\( \Theta^+ \) in a conventional correlated perturbative chiral quark model (CPxQM)

A. R. Haghpayma

Department of Physics, Ferdowsi University of Mashhad
Mashhad, Iran

Abstract

Assuming a conventional correlated perturbative chiral quark model (CPxQM), we suggest that the \( \Theta^+ \) baryon is a bound state of two vector diquarks and a single antiquark, the spatial wave function of these diquarks has a \( P^- \) wave and a \( S^- \) wave in the angular momentum in the first and second version of our model respectively, as the result of these considerations we construct the orbital - colour - flavour - spin symmetry of \( Q\bar{Q} \) contribution of quarks and by imposing considerations such as HF interactions and suitable diquark currents we leads to a conventional pentaquark interpolating field which may be suitable for QSR analysis. In order to get an idea of the general features of the spectrum we have calculated the mass spectrum of exotic pentaquark states with the Gürsey Radicati mass formula.

Pentaquark baryons may be pure exotic or Crypto- exotic the pure exotic states can easily be identified by their unique quantum numbers, but the crypto- exotic states are hard to be identified as their quantum numbers can also be generated by three - quark states, therefore, it is crucial to have careful analyses for their decay channels.

Quark models have provided a cornerstone for hadron physics and one can studying the structure of pentaquark baryons in a naive constituent quark model \((^{6S}_{15})\).

In Ref \([2]\) barliner and lipkin suggested a triquark - diquark model where for example, \( \phi^+ \) is a system of \( (u\bar{u}) + (d\bar{d}) \).

In Ref \([3]\) jaff and wiczek presented a diquark - diquark- antiquark model so \( \phi^+ \) is \( (u\bar{u}) - (d\bar{d}) - (\bar{s}\bar{s}) \), in this model they also considered the mixing of the pentaquark antidecuplet with the pentaquark octet, which makes it different from the \( SU(3) \) soliton models where the octet describes the normal three - quark baryon octet.

More predictions based on quark models can be found in Ref \([1]\).

In order to investigate mass, width, reaction channels and other properties of pentaquarks a set of its wave functions in quark model is required \((^{6S}_{15})\).

By introducing the quark and antiquark operators and by taking direct product of two diquarks and one antiquark the \( \phi^+ \) state can be represented by \( SU(3) \) tensors.

The \( SU(3) \) symmetric lagrangian for pentaquark baryons and their interactions with other multiplets can be constructed by imposing dynamical interactions between quarks, the symmetry breaking can be included.

We denote a quark with \( q \), and antiquark with \( \bar{q} \) in which \( i = 1, 2, 3 \) denote u, d, s and impose normalization relations as:
\[
\langle q_i, q_j | q_k, q_l \rangle = \delta_{ij} \delta_{kl}.
\]

We consider a tensor \( T^{(2)}_{ij, kl} \) which is completely symmetric in upper and lower indices and traceless on every pair of indices.

We introduce \( S_{ij} = \frac{1}{\sqrt{2}}\left(\delta_{ij} + \delta_{ji}\right) \) and \( A_{ij} = \frac{1}{\sqrt{3}}\left(\delta_{ij} - \delta_{ji}\right) \) then for a quark and an antiquark we would have:

\[
\begin{align*}
\text{quark: } & T_i = i\delta_{ij}A_{jk}, \quad i, j, k = 1, 2, 3 \\
\text{antiquark: } & T^\prime = i\delta_{ij}A_{jk}, \quad i, j, k = 1, 2, 3
\end{align*}
\]

for \( q^\prime \) we have:

\[
(3 \otimes 3 \otimes 3 \otimes 3) = (6 \otimes \bar{3}) \otimes \bar{6} = (6 \otimes 6) \otimes (\bar{3} \otimes 3) - (15, 15) \otimes (\bar{3} \otimes 3)
\]

and for \( q^\prime \) we have:

\[
(3 \otimes 3 \otimes 3 \otimes 3) = (6 \otimes 6) \otimes (\bar{3} \otimes 3) = (6 \otimes 6) \otimes (\bar{3} \otimes 3)
\]

The tensor notation of all of these multiplets can be constructed as \( T_i \) and \( T^\prime_i \).

For example the tensor notations for 10 and 8 are:

\[
\begin{align*}
10: &\quad T^{10}_{ij} = \frac{\epsilon^1_1}{\sqrt{6}}(S^{(1)}_{ij} + S^{(4)}_{ij} + \sqrt{2}S^{(8)}_{ij}) + \frac{\epsilon^2_1}{\sqrt{3}}(T^{(1)}_{ij} + T^{(8)}_{ij} + T^{(10)}_{ij}), \\
8: &\quad T^{8}_{ij} = \frac{\epsilon^1_1}{\sqrt{6}}(S^{(1)}_{ij} - \frac{1}{\sqrt{2}}S^{(8)}_{ij}) + \frac{\epsilon^2_1}{\sqrt{3}}(T^{(1)}_{ij} - \frac{1}{\sqrt{2}}T^{(8)}_{ij} + T^{(10)}_{ij}) + \frac{\epsilon^1_1}{\sqrt{6}}(S^{(1)}_{ij} - \frac{1}{\sqrt{2}}S^{(8)}_{ij}) + \frac{\epsilon^2_1}{\sqrt{3}}(T^{(1)}_{ij} - \frac{1}{\sqrt{2}}T^{(8)}_{ij} + T^{(10)}_{ij})
\end{align*}
\]

The nomenclature for pentaquark states based on hypercharge is as follow:

\[
\begin{align*}
Y = -2, &\quad \Theta = 0, \quad \Theta = \Sigma, \quad Y = -2, \quad \Xi, \\
Y = 1, &\quad N, \Delta = -1 \quad \Xi, \quad Y = -3, \quad \Omega
\end{align*}
\]

\[\text{e-mail: haghpeima@wali.um.ac.ir}\]
which in the notation of the 

\[ Y = p_1 - q_1 + p_2 - q_2 + \frac{1}{2}(p - q) \]

\[ l_1 = \frac{1}{2}(p_1 - q_1 - q_2), \quad p_1 + p_2 = p \text{ and } q_1 + q_2 = q \]

\[ Q = l_1 + Y/2 \]

We can construct the SU(3) symmetry lagrangians for example:

\[ L_{\text{SU}(3)} = \partial \cdot \theta + \frac{1}{8} \partial \cdot M \]

And mass terms, for example:

\[ h_k = -\frac{1}{3} \partial \theta Y \partial Y \]

\[ h_Q = -\frac{1}{2} \partial \cdot M \]

for pentaquark octet and antidecuplet respectively.

The pentaquark wave function contains contributions connected to the spatial degrees of freedom and the internal degrees of freedom of color, flavour and spin, the pentaquark wave function should be a color singlet as all physical states, and should be antisymmetric under permutation of the four quarks (pauli principle). In order to classify quark and antiquark in SU(3), SU(2) we introduce the notations:

\[ SU_6(6) \supset SU_3(3) \otimes SU_2(2) \]

\[ \text{quark} \quad [1] \supset [1] \otimes [1] \]

\[ \text{antiquark} \quad [1111] \supset [11] \otimes [1] \]

Thus we can take the outer product of quark and antiquark representations we have:

\[ \text{For } q^2 SU(6) \quad [1] \otimes [1] \]

\[ \text{For } q^2 SU(6) \quad [1] \otimes [1] \]

One can decompose these multiplets:

| Spin-flavour decomposition of \( q^4 \) states |
| --- |
| \( D_3 \) | \( T_3 \) | \( SU_6(6) \supset SU_3(3) \otimes SU_2(2) \) |
| \( A_1 \) | \( A_1 \) | \( [1111] \) | \( [4] \) | \( [4] \) | \( [4] \) | \( [31] \) |
| \( A_1 + E \) | \( F_2 \) | \( [31] \) | \( [31] \) | \( [31] \) | \( [4] \) |
| \( E + A_2 \) | \( F_1 \) | \( [22] \) | \( [22] \) | \( [22] \) | \( [22] \) | \( [31] \) |
| \( A_2 \) | \( A_2 \) | \( [211] \) | \( [211] \) | \( [211] \) | \( [211] \) | \( [31] \) |

For \( q \ q \) SU(6) :

\[ [1] \otimes [1] \otimes [1] \otimes [1] \otimes [1111] \]

For \( q^2 SU(3) \) :

\[ [1] \otimes [1] \otimes [1] \otimes [1] \otimes [11] \]

Thus the total orbital - color - flavor - spin wave function of \( q \) is

\[ [1111] \]

For the explicit pentaquark wave functions see Ref [8].

This results are in agreement with the reduction of the color-spin SU(6) algebra of Ref [7].

The spin-flavour part has to be combined with the color part and orbital part in such a way that the total pentaquark wave function is a \( [221] \) color singlet state, and that the four quarks obey the pauli principle, i.e. are antisymmetric under any permutation of the four quarks, because of the color wave function of an antiquark that is a \( [11] \) anti-triplet, the color wave function of the four-quark configuration is a \( [211] \) triplet, the total \( q^4 \) wave function is antisymmetric, hence the orbital - spin - flavour part is a \( [31] \) state which is obtained from the color part by interchanging rows and columns, now if 4 - quarks are in a p - wave state, there are several allowed SU(3) representations which are:

\[ [4], [31], [22], [21] \]

Thus the total orbital - color - flavor - spin wave function of \( q \) is

\[ [1111] \]

For the explicit pentaquark wave functions see Ref [8].

Now we can impose an specific dynamical model to this general constituent quark model.

Assuming a conventional correlated perturbative chiral quark model (CPQG), we suggest that the \( q^4 \) baryon is a bound state of two diquarks and a single antiquark, the spatially wave function of these diquarks has a p-wave and a s-wave in angular momentum in the first and second version of our model respectively.

The \( [2] \) flavour symmetry of each diquark leads to \( [22] \) flavour symmetry, the \( [2] \) spatial symmetry of each diquark leads to \( [211] \) and \( [31] \) spin symmetry for \( q^4 \) in the first and second version of our model respectively. Thus the color symmetry of each diquark is \( [11] \) and for the first version of our model we assume \( [211] \) for one of the diquark pairs this leads to \( [211] \) color symmetry for \( q^4 \). The orbital symmetry of each diquark is \( [2] \) and for the first version we assume \( [31] \) for one of the diquark pairs, this leads to \( [31] \) and \( [4] \) orbital symmetry for \( q^4 \) in the first and second version of our model respectively.

As a result of these considerations we would have for \( q^4 \) contribution of quarks \( [1111] \) and \( [4] \) and for \( q^4 \) for \( q^4 \) the same for two versions and lead to a totally antisymmetric wave function for \( q^4 \) due to pauli principle.

The flavour symmetry contribution for \( q^4 \) configuration comes from the decomposition formula \( (2) \) for \( q^4 \) in which we have two \( n_1 \), the second one is used in J W model and the first one is used in our model due to vector diquark contribution of it.

Thus the tensor notations for 10 in our model comes from Eq (3) with \( C_1 = 0 \) and \( C_2 = 0 \) also the tensor notations for 8 in our model comes from Eq (4) with \( C_1 = 0 \) and \( C_2 = 0 \).

Thus inserting 10 tensor notation into \( L_m \), symmetry lagrangian leads to SU(3) symmetry interactions which experimental evidence of them would be explored.

Moreover inserting 8 and 10 tensor notations into conventional quark model mass formulas (6) and (7) respectively leads to 10, and 8 mass terms and spacing rules for the introduced particles in these multiplets in terms of experimental parameters (a, b, and c) of model.

In our model the diquarks are in 6 and 3 symmetry configurations and the hyperfine interactions (color - spin and flavour - spin) leads to a diquark mass that is greater than the masses which predicted in the models that the diquarks are in 3, and 3 symmetry configurations.
Thus the difference between the mass of a diquark and an antiquark in our model is larger than the models which based on scalar diquark instead of vector ones, and thus leads to the current experimental perspective in which no one has found the supersymmetry partners of pentaquark states.

In our model we have used diquark ideas in the chiral limit diquark correlations in the relativistic region, the first version of model yields the positive parity for pentaquarks and this is in agreement with lattice calculations in which there are attempts to find that states which have the maximum overlap with the positive parity.

In addition to usual HF interactions between quarks in a diquark, these interactions exist between the antiquark $\bar{s}$ and each of the diquarks due to vector configuration of them in our model, these interactions are absent in the scalar diquark models in which there is only electromagnetic interaction between antiquark and each diquark in a pentaquark.

By imposing considerations such as these HF interactions and suitable diquark currents we leads to a conventional pentaquark interpolating field which may be used for QSR analysis.

For example for a diquark with $|2\bar{s}\rangle$ and $|2\bar{s}\rangle$ spin - flavour configurations and $\ell = 0$ angular momentum one may use:

$$V^{(\alpha)} = \sum_{\alpha \beta \gamma \delta} \left[ \begin{array}{c} a'(\alpha)C_{\beta}(x)C_{\delta}(x) \end{array} \right] \left[ \begin{array}{c} u'\beta(x)u\gamma(x) \end{array} \right]$$

and if the diquark angular momentum is $\ell = 1$ one can use:

$$V^{(\alpha)} = \sum_{\alpha \beta \gamma \delta} \left[ \begin{array}{c} a'(\alpha)C_{\beta}(x)C_{\delta}(x) \end{array} \right] \left[ \begin{array}{c} u'\beta(x)C_{\gamma}(x) \end{array} \right]$$

as diquark currents.

Thus for the first and second version of our model we can consider the following interpolating fields respectively.

$$I_1 = \sum_{\alpha \beta \gamma \delta} \left[ \begin{array}{c} a'(\alpha)C_{\beta}(x)C_{\delta}(x) \end{array} \right] \left[ \begin{array}{c} u'\beta(x)C_{\gamma}(x) \end{array} \right]$$

$$I_2 = \sum_{\alpha \beta \gamma \delta} \left[ \begin{array}{c} a'(\alpha)C_{\beta}(x)C_{\delta}(x) \end{array} \right] \left[ \begin{array}{c} u'\beta(x)C_{\gamma}(x) \end{array} \right]$$

Considering flavour configuration of $q^3$, one can see that there are 15 and 15 multiplets which comes from 6 × 6 normal products and this leads to two 45 multiplets for flavour configurations of $q^5$ in which there is octet, decuplet, 27plet and 35plet.

The tensor notations of 15 and 15 are $T_{\text{15},\text{15}}$ and $S_{\text{15},\text{15}}$ respectively.

$$T_{\text{15},\text{15}} = \sum_{\alpha \beta \gamma \delta} \left[ \begin{array}{c} a'(\alpha)S_{\beta\gamma\delta}(x)S_{\alpha\beta\gamma\delta}(x) \end{array} \right]$$

$$S_{\text{15},\text{15}} = \sum_{\alpha \beta \gamma \delta} \left[ \begin{array}{c} a'(\alpha)S_{\beta\gamma\delta}(x)S_{\alpha\beta\gamma\delta}(x) \end{array} \right]$$

Thus by multiplying to tensor notation of $q^5$ we leads to tensor notations of $(8, 10, 27, 35)$ plets.

In fact there is nothing in quark model to prevent us for constructing such multiplets in which they have vector diquarks and the discovery of them will be evidence supporting the diquark model.

Briefly the spin - flavour - color and parity of our model for the first version and second one are as follows:

$$|QQ\bar{s}\rangle = |2\bar{s}\rangle + |2\bar{s}\rangle + |2\bar{s}\rangle$$

$$|QQ\bar{s}\rangle = |2\bar{s}\rangle + |2\bar{s}\rangle + |2\bar{s}\rangle$$

We have considered $|4\bar{s}\rangle$ and $|31\bar{s}\rangle$ for the flavour - spin configurations of $q^5$ in the first and second version of our model respectively, this leads to $|51111\rangle$ and $|42211\rangle$ for the flavour - spin configurations of $q^5$, but if one assume the angular momentum $\ell = 1$ for the four quarks $\bar{s}$ there are several allowed SU(6) representations for $q^5$ which are $|51111\rangle$ $|42211\rangle$ $|33311\rangle$ $|32221\rangle$ based on $|4\bar{s}\rangle$, $|31\bar{s}\rangle$, $|22\bar{s}\rangle$ and $|21\bar{s}\rangle$ SU(6) representations for $q^5$ respectivley.

In order to get an idea of the general features of the spectrum we have calculated the mass spectrum of exotic pentaquark states with the Gürsey - Radicati mass formula $\Xi$:

$$M = M_0 + M_{\text{orb}} + M_{\text{orb}}$$

where $M_0 = -AC_{\text{SU}(6)} + B_{\text{SU}(6)} + C(s + 1) + DY + E[(H + 1) - 1\frac{1}{2}]$.

The first two terms represent the quadratic Casimir operators of SU(6) spin - flavour and SU(3) flavour groups and $\Xi$, $Y$, and $E$ denote the spin, hypercharge and isospin respectively.

The last two terms in Eq. (19) correspond to the Gell-Mann - Okubo mass formula that describes the splitting with in a flavour multiplet.

Neglecting the hyperfine interaction radial dependence the matrix element of these interactions depends on the Casimirs of the SU(6) and SU(3) and the SU(2) groups, also the spatial dependence of the SU(6) breaking part has been neglected in Eq. (20).

The coefficients $B$, $C$, $D$ and $E$ are determined from the three - quark spectrum [3]:

$$B = 21.2\text{MeV}$$

$$C = 38.3\text{MeV}$$

$$D = 197.3\text{MeV}$$

$$E = 38.5\text{MeV}$$

The eigenvalues of the Casimirs for the qq or qqqq systems are as follows:

|Spin-flavour $C_{\text{SU}(6)}$ flavour $C_{\text{SU}(3)}$| Eigenvalues |
|---|---|
|51111| 81/4 | 51111 | 12 |
|41111| 45/4 | 41111 | 8 |
|42111| 65/4 | 43111 | 6 |
|32221| 33/4 | 41111 | 6 |
|33311| 57/4 | 33311 | 3 |
|32221| 49/4 | 32221 | 0 |
|22221| 21/4 | 22221 | 0 |
|22221| 33/4 | 22221 | 0 |

If we neglect $M_{\text{orb}}$ and $AC_{\text{SU}(6)}$ in Eq. (19) and using $M_0$ in order to normalize the energy scale to the observed mass of the $\eta'$ (1540).

We find that a flavour anti-decuplet [33] state with spin S=1/2 and isospin I = 0 which has $|4\bar{s}\rangle$ flavour - spin configuration based on q | 22 $\rangle$ flavour and $|22\rangle$ spin configuration and lead to $|51111\rangle$ for qq which we have introduced in the first version of our model is the lowest pentaquark state.

The state is in agreement with the available experimental data which indicate that the $\Theta^+$ (1540) is an isosinglet, this means that the contributions of $M_{\text{orb}}$ and $AC_{\text{SU}(6)}$ SU(6) terms in Eq. (19)must be negligible and this is a good test for the values of exciting energy of the orbital degrees of freedom for q and the underlying dynamical structure.

However, one can see by neglecting this two terms the mass spectrum of a $|51111\rangle$ state with spin S=1/2 and isospin I = 0 which has $|4\bar{s}\rangle$ flavour - spin configuration based on q | 22 $\rangle$ flavour and $|22\rangle$ spin configuration and lead to $|51111\rangle$ for qq which we have introduced in the first version of our model is the lowest pentaquark state.

The state is in agreement with the available experimental data which indicate that the $\Theta^+$ (1540) is an isosinglet, this means that the contributions of $M_{\text{orb}}$ and $AC_{\text{SU}(6)}$ SU(6) terms in Eq. (19)must be negligible and this is a good test for the values of exciting energy of the orbital degrees of freedom for q and the underlying dynamical structure.

The hyperfine interactions between two quarks in diquarks and the confinement between quarks are the underlying interactions which have ignored in this general study of the structure of the spectrum.

We have mentioned that one find $(8, 10, 27, 35)$ plets by decomposition of 45 multiplets for flavour configurations of $q^5$.

The SU(3) configurations of these multiplets are $|321\rangle$, $|411\rangle$, $|42\rangle$ and $|51\rangle$ and the SU(6) configurations of them are:

- $|33311\rangle$ $|32211\rangle$, $|41111\rangle$, and $|51111\rangle$. 

- $|33111\rangle$, $|32211\rangle$, $|41111\rangle$, and $|51111\rangle$. 

- $|33111\rangle$, $|32211\rangle$, $|41111\rangle$, and $|51111\rangle$. 

- $|33111\rangle$, $|32211\rangle$, $|41111\rangle$, and $|51111\rangle$. 

We do not give the full list of these pentaquark masses and one can read them directly from the results presented in Tab. 1 for $C_{2W}(Q)$ and $C_{3W}(R)$ and by finding their quantum numbers according to tensor notations of them and using of Eq (20) for their masses.

However, observation of other pentaquark states in higher multiplets will help us to understand the structure of pentaquark baryons: diquark-quark-antiquark models.

References

[1] R. Bijker, M. M. Giannini, and E. Santopinto, hep-ph/0310281.
Fl. Stancu and D. O. Riska, Phys. Lett. B 575, 242 (2003).
C. E. Carlson, C. D. Carone, H. J. Kwee, and V. Nazaryan, Phys. Lett. B 579, 101 (2004).
S. M. Gerasyuta and V. I. Kochkin, hep-ph/0310227.
N. I. Kochev, H. J. Lee, and V. Vento, hep-ph/0404055.
[2] M. Karliner and H. J. Lipkin, arXiv:hep-ph/0307243.
[3] R. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003).
[4] F. E. Close and J. J. Dudek, Phys. Lett. B 586, 75 (2004).
S. H. Lee, H. Kim, and Y. Oh, hep-ph/0402135.
D. Diakonov and V. Petrov, hep-ph/0310212.
R. Bijker, M. M. Giannini, and E. Santopinto, hep-ph/0310281.
Fl. Stancu and D. O. Riska, Phys. Lett. B 575, 242 (2003).
C. E. Carlson, C. D. Carone, H. J. Kwee, and V. Nazaryan, Phys. Lett. B 575, 101 (2004).
[5] Yongseok Oh et al, arXiv:hep-ph/0405010.
[6] F. E. Low, Symmetries and Elementary Particles, (Gordon and Breach, New York, 1967).
[7] H. Høgaasen and P. Sorba, Nucl. Phys. B 145, 119 (1978); M. de Crombrugghe, H. Høgaasen and P. Sorba, Nucl. Phys. B 156, 347 (1979).
[8] R. Bijker, M. M. Giannini and E. Santopinto, in preparation.
E. Santopinto, F. Iachello and M. M. Giannini, Eur. Phys. J. A, 307 (1998).
[9] F. G. Gursey and L. A. Radicati, Phys. Rev. Lett. 13, 173 (1964).
R. Bijker, F. Iachello and A. Leviatan, Ann. Phys. (N.Y) 236, 69 (1994).
R. Bijker, F. Iachello and A. Leviatan, Ann. Phys. (N.Y) 284, 89 (2000).
[10] C. Helmimen and D. O. Riska, Nucl. Phys. A 699, 624 (2002).
[11] B. K. Jennings and K. Maltman, hep-ph/0308286.
[12] M. M. Giannini, E. Santopinto and A. Vassallo, to be published.