A Tabula Rasa Approach to Sporadic Location Privacy

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Abstract—Attacks and defenses in the location privacy literature largely consider that users’ data available for training wholly characterizes their mobility patterns. Thus, they hardwire this information in their models. We show that, in practice, training information cannot capture users’ behavior with perfect certainty, and hence state-of-the-art defenses overestimate the level of privacy they provide. To tackle this problem, we propose a new model for user mobility in sporadic scenarios, that we call the blank-slate model. This model acknowledges the uncertainty of designers about real user behavior: it starts as a “blank page”, and learns the actual user behavior as she queries the service provider. We use the blank-slate model to develop both an attack, the Profile Estimation Based Attack (PEBA), and a design technique, called output-feedback, that can be used to create LPPMs that progressively learn the user behavior. Using real data, we empirically show that our proposals outperform state-of-the-art mechanisms designed based on previous hardwired models. Our learning LPPMs do not require bootstrapping with training data and are easy to compute, so they could be easily deployed in current location privacy solutions.

Index Terms—location privacy, LPPM design, mobility models

I. INTRODUCTION

In the last decade, the research community has progressed significantly in the theoretical study and development of Location Privacy-Preserving Mechanisms (LPPMs) [1]–[4], including a number of optimal defense proposals [2]–[9]. In practice, however, there are few LPPMs deployed, and the most popular ones are either not effective at protecting the user’s privacy [10] or implement simple algorithms [11] that are known to be suboptimal [8] and provide little privacy guarantees [12]. A potential reason for this deployment gap is the fact that the latest LPPMs proposed by academia require mobility traces to bootstrap the design, and hardwire the characteristics of this training set into the resulting defenses. However, gathering mobility data is complicated in practice, and the LPPMs designed in this manner are heavily tailored to this data and might not provide enough privacy for users that deviate from this behavior.

Chatzikokolakis et al. have recently acknowledged part of this problem in [8], where they claim that a fair assessment of LPPMs requires the separation between the training dataset used for design, and the testing dataset used for evaluation. Even though the training/testing separation is a step forward towards ensuring that LPPMs will perform well in real conditions, these works inherit the hardwired mobility models from previous works. Hence, the LPPMs that they evaluate provide privacy guarantees that are tied to the training set and cannot handle discrepancies that appear on the testing data.

In this work, we aim at designing LPPMs that are effective even when the testing data cannot be fully characterized a-priori by the training data. We focus on sporadic scenarios [2], [5] where users query the Location Based Service (LBS) occasionally, e.g., location check-in, location-tagging, or applications for finding nearby points-of-interest or friends. In this setting, we make three contributions:

- We propose the blank-slate model for user mobility in sporadic location privacy. Contrary to the hardwired model [1]–[9], this model treats the user mobility as an unknown variable that is learned a-posteriori as the user queries the LBS. Therefore, it captures the unpredictability of user movements and enables the design of LPPMs that are able to learn users’ behavior.
- We develop the Profile Estimation-Based Attack (PEBA) that learns the user mobility profile on the fly. We show that, when evaluated on data not used for training, PEBA outperforms optimal attacks developed for hardwired models of user mobility [8], [9].
- We develop output-feedback design, a new technique to build LPPMs. This technique uses past obfuscated locations to aid the generation of new obfuscations. It produces learning LPPMs that do not need a bootstrapping phase with training data, but start as a clean slate and learn the user behavior a-posteriori. We show that learning LPPMs outperform previous works designed considering a hardwired model for user mobility in practical scenarios where training and testing sets are different, and empirically quantify this improvement.

Our work is a departure from the trend of location privacy works that derive from [1]. We advocate that, counterintuitively, attacks and defenses designers have to abandon the idea, inspired by Kerckhoffs’s principle, that the real mobility model of the users is known. By acknowledging that one cannot fully ascertain users’ behavior based on side-information, and incorporating this idea into the mobility model itself, we build attacks and defenses that perform better in practice. We do not claim that our mobility model is optimal, but we hope that it fosters new research avenues that improve the performance of privacy-preserving mechanisms in realistic scenarios. Furthermore, the fact that our learning LPPMs do
not rely on training data could simplify the implementation of proposed theoretical LPPMs into real-life solutions, and close the gap between theory and practice in location privacy.

II. OVERVIEW OF THE LOCATION PRIVACY PROBLEM

In this section, we first provide an abstraction of the location privacy problem and introduce our notation. Then, we present the framework that we use throughout this paper for design and evaluation of LPPMs.

A. Problem Statement and Notation

As in previous works [1], [5], [8], [9], we consider the scenario where an individual, the user, sends queries to an LBS provider and receives responses with the information she desires. We consider that there is a passive adversary observing the locations inside the user queries. This adversary can be an honest-but-curious LBS or an eavesdropper, and her goal is to infer private information from the user queries [13], [14]. In order to avoid this privacy violation, the user generates fake locations using an LPPM, and sends these fake locations in the queries. By doing so, the user trades in quality of service for privacy.

We illustrate the location privacy problem in Fig. 1. We use \( \rho \) to denote the total number of queries sent by the user to the LBS, and refer to each query by its query number \( r \in \{1, 2, \ldots, \rho\} \). We use \( x^r \in \mathcal{X} \) to denote the real location associated with the \( r \)-th query, i.e., the location the user wants to query about. We use \( x \equiv [x^1, \ldots, x^\rho] \in \mathcal{X}^{\rho} \) to denote the vector of all the real locations, and \( x^r \equiv [x^1, \ldots, x^r] \in \mathcal{X}^{r} \) to denote the vector of all the real locations up to query number \( r \). Likewise, we use \( z^r \in \mathcal{Z} \) to denote the \( r \)-th fake location reported and define the vectors \( z \) and \( z^r \). The real and fake locations are also called input and output locations respectively. Finally, we use \( \hat{x}^r \in \hat{\mathcal{X}} \) to denote the adversary’s estimation of \( x^r \).

In this work, we assume that \( \mathcal{X} \) is a discrete set of points of interest, denoted as \( \mathcal{X} \equiv \{x_1, x_2, \ldots, x_{|\mathcal{X}|}\} \). Also, for simplicity we consider that \( \mathcal{Z} \equiv \mathbb{R}^2 \) is the whole plane, but all of our findings can be extended to other scenarios (e.g., \( \mathcal{Z} \) is a discrete set of cloaking regions [15], or a powerset of points of interest [6]). We also assume that \( \hat{\mathcal{X}} \equiv \mathbb{R}^2 \), i.e., the adversary can produce estimations in any point in the map. Finally, we use \( p \) generally to denote the probability mass function of a discrete random variable, or the probability density function when the variable is continuous. \( E(\cdot) \) and \( H(\cdot) \) denote the expectation and the entropy.

Now, we explain how real, obfuscated, and estimated locations are generated. The real locations \( x \) are chosen by the user as she queries the LBS. We assume that the user makes a sporadic usage of the LBS. This means that the real locations of any two queries (e.g., \( x^r \) and \( x^s \), with \( r \neq s \)) are not temporally dependent. Mathematically, we can write this as \( p(x) = p(\text{perm}(x)) \) for any permutation \( \text{perm}{}(\cdot) \) of the vector \( x \). This assumption is appropriate when the user check-ins are enough separated in time (e.g., a sporadic usage of the service) or when the user only checks-in a single time [3], [5], [6], [8], [9], [16], [17].

In order to generate obfuscated locations \( z \) from the real locations \( x \) the user employs an LPPM \( f \). We study the online location privacy setting, in which the user expects to get the service from the LBS right away. In this case, the LPPM is modeled as a probabilistic function that maps a real location \( x^r \in \mathcal{X} \), and possibly other information available to the user up to that point (i.e., \( x^{r-1} \) and \( z^{r-1} \)), to a value \( z^r \in \mathcal{Z} \). We use \( f \) to denote the probability density function that characterizes the LPPM. Hence, we can write \( p(z|x) \) as

\[
p(z|x) = \prod_{r=1}^{\rho} p(z^r|z^{r-1}, x) = \prod_{r=1}^{\rho} f(z^r|z^{r-1}, x^r).
\] (1)

where the first equality is the chain rule of probability and the second equality reflects the online setting assumption, i.e., the user generates \( z^r \) given \( x^r \) and \( z^{r-1} \), but independently of future locations \( x^{r+1}, x^{r+2}, \ldots \). In our case, since \( f \) outputs locations in \( \mathbb{R}^2 \), we also refer to it as the obfuscation mechanism.

Finally, the adversary generates the estimated locations using an attack \( h \). We assume that the adversary knows the obfuscation mechanism \( f \) and she uses it to design her attack \( h \). We treat \( h \) as a deterministic function that takes a vector of obfuscated locations \( z^r \) and produces an estimation \( \hat{x}^r \) of a (possibly past) real location \( x^r \). We use \( \hat{x}^r(z^r) \) to denote the estimate produced from \( z^r \) using \( h \). We do not consider randomized attacks, since the goal of the adversary is to choose her estimation so as to minimize a specific privacy metric, which can be achieved with deterministic attacks.

LPPM types: Depending on how much information they use to generate obfuscated locations, LPPMs can offer stronger privacy guarantees at cost of complexity in the design. In this paper we study the following LPPM types that can accommodate all previous proposals in the literature:

1) Full LPPMs are the most generic LPPM in the online location privacy setting, i.e., \( f(z^r|z^{r-1}, x^r) \). They generate each obfuscation location \( z^r \) randomly using all the information available to the user, i.e., the previous and current input locations \( x^r \), and the previously released obfuscated locations \( z^{r-1} \).
TABLE I
SUMMARY OF NOTATION

| Symbol | Meaning |
|--------|---------|
| $\rho$ | Total number of queries. |
| $x^r$ | Real location of the user in the $r$-th query. |
| $z^r$ | Obfuscated location of the user in the $r$-th query. |
| $x'$ (or $z'$) | Vector of real (or obfuscated) locations up to query $r$. |
| $x$ (or $z$) | Vector of all real (or obfuscated) locations. |
| $X$ (or $Z$) | Set of all possible real (or obfuscated) locations. |
| $h$ | Adversary’s attack. |
| $\hat{x}^r(z')$ | Adversary’s estimate of the real location $x^r$ using $z'$. |
| $f$ | LPPM or obfuscation mechanism (pdf that generates $z'$). |
| $f(z'|x^{r-1},x^r)$ | Full LPPM. |
| $f(z'|x^{r-1},z^r)$ | Output-based LPPM. |
| $f(z'|x^r)$ | Memoryless LPPM. |
| $d_Q(x',z^r)$ | Quality loss when reporting $z^r$ given $x'$. |
| $Q(f,s)$ | Average quality loss metric at query number $r$. |
| $d_P(f,x',z^r)$ | Adv. error when the adversary estimates $x^r$ as $z'$. |
| $P_{adv}(f,h,r,s)$ | Avg. adv. error of $\hat{x}^r$ given $z'$ and attack $h$. |
| $P_{ce}(f,r,s)$ | Conditional entropy of $x^r$ given $z'$. |
| $\epsilon$ | Geo-ind parameter. |

2) Output-based LPPMs, $f(z'|x^{r-1},x^r)$, generate the obfuscated location randomly using only the current real location $x^r$ and all the previous obfuscated locations $z^{r-1}$. These are a sub-type of full LPPMs.

3) Memoryless LPPMs, $f(z'|x^r)$, generate each obfuscated location randomly using the current real location $x^r$ only. These are a sub-type of output-based LPPMs.

We note that the framework in [1] considers LPPMs of the full type in its theoretical setup, but the evaluation studies only memoryless LPPMs. Memoryless LPPMs are used in sporadic location privacy and works that study the geoindistinguishability privacy notion [3], [5], [6], [8], [9], [16].

The notation used in the paper is summarized in Table I.

B. Design and Evaluation Framework

We now describe a framework that instantiates the abstraction above. This framework extends ideas from [1], [8]. It consists of two steps: first, the design step, where the user designs the LPPM $f$ and the adversary designs her attack $h$. Second, the evaluation step, where the performance of $f$ and $h$ is evaluated. The framework is represented in Fig. 2.

**Design Step:** In this step, the user and the adversary study the location privacy problem, building the LPPM $f$ and the attack $h$, respectively. We assume that they have access to a training set, which might be different for each of them. They derive their designs according to some performance requirements, in terms of privacy and utility metrics. In this work, we consider that the adversary wants to find the attack that minimizes a certain privacy metric, and the user wants to maximize privacy while keeping a minimum utility level. In order to compute these metrics, the user and the adversary need a model for the joint distribution $p(x,z) = p(x) \cdot p(z|x)$. The first term, $p(x)$, is the joint distribution of the real locations of the user. User and adversary derive this distribution by training their mobility model with their training set information. The second term, $p(z|x)$, is determined by the LPPM $f$, as in [1]. The obfuscation mechanism $f$ is chosen by the user and it is assumed to be known by the adversary. We also assume that the adversary knows the mobility model $p(x)$ employed by the user.

**Evaluation Step:** In this step, the performance of $f$ and $h$ is assessed using a testing set. The testing set contains real traces of locations $x$ from a sporadic location privacy scenario. The outputs $z$ are probabilistically generated using $f$ and $x$. Then, the estimations $\hat{x}^s (s = 1, 2, \cdots)$ are calculated using $h$ and $z$. The privacy and utility performance of the LPPM is assessed empirically based on $x$, $z$ and $\hat{x}^s$.

Note that there is a fundamental difference between the design and the evaluation steps, regarding the treatment of the real locations $x$. The design step is carried out by studying the problem analytically, and this is done by assuming a particular mobility model for the real locations $p(x)$. The evaluation step, on the contrary, is carried out empirically with real samples of $x$. Ideally, the user and adversary want their mobility models to closely resemble real user behavior, so that their theoretical analyses translate well into practice. However, finding a realistic model for $x$ is a very complicated task due to the unpredictability and complexity of user behavior. Notice that this is not an issue for the generation of $z$. This is because these samples are generated by $p(z|x)$, which is completely characterized by the obfuscation mechanism and is the same in the design and evaluation steps.

**Main Differences with Previous Work:** This framework takes ideas from the literature, but also adds some contributions. The framework by Shokri et al. [1] considers an adversary that designs her attack based on side-information (i.e., a training set), but they evaluate LPPMs using the same data, and thus do not separate training and testing sets. This separation is considered for the first time in the evaluation in [8].

In this work, we put this idea into context by adding the training/testing separation as part of the framework. We also
consider the selection of a model for the real locations \( p(x) \) as a crucial part of the designing step (in previous works, it is assumed given). Finding a suitable theoretical model for the user mobility \( p(x) \) and fitting it to the training data is part of solving this location privacy problem. However, the performance is evaluated using testing data, and we cannot take for granted that \( x \) will follow the theoretical model that the user/adversary considered for design.

Additionally, our framework accommodates adversaries with different amounts of knowledge. We can model an omniscient adversary, i.e., an adversary that trains her model on evaluation data \([1]\), by making the adversary training set equal to the testing set. However, we can also model less powerful and more realistic adversaries. Judging the performance of an LPPM solely based on how well it fares against an omniscient adversary \([1]\) is debatable, but we do not enter that discussion in this work. From the defense design perspective, however, we show that optimizing the LPPM to perform well against a realistic adversary yields an LPPM that performs better in practice, even against an omniscient adversary.

C. Performance Metrics

The performance of LPPMs is quantified using privacy and utility (or quality loss) metrics. In this work, we take a selection of the most relevant metrics in the location privacy literature \([1]–[5], 8, 9, 17, 18\). We consider one quality loss metric, the average loss, and three privacy metrics: the average adversary error, the conditional entropy, and geo-indistinguishability. These privacy metrics provide a comprehensive picture of the privacy properties of LPPMs \([9]\). We define these metrics below, and explain how to compute them analytically given a model of \( p(x) \). Later, in Section VI, we explain how to evaluate them empirically.

1) Quality Loss Metrics:

Average Quality Loss. The average quality loss measures how much quality the user loses on average by reporting obfuscated locations instead of real ones \([2], 3, 5, 8, 9, 17\). Let \( d_Q(x, z) \) be a point-to-point distance function that measures the loss incurred by revealing \( z \) when the real location is \( x \). The average loss at query \( r \) given LPPM \( f \) is

\[
Q_\text{ave}(f, r) = \mathbb{E}\{d_Q(x^r, z^r)\},
\]

(2)

where the expectation is taken over realizations of \( x^r \) and \( z^r \). Given a distribution \( p(x) \), we can compute this metric as

\[
Q_\text{ave}(f, r) = \int \sum_{x^r \in X} p(x^r) \cdot p(z^r | x^r) \cdot d_Q(x^r, z^r) \cdot dz^r,
\]

(3)

where \( p(x^r) \) and \( p(z^r | x^r) \) can be obtained analytically from \( p(x) \) and \( p(z \perp x) \).

The typical choice for the distance function \( d_Q(\cdot) \) is the Euclidean distance. However, \( d_Q(\cdot) \) can be tailored to the particular application where we want to provide location privacy (e.g., the Manhattan distance, the time-distance of two points \( x \) and \( z \) in the map, a semantic distance related to the location tags of \( x \) and \( z \), etc).

2) Privacy Metrics:

Average Adversary Error. The average adversary error is defined as the mean error incurred by an adversary that estimates the user real locations using an attack \( h \) \([1]–[5], 8, 9, 17\). Let \( d_P(x, \hat{x}) \) be a function that quantifies how much privacy the user has when her real location is \( x \) and the location estimated by the adversary is \( \hat{x} \). Typically, \( d_P(\cdot) \) is the Euclidean distance, but it can adapted to a particular application. Consider that the adversary has observed \( r \) outputs \( (z^s) \) and wants to estimate the location \( z^s \) with \( s \leq r \). For this, she uses an attack \( h \) that produces an estimation \( \hat{x}(z^s) \). The average adversary error at query \( r \) regarding \( x^s \) can be defined as

\[
P_{AE}(f, h, r, s) = \mathbb{E}\{d_P(x^s, \hat{x}(z^s))\},
\]

(4)

where the expectation is taken over \( x^s \) and \( z^s \) (the attack is deterministic, i.e., \( \hat{x} \) is a function of \( z^s \)). Given a mobility model \( p(x) \), this metric can be computed analytically as

\[
P_{AE}(f, h, r, s) = \int \sum_{x^s \in X} p(x^s) p(z^s | x^s) dP(x^s, \hat{x}(z^s)) dz^s.
\]

(5)

Conditional Entropy. The conditional entropy \([1], 9\) measures how much uncertainty an adversary has about the real locations of the user, when she observes the outputs and knows the real model of \( x \). The conditional entropy of \( x^s \) given \( z^r \) (with \( s \leq r \)) when the LPPM is \( f \) is

\[
P_{CE}(f, r, s) = H(x^s | z^r) = -\mathbb{E}\{\sum_{x^s \in X} p(x^s | z^r) \log_2 p(x^s | z^r)\}
\]

(6)

where the expectation is computed over \( z^r \). Analytically, this can be computed given \( p(x) \) and \( f \) as

\[
P_{CE}(f, r, s) = -\int \sum_{x^s \in X} p(z^r) p(x^s | z^r) \log_2 p(x^s | z^r) dz^r.
\]

(7)

where \( p(x^s | z^r) \) is computed from \( p(x^s) \) and \( p(z^r \perp x^s) \) using Bayes’ rule.

Note that the conditional entropy does not only depend on the LPPM \( f \), but also on the entropy of the real locations \( H(x^s) \). In order to get a full picture of the information-theoretic properties of an LPPM, it is important to also evaluate the maximum amount of conditional entropy achievable, which is \( H(x^s) \). Alternatively, this information is given by the mutual information, i.e., \( I(x^s; z^r) = H(x^s) - H(x^s | z^r) \).

As mentioned in \([1], 9\), the conditional entropy is only useful when the distribution \( p(x) \) is the actual distribution followed by the inputs. Note that this metric is independent of the adversary attack.

Geo-Indistinguishability. Geo-Indistinguishability \([17]\) (from now on, geo-ind) is an extension of the differential privacy notion to location privacy. Formally, an LPPM \( f \) guarantees \( \epsilon \)-geo-ind for a single query if, for every pair of real locations \( x, \tilde{x} \in X \), the following relation holds:

\[
\max_{z \in Z} \frac{f(z|x)}{f(z|\tilde{x})} \leq e^{\epsilon} d_Z(x, \tilde{x}),
\]

(8)
where \( d_2(\cdot) \) is the Euclidean distance. This condition indicates that two locations \( x \) and \( \hat{x} \) that are close generate outputs \( z \) with a similar likelihood, and therefore they are statistically indistinguishable for an adversary observing \( z \).

The privacy parameter is \( \epsilon \), which is typically chosen as a ratio \( \epsilon = l/r \). In this ratio, \( l \) is the privacy level that we want to guarantee (e.g., \( l = \log 2 \) in [17]) and \( r \) is a radius around the user’s location where that privacy level is guaranteed. Note that small values of \( \epsilon \) force the inequality (8) to become tighter, and thus \( f(z|x) \) and \( f(z|\hat{x}) \) become closer. This means that small values of \( \epsilon \) provide more indistinguishability than large values. This condition is adversary-agnostic, i.e., independent of the adversary attack or her knowledge of the distribution of the real locations.

The condition above is only guaranteed when a single location is released. In [17], Andrés et al. argue that, by applying an \( \epsilon \)-geo-ind LPPM individually to \( r \) queries, the overall geo-ind level is \( \epsilon' = r \cdot \epsilon \). This is due to the fact that input locations \( x^r \) might be correlated, and therefore an output \( z^r \) still leaks information about past inputs \( x^s \) with \( s \leq r \). This means that the privacy parameter \( \epsilon \) does not scale well with \( r \). Despite some attempts at mitigating this scalability issue [18], providing geo-ind in traces of locations at a reasonable utility cost is still an open problem.

### III. Mobility Models for LPPM Design

In this section, we study two different mobility models for sporadic location privacy. We describe these models and explain how they use the training set information to provide a characterization of \( p(x) \). We do not claim that there is a correct mobility model for \( p(x) \) that the user/adversary should follow. However, it is true that LPPMs and attacks optimized with a certain model will perform better when the evaluation data follows such model. Therefore, models that are closer to real behavior are more useful, since they can be used to develop tools that are optimized for real scenarios.

We consider two mobility models: the hardwired model and the blank-slate model. Both models characterize \( p(x) \) using the mobility profile, denoted by \( \pi \). The mobility profile is an abstraction that represents the long-term user behavior, i.e., the probability with which the user visits each location \( x \in X \). We use \( \pi(x) \) to denote the probability that the user’s real location is \( x \) given the profile \( \pi \).

Both the hardwired and the blank-slate models assume that the input locations of the user are generated following the distribution provided by the mobility profile. The main difference between them is the fact that the hardwired model considers that this mobility profile is known, while in the blank-slate model the user profile is unknown.

#### A. Hardwired Model of User Mobility

We refer to the state-of-the-art treatment of mobility as hardwired model. This model was originally proposed by Shokri et al. in [1] for non-sporadic location privacy and was adapted to sporadic location privacy in most of the following works [2], [5], [8], [9], [19]. We depict this model in Fig. 3.

The hardwired model considers that each real location of a user is an i.i.d. sample of the mobility profile \( \pi \) defined over \( X \), i.e., \( p(x) = \prod_{r=1}^{\rho} \pi(x^r) \). The training data is used to compute the mobility profile \( \pi \).

This means that the mobility profile is hardwired into the model a-priori (c.f. [1], [8]). In a practical scenario, however, it is most likely that the actual input locations of the user (i.e., the ones in the testing set) will not follow a distribution that can be anticipated from some training data. This is because human behavior is complex and hard to predict.

#### B. Blank-slate Model of User Mobility

The blank-slate model of user mobility is depicted in Fig. 4. Let \( F_\pi \) be a family of probability distributions over \( X \), and let \( p(\pi) \) be a distribution over \( \pi \in F_\pi \). In the blank-slate mobility model, the user is assigned at random a profile \( \pi \) with probability \( p(\pi) \), and her real locations are i.i.d. samples of the distribution given by this profile. Formally,

\[
p(x) = \sum_{\pi \in F_\pi} p(\pi) p(x|\pi) = \sum_{\pi \in F_\pi} p(\pi) \prod_{r=1}^{\rho} \pi(x^r) .
\]

The training set is used to compute \( p(\pi) \) and the family \( F_\pi \).

The blank-slate model enables the design of LPPMs and attacks that learn the real user behavior a-posteriori. The model starts as a tabula rasa, with only probabilistic information about \( \pi \) that is computed from the training set (i.e., \( p(\pi) \)). Afterwards, the model acquires additional information about this mobility profile from the observations (\( x \) and \( z \) from the testing set). By assuming this mobility model, the user/adversary will adapt her knowledge of \( \pi \) after each query, and use that knowledge to tailor her LPPM/attack to the real user behavior.

Note that, as a consequence of (9), \( p(x) \neq \prod_{r=1}^{\rho} p(x^r) \), i.e., the input samples are statistically dependent. This dependence comes from the fact that \( \pi \) is unknown: observing a particular \( x^s \) gives information about \( \pi \), and this affects the probability of every other \( x^r \) with \( r \neq s \). However, note that this statistical dependence is not temporal, i.e., \( p(x) = p(\text{perm}(x)) \) for any permutation \( \text{perm}(\cdot) \).

### Previous “Learning” Non-Sporadic Location Privacy Models

Previous work already proposed the use of released information (e.g., \( z^r \)) to design LPPMs and attacks in the framework of non-sporadic location privacy (e.g., Markov mobility models [7]). However, these works models are still
hardwired. For instance, in [7], the Markov transition matrix that models mobility patterns is bootstrapped from the training data and never changed during LPPM and attack design.

The novelty of our work lies in the fact that the blank-slate model has unknown parameters that are (re-)estimated as the user queries the LBS. As explained above, this uncertainty unveils previously unknown dependencies between the real locations that can be exploited by an adversary or the LPPM designer. This is in contrast with correlations considered in previous work [4], [7] which are temporal, i.e., based on the fact that user’s current location depends on where she was in the past.

An interesting future line of work is to extend the blank-slate model to non-sporadic location privacy (e.g., a Markov model with unknown transition matrix) since it might reveal vulnerabilities of current non-sporadic LPPMs.

IV. REVISITING ATTACK DESIGN

In this section, we study the adversary’s design step, where she develops an attack h given a mobility model $p(x)$. We follow the approach in [1], where the adversary computes her estimation so as to minimize the average estimation error $P_{\text{AE}}(f, h, r, s)$. The general procedure to compute an optimal attack is to first compute the posterior

$$p(x^s|z^r) = \frac{p(z^r|x^s) \cdot p(x^s)}{p(z^r)} = \prod_{l=1}^{r} \frac{p(z^{|l-1|}, x^s)}{p(z^r)},$$

and then choose the estimate $\hat{x}^s$ that minimizes, on average, $d_P(x^s, \hat{x}^s)$, i.e.,

$$\hat{x}^s(z^r) = \arg\min_{\hat{x}^s} \sum_{x^s \in \mathcal{X}} p(x^s|z^r) \cdot d_P(x^s, \hat{x}^s).$$

This last operation is relatively easy to carry on (depending on the distance function $d_P(\cdot)$ used and the size of $\mathcal{X}$), and the main computational cost of the attack comes from calculating the posterior $p(x^s|z^r)$. Note that we can replace $p(x^s|z^r)$ in (11) for just the numerator of (10), since the normalization term $p(z^r)$ does not affect the minimization. This means that the hardest task of the adversary is to compute, for each value of $x^s \in \mathcal{X}$, a parameter $v(x^s)$ equal to

$$v(x^s) \triangleq \prod_{l=1}^{r} v_l(x^s) \cdot p(x^s), \quad \text{with} \quad v_l(x^s) \triangleq p(z^{|l-1|}, x^s).$$

We explicitly write these values as $v(x^s)$ to emphasize that they are a function of $x^s$ and that, in order to derive the optimal attack, the adversary must compute their value for each $x^s \in \mathcal{X}$. The computational cost of this task depends on the mobility model assumed by the adversary and the type of obfuscation mechanism the adversary is attacking (see Sect. IV-A). Following, we overview optimal attack design in the hardwired model, and then we study the case of attack design in the blank-slate model.

A. Hardwired model: Previous Optimal Attacks

When the adversary assumes the hardwired model, $p(x^s)$ is given directly by the mobility profile $\pi$, i.e., $p(x^s) = \pi(x^s)$. Also, we can disregard $v_l(x^s)$ for $l \geq s$, as in that case $p(z^{|l-1|}, x^s) = p(z^{|l-1|})$ is independent of $x^s$.

Full LPPM. We can write $v_l(x^s)$ as

$$v_l(x^s) = \sum_{x^{s-r} \in X^{-r}} f(z^{|l-1|}, x^{s-r}, x^s) \prod_{m=1}^{l} \pi(x^m),$$

where $X^{-r} \triangleq [x^1, x^2, \ldots, x^{s-r}, x^{s+1}, \ldots, x^r]$. The amount of values that we have to add in the summation is $|\mathcal{X}|^{l-1}$, so finding out the optimal attack is unfeasible even for a low value of $l$.

Output-based and memoryless LPPMs. In the particular case where the generation of each obfuscated $z^l$ does not depend on any real location other than $x^l$, deriving the optimal attack becomes much easier. In this case, $z^l$ is conditionally independent of $x^s$ (for $l \neq s$) when $z^{|l-1|}$ is given, and therefore $p(z^{|l-1|}, x^s) = p(z^{|l-1|})$. Therefore, we can rewrite (12) as just

$$v(x^s) = f(z^r|x^{s-1}, x^s) \cdot \pi(x^s).$$

The optimal attack against memoryless LPPMs can be found by replacing $f(z^r|x^{s-1}, x^s)$ for $f(z^r|x^s)$ in (14) (recall that memoryless LPPMs are a sub-type of output-based LPPMs). Optimal attack design against memoryless LPPMs has been widely studied [5], [8], [9], due to the fact that most of the LPPMs proposed in sporadic location privacy are in fact memoryless [5], [6], [9], [16].

B. Blank-slate model: Profile-Estimation Based Attack (PEBA)

In the blank-slate model, designing optimal attacks is computationally unmanageable. Even when $f$ is an output-based or memoryless LPPM, the fact that real locations are correlated creates dependencies between $z^l$ and $x^l$ even when $l \neq s$ (the value of $x^s$ affects the distribution of $x^l$ and thus the distribution of $z^l$). This makes finding the optimal attack mathematically intractable, regardless of the category of the LPPM.

Thus, we propose a sub-optimal attack that trades in performance for efficiency. We call this attack the Profile Estimation-Based Attack (PEBA). PEBA sidesteps the computational issues of optimal attack design by decomposing the estimation problem of $x^s$ given $z^r$ into two steps:

1) First, estimate the mobility profile of the user given her obfuscated locations $z^r$. In this paper, for simplicity we decide to use the Maximum Likelihood Estimator (MLE) of the mobility profile, that we denote by $\hat{\pi}^r_{\text{ML}}$.

2) Second, estimate the real location $\hat{x}^s$ assuming it follows the distribution given by $\hat{\pi}^r_{\text{ML}}$ (smoothened for lower values of $r$) and using the obfuscated locations $z^r$.

Note that we only derive this attack for output-based and memoryless LPPMs. We do this for two reasons: first, for
tractability, as attacking full-type LPPMs is computationally challenging. Second, because we do not evaluate full-type LPPMs in Sect. V (we did not find any full-type mechanism in the sporadic location privacy literature).

Step 1: Mobility Profile Estimation. We derive the Maximum Likelihood Estimator (MLE) of the mobility profile, as opposed to using the Maximum a Posteriori (MAP) approach, for computational and analytical simplicity. Our attack is equivalent to the MAP attack for an adversary that assumes that the distribution of mobility profiles \( p(\pi) \) is uniform.

We use \( \pi_i \equiv p(x = x_i) \) to denote the probability mass function by \( \pi \). Let \( \mathcal{P} \) be the set of all the possible mobility profiles, i.e., \( \mathcal{P} \triangleq \{ \pi \mid \sum_{i=1}^{|X|} \pi_i = 1, \; \pi_i \geq 0 \} \). The MLE of \( \pi \) is defined as

\[
\hat{\pi}_{ML} = \arg\max_{\pi \in \mathcal{P}} p(z|\pi). \tag{15}
\]

An efficient iterative way of computing this estimator is the Expectation-Maximization (EM) method [20]. Instead of maximizing \( p(z|\pi) \), we rely on \( x \) as auxiliary data and define a \( Q \) function as

\[
Q(\pi, \pi^t) = E\{\log p(x|\pi)|Z = z, \Pi = \pi^t\}. \tag{16}
\]

The EM method iterates over two steps: first, compute \( Q(\pi, \pi^t) \) (E-step), and then find \( \pi^{t+1} \) as the profile \( \pi \) that maximizes \( Q(\pi, \pi^t) \) (M-step). We expand \( Q \) as

\[
Q(\pi, \pi^t) = E\{\log p(x|\pi)|Z = z, \Pi = \pi^t\} = \sum_{r=1}^{\rho} E\{\log p(x^r|\pi)|Z = z, \Pi = \pi^t\} = \sum_{i=1}^{X} \log \pi_i \cdot p(x_i^r|z, \pi^t) = \sum_{i=1}^{X} \log \pi_i \cdot \left[ \sum_{r=1}^{\rho} p(x_i^r|z, \pi^t) \right]. \tag{17}
\]

In order to find the \( \pi \in \mathcal{P} \) that maximizes \( Q(\pi, \pi^t) \), we build the Lagrange multipliers function

\[
L(\pi, \lambda, \mu) = Q(\pi, \pi^t) + \lambda \left( \sum_{i=1}^{X} \pi_i - 1 \right) + \sum_{i=1}^{X} \mu_i \pi_i, \tag{18}
\]

where the term with \( \lambda \) corresponds to the constraint \( \sum_{i=1}^{X} \pi_i = 1 \) and the terms with \( \mu_i \) correspond to \( \pi_i \geq 0 \). We take \( \mu_i = 0 \) for the non-negativity constraints, and by solving \( \partial L / \partial \pi_i = 0 \) and \( \partial L / \partial \lambda = 0 \) we obtain the maximum, which gives us the update rule

\[
\pi_i^{t+1} = \frac{1}{\rho} \sum_{r=1}^{\rho} p(x_i^r|z^r, \pi^t) = \frac{1}{\rho} \sum_{r=1}^{\rho} \pi_i^t \cdot f(z^r|x_i^{r-1}, z^r), \tag{19}
\]

Following [21], we can see that this solution is the global maximum of \( Q(\pi, \pi^t) \), since it meets the KKT conditions, \( Q(\pi, \pi^t) \) is strictly concave on \( \pi \) (it is a weighted sum of logarithms) and \( \mathcal{P} \) is a convex set.

Summarizing, in order to compute the MLE of the mobility profile, one proceeds as follows. First, define an initial profile \( \pi^0 \). Then, follow the update rule given by (19) until convergence (i.e., until \( \pi^t \) to \( \pi^{t+1} \) is small enough). This algorithm is ensured to converge to the MLE for memoryless and output-based LPPMs, as we prove in Appendix A. The computational cost of the attack depends on the particular obfuscation mechanisms being used.

Step 2: Real location estimation given \( \hat{\pi}_{ML}^r \). We explain now how we estimate each real location \( x^s \) using \( \hat{\pi}_{ML}^r \) and \( z^r \) in our PEBA implementation. Let \( \hat{\pi}_{avg} \) be the average mobility profile of all the users (computed from the training set). First, we run the MLE of the mobility profile, using \( \hat{\pi}_{avg} \) as initial profile for the iteration (19). Then, we perform a normalization step to reduce the estimation noise for low \( r \). We build the adversary estimation of the mobility model \( \hat{x} \) by doing

\[
\hat{x}^r = \frac{1}{\rho^{0.5}} \cdot \hat{\pi}_{avg} + \left( 1 - \frac{1}{\rho^{0.5}} \right) \cdot \hat{\pi}_{ML}^r. \tag{20}
\]

We picked the coefficient \( \rho^{0.5} \) heuristically, with the idea that we want the effect of \( \hat{\pi}_{avg} \) to fade out fast as \( r \) increases.

We use \( \hat{x}^s \) in (20) to build the posterior

\[
p(x^s|z^r, \hat{x}^r) = p(z^r|x^s, \hat{x}^r) \cdot \hat{x}^r(x^s)/p(z^r) = \prod_{l=1}^{r} p(z^l|x_{l-1}^s, x^s, \hat{x}^r) \cdot \hat{x}^r(x^s)/p(z^r). \tag{21}
\]

The terms \( p(z^l|x_{l-1}^s, x^s, \hat{x}^r) \) for \( l < s \) can be disregarded because they are conditionally independent of \( x^s \), since we are in the online setting (1). The cost of computing \( p(x^s|z^r, \hat{x}^r) \) depends on the specific LPPM under attack.

Once the posterior is computed, the PEBA estimate of \( x^s \) can be calculated as

\[
\hat{x}^s(z^r) = \arg\min_{\hat{x}^r} \sum_{x \in \mathcal{X}} p(x^s|z^r, \hat{x}^r) \cdot d_P(x^s, \hat{x}^r). \tag{23}
\]

For example, if \( d_P(\cdot) \) is the Euclidean distance, \( \hat{x}^s \) is the geometric median of \( p(x^s|z^r, \hat{x}^r) \).

V. REVISITING LPPM DESIGN

In this section, we study the design of LPPMs \( f \) using a mobility model \( p(x) \) and some privacy and utility requirements. First, we overview previous LPPMs designed under the hardwired model of user mobility. Then, we explain how we design LPPMs leveraging the blank-slate model.

A. Hardwired model: Previous LPPMs

In the literature, we find many works that study LPPM design under the hardwired model for user mobility. Most works consider that the LPPM belongs to the memoryless type \( f(z^r|x^r) \), either for tractability [3, 9] or because they focus on single queries [2, 8]. In Appendix B we formally prove

\footnote{The choice of this coefficient only affects estimations for low values of \( r \). This choice does not affect our evaluation, since the smallest values of \( r \) we take are \( r = 1 \) and \( r = 50 \) (in the former \( \hat{x}^1 = \hat{\pi}_{avg} \) and in the latter \( \hat{x}^{50} \approx \hat{\pi}_{ML}^r \).}
that, in the hardwired model, a properly designed LPPM of the memoryless type does not provide less privacy than an LPPM of the full type \( f(z'|z^{r-1}, x^r) \). This is true for all the privacy metrics we consider in this work: the average adversary error, the conditional entropy and geo-ind. This means that, in the hardwired model, considering full-type or output-based LPPMs just complicates the problem and does not provide any advantage over memoryless LPPMs.

We now summarize the current state-of-the-art proposals of LPPM design under the hardwired model.

**Optimal average adversary error.** Shokri et al. provide a technique to design optimal LPPMs against the average adversary error, given any pairs of functions \( d_P(\cdot) \) and \( d_Q(\cdot) \). This approach consists on solving a linear program, which can only be done, for computational reasons, if the spaces of real (\( \mathcal{X} \)) and obfuscated (\( \mathcal{Z} \)) locations are discrete.

The optimal remapping techniques in [8] can be used to design optimal LPPMs in terms of the average error when \( d_P(\cdot) \equiv d_Q(\cdot) \), even when \( \mathcal{Z} \equiv \mathbb{R}^2 \) (c.f. [9]). There is no general technique to design an optimal LPPM for generic \( d_P(\cdot) \) and \( d_Q(\cdot) \) when \( \mathcal{Z} \) or \( \mathcal{X} \) are continuous.

**Optimal conditional entropy.** The only LPPM specifically designed to maximize the conditional entropy is the exponential posterior LPPM proposed in [9] by Oya et al. This LPPM is designed following a variation of the Blahut-Arimoto algorithm, and is optimal in terms of \( P_{CE} \) if \( \mathcal{Z} \) is discrete. The authors also provide a sub-optimal technique to achieve large entropy even when \( \mathcal{Z} \) is continuous.

**Optimal geo-ind.** Bordeneb et al. compute optimal geo-indistinguishability LPPMs for individual locations by solving a linear program [2]. This program can only be solved if both \( \mathcal{X} \) and \( \mathcal{Z} \) are discrete, although its computational cost does not scale well with the sizes of \( \mathcal{X} \) and \( \mathcal{Z} \). The state-of-the-art geo-ind LPPM for the release of individual locations in \( \mathcal{Z} \equiv \mathbb{R}^2 \) is to add 2-dimensional Laplacian noise to the input and then perform an optimal remapping [8]. There have been attempts at providing geo-ind for traces of locations, but despite some efforts [13], the privacy level does not scale well with the number of queries.

**Remapping techniques.** In [8], Chatzikokolakis et al. propose a technique called **optimal remapping** that provides an average loss improvement for any LPPM, without reducing privacy. This method was proposed for memoryless LPPMs, and we generalize it for full-type LPPMs. Let \( \hat{f} \) be an obfuscation mechanism, and let \( \tilde{z'} \) be an obfuscated location generated from \( z' \) using such LPPM. Before reporting \( \tilde{z'} \), the user can compute the posterior \( p(z'|\tilde{z'}, z'^{-1}) \) using \( p(x) \) and \( \hat{f} \). With this posterior, she can compute an alternative obfuscated location \( z'' \):

\[
z'' = \arg\min_{z'} \sum_{x' \in \mathcal{X}} p(x'|\tilde{z'}, z'^{-1}) \cdot d_Q(x', z'').
\]  

(24)

By reporting \( z'' \) (instead of \( \tilde{z'} \)), the user achieves a reduction on her average loss (if her model of \( p(x) \) used to compute (24) is close to her real behavior). Also, note that no information about the previous or current input is used in the remapping (since the posterior is computed only depending on previous outputs and the user’s \( p(x) \), which are known to the adversary). This means that, by performing this “remapping” from \( \tilde{z'} \) to \( z'' \), the privacy of the resulting LPPM cannot decrease.

**B. Blank-slate model: Output-feedback LPPM Design**

Designing optimal LPPMs under the blank-slate model is cumbersome. One of the reasons for this is that, contrary to the hardwired model case, memoryless LPPMs do not perform better than output-based or full LPPMs under the blank-slate model. Therefore, we disregard optimal LPPM design and propose instead a sub-optimal technique to design LPPMs under this model. In a nutshell, output-feedback design starts from an optimal LPPM derived from the hardwired model, but then learns the user’s real behavior as information is released and thus the effect of training information vanishes quickly. As it learns information tailored to the user, we expect that it outperforms the initial hardware-based LPPM. We confirm this intuition in the comparison between hardwire and blank-slate approaches in Sect. [VI] below.

First, we indicate that we design output-based LPPMs only, instead of full LPPMs. The reason for this is that, even though a full-type LPPM gives the user more degrees of freedom to improve her privacy (she can work with \( x^{r-1} \)), it also entails privacy risks. Using \( x^{r-1} \) to generate \( z' \) means that every obfuscated location \( z' \) leaks information about all previous real locations. Since we cannot quantify this leakage due to the computational intractability of current attacks against full-type LPPMs, the only approach to provide quantifiable privacy guarantees is to design output-feedback LPPMs.

Our technique to design output-based LPPMs is called **output-feedback** design (Fig. 5). This technique leverages the PEBA to improve any of the known LPPMs derived for the hardwired model. The resulting LPPM learns the user actual behavior using the information from the past released locations. We describe the process of generating \( z'' \) given \( x' \) and \( z'^{-1} \) in the following steps:

1. Choose a memoryless LPPM \( \tilde{f}(z'|x') \) designed in the hardwired model according to some desired privacy and quality loss requirements.
2. Generate a temporary output \( \tilde{z'} \) using this LPPM and the real location \( x' \).
3. Using only the current \( \tilde{z'} \) and past generated outputs \( z'^{-1} \), compute the MLE estimator of the mobility profile \( \hat{\pi}_{ML,d} \), as explained for PEBA in Section [IV-B].
4. Report the obfuscated location \( z'' \) obtained by applying the optimal remapping to \( \tilde{z'} \) using \( \hat{\pi}_{ML,d} \) (or an smoothened version such as (20)).

**VI. COMPARISON: BLANK-SLATE VS. HARDWIRED MODELS**

In this section, we aim at validating that LPPMs and attacks developed using the blank-slate model are superior to those developed state-of-the-art hardwired models. For this purpose, we consider three different types of LPPMs (both in their
learning and vanilla flavors) and 3 different attacks. First, we explain how to compute the privacy and utility metrics empirically, then we introduce our experimental setup, and finally we present our results.

A. Computing the Performance Metrics in Practice.

As explained in Section III (see Fig. 2), the actual performance of the LPPMs is evaluated empirically, with samples of $x$, $z$ and adversary estimations. Assume that we have $N$ samples of pairs $(x, z)$, and let $E_{\text{emp}} \{ \cdot \}$ denote the empirical mean computed over these realizations.

a) Average Quality Loss: The average loss in query $r$ is computed as

$$Q_{\text{prac}}(f, r) = E_{\text{emp}} \{ d_Q(x^r, z^r) \}. \quad (25)$$

b) Average Adversary Error: For each realization of $x$ and $z$, we obtain the adversary estimation $\hat{x}^s(z^r)$, and then compute the average adversary error as

$$P_{AE}^{\text{prac}}(f, h, r, s) = E_{\text{emp}} \{ d_P(x^s, \hat{x}^s(z^r)) \}. \quad (26)$$

c) Conditional Entropy: The conditional entropy metric is only useful when computed with the true distribution of the inputs $f$. This distribution is not available in practice (i.e., we have $x$ but not $p(x)$). Therefore, we compute $P_{CE}$ by using an empirical estimation of $p(x)$. This empirical estimation is given by a mobility profile $\pi$ that is computed after each query $r$ as the normalized histogram of past real locations $x^r$. Note that we could have used the raw trace $x$ to compute this histogram. However, this information is not available in a practical scenario (it would require looking into the “future” real locations), so we do not take this approach. The empirical way of computing $P_{CE}$ that we propose could be computed in practice by a user that wishes to assess her performance.

We follow these steps: after each query $r$, we compute a normalized histogram of current and past real locations $\pi_{h1}^r = \text{hist}(x^r)/r$. Then, we compute the posterior $p(x^s|z^r)$ as

$$p(x^s|z^r, \pi_{h1}^r) = p(x^s|z^r, \pi_{h1}^r) = \frac{f(z^r|x^r, x^s) \cdot p(x^r|\pi_{h1}^r)}{p(z^r|x^r-1, \pi_{h1}^r)}. \quad (27)$$

Using samples from these posteriors, generated in different realizations of the experiment, we compute the entropy as

$$P_{CE}^{\text{prac}}(f, h, r, s) = -E_{\text{emp}} \{ \sum_{x^r \in \mathcal{X}} p(x^s|z^r, \pi_{h1}^r) \log p(x^s|z^r, \pi_{h1}^r) \}. \quad (28)$$

We can see this entropy as the uncertainty of an omniscient adversary that has access to the histogram of real locations. Likewise, we compute $H(x^r)$ using $\pi_{h1}^r$, and use it as a reference of the maximum $P_{CE}$ that can be achieved by any LPPM. Note that, in the first query, $H(x^s) = 0$, and thus $P_{CE}^{\text{prac}} = 0$. As $r$ grows, the histogram of past locations gets more varied and thus $H(x^s)$ grows; this gives room for the conditional entropy to grow.

d) Geo-indistinguishability: Geo-indistinguishability is not, strictly speaking, a metric, but a condition that an LPPM can satisfy. Each obfuscation mechanism guarantees a particular level $\epsilon$ of geo-ind in theory, and the same level of geo-ind holds in practice (i.e., in the evaluation). Recall that, after $r$ queries using a $\epsilon$-geo-ind LPPM, the overall geo-ind guarantee is only $r \cdot \epsilon$.

B. Experimental Setup

We follow the procedure in [8, 9] and take user check-ins from Gowalla\(^2\) and Brightkite\(^3\) real-world datasets in the San Francisco region, defined by the latitude coordinates 37.5395 and 37.7910 and longitude coordinates $-122.5153$ and $-122.3780$. We split each of these datasets into a training and a testing set. We build the testing set in each dataset by taking 16 users that have at least $\rho = 300$ check-ins. We select users so that they have enough variability in their traces so as to achieve a maximum $P_{AE}$ of 2km. This enables us to study the privacy vs. quality loss trade-offs, which we cannot do with users that, for example, only check-in in one or two locations. Consequently, we use roughly 14% and 20% of the data as testing set in Gowalla and Brightkite, respectively, and we use the rest as training set.

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\(^2\)https://snap.stanford.edu/data/loc-gowalla.html

\(^3\)https://snap.stanford.edu/data/loc-brightkite.html
Since we want to evaluate the attacks and defenses in the sporadic location privacy scenario exclusively, we need to remove temporal dependencies in our data. We do this by generating each sample of $x$ as a random permutation of the original trace of a user (up to $p = 300$ queries). The samples of $z$ are generated from $x$ by following the LPPM. We tune the number of samples $N$ that we use to compute the metrics depending on the LPPM used, ensuring that our metrics are within a 50 meter 95\% confidence margin while establishing a maximum of 10,000 repetitions.

In order to compare the blank-slate and hardwired models, we evaluate 6 obfuscation mechanisms and 3 attacks, which are summarized in Tables I and II respectively. In the tables and our description of attacks/defenses below, HW and BS stand for HardWired model and Blank-Slate model, respectively. All of our experiments are conducted using Matlab.

We use the training set to build an average mobility profile $\pi_{\text{train}}$. We do this by normalizing the histogram of check-ins of the users in the training set [8], [9]. We use this profile as $\pi_{\text{avg}}$ in (20) to smoothen the mobility profile in PEBA and output-feedback techniques, as explained in Sect. IV-B. Note that we do not need to infer $F_\pi$ and $p(\pi)$ from the training set, as these are not used by our attacks and defenses.

We evaluate the following defenses:

- **Lap** [HW] is the LPPM that results from adding 2-dimensional Laplacian noise to the real location and then performing an optimal remapping using $\pi_{\text{train}}$. This LPPM is the state-of-the-art of geo-ind in sporadic location privacy [8] (see [17] for the description of the implementation).

- **LH** [HW] refers to location hiding: for each input location $x^r$, the user chooses randomly between revealing her real location $z^r = x^r$ or not revealing any information. We model the second case as revealing a fixed location in the map that minimizes the average loss of the user (computed with $\pi_{\text{train}}$ as explained in [9]).

- **ExPost** [HW] denotes the exponential posterior LPPM. This LPPM tries to maximize the conditional entropy metric and leverages remapping techniques to optimize its performance in terms of $P_{AE}$ [9].

- **Lap+, LH+ and ExPost+** [BS] are the enhanced versions of the previous vanilla LPPMs. We compute these learning LPPMs by applying our output-feedback technique explained in Sect. IV-B to Lap, LH and ExPost respectively.

We evaluate the following attacks:

- **OptHW** [HW] is the optimal attack in the hardwired model [5], [8], [9]. The adversary observes $z^r$ and computes $\tilde{x}^r$ using the mobility profile $\pi_{\text{train}}$ as explained in Sect. IV-A.

- **PEBA** [BS] is the profile-estimation based attack explained in Sect. IV-B, implemented against memoryless

and output-based LPPMs. We perform the EM iteration until the maximum absolute difference between $\pi^n$ and $\pi^{n+1}$ is below 0.005 or until we have iterated more than 50 times, whichever is met first.

- **Omni** [HW] is the optimal attack in the hardwired model carried out by an omniscient adversary that builds the mobility prior from the testing set. We use the same implementation as OptHW, using $\pi_{\text{test}}$ instead of $\pi_{\text{train}}$. This is the only case in the paper where we use the testing set for training. This attack provides a lower bound for privacy.

Note that the attacks are only used to compute the average adversary error metric $P_{AE}$, while all the other privacy metrics are adversary-agnostic. All the mechanisms we evaluate use remapping techniques (Sect. V-A) to generate the obfuscated locations. Therefore, in order to derive attacks against these mechanisms (i.e., to compute the posterior of the real locations), the adversary has to revert these remappings. We have verified empirically that in our scenario (Euclidean distance used for $d_U(\cdot)$), the remapping can be reverted. However, reverting a remapping with high accuracy is very time consuming. In order to speed up our simulations, we give the adversary the temporary locations $z^r$ to compute the posterior $p(x^s|z^r)$ faster. This does not affect the results of our experiments.

### C. Performance: Average Adversary Error ($P_{AE}$)

We evaluate the performance of the 6 LPPMs and 3 attacks. We compare $P_{AE}(f, h, r, s)$ to $\overline{Q}(f, s)$ using the Euclidean distance for the functions $d_Q(\cdot)$ and $d_P(\cdot)$. Recall that $r$ denotes the number of queries observed by the adversary, and $s$ denotes query number of the real location $x^r$ that the adversary estimates. First, we see that the relationship between $P_{AE}(f, h, r, s)$ and $\overline{Q}(f, s)$ is highly linear. For instance, Figure 6 shows the performance of Lap against PEBA for a single user of Brightkite dataset. Each point is value of $P_{AE}(f, h, r, r)$ and $\overline{Q}(f, r)$, for $r = 1, 50, 100, 200, 300$. The dotted lines are regression lines for each value of $r$. The slopes of these lines reflect how much privacy the user gets, on average, for each unit of quality loss. We verified that other users, LPPMs and attacks exhibit the same linear behavior (also for $r \neq s$).

Thus, we use the average slope (computed over all users) of these regression lines as an indicator of the performance of the LPPM. The user is interested in LPPMs that result on a slope as large as possible. We now show the results for the vanilla LPPMs (state-of-the-art) and then the learning LPPMs derived using the output-feedback technique.

**Vanilla LPPMs** [HW]. Figure 7 shows the performance of Lap, LH and ExPost against all three attacks. Since these LPPMs are memoryless, their average performance is the same in all queries $s$. Thus, we show the results for $s = r$ without loss of generality. First, we note that OptHW is the attack that performs the worst. This is because the considered LPPMs are designed assuming the same mobility model as OptHW, and thus they achieve the maximum possible privacy, i.e., $P_{AE} = Q$ (cf. [8], [9]). At the other end of the spectrum lies Omni, that reduces privacy the most. This is expected, as this...
attacker has been trained with the evaluation data. Note that OptHW and Omni are both constant with \( r \), since they are based on hardwired mobility models that do not update as more obfuscated locations \( z' \) are available. Finally, PEBA lies in between: it performs as OptHW in the first query but, as more queries are observed, it reduces the users’ privacy. This improvement is achieved because PEBA learns the real user behavior. These conclusions are qualitatively the same in both datasets, though the decrease of privacy is more significant in Brightkite dataset. This is because users in Brightkite have a more skewed behavior (cf. [9]), and thus they stand out more and their mobility behavior is easier to estimate.

Comparing between defenses, we see that ExPost provides the best protection against PEBA, followed by Lap and then LH. It is interesting to note that LH is especially weak against PEBA: after observing \( r = 50 \) output locations, the adversary’s performance is almost the same as that of Omni, i.e., the adversary is able to figure out the histogram of the real locations of the user.

These results reveal an important fact: previous analyses of LPPMs are either overestimating privacy, because they are evaluated against OptHW [6], [9], or reporting the performance of an unrealistic adversary with access to the testing set, Omni [1]. [8]–[7], [16], [19].

**Learning LPPMs [BS].** Figure 8 shows the performance of Lap+, LH+ and ExPost+. Since these LPPMs learn the user behavior as the number of queries increases, we plot the protection achieved at the location in the \( s \)-th query (varying \( s = 1, 50, 100, 200, 300 \)). OptHW and Omni are independent of the number of queries observed by the adversary \( r \). Thus, we just plot them for \( r = 300 \), and vary \( r \) for PEBA (always keeping \( r \geq s \), since the adversary cannot estimate an \( x^* \) that has not happened yet).

The results for \( s = 1 \) correspond to the slopes in Fig. 7 because the learning LPPMs are the same as the vanilla LPPMs in the first query. As expected, as \( s \) increases, the protection achieved by the learning LPPMs increases, whilst the protection of the vanilla LPPMs remains constant. This happens across all the LPPMs we evaluate, and against all attacks, and confirms that learning LPPMs outperform hardwired-based LPPMs in terms of the average adversary error privacy. Note that, since LH is the LPPM that leaks more information about the user’s real mobility profile, it is also the LPPM that most benefits from the output-feedback technique: LH+ improves considerably the performance of vanilla LH after \( s = 50 \) queries. Lap+ achieves a more modest improvement, and ExPost+ a slightly smaller one. This is because the range of improvement is limited: none of these LPPMs can go beyond a slope of 1 against PEBA. This is because output-feedback techniques use the mobility profile estimation of PEBA. Therefore, they achieve \( P_{AE} = Q \) when \( r = s \) (i.e., slope of 1), but as the adversary gathers more information \( r > s \) privacy decreases \( P_{AE} < Q \) (i.e., slope smaller than 1). The results in Gowalla and Brightkite are qualitatively the same. Thus, we believe our conclusions can be extrapolated to other cases.

Finally, note that the performance of these LPPMs against Omni also increases with \( s \). This means that learning LPPMs achieve more protection against the omniscient adversary than state-of-the-art LPPMs. This is because learning LPPMs are built using a mobility model that is more realistic than the hardwired model used in state-of-the-art proposals.

**D. Performance: Conditional Entropy (\( P_{CE} \)).**

Figure 9 shows an example of the performance of Lap+ for \( r = 300 \) and \( s = 50 \) for 7 different users of Gowalla dataset. We see that the behavior is not linear, but follows a concave curve that can be well modeled by a smoothing spline.

We choose the average \( P_{CE} \) at the point \( Q = 0.5 \km \) determined by the splines as an overall descriptor of the performance of the LPPMs in terms of conditional entropy. We verified that the LPPMs behavior is the same for other values of \( Q \) (e.g., \( Q = 0.25 \km \) or \( Q = 0.75 \km \)).

Figure 10 shows the average conditional entropy \( P_{CE}(f, r, s) \) of \( x^* \) given \( z' \) for different values of \( s \) and \( r \), when \( Q(f, s) = 0.5 \km \). Each plot shows the results for a different LPPM (vanilla and learning versions). The continuous black line shows the maximum entropy achievable (i.e., \( H(x^*) \)), computed from \( \mu_{s} \), as explained in Section VI-A. This entropy starts at 0 because the user has only visited one location at \( r = 1 \), and grows with \( r \) as the user visits more locations. The dashed black line is the performance of the vanilla versions (recall that they are independent of \( s \)). The colored lines show the entropy of learning LPPMs as \( s \) increases. As expected, their conditional entropy increases with \( s \), because these LPPMs adjust to the behavior of the user. However, the performance improvement is almost insignificant in Gowalla, and slightly larger in Brightkite. The reason for this modest increase is that the entropy is concave with the quality loss (see Fig. 9). Hence, the significant utility improvement of learning LPPMs only translates into a small entropy increase.

The results for Lap+ in Brightkite are particularly interesting. The performance of Lap in terms of conditional entropy is slightly worse than the performance of ExPost. This is not surprising, as ExPost is designed to maximize \( P_{CE} \). However, the performance of the enhanced version Lap+ is on par with ExPost+, especially after \( s = 300 \) queries. This means that one can use Lap+ instead of ExPost+ or ExPost towards maximizing entropy. This is very useful, as Lap+ is computationally faster than ExPost+ (cf. [9]) and it also provides geo-indistinguishability guarantees (see below).

Finally, as explained in [9], the conditional entropy reveals that LH and LH+ are very binary LPPMs, i.e., they either provide no privacy (when the user reveals her location) or no utility (when the user reports the center of the map).

**E. Performance: Geo-indistinguishability (\( \epsilon \)).**

Finally, we evaluate the trade-off between the geo-ind parameter \( \epsilon \) and the average loss \( \overline{Q}(f, s) \). In [9], Oya et al. show
Fig. 6. Performance of PEBA (different values of $r$) against Lap in Brightkite dataset, for a single user.

Fig. 7. Privacy versus quality loss performance, shown as the average slope of the regression lines of $P_{AE}$ vs. $Q$ of three state-of-the-art vanilla LPPMs, against the three attacks we consider. We vary the number queries observed by the adversary $r$.

Fig. 8. Privacy versus quality loss performance, shown as the average slope of the regression lines of $P_{AE}$ vs. $Q$ of the learning LPPMs (created using output-feedback design), against the three attacks we consider. We vary the number of queries observed by the adversary $r$ and the user $s$.

Fig. 9. $P_{CE}(f,r,s)$ of Lap+ with $r = 300$ and $s = 50$ for 7 users of the testing set, and smoothing splines for each user (different colors represent different users). Gowalla dataset.

Fig. 10. $P_{CE}(f,50,r)$ of Lap+ against the three attacks we consider. We vary the number of queries observed by the adversary $r$.

Fig. 11. $\epsilon$ for both vanilla and learning Laplacian mechanisms. Recall that lower values of $\epsilon$ denote more indistinguishability, i.e., more privacy. The curves correspond to smoothing splines, and the dots are the empirical values from our experiments. As before, at the first query $s = 1$, the performance of Lap and Lap+ is the same. As the user performs more queries, the average loss of Lap+ decreases without affecting $\epsilon$, and thus the privacy and quality loss trade-off improves.

The increase of privacy (or decrease of $\epsilon$) is also an increase of the geo-ind privacy radius. As we explained before, $\epsilon$ can be defined as a ratio $l/r$, where $l$ is the geo-ind privacy level and $r$ is the radius around the user’s location where that privacy level is guaranteed. Based on the smoothing splines in Fig. 11 we compute how much the privacy radius increases (keeping the privacy level $l$ constant) by using Lap+ compared to Lap (see Fig. 12). The results are noisy for the first $Q$ values (since we are extracting them from smoothing splines, and the $\epsilon$ vs. $Q$ curves are concave), and could be improved by carrying out simulations for more values of $\epsilon$. Still, we can appreciate that the increase of the privacy radius in Lap+ is significant, especially in Brightkite. In this dataset, after learning for $s = 300$ queries, Lap+ achieves an increase of $12$.
the radius by a factor between 1.4 and 2, compared to Lap. These experiments confirm that, in terms of geo-ind, the blank-slate Laplacian LPPM improves over the hardwired version.

F. Summary of results.

Our experiments show that, for all the considered privacy metrics, the learning LPPMs are consistently superior to the vanilla LPPMs. Similarly, the PEBA attack outperforms the optimal attack in the hardwired model OptHW. These cases illustrate the advantage of the blank-slate model over hardwiring. By enabling the algorithms behind attacks and defenses to learn and adapt themselves to the observation, one obtains better results. In fact, they can only be outperformed in unrealistic cases, like the Omni attack that is trained using the testing data directly.

Another advantage of our learning LPPMs is that, since they quickly forget about their initial configuration (see (20)), they do not need to be bootstrapped with training data. For instance, they could be initialized with a uniform prior, as soon the new observations override this initialization. This is particularly important at the time of deploying these mechanisms, since developers and users may not necessarily have any information about prior users’ behavior.

Finally, although we have only evaluated three LPPMs, we conjecture that any other LPPM developed under the hardwired model can be enhanced with our output-feedback design technique.

VII. RELATED WORK

Early surveys of location privacy attacks, defenses and privacy metrics, by Decker [22] and Krumm [23], do not include any discussion about modeling user mobility.
A first explicit modeling appears in [1], where Shokri et al. propose a framework to evaluate location privacy mechanisms. In their framework instantiation, they consider a Markov hardwired model for user mobility, and in their evaluation they effectively merge training and testing sets. A number of follow-ups also hardwire the mobility model using the evaluation data itself. In sporadic location privacy, this methodology was used to design and evaluate LPPMs according to different privacy notions. First, it was used to find optimal LPPMs in terms of the average adversary error, either by reporting individual locations [5], using dummy check-ins [6], or in combination with geo-ind guarantees [16]. Second, hardwired user mobility models are used to obtain utility improvements and derive optimal geo-indistinguishability LPPMs [2], or to evaluate a semantic variation of this notion [3]. Non-sporadic location privacy works also train their mobility models on the evaluation data, but adopt a Markov model for user mobility to account for temporal correlations [4]. These non-sporadic models are also hardwired, in the sense that their parameters are inferred once from the training data, and never modified a-posteriori during the evaluation.

Chatzikokolakis et al. are the first to explicitly separate data used to design LPPMs and to evaluate them [8], in the context of geo-indistinguishability. Subsequently, Oya et al. [9] used the same approach to design optimal LPPMs in terms of the conditional entropy.

To the best of our knowledge, our work is the first to consider a blank-slate model for user mobility, which allows us to design LPPMs and attacks that learn the model parameters during the evaluation. Our experiments show that such an approach brings both privacy and utility improvements compared to the hardwired approach, when the data used for evaluation is different from the data used to design the defenses.

VIII. CONCLUSIONS

Previous Location Privacy-Preserving Mechanisms (LPPMs) assume that training data can completely characterize user mobility behavior, and hardwire this information at their core. As a result, they overestimate the privacy they provide to users that deviate from the behavior learned in training.

We proposed a new model for user mobility that, instead of hard-wiring the user mobility profile, considers it as an unknown variable that has to be learned. We leverage this blank-slate model to propose a new attack and learning LPPMs, and show using real data that these proposals outperform previous designs.

Even though we have focused on sporadic location privacy, our results put in question proposals to design and evaluate LPPMs in non-sporadic scenarios which also use hardwired models and, in many cases, use the same data for design and evaluation. Furthermore, the problem identified in this paper is not unique to the location privacy domain. More generally, to build privacy enhancing technologies that provide strong privacy guarantees in real cases, we have to embrace that training information cannot always fully capture real user behavior. We believe that blank-slate models, that incorporate the uncertainty about real user behavior, are a promising approach to improve the protection provided by privacy mechanisms not only in location privacy but in a broader type of privacy problems.

APPENDIX

A. Convergence of the EM sequence to the MLE of the mobility profile.

We prove the convergence of the EM iteration in [19] to the maximum likelihood estimator of the mobility profile, for memoryless and output-based LPPMs only. Let \( \mathcal{P} \) be the probability simplex, i.e., the set of valid mobility profiles \( \mathcal{P} = \{ \pi | \sum_{i=1}^{|X|} \pi_i = 0, \pi_i \geq 0 \} \). Then, the MLE is

\[
\hat{\pi}_{ML} = \underset{\pi \in \mathcal{P}}{\text{argmax}} \log p(z|\pi).
\] (29)

In [20], [24], authors show that if the likelihood function (i.e., \( \log p(z|\pi) \)) has a unique global maximum over \( \mathcal{P} \) and the derivatives \( \partial Q(\pi, \pi^t)/\partial \pi \) are continuous over \( \pi \) and \( \pi^t \), then any EM sequence \( \{ \pi^0, \pi^1, \pi^2, \ldots \} \) computed as in [19] converges to the unique global maximum \( \hat{\pi}_{ML} \). We now prove that our problem meets these requirements, and refer to [20], [24] for the complete details of the proof.

First, we prove that \( \log p(z|\pi) \) is strictly concave and has a unique global maximum over \( \mathcal{P} \). By definition, it is easy to see that \( \mathcal{P} \) is convex, i.e., given two profiles \( \pi, \pi' \in \mathcal{P} \), we can check that \( \pi'' = \lambda \pi + (1-\lambda)\pi' \in \mathcal{P} \) for \( \lambda \in [0, 1] \). On the other hand, we can write \( \log p(z|\pi) = \sum_{t=1}^{r} \log p(z^{t}|z^{r-1}, \pi) \) and show that

\[
p(z^t|z^{r-1}, \pi) = \sum_{i=1}^{|X|} f(z^t|z^{r-1}, x^r = x_i) \cdot \pi_i,
\] (30)

where \( f(z^t|z^{r-1}, x^r = x_i) \) is given by the LPPM (it does not require \( \pi \) for its computation, since it is an output-based LPPM). This means that \( p(z^t|z^{r-1}, \pi) \) is linear with \( \pi \), and therefore \( \log p(z^t|z^{r-1}, \pi) \) is strictly concave. This implies that \( \log p(z|\pi) \) is also strictly concave, since it is the sum of strictly concave functions. Since \( \mathcal{P} \) is a convex set, then \( \log p(z|\pi) \) has a unique global maxima over \( \mathcal{P} \).

On the other hand, it is easy to see that the derivatives \( \partial Q(\pi, \pi^t)/\partial \pi \) are continuous over \( \pi \) and \( \pi^t \) (note that \( \pi_i \in [0, 1] \)), which concludes the proof.

The proof for memoryless LPPMs is the same, since they are a sub-type of output-based LPPMs. This result is not true, however, for full-type LPPMs, since \( p(z^t|z^{r-1}, x^r = x_i) \) depends on the mobility profile (through all the other inputs).

B. Privacy performance of the memoryless LPPM in the hardwired model of user mobility.

Consider the full-type LPPM \( f(z^t|z^{r-1}, x^r) \), and a memoryless-type LPPM that we denote by \( f^* \), defined as

\[
f^*(z^t|x^r) \doteq \int_{z^{r-1} \in X^{r-1}} p(x^{r-1}, z^{r-1}|x^r) \cdot f(z^t|z^{r-1}, x^r) dz^{r-1}.
\] (31)
The average loss of \( f \) and \( f^* \) is the same, i.e., \( \mathbb{Q}(f, r) = \mathbb{Q}(f^*, r) \) due to the linearity of this metric. Then, by proving that \( f^* \) does not achieve lower privacy than \( f \), we prove that the privacy and quality loss trade-off of \( f^* \) is not worse than that of \( f \). We show this for the three privacy notions that we consider in this work: the average adversary error, the conditional entropy and geo-indistinguishability. For these proofs, we use \( p^* \) and \( H^* \) to denote the probabilities and entropy referred to the case where the LPPM used is \( f^* \). Also, we use \( z^{-s} = \{z^1, z^2, \ldots, z^{-s-1}, z^{-s+1}, \ldots, z^r\} \).

**Worst-Case Average Adversary Error.** In terms of \( P_{AE} \), we show that \( \min_h P_{AE}(f, h, r, s) \leq \min_h P_{AE}(f^*, h, r, s) \), i.e., that \( f^* \) does not achieve less privacy than \( f \) against an optimal adversary that minimizes \( P_{AE} \):

\[
\min_h P_{AE}(f, h, r, s) = \int z^* \left[ \sum_{x^* \in \mathcal{X}} \pi(x^*) p(z^* | x^*) d_P(x^*, \hat{x}^*) \right] dz^*
\]

\[
\leq \int z^* \left[ \sum_{x^* \in \mathcal{X}} \pi(x^*) p(z^* | x^*) d_P(x^*, \hat{x}^*) \right] dz^*
\]

\[
= \int \left[ \sum_{x^* \in \mathcal{X}} \pi(x^*) p(z^* | x^*) d_P(x^*, \hat{x}^*) \right] dz^*
\]

\[
= \int \left[ \sum_{x^* \in \mathcal{X}} \pi(x^*) f^*(z^* | x^*) d_P(x^*, \hat{x}^*) \right] dz^*
\]

\[
= \min_h P_{AE}(f^*, r, s)
\]

Step (a) comes from splitting the integral over \( z^* \) into two integrals: one over \( z^* \) and the other over the complement. Then, computing the integral (over \( z^{-s} \)) of the minima over \( \hat{x}^* \) is smaller or equal than computing the minimum of the integral. Step (b) follows from the fact that \( z^* \) is independent of \( z^{-s} \) and \( x^* \) in the hardwired model and with a memoryless LPPM \( f^* \).

**Conditional Entropy.** We prove that \( P_{CE}(f, r, s) \leq P_{CE}(f^*, r, s) \).

\[
P_{CE}(f, r, s) = H(x^* | z^*) \leq \sum_{s=1}^{r} H(z^s | x^s) \leq \sum_{s=1}^{r} H^*(x^s | z^s)
\]

\[
= \sum_{s=1}^{r} H^*(x^s | z^s) = P_{CE}(f^*, r, s)
\]

Step (a) comes from the fact that conditioning does not increase entropy. In (b), we use that \( p(z^s, x^s) = p^*(z^s, x^s) \), and therefore \( H^*(x^s | z^s) = H^*(x^s | z^s) \). Finally, in (c) we use that, given \( f^* \) and a known mobility profile, \( x^* \) is independent of the all the outputs of other queries \( z^* \) with \( f \neq s \).

**Geo-Indistinguishability.** Since the geo-ind condition (5) only depends on the probabilities \( p(z^* | x^*) \), which are the same in \( f \) and \( f^* \), using \( f^* \) is not worse than \( f \) towards guaranteeing geo-ind for a single sample.
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