Describe the Mathematical Model for Exchanging Waves Between Bacterial and Cellular DNA

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Abstract: In this article, we have shown that bacterial DNA could act like some coils which interact with coil-like DNA of host cells. By decreasing the separating distance between two bacterial cellular DNA, the interaction potential, entropy, and the number of microstates of the system grow. Moreover, the system gives its energy to the medium and the temperature of the host body grows. This could be seen as fever in diseases. By emitting some special waves and changing the temperature of the medium, the effects of bacterial waves could be reduced and bacterial diseases could be controlled. Many investigators have shown that bacterial DNA could emit or absorb electromagnetic waves. One of the main experiments about bacterial waves has been done by Montagnier and his group. They have shown that the genomic DNA of most pathogenic bacteria includes sequences that are able to emit electromagnetic waves. The results have shown that wave affects the crucial physicochemical processes in both Gram-positive and Gram-negative bacteria. The emphasis in this survey is on the development of controlling model equations and computer emulation of the model equations rather than on mathematical methods for solving the model equations and differential equations of epidemics.

Keywords: DNA; bacteria; diseases; inductor; magnetic field

1 Introduction

Up to date, many investigators have shown that bacterial DNA could emit or absorb electromagnetic waves. One of the main experiments about bacterial waves has been done by Montagnier and his group. They have shown that the genomic DNA of most pathogenic bacteria includes sequences that are able to emit electromagnetic waves \cite{1}. They have described the experimental conditions by which diluted aqueous solutions of some bacterial DNA emit electromagnetic waves \cite{2}. Other groups have considered the effect of millimeter waves on the survival of UVC-exposed \textit{Escherichia coli} \cite{3}. Moreover, other scientists have argued that extremely high-frequency electromagnetic radiation enforces the bacterial effects of inhibitors and antibiotics. They have shown that the radiation of bacteria might lead to changed metabolic pathways.

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Fractional order differential equations (FDEs) are usually used to model systems that have a memory that occurs in sundry physical phenomenas, models in the thermoelasticity field, and biological ideals see ([9–11], [12–14], [15–17], [18–20], [21–23], [24–26], [27–29], [30–32]).

This study allows the notion of mathematical biology and a preamble to mathematical modeling and fact for biological and biomedical systems ([33–35], [36–38], [39–41]). Models include the formation of animal coat patterns, the spread of diseases through the community, the interaction between pathogens and the immune system of the body, the growth of tumors, nerve cell signaling, population dynamics, pharmacokinetics, and bacterial growth ([42–44], [44–46]).

The emphasis in this study is on the development of the governing model equations and computer simulations of the model equations rather than on mathematical methods for solving the model equations. Differential equations of epidemics.

The outline of this paper is as follows: In Section 2, we propose a mathematical model for exchanging waves between bacterial and cellular DNA. In Section 3, we explain how this model can help us in curing diseases. The last section is devoted to a summary and conclusions.

2 A Quantum Model for Exchanging Waves Between Bacterial and Cellular DNA

Bacterial DNA divide into two groups: 1. Chromosomal DNA, and 2. Non-chromosomal DNA (plasmid). Both of these genetic matters are formed from charged particles and according to the laws of physics, by any motion, some waves are emitted. Especially, plasmids have a structure like round coils and emit some magnetic fields. On the other hand, a DNA within the cell acts like an inductor and emits some magnetic field.

By exchanging these magnetic fields, the bacterial coil and DNA inductors interact with each other (See Fig. 1).

This interaction causes the absorption of bacterial genetic matter by host cells and the emergence of infectious. For a coil, the magnetic field can be obtained from the below equation:

\[ B \simeq \frac{\mu_0 IN}{2} \frac{a^2}{\left[ a^2 + z^2 \right]^{3/2}}, \]  

(1)
where $I$ is the current, $a$ is the radius of the loop, $N$ is the number of loops, and $z$ is the separation distance from the center of the coil or inductor to the desired point. For hexagonal and pentagonal bases within a DNA, the current along the plane of the molecule could be obtained as:

\[ I_5 \simeq \sum_{i=1}^{5} q_i v_i \sin \left( \frac{2\pi}{5} \right), \tag{2} \]

\[ I_6 \simeq \sum_{i=1}^{6} q_i v_i \sin \left( \frac{2\pi}{6} \right), \tag{3} \]

where $I_{5/6}$ are the currents of hexagonal/pentagonal molecules, $q_i$ is the electric charge of atom $i$ and $v_i$ is its velocity. Putting Eqs. (2) and (3) in Eq. (1), we obtain:

\[ B \simeq \sum_{j=1}^{N} \frac{\mu_0 [I_{5,j} + I_{6,j}]}{2} \frac{a_j^2}{\left[ a_j^2 + z_j^2 \right]^{3/2}} \]

\[ \simeq \sum_{j=1}^{N} \frac{\mu_0 \left[ \sum_{i=1}^{5} q_{ij} v_{ij} \sin \left( \frac{2\pi}{5} \right) + \sum_{i=1}^{6} q_{ij} v_{ij} \sin \left( \frac{2\pi}{6} \right) \right]}{2} \frac{a_j^2}{\left[ a_j^2 + z_j^2 \right]^{3/2}}. \tag{4} \]

Figure 1: The interaction between bacterial and cellular DNA
Using the above equation, we can calculate the interaction potential between two cellular and bacterial DNA:

\[
V_{\text{Cellular--bacteria}} \simeq \frac{B_{\text{bacteria}} - B_{\text{cellular}}}{2\mu_0} \left[ \sum_{j=1}^{N} \mu_0 \left[ \sum_{i=1}^{5} q_{ij}^{\text{bacteria, bacteria}} \sin \left( \frac{2\pi}{5} \right) + \sum_{i=1}^{6} q_{ij}^{\text{bacteria, bacteria}} \sin \left( \frac{2\pi}{6} \right) \right] \right] \left[ \frac{\left( d_j^{\text{bacteria}} \right)^2}{\left[ \left( d_j^{\text{cellular}} \right)^2 + z_j^2 \right]^{3/2}} \right]
\]

(5)

The above potential shows that bacterial and cellular DNA could absorb or repel each other. The negative or positive signs of potential depend on the number of positive and negative charges on bacterial and cellular DNA. For large distances, we can rewrite the above equation as:

\[
V_{\text{Cellular--bacteria}} \simeq \frac{Y_{\text{bacteria}} Y_{\text{Cellular}}}{z_6^{\text{Cellular--bacteria}}},
\]

(6)

where

\[
Y_{\text{bacteria}} \simeq \sum_{j=1}^{N} \left[ d_j^{\text{bacteria}} \right]^2 \mu_0 \left[ \frac{\sum_{i=1}^{5} q_{ij}^{\text{bacteria, bacteria}} \sin \left( \frac{2\pi}{5} \right) + \sum_{i=1}^{6} q_{ij}^{\text{bacteria, bacteria}} \sin \left( \frac{2\pi}{6} \right) }{2} \right]
\]

(7)

and

\[
Y_{\text{Cellular}} \simeq \sum_{j=1}^{M} \left[ d_j^{\text{cellular}} \right]^2 \mu_0 \left[ \frac{\sum_{i=1}^{5} q_{ij}^{\text{cellular, cellular}} \sin \left( \frac{2\pi}{5} \right) + \sum_{i=1}^{6} q_{ij}^{\text{cellular, cellular}} \sin \left( \frac{2\pi}{6} \right) }{2} \right]
\]

(8)

Above potential causes that bacterial DNA are absorbed by cellular DNA. The wave equation for the motion of bacterial DNA in flat space-time can be written as:

\[
- \frac{\partial^2 \text{DNA}_{Bacteria}}{\partial \tau^2} + \frac{\partial^2 \text{DNA}_{Bacteria}}{\partial \sigma^2} = 0.
\]

(9)

To regard to the interacting potential, we should use of replacements:

\[
\rho = \rho_0 \sigma^2 = \rho_0 \frac{\sigma^2}{w^2},
\]

\[
w = \frac{3\tau_3 + [3 - 3\tau_3] V_{\text{Cellular--bacteria}}}{V_{\text{Cellular--bacteria}}}
\]
\[ 3\tau_3 + [3 - 3\tau_3] \frac{Y_{\text{bacteria}} Y_{\text{Cellular}}}{z^6_{\text{Cellular--bacteria}}} = \frac{\dot{z}_{\text{Cellular--bacteria}}}{z^7_{\text{Cellular--bacteria}}} \frac{Y_{\text{bacteria}} Y_{\text{Cellular}}}{z^8_{\text{Cellular--bacteria}}} , \]

\[ \tau = \gamma \int_0^t \frac{w}{w'} d\tau' - \gamma \sigma^2 \frac{\sigma^2}{2} . \] (10)

Substituting the above equation in Eq. (9) gives:

\[ \left\{ \left[ \frac{\partial \tau}{\partial \tau} \right]^2 - \left( \frac{\partial \tau}{\partial \sigma} \right)^2 \right\} \frac{\partial^2}{\partial \tau^2} + \left\{ \left( \frac{\partial \rho}{\partial \sigma} \right)^2 - \left( \frac{\partial \rho}{\partial \tau} \right)^2 \right\} \frac{\partial^2}{\partial \rho^2} \right\} DNA_{\text{Bacteria}} = 0. \] (11)

We compare the above equation with the general equation for scalar fields and write:

\[ (-g)^{-1/2} \frac{\partial}{\partial x_\mu} \left( (-g)^{1/2} g^{\mu\nu} \right) \frac{\partial}{\partial x_\nu} DNA_{\text{Bacteria}} = 0, \] (12)

where

\[ g^{\tau\tau} \sim -\frac{1}{\beta^2} \left( \frac{w'}{w} \right)^2 \frac{\left( 1 - \frac{w}{w'} \right)^2 \frac{1}{\sigma^4}}{\left( 1 + \left( \frac{w}{w'} \right)^2 \frac{1 + \gamma^{-2}}{\sigma^4} \right)}^{1/2} \]

\[ = -\frac{1}{\beta^2} \left[ 1 + \frac{\dot{z}_{\text{Cellular--bacteria}}}{z^2_{\text{Cellular--bacteria}}} \right] \left[ 3\tau_3 + [3 - 3\tau_3] \frac{Y_{\text{bacteria}} Y_{\text{Cellular}}}{z^6_{\text{Cellular--bacteria}}} \right]^2 \]
The above equation is very similar to the wave equation of a particle in curved spacetime. This is because that DNA is a long object 7 meters long which is compacted in less than micrometers. Thus, this system could be similar to a black string with the below metric \([9–22]\):

\[
\begin{align*}
\text{d}s^2 &= D^{-1/2} H^{-1/2} \left(-f \text{d}t^2 + \text{d}x_1^2\right) + D^{1/2} H^{1/2} \left(\text{d}x_2^2 + \text{d}x_3^2\right) + D^{-1/2} H^{1/2} \left(f^{-1} r^2 \text{d}r^2 + r^2 \text{d}\Omega_5^2\right),
\end{align*}
\]  \(13\)

where

\[
\begin{align*}
f &= 1 - \frac{r_0^4}{r^4}, \quad H = 1 + \frac{r_0^4}{r^4} \sinh^2 \alpha, \quad D^{-1} = \cos^2 \varepsilon + H^{-1} \sin^2 \varepsilon, \quad \cos \varepsilon = \frac{1}{\sqrt{1 + \frac{\beta^2}{\sigma^4}}},
\end{align*}
\]  \(15\)
Now, we can arrange the parameters of this black string in terms of the parameters of the bacterial cellular system:

\[ f = 1 - \frac{r_0^4}{r^4} \sim 1 - \left( \frac{w}{w'} \right)^2 \frac{1}{\sigma^4} = \]

\[
1 - \left( \frac{w}{w'} \right)^2 \left( \frac{3\tau_3 + [3 - 3\tau_3]}{z_{\text{Cellular} - \text{bacteria}}} \right)^{-2} \left[ \frac{\dot{z}_{\text{Cellular} - \text{bacteria}} Y_{\text{bacteria} Y_{\text{Cellular}}}}{z_{\text{Cellular} - \text{bacteria}}} \right]^{-2}
\]

\[
\mathcal{T} = 1 + \frac{r_0^4}{r^4} \sin^2 \alpha \sim 1 + \left( \frac{w}{w'} \right)^2 \left( 1 + \gamma^{-2} \right) \frac{1}{\sigma^4}
\]

\[
1 + \left( \frac{w}{w'} \right)^2 \left( \frac{3\tau_3 + [3 - 3\tau_3]}{z_{\text{Cellular} - \text{bacteria}}} \right)^{-2} \left[ \frac{\dot{z}_{\text{Cellular} - \text{bacteria}} Y_{\text{bacteria} Y_{\text{Cellular}}}}{z_{\text{Cellular} - \text{bacteria}}} \right]^{-2}
\]

\[
D^{-1} = \cos^2 \varepsilon + \mathcal{T}^{-1} \sin^2 \varepsilon \simeq 1
\]

\[
\Rightarrow r \sim \sigma, r_0 \sim \left( \frac{w}{w'} \right)^2 \left( \frac{3\tau_3 + [3 - 3\tau_3]}{z_{\text{Cellular} - \text{bacteria}}} \right)^{-2} \left[ \frac{\dot{z}_{\text{Cellular} - \text{bacteria}} Y_{\text{bacteria} Y_{\text{Cellular}}}}{z_{\text{Cellular} - \text{bacteria}}} \right]^{-1/2}, (1 + \gamma^{-2}) \sim \sinh^2 \alpha
\]
For a bacterial cellular system, the temperature could be obtained from the below equation:

\[ T_{\text{bacterial–cellular–system}} = \frac{1}{\pi r_0 \cosh \alpha} = \frac{\gamma}{\pi} \left( \frac{w'}{w} \right)^{1/2} \]

\[ \sim \frac{\gamma'}{\pi} \left( 1 + \frac{\dot{z}_{\text{Cellular–bacteria}}}{z_{\text{Cellular–bacteria}}} \right) \left( \frac{3 \tau_3 + [3 - 3 \tau_3] Y_{\text{bacteria}} Y_{\text{Cellular}}}{z^6_{\text{bacteria–bacteria}}} \right)^{1/2} \]  
\[ \left( 3 \tau_3 + [3 - 3 \tau_3] Y_{\text{bacteria}} Y_{\text{Cellular}} \right)^{1/2} \left( 9 + \frac{\gamma - 2}{4} \right)^{-1/2} \]  
\[ 3 \tau_3 + [3 - 3 \tau_3] Y_{\text{bacteria}} Y_{\text{Cellular}} \]

The above equation shows that the temperature of the cellular bacteria system depends on the separation distance between bacteria and cells and by decreasing it, more interactions between DNA occur and consequently, the system gives its energy to the medium and becomes cooler, however, the medium around it becomes hotter. This may be a reason for the emergence of fever in diseases.

\[ T_{\text{medium/fever}} \sim T_{\text{bacterial–cellular–system}}^{-1} \]  

(17)

Now, we calculate the total potential of the system:

\[ E_{\text{bacterial–cellular–system}} = \int d\sigma T^{00} = \int d\sigma \frac{\pi^2}{2} T_0^4 \left( 5 + 4 \sinh^2 \alpha \right) \sim \]  
\[ \frac{\pi^2}{2} \left( \frac{w}{w'} \right)^{1/2} \left( 9 + \gamma^{-2} \right) \sim \]  
\[ \left( 9 + \gamma^{-2} \right)^{-1/2} \]  

(19)

The above equation shows that by decreasing the separation distance between bacterial and cellular DNA, the interaction between charges increases, and the energy of the system grows.
Using Eq. (19), we can obtain the entropy of the system.

\[
S_{\text{bacterial-cellular-system}} \sim \frac{E_{\text{bacterial-cellular-system}}}{T_{\text{bacterial-cellular-system}}}. \tag{20}
\]

This entropy depends on the separation distance between bacterial and cellular DNA and by its decreasing growth. This means that the number of microstates of this system depends on the interaction between DNA. By closing bacterial DNA, they could exchange more waves with DNA of host cells and the energy of the system and its entropy increase.

### 3 Controlling Diseases by Exchanging Waves Between Bacterial and Cellular DNA

To control bacterial diseases, we need to obtain the number of micro states. We can write:

\[
S_{\text{bacterial-cellular-system}} \sim K \ln \left( \Omega_{\text{bacterial-cellular-system}} \right) \Rightarrow \Omega_{\text{bacterial-cellular-system}} \propto e^{\frac{E_{\text{bacterial-cellular-system}}}{T_{\text{bacterial-cellular-system}}}}. \tag{21}
\]

These micro states could be removed by emitting some waves which their energy and temperature can be obtained from the below equation:

\[
1 = \frac{\Omega_{\text{bacterial-cellular-system}}}{E_{\text{bacterial-cellular-system}}} \times \frac{\Omega_{\text{wave}}}{E_{\text{wave}}} \times e^{\frac{E_{\text{wave}}}{T_{\text{medium}}}}. \tag{22}
\]

Consequently, the energy of the wave and the temperature of the medium should have a reverse relation with the energy and temperature of the bacterial cellular systems:

\[
\frac{E_{\text{bacterial-cellular-system}}}{T_{\text{bacterial-cellular-system}}} = \left[ \frac{E_{\text{wave}}}{T_{\text{medium}}} \right]^{-1}. \tag{23}
\]

The above equation is in agreement with experiments. For example, in a bacterial disease, the temperature of its host body should be reduced. If we could measure the energy and temperature of the bacterial cellular system, we can obtain the energy and temperature of waves, which could help us to remove the effects of bacteria waves. To examine the model, we can measure the radiated waves from bacterial DNA within the milk. We can put a vessel of milk at a temperature around 38°C for some hours. We put this vessel in a coil and connect it to a generator.

**Fig. 2:** Measured current in terms of time from one end. Then, we can connect it to a scope like an oscilloscope or radio sky pipe and an amperemeter from another end. We can measure the differences between input and output currents (See Fig. 2).

In Fig. 3, we bring some results. It is clear that bypassing time, the more bacteria in a vessel of milk grow, the more waves are emitted, and consequently the observed current increases. The emitted waves interact with free electrons along metal wires and cause their motion and the emergence of some extra currents.
4 Results and Discussion

A DNA is formed from hexagonal pentagonal molecules and each molecule is formed from charged particles. By any motion of charge, some waves have emerged. The shape of these waves depends on the shape of their DNA sources. For example, some of the bacterial DNA like those in plasmid have a round shape like round coils. These DNA coils send some waves which are absorbed by DNA inductors within host cells. These exchanging of waves lead to the absorption
of bacteria by host cells. By emitting some waves, we can cancel bacterial waves and prevent some bacterial diseases.

5 Conclusions

In this paper, some of the DNA within bacteria have the shape of coils. These DNA are formed from charged particles and by their motion, some currents have emerged. These currents emit some electromagnetic waves. These waves could be taken by cellular DNA and consequently, a cellular-bacterial system is formed. By closing bacterial DNA towards cellular DNA, the interacting potential grows, and the number of microstates increases. However, this system gives its energy to the medium and causes the growth of temperature and the emergence of loss. To control bacterial diseases, we should reduce temperature or emit some waves to cancel the bacterial waves. We design a circuit and measure the radiated currents by bacteria within the milk. This investigation permits a thought of numerical science and a prelude to numerical displaying and reality for organic and biomedical frameworks. Models incorporate the development of creature coat designs, the spread of sicknesses through the local area, the association between microbes and the safe arrangement of the body, the development of tumors, nerve cell flagging, populace elements, pharmacokinetics, and bacterial development.

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