INTRODUCTION

Supersymmetric axion models are a possible solution to the strong CP problem [1]. In the axion and axino they have two potential promising dark matter (DM) candidates. A strength of the Minimal Supersymmetric Standard Model (MSSM) is that it is a complete model, *i.e.* in principle it should explain all phenomena from low–energy physics like the anomalous magnetic moment of the muon, $a_\mu$, to the LHC bounds and potential observations at the TeV scale, as well as the dark matter observations and direct detection constraints. If the axion or axino are to be considered as contributing to the DM they should be included in a complete self-consistent model, including CP violation. This is ultimately our goal. R-parity violating theories [2], that without an axion supermultiplet would lack a DM candidate, would be of particular interest in this context. In such theories the axino mixes with the neutrinos and neutralinos [3]. It is thus only in a complete model that potential correlations between the axion/axino sector and the more readily observable neutrino/neutralino sector can be determined. We will turn our attention to an R-parity violating SUSY model with an axion multiplet in a future publication.

The aim of the current paper is to point out a model building issue that is already present in simpler R-parity conserving models and which seems to have been overlooked in previous literature [4,17]. We have found, when extending the MSSM to include the minimal supersymmetric DFSZ axion model [18], that the scalar potential and scalar mass spectrum are not self consistent. The problem is related to the minimization of the scalar potential, which leads to a negative squared mass for the axion, the scalar partner of the axion. Thus one is forced to extend the model and include an explicit sector to spontaneously break the PQ symmetry. Only then the scalar spectrum is self consistent. We distinguish two cases: (1) the SUSY breaking scale is lower than the PQ breaking scale, and (2) the scales are comparable. We find that the mass of the axino, the fermionic partner of the axion, is very light in (1), while it is generically of the order of the other soft SUSY breaking masses in (2). We have implemented SUSY breaking via generic soft breaking terms, and thus make no explicit statement about the form and mediation of SUSY breaking. Having implemented this explicit extension, SUSY models formulated at the unification scale can be extended to include the axion superfield and can be consistently connected to the low–energy observable scale $O(1$ TeV$)$.

THE MINIMAL INCONSISTENT MODEL

It is not straightforward to embed the DFSZ axion in supersymmetric models. The trouble is that the non-supersymmetric DFSZ model [19,20] contains a term $g \varphi^2 H_1^2 H_d$ in the scalar potential, where the phase of the complex scalar field $\varphi$ is the axion, $H_u,d$ are two complex Higgs doublets, and $g$ is a dimensionless real coupling constant. In supersymmetry this term can not be obtained from a renormalizable superpotential. There are two solutions in the literature. In Ref. [21] the requirement of renormalizability was dropped. One can write a higher dimensional operator in the superpotential of the form $\frac{g}{M_{Pl}} \hat{A} \hat{A} \hat{H}_u \hat{H}_d$. The hat here denotes a superfield. The axion field is the CP-odd scalar component of $\hat{A}$. The choice of this operator also offers an answer to the $\mu$ problem [21]. The second solution is a simple recipe proposed in Ref. [21], which we review in this work. The inconsistency we point out for the latter applies to both models in their minimal version.

In the ansatz in Ref. [18] the $\mu$–term in the superpotential is replaced by

$$c_1 \hat{A} \hat{H}_u \hat{H}_d.$$

(1)
Here the superfields are gauge eigenstates and $c_1$ is a dimensionless coupling constant. We will see that the physical axion is a linear combination of the CP odd scalar components of $A$, $H_u$ and $H_d$, in complete analogy with the non-SUSY model of Ref. [19]. In order to compute the physical spectrum, one assumes that the scalar part of the superfield $A$, $A$, gets a vacuum expectation value (VEV) $(A) \sim f_a$, where $f_a \sim 10^{12}$ GeV is the PQ breaking scale [22]. We will see that simply assuming such a VEV without specifying explicitly the PQ breaking mechanism and stabilizing the PQ potential leads to an inconsistency in the model.

The $\mu$-term is effectively $\mu_{\text{eff}} = c_1(A)$, and thus the coupling $c_1$ has to be of order $10^{-10}$ for $\mu_{\text{eff}}$ to be of order the electroweak scale. It is not the aim of this note to address the $\mu$ problem [21, 23]. We just remark, as the authors of Ref. [18], that $c_1$ must be so tiny for the axion to be invisible.

The full superpotential, assuming R-parity conservation, reads

$$W = Y_u \hat{U} \hat{Q} \hat{H}_u + Y_d \hat{D} \hat{Q} \hat{H}_d + Y_e \hat{E} \hat{L} \hat{H}_d + c_1 \hat{A} \hat{H}_u \hat{H}_d . \quad (2)$$

$Y_{u,d,e}$ are $3 \times 3$ Yukawa matrices. We have suppressed generation and $SU(2)$ and $SU(3)$ gauge indices. The superfield $\hat{A}$ and the Standard Model superfields carry PQ charges, as is distinctive of DFSZ axion models, such that each term is invariant under the global $U(1)_{\text{PQ}}$. The scalar potential is given by

$$V = V_{\text{soft}} + V_F + V_D \quad (3)$$

where

$$V_F = \sum_\phi \left| \frac{\partial W(\phi)}{\partial \phi} \right|^2 , \quad (4)$$

$$V_D = \frac{1}{2} \left( \Phi T^{i,0}_\phi \Phi^* \right) \left( \Phi T^{j,0}_\phi \Phi^* \right) , \quad (5)$$

$$V_{\text{soft}} = T_u \tilde{u} \tilde{q} H_u + T_d \tilde{d} \tilde{q} H_d + T_e \tilde{e} \tilde{\nu} H_d + T_c_1 \hat{A} H_u \hat{H}_d + m_d^2 |A|^2 + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + \tilde{\phi}^* m_\phi^2 \tilde{\phi} , \quad (6)$$

with $\tilde{\phi} \equiv \{ \tilde{e}, \tilde{d}, \tilde{u}, \tilde{q} \}$, as well as $A$, $H_u$, $H_d$ the scalar components of the respective superfields. Here $W(\phi)$ denotes the superpotential evaluated as a function of scalar fields. $T_{u,d,e,c_1}$ are the trilinear soft breaking terms [24], elsewhere often denoted $A$. $T^{i,0}_\phi$ are the gauge generators. To avoid clutter we take the soft parameters to be real in the following equations. This restriction does not affect our conclusions. The conventional $B_\mu$-term resulting from Eq. (4) is given by $B_\text{eff} = T_{c_1}(A)$.

The parameters have to fulfill the following tadpole equations to minimize the scalar potential

$$\frac{\partial V}{\partial \phi_{d}} |_{\phi_{d} = 0} = m_d^2 v_d + \frac{1}{8} [4 c_d^2 v_d (v_d^2 + f_d^2) - 4 \sqrt{2} v_d B_{\text{eff}} + v_d (g_d^2 + g_d^2) (v_d^2 - v_u^2)] = 0 , \quad (7)$$

$$\frac{\partial V}{\partial \phi_{a}} |_{\phi_{a} = 0} = m_a^2 v_a + \frac{1}{8} [4 c_a^2 v_a (v_a^2 + f_a^2) - 4 \sqrt{2} v_a B_{\text{eff}} - v_a (g_a^2 + g_a^2) (v_a^2 - v_u^2)] = 0 , \quad (8)$$

$$\frac{\partial V}{\partial \phi_{a}} |_{\phi_{a} = 0} = f_a m_a^2 + \frac{1}{2} (\mu_{\text{eff}} (v_a^2 + v_u^2) - \sqrt{2} v_a v_u B_{\text{eff}}) = 0 . \quad (9)$$

Here we have parametrized the scalar fields as in Ref. [13]:

$$H_d = \frac{1}{\sqrt{2}} (\phi_d + i \sigma_d + v_d) , \quad H_u = \frac{1}{\sqrt{2}} (\phi_u + i \sigma_u + v_u) , \quad A = \frac{1}{\sqrt{2}} (\phi_a + i \sigma_a + f_a) . \quad (10)$$

The derivatives in Eqs. (7)-(9) are evaluated at the minimum, where $\phi_{d,u,a} = \sigma_{d,u,a} = 0$. All tadpole equations and mass matrices have been calculated with the public code SARAH [25, 26].

Upon closer examination, Eq. (9) presents a problem. In order to solve the hierarchy problem, the scale of the soft SUSY breaking terms $M_{\text{SUSY}}$, should be of order $M_W$. One would expect that $m_a \sim M_{\text{SUSY}}$. For proper electroweak symmetry breaking, we must also have $\mu_{\text{eff}}^2$, $B_{\text{eff}} = O(M_W^2)$. Under these conditions Eq. (9) is not soluble. Let us then fix $\mu_{\text{eff}}^2$ and $B_{\text{eff}}$ at $M_{\text{SUSY}}$ and solve for $m_a$. Then $m_a \sim M_{\text{SUSY}}/f_a$ is tiny. This has an important consequence: it leads to a negative squared mass eigenvalue for the scalar field that we can identify as the saxion.

Before we show this let us check briefly the CP odd scalar sector. After replacing the soft mass terms with the solutions of the tadpole equations the mass matrix squared in the basis $(\sigma_d, \sigma_u, \sigma_a)$ reads in the Landau gauge

$$M^2_{\text{CP,odd}} = \begin{pmatrix} B_{\text{eff}} (\beta_{d} - 1) & B_{\text{eff}} (\beta_{u} - 1) & B_{\text{eff}} (\beta_{a} - 1) \\ B_{\text{eff}} (\beta_{d} - 1) & B_{\text{eff}} (\beta_{u} - 1) & B_{\text{eff}} (\beta_{a} - 1) \\ B_{\text{eff}} (\beta_{d} - 1) & B_{\text{eff}} (\beta_{u} - 1) & B_{\text{eff}} (\beta_{a} - 1) \end{pmatrix} . \quad (11)$$

Here we have written $t_{d} \equiv \tan \beta \equiv \frac{v_d}{v_u}$ for the ratio of the vacuum expectation values. The matrix Eq. (11) has two eigenvalues which are exactly zero. One is associated with the Goldstone boson which gets absorbed by the massive Z boson. The other is associated with the axion, the Goldstone boson of the spontaneously broken (global) PQ symmetry. This represents a check that the SUSY breaking effects have not spoiled the Goldstone theorem [30]. The third eigenvalue is the mass squared of the
physical CP odd Higgs boson

\[ m_A^2 = B_{\text{eff}} \left[ t_\beta + \frac{1}{t_\beta} + \frac{t_\beta v^2}{f_\beta^2} \right], \]  

(12)

which is the same as in the MSSM, apart from the small correction given by the last term.

Let us turn now to the scalar mass matrix squared for the CP even states. After rotating the Higgs fields \( (\phi_d, \phi_u) \rightarrow (h, H) \), it reads in the basis \( (h, H, \phi_u) \)

\[
\mathcal{M}_{CP}^2 = \begin{pmatrix}
\frac{1}{2} (g_1^2 + g_2^2) t_\beta + \frac{\nu_\text{eff}^2}{2} (t_\beta - 1) & 2 \frac{\nu_\text{eff}^2}{f_\beta} (t_\beta - 1) & 2 \frac{\nu_\text{eff}^2}{f_\beta} (t_\beta - 1) \\
2 \frac{\nu_\text{eff}^2}{f_\beta} (t_\beta - 1) & \left( \frac{g_1^2 + g_2^2}{t_\beta} + B_{\text{eff}}^2 t_\beta \right) v & \left( \frac{g_1^2 + g_2^2}{t_\beta} + B_{\text{eff}}^2 t_\beta \right) v \\
2 \frac{\nu_\text{eff}^2}{f_\beta} (t_\beta - 1) & \left( \frac{g_1^2 + g_2^2}{t_\beta} + B_{\text{eff}}^2 t_\beta \right) v & \left( \frac{g_1^2 + g_2^2}{t_\beta} + B_{\text{eff}}^2 t_\beta \right) v \\
\end{pmatrix}.
\]

Neglecting the entries with a \( 1/f_\beta^2 \) suppression and approximating \( t_\beta + 1 = t_\beta - 1 = t_\beta^2 \), this matrix has the form

\[
\begin{pmatrix}
\frac{1}{2} (g_1^2 + g_2^2) t_\beta v^2 & (g_1^2 + g_2^2) t_\beta v^2 - 4 \frac{\nu_\text{eff}^2}{f_\beta} t_\beta v & (g_1^2 + g_2^2) t_\beta v^2 - 4 \frac{\nu_\text{eff}^2}{f_\beta} t_\beta v \\
(g_1^2 + g_2^2) t_\beta v & (g_1^2 + g_2^2) t_\beta v^2 & (g_1^2 + g_2^2) t_\beta v^2 \\
(g_1^2 + g_2^2) t_\beta v & (g_1^2 + g_2^2) t_\beta v^2 & (g_1^2 + g_2^2) t_\beta v^2 \\
\end{pmatrix}
\]

The determinant is given by

\[
-\frac{1}{4 f_\beta^4} \left\{ v^4 \left[ (g_1^2 + g_2^2) B_{\text{eff}} t_\beta + 4 \frac{\nu_\text{eff}^2}{f_\beta} t_\beta \right]^2 + 16 B_{\text{eff}}^2 v^2 \left( B_{\text{eff}} - \frac{\nu_\text{eff}^2}{f_\beta} t_\beta \right) \right\}.
\]

(15)

\( B_{\text{eff}} \) must be positive, otherwise the mass of the charged Higgs would be below the \( W \) boson mass. Hence the determinant is always negative and the saxion is a tachyon. We conclude that this model is not consistent. As the issue can be traced back to the minimization condition, Eq. (13), corresponding to the PQ-breaking VEV, one can fix the problem by adding terms in the superpotential to spontaneously break the PQ symmetry and stabilize the PQ breaking scale.

**A SELF-CONSISTENT MODEL**

We add the following terms [31] to the superpotential in Eq. (2)

\[
W_{\text{PQ}} = \lambda \tilde{\chi} \left( \tilde{A} \tilde{A} - \frac{1}{4} f_\alpha^2 \right),
\]

(16)

with the distinct superfields \( \tilde{A}, \tilde{\chi} \), as well as \( \tilde{\chi} \). \( \tilde{A} \) carries a PQ charge opposite to \( A \), while \( \tilde{\chi} \) is PQ neutral. Assuming that an R symmetry forbids terms quadratic and cubic in \( \chi \), we have written all the terms consistent with the gauge and PQ symmetries, as well as with R-parity. After electroweak symmetry breaking (EWSB) \( \chi \) gets a VEV, \( v_\chi \), thus the R symmetry is broken. The corresponding R-axion has a mass of order \( M_{\text{SUSY}} \) because the R-symmetry is also explicitly broken by the soft terms. Beyond those in Eq. (3) we have the soft-breaking terms

\[
V_{\text{PQ soft}} = T_{\lambda\chi} A\bar{A} - L_{V\chi} + m_\chi^2 |A|^2.
\]

(17)

The trilinear and linear terms, with coefficients \( T_a \) and \( L_{V\chi} \), will play an important role when we discuss the mass of the axino. Note that one expects \( L_{V\chi} \sim M_{\text{SUSY}} f_\alpha^2 \). We have not written the soft term \( m_\chi^2 |\chi|^2 \) because it is negligible, as the \( F \)-terms from the superpotential Eq. (16) already produce a mass for \( \chi \) of order \( f_\alpha \).

After PQ and EW breaking the fields \( H_d, H_u, A, \tilde{A} \) and \( \chi \) receive VEVs:

\[
A = \sqrt{2} (\phi_u + i \sigma_u + v_u), \quad \tilde{A} = \sqrt{2} (\phi_u + i \sigma_u + v_u), \quad \chi = \sqrt{2} (\phi_u + i \sigma_u + v_u).
\]

(18)

with \( v_u v_\chi = \frac{1}{4} f_\alpha^2 \). The fields \( H_u, H_d \) are parametrized as in Eq. (16). The tadpole equations for \( \phi_u, \phi_d \) and \( \chi \) read

\[
\frac{\partial V}{\partial \phi_u} = m_u^2 v_u + \frac{1}{4} \left[ 2 v_u (c_u^2 v_d^2 + v_d^2) \right] + 2 \sqrt{2} (v_\chi v_u T_\chi - v_d v_u T_{c1}) + 2 \lambda^2 v_u v_\chi^2 = 0,
\]

(19)

\[
\frac{\partial V}{\partial \phi_d} = m_d^2 v_d + \frac{1}{4} \left[ -2 \lambda c_1 v_u v_\chi v_d + 2 \lambda^2 v_u v_\chi^2 \right] + 2 \sqrt{2} v_\chi v_u T_\chi = 0,
\]

(20)

\[
\frac{\partial V}{\partial \phi_\chi} = -\sqrt{2} L_{V\chi} + \frac{1}{2} \left[ -\lambda c_1 v_d v_u v_\chi + \sqrt{2} v_u v_\chi T_\lambda \right. \\
+ \left. v_\chi \lambda^2 (v_d^2 + v_u^2) \right] = 0.
\]

(21)
We can consistently solve these equations keeping all the soft parameters at the mass scale. In particular we can solve the last equation for $v_\chi$ and find
\[
v_\chi = \frac{2\sqrt{2}}{\lambda^2(v_u^2 + v_d^2)} L^0 - \frac{\sqrt{2} v_u v_\chi}{\lambda^2(v_u^2 + v_d^2)} T_\lambda + O\left(\frac{M_{\text{SUSY}}^2}{f_\chi}\right). \quad (22)
\]
This result will be important for the discussion below on the axino mass.

First we comment on the scalar masses in our model. Eq. (11) introduces a mixing between the MSSM Higgs sector and the axion sector. It turns out that the correction to the light Higgs mass, $m_h$, is of order $v_u^2 f_\chi$, which is negligible. As a consequence the usual upper limit $m_h < m_Z$ holds at tree level. On the other hand the tree-level mass of the heavy Higgs, $m_H$, is modified as
\[
m_H^2 = \frac{(2 B_{0\chi} + \sqrt{2} \bar{f}_0 \chi \lambda)}{\sin(2\beta)} + \frac{v_u^2}{4} \sin^2(2\beta) \left( g_1^2 + g_2^2 \right), \quad (23)
\]
and can therefore be potentially different from the MSSM. If we neglect the small mixing between the MSSM and the axion sector the three squared mass eigenvalues stemming from the mixing among ($\phi_u, \phi_d, \phi_\chi$) are given by
\[
\frac{4 L^0 T_\lambda + T_\lambda^2 f_\chi^2}{f_\chi^2 \lambda^2}, \quad f_\chi^2 \lambda^2 + 4 \sqrt{2} L^0 V. \quad (24)
\]
The first is the smaller one and we can associate it with the axion mass squared, which we see is of order $M_{\text{SUSY}}^2$. The other two scalars have a mass of order $f_\chi$. In the scalar CP-odd sector we find a massless axion as expected.

The neutralino mass matrix reads, in the basis $(\lambda_\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{A}, \tilde{A}, \tilde{\chi})$
\[
\begin{pmatrix}
M_1 & 0 & \frac{1}{2} g_1 v_\chi u & -\frac{1}{2} g_1 v_d & 0 & 0 \\
0 & M_2 & \frac{1}{\sqrt{2}} g_2 v_u & \frac{1}{\sqrt{2}} g_2 v_d & 0 & 0 \\
\frac{1}{2} g_1 v_u & \frac{1}{\sqrt{2}} g_2 v_u & 0 & \frac{1}{\sqrt{2}} v_\chi & -\frac{1}{\sqrt{2}} v_d & 0 \\
-\frac{1}{2} g_1 v_d & \frac{1}{\sqrt{2}} g_2 v_d & \frac{1}{\sqrt{2}} v_d & 0 & \frac{1}{\sqrt{2}} v_\chi & 0 \\
0 & 0 & -\frac{1}{\sqrt{2}} v_\chi & 0 & \frac{1}{\sqrt{2}} v_\lambda & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} v_\chi & \frac{1}{\sqrt{2}} v_\lambda \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} v_d & \frac{1}{\sqrt{2}} v_\lambda \\
0 & 0 & 0 & 0 & 0 & \lambda_\tilde{A}
\end{pmatrix}. \quad (25)
\]

In the limit $v_u = v_d = \frac{f_\chi}{\sqrt{2}}$, $c_1 v_u \rightarrow 0$, $c_1 v_d \rightarrow 0$ the lower right $3 \times 3$ block has the singular values
\[
-\frac{1}{\sqrt{2}} v_\chi \lambda, \quad \frac{1}{2} \sqrt{2} \left( \pm \frac{\sqrt{2}}{v_\chi} + 4 f_\chi^2 \lambda + v_\chi \lambda \right). \quad (26)
\]
The first is associated with the physical axino. Its mass is proportional to $v_\chi$, therefore of order $M_{\text{SUSY}}$. In this model we also have an extra handle on the axino mass. We can relax the assumption $v_u = v_d = \frac{f_\chi}{\sqrt{2}}$ and consider a hierarchy between the two VEVs, for example $v_d \gg v_u$. If we do so we find that the axino mass becomes lighter. In the limit $v_d \rightarrow 0$, keeping fixed $v_u v_d = 1/2 f_\chi^2$, the axino mass tends to zero.

**THE AXINO MASS**

In the above discussion we have parametrized the SUSY breaking effects in the soft terms and made no reference to the SUSY breaking scale, $\sqrt{F}$. However note that the field $\tilde{\chi}$ and a linear combination of $\tilde{A}, \tilde{A}$ have masses of order $f_\chi$. If SUSY is broken at a much lower scale, $\sqrt{F} \ll f_\chi$, it is sensible to first integrate out these heavy fields in the supersymmetric limit and then introduce the soft breaking terms. In the next section, we show what happens to the axino mass when we follow such a procedure. Then we comment on the case where $\sqrt{F} \geq f_\chi$, where the heavy fields can no longer be integrated out before SUSY breaking.

**Low scale SUSY breaking**

If $\sqrt{F} \ll f_\chi$ SUSY is still unbroken at the PQ scale and we can perform the following redefinitions of the su-
perfields

\[ \hat{\Phi} \rightarrow \hat{\Phi} \]  
\[ \hat{\Phi} \rightarrow \left( \frac{1}{2} f_a + \frac{1}{\sqrt{2}} \hat{\Phi}_H \right) e^{\sqrt{2} \hat{s}_a} \]  
\[ \hat{\Phi} \rightarrow \left( \frac{1}{2} f_a + \frac{1}{\sqrt{2}} \hat{\Phi}_H \right) e^{-\sqrt{2} \hat{s}_a} \]  

Plugging into Eq. (16), we see that the superfields \( \hat{\Phi} \) and \( \hat{\Phi}_H \) have masses of order \( f_a \), while \( \hat{\Phi}_a \) is massless. The latter is the axion superfield. This parametrization \[ \hat{s}_a \] is useful because it makes it obvious that the original PQ transformation \( \hat{A} \rightarrow e^{i\alpha} \hat{A} \) is now encoded in \( \hat{\Phi}_a \rightarrow \hat{\Phi}_a + i \sqrt{2} \alpha f_a \). We recognize here the shift symmetry typical of axions that must be respected in the low energy theory. Let’s consider the superpotential in terms of the new superfields

\[ W_2 = \frac{\lambda}{2} \Phi_H (\hat{\Phi}_H + \sqrt{2} f_a) \]  
\[ + c_1 \left( \frac{1}{2} f_a + \frac{1}{\sqrt{2}} \hat{\Phi}_H \right) e^{\sqrt{2} \hat{s}_a} \hat{H}_u \hat{H}_d. \]  

We can integrate out the heavy fields in a supersymmetric fashion using their equations of motion: \( \frac{\partial W}{\partial \hat{\Phi}_H} = 0 \) and \( \frac{\partial W}{\partial \hat{s}_a} = 0 \). We find the following effective superpotential

\[ W_{\text{eff}} = \mu_{\text{eff}} \hat{H}_u \hat{H}_d + \frac{c_1}{\sqrt{2}} \Phi_H \hat{H}_u \hat{H}_d + \frac{c_1}{2} \sum_{n \geq 2} \left( \frac{\sqrt{2} \Phi_H}{f_a} \right)^n \hat{H}_u \hat{H}_d. \]  

In the first term we have \( \mu_{\text{eff}} = \frac{c_1}{2} f_a \), while the last term contains higher dimension operators that we can safely neglect because they are suppressed by increasing negative powers of \( f_a \). Now we can consider the effects of SUSY breaking. The soft terms for the low energy field content read

\[ V_{\text{soft}} = T_u \hat{u} \hat{H}_u + T_d \hat{d} \hat{H}_d + T_e \hat{e} \hat{H}_d + T_{c_1} \hat{\Phi}_H \hat{H}_u \hat{H}_d \]  
\[ + B_{\text{eff}} \hat{H}_u \hat{H}_d + m_1^2 [H_u]^2 + m_2^2 [H_d]^2 + \frac{\lambda}{2} m_3^2 \hat{\Phi}_H \]  
\[ + m_4^2 (\Phi_1 + \Phi_2). \]  

The form of the last term is dictated by the shift symmetry. Note that \( m_4^2 |\Phi_a|^2 \) would violate such a symmetry. Indeed the term in Eq. (33) gives a mass to the saxion, the real part of \( \Phi_a \), but leaves the axion massless. We parametrize the fields \( H_u \) and \( H_d \) as in Eq. (10), but do not assign a VEV to the field \( \Phi_a \), as that would break the shift symmetry. Thus we write \( \Phi_a = \frac{1}{\sqrt{2}} (\phi_a + i \sigma_a) \).

The tadpole equations read

\[ \partial V \bigg|_{\phi_a = 0} = m_2^2 v_d + \frac{1}{8} \left( 2c_1^2 v_a^2 + 8 \mu_{\text{eff}} v_d - 8 v_d B_{\text{eff}} \right) \]  
\[ + v_d \left( g_1^2 + g_2^2 \right) (v_d^2 - v_u^2) = 0, \]  
\[ \partial V \bigg|_{\phi_a = 0} = m_2^2 v_u + \frac{1}{8} \left( 2c_1^2 v_u^2 + 8 \mu_{\text{eff}} v_u - 8 v_d B_{\text{eff}} \right) \]  
\[ - v_u \left( g_1^2 + g_2^2 \right) (v_d^2 - v_u^2) = 0, \]  
\[ \partial V \bigg|_{\phi_a = 0} = v_d v_u T_{c_1} - c_1 \mu_{\text{eff}} (v_d^2 + v_u^2) = 0. \]  

We can see at this stage that the issue which made the model of the first section inconsistent is no longer present. The parameter \( m_a \) is absent from these equations. Thus we can retain all the soft masses at the \( M_{\text{SUSY}} \) scale. We find that the Higgs masses are the same as in the MSSM, up to tiny corrections proportional to the small parameter \( c_1 \), and the axion is massless. The saxion mass is \( m_a \sim M_{\text{SUSY}} \) and deserves a comment. In Ref. \[ 33 \] the authors claimed that in theories with spontaneously broken SUSY with \( \sqrt{f_a} \ll f_a \) the saxion mass is at most \( M_{\text{SUSY}}^2 / f_a \). Their result relies on the assumption that the supertrace sum rule \[ 33 \] holds. The inclusion of the explicit soft SUSY breaking terms violates this assumption, and our saxion mass comes indeed from the soft term. Therefore our result is not in conflict with Ref. \[ 33 \].

Let us consider the fermions. In the basis \( \left( \lambda_B, \hat{W}^0, \hat{H}_u^0, \hat{H}_d^0, \hat{\Phi}_a \right) \) the \( 5 \times 5 \) neutralino mass matrix reads

\[ m_{\chi_0} = \begin{pmatrix} M_1 & 0 & \frac{1}{2} g_1 v_u & \frac{1}{2} g_1 v_d & 0 \\ 0 & M_2 & -\frac{1}{2} g_2 v_u & -\frac{1}{2} g_2 v_d & 0 \\ \frac{1}{2} g_1 v_u & -\frac{1}{2} g_2 v_u & -\mu_{\text{eff}} - \frac{1}{2} c_1 v_d & 0 & 0 \\ \frac{1}{2} g_1 v_d & -\frac{1}{2} g_2 v_d & 0 & -\mu_{\text{eff}} - \frac{1}{2} c_1 v_d & 0 \\ 0 & 0 & -\frac{1}{2} c_1 v_d & -\frac{1}{2} c_1 v_u & 0 \end{pmatrix}. \]  

The smallest eigenvalue here is of order \( c_1 v \), with \( v \) of order the EWSB VEV, and it corresponds to the axino.
High scale SUSY breaking

If \( \sqrt{F} \geq f_a \) the SUSY breaking effects are already present at the PQ scale and the procedure we employed in the previous section of integrating out the heavy fields supersymmetrically is no longer applicable. Keeping all the fields in the game we end up with an axino mass of order \( M_{\text{SUSY}} \), as we have seen in Eq. (26). We have checked that after integrating out the heavy fields, which now has to be done component by component as SUSY is broken, the axino mass is unchanged.

Comments

The axino mass has been widely discussed in the literature. Tamvakis and Wyler \[33\] showed that in models with global SUSY the axino mass would be at most of order \( O \left( \frac{M_{\text{SUSY}}}{f_a} \right) \) after SUSY breaking. Chun, Lukas, Kim and Nilles \[35, 36\] found that in models with local SUSY, i.e. supergravity, the axino mass can have a wider range and can be as large as the gravitino mass, \( m_{3/2} \). The results of this work agree with those statements. Indeed a low SUSY breaking scale, for which we find a light axino, is typical of models with global SUSY, while a higher scale, \( \sqrt{F} \geq f_a \), for which we find a heavier axino, is representative of supergravity. In the latter case we can identify the scale of our soft terms with the gravitino mass, \( M_{\text{SUSY}} \sim m_{3/2} \sim \frac{f_p}{M_p} \), with \( M_p \) the Planck mass.

We emphasize, however, that the distinction between models of global SUSY breaking and supergravity is not strictly related to the scale \( \sqrt{F} \). Recently, for example, gauge mediation models with a high scale, \( \sqrt{F} > f_a \), have been considered (see e.g. \[37\]). Our statements on the axino mass only refer to the relative size of the scales \( \sqrt{F} \) and \( f_a \) and make no explicit reference to the SUSY breaking mechanism.

CONCLUSION

We have pointed out that the minimal SUSY model with a DFSZ axion proposed in the literature is inconsistent, as it suffers from a tachyonic saxion. The issue is solved if one considers an extended superpotential which stabilizes the PQ scale. We have then considered two cases: one where the SUSY breaking scale is much lower than the PQ breaking scale, the other where the two scales are comparable. In both cases the axino remains massless, as it should, and the saxion gets a mass of order \( M_{\text{SUSY}} \) (or \( m_{3/2} \)), roughly in the TeV range. The axino mass is dramatically different depending on the scenario. In the first one (\( \sqrt{F} \ll f_a \)) it is very light, below the keV scale, while in the second one (\( \sqrt{F} \geq f_a \)) it can be as large as the saxion mass. These results are in agreement with previous statements in the literature. Furthermore, in the second case, the mixing between the new states and the MSSM Higgs sector doesn’t affect the mass of the light Higgs but can change the prediction for the heavy Higgs mass.

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[38] $\mathcal{M}_{SUSY}$ is the scale of the soft terms and is typically in the TeV range. It should not be confused with the SUSY breaking scale $\sqrt{F}$ which can be much higher, depending on the mechanism that mediates the SUSY breaking.
[39] We are neglecting here QCD instanton effects that generate a small axion mass.