Scaling of heights and widths of laminar jet diffusion flames under subatmospheric pressures

Pingchuan Ma¹, Haihang Li², Jian Wang¹

1 State Key Laboratory of Fire Science, University of Science and Technology of China, Hefei, China.
2 China Jiliang University

Abstract. The luminous shapes were presented to examine the scaling of flame heights and widths of laminar jet diffusion flames of burning methane, ethylene, and propane in a subatmospheric pressure chamber. All the flames examined were stable with no soot emitting. The Reynolds scaling mentioned below was generally suitable for the three hydrocarbon fuels although slopes were becoming steeper as soot formation elongated the flame height at high $Re$ under high pressure. The regions of Froude number where buoyancy is dominant or buoyancy is unimportant were demonstrated by following Reynolds–Froude height scaling of Altenkirch’s model. In Froude scaling of width, the unchanged slopes between dimensionless width and $Fr$ with pressure increase indicated the good linearity of flame width changing on pressure. Finally, regression of the form for flame width was also considered.

1. Introduction
Flame height and width are essential properties of diffusion flames to be modeled under different pressures [1-6] and gravity levels [2,7]. Linear scaling between normalized height and dimensionless numbers was commonly used in those studies [8-10]. $L/d \propto Re$ is the classical expression to expound the proportionality between flame height and fuel mass flow rate [9-11] which derived by Jost [12] through equating diffusion time with convection time according to Einstein’s diffusion equation. But the expression cannot reflect the effect of pressure on flame height since Reynoldes number is independent with pressure.

By adding Froude number, $Fr = u^2/gd \propto P^{-2}$, Altenkirch et al. [5] introduced both Reynolds and Froude numbers to discuss the role of buoyancy at elevated gravity. After obtaining the slopes of $(L/d)Re^{-2/3}Fr^{-1/3}$ versus $Fr$, Altenkirch et al. [5] found regions where buoyancy was dominant and where buoyancy was not important. But Sunderland et al. [10] concluded that the Reynolds-Froude height scaling of Altenkirch et al. [5] is more appropriate for slot burner flames because it neglects transverse curvature effect.

Subatmospheric and superatmospheric conditions also provide various buoyancy levels for testing the above scaling laws. Recently, Hu et al. [13] scaled normalized flame height with heat release rate at two different atmospheric pressures in a cubic fire enclosure. At reduced pressures luminous flame heights and widths were considered to examine the relative merits of Reynolds, Froude and Reynolds-Froude scaling laws in this paper.

2. Experimental setup

2.1. Subatmospheric pressure chamber and burner
The experiments were conducted in a subatmospheric pressure chamber of $3 \times 2 \times 2$ m in the internal size. Low pressures can be obtained by depressurizing the sealed chamber space. An observation window was embedded on the chamber wall to facilitate observation on ignition and flame stability. The burner configuration is Burke-Schumann type and made of stainless steel, but no air co-flow was used during the experiments. The fuel tube of the burner has an inner diameter of 5.0 mm with a tapered exit to reduce the formation of turbulent eddies. Sintered metal foam elements placed in the fuel nozzle minimize the instabilities in the initial fuel flow and create a top-hat velocity profile on the exit.

### 2.2. Mass flow controller

Methane, propane, and ethylene were selected (all with purities $\geq 99.9\%$) as studied fuels with increasing carbon ratio. Fuel was supplied from compressed cylinder into the burner and its flow rate was metered by a thermal based mass flow controller (Sevenstar D07-19B). The range of the controller is 0-1000 sccm with 0.1 sccm resolution. The accuracy of the mass flow rate measured is within $\pm 1\%$ and the repeatability is $\pm 0.2\%$.

### 2.3. Fuel flow rates

The volumetric flow rate (sccm) of the fuel is defined as the gas jet volume under the standard condition (1.0 atm, 0 °C). And the mass flow rates of the fuels maintain constant at all pressures, Table 1. The fuel mass flow rates were designed after trial tests to insure soot was completely oxidized within the visible flame envelope and the flame was relatively stable at all pressures considered. The estimated Reynolds numbers ($Re$) indicate that all the examined flames were in laminar status. The flames remained stable for a significant amount of time, but spontaneous fluctuations in the form of tip flicking occurred at a regular interval throughout the experiment. The flame images of stable periods were analyzed to determine the flame shape (height and width) for every test.

#### Table 1. Summary of test parameters.

| Parameter   | Fuel flow rate $^{1}$, sccm | Exit velocity, m/s | $Re$     | $Fr$   |
|-------------|-----------------------------|--------------------|----------|--------|
| Methane     | 25.0-150.0                  | 0.021-0.425        | 7.4-44.4 | 0.009-3.7 |
| Ethylene    | 12.5-150.0                  | 0.011-0.425        | 7.1-84.7 | 0.002-3.7 |
| Propane     | 6.25-100.0                  | 0.005-0.283        | 7.0-111.4| 0.0006-1.6|

1 Volumetric fuel flow rate under 1.0 atm and 0 °C (same in this paper without special note).

### 3. Results and discussion

#### 3.1. Flame height

As mentioned in the Introduction, many theoretical analyses [12] and experimental results [9-11] have reached a consensus about the Reynolds scaling of

$$L/d \propto Re. \tag{1}$$

For a given fuel mass flow rate, Eq. (1) indicates that flame height is independent of pressure on account of $Re = \rho ud/\mu = 4\pi \mu d P^{0.5}$. However, many experiment data have reported that flame height changed with pressure [7,14, 15]. So this prediction of independence from pressure is of particular interest here.

Figure 1 shows the relationships between normalized flame height $L/d$ and Reynolds number $Re$. This type of coordinate axis followed a long tradition [9-11]. We can imagine that the decreasing air entrainment by buoyancy under reduced pressure should make the flame become longer. This speculation can be confirmed by data of relatively low Reynolds (i.e. low flow rate). As Reynolds increased, this shrank difference giving a concentrating appearance of scatters. This happens because Froude number (the ratio of momentum to buoyancy forces) generally increases with $Re$ [9], which
weakening the effect of buoyancy at higher $Re$. This can be seen in Figure 1 of Reynolds number less than or equal to 33, 42, and 42 for methane, ethylene, and propane flames respectively. In fact, the increasing soot formed at higher $Re$ contributes a great deal to the narrowing of the flame height differences. As shown in Figure 1, when $Re$ exceeded the above values, the relative size of flame height between lower pressure and higher pressure changed. This is mainly because at higher fuel flow rate, the excessive soot generated under higher pressure greatly elongated the flame height [16]. This phenomenon was also observed under elevated pressures in [17], where soot volume fraction is proportional to pressure of 1.2 power. And under some moderate fuel flow rates, flame heights increased with pressure below their smoke points [18]. The combined effect of increased buoyancy and soot formation made the slopes of all the hydrocarbon fuels bigger as pressure increase, Figure 1 When the chamber pressure increased from 0.3 to 1.0 atm, the average slope of the three fuels changed from about 0.71 to 1.09.

![Figure 1](image_url)

**Figure 1.** Normalized flame height as a function of Reynolds number for $CH_4$, $C_2H_4$, $C_3H_8$ flames under subatmospheric pressures. The lines and equations shown are best fits predicted on unity slope.
Altenkirch et al. [5] simplified the laminar gas jet diffusion flame as a cylinder and used a free convection boundary layer method. By establishing the balance of the oxidizer and fuel, it finally led to

\[ \frac{L}{d} \sim \frac{Fr^{2/3}}{Re^{1/3} (W/d)^{2/3}}. \]  

(2)

Suggesting \( W/d \) is constant, Altenkirch et al. [5] arrived two limits [10] using their results at elevated gravity:

- Nonbuoyant limit (large \( Fr \)), \( \frac{L}{d} \sim Fr^{2/3} \);  
- Buoyant limit (small \( Fr \)), \( \frac{L}{d} \sim Re^{2/3} Fr^{1/3} \).  

(3a) (3b)

Figure 2 shows the relationship between \( (L/d)Re^{2/3} Fr^{-1/3} \) and \( Fr \), this type of coordinate axis also followed a long tradition as [5, 10]. It can be seen that the linearity of methane is the best among the three fuels, and the divergence seemed to increase with increasing sooting tendency of hydrocarbons. The fitting slopes have also been found by [10], whose values were -0.26 and -0.23 for methane and ethane, respectively. And the average slope for methane, ethane, and propane was -0.25 in [5] and -0.23 in [9].

As for methane flames, the slopes changed from -0.35 to -0.18 as pressure increased from 0.3 to 1.0 atm. This can be illustrated by Froude number changing in Figure 2: The slope at the largest \( Fr \) was about -1/3 showing \( (L/d)Re^{2/3} Fr^{-1/3} \) is changing with \( Fr^{-1/3} \). The largest contribution of \( Fr \) at 0.3 atm indicates buoyancy is unimportant in determining flame height. The phenomenon observed in [18-21] in which flame heights is constant in different pressure is this case. In stark contrast to the largest \( Fr \), \( (L/d)Re^{2/3} Fr^{-1/3} \) remained nearly constant when \( Fr<0.02 \) indicating buoyancy dominant tendency was reached [5].

With regard to ethylene and propane flames, the absolute slopes between \( (L/d)Re^{2/3} Fr^{-1/3} \) and \( Fr \) were becoming smaller at higher pressures resulting from the increasing buoyancy effects. Similarly to methane flames, \( (L/d)Re^{2/3} Fr^{-1/3} \) kept nearly constant when \( Fr < 0.01 \) reflecting buoyancy became dominant in this region for propane flames. But for propane flames, the data of the smallest fuel flow rate above the blue dotted line did not meet the level off trend. The flame images of this flow rate were found to be too crescent that the assumption of cylindrical shape of flame is no longer reasonable. This disarray distribution phenomenon was also observed for ethylene flames. And for ethylene we might infer that \( (L/d)Re^{2/3} Fr^{-1/3} \) would be level off below a certain \( Fr \) if the burner diameter was suitable.
Figure 2. Flame height relationship for CH$_4$, C$_2$H$_4$, C$_3$H$_8$ flames under subatmospheric pressures. The lines and equations shown are best fits, where $y$ is defined as $(L/d)Re^{-2/3}Fr^{-1/3}$.

3.2. Flame width
Davis et al. [22] reported that Froude number is the appropriate scaling parameter for gravitational variations in laminar jet diffusion flames. We will verify whether this scaling relationship is suitable for pressure change.

Figure 3 shows the relationship between $W/d$ and $Fr$ for the present flames. The sooting tendency of methane, propane, and ethylene can be also seen in normalized width verse $Fr$, which can be demonstrated by increased scattering level in data. The power of $W/d$ dependence on $Fr$ were 0.16, 0.13, and 0.13 for methane, ethylene, and propane respectively in Figure 3, approximating to 0.16 in [9] averaged from methane, ethane, and propane. Besides, the slopes between $W/d$ and $Fr$ of all the
fuels were not changing with pressure and the level off phenomenon didn’t happen as in Froude scaling of height, both of which were indicating the good linearity of flame width changing on pressure.

![Normalized flame width as a function of Froude number for CH4, C2H4, C3H8 flames under subatmospheric pressures. The lines and equations shown are best fits.](image)

**Figure 3.** Normalized flame width as a function of Froude number for CH4, C2H4, C3H8 flames under subatmospheric pressures. The lines and equations shown are best fits.

Following the correlation of flame widths in [9] and references therein, regressions of the form $W/d \propto Fr^{a}Re^{b}$ were also considered in this paper. As mentioned in [9], several theories predicted $a = \alpha - \beta = 0.25$, or $W/d \propto St^{0.25}$. According to the multiple linear regression of methane, ethylene,
and propane flames, the values of $a$ were 0.1964, 0.2175, and 0.2047, $b$ were -0.1434, -0.2420, -0.1973 here.

4. Conclusions
The luminous shapes of laminar jet diffusion flame were measured to examine the scaling of flame height and width in a subatmospheric pressure chamber. The mass flow rates of methane, ethylene, and propane were kept constant at all pressures and the corresponding flames were no soot emitting. The major conclusions were as follows:

- The Reynolds scaling of $L/d \propto Re$ was generally suitable for the three hydrocarbon fuels. Under reduced pressure, the air density will become lower, which contributes to a weaker buoyancy. And the decreasing air entrainment by a weaker buoyancy made the flame longer at relatively low $Re$. As $Re$ increased, the weakening effect of buoyancy showed a concentrating height difference. And excessive soot generated under higher pressure greatly elongated the flame height at the higher Reynolds number.

- The $-1/3$ power of $Fr$ contribution to $(L/d)Re^{-2/3}Fr^{-1/3}$ indicates that buoyancy is unimportant in determining flame height at the largest $Fr$ and at 0.3 atm. Thus, for a relative larger $Fr$, flame heights of a laminar diffusion flame will not change with pressure. Oppositely, at the smallest $Fr$, $(L/d)Re^{-2/3}Fr^{-1/3}$ remained nearly constant where buoyancy dominant tendency was reached.

- In Froude scaling of width, the slopes between $W/d$ and $Fr$ of all the fuels were not changing with pressure and the level off phenomenon didn’t happen, both of which were indicating the good linearity of flame width changing on pressure. Finally, regression of the form $W/d \propto Fr^a Re^b$ for flame width was also found rational here.

5. Reference
[1] S.P. Burke, Schumann, T.E.W. 1928 Diffusion Flames Ind. Eng. Chem. 20(10) 998-1004.
[2] J.-M. Most, P. Mandin, J. Chen, P. Joulain 1996 Influence of pressure and pressure on pool fire-type diffusion flames  Proc. Combust. Inst. 26(1) 1311-1317.
[3] T. YUAN, DUROX, D., VILLERMAUX, E. 1993 The Effects of Ambient Pressure Upon Global Shape and Hydrodynamic Behavior of Buoyant Laminar Jet Diffusion Flames Combust. Sci. Technol. 92 69-86.
[4] J. Gong, X. Zhou, Z. Deng, L. Yang 2013 Influences of low atmospheric pressure on downward flame spread over thick PMMA slabs at different altitudes Int J Heat Mass Tran 61 191-200.
[5] R.A. Altenkirch, Eichhorn, R., Hsu, N.N., Brancic, A.B., Cevallos, N.E. 1977 Characteristics of laminar gas jet diffusion flames under the influence of elevated gravity, Proc. Combust. Inst. 16(1) 1165-1174.
[6] Thomson, K. A., Gülder, Ö. L., Weckman, E. J., Fraser, R. A., Smallwood, G. J., & Snelling, D. R. 2005 Soot concentration and temperature measurements in co-annular, nonpremixed CH4/air laminar flames at pressures up to 4 MPa. Combustion and Flame 140(3) 222-232.
[7] J. Baker, Siredddy, K., Varagani, R. 2003 Buoyancy-Controlled Laminar Diffusion Slot Flame Heights: A Comparison of Theoretical Predictions and Microgravity Results Microgravity Sci. Tec. 14(4) 27-35.
[8] F. Khalidi, Gillon, P. 2003 Laminar jet diffusion flame behavior under a strong magnetic field gradient Proceedings of the European Combustion Meeting (Orléans, Frankreich) pp. 1-6.
[9] P.B. SUNDERLAND, MENDELSON, B. J., YUAN, Z.-G., URBAN, D. L. 1999 Shapes of Buoyant and Nonbuoyant Laminar Jet Diffusion Flames Combust. Flame 116 376-386.
[10] P.B. Sunderland, J.E. Haylett, D.L. Urban, V. Nayagam 2008 Lengths of laminar jet diffusion flames under elevated gravity Combust. Flame 152 60-68.
[11] J.R. Camacho, Choudhuri, Ahsan R. 2006 Shapes of elliptic methane laminar jet diffusion flames J. Eng. Gas. Turb. Power 128 1-7.
[12] D. Drysdale 1999 An Introduction to Fire Dynamics ed John Wiley & Sons Ltd (Chichester, England) chapter2 pp65-71.
[13] L.H. Hu, F. Tang, M.A. Delichatsios, Q. Wang, K.H. Lu, X.C. Zhang 2013 Global behaviors of enclosure fire and façade flame heights in normal and reduced atmospheric pressures at two altitudes Int J Heat Mass Tran 56(1–2) 119-126.
[14] P.M. Mandatori, O.L. Gülder 2011 Soot formation in laminar ethane diffusion flames at pressures from 0.2 to 3.3 MPa Proc. Combust. Inst. 33 577-584.
[15] G. Intasopa 2011 Soot Measurements in High-Pressure Diffusion Flames of Gaseous and Liquid Fuels (Toronto, University of Toronto) chapter4 pp36-37.
[16] I.M. Miller, H.G. Maahs 1997 High-pressure flame system for pollution studies with results for methane-air diffusion flames (Washington National Aeronautics and Space Administration.) pp 121-124
[17] [L.L. McCrain, Roberts, W.L. 2005 Measurements of the soot volume field in laminar diffusion flames at elevated pressures, Combust. Flame, 140(1-2) 60-69.
[18] A.E. Karatas, Gülder, Omer L. 2012 Soot formation in high pressure laminar diffusion flames, Prog. Energy Combust., 38 818-845.
[19] Daca, A. E., & Gülder, Ö. L. 2017 Soot formation characteristics of diffusion flames of methane doped with toluene and n-heptane at elevated pressures. Proceedings of the Combustion Institute 36(1) 737-744.
[20] Bento, D. S., Thomson, K. A., & Gülder, Ö. L. 2006 Soot formation and temperature field structure in laminar propane–air diffusion flames at elevated pressures. Combustion and Flame 145(4) 765-778.
[21] Karataş, A. E., Intasopa, G., & Gülder, Ö. L. 2013 Sooting behaviour of n-heptane laminar diffusion flames at high pressures. Combustion and Flame 160(9) 1650-1656.
[22] R.W. DAVIS, MOORE, E.F., SANTORO, R.J., NESS, J.R.,1990 Isolation of Buoyancy Effects in Jet Diffusion Flame Experiments, Combust. Sci. Technol., 73 625-635.

Acknowledgments
This research was supported by the Open Foundation of State Key Laboratory of Fire Science (No. HZ2016-KF13).