The influence of boundary conditions on the form of the optical beam in the array of coupled optical waveguides.

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Abstract

We investigate the optical beam behavior in the periodical array of the coupled optical waveguides with the monotonic change of the refractive index in the transverse direction. We consider the dependence of the form of the optical beam on the boundary conditions. It is well known that if the input wave packet is wide enough, the optical Bloch oscillations occur, while for the enough narrow input wave packet the breathing mode is observed. We show that if the input wave packet is neither too wide nor too narrow, the optical beam takes a peculiar form which can be considered neither as the Bloch oscillations nor as the breathing mode. We qualitatively explain the transformation of this intermediate form of the optical beam when the width of the input wave packet changes.
Figure 1: The array of waveguides and the optical beam: (a) Bloch oscillations, (b) breathing mode.

I. INTRODUCTION

The optical waveguides are the inherent component of the optical and optoelectronic devices, indispensable for the optical signals transmission between different parts of the system. The interaction of different waveguides is usually an undesirable effect which distorts the transmitted signal. Thus, the isolation of the waveguides is an important problem for the optical devices design.

However, in some cases, the interaction of the waveguides can cause unexpected phenomena useful in practice. Those phenomena can occur in periodic arrays of waveguides, which are a special kind of low-dimensional photonic crystals. The main feature of these systems is a band structure of the optical spectrum which defines their peculiar properties [1–4].

Among the most famous phenomena which arise due to the band structure of the periodic arrays there are the optical Bloch oscillations and breathing modes (see, for example, [5–10]). These are two different forms of the optical beam which occur in the arrays with the monotonic change of the refractive index of the waveguides in the transverse direction. In the case of the optical Bloch oscillations, the width of the light beam is substantially constant, but the beam path possesses the curved oscillatory form (see Fig. 1(a)). In the case of the breathing mode, the beam path is straight, but the width of the beam changes periodically, i.e. the optical beam periodically spreads and focuses (see Fig. 1(b)).

The form of the optical beam depends on the boundary conditions at the edge of the array (the plane $z = 0$ in Fig. 1). The boundary conditions are specified with the input wave packet. The optical Bloch oscillations arise if the input wave packet possesses the Gaussian form with the width $W \gg a$, $a$ being the period of the array. The breathing mode arises if the input wave packet excites a single waveguide only, i.e. $W \lesssim a$.

The purpose of this work is to find the form of the optical beam for the intermediate case
When the optical beam behavior can be considered neither as the Bloch oscillations nor as the breathing mode. For some values of the input wave packet width, we obtain the peculiar form of the optical beam which were unknown before.

We apply the multiple scattering formalism for the numerical simulation. This method was used earlier in our works [11, 12] for the theoretical investigation of the Bloch oscillations.

This paper is organized as follows. In Sec. II we describe the multiple scattering formalism. In Sec. III we present the algorithm for calculating the form of the optical beam in an array of waveguides for the specified boundary conditions. In Sec. IV we apply the method explained in the previous section for calculating the optical beam form for the boundary conditions possessing the form of the Gaussian wave packet. Varying the boundary conditions, we obtain the different form of the optical beam. The obtained results are summarized in Sec. V.

II. MULTIPLE SCATTERING FORMALISM

Let us consider the array of $N$ parallel infinite cylindrical dielectric waveguides parallel to the $z$-axis. All the waveguides are assumed to possess the same radius $R$ but different refractive indices $n_j$, $j$ being the waveguide number. The refractive index of the environment is $n_e$. The permeability of the waveguide material and the environment is unity.

Suppose that a guided mode with a frequency $\omega$ is excited within the array. Then, all the components of the electromagnetic field are proportional to $e^{-i\omega t + i\beta z}$.

Let us consider the field of the guided mode inside of the array. The field of the guided mode is finite in any point inside the waveguide. So, the electromagnetic field inside of the $j$-th waveguide may be represented in the form

\[
\begin{align*}
\hat{E}_j(r) &= e^{-i\omega t + i\beta z} \sum_{m=0, \pm 1, \ldots} e^{im\phi_j} \left( c_{jm} \tilde{N}_{\omega_j \beta m}(\rho_j) - d_{jm} \tilde{M}_{\omega_j \beta m}(\rho_j) \right), \\
\hat{H}_j(r) &= e^{-i\omega t + i\beta z} n_j \sum_{m=0, \pm 1, \ldots} e^{im\phi_j} \left( c_{jm} \tilde{M}_{\omega_j \beta m}(\rho_j) + d_{jm} \tilde{N}_{\omega_j \beta m}(\rho_j) \right),
\end{align*}
\]

(1)

Here $\omega_j = n_j \omega$, and $\rho_j$, $\phi_j$ are the cylindrical coordinates of the vector $r - r_j$, where $r_j$ is the coordinates of the axis of the $j$-th waveguide. The vector cylinder harmonics $\tilde{M}_{\omega_j \beta m}(\rho_j)$ and $\tilde{N}_{\omega_j \beta m}(\rho_j)$ are defined as follows

\[
\tilde{N}_{\omega_j \beta m}(\rho_j) = e_r \frac{i \beta}{\kappa_j} J'_m(\kappa_j \rho_j) - e_\phi \frac{m \beta}{\kappa_j^2 \rho_j} J_m(\kappa_j \rho_j) + e_z J_m(\kappa_j \rho_j),
\]

(2)
\[ \tilde{M}_{\omega, \beta m}(\rho_j) = e_r \frac{m \omega_j}{\kappa_j^2 \rho_j} J_m(\kappa_j \rho_j) + e_\phi \frac{i \omega_j}{\kappa_j} J'_m(\kappa_j \rho_j), \]  
(3)

where \( \kappa_j = \sqrt{\omega_j^2 - \beta^2} \), \( J_m(\kappa_j \rho_j) \) is the Bessel function, and the prime means the derivative with respect to the argument \( \kappa_j \rho_j \).

Let us turn to the electromagnetic field outside of the array. This field is the sum of the contributions of all the waveguides,

\[ E(r) = \sum_{j=1}^{N} E_j(r), \quad H(r) = \sum_{j=1}^{N} H_j(r). \]  
(4)

The contribution induced by the \( j \)-th waveguide and vanishing at \( \rho_j \to \infty \) may be represented in the form

\[ E_j(r) = e^{-i \omega t + i \beta z} \sum_m e^{im \phi_j} \left( a_{jm} N_{\omega, \beta m}(\rho_j) - b_{jm} M_{\omega, \beta m}(\rho_j) \right), \]

\[ H_j(r) = e^{-i \omega t + i \beta z} n_e \sum_m e^{im \phi_j} \left( a_{jm} M_{\omega, \beta m}(\rho_j) + b_{jm} N_{\omega, \beta m}(\rho_j) \right), \quad \rho_j > R. \]  
(5)

Here \( n_e = n_c \omega \). The vector cylinder harmonics \( \tilde{M}_{\omega, \beta m}(\rho_j) \) and \( \tilde{N}_{\omega, \beta m}(\rho_j) \) are

\[ N_{\omega, \beta m}(\rho_j) = e_r \frac{i \beta}{\kappa_e} H_m(\kappa_e \rho_j) - e_\phi \frac{m \beta}{\kappa_e^2 \rho_j} H_m(\kappa_e \rho_j) + e_z H_m(\kappa_e \rho_j), \]  
(6)

\[ M_{\omega, \beta m}(\rho_j) = e_r \frac{m \omega_e}{\kappa_e^2 \rho_j} H_m(\kappa_e \rho_j) + e_\phi \frac{i \omega_e}{\kappa_e} H'_m(\kappa_e \rho_j), \]  
(7)

where \( \kappa_e = \sqrt{\omega_e^2 - \beta^2} \), and \( H_m(\kappa_e \rho_j) \) is the Hankel function of the first kind. Note that, for \( \beta = 0 \), Eqs. (11) and (5) transform into the corresponding expressions in [13], however different notations are used there.

Below we consider the simplest approximation to these equations, namely, the zero-harmonic approximation. This means that in Eqs. (11) and (5) only the terms with \( m = 0 \) are taken into account. In this approximation there are two kinds of the guided modes, namely the transverse magnetic (TM) and transverse electric (TE) modes. For the TM-mode \( b_{j0} = d_{j0} = 0 \), and for the TE-mode \( a_{j0} = c_{j0} = 0 \). As an example, let us consider the TM-modes.

\[ \tilde{E}_j(r) = c_j \tilde{N}_{\omega, \beta 0}(\rho_j), \quad \tilde{H}_j(r) = c_j \tilde{M}_{\omega, \beta 0}(\rho_j) \quad \rho_j < R. \]  
(8)

\[ E_j(r) = a_j N_{\omega, \beta 0}(\rho_j), \quad H_j(r) = a_j M_{\omega, \beta 0}(\rho_j) \quad \rho_j > R. \]  
(9)

Here and below, \( a_j \) and \( c_j \) stand for \( a_{j0} \) and \( c_{j0} \), and the factor \( e^{-i \omega t + i \beta z} \) is omitted.
To derive the equations that determine the partial amplitudes $a_j$ and $c_j$, one should use the boundary conditions on the surface of every waveguide. The boundary conditions on the surface of the $j$-th waveguide connect the field $\vec{E}_j(r)$, $\vec{H}_j(r)$ inside the $j$-th waveguide and the field $E(r)$, $H(r)$ outside. In general, there are four independent boundary conditions. However, for $m = 0$ TM-modes and only two boundary conditions are required:

$$[E(R_j)]_z = [\tilde{E}_j(R_j)]_z, \quad [H(R_j)]_\phi = [\tilde{H}_j(R_j)]_\phi,$$

where $R_j$ is the radius-vector of a point on the surface of the $j$-th waveguide.

The boundary conditions on the surface of the $j$-th waveguide can be expressed in the most convenient form if the contributions of all the waveguides to the field outside the array are expressed in terms of the same argument $\rho_j$. For this propose we apply the following relations:

$$N_{\omega, \beta_0}(\rho_l) \approx U_{lj}(\omega, \beta) \tilde{N}_{\omega, \beta_0}(\rho_j),$$

$$M_{\omega, \beta_0}(\rho_l) \approx U_{lj}(\omega, \beta) \tilde{M}_{\omega, \beta_0}(\rho_j), \quad l \neq j,$$

where $U_{lj}(\omega, \beta) = H_0(\kappa_e r_{lj})$, $H_0$ being the Hankel function of the first kind and $r_{lj}$ being the distance between the axes of the $j$-th and the $l$-th waveguides. The relations (11) follow from the Graf theorem (see [14]) in the zero-harmonic approximation.

Thus, it follows from (9) that

$$E_l(r) = a_j U_{lj}(\omega, \beta) \tilde{N}_{\omega, \beta_0}(\rho_j), \quad H_l(r) = a_j n_e U_{lj}(\omega, \beta) \tilde{M}_{\omega, \beta_0}(\rho_j).$$

Substituting (12) to (4), one gets

$$E(r) = a_j N_{\omega, \beta_0}(\rho_j) + \sum_{l \neq j} a_l U_{lj}(\omega, \beta) \tilde{N}_{\omega, \beta_0}(\rho_j),$$

$$H(r) = a_j n_e M_{\omega, \beta_0}(\rho_j) + \sum_{l \neq j} a_l n_e U_{lj}(\omega, \beta) \tilde{M}_{\omega, \beta_0}(\rho_j).$$

So, the boundary conditions (10) take the form

$$a_j H_0(\kappa_e R) + \sum_{l \neq j} a_l U_{lj}(\omega, \beta) J_0(\kappa_e R) = c_j J_0(\kappa_j R),$$

$$a_j \frac{n_e \omega_e}{\kappa_e} H_0'(\kappa_e R) + \sum_{l \neq j} a_l U_{lj}(\omega, \beta) \frac{n_e \omega_e}{\kappa_e} J_0'(\kappa_e R) = c_j \frac{n_j \omega_j}{\kappa_j} J_0'(\kappa_j R).$$

Eqs. (14) lead to the following system of equations:

$$\frac{a_j}{a_j(\omega, \beta)} - \sum_{l \neq j} U_{jl}(\omega, \beta) a_l = 0,$$
\[ c_j = \bar{c}_j(\omega, \beta) a_j. \]  

(16)

Here
\[ \bar{a}_j(\omega, \beta) = \frac{n_e^2 \kappa_e J_0'(\kappa_e R) J_0(\kappa_e R) - n_e^2 \kappa_e J_0(\kappa_e R) J_0'(\kappa_e R)}{n_e^2 \kappa_e J_0(\kappa_e R) H_0'(\kappa_e R) - n_e^2 \kappa_e J_0'(\kappa_e R) H_0(\kappa_e R)}, \]

(17)

\[ \bar{c}_j(\omega, \beta) = \frac{n_e^2 \kappa_e \{ H_0(\kappa_e R) J_0'(\kappa_e R) - H_0'(\kappa_e R) J_0(\kappa_e R) \}}{n_e^2 \kappa_e J_0(\kappa_e R) J_0'(\kappa_e R) - n_e^2 \kappa_e J_0'(\kappa_e R) H_0(\kappa_e R)}. \]

(18)

III. METHOD FOR THE OPTICAL BEAM CALCULATION.

The system of equations (15) describes the guided modes of the array of waveguides. This system possesses the nontrivial solution only if the determinant of the matrix of this system vanishes,

\[ \text{det} \left| \frac{\delta_{jl}}{\bar{a}_j(\omega, \beta)} - U_{jl}(\omega, \beta) \right| = 0. \]

(19)

This equation allows to obtain the propagation constants \( \beta_n \) of the guided modes for the given frequency \( \omega \), \( n \) being the number of a guided mode. There are \( N \) solutions of Eq. (19).

Let \( a_j(\beta_n) \) be the normalized solution of Eq. (15), \( \sum_{j=1}^{N} |a_j(\beta_n)|^2 = 1 \). The guided mode of the frequency \( \omega \) is a superposition of the modes with the different \( \beta_n \):

\[ E(t, r) = e^{-i\omega t} \sum_n C_n e^{i\beta_n z} \sum_{j=1}^{N} a_j(\beta_n) N_{\omega, \beta_n, 0}(\rho_j), \]

(20)

\[ H(t, r) = n_e e^{-i\omega t} \sum_n C_n e^{i\beta_n z} \sum_{j=1}^{N} a_j(\beta_n) M_{\omega, \beta_n, 0}(\rho_j). \]

The coefficients \( C_n \) determine the superposition.

The functions \( N_{\omega, \beta_n, 0}(\rho_j), M_{\omega, \beta_n, 0}(\rho_j) \) vanish rapidly as \( \rho_j \) increases. So, the field near the \( j \)-th waveguide is mainly determined by the partial amplitudes \( a_j(\beta_n) \). Thus, for the field near the \( j \)-th waveguide one can retain in (20) only the terms which contain \( a_j(\beta_n) \):

\[ E(t, r) = e^{-i\omega t} \sum_n C_n e^{i\beta_n z} a_j(\beta_n) N_{\omega, \beta_n, 0}(\rho_j), \]

(21)

\[ H(t, r) = n_e e^{-i\omega t} \sum_n C_n e^{i\beta_n z} a_j(\beta_n) M_{\omega, \beta_n, 0}(\rho_j). \]

If the waveguides interact weakly \( (U_{ij}(\omega, \beta) \ll 1/\bar{a}_j(\omega, \beta)) \) and the difference between the waveguides is negligible \( (n_j - n_j-1 \ll n_j) \), all the values \( \beta_n \) are close to each other. For this case,
\( N_{\omega,\beta_0}(\rho_j) \) and \( M_{\omega,\beta_0}(\rho_j) \) in (21) may be approximately replaced with \( N_{\omega,\bar{\beta}_0}(\rho_j) \) and \( M_{\omega,\bar{\beta}_0}(\rho_j) \), where \( \bar{\beta} = \left( \sum_n \beta_n \right)/N \). Thus, instead of (21) one obtains

\[
E(t, r) = e^{-i\omega t} \sum_n C_n e^{i\beta_n z} a_j(\beta_n) N_{\omega,\bar{\beta}_0}(\rho_j) = e^{-i\omega t} A_j(z) N_{\omega,\bar{\beta}_0}(\rho_j),
\]

\[
H(t, r) = n_e e^{-i\omega t} \sum_n C_n e^{i\beta_n z} a_j(\beta_n) M_{\omega,\bar{\beta}_0}(\rho_j) = n_e e^{-i\omega t} A_j(z) M_{\omega,\bar{\beta}_0}(\rho_j).
\]

In (22) the modal amplitudes \( A_j(z) \) are introduced,

\[
A_j(z) = \sum_n C_n e^{i\beta_n z} a_j(\beta_n). \tag{23}
\]

For the case of weakly interacting nearly identical waveguides, the modal amplitudes \( A_j(z) \) represent the behavior of the guided modes properly. Below we define the optical excitation intensity at the point with the coordinate \( z \) of the \( j \)-th waveguide as \( |A_j(z)| \).

The coefficients \( C_n \) are obtained from the boundary condition at \( z = 0 \):

\[
\sum_n C_n a_j(\beta_n) = A_j(0). \tag{24}
\]

The system of equations (24) allows to obtain the coefficients \( C_n \) for given \( A_j(0) \).

The boundary condition \( A_j(0) \) are determined with the input wave packet which possesses the Gaussian form. Thus,\n
\[
A_j(0) = e^{-\frac{a^2}{2\sigma^2} + ik_0 a j}. \tag{25}
\]

This means that the input wave packet approximately illuminates the ends of the waveguides with the numbers \(-\sigma < j < \sigma\), and the width of the input wave packet is \( W = 2\sigma a \). The phase difference between the amplitudes taken at the ends of the nearest waveguides is \( k_0 a \).

Thus, the guided mode can be found as follows:

1) Calculate numerically the set of propagating constants \( \beta_n \) using Eq. (19);
2) Obtain the amplitudes \( a_j(\beta_n) \) for every \( \beta_n \) using Eq. (15);
3) For the given boundary conditions find the coefficients \( C_n \) using Eq. (24);
4) Calculate the function \( A_j(z) \) by means of Eq. (23).

IV. THE OPTICAL BEAM IN THE ARRAY OF WAVEGUIDES FOR DIFFERENT BOUNDARY CONDITIONS.

We apply the developed technique for calculating the optical beam in an array represented in Fig. 1. We consider a sample that can be fabricated by means of the technology represented in [15].
Figure 2: The optical beam in the array for different boundary conditions. The period of the array \( a = 3R \). The width of the input wave packet: (a) \( \sigma = 3.0 \), (b) \( \sigma = 1.5 \), (c) \( \sigma = 1.0 \), (d) \( \sigma = 0.5 \).

In these works a new method to fabricate low bend loss femtosecond-laser written waveguides is explained. The parameters taken for the numerical simulation correspond approximately to the parameters of the arrays of waveguides reported in work [15].

The wavelength of the laser source in [15] is \( \lambda = 1550 \) nm. We take the waveguide radius \( R = 5\lambda = 7750 \) nm. The refractive index of the environment is \( n_e = 1.4877 \), and the refractive index of the waveguide \( j = 0 \) (the central waveguide of the array) is \( n_0 = n_e + 5 \times 10^{-3} \). These parameters also approximately correspond to the experiments reported in [15]. For our calculations we assume that the variation of refractive indices between the nearest waveguides is \( \delta n = n_j - n_{j-1} = 5 \times 10^{-6} \).

We take the arrays with two different periods \( a_1 = 3R \) and \( a_2 = 2.5R \).

We produce the calculation for different values of the input wave packet width: \( \sigma_1 = 3.0 \), \( \sigma_2 = 1.5 \), \( \sigma_3 = 1.0 \), \( \sigma_4 = 0.5 \). In all cases \( k_0 = 0 \).

The results of the numerical simulation for \( a_1 = 3R \) and \( a_2 = 2.5R \) are presented in Fig. 2 and Fig. 3 correspondingly.
Figure 3: The optical beam in the array for different boundary conditions. The period of the array $a = 2.5R$. The width of the input wave packet: (a) $\sigma = 3.0$, (b) $\sigma = 1.5$, (c) $\sigma = 1.0$, (d) $\sigma = 0.5$.

The figures show that generally similar qualitative results are obtained for the both selected periods $a$ of the array. Namely, $\sigma = 3$ (i.e. the input wave packet illuminates nearly seven waveguides) is enough wide for the optical Bloch oscillations occur. At the same time, the optical beam obtained for $\sigma = 0.5$ doubtless can be considered as the breathing mode.

But for $\sigma = 1.5$ and $\sigma = 1.0$ the form of the optical beam does not relate any of these phenomena. The most exactly, the obtained picture can be defined as an “asymmetrical breathing modes”. In fact, the form of the optical beam resembles the breathing mode, but the left side of the beam is brighter then the right side, and the intensity of the beam gradually decreases with the shift to the right. When $\sigma$ increases, the intensity reduction in the transverse direction becomes faster. Thus, as $\sigma$ becomes large enough, the optical beam concentrates along the oscillating curve, which for small $\sigma$ forms the left edge of the breathing mode. Thus, the “asymmetrical breathing mode” turns into the optical Bloch oscillations.
V. CONCLUSION

In this paper we investigated the optical beam behavior in the array of the coupled optical waveguides with the monotonic change of the refractive index in the transverse direction. We considered the optical beam form dependence on the boundary conditions, i.e. on the form of the input wave packet.

For this purpose we applied the multiple scattering formalism based on the macroscopic electrodynamics approach. For the simplicity, we used the zero-harmonic approximation taking into account the TM-modes only. The MSF is the basis of the numerical algorithm for calculating the form of the optical beam for the specified boundary conditions at $z = 0$.

We chose the boundary conditions possessing the form of the Gaussian wave packet. Varying the width of the input wave packet, we observed the change of the optical beam excited in the array.

As expected, for the input wave packet being narrow enough, the breathing mode was observed, while for the case of enough large input wave packet, the optical Bloch oscillation occurred. But for the intermediate case, the optical beam takes an unexpected form which can be considered neither as the Bloch oscillations nor as the breathing mode. The obtained form of the optical beam can be defined as an “asymmetrical breathing mode”. We qualitatively described the optical beam transformation when the width of the input wave packet changes. We showed that the breathing mode and the optical Bloch oscillations are the limiting cases of the “asymmetrical breathing mode”.

Acknowledgments

The study is supported by the Russian Fund for Basic Research (Grant 14-29-08165).

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