Vacuum expectation value renormalization in the Standard Model and beyond

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We show how the renormalization constant of the Higgs vacuum expectation value, fixed by a tadpole condition, is responsible for gauge dependences in various definitions of parameters in the $R_\xi$-gauge. Then we show the relationship of this renormalization constant to the Fleischer-Jegerlehner (FJ) scheme, which is used to avoid these gauge dependences. In this way, we also present a viewpoint on the FJ-scheme complementary to the ones already existing in the literature. Additionally, we compare and discuss different approaches to the renormalization of tadpoles by identifying the similarities and relations between them. The relationship to the Higgs background field renormalization is also discussed.

I. INTRODUCTION

In modern particle physics, high precision calculations are of increasing importance for finding signs of new physics in the comparisons of theory predictions to experimental data. An integral part of these calculations is the subject of renormalization. Even though the main principles of renormalization are well understood (see e.g. [1] for a recent review of electroweak radiative corrections) and represent a standard textbook subject, some subtleties remain being actively discussed. One of these is the subject of vacuum expectation value (VEV) renormalization in conjunction with so called tadpole schemes. For existing examples of discussions in the literature, see e.g. [2–8] or [1] for a list of different tadpole schemes. However, we find that a more detailed and unified exposition of the relationships between such schemes is still missing in the literature. Hence, in this paper, we want to elucidate the relation between the renormalization of vacuum expectation values, tadpole schemes, gauge dependences and the special role of Goldstone boson tadpoles in this respect. More specifically, we show the connections between methods that are commonly used in precision calculations for the Standard Model (SM) as e.g. in [1] and more formal discussions of VEV-renormalization in general gauge theories as e.g. in [9, 10].

We want to clarify that an independent VEV (or tadpole) renormalization constant is necessary in addition to the renormalization of the parameters and fields of the unbroken theory in order to render the full theory finite (this was already noted in [3, 11]). In the broken phase, the usage of the VEV in gauge fixing functions affects the global symmetry properties of the theory [9], which leads to the need of this additional degree of freedom. Hence in spontaneously broken gauge theories such as the SM, this can be understood as an artifact of the gauge-fixing procedure rather than a direct consequence of the mechanism of spontaneous symmetry breaking itself. This introduction affects definitions of parameters, leading to gauge dependencies in some of them. In principle, the gauge dependences will always cancel in the observables, however, they are not a feature one usually wants in parameter definitions. The Fleischer-Jegerlehner tadpole scheme (FJ-scheme) [12] was proposed to avoid these spurious gauge dependences. This becomes even more important if one goes beyond the Standard Model (BSM), where the usual on-shell(OS) renormalization is not possible (see [13] for a discussion of the problems arising here). This explains a renewed attention to the subject in the context of the two Higgs doublet models [5–8, 14].

The FJ-scheme is closely related to the aforementioned additional independent VEV-renormalization constant. As we will see, the FJ-scheme makes sure that this degree of freedom would not enter the parameter and counterterm definitions and hence, allows for gauge-independent definitions. This suggests to look at the FJ-scheme as simply being a convenient set of counterterm redefinitions and allows to avoid arguments about proper VEV [12] or the correct one-loop minimum [1, 5, 8, 14].

We keep our presentation at one-loop level and study only the Standard model (SM), which is inevitably the integral part of many BSM models. Considering gauge dependences and mass parameters, it is proven that the exact poles of the propagators are gauge-independent at all loops [15]. The scheme that uses the exact pole masses is called the complex mass scheme (CMS) [16–21]. In contrast, the OS scheme gives gauge dependent mass definitions for unstable particles at two loops already [16]. To get gauge independent mass counterterms by using an FJ scheme at more than one loop, one then has to rely on the gauge independent definition of the renormalized mass, which is given by the CMS and not the OS scheme,

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when the particle is unstable. Nevertheless, at one loop they give equivalent mass definitions and hence we will frequently use the term OS having this equivalence in mind. On the other hand, the FJ scheme can be used to cancel only the ”spurious” gauge dependencies, which appear because of the gauge fixing procedure, while not touching the gauge dependencies that appear in the OS masses for unstable particles.

We start our presentation in Sec. II, by explaining why it is necessary to have an additional counterterm in the spontaneously broken phase of the SM as compared to the unbroken phase and introduce the tadpole condition to fix the former. Then, in Sec. III, we present the translations between the renormalization constants of different parameter sets that are used as independent in the renormalization procedure. In particular, we show the relations between renormalization constants of symmetry-based (or ”original”) parameters of the theory, to the ones used in the usual OS scheme in [22]. These translations illuminate the gauge dependences in the definitions of the usual mass renormalization constants. We discuss the tadpole scheme presented in [22] which was later called parameter-renormalized tadpole scheme (PRTS) [1]. In Sec. IV we show how to redefine counterterms to be gauge-independent via the FJ-scheme. This section also relates the FJ-scheme to the findings and interpretations presented in [9]. In addition, we comment on the usage of the prescription that we consider somewhat too briefly discussed in the existing literature. In Sec V, we make comparisons of the tadpole schemes and relate our approach to the interpretations found in the literature on the matter. To illustrate the differences between the schemes, we compare VEV-renormalization constants numerically and comment on the outcomes. We conclude our presentation in Sec. VI Some details of our calculations can be found in the appendices. These include a short note on the construction of the $R_\xi$-gauge in the background field formalism in App. A; the calculation of the gauge-dependent divergences, using the background fields in the SM (an adaptation from [9]) in App. B an example and consequences of a different choice of an arbitrary renormalization condition in App C; explicit divergences of renormalization constants in App. D and numerical input values that we used to calculate VEV-renormalization constants in App. E.

The one-loop relations and expressions presented in the paper have been explicitly checked using the native FeynArts [23] SM-file (both with and without background fields) and with FeynCalc [24–26] and FromCalc [27, 28] for cross-checking.

II. AN ADDITIONAL COUNTERTERM IN $R_\xi$

A. Necessity of a VEV-counterterm

Consider the usual Higgs potential:

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2. \quad (1)$$

The neutral component of the Higgs doublet $\phi$ acquires a VEV $v$ as in

$$\phi = \left( \frac{1}{\sqrt{2}} \left( G_W^+ + h + i G_Z \right) \right), \quad (2)$$

where $G_W^+$ and $G_Z$ are the Goldstone boson fields and $h$ is the physical Higgs field. The minimum condition for Eq. (1) gives

$$\frac{\partial V}{\partial h} |_{h = G_{W/Z} = 0} = 0 \Rightarrow v^2 = -\frac{\mu^2}{\lambda}, \quad (3)$$

which leads to the Higgs mass being:

$$m_h^2 = 2 \lambda v^2 = -2 \mu^2. \quad (4)$$

Using multiplicative renormalization constants, i.e.

$$p \rightarrow Z_p p, \; f \rightarrow \sqrt{Z_f} f \quad (5)$$

for all parameters $p$ and fields $f$ of the theory, the potential in Eq. (1) can be renormalized in the unbroken phase as

$$V^R(\phi) = Z_{\mu^2} Z_{\phi^2} \phi^\dagger \phi + Z_{\lambda} Z_{\lambda}^2 \lambda (\phi^\dagger \phi)^2. \quad (6)$$

In a gauge theory, it is customary to use the Faddeev-Popov procedure to cancel out unphysical degrees of freedom in the gauge sector. This comprises the introduction of gauge-fixing functions which do not affect the S-matrix [29], but they may induce divergence structures that can only be renormalized via the introduction of additional renormalization constants as we will discuss later in this section. In the $R_\xi$-gauge, the additional divergences effectively come about via the Goldstone-boson masses $\xi \sqrt{m_{Z_{\phi}}^2}$. The gauge-fixing functions act in such a way that quadratic mixing terms between scalars and vector bosons are eventually canceled and propagator mixing need not be taken into account in perturbative calculations. At the same time this breaks the $SU(2)$-gauge invariance of the scalar sector by construction. In the SM (and many extensions thereof), the vector boson masses are in turn fixed via the gauge couplings and the VEV. This serves as an implication as to why a VEV-counterterm is intrinsically related to the aforementioned additional divergences. Moreover, one can view the Goldstone boson mass terms as a modification of the scalar potential. Then, the renormalization constants of the unbroken phase, introduced in Eq. (6) can only absorb
divergences in the $\xi = 0$ gauge. From the minimum condition for the renormalized scalar potential one finds
\[
\frac{\partial V^R}{\partial h}|_{h=G_{W,Z}=0} = 0 \Rightarrow v^2 Z_\phi = -\frac{Z_\mu^2 \mu^2}{Z_\lambda \lambda}, \text{ when } \xi = 0.
\] (7)

This implies that there is no independent renormalization constant $Z_v$, or in other words that the VEV renormalizes as the Higgs doublet field, i.e.
\[
Z_v = \sqrt{Z_\phi}, \text{ for } \xi = 0. \tag{8}
\]

It was explicitly shown in [9], that Eq. (8), can not hold in the $R_\xi$-gauge for $\xi \neq 0$ in the broken phase as the divergences absorbed in $Z_\mu$ and $\sqrt{Z_\phi}$ can not be the same in that case. Neither can $Z_v$ be expressed solely in terms of shifts of the potential parameters $Z_\mu^2$ and $Z_\lambda$ as found in [7]. This fact was already noted in [3]. Here, it was shown that tadpole contributions resulting from the gauge-fixing 1 can not be interpreted as a shift in the scalar potential, i.e. a shift of $\mu$ and $\lambda$ – in contrast to all other tadpole contributions. This is equivalent to the finding that, in a general $R_\xi$-gauge, $Z_\lambda$ needs to act as an independent renormalization constant in order to render the theory finite.2

A rather recent explanation of the need for the mentioned additional counterterm in spontaneously broken gauge theories is given in [9]. Here we will adapt their findings and interpretation to the SM. The argument of why the renormalization of only fields and parameters is not enough to absorb the divergences is the following: the gauge-fixing terms in the $R_\xi$-gauge breaks the global invariance under $SU(2)$, hence one should expect divergences that are $SU(2)$ breaking. One then in principle should introduce all possible $SU(2)$ breaking counterterms additionally to the $SU(2)$ preserving ones. However, ref. [9] shows that one needs only one additional degree of freedom to absorb the divergences coming from Goldstone boson mass terms. This is proven with the help of background fields which allows to restore the global invariance under the $SU(2)$ in case of
\[
\xi = \xi_W = \xi_Z = \xi_A. \tag{9}
\]

Then, the global invariance restricts possible divergences and hence, all the divergences can be absorbed into the redefinitions of the fields, background fields and parameters. To see this, we follow [9] and write the Higgs doublet as:
\[
\phi \to \phi + \hat{\phi} = \left( \frac{1}{\sqrt{2}} \frac{G_W}{(h + iG_Z)} \right) + \left( \frac{1}{\sqrt{2}} \frac{G_W^+}{(v + \tilde{h} + iG_Z)} \right), \tag{10}
\]

where background fields are denoted by hats. The $R_\xi$-gauge-fixing function is modified in such a way that the gauge fixing and the ghost part of the Lagrangian are invariant under the global gauge transformation.3 An explicit construction of the gauge-fixing functions using background fields in the SM is explained in App. A. In this case, the counterterms appear from the $Z$–factors not only of quantum, but also of background fields, hence the VEV-renormalization constant is now interpreted as the background field renormalization constant and is independent from the quantum field renormalization constant. Since the difference in the divergences between the renormalization of quantum field vs. the renormalization of the background field appears only in $\xi \neq 0$, it is convenient to factor the renormalization of this difference from the common field renormalization constant. Thus we write the renormalization transformation for the quantum and the background field doubles as:
\[
\phi + \hat{\phi} \to \sqrt{Z_\phi} \left( \frac{1}{\sqrt{Z_\phi}} \phi + \sqrt{Z_\phi} \hat{\phi} \right). \tag{11}
\]

Using the parametrization of Eq. (11), only $\hat{Z}_\phi$ has gauge-dependent UV-divergences, while the UV divergent terms in all the rest of the independent renormalization constants are the same as for the $\xi = 0$ case. As was shown in [9], the renormalization constant $\hat{Z}_\phi$4 is conveniently determined by the unphysical two-point Greens function of the BRST source and the field that corresponds to the BRST variation of the scalar field, and is shown to be $\propto \xi$. We present the calculation of this term, adapted for the Standard Model in App. B

For convenience, we define the $Z$-factors for the Higgs field $h$, its background field $\tilde{h}$ and VEV $v$:
\[
h \to \sqrt{Z_h} h, \quad \tilde{h} \to \sqrt{Z_{\tilde{h}}} \tilde{h}, \quad v \to Z_v v. \tag{12}
\]

They do not represent independent renormalization constants and are expressed in terms of the ones introduced

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1 These are precisely the Goldstone-boson tadpoles, because all other gauge-parameter dependent contributions cancel.

2 Note that this stands for rendering all connected $n$-point Greens functions finite, including the 1-point functions. In principle, one can render all $n$-point functions finite for $n > 1$ without an independent $Z_v$ though.

3 This set-up is also related to a more formal studies of algebraic renormalization [30–35]. In these studies, the term rigid symmetry is used instead of global symmetry, but the meaning is the same.

4 Due to our choice to include $\hat{Z}_\phi$ in the quantum field renormalization (Eq. (11) or Eq. (13a)), our $\hat{Z}_\phi$ differs from the one presented in [9] by a square root, i.e.
\[
\hat{Z}_\phi^{\text{ours}} = \sqrt{Z_\phi^{\text{ref}}} \tag{9}.
\]

This can be directly seen, comparing the last term of Eq. (B11) with the Eq. (18) of [9].
The renormalization constant $\hat{Z}_\phi$ contains divergences which are purely induced by the symmetry breaking effect of the $R_\xi$-gauge. Adding the background fields restores the global symmetry and forbids the appearance of any additional counterterms, as long as Eq. (9) holds. Background fields do not affect the Green’s functions of the quantum fields (they do not appear as internal lines in the respective Feynman diagrams). This means that the divergence structure of the quantum fields does not change when the background fields are set to zero. Hence, the symmetry argument that this formalism provides, shows that the single additional constant $\hat{Z}_\phi$ is enough to absorb the $R_\xi$-gauge induced divergences even without the background fields, as long as Eq. (9) holds.

An explicit calculation shows that at one-loop it is enough to use a single additional counterterm even without the equality of the gauge parameters of Eq. (9). We will keep our presentation at the one-loop level, hence one is safe to have $\xi_W \neq \xi_Z$ in all of our expressions. However, it is not clear if this can strictly hold at higher loop orders, when $\xi_Z \neq \xi_A \neq \xi_W$, as the global $SU(2)$ invariance in the background field formulation is then lost. On the other hand, if one could construct the gauge-fixing function which preserves the global invariance when Eq. (9) does not hold, one can prove the same for $\xi_Z \neq \xi_A \neq \xi_W$. Such a gauge fixing function was introduced in [35], yet it is unclear how this would impact our results as it is slightly different from the usual $R_\xi$-gauge. Hence for consistency, we will use a single gauge parameter as in Eq. (9).

As a last note in this section, we would like to stress the differences in the use of background fields in [9] and [36]. It might appear as if the authors state exact opposites, namely that a VEV-counterterm is strictly necessary and non-zero versus the statement that no VEV-renormalization in addition to the Higgs field renormalization is needed. However, both statements are not contradictory as the respective contexts differ. In [36], the authors do not renormalize quantum fields at all as they are interested only in the Green’s functions of the background fields. Then, the statement that no genuine VEV-counterterm is needed translates to the fact that no renormalization in addition to Eq. (13b) is necessary. In [9], however, the focus lies on the renormalization of quantum fields, while the background fields are still used to preserve the symmetry structure of the theory. Then, the $\hat{Z}_\phi$ in Eq. (13b) is interpreted as an additional counterterm to the one of Eq. (13a), due to a mismatch between quantum field renormalization and the renormalization of its VEV. In their notation, $\delta v \neq 0$ as they parameterized it in a relationship with the quantum instead of the background field renormalization as compared to [36].

### B. Tadpole condition and $\hat{Z}_\phi$

We now introduce the renormalization transformation for all parameters $p$ of the SM by

$$p \rightarrow (1 + \delta_p)p.$$  

(14)

Note that, with the background field renormalization being dimensionless, we choose all parameter renormalization constants including the one of the VEV to be dimensionless as well for simplicity. The translation to the dimensionful constants can be easily done by replacing

$$\delta_p \rightarrow \frac{\delta_p}{p}$$  

(15)

in all our expressions.

For the field renormalization constants we use the parametrization of Eq. (11) and write them as

$$\hat{Z}_\phi = 1 + \delta_\phi, \quad \hat{Z}_\phi = 1 + \hat{\delta}_\phi.$$  

(16)

The Higgs-field and VEV-renormalization constants are

$$Z_h = 1 + \delta_h, \quad Z_v = 1 + \delta_v.$$  

(17)

Then, from Eq. (13a) and Eq. (13b) we have:

$$\delta_h = \delta_\phi - \hat{\delta}_\phi, \quad \delta_v = \frac{1}{2}(\hat{\delta}_\phi + \delta_\phi).$$  

(18)

By inserting these redefinitions into Eq. (1) collecting all the terms linear in $h$, and using the tree-level minimum condition Eq. (3) we get a counterterm for the one-point function of $h$, i.e.

$$\delta t_h = -\lambda h^3 \left( \delta_\lambda - \delta_{1 \mu^2} + \hat{\delta}_\phi + \delta_\phi \right).$$  

(19)

By virtue of the *tadpole condition*, i.e.

$$\delta t_h + T_h = 0,$$  

(20)

where $T_h$ are the one-loop tadpole contributions, linear shifts to the VEV of the physical scalar field $h$ are canceled. One can understand the relevance of employing this condition when working with the 1PI generating functional which is defined as the Legendre transformation of the connected generating functional. This transformation is only well-defined for vanishing linear terms.

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5 Both references use the notation $\delta v$ for different quantities leading to a potential confusion.
of the fields though, making the condition Eq. (20) essential [15, 37, 38]. This is useful when deriving functional identities such as Slavnov-Taylor [39] or Nielsen 40]. Thus in all “tadpole schemes”, Eq. (20) is used. As we will shortly see, the differences between tadpole schemes are mainly ones of how renormalization constants are defined without really altering the tadpole condition Eq. (20).

First, we focus on $\delta_\phi$. As discussed before, its divergence structure can be calculated with the help of BRST-sources [9] and is found to be equivalent to that of the Goldstone-boson tadpoles. Alternatively, one can explicitly check the divergence structures in the Higgs one, two and four-point functions to arrive at the same conclusion [7]. We checked it both ways, and present the calculation of the divergences following [9] in App. B while the ones of the Higgs 1,2,3 and 4-point functions are given in App. D. The result is

$$\delta_\phi|_{\text{UV}} = \frac{2}{4-D} \cdot \frac{\xi}{16\pi^2 v^2} \left(2m_H^2 + m_Z^2\right),$$

where $D$ is the number of space-time dimensions. This term alone cancels all the gauge-dependent divergences coming from the tadpole condition, Eq. (20), since the only $\xi$-dependent terms in Eq. (20) are given by

$$T_h^{(\xi)} = G^{\xi}_{\phi} + G^\phi_{\xi} = \frac{1}{16\pi^2} \frac{2\lambda v^2}{v} \left[A_0(\xi m_W^2) + \frac{1}{2}A_0(\xi m_Z^2)\right],$$

where $(\xi)$ denotes that we take only the gauge-dependent tadpoles and $A_0$ is a one-point Passarino-Veltman function [41]. Checking the UV divergences in Eq. (22), we see that the gauge-dependent UV-divergences from Eq. (20) cancel as

$$T_h^{(\xi)}|_{\text{UV}} - \lambda v^3 \delta_\phi|_{\text{UV}} = 0.$$  

An inspection of the divergences of 1, 2 and 3-point functions show us that there are no gauge-dependent divergences in other renormalization constants (see App. D): 

$$\frac{\partial}{\partial \xi} \delta_\lambda|_{\text{UV}} = \frac{\partial}{\partial \xi} \delta_\mu|_{\text{UV}} = \frac{\partial}{\partial \xi} \delta_\phi|_{\text{UV}} = 0.$$  

Note that $\delta_\phi|_{\text{UV}}$ vanishes when $\xi \to 0$, while the gauge-independent part is obviously untouched by this limit. Hence we can conclude that $\delta_\phi$ renormalizes purely the contributions induced by the gauge-fixing procedure in $R_\xi$-gauge. It is therefore only necessary as an independent renormalization constant when $\xi \neq 0$. In this sense, it is the minimal addition to the set of renormalization parameters of the unbroken theory in order to render the theory finite.

III. TRANSLATION OF RENORMALIZATION CONSTANTS

Before a renormalization procedure is carried out, one has to choose a set of independent parameters of the model to be renormalized. In the SM, one usually chooses masses as independent parameters in the OS scheme while in studies of more general models, it can be convenient to use the set of “original” parameters and the VEVs, especially when the use of an MS scheme cannot be avoided. For the comparability of different choices, it is instructive to have a translation between the sets of renormalization constants. To get these relations, consider we have a parameter set $\{p\}$ related at tree-level to a parameter set $\{p'\}$ by some function $f$:

$$p'_i = f_i(\{p\}).$$

Introducing the renormalization transformation as in Eq. (14) and expanding it to the first order in $\delta$’s will induce one-loop relations between the renormalization constants:

$$\delta_{p'_i} = \frac{1}{f_i(\{p\})} \cdot (\delta_{p_j} \frac{\partial}{\partial p_j} f(\{p\}).$$

By using the relations

$$m_h^2 = \mu^2 + 3\lambda v^2, \quad m_W = \frac{v}{2} g_2, \quad m_Z = \frac{v}{2} \sqrt{g_1^2 + g_2^2},$$

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}, \quad t_h = -v(\mu^2 + v^2 \lambda).$$

in Eq. (26), we get the relations between the renormalization constants of the PRTS [22], usually used with an OS scheme, to the ones of the “original” parameters, namely

$$\{t_h, m_h, m_W, m_Z, e\} \leftrightarrow \{v, \mu^2, \lambda, g_1, g_2\},$$

where $g_1$ and $g_2$ are couplings of the $U(1)$ and $SU(2)$ group, respectively. The results are shown in the first column of Table I, where $\delta_\xi$ is expressed in terms of the field renormalization constants of Eq. (18). This is to show where the gauge-fixing induced $\xi$-dependences appear via $\delta_\phi$. The meaning of $\Delta$ and an “FJ-scheme” also shown in Table I will become clear in the next section.

Inspecting the first column of the Table I, one clearly sees that $\delta_\phi$ enters the definition of the usual mass counterterms. As discussed in a previous section, and shown in App. B this renormalization constant has gauge-dependent divergences. This means that the mass counterterms are necessarily gauge-dependent if one defines them as in the PRTS. In the next section, we will present the FJ-scheme and show its role in the cancellation of gauge dependences by virtue of the renormalization constant redefinitions shown in Tab. I.
Table I: The mass, VEV and electric charge renormalization constants are expressed in terms of the renormalization constants of gauge couplings $g_1, g_2$ and potential parameters $\lambda, \mu$ together with the (background) field renormalization constants in the two tadpole schemes. $\Delta$ is the “FJ term” used to relate counterterms from the usual tadpole scheme to the FJ-scheme (see Sec. IV).

| Usual tadpole scheme [22] | FJ-scheme [12] |
|---------------------------|----------------|
| $\Delta = 0$              | $\Delta = -\frac{\delta t_\mu}{v m_Z^2} = \frac{1}{2} \left( \delta \lambda - \delta \mu^2 + \delta \phi + \hat{\delta} \phi \right)$ |
| $\delta_e = \frac{1}{2} \left( \hat{\delta} \phi + \hat{\delta} \phi \right)$ | $\delta_e|_{pJ} = \frac{1}{2} \left( \delta \mu^2 - \delta \lambda \right)$ |
| $\delta_{M_W^2} = \frac{g^2}{2} \left( \delta \lambda + \delta \phi + \hat{\delta} \phi \right) - \frac{1}{2} \delta \mu^2$ | $\delta_{M_W^2}|_{pJ} = \delta \mu^2$ |
| $\delta_{M_Z^2} = 2 \delta g_2 + \delta \phi + \hat{\delta} \phi$ | $\delta_{M_Z^2}|_{pJ} = 2 \delta g_2 + \delta \mu^2 - \delta \lambda$ |
| $\delta_{m_f} = \delta_y + \frac{1}{2} \left( \hat{\delta} \phi + \hat{\delta} \phi \right)$ | $\delta_{m_f}|_{pJ} = \delta y + \frac{1}{2} (\delta \mu^2 - \delta \lambda)$ |
| $\delta \mu = \lambda \nu (\delta \lambda - \delta \mu^2 + \hat{\delta} \phi + \hat{\delta} \phi)$ | $\delta_e = \frac{1}{g_1^2 + g_2^2} (g_1^2 \delta g_2 + g_2^2 \delta g_1)$ |

IV. FJ-SCHEME AND GAUGE DEPENDENCIES

The FJ-scheme is a prescription of including tadpole contributions in perturbative calculations, such that mass and coupling counterterms can be defined in a gauge-invariant way. As we saw in the previous section, the usual tadpole scheme or PRTS (as e.g. in [22]) introduces gauge dependences in mass counterterms via the additional counterterm $\delta_{\phi}$, since it has gauge-dependent UV-divergences. To clarify also the finite gauge dependences, consider the one-loop corrected fermion mass, $m_{\text{pole}}$, in a bare perturbation theory:

$$ m_{\text{pole}} = m_{\text{bare}} + i \left( \begin{array}{c} \text{fermion} \cr \text{fermion} \cr \text{fermion} \cr \text{fermion} \end{array} \right) _{p=m_{\text{bare}}} \tag{29} $$

The expression Eq. (29) is gauge-independent as it is a one-loop pole mass, while $m_{\text{bare}}$ is gauge-independent by principle. Thus the mass shift, i.e. the term in the parenthesis in Eq. (29), is gauge independent. Writing the bare mass parameter as

$$ m_{\text{bare}} = m_{\text{ren}} + \delta_m \cdot m_{\text{ren}} \tag{30} $$

will generate a counterterm for the two-point function. However, a tadpole also enters the mass shift, and hence, the tadpole counterterm, Eq. (19), enters the expression of a one-loop mass. Diagrammatically, the loop corrected mass in terms of the renormalized quantities and counterterms is

$$ m_{\text{pole}} = m_{\text{ren}} + i \left( \begin{array}{c} \text{fermion} \cr \text{fermion} \cr \text{fermion} \cr \text{fermion} \end{array} \right) _{p=m_{\text{bare}}} \tag{31} $$

The second line of Eq. (31) vanishes if the tadpole condition of Eq. (20) is used. Since the tadpoles contain the gauge-dependent contributions of Eq. (22), $\delta_m$ becomes gauge-dependent. The pole mass is always gauge-independent, hence the bare mass of the Eq. (30) becomes gauge-dependent too. In other words, bare parameter of Eq. (29) is inconsistent with Eq. (30) used with Eq. (31).

For the sake of the argument consider a case in which we do not renormalize tadpoles at all (this leads to UV divergent 1 point function for the Higgs). Then we have:

$$ m_{\text{pole}} = m_{\text{ren}} + i \left( \begin{array}{c} \text{fermion} \cr \text{fermion} \cr \text{fermion} \cr \text{fermion} \end{array} \right) _{p=m_{\text{bare}}} \tag{32} $$
where the counterterm is marked by the hexacross to indicate that no tadpole renormalization is done. This definition of a mass counterterm coincides with the definition in the FJ-scheme [12]. Hence here and everywhere else in our paper, we indicate a counterterm in an FJ scheme by a \("FJ\)" after the symbol. Requiring the OS condition, which enforces \(m_{pole} = m_{ren}\), Eq. (32) leads to a gauge-independent mass renormalization constant \(\delta_m|_{FJ}\). Then both bare and renormalized masses are gauge-independent and hence, the FJ-scheme keeps the definition of the bare mass parameter of Eq. (29) untouched.

Comparing Eq. (31) with Eq. (32), we conclude that:

\[
-\imath\delta_m|_{FJ} \cdot m_{ren} = -\imath\delta_m \cdot m_{ren} + \frac{1}{vM_H^2} \cdot m_{ren}
\]  

(33)

We can write down this relation in terms of the renormalization constants of the original parameters. To do this, we write the bare fermion mass parameter in terms of the bare Yukawa coupling and the VEV of the Higgs:

\[
m_{bare} = \frac{1}{\sqrt{2}} y_{bare} v_{bare}
\]  

(34)

and, using Eq. (26) together with Eq. (18) we get the expression for the \(\delta_m\) of the fermion, shown in the first column of Tab. I, which is:

\[
\delta_m = \delta_y + \frac{1}{2} \delta_\phi + \frac{1}{2} \delta_\mu^2.
\]  

(35)

Inserting this expression and the expression for the tadpole counterterm from Eq. (19) into Eq. (33) we get:

\[
\delta_m|_{FJ} = \delta_y - \frac{1}{2} \delta_\lambda + \frac{1}{2} \delta_\mu^2,
\]  

(36)

which is also shown in the second column of Tab. I We see that \(\delta_\phi\) disappears from the definition of the counterterm in the FJ-scheme. This shows us that the FJ counterterm satisfies the necessary condition to be gauge-independent, and hence allows the consistency of the scheme.

In the current literature, the FJ scheme is usually introduced by a notion of “shifting the bare VEV to the true minimum” [1, 5, 8, 14]. We, however, look at this scheme simply as a convenient redefinition of renormalization constants, as shown above for the fermion mass renormalization. As a set of renormalization constants to begin with, we choose the ones of [22], as it is a widely used set-up. The relations of those constants to the “original” parameters of the model are given in the first column of Tab. I To redefine renormalization constants from the usual scheme to the FJ scheme we introduce an \(FJ\) term \(\Delta\) by adding and subtracting a shift of the VEV of the already renormalized theory:

\[
v \rightarrow v (1 + \Delta - \Delta).
\]  

(37)

Going back to the example of a fermion mass, we now modify Eq. (31) to:

\[
m_{pole} = m_{ren} + \imath \left( -im(\delta_m - \Delta) + \hat{\delta} \cdot \vec{\lambda}_m + \hat{\delta} \cdot \vec{\mu} \right)_{p=m_{bare}}.
\]  

(38)

To get Eq. (32), i.e. the mass renormalization constant in the FJ scheme, one sets the first two terms of the second line of Eq. (38) to zero. Together with the tadpole condition of Eq. (20) it gives:

\[
\Delta = -\frac{\delta t_h}{vM_H^2} = \frac{T_h}{vM_H^2} = \frac{1}{2} \left( \delta_\lambda - \delta_\mu^2 + \delta_\phi + \bar{\delta}_\phi \right).
\]  

(39)

It is then easy to get the relationships between the usual mass renormalization constants and the ones of the FJ-scheme by Eq. (26), replacing \(\delta_v\) with \(\delta_v|_{FJ}\), which is defined as:

\[
\delta_v|_{FJ} = \delta_v - \Delta.
\]  

(40)

This leads to:

\[
\delta_m|_{FJ} = \delta_m - \Delta,
\]  

(41)

\[
\delta_{m_\lambda}|_{FJ} = \delta_{m_\lambda} - 2\Delta,
\]  

(42)

\[
\delta_{m_\mu^2}|_{FJ} = \delta_{m_\mu^2} - 3\Delta.
\]  

(43)

Only the last expression is not completely straightforward, as one should be careful to require the tree-level tadpole condition only after the expression is derived. As an example, we show this for the Higgs mass counterterm, i.e.

\[
\delta_{m_\lambda}|_{FJ} = \frac{1}{3\lambda v^2 + \mu^2} \sum_{p=v,\mu,\lambda} (\delta_{pp}) \left. \frac{\partial}{\partial p} (3\lambda v^2 + \mu^2) \right|_{p^2=\lambda v^2} = \frac{1}{3\lambda v^2 + \mu^2} \left. \left[ 6\lambda v^2 (\delta_v - \Delta) + 3\lambda v^2 \delta_\lambda + \mu^2 \delta_\mu^2 \right] \right|_{\mu^2=\lambda v^2} = \left[ \frac{3}{2} (\delta_\phi + \bar{\delta}_\phi) + \frac{3}{2} \delta_\lambda - \frac{1}{2} \delta_\mu^2 \right] - 3\Delta.
\]  

(44)

where in the second line we inserted the parameters and renormalization constants, using Eq. (40). In the last line, we required the tree-level tadpole condition and expressed the VEV-renormalization constant in terms of Eq. (18). Comparing the last line to the usual mass renormalization constant from Tab. I, we see that we get Eq. (43).

The results for all the FJ mass renormalization constants in terms of the original parameters of the SM is shown in the second column of Tab. I. Notice that none of the mass renormalization constants in an FJ-scheme has \(\delta_\phi\) in their definitions, presenting yet another check
of consistency for the FJ-schemes’ implementation. In addition, it presents a nice relationship to the interpretation of the VEV counterterm as the renormalization of the background field from ref. 9 or App. B.

It is rather straightforward to generalize this procedure to BSM models with altered scalar sectors and we refer to the existing presentations of 1, 8. The Two-Higgs Doublet Model is one example. Here, the easy way to generalize the FJ procedure is to choose the Higgs basis 42, 43]. Then the renormalization constants shown in Eq. (18), apply for the VEV and the Higgs in the Higgs basis identically to the SM counterparts and δφ has exactly the same divergences as in the SM, shown in Eq. (21). Then one can use the same Eqs. (41 42, 43) with ∆ being identified analogously to Eq. (39):

\[ \Delta = -\frac{\delta \phi}{v} \frac{T_h}{v m_h^2}, \]

(45)

where \( T_h \) and \( \delta \phi \) are now to be interpreted as the tadpole contributions and the tadpole counterterm of the extended scalar sector in the Higgs basis.

As a word of warning, we want to make a comment about the potential inclusion of finite loop-contributions in \( \delta \phi \). As discussed in Sec. II, \( \delta \phi \) absorbs the gauge-dependent divergences coming from the introduction of the \( R_\xi \)-gauge. In this section we saw that the gauge-independent definition of mass renormalization constants in the FJ-scheme do not have this troublesome \( \delta \phi \) in their definitions. In addition, the gauge-dependent tadpoles are the only gauge-dependent terms in the FJ-term ∆. Thus, looking at Eq. (23) it is tempting to promote this relation to a condition on finite terms, so that \( \delta \phi \) absorbs the full Goldstone tadpoles. This would allow to state that the absence of \( \delta \phi \) in the definitions of mass counterterms is a necessary and sufficient condition to have bare mass parameters gauge-independent and thus to identify the gauge dependence of ∆ with the single renormalization constant \( \delta \phi \) instead of combination of them, shown in Eq. (39). However, this condition might lead to unexpected and impractical results. For instance, as shown in App. C Eq. (23) imposed on finite parts is in conflict with the OS charge renormalization condition and leads to a gauge dependent charge renormalization constant\(^6\). In contrast, just applying the OS renormalization with the FJ scheme in its original form, all parameter renormalization constants become gauge-independent.

Apart from straightforwardly using the FJ-scheme in the sense of defining parameters, one can also use the prescription to account for gauge dependences after the renormalization in some specific scheme is already done. As an example, consider that the renormalization is done in \( \overline{\text{MS}} \). Then the relations between counterterms of Eqs. (41 42, 43) give us the relations between the \( \overline{\text{MS}} \) renormalized mass parameters of fermion f, vector V and the Higgs h:

\[ m_{f,V}^{\overline{\text{MS}}} |_{\text{FJ}} = m_{f,V}^{\overline{\text{MS}}} (1 - \Delta_{\text{finite}}), \]

(46)

\[ m_{h}^{\overline{\text{MS}}} |_{\text{FJ}} = m_{h}^{\overline{\text{MS}}} (1 - \frac{3}{2} \Delta_{\text{finite}}). \]

(47)

The \( \overline{\text{MS}} \) mass parameters in the FJ-scheme are gauge-independent. This means that using these equations, it is now possible to compare calculation results in different gauges using the usual gauge-dependent \( \overline{\text{MS}} \) masses. From Eq. (46) and Eq. (47), we get that if we change the gauge parameter value from \( \xi \) to \( \xi' \), we need to change the \( m_{\overline{\text{MS}}} \) masses accordingly:

\[ \frac{m_{f,V}^{\overline{\text{MS}}} (\xi')} {m_{f,V}^{\overline{\text{MS}}} (\xi)} = 1 - \frac{\Delta_{\text{finite}} (\xi)} {\Delta_{\text{finite}} (\xi')} \approx 1 + \Delta_{\text{finite}} (\xi') - \Delta_{\text{finite}} (\xi), \]

(48)

\[ \frac{m_{h}^{\overline{\text{MS}}} (\xi')} {m_{h}^{\overline{\text{MS}}} (\xi)} \approx 1 + \frac{3}{2} \Delta_{\text{finite}} (\xi') - \frac{3}{2} \Delta_{\text{finite}} (\xi), \]

(49)

where the one-loop approximation is assumed. The calculated pole mass with the gauge parameter value \( \xi \) has to be numerically the same as the calculated pole mass with the value \( \xi' \), if the input masses are adjusted for the corresponding gauge parameter values as in Eq. (48) and Eq. (49). Note that the gauge-dependent terms in \( \Delta_{\text{finite}} \) are only the finite parts of the Goldstone boson tadpoles, shown in Eq. (22). This means that if one just wants to check gauge dependences, it is enough to account only for the Goldstone tadpoles in \( \Delta_{\text{finite}} \), as everything else will be gauge-independent. Keeping track of the FJ-term allows for comparisons of various calculations both in different schemes and/or in different gauges. Furthermore, it allows an easy accounting of gauge dependences in an already gauge-dependent scheme, thus giving a possibility for additional consistency checks.

V. DIRECT COMPARISON OF VEV-SCHEMES

In this section, we want to show the relation between various known VEV-schemes and their interpretations. For clarity, we repeat Eq. (13b) in terms of counterterms, i.e.

\[ \delta_v = \frac{1}{2} \delta_h = \frac{1}{2} \left( \delta_\phi + \hat{\delta}_\phi \right) = \frac{1}{2} \delta_h + \hat{\delta}_\phi. \]

(50)

This shows that \( \hat{\delta}_\phi \) can be thought of as the difference between VEV- and the scalar field renormalization while \( \delta_v \)

---

\(^6\) Note that there are finite gauge-dependent contributions, which carry gauge-independent UV divergent terms e.g. a Passarino-Veltmann function \( B_0(p^2, m^2, m^2, m'^2) \), which can cancel its divergences with any gauge-independent \( B_0 \) function leading to finite results. Hence one should be careful in making conclusions about the gauge dependencies from the UV terms.
can be interpreted as the Higgs background field renormalization. Moreover, as we have seen in the previous sections, $\delta v$ represents the minimal addition to the set of renormalization constants of the unbroken theory in order to render the theory finite. Here, we show explicitly how this compares to the application of the FJ-scheme and the PRTS, as discussed in [1]. To relate the discussion of the previous section to other common presentations of the FJ-scheme, we recover dimensionful renormalization constants via Eq. (15) and insert (in principle arbitrary) VEV-shifts of the bare VEV via

$$v_{\text{bare}} = v_{\text{bare}|\text{FJ}} + \Delta v.$$  

(51)

By setting

$$\Delta v = \frac{T_h}{m_h^2}, \quad iT_h = \frac{i}{h},$$  

(52)

the full tadpole contributions are absorbed in the VEV-shift. Note that Eq. (52) is the dimensionful equivalent of Eq. (39). Introducing this shift in the Lagrangian generates a tadpole counterterm whereby Eq. (52) and the usual tadpole-condition of Eq. (20) become equivalent. We also note that Eq. (51) is defined in terms of the bare VEV, indicating that in the FJ scheme, one uses $v_{\text{bare}|\text{FJ}}$ in the parameter relations of Eq. (27). Therefore, it still needs renormalization, i.e.

$$v_{\text{bare}|\text{FJ}} = v + \delta v_{\text{FJ}}.$$  

(53)

Then, in the FJ-scheme, the resulting relation between the bare VEV that we initially started with and the renormalized VEV is

$$v_{\text{bare}} = v + \Delta v + \delta v_{\text{FJ}}.$$  

(54)

In non-FJ-schemes such as the PRTS, the bare and renormalized VEVs are related by a single renormalization constant $\delta v$, i.e.

$$v_{\text{bare}} = v + \delta v.$$  

(55)

Comparing these two equations with Eq. (40) and Eq. (39), we see that shifting the bare VEV as in Eq. (53) is equivalent to the redefinitions of counterterms as presented in Section IV. From Eq. (32) and Eq. (38) we see that it is also equivalent to including tadpoles in all of the diagrams as if they were not renormalized at all.

In the SM, one can use the relations Eq. (27) to define the VEV-counterterm in terms of mass and the renormalization of the weak coupling constant $g_2$:

$$\delta v = \delta v(\delta g, \delta m_W^2) = v \left( \frac{\delta m_W^2}{2m_W^2} - \frac{\delta g_2}{g_2} \right),$$  

(56)

which can be used in both of the tadpole schemes. This makes the determination of the VEV-counterterm possible in terms of physical observables that are known to high accuracy. Note that this step is rarely mentioned explicitly in the literature because a VEV-counterterm by itself is usually not the focus point there. If $\delta g_2$ is determined via the charge-renormalization $\delta e$ of [1] using the relations of Eq. (27), it is not affected by the choice of the tadpole scheme. For mass renormalization, tadpole contributions are implicitly taken into account in the FJ-scheme via the VEV-shifts. Effectively, this means that when one determines the counterterms of Eq. (56), then Eq. (32) translates to

$$\delta m_W^2 = \text{Re} \left[ - i \left( \begin{array}{c} \bullet \hline w \end{array} + \begin{array}{c} \bullet \hline w \end{array} \right) \right]_{p^2 = m_W^2}^{\text{transverse}},$$  

(57)

for the on-shell $W$-boson mass counterterm. This definition is gauge-independent. In contrast to that, no tadpole contributions enter said counterterm at all in the PRTS, i.e.

$$\delta m_W^2 = \text{Re} \left[ - i \left( \begin{array}{c} w \hline \bullet \end{array} \right) \right]_{p^2 = m_W^2}^{\text{transverse}},$$  

(58)

which is a gauge-dependent quantity. Hence using Eq. (57) or Eq. (58) in Eq. (56) defines the VEV counterterm in the FJ or the PRTS scheme respectively.

Inserting the FJ mass counterterm of Eq. (57) into Eq. (56) gives the FJ VEV-counterterm. This counterterm can be expressed in terms of the scalar potential parameter renormalization (see Tab. I), viz.

$$\frac{\delta v_{\text{FJ}}}{v} = \frac{1}{2} (\delta v^2 - \delta \lambda).$$  

(59)

This shows that in the FJ-scheme, one recovers the tree-level relation between the VEV and the parameters of the scalar potential, so that the VEV renormalization can be fully expressed in terms of shifts of the potential parameters $\mu^2$ and $\lambda$. In this sense, the shift of Eq. (52) can be paraphrased as shifting the VEV to the correct one-loop minimum. Or, as presented in [12], one chooses the proper VEV. However, there is a potential for confusion due to the fact that VEV shifts induced by Goldstone boson tadpoles can not be interpreted as contributions to a one loop potential in a strict sense, as explained in [3]. In any case, one can simply look at it as being a set of counterterm redefinitions, as explained in Section IV. In contrast, one looses this immediate interpretation in the PRTS where the respective VEV-counterterm is a gauge-dependent quantity that can not solely be expressed as a shift of the potential.

Now, we want to compare the VEV-counterterms (and shifts) of the FJ-scheme, the PRTS and the one defined via the background field method in Eq. (50). As discussed in Sec. II B, there are various ways to fix the latter, or in turn $\delta \phi$. One of them is via an OS condition on the renormalized two-point function of the physical
Higgs field and the Higgs background field, i.e.
\[ \frac{\partial}{\partial p^2} \sum_{i \neq j} v_i | p^2 = m_h^2 | = 0, \quad \frac{\partial}{\partial p^2} \sum_{i \neq j} v_i | p^2 = m_h^2 | = 0. \] (60)

This yields
\[ \delta \phi |_{OS} = \frac{1}{2} (\delta \phi |_{OS} - \delta \phi |_{\text{BG}}) \]
\[ = \frac{1}{4} \xi g^2 \frac{1}{(4\pi)^2} \left[ 2B_0 \left( \frac{m_h^2}{4}, \frac{m_W^2}{4}, \frac{m_Z^2}{4} \right) \right] + \frac{1}{\cos^2 \theta_W} B_0 \left( \frac{m_h^2}{4}, \frac{m_W^2}{4}, \frac{m_Z^2}{4} \right). \] (61)

Remarkably, it gives the same functional expression as the unphysical Green’s function used to check the divergences in Eq. (B12), except that the subtraction point is \( p^2 = m_h^2 \) instead of 0. Moreover, we can also define the background VEV counterterm directly via the Higgs field renormalization, i.e.
\[ \delta v_{\text{BG}} \]
\[ = \frac{1}{2} \delta \phi |_{OS}. \] (62)

Now, using Eq. (40) and Eq. (50), we can relate the presented three different definitions of the VEV counterterms, i.e.
\[ \delta v_{\text{PRTS}} = \delta v_{\text{BG}} + \Delta v \hat{=} \delta v_{\text{BG}}. \] (63)

Note that the second equal sign only holds for the UV-parts of the counterterms, while the first equality holds for the finite parts also. This relation is interesting because it compares the three rather different approaches: in the PRTS, one uses a tadpole counterterm which implicitly utilizes the additional degree of freedom \( \delta \phi \) to get rid of tadpole contributions completely; in the FJ-scheme, one uses the same degree of freedom to carry out an infinite shift of the VEV by tadpole contributions to the one-loop minimum, then an on-shell renormalization of the bare VEV; while on the r.h.s., one merely has the field renormalization of the Higgs background field and fixes it by a residue condition.

Using the numerical values, presented in App. E, the expressions yield
\[ \delta v_{\text{fin}} |_{\text{PRTS}} = 10.155 \text{ GeV}, \] (64a)
\[ \delta v_{\text{fin}} |_{\text{FJ}} = -138.457 \text{ GeV}, \] (64b)
\[ \Delta v_{\text{fin}} = 148.612 \text{ GeV}, \] (64c)
\[ \delta v_{\text{fin}} |_{\text{BG}} = 15.250 \text{ GeV}. \] (64d)

With all of these quantities except for Eq. (64b) being gauge-dependent, one should not put too much of an interpretation to these values. Nevertheless, it is interesting to see that Eq. (64b) and Eq. (64c) show a large cancellation when combined to give Eq. (64a). This cancellation would be absent if one were to define \( \delta v_{\text{fin}} |_{\text{BG}} \) in an \( \overline{\text{MS}} \)-scheme. We will shortly discuss this point at the end of this section. Moreover, one can see that \( \delta v_{\text{fin}} |_{\text{BG}} \) differs from the VEV-shift in the PRTS scheme. This indicates that the renormalization conditions used in the PRTS are incompatible with the Eq. (61). More specifically, instead of the condition on the right hand side of the Eq. (60), the charge renormalization condition is used in the conventional PRTS. Hence the difference of Eq. (64a) versus Eq. (64d) means that the charge renormalization condition is in conflict with Eq. (60). This is also explained in the App. C, where we have shown that Eq. (61) (although evaluated at \( p^2 = 0 \) instead of \( p^2 = m_h^2 \), but it does not change the argument) leads to a gauge-dependent charge renormalization constant while it is gauge-independent in the PRTS.

As a final note, we discuss the VEV-renormalization of [7] in comparison to the aforementioned approaches. In [7], VEV-counterterms similar to the ones of [9] are used (which in turn correspond to the divergent parts of \( \delta \phi \)) in the multi-Higgs Doublet Model. In addition, finite VEV-shifts are introduced which contain only the finite parts of all tadpole contributions with the goal of having gauge-independent one-loop masses. The VEV-shifts of [7] correspond to the finite parts of Eq. (64c) while there would be no equivalent to Eq. (64b), because all parameters are renormalized in an \( \overline{\text{MS}} \)-scheme. As noted above, cancellations similar to the ones between Eq. (64b) and Eq. (64c) can not occur and therefore, the finite tadpole contributions are expected to yield large numerical corrections. In general, this is important in BSM theories where one can not fix all model parameters via process-independent physical observables. Here, it is known that tadpole contributions can be a source of numerical instabilities [13]. The potentially missing cancellations mentioned above seem to be in accordance with those, but it would be interesting to study more explicitly how closely these findings are related.

VI. CONCLUSIONS

The symmetry breaking effect of the \( R_\xi \)-gauge fixing leads to the necessity of a renormalization constant in addition to the ones for parameters and fields. By adapting the findings of [9] to the SM, we showed explicitly how this independent renormalization constant is related to the Higgs background field renormalization and to Goldstone boson tadpoles. Effectively this degree of freedom was used in the tadpole condition already in [44], yet we wanted to emphasize its origin lying in the gauge fixing as opposed to being a direct consequence of spontaneous symmetry breaking. We have shown how this degree of freedom leads to gauge dependences in all the counterterms it enters, such as the mass counterterms in Tab. I
The FJ-scheme [12] manages to avoid these gauge dependences in parameter and counterterm definitions. The scheme was originally presented with arguments about using the proper VEV, while in the recent literature the notion of true one-loop minimum [1] was employed. We, however, avoided both of these notions in our presentation and showed that one can look at this scheme as being simply a set of convenient redefinitions of counterterms.

The global symmetry argument which let us claim that we need only one additional renormalization constant $\delta_\phi$ breaks down whenever $\xi_W \neq \xi_Z$. On the other hand, the FJ-scheme generalizes to any loop-order straightforwardly also when $\xi_W \neq \xi_Z$, even though it implicitly uses the degree of freedom of the Higgs background field renormalization. This hints at the possibility that a single renormalization constant $\delta_\phi$ is enough also in this case, yet it is unclear whether there exists a rigorous symmetry argument for that. Hence, there is a subtle interplay between the VEV-renormalization, interpreted as the renormalization of the Higgs background field, and the FJ-scheme.

We used the SM as a playground to test various aspects of the renormalization in the $R_\xi$-gauge with a special emphasis on tadpole conditions and the connection between different approaches to the subject. This becomes especially relevant in BSM models with extended scalar sectors, where e.g. numerical effects of tadpole contributions and the discussion of gauge-dependences in mixing angles remain actively discussed [13, 45]. We advocate the use of the FJ prescription for keeping track of gauge dependences in intermediate expressions of perturbative calculations. We find that this specific advantage of working with the FJ scheme is not emphasized enough in the existing literature, even though this feature can be very useful for consistency checks in practical calculations.

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Appendix A: Gauge fixing functions

In the background field formalism, the gauge-fixing functions for the $U(1)$ and $SU(2)$ gauge fixing parts, respectively, are given by:

\[ F_B = \partial^\mu B_\mu - i\xi g_1 \left( \hat{\phi} \frac{Y}{2} \hat{\phi} - \phi \frac{Y}{2} \phi \right), \quad (A1) \]

\[ F_W = \partial^\mu W_\mu^i - i\xi g_2 \left( \frac{1}{2} \delta^i_\phi \phi - \phi \frac{1}{2} \phi \right), \quad (A2) \]

where $\phi$ is the Higgs doublet field and $\hat{\phi}$ is its background field. Both fields, $\phi$ and $\hat{\phi}$, transform in the same way under the global gauge transformation, hence it is easy to show that Eq. (A1) is invariant under the gauge transformation, while $F_W^i$ of Eq. (A2) transforms as a vector in the adjoint representation of $SU(2)$. The antighost $\bar{c}$ also transforms as a vector in the adjoint representation, hence the gauge fixing term,

\[ \mathcal{L}_G = s \left[ \bar{c} \left( F^1 + \frac{\xi}{2} B^1 \right) \right], \quad (A3) \]

is invariant. Mass eigenstate gauge-fixing functions are recovered by:

\[ F_A = F^A_W s_W + F_B c_W, \quad (A4) \]

\[ F_Z = F^Z_W c_W - F_B s_W, \quad (A5) \]

\[ F_W = \frac{1}{\sqrt{2}} \left( F^1 + i F^2 \right), \quad (A6) \]

where $s_W$ and $c_W$ are sine and cosine of the Weinberg angle, respectively. Taking the limit $\hat{\phi} \to \left( \begin{array}{c} 0 \\ \frac{\xi}{2} \end{array} \right)$ in Eq. (A1) and Eq. (A2) and inserting them into Eq. (A4), Eq. (A5), and Eq. (A6), we recover the usual $R_\xi$ gauge-fixing functions:

\[ F_A = \partial^\mu A_\mu, \quad F_Z = \partial^\mu Z_\mu - \xi m_Z G_Z, \]

\[ F_W^\pm = \partial^\mu W^\pm_\mu \mp i \xi m_W G^\pm_\mu. \quad (A7) \]

Appendix B: Calculation using BRS-sources

Following [9] we can calculate divergences of $\delta_\phi$ directly from the unphysical Green’s functions that include BRS sources. The Higgs doublet of the SM is decomposed into quantum and background field:

\[ \Phi = \phi + \hat{\phi}. \quad (B1) \]

The fields $\Phi$ and $\hat{\phi}$ correspond to $\phi^\text{eff}$ and $\hat{\phi} + \tilde{v}$ of [9] respectively. The BRS transformation of the background field is postulated to be in a “contractible pair” with another background field $\tilde{q}$:

\[ s\hat{\phi} = \hat{q}_\phi, \quad s\tilde{q}_\phi = 0, \quad (B2) \]

so neither of $\hat{\phi}$, nor $\tilde{q}$ contribute to the BRS cohomology [46]. In other words, they do not contribute to the physical spectrum of the theory. The BRS transformation for the field $\Phi$ is

\[ s\Phi = g_2 \frac{\alpha_k}{2} \left( \phi + \hat{\phi} \right) c_k + \frac{i}{2} g_1 \left( \phi + \hat{\phi} \right) c_B. \quad (B3) \]
\[ \hat{q}_h = -i \delta_\phi. \]  

(B6c)

The renormalization transformations of field and background field from Eq. (11) are

\[ \phi \to \sqrt{Z_{\phi}} \bar{Z}_{\phi} \phi, \quad \hat{\phi} \to \sqrt{\bar{Z}_{\phi} Z_{\phi}} \hat{\phi}, \]  

(B7)

thus the introduced “technical” background field \( \hat{q} \) transform as:

\[ s \hat{\phi} \to s \sqrt{Z_{\phi}} \bar{Z}_{\phi} \hat{\phi} = \sqrt{Z_{\phi} Z_{\phi}} \hat{q} \Rightarrow \hat{q} \to \sqrt{\bar{Z}_{\phi} Z_{\phi}} \hat{q}. \]  

(B8)

BRS sources transform as the inverse renormalization transformation of the corresponding field. Then the relation

\[ \frac{\delta \Gamma}{\delta K_\phi} \equiv \langle s\phi \rangle, \]  

(B9)

where \( \Gamma \) is the effective vertex functional, is unchanged after the renormalization transformation. From Eq. (B7) we get that the transformation for the BRS source of the Higgs doublet quantum field is:

\[ K_\phi \to \sqrt{Z_{\phi}} / \bar{Z}_{\phi} K_\phi. \]  

(B10)

Including all these renormalization transformations into Eq. (B5) we get:

\[ \mathcal{L}_K = K_\phi^{-1} \left( g_{2i} \sigma^k \left( \phi + \bar{Z}_\phi \phi \right) c_k + i \frac{\gamma_1}{2} \left( \phi + \bar{Z}_\phi \phi \right) c_B - \hat{q}_\phi \right), \]  

(B11)

\[ - \bar{Z}_\phi K_\phi^{-1} \hat{q}_\phi + h.c. \]

The last term gives the counterterm for an unphysical Green’s function that includes only \( \bar{Z}_\phi \). We will look at the Green’s function \( \Gamma_{\hat{q}_h K_h} \), where \( h \) is the Higgs field component of the doublet. The counterterm of this Green’s function is shown in Fig. B6c. To calculate this Green’s function at one-loop, one only needs the Feynman rules for interactions between \( \hat{q}_h, K_\phi \) and \( c \), which can be read out from Eq. (B11) and Eq. (A3), and are shown in Fig. 1. The loop diagram that we will need to calculate is shown in Fig. 2. The result of the sum of Fig. B6c and Fig 2 is:

\[ \Gamma_{\hat{q}_h K_h}^{[1]} = - i \delta_\phi + \frac{1}{4 \xi g_2^2} \frac{1}{(4\pi)^2} \left[ 2 B_0 \left( 0, \xi m_W^2, \xi m_Z^2 \right) + \frac{1}{\cos^2 \theta_W} B_0 \left( 0, \xi m_W^2, \xi m_Z^2 \right) \right] \]  

(B12)

or, using \( A_0 (m^2) = m^2 \left( 1 + B_0 \left( 0, m^2, m^2 \right) \right), m_W = \frac{g_W}{2} \) and \( m_{W} = \cos \theta_W m_Z \):

\[ \Gamma_{\hat{q}_h K_h}^{[1]} = - \delta_\phi + \frac{1}{(4\pi)^2} \left[ 2 \left( A_0 (m_W^2 \xi) - \xi m_W^2 \right) + \left( A_0 (m_Z^2 \xi) - \xi m_Z^2 \right) \right]. \]  

(B13)

This fixes the divergences of \( \delta_\phi \). Note that these are the same divergences that come from the Goldstone tad-

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**Figure 1: Feynman rules for calculating \( \delta_\phi \).**

**Figure 2: One loop Feynmann diagram for calculating \( \delta_\phi \).**

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pole, Eq. (22), which means that \( \delta_\phi \) indeed absorbs all the gauge-dependent divergences in the tadpole condition Eq. (20).

**Appendix C: Gauge dependence in the background-field-modified OS scheme**

In this section, we present what happens if we promote Eq. (23) to a renormalization condition on finite parts, namely:

\[
T_h^{(\xi)} - \lambda v^3 \delta_\phi = 0. \tag{C1}
\]

In principle, this is a valid renormalization condition, which allows to absorb all the tadpole finite gauge dependences into \( \delta_\phi \). One can verify that the FJ-term \( \Delta \) is

\[
\Delta = \frac{1}{2} \delta_\phi + \text{gauge-independent}, \tag{C2}
\]

when Eq. (C1) holds. This allows for a more direct interpretation of an FJ term in the sense of a background field renormalization also in finite parts, leading to a gauge-independent OS mass renormalization constant. Also, the form of Eq. (C2) gives a possibility to modify the FJ procedure to include only the gauge-dependent term, namely \( \delta_\phi \). However, we will show that this choice, together with the OS conditions on the two point functions, leads to a gauge-dependent charge renormalization constant and cannot be used with the usual charge renormalization condition presented in e.g. [1, 22, 29].

To show this, we first need to get the gauge-dependent part of \( \delta_\phi \) in this scheme. For that we look at the gauge-dependent part of the Higgs self energy, which can be written as:

\[
\Sigma_\xi = \frac{1}{(4\pi v)^2} \left( p^2 - m_H^2 \right) (f_Z + 2f_W) + m_h^2 \frac{1}{(4\pi v)^2} \left( A_0 (m_Z^2 \xi Z) + 2 A_0 (m_W^2 \xi W) \right), \tag{C3}
\]

\[
f_V = A_0 (m_Z^2 \xi Z) - \frac{1}{2} (p^2 + m_h^2) B_0 (p^2, m_Z^2 \xi Z, m_W^2 \xi W). \tag{C4}
\]

The gauge dependent part of the renormalized one-loop self energy function is

\[
\Sigma_{\xi}^R = \delta_h |p^2 - \left( \delta_{m_h^2} + \delta_h \right) |m_H^2 + \Sigma_\xi, \tag{C5}
\]

where we write \( | \xi \) to denote the gauge-dependent terms of the renormalization constants. The OS conditions give:

\[
\frac{\partial}{\partial p^2} \Sigma_{\xi}^R |_{p^2 = m_H^2} = 0, \quad \Sigma_{\xi}^R |_{p^2 = m_H^2} = 0. \tag{C6}
\]

Inserting Eq. (C3), Eq. (C4), Eq. (C5) into OS conditions Eq. (C6) to check the gauge-dependent parts, we get the gauge dependences of mass and field renormalization constants of the OS scheme (i.e. in the tadpole scheme of ref. [22]):

\[
\delta_m^R |_\xi = \frac{1}{(4\pi v)^2} \left( A_0 (m_Z^2 \xi Z) + 2 A_0 (m_W^2 \xi W) \right), \tag{C7}
\]

\[
\delta_h |_\xi = - \frac{1}{(4\pi v)^2} \left[ A_0 (m_Z^2 \xi Z) + 2 A_0 (m_W^2 \xi W) \right] + \frac{1}{(4\pi v)^2} m_h^2 \left[ B_0 (p^2, m_Z^2 \xi Z, m_W^2 \xi W) + 2 B_0 (p^2, m_W^2 \xi W, m_W^2 \xi W) \right]. \tag{C8}
\]

By using the expression for \( \Delta \) from Eq. (C2) and \( \delta_\phi \), fixed by Eq. (C1), we see that the FJ-OS mass counterterm, as defined in Eq. (43), is truly gauge independent:

\[
\left( \delta_m^R |_{FJ} \right) |_\xi = \delta_m^R |_\xi - 3 \Delta = \delta_m^R |_\xi - \frac{3}{2} \delta_\phi = 0. \tag{C9}
\]

From Eq. (18) we can get the field renormalization part \( \delta_\phi \) that does not have gauge-dependent divergences. However, it turns out that in the OS, the finite part of \( \delta_\phi \) is gauge-dependent:

\[
\delta_\phi |_\xi = \delta_h |_\xi + \delta_\phi = \frac{1}{(4\pi v)^2} m_h^2 \left[ B_0 (p^2, m_Z^2 \xi Z, m_W^2 \xi W) + 2 B_0 (p^2, m_W^2 \xi W, m_W^2 \xi W) \right]. \tag{C10}
\]

Note that the divergences in this term are gauge-independent as it should be.

Now using Eq. (C1) together with Eq. (39) we see that we must have:

\[
\delta_\lambda |_\xi + \delta_\phi |_\xi - \delta_{m^2} |_\xi = 0. \tag{C11}
\]

From the fact that the FJ mass renormalization constant of the Higgs coincides with \( \delta_{m^2} \) (see. Tab I) and is gauge-independent, the gauge dependence of \( \delta_{\lambda} \) is:

\[
\delta_{\lambda} |_\xi = - \delta_\phi |_\xi. \tag{C12}
\]

From Tab. I, we see that \( \delta_{\lambda} \) enters the definition of the FJ mass renormalization constants, which are gauge-independent. Hence from the gauge independence of \( \delta_{M_Z^2} |_{FJ}, \delta_{M_W^2} |_{FJ} \) and \( \delta_{m^2} |_{FJ} \) we get

\[
0 = 2 \delta_{g_{1,2}} |_\xi - \delta_{\lambda} |_\xi = 2 \delta_{g_{1,2}} |_\xi + \delta_\phi |_\xi, \tag{C13}
\]

\[
0 = \delta_{g} |_\xi - \frac{1}{2} \delta_{\lambda} |_\xi = \delta_{g} |_\xi + \frac{1}{2} \delta_\phi |_\xi. \tag{C14}
\]

which leads to

\[
\frac{1}{g_1^2 + g_2^2} (g_2^3 \delta_{g_{21}} |_\xi + g_2^2 \delta_{g_{12}} |_\xi + \delta_\phi |_\xi = \delta_{e} |_\xi + \delta_\phi |_\xi = 0. \tag{C15}
\]
From Eq. (C15), we see that the charge renormalization constant $\delta_q$ is gauge dependent, because it needs to cancel the gauge-dependence of $\delta_{\partial_\xi}$ given in Eq. (C10). Note that this is solely because we enforced the Eq. (C1). Thus, we have two tadpole conditions, pole and residue conditions for $W, Z$ and Higgs two point function, which are in total 8 conditions, hence fully determines $g_1, 2, \delta_\lambda, \delta_{\mu,2}, \delta_\phi, \hat{\delta}_\phi$ and field renormalization constants of $Z$ and $W$ bosons. This means that there is no freedom left to impose a charge renormalization condition as in e.g. 1, 22, 29, which would give a gauge-independent charge renormalization constant otherwise. Nevertheless, it is interesting to see that the gauge independent definition of the charge renormalization constant is possible also in this “scheme” by absorbing $\hat{\delta}_\phi$ into their definitions, as suggested by Eq. (C15). Hence in principle it is possible to use Eq. (C1) instead of the usual charge renormalization condition and even define a gauge-independent charge renormalization constant. Yet to understand all the consequences of this unconventional choice a more thorough study is needed which is beyond the scope of this work.

Appendix D: Explicit divergences

We used the SM-model file from FeynArts together with FormCalc to get the explicit expressions for the divergences. The divergences of 1, 2, 3 and 4 point function of the Higgs in the SM respectively are:

\[
\Gamma_h^{UV} = v^3 (A + G + S), \quad (D1)
\]
\[
\Gamma_{hh}^{UV} = p^2 \left( \frac{1}{v^2} B + \frac{1}{2m_h^2} G \right) + v^2 (3A + 5S + G), \quad (D2)
\]
\[
\Gamma_{hhh}^{UV} = 6v (A + 2S - G), \quad (D3)
\]
\[
\Gamma_{hhhh}^{UV} = 6 (A + 2S - 2G), \quad (D4)
\]

where we abbreviated

\[
A = -\frac{1}{(4\pi v^2)^2} \left[ 4 \left( \sum_{f=e,\mu,\tau} m_f^4 + 3 \sum_{q=u,d,s,c,t,b} m_q^4 \right) \right], \quad (D5)
\]
\[
B = \frac{1}{(4\pi v^2)^2} \left[ 2 \left( \sum_{f=e,\mu,\tau} m_f^2 + 3 \sum_{q=u,d,s,c,t,b} m_q^2 \right) \right], \quad (D6)
\]
\[
S = \frac{3}{2} \frac{m_h^4}{(4\pi v^2)^2}, \quad G = \frac{1}{2} \frac{m_h^2}{(4\pi v^2)^2} (2m_W^2 \xi + m_Z^2 \xi), \quad (D7)
\]

and omitted the global factor of $\frac{1}{4D}$, where $D$ is the spacetime dimension close to 4. The divergences in $A$ and $B$ come from loop diagrams with ghosts, vectors and fermions. Note that neither $A$ nor $B$ are gauge-dependent, since ghost and vector boson gauge dependences cancel exactly. The divergences abbreviated as $S$ come from diagrams with the Higgs boson loop contribution. Finally, the only gauge-dependent UV divergent term, $G$, corresponds to divergences of the Goldstone boson loop and vanishes in case of $\xi \to 0$. Note that we wrote all these functions in terms of 4 abbreviated constants, thus we need 4 independent conditions and 4 degrees of freedom to uniquely fix them. In case of $\xi \to 0$, the number is reduced to only 3. The four degrees of freedom are, $\mu, \lambda, \delta_\phi$ and $\hat{\delta}_\phi$. They appear in the counterterms of 1, 2, 3 and 4 point function of the Higgs respectively:

\[
\delta \Gamma_h = -\frac{1}{2} m_h^2 v \left( \delta_\lambda - \delta_{\mu,2} + \tilde{\delta}_\phi + \hat{\delta}_\phi \right), \quad (D10)
\]
\[
\delta \Gamma_{hh} = -\frac{1}{2} m_h^2 \left( 3\delta_\lambda - \delta_{\mu,2} + 5\tilde{\delta}_\phi + \hat{\delta}_\phi \right) + p^2 \left( \hat{\delta}_\phi - \hat{\delta}_\phi \right), \quad (D11)
\]
\[
\delta \Gamma_{hhh} = -3 \frac{m_h^2}{v} \left( \delta_\lambda + 2\tilde{\delta}_\phi - \hat{\delta}_\phi \right), \quad (D12)
\]
\[
\delta \Gamma_{hhhh} = -\frac{3}{v^2} \left( \delta_\lambda + 2\tilde{\delta}_\phi - 2\hat{\delta}_\phi \right). \quad (D13)
\]

To make sure that the renormalized n-point functions are finite, we solve four equations:

\[
\delta \Gamma_h^{UV} + \Gamma_h^{UV} = 0, \quad (D14)
\]
\[
\delta \Gamma_{hh}^{UV} + \Gamma_{hh}^{UV} = 0, \quad (D15)
\]
\[
\frac{\partial}{\partial p^2} (\delta \Gamma_{hh}^{UV} + \Gamma_{hh}^{UV}) = 0, \quad (D16)
\]
\[
\delta \Gamma_{hhh}^{UV} + \Gamma_{hhh}^{UV} = 0. \quad (D17)
\]

They give us

\[
\delta_\mu^2 = B + \frac{1}{\lambda} S, \quad \delta_\lambda = 2 \left( B + \frac{1}{\lambda} S \right) + \frac{1}{\lambda} A, \quad (D18)
\]
\[
\hat{\delta}_\phi = \frac{1}{\lambda} G, \quad \tilde{\delta}_\phi = -B, \quad \lambda = \frac{m_h^2}{2v^2}. \quad (D19)
\]

Inserting these expressions into Eq. (D13) we automatically get

\[
\delta \Gamma_{hhhh}^{UV} + \Gamma_{hhhh}^{UV} = 0, \quad (D19)
\]

where $\Gamma_{hhhh}^{UV}$ is given in Eq. (D4).

The VEV counterterms, presented in Sec. V have divergences, which can be read out from Tab. I inserting the expressions for the renormalization constants, shown
in Eq. (D18):
\[
\delta v = \frac{1}{2} v \left( \delta_\phi + \delta_\phi \right) = \frac{1}{2} v \left( \frac{1}{\lambda} G - B \right), \tag{D20}
\]
\[
\delta v|_{\varepsilon_J} = \frac{1}{2} v \left( \delta_{\mu^2} - \delta_\lambda \right) = -\frac{1}{2} v \left( B + \frac{1}{\lambda} S + \frac{1}{\lambda} A \right), \tag{D21}
\]
\[
\delta v|_{\varepsilon_J} = \delta v + \Delta v
\Rightarrow \Delta v = -\frac{1}{2} v \left( \frac{1}{\lambda} S + \frac{1}{\lambda} A + \frac{1}{\lambda} G \right) = -\frac{1}{2} \frac{1}{m_h^2} \Gamma_{UV}. \tag{D22}
\]

**Appendix E: Input values**

Here we present all the numerical input parameters that we used, when evaluating the Eqs. (64a - 64d):

\[\xi = 1\]
\[m_W = 80.398 \text{ GeV}, m_Z = 91.1876 \text{ GeV},\]
\[m_h = 125.9 \text{ GeV}, v = 246.221 \text{ GeV},\]
\[m_c = 0.00501999 \text{ GeV}, m_\mu = 0.105658 \text{ GeV},\]
\[m_t = 1.77684 \text{ GeV},\]
\[m_u = 0.19 \text{ GeV}, m_c = 1.4 \text{ GeV}, m_t = 172.5 \text{ GeV},\]
\[m_d = 0.19 \text{ GeV}, m_s = 0.19 \text{ GeV}, m_b = 4.75 \text{ GeV},\]
\[e = 0.308147.\]

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