Modified perturbation theory approach for $t\bar{t}$ production and decay

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Abstract

The modified perturbation theory (MPT), based on direct expansion of probabilities instead of amplitudes, allows one to avoid divergences in the phase-space integrals resulting from production and decay of unstable particles. In the present paper the range of applicability of MPT is determined numerically in the case of the process $e^+e^-\rightarrow (\gamma,Z)\rightarrow t\bar{t}\rightarrow W^+bW^-\bar{b}$. It is shown that with the complete expansion in powers of the coupling constant (without Dyson resummation) MPT operates best at the energies located near the maximum of the cross-section and slightly above the maximum. In this region the MPT expansion within the next-to-leading order considerably exceeds in accuracy well-known DPA approach.

1 Introduction

The problem of high-precision description of the processes with production and decay of unstable particles is consistently of great importance, in particular, in view of the forthcoming time of electron linear colliders (TESLA, NLC, CLIC). The solving of this problem requires methods maintaining gauge invariance. One such method is the pole scheme [1] based on expansion of the amplitude in Laurent series around the mass-shell of unstable particles. In the case of pair production of unstable particles the leading order (LO) of this expansion is known as the double pole approximation (DPA). In the case of W-boson pair production studied at LEP2 the DPA was the sole approach where the one-loop radiative corrections had been actually evaluated [2].

Nevertheless, the DPA itself does not provide a high enough precision. At LEP2, while calculating the one-loop corrections, this difficulty was concealed by the additional small factor $\alpha/\pi$ which diminished the systematic error to the acceptable level. However in the Born approximation the systematic error was too large, such large that it forced one to use the gauge-dependent CC03 off-shell cross-section instead of the DPA Born cross-section required conceptually and for preserving gauge-invariance. As a whole the situation was considered acceptable for the needs of LEP2 [3]. However it hardly probable can be considered acceptable, both at the Born and the one-loop level, in the case of the high-precision calculations needed for the experiments on future linear colliders [4].

Fortunately the pole scheme is not the sole approach that maintains the gauge-invariance. This property should be inherent in the modified perturbation theory (MPT) [4,5,6], as
well, which is based on direct expansion of probabilities in powers of the coupling constant instead of amplitudes. The expansion of probabilities in MPT is made possible due to the applying of the methods of distributions [7] which allow one to escape the divergences in the phase-space integrals. As a result the probability in MPT is represented as an expansion in a power series in the coupling constant. This means that one can expect the gauge invariance of each term of the expansion, at least, if to believe in gauge invariance of the exact result.

Unfortunately, the above picture not everywhere is completely legitimate since the MPT series not everywhere is well-defined and not everywhere has admissible convergence properties. In fact, the MPT series is ill-defined when the energy takes a value of the threshold of the production of unstable-particles [8]. (It is worth emphasizing the difference between the threshold and the mass shell in the case of the pair, or multiple production of unstable particles.) One way to solve this problem is to partially resum the MPT series, by the analogy with the Dyson resummation in the amplitude. The result of such resummation can be different from that in the conventional approach where the Dyson resummation is carried out in the amplitude and only then the probability is calculated (see discussion in [5, 6]). Nevertheless, the gauge invariance in MPT after the Dyson resummation all the same can be broken, and so the question about the gauge cancellations in each case should be additionally examined. This problem should not arise in the MPT series with the complete expansion in the coupling constant. So, it is important to find a range of energies where the MPT series well converges. If there is such a range and if the MPT is able to give there more precise description than the DPA approach, then the MPT is preferable to carry out the high-precision calculations in this range of energies.

The searching for this range of energies is the main aim of the present paper. The study is carried out on an example of the process \( e^+e^- \to (\gamma, Z) \to t\bar{t} \to W^+bW^-\bar{b} \) which is a part of the more general process \( e^+e^- \to 6f \) planed for future linear colliders. In the present paper the above-mentioned process is considered in the Born approximation for the cross-section but with the taking into consideration of the one-loop and two-loop corrections in the Dyson resummed propagators before applying the MPT expansion. This is the necessary first step for a key solution of all the problem. By the subsequent step the similar examination should be carried out in more realistic case which would more precisely simulate the complete process \( e^+e^- \to 6f \) with taking into consideration the other loop corrections. Some aspects of this consideration concerning, in particular, the non-factorizable corrections, have already been discussed in [5, 6].

The structure of the present paper is as follows. In the next Section we discuss the details of the definition of the Born cross-section for the process \( e^+e^- \to (\gamma, Z) \to t\bar{t} \to W^+bW^-\bar{b} \) with the Dyson resummation in the \( t \)-quark propagators. In Section 3 the same process is considered in MPT approach within the next-to-leading-order (NLO) approximation. In Section 4 we present the results of numerical calculations and compare the “exact” result of Section 2 with the results of MPT and DPA approaches. Section 5 summarizes the main results of the paper.
2 The statement of the problem

The total cross-section of the unpolarized process $e^+e^−\rightarrow(γ, Z)\rightarrow t\bar{t}\rightarrow W^+bW^−\bar{b}$ (see Fig. 1) under the supposition of stability of $W$ and $b$, with massive $W$ bosons and massless $b$ quarks reads as follows:

$$\sigma(s) = \int \frac{dz}{4M_W^2/s} \phi(z; s) \tilde{\sigma}(zs), \quad (1)$$

$$\tilde{\sigma}(s) = \int_{M_W^2}^{(\sqrt{s}-M_W)^2} \int_{M_W^2}^{(\sqrt{s}-\sqrt{s_+})^2} ds_+ \tilde{\sigma}(s; s_+, s_-). \quad (2)$$

Here $\phi(z; s)$ is the flux function describing the large QED corrections that arise due to virtual and real photon radiation from initial states,

$$\phi(z; s) = \beta_e(1-z)^{(\beta_e-1)} - \frac{1}{2}\beta_e(1+z), \quad \beta_e = \frac{2\alpha}{\pi} \left(\ln \frac{s}{m_e^2} - 1 \right). \quad (3)$$

Variables $s_\pm$ stand for the invariant masses of the final-state systems $W^+b$ and $W^−\bar{b}$. The quantity $\tilde{\sigma}(s; s_+, s_-)$ is the exclusive cross-section of the off-shell process with the Dyson resummation in $t$-quark propagators. In general case $\tilde{\sigma}(s; s_+, s_-)$ can be written as

$$\tilde{\sigma}(s; s_+, s_-) = \frac{1}{s^2} \sqrt{\lambda(s; s_+, s_-)} F(s; s_+, s_-) \rho(s_+) \rho(s_-). \quad (4)$$

In formula (4) we have explicitly extracted the density of flow of the colliding particles and the $t\bar{t}$ phase-volume factor. Function $F$ is to be determined by direct calculations in the conventional perturbation theory with taking into account the Breit-Wigner factors which are chosen in the form

$$\rho(s) = \frac{M\Gamma_0}{\pi} \times |\Delta(s)|^2. \quad (5)$$

Here $M$ is the mass of $t$ quark, $\Gamma_0$ is its Born width, and $\Delta(s)$ is the scalar factor of the Dyson resummed propagator of $t$ quark. In general case we have

$$\Delta(s) = \frac{1}{s - M^2 + \text{Re}\Sigma(s) + i \text{Im}\Sigma(s)}. \quad (6)$$

![Figure 1: The process $e^+e^- \rightarrow (\gamma, Z) \rightarrow t\bar{t} \rightarrow W^+bW^−\bar{b}$.](image-url)
where \( \Sigma(s) \) is the scalarized self-energy. We suppose that \( \Sigma(s) \) is determined in the on-mass-shell (OMS) scheme of UV renormalization, more precisely, in some variant of its generalization to the unstable-particle case. Actually there are many such variants. The conventional one [9] implies the gauge-dependence in the renormalized mass at the two-loop order. The variants free from this defect are considered in [10, 11].

It is well-known that the self-energy in the denominator of a Dyson resummed propagator has to be determined with the exceeding precision with respect to that of the cross-section. For example, if \( \hat{\sigma}(s; s_+, s_-) \) is to be determined within the one-loop precision, then \( \Sigma(s) \) in (6) has to be determined at least within the two-loop precision, etc. This rule follows from the relation \( s - M^2 = O(\alpha) \) which takes place in the resonance region, so that \( \Sigma(s) \) becomes comparable with \( s - M^2 \). Outside the resonance region the above-mentioned rule is irrelevant.

Further on we are interested in the Born approximation for \( \hat{\sigma}(s; s_+, s_-) \), determined with the Dyson resummation implemented within the NLO precision. This particular quantity is a part of the complete cross-section determined with NLO precision. So, we have to define \( \Sigma(s) \) in propagator (6) in the resonance region within the two-loop precision and outside the resonance region within the one-loop precision.

In respect to the imaginary part of \( \Sigma(s) \) the above requirement can be satisfied by putting

\[
\text{Im}\Sigma(s) = \sqrt{s} \Gamma(s),
\]

where \( \Gamma(s) \) is the width determined within the one-loop precision (two-loop precision for \( \text{Im}\Sigma \)) and multiplied by the Born \( s \)-dependent factor. In other words, by definition we put

\[
\Gamma(s) = \Gamma(M^2) \times f(s)/f(M^2),
\]

where \( f(s) = \sqrt{s} \lambda(s; M_W^2, 0) \times (1 + M_W^2/s - 2M_W^4/s^2) \) [12], and \( \Gamma(M^2) \) is the sum of the Born and the one-loop contributions to the on-shell width,

\[
\Gamma(M^2) = \Gamma_0 + \alpha \Gamma_1.
\]

Notice that the self-energy within the same precision can be written as \( \Sigma(s) = \alpha \Sigma_1(s) + \alpha^2 \Sigma_2(s) \) where \( \alpha \Sigma_1 \) and \( \alpha^2 \Sigma_2 \) are the one- and two-loop contributions, respectively.

The real part of the renormalized self-energy in the most general case can be written as

\[
\text{Re}\Sigma(s) = r^{(0)} + (s - M^2) r^{(1)} + \frac{1}{2} (s - M^2)^2 r^{(2)}(s).
\]

Here \( r^{(0)} \) and \( r^{(1)} \) are numerical coefficients, and \( r^{(2)}(s) \) is a function finite at \( s = M^2 \). The coefficients \( r^{(0)} \) and \( r^{(1)} \) are dependent on the UV renormalization scheme. In the framework of conventional generalization of the OMS scheme [9] they are zero,

\[
r^{(0)} = 0, \quad r^{(1)} = 0.
\]

In the generalized by [10] OMS scheme they are equal to

\[
r^{(0)} = -\alpha^2 \text{Im}\Sigma_1(M^2) \text{Im}\Sigma'_1(M^2) + O(\alpha^3), \quad r^{(1)} = 0.
\]

In the so called OMS scheme [11] they are

\[
r^{(0)} = -\alpha^2 \text{Im}\Sigma_1(M^2) \text{Im}\Sigma'_1(M^2) + O(\alpha^3), \quad r^{(1)} = -\frac{1}{2} \alpha^2 \text{Im}\Sigma_1(M^2) \text{Im}\Sigma''_1(M^2) + O(\alpha^3).
\]
Formulas (7)–(13) completely determine the structure of the Dyson resummed propagator. In what follows we put $r^{(2)} = 0$ which is reasonable in the context of verification of the MPT approach in the LO+NLO approximation. For determinancy, we adhere to the OMS scheme of UV renormalization.

### 3 The LO and NLO in MPT

The essence of the MPT approach [4] factually consists in expanding in the sense of distributions [7] of the propagator squared of unstable particle $|\Delta(s)|^2$ into an asymptotic series in powers of the coupling constant. Up to $O(\alpha)$ corrections this series looks as follows:

$$|\Delta(s)|^2 = \frac{\pi}{\alpha \text{Im}\Sigma_1(M^2)} \delta(s - M^2) - \frac{\text{Im}\Sigma_2(M^2)}{[\text{Im}\Sigma_1(M^2)]^2} \delta(s - M^2) + \text{VP} \frac{1}{(s - M^2)^2} + O(\alpha). \tag{14}$$

Here $\delta(\cdots)$ is the $\delta$-function, $\text{VP}$ means the principal-value prescription. Each term of expansion (14) is to be integrated with some weight function when substituting into the formulas for the cross-section. The first term in (14) in view of (7) represents the well-known formula for the Breit-Wigner resonance in the narrow-width approximation. The second and the third terms describe the NLO corrections. The omitted terms contain the $\delta$-function, its derivatives, and the $\text{VP}$-terms with the poles of the higher orders.

Further on we consider the LO and the NLO approximation only, i.e. we restrict ourselves by the analysis of the terms displayed by formula (14). Correspondingly, the cross-section $\hat{\sigma}(s)$ can be represented as

$$\hat{\sigma}(s) = \hat{\sigma}_0(s) + \hat{\sigma}_1(s), \tag{15}$$

where $\hat{\sigma}_0(s)$ and $\hat{\sigma}_1(s)$ stand for the LO and NLO approximations. Combining (4), (5) and (14), we get

$$\hat{\sigma}_0(s) = \theta(s - 4M^2) \times \frac{1}{s^2} \sqrt{\lambda(s; M^2, M^2)} F(s; M^2, M^2). \tag{16}$$

The $\hat{\sigma}_1(s)$ can be represented in a form of sum of two terms, $\hat{\sigma}_1(s) = \hat{\sigma}_1^{an}(s) + \hat{\sigma}_1^{VP}(s)$. Here the first term, the so-called “anomalous” term [5, 6], differs on a factor from $\hat{\sigma}_0(s)$,

$$\hat{\sigma}_1^{an}(s) = -2 \frac{\alpha \Gamma_1}{\Gamma_0} \hat{\sigma}_0(s). \tag{17}$$

The second term includes the $\text{VP}$ contribution,

$$\hat{\sigma}_1^{VP}(s) = 2 \frac{M \Gamma_0}{\pi} \theta(s - (M + M_W)^2) \times \int_{M^2_W} (s - (M + M_W)^2)^2 \frac{1}{s^2} \sqrt{\lambda(s; M^2, \tilde{s})} F(s; M^2, \tilde{s}) \times \text{VP} \frac{1}{(\tilde{s} - M^2)^2}. \tag{18}$$

Factor 2 in both formulas, (17) and (18), appears in view of the pairness of production of unstable particles in the given process.
It is worth noticing that $\hat{\sigma}_1^{VP}(s)$, in contrast to $\hat{\sigma}_0(s)$ and $\hat{\sigma}_1^{an}(s)$, includes nonzero contribution below the threshold.\footnote{Nevertheless this contribution becomes zero below $\sqrt{s} = M + M_W$ since in NLO one of two $t$ quarks is produced on-shell. The NNLO approximation contains contributions with two $t$ quarks produced off-shell and these contributions are nonzero till the low physical threshold, $\sqrt{s} = 2M_W$ with the massless $b$ quarks.} The principal-value symbol in (18) at its calculating can be omitted because the double-pole singularity turns out to be beyond the integrating area. Correspondingly, at $s \to 4M^2$ from below, the function $\hat{\sigma}_1^{VP}(s)$ possesses a singular behavior,

$$\hat{\sigma}_1^{VP}(s) \bigg|_{s \to 4M^2} = \frac{\Gamma_0}{(2M)^2} F(4M^2, M^2, M^2) \left(4M^2 - s\right)^{-1/2}. \quad (19)$$

At $s > 4M^2$ the double pole in (18) gets into the integration area and is regularized by the principal value. Correspondingly, at $s \to 4M^2$ from above, integral (18) takes a finite value.

Substituting (16)–(18) into formula (1) and calculating the convolution, we get the total cross-section $\sigma(s)$ with taking into consideration the effect of the initial state radiation. In the case of $\hat{\sigma}_0(s)$ and $\hat{\sigma}_1^{an}(s)$, in so doing, we do not meet any problems. Below the threshold their contributions are zero due to the $\theta$-function in (16), and while going over the threshold $s = 4M^2$ the nonzero contributions are switching-off continuously. In the case of $\hat{\sigma}_1^{VP}(s)$ we have completely different situation because of singularity (19). Really, though this singularity is integrable the contribution of $\hat{\sigma}_1^{VP}(s)$ to $\sigma(s)$ turns out to be divergent at $s \to 4M^2$ (from above and below) as $|s - 4M^2|^{\beta-1/2}$. Such a behavior is the consequence of a superposition of singularities in the flux function $\phi(z; s)$ at $z \to 1$ and in the function $\hat{\sigma}_1^{VP}(zs)$ at $zs \to 4M^2$. The occurrence of the divergence in $\sigma(s)$ means that its expansion in powers of $\alpha$ becomes formal and certainly has bad convergence properties in a neighborhood of the threshold.

Actually, the latter property is a common one for all the processes with production and decay of unstable particles. The appearance of the divergencies in the MPT series is a manifestation of a nonanalyticity in the coupling constant occurring on the threshold in the cross-section $\sigma(s)$. In turn, the occurrence of the nonanalyticity is conditioned by the convolution of the hard-scattering cross-section $\hat{\sigma}(s)$ with the photon flux function $\phi(z; s)$ which is singular and includes a nonanalyticity in the missing energy in the limit of the soft photons (at $z \to 1$). The fact of presence of the nonanalyticity in the coupling constant in the convoluted cross-section at the threshold can be verified by direct calculation, most easily in the case of a single production and decay of unstable particles.

As mentioned above, a way to solve the problem is to Dyson resum the MPT series. This camouflages the nonanalyticity in the coupling constant and the cross-section becomes a regular function at the threshold. However after the Dyson resummation the property of the gauge cancellations becomes not apparent. So, it makes sense to take advantage of another strategy, which is to shift from the threshold and to not consider this region at all while considering the convoluted cross-section. In this case all terms of the complete MPT expansion of $\sigma(s)$ appear to be finite and it becomes possible to state a problem about the precision of approximation of the series by a finite number of its terms. The precision of such an approximation is determined by the first discarded term. For example, the error of the approximation of $\sigma(s)$ by $\sigma_0(s)$ can be estimated as $O(\sigma_1(s))$. More precisely, there is an asymptotic estimate $\sigma(s) = \sigma_0(s) + O(\sigma_1(s))$ which determines the LO approximation of $\sigma(s)$. The LO+NLO approximation is determined by the sum of two terms, $\sigma(s) = \sigma_0(s) + \sigma_1(s)$, with an error $O(\sigma_2(s))$ which is to be determined in the framework of NNLO approximation.
The definition of $\hat{\sigma}_2(s)$ which is necessary for calculating $\sigma_2(s)$ is reduced to the integrals of the above type and also to the double integral which corresponds to product of the $VP$-terms of formula (14). The calculating of the latter integral is a nontrivial task since a naive calculation leads to an occurrence of a nonintegrable singularity in $\hat{\sigma}_2(s)$ at $s \rightarrow 4M^2$. The occurrence of a nonintegrable singularity means that the function $\hat{\sigma}_2(s)$ should be considered as a distribution, by the complete analogy to the case of a single $|\Delta(s)|^2$. The solution of this problem, however, is beyond the scope of the present paper. So, in the next Section we carry out the “phenomenological” determination of the error $O(\sigma_2(s))$ basing on the direct evaluation of $\sigma(s)$ and $\sigma_0(s) + \sigma_1(s)$.

4 Numerical results and discussion

On the basis of the above consideration we are able to begin the searching for the ranges of $s$ where the LO and NLO approximations of MPT operate satisfactorily. First we find $s$ where $\sigma_1(s)$ is small enough. (Below we determine the meaning of the term “small enough”.) Thus we determine the range of satisfactory action of the LO approximation. Then we compare $\sigma_0(s) + \sigma_1(s)$ with $\sigma(s)$ calculated by means of formulas of Section 2 and find the values of $s$ where $\sigma(s) - [\sigma_0(s) + \sigma_1(s)]$ is small enough. This will be the “phenomenological” determination of the range of applicability of LO+NLO approximation.

We use $M = 175$ GeV, $M_W = 80.4$ GeV, and the following input-data for the width [13]:

$$\Gamma_0 = 1.56 \text{ GeV}, \quad \Gamma_0 + \alpha\Gamma_1 = 1.45 \text{ GeV}. \quad (20)$$

Function $F(s; s_+, s_-)$ introduced by formula (4) we calculate by means of CompHEP package [14] with the subsequent phase-space integration as described in [15]. The results of calculations of $\sigma(s)$, $\sigma_0(s)$ and $\sigma_0(s) + \sigma_1(s)$ are represented in the vicinity of the threshold by Fig.2a and above the threshold by Fig.3a. The results for the DPA cross-section are represented by the same figures, where $\sigma_{DPA}(s)$ is determined by the same formulas that were used for $\sigma(s)$, but with substituting $F(s; M^2, M^2)$ for $F(s; s_+, s_-)$ and $|s - M^2 + i\Gamma|^2$ for $|\Delta(s)|^2$. As the “exact” $\Gamma$ we use $\Gamma_0 + \alpha\Gamma_1 + \alpha^2\Gamma_2 = 1.42$ GeV [13]. The $\sigma_0(s)$, $\sigma_0(s) + \sigma_1(s)$ and $\sigma_{DPA}(s)$ in percentage of $\sigma(s)$ are represented by Fig.2b and Fig.3b. In the same place we represent the results for $\sigma_1(s)/\sigma(s)$, more precisely for $100% + \sigma_1(s)/\sigma(s)\%$ where 100% is added for the convenience of representation of the results on the same figures.

For the analysis of the above results a criterion of smallness of the corrections must be fixed. For this purpose we again make use of the input-data (20) where $\Gamma_0$ and $\alpha\Gamma_1$ are the LO and NLO approximations for the width. In what follows we consider $\Gamma_0$ and $\alpha\Gamma_1$ by typical representatives for the LO and NLO approximations and use them for the definition of the corresponding scales. In so doing, we also take into consideration the factor 2 that arises in the cross-section by virtue of the pairness of the production of unstable particles. So, $\sigma_1(s)$ will be considered small if the ratio $\sigma_1(s)/\sigma(s)$ will be no more or about $2 \times \alpha\Gamma_1/\Gamma = 15.5\%$. For the correction $\sigma_2(s)$ the analogous estimate makes up $2 \times \alpha^2\Gamma_2/\Gamma = 4.2\%$. The “phenomenological” criteria are as follows: the LO approximation

\footnote{Unfortunately, $\alpha^2\Gamma_2$ is determined not finally because only the QCD two-loop corrections have been taken into consideration. This fact introduces some conditionality into the subsequent discussion.}
could deviate from the exact result by no more or about $2 \times |\Gamma - \Gamma_0|/\Gamma = 19.7\%$. For the LO+NLO approximation the similar estimate makes up $2 \times |\Gamma - \Gamma_0 - \alpha \Gamma_1|/\Gamma = 4.2\%$.

By combining the above results we see that the correction $\sigma_1(s)$ is small enough ($\leq 15.5\%$), beginning practically with the threshold, more precisely with $\sqrt{s} = 350.8$ GeV, and persisting up to the end of the tested area, $\sqrt{s} = 1$ TeV. In the same region also the quantity $\sigma(s) - \sigma_0(s)$ is small ($\leq 19.7\%$), which is responsible for the “phenomenological” criterion of the applicability of LO.

As for the LO+NLO approximation, until the correction $\sigma_2$ is calculated we have the “phenomenological” criterion only. According to this criterion the LO+NLO approximation is good in two narrow regions placed in the neighborhood of the threshold, and in a broad region $450$ GeV $\leq \sqrt{s} \leq 650$ GeV. The first two regions seemingly are not too reliable in view of the vicinity of the “cutoff” of LO and of the large NLO corrections. The third, broad region satisfies to all criteria. Moreover, at the larger energies the exceeding of the LO+NLO approximation of the 4.2%-limits is hardly noticeable. For instance, at the end of the tested area $\sqrt{s} = 1$ TeV the discrepancy $\sigma(s) - [\sigma_0(s) + \sigma_1(s)]$ makes up only 6.3% of $\sigma(s)$, which practically falls under the definition “about” 4.2%. With this stipulation it is possible to say that the LO+NLO approximation operates satisfactorily from 450 GeV up to the end of the tested area.

In the practically interesting region for the linear colliders, at $\sqrt{s} = 500$ GeV, the LO+NLO approximation deviates from the “exact” $\sigma(s)$ on 0.2% only. This means that the MPT approach works very well in this region. For comparison, the deviation of DPA from the “exact” result at $\sqrt{s} = 500$ GeV makes up 6.4%, which is more than by factor 10 as greater. At the other interesting point $\sqrt{s} = 800$ GeV the convergence property of MPT changes to the worse and the precision of the LO+NLO approximation becomes 5.5% against 2.9% of DPA. Nevertheless a possibility of the practical application of MPT in this region is not closed since the results in MPT can be further improved by means of evaluating the NNLO correction. For getting greater clearness in this question also the two-loop correction to the width of $t$ quark is necessary.

In closing this Section it is worth mentioning that the above results are found rather stable in a relative calculus with respect to varying the function $F(s; s_+, s_-)$. This gives a ground to expect that the similar results can take place in other processes with the pair production of unstable particles. Moreover, one can expect that our results will remain in force with taking into account the loop corrections in the function $F(s; s_+, s_-)$, including the case when just the loop corrections are discussed. If this is the case, then the MPT opens an opportunity to unprecedentedly increase the precision of the description of the processes with the pair production and decay of unstable particles at the energies near the maximum of the cross-section.

5 Conclusion

The analysis of this paper shows that the MPT approach considered with the complete expansion in powers of the coupling constant (without Dyson resummation) operates satisfactorily in the range of the maximum of the cross-section and somewhat higher in the energies. This result is obtained in the case of the pair production of $t$ quarks in the reaction...
of $e^+e^-$ annihilation within the LO+NLO approximation, and can be further improved by attracting the higher orders of the MPT expansion. At the point $\sqrt{s} = 500$ GeV, designed by TESLA and other linear-collider projects, the LO+NLO approximation of MPT more than on one decimal order exceeds in accuracy the DPA approach.

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Figure 2: The behavior of $\sigma(s)$, $\sigma_0(s)$, $\sigma_0(s) + \sigma_1(s)$ and $\sigma_{\text{DPA}}(s)$ in a neighborhood of the threshold. In Fig.2a the results are presented in absolute arbitrary units, in Fig.2b in percentage of $\sigma(s)$. In Fig.2b also the results for $\sigma_1(s)$ in percentage of $\sigma(s)$ are presented with adding 100% for the convenience of representation of the results on the same graph. The filled area designates a 4.2%-corridor of the applicability of the LO+NLO approximation.
Figure 3: The behavior of $\sigma(s)$, $\sigma_0(s)$, $\sigma_0(s) + \sigma_1(s)$ and $\sigma_{\text{DPA}}(s)$ above the threshold. The notation is the same that on Fig.2.