Evaluation of an error correction function using correlation characteristics of spread codes in CDMA-based transmission methods

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Abstract. In previous researches, we have proposed novel modulation methods such as the CDMA-QAM method in which Code-Division Multiple Access (CDMA) and Quadrature Amplitude Modulation (QAM) are combined, and applied these methods for improvement of railway signalling systems. Although a rail is used as a transmission medium, there are severe conditions for transmission, such as strong attenuation at frequencies above about 10 kHz. To overcome this restriction, we attempted to improve the S/N characteristics in our proposed method. Specifically, we focused on correlation characteristics of spread codes which are used in CDMA. In this paper, we show that an error correction function that does not require additional bits for such correction can be realized by using the correlation characteristics. We evaluated the effectiveness of this correction method and conducted computer simulations in which the method was employed for CDMA-based transmission.

1. Introduction
To date, there have been many studies on improved railway signalling systems[1-3], and in one such study, a method for improving the transmission characteristics has been investigated[4]. In that study, although rails are used as a transmission medium, there are severe restrictions in the transmission environment. Figure 1 shows an example of the frequency characteristics of a rail serving as a transmission medium. As shown in this figure, since there is strong attenuation at frequencies above about 10 kHz, the audio frequency (AF) band must be used for signal transmission[5]. In addition, there is a return current in the rails for driving trains, which is generated by full-wave rectification of three-phase alternating current. The return current includes strong sixth harmonic components of the power frequency, and we must consider the influence of these in actual data transmission.

To overcome these restrictions, we have attempted to improve the S/N characteristics. In previous research, we proposed a method in which Code-Division Multiple Access (CDMA) and Quadrature Amplitude Modulation (QAM) are combined; it is called the CDMA-QAM method. This approach has the benefit that CDMA can reduce errors even if they are present with QAM demodulation. Since QAM does not directly allocate the transmitted data but allocates the multiplexed CDMA signal, so long as the QAM errors can be absorbed by the CDMA demodulation, they can be corrected[6].
In this study, we investigated an error correction function for CDMA[7] in which there are no additional bits. Although additional bits are generally employed in error correction methods such as Low Density Parity Check (LDPC)[8], we focused on the correlation characteristics of spread codes which are used for CDMA-based transmission methods. Based on this idea, since we conducted computer simulations, we show the effectiveness of the proposed method.

2. Overview of CDMA-QAM method

Figure 2 shows a block diagram of a CDMA-QAM transmitter. In the CDMA-QAM method, a QAM symbol allocated based on a multiplexed CDMA signal is transmitted. The multiplexed CDMA signal is generated by subjecting all channels to CDMA modulation and adding them. Figure 3 shows amplitude data allocation of each symbol in the CDMA-QAM method. In this method, 64-channel base signals are multiplexed by 64 orthogonal codes (Walsh codes). Since each of the 64 Walsh codes is orthogonal to all others, the signals are channelized into 64 orthogonal signals. As a result, the multiplexed signal is allocated to 64 QAM symbols, according to the amplitude of the multiplexed CDMA signal, between 0 and 63, and is transmitted on the transmission line. A benefit of this proposed method is that CDMA can reduce errors even if a QAM symbol error occurs. Since QAM does not directly allocate the transmitted data but the multiplexed CDMA signal, QAM errors can be absorbed by the CDMA demodulation, making recovery possible[6].

3. Error correction function using correlation characteristics of spread codes

We considered the correlation value obtained in the receiver at a certain time. The correlation value is 64 or -64; the absolute value is the same as the code length when spread codes are synchronized, and the sign depends on the transmission data. In practice, however, the correlation value includes both signal and noise components. Based on these characteristics, we derived an equation for noise estimation. Letting the i-th channel of the Walsh codes, the j-th multiplexed signal, and the j-th noise component be denoted by $h_i, s_j$ and $n_j$, respectively, the correlation value is expressed by:

$$c_{i,j} = h_{i,1}(s_{j-63} + n_{j-63}) + h_{i,k}(s_{j+k-64} + n_{j+k-64}) + h_{i,64}(s_j + n_j).$$

(1)

![Figure 2. Block diagram of CDMA-QAM transmitter.](image-url)
Then, letting the demodulated data be denoted by $d_{ij}$ yields the following variation of equation (1):

$$\sum_{k=1}^{64} h_{i,k} h_{j+k-64} = c_{ij} - 64d_{ij}. \quad (2)$$

Considering that 64 channels are multiplexed, the following equation is obtained from equation (2):

$$\begin{pmatrix} h_{1,1} & \cdots & h_{1,64} \\ \vdots & \ddots & \vdots \\ h_{64,1} & \cdots & h_{64,64} \end{pmatrix} \begin{pmatrix} h_{1j-63} \\ \vdots \\ n_j \end{pmatrix} = \begin{pmatrix} c_{1,j} - 64d_{1j} \\ \vdots \\ c_{64,j} - 64d_{64j} \end{pmatrix}. \quad (3)$$

Details of symbol errors can be obtained from equation (3).

Here, since all received data must be correct in this estimation, we considered that error correction is realized if the exactness of this estimation can be evaluated. On the other hand, the probability that QAM symbol errors occur near the transmitted symbol is high. Paradoxically, a received symbol that is distant from the transmitted symbol is most likely to be wrong. For example, if a 1-bit intentional error occurs in the demodulated data and then we recalculate equation (3), the multiplexed signals are allocated to a symbol farther from a natural symbol. Therefore, among all the reverse patterns, the pattern having the smallest norm indicates proper correction, that is, reliable demodulation data. Hence, we employed norm characteristics for the evaluation of this estimation. In a previous study, we verified that it is possible to perfectly correct errors of 3 bits or less [9].

4. Evaluation of the proposed method by using computer simulation

![Frequency distribution of amplitude of multiplexed CDMA signal.](image)
Based on the principle described above, we conducted computer simulations to evaluate the effectiveness of the proposed method. Figure 4 shows the frequency distribution of the amplitudes of the multiplexed CDMA signal. In this figure, the frequency distribution is inhomogeneous and centered at 32 (almost the median amplitude value of the multiplexed signal). Therefore, we designed a new symbol allocation scheme in which the amplitude range of the multiplexed CDMA signal is limited to 32 values from 16 to 47. In addition, since all orthogonal codes that are employed as the spread codes of CDMA include 1 bits in the even-numbered pieces, the amplitude of the multiplexed CDMA signal can be fixed to an even number if the number of 1 bits included in the transmission data of 64 channels is an even number. Therefore, since the amplitude of the multiplexed CDMA signal can be fixed to an even number by defining the transmission data of channel No. 64 as a balance bit, the number of multiplexed CDMA signals can be further reduced to half, i.e., 16 [9].

![Figure 4](image4.png)

**Figure 4.** The frequency distribution of the amplitudes of the multiplexed CDMA signal.

**Figure 5.** Allocation of QAM symbols after the number of symbols was reduced.

Based on this idea, we conducted re-allocation of QAM symbols as shown in figure 5. And, in previous study, we verified that the I channel is more important than the Q channel for bit error rate (BER) characteristics. This is because the I channel is allocated the three higher-order bits of the multiplexed CDMA signal. Based on this requirement, we designed a symbol placement scheme in which the symbol interval of the I channel is wider than that of the Q channel[6]. Figure 6 shows a comparison of the BER characteristics with white noise in cases where the proposed error correction (EC) function was applied and not applied, where the I/Q ratio is 3:1. From this figure, we verified that the BER characteristics were improved dramatically near $E_b/N_0 = 10$ dB. It is considered that error correction function of the proposed method is effective because the number of error bits is 3 bits or less that errors are perfectly corrected as shown previous chapter.

5. Another CDMA-based transmission method using multi-value PSK

In the previous section, we evaluated the CDMA-QAM method. However, QAM has an amplitude component for data transmission, and this is a disadvantage for railway signalling systems using rails that have strong attenuation, as shown in figure 1.

![Figure 5](image5.png)

**Figure 5.** Allocation of QAM symbols after the number of symbols was reduced.

![Figure 6](image6.png)

**Figure 6.** Comparison of BER characteristics using QAM-based methods.
Therefore, we considered the application of a multi-value Phase Shift Keying (PSK) method that has no amplitude component. It is called the CDMA-PSK method. Figure 7 shows symbol allocation for multi-value PSK. In this figure, we can allocate the multiplexed CDMA signal without an amplitude component. Here, the influence of the BER characteristics, given by the symbol error between the symbol of 0 (16) and that of 15 (46), is much larger than other symbol errors. Based on this idea, we omitted these two symbols, and only 14 symbols (from 1 to 14) were used in the CDMA-PSK method. Figure 8 shows a comparison of the BER characteristics with white noise. From this figure, we verified that the CDMA-PSK method realized improved BER characteristics compared with the theoretical multi-value PSK. However, since the effectiveness of the proposed error correction method was small, optimization suitable for the CDMA-PSK method is required.

6. Conclusion
In this study, we proposed an error correction method using the correlation characteristics of spread codes. It has the advantage that efficient communication is realized because there are no additional bits for error correction function. And we evaluated the effectiveness by conducting computer simulation in which the proposed method was applied to CDMA-QAM transmission. As a result of computer simulation, we verified that the BER characteristics were improved dramatically by applying the proposed method near Eb/N0 = 10 dB.

In addition, we proposed a CDMA-PSK method that has no amplitude component. And we verified that the CDMA-PSK method realized improved BER characteristics compared with the theoretical multi-value PSK as a result that the fundamental performance of the proposed error correction method was evaluated.
In future works, we will conduct hardware development using an embedded device such as FPGA after we optimized the proposed method to apply it to the CDMA-PSK method. And we will evaluate the effectiveness of the proposed method against transmission characteristics and noise in an actual railway signalling system.

Acknowledgements
This work was supported by JSPS KAKENHI Grant-in-Aid for Young Scientists (B), Grant Number JP 17K18153.

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