Reconnection via the Tearing Instability

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ABSTRACT

We discuss the role of tearing instabilities in magnetic reconnection. In three dimensions this instability leads to the formation of strong Alfvenic waves that remove plasma efficiently from the reconnection layer. As a result the instability proceeds at high rates while staying close to the linear regime. Our calculations show that for a resistive fluid the reconnection speed scales as \(V_A Re^{-3/10}\), where \(V_A\) is the Alfven velocity and \(Re\) is the magnetic Reynolds number. In the limit of vanishing resistivity, tearing modes proceed at a non-zero rate, driven by the electron inertia term, giving rise to a reconnection speed \(\sim V_A(c/\omega_p L_x)^{3/5}\), where \(\omega_p\) is the plasma frequency and \(L_x\) is the transverse scale of the reconnection layer. Formally this solves the problem of fast reconnection, but in practice this reconnection speed is small.

Subject headings: Magnetic fields; Galaxies: magnetic fields, ISM: molecular clouds, magnetic fields

1. Introduction

Reconnection of magnetic field lines is a problem that has been hotly debated for the last forty years. Its critical importance stems from the fact that understanding the origin and evolution of large scale magnetic fields is impossible without a knowledge of the mobility and reconnection of magnetic fields. Standard dynamo theories employ the concept of turbulent diffusion to circumvent the problems associated with the high conductivity of astrophysical plasmas (see Parker 1979, Moffat 1978, Krause & Radler 1980). Without some sort of enhanced diffusion flux freezing would be an excellent approximation to the motion of the magnetic field in a highly conducting fluid. To change magnetic field topology, to form large scale fields from the small scale loops produced by turbulent motions, one needs to invoke Ohmic diffusion in some way. As this is usually very slow in astrophysical
contexts, the turbulent diffusion paradigm appeals to the notion that whenever field lines are properly intermixed, Ohmic dissipation may be enhanced by introducing a very small magnetic correlation scale.

However, this concept has been widely criticized as ill-founded (e.g. Parker 1992, Zweibel 1998). Strong large scale magnetic fields should prevent magnetic fields of opposite polarity from intermixing by turbulent hydrodynamic motions. Both numerical and analytic studies (see Cattaneo & Vainshtein 1991, Kulcsrud & Anderson 1992) confirm that the traditional (Ruzmaikin et al. 1988) theory of kinematic dynamos is seriously and fundamentally flawed. On the other hand, observations of the solar corona and chromosphere seem to show that reconnection often takes place at speeds of \( \sim 0.1V_A \) (cf. Dere 1996, Innes et al. 1997 and references contained therein). Evidently at least some astrophysical plasmas can undergo reconnection on short time scales.

Unfortunately, current proposed solutions to this problem are not satisfactory. The widely cited work by Parker (1992) assumes that the galactic dynamo depends on reconnection in the galactic halo, where it is driven by cosmic rays. This leaves the problem of reconnection in other astrophysical contexts. Moreover it is far from obvious that reconnection in the galactic halo can provide the basis for an efficient galactic dynamo.

One of us (Vishniac 1995a) has proposed that rapid reconnection of magnetic flux follows from the formation of intense flux tubes in a turbulent plasma. However, this assumes that reconnection is initially rapid enough to allow the formation of such structures in a small number of dynamical time scales and that the plasma has a high \( \beta \) so that the magnetic field can be distributed intermittently. The first assumption may reasonably be regarded as ignoring the question of fast reconnection, since the conditions for flux tube formation are only marginally weaker than the conditions for fast reconnection when the field is not intermittent. Even granting these assumptions, while flux tubes formation certainly takes place in the solar convection zone and may also be relevant to accretion discs (Vishniac 1995b), it does not seem to provide a universal solution (Lazarian & Vishniac 1996). For instance, ambipolar diffusion can infiltrate material into flux tubes and suppress turbulent pumping. Estimates of the reconnection rate of such flux tubes using the Sweet-Parker reconnection process (Sweet 1958, Parker 1957) do not guarantee reconnection in less than an eddy turn over time under these conditions.

In our earlier paper (Vishniac & Lazarian 1998) we studied the role of ambipolar diffusion in reconnection adopting a simple two dimensional Sweet-Parker (Sweet 1958, Parker 1957) geometry. The reconnection speed was shown to be enhanced, but the level of the enhancement was neither sufficient to account for efficient turbulent pumping nor to satisfy the requirements of the galactic dynamo. In addition, this process is not relevant
to reconnection in the solar chromosphere and corona. These considerations motivate our current study of enhanced reconnection.

In this paper we calculate reconnection speeds in the presence of the tearing instability, when the three dimensional structure of the reconnection region is properly accounted for.

The tearing mode instability, which is a particular type of resistive instability, was quantitatively described by Furth, Killeen, and Rosenbluth (1963). It is a generic instability which has been cited in many contexts, and in particular, has often been discussed as a means of explaining reconnection rates associated with solar flaring (e.g. Bulanov, Sakai, & Syrovatskii 1979, Dere 1996, Glukhov 1996).

One of the problems with the customary treatment of the tearing modes is that in a two dimensional treatment, after a short period of linear growth, they enter a nonlinear stage that depends on the still unclear, but probably slow, evolution of magnetic islands formed during the linear stage (see Manheimer & Lashmore-Davis 1984). In this paper we note that the case of island formation is really singular and that in the generic case, where the opposing field lines are not perfectly anti-parallel, magnetic islands do not form. Instead, strong Alfvénic waves are produced, which carry fluid away from the reconnection zone so that the instability starts again for new portions of magnetic flux.

The speed of reconnection is determined by the most rapidly growing tearing modes, which reach the end of their linear growth in the time required to eject magnetic flux in the transverse direction. Slower modes do not have time to develop as the fluid and the flux are carried away. Consequently, the instability just barely reaches the nonlinear stage, and a linear analysis of the problem is adequate for a qualitative analysis.

A peculiar feature of tearing modes is that they persist as fluid resistivity vanishes. This formally solves the problem of “fast” dynamo, i.e. the dynamo in fluid with resistivity tending to zero, but leaves us with the problem of real world astrophysical dynamo as the reconnection rates that we find are small.

In what follows we discuss the physics of the tearing instability (§2), reconnection in resistive fluids (§3), and very highly conducting fluids (§4). A summary of our results is presented in section 5.

2. Tearing modes

The tearing instability arises from the decoupling of magnetic field lines from the fluid. This can be due to non-zero resistivity, electron inertia or electron shear viscosity. If two
opposite magnetic flux regions are brought into contact, the instability forms magnetic ‘islands’, as shown in Fig. 1. In the presence of a shared component of magnetic field perpendicular to the plane of the figure it is easy to see that nonlinear Alfven waves rather than islands are formed. Although the projection of these waves to the plane of Fig. 1 looks like islands, the dynamics of the waves is radically different from that of islands. The latter stagnate as the instability enters its nonlinear regime, while the former efficiently drive fluid away from the reconnection zone as the Alfven waves propagate away from the reconnection zone. We note that the formation of islands also suggests the illusion that isolated loops of magnetic flux can leave the reconnection zone in any direction, ejected by a local pressure excess, whereas the reality is that any such motion would involve radical distortions of the magnetic fields, and can be ruled out on energetic grounds.

The classic study by Furth, Killeen, & Rosenbluth 1963 (hereinafter FKR) concluded that the tearing mode growth rate at low wave numbers is

\[
\gamma = \left( \frac{S}{\alpha} \right)^{\frac{2}{5}} \frac{\eta}{a^2},
\]

where

\[
S \equiv \frac{V_A a}{\eta},
\]

\[
\alpha \equiv k a,
\]

\( \eta \) is the resistivity, \( a \) is the current sheet thickness, and \( k \) is the transverse wavenumber of the tearing mode. This result has been confirmed by all subsequent work. We see that the fastest growing modes are those with the longest transverse wavelength.

There is a controversy, however, on the the minimum wavenumber of the growing modes. FKR conclude that the instability only exists for

\[
S^{-1/4} < \alpha < 1.
\]

Using the fastest growing mode, which also gives the fastest reconnection speed and so can be assumed to dominate transport, we get

\[
\gamma \approx S^{1/2} \frac{\eta}{a^2},
\]

On the other hand, Van Hoven & Cross (1971) (hereinafter VHC) point out that this solution assumes infinite magnetic fields far from the origin. They solve numerically for the minimum transverse wavenumber and the fastest growing modes and suggest another scaling, equivalent to:

\[
\alpha > S^{-3/7},
\]
so that
\[ \gamma \approx S^{4/7} \frac{\eta}{a^2}. \] 

The difference between VHC and FKR can be understood in physical terms. Sharp gradients in the former treatment allow the instability to proceed up to the largest scales. Ultimately, the limiting scale is set by the condition that \( kV_A > \gamma \). Combining this criterion with equation (1) we recover equation (7) for the maximum growth rate. (VHC actually quote an exponent of 0.57. Here we have taken the liberty of replacing that with the numerically indistinguishable, but physically motivated value of \( 4/7 \).) On the other hand, in FKR’s treatment gradients are artificially reduced so that the instability stops at smaller scales. This suggests that the work of VHC is more realistic, as gradients sharpen in the course of reconnection with \( V_{rec} < V_A \). In what follows we will adopt Eq. (7), and show the results of using Eq. (5) in the Appendix. We note that Bulanov et al (1979) proposed simply using \( L_x \), the transverse scale of the reconnection region, as the limiting transverse wavelength. For our purposes this is exactly the same as using the results of VHC.

If the fluid conductivity is high, electron inertia and electron shear viscosity can generate the tearing instability (see Manheimer & Lashmore-Davies 1984). A simple replacement \( \eta \rightarrow (m_e c^2 \gamma)/(n_e e^2) \) should be used to account for electron inertia, while a more elaborate treatment is required for electron shear viscosity.

3. Tearing Reconnection

Imagine the usual situation for Sweet-Parker reconnection. We have two volumes containing magnetic fields with strongly differing directions. To linear order and disregarding viscosity effects, we can ignore the magnetic field component that is shared by both regions, so that the problem reduces to the one considered by FKR, except that the shared field component causes the unstable intermediate layer to shed matter out both ends at a speed \( \sim V_A \) and with a local shear \( \sim V_A/L_x \). Magnetic tension pulls reconnected magnetic field lines with the entrained conducting fluid out of the reconnection zone.

This instability will be suppressed if the transverse shear exceeds \( \gamma \). When the instability exists it mixes together magnetic fields with opposing polarities, thereby increasing the current layer thickness \( a \) (see Fig. 1). Therefore we have a dynamic

\[ \text{The shared component of magnetic field decreases the viscosity of the plasma in the reconnection layer (see Glukhov 1996).} \]
equilibrium when \cite{Bulanov_Sakai_Syrovatskii_1979}:

\[ \frac{V_A}{L_x} \approx \gamma, \]  

(8)

where \( \gamma \) is the maximum growth rate, attained by the longest transverse wavelength modes. The resulting reconnection speed is \( \sim a\gamma \), where we take \( a \) as a characteristic scale of the amplitude of the most rapidly growing tearing perturbations \cite{Bulanov_Sakai_Syrovatskii_1979}. Expression (8) implies that

\[ \frac{a}{L_x} = S^{-3/7}. \]  

(9)

If we define the magnetic Reynolds number as

\[ Re \equiv \frac{L_x V_A}{\eta} = \frac{L_x}{a} S, \]  

(10)

then we obtain

\[ S = Re^{7/10}. \]  

(11)

This gives

\[ a = L_x Re^{-3/10}, \]  

(12)

and the reconnection speed is

\[ V_{rec} = a\gamma = V_A Re^{-3/10}. \]  

(13)

This rate is significantly faster than conventional Sweet-Parker reconnection speeds although still small when \( Re \) is large, as it is in most astrophysical plasmas.

We note, that although \( \gamma \) scales as \( \eta^{3/5} \) the scaling of reconnection rate is different. This is the consequence of the fact that the thickness of current layer is also a function of \( \eta \). Our treatment is only self-consistent if \( k L_x \geq 1 \). Since

\[ ka = S^{-3/7}, \]  

(14)

this is the same as requiring

\[ L_x \geq a S^{3/7} = L_x Re^{-3/10} Re^{3/10} = L_x \]  

(15)

which is always true. We see from this that adopting an upper limit of \( L_x \) for the transverse wavelength is equivalent to using VHC’s result.
4. Reconnection in High Conductivity Plasmas

The tearing instability persists in the limit $\eta \to 0$. In this case the electron inertia term substitutes for the resistivity. This was shown using the Vlasov equation (see Hoh 1966) and confirmed by Cross and Van Hoven (1976) using magnetohydrodynamic theory.

There is some controversy over whether or not the electron inertia term can actually change the topology of magnetic field lines (see Shivamogy 1997). However, this argument seems to be of purely academic interest. The development of the tearing instability in the presence of the electron inertia term results in sharp current gradients and therefore any residual fluid resistivity is sufficient to enable the actual reconnect ion of the field lines.

In other words, if the conductivity is high the term $\omega (c/\omega_p)^2$, where $\omega_p$ is the plasma frequency and $\omega \approx \gamma$, should be substituted instead of $\eta$ in the expression for the growth rate (5):

$$\gamma \approx \frac{V_A}{a} \left( \frac{c}{\omega_p a} \right)^2.$$  \hspace{1cm} (16)

Since $\gamma \approx V_A/L_x$ this implies

$$a \approx L_x^{2/5} \left( \frac{c}{\omega_p} \right)^{3/5}.$$  \hspace{1cm} (17)

As a result the reconnection rate $V_{rec} = a\gamma$ is

$$V_{rec} \approx V_A \left( \frac{c}{L_x \omega_p} \right)^{3/5},$$  \hspace{1cm} (18)

which constitutes the minimal reconnection rate achievable in plasma with $\eta \to 0$.

For typical parameters of the cold interstellar medium

$$\omega_p \approx 10^{3.5} (n_e/0.003 \text{ cm}^{-3})^{1/2} \text{ s}^{-1},$$  \hspace{1cm} (19)

where $n_e$ is electron density. Therefore

$$\frac{c}{L_x \omega_p} \approx 3 \times 10^{-12} (n_e/0.003 \text{ cm}^{-3})^{-1/2} (L_x/1 \text{ pc})^{-1},$$  \hspace{1cm} (20)

and reconnection velocities will be $\sim 10^{-7}$ of the Alfven speed. We see that although $V_{rec}$ does not go to zero with the conductivity, the minimum reconnection speed is slow indeed.

Finally, we note that if tearing modes dominate reconnection, it may be seen that the ratio of the reconnection rates in the collisionless and resistive regimes is

$$\frac{V_{rec,\text{inertia}}}{V_{rec,\text{coll}}} \approx \left[ \frac{V_{A\text{col}}}{L_x} \right]^{3/10},$$  \hspace{1cm} (21)
where $t_{\text{col}}$ is the electron collision time. Naturally, the higher the magnetic field, the more important the electron inertia term becomes. However, the most important point is that the electron inertia term is important only when electrons do not collide in a shearing time $L_x/V_A$. In practice this is equivalent to saying that this term is almost never important in an astrophysical context.

5. Summary and Conclusions

In this paper we have shown that three dimensional tearing modes in resistive fluids lead to reconnection speeds that scale as the magnetic Reynolds number to the minus three tenths power, compared to the scaling $Re^{-1/2}$ in the Sweet-Parker model. In addition, we have found that unlike standard Sweet-Parker reconnection, reconnection involving tearing modes persists as the fluid resistivity tends to zero while electron inertia drives tearing modes. This formally solves the problem of “fast” dynamo, but is of marginal assistance to actual astrophysical dynamos as the tearing reconnection is not fast.

The treatment so far ignores viscosity effects. Electron viscosity can initiate tearing modes on its own. The corresponding term in Ohm’s law is proportional to the second derivative of the current. In the case of tearing modes driven by electron inertia this term may become important when sharp current gradients form. The energy deposited is dissipated by viscous dissipation.

In a recent study Glukhov (1996) has shown that viscosity plays an important role when the current sheet is neutral, i.e. there is no magnetic field in the direction of current. We believe that such situations are rather singular and, in general, there will be a component of magnetic field along current. Moreover, a neutral current sheet is likely to be subjected to a strong interchange instability and therefore its response to slower tearing instability is only of academic interest.

In the present paper we do not treat the highly controversial case of “forced reconnection” involving tearing modes (see Hassam 1992) and follow the line of reasoning adopted in FKR and later studies (e.g. Kulsrud & Hahm 1982).

Finally, we note that our conclusions differ quite dramatically from those of Strauss (1988), who claimed that tearing modes would lead to fast reconnection, i.e. $V_{\text{rec}} \sim V_A$ on the basis of a weakly nonlinear analysis of tearing mode interactions. However, there is less to this disagreement than would appear at first glance. We agree that tearing modes will grow to the point of marginal nonlinearity. We do not agree that this implies fast reconnection. Instead, global constraints play a dominant role in setting $V_{\text{rec}}$, as they do in
the usual Sweet-Parker argument without tearing modes.

We conclude that by themselves tearing modes do not look like a panacea for the problems of the astrophysical dynamo. Indeed, although the enhancement of reconnection speeds is substantial in numerical terms, the consequent reconnection rates are not sufficient to support contemporary dynamo theories. Reconnection in the presence of the tearing modes is still too slow.

Is this a crisis? Probably not. Our present model is still too simple to treat realistic reconnection geometries. In our next paper we will show that the reconnection speeds are substantially enhanced in the presence of MHD turbulence.

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A. FKR Treatment

FKR assume that the magnetic field strength increases linearly with distance to the neutral line, while a more realistic structure of the background magnetic field is discussed by VHC. In the text we use the latter result. What is more important to our analysis is that tearing modes happen with very similar growth rates in quite different magnetic configurations. Therefore our results are robust. Below we present the reconnection speeds based on the work of FKR.

In this case instead of Eq. (8) we get

$$\frac{V_A}{L_x} \approx \frac{S^{1/2} \eta}{a^2},$$

which means that

$$\frac{a}{L_x} \approx S^{-1/2}.$$  \hspace{1cm} (A2)

Therefore $Re = (L_x/a)S \approx S^{3/2}$, which means that

$$a \approx L_x Re^{-1/3}$$  \hspace{1cm} (A3)

and

$$V_{rec} \approx V_A Re^{-1/3},$$  \hspace{1cm} (A4)
which should be compared with Eq. (13).

In the limit of negligible resistivity

$$\gamma \approx \frac{V_A L_x}{a^2} \left( \frac{c}{\omega_p a} \right)^2,$$

and by equating $\gamma$ to $V_A / L_x$ we get

$$a \approx \frac{L_x^{3/5}}{\left(\frac{c}{\omega_p}\right)^{2/5}}.$$  \hspace{1cm} (A6)

Finally,

$$V_{rec} \approx V_A \left( \frac{c}{L_x \omega_p} \right)^{2/3},$$

which differs insubstantially from our estimate (13).

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Fig. 1.— A schematic of a reconnection region.