The Radiative $Z_2$ Breaking Twin Higgs

Jiang-Hao Yu

1 Amherst Center for Fundamental Interactions, Department of Physics, University of Massachusetts-Amherst, Amherst, MA 01003, U.S.A.

In twin Higgs model, the Higgs boson mass is protected by a $Z_2$ symmetry. The $Z_2$ symmetry needs to be broken either explicitly or spontaneously to obtain misalignment between electroweak and new physics vacua. We propose a novel $Z_2$ breaking mechanism, in which the $Z_2$ is spontaneously broken by radiative corrections to the Higgs potential. Two twin Higgses with different vacua are needed, and vacuum misalignment is realized by opposite but comparable contributions from gauge and Yukawa interactions to the potential. Due to fully radiative symmetry breaking, the Higgs sector is completely determined by twin Higgs vacuum, Yukawa and gauge couplings. There are eight pseudo-Goldstone bosons: the Higgs boson, inert doublet Higgs, and three twin scalars. We show the 125 GeV Higgs mass and constraints from Higgs coupling measurements could be satisfied.

In this work, we propose a novel approach to spontaneously break the $Z_2$ symmetry: the radiative $Z_2$ breaking mechanism. The Higgs potential is fully generated from gauge and Yukawa corrections, and the $Z_2$ symmetry is broken spontaneously and radiatively. The radiatively generated Higgs potential is parametrized as

$$V(h) \sim \frac{g_{SM} m_h^2}{16 \pi^2} \left( -a |h|^2 + b |h|^4 \right),$$

where $g_{SM}$ is a typical SM coupling and $m_h$ is the mass scale of twin partner. Typically radiative corrections have $a$ and $b$ at the same order, which only induce symmetric vacua $v = f$. To realize vacuum misalignment $v < f$, we need either $a$ is suppressed or $b$ is enhanced. For example, in littlest Higgs the quartic term $b$ is enhanced via adding tree-level quartic terms by hand. Without adding terms by hand, we could utilize possibly large cancellation among radiative corrections to suppress quadratic term $a$. In the original twin Higgs, we note that gauge and Yukawa corrections to the quadratic term $a$ have opposite sign. However, a large cancellation cannot happen because gauge corrections are much smaller than Yukawa ones. Interestingly, gauge corrections can be enhanced by introducing a second twin Higgs with global symmetry breaking scale $f' \gg f$. This causes comparable but opposite gauge and Yukawa corrections, and leads to vacuum misalignment with a moderate tuning between $v$ and $f$. Thus, a different but more minimal spontaneous $Z_2$ breaking mechanism is naturally realized without introducing either a soft breaking term or a bilinear tadpole term.

Two $U(4)$ invariant Higgs fields are introduced as

$$H_1 \equiv \begin{pmatrix} H_{1A} \\ H_{1B} \end{pmatrix}, \quad H_2 \equiv \begin{pmatrix} H_{2A} \\ H_{2B} \end{pmatrix},$$

where two twin Higgs doublets $H_{1B}$ and $H_{2B}$ are in twin sector. The $Z_2$ symmetry maps the twin Higgses into visible Higgses: $H_{1B} \mapsto H_{1A}, H_{2B} \mapsto H_{2A}$. The scalar potential, which respects both $Z_2$ and global $U(4)_1 \times U(4)_2$ symmetries, reads

$$V(H_1, H_2) = -\mu_1^2 |H_1|^2 - \mu_2^2 |H_2|^2$$
$$+ \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2.$$ (3)  

The Higgs sector is weakly gauged under both the SM and the mirror SM gauge symmetries. After symmetry breaking \((H_i) \equiv f_i\) \((i = 1, 2)\), the symmetries of the Lagrangian have

\[
\text{global symmetry: } U(4) \times U(4) \to U(3) \times U(3), \\
\text{gauge symmetry: } [SU(2) \times U(1)]_{A,B} \to [SU(2) \times U(1)]_A.
\]

In nonlinear \(\sigma\) Lagrangian, assuming radial modes in \(H_i\) are decoupled, the fields \(H_i\) are parametrized as

\[
H_i = \exp \left[ \frac{i}{f_i} \begin{pmatrix} 0_{2 \times 2} & h_{1i} \cr 0_{2 \times 1} & C_i \end{pmatrix} \right] \begin{pmatrix} 0_{1 \times 2} \cr 0 \end{pmatrix}, \quad \frac{1}{f_i}, \quad \text{where}\ 14 \text{ GBs } h_i, C_i, N_i(i = 1, 2) \text{ are generated.}
\]

There are two ways to incorporate fermions. In the “mirror fermion” assignment \([5,6]\), the SM fermions have mirror fermions: \(q_A(3,2;1,1) \leftrightarrow q_B(1,1;3,2)\) and \(t_A(3,1;1,1) \leftrightarrow t_B(1,1;3,1)\), with quantum number assignment \([SU(3), SU(2)]_{A,B}\). The general top-Yukawa Lagrangian reads

\[
- \mathcal{L}_{\text{Yuk}} = y_1 \left( H^+_1 q_A t_A + H^+_1 q_B t_B \right) + (1 \leftrightarrow 2) + \mathcal{H}(6)
\]

To avoid Higgs mediated flavor changing neutral current in \(A\) sector, similar to the two Higgs doublet model (2HDM), either the discrete \(Z_2\) symmetry or aligned Yukawa structure \([12]\) are imposed. We will discuss the following two Yukawa structures: Type-I Yukawa structure \((y_2 = 0)\), and a special aligned Yukawa structure \(y_2 = \epsilon y_1\) \((\epsilon \ll 1)\) \([13]\). In the “\(U(4)\) fermion” assignment \([5]\), the following \(U(4)\) fermions are introduced:

\[
Q = q_A + q_B + q_A(3,1;1,2) + q_B(2,1;3,1), \\
U = t_A(3,1;1,1) + t_B(1,1;3,1).
\]

The top Yukawa Lagrangian is

\[
- \mathcal{L}_{\text{Yuk}} = y_1 H^+_1 Q \bar{U} + y_2 H^+_2 Q \bar{U} + M_{q_A,B} q_{A,B} + h.c.(8)
\]

Here either Type-I or aligned Yukawa structure is used.

The global \(U(4) \times U(4)\) symmetry is weakly broken by the radiative corrections from the gauge and Yukawa interactions. The dominant radiative corrections to the scalar potential are written as

\[
V_{\text{loop}} = \delta_1|H_1|^4 + \delta_2|H_2|^4 + \delta_3|H_1|^2|H_2|^2 + \delta_4|H^+_1 H_2|^2 + \delta_5 \left( \frac{1}{2} \left( |H^+_1 H_2|^2 + h.c. \right) \right) + (A \leftrightarrow B).
\]

Here for gauge bosons \(W_{A,B}\) and \(Z_{A,B}\), the one-loop corrections are

\[
\delta^{B}_{12} \simeq - \frac{1}{16\pi^2} \left( \frac{9}{8} g^4 + \frac{3}{4} g^2 g' + \frac{3}{8} g'^4 \right) \log \frac{\Lambda}{f},
\]

\[
\delta^{B}_{3} \simeq - \frac{1}{16\pi^2} \left( \frac{9}{4} g^4 - \frac{3}{2} g^2 g' + \frac{3}{2} g'^4 \right) \log \frac{\Lambda}{f},
\]

\[
\text{where } f \equiv \sqrt{f_1^2 + f_2^2} \text{ and } \Lambda \equiv 4\pi f. \text{ For fermions, radiative corrections depend on the fermion assignment and Yukawa structure. In the Type-I Yukawa structure, we obtain [5]}
\]

\[
\delta^F \simeq \left\{ \begin{array}{ll}
\frac{3g^4}{16\pi^2} \log \frac{\Lambda^2}{f^2}, & \text{(mirror fermion)} \\
\frac{3g^4}{8\pi^2} x^2 \left[ x \log \frac{x^2 + z}{z} - (x \leftrightarrow z) \right], & \text{(U(4) fermion)}
\end{array} \right.
\]

\[
\text{where } x = y_f^2 \text{ and } z = M^2, \text{ and } \delta^F_{-7} = 0. \text{ In the aligned Yukawa structure with } y_2 \ll y_1, \text{ we have}
\]

\[
\delta^F_{1,2} \simeq \frac{3g^4}{16\pi^2} \log \frac{\Lambda^2}{f^2}, \quad \delta^F_{-5} \simeq \frac{3g^4}{16\pi^2} \log \frac{\Lambda^2}{f^2},
\]

\[
\delta^F_{6} \simeq \frac{3g^4}{16\pi^2} \log \frac{\Lambda^2}{f^2}, \quad \delta^F_{-7} \simeq \frac{3g^4}{16\pi^2} \log \frac{\Lambda^2}{f^2}.
\]

The radiatively generated scalar potential in Eq.9 further triggers electroweak symmetry breaking and induces VEVs for the GBs \(h_{1,2}\) in visible sector. The VEVs of the fields \(H_{1,2}\) are parametrized as

\[
\langle H_1 \rangle = \begin{pmatrix} 0 \\
\sin \theta_1 f_1 \cos \theta_2 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\
\sin \theta_2 \cos \theta_1 \end{pmatrix}
\]

\[
\text{where } \theta_1 = \frac{\sin \theta_1}{f_1}, \quad \theta_2 = \frac{\sin \theta_2}{f_2}. \text{ Similar to 2HDM, } \tan \beta = \frac{\theta_2}{\theta_1}, \quad \delta_{45} = \delta_4 + \delta_5 \text{ and } \delta_{345} \equiv \delta_3 + \delta_4 + \delta_5 \text{ are used. Imposing tadpole conditions on Eq.9 determine } \theta_1, \theta_2. \text{ We will neglect } \delta_{6,7} \text{ terms, because either } \delta_{6,7} = 0 \text{ in Type-I or } \delta_{6,7} \ll \delta_{-5} \text{ (} y_2 \ll y_1 \text{) in aligned Yukawa structure. The tadpole conditions are}
\]

\[
\sin \theta_1 + \Omega_1 \sin \theta_2 + \Omega_2 \sin 2(\theta_1 + \theta_2) = 0, \\
\sin 4\theta_1 - \Omega_1 \sin 4\theta_2 + \Omega_2 \sin 2(\theta_1 - \theta_2) = 0.
\]

\[
\text{where } \Omega_1 = t_0^2 \delta_{2}/\delta_1 \text{ and } \Omega_2 = t_0^2 \delta_{345}/\delta_1. \text{ We are interested in the region } \Omega_{1,2} < 0 \text{ because } \delta_1 > 0, \delta_{2,5} < 0. \text{ If } |\Omega_1 + \Omega_2| > 1 \text{ the two conditions are symmetric under } \theta_2 \leftrightarrow -\theta_2. \text{ While if } |\Omega_1 + \Omega_2| < 1 \text{ they are symmetric under } \theta_1 \leftrightarrow -\theta_1. \text{ The solutions should be}
\]

\[
\left\{ \begin{array}{ll}
\theta_2 = 0, & \theta_1 < \pi/4, \text{ for } |\Omega_1 + \Omega_2| > 1 \\
\theta_1 = 0, & \theta_2 < \pi/4, \text{ for } |\Omega_1 + \Omega_2| < 1.
\end{array} \right.
\]

Thus only one \(H_1\) further generates a VEV after radiative symmetry breaking. We plot the (\(\theta_1, \theta_2\)) contours imposed by tadpole conditions for different (\(\Omega_1, \Omega_2\)) in Fig.1. We note that \(\Omega_2\) alone could determine \(\theta_1\) which is intersection point between solid and dashed curves, while \(\Omega_1\) only controls the convex behavior of the curves.

To obtain the electroweak vacuum \(v < f, \theta_i < \pi/4\) is required, which spontaneously breaks the \(Z_2\) symmetry.
This implies \(|\Omega_1 + \Omega_2| > 1\) and \(\theta_2 = 0\) in Eq. 15. The electroweak vacuum thus has \(v = f_1 \sin \theta_1 = 174\) GeV. The tadpole conditions reduce to one condition

\[
\sin^2 \theta_1 = \frac{v^2}{f_1^2} \equiv \frac{1}{2} \left( 1 + \frac{\delta_{345} \cos^2 \left( \frac{\theta_1}{f_1} \right)}{2\delta_1} \right).
\] (16)

Because of \(\delta_1 < 0, \delta_{345} > 0\), we have \(\theta_1 < \pi/4\). Furthermore, if \(t_\beta\) is large \((f_2 > f_1)\), \(\theta_1\) could be much smaller than \(\pi/4\). Therefore, \(t_\beta\) controls the tuning between \(v\) and \(f_1\), and it is natural to realize such tuning by setting \(f_2 \gg f_1\). Let us understand the purely radiative breaking mechanism physically. The leading terms in the Higgs potential can be parameterized by

\[
V(h_1) = f_1^2 \delta_1 \left[ \sin^4 \left( \frac{h_1}{f_1} \right) + \cos^4 \left( \frac{h_1}{f_1} \right) + \frac{f_1^4 t_\beta^2 \delta_{345} \cos^2 \left( \frac{h_1}{f_1} \right)}{\delta_1} \right],
\]

\[
\simeq -(2 + \Omega_2) \delta_1 f_1^2 |h_1|^2 + \frac{8 + \Omega_2}{3} \delta_1 |h_1|^4.
\] (17)

Both the quadratic and quartic terms are loop-suppressed. As mentioned in introduction, if the quadratic term is much smaller than the quartic term, the electroweak VEV could be obtained. Here let us expand the Higgs quartic term in Eq. 17.

\[
\mu_{h_1}^2 = 2\delta_1 f_1^2 + \delta_{345} t_\beta f_1^2.
\] (18)

Since the Yukawa and gauge corrections have \(\delta_1 > 0\) and \(\delta_{345} < 0\) respectively, the Higgs mass squared is suppressed by cancellation between Yukawa and gauge corrections. Note \(t_\beta\) plays an important role: only when \(t_\beta\) is not so small, cancellation in quadratic term is adequate. This implies a moderate tuning \(f_1 < f_2\), which induces tuning between Yukawa and gauge corrections correspondingly. As a measure of the naturalness, the estimation of the fine-tuning is

\[
\Delta = \left| \frac{2\delta m^2}{\mu_{h_1}^2} \right|^{-1} \simeq \left| \frac{3g_2^2 m_t^2}{4\pi^2 m_Z^2} \right|^{-1} \frac{2v^2}{f_1^2}.
\] (19)

Unlike the soft breaking or tadpole breaking mechanism, the tuning is realized via hierarchy between \(f_1\) and \(f_2\). From Eq. 10 for a level of tuning \(10\%\), \(t_\beta \simeq 3\) \((f_2 \simeq 3f_1)\).

The purely radiative symmetry breaking only generate VEV \((H_{1A})\), but not \((H_{2A})\). Zero \(H_{2A}\) VEV \((\theta_2 = 0)\) implies that the second Higgs \(H_{2A}\) in \(A\) sector is inert Higgs doublet \(15\), which does not mix with \(H_{1A}\). In visible sector, particles in \(H_{1A}\) are identified as GBs \(h_i\). Among them, \((\phi_{0,2})\) in \(H_{1A}\) and \((H^\pm, A^0)\) in \(H_{2A}\) have

\[
m_{y_{0}}^2 = m_{y_2}^2 = 0, \quad \text{(exact GBs eaten by } W_A, Z_A),
\]

\[
m_{H^+}^2 \simeq 2\delta_{1} f_1^2 t_\beta \cos^2 \theta_1 - 2\delta_2 f_2^2 - 4\delta_3 f_1^2 \sin^2 \theta_1,
\]

\[
m_{A^0}^2 \simeq 2\delta f_1^2 t_\beta \cos^2 \theta_1 - 2\delta_2 f_2^2 - 2\delta_3 f_1^2 \sin^2 \theta_1.
\] (20)

And two CP-even GBs \(h_1\) in \(H_{1A}\) and \(h_2\) in \(H_{2A}\), which do not mix together due to zero \(H_{2A}\) VEV, have

\[
m_{h_1}^2 \simeq 8\delta_1 f_1^2 \sin^2 \theta_1,
\]

\[
m_{h_2}^2 \simeq -2\delta_2 f_2^2 + 2\delta_1 f_1^2 \cos^2 \theta_1/t_\beta.
\] (21)

Here \(h_1\) is identified as the SM Higgs boson. However, in twin sector \(B\), the GBs in \(H_{1B}\) and \(H_{2B}\) are mixed due to the VEVs \(f_{1,2}\). The rotation angle \(\beta_B\) between \(C^+_1(N^1)\) and \(C^+_2(N^2)\) is defined as \(\tan \beta_B = t_\beta / \cos \theta_1\). Performing rotation to mass basis, we obtain

\[
m_{y_0}^2 = m_{y_2}^2 = 0, \quad \text{ (exact GBs eaten by } W_B, Z_B),
\]

\[
m_{H^+}^2 = -\left( \delta_{15} + \delta_7 \tan \beta_B \right) f_1^2 \left( \cos^2 \theta_1 + t_\beta^2 \right),
\]

\[
m_{A^0}^2 = -\left( 2\delta_5 + \delta_7 \tan \beta_B \right) f_1^2 \left( \cos^2 \theta_1 + t_\beta^2 \right)^2 / \cos^2 \theta_1.
\] (22)

Fig. 2 shows mass spectra of the pGBs in two cases. In “mirror fermion” case, the pGB masses only depend on single parameter \(\theta_1\). Thus the requirement of a 125 GeV Higgs mass determines \(\theta_1 = 0.57\), which corresponds to \(t_\beta = 2\). In “U(4) fermion” case, mass spectra depend on both \(\theta_1\) and vectorlike fermion \(\tilde{q}_A\) mass \(M\), which should have \(M \leq 4\pi f\). As the \(M\) takes smaller value than \(4\pi f\), the \(\theta_1\), obtained from the 125 GeV Higgs mass condition, gets smaller value. However, when \(M = 8f\), \(\theta_1\) reaches zero, which put a lower cutoff for \(M\). In the following, we take \(M = 10f\) as the benchmark point.

The current limits on NP searches at the LHC put very strong constraints on new particles. New particles
in a sector are the inert Higgses $H^\pm, h_2, A^0$, which is typically tightly constrained [15]. However, if the inert Higgses have nearly degenerate masses, it happens to be very difficult to probe this compressed parameter region at the LHC, which has been studied in Ref. [16]. For particles in twin sector, it is harder to directly probe them due to zero SM charges. However, because of twin colorness, there are rich twin hadron phenomenology, which has been discussed in Ref. [17, 18]. For simplicity, in the following we adopt minimal twin matters: fraternal twin Higgs [8], in which only the third generation twin fermions are introduced, and typically twin lepton is identified as dark matter candidate. In this scenario, the twin photon and $A^0$ could be either massless or massive depending on gauge and fermion assignments. For example, the aligned Yukawa structure could lift the $A^0$ mass from zero value. If they are massless, they should contribute to dark radiation. Depending on temperature of thermal decoupling between visible and twin sector [13], the number of effective neutrino species $\Delta N_{\text{eff}}$ could be adjusted to be within the range of recent Planck measurement $0.11 \pm 0.23$ [19]. We leave the detailed discussion in future study [20].

The measured Higgs production and decay cross sections, and the upper limits on Higgs invisible decays at the LHC [21] also provide strong constraints on model parameters. The tree-level couplings of the Higgs boson to fermions and bosons in $A(B)$ sector are altered by a factor $\cos \theta_1$ (sin $\theta_1$) relative to SM. We assume masses of twin particles are altered by a factor $\cot \theta_1$ relative to SM. In “$U(4)$ fermion” case, there are also a heavier vectorlike top $T$ which mixes with the top quark through mixing angle $\cos \theta_R = y_t f / \sqrt{M^2 + y_t^2 f^2}$. This modifies the top-Higgs coupling to $y_t \cos \theta_1 \cos \theta_R$, and a new $TTh$ coupling has $y_t \sin \theta_1 \sin \theta_R$. We calculate various Higgs signal strengths $\mu_{pp \to h_1 \to VV} = \sigma(pp \to h_1)Bf_{h_1 \to VV}/\sigma_{\text{SM}}Bf_{\text{SM}}$, and invisible decay width. Based on Higgs signal strengths at the 8 TeV LHC with 20.7 fb$^{-1}$ data [21], we perform a global fit on model parameters [20]. Fig. 3 (left) shows the allowed contours on $(\theta_1, M/f_1)$ at 68%, 95%, 99% confidence levels (CLs) in “$U(4)$ fermion” case. Fig. 3 (right) plots the signal strength of gluon fusion $gg \to h_1 \to VV/ff$, and invisible decay width in two assignments. We list our global $\chi^2$-fitting results on Higgs signal strengths in “mirror fermion” (“$U(4)$ fermion” with $M = 10f_2$) assignments:

$$\theta_1 \equiv \frac{v}{f_1} < 0.25 \pm 0.31 \, \% \quad 95\% \, \text{CL.}$$

This limit rules out the whole parameter region of the “mirror fermion” case, but “$U(4)$ fermion” case is still viable. The electroweak precision test put additional constraints on model parameters. The contribution from inert Higgs doublet is negligble due to their degenerated masses. Thus the dominant Logarithmic contributions to $S$ and $T$ parameters [22] are $\alpha T(\alpha S) \sim \mp \sin \theta_1 \log \frac{M}{m}$.

We also estimate the levels of tuning of about 10% and 30%, which are shown in Fig. 3 (right). The high luminosity LHC will improve sensitivity of signal strengths to around 5% assuming current uncertainty with 3 ab$^{-1}$ luminosity [22]. This indicate that we could probe this model with about 12% tuning by the end of high luminosity LHC run.

In summary, we have investigated a minimal two twin Higgs model, in which the Higgs boson is a pseudo-Goldstone boson after symmetry breaking $[U(4) \times U(4)] \rightarrow [U(3) \times U(3)]$. The $\mathbb{Z}_2$ symmetry, which protects the Higgs mass against quadratic divergence, is spontaneously broken by radiatively generated Higgs potential. The vacuum misalignment $v < f$ is realized radiatively via cancellation of gauge and Yukawa corrections to the Higgs mass term. This minimal setup for spontaneous $\mathbb{Z}_2$ breaking has less parameters and it has predictive but rich phenomenology.

Acknowledgements - The author thanks Can Kilic and Nathaniel Craig for valuable discussions. This work was supported by DOE Grant [de-sc0011005].

[1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716, 1 (2012). S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012).
[2] D. B. Kaplan and H. Georgi, Phys. Lett. B 136, 183 (1984). K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719, 165 (2005).
[3] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207, 034 (2002).
[4] R. Barbieri and A. Strumia, hep-ph/0007265
[5] Z. Chacko, H. S. Goh and R. Harnik, Phys. Rev. Lett. 96, 231802 (2006); JHEP 0601, 108 (2006).
[6] N. Craig, S. Knapen and P. Longhi, Phys. Rev. Lett. 114, no. 6, 061803 (2015); JHEP 1503, 106 (2015).
[7] G. Burdman, Z. Chacko, R. Harnik, L. de Lima and C. B. Verhaaren, Phys. Rev. D 91, no. 5, 055007 (2015).
[8] N. Craig, A. Katz, M. Strassler and R. Sundrum, JHEP 1507, 105 (2015).
[9] Z. Chacko, Y. Nomura, M. Papucci and G. Perez, JHEP 0601, 126 (2006). H. S. Goh and S. Su, Phys. Rev. D 75, 075010 (2007).
[10] S. Chang, L. J. Hall and N. Weiner, Phys. Rev. D 75, 035009 (2007); N. Craig and K. Howe, JHEP 1403, 140 (2014).
[11] H. Beauchesne, K. Earl and T. Grgoire, JHEP 1601, 130 (2016).
[12] R. Harnik, K. Howe and J. Kearney, arXiv:1603.03772 [hep-ph].
[13] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, Phys. Rept. 516, 1 (2012).
[14] A. Pich and P. Tuzon, Phys. Rev. D 80, 091702 (2009).
[15] R. Barbieri, L. J. Hall and V. S. Rychkov, Phys. Rev. D 74, 015007 (2006).
[16] N. Blinov, J. Kozaczuk, D. E. Morrissey and A. de la Puente, Phys. Rev. D 93, no. 3, 035020 (2016).
[17] H. C. Cheng, S. Jung, E. Salvioni and Y. Tsai, JHEP 1603, 074 (2016).
[18] N. Craig and A. Katz, JCAP 1510, no. 10, 054 (2015); I. Garcia Garca, R. Lasenby and J. March-Russell, Phys. Rev. D 92, no. 5, 055034 (2015); Phys. Rev. Lett. 115, no. 12, 121801 (2015).
[19] P. A. R. Ade et al. [Planck Collaboration], arXiv:1502.01589 [astro-ph.CO].
[20] J. H. Yu, in preparation.
[21] G. Aad et al. [ATLAS Collaboration], Eur. Phys. J. C 76, no. 1, 6 (2016); V. Khachatryan et al. [CMS Collaboration], Eur. Phys. J. C 75, no. 5, 212 (2015).
[22] ATLAS Collaboration, ATL-PHYS-PUB-2013-014, CERN, Geneva, Oct, 2013.