Dynamic sine calibration of pressure transducers

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Abstract. Pressure transducers are frequently used in dynamic environments such as combustion engines, aerospace applications or industrial process control. Like for every measurement task it is essential to know the uncertainty of the obtained measurement result, even in a dynamic environment. But most pressure sensors are only calibrated by static calibration methods. So the frequency response of the transducer and the uncertainty for the dynamic measurement result cannot be quantified. Furthermore it is remarkable that there is no standardized method to quantify the frequency response of a pressure transducer. Hence no national standard or traceability for periodic, dynamic pressure calibration exists. Due to this lack of investigation methods the author has made an effort to develop a primary method to calibrate pressure transducers dynamically. This method allows to measure the frequency response with sufficient dynamic pressure amplitudes up to 1.2 MPa. Due to the construction of the pressure generator frequencies up to 10 kHz can be reached. In the paper the pistonphone based apparatus is presented.

1. Introduction
Measuring dynamic pressure is an essential task when it comes to automotive, fluidic or aerospace applications. The pressure pulsation has to be measured with high accuracy in respect to the frequency and the amplitude of the pulsation. If the pressure is measured correctly the combustion processes in an automotive can be understood or damages can be prevented. But for a measurement of pressure in a dynamical environment, the frequency response of the transducer has to be known. Hence the transducer has to be calibrated by means of dynamic calibration methods.

Some dynamic pressure calibration methods are known. For example shock tubes and fast opening devices. But they work with pressure amplitudes of several MPa and in an aperiodic manner. If converting the aperiodic time based results to the frequency domain a high measurement uncertainty has to be accepted. Periodic pressure calibration systems are widely used for microphones. But this type of calibration deals with pressure amplitudes obviously lower than reached by industrial pressure sensors.

That is why the author has made an effort to develop a periodic calibration device and a primary calibration method, to investigate the frequency response of pressure transducers. The device can be operated in a frequency range of \( f = 0 \ldots 10 \text{ kHz} \) and pressure amplitudes higher than 1 MPa.

2. Design
The design is based on a pistonphone. The device consists of a piezoelectric actuator which is located between a housing and a piston. The piston compresses the enclosed liquid. Figure 1 shows the design and the mechanical model.
Figure 1. Design of pressure generator and mechanical model

From the mechanical model it is obvious that the pressure generator is not mounted to a fundament, but build up as a free two mass oscillator. The fluid and the piezoelectric drive can be modelled as a spring-damping-system, while the stiffness of the fluid and the actuator governs the behavior. Both, the fluid and the actuator connect the two masses, so they can be summed to a resultant stiffness $c = c_{\text{fluid}} + c_{\text{piezo}}$ and damping factor $d = d_{\text{fluid}} + d_{\text{piezo}}$. The system can be described by the Newtonian equation of motion.

\[
\begin{bmatrix}
F_{\text{Piezo}} \\
-F_{\text{Piezo}}
\end{bmatrix} =
\begin{bmatrix}
m_h & 0 \\
0 & m_p
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_h \\
\ddot{x}_p
\end{bmatrix} +
\begin{bmatrix}
d & -d \\
-d & d
\end{bmatrix}
\begin{bmatrix}
\dot{x}_h \\
\dot{x}_p
\end{bmatrix} +
\begin{bmatrix}
c & -c \\
-c & c
\end{bmatrix}
\begin{bmatrix}
x_h \\
x_p
\end{bmatrix}
\tag{1}
\]

The force $F_{\text{Piezo}}$ is generated by the piezo. The force-voltage dependency of the piezo can be evaluated with the piezo constant, since the device is operated in a low voltage mode. The displacement amplitudes of the piston and housing are limited by the dynamic motion and the power limit of the actuator. With the current setup the maximum dynamic peak pressure is $p_{\text{peak}} = 1.2 \text{ MPa}$.

3. Measurement model
3.1 Basic model

The fundamental idea is to calculate the dynamic pressure based on the motion of the piston and the housing. With the mass balance applied on the fluid filled volume, the pressure can be calculated

\[
C_H \frac{dp}{dt} = -\frac{dV}{dt} + \frac{1}{\rho} \frac{dm}{dt} + V \gamma \frac{dT}{dt}
\tag{2}
\]

Where $p$ is the pressure, $t$ is the time, $C_H$ is the hydraulic capacity, $V$ is the initial volume, $\rho$ is the density, $m$ is the mass, $\gamma$ is the heat expansion coefficient and $T$ is the temperature. Since the fluid chamber is housed, the mass flow term is neglected. And further the temperature dependency is neglected in a first step. The equation then simplifies to

\[
C_H \frac{dp}{dt} = -\frac{dV}{dt}
\tag{3}
\]

\[
dp = -\frac{1}{C_H}dV
\tag{4}
\]

and after integration to

\[
\Delta p = -\frac{1}{C_H} \Delta V = -K \frac{\Delta V}{V} = -K \frac{A \Delta x}{V}, \quad \Delta x = x_{\text{piston}} - x_{\text{housing}}
\tag{5}
\]

Where $K$ is the bulk modulus of the fluid, $A$ is the area of the piston and $x$ is the difference of the displacements of the piston and the housing. For the pressure calculation the three constants bulk modulus, piston area and the initial volume have to be measured. Under certain circumstances, explained
in section 3.2, the three parameters can be regarded as constant with respect to the frequency. Hence they can be concentrated in a constant C1 and evaluated in a static measurement.

\[
\Delta p = C_1 \Delta x \\
C_1 = \frac{\Delta p}{\Delta x}
\]  

To measure the displacement of the piston a laser interferometer is used. The pressure is measured by a statically calibrated reference transducer. With the measurement of the constant C1 all physical dependencies of the basic measurement model are defined.

The basic measurement model can be regarded as a primary calibration method according to VIM [1]. Further all quantities which are measured are traceable to national standards.

3.2 Thermophysical influences

3.2.1 Bulk modulus. As mentioned in section 3.1 the parameters bulk modulus, piston area and volume are regarded as constant with respect to frequency. Due to the design of the system, no major influence of modal oscillations of the piston and the fluid filled chamber respective the housing are expected. Then it is obvious, that the piston area and the initial volume can be regarded as constants. But when it comes to the bulk modulus an isothermal and adiabatic bulk modulus have to be distinguished. The isothermal bulk modulus can be regarded as the static one, which is shown in formula 5. However the adiabatic bulk modulus is the dynamic one and formula one has to be expanded by the temperature term.

\[
C_H \frac{dp}{dt} = -\frac{dV}{dt} + V \gamma \frac{dT}{dt} \\
dp = \frac{A dx}{V \left( \gamma \frac{dT}{dp} - \frac{1}{K} \right)}
\]  

For an adiabatic compression the produced heat cannot be absorbed by the housing, but remains in the fluid. The pressure is then increased by the produced heat. Modelling the frequency dependency of this two different states and their transition is complex due to the difficult description of the thermal conductivities and resistances. For this reason the author decided to minimize the ratio between the isothermal and adiabatic compression and use a constant bulk modulus over frequency.

3.2.2 Numerical evaluation and comparison. The pressure and temperature change for the adiabatic process are calculated by:

\[
dp = \frac{A dx}{V \left( \gamma \frac{dT}{dp} - \frac{1}{K} \right)}
\]

\[
dT = \frac{\gamma T dp}{\rho cp}
\]

and for the isothermal compression by

\[
\Delta p = -K \frac{A \Delta x}{V}
\]

The calculation is done for the standard hydraulic oil HLP46 and for water. The data for the HLP46 oil is taken from approximations like the IFAS Modell for the bulk modulus [3], [4]. The properties of water are taken from the IAPWS-IF97 formulation [5]. The initial conditions are set to a temperature of
$T = 20\, ^\circ C$ and a static pre-pressure of $p_{\text{static}} = 3\, \text{MPa}$. In a second step the initial temperature is varied to $T = 4\, ^\circ C$. Then for the heat expansion coefficient equation 13 applies

$$\lim_{T \to 4\, ^\circ C} \gamma (p_{\text{static}} = 3\, \text{MPa}) \approx 0$$  \hspace{1cm} (13)

As a consequence the temperature to pressure gradient (equation 11) minimizes. Figure 2 shows the results for the simulation.

![Figure 2. Pressure to piston displacement and comparison of adiabatic to isothermal compression for different working fluids and initial conditions](image)

Figure 2 shows that water is the much better working fluid. The deviation between the adiabatic and the isothermal compression is less than 0.7%. This ratio is much higher for the hydraulic oil HLP46 with 12.8%. If the initial conditions are varied to a temperature of $T = 4\, ^\circ C$ even the systematic deviation of 0.7 % can be reduced to almost 0%. As a consequence the validity of the assumptions of the basic measurement model is shown.

4. Conclusions

In the paper a calibration device and a primary, dynamical calibration method for pressure transducers are presented. All measurands used for this calibration are traceable to national standards. The new developed periodic calibration method can also be regarded as a primary method since the dynamic pressure is traced back to the oscillation of a piston.

The theoretical investigation of the basic measurement model is very promising. Further some influences on the measurement uncertainty were already analyzed and the thermophysical influences are presented and quantified in this paper. This promising approach is also confirmed by the first measurement results. Future work has to be done to study some uncertainty components in more detail and to verify the frequency dependency of the bulk modulus. Secondly comparison measurements have to be performed with established dynamical calibration methods like the shock tube method.

References

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