An analytical calculation and applications of the kinematic characteristics of the motor vehicle movement at an oblique hitting the side of a cable barrier

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Abstract. An analytical calculation of the kinematic characteristics of a motor vehicle allows both checking the correctness of the developed FE model of cable barriers and can be used to study the movement of a motor vehicle directly during the construction of the FE model. In the article it has been provided recommendations on the procedure for conducting a virtual test in order to verify the correctness of the construction of the finite element (FE) model of the cable barriers, and also it has been analyzed the possibility of using analytical trajectories in the study of passive safety of the driver/passenger. FE modeling has been performed using the LS-Dyna multipurpose finite element complex. The present work continued researches which has been published as [1] and [2].

Keywords: cable barrier, mathematical modeling, analytical construction, trajectory.

1. Introduction

The road infrastructure of the world continues to develop very actively, therefore, strict laws are constantly being developed and adopted in various countries in the field of organizing traffic flows. Nonetheless cars are still related to the area of increased danger to humans.

Recently, as an element of active safety, cable road barriers was gaining popularity. It was installed on the dividing lane of highways to prevent deliberate and unintentional crossings of vehicles across the road; to separate traffic flows in opposite directions; as well as on the side of the road – to prevent deliberate and unintentional trips of vehicles outside the highway.

The main advantage of cable barrier is the low value of the index of the severity of the injury, relatively low cost, lack of snow retention. [3]. The process when a motor vehicle has been described with oblique collision with side cable barrier has a difficult interaction structure. For the first a cable has a high degree of geometric non-linearity. Then racks and soil have a high degree of physical non-linearity. At least, all emerging processes are transient. [3]. Also, an analytical calculation of the movement of motor vehicle during collisions with cable barriers is associated with the need to take into account a large number of factors characterizing elements of cable barriers Also it possible to use string oscillation equations like it has been done for cable-stayed structures [4, 6-9] and analytical calculation of air finishers [6, 7]. But it is because of complex cable barrier structure like way has

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issued problem with formulation of crane conditions. [3-10]. All of the above has been lead to use Finite Element modeling (for example, with DYNA, MARK, NASTRAN and so on) for research cable barrier structure and the crash test process. [3, 11].

Other possible way of research is simplification of the mechanical system: replacement of the cable with some physical models that adequately imitate the cable properties which necessary for the task and neglect the insignificant ones in the context of the problem under consideration. [10]. Such way also has been used in [1] и [2] and in present work.

Also it should be noted, that oblique collision of a motor vehicle with side cable barrier is impotent part of field tests to determine the level of active and passive a motor vehicle safety. The using of an analytical description of a motor vehicle, regardless of the complexity of the engineering design of the cable barrier, greatly facilitates the work on assessing the internal passive safety of the driver/passenger, and also it allows to verify the correctness of the developed FE model for virtual crash tests.

2. Recommendations for conducting virtual tests in order to verify the correctness of the construction of the finite element (FE) model of a cable barriers.

As an example, it has been considered two virtual tests, FE models of which have been developed by I. Karpov, a graduate student of the Department of Structural Mechanics, MADI. For these models, it has been constructed the analytical kinematic characteristics of a motor vehicle and compare them with the results of the virtual crash tests.

The angle between the longitudinal axis and the direction of movement of the test ATS used to point it is 20 ° according to GOST P 52721-2007 (ГОСТ Р 52721-2007) (pp. 6.1-6.6). [12].

2.1. Calculation of model No. 1.

A virtual test has been conducted for a motor vehicle weighing 1050 kg at an initial contact speed v₀ = 25 m/c. According to the results of virtual test, the contact time τ of motor vehicle and cable barrier has been taken about 0,7 s, the maximal dynamic deflection Y was 2,5 m, the length of the interaction path was x(τ) ≈ 15 m. Figure 1 presents the results of virtual crash test.

![Figure 1](image-url)

**Figure 1.** The results of virtual crash test for model No. 1: red line (A) – y-displacement, m; green line (B) – x-displacement, m. (Blue line (C) – z-displacement. It was not necessary for the calculations).

For the first, it needs to find conditional cable frequency p according algorithm of calculation, which has been presented in [1]:

...
According the same algorithm amplitude of oscillation conditional cable was equal to:

\[ A_0 = \frac{v_0 \sin \alpha}{p} = \frac{8.55}{4.489} = 1.905, \text{m.} \]

Obtained value \( A_0 \) was less than the maximal dynamic deflection (2.5 m). But, in the [1] it has been showed that condition

\[ \frac{A_0}{Y^* \sin pt^*} > 1 \]

is necessary for the existence of a solution to the equation

\[ \frac{pt^*}{\tan pt^*} = \ln \left( \frac{A_0}{Y^* \sin pt^*} \right). \]

Here, \( t^* \) is the moment of time when the maximal dynamic deflection has been reached.

Anyway, for checking the operation of the algorithm, it has been found a solution to this equation, which in the present problem took the form:

\[ \frac{\zeta}{\tan \zeta} = \ln(0.762 \sin \zeta), \zeta = pt^*. \]

The solution was:

\[ \zeta = 1.73. \]

Further it has been calculated:

\[ \varepsilon = \frac{p}{\tan pt^*} = \frac{p}{\tan(1.73)} = -0.388, \frac{1}{s}. \]

Thus, a sufficient condition has been violated:

\[ \tan pt^* > 0. \]

Because of it, a motor vehicle displacements has been calculated by the second way [1]. According this way of algorithm [1] conditional cable frequency \( p \) was equal to:

\[ p = \frac{v_0 \sin \alpha}{Y^*} = \frac{8.55}{2.5} = 3.42, \frac{1}{s}. \]

A rather large discrepancy has been obtained when calculating the frequency for both forms. However, both formulas should give fairly close results. The discrepancy can be explained by the inaccuracy of taking motion data. Inaccuracy manifests itself in the first place in the value of time \( \tau \).

Compare both formulas for calculating the frequency, and reveal the pattern of correction of contact time:

\[ \frac{v_0 \sin \alpha}{Y^*} = \frac{\pi}{\tau}, \]

ore

\[ \tau = \frac{\pi}{A_0} = \frac{\pi}{v_0 \sin \alpha} Y^*, s. \]

In the context of the calculation in question, it has been obtained:

\[ \tau = \frac{\pi}{v_0 \sin \alpha} Y^* = 0.919, s; \]

and

\[ p = \frac{\tau}{\pi} = 3.42, \frac{1}{s}. \]
Further, for $A_0$ it has gave:

$$A_0 = \frac{v_0 \sin \alpha}{p} = 2.5, \text{ m.}$$

As a result, the expressions of displacements have took the form (figure 2):

$$\begin{align*}
x(t) &= -0.375 \sin 3.42t + 24.775 \cdot t, \text{ m;} \\
y(t) &= 2.5 \sin 3.42t, \text{ m.}
\end{align*}$$

Figure 2. The results of analytical calculation (harmonic construction algorithm [1]) for model 1: red line (dotted line) – $y$-displacement, m; green line (solid line) – $x$-displacement, m.

Compared with the FE model, the constructed mathematical model has given displacements with an error of about 2-7%.

The method of polynomial quadratic approximation [2] has been not applicable, since it did not satisfy the necessary condition ($\varepsilon < 0$).

Figure 3 shows the results of using the polynomial cubic approximation [2] with $\tau = 0.6$, s. However, this calculation also has not given a correct result, since the maximum deviation of the $y$-displacement did not reach the required 2.5 m. Such a result with an external, as it were, successful construction, obtained if the determinant is too close to the zero value. (In this case, the determinant was $\Delta = -0.033$). [2].
Figure 3. The results of analytical calculation (cubic polynomial construction algorithm) for model No. 1: red line (dotted line) – $y$-displacement, m; green line (solid line) – $x$-displacement, m.

Thus, none of the methods has gave a correct analytical approximation of displacement, although the angle of inclination of the line of displacements along the $x$ axis with the horizon in all cases was about 87.5°.

The virtual test was carried out incorrectly; therefore, no conclusions can be drawn on the correct construction of the FE model.

2.2. Calculation of model No. 2.

A virtual test has been conducted for a motor vehicle weighing 850 kg at an initial contact speed $v_0 = 25 \cdot 10^3$ mm/s. According to the results of virtual test, the contact time $\tau$ of motor vehicle and cable barrier has been taken about 0.565 s.

According to the virtual test data for the motor vehicle, the following data has been obtained for the speeds and displacements of the points (figure 4): the first pair of points (points A and B), the the center of gravity point C, the back point of the point (points D and E). (Tables 1 and 2; data acquisition step was equal to $0,4999499768 \cdot 10^{-2}$, s.).

Figure 4. Points of a motor vehicle, which have been selected for kinematic analysis.
Table 1. Points displacements according to virtual crash test: first pair (points \(A\) and \(B\)), center of gravity point \(C\), back pair of points (points \(D\) and \(E\)).

| First pair (points \(A\) and \(B\)) | Center of gravity point \(C\) | Back pair of points (points \(D\) and \(E\)) |
|-------------------------------------|--------------------------------|-----------------------------------------------|
| \(x, \text{mm} \cdot 10^4\) | \(-y, \text{mm} \cdot 10^2\) | \(x, \text{mm} \cdot 10^4\) | \(-y, \text{mm} \cdot 10^2\) |
| 1. | 0.0000000000 | 0.0000000000 | 0.0000000000 | 0.0000000000 |
| 2. | 0.011735742188 | 0.42724914551 | 0.42716430664 | 0.42816894531 |
| 3. | 0.023488085938 | 0.85490905762 | 0.85421142578 | 0.85806762695 |
| 4. | 0.035287304688 | 1.2813952637 | 1.2809692383 | 1.2864416504 |
| 5. | 0.047186132813 | 1.6941937256 | 1.7066859355 | 1.712830213 |
| 6. | 0.058922656250 | 2.0881134033 | 2.1319311523 | 2.1394750977 |
| 7. | 0.070482421875 | 2.4871514893 | 2.5571520996 | 2.5664038086 |
| 8. | 0.082136132813 | 2.8929064941 | 2.9807263184 | 2.9910021973 |
| 9. | 0.093833203125 | 3.2908404541 | 3.4015979004 | 3.4170751953 |
| 10. | 0.10591445313 | 3.6792816162 | 3.8197412109 | 3.8486376953 |
| 11. | 0.11794902344 | 4.0100405884 | 4.2337933350 | 4.2826110840 |
| 12. | 0.13020908203 | 4.3164981079 | 4.6429376221 | 4.7162646484 |
| 13. | 0.14213574219 | 4.6066687012 | 5.0470452881 | 5.1548767090 |
| 14. | 0.15397841797 | 4.8462838745 | 5.4430548096 | 5.5972607422 |
| 15. | 0.16570996094 | 5.013294678 | 5.8271038818 | 6.0421716309 |
| 16. | 0.17708505859 | 5.1333898926 | 6.2046942139 | 6.5009326172 |
| 17. | 0.18817021484 | 5.2318072510 | 6.5761364746 | 6.9688195801 |
| 18. | 0.19944218750 | 5.3511169434 | 6.9408502197 | 7.4412304688 |
| 19. | 0.21087255859 | 5.4970251465 | 7.2987225342 | 7.9208544922 |
| 20. | 0.22278652344 | 5.6646405029 | 7.6523266602 | 8.4064367676 |
| 21. | 0.23509130859 | 5.8772161865 | 8.000795605 | 8.9014618116 |
| 22. | 0.24699335938 | 6.1782956506 | 8.3366925049 | 9.4018176270 |
| 23. | 0.25467841797 | 6.2451055908 | 8.6587805176 | 9.9043231201 |
First pair (points \(A\) and \(B\))

| \(x, \text{mm} \cdot 10^4\) | \(-y, \text{mm} \cdot 10^2\) |
|---------------------------|-----------------|
| 24. 0.26590673828 6.3659613037 | 0.26950859375 8.9724877930 |
| 25. 0.27767089844 6.4696960449 | 0.27423964844 10.905906982 |
| 26. 0.28959960938 6.5938940430 | 0.28502988821 11.407419434 |
| 27. 0.3016039844 6.6850042725 | 0.29575683594 11.911207275 |
| 28. 0.3135662109 6.8124182129 | 0.30628906250 12.414035645 |
| 29. 0.3259010742 6.9219647217 | 0.31673769531 12.916185303 |
| 30. 0.33744785156 6.9999383545 | 0.32723691406 13.408842773 |
| 31. 0.34920966797 7.0548278809 | 0.33785761719 13.898502197 |
| 32. 0.36095673828 7.082365723 | 0.35830664064 11.162287598 |
| 33. 0.37258808594 7.0955639648 | 0.36981308594 11.386523438 |
| 34. 0.38403291016 7.0962664795 | 0.38095673828 11.159687744 |
| 35. 0.39537080078 7.0784619141 | 0.39211386719 11.790001221 |
| 36. 0.40685654297 7.0457293701 | 0.40328808594 11.971683350 |
| 37. 0.41758945313 6.9988562012 | 0.41441015625 12.147197266 |
| 38. 0.42806738281 6.9538006592 | 0.42548642578 12.320817871 |
| 39. 0.43849667969 6.8861230469 | 0.43651376953 12.492757568 |
| 40. 0.44861074219 6.8527166748 | 0.44752607422 12.663985596 |
| 41. 0.45898906250 6.8236669922 | 0.45849638672 12.829438477 |
| 42. 0.46959189453 6.7478466797 | 0.46941796875 12.982799072 |
| 43. 0.48044902344 6.6888824463 | 0.48035839844 13.127622070 |
| 44. 0.49134013672 6.6301873779 | 0.49127792969 13.259077148 |
| 45. 0.50204892578 6.5736145020 | 0.50217392578 13.374978027 |
| 46. 0.51262988281 6.5006811523 | 0.51305156250 13.476555176 |
| 47. 0.52311259766 6.3889666748 | 0.52392939453 13.563979492 |
| 48. 0.53202607422 6.3948315430 | 0.53474033203 13.636383057 |
# First pair (points $A$ and $B$)

| $x$, mm·$10^4$ | $-y$, mm·$10^2$ |
|----------------|-----------------|
| 49. 0.54105947266 | 6.4408532715    |
| 50. 0.55099248047 | 6.4445196533    |
| 51. 0.56204804688 | 6.4477313232    |
| 52. 0.57397929688 | 6.4904107666    |
| 53. 0.58581630859 | 6.5139556885    |
| 54. 0.59803466797 | 6.521172646     |
| 55. 0.6041474609  | 6.4988488770    |
| 56. 0.62283291016 | 6.4709564209    |
| 57. 0.63473535156 | 6.445794189     |
| 58. 0.64655566406 | 6.441749023     |
| 59. 0.65839472666 | 6.4441064453    |
| 60. 0.66973613281 | 6.442627686     |
| 61. 0.68056386719 | 6.4328570557    |
| 62. 0.69116328125 | 6.442608426     |
| 63. 0.70201152344 | 6.4730926514    |
| 64. 0.71261708984 | 6.5221716309    |
| 65. 0.72335068359 | 6.5659600830    |
| 66. 0.73369892578 | 6.5978259858    |
| 67. 0.74410537109 | 6.6134997559    |
| 68. 0.75462587891 | 6.6403887939    |
| 69. 0.76538623047 | 6.6789794922    |
| 70. 0.77611552734 | 6.7261590576    |
| 71. 0.78658925781 | 6.7768920898    |
| 72. 0.79671972656 | 6.8099963379    |
| 73. 0.80669951172 | 6.8183300781    |

# Center of gravity point $C$

| $x$, mm·$10^4$ | $-y$, mm·$10^2$ |
|----------------|-----------------|
| 13.695340576   | 0.53172958984   |
| 13.744777359   | 0.52370214844  |
| 13.789904785   | 0.55280595703  |
| 13.832406006   | 0.56344570313  |
| 13.868077393   | 0.57453896484  |
| 13.892197266   | 0.58509677734  |
| 13.90522705    | 0.59576074219  |
| 13.921384277   | 0.60648398438  |
| 13.923502197   | 0.61699365234  |
| 13.916580811   | 0.62743281250  |
| 13.904781494   | 0.63811279297  |
| 13.890705566   | 0.64874042969  |
| 13.873215332   | 0.65944326172  |
| 13.841600342   | 0.67003369141  |
| 13.800437012   | 0.68066064453  |
| 13.758641357   | 0.6914233984   |
| 13.717397461   | 0.70205566406  |
| 13.670294189   | 0.71276005859  |
| 13.608650066   | 0.72340761719  |
| 13.534730225   | 0.73398251953  |
| 13.447839355   | 0.74453115234  |
| 13.351826172   | 0.75511923828  |
| 13.252973633   | 0.76565214844  |
| 13.157325439   | 0.77626142578  |
| 13.064448242   | 0.78693662109  |

# Back pair of points (points $D$ and $E$).

| $x$, mm·$10^4$ | $-y$, mm·$10^2$ |
|----------------|-----------------|
| 20.726259766  | 0.53172958984   |
| 20.937326660  | 0.54237021484   |
| 21.106718750  | 0.55280595703   |
| 21.269436035  | 0.56344570313   |
| 21.401984863  | 0.57453896484   |
| 21.464702148  | 0.58509677734   |
| 21.540268555  | 0.59576074219   |
| 21.573718262  | 0.60648398438   |
| 21.588176270  | 0.61699365234   |
| 21.584055176  | 0.62743281250   |
| 21.569438477  | 0.63811279297   |
| 21.541318359  | 0.64874042969   |
| 21.496013184  | 0.65944326172   |
| 21.444379883  | 0.67003369141   |
| 21.370612793  | 0.68066064453   |
| 21.297770996  | 0.6914233984    |
| 21.208952637  | 0.70205566406   |
| 21.088505859  | 0.71276005859   |
| 20.977731934  | 0.72340761719   |
| 20.865014648  | 0.73398251953   |
| 20.737910156  | 0.74453115234   |
| 20.600954590  | 0.75511923828   |
| 20.456986084  | 0.76565214844   |
| 20.316046143  | 0.77626142578   |
| 20.170590820  | 0.78693662109   |
| First pair (points A and B) | Center of gravity point C | Back pair of points (points D and E). |
|-----------------------------|---------------------------|--------------------------------------|
| \(x, \text{mm} \cdot 10^4\) | \(-y, \text{mm} \cdot 10^2\) | \(x, \text{mm} \cdot 10^4\) | \(-y, \text{mm} \cdot 10^2\) |
| 74. | 0.81651787109 | 6.8042810059 | 0.81170126953 | 12.973468018 | 0.79751357422 | 20.002503662 |
| 75. | 0.82290156250 | 6.8346539307 | 0.82227978516 | 12.883597412 | 0.80802490234 | 19.822410889 |
| 76. | 0.83085312500 | 6.7743542480 | 0.83276138359 | 12.792709961 | 0.81863193359 | 19.648481445 |
| 77. | 0.84055917969 | 6.7756951904 | 0.84320673828 | 12.697185059 | 0.82927929688 | 19.473464355 |
| 78. | 0.85028486328 | 6.7711938477 | 0.8536830078 | 12.59722900 | 0.83987666016 | 19.295021973 |
| 79. | 0.85991835938 | 6.8006359863 | 0.86410800781 | 12.496519775 | 0.85030644531 | 19.115434570 |
| 80. | 0.87147822266 | 6.8140167236 | 0.87448320313 | 12.397192383 | 0.86066054688 | 18.932738037 |
| 81. | 0.88402919922 | 6.8291760254 | 0.88481005859 | 12.296574707 | 0.8709071484 | 18.747817383 |
| 82. | 0.89563906250 | 6.8314605713 | 0.89513623047 | 12.196097412 | 0.88103769531 | 18.569421393 |
| 83. | 0.9084009766 | 6.8276232910 | 0.90544394531 | 12.096650391 | 0.89120214844 | 18.397570801 |
| 84. | 0.92036933594 | 6.8712963867 | 0.91563564453 | 11.998245850 | 0.90140273438 | 18.211265869 |
| 85. | 0.93214804688 | 6.8912194824 | 0.92578398438 | 11.899575195 | 0.91162988281 | 18.016169434 |
| 86. | 0.94407558594 | 6.8808349609 | 0.93593866719 | 11.794100342 | 0.92175976563 | 17.823281250 |
| 87. | 0.95508730469 | 6.8731860352 | 0.94613193359 | 11.683602295 | 0.93175175781 | 17.63836934 |
| 88. | 0.96521953125 | 6.8491229248 | 0.95634521484 | 11.571904297 | 0.94179775391 | 17.460300293 |
| 89. | 0.97537792969 | 6.8210723877 | 0.96652119141 | 11.460773926 | 0.95193369141 | 17.279895020 |
| 90. | 0.98445660463 | 6.7949829102 | 0.97668759766 | 11.355078125 | 0.96214775391 | 17.107062988 |
| 91. | 0.99411142578 | 6.7427233887 | 0.98686884766 | 11.25531563 | 0.9724355078 | 16.950795898 |
| 92. | 1.0039439453 | 6.6933648682 | 0.9970529297 | 11.160751953 | 0.98267802734 | 16.792612305 |
| 93. | 1.0138986328 | 6.6342120361 | 1.0072476563 | 11.066726074 | 0.99292714844 | 16.627543945 |
| 94. | 1.0235517578 | 6.5658294678 | 1.0174671719 | 10.970548096 | 1.0031154297 | 16.454215088 |
| 95. | 1.0330688477 | 6.5005328369 | 1.0276739258 | 10.873342285 | 1.0132407227 | 16.278198242 |
| 96. | 1.0427845703 | 6.4116125488 | 1.0377941406 | 10.774819336 | 1.0234355469 | 16.105628662 |
| 97. | 1.0529701172 | 6.3205737305 | 1.0478781250 | 10.673626709 | 1.0336921875 | 15.919086914 |
| 98. | 1.0630866211 | 6.2386499023 | 1.0579440430 | 10.569880371 | 1.0438461914 | 15.727788086 |
Table 2. Points speed according to virtual crash test: first pair (points $A$ and $B$), center of gravity point $C$, back pair of points (points $D$ and $E$).

| First pair (points $A$ and $B$) | Center of gravity point $C$ | Back pair of points (points $D$ and $E$). |
|----------------------------------|-------------------------------|---------------------------------|
| $x$, mm $\cdot 10^4$ | $-y$, mm $\cdot 10^2$ | $x$, mm $\cdot 10^4$ | $-y$, mm $\cdot 10^2$ |
| 99. 1.0733368164 6.1494122314 | 1.0679505859 10.468197021 | 1.0539175781 15.543571777 |
| 100. 1.0835636719 6.0758154297 | 1.0779264648 10.366904799 | 1.0639434570 14.845306396 |
| 101. 1.0937746094 6.0139105225 | 1.0879120117 10.262816162 | 1.0739454102 14.681722412 |
| 102. 1.1041771484 5.9437353516 | 1.0979027344 10.155823975 | 1.0839296868 14.52030176 |
| 103. 1.1141308594 5.8589379883 | 1.1079117188 10.045789795 | 1.1040048828 14.359140625 |
| 104. 1.1238710938 5.7496398926 | 1.1179051758 9.9336279297 | 1.114326172 14.2048340 |
| 105. 1.1335179688 5.6317346191 | 1.1278786133 9.8206072998 | 1.114326172 13.70438232 |
| 106. 1.1431992188 5.5252899170 | 1.137861641 9.7084423828 | 1.124447266 13.54418701 |
| 107. 1.1534025391 5.426330078 | 1.147882227 9.5993457031 | 1.1345375000 14.19343994 |
| 108. 1.1639262695 5.3226586914 | 1.1578958984 9.4953594791 | 1.147015625 14.02048340 |
| 109. 1.1741462891 5.2270507813 | 1.167911788 9.3939868164 | 1.1548791016 13.85799072 |
| 110. 1.1842718750 5.1373718262 | 1.177919219 9.2946093750 | 1.1650471680 13.70438232 |
| 111. 1.1933393555 5.0381927490 | 1.1879085938 9.1957836914 | 1.1751407227 13.54418701 |
| 112. 1.2031646484 4.8952273560 | 1.197894375 9.0971765137 | 1.1851384766 13.38697201 |
| 113. 1.2132005859 4.7502304077 | 1.2078829578 9.0030029297 | 1.195059063 13.22875610 |
| 114. 1.2235056641 4.6589648438 | 1.217838867 8.9120971680 | 1.2049483398 13.06854492 |

1. $2.3500000000$ $-8.5500000000$ $2.3500000000$ $-8.5500000000$ $2.3500000000$ $-8.5500000000$
2. $2.3497181641$ $-8.6555458984$ $2.3500300781$ $-8.5378496094$ $2.3410730469$ $-8.5510371094$
3. $2.3503292969$ $-8.3975136719$ $2.3501710938$ $-8.536982578$ $2.3524574219$ $-8.595113281$
4. $2.3904964844$ $-8.5446308594$ $2.3500402344$ $-8.5246044922$ $2.3534710938$ $-8.5526591797$
5. $2.330605469$ $-8.2488955078$ $2.3476744141$ $-8.5015761719$ $2.3511623047$ $-8.5470058594$
|   | First pair (points A and B) | Center of gravity point C | Back pair of points (points D and E). |
|---|-----------------------------|---------------------------|--------------------------------------|
|   | \( v_x, \frac{\text{mm}}{s} \cdot 10^4 \) | \( v_y, \frac{\text{mm}}{s} \cdot 10^3 \) | \( v_x, \frac{\text{mm}}{s} \cdot 10^4 \) | \( v_y, \frac{\text{mm}}{s} \cdot 10^3 \) | \( v_x, \frac{\text{mm}}{s} \cdot 10^4 \) | \( v_y, \frac{\text{mm}}{s} \cdot 10^3 \) |
| 6. | 2.3911929688 | -8.2400664063 | 2.3487478516 | -8.5050654297 | 2.3517548828 | -8.5506806641 |
| 7. | 2.2504242188 | -7.8712382813 | 2.3529462891 | -8.503435938 | 2.3426978516 | -8.4919589844 |
| 8. | 2.3737628906 | -8.1691635742 | 2.3482187500 | -8.4408203125 | 2.3420771519 | -8.495765625 |
| 9. | 2.3348908203 | -7.9350426988 | 2.3468988281 | -8.3882382813 | 2.3531248047 | -8.596534766 |
| 10. | 2.3844009766 | -7.0935190430 | 2.374580078 | -8.3292871094 | 2.3395546875 | -8.6673759766 |
| 11. | 2.5130523438 | -5.9837382813 | 2.3483486328 | -8.2263291016 | 2.3222822266 | -8.664279297 |
| 12. | 2.3666103516 | -5.9411293945 | 2.3517601563 | -8.1417539063 | 2.3137687500 | -8.7038574219 |
| 13. | 2.4076935547 | -5.3267885742 | 2.3555451172 | -8.0092133789 | 2.3005273438 | -8.8088466797 |
| 14. | 2.333037109 | -3.7372858887 | 2.349482734 | -7.7844262695 | 2.2907804688 | -8.845260938 |
| 15. | 2.4132353516 | -1.2575129395 | 2.3505066406 | -7.6228632813 | 2.2770000000 | -8.992678109 |
| 16. | 2.2560626953 | -2.382719727 | 2.3572451172 | -7.4964386762 | 2.285591172 | -9.3279482422 |
| 17. | 2.1269867188 | -3.6772185059 | 2.3479044922 | -7.4491894531 | 2.2662019531 | -9.389709609 |
| 18. | 2.2693421875 | -2.8691623535 | 2.3459087891 | -7.2334106445 | 2.2365785156 | -9.508983475 |
| 19. | 2.303902344 | -2.7680339355 | 2.3438148438 | -7.1158779297 | 2.2229646848 | -9.665033203 |
| 20. | 2.609427734 | -3.7204082031 | 2.3316408203 | -7.0454426891 | 2.2072656250 | -9.7674921875 |
| 21. | 2.367439609 | -6.4763813477 | 2.3280130859 | -6.8827377930 | 2.195617578 | -9.983906250 |
| 22. | 2.0339531250 | -7.3022729492 | 2.3262646484 | -6.5856147461 | 2.1824865234 | -10.004263672 |
| 23. | 1.8403169922 | -1.1879824219 | 2.3027365234 | -6.2727148438 | 2.1806009766 | -10.039356445 |
| 24. | 2.4630705078 | -1.5435731201 | 2.6278246094 | -6.3115229492 | 2.1747833984 | -10.02610352 |
| 25. | 2.275509766 | -2.7614809570 | 2.2435986328 | -5.9984057617 | 2.1656676969 | -10.01840391 |
| 26. | 2.3503693359 | -1.9545184326 | 2.2160511719 | -5.8212788086 | 2.1621347656 | -10.0551359 |
| 27. | 2.1237365234 | -1.7325063477 | 2.2116699219 | -5.7461450195 | 2.1249212891 | -10.090327148 |
| 28. | 2.5710923828 | -2.129314766 | 2.2305244141 | -5.5509116211 | 2.0911312500 | -10.025192383 |
| 29. | 2.4631345703 | -2.0570810547 | 2.2283392578 | -5.3469921875 | 2.0910117188 | -9.9863125000 |
| 30. | 2.2149314453 | -1.6782648926 | 2.2181839844 | -5.0546855469 | 2.1079232302 | -9.7703134766 |
| 31. | 2.3363052734 | -0.74081164551 | 2.2204460938 | -4.9181113281 | 2.137917266 | -9.8642929688 |
| 32. | 2.4000728516 | -0.52847991943 | 2.2453542969 | -4.5546484375 | 2.1459798828 | -9.9684394531 |
| First pair (points A and B) | Center of gravity point C | Back pair of points (points D and E) |
|-----------------------------|---------------------------|-------------------------------------|
| $v_x, \frac{mm}{s} \cdot 10^4$ | $v_y, \frac{mm}{s} \cdot 10^3$ | $v_x, \frac{mm}{s} \cdot 10^4$ | $v_y, \frac{mm}{s} \cdot 10^3$ | $v_x, \frac{mm}{s} \cdot 10^4$ | $v_y, \frac{mm}{s} \cdot 10^3$ |
| 33. | 2.2185144531 | 1.1554492950 | 2.317820313 -4.3730449219 | 2.1451955078 -9.7264970703 |
| 34. | 2.3495904297 | 0.066588577271 | 2.2269255859 -4.0297058105 | 2.1309275391 -9.609514684 |
| 35. | 2.3153357422 | 0.44723937988 | 2.2339503906 -3.7965722666 | 2.1247644531 -9.3637470703 |
| 36. | 2.2129755859 | 0.79397991943 | 2.2321394531 -3.5398688965 | 2.1482406250 -9.0140410156 |
| 37. | 2.1964667969 | 0.64619403076 | 2.2206126953 -3.4560539551 | 2.1320431641 -8.6315820313 |
| 38. | 2.1011707031 | 1.3211889648 | 2.2139580078 -3.4073767090 | 2.1200367188 -8.4032255859 |
| 39. | 2.0595019531 | 1.1604498291 | 2.2049853516 -3.3858393555 | 2.153517734 -8.0411162109 |
| 40. | 2.059912109 | 0.27432492065 | 2.1979720703 -3.3866560059 | 2.1329845703 -7.9941567383 |
| 41. | 2.1315636719 | 1.1619736328 | 2.1895158203 -3.2081972727 | 2.1609482422 -7.3785864258 |
| 42. | 2.1390564453 | 1.8520682373 | 2.1811492188 -3.0014074707 | 2.2224207031 -7.0625097656 |
| 43. | 2.1930156250 | 1.0237455444 | 2.1895710938 -2.7663376465 | 2.1123541016 -6.7190439453 |
| 44. | 2.2012392578 | 1.6611018066 | 2.1814810547 -2.4988967285 | 2.1233060547 -6.0310165234 |
| 45. | 2.1367654297 | 1.4245677490 | 2.1775808594 -2.1699145508 | 2.1715361328 -6.0089243164 |
| 46. | 2.1157267578 | 1.5491402588 | 2.175533984 -1.8767637939 | 2.1807857422 -5.186938964 |
| 47. | 2.0161070313 | 0.83905914307 | 2.1720195131 -1.6231989746 | 2.221828906 -5.7759936253 |
| 48. | 1.8617441406 | -0.24874313354 | 2.1478433594 -1.2987268066 | 2.2256804688 -5.0692942192 |
| 49. | 1.7944931641 | -1.1360129395 | 2.1296917969 -1.0414846191 | 2.1352925784 -4.6386069336 |
| 50. | 2.1345740234 | 0.057340698242 | 2.1051185547 -0.92657525635 | 2.0956458984 -3.5658488770 |
| 51. | 2.4405929688 | -0.6557355957 | 2.0970845703 -0.8854935059 | 2.0668859375 -3.2689152832 |
| 52. | 2.3719763672 | -0.64244293213 | 2.1001390625 -0.81822332764 | 2.1572910156 -3.1894394531 |
| 53. | 2.305413672 | -0.09341311317 | 2.1077384766 -0.60312591553 | 2.1501080078 -2.0460627441 |
| 54. | 2.5454982422 | 1.7104362488 | 2.1278564453 -0.37877923584 | 2.1141517119 -1.5966945801 |
| 55. | 2.4588585938 | 0.33533041382 | 2.1126802734 -0.28635156250 | 2.1728123047 -1.303500000 |
| 56. | 2.3524146484 | 1.2773712158 | 2.0997888672 -0.155532313232 | 2.1220757813 -0.33206637573 |
| 57. | 2.3380714844 | 0.32587554932 | 2.1152750000 -0.063586757660 | 2.1047178785 -0.11251281738 |
| 58. | 2.3739515625 | -0.20593342590 | 2.1297894531 -0.2358678369 | 2.2135675781 -0.21145855713 |
| 59. | 2.2779357422 | -0.47477972412 | 2.1433677734 -0.3072027827 | 2.1427048828 -0.54630157471 |
| First pair (points A and B) | Center of gravity point C | Back pair of points (points D and E) |
|-----------------------------|---------------------------|--------------------------------------|
| \( v_x, \frac{\text{mm}}{s} \cdot 10^4 \) | \( v_y, \frac{\text{mm}}{s} \cdot 10^3 \) | \( v_x, \frac{\text{mm}}{s} \cdot 10^4 \) | \( v_y, \frac{\text{mm}}{s} \cdot 10^3 \) |
| 60. | 2.2232781250 | 2.1485318359 | 0.28332723999 | 2.1129375000 | 0.85822668457 |
| 61. | 2.1163406250 | 0.010626397133 | 2.1510873047 | 0.47044311523 | 2.1391193359 | 1.0590959473 |
| 62. | 2.1390232442 | -0.7107125444 | 2.1456875000 | 0.78031890869 | 2.1377154297 | 1.8176621094 |
| 63. | 2.1290462891 | -1.1413905029 | 2.1470468750 | 0.85190435791 | 2.1032578125 | 1.5815295410 |
| 64. | 2.1398431641 | -0.8880242920 | 2.1521914063 | 0.87735595703 | 2.0675128906 | 2.1351279297 |
| 65. | 2.0860910156 | -0.30380371094 | 2.1502912109 | 1.0456063232 | 2.1571164063 | 2.3543481445 |
| 66. | 2.0936654297 | -0.3474855619 | 2.1479105469 | 1.3830023193 | 2.1309486328 | 2.0621743164 |
| 67. | 2.1058990234 | 0.74280944824 | 2.1422177334 | 1.6171040039 | 2.0667810547 | 2.549785156 |
| 68. | 2.1852447266 | -0.65991638184 | 2.1395195313 | 1.8458696289 | 2.109443359 | 2.6291000977 |
| 69. | 2.1208880859 | -0.97926971436 | 2.1386921875 | 1.9612816162 | 2.1247441406 | 2.8281806641 |
| 70. | 2.0640798828 | -0.8949516748 | 2.1317316406 | 1.9525555420 | 2.0986451172 | 2.9142971191 |
| 71. | 1.9797785156 | -0.36758981323 | 2.1316871094 | 1.8677653809 | 2.1165974609 | 2.8669692662 |
| 72. | 2.0754289063 | 0.052084087327 | 2.128963359 | 1.8339897461 | 2.1302164063 | 3.0945874023 |
| 73. | 0.9.4561103516 | 0.052084087327 | 2.128963359 | 1.8339897461 | 2.1302164063 | 3.0945874023 |
| 74. | 1.5611399414 | 1.3096927490 | 2.1093292696 | 1.7648854980 | 2.106207344 | 3.6279384766 |
| 75. | 1.510281445 | -0.32684902954 | 2.087894219 | 1.8905595703 | 2.1248398438 | 3.507794336 |
| 76. | 1.8629617188 | 0.68197686768 | 2.0952492188 | 1.9423402100 | 2.1154863281 | 3.634210492 |
| 77. | 1.2957769531 | 0.1972396851 | 2.0915591797 | 2.0466858759 | 2.0928250000 | 3.5193109710 |
| 78. | 2.0012609375 | -0.47319622803 | 2.0765001953 | 1.9769492188 | 2.0769261719 | 3.5991892090 |
| 79. | 2.6633626953 | 0.19673172747 | 2.0683945341 | 1.9528485107 | 2.0507265758 | 3.619344283 |
| 80. | 2.1959064453 | 0.98487045288 | 2.0629580078 | 2.042158693 | 2.0255488281 | 3.6609460449 |
| 81. | 2.5944242188 | 0.52107226563 | 2.067545313 | 1.9819887695 | 2.0195445313 | 3.4508698730 |
| 82. | 2.703105894 | -0.13698130798 | 2.0529554688 | 2.0023527175 | 2.0255144531 | 3.5069333496 |
| 83. | 2.488860156 | -1.3532128806 | 2.0289538984 | 1.9592360840 | 2.0304070301 | 3.8553491211 |
| 84. | 2.6356169922 | 0.29762765503 | 2.0285435547 | 1.994096680 | 2.0472105469 | 3.893498730 |
| 85. | 1.7967576172 | 1.2861058350 | 2.0327158203 | 2.1880737305 | 2.0143554688 | 3.7448010254 |
|   | First pair (points $A$ and $B$) | Center of gravity point $C$ | Back pair of points (points $D$ and $E$). |
|---|---------------------------------|-----------------------------|------------------------------------------|
|   | $v_x, \text{ mm s}^{-1} \cdot 10^3$ | $v_y, \text{ mm s}^{-1} \cdot 10^3$ | $v_x, \text{ mm s}^{-1} \cdot 10^3$ | $v_y, \text{ mm s}^{-1} \cdot 10^3$ | $v_x, \text{ mm s}^{-1} \cdot 10^3$ | $v_y, \text{ mm s}^{-1} \cdot 10^3$ |
| 87 | 2.5708314453 | 1.2086445313 | 2.0437714844 | 2.2316730957 | 2.0040162109 | 3.6060300293 |
| 88 | 2.0637345703 | 0.5047498471 | 2.0393378906 | 2.2509140625 | 2.0278975816 | 3.5957871094 |
| 89 | 1.9594548828 | 0.22800660706 | 2.0313355469 | 2.1550092773 | 2.0366777344 | 3.6236679688 |
| 90 | 1.9789615234 | 0.87765234375 | 2.0352787109 | 2.0492724609 | 2.0533599609 | 3.2321086426 |
| 91 | 1.9525539063 | 1.0362442627 | 2.0355814453 | 1.9410322266 | 2.0462062500 | 3.1268613281 |
| 92 | 1.9679597656 | 1.3187076146 | 2.0387035156 | 1.8790773926 | 2.0483410156 | 3.2311374512 |
| 93 | 1.9128515625 | 1.2674539795 | 2.0434101563 | 1.8964530029 | 2.0440314453 | 3.3464760742 |
| 94 | 1.8796933594 | 1.2852348633 | 2.0453009766 | 1.9482075195 | 2.0325021484 | 3.5757902832 |
| 95 | 1.8198894531 | 1.6047843018 | 2.0319537109 | 1.9415271000 | 2.0271925781 | 3.4157766113 |
| 96 | 2.0929984375 | 2.0260950928 | 2.0166214844 | 1.9941793213 | 2.0502490234 | 3.5645390625 |
| 97 | 1.9904955078 | 1.4561474609 | 2.0153091797 | 2.0448959961 | 2.0395167979 | 3.8365083008 |
| 98 | 2.1014478516 | 1.9770104980 | 2.0071826172 | 2.0541591797 | 2.0288007812 | 3.7335964355 |
| 99 | 2.0630322266 | 1.6719705811 | 1.9988968750 | 2.0373404541 | 2.0141015625 | 3.6517588887 |
| 100 | 2.1351193359 | 1.4529450684 | 1.9936090363 | 2.0360212402 | 1.9977343750 | 3.6391123047 |
| 101 | 2.0903882813 | 1.0358211670 | 1.9979041016 | 2.0993269043 | 1.9935488281 | 3.4998933105 |
| 102 | 2.0535283203 | 1.4383634033 | 2.0001796875 | 2.1787739258 | 2.0040197266 | 3.3543510742 |
| 103 | 2.0492578125 | 1.9754842529 | 2.0016636719 | 2.2210395508 | 2.0266943359 | 3.2804709473 |
| 104 | 1.9095013672 | 2.3883273926 | 1.9951074219 | 2.2455275879 | 2.0363240234 | 3.2512861328 |
| 105 | 1.944156250 | 2.1756193848 | 1.9944352500 | 2.2680755666 | 2.0273894531 | 3.2158173828 |
| 106 | 1.9192414063 | 2.004102783 | 2.006423828 | 2.2126501465 | 2.0091476563 | 3.2607297363 |
| 107 | 2.0726523438 | 2.1180705566 | 2.002630469 | 2.1283324285 | 2.0218271484 | 3.4104054140 |
| 108 | 2.1024945313 | 1.8463615723 | 2.0043001953 | 2.0607863770 | 2.0409355469 | 3.3888381348 |
| 109 | 2.1112583984 | 1.3313614502 | 2.0039183594 | 2.0044637451 | 2.0398616414 | 3.1735908203 |
| 110 | 1.9721968441 | 1.1226138916 | 1.9984845703 | 1.9723289795 | 2.0331263282 | 3.1386374512 |
| 111 | 1.7103490234 | 2.0859785156 | 1.997011788 | 1.9925576172 | 2.0074123047 | 3.1197929688 |
| 112 | 2.0978136719 | 3.4147412109 | 1.9986835938 | 1.9433275146 | 1.9931824129 | 3.1655278320 |
| 113 | 1.9116576172 | 2.5443457031 | 1.9981652344 | 1.8272825928 | 1.9801757813 | 3.1458527832 |
During the virtual test, the vehicle has been located at some distance from the cable barrier. To construct analytical displacements, it has been considered two ways of the approach.

**2.2.1. First way of the approach for model 2.**

It has been constructed the analytical displacements so that for the moment of the start of the movement it has been taken a real zero count – that was, a moment that coincided with the zero count of the virtual test. With the proposed organization of the test, analytical construction using the harmonic method [1] was impossible, since the method assumes that the movement of the point under study ceases at the level of the point of origin of movement. That was, the movement of the motor vehicle along the transverse \( y \) axis was close to the half-cycle of the oscillation.

The application of the polynomial quadratic method [2] was extremely cumbersome, because it has lead to the solution of a system of nonlinear equations.

Thus, the analytical construction has been carried out by the cubic polynomial method [2]. For this, a correction of the system of linear algebraic equations has been carried out as follows:

\[
\begin{align*}
\Delta &= \tau t^4 - 2 \tau^2 t^3 + \tau^3 t^2, \\
\alpha_1 &= v_0 \sin \alpha; \\
\alpha_1 \tau + \alpha_2 \tau^2 + \alpha_3 \tau^3 &= Y_k; \\
\alpha_1 t^* + \alpha_2 t^* \tau^2 + \alpha_3 t^* \tau^3 &= Y^*; \\
\alpha_1 + 2 \alpha_2 t^* + 3 \alpha_3 t^* \tau^2 &= 0.
\end{align*}
\]

Here \( Y_k \) – \( y \)-coordinate of the motor vehicle at the end of the movement. System decision regarding coefficients \( \alpha_1, \alpha_2, \alpha_3 \) was:

\[
\begin{align*}
\alpha_1 &= t^* \frac{2Y^* t^* + Y_k t^* \tau^3 - 3Y^* \tau^2 t^*}{\Delta}; \\
\alpha_2 &= -Y^* \tau^3 - 2Y_k \tau^2 t^* t^2; \\
\alpha_3 &= \frac{Y^* \tau^2 + Y_k t^* \tau^2 - 2Y^* \tau t^*}{\Delta}.
\end{align*}
\]

Since the conditions were met \( t \neq 0, \tau^2 + t^*^2 - 2 \tau t^* \neq 0, \tau \neq 0 \), then a solution to the system of linear algebraic equations existed and was unique.

For the first pair of points (points \( A \) and \( B \)), the obtained analytical displacements had the form:

\[
\{ x(t) = -0.15((8.55t - 21.707t^2 + 14.918t^3) \cdot 10^3 + 24.775t \cdot 10^3, \ mm; \\
(8.55t - 21.707t^2 + 14.918t^3) \cdot 10^3, \ mm
\}
\]

The displacements along the \( x \) and \( y \) axes and the velocities for this pair have been shown in figures 5 and 6, respectively.
Figure 5. Points $A$ and $B$ (solid line) and $y$ (dotted line) displacements, mm: grey lines – according to the virtual crash test; black lines – according to the analytical calculation.

Figure 6. Point $A$ and $B$ (solid line) and $y$ (dotted line) velocities, mm/s: grey lines – according to the virtual crash test; black lines – according to the analytical calculation.
In this case, the results of the virtual test reflected bumper vibrations. The incorrectness of the constructed model has associated either with the FE model of the cable barrier, or in organized communications in the nodes of the FE model of the motor vehicle.

For the center of gravity of the vehicle (point C), the constructed analytical approximation of displacements had the form:

\[
\begin{align*}
    x(t) &= -0.15 \cdot ((8.55t - 15.454t^2 + 5.538t^3) \cdot 10^3 + 24.775t \cdot 10^3, \text{mm}; \\
    y(t) &= (8.55t - 15.454t^2 + 5.538t^3) \cdot 10^3, \text{mm}.
\end{align*}
\]

Point C displacements and velocities along the x and y axes have been shown in figures 7 and 8, respectively.

![Figure 7](image)

**Figure 7.** Point C x (solid line) and y (dotted line) displacements, mm: grey lines – according to the virtual crash test; black lines – according to the analytical calculation.
Figure 8. Point C x (solid line) and y (dotted line) velocities, mm/s: grey line – according to the virtual crash test; black line – according to the analytical calculation.

Points D and E displacements were equal to:

\[
\begin{align*}
    x(t) &= -0.15 \cdot (8.55t - 0.398t^2 - 18.842t^3) \cdot 10^3 + 24.775t \cdot 10^3, \text{ mm;} \\
    y(t) &= (8.55t - 0.398t^2 - 18.842t^3) \cdot 10^3, \text{ mm.}
\end{align*}
\]

Points D and E displacements and velocities along the x and y axes have been shown in figures 9 and 10, respectively.

Figure 9. Points D and E x (solid line) and y (dotted line) displacements, mm: grey lines – according to the virtual crash test; black lines – according to the analytical calculation.
Figure 10. Points D and E (solid line) and y (dotted line) velocities, mm/s: grey lines – according to the virtual crash test; black lines – according to the analytical calculation.

For points D and E, the time offset error of the maximum deviation along the y axis was 21.4%. For all points, the error of the final analytically calculated displacements along the x axis were 20-23%.

2.2.2. Second way of the approach for model 2

The greatest error in the analytical construction was associated with the construction of displacements for points D and E. Therefore, it needs to reconstruct for these points. To do this, for the beginning of a new zero point in time $\hat{t}_0$ it has been taken the point in time at which the motor vehicle at the beginning of the movement was as close as possible to the deviation along the y axis at the level of the final value of this coordinate at the end of the movement. According to the virtual test (table 1), the final value of the deviation along the y axis for points D и E was $Y_k = 1306.8544922$, mm.

Close to this value was $y = 1291.6185303$, mm (29th step, $t = 0.1399995$, s) and $y = 1340.8842773$, mm (30th step, $t = 0.1449999$, s). As a new start $\hat{t}_0$ it has been taken:

$$\hat{t}_0 = \frac{0.1399995 + 0.1449999}{2} = 0.1424997, \text{s};$$

for a new level of the new coordinate system it has been taken:

$$\hat{Y}_0 = \frac{1291.6185303 + 1340.8842773}{2} = 1316.2514, \text{mm}.$$

Thus, the new maximum deviation value $\hat{Y}^*$ along y axis was:

$$\hat{Y}^* = Y^* - \hat{Y}_0 = 2158.405 - 1316.2514 = 851.55, \text{mm}.$$

The new period of contact between the motor vehicle and the cable barrier was equal to:

$$\hat{t} = 0.565 - 0.1424997 = 0.4224996, \text{s}.$$

An analytical construction of the trajectory has been done using the harmonic construction algorithm [1], neglecting the loss of speed over the passed time interval. To do this, it has been calculated:

$$p = \frac{\pi}{\hat{t}} = 7.436, \frac{1}{\text{s}};$$
The necessary condition has been met: $A_0 > \hat{Y}$. So, the time moment $\hat{t}^*$ of the maximum dynamic deflection along $y$ axis (with new time scale) has been found solving the equation:

$$\frac{7,436 \hat{t}^*}{tg(7,436 \hat{t}^*)} = \ln(1,667 \sin(7,436 \hat{t}^*)).$$

The result of solution was:

$$\hat{t}^* = 0,062, s.$$

Thus parameter $\varepsilon$ has been counted:

$$\varepsilon = \frac{p}{tg(7,436 \hat{t}^*)} = 2,721, \frac{1}{s}.$$

According to new time scale the equations of displacements were:

$$\begin{cases} y(t) = 1420e^{-2.721t \sin(7,436t)}, \text{mm}; \\ x(t) = -0,15y(t) + 24775t, \text{mm}. \end{cases}$$

For convenient comparison with the data of a virtual test, the correction of the obtained equations of displacements has been done:

$$\begin{cases} \hat{y}(t) = 1420e^{-2.732(t-0.142)} \sin(7,436(t-0.142)) - 1306,85, \text{mm}; \\ \hat{x}(t) = -0,15\hat{y}(t) + 24775(t-0.142) + 3167,377, \text{mm}. \end{cases}$$

That is, to transfer the obtained analytical functions to the old frame of reference, the following changes has been made:

$$\begin{cases} y(t) = A_0e^{-\varepsilon(t-t_0)} \sin(7,436(t-t_0))) - Y_k, \text{mm}; \\ x(t) = -0,15y(t) + 24775(t-t_0)) + \hat{x}(t_0), \text{mm}. \end{cases}$$

The results of the analytical calculation of displacements and speeds have been presented in figures 11 and 12 (respectively) in comparison with the results of the virtual test and the results of the analytical calculation which has been done in 2.2.1.

The analytical construction for points $D$ and $E$ in the second version of the calculation have gave good convergence with the data of the virtual calculation: there was a complete coincidence at the time of the maximum deviation of the motor vehicle along the $y$ axis. The error of the final displacement along the $y$ axis was 13.22%.
Figure 11. Points D and E: x (solid line) and y (dotted line) displacements, mm: grey lines – according to the virtual crash test; black lines – according to the analytical calculation (first way of approach); dot-and-dash lines – according to the analytical calculation (second way of approach).

Figure 12. Points D and E: x (solid line) and y (dotted line) velocities, mm/s: grey lines – according to the virtual crash test; black lines – according to the analytical calculation; dot-and-dash lines – according to the analytical calculation (second way of approach).
As a result of the calculation, it can be assumed that the development of the FE model has been carried out quite correctly.

In the above virtual experiments (model No. 1 and model No. 2), a motor vehicle has approached the point of contact from a certain distance with the stated initial speed $v_0 = 25 \text{ m/s}$. But when directly at the beginning of the contact, the speed has been lost. Therefore, there was a mismatch between the analytical and calculated displacements, as well as the moment of maximum deviation along the $y$ axis.

In the case of model No. 2, the results of the virtual test reflected the vibrations of the bumper. A certain incorrectness of the data has been associated with the organization of connections in the nodes of the FE model of a motor vehicle.

In total, according to the analysis of FE models, it is possible to conclude that when conducting a virtual test in order to verify the correctness of the constructed FE model, it is necessary to organize the test in such a way that the start time of the test coincides with the moment of contact of the motor vehicle and the cable barrier.

3. The use of analytical displacements in the research of passive safety of the driver/passenger.

To analyze the passive safety of the driver/passenger in order to ensure human viability during oblique collision of a motor vehicle with side road cable barrier, the LS-DYNA calculation package was chosen. [10].

Livermore Software Technology Corporation (LSTC), an LS-DYNA provider, offers a range of virtual dummies for examining passive driver / passenger safety. To study internal passive safety, a 3D model of the Hybrid III mannequin, height 160 cm, weight 50 kg, was recommended as a human test model. [13].

To conduct a study on ensuring human viability during oblique collision of a vehicle, the trajectories, speeds, and accelerations of a motor vehicle, constructed using analytical calculations for a motor vehicle of the Mercedes-Benz-0345 type (at a speed of about 69.4 km/h as a initial speed) were taken in comparison with the kinematic characteristics of a motor vehicle obtained during the virtual test. The analytical and calculated trajectories were given the movement of the Hybrid III virtual dummy (height 160 cm, weight 50 kg – figure 14), with an initial relative speed of 0 m/s.

FE model of crash-test has been constructed by I. Karpov (graduate student of the Department of Structural Mechanics, MADI), A. Gasaniev (master student of the Department of Structural Mechanics, MADI, graduation 2018) and the author of the article.

As a result, in two versions, the displacements and accelerations of the mannequin's head were calculated. Graphs of relative displacements and accelerations has been given in tables 3 and 4, respectively.
Table 3. Relative displacements according to a data of the analytical calculation and the virtual crash test.
Relative displacements of virtual dummy obtained using analytically constructed displacements
Along the longitudinal axis (x axis) of movement of a motor vehicle.
Along the transverse axis (y axis) of movement of a motor vehicle.

Table 4. Relative acceleration according to data of analytical calculation and the virtual crash test.
Relative accelerations obtained using analytically constructed displacements of a motor vehicle.
Relative accelerations obtained using displacements of a motor vehicle obtained as a result of a virtual crash test.

In both cases, the HIC index has been derived. [14]. Head injury criterion (HIC) is an indicator of the likelihood of a head injury caused by a blow. HIC is used to assess the safety associated with a motor vehicles, personal protective equipment and sports equipment and is calculated by the formula:
Here $t_1$ and $t_2$ – start and end time (in seconds) selected to maximize HIC, and acceleration $j(t)$ measured in gs (standard gravity acceleration). [14].

Time duration $(t_2 - t_1)$ limited to a maximum value of 36 ms, usually 15 ms. This means that the HIC includes the effects of head acceleration and duration of acceleration. Large accelerations can be tolerated for a very short time.

With a HIC of 1000, there is a 18% chance of a severe head injury, a 55% chance of a fairly serious injury, and a 90% chance of a moderate head injury in an adult. A HIC value of 700 is the maximum allowable and implies a 5 percent risk of serious injury. [14].

According to table 6, the HIC index has been produced. The time duration $(t_2 - t_1)$ has been taken for 15 ms. Using analytically obtained displacements of a motor vehicle, the HIC was 168.7; when using the displacements calculated according to the results of the virtual crash test – 172.2. The relative error of calculations using the analytical model of a motor vehicle was 2%.

Thus, analytically obtained displacements of a motor vehicle can be used in the analysis of passive safety of the driver / passenger.

4. Conclusion

In the article possible ways to verify the construction of FE models have been presented. The check has been based on the use of an analytical calculation of the displacements of a motor vehicle.

Analytical calculations were on algorithms, which has been obtain in [1] and [2]: harmonic construction algorithm and cubic polynomial construction algorithm. The correction of both algorithms has been done in 2.2.1 and 2.2.2.

Also it has been noted that quadratic polynomial construction algorithm [2] was extremely cumbersome, because it has lead to the solution of a system of nonlinear equations.

Finally, recommendations on the procedure for conducting a virtual test in order to verify the correctness of the construction of the FE model of the cable barriers have been provided.

It has been also shown that analytically calculated displacements of a motor vehicle can be used in the study of passive safety of the driver / passenger.

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