THE INTERFACE OF COSMOLOGY
WITH STRING AND M(ILLENNIUM) THEORY

Damien A. Easson

Brown University, Department of Physics,
Providence, RI 02912, USA

ABSTRACT

The purpose of this review is to discuss recent developments occurring at the interface of cosmology with string and M-theory. We begin with a short review of 1980s string cosmology and the Brandenberger-Vafa mechanism for explaining spacetime dimensionality. It is shown how this scenario has been modified to include the effects of p-brane gases in the early universe. We then introduce the Pre-Big-Bang scenario (PBB), Hořava-Witten heterotic M-theory and the work of Lukas, Ovrut and Waldram, and end with a discussion of large extra dimensions, the Randall-Sundrum model and Brane World cosmologies.

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## Contents

1 Introduction .......................................................... 2

2 M-Theory .............................................................. 3

3 Superstrings and Spacetime Dimensionality .......................... 4
   3.1 Duality .............................................................. 5
   3.2 Thermodynamics of Strings ....................................... 5
   3.3 The BV Mechanism and the Early Universe ....................... 6
   3.4 The Dimensionality Problem .................................... 8
   3.5 Brane Gases and the ABE Mechanism ............................ 10

4 Pre-Big-Bang .......................................................... 15
   4.1 Introduction ....................................................... 15
   4.2 More on Duality .................................................. 17
   4.3 PBB-Cosmology ................................................... 19

5 Cosmology and Heterotic M-Theory ................................... 21
   5.1 Hořava-Witten Theory ........................................... 21
   5.2 Five-Dimensional Effective Theory .............................. 23
   5.3 Three-Brane Solution ............................................ 24
   5.4 Cosmological Domain-Wall Solution ............................ 25

6 Large Extra Dimensions ................................................ 26
   6.1 Motivation and the Hierarchy Problem ........................... 26
   6.2 Randall-Sundrum I ............................................... 28
   6.3 Randall-Sundrum II ............................................... 32
   6.4 RS and Brane World Cosmology ................................... 34
   6.5 Supersymmetry .................................................... 38

7 Conclusions ............................................................ 39
1 Introduction

In recent years there have been many exciting advances in our understanding of M-theory – our best candidate for the fundamental theory of everything. The theory claims to describe physics appropriately in regions of space with high energies and large curvature scales. As these characteristics are exactly those found in the initial conditions of the universe it is only natural to incorporate M-theory into models of early universe cosmology.

The necessity to search for alternatives to the Standard Big-Bang (SBB) model of cosmology stems from a number of detrimental problems such as the horizon, flatness, structure formation and cosmological constant problems. Although inflationary models have managed to address many of these issues, inflation, at least in its current formulation, does not explain everything. In particular, inflation fails to address the fluctuation, super-Planck scale physics, initial singularity and cosmological constant problems as discussed in [2].

At the initial singularity, physical invariants such as the Ricci scalar, $R$, blow up. Other measurable quantities, for example temperature and energy density also become infinite. From the Hawking-Penrose singularity theorems we know that such spacetimes are geodesically incomplete. So, when we ask the question of how the universe began, the inevitable and unsatisfactory answer is that we don’t know. The physics required to understand this epoch of the early universe is necessarily rooted in a theory of quantum gravity. Presently, string theory is the only candidate for such a unifying theory. It is therefore logical to study the ways in which it changes our picture of cosmology. Although an ambitious aspiration, we hope that M-theory will solve the above mentioned dilemmas and provide us with a complete description of the evolution of the universe.

In this analysis, we must proceed with caution. Our present understanding of M-theory is extremely limited, as is our understanding of cosmology before the first $10^{-43}$ seconds. Nevertheless, it is clear that the study of string cosmology is essential to the development of string theory, and extremely important for our understanding of the early universe.

The purpose of this article is to introduce some of the most promising work and themes under investigation in string cosmology. We begin with a brief, qualitative introduction to M-theory in Section 2.

In Section 3 we review the work of Brandenberger and Vafa [1] in which the 1980s version of string theory is used to solve the initial singularity problem and in an attempt to explain why we live in four macroscopic dimensions despite the fact that string theory seems to predict the wrong number of dimensions, namely ten. We then explain how this scenario has been updated in order to include the effects of $p$-branes [19].
Section 4 provides a brief introduction to the Pre-Big-Bang scenario [21]-[26]. This is a theory based on the low energy effective action for string theory, developed in the early 1990s by Gasperini and Veneziano.

Another promising attempt to combine M-theory with cosmology, that of Lukas, Ovrut and Waldram [41], is presented in Section 5. Their work is based on the model of heterotic M-theory constructed by Hořava and Witten and is inspired by eleven dimensional supergravity, the low energy limit of M-theory. The motivation for this work was to construct a toy cosmological model from the most fundamental theory we know.

The final section (6) reviews some models involving large extra dimensions. This section begins with a short introduction to the hierarchy problem of standard model particle physics and explains how it may be solved using large extra dimensions. “Brane World” scenarios are then discussed focusing primarily on the models of Randall and Sundrum [52, 53], where our four dimensional universe emerges as the world volume of a three brane. The cosmologies of such theories are reviewed, and we briefly comment on their incorporation into supergravity models, string theory and the AdS/CFT correspondence.

The sections in this review are presented more or less chronologically.

2 M-Theory

For several years now, we have known that there are five consistent formulations of superstring theory. The five theories are ten-dimensional, two having $N = 2$ supersymmetry known as Type IIA and Type IIB and three having $N = 1$ supersymmetry, Type I, $SO(32)$ heterotic and $E_8 \times E_8$ heterotic. Recently, duality symmetries between the various theories have been discovered, leading to the conjecture that they all represent different corners of a large, multidimensional moduli space of a unified theory named, M-theory. Using dualities we have discovered that there is a sixth branch to the M-theory moduli space (see Fig. (2)) corresponding to eleven-dimensional supergravity.

\footnote{This review is in no way comprehensive. As it is impossible to discuss all aspects of string cosmology I have included a large list of references at the end. Some of the topics I will not cover in the text may be found there. For discussions of p-brane dynamics and cosmology see [176]-[178], [182]. For recent reviews on other cosmological aspects of M-theory see [179, 180]. For some ideas on radically new cosmologies from M-theory see e.g. [182]-[187].}
Figure 2. This is a slice of the eleven-dimensional moduli space of M-theory. Depicted are the five ten-dimensional string theories and eleven-dimensional supergravity, which is identified with the low energy limit of M-theory.

It is possible that using these six cusps of the moduli space we have already identified the fundamental degrees of freedom of the entire nonperturbative M-theory, but that their full significance has yet to be appreciated. A complete understanding and consistent formulation of M-theory is the ultimate challenge for string theorists today and will take physicists into the new M(illennium).

3 Superstrings and Spacetime Dimensionality

Perhaps the greatest embarrassment of string theory is the dimensionality problem. We perceive our universe to be four dimensional, yet string theory seems to naively predict the wrong number of dimensions, namely ten. The typical resolution to this apparent conflict is to say that six of the dimensions are curled up on a Planckian sized manifold. The following question naturally arises, why is there a six/four dimensional split between the small/large dimensions? Why not four/six, or seven/three? Although there is still no official answer to this question, a possible explanation emerges from cosmology and the work of Brandenberger and Vafa \[1\] which we will summarize in this section. We will then show how it is possible to generalize this scenario of the 1980s to incorporate our current understanding of string theory \[19\].
3.1 Duality

Before diving into the specifics of the BV model we review some basics of string dualities and thermodynamics. Consider the dynamics of strings moving in a nine-dimensional box with sides of length $R$. We impose periodic boundary conditions for both bosonic and fermionic degrees of freedom, so we are effectively considering string propagation in a torus. What types of objects are in our box? For one, there are oscillatory modes corresponding to vibrating stationary strings. Then, there are momentum modes which are strings moving in the box with fourier mode $n$ and momentum

$$p = n/R.$$  \hspace{1cm} (3.1)

There are also winding modes which are strings that stretch across the box (wrapped around the torus) with energy given by

$$\omega = mR,$$  \hspace{1cm} (3.2)

where $m$ is the number of times the string winds around the torus.

We now make the remarkable observation, that the spectrum of this system remains unchanged under the substitution

$$R \rightarrow \frac{1}{R},$$  \hspace{1cm} (3.3)

(provided we switch the roles of $m$ and $n$). This symmetry is known as T-duality and is a symmetry of the entire M-theory, not just the spectrum of this particular model. T-duality leads us to the startling conclusion that any physical process in a box of radius $R$ is equivalent to a dual physical process in a box of radius $1/R$. In other words, one can show that scattering amplitudes for dual processes are equal. Hence, we have discovered that distance, which is an invariant concept in general relativity (GR), is not an invariant concept in string theory. In fact, we will see that many invariant notions in GR are not invariant notions in string theory. These deviations from GR are especially noticeable for small distance scales where the Fourier modes of strings become heavier (3.1) and less energetically favorable, while the winding modes become light (3.2) and are therefore more easy to create.

3.2 Thermodynamics of Strings

Before discussing applications of t-duality to cosmology let us review a few useful calculations of string thermodynamics. The primary assumption we will make for the following
discussion is that the string coupling is sufficiently small so that we may ignore the gravitational back reaction of thermodynamical string condensates on the spacetime geometry.

String thermodynamics predicts the existence of a maximum temperature known as the Hagedorn temperature \( T_H \) above which the canonical ensemble approach to thermodynamics breaks down. This is due to the divergence of the partition function because of string states which exponentially increase as

\[
d(E) \propto E^{-p} \exp(\beta_H E),
\]

where \( p > 0 \). The partition function is easily calculated,

\[
Z = \sum_i \exp(-\beta E_i),
\]

which diverges for \( \beta < \beta_H \), or \( T > T_H \).

### 3.3 The BV Mechanism and the Early Universe

Consider the following toy model of a superstring-filled early universe. Besides the assumption of small coupling stated in section 3.2, we also assume that the evolution of the universe is adiabatic and make some assumptions about the size and shape of the universe.

Before the work of Brandenberger and Vafa, it was typical to speak about the process of “spontaneous compactification” of six of the ten dimensions predicted by string theory in order to successfully explain the origins of a large, 3+1 dimensional universe. Brandenberger and Vafa proposed that, from a cosmological perspective, it is much more logical to consider the decompactification of three of the spatial directions. In other words, one starts in a universe with nine dimensions, each compactified close to the Planck length and then, for one reason or another, three spatial dimensions grow large.

The toy model of the early universe considered here is a nine dimensional box with each dimension having equal length, \( R \). The box is filled with strings and periodic boundary conditions are imposed as described in Section (3.1).

In the SBB model it is possible to plot the scale factor \( R \) vs. \( t \) using the Einstein equations (Fig. (3.3)(a)). For the radiation dominated epoch, \( R \propto t^{1/2} \). Furthermore, it is possible to plot \( R \) vs. the temperature \( T \), where \( T \propto 1/R \) (Fig. (3.3)(b) and (c)). In string theory we have no analogue of Einstein’s equations and hence we cannot obtain a plot of the scale factor, \( R \) vs. \( t \). On the other hand, we do know the entire spectrum of string states and so we can obtain an analogue of the \( R \) vs. \( T \) curve (see Fig. (3.3)(d)). Note

\[\text{3For more on string thermodynamics see e.g. \cite{[1]-[14]}.}\]
that the region of Fig. (3.3)(d) near the Hagedorn temperature is not well understood, and canonical ensemble approaches break down. Fortunately, the regions to the left and right of $T_H$ are connected via dualities. The interested reader should see e.g. [5]-[14] for more modern investigations of the Hagedorn transition.

Recall, that in General Relativity the temperature $T$ goes to infinity as the radius $R$ decreases. As we have already mentioned, string theory predicts a maximum temperature, $T_H$ and therefore one should expect the stringy $R$ vs. $T$ curve to be drastically altered. Furthermore, we found that string theory enjoys the $R \rightarrow 1/R$ symmetry which leads to a $\ln R \rightarrow -\ln R$ symmetry in Fig. (3.3)(d). For large values of $R$, $R \propto 1/T$ is valid since the winding modes are irrelevant and the theory looks like a point particle theory. For small $R$ the $T - R$ curve begins to flatten out, approach the Hagedorn temperature and then as we continue to go to smaller values of $R$ the temperature begins to decrease. This behavior is a consequence of the T-duality of string theory. As $R$ shrinks, the winding modes which are absent in point particle theories become lighter and lighter, and are therefore easier to produce. Eventually, (with entropy constant) the thermal bath will consist mostly of winding modes, which explains the decrease in temperature once one continues past $T_H$ to smaller values of $R$. 
3.3 An observer traveling from large $R$ to small $R$, actually sees the radius contracting to $R = 1$ (in Planck units) and then expanding again. This makes us more comfortable with the idea of the temperature beginning to decrease after $R = 1$. The reason for this behavior is that the observer must modify the measuring apparatus to measure distance in terms of light states. The details for making this change of variables are described in [1].

Hence, the observer described above encounters an oscillation of the universe. This encourages one to search for cosmological solutions in string theory where the universe oscillates from small to large, eliminating the initial and final singularities found in (SBB) models.

3.4 The Dimensionality Problem

We are now ready to ask the question, how can superstring theory, a theory consistently formulated in ten dimensions give rise to a universe with only four macroscopic dimensions? This is equivalent within the context of our toy model to asking why should three of the
nine spatial dimensions of our box “want” to expand? To address this question, note the following observation: winding modes lead to negative pressure in the thermal bath. To understand this, recall that as the volume of the box increases, the energy in the winding modes also increases \((3.2)\). Thus the phase space available to the winding modes decreases, which brings us to the conclusion that winding modes would “like” to prevent expansion. The point is that it costs a lot of energy to expand with winding modes around. Thermal equilibrium demands that the number of winding modes must decrease as \(R\) increases (since the winding modes become heavier). Therefore, we conclude that expansion can only occur when the system is in thermal equilibrium, which favors fewer of the winding states as \(R\) increases. If, on the other hand, the winding modes are not in thermal equilibrium they will become plentiful and thus any expansion will be slowed and eventually brought to a halt.

Thermal equilibrium of the winding modes requires string interactions of the form

\[
W + \bar{W} \leftrightarrow unwound states.
\]  
(3.6)

Here \(W\) is a winding state and \(\bar{W}\) is a winding state with opposite winding as depicted in Fig. (3.4).

![Figure 3.4: Strings that interact with opposite windings become unwound states.](image)

In order for such processes to occur, the strings must come to within a Planck length of one another. As the winding strings move through spacetime they span out two dimensional world sheets. In order to interact, their worldsheets must intersect, but in a nine dimensional box the strings will probably not intersect because \(2 + 2 < 9 + 1\). Since there is so much room in the box, the strings will have a hard time finding one another in order for their worldsheets to intersect and therefore it is unlikely that they will unwind. If the winding
strings do not unwind, and the box starts to expand, the winding states will fall out of thermal equilibrium and the expansion will be halted.

The conclusion is that the largest spacetime dimensionality consistent with maintaining thermal equilibrium is four. Since, $2 + 2 = 3 + 1$, and therefore the largest number of spatial dimensions which can expand is three. In the next section we will see how this scenario can be incorporated into our current understanding of string theory.

### 3.5 Brane Gases and the ABE Mechanism

Recent developments in M/string theory have revealed that strings are not the only fundamental degrees of freedom in the theory. The spectrum of fundamental states also includes higher dimensional extended objects known as D-branes. Here we will examine the way in which the BV scenario unfolds in the presence of D-branes in the early universe as constructed by Alexander, Brandenberger and Easson (ABE) [19]. Specifically, we are interested in finding out if the inclusion of branes affects the cosmological implications of [1]. Note that this approach to string cosmology is in close analogy with the starting point of the standard big-bang model and is very different from other cosmological models which have attempted to include D-branes, for example the brane-world scenarios discussed in Sections 5 and 6. However, possible relations between this model and brane-world scenarios will be discussed later.

Our initial state will be similar to that of [1]. We assume that the universe started out close to the Planck length, dense and hot and with all degrees of freedom in thermal equilibrium. As in [1], we choose a toroidal geometry in all spatial dimensions. The initial state will be a gas composed of the fundamental branes in the theory. We will consider 11-dimensional M-theory compactified on $S^1$ to yield 10-dimensional Type II-A string theory. The low-energy effective theory is supersymmetrized dilaton gravity. Since M-theory admits the graviton, 2-branes and 5-branes as fundamental degrees of freedom, upon the $S^1$ compactification we obtain 0-branes, strings (1-branes), 2-branes, 4-branes, 5-branes, 6-branes and 8-branes in the 10-dimensional universe.

The details of the compactification will not be discussed here, however we will briefly mention the origins of the above objects from the fundamental eleven-dimensional, M-theory perspective. The 0-branes of the II-A theory are the BPS states of nonvanishing $p_{10}$. In M-theory these are the states of the massless graviton multiplet. The 1-brane of the II-A theory is the fundamental II-A string which is obtained by wrapping the M-theory supermembrane around the $S_1$. The 2-brane is just the transverse M2-brane. The 4-branes are wrapped
M5-branes. The 5-brane of the II-A theory is a solution carrying magnetic NS-NS charge and is an M5-brane that is transverse to the eleventh dimension. The 6-brane field strength is dual to that of the 0-brane, and is a KK magnetic monopole. The 8-brane is a source for the dilaton field \[4\].

The low-energy bulk effective action for the above setup is

\[
S_{\text{bulk}} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} e^{-2\phi} [R + 4G^\mu\nu\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha}], \tag{3.7}
\]

where \(G\) is the determinant of the background metric \(G_{\mu\nu}\), \(\phi\) is the dilaton, \(H\) denotes the field strength corresponding to the bulk antisymmetric tensor field \(B_{\mu\nu}\), and \(\kappa\) is determined by the 10-dimensional Newton constant in the usual way.

For an individual \(p\)-brane the action is of the Dirac-Born-Infeld form

\[
S_p = T_p \int d^{p+1}\xi e^{-\phi} \sqrt{-\det(g_{mn} + b_{mn} + 2\pi\alpha' F_{mn})} \tag{3.8}
\]

where \(T_p\) is the tension of the brane, \(g_{mn}\) is the induced metric on the brane, \(b_{mn}\) is the induced antisymmetric tensor field, and \(F_{mn}\) the field strength tensor of gauge fields \(A_m\) living on the brane. The total action is the sum of the bulk action (3.7) and the sum of all of the brane actions (3.8), each coupled as a delta function source (a delta function in the directions transverse to the brane) to the 10-dimensional action.

In the string frame the tension of a \(p\)-brane is

\[
T_p = \frac{\pi}{g_s} (4\pi^2\alpha')^{-(p+1)/2}, \tag{3.9}
\]

where \(\alpha' \sim l_{st}^2\) is given by the string length scale \(l_{st}\) and \(g_s\) is the string coupling constant.

In order to discuss the dynamics of this system, we will need to compute the equation of state for the brane gases for various \(p\). There are three types of modes that we will need to consider. First, there are the winding modes. The background space is \(T^9\), and hence a \(p\)-brane can wrap around any set of \(p\) toroidal directions. These modes are related by t-duality to the momentum modes corresponding to center of mass motion of the branes. Finally, the modes corresponding to fluctuations of the branes in the transverse directions are (in the low-energy limit) described by scalar fields on the brane, \(\phi_i\). There are also bulk matter fields and brane matter fields.

We are mainly interested in the effects of winding modes and transverse fluctuations to the evolution of the universe, and therefore we will neglect the antisymmetric tensor field \(B_{\mu\nu}\). We will take our background metric with conformal time \(\eta\) to be

\[
G_{\mu\nu} = a(\eta)^2 \text{diag}(-1, 1, \ldots, 1), \tag{3.10}
\]
where $a(\eta)$ is the cosmological scale factor.

If the transverse fluctuations of the brane and the gauge fields on the brane are small, the brane action can be expanded as

$$S_{\text{brane}} = T_p \int d^{p+1} \zeta a(\eta)^{p+1} e^{-\phi}$$

$$e^{\frac{1}{2} \text{tr} \log \left( 1 + \partial_m \phi_i \partial_n \phi_i + a(\eta)^{-2} \alpha' F_{mn} \right)}$$

$$= T_p \int d^{p+1} \zeta a(\eta)^{p+1} e^{-\phi}$$

$$\left( 1 + \frac{1}{2} (\partial_m \phi_i)^2 - \pi^2 \alpha'/a^{-4} F_{mn} F^{mn} \right).$$

(3.11)

The first term in the parentheses in the last line represents the brane winding modes, the second term corresponds to the transverse fluctuations, and the third term to brane matter. In the low-energy limit, the transverse fluctuations of the brane are described by a free scalar field action, and the longitudinal fluctuations are given by a Yang-Mills theory. The induced equation of state has pressure $p \geq 0$.

To find the equation of state for the winding modes, we use equation (3.11) to get

$$\tilde{p} = w_p \rho \text{ with } w_p = -\frac{p}{d}$$

(3.12)

where $d$ is the number of spatial dimensions (9 in our case), and where $\tilde{p}$ and $\rho$ stand for the pressure and energy density, respectively.

Fluctuations of the branes and brane matter are given by free scalar and gauge fields on the branes. These may be viewed as particles in the transverse directions extended in brane directions. Therefore, the equation of state is simply that of ordinary matter,

$$\tilde{p} = w \rho \text{ with } 0 \leq w \leq 1.$$  

(3.13)

From the action (3.11) we see that the energy in the winding modes will be

$$E_p(a) \sim T_p a(\eta)^p,$$

(3.14)

where the constant of proportionality is dependent on the number of branes.

The equations of motion for the background are given by [15, 21]

$$- d\dot{\lambda}^2 + \dot{\phi}^2 = e^\phi E$$

(3.15)

$$\ddot{\lambda} - \dot{\phi} \dot{\lambda} = \frac{1}{2} e^\phi P$$

(3.16)

$$\ddot{\phi} - d\dot{\lambda}^2 = \frac{1}{2} e^\phi E,$$

(3.17)

\[\text{Note that the above result is still valid when brane fluctuations and fields are large.}\]
where \( E \) and \( P \) denote the total energy and pressure, respectively,

\[
\lambda(t) = \log(a(t)),
\]

and \( \varphi \) is a shifted dilaton field which absorbs the space volume factor

\[
\varphi = 2\phi - d\lambda.
\]

The matter sources \( E \) and \( P \) are made up of all the components of the brane gas:

\[
E = \sum_p E^w_p + E^{nw}_w, \\
P = \sum_p w_p E^w_p + wE^{nw}_w,
\]

where the superscripts \( w \) and \( nw \) stand for the winding modes and the non-winding modes, respectively. The contributions of the non-winding modes of all branes have been combined into one term. The constants \( w_p \) and \( w \) are given by (3.12) and (3.13). Each \( E^w_p \) is the sum of the energies of all of the brane windings with fixed \( p \).

We may now draw the comparison between the ABE mechanism and [1]. First of all we see that both t-duality and limiting Hagedorn temperature are still manifest once we include the \( p \)-branes [19]. Therefore, there is no physical singularity as \( R \to 0 \). What about the de-compactification mechanism described in section (3.3)? Recall that our initial conditions are in a hot, dense regime near the self dual point \( R = 1 \). All the modes (winding, oscillatory and momentum) of all the \( p \)-branes will be excited. By symmetry, we assume that there are equal numbers of winding and anti-winding modes in the system and hence the total winding numbers cancel as in [1].

Now assume that the universe begins to expand in all directions. The total energy in the winding modes increases with \( \lambda \) as (3.14), so the largest \( p \)-branes contribute the most. The classical counting argument discussed in [1] is easily generalized to our model. When winding modes meet anti-winding modes, the branes unwind (recall Fig. (3.4)) and allow a certain number of dimensions to grow large.

Consider the probability that the world-volumes of two \( p \)-branes in spacetime will intersect. The winding modes of \( p \)-branes are likely to interact in at most \( 2p + 1 \) spatial dimensions.\footnote{To see this, consider the example of two particles (0-branes) moving through a space of dimension \( d \). These particles will definitely interact (assuming the space is periodic) if \( d = 1 \), whereas they probably will not find each other in a space with \( d > 1 \).}
Since we are in $d = 9$ spatial dimensions the $p = 8, 6, 5, 4$ branes will interact and hence unwind very quickly. For $p < 4$ a hierarchy of dimensions will be allowed to grow large. Since the energy contained in the winding modes of 2 branes is larger than that of strings (see (3.14)) the 2 branes will have an important effect first. The membranes will allow a $T^5$ subspace to grow large. Within this 5 dimensional space the 1-branes will allow a $T^3$ subspace to become large. We therefore reach the conclusion that the inclusion of D-branes into the spectrum of fundamental objects in the theory will cause a hierarchy of subspaces to become large while maintaining the results of the BV scenario, explaining the origin of our $3 + 1$-dimensional universe.

Let us summarize the evolution of the ABE universe. The universe starts out in an initial state close to the self-dual point ($R = 1$), a 9-dimensional toroidal space, hot, dense and filled with particles, strings and $p$-brane gases. The universe then starts to expand according to the background equations of motion (3.15 - 3.17). Branes with the largest value of $p$ will have an effect first, and space can only expand further if the winding modes annihilate. The 8, 6, 5 and 4-branes winding modes annihilate quickly, followed by the 2-branes which allow only 5 spatial dimensions to become large. In this $T^5$ the strings allow a 3-dimensional subspace to become large. Hence, it is reasonable to hypothesize the existence of a $5 + 1$-dimensional effective theory at some point in the early history of the universe. In particular, one is tempted to draw a relation between this 5-dimensional picture and the scenario of large extra dimensions proposed in [59].

There are several problems with the toy model analyzed above. Most of these have already been mentioned by the authors. First, the strings and branes are treated classically. Quantum effects will cause the strings to take on a small but finite thickness [16], although in our case we are restricted to energy densities lower than the typical string density, and hence the effective width of the strings is of string scale [17]. This presumably will also apply to the branes, although there is no current, consistent quantization scheme developed for branes.

In this scenario there is a brane problem. This is a new problem for cosmological theories with stable branes analogous to the domain wall problem in cosmological scenarios based on quantum field theories with stable domain walls. However, we have found background solutions in our models which approach a point of loitering [21]. Loitering occurs if at some point in the evolution of the universe the size of the Hubble radius extends larger than the physical radius. Such a phase in the background cosmological evolution will naturally solve the brane problem.
The toroidal topology of the compactified manifold was chosen for simplicity. It is important from the point of view of string theory to consider how things would change if this manifold was a Calabi-Yau space. Calabi-Yau three-folds do not admit one cycles for strings to wrap around, although they are necessary if the four-dimensional low energy effective theory is to have $N = 1$ supersymmetry. Note that in cosmology we do not necessarily expect $N = 1$ supersymmetry. In particular, maximal supersymmetry is consistent with the toroidal background used.

Also, it was argued in [18] that M-theory should not be formulated in a spacetime of definite dimension or signature. In other words, we must ultimately be able to explain why there is only one time dimension.

Although there is no horizon problem present in this scenario since the universe starts out near the string length and hence there are no causally disconnected regions of space, other problems solved by inflation such as the flatness and structure formation problems are still present. Other less significant concerns are stressed in [19]. This scenario provides a new method for studying string cosmology which is similar to the SBB model and utilizes $p$-branes in a very different way from scenarios involving large extra dimensions.

## 4 Pre-Big-Bang

The next attempt to marry cosmology with string theory we will review was proposed in the early 1990s by Veneziano and Gasperini [21]-[25].

### 4.1 Introduction

The Pre-Big-Bang (PBB) model is based on the low energy effective action of string theory, which in $d$ spatial dimensions is given by

$$ S = -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x \sqrt{-g} e^{-\varphi} \left[ R + (\partial \mu \varphi)^2 + \cdots \right], \quad (4.1) $$

where $\varphi$ is the dilaton and $\lambda_s$ is the string length scale. The qualitative differences between the PBB model, and the SBB model based on the Einstein-Hilbert action,

$$ S = -\frac{1}{2\lambda_p^{d-1}} \int d^{d+1}x \sqrt{-g} R, \quad (4.2) $$

are most easily visualized by plotting the history of the curvature of the universe (see Fig. [3]) according to each theory. In the SBB scenario the curvature increases as we go

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6For an updated collection of papers on this model see [http://www.to.infn.it/~gasperin](http://www.to.infn.it/~gasperin)
back in time, eventually reaching an infinite value at the Big-Bang singularity. In standard inflationary models the curvature reaches some fixed value as \( t \) decreases at which point the universe enters a de Sitter phase. It has been shown however that such an inflationary phase cannot last forever, for reasons of geodesic completeness, and that the initial singularity problem still remains [27, 2]. The cosmology generated by (4.1) differs drastically from the standard scenarios. The action (4.1) without the “…” terms does not realize the PBB scenario, as we will discuss below. In the PBB model, as one travels back in time the curvature increases as in the previously mentioned models, but in the PBB a maximum curvature is reached at which point the curvature and temperature actually begin to decrease. Although we will examine the details of how this occurs below, a few simple considerations make us feel more comfortable with this picture.

For one, string theory predicts a natural cut-off length scale,

\[
\lambda_s = \sqrt{\frac{\hbar}{T}} \sim 10 l_{pl} \sim 10^{-32} \text{cm},
\]

where \( T \) is the string tension and \( l_{pl} \) is the Planck length. So it is natural from the point of view of strings to expect a maximum possible curvature. Logically, as we travel back in time there are only two possibilities if we want to avoid the initial singularity. Either the curvature starts to grow again before the de Sitter phase, in which case we are still left with a singularity shifted earlier in time, or the curvature begins to decrease again, which is what happens in the PBB scenario (Fig. (6)c). This behavior is a consequence of scale-factor duality.
4.2 More on Duality

To demonstrate the enhanced symmetries present in the PBB model we will examine the consequences of scale-factor duality. The Einstein-Hilbert action is invariant under time reversal. Hence, for every solution $a(t)$ there exists a solution $a(-t)$. Or in terms of the Hubble parameter $H(t) = \dot{a}(t)/a(t)$, for every solution $H(t)$ there exists a solution $-H(-t)$. Thus, if there is a solution representing a universe with decelerated expansion and decreasing curvature ($H > 0$, $\dot{H} < 0$) there is a “mirror” solution corresponding to a contracting universe ($H(-t)$, $H < 0$).

The action of string theory is not only invariant under time reversal, but also under inversion of the scale factor $a(t)$, (with an appropriate transformation of the dilaton). For every cosmological solution $a(t)$ there is a solution $\tilde{a} = 1/a(t)$, provided the dilaton is rescaled, $\varphi \rightarrow \tilde{\varphi} = \varphi - 2d \ln a$. Hence, time reversal symmetry together with scale-factor duality imply that every cosmological solution has four branches, Fig. 4.2. For the standard scenario of decelerated expansion and decreasing curvature ($H(t) > 0$, $\dot{H}(t) < 0$) there is a dual partner solution describing a universe with accelerated expansion parameter $\tilde{H}(t)$ and growing curvature $\dot{\tilde{H}}(-t)$.
We will now show how one can create a universe from the string theory perturbative vacuum, that today looks like the standard cosmology. This problem is analogous to finding a smooth way to connect the Pre-Big-Bang phase with a Post-Big-Bang phase, or how to successfully connect the upper-left side of Fig. (4.2) to the upper-right side. In general, the two branches are separated by a future/past singularity and it appears that in order to smoothly connect the branches of growing and decreasing curvature one requires the presence of higher order loop and/or derivative corrections to the effective action (4.1). This cancer of the PBB model is known as the Graceful Exit Problem (GEP) and is the subject of many research papers (see [25, 26] for a collection of references).

One example of how the GEP can be solved is given in [28]. In this work we consider a theory obtained by adding to the usual string frame dilaton gravity action specially constructed higher derivative terms motivated by the limited curvature construction of [29]. The action is (4.1) with the “…” term being replaced by the constructed higher derivative terms. In this scenario all solutions of the resulting theory of gravity are nonsingular and for initial conditions inspired by the PBB scenario solutions exist which smoothly connect a “superinflationary” phase with $\dot{H} > 0$, to an expanding FRW phase with $\dot{H} < 0$, solving the GEP in a natural way.
4.3 PBB-Cosmology

Here we examine cosmological solutions of the PBB model. By adding matter in the form of a perfect fluid to the effective action (4.1) (without the “…” terms) and taking a Friedmann-Robertson-Walker background with \( d = 3 \), we vary the action to get the equations of motion for string cosmology,

\[
\begin{align*}
\dot{\phi}^2 - 6H\dot{\phi} + 6H^2 &= e^\phi \rho, \\
\dot{H} - H\dot{\phi} + 3H^2 &= \frac{1}{2}e^\phi p, \\
2\ddot{\phi} + 6H\dot{\phi} - \dot{\phi}^2 - 6\dot{H} - 12H^2 &= 0.
\end{align*}
\]

As an example, for \( p = \rho/3 \) the equations with constant dilaton are exactly solved by

\[
a \propto t^{1/2}, \quad \rho \propto a^{-4}, \quad \phi = \text{const.},
\]

which is the standard scenario for the radiation dominated epoch, having decreasing curvature and decelerated expansion:

\[
\dot{a} > 0, \quad \ddot{a} < 0, \quad \dot{H} < 0.
\]

But there is also a solution obtained from the above via time translation and scale-factor duality,

\[
t \to -t, \quad a \propto (-t)^{-1/2}, \quad \phi \propto -3\ln(-t), \quad \rho = -3p \propto a^{-2}.
\]

This solution corresponds to an accelerated, inflationary expansion, with growing dilaton and growing curvature:

\[
\dot{a} > 0, \quad \ddot{a} > 0, \quad \dot{H} > 0.
\]

Solutions with such behavior are called “superinflationary” and are located in the upper left quadrant of Fig. [12].

Let us briefly review the history of the universe as predicted by the PBB scenario. Recall, that in the SBB model the universe starts out in a hot, dense and highly curved regime. In contrast, the PBB universe has its origins in the simplest possible state we can think of, namely the string perturbative vacuum. Here the universe consists only of a sea of dilaton and gravitational waves. It is empty, cold and flat, which means that we can still trust calculations done with the classical, low-energy effective action of string theory.

In [30], the authors showed that in a generic case of the PBB scenario, the universe at the onset of inflation must already be extremely large and homogeneous. In order for
inflation to solve flatness problems the initial size of a homogeneous part of the universe before PBB inflation must be greater than $10^{19} l_s$. In response, it was proposed in [35] that the initial state of the PBB model is a generic perturbative solution of the tree-level, low-energy effective action. Presumably, quantum fluctuations lead to the formation of many black holes (Fig. (4.3)) in the gravi-dilaton sector (in the Einstein frame). Each such singular space-like hypersurface of gravitational collapse becomes a superinflationary phase in the string frame [33, 34, 32, 35]. After the period of dilaton-driven inflation the universe evolves in accordance with the SBB model.

![Figure 4.3](image)

Figure 4.3: A $2 + 1$ dimensional slice of the string perturbative vacuum giving rise to black hole formation in the Einstein frame.

To conclude let us mention a few benefits of the PBB scenario. For one, there is no need to *invent* inflation, or fine tune a potential for the inflaton. This model provides a “stringy” realization of inflation which sets in naturally and is dilaton driven. Pair creation (quantum instabilities) provides a mechanism to heat up an initially cold universe in order to produce a hot big-bang with homogeneity, isotropy and flatness. This scenario also has observable consequences.

Problems with this scenario include the graceful exit problem, mentioned above. This is the problem of smoothly connecting the phases of growing and decreasing curvature, a process that is not well understood and requires further investigation. Most cosmological models require a potential for the dilaton to be introduced by hand in order to freeze the dilaton at late times. In general it is believed that the dilaton should be massive today, otherwise we would notice its effects on physical gauge couplings.

Inclusion of a non-vanishing $B_{\mu\nu}$ into the action (1.1) greatly reduces the initial conditions which give rise to inflation [26]. Also the initial collapsing region must be sufficiently large and weakly coupled. Lastly, the dimensionality problem is still present in this model.


5 Cosmology and Heterotic M-Theory

In this section we will focus on the work of Lukas, Ovrut and Walram (LOW)\cite{LOW} in 1998, which is based on the heterotic M-theory of Hořava and Witten \cite{HoravaWitten1,HoravaWitten2,HoravaWitten3}. Their motivation was to see if it is possible to construct a realistic, cosmological model starting from the most fundamental theory we know.

5.1 Hořava-Witten Theory

In 1996, Hořava and Witten showed that eleven-dimensional M-theory compactified on an $S^1/Z_2$ orbifold with a set of $E_8$ gauge supermultiplets on each ten-dimensional orbifold fixed plane can be identified with strongly coupled $E_8 \times E_8$ heterotic string theory\cite{HoravaWitten1,HoravaWitten2}. The basic setup is that of Fig. (5.1), where the orbifold is in the $x^{11}$ direction and $x^{11} \in [-\pi \rho, \pi \rho]$ with the endpoints being identified. The orbifolding with $Z_2$ leads to the symmetry $x^{11} \rightarrow -x^{11}$. It has been shown that this M-theory limit can be consistently compactified on a deformed Calabi-Yau three-fold resulting in an $N=1$ supersymmetric theory in four dimensions (see fig.(5.2)). In order to match (at tree level) the gravitational and grand-unified gauge couplings one finds the requirement $R_{\text{orb}} > R_{\text{CY}}$, where $R_{\text{orb}}$ is the radius of the orbifold and $R_{\text{CY}} \approx 10^{16}$GeV is the radius of the Calabi-Yau space. This picture leads to the conclusion that the universe may have gone through a phase in which it was effectively five-dimensional, and therefore provides us with a previously unexplored regime in which to study the early universe.

![Figure 5.1](https://example.com/figure5.1.png) Figure 5.1 The Hořava-Witten scenario. One of the eleven-dimensions has been compactified onto the orbifold $S^1/Z_2$. The manifold is $\mathcal{M} = \mathbb{R}^{10} \times S^1/Z_2$. 

21
Here we construct the five-dimensional effective theory via reduction of Hořava-Witten on a Calabi-Yau three-fold, and then show how this can lead to a four-dimensional toy model for a Friedmann-Robertson-Walker (FRW) universe.

We start with an eleven-dimensional action with bosonic contribution

\[ S = S_{\text{SUGRA}} + S_{\text{YM}}, \]  

(5.1)

where \( S_{\text{SUGRA}} \) is the action of eleven-dimensional supergravity

\[
S_{\text{SUGRA}} = -\frac{1}{2\kappa^2} \int_{M^{11}} \sqrt{-g} \left[ R + \frac{1}{24} G_{IJKL} G^{IJKL} \right. \\
+ \left. \frac{\sqrt{2}}{128} \epsilon^{I_1 \cdots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \cdots I_7} G_{I_8 \cdots I_{11}} \right],
\]

(5.2)

and \( S_{\text{YM}} \) are two \( E_8 \) Yang-Mills theories on the ten-dimensional orbifold planes

\[
S_{\text{YM}} = -\frac{1}{8\pi\kappa^2} \left( \frac{\kappa}{4\pi} \right)^{2/3} \int_{M^{(i)}_{10}} \sqrt{-g} \left\{ \text{tr}(F^{(1)})^2 - \frac{1}{2} \text{tr}R^2 \right\}
- \frac{1}{8\pi\kappa^2} \left( \frac{\kappa}{4\pi} \right)^{2/3} \int_{M^{(2)}_{10}} \sqrt{-g} \left\{ \text{tr}(F^{(2)})^2 - \frac{1}{2} \text{tr}R^2 \right\}.
\]

(5.3)

The values of \( I, J, K, \ldots = 0, \ldots, 9, 11 \) parametrize the full eleven-dimensional space \( M_{11} \), while \( \bar{I}, \bar{J}, \bar{K}, \ldots = 0, \ldots, 9 \) are used for the ten-dimensional hyperplanes, \( M^{(i)}_{10}, i = 1, 2 \), orthogonal to the orbifold. The \( F_{IJ}^{(i)} \) are the two \( E_8 \) gauge field strengths and \( C_{IJK} \) is the 3-form with field strength given by \( G_{IJKL} = 24 \partial_{[I} C_{JKL]} \). In order for this theory to be supersymmetric and anomaly free the Bianchi identity for \( G \) must pick up the following correction,

\[
(dG)_{1IJKL} = -\frac{1}{2\sqrt{2\pi}} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left\{ J^{(1)} \delta(x^{11}) + J^{(2)} \delta(x^{11} - \pi\rho) \right\}_{IJKL}
\]

(5.4)

where the sources are

\[ J^{(i)} = \text{tr}F^{(i)} \wedge F^{(i)} - \frac{1}{2} \text{tr}R \wedge R. \]

(5.5)

Now, we search for solutions to the above theory which preserve four of the thirty-two supercharges and, when compactified, lead to four dimensional, \( N = 1 \) supergravities. To begin, consider the manifold \( \mathcal{M} = \mathbb{R}^4 \times X \times S^1 / \mathbb{Z}_2 \), where \( \mathbb{R}^4 \) is four-dimensional Minkowski space and \( X \) is a Calabi-Yau three-fold. Upon compactification onto \( X \), we are left with a five-dimensional effective spacetime consisting of two copies of \( \mathbb{R}^4 \), one at each of the orbifold fixed points, and the orbifold itself (see fig. (5.2)). On each of the \( \mathbb{R}^4 \) planes there is a gauge group \( H^{(i)}, i = 1, 2 \), and \( N = 1 \).
In the next section we construct the five-dimensional effective theory.

5.2 Five-Dimensional Effective Theory

As we have discussed, according to the model presented above, there is an epoch when the universe appears to be five dimensional. Hence, it is only natural to try to find the action for this five-dimensional effective theory. Let us identify the fields in the five-dimensional bulk. First, there is the gravity multiplet \((g_{\alpha\beta}, A_{\alpha}, \psi_{i}^{\alpha})\), where \(g_{\alpha\beta}\) is the graviton, \(A_{\alpha}\) is a five-dimensional vector field, and the \(\psi_{i}^{\alpha}\) are the gravitini. The indices \(\alpha, \beta = 0, ..., 3, 11\) and \(i = 1, 2\). There is also the universal hypermultiplet \(q \equiv (V, \sigma, \xi, \bar{\xi}, \zeta^{i})\). Here \(V\) is a modulus field associated with the volume of the Calabi-Yau space, \(\xi\) is a complex scalar zero mode, \(\sigma\) is a scalar resulting from the dualization of the three-form \(C_{\alpha\beta\gamma}\), and the \(\zeta^{i}\) are the hypermultiplet fermions.

It is now possible, using the action (5.1) to construct the five-dimensional effective action of Ho\'rava-Witten theory,

\[
S_{5} = S_{\text{grav}} + S_{\text{hyper}} + S_{\text{bound}}, 
\]

where,

\[
S_{\text{grav}} = -\frac{v}{2\kappa^{2}} \int_{M_{5}} \sqrt{-g} \left[ R + \frac{3}{2} (F_{\alpha\beta})^{2} + \frac{1}{\sqrt{2}} \epsilon^{\alpha\beta\gamma\delta\epsilon} A_{\alpha} F_{\beta\gamma} F_{\delta\epsilon} \right], 
\]

\[
S_{\text{hyper}} = -\frac{v}{2\kappa^{2}} \int_{M_{5}} \sqrt{-g} \left[ 4 h_{\mu\nu} \nabla_{\alpha} q^{\mu} \nabla^{\alpha} q^{\nu} + \frac{\alpha^{2}}{3} V^{-2} \right], 
\]
\[ S_{\text{bound}} = -\frac{v}{2\kappa^2} \left[ \mp 2\sqrt{2} \sum_{i=1}^{2} \int_{M_4^{(i)}} \sqrt{-g} \alpha V^{-1} \right] \]
\[ = -\frac{v}{8\pi\kappa^2} \left( \frac{\kappa}{4\pi} \right)^{2/3} \sum_{i=1}^{2} \int_{M_4^{(i)}} \sqrt{-g} V \left( F^{(i)}_{\mu\nu} \right)^2. \] (5.9)

In the above, \( v \) is a constant that relates the five-dimensional Newton constant, \( \kappa_5 \), with the eleven-dimensional Newton constant, \( \kappa \), via \( \kappa^2 = \kappa^2 / v \). The metric \( h_{\mu\nu} \) is the flat space metric and \( \alpha \) is a constant. Higher-derivative terms have been dropped and this action provides us with a minimal \( N = 1 \) supergravity theory in the five-dimensional bulk.

This theory admits a three-brane domain wall solution with a world-volume lying in the four uncompactified dimensions [41]. In fact, a pair of domain walls is the vacuum solution of the five-dimensional theory which provides us with a background for reduction to a \( d = 4 \), \( N = 1 \) effective theory. This solution will be the topic of the next section.

### 5.3 Three-Brane Solution

In order to find a pair of three-branes solution we should start with an ansatz for the five-dimensional metric of the form
\[
ds_5^2 = a(y)^2 dx^\mu dx^\nu \eta_{\mu\nu} + b(y)^2 dy^2
\]
\[ V = V(y), \] (5.10)

where \( y = x^{11} \). By using the equations of motion derived from the action (5.6) we find
\[ a = a_0 H^{1/2} \]
\[ b = b_0 H^2 \]
\[ V = b_0 H^3, \] (5.11)

where \( H \equiv \frac{\sqrt{2}}{3} \alpha_0 |y| + c_0 \), and \( a_0, b_0 \) and \( c_0 \) are all constants. Using the equations of motion derived by varying the action with respect to \( g_{\mu\nu} \) of (5.10), we arrive at a differential equation which leads to
\[ \partial_y^2 H = \frac{2\sqrt{2}}{3} \alpha_0 \left( \delta(y) - \delta(y - \pi \rho) \right). \] (5.12)

A detailed derivation of this equation is discussed in [41]. Clearly, (5.12) represents two parallel three-branes located at the orbifold planes, as in Fig.(5.2). This solves the five-dimensional theory exactly and preserves half of the supersymmetries, with low-energy gauge and matter fields carried on the branes. This prompts us to find realistic cosmological models from the above scenario where the universe lives on the world-volume of a three-brane.
5.4 Cosmological Domain-Wall Solution

In order to construct a dynamical, cosmological solution, the solutions in (5.11) are made to be functions of time $\tau$, as well as the eleventh dimension $y$,

$$ds_5^2 = -N(\tau, y)d\tau^2 + a(\tau, y)^2 dx^m dx^n \eta_{mn} + b(\tau, y)^2 dy^2$$  
$$V = V(\tau, y).$$ (5.13)

Here we have introduced a lapse function $N(\tau, y)$. Because this ansatz leads to a very complicated set of non-linear equations we will seek a solution based on the separation of variables. Note, there is no a priori reason to believe that such a solution exists, but we will see that one does. Separating the variables $\tau$ and $y$,

$$N(\tau, y) = n(\tau) a(y)$$  
$$a(\tau, y) = \alpha(\tau) a(y)$$  
$$b(\tau, y) = \beta(\tau) b(y)$$  
$$V(\tau, y) = \gamma(\tau) V(y).$$ (5.14)

Since this article is intended only as an elementary review we will not repeat the details involved in solving the above system. For our purposes it suffices to say that the equations take on a particularly simple form when $\beta = \gamma$ and with the gauge choice of $n = const.$ In this gauge, $\tau$ becomes proportional to the comoving time $t$, since $dt = n(\tau)d\tau$. A solution exists such that

$$\alpha = A |t - t_0|^p$$  
$$\beta = B |t - t_0|^q,$$ (5.15)

where

$$p = \frac{3}{11}(1 \pm \frac{4}{3\sqrt{3}})$$  
$$q = \frac{2}{11}(1 \pm 2\sqrt{3}),$$ (5.16)

and $A, B$ and $t_0$ are arbitrary constants. This is the desired cosmological solution. The $y$-dependence is identical to the domain wall solution (5.12) and the scale factors evolve with $t$ according to (5.13). The domain wall pair remain rigid, while their sizes and the separation between the walls change. In particular, $\alpha$ determines the size of the domain-wall world-volume while $\beta$ gives the separation of the two walls. In other words, $\alpha$ determines the size
of the three-dimensional universe, while $\beta$ gives the size of the orbifold. Furthermore, the $d = 4$ world-volume of the three-brane universe exhibits $N = 1$ SUSY (of course SUSY is broken in the dynamical solution) and a particular solution exists for which the domain wall world-volume expands in a FRW-like manner while the orbifold radius contracts.

Although the above model provides an intriguing use of M-theory in an attempt to answer questions about early universe cosmology there are still many problems to be worked out. Foremost, these are vacuum solutions, devoid of matter and radiation. There is no reason to think that, of all the solutions, the one which matches our universe (expanding domain-wall, shrinking orbifold) should be preferred over any other. This problem is typical of many cosmological models, however. The Calabi-Yau (six-dimensional) three-fold is chosen by hand in order to give four noncompact dimensions. Hence, the dimensionality problem mentioned in Section 3 is still present in this model. Stabilization of moduli fields, including the dilaton has recently been addressed in [47]. There are no cosmological constants in the model. There is also no natural mechanism supplied for SUSY breaking on the domain wall, and currently no discussion of inflationary dynamics. For more on heterotic M-theory and cosmology see, [36]-[51].

6 Large Extra Dimensions

This section provides a brief discussion of scenarios involving large extra dimensions, focusing primarily on the models of Randall and Sundrum (RSI and RSII) [52, 53]. The RSI model is similar in many respects to that of the Lukas, Ovrut and Waldram scenario discussed in section 3, although its motivation is quite different. In the LOW construction the motivation was to construct a cosmology out of the fundamental theory of everything. In the RSI model the motivation is to construct a cosmology in which the Hierarchy problem of the Standard Model (SM) is solved in a natural way. Some earlier proposals involving large extra dimensions include [50]-[54]. Also see the extensive set of references in [61].

6.1 Motivation and the Hierarchy Problem

There is a hierarchy problem in the Standard Model because we have no way of explaining why the scales of particle physics are so different from those of gravity. Many attempts to solve the hierarchy problem using extra dimensions have been made before, see for example [50] and [60]. If spacetime is fundamentally $(4 + n)$-dimensional then the physical Planck

\footnote{The distinction between RSI and RSII models will be clarified below.}
mass  

\[ M_{pl}^{(4)} \simeq 2 \times 10^{18} \text{GeV}, \]  

(6.1)

is actually dependent on the fundamental \((4 + n)\)-dimensional Planck mass \(M_{pl}\) and on the geometry of the extra dimensions according to

\[ M_{pl}^{(4)} = M_{pl}^{n+2} V_n, \]  

(6.2)

Here \(V_n\) is the volume of the \(n\) compact extra dimensions. Because we have not detected any extra dimensions experimentally, the compactification scale \(\mu_c \sim 1/V_n^{1/n}\) would have to be much smaller than the weak scale, and the particles and forces of the SM (except for gravity) must be confined to the four-dimensional world-volume of a three-brane (See Fig. (6.1)).

We see from (6.2) that by taking \(V_n\) to be large enough it is possible to eliminate the hierarchy between the weak scale \(v\) and the Planck scale. Unfortunately, in this procedure a new hierarchy has been introduced, namely the one between \(\mu_c\) and \(v\). Randall and Sundrum proposed the following: We assume that the particles and forces of the SM with the exception of gravity are confined to a four-dimensional subspace of the \((4 + n)\)-dimensional spacetime. This subspace is identified with the world-volume of a three-brane and an ansatz for the metric is made. Randall and Sundrum’s proposal is that the metric is not factorizable, but the four-dimensional metric is multiplied by a “warp” factor that is
exponentially dependent upon the radius of the bulk, fifth dimension. The metric ansatz is

\[ ds^2 = e^{-2kr\phi} \eta_{\mu\nu} \, dx^\mu \, dx^\nu + r_c^2 \, d\phi^2, \]

where \( k \) is a scale of order the Planck scale, \( \eta_{\mu\nu} \) is the four-dimensional Minkowski metric and \( 0 \leq \phi \leq \pi \) is the coordinate for the extra dimension. Randall and Sundrum have shown that this metric solves the Einstein equations and represents two three-branes with appropriate cosmological constant terms separated by a fifth dimension. The above scenario, in addition to being able to solve the hierarchy problem (see section 6.2.1), provides distinctive experimental signatures. Coupling of an individual Kaluza-Klein (KK) excitation to matter or to other gravitational modes is set by the weak and not the Planck scale. There are no light KK modes because the excitation scale is of the order a TeV. Hence, it should be possible to detect such excitations at accelerators (such as the LHC). The KK modes are observable as spin 2 excitations that can be reconstructed from their decay products. For experimental signatures of KK modes within large extra dimensions see e.g. \[172, 173, 174\].

6.2 Randall-Sundrum I

The basic setup for the RSI model is depicted in Fig. (6.2). The angular coordinate \( \phi \) parameterizes the fifth dimension and ranges from \( -\pi \) to \( \pi \). The fifth dimension is taken as the orbifold \( S^1/Z_2 \) where there is the identification of \( (x, \phi) \) with \( (x, -\phi) \). The orbifold fixed points are at \( \phi = 0, \pi \) and correspond with the locations of the three-brane boundaries of the five-dimensional spacetime. Note the similarities of this model with the LOW model of Section 5. One difference is that we are now considering nonzero vacuum energy densities on both the visible and the hidden brane and in the bulk.
The Randall-Sundrum scenario. The fifth dimension is compactified onto the orbifold $S^1/Z_2$.

The action describing the scenario is

$$S = S_{\text{grav}} + S_{\text{vis}} + S_{\text{hid}}$$

(6.4)

where

$$S_{\text{grav}} = \int d^4x \int_{-\pi}^{\pi} d\varphi \sqrt{-G} \left( -\Lambda + 2M^3R \right)$$

$$S_{\text{vis}} = \int d^4x \sqrt{-g_{\text{vis}}} (\mathcal{L}_{\text{vis}} + V_{\text{vis}})$$

$$S_{\text{hid}} = \int d^4x \sqrt{-g_{\text{hid}}} (\mathcal{L}_{\text{hid}} + V_{\text{hid}}).$$

(6.5)

Here, $M$ is the Planck mass, $R$ is the Ricci scalar, $g_{\text{vis}}$ and $g_{\text{hid}}$ are the four-dimensional metrics on the visible and hidden sectors respectively and $V_{\text{vis}}$, $\Lambda$ and $V_{\text{hid}}$ are the cosmological constant terms in the visible, bulk and hidden sectors. The specific form for the three-brane Lagrangians is not relevant for finding the classical five-dimensional, ground state metric. The five-dimensional Einstein equations for the above action are

$$\sqrt{-G} \left( R_{MN} - \frac{1}{2} G_{MN} R \right) = -\frac{1}{4M^3} \left[ \Lambda \sqrt{-G} G_{MN} 

+ V_{\text{vis}} \sqrt{-g_{\text{vis}}} g_{\mu\nu}^{\text{vis}} \delta_M^\mu \delta_N^\nu \delta(\varphi - \pi) 

+ V_{\text{hid}} \sqrt{-g_{\text{hid}}} g_{\mu\nu}^{\text{hid}} \delta_M^\mu \delta_N^\nu \delta(\varphi) \right].$$

(6.6)

We now assume that a solution exists which has four-dimensional Poincaré invariance in the $x^\mu$ directions. A five-dimensional ansatz which obeys the above requirements is

$$ds^2 = e^{-2\sigma(\varphi)} g_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\varphi^2.$$
Substituting this ansatz into (6.6) reduces the Einstein equations to
\[
\frac{6\sigma'^2}{r_c^2} = -\frac{\Lambda}{4M^3}, \quad (6.8)
\]
\[
\frac{3\sigma''}{r_c^2} = \frac{V_{hid}}{4M^3r_c}\delta(\varphi) + \frac{V_{vis}}{4M^3r_c}\delta(\varphi - \pi). \quad (6.9)
\]
Solving (6.8) consistently with orbifold symmetry \(\varphi \to -\varphi\), we find
\[
\sigma = r_c|\varphi|\sqrt{-\frac{\Lambda}{24M^3}}, \quad (6.10)
\]
which makes sense if \(\Lambda < 0\). With this choice, the spacetime in the bulk of the theory is a slice of an \(AdS_5\) manifold. Also, to solve (6.9) we should take
\[
V_{hid} = -V_{vis} = 24M^3k \quad \Lambda = -24M^3k^2. \quad (6.11)
\]
Note that the boundary and bulk cosmological terms are dependent upon the single scale factor \(k\), and that the relations between them are required in order to get four-dimensional Poincaré invariance.

Further connections with the LOW scenario of Section 5 are now visible. The exact same relations given in (6.11) arise in the five-dimensional Hořava-Witten effective theory if one identifies the expectation values of the background three-form field as cosmological terms [39].

We want the bulk curvature to be small compared to the higher dimensional Planck scale in order to trust the solution and thus, we assume \(k < M\). The bulk metric solution is therefore,
\[
d s^2 = e^{-2kr_c|\varphi|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\varphi^2. \quad (6.12)
\]
Since \(r_c\) is small but still larger than \(1/k\), the fifth dimension cannot be experimentally observed in present or future gravity experiments. This prompts us to search for a four-dimensional effective theory.

### 6.2.1 Four-Dimensional Effective Theory

In our four-dimensional effective description we wish to find the parameters of this low-energy theory (e.g. \(M_{pl}^{(4)}\) and mass parameters of the four-dimensional fields) in terms of the five-dimensional, fundamental scales, \(M\), \(k\) and \(r_c\). In order to find the four-dimensional theory one identifies massless gravitational fluctuations about the classical solution 6.12 which correspond to the gravitational fields for the effective theory. These are the zero
modes of the classical solution. The metric of the four-dimensional effective theory is of the form
\[ \bar{g}_{\mu\nu}(x) \equiv \eta_{\mu\nu} + \tilde{h}_{\mu\nu}(x), \tag{6.13} \]
which is locally Minkowski. Here, \( \tilde{h}_{\mu\nu}(x) \) represents the tensor fluctuations about Minkowski space and gives the physical graviton of the four-dimensional effective theory. By substituting the metric (6.13) for \( \eta_{\mu\nu} \) in (6.12) and then using the result in the action (6.5) the curvature term becomes
\[ S_{\text{eff}} \propto \int d^4x \int_{-\pi}^{\pi} d\varphi 2 M^3 r_c e^{-2 k r_c |\varphi|} \sqrt{-\bar{g}} \bar{R}, \tag{6.14} \]
where \( \bar{R} \) is the four-dimensional Ricci scalar made out of \( \bar{g}_{\mu\nu}(x) \). We focus on the curvature term so that we may derive the scale of the gravitational interactions. The effective fields depend only on \( x \), and hence it is possible to perform the integration over \( \varphi \) explicitly, obtaining the four-dimensional effective theory \[52\]. Using the result one may derive an expression for the four-dimensional Planck mass in terms of the fundamental, five-dimensional Planck mass
\[ M_{pl}^{(4)} = M^3 r_c \int_{-\pi}^{\pi} d\varphi e^{-2 k r_c |\varphi|} = \frac{M^3}{k} [1 - e^{-2 k r_c \pi}], \tag{6.15} \]
Notice that \( M_{pl}^{(4)} \) depends only weakly on \( r_c \) in the large \( k r_c \) limit.

From the fact that \( g^{\text{vis}}_{\mu\nu}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \varphi = \pi) \) and \( g^{\text{hid}}_{\mu\nu}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \varphi = 0) \) we find,
\[ \bar{g}_{\mu\nu} = g^{\text{hid}}_{\mu\nu}, \tag{6.16} \]
but
\[ \bar{g}_{\mu\nu} = g^{\text{vis}}_{\mu\nu} e^{2 k r_c \pi}. \tag{6.17} \]
It is now possible to find the matter field Lagrangian of the theory. With proper normalization of the fields one can determine physical masses. Let us consider the example of a fundamental Higgs field. The action is
\[ S_{\text{vis}} \simeq \int d^4x \sqrt{-g_{\text{vis}}} \left( g^{\mu\nu}_{\text{vis}} D_{\mu} H^\dagger D_{\nu} H - \lambda \left( |H|^2 - v_0^2 \right)^2 \right), \tag{6.18} \]
which contains only one mass parameter \( v_0 \). Using (6.17) the action becomes
\[ S_{\text{eff}} \simeq \int d^4x \sqrt{-\bar{g}} e^{-4 k r_c \pi} \left( \bar{g}^{\mu\nu} e^{2 k r_c \pi} D_{\mu} H^\dagger D_{\nu} H - \lambda \left( |H|^2 - v_0^2 \right)^2 \right), \tag{6.19} \]
and after wavefunction renormalization, \( H \to e^{k r_c \pi} H \), we have
\[ S_{\text{eff}} \simeq \int d^4x \sqrt{-\bar{g}} \left( \bar{g}^{\mu\nu} D_{\mu} H^\dagger D_{\nu} H - \lambda \left( |H|^2 - e^{-2 k r_c \pi} v_0^2 \right)^2 \right). \tag{6.20} \]
This result is completely general. The physical mass scales are set by a symmetry-breaking scale,
\[ v \equiv e^{-kr_c} v_0, \]  
and hence any mass parameter \( m_0 \) on the visible three-brane is related to the fundamental, higher-dimensional mass via
\[ m \equiv e^{-kr_c} m_0. \]  
Note that if \( e^{kr_c} \sim 10^{15} \), TeV scale physical masses are produced from fundamental mass parameters near the Planck scale, \( 10^{19} \text{GeV} \). Therefore, there are no large hierarchies if \( kr_c \approx 50 \).

### 6.3 Randall-Sundrum II

In the RSI scenario described in the last section our universe was identified with the negative tension brane while the brane in the hidden sector had positive tension (Eq. (6.11)). In this model it was shown that the hierarchy problem may be solved. Unfortunately, there are several problems with the idea that the universe we live in can be a negative tension brane. For one, the energy density of matter on such a brane would be negative and gravity repulsive \([115, 114, 111]\). Life is more comfortable on a positive tension brane since the D-branes which arise as fundamental objects in string theories are all positive tension objects and the localization of matter and gauge fields on positive tension branes is well understood within the context of string theory.

For the above reasons Randall and Sundrum suggested a second scenario (RSII) in which our universe is the positive tension brane and the hidden brane has negative tension \([53]\). In this case the boundary and bulk cosmological constants are related by
\[ V_{\text{vis}} = -V_{\text{hid}} = 24M^3k \]
\[ \Lambda = -24M^3k^2, \]  
as opposed to the realtion in RSI, Eq. (6.11).
gravitational fluctuations,

\[
\left( \partial_\mu \partial^\mu - \partial_i \partial^i + V(z_i) \right) \hat{h}(x^\mu, z_i) = 0. \tag{6.24}
\]

This has a non-trivial potential term \( V \) resulting from the curvature, \( \mu \) runs from 0 to 3 and \( i \) labels the extra dimensions. It is possible to write \( \hat{h} \) as a superposition of modes

\[
\hat{h} = e^{ip\cdot x} \hat{\psi}(z) \text{ where } \hat{\psi} \text{ is an eigenmode of the equation}
\]

\[
\left( -\partial_i \partial^i + V(z) \right) \hat{\psi}(z) = -m^2 \hat{\psi}(z), \tag{6.25}
\]

in the extra dimensions and \( p^2 = m^2 \). Hence, the higher-dimensional gravitational fluctuations are Kaluza-Klein reduced in terms of four-dimensional KK states with mass \( m^2 \) given by the eigenvalues of \((6.25)\). The zero mode that is also a normalizable state in the spectrum of Eq. \((6.25)\) is the wave function associated with the four-dimensional graviton. This state is a bound state whose wave function falls off rapidly away from the 3-brane. Such behavior corresponds to a 3-brane acting as a positive tension source on the right hand side of Einstein’s equations.

The procedure of RSII is to decompactify the orbifold of RSI (i.e. consider \( r_c \to \infty \)) taking the hidden, negative tension brane off to infinity. In doing this, one obtains an effective four-dimensional theory of gravity where the setup is a single three-brane with positive tension embedded in a five-dimensional bulk spacetime. On this brane one can compute an effective nonrelativistic gravitational potential between two particles of masses \( m_1 \) and \( m_2 \) which is generated by exchange of the zero-mode and continuum Kaluza-Klein mode propagators. The potential behaves as

\[
V(r) = G_N \frac{m_1 m_2}{r} \left( 1 + \frac{1}{r^2} k^2 \right). \tag{6.26}
\]

Here the leading term is the usual Newtonian potential and is due to the bound state mode. The KK modes generate the \( 1/r^3 \) correction term which is heavily suppressed for \( k \) of order the fundamental Planck scale and \( r \) of the size tested with gravity. The propagators calculated in \([53]\) are relativistic and hence, going beyond the nonrelativistic approximation one recovers all the proper relativistic corrections with negligible corrections from the continuum modes.

Let us compare the RSI and RSII models. In RSI, the solution to the hierarchy problem requires that we are living on a negative tension brane. The positive tension brane has no such suppression of its masses and is therefore often referred to as the “Planck” brane, which is hidden from the visible brane. Serious arguments against this scenario are that the negative tension “TeV” brane seems physically unacceptable.
In RSII, the visible brane is taken as the positive tension brane while the TeV brane is sent off to infinity. In this model the proper Newtonian gravity is manifest on the visible brane, but the hierarchy problem is not addressed.

Although more successful as a potential physical model of our universe than its predecessor RSI, RSII seems to lack the elegant solution to the hierarchy problem made possible by considering the universe as a negative tension brane. Recent work however suggests that by including quantum effects (analogous to the Casimir effect) it is possible to solve the hierarchy problem on the visible brane having either positive or negative tension [83]. If the Casimir energy is negative and one accepts a degree of fine tuning of the tension on the hidden brane it is possible to obtain a large enough warp factor to explain the hierarchy on the visible brane having either positive or negative tension. Further work on this scenario is needed however including a study of the stability of this model against perturbations.

6.4 RS and Brane World Cosmology

The next obvious step is to consider the cosmologies of the RS model discussed above. There has been an extensive amount of work done in these areas and the reader is invited to examine the references at the end of the review related to Randall-Sundrum and “brane world” cosmologies [62] - [138] for a comprehensive study. Due to the vast number of cosmological models discussed in the literature we will review only the basics and focus on the problems of brane world cosmologies while mentioning potential resolutions and future work, referencing various relevant authors. Much of the discussion in this section closely parallels the excellent review of J. Cline [97].

We begin by considering the cosmological expansion of 3-brane universes in a 5-dimensional bulk with a cosmological constant as discussed by Binétruy, Deffayet, Ellwanger and Langlois (BDEL) [117]. Note that in an earlier work [116], BDL considered the solutions to Einstein’s equations in five dimensions with an $S_1/Z_2$ orbifold and matter included on the two branes but with no cosmological constants on the branes or in the bulk. They found that the Hubble expansion rate of the visible brane was related to the energy density of the brane quadratically opposed to the standard Friedmann equation, $H^2 \propto \rho$. We will show this explicitly below. The altered expansion rate proved to be incompatible with nucleosynthesis constraints.

When the analysis was applied to the RSII scenario one does in fact reproduce the ordinary FRW universe on the positive tension, Planck brane [113, 114]. Note however, that in the RSII scenario on the negative tension brane where the hierarchy problem is
solved the Friedmann equation has a critical sign difference.

In the BDEL model the authors consider five-dimensional spacetime metrics of the form
\[
ds^2 = \tilde{g}_{AB} \, dx^A \, dx^B = g_{\mu\nu} \, dx^\mu \, dx^\nu + b^2 \, dy^2
\]  
(6.27)
where \( y \) is the coordinate associated with the fifth dimension. The visible universe is taken to be the hypersurface at \( y = 0 \). The metric is taken to be
\[
ds^2 = -n^2(\tau, y) \, d\tau^2 + a^2(\tau, y)g_{ij} \, dx^i \, dx^j + b^2(\tau, y) \, dy^2,
\]  
(6.28)
where \( \gamma_{ij} \) is a maximally symmetric three-dimensional metric (\( k = -1, 0, 1 \) will parametrize the spatial curvature), and \( n \) is a lapse function.

The five-dimensional Einstein equations have the usual form
\[
\tilde{G}_{AB} \equiv \tilde{R}_{AB} - \frac{1}{2} \tilde{R} \tilde{g}_{AB} = \kappa^2 \tilde{T}_{AB},
\]  
(6.29)
where \( \kappa \) is related to the five-dimensional Newton’s constant \( G_{(5)} \) and the five-dimensional reduce Planck mass \( M_{(5)} \) by
\[
\kappa^2 = 8\pi G_{(5)} = M_{(5)}^{-3}.
\]  
(6.30)
Using the ansatz (6.28) one finds the non-vanishing components of the Einstein tensor to be
\[
\tilde{G}_{00} = 3 \left\{ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{n^2}{b^2} \left( \frac{a''}{a} + \frac{a'}{a} \left( \frac{a'}{a} - \frac{b'}{b} \right) \right) + k \frac{n^2}{a^2} \right\},
\]  
(6.31)
\[
\tilde{G}_{ij} = \frac{a^2}{b^2} \gamma_{ij} \left\{ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} + \frac{\dot{n}'}{n} \right) - \frac{\dot{b}}{b} \left( \frac{n'}{n} + \frac{2\dot{a}'}{a} \right) + 2 \frac{a''}{a} + \frac{n''}{n} \right\}
\]  
+ \frac{a^2}{n^2} \gamma_{ij} \left\{ \frac{\ddot{a}}{a} \left( -\frac{\dot{a}}{a} + \frac{2\dot{n'}}{n} \right) - 2 \frac{\dot{a}'}{a} + \frac{\dot{b}}{b} \left( -\frac{2\dot{a}}{a} + \frac{\dot{n}}{n} \right) - \frac{\dot{b}}{b} \right\} - k \gamma_{ij},
\]  
(6.32)
\[
\tilde{G}_{05} = 3 \left\{ \frac{n'}{n} \frac{\dot{a}}{a} + \frac{\dot{a}'}{a} \frac{b}{b} - \frac{\dot{a}}{a} \right\},
\]  
(6.33)
\[
\tilde{G}_{55} = 3 \left\{ \frac{a'}{a} \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{b^2}{n^2} \left( \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\dot{a}}{a} \right) - k \frac{b^2}{a^2} \right\}.
\]  
(6.34)
Here the prime indicates differentiation with respect to \( y \) and dot indicates differentiation with respect to \( \tau \).

The energy-momentum tensor can be described in terms of the fields living on the visible brane world-volume \( T^A_B \) and the fields living in the bulk space (or on other branes) \( \tilde{T}^A_B \). We have
\[
T^A_B = \frac{\delta(y)}{b} \, \text{diag}(-\rho, p, p, p, 0),
\]  
(6.35)
where the energy density \( \rho \) and pressure \( p \) are independent of the position in the brane in order to recover a homogeneous cosmology on the brane. The total energy-momentum tensor is then

\[
\tilde{T}^A_B = T^A_B + \tilde{T}^A_B.
\]  

(6.36)

Note that in reality the brane would have some thickness in the fifth dimension determined by the fundamental scale of the underlying theory. However, the presence of the delta function in (6.35) (the “thin-brane” approximation) should be valid when the energy scales are much smaller than the fundamental scale. In what follows unless otherwise mentioned we will take \( k = 0 \) as in [116].

From the Bianchi identity \( \nabla_A \tilde{G}^A_B = 0 \) and the Einstein equations (6.29) an equation of conservation is obtained,

\[
\dot{\rho} + 3(p + \rho) \frac{\dot{a}}{a} = 0,
\]  

(6.37)

which matches the usual four-dimensional equation of energy density conservation in standard cosmology. Here \( a_0 \) is the value of \( a \) on the brane.

To find a solution of Einstein’s equations (6.29) in the vicinity of the visible brane at \( y = 0 \) one must deal with the delta function sources. The details may be found in [117]. From the 55 component equation (6.34) one finds a new Friedmann-like equation

\[
\frac{\dot{a}_0^2}{a_0^2} + \frac{\ddot{a}_0}{a_0} = -\frac{\kappa^4}{36} \rho (\rho + 3p) - \frac{\kappa^2}{3b_0^2} \tilde{T}_{55}.
\]  

(6.38)

Immediately, and as mentioned above one sees the unusual quadratic dependence \( H^2 \propto \rho^2 \).

Note that if one allows for a cosmological constant in the bulk \( \Lambda \) and extra energy densities on the Planck and TeV branes (\( \rho_P \) and \( \rho_T \), respectively), in addition to the respective tensions \( \sigma \) and \( -\sigma \) one finds a Friedmann-like equation of the form

\[
H^2 = \frac{(\sigma + \rho_P)^2}{36M^6} + \frac{\Lambda}{6M^3} = \frac{(-\sigma + e^{-2kb} \rho_T)^2}{36M^6} + \frac{\Lambda}{6M^3}.
\]  

(6.39)

When the tension \( \sigma \) is fine tuned to cancel the contribution from \( \Lambda \) in the limit \( \rho_i = 0 \), it is possible to recover the correct FRW behavior \( H \propto \sqrt{\rho} \) at leading order in \( \rho \) [115, 114, 97]. Interestingly, this fine tuning is exactly that required by RS to obtain a static solution. Unfortunately while the cosmology on the Planck brane appeared normal, the energy density on the TeV brane, where the hierarchy problem is solved, is negative which is physically unacceptable as we mentioned above.

\[8^6\text{Now we have switched to the notation of [97] but here } k \text{ is the } k \text{ introduced in our discussion of RSI, Section (6.2).}\]
Exciting new developments have shown that when the radion is stabilized the previously mentioned unconventional cosmologies in the RS model disappear \cite{115}. By assuming that a 5-dimensional potential $U(b)$ is generated by some mechanism (e.g. \cite{138}) in the 5-dimensional theory, the nonvanishing equations of motion in the bulk (with cosmological constant $\Lambda$) reduce to

$$
\tilde{G}_{00} = \kappa^2 n^2 (\Lambda + U(b)),
$$

$$
\tilde{G}_{ii} = -\kappa^2 a^2 (\Lambda + U(b)),
$$

$$
\tilde{G}_{55} = -\kappa^2 b^2 (\Lambda + U(b) + U'(b)).
$$

(6.40)

Here $\tilde{G}_{AB}$ is given by (6.31)-(6.34) with $k = 0$. Let us introduce notation $m_0$ such that the static RS solution is recovered when $V_p = -V_T = 6m_0/\kappa^2$ and $\Lambda = -6m_0^2/\kappa^2$. We take the locations of the Planck and TeV branes to be at $y = 0$ and $y = 1/2$, respectively. To simplify the solution of (6.40) the radion is assumed to be very heavy and near its minimum $U \approx M_5^2 ((b - b_0)/b_0)^2$. Here $b_0$ is the stabilized value of $b$ and $M_5$ is proportional to the radion mass $m_{rad}$. Now one may perturb around the RS solution, with cosmological constants $\delta V_p$ and $\delta V_T$ instead of matter densities. Using the ansatz

$$
a(t, y) = e^{Ht - |y|m_0b_0} (1 + \delta a(y)),
$$

$$
n(t, y) = e^{-|y|m_0b_0} (1 + \delta a(y)),
$$

$$
b = b_0,
$$

(6.41)

it is possible to derive the Friedmann equation

$$
H^2 = \frac{\kappa^2 m_0}{3(1 - \Omega_0)} (\delta V_p + \delta V_T \Omega_0^4),
$$

(6.42)

where $\Omega_0 \equiv e^{-m_0b_0/2}$. Note that (6.42) is the standard Hubble law with correct normalization for the physically observed energy density $\rho = \delta V_p + \delta V_T \Omega_0^4$.

The constraint between the matter on the two branes was a consequence of trying to find a static solution to the radion equations of motion without actually providing a mechanism for stabilization. Once such a mechanism is introduced the constraint vanishes as described above, and the ordinary 4-dimensional FRW behavior is recovered at low temperatures if the radion has a mass of order the weak scale. It was suggested in \cite{115} that matter on the hidden brane or in the bulk may be a dark matter candidate.

As we have already discussed above, it seems unlikely that the RSI scenario as presented in \cite{52} can provide a physically realistic cosmological model as the energy density on the TeV brane is negative. The RSII model, having non-compact extra dimension, has greater
success as a cosmological model in that it correctly reproduces the conventional cosmology on the visible brane (see e.g. [117]). Other variations of both RSI and RSII and alternative brane world models have also produced correct cosmological behaviors of our universe. The reader is referred to the review [97] for a detailed summary of work on RS cosmology.

Important problems and challenges which need to be explained in brane world scenarios include the stabilization of the radius of the extra dimension and the radion field [132]-[138], inflation [121]-[131], incorporation into supergravity models [139]-[154], string theory and the AdS/CFT correspondence [155]-[165]. In particular see the review [155] and the references therein. For more on the cosmological constant and brane worlds see [99, 148] and [166]-[171]. For early versions of brane world scenarios see [54]-[61]. Experimental predictions are discussed in e.g. [172]-[174]. Cosmologies of brane world scenarios are analyzed in [62]-[120].

6.5 Supersymmetry

We will have only a few comments in this section, as the work in this area is still too new to review. There have been a number of attempts to include supersymmetry into the RS and brane world scenarios [139]-[154]. Supersymmetry may play an important role in many aspects of brane world models such as fine-tuning between bulk and brane cosmological constants and the stabilization of the fifth dimension (BPS vacua are stable against perturbations). Furthermore, supersymmetry and supergravity are critical aspects of string theory and hence it should be expected that they will play an integral role in string theory realizations of brane world scenarios.

Although there was legitimate concern that brane world models may be impossible to realize as BPS or non-BPS configurations of a supersymmetric theory [153, 150], recent work has found a way to circumvent these no-go theorems (see, e.g. [146]). In [146] the authors obtain the original Randall-Sundrum configuration from type IIB supergravity. This is achieved by considering a solution to the $D = 10$ type IIB supergravity equations which has a 5D interpretation. Note however that this is not fully a $D = 5$ solution as it requires the $S^5$ massive Kaluza-Klein breathing mode. Breathing modes of sphere reductions are often useful in supporting domain walls [146, 163, 141, 154]. In this model it is possible to recover the single brane RSII model by pushing the hidden brane off to the Cauchy horizon of AdS. Another pleasing feature of this model is that the D3-brane configuration is dynamically stable.

Another interesting work provides a supersymmetric version of the minimal RS model
in which the branes are singular \[151\].

Note that not all scenarios involving large extra dimensions rely on supersymmetry, such as the ADD model described in \[59\]. The ADD scenario is not without its own troubles however as it has light KK gravitinos which could cause drastic problems with nucleosynthesis and the cosmic gamma ray background \[97\].

As an increasing number of works on the supersymmetrization of the RS model become available we will no doubt gain a better understanding of how this configuration should be assimilated into models of M/superstring theory and the AdS/CFT correspondence.

7 Conclusions

In this review we have discussed a number of intriguing approaches to string and M-theory cosmology. While the past few years have shown a considerable increase in our understanding of M-theory, there is still plenty of room for further research.

Perhaps the greatest advances have come from the discovery of duality symmetries in the M-theory moduli space, D-branes, the AdS/CFT correspondence and the development of Matrix theory. As demonstrated in this review we have taken the first steps to incorporate this new knowledge into cosmology. M-theory provides an innovative framework in which to study the early Universe and to search for alternatives to the Standard Big-Bang and Inflationary models. Conversely, cosmology is essential to our study of M-theory, since couplings and masses set by the vacuum state of string theory must agree with those observed in our Universe. The amalgamation of M-theory and cosmology may reveal the answers to a number of tantalizing questions and provide the tools to probe the earliest moments of creation.

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