The $b \to s\gamma\gamma$ transition in softly broken supersymmetry

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Abstract

We study the effect of supersymmetric contributions to the effective quark transition $b \to s\gamma\gamma$, including leading order QCD effects. We apply the discussion to the decay $B_s \to \gamma\gamma$. Even though one-particle irreducible contributions could play a role, numerical cancelations make the amplitude for the two-photon emission strongly correlated to the $b \to s\gamma$ amplitude which is sharply constrained by experiment. A quite general statement follows: as long as non standard physics effects appear only in the matching of the Wilson coefficients of the standard effective operator basis, the deviations from the standard model expectations of the decay rates induced by $b \to s\gamma\gamma$ are bound to follow closely the corresponding deviations on $b \to s\gamma$. Effects of new physics are therefore bound to be small.

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I. INTRODUCTION

The rare flavour changing transition $b \to s \gamma \gamma$ has recently attracted new interest in view of the planned experiments at the upcoming KEK and SLAC B-factories and existing hadronic accelerators, which may test branching fractions as low as $10^{-8}$ times the B meson decay width.

Rare B-physics potentially provides us with valuable redundancy tests of the flavour structure of the standard model (SM) and complementary information on the related CP violation. It is also most sensitive to the “heavy” sector of the SM particle spectrum and it is the preferred low-energy laboratory for virtual signals of exotic physics.

In recent years the $b \to s \gamma$ decay has provided us with the first experimental evidence of “penguin” physics and the experimental bounds on the $B \to X_s \gamma$ decays have shown to provide sharp constraints on the physics beyond the SM.

The fact that the $b \to s \gamma$ transition has been already experimentally observed is related to the peculiar enhancement of the electroweak amplitude that arises at the two-loop level due to a large logarithmic QCD mixing with the effective $b \to s \bar{c}c$ operator \cite{1,2}. The study of the QCD leading logarithmic (LO) resummation for $b \to s \gamma$ (and $b \to s$ gluon) has been very recently extended to the next-to-leading order \cite{3,4} thus reducing the theoretical estimated error for the inclusive rate below the 10% threshold.

In order to produce similar studies for the $b \to s \gamma \gamma$ transition it is extremely helpful to observe that, by the use of an extension of Low’s low energy theorem \cite{6,7} or, alternatively, by applying the equation of motions, the $b \to s \gamma \gamma$ quark operator can be expanded at $O(G_f)$ on the standard operator basis needed for $b \to s \gamma$.

Three groups have recently presented a QCD LO calculation of the two photon transition \cite{8,9,10} thus improving on the previous electroweak calculations \cite{11,12}.

More recently a study of the $b \to s \gamma \gamma$ decay in two-Higgs doublet extensions of the SM has appeared \cite{13}.

The purpose of the present paper is to study the influence on the $b \to s \gamma \gamma$ transition
of softly broken supersymmetry. We find actually the results of our analysis having a more
general impact on the possible effects of new physics for the radiative two-photon decay. At
the LO the short-distance part of the $b \to s\gamma\gamma$ amplitude turns out to be controlled by the
one-photon radiative component. The higher dimension one-particle irreducible contribu-
tions present in the $b \to s\gamma\gamma$ amplitude, potentially large because of the $1/m_c^2$ dependence,
turn out to remain subleading because of accidental cancelations. This result remains true
when studying two-Higgs doublet extensions of the SM where the additional charged Higgs
contribution to the $b \to s\gamma$ component of the two-photon amplitude adds coherently to
the SM one. On the other hand, in supersymmetric theories there are potentially large
destructive effects related to the exchange of superpartners, which may reduce the size of
the one-particle reducible part of the $b \to s\gamma\gamma$ amplitude thus affecting the relative weight
between the latter and the one-particle irreducible components.

Nevertheless, the present experimental constraints on $b \to s\gamma$ induced decays are tight,
and by studying the effects of low energy supersymmetry as a paradigmatic case, the present
analysis shows generally that, as long as extensions of the SM affect only the short distance
Wilson coefficients of the standard effective Hamiltonian, the $b \to s\gamma\gamma$ induced decays are
severely constrained by the present bounds on the inclusive $B \to X_s\gamma$ decay [14]

\[
BR(B \to X_s\gamma) = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4}.
\]  

(1)

Since this result is consistent within 30% with the next-to-leading order (NLO) SM calcula-
tion [4]

\[
BR(B \to X_s\gamma) = (3.28 \pm 0.33) \times 10^{-4},
\]

(2)

one may not expect much larger deviations of the two-photon decay rates from the cor-
responding SM estimates. Were one to invoke supersymmetry to reduce by 30% the SM
prediction of $\Gamma(B \to X_s\gamma)$ then the SUSY rates related to $b \to s\gamma\gamma$ will unlikely exceed
the corresponding SM expectations. Potential implications of a (albeit challenging) NLO
analysis of the two-photon amplitude in extensions of the SM are commented upon in the
conclusions.
Although we shall apply our results to the $B_s \rightarrow \gamma\gamma$ decay we are not interested in predicting absolute decay rates and thus we will not discuss the uncertainties related to hadronization, which are not affected by short-distance new physics. Our main purpose is to study the deviations of the quark amplitudes from the SM predictions.

On the basis of these considerations and at present time, it is not worthy the effort to perform a detailed analysis of a specific supersymmetric model. We will investigate the main features of the SUSY amplitudes by means of relatively simple limiting cases in the supersymmetric and soft breaking parameter space.

II. EFFECTIVE QUARK LAGRANGIAN: OPERATOR BASIS AND COEFFICIENTS

At the LO in QCD the effective Hamiltonian for $b \rightarrow s\gamma$ closes on a basis of eight effective operators

$$H_{eff} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i(\mu),$$

where

$$O_1 = (\bar{s}_i c_j)_{V-A}(\bar{c}_j b_i)_{V-A}$$
$$O_2 = (\bar{s}_i c_i)_{V-A}(\bar{c}_j b_j)_{V-A}$$
$$O_3 = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}$$
$$O_4 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}$$
$$O_5 = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}$$
$$O_6 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}$$
$$O_7 = \frac{e}{8\pi^2} \bar{s}_i \sigma^{\mu\nu} (m_s(1-\gamma_5) + m_b(1+\gamma_5)) b_i F_{\mu\nu}$$
$$O_8 = \frac{g}{8\pi^2} \bar{s}_i \sigma^{\mu\nu} (m_s(1-\gamma_5) + m_b(1+\gamma_5)) T^a_{ij} b_j G_{\mu\nu}^a .$$

In eq. (4) $i, j = 1, 2, 3$ are color indices, $a = 1, ..., 8$ labels the $SU(3)$ generators, and $V \pm A \equiv 1 \pm \gamma_5$. The sum runs over the active quark flavours $u, d, s, c, b$. 
Having factored out the relevant Kobayashi-Maskawa (KM) mixings, the LO matching of the Wilson coefficients $C_i(\mu)$ at the scale $m_W$ is given in the SM by

\[
C_i^{SM}(m_W) = 0, \quad i = 1, 3, 4, 5, 6
\]  

\[
C_2^{SM}(m_W) = 1
\]  

\[
C_7^{SM}(m_W) = \frac{3x^3 - 2x^2}{4(x - 1)^4} \log x - \frac{8x^3 + 5x^2 - 7x}{24(x - 1)^3}
\]  

\[
C_8^{SM}(m_W) = \frac{-3x^2}{4(x - 1)^4} \log x - \frac{x^3 - 5x^2 - 2x}{8(x - 1)^3}
\]

where $x = m_t^2/m_W^2$.

Large $(\alpha_s \log \mu)^n$ corrections to the weak amplitudes are resummed via the renormalization group (RG) equations by evolving the Wilson coefficients at the typical scale of the process ($\mu \approx m_b$). While the anomalous dimension matrix of the first six operators is regularization scheme independent, the operator mixings between $O_1, \ldots, O_6$, and the dipole penguins $O_7,8$ are generally regularization-scheme dependent since they arise in LO at the two-loop level. However, the finite one-loop matrix elements of $b \to s\gamma$ (and $b \to s\gamma$) generated by the insertion of the $O_{5,6}$ gluonic penguins carry a compensating scheme-dependence such that the total physical amplitudes are independent on the regularization scheme. In view of this, one defines the so called “effective” Wilson coefficients \cite{15,16}, $C_7^{eff}(\mu)$ and $C_8^{eff}(\mu)$, for which the LO RG running is scheme independent. As an example, for $C_7(\mu)$ one finds the LO expression

\[
C_7^{eff}(\mu) = \eta^{21} C_7(m_W) + \frac{8}{3} \left( \eta^{21} - \eta^{26} \right) C_8(m_W) + C_2(m_W) \sum_{i=1}^{8} h_i \eta^{a_i}
\]

where $\eta = \alpha_s(m_W)/\alpha_s(\mu)$ and the numbers $h_i$ and $a_i$ are given in the appendix of ref. \cite{16}. The sum of all $h_i$ is zero and $C_i^{eff}(m_W) = C_i(m_W)$. For notational convenience, the superscript “eff” on $C_{7,8}(\mu)$ will be henceforth omitted.
III. THE $B_S \to \gamma\gamma$ DECAY

The quark $b \to s\gamma\gamma$ transition induces at the hadronic level two interesting rare decay modes of the $B$ mesons: $B_{u,d} \to X_s \gamma\gamma$ and $B_s \to \gamma\gamma$, where $X_s$ represents strange mesonic states. Both these decay modes and their features at the hadronic level within the SM have been widely studied in the literature \[17\].

We apply the analysis of the QCD corrected supersymmetric $b \to s\gamma\gamma$ amplitude to the discussion of the two body $B_s \to \gamma\gamma$ decay, whose total rate can be cast in a simple and compact form; this allows us to carry out the present analysis without unnecessary complications. Our conclusions are based on short-distance properties of the quark transition that hold as well for the $B_{u,d} \to X_s \gamma\gamma$ decay, even though the detailed form of the amplitude differs from that of the $B_s \to \gamma\gamma$ decay.

The total $B_s \to \gamma\gamma$ amplitude can be separated into a CP-even and a CP-odd part

$$\mathcal{A}(B_s \to \gamma\gamma) = M^+ F_{\mu\nu} F^{\mu\nu} + i M^- F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (10)$$

According to the notation of ref. \[9\] one finds

$$M^+ = -\frac{4\sqrt{2}\alpha_{em} G_F}{9\pi} f_{B_s} m_{B_s} V_{ts}^* V_{tb} \left( B(\mu) m_b K(m_b^2) + \frac{3C_7(\mu)}{8\Lambda} \right), \quad (11)$$

and

$$M^- = \frac{4\sqrt{2}\alpha_{em} G_F}{9\pi} f_{B_s} m_{B_s} V_{ts}^* V_{tb} \left( \sum_q m_{B_s} A_q(\mu) J(m_q^2) + m_b B(\mu) L(m_b^2) + \frac{3C_7(\mu)}{8\Lambda} \right), \quad (12)$$

where $\bar{\Lambda} = m_{B_s} - m_b$ and

$$A_u = (C_3 - C_5) N_c + (C_4 - C_6)$$
$$A_d = \frac{1}{4} ((C_3 - C_5) N_c + (C_4 - C_6))$$
$$A_c = (C_1 + C_3 - C_5) N_c + (C_2 + C_4 - C_6)$$
$$A_s = A_b = \frac{1}{4} ((C_3 + C_4 - C_5) N_c + (C_3 + C_4 - C_6))$$
$$B = -\frac{1}{4} (C_6 N_c + C_5), \quad (13)$$
are combinations of Wilson coefficients evaluated at the scale $\mu \approx m_b$.

The functions $J(m^2)$, $K(m^2)$ and $L(m^2)$ are defined by

$$J(m^2) = I_{11}(m^2),$$
$$K(m^2) = 4 I_{11}(m^2) - I_{00}(m^2),$$
$$L(m^2) = I_{00}(m^2),$$

(14)

with

$$I_{pq}(m^2) = \int_0^1 dx \int_0^{1-x} dy \frac{x^p y^q}{m^2 - 2xy k_1 \cdot k_2 - i\varepsilon},$$

(15)

and $2 \cdot k_1 \cdot k_2 = m_{B_s}^2$.

The decay width for $B_s \to \gamma \gamma$ is then given by

$$\Gamma(B_s \to \gamma \gamma) = \frac{m_{B_s}^3}{16\pi}(|M^+|^2 + |M^-|^2),$$

(16)

and, from the measured $B_s$ lifetime, the corresponding branching ratio is finally obtained

$$BR(B_s \to \gamma \gamma) = \frac{\Gamma(B_s \to \gamma \gamma)}{\Gamma(B_s)}. $$

(17)

Coming to the inclusive $B_s \to X_s \gamma$ decay it is convenient to use the approximate equality

$$BR(B \to X_s \gamma) \simeq \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\bar{\nu}_e)} BR(B \to X_s ce\bar{\nu}_e),$$

(18)

where

$$\frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\bar{\nu}_e)} = \frac{|V_{ts} V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_{em}}{\pi g(z)} |C_7(\mu)|^2,$$

(19)

which minimizes the uncertainties related to the bottom quark mass and KM mixings. In eq. (19) the function $g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \log z$ is the phase space factor in the semileptonic decay and $z = m_c/m_b$.

In Figs. 1 and 2 we show the LO results for the SM $B_s \to \gamma \gamma$ and $B \to X_s \gamma$ branching ratios, as a function of the renormalization scale $\mu$ and of $m_t$. Our numerical results are obtained using the values given in Table I for the other input parameters.
Fig. 1. $BR(B_s \to \gamma\gamma)_{SM} \times 10^7$ as a function of $\mu$ and $m_t$ (GeV) for central values of the other input parameters (Table I).

Fig. 2. $BR(B \to X_s\gamma)_{SM} \times 10^4$ as a function of $\mu$ and $m_t$ (GeV) for central values of the other input parameters (Table I).

From a direct comparison of Figs. 1 and 2 it appears clear that the $B_s \to \gamma\gamma$ decay rate is dominated by the $C_7$ component. This is related to the fact that the one-particle irreducible contributions arising from the operators $Q_{1,2}$, which could be potentially large due to the $1/m_c^2$ dependence of eq. (15), appear in eq. (13) ($A_c$) via the combination $N_c C_1 + C_2$ which is numerically suppressed [9]. Notice that the scale dependence represents the largest source of uncertainty of the LO calculation [16,18]. We have here shown the range $2.5$ GeV $< \mu < 10$ GeV.

As a reference for the following analysis, the SM central values for the LO QCD corrected decay rates at the scale $\mu = m_b$ are given by

$$BR(B \to X_s\gamma)_{SM} = 2.5 \times 10^{-4}$$
and

\[ BR(B_s \rightarrow \gamma\gamma)_{SM} = 4.4 \times 10^{-7}. \] \hspace{1cm} (21)

The prediction in eq. (21) compares to the present experimental bound [19]

\[ BR(B_s \rightarrow \gamma\gamma) < 1.48 \times 10^{-4} \] \hspace{1cm} (22)

which is about three orders of magnitude away from the needed sensitivity.

**IV. THE \( B_s \rightarrow \gamma\gamma \) DECAY IN SOFTLY BROKEN SUPERSYMMETRY**

In a wide class of realistic SUSY models the global supersymmetry breaking is a consequence of the spontaneous breaking of an underlying \( N=1 \) supergravity theory (for reviews see ref. [20]). The locally supersymmetric lagrangian is supposed to undergo a spontaneous breaking in the so called hidden sector, and the effects of this breaking are communicated to the observable sector through gravitational effects. A renormalizable theory is obtained in the limit in which the Planck mass goes to infinity. By doing so one is left with an effective globally supersymmetric lagrangian and explicit soft breaking terms. In our present study we shall consider the following gauge invariant soft breaking Lagrangian:

\[ \mathcal{L}_{\text{soft}} = -\mathcal{M}^2 - (\hat{M} + S + \text{h.c.}) \] \hspace{1cm} (23)

where \( \mathcal{M}^2 \) is a common mass term for all the scalar components \( z_i \) in the theory

\[ \mathcal{M}^2 \equiv \sum_i \bar{m}^2 z_i^* z_i, \] \hspace{1cm} (24)

\( \hat{M} \) is a mass term for the gauginos \( \lambda_\alpha, \alpha = 1, 2, 3 \) considered as Weyl fields

\[ \hat{M} \equiv -\frac{M_\alpha}{2} \lambda_\alpha \lambda_\alpha, \] \hspace{1cm} (25)

and \( S \) is the scalar analogue of the superpotential

\[ S = \bar{m} \left[ -A_U h_U H_2 \bar{Q} \bar{U}^c + A_D h_D H_1 \bar{Q} \bar{D}^c + A_E h_E H_1 \bar{L} \bar{E}^c + B \mu H_1 H_2 \right], \] \hspace{1cm} (26)
where with standard notation $h_{U,D,E}$ are the $3 \times 3$ Yukawa matrices for the quarks and charged leptons. The soft breaking parameters $A_i$ and $B$ are dimensionless numbers of order unity.

The $b \to s \gamma$ and $b \to s \gamma \gamma$ transitions can proceed in the SUSY model via five different intermediate particles exchanges:

1. Charged gauge bosons ($W^-$) + up-quarks
2. Charged Higgs bosons ($H^-$) + up-quarks
3. Charginos ($\chi^-$) + up-squarks
4. Gluinos ($\tilde{g}$) + down-squarks
5. Neutralinos ($\chi^0$) + down-squarks

The total amplitude is the sum of all these contributions. The complete analytic expressions for the various components are found in ref. [21].

An effective $b - s$ flavour changing transition induced by $W^-$ exchange is the only way through which the decays proceed in the SM. A two-Higgs doublet extension of the SM would include the first two contributions, while the last three are genuinely supersymmetric in nature.

Gluinos and neutralinos can mediate flavour changing interactions only via renormalization effects which are crucially dependent on the detailed structure of the model. Their consideration is beyond the scope of the present work and our results do not presently justify a more detailed analysis. We shall discuss the features of the inclusion of the first three contributions in the matching of the Wilson coefficients.

The supersymmetric Wilson coefficients are then given by

$$C^{SUSY}_{7,8}(m_W) = C^{SM}_{7,8}(m_W) + C^H_{7,8}(m_W) + C^X_{7,8}(m_W) ,$$

while at the LO the matching conditions in eqs. (5)–(6) remain unaffected.
From the results of ref. [21] and comparing with eq. (3) we obtain the following contributions to the $C_{7,8}(m_W)$ coefficients:

$$C_{7,8}^{H}(m_W) = \frac{1}{2} \frac{m_t^2}{m_H^2} \left[ \frac{1}{\tan^2 \beta} f_{7,8}^{(1)} \left( \frac{m_t^2}{m_H^2} \right) + f_{7,8}^{(2)} \left( \frac{m_t^2}{m_H^2} \right) \right], \quad (28)$$

induced by charged Higgs exchange and

$$C_{7,8}^{\chi}(m_W) = -\frac{1}{V_{ts}^* V_{tb}} \sum_{j=1}^{6} \sum_{k=1}^{6} \frac{m_W^2}{\tilde{m}_{\chi j}^2} \left[ (G_{UL}^{jk} - H_{UR}^{jk})(G_{UL}^{*jk} - H_{UR}^{*jk}) f_{7,8}^{(1)} \left( \frac{\tilde{m}_{uk}^2}{\tilde{m}_{\chi j}^2} \right) \right] - H_{UL}^{jk} (G_{UL}^{*jk} - H_{UR}^{*jk}) \frac{\tilde{m}_{\chi j}}{m_b} \frac{\tilde{m}_{uk}^2}{\tilde{m}_{\chi j}} f_{7,8}^{(3)} \left( \frac{\tilde{m}_{uk}^2}{\tilde{m}_{\chi j}^2} \right), \quad (29)$$

induced by chargino exchange. We have found convenient for the present discussion to introduce the functions $f_{7,8}^{(n)}$ according to the notation of ref. [22]

$$f_{7}^{(1)}(x) = \frac{(7 - 5x - 8x^2)}{36(x - 1)^3} + \frac{x(3x - 2)}{6(x - 1)^4} \log x \quad (30)$$

$$f_{7}^{(2)}(x) = \frac{(3 - 5x)}{6(x - 1)^2} + \frac{(3x - 2)}{3(x - 1)^3} \log x \quad (31)$$

$$f_{7}^{(3)}(x) = (1 - x) f_{7}^{(1)}(x) - \frac{x}{2} f_{7}^{(2)}(x) - \frac{23}{36} \quad (32)$$

$$f_{8}^{(1)}(x) = \frac{(2 + 5x - x^2)}{12(x - 1)^3} - \frac{x}{2(x - 1)^4} \log x \quad (33)$$

$$f_{8}^{(2)}(x) = \frac{(3 - x)}{2(x - 1)^2} - \frac{1}{(x - 1)^3} \log x \quad (34)$$

$$f_{8}^{(3)}(x) = (1 - x) f_{8}^{(1)}(x) - \frac{x}{2} f_{8}^{(2)}(x) - \frac{1}{3} \quad (35)$$

These functions have simple and obvious relations with the functions $F_n(x)$ defined originally in ref. [21], to which we refer the reader for all details.

In eq. (29) $j = 1, 2$ is the label of the chargino mass eigenstates and $k = 1, ..., 6$ is the analogous label for the up-squarks; the matricial couplings $G_{UL}$ arise from charged gaugino-squark-quark vertices, whereas $H_{UL}$ and $H_{UR}$ are related to the charged higgsino-squark-quark vertices. These couplings contain among else the unitary rotations $U$ and $V$ which diagonalize the chargino mass matrix

$$U^* \begin{pmatrix} M_2 & m_W \sqrt{2} \sin \beta \\ m_W \sqrt{2} \cos \beta & -\mu \end{pmatrix} V^{-1} = \begin{pmatrix} \tilde{m}_{\chi_1} & 0 \\ 0 & \tilde{m}_{\chi_2} \end{pmatrix}, \quad (36)$$
where $M_2$ is the weak gaugino mass and $\mu$ the Higgs mixing parameter. The sign of the $\mu$ entry is defined accordingly to the Feynman rules used in obtaining the above results (see the comment following eq. (15) in ref. [23]).

Due to the relevance of the chargino amplitude for the present discussion, it is worth trying to have a better understanding of the nature of the features exhibited by this amplitude.

An explicit $\tan \beta$ dependence is found in $H_{UL}$ and $H_{UR}$ where quark Yukawa couplings are present; more precisely, $H_{UL}$ is proportional to the down-quark Yukawa coupling, which grows with $\tan \beta$ as $1/\cos \beta$, whereas $H_{UR}$ contains the up-quark Yukawa coupling, that approaches in the large $\tan \beta$ limit a constant value ($\propto 1/\sin \beta$). It is in fact the contribution in the second line of eq. (29) that determines the behaviour of the amplitude in the large $\tan \beta$ regime. Detailed studies of this feature of the chargino amplitude are available in the literature [24].

In order to allow for an analytic and more transparent discussion of the chargino component we resort to simplified assumptions on the squark mass spectrum which reproduce with good approximation the global features of the model. In this we follow closely the analysis of ref. [22]. We assume that all squarks, other than the two scalar partners of the top quark, are degenerate with the soft breaking mass $\tilde{m}$. The remnant $2 \times 2$ top squark mass matrix is diagonalized by an orthogonal matrix $T$ such that:

$$
T \begin{pmatrix}
\bar{m}^2 + m_t^2 & (A_t \bar{m} + \mu/\tan \beta) m_t \\
(A_t \bar{m} + \mu/\tan \beta) m_t & \bar{m}^2 + m_t^2
\end{pmatrix} T^{-1} = \begin{pmatrix} \bar{m}^2_{t_1} & 0 \\ 0 & \bar{m}^2_{t_2} \end{pmatrix},
$$

(37)

where $A_t$ is the supersymmetry-breaking trilinear coupling. The sign of the $\mu$ term is consistent with eq. (36).

With these assumptions eq. (29) can be written as

$$
C_{7,8}^\chi(m_W) = \sum_{j=1}^{2} \left[ \frac{m_W^2}{\bar{m}_{\chi_j}^2} \left| V_{j1}\left| f_{7,8}^{(1)} \left( \frac{\bar{m}^2_{\chi_j}}{m_W^2} \right) \right| \right|^2 - \sum_{k=1}^{2} \left| V_{j1} T_{k1} - V_{j2} T_{k2} \frac{m_t}{\sqrt{2} m_W \sin \beta} \right|^2 f_{7,8}^{(1)} \left( \frac{\bar{m}_{t_k}^2}{\bar{m}_{\chi_j}^2} \right) \right],
$$

12
\[
- \frac{U_{j2}}{\sqrt{2}} \cos \beta \tilde{m}_{\chi_j} \left[ V_{j1} f_{T,8}^{(3)} \left( \frac{\tilde{m}_t^2}{\tilde{m}_{\chi_j}^2} \right) + \sum_{k=1}^{2} \left( V_{j1} T_{k1} - V_{j2} T_{k2} \frac{m_t}{\sqrt{2} m_W \sin \beta} \right) T_{k1} f_{T,8}^{(3)} \left( \frac{\tilde{m}_{t_k}^2}{\tilde{m}_{\chi_j}^2} \right) \right] \right], \tag{38}
\]

A. Four exemplifying cases

We are now ready to investigate the effects of the SUSY matchings on the \(b \to s \gamma \gamma\) transition. We will show our results by plotting the ratios of SUSY versus SM decay rates for central values of the SM input parameters while varying the unknown SUSY parameters. We investigate four limiting cases which span the global features of the new amplitudes.

First, we take \(M_2 = \mu = A_t = 0\) and fix \(\tan \beta = 1\) (case 1). In this approximation we have

\[
U = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 \\
-1 & 1
\end{pmatrix}, \quad V = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix}, \quad \tilde{m}_{\chi_{1,2}} = m_W, \tag{39}
\]

\[
T = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}, \quad \tilde{m}_{t_{1,2}}^2 = \tilde{m}_t^2 + m_t^2. \tag{40}
\]

The Wilson coefficients \(C_{7,8}^Y\) can be simply written as:

\[
C_{7,8}^Y (m_W) = z \left[ f_{T,8}^{(1)} (z) + \frac{1}{2} f_{T,8}^{(2)} (z) \right] - (2x + z) f_{T,8}^{(1)} (x + z) - \frac{x + z}{2} f_{T,8}^{(2)} (x + z), \tag{41}
\]

where

\[
x = \frac{m_t^2}{m_W^2}, \quad z = \frac{\tilde{m}_t^2}{m_W^2}. \tag{42}
\]

As remarked in ref. \[22\], \(C_{7,8}^{SUSY} (m_W)\) shows an exact cancellation in the supersymmetric limit, \(z \to 0\) and \(m_H \to m_W\). This is a consequence of the fact that any magnetic moment transition vanishes in exact supersymmetry \[23\]. Therefore non-vanishing contributions to the \(C_{7,8}^{SUSY}\) coefficients arise due to the presence of the soft breaking terms.

We define
\[ R_{\gamma\gamma} = \frac{BR(B_s \to \gamma\gamma)_{\text{SUSY}}}{BR(B_s \to \gamma\gamma)_{\text{SM}}} \],

and

\[ R_{\gamma} = \frac{BR(B \to X_s\gamma)_{\text{SUSY}}}{BR(B \to X_s\gamma)_{\text{SM}}} \],

where the SM decay rates are those given in eqs. (20)–(21), obtained using the central values of the SM input parameters.

Fig. 3. Case 1. \( R_{\gamma\gamma} \) as a function of \( \tilde{m} \) and \( m_H \) (GeV), for degenerate chargino masses \( \tilde{m}_{\chi_{1,2}} = m_W \).

Fig. 4. Case 1. The allowed range for \( R_{\gamma\gamma} \) is shown as a function of \( \tilde{m} \) and \( m_H \) (GeV), by constraining the \( BR(B \to X_s\gamma)_{\text{SUSY}} \) to vary within a ±30% from its SM value.

In Fig. 3 we show \( R_{\gamma\gamma} \) as a function of the charged Higgs mass and the scalar soft breaking mass in a few hundred GeV range. In Fig. 4 the same range is spanned assuming the constraint

\[ 0.7 < R_{\gamma} < 1.3 \].

(45)
We see that $R_{\gamma\gamma}$ as well is bound to vary in approximately the same range. The study of the ratio $R_{\gamma\gamma}/R_\gamma$ in the same region shows deviations of at most $\pm 4\%$ from unity, which shows the strong correlation between the two decays.

By releasing the constraint $A_t = 0$, while holding $M = \mu = 0$ and $\tan \beta = 1$, we allow for a mass splitting of the stop eigenstates (case 2). This corresponds to having

$$T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad \tilde{m}_{t_{1,2}}^2 = \tilde{m}_t^2 + m_t^2 \pm A_t \tilde{m}_t m_t. \quad (46)$$

The chargino contribution to $C_{7,8}$ becomes:

$$C^\chi_{7,8}(m_W) = z f^{(1)}_{7,8}(z) + \frac{z}{2} f^{(2)}_{7,8}(z) - \frac{x}{2} f^{(1)}_{7,8}(w_k) + \frac{w_k}{4} f^{(2)}_{7,8}(w_k) \right]. \quad (47)$$

where $w_{1,2} = x + z \pm A_t \tilde{m}_t m_t / m_W^2$.

Fig. 5. Case 2. $R_{\gamma\gamma}$ is shown as a function of $\tilde{m}$ (GeV) and $A_t$ for $m_H = 150$ GeV.

In Fig. 5 we plot $R_{\gamma\gamma}$ as a function of $A_t$ and $\tilde{m}$ for fixed $m_H = 150$ GeV, under the requirement that the lightest stop mass is always above 45 GeV. As we see, releasing the stop squark degeneracy while keeping charginos degenerate does not sizeably modify the features shown in case 1, and the same conclusions apply.

Next we consider $M = \mu = A_t = 0$, and arbitrary $\tan \beta$ (case 3). The chargino component reduces to:

$$C^\chi_{7,8}(m_W) = -\frac{x + z}{4 \cos^4 \beta} \left[ f^{(1)}_{7,8} \left( \frac{x + z}{2 \cos^2 \beta} \right) + \frac{1}{2} f^{(2)}_{7,8} \left( \frac{x + z}{2 \cos^2 \beta} \right) \right] - \frac{x}{4 \sin^4 \beta} f^{(1)}_{7,8} \left( \frac{x + z}{2 \sin^2 \beta} \right)$$

$$+ \frac{z}{4 \cos^4 \beta} \left[ f^{(1)}_{7,8} \left( \frac{z}{2 \cos^2 \beta} \right) + \frac{1}{2} f^{(2)}_{7,8} \left( \frac{z}{2 \cos^2 \beta} \right) \right]. \quad (48)$$
In this case, the chargino degeneracy is lifted, while keeping the degeneracy in the squark sector. The chargino contribution becomes dependent on tan $\beta$. On the other hand, as can be verified by means of eqs. (30)–(35), the tan $\beta$ dependence of eq. (48) in the large tan $\beta$ limit is only logarithmic. As we will see later, for the SUSY amplitude to exhibit a stronger tan $\beta$ dependence $A_t \neq 0$ is required as well.

Fig. 6. Case 3. The allowed range for $R_{\gamma\gamma}$ is shown as a function of $\tilde{m}$ and $m_H$ (GeV) for tan $\beta = 10$, imposing the constraint of eq. (45) on $BR(B \to X_s\gamma)_{SUSY}$.

Fig. 7. Case 3. The allowed range for $R_{\gamma\gamma}$ is shown as a function of $\tilde{m}$ (GeV) and tan $\beta$ for $m_H = 150$ GeV, imposing the constraint of eq. (45) on $BR(B \to X_s\gamma)_{SUSY}$.

In Figs. 6 and 7 we show as a function of different SUSY parameters the allowed range for $R_{\gamma\gamma}$ once the constraint on $BR(B \to X_s\gamma)_{SUSY}$ in eq. (45) is imposed. $R_{\gamma\gamma}$ is always bound to vary within $\pm 30\%$ from its SM expectation with high correlation to $R_{\gamma\gamma}$.

Finally we consider the case for which $M_2$, $\mu$ and $A_t$ are different from zero and tan $\beta \gg 1$ (case 4). In fact, the part of the chargino contribution which leads the large tan $\beta$ behaviour...
vanishes when either squarks or charginos are degenerate, as one can verify from the simplified form of eq. (38).

![Diagram](image.png)

Fig. 8. Case 4. The potential enhancement of $R_{\gamma\gamma}$ is shown as a function of $\tilde{m}$ and $m_H$ (GeV) for $\tan \beta = 15$ (upper surface) and 10 (lower surface).

An analytic approximation of the $H_{UL}H_{UR}^*$ component of the chargino amplitude can be derived which shows explicitly its interesting features [23]. We assume the chargino mass matrix in eq. (36) to be approximately diagonal:

$$M_\chi \approx \text{diag}(M_2, -\mu)$$  \hspace{1cm} (49)

This approximation holds effectively when $|M_2^2 - \mu^2| = O[\max(M_2^2, \mu^2)] \gg m_W^2$ and $(M_2^2, \mu^2) \gtrsim m_W^2$. It is important to notice that these requirements, and therefore the approximation of eq. (49) is consistent with one of the eigenvalues, say $|\mu|$, being of the order of $m_W$, while the other $|M_2|$ is much heavier. The chargino mass matrix already being diagonal, the approximate mass eigenvalues are simply given by the absolute values of the parameters $M_2$ and $\mu$, and the two unitary rotations which “diagonalize” the chargino mass matrix can be written as:

$$U \approx \text{diag}(\text{sign}[M_2], -\text{sign}[\mu]) ,$$  \hspace{1cm} (50)

$$V \approx 1$$

In this approximation, the matrix $T$ and the stop mass eigenstates are given by eq. (10).

Using eq. (10) and eqs. (19)–(50) we obtain a simple expression for the part of the chargino component relevant for large $\tan \beta$: 
where $\tilde{m}_\chi^2 = |\mu|$ is the lightest chargino eigenvalue. Notice that the amplitude in eq. (51) depends on the signs of both $\mu$ and the trilinear soft breaking parameter $A_t$ (changing the sign of the latter amounts to interchanging the two stop mass eigenvalues). One also verifies that the amplitude vanishes for either $\mu = 0$ or $A_t = 0$ as it should.

As already mentioned, at variance with the case 3, the leading behaviour of the chargino amplitude is linear with $\tan \beta$. This may in general be the source of large deviations of the SUSY rates from the corresponding SM expectations. In Fig. 8 we show the ratio of the SUSY to SM branching ratio for $B_s \to \gamma\gamma$ as a function of $m_H$ and $\tilde{m}$ for $\tan \beta = 10$ and 15. In the example shown we have chosen $\mu = m_W$ and $A_t = 1.5$. We see the potential large enhancements which arise for large $\tan \beta$ from this component of the chargino amplitude. On the other hand, imposing the constraint in eq. (45) allows only those regions of the lower surface for which $R_{\gamma\gamma}$ varies approximately in the range $0.7 - 1.3$.

Globally, in the tested region of parameters the deviations of the $B_s \to \gamma\gamma$ decay rate from the SM expectations are confined to be within 10% from the corresponding deviations for the $B \to X_s\gamma$ decay.

At the next-to-leading order one may try to devise models that enhance the matchings of the $O_{3-6}$ penguin operators (which are vanishing at the LO) keeping the $O_{7,8}$ Wilson coefficients “under control”. As unlikely as this may be, numerically it is anyhow difficult to expect drastic deviations from the SM predictions, due to the subleading role of the $O_{3-6}$ operators (analogous considerations apply to the electroweak penguin operators, which we have neglected in the LO analysis).

We conclude that in order to disentangle new physics effects from a comparison of the two $b \to s$ radiative decays a precision below 10% is required both on the theoretical and experimental sides. Due to the smallness of the two-photon rates and to the theoretical uncertainties related to long-distance physics it shows as a truly challenging task.
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| Parameter | Value |
|-----------|-------|
| $\alpha_s(m_Z)$ | 0.118 |
| $\alpha_{em}$ | 1/129 |
| $m_Z$ | 91.19 GeV |
| $m_W$ | 80.33 GeV |
| $m_t$ | 175 GeV |
| $m_b$ | 4.8 GeV |
| $m_c$ | 1.4 GeV |
| $m_s$ | 0.150 GeV |
| $|V_{ts}^* V_{tb}|/|V_{cb}|$ | 0.976 |
| $|V_{ts}^* V_{tb}|$ | $4 \times 10^{-2}$ |
| $m_{B_s}$ | 5.37 GeV |
| $f_{B_s}$ | 0.2 GeV |
| $\Gamma(B_s)$ | $4.09 \times 10^{-13}$ GeV |
| $BR(B \to X_c e\bar{\nu}_e)$ | $10.4 \times 10^{-2}$ |

**TABLE I.** Values of the input parameters used in the numerical calculations.