The importance of the parallel momentum balance on the bootstrap current evaluation in non-axisymmetric systems is demonstrated by the benchmarks among the local drift-kinetic equation solvers, i.e., the Zero-Orbit-width (ZOW) model, DKES, and PENTA. The ZOW model is extended to include the ion parallel mean flow effect on the electron-ion parallel friction. Compared to DKES code in which only the pitch-angle-scattering term is included in the collision operator, PENTA code employs the Sugama-Nishimura method to correct the momentum balance. The ZOW model and PENTA codes, both of which conserve the parallel momentum in like-species collisions and include the electron-ion parallel frictions, agree each other well on the calculations of the bootstrap current. The DKES results without the parallel momentum conservation deviates significantly from those from the ZOW model and PENTA. This work verifies the reliability of the bootstrap current calculation with the ZOW model and PENTA for the helical plasmas.

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The study of the bootstrap current is necessary to reproduce accurately the MHD equilibrium for high-beta plasmas. For the axisymmetric magnetic geometry, reliable analytic formulas of bootstrap current are available [1]. For the non-axisymmetric system, one needs to rely on numerical methods to evaluate the bootstrap current, which is complicatedly dependent on the magnetic geometry, the collision frequency, and the radial electric field. The past studies [2] presented the benchmark between the Monte-Carlo global model VENUS+δf and the local semi-analytical solution SPBSC [3] in LHD. The bootstrap current between the VENUS+δf and the SPBSC codes shows a systematic difference. Although the difference may be caused in part by the finite-orbit-width effect, a missing discussion in that paper is about the treatment of collision term. The VENUS+δf code did not treat the friction force between electrons and ions, while SPBSC solved the balance between parallel viscosity and friction force as shown in later in Eq. (3) by analytic formula. In order to carry out a more direct investigation on the impact of the parallel friction on the bootstrap current calculations, this paper performs the benchmark among the ZOW model [4], DKES [5], and PENTA [6], which are all based on local neoclassical models.

The ZOW model [7] solves the radially-local drift-kinetic equation by the δf Monte-Carlo method, and the parallel friction $F_{||}$ is treated as follows. For the like-species collisions, the linearized collision operators are employed and this satisfies the parallel momentum balance, i.e., $\mathcal{F}_{||ee} = \mathcal{F}_{||ii} = 0$. For ion, the ion-electron friction $\mathcal{F}_{||ie}$ is neglected because of the large mass ratio, $m_e/m_i \ll 1$. For electron, in the previous work, the electron-ion collision was only approximated as the pitch-angle scattering in the their collision operators. The study of the bootstrap current is necessary to reproduce accurately the MHD equilibrium for high-beta plasmas. For the axisymmetric magnetic geometry, reliable analytic formulas of bootstrap current are available [1]. For the non-axisymmetric system, one needs to rely on numerical methods to evaluate the bootstrap current, which is complicatedly dependent on the magnetic geometry, the collision frequency, and the radial electric field. The past studies [2] presented the benchmark between the Monte-Carlo global model VENUS+δf and the local semi-analytical solution SPBSC [3] in LHD. The bootstrap current between the VENUS+δf and the SPBSC codes shows a systematic difference. Although the difference may be caused in part by the finite-orbit-width effect, a missing discussion in that paper is about the treatment of collision term. The VENUS+δf code did not treat the friction force between electrons and ions, while SPBSC solved the balance between parallel viscosity and friction force as shown in later in Eq. (3) by analytic formula. In order to carry out a more direct investigation on the impact of the parallel friction on the bootstrap current calculations, this paper performs the benchmark among the ZOW model [4], DKES [5], and PENTA [6], which are all based on local neoclassical models.

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mean flow is determined so that the total ion parallel viscosity vanishes; \[
\langle B \cdot \nabla \cdot (P_{\text{CGL}} + \Pi_2) \rangle_\parallel \approx 0,
\]
Eq. (4). In Fig. 1 (a), the ZOW model and PENTA agree with each other well even though the $F_{||\text{ie}}$ is absent in the ZOW model. This suggests that the friction $F_{||\text{ie}}$ is in fact negligible as it is expected from the large mass ratio $m_e/m_i \ll 1$. The gap between the results from DKES and PENTA indicates that it is necessary to maintain the momentum conservation of the like-species collisions even in the helical plasmas. For the electrons, in Fig. 1 (b), there are two results from the ZOW model in order to examine the impact of the ion parallel mean flow $U_{||i}$. The electron parallel momentum equation depends on the balance, \[
\langle B \cdot \nabla \cdot (P_{\text{CGL}} + \Pi_2) \rangle_\parallel \approx \langle F_{||\text{ie}}B \rangle.
\]
Eq. (5). In Fig. 1 (b), the friction $F_{||\text{ie}}$ with the finite $U_{||i}$ gives rise to the gap between the two ZOW simulation results. The ZOW result with finite $U_{||i}$ agrees with the PENTA’s one. These models both maintain the parallel momentum conservation with finite $U_{||i}$ and the momentum correction of the like-species collision, respectively. For the ZOW model, it is obvious that the correct ion parallel mean flow is necessary to improve the collision operator on the electron parallel flow calculation. For the electron, there is also the large gap between the results from DKES and PENTA as in the ion simulations.

In Fig. 2, the radial profile of the bootstrap current in the FFHR-d1 case at the ambipolar condition is estimated by the three codes. The bootstrap current from the ZOW model with finite $U_{||i}$ agrees with PENTA. In the previous studies [4, 7] it is found that neglecting the tangential magnetic drift in DKES and PENTA causes the overestimation of the ion radial particles flux when $E_r$ is small. This results in the difference in the ambipolar-$E_r$ values as shown in Fig. 1. However, in the present case,
since $\langle B_{\parallel} \rangle = \langle B_{n\parallel}(U_{\parallel} - U_{\parallel e}) \rangle$ from the ZOW model and PENTA have very weak dependence on $E_r$, the bootstrap current from these two codes agrees each other. The DKES result shows approximately 10 times larger magnitude of the bootstrap current than those from PENTA and the ZOW models.

It is well-known that the pitch-angle scattering operator is enough to evaluate the radial neoclassical fluxes in helical plasmas. However, it is insufficient for the bootstrap current calculation. The present study shows that both the momentum conservation in the like-species collision and the friction acting on the electrons are important physics to estimate the bootstrap current correctly, even in helical plasmas. This work is also the first report of verification of the ZOW model and PENTA for bootstrap current calculations. These two codes will serve to improve the accuracy of the bootstrap current calculation in general helical plasmas.

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