On Brownian Motion of Helium Ions in the Ballistic Regime

A.Kleymenicheva, V.Shikin
Institute of Solid State Physics of RAS, 142432, Chernogolovka, Russia

Abstract

Discussed in the paper is the possibility of introducing the concept of Brownian motion of various mesoparticles in the ballistic regime. The case in point is the effect of collisions between thermal excitations in the liquid and the test mesoparticle (allowing to trace its position) on the thermal motion of the latter. The standard criterion for the brownian nature of the particle motion assumes that the inequality \( l_s < R_i \) is satisfied and, consequently, a large number of collisions with the averaged response of the mesoparticle to such a bombardment (here \( l_s \) is the mean free path of helium excitations, \( R_i \) is the effective mesoparticle radius). However, the opposite limit \( l_s > R_i \), which we refer to as the ballistic regime, is also possible. It is characterized with the specific features in the behavior of mesoparticles. The emphasis is made on the already available evidences indicating this behavior in the thermal motion of helium ions.

PACS:61.20.Qg

The term Brownian is used to denote the thermal motion of various mesoscopic particles (observed with appropriate techniques) due to their interaction with thermal excitations in the environment. The pioneering works [1, 2] on the diffusion-type behavior of brownian particles laid the foundation of classical non-equilibrium thermodynamics [3, 4].

A rather similar problem on the mobility \( \mu_i \) of charged particles in the external field contains two limiting cases. In the first one, which is hydrodynamic in nature, when \( l_s < R_i \), the friction force is given the well-known Stokes formula \( F_{st} \) [5] (here \( l_s \) is the typical free path of the quasiparticles which govern the medium viscosity, and \( R_i \) is the radius of the moving sphere)

\[
F_{st} = 6\pi R_i \eta V,
\]

where \( V \) is the sphere velocity relative to the liquid whose viscosity is \( \eta \). The numerical coefficient in Eq. (1) may vary according to the imposed boundary conditions (e.g., for the Ryabchinsky problem this coefficient is 4 [5]).

The mobility \( \mu_i \) of the ion of radius \( R_i \) arising from the condition \( F_{st} = eE \), is

\[
\mu_i = V/E = e/(6\pi R_i \eta)
\]

where \( E \) is the driving electric field pulling the charged sphere through the viscous liquid.
In the opposite limit \( l_s > R_i \), the so-called ballistic regime is realized. Here the ion mobility is governed by the thermal excitations kinetics. For ions in liquid helium in the roton temperature range

\[
\frac{e}{\mu_{\text{rot}}} \simeq \sqrt{\frac{\pi}{2}} R_i^2 v_{\text{rot}}.
\] (3)

Here \( v_{\text{rot}} \) is the roton thermal velocity. The difference between (2) and (3) is obvious. It is easily verified in the experiments with helium ions (e.g., see Ref. [6]).

The question is what happens with thermal motion of mesoparticles in the transition domain \( l_s < R_i \rightarrow l_s > R_i \)? For helium ions in liquid helium this crossover is easily realized already in the temperature range of \( \leq 2K \) (just as for the the transition from (2) to (3) in the problem of finding the particle mobility).

Qualitatively, the important ballistic alternative to the diffusion-type brownian motion occurs in the dissipationless limit \( l_s \gg R_i \), or, which is the same, \( l_s \rightarrow \infty \). In that case the ions or neutral impurity particles form a low-density gas (suspension) in the solvent and obey Maxwell velocity distribution since they practically behave as free particles. We believe that the observation of free thermal motion of impurity particles of any origin is sufficient to demonstrate the existence of the ballistic regime in the brownian motion problem.

1. The simplest observable manifestation of the ballistic nature of the thermal motion of impurities is the finite value of the osmotic pressure \( \delta P(c) \neq 0 \) in various classic liquid solutions,

\[
\delta P(c) = TC.
\] (4)

Here \( T \) is the temperature and \( C \) the impurity concentration. Formally, the definition (4) arises when manipulating with ideal chemical potentials of the solvent and the solved substance at the semipermeable membrane [7]. Actually, the osmotic pressure is a common practice when dealing with various (not necessarily ideal) solutions.

2. More prominently the ballistic motion of ions manifests itself in Doppler shift of optical spectra of the atoms of different alkali metals artificially injected in liquid helium. The main goal of these experiments is the study of the compressing effect of surrounding helium on the external shells and, consequently, on the luminescence spectrum of alkali atoms excited in the superfluid liquid [8-10]. The expected shift of optical lines should be accompanied with their broadening of different origin, including thermal Doppler effect.

The frequency \( \omega_0 \) of radiation emitted by the atom whose velocity component along the observation direction equals \( v \) is shifted, according to the Doppler principle, by \( \omega_0 v/c \)

\[
\omega = \omega_0 (1 + \frac{v}{c}), \quad \text{or} \quad v = (\omega - \omega_0)/c
\] (4)

where \( c \) is the light velocity. Assume now that velocity distribution of emitting atoms is defined by function \( W(v) \). Bearing in mind Eq. (4), one has

\[
I(\omega)d\omega = W(c, \frac{\omega - \omega_0}{\omega_0}) \frac{c}{\omega_0} d\omega
\] (5)

For the Maxwell distribution

\[
W(v)dv = \frac{1}{\sqrt{\pi}} \exp \left[-(v/v_T)^2\right] dv/v_T, \quad v_T = \sqrt{2T/M_i}
\] (6)
Eq. (5) yields

$$I(\omega) = \frac{1}{\sqrt{\pi}} \exp \left[ - \left( \frac{\omega - \omega_0}{\Delta \omega_D} \right)^2 \right] d\omega, \quad \Delta \omega_D = \omega_0 v_T/c \tag{7}$$

The frequency dependence $I(\omega)$ (7) is symmetric about $\omega_0$. The Doppler broadening width is defined by the parameter $\Delta \omega_D$ which explicitly depends on the emitter effective mass $M_i$.

The broadening of type (7) is known to yield one of the most substantial contribution to the line width in optical spectra for normal gaseous media [11]. For the low-temperature gas of alkali atoms in liquid helium [10] the optical lines broadening mechanisms are currently under study. Doppler broadening should also be present among them.

This work was partly supported by the RFBR grant N 12-02-00229 and the Program of the Presidium RAS “Disordered systems”.

References

[1] A.Einstein, Annalen der Physik 17, (1905), 549
[2] M. von Smoluchowsky, Ann. Phys. (Leipzig) 21, (1906), 756
[3] S.Chandrasekhar. Stochastic Problems in Physics Astronomy. Moscow, Izdat. Inostr. Lit., 1947, 168 pp. (in Russian)
[4] I.Prigogine, D.Kondepudi. Contemporary Thermodynamics. Moscow, Mir, 2002, 462 pp.
[5] L.Landau, E.Lifshits, Hydrodynamics. Moscow, Nauka, 1986. (in Russian)
[6] A.Dahm, T.Sanders, JLTP, 2, (1970), 199
[7] L.Landau, E.Lifshits, Statistical Physics. Moscow, Nauka, 1975. (in Russian)
[8] A.Khrapak, I.Yakubov, Electrons in dense gases and plasma, Nauka, Moscow 1981, 282 p.
[9] Electronic Excitations in Liquid Rare Gases, Edited by W.Schmidt and E.Illenberger; American Sci.Publishers 25650, North Lewis Way, USA, 470 pp.
[10] B.Tabbert, G.zuPutlitz, JLTP 109, (1997), 653
[11] I. Sobelman, Introduction to Atomic Spectra Theory. Moscow, FizmatGiz, 1963, 640 pp. (in Russian)