CPT AND LORENTZ VIOLATION IN NEUTRAL-MESON OSCILLATIONS

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The status of CPT tests with neutral mesons is reviewed in the context of quantum field theory and the Lorentz- and CPT-violating standard-model extension.

1 Introduction

Local relativistic quantum field theories, including the standard model of particle physics, are known to be invariant under Lorentz and CPT transformations. This symmetry is consistent with the results of numerous sensitive laboratory tests. Although no definitive violation has been discovered to date, there are many reasons to undertake careful theoretical studies of possible mechanisms and descriptions of Lorentz and CPT violation. One basic motivation is that a comparative and quantitative interpretation of the numerous experimental tests requires a comprehensive theoretical framework within which violations are both allowed and internally consistent. A more ambitious motivation is that suppressed Lorentz and CPT violation might arise from a fundamental theory at the Planck scale but nonetheless be observable with existing technology in experiments of exceptional sensitivity.

At the 1998 Bloomington conference on CPT and Lorentz symmetry, I discussed the possibility that Lorentz and CPT symmetry might be broken by physical effects arising in a theory underlying the standard model, including string theory. I also described the general standard-model extension allowing Lorentz and CPT violation and summarized some of the experiments that had already been performed to test it at that time. In the intervening three years, substantial advances have been made on both the theoretical and experimental fronts, many of which are discussed in other presentations at this meeting. In particular, experimental tests of the standard-model extension now include studies of neutral-meson oscillations, comparative tests of QED in Penning traps, spectroscopy of hydrogen and antihydrogen, measurements of muon properties, clock-comparison experiments, tests with spin-polarized matter, measurements of cosmological birefringence, studies of neutrinos, and observations of the baryon asymmetry. These experiments

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measure coefficients for Lorentz and CPT violation in the standard-model extension and are probing the Planck scale.

This talk focuses on the theoretical issues involving tests of the standard-model extension using neutral-meson oscillations. Meson interferometry is a sensitive tool for both CPT and Lorentz violation. Any indirect CPT violation in a neutral-meson system can be parametrized with a complex quantity, denoted in this talk by $\xi_P$, where $P$ is one of the neutral mesons $K$, $D$, $B_d$, $B_s$. The talk outlines the formalism involving $\xi_P$, describes the calculation of $\xi_P$ in the general Lorentz-violating standard-model extension, and briefly considers some implications for experiment. Reports on the latest experimental results in the $K$, $D$, and $B_d$ systems are being presented separately at this conference. The reader may also find of interest some related recent analyses of possible classical analogues for CPT violation in neutral mesons which fall outside the scope of this talk.

2 Setup

Any neutral-meson state is a linear combination of the Schrödinger wave functions for the meson $P^0$ and its antimeson $\overline{P^0}$. If this state is viewed as a two-component object $\Psi(t)$, its time evolution is controlled by a $2\times2$ effective Hamiltonian $\Lambda$ according to the Schrödinger-type equation

$$i\partial_t \Psi = \Lambda \Psi.$$

Note that the effective Hamiltonian is different for each neutral-meson system, but for simplicity a single symbol is used here.

The eigenstates of $\Lambda$ are the physical propagating states of the neutral-meson system, denoted here as $|P_a\rangle$ and $|P_b\rangle$. These states develop in time according to

$$|P_a(t)\rangle = \exp(-i\lambda_a t)|P_a\rangle, \quad |P_b(t)\rangle = \exp(-i\lambda_b t)|P_b\rangle,$$

as usual. The complex parameters $\lambda_a$, $\lambda_b$ in these equations are the eigenvalues of $\Lambda$, and they are comprised of the physical masses $m_a$, $m_b$ and decay rates $\gamma_a$, $\gamma_b$ of the propagating particles:

$$\lambda_a \equiv m_a - \frac{1}{2}i\gamma_a, \quad \lambda_b \equiv m_b - \frac{1}{2}i\gamma_b.$$

For practical purposes, it is convenient to work instead with the sum and difference of the eigenvalues, defined as

$$\lambda \equiv \lambda_a + \lambda_b = m - \frac{1}{2}i\gamma,$$

$$\Delta \lambda \equiv \lambda_a - \lambda_b = \Delta m - \frac{1}{2}i\Delta \gamma.$$
In these equations, $m = m_a + m_b$, $\Delta m = m_b - m_a$, $\gamma = \gamma_a + \gamma_b$, and $\Delta \gamma = \gamma_a - \gamma_b$.

Since the effective hamiltonian is a $2 \times 2$ complex matrix, it consists of eight independent real quantities for each meson system. Four of these can be specified in terms of the masses and decay rates. Three of the others determine the extent of indirect CP violation in the neutral-meson system. If (and only if) the difference $\Delta \Lambda \equiv \Lambda_{11} - \Lambda_{22}$ of diagonal elements of $\Lambda$ is nonzero, then the meson system exhibits indirect CP violation. Also, indirect T violation occurs if (and only if) the magnitude of the ratio $|\Lambda_{21}/\Lambda_{12}|$ of the off-diagonal components of $\Lambda$ differs from 1. The effective hamiltonian thus contains two real parameters for CPT violation and one real parameter for T violation. The remaining parameter of the eight in $\Lambda$ can be taken as the relative phase between the off-diagonal components of $\Lambda$. It is physically irrelevant because it can be freely changed by shifting the phases of the $P^0$ and $\bar{P}^0$ wave functions by equal and opposite amounts. Such shifts are allowed because the wave functions are strong-interaction eigenstates. If the $P^0$ wave function is shifted by a phase factor $\exp(i\chi)$, the off-diagonal elements of $\Lambda$ shift by equal and opposite phases $\exp(\pm 2i\chi)$.

3 Formalism

For applications to the heavy-meson systems, where less is known about CPT and T violation than in the $K$ system, it is desirable to adopt a general parametrization of the effective hamiltonian $\Lambda$ that is independent of phase conventions, valid for arbitrary size CPT and T violation, model independent, and expressed in terms of mass and decay rates insofar as possible. An analysis shows that a practical parametrization permitting the clean representation of CPT- and T-violating quantities can be obtained by expressing the two diagonal elements of $\Lambda$ as the sum and difference of two complex numbers, and the two off-diagonal elements as the product and ratio of two other complex numbers. A general expression for $\Lambda$ can therefore be taken as:

$$\Lambda = \frac{1}{2} \Delta \lambda \begin{pmatrix} U + \xi & VW^{-1} \\ VW & U - \xi \end{pmatrix},$$

where $UVW\xi$ are complex numbers that are dimensionless by virtue of the prefactor $\Delta \lambda$. Imposing that the trace of $\Lambda$ is $\text{tr} \Lambda = \lambda$ and that its determinant is $\det \Lambda = \lambda_a \lambda_b$ fixes the complex parameters $U$ and $V$:

$$U \equiv \lambda/\Delta \lambda, \quad V \equiv \sqrt{1 - \xi^2}. \quad (6)$$
The CPT and T properties of the effective Hamiltonian \( H \) are contained in the complex numbers \( W = w \exp(i\omega), \xi = \text{Re} \xi + i\text{Im} \xi \). Of the four real components, the argument \( \omega \) of \( W \) is physically irrelevant and can be freely dialed by the wave-function phase shifts described above. The remaining three components are physical, with \( \text{Re} \xi \) and \( \text{Im} \xi \) governing CPT violation and the modulus \( w = |W| \) of \( W \) governing T violation. They are related to the components of \( \Lambda \) by

\[
\xi = \frac{\Delta \lambda}{\Delta \lambda}, \quad w = \sqrt{|\Lambda_{21}/\Lambda_{12}|}.
\]

If CPT is preserved \( \text{Re} \xi = \text{Im} \xi = 0 \), while if T is preserved \( w = 1 \).

The eigenstates of \( \Lambda \), which are the physical states of definite masses and decay rates, can be written as

\[
|P_a\rangle = N_a (|P^0\rangle + A|\bar{P}^0\rangle), \quad |P_b\rangle = N_b (|P^0\rangle + B|\bar{P}^0\rangle),
\]

with

\[
A = (1 - \xi)W/V, \quad B = -(1 + \xi)W/V.
\]

If unit-normalized states are desired, the normalizations \( N_a, N_b \) in Eq. (8) take the form

\[
N_a = \exp(i\eta_a)/\sqrt{1 + |A|^2}, \quad N_b = \exp(i\eta_b)/\sqrt{1 + |B|^2},
\]

where \( \eta_a, \eta_b \) are free phases that play no role in what follows. For the special case with no CPT or T violation \( (\xi = 0, w = 1) \), the states \( |P_a\rangle, |P_b\rangle \) are CP eigenstates. If the choice of phase convention \( \omega = \eta_a = \eta_b = 0 \) is imposed, Eq. (8) reduces to the usual form, \( |P_{a,b}\rangle = (|P^0\rangle \pm |\bar{P}^0\rangle)/\sqrt{2} \).

As an aside, note that the \( w\xi \) formalism above can be related to other formalisms used in the literature provided appropriate assumptions about the phase conventions and the smallness of CP violation are made. For instance, in the \( K \) system the widely adopted formalism involving \( \epsilon_K \) and \( \delta_K \) is phase-convention dependent and can be applied only if CPT and T violation are small. Under the assumption of small violation and in a special phase convention, \( \delta_K \) is related to \( \xi_K \) by \( \delta_K \approx 2\xi_K \).

For the heavy meson systems \( D, B_d, B_s \), the \( w\xi \) formalism appears simpler to use than other formalisms. The three parameters for CP violation \( w, \text{Re} \xi, \text{Im} \xi \) are dimensionless and independent of assumptions about the size of violations or about the choice of phases. Since they are phenomenologically introduced, they contain no model dependence. However, it is crucial to note that they need not be constant numbers. In fact, as outlined in the
next section, the assumption of constant $\xi$ often adopted for experimental and theoretical analyses represents a strong constraint on the generality of the formalism. Moreover, according to the CPT theorem, the assumption of constant $\xi$ is inconsistent with the underlying basis of Lorentz-invariant quantum field theory. If instead Lorentz violation is allowed within quantum field theory, then $\xi$ is found to vary with the meson 4-momentum. Although this may seem surprising at first sight, in fact unconventional behavior for $\xi$ is to be expected because CPT violation is a fundamental effect.

4 Theory

The standard-model extension provides a general quantitative microscopic framework in the context of conventional quantum field theory within which to study various effects of Lorentz and CPT violation. As noted above, many experiments with systems other than neutral mesons have been performed to measure coefficients in this theory. However, to date none of these experiments has sensitivity to the same sector of the standard-model extension as neutral-meson oscillations, basically because only the latter involve flavor changes.

The dominant CPT-violating contributions to $\Lambda$ can be calculated perturbatively in the coefficients for CPT and Lorentz violation that appear in the standard-model extension. These contributions are expectation values of perturbative interactions in the hamiltonian for the theory, evaluated with unperturbed wave functions $|P^0 \rangle$, $|\overline{P}^0 \rangle$ as usual. The hermiticity of the perturbation hamiltonian guarantees real contributions.

To find an expression for the parameter $\xi$, one needs to derive the difference $\Delta\Lambda = \Lambda_{11} - \Lambda_{22}$ of the diagonal terms of $\Lambda$. A calculation yields

$$\Lambda \approx \beta^\mu \Delta a_\mu,$$

where $\beta^\mu = \gamma(1, \vec{\beta})$ is the four-velocity of the meson state in the observer frame. In this equation, $\Delta a_\mu = r_{q_1} a^{q_1}_\mu - r_{q_2} a^{q_2}_\mu$, where $a^{q_1}_\mu$, $a^{q_2}_\mu$ are coefficients for CPT and Lorentz violation for the two valence quarks in the $P^0$ meson. They have mass dimension one, and they arise from lagrangian terms of the form $-a^q_\mu \overline{q} \gamma^\mu q$, where $q$ specifies the quark flavor. The quantities $r_{q_1}$, $r_{q_2}$ emerge from normalization and quark-binding effects.

Among the consequences of Lorentz and CPT violation are the 4-velocity and hence 4-momentum dependence appearing in Eq. (11). These establish the failure of the standard assumption of constant parameter $\xi$ for CPT violation. In particular, the appearance of the 4-velocity implies that CPT observables will typically vary with the magnitude and orientation of the meson momentum. This can have major consequences for experimental analyses, since the
meson momentum spectrum and angular distribution now contribute directly in determining the experimental CPT reach.\(^{10\,\text{a},\,11\,\text{a},\,12\,\text{a}}\)

A crucial effect of the 4-momentum dependence is the appearance of sidereal variations in some CPT observables.\(^{10\,\text{a},\,11\,\text{a},\,12\,\text{a}}\) The point is that the vector \(\Delta \vec{a}\) is constant, while the Earth rotates in a celestial equatorial frame. Since a laboratory frame is adopted for the derivation of Eq. (11), and since this frame is rotating, observables can exhibit sidereal variations. To display explicitly this sidereal-time dependence, one can convert the expression (11) for \(\Delta \Lambda\) from the laboratory frame to a nonrotating frame. Denote the spatial basis in the laboratory frame by \((\hat{x}, \hat{y}, \hat{z})\) and that in the nonrotating frame by \((\hat{X}, \hat{Y}, \hat{Z})\). Choose the \(\hat{z}\) axis in the laboratory frame for maximal convenience: for example, the beam direction is a natural option for the case of collimated mesons, while the collision axis could be adopted in a collider. Define the nonrotating-frame basis \((\hat{X}, \hat{Y}, \hat{Z})\) to be consistent with celestial equatorial coordinates, with \(\hat{Z}\) aligned along the Earth’s rotation axis. Assume \(\cos \chi = \hat{z} \cdot \hat{Z}\) is nonzero, as required for the observation of sidereal variations. It follows that \(\hat{z}\) precesses about \(\hat{Z}\) with the Earth’s sidereal frequency \(\Omega\). The complete transformation between the two bases is in the literature.\(^{17\,\text{a}}\) In particular, any coefficient \(\vec{a}\) for Lorentz violation with laboratory-frame components \((a^1, a^2, a^3)\) has associated nonrotating-frame components \((a^X, a^Y, a^Z)\). This transformation determines the sidereal variation of \(\Delta \vec{a}\) and hence of \(\Delta \Lambda\). The entire momentum and sidereal-time dependence of the parameter \(\xi\) for CPT violation in any \(P\) system can then be extracted.

To express the final answer for \(\xi\), define \(\theta\) and \(\phi\) to be standard polar coordinates about the \(\hat{z}\) axis in the laboratory frame. These angles reduce to the usual detector polar coordinates if the \(\hat{z}\) axis is chosen along the detector axis. In general, the laboratory-frame 3-velocity of a \(P\) meson can be written as \(\vec{\beta} = \beta(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)\). The magnitude of the momentum is given by \(p \equiv |\vec{p}| = \beta m \gamma(p)\), where \(\gamma(p) = \sqrt{1 + p^2/m^2_p}\) as usual. In terms of these quantities and the sidereal time \(\hat{t}\), the result for \(\xi\) becomes:\(^{12\,\text{a}}\)

\[
\xi \equiv \xi(\hat{t}, \vec{p}) \equiv \xi(\hat{t}, p, \theta, \phi) = \frac{\gamma(p)}{\Delta \lambda} \{ \Delta a_0 + \beta \Delta a_Z (\cos \theta \cos \chi - \sin \theta \cos \phi \sin \chi) \\
+ \beta [\Delta a_Y (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) - \Delta a_X \sin \theta \sin \phi] \sin \Omega \hat{t} \\
+ \beta [\Delta a_X (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) + \Delta a_Y \sin \theta \sin \phi] \cos \Omega \hat{t} \}. \tag{12}
\]
Experimental Tests

The experimental challenge is to measure the four independent coefficients $\Delta a_\mu$ for CPT violation allowed by quantum field theory. The result (12) shows that suitable binning of data in sidereal time, momentum magnitude, and orientation has the potential to extract four independent bounds from any observable that depends on $\xi$. Note that each neutral-meson system can have different values of these coefficients. Since the physics of each system is distinct by virtue of the distinct masses and decay rates, a complete experimental analysis of CPT violation requires four independent measurements in each system.

Consider the special case of semileptonic decays into a final state $f$ or its conjugate state $\bar{f}$. For simplicity, disregard any violations of the $\Delta Q = \Delta S$, $\Delta Q = \Delta C$, or $\Delta Q = \Delta B$ rules. Then, the basic transition amplitudes can be taken as $\langle f | T | P^0 \rangle = F$, $\langle f | T | P^0 \rangle = \bar{F}$, $\langle f | T | P^0 \rangle = \langle f | T | P^0 \rangle = 0$. The standard procedure can be applied to obtain the various time-dependent decay amplitudes and probabilities. Since the meson decays quickly relative to the Earth’s sidereal period, the dependence of $\xi$ on the meson proper time $t$ can be neglected. The decay probabilities depend on the proper time, as usual, but in the presence of CPT violation they also acquire sidereal time and momentum dependences from those of $\xi(\hat{t}, \vec{p})$.

To illustrate the resulting effects for the case of uncorrelated mesons, suppose direct CPT violation is negligible, so that $F^* = \bar{F}$. An appropriate asymmetry sensitive to CPT violation is then

$$A_{\text{CPT}}(t, \hat{t}, \vec{p}) = \frac{\bar{P}_F(t, \hat{t}, \vec{p}) - P_F(t, \hat{t}, \vec{p})}{\bar{P}_F(t, \hat{t}, \vec{p}) + P_F(t, \hat{t}, \vec{p})} = \frac{2\text{Re}\xi \sinh \Delta t/2 + 2\text{Im}\xi \sin \Delta mt}{(1 + |\xi|^2) \cosh \Delta t/2 + (1 - |\xi|^2) \cos \Delta mt}. \quad (13)$$

This is understood to depend on $\hat{t}$, $\vec{p}$ through $\xi(\hat{t}, \vec{p})$. Independent measurements of the four coefficients $\Delta a_\mu$ can be obtained by various suitable averagings over $t$, $\hat{t}$, $\vec{p}$, $\theta$, $\phi$, either before or after constructing the asymmetry (13). For example, if data are binned in $\hat{t}$ then it follows from Eq. (12) that measurements of the CPT coefficients $\Delta a_X$ and $\Delta a_Y$ are possible. As another example, binning in $\theta$ separates the spatial and timelike components of $\Delta a_\mu$.

To date, these ideas have been applied in experiments with the $K$ and $D$ systems. For the $K$ system, two independent CPT measurements of different combinations of the coefficients $\Delta a_\mu$ have been obtained [14, 15] one about $10^{-20}$ GeV on a linear combination of $\Delta a_0$ and $\Delta a_Z$, and the other about...
$10^{-21}\text{ GeV}$ on a combination of $\Delta a_X$ and $\Delta a_Y$. The experiments in question were performed with mesons highly collimated in the laboratory frame. In this situation, $\xi$ simplifies because the 3-velocity takes the form $\vec{\beta} = (0, 0, \beta)$. Binning in $t$ provides sensitivity to the equatorial components $\Delta a_X$, $\Delta a_Y$, while averaging over $t$ eliminates them altogether. For the $D$ system, preliminary sensitivity results for two independent bounds have also been obtained by the FOCUS experiment.\footnote{Note that CPT bounds in the $D$ system are unique in that the valence quarks involved are the $u$ and the $c$, whereas the other neutral mesons involve the $d$, $s$, and $b$.} A different illustration is provided by the case of correlated meson pairs produced by quarkonium decay into $ff$. The double-decay probability is a function of the proper decay times $t_1$, $t_2$, the momenta $\vec{p}_1$, $\vec{p}_2$, and the sidereal time $\hat{t}$. The CPT properties of the two mesons in each decay typically are distinct because the corresponding parameters $\xi_1$ and $\xi_2$ differ. Since the time sum $t = t_1 + t_2$ is typically unobservable in practice, an integration over $t$ is appropriate in deriving the relevant probability $\Gamma_{ff}$. It is then natural to define a CPT-sensitive asymmetry $A_{ff}^{\text{CPT}}$ as a function of the difference $\Delta t = t_1 - t_2$ and the sum $\xi_1 + \xi_2$:

$$A_{ff}^{\text{CPT}}(\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) = \frac{\Gamma_{ff}(\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) - \Gamma_{ff}(-\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2)}{\Gamma_{ff}(\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) + \Gamma_{ff}(-\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2)}$$

$$= \frac{-\text{Re} (\xi_1 + \xi_2) \sinh \frac{1}{2} \Delta \gamma \Delta t - \text{Im} (\xi_1 + \xi_2) \sin \Delta m \Delta t}{\cosh \frac{1}{2} \Delta \gamma \Delta t + \cos \Delta m \Delta t}. \quad (14)$$

As in the previous asymmetry, $\xi_1$, $\xi_2$ are understood to have sidereal-time and momenta dependences, so the attainable CPT reach can depend on the specific experiment. Suppose, for example, that the quarkonium is created at rest in a symmetric collider. The sum $\xi_1 + \xi_2 = 2 \gamma(p) \Delta a_0 / \Delta \lambda$ is then independent of $\Delta a$, and direct fitting of the data binned in $\Delta t$ allows a measurement of $\Delta a_0$. If instead the quarkonium is created in an asymmetric collider, then $\xi_1 + \xi_2$ could be sensitive to all four coefficients $\Delta a_\mu$ for that neutral-meson system. This implies that appropriate data binning would allow up to four independent CPT measurements. The existing asymmetric $B_d$ factories BaBar and BELLE can undertake measurements of these types.

Acknowledgments

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