Beamspace MUSIC Algorithm based on Space-time Array Signal Model

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Abstract. This paper is devoted to the direction-of-arrival (DOA) algorithm in the moving array scenario. A beamspace MUSIC algorithm based on space-time signal model is proposed to cope with this scenario. First, the DOA information contained in Doppler shift is used to combine with the array spatial information, thus constructing a space-time signal model. Second, allowing for the considerable dimension and complexity of the space-time model, we transform the array sensor data into beamspace to reduce the computation load. The introduction of Doppler information leads to significant enhancement in algorithm performance. Simulation results demonstrate the promotion of proposed algorithm.

1. Introduction

Direction-of-arrival (DOA) estimation using an antenna array is an important problem across a number of applications. Its performance mainly depends on the array aperture. In the modern communication system, the array is always disposed on mobile carriers such as ships, aeroplanes and satellites to obtain better manoeuvrability [1]. Because of the limitation of the carrier size, the array aperture is restricted which will degrade the DOA performance [2].

To improve DOA estimation performance, researchers have put forward various modified methods. Literature [3-5] improve DOA performance by making use of non-circularity, cyclostationarity and fourth-order cumulant of the signal, respectively. However, in essence, these techniques merely use the properties of signals to improve DOA performance, but not solve the conflict between the array aperture and DOA performance in moving array scenario.

Except for above methods, DOA estimation utilizing Doppler information has drawn increasing attention. One of the representative methods is synthetic aperture (SA) technique, which includes ETAM [6-7] and FFTSA [8] algorithms. It has been widely used for moving array and provides improved performance. However, this technique has strict demand for the motion attitude of the array, so it is only suitable for specific scenarios. Another type of method is space-time signal processing. This technique exploits the DOA information in space and time domain simultaneously and constructs a space-time signal model [9]. Algorithm based on this signal model has been proposed in [10] and demonstrates good performance. However, to the best of our knowledge, existing space-time processing methods mainly focus on the scenario where a static array detects a moving target [11]. Methods intended for moving array have not been addressed often.
According to the moving array scenario assumed in this paper, we construct a space-time signal model based on existing methods, so that the spatial and Doppler information has been combined and embedded in the space-time manifold vector. To achieve high accuracy and resolution estimation, we adopt MUSIC algorithm. Because of the accumulation of temporal data, the dimension of space-time model grows manyfold compared with that of conventional signal model, which causes heavy computation load. Thus we utilize beamspace MUSIC algorithm to reduce the dimension of the subspace without degradation of performance. Finally, numerical simulations are presented to illustrate the reliability and effectiveness of proposed algorithm.

2. Scenario description and problem formulation

2.1. Scenario description and assumptions

Assume that a $M$ sensors uniform linear array with an inter-element spacing $d$ is equipped on a moving platform, which moves along its baseline at velocity $v$. $P$ far-field narrowband signals of DOA $\theta_p$, $1 \leq p \leq P$, is received by the array. The carrier frequencies of signals are $f_0$, which is known to the receiver. The Doppler shift caused by array motion can be expressed as

$$ f(\theta_p) = f_0 v \sin \theta_p / c $$

where $c$ is the velocity of electromagnetic wave.

Then we can represent the received signal of the $m$th sensor at time $t$ as

$$ x_m(t) = \sum_{p=1}^{P} s_p e^{j2\pi f_0 \left( \frac{\sin \theta_p}{c} \right) t - \frac{(m-1)d}{c} \sin \theta_p} + n_m(t) $$

Then during the interval $[t-q, t]$ where $q \ll c$ and $f_0$, is a prior knowledge, we can simplify equation (2) as

$$ x(t) = \sum_{p=1}^{P} a(\theta_p) s_p e^{j2\pi f_0 \left( \frac{\sin \theta_p}{c} \right) t} + n(t) $$

where $a(\theta_p) = \left[ 1, e^{-j2\pi f_0 \frac{d}{c} \sin \theta_p}, \cdots, e^{-j2\pi f_0 \frac{(M-1)d}{c} \sin \theta_p} \right]^T$ is the spatial manifold vector, $n(t)$ is the noise vector. We assume that the signals are independent.

2.2. Problem formulation for space-time model

From the signal model in equation (3), we observe that Doppler shift contains DOA information, which is useful for DOA estimation. Besides, the time domain samples constitute a geometric sequence, which is similar to spatial manifold. Thus, we consider constructing a space-time signal model to make full use of Doppler information and acquire better DOA estimation performance.

Assuming that the total sample number is $K$, we can divide the samples into $Q$ equal intervals and every interval has $N = K/Q$ samples. Then during the $q$th interval, the array output vector at the $n$th sample moment can be expressed as
\begin{equation}
x_s(q) = \sum_{p=1}^{P} s_p \mathbf{a}^\top(\theta_p) e^{-j2\pi f_0 \frac{\sin \theta_p}{c} \left( [q-1]N \sin \phi + (n-1)T_i \right)} + n_n(q)
\end{equation}

where 
\begin{equation}
l_p(q) = \exp \left( -j2\pi f_0 \frac{\sin \theta_p}{c} (q - 1) NT_s \right)
\end{equation}

and 
\begin{equation}
\beta_p(q) = s_p l_p(q). \quad T_s \text{ represents sample interval.}
\end{equation}

From equation (4), it can be observed that the phases of the samples in an interval can form a vector as 
\begin{equation}
b(\theta_p) = \left[ 1, \exp \left( -j2\pi f_0 \frac{\sin \theta_p}{c} T_s \right), \ldots, \exp \left( -j2\pi f_0 \frac{\sin \theta_p}{c} (N - 1)T_s \right) \right]^\top
\end{equation}

The vector \( b(\theta_p) \) has similar structure with the spatial manifold vector \( \mathbf{a}(\theta_p) \), so it can be defined as temporal manifold vector. Then we use the array output vector at every moment in an interval to construct an \( MN \)-dimensional vector 
\begin{equation}
y(q) = \left[ x_1^\top(q), x_2^\top(q), \ldots, x_N^\top(q) \right]^\top.
\end{equation}

\( y(q) \) is the space-time observed data and its expression is 
\begin{equation}
y(q) = \sum_{p=1}^{P} \left[ b(\theta_p) \otimes \mathbf{a}(\theta_p) \right] \beta_p(q) + \mathbf{\mu}(q)
\end{equation}

where \( \otimes \) represents Kronecker product. \( \mathbf{g}(\theta_p) = b(\theta_p) \otimes \mathbf{a}(\theta_p) \) is the space-time manifold vector and \( \mathbf{\mu}(q) = \left[ \mathbf{\mu}_1(q), \mathbf{\mu}_2(q), \ldots, \mathbf{\mu}_N(q) \right]^\top \) is the space-time noise vector. For simplicity, we can rewrite equation (6) as 
\begin{equation}
y(q) = (\mathbf{B} \circ \mathbf{A}) \mathbf{\beta}(q) + \mathbf{\mu}(q)
\end{equation}

where \( \circ \) denotes Khatri-Rao product. \( \mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_P)] \), \( \mathbf{B} = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \ldots, \mathbf{b}(\theta_P)] \) and \( \mathbf{\beta}(q) = [\beta_1(q), \beta_2(q), \ldots, \beta_P(q)]^\top \). \( \mathbf{G} = [\mathbf{g}(\theta_1), \mathbf{g}(\theta_2), \ldots, \mathbf{g}(\theta_P)] \), and it is an \( MN \times P \)-dimensional space-time manifold matrix.

In next section, we will come up with an algorithm to estimate the DOA based on the proposed signal model.

3. Beamspace MUSIC algorithm for DOA estimation

In this section, we consider using MUSIC algorithm to estimate the DOAs of signals. However, equation (7) indicates that the dimension of proposed signal model is \( N \) times as that of conventional model, which will increase the complexity of algorithm and computation load. With this in mind, we switch observed signal to beamspace to reduce the complexity [12].

The principle of beamspace processing is demonstrated in figure 1. The array sensors are passed through beamformers, then the beam data is used to estimate DOAs [13].
Figure 1. Principle of beamspace DOA estimation

From section 2, we know that the space-time manifold vector $g(\theta_p)$ is

$$g(\theta_p) = \begin{bmatrix} 1, e^{-j2\pi f_0 \frac{d \sin \theta_p}{c}}, \ldots, e^{-j2\pi f_0 \frac{(M-1)d \sin \theta_p}{c}}, \ldots, e^{-j2\pi f_0 \frac{v_T \sin \theta_p}{c}}, \\ e^{-j2\pi f_0 \frac{d \sin \theta_p}{c}}, \ldots, e^{-j2\pi f_0 \frac{(M-1)d \sin \theta_p}{c}}, \ldots, e^{-j2\pi f_0 \frac{v_T \sin \theta_p}{c}}, \\ e^{-j2\pi f_0 \frac{d \sin \theta_p}{c}}, \ldots, e^{-j2\pi f_0 \frac{(M-1)d \sin \theta_p}{c}}, \ldots, e^{-j2\pi f_0 \frac{v_T \sin \theta_p}{c}}, \ldots \\ \end{bmatrix}^T,$$

(8)

According to the form of $g(\theta_p)$, we can define a beamforming weight vector as

$$w(u) = \begin{bmatrix} 1, e^{-j2\pi u \frac{d}{c}}, \ldots, e^{-j2\pi u \frac{(M-1)d}{c}}, \ldots, e^{-j2\pi u \frac{v_T}{c}}, \\ e^{-j2\pi u \frac{d}{c}}, \ldots, e^{-j2\pi u \frac{(M-1)d}{c}}, \ldots, e^{-j2\pi u \frac{v_T}{c}}, \\ e^{-j2\pi u \frac{d}{c}}, \ldots, e^{-j2\pi u \frac{(M-1)d}{c}}, \ldots, e^{-j2\pi u \frac{v_T}{c}}, \ldots \\ \end{bmatrix}^T,$$

(9)

where $u = \sin \theta$, and the interval $-1 \leq u \leq 1$ is corresponding to the angular interval $-90^\circ \leq \theta \leq 90^\circ$. $w(u)$ represents the beamformer of the beam whose main lobe points at $\theta$. Based on this, $D$ adjacent beamforming vectors of the structure in (9) with respective pointing angles equispaced by $\Delta = 2/MN$ can make up a $MN \times D$ beamforming matrix

$$W = \frac{1}{\sqrt{MN}} \begin{bmatrix} w(r \frac{2}{MN}), w((r+1) \frac{2}{MN}), \ldots, w((r+D-1) \frac{2}{MN}) \end{bmatrix}^T,$$

(10)

Note that $w(u)$ does not possess Vandermonde structure, hence the columns of $W$ are not orthonormal. We can orthogonalize $W$ and obtain the transform matrix as

$$H = W (W^HW)^{-\frac{1}{2}},$$

(11)

Utilizing matrix $H$, the observed data $y(q)$ can be transformed into

$$z(q) = H^Hy(q),$$

(12)

It is noteworthy that when $H = I$, the beamspace data $z(q)$ is equivalent to sensor data $y(q)$. Besides, equation (12) also indicates that the dimension of observed data is reduced from $MN \times 1$ to...
The corresponding covariance matrix is
\[ R_z = H^H R_y H = H^H G R_p G^H H + \sigma^2 I \]  
(13)
where \( R_y \) and \( R_p \) are the covariance matrices of \( y(q) \) and \( \beta(q) \), respectively. \( \sigma^2 \) is the power of noise. In light of the procedure of MUSIC algorithm, the noise subspace \( \tilde{E}_\mu \) can be obtained through eigen decomposition of \( R_z \). Then the beamspace MUSIC spatial spectrum is formulated as
\[ \hat{S}_{\text{MUSIC}}(\theta) = \left[ g^H(\theta) H \tilde{E}_\mu H^H g(\theta) \right]^{-1} \]  
(14)
Finally, \( \hat{\theta} \) can be estimated by searching the maximum of \( \hat{S}_{\text{MUSIC}}(\theta) \) as follow
\[ \hat{\theta} = \arg \max_\theta \hat{S}_{\text{MUSIC}}(\theta) \]  
(15)
As is well known, the main computation load of MUSIC algorithm is centered on eigen decomposition. The complexity of original MUSIC algorithm for space-time signal model is \( O(M^3 N^3) \), while beamspace MUSIC algorithm requires only \( O(D^3) \), which is significantly reduced.

4. Numerical results
In this section, numerical simulations of the proposed algorithm (BMUSIC-ST) are presented and compared with conventional spatial MUSIC (MUSIC-spatial), space-time MUSIC (MUSIC-ST) and ETAM algorithm. In simulations, the resolution and root mean square error (RMSE) are exploited to evaluate the performance of algorithm.

In the first scenario, we consider an 8 sensors uniform linear array with half-wavelength inter-element spacing. The array moves at velocity 200m/s.

Figure 2 demonstrates the comparison versus signal-to-noise ratio (SNR) with 900 signal samples based on 300 Monte Carlo trials. We assume that two narrow-band signals arrive from 20° and 23°, respectively. Their center frequencies are 300MHz and the dimension of vector \( b(\theta) \) (temporal manifold vector) is 18 ( \( N = 18 \)). From the figure, we can observe that BMUSIC-ST and MUSIC-ST can absolutely distinguish two signals at -4dB and 6dB, respectively, while spatial MUSIC algorithm cannot achieve this until 14dB. It can be concluded that algorithms based on space-time model possess better resolution than those based on conventional model. Besides, obviously, the resolution of BMUSIC algorithm is superior to that of MUSIC algorithm.

![Figure 2. Resolution probabilities versus SNR](image-url)
In figure 3, we present the performance comparison versus SNR amongst four algorithms to examine the accuracy of proposed algorithm. Assume that a narrow-band signal impinges on the array from 15°. Other conditions remain the same as those in figure 2. In particular, we introduce sensor errors in figure 3(b), where the gain and phase errors of each sensor are drawn from the interval $[-0.5 \text{dB}, 0.5 \text{dB}]$ and $[-20^\circ, 20^\circ]$, respectively. In figure 3(a), it can be seen that the performances of BMUSIC-ST and MUSIC-ST almost coincide, and the two algorithms outperform the other comparison algorithms. As the SNR increases, the RMSEs of BMUSIC-ST and MUSIC-ST approaches the associated CRB. In figure 3(b), we can observe that the performances of all the algorithms are degraded by the presence of sensor errors. However, BMUSIC-ST and MUSIC-ST still exhibit better accuracy and robustness than other algorithms under the influence of sensor errors.

![Figure 3. RMSEs and CRB versus SNR: (a) without sensor errors, (b) with sensor errors](image)

Next, we investigate the impact of temporal manifold vector dimension $N$ on the performance of proposed algorithm. Consider a 9 sensors uniform linear array, which moves at 200m/s. In figure 4, the DOAs of two narrow-band signals are set to be 20° and 23°, respectively. For each SNR, the probabilities are calculated by 300 Monte Carlo runs for 1200 snapshots. Other conditions stay unchanged. We can find that the resolution is improved as $N$ increases. In figure 5, we simulate a signal impinging from 15°, and other conditions are the same as those in figure 4. It can be seen that the RMSE reduces as $N$ increases. From figure 4 and 5, we can conclude that the estimation accuracy and resolution can be enhanced when temporal manifold vector dimension increases.

![Figure 4. Resolution probabilities versus SNR for different temporal manifold vector dimensions](image)

![Figure 5. RMSEs versus SNR for different temporal manifold vector dimensions](image)
5. Conclusions
In this paper, we propose a beamspace MUSIC algorithm based on moving array. First, a space-time signal model is established, which combines spatial information with temporal information and makes it possible to acquire acceptable DOA estimation with limited array aperture. Then the beamspace MUSIC algorithm is proposed for fast and high-accuracy DOA estimation. Numerical results demonstrated that the algorithms based on space-time model outperform other contrastive algorithms. Besides, beamspace MUSIC algorithm shows similar accuracy and superior resolution compared with original MUSIC algorithm.

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