THE QUASIPARTICLE STRUCTURE OF HOT GAUGE THEORIES

Edmond Iancu

Service de Physique Théorique, CE-Saclay
91191 Gif-sur-Yvette, France

Abstract

The study of the ultrarelativistic plasmas in perturbation theory is plagued with infrared divergences which are not eliminated by the screening corrections. They affect, in particular, the computation of the lifetime of the elementary excitations, thus casting doubt on the validity of the quasiparticle picture. We show that, for Abelian plasmas at least, the infrared problem of the damping rate can be solved by a non-perturbative treatment based on the Bloch-Nordsieck approximation. The resulting expression of the fermion propagator is free of divergences, and exhibits a non-exponential damping at large times: $S_R(t) \sim \exp\{-\alpha T \ln \omega_p t\}$, where $\omega_p = gT/3$ is the plasma frequency and $\alpha = g^2/4\pi$.

Invited talk at the Workshop on Quantum Chromodynamics, 3-8 June, 1996, American University of Paris, France.

\textsuperscript{a}Laboratoire de la Direction des Sciences de la Matière du Commissariat à l’Energie Atomique
The study of the ultrarelativistic plasmas in perturbation theory is plagued with infrared divergences which are not eliminated by the screening corrections. They affect, in particular, the computation of the lifetime of the elementary excitations, thus casting doubt on the validity of the quasiparticle picture. We show that, for Abelian plasmas at least, the infrared problem of the damping rate can be solved by a non-perturbative treatment based on the Bloch-Nordsieck approximation. The resulting expression of the fermion propagator is free of divergences, and exhibits a non-exponential damping at large times: $$S_R(t) \sim \exp\{-\alpha T \ln \omega_p t\}$$, where $$\omega_p = gT/3$$ is the plasma frequency and $$\alpha = g^2/4\pi$$.

1 Introduction

The study of the elementary excitations of ultrarelativistic plasmas, such as the quark-gluon plasma, has received much attention in the recent past. The physical picture which emerges, for both Abelian and non-Abelian gauge theories, is that of a system with two types of degrees of freedom: i) the plasma quasiparticles, whose energy is of the order of the temperature $$T$$; ii) the collective excitations, whose typical energy is $$gT$$, where $$g$$ is the gauge coupling, assumed to be small: $$g \ll 1$$ (in QED, $$g = e$$ is the electric charge).

For this picture to make sense, however, it is important that the lifetime of the excitations be large compared to the typical period of the modes. Information about the lifetime is obtained from the retarded propagator. A usual expectation is that $$S_R(t, \mathbf{p})$$ decays exponentially in time, $$S_R(t, \mathbf{p}) \sim e^{-iE (\mathbf{p}) t} e^{-\gamma (\mathbf{p}) t}$$, where $$E (\mathbf{p}) \sim T$$ or $$gT$$ is the average energy of the excitation, and $$\gamma (\mathbf{p})$$ is the damping rate. The exponential decay may then be associated to a pole of the Fourier transform $$S_R(\omega, \mathbf{p})$$, located at $$\omega = E - i\gamma$$:

$$S_R(\omega, \mathbf{p}) = \int_{-\infty}^{\infty} dt e^{-i\omega t} S_R(t, \mathbf{p}) \sim \frac{1}{\omega - (E (\mathbf{p}) - i\gamma (\mathbf{p}))}.$$  \hspace{1cm} (1)

The quasiparticles are well defined if the damping rate $$\gamma$$ is small compared to the energy $$E$$. If this is the case, the respective damping rates can be computed from the imaginary part of the on-shell self-energy, $$\Sigma(\omega = E(\mathbf{p}), \mathbf{p})$$.

Previous calculations suggest that $$\gamma \sim g^2 T$$ for both the single-particle and the collective excitations. In the weak coupling regime $$g \ll 1$$, this is indeed small compared to the corresponding energies (of order $$T$$ and $$gT$$, respectively), suggesting that the quasiparticles are well defined, and the collective modes are weakly damped. However, the computation of $$\gamma$$ in perturbation theory is plagued with infrared (IR) divergences, which casts doubt on the validity of these statements.
known that most of these divergences are actually removed by screening effects which are generated by the collective motion of the thermal particles. In ultrarelativistic plasmas, the screening effects manifest themselves over typical space-time scales $\sim 1/gT$. Their inclusion in the perturbative expansion — which is achieved in a gauge-invariant way by resumming the so-called “hard thermal loops” (HTL) of Braaten and Pisarski — greatly improve the infrared behavior, and yields IR-finite results for the transport cross-sections, and also for the damping rates of the excitations with zero momentum.

However, the HTL resummation is not sufficient to render finite the damping rates of the excitations with non-vanishing momenta. The remaining infrared divergences are due to collisions involving the exchange of longwavelength, quasistatic, magnetic photons (or gluons), which are not screened in the hard thermal loop approximation. Such divergences affect the computation of the damping rates of charged excitations (fermions and gluons), in both Abelian and non-Abelian gauge theories. Furthermore, the problem appears for both soft ($p \sim gT$) and hard ($p \sim T$) quasiparticles. In QCD this problem is generally avoided by the ad-hoc introduction of an IR cut-off (“magnetic screening mass”) $\sim g^2 T$, which is expected to appear dynamically from gluon self-interactions. In QED, on the other hand, it is known that no magnetic screening can occur, so that the solution of the problem must lie somewhere else.

We have shown recently that, for Abelian plasmas, the divergences can be eliminated through a non-perturbative treatment, which involves a soft photon resummation à la Bloch-Nordsieck. We have thus obtained the large-time decay of the fermion propagator, which is not of the exponential type alluded to before, but of the more complicated form

$$S_R(t) \sim e^{-\frac{1}{2} q^2 \Pi_l(q^2)} e^{-\frac{1}{2} q^2 \Pi_l(q^2)} e^{\frac{-\alpha}{2} T t \ln \omega_p t},$$

where $\alpha = g^2/4\pi$ and $\omega_p \sim gT$ is the plasma frequency. Accordingly, the Fourier transform $S_R(\omega)$ is analytic in the vicinity of the mass-shell. In what follows, I will briefly discuss these results, their derivation and their consequences.

### 2 Perturbation theory for $\gamma$

Let me first recall how the infrared problem occurs in the perturbative calculation of the damping rate $\gamma$. For simplicity, I consider an Abelian plasma, as described by QED, and compute the damping rate of a hard electron, with momentum $p \sim T$ and energy $E(p) = p$.

To leading order in $g$, and after the resummation of the screening corrections, $\gamma$ is obtained from the imaginary part of the effective one-loop self-energy in Fig. 1. The blob on the photon line in this figure denotes the effective photon propagator in the HTL approximation, commonly denoted as $^{*}D^{\mu\nu}(q)$. In the Coulomb gauge, the only non-trivial components of $^{*}D^{\mu\nu}(q)$ are the electric (or longitudinal) one $^{*}D^{00}(q) \equiv ^{*}D_l(q)$, and the magnetic (or transverse) one $^{*}D^{ij}(q) = (\delta^{ij} - \hat{q}_i \hat{q}_j)^{*}D_l(q)$, with

$$^{*}D_l(q,0) = \frac{-1}{q^2 - \Pi_l(q,0)}, \quad ^{*}D_l(q,0) = \frac{-1}{q^2 - \Pi_l(q,0)}, \quad ^{*}D_l(q,0) = \frac{-1}{q^2 - \Pi_l(q,0)}, \quad ^{*}D_l(q,0) = \frac{-1}{q^2 - \Pi_l(q,0)},$$

where $\Pi_l$ and $\Pi_t$ are the respective pieces of the photon polarisation tensor. Physically, the on-shell discontinuity of the diagram in Fig. accounts for the scattering of the incoming electron off the thermal fermions, with the exchange of a soft, dressed, virtual photon.
Figure 1: The resummed one-loop self-energy

The corresponding interaction rate is simply computed as \( \gamma = \sigma \rho \), where \( \rho \sim T^3 \) is the density of the scatterers, and \( \sigma = \int d^2 q (d\sigma/dq^2) \), with \( q \) denoting the momentum of the exchanged (virtual) photon. For a bare (i.e., unscreened) photon, the Rutherford formula yields \( d\sigma/dq^2 \sim g^4/q^4 \), so that \( \gamma \sim g^4 T^3 \int (d\sigma/dq^3) \) is quadratically infrared divergent. Actually, the screening effects included in \( \Pi_{l,t} \) soften this IR behaviour. We have, in the kinematical regime of interest, \( (\omega_p = eT/3) \)

\[
\Pi_l(q_0 \ll q) \simeq 3\omega_p^2 \equiv m_D^2, \quad \Pi_t(q_0 \ll q) \simeq -i \frac{3\pi}{4} \omega_p^2 q_0 q. \tag{3}
\]

We see that screening occurs in different ways in the electric and the magnetic sectors. In the electric sector, the familiar static Debye screening provides an IR cut-off \( m_D \sim gT \).

Accordingly, the electric contribution to \( \gamma \) is finite, and of the order \( \gamma_l \sim g^4 T^3/m_D^2 \sim g^2 T \).

In the magnetic sector, screening occurs only for nonzero frequency \( q_0 \). This comes from the imaginary part of the polarisation tensor, and corresponds to the absorption of the space-like photons \( (q_0^2 < q^2) \) by the hard thermal fermions (Landau damping [4]). This “dynamical screening” is not sufficient to completely remove the IR divergence of \( \gamma_t \), which is just reduced to a logarithmic one:

\[
\gamma_t \sim g^4 T^3 \int_0^\infty dq \int_{-q}^q dq_0 \left| \star D_t(q_0, q) \right|^2 \sim g^2 T \int_0^{\omega_p} dq q \left| \star D_t(q_0, q) \right|^2 \sim g^2 T \int_0^{\omega_p} dq q. \tag{4}
\]

With an IR cut-off \( \mu \), \( \gamma_t \sim g^2 T \ln(\omega_p/\mu) \). The remaining logarithmic divergence is due to collisions involving the exchange of very soft, quasistatic \( (q_0 \to 0) \), magnetic photons, which are not screened by plasma effects. To see that, note that the IR contribution to \( \gamma_t \) comes from momenta \( q \ll gT \), where \( \left| \star D_t(q_0, q) \right|^2 \) is almost a delta function of \( q_0 \):

\[
\left| \star D_t(q_0, q) \right|^2 \simeq \frac{1}{q^4 + (3\pi \omega_p^2 q_0/4q)^2} \to q \to 0 \frac{4}{3q\omega_p^2} \delta(q_0). \tag{5}
\]

This is so because, as \( q_0 \to 0 \), the imaginary part of the polarisation tensor vanishes linearly (see the second equation [3]), a property which can be related to the behaviour of the phase space for the Landau damping processes. Since energy conservation requires \( q_0 = q \cos \theta \), where \( \theta \) is the angle between the momentum of the virtual photon \( (q) \) and that of the incoming fermion \( (p) \), the magnetic photons which are responsible for the singularity are emitted, or absorbed, at nearly 90 degrees.
3 A non perturbative calculation

The IR divergence of the leading order calculation invites to a more thorough investigation of the higher orders contributions to $\gamma$. Such an analysis reveals strong, power-like, infrared divergences, which signal the breakdown of the perturbation theory. To a given order in the loop expansion, the most singular contributions to $\gamma$ arise from the quenched (no internal fermion loops) self-energy diagrams in Fig. 2, where all the internal photon lines are of the magnetic type. As in the one loop calculation, the leading divergences arise, in all orders, from the kinematical regime where the internal photons are soft ($q \to 0$) and quasistatic ($q_0 \to 0$). Physically, these divergences come from multiple magnetic collisions.

This peculiar kinematical regime can be conveniently exploited in the imaginary time formalism, where the internal photon lines carry only discrete (and purely imaginary) energies, of the form $q_0 = i\omega_n = i2\pi nT$, with integer $n$ (the so-called Matsubara frequencies). Note that the non-static modes, with $n \neq 0$, are well separated from the static one $q_0 = 0$ by a gap of order $T$. In this formalism, all the leading IR divergences of the damping rate — which, I recall, arise from the kinematical limit $q_0 \to 0$ — are concentrated in diagrams in which the photon lines are static, i.e., they carry zero Matsubara frequency. Note that, for these diagrams, all the loop integrations are three-dimensional (they run over the three-momenta of the internal photons), so that the associated IR divergences are those of a three-dimensional gauge theory. This clearly emphasizes the non perturbative character of the leading IR structure.

In what follows, we restrict ourselves to these diagrams, and compute their contribution to the fermion propagator near the mass-shell, in a non perturbative way. This can be “exactly” done in the Bloch-Nordsieck approximation, which is the relevant approximation for the infrared structure of interest. Namely, since the incoming fermion is interacting only with very soft ($q \to 0$) static ($q_0 = 0$) magnetic photons, its trajectory is not significantly deviated by the successive collisions, and its spin state does not change. Thus, we can ignore the spin degrees of freedom, which play no dynamical role, and we can assume the fermion to move along a straightline trajectory with constant velocity $v$ (for the ultrarelativistic hard fermion, $|v| = 1$; more generally, for the soft excitations, $v(p) \equiv \partial E(p)/\partial p = v(p)\hat{p}$ is the corresponding group velocity, with $|v(p)| < 1$). Under these assumptions, the fermion

Figure 2: A generic $n$-loop diagram (here, $n = 6$) for the self-energy in quenched QED.
The propagator can be easily computed as

\[ S_R(t, p) = i \theta(t) e^{-iE(p)t} \Delta(t), \]  

where

\[ \Delta(t) = \exp \left\{ -g^2 T \int_{\omega_p} \frac{d^3q}{(2\pi)^3} \frac{1}{q^2} \frac{1 - \cos t(v(p) \cdot q)}{(\hat{p} \cdot q)^2} \right\}, \]

contains all the non-trivial time dependence. The integral in eq. (7) is formally identical to that one would get in the Bloch-Nordsieck model in 3 dimensions. Note, however, the upper cut-off \( \omega_p \sim gT \), which occurs for the same reasons as in eq. (4). Namely, it reflects the dynamical cut-off at momenta \( \sim gT \), as provided by the Landau damping.

The integral over \( q \) has no infrared divergence, but one can verify that the expansion of \( \Delta(t) \) in powers of \( g^2 \) generates the most singular pieces of the usual perturbative expansion for the self-energy. Because our approximations preserve only the leading infrared behavior of the perturbation theory, eq. (6) describes only the leading large-time \( (t \gg 1/gT) \) behavior of \( \Delta(t) \). This is gauge independent and of the form (we set here \( \alpha = g^2/4\pi \) and \( v(p) = 1 \) to simplify writing)

\[ \Delta(\omega_p t \gg 1) \simeq \exp(-\alpha T t \ln \omega_p t). \]  

Thus, contrary to what perturbation theory predicts, \( \Delta(t) \) is decreasing faster than any exponential. It follows that the Fourier transform

\[ S_R(\omega, p) = \int_{-\infty}^{\infty} dt e^{-i\omega t} S_R(t, p) = i \int_0^{\infty} dt e^{i(\omega - E(p) + i\eta)} \Delta(t), \]

exists for any complex (and finite) \( \omega \). Thus, the retarded propagator \( S_R(\omega) \) is an entire function, with sole singularity at \( \text{Im} \omega \to -\infty \). The associated spectral density \( \rho(\omega, p) \) (proportional to the imaginary part of \( S_R(\omega, p) \)) retains the shape of a resonance strongly peaked around the perturbative mass-shell \( \omega = E(p) \), with a typical width of order \( \sim g^2 T \ln(1/g^2) \).
and governed by the plasma frequency, according to our result (8): \[ \Delta(t) \sim \exp(-\alpha T \ln \omega_p t). \]

Thus, at least within this limited model, which is QED with a “magnetic mass”, the time behavior in the physical regime remains controlled by the Bloch-Nordsieck mechanism. But, of course, this result gives no serious indication about the real situation in QCD, since it is unknown whether, in the present problem, the effects of the gluon self-interactions can be simply summarized in terms of a magnetic mass.

References

1. J.P. Blaizot, J.-Y. Ollitrault and E. Iancu, *Collective Phenomena in the Quark-Gluon Plasma*, in *Quark-Gluon Plasma 2*, ed. R.C. Hwa (World Scientific, Singapore, 1996).
2. M. Le Bellac, *Recent Developments in Finite Temperature Quantum Field Theories* (Cambridge University Press, 1996).
3. R.D. Pisarski, *Phys. Rev. Lett.* **63** (1989) 1129; E. Braaten and R.D. Pisarski, *Phys. Rev. Lett.* **64** (1990) 1338; *Phys. Rev.* **D42** (1990) 2156; *Nucl. Phys.* **B337** (1990) 569.
4. V.V Lebedev and A.V. Smilga, *Phys. Lett.* **B253** (1991) 231; *Ann. Phys.* **202** (1990) 229; *Physica* **A181** (1992) 187.
5. C.P. Burgess and A.L. Marini, *Phys. Rev.* **D45** (1992) 482; F. Flechsig, H. Schulz and A.K. Rebhan, *Phys. Rev.* **D52** (1995) 2994.
6. R.D. Pisarski, *Phys. Rev.* **D47** (1993) 5589; T. Altherr, E. Petitgirard and T. del Rio Gaztelurrutia, *Phys. Rev.* **D47** (1993) 703; H. Heiselberg and C.J. Pethick, *Phys. Rev.* **D47** (1993) R769; A. Niégawa, *Phys. Rev. Lett.* **73** (1994) 2023; K. Takashiba, [hep-ph/9501223](http://arxiv.org/abs/hep-ph/9501223) (unpublished).
7. S. Peigné, E. Pilon and D. Schiff, *Z. Phys.* **C60** (1993) 455; A.V. Smilga, *Phys. Atom. Nuclei* **57** (1994) 519; R. Baier and R. Kobes, *Phys. Rev.* **D50** (1994) 5944.
8. G. Baym, H. Monien, C.J. Pethick, and D.G. Ravenhall, *Phys. Rev. Lett.* **64** (1990) 1867.
9. E. Braaten and M.H. Thoma, *Phys. Rev.* **D44** (1991) 1298.
10. R. Kobes, G. Kunstatter and K. Mak, *Phys. Rev.* **D45** (1992) 4632; E. Braaten and R.D. Pisarski, *Phys. Rev.* **D46** (1992) 1829.
11. E. Fradkin, *Proc. Lebedev Phys. Inst.* **29** (1965) 7; J.P. Blaizot, E. Iancu and R. Parwani, *Phys. Rev.* **D52** (1995) 2543.
12. J.P. Blaizot and E. Iancu, *Phys. Rev. Lett.* **76** (1996) 3080 and [hep-ph/9607303](http://arxiv.org/abs/hep-ph/9607303), to appear in *Phys. Rev.* **D**.
13. H.M. Fried, *Functional Methods and Models in Quantum Field Theory* (the MIT Press, Boston, 1972).
14. E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics* (Pergamon Press, Oxford, 1981).
15. J.P. Blaizot and E. Iancu, *Nucl. Phys.* **B459** (1996) 559.
16. C. DeTar, *Quark-Gluon Plasma in Numerical Simulations of Lattice QCD*, in *Quark-Gluon Plasma 2*, ed. R.C. Hwa (World Scientific, Singapore, 1996).