Notes on D-instanton correction to $AdS_5 \times S^5$ geometry

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Abstract

We show that the D-instanton in the $AdS_5 \times S^5$ background is a wormhole connecting the background $AdS_5 \times S^5$ to the flat space $R^{10}$ located at the position of the D-instanton. By a $SL(2,R)$ rotation of type IIB theory, we can make the global geometry flat in string frame. We also find that, due to the tight relation between the dilaton and the axion, there is no $SL(2,R)$ element that takes strong string coupling to weak one without making the axion ill defined. We also discuss the case of $AdS_3$ as well as the instanton gases. A subtlety on the D-instanton at the boundary or at the horizon is discussed.

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Recently, the proposal of Maldacena [1] on the duality between the string theory in an anti de Sitter(AdS) space and the conformal field theory at the boundary brought an explosive interests, and by now refinements and significant evidences[2, 3, 4, 5, 6, 7, 8, 9] were established. The correspondence open the possibility to study non-perturbative QCD in four dimension in terms of the semi-classical supergravity. More recently, the question of instanton effects in these contexts [12, 13, 14, 15, 16] was raised and it was shown that the D-instanton in the $AdS_5$ background is relevant to the instanton of the super Yang-Mills theory living at the boundary.

In this paper, we point out that the metric for the D-instanton inside the bulk of $AdS_5 \times S^5$ background [14] is a wormhole between background $AdS_5 \times S^5$ and the flat space $R^{10}$ located at the position of the D-instanton. One can make the result more surprising by using the $SL(2, R)$ symmetry of the type IIB theory. In fact we will show that one can make the geometry globally flat by an appropriate $SL(2, R)$ rotation. On the other hand, the dilaton diverge at the D-instanton position. Ordinary recipe for this is going to the S-dual picture by duality rotation. However, we will see that, due to the tight relation between the dilaton and the axion, there is no $SL(2, R)$ element that rotate strong string coupling to weak one without making the axion diverge.

To set the notation and for the later use, we give a brief review on D-instanton solution of type IIB supergravity in the flat background [18] as well as that in the $AdS_5 \times S^5$ background. We start by D-instanton in flat background. Here we only consider the Ramond-Ramond (RR) pseudo-scalar axion $\chi$, the dilaton $\phi$ and the metric $g_{\mu\nu}$. The bosonic part of type IIB action in Einstein frame is given by

$$S_M = \int d^{10}x \sqrt{-g} \left[ R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} e^{2\phi} (\partial \chi)^2 \right], \quad (1)$$

in Minkowski space. After the Wick rotation, the Euclidean action reads

$$S_E = \int d^{10}x \sqrt{-g} \left[ R - \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} e^{2\phi} (\partial \chi)^2 \right]. \quad (2)$$

Notice that $\chi \rightarrow i\chi$ under the Wick rotation, since $\chi$ is a pseudo-scalar. The equations of motion are given by

$$0 = R_{\mu\nu} - \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} e^{2\phi} \partial_\mu \phi \partial_\nu \phi,$$

$$0 = \partial_\mu (\sqrt{g}g^{\mu\nu} e^{2\phi} \partial_\nu \phi).$$
\[ 0 = e^{2\phi}(\partial \chi)^2 + \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} g^{\mu \nu} \partial_\nu \phi). \]  

(3)

For the flat background, \( g_{\mu \nu} = \delta_{\mu \nu} \), the solution is given by

\[ \pm \chi + \alpha = e^{-\phi} = \frac{1}{H} \]  

(4)

where \( \alpha \) is constant and \( H \) is a harmonic function i.e. \( \partial^2 H = 0 \). Assuming the spherical symmetry, it is given by

\[ H = h + \frac{Q}{r^8}, \]  

(5)

where \( h = e^{\phi_\infty} \) and \( Q \) is the Noether charge defined by

\[ Q = \pm \frac{1}{8 \Omega_9} \int_{\partial M} e^{2\phi} \partial \chi := N_1 \alpha^4 \]  

(6)

where \( \Omega_9 = 2\pi^{5/2}/24 \) is the volume of the nine-sphere. This single instanton solution is evidently singular at \( r = 0 \) in the Einstein frame. However, in string frame the metric become

\[ ds^2 = \sqrt{H}[dr^2 + r^2 d\Omega_9^2] = \sqrt{hr^4 + \frac{Q}{r^4}} \left[ \left( \frac{dr}{r} \right)^2 + d\Omega_9^2 \right]. \]  

(7)

Note that the solution is invariant under the inversion transformation

\[ r \rightarrow (\frac{Q}{h})^{1/4} \frac{1}{r}. \]  

(8)

It corresponds to a wormhole connecting two asymptotically flat Euclidean regions. The wormhole-throat has minimal diameter at \( r = r_{\text{min}} \) given by

\[ r_{\text{min}}^8 = \frac{Q}{h}. \]  

(9)

Under the inversion (8), the asymptotic flat regions at \( r = 0 \) and \( r = \infty \) are mapped onto each other.

We now turn to the D-instanton in AdS\(_5 \times S^5\) background. We first describe the background geometry as the near horizon geometry of D3 branes. The \( Dp \)-brane as a supergravity solution can be characterized in terms of \( H_p(x_\perp) \), a harmonic function of the coordinates perpendicular to the world volume of Dp-brane. In fact, \( H_p \) depends only on the radial coordinate \( r = \sqrt{x_{p+1}^2 + \cdots + x_9^2} \) and is given by

\[ H_p = 1 + \frac{Q_p}{r^{7-p}}. \]  

(10)
where the charge $Q_p$ is
\[ Q_p = g_s (2\pi)^{(5-p)/2} (2\pi \alpha')^{(7-p)/2} [2\pi (7-p)/2 / \Gamma((7-p)/2)]^{-1}. \] (11)

In string frame, the Dp-brane metric in Euclidean version is
\[ ds_p^2 = H_p^{-1/2} (dx_0^2 + \cdots + dx_p^2) + H_p^{1/2} (dx_{p+1}^2 + \cdots + dx_9^2) \] (12)
with the dilaton field \( \phi \) given by
\[ e^{2\phi} = H_p^{(3-p)/2}. \] (13)

Therefore the string background describing \( N \) D3-branes is given by
\[ ds_3^2 = H_3^{-1/2} d\vec{x}^2 + H_3^{1/2} (dr^2 + r^2 d\Omega_5^2). \] (14)

In the decoupling limit (\( \alpha' \to 0, u = \frac{x}{\alpha'} \) = fixed), the constant term in the harmonic function \( H_3 \) can be neglected. After rescaling the \( u \to \lambda^{-1} u \) and \( \vec{x} \to \lambda \vec{x} \) by a constant factor \( \lambda^4 = 4\pi gN \), the metric can be written
\[ ds_3^2 = \alpha' \sqrt{4\pi gN} \left[ u^2 d\vec{x}^2 + \frac{du^2}{u^2} + d\Omega_5^2 \right], \] (15)
which is the metric of \( AdS_5 \times S^5 \). In terms of the variable \( z = 1/u \), the metric is
\[ ds_3^2 = \alpha' \sqrt{4\pi gN} \left[ \frac{1}{z^2} (d\vec{x}^2 + dz^2 + z^2 d\Omega_5^2) \right]. \] (16)

The boundary of \( AdS_5 \) is at \( u = \infty \) or equivalently at \( z = 0 \).

Now we consider D-instanton in the \( AdS_5 \times S^5 \) background. The equations of motion in \( AdS \) background are equal to (3), except that the first equation in (3) is replaced \( [13] \) by
\[ 0 = R_{\mu\nu} - \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} e^{2\phi} \partial_\mu \chi \partial_\nu \chi - \frac{1}{6} F_{\mu\delta\phi\lambda\sigma} F_{\nu\phi\lambda\sigma} \] (17)
where \( F_{\mu\nu\rho\lambda\sigma} \) is a self-dual gauge field strength. There are two different solutions of D-instanton in the bulk of the \( AdS_5 \). The first solution is pointlike in \( AdS_5 \) and uniformly spread over \( S^5[13] \) while the second is pointlike in ten dimensional \( AdS_5 \times S^5 \) space \( [14] \). These two solutions agree only at the boundary of the \( AdS_5 \) and equal to the solution given by \( [15] \). Here we use the solution given in the ref.\([14]\). The solution spread over \( S^5 \) can be regarded as superposition of the former. The supergravity solution for D-instanton embedded in the bulk of the \( AdS_5 \times S^5 \) is given by the metric of the following form,
\[ ds^2 = H_{-1}^{1/2} \left[ \frac{1}{z^2} (d\vec{x}^2 + dz^2 + z^2 d\Omega_5^2) \right], \] (18)
where we regarded $z$ as the length of the six vector $\vec{z}$ of the inverted transverse space of D3, i.e., $\vec{z} = x_\perp / u^2$. Here, we have taken off the constant factor $(\alpha' \sqrt{4\pi gN})$. The associated one and five form R-R field strengths and dilaton field are given by

$$
F^{(1)} = d(H_{-1})^{-1}, \\
F^{(5)} = d(1/z^4)(dx_0 \wedge \cdots \wedge dx_3), \\
e^\phi = H_{-1}. 
$$

(19)

Notice that $H_{-1}$ is a harmonic function satisfying the laplace equation in the $AdS_5 \times S^5$

$$\frac{1}{\sqrt{g}} \partial_\mu (g^{\mu\nu} \sqrt{g} \partial_\nu H_{-1}) = 0.$$  

(20)

Here, we give some detail of the derivation of $H_{-1}$ since it does not appear elsewhere. Since the metric (16) is conformaly related to the flat metric,

$$ds^2 = \Gamma(z)[d\vec{x}^2 + dz^2 + z^2d\Omega_5^2]$$

(21)

with $\Gamma(z) = 1/z^2$, we look for an laplace equation in the flat metric by changing the variable

$$H_{-1}(\vec{x}, \vec{z}) = G(z)H(\vec{x}, \vec{z}),$$

(22)

where $G(z)$ is a function of $z$ only. Then the laplace equation for $H_{-1}$ becomes

$$0 = \frac{1}{\Gamma}[G\Delta_0 H + 2\delta^{\mu\nu}\{\partial_\mu G + 2G(\partial_\mu \log \Gamma)\}\partial_\nu H + \{\Delta_0 G + 4\delta^{\mu\nu}(\partial_\mu G)(\partial_\nu \log \Gamma)\}H]$$

(23)

where $\Delta_0$ is a laplacian operator in ten-dimensional flat space. If $\Gamma$ and $G$ satisfies the following two equations

$$\Delta_0 G + 4\delta^{\mu\nu}(\partial_\mu G)(\partial_\nu \log \Gamma) = 0$$

$$\partial_\mu G + 2G(\partial_\mu \log \Gamma) = 0,$$

(24)

then the equation (23) is reduced to the flat space laplace equation:

$$\Delta_0 H(\vec{x}, \vec{z}) = 0,$$

(25)

whose solution can be readily written by

$$H(\vec{x}, \vec{z}) = c_1 + \frac{c_2}{[(\vec{x} - \vec{x}_0)^2 + (\vec{z} - \vec{z}_0)^2]^4}.$$  

(26)
where \((\vec{x}_0, \vec{z}_0)\) is the location of the D-instanton. Now for the given \(\Gamma\), the solution of (24) is easily found and given by

\[
G(z) = c_3 z^4. \tag{27}
\]

Therefore the solution \(H_{-1}\) can be written as

\[
H_{-1}(\vec{x}, \vec{z}) = h + \frac{N_{-1} z_0^4 z^4}{[ (\vec{x} - \vec{x}_0)^2 + (\vec{z} - \vec{z}_0)^2]^4} \tag{28}
\]

Here \(z_0^4\) factor is added for the dimensional reason. Notice that the term like \(c'(z/z_0)^4\) could be added to above solution. However, since it can be regarded as the large \(z_0\) limit of the second term, we deleted it. A few important remarks regarding the role of \(z_0\) should be made here.

- While \(\vec{x}_0\) could be set to zero without changing the geometry, changing \(z_0\) changes the geometry. This is because the presence of the D3 branes breaks the translational symmetry in the directions transverse to them.

- \(z_0\) is the only scale that appears in the near horizon geometry where \(\alpha'\) is taken to be zero. This scale is introduced by Higgsing D3 and D(-1) by separation \(z_0 = 1/u_0\), and it can be interpreted as the size of the D-instanton. In fact it is interpreted as the size of the Yang-Mills instanton of the boundary theory\[13, 14\].

- The solution is \(\alpha'\) independent. This should be so since we already took the near horizon geometry. For this it is crucial to have a scale \(z_0\). We discuss the problem that rises in the absence of \(z_0\) in the next item.

- By considering the near horizon geometry of the D(-1)-D3 system (D(-1) contained in the D3), one can get the solution \(H_{-1} = 1 + c\alpha'^4/r^4 = 1 + cz^4\). The problem is that \(c\) is a dimensionful constant but we do not have any scale other than \(\alpha'\). So, \(c \sim \alpha'^{-2}\) by counting the dimension. Then, the constant piece is negligible in the decoupling limit and \(H_{-1} \sim z^4\). Then it is easy to see that the metric (18) becomes globally flat. Looking from the AdS point of view, this is not surprising since in this case the D-instanton size is of string length which is regarded as infinitely large in the decoupling limit. Namely since \(u = r/\alpha'\) fixed and \(\alpha' \to 0\), \(l_s \sim \sqrt{\alpha'} >> r \sim \alpha'u\). However, it seems to be puzzling from the observer inside the D3 brane if we regards the D-instanton as a localized object inside the D3 brane. How can a localized object
change the global geometry? There are two possible resolution to this. The first one
is to regard the solution \( H_{-1} = z^4 \) not for the point like D-instanton but for the
D-instantons spread over the world volume of D3 just as the solution in [13] for the
D-instanton spread over the \( S^5 \). In fact, from ten dimensional point of view, the power
\( r^{-4} \) for the harmonic function is acceptable only if the source is four dimensional. In
this case \( z^4 \) term dominate the constant piece and the global geometry changes from
\( AdS_5 \times S^5 \) to flat space. The second way to resolve the problem is to regard the \( 1 + cz^4 \)
as the \( z_0 \rightarrow \infty \) limit of (28). Since \( z_0 \) is the D-instanton size, it corresponds to a
very large instanton. However, notice that the \( z^4 \) term in this case is negligible since
\( H_{-1} \approx h + N_1(z/z_0)^4 \rightarrow h \). Similar comments can be applied to the D-instanton at the
boundary, which can be obtained from \( 1 + cz^4 \) by taking the inversion \( z \rightarrow z/(z^2 + \bar{x}^2) \).
It should be considered as the \( z_0 \rightarrow 0 \) limit of (28), therefore it is a very small instanton.
Here also the contribution of this small instanton to the geometry change is negligible
since \( H_{-1} \approx h + z_0^4 \cdot O(1) \rightarrow h \).

Let’s discuss the instanton-correction to \( AdS_5 \times S^5 \) geometry. We start the discussion
by observing that near the D-instanton position \((\vec{x}_0, \vec{z}_0)\), the string frame metric becomes
flat:
\[
\begin{align*}
    ds^2 & \sim \frac{d\vec{x}^2 + d\vec{z}^2}{((\vec{x} - \vec{x}_0)^2 + (\vec{z} - \vec{z}_0)^2)^{1/2}} & (29) \\
    & = \frac{1}{w^2} (dw^2 + w^2 d\Omega_9^2) & (30) \\
    & = d\vec{X}^2, & (31)
\end{align*}
\]
where \( w = \sqrt{(\vec{x} - \vec{x}_0)^2 + (\vec{z} - \vec{z}_0)^2} \) is the ten-dimensional radius and
\[
\vec{X} = \left( \frac{\vec{x}}{w^2}, \frac{\vec{z}}{w^2} \right) = \left( \frac{\vec{x}}{\vec{x}_0^4/w^4 + \vec{x}^2}, \frac{\vec{z}}{\vec{z}_0^4/w^4 + \bar{x}^2} \right). (32)
\]
Notice that \( H_{-1} \) is constant far from the D-instanton, especially near the boundary, \( z \rightarrow 0 \).
Therefore what we have shown is that the D-instanton solution is a wormhole solution
connecting the asymptotic \( AdS_5 \times S^5 \) space and a flat space at the D-instanton position.
See. figure 1.

In fact, one can take advantage of the \( SL(2, \mathbb{R}) \) invariance of the Type IIB theory to
make the whole geometry flat. Introducing a scalar \( S_\pm \) by

\[
S_\pm = \chi \pm e^{-\phi} \quad (33)
\]
the action and equations of motion of type IIB theory are invariant under the \( SL(2,\mathbb{R}) \) transformations
\[
S_\pm \rightarrow \frac{aS_\pm + b}{cS_\pm + d}, \quad ad - bc = 1
\] (34)
with the two generators
\[
\Omega_1 : S_\pm \rightarrow S_\pm + 1, \quad \Omega_2 : S_\pm \rightarrow -\frac{1}{S_\pm}.
\] (35)
Using these transformations we can change arbitrarily the parameters \( \alpha \) and \( h \) characterizing the D-instanton solution (but not \( Q \)). In particular, we can transform the constant part of the harmonic function \( H_{-1} \) to zero by any \( SL(2,\mathbb{R}) \) transformation of the form
\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix}
-c\alpha + \frac{h}{2c} & -c\alpha^2 + \frac{2\alpha}{h} - \frac{ha}{2c} \\
c & c\alpha - \frac{2c}{h}
\end{pmatrix},
\] (36)
with arbitrary \( c \) and the dilaton becomes
\[
e^\phi = \frac{N_{-1}'z_0^4 z^4}{[(\vec{x} - \vec{x}_0)^2 + (\vec{z} - \vec{z}_0)^2]^4},
\] (37)
where \( N_{-1}' = N_{-1}(2c/h)^2 \). If we require \( N_{-1}' = N_{-1} \), then we get \( c = \pm h/2 \). Now, the metric is flat \( globally \). This is surprising. It indicates that the special \( SL(2,\mathbb{R}) \) rotation effectively lead us to the near horizon geometry of the D-instanton. One remark is
that above statement is approximation, since the $SL(2,R)$ is broken to $SL(2,Z)$ due to the quantum effect.

Next, we observe that the instanton correction ruins the conformal invariance of the D3 system. That is, in the super-Yang-Mills theory on the world volume of the Dp-brane, the dimensionless effective coupling constant $g_{eff}$ is given by

$$g_{eff}^2 = Ng_{YM}u^{p-3}. \tag{38}$$

Notice that for $p = 3$, $g_{eff}$ does not run as the energy scale ($u = r/\alpha'$) changes, implying the conformal invariance. The supergravity solution in the decoupling limit is reliable for $gN >> 1$. On the other hand, after the D-instanton insertion, the string coupling $g_s = e^\phi$ runs as $u$ changes. That is, the instanton correction ruins the conformal invariance of Yang-Mills system at the boundary. Far from the D-instanton position, the metric is just the AdS background and the dilaton is small therefore the system can be described by the type IIB supergravity. Notice, however, the dilaton diverge near the D-instanton position. There, one might expect that this system could be described by the S-dual of the type IIB supergravity. In the absence of the axion, the inversion of the coupling ($g_s \to 1/g_s$) can be obtained by a $SL(2,R)$ rotation $\Omega_2$ in (35). In our case, the axion and the dilaton is tightly related by

$$\chi + \alpha = e^{-\phi}. \tag{39}$$

Due to this relation, the axion and the dilaton transform under a general $SL(2,R)$ rotation

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

into

$$\chi \to \chi' = \frac{(ad + bc - 2ac\alpha)e^{-\phi} + (b - a\alpha)(d - c\alpha)}{(d - c\alpha)^2 + 2c(d - c\alpha)e^{-\phi}},$$

$$e^\phi \to e^{\phi'} = (d - c\alpha)^2e^\phi + 2c(d - c\alpha). \tag{40}$$

From these, the followings can be shown easily:

- It is impossible to find a $SL(2,R)$ rotation which transforms away $\chi$.
- Starting from our solution, it is impossible to get strong and weak exchange $e^\phi \to e^{-\phi}$.
- With the choice $d = c\alpha$, the dilaton vanishes. Therefore the dyonic string, whose electric and magnetic charge is $(n_e, n_m) = (a, c)$, becomes free near the D-instanton. However, the axion blows up in this case.
Figure 2: Multi-D-instanton solution generate multi-wormholes.

All these unusual properties come from the ansatz (39). Consequently, the physical interpretation of the divergence of the dilaton and the flatness of the geometry near D-instanton is not very clear, yet. Nevertheless, it is well established [12, 13, 14, 17] that the D-instanton holographically projected corresponds to the Yang-Mills instanton.

What happen if we include the multi-D-instanton solutions? Multi-instanton solution in $AdS_5 \times S^5$ can be given by

$$H_{-1} = h + \sum_i \frac{N_{-1} z_0^4 z_i^4}{((\vec{x} - \vec{x}_i)^2 + (\vec{z} - \vec{z}_i)^2)^4} \quad (41)$$

For the dilute instanton gas, that is, when instantons are well separated, multiple wormholes are generated. See figure 2. However, contrary to the one instanton case, one can not make the string frame metric globally flat by SL(2, R) rotation.

One may ask whether the (local) flatness near the D-instanton is general feature of the near horizon geometry of any D-p-brane. In order to answer this, we describe D-instanton in $AdS_3 \times S^3$, which is a near horizon geometry of the D1-D5 system. Its metric is given by

$$ds_6^2 = \frac{1}{\bar{z}^2} (dx_0^2 + dx_1^2 + dz^2) + d\Omega_3^2 \quad (42)$$

Here $\bar{z}$ is the four dimensional vector transverse to the D1- and D5-brane. Now, we add D-instanton to this background. The metric of the resulting system is given by

$$ds^2 = H_{-1}^{1/2} \left[ \frac{1}{\bar{z}^2} (dx_0^2 + dx_1^2 + dz^2) \right] \quad (43)$$
where $H_{-1}(\vec{x}, \vec{z})$ is a harmonic function. $H$ can be found in similar way as before

$$H_{-1} = h + \frac{N_{-1}z_0^2z^2}{[(\vec{x} - \vec{x}_0)^2 + (\vec{z} - \vec{z}_0)^2]^2}. \tag{44}$$

Notice that the metric with the above $H_{-1}$ given in (43) is not flat near the D-instanton position. Therefore, the (local) flatness of the geometry after the instanton correction is very special for the $AdS_5 \times S^5$ geometry.

So far, we have shown that the effect of adding D-instanton to the $AdS_5 \times S^5$ background is to make the string frame geometry locally flat near the D-instanton position. By a $SL(2, \mathbb{R})$ rotation of type IIB theory, we could make the global geometry flat in string frame. A subtlety on the D-instanton at the boundary or at the horizon was discussed. We also found that, due to the tight relation between the dilaton and the axion, there is no $SL(2, \mathbb{R})$ element that rotates strong string coupling to weak one without making the axion ill defined. We also discussed the case of $AdS_3$ as well as the instanton gases.

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