A self-calibration method for gyro scale factor asymmetry in rotational inertial navigation system

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Abstract. In the rotational inertial navigation system (RINS), the inertial measurement unit (IMU) rotates around the motor shaft, which will stimulate the gyro scale factor asymmetry, so it needs to be effectively calibrated and compensated. In this paper, the influence of gyro scale factor asymmetry on angular velocity error, angle error and velocity error is analysed, and a self-calibration method for it is proposed in the RINS using velocity errors as measurements. A self-calibration rotation strategy is designed for the dual-axis RINS so as to carry out self-calibration for the scale factor asymmetry of x-gyro, y-gyro and z-gyro. The feasibility of this scheme is verified by theoretical analysis and simulation. Experiments are carried out on a set of dual-axis RINS based on fiber optic gyroscope, and the scale factor asymmetry calibration results of three gyros are 2.56 ppm, 10.01 ppm and 0.88 ppm, respectively. In the navigation experiment, the navigation error is improved by 57.45% after compensating the gyro scale factor asymmetry error, which fully illustrates the significance of the proposed self-calibration method in improving the navigation performance of RINS.

1. Introduction
The rotary inertial navigation (RINS) is an inertial navigation system (INS) developed on the basis of strap-down inertial navigation system (SINS). It offsets the constant error of inertial devices by rotating inertial measurement unit (IMU), which can improve the navigation accuracy during long voyage [1]. But the RINS puts forward high requirements to the gyro scale factor asymmetry. If there is gyro scale factor asymmetry, errors will be caused in the process of rotational modulation. If single-axis RINS runs in continuous positive and negative rotation with the angular velocity of 6 °/s, the gyro scale factor asymmetry error of 1 ppm (relative error) will cause the longitude and latitude error of about 0.2 ’ in 2 hours. As for the navigation accuracy of 1 n mile in 24 hours, the gyro scale factor asymmetry should be less than 0.23 ppm [2]. Especially, the faster the rotation speed is, the larger the angle error accumulated in the same time will be, the larger the velocity error will be, and the more obvious the harm of gyro scale factor asymmetry will be.

Therefore, it is necessary to calibrate and compensate the gyro scale factor asymmetry. While the calibration methods for the gyro scale factor asymmetry mainly include two types: discrete calibration and system-level calibration. The discrete calibration is using the output of the gyro as the observation measurement directly. The system-level calibration is using the navigation error, which is obtained by navigation calculation with the output of the inertial device, as the measurement to determine the error parameter [3]. In the national military standard, a discrete calibration method for measuring the gyro scale factor asymmetry is provided, that is, the gyro scale factors in the input of positive and negative angular velocity are respectively determined, and then the gyro scale factor asymmetry is calculated.
according to the equations (1) ~ (2).

\[ K = \frac{K_{+} + K_{-}}{2} \]  
(1)

\[ K_{a} = \frac{K_{+} - K_{-}}{K} \]  
(2)

In some references which study the gyro scale factor asymmetry, the discrete calibration methods are mostly adopted. Zhan puts forward an experimental scheme for measuring the gyro scale factor asymmetry without knowing the gyro drift and the Earth's rotation component [4]. Yang proposes a method for measuring the gyro scale factor asymmetry based on angular rate integral. The asymmetry of the high-precision fiber optic gyro scale factor is less than 1 ppm. Li proposes an angular increment method comparing with the angular rate method. Gyro scale factor asymmetry errors of the two methods are both in the range of 1 ppm, and the angular increment method is better than angular rate method [5]. The disadvantage of discrete calibration is that it requires high accuracy of the turntable. The accuracy of calibration depends directly on the accuracy of the turntable, and calibration requires the turntable to provide an accurate attitude reference and angular rate reference [6].

Therefore, it is of great significance to study the system-level self-calibration that does not rely on the high-precision turntable. The RINS can realize self-calibration through its own rotating mechanism. We can use different rotation strategies to stimulate various errors, and estimate the errors by least square estimation (LSE) or Kalman filtering (KF). However, most self-calibration methods, such as the classic nineteen-position calibration method [7-9], only estimate gyro drift, gyro scale factor error, gyro installation angle error, accelerometer bias, accelerometer scale factor error, and accelerometer installation angle error. The gyro scale factor asymmetry error is not estimated. Because the gyro scale factor asymmetry is coupled with the gyro drift, it cannot be estimated directly. Therefore, this paper proposes a system-level self-calibration method for gyro scale factor asymmetry in RINS.

The rest of the paper is organized as follows. The basic structure of the system, coordinate systems and error parameters are introduced in section 2. In section 3, error model, rotation strategy, and filtering model are described in detail. MATLAB Simulation and Experimental verification are discussed in section 4. Finally, conclusions are drawn in section 5.

2. System structure and definition

2.1. Introduction to the basic structure of the system

The structure of the dual-axis RINS is shown in figure 1.

![Figure 1. Structure of dual-axis RINS.](image)

The dual-axis RINS consists of an IMU, two angle sensors, two torque motors, two frames, and so on. The innermost IMU consists of three gyroscopes and three accelerometers for measuring angular
velocity and acceleration information. The torque motors are used to drive the rotation of the gimbals [10]. Angle sensors are used to sense relative rotational angles and serve as feedback for motor control.

2.2. Coordinate system definition
For convenience of description, we define some coordinate systems. The body frame (b-frame) is defined as right-forward-upward of the carrier. The navigation frame (n-frame) is defined as east-north-up. The platform frame (p-frame) is defined as follows. The $Z_p$ axis of the p-frame is defined to coincide with the inner rotation axis, the $X_p$ axis is defined by the projection of $x$-gyro in the normal plane of $Z_p$, and the $Y_p$ axis is defined according to the right-hand rule. Furthermore, this coordinate system rotates with the rotation of IMU [11]. In addition, two non-orthogonal frames are defined, namely accelerometer frame (ma-frame) and gyro frame (mg-frame), which are defined by sensitive axis direction of accelerometer and gyro respectively.

2.3. Error definition
Inertial device's own errors include: gyro drift $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$, gyro scale factor error $\Delta K_{g_\alpha}$, $\Delta K_{g_\beta}$, $\Delta K_{g_\gamma}$, accelerometer bias $\nu_x$, $\nu_y$, $\nu_z$, accelerometer scale factor error $\Delta K_{a_\alpha}$, $\Delta K_{a_\beta}$, $\Delta K_{a_\gamma}$. Moreover, assuming that gyro scale factor error of positive rotation is $\Delta K_{g+}$ and gyro scale factor error of negative rotation is $\Delta K_{g-}$, then the asymmetry error of the gyro scale factor is $\Delta K_{g_\alpha}$, which is shown in equation (3).

$$\Delta K_{g_\alpha} = \Delta K_{g+} - \Delta K_{g-}$$

(3)

The installation errors in IMU include: gyro installation error matrix $C_{mg}^p$, which can be expressed as equation (4), and accelerometer installation error matrix $C_{ma}^p$, which can be expressed as equation (5). Installation errors schematic diagrams of gyros and accelerometers are shown in figure 2(a) and figure 2(b) respectively.

$$C_{mg}^p = \begin{bmatrix} 1 & 0 & \beta_{gx} \\ \alpha_{gy} & 1 & -\beta_{gy} \\ -\delta_{gx} & \delta_{gy} & 1 \end{bmatrix}$$

(4)

$$C_{ma}^p = \begin{bmatrix} 1 & -\alpha_{as} & \beta_{as} \\ \alpha_{ay} & 1 & -\beta_{ay} \\ -\delta_{as} & \delta_{ay} & 1 \end{bmatrix}$$

(5)

Figure 2. Installation errors of gyros and accelerometers.
3. Self-calibration model

3.1. Error model
The calibration method of common error parameters is very mature, so it does not be introduced any more. This paper focuses on gyro scale factor asymmetry errors. It’s assumed that twenty-three error items have been calibrated, which include 3 gyro drifts, 3 gyro scale factor errors, 3 accelerometer biases, 3 accelerometer scale factor errors, 5 gyro installation errors, and 6 accelerometer installation. There are only 3 gyro scale factor asymmetry errors left. If the system is in the east-north-up state, we take the x-gyro as an example. Supposing that the rotational angular velocity is $\omega_e$, theoretically, the measured output of gyros in this process is as shown in equation (6). Where, $\omega_{h_e}$ and $\omega_{v_e}$ respectively represent the north and the up components of the angular velocity of the Earth’s rotation.

$$\omega^m_e = \begin{bmatrix} \omega_h \\ \omega_v \cos \omega_t + \omega_e \sin \omega_t \\ \omega_v \cos \omega_t - \omega_h \sin \omega_t \end{bmatrix}$$

(6)

After considering the gyro scale factor asymmetry errors, the gyro scale factor error in positive rotation is $\Delta K_{g^+}$, and the gyro scale factor error in negative rotation is $\Delta K_{g^-}$. In the process of positive rotation, the angular velocity error in n-frame is shown in equation (7).

$$\Delta \omega^m_e = C_p^e C_m^e \omega^m_e$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \omega_t & -\sin \omega_t \\ 0 & \sin \omega_t & \cos \omega_t \end{bmatrix} \begin{bmatrix} \Delta K^+_{g^e} & 0 & 0 \\ 0 & \Delta K^+_{g^e} & \omega_h \cos \omega_t + \omega_e \sin \omega_t \\ 0 & 0 & \Delta K^+_{g^e} \omega_v \end{bmatrix} \begin{bmatrix} \omega_h \\ \omega_v \cos \omega_t + \omega_e \sin \omega_t \\ \omega_v \cos \omega_t - \omega_h \sin \omega_t \end{bmatrix}$$

(7)

$$= \begin{bmatrix} \Delta K^+_{g^e} \omega_h \\ \Delta K^+_{g^e} \omega_v \cos \omega_t + \omega_e \sin \omega_t \\ \Delta K^+_{g^e} \omega_v \cos \omega_t - \omega_h \sin \omega_t \end{bmatrix}$$

Set the angle of rotation as $\phi$, the east angle error as $\Delta \phi_e$, the north angle error as $\Delta \phi_n$, then:

$$\begin{bmatrix} \Delta \phi_e \\ \Delta \phi_n \end{bmatrix} = \begin{bmatrix} \int_{0}^{2\pi} \Delta \omega_e dt \\ \int_{0}^{2\pi} \Delta \omega_n dt \end{bmatrix}$$

(8)

According to equation (8), if the IMU rotates one circle around x-gyro in positive direction, that is $\phi = 2\pi$, there are:

$$\begin{bmatrix} \Delta \phi_e^+ = \int_{0}^{2\pi} \Delta \omega_e dt \\ \Delta \phi_n^+ = \int_{0}^{2\pi} \Delta \omega_n dt \end{bmatrix}$$

(9)

$$= 2\pi \Delta K^+_{g^e}$$

If we continue to rotate one circle around x-gyro in negative direction, similarly, there are:

$$\begin{bmatrix} \Delta \phi_e^- = 2\pi \Delta K^+_{g^e} \\ \Delta \phi_n^- = 0 \end{bmatrix}$$

(10)

It can be seen from the above analysis that, after one circle of positive and negative rotation around the x-gyro, the angle error caused by the gyro scale factor asymmetry is:

$$\begin{bmatrix} \Delta \phi_e = 2\pi \Delta K^+_{g^e} \\ \Delta \phi_n = 0 \end{bmatrix}$$

(11)
Theoretically, if there is no gyro scale factor asymmetry error, the angle error caused by the gyro scale factor error will be offset after a positive and negative rotation, which is shown in figure 3. Because of the gyro scale factor asymmetry error, the angle error cannot be offset after a positive and negative rotation, which is shown in figure 4.

\[
\begin{align*}
\Delta V_E &= -\int_0^t \Delta \phi_N g dt = 0 \\
\Delta V_N &= \int_0^t 2\pi K_{\text{out}} g dt = 2\pi K_{\text{out}} gt
\end{align*}
\]

After a positive and negative rotation, the IMU keeps still for navigation. At the stage of stationary navigation, the eastward velocity error and the northward velocity error caused by the angle error are shown in equation (12), where \(g\) is the gravitational acceleration.

3.2. Rotation strategy and filtering model

According to the above analysis, when the IMU rotates a normal-reverse rotation around one gyro, the scale factor asymmetry error of the corresponding gyro will be stimulated. So the following rotation strategy is designed.

Firstly, the IMU rotates a normal-reverse rotation around y-gyro, and it keeps still for 200 seconds, causing the eastward velocity error. Secondly, the inner frame is rotated by 90 degrees, and the IMU rotates a normal-reverse rotation around x-gyro, and keeps still for 200 seconds, causing the eastward velocity error. Thirdly, the outer frame is rotated by 90 degrees, and the IMU rotates a normal-reverse rotation around z-gyro, and keeps still for 200 seconds, causing a northward velocity error.

A schematic diagram of this rotation strategy is shown in figure 5.
the state variable, and H is the observation matrix. $t_k$ means current moment and $t_{k-1}$ represents previous moment.

\[
\begin{align*}
    p(t_k) &= p(t_{k-1}) - p(t_{k-1}) \cdot H(t_k) \cdot (I + H(t_k) \cdot p(t_{k-1}) \cdot H(t_k)^T)^{-1} \cdot H(t_k) \cdot p(t_{k-1}) \\
    X(t_k) &= X(t_{k-1}) + p(t_k) \cdot H(t_k)^T \cdot (Z(t_k) - H(t_k) \cdot X(t_{k-1}))
\end{align*}
\]  

(13)

According to the above formula, once the measurement Z and the observation matrix H are determined, the state variable X can be estimated if given an initial value. For the error model in this paper, the state variable X is the gyro scale factor asymmetry error to be estimated, the measurement Z is the velocity error, and the observation matrix is the equation (14).

\[
H = 2\pi g t
\]  

(14)

The scale factor asymmetry error of three gyros are respectively calibrated in steps. According to the designed self-calibration rotation strategy, the scale factor asymmetry error of y-gyro is calibrated firstly, then the scale factor asymmetry error of x-gyro is calibrated, finally the scale factor asymmetry error of z-gyro is calibrated.

4. Simulation and experiment

4.1. MATLAB Simulation

We simulate with MATLAB. Only the gyro scale factor asymmetry error is added when the simulation is performed. The simulation results are shown in table 1. The first column is the gyro scale factor asymmetry error added when simulating, the second column is the gyro scale factor asymmetry calibrated. The average accuracy of the simulation calibration is 98.31%. The estimation curves of gyro scale factor asymmetry error are shown in figure 6 when simulating. The simulation results show that the calibration has good accuracy and the estimation curve has good convergence, which prove the feasibility of this calibration scheme.

![Figure 6. Estimate curves.](image)
4.2. Experimental verification

In order to verify the practical feasibility of the calibration scheme, we carry out experiments on a set of existing dual-axis RINS based on fiber optic gyro. The system consists of three fiber optic gyros, three quartz flexible accelerometers, two DC brushless torque motors, and two optoelectronic encoders.

In order to obtain 23 error parameters except the gyro scale factor asymmetry errors, the commonly used nineteen-position calibration method is used to obtain the 23 error parameters. The outputs of the gyros and accelerometers are compensated according to the 23 error parameters, and then the designed self-calibration experiment of the gyro scale factor asymmetry is performed.

The self-calibration experimental process of the gyro scale factor asymmetry is shown in figure 7. The fiber-optic gyro RINS is placed on the platform which keeps approximate horizontal. The system is placed in east-north-up, and it is powered by a DC stabilized voltage supply. The raw data of gyros and accelerometers are collected by LabVIEW on the computer at the frequency of 200 Hz. The raw data are processed offline with MATLAB. After the system is initially powered up, it returns to the system zero position firstly, then it performs a stationary coarse alignment to obtain the initial attitude angle of the system, finally it executes the designed self-calibration rotation strategy. The rotation angular velocity is set to 6 °/s, and the entire calibration process is about 16 minutes. We repeat the calibration experiment five times.

![Figure 7. Experiment.](image)

The five groups of experimental results are shown in table 2. The first column stands for the number of experiment, and the remaining three columns stand for scale factor asymmetry errors of the x, y, and z gyro in each group of calibration experiment. The sixth row represents the average of five groups of calibration results, and the calibration results of three gyro scale factor asymmetry errors are 2.56 ppm, 10.01 ppm, and 0.884 ppm, respectively. The last row represents the standard deviation of five groups of calibration results. It can be concluded from the results of the standard deviation that the calibration scheme has good calibration repeatability. Among them, the first group of experiment of the gyro scale factor asymmetry error estimation curve is shown in figure 8, and the error can converge to a stable value finally.

| Experiment number | $\Delta K_{gax}$ (ppm) | $\Delta K_{gay}$ (ppm) | $\Delta K_{gaz}$ (ppm) |
|-------------------|------------------------|------------------------|------------------------|
| 1                 | 2.81                   | 10.3                   | 0.947                  |
| 2                 | 2.36                   | 9.4                    | 0.934                  |
| 3                 | 2.352                  | 10.53                  | 0.752                  |
| 4                 | 2.648                  | 9.62                   | 0.863                  |
| 5                 | 2.63                   | 10.2                   | 0.924                  |
| Mean value        | 2.56                   | 10.01                  | 0.884                  |
| Standard deviation| 0.199                  | 0.478                  | 0.072                  |
In order to verify the correctness of the calibration results, after compensating the gyro scale factor asymmetry errors, a two-hour static navigation experiment is carried out. During the navigation experiment, the IMU rotates around x-gyro in continuous normal-reverse rotation. Besides, the compensation method of gyro scale factor asymmetry error is shown in equations (15) ~ (16).

\[ \Delta K_x = \Delta K_x + \frac{1}{2} \Delta K_{gy} \]  
\[ \Delta K_y = \Delta K_y - \frac{1}{2} \Delta K_{gy} \]  

In the process of static navigation, the comparisons of velocity error between uncompensated and compensated are shown in figure 9. The first figure is eastward velocity error and the second figure is northward velocity error. Since the IMU rotates around x-gyro in continuous normal-reverse rotation during the navigation, the eastward velocity error before and after compensation is approximately unchanged, and the compensated northward velocity error is better than uncompensated northward velocity error. The static navigation experiment results are shown in table 3. The second row is the root mean square (RMS) of eastward velocity error, which is approximately the same before and after compensation. The third row is the RMS of northward velocity error, and the compensated result is 40.6% higher than the uncompensated. The fourth row is the circular error probable (CEP), and the compensated result is 57.45% higher than the uncompensated. The improvement of navigation error proves the correctness of this calibration result and calibration scheme.
### Table 3. The results of 2-hour navigation.

|               | Uncompensated | compensated | Improved |
|----------------|---------------|--------------|----------|
| VE(m/s)        | 0.044         | 0.044        | 0%       |
| VN(m/s)        | 0.133         | 0.079        | 40.6%    |
| CEP(n mile/h)  | 0.188         | 0.080        | 57.45%   |

5. Conclusion

In this thesis, the influence of gyro scale factor asymmetry on angular velocity error, angle error and velocity error is analysed. The self-calibration rotation strategy is designed for dual-axis RINS to calibrate the scale factor asymmetry errors of x-gyro, y-gyro, and z-gyro. We adopt RLS to estimate them. The feasibility of this scheme is verified by theoretical analysis and simulation. The experiment is carried out on a set of dual-axis RINS based on fiber optic gyro. The calibration results of scale factor asymmetry of three gyros are 2.56ppm, 10.01ppm and 0.884 ppm respectively. In the navigation experiment, after compensating gyro scale factor asymmetry errors, the navigation error is improved by 57.45%, which proves the correctness of the calibration result and the calibration scheme.

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