Energy-Efficient and Thermal-Aware Resource Management for Heterogeneous Datacenters

Hongyang Sun, Patricia Stolf, Jean-Marc Pierson, and Georges Da Costa

IRIT, University of Toulouse
118 Route de Narbonne, F-31062 Toulouse Cedex 9, France
{sun, stolf, pierson, dacosta}@irit.fr

Abstract: We propose in this paper to study the energy-, thermal- and performance-aware resource management in heterogeneous datacenters. Witnessing the continuous development of heterogeneity in datacenters, we are confronted with their different behaviors in terms of performance, power consumption and thermal dissipation: Indeed, heterogeneity at server level lies both in the computing infrastructure (computing power, electrical power consumption) and in the heat removal systems (different enclosure, fans, thermal sinks). Also the physical locations of the servers become important with heterogeneity since some servers can (over)heat others. While many studies address independently these parameters (most of the time performance and power or energy), we show in this paper the necessity to tackle all these aspects for an optimal resource management of the computing resources. This leads to improved energy usage in a heterogeneous datacenter including the cooling of the computer rooms. We build our approach on the concept of heat distribution matrix to handle the mutual influence of the servers, in heterogeneous environments, which is novel in this context. We propose a heuristic to solve the server placement problem and we design a generic greedy framework for the online scheduling problem. We derive several single-objective heuristics (for performance, energy, cooling) and a novel fuzzy-based priority mechanism to handle their tradeoffs. Finally, we show results using extensive simulations fed with actual measurements on heterogeneous servers.

Keywords: Datacenter heterogeneity; online scheduling; server placement; cooling; multi-objective optimization.

1 Introduction

The last years have witnessed the development of heterogeneity in clusters and datacenters. Two main reasons have led to this situation today. The first one is due to the maintenance and evolution of the components in the datacenters: different generations of computers are commonly seen in production datacenters since the owners are not changing everything at each update. The second reason is driven by the idea that heterogeneity might be the key to achieving energy-proportional computing [5, 9], especially for high-performance computing applications.

Many recent studies alert dramatically on the energy consumption of the datacenters. For instance, Koomey’s report [21] claims that today’s datacenters are consuming nearly 2% of the global energy, and up to half of that is spent on cooling-related activities [33]. This results generally in very poor Power Usage Effectiveness (PUE).

In this paper, we study the multi-objective resource management problem for heterogeneous datacenters. Besides the performance criterion, we also consider the energy consumption of the servers and their thermal impact on the datacenter cooling. The aim of our work is to optimize these objectives and to explore their tradeoffs. In particular, the energy consumption is partly due to the cooling efficiency in the datacenter [25, 38], which is related to both the physical placement of the servers and the scheduling strategies when jobs dynamically enter and leave the system. The latter also affects the performance and the energy consumed by the servers.

Server placement in a computer room has been relatively less studied, especially its impact on the cooling efficiency. The reason for this lack of attention is mainly due to the fact that, when servers are homogeneous, their relative positions have no impact on the performance and computing energy. However, server placement can have an impact on the cooling infrastructure. The main observation is that one server might contribute to the temperature raise at the inlets of the other servers, due to the recirculation of heat in a datacenter. Such mutual influence can be modeled by a heat distribution matrix among the servers. If one wants to keep the inlet temperature under a given threshold, the supplied air temperature has to be decreased accordingly by
the cooling system, which in turn increases its energy consumption. In the presence of heterogeneous servers with different power consumptions and hence heat dissipation, the problem of finding the optimal placement becomes complicated and, to the best of our knowledge, has not been studied. Since it is not feasible to change dynamically the positions of the servers in a datacenter, we focus on static placement to minimize the cooling cost induced by different configurations.

With a given server placement, the traditional problem of job scheduling in the heterogeneous environment remains. Many previous work (e.g., \cite{4, 40}) considered only the performance criterion and hence focused on the jobs’ execution times. In order to address the power consumption issue in datacenters, however, application scheduling must employ a multi-objective approach by considering performance, energy and cooling together. To account for the fact that a scheduler has no future knowledge (jobs arrive over time), we need an online scheduling strategy. Instead of designing different independent algorithms, we design a greedy online scheduling framework that can be adapted easily by redefining the cost function, from a single objective to two or more objectives. To tackle the energy-performance tradeoff, we further introduce a fuzzy-based priority approach, which allows to explore the potential improvement in one objective while relaxing the other objective up to an acceptable range. This approach can be extended to incorporate more than two objectives in the framework. Its principle is not limited to the case at hand and can potentially be applied to other multi-objective optimization problems.

The main contributions of this paper are the following:

- A static server placement heuristic to reduce the cooling cost for the servers in a datacenter.
- A greedy scheduling framework and several cost functions to tackle single-objective scheduling (for performance, energy, and cooling).
- A fuzzy-based priority approach to handle the tradeoff between two conflicting objectives, and its extension to multi-objective optimization.

These proposals are supported by extensive simulations conducted using real hardware specifications and software benchmarks, as well as experimentally verified cooling model and heat distribution matrix \cite{39, 38}. Specifically for the hardware, a server system with high packing density and integrated cooling support is chosen for the experiments, which we believe represents well an emerging class of highly integrated energy-efficient servers. The results demonstrate the flexibility of our scheduling framework and confirm the effectiveness of the fuzzy-based approach for exploring the energy-performance tradeoff in heterogeneous datacenter environments. Our static server placement heuristic is also shown to provide much improved thermal distribution leading to significant reduction in cooling cost.

The rest of this paper is organized as follows. Section 2 formally states the system model and the scheduling problem. Section 3 describes our greedy server placement heuristic. Section 4 presents the job scheduling framework, various cost functions and the fuzzy-based priority approach. The simulation results are shown in Section 5. Section 6 reviews some related work, and Section 7 summarizes the paper and addresses future directions.

2 Problem Statement

2.1 System Model

Motivated by the placement of physical servers and the scheduling of High-Performance Computing (HPC) applications in heterogeneous datacenters, we consider the following system model: A set $\mathcal{M} = \{M_1, M_2, \ldots, M_m\}$ of $m$ servers (or machines) needs to be placed inside a computer room (or datacenter) with a set of $m$ rack slots, denoted by $S = \{S_1, S_2, \ldots, S_m\}$. Each server $M_j \in \mathcal{M}$ consists of $L_j$ processors of the same type (possibly on different boards), but the type and the number of processors may vary for different servers, rendering the system heterogeneous. Each server consumes a base power $U^{base}$ to support the basic operations of the infrastructure backbone, such as monitoring, networking and cooling (for instance fans). A set $\mathcal{J} = \{J_1, J_2, \ldots, J_n\}$ of $n$ jobs arrive at the system over time, and they need to be assigned in an online manner to the servers. Each job $J_i \in \mathcal{J}$ has a release time $r_i$ and a processor requirement $l_i$ that must be granted in order to run on any server. To execute job $J_i$ on server $M_j$ incurs a processing time $P_{i,j}$ and a power consumption $U_{i,j}$, both of which are server-dependent and become known upon the job’s arrival by prior profiling of the applications. In particular, the profiled application power consumption is assumed to include the leakage power.

\footnote{In this paper, we assume that the number of rack slots is equal to the number of servers to be placed, which represents a common scenario in small- and medium-size datacenters.}
2.2 Scheduling Model

We study two orthogonal problems that deal with the placements of hardware and software, respectively. We call the two problems static server placement and online job scheduling. The former concerns the positioning of physical servers in the datacenter, which as explained in Section 3 will have an impact on the cooling energy in heterogeneous environment. The latter concerns the dynamic assignment of workloads to the servers, which will impact energy (due to both computing and cooling) as well as performance.

For the first problem of static server placement, each server needs to be physically and statically placed in advance to one of the available rack slots in the datacenter. In particular, we are looking for a mapping \( \sigma : \{1, 2, \ldots, m\} \to \{1, 2, \ldots, m\} \) from rack slots to servers so that each slot \( S_k \) is filled with a server \( M_{\sigma(k)} \).

The objective is to minimize the cooling cost. More details about this problem will be described in Section 3.

Given a particular server placement, an online scheduling strategy is then required to assign the jobs to the servers for execution. Specifically, each arrived job \( J_i \in J \) must be assigned irrevocably to a server with at least \( l_i \) idle processors, and without any knowledge of the future arriving jobs. Once the job has been assigned, no preemption or migration is allowed, which is typically assumed for HPC applications since they tend to incur a significant cost in terms of data reallocation.

At any time \( t \), the total computing power of server \( M_j \) is the sum of its base power and the power consumed for executing all jobs assigned to it, i.e.,

\[
U_{j}^{\text{comp}}(t) = U_{j}^{\text{base}} + \sum_{i=1}^{n} \delta_{i,j}(t) \cdot U_{i,j} ,
\]

where \( \delta_{i,j}(t) \) is a binary variable that takes value 1 if job \( J_i \) is running on server \( M_j \) at time \( t \) and 0 otherwise.

In order to optimize performance, we do not allow processor sharing among the jobs. Thus, each server at any time can only host a subset of the jobs whose total processor requirements are no more than the server’s total number of available processors, i.e., \( \sum_{i=1}^{n} \delta_{i,j}(t) \cdot l_i \leq L_j \) for all \( 1 \leq j \leq m \) at all time \( t \).

2.3 Cooling Model

To characterize the cost of cooling, we consider a standard datacenter layout, where server racks are organized in rows with alternating cold and hot aisles. The computer room air conditioning (CRAC) unit supplies cool air to the cold aisles through raised floor vents. Each server \( M_j \in M \) in the racks is oriented such that it draws cool air with temperature \( T_{j}^{\text{in}} \) from the inlet and dissipates hot air with temperature \( T_{j}^{\text{out}} \) to the outlet. Assuming that the computing power consumed by a server is completely transformed into heat, the relationship between the power consumption and the inlet/outlet temperature of server \( M_j \) at any time \( t \) can be characterized by

\[
T_{j}^{\text{out}}(t) = T_{j}^{\text{in}}(t) + K_j \cdot U_{j}^{\text{comp}}(t) ,
\]

where \( K_j = p f_j c \), with \( p \) denoting the air density (in \( \text{kg/m}^3 \)), \( f_j \) the airflow rate of server \( M_j \) (in \( \text{m}^3/\text{s} \)), and \( c \) the air heat capacity\(^2\) (in \( J/(\text{C} \cdot \text{kg}) \)).

Due to complex airflow patterns, typical datacenters experience the so-called heat recirculation phenomenon, where the hot air from the server outlets recirculates in the room and is mixed with the supplied cool air from the CRAC, causing the temperature at the server inlets to be higher than that of the supplied air. Prior studies\(^3\) have characterized this phenomenon with a heat distribution matrix \( D \) by assuming a fixed airflow pattern in the room and conservation of energy as described by Equation (2). We adopt this approach here. Let each element \( d_{j,k} \in D \) represent the temperature increase at the inlet of server \( M_j \) per unit of power consumed by server \( M_k \). Combining the heat contributions from all servers, the inlet temperature of server \( M_j \) at time \( t \) is given by the following equation:

\[
T_{j}^{\text{in}}(t) = T^{\text{sup}}(t) + \sum_{k=1}^{m} d_{j,k} \cdot U_{k}^{\text{comp}}(t) ,
\]

where \( T^{\text{sup}}(t) \) denotes the supplied air temperature at time \( t \), which should be adjusted to prevent the inlet temperature of any server from going beyond a redline temperature \( T^{\text{red}} \); otherwise, the electronic components

\(^2\)The air heat capacity specifies the energy required to change the temperature of one unit mass of air by one unit degree.

\(^3\)Technically speaking, \( d_{j,k} \) represents the temperature increase for the server at slot \( S_j \) due to the power consumption by the server at slot \( S_k \). For convenience, we simply assume that the servers are renamed such that server \( M_j \) is placed in slot \( S_j \) for all \( 1 \leq j \leq m \).
may not work reliably or are at risk of being damaged. Hence, the supplied air temperature should be set at most to

\[ T_{\text{sup}}(t) = T_{\text{red}} - \max_{j=1,m} \sum_{k=1}^{m} d_{j,k} \cdot U_{k}^{\text{comp}}(t) . \]  

The cooling cost is specified as

\[ U_{\text{cool}}(t) = \frac{\sum_{j=1}^{m} U_{j}^{\text{comp}}(t)}{\text{CoP}(T_{\text{sup}}(t))} , \]

where CoP is the coefficient of performance, defined as the ratio of the amount of heat to be removed to the energy that needs to be consumed in order to perform the cooling [25]. This coefficient characterizes the efficiency of the CRAC unit, and is an increasing (usually non-linear) function of the supplied air temperature. Intuitively, it means that the CRAC unit needs to work harder and thus consumes more energy in order to provide cooler air to the computer room.

### 2.4 Optimization Objectives

We consider the following bi-objective optimization problem: optimizing the performance of the jobs and minimizing the energy consumption of the datacenter, due to both computing and cooling.\(^4\)

For performance, we use the average response time of the jobs as the metric, and it is defined as

\[ R_{\text{ave}} = \frac{1}{n} \sum_{i=1}^{n} (c_i - r_i) , \]

where \(c_i\) and \(r_i\) denote the completion time and release time of job \(J_i\), respectively.

The energy consumption comes from two sources: computing and cooling. The one due to computing is given by the total computing power of all servers integrated over time, i.e.,

\[ E_{\text{comp}} = \int_{t_1}^{t_2} \sum_{j=1}^{m} U_{j}^{\text{comp}}(t) \, dt , \]

where \([t_1, t_2]\) denotes the interval of interest, during which all jobs arrive and complete their executions. This computing energy can be further divided into two parts, namely, the static part due to the base power consumption, i.e.,

\[ E_{\text{stat comp}} = (t_2 - t_1) \cdot \sum_{j=1}^{m} U_{j}^{\text{base}} , \]

and the dynamic part due to the power consumed for executing the jobs, i.e.,

\[ E_{\text{dync comp}} = \sum_{i=1}^{n} \sum_{j=1}^{m} \delta_{i,j} \cdot P_{i,j} \cdot U_{i,j} , \]

where \(\delta_{i,j} = 1\) if job \(J_i\) is assigned to server \(M_j\) and \(0\) otherwise.

The energy spent on cooling is the total cooling power integrated over time, i.e.,

\[ E_{\text{cool}} = \int_{t_1}^{t_2} U_{\text{cool}}(t) \, dt , \]

and as with computing energy, cooling energy can also be broken into a static part and a dynamic part. Specifically, the static part is the cooling energy that will be spent during interval \([t_1, t_2]\) even if no job arrives, i.e.,

\[ E_{\text{stat cool}} = \int_{t_1}^{t_2} \frac{\sum_{j=1}^{m} U_{j}^{\text{base}}(t)}{\text{CoP}(T_{\text{red}} - \max_{j} \sum_{k=1}^{m} d_{j,k} \cdot U_{k}^{\text{base}}(t))} \, dt , \]

and the dynamic part is the difference between the total cooling energy and the static one, i.e.,

\[ E_{\text{dync cool}} = E_{\text{cool}} - E_{\text{stat cool}} . \]

\(^4\)The energy consumed by other parts of the datacenter, such as lighting, are ignored, since they are insignificant compared to the computing and cooling energy.
In this paper, we assume that all servers are turned on all the time to sustain the servers’ infrastructure backbone, so the static energy due to both computing and cooling is independent of the workload and the job scheduling strategy. On the other hand, the total dynamic energy given by

$$E_{total}^{\text{dyn}} = E_{\text{comp}}^{\text{dyn}} + E_{\text{cool}}^{\text{dyn}}$$  \hspace{1cm} (13)$$

is closely related to job scheduling, and it will be the focus of this study.

Due to the heterogeneity of the servers in the datacenter, different job scheduling strategies may result in very different job response time, computing energy and cooling cost. While a specific scheduling strategy may optimize one objective, these different objectives can be conflicting with each other, making the optimization difficult. In Section 4, we will propose and evaluate online scheduling algorithms to address both performance and energy as well as to deal with their tradeoffs.

3 Static Server Placement and A Greedy Heuristic

In this section, we consider the problem of static server placement. We first motivate the study from the perspective of cooling in heterogeneous datacenters. We then formulate the problem and present a greedy heuristic.

3.1 Motivation

The literature contains extensive studies on virtual machine placement (e.g., [6, 15, 44]) for datacenters, but the placement of physical servers has received little attention. There are two main reasons. First, many traditional datacenters are homogeneous, so different placements of identical servers do not make a difference. Second, traditional metrics such as job performance and energy consumption (due to computing) are independent of the servers’ relative positions, so they are unaffected by the different placement configurations.

As far as the cooling cost is concerned for heterogeneous datacenters, however, the placement of the physical servers will have an impact. In particular, the studies in [39, 38] have shown that the heat recirculation phenomenon in typical datacenters exhibits the following properties:

(1). Different rack positions tend to behave differently in terms of heat recirculation. Typically, servers located at the upper parts of the racks “inhale” more recirculated hot air while servers located at the lower parts “contribute” more hot air to recirculate in the room.

(2). In a closed computer room with fixed locations of all major objects and without moving objects, the airflow pattern that characterizes the heat recirculation among different rack positions is relatively stable.

While the first property suggests that the heat distribution matrix tends to be highly asymmetric, the second property assures that the matrix does not change significantly with different workloads in the servers or different positions of the servers. In the next section, we will rely on workload placement (or job scheduling) techniques to manage the cooling cost together with other objectives. Here, we focus on arranging the positions of the servers with different power profiles. The goal is to reduce the maximum inlet temperature of the servers so as to minimize the cooling cost under a given load condition.

To illustrate the effectiveness of this approach, consider a simple datacenter with two servers, two rack slots, and the following heat distribution matrix:

$$D = \begin{bmatrix} 0.002 & 0.004 \\ 0.001 & 0.002 \end{bmatrix}.$$  

Suppose the two servers consume an average power of 100W and 200W, respectively. By placing the first server in slot 1 and the second server in slot 2, their inlet temperatures increase by 1°C and 0.5°C respectively according to Equation (3). By simply swapping the positions of the two servers, their temperature increases will now become 0.4°C and 0.8°C. The 0.2°C difference in the maximum inlet temperature of these two configurations directly determines the temperature of the supplied air by Equation (4), and therefore impacts the cooling cost. For instance, consider a redline temperature of 25°C and the following CoP model for a water-chilled CRAC unit in an HP datacenter [25, 38]:

$$\text{CoP}(T) = 0.0068T^2 + 0.0008T + 0.458.$$  \hspace{1cm} (14)$$

According to Equations (4) and (5), the cooling costs for the two placement configurations are 68.275W and 67.269W, respectively. The impact will be more significant with a lower redline temperature or a more skewed heat distribution matrix, or when the servers are consuming more power. The problem will also become more challenging when there is a large number of servers/positions, since exhaustive search will no longer be possible. The next subsection considers this general case and proposes a heuristic algorithm for the problem.
server’s typical power consumption during runtime. We call this static value the average power it consumes when executing historical workloads. This provides a reasonable estimation on the time. To cope with this uncertainty, we characterize the power consumption of each server statically using the sensor-based approach \[39\], the latter essentially depends on the servers’ workloads, which can vary with the power consumption profile of all servers. While the former is relatively stable and can be measured using sensors, the latter is introduced to meet the dynamic needs of the datacenter. As we have seen previously, this is determined by both the heat-distribution matrix and the workloads of the servers.

To reduce the cooling cost, we should minimize the maximum temperature increase at the inlet of any server in the datacenter. As we have seen previously, this is determined by both the heat-distribution matrix and the reference power vector. Appendix provides the NP-hardness proof.

Let \( U_{j}^\text{ref} \) denote the reference power of server \( M_j \in \mathcal{M} \). The static server placement problem can then be formulated as follows: find a mapping \( \sigma : \mathcal{M} \rightarrow \mathcal{S} \) from rack slots to servers, so as to minimize:

\[
\text{max } D \cdot U_{\sigma(k)}^\text{ref},
\]

where \( U_{\sigma}^\text{ref} = [U_{\sigma(1)}^\text{ref}, U_{\sigma(2)}^\text{ref}, \ldots, U_{\sigma(m)}^\text{ref}]^T \). Finding the optimal placement turns out to be a NP-hard problem for arbitrary heat-distribution matrix and reference power vector. Appendix provides the NP-hardness proof.

Given the hardness result, we design a heuristic algorithm for the static server placement problem based on a greedy allocation strategy. Algorithm 1 presents the pseudocode of our Greedy Server Placement (GSP) heuristic.

First, GSP sorts the servers in descending order of reference powers (Line 1). Since the servers that consume more power on average will have larger contributions to the temperature increases at all inlets, they are placed first to have more flexibility in the slot selection and so to avoid high peak temperature. Let \( T_{\text{incr}} \) denote the existing temperature increase at the inlet of slot \( S_l \), and it is initially set to zero for all inlets (Line 2). Let \( T_{\text{incr}}^{\text{max}}(k) \) denote the maximum temperature increase if the next server \( M_j \in \mathcal{M} \) is placed in slot \( S_k \), i.e.,

\[
T_{\text{incr}}^{\text{max}}(k) = \max_{l=1..m} \left( T_{\text{incr}} + d_{l,k} \cdot U_j^\text{ref} \right).
\]

Server \( M_j \) will be placed in one of the remaining slots \( S_k \in \mathcal{S} \) that minimizes the maximum temperature increase, i.e., \( k^* = \text{argmin}_{k} T_{\text{incr}}^{\text{max}}(k) \). The temperature increase at all inlets will then be updated and the filled slot \( S_k \) will be removed from the available set \( \mathcal{S} \) (Lines 12 and 13). The algorithm iterates over all servers and terminates after the last one is placed.

For the complexity of the algorithm, sorting and initialization takes \( O(m \log m) \) time. In the iteration, placing each server incurs \( O(m^2) \) time as all remaining slots are examined to determine the maximum temperature increase at all inlets. Therefore, the overall complexity is \( O(m^3) \). This is reasonable even for a large number of servers, since the process is performed relatively infrequently: new placement of the servers is only necessary if there are significant alteration to the datacenter layout or when some servers are removed and new ones are introduced.

### Algorithm 1 Greedy Server Placement (GSP)

**Input:** The set \( \mathcal{M} = \{M_1, M_2, \ldots, M_m\} \) of \( m \) servers, and the reference power \( U_j^\text{ref} \) of each server \( M_j \in \mathcal{M} \); the set \( \mathcal{S} = \{S_1, S_2, \ldots, S_m\} \) of \( m \) rack slots, and the heat distribution matrix \( D \).

**Output:** A mapping \( \sigma \) from rack slots to servers.

1. Sort the servers in descending order of reference power, i.e., \( U_1^\text{ref} \geq U_2^\text{ref} \geq \cdots \geq U_m^\text{ref} \).
2. Initialize \( T_{\text{incr}}^{\text{max}}(l) = 0 \) for all \( 1 \leq l \leq m \).
3. for each server \( M_j \in \mathcal{M} \) do
   4. \( k^* = 0 \) and \( T_{\text{incr}}^{\text{max}}(k^*) = \infty \).
   5. for each slot \( S_k \in \mathcal{S} \) do
      6. \( T_{\text{incr}}^{\text{max}}(k) = \max_{l=1..m} \left( T_{\text{incr}}^{\text{max}}(l) + d_{l,k} \cdot U_j^\text{ref} \right) \)
      7. if \( T_{\text{incr}}^{\text{max}}(k) < T_{\text{incr}}^{\text{max}}(k^*) \) then
         8. \( T_{\text{incr}}^{\text{max}}(k^*) = T_{\text{incr}}^{\text{max}}(k) \) and \( k^* = k \).
   9. end if
10. end for
11. Place server \( M_j \) to slot \( S_{k^*} \), i.e., \( \sigma(k^*) = j \).
12. Update \( T_{\text{incr}}^{\text{max}} = T_{\text{incr}}^{\text{max}} + d_{l,k^*} \cdot U_j^\text{ref} \) for all \( 1 \leq l \leq m \).
13. Update \( \mathcal{S} = \mathcal{S} \setminus S_{k^*} \).
14. end for

#### 3.2 Greedy Heuristic

To reduce the cooling cost, we should minimize the maximum temperature increase at the inlet of any server in the datacenter. As we have seen previously, this is determined by both the heat-distribution matrix and the power consumption profile of all servers. While the former is relatively stable and can be measured using sensors, the latter is introduced to meet the dynamic needs of the datacenter. As we have seen previously, this is determined by both the heat-distribution matrix and the reference power vector. Appendix provides the NP-hardness proof.

Given the hardness result, we design a heuristic algorithm for the static server placement problem based on a greedy allocation strategy. Algorithm 1 presents the pseudocode of our Greedy Server Placement (GSP) heuristic.

First, GSP sorts the servers in descending order of reference powers (Line 1). Since the servers that consume more power on average will have larger contributions to the temperature increases at all inlets, they are placed first to have more flexibility in the slot selection and so to avoid high peak temperature. Let \( T_{\text{incr}} \) denote the existing temperature increase at the inlet of slot \( S_l \), and it is initially set to zero for all inlets (Line 2). Let \( T_{\text{incr}}^{\text{max}}(k) \) denote the maximum temperature increase if the next server \( M_j \in \mathcal{M} \) is placed in slot \( S_l \), i.e.,

\[
T_{\text{incr}}^{\text{max}}(k) = \max_{l=1..m} \left( T_{\text{incr}} + d_{l,k} \cdot U_j^\text{ref} \right).
\]

Server \( M_j \) will be placed in one of the remaining slots \( S_k \in \mathcal{S} \) that minimizes the maximum temperature increase, i.e., \( k^* = \text{argmin}_{k} T_{\text{incr}}^{\text{max}}(k) \). The temperature increase at all inlets will then be updated and the filled slot \( S_k \) will be removed from the available set \( \mathcal{S} \) (Lines 12 and 13). The algorithm iterates over all servers and terminates after the last one is placed.

For the complexity of the algorithm, sorting and initialization takes \( O(m \log m) \) time. In the iteration, placing each server incurs \( O(m^2) \) time as all remaining slots are examined to determine the maximum temperature increase at all inlets. Therefore, the overall complexity is \( O(m^3) \). This is reasonable even for a large number of servers, since the process is performed relatively infrequently: new placement of the servers is only necessary if there are significant alteration to the datacenter layout or when some servers are removed and new ones are introduced.

6
Algorithm 2 Greedy Scheduling Framework (GSF)

**Input:** Job queue $Q$, and for each job $J_i \in Q$, the processor requirement $l_i$, processing time $P_{i,j}$ and power consumption $U_{i,j}$; Server set $M$, and for each server $M_j \in M$, the number $L_j$ of available processors, which is initialized to $L_j = L_j$.

**Output:** Assignments of the newly arrived job and the jobs in $Q$ to the servers in $M$.

1: if a new job $J_i$ arrives then
2: $j^* = 0$ and $H_{i,j^*} = \infty$
3: for each server $M_j \in M$ do
4: if $L_j \geq l_i$ & $H_{i,j} < H_{i,j^*}$ then
5: $H_{i,j^*} = H_{i,j}$ and $j^* = j$
6: end if
7: end for
8: if $H_{i,j^*} \neq \infty$ then
9: Assign job $J_i$ to server $M_{j^*}$
10: Update $L_{j^*} = L_{j^*} - l_i$
11: else
12: Put job $J_i$ in queue $Q$ in shortest job first order
13: end if
14: else if a job $J_i$ completes execution on server $M_j$ then
15: Update $L_j = L_j + l_i$
16: for each job $J_k \in Q$ do
17: if $L_j \geq l_k$ then
18: Assign job $J_k$ to server $M_j$
19: Update $L_j = L_j - l_k$
20: end if
21: end for
22: end if

4 Online Job Scheduling and a Fuzzy-Based Priority Approach

Once the servers have been placed in a datacenter, they will start operation by executing the applications or jobs. In practice, jobs are submitted by different users over time, so each job must be assigned to a server without knowing future job arrivals. This section considers online job scheduling under a given server placement to optimize performance and energy, and to deal with their tradeoffs.

4.1 Greedy Scheduling Framework

All online scheduling algorithms described in this section fall under a Greedy Scheduling Framework (GSF), which is evoked whenever a new job arrives or an existing job completes execution. Algorithm 2 presents the pseudocode of this framework.

The variable $H_{i,j}$ shown in the pseudocode represents the cost of assigning job $J_i$ to server $M_j$. Specifically, $H_{i,j}$ can be a single-objective cost function of job response time, energy consumption, etc. (see Section 4.2), or it can be a composite cost function of two or more objectives (see Section 4.3).

For each newly arrived job $J_i$, among the servers that have sufficiently available processors to host it, the server with the minimum cost in terms of $H_{i,j}$ will be chosen for assigning the job (Lines 2-9). This makes the scheduling framework greedy. If no server has enough processors to host it, the job will be put in a waiting queue $Q$ in Shortest Job First (SJF) order [35], which is known to optimize the average response time (Line 12). Note that although the processing times of the jobs are server-dependent, their relative sizes are assumed to be consistent on different servers, i.e., a fast server is fast for all jobs. Hence, SJF can be realized by using any server as the reference for comparing the jobs’ processing times. When a job completes execution on a server and therefore releases the occupied processors, the waiting jobs in the queue will be tested in sequence to see if they can be assigned to this server (Lines 16-18). Whenever a job is assigned or a running job completes execution, the number of available processors on the server will be updated (Lines 10, 15, 19). Under this greedy scheduling framework, the assignment of each job takes $O(m)$ time, so the overall complexity is $O(mn)$ for assigning $n$ jobs.

The next two subsections will describe heuristic algorithms that minimize different single- and multi-objective cost functions depending on the optimization criteria.
4.2 Single-Objective Scheduling

Single-objective scheduling considers one optimization criterion when deciding where to assign each job. In this subsection, we will present several single-objective scheduling heuristics. Some of them will also be used as the base algorithms for designing the more complex multi-objective scheduling heuristics in the next subsection.

First, the following describes some single-objective heuristics proposed in the literature [25, 38].

- **Uniform**: Assign each job randomly to a server according to the uniform distribution.
- **MinHR**: Assign each job to a server that contributes minimally to the heat recirculation in the room. The cost function is defined as
  \[ H_{i,j}^{HR} = \sum_{k=1}^{m} d_{k,j} \, . \]  
  (17)
- **CoolestInlet**: Assign each job to a server with the lowest temperature at its inlet. The cost function is defined as
  \[ H_{i,j}^{CI} = T_{j}^{in} \, , \]  
  (18)
  where \( T_{j}^{in} \) denotes the current temperature at the inlet of server \( M_j \).

Note that, in [25, 38], these heuristics were applied in the offline setting, where the information of all jobs is available to the scheduler. Here, they are cast as online heuristics. While the aim of Uniform is to balance the workload on all servers, MinHR and CoolestInlet attempt to minimize the overall heat recirculation and to achieve a uniform temperature distribution, respectively. However, these heuristics were proposed for the homogeneous datacenter environments, and therefore do not consider job-specific characteristics. The following heuristics take job-dependent information into account by minimizing the performance, energy consumption, and temperature, respectively.

- **Perf-Aware**: Assign job \( J_i \) to a server that renders the minimum response time. The cost function is defined as
  \[ H_{i,j}^{P} = P_{i,j} \, , \]  
  (19)
  where \( P_{i,j} \) denotes the execution time of job \( J_i \) on server \( M_j \).
- **Energy-Aware**: Assign job \( J_i \) to a server that incurs the minimum dynamic energy consumption due to both computing and cooling. The cost function is defined as
  \[ H_{i,j}^{E} = E_{total}^{dyn} (\delta_{i,j} = 1) \, , \]  
  (20)
  where \( E_{total}^{dyn} \) is the total dynamic energy defined in Equation (13), and it is evaluated based on the currently running jobs and with job \( J_i \) assigned to server \( M_j \), i.e., \( \delta_{i,j} = 1 \).
- **Thermal-Aware**: Assign job \( J_i \) to a server that minimizes the maximum inlet temperature. The cost function is defined as
  \[ H_{i,j}^{T} = \max_{k=1}^{m} \left( T_{k}^{in} + \sum_{k=1}^{m} d_{k,j} \cdot U_{i,j} \right) \, , \]  
  (21)
  where \( T_{k}^{in} \) denotes the current temperature at the inlet of server \( M_k \), and \( U_{i,j} \) denotes the power consumption of job \( J_i \) on server \( M_j \).

Except for Uniform, all heuristics above break the tie by randomly selecting a server with the best cost function. The difference between CoolestInlet and Thermal-Aware is that the former considers the current inlet temperature before the job is assigned, whereas the latter considers the resulting temperature if the job is assigned to the server. Note that all of these heuristics make greedy decisions locally for each arriving job, so they are not guaranteed to provide the optimal global cost.

4.3 Multi-Objective Scheduling with Fuzzy-Based Priority

Scheduling jobs to optimize two or more objectives usually require exploring the tradeoff between the conflicting goals. In this subsection, we propose a novel fuzzy-based priority approach to handle such a tradeoff.


Fuzzy-Based Priority for Bi-Objective Scheduling  We first consider optimizing two objectives, for which we define the following composite cost function:

\[
H_{i,j}^{X,Y} = \langle \Pi_{i,j}^X(f), H_{i,j}^Y \rangle .
\]  

(22)

In this case, the objectives \(X\) and \(Y\) are considered one after another by first selecting all servers that offer the best performance in terms of \(X\), and then selecting among this subset any server that offers the best performance in terms of \(Y\). To avoid depriving the second objective altogether, a fuzzy factor \(f\), where \(f \in [0, 1]\), is used to relax the selection criterion for the first objective up to a predefined margin (in percentage). The purpose is to explore any potential improvement for \(Y\) while maintaining the performance for \(X\) within a user-defined range of acceptance. The approach will be particularly effective if a small compromise in \(X\) can lead to a large improvement in \(Y\). Setting \(f = 0\) indicates the high importance of \(X\) that should not be compromised at all, while setting \(f = 1\) suggests that \(X\) does not matter in the optimization. Varying \(f\) in between gives the user a flexible and intuitive way to specify the tradeoff between the two objectives.

To implement the fuzzy-based priority approach in the online Greedy Scheduling Framework (GSF) as shown in Algorithm 2, the cost function for the first objective \(X\) needs to be normalized between 0 and 1 in order to take the fuzzy factor into account, i.e.,

\[
\Pi_{i,j}^X = \frac{H_{i,j}^X - H_{i,j}^{X,min}}{H_{i,j}^{X,max} - H_{i,j}^{X,min}} ,
\]  

(23)

where \(H_{i,j}^{X,min}\) and \(H_{i,j}^{X,max}\) denote the minimum and maximum costs in terms of objective \(X\) among all available servers to assign job \(J_i\), and they can be easily collected by a linear scan of the available servers. The implementation then relies on the following rule for comparing the relative cost of assignment on any two servers.

**Fuzzy-Based Priority Rule (for Two Objectives):** The costs incurred by assigning job \(J_i\) to any two servers \(M_{j_1}\) and \(M_{j_2}\) satisfy \(H_{i,j_1}^{X,Y} < H_{i,j_2}^{X,Y}\) if and only if one of the following conditions holds:

- \(\Pi_{i,j_1}^X \leq f < \Pi_{i,j_2}^X\), or
- \(\Pi_{i,j_1}^X \leq f < \Pi_{i,j_2}^X \land \Pi_{i,j_2}^X \leq f < H_{i,j_1}^Y < H_{i,j_2}^Y\), or
- \(\Pi_{i,j_1}^X < \Pi_{i,j_2}^X \land \Pi_{i,j_2}^X \leq f < H_{i,j_1}^Y = H_{i,j_2}^Y\), or
- \(f < \Pi_{i,j_1}^X < \Pi_{i,j_2}^X\), or
- \(f < \Pi_{i,j_1}^X = \Pi_{i,j_2}^X \land H_{i,j_1}^Y < H_{i,j_2}^Y\).

This rule can be applied to optimize any two objectives, as long as they have well-defined cost functions, such as the ones given in Section 4.2. The value of the fuzzy factor as well as the priority depend on the relative importance of the two objectives to optimize, which can be determined by the user or the system administrator.

**Extension to Multi-Objective Scheduling** The fuzzy-based priority approach can be extended to include more than two objectives. As in the bi-objective case, we can optimize a sequence of objectives one after another, while using a (possibly different) fuzzy factor to specify the acceptable range for each objective. The following illustrates this method with a composite cost function consisting of \(s\) objectives:

\[
H_{i,j}^{X_1,X_2,\ldots,X_s} = \langle \Pi_{i,j}^{X_1}(f_1), \Pi_{i,j}^{X_2}(f_2), \ldots, H_{i,j}^{X_s} \rangle .
\]  

(24)

In this case, the servers that are ranked among the top \(f_1\) percent in terms of objective \(X_1\) will be selected first. Then, within this subset, the ones that fall into the top \(f_2\) percent in terms of objective \(X_2\) will be further selected. This process continues until the \((s-1)\)-th objective is considered. Finally, a server that survives the first \(s-1\) rounds of selection and has the best performance in terms of the last objective \(X_s\) will be chosen as the final winner.

Again, the order of the priorities and the values of the fuzzy factors should be determined by the relative importance of different objectives to optimize.

**Comparison with Other Approaches** We now comment on the similarities and differences of the fuzzy-based priority approach in comparison with a few other multi-objective optimization approaches commonly found in the literature. Figure 1 illustrates the basic principles of these approaches using bi-objective scheduling as an example. Section 5 describes some related work on the applications of these approaches in multi-objective scheduling.
Fuzzy-based priority range specified by fuzzy factor $f$ in fuzzy-based priority approach

Simple priority

Fuzzy-based priority

Constraint optimization

Weighted sum (equal weights for $X$ & $Y$)

Pareto frontier

range specified by fuzzy factor $f$ in fuzzy-based priority approach

strict constraint on $H_N$ in constraint optimization approach

Figure 1: Comparison of the fuzzy-based priority approach with four other approaches in bi-objective scheduling. Each dot represents a potential solution, and the solution returned by each approach is indicated.

(1). **Simple priority.** This is a special case of the fuzzy-based priority approach with fuzzy factor $f = 0$. It is usually applied in settings where strict priorities are imposed on different objectives. This approach provides better result for the first objective, but may lead to much worse performance for the second one. In contrast, the fuzzy-based priority approach is more effective in settings with soft (or non-strict) priorities, especially if an objective with slightly lower priority can be significantly improved with just a little compromise for a high-priority objective.

(2). **Pareto frontier.** This approach returns a set of non-dominated solutions\(^5\) to the user instead of only one solution. It is widely applied in offline settings to quantify the tradeoffs among different objectives. In the context of online scheduling, however, multiple solutions are hard to maintain over time, and one of the intermediate solutions must be selected on-the-fly in order to decide where each job should be assigned.

(3). **Constraint optimization.** This approach optimizes one objective subject to certain constraints imposed on the other(s). It is commonly applied in environments with strict or clearly-defined requirements, e.g., job deadline or energy budget. Instead of using an absolute value as the constraint, the fuzzy-based priority approach specifies the constraint as a relative threshold, i.e., fuzzy factor, in terms of percentage.

(4). **Weighted sum.** This approach transforms multiple objectives into a single one by optimizing a weighted combination. Although priorities are not explicitly specified, it uses weights to indicate the relative importance of the objectives. As different objectives can have different units, they are often normalized in order to be combined. However, it may not be intuitive to set the values of the weights, e.g., for time and energy.

Compared to simple priority and constraint optimization, fuzzy-based priority is particularly suitable for scheduling HPC applications in datacenters, where no strict constraints or priority are normally imposed on job performance or energy consumption. Compared to weighted sum, fuzzy-based priority provides an intuitive alternative to describing the tradeoffs while specifying soft preference of the user on the priority of the objectives. Setting an appropriate fuzzy factor encodes such preference in an online manner. As shown in Figure 1, the solution returned by fuzzy-based priority (and other approaches) when scheduling an individual job actually lies on the pareto frontier.

## 5 Performance Evaluations

In this section, we will evaluate the proposed online scheduling heuristics with the fuzzy-based priority approach and the greedy heuristic for server placement. The evaluations are performed by simulation using the Data Center Workload and Resource Management Simulator (DCworms)\(^22\).

### 5.1 Simulation Setup

**Datacenter Configuration** We simulate a datacenter with 50 servers and the same configuration as the one considered in\(^{39,38}\), which essentially determines the heat distribution matrix. Specifically, the datacenter

---

\(^5\)A solution is called non-dominated if no other solution has better performance in terms of all the objectives.
Figure 2: The performance and energy indices of 500+ processors released by Intel between 2009 and 2013. Five processors (marked) in the pareto frontier are selected for our simulation.

consists of two rows of racks in a typical cold aisle and hot aisle layout. The cool air is supplied by the CRAC unit from the cold aisle between the two rows. Each row has five racks and each rack contains five servers. The server platform used in the simulation is based on Christmann’s Resource Efficient Cluster Server (RECS) unit [8], which is a multi-node computer system consisting of 18 processors. The datacenter consists of 900 processors in total. The RECS platform is chosen because it represents an emerging class of high-density and energy-efficient servers with built-in power and temperature sensors and integrated cooling support.

Table 1 shows the parameters used in the simulation, whose values are based on real measurements in a RECS unit. From the first three parameters, the heat distribution matrix D used in this paper is derived by adopting the same airflow pattern as the one measured in [39, 38]. The Coefficient of Performance (CoP) is based on the one in an HP datacenter [25] as shown by Equation (14).

Table 1: Values of the parameters used in the simulation.

| Parameter               | Value                      |
|-------------------------|----------------------------|
| air density (p)         | 1.168 kg/m³                |
| air flow rate \( f_j \) | 0.1 m³/s                   |
| air heat capacity \( c \) | 1004 J/(°C kg)             |
| base power \( U_{base} \) | 130 W                      |
| redline temperature \( T_{red} \) | 25°C                      |

Processor Types To construct a heterogeneous datacenter, we select a set of five nondominated processors in terms of performance and energy indices (the smaller the better). The performance index of a processor is calculated as the reciprocal of its performance score measured by the passmark software [28], which synthesizes thousands of benchmark results as the processor’s performance indicator. The energy index is simply the product of the processor’s performance index and its Thermal Design Power (TDP), which gives a relative indicator (compared to other processors) on the average energy the processor consumes when running typical benchmarks.

Figure 2 plots the two indices for more than 500 types of processors released by Intel between 2009 and 2013, among which five processors in the pareto frontier are selected (marked in the figure). Table 2 shows the passmark scores and TDPs of the five selected processors. We choose these processors because they form a nondominated set, making the scheduling problem non-trivial. In this case, no processor is dominated by others in terms of both performance and energy consumption; hence tradeoff exists when assigning a job to different processor types. In the simulation, each type of processor makes up 10 RECS servers with 180 computing nodes in total.

Table 2: Passmark scores (as of January 2014) and TDPs of five types of processors used in the simulation.

| Processor Type              | Passmark | TDP(W) |
|----------------------------|----------|--------|
| Intel Corei7_4770R         | 10381    | 65     |
| Intel Corei7_4960HQ        | 10310    | 47     |
| Intel Corei7_4600U         | 4498     | 15     |
| Intel XeonE5_2697v2        | 19125    | 130    |
| Intel XeonE3_1230Lv3       | 7344     | 25     |
Benchmarks and Workloads  The benchmarks used in the simulation consist of the following high-performance computing applications, which are included in DCWorms.

- **fft**: a program to compute Fast Fourier Transforms.
- **c-ray**: a raytracing software.
- **abinit**: a tool to compute material properties at the atom level.
- **linpack**: a library for performing numerical linear algebra.
- **tar**: a program to create and manipulate tar archives.

These benchmarks exhibit a large spectrum of behaviors, from CPU intensive to memory intensive, to communication and I/O intensive. More explanation and rationale of this choice can be found in [10]. To profile the execution time and power consumption of these benchmarks, an application-specific approach [22] was adopted. Specifically, average measurements are collected for each application with different input parameters on Intel Core i7_2715QE, a less powerful processor available in our RECS testbed. The results are then translated to our target platforms using the relative performance and power indicators as shown in Table 2. Table 3 details the average execution time and the corresponding power consumption of the benchmarks on each of the five selected processors.

| Benchmark | Corei7 4770R | Corei7 4900HQ | Corei7 4600U | XeonE5 2697v2 | XeonE3 1230Lv3 |
|-----------|---------------|---------------|--------------|---------------|---------------|
| fft       | 3400          | 3450          | 7850         | 1850          | 4800          |
|           | 62.27         | 45.03         | 14.37        | 124.54        | 23.95         |
| c-ray     | 1150          | 1200          | 2700         | 650           | 1650          |
|           | 33.70         | 24.37         | 7.78         | 67.41         | 12.96         |
| abinit    | 1700          | 1750          | 3950         | 950           | 2450          |
|           | 36.11         | 26.11         | 8.33         | 72.22         | 13.89         |
| linpack   | 3350          | 3400          | 7700         | 1850          | 4750          |
|           | 53.81         | 38.91         | 12.42        | 107.61        | 20.69         |
| tar       | 2000          | 2050          | 4600         | 1100          | 2800          |
|           | 50.92         | 36.82         | 11.75        | 101.83        | 19.58         |

Each job is randomly selected from one of these benchmarks and the number of processors it requires is randomly generated from 1 to 8 with uniform distribution. Following the definition in [11], the system load $\rho$ is defined to be

$$\rho = \frac{\lambda \cdot E[P]}{\sum_{j=1}^{m} L_j},$$

where $\lambda$ is the arrival rate (in #jobs per hour), $E[P]$ is the average sequential execution time of the jobs on all processor types (roughly 4.5 hours) and $\sum_{j=1}^{m} L_j$ is the total number of processors, which is 900 in the simulation. Jobs arrive according to the Poisson process, and the arrival rate $\lambda$ is increased from 20 to 200 with a fixed arrival duration of 8 hours. The total number of jobs ranges from 160 to 1600, and the system load is between 0.1 and 1.

5.2 Simulation Results

This section presents the simulation results. First, we evaluate the performance of various online scheduling heuristics with a fixed placement for the servers. We then study the impact of different placement configurations on the performance of the scheduling heuristics. All results are obtained by carrying out the experiments 10 times and taking the average.

5.2.1 Results of Single-Objective Scheduling Heuristics

We first evaluate the online scheduling heuristics for a single objective. The results are used as references for exploring the energy-performance tradeoff in the next subsection. In both cases, the server placement is fixed with each type of processor occupying 10 contiguous server slots over two racks, according to the order specified in Table 2.
Figure 3: Performance of six single-objective online scheduling heuristics. The legend applies to all subfigures.

Figure 4: Average supply temperature of the heuristics.

Six heuristics presented in Section 4.2 are evaluated, namely, Uniform, MinHR, CoolestInlet, Perf-Aware, Energy-Aware and Thermal-Aware. Figure 3 presents the results of these heuristics. As we can see in Figure 3(a), Perf-Aware has significantly better average job response time compared to the other heuristics, especially under light system loads. This is because all jobs in Perf-Aware are assigned to high-performance (faster) processors before slower ones whenever possible. For the same reason, Perf-Aware also has better makespan (completion time of the last finished job) and processor utilization (ratio between the utilized processor cycles and all processor cycles during the simulation period), as shown in Figures 3(b) and 3(c). Note that the processor utilizations remain under 70% even when the system load reaches 1. This is partly due to the fragmented processors in some servers that cannot be utilized because a ready job simply has higher processor requirement.

Figure 3(d) compares the total (dynamic) energy consumption of the scheduling heuristics, and Figures 3(e) and 3(f) show the energy consumed for computing and cooling, separately. For all heuristics, the energy consumption increases with the system load or the total number of jobs in the arrival interval. Energy-Aware consumes less total energy compared to the other heuristics, since jobs are assigned to processors with better energy efficiency. The improvement is more significant in terms of computing energy. For the cooling part, MinHR and Thermal-Aware consumes roughly the same energy as Energy-Aware, since they are designed to minimize the heat recirculation and the maximum inlet temperature, which in turn increases the supplied temperature in the room and hence directly impacts the cooling cost. Figure 4 shows the average supply temperature of the different scheduling heuristics in the simulation period. Indeed, Thermal-Aware and MinHR are better than Energy-Aware in terms of the average supply temperature by up to 1.3°C and 1.6°C, respectively.

As the system load increases further and hence the processor utilization becomes higher, the performance...
Figure 5: Bi-objective scheduling for $H_{i,j}^{E,P} = (\left\langle E_{i,j}(f) \right\rangle, H_{i,j}^P)$ with different fuzzy factors at three system loads. The legend applies to all subfigures.

Figure 6: Energy-performance tradeoff curve for $H_{i,j}^{E,P} = (\left\langle E_{i,j}(f) \right\rangle, H_{i,j}^P)$ at three system loads. The legend applies to all subfigures.

of all heuristics tend to converge, since all servers are roughly equally loaded under all heuristics. In particular for Energy-Aware, some jobs are forced to be assigned to the high-performance servers since the energy-efficient ones are all occupied, resulting in improved average job response time.

5.2.2 Energy-Performance Tradeoff with Fuzzy-Based Priority

We now evaluate the effectiveness of the fuzzy-based priority approach for exploring the energy-performance tradeoff in online scheduling. To this end, we consider the composite cost function $H_{i,j}^{E,P} = (\left\langle E_{i,j}(f) \right\rangle, H_{i,j}^P)$ that optimizes the energy consumption followed by the job response time.

Figure 5 shows the results of minimizing $H_{i,j}^{E,P}$ when the fuzzy factor $f$ is increased from 0 to 1 at three different system loads (0.2, 0.5 and 0.8). The values of both objectives are plotted as a function of $f$, with energy consumption shown on the left Y axis and average response time on the right. In addition, the figure also shows the results when $f = -1$ and $f = 2$, denoting the cases where the scheduling decision is based solely on the first objective (energy) and the second objective (response time). The two cases are equivalent to the single-objective heuristics Energy-Aware and Perf-Aware, respectively.

As we can see, the average response time improves with increased fuzzy factor at the expense of the energy consumption under all system loads. However, the improvement can be significant even before major compromise in energy consumption is observed. For instance, at medium load ($\rho = 0.5$), the response time is reduced by about 1000 when $f$ reaches 0.6 without much increase in the energy consumption. Similar results can also be observed at light load and heavy load. The fuzzy-based priority approach can take advantage of such characteristics by setting suitable fuzzy factors in order to achieve desirable energy-performance tradeoff in the online setting.

Figure 6 shows the energy-performance tradeoff curve for $H_{i,j}^{E,P} = (\left\langle E_{i,j}(f) \right\rangle, H_{i,j}^P)$ obtained by varying the fuzzy factor from 0 to 1. The results of the six single-objective heuristics are also shown in the figure under the respective load. We can see that MinHR and Thermal-Aware lie around the curve (or even slightly to the left of the curve in the case of MinHR), indicating that they achieve fairly efficient tradeoffs between job response time and energy consumption. On the other hand, Uniform and CoolestInlet are completely dominated by the curve, which suggests that they provide less attractive tradeoff results.

Figure 7 plots the tradeoff curves achieved by optimizing the heat recirculation and the maximum inlet temperature followed by the job response time, i.e., with cost functions $H_{i,j}^{HR,P} = (\left\langle H_{i,j}^{HR}(f) \right\rangle, H_{i,j}^P)$ and $H_{i,j}^{T,P} = $
Figure 7: Energy-performance tradeoff curves for $H^T_{i,j}$, $H^{E,P}_{i,j}$, and $H^{H,R,P}_{i,j}$ at three system loads. The legend applies to all subfigures.

The results under three different system loads are shown alongside the ones for $H^{E,P}_{i,j}$. The curves indicate that the two heuristics are able to provide better tradeoffs in the medium to high energy range (e.g., between 150 and 220 for $\text{MinHR}$ at $\rho = 0.5$) while the tradeoff remains efficient for the cost function $H^{E,P}_{i,j}$ when the energy consumption is close to the minimum. The results demonstrate the flexibility of the fuzzy-based priority approach in exploring the energy-performance tradeoff in online scheduling. The approach can be potentially applied to other multi-objective optimization problems.

5.2.3 Evaluation of Server Placement Strategies

We now study the impact of server placement on the performance of the online scheduling heuristics. Besides the simple location-based placement used in the previous evaluations, which we call LOC, we generate three additional placements for the servers. One is based on our GSP heuristic and the other two are based on its variations. We call the three placement configurations GSP1, GSP2, and GSP3, respectively. The two variants (GSP2 and GSP3) are obtained in a similar fashion as GSP1. In particular, in GSP2 the servers are sorted in ascending order of reference power instead of descending order, and in GSP3 each server is assigned to a remaining rack slot that maximizes the maximum inlet temperature instead of minimizing it. Apparently, these two heuristics are counter-intuitive and are expected to provide undesirable configurations. The purpose of including them is to demonstrate the impact of different server placements on a scheduling algorithm’s performance, especially on the cooling cost.

Figure 8 shows the inlet temperature distribution of the 50 servers under the four placement configurations. In all cases, each processor is loaded with the average power consumption of the benchmarks shown in Table 3. As we can see, GSP1 has better thermal balance than the other configurations. Specifically, it improves LOC by about 8°C in terms of the maximum inlet temperature and improves GSP2 and GSP3 by over 14°C and 16°C, respectively.

Figures 9 and 10 show the performance of Perf-Aware and Energy-Aware under the four server placements at different system loads. In both heuristics, job response time and computing energy are not affected by different configurations. However, GSP1 has reduced cooling energy compared to the other configurations. This is particularly evident under heavy system load, where all servers are almost fully and equally loaded, thus their power consumption ratios match closely those of the average values used in the server placement heuristic. Under light system load, however, the servers could experience unbalanced loads, which causes their power consumption ratios to deviate from those of the average values. As a result, the advantage of GSP1 becomes smaller or even diminishes, but since the overall energy consumption is small in this case, the impact of server placement is not significant.

Quite similar effect on the cooling energy can be observed for Thermal-Aware and MinHR as shown in Figures 11 and 12. Notice that, for these two heuristics, different server placements also lead to a tradeoff between job response time and computing energy. To further investigate the tradeoff efficiency, Figure 13 shows the energy-performance tradeoff curves for three heuristics with cost functions $H^{E,P}_{i,j}$, $H^{H,R,P}_{i,j}$ and $H^{T,P}_{i,j}$ at load $\rho = 0.8$ under different server placements. We can see that, although the tradeoff remains, in all cases GSP1 provides the best cooling energy and hence improves the overall tradeoff efficiency. Note that MinHR and Perf-Aware behave exactly the same under GSP1, since servers with faster processors and hence more power consumptions are placed in the slots with less heat recirculation. Therefore, the same performance and energy are observed for $H^{H,R,P}_{i,j}$ regardless of the fuzzy factor, as shown in Figure 13(b).

The results confirm that strategic server placement indeed improves the thermal balance in a heterogeneous...
Figure 8: Inlet temperature distribution of the 50 servers under four different server placements. The maximum inlet temperature of each placement is indicated in the legend and by the horizontal line.

Figure 9: Performance of Perf-Aware under different server placements and system loads. The legend applies to all subfigures.

datacenter, which helps reduce the cooling cost. This is achieved with little impact on the job response time and computing energy, or the tradeoff between them.

6 Related Work

In this section, we review some related work in the literature on multi-objective scheduling and thermal-aware scheduling for datacenters.

Multi-objective scheduling  Scheduling with multiple conflicting objectives has attracted much attention in many optimization problems. Section 4.3 described a few commonly used approaches. The following reviews some applications of these approaches in various problem domains.

(1) Simple priority. This is a simple priority-based approach to optimize multiple objectives in sequence. Assayad et al. [2] introduced a bi-criteria compromise function to set priorities between makespan and reliability for scheduling real-time applications. To minimize carbon emission and to maximize profit, two-step policies were proposed by Garg et al. [18] to map applications to heterogeneous datacenters based on the relative priority of the two objectives. Du et al. [12] proposed heuristics to optimize the QoS for interactive services before considering energy consumption on multicore processors with DVFS (Dynamic Voltage & Frequency Scaling) capability.

(2) Pareto frontier. This approach is often used in the offline setting to generate a set of nondominated solutions. Durillo et al. [13] applied this technique to tradeoff makespan and energy consumption for heterogeneous servers. Torabi et al. [41] used particle swarm optimization to approximate the pareto frontier for the unrelated machine scheduling problem with uncertainties in the inputs. Gao et al. [15] utilizes ant colony
optimization to obtain the pareto frontier for resource wastage and power consumption in virtual machine placement. Evolutionary algorithms were employed in [15, 17] to obtain a set of alternative solutions for scheduling scientific workloads in the Grid environment.

(3). Constraint optimization. This approach optimizes one objective subject to constraint(s) on the other(s). Rizvandi et al. [31] applied it to minimize the energy consumption subject to the makespan achieved in an initial schedule. A mixed integer programming model was used by Petrucci et al. [29] to reduce the power consumption of virtualized servers subject to QoS requirements. Fard et al. [14] developed a double strategy to minimize the Euclidean distance between the generated solutions to a set of user-specified constraints in a four-objective optimization problem. The authors in [19] applied $\epsilon$-constraint method to cloud scheduling, which optimizes each objective in turn with upper bounds specified for the others.

(4). Weighted combination. This approach combines multiple objectives into a single one. Lee and Zomaya [23] used DVFS to tradeoff makespan with energy consumption by considering a weighted sum of the two objectives. The same technique was used by the authors of [1, 36] in an online manner to minimize a combined objective of job response time and energy. A similar approach was taken by Sheikh and Ahmad [34], who considered an additional objective of peak temperature in a multicore system, and hence optimizing three objectives at the same time. Instead of summation, some work (e.g., [7, 30]) also used energy-delay product as a metric for scheduling applications in heterogeneous multicore systems.

Compared to these approaches, our fuzzy-based priority approach provides a rather flexible solution to handling two or more conflicting objectives. Although multi-objective scheduling with “fuzzy” or “good enough” solutions [44, 46] are known in the pareto-based approach, our fuzzy method is novel when (soft) priorities exist between different objectives. The principle can be potentially applied to other multi-objective optimization problems.

Thermal-Aware Scheduling As cooling energy constitutes a significant fraction of the total energy consumption in today’s large-scale datacenter, thermal-aware scheduling for this environment has been the focus of many research in recent years.

Wang et al. [42, 43] considered thermal-aware workload placement in datacenters to reduce the server temperatures characterized by an RC-model, while minimizing the job response time. They proposed simple
heuristics that allocate “hot” jobs to “cool” computing nodes, as well as backfilling techniques for scheduling parallel applications. In their study, the thermal map of the datacenter is assumed to be available through ambient and on-board sensors.

Moore et al. [25] first introduced the concept of heat recirculation effect and proposed workload placement algorithms, including MinHR, to reduce the recirculation of heat and the cooling cost in a datacenter. A prediction tool called Weatherman [24] was used to predict the datacenter thermal map using machine learning techniques. The authors showed that the tool accurately predicts the heat distribution of the datacenter without the need of static thermal configuration, and a scheduling algorithm based on Weatherman achieves similar performance as MinHR.

Tang et al. [38] also studied the problem of minimizing the cooling cost in datacenters with heat recirculation consideration. Based on an abstract heat flow model, they characterized the thermal behavior of datacenters via a heat distribution matrix. The model was validated by computational fluid dynamics (CFD) simulations in [39, 32]. They proposed offline scheduling solutions by using genetic algorithms and quadratic programming, which were evaluated using the heat distribution matrix captured for a small-scale datacenter. The same matrix is used in this paper for evaluating our online scheduling heuristics.

Instead of minimizing only the cooling cost, Pakbaznia and Pedram [27] considered minimizing the total energy of a datacenter from both computation and cooling. They showed that performing consolidation to turn off idle servers together with job scheduling to account for the heat recirculation can significantly reduce the total power usage. Banerjee et al. [3] further considered cooling-aware scheduling workload placement by exploring the dynamic cooling behavior of the CRAC unit in a datacenter.

While the above results considered only the energy consumption of a datacenter, the following also takes application performance into consideration. Mukherjee et al. [26] considered a similar problem as in [27] but further took the temporal dimension of the job placements into account. They formulated the problem as a non-linear program and proposed both offline and online heuristics to minimize the total energy subject to the deadline constraint for the jobs. Sansottera and Cremonesi [32] considered a datacenter environment hosting web services, and presented heuristics to minimize the total energy subject to service response time constraints. Kaplan et al. [20] studied the dual optimization of cooling and communication costs for HPC applications in a

---

**Figure 12:** Performance of MinHR under different server placements and system loads. The legend applies to all subfigures.

**Figure 13:** Energy-performance tradeoff curves for $H_{i,j}^{E,P}$, $H_{i,j}^{HR,P}$ and $H_{i,j}^{T,P}$ under four different server placements at load $\rho = 0.8$. The legend applies to all subfigures.
datacenter. They proposed a heuristic algorithm that achieves a good tradeoff between the two objectives, and subject to reliability constraint specified by the processor junction temperature.

In contrast to the previous work, which focused on either offline scheduling or homogeneous datacenters, we studied the problem of online scheduling for heterogeneous datacenters with both energy and performance considerations, as well as their tradeoffs. Furthermore, we considered static server placement to balance the thermal distribution in the presence of nonuniform heat distribution matrix. In our previous work [37], we have applied this concept to the arrangement of computing nodes in a smaller scale problem (at the server level). To the best of our knowledge, no prior work has considered this problem for heterogeneous datacenters.

7 Conclusion and Future Work

In this paper, we have considered the energy-efficient and thermal-aware placements for both servers and workloads in heterogeneous datacenters. For the static server placement problem, we have shown that it is NP-hard and presented a greedy heuristic. To schedule the workloads, we have presented a greedy scheduling framework, which can be applied in an online manner with any well-defined cost function. Moreover, a novel fuzzy-based priority approach was proposed to simultaneously optimize two or more conflicting objectives. Simulations were conducted for a heterogeneous datacenter with heat recirculation effect. The results demonstrated the effectiveness of the proposed approaches for exploring the energy-performance tradeoff with cooling consideration. Our static server placement heuristic was also shown to provide better thermal balance, which directly leads to reductions in cooling cost.

For future work, other resource management techniques, such as DVFS or server consolidation, can be applied to achieve better energy and thermal efficiency. In this context, the tradeoff between the computing energy and cooling energy can be explored, possibly with the fuzzy-based priority approach. For the static server placement problem, it will be useful to design better heuristic solutions or good approximation algorithms, and to consider large datacenters with more rack slots than servers, which will provide additional space for optimization. Finally, we considered server placement and job scheduling separately in this paper; it may be helpful to consider the two aspects jointly to achieve further energy savings.

Acknowledgment

This research was funded by the European Commission under contract 288701 through the project CoolEmAll.

References

[1] L.H. Andrew, M. Lin, and A. Wierman. Optimality, fairness, and robustness in speed scaling designs. In Proceedings of ACM SIGMETRICS, pp. 37-48, 2010.
[2] I. Assayad, A. Girault, and H. Kalla. A bi-criteria scheduling heuristic for distributed embedded systems under reliability and real-time constraints. In Proceedings of the IEEE/IFIP International Conference on Dependable Systems and Networks (DSN), page 347, 2004.
[3] A. Banerjee, T. Mukherjee, G. Varsamopoulos, and S. K. Gupta. Integrating cooling awareness with thermal aware workload placement for HPC data centers. Sustainable Computing: Informatics and Systems, 1(2):134-150, 2011.
[4] J. Barbosa, and B. Moreira. Dynamic job scheduling on heterogeneous clusters. In Proceedings of the IEEE International Symposium on Parallel and Distributed Computing (ISPDCE), pp. 3-10, 2009.
[5] L.A. Barroso, and U. Holzle. The case for energy-proportional computing. Computer, 40(12):33-37, 2007.
[6] D. Borgetto, H. Casanova, G. Da Costa, J.-M, Pierson, Energy-aware service allocation. Future Generation Computer Systems, 28(5):769-779, 2012.
[7] J. Cong, and B. Yuan. Energy-efficient scheduling on heterogeneous multi-core architectures. In Proceedings of the International Symposium on Low Power Electronics and Design (ISLPED), pp. 345-350, 2012.
[8] Christmann. Description for Resource Efficient Computing System (RECS). [http://shared.christmann.info/download/project-recs.pdf](http://shared.christmann.info/download/project-recs.pdf) 2009.
[9] G. Da Costa. Heterogeneity: The key to achieve power-proportional computing. In Proceedings of the IEEE/ACM International Symposium on Cluster, Cloud and Grid Computing (CCGrid), pp. 656-662, 2013.

[10] G. Da Costa, T. Zilio, M. Jarus, and A. Oleksiak. Energy-and heat-aware HPC benchmarks. In Proceedings of the International Conference on Cloud and Green Computing (CGC), pp. 435-442, 2013.

[11] A.B. Downey. A parallel workload model and its implications for processor allocation. In Proceedings of the ACM Symposium on High-Performance Parallel and Distributed Computing (HPDC), pp. 112-123, 1997.

[12] Z. Du, H. Sun, Y. He, Y. He, D.A. Bader, and H. Zhang. Energy-efficient scheduling for best-effort interactive services to achieve high response quality. In Proceedings of the IEEE International Parallel & Distributed Processing Symposium (IPDPS), pp. 637-648, 2013.

[13] J.J. Durillo, V. Nae, and R. Prodan. Multi-objective workflow scheduling: An analysis of the energy efficiency and makespan tradeoff. In Proceedings of the IEEE/ACM International Symposium on Cluster, Cloud and Grid Computing (CCGrid), pp. 203-210, 2013.

[14] H.M. Fard, R. Prodan, J. Barriomeu, and T. Fahringer. A multi-objective approach for workflow scheduling in heterogeneous environments. In Proceedings of the IEEE/ACM International Symposium on Cluster, Cloud and Grid Computing (CCGrid), pp. 300-309, 2012.

[15] Y. Gao, H. Guan, Z. Qi, Y. Hou, and L. Liu. A multi-objective ant colony system algorithm for virtual machine placement in cloud computing. Journal of Computer and System Sciences, 79(8):1230-1242, 2013.

[16] M.R. Garey and D.S. Johnson. Computers and Intractability: A Guide to the Theory of NP-Completeness. First Edition, W. H. Freeman, 1979.

[17] R. Garg and A.K. Singh. Reference Point Based Multi-Objective Optimization to Workflow Grid Scheduling. International Journal of Applied Evolutionary Computation, 3(1):80-99, 2012.

[18] S.K. Garg, C. Yeo, A. Anandasivam, and R. Buyya. Environment-conscious scheduling of HPC applications on distributed Cloud-oriented data centers. Journal of Parallel and Distributed Computing, 71(6):732-749, 2011.

[19] L. Grandinetti, O. Pisacane, and M. Sheikhfalahi. An approximate $\epsilon$-constraint method for a multi-objective job scheduling in the cloud. Future Generation Computer Systems, 29(8):1901-1908, 2013.

[20] F. Kaplan, J. Meng, and A.K. Coskun. Optimizing communication and cooling costs in HPC data centers via intelligent job allocation. In Proceedings of the International Green Computing Conference (IGCC), pp. 1-10, 2013.

[21] J. Koomey. Worldwide electricity used in data centers. Environmental Research Letters, Vol. 3, 034008, 2008.

[22] K. Kurowski, A. Oleksiak, W. Piatek, T. Piontek, A. Przybyszewski, and J. Weglarz. DCworms – A tool for simulation of energy efficiency in distributed computing infrastructures. Simulation Modelling Practice and Theory, 39:135-151, 2013.

[23] Y. Lee and A.Y. Zomaya. Energy conscious scheduling for distributed computing systems under different operating conditions. IEEE Transactions on Parallel and Distributed Systems, 22(8):1374-1381, 2011.

[24] J. Moore, J. S. Chase, and P. Ranganathan. Weatherman: Automated, online and predictive thermal mapping and management for data centers. In Proceedings of the International Conference on Autonomic Computing (ICAC), pp. 155-164, 2006.

[25] J. Moore, J. Chase, P. Ranganathan, and R. Sharma. Making scheduling “cool”: temperature-aware workload placement in data centers. In Proceedings of the Annual Conference on USENIX Annual Technical Conference (ATEC), pp. 5-5, 2005.

[26] T. Mukherjee, A. Banerjee, G. Varsamopoulos, S. Gupta, and S. Rungta. Spatio-temporal thermal-aware job scheduling to minimize energy consumption in virtualized heterogeneous data centers. Computer Networks, 53(17):2888-2904, 2009.

[27] E. Pakbaznia and M. Pedram. Minimizing data center cooling and server power costs. In Proceedings of the International Symposium on Low Power Electronics and Design (ISLPED), pp. 145-150, 2009.
[28] Passmark® results. [http://www.cpubenchmark.net/]

[29] V. Petrucci, E. V. Carrera, O. Loques, J. C. B. Leite, and D. Mosse. Optimized management of power and performance for virtualized heterogeneous server clusters. In *Proceedings of the IEEE/ACM International Symposium on Cluster, Cloud and Grid Computing (CCGrid)*, pp. 23-32, 2011.

[30] V. Petrucci, O. Loques, and D. Mosse. Lucky scheduling for energy-efficient heterogeneous multi-core systems. In *Proceedings of the USENIX conference on Power-Aware Computing and Systems (HotPower)*, pp. 7-7, 2012.

[31] N.B. Rizvandi, J. Taheri, A.Y. Zomaya and Y. Lee. Linear combinations of DVFS-enabled processor frequencies to modify the energy-aware scheduling algorithms. In *Proceedings of the IEEE/ACM International Symposium on Cluster, Cloud and Grid Computing (CCGrid)*, pp. 388-397, 2010.

[32] A. Sansottera, and P. Cremonesi. Cooling-aware workload placement with performance constraints. *Performance Evaluation*, 68(11):1232-1246, 2011.

[33] R. Sawyer. Calculating Total Power Requirements for Data Centers. *White Paper*, 2004.

[34] H.F. Sheikh and I. Ahmad. Simultaneous optimization of performance, energy and temperature for DAG scheduling in multi-core processors. In *Proceedings of the International Green Computing Conference (IGCC)*, pp. 1-6, 2012.

[35] A. Silberschatz, P.B. Galvin, and G. Gagne. Operating Systems Concepts. *7th Edition*, Wiley, 2005.

[36] H. Sun, Y. Cao, and W.-J. Hsu. Non-clairvoyant speed scaling for batched parallel jobs on multiprocessors. In *Proceedings of the ACM Conference on Computing Frontiers*, pp. 99-108, 2009.

[37] H. Sun, P. Stolf, J.-M. Pierson, and G. Da Costa. Multi-Objective Scheduling for Heterogeneous Server Systems with Machine Placement. In *Proceedings of the IEEE/ACM International Symposium on Cluster, Cloud and Grid Computing (CCGrid)*, 2014.

[38] Q. Tang, S. Gupta, and G. Varsamopoulos. Energy-efficient thermal-aware task scheduling for homogeneous high-performance computing data centers: A cyber-physical approach. *IEEE Transactions on Parallel and Distributed Systems*, 19(11):1458–1472, 2008.

[39] Q. Tang, T. Mukherjee, S. Gupta, and P. Cayton. Sensor-based fast thermal evaluation model For energy efficient high-performance datacenters. In *Proceedings of the International Conference on Intelligent Sensing and Information Processing*, pp. 203–208, 2006.

[40] H. Topcuoglu, H. S. Hariri, and M.-Y. Wu. Performance-effective and low-complexity task scheduling for heterogeneous computing. *IEEE Transactions on Parallel and Distributed Systems*, 13(3):260-274, 2002.

[41] S.A. Torabi, N. Sahebjamnia, S.A. Mansouri, M.A. Bajestani. A particle swarm optimization for a fuzzy multi-objective unrelated parallel machines scheduling problem. *Applied Soft Computing*, 13(12):4750-4762, 2013.

[42] L. Wang, G. von Laszewski, J. Dayal, X. He, A. J. Younge, and T. R. Furlani. Towards thermal aware workload scheduling in a data center. In *Proceedings of the International Symposium on Pervasive Systems, Algorithms, and Networks (ISPAN)*, pp. 116-122, 2009.

[43] L. Wang, S.U. Khan, and J. Dayal. Thermal aware workload placement with task-temperature profiles in a data center. *The Journal of Supercomputing*, 61(3):780-803, 2012.

[44] J. Xu and J. A. B. Fortes. Multi-objective virtual machine placement in virtualized data center environments. In *Proceedings of the IEEE/ACM International Conference on Green Computing and Communications & International Conference on Cyber, Physical and Social Computing (GREENCOM-CPSCOM)*, pp. 179-188, 2010.

[45] J. Yu, M. Kirley, and R. Buyya. Multi-objective planning for workflow execution on Grids. In *Proceedings of the IEEE/ACM International Conference on Grid Computing*, pp. 10-17, 2007.

[46] F. Zhang, J. Cao, K. Li, S.U. Khan, and K. Hwang. Multi-objective scheduling of many tasks in cloud platforms. *Future Generation Computer Systems*, 2013.
Appendix. NP-Hardness Proof of the Static Server Placement Problem

Claim 1 The static server placement problem is NP-hard.

Proof. We reduce the 3-partition problem to the static server placement problem. In 3-partition, a finite set $A = \{a_1, a_2, \cdots, a_n\} \subset \mathbb{Z}^+$ of $n = 3h$ positive integers is given, and the sum of the integers is $\sum_{j=1}^{n} a_j = h \cdot B$. The question is whether $A$ can be partitioned into $h$ disjoint subsets such that the sum of the numbers in each subset is equal to $B$. The problem is known to be NP-hard even if every integer in $A$ is strictly between $B/4$ and $B/2$, so each subset must contain exactly three numbers [10].

Given an instance $A$ of the 3-partition problem, where each integer $a_j \in A$ satisfies $B/4 < a_j < B/2$, we construct an instance of the static server placement problem as follows. Let $m = n = 3h$, and assign $U^\text{ref}_j = a_j, \forall j = 1, \cdots, n$.

The heat-distribution matrix $D$ is specified by setting 

$$d_{3l,3l-2} = d_{3l,3l-1} = d_{3l,3l} = 1, \forall l = 1, \cdots, h,$$

and setting all the other elements to zero.

Suppose $\sigma^*$ is an optimal mapping for the server placement instance constructed above. The temperature increase at the inlet of slot $S_k$, where server $M_{\sigma^*(k)}$ is placed, is given by

$$T^\text{incr}_k = \begin{cases} a_{\sigma^*(k-2)} + a_{\sigma^*(k-1)} + a_{\sigma^*(k)}, & \text{if } k \mod 3 = 0 \\ 0, & \text{otherwise} \end{cases}.$$ 

The maximum temperature increase at any inlet is therefore

$$T^\text{incr}_{\text{max}} = \max_{k=3,6,\cdots,3h} \left( a_{\sigma^*(k-2)} + a_{\sigma^*(k-1)} + a_{\sigma^*(k)} \right).$$ 

This leads to the conclusion that the server placement instance has a maximum temperature increase of $B$ if and only if $A$ can be partitioned into $h$ disjoint subsets, where the sum of the numbers in each subset is also $B$. \qed