Improvement of solution using local search method by perturbation on VRPTW variants

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Abstract. One application of graph theory is to optimize the distribution problem. This problem can be solved using Vehicle Routing Problem with Time Window (VRPTW) model and its variants such as VRPTW, CVRPTW dan OVRPTW. This article comprehends the improvement of the solution with the local search method using perturbation on those variants. There are three parts in the method: generating an initial solution, improvement using local search, and perturbation. The initial solution was generated using the sequential insertion algorithm, the local search process used inter-route and intra-route operators, and the perturbation using ejection chain and double swap. Result of experiments showed that perturbation using double swap gave a better solution than ejection chain. This caused by two-times movement in the double swap that could examine all optimal solution possibilities. An example of implementation the VRPTW variant on distribution optimization is given in this article.

1. Introduction

The distribution process is one of the processes that have an important role in carrying out economic activities. Distribution is efficient or optimum if the distance of the distribution route is minimum. One method to solve the problem of optimum distribution is the Vehicle Routing Problem (VRP). Mathematically, VRP deals with the problem of determining the optimal route by considering capacity of the vehicle, which aims to minimize the number of vehicles to be used and the total distance traveled. The main process is determining the route, started from the depot (distribution center) to visit all existing customers exactly once, then returns to the same depot.

Vehicle Routing Problem with Time Windows (VRPTW) is one variant of VRP with additional time windows constraints for each customer. The application of VRPTW to complete distribution optimization using VND (Variable Neighborhood Descent) has been discussed in [1]. VRPTW solution with memetic algorithm has been discussed in [2], and by using VNS (Variable Neighborhood Search) metaheuristic is discussed in [3].

The VRPTW variant that can also be used to complete distribution optimization and interesting to be discussed is the Capacitated Vehicle Routing Problem with Time Window (CVRPTW). In the real problem application, many shipping service companies use a number of similar vehicles and the shipping is divided into several shifts and there is a time limit for each shift. Modeling CVRPTW as a distribution system consists of a main depot and a number of vehicles with the same capacity, to serve a number of scattered customers. Each customer has certain time limit, their request is less than the capacity of the vehicle, and each customer is visited once by one vehicle. Some research related to CVRPTW problems, for example real time stochastic settlement [4] and local search analysis [5].

In certain situations, the distributor does not need a vehicle to return to the depot. For example, in the case of a company that does not have its own vehicle, the company will rent a vehicle to carry out the distribution process. After completing the distribution process the vehicle does not need to return to the depot but instead returns directly to the vehicle rental location. Such problems can be solved by
using Open Vehicle Routing Problem (OVRP). Research on OVRP can be seen in [6], [7], [8] dan [9]. One variant of VRPTW in the OVRP case that is often discussed is the Open Vehicle Routing Problem with Time Windows (OVRPTW) which adds a time windows variable to the process. Research on the completion of OVRPTW with Iterated Local Search can be seen in [10] [11].

The heuristic method for solving the VRPTW variant is a local search method that repeats iteratively to obtain an optimal solution. Implementation with effective local search methods for finding solutions can be seen in [12]. The discussion on local search methods became an interesting discussion of researchers not only on the VRPTW variant but also other VRP variants. This discussion can be seen in [13], [14], [15], [16], [17], [18], [19].

In the method, after searching for a solution in the local search stage, next stage is improvement with the perturbation stage. The perturbation stage of the local search stage aims to direct the continued search of existing solutions so as to obtain an even better optimum solution [11]. There are several types of perturbations that can be used to improve solutions such as ejection chains, double swaps, and double bridges can be seen in [11], [20], [21], [22], [23]. This article discusses improvement of solution using local search methods by perturbation on VRPTW, CVRPTW and OVRPTW. A computer program made by Delphi was made to help finding the solution.

VRPTW, CVRPTW and OVRPTW problems can be solved using local search algorithm. VRPTW solution with local search can be seen in [5], using VND as in [1], using VNS in [3]. The CVRPTW solution with local search can be seen in [5] also, and OVRPTW solution with local search can be seen in [11].

2. Local Search Algorithm

Local search algorithm is a method of finding new solutions from the initial predetermined solution. The main principle of this local search is done by changing the structure of the neighborhood of existing solutions. If there is a better solution found, then this solution will update the previous solution, and becoming a new initial solution for the next iteration. Improvements to the neighborhood structure can be done inter-route (structural changes between routes) and intra-route (structural changes on the route itself). Here are some changes in the structure of neighborhood inter-route and intra-route:

2.1. Changes in inter-route structure

- **Shift (1, 0):** Move one customer-i from one route to another.
- **Swap (1, 1):** Swaps the order between one customer-i from route-1 to one customer-j from route-2.
- **Shift (2, 0):** Moves two adjacent customer-i and j from one route to another.
- **Swap (2, 1):** Exchange order between two customer-i and j adjacent from route-1 to one customer-k from route-2.
- **Swap (2, 2):** Exchange the order between two adjacent customer-i and j from route-1 with two customers, k and l, connected from route-2.
- **Cross:** Removes sides (i, j) of route-1 and sides (k, l) from route-2. Then form the new side into (i, l) and paste it on route-1, and (k, j) and paste it on route-2.

2.2. Changes in intra-route structure:

- **Or-opt:** One to three adjacent customers are deleted, then the deleted customer is reinserted into different positions on the same route.
- **2-opt:** Two non-adjacent sides are deleted, then two other sides are added so that they form a new route.
- **Exchange:** Permutation between two points or customers. This Exchange is shaped like Swap (1,1) but is in an intra-route version or occurs in the same route.
- **Reinsertion:** One customer in the route is deleted, then the customer is reinserted into a different position on the same route.
There are various procedures for implementing local search regarding changes in the structure of the neighborhood, including Variable Neighborhood Search (VNS), Variable Neighborhood Descent (VND), and Random Variable Neighborhood Descent (RVND).

3. Discussion
Refer to the previous research: the local search for VRP [13], efficient local search strategies [17], enhanced multi-directional local search [18], and a hybrid iterative local search algorithm [19] these results support the idea of solution using local search for VRPTW variant. Iterated local search algorithm with ejection chains for OVRPTW [11], best solutions to VRPSPD by a perturbation based algorithm [22] and perturbed decomposition algorithm for TSP variant [23] these results support the idea of improving by perturbation. To solve the VRPTW variant problem starts from building the initial solution. The steps to building an initial solution are explained as follows.

3.1. Building Initial Solutions Using Sequential Insertion Algorithms
The sequential insertion algorithm can be used to build the initial solution for the VRPTW. The application of sequential insertion algorithm to OVRPTW problems differs from VRPTW and CVRPTW problems. In the case of OVRPTW it does not form a cycle but forms a path. The steps of the sequential insertion algorithm in solving VRPTW variant problems are as follows.

Step 1 Select the starting point as a depot
Step 2 Insertion point
- List all customers who have not entered the route. If the list of customers who have not entered the route already exists, then proceed to step 4.
- Select customer j from the list closest to the depot and enter it into the current sub-route, so that a subroute (i-j) is obtained. Calculate whether the addition of customer j meets the vehicle capacity constraints and the time requirement. If it fulfills, then proceed to step 2 and if it does not meet then the insertion is canceled and proceed to step 3.
- Select the next customer from the list of customers who have not entered the route and it has the closest distance from any customer in the current sub-route. After the new customer is selected, then list all possible routes formed by inserting a new customer into the current subroute. From these possible routes, select the minimum length and calculation time (travel time, waiting time, and service time).
- Repeat step 3 until there is a customer insertion that violates the vehicle capacity or time constraints. If there is a violation, then the insertion point is canceled and proceed to step 3.
Step 3 Repeat steps 1 and 2 if there are still a list of customers who have not entered the route.
Step 4 If there is no more customer who has not entered the route, the route search is complete.
STOP
After obtaining the initial solution using the sequential insertion algorithm, the next step is to make changes to the neighbor structure in the local search stage by using the RVND procedure.

3.2. Local Search using RVND
Local search is a series of experiments carried out repeatedly to improve the current solution by changing the structure of its neighborhood. The process used in this local search is Randomized Neighborhood Variable Neighborhood Descent (RVND). RVND is basically VND by using a randomly chosen neighborhood sequence. The improvement of neighborhood structure in RVND can be in the form of inter-route and intra-route.

In inter-route improvement the solution is said to be feasible if the total length is smaller than the current total length. Whereas in the intra-route improvement, the solution is said to be feasible if the distance of each route produced is smaller than the current distance of each route. In addition, inter-route and intra-route improvements should not exceed the capacity constraints and Time Windows.

Below here are the steps of RVND at the local search stage to solve the VRPTW variant problem.
Step 1. An initial solution is given using a sequential insertion algorithm and a list of neighborhood moves to be used, i.e.

\[ NL = \{ \text{Shift}(1,0), \text{Swap}(1,1), \text{Shift}(2,0), \text{Swap}(2,1), \text{Swap}(2,2), \text{Cross} \} \]

\[ NL' = \{ \text{Or-opt}, \text{2-opt}, \text{Exchange}, \text{Reinsertion} \} \]

Step 2

Choose \( N^{(\eta)} \in NL \) in random order

- If inter-route improvement (\( NL \)) can produce a better solution than the current solution, then update the current solution with a new solution. The process is continued by carrying out intra-route improvement (\( NL' \)). Choose \( N'^{(\eta)} \in NL' \) randomly.
  
  - If intra-route improvement (\( NL' \)) results in a solution that is better than the current solution, then update the current solution with the new solution.
  
  - If intra-route improvement (\( NL' \)) does not produce a better solution than the current solution, then delete \( N'^{(\eta)} \) from \( NL' \)

The process is performed until \( NL' \) is empty.

- If inter-route improvement (\( NL \)) does not produce a better solution, then delete \( N^{(\eta)} \) from \( NL \) and continue for other types of inter-route movement that are also randomly selected without having to make an intra-route move again

Step 3

The process is performed until all list in inter-route is examined.

STOP

After the local search using RVND is complete, the next step is the perturbation stage. In the perturbation stage, 2 kinds of perturbation methods (Ejection Chain and Double Swap) is applied

3.3. Improvement with Perturbation

The perturbation stage in this discussion is carried out by applying two types of perturbation namely Ejection Chain and Double Swap. The following are the second steps to the type of perturbation.

3.3.1. Ejection chain

Ejection chain is a method to eject a customer from a route in chaining. Here steps of ejection chain briefly:

Step 1 Given an initial solution from local search method

Step 2 The first chain starts from route 1. Randomly select a customer from route-1, and then the point customer moves and ejects other customers from routes that are also randomly selected.

Step 3 An ejected customer from route-2 moves to route-3, subsequently ejects next randomly chosen customer from route-3. This process is carried out continuously

Step 4 The chain ends when the customer from the last route is inserted to the first route by occupying the position of the customer from route 1 that moved earlier.

Steps 2 to 4 are carried out repeatedly until a better solution is found or up to the maximum trial limit specified. The maximum number of routes for implementing an ejection chain is 12 routes, because based on previous research, the application of ejection chains on more than 12 routes has less effect.

3.3.2. Double Swap

The main concept of double swap is to make Swap(1,1) changes which are carried out 2 times in one trial, and the customer that does the double swap is chosen randomly. Following are the steps of the double swap method in brief

Step 1 Given a current solution from local search method.

Step 2 Pick a random customer from route 1, then exchange with a randomly chosen customer from route 2.

Step 3 Take another customer from route 1 randomly and different to customer from step 2, then exchange with one customer from route 2 that is also taken randomly and differently from step 2.
Similar to the ejection chain method, steps 2 and 3 are carried out until a better solution is found (a feasible solution) or up to the maximum trial limit specified.

The acceptance stage of the optimum criteria is the stage of selecting a new solution that is better than the previous solution in the process of improving local search and perturbation. If there are more than one feasible solution, then the role of the criteria acceptance stage is to choose the best feasible solution. This aims to avoid a decrease in the quality of the solution.

Following are the steps of the local search method with perturbation improvements to solve the VRPTW variant problem.

• **Initialization stage:**
  An initial solution is given using a sequential insertion algorithm

• **Local Search method**
  RVND procedure is applied to move into other neighborhood, both for inter-route ($NL$) and intra-route ($NL'$) movement. The movement must confirm to the VRPTW constraints that are capacity of vehicle and time windows.

• **Perturbation: Ejection Chain dan Double Swap**
  The perturbation step was explained earlier. The experiments carried out in this perturbation stage were carried out randomly and if a better solution was found than before, the experiment stopped. In order for this method to be applied, there are at least 2 routes with 2 customers for each route.

• **Acceptance of criteria (optimal conditions).**
  Selection of the best solution from the iterations that have been done.

4. **Implementation**

A computer program made by Delphi was made to help finding the solution. The distance of the depot to the customer and between customers the real problem is shown in figure 1. The route of the results of the iteration process with delphy is shown in figure 2. The graph model obtained from the iteration process in the program can be seen in Figure 3.
5. Conclusion
The VRPTW variant can be solved using the local search method. The completion of the VRPTW variant has 4 main stages: initial solution search, local search method, improvement with perturbation, and acceptance of criteria (optimal conditions). The initial solution search is performed using a sequential insertion algorithm. The sequential insertion algorithm used in the VRPTW is carried out by performing various insertion possibilities from the nearest selected point and then selecting the route that has the shortest distance. The local search stage is carried out using the RVND process, which is the random movement of neighborhood structures. In the repair phase with perturbation, it uses two ejection chain and double swap procedures. Then from repeating the local search and perturbation stages, the most optimum solution will be chosen at the criterion acceptance stage. The local search algorithm can be improved by using perturbation method on the VRPTW variant problem. As a result of conducting this research, we propose that improving solution by perturbation for other VRP variants.

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