Decoupling of superconducting layers in magnetic superconductor RuSr$_2$GdCu$_2$O$_8$

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We propose the model for magnetic properties of the magnetic superconductor RuSr$_2$GdCu$_2$O$_8$, which incorporates the theory of the superconducting/ferromagnetic multilayers. The transition line $T_d(h)$, on which the Josephson coupled superconducting planes are decoupled, i.e. $j_c(T_d) = 0$, is calculated as a function of the exchange energy $h$. As the result of this decoupling a nonmonotonic behavior of magnetic properties, like the lower critical field $H_{c1}$, Josephson plasma frequency, etc. is realized near (or by crossing) the $T_d(h)$ line. The obtained results are used in analyzing the newly discovered antiferromagnetic ruthenocuprate RuSr$_2$GdCu$_2$O$_8$ with possible weak ferromagnetic order in the RuO planes.
I. INTRODUCTION

The physics of magnetic superconductors is interesting due to competition of magnetic order and singlet superconductivity in bulk materials. The problem of their coexistence was first set up theoretically in the pioneering work by V. L. Ginzburg in 1956, while the experimental progress in the field begun after the discovery of ternary rare earth (RE) compounds (RE)Rh$_4$B$_4$ and (RE)Mo$_6$X$_8$ (X=S,Se) with a regular distribution of localized RE magnetic moments. It turned out that in many of these systems superconductivity (with the critical temperature $T_c$) coexists rather easily with antiferromagnetic (AF) order (with the critical Néel temperature $T_N$), where usually the situation with $T_N < T_c$ is realized. Due to their antagonistic characters singlet superconductivity and ferromagnetic order cannot coexist in bulk samples with realistic physical parameters. However, under certain conditions the ferromagnetic order is transformed, in the presence of superconductivity, into a spiral or domain-like structure - depending on the type and strength of magnetic anisotropy in the system. As the result of this competition, these two orderings coexist in a limited temperature interval $T_{c2} < T < T_m$ (the reentrant behavior) in ErRh$_4$B$_4$ and HoMo$_6$S$_8$, or even down to $T = 0$ K in HoMo$_6$Se$_8$, where $T_m$ is the critical temperature for the existence of the inhomogeneous magnetic order. The coexistence region in ErRh$_4$B$_4$ is narrow where $T_c = 8.7$ K, $T_m \approx 0.8$ K, $T_{c2} \approx 0.7$ K, while for HoMo$_6$S$_8$ it is even narrower with $T_c = 1.8$ K, $T_m \approx 0.74$ K, $T_{c2} \approx 0.7$ K - see Refs. [2], [3]. In most of the new quaternary rare-earth compounds (RE)Ni$_2$B$_2$C the antiferromagnetic order and superconductivity coexist up to $T = 0$ K [4], while in HoNi$_2$B$_2$C an additional oscillatory magnetic structure is realized in a limited temperature interval. This oscillatory magnetic structure competes strongly with superconductivity giving rise to reentrant behavior in this compound. Recently the Pobell’s group in Bayreuth made a remarkable discovery of the coexistence of superconductivity and nuclear magnetic order in AuIn$_2$ with $T_c = 0.207$ K and $T_m = 35 \mu$ K. This exciting phenomenon was explained in Ref. [5] where it is argued that superconductivity can coexist either with spiral or domain-like nuclear magnetic ordering only, depending on the strength of magnetic anisotropy in this cubic system. Important contribution to the physics of magnetic superconductors has been done in Ref. [6], where for the first time was proposed the coexistence of weak-ferromagnetism and superconductivity. In such a case the spontaneous vortex state due to weak-ferromagnetism is also possible.

We point out that in the above cited magnetic superconductors the exchange interaction (between localized magnetic moments and conduction electrons) influences superconductivity much stronger than the electromagnetic interaction. The latter is due to the localized magnetic moments which create dipolar magnetic field, thus affecting the orbital motion of superconducting electrons.

Recently, a new class of magnetic superconductors based on layered perovskite ruthenocuprate compound RuSr$_2$GdCu$_2$O$_8$ comprising CuO$_2$ bilayers and RuO monolayers has been synthesized [7]. This compound belongs also to the class of high-$T_c$ superconductors (HTS). A subsequent study of transport and magnetic properties has revealed that it exhibits some kind of ferromagnetic order at the critical temperature $T_N = (133 - 137)$ K. The polarized neutron scattering measurements [8] show that the magnetic structure (which appears at $T_N$) is predominantly antiferromagnetic with a Ru
magnetic moment $\mu_{\text{Ru}} \approx 1.18 \mu_B$ along the $c$-axis at low temperature. The same measurements put on an upper limit $\sim 0.1 \mu_B$ to any net ferromagnetic zero-field Ru moment. Concerning the last point the important results came from magnetization measurements first reported in Ref. [9], which show a hysteresis loop and remanent magnetization. The latter hints to existence of a ferromagnetic component in the system. Recent magnetization measurements on RuSr$_2$EuCu$_2$O$_8$ [11] give evidence for a small ferromagnetic component, which lies probably parallel to the RuO plane, with the magnetic moment (per Ru) $\sim 0.05 \mu_B$ at 5 K consistent with the neutron scattering data [10]. Note that the smaller value of magnetic moment ($0.05 \mu_B$) in this compound tells us that in the Gd-compound some admixture of the large Gd moment might take place. This conclusion is also confirmed by the zero-field muon spin rotation (ZF-$\mu$SR) measurements [12] which provide important evidence that the magnetic order is homogeneous on a microscopic scale and accounts for most of the sample volume. At lower temperatures the superconductivity sets in at $T_c = (35 - 45)$ K without affecting the AF order [10], [12] notably. This fact means that superconductivity - which is realized predominantly in the CuO$_2$ planes, and magnetic order - which is present only in the RuO planes, interact rather weakly, i.e. these two orders are separated spatially. Recently, it was reported [13] that in Ru$_{1-x}$Sr$_2$GdCu$_{2+x}$O$_{8-y}$ the highest superconducting critical temperature reaches 72 K for $x = 0.3 - 0.4$, while there is no sign of the weak-ferromagnetic (WF) component in the RuO planes.

It seems that RuSr$_2$GdCu$_2$O$_8$ has very interesting magnetic properties, which might result in the absence of Meissner phase in some samples [14], [15], while in some others it is realized [9], [16]. (This problem will be briefly discussed in Section IV.)

In this paper we propose a model of layered magnetic superconductor with weak-ferromagnetism, which might be relevant for the RuSr$_2$GdCu$_2$O$_8$ compound - the SWF model. This model, studied in Section II, assumes the existence of S/F multilayers with small hopping parameter $t$ between S (superconducting) and F (ferromagnetic) planes along the $c$-axis, i.e. $t < T_c$. As a result the small $t$ gives rise to an effective Josephson coupling current $j_c$ between superconducting planes. It turns out that $j_c$ is suppressed by the exchange field - present in the F-plane only, which causes drastic changes of magnetic properties. The Gibbs free-energy $\mathcal{G}$ of such a magnetic superconductor with both AF and WF orderings in external magnetic field $\mathbf{H}$ is formulated in Section III. Based on it the lower critical field $H_{c1}$ is also studied there. The estimation of theoretical parameters of the SWF model from the experimental results in RuSr$_2$GdCu$_2$O$_8$ is done in Section IV, where the obtained results are discussed too.

II. MODEL FOR S/F ATOMIC MULTILAYER AND JOSEPHSON CURRENT

As was mentioned above we consider the magnetic superconductor RuSr$_2$GdCu$_2$O$_8$ as a prototype for S/F atomic multilayers by assuming good conduction in CuO$_2$ planes - with the quasiparticle spectrum $\xi_S(p)$, and a small hopping parameter $t < T_c$ between the S- and F-planes (i.e., along the $c$-axis). The second assumption is related to the existence of WF order (with the magnetization $\mathbf{M}$ lying in the RuO planes) which gives rise to an effective exchange field parameter $h = h e_{ab}$. The latter affects spins of conduction electrons with the dispersion $\xi_F(p)$ moving in the normal conducting RuO planes. (The
The parameter $h$ can be related to an effective spontaneous spin $S_{\text{eff}}$ (magnetization normalized to saturation magnetization) in the $ab$-plane, i.e. $h = J^{ab}S_{\text{eff}}$ — see also Section IV.

The electronic part of the SWF model is similar to the model in the Ref. \cite{17} and in what follows the same notation is used. According to this model the elementary cell of the superlattice consists of one superconducting and one ferromagnetic layer which are both metallic. For simplicity it is supposed here that both layers have similar quasiparticle energy spectra, i.e. $\xi(p) \equiv \langle \xi_S(p) \rangle \approx \xi_F(p))$. It is also assumed, that the superconductivity is realized in S-planes ($\text{CuO}_2$ planes) with pairing coupling $g(p)$ (having in mind application to the HTS compound RuSr$_2$GdCu$_2$O$_8$ the clean limit, $\xi_0 \ll l$, is supposed). The Hamiltonian of the system is given by
\begin{align*}
H &= H_0 + H_{\text{int1}} + H_{\text{int2}}, \\
H_0 &= \sum_{\mathbf{p},n,i,\sigma} \xi(p) a_{n,i,\sigma}^\dagger(p) a_{n,i,\sigma}(p) + t \left[ a_{n+1,i,\sigma}^\dagger(p) a_{n+1,i,\sigma}(p) + a_{n-1,i,\sigma}^\dagger(p) a_{n-1,i,\sigma}(p) + h.c. \right], \\
H_{\text{int1}} &= \frac{1}{2} \sum_{\mathbf{p}_1, \mathbf{p}_2, n, \sigma} g(p_1 - p_2) a_{n,i,\sigma}^\dagger(p_1) a_{n,i,-\sigma}^\dagger(-p_1) a_{n,i,-\sigma}(-p_2) a_{n,i,\sigma}(p_2), \\
H_{\text{int2}} &= - \sum_{\mathbf{p}, n, \sigma} h \sigma a_{n,i,\sigma}^\dagger(p) a_{n,i,-\sigma}(p),
\end{align*}
where $a_{n,i,\sigma}^\dagger(p)$ is the creation operator of an electron with spin $\sigma$ (the quantization axis is parallel to the $ab$-plane) in the $n$-th elementary cell and momentum $\mathbf{p}$ in the layer $i$ is parallel to the $ab$-plane, where $i = 1$ for the S layer, and $i = -1$ for the F layer. Since the obtained results below are qualitatively similar for $s$- and $d$-wave pairing, the calculations were done for $s$-wave pairing where $g(p) = g_0$ is constant, while quantitative changes due to $d$-wave pairing are discussed below and in Section IV.

By assuming that the order parameter changes from cell to cell in the manner $\Delta_n = |\Delta| e^{i\varphi_n}$ (with $\varphi_n = kn$ in absence of orbital effects) the quasiparticle Green’s functions are obtained in the standard way \cite{17}. The self-consistency equation for the order parameter $|\Delta|$ reads \cite{17}
\begin{equation}
\frac{1}{\Lambda} = T \sum_{\omega} \int_{-\infty}^{\infty} d\xi \int_{0}^{2\pi} \frac{dq}{2\pi} \frac{\bar{\omega} + \bar{\omega}_-}{|\Delta|^2 |\Delta| - (\omega - \bar{\omega}) - |T_{q+k}|^2 (\omega + \bar{\omega} - |T_q|^2)}, \tag{1}
\end{equation}
where $\Lambda = g_0 \rho(0)$ and $\rho(0) = m_\parallel/2\pi$ is the electron density of states at the Fermi level in the normal state, $\omega_\pm = i\omega \pm \xi(p)$, $\bar{\omega}_\pm = \omega_\pm + h$, $\omega = \pi T(2n + 1)$. The quasimomentum $q$ lies in the direction perpendicular to the layers, and $T_q = 2t \cos(q/2)e^{iq/2}$ and $T_{q+k} = 2t \cos((q + k)/2)e^{i(q+k)/2}$.

The free energy $\mathcal{F}$ in the superconducting state is obtained by using the following relation
\begin{equation}
\frac{\partial \mathcal{F}}{\partial |\Delta|} = \frac{|\Delta|}{\Lambda} - \frac{T \rho(0)}{2\pi} \sum_{\omega} \int_{0}^{\infty} \int_{0}^{2\pi} d\xi dq F_{11}^\dagger, \tag{2}
\end{equation}
where the expression for $F_{11}^\dagger$ is obtained in \cite{17}.
In order to study transport and magnetic properties in magnetic field we need to know the supercurrent $j_z$ flowing across the layers (along the $c$-axis in RuSr$_2$GdCu$_2$O$_{8}$). In this case, the vector potential $A_z = A_z e_z$ enters the Hamiltonian through the substitution $t \rightarrow te^{i e A_z / c}$, where $d$ is S-F interlayer distance, and the part of the Hamiltonian depending on $A_z$ is given by

$$H_A = \sum_{p,n,i,\sigma} t[a^\dagger_{n,1,\sigma}(p)a_{n,-1,\sigma}(p)e^{i e A_z / c} + a^\dagger_{n+1,-1,\sigma}(p)a_{n,1,\sigma}(p)e^{i e A_z / c} + h.c.]$$

The supercurrent across the planes is obtained by the standard procedure

$$j_z = -\frac{c}{2d} \frac{\delta H_A}{\delta A_z}.$$ 

Note that the Josephson supercurrent in the S/F superlattice is carried by Andreev bound states, similarly to the S/N case. In S layer the supercurrent is carried by Cooper pairs, but in N layer it flows via quasiparticles, which recondense in the next S layer; bound states represent this process [18].

In the case of small hopping parameter $t \ll T_c$ the Josephson current along the $c$-axis is obtained in leading order (proportional to $t^4$) by standard perturbation theory. After the integration over the energy $\xi$ it reads

$$j_z = 4 e \pi |\Delta|^2 t^4 \rho(0) T \sum_{\omega > 0} \left\{ 4\omega - \frac{5h^4 + 6h^2|\Delta|^2 + |\Delta|^4 - 4h^2 \omega^2}{R^2(\omega)(\omega^2 + h^2)} \right\} \sin(k)$$

$$= j_c \sin k,$$

where $R(\omega) = (|\Delta|^2 + h^2)^2 + 4h^2\omega^2$ and $\Omega(\omega) = \sqrt{\omega^2 + |\Delta|^2}$. As in the standard Josephson effect, the supercurrent $j_z$ is proportional to $\sin k$, $k$ being the phase difference between $n$- and $n + 1$ S-layers.

In what follows we calculate numerically the critical current $j_c$ (in Eq.(3)) at any point of the phase diagram $(T, h)$ by replacing $|\Delta| \rightarrow \Delta_0(T)$, where $\Delta_0(T)$ is given by the BCS theory. The latter is correct due to the smallness of $t$, in which case $T_c$ is practically unaffected by the exchange field, i.e. $T_c \approx T_{c0}$ up to the second order terms in $t/T_{c0}$.

From Eq.(3) it comes out in particular, that near $T_c$ and for $h = 0$ one has $j_c > 0$, while $j_c < 0$ for $h \gg T_c$. The change of sign of $j_c$ (near $T_c$), which corresponds to the transition from $k = 0$ to $k = \pi$ in the ground state, occurs at $h_c = 3.77T_{c0}$, in accordance with the calculation in [17]. At low temperatures, $T \rightarrow 0$, $j_c$ goes to zero at $h/\Delta_0(0) \approx 1/2$, which just corresponds to $h_{c0} = 0.87T_{c0}$ at $T = 0$, again in accordance with Ref. [17]. Note that the same approach if applied to d-wave pairing [19] gives $h_{c0}^{(d)} = 0.6T_{c0}$ at $T = 0$. The sign-change of $j_c$ is related to the transition from the “0”-phase to “π”-phase. This transition goes smoothly if we take into account the higher order term ($\sim t^8 \cos 2k$) in the free-energy, in fact it means that the width of the region $\Delta h$ where the transition from “0”-phase to “π”-phase occurs is of the order of $\Delta h \sim t^4 / T_{c0}^3$. In the case of weak hopping $t \ll T_{c0}$ this region is very narrow and we may define the decoupling line $j_c(T_d, h) = 0$.
which results in the $(T_d, h)$ phase diagram shown in Fig.1.

![Fig. 1](image1.png)

**FIGURE 1.** The $(T, h)$-phase diagram for the case $t \ll T_{c0}$. The Josephson decoupling line $j_c(T, h) = 0$ - black line.

The temperature dependence of the Josephson penetration depth $\lambda_J = \sqrt{c\phi_0 / 8\pi^2 |j_c| (2d)}$ [20], where $\phi_0$ is the flux quantum, is shown in Fig.2 for various $h \neq 0$. Here $2d$ is the period of the multilayer. One should note its nonmonotonic behavior if $h \neq 0$, particularly when $h \sim T_c$. Based on these results one can analyze some magnetic properties, like the lower critical field $H_{c1}$ in the $ab$-plane.

![Fig. 2](image2.png)

**FIGURE 2.** The $T$-dependence of the Josephson penetration depth $\lambda_J(T, h)$ for various $h$. We defined 
\[ \Delta_0 = 1.76T_{c0}, \quad j_{c0} = e\rho(0)t^4/\Delta_0^2, \quad \text{and} \quad \lambda_{J0} = \sqrt{c\phi_0 / 16\pi^2 d_j c_0}. \]

III. GIBBS ENERGY AND IN-PLANE CRITICAL FIELD $H_{C1}$
A. Gibbs energy

In order to calculate the lower critical field \( H_c \) (and the possible absence of Meissner phase \([14], [15]\)) we need the Gibbs energy functional \( G \). Having in mind the application to the RuSr\(_2\)GdCu\(_2\)O\(_8\) we assume, according to the neutron scattering data \([10]\), that in the magnetic subsystem (F-layers coinciding with RuO planes) AF order with spins along the \( c \)-axis is realized at \( T_N \gg T_c \). The AF order parameter is \( \mathbf{L} = L_z \mathbf{e}_z \). The magnetization measurements \([9], [11]\) imply WF order with the magnetization lying (most probably) parallel to the \( ab \)-planes and with the effective moment, \(|M|/n_{Ru} = \mu_{eff} < 0.1 \mu_B\), where \( n_{Ru} \) is the density of the magnetic Ru ions. In order to construct the magnetic free-energy it is necessary to know the symmetry of the system as well as orientations of the easy axis in different sublattices. So, for instance if the magnetic anisotropy energy on different sublattices are unequal, then one expects the WF order to be realized. However, at present there are no sufficient experimental data on the local lattice distortion which might favor WF order and accordingly the preferred direction of \( M \). The above discussed experiments \([10], [12], [9], [11]\) suggest only that \( M \) is in the \( ab \)-plane, i.e. \( M = M_x \mathbf{e}_x \), where \( \mathbf{e}_y \) is in the \( ab \)-plane. As the result of this analysis the SWF model contains the following order parameters: \( L_z \) for the AF order, \( M_x \) for the WF order and \( \Delta_n(x,y) \) for the S order.

In the applied magnetic field \( \mathbf{H} \) the Gibbs energy of the layered magnetic superconductor reads - see also \([3], [8]\),

\[
G[\Delta_n, \mathbf{L}, \mathbf{M}, \mathbf{B}; \mathbf{H}] = \int dV \left( \mathcal{F}_M[\mathbf{L}, \mathbf{M}] + \frac{(\mathbf{B} - 4\pi \mathbf{M})^2}{8\pi} - \frac{\mathbf{B}\mathbf{H}}{4\pi} \right) + \sum_n (2d) \int dxdy \mathcal{F}_S[\Delta_n, \mathbf{A}],
\]

where \( \Delta_n \equiv \Delta_n(x,y) \), and \( \mathbf{L}, \mathbf{M}, \mathbf{B} \) are also coordinate dependent. The magnetic field \( \mathbf{B} = \text{rot} \mathbf{A} \) is due to the dipolar field created by the magnetic moments, the external magnetic field and the superconducting screening current. The vector potential \( \mathbf{A} = A_{ab} + A_c = A_{ab} \mathbf{e}_{ab} + A_z \mathbf{e}_z \) contains the component, \( A_{ab} \), in the \( ab \)-plane, and \( A_z \) along the \( c \)-axis.

The magnetic free-energy density functional \( \mathcal{F}_M[\mathbf{L}, \mathbf{M}] \) which mimics the experimental results in RuSr\(_2\)GdCu\(_2\)O\(_8\) with \( \mathbf{L} = L_z \mathbf{e}_z \) and \( \mathbf{M} = M_x \mathbf{e}_x \) is given by the following phenomenological expression

\[
\mathcal{F}_M[\mathbf{L}, \mathbf{M}] = \frac{\alpha}{2} \mathbf{L}^2 + \frac{\beta}{4} \mathbf{L}^4 + \frac{\delta}{2} \mathbf{M}^2 - \gamma L_z M_x + F_a(\mathbf{L}, \mathbf{M}) + F_g(\nabla \mathbf{L}, \nabla \mathbf{M}) + \ldots
\]

Although this expression is quantitatively correct near the AF transition at \( T_N \) it is also suitable for semiquantitative analysis even below superconducting transition temperature \( T_c \), due to the smallness of \( \mathbf{M} \) and \( \gamma \). The first two terms describe the AF order \((\alpha = \alpha'(T - T_N) < 0, \beta > 0)\), the third \((\delta > 0)\) and fourth \((\sim \gamma)\) terms describe the induced WF by the AF order. The parameters \( \alpha \) and \( \delta \) are due to the exchange interaction (between Ru spins in RuSr\(_2\)GdCu\(_2\)O\(_8\)), where one has \( \alpha' \sim 1/\theta_{em} \) and \( \delta \sim T_N/\theta_{em} \) with
\[ \theta_{em} = 2\pi \mu_B^2 \sim 1 \text{K} \]. The unknown anisotropy term \( F_a \) fixes the direction of \( \mathbf{L} \) and \( \mathbf{M} \), i.e. \( L_z, M_x \).

Since in the following we analyze the lower critical field along the \( ab \)-plane, with characteristic length-scales \( \lambda_{ab}, \lambda_J \gg \xi_{ab}, d_{Ru-Ru} \), where \( \xi_{ab} \) is the coherence length and \( d_{Ru-Ru} \) is the Ru-Ru distance, it is justified to omit the gradient term \( F_g(\nabla L, \nabla M) \). By minimizing \( F_M[\mathbf{L}, \mathbf{M}] \) with respect to \( L_z \) and \( M_x \) one gets \( (T_c < T < T_N) \)

\[ M_x^0 = \frac{\gamma}{\delta} L_z. \quad (8) \]

The neutron scattering and magnetization measurements give limits for \( (L_z/n_{Ru}) \sim (1 - 1.2) \mu_B \), and \( (M_x^0/n_{Ru}) < (0.05 - 0.1) \mu_B \), which implies an upper limit for \( \gamma \), i.e. \( (\gamma/\delta) \lesssim (0.05 - 0.1) \).

According to experiments [10], [12] the AF (and WF) ordering is practically unaffected by the appearance of superconductivity, then it is reasonable to neglect the effect of superconductivity on the exchange (RKKY) interaction in \( F_S[\Delta_n, \mathbf{A}] \). Therefore we keep in \( F_S[\Delta_n, \mathbf{A}] \) the electromagnetic interaction between superconducting electrons and magnetic order only

\[ F_S[\Delta_n, \mathbf{A}] = F_S[|\Delta_n|, 0] + \left( \frac{4\pi \lambda_{ab}}{c} \right)^2 j_{ab}^2 + \frac{j_c \Phi_0}{2\pi c} (1 - \cos \chi_{n,n+1}), \quad (9) \]

where \( F_S[|\Delta_n|, 0] \) is the condensation energy and \( \Delta_n = |\Delta_n| \exp(i\varphi_n) \). Note that the exchange interaction between conduction electrons and localized Ru moments affects superconductivity by renormalizing \( j_c \), which is function of \( h \).

The current in the \( ab \)-plane \( j_{ab} \) reads

\[ j_{ab} = -\frac{c}{4\pi \lambda_{ab}^2} (A_{ab} - \frac{\phi_0}{2\pi} \nabla_{ab} \varphi_n), \quad (10) \]

where \( \lambda_{ab} \) is the bulk London penetration depth in the \( ab \) superconducting layers (we assumed \( \lambda_a = \lambda_b \equiv \lambda_{ab} \)). The last term depends on the gauge invariant phase \( \chi_{n,n+1} \)

\[ \chi_{n,n+1} = \varphi_{n+1} - \varphi_n - \frac{2\pi A_z d}{\phi_0}, \quad (11) \]

which characterizes the effective Josephson coupling between two neighboring S-planes with the distance \( 2d \). It is due to the hopping between S- and F-planes and \( j_c = j_c(T, h) \) is determined by Eq.(5).

**B. Lower critical field \( H_{c1} \)**

Let us calculate the lower critical field \( H_{c1}^{ab} \) for the case when the magnetic field \( \mathbf{H} \) and the single vortex are along the magnetization \( \mathbf{M} = M_x \mathbf{e}_z \), i.e. \( \mathbf{H} = H_x \mathbf{e}_x \) and \( \mathbf{B} = B_x \mathbf{e}_z \). By the standard minimization procedure of the Gibbs free-energy \( G[\Delta_n, \mathbf{L}, \mathbf{M}, \mathbf{B}; \mathbf{H}] \) with respect to \( \Delta_n, \mathbf{L}, \mathbf{M}, \mathbf{B} \), and by assuming the continuum limit \( \mathcal{C} \), one gets the complete set of equations for these quantities as well as the Gibbs free-energy of the vortex \( G_v \) - see also [3], [8].
\[(\delta + 4\pi)M_x - \gamma L_z - B_x = 0.\]  

(12)

The Maxwell equation for the magnetic field \(B\) reads

\[
\text{rot}(B - 4\pi M) = \frac{4\pi}{c} j_s,
\]

(13)

where

\[
j_s = j_{ab} + j_z. \tag{14}\]

The in-plane current \(j_{ab}\) is given by Eq. (10) while \(j_z\) is the Josephson current between planes

\[
j_z = j_z e_z \sin \chi_{n,n+1}. \tag{15}\]

The phase \(\chi_{n,n+1}\) is given by Eq. (11). In the following we assume that the vortex axis, \(B\) and the external field \(H\) are along the \(x\)-axis. By the standard procedure we get the equation for the single vortex (centered on the origin)

\[
\lambda_{ab}^2 \frac{\partial^2 B_x}{\partial z^2} + \lambda_j^2 \frac{\partial^2 B_x}{\partial y^2} - \frac{B_x}{p} = 0. \tag{16}\]

The parameter \(p = \delta/(\delta + 4\pi)\) takes into account the additional screening due to the appearance of the WF order. After straightforward transformations the Gibbs energy of the vortex, \(G_v\), has the form

\[
G_v = \frac{p^2}{8\pi} \int dxdy \left\{ B_x^2 + \lambda_{ab}^2 \left( \frac{\partial B_x}{\partial z} \right)^2 + \lambda_j^2 \left( \frac{\partial B_x}{\partial y} \right)^2 \right\} - \frac{\Phi_0 \tilde{H}_{c1}}{4\pi}, \tag{17}\]

where

\[
\tilde{H}_{c1} = H_{ext} + 4\pi p M_0^\eta. \tag{18}\]

\(M_0^\eta\) is approximately given by Eq. (8).

The Eq. (18) can be generalized by taking into account the nonlinear core effects (20). In that case the solution is

\[
B_x(y,z) = \frac{\Phi_0}{2\pi p \lambda_{ab} \lambda_j} K_0\left(\frac{R}{\lambda_j}\right). \tag{19}\]

where

\[
R = \sqrt{y^2/\lambda_j^2 + z^2/\lambda_{ab}^2} \text{ and } K_0 \text{ is the Bessel function of the zeroth order of an imaginary argument. Inserting such a solution into Eq. (17) a straightforward calculation gives the lower critical field } \tilde{H}_{c1} \text{ from the condition } G_v = 0 \text{ as}
\]

\[
H_{ext} + 4\pi p M_0^\eta = \tilde{H}_{c1} \approx \frac{p \Phi_0}{4\pi \lambda_j \lambda_{ab}} \ln \frac{\lambda_{ab} \sqrt{p}}{d}, \tag{19}\]

where \(M_0^\eta = (\gamma/\delta)L_z\). We stress that the logarithmic factor in Eq. (19) is due to the nonlinear core effects of the Josephson vortex (20). Note that in systems with \(T_N \gg \theta_{em}\), like in RuSr\(_2\)GdCu\(_2\)O\(_8\), one has \(p \sim 1\).
From (19) it is seen that for

\[ M_x^0 > \frac{\Phi_0}{16\pi^2 J_{J} \lambda_{ab}} \ln \frac{\lambda_{ab} \sqrt{p}}{d} \] \tag{20} \]

spontaneous vortices appear in the system. This condition is more easily realized near the “0”-to-“π” transition (decoupling) line \( T_d(h) \), i.e. when \( \lambda_J \) is significantly increased. This means that in systems where the exchange parameter fulfills the condition \( 0.87 T_c < h < 3.77 T_c \) for s-wave pairing, while for d-wave pairing \( 0.87 T_c \) is replaced by \( 0.6 T_c \), then by lowering the temperature the \( \tilde{H}_{c1}(T, h) \) shows pronounced nonmonotonic behavior reaching minimum at the “0”-“π” boundary line as it is seen in Fig.3.

**FIGURE 3.** The \( T \)-dependence of the lower critical field, \( \tilde{H}_{c1}(T, h) \), in the \( ab \)-plane for various \( h \). We defined \( \tilde{H}_{c10} = (p\Phi_0 / 4\pi J_0 \lambda_{ab}) \ln \left( \lambda_{ab} \sqrt{p} / d \right) \).

### IV. COMPARISON WITH THE EXPERIMENT AND DISCUSSION

Let us discuss some relevant points related to the interpretation of the obtained results on RuSr\(_2\)GdCu\(_2\)O\(_8\).

(i) In order to analyze the magnetic properties the value of the exchange field parameter \( h = J^{ab} S_{\text{eff}} \) is needed. If one takes the experimental value \( S_{\text{eff}} \sim 0.1 \) one gets \( \gamma / \delta \sim 0.1 \) since \( S_{\text{eff}} \approx \gamma / \delta \). However, at present we do not know the relation between \( J^{ab} \) and \( T_N \). As \( d_{Ru-Ru}^{ab} \ll d_{Ru-Ru}^c \) one expects that the coupling of spins along the \( c \)-direction, \( J^c \) is much smaller than along the \( ab \)-plane, \( J^{ab} \). In such a situation one has \( T_N \sim J^{ab} / \ln(J^{ab} / J^c) \). Then, \( J^{ab} > T_N \) and \( h \sim 10 - 20 \) K. We pay attention that there are evidences that in the underdoped HTS materials d-wave pairing is realized [21]. In that case the point on the phase diagram \( j_c(T = 0, h_c) = 0 \) is realized for \( h_c^{(d)} = 0.6 T_c \). According to the specific heat measurements [12] in RuSr\(_2\)GdCu\(_2\)O\(_8\) with \( T_c = 30 - 40 \) K this compound behaves like an underdoped HTS material. If it is so it gives \( h_c^{(d)} \sim 20 \) K, i.e. \( h \) is near to \( h_c \) and a nonmonotonic behavior of \( \tilde{H}_{c1} \) is expected as shown in Fig.3.
(ii) If $M^0_x$ fulfills Eq. (20) then there is a spontaneous vortex state and the Meissner effect is absent. In opposite case the Meissner state is realized.

(iii) It may happen that $M^0_x < \tilde{H}_{c1}$ in some temperature intervals and $M^0_x > \tilde{H}_{c1}$ in the interval between, which case corresponds to a reentrant behavior.

(iv) At present the origin and the magnitude of the parameter $\gamma$ in Eqs. (7,8) is unknown. However, it may also happen that in polycrystalline samples strains induce additional changes of this quantity. A drastic case might be realized if the symmetry of the crystal implies that $\gamma = 0$. Even in that case strains in samples, for instance the component $\sigma_{xy}$, can induce a magnetic moment in piezomagnetic systems, i.e. $M^0_y \sim \sigma_{xy} L_z$ thus producing weak ferromagnetism. If strains in a sample are such that $M^0_y > \tilde{H}_{c1}$ then the Meissner phase is not realized as reported in [14], [15]. In such a way one could reconcile the opposite claims on existence [3], [10] and nonexistence [14], [15], of the Meissner phase in differently prepared samples of RuSr$_2$GdCu$_2$O$_8$.

(v) Based on the above analysis one expects that dynamical properties of such a system are very exotic. For systems near the decoupling line $j_c(T_d, h) = 0$ there is a significant reduction of the Josephson plasma frequency $\omega_{0, SF} \sim \sqrt{j_c} \ll \omega_{0, JJ}$ [22] (compared to standard Josephson junction with $\omega_{0, JJ}$) for the waves propagating along the $xy$-planes in the $S/F$ superlattice

$$\omega^2_{SF} = \omega^2_{0, SF} + v^2_{SF}q^2.$$ 

Due to the microscopic character of the S/F superlattice one expects that $v^2_{SF} \gg v^2_{JJ}$ where $v_{JJ}$ is the phase velocity for the Josephson junction made from bulk superconductors. This means that in a S/F superlattice, like for instance in RuSr$_2$GdCu$_2$O$_8$, it is possible to tune $\omega^2_{0, SF}$ nonmonotonically and also to extract the radiation with much higher intensity than in the single Josephson junction. This is a matter of future investigations.

In conclusion, we have shown that in a S/F superlattice with the exchange field $h \sim T_c$ acting in F-planes only a nontrivial and nonmonotonic behavior of magnetic properties, like the lower critical field $H_{c1}$, is realized. This property is due to the decrease of the effective Josephson coupling between S-planes by increasing $h$.

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