The circle electromagnetic pulsar

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Abstract
The power spectrum formula of the synchrotron radiation generated by the electron and positron moving along the concentric circles at the opposite angular velocities in homogeneous magnetic field is derived in the Schwinger version of quantum electrodynamics. The asymptotical form of this formula is found. The spectrum depends periodically on time which means that the system composed from electron, positron and magnetic field forms the pulsating system. The similar calculations are performed in the case that the circles of motion are excentric.

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1 Introduction
The production of photons by circular motion of charged particle in accelerator is one of the most interesting problems in the classical and quantum electrodynamics.

In this paper we are interested in the synergic photon production initiated by the circular motion of electron and positron in the homogenous magnetic field. It is supposed that electron and positron are moving at the opposite angular velocities. This process is the generalization of the one-charge synergic synchrotron-Čerenkov radiation which has been calculated in source theory two decades ago by Schwinger et al. [1]. We will follow also the article [2] as the starting point. Although our final problem is the radiation of the two-charge system in vacuum, we consider, first in general, the presence of dielectric medium, which is represented by the phenomenological index of refraction $n$. Introducing the phenomenological constant enables to consider also the Čerenkovian processes.

We will investigate here how the original Schwinger et al. spectral formula of the synergic synchrotron-Čerenkov radiation of the charged particle is modified if the electron and positron are moving along the concentric and excentric circles at the opposite angular velocities. This problem is an analogue of the linear problem solved recently by author [3] also in source theory.

We apply the Schwinger methods [4–6] which was initially used in a description of the particle physics situations occurring in high-energy physics experiments. It enables
simplification of the calculations in the electrodynamics and gravity where the interactions are mediated by the photon or graviton, respectively. It simplifies particularly the calculations with radiative corrections [6,7].

2 Formulation of a problem

The basic formula of the Schwinger source theory is the so called vacuum to vacuum amplitude:

$$\langle 0_+|0_- \rangle = e^{\frac{i}{\hbar}W}, \tag{1}$$

where in case of the electromagnetic field in the medium, the action $W$ is given by the following formula:

$$W = \frac{1}{2c^2} \int (dx)(dx')J^\mu(x)D_{+\mu\nu}(x-x')J^\nu(x'), \tag{2}$$

where

$$D^{\mu\nu}_+(x-x') = \frac{\mu}{c}[g^{\mu\nu} + (1-n^{-2})\beta^\mu\beta^\nu]D_+(x-x'), \tag{3}$$

where $\beta^\mu \equiv (1,0), J^\mu \equiv (c\varrho,J)$ is the conserved current, $\mu$ is the magnetic permeability of the medium, $\varepsilon$ is the dielectric constant of the medium and $n = \sqrt{\varepsilon\mu}$ is the index of refraction of the medium. Function $D_+$ is defined as follows [1]:

$$D_+(x-x') = \frac{i}{4\pi^2c} \int_0^\infty d\omega \frac{\sin n\omega}{|x-x'|} e^{-i\omega|t-t'|}. \tag{4}$$

The probability of the persistence of vacuum follows from the vacuum amplitude (1) in the following form:

$$|\langle 0_+|0_- \rangle|^2 = e^{-\frac{2}{\hbar}\text{Im}W}, \tag{5}$$

where $\text{Im} W$ is the basis for the definition of the spectral function $P(\omega, t)$ as follows:

$$-\frac{2}{\hbar}\text{Im} W \overset{d}{=} - \int dt d\omega \frac{P(\omega, t)}{\hbar\omega}. \tag{6}$$

Now, if we insert eq. (2) into eq. (6), we get after extracting $P(\omega, t)$ the following general expression for this spectral function:

$$P(\omega, t) = -\frac{\omega}{4\pi^2n^2} \int dx dx'dt' \left[ \frac{\sin n\omega}{x-x'} \right] \times \cos[\omega(t-t')] \left[ g(x, t)g(x', t') - \frac{n^2}{c^2}J(x, t) \cdot J(x', t') \right]. \tag{7}$$

Let us recall that the last formula can be derived also in the classical electrodynamical context as it is shown for instance in the Schwinger article [8]. The derivation of the power spectral formula from the vacuum amplitude is more simple.
The power spectral formula of motion of opposite charges

Now, we will apply the formula (7) to the two-body system with the opposite charges moving at the opposite angular velocities in order to get in general synergic synchrotron-Čerenkov radiation of electron and positron moving in a uniform magnetic field.

While the synchrotron radiation is generated in a vacuum, the synergic synchrotron-Čerenkov radiation can produced only in a medium with dielectric constant $n$. We suppose the circular motion with velocity $v$ in the plane perpendicular to the direction of the constant magnetic field $H$ (chosen to be in the $+z$ direction).

We can write the following formulas for the charge density $\varrho$ and for the current density $J$ of the two-body system with opposite charges and opposite angular velocities:

\[
\varrho(x, t) = e\delta(x - x_1(t)) - e\delta(x - x_2(t))
\]

and

\[
J(x, t) = ev_1(t)\delta(x - x_1(t)) - ev_2(t)\delta(x - x_2(t))
\]

with

\[
x_1(t) = x(t) = R(i\cos(\omega_0 t) + j\sin(\omega_0 t)),
\]

\[
x_2(t) = R(i\cos(-\omega_0 t) + j\sin(-\omega_0 t)) = x(-\omega_0 t) = x(-t).
\]

The absolute values of velocities of both particles are the same, or $|v_1(t)| = |v_2(t)| = v$, where $(H = |H|, E = \text{energy of a particle})$

\[
v(t) = dx/dt, \quad \omega_0 = v/R, \quad R = \frac{\beta E}{eH}, \quad \beta = v/c, \quad v = |v|.
\]

After insertion of eqs. (8)-(9) into eq. (7), and after some mathematical operations we get

\[
P(\omega, t) = -\frac{\omega}{4\pi^2} \frac{\mu}{n_e^2} \frac{e^2}{2} \int_{-\infty}^{\infty} dt' \cos(t - t') \sum_{i,j=1}^{2} (-1)^{i+j} \left[ 1 - \frac{v_i(t) \cdot v_j(t')}{c^2 n^2} \right] \times
\]

\[
\left\{ \frac{\sin \frac{n_0}{e} |x_i(t) - x_j(t')|}{|x_i(t) - x_j(t')|} \right\}.
\]

Using $t' = t + \tau$, we get for

\[
x_i(t) - x_j(t') \overset{d}{=} A_{ij},
\]

\[
|A_{ij}| = [R^2 + R^2 - 2RR \cos(\omega_0 \tau + \alpha_{ij})]^{1/2} = 2R \left| \sin \left( \frac{\omega_0 \tau + \alpha_{ij}}{2} \right) \right|,
\]
where \( \alpha_{ij} \) were evaluated as follows:

\[
\begin{align*}
\alpha_{11} &= 0, & \alpha_{12} &= 2\omega_0 t, & \alpha_{21} &= 2\omega_0 t, & \alpha_{22} &= 0.
\end{align*}
\] (16)

Using

\[
\mathbf{v}_i(t) \cdot \mathbf{v}_j(t + \tau) = \omega_0^2 R^2 \cos(\omega_0 \tau + \alpha_{ij}),
\] (17)

and relation (15) we get with \( v = \omega_0 R \)

\[
P(\omega, t) = -\frac{\omega}{4\pi^2 n^2} \mu e^2 \int_{-\infty}^{\infty} d\tau \cos \omega \tau \sum_{i,j=1}^{2} (-1)^{i+j} \left[ 1 - \frac{n^2}{c^2} v^2 \cos(\omega_0 \tau + \alpha_{ij}) \right] \times \frac{1}{2R} \left\{ \sin \left( \frac{2R \omega t + \alpha_{ij}}{c} \right) \right\}.
\] (18)

Introducing new variable \( T \) by relation

\[
\omega_0 \tau + \alpha_{ij} = \omega_0 T
\] (19)

for every integral in eq. (18), we get \( P(\omega, t) \) in the following form

\[
P(\omega, t) = -\frac{\omega}{4\pi^2 n^2} \mu e^2 \frac{\mu}{2R n^2} \int_{-\infty}^{\infty} dT \sum_{i,j=1}^{2} (-1)^{i+j} \cos(\omega T) \left\{ \frac{1}{2R} \left[ \sin \left( \frac{2R \omega T + \alpha_{ij}}{c} \right) \right] \right\}.
\] (20)

The last formula can be written in the more compact form,

\[
P(\omega, t) = -\frac{\omega}{4\pi^2 n^2} \mu e^2 \frac{\mu}{2R} \sum_{i,j=1}^{2} (-1)^{i+j} \left\{ P_{1(ij)} - \frac{n^2}{c^2} v^2 P_{2(ij)} \right\},
\] (21)

where

\[
P_{1(ij)} = J_{1a}^{(ij)} \cos \frac{\omega}{\omega_0} \alpha_{ij} + J_{1b}^{(ij)} \sin \frac{\omega}{\omega_0} \alpha_{ij},
\] (22)

and

\[
P_{2(ij)} = J_{2A}^{(ij)} \cos \frac{\omega}{\omega_0} \alpha_{ij} + J_{2B}^{(ij)} \sin \frac{\omega}{\omega_0} \alpha_{ij},
\] (23)

where

\[
J_{1a}^{(ij)} = \int_{-\infty}^{\infty} dT \cos \omega T \left\{ \frac{1}{2R} \sin \left( \frac{2R \omega T + \alpha_{ij}}{c} \right) \right\}.
\] (24)
\[ J_{1b}^{(ij)} = \int_{-\infty}^{\infty} dT \sin \omega T \left\{ \frac{\sin \left( \frac{2Rn \omega}{c} \sin \left( \frac{\omega T}{2} \right) \right)}{\sin \left( \frac{\omega T}{2} \right)} \right\}; \quad (25) \]

\[ J_{2A}^{(ij)} = \int_{-\infty}^{\infty} dT \cos \omega_0 T \cos \omega T \left\{ \frac{\sin \left( \frac{2Rn \omega}{c} \sin \left( \frac{\omega T}{2} \right) \right)}{\sin \left( \frac{\omega T}{2} \right)} \right\}; \quad (26) \]

\[ J_{2B}^{(ij)} = \int_{-\infty}^{\infty} dT \cos \omega_0 T \sin \omega T \left\{ \frac{\sin \left( \frac{2Rn \omega}{c} \sin \left( \frac{\omega T}{2} \right) \right)}{\sin \left( \frac{\omega T}{2} \right)} \right\}; \quad (27) \]

Using

\[ \omega T = \phi + 2\pi l, \quad \phi \in (-\pi, \pi), \quad l = 0, \pm 1, \pm 2, \ldots; \quad (28) \]

we can transform the \( T \)-integral into the sum of the telescopic integrals according to the scheme:

\[ \int_{-\infty}^{\infty} dT \rightarrow \frac{1}{\omega_0} \sum_{l=-\infty}^{l=\infty} \int_{-\pi}^{\pi} d\phi. \quad (29) \]

Using the fact that for the odd functions \( f(\phi) \) and \( g(l) \), the relations are valid

\[ \int_{-\pi}^{\pi} f(\phi) d\phi = 0; \quad \sum_{l=-\infty}^{l=\infty} g(l) = 0, \quad (30) \]

we can write

\[ J_{1a}^{(ij)} = \frac{1}{\omega_0} \sum_{l} \int_{-\pi}^{\pi} d\phi \left\{ \cos \frac{\omega}{\omega_0} \phi \cos 2\pi l \frac{\omega}{\omega_0} \right\} \left\{ \frac{\sin \left( \frac{2Rn \omega}{c} \sin \left( \frac{\phi}{2} \right) \right)}{\sin \left( \frac{\phi}{2} \right)} \right\}; \quad (31) \]

\[ J_{1b}^{(ij)} = 0. \quad (32) \]

For integrals with indices A, B we get:

\[ J_{2A}^{(ij)} = \frac{1}{\omega_0} \sum_{l} \int_{-\pi}^{\pi} d\phi \cos \phi \left\{ \cos \frac{\omega}{\omega_0} \phi \cos 2\pi l \frac{\omega}{\omega_0} \right\} \left\{ \frac{\sin \left( \frac{2Rn \omega}{c} \sin \left( \frac{\phi}{2} \right) \right)}{\sin \left( \frac{\phi}{2} \right)} \right\}; \quad (33) \]

\[ J_{2B}^{(ij)} = 0, \quad (34) \]

So, the power spectral formula (21) is of the form:

\[ P(\omega, t) = -\frac{\omega}{4\pi^2 n^2 2R} \sum_{i,j=1}^{2} (-1)^{i+j} \left\{ P_1^{(ij)} - n^2 \beta^2 P_2^{(ij)} \right\}; \quad \beta = \frac{v}{c}, \quad (35) \]

where

\[ P_1^{(ij)} = J_{1a}^{(ij)} \cos \frac{\omega}{\omega_0} \alpha_{ij}, \quad (36) \]
Using the Poisson theorem
\[
\sum_{l=-\infty}^{\infty} \cos 2\pi \frac{\omega}{\omega_0} l = \sum_{k=-\infty}^{\infty} \omega_0 \delta(\omega - \omega_0 l),
\]
we get for \(J_{1a}^{(ij)}\) and \(J_{2A}^{(ij)}\) \((z = 2\ln \beta)\):

\[
J_{1a}^{(ij)} = \sum_l \int_{-\pi}^\pi d\varphi \cos \varphi \cos l\varphi \left\{ \frac{\sin(z \sin(\varphi/2))}{\sin(\varphi/2)} \right\},
\]
\[
J_{2A}^{(ij)} = \sum_l \int_{-\pi}^\pi d\varphi \cos \varphi \sin l\varphi \left\{ \frac{\sin(z \sin(\varphi/2))}{\sin(\varphi/2)} \right\}.
\]

Using the definition of the Bessel functions \(J_{2l}\) and their corresponding derivation and integral

\[
\frac{1}{2\pi} \int_{-\pi}^\pi d\varphi \cos \left( z \sin \frac{\varphi}{2} \right) \cos l\varphi = J_{2l}(z),
\]
\[
\frac{1}{2\pi} \int_{-\pi}^\pi d\varphi \sin \left( z \sin \frac{\varphi}{2} \right) \cos l\varphi = -J_{2l}'(z),
\]
\[
\frac{1}{2\pi} \int_{-\pi}^\pi d\varphi \sin \left( z \sin \frac{\varphi}{2} \right) \sin(\varphi/2) \cos l\varphi = \int_0^z J_{2l}(x) dx,
\]
and using equations

\[
\sum_{i,j=1}^{2} (-1)^{i+j} \cos \frac{\omega}{\omega_0} \alpha_{ij} = 2(1 - \cos 2\omega t) = 4 \sin^2 \omega t,
\]

we get the following final form of the partial power spectrum generated by motion of two-charge system moving in the cyclotron:

\[
P_l(\omega, t) = \sum_{l=1}^{\infty} \delta(\omega - l\omega_0) P_l,
\]

\[
P_l(\omega, t) = [4(\sin \omega t)^2] \frac{e^2}{\pi n^2} \frac{\omega \mu \omega_0}{v} \left( 2n^2 \beta^2 J_{2l}(2\ln \beta) - (1 - n^2 \beta^2) \int_0^{2\ln \beta} dx J_{2l}(x) \right).
\]

So we see that the spectrum generated by the system of electron and positron is formed in such a way that the original synchrotron spectrum generated by electron is modulated by function \(4 \sin^2(\omega t)\). This formula is analogous to the formula derived in [3] for the linear motion of the two-charge system emitting the Čerenkov radiation. The derived
formula involves also the synergic process composed from the synchrotron radiation and the Čerenkov radiation for electron velocity \( v > c/n \) in a medium.

Our goal is to apply the last formula in situation where there is a vacuum. In this case we can put \( \mu = 1, n = 1 \) in the last formula and so we have

\[
P_l(\omega, t) = 4 \sin^2 (\omega t) \frac{e^2 \omega \omega_0}{v} \left( 2\beta^2 J_0'(2l\beta) - (1 - \beta^2) \int_0^{2l\beta} dx J_2(x) \right). \tag{47}
\]

So, we see, that final formula describing the opposite motion of electron and positron in accelerator is of the form

\[
P_l(\omega, t) = 4 \sin^2 (\omega t) P_{l(\text{electron})}(\omega), \tag{48}
\]

where \( P_{\text{electron}} \) is the spectrum of radiation only of electron. The result is suprising because we naively expected that the total radiation of the opposite charges should be

\[
P_l(\omega, t) = P_{l(\text{electron})}(\omega, t) + P_{l(\text{positron})}(\omega, t). \tag{49}
\]

So, we see that the resulting radiation can not be considerred as generated by the isolated particles but by a synergical production of a system of particles and magnetic field. At the same time we cannot interprete the result as a result of interference of two sources because the distance between sources radically changes and so, the condition of an interference is not fulfilled.

The classical electrodynamics is not broken by this formula but what is broken is our naive image on the processes in the magnetic field. From the last formula also follows that at time \( t = \pi k/\omega \) there is no radiation of the frequency \( \omega \). At every frequency \( \omega \) the spectrum oscillates with frequency \( \omega \). If the radiation were generated not synergically, then the spectral formula would be composed from two parts corresponding to two isolated sources.

### 4 The more realistic situation

The situation which we have analysed was the ideal situation where the angle of collision of positron and electron was equal to \( \pi \). Now, the question arises what is the modification of a spectral formula when the collision angle between particles differs from \( \pi \). The goal of this article is to formulate this problem using the above mathematical approach. It can be easily seen that if the second particle follows the shifted circle trajectory, than the collision angle differs from \( \pi \). Let us suppose that the the circle center of the second particle has coordinates \((a,0)\). It can be easy to see from the geometry of the situation and from the planimetry that the collision angle is \( \pi - \alpha, \alpha \approx \tan \alpha \approx a/R \) where \( R \) is a radius of the first or second circle. The same result follows from the analytical geometry of the situation.
While the equation of the first particle is the equation of the original trajectory, or this is eq. (10)

\[ x_1(t) = x(t) = R(i \cos(\omega_0 t) + j \sin(\omega_0 t)), \] (50)

the equation of a circle with a shifted center is as follows

\[ x_2(t) = x(t) = R(i(\frac{a}{R} + \cos(-\omega_0 t)) + j \sin(-\omega_0 t)) = x(-t) + ia, \] (51)

The absolute values of velocities of both particles are equal and the relation (12) is valid. Instead of equation (14) we have for radius vectors of particle trajectories:

\[ x_i(t) - x_j(t') = B_{ij}, \] (52)

where \( B_{11} = A_{11}, B_{12} = A_{12} - ia, B_{21} = A_{21} + ia, B_{22} = A_{22}. \)

In general, we can write the last information on coefficients \( B_{ij} \) as follows:

\[ B_{ij} = A_{ij} + \varepsilon_{ij}ia, \] (53)

where \( \varepsilon_{11} = 0, \varepsilon_{12} = -1, \varepsilon_{21} = 1, \varepsilon_{22} = 0. \)

For motion of particles along trajectories the absolute value of vector \( A_{ij} \gg a \) during the most part of the trajectory. It means, we can determine \( B_{ij} \) approximatively. After elementary operations, we get:

\[ |B_{ij}| = (A_{ij}^2 + 2|A_{ij}|\varepsilon_{ij}a \cos \varphi_{ij} + a^2\varepsilon_{ij}^2)^{1/2}, \] (54)

where \( \cos \varphi_{ij} \) can be expressed by the \( x \)-component of vector \( A_{ij} \) and \( |A_{ij}| \) as follows:

\[ \cos \varphi_{ij} = \frac{(A_{ij})_x}{|A_{ij}|} \] (55)

After elementary trigonometric operations, we derive the following formula \( (A_{ij})_x \).

\[ (A_{ij})_x = 2R \sin \frac{2\omega_0 t + \omega_0 \tau}{2} \sin \frac{\omega_0 \tau}{2} \] (56)

Then, using equation (56), we get with \( \varepsilon = a/R. \)

\[ |B_{ij}| = 2R(sin^2 \frac{\omega_0 \tau + \alpha_{ij}}{2} + \varepsilon \varepsilon_{ij} \sin \frac{2\omega_0 t + \omega_0 \tau}{2} \sin \frac{\omega_0 \tau}{2} + \varepsilon^2 \varepsilon_{ij}^2)^{1/2}, \] (57)

In order to perform the \( \tau \)-integration the substitution must be introduced. However, the substitution \( \omega_0 \tau + \alpha_{ij} = \omega_0 T \) does not work. So we define the substitution \( \tau = \tau(T) \) by the following transcendental equation (we will neglect the term with \( \varepsilon^2 \)):

\[ \left[ \sin^2 \frac{\omega_0 \tau + \alpha_{ij}}{2} + \varepsilon \varepsilon_{ij} \sin \frac{\omega_0 t + \omega_0 \tau}{2} \sin \frac{\omega_0 \tau}{2} \right]^{1/2} = \sin \frac{\omega_0 T}{2} \] (58)
Or, after some trigonometrical modifications and using the approximative formula 

\[(1 + x)^{1/2} \approx 1 + x/2 \text{ for } x \ll 1\]

\[\left(\frac{\omega_0 \tau + \alpha_{ij}}{2}\right) + \frac{\varepsilon}{2} \varepsilon_{ij} \sin \left(\frac{2\omega_0 \tau + 2\omega_0 t - \alpha_{ij}}{2}\right) = \sin \frac{\omega_0 T}{2}\]  

(59)

We see that for \(\varepsilon = 0\) the substitution is \(\omega_0 \tau + \alpha_{ij} = \omega_0 T\). The equation (58) is the transcendental equation and the exact solution is the function \(\tau = \tau(T)\). We are looking for the solution of equation (59) in the approximative form using the approximation \(\sin x \approx x\).

Then, instead of (59) we have:

\[\left(\frac{\omega_0 \tau + \alpha_{ij}}{2}\right) + \frac{\varepsilon}{2} \varepsilon_{ij} \left(\frac{2\omega_0 \tau + 2\omega_0 t - \alpha_{ij}}{2}\right) = \frac{\omega_0 T}{2}\]  

(60)

Using substitution

\[\omega_0 \tau + \alpha_{ij} = \omega_0 T + \omega_0 \varepsilon A\]  

(61)

in eq. (60) we get, to the first order in \(\varepsilon\)-term:

\[A = -\frac{\varepsilon_{ij}}{2\omega_0} (\omega_0 T - 2\alpha_{ij} + 2\omega_0 t)\]  

(62)

Then, after some algebraic manipulation we get:

\[\omega_0 \tau + \alpha_{ij} = \omega_0 T(1 - \varepsilon \varepsilon_{ij}) - \varepsilon \varepsilon_{ij} \omega_0 t(-1)^{i+j}\]  

(63)

and

\[\omega \tau = \omega T(1 - \varepsilon \varepsilon_{ij}) - \frac{\omega}{\omega_0} (\varepsilon \varepsilon_{ij}(-1)^{i+j} \omega_0 t + \alpha_{ij})\]  

(64)

For small time \(t\), we can write approximately:

\[\cos(\omega_0 \tau + \alpha_{ij}) \approx \cos \omega_0 T(1 - \frac{\varepsilon}{2} \varepsilon_{ij})\]  

(65)

and from eq. 64

\[d\tau = dT(1 - \frac{\varepsilon}{2} \varepsilon_{ij})\]  

(66)

So, in case of the excentric circles the formula (64) can be obtained from nonperturbative formula (20) only by transformation

\[T \rightarrow T(1 - \frac{\varepsilon}{2} \varepsilon_{ij}); \quad \alpha_{ij} \rightarrow (\varepsilon \varepsilon_{ij}(-1)^{i+j} \omega_0 t + \alpha_{ij}) = \tilde{\alpha}_{ij}\]  

(67)

excepting specific term involving sine functions.

Then, instead of formula (20) we get:
\[ P(\omega, t) = -\frac{\omega}{4\pi^2} \frac{e^2 \mu}{2R n^2} \int_{-\infty}^{\infty} dT \sum_{i,j=1}^{2} (-1)^{i+j} \times \]

\[ \cos(\omega T - \frac{\omega}{\omega_0} \tilde{\alpha}_{ij}) \left[ 1 - \frac{c^2}{n^2} v^2 \cos(\omega_0 T) \right] \left\{ \frac{\sin \left[ \frac{2R n \omega}{c} \sin \left( \frac{\omega_0 T}{2} \right) \right]}{\sin \left( \frac{\omega_0 T}{2} \right)} \right\} . \] (68)

where \( \tilde{T} = T(1 - \frac{\varepsilon}{2} \varepsilon_{ij}) \). We see that only \( \tilde{T} \) and the \( \alpha \) term are the new modification of the original formula (20).

However, because \( \varepsilon \) term in the sine functions is of very small influence on the behaviour of the total function for finite time \( t \), we can neglect it and write approximatively:

\[ P(\omega, t) = -\frac{\omega}{4\pi^2} \frac{e^2 \mu}{2R n^2} \int_{-\infty}^{\infty} dT \sum_{i,j=1}^{2} (-1)^{i+j} \times \]

\[ \cos(\omega T - \frac{\omega}{\omega_0} \tilde{\alpha}_{ij}) \left[ 1 - \frac{c^2}{n^2} v^2 \cos(\omega_0 T) \right] \left\{ \frac{\sin \left[ \frac{2R n \omega}{c} \sin \left( \frac{\omega_0 T}{2} \right) \right]}{\sin \left( \frac{\omega_0 T}{2} \right)} \right\} . \] (69)

So, we see that only difference with the original radiation formula is in variable \( \tilde{\alpha}_{ij} \).

It means that instead of sum (44) we have the following sum:

\[ \sum_{i,j=1}^{2} (-1)^{i+j} \cos \frac{\omega}{\omega_0} \tilde{\alpha}_{ij} = 2(1 - \cos 2\omega t \cos \varepsilon \omega t) \] (70)

It means that the one electron radiation formula is not modulated by the formula (44) but by the formula (70) and the final formula of for the power spectrum is as follows:

\[ P_l(\omega, t) = 2(1 - \cos 2\omega t \cos \varepsilon \omega t) P_{l(electron)}(\omega) , \] (71)

For \( \varepsilon \to 0 \), we get the original formula (48).

5 Discussion

We have derived in this article the power spectrum formula of the synchrotron radiation generated by the electron and positron moving at the opposite angular velocities in homogeneous magnetic field. This article is an extended version of the previous article [9]. We have used the Schwinger version of quantum field theory, for its simplicity. It is suprising that the spectrum depends periodically on time which means that the system composed from electron, positron and magnetic field behaves as a pulsar. While such pulsar can be represented by a terrestrial experimental arrangement it is possible to consider also the cosmological existence in some modified form.

To our knowledge, our result is not involved in the classical monographies on the electromagnetic theory and at the same time it was not still studied by the accelerator experts investigating the synchrotron radiation of bunches. This effect was not described
in textbooks on classical electromagnetic field and on the synchrotron radiation. We hope that sooner or later this effect will be verified by the accelerator physicists.

The radiative corrections obviously influence the synergic spectrum of photons [2,7]. However, the goal of this article is restricted only to the simple processes.

The particle laboratory LEP in CERN used instead of single electron and positron the bunches with $10^{10}$ electrons or positrons in one bunch of volume $300\mu m \times 40\mu m \times 0.01$ m. So, in some approximation we can replace the charge of electron and positron by the charges $Q$ and $-Q$ of both bunches in order to get the realistic intensity of photons. Nevertheless the synergic character of the radiation of two bunches moving at the opposite direction in a magnetic field is conserved. The more exact description can be obtained in case we consider the internal structure of both bunches. But this is not goal of our article.

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