Fast Track Communications

Conformal Carroll groups and BMS symmetry

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Received 4 March 2014
Accepted for publication 24 March 2014
Published 16 April 2014

Abstract
The Bondi–Metzner–Sachs group is shown to be the conformal extension of Lévy-Leblond’s ‘Carroll’ group. Further extension to the Newman–Unti group is also discussed in the Carroll framework.

Keywords: BMS symmetry, Newman–Unti group, conformal Carroll group, Carroll manifolds

PACS numbers: 04.20.−q, 04.20.Ha, 02.40.−k, 02.20.Sv, 02.20.Tw

1. Introduction
There has recently been a resurgence of interest in both the Bondi–Metzner–Sachs (BMS) group [1–4]) and, independently, in Lévy-Leblond’s ‘Carroll’ group [5–7].

The BMS group arose as the asymptotic symmetry group of a four-dimensional asymptotically flat spacetime representing an isolated time-dependent system emitting gravitational radiation [1]. It had been anticipated, as is the case for asymptotically flat time independent isolated systems, that the asymptotic symmetry group would be the Poincaré group and it came as a surprise that in the presence of gravitational radiation it is impossible to isolate a unique asymptotic Poincaré group, but rather the asymptotic symmetries constitute an infinite-dimensional group which contains many copies of the Poincaré group, none of which being invariant.

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The BMS group may be thought of as acting on future (or past) null infinity \( \mathcal{I}^\pm \). The latter are null hypersurfaces contained in the conformal boundary of an asymptotically flat spacetime. Topologically \( \mathcal{I}^\pm = S^2 \times \mathbb{R} \), where the \( S^2 \) factor corresponds to the 2-sphere of asymptotic directions and the \( \mathbb{R} \) factor to retarded (advanced) time. Thus local charts \( \theta, \phi, u \) or \( \theta, \phi, v \) may be introduced, where \( \theta, \phi \) parametrize the null generators and \( u, v \) are affine parameters along the null generators. From now on we shall consider only \( \mathcal{I}^+ \) since the story for \( \mathcal{I}^- \) is identical.

Let us recall that \( \mathcal{I}^+ \) admits a degenerate conformal structure for which we may take a representative metric of the form \( ds^2 = 0 \times du^2 + d\theta^2 + \sin^2 \theta \, d\phi^2 \), where the non-vanishing summand is the standard round metric on the unit 2-sphere which we wish to think of as the Riemann sphere \( \mathbb{C} \cup \{\infty\} \). To this end we introduce stereographic coordinates \( \zeta = e^{i\theta} \cot(\theta/2) \) in terms of which \( d\zeta^2 = 4(1 + \zeta \bar{\zeta})^{-2} \, d\zeta \). The identity component of the Lorentz group is isomorphic to \( \text{PSL}(2, \mathbb{C}) = \text{SL}(2, \mathbb{C})/\mathbb{Z}_2 \) and acts as conformal transformations of the Riemann sphere. Specifically, if \( a, b, c, d \in \mathbb{C} \) are such that \( ad - bc = 1 \), then for

\[
\zeta' = \varphi(\zeta) = \frac{a\zeta + b}{c\zeta + d},
\]  

we have

\[
\frac{d\zeta' d\bar{\zeta}'}{(1 + \zeta' \bar{\zeta}')^2} = \Omega^2(\zeta, \bar{\zeta}) \frac{d\zeta d\bar{\zeta}}{(1 + \zeta \bar{\zeta})^2},
\]

where

\[
\Omega(\zeta, \bar{\zeta}) = \frac{1 + \zeta \bar{\zeta}}{|a\zeta + b|^2 + |c\zeta + d|^2}.
\]

The infinite-dimensional abelian group \( T \) of supertranslations acts on the 2-sphere sections (known as ‘cuts’) of the product as \( u \mapsto u' = u + \alpha(\zeta, \bar{\zeta}) \) preserving the conformal structure, where \( \alpha(\zeta, \bar{\zeta}) \) is a smooth real valued function on the Riemann sphere which transforms as a scalar of weight 1 under conformal transformations of \( S^2 \) [8, 9]. The standard BMS group is thus the semi-direct product of \( \text{PSL}(2, \mathbb{C}) \) with \( T \equiv C^\infty(S^2, \mathbb{R}) \) and acts on \( \mathcal{I}^+ \) as \( (\zeta, u) \mapsto (\zeta', u') \) with \( \zeta' \) as in equation (1.1), and

\[
u' = \Omega(\zeta, \bar{\zeta})[u + \alpha(\zeta, \bar{\zeta})].
\]

If \( \alpha \) is expanded in spherical harmonics and only the \( l = 0 \) and \( l = 1 \) terms retained, one obtains a closed subgroup isomorphic with the Poincaré group. Now, the sum of the \( l = 0 \) and \( l = 1 \) representations of \( \text{SO}(3) \) transform as the \( D^{2+1}\bar{1} \) representation of \( \text{PSL}(2, \mathbb{C}) \), i.e., as the defining representation of the Lorentz group and as a scalar of weight 1 under conformal transformations. However the Poincaré subgroup so defined is not an invariant subgroup [10]. In that case the generalized BMS group (and also its Lie algebra of vector fields) will be much larger than the Poincaré subgroup defined above.

Rather weaker versions of the BMS group may be defined. For example, the Newman–Unti (NU) group is defined by choosing \( a, b, c, d \in \mathbb{C} \) such that \( ad - bc = 1 \), and set

\[
\zeta' = \frac{a\zeta + b}{c\zeta + d}, \quad u' = f(\zeta, \bar{\zeta}, u),
\]

where \( f \) must decrease with \( u \) at fixed \( \zeta \) [10]. The NU group preserves the conformal geometry, but not what is called the strong conformal geometry [10]. There is also an intermediate version, for which

\[
f(\zeta, \bar{\zeta}, u) = \beta(\zeta, \bar{\zeta})[u + \alpha(\zeta, \bar{\zeta})].
\]

Taking \( \beta(\zeta, \bar{\zeta}) = \Omega(\zeta, \bar{\zeta}) \) gives the general element of the BMS group.
The obvious generalization is to replace $S^2$ by $S^d$ and $O(3, 1)$ by $O(d + 1, 1)$ [11], but since the asymptotics of solutions of the Einstein’s equations in greater than four spacetime dimensions differ somewhat from the four-dimensional case [12], its physical relevance remains unclear. More generally, one may clearly consider the group obtained by replacing the Riemann sphere by any closed Riemannian manifold $(\Sigma, \hat{g})$. In general this will have no proper conformal isometries and so one gets the semi-direct product of the isometry group of $(\Sigma, \hat{g})$ with $C^\infty(\Sigma, \mathbb{R})$. I f $\Sigma$ is non-compact, for example the Euclidean plane $\mathbb{E}^2 \equiv \mathbb{C}$, then if one is unconcerned about global issues, in a formal sense any holomorphic map is conformal. Similarly for the cylinder $\mathbb{E}^2 \setminus \{0\} \equiv \mathbb{C} \setminus \{0\} \equiv S^1 \times \mathbb{R}$. In that case the generalized BMS group and its Lie algebra will be much larger than that defined above. It is this type of infinite-dimensional extension of the standard BMS group which has figured in much of the current literature [2, 3].

Our purpose being to relate all these groups to some particular overgroups of the Carroll group, let us recall that the latter, originally introduced as an unusual contraction of the Poincaré group [5], is highlighted by that a ‘Carrollian boost’ with parameter $b \in \mathbb{R}^3$ transforms ‘Carrollian time’ $u$ alone, according to

$$ x' = x, \quad u' = u - b \cdot x $$

where $x \in \mathbb{R}^3$, instead of the familiar Galilean action $x' = x + bt, t' = t$ on non-relativistic spacetime.

The aim of the present communication is thus to show that the BMS and NU groups can be understood as conformal symmetries, namely as conformal Carroll groups introduced in this paper, associated with Carroll manifolds [6, 7]. Further extension to the NU group [10] is also discussed.

2. Carroll group and manifold

In [7] a definition independent of relativity and group contraction has been put forward. It is based on defining a Carroll manifold analogous to a Newton–Cartan manifold [7, 13] as a $(d + 1)$-dimensional manifold $C$ (for Carroll), endowed with a twice-symmetric covariant, positive tensor field, $g$, whose kernel is generated by a nowhere vanishing vector field $\xi$. The stronger definition proposed in [7] requires a Carroll manifold to carry, in addition, a symmetric affine connection, compatible with both $g$ and $\xi$. The degeneracy of the ‘metric’ $g$ implies that the connection is not uniquely defined or may let alone exist, see section 4 below. In this communication we stick essentially to our new and less restrictive definition given here.

(i) The standard Carroll structure is given by $C^{d+1} = \mathbb{R}^d \times \mathbb{R}$, with $g = \delta_{AB} \, dx^A dx^B$, and $\xi = \partial/\partial u$, where $A, B = 1, \ldots, d$ are spatial indices, and $u = x^{d+1}$ is the “Carrollian ‘time’”-coordinate with dimension action/mass.

(ii) More general Carroll manifolds can be constructed out of hypersurfaces $\Sigma$ with Riemannian metric $\hat{g}$, namely as $C = \Sigma \times \mathbb{R}$ and $g = \hat{g}_{AB}(x) \, dx^A dx^B$, and $\xi = \partial/\partial u$. In both cases, $C$ can also be endowed with a compatible connection, e.g., $\Gamma^i_{ij} = 0$ for all $i, j, k = 1, \ldots, d + 1$ in case (i). The non-vanishing components of the Carroll connection of case (ii) being $\Gamma^C_{AB} = \hat{\Gamma}^C_{AB}$ (the Levi-Civita connection of $\hat{g}$), the $\Gamma^u_{AB}$ remaining arbitrary. Yet another important example will be studied in section 4 below.

3. Conformal Carroll transformations

Inspired by the definition of relativistic, and also non-relativistic [13], conformal transformations, we now introduce for, a given Carroll manifold, the conformal Carroll
group of level \( N \), \( \text{CCarr}_N(C, g, \xi) \), of those transformations which preserve the tensor field \( g \otimes \xi \otimes ^N \) canonically associated with our Carroll structure, i.e., of all transformations, \( a \), which satisfy

\[
a^* g = \Omega^2 g \quad \text{and} \quad a^* \xi = \Omega^{-2/N} \xi
\]

for some positive function \( \Omega \) on \( C \) and positive integer \( N \). The Lie algebra of infinitesimal conformal Carroll transformations, \( \text{ccarr}_N(C, g, \xi) \), is spanned, accordingly, by vector fields \( X \) such that

\[
L_x g = \lambda \ g \quad \text{and} \quad L_x \xi = -\frac{\lambda}{N} \xi
\]

for some function \( \lambda \) on \( C \). In the flat case (i) our formulae yield \([14]\]

\[
X = (\omega^B A^B + \chi^A + (\gamma - 2\kappa B B) x^A + \kappa^A \chi B B) \frac{\partial}{\partial x^A} + \left( \frac{2}{N} (\chi - 2\kappa B B) u + T(x) \right) \frac{\partial}{\partial u}
\]

where \( \omega \in \text{so}(d), \chi, \kappa \in \mathbb{R}^d \), and \( \gamma \in \mathbb{R} \), with \( T \in C^\infty(\mathbb{R}^d, \mathbb{R}) \). In view of (3.1), the supertranslation \( T \) in (3.3) has conformal weight \(-2/N\), and should therefore be regarded as a density with weight \( v = -2/(Nd) \). Hence, \( \text{ccarr}_N(d+1) \) is the semi-direct product of the conformal Lie algebra \( \text{so}(d+1, 1) \) with \( \nu \)-densities on \( \mathbb{R}^d \).\(^{5}\) Due to the degeneracy of the Carroll ‘metric’, \( g \), our conformal Carroll Lie algebras are infinite-dimensional, owing to supertranslations represented by \( T \). In view of (3.3), the quantity \( z = 2/N \) is the associated dynamical exponent. The value \( N = 2 \) is particularly interesting: space and ‘time’ are then equally dilated so that the dynamical exponent is \( z = 1 \).\(^{6}\) Moreover, in \( d = 1 \) space dimension, interchanging position and time, \( x \leftrightarrow s \) and renaming \( s \) as \( t \), our conformal Carroll algebra becomes precisely the Conformal Galilei algebra \( \text{CGA} \) \([13, 15]\). Note that the interchange also swaps Carrollian (1.7) and ordinary Galilean boosts.

Requiring \( \Omega = 1 \), i.e., \( \lambda = 0 \) in (3.2) would yield the ‘isometry group’ of the Carroll structure \( (g, \xi) \). It is infinite-dimensional owing to the presence of supertranslations. Requiring, in addition, the preservation of a Carroll connection would reduce this to a finite-dimensional group; for, e.g., the flat Carroll structure (i) we get the usual Carroll group \([5]\), denoted by \( \text{Carr}(d + 1) \) in \([7]\). The Carroll Lie algebra, \( \text{ccarr}(d + 1) \), is spanned by the vector fields

\[
X = (\omega^B A^B + \gamma^A + (\sigma - \kappa A A) x^A) \frac{\partial}{\partial x^A} + (\sigma - \beta A A) \frac{\partial}{\partial u},
\]

where \( \omega \in \text{so}(d), \beta, \gamma \in \mathbb{R}^d \), and \( \sigma \in \mathbb{R} \).

In case (ii), the general expression of a conformal Carroll vector field in \( \text{ccarr}_N(C, g, \xi) \) is

\[
X = Y + \left( \frac{\lambda}{N} u + T(x) \right) \frac{\partial}{\partial u},
\]

where \( Y = \gamma^A(x) \partial/\partial x^A \) is a conformal vector field of \( (\Sigma, \tilde{g}) \), i.e., such that \( L_x \tilde{g} = \lambda \tilde{g} \), hence with \( \lambda = (2/d) \tilde{\nabla}_A \gamma^A \), and \( T \) is a real function on \( \Sigma \). Integration of the vector field (3.5) readily yields the group action \( (x, u) \mapsto (x', u') \), where

\[
x' = \varphi(x), \quad u' = \Omega^{2/N}(x)[u + \alpha(x)],
\]

\(^{5}\) The canonical \( \nu \)-densities of a Riemannian manifold \( (\Sigma, \tilde{g}) \) are locally of the form \( f \det (g_{AB})^{\nu/2} \) with \( f \) a smooth function on \( \Sigma \).

\(^{6}\) The special value \( z = 1 \) corresponds to group contraction from the relativistic conformal group, analogous to that in the Galilean case \([15]\). Accordingly, the conformal Carroll invariant \( g \otimes \xi \otimes \xi \) can be regarded as a limit as \( c \downarrow 0 \) of the conformal invariant \( c^2 G \otimes G^{-1} \) of a Lorentz manifold \( (M, G) \) for which the Carroll manifold is an ultra-relativistic asymptote.
with $\varphi \in \text{Conf}(\Sigma, \hat{g})$, and $\alpha \in C^\infty(\Sigma, \mathbb{R})$. Putting $a = (\varphi, \alpha)$, we readily find the group law of the conformal Carroll group of level $N$; if $a'' = da$, we end up with the group law
\[
\varphi'' = \varphi' \circ \varphi, \quad \alpha'' = \Omega^{2/N} \varphi^* \alpha + \alpha.
\] (3.7)

Notice, again, that supertranslations are actually densities of conformal weight $-2/N$. The conformal Carroll transformations of $C = \Sigma \times \mathbb{R}$ belong therefore to the semi-direct product of conformal transformations of $(\Sigma, \hat{g})$ with ‘supertranslations’ of $\Sigma$,
\[
\text{CCarr}_N(C, g, \xi) \equiv \text{Conf}(\Sigma, \hat{g}) \ltimes T,
\] (3.8)
where $T$ is a shorthand for supertranslations (mathematically, $-2/(Nd)$-densities on $\Sigma$).

If, for example, $\Sigma = S^1$ and $g = d\theta^2$, conformal Carroll transformations of level $N$ will be given by the semi-direct product of the conformal transformations of the circle, $\text{Diff}(S^1)$, and supertranslations with weight $\nu = -2/N$. They are generated by the vector fields
\[
X = Y(\theta) \frac{\partial}{\partial \theta} + \left( \frac{2}{N} \nu'(\theta) u + T(\theta) \right) \frac{\partial}{\partial u}.
\] (3.9)

Considering instead $\Sigma = S^2$ endowed with its round metric allows us to conclude that the conformal Carroll transformations of level $N = 2$ are the semi-direct product of the conformal group of $S^2$ with supertranslations,
\[
\text{CCarr}_2(S^2 \times \mathbb{R}, g, \xi) \equiv \text{PSL}(2, \mathbb{C}) \ltimes T
\] (3.10)
with $T$ the $-1/2$ densities on the two-sphere. This is, precisely, the BMS(4) group [1] whose group law is given by (3.7). Its action on our Carroll manifold can be read off from (3.6) with $N = 2$, and yielding equations (1.1) and (1.4).

Then, for $\Sigma = S^d$, the group $\text{SL}(2, \mathbb{C})$ is simply replaced by (the neutral component of) $\text{O}(d+1, 1)$, and Carroll isometries readily identified with the semi-direct product of the orthogonal group $\text{O}(d+1)$ with supertranslations—and is therefore still infinite-dimensional.

We therefore claim that $\text{CCarr}_2(C, g, \xi) \equiv \text{BMS}(d + 2)$.

4. The light-cone as Carroll a manifold and the BMS group

To present our third example of a Carroll manifold, we deal with Minkowski spacetime $\mathbb{R}^{d+1,1}$ endowed with the metric $G = \text{diag}(1, \ldots, 1, -1)$, and look at the punctured future light-cone, $C = T^+$, of future pointing null vectors, described (non canonically) by those vectors $(x, t) \in \mathbb{R}^{d+1} \times \mathbb{R}$ such that $t \equiv |x| > 0$. Let us then consider the symmetric tensor, $g$, inherited on $C$ from the Minkowski metric $G$. Using coordinates, and since $t > 0$ can be viewed as a (global) radial coordinate on $C$, let the unit vector $x = x/t$ denote the direction of $z$. Then $g = t^2 |dz|^2 = t^2 \hat{g}$, where $\hat{g}$ is the usual round metric of $S^d$ (the directions of light rays). Let us insist that, the above decomposition of null vectors being non-canonical, ‘projecting’ $g$ to the ‘celestial sphere’, $S^d$, only defines a conformal class, $[g]$, of metrics of the latter. The kernel of $g$ is seen to be spanned by the restriction, $\xi$, to $C$ of the Euler vector field of $\mathbb{R}^{d+1,1}$, namely $\xi = z \partial_z = \partial_u$, where $u = \log t$.

The light-cone $C = T^+$ is, hence, an intrinsically defined Carroll manifold. Can it be endowed with a compatible connection? Crucially for us, the answer is no! To see this, choose a coordinate system $(x^A, u)$ on $C$, such that $g_{AB} = \epsilon^{2u} \hat{g}_{AB}$, $g_{Au} = 0$, $g_{su} = 0$, $\xi^s = 0$, $\xi^u = 1$. Then a symmetric affine connection $\nabla$ on $C$ should satisfy both conditions $\langle \nabla_u g_{AB} \rangle_{uA} = (\nabla_A g)_{uA} = 0$, which are readily seen to be contradictory.

Providentially enough, our definition for a conformal Carroll transformation does not involve the connection, though. Then a calculation analogous to the proof of equation (3.8) in the case (ii) shows that $L_\xi g = \lambda \cdot g$ requires $\partial_u X^A = 0$ as well as $L_T \hat{g} = (\lambda - 2X^u) \hat{g}$. Using
the second condition \( L_{\xi} \xi = -\left(\lambda / N\right) \xi \) in (3.2) allows us to deduce that the conformal Carroll group of the punctured future light-cone is (3.10, i.e., for \( N = 2 \), the BMS group).

The condition \( \lambda = 0 \) would fix the supertranslations as \( T = \sum u \frac{\partial}{\partial u} \) with \( u = \frac{1}{2} (L_{\xi} \xi) / \xi \), while leaving the space-part, \( Y \), conformal; the Carroll ‘isometries’ of the light-cone span therefore the conformal group, \( O(d+1,1) \), of the celestial sphere, \( \mathbb{S}^d \), with rigidly fixed ‘compensating’ supertranslations.

### 5. Newman–Unti groups of the light-cone

Our formalism allows us to define the NU group of a Carroll manifold, and of the light-cone in particular.

The NU group is spanned of those (local) diffeomorphisms \( a \) of \( C \) which preserve the sole degenerate ‘metric’, \( g \), up to a conformal factor, namely \( a^* [g] = [g] \). This entails that the direction of \( \xi \) is automatically preserved (since \( a^* \xi \) lies again in the kernel of \( g \)). Its Lie algebra consists, hence, of all vector fields \( X \) on \( C \) such that \( L_X g = \lambda g \), the condition \( L_X \xi = \mu \xi \) being automatically satisfied.

If \( C \) is the light-cone \( \mathbb{I}^+ \) of \( \mathbb{R}^{d+1} \), we find that
\[
X = Y + X^u (x,u) \frac{\partial}{\partial u},
\]
(5.1)
with \( Y = Y^A (x) \partial / \partial x^A \) a conformal vector field of \( \mathbb{S}^d \) and \( X^u \in C^\infty (C, \mathbb{R}) \), is, this time, an arbitrary function of the \( x^A \) and \( u \). The NU group of the light-cone \( C \) is, therefore,
\[
 NU \equiv \text{Conf}(\mathbb{S}^d) \ltimes C^\infty (C, \mathbb{R}),
\]
(5.2)
consistently with equations (1.5) and (1.6). We notice that equation (1.6) corresponds in fact to the ‘intermediate’ Lie subalgebra of the NU Lie algebra defined by \( L_{\xi} g = \lambda g \), and \( (L_{\xi})^2 X = 0 \). For the light-cone, it consists in those vector fields \( X = Y + X^u \partial / \partial u \) with, just as before, \( Y \in \text{conf}(\mathbb{S}^d) \), and \( (\partial_u)^2 X^u = 0 \), i.e., such that \( X^u = S [u + T] \), where \( S \) and \( T \) remain arbitrary functions on \( \mathbb{S}^d \), see equation (1.6).

Referring to equations (3.5) and (5.1) giving the generators of the conformal Carroll Lie algebras previously studied, we can highlight, in the case \( N = 2 \), the interesting array of nested Lie groups
\[
 NU_1 \subset \text{BMS} \subset NU_2 \subset \ldots \subset NU.
\]
(5.3)
We finally notice that \( NU_1 = \text{CCarr} \).

### 6. Concluding remarks

The basic result of this paper is that the future null conformal boundary \( \mathbb{I}^+ \) of an asymptotically flat spacetime emitting gravitational radiation is a Carroll manifold [7] and its asymptotic symmetries, i.e., elements of the BMS group [1], constitute the associated conformal Carroll group. Originally introduced as the limit of the Poincaré group as the speed of light tends to zero [5], Carroll groups and Carroll manifolds have found applications in the study of velocity-dominated spacetimes and physics on branes which approach the speed of light. They are the analogue of Newton–Cartan manifolds, which arise when the speed of light tends to infinity [7]. As a null hypersurface in the conformal compactification of spacetime \( \mathbb{I}^+ \)

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7 As explained before, for \( d = 1 \) and \( N = 2 \) the conformal Carroll group is the same as CGA with \( z = 1 \), as seen by interchanging position and time, cf [3].

8 We note that the \( X^u \) component of the generators (5.1) of the Lie algebra of \( NU_k \) is a polynomial of degree \( k - 1 \) in \( u \).
carries an induced metric, \( g \), which is degenerate with a one-dimensional kernel spanned by \( \xi \) which is tangent to its null generators. It also carries a so-called strong conformal structure [10]. Depending upon how much of this structure one requires to be preserved one may obtain different symmetry groups. Dropping the requirement that the strong conformal structure be preserved leads to a larger group, namely to the NU group [10], which is now seen to fit into the general theory of Carroll manifolds and their symmetries.

As we recalled in the introduction, there has been a considerable revival of interest recently in the BMS group in connection with its application to conformal field theory [2–4]. We hope that the clarification brought about in the present paper will further advance this study.

Acknowledgments

We are indebted to G Barnich whose advice allowed us to correct a misinterpretation. CD acknowledges ancient and enlightening discussions with F Ziegler about the BMS group. GWG would like to thank KITP, Santa Barbara for its hospitality during its *Bits and Branes* program (2012), which provided a stimulus for this work. He is grateful also to the Laboratoire de Mathématiques et de Physique Théorique de l’Université de Tours for hospitality, and the Région Centre for a ‘Le Studium’ research professorship.

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