Measurement of the $\gamma\gamma^* \rightarrow \eta_c$ transition form factor

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We study the reaction $e^+e^-\rightarrow e^+e^-\eta_c$, $\eta_c\rightarrow K_S K^\pm \pi^\mp$ and obtain $\eta_c$ mass and width values $2982.2\pm0.4\pm1.6$ MeV/$c^2$ and $31.7\pm1.2\pm0.8$ MeV, respectively. We find $\Gamma(\eta_c\rightarrow \gamma\gamma) B(\eta_c\rightarrow KK\pi) = 0.374\pm0.009\pm0.031$ keV, and measure the $\gamma\gamma^*\rightarrow \eta_c$ transition form factor in the momentum transfer range from 2 to 50 GeV$^2$. The analysis is based on 469 fb$^{-1}$ of integrated luminosity collected at PEP-II with the BABar detector at $e^+e^-$ center-of-mass energies near 10.6 GeV.

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I. INTRODUCTION

In this paper we study the process

$$e^+e^-\rightarrow e^+e^-\eta_c,$$ (1)

where the $\eta_c$ meson ($J^{PC} = 0^{-+}$), the lowest lying charmonium state, is produced via the two-photon production mechanism illustrated in Fig. 1. We measure the differential cross section for this process in the single-tag mode where one of the outgoing electrons$^1$ is detected (tagged), while the other electron is scattered at a small angle and hence is undetected (untagged). The tagged electron emits a highly off-shell photon with squared momentum transfer $q_1^2 = -Q^2 = (p' - p)^2$, where $p$ and $p'$ are the four-momenta of the initial- and final-state electrons. The momentum transfer squared to the untagged electron ($q_2^2$) is near zero. The differential cross section $d\sigma(e^+e^-\rightarrow e^+e^-P)/dQ^2$ for pseudoscalar meson ($P$) production depends on only one form factor $F(Q^2)$.

$^1$ Unless otherwise specified, we use the term “electron” for either an electron or a positron.
which describes the $\gamma\gamma^* \rightarrow P$ transition. To relate the differential cross section to the transition form factor we use the formulae for the $e^+e^- \rightarrow e^+e^-\pi^0$ cross section in Eqs. (2.1) and (4.5) of Ref. [1].

According to perturbative QCD (pQCD), the transition form factor can be presented as a convolution of a calculable hard scattering amplitude for the nonperturbative light-cone wave function. The measurement of the form factor allows us to test the pQCD prediction and to obtain information on the shape of the $\eta_c$ wave function. The $Q^2$ dependence of the form factor is studied theoretically in Refs. [3, 4] using pQCD, and in Ref. [5] using the lattice QCD approach.

The $\eta_c$ transition form factor was measured by the L3 Collaboration [6] with very small data sample. In this paper we study the $e^+e^- \rightarrow e^+e^-\eta_c$ reaction in the $Q^2$ range from 2 to 50 GeV$^2$. The $\eta_c$ is observed via the $\eta_c \rightarrow K\ell\pi^\pm$ decay, which allows the $\eta_c$ to be selected with relatively low background. The $\eta_c$ two-photon width and branching fractions are not well measured [7]. Therefore, we also study no-tag data ($Q^2 \sim 0$), measure the product $\Gamma(\eta_c \rightarrow \gamma\gamma)B(\eta_c \rightarrow K\ell\pi)$, and normalize the transition form factor $F(Q^2)$ to $F(0)$. The measured values of the $\eta_c$ mass and width obtained in different experiments have a large spread [8]. The high statistics no-tag data sample allows us to extract precise values for these parameters.

II. THE BABAR DETECTOR AND DATA SAMPLES

We analyze a data sample corresponding to an integrated luminosity of 469 fb$^{-1}$ recorded with the BABAR detector [8] at the PEP-II asymmetric-energy storage rings at SLAC. At PEP-II, 9 GeV electrons collide with 3.1 GeV positrons to yield an $e^+e^-$ center-of-mass (c.m.) energy of 10.58 GeV (the $\Upsilon(4S)$ resonance). About 10% of the data used in the present analysis were recorded about 40 MeV below the resonance.

Charged-particle tracking is provided by a five-layer silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH) operating in a 1.5-T axial magnetic field. The transverse momentum resolution is 0.47% at 1 GeV/c. Energies of photons and electrons are measured with a CsI(Tl) electromagnetic calorimeter (EMC) with a resolution of 3% at 1 GeV. Charged-particle identification is provided by specific ionization measurements in the SVT and DCH, and by an internally reflecting ring-imaging Cherenkov detector (DIRC). Electron identification also uses shower shape in the EMC and the ratio of shower energy to track momentum. Muons are identified in the solenoid’s instrumented flux return, which consist of iron plates interleaved with resistive plate chambers.

The signal process is simulated with the Monte Carlo (MC) event generator GGResRc. It uses the formula for the differential cross section for pseudoscalar meson production from Ref. [1]. Two samples of signal events are produced: one for no-tag measurement without any kinematic restrictions, and the other with the restrictions on the momentum transfer values to the electrons $Q^2 = -q_1^2 > 1.5$ GeV$^2$ and $-q_2^2 < 1$ GeV$^2$. The restriction on $Q^2$ for the tagged electron corresponds to the detector acceptance. The experimental criteria providing these restrictions for data events will be described in Sec. III. In the simulation of no-tag events we use the form factor

$$F(q_1^2, q_2^2) = \frac{F(0,0)}{(1 - q_1^2/m_{J/\psi}^2)(1 - q_2^2/m_{J/\psi}^2)}$$

expected in the vector dominance model. The form factor is fixed to the constant value $F(0,0)$ in the simulation of single-tag events. The produced $\eta_c$ decays into the $K\ell\pi^\pm$ final state. The simulation uses a three-body phase space distribution to describe this decay.

The GGResRc event generator includes next-to-leading-order radiative corrections to the Born cross section calculated according to Ref. [9]. In particular, it generates extra photons emitted by the initial- and final-state electrons. The formulae from Ref. [9] are modified to take into account the hadron contribution to the vacuum polarization diagrams. The maximum energy of the photon emitted from the initial state is restricted by the requirement $E_\gamma < 0.05\sqrt{s}$, where $\sqrt{s}$ is the $e^+e^-$ c.m. energy. The generated events are subjected to detailed detector simulation based on GEANT4 [3] and are reconstructed with the software chain used for the experimental data. Temporal variations in the detector performance and beam background conditions are taken into account.

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2 The use of charge conjugate reactions is implied throughout unless explicitly stated otherwise.

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3 Throughout this paper an asterisk superscript denotes quantities in the $e^+e^-$ c.m. frame. In this frame the positive $z$-axis is defined to coincide with the $e^-$ beam direction.
The processes with a $J/\psi$ in the final state (Fig. 2), with $J/\psi$ decaying into $\eta\gamma$, can imitate the process under study. The initial state radiation (ISR) process (Fig. 2(a)) contributes to the no-tag mode, while the $J/\psi$ bremsstrahlung process (Fig. 2(b)) contributes background to the single-tag mode. We simulate both processes with $J/\psi$ decaying to $K_S K^\pm \pi^\mp$ and also to $\eta\gamma$ followed by $\eta \rightarrow K_S K^\pm \pi^\mp$. To estimate a possible background from other two-photon processes we simulate the reaction $e^+e^- \rightarrow e^+e^- \eta, \pi^0$. These events are generated with an isotropic distribution of the final state mesons in the $\eta, \pi^0$ rest frame.

III. EVENT SELECTION

We select $e^+e^- \rightarrow e^+e^- \eta_c$ candidates in the no-tag and single-tag modes, with zero and one detected electron, respectively. The decay mode $\eta_c \rightarrow K_S K^\pm \pi^\mp (K_S \rightarrow \pi^+\pi^-)$ is used to reconstruct $\eta_c$ meson candidates.

Events are selected with at least four (five for single-tag mode) charged-particle tracks. Since a significant fraction of events contain beam-generated spurious track and photon candidates, one extra track and any number of extra photons are allowed in an event. The tracks corresponding to the charged kaon and pion must be oppositely charged, and must extrapolate to the interaction region. The kaon is required to be positively identified, while the pion track is required to be inconsistent with the kaon hypothesis.

The track identified as an electron must originate from the interaction region and be in the polar angle range $0.387 < \theta < 2.400$ in the lab frame (0.64–2.69 in the $e^+e^-$ c.m. frame). The latter requirement is needed for good electron identification. To recover electron energy loss due to bremsstrahlung, both internal and in the detector material before the DCH, the energy of any EMC shower close to the electron direction is combined with the measured energy of the electron track. The resulting c.m. energy of the electron candidate must be greater than 1 GeV.

A $K_S$ candidate is formed from a pair of oppositely charged tracks fitted to a common vertex, and yielding an invariant mass value in the range 487.5–507.5 MeV/c^2, when the charged-pion mass is assigned to each track. The candidate is then refitted with a $K_S$ mass constraint to improve the precision of its momentum measurement. To suppress combinatorial background, the angle between the $K_S$ candidate momentum and the line connecting its production and decay vertices ($\psi_{K_S}$) is required to satisfy $\cos \psi_{K_S} > 0.95$.

An $\eta_c$ candidate is formed from $K_S$, $K^\pm$, and $\pi^\mp$ candidates fitted to a common vertex. The preliminary selection criterion for no-tag events requires $|\cos \theta_{c\eta}^*| > 0.95$, where $\theta_{c\eta}^*$ is the polar angle of the candidate $\eta_c$ in the $e^+e^-$ c.m. frame. Figure 3 shows the distribution of the invariant mass of the pions forming a $K_S$ candidate ($M_{2\pi}$) for events satisfying this criterion. The shaded histogram demonstrates the effect of the requirement $\cos \psi_{K_S} > 0.95$. The transverse momentum of the $\eta_c$ candidate in the $e^+e^-$ c.m. frame is restricted to the range $p^*_t < 0.25$ GeV/c. The invariant mass distribution for $\eta_c$ candidates is shown in Fig. 4 where for events with more than one $\eta_c$ candidate (about 0.4% of signal events), the candidate with the smallest value of $p^*_t$ is selected. The $\eta_c$ peak from two-photon production and the $J/\psi$ peak from the ISR process $e^+e^- \rightarrow J/\psi \gamma$ are clearly seen in the invariant mass distribution. The shaded histogram shows the distribution for candidates rejected by the requirement $p^*_t < 0.25$ GeV/c. This requirement limits the momentum transfers to the electrons. The value of the effective threshold for $q_i^2 (i = 1, 2)$ is determined from the dependence of the detection efficiency on $\max(-q_1^2, -q_2^2)$ and is about 0.1 GeV^2. Such a low $q^2$ threshold yields a model-independent extraction of $\Gamma(0, 0)$ from the no-tag data. We note that $d\sigma/dq_1^2 dq_2^2 \propto 1/(q_1^2 q_2^2)$ at small $|q_1^2|$ and $|q_2^2|$.

For single-tag events we combine an $\eta_c$ candidate with an electron candidate and require $|\cos \theta_{e\eta_c}^*| > 0.95$, where $\theta_{e\eta_c}^*$ is the polar angle of the momentum vector of the $e\eta_c$ system in the $e^+e^-$ c.m. frame. The transverse momentum of the $e\eta_c$ system is restricted to $p^*_t < 0.25$ GeV/c. The $p^*_t$ distribution for data candidates is shown in Fig. 5 where the shaded histogram is the corresponding distribution for simulated signal events. The condition on $p^*_t$ limits the value of the momentum transfer

![Feynman Diagrams](image-url)
FIG. 3: The $M_{2\pi}$ distribution for $K_S$ candidates in the no-tag data sample. The shaded histogram shows events rejected by the requirement $\cos \psi_{K_S} > 0.95$. The arrows indicate the region used to select event candidates.

FIG. 5: The transverse momentum distribution for $e\eta_c$ data candidates. The shaded histogram is for simulated signal events. Data candidates for which $p_T^* < 0.25$ GeV/c (indicated by the arrow) are retained.

FIG. 4: The distribution of the invariant mass of $\eta_c$ candidates in the no-tag data sample. The shaded histogram shows events rejected by the requirement $p_T^* < 0.25$ GeV/c.

to the untagged electron ($q^2$). The effective $q^2$ threshold determined from simulation is about 0.1 GeV$^2$.

The emission of extra photons from the electrons involved leads to a difference between the measured and actual values of $Q^2$. In the case of ISR, $Q^2_{\text{meas}} = Q^2_{\text{true}}(1 + r_\gamma)$, where $r_\gamma = 2E^*_e/\sqrt{s}$. To restrict the energy of the ISR photon we use the parameter

$$r = \frac{\sqrt{s} - E^*_e}{|p^*_e|},$$

(3)

where $E^*_e$ and $p^*_e$ are the c.m. energy and momentum of the detected $e\eta_c$ system. In the ISR case this parameter coincides with $r_\gamma$ defined above. The $r$ distributions for data and simulated signal are shown in Fig. 6. Candidates with $-0.02 < r < 0.03$ are retained. We note that the condition on $r$ ensures compliance with the restriction $r_\gamma < 0.1$ used in the MC simulation.

FIG. 6: The distribution of $r$ defined in Eq.(3) for data (points with error bars) and simulated signal events (shaded histogram). The arrows indicate the region used to select candidate events ($-0.02 < r < 0.03$).

For two-photon events with a tagged positron (electron), the momentum of the detected $e\eta_c$ system in the $e^+e^-$ c.m. frame has a negative (positive) z-component, while events resulting from $e^+e^-$ annihilation are produced symmetrically. To suppress the $e^+e^-$ annihilation background, event candidates with the wrong sign of the momentum z-component are removed.

The distribution of the invariant mass of $\eta_c$ candidates for single-tag events satisfying the selection criteria described above is shown in Fig. 7. For events with more than one $\eta_c$ candidate, the candidate with smallest $p^*_e$ is selected. Signals corresponding to $\eta_c$ and $J/\psi$ production are seen clearly in the mass spectrum. The $J/\psi$ events are from the process $e^+e^- \rightarrow e^+e^- J/\psi$ (see Fig. 2(b)). The shaded histogram in Fig. 7 shows the distribution for candidates with the wrong sign of the $e\eta_c$ momentum.
z-component. Since the numbers of events from $e^+e^-$ annihilation in the wrong- and right-sign data samples are expected to be approximately the same, this shows that the background from $e^+e^-$ annihilation peaking at the $\eta_c$ mass is small.

IV. FITTING THE $K_S K^\pm \pi^\mp$ MASS SPECTRUM FOR NO-TAG EVENTS

The mass spectrum for no-tag events exhibits the $\eta_c$ and $J/\psi$ peaks corresponding to the two-photon and ISR production mechanisms, respectively. The c.m. momentum $p^*$ of the $K_S K^\pm \pi^\mp$ system for ISR events is equal to $(\sqrt{s}/2)(1 - M^2_{K\pi}/s)$. In the mass region under study the detector acceptance strongly limits the efficiency for ISR events. Due to the asymmetry of the acceptance most of the detected ISR events have positive $\cos \theta^*_{\eta_c}$. It follows that the ISR events can be selected by requiring:

$$p^*/(1 - M^2_{K\pi}/s) > 5.1 \text{ GeV}/c, \ \cos \theta^*_{\eta_c} < 0.\ (4)$$

The mass distribution for the events satisfying this condition is shown by dashed histogram in Fig. 7. The selected event sample contains mostly ISR events with very little two-photon $\eta_c$ admixture. The $K_S K^\pm \pi^\mp$ mass distribution for the events not satisfying Eq. (4) is shown by the solid histogram in Fig. 7. The remaining $J/\psi$ events are from the ISR process with more than one photon emitted from the initial state.

To determine the $\eta_c$ mass and width, and the number of events containing an $\eta_c$, a binned likelihood fit is made to the distributions in Fig. 7 using a function consisting of signal ($\eta_c$ and $J/\psi$) and background distributions. The bin size used in the fit is chosen to be 2.5 MeV/$c^2$. The $J/\psi$ line shape is represented by the detector resolution function for ISR events. The $\eta_c$ line shape is described by a Breit-Wigner function convolved with the detector resolution function corresponding to two-photon production. In each case, the detector resolution function is obtained using MC simulation of the detector response. We use the nonrelativistic Breit-Wigner form

$$\frac{(\Gamma/2)^2}{(M_0 - M)^2 + (\Gamma/2)^2};\ (5)$$

where $M$ is the $K_S K^\pm \pi^\mp$ invariant mass, and $M_0$ and $\Gamma$ are the $\eta_c$ mass and width. The changes in the values of the parameters, if a relativistic Breit-Wigner function is used, are negligible.

The detector resolution functions are determined from the distributions of the difference between measured and true simulated $K_S K^\pm \pi^\mp$ mass for the processes $e^+e^- \rightarrow e^+e^- \eta_c$, $\eta_c \rightarrow K_S K^\pm \pi^\mp$ and $e^+e^- \rightarrow J/\psi \gamma$, $J/\psi \rightarrow K_S K^\pm \pi^\mp$ shown in Fig. 8(a) and Fig. 8(b), respectively. The distributions are fit with the following function:

$$F(x) = A[G(x) \sin^2 \zeta + B(x) \cos^2 \zeta],\ (6)$$

where

$$G(x) = \exp \left(-\frac{(x - a_1)^2}{2\sigma^2}\right),\ (7)$$

$$B(x) = \begin{cases} \frac{(\Gamma/2)^2}{(x-a_1)^2 + (\Gamma/2)^2} & \text{if } x < a_2; \\ \frac{(x-a_2)^2 + (\Gamma/2)^2}{(x-a_2)^2 + (\Gamma/2)^2} & \text{if } x \geq a_2, \end{cases}\ (8)$$

$A$, $\zeta$, $a_1$, $\sigma$, $a_2$, $\Gamma_1$, $\beta_1$, $\Gamma_2$, $\beta_2$ are free fit parameters. The $B(x)$ term is added to the Gaussian function to describe the asymmetric power-law tails of the $\Delta m$ distributions.

When used in data, the resolution $\sigma$ in the Gaussian term of Eq. (5), is modified to take into account a possible difference between data and simulation:

$$\sigma^2 = \begin{cases} \sigma_{MC}^2 - \Delta \sigma^2 & \text{if } \Delta \sigma < 0; \\ \sigma_{MC}^2 + \Delta \sigma^2 & \text{if } \Delta \sigma \geq 0. \end{cases}\ (9)$$
The distribution of the difference ($\delta m$) between measured and true $K_S K^\pm \pi^\mp$ mass for simulated events (a) for the two-photon process $e^+ e^- \rightarrow e^+ e^- \eta_c$, and (b) for the ISR process $e^+ e^- \rightarrow J/\psi \gamma$. The curves correspond to fits defined in the text.

The parameter $\sigma_{MC}$ is found to be 7.8 MeV/c$^2$ for the $J/\psi$ and 7.6 MeV/c$^2$ for the $\eta_c$. The parameter $\Delta \sigma$ is determined from the fit to the measured $K_S K^\pm \pi^\mp$ mass spectra.

The background distribution is described by a second-order polynomial. Both spectra shown in Fig. 8 are fit simultaneously with 14 free parameters: the $J/\psi$ peak position ($m_{J/\psi}$), the difference between the $J/\psi$ and $\eta_c$ mass values ($\Delta m$), the $\eta_c$ width ($\Gamma$), the numbers of $\eta_c$ and $J/\psi$ events, $\Delta \sigma$, and the background parameters for both spectra. The fitted curves are shown in Fig. 10. For the full mass range, 2.7–3.3 GeV/c$^2$, the $\chi^2$ values corresponding to the $\eta_c$ and $J/\psi$ distributions (solid and dashed in Fig. 8) are 230 and 198, respectively, for a total number of degrees of freedom $2 \times 240 - 14$. The resulting parameter values are as follows:

$$
\begin{align*}
\Delta m &= 114.7 \pm 0.4 \text{ MeV/c}^2, \\
\Gamma &= 31.7 \pm 1.2 \text{ MeV}, \\
N_{\eta_c} &= 14450 \pm 320, \\
\Delta \sigma &= -0.4 \pm 2.5 \text{ MeV/c}^2, \\
m_{J/\psi} &= 3095.8 \pm 0.3 \text{ MeV/c}^2.
\end{align*}
$$

The mass resolution for the $J/\psi$ in data is found to be consistent with the prediction of MC simulation. The fitted value of the $J/\psi$ mass is shifted by $-1.1 \pm 0.3$ MeV/c$^2$ from the nominal $J/\psi$ mass value $[2]$. It is assumed that this mass scale shift does not affect the mass difference $\Delta m$. Since the momentum distributions for $J/\psi$ and $\eta_c$ events are different, and the MC simulation of the detector response is not perfect, we test this assumption as follows. The no-tag event sample was divided into three subsamples with approximately equal statistics but with different laboratory $z$ momentum ($p_z$) of $\eta_c$ candidates. The average $p_z$ values in the subsamples are 3.2, $-0.4$, and $-1.3$ GeV/c, while the $J/\psi$ momentum is peaked at $p_z = -2.34$ GeV/c. The fitted values of the $\Delta m$ parameter for the three subsamples are found to be shifted relative to the nominal fit value by $0.5 \pm 0.6$, $-0.6 \pm 0.6$, and $0.2 \pm 0.6$ MeV/c$^2$, respectively. We do not observe any significant dependence of the $\Delta m$ parameter on the $\eta_c$ momentum direction and absolute value. Nevertheless, the shift value at the maximum difference between the $\eta_c$ and $J/\psi$ momenta, 0.5 MeV/c$^2$, is taken as an estimate of the $\Delta m$ systematic uncertainty due to the difference of the $J/\psi$ and $\eta_c$ momentum distributions.

To estimate the uncertainty of the fit parameters due to the assumed background shape, the second-order polynomial describing background in Fig. 10(a) is replaced by an exponential function, and the changes in the parameter values are considered to be measures of their associated systematic uncertainties. This yields $\Delta \Gamma = 0.8$ MeV and $\Delta N_{\eta_c} = 400$.

The MC simulation uses a phase space distribution for $\eta_c \rightarrow K_S K^\pm \pi^\mp$ decay. This can lead to distortion of the resolution function and a systematic change in the detection efficiency determined from simulation. In order to address this issue, a study of the Dalitz plot distribution for $\eta_c \rightarrow K_S K^\pm \pi^\mp$ decay was performed. The Dalitz plots for data events from the $\eta_c$ signal ($2.94 < M_{K\bar{K}} < 3.02$ GeV/c$^2$) and sideband ($2.90 < M_{K\bar{K}} < 2.94$ GeV/c$^2$ and $3.02 < M_{K\bar{K}} < 3.06$ GeV/c$^2$) regions are shown in Fig. 11 and their projections are shown in Fig. 12.

Figure 13 shows the distributions of $\cos \theta_K$, where $\theta_K$ is the charged-kaon polar angle in the $K_S K^\pm \pi^\mp$ rest frame, for events from the $\eta_c$ region after background subtraction, and for events from the sidebands with the $\eta_c$ contribution subtracted. The $\cos \theta_K$ distribution for $\eta_c$ events closely resembles the distribution in the signal.
FIG. 10: The \(K_S K^\pm \pi^\mp\) invariant mass distribution and fitted curve for no-tag data events that (a) fail Eq. (4), and (b) satisfy Eq. (4) for ISR event selection.

FIG. 11: The Dalitz plots for data events from (a) the \(\eta_c\) signal region \(2.94 < M_{K\bar{K}\pi} < 3.02\) GeV/\(c^2\), and (b) the combined sideband regions, \(2.90 < M_{K\bar{K}\pi} < 2.94\) GeV/\(c^2\) and \(3.02 < M_{K\bar{K}\pi} < 3.06\) GeV/\(c^2\).

MC simulation shown by the dashed histogram in Fig. 13.

Since the MC simulation uses a phase space decay model, this suggests that in the \(K_S K^\pm \pi^\mp\) mode the \(\eta_c\) decays predominantly via the scalar \(K_0^*(1430)\) meson, i.e., \(\eta_c \rightarrow K_0^*(1430) \bar{K}\).

It should be noted that a significant part of the non-\(\eta_c\) background comes from \(\gamma \gamma \rightarrow K_S K^\pm \pi^\mp\), and so can interfere with the signal process \(\gamma \gamma \rightarrow \eta_c \rightarrow K_S K^\pm \pi^\mp\). From the Dalitz plot, the background process seems to proceed mainly via the intermediate \(K^*(892)\bar{K}\) and \(K^*(1430)\bar{K}\) states, where the \(K^*(1430)\) may be either the scalar (\(K_0^*(1430)\)) or tensor (\(K_2^*(1430)\)) state. Interference between the \(I = 0\) and \(I = 1\) background amplitudes appears to lead to the suppression of neutral \(K^*(892)\) production (observed in Fig. 12(a) with respect to Fig. 12(b)). The \(\eta_c\) signal distribution in Fig. 13 actually represents the \(\cos \theta_K\) dependence of the detector acceptance. The \(\cos \theta_K\) distribution for non-\(\eta_c\) events corrected for this acceptance will be peaked near \(\pm 1\). This means that the kaon in continuum is dominantly produced with large orbital momentum. Therefore, interference between the \(S\)-wave \(\eta_c\) decay amplitude and the \(\gamma \gamma \rightarrow K_S K^\pm \pi^\mp\) nonresonant amplitude is expected to be small.

To estimate possible shifts of the \(\eta_c\) parameter values due to interference, an interference term is introduced.
into the fitting function through the following:

$$\frac{\Gamma/2}{M_0 - M - i\Gamma/2} + A\sqrt{\frac{P_2(M)}{P_2(M_0)}}^2,$$

where the Breit-Wigner function describes the \(\eta_c\) amplitude, \(P_2(M)\) is a second-order polynomial describing the mass dependence of the nonresonant intensity, and \(A\) is the value of this amplitude at the \(\eta_c\) mass. The \(P_2(M)\) coefficients are chosen to be equal to the coefficients of the second order polynomial describing the nonresonant background. A comparison of the Dalitz plot distributions for \(\eta_c\) and non-\(\eta_c\) events indicates that the maximal interference should occur in the vicinity of \(M^2(K\pi) \approx 2\) GeV\(^2/c^4\), where in both signal and background the quasi-two-body final states, \(K_0^{*}(1430)\bar{K}\) and \(K_1^{*}(1430)\bar{K}\), dominate. Therefore, no significant phase shift is expected between the two amplitudes, and so parameter \(A\) is chosen to be real. From the fit, \(A = 0.03 \pm 0.01\) and there are insignificant changes in the values of parameters \(\Gamma\) and \(N_{\eta_c}\). The value of \(\Delta m\) changes by 1.5 MeV/c\(^2\), and so this is considered to provide an estimate of the systematic uncertainty due to possible interference between the \(\eta_c\) and nonresonant amplitudes.

To take into account the difference between data and simulation of the \(\eta_c \rightarrow K_S K^{\pm} \pi^\mp\) decay dynamics, the Dalitz plot is divided into 26 cells as shown in Fig. 11(a). For each cell the \(K_S K^{\pm} \pi^\mp\) mass spectrum is fit using the fitting function described above, and the number of \(\eta_c\) events is determined. This experimental Dalitz plot distribution corrected for detection efficiency is used to reweight the signal simulation. The reweighting changes the resolution function and the fit parameter values insignificantly.

Thus, from the fit to the \(K_S K^{\pm} \pi^\mp\) mass spectrum for no-tag events the following \(\eta_c\) parameter values are obtained:

$$\Delta m = 114.7 \pm 0.4 \pm 1.6\text{ MeV}/c^2,\quad \Gamma = 31.7 \pm 1.2 \pm 0.8\text{ MeV},\quad N_{\eta_c} = 14450 \pm 320 \pm 400.$$  

(12)

When the nominal value of the \(J/\psi\) mass is used, the \(\eta_c\) mass becomes 2982.2 \pm 0.4 \pm 1.6 MeV/c\(^2\). The results for the mass and width are in agreement with the previous BABAR measurement obtained using 88 fb\(^{-1}\) data: \(m_{\eta_c} = 2982.5 \pm 1.1 \pm 0.9\) MeV/c\(^2\) and \(\Gamma = 34.3 \pm 2.3\) MeV. The systematic uncertainty reported here is greater than that reported in the previous analysis as we are now allowing for the possibility that there exists a \(J^P = 0^-\) continuum \(K_S K^{\pm} \pi^\mp\) amplitude which interferes with the \(\eta_c\) amplitude.
A function consisting of a sum of binned likelihood fit is performed to the spectrum using $\eta$ over background. To determine the number of $K_S^0 K_L^0$ events, a fitting procedure similar to that described above is applied in each of the eleven $Q^2$ intervals. The number of $\eta_c$ events obtained from the fit using a quadratic background is listed for each $Q^2$ interval in Table I. The difference between the fit results for the two background hypotheses is used as an estimate of the systematic uncertainty associated with the assumed background form.

VI. PEAKING BACKGROUND ESTIMATION AND SUBTRACTION

Background containing true $\eta_c$'s might arise from $e^+e^-\rightarrow J/\psi \eta_c$ annihilation processes and two-photon processes with higher multiplicity final states. The processes with a $J/\psi$ in the final state considered in previous sections are also sources of peaking background because of the relatively large branching fraction for the decay $J/\psi \rightarrow \eta_c \gamma$. For no-tag events the most discriminating variable between signal and background is the $\eta_c$ candidate transverse momentum ($p_T^{\eta_c}$). In particular it is expected that background from $e^+e^-\rightarrow J/\psi \eta_c$ annihilation increases rapidly with transverse momentum. Figure 16 shows the $p_T^{\eta_c}$ distribution for no-tag data events containing an $\eta_c$. In each $p_T^{\eta_c}$ interval the number of $\eta_c$ events is determined from the fit to the $K_S^0 K_L^0$ mass spectrum. It is seen that the data distribution is in good agreement with signal MC simulation. A conservative upper limit on the level of $e^+e^-\rightarrow J/\psi \eta_c$ background is obtained by fitting a sum of the MC signal distribution and a constant background to the data $p_T^{\eta_c}$ distribution. The number of background events with $p_T^{\eta_c} < 0.25$ GeV/$c$ is found to be $110 \pm 150$.

The two-photon background from the process $e^+e^-\rightarrow 2\gamma(e^-\eta_c \pi^0 \pi^0)$ is studied using a special selection. From the sample of events satisfying preliminary selection criteria, events with two or more extra photons are selected with the energy of each photon required to be greater than 50 MeV. Two photons with total energy greater than 0.2 GeV, and invariant mass in the range 0.07–0.20 GeV are taken from the fit to the mass spectrum for single-tag data events with $2 < Q^2 < 50$ GeV.$^2$. The solid curve is the fit result. The dashed curve represents non-peaking background.

FIG. 14: The $K_S^0 K_L^0$ mass spectrum for single-tag data events with $2 < Q^2 < 50$ GeV.$^2$. The number of $\eta_c$ events, a binned likelihood fit is performed to the spectrum using a function consisting of a sum of $\eta_c$, $J/\psi$, and background distributions. The bin size used in the fit is chosen to be 2.5 MeV/$c^2$. The mass resolution line shape is described by the function of Eq. (11) with parameters determined from the signal simulation reweighted to reproduce the $Q^2$ dependence observed in data. This resolution function and its convolution with a Breit-Wigner function are used to describe the $J/\psi$ and $\eta_c$ line shapes, respectively. The background distribution is described by either a second order polynomial or an exponential function. The fit result is shown in Fig. 14 for a quadratic background. The fitted $\eta_c$ parameter values, $\Delta m = 111.2 \pm 2.0$ MeV/$c^2$ and $\Gamma = 31.9 \pm 4.3$ MeV, are in agreement with the results obtained for no-tag events, and the total $\eta_c$ signal is $530 \pm 41$ events. The difference in signal yield for the two background hypotheses is 17 events.

A fitting procedure similar to that described above is applied in each of the eleven $Q^2$ intervals indicated in Table I. The parameters of the mass resolution function are taken from the fit to the mass spectrum for simulated events in the corresponding $Q^2$ interval. The $\eta_c$ and $J/\psi$ masses are fixed at the values obtained from the fit to the spectrum of Fig. 14 while the $\eta_c$ width is taken from the fit to the no-tag data. The free parameters in the fit are the numbers of $\eta_c$ and $J/\psi$ events, and two or three additional parameters depending upon the description of the background shape (quadratic or exponential). The $K_S^0 K_L^0$ mass spectra and fitted curves (quadratic background) for three representative $Q^2$ intervals are shown in Fig. 14. The number of $\eta_c$ events obtained from the fit using a quadratic background is listed for each $Q^2$ interval in Table I. The difference between the fit results for the two background hypotheses is used as an estimate of the systematic uncertainty associated with the assumed background form.
FIG. 15: The $K_\pm K^\pm \pi^\mp$ mass spectra for single-tag data events from three representative $Q^2$ intervals. In each figure the solid curve represents the fit result, and the dashed curve indicates non-peaking background.

TABLE I: The $Q^2$ interval, number of events with $\eta_c$ obtained from the fit ($N_{\eta_c}$), number of background events from $J/\psi \rightarrow \eta_c \gamma$ decay ($N_{bkg}$), efficiency correction ($\delta_{\text{total}}$), and number of signal events corrected for data/MC difference and resolution effects ($N_{\text{corr}}^{\text{unfolded}}$). The quoted errors on $N_{\eta_c}$ and $N_{\text{corr}}^{\text{unfolded}}$ are statistical and systematic.

| $Q^2$ interval (GeV$^2$) | $N_{\eta_c}$       | $N_{bkg}$       | $\delta_{\text{total}}$(%) | $N_{\text{corr}}^{\text{unfolded}}$ | $\varepsilon$(%) |
|--------------------------|--------------------|-----------------|-----------------------------|-------------------------------------|------------------|
| no-tag                   | 14450 ± 320 ± 400  | 730 ± 240       | -2.6                        | 14090 ± 330 ± 480                   | 14.5             |
| 2-3                      | 41.0 ± 8.6 ± 1.3   | 0.7 ± 0.4       | -0.1                        | 39.9 ± 9.0 ± 1.4                    | 2.0              |
| 3-4                      | 56.2 ± 10.5 ± 4.0  | 0.6 ± 0.4       | 0.0                         | 55.3 ± 10.9 ± 4.2                   | 4.9              |
| 4-5                      | 65.0 ± 10.9 ± 1.1  | 0.1 ± 0.4       | -0.1                        | 64.8 ± 11.5 ± 1.2                   | 9.1              |
| 5-6                      | 52.6 ± 9.6 ± 0.6   | 0.5 ± 0.4       | -0.4                        | 51.8 ± 10.3 ± 0.8                   | 12.1             |
| 6-8                      | 90.9 ± 12.2 ± 4.6  | 1.3 ± 0.8       | -0.4                        | 90.3 ± 12.8 ± 4.9                   | 14.0             |
| 8-10                     | 60.9 ± 10.9 ± 2.8  | 0.9 ± 0.6       | -0.8                        | 61.3 ± 11.7 ± 3.1                   | 17.9             |
| 10-12                    | 34.8 ± 7.3 ± 1.8   | 1.0 ± 0.6       | -1.0                        | 33.5 ± 7.9 ± 2.1                    | 21.4             |
| 12-15                    | 42.3 ± 8.7 ± 2.1   | 1.9 ± 0.8       | -1.3                        | 41.2 ± 9.4 ± 2.4                    | 23.0             |
| 15-20                    | 45.5 ± 7.9 ± 1.0   | 2.4 ± 1.0       | -1.0                        | 44.3 ± 8.5 ± 1.5                    | 23.8             |
| 20-30                    | 23.7 ± 6.6 ± 0.6   | 1.6 ± 0.7       | -1.0                        | 22.5 ± 6.9 ± 1.0                    | 24.7             |
| 30-50                    | 10.8 ± 4.5 ± 0.1   | 0.9 ± 0.5       | -1.3                        | 10.3 ± 4.8 ± 0.5                    | 24.5             |

FIG. 16: The distribution of the $\eta_c$ candidate transverse momentum for no-tag data events containing an $\eta_c$ (points with error bars), and for simulated signal events (histogram).

FIG. 17: The $\pi^0$ candidate mass spectrum for the selected $e^+e^- \rightarrow e^+e^- K_\pm K^\pm \pi^\mp \pi^0$ data events in no-tag mode.
the two-photon mass spectrum, nor is there an \( \eta_c \) signal in the \( K_S K^{\pm} \pi^{\mp} \) mass spectrum. The \( e^+ e^- \to e^+ e^- \eta_c \eta_c \) background is therefore considered negligible.

Background from both sources, \( e^+ e^- \) annihilation and two-photon processes, does not exceed 490 events (90% CL), i.e., 3.5% of the total number of \( \eta_c \) events. This number is considered to provide an estimate of the systematic uncertainty due to possible \( e^+ e^- \) annihilation and two-photon background.

The total number of \( e^+ e^- \to J/\psi \gamma, J/\psi \to K_S K^{\pm} \pi^{\mp} \) events found in the no-tag event sample is 3170 \( \pm \) 100. From simulation the ratio of the detection efficiencies \( \varepsilon(e^+ e^- \to J/\psi \gamma, J/\psi \to \eta_c \gamma, \eta_c \to K_S K^{\pm} \pi^{\mp})/\varepsilon(e^+ e^- \to J/\psi \gamma, J/\psi \to K_S K^{\pm} \pi^{\mp}) \) is found to be 1.18 \( \pm \) 0.01. Taking into account the ratio of the branching fractions

\[
b = \frac{B(J/\psi \to \eta_c \gamma) B(\eta_c \to K K \pi)}{B(J/\psi \to K K \pi)} = 0.20 \pm 0.07 \quad (13)
\]

the corresponding background contribution to the \( \eta_c \) peak from the ISR process is found to be 730 \( \pm \) 240 events.

For single-tag events, the background from \( e^+ e^- \) annihilation can be estimated using events with the wrong sign of the \( e^+ \eta_c \) momentum \( z \)-component. The \( K_S K^{\pm} \pi^{\mp} \) mass spectrum for the wrong-sign events is shown in Fig. 17 together with the spectrum for right-sign events. Assuming that the numbers of background events for the wrong- and right-sign data samples are approximately the same, a fit to the mass spectrum for wrong-sign events yields 1.4 \( \pm \) 3.0 \( e^+ e^- \) annihilation events peaking at the \( \eta_c \) mass.

To estimate the two-photon background in the single-tag event sample, the result obtained for no-tag events is used. Assuming that the signal and background \( Q^2 \) dependences are approximately the same, it is estimated that the number of two-photon background events is 5.7 \( \pm \) 3.0. The total background from \( e^+ e^- \) annihilation and two-photon processes does not exceed 13 events (90% CL), or 2.5% of the total number of \( \eta_c \) events in the single-tag event sample. This number is considered to provide an estimate of the systematic uncertainty due to possible \( e^+ e^- \) annihilation and two-photon background.

The background from the process \( e^+ e^- \to e^+ e^- J/\psi \) (Fig. 2b)), \( J/\psi \to \eta_c \gamma, \eta_c \to K_S K^{\pm} \pi^{\mp} \) is estimated from the measured \( Q^2 \) distribution \( (N_{J/\psi,i}) \) for the \( e^+ e^- \to e^+ e^- J/\psi, J/\psi \to K_S K^{\pm} \pi^{\mp} \) events as \( \kappa_i b N_{J/\psi,i} \), where \( b \) is the ratio of the branching fractions defined in Eq.(13), and \( \kappa_i \) is the ratio of the detection efficiencies for the \( J/\psi \to \eta_c \gamma, \eta_c \to K_S K^{\pm} \pi^{\mp} \) and \( J/\psi \to K_S K^{\pm} \pi^{\mp} \) decay modes. The coefficient \( \kappa_i \) varies from 0.7 to 0.5 in the \( Q^2 \) range of interest. The estimated background contributions resulting from \( J/\psi \to \eta_c \gamma \) decay are listed in Table II. The fraction of background in the \( e^+ \eta_c \) data sample changes from about 1.0% for \( Q^2 < 10 \text{ GeV}^2 \) to about 5% at \( Q^2 \approx 30 \text{ GeV}^2 \).

VII. DETECTION EFFICIENCY

The detection efficiency is determined from MC simulation as the ratio of the true \( Q^2 \) distributions computed after and before applying the selection criteria. The \( Q^2 \) dependence of the detection efficiency is shown in Fig. 19. The detector acceptance limits the detection efficiency at small \( Q^2 \). The cross section is measured in the region \( Q^2 > 2 \text{ GeV}^2 \) where the detection efficiency is greater than 2%. The asymmetry of the \( e^+ e^- \) collisions at PEP-
leading to different efficiencies for events with electron and positron tags. The \( Q^2 \) range from 2 to 6 GeV\(^2\) is measured only with the positron tag. For no-tag events the detection efficiency is 0.1446 ± 0.0023. The efficiency is calculated using simulated events reweighted according to the Dalitz plot distribution observed in data. For no-tag events the relative difference between detection efficiencies calculated with and without weighting is found to be −(1.1 ± 1.6)%. The quoted error is determined by the statistical errors of the measured Dalitz plot distribution. The corresponding relative difference for single-tag events is shown in Fig. 20 as a function of \( Q^2 \). The detection efficiency has only a very weak dependence on the dynamics of \( \eta_c \) decay.

Possible sources of systematic uncertainty due to differences between data and simulated detector response are now considered. For no-tag events the MC simulation predicts a significant loss of signal events, \( (5.3 ± 0.1)\% \), due to background filters used in event reconstruction. The filter inefficiency can be measured in data using a special sample of prescaled events that does not pass the background filters. The filter inefficiency obtained in data is \( (7.5 ± 1.2)\% \). The difference \( \delta = -(2.2 ± 1.2)\% \) is used to correct the number of signal events. For single-tag events the presence of the additional electron leads to a significantly smaller filter inefficiency. The simulation predicts a filter inefficiency of \( (0.57 ± 0.02)\% \), which is about ten times smaller than for no-tag events. We conclude that this source of systematic uncertainty is negligible for single-tag events.

To study the possible systematics for no-tag events due to selection criteria, the mass window for the \( K_S \) candidate is increased from 487.5–507.5 MeV/c\(^2\) to 475.0–520.0 MeV/c\(^2\), the limit on the \( \eta_c \) candidate transverse momentum is changed from 0.25 to 0.50 GeV/c, and the polar angle restriction is set to \( 0.387 < \theta < 2.400 \) rad for all four charged-particle tracks in an event. The last mod-

FIG. 20: The \( Q^2 \) dependence of the relative difference between detection efficiencies determined from MC simulation with and without Dalitz-plot reweighting.

FIG. 21: The \( Q^2 \) dependence of the data-MC double ratio for events with and without the polar angle restriction \( 0.387 < \theta < 2.400 \) rad for all four charged-particle tracks from the \( \eta_c \) decay.

FIG. 22: The correction to the MC-estimated kaon identification efficiency as a function of \( Q^2 \).

FIG. 23: The correction to the MC-estimated electron identification efficiency as a function of \( Q^2 \).

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The efficiency correction for no-tag events is 6.2%. The data-MC simulation difference in the kaon identification is estimated using the mass window (0.993 ± 0.005), and for the condition on $p_T^e$ (1.002 ± 0.009). A significant deviation from unity (5.9 ± 1.8)% is observed for the polar angle restriction. This deviation is taken as an estimate of the systematic uncertainties due to imperfect simulation of the $\eta_c$ selection criteria close to the limits of the fiducial tracking region.

The systematics due to a possible difference between the data and simulation in the charged-particle track reconstruction for pions and kaons is estimated to be about 0.35% per track, so that the total uncertainty is estimated to be 1.4%. The data-MC simulation difference in the kaon identification is estimated using the identification efficiencies measured for kaons from the $D^{*+} \rightarrow D^0\pi^+$, $D^0 \rightarrow \pi^+K^-$ decay sequence. The efficiency correction is found to be −0.4%. The systematic uncertainty associated with this correction is taken to be equal to the value of the correction, 0.4%. The total efficiency correction for no-tag events is −2.6%, and the total systematic error associated with the efficiency is about 6.2%.

The polar angle restriction described above is also tested for single-tag events, and Fig. 24 shows the $Q^2$ dependence of the data-MC double ratio. No significant $Q^2$ dependence is observed, and the average value is 1.057 ± 0.032, which is very close to the value for no-tag events. For single-tag events the systematic uncertainty due to the $\eta_c$ selection criteria is estimated to be 5.7%. The efficiency correction ($\delta_1$) for kaon identification for single-tag events is shown in Fig. 24 as a function of $Q^2$, and this results in an associated systematic uncertainty of 0.5%.

The data-MC simulation difference in electron identification is estimated using the identification efficiencies measured for electrons from radiative Bhabha events. The efficiency correction ($\delta_2$) is shown as a function of $Q^2$ in Fig. 25. The associated systematic uncertainty does not exceed 0.5%. The systematic uncertainty due to data-MC simulation difference in the electron track reconstruction is about 0.1%. To estimate the effect of the requirement $-0.02 < r < 0.03$ (see Fig. 3), events with $0.03 < r < 0.06$ are studied. The data-MC simulation difference double ratio defined by Eq. (14) is found to be consistent with unity, $(0.99 ± 0.02)$, so that the simulation reproduces the shape of the $r$ distribution very well.

The main sources of systematic uncertainty associated with detection efficiency are summarized in Table II for the no-tag and single-tag samples. The values of the detection efficiency and total efficiency correction $\delta_{\text{total,i}} \approx \delta_{1,i} + \delta_{2,i}$ for different $Q^2$ intervals are listed in Table II. The data distribution is corrected as follows:

$$N_{\text{corr},i} = N_i/(1 + \delta_{\text{total,i}}),$$

where $N_i = N_{\eta_c,i} - N_{\text{bkg},i}$ is the number of signal events in the $i$th $Q^2$ interval.

**VIII. CROSS SECTION AND FORM FACTOR**

The Born differential cross section for $e^+e^- \rightarrow e^+e^−\eta_c, \eta_c \rightarrow K\bar{K}\pi$ is

$$d\sigma/dQ^2 = \left(\frac{dN/dQ^2}_{\text{unf,corr}}\right) \times \frac{B(\eta_c \rightarrow K\bar{K}\pi)}{B(\eta_c \rightarrow KS\pi^+\pi^-)} ,$$

where $(dN/dQ^2)_{\text{unf,corr}}$ is the mass spectrum corrected for data-MC simulation difference and unfolded for detector resolution effects, explained below, $L$ is the total integrated luminosity, $\varepsilon$ is the $Q^2$-dependent detection efficiency, $R$ is a radiative correction factor accounting for distortion of the $Q^2$ spectrum due to the emission of photons from the initial-state particles, and for vacuum polarization effects. The ratio $B(\eta_c \rightarrow K\bar{K}\pi)/B(\eta_c \rightarrow KS\pi^+\pi^-)$ is expected to be 3 from isospin relations.

The radiative correction factor is determined using simulation at the generator level (with no detector simulation). The $Q^2$ spectrum is generated using only the pure Born amplitude for the $e^+e^- \rightarrow e^+e^-\eta_c$ process, and then using a model with radiative corrections included. The $Q^2$ dependence of the radiative correction factor, evaluated as the ratio of the second spectrum to the first, is shown in Fig. 24 and is fitted using the function $a/(1 + bQ^2)$. The accuracy of the radiative correction calculation is estimated to be 1% [8]. It should be noted that the value of $R$ depends on the requirement on the extra photon energy. The $Q^2$ dependence obtained corresponds to the condition $r = 2E_{\gamma}/\sqrt{s} < 0.1$ imposed in the simulation.

The corrected and unfolded $Q^2$ distribution $(dN/dQ^2)_{\text{corr}}$ is obtained from the measured distribution by dividing by the efficiency correction factor (see Eq. (15)) and unfolding the effect of $Q^2$. 

---

| Source               | No-tag, % | Single-tag, % |
|----------------------|-----------|---------------|
| trigger, filters     | 1.2       | −             |
| $\eta_c$ selection  | 5.9       | 5.7           |
| track reconstruction | 1.4       | 1.5           |
| $K^\pm$ identification | 0.4     | 0.5           |
| $\epsilon^+$ ident  | −         | 0.5           |
| **total**            | 6.2       | 5.9           |
errors on the efficiency correction and MC simulation.
The detection efficiency is determined, with the restrictive correction factor (1%), and the error on the inefficiency (5.9%), background subtraction (2.5%), the results from the systematic uncertainties on the detection only are listed in Table III, where the first error is statistical and systematic errors (by means of the measured distribution. The procedure changes the shape by applying the inverse of the migration matrix to the model cross section on $Q^2$ measured differential cross section is shown in Fig. 25. In the case of extra photon emission, $Q^2_{\text{true}}$ is calculated as $-\left(p-p'-k\right)^2$, where $k$ is the photon four-momentum; $\varepsilon$ and $R$ in Eq. (10) are functions of $Q^2_{\text{true}}$. As the chosen $Q^2$ interval width significantly exceeds the resolution for all $Q^2$, the migration matrix is nearly diagonal, with values of diagonal elements $\sim 0.95$, and of the next-to-diagonal $\sim 0.02$. The true $Q^2$ distribution is obtained by applying the inverse of the migration matrix to the measured distribution. The procedure changes the shape of the $Q^2$ distribution insignificantly, but increases the errors (by $\approx 10\%$) and their correlations. The corrected $Q^2$ spectrum ($\Delta N_{\text{unfolded}}$) is listed in Table II.

The values of the differential cross sections obtained are listed in Table III, where the first error is statistical and the second systematic. The latter includes only $Q^2$-dependent errors, namely, the systematic uncertainty in the number of signal events and the statistical errors on the efficiency correction and MC simulation. The $Q^2$-independent systematic error is 6.6%; this results from the systematic uncertainties on the detection efficiency (5.9%), background subtraction (2.5%), the radiative correction factor (1%), and the error on the integrated luminosity (1%) combined in quadrature. The MC simulation for single-tag events is performed, and the detection efficiency is determined, with the restriction that the momentum transfer to the untagged electron be greater than $-1$ GeV$^2$, so that the cross section is measured for the restricted range $|q^2| < 1$ GeV$^2$. The measured differential cross section is shown in Fig. 25.

Because of the strong nonlinear dependence of the cross section on $Q^2$, the value of $Q^2$ corresponding to the measured cross section differs slightly from the center of the $Q^2$ interval. The measured cross section is described by a smooth function, which is then used to reweight the simulated $Q^2$ distribution and calculate the weighted average value $(\bar{Q}^2)$ for each $Q^2$ interval. The values of $\bar{Q}^2$ obtained are listed in Table IV.

The no-tag event sample is used to obtain the total cross section for the reaction $e^+e^- \rightarrow e^+e^- \eta_c$, $\eta_c \rightarrow K\bar{K}\pi$:

$$\sigma_{\text{total}} = \frac{N_{\text{corr}}}{\varepsilon L} \frac{B(\eta_c \rightarrow K\bar{K}\pi)}{B(\eta_c \rightarrow K\bar{S}\pi^\pm\pi^\mp) B(K\bar{S} \rightarrow \pi^+\pi^-)}.$$  

In the no-tag mode the radiative correction is expected to be less than 1% [12], and so the associated systematic uncertainty is assigned conservatively as 1%. Taking $N_{\text{corr}}$ and $\varepsilon$ from Table II, the value obtained from Eq. (18) is

$$\sigma_{\text{total}} = 0.900 \pm 0.021 \pm 0.074 \text{ pb.}$$  

The quoted errors are statistical and systematic, respectively. The latter is 8.1% and includes the systematic uncertainty in the number of signal events (3.3%), the statistical and systematic errors on the detection efficiency (1.6% and 6.2%, respectively), on the background subtraction (3.5%), on the radiative correction (1%), and the error on the integrated luminosity (1%). Using MC simulation, the calculated total cross section corresponding to $\Gamma(\eta_c \rightarrow \gamma\gamma) B(\eta_c \rightarrow K\bar{K}\pi) = 1$ keV is found to be 2.402 pb, and hence from Eq. (19) the value

$$\Gamma(\eta_c \rightarrow \gamma\gamma) B(\eta_c \rightarrow K\bar{K}\pi) \approx 0.374 \pm 0.009 \pm 0.031 \text{ keV.}$$  

![Figure 24: The $Q^2$ dependence of the radiative correction factor.](image-url)
is obtained. This result agrees with the Particle Data Group value 0.44 ± 0.05 keV [7], and also with the recent CLEO measurement 0.407 ± 0.022 ± 0.028 keV [13].

To extract the transition form factor, the measured and calculated $Q^2$ spectra are compared. The simulation for single-tag events uses a constant form factor value, so that the measured normalized form factor is determined from

$$|F(Q^2)/F(0)| = \frac{\langle d\sigma/dQ^2 \rangle_{\text{data}}}{N_{\text{data}}^{\text{no-tag}}} \times \frac{\varepsilon_{\text{no-tag}}^{\text{MC}}}{\varepsilon_{\text{single-tag}}^{\text{MC}}}. \tag{21}$$

The normalized form factor is proportional to the ratio of the number of single-tag events to the number of no-tag events. It is expected that part of the systematic uncertainty, in particular that associated with the detection efficiency, cancels in this ratio. However, the single-tag data sample is not large enough for a detailed study of the $Q^2$-dependence of the observed data-MC simulation difference in detector response. Therefore, this uncertainty is estimated conservatively to be equal to the corresponding systematic uncertainty for the single-tag events (6%). The values of the form factor obtained are listed in Table III and shown in Fig. 26 with the statistical and $Q^2$-dependent systematic errors combined.

The $Q^2$-independent systematic error on the form factor is 4.3%; this value combines in quadrature the systematic uncertainty on detection efficiency, the uncertainty on the number of no-tag events, the statistical error on the detection efficiency for no-tag events, the uncertainties associated with the background subtraction, and the uncertainty on the radiative correction.

The form factor data of Fig. 26 are well described by the monopole form

$$|F(Q^2)/F(0)| = \frac{1}{1 + Q^2/\Lambda}. \tag{22}$$

as shown by the solid curve. The corresponding fitted value of the pole parameter $\Lambda$ is

$$\Lambda = 8.5 \pm 0.6 \pm 0.7 \ \text{GeV}^2, \tag{23}$$

where the second quoted error is due to the 4.3% $Q^2$-independent systematic error on the measurements. This value of the pole parameter is in reasonable agreement with that expected from vector dominance, namely $\Lambda = m_{J/\psi}^2 = 9.6 \ \text{GeV}^2$, and in good agreement with the lattice QCD calculation, $\Lambda = 8.4 \pm 0.4 \ \text{GeV}^2$ [8]. The dotted curve in Fig. 26 shows the result of the leading-order pQCD calculation from Ref. 3. The data lie systematically below this prediction, but within the theoretical uncertainty quoted in Ref. 3.

IX. SUMMARY

The reaction $e^+e^- \rightarrow e^+e^-\eta_c$, with $\eta_c \rightarrow K_S K^\pm \pi^\mp$, has been studied in the no-tag and single-tag modes. We measure the following values for the $\eta_c$ mass and width:

$$m_{\eta_c} = 2982.2 \pm 0.4 \pm 1.6 \ \text{MeV}/c^2,$$
$$\Gamma = 31.7 \pm 1.2 \pm 0.8 \ \text{MeV}. \tag{24}$$

These results agree with earlier BaBar measurements [11] and supersede them.
We have also measured the total cross section \( \sigma(e^+e^- \rightarrow e^+e^-\eta_c)B(\eta_c \rightarrow K\bar{K}\pi) \) and differential cross section \( (d\sigma/dQ^2)B(\eta_c \rightarrow K\bar{K}\pi) \). From these data we determine the value

\[
\Gamma(\eta_c \rightarrow \gamma\gamma)B(\eta_c \rightarrow K\bar{K}\pi) = 0.374 \pm 0.009 \pm 0.031 \text{ keV}
\]

and measure the normalized \( \gamma\gamma^* \rightarrow \eta_c \) transition form factor \( |F(Q^2)/F(0)| \) for the momentum transfer range from 2 to 50 GeV\(^2\). The latter is well described by the simple monopole form of Eq.(22) with \( \Lambda = 8.5 \pm 0.6 \pm 0.7 \text{ GeV}^2 \) in agreement with both the vector dominance expectation and the QCD prediction.

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