The QCD renormalization scale stability of high twists and $\alpha_s$ in deep inelastic scattering

S.I. Alekhin

Institute for High Energy Physics, 142281 Protvino, Russia

Abstract

A sensitivity of twist-4 and $\alpha_s$ values extracted in the NLO QCD analysis of non-singlet SLAC-BCDMS-NMC deep inelastic scattering data to the choice of QCD renormalization scale (RS) is analysed. It is obtained that the high twist (HT) contribution to structure function $F_2$, is retuned with the change of RS. This retuning depends on the choice of starting QCD evolution point $Q_0$ and $x$. At $Q_0 \gtrsim 10 \text{ GeV}^2$ the HT contribution to $F_2$ is retuned at small $x$ and almost not retuned at large $x$; at small $Q_0$ it exhibits approximate RS stability for all $x$ in question. The HT contribution to $F_1$ is RS stable for all $Q_0$ and $x$. The RS sensitivity of $\alpha_s$ also depends on the choice of $Q_0$: at large $Q_0$ this sensitivity is weaker, than at small one. For $Q_0^2 = 9 \text{ GeV}^2$ the value $\alpha_s(M_Z) = 0.1183 \pm 0.0021(\text{stat + syst}) \pm 0.0013(\text{RS})$ is obtained. Connection with the higher order QCD corrections is discussed.
1. Interest to the quantitative description of high twist (HT) contribution to the deep inelastic scattering (DIS) cross sections has been increased recently, in particular, due to the development of infrared renormalon (IRR) model (see e.g. review [1]). Within this model one can derive the $x$-shape of HT contribution from the $x$-shape of leading twist (LT) structure functions. This connection allows for to obtain precise predictions for the HT contribution since the LT contribution can be rather precisely determined from experimental data. Meanwhile the experimental determination of HT contribution is not direct and is based on fitting a combination of log- and power-like terms to the data. If the data accuracy is not high enough, the correlation between these log- and power-like terms can be large, that was explicitly shown in the combined SLAC-BCDMS data analysis of Ref. [2]. If the large correlations occur, the separation of terms is unstable with respect to the various inputs of fit and thus it becomes important to study the stability of HT separation. One of the poorly defined ansatz of a QCD analysis of DIS data is the choice of renormalization scale (RS). The uncertainty due to RS variation is connected with the account of higher order (HO) QCD corrections since in the analysis with complete account of HO terms, the RS dependence of fitted parameters should vanish and thus the observed RS dependence can be used for the estimation of HO terms effect. Earlier we reported the results of NLO QCD analysis of high $x$ SLAC-BCDMS-NMC data [3]. A short communication on the RS dependence of HT contribution and $\alpha_s$ obtained in this analysis was reported in Ref. [4]. In this paper more detailed study of this dependence is given.

2. Our approach used for the study of RS stability of DGLAP evolution equation in NLO QCD is the same as described in Refs. [5, 6]. Within this approach the RS of QCD evolution is changed from $Q$ to $k_R Q$, where $k_R$ is arbitrary parameter, conventionally varied from 1/2 to 2. This approach contains certain simplification since the change of scale can depend on $x$, but in our analysis this effect is not so essential due to the limited range of $x$. For the nonsinglet case NLO DGLAP equation with an arbitrary choice of RS looks as follows:

$$Q \frac{\partial q^{NS}(x, Q)}{\partial Q} = \frac{\alpha_s(k_R Q)}{\pi} P_{q\bar{q},(0)}^{NS} \otimes q^{NS} + \frac{\alpha_s^2(k_R Q)}{2\pi^2} \left[ P_{q\bar{q},(1)}^{NS} \otimes q^{NS} + \ln(k_R) \beta_0 P_{q\bar{q},(0)}^{NS} \otimes q^{NS} \right]$$

(1)

where $P \otimes q = \int_x^1 dz P(z)q(x/z, Q)$ denotes Mellin convolution; $q^{NS}$ is the evolved distribution; $P^{NS,(0)}$ and $P^{NS,(1)}$ are the LO and NLO parts of splitting function; $\alpha_s(Q)$ is the running strong coupling constant; and $\beta_0$ is the regular coefficient of renormgroup equation for $\alpha_s$:

$$Q \frac{d\alpha_s}{dQ} = \frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3.$$

The equation (1) was solved with the help of direct integration method implemented in the code used earlier [2]. In the analysis of Ref. [3] we used combined SLAC-BCDMS-NMC proton-deuteron data [4] with the cuts $x \geq 0.3$ to reduce QCD evolution to the nonsinglet case and $x \leq 0.75$ to reject the region where a nuclear effects in deuterium can be significant. The initial scale of evolution was chosen equal to $Q_0^2 = 9$ GeV$^2$ to provide comparability with the earlier results of Ref. [2]. A complete account of point-to-point correlations due to systematic errors was made through the covariance matrix approach, similarly to our earlier analysis of Ref. [5]. The formula were fitted to the cross section data to allow for simultaneous and unbiased determination of a twist-4 contribution to structure functions $F_L$. 
Figure 1: The values of proton and deuterium $H_2(x)$ for different values of $k_R$ (open circles: $k_R = 1/2$; full circles: $k_R = 1$).

Figure 2: The dependence of proton $\Delta q^{NS}$ on $\alpha_s^2$ at $Q_0^2 = 9$ GeV$^2$ (full lines: $\log_2 k_R = -1$; dashed lines: $\log_2 k_R = -1/2$).

and $F_2$:

$$ \frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2(s - M^2)}{Q^4} \left[ \left( 1 - y - \frac{(Mxy)^2}{Q^2} \right) F_2^{HT}(x, Q) + \left( 1 - 2\frac{m_l^2}{Q^2} \right) \frac{y^2}{2} 2xF_1^{HT}(x, Q) \right], $$

$$ 2xF_1^{HT}(x, Q) = F_2^{HT}(x, Q) - F_L^{HT}(x, Q), $$

$$ F_{2,1L}^{HT}(x, Q) = F_{2,1L}^{TMC}(x, Q) + H_{2,1L}(x) \frac{1}{Q^2}, $$

where $F_{2,1L}^{TMC}$ are the LT contributions obtained as a result of integration of Eqn. (1) with the account of target mass corrections [3]; $s$ is the total c.m.s. energy; $m_l$ is the scattered lepton mass; and $y$ is the regular lepton scattering variable. The values of functions $H_{2,1L}(x)$ at $x = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$ were fitted, between these points $H_{2,1L}(x)$ were linearly interpolated. The functions $H_2$ for proton and deuterium were fitted independently, while the functions $H_L$ for proton and deuterium due to limited accuracy of data, turned out to be compatible within errors and all fits were performed under constraint $H_L^p(x) = H_L^d(x)$.

3. The proton and deuterium $H_2(x)$ for $k_R = 1$ and $k_R = 1/2$ are given in Fig. 2. One can see that they depend on $k_R$ at $x \sim 0.3$ and practically does not depend at $x \sim 0.8$. 
To give explanation of this behaviour recall the basic properties of solutions to the DGLAP evolution equations. After linearization on $\ln k_R$ Eqn. (1) can be analytically solved in the Mellin momentum space:

$$M^{\text{NS}}(n, Q) = M^{\text{NS}}(n, Q_0)M^{\text{NLO}}(n, \alpha) \exp \left[ g(n) \left( \alpha^2 - \alpha_0^2 \right) \ln(k_R) \right],$$

where $\alpha \equiv \alpha_s(Q)$, $\alpha_0 \equiv \alpha_s(Q_0)$; $M^{\text{NS}}(n, Q)$ are Mellin moments of $q^{\text{NS}}$; $M^{\text{NLO}}(n, \alpha)$ defines NLO evolution of these moments; $g(n)$ is a linear function of Mellin moments of the splitting functions $P_{qq}^{\text{NS},(0)}$ and $P_{qq}^{\text{NS},(1)}$. Introduce a function $\Delta q^{\text{NS}}(k_R) \equiv q^{\text{NS}}(k_R) - q^{\text{NS}}(k_R = 1)$ that is convenient to study RS dependence. The Mellin moments of this function $M^\Delta(n, Q)$ are given by

$$M^\Delta(n, Q) = M^{\text{NS}}(n, Q_0)M^{\text{NLO}}(n, \alpha) \left\{ \exp \left[ g(n) \left( \alpha^2 - \alpha_0^2 \right) \ln(k_R) \right] - 1 \right\}. \quad (2)$$

In Fig. 3 the precise dependence of proton $\Delta q^{\text{NS}}$ on $\alpha_s^2(Q)$ for different $k_R$, obtained as the result of numerical integration of Eqn. (1) at $Q_0^2 = 9$ GeV$^2$, is given. The range of $\alpha_s$ in the figure correspond to the variation of $Q^2$ from 1 to 9 GeV$^2$, i.e. the region where the HT contribution is most significant. It is evident that for all values of $x$ in question $\Delta q^{\text{NS}}$ is approximately proportional to $\ln k_R$. This means that the linearization of Eqn. (1) is justified and that in Eqn. (2) the part of exponent containing $\ln k_R$ can be expanded, so that

$$\Delta q^{\text{NS}}(x, Q) \approx \ln(k_R) \left( \alpha^2 - \alpha_0^2 \right) \tilde q_{\text{NLO}}(x, \alpha), \quad (3)$$

where $\tilde q_{\text{NLO}}(x, \alpha)$ is the Mellin inverse of the product $M^{\text{NS}}(n, Q_0)M^{\text{NLO}}(n, \alpha)g(n)$.

The $Q$-behaviour of $\tilde q_{\text{NLO}}$ is defined by $M^{\text{NLO}}(n, \alpha)$. At $x = 0.3$ the function $\tilde q_{\text{NLO}}$ weakly depends on $\alpha$ (see Fig. 2). This is consequence of the well known effect, that the non-singlet QCD evolution has stationary point at $x \approx 0.1$ due to the fermion conservation. In the vicinity of stationary point scaling violation is small, but, due to at large $n$ the function $M^{\text{NLO}}(n, \alpha)$ rises with $\alpha$ faster, than at low ones, at large $x$ the scaling violation is more pronounced and the function $\Delta q^{\text{NS}}$ rises with $\alpha$ significantly faster than $\alpha^2$.

\footnote{Here and below we do not give the results for deuterium since they are similar to the proton ones.}
Figure 4: The value of $\alpha_s$ at starting evolution point $Q_0^{(n)}$, that provides the power-behaviour simulation of factors $[\alpha_s^n(Q^2) - \alpha_s^n(Q_0)]$ in the region from $Q_1$ to $Q_2$ (full line: $n = 1$, dashed line: $n = 2$, dotted line: $n = 3$). The value of $\alpha_s(Q_2)$ is also given for comparison (dashed-dotted line).

Figure 5: The dependence of $|Q^2\Delta F_2^p| [\text{GeV}^2]$ on $Q^2$ at $k_R = 1/2$ and at different values of $x$ and $Q_0$ (full lines: $x = 0.3$; dashed lines: $x = 0.8$).

In the NLO non-singlet approximation the LT contribution to $F_2$ is given by

$$F_2^{LT} = q^{NS} + \frac{\alpha_s}{2\pi} q^{NS} \otimes C^{NS,(1)}_2,$$

where $C^{NS,(1)}_2$ is the NLO coefficient function. Since the second term of above expression is suppressed with respect to the first one at moderate $x$, the $Q$-behaviour of function

$$\Delta F_2(k_R) \equiv F_2^{LT}(k_R) - F_2^{LT}(k_R = 1)$$

at $x = 0.3$ approximately coincides with the ones of $\Delta q^{NS}$. The $Q$-behaviour of the factor $[\alpha_s^2(Q^2) - \alpha_s^2(Q_0)]$, coming to Eqn. (3) depends on $Q_0$. If $Q \ll Q_0$ this factor is $\sim 1/\ln^2 Q$, i.e. falls with $Q$ slower, than $1/Q^2$. Meanwhile in the vicinity of $Q_0$, where this factor vanishes, its $Q$-dependence is steeper and it can simulate $1/Q^2$ behaviour in a rather wide range of $Q$ (see Fig. 4). One can easily show that the value of $\alpha_s$ at a starting evolution point $Q_0^{(n)}$, that provides the $1/Q^2$ behaviour simulation of the
Figure 6: The values of $H_2^P(x)$ for different choices of $Q_0$ and RS (open circles: $k_R = 1/2$; full circles: $k_R = 1$; squares: $k_R = 2$).

Figure 7: The dependence of $\Delta \alpha_{RS} (k_R) \equiv \alpha_s(M_Z) |_{k_R} - \alpha_s(M_Z) |_{k_R=1}$ on the choice of renormalization scale for different $Q_0$ (open circles: $Q_0^2 = 50$ GeV$^2$, full circles: $Q_0^2 = 1$ GeV$^2$). For comparison are also given the $\Delta \alpha_{RS} (k_R)$ values obtained in the fits at $Q_0^2 = 50$ GeV$^2$ with HT fixed at the values obtained in fits with $k_R = 1$ (squares). Error bars are not given.

The dependence of $\alpha_s(Q(2))$ on $Q_2$ for $Q_1 = 1$ GeV$^2$ obtained with the help of Eqn. (4) is given in Fig. [4]. For the $Q^2$ interval from 1 to $\sim 10$ GeV$^2$, where the HT are mostly significant, $\alpha_s(Q(2)) \sim 0.2$, that corresponds to $[Q_0^{(n)}]^2 \sim 40$ GeV$^2$. Due to the weak dependence of $\tilde{\alpha}_{NLO}$ on $Q$ at small $x$, the function $\Delta F_2$ also simulates $1/Q^2$ behaviour at $x = 0.3$, if the starting evolution point is chosen equal to $Q_0^{(2)}$. At fixed $Q_0$ this simulation, in general, becomes less probable with the rise of $x$ just due to the steeper rise of $\Delta q^{NS}$ with $\alpha$ at large $x$.

The behaviour of $Q^2 \Delta F_2^0$ at $k_R = 1/2$ for different $x$ and $Q_0$ is given in Fig. [5]. At $Q_0^2 = 50$ GeV$^2$ and $x = 0.3$ the function $\Delta F_2$ simulates the $1/Q^2$ behaviour about perfectly

\[ \alpha_s(Q(n)) = \left[ \frac{Q_2^2 \alpha_s^n(Q_2^2) - Q_1^2 \alpha_s^n(Q_1^2)}{Q_2^2 - Q_1^2} \right]^{1/n}. \]
in the total $Q$ region relevant for HT determination. This leads to the $H_2$ retuning in the fits with different $k_R$, since $\Delta F_2$ is compensated by the additional contribution to $H_2$ with the sign opposite to $\Delta F_2$ (see Fig. 4). At $x = 0.8$, due to the fall of $\hat{q}_{\text{NLO}}$ with $Q$, the simulation is much worse. If $Q_0^2 = 1 \text{ GeV}^2$, the absolute value of $Q^2 \Delta F_2$ steeply rises at $x = 0.3$ and small $Q$ due to the factor $[\alpha_s^2(Q^2) - \alpha_s^2(Q_0)]$. At $x = 0.8$ this rise is not so steep because of the fall of $\hat{q}_{\text{NLO}}$ with $Q$, but it cannot suppress the general rise. As a consequence, at small $Q_0$ the function $\Delta F_2$ cannot simulate $1/Q^2$ behaviour at all $x$ and it should be at least partially compensated by the change of $Q$-dependence of the LT contribution. The remnant HT retuning is still possible if the $Q$-dependence of $\Delta F_2$ at some $x$ is more similar to $1/Q^2$, than to the $Q$-dependence of $\partial F_2/\partial \alpha_s(M_Z)[4]$. The explicit tracing of the balance between the $H_2$ retuning and the change of LT contribution is not so simple due to $\alpha_s(M_Z)$ is determined by data at all $x$ and $Q$. Anyway, as one can see from Fig. 4 at small $Q_0$ the function $H_2$ is retuned with the change of $k_R$ significantly smaller than at large $Q_0$. At $Q_0^2 = 9 \text{ GeV}^2$ the retuning effect is almost the same as for $Q_0^2 = 50 \text{ GeV}^2$. This is natural since the $\Delta F_2$ behaviour at small $Q$ weakly depends on $Q_0$, if $Q_0$ is large, and the data are not very sensitive to the HT contribution at $Q^2 \gtrsim 10 \text{ GeV}^2$. Thus the choice of $Q_0$ have small effect on the fitted HT contribution if $Q_0^2 \gtrsim 10 \text{ GeV}^2$.

The RS stability of $\alpha_s$ also depends on the choice of $Q_0$. In the fit with HT fixed $\Delta F_2$ is compensated by the change of LT contribution, that leads to the shift of $\alpha_s$. If $H_2$ is released in the fit, the shift of $\alpha_s$ can change due to the partial compensation of $\Delta F_2$ by the change of $H_2$. Since the $1/Q^2$ simulation of $\Delta F_2$ depends on $Q_0$, the $\alpha_s$ dependence on $k_R$ changes with $Q_0$ as well as $H_2$ (see Fig. 4). At large $Q_0$ the $\Delta F_2$ slope on $k_R$ is negative at small $Q$. As a result, in the fit with HT released the $H_2$ slope on $k_R$ is positive. Correspondingly the $Q$-dependence of LT contribution after HT releasing becomes weaker at large $k_R$ and steeper at small ones, i.e. the slope of fitted $\alpha_s(M_Z)$ value on $k_R$ decreases as compared to the fit with HT fixed. In the fit with HT fixed the $\alpha_s(M_Z)$ slope on $k_R$ is positive and thus the RS uncertainty on $\alpha_s$ at large $Q_0$ becomes smaller. At small $Q_0$ the $H_2$ slope on $k_R$ at $x \approx 0.5 \div 0.7$ is negative. The data from this region of $x$ have the largest impact on the $\alpha_s$ determination and thus the $H_2$ releasing leads to the increase of RS error on $\alpha_s$, although the scale of effect is smaller as compared with the fit at large $Q_0$ due to the weakness of HT retuning at small $Q_0$. In our analysis

$$\alpha_s(M_Z) = 0.1151 \pm 0.0015(\text{stat + syst}) \pm 0.0045(\text{RS})$$

for $Q_0^2 = 1 \text{ GeV}^2$ and

$$\alpha_s(M_Z) = 0.1183 \pm 0.0021(\text{stat + syst}) \pm 0.0013(\text{RS})$$

for $Q_0^2 = 9 \text{ GeV}^2$, where the RS error is estimated as the half of $\alpha_s(M_Z)$ spread with the change of $k_R$ from 1/2 to 2; the central value is shifted to the centre of this spread. The values of $\chi^2$ are approximately the same for different $k_R$, but in view of that at small $Q_0$ the total $\alpha_s(M_Z)$ error is about two times larger, than at large one, we consider the $\alpha_s$ value determined from the fit with large $Q_0$ as more reliable.

The function $\Delta q^{\text{NS}}$, by definition, is connected with the NNLO QCD corrections to evolved distributions. A natural assumption is that the exponent in NNLO part of moment expression can be expanded, similarly to the exponent in Eqn. (2), and the NNLO

\[ \]
The comparison of $Q^2 \Delta F_2^p(k_R = 1/2)$ and $Q^2 \Delta F_L^p(k_R = 1/2)$ dependence on $Q^2$.

contribution is $\sim [\alpha_s^2(Q^2) - \alpha_s^2(Q_0)]$ as well as $\Delta q^{NS}$. One can see from Fig. 8 that the factor $[\alpha_s^3(Q^2) - \alpha_s^3(Q_0)]$ coming to N$^3$LO contribution also can simulate $1/Q^2$ behaviour. Moreover, the region of $Q$ where the simulation is possible widens from the lower orders to the higher ones. This gives an indication that the retuning of HT contribution after the accounting of HO QCD corrections to DGLAP kernel would exhibit the same properties as the RS retuning in NLO. Note that if $Q_2 \gg Q_1$, then $\alpha_s(Q_0^{(n)}) \sim \alpha_s(Q_2)$ for all $n$ (see Eqn. 4 and Fig. 4). This means, that in the analysis with simultaneous determination of $\alpha_s$ and HT at large $Q_0$ the contribution to $F_2$ due to the HO QCD corrections to DGLAP kernel can be merely fitted together with the HT, if $g(n)$ and the Mellin moments of HO splitting functions have the similar large $n$ asymptotes. The fitting of HO corrections sure leads to the increase of $\alpha_s$ error. Meanwhile the reduction of RS uncertainty, that is one of the dominant sources of $\alpha_s$ error, is larger and, as one can see from the above, the total $\alpha_s$ error becomes smaller.

One can see from Fig. 8, that at $Q_0^2 = 50$ GeV$^2$ the fitted $\alpha_s(M_Z)$ value is a nonlinear function of $\ln k_R$ contrary to the fits at $Q_0^2 = 1$ GeV$^2$. The reason of this difference is the large correlation between $\alpha_s$ and $H_2$ (c.f. Ref. [2]), that depends on how well $\partial F_2/\partial \alpha_s(M_Z)$ can simulate $1/Q^2$ behaviour. With the rise of $Q_0$ this correlation increases. For example the correlation coefficient $\rho_{0.5}$ for $\alpha_s(M_Z)$ and $H_2(x = 0.5)$ at $k_R = 1/2$ is $-0.82$ for $Q_0^2 = 1$ GeV$^2$ and $-0.97$ for $Q_0^2 = 50$ GeV$^2$. As a consequence, at large $Q_0$ a small nonlinearity of $q^{NS}$ on $\ln k_R$ better manifests and takes non-negligible effect on the fitted parameters values (remind that the effective amplification of nonlinear effects in a fit is proportional to $1/(1 - \rho^2)$). For the comparison, the fits with HT fixed exhibit almost linear dependence of $\alpha_s(M_Z)$ on $\ln k_R$ (see Fig. 8). One of the reflections of this nonlinearity is that for $Q_0^2 = 50$ GeV$^2$ the difference between $H_2(x)$ at $k_R = 2$ and at $k_R = 1$ is small (see Fig. 8). This is unpleasant feature of the analysis since the variation range of $k_R$ is conventional and the nonlinearity does not allow to rescale the RS uncertainty. One of the possible ways to suppress the nonlinear effects is to decrease the correlation between HT contribution and $\alpha_s$, e.g. adding more data to the analysis. From another side this correlation leads to the reducing of RS error on $\alpha_s$ and it is necessary to keep a balance between the linearity and the size of RS error.

\footnote{In the fit at $Q_0^2 = 50$ GeV$^2$ with HT fixed the $\alpha_s(M_Z)$ error is 0.0004 as compared with 0.0021 in the fit with HT released.}
In NLO QCD the LT contribution to structure function $F_L$ is proportional to the Mellin convolution of $q^{\text{NS}}$ with the NLO coefficient function $C^{\text{NS},(1)}_L$:

$$F^\text{LT}_L = \frac{\alpha_s}{2\pi} q^{\text{NS}} \otimes C^{\text{NS},(1)}_L.$$  

Due to the convolution smearing a function $\Delta F_L(k_R) \equiv F^\text{LT}_L(k_R) - F^\text{LT}_L(k_R = 1)$ depends on $Q$ steeper, than $\Delta F_2$. One can see, that at $x = 0.3$ and large $Q_0$ the function $Q^2 \Delta F_L(k_R = 1/2)$ falls in about two times in the region of $Q^2 = 1 \div 3 \text{ GeV}^2$ (see Fig. 8). At large $x$ and low $Q_0$ it falls even more steeply. As a result, $H_L(x)$ exhibits only weak dependence on $k_R$ for all $x$ and $Q_0$ (see Fig. 9).

4. In summary, we can conclude that the HT contribution to structure function $F_2$ extracted in the NLO QCD analysis of nonsinglet SLAC-BCDMS-NMC data is retuned with the RS change. This retuning depends on the choice of starting evolution point $Q_0$ and $x$. At $Q_0 \gtrsim 10 \text{ GeV}^2$ the HT contribution to $F_2$ is retuned at small $x$ and not almost retuned at large $x$; at small $Q_0$ it exhibits approximate RS stability for all $x$ in question. The RS sensitivity of $\alpha_s$ also depends on the choice of $Q_0$: at large $Q_0$ this sensitivity is weaker, than at small ones. The HT contribution to $F_L$ is RS stable for all $Q_0$ and $x$.

The RS stability of HT contribution is important for clarification of their nature: due to the both HT and HO corrections fall with $Q$, it was often claimed that the extracted HT terms can contain contribution from HO. The HT absorption by NNLO correction was observed in the analysis of neutrino structure function $x F_3$ [10], although the effect was smashed due to low accuracy of the data. Our results indicate, that in the analysis of high statistical charged leptons DIS data the unambiguous separation of twist-4 contribution and NNLO QCD corrections to DGLAP kernel is possible if $Q_0$ is low. This conclusion is especially important because no complete NNLO calculation of the splitting functions is available up to now and it is impossible to perform exact direct clarification of this point.

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