The formation and decay of superheavy nuclei produced in $^{48}Ca$-induced reactions

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Abstract

The formation of superheavy nuclei in $^{48}Ca+^{232}Th$, $^{238}U$, $^{242,244}Pu$ and $^{248}Cm$ reactions and their subsequent decay are studied within the quantum mechanical fragmentation theory (QMFT) and the QMFT based preformed cluster-decay model (PCM) of Gupta and collaborators. According to QMFT, all these $^{48}Ca$-induced reactions are cold fusion reactions with relative excitation energies larger than for the $Pb$-induced cold fusion reactions and smaller than for the lighter beam i.e. $Mg$, $Si$ or $S$-induced hot fusion reactions. The same reactions were first suggested by Gupta et al. in 1977 on the basis of QMFT, and this study re-establishes the same result. In fact, for such heavy isotopes of $Z=110$ to 116, $^{50}Ca$ is shown to be a better beam for cold fusion, but $^{50}Ca$ is a radioactive nucleus. The $\alpha$-decay half-lives of these nuclei after 3n and/or 4n evaporations, i.e. of the evaporation residues of these compound systems, calculated on PCM compare reasonably well with experiments published by Dubna group and another recent calculation. As expected for such rare decays, PCM calculations show that the $\alpha$-preformation factors are small, $\sim 10^{-8}$ to $10^{-10}$. The possible competition of $\alpha$-decays with heavy cluster emissions from these superheavy nuclei is also probed from the point of view of searching for new nuclear structure information and possible future experiments with such exotic nuclei. The decay half-lives for some clusters are in fact shown to be lower than the limits of experiments for nuclei with enough available atoms.
1 Introduction

The formation of new and superheavy elements through the use of highly neutron rich beam of $^{48}$Ca on neutron rich actinide targets, such as $^{244}$Pu, $^{248}$Cm and $^{252}$Cf, was first suggested by Flerov [1] and was used at his JINR Dubna U-300 heavy-ion cyclotron for the successful synthesis of $^{252}$No in a cold (17-18 MeV excitation) 2n emission reaction with $^{206}$Pb target [2]. However, the same could not be continued to heavier elements due to the poor sensitivity level of the equipment at that time [3]. Theoretically, Nix [4] later suggested the use of very asymmetric target-projectile combinations, such as $^{250}$Cm and $^{257}$Fm with $^{48}$Ca, on the basis of the kinetic energy calculations made for different nuclear shape configurations by using an idealised liquid drop model. In a more complete theory, called quantum mechanical fragmentation theory (QMFT) [5, 6, 7, 8, 9, 10, 11, 12], based on two centre shell model and Strutinsky renormalization procedure, Gupta, Sândulescu and Greiner [11] also suggested the use of $^{48}$Ca beam, on the minimum energy considerations, with even-even $^{196-204}$Hg, $^{204-208}$Pb, $^{232}$Th, $^{234-238}$U, $^{240-244}$Pu and $^{244-248}$Cm targets for synthesizing the various isotopes of even Z=100, 102, and 110-116 (only even Z nuclei were studied). The calculations also showed that for the heavier mass isotopes at the minimum in energy either the projectile is not $^{48}$Ca or the corresponding target nucleus is not stable in nature. On the other hand for more neutron-deficient isotopes, the potential energy surfaces tend to become flat and $^{48}$Ca is no more a favoured nucleus since other new minima start to develop. Furthermore, for Z=104-108, the targets to be used with $^{48}$Ca projectile are the unstable, very difficult to handle Po, Rn and Ra nuclei. The key consideration behind the QMFT is the shell closure effects of one or both the reaction partners and hence of ”cold reaction valleys” or the reaction valleys leading to cold fusion. Four or five such cold reaction valleys were always found to exist, $^{48}$Ca being one of them. Their relative excitations would mean the cases of ”cold”, ”warm/tepid” and ”hot” fusion reactions. In other words, all cases of potential energy minima are in fact of ”cold” reactions with different relative excitation energies, and the real ”hot” reactions are the ones coming from outside the potential energy minima.

More recently, an intense $^{48}$Ca beam at low consumption of material in the ion source was developed [13] and used with the upgraded Dubna cyclotron U-400 and new recoil mass separators (VASSILISSA or GNS) for the formation of
superheavy elements 110 to 116 [14, 15, 16, 17, 18, 19]. The targets used are $^{232}Th$, $^{238}U$, $^{242,244}Pu$ and $^{248}Cm$, and, for near the Coulomb barrier energies, the excitation energy $E^* \sim 30-35$ MeV, in between the one for cold ($E^* \sim 10-20$ MeV) and hot ($E^* \sim 40-50$ MeV) fusion reactions. Thus, the $^{48}Ca$-induced reactions are referred to as ’’warm/ tepid” fusion reactions, in between the ’’cold” and ”hot” fusion reactions. For some $^{48}Ca$-induced reactions, $E^* \sim 40$ MeV also. Thus, the resulting compound systems de-excite by 3n and/ or 4n evaporation, compared to 1n and 2n in cold and 5n in hot fusion reactions. The cold fusion reactions are based on $Pb$ and $Bi$ targets with $Ti$ to $Zn$ beams, and the hot fusion reactions use heavy actinide targets from $Th$ to $Cf$ with very light $C$ to $S$ projectiles. In all the ”warm” fusion reactions studied here, the final nuclei formed after the evaporation of neutrons (and $\gamma$-ray emission), the evaporation residues EVRs, are relatively long-lived nuclei and decay only via $\alpha$-particles, ending in spontaneous fission (SF) of the last nucleus at the edge of the stability region. This is the complete fusion reaction channel, giving the $\alpha$-genetically related nuclei, called $\alpha$-decay chain. Another decay channel for the excited compound system is that of fission-like decays: the fusion-fission, quasi-fission, etc., which has also been of interest both experimentally [20, 21, 22, 23] and theoretically [24, 25, 26]. It may be relevant to mention here that the fusion-fission process in hot superheavy nuclei could also be treated as a dynamical cluster decay process [27] advanced recently for light and medium mass hot compound systems.

In this paper, we first re-investigate the use of the $^{48}Ca$ beam for the synthesis of superheavy elements within the dynamical fragmentation theory via cold fusion reactions. This has become essential because the binding energy data has improved considerably since its last use [11] by some of us in 1977. Specifically, we consider the compound systems $^{280}110^*$, $^{286}112^*$, $^{290,292}114^*$ and $^{296}116^*$, formed in reactions of $^{48}Ca$ beam on $^{232}Th$, $^{238}U$, $^{242,244}Pu$ and $^{248}Cm$ targets at various incident energies. As already stated above, the excitation energy $E^* = 32-35$ MeV in these reactions and hence they all de-excite by 3n and/ or 4n evaporation, to give $^{277}110$, $^{283}112$, $^{287,288,289}114$ and $^{292}116$ parents. Then, as a second aim of this paper, we investigate the observed $\alpha$-decay characteristics of these nuclei within the preformed cluster decay model (PCM) of Gupta and Collaborators [28, 29, 30, 31, 32, 33] which is also based on the dynamical fragmentation theory. Furthermore, a possible branching of $\alpha$-decay to some (theoretically) most probable heavy cluster decays is also studied with a view to look for some new or known magic daughters.
in this exotic process of, so-called, cluster radioactivity.

The paper is organised as follows. A brief description of the fragmentation theory and PCM is given in section 2. The results of our calculations, compared with Dubna experiments and other recent works, are presented in section 3. A summary of our results is added in section 4.

2 The Theory

2.1 Quantum mechanical fragmentation theory

The quantum mechanical fragmentation theory (QMFT) is a dynamical theory of heavy-ion collisions, specifically the three cold processes of fusion, fission and cluster radioactivity, worked out in terms of the coordinates of mass (and charge) asymmetry \( \eta = (A_1 - A_2)/A \) (and \( \eta_Z = (Z_1 - Z_2)/Z \)), the relative separation distance \( R \), the deformations \( \beta_1 \) and \( \beta_2 \) of two nuclei (or, in general, fragments), and the neck parameter \( \epsilon \). Taking motions in \( \eta \) and \( \eta_Z \) as weakly coupled, the time-dependent Schrödinger equation in \( \eta \),

\[
H \Psi(\eta, t) = i\hbar \frac{\partial}{\partial t} \Psi(\eta, t),
\]

is solved for \( R(t) \) treated classically and the other coordinates \( \beta_1, \beta_2 \) and \( \epsilon \) fixed by minimizing the collective potential in these coordinates [34, 35]. We find that for a given compound system (formed by different target + projectile combinations), a few nucleon to a large mass transfer occurs for the target + projectile combinations coming from outside the potential energy minima (the real ”hot” combinations), whereas the same is zero (no transfer at all) for the target + projectile combinations referring to potential energy minima (the ”cold” combinations); see Fig. 14 in [35]. This means that for cold reaction partners, the two nuclei stick together and form a deformed compound system, as is illustrated in Fig. 2 of [9]. On the other hand, a few nucleon transfer may occur if only a ”conditional” saddle is formed (see Figs. 1 and 2 in [36]). Eq. (1) is solved for only a small number of heavy systems, since its solution is very much computer-time consuming. However, certain simplifications, discussed in the following, have resulted from actual calculations performed over the years which seem to work rather nicely.

The dynamical QMFT establishes the best target + projectile combination, with the variation of relative separation coordinate \( R \) allowing to pass continuously
from a separated pair of nuclei to a cool compound nucleus [8, 9] or any other process like fission or deep inelastic collision, etc., [25, 26]. However, the potentials V(R, η) and V(R, ηZ), calculated within the Strutinsky renormalization procedure (V=V_{LDM} + δU) by using the appropriate liquid drop model for V_{LDM} [37] and the asymmetric two-center shell model (ATCSM) of Maruhn and Greiner [38] for the shell effects δU, show that the motions in both η and ηZ are much faster than the R-motion. This means that both the potentials V(R, η) and V(R, ηZ) are nearly independent of the R-coordinate (see e.g. Fig. 1 in [8] or [24]) and hence R can be taken as a time-independent parameter. Then, the time-dependent Schrödinger equation (1) in η reduces to the stationary Schrödinger equation in η,

\begin{equation}
\left\{-\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} + V_R(\eta)\right\} \Psi_{R}^{(\nu)}(\eta) = E_{R}^{(\nu)} \Psi_{R}^{(\nu)}(\eta),
\end{equation}

where R is fixed at the post-saddle point, justified by many good fits to both fission and heavy-ion collision data [39] and by an explicit, analytical solution of time-dependent Schrödinger equation in ηZ coordinate [40]. An interesting result of these calculations is that the mass (and charge) distribution yields,

\begin{equation}
Y(A_i) = |\psi_R(\eta(A_i))|^2 \sqrt{B_{\eta\eta}\frac{2}{A}}, \quad (i=1,2)
\end{equation}

(and Y(ηZ)) are nearly insensitive to the detailed structure of the cranking masses B_{ηη}, calculated consistently by using ATCSM. The nuclear temperature effects are included here through a Boltzmann-like function

\begin{equation}
|\psi_R|^2 = \sum_{\nu=0}^{\infty} |\psi_R^{(\nu)}|^2 \exp\left(-\frac{E_{R}^{(\nu)}}{T}\right),
\end{equation}

with the temperature T (in MeV) defined, via the compound nucleus excitation energy, as

\begin{equation}
E^* = E_{cm} + Q_{in} = \frac{A}{9}T^2 - T.
\end{equation}

Q_{in} is the Q-value for reaction partners. Also, the shell corrections δU are T-dependent, but this could not be included here since we are using the experimental binding energies where δU are there in them but could not be separated out in a model independent way. However, the T-values involved here are small, ∼1 MeV only.

Thus, the static potentials V(η) (and V(ηZ)) contain all the important information of a colliding or fissioning system. Furthermore, since these potentials are
nearly independent of the choice of R-value (as discussed above), they are calculated at some critical distance \( R = C_t = C_1 + C_2 \) where the two nuclei come in close contact with each other. This means fixing the neck parameter \( \epsilon = 1 \) and the potential \( V(\eta, \eta Z) \) given simply as the sum of the ground state binding energies \( B_i \) of two nuclei, the Coulomb interaction \( E_c \) between them plus the additional attraction due to nuclear proximity \( V_P \) [41],

\[
V(C_t, \eta, \eta Z) = -\sum_{i=1}^{2} B(A_i, Z_i, \beta_i) + \frac{Z_1 Z_2 e^2}{C_t} + V_P. \tag{6}
\]

The \( C_i \), defining \( C_t \), are the Süssmann central radii \( C_i = R_i - (1/R_i) \), with the radii \( R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{1/3} \text{ fm} \). The binding energies \( B_i \) are from the 1995 experimental compilation of Audi and Wapstra [42] and from 1995 calculations of Möller et al. [43] whenever not available in [42]. The charges \( Z_1 \) and \( Z_2 \) are determined by minimizing the potential in \( \eta Z \) coordinate, which automatically minimizes the \( \beta_i \) coordinates. Note that the minimized \( \beta_i \)'s are not always for the spherical nuclei since the total binding energy \( B_1 + B_2 \) of the reaction partners is minimized and not their individual \( B_1 \) or \( B_2 \). Also, it may be pointed out that \( V_P \), the additional attraction between nuclear surfaces, was not added in our earlier calculations, whose effect is to change the relative heights of the potential energy minima, and hence the relative excitations, but not their positions [36]. The positions of the minima are due to shell effects only. Also, the role of deformation in both \( E_c \) and \( V_P \) is neglected here because their combined effect is shown [44] to lower the interaction barriers but not the relative formation yields. In other words, Eq. (6), without \( V_P \), formed the basis of our first calculation on "cold fusion" reaction valleys [8, 10, 11, 12], which was later optimized by adding the requirements of smallest interaction barrier, largest interaction radius and non-necked (no saddle) nuclear shapes [9]. Like necked-in shapes are known [6, 7] to witness the preformation of fission fragments, non-necked shapes are the signatures of cold fusion of two nuclei [9]. The contribution due to \( V_P \) is, however, added in the present calculations.

### 2.2 Preformed cluster-decay model

The preformed cluster-decay model (PCM) is based on the QMFT and hence uses the same coordinates as are introduced above. In a PCM, the decay constant \( \lambda \),
or the decay half-life $T_{1/2}$, is defined as

$$\lambda = \frac{\ln 2}{T_{1/2}} = P_0 \nu_0 P,$$

(7)

with $P_0$ as the cluster (and daughter) preformation probability in the ground state of nucleus and $P$ the barrier penetrability which refer, respectively, to the $\eta$ and $R$ motions. The $\nu_0$ is the barrier assault frequency. In principle, the coordinates $\eta$ and $R$ are coupled, but Eq. (7) is always written in a decoupled approximation, which further justifies our simplifying conditions of the last subsection.

The $P_0$ are the solutions (3) of the stationary Schrödinger equation (2) in $\eta$ for the ground-state $\nu = 0$, i.e. $P_0 = Y^0(\eta)$. The mass parameters $B_{\eta \eta}(\eta)$ are the classical hydrodynamical masses of Kröger and Scheid [45] since the two masses (cranking and hydrodynamical) are shown to give similar results for heavier nuclei.

The $P$ is the WKB tunnelling probability, calculated for the tunnelling path shown in Fig. 1, as $P = P_i P_b$ with

$$P_i = \exp\left[-\frac{2}{\hbar} \int_{R_a}^{R_i} \left\{2\mu[V(R) - V(R_i)]\right\}^{1/2} dR\right],$$

(8)

$$P_b = \exp\left[-\frac{2}{\hbar} \int_{R_i}^{R_b} \left\{2\mu[V(R) - Q]\right\}^{1/2} dR\right],$$

(9)

with the first turning point $R_a = C_t$ and the second turning point $R_b$ defined by $V(R_b) = Q$-value for the ground-state ($\alpha$ or cluster) decay. These integrals are solved analytically [29]. This choice of $R_a = C_t (= C_1 + C_2)$, instead of $R = R_0$, the compound nucleus radius, assimilates to a good extent the effects of both the deformations of two fragments and neck formation between them [44]. In other words, the two-centre nuclear shape is simulated here through a neck-length parameter which for actinides is nearly zero [44]. We have taken it to be the same for superheavy nuclei. The role of deformation in $V(R)$ is to lower the interaction barriers [44], which is achieved here by raising $R_a$ from $R_0$ to $C_t$.

The assault frequency $\nu_0$ in (7) is given simply as

$$\nu_0 = \frac{(2E_2/\mu)^{1/2}}{R_0},$$

(10)

with $E_2 = (A_1/A)Q$, the kinetic energy of the lighter fragment, for the $Q$-value shared between the two products as inverse of their masses. $\mu$ is the reduced mass here. Eq. (10) usually results in $\nu_0 \approx 2.7 \times 10^{21} \text{s}^{-1}$, whereas the more often used
value in literature is higher for even parents and lower for odd parents [32]. We choose to use here $\nu_0 = 2.7 \times 10^{21}\text{s}^{-1}$ for odd parents and $\nu_0 = 2.7 \times 10^{23}\text{s}^{-1}$ for even parents.

3 Calculations

The superheavy nucleus formation process: Figure 2 shows our calculated fragmentation potentials $V(\eta)$ at a fixed $R = C_t$-value, based on Eq. (6), for some excited compound systems $^{296}116^*$, $^{292}114^*$, $^{286}112^*$ and $^{280}110^*$ formed via $^{48}Ca$-induced reactions. We find in Fig. 2 that in each case there is a minimum corresponding to $^{50}Ca$ nucleus (the $^{48}Ca$ minimum lies in its immediate neighbourhood but at somewhat higher energy), which is in addition to other minima at symmetric (or nearly symmetric), the $^{208}Pb$ and $^{86}Kr$ or their neighbouring nuclei, respectively, with $Z = 80 \pm 2$ and $36 \pm 2$ (the $Pb$ and $Kr$ minima merge into one minimum for $Z \geq 116$ elements) and the ones corresponding to the super-asymmetric combinations using heavier actinides with much lighter beams. The $^{48}Ca$ minimum is deepest only for the lighter isotopes of these nuclei, e.g. for $^{290}116$, $^{284}114$, $^{278}112$ and $^{270}110$ (see Fig. 1 and Table 1 in [46]). Apparently, all these minima correspond to at least one closed shell nucleus and hence to ”cold” reaction valleys. Also, the relative excitation energies are different, the $^{48}Ca$ minimum always lying higher than the $^{208}Pb$ (and/or $^{86}Kr$) and the symmetric (or near symmetric) minima, but deeper than the ones corresponding to many heavier actinides with lighter beams of $Mg$, $Si$, $S$, etc.,. The excitation energy can be determined by the hight of the minimum with respect to the ground state of the compound system. Knowing that temperature effects are not added here in Fig. 2, the approximate excitation energy $E^* \sim 30\text{ MeV}$ for $Ca$ minima, to be compared with $E^* \sim 10\text{ MeV}$ for $Pb$ minima and $\sim 40\text{ MeV}$ for heavy actinides. For lighter elements ($Z=102$ and 104), the $^{48}Ca$ minimum is deeper than the $Pb$ and $Kr$ minima, and its relative depth is only 15 to 20 MeV [11, 46]. This means to say that $Ca$-induced reactions for heavy elements ($Z > 106$) give rise to ”cold” compound systems with an ”intermediate” amount of excitation energy, and hence the name ”warm or tepid” with respect to ”cold and hot” fusion. The number of neutrons emitted in $^{48}Ca$-induced reactions (3n or 4n) also lie in between the cold (1n or 2n) and hot (5n) fusion reactions. In other words, according to the QMFT, the $^{48}Ca$ beams are as good as the $^{208}Pb$ beams (or the lighter ones like $Mg$, $Si$ and $S$) for forming
cold compound nuclei with relatively higher (or lower) excitation energies. The calculated interaction barriers and nuclear shapes also support that Ca-induced reactions are as good as the Pb-induced ones [9, 26, 46]. Finally, it may be relevant to note that the $^{50}$Ca nucleus, preferred for cold compound nucleus formation in heavier isotopes of superheavy nuclei studied here, is radioactive whereas the $^{48}$Ca is not a radioactive nucleus. The predicted use of radioactive nuclei (as a beam and/or a target) for heavier isotopes of superheavy nuclei is a general result of the QMFT [46].

We have also calculated the fragment mass distribution yields $Y(\eta)$, using Eq. (3), taking the view that, since fragments related to the minimum in the potential $V(\eta)$ are more probable, the yields $Y(\eta)$ must give the intermediate (two) fragment formation yields or, in short, the formation yields for a cool compound nucleus [47], where the contribution of barrier penetration is not included. Fig. 3 illustrates our calculation for two compound systems $^{296}_{116}^*$ and $^{292}_{114}^*$. For the mass parameters, we have used the classical hydrodynamical masses [45]. Excluding the cases of reactions involving symmetric combinations, in view of their forming necked-in shapes, the formation yields for $^{48}$Ca-induced (here $^{50}$Ca) cold fusion reactions are apparently larger than for the light mass beams but smaller than for Pb-induced reactions. The contribution of penetrability $P$ would, however, quantify these results.

The $\alpha$-decay process in superheavy nuclei: Table 1 and Fig. 4 show the results of our calculation for $\alpha$-decay half-lives, in terms of $\log_{10}T_{1/2}^{\alpha}$ (s), for the measured complete decay chains of $^{277}_{110}$, $^{287,288,289}_{114}$ and $^{292}_{116}$ parents (no $\alpha$-decay of $^{283}_{112}$ parent is observed), compared with the experimental data published by Dubna group and a very recent calculation of Royer and Gherghescu [48, 49] based on the generalized liquid drop model (GLDM). The Dubna data is for $\alpha$-particle energies $E_\alpha$ and decay times $\tau$, which agree with $Q_\alpha$-values (within less than 150KeV) and $\alpha$-decay half-lives $T_{1/2}^{\alpha}$ (within less than one order of magnitude), respectively, via the well known formula of Viola and Seaborg [50] whose parameters are fitted to measured $Q_\alpha$ and $T_{1/2}^{\alpha}$ values of 58 even-even $Z > 82$, $N > 126$ nuclei [51]. We have, therefore, used this data to compare with the calculated $Q_\alpha$ and $T_{1/2}^{\alpha}$-values. The resulting differences are of 0.2-0.4 atomic mass numbers (compare the resulting 114.4$_{+0.5}^{+1.6}$ and 110.2$_{-0.8}^{+1.5}$ [14], respectively, with $A=114$ and 110). Wherever more than one chain is observed, we present the data for all chains. Table 1 also lists the respective $Q_\alpha$-values and the preformation factor $P_0$.
and penetrability $P$ calculated on PCM. The PCM calculated $Q$-values ($Q_{\alpha}^{cal}$) are based on experimental [42] and Möller et al. [43] binding energies, whereas that of GLDM on Thomas-Fermi (TF) model [52]. The $Q_{\alpha}^{exp}$ are taken to be the experimental $E_{\alpha}$-values measured in respective $\alpha$-decay chains. In order to illustrate the role of $Q$-value, we have also calculated the decay half-lives within PCM using the experimental $Q_{\alpha}^{exp}$-values, and compared with the results of our calculations using $Q_{\alpha}^{cal}$.

We first notice in Table 1 that a decrease in $Q$-value by about 0.5 MeV increases the $T_{1/2}$-value by about one order. This is also illustrated in Fig. 4 for $^{277}110$ and $^{288,289}114$ decay chains. Apparently, in one case ($Z=114$ decay chains) use of experimental $Q$-values improve the comparisons, whereas in the other case ($^{277}110$ decay chain) this spoils it. In general, the comparisons of $T_{1/2}$-values for the two models with experiments are within experimental errors, i.e. within less than two orders of magnitude. In some isolated cases, like for $^{269}106$ nucleus in $^{277}110$ decay chain, the trends of experiments are different from calculations and hence need further checking. In total, the data obtained do not show any strange behaviour and hence do not call for any special criticism [53].

The interesting result to note in Fig. 4 is that both the GLDM and PCM calculations give similar trends, which is due to their used $Q$-values. This is illustrated in Fig. 5, where the calculated $Q$-values on two models are compared with experiments. The experimental $E_{\alpha}$-values (taken as $Q_{\alpha}^{exp}$-values) are in general higher and hence are the cause of one to two orders of magnitude deviations between the calculated $T_{1/2}$ and measured $\tau$-values.

Finally, in Table 1, we have a look at $P_0$ and $P$, calculated on PCM. The $P_0$ are of the order of $10^{-8}$ to $10^{-10}$ which means that all these $\alpha$-decays are rare in nature, as is actually the case. In GLDM, $P_0 = 1$, which means that the $P$ calculated on PCM should be higher, due to either the higher $Q$-values or lower barriers. However, Fig. 5 shows that the $Q$-values used in PCM are lower than those used in GLDM. Also, it may be noted that $\nu_0 = 10^{20} s^{-1}$ in GLDM, an order of one and three lower from that used in PCM, respectively, for odd and even parents. All these factors have a combined effect on the nature of predictions made by a particular model.

**The heavy cluster-decay process in superheavy nuclei:** Figure 6 shows the results of our calculations for some heavy cluster decays whose preformation factors $P_0$ are the largest and hence refer to the minima in fragmentation potentials $V(\eta)$,
for each of the parents in the $\alpha$-decay chain of, say, $^{277}110$ nucleus. The heaviest cluster included here in the calculations is with $Z=20$ ($^{49-51}Ca$) because an earlier calculation on PCM shows that the two processes of cold fission and cluster decay become indistinguishable for clusters heavier than of mass $A_2 \approx 48$ [31]. The results of other decay chains are not presented hence since this study at present is more of an academic interest than for comparisons with experiments.

First of all we notice in Fig. 6 that, almost independent of the parent mass, the $Q$-value increases as the size of cluster increases. On the other hand, $P$ and $P_0$ are further smaller than for $\alpha$-decay, but present an interesting result. We notice that though $^{10}Be$ is best preformed in all parents, its $P$-values are very small. Such a result is important since $T_{1/2}$ is a combined effect of both $P_0$ and $P$ ($\nu_0$ being constant). For $^{10}Be$, the $T_{1/2}$ is much larger than for other clusters such that the studied parents could be said stable against $^{10}Be$ decay. In other words, in addition to $\alpha$-decay and fission, $^{14}C$, $^{34}Si$ and/or $Ca$ clusters present the best possible cases of cluster decays for any of the parents of $^{277}110$ $\alpha$-decay chain. There seems to be no new shell stabilizing effects in the considered decays, except for $^{10}Be$ decays of $^{269}106$ or $^{273}108$ nuclei, possibly due to the known deformed magicity of these superheavy nuclei [54]. Interesting enough, the calculated half-lives for $^{49-51}Ca$ decays lie far below the present limits of experiments, which go upto $\sim 10^{28}$ s [32] for nuclei where enough atoms are available. The closed shell effects of a cluster, however, are not yet observed in the exotic cluster radioactivity studies.

4 Summary of our results

We have re-investigated the problem of synthesizing superheavy elements via the use of $^{48}Ca$ beam, within the quantum mechanical fragmentation theory (QMFT) that was first used by Gupta, Sândulescu and Greiner [11] in 1977. The most probable, specific reaction partners were suggested and some of these reactions with $^{48}Ca$ beam on targets of $^{232}Th$, $^{238}U$, $^{242,244}Pu$ and $^{248}Cm$ nuclei are recently used successfully at the JINR Dubna cyclotron U-400. According to the QMFT, all these $^{48}Ca$-induced reactions, resulting in different excited compound systems, are cold fusion reactions with an ”intermediate” amount of excitation energy, compared to other cold fusion reactions based on $Pb$ beams or light nuclei beams such as $Mg$, $Si$, and $S$. The main result of the QMFT is that these three types of reactions with different excitation energies refer to three different minima in a
potential energy surface of the same excited compound system having different depths, and are due to the shell effects of one or both the reaction partners. The re-calculated potential energy surfaces using recent binding energy data re-ensure the 1977 result. In fact, the QMFT favours the use of $^{50}\text{Ca}$ beam for these heavy isotopes and $^{48}\text{Ca}$ for the lighter isotopes like $^{276}\text{110}, ^{278}\text{112}, ^{284}\text{114}$ and $^{290}\text{116}$. However, $^{50}\text{Ca}$ is a radioactive nucleus and hence at present difficult to handle it as a beam. Though the experimental physics of radioactive nuclear beams is quite different, and at present difficult, the QMFT treat them at par with stable nuclei.

We have also studied the $\alpha$-decay chains of the superheavy nuclei $^{277}\text{110}, ^{287,288,289}\text{114}$ and $^{292}\text{116}$, obtained in ground-states after neutron evaporations from the above mentioned excited compound systems. The model used is the preformed cluster-decay model (PCM) of Gupta, also based on the QMFT. The PCM calculations are then compared with Dubna experimental data and another recent calculation. Both the calculations predict similar trends for $\alpha$-decay half-lives which is due to their used Q-values. The experimental $E_\alpha$-values (taken as $Q_\alpha$-values) are in general higher and show different trends for atleast some cases and are the cause for the missing comparisons between calculations and data by about two orders of magnitude. An assuring result of PCM is that $\alpha$-particle preformation factors $P_0$ are small, $\sim 10^{-8}$ to $10^{-10}$, required for such rare decays. The other calculation uses $P_0 = 1$.

Finally, the branching of $\alpha$-decays to other (theoretically possible) cluster decays are also studied for the $^{277}\text{110}$ nucleus. Interesting enough, some clusters like $^{49–51}\text{Ca}$ have their predicted decay half-lives within the present experimental limits for nuclei with enough available atoms. Note that here, the not yet observed, shell effects of clusters are important. Also, the shell stabilizing effects of deformed and weakly deformed $Z=108$ and 106 nuclei are present in these calculations. This means that at present the study of cluster decay of superheavy nuclei is mainly of theoretical interest to look for new or already present nuclear structure information and perhaps for also pointing out some future possibilities with exotic superheavy nuclei.

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Table 1: The α-decay half-lives \( \log_{10} T_{\frac{1}{2}}^{\alpha} (s) \) and other characteristic quantities for ground-state decays of superheavy nuclei with \( Z=110-116 \) calculated on PCM, and compared with Dubna data (the α-particle energies \( E_\alpha \) and decay times \( \tau \), taken as \( Q_{\alpha}^{\text{expt.}} \) and \( T_{\frac{1}{2}}^{\alpha} \), respectively) and GLDM calculations. For PCM, the calculations are made at \( R_\alpha = C_1 + C_2 = C_t \), using binding energies from Audi-Wapstra and Möller et al. An increase or decrease in \( R_\alpha \)-value does not improve the PCM results.

| Parent | PCM | GLDM | Experiments |
|--------|-----|------|-------------|
|        | case | \( Q_{\alpha}^{\text{expt.}} \) | \( Q_\alpha \) | \( \log_{10} T_{\frac{1}{2}}^{\alpha} \) | \( E_\alpha \) | \( \log_{10} \tau \) | chain no. |
|        | \( (\text{MeV}) \) | \( (s) \) | \( (\text{MeV}) \) | \( (s) \) | \( (\text{MeV}) \) | \( (s) \) |             |
| 277110 | cal. | 10.696 | 5.96\times 10^{-9} | 1.69\times 10^{-14} | 0.406 | 10.89 | -4.13 | 1 |
|        | expt. | 10.310 | 5.96\times 10^{-9} | 3.69\times 10^{-15} | 1.067 | 10.31 | 0.298 | 1 |
| 273108 | cal. | 9.426 | 1.14\times 10^{-9} | 1.71\times 10^{-16} | 3.121 | 9.61 | -1.36 | escape 2.581 |
| 269106 | cal. | 7.726 | 1.53\times 10^{-10} | 1.93\times 10^{-20} | 7.939 | 7.93 | 3.59 | 3.828 |
|        | expt. | 7.46 | 1.53\times 10^{-10} | 2.56\times 10^{-21} | 8.816 | 8.81 | 3.828 | 3.828 |
| 287114 | cal. | 9.306 | 3.35\times 10^{-10} | 3.80\times 10^{-18} | 5.304 | 9.39 | 1.16 | 9.81 | 0.384 |
|        | expt. | 9.80 | 3.29\times 10^{-9} | 5.74\times 10^{-17} | 1.133 | 9.81 | -0.509 | 1 |
| 288114 | cal. | 9.166 | 3.29\times 10^{-9} | 1.74\times 10^{-14} | 2.652 | 9.39 | 2.13 | 9.81 | 1.013 |
|        | expt. | 9.80 | 3.29\times 10^{-9} | 5.74\times 10^{-17} | 1.133 | 9.81 | 1.013 | 1 |
| 284112 | cal. | 8.696 | 2.49\times 10^{-9} | 3.24\times 10^{-19} | 3.503 | 9.81 | 1.255 | 1 |
|        | expt. | 9.80 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 1.255 | 1 |
| 289114 | cal. | 8.546 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 2.16 | 9.81 | 1.483 |
|        | expt. | 8.546 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 2.16 | 9.81 | 1.483 |
| 285112 | cal. | 8.546 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 2.16 | 9.81 | 1.483 |
|        | expt. | 8.546 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 2.16 | 9.81 | 1.483 |
| 281110 | cal. | 8.546 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 2.16 | 9.81 | 1.483 |
|        | expt. | 8.546 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 2.16 | 9.81 | 1.483 |
| 282116 | cal. | 8.546 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 2.16 | 9.81 | 1.483 |
|        | expt. | 8.546 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 2.16 | 9.81 | 1.483 |
| 284112 | cal. | 8.546 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 2.16 | 9.81 | 1.483 |
|        | expt. | 8.546 | 2.49\times 10^{-9} | 4.53\times 10^{-18} | 2.358 | 9.81 | 2.16 | 9.81 | 1.483 |

* This chain needs a further careful experimental investigation [14].

† These isotopes are also observed in an independent decay chain of \(^{288}_{114}\), shown above.
Figure Captions

Fig. 1. The scattering potential for α-decay of $^{292}_{116}$ nucleus, calculated as the sum of Coulomb and nuclear proximity potential. The tunnelling path for PCM is shown, with the first and second turning points $R_a$ and $R_b$ also marked.

Fig. 2. The fragmentation potentials $V(A_2)$, calculated at $R = C_t$, for various excited compound systems with $Z=110-116$. $A_2$ is the mass of the lighter nucleus. The heavier nucleus is shown in parentheses for some cases only. The ground-state (g.s.) of the compound system is also marked.

Fig. 3. The calculated formation yields as a function of the mass of reaction partners for the compound systems $^{296}_{116}*$ and $^{292}_{114}*$ at temperatures corresponding to $E_{cm} = 201.1$ and $197.2$ MeV.

Fig. 4. The logarithm of α-decay half-lives calculated on PCM, and compared with the GLDM and experimental data (decay times $\tau$), plotted as a function of the parent nucleus mass for various α-decay chains. The data for all the observed decay chains are shown and the chains are identified by numbers. For PCM, a calculation for $Q_{\alpha}^{exp}$ is also shown for $^{288,289,114}$ and $^{277,110}$ nuclei.

Fig. 5. The Q-values used in PCM and GLDM, compared with those measured ($E_\alpha$ energies) in α-decay chains, plotted as a function of the parent nucleus mass. More than one data plotted means more than one observed chain, with the same chain identification as in Fig. 4.

Fig. 6. The PCM calculated Q-values, penetrability $P$, preformation factors $P_0$, and decay half-lives for some cluster decays of the parents of α-decay chain for $^{277}_{110}$, plotted as a function of the parent nucleus mass.
Fig. 1 Kumar et al. 'The formation and decay of Superheavy...'
$T = 1.03 \text{ MeV}$

$R = C_t$

$T = 1.07 \text{ MeV}$
Fig. 5 Kumar et al. 'The formation and decay of Superheavy...'
Fig. 6 Kumar et al. 'The formation and decay of Superheavy...'