Modeling, Simulation and Uncertain Optimization of the Gun Engraving System

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Abstract: The system designed to accomplish the engraving process of a rotating band projectile is called the gun engraving system. To obtain higher performance, the optimal design of the size parameters of the gun engraving system was carried out. First, a fluid–solid coupling computational model of the gun engraving system was built and validated by the gun launch experiment. Subsequently, three mathematical variable values, like performance evaluation indexes, were obtained. Second, a sensitivity analysis was performed, and four high-influence size parameters were selected as design variables. Finally, an optimization model based on the affine arithmetic was set up and solved, and then the optimized intervals of performance evaluation indexes were obtained. After the optimal design, the percent decrease of the maximum engraving resistance force ranged from 6.34% to 18.24%; the percent decrease of the maximum propellant gas temperature ranged from 1.91% to 7.45%; the percent increase of minimum pressure wave of the propellant gas ranged from 0.12% to 0.36%.

Keywords: fluid-solid coupling computational model; sensitivity analysis; interval uncertain optimization; multiple objective optimizations; optimal design of size parameters

1. Introduction

The gun launch process can be simply introduced as follows: first, the propellant particles burn and produce propellant gas, and then the propellant gas pushes projectile base to move along translation–rotation trajectory to the projectile leaves the muzzle. The gun launch process can be divided into three steps, namely the propellant combustion, the engraving process of the projectile rotating band and the high-speed movement of the projectile. As for the engraving process of the projectile rotating band (engraving process for short in this paper), it has two main functions. One is making the projectile rotate and improving the movement stability of the projectile in the air, and the other one is avoiding the spillage of the propellant gas.

The system, which is designed to accomplish the engraving process of the rotating band, is called a gun engraving system, and it consists of two parts, namely the propellant part and the mechanical part. Many researchers have studied the gun engraving system model and engraving process in relation to the research of the mechanical part of the gun engraving system. Various types of rotating bands were manufactured by Wu et al. [1–5], and the gas gun was used to conduct a series of contrast tests. He investigated the influence factors of the engraving process, which include load type, shape and dimension of the rotating band. By proposing the dynamics calculation model, Sudarsan et al. [6] studied the short start pressure of the engraving process on different calibers of the gun and discussed the dependent parameters. Sun et al. [7] established the three-dimensional finite element model of the gun engraving system and simulated the dynamic response of the rotating
band on different charges. In relation to the research of the gun propellant combustion and load calculation. Measuring the propellant combustion law by closed bomb vessel, Monreal-González et al. [8,9] proposed an interior ballistics model of two-phase flow and the calculated pressure data matched well with the experimental firing data. The two-phase flow model, which is described propellant combustion characteristics, was built by Oton-Martinez et al. [10], and the relative variable values obtained by his model were consistent with the data obtained by IBHVG2 software and experiment. MacCormack-TVD and Rusanov numerical schemes were employed by López et al. [11] to solve the propellant detonation model, and the contrastive results showed that MacCormack-TVD is suitable for solving this kind of detonation problem. Although many researchers have studied either the mechanical or the propellant part of gun engraving systems, respectively, the separate models cannot describe the interactive characteristic of each part. In this paper, the fluid–solid coupling model of the gun engraving system, which consists of both mechanical part and propellant part, is established.

The size parameter of an object is an important influence factor on its performance. Moreover, the relationship between the size parameter of the object and its performance is widely studied in the traditional design fields such as welding engineering [12] and newly developing design fields such as biomechanical engineering [13]. Likewise, the size parameters of the gun engraving systems also play an important role in its performance. Furthermore, there exists uncertainty due to manufacturing errors in the gun engraving systems. The values of these uncertainties may be tiny, but their coupling effect will produce a critical impact on the gun engraving system. Thereby, considering the parameter uncertainties, it is necessary to carry out the size parameter optimization of the gun engraving system for higher engraving process performance. Although there is little research in this area, there are many efforts in other fields. At present, there are mainly random programming methods, fuzzy programming methods and interval optimization methods in the field of uncertainty optimization problems [14]. In interval optimization, the uncertain parameters are described by interval numbers. Only the upper and lower bounds of the parameters need to be known; the precise probability distribution or the fuzzy membership function is unnecessary. Therefore, the interval optimization method is employed in this article. The traditional interval optimization method [15,16] is a two-layer nested method, which is very time-consuming due to the inner layer optimization. Transforming the two-layer nested optimization into a single-layer one is an efficient solution to avoid inner layer optimization. Jiang et al. [15] transformed the nonlinear objective function and constraints into an approximate linear model through first-order Taylor series expansion using the sequential linear programming method. The intervals of the objective function and constraints were obtained; thus, the problem was transformed into a single-layer deterministic problem. Chen et al. [17] studied the uncertain optimization problem of a multi-degree-of-freedom vibration system. By combining interval extension and first-order Taylor expansion, the boundary of nonlinear function under variable disturbance was approximately solved so as to convert the problem into a single-layer optimization problem. However, using first-order Taylor expansion to transfer nonlinear problems into linear problems will bring errors that cannot be ignored. Hence, Liu and Yan [18] proposed an improved Taylor expansion based method considering monotonicity. Zhou and Li [19] proposed the concept of Utopian Point based on the sequential quadratic programming method. The upper and lower bounds of the corresponding inner optimization can be obtained only by performing matrix operations, thus transform the problem into a single-layer deterministic problem. Wu et al. [20] proposed an interval uncertainty optimization method combining the Chebyshev surrogate model and higher-order Taylor expansion. It could effectively inhibit the interval expansion problem caused by interval arithmetic and could directly calculate the upper and lower bounds of interval functions. Wang et al. [21,22] introduced the feedforward neural networks to compute the derivative information in Taylor expansion, thus proposed a novel interval optimization algorithm. However, the calculation of the abovementioned optimization methods is toughly complicated. In this paper, a nonlinear
uncertainty optimization method based on an affine algorithm is introduced. By using affine arithmetic to calculate the uncertain objective function and constraint, its interval bounds can be calculated directly so that the two-layer nested uncertain optimization can be transformed to a single-layer uncertain optimization problem.

Multiobjective optimal design can improve the whole system performance significantly [23–26], and the optimal design of the gun engraving system also belongs to the category of multiobjective optimal design. On the other hand, high-efficiency [27,28] and low-cost [29] are also pursued by designers in the field of optimal design. The former optimal design of the gun engraving system needs a series of launch experiments, and therefore it costs much budget and time. For obtaining higher performance, this paper will provide an optimal design scheme of the gun engraving system, and its process is exhibited in Figure 1. This paper is organized as follows: In Section 2, the fluid–solid coupling computational model of the gun engraving system is set up, and the dynamic response of the engraving process is analyzed. Then the twelve related size parameters are selected for sensitivity analysis in Section 3. In Section 3, the sensitivity analysis on the engraving resistance force $f_e$, temperature $T$ and the pressure wave of the propellant gas $\Delta p$ are performed. Then four high-influence size parameters are selected as design variables for optimization in Section 5. In Section 4, the affine arithmetic-based method is validated by the numerical example and is adopted for optimization in Section 5. In Section 5, the optimization of the gun engraving system is carried out, and then the optimized intervals of performance evaluation indexes are obtained. In addition, this paper has two main novelty contents. One is that building a fluid–solid coupling computational model of the gun engraving system, which can take the fluid–solid interaction into account and analyze the comprehensive performance. The other one is that the affine arithmetic-based method is adopted for optimization, which has high accuracy and high computational efficiency.

Figure 1. Flow chart of the present work in this paper.

2. Modeling of the Gun Engraving System and Experimental Investigation of Engraving Process

2.1. Introduction of Engraving Process

On the structural design of a gun, usually, there are one or two rotating bands encircling the projectile and dozens of spiral riflings attaching at the bore surface, as exhibited in Figure 2. For most types of rotating bands, their materials are copper alloy because the strength of the copper alloy is lower than that of steel, which is adopted by the gun barrel and riflings.
Figure 2. Exhibition of physical pictures of the gun engraving system.

Figure 3 represents the size comparison among the first rotating band diameter $D_r$, the second rotating band diameter $D_l$ and barrel bore diameter $D_b$. As can be seen, the value of $D_l$ (167 mm) is higher than the value of $D_r$ (164 mm). This design purpose is to generate a contact surface between the second rotating band and forcing cone before the projectile moves, which can ensure the location of the projectile.

In addition, the values of $D_r$ (164 mm) and $D_l$ (167 mm) is higher than the value of $D_b$ (160 mm). Due to the size relationship, there exists a contact surface among these two rotating bands, the riflings and the bore surface after the projectile moves.

Figure 4 represents the schematic diagram of the gun engraving system, which has the section of the gun barrel, and here we amplify and straighten one spiral rifling for better display. This spiral rifling can be divided into two segments, namely the initial segment and the formal segment. The rifling height at the initial segment gradually increases for the sake of getting a smooth and stable engraving process.

The engraving process can be described as follows: First, the projectile is pushed by the propellant gas. Subsequently, the contact occurs between the rotating band, riflings and bore surface. Thus, the projectile starts to translate and rotate. When the rotating band moves across the initial segment of riflings, the engraving process finishes, and at this time point, the velocity of the projectile is called engraving-completion velocity. After the engraving process, the rotating band is engraved, as exhibited in Figure 2.
2.2. Investigation of Engraving Process by Real Gun Launch Experiment

To investigate the engraving process, previous experiments have utilized simplified instruments. In this paper, considering the validation of the model of the gun engraving system, a real gun launch experiment was carried out. In Figure 5, the schematic diagram of the experiment is exhibited in the center with a black dotted line, and the red arrows point to the corresponding physical pictures.

![Figure 5](image)

**Figure 5.** Schematic diagram and the physical picture of real gun launch experiment.

The launch experiment is described below. A hole was dug throughout the gun barrel, and the pressure sensor was fixed in this hole to measure the propellant gas pressure at the projectile base. The measured data were saved in a data processing computer, as shown in Figure 6a. Furthermore, the projectile velocity was also obtained. More specifically, we connected the projectile rigidly with a calibrated bar, which can move with the projectile. A high-speed camera was used to collect the movement data of the calibrated bar. Moreover, the velocity of the calibrated bar is equal to that of the projectile. Finally, the projectile velocity is shown in Figure 6b.

![Figure 6](image)

**Figure 6.** (a) Pressure at the projectile base vs. projectile displacement; (b) projectile velocity vs. projectile displacement.
2.3. Modeling of the Gun Engraving System and Results of Dynamic Response

As mentioned above, the gun engraving system contains the propellant part and the mechanical part. Correspondingly, the fluid–solid coupling model of the gun engraving system contains two submodels, namely the launch-load model (the fluid submodel) and the finite element model (the solid submodel).

2.3.1. Launch-Load Model and its Calculation Results

The force exerted on the projectile base comes from the propellant gas, and the interior ballistics model is used to describe the propellant combustion law and obtain the state value of the propellant gas [30], especially the pressure data. Based on multiphase flow theory [31], we have made minor modifications in the original launch-load model, which can be found in reference [30]. The minor modified launch-load model has five main equations, as expressed below.

The mass conservation equation of the propellant gas:

\[
\frac{\partial (\phi \rho_g u_g)}{\partial t} + \frac{\partial (\phi \rho_g u_g^2)}{\partial x} = m_c \tag{1}
\]

The momentum conservation equation of the propellant gas:

\[
\frac{\partial (\phi \rho_g u_g^2)}{\partial t} + \frac{\partial (\phi \rho_g u_g^2 u_p)}{\partial x} + \phi \frac{\partial p}{\partial x} = -f_s + m_c u_p \tag{2}
\]

The energy conservation equation of the propellant gas:

\[
\frac{\partial (\phi \rho_g u_g (e_g + u_g^2/2))}{\partial t} + \frac{\partial (\phi \rho_g u_g (e_g + u_g^2/2 + u_p^2/2))}{\partial x} + \phi \frac{\partial p}{\partial x} = -Q_p - f_s u_p + m_c \left( e_p + \frac{p}{\rho_p} + \frac{u_p^2}{2} \right) \tag{3}
\]

The mass conservation equation of solid-phase propellant:

\[
\frac{\partial [(1 - \phi) \rho_p]}{\partial t} + \frac{\partial [(1 - \phi) \rho_p u_p]}{\partial x} = -m_c \tag{4}
\]

The momentum conservation equation of solid-phase propellant:

\[
\frac{\partial [(1 - \phi) \rho_p u_p]}{\partial t} + \frac{\partial [(1 - \phi) \rho_p u_p^2]}{\partial x} + (1 - \phi) \frac{\partial p}{\partial x} + \frac{\partial [(1 - \phi) R_p]}{\partial x} = f_s - m_c u_p \tag{5}
\]

The symbol meanings in the above equations are listed in Table 1. In addition, MacCormack [32] difference scheme is used for solving the equations, and it has second-order accuracy, as expressed in Equation (6).

\[
\begin{align*}
U^{n+1}_j &= U^n_j - \frac{\Delta t}{\Delta x} \left( F^n_{j+1} - F^n_j \right) + \Delta t H^n_j \\
\bar{U}^{n+1}_j &= \bar{U}^{n+1}_{j-1} - \frac{\Delta t}{\Delta x} \left( F^{n+1}_{j+1} - F^{n+1}_{j-1} \right) + \Delta t \bar{H}^{n+1}_j \\
U^{n+1} &= \frac{1}{2} \left( U^n + \bar{U}^{n+1} \right)
\end{align*}
\tag{6}
\]

After the calculation, the three-dimensional displacement–time–temperature data of the propellant gas and the two-dimensional time–temperature data of the propellant gas at the point where \( x = 0.9 \) m are shown in Figure 7a,b, respectively. Here the displacement represents the distance from the propellant gas to the gun breech, and it will increase after the projectile moves. Hence, Figure 7a,b contains the blank space at each right side, and the curves represent the trajectory of the projectile.
Table 1. The symbol meanings in the model of the launch load.

| Symbol | Meaning                      | Symbol | Meaning                                           |
|--------|------------------------------|--------|---------------------------------------------------|
| $x$    | Displacement                 | $t$    | Time                                              |
| $\varphi$ | Porosity                    | $m_c$  | Generation mass of the propellant gas             |
| $\rho_g$ | Propellant gas density      | $\rho_p$ | Solid propellant density                         |
| $u_g$  | Propellant gas velocity      | $u_p$  | Solid propellant velocity                         |
| $e_g$  | Internal energy of the propellant gas | $e_p$  | Internal energy of the solid propellant           |
| $f_s$  | Interphase resistance        | $Q_p$  | Interphase heat transfer                          |
| $R_p$  | Particle stress              |        |                                                   |

Figure 7. (a) Displacement–time–temperature data of the propellant gas; (b) time–temperature data of the propellant gas at 0.9 m.

The high-temperature of the propellant gas causes the erosion phenomenon on the internal surface of the gun barrel, and the erosion degree affects the service life of the gun barrel. Hence the temperature value of the propellant gas $T$ is the first performance evaluation index during the engraving process. In this paper, there are totally three performance evaluation indexes, and they are described in Figure 8.

Figure 8. Three performance evaluation indexes and their corresponding physical effects.

The three-dimensional displacement–time–pressure data of the propellant gas and the two-dimensional time versus pressure wave data of the propellant gas are shown in Figure 9a,b, respectively. The pressure wave is calculated by the way that pressure at breech minus that at the projectile base, and the minimum value of pressure wave represents the propellant safety. The higher the minimum value of the pressure wave is, the better the combustion stability of the propellant is. Hence, the value of pressure wave $\Delta p$ is the second performance evaluation index, which is used to evaluate the combustion safety during the engraving process.
Figure 9. (a) Displacement–time–pressure data of the propellant gas; (b) time–pressure wave data of the propellant gas.

The displacement–pressure data at the projectile base, which are extracted from Figure 9a, are shown in Figure 10. As can be seen, the calculation data match well with the experimental data from Figure 6a. Hence, the accuracy of the launch-load model is validated.

Figure 10. Comparison of displacement–pressure data of the projectile by calculation and experiment.

2.3.2. Finite Element Model and its Simulation Results

The finite element model of the mechanical part of the gun engraving system is built by Hypermesh software, as shown in Figure 11. Here, the cross-section of the rotating bands is enlarged for better display, as shown in Figure 12. Specifically, this model contains four components, namely the gun barrel, the rifling, the projectile and the rotating band. In the model, there are 48 riflings and 2 rotating bands in total. In addition, the whole finite element model has 1,443,203 elements and 1,285,561 nodes, and all element types are hexahedron for computational accuracy. Peculiarly, considering the large deformation of the rotating bands, the element sides of the rotating bands range from 0.3 mm to 0.5 mm, while the element sides of other components are from 3 mm to 6 mm. The propellant gas pressure, which is calculated by the launch-load model in Section 2.3.1, is applied on the projectile base by Vuamp subroutine in Abaqus software. The Vuamp function offers a data exchange interface between the launch-load model and the finite element model. Hence, the integrated gun engraving system model is a two-way fluid–solid coupling model, and there is an interaction between the launch-load model and the finite element model. Finally, the dynamic response data of the rotating band and projectile are calculated by Abaqus explicit solver.
Figure 11. Three-dimensional finite element model of the gun engraving system.

Figure 12. Cross-section of finite element model of the rotating bands.

The basic material parameters are listed in Table 2. Considering the large deformation of rotating bands, the Johnson–Cook material constitutive model and Johnson–Cook material failure model are used to describe the material behavior of rotating bands, as expressed by Equations (7) and (8), respectively. Here $\sigma$ is equivalent stress, $\varepsilon$ is equivalent plastic strain, $\dot{\varepsilon}$ is equivalent strain rate, $\dot{\varepsilon}_0$ is reference strain rate, $T$ is temperature variable, and $\varepsilon_d$ is the failure strain. The Johnson–Cook material parameters of the rotating bands are listed in Table 3.

Table 2. Basic material parameters of finite element model.

| Component       | Material Name | Young’s Modulus | Poisson’s Ratio | Density       |
|-----------------|---------------|-----------------|----------------|---------------|
| Projectile      | steel         | 192 GPa         | 0.31           | 5421 kg·m$^{-3}$ |
| Gun barrel      | steel         | 210 GPa         | 0.31           | 7832 kg·m$^{-3}$ |
| Riflings        | steel         | 210 GPa         | 0.31           | 7832 kg·m$^{-3}$ |
| Rotating bands  | copper        | 125 GPa         | 0.34           | 8962 kg·m$^{-3}$ |

Table 3. Johnson–Cook material parameters of the rotating bands.

| $A$  | $B$  | $C$  | $m$  | $n$  | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ |
|------|------|------|------|------|-------|-------|-------|-------|-------|
| 90   | 292  | 0.025| 1.09 | 0.31 | 0.54  | 4.89  | 3.03  | 0.014 | 1.12  |

\[
\sigma = (A + Be^n) \left[ 1 + C \ln(\dot{\varepsilon}/\dot{\varepsilon}_0) \right] \left( 1 - T^m \right) \tag{7}
\]

\[
\varepsilon_d = \left( d_1 + d_2\varepsilon^{-d_7} \right) \left[ 1 + d_4 \ln(\dot{\varepsilon}/\dot{\varepsilon}_0) \right] \left( 1 + d_5 T \right) \tag{8}
\]

Figures 13–15 show the stress evolution of riflings and rotating bands during the engraving process, which is simulated by the Abaqus explicit solver. At the right side of each figure, the riflings are hidden, and a half-section of the rotating bands are displayed. The engraving process can be divided into three periods, which is corresponding to Figures 13–15.
Figure 13. Appearance and stress distribution of the rotating bands and riflings in the first period.

Figure 14. Appearance and stress distribution of the rotating bands and riflings in the second period.

Figure 15. Appearance and stress distribution of the rotating bands and riflings in the third period.
In the first period, the initial segments of 48 riflings are cutting the first rotating band. Thus, the high stress produces plastic and damage deformation of the first rotating band, so the corresponding 48 rotating band grooves form, and the rotating bands begin to rotate. As for the second rotating band, it is squeezed by the bore surface and also undergoes plastic deformation, as shown in the brown wireframe in Figure 13. Hence, the second rotating band can avoid the spillage of the propellant gas.

In the second period, the initial segments of 48 riflings are continuously cutting these two rotating bands. Since the height of initial segments of riflings gradually increases, so the depth of grooves at the first rotating band is deeper than that at the second rotating band, as exhibited in Figure 14.

In the third period, the initial segments of 48 riflings have cut these two rotating bands completely, and the whole engraving process finishes. After the engraving process, no obvious changes in groove shape will take place until the rotating bands and projectile leave the muzzle.

The engraving resistance force \( f_e \) is calculated by Equation (9), and its value is shown in Figure 16a. Here \( s \) is the projectile base area, \( p_b \) is the projectile base pressure, which is calculated by the launch-load model, \( m \) is the projectile mass, \( a \) is the projectile acceleration, which is simulated by Abaqus explicit solver.

\[
f_e = s \cdot p_b - ma
\]  

Figure 16. (a) Engraving resistance force vs. projectile displacement; (b) engraving-completion velocity of the projectile vs. projectile displacement.

Figure 16b shows engraving-completion velocity data of the projectile, which is simulated by Abaqus explicit solver. As can be seen, the engraving-completion velocity of the projectile by simulation is 111.46 m·s\(^{-1}\), and that by the experiment is 114.13 m·s\(^{-1}\). Compared with the data, the error is \(-2.58\%\). Therefore, the accuracy of the finite element model is validated.

From the above simulation results of the finite element model, the interaction between rotating bands, riflings and bore surface produces cut phenomenon and squeezing phenomenon, which complete the engraving functions. On the other hand, the cut and squeezing phenomenon also generates the engraving resistance force, which is opposite to the direction of the propellant gas force and will impede the projectile motion. Because the smaller the engraving resistance force is, the higher engraving-completion velocity and the higher muzzle velocity are. So, the engraving resistance force \( f_e \) is the third performance evaluation index during the engraving process. Furthermore, the size parameters of the gun engraving system have an effect on the engraving resistance force and the motion state of the projectile. Subsequently, by determining the space volume of the propellant combustion, the state of the projectile motion can also affect the temperature value and
the pressure wave value of the propellant gas. In conclusion, the size parameters of the
gun engraving system have an effect on all three performance evaluation indexes, and it is
necessary to optimize size parameters for better performance.

3. Sensitivity Analysis of Size Parameters of the Gun Engraving System

3.1. The Meaning and Range of Size Parameters

In the optimal design process, the less modification and the more performance
improvement are pursued by designers. Therefore, before optimization, sensitivity analysis
of size parameters of the gun engraving system is needed and will be carried out in this
section. Figures 17 and 18 present the meanings of size parameters from $E_1$ to $E_{11}$. Additionally $E_{12}$ is the forcing cone angle. The nominal values and tolerance intervals of size
parameters are listed in Table 4.

![Figure 17. Schematic diagram of the position of size parameters on the cross profile of the
rotating bands.](image)

![Figure 18. Schematic diagram of the position of size parameters on the initial segment of rifling.](image)

Table 4. The nominal values and tolerance intervals of size parameters.

| Size Parameters | Initial Value | Lower Bound | Upper Bound |
|-----------------|--------------|-------------|-------------|
| $E_1$(mm)       | 6.3          | 5.7         | 6.8         |
| $E_2$(mm)       | 3.2          | 2.9         | 3.6         |
| $E_3$(mm)       | 3.1          | 2.8         | 3.5         |
| $E_4$(mm)       | 25.2         | 23.9        | 26.7        |
| $E_5$(mm)       | 8.3          | 7.6         | 8.9         |
| $E_6$(mm)       | 3.2          | 2.9         | 3.6         |
| $E_7$(mm)       | 3.1          | 2.8         | 3.5         |
| $E_8$(mm)       | 24.9         | 23.7        | 26.3        |
| $E_9$(mm)       | 49.6         | 47.1        | 52.1        |
| $E_{10}$(mm)    | 2.5          | 2.2         | 2.9         |
| $E_{11}$(mm)    | 3.8          | 3.4         | 4.3         |
| $E_{12}$(°)     | 3.3          | 3.1         | 3.6         |

3.2. The Method and Process of Sensitivity Analysis

The sensitivity analysis process denoted in Figure 19 is illustrated as follows:
Step 1: the fluid–solid coupling model of the gun engraving system is built.

Step 2: the upper limits $X^\text{max}_k$ and lower limits $X^\text{min}_k$ of the twelve size parameters $X = (X_1, X_2, \ldots, X_k, \ldots, X_n)$ are determined, respectively.

Step 3: the size parameters matrix $D_{m \times n}$ is obtained by the OLHD (optimal Latin hypercube design) methods [33].

Step 4: the size parameters matrix is brought into the fluid–solid coupling model of the gun engraving system until the end of the simulation.

Step 5: the polynomial regression model is employed to reflect the response relationship of the gun engraving system model, as follows:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik} + \varepsilon - N \left(0, \sigma^2 \right)$$ (10)

The unknown parameter $\beta_0, \beta_1, \beta_2, \ldots, \beta_k, \sigma^2$ can be estimated by ordinary least squares. The principle is to minimize the sum of the squares of the residuals:

$$Q = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_{i1} - \beta_2 x_{i2} - \cdots - \beta_k x_{ik})^2$$ (11)
This least-squares method should satisfy the equation $Q(\beta_0, \beta_1, \beta_2, \ldots, \beta_k) = \min Q(\beta_0, \beta_1, \beta_2, \ldots, \beta_k)$. The partial derivative of Equation (11) is set to 0:

$$
\begin{align*}
\frac{\partial Q}{\partial \beta_0} &= -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_{1i} - \beta_2 x_{2i} - \cdots - \beta_k x_{ki}) = 0 \\
\frac{\partial Q}{\partial \beta_1} &= -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_{1i} - \beta_2 x_{2i} - \cdots - \beta_k x_{ki}) x_{1i} = 0 \\
&\vdots \\
\frac{\partial Q}{\partial \beta_k} &= -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_{1i} - \beta_2 x_{2i} - \cdots - \beta_k x_{ki}) x_{ki} = 0
\end{align*}
$$

(12)

The equivalent form of Equation (12) is expressed as follows:

$$
\begin{align*}
\beta_0 n + \beta_1 \sum_{i=1}^{n} x_{1i} + \beta_2 \sum_{i=1}^{n} x_{2i} + \cdots + \beta_k \sum_{i=1}^{n} x_{ki} &= \sum_{i=1}^{n} y_i \\
\beta_0 \sum_{i=1}^{n} x_{1i} + \beta_1 \sum_{i=1}^{n} x_{1i}^2 + \beta_2 \sum_{i=1}^{n} x_{1i} x_{2i} + \cdots + \beta_k \sum_{i=1}^{n} x_{1i} x_{ki} &= \sum_{i=1}^{n} x_{1i} y_i \\
&\vdots \\
\beta_0 \sum_{i=1}^{n} x_{ki} + \beta_1 \sum_{i=1}^{n} x_{1i} x_{ki} + \beta_2 \sum_{i=1}^{n} x_{2i} x_{ki} + \cdots + \beta_k \sum_{i=1}^{n} x_{ki}^2 &= \sum_{i=1}^{n} x_{ki} y_i
\end{align*}
$$

(13)

The matrix operation method is adopted to solve the normal Equation (13):

$$
\begin{align*}
\begin{bmatrix}
1 & x_{11} & x_{21} & \cdots & x_{k1} \\
1 & x_{12} & x_{22} & \cdots & x_{k2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & x_{1n} & x_{2n} & \cdots & x_{kn}
\end{bmatrix}
\begin{bmatrix}
\hat{\beta}_0 \\
\hat{\beta}_1 \\
\hat{\beta}_2 \\
\vdots \\
\hat{\beta}_n
\end{bmatrix}
= 
\begin{bmatrix}
\hat{y}_1 \\
\hat{y}_2 \\
\vdots \\
\hat{y}_n
\end{bmatrix}
\end{align*}
$$

(14)

According to the operation rules of the matrix, Equation (12) can be expressed as a matrix form:

$$
x^T x \beta = x^T y
$$

(15)

Then, Equation (16) can be obtained by left multiplying $(x^T x)^{-1}$:

$$
(\hat{\beta}_0, \hat{\beta}_1, \cdots, \hat{\beta}_k)^T = \left(x^T x\right)^{-1} x^T y
$$

(16)

Step 6: Since the parameters are normalized before analysis, the fitted model coefficients can fairly reflect the contribution of input variables to the response. The coefficients of the polynomial regression model are transformed into contribution ratio by Equation (17):

$$
N_i = 100\hat{\beta}_i / \sum_{j=1}^{k} |\hat{\beta}_i|
$$

(17)

Step 7: In order to better characterize the contribution rate of each parameter, the sensitivity results on the performance evaluation indexes $T$, $\Delta p$, $f_c$ are taken as absolute values. The comprehensive contribution ratio is expressed by Equation (18):

$$
\chi_i = \frac{|N_1|}{n} + \frac{|N_2|}{n} + \cdots + \frac{|N_m|}{n}
$$

(18)

3.3. The Results of Sensitivity Analysis

As described in Figure 8, there are totally three performance evaluation indexes during the engraving process. Figure 20a–c show sensitivity analysis results on these three performance evaluation indexes, respectively, and Figure 20d shows sensitivity analysis results on the comprehensive performance index. In Figure 20d, the dotted line is a separate
line whose contribution rate is 10%. Moreover, the size parameters whose contribution rate is more than 10% are defined as high-influence size parameters. As can be seen, the contribution rates of \( E_5, E_8, E_{10} \) and \( E_{11} \) are more than 10%, and so these four size parameters are taken as design variables for optimization in Section 5.

![Figure 20](image_url)

Figure 20. Sensitivity analysis results on (a) engraving resistance force (b) pressure wave of the propellant gas (c) propellant gas temperature (d) three performance evaluation indexes.

4. The Interval Uncertain Optimization Method Based on Affine Arithmetic

4.1. Nonlinear Interval Uncertain Optimization Problem

The nonlinear interval uncertain optimization can be expressed as follows [16]:

\[
\begin{align*}
\min & \quad f(X, U) \\
\text{s.t.} & \quad g_i(X, U) = b^I_i = [b^L_i, b^R_i] \\
& \quad i = 1, 2, \ldots, l, X \in \Omega^n, U \in U^I = [U^L, U^R], U_i \in U^I_i = [U^L_i, U^R_i], i = 1, 2, \ldots, q
\end{align*}
\]  

(19)

where \( X \) is the n-dimensional design vector whose value range is \( \Omega^n \). \( U \) is a q-dimensional uncertain vector, which is described by a q-dimensional interval vector \( U^I \). \( f \) and \( g \) are objective function and the \( i \)-th constraint, respectively. For the optimization model of the gun engraving system, the engraving resistance force \( f_e \) is taken as an objective function. Because \( f_e \) can affect the engraving-completion velocity of the projectile. Moreover, the engraving-completion velocity of the projectile is the most crucial index for designers. Additionally, the temperature of the propellant gas \( T \) can affect the erosion degree and service life of the gun barrel. The pressure wave of the propellant gas \( \Delta p \) can affect the propellant combustion stability. Both \( T \) and \( \Delta p \) should be in a feasible scope. Therefore, \( T \) and \( \Delta p \) are taken as a constraint. \( l \) is the number of constraints. They are all continuous nonlinear functions of \( X \) and \( U \), and at least one of them is a nonlinear function of \( U \). \( b^I_i \) is the allowable interval of the \( i \)-th uncertain constraint, and it can also be a real number in practical problems.
Since the objective function and the constraint are both continuous functions of $U$, for any design variable $X$ that is determined, the values of the objective function and the constraint are also intervals.

### 4.2. Affine Arithmetic

In interval arithmetic, two interval numbers are set as $A^I = [A^L, A^R], B^I = [B^L, B^R]$, and the basic operation rules are as follows:

$$
\begin{align*}
A^I + B^I &= [A^L + B^L, A^R + B^R] \\
A^I - B^I &= [A^L - B^L, A^R - B^R] \\
A^I \times B^I &= \min(A^I B^L, A^L B^R, A^R B^L, A^R B^R), \\
&\max(A^L B^L, A^L B^R, A^R B^L, A^R B^R) \\
A^I \div B^I &= [A^L, A^R] \div [B^L, B^R] = [A^L, A^R] \times [\frac{1}{B^R}, \frac{1}{B^L}], 0 \notin [B^L, B^R]
\end{align*}
$$

The interval arithmetic can directly calculate the interval number. Although the calculation is simple, it will bring about the interval expansion phenomenon. When calculating the function value range, the calculation result interval is amplified. For example, calculate $x^2$, where $x = [-1, 1]$. The resulting interval obtained from Equation (19) is $x^2 = [-1, 1]$, while the actual result is $x^2 = [0, 1]$. The root of the interval expansion problem comes from the correlation between interval numbers.

The affine arithmetic is similar to interval arithmetic, but the affine arithmetic considers the correlation of variables in the calculation, so it can well suppress the interval expansion. Moreover, when the same variable appears in different terms of the function, the affine arithmetic can retain its complete correlation to the greatest extent, while the interval arithmetic will cause serious information loss. The affine form of the uncertain quantity $u$ is denoted as $\hat{u}$:

$$
\hat{u} = u_0 + u_1 \varepsilon_1 + u_2 \varepsilon_2 + \cdots + u_t \varepsilon_t
$$

where $u_0$ is the central value of the affine afform, and the $\varepsilon_i$ are noise symbols, whose values are unknown but definitely lie in the interval $[-1, 1]$, correspondingly, $u_i$ are the known real coefficients of the noise symbols. When $|\varepsilon_i| = 1$, the affine form obtains its maximum and minimum values, which can be expressed as the interval:

$$
\hat{u} = \left[ u_0 - \sum_{i=1}^{t} |u_i|, u_0 + \sum_{i=1}^{t} |u_i| \right]
$$

For interval uncertain optimization problems, the uncertain interval quantity can be expressed as:

$$
\hat{u}_i = u_{i0} + \Delta u_i \cdot \varepsilon_i
$$

where $u_{i0}$ and $\Delta u_i$ are the central value and radius of the interval, respectively.

Affine afform $\beta$, and $\hat{\beta} = x_0 + x_1 \varepsilon_1 + x_2 \varepsilon_2 + \cdots + x_t \varepsilon_t$, $\hat{\beta} = y_0 + y_1 \varepsilon_1 + y_2 \varepsilon_2 + \cdots + y_t \varepsilon_t$, $\eta, \xi \in R$. The basic affine operation rules are as follows:

$$
\begin{align*}
\hat{x} \pm \hat{y} &= (x_0 \pm y_0) + (x_1 \pm y_1) \varepsilon_1 + (x_2 \pm y_2) \varepsilon_2 + \cdots + (x_t \pm y_t) \varepsilon_t \\
\eta \hat{x} &= \eta x_0 + \eta x_1 \varepsilon_1 + \eta x_2 \varepsilon_2 + \cdots + \eta x_t \varepsilon_t \\
\hat{x} \pm \hat{\xi} &= (x_0 \pm \xi_0) + x_1 \varepsilon_1 + x_2 \varepsilon_2 + \cdots + x_t \varepsilon_t \\
\hat{x} \cdot \hat{y} &= x_0 y_0 + \sum_{i=1}^{t} (x_0 y_i + y_0 x_i) \varepsilon_i + \sum_{i=1}^{t} x_i \varepsilon_i \sum_{i=1}^{t} y_i \varepsilon_i
\end{align*}
$$

(24)
The quadratic term $\varepsilon_i \cdot \varepsilon_i$ appears in the multiplication operation. $\varepsilon_i^2 = [0, 1]$ can be obtained from $\varepsilon_i = [-1, 1]$, so the upper and lower bounds of the affine operation $\hat{x} \cdot \hat{y}$ can be expressed as interval $\hat{x} \cdot \hat{y} \in [a, b]$, where:

$$
\begin{align*}
  a &= x_0y_0 - \left| \sum_{i=1}^{1} (x_0y_i + y_0x_i) \varepsilon_i \right| \\
  b &= x_0y_0 + \left| \sum_{i=1}^{1} (x_0y_i + y_0x_i) \varepsilon_i \right| + \left| \sum_{i=1}^{1} x_iy_i \right|
\end{align*}
$$

(25)

4.3. An Optimization Method Based on Affine Arithmetic

In this work, a nonlinear interval uncertain optimization method based on affine arithmetic is adopted, and the phrase “affine arithmetic-based method” is used for calling this method. First, the uncertain variables in the objective function and constraint are rewritten into the affine form. Second, the affine arithmetic is utilized to obtain the interval of objective function and constraint of the design variable $X$, and then the uncertain problem is transformed into a deterministic problem. Finally, the genetic algorithm is used to solve the problem.

The details of the approach are summarized below.

First, let $\hat{U}_i = U_{i0} + \Delta U_i \varepsilon_i$. Then, the objective function is rewritten into affine form.

$$
f(X) = d_0 + \sum_{i=1}^{m} d_i \varepsilon_i
$$

(26)

where $d_0$ is the central value of the affine form, $d_i$ are the known real coefficient, $\varepsilon_i$ are the noise symbol. Therefore, the interval of the objective function at the design variable $X$ can be obtained:

$$
\begin{align*}
  f^L(X) &= d_0 - \sum_{i=1}^{m} |d_i| \\
  f^R(X) &= d_0 + \sum_{i=1}^{m} |d_i|
\end{align*}
$$

(27)

Second, the constraints are rewritten in affine form:

$$
g_i(X) = c_0 + \sum_{i=1}^{n} c_i \varepsilon_i
$$

(28)

where $c_0$ is the central value of the affine form, $c_i$ are the known real coefficient, $\varepsilon_i$ are the noise symbol. The interval of the constraint $g_i$ at the design variable $X$ can be obtained:

$$
\begin{align*}
  g_i^L(X) &= c_0 - \sum_{i=1}^{n} |c_i| \\
  g_i^R(X) &= c_0 + \sum_{i=1}^{n} |c_i|
\end{align*}
$$

(29)

The above uncertain problem can be rewritten as:

$$
\begin{align*}
  \min_X & \quad f(X) = d_0 + \sum_{i=1}^{m} d_i \varepsilon_i \\
  \text{s.t.} & \quad g_i(X) = c_0 + \sum_{i=1}^{n} c_i \varepsilon_i = b_i = [b_i^L, b_i^R], i = 1, 2, \ldots, l
\end{align*}
$$

(30)
It can be further rewritten as:

\[
\begin{align*}
\min_{\mathbf{X}} f(\mathbf{X}) &= d_0 + \sum_{i=1}^{m} d_i \varepsilon_i \\
\text{s.t. } g^L_i(\mathbf{X}) &= c_0 - \sum_{i=1}^{n} |c_i| \geq b_i^L \\
g^R_i(\mathbf{X}) &= c_0 + \sum_{i=1}^{n} |c_i| \leq b_i^R, i = 1, 2, \ldots, l
\end{align*}
\]

At this point, the uncertain optimization problem is transformed into a single-layer deterministic optimization problem, which can be solved by existing sequential quadratic programming (SQP), genetic algorithm (GA) and other intelligent optimization algorithms. This method is further illustrated with a numerical example below.

This numerical example is a nonlinear interval optimization problem with three uncertain variables, and it is from a doctoral thesis [34]. In addition, all the parameters, which we set, are the same as the parameters in reference [34].

\[
\begin{align*}
\min_{\mathbf{X}, \mathbf{U}} f(\mathbf{X}, \mathbf{U}) &= 130.0 - U_1^2(X_1 + 2) - U_2X_2^2 - U_3^2X_3^2 \\
\text{s.t. } g_1(\mathbf{X}, \mathbf{U}) &= U_1X_1^2 - U_2X_2 + U_3X_3 + 1 - 15, 20 \\
g_2(\mathbf{X}, \mathbf{U}) &= U_1X_1 + U_2X_2 + U_3X_3^2 - 1.0 \leq X_1 \leq 5.0, -3.0 \leq X_2 \leq 6.0, -2.0 \leq X_3 \leq 7.0 \\
U_1 &\in [1.0, 1.3], U_2 \in [0.9, 1.1], U_3 \in [1.2, 1.4]
\end{align*}
\]

Step 1: rewriting the interval quantity into affine quantity.
Let \( \hat{U}_1 = u_1 + \Delta u_1 \cdot \varepsilon_1, \hat{U}_2 = u_2 + \Delta u_2 \cdot \varepsilon_2, \hat{U}_3 = u_3 + \Delta u_3 \cdot \varepsilon_3. \)
Step 2: computing the affine form of the objective function.
The nonlinear term of the uncertain variables in the objective function are \( \hat{U}_1^2 \) and \( \hat{U}_2^2 \):

\[
\begin{align*}
\hat{U}_1^2 &= u_1^2 + 2u_1\Delta u_1\varepsilon_1 + \Delta u_1^2\varepsilon_1^2 \\
\hat{U}_2^2 &= u_2^2 + 2u_2\Delta u_2\varepsilon_2 + \Delta u_2^2\varepsilon_2^2
\end{align*}
\]

The objective function can be rewritten as:

\[
f(\mathbf{X}) = (-2\Delta u_1 \cdot u_1(X_1 + 2))\varepsilon_1 + (-\Delta u_1^2(X_1 + 2))\varepsilon_1^2 + (-\Delta u_2X_2^2)\varepsilon_2 + (-\Delta u_3X_3^2)\varepsilon_3^2 \\
+ (-2\Delta u_3 \cdot u_3X_3^2)\varepsilon_3 + 130 + (-X_1 - 2)u_1^2 - u_2X_2^2 - u_3^2X_3^2
\]  

Since \( \varepsilon_1, \varepsilon_3 \in [-1, 1] \), the value interval of even term \( \varepsilon_1^2, \varepsilon_3^2 \) are \([0, 1]\). The interval of the objective function can be obtained as:

\[
\begin{align*}
\begin{cases}
\ell(\mathbf{X}) = 130 + (-X_1 - 2)u_1^2 - u_2X_2^2 - u_3^2X_3^2 - |2\Delta u_1 \cdot u_1(X_1 + 2)| \\
\bar{\ell}(\mathbf{X}) = 130 + (-X_1 - 2)u_1^2 - u_2X_2^2 - u_3^2X_3^2 + |2\Delta u_1 \cdot u_1(X_1 + 2)|
\end{cases}
\end{align*}
\]

Step 3: computing the affine form of the constraints.
The nonlinear term of the uncertain variables in the objective function are \( \hat{U}_2^2 \) and \( \hat{U}_3^2 \):

\[
\hat{U}_2^2 = u_2^2 + 2u_2\Delta u_2\varepsilon_2 + \Delta u_2^2\varepsilon_2^2
\]  

The constraint \( g_1(\mathbf{X}, \mathbf{U}) \) can be rewritten as:

\[
g_1(\mathbf{X}) = \Delta u_1X_1^2\varepsilon_1 - 2\Delta u_2u_2X_2\varepsilon_2 - \Delta u_2X_2\varepsilon_2^2 \\
+ \Delta u_3X_3\varepsilon_3 + u_1X_1^2 - u_2^2X_2 + u_3X_3
\]
Since $\varepsilon_2 \in [-1, 1]$, the value interval of even term $\varepsilon_2^2$ is $[0, 1]$. The interval of constraint $g_1(X,U)$ can be obtained as:

\[
\begin{align*}
   g^{L}_1(X) &= u_1X_1^2 - u_2X_2 + u_3X_3 - |\Delta u_1X_1^2| - |2\Delta u_2X_2| - |\Delta u_3X_3| \\
   g^{R}_1(X) &= u_1X_1^2 - u_2X_2 + u_3X_3 + |\Delta u_1X_1^2| + |2\Delta u_2X_2| + |\Delta u_3X_3|
\end{align*}
\] (38)

Similarly, the affine form of the constraint $g_2(X,U)$ can be obtained as:

\[
g_2(X) = \Delta u_1X_1 + \Delta u_2X_2 + 2\Delta u_3X_3\varepsilon_3 + \Delta u_3X_3^2\varepsilon_3^2 + u_1X_1 + u_2X_2 + 1
\] (39)

Its value interval is:

\[
\begin{align*}
   g^{L}_2(X) &= -|\Delta u_1X_1| - |\Delta u_2X_2| - |2\Delta u_3X_3X_3^2| + u_1X_1 + u_2X_2 + 1 \\
   g^{R}_2(X) &= |\Delta u_1X_1| + |\Delta u_2X_2| + |2\Delta u_3X_3X_3^2| + |\Delta u_3X_3^2| + u_1X_1 + u_2X_2 + 1
\end{align*}
\] (40)

Step 4: deterministic transformation.

The original uncertain optimization problem can be rewritten as the optimization problem in the following affine form:

\[
\begin{align*}
   \min f(X) \\
   \text{s.t.} \quad g^{L}_1(X) \geq 6.2 \\
   \quad g^{R}_1(X) \leq 7.3 \\
   \quad g^{L}_2(X) \geq 15
\end{align*}
\] (41)

According to the interval order model proposed in Reference [16], the above optimization problem can be transformed into the following deterministic optimization problem:

\[
\begin{align*}
   \min & \left( f^c(X,U), f^w(X,U) \right) \\
   \text{s.t.} & \quad g^{L}_1(X) \geq 6.2 \\
   & \quad g^{R}_1(X) \leq 7.3 \\
   & \quad g^{L}_2(X) \geq 15 \\
   f^w(X) &= f^d(X,U) - f^c(X,U) \\
   f^c(X) &= f^d(X,U) + f^d(X,U)
\end{align*}
\] (42)

So far, the nonlinear interval uncertainty optimization problem has been transformed into a deterministic optimization problem. The linear weighting method can be used to further convert the problem into a single-objective optimization problem:

\[
\begin{align*}
   \min & \quad f_d(X) = (1 - \beta)(f^c(X) + \xi) / \phi + \beta(f^c(X) + \xi) / \psi \\
   \text{s.t.} & \quad g^{L}_1(X) \geq 6.2 \\
   & \quad g^{R}_1(X) \leq 7.3 \\
   & \quad g^{L}_2(X) \geq 15
\end{align*}
\] (43)

where $f_d(X)$ is the multiobjective evaluation function, $0 \leq \beta \leq 1$ is the multiobjective weight coefficient, $\xi$ is the parameter that guarantees $f^c(X) + \xi$ and $f^w(X) + \xi$ to be nonnegative, $\phi$ and $\psi$ are regularization factors of a multiobjective function.

In order to be consistent with reference [34], the regularization factors $\phi$, $\psi$ and $\xi$ are 1.33, 0.34 and 0.0, respectively. GA [35] is used to solve the problem, and the results are listed in Table 5.

The calculation results obtained by the two-layer nested method are regarded as the reference solution for interval uncertain optimization problem. Comparing with the results by these two methods, the max error is only 1.6%. It indicates that the affine arithmetic-based method has high accuracy. Furthermore, the affine arithmetic-based method costs only 1.944 s to complete the numerical example, while the two-layer nested method costs 544.48 s. It also indicates that the affine arithmetic-based method has obvious improvement in computational efficiency.
Table 5. Comparison of optimization results.

| Method                            | Interval of Objective Function | Interval of Constraint $\eta_1$ | Interval of Constraint $\eta_2$ | Computation Time of Numerical Example |
|-----------------------------------|--------------------------------|---------------------------------|---------------------------------|--------------------------------------|
| Two-layer nested method           | [73.01, 88.58]                 | [6.19, 7.26]                    | [40.47, 54.80]                  | 544.48 s                             |
| Affine arithmetic-based method    | [72.94, 88.52]                 | [6.21, 7.30]                    | [40.43, 54.99]                  | 1.944 s                              |
| Errors                            | 0.38%                          | 0.32%                           | 1.6%                            | -                                    |

5. Uncertain Optimization of the Gun Engraving System

5.1. Optimization Model

As described in Figure 8, there are totally three performance evaluation indexes during the engraving process. Because the engraving-completion velocity of the projectile is the most crucial index for designers, thus the engraving resistance force $f_c$ is taken as the objective function. Additionally, the temperature $T$ and the pressure wave of the propellant gas $\Delta p$ are taken as constraints.

The optimization model of the gun engraving system, which is as expressed in Equation (44), contains four design variables, namely the rifling width $X_1$, the rifling height $X_2$, the total height of the second rotating band $X_3$, the width of the second rotating band $X_4$. In addition, four uncertain parameters, namely the propellant length $U_1$, the propellant thickness $U_2$, the projectile mass $U_3$, the propellant mass $U_4$, are taken into consideration.

\[
\begin{align*}
\min_{X} & \quad f_c(X, U) \\
\text{s.t.} & \quad T(X, U) \leq \eta_1 \quad \Delta p(X, U) \leq \eta_2 \\
\quad X & \in \Omega
\end{align*}
\]  

(44)

where $\eta_1 = [293, 2700], \eta_2 = [-5, 0]$  
$7.6 \leq X_1 \leq 8.9, 23.7 \leq X_2 \leq 26.3, 2.2 \leq X_3 \leq 2.9, 3.4 \leq X_4 \leq 4.3$  
$U_1 \in [79.8, 80.2], U_2 \in [2.0, 2.4], U_3 \in [45.3, 45.7], U_4 \in [16.8, 17.2]$

According to the interval order model [16], the uncertain optimization model above is converted into a deterministic multiobjective optimization model shown as follows:

\[
\begin{align*}
\min f_c^L(X, U), f_c^R(X, U) \\
\text{s.t.} & \quad g_1^L(X, U) \leq \eta_1^L, g_1^R(X, U) \leq \eta_1^R \\
& \quad g_2^L(X, U) \leq \eta_2^L, g_2^R(X, U) \leq \eta_2^R \\
\quad X & \in \Omega, U \in U^I
\end{align*}
\]  

(45)

where  
$g_c^L(X, U) = f_c^L(X, U) + f_c^R(X, U)$  
$g_c^R(X, U) = f_c^R(X, U) - f_c^L(X, U)$  
$g_c^L(X, U) = \eta_1^L - \eta_1^R$  
$g_c^R(X, U) = \eta_2^L - \eta_2^R$

$\eta_1 = [293, 2700], \eta_2 = [-5, 0]$  
$7.6 \leq X_1 \leq 8.9, 23.7 \leq X_2 \leq 26.3, 2.2 \leq X_3 \leq 2.9, 3.4 \leq X_4 \leq 4.3$  
$U_1 \in [79.8, 80.2], U_2 \in [2.0, 2.4], U_3 \in [45.3, 45.7], U_4 \in [16.8, 17.2]$

The model cannot be directly processed by the affine arithmetic. Therefore, the response surface method (RSM) [36] is employed to construct the explicit polynomial expressions of the objective function and constraint. Thereby, the intervals of objective functions and constraints can be obtained by affine arithmetic. Finally, the uncertain problem is transferred into a deterministic problem that can be easily solved.
5.2. Optimization Results

The NSGA-II [37] exhibits good global convergence without falling into the local optima and has strong robustness, which is especially suitable for multiobjective optimization problems. Therefore, it is used as the optimization operator. The population size is set as 100, and the stopping criterion is set as 100 generations. Finally, the Pareto front is obtained, as shown in Figure 21.

![Figure 21. The Pareto front of optimization results.](image)

All solutions in the Pareto front are feasible. The choice of the final solution depends on the preference of the decision-maker. Optimizing \( f_C(X) \) is for improving the average design performance of an objective function under uncertainty. Minimizing \( f_W(X) \) can reduce the sensitivity of the objective function to uncertainty, and thereby can ensure design robustness. Therefore, considering both the average design performance and the robustness, the “A” point in Figure 21 is adopted as the final optimal solution. The corresponding size parameters data are listed in Table 6.

Table 6. Comparison of design variables before and after optimization.

|       | \( X_1 \) | \( X_2 \) | \( X_3 \) | \( X_4 \) |
|-------|-----------|-----------|-----------|-----------|
| Initial value | 8.3 mm | 24.9 mm | 2.5 mm | 3.8 mm |
| Optimized value | 8.7 mm | 26.2 mm | 2.3 mm | 3.5 mm |

Finally, by substituting the optimized size parameters data into the model of the gun engraving system, the initial and optimized interval of performance evaluation indexes are obtained, as shown in Figures 22–24. Additionally, the detailed performance evaluation indexes of point A are listed in Table 7. As can be seen, after optimization, \( f_e \) decreases, and the percent decrease of \( f_{\text{emax}} \) ranges from 6.34% to 18.24%. \( T \) also decreases, and the percent decrease of \( T_{\text{max}} \) ranges from 1.91% to 7.45%. At the same time, the change of \( \Delta p \) is not obvious and the percent increase of \( \Delta p_{\text{min}} \) ranges slightly from 0.12% to 0.36%. The optimization results mean that, first, the engraving-completion velocity of the projectile raises. Then the erosion degree of the gun barrel declines during the engraving process, and the service life of the gun barrel improves. Finally, the combustion stability of the propellant has no obvious change.
Figure 22. The initial and optimized interval of the engraving resistance force.

Figure 23. The initial and optimized interval of the propellant gas temperature.

Figure 24. The initial and optimized interval of pressure wave of the propellant gas.
Table 7. Detailed performance evaluation indexes of point A.

|                      | \( f_{\text{emax}} \)   | \( T_{\text{max}} \)   | \( \Delta p_{\text{min}} \) |
|----------------------|--------------------------|-------------------------|-----------------------------|
| Initial upper bound  | \( 1.48 \times 10^6 \) N | 2611.43 K               | \(-4.218\) Mpa              |
| Initial lower bound  | \( 1.42 \times 10^6 \) N | 2519.07 K               | \(-4.211\) Mpa              |
| Optimized upper bound| \( 1.33 \times 10^6 \) N | 2471.08 K               | \(-4.206\) Mpa              |
| Optimized lower bound| \( 1.21 \times 10^6 \) N | 2416.75 K               | \(-4.203\) Mpa              |
| Change percent       | \((-18.24\%, -6.34\%)\) | \((-7.45\%, -1.91\%)\) | \((0.12\%, 0.36\%)\)       |

6. Conclusions

In this paper, first, a computational model of the launch load and the finite element model of the gun engraving system was established and validated by the gun launch experiment. Then by solving the launch-load model, two performance evaluation indexes, namely the propellant gas temperature and the pressure wave of the propellant gas, were obtained. By solving the finite element model, the engraving resistance force, as the third performance evaluation index, was obtained. Second, the sensitivity analysis was performed, and four high-influence size parameters were selected as design variables. Finally, an interval uncertain optimization method based on affine arithmetic was introduced and validated by the numerical example. The validation results showed this method had high accuracy and high computational efficiency. Then the uncertain optimization on three performance evaluation indexes was carried out. From the simulation results and optimization results, two main conclusions can be drawn:

1. During the engraving process, the projectile rotating bands are squeezed by the bore surface and cut by riflings, which will lead to the plastic deformation and damage of the rotating bands. The functions of this process are making the projectile rotate and avoiding the spillage of the propellant gas. On the other hand, the cut and squeezing process also generates the engraving resistance force, which will impede the projectile motion. By the mechanism analysis of the engraving process, twelve related size parameters are selected for the sensitivity analysis.

2. After the optimal design of size parameters of the gun engraving system, the percent decrease of maximum engraving resistance force ranges from 6.34% to 18.24%, which means an increase of the engraving-completion velocity of the projectile. The percent decrease of maximum propellant gas temperature ranges from 1.91% to 7.45%, which means the improvement in erosion degree and service life of the gun barrel. The percent increase of the minimum pressure wave of the propellant gas ranges from 0.12% to 0.36%, which means that the combustion stability of the propellant has no obvious effect.

The present work provides an optimal design scheme with high-efficiency and low-cost. It would be suitable for not only gun engraving systems but also other industrial products.

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