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An extended state observer based U-model control of the COVID-19

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A B S T R A C T

The coronavirus disease 2019 (COVID-19) is a new, rapidly spreading and evolving pandemic around the world. The COVID-19 has seriously affected people’s health or even threaten people’s life. In order to contain the spread of the pandemic and minimize its impact on economy, the tried-and-true control theory is utilized. Firstly, the control problem is clarified. Then, by combining advantages of the U-model control and the extended state observer (ESO), an extended state observer-based U-model control (ESOUC) is proposed to generate a population restriction policy. Closed-loop stability of the regulation system is also proved. Two examples are considered, and numerical simulation results show that the ESOUC can suppress the COVID-19 faster than the linear active disturbance rejection control, which benefits controlling the infectious disease and the economic recovery. The ESOUC may provide a feasible non-pharmaceutical intervention in the control of the COVID-19.

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1. Introduction

When we write this paper, a vast majority of people from all over the world are experiencing physical-distancing policies or confining to homes so as to contain the coronavirus disease 2019 (COVID-19), which is one of the worst pandemics. The COVID-19 is emerging and rapidly evolving. It results in severe acute respiratory syndrome and substantial morbidity and mortality. According to a report of the World Health Organization dated by May 16th, 2020, the number of the confirmed infections had increased to 4.65 million cases, with 1.7 million recovered patients and 300,000 deaths due to the COVID-19 in 213 countries [1]. Harsh realities, such as unknown nature, ambiguous origin, high transmission rate, no exact medicine, and no suitable vaccine, make the COVID-19 become a global pandemic. Most of the governments take various stringent lockdown policies to stop spread of the coronavirus. For example, in China, all clustering social activities have been ceased, students from all levels have been taking classes on line at home, people are encouraged to stay at home to reduce the possibility of the infection. The containment strategies do work, and most governments, including Germany, Italy and South Korea, take similar lockdown measures to limit the spread of the COVID-19, and they have passed the peak of cumulative infectious cases [1]. However, consequently, global economy plummets, industries ground to a halt and millions of people are out of work.

Based on above harsh realities, it is natural to ask what is an effective approach to exit such a dilemma? When is safe to apply it? and how to apply is suitable? Therefore, experts from different areas, like epidemiologists, statisticians, biologists, doctors, and health officials, are focusing on those problems. Actually, to deal with the COVID-19, effective medicines and vaccines are critical. However, so far, there are no effective vaccines. Additionally, parts of rehabilitative patients re-infected. In other words, effective medicines are still necessary, but the nature of the coronavirus is also a mystery. Therefore, besides the medicines and vaccines, it is crucial to devise a feasible and effective mitigation or control strategies to contain the COVID-19, and treat the infected patients. Moreover, the COVID-19 pandemic is an unstable open-loop system, the infected people will increase exponentially, if no effective measure is applied. Therefore, although the COVID-19 pandemic is not a typical engineering problem, feedback, a classic and effective approach in system and control, can also be taken to stabilize and diminish the propagation of the coronavirus [2]. Then, outbreak of the COVID-19 will be under control, the long-term response of the COVID-19 can be adeptly managed, and finally, maximum survival and minimum damage to the economy can also be acquired [2].

To achieve those targets, a computational model of the disease is necessary, since it does help to analyze and predict the pandemic. Furthermore, a model also provides an access to the
closed-loop control. Then, evaluations on infectious disease control or mitigation strategies before they are applied become possible. Based on some assumptions, several models have been established to describe dynamical behaviors of the COVID-19 [3–9].

On the basis of these models, systematically designed control strategies have been proposed. Optimal control is designed to minimize the infected people and the associated cost [8]. Genetic algorithm is also taken to obtain an optimal solution to address the COVID-19 [9]. So far, most of the reported methods to control the COVID-19 are based on the infectious disease model, and an optimization algorithm is employed to find an optimal policy. However, according to a recent research paper from Imperial College London, large uncertainties exist during the transmission of the COVID-19 [2]. Uncertainties will result in rebound spikes [2]. Therefore, an effective policy, which is able to lessen the influence of the uncertainties as much as possible, would be invaluable, since it could save countless lives. Therefore, uncertainties are taken into consideration in the control of the COVID-19 [10].

Simultaneously, it is worth pointing out that although non-pharmaceutical interventions (NPIs), like closing borders, enforcing lockdowns, do suppress the spread of the COVID-19, they also give rise to economic recession. Therefore, it is necessary for the policymakers to take account of both disease transmission and economic developments. Then, a better NPI which mitigates or suppresses the pandemic faster and deal with the nonlinearities and uncertainties of the COVID-19 better, can reduce the impact of the pandemic on both death rates and economy. It means that a control strategy, which can shorten the transient time of the closed-loop system (suppress the pandemic in a shorter time), diminish the influence of the pandemic on economy to a great extent, and achieve better control on epidemic even in presence of various uncertainties and disturbances, is more suitable or necessary to suppress the COVID-19.

The U-model control (UC), proposed by Zhu in 2002 [11], is a proper control strategy. Its key point is that, by getting an inversion, a plant can be reshaped into ‘1’. Then, ideally, no phase delay exists between inputs and outputs of a plant. Accordingly, the system response becomes much faster. In addition, one can preset the closed-loop system performance on the basis of the reshaped plant (namely the unit). Based on key point of the UC, some new methods have been proposed. For example, the U-model based fuzzy PID control [12], the U-model based adaptive control [13], and the U-model based predictive control [14] have been proposed and satisfied results have been achieved. However, there are also some disadvantages on the UC [15]. First, for a complex plant, it is not easy to obtain its inverse. Secondly, current results on the UC are almost discrete-time, few results are in continuous-time form. Finally, the UC does not take uncertainties and external disturbances into consideration.

In order to overcome the disadvantages of the UC, an extended state observer (ESO) is employed. It makes full use of inputs and outputs of a plant to reconstruct system states and the total disturbance. Then, based on the estimation of the total disturbance, the total disturbance can be canceled out, and influence of both disturbances and uncertainties on system outputs can be minimized to a great extent [16]. For desired closed-loop dynamics, in recent years, based on the ESO, the linear active disturbance rejection control (LADRC) has been applied in many fields [17–19]. Here, an ESO is also utilized. By estimating and compensating the total disturbance, dynamics of an nth order plant (no matter it is linear/nonlinear, time varying/time invariant) can be reshaped into pure integrators in series (1/sn), and its inversion becomes s

\[ S = \frac{1}{s} \]

Then, disadvantages of the UC mentioned above are overcome. Simultaneously, faster response of the UC can also be utilized. Thus, by integrating advantages of the UC and the ESO, an extended state observer-based U-model control (ESOUC) is put forward. Based on the ESOUC and the full understanding of the dynamics of the epidemic, a more promising population restriction policy is produced to contain the COVID-19 pandemic in a shorter time, and impact of the pandemic on economy is also minimized.

The rest of this paper is organized as follows. Section 2 presents a model of the COVID-19, and clarifies the control problem. Section 3 presents the control strategy, including its design and analysis. Section 4 shows two numerical examples to demonstrate the developed strategy. Finally, Section 5 draws conclusions.

2. A mathematical model and the problem statements

In this section, a mathematical model of the COVID-19 is presented to establish a basis for the numerical research on containing the pandemic. Then, some basic issues for the control of the COVID-19 are clarified.

2.1. A model of the COVID-19

Here, a mathematical model of the COVID-19 proposed in Ref. [8] is taken, since it includes a quarantined class and governmental intervention measures to mitigate the transmission of the pandemic. Its flow diagram is shown in Fig. 1 [8].

In this model, population can be divided into five classes, i.e., the susceptible class \( S(t) \), the exposed class \( E(t) \), the quarantined class \( Q(t) \), the recovered class \( R(t) \), and the removed class \( R(t) \).

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\[ \begin{align*}
\dot{S} &= A - \beta(1 - \rho_1)(1 - \rho_2)SE + b_1Q - dS - pMS \\
\dot{E} &= \beta(1 - \rho_1)(1 - \rho_2)SE - b_2E - aE - \sigma E - dE \\
\dot{Q} &= b_2E - b_1Q - cQ - dQ \\
\dot{l} &= \alpha E + cQ - nI - dI - dI \\
\dot{R} &= nI + \sigma E - dR + pMS
\end{align*} \]

Table 1 shows parameters of the model (1).

| Parameter | Description |
|-----------|-------------|
| \( \alpha \) | Transition rate from exposed to quarantined |
| \( b_1 \) | Movement rate from susceptible to exposed |
| \( b_2 \) | Movement rate from exposed to quarantined |
| \( c \) | Recovery rate from quarantined |
| \( d \) | Mortality rate |
| \( p \) | Proportion of infected cases that are quarantined |
| \( \sigma \) | Removal rate from exposed to recovered |
| \( \rho_1 \) | Proportion of exposed individuals that are quarantined |
| \( \rho_2 \) | Proportion of quarantined individuals that are recovered |

System (1) has two equilibria. One is the disease-free equilibrium (DFE) \( E_0(S_0, 0, 0, 0, R_0) \), and the other is the endemic equilibrium (EE) \( E_1(S_1, E_1, Q_1, R_1) \). Here, \( S_0 = \frac{A}{d + pM} \), \( S_1 = \frac{A}{d + pM} \).
Table 1

| Parameters | Descriptions |
|------------|--------------|
| $A$        | The recruitment to the susceptible population |
| $\beta$    | The disease transmission rate |
| $\rho_1$   | The portion of $E$ contact with $E$ |
| $\rho_2$   | The portion of $S$ contact with $E$ |
| $d$        | The natural death rate |
| $b_1$      | The rates that Q becomes S |
| $b_2$      | The rates that E becomes Q |
| $\alpha$   | The rates that E becomes I |
| $\eta$     | The natural recovery rate of the hospitalized infected people |
| $\sigma$   | The natural recovery rate of the exposed class E |
| $c$        | The rate that Q becomes I |
| $\delta$   | The COVID-19 induced death rate |
| $M$        | The awareness due to the media coverage |
| $p$        | The implementation rate of $M$ |

\[
\frac{b_2 + u + \alpha + d}{\beta(1 - \rho_1)(1 - \rho_2)}, \quad \text{and} \quad R_0 = \frac{A\beta(1 - \rho_1)(1 - \rho_2)}{(d + pM)b_2 + \alpha + \sigma + d} \quad [8].
\]

If the system runs at the DFE, it means the infection vanishes, and if the system runs at the EE, it means the infection is also prevalent.

In addition, for the two equilibrium points of the system (1), one has that the DFE $E_0$ is locally asymptotically stable if the pandemic cannot invade the healthy population [8]. At the same time, the EE $E_1$ is also asymptotically stable when the epidemic is still able to invade the healthy population, or the invasion is always possible [8]. Obviously, if one wants to contain the pandemic, system (1) should run at the DFE and make the pandemic cannot invade the healthy population. In following sections, problems including how to drive the system to run at the DFE and how to make the disease cannot invade the healthy population are discussed.

2.2. Problem statements

Unlike an electromechanical system, the control of a pandemic is somewhat special. Thus, before designing a control system to suppress the infectious disease, four issues should be clarified.

(1). What is the goal of the control?
(2). What is the reference signal?
(3). What is the controlled variable?
(4). What is the physical interpretation of the control signal?

To answer these questions, firstly, a critical parameter should be introduced. It is well known that a key issue on any epidemic is its ability to invade the healthy people. Generally, a threshold parameter, which is known as the basic reproduction number $R_0$, is defined. It is time-varying. Various factors, such as the density of a community, the general health of its populace, medial infrastructure and (or) resource, and details of the community's response, decide its value [2]. According to Ref. [8], the basic reproduction number $R_0$ of model (1) is

\[
R_0 = \frac{A\beta(1 - \rho_1)(1 - \rho_2)}{(d + pM)b_2 + \alpha + \sigma + d}.
\]

If $R_0 > 1$, every infected individual produces more than one new infection. It means that the disease is still able to invade the healthy people. On the contrary, when $R_0 < 1$, an infected person produces less than one new infected individual during the infectious period. It signifies that, in this case, the disease cannot invade the healthy population any more. Therefore, $R_0$ is an indicator that the number of new infections or secondary cases, on average, is produced by a typical infective person during the whole course of his/her illness. Then, $R_0$ is utilized to reflect the expectation of the spreading disease [8].

According to Ref. [2], for an ordinary seasonal influenza, $R_0 = 1.3$, however, for the COVID-19, $R_0 = 2.6$ in Wuhan, China. For the outbreaks in Italy, $R_0$ ranges from 2.76 to 3.25. Thus, reducing $R_0$ to be less than 1 is crucial to suppress the COVID-19. In other words, the goal of the infectious-disease intervention is to reduce $R_0$ below 1, even if large uncertainties exist.

In Ref. [8], awareness $M(t)$ due to the media coverage is taken as a control strategy. However, in this paper, instead of $M(t)$, a population restriction policy is generated to suppress the contagion. In this case, $M(t)$ is chosen as a fixed value, which means the awareness of media coverage reaches its maximum benefit. Considering that the parameter $A$ represents the daily recruitment, and it has a positive impact on $R_0$ [8], reducing $A$ is an alternative way to control the pandemic. Thus, a population restriction policy $u$ is designed to reduce $A$, and $(A - u)$ is a new recruitment. Then, the controlled system is shown in (3)

\[
\begin{align*}
\dot{S} &= (A - u) - \beta(1 - \rho_1)(1 - \rho_2)SE + b_1Q - dS - pMS \\
\dot{E} &= \beta(1 - \rho_1)(1 - \rho_2)SE - b_2E - \alpha E - \sigma E - dE \\
\dot{Q} &= b_2E - b_1Q - cQ - dQ \\
\dot{I} &= \alpha E + cQ - \eta I - dI - \delta I \\
\dot{R} &= \eta I + \sigma E - dR + pMS
\end{align*}
\]

Theoretically, when $u = A$, it means that the city is completely lockdown. However, it definitely results in economic decline more seriously. Therefore, a systematically designed strategy (or an NPI) is essential to provide a scientific regulation and decrease $R_0$.

Note that $S_0/S_1 = R_0$, and when $R_0 < 1$, model (1) is local asymptotically stable at the DFE $E_0$ [8]. Therefore, a reference value $R_0^* < 1$ can be set for $R_0$, and $S_0^* = R_0^*S_1$. Then, let the susceptible $S(t)$ be a controlled variable, and its reference signal is $S_0^*$, which is determined by the expected basic reproduction number $R_0^*$. Finally, spread of the COVID-19 is controlled by a population restriction policy $u$. Then, no matter what initial values $[S(0), E(0), Q(0), I(0), R(0)]^T$ are, with the help of a population restriction policy $u$, the controlled system (3) runs at the DFE.

So far, four issues on controlling the pandemic have been clarified, and the population restriction policy is designed and analyzed in following section.

3. Population restriction policy

Here, in order to suppress the pandemic as soon as possible, the ESOU UC is designed. Utilizing advantage of an ESO, uncertainties existing in the transmission of the COVID-19 can be addressed effectively. Simultaneously, advantage of the UC makes the response of the closed-loop system be faster. Thus, the ESOU UC is able to generate a proper population restriction policy, so that the spread of the pandemic can be maintained at a desired level in a shorter period.

3.1. Brief introduction to the U-model control

Key idea of the UC is to find an inversion of a plant so as to make the plant be unit. Then, the desired closed-loop performance can be specified by determining/designing a controller including the inversion of the plants/processes. Structure of the UC is presented in Fig. 2. In which, $r$ is a reference signal, $y$ is the system output, $G_p$ and $G_c$ are system model and its inversion. The transfer relationship of the forward path in Fig. 2 is $G_cG_p^{-1}G_u$, and an equivalent structure of Fig. 2 is shown in Fig. 3. Then, one can obtain the desired closed-loop dynamics by designing $G_c$.

From Fig. 3, one can see that the reshaped controlled plant is “1”. It depicts a fact that there are no amplitude attenuation and phase delay for the reshaped controlled plant. Then, faster system response can be guaranteed. It is perfect in any control system, but the question is that how to get a $G_p^{-1}$? Actually, it is still a challenge, especially for a nonlinear time-varying uncertain
system, so does the dynamics of the COVID-19. Thus, the key point in realizing the UC is to find an effective way to obtain the inversion of a controlled plant, no matter it is a linear or nonlinear, time invariant or time varying plant.

Thanks to the ESO, a feasible way to dynamically shape any system to be a pure integrators chain is provided. Then, any system to be a pure integrators chain is provided. Then, any nonlinear, time invariant or time varying plant.

Based on states of the ESO (5), a control law is

\[ u = u_0 - z_2 \tag{6} \]

Substituting (6) into (4), one has

\[ \dot{S} = u_0 + f - z_2 \approx u_0 \tag{7} \]

Here, \( u_0 \) is an auxiliary control law which is needed to be designed. Obviously, system (1) is approximately converted to be a linear system, and its inversion is a differentiator. Then, the ESOUC can be designed.

Remark 1. Key point of the LADRC is to dynamically reshape a controlled plant, no matter it is a linear or nonlinear, time-invariant or time-varying system, to be pure integrators in series. Then, one can design the closed-loop system dynamics as desired. Generally, a proportional differential (PD) controller is enough in engineering. For more details on the extended state observer and the active disturbance rejection control, one may refer to Refs. [16–21].

3.3. An extended state observer-based U-model control

Based on the UC and the ESO, an ESOUC can be designed. Structure of the ESOUC is presented in Fig. 4.

By an ESO, the total disturbance can be reconstructed. Based on the reconstructed total disturbance, the controlled system can be dynamically linearized to be an integrator \( s^{-1} \) (see the green background box, here \( s \) is the Laplace operator). Then, according to the idea of the UC, a controller includes two parts, one is a differentiator \( s \), which is the inversion of the dynamically linearized plant, and the other is \( G_c \), which is designed to achieve the desired closed-loop performance. If an ESO works as expected, Fig. 4 will be equivalent to Fig. 3. Here, \( G_c \) is designed as

\[ G_c = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} \tag{8} \]

where \( \zeta, \omega_n \) are tunable parameters, and they are determined according to performance requirements of the closed-loop system.

Remark 2. To realize the ESOUC, instead of using a PD controller, a controller composed of \( G_c \) and a pure differentiator \( s \) is designed. Similar to the realization of the LADRC, based on the structure of the ESOUC shown in Fig. 4, the ESOUC can be implemented to control the COVID-19, or other dynamic plants/processes.

Remark 3. A pure differentiator given in Fig. 4 is unrealizable in practice, but it can be realized by virtue of \( G_c \), if the numerator order \( m \) and denominator order \( n \) of \( G_c \) satisfy \( n - m > 1 \).

Remark 4. Based on the structure of the ESOUC, the closed-loop system is equivalent to the structure presented in Fig. 3. Then, one can pre-specify desired closed-loop dynamics, including the overshoot, the rising time, and the settling time, by choosing proper \( \zeta \) and \( \omega_n \). It is indeed important to contain the spread of the pandemic in a shorter time and reduce its influence on economy. Similarly, such a control strategy is also helpful to control an electromechanical system satisfactorily.

So far, the ESOUC is introduced, and the population restriction policy can be generated by the ESOUC. Before it can be applied in the control of the COVID-19, the stability of the closed-loop system should be analyzed.

3.4. Stability analysis

System (1) is locally asymptotically stable at the DFE \( E_0 \), if \( R_0 < 1 \) [8]. Therefore, \( S_0^* = R_0^* S_1 \) is taken as the set-value for the susceptible \( S(t) \). By virtue of the regulation policy \( u \), \( S(t) \) tracks \( S_0^* \), and the system runs at the DFE \( E_0 \). Then, the controlled COVID-19 system will be stable, if the controlled susceptible \( S(t) \) is stable. Next, the stability of the controlled susceptible \( S(t) \) is proved.

Firstly, convergent of an ESO should be guaranteed. Here, let change rate of the total disturbance be \( f \), one has a conclusion that a proper observer bandwidth \( \omega_n \) guarantees bounded estimation errors in a finite time, when \( f \) is bounded [21]. In view
of the solutions of model (1) are uniformly bounded [8], and all variables in the total disturbance $f$ will not change suddenly, then $\dot{f}$ is also bounded. Thus, the ESO (5) definitely converges in a finite time.

Secondly, based on the convergence of the ESO, the closed-loop stability is discussed. Transfer function of the ESO (5) is

$$
\begin{align*}
Z_1(s) &= \frac{\alpha_0^2 + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} S(s) + \frac{b}{s^2 + \omega_n^2} U(s) \\
Z_2(s) &= \frac{\alpha_0^2 + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} S(s) - \frac{b\omega_n^2}{s^2 + \omega_n^2} U(s)
\end{align*}
$$

Substituting (9) into (6), and on the basis of the relationship depicted in Fig. 4, one has

$$
u(s) = \frac{1}{b} \left( \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \right) \frac{\alpha_0^2 + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} S(s)
$$

Then, an equivalent structure of the closed-loop system can be shown in Fig. 5. Here,

$$
G(s) = \frac{(s^2 + 2\zeta\omega_n s + \omega_n^2)}{(s^2 + 2\zeta\omega_n s + \omega_n^2) + \omega_n^2} S(s)
$$

$$
H(s) = \frac{(s + 2\zeta\omega_n s + \omega_n^2)}{(s + 2\zeta\omega_n s + \omega_n^2)}
$$

For the system shown in Fig. 5, one has the closed-loop transfer function between $S_0$ and $S$

$$
G_{CL}(s) = s G_c \frac{s + 2\zeta\omega_n s + \omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{\alpha_0^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
$$

It is a typical second-order system, and there exist two closed-loop poles $s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$. Then, one may choose proper or optimized $\omega_n$, $\zeta > 0$ to stabilize the system and achieve desired closed-loop system performance.

Thus, the susceptible $S(t)$ can be stabilized, and the closed-loop stability is also guaranteed.

4. Simulation results

Here, simulations are performed to confirm the strategy. Two examples in Ref. [8] are also taken in this paper. In the first example, some uncertainties on different classes of population are introduced to verify the robustness of the population restriction policies. Then, the case of Delhi, one of the states in India, is considered, and the strategy is also demonstrated. In all examples, the simulation step size is set to be $1 \times 10^{-4}$.

4.1. Example 1

In this example, parameters listed in Table 2 are taken from Ref. [8]. Two uncertain population changing cases are considered. In Case I, constant uncertainties are introduced, and in Case II, time-varying uncertainties are taken into consideration. The controlled system including population uncertainties is described as

$$
\begin{align*}
\dot{S} &= (A - \mu) - \beta(1 - \rho_1)(1 - \rho_2)SE + b_1 Q - dS - pMS + d_1 \\
\dot{E} &= \beta(1 - \rho_1)(1 - \rho_2)SE - b_2 E - \alpha E - \sigma E - dE + d_2 \\
\dot{Q} &= b_2 E - b_3 Q - cQ - dQ + d_3 \\
\dot{l} &= \alpha E + cQ - \eta l - dl - d_1 \\
\dot{R} &= \eta l + \sigma E - dR + pMS + d_4
\end{align*}
$$

Remark 5. Here, population uncertainties, coming from the susceptible $S(t)$, the exposed $E(t)$, the quarantined $Q(t)$, and the recovered $R(t)$, are considered. Considering that non-native population are not able to become the hospitalized infected population $I(t)$ directly, population uncertainties are not introduced into the Class $I(t)$.

Case I: Constant uncertainties on population

In this case, population changes are described as four kinds of constant uncertain events $d_1$–$d_4$.

1. $d_1 = 5 \times 1(t - 50)$, which means, from the 50th day, five susceptible people join the system.
2. $d_2 = 5 \times 1(t - 100)$, which means, from the 100th day, five exposed people join the system.
3. $d_3 = 5 \times 1(t - 150)$, which means, from the 150th day, five people join the system and they are immediately quarantined.
4. $d_4 = -5 \times 1(t - 200)$, which means, from the 200th day, five people are recovered and removed from the system.

Table 3 presents parameters of the ESOU and the LADRC. Here, $\omega_h$ is the control bandwidth in the LADRC [20].

System initial condition is $[S(0), E(0), Q(0), I(0), R(0)]^T = [500, 10, 5, 1, 0]^T$. Initial values of an ESO is set to be $z_e(0) = S(0), z_q(0) = f(0) = A - \beta(1 - \rho_1)(1 - \rho_2)SE + b_1 Q - dS - pMS(0)$, then, the ESO works better.

The initial basic reproduction number $R_0(0) = 1.8204$. After regulation, one expects the final basic reproduction number $R_0(t)$
Fig. 7. Dynamic evolution of the five classes population (Example I Case I).

Based on the parameters listed in Table 3, the population restriction policies (regulation signals) generated by the ESOUC and the LADRC are shown in Fig. 6. It provides the suggested population restriction policies per day. $(A - u)$ represents a new recruitment, or the maximum of the recruitment. It is worth pointing out that faster regulation policy generated by the ESOUC promises a faster response to the desired $R_0^*$ or the new equilibrium point. Changes of the five classes population are shown in Fig. 7. From the evolution of the susceptible people, one can see that, with the help of the control strategies, a new equilibrium point is achieved. By virtue of the regulation policies, the exposed, the quarantine, and the hospitalized infected people reduce sharply. It signifies that, by restriction policies, the risk of infection reduces greatly. Simultaneously, when constant uncertainties exist, one can find that the ESOUC and the LADRC are also able to overcome the uncertainties and reduce the risk of infection.

In addition, in face of the uncertainties $d_1 \sim d_4$, the restriction policy $u$ (shown in Fig. 6) adapts immediately to those uncertainties. However, considering delay on the influence of the control, $E(t)$, $Q(t)$ and $I(t)$ shown in Fig. 7(b)∼(d) increase. From the controlled system (12), one can also find that, when uncertainties $d_1$ are introduced to $Q(t)$, both $Q(t)$ and $I(t)$ rise. Fig. 7(c)∼(d) confirm the rising $Q(t)$ and $I(t)$. Since uncertainties $d_1 \sim d_3$ last until the end of the simulations, $E(t)$, $Q(t)$ and $I(t)$ are not zero at the end of the simulations. Nevertheless, as a result of the regulation policy $u$, after the transient process, the basic reproduction number $R_0$s are always less than one. It means that the pandemic cannot invade population any more, even if constant uncertainties exist.

The advantage of the ESOUC can be found in Fig. 8, which shows the evolution of the basic reproduction number $R_0$. One can see that the ESOUC is able to contain the pandemic faster, which is more helpful to reduce both death rates and economic impacts.

To show the advantage of the ESOUC clearer, quantitative comparisons of $R_0$ between the ESOUC and the LADRC are listed in Table 4. From the table, one can see that the ESOUC just takes 20 days to reduce $R_0$ from 1.8204 to the expected value 0.5. Comparatively, the LADRC needs 49 days to lower $R_0$ from 1.8204 to 0.5109. Obviously, the suppression period is shortened by 29 days. Therefore, the ESOUC can provide a faster and more accurate population restriction policy to achieve an expectant suppression of the pandemic. Additionally, from the 50th day, some constant uncertainties result in fluctuations of $R_0$. However, since the same ESO and the same bandwidth are adopted in the ESOUC and LADRC, the estimation ability of the total disturbance is also the same. Thus, the maximum fluctuation of the ESOUC and the LADRC are both 0.178.

Case II: Time-varying uncertainties on population

Considering that, in most cases, uncertainties are time-varying. Therefore, in this case, the time-varying uncertainties are added to the system. Here, $d_1 \sim d_4$ in controlled system (12) are designed...
Table 6. Parameter values of Delhi.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| $A$        | 2144   | $\alpha$   | 0.25   |
| $\beta$    | 1.5    | $\eta$     | 1/14   |
| $\rho_1$   | 0.72   | $\sigma$   | 0.14029|
| $\rho_2$   | 0.82   | $\zeta$    | 0.12   |
| $d_1$      | 0.0036 | $\delta$   | 0.0232 |
| $b_1$      | 0.045185 | $M$   | 0.925161|
| $b_2$      | 0.78529 | $\rho$     | 0.923769|

Table 7. Parameters of the ESOUC and the LADRC (Example II).

|                | $b$   | $\omega_c$ | $\omega_1$ | $\omega_2$ | $\zeta$ |
|----------------|-------|------------|------------|------------|---------|
| ESOUC          | $-1$  | 20         | $-$        | 1          | 1       |
| LADRC          | $-1$  | 20         | 0.4        | $-$        | $-$     |

Fig. 9. Input population restriction policies (Example I Case II).

### 4.2. Example II

In this example, data of Delhi, a northern state of India, are utilized, and they are given in Table 6. Here, except $\beta$ and $c$ are estimated, other parameters are taken from Ref. [8]. $[S(0), E(0), Q(0), I(0), R(0)]^\top = [134555, 214, 845, 2, 0]^\top$ [8].

Table 7 lists parameters of the ESOUC and the LADRC. Initial values of an ESO are also set to be $z_1(0) = S(0), z_2(0) = f(0) = A - \beta(1 - \rho_1)(1 - \rho_2)S(0)E(0) + b_1Q(0) - dS(0) - pMS(0)$, such that the ESO works better.

Similarly, from model (1), one can find that, for a period of time, the recovery rate of the recovered $R(t)$ is positive, and then it becomes negative. Thus, the recovered $R(t)$ presented in Fig. 13(e) rises first, then declines until $dt = pMS$.

Quantitative comparisons of $R_0$ are shown in Table 8. The ESOUC takes 71 days to control $R_0$ to arrive at the expected value 0.5, while the LADRC needs 86 days. It also shows the advantage of the ESOUC on suppressing the pandemic.

### Remark 7.

Maximum fluctuation of $R_0$ depicts the fluctuations of $R_0$ in presence of external uncertainties and/or disturbances. In this case, based on the real data of Delhi, the uncertainties or disturbances are not introduced. Therefore, there is no maximum fluctuation of $R_0$ in Table 8.

From the results of both cases, one can see that the population restriction policy produced by the ESOUC is more suitable to contain the contagion, even if time-varying uncertainties exist.
constant uncertainties (see Fig. 6) and the time-varying uncertainties (see Fig. 9) on different classes population. In other words, the population restriction policies adapt to those uncertainties. As a result of the changing population regulation policies, the basic reproduction number $R_0$ is always in a satisfactory level, even if lasting uncertainties exist on different classes population. It means that both ESOUC and LADRC can contain the pandemic and treat the infected patients, and they are robust enough to the uncertainties. However, comparatively, the ESOUC is more suitable to obtain a regulation policy for its faster system responses.

5. Conclusions

The COVID-19 pandemic has been partially pausing plenty of aspects in societies, and it has become a threat to the world. To tackle it effectively, increasing collaboration across disciplines and sectors is indispensable. In this paper, the ESOUC is proposed to generate an NPI to contain the spread of the COVID-19, and treat the infected patients. The generated policy takes both dynamics and uncertainties into account, simulation results show that the basic reproduction number does reduce to the expected value, and the high and sharp curve are flattened. In addition, the results also show that the ESOUC can suppress the pandemic faster. It signifies that the ESOUC may be a more feasible strategy to minimize both death rates and economic impacts.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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| Table 8 | Comparison of $R_0$ between the ESOUC and the LADRC (Example II). |
|---------|-------------------------------------------------------------------|
| Initial $R_0(0)$ | Final $R_0(t)$ | Stable time of $R_0$ (days) | Maximum fluctuation of $R_0$ |
| ESOUC 160.1623 | 0.5000 | 71 | – |
| LADRC 160.1623 | 0.5000 | 86 | – |
Fig. 13. Dynamical evolution of the five classes population (Example II).

Fig. 14. Evolution of the basic reproduction number by the ESOUC and the LADRC (Example II).

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