CoMP Transmission in Downlink NOMA-Based Indoor VLC Cellular Systems

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Abstract—In this paper, we investigate the dynamic power allocation for a visible light communication (VLC) cellular system consisting of two coordinating attocells, each equipped with one access-point (AP). The joint-transmission coordinated multipoint (JT-CoMP) between the two cells is introduced to assist users experiencing high inter-cell-interference (ICI), especially the ones located at the edge of both cells, where each cell invokes non-orthogonal-multiple-access (NOMA) to serve its associated users. A power allocation framework is formulated as an optimization problem with the objective of maximizing the network sum-rate while guaranteeing a certain quality-of-service (QoS) for each user. The formulated optimization problem is not concave, which is difficult to be solved directly unless using heuristic methods, which comes at the expense of high computational complexity. To overcome this issue, an optimal and low complexity power allocation scheme is derived. In the simulation results, the performance of the proposed CoMP-assisted NOMA scheme is compared with those of the CoMP-assisted orthogonal-multiple-access (OMA) scheme, the NOMA scheme and the OMA scheme where the superiority of the proposed scheme is demonstrated. Finally, the performance of the proposed scheme and the considered baselines is evaluated while varying the coverage area of each attocell and the distance between the APs.

Index Terms—Coordinated Multi-Point (CoMP), Inter-Cell Interference (ICI), Non-Orthogonal Multiple Access (NOMA), Visible Light Communication (VLC).

I. INTRODUCTION

A. Motivation

As the fifth generation (5G) wireless networks are currently under deployment, researchers from both academia and industry started shaping their vision on how the upcoming sixth generation (6G) should be [1]. The main goals of 6G networks are not only to fill the gap of the original and unfulfilled promises of 5G or to keep up with the continuous emergence of the Internet of Things (IoTs) networks but also to handle the exponential increase of the number of devices connected to the Internet, which is predicted to reach 29.3 billion devices in 2023 [2], [3]. Therefore, 6G networks must urgently provide high data rates, seamless and massive connectivity, ubiquitous coverage and ultra-low latency communications in order to reach the preset targets [3]. Due to this, researchers from industry and academia are trying to explore new network architectures, new transmission techniques and higher frequency bands, such as the millimeter wave (mmWave), the terahertz (THz), the infrared, and the visible light bands to meet these high requirements [3].

VLC is an emerging high speed optical wireless communication technology, that uses the visible light as the propagation medium in the downlink for the purposes of illumination and wireless communication. VLC operates in the visible light spectrum to communicate data between access points (APs) and users using the existing illumination component, such as the Light Emitting Diodes (LEDs). VLC offers a number of important benefits that have made it favorable for 6G networks [4] such as the large unregulated visible light, which translates into higher data rates and higher connectivity in comparison to traditional radio-frequency (RF) networks [5], the high energy efficiency [6], the straightforward deployment that uses off-the-shelf LEDs and photodetector (PD) devices at the transmitter and receiver ends respectively, and the enhanced security since light does not penetrate through opaque objects [7].

As a wireless broadband technology, VLC must support a high number of users with simultaneous network access [8]. Traditional multiple access techniques, referred to as orthogonal multiple access (OMA) techniques, allocate the available resources to coexisting users in an orthogonal manner in order to cancel the inter-user interference. Such multiple access techniques include code division multiple access (CDMA), time division multiple access (TDMA) and frequency division multiple access (FDMA) [8]. The main drawback of the aforementioned techniques is that the maximum number of users that can be served is limited by the number of available orthogonal resources. In other words, the OMA techniques cannot provide sufficient resource reuse when the number of coexisting users is high. This makes OMA techniques unable to support a massive number of users, and hence, unable to provide a massive connectivity which is one of the main requirements of 6G networks.

In an effort to increase the throughput and improve the fairness of VLC systems, the non-orthogonal multiple access (NOMA) technique was introduced in the recent literature. In contrast to OMA techniques, NOMA allows multiple users to exploit the same time/frequency resource blocks at the expense of some inter-user interference (IUI), leading to an efficient resource utilization. Downlink NOMA relies on the superposition coding (SC) concept at the transmitter side to multiplex the data streams of different users in the power domain, and on the successive interference cancellation (SIC) concept at the end users to decode their received data [9]. NOMA operates by allocating power to users based on their channel gain. Low power is allocated to users with high channel gain, referred to as strong users, and high power is allocated to users with low channel gain, referred to as weak users.
users. Strong users first apply SIC to decode the weak users’ data and then decode their data, whereas weak users proceed to decode their data directly, and hence, suffer from inter-user interference resulting from the superposition of the strong users’ data.

Despite the great benefits that VLC offers, it suffers from various shortcomings that make the current technology still far from satisfying the demands of 6G networks. The first limitation is the short communication range resulting from the short wavelengths of visible light waves. This results in high propagation losses as the visible light can be easily blocked by obstacles and the VLC channel gain significantly deteriorates when the distance between transmitting and receiving devices increases \([10]–[12]\). In addition, unlike conventional RF wireless systems, the VLC channel is not isotropic, meaning that the orientation of the transmitting and receiving devices affects the channel gain significantly \([10]–[12]\). As a result, the VLC channel quality fluctuates and the performance of advanced multiple access techniques, such as NOMA, is affected when applied to VLC systems \([10]–[12]\).

### B. Related Works

A large body of work has been produced in the application of NOMA in VLC systems, such as \([13]–[16]\) to name a few. In \([13]–[14]\), it was shown that NOMA outperforms the traditional orthogonal frequency division multiple access (OFDMA) scheme in an indoor VLC system model serving multiple users with fixed positions and orientations. Both works prove the superiority of NOMA over OMA for stationary users but refrain from studying the performance of NOMA for mobile users with random position and orientation. In \([15]\), NOMA was investigated in a downlink multi-user VLC system model with mobile and randomly oriented users, where the sum rate and outage probability were derived. In \([16]\), a system model consisting of one AP serving multiple users simultaneously was established, where users are served using the cooperative NOMA (C-NOMA) scheme.

C-NOMA is an enhanced version of NOMA that takes advantage of the desirable attributes of NOMA and device-to-device (D2D) communication. In a practical scenario, and by exploiting the SIC capabilities, each strong user can act as a relay to assist the communication between the transmitter and the weak user through an RF D2D link. Hence, each weak user receives two versions of his data, one through the VLC link coming from the transmitter and one from the RF link coming from the associated strong user. In \([16]\), the VLC system sum rate was maximized by optimizing the strong/weak user pairing, the VLC/RF link selection, and the transmitted power, and it was shown that C-NOMA outperforms the conventional NOMA scheme. However, the main drawback with the works in \([13]–[16]\) is that the performance of NOMA was investigated in a single-cell setup, and the extension to multi-cell configuration was not considered.

For 5G wireless networks and beyond, cell densification has been demonstrated to be an effective method to increase the network capacity. The main motivation behind cell densification is reducing the path loss and allowing the reuse of partial or total spectrum by small cells within a given coverage area. In the context of VLC, the concept of optical attocell was first introduced in \([17]\). However, similar to the cell densification concept, the main drawback of the use of multiple optical attocells in indoor environments is the severe inter-cell interference (ICI). Precisely, an indoor environment can be composed of multiple optical attocells, each having a radius of around 3 m \([14]\). These optical attocells are adjacent to each other, and hence, when the optical attocells are exploiting the same frequency resources, the users within one attocell can experience severe ICI from adjacent cells.

The application of NOMA in multicell VLC systems was studied in \([18]–[19]\). In \([18]\), a user grouping scheme based on users locations was proposed to reduce the ICI effects in NOMA-based multi-cell VLC networks. With the residual interference from the SIC process in NOMA taken into account, the power allocation within each attocell was optimized to improve the achievable rate per user under a quality-of-service (QoS) constraint. Recently, a multi-cell VLC system was considered in \([19]\), where each attocell consists of an AP that serves two users coexisting within its coverage using C-NOMA. However, the main drawback of the schemes proposed in \([18]–[19]\) is that the VLC users still suffer from the ICI effects since no ICI mitigation techniques were employed. In order to mitigate the ICI effects, the concept of cooperative cellular systems has been introduced in practical VLC systems, where multiple VLC APs coordinate together in serving multiple users within the resulting illuminated area \([20]–[24]\). In this context, the users that are highly prone to ICI effects can be jointly served by a set of adjacent APs. This ICI mitigation technique is referred to as the joint transmission (JT) coordinated multipoint (CoMP) or the coordinated broadcasting technique \([20]–[24]\).

The performance of CoMP transmission in VLC systems was investigated in the literature \([20]–[24]\). In \([20]\), it was demonstrated that the JT CoMP can achieve higher signal-to-interference-plus-noise ratios (SINRs) for ICI prone users in comparison to the frequency reuse (FR) technique. In \([21]\), two linear precoders based on the minimum mean square error (MMSE) method were proposed to minimize the mean-square error (MSE) in multiple coordinated VLC attocells under imperfect channel state information (CSI), where all the LED transmitters are assumed to be coordinated through an optical power line communication (PLC) link. Considering multi-user multi-cell multiple-input multiple-output (MIMO) VLC systems, a coordinated zero-forcing (ZF) precoding technique was proposed in \([23]\) in a way to cancel the ICI and to maximize the achievable sum rate of the cellular users. For the same system, a joint precoder and equalizer design based on interference alignment is proposed in \([24]\) to mitigate both the IUI and the ICI under imperfect CSI, where the MSE is minimized at the received side under optical power constraints. The main drawback of the proposed precoding techniques in \([21]–[24]\) is that the maximum number of users that can be served simultaneously is limited by the number of the APs, and hence, the proposed schemes are unable to support a massive number of VLC users.

Based on the above background, one can see that NOMA...
is an auspicious multiple access technique that can boost the network connectivity, while the CoMP technique is an effective ICI mitigation technique. Motivated by this, it is expected that the integration between both techniques can improve the performance of multi-user multi-cell VLC systems and can make the VLC technology a promising candidate for 6G wireless networks. To the best of our knowledge, the integration between NOMA and CoMP in multi-user multi-cell VLC systems was not studied in the literature, which is the focus of this paper.

C. Contributions and Outcomes

In this paper, we consider a downlink VLC system consisting of two adjacent VLC attocells that utilize the same frequency resources. Each attocell contains one AP used for illumination and data communication simultaneously. The coverage area of the two attocells is overlapped. Within this system, three stationary VLC users are communicating simultaneously with the two APs, where each user is equipped with a randomly oriented user-equipment (UE). The first and second UEs are located near the center of each attocell, whereas the third UE is located at the edge of the attocells, i.e., in the area of overlapping coverage. Therefore, the first and second UEs are associated to the first and second APs, respectively, whereas the third UE is associated to both APs. In this setup, each attocell employs NOMA to serve its associated UEs. Hence, the first and second UEs are the strong NOMA UEs in their respective cells, whereas the third UE is the weak UE in both cells. Since the two attocells utilize the same frequency resources, the VLC users suffer from ICI. To overcome this issue, the CoMP technique is employed to mitigate the ICI at the weak UE, whereas the ZF precoding technique is used to cancel the ICI at the strong UEs.

For the considered CoMP-assisted two-cell NOMA network, an optimization framework that maximizes the network sum-rate (under QoS, SIC and power constraints) is formulated as an optimization problem. Then, an optimal and low-complexity power control policy is derived. In the simulation results, the optimality of the proposed low-complexity power control policy is verified. In addition, three baseline schemes are considered for comparison purpose, which are the CoMP-assisted FDMA scheme, the NOMA scheme and the FDMA scheme, and our results demonstrate the superiority of the proposed CoMP-assisted NOMA scheme. Finally, the performance of the proposed scheme and the considered baselines is evaluated while varying the coverage area of each attocell and the distance between the APs.

D. Outline and Notations

The rest of this paper is organized as follows. Section II introduces the system model and the transmission strategy. Section III presents the problem formulation and the proposed solution. Section IV and V present the simulation results and the conclusion, respectively.

Upper case bold characters denote matrices, whereas lower case bold characters denote vectors. $(\cdot)\T$ and $(\cdot)^{-1}$ denotes the transpose and the inverse operators, respectively. $\|\cdot\|_\infty$ denotes the infinity norm operator. For every positive real number $a$, the function $\text{rect}(\cdot/a)$ denotes the rectangular function within $[0,a]$. For every real numbers $a$ and $b$, $\min(a,b)$ and $\max(a,b)$ denote the maximum between $a$ and $b$, respectively. Finally, for $N \in \mathbb{N}$, $\mathbf{0}_N$ and $\mathbf{1}_N$ denote the all zeros and all ones $N \times 1$ vector, respectively.

II. SYSTEM MODEL

A. System Setup

The system model considered in this paper is shown in Fig. 1 where two APs, each equipped with a set of LEDs, are installed at the ceiling of an indoor environment at a height $H$ from the ground. Hence, the circular coverage area of each AP has a radius $R = H \times \tan(\Phi_{1/2})$, where $\Phi_{1/2}$ represents the half-power semi-angle of the LEDs. The two APs are jointly monitored by a VLC controller and they share the same frequency bandwidth $B$. Two UEs, denoted by $SU_a$ and $SU_b$, respectively, are located around the centers of the first and second cells, respectively, i.e., around the centers of the coverage areas of AP$_1$ and AP$_2$, respectively. On the other hand, since the two APs are adjacent, their resulting coverage areas may be overlapping. The area of the resulting overlapping region depends on the radius of each cell $R$, and hence depends on the height $H$ and the half-power semi-angle $\Phi_{1/2}$, as well as the distance between the two APs, which is denoted by $d_{AP}$. In this context, one UE, denoted by $WU$, is located within the edge of both cells, i.e., within the overlapping region between the coverage areas of AP$_1$ and AP$_2$.

B. Transmission and Reception Models

Since they are located around the centers of their associated cells, the UEs $SU_a$ and $SU_b$ are associated with AP$_1$ and AP$_2$. In practical VLC systems, the number of deployed APs might be greater than two. In such a case, the APs can be clustered into pairs of adjacent APs and the proposed CoMP-assisted NOMA scheme can be applied within each pair. In addition, the main reason behind limiting the number of APs to two in this study is that this paper proposes a proof of concept of the CoMP-assisted NOMA scheme for VLC systems.

![Fig. 1: System Model.](image-url)
AP\(_2\), respectively. However, the UE WU is associated with both APs since it is located within the intersection of their coverage areas. In this considered cellular system, the VLC controller applies NOMA to serve the UEs within each cell, where SU\(_a\) and WU are the NOMA UEs associated with AP\(_1\) and SU\(_b\) and WU are the NOMA UEs associated with AP\(_2\). In this context, when applying the NOMA principle, SU\(_a\) and SU\(_b\) are considered as strong users in their respective cells since they are located around their centers, whereas WU is considered as the weak user. Based on this, the 2 × 1 vector of positive-valued signals \(s\) that should be broadcast by AP\(_1\) and AP\(_2\) is expressed as

\[
s = \mathbf{Wx} + I_{DC}\mathbf{1}_2, \tag{1}
\]

where \(\mathbf{W}\) is the 2 × 2 precoding matrix of the considered cellular system, which should be designed by the VLC controller, \(\mathbf{x} = [x_1, x_2]\) is the 2 × 1 vector of the superimposed messages of the NOMA UEs, and \(I_{DC}\) represents the electrical direct-current (DC) provided to each AP\(_2\). In this case, the signals \(x_1\) and \(x_2\) are expressed, respectively, as

\[
x_1 = (1 - \alpha_1)I_{DC}s_a + \alpha_1I_{DC}s_w, \tag{2a}
\]

\[
x_2 = (1 - \alpha_2)I_{DC}s_b + \alpha_2I_{DC}s_w, \tag{2b}
\]

where \(s_a\), \(s_b\), and \(s_w\) represent the messages intended to SU\(_a\), SU\(_b\), and WU, respectively, and \(\alpha_1, \alpha_2 \in [0, 1]\) represent the DC factors assigned by AP\(_1\) and AP\(_2\) to the WU, respectively, which should also be designed by the VLC controller. In addition, for all \(k \in \{a, b, w\}\), the message \(s_k\) should satisfy \(s_k \in [−\nu, \nu]\), where \(\nu \in [0, 1]\) denotes the modulation index of the VLC system \([26]\).

One operating constraint in VLC systems is the peak-power constraint, also known as the amplitude constraint, which is widely adopted in the literature \([27]–[29]\). In fact, typical LEDs suffer from nonlinear distortion and clipping effects. Hence, in order to maintain a linear current to light conversion and to avoid clipping distortion, a peak constraint is imposed on the emitted optical power from the APs \([7]\). In the context of this paper, this constraint is expressed as \(||\mathbf{Wx}||_\infty \leq \nu_{DC}\). Now, since for all \(k \in \{a, b, w\}\), \(|s_k| \leq \nu\) and for all \(i \in 1, 2, \alpha_i \in [0, 1]\), we have \(||\mathbf{x}||_\infty \leq \nu_{DC}\). Hence, by imposing \(||\mathbf{W}||_\infty \leq 1\), the peak-power constraint of the considered cellular system is satisfied.

For all \(i \in 1, 2\), let \(h_{i,a}, h_{i,b}\), and \(h_{i,w}\) denote the positive-valued downlink VLC channel gains from AP\(_i\) to SU\(_a\), SU\(_b\), and WU, respectively, where the expression of each channel gain can be found in Appendix A. Therefore, the received signals at SU\(_a\), SU\(_b\), and WU can be expressed as \([27]\)

\[
y = \mathbf{HWx} + \mathbf{n}, \tag{3}
\]

where \(y = [y_a, y_b, y_w]^T\) is the vector of received signals, such that \(y_a, y_b\), and \(y_w\) are the received signals at SU\(_a\), SU\(_b\), and WU, respectively, \(\mathbf{H}\) denotes the channel matrix of the considered cellular system, that is expressed as

\[
\mathbf{H} = \begin{bmatrix}
h_{1,a} & h_{2,a} \\
h_{1,b} & h_{2,b} \\
h_{1,w} & h_{2,w}
\end{bmatrix}, \tag{4}
\]

\(\mathbf{n} = [n_a, n_b, n_w]^T\) in which \(n_a, n_b\), and \(n_w\) represent the additive white Gaussian noise (AWGN) experienced at SU\(_a\), SU\(_b\), and WU, respectively. In addition, for all \(k \in \{a, b, w\}\), the noise \(n_k\) is \(\mathcal{N}(0, \sigma^2)\) distributed, where \(\sigma^2 = N_0B\) is the noise power, in which \(N_0\) is the noise power spectral density and \(B\) is the bandwidth of the cellular system.

One key parameter in the considered cellular system is the design of the precoding matrix \(\mathbf{W}\) at the VLC controller, in a way that boosts the overall performance of the system. As discussed above, and as it can be seen from Fig. 1, the considered cellular system suffers from ICI. In fact, since both APs are exploiting the same frequency bandwidth, the strong UE at each cell suffers from ICI that is broadcast from the other cell. Precisely, SU\(_a\), which is associated to cell 1 and served by AP\(_1\), is experiencing ICI generated from AP\(_2\) through the wireless channel \(h_{2,a}\), and SU\(_b\), which is associated to cell 2 and served by AP\(_2\), is experiencing ICI generated from AP\(_1\) through the wireless channel \(h_{1,b}\). One way to overcome this issue is the cancel the ICI realizations at both strong UEs through ZF precoding. As such, by taking into account the imposed amplitude constraint \(||\mathbf{W}||_\infty \leq 1\), the ZF precoding matrix for the considered cellular system can be expressed as \([30]\)

\[
\mathbf{W} = \frac{\mathbf{H}_{a,b}^{-1}}{||\mathbf{H}_{a,b}^{-1}||_\infty}, \tag{5}
\]

where the matrix \(\mathbf{H}_{a,b}\) is given by

\[
\mathbf{H}_{a,b} = \begin{bmatrix} h_{1,a} & h_{2,a} \\ h_{1,b} & h_{2,b}\end{bmatrix}. \tag{6}
\]

Based on this, the received signals at SU\(_a\) and SU\(_b\) are expressed, respectively, as

\[
y_a = \frac{1}{||\mathbf{H}_{a,b}^{-1}||_\infty} x_1 + n_a, \tag{7a}
\]

\[
= (1 - \alpha_1)I_{DC}^{-1}s_a + \frac{\alpha_1I_{DC}}{||\mathbf{H}_{a,b}^{-1}||_\infty}s_w + n_a, \tag{7b}
\]

\[
y_b = \frac{1}{||\mathbf{H}_{b,a}^{-1}||_\infty} x_2 + n_b,
\]

\[
= (1 - \alpha_2)I_{DC}^{-1}s_b + \frac{\alpha_2I_{DC}}{||\mathbf{H}_{b,a}^{-1}||_\infty}s_w + n_b,
\]

whereas the received signal at the WU is expressed as

\[
y_w = \frac{h_{1,w}}{||\mathbf{H}_{a,b}^{-1}||_\infty} x_1 + \frac{h_{2,w}}{||\mathbf{H}_{b,a}^{-1}||_\infty} x_2 + n_w,
\]

\[
= h_{1,w}(1 - \alpha_1)I_{DC}^{-1}s_a + \frac{h_{2,w}(1 - \alpha_2)I_{DC}}{||\mathbf{H}_{b,a}^{-1}||_\infty}s_b \tag{8}
\]

\[
+ (h_{1,w}\alpha_1 + h_{2,w}\alpha_2)I_{DC}^{-1}s_w + n_w.
\]

\(\text{C. Rate Analysis}\)

The data rates received at the UEs are governed by the NOMA technique employed by the APs. Accordingly, SU\(_a\),
and $SU_b$, will first use SIC to decode the message of $WU$ and then decode their own messages. In this context, and according to the NOMA principle [16], [19], in order for the strong UEs to be able to perform SIC, the power assigned by the APs to $WU$ should be higher than that assigned to $SU_a$ and $SU_b$, i.e., for all $i \in \{1, 2\}$, $\frac{P_i}{\sigma_i^2 H_{i,c}} \geq \frac{P_{WU}}{\sigma_{WU}^2 H_{WU,c}}$, which is guaranteed if and only if $0.5 \leq \alpha_i$, for all $i \in \{1, 2\}$. Based on this, the rate of $SU_a$ to decode the message of $WU$ is expressed as

$$R_{a \rightarrow W}(\alpha_1) = \frac{1}{2} \log \left( 1 + \frac{\alpha_1^2}{\gamma_{RX} + (1 - \alpha_1)^2} \right), \quad (9)$$

where $\gamma_{RX} = \frac{c \alpha^2 \sigma_{WU}^2}{3 \alpha^2 \|H_{WU,c}\|^2}$ and $c = \frac{1}{2\pi}$. After this, $SU_a$ cancels the message of $WU$ from its reception and decodes its own interference-free message. Hence, the achievable rate of $SU_a$ to decode its own message is expressed as

$$R_{a}(\alpha_1) = \frac{1}{2} \log \left( 1 + \gamma_{RX} (1 - \alpha_1)^2 \right). \quad (10)$$

Similarly, $SU_b$ will first decode the message of $WU$ using SIC, then cancel it from its own reception, and finally, decodes its own interference-free message. Thus, the achievable rate of $SU_b$ to decode the message of $WU$ is expressed as

$$R_{b \rightarrow W}(\alpha_2) = \frac{1}{2} \log \left( 1 + \frac{\alpha_2^2}{\gamma_{RX} + (1 - \alpha_2)^2} \right), \quad (11)$$

and the achievable rate of user $SU_b$ to decode its own message is expressed as

$$R_{b}(\alpha_2) = \frac{1}{2} \log \left( 1 + \gamma_{RX} (1 - \alpha_2)^2 \right). \quad (12)$$

On the other hand, following the NOMA principle, $WU$ will treat the messages of $SU_a$ and $SU_b$ as noise and decode directly its own message. Based on this, the achievable rate of the $WU$ to decode its own message is expressed as

$$R_{W}(\alpha_1, \alpha_2) = \frac{1}{2} \log \left( 1 + \frac{(h_{1,w} \alpha_1 + h_{2,w} \alpha_2)^2}{\gamma_{RX} + h_{1,w}^2 (1 - \alpha_1)^2 + h_{2,w}^2 (1 - \alpha_2)^2} \right), \quad (13)$$

III. PROBLEM FORMULATION AND PROPOSED SOLUTION

A. Problem Formulation

The goal of this paper is to maximize the sum rate of the considered VLC cellular system under the CoMP-NOMA scheme, i.e., to maximize $R_{CN}(\alpha_1, \alpha_2) = R_{a}(\alpha_1) + R_{b}(\alpha_2) + R_{W}(\alpha_1, \alpha_2)$, while a target QoS is guaranteed for each UE in terms of its minimum achievable rate threshold, denoted by $R_{th}$. Such objective can be reached by solving the following optimization problem.

$$\mathcal{P} : \begin{align*}
R_{CN} &= \max_{\alpha_1, \alpha_2} R_{a}(\alpha_1) + R_{b}(\alpha_2) + R_{W}(\alpha_1, \alpha_2), \quad (14a) \\
\text{s.t.} \quad &0.5 \leq \alpha_1 \leq 1, \quad (14b) \\
&0.5 \leq \alpha_2 \leq 1, \quad (14c) \\
&R_{a}(\alpha_1) \geq R_{th}, \quad (14d) \\
&R_{a \rightarrow W}(\alpha_1) \geq R_{th}, \quad (14e) \\
&R_{b}(\alpha_2) \geq R_{th}, \quad (14f) \\
&R_{b \rightarrow W}(\alpha_2) \geq R_{th}, \quad (14g) \\
&R_{W}(\alpha_1, \alpha_2) \geq R_{th}. \quad (14h)
\end{align*}$$

Based on the rate expressions presented in [9]-[13], problem $\mathcal{P}$ is a non-linear non-convex problem that cannot be solved in a straightforward manner. Alternatively, we propose in the following part an efficient and low complexity approach to solve problem $\mathcal{P}$.

B. Proposed Solution

First, in order to be able to solve problem $\mathcal{P}$, the conditions under which at least one feasible solution exists must be derived. In this case, the feasibility conditions of problem $\mathcal{P}$ are presented in the following theorem.

Theorem 1. Problem $\mathcal{P}$ is feasible if and only if the following conditions hold:

- Condition 1: $R_{th} \leq \frac{1}{2} \log \left( 1 + \frac{\gamma_{RX}}{4} \right)$, \quad (15a)
- Condition 2: $\alpha_{\min} \leq \alpha_{\max}$, \quad (15b)
- Condition 3: $f(\alpha_{\max}) \geq 0$, \quad (15c)

where $\alpha_{\min}$ and $\alpha_{\max}$ are expressed, respectively, as

$$\alpha_{\min} = \begin{cases} 
\max(0.5, \alpha_{0,2}), & \text{if } 0 < t < 1, \\
0.5, & \text{otherwise},
\end{cases} \quad (16)$$

$$\alpha_{\max} = \begin{cases} 
\left(1 - \sqrt{\frac{t}{\gamma_{RX}}}\right), & \text{if } 0 < t < 1, \\
\min \left(1 - \sqrt{\frac{t}{\gamma_{RX}}}, \alpha_{0,1}\right), & \text{otherwise},
\end{cases} \quad (17)$$

in which $\alpha_{0,1}$ and $\alpha_{0,2}$ are given by

$$\alpha_{0,1} = \frac{-2t - \sqrt{4t \left(\frac{1 - t}{\gamma_{RX}} + 1\right)}}{2(1 - t)}, \quad (18a)$$

$$\alpha_{0,2} = \frac{-2t + \sqrt{4t \left(\frac{1 - t}{\gamma_{RX}} + 1\right)}}{2(1 - t)}, \quad (18b)$$

and the function $f(\cdot)$ is expressed, $\forall x \in \mathbb{R}$, as

$$f(x) = x^2 (h_{1,w} + h_{2,w})^2 - (1 - x)^2 t (h_{1,w}^2 + h_{2,w}^2) - \frac{t}{\gamma_{RX}}. \quad (19)$$

Proof. See Appendix B.

Based on Theorem 1 and its proof in Appendix B, the feasibility region of the optimization variables $(\alpha_1, \alpha_2)$ is defined by the set

$$\Omega = \left\{ (\alpha_1, \alpha_2) \in [\alpha_{\min}, \alpha_{\max}]^2 \mid g(\alpha_1, \alpha_2) \geq 0 \right\}, \quad (20)$$

where the function $g(\cdot, \cdot)$ is expressed, $\forall (\alpha_1, \alpha_2) \in \mathbb{R}^2$, as

$$g(\alpha_1, \alpha_2) = (h_{1,w} \alpha_1 + h_{2,w} \alpha_2)^2 - th_{1,w}^2 (1 - \alpha_1)^2 - th_{2,w}^2 (1 - \alpha_2)^2 - \frac{t}{\gamma_{RX}}. \quad (21)$$
Now that the feasibility conditions are set, our objective is to find the optimal solution of problem $\mathcal{P}$, i.e., the optimal values of the power allocation factors $\alpha_1$ and $\alpha_2$ that maximize the network sum-rate $R_{\text{sum}} = R_a + R_b + R_w$. In this setup, the optimal power allocation strategy is to maximize the achievable rates of the strong UEs, $SU_a$ and $SU_b$, while guaranteeing the required rate threshold $R_{th}$ for the IWU [31]. Therefore, since the expressions of the achievable rates $R_a$ and $R_b$ are decreasing functions of $\alpha_1$ and $\alpha_2$, respectively, and that

$$R_w = R_{th} \Leftrightarrow g(\alpha_1, \alpha_2) = 0,$$

then the optimal power allocation strategy is the one that satisfies the equality $g(\alpha_1, \alpha_2) = 0$ with the lowest possible values of $\alpha_1$ and $\alpha_2$ within the feasibility region. Based on this, the optimal power allocation factors $\alpha_1$ and $\alpha_2$ can be graphically obtained by investigating the intersection of the line $\Delta$, that has $g(\alpha_1, \alpha_2) = 0$ as an equation, with the square boundaries of $\alpha_1$ and $\alpha_2$ given by $[\alpha_{\text{min}}, \alpha_{\text{max}}]^2$.

Fig. 2 presents all possible cases of intersection between the line $\Delta$ and the boundaries $[\alpha_{\text{min}}, \alpha_{\text{max}}]^2$. In the following, we will determine the optimal power allocation factors $\alpha_1$ and $\alpha_2$ for each case.

- 1<sup>st</sup> case: $g(\alpha_{\text{min}}, \alpha_{\text{min}}) \geq 0$:
  As shown in Fig. 2a, if $g(\alpha_{1,\text{min}}, \alpha_{2,\text{min}}) \geq 0$, then the whole region $[\alpha_{\text{min}}, \alpha_{\text{max}}]^2$ contains feasible solutions for problem $\mathcal{P}$. In addition, the optimal values of the power allocation fractions are $(\alpha_1^*, \alpha_2^*) = (\alpha_{1,\text{min}}, \alpha_{2,\text{min}})$, since they are the lowest possible values within the feasibility region.

- 2<sup>nd</sup> case: $g(\alpha_{\text{min}}, \alpha_{\text{min}}) \leq 0$ and $g(\alpha_{\text{max}}, \alpha_{\text{max}}) \geq 0$:
  In this case, and as shown in Figs. 2b-2e, the optimal solution $(\alpha_1^*, \alpha_2^*)$ lies within the segment $[p_1, p_2]$, where $p_1$ and $p_2$ are the exact two intersection points between the line $\Delta$ and the boundaries of the square region $[\alpha_{\text{min}}, \alpha_{\text{max}}]^2$. The coordinates of the intersection points $p_1$ and $p_2$ are given in Table I where $y_1$, $y_2$, $y_3$ and $y_4$ are the solutions of the equations

  $$g(\alpha_{\text{min}}, y_1) = 0,$$
  $$g(y_2, \alpha_{\text{min}}) = 0,$$
  $$g(y_3, \alpha_{\text{max}}) = 0,$$
  $$g(\alpha_{\text{max}}, y_4) = 0.$$  

Consequently, in order to determine $(\alpha_1^*, \alpha_2^*)$, we opt for a discrete line search technique within the segment $[p_1, p_2]$. Let $K \in \mathbb{N}$ be the number of discrete points within $[p_1, p_2]$, and let

$$Q = \left\{ q_i = z_1 + \frac{z_2 - z_1}{K - 1} \times i \mid i \in [0, K - 1] \right\},$$

be the set of the abscissa of the $K$ equidistant discrete points within the segment $[p_1, p_2]$, where $z_1$ and $z_2$ represent the abscissa of the intersection points $p_1$ and $p_2$ provided in Table I. Based on this, the discrete line search technique within the segment $[p_1, p_2]$ works as follows. For all $i \in [0, K - 1]$, we calculate $\alpha_1^* = q_i = z_1 + \frac{z_2 - z_1}{K - 1} \times i$. Then, we determine the value of $\alpha_2^*$ that satisfies the equation $g(\alpha_1^*, \alpha_2^*) = 0$. Afterwards, we calculate the corresponding network sum rate $R_i(\alpha_1^*, \alpha_2^*) = R_a(\alpha_1^*) + R_b(\alpha_2^*) + R_w(\alpha_1^*, \alpha_2^*)$. Finally, the optimal power allocation fractions $(\alpha_1^*, \alpha_2^*)$, i.e., the solution of problem $\mathcal{P}$, is the one that achieves the highest network sum-rate, i.e.,

$$\left(\alpha_1^*, \alpha_2^*\right) = \arg\max R_i(\alpha_1^*, \alpha_2^*),$$

which can be obtained through a brute force search over the set $\{R_i(\alpha_1^*, \alpha_2^*) \mid i \in [0, K - 1]\}$.

- 3<sup>rd</sup> case: $g(\alpha_{\text{max}}, \alpha_{\text{max}}) \leq 0$: In this case, there are no points within $[\alpha_{\text{min}}, \alpha_{\text{max}}]^2$ that satisfy $g(\alpha_1, \alpha_2) \geq 0$, and therefore, as shown in Fig. 2f, problem $\mathcal{P}$ is unfeasible.

The optimality of the proposed solution will be verified in the simulation results section. Moreover, since the proposed solution is based on a discrete line search technique over a set of $K$ points, its computational complexity is $\mathcal{O}(K)$, i.e., linear complexity. This fact demonstrates the low complexity of the proposed solution.

IV. Simulation Results

In this section, our objective is to evaluate the performance of the proposed CoMP-NOMA scheme for indoor VLC cellular systems through extensive simulations.

A. Simulations Parameters

In this paper, we consider a cellular system consisting of two adjacent attocells. Each attocell is equipped with an AP that is oriented vertically downward and located at a height $H = 3$ m from the ground. The radius of the coverage area of each cell depends of the half power semi-angle of the LEDs $\Phi_{1/2}$, and its value will be defined within the following paragraphs. Within this cellular system, three VLC UEs, that are each equipped with a single PD, are communicating with the two APs. Each UE has a random orientation that is generated using the measurements-based orientation models proposed in [11], [12], [32]. Moreover, the first and second UEs are randomly located around the center of the first and the second attocells, respectively, whereas the third UE is randomly located near the edge of both attocells, i.e., at the overlapped area between the coverage regions of both attocells. Therefore, using the

| TABLE I: Coordinates of the intersection points |
|-----------------------------------------------|
| case                                         |
| $g(\alpha_{\text{min}}, \alpha_{\text{min}}) \leq 0$, $g(\alpha_{\text{min}}, \alpha_{\text{max}}) \geq 0$ and $g(\alpha_{\text{max}}, \alpha_{\text{min}}) \geq 0$ | $(\alpha_{\text{min}}, y_1)$, $(y_2, \alpha_{\text{min}})$ |
| $g(\alpha_{\text{min}}, \alpha_{\text{min}}) \leq 0$, $g(\alpha_{\text{min}}, \alpha_{\text{max}}) \leq 0$ and $g(\alpha_{\text{max}}, \alpha_{\text{min}}) \geq 0$ | $(y_3, \alpha_{\text{max}})$, $(y_2, \alpha_{\text{min}})$ |
| $g(\alpha_{\text{min}}, \alpha_{\text{min}}) \leq 0$, $g(\alpha_{\text{min}}, \alpha_{\text{max}}) \geq 0$ and $g(\alpha_{\text{max}}, \alpha_{\text{min}}) \leq 0$ | $(\alpha_{\text{min}}, y_1)$, $(\alpha_{\text{max}}, y_4)$ |
| $g(\alpha_{\text{min}}, \alpha_{\text{min}}) \leq 0$, $g(\alpha_{\text{min}}, \alpha_{\text{max}}) \leq 0$ and $g(\alpha_{\text{max}}, \alpha_{\text{max}}) \leq 0$ | $(y_3, \alpha_{\text{max}})$, $(\alpha_{\text{max}}, y_4)$ |
The proposed CoMP-NOMA scheme, the first and second UEs are associated to the first and second attocells, respectively, and they are each communicating with the AP of its associated attocell, whereas in order to cancel the inter-cell interference, the third UE is associated to the two attocells and it is jointly served with the two APs. The parameters used throughout the paper are shown in Table II. The central processing unit (CPU) of the machine on which all the simulations were performed was an Intel Core i5 from the second generation that has a dual-core, a basic frequency of 2.40 GHz and a maximum turbo frequency of 3.40 GHz.

For comparison purposes, three baselines are considered, which are

- CoMP-assisted OMA: the two cells are coordinating together to serve the cell edge UE, whereas, unlike the proposed scheme, each cell adopts frequency division multiple access (FDMA) to serve its associated UEs. The expressions of the received signals at the UEs and their achievable rates are presented in Appendix C.
- NOMA: each cell adopts NOMA to serve its associated UEs. In addition, the two cells exploit the same frequency bandwidth but without any coordination between them, unlike the proposed scheme.
- OMA: each cell adopts OMA to serve its associated UEs. In addition, the two cells exploit the same frequency bandwidth but without any coordination between them, unlike the CoMP-assisted OMA scheme.

Fig. 3 presents the average achievable sum rate per cell, achieved by the proposed CoMP-assisted NOMA scheme, the CoMP-assisted OMA scheme, the NOMA scheme, and the OMA scheme, versus the required rate threshold per UE $R_{th}$ for different values of the distance between the APs $d_{AP}$ and the LEDs half-power semi-angle $\Phi_{1/2}$. For the proposed CoMP-assisted NOMA scheme, the results of both the numerical solution and the proposed solution of the power allocation coefficients are presented. The numerical solution is obtained by solving problem $\mathcal{P}$ using an off-the-shelf optimization solver, whereas the closed form solution is
Fig. 3: Average achievable sum rate per cell, achieved by the proposed CoMP-assisted NOMA scheme, the CoMP-assisted OMA scheme, the NOMA scheme, and the OMA scheme, versus the required rate threshold per UE $R_{th}$ for different values of the distance between the APs $d_{AP}$ and the LEDs half-power semi-angle $\Phi_{1/2}$.

obtained through theorem 1.\(^3\)

Fig. 3 shows that the proposed solution of the power allocation coefficients of the proposed CoMP-assisted NOMA scheme matches perfectly the numerical solution, which demonstrates its optimality. Moreover, for all the considered cases of $(d_{AP}, \Phi_{1/2})$, we notice that the average sum rates per cell achieved by all the considered schemes decrease when the rate threshold $R_{th}$ increases. This is basically due to the fact that as $R_{th}$ increases, the number of UEs that satisfy the target QoS decreases, and therefore, the number of UEs that can be served by the two APs decreases. In addition, for all the considered cases of $(d_{AP}, \Phi_{1/2})$, one can note that the proposed CoMP-NOMA scheme outperforms all the considered baselines, which demonstrates the capability of integrating CoMP with NOMA in beating the ICI effects and increasing the network sum-rate simultaneously.

For all the considered cases of $(d_{AP}, \Phi_{1/2})$, one can note that the proposed NOMA scheme outperforms the OMA scheme, which validates the spectral efficiency of NOMA compared to OMA, even in the presence of ICI. On the other hand, for the case when $d_{AP} = 4m$, and as depicted in Figs 3a and 3b, we remark that the CoMP-assisted OMA scheme outperforms the NOMA scheme when the rate threshold $R_{th}$ is low. However, starting from a certain $R_{th}$, which is approximately 1.55 nats/s/Hz in this context, we remark that the NOMA scheme outperforms the CoMP-OMA scheme. In fact, when the rate threshold $R_{th}$ is low, lower fractions of bandwidth are needed to meet the QoS of the cellular UEs, especially the cell-edge UE, and hence, the CoMP-OMA scheme outperforms the NOMA scheme due to the potential of the coordinating broadcasting from the APs in beating the ICI effects, which is not the case for the NOMA scheme. On the other hand, as the rate threshold $R_{th}$ increases, more fractions of bandwidth are needed to meet the QoS of the cellular UEs, especially the cell-edge UE, and hence, the NOMA scheme outperforms the CoMP-OMA scheme due to its higher spectral efficiency. Now, considering the case when the distance between the APs is $d_{AP} = 4m$, and as depicted in Figs 3a and 3b we remark that the NOMA scheme outperforms the CoMP-OMA scheme due to its higher spectral efficiency. Now, considering the case when the distance between the APs is $d_{AP} = 4m$, and as depicted in Figs 3a and 3b we remark that the NOMA scheme outperforms the CoMP-OMA scheme due to its higher spectral efficiency.

\(^3\)The adopted solver is fmincon, which is a predefined MATLAB solver. In addition, 100 distinct initial points were generated in order to converge to the optimal solution.
CoMP-OMA scheme for the considered values of the LEDs half power semi-angle, i.e., $\Phi_{1/2} = 45^\circ$ and $\Phi_{1/2} = 60^\circ$. In fact, when the distance from the APs increases, i.e., from 4m to 5m in this context, the effects of the ICI decreases, and hence, the NOMA scheme becomes less prone to ICI and can outperform the CoMP-OMA scheme due to its higher spectral efficiency.

In Fig. 3 one can note that the average achievable sum rates achieved by all the considered schemes decrease as the distance between the APs $d_{AP}$ decreases and/or the LEDs half power semi-angle $\Phi_{1/2}$ increases. In fact, when $d_{AP}$ decreases and/or $\Phi_{1/2}$ increases, the coverage area of each AP increases, and hence, the area of the overlapping region between the two APs increases. Therefore, the ICI effects from one AP to others increases, which explains the performance degradation for the NOMA and the OMA schemes. On the other hand, despite the exploitation of the coordinated broadcasting technique between the two APs, the performance degradation of the CoMP-NOMA and the CoMP-OMA schemes is resulting from the use of the ZF precoding and the peak-power constraint imposed on VLC systems. Specifically, the use of the CoMP technique and the ZF precoding eliminates the ICI effects at the cell-edge UE and the cell-center UEs, respectively. However, as the coverage area of each AP increases, the channel coefficients from the two APs to the cell-center UEs increases, and hence, the coefficients of the channel matrix $H_{-,b}$ in (6) increases. Hence, the multiplicative term $\frac{1}{||H_{-,b}||}$, which is required to make the ZF precoding matrix satisfy the peak-power constraint at the LEDs of the APs, decreases. Therefore, when the distance between the APs $d_{AP}$ increases and/or the LEDs half power semi-angle $\Phi_{1/2}$ increases, the average received SNR at the UEs $\gamma_{RX} = \frac{cel^2}{3\sigma^2||H_{-,b}||^2}$ decreases, which explains the performance degradation for the CoMP-NOMA and the CoMP-OMA schemes, despite the use of ICI mitigation techniques.

V. Conclusion

This paper studies the performance of CoMP transmission in downlink multi-cell NOMA VLC systems. For a system consisting of two adjacent attocells serving three coexisting users, an optimal and low-complexity power control policy that maximizes the network sum-rate while guaranteeing target quality-of-service (QoS) and SIC constraints at the end users is derived. In the simulation results, the optimality of the derived power control policy is verified and the performance of the proposed CoMP-assisted NOMA scheme is compared with those of the CoMP-assisted FDMA scheme, the NOMA scheme and the OMA scheme, where the superiority of the proposed scheme is demonstrated. The extension of the proposed CoMP-assisted NOMA scheme to multiple coordinating attocells, i.e., a number of APs higher than two, can be considered as a potential future research direction. In such a case, although the dynamic power control and the interference management become more challenging, it is expected that a performance enhancement of VLC cellular systems can be achieved.

ACKNOWLEDGMENT

S. Fayad, M. A. Arfaoui and C. Assi acknowledge the financial support from the Natural Sciences and Engineering Research Council of Canada (NSERC), Fonds Qu éb è cois de la Recherche sur la Nature et les Technologies (FQRNT) and from Concordia University.

APPENDIX A

VLC Channel Model

The VLC channel gain between a VLC AP and a UE is expressed as $h = TR_d \eta L$, where $\eta$ (W/A) is the current-to-power conversion efficiency of the LEDs, $R_d$ (A/W) is the responsivity of the PD of the UE, $T$ (V/A) is the transimpedance amplifier gain and $L$ is the path gain between the AP and the UE. In the addition, the path gain $L$ is expressed as $[35]

$$L = \frac{(m + 1)A}{2\pi d^2} \cos^m(\phi) \cos^2(\psi) \text{rect}\left(\frac{\psi}{\Psi}\right),$$

where $m = -1/\log_2(\cos(\Phi_{1/2}))$ is the Lambertian emission order of the LEDs, $A$ is the area of the PD, $\phi$ is the angle of radiance, $\psi$ is the incidence angle, $\Psi$ is the FOV of the PD of the UE and $d$ is the distance between the AP and the UE.

APPENDIX B

Proof of Theorem 1

We start by checking the feasibility conditions of (14d) and (14f). By substituting $R_{th}$ with its expression into constraint (14d), and solving the resulting inequality, we obtain

$$\alpha_1 \leq 1 - \sqrt{\frac{t}{\gamma_{RX}}},$$

where $t = e^{2R_{th}} - 1$. However, from constraint (14b), $\alpha_1$ should be higher than 0.5. Thus, $\alpha_1$ should satisfy the inequalities

$$0.5 \leq 1 - \sqrt{\frac{t}{\gamma_{RX}}},$$

which hold if and only if $t \leq \frac{\gamma_{RX}}{4}$, i.e.,

$$R_{th} \leq \frac{1}{2} \log (1 + \frac{\gamma_{RX}}{4}).$$

Similarly, in order for constraint (14f) to be feasible, the inequality in (30) should also hold, which constitutes the first feasibility condition of problem $P$.

Now we move to constraints (14e) and (14g). By substituting $R_{th}$ with its expression into constraint (14e), and solving the resulting inequality, we obtain

$$(1 - t)\alpha_2^2 + (2t)\alpha_1 + (-t - \frac{t}{\gamma_{RX}}) \geq 0,$$

The inequality in (31) is a quadratic inequality of the form $A\alpha_2^2 + B\alpha_1 + C \geq 0$. In order to determine its feasibility, we calculate the discriminant $\delta$, which is given by

$$\delta = 4t \left(\frac{1}{\gamma_{RX}} + 1\right),$$

where $\alpha_2$ is the sum-rate of the cell-center UEs, $\alpha_1$ is the sum-rate of the cell-edge UEs, $\gamma_{RX}$ is the average received SNR at the UE, $m$ is the order of the LEDs, $A$ is the area of the PD, $\phi$ is the angle of radiance, $\psi$ is the incidence angle, $\Psi$ is the FOV of the PD of the UE and $d$ is the distance between the AP and the UE.
The discriminant $\delta$ is positive if and only if $t \leq 1 + \gamma_{RX}$. Or, as demonstrated above, since $t \leq \frac{2\max}{\alpha} < 1 + \gamma_{RX}$, the discriminant $\delta$ is always positive. Based on this, the roots of the quadratic expression in (31) are given in (18) in Theorem 1. Consequently, the inequality in (31) holds if and only if
\[
\alpha_1 \in \left\{ [0, \alpha_{\min}], \quad \text{if}, \quad 0 < t < 1, \right. \]
\[
\alpha_1 \in \left\{ [0, +\infty), \quad \text{if}, \quad t = 1, \right. \]
\[
\alpha_1 \in \left\{ [\alpha_{\min}, 0], \quad \text{if}, \quad t > 1, \right. \] otherwise.
\]
Now, note that constraint (14g) is similar to constraint (14e), with $\alpha_1$ being replaced by $\alpha_2$. Therefore, based on the above analysis, $\alpha_1$ and $\alpha_2$ have the same bounds, which are expressed as
\[
\alpha_{\min} \leq \alpha_1 \leq \alpha_{\max},
\]
\[
\alpha_{\min} \leq \alpha_2 \leq \alpha_{\max},
\]
where the expressions of $\alpha_{\min}$ and $\alpha_{\max}$ are given respectively in (16) and (17) in Theorem 1. Consequently, constraints (14e) and (14g) are feasible if and only if $\alpha_{\min} \leq \alpha_{\max}$, which constitutes the second feasibility condition of problem $\mathcal{P}$.

Finally, we focus on constraint (14h). By substituting $R_w$ with its expression into constraint (14h), and solving the resulting inequality, we obtain the inequality
\[
g(\alpha_1, \alpha_2) \geq 0,
\]
where the function $(\alpha_1, \alpha_2) \mapsto g(\alpha_1, \alpha_2)$ is expressed as
\[
g(\alpha_1, \alpha_2) = (h_{1, w_{\alpha_1}} + h_{2, w_{\alpha_2}})^2 - th_{1, w_{(1-\alpha_1)}}^2
\]
\[
- th_{2, w_{(1-\alpha_2)}}^2 - \frac{t}{\gamma_{RX}}.
\]
Obviously, the inequality in (35) is feasible if and only if it is satisfied by the highest values of $\alpha_1$ and $\alpha_2$, i.e., $\alpha_1 = \alpha_{\max}$ and $\alpha_2 = \alpha_{\max}$, which constitutes the third and last feasibility condition of problem $\mathcal{P}$.

APPENDIX C

CoMP-assisted OMA: Transmission Model and Rate Analysis

In the CoMP-OMA scheme, the two APs, which exploits the same frequency bandwidth, are coordinating together to serve the cell edge UE, whereas, unlike the proposed scheme, each AP adopts frequency division multiple access (FDMA) to serve its associated UEs. In this context, let $\beta B$ and $(1 - \beta) B$ be the sub-bandwidths that the APs use to serve the cell-edge UE and the cell-center UEs, respectively, where $\beta \in [0, 1]$ denotes the bandwidth allocation factor. Based on this, the received signals at $SU_a$ and $SU_b$ within the sub-bandwidth $(1 - \beta) B$ are expressed, respectively, as
\[
y_a = \frac{(1 - \alpha_1)I}{\|H^a_{1, w_{\alpha_1}}\|} s_a + n_a,
\]
\[
y_b = \frac{(1 - \alpha_2)I}{\|H^a_{1, w_{\alpha_2}}\|} s_b + n_b,
\]
whereas the received signal at the $WU$ within the sub-bandwidth $\beta B$ is expressed as
\[
y_w = \frac{I(h_{1, w_{\alpha_1}} + h_{2, w_{\alpha_2}})}{\|H^b_{1, w_{\alpha_1}}\|} s_w + n_w,
\]
in which $\alpha_1, \alpha_2 \in [0, 1]$ represent the DC factors assigned by $\text{AP}_1$ and $\text{AP}_2$ to the $WU$, respectively, within the sub-bandwidth $\beta B$, and $(1 - \alpha_1)$ and $(1 - \alpha_2)$ represent the DC factors assigned by $\text{AP}_1$ and $\text{AP}_2$ to $SU_a$ and $SU_b$, respectively, within the sub-bandwidth $(1 - \beta) B$.

The achievable rates of $SU_a$ and $SU_b$ are expressed, respectively, as
\[
R_a(\alpha_1, \beta) = \frac{(1 - \beta)}{2} \log \left( 1 + \frac{\gamma_{RX} (1 - \alpha_1)^2}{1 - \beta} \right),
\]
\[
R_a(\alpha_2, \beta) = \frac{(1 - \beta)}{2} \log \left( 1 + \frac{\gamma_{RX} (1 - \alpha_2)^2}{1 - \beta} \right),
\]
whereas the achievable rate of the $WU$ is expressed as
\[
R_w(\alpha_1, \alpha_2, \beta) = \frac{\beta}{2} \log \left( 1 + \frac{\gamma_{RX} (h_{1, w_{\alpha_1}} + h_{2, w_{\alpha_2}})^2}{\beta} \right).
\]
With the goal of maximizing the sum rate of the VLC cellular system under the CoMP-OMA scheme, i.e., $R_{\text{CO}}(\alpha_1, \alpha_2, \beta) = R_a(\alpha_1, \beta) + R_b(\alpha_2, \beta) + R_w(\alpha_1, \alpha_2, \beta)$, with target QoS constraints at the cellular UEs, the following optimization problem should be solved
\[
\mathcal{P} : R_{\text{CO}}^* = \max_{\alpha_1, \alpha_2, \beta} R_a(\alpha_1, \beta) + R_b(\alpha_2, \beta) + R_w(\alpha_1, \alpha_2, \beta),
\]
s.t.
\[
\alpha_1, \alpha_2, \beta \in [0, 1],
\]
\[
R_a(\alpha_1, \beta) \geq R_{\text{th}},
\]
\[
R_b(\alpha_2, \beta) \geq R_{\text{th}},
\]
\[
R_w(\alpha_1, \alpha_2, \beta) \geq R_{\text{th}},
\]

REFERENCES

[1] W. Saad, M. Bennis, and M. Chen, “A vision of 6G wireless systems: Applications, trends, technologies, and open research problems,” IEEE network, vol. 34, no. 3, pp. 134–142, Oct. 2019.

[2] U. Cisco. “Cisco annual internet report (2018–2023) white paper,” [Online]. Available: https://www.cisco.com/c/en/us/solutions/collateral/ executive-perspectives/annual-internet-report/whitepaper-c11-714940.html, Mar. 2020.

[3] Z. Zhang, Y. Xiao, Z. Ma, M. Xiao, Z. Ding, X. Lei, G. K. Karagiannidis, and P. Fan, “6G wireless networks: Vision, requirements, architecture, and key technologies,” IEEE Vehicular Tech. Mag., vol. 14, no. 3, pp. 28–41, Jul. 2019.

[4] K. David and H. Berndt, “6G vision and requirements: Is there any need for beyond 5G?” IEEE Vehicular Tech. Mag., vol. 13, no. 3, pp. 72–80, Sep. 2018.

[5] H. Haas, L. Yin, Y. Wang, and C. Chen, “What is LiFi?” Journal of lightwave tech., vol. 34, no. 6, pp. 1533–1544, Mar. 2015.

[6] I. Tavakkolnia, C. Chen, R. Bian, and H. Haas, “Energy-efficient adaptive MIMO-VLC technique for indoor LiFi applications,” in Proc. IEEE ICT, Saint-Malo, France, Sep. 2018.

[7] M. A. Arfaoui, M. D. Soltani, I. Tavakkolnia, A. Ghrayeb, M. Safari, C. Assi, and H. Haas, “Physical layer security for visible light communication systems: A survey,” IEEE Commun. Surveys & Tutorials, vol. 22, no. 3, pp. 887 – 908, Apr. 2020.

[8] M. Obeed, A. M. Salhab, M.-S. Alouini, and S. A. Zummo, “On optimizing VLC networks for downlink multi-user transmission: A survey,” IEEE Communications Surveys & Tutorials, vol. 21, no. 3, pp. 2947 – 2976, Mar. 2019.

[9] X. Zhang and M. Haenggi, “The performance of successive interference cancellation in random wireless networks,” IEEE Trans. on Inform. Theory, vol. 60, no. 10, pp. 6368–6388, Jul. 2014.
[10] Z. Zeng, M. D. Soltani, M. Safari, and H. Haas, “Angle diversity receiver in LiFi cellular networks,” in Proc. IEEE ICC Workshops, Shanghai, China, May. 2019.

[11] M. D. Soltani, M. A. Arfaoui, I. Tavakkolnia, A. Ghrayeb, M. Safari, C. M. Assi, M. O. Hasna, and H. Haas, “Bidirectional optical spatial modulation for mobile users: Toward a practical design for li systems,” IEEE Journal on Selected Areas in Communications, vol. 37, no. 9, pp. 2069–2086, Aug. 2019.

[12] M. A. Arfaoui, M. D. Soltani, I. Tavakkolnia, A. Ghrayeb, C. Assi, M. Safari, and H. Haas, “Measurements-based channel models for indoor LiFi systems,” IEEE Trans. on Wireless Commun., vol. 20, no. 2, pp. 827–842, Oct. 2020.

[13] R. C. Kizilirmak, C. R. Rowell, and M. Uysal, “Non-orthogonal multiple access (NOMA) for indoor visible light communications,” in Proc. IEEE IJWOW, Istanbul, Turkey, Sep. 2015.

[14] L. Yin, W. O. Popoola, X. Wu, and H. Haas, “Performance evaluation of non-orthogonal multiple access in visible light communication,” IEEE Trans. on Commun., vol. 64, no. 12, pp. 5162–5175, Sep. 2016.

[15] Y. Yapıcı and I. Güvenci, “NOMA for VLC downlink transmission with random receiver orientation,” IEEE Trans. on Commun., vol. 67, no. 8, pp. 5558–5573, Aug. 2019.

[16] M. A. Arfaoui, M. D. Soltani, I. Tavakkolnia, A. Ghrayeb, C. Assi, M. Safari, and H. Haas, “Invoking deep learning for joint estimation of indoor LiFi user position and orientation,” IEEE J. on Selected Areas in Commun., Mar. 2021 (Early Access).

[17] H. Haas, “High-speed wireless networking using visible light,” Spie Newsroom, vol. 1, no. 1, pp. 1–3, Apr. 2013.

[18] X. Zhang, Q. Gao, C. Gong, and Z. Xu, “User grouping and power allocation for NOMA visible light communication multi-cell networks,” IEEE commun. letters, vol. 21, no. 8, pp. 777–780, Dec. 2016.

[19] M. Obeed, H. Dahrouj, A. M. Salhab, A. Chaaban, S. A. Zrumo, and M.-S. Alouini, “User pairing, link selection and power allocation for cooperative NOMA hybrid VLC/RF systems,” IEEE Trans. on Wireless Commun., vol. 20, no. 3, Nov. 2020.

[20] C. Chen, D. Tsonev, and H. Haas, “Joint transmission in indoor visible light communication downlink cellular networks,” in Proc. IEEE Globecom Workshops, Atlanta, GA, USA, Dec. 2013.

[21] H. Ma, L. Lampe, and S. Hranilovic, “Coordinated broadcasting for multiuser indoor visible light communication systems,” IEEE Trans. on Commun., vol. 63, no. 9, pp. 3313–3324, Jul. 2015.

[22] H. Ma, L. Lampe, and S. Hranilovic, “Integration of indoor visible light and power line communication systems,” in Proc. IEEE ISPLC, Johannesburg, South Africa, Mar. 2013.

[23] T. V. Pham, H. Le Minh, and A. T. Pham, “Multi-cell VLC: Multi-user downlink capacity with coordinated precoding,” in Proc. ICC Workshops. Paris, France: IEEE, May 2017.

[24] H. Yang, C. Chen, W.-D. Zhong, and A. Alphones, “Joint precoder and equalizer design for multi-user multi-cell MIMO VLC systems,” IEEE Trans. on Vehicular Tech., vol. 67, no. 12, pp. 11354–11364, Oct. 2018.

[25] M. A. Arfaoui, M. D. Soltani, I. Tavakkolnia, A. Ghrayeb, C. Assi, M. Safari, and H. Haas, “SNR statistics of indoor mobile VLC users with random device orientation,” in Proc. IEEE ICC Workshops, Shanghai, China, May 2019.

[26] A. Mostafap and L. Lampe, “Physical-layer security for MISO visible light communication channels,” IEEE J. on Selected Areas in Commun., vol. 33, no. 9, pp. 1806–1818, May. 2015.

[27] M. A. Arfaoui, A. Ghrayeb, and C. M. Assi, “Secrecy performance of multi-user MISO VLC broadcast channels with confidential messages,” IEEE Trans. on Wireless Commun., vol. 17, no. 11, pp. 7789–7800, Sep. 2018.

[28] M. A. Arfaoui, H. Zaid, Z. Rezki, A. Ghrayeb, A. Chaaban, and M.-S. Alouini, “Artificial noise-based beamforming for the MISO VLC wiretap channel,” IEEE Trans. on Commun., vol. 67, no. 4, pp. 2866–2879, Dec. 2018.

[29] M. A. Arfaoui, A. Ghrayeb, and C. M. Assi, “Secrecy performance of the MIMO VLC wiretap channel with randomly located eavesdropper,” IEEE Trans. on Wireless Commun., vol. 19, no. 1, pp. 265–278, Oct. 2019.

[30] T. V. Pham, H. Le-Minh, and A. T. Pham, “Multi-user visible light communication broadcast channels with zero-forcing precoding,” IEEE Trans. on Commun., vol. 65, no. 6, pp. 2509–2521, Apr. 2017.

[31] P. Huu, M. A. Arfaoui, S. Sharafeddine, C. M. Assi, and A. Ghrayeb, “A low-complexity framework for joint user pairing and power control for cooperative NOMA in 5G and beyond cellular networks,” IEEE Trans. on Commun., vol. 68, no. 11, pp. 6737–6749, Jul. 2020.

[32] M. D. Soltani, A. A. Purwita, Z. Zeng, H. Haas, and M. Safari, “Modeling the random orientation of mobile devices: Measurement, analysis and lifi use case,” IEEE Trans. on Commun., vol. 67, no. 3, pp. 2157–2172, Nov. 2018.

[33] S. Ebbespen, P. Kiwitz, and L. Guzzella, “A generic particle swarm optimization Matlab function,” in Proc. Amer. Control Conf. (ACC), Montreal, QC, Canada, Jun. 2012.

[34] M. A. Arfaoui, M. D. Soltani, I. Tavakkolnia, A. Ghrayeb, C. Assi, M. Safari, and H. Haas, “Modeling the random orientation of mobile devices: Measurement, analysis and lifi use case,” IEEE Trans. on Commun., vol. 67, no. 3, pp. 2157–2172, Nov. 2018.