Generation of Magnetic Fields and Gravitational Waves at Neutrino Decoupling

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We show that an inhomogeneous cosmological lepton number may have produced turbulence in the primordial plasma when neutrinos entered (almost) free-streaming regime. This effect may be responsible for the origin of cosmic magnetic fields and give rise to a detectable background of gravitational waves. An existence of inhomogeneous lepton asymmetry could be naturally generated by active-sterile neutrino oscillations or by some versions of Affleck and Dine baryogenesis scenario.

The idea that the early universe went through one or more turbulent phases is recurrent in the scientific literature since the beginning of the modern cosmology. Among other reasons, cosmic turbulence was often invoked to explain the origin of the magnetic fields (MFs) observed in most spiral galaxies and galaxy clusters \cite{ref1,ref2}. One of the first attempts \cite{ref3} was based on the observation that in the radiation era a weak MF should be generated by turbulent eddies because the rotational velocities of relativistic electrons and non-relativistic ions would change differently during universe expansion. Subsequently, turbulence may have induced a Magneto-Hydro-Dynamical (MHD) dynamo which amplified exponentially the field until equipartition between the plasma turbulent kinetic energy and the MF energy is eventually reached. This nice scenario, however, remains unaccomplished in the absence of a plausible mechanism of turbulence generation. Since gravitational forces conserve angular moment, vorticity must have been produced by non-gravitational forces. Primordial phase transitions might have produced MFs if they were first order \cite{ref4}. The idea is that the expanding bubble walls between two phases give rise to small electric currents which power a seed MF. Turbulence appears when bubbles collide at the end of the phase transition, or hydrodynamic instabilities develop on the bubble walls, producing MHD amplification of the seed field. Several applications of this idea have been studied for the case of the quark-hadron (QHPT) and electroweak (EWPT) phase-transitions \cite{ref2} and refs. therein). Unfortunately, it is still unclear if any of these transitions can really be first order. Furthermore, the major problem with this kind of scenario, is that it can hardly account for large scale MFs. The reason is that the comoving coherence length of the MF is at most given by the Hubble horizon at the phase transition which is much smaller than a typical galaxy size. Although the coherence length may grow due to MHD effects this generally happens to the expenses of the MF strength. Careful studies showed that the EWPT and QHPT cannot account for the galactic and cluster MFs \cite{ref3}. To overcome the small scale problem several mechanisms of generation of seed magnetic fields at inflationary stage have been proposed which are operative if conformal invariance of electromagnetic interaction is broken \cite{ref4} (for more references see the review \cite{ref2}).

In this Letter we propose a new mechanism for the generation of cosmic MFs. In contrast to many previously considered mechanisms it does not demand any new strong physical assumptions and can be realized, in particular, in almost minimal standard model of particle physics. It might operate at neutrino decoupling epoch, $T \sim 1-2 \text{ MeV}$, i.e. when the Hubble horizon was considerably larger than what it was at EWPT and at QHPT. Our basic assumption is that the net lepton number density $\langle N_a(x) \equiv n_{\nu_a}(x) - n_{\bar{\nu}_a}(x), \ a = e, \mu, \tau \rangle$ of one, or more, neutrino species was not uniform before neutrino decoupling and changed in space over some characteristic scale $\lambda$ which could be smaller than the Hubble horizon at the decoupling time.

As a result, when the neutrino mean free path $\ell_\nu(T)$ grew and became comparable to $\lambda$, neutrino currents should be developed along the density gradients. We will show that elastic scattering of the diffusing neutrinos on electrons and positrons would be able to accelerate the electron-photon fluid producing vorticity in the plasma. Turbulence may have developed by this process in the short interval of time during which the random forces due to neutrino elastic scattering overcome the shear viscosity force. Depending on the amplitude and wavelength of the fluctuations of $N_a(x)$, this period could be sufficient for the MHD engine to generate magnetic field in equipartition with the turbulent kinetic energy. The seed field required to initiate the process arises naturally as a consequence of the difference between the $\nu_e e^-$ and $\nu_\mu e^+$ cross sections and of the neutrino-antineutrino local asymmetry. Furthermore, we will show that turbulence should give rise to a background of gravitational waves.

The first step of our computation is to determine the
neutrino momentum flux produced by the inhomogeneous $N_\nu(x)$. Starting form the Boltzmann equation we obtain the equation describing evolution of the average flux of $i$-th component of neutrino momentum

$$\frac{\partial}{\partial t} K_i(x,t) + 4H K_i(x,t) + \frac{\partial}{\partial x_j} K_{ij}(x,t) = -\tau_{\nu}^{-1} K_i \quad (1)$$

where

$$K_i = \int k_i f_\nu(E,k) \frac{d^3k}{(2\pi)^3}, \quad K_{ij} = \int \frac{k_i k_j}{E} f_\nu(E,k) \frac{d^3k}{(2\pi)^3}$$

In the above $E$ and $k$ are respectively the neutrino energy and spatial momentum, $H$ is the universe expansion rate, $\tau_{\nu}$ is the effective weak interaction time, and $f_\nu(E,k,x,t)$ is the neutrino distribution function. As we see in what follows, the source of the magnetic field is proportional to curl of electric current, $\nabla \times J$, which in turn is proportional to the local vorticity of the source term $\partial_t K_{ij}$ in Eq.(1). The latter is nonvanishing for anisotropic random initial distribution of neutrino leptonic charge and is numerically close to $K_{ij}$ divided by $\lambda$. Hence, Eq.(1) becomes

$$\frac{\partial}{\partial t} K_\lambda(x,t) = \frac{\dot{V}_\lambda}{\lambda} \left( \frac{\delta n_{\nu}}{n_{\nu}} \right) \exp \left( -\int_0^t \frac{E_{\nu}(t')}{\lambda^2} dt' \right)$$

$$- 4H K_\lambda(x,t) - \tau_{\nu}^{-1} K_\lambda(x,t), \quad (2)$$

where $K(x,t) \equiv K/\rho_\nu$ is the specific momentum flux on the length-scale $\lambda$, and $\dot{V}_\lambda$ is a unit vector parallel to the initial value of the vector $\partial_t K_{ij}$, which has a non-zero vorticity, i.e. $\nabla \times \dot{V} \neq 0$. The exponential in the first term of the r.h.s. of Eq.(2) has been inserted to account for the damping of the fluctuations due to neutrino diffusion. The last term in the r.h.s. of Eq.(2) represents the force per unit mass between the drifting neutrinos and the electron-positron fluid due to their scattering. It is understood that $\lambda$ changes with time due to universe expansion. Since at $T \sim 1$ MeV, the Compton scattering rate was much larger than the universe expansion rate $H$, electrons, positrons and photons formed a (almost) single relativistic fluid at neutrino decoupling time (the role of baryons was negligible at this epoch). We assume the electron-photon fluid was approximately homogeneous before neutrino decoupling. This is possible if the local excess of neutrinos of a given species is balanced by a deficiency of neutrinos of a different species (also sterile). The Euler equation of the fluid is

$$\frac{\partial}{\partial t} \mathbf{v} \simeq \tau_{\nu e}^{-1} \frac{\rho_{\nu}}{(\rho + p)^2} (K_{\nu} + K_{\bar{\nu}}) - H \mathbf{v} + \eta \nabla^2 \mathbf{v}, \quad (3)$$

where $\rho$ and $p$ are the energy density and pressure of the electron-photon fluid, $\gamma = \sqrt{1 + \eta^2}$ is the Lorentz factor, $\tau_{\nu e} \equiv \tau_{\nu e} + \tau_{\bar{\nu} e}$ is the neutrino-electron(positron) mean collision time, and $\eta = 4\rho_{\nu}E_{\nu}/15(\rho + p)$ is the shear viscosity due to neutrino diffusion. This approximation for the viscosity is valid in the limit when the neutrino mean free path is smaller than the characteristic scale of the problem. In our case this is not always true and the viscosity can be smaller. In Fig.1 the macroscopic velocity $v$ of the electron-photon fluid (continuous line) is presented as a function of the ratio $x \equiv t/t_d$ ($t_d$ is the neutrino decoupling time). It is obtained by numerical solution of the coupled set of Eqs. (2) and (3) for a particular choice of $\delta n_{\nu}$ and characteristic size of the fluctuations (see the figure caption). It is visible from this figure that the velocity grows rapidly when the neutrino mean free path becomes comparable to the fluctuations size $\lambda$ and it is suppressed soon later when the viscosity on that length scale becomes dominant.

![FIG. 1. The local fluid velocity $v$ (continuous line), the specific neutrino momentum flux $K$ (dashed line), and the ratio $b = |\mathbf{e}|B/T^2$ (dotted-dashed line) as functions of the time parameter $x$. We assumed here $\delta n_{\nu}/n_{\nu} = 1$ over a comoving length scale $\lambda_d \equiv (\lambda/H^{-1})_{t=t_d} = 10^{-2}$. The quantities plotted here represent averages on the same scale.](image_URL)

In this Letter we do not investigate the velocity and MF power spectrum. We only observe here that due to the rapid grow of the viscosity with time, the turbulence spectrum may be quite different from the Kolmogoroff’s one.

We now turn to the problem of magnetic field generation. Starting from Maxwell equations in the primordial plasma and neglecting terms proportional to the inverse of the electric conductivity we obtain after simple algebra

$$\partial_t \mathbf{B} + 2H \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \kappa^{-1} \nabla \times \mathbf{J}_{\text{ext}} \quad (4)$$

where $\kappa \sim T/\alpha$ is the electric conductivity of the relativistic cosmological plasma ($\alpha$ is the fine structure constant), and $2HB$ is the damping term related to the cosmological expansion. The validity of our approximation can be easily verified by noting that the magnetic diffusion length $L_{\text{diff}}(t) = \sqrt{t/4\pi \kappa}$ exceeds safely the characteristic length of the relevant process, namely the
neutrino mean-free-path $\ell_n(t) \sim 1\ s\ (t/t_d)^{3/2}$. Using the expression of the electric conductivity for relativistic plasma presented above we find that the condition $L_{\text{diff}}(t) \gg \ell_n(t)$ is comfortably fulfilled at any time essential for the presented mechanism. We also checked that relativistic corrections to Eq. (3) do not affect significantly our results even if $v \approx 1$. Thanks to the absence of magnetic diffusion, the first term in the r.h.s. of Eq. (4) ensures an exponential amplification of any preexistent seed field which is accounted by the the term $\kappa^{-1}\nabla \times J_{\text{ext}}$ in Eq. (4). Interestingly, neutrino diffusion in the presence of a inhomogeneous neutrino-antineutrino asymmetry naturally provides such a seed. The relevant point here is that, due to charge symmetry breaking in the standard model, the neutrino-electron and the neutrino-positron weak cross sections are different. As a consequence electrons and positrons are subject to different forces in the presence of a net flux of neutrinos and this gives rise to a non-zero electric current. The latter can be estimated as follows. The equation of motion of electron (or positron) in plasma is $\dot{p}_e = \Phi_\nu \sigma_{\nu e} \Delta p - p_e/\tau_e$ where $\Phi_\nu = \delta n_\nu$ is the neutrino flux with $\delta n_\nu$ being an excess of neutrino number density, $\Delta p \sim T$ is the transferred momentum, and $\tau_e = (\sigma_T n_e)^{-1}$ is the characteristic damping time due to $e^\pm$ interactions with photons, $\sigma_T = 8\pi^2/3m_e^2$ is the Thomson cross-section and $n_e = 0.24T^3$ is the photon number density.

Since the $\nu e^-$ and $\nu e^+$ cross-sections are different, $\sigma_{\nu e^-} = G_F^2 s/\pi \approx 10G_F^2 T^2$ and $\sigma_{\nu e^+} = G_F^2 s/3\pi \approx 3G_F^2 T^2$ (barring finite electron mass corrections), the drift velocities of electrons and positrons would be different and a non-zero electric current would be induced by neutrino flux. Using the previous expressions the difference of $(e^- - e^+)$-velocities can be estimated as $\Delta v \approx 2 \times 10^{-19} (\delta n_\nu/n_\nu)(T/\text{MeV})^3$. Correspondingly the electric current induced by neutrinos is given by:

$$J_{\text{ext}} = 4 \times 10^{-20} e T^3 \left( \frac{T}{\text{MeV}} \right)^3 \frac{(\delta n_\nu/n_\nu)}{\lambda},$$

(5)

which corresponds to a seed field with strength $B_\lambda^{\text{seed}} \approx 10^{-22} \left( \frac{T}{\text{MeV}} \right)^2$ at a time $t/\lambda \sim 1$. We are now able to solve numerically Eq. (4) by coupling this equation to Eqs. (2). Instead of the absolute value of $B$ it is convenient to focus on the ratio $b = |eB|/T^2$ which is a constant quantity for a frozen-in MF in the absence of entropy production. From Fig.1 it is evident at a glance the huge amplification of $b$ which takes place in the short interval of time during which fluid “turbulence” is active. A suitable cutoff has been used in our computation to account for the saturation of the amplification process which has to come in when the MF approaches energy equipartition with the fluid motion, i.e. when $B^2/4\pi \sim (\rho + p) u^2$. We think that a more elaborate treatment of non-linear MHD effects might only slightly change the knee of $b(x)$ without affecting significantly our main results. The final value of $b$ depends on the rapidity of the amplification process. Since $b(x) \propto \exp (v(x)x/\lambda)$, it is clear that equipartition will be reached more quickly, i.e. when $v$ is yet not suppressed by viscosity, for fluctuations having large amplitude and small sizes. In Fig.2 we present the final value of $b$ as a function of $\lambda_d$ for several values of $(\delta n_\nu/n_\nu)$. In principle the requirement of a successful BBN put an upper limit on the strength of the MFs, which is roughly $b \lesssim 0.1$ [3]. This bound, however, was obtained under the assumption that the MF was uniform over the Hubble volume at BBN time and, depending on the details of the MF power spectrum, it may not apply to our case.

![FIG. 2. The ratio $b = |eB|/T^2$ on the comoving length scale $\lambda_d$ for values of the neutrino number density contrast: starting from the right curve $(\delta n_\nu/n_\nu)\lambda_d = 1$, $10^{-1}$, $10^{-2}$.](image)

In principle inhomogeneous MFs can be subject to dissipation due to their back-reaction on the plasma. In fact, tangled MFs give rise to an anisotropic pressure that could induce plasma oscillations (mainly Alfvén waves) about a force-free configuration. These oscillations might be rapidly damped because of the high fluid viscosity which implies a dissipation of magnetic energy [13]. In our case, it is easy to verify that during the neutrino decoupling process the oscillation frequency of Alfvén waves is always smaller than the characteristic rate at which MFs are produced which is $\lesssim H$ so that these MHD modes are not excited. MF dissipation could also take at photon decoupling. However, it was showed in Ref. [10] that such an effect is negligible for MFs extending on galactic scale if their present time strength is smaller than $10^{-9}$ Gauss.

The generation mechanism that we have discussed gives rise to MFs of intensity $B_0 = b\ 8 \times 10^{-6}$ Gauss at the present time with a coherence length $\lambda_0 = \lambda_d \ r_{\nu}(t_d) (T_d/T_0) \approx 10^2 \lambda_d$ pc. Galactic MFs are observed with characteristic strength of the order of $1\ \mu G$ extending over scales $\sim 1$ kpc [1]. Taking into account
flux conservation during the protogalaxy collapse, the primordial origin of galactic fields would require a protogalactic field with the strength $\sim 10^{-10}$ Gauss and the coherence length of 0.1 Mpc [11]. Although this scale is much larger than the coherence length predicted by our model, it is natural to expect that some homogenization could take place during galaxy formation. Since the field orientation is random over scales larger than $\lambda_0$, the predicted mean field on the protogalactic scale will be obtained by a suitable volume average [11].

$$B(0.1 \text{ Mpc}) \simeq B_0 \left( \frac{\lambda_0}{0.1 \text{ Mpc}} \right)^{3/2} \simeq 10^{-10} b \lambda_d^{3/2} \text{ G.} \quad (6)$$

From this equation and our previous results (see Fig.2) we find that galactic MFs may be a product of neutrino number fluctuations with the amplitude $\sim 1$ extending over scales comparable to the Hubble horizon at neutrino decoupling. It is remarkable that MFs with this intensity may have produced observable effects on the CMB and have interesting consequences for structure formation [2]. Fluctuations with a smaller amplitude could still have played a role in the generation of galactic MFs if a galactic dynamo, or even much less efficient amplification processes, took place after or during galaxy formation.

Another interesting consequence of stirring the primordial plasma by neutrino inhomogeneous diffusion is the production of gravitational waves (GW). The generation of a cosmic background of GW by turbulence produced at the end of a first-order phase transition was discussed in Ref. [11]. In our case GW with the largest amplitude are produced with a comoving wavelength $\lambda_0 = \lambda_d 10^2$ pc which corresponds to the present time frequency $\omega_0 \simeq 10^{-9} \lambda_d^{-1}$ Hz. The GW production took place when $v_{\nu} (x_s) = \lambda (x_s)$, i.e. $x_s = \sqrt{\delta m^2}$, and it lasted for a time interval comparable to $H(x_s)$. Following ref. [11] we estimate the energy density parameter of GW with frequency $\omega_0$ to be

$$\Omega_{GW} h^2 \simeq 10^{-5} H(x_s) \lambda(x_s) v^6(x_s) \simeq 3 \times 10^{-5} \lambda_d^{3/4} v^6(x_s). \quad (7)$$

From our previous results it follows that a GW background produced by neutrino number fluctuations of amplitude $\lesssim 1$ with $\lambda_d < 10^{-3}$ may be detectable by future GW space based observatories [12]. As tangled MFs can also act as a source of GW [13], a further, and perhaps dominant, contribution to the GW background may come from MFs produced during the decoupling process.

We conclude this Letter by observing that large neutrino number density fluctuations, as those required to power the effects discussed above, might be generated by several mechanisms. One possible way to generate inhomogeneous and large lepton asymmetry at large scales together with a small baryon asymmetry could be achieved [14,15] in the frameworks of Affleck-Dine [10] baryo(lepto)-genesis scenario. Another very interesting mechanism based on active-sterile neutrino oscillations was proposed recently by DiBari [7]. Such a mechanism naturally gives rise to domains where neutrinos, or antineutrinos, of a given species are strongly converted into sterile neutrinos (or sterile antineutrinos), hence $\delta n_{\nu}/n_{\nu} \sim 1$. The typical domain size is determined by the neutrino mean free path at the critical temperature at which neutrino conversion takes place $T_c \simeq 15$ MeV ($|\delta m^2|/eV^2)^{1/2}$, which implies $\lambda_d \simeq 10^{-3}$ ($|\delta m^2|/eV^2)^{-2/3}$ A nice feature of this scenario is that it practically does not need to invoke new physics, except for only one that there exists a sterile neutrino mixed with an active one. In this case very small inhomogeneities in the baryon number density, which is known to exist in the early universe, would give rise to a very strong amplification of initially negligible lepton asymmetry. Since the initial excess of energy density of active neutrinos is exactly compensated by the deficit in sterile ones such a model does not suffer from a possible distortion of approximate isotropy of the cosmic microwave background radiation. Both mechanisms discussed here produce isocurvature fluctuations in the neutrino fluid. Chaotic vector and tensor perturbations, which may be absent initially, are produced when neutrino start to diffuse according to the mechanism discussed in this Letter.

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[1] P.P. Kronberg, Rep. Prog. Phys. 57, 325 (1994).
[2] D. Grasso and H.R. Rubinstein, Phys. Rept. 348, 161 (2001).
[3] E.R. Harrison, Mon. Not. Roy. Astron. Soc., 147, 279 (1970); Phys. Rev. Lett. 30, 188 (1973).
[4] C.H. Hogan, Phys. Rev. Lett. 51, 1488 (1983).
[5] D.T. Son, Phys. Rev. D59, 063008 (1999).
[6] M.S. Turner and L.M. Widrow, Phys. Rev. D37, 2743 (1988).
[7] A.D. Dolgov, Phys. Rev. D48, 2499 (1993).
[8] J. Ahonen and K. Enqvist, Phys. Lett. B382, 40 (1996); G. Baym and H. Heiselberg, Phys. Rev. D59, 5254 (1999).
[9] D. Grasso and H.R. Rubinstein, Phys. Lett. B379, 73 (1996).
[10] K. Jedamzik, V. Katalinić and A. Olinto, Phys. Rev.
11] M. Kamionkowski, A. Kosowsky and M. Turner, Phys. Rev. D49, 2837 (1994).
[12] M. Maggiore, Phys. Rept., 331, 283 (2000).
[13] R. Durrer, P.G. Ferreira and T. Kahniashvili, Phys. Rev. D61, 043001 (2000).
[14] A.D. Dolgov, Phys. Rept. 222, 309 (1992).
[15] A.D. Dolgov and D.P. Kirilova, J. Moscow Phys. Soc. 1, 217 (1991).
[16] I. Affleck and M. Dine, Nucl. Phys. B249, 361 (1985).
[17] P. Di Bari, Phys. Lett. B482, 150 (2000).