Noise Kernel for Self-similar Tolman Bondi Metric: Fluctuations on Cauchy Horizon

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We attempt to calculate the point separated Noise Kernel for self similar Tolman Bondi metric, using a method similar to that developed by Eftekharzadeh et al. for ultra-static spacetimes referring to the work by Page. In case of formation of a naked singularity, the Noise Kernel thus obtained is found to be regular except on the Cauchy horizon, where it diverges. The behavior of the noise in case of the formation of a covered singularity is found to be regular. This result seemingly renders back reaction non-negligible which questions the stability of the results obtained from the semiclassical treatment of the self similar Tolman Bondi metric.

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I. INTRODUCTION

Cosmic censorship ever since its proposition in 1969 has been an issue studied mostly within the framework of classical relativity. This includes important conjectures like the hoop conjecture, studies on stability of scalar fields on spherically symmetric collapse metrics as well as particular examples of gravitational collapse metrics. Some studies have addressed gravitational collapse at the semiclassical level though most of the work concerns 1+1 dimensional metrics. In the semiclassical theory, the quantum expectation of the energy momentum tensor (also referred to as RSET or renormalized stress energy tensor) is central to studying quantum fields on spacetimes. One line of investigation in applying this to gravitational collapse is to question the features like trapped surfaces which arise earlier and the final fate of the collapse. Another line is to work out the quantum stress tensor when one employs as a background spacetime the classical relativistic examples of gravitational collapse and study its effects. In the later in particular, the metric representing pressureless fluid collapse in spherical symmetry, the Lemaître Tolman Bondi (LTB) metric is a common choice. We compare with this line of investigation in this work.

The classical Lemaître Tolman Bondi (LTB) metric exhibits both naked and covered singularities which can be easily related to data on the initial Cauchy surface. The transition from covered to naked singularities appears to occur on account of inhomogeneities in initial data. Further, at the semiclassical level it appears that this increasing inhomogeneity causes a drastic change in the behavior of the quantum stress tensor. In fact, it is found that the quantum stress tensor diverges in the naked singular case where the Cauchy horizon is expected to form whereas there is no divergence anywhere in the covered singularity case. This divergence is physically interpreted as an energy burst on the Cauchy horizon.

It is still not clear if such a behavior is generic as the results obtained are in fact exact in 1+1 dimensional cases and for a few cases of background metrics. Also the calculations involve a background metric which remains unaffected by the quantum stress tensor. In other words, the backreaction is not implicitly incorporated in the results. However, physical arguments are offered for the divergence which rely on high curvature regions of spacetime causing particle production. Such regions would not be exposed in the covered case as against the naked case and it is likely that the semiclassical divergence is actually a general feature.

Semiclassical gravity is however fraught with its share of interpretational issues apart from operational difficulties like backreaction. Since the idea by Leon Rosenfeld of using the quantum stress tensor on the right of the Einstein field equations, several issues have been raised. At least some of them involve serious objections to employing an averaged quantity for the stress tensor. Indeed one can imagine that in situations where fluctuations are important, the use of the semiclassical quantum averaged stress tensor would not be justified for any physical interpretation like the one above. So for a complete consideration it looks natural to address the fluctuations as well, whenever they can be argued to be significant.

Stochastic gravity seeks to remedy this situation, addressing the effect of including fluctuations of the energy momentum tensor and possibly its backreaction on the metric as well. This is accomplished using the Einstein Langevin Equation. The central object in the calculations of stochastic gravity is the noise Kernel which gives an idea of the stochastic source, that being in addition to the quantum stress tensor source. The Noise Kernel is the vacuum expectation value of symmetrized stress-energy bi-tensor for a quantum field in curved spacetime. It characterizes fluctuations of stress tensor which play as mentioned above, a significant role in analysis of quantum effects in curved spacetime. These fluctuations lead to fluctuations in the metric, themselves understood in two parts as induced or passive fluctuations. Induced fluctuations have played an important role in many studies involving backreaction. Backreaction problems in gravity and cosmology for example have been addressed using Einstein-Langevin equations.

This far the effect of quantum fluctuations has not been studied in the context of gravitational collapse, the physical scenario for studying cosmic censorship. This is firstly because of the operational difficulty in calculating the central quantity, the Noise Kernel which gives an idea of the stochastic source term. Secondly, the quantum stress tensor does not add significantly to the classical source for most of the spacetime regions. This would support any interpretation ignoring backreaction effects of the quantum stress especially regarding the effects on quantum fields like pair creation or Hawking radiation (at ‘late’ times) However, it is physically important to assure that the quantum averaged source is indeed a good approximation to the quantum field contribution to the source. Any effect of fluctuations, which appear as the stochastic source described above would have to be negligible. However, it has been a challenge in stochastic gravity to come up with a method to calculate the Noise Kernel for specific background metrics. There are various approaches that have been used for such calculations, in the case of Minkowski, de-Sitter, anti-deSitter and Schwarzschild Spacetimes in the coincidence limit. Recently, Eftekharzadeh et. al. have devised a method to compute Noise Kernel for conformally invariant quantum fields in Schwarzschild spacetime under the Gaussian approximation. This is based on method developed by Page and provides a possibly general way to estimate the Noise Kernel approximately.

We seek to apply stochastic gravity techniques in the study of cosmic censorship to the gravitational collapse
scenario, particularly to the LTB metric. This metric has been hitherto studied only at the level of quantum fields on spacetime. It has been demonstrated that stress energy tensor diverges at the Cauchy horizon, despite Cauchy horizon being perfectly regular \cite{5}. Such results do not take into account backreaction of the quantum fields on spacetime. The motivation of calculating Noise Kernel in LTB background is to test the validity of the above mentioned result in view of backreaction. Our aim in carrying out this work has been to investigate this in the context of gravitational collapse and suggest interpretations of the particular calculations for the Noise Kernel. In this paper, we employ a similar method to that of \cite{19} and compute the Noise Kernel on self similar Tolman Bondi metric background for conformally invariant fields under Gaussian approximation. We obtain the Noise Kernel in this case which diverges at the Cauchy horizon, seemingly making backreaction important for consideration and hence questioning any attempt to study the spacetime in a semi-classical or perturbative manner.

The paper is organized as follows. In section II we introduce the building blocks of stochastic gravity, the Noise Kernel and the required Wightman function. Section III describes the conformal rescaling of the Tolman Bondi metric and its conformally equivalent ultra-static form is obtained. We obtain the expansions of Synge function and Van Vleck Morette determinant in the coordinates of this ultra-static metric. In section IV we relate the Noise Kernel of this metric to that of self-similar Tolman Bondi. We argue here for a plausible divergence. In the end we summarize the results and have a discussion of interpretation of the divergence.

II. NOISE KERNEL IN ULTRA-STATIC SPACETIMES

The Noise Kernel is introduced briefly and related to the contribution of stress fluctuations to the source in Einstein equations. The nature of the assumed quantum state is described for this situation. Its relation to the Noise Kernel is mentioned and its expression is provided in case of an ultra-static spacetime.

A. Einstein Langevin Equation and Noise Kernel

The Noise Kernel for quantized matter field is given by

\[ N_{abc'd'}(x, x') = \frac{1}{2} \langle \{ \hat{t}_{ab}(x), \hat{t}_{c'd'}(x') \} \rangle \quad (1) \]

where

\[ \hat{t}_{ab}(x) = \hat{T}_{ab}(x) - \langle \hat{T}_{ab}(x) \rangle \]

and \( < \cdots > \) is the quantum expectation value taken with respect to a normalized state. \( \hat{T}_{ab} \) is the stress tensor operator of the field. The classical stress tensor of a conformally invariant scalar field \( \phi \) is given by

\[ T_{ab} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi + \frac{1}{6} (g_{ab} \Box - \nabla_a \nabla_b + 4 G_{ab}) \phi^2 \quad (2) \]

As fit for doing quantum field theory on curved space-time, the field \( \phi \) is raised to the level of an operator whereas \( g_{\mu \nu} \) is to be treated classically.

The Noise Kernel embodies the contributions of the higher correlation functions in the quantum field on account of which it may be used to interpret issues related to the quantum nature of space-time. Two point functions of energy momentum tensor would involve fourth order correlations of the quantum field, for example, which would affect the coherence of the geometry \cite{21}. If we were to employ the Einstein Langevin equations (often referred to as semiclassical Einstein Langevin equations)

\[ G_{\mu \nu} = < \hat{T}_{\mu \nu} > + \xi_{\mu \nu}, \quad (3) \]

where \( \xi_{\mu \nu} \) is a random variable (tensor) which is closely related to the Noise Kernel. The relations

\[ < \xi_{\mu \nu}(x) >_s = 0 \]

\[ < \xi_{\alpha \beta}(x) \xi_{\mu \nu'}(x') >_s = N_{\alpha \beta \mu \nu'}(x, x') \quad (4) \]

completely characterize the variable \( \xi_{\alpha \beta}(x) \). The statistical expectation \( < >_s \) is taken over various stochastic realizations of the Gaussian source \( \xi_{\mu \nu}(x) \). Each of the realizations would lead to a metric solution

\[ h_{\alpha \beta}(x) = h_{\alpha \beta}^{(0)}(x) + 8\pi \int d^4 y' \sqrt{-g(y')} G_{\alpha \beta}(x, y') \xi_{\gamma \delta'}(y') \quad (5) \]
where $h^{(0)}_{\alpha\beta}$ is the solution to the semiclassical equation

$$G_{\mu\nu} = \langle \hat{T}_{\mu\nu} \rangle$$

and $G^{(ret)}_{\alpha\beta\gamma' \delta'}$ serves as the retarded propagator of the semiclassical Einstein Langevin equations with vanishing initial conditions. The stochastic realization would contain information about coherence of geometry, $\xi^{\mu\nu}(x)$ being treated as a random variable.

B. The quantum state for the Noise Kernel: Wightman function

Before interpreting physically any particular noise calculation, the nature of the quantum state of the field needs to be taken into account. The state we consider in our calculations turns out to be Hadamard in addition to our assumptions of it being quasi-free and thermal. We discuss these three qualifiers of the state below.

Firstly, the Hadamard nature of the state guarantees that the stress tensor is well defined within the maximal Cauchy development and obeys the Wald axioms \[2\]. We will then analytically continue the state across the Cauchy horizon. This consideration can be straightforwardly generalized to Schwarzschild spacetime as considered in \[19\]. Operationally, we are not interested in the stress tensor and would not require any coincidence limits, even for the Noise Kernel for addressing issues of fluctuations. However, the fluctuations obtained would be specific to the quantum state and a physically reasonable choice of state is warranted. Hence the Hadamard nature of the state is accorded physical significance in the discussion of our results.

In our calculations below, we have emphasized the singularity structure of the symmetrized form of the two point function we use. That being represented by a Hadamard expansion, we conclude that the quantum state being used is Hadamard. The expressions we use are for a general quasi-static spacetime and based on the approximation carried out by Page and so the conclusion would be more general. However, a careful interpretation of the divergent expression in $1/\sigma$ with $\sigma^2$ being the Synge function, is required to support the claim that the state is Hadamard. Such an interpretation in the sense of distributions is provided by Kay and Wald \[22\]. A rigorous definition is provided by Radzikowski \[23\] based on micro-local analysis but we do not attempt to relate to that in this paper. Secondly, we operationally assume that the Wightman two point function defined as

$$G(x, x') = \langle \phi(x)\phi(x') \rangle$$

determines the quantum state of the field. Indeed, that is true if one is working with quasi-free (or Gaussian) states. Quasi-free states have been extensively used \[2\] in curved spacetime. They conveniently lead to Fock space representations of one particle Hilbert spaces via the GNS construction on the Weyl algebra of observables. One advantage of this is that can admit a concept of particles in a limited manner, despite having curved spacetime at hand. For example, large fluctuations implied by the Noise Kernel could be interpreted as highly fluctuating particle creation.

Thirdly, it is assumed that the state is thermal or K.M.S. with temperature $\kappa$. It turns out that the Wightman function can be approximated for thermal states in a particular manner called Gaussian approximation. We employ the related expression in the last subsection for ultra-static spacetimes.

Our results would thus be applicable to a quasi-free thermal Hadamard state of a conformally invariant field. The state specific information is not contained in the singularity structure. So changing to a different quasi-free Hadamard state would amount to adding an analytic function to the symmetrized two point function. We find that the Noise Kernel expression would amount to adding an analytic function to the symmetrized two point function. We find that the Noise Kernel expression would not change at the $\kappa^0$ level. The divergence on Cauchy horizon, being at all orders in $\kappa$, is expected to persist for any quasi-free Hadamard state.

C. Noise Kernel in terms of the Wightman function

The properties and the expressions of a Noise Kernel, given a quasi-free (or Gaussian) state of quantum matter field, are obtained in \[24\]. Further, for conformally invariant fields the Noise Kernel will be given are given in terms of the Wightman function as \[19\]

$$N_{abc'd'} = \text{Re} \left\{ \hat{K}_{abc'd'} + g_{ab} \hat{K}_{c'd'} + g_{c'd'} \hat{K}_{ab} + g_{ab} g_{c'd'} \hat{K} \right\}$$ \[6\]
where

\begin{equation}
9K_{ab'd'} = 4 \left( G_{bc} G_{d'a} + G_{cb} G_{d'a} \right) + G_{bc} G_{a'd'} + G_{cb} G_{a'd'} - 2 \left( G_{bd} G_{ac'd'} + G_{cd} G_{ab'd'} + G_{bd} G_{ac'd'} + G_{cd} G_{ab'd'} \right) + 2 \left( G_{b'd'} G_{c'a} R_{ab} R_{cd} + G_{c'd'} G_{b'a} R_{ab} R_{cd} \right) - (G_{ab} R_{c'd'} + G_{c'd'} R_{ab}) G + \frac{1}{2} R_{c'd'} R_{ab} \right. \tag{7a}
\end{equation}

\begin{equation}
36K'_{ab} = 8 \left( -G_{dp} G_{q'a} + G_{dp} G_{qa'} + G_{dp} G_{qa'} \right) + 4 \left( G_{dp} G_{ai} - G_{dp} G_{ia} \right) R_{ab} - R' R_{ab} G^2 \tag{7b}
\end{equation}

\begin{equation}
36K = 2 G_{pq'} G_{p'q} + 4 \left( G_{p'q} G_{pq} + G_{p'q} G_{pq} \right) - 4 \left( G_{p'} G_{pq} + G_{p} G_{pq} \right) + 2 \left( R G_{p'} G_{pq} + R' G_{pq} G_{pq} \right) G + \frac{1}{2} R R' G^2 \tag{7c}
\end{equation}

are given in terms of the Wightman function. Primes on indices represent the point \( x' \) while the entities at \( x \) are the unprimed ones. \( R_{ab} \) is the Ricci tensor and \( R \) is the Riemann curvature.

D. Wightman function in ultra-static spacetime

Next, we give a short review of the method used in [19] for ultra-static metric. The metric in a static spacetime takes the form

\begin{equation}
\frac{d}{d\tau} (\vec{x}) d\tau^2 + g_{ij}(\vec{x}) dx^i dx^j \tag{8}
\end{equation}

This can be transformed into an ultra-static form, called the optical metric, by a conformal transformation. This optical metric takes the following form

\begin{equation}
\frac{d}{dt} (\vec{x}) dt^2 + g_{ij}(\vec{x}) dx^i dx^j \tag{9}
\end{equation}

where the metric functions \( g_{ij} \) are independent of time \( t \). The Synge function (half of the square of the proper distance of the shortest geodesic between two points) can thus in ultra-static space-time take the form

\[ \sigma(x, x') = \frac{1}{2} ((t - t')^2 - r)^2 \]

where \( r^2 \) is twice the spatial part of the Synge function and it depends only on spatial co-ordinates as in [19].

The Wightman function in an optical background metric for a thermal (K.M.S.) state can be calculated under the Gaussian approximation [19]. This approximation was first carried out by Page using the Schwinger De-Witt expansion in the context of calculating quantum stress tensors. In this scheme

\[ \mathcal{G}(\delta t, \vec{x}, \vec{x}') = \frac{\kappa \sinh \kappa r}{8\pi^2 r[\cosh(\kappa r) - \cosh(\kappa \delta t)]} U(\delta t, \vec{x}, \vec{x}'), \tag{10} \]

where

\[ T = \frac{\kappa}{2\pi} \]

is the temperature of the thermal state considered[1] and

\[ \delta t = t - t' \]

[1] In Schwarzschild spacetime, the \( \kappa \) has been chosen to be the surface gravity on the event horizon null surface. However, one can work with a general thermal state without ascribing this significance to it.
while

\[ U(x, x') \equiv \Delta^{1/2}(x, x') \]  
\[ \Delta(x, x') \equiv \frac{1}{\sqrt{-g(x)} \sqrt{-g(x')}} \text{det}(\sigma_{ab}) \]

where \( \Delta(x, x') \) is known as the Van Vleck Morette determinant.

### III. CONFORMALLY OPTICAL FORM OF SELF SIMILAR TOLMAN BONDI METRIC

In order to evaluate the Noise Kernel for Schwarzschild spacetime Eftekharzadeh et al. [19] have used an approach of conformally rescaling the metric to the form of an ultra-static (optical) metric. First the Noise Kernel is evaluated in this ultra-static spacetime. Then the Noise Kernel of Schwarzschild spacetime is obtained by just rescaling back the result in the optical case. We will be applying the similar technique to self-similar Tolman Bondi (TB) metric, which is given by the line element

\[ ds^2 = dt^2 - R'^2 dr^2 - R^2 d\theta^2 - R^2 \sin^2 \theta d\phi^2 \]

where \( R(t, r) \) is the area radius

\[ R^{3/2}(t, r) = r^{3/2}(1 - \frac{3}{2} t \sqrt{\lambda}) \]

is characterized by a dimensionless parameter \( \lambda \). The collapse rate can also be given as in terms of this parameter

\[ \dot{R} = -r \sqrt{\frac{\lambda}{R^2}} \]

The initial data for this metric is regular and a curvature singularity gets eventually formed at \( r = 0 \). The singularity is naked if \( \lambda < 1/8 \) and is covered otherwise.

For conformal transformation is it useful to work in a new coordinate system \( (t, z, \theta, \phi) \) where \( z = t/r \). Then

\[ R^{3/2}(z) = r^{3/2}(1 - \frac{3}{2} z \sqrt{\lambda}) \]
\[ R'(z) = \frac{1 - \frac{3}{2} z \sqrt{\lambda} \sqrt{\lambda}}{(1 - \frac{3}{2} z \sqrt{\lambda})^{1/3}} \]

The self similar TB metric can now be put in the conformally ultrastatic form as

\[ ds^2 = \Omega^2[dT^2 - f_1(z)^2 dz^2 - z^2 f_2(z)^2 d\Omega^2], \]

where

\[ \Omega^2 = \left(1 - \frac{R'^2}{z^2}ight) t^2 \]
\[ f_1(z)^2 = \left(\frac{R'}{z^2 - R'^2}\right)^2 \]
\[ f_2(z)^2 = \frac{(1 - \frac{3}{2} z \sqrt{\lambda})^{4/3}}{z^2(z^2 - R'^2)} \]

and

\[ dT = \frac{dt}{t} + \frac{R'^2}{z(z^2 - R'^2)} dz. \]
Since the metric \((19)\) is in conformally related to an ultra-static metric, the Synge function can readily be obtained in the following form as is done in \([19]\) for Schwarzschild metric in optical form. We apply the same procedure for TB metric in the optical form. The Synge function expansion used here becomes

\[
\sigma = \sum_{ijk} s_{ijk}(z) \delta T^{2i} \eta^j \delta z^k
\]

where

\[
\delta T = (T - T'), \\
\delta z = (z - z'), \\
\eta + 1 = \cos(\theta) \cos(\theta') + \sin(\theta) \sin(\theta') \cos(\phi - \phi').
\]

The expression for Synge function is obtained as

\[
\sigma(x, x') = \frac{1}{2} \left( \delta T^2 - \delta z^2 f_1(z)^2 + 2\eta z^2 f_2(z)^2 \right) - 2\delta z \eta (z f_2(z)^2 + z^2 f_2(z) f_1'(z)) + \delta z^3 f_1(z) f_1''(z) + O((x - x')^4).
\]

The function \(U(x, x')\) present in the Wightmann function \((10)\) can be expanded in powers of \((x - x')\) using the Synge function \([11], [12]\) as follows:

\[
U(x, x') = 1 + \delta z^2 \left[ \frac{f_1'(z)}{6z f_1(z)} - \frac{f_2'(z)}{3z f_2(z)} + \frac{f_1(z) f_2'(z)}{6 f_1(z) f_2(z)} \right] - \eta \left[ \frac{1}{6} - \frac{f_2'(z)^2}{6 f_2(z)^2} - \frac{z f_2(z)^2 f_1'(z)}{6 f_1(z)^2} - \frac{2 z f_2(z) f_1'(z)}{3 f_1(z)^2} \right]
\]

\[
- \frac{z^2 f_2(z) f_1'(z) f_2'(z)}{6 f_1(z)^3} - \frac{z^2 f_2(z)^2 f_1'(z)}{6 f_1(z)^2} - \frac{z^2 f_2(z) f_1''(z)}{6 f_1(z)^2}
\]

\[
+ O((x - x')^3)
\]

With \(U\) and the Synge function obtained for the optical self-similar TB spacetime we can now evaluate the Wightman function which will be used to obtain the Noise Kernel.

### IV. NOISE KERNEL EXPRESSION FOR (SELF SIMILAR) TOLMAN BONDI SPACETIME

The Wightmann function for a thermal state with temperature \(\kappa\) in the Gaussian approximation on ultra-static spacetime is given by \([19]\).

\[
\mathcal{G}(x, x') = \frac{1}{8\pi} \frac{1}{\sigma} + \frac{\kappa^2}{6} - \frac{\kappa^4}{180} (2\delta T^2 + \sigma) + O((x - x')^4) U(x, x')
\]

The Noise Kernel, when points are separated can now be computed using the above Green’s function, in usual way as given in \([19]\). Here we will present one component of the Noise Kernel for demonstrating some important results for our metric. The expression for Noise Kernel(component given by \(N_{TTTT}\)) for points separated in \(T\), where \(\delta z = \eta = 0\) and \(\delta T \neq 0\). The noise Kernel \(N_{TTTT}\) can be expanded in terms of various powers of \(\kappa\). These coefficients in such an expansion are presented in the tabular form below:

\[\text{\footnote{At this point we note that the } U \text{ above is analytic in its domain and the prefactor contains the singularity structure. The same singularity structure would present in the symmetrized two point function. We have a Hadamard singularity structure above, in fact without any } \log \sigma \text{ term.}}\]
\[ \frac{z^4 f^2}{N^{2/3} (t - \delta t)} \left\{ \frac{13}{12} \pi^3 + \left[ -4 f^4_1 + 3z^2 f^2_1 f^2_1 + 2zf_1 f_2 \{-2zf_1 f_2 + f_2 \right\} \\
\left( -2f_1 + z f_1'^1 \right) + 4 f^4_1 \{2 f^2_2 + 2z^2 f^2_1 + z f_2 (6 f_2' + z f_2'^1)\}/[728 T^6 \pi^3 z^2 f^2_1 f^2_2] \\
+ [27 f^4_1 + 50z f^4_1 f_1' (f_1 + z f_2') + 360z^3 f^2_1 f^2_1 (f_2 + z f_2')] \\
- 10 f^4_1 [3 f^2_2 + 3z^2 f^2_1 + z f_2 (16 f_2' + 5z f_2'^1)] - 5z^2 f_1 f_2' f_1 (z f_1'^1 f_2^2 + \\
f^2_2 f_1' + 48 f_1 f_1'^1 + 2zf_2 [24z f^2_1 f_1' + f (73 f_2 + 36z f_2')] + \\
2zf_1 f_2 [-49z^3 f_1 f_1'^3 + z^2 f_2 f_2' [12 z f_1'' (f_2 - f_3 (209 f_2') + \\
31z f_1'' + f_2 (5f_1' + z f_1'' + z f_1'') + z f_2^{(3)}] + f_2 [(-209 f_2' + z (185 f_2' + 72z f_2^{(3)}))]/[259208 T^4 \pi^3 z^4 f^2_1 f^2_2] \\
3z^4 f_1'' + 2z^3 f_2 f_2' [104 f_4' + 49z f_2'' + z^2 f_2' [678 f_2' + 67z^2 f_2'' - 8z f_2' (-49 f_2' + \\
3z f_2^{(3)}]] + 2zf_2 [104 f_4' + z (13 f_2'' - 12z (5 f_2^{(3)} + z f_2^{(4)})/ [259206 T^4 \pi^3 z^4 f^2_1 f^2_2] \\
+ 2f_1 [5 f_2' + 5z^2 f_2'' - 2zf_2 (3 f_2' + 4z f_2'^1)]/ [259206 T^4 \pi^3 z^4 f^2_1 f^2_2] \\
+ O(1) \] \\
\] \\
\]

The above expansion for the Noise Kernel has been obtained after conformal transformation of the optical form back to the original form \[ \text{13} \]. We have displayed the expression of \[ N_{TT T'} \] for points separated in \[ T \] co-ordinate only. This turns out to be interesting as it yields a divergence at the Cauchy horizon with leading orders in separation. This takes place despite the Cauchy horizon itself being regular in that there is no curvature singularity along it.

### A. Structure for naked singularity and comparison with covered case

In case of the naked singular solution, the Cauchy Horizon is given by the smaller root \( z_- \) of \( (z^2 - R' (z)^2) \) for self similar TB metric \[ \text{3} \]. This leads to divergence of the factors \( f_1 (z) \) and \( f_2 (z) \) if evaluated on the Cauchy Horizon. We could choose \( T \) for the point on the Cauchy Horizon and \( T' \) for the one elsewhere. The behavior thus obtained would be clearly of the point separated Kernel.

The terms for various orders of \( \kappa \) displayed above diverge on the account of the appearance of \( f_1 (z) \) in the factor (the appearance of \( f_1 (z) \sim z^- \) and \( f_2 (z) \sim \sqrt{z} \) occurring in terms in the brackets essentially does not contribute any singular structure) leading to our main result, the divergence of the Noise Kernel on the Cauchy Horizon. It should be noted that the above divergence has been obtained using metric \[ \text{19} \]. We have been constrained by our approach to use such a metric in coordinates ill behaved at the Cauchy horizon \[ \text{17} \]. Suitable co-ordinate transformations can be used to remedy this. From the metric one can observe that the coordinate transformation factors acting on \( N_{TT T'} \) above would need to diverge at the Cauchy Horizon as \( z_- \) similar to \[ \text{3} \]. The divergence of the expression of the Noise Kernel above is rather enhanced if the metric is demanded to be regular at the Cauchy horizon.

In the case of covered singularity \( (z^2 - R' (z)^2) \) has no real roots. So the factors of \( f_1 (z) \) and \( f_2 (z) \) do not yield divergences. Nor do the rest of the factors. The metric used is also regular everywhere.

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3. The metric in the earlier \( (t, r, \theta, \phi) \) co-ordinate system is perfectly regular on Cauchy horizon and we have conformally rescaling back our results. Our results would thus hold for any other metric related to it by regular coordinate system transformations.
B. Singularity structure

The issue of regularity also occurs in the noise expressions of \[19\] as the Schwarzschild metric used is expressed in the usual coordinates rather than say Kruskal coordinates at the event horizon. For interpreting the expressions on the event horizon, further coordinate transformations would be necessary on the Noise Kernel.

The above analysis is restricted to the Noise Kernel with only \(\delta T \neq 0\). We obtain a similar divergence on Cauchy Horizon (in the naked singular case) for \(\delta z \neq 0\) separation as well. The corresponding noise expressions however are not short enough for explicit display in this communication.

Our results above for the Noise Kernel would be applicable to a particular quasi-free thermal Hadamard state of a conformally invariant field on self similar LTB spacetime. It would be of interest to examine if this could be applicable to any quasi-free thermal Hadamard state.

The state specific information is not contained in the singularity structure. So changing to a different quasi-free Hadamard state would amount to adding an analytic function to the symmetrized two point function. If the above expressions of the Noise Kernel are examined, we find that the Noise Kernel expression would not change at the \(\kappa^0\) level. The divergence on Cauchy horizon (in the naked singular case) being at all orders in \(\kappa\), is expected to persist for any quasi-free Hadamard state.

V. RESULTS AND DISCUSSION

We have thus obtained the Noise Kernel for a conformally coupled scalar field in the self similar Tolman Bondi metric in an approximate form for a quasi-free thermal Hadamard state. We seek to interpret this result below.

We know that the Noise Kernel affects the induced fluctuations of the metric. In equation (5), however the \(G_{\text{ret}}\) function required would need a background metric for calculating it. Unfortunately, the largely fluctuating stochastic source as implied by the Noise Kernel for naked singular metric does not allow us to treat the backreaction as negligible. So, our results cannot easily be extended for studying the induced fluctuations on Cauchy Horizon.

At the same time, we note that Noise Kernel is not very large till near the Cauchy Horizon (in the naked singularity case) and so the background appears to be a good approximation despite backreaction.

Despite the above difficulty regarding the metric fluctuations, the source fluctuations can still be directly used for the purpose of interpreting. The semiclassical interpretation of the stress tensor divergence on Cauchy Horizon suggested that an energy burst on Cauchy horizon is plausible. Such an interpretation would need to be reconsidered in view of our analysis. The highly fluctuating source near the Cauchy horizon would render it difficult to interpret any of its stochastic realizations as physically significant, in particular the semiclassical one of an energy burst on Cauchy Horizon.

The covered singularity case is in contrast with the above results. This is similar to the difference seen in the semiclassical behavior of the metric. In the latter analysis, the quantum stress tensor diverges on the Cauchy horizon in the naked singular case against regular behavior in the covered case. The contrast is borne out by analysis of fluctuations as well.

The divergence of quantum stress tensor in the semiclassical analysis has been attributed to the fact that high curvature regions are exposed in the naked singular case leading to high energy effects. It would seem that high curvatures lead to diverging noise as well. Thus the contrast in the behavior of local quantum fields seems to be tied to the exposure of high curvature regions, or the lack of it. We suggest that this plausible connection could be investigated further.

Finally, the stochastic source fluctuating highly near the Cauchy Horizon, could be interpreted for its effect as an environment (quantum fields) on the system (background metric) \[37\]. The spacetime could be highly sensitive to such environmental decoherence effects very near the Cauchy Horizon. In particular, as we have a quasi-free state, one could interpret this for example as spacetime reaction effects of particle creation. We suggest that this issue could be pursued separately.

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[1] Birrell N. D. and Davies P. C. W. *Quantum fields in curved spacetime* (Cambridge University Press, 1984).
[2] Robert Wald *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics* (University of Chicago Press, 1994).
[3] Carlos Barcelo, Stefano Liberati, Sebastiano Sonego, Matt Visser, Phys.Rev.D 77:044032 (2008).
[4] James Mattingly *Singularities and Scalar Fields. Matter Theory and General Relativity*. Proceedings of the Seventh Biennial Meeting of the Philosophy of Science Association, 2000, Vancouver.
[5] Sukruatu Barve, T.P.Singh Cenalo Vaz Phys.Rev D 62, 084021 (2000).
   See also
   Sukruatu Barve, T.P.Singh, Cenalo Vaz and Louis Witten Phys.Rev D 58, 104018 (1998).
[6] James Mattingly *Is quantum gravity necessary?* Proceedings of the 5th International Conference on the History and Foundations of General Relativity, 1999.
[7] Bei Lok Hu and Enric Verdaguer, Living Rev. Relativity 11, 3 (2008).
[8] E.Verdaguer XXIXth Spanish Meeting (ERE 2006), Journal of Physics: Conference Series 66, 012006 (2007).
[9] R. Martin and E. Verdaguer , Phys. Rev D 60, 084008 (1999).
[10] N.G Phillips and B.L.Hu, Phys Rev D 63, 104001 (2001).
[11] Sukanya Sinha, Alpan Raval and B.L.Hu Foundations of Physics, Vol. 33, No. 1 January 2003.
[12] Klaus Fredenhagen and Rudolf Haag Comm. Math. Phys. 127, No.2, 273 (1990).
[13] G.Perez-Nadal, A.Roura and E. Verdaguer, JCAP 05, 036 (2010).
[14] C.H.Fleming, A.Roura and B.L.Hu, arXiv:1004.1603.
[15] N.G. Phillips and B.L.Hu, Phys. Rev D 67, 104002 (2003).
[16] A.Roura and E. Verdaguer, Int. J. Theor. Phys. 38, 3123 (1999).
[17] G.Perez-Nadal, A.Roura and E. Verdaguer, Journal of Cosmology and Astroparticle Physics. Volume 2010 (2009).
[18] H.T.Cho and B.L.Hu Journal of Physics: Conference series, Volume 330, conference 1.(2011).
[19] A.Eftekhazadeh, Jason D.Bates, Albert Roura, Paul R. Anderson and B.L.Hu Phys. Rev. D 85,044037 (2012).
[20] D.N.Page, Phys. Rev D, 25, 1499 (1982).
[21] B. L. Hu, Int.J.Theor.Phys. 41 2091-2119 (2002)
[22] Bernard S. Kay, Robert M. Wald, Phys. Rep. 19749-136 (1991).
[23] Marek J. Radzikowski, Comm. Math. Phys. 179529-553 (1996).
[24] N.G.Phillips and B.L.Hu Phys. Rev. D 63, 104001 (2001).