Covert Communication Achieved by A Greedy Relay in Wireless Networks

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Abstract—Covert communication aims to hide the very existence of wireless transmissions in order to guarantee a strong security in wireless networks. In this work, we examine the possibility and achievable performance of covert communication in one-way relay networks. Specifically, the relay is greedy and opportunistically transmits its own information to the destination covertly on top of forwarding the source’s message, while the source tries to detect this covert transmission to discover the illegitimate usage of the resource (e.g., power, spectrum) allocated only for the purpose of forwarding source’s information. We propose two strategies for the relay to transmit its covert information, namely fixed-rate and fixed-power transmission schemes, for which the source’s detection limits are analysed in terms of the false alarm and miss detection rates and the achievable effective covert rates from the relay to destination are derived. Our examination determines the conditions under which the fixed-rate transmission scheme outperforms the fixed-power transmission scheme, and vice versa, which enables the relay to achieve the maximum effective covert rate. Our analysis indicates that the relay has to forward the source’s message to shield its covert transmission and the effective covert rate increases with its forwarding ability (e.g., its maximum transmit power).

Index Terms—Physical layer security, covert communication, wireless relay networks, detection, transmission schemes.

I. INTRODUCTION

A. Background and Related Works

Security and privacy are critical in existing and future wireless networks since a large amount of confidential information (e.g., credit card information, physiological information for e-health) is transferred over the open wireless medium [2]–[4]. Against this background, conventional cryptography [5], and information-theoretic secrecy technologies [6]–[11] have been developed to offer progressively higher levels of security by protecting the content of the message against eavesdropping. However, these technologies cannot mitigate the threat to a user’s security and privacy from discovering the presence of the user or communication. Meanwhile, this strong security (i.e., hiding the wireless transmission) is desired in many application scenarios of wireless communications, such as covert military operations and avoiding to be tracked in vehicular ad hoc networks. As such, the hiding of communication termed covert communication or low probability of detection communication, which aims to shield the very existence of wireless transmissions against a warden to achieve security, has recently drawn significant research interests and is emerging as a cutting-edge technique in the context of wireless communication security [12]–[14].

Although spread-spectrum techniques are widely used to achieve covertness in military applications of wireless communications [15], many fundamental problems have not been well addressed. This leads to the fact that when, or the probability that the spread-spectrum techniques fail to hide wireless transmissions is unknown, significantly limiting its application. The fundamental limit of covert communication has been studied under various channel conditions, such as additive white Gaussian noise (AWGN) channel [16], binary symmetric channel [17], discrete memoryless channel [18], and multiple input multiple output (MIMO) AWGN channel [19]. It is proved that $O(\sqrt{n})$ bits of information can be transmitted to a legitimate receiver reliably and covertly in $n$ channel uses as $n \to \infty$. This means that the associated covert rate is zero due to $\lim_{n \to \infty} O(\sqrt{n})/n \to 0$. Following these pioneering works on covert communication, a positive rate has been proved to be achievable when the warden has uncertainty on his receiver noise power [20]–[22], the warden does not know when the covert communication happens [23], or an uniformed jammer comes in to help [24], [25]. Most recently, [26] has examined the impact of noise uncertainty on covert communication by considering two practical uncertainty models in order to debunk the myth of this impact. In addition, the effect of the imperfect channel state information and finite blocklength (i.e., finite $n$) on covert communication has been investigated in [27] and [28], respectively.

B. Motivation and Our Contributions

The ultimate goal of covert communication is to establish shadow wireless networks [13], in which each hop transmission should be kept covert to enable the end-to-end covert
communication, in order to guarantee the “invisibility” of the transmitters. Following the previous works that only focused on covert transmissions in point-to-point communication scenarios, in this work, for the first time, we consider covert communications in the context of one-way relay networks. This is motivated by the scenario where the relay (R) tries to transmit its own information to the destination (D) on top of forwarding the information from the source (S) to D, while S forbids R’s transmission of its own message, since the resource (e.g., power, spectrum) allocated to R by S is dedicated to forwarding the information from the source (S) to D, while S acts as the warden trying to keep covert from S, where S acts as the warden trying to solely use on aiding the transmission from S to D. As such, R’s transmission of its own message to D should be kept covert from S, where S acts as the warden trying to detect this covert communication. Our main contributions are summarized below.

- We examine the possibility and achievable performance of covert communications in one-way relay networks. Specifically, we propose two strategies for R to transmit the covert information to D, namely the fixed-rate and fixed-power transmission schemes, in which the transmission rate and transmit power of the covert message are fixed and to be optimized regardless of the channel quality from R to D, respectively. We also identify the necessary conditions that the covert transmission from R to D can possibly occur without being detected by S with probability one and clarify how R hides its covert transmission in forwarding S’s message to D in these two schemes.

- We derive the detection limits at S in terms of the false alarm rate $\alpha$ and miss detection rate $\beta$ are in closed-from expressions for the proposed two transmission schemes. Then, we determine the optimal detection threshold at S, which minimizes the detection error $\xi = \alpha + \beta$ and obtain the associated minimum detection error $\xi^\ast$. Our analysis leads to many useful insights. For example, we analytically prove that $\xi^\ast$ increases with R’s maximum transmit power, which indicates that boosting the forwarding ability of R also increases its capacity to perform covert transmissions. This demonstrates a tradeoff between the achievable effective covert rate and R’s ability to aid the transmission from S to D.

- We analyze the effective covert rates achieved by these two schemes subject to the covert constraint $\xi^\ast \geq 1 - \epsilon$, where $\epsilon \in [0, 1]$ is predetermined to specify the covert constraint. Our analysis indicates that the achievable effective covert rate approaches zero as the transmission rate from S to D approaches zero, which demonstrates that covert transmission from D to R is only feasible with the legitimate transmission from S to D as the shield. Our examination shows that the fixed-rate transmission scheme outperforms the fixed-power transmission scheme under some specific conditions, and vice versa. Our examination enables R to switch between these two schemes in order to achieve a higher effective covert rate.

The rest of this paper is organized as follows. Section II details our system model and adopted assumptions. Section III and IV present the fixed-rate and fixed-power transmission schemes, respectively. Thorough analysis on the performance of these two transmission are provided in these two sections as well. Section V provides numerical results to confirm our analysis and provide useful insights on the impact of some parameters. Section VI draws conclusions.

**Notation:** Scalar variables are denoted by italic symbols. Vectors are denoted by boldface symbols. Given a complex number, $\lvert \cdot \rvert$ denotes the modulus. Given a complex vector, $(\cdot)\dagger$ denotes the conjugate transpose. $\mathbb{E}[\cdot]$ denotes expectation operation.

II. SYSTEM MODEL

A. Considered Scenario and Adopted Assumptions

As shown in Fig. 1 in this work we consider a one-way relay network, in which S transmits information to D with the aid of R, since a direct link from S to D is not available. As mentioned in the introduction, S allocates some resource to R in order to seek its help to relay the message to D. However, in some scenarios R may intend to use this resource to transmit its own message to D as well, which is forbidden by S and thus should be kept covert from S. As such, in the considered system model S is also the warden to detect whether R transmits its own information to D when it is aiding the transmission from S to D.

We assume the wireless channels within our system model are subject to independent quasi-static Rayleigh fading with equal block length and the channel coefficients are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero-mean and unit-variance. We also assume that each node is equipped with a single antenna. The channel from S to R is denoted by $h_{sr}$ and the channel from R to D is denoted by $h_{rd}$. We assume R knows both $h_{sr}$ and $h_{rd}$ perfectly, while S only knows $h_{sr}$ and D only knows $h_{rd}$. Considering channel reciprocity, we assume the channel from R to S (denoted by $h_{rs}$) is the same as $h_{sr}$ and thus it is perfectly known by S. We further assume that R operates in the half-duplex mode and thus the transmission from S to D occurs in two phases: phase 1 (S transmits to R) and phase 2 (R transmits to D).

B. Transmission from Source to Relay (Phase 1)

In phase 1, the received signal at R is given by

$$y_r[i] = \sqrt{P_s} h_{sr} x_s[i] + n_r[i],$$

where $P_s$ is the fixed transmit power of source, $x_s$ is the transmitted signal by S satisfying $\mathbb{E}[x_s[i]x_s^\dagger[i]] = 1, i = 1, 2, \ldots, n$. 

![Covert communication in one-way relay networks.](image-url)
is the index of each channel use (n is the total number of channel uses in each phase), and \( n_c[i] \) is the AWGN at relay with \( \sigma_n^2 \) as its variance, i.e., \( n_c[i] \sim \mathcal{N}(0, \sigma_n^2) \).

In this work, we consider that R operates in the amplify-and-forward mode and thus R will forward a linearly amplified version of the received signal to D in phase 2. As such, the forwarded (transmitted) signal by R is given by

\[
 x_r[i] = G y_r[i] = G(\sqrt{P_r}h_{sr}x_b[i] + n_r[i]),
\]

which is a linear scaled version of the received signal by a scalar \( G \). In order to guarantee the power constraint at R, the value of G is chosen such that \( E[x_r[i]|x_r^t[i]] = 1 \), which leads to \( G = 1/\sqrt{P_r|h_{sr}|^2 + \sigma_n^2} \).

In this work, we also consider that the transmission rate from S to D is predetermined, which is denoted by \( R_{sd} \). We also consider a maximum power constraint at R, i.e., \( P_r \leq P_{r}^{\text{max}} \). As such, although R knows both \( h_{sr} \) and \( h_{rd} \) perfectly, transmission outage from S to D still incurs when \( C_{sd}^\text{max} < R_{sd} \), where \( C_{sd}^\text{max} \) is the channel capacity from S to D for \( P_r = P_{r}^{\text{max}} \). Then, the transmission outage probability is given by \( \delta = \mathcal{P}(C_{sd}^\text{max} < R_{sd}) \), which has been derived in a closed-form expression [29]. We assume that all the nodes in the network do not transmit signals when the outage occurs. In practice, the pair of \( R_{sd} \) and \( \delta \) determines the specific aid (i.e., the value of \( P_{r}^{\text{max}} \)) required by S from R, which relates to the amount of resource allocated to R by S as a payback. In this work, we assume both \( R_{sd} \) and \( \delta \) are predetermined, which leads to a predetermined \( P_r^{\text{max}} \).

### C. Transmission Strategies at Relay (Phase 2)

In this subsection, we detail the transmission strategies of R when it does and does not transmit its own information to D. We also determine the condition that R can transmit its own message to D without being detected by S with probability one, in which the probability to guarantee this condition is also derived.

1) Relay’s Transmission without Covert Message: In the case when relay does not transmit its own message (i.e., covert message) to Bob, it only transmit \( x_r \) to D. Accordingly, the received signal at D is given by

\[
y_d[i] = \sqrt{P_r}h_{rd}x_r[i] + n_d[i]
\]

\[
y_d[i] = \sqrt{P_r}h_{rd}\sqrt{P_r}h_{sr}x_b[i] + \sqrt{P_r}h_{rd}n_r[i] + n_d[i],
\]

where \( P_r \) is the transmit power of R, \( x_b \) is the embedded message, and \( n_d[i] \) is the AWGN at D with \( \sigma_n^2 \) as its variance, i.e., \( n_d[i] \sim \mathcal{N}(0, \sigma_n^2) \). Accordingly, the signal-to-noise ratio (SNR) at the destination for \( x_b \) is given by

\[
\gamma_d = \frac{P_r|h_{sr}|^2P_r^{0.5}|h_{rd}|^2G^2}{P_r|h_{rd}|^2G^2\sigma_r^2 + \sigma_d^2} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1},
\]

where \( \gamma_1 \triangleq (P_r|h_{sr}|^2)/\sigma_r^2 \) and \( \gamma_2 \triangleq (P_r^{0.5}|h_{rd}|^2)/\sigma_d^2 \).

For a predetermined \( R_{sd} \), R does not have to adopt the maximum transmit power for each channel realization in order to guarantee a specific transmission outage probability. When the transmission outage occurs (i.e., \( C_{sd}^\text{max} < R_{sd} \) occurs), R will not transmit (i.e., \( P_r = 0 \)). When \( C_{sd}^\text{max} \geq R_{sd} \), R only has to ensure \( C_{sd} = R_{sd} \), where \( C_{sd} = 1/2\log_2(1 + \gamma_d) \). Then, following (4) the transmit power of R when \( C_{sd}^\text{max} \geq R_{sd} \) is given by

\[
P_r^{1} = \frac{\mu\sigma_r^2}{|h_{rd}|^2},
\]

where \( \mu \triangleq \frac{(P_r|h_{sr}|^2 + \sigma_n^2)(2^{2R_{sd}} - 1)}{|P_r|h_{sr}|^2 - \sigma_n^2(2^{2R_{sd}} - 1)} \).

Noting \( \gamma_d < \gamma_1 \), we have \( 1/2\log_2(1 + \gamma_1) > R_{sd} \) when \( C_{sd} = R_{sd} \). As such, \( \mu \) given in (5) is nonnegative. Following (4), we note that \( C_{sd}^\text{max} \geq R_{sd} \) requires \(|h_{rd}|^2 \geq \mu\sigma_r^2/P_r^{1}\). As such, the transmit power of R without covert message is given by

\[
P_r^{1} = \begin{cases} \frac{\mu\sigma_r^2}{|h_{rd}|^2}, & |h_{rd}|^2 \geq \frac{\mu\sigma_r^2}{P_r^{1}}, \\ 0, & |h_{rd}|^2 < \frac{\mu\sigma_r^2}{P_r^{1}}. \end{cases}
\]

2) Relay’s Transmission with Covert Message: In the case when R transmits the covert message to D on top of forwarding \( x_b \), the received signal at D is given by

\[
y_d[i] = \sqrt{P_r}h_{rd}\sqrt{P_r}h_{sr}x_b[i] + \sqrt{P_r}h_{rd}x_c[i] + \sqrt{P_r}h_{rd}n_r[i] + n_d[i].
\]

where \( P_r^1 \) is R’s transmit power of \( x_c \) in this case and \( P_r^\Delta \) is R’s transmit power of the covert message \( x_c \) satisfying \( \mathbb{E}[x_c[i]|x_r^t[i]] = 1 \). We note that the covert transmission from R to D should not affect the transmission from S to D. Otherwise, S can easily observe this covert transmission. As such, here we assume D always first decodes \( x_b \) with \( x_c \) as interference. Following (7), the signal-to-interference-plus-noise ratio (SINR) for \( x_c \) is given by

\[
\gamma_d = \frac{P_r^{0.5}|h_{rd}|^2P_r^1|h_{rd}|^2G^2}{P_r^{0.5}|h_{rd}|^2G^2\sigma_r^2 + P_r^{\Delta}|h_{rd}|^2 + \sigma_d^2} \]

\[
= \frac{\gamma_1\gamma_3}{\gamma_3 + (\gamma_1 + 1)(\gamma_3P_r^\Delta/P_r^1 + 1)},
\]

where \( \gamma_3 \triangleq (P_r^{0.5}|h_{rd}|^2)/\sigma_d^2 \). We will determine \( P_r^{0.5} \) based on different transmission strategies of the covert message from R to D.

### D. Decoding of the Covert Message

As discussed above, the covert transmission from R to D should not affect the transmission from S to D. We also note that this covert transmission cannot happen when the transmission outage from S to D occurs. This is, for example, due to the fact that when the transmission outage occurs R will request a retransmission from S, which enables S to detect R’s covert transmission with probability one if the covert transmission happened. Therefore, the covert transmission from R to D only occur when the successful transmission from S to D is guaranteed (i.e., when \( x_b \) is successfully decoded at D). As such, when the covert message is transmitted by R, D first decodes \( x_b \) and subtracts the corresponding component from its received signal \( y_d \) given in (7). Then, the effective received signal used to decode the covert message \( x_c \) is given by

\[
\hat{y}_d[i] = \sqrt{P_r^\Delta}h_{rd}x_c[i] + \sqrt{P_r}h_{rd}Gn_r[i] + n_d[i].
\]

Then, following (8) the SINR for \( x_c \) is

\[
\gamma_c = \frac{P_r^{0.5}|h_{rd}|^2P_r^{\Delta}|h_{rd}|^2G^2\sigma_r^2 + \sigma_d^2}{P_r^{0.5}|h_{rd}|^2G^2\sigma_r^2 + \sigma_d^2}.
\]
E. Binary Detection at Source and the Covert Constraint

In this subsection, we present the optimal detection strategy adopted by S (i.e., Source).

In phase 2 when R transmits to D, S is to detect whether R transmits the covert message $x_c$ on top of forwarding S’s message $x_s$ to D. R does not transmit $x_c$ in the null hypothesis $H_0$ while it does in the alternative hypothesis $H_1$. Then, the received signal at S in phase 2 is given by

$$y_s[i] = \begin{cases} \sqrt{P_{h^0}}h_rx_s[i] + n_s[i], & H_0, \\ \sqrt{P_{h^0}}h_rx_s[i] + \sqrt{P_D}h_rx_c[i] + n_s[i], & H_1. \end{cases}$$

(11)

where $n_s[i]$ is the AWGN at S with $\sigma^2_s$ as its variance. We note that neither $P_{h^0}$ nor $P_{h^1}$ is known at S since it does not know $h_{rd}$, while the statistics of $P_{h^0}$ and $P_{h^1}$ are known since the distribution of $h_{rd}$ is publicly known. The ultimate goal of S is to detect whether $y_s$ comes from $H_0$ or $H_1$ in one fading block. As proved in [27], the optimal decision rule that minimizes the detection error at S is given by

$$T \geq \tau, \quad \text{if} \quad \frac{P_{h^1}}{P_{h^0}} \leq \frac{\tau}{\rho},$$

(12)

where $T = 1/n \sum_{i=1}^{n} |y_s[i]|^2$, $\tau$ is a predetermined threshold, $D_1$ and $D_0$ are the binary decisions that infer whether R transmits covert message or not, respectively. In this work, we consider infinite blocklength, i.e., $n \to \infty$. As such, we have

$$T = \begin{cases} P_{h^1}|h_{rs}|^2 + \sigma^2_s, & H_0, \\ P_{h^1}|h_{rs}|^2 + P_{\Delta}|h_{rs}|^2 + \sigma^2_s, & H_1. \end{cases}$$

(13)

The detection performance of S is normally measured by its detection error, which is defined as

$$\xi \triangleq \alpha + \beta,$$

(14)

where $\alpha = P(D_1|H_0)$ is S’s false alarm rate and $\beta = P(D_0|H_1)$ is S’s miss detection rate. In covert communications, it is required that $\xi \geq 1 - \epsilon$, where $\epsilon \in [0, 1]$ is predetermined to specify the covert constraint. In practice, it is hard, if not impossible, to know $\xi$ at R since the threshold $\tau$ adopted by S is unknown. In this work, we consider the worst-case scenario where $\tau$ is optimized to minimize $\xi$. As such, the covert constraint considered in this work is $\xi^* \geq 1 - \epsilon$, where $\xi^*$ is the minimum detection error achieved at S.

III. FIXED-RATE TRANSMISSION SCHEME

In this subsection, we consider the fixed-rate transmission scheme, in which R transmits covert message to D with a constant rate if possible. To this end, R varies its transmit power as per $h_{rd}$ such that $P_{\Delta}|h_{rd}|^2$ is fixed as $Q$. Specifically, we first determine R’s transmit power in $H_1$ and then analyze the detection performance at S, based on which we also derive S’s optimal detection threshold. Furthermore, we derive the effective covert rate achieved by the fixed-rate transmission scheme.

A. Transmit Power at Relay under $H_1$

Following (8) and defining $Q = P_{\Delta}|h_{rd}|^2$, in order to guarantee $C_{sd} = R_{sd}$ under $H_1$, $P_r^1$ is given as

$$P_r^1 = \frac{\mu(Q + \sigma^2)}{|h_{rd}|^2},$$

(15)

which requires $C_{sd}^* \geq R_{sd}$ that leads to $|h_{rd}|^2 \geq (\mu^2 + \mu Q + Q)/P_{\max}$. Considering the maximum power constraint at R (i.e., $P_r^1 + P_D \leq P_{\max}$ under this case), R has to give up the transmission of the covert message (i.e., $P_D = 0$) when $P_r^1 > P_{\max} - P_D$ and sets $P_r^1$ the same as $P_0^0$ given in (9). This is due to the fact that S knows $h_{rs}$ and it can detect with probability one when the total transmit power of R is greater than $P_{\max}^r$. Then, the transmit power of $x_c$ under $H_1$ for the fixed-rate transmission scheme is given by

$$P_r^1 = \begin{cases} \frac{\mu(Q + \sigma^2)}{|h_{rd}|^2}, & |h_{rd}|^2 \geq (\mu^2 + \mu Q + Q)/P_{\max}, \\ \frac{\mu^2}{|h_{rd}|^2}, & |h_{rd}|^2 < \frac{\mu^2}{|h_{rd}|^2}. \end{cases}$$

(16)

As per (10), we note that R also not transmit covert message as well when it cannot support the transmission from S to D (i.e., $|h_{rd}|^2 < \mu^2/P_{\max}$). This is due to the fact that a transmission outage occurs when $|h_{rd}|^2 < \mu^2/P_{\max}$ and D will request a retransmission from S, which enables S to detect R’s covert transmission with probability one if the covert transmission happened. In summary, S cannot detect R’s covert transmission with probability one (R could possibly transmit covert message without being detected) only when the condition $|h_{rd}|^2 \geq (\mu^2 + \mu Q + Q)/P_{\max}$ is guaranteed. We denote this necessary condition for covert communication as $C$. Considering Rayleigh fading, the cumulative distribution function (cdf) of $|h_{rd}|^2$ is given by $P_{h_{rd}^2}(x) = 1 - e^{-x}$ and thus the probability that C is guaranteed is given by

$$P_c = \exp \left\{ -\frac{\mu^2}{P_{\max}} \right\}.$$ 

(17)

We note that $P_c$ is a monotonically decreasing function of $Q$ (and the covert transmission rate), which indicates that the probability that R can transmit covert message (without being detected with probability one) decreases as $Q$ increases.

B. Detection Performance at Source

In this subsection, we derive S’s false alarm rate, i.e., $\alpha$, and miss detection rate, i.e., $\beta$.

**Theorem 1:** When the condition $C$ is guaranteed, for a given $\tau$, the false alarm and miss detection rates at S are derived as

$$\alpha = \begin{cases} 1, & \tau < \sigma^2_s, \\ \frac{1}{1 - P_c^{-1}\kappa_1(\tau)}, & \sigma^2_s \leq \tau \leq \rho_1, \\ 0, & \tau > \rho_1. \end{cases}$$

(18)

$$\beta = \begin{cases} 0, & \tau \geq \sigma^2_s, \\ \frac{1}{P_c^{-1}\kappa_2(\tau)}, & \sigma^2_s \leq \tau \leq \rho_2, \\ 1, & \tau > \rho_2. \end{cases}$$

(19)
where
\[
\begin{align*}
\rho_1 & \triangleq P_r^{\max} |h_{rs}|^2 \frac{\mu \sigma_2^2}{\mu \sigma_2^2 + \mu Q + Q} + \sigma_s^2, \\
\rho_2 & \triangleq P_r^{\max} |h_{rs}|^2 + \sigma_s^2, \\
\kappa_1(\tau) & \triangleq \exp \left\{ \frac{\mu \sigma_2^2 |h_{rs}|^2}{\tau - \sigma_s^2} \right\}, \\
\kappa_2(\tau) & \triangleq \exp \left\{ \frac{- (\mu \sigma_2^2 + \mu Q + Q) |h_{rs}|^2}{\tau - \sigma_s^2} \right\}.
\end{align*}
\]

Proof: Considering the maximum power constraint at R under \( H_0 \) (i.e., \( P_r^1 \leq P_r^{\max} \)) and following (16), (17), and (18), the false alarm rate under the condition \( C \) is given by
\[
\alpha = P \left[ \frac{\mu \sigma_2^2}{|h_{rs}|^2} |h_{rs}|^2 + \sigma_s^2 \geq \tau | C \right] = \begin{cases} 
1, & \tau < \sigma_s^2, \\
\frac{\rho_2}{\rho_1} \frac{\lambda^{\mu \sigma_2^2 + \mu Q + Q} |h_{rs}|^2}{\tau - \sigma_s^2}, & \sigma_s^2 \leq \tau \leq \rho_1, \\
0, & \tau > \rho_1. 
\end{cases}
\]

Then, substituting \( F_{|h_{rs}|^2}(x) = 1 - e^{-x} \) into the above equation (\( h_{rs} \) is perfectly known by S and thus it is not a random variable here) we achieve the desired result in (18).

Considering the maximum power constraint at R under \( H_1 \) (i.e., \( P_r^1 + P_{\Delta} \leq P_r^{\max} \)) and following (12), (13), and (16), the miss detection rate under the condition \( C \) is given by
\[
\beta = P \left[ \frac{\mu (Q + \sigma_2^2) |h_{rs}|^2 + Q |h_{rs}|^2}{\tau - \sigma_s^2}, \frac{\sigma_s^2}{\tau} \leq \sigma_s^2, \\
\frac{\rho_2}{\rho_1} \frac{\lambda^{\mu \sigma_2^2 + \mu Q + Q} |h_{rs}|^2}{\tau - \sigma_s^2}, & \sigma_s^2 \leq \tau \leq \rho_2, \\
0, & \tau > \rho_2. 
\end{cases}
\]

Then, substituting \( F_{|h_{rs}|^2}(x) = 1 - e^{-x} \) into (22) we achieve the desired result in (19).

We note that the false alarm and miss detection rates given in Theorem 1 are functions of the threshold \( \tau \) and we next examine how S sets the value of it in order to minimize its detection error in the following subsection.

C. Optimization of the Detection Threshold at Source

In this subsection, we derive the optimal value of the detection threshold \( \tau \) that minimizes the detection error \( \xi \) for the fixed-rate transmission scheme.

Theorem 2: The optimal threshold that minimizes \( \xi \) for the fixed-rate transmission scheme is given by
\[
\tau^* = \min \left\{ \frac{(\mu + 1)Q |h_{rs}|^2}{\ln \left( 1 + \frac{\mu + 1)Q |h_{rs}|^2}{\mu \sigma_2^2} \right)}, \rho_1 \right\}.
\]

Proof: Since \( \rho_2 > \rho_1 \) as given in Theorem 1 following (18) and (19), we have the detection error at S as
\[
\xi = \begin{cases} 
1, & \tau \leq \sigma_s^2, \\
1 - P_c^{-1}(\kappa_1(\tau) - \kappa_2(\tau)), & \sigma_s^2 < \tau \leq \rho_1, \\
1 - P_c^{-1}(\kappa_1(\tau) - \kappa_2(\tau)), & \rho_1 \leq \tau < \rho_2, \\
1, & \tau \geq \rho_2.
\end{cases}
\]

We first note that \( \xi_1 = 1 \) is the worst case for S and thus S does not set \( \tau \leq \sigma_s^2 \) or \( \tau > \rho_2 \). We also note that \( \xi\) given in (24) is a continuous function of \( \tau \) following Theorem 1. Following (24), we derive the first derivative of \( \xi \) with respect to \( \tau \) when \( \rho_1 \leq \tau < \rho_2 \) as
\[
\frac{\partial \xi}{\partial \tau} = \frac{P_c^{-1}(\mu \sigma_2^2 + \mu Q + Q) |h_{rs}|^2}{(\tau - \sigma_s^2)^2} \kappa_2(\tau) > 0.
\]

This demonstrates that \( \xi \) is an increasing function of \( \tau \) when \( \rho_1 \leq \tau < \rho_2 \). Thus, S will set \( \tau \) as the threshold to minimize \( \xi \) if \( \rho_1 \leq \tau < \rho_2 \). We next derive the first derivative of \( \xi \) with respect to \( \tau \) for \( \sigma_s^2 < \tau \leq \rho_1 \). We note that \( \frac{\partial \xi}{\partial \tau} = \frac{P_c^{-1}(\mu \sigma_2^2 + \mu Q + Q) |h_{rs}|^2}{(\tau - \sigma_s^2)^2} \kappa_2(\tau) \) > 0 when \( \sigma_s^2 < \tau \) and \( \kappa_2(\tau) > 0 \) as given in Theorem 1. As such, without the constraint \( \tau \leq \rho_1 \), the value of \( \tau \) that ensures \( \frac{\partial \xi}{\partial \tau} < 0 \) in (26) is given by
\[
\tau^* = \frac{(\mu + 1)Q |h_{rs}|^2}{\ln \left( 1 + \frac{(\mu + 1)Q |h_{rs}|^2}{\mu \sigma_2^2} \right)} + \sigma_s^2.
\]

We note that \( \frac{\partial \xi}{\partial \tau} > 0 \) for \( \tau < \tau^* \), which satisfies the constraint \( \tau > \sigma_s^2 \). We note that \( \frac{\partial \xi}{\partial \tau} < 0 \), for \( \tau \geq \tau^* \), and \( \frac{\partial \xi}{\partial \tau} = 0 \), for \( \tau > \tau^* \). This is due to the term \( \exp \left\{ (\mu + 1)Q |h_{rs}|^2/(\tau - \sigma_s^2) \right\} \) in (26) is monotonically decreasing with respect to \( \tau \). This indicates that \( \tau^* \) minimizes \( \xi \) without the constraint \( \tau \leq \rho_1 \). As such, if \( \tau^* \leq \rho_1 \), the optimal threshold is
\[
\tau^* = \tau^*.
\]

If \( \rho_1 < \tau^* \), following (25) and noting \( xi \) is a continuous function of \( \tau \), we can conclude that the optimal threshold is
\[
\tau^* = \rho_1.
\]

This completes the proof of Theorem 2.

Following Theorem 2, we obtain the minimum detection error at S in the following corollary.

Corollary 1: The minimum value of \( \xi \) at S is
\[
\xi^* = \begin{cases} 
1 - \exp \left\{ \frac{\mu \sigma_2^2 + \mu Q + Q}{P_r^{\max}} Q |h_{rs}|^2 \right\}, & 1 - \exp \left\{ \frac{\mu \sigma_2^2 + \mu Q + Q}{P_r^{\max}} Q |h_{rs}|^2 \right\}, \\
\frac{f_1(Q)}{f_2(Q)} - \frac{\mu \sigma_2^2 + \mu Q + Q}{\mu \sigma_2^2}, & \tau = \tau^*, \\
\frac{f_1(Q)}{f_2(Q)} - \frac{\mu \sigma_2^2 + \mu Q + Q}{\mu \sigma_2^2}, & \tau = \rho_1.
\end{cases}
\]

Proof: Substituting \( \tau^* \) into (24), we obtain the minimum value of \( \xi \) as \( \xi^* = 1 - P_c^{-1}[\kappa_1(\tau^*) - \kappa_2(\tau^*)] \), which completes the proof of Corollary 1.
Base on Theorem 1, Theorem 2 and Corollary 1, we draw the following useful insights.

Remark 1: We can conclude that $\xi^*$ monotonically decreases with $Q$. Based on (30), this conclusion is true for $\tau^* = \rho_1$. We now prove this conclusion for $\tau^* = \tau^\dagger$. To this end, we next prove that $f_3(Q)$ in (30) monotonically increases with $Q$, since both $f_1(Q)$ and $f_2(Q)$ in (30) are monotonically increasing functions of $Q$. Setting $(\mu + 1)Q/\mu \sigma_d^2 = x$, we have $f_3(Q) = f_3(x)$, where

$$f_3(x) = (1 + x)^{-1/3}. \quad (31)$$

In order to determine the monotonicity of $f_3(x)$ with respect to $x$, we derive its first derivative as

$$\frac{\partial f_3(x)}{\partial x} = \exp \left\{ -\frac{\ln(1 + x)}{x} \right\} \frac{(1 + x) \ln(1 + x) - x}{x^2(1 + x)}. \quad (32)$$

We note that whether $\frac{\partial f_3(x)}{\partial x} > 0$ or $\frac{\partial f_3(x)}{\partial x} < 0$ depends on $\ln(1 + x) - x$. As such, we derive the first derivative of $g(x)$ with respect to $x$ as

$$\frac{\partial g(x)}{\partial x} = \ln(1 + x). \quad (33)$$

Noting that $x \geq 0$ and $\frac{\partial g(x)}{\partial x} \geq 0$, we conclude that $g(x)$ monotonically decreases with $x$. Then, we have $\frac{g(x)}{\partial x} = 0$ and thus $\frac{\partial f_3(x)}{\partial x} \geq 0$. This leads to that $f_3(Q)$ monotonically increases with $Q$ and thus $\xi^*$ monotonically decreases with $Q$ for $\tau^* = \tau^\dagger$.

Remark 2: We also conclude that $\xi^* \rightarrow 0$ when $Q \rightarrow \infty$. This follows from (30) for $\tau^* = \rho_1$, since when $Q \rightarrow \infty$ we have $\rho_1 \rightarrow \sigma_d^2$ as per (20) and thus $\rho_1 \leq \tau^\dagger$ (then $\tau^* = \rho_1$). These conclusions indicate that the covert constraint $\xi^* \geq 1 - \epsilon$ determines an upper bound on $Q$, which is denoted by $Q^*$ and achieved by solving $\xi^* = 1 - \epsilon$.

Remark 3: The minimum detection error $\xi^*$ increases with $P_r^\text{max}$ as shown in (31), but $\xi^* \rightarrow 1$ when $P_r^\text{max} \rightarrow \infty$. This later conclusion follows from (30) for $\tau^* = \tau^\dagger$, since when $P_r^\text{max} \rightarrow \infty$ we have $\rho_1 \rightarrow \infty$ as per (20) and thus $\rho_1 \geq \tau^\dagger$ (then $\tau^* = \tau^\dagger$). This result demonstrates that Willie can still possibly detect the covert communication even when R does not have the maximum power constraint, where Willie’s detection performance still depends on other system parameters (i.e., $\mu$, $\sigma_d^2$, and $Q$).

Remark 4: We have $\xi^* \rightarrow 0$ when $R_{sd} \rightarrow 0$ or $R_{sd} \rightarrow \infty$. As $R_{sd} \rightarrow 0$, as per $[5]$ we have $\mu \rightarrow 0$ and thus $\rho_1 \rightarrow \sigma_d^2$ (then $\tau^* = \rho_1$) following (20). Then, from (30) for $\tau^* = \rho_1$ we can see that $\xi^* \rightarrow 0$ as $\mu \rightarrow 0$. As $R_{sd} \rightarrow \infty$, following (5) again we note that $\mu$ will be negative and thus the transmission from S to D fails, which leads to $\xi^* \rightarrow 0$ as discussed in Section III-A. This result means that there exists an optimal value of $R_{sd}$ that maximizes $\xi^*$ and thus maximizes the effective covert rate for given other system parameters. We will numerically examine the impact of $R_{sd}$ on covert communications in Section V.

**D. Optimization of Effective Covert Rate**

In this section, we examine the effective covert rate achieved in the considered system subject to a covert constraint.

1) **Effective Covert Rate**:

From (10), the SINR of $x_c$ at D in the fixed-rate transmission scheme is given as

$$\gamma_c = \frac{P_d |h_{rd}|^2}{P_r |h_{rd}|^2 2 \sigma_r^2 + \sigma_d^2} = \frac{\mu Q + \sigma_d^2}{\eta |h_{sr}|^2 2 \sigma_r^2 + \sigma_d^2} = \frac{Q}{Q + \sigma_d^2} \left( \frac{\mu Q + \sigma_d^2}{\eta |h_{sr}|^2 + \sigma_d^2} \right) \quad (34)$$

where $\eta \triangleq P_d/\sigma_d^2$. Then, the covert rate achieved by R is $R_c = \log_2 (1 + \gamma_c)$. As such, we can see that the covert rate is fixed when $Q$ is fixed as per (34). We next derive the effective covert rate, i.e., the covert rate averaged over all realizations of $|h_{rd}|^2$, in the following theorem.

**Theorem 3**:

The achievable effective covert rate $\overline{R}_c$ by R in the fixed-rate transmission scheme is derived as a function of $Q$ given by

$$\overline{R}_c = R_c P_c$$

$$\text{min} \left\{ \log_2 \left( 1 + \frac{\mu Q + \sigma_d^2}{\eta |h_{sr}|^2 + \sigma_d^2} \right) \times \left\{ -\frac{\mu \sigma_d^2 + \mu Q + Q}{P_r^\text{max}} \right\}, \gamma_c \right\} \quad (35)$$

Based on Theorem 3, we note that $\overline{R}_c$ is not an increasing function of $Q$ and thus $\overline{R}_c$, since as $Q$ increases $R_c$ increases as per (25) while $P_c$ (i.e., the probability that the condition C is guaranteed) decreases following (17). This indicates that there may exist an optimal value of $Q$ (equivalently $R_c$) that maximizes the effective covert rate, which motivates our following optimization of $Q$ in the considered system model.

2) **Maximization of $\overline{R}_c$ with the Covert Constraint**:

Following Theorem 2, the optimal value of $Q$ that maximizes $\overline{R}_c$ subject to the covert constraint $\xi^* \geq 1 - \epsilon$ can be obtained through

$$Q^* = \arg\max_{0 \leq Q \leq Q^*} \overline{R}_c \quad (36)$$

where $Q^*$ is the upper bound on $Q$ determined by the covert constraint as discussed in Remark 1. We note that the optimization problem (36) is of one dimension, which can be solved by efficient numerical search. The maximum value of $\overline{R}_c$ is then achieved by substituting $Q^*$ into (35), which is denoted by $\overline{R}_c$.

**IV. FIXED-POWER TRANSMISSION SCHEME**

In this section, we consider the fixed-power transmission scheme, in which R transmits covert message to D with a constant transmit power if possible. Specifically, we first determine R’s transmit power in $\mathcal{H}_1$ and then analyze the detection performance at S, based on which we also derive S’s optimal detection threshold. Furthermore, we derive the effective covert rate achieved by the fixed-power transmission scheme.
A. Transmit Power at Relay

Following (5), when \( C_{sd} = R_{sd} \) we have

\[
P^1_r = \mu P_\Delta + \frac{\mu \sigma_s^2}{|h_{rd}|^2}.
\]

We note that \( C_{sd} = R_{sd} \) requires \( C_{sd} \geq R_{sd} \) and thus \( |h_{rd}|^2 \geq \mu \sigma_3^2/P_{\text{max}} - (\mu + 1)P_\Delta \). Considering the maximum power constraint at \( R \) (i.e., \( P^1_r + P_\Delta \leq P_{\text{max}} \) under this case), \( R \) has to give up the transmission of the covert message (i.e., \( P_\Delta = 0 \)) when \( P^1_r > P_{\text{max}} - P_\Delta \) and sets \( P^1_r \) the same as \( P^0_r \) given in (6). This is due to the fact that \( S \) knows \( h_{rs} \) and it can detect the covert transmission with probability one when the total transmit power of \( R \) is greater than \( P_{\text{max}} \). Then, the transmit power of \( x_r \) under \( H_1 \) for the fixed-power transmission scheme is given by

\[
P^1_r = \begin{cases} \mu P_\Delta + \frac{\mu \sigma_s^2}{|h_{rd}|^2}, & |h_{rd}|^2 \geq \frac{P_{\text{max}} - (\mu + 1)P_\Delta}{\mu \sigma^2_s}, \\ \frac{\mu \sigma^2_s}{|h_{rd}|^2}, & \frac{P_{\text{max}} - (\mu + 1)P_\Delta}{\mu \sigma^2_s} \leq |h_{rd}|^2 < \frac{P_{\text{max}} - (\mu + 1)P_\Delta}{\mu \sigma^2_s}, \\ 0, & |h_{rd}|^2 < \frac{P_{\text{max}} - (\mu + 1)P_\Delta}{\mu \sigma^2_s}. \end{cases}
\]

As per (38), we note that \( R \) also does not transmit covert message when it cannot support the transmission from \( S \) to \( D \) (i.e., when \( |h_{rd}|^2 < \mu \sigma^2_3/P_{\text{max}} \)). This is due to the fact that a transmission outage occurs when \( |h_{rd}|^2 < \mu \sigma^2_3/P_{\text{max}} \) and \( D \) would request a retransmission from \( S \), which enables \( S \) to detect \( R \)'s covert transmission with probability one if this covert transmission happened. In summary, \( S \) cannot detect \( R \)'s covert transmission with probability one (\( R \) could possibly transmit covert message without being detected) only when the condition \( |h_{rd}|^2 \geq \mu \sigma^2_3/P_{\text{max}} - (\mu + 1)P_\Delta \) is guaranteed. We again denote this necessary condition for covert communication as \( C \). Noting \( F_{|h_{rd}|^2}(x) = 1 - e^{-x} \), the probability that \( C \) is guaranteed is given by

\[
P_c = \exp \left\{ -\frac{\mu \sigma^2_s}{P_{\text{max}} - (\mu + 1)P_\Delta} \right\}.
\]

We note that \( P_c \) is a monotonically decreasing function of \( P_\Delta \), which indicates that the probability that \( R \) can transmit covert message (without being detected with probability one) decreases as \( P_\Delta \) increases. Following (37) and noting \( P^1_r + P_\Delta \leq P_{\text{max}} \), we have \( P_{\text{max}} > (\mu + 1)P_\Delta \) and thus \( 0 \leq P_c \leq 1 \).

B. Detection Performance at Source

In this subsection, we derive \( S \)'s false alarm rate, i.e., \( \alpha = P(D_1|H_0) \), and miss detection rate, i.e., \( \beta = P(D_0|H_1) \).

**Theorem 4:** When the condition \( C \) is guaranteed, for a given \( \tau \), the false alarm and miss detection rates at \( S \) are derived as

\[
\alpha = \begin{cases} 1, & \tau < \sigma_s^2, \\ 1 - P_c^{-1} \kappa_3(\tau), & \sigma_s^2 \leq \tau \leq \rho_3, \\ 0, & \tau > \rho_3, \end{cases}
\]

\[
\beta = \begin{cases} 0, & \tau < \rho_4, \\ P_c^{-1} \kappa_4(\tau), & \rho_4 \leq \tau \leq \rho_5, \\ 1, & \tau > \rho_5. \end{cases}
\]

**Proof:** Considering the maximum power constraint at \( R \) under \( H_0 \) (i.e., \( P^0_r \leq P_{\text{max}} \)) and following (6), (12), and (13), the false alarm rate under the condition \( C \) is given by

\[
\alpha = P \left\{ \frac{\mu \sigma^2_s}{|h_{rs}|^2} \geq \tau | C \right\} = \begin{cases} 1, & \tau < \rho^*_s, \\ \frac{1}{\rho_s} \left\{ \frac{\mu \sigma^2_s}{|h_{rs}|^2} \geq \tau \right\}, & \sigma_s^2 \leq \tau \leq \rho_3, \\ 0, & \tau > \rho_3. \end{cases}
\]

Then, substituting \( F_{|h_{rs}|^2}(x) = 1 - e^{-x} \) into the above equation we achieve the desired result in (40).

We first clarify that we have \( \rho_1 < \rho_5 \) due to \( P_{\text{max}} > (\mu + 1)P_\Delta \) as discussed after (39). Then, considering the maximum power constraint at \( R \) under \( H_1 \) (i.e., \( P^1_r + P_\Delta \leq P_{\text{max}} \)) and following (12), (13), and (38), the miss detection rate under the condition \( C \) is given by

\[
\beta = P \left\{ \left( \frac{\mu \sigma^2_s}{|h_{rs}|^2} + (\mu + 1)P_\Delta \right) \geq \tau | C \right\} = \begin{cases} 1, & \tau < \rho_4, \\ \frac{1}{\rho_s} \left\{ \frac{\mu \sigma^2_s}{|h_{rs}|^2} \geq \tau \right\}, & \rho_4 \leq \tau \leq \rho_5, \\ 0, & \tau > \rho_5. \end{cases}
\]

Then, substituting \( F_{|h_{rs}|^2}(x) = 1 - e^{-x} \) into the above equation we achieve the desired result in (41).

We note that the false alarm and miss detection rates given in Theorem 4 are functions of the threshold \( \tau \) and we examine how \( S \) sets the value of it in order to minimize its detection error in the following subsection.

C. Optimization of the Detection Threshold at Source

In this subsection, we first derive a constraint (i.e., an upper bound) on \( P_\Delta \) to ensure a non-zero detection error at \( S \). Then, under this constraint we derive the lower and upper bounds on the optimal value of \( \tau \) that minimizes the detection error \( \xi \) for the fixed-power transmission scheme.

**Theorem 5:** \( R \)'s transmit power of the covert message \( P_\Delta \) should satisfy

\[
P_\Delta \leq P_\Delta^* = \frac{P_{\text{max}}}{|2(\mu + 1)|}
\]

in order to guarantee \( \xi > 0 \) and when \( P_{\text{max}} \) is guaranteed the optimal \( \tau \) at \( S \) that minimizes \( \xi \) should satisfy \( \rho_4 \leq \tau^* \leq \rho_5 \).
Proof: When $\rho_3 < \rho_4$ that requires $P_\Delta > P_\Delta^{\text{max}}/[2(\mu + 1)]$ as per Theorem 4, following (40) and (41), we have
\[
\xi = \begin{cases} 
1, & \tau \leq \sigma_1^2, \\
1 - P_c^{-1} \kappa_3(\tau), & \sigma_1^2 < \tau < \rho_3, \\
0, & \rho_3 \leq \tau \leq \rho_4, \\
P_c^{-1} \kappa_4(\tau), & \rho_4 < \tau < \rho_5, \\
1, & \tau \geq \rho_5. 
\end{cases}
\]
This indicates that $S$ can simply set $\tau \in [\rho_3, \rho_4]$ to ensure $\xi = 0$ when $P_\Delta > P_\Delta^{\text{max}}/[2(\mu + 1)]$, i.e., $S$ can detect the covert transmission with probability one. As such, $P_\Delta$ should satisfy (43) in order to guarantee $\xi > 0$.

We next prove $\rho_4 \leq \tau^* < \rho_3$. When $P_\Delta \leq P_\Delta^{\text{max}}/[2(\mu + 1)]$, i.e., $\rho_4 < \rho_3$, following (40) and (41), we have
\[
\xi = \begin{cases} 
1, & \tau \leq \sigma_2^2, \\
1 - P_c^{-1} \kappa_3(\tau), & \sigma_2^2 < \tau \leq \rho_4, \\
1 - P_c^{-1} \kappa_4(\tau) - \kappa_4(\tau), & \rho_4 < \tau < \rho_3, \\
P_c^{-1} \kappa_4(\tau), & \rho_3 \leq \tau < \rho_5, \\
1, & \tau \geq \rho_5. 
\end{cases}
\]
due to $\rho_5 > \rho_3$ as shown in Theorem 4. Obviously, Willie will not set $\tau \leq \sigma_2^2$ or $\tau \geq \rho_5$, since $\xi = 1$ is the worst case for Willie.

For $\sigma_2^2 < \tau \leq \rho_4$, we derive the first derivative of $\xi$ with respect to $\tau$ as
\[
\frac{\partial(\xi)}{\partial \tau} = -\frac{\mu \sigma_2^3}{(\tau - \sigma_2^2)^2} \kappa_3(\tau) < 0.
\]
This demonstrates that $\xi$ is a decreasing function of $\tau$ when $\sigma_2^2 < \tau \leq \rho_4$. For $\rho_3 < \tau < \rho_5$, we derive the first derivative of $\xi$ with respect to $\tau$ as
\[
\frac{\partial(\xi)}{\partial \tau} = \frac{\mu \sigma_2^2}{(\tau - \rho_4)^2} \kappa_4(\tau) > 0.
\]
This proves that $\xi$ is an increasing function of $\tau$ when $\rho_3 \leq \tau < \rho_4$. Noting that $\xi$ is a continuous function of $\tau$ and considering (47) and (48), we can conclude that $\tau^*$ should satisfy $\rho_4 \leq \tau^* < \rho_3$, no matter what is the value of $\xi$ for $\rho_3 < \tau < \rho_5$.

1) Effective Covert Rate: Following (10), the SINR at destination for covert communication is given as
\[
\gamma_c = \frac{P_\Delta |h_{rd}|^2}{P_r |h_{rd}|^2 G^2 \sigma_d^2 + \sigma_d^2} = \frac{P_\Delta (\eta |h_{sr}|^2 + 1) |h_{rd}|^2}{\mu P_\Delta |h_{rd}|^2 + (\eta |h_{sr}|^2 + \mu + 1) \sigma_d^2}.
\]
Then, the covert rate achieved by $R$ is $R_c = \log_2(1 + \gamma_c)$. We next derive the effective covert rate, i.e., averaged $R_c$ over all realizations of $|h_{rd}|^2$, in the following theorem.

Theorem 6: The achievable effective covert rate $\overline{R}_c$ by $R$ with the fixed-power transmission scheme is derived as a function of $P_\Delta$ given by
\[
\overline{R}_c = \frac{1}{\ln 2} \exp \left\{ -\frac{\mu \sigma_2^2}{P_\Delta^{\text{max}} - (\mu + 1) P_\Delta} \right\} \times \left[ \ln \left( \frac{\beta_1}{\beta_2} \right) + e^{\beta_2} \text{Ei} \left( -\frac{\beta_2}{\alpha_2} \right) - e^{\beta_1} \text{Ei} \left( -\frac{\beta_1}{\alpha_1} \right) \right],
\]
where
\[
\begin{align*}
\beta_1 &= \frac{\eta |h_{sr}|^2 + \mu + 1}{P_\Delta^{\text{max}} - (\mu + 1) P_\Delta} \sigma_d^2, \\
\beta_2 &= \frac{\eta |h_{sr}|^2 + \mu + 1}{P_\Delta^{\text{max}} - (\mu + 1) P_\Delta} + \mu P_\Delta, \\
\alpha_1 &= \frac{P_\Delta (\eta |h_{sr}|^2 + 1)}{P_\Delta^{\text{max}} - (\mu + 1) P_\Delta}, \\
\alpha_2 &= \mu P_\Delta.
\end{align*}
\]
and the exponential integral function $\text{Ei}(\cdot)$ is given by
\[
\text{Ei}(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt, \quad [x < 0].
\]

Proof: A positive covert rate is only achievable under the condition $C$ and thus $\overline{R}_c$ is given by
\[
\overline{R}_c = \int_0^\infty \frac{1}{|h_{rd}|^2} R_c f(|h_{rd}|^2) d|h_{rd}|^2 = \frac{1}{\ln 2} \exp \left\{ -\frac{\mu \sigma_2^2}{P_\Delta^{\text{max}} - (\mu + 1) P_\Delta} \right\} \times \int_0^\infty \ln \left( \frac{\beta_1}{\beta_2} \right) e^{-\beta_2 x} dx,
\]
where $\beta_1$ is achieved by setting $x = |h_{rd}|^2 - \mu \sigma_2^2/[P_\Delta^{\text{max}} - (\mu + 1) P_\Delta]$. We then solve the integral in (53) with the aid of (40), Eq. (4.337.1)
\[
\int_0^\infty e^{-\beta_2 x} \ln(\theta + x) dx = \frac{1}{\mu} \ln \theta + e^{\nu \theta} \text{Ei}(-\theta \nu),
\]
and achieve the result given in (51).

Based on Theorem 5, we note that $\overline{R}_c$ is not an increasing function of $P_\Delta$, since as $P_\Delta$ increases $R_c$ increases but $P_c$ (i.e., the probability that the condition $C$ is guaranteed) decreases. This motivates our following optimization of $P_\Delta$ in order to maximize the effective covert rate subject to the covert constraint.

D. Optimization of Effective Covert Rate

In this section, we examine the effective covert rate achieved by the fixed-power transmission scheme subject to the covert constraint.
2) Maximization of $\bar{R}_c$ with the Covert Constraint: Following Theorem 3, the optimal value of $P_\Delta$ that maximizes $\bar{R}_c$ subject to this constraint can be obtained through

$$P_\Delta^* = \arg\max_{0 \leq P_\Delta \leq P_\Delta^0} \bar{R}_c$$

s.t. $\xi^* \geq 1 - \epsilon$.

We note that this is a two-dimensional optimization problem that can be solved by efficient numerical searches. Specifically, for each given $P_\Delta, \xi^*$ should be obtained based on (49) where $\tau^*$ is also numerically searched. We note that the numerical search of $P_\Delta^*$ and $\tau^*$ is efficient since their lower and upper bounds are explicitly given. The maximum value of $\bar{R}_c$ is denoted by $\bar{R}_c^*$.

V. NUMERICAL RESULTS

In this section, we first present numerical results to verify our analysis on the performance of covert communications in relay networks. Then, we provide a thorough performance comparison between the fixed-rate and fixed-power transmission schemes. Based on our examination, we draw many useful insights with regard to the impact of some system parameters (e.g., $P_r^\text{max}$, $R_{sd}$, and $\epsilon$) on covert communications in wireless relay networks.

A. Fixed-Rate Transmission Scheme

Fig. 2 shows S’s minimum detection error $\xi^*$ versus $\mu$ for the fixed-rate transmission scheme, which is achieved using (30). In this figure, we first observe that $\xi^*$ is not a monotonic function of $\mu$. We also observe that $\xi^* \to 0$ as $\mu \to \infty$. This is due to the fact that $\tau^* \to \infty$ when $\mu \to \infty$ according to (27), which leads to $\tau^* = \rho_1$ in (30) and thus $\xi^* \to 0$ accordingly. This observation demonstrates that the covert transmission from R to D becomes easier to be detected when $\mu$ is sufficiently large. Following (5), we note that the transmission from S to D fails when $\mu$ is small, which leads to $\xi^* \to 0$ as discussed in Section III-A. Finally, we observe that $\xi^*$ is not a monotonic function of $\sigma_d^2$ as well.

In Fig. 3 we plot the effective covert rate $\bar{R}_c$ versus $Q$, in which we also show the upper bound on $Q$ determined by the covert constraint $\xi^* \geq 1 - \epsilon$ (denoted by $Q^*$ and marked by red circle in this figure). We first observe that $\bar{R}_c$ is not a monotonically increasing function of $P_\Delta$ without the constraint $\xi^* \geq 1 - \epsilon$. This is due to the fact that as $Q$ increases the probability to guarantee the condition $\xi^* \geq 1 - \epsilon$, i.e., $P_c$ given in (17), decreases, while the covert rate $R_c$ increases. In addition, we observe that $\bar{R}_c$ without $\xi^* \geq 1 - \epsilon$ increases as $|h_{sr}|^2$ increases. This is due to the fact that as $|h_{sr}|^2$ increases $\mu$ as given in (5) decreases, which leads to that $P_c$ increases, i.e., the probability that R can conduct covert transmission increases, while $\gamma_c$ given in (34) also increases with $|h_{sr}|^2$ (effectively $R_c$ increases with $|h_{sr}|^2$), since the term $\mu/\eta |h_{sr}|^2 + 1$ in $\gamma_c$ decreases with $|h_{sr}|^2$ as per (5). Furthermore, we observe that $Q^*$ marked by red circle increases with $|h_{sr}|^2$ as well. As such, following the last two observations we can conclude that the achievable effective covert rate with the covert constraint $\xi^* \geq 1 - \epsilon$ increases with $|h_{sr}|^2$. Intuitively, this is due to the fact that as $|h_{sr}|^2$ increases, R has a higher chance to support the transmission of $x_0$ and perform covert transmission, resulting in that from S’s point of view the possible transmit power range of R used to transmit $x_0$ increases (i.e., transmit power uncertainty increases). Finally, we observe that $R_{sd}$ has a significant impact on both $\bar{R}_c$ and $Q^*$, which can be explained by our Remark 4.

B. Fixed-Power Transmission Scheme

In Fig. 4 (a), we plot the minimum detection error $\xi^*$ versus R’s maximum transmit power $P_r^\text{max}$ and observe that $\xi^*$ increases with $P_r^\text{max}$. This shows that the covert transmission becomes easier as the desired performance of the normal transmission increases, since the transmission outage...
probability decreases with $P_r^{\max}$ for a fixed $R_{sd}$. We also observe $\xi^*$ does not approach 1 (but a specific value that is lower than 1) as $P_r^{\max} \to \infty$, which is the same as the result discussed in Remark 1 for the fixed-rate transmission scheme. This observation demonstrates that the covert transmission can still be possibly detected by $S$ even without the maximum power constraint at $R$. In Fig. 4 (b), we plot $\xi^*$ versus the transmission rate from $S$ to $D$ (i.e., $R_{sd}$). We first observe that $\xi^*$ is not a monotonic function of $R_{sd}$ and $\xi^* \to 0$ as $R_{sd} \to 0$ or $R_{sd} \to \infty$. This observation indicates that there may exist an optimal value of $R_{sd}$ that maximizes $\xi^*$, which will be examined in the following. In Fig. 4 we finally observe that $\xi^*$ is not a monotonic function of $\sigma_d^2$.

Fig. 5 shows the effective covert rate $R_c$ versus $P_\Delta$ for the fixed-power transmission scheme, in which we also show the upper bound on $P_\Delta$ determined by the covert constraint $\xi^* \geq 1 - \epsilon$ (denoted by $P_\Delta^*$ and marked by red circle in this figure). We first observe that $R_c$ is not a monotonically increasing function of $P_\Delta$ without the covert constraint $\xi^* \geq 1 - \epsilon$. This is due to the fact that the probability to guarantee the condition $C$, i.e., $P_c$ given in (39), decreases with $P_\Delta$, while $\gamma_c$ given in (50) increases with $P_\Delta$ (effectively the covert rate $R_c$ increases with $P_\Delta$). In addition, we observe that $R_c$ without $\xi^* \geq 1 - \epsilon$ increases with $|h_{sr}|^2$. This is due to the fact that as $|h_{sr}|^2$ increases $\mu$ given in (5) decreases, which leads to that $P_c$ increases in (39) increases, while $\gamma_c$ given in (50) increases with $|h_{sr}|^2$ as well. Finally, we observe that $P_\Delta$ increases with $|h_{sr}|^2$ as well. As such, following the last two observations we can conclude that the maximum effective covert rate achieved subject to $\xi^* \geq 1 - \epsilon$ increases with $|h_{sr}|^2$. This conclusion is the same as that drawn from Fig. 4 for the fixed-rate transmission scheme and the reason is similar as well.

C. Performance Comparison between the Fixed-Rate and Fixed-Power Transmission Schemes

Fig. 6 illustrates $R_c$ versus $P_r^{\max}$ under different value of $P_s$, where $\sigma_d^2 = \sigma_d^2 = 0$ dB, $\epsilon = 0.1$, $R_{sd} = 1$, and $|h_{sr}|^2 = 1$.

Fig. 7 illustrates $R_c$ versus $P_r^{\max}$ with different values of $P_s$ for the fixed-rate and fixed-power transmission schemes using (56) and (55), respectively. In this figure, we first observe that for both schemes $R_c$ monotonically increases as $P_r^{\max}$ or $P_s$ increases, which demonstrates that the covert message becomes easier to be transmitted when more power is available at $S$ or $R$. In Fig. 6 we also observe that the fixed-power transmission scheme outperforms the fixed-rate transmission scheme when $P_s$ or $P_r^{\max}$ is in the low regime. However, when $P_s$ and $P_r^{\max}$ are larger than some specific values (e.g., when $P_s \geq 10$ dB and $P_r^{\max} \geq 10$ dB), the performance of fixed-rate transmission scheme is better than that of the fixed-power transmission scheme. This is mainly due to the
fact that the transmit power constraints are not limits of the covert transmission when \( P_s \) or \( P_{s_{\max}} \) is large, and thus under this case selecting a proper covert transmission rate (in the fixed-rate transmission scheme) can gain more benefit. We note that this observation demonstrates the significance of our work, since with our analysis R can easily determine which transmission is better under the specific system settings.

Fig. 7 shows \( R_c^* \) versus \( R_{sd} \) with different values of \( \epsilon \). In this figure, we first observe that \( R_c^* \) first increases and then decreases as \( R_{sd} \) increases, which confirms that an optimal value of \( R_{sd} \) that maximizes \( R_c^* \) exists. We also observe that \( R_c^* \to 0 \) when \( R_{sd} \to 0 \) or \( R_{sd} \to \infty \). This can be explained by our Remark 4 for the fixed-rate transmission scheme and Fig. 4(b) for the fixed-power transmission scheme. In Fig. 7 we further observe that the achieved \( R_c^* \) decreases significantly as \( \epsilon \) decreases, which demonstrates that it is the covert constraint \( \xi^* \leq 1 - \epsilon \) that mainly limits the performance of the covert transmission. Finally, we observe that the fixed-rate transmission scheme outperforms the fixed-power transmission scheme when \( R_{sd} \) is low. This observation is consistent with that found in Fig. 6 since the transmit power (e.g., \( P_s, P_{s_{\max}} \)) is relatively large for the covert transmission when \( R_{sd} \) is low.

VI. CONCLUSION

This work examined covert communication in one-way relay networks over Rayleigh fading channels, in which R opportunistically transmits its own information to the destination covertly on top of forwarding S’s message, while S tries to detect this covert transmission. Specifically, we proposed the fixed-rate and fixed-power transmission schemes for R to convey covert information to D. We analyzed S’s detection limits of the covert transmission from R to D in terms of the detection error and determined the achievable effective covert rates subject to \( \xi^* \geq 1 - \epsilon \) for these two schemes. Our examination shows that the fixed-rate transmission scheme outperforms the fixed-power transmission scheme under some specific conditions, and otherwise the comparison result in on the contrary. As such, our conducted analysis enabled R to switch between these two strategies to achieve the maximum covert rate. Our investigation also demonstrates that covert communication in the considered relay networks is feasible and the effective covert rate achieved by R increases with its forwarding ability.

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