Natural Quasi-Rendezvous and Evasive Maneuvers assisted by Atmospheric Drag

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Abstract. The study of evasive maneuvers of spacecraft in the face of the danger of collision with space debris is essential and important today due to the growth of missions at low altitudes (LEO). Bodies that orbit this region are affected by the Earth’s drag force. In this work, we present the results of numerical simulations for collision dynamics between spatial objects (operational vehicle and space debris) and investigate evasive maneuvers of the vehicle under the possibility of collision with a debris in atmospheric drag environment. We tested the evasive maneuvers for collision conditions in this environment using the exhaust velocity of the vehicle's propulsion system. We find three important results: 1) evasive maneuvers are possible and the atmospheric drag can contribute to the removal of the collision objects in small and medium intervals of time; 2) for very small initial velocities, there are natural quasi-Rendezvous, under the action of gravity alone and drag; 3) propulsion in conjunction with drag can enable evasive maneuvers for various spatial debris sizes.

1. Introduction

We call space debris (DE) any non-operational space body that is orbiting the Earth at various altitudes, due to its capture by the terrestrial gravitational field. These bodies can be natural (cosmic dust, meteoroids, remainders of gases left by celestial bodies, etc.) or unnatural (scrapping of space missions such as screws, pieces of paint, etc.). The latter come from more than 60 years of space activities. Many non-operational vehicles that have become DE may have their structures deteriorated, leading to leaks and possible fragmentation and/or explosions. These explosions generate even debris. An operational vehicle can become a debris because of technical problems during its mission, among them, the impossibility of subsystems affected by sub-millimeter debris. Dissipative forces, for example, atmospheric drag, can remove LEO operating vehicles from their nominal orbits and cause them to collide with other vehicles or even with DE pre-existing in that orbit. Some ESA statistical models for a considerable amount of DE in orbit show that there are 29,000 DE of size greater than 10 cm, 750,000 of between 1 and 10 cm and 166 million of DE between 1 mm and 1 cm [2]. The total
mass of these bodies would total about 7,500 tons. Several accidents are related to the presence of DE in Earth orbits. An important example was the collision at 11.7 km/s of Iridium-33 (of private American company) and the Russian military satellite Kosmos 2251 that happened in 2009, producing approximately 2,300 fragments. Other studies have shown that the increase in the likelihood of collisions between DE in a chain reaction would result in an exponential growth of orbiting fragments. These data were compared with the 2010 observations and it was observed that there was a rate of 320 objects produced per year and no significant collisions have occurred since 2010 to 2018 [3], [4]. They demonstrated that the energy involved in these collision events would be related to high relative velocities of impact, of the order of 10 km/s [5]. Several disasters involving satellites and that aggravated the problem of space pollution were listed in [6]. In 2007, an incident involving a Chinese anti-satellite gun test occurred. With the weapon, they destroyed one of their own satellites, generating thousands of new space debris [7]. The possibility of an increase in a space arms race to neutralize an enemy satellite only aggravates the rate of increased DE and the risk of collisions in the space activity environment.

The drag force is a disturbing force that acts on the DE that are in LEO and could destroy them [8]. Eventually, some of these objects can reach the Earth's surface, for example, the second stage of the Delta II rocket that crashed in South Africa in 2000. This force does not operate in other operational regions such as MEO and GEO. The DE of these regions re-enter the Earth's atmosphere in decades or centuries. During space missions, the operational vehicle may cross the orbit of a space debris and may collide with it. This fact, more and more frequent, due to the accumulation in clouds of DE, justifies the study of evasive maneuvers of the space vehicles to escape of collisions. In this work, we first studied the natural Quasi-Rendezvous phenomenon between a vehicle and a space debris, both under the action of the terrestrial gravitational force and the DE subject to atmospheric drag. Soon after, with the collision conditions between the objects, we studied the feasibility of evasive maneuvers of the space vehicle, based on technological parameters that characterize its propulsion system.

2. Mathematical model

The relative dynamics between a vehicle and a spatial debris can be described by the equations of Clohessy-Witshire [9]. We adapted this dynamic to study evasive maneuvers implemented by a propulsion system, whose mass of fuel decreased exponentially [10]. In this work, we include too the atmospheric drag force with constant density. The equations of relative dynamics, considering these conditions and the near objects in relation to the distance of the vehicle to the center of the Earth ($r << R$, Figure 1), are:

\[ \ddot{x} - 2\omega \dot{y} - 3\omega^2 x = -A[\dot{x} - y(\omega - \omega_e)] - v_{ex} \frac{dln[m_o(\chi + e^{-\gamma t})]}{dt} \]  

\[ \ddot{y} + 2\omega \dot{x} = -A[2\dot{y} - (R - 2x)(\omega - \omega_e)] - v_{ey} \frac{dln[m_o(\chi + e^{-\gamma t})]}{dt} \]  

\[ \ddot{z} + \omega^2 z = -A\dot{z} - v_{ez} \frac{dln[m_o(\chi + e^{-\gamma t})]}{dt} \]

with,

\[ A = \frac{1}{2} \frac{C_S D}{m_o} \rho R (\omega - \omega_e) \]
\( R \) is the position of the vehicle in relation to the center of the Earth, \( \omega \) is the vehicle's angular velocity, \( \omega_e \) is the rotation's Earth velocity, \( C \) is the Drag coefficient, \( \rho \) is the atmospheric density, \( S_D \) is the area impact section debris, \( m_D \) is the debris mass.

The propellant will vary during the operation of the evasive maneuver. In this work, we will use model for mass variation [10],

\[
M(t) = m_0 \left( \chi + e^{-\gamma t} \right) \tag{5}
\]

\( M(t) \) is the total mass of the vehicle (mass of the space vehicle body added to the mass of the propellant). We call technological parameters: \( \gamma > 0 \) - motor power factor; \( \chi > 1 \) - mass factor, i.e., the ratio between \( M_0 \) (initial vehicle mass), \( m_0 \) (initial mass of the propellant) and \( v_{ex}, v_{ey}, v_{ez} \) - components Cartesian of \( v_e \) - exhaust gas velocity.

Figure 1 - Reference system centered on the satellite in relation to the center of the Earth.

2.1 Relative Dynamics - Equation Solution

With some simple algebraic calculations, Equations (1) - (3) will yield two coupled equations in the \( \dot{x} \) and \( \dot{y} \) components. Uncoupling is obtained by introducing a constant \( \lambda_n \), which appears in the final solution, Equations (6) - (8). The \( z \) component is not coupled. These three resulting differential equations are inhomogeneous and were solved by direct integration, using second order differential equation solution methods found in the literature. The solution provides the Cartesian components of the relative position vector between vehicle and debris as function of time. This model considers the gravitational force of the Earth acting on the two space objects, the atmospheric drag force on the debris and the propulsive force on the vehicle.

\[
X(t) = D_x e^{\left(\frac{\left(k_4 + a_2\right) t}{2}\right)} + D_4 e^{-\left(\frac{\left(k_4 + a_2\right) t}{2}\right)} - \frac{e^{\left(\frac{\left(k_4 + a_2\right) t}{2}\right)}}{2k_4} \sum_{n=1}^{\infty} \left( -1 \right)^n \frac{1}{n^2} \frac{\ln(\chi + 1) - \ln(\chi + e^{-\gamma t})}{-k_4 + \frac{a_2}{2} - \gamma n} \tag{6}
\]

\[
Y(t) = D_1 e^{\left(\frac{\left(k_3 + a_1\right) t}{2}\right)} + D_2 e^{-\left(\frac{\left(k_3 + a_1\right) t}{2}\right)} - \frac{e^{\left(\frac{\left(k_3 + a_1\right) t}{2}\right)}}{2k_3} \sum_{n=1}^{\infty} \left( -1 \right)^n \frac{1}{n^2} \frac{\ln(\chi + 1) - \ln(\chi + e^{-\gamma t})}{-k_3 + \frac{a_1}{2} - \gamma n} \tag{7}
\]
\[ Z(t) = e^{-\frac{\lambda}{2}t}[C_1 \cos(k_2 t) + C_2 \sin(k_2 t)] e^{\frac{v_{exA}}{2}t}\ln\left((m_o(\chi + e^{-\lambda t}))\sin(k_2 t)\right) - \]
\[\frac{v_{exA}}{2k_2} - e^{-\frac{\lambda}{2}t}\left(\frac{A}{2} \sin(k_2 t) + k_2 \cos(k_2 t)\right) + k_2^2 \left[\frac{\ln(x \gamma^2_c)}{2} - \frac{v_{exA} \ln(x \gamma^2_c)}{2} - e^{-\frac{\lambda}{2}t}\left(\frac{A}{2} \sin(k_2 t) - \frac{A}{2} \cos(k_2 t)\right) + k_2 e^{-\lambda t}\right] \tag{8}\]

These equations depend on the initial conditions \((x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0)\) of collision when the objects are subject only to gravitational force. They also depend on the technological parameters of the propulsive force \((\gamma, \chi, v_p)\) and the constants that characterize the atmospheric drag force. All these constants are shown in the Appendix and Equation (4). The constants \(C_1\) and \(C_2\) are arbitrary. The solution is also a function of the coupling constant \(\lambda\). Its value can be found by experimental calibration of the solution. Equations (6) - (8) determine the Cartesian components of \(r(t)\), the final relative position between the spatial objects at each instant. The value of \(r(t)\) will be determined by the combination of these factors and the initial conditions. If the combination yields a nonzero \(r(t)\) value, the evasive maneuver was a success.

### 2.2 Collision Conditions

If we consider the vehicle and the DE as spherical bodies, average radii \(r_v\) and \(r_{DE}\), respectively, the collision conditions will occur:

\[ r(t_c) = d(t_c) + r_v + r_{DE} \Rightarrow r(t_c) - (r_v + r_{DE}) = d(t_c) \leq 0 \tag{9}\]

The collision occurs for any values of their relative velocities, at a given time, \(t_c\). In the Equation (9), \(d(t_c)\) is the distance between the edges of the spheres.

Thus, for a given pair of angles \((0^\circ \leq \phi \leq 180^\circ\) - out-plane angle, and \(0^\circ \leq \theta < 360^\circ\) - in-plane angle) there will be initial relative positions \((x_0, y_0, z_0)\) and velocities \((\dot{x}_0, \dot{y}_0, \dot{z}_0)\) that are favorable to collision between colliding objects. The initial relative positions are obtained by taking the center of the vehicle as its origin and the sphere of radius \(r_o = (x_0, y_0, z_0)\) centered on it. That is,

\[ x_0 = r_o \sin \phi \cos \theta \tag{10}\]
\[ y_0 = r_o \sin \phi \sin \theta \tag{11}\]
\[ z_0 = r_o \cos \phi \tag{12}\]

In general, the operational vehicle would have the time \(t_c\) to escape the collision with a spatial debris, performing an evasive maneuver. If the dimensions of these bodies are negligible, the collision condition will be:

\[ r(t_c) = 0 \Rightarrow x(t_c) = 0, y(t_c) = 0, z(t_c) = 0 \tag{13}\]

### 3. Numerical Results

In this section we present the results of numerical simulations for collision dynamics and evasive maneuvers between spatial objects.
3.1 Collision Possibilities and Natural Quasi-Rendezvous

The following graphs (Figure 2) show the possibilities of collision between the space vehicle and the space debris under the exclusive action of the terrestrial gravitational force for several initial positions relative to each other. This set of initial conditions was identified as the configurations that the collisional system (vehicle-debris) can take, considering all pair of angles \((\phi, \theta)\). That is, they are the states that the system can take on a collision course for relative distances between 3 and 50 km. The number of states is called “collision possibilities”. This distribution of the configurations was simulated for a time interval between 1 and 3,000 s, and a time step 1 s, which we found considering the situation where the vehicle is at the center of the reference system and the debris are initially on a sphere of radius variable from 3 to 50 km and is approaching the vehicle with an initial relative velocity that can vary between 0 and 20 km/s. The simulations were performed for altitude 220 km.

In the upper right corner of the Figure 2, there is lower initial relative velocity ranges favorable for collision. The range is between 0 and 0.1 km/s and the collision possibilities the order of \(10^8\). In this range, we say that the encounter between the objects is not catastrophic (typical collision). For this speed range, we define natural quasi-Rendezvous to meet the two space objects at very small speeds and only subject to natural forces. It is a smooth encounter between the objects, whose energy is not enough to generate significant damages to the vehicle. The Space Clean maneuvers and landing or collision of space vehicles with asteroids are possible applications for the natural quasi-Rendezvous. In the maneuvers it is necessary to have a smooth encounter between the space objects, even with small relative velocity between them. Space Clean maneuvers are Earth-orbiting DE rescue maneuvers, captured by towed vehicle, which force the re-entry of DE into atmospheric drag regions.

In addition, we observed that for large initial distances between objects, the possibility of collision increases, but for small relative initial velocities. The collision time, \(t_c\), is fixed and therefore the larger initial distances will result in lower initial velocities and thus the collision configurations are established. The angular momentum of the vehicle-DE system for dynamics subjected only to gravitational force is conserved. In LEO the orbital periods are shorter with respect to another operating regions, facts that select the slower speeds as more favorable to collisions.

![Figure 2 - Possibilities of Collision Vs. Relative initial speeds.](image-url)
3.2 Collision Possibilities and Atmospheric Drag
Figure 3, below, shows the relation between the objects final relative position and direction angles. We simulate several pairs of angles. We observe that in the maneuver without the drag (Drag out), the collision occurs until the time $t_c = 600$ s, whereas with the drag (Drag in) the objects distance themselves. The final distance in this period depends on the combination of direction angles. On the other hand, this non-collision condition can be overcome, even with drag, when longer period and for small-sized debris. This result can be seen in Figure 4, below. In these simulations we used the values of the quantities: $C = 2.2; \rho = 2.789 \times 10^{-10}$ kg/m$^3$, $S_D = 3.1416 \times 10^{-10}$ km$^2$, $m_D = 10^{-2}$ kg.

![Figure 3 - Maneuvers with and without Atmospheric Drag vs. Collision time](image1)

![Figure 4 - Maneuvers with atmospheric drag vs. collision time](image2)

The results show that the effect of drag is to push objects away and then to collide, depending on the minimum size of the debris. This separation does not last for long. The graphics in the Figure 4 show results of an initial condition configuration with different collision time, the first at 2,500 s and the second at 3,000 s. We can see change in the values of the final relative distances. In the first, its maximum occurs before 1,300 s and, in the second, after 1,500 s.
3.3 Evasive Maneuvers
With the propulsion system of the vehicle it is possible to carry out evasive maneuvers in the face of the possibility of collision with the debris. The exhaust velocity of the engine exaggerates the effect of the acceleration the atmospheric drag causes in the bodies, separating them, as its module grows. This phenomenon can be seen in Figure 5. Note that for small speeds (black color line) collision occurs, but as the velocity modulus increases, the spacing of objects increases. This phenomenon is independent if propulsion or retro propulsion occurs. These results were obtained for the quantities: $\gamma = 10^{-6}$ Hz, $\chi = 10$, $m_o = 4.5$ kg, $(\phi, \theta) = (57^o, 68^o)$.

![Figure 5 - Relative final position vs. Exhaust Speed (Propulsion and Retro-Propulsion).](image)

Figure 5 shows the final relative positions as a function of the direction angles, $(\phi, \theta)$. In the dynamics with atmospheric drag, Figure 6(a), there is no collision, unless the debris dimensions are up to 50 cm, in the blue region that presents final relative positions up to 0.7075 km. That is, the effect of atmospheric drag favored the spacing of objects for any combination of these initial angles. This separation allows the migration of the debris to other regions, among them, that of reentry in the terrestrial atmosphere. Figure 6(b) shows the maneuver with propulsion. There is also no collision for debris up to 50 cm in the blue region with final relative positions up to 2,425 km.

![Figure 6 - Distribution of the final relative positions vs. initial angles.](image)
4. Conclusions

We studied evasive and Rendezvous maneuvers in an atmospheric drag environment with constant density, acting on the spatial debris. We note that the atmospheric drag favored the spacing of objects for a period and then approaches them in a collision course. This condition depends on a minimum approximation between the objects that can be filled by the debris. This condition also depends on the distribution of steering angles. In addition, we obtained possibility of collision with very low speeds, which we call natural quasi-Rendezvous. Propelled maneuvers are assisted by drag away the collision objects, depending the DE sizes. These results can be used in the analysis of various space missions, for example Space Clean maneuvers, asteroid vehicle landing or collision maneuvers.

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Appendix

\[ \alpha_1 = 2(A - \omega), \]
\[ \alpha_2 = A - 2\omega, \]
\[ \alpha_3 = A(\omega - \omega_e) + 3\omega^2 \]
\[ k_2 = \sqrt{\left(\frac{A}{2}\right)^2 - \omega^2}, \]
\[ k_3 = \sqrt{\left(\frac{\alpha_1}{2}\right)^2 - A(\omega - \omega_e)}, \]
\[ k_4 = \sqrt{\left(\frac{\alpha_2}{2}\right)^2 + \alpha_3}, \]
\[ D = AR(\omega - \omega_e), \]
\[ D_1 = y_0 + D_2, \]
\[ D_2 = -\frac{1}{2k_3} \left( k_3 - \frac{\alpha_1}{2} \right) x_0 + \frac{1}{2k_3} \frac{y^2}{(k_3 - \frac{\alpha_1}{2})^2} + v_{ex} \frac{1}{2k_3} \left( k_3 - \frac{\alpha_1}{2} \right) \ln(m_0 \chi) - \frac{v_{ex}}{2k_3} \ln(m_0 (\chi + 1)) + 1) \left( k_3 - \frac{\alpha_1}{2} \right) + \frac{\gamma}{(\chi + 1)} \right] + v_{ey} \left[ \frac{k_3 - \frac{\alpha_1}{2}}{2k_3} \ln \left( \frac{\chi + 1}{\chi} \right) \right] \]
\[ D_3 = x_0 + D_4, \]
\[ D_4 = -\frac{1}{2k_4} \left( k_4 - \frac{\alpha_2}{2} \right) x_0 + \frac{1}{2k_4} \frac{y^2}{(k_4 - \frac{\alpha_2}{2})^2} + v_{ex} \frac{1}{2k_4} \left( k_4 - \frac{\alpha_2}{2} \right) \ln(m_0 \chi) - \frac{v_{ex}}{2k_4} \ln(m_0 (\chi + 1)) + \left( k_4 - \frac{\alpha_2}{2} \right) + \frac{\gamma}{(\chi + 1)} \right] + v_{ey} \left[ \frac{k_4 - \frac{\alpha_2}{2}}{2k_4} \ln \left( \frac{\chi + 1}{\chi} \right) \right] \]