D-branes as coherent states in the open string channel.

M. Botta Cantcheff ‡
‡ Instituto de Física La Plata, CONICET, UNLP
CC 67, Calles 49 y 115, 1900 La Plata, Buenos Aires, Argentina

Abstract

We show that bosonic D-brane states may be represented as coherent states in an open string representation. By using the Thermo Field Dynamics (TFD) formalism, we may construct a condensed state of open string modes which encodes the information on the D-brane configuration.

We also introduce a construction alternative to TFD, which does not requires to assume thermal equilibrium. It is shown that the dynamics of the system combined with geometric properties of the duplication rules of TFD is sufficient to obtain the thermal states and their analytic continuations in a geometric fashion. We use this approach to show that bosonic D-brane state in the open string sector may also be built as boundary states in a special sense.

Some implications of this study on the interpretation of the open/closed duality and on the kinematic/algebraic structure of an open string field theory are also commented.

1 Introduction

D-branes states may be constructed as boundary states in the closed string Hilbert space by using the so-called world sheet duality [1], however since one may define D-branes as the surfaces where the open strings end, it is natural to ask for this definition also in the open string description. In this sense, it has been shown that some exact classical solutions of Vacuum string field theory, which is a simplification of the Witten’s open string field theory, represent D-branes [2]. Other approaches addressed to define boundary states in the open channel where recently proposed [3]. Apart from this, the possibility of describing D-branes in terms of open string states, in contrast to the traditional approaches (in the closed string channel) is specially motivating since it might shed some light on the open/closed duality, intimately related to the AdS/CFT conjecture.

On the other hand, by virtue of the microscopical description of the black hole entropy [4], one of the most interesting problems concerning D-branes is the development of a model where their thermodynamical properties and microscopical structure be clarified. The framework presented in this article is devoted to the two purposes simultaneously.

The problem with defining D-brane states in the open string channel is that the boundary conditions are imposed on the operators of the theory rather on particular states, which could be interpreted as D-brane states in an open string Fock space. However, we claim that this difficult may be solved in the context of the so called Thermo Field Dynamics (TFD), developed by Takahashi and Umezawa [5, 6, 7, 8, 9, 10], where an identical but fictitious copy of the system is introduced. In this framework, variables and degrees of freedom are duplicated so as the original Hilbert space state, then it seems to be possible to construct a state (or a family of states) to describe an open-string + brane system.

Thermo Field Dynamics is a real time approach to quantum field theory at finite temperature [11, 12]. In this formalism one canonically quantizes the fields as operators on a thermal Hilbert space and the statistical average of an operator Q is defined as its expectation value in a thermal vacuum state:

$\frac{\text{Tr}[Qe^{-\beta H}]}{Z} = \langle 0(\beta) | Q | 0(\beta) \rangle.$

(1)
The Hamiltonian evolution of these thermal fields is given by the operator $H - \tilde{H}$, where $H$ and $\tilde{H}$ denote the Hamiltonian of the original system and its non-physical copy respectively. So the fundamental state encoding the statistical information may be represented as follows

$$|0(\beta)\rangle = Z^{-1/2} \sum_n e^{-\beta E_n/2} |n\rangle \tilde{|n\rangle},$$

(2)

where $|n; \tilde{n}\rangle$ denotes the $n^{th}$ energy eigenvalue of the two systems.

The idea of using TFD to study D-branes at finite temperature came up in Refs. [13, 14, 15, 16, 17, 18], where thermal boundary states are constructed in the closed string channel by considering string coordinates as thermal fields. In contrast, one may compute the free energy of the open string and obtain the self-energy of the D-brane by using the finite temperature dualities [19]. However this method does not provide us the representation of the D-brane as a (thermal) state in the open string channel. The general goal of this work is precisely obtain such D-brane states at arbitrary finite temperature in general.

The paper is organized as follows. In Section 2 we briefly describe the TFD formalism and propose that a Dp-brane states may be built as the fundamental one in a thermal Hilbert space of open strings. We show this statement using the standard representation of boundary states in Subsection 2.1. In Section 3 we develop a geometric approach in order to formulate D-branes as boundary states in the open string channel. Finally, in Section 4 we present the conclusions of this approach and discuss the main consequences and perspectives.

2 Thermo Field Dynamics and D-brane states

Let us consider the thermodynamics of an open string, that is, to consider a bosonic open string in contact with a thermal reservoir at temperature $\beta^{-1}$. The partition function in the canonical ensemble is

$$Z_o(\beta) = \text{tr} e^{-\beta H_o} = \int DX e^{-S_{W_o}[X]}$$

(3)

where the Euclidean, two-dimensional world sheet manifold is $W_o \sim S^1_\beta \times [0, \pi]$ ($\beta$ denotes the circle length) where the path integral is realized by summing over histories of the open string fields $X : W_o \rightarrow M$ satisfying specific boundary conditions which encodes the information about the Dp-brane. Since $\partial W_o = S^1_\beta \cup S^1_{\beta,\pm}$ where the circles $S^1_{\beta,\pm}$ correspond to the points 0, $\pi$ respectively and the corresponding boundary conditions are:

$$X_i |_{\sigma=0,\pi} = x^i_{\pm} \quad i = p + 1, \ldots, 25$$

$$\partial_\sigma X^a |_{\sigma=0,\pi} = 0 \quad a = 0, 1, \ldots, p$$

(4)

(5)

For simplicity, we assume here that both string endpoints belong to the same Dp-brane. If one assumes that an open string interchanges energy-momentum only through its endpoints, then the reservoir should be thought to be placed in the same region that these two points are confined. This is precisely the Dp-brane surface. So a priori, one could be tempted to identify the Dp-brane with the macroscopic system which acts as the thermal reservoir in itself.

Let us observe that the density matrix $\rho = Z^{-1} e^{-\beta H_o}$ describes the thermal (mixed) state the bosonic open string attached to the Dp-brane in equilibrium, and it encodes the full information about this system in the open channel. Therefore, by virtue of the TFD approach, the thermal state given by this density matrix $\rho$ is equivalent to a pure state (the thermal vacuum) in the tensor product of two copies of the quantum Hilbert space of open string, which encloses the quantum and thermal information of the D-brane. In this way, we have a straightforward procedure to construct a representation of a D-brane as a well defined coherent state directly in the open channel and we do not need to use the standard world-sheet transformation [1] to construct the boundary states in the closed picture. For completeness however, in Subsection 2.1, we will show that in fact this thermal vacuum corresponds to a boundary state via the world-sheet correspondence.
As mentioned before, the TFD algorithm consists first in duplicating the degrees of freedom of the system. To this end a copy of the original Hilbert space may be constructed with a set of operators of creation/annihilation that have the same commutation properties as the original ones. The total Hilbert space is the tensor product of the two spaces $\mathcal{H}_s \otimes \mathcal{H}_o$, where in this case $\mathcal{H}_s$ denotes the physical quantum states space of the bosonic open string whose endpoints are in contact with Dp-brane described by $(4, 5)$.

From now on let us adopt the light cone gauge and use the capital indices $I; J; K... = 1, ..., D - 1$ to denote the physical components $(\mu = I, +, -)$ of the string embedding. The general solution, for the open string coordinates satisfying these boundary conditions reads

$$X^I(t, \sigma) = x^I_+(\pi - \sigma)/\pi + x^I_+ (\sigma/\pi) - \sqrt{2}\alpha' \sum_{n \neq 0} \left( \frac{1}{n} \alpha_n^I e^{-int} \sin(n\sigma) \right)$$  \hspace{1cm} (6)

$$X^a(t, \sigma) = x^a + 2\alpha' p^a t + 2i\alpha' \sum_{n \neq 0} \left( \frac{1}{n} \alpha_n^a e^{-int} \cos(n\sigma) \right)$$  \hspace{1cm} (7)

The solution correspondent to the non-physical string $\tilde{X}(\tilde{t}, \tilde{\sigma})$, with the same boundary conditions, may also be expanded in this basis of solutions. Since introducing finite temperature breaks the Lorentz invariance, we consider these solution in the zero-momentum frame: $p^a = \tilde{p}^a = 0$. Next, these fields are canonically quantized and, according to the TFD rules $[20]$ the operators of the two system are built commuting among themselves, so the doubled system is described by two independent strings defining two world-sheets.

The Fourier modes may be redefined as,

$$a^I_n = \frac{\alpha^I_n}{\sqrt{n}}$$ \hspace{1cm} (8)

$$a^{\dagger I}_n = \frac{\alpha^{\dagger I}_n}{\sqrt{n}}$$

so as the tilde oscillators, in order to satisfy the extended algebra:

$$[a^I_n, a^J_m] = [\tilde{a}^I_n, \tilde{a}^J_m] = \delta_{n,m} \delta^{IJ},$$

$$[\tilde{a}^{\dagger I}_n, \tilde{a}^J_m] = [a^{\dagger I}_n, a^J_m] = [a^I_n, \tilde{a}^{\dagger J}_m] = 0.$$  \hspace{1cm} (9)

The standard vacuum in this extended theory is defined by

$$a^I_n \ket{0} = \tilde{a}^I_n \ket{0} = 0,$$ \hspace{1cm} (10)

for $n > 0$ and $\ket{0} = \ket{0} \otimes \ket{0}$ as usual. However, the physical thermal fundamental state shall be obtained from this through a Bogoliubov transformation, $e^{-iG}$, which entangles the states of the two independent Hilbert spaces. This is given by the following relation

$$\ket{0(\theta)} = e^{-iG} \ket{0} = \prod_{n=1}^{D-2} \left( \frac{1}{\cosh(\theta_n)} \right)^{D-2} e^{\tan(\theta_n) \delta_{IJ} a^{\dagger I}_n a^J_n} \ket{0}.$$  \hspace{1cm} (11)

Here $\theta$ denotes the set of transformation parameters. By applying the operator (4) to the state $\ket{0}$ one may see that this state encloses the specific values of the endpoints position, and furthermore this is not modified by the Bogoliubov transformation.

The thermal creation and annihilation are also transformed according to

$$a^I_n(\theta_n) = e^{-iG} a^I_n e^{iG} = \cosh(\theta_n) a^I_n - \sinh(\theta_n) \tilde{a}^{\dagger I}_n$$  \hspace{1cm} (12)

$$\tilde{a}^I_n(\theta_n) = e^{-iG} \tilde{a}^I_n e^{iG} = \cosh(\theta_n) \tilde{a}^I_n - \sinh(\theta_n) a^{\dagger I}_n$$  \hspace{1cm} (13)
As the Bogoliubov transformation is canonical, the thermal operators obey the same commutation (9). These operators annihilate the state written in (11) defining it as the vacuum. By using the Bogoliubov transformation, the relations
\[
a_n^I(\theta_n)|0(\theta)\rangle = \sim a_n^I(\theta_n)|0(\theta)\rangle = 0,
\]
give rise to the so-called thermal state conditions:
\[
[\sim a_n^I(\theta_n) - \tanh(\theta_n)a_n^J] |0(\theta)\rangle = 0,
\]
\[
[a_n^I(\theta_n) - \tanh(\theta_n)\sim a_n^J] |0(\theta)\rangle = 0,
\]
Then, the physical open string Fock space is constructed by applying the thermal creation operators to the vacuum (11) that may consistently be identified with the D-brane state. In fact, if the modes of the open string attached to the D-brane are created from this state, precisely in absence of such string excitations, the D-brane on its own must be associated to the fundamental state.

Finally, the thermal open string vacuum is completely defined by minimizing the free energy
\[
F = U - \frac{1}{\beta} S \tag{17}
\]
with respect to the transformation’s parameters \(\theta\)'s [5]. Here \(U\) is given by computing the matrix elements of the open string Hamiltonian in the thermal vacuum and \(S\) is the expectation value of the entropy operator \(K \equiv -\sum_{n=1} N_n \ln N_n\) in this state. The number operator is defined by
\[
N_n = a_n^I a_n^J \delta_{IJ},
\]
whose expectation value is proportional to \(\sinh^2 \theta_n\), therefore we get
\[
K = -\sum_{n=1} \left\{ a_n^I a_n^J \delta_{IJ} \ln \left( \sinh^2 (\theta_n) \right) - a_n^I a_n^J \delta_{IJ} \ln \left( \cosh^2 (\theta_n) \right) \right\}. \tag{19}
\]
Therefore, the solution for the angular parameters \(\theta_n\) is given by the Bose-Einstein distribution:
\[
\sinh^2 \theta_n = (e^{\beta E_n} - 1)^{-1}. \tag{20}
\]
Notice that, although expressed in the open sector, the thermal state conditions (15, 16) together with the thermal equilibrium requirement (in order to fix the free parameters \(\theta_n\)) determine the state of the system so as a D-brane state is built in the closed channel. So in this sense, we could see (15, 16) as a sort of boundary state condition in the open channel. In Section 5 we will construct a geometrical approach for boundary states in the open channel, where this interpretation arises explicitly.

It is easy to see that thermal states are not eigenstates of the original Hamiltonian but they are eigenstates of the combination:
\[
\widehat{H} = H - \sim H, \tag{21}
\]
in such a way that \(\widehat{H}\) plays the rôle of the Hamiltonian, generating temporal translation in the thermal Fock space. Let us point out that the physical variables are described by the non-tilde operators. This is then the Hamiltonian which governs the dynamical evolution of the D-brane and its excitations, which correspond to an open string attached to it.

The state (11) describes a condensate of entangled open string modes localized on the D-brane surface. Since this is a coherent state, it constitutes a macroscopical object (see reference [8]) which may be identified with the D-brane. So we conclude this part by emphasizing that (11) describes the microscopical structure of the D-brane in terms of open string modes.

**Analytical Continuation:** Let us remark that despite D-brane states have been constructed here at arbitrary finite temperature, the parameter \(\beta\) can be analytically continued to the complex plane and in particular to purely imaginary values, \(\beta = i\lambda\), \(\lambda \in \mathbb{R}\) (see ref. [21]) in order to describe states without temperature. In this context however, the quantity \(\tau\) should be not interpreted as a time evolution parameter in order to describe stationary states, but, as clarified in the construction of Section 5, it could be seen as sort of “time delay” between the physical system and its copy.
2.1 D-branes States from the current Closed String Description

We found a way to represent a D-brane as an state in the open channel and we do not need to transform the problem by going to the closed representation; however by using the world sheet duality [1], one may verify that the thermal vacuum (11) consistently corresponds to a current boundary state in the closed channel.

Let us first motivate our proposal in the context of this duality. In fact, the interaction between two D-branes is given by the vacuum fluctuations of an open string ending on them and propagating in a loop with periodic Euclidean time $t \in [0, \beta]$ (the Casimir effect). Graphically, the topology of this open string world-sheet is a cylinder ending on the two branes. Since the theory is conformally invariant, one can find a conformal transformation such that the world-sheet coordinates are exchanged and the cylinder corresponds to the tree diagram of a boundary state of closed string being created in one of the D-branes, propagated for a while, and annihilated on the other brane. These boundary states are identified with the D-branes states in the closed string channel. The crucial observation of our work is that one may avoid the world sheet transformation in this algorithm, and to recognize the D-brane states in the open sector. The second remarkable point in this observation is that, roughly speaking, the one-loop cylinder diagram in fact characterizes a thermal state by virtue of the Euclidean period. Let us now show this in detail:

Consider a particular initial configuration of a closed string,
\[
\partial_t X^a|_{t=0} = 0 \quad a = 0, 1, \ldots, p
\]
\[
X^i|_{t=0} = x^i \quad i = p + 1, \ldots, 25.
\] (22, 23)
Because the operators $\partial_t X^a$, $X^i$ commute among themselves at the same time, a specific configuration of these constitutes a definite state in the quantum Hilbert space of closed string. This is called a boundary state which is interpreted as the Dp-brane state in itself. It may be expressed as a coherent state of closed string modes and its form is remarkably similar to (11) [1].

The transition amplitude from this state, correspondent to the initial configuration (22, 23) and denoted by $|B_p(t = 0)\rangle$, into a final one $|B_p(t = -i\pi)\rangle$ (defined by conditions similar to (22, 23)) through an imaginary time interval $-i\pi$, is given by
\[
\langle B_p(t = -i\pi)|e^{-\pi H_c}|B_p(t = 0)\rangle = \int_{\Gamma} DX e^{-S_{Wc}[X]}
\] (24)
where $H_c$ is the closed string hamiltonian. This may be represented as a sum over histories as expressed in the right hand side of this identity, where $W_c \sim S^1_\beta \times [0, \pi]$ is the closed world-sheet topology whose boundary are two circles that we denote by $S^1_{\beta, \pm}$. Then $\Gamma$ represents the set of histories of one closed string ending on these two circles whose states are fixed by configurations as (22, 23).

Then, by considering the world sheet transformation $\sigma, t \rightarrow t$, $\sigma, W_c$ transforms into $W_c$, and the above sum over histories coincides with (3). Finally, one straightforwardly obtains the state (11) according to the procedure previously shown, which encodes the information of the boundary state. In fact, this turns out clear that the state $|0(\beta)\rangle$, so as the boundary state $|B_p\rangle$, may both be used to calculate the significant observables/amplitudes in the respective representations, and they are corresponding in the proper sense.

If one inserts any operator $Q$ as in eq. (1), the quantity $\langle 0(\beta)|Q|0(\beta)\rangle$, must be identified with the amplitude
\[
A_{B_p}[Q_c] = \int_{\Gamma[B_p]} DX Q_c e^{-S_{Wc}[X]},
\] (25)
computed in the closed channel. This integral is a sum over all worldsheet embeddings subject to the boundary conditions (22, 23) as discussed above. In particular, if we take $Q$ to be a product of operators $X(\sigma_i, t_i)$, $i = 1, \ldots, n$ valued on a collection of $n$ different world-sheet points, we get the $n$-point thermal Green function of the open string. On the other hand, $Q_c$ corresponds to the same object in the closed

\footnote{Or, according to the case analyzed in this paper, the self-interaction of only one D-brane.}

\footnote{Transition amplitudes between different D-brane states will be considered in a forthcoming paper.}

\footnote{The propagators in the thermal vacuum}
channel, the \( n \)-point correlation function for \( n \) points of the closed string, which are defined by exchange of the world-sheet coordinates: \((\sigma_i, t_i)_{\text{open}} \rightarrow (t_i, \sigma_i)_{\text{closed}}\), according to discussion above. This prescription may be extended to consider products of different derivatives of these operators. This completes the argument on our initial statement.

3 Geometric formulation and boundary states in the open string representation.

The goal of this section is to show that D-brane states may be constructed as boundary states even in the open sector in an appropriate sense. In other words, the boundary state conditions whose solution is \( |B_{\text{open}}\rangle \) may be imposed also in the open string channel as conditions on states rather than operators in a way similar to the Gupta-Bleuler standard procedure. In fact, these conditions consist in a fixing of non-physical variables in terms of physical ones\(^5\) (on their respective spatial boundaries), which is analogous to a gauge fixing. So in this study, we rigorously refer to boundary state in the context of open string in this precise sense; however, we would like to emphasize here that despite these states may be fixed initially\(^6\) and, as argued before they carry the same information as the closed string boundary states, it should be clearly differentiated from the concept of “boundary state in the open string channel” properly introduced in Ref. [3], where such states may actually describe the emission and absorption of the (open) strings by D-branes as the usual boundary state does for the closed strings.

To do this we introduce a purely geometrical approach which only requires the duplication structure of TFD and where the thermodynamic concepts may be ignored. The dynamical information of the system is sufficient to determine these states. This technique is interesting in itself because it seems to be applicable to other situations with boundary conditions (e.g. [21]), it is similar to TFD but incorporates some new ingredients related with the dynamical properties of the system and with its geometry.

Note that the main idea underlying the TFD approach to this problem (emphasized in this new construction) is the possibility of describing the contact of one open string with a D-brane by effectively substituting such object, whose dynamics and degrees of freedom are unknown in principle, by another fictitious string. In fact, it seems to be natural to think that the effective degrees of freedom of the brane, which are activated by the energy-momentum exchange with the physical string, be in correspondence with a single-string degrees of freedom. This is what we call the fictitious string or “hole” (as used in the TFD literature).

In geometric terms, we may represent a string ending on the D-brane surface, while the fictitious string lives on the other side [22] and ends on the same brane as required by the TFD duplication rules. The group of invariance of the string attached to a Dp-brane is \( G_p \equiv SO(1, p) \times SO(D - p) \) and another equal symmetry \( \tilde{G}_p \) should be attributed to the fictitious string variables.

The boundary conditions to quantize the open string are

\[
X^i|_{\sigma=0, \pi} = x^i \quad i = p + 1, \ldots, 25
\]

\[
\partial_\sigma X^a|_{\sigma=0, \pi} = 0 \quad a = 0, 1, \ldots, p
\]

and they must be the same for \( \tilde{X} \), in order to have a copy of the original system as required by TFD:

\[
\tilde{X}^i|_{\tilde{\sigma}=0, \pi} = x^i \quad i = p + 1, \ldots, 25
\]

\[
\partial_{\tilde{\sigma}} \tilde{X}^a|_{\tilde{\sigma}=0, \pi} = 0 \quad a = 0, 1, \ldots, p
\]

which defines another string in contact with a Dp-brane in the same position \( x^i \). Once more we assume the Hilbert space \( \mathcal{H}_o \otimes \tilde{\mathcal{H}}_o \). By using a part of the symmetry \( \tilde{G}_p \) we may translate its endpoint along the Dp-brane hyperplane to coincide with those of the physical string. So we may see this procedure as a

\(^5\)which may alternatively be interpreted as a selection of physical states, that in this case shall describe the D-brane and its excitation (open) modes at finite temperature.

\(^6\)Then they are preserved by the evolution of the system, generated by the total Hamiltonian (Eq (21))
sort of gauge fixing which according to the Gupta-Bleuler prescription, this shall to be imposed on states as follows:

\[ \tilde{X}^\mu(\tilde{\sigma}, \tilde{t})|_{\sigma=0,\pi} - X^\mu(\sigma, t)|_{\sigma=0,\pi} \ |B_{\text{open}}\rangle = 0 , \]  

(30)

whose components \( \mu = i = p + 1, \ldots, 25 \) are trivially satisfied due to (26) and (28), furthermore this manifestly implies that the respective RHS of these two conditions must coincide. On the other hand, in order to ensure the smooth gluing of both open strings in their respective endpoints, we shall require the continuity of the first derivative with respect to the respective strings parameters in their boundaries. This condition is:

\[ \partial_\sigma \tilde{X}^\mu(\tilde{\sigma}, \tilde{t})|_{\sigma=0,\pi} - \partial_\sigma X^\mu(\sigma, t)|_{\sigma=0,\pi} \ |B_{\text{open}}\rangle = 0 , \]  

(31)

whose components \( \mu = a = 0, \ldots, p + 1 \) trivially annihilate all the states of the Hilbert space by virtue of (27) and (29). Then (30) and (31) constitute \( D \) non trivial conditions on the D-brane states \( |B_{\text{open}}\rangle \) (\( D - 2 \), considering only the physical components in the light cone gauge).

Let us remark that the tilde variables were fixed in terms of the non-tilde ones by these conditions, which clearly break the symmetry \( G_p \times \hat{G}_p \) into the original group \( G_p \).

The map between the tilde and non-tilde operators is defined by the following tilde (or dual) conjugation rules [20]:

\[
\begin{align*}
(XY)^\dagger &= \tilde{X}\tilde{Y}, \\
(cX + Y)^\dagger &= c^* \tilde{X} + \tilde{Y}, \\
(X^\dagger)^\dagger &= (\tilde{X})^\dagger, \\
(\tilde{X})^\dagger &= X, \\
[\tilde{X}, Y] &= 0.
\end{align*}
\]

Considering the solutions (6) and (7) in the light cone gauge, one may use these rules to construct the fictitious copy of the string \( \tilde{X}^\dagger \) defined on an independent ([22]) world sheet manifold whose coordinates are \( \tilde{t}, \tilde{\sigma} \), and we get:

\[
\tilde{X}^i(\tilde{t}, \tilde{\sigma}) = x^i_-(\pi - \tilde{\sigma})/\pi + x^i_+(\tilde{\sigma}/\pi) - \sqrt{2\alpha'} \sum_{n \neq 0} \left( \frac{1}{n} \tilde{\alpha}_n^i e^{int} \sin(n\tilde{\sigma}) \right),
\]

(33)

\[
\tilde{X}^a(\tilde{t}, \tilde{\sigma}) = \tilde{x}^a + 2\alpha' \tilde{p}^a \tilde{t} - 2i\alpha' \sum_{n \neq 0} \left( \frac{1}{n} \tilde{\alpha}_n^a e^{int} \cos(n\tilde{\sigma}) \right).
\]

(34)

If we finally redefine the mode number \( n \rightarrow -n \) \(^7\) in each term of this expression, the solution reads:

\[
\begin{align*}
\tilde{X}^i(\tilde{t}, \tilde{\sigma}) &= x^i_-(\pi - \tilde{\sigma})/\pi + x^i_+(\tilde{\sigma}/\pi) - \sqrt{2\alpha'} \sum_{n \neq 0} \left( \frac{1}{n} \tilde{\alpha}_n^i e^{-int} \sin(n\tilde{\sigma}) \right), \\
\tilde{X}^a(\tilde{t}, \tilde{\sigma}) &= \tilde{x}^a + 2\alpha' \tilde{p}^a \tilde{t} + 2i\alpha' \sum_{n \neq 0} \left( \frac{1}{n} \tilde{\alpha}_n^a e^{-int} \cos(n\tilde{\sigma}) \right). \quad (35) \\
\end{align*}
\]

Although the time evolution of both strings is respectively given by the independent time parameters \( t \) and \( \tilde{t} \), they both shall parameterize the same time direction (in the target) in order to preserve the gauge choice; then in particular, we may define them up to a general shift \( \tilde{t} \equiv t + \tau \). In the light cone frame both time parameters are given by the coordinate \( X^+ \) and they only can be related up to a additive number. Below, we will discuss on the important meaning of this time delay between both strings.

An appropriate initial choice of the non-physical/physical gluing of the variables in the boundary of the system according to the conditions (30) and (31) shall in fact determine boundary states, then these

\(^7\)Notice that this change is related to the structure of the Hamiltonian (21) for the duplicated system.
states shall be built such that these conditions be satisfied for all \( t \). By requiring this in the expansion in modes of (30, 31), results:

\[
\tilde{a}_n^I - e^{in\tau} a_n^I |_{B_{\text{open}}} = 0, \\
x^a - \tilde{x}^a + 2\alpha' \tilde{p}^a \tau |_{B_{\text{open}}} = 0, \\
\tilde{p}^a - p^a |_{B_{\text{open}}} = 0,
\]

where we have used (8) to define the canonical creation/annihilation operators. Then the solution of these equations may be expressed as:

\[
|_{B_{\text{open}}} = N' \delta(x^a - \tilde{x}^a + 2\alpha' \tilde{p}^a \tau) \delta(\tilde{p}^a - p^a) \prod_{l,n>0} e^{q_n a_{n,l}^I \tilde{a}_{n,l}^I} \langle 0 \rangle , 
\]

where \( q_n = e^{i n \tau}, N' \equiv N \delta(X^i|_{\sigma=0,x} - x_+^i)\delta(\tilde{X}^i|_{\sigma=0,x} - x_-^i) \) and \( N \) is the normalization constant\(^8\).

Now we remarkably observe that the time shift \( \tau \) may be an arbitrary complex number. In particular if this is taken to be a purely imaginary number \( \tau \equiv -i\beta/2 \), one may define the physical time \( t \) parameter to be real and in this case, the evolution of fictitious system will be parameterized precisely by the real part of \( \tilde{t} \) (where \( \tilde{t} \equiv I m(\tilde{t}) \)).

If \( \tau \equiv -i\beta/2 \), and if one assumes reality conditions also for the fictitious variables, equation (38) splits into the equations:

\[
x^a - \tilde{x}^a |_{B_{\text{open}}} = 0, \\
\tilde{p}^a |_{B_{\text{open}}} = 0 .
\]

Thus the solution of (37, 39, 41, 42) may be written as:

\[
|_{B_{\text{open}}} = N' \delta(x^a - \tilde{x}^a)\delta\tilde{p}^a(\tilde{p}^a) \prod_{l,n>0} e^{q_n a_{n,l}^I \tilde{a}_{n,l}^I} \langle 0 \rangle .
\]

In this case \( q_n = e^{-n\beta/2} \). Defining \( \theta_n = \text{tanh}^{-1} q_n(\tau) \), the string modes occupation number is given by \( N_n = \sinh^2 \theta_n \), which agrees with a Bose-Einstein distribution of string modes at the temperature \( \beta^{-1} \), and this state remarkably coincides with (11) found in Section 2. Therefore, we showed here that the effect produced by the brane on the open string at finite temperature is equivalent to the effect due to another string, joined to the first one through their boundaries, but forwarded by an imaginary time interval (which in this formalism need not to be compactified to the circle as usual\(^9\)). Furthermore this time delay may be interpreted in terms of the temperature of the brane.

As pointed out in Section 2, one may consider the analytical continuation of the parameter \( \tilde{t} \) to take real values and the boundary states are given by (40). Although this does not arise from the traditional TFD construction, we may observe here that this number should not be interpreted as a time evolution parameter (as often believed) but as a relative delay.

Finally, it is interesting to remark that this may be seen as the geometrical representation of the postulate of non-physicalness of the tilde system, since the “world-manifold” associated with it, is causally disconnected of the parallel world-manifold of the real system due to the imaginary time delay between them. However there is quantum entanglement between them which give place to thermal states. In contrast if \( \tau \in \mathbb{R} \), in principle both systems may superpose and to cause interference among them. This may be describing dynamical (out of the thermal equilibrium) effects of the brane. We furthermore believe that this also open the possibility of studying interactions at finite temperature\(^{25}\).

These are some of the new remarks and perspectives emphasized by this formulation and shall be studied in detail elsewhere\(^{10}\).

---

\(^8\)Note that the centers of mass of both strings follow the same trajectory but the position of one of them is forwarded by \( 2\alpha' \tilde{p}^a \tau \) with respect to the other one.

\(^9\)In particular in the imaginary time approach\(^{23}\) [24].

\(^{10}\)We shall to cite that the construction presented here and the geometric description of thermal closed string given in\(^{26}\) have some similar aspects.


4 Concluding Remarks and Outlook

In this work we obtained the open string representation of a bosonic Dp-brane state and its generalization at finite temperature. A remarkable strength of this description is that the Dp-branes are emphasized as vacuum states of an open string Fock space. On the other hand, this approach handles a model for Dp-branes where its macroscopical nature is manifest [8]. Notice in particular that if we consider an ensemble of open strings attached to the same Dp-brane, then in the thermodynamic limit this may be seen as a sort of medium extended on $p$ spacial dimensions, filled with open string modes.

Let us observe that the configuration discussed in Section 5 constitutes a composite between two open strings (so one may say that $|B_{\text{open}}\rangle$ represent composite states) and notice in addition, that from a strictly topological point of view, they two form a closed string. This observation could help to understand what is the manifest correspondence between both open/closed representations of D-brane states in the TFD language and this furthermore gives rise to a more deep question: may closed string states be viewed as bound states of open strings?

In this sense, we believe that the open/closed duality could be properly interpreted as a correspondence between closed string states and thermal (mixed) states of open strings. In fact, according to the arguments explained in Section 3, if we search for an open string configuration at thermal equilibrium, we generically may read it as a path integral formulation of a closed string theory. On the other hand, let us remark that this fact suggests that the kinematical structure of an open string field theory shall require a $C^*$ algebra [27, 28], and indicates how the closed string sector could be recovered in such theory.

In future works we will investigate the thermal stability of the D-brane configurations so as the possibility of describing black branes using these ideas. According to the references [29, 30, 31], The infrared behavior of theories whose dual bulk-gravities contain a black brane is governed by hydrodynamics, and the main observation in this sense is the existence of an universal value for the ratio of shear viscosity to entropy density [32] which should be investigated in the context of an appropriate microscopical model.

Finally, the connection of our results with the closed string description of thermal D-branes [13, 14, 15, 16, 17, 18] should be clarified.

5 Acknowledgements

The author would like to thank to A. Gadelha, N. Grandi and G. Silva for fruitful conversations on the subject of this paper. J. A. Helayel-Neto, R. Scherer Santos and N. Quiroz Perez, are specially acknowledged for useful comments and observations. This work was supported by CONICET.

References

[1] D-branes in string theory, P. Di Vecchia, hep-th/9912161, hep-th/9912275; Lectures on non-BPS Dirichlet branes, M. R Gaberdiel, Class. Quant. Grav. 17 (2000) 3483, hep-th/0005029; D-branes and boundary states in closed string theories, B. Craps, hep-th/0004198.

[2] L. Rastelli, A. Sen and B. Zwiebach, Vacuum string field theory, arXiv: hep-th/0106010.

[3] Boundary state in open string channel and open/closed string field theory H. Isono, Y. Matsuo, hep-th/0511203; Y. Imamura, H. Isono, Y. Matsuo, Prog. Theor. Phys. 115 (2006) 979 (hep-th/0512098)

[4] A. Stromminger and C. Vafa, Phys Lett B 379 (1996) 99.

[5] Y. Takahashi and H. Umezawa, Coll. Phenomena 2 (1975) 55 (Reprinted in Int. J. Mod. Phys. 10 (1996) 1755).

[6] H. Umezawa, H. Matsumoto, M. Tachiki, Thermofield Dynamics and Condensed States (North-Holland, Amsterdam, 1982).
[7] A. Mann, M. Revzen, H. Umezawa and Y. Yamanaka, Phys. Lett. A 140 (1989) 475.
[8] H. Umezawa, *Advanced Field Theory: Micro, Macro and Thermal Physics* (AIP, New York, 1993).
[9] A. E. Santana, F. C. Khanna, Phys. Lett. A 203 (1995) 68.
[10] A. E. Santana, F. C. Khanna, H. Chu and C. Chang, Annals Phys. 249 (1996) 481.
[11] R. Kobes, Phys. Rev. D 42 (1990) 562.
[12] M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, 1996).
[13] I. V. Vancea, Phys. Lett. B 487 (2000) 175.
[14] M. C. B. Abdalla, A. L. Gadelha and I. V. Vancea, Phys. Rev. D 64 (2001) 086005.
[15] M. C. B. Abdalla, E. L. Graca and I. V. Vancea, Phys. Lett. B 536 (2002) 144.
[16] M. C. B. Abdalla, A. L. Gadelha and I. V. Vancea, Phys. Rev. D 66 (2002) 065005.
[17] M. C. B. Abdalla, A. L. Gadelha and I. V. Vancea, Int. J. Mod. Phys. A 18 (2003) 2109.
[18] M. C. B. Abdalla, A. L. Gadelha and I. V. Vancea, arXiv:hep-th/0308114.
[19] M. A. Vazquez-Mozo, Phys. Lett. B 388 (1996) 494.
[20] A. E. Santana, A. Matos Neto, J. D. M. Vianna, F. C. Khanna, Physica A 280 (2000) 405.
[21] J. C. da Silva, F. C. Khanna, A. Matos Neto and A. E. Santana, Phys. Rev. A 66 (2002) 052101.
[22] R. Laflamme, Nucl. Phys. B 324 (1989) 233.
[23] T. Matsubara, Prog. Theor. Phys. 14 (1955) 351.
A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particles Systems* (McGraw-Hill, New York, 1971).
[24] A. Das, *Finite Temperature Field Theory* (W. Scientific, Singapore, 1977).
[25] L. Alvarez-Gaume, C. Gomez, G. W. Moore and C. Vafa, Nucl. Phys. B 303 (1988) 455.
[26] M. C. Abdalla, A. L. Gadelha and D. L. Nedel Phys. Lett. B 613 (2005) 213.
[27] R. Haag, *Local Quantum Physics: Fields, Particles, Algebras* (Springer-Verlag, New York, 1992).
[28] G. G. Emch, *Algebraic Methods in Statistical and Quantum Field Theory* (John Wiley, New York, 1972).
[29] G. Policastro, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 87 (2001) 081601 [arXiv:hep-th/0104066].
[30] G. Policastro, D. T. Son and A. O. Starinets, JHEP 0209 (2002) 043 [arXiv:hep-th/0205052].
[31] C. P. Herzog, JHEP 0212 (2002) 026 [arXiv:hep-th/0210126].
[32] P. Kovtun, D. T. Son and A. O. Starinets, JHEP 0310 (2003) 064 [arXiv:hep-th/0309213].