. Biom Biostat Int J. 2016;3(4):125–135. DOI: 10.15406/bbij.2016.03.00072
Table B3 The number of leaf spot grade of Ichinose variety of Mulberry, reported by Khurshid

| Number of Leaf Spot Grade | Observed Frequency | Expected Frequency | 
|---------------------------|--------------------|--------------------|
|                           |                    | ZTPD | ZTPLD | ZTPSD |
| 1                         | 18                 | 14.2 | 23.0  | 21.7  |
| 2                         | 15                 | 18.7 | 16.3  | 16.5  |
| 3                         | 10                 | 16.5 | 11.1  | 11.6  |
| 4                         | 14                 | 10.9 | 7.3   | 7.8   |
| 5                         | 13                 | 9.7  | 12.3  | 12.4  |
| Total                     | 70                 | 70.0 | 70.0  | 70.0  |
| ML Estimate               |                    | $\hat{\theta} = 2.639984$ | $\hat{\theta} = 0.781902$ | $\hat{\theta} = 1.056454$ |
| d.f.                      |                    | 6.311 | 7.476 | 5.943 |
| P-value                   |                    | 0.0974 | 0.0582 | 0.1144 |

Table B4 The number of leaf spot grade of Kokuso-20 variety of Mulberry, reported by Khurshid

| Number of Leaf Spot Grade | Observed Frequency | Expected Frequency | 
|---------------------------|--------------------|--------------------|
|                           |                    | ZTPD | ZTPLD | ZTPSD |
| 1                         | 37                 | 28.5 | 36.7  | 35.9  |
| 2                         | 16                 | 26.7 | 21.4  | 21.8  |
| 3                         | 15                 | 16.7 | 12.0  | 12.4  |
| 4                         | 8                  | 6.6  | 6.8   | 6.8   |
| 5                         | 8                  | 7.3  | 7.1   | 7.1   |
| Total                     | 84                 | 84.0 | 84.0  | 84.0  |
| ML Estimate               |                    | $\hat{\theta} = 1.874567$ | $\hat{\theta} = 1.130211$ | $\hat{\theta} = 1.470098$ |
| d.f.                      |                    | 8.329 | 2.477 | 2.446 |
| P-value                   |                    | 0.0155 | 0.4795 | 0.4851 |

Citation: Shanker R, Fesshaye H. On zero-truncation of poisson, poisson-lindley and poisson-sujatha distributions and their applications. Biom Biostat Int J. 2016;3(4):125–135. DOI: 10.15406/bbij.2016.03.00072
Table B5 The number of counts of sites with particles from Immunogold data reported by Mathews & Appleton

| Number of Sites with Particles | Observed Frequency | Expected Frequency |
|-------------------------------|--------------------|--------------------|
|                              |                    | ZTPD | ZTPLD | ZTPSD |
| 1                             | 122                | 115.9 | 124.8 | 124.4 |
| 2                             | 50                 | 57.4  | 46.8  | 47.0  |
| 3                             | 18                 | 18.9  | 17.1  | 17.2  |
| 4                             | 4                  | 4     |       |       |
| 5                             | 4                  | 4     |       |       |
| Total                         | 198                | 198.0 | 198.0 | 198.0 |
| ML Estimate                   | \( \hat{\theta} = 0.990586 \) | \( \hat{\theta} = 2.18307 \) | \( \hat{\theta} = 2.614691 \) |
| d.f.                          | 2                  | 2     | 2     |
| P-value                       | 0.3430             | 0.7749 | 0.7945 |

Table B6 The number of snowshoe hares counts captured over 7 days, reported by Keith & Meslow

| Number of Times Hares Caught | Observed Frequency | Expected Frequency |
|------------------------------|--------------------|--------------------|
|                              |                    | ZTPD | ZTPLD | ZTPSD |
| 1                            | 184                | 176.6 | 182.6 | 182.6 |
| 2                            | 55                 | 66.0  | 55.3  | 55.3  |
| 3                            | 14                 | 16.4  |       | 16.4  |
| 4                            | 4                  | 4     |       |       |
| 5                            | 4                  | 4     |       |       |
| Total                        | 261                | 261.0 | 261.0 | 261.0 |
| ML Estimate                  | \( \hat{\theta} = 0.756171 \) | \( \hat{\theta} = 3.320063 \) |
| d.f.                         | 1                  | 2     | 2     |
| P-value                      | 0.1175             | 0.7371 | 0.7501 |

**Concluding remarks**

In this paper, the nature and behavior of ZTPD, ZTPLD and ZTPSD have been studied by drawing different graphs of their probability functions for the different values of their parameter. A very simple and easy method for finding moments of ZTPSD has been suggested. An attempt has been made to study the goodness of fit of ZTPD, ZTPLD and ZTPSD to count data relating to demography and biological sciences and it has been observed that ZTPSD is a better model than the ZTPD and ZTPLD in almost all data-sets relating to mortality, migration and biological sciences.

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Conflict of interest
None.

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