The effect of beam related systematics, namely uncertainty in the beam energy and differential luminosity spectrum measurements and beam energy spread on the precision of the Higgs boson mass measurement at a future linear $e^+e^-$ collider is investigated.

A future linear $e^+e^-$ collider will be an ideal tool to measure Higgs boson properties. A number of studies have been performed to examine physics potential of the linear collider in terms of attainable statistical errors on the observable quantities such as Higgs boson mass, decay branching fractions and production rate. However, most of these studies did not take into account possible systematic effects which may influence the precision of these measurements. In this note possible impact of the beam related systematic errors on the Higgs boson mass measurement is discussed. Previous analysis has demonstrated that Higgs boson mass can be measured with the statistical precision of 40 MeV for Higgs boson mass of 120 GeV. In our study this analysis has been complemented with the simulation of the following effects:

- uncertainty in the beam energy measurement;
- beam energy spread;
- uncertainty in the differential luminosity spectrum measurements.

The Higgs boson mass is measured exploiting Higgs-strahlung process, in which Higgs boson is produced in association with the Z boson. The investigated topologies include the following final states: $HZ \rightarrow b\bar{b}q\bar{q}$, $HZ \rightarrow b\bar{b}e^+e^-$ and $HZ \rightarrow b\bar{b}\mu^+\mu^-$. The analysis is performed for the centre-of-mass energy of 350 GeV and integrated luminosity of 500 fb$^{-1}$. Signal and main background processes, $W^+W^-$, $ZZ$ and $q\bar{q}$ production, are generated using PYTHIA. Detector response is simulated using fast parametric Monte Carlo program SIMDET. Beamstrahlung is accounted for using CIRCE.
Analysis proceeds as follows. Initially selection of specific final state is applied, exploiting event shape variables and b-tag information. In the case of the $HZ \to b\bar{b} e^+ e^-$ and $HZ \to b\bar{b} \mu^+ \mu^-$ channels, the presence of two isolated leptons, $e^+ e^-$ or $\mu^+ \mu^-$, with invariant mass compatible with the mass of $Z$ is required. Hadronic part of an event is then clustered into 4 or 2 jets for $HZ \to b\bar{b}q \bar{q}$ and $HZ \to b\bar{b}\ell^+ \ell^-$ channels, respectively. Kinematical fit, imposing four momentum conservation is performed to improve the Higgs boson mass resolution and using energy and angular resolution functions for jets and leptons derived from Monte Carlo studies. For the $HZ \to b\bar{b}q \bar{q}$ channel in addition to the four momentum conservation requirement invariant mass of the two jets assigned for the $Z$ boson is constrained to the $Z$ mass. The Higgs boson mass is reconstructed after kinematical fit as the invariant mass of the two jets assigned for the Higgs boson. Higgs boson mass and corresponding statistical error are obtained from the fit of resulting reconstructed Higgs boson mass spectrum with the superposition of background and signal distributions. Detailed description of the analysis can be found in Reference 1.

The impact of the uncertainty in the beam energy measurement is estimated in the following way. At the stage of generating signal samples both positron and electron beam energies are artificially shifted with respect to the nominal value of 175 GeV. The shifts to the beam energies are varied from -100 to 100 MeV in 25 MeV steps. Since the kinematical fit uses the nominal value for the centre-of-mass energy of 350 GeV, the shift in the beam energy will result also in the shift in the measured Higgs boson mass. As an example Figure 1 shows the distributions of fitted values of Higgs boson mass in the $HZ \to b\bar{b}q \bar{q}$ channel for three scenarios: when both electron and positron beam energies are overestimated by 25 MeV, when they are underestimated by 25 MeV and when no shifts are introduced to the beam energies. In each of the three considered cases the distributions are obtained from 200 statistically independent signal samples. Corresponding systematic error on the Higgs boson mass is found to depend linearly on the uncertainty in the beam energy:

- $\delta(m_H) \sim 0.8 \cdot \delta(E_b)$ for the $HZ \to b\bar{b}q \bar{q}$ channel,
- $\delta(m_H) \sim \delta(E_b)$ for the $HZ \to b\bar{b}\ell^+ \ell^-$ channels.

At this point, one can conclude that in order to keep systematic error at the level of statistical one, the beam energy must be measured with relative precision of $10^{-4}$.

To estimate the impact of beam energy spread, Gaussian smearing of beam energy has been applied at the stage of generating signal events. As an example, Figure 2 shows reconstructed Higgs boson mass spectrum in the sample
of $HZ \rightarrow b\bar{b}q\bar{q}$ events for the case of ideal monochromatic beams and for the case of 0.5% energy spread for both electron and positron beams. In the latter case the statistical precision in the Higgs boson mass measurement degrades from 45 to 50 MeV in the $HZ \rightarrow b\bar{b}q\bar{q}$ channel and from 70 to 80 MeV in the $HZ \rightarrow b\bar{b}e^+\ell^-$ channel. For the TESLA machine the expected energy spread amounts to 0.15% for electron beam and 0.032% for positron beam. In this scenario no significant degradation of the precision in the Higgs boson mass measurement is observed.

The energy spectra of colliding electron and positron at linear collider will be significantly affected by photon radiation of an electron/positron in one bunch against coherent field of opposite bunch. This effect is referred to as beamstrahlung. To have a fast simulation of beamstrahlung the program CIRCE has been written which assumes that the beamstrahlung in the two beams is equal and uncorrelated between the beams and parameterizes beam
energy according to

\[ f(x) = a_0 \delta(1 - x) + a_1 x^{a_2}(1 - x)^{a_3}, \]

where \( x \) is the ratio of colliding electron/positron energy to initial energy of undisrupted beam. Parameters \( a_i \) depend on operational conditions of linear collider. Normalization condition, \( \int f(x) \, dx = 1 \), fix one these parameters, leaving only three of them independent. The default parameters for the TESLA machine operated at the centre-of-mass energy of 350GeV are:

\[ a_0 = 0.55, \quad a_1 = 0.59, \quad a_2 = 20.3, \quad a_3 = -0.63. \]

It was shown that from the analysis of acollinearity spectrum in bhabha events, parameters \( a_i \) can be determined with a precision better than 1%. To visualize an effect of the uncertainty in determination of parameters \( a_i \) in figure 3 we show distribution of beam energy spectrum for nominal values of parameters \( a_i \) and for the cases when parameter \( a_0 \) is shifted by \( \pm10\% \) from its nominal value. Figure 4 presents corresponding Higgs boson mass spectra for the sample...
of $HZ \to b\bar{b}\ell^+\ell^-$ events. An uncertainty of 10% in the determination of parameters $a_i$ results in a systematic error of $\mathcal{O}(10\text{MeV})$ on the Higgs boson mass in both $HZ \to b\bar{b}\ell^+\ell^-$ and $HZ \to b\bar{b}q\bar{q}$ channels. The error is reduced to $\mathcal{O}(1\text{MeV})$ if parameters $a_i$ are measured with an accuracy of 1%.

References

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