Intelligent Control of Performance Constrained Switched Nonlinear Systems With Random Noises and Its Application: An Event-Driven Approach

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Abstract—In this paper, the adaptive fuzzy control of switched stochastic nonlinear systems with set-time prescribed performance based on event-driven mechanism is studied. The creative part of this paper is that based on the set-time performance function, a modified event-triggered strategy that considers asynchronous switching to deteriorate system performance without strict assumptions is presented, which avoids Zeno behavior and saves communication resources. Then, by using backstepping recursive design technique, Itô’s differential lemma and mode-dependent average dwell time (MDADT) method, a novel adaptive performance control scheme is proposed, which can ensure that all the variables in the system are semiglobally uniformly ultimately bounded (SGUUB) in probability and the tracking error gets into prescribed boundary no later than an arbitrarily adjusted setting time. Finally, the proposed algorithm is applied to a RLC circuit and its practicability is verified via simulation results.

Index Terms—Fuzzy logic system, event-triggered scheme, multiple Lyapunov function techniques, switched nonlinear system.

I. INTRODUCTION

IN RECENT decades, the electrical circuits and its control methods have been researched deeply in reports [1]–[4]. It is worth noting that the existence of nonlinear dynamics in electrical circuit systems can not be ignored, so the controller design of nonlinear systems has aroused great interest of scholars, and massive excellent achievements have emerged based on neural network or fuzzy approximation approach [5]–[9]. However, the above control schemes are mainly applied for nonstochastic nonlinear systems. An enormous number practical engineering systems are subject to stochastic uncertainty, such as biological system, financial system and chemical reaction process and so on. For nonlinear multi-input and multi-output (MIMO) stochastic systems, an adaptive tracking control scheme based on a new stochastic finite-time stability theorem was proposed in [10]. Then, Liu et al. [11] studied the control design of nonlinear stochastic systems with state constraints for the first time by constructing two different forms of barrier Lyapunov functions. For discrete-time stochastic nonlinear systems, an adaptive neural control scheme that mitigates the communication burden and improves the tracking accuracy was developed in [12]. Furthermore, for stochastic systems with unmeasurable states, some effective state observers were elegantly designed in [13]–[15] to estimate the unmeasured states.

Unexceptionally, the above researches are both interesting and challenging, but their conclusions are only valid for nonswitched systems. Due to the fact that most systems are difficult to be described by one model in practice. Multi-model switching control have capacious developed foreground in practical systems. For the stability analysis and controller design of switched systems, massive outstanding achievements have been popping up (see [16]–[24]). Especially, the MDADT of milestone was proposed in [24] to analyze the stability of switched systems, which aroused the attention of many scholars. Since then, such method is extended to many kinds of switched systems to relax the restrictions of switching signals and realize the stability of system. To just name a few, Yang et al. [25] developed a transition probability-based MDADT switching mechanism for dynamic systems with mixed delays by designing a multiple Lyapunov-Krasovskii functional. In [26], the exponential stability was studied for discrete-time switched positive systems under the framework of MDADT. It was first reported in [27] that the adaptive control scheme for switched nonlinear lower triangular systems under MDADT switching. Nevertheless, up to now, the investigation of the adaptive control for switched nonlinear systems with random noises under MDADT switching is seldom.

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Furthermore, the transient performance of controlled systems is not considered in the above articles. The prescribed performance control (PPC) method was first proposed in [28] and quickly applied to various nonlinear systems, such as large-scale nonlinear systems [29], MIMO nonlinear systems [30], and stochastic nonlinear systems [31], etc. Subsequently, for the convergence time of closed-loop systems, their finite-time adaptive neural networks and fuzzy PPC methods were studied in [32], [33], which effectively solved the problems of slow convergence and low accuracy of traditional adaptive PPC method. Although the above PPC schemes have satisfactory control effects, they have a common disadvantage, that is, the initial value of the performance function depends on the reference signal and the system output. But many industrial systems do not have constraints at the initial time, after the system runs for a certain time, there will be constraints on the system performance, that is, in \([0, T]\), there are no constraints on the system; and after \(t > T\), the system has constraints. Therefore, how to design an effective adaptive PPC scheme to deal with this more complex constraint situation is worthy of further research.

On the other hand, event-triggered communication control (ETCC) has attracted widespread attention due to its important role in networked control system [34]–[38]. For the ETCC of the switched systems, an enormous challenge is that the asynchronous switching between the subsystem and the controller is proving elusive. Asynchronous switching is caused by the switch within two consecutive triggering instants. Most of the existing results evade this problem or make strict assumptions about the maximum asynchronous duration, resulting in a lot of restrictions on the applicability of the results in practice, e.g., [39]–[41]. Recently, some excellent reports [42], [43] have been published to solve asynchronous switching to ensure system performance. Unfortunately, the above schemes do not consider stochastic disturbances. In other words, these event-triggered controllers do not be directly applied to switched stochastic nonlinear systems.

In conclusion, we find that the event-triggered fuzzy control methods for switched stochastic nonlinear systems are numbered. Also, the existing methods do not ensure the transient performance of the controlled plant under asynchronous switching. In this paper, a fuzzy set-time PPC scheme is proposed for switched stochastic nonlinear systems. The innovations of this article can be embodied in the following points.

1) By introducing the set-time performance function into the controller design, the proposed adaptive fuzzy set-time PPC scheme not only ensures that the tracking error gets into the predefined constraint region no later than a settable time \(T\), but also eliminates the “initial condition” of the constrained variable \(e_1\) in the traditional PPC scheme.

2) A novel mode-dependent event-triggered mechanism (MDETM) is designed for switched nonlinear systems with random noises considering the impact of asynchronous switching on system performance. The proposed control scheme achieves the expected control effect while mitigating the communication burden.

3) By using the lower bound of the control gain functions of each subsystem, the individual Lyapunov function is constructed, and a novel event-triggered fuzzy performance controller is designed so that all the variables in the system are bounded.

II. PRELIMINARIES AND PROBLEM FORMULATIONS

A. Basic Knowledge

**Definition 1** [13]: Consider the stochastic system \(dx = f(x(t))dt + g(x(t))dw\). Define the differential operator \(\ell\) for \(C^2\) function \(V(x)\) as:

\[
\ell V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} \text{Tr} \left\{ \frac{\partial^2 V}{\partial x^2} g^T g \right\}
\]

where \(\text{Tr}(A)\) is the trace of \(A\).

**Lemma 1** [10]: Let \(f(Z)\) be a continuous function defined on a compact set \(\tilde{\Omega}\). Then for any \(\tau > 0\), there exists a fuzzy systems \(\psi^T S(Z)\) such that

\[
\sup_{Z \in \tilde{\Omega}} |f(Z) - \psi^T S(Z)| \leq \tau.
\]

**Lemma 2** [8]: For \(\forall \omega_1 > 0\) and \(\omega_2 \in R\), the following result hold

\[
0 \leq |\omega_2| - \omega_2 \tanh(\frac{\omega_2}{\omega_1}) \leq 0.2785\omega_1.
\]

B. Problem Statement

Consider switched Itô stochastic nonlinear systems

\[
\begin{align*}
\dot{x}_i &= (l_{i,\sigma(t)}(\tilde{x}_i)x_i + f_{i,\sigma(t)}(\tilde{x}_i))dt + g_{i,\sigma(t)}^T(\tilde{x}_i)d\omega_i, \quad i = 1, 2, \ldots, n - 1, \\
\dot{x}_n &= (l_{n,\sigma(t)}(\tilde{x}_n)u + f_{n,\sigma(t)}(\tilde{x}_n))dt + g_{n,\sigma(t)}^T(\tilde{x}_n)dw, \\
y &= x_1
\end{align*}
\]

where \(\tilde{x}_i = [x_1, \ldots, x_i]^T \in \mathbb{R}^i\) and \(y \in \mathbb{R}\) are the system states, output, respectively. \(\sigma(t) : [0, \infty) \to M = \{1, 2, \ldots, d\}\) is a switching signal. \(l_{i,p}(\tilde{x}_i)\) is known control gain function. \(f_{i,p}(\tilde{x}_i)\) and \(g_{i,p}(\tilde{x}_i)(1 \leq i \leq n, p \in M)\) are unknown smooth nonlinear functions satisfying local Lipschitz. \(w \in \mathbb{R}^d\) denotes standard Brownian motion.

**Remark 1**: The above-mentioned switched stochastic nonlinear system can be applied to the RLC circuit with stochastic perturbations in the capacitor and the inductor. For example,
a RLC circuit is shown in Fig. 1, where $L$ is the inductor, $R$ the resistor, $C_1, C_2$ the two mutual switching capacitors.

Define the tracking error as $e_1 = y - y_d$ with $y_d$ being the reference signal. In this paper, the tracking error need to satisfy

$$-\zeta_1(t) < e_1 < \zeta_1(t), \quad t \geq T > 0$$

where $T$ is a time parameter, $\zeta_1(t)$ is called the set-time performance function and is defined as

$$\zeta_1(t) = (\zeta_0 - \zeta_\infty)e^{-\kappa_1(t-T)} + \zeta_\infty$$

where $\zeta_0 > \zeta_\infty > 0$, $\kappa_1 \geq 0$ are the design parameters.

Remark 2: Whether it is the traditional PPC schemes proposed in [29]–[31] or the finite-time PPC schemes proposed in [32], [33], the performance function requires "initial condition", that is, $\zeta_1(0)$ satisfies $-\zeta_1(0) < e_1(0) < \zeta_1(0)$. Obviously, $\zeta_1(0)$ introduced in this paper is independent of the initial conditions of the system output and the desired signal.

Our control objectives are as follows:

1) All signals of the controlled systems are SGUUB in probability under MADT method;
2) The tracking error $e_1$ gets into a prescribed boundary no later than a settable time $T$;
3) The designed MDET is Zeno-free.

To this end, the following mapping is proposed:

$$\chi_1 = \tanh(e_1)$$

 meanwhile, the following indirect performance function $\zeta_2(t)$ is adopted

$$-\zeta_2(t) < \chi_1 < \zeta_2(t), \quad t \geq 0$$

where

$$\zeta_2(t) = Ne^{-\kappa_2t} \frac{s(t) - s_i}{s_0} + \tanh(\zeta_1),$$

$$s(t) = \begin{cases} (s_0 - \frac{t}{T})e^{1 - \frac{t}{T}}, & 0 \leq t < T, \\ s_i, & t \geq T \end{cases}$$

and $N \geq 1$, $\kappa_2 \geq 0$, $s_0 \geq 0$, $s_i \geq 0$ are the design parameters.

Remark 3: According to the expression of $\zeta_2$, it can be seen from $N \geq 1$, $t = 0$, we have $\zeta_2(0) \geq 1$. Then from $-1 < \chi_1(0) < 1$, it follows that $-\zeta_2(0) \leq \chi_1(0) \leq \zeta_2(0)$. And the proposed method removes the "initial condition" imposed on the tracking error $e_1$.

Specially, the following assumptions are imposed.

Assumption 1: (Slow Switching)

1) There exists a number $\tau^*_s > 0$ (called a dwell time) such that any two switches are separated by at least $\tau^*_s > 0$;
2) There exist numbers $\tau_{ap} > \tau^*_s$ (called a mode-dependent average dwell time) and $N_{0p} \geq 1$ such that

$$N_{0p}(T, t) \leq N_{0p} + \frac{T_p(T, t)}{\tau_{ap}}, \quad \forall T \geq t \geq 0$$

where $N_{0p}(T, t)$ is the numbers of times the $p$th subsystem is activated on $[t, T]$, $T_p(T, t)$ is the total running time of the $p$th subsystem on $[t, T]$.

Assumption 2: There are two constants $b_{i,m}^0, b_{i,M}^0$ such that $0 < b_{i,m}^0 \leq |l_{i,p}(\tilde{x}_i)| \leq b_{i,M}^0$. Without losing generality, we assumes that $\text{sign}(l_{i,p}(\tilde{x}_i)) > 0$.

Assumption 3: The reference signal $y_d(t)$ and its derivatives $y^{(i)}_d(t)$, $i = 1, 2, \cdots, n$ are known and bounded.

III. ADAPTIVE FUZZY CONTROL DESIGN SCHEME

The development of backstepping design starts by defining the following coordinate transformations

$$\begin{cases} z_1 = \tan(\frac{\pi \chi_1}{2\zeta_2^2}), \\ z_i = x_i - a_{i-1}, \quad i = 2, 3, \cdots, n \end{cases}$$

where $a_{i-1}$ denotes the virtual control signal. To simplify the backstepping process, the virtual signal $a_i$ and the adaptive law $\hat{W}_i$ are chosen as

$$a_1 = -\frac{\rho^2}{\rho_1}c_1z_1 - \frac{\zeta_1^2}{\pi a_1^2}\hat{W}_1^Tz_1(1)S_1z_1(1),$$

$$a_i = -c_iz_i - \frac{\zeta_1^2}{\pi a_2^2}\hat{W}_1^Tz_1(1)S_i(1), \quad i = 2, 3, \cdots, n,$$

$$\dot{\hat{W}}_i = \frac{b_{i,m}^0}{\rho_2}S_i^T(1)S_i(1) - l_i\hat{W}_i, \quad i = 1, 2, \cdots, n$$

where $\rho_1 = 1 - \tanh^2(e_1)$, $\rho_2 = \frac{1}{\zeta_1^2}\cos^2(\frac{\pi \chi_1}{2\zeta_2^2})$, $a_i$, $c_i$, $l_i$ represent positive design parameters. $Z_i = [x_1, \tilde{z}_2, \tilde{z}_3, \tilde{y}_d]^T$, $Z_1 = [\tilde{x}_1, \tilde{W}_2, \cdots, \tilde{W}_{i-1}, \tilde{z}_2, \tilde{z}_3, \cdots, \tilde{z}_i, \tilde{y}_d, \tilde{y}_d(1)]^T(i \geq 2)$. Define $W_i = \max_{p \in M} \{|y_{i,p}(\tilde{x}_i)|\}$ with $b_{i,min} = \min_{p \in M}|b_{i,m}^0|$. $\hat{W}_i = W_i - \hat{W}_i$, $\hat{W}_i$ is the estimation of $W_i$.

**Step 1:** From (3) and (9), it follows that

$$d\dot{z}_1 = \frac{\pi}{2\rho_2}(\rho_1 l_1, p(\tilde{x}_1))(z_2 + a_1) + \rho_1 (f_1, p(\tilde{x}_1) - \tilde{y}_d)$$

$$- \frac{\chi_1^2}{\zeta_2}\hat{W}_1^Tz_1d\dot{t} + \frac{\rho_1}{2\rho_2}g_1, p(\tilde{x}_1)d\dot{w}.$$ (13)

The Lyapunov function candidate for the $p$th switching subsystem is defined as

$$V_{1,p} = \frac{1}{4\zeta_2^4} + \frac{b_{i,m}^0}{2l_1}\hat{W}_1^2.$$ (14)

Thus, $\ell V_{1,p}$ is given as

$$\ell V_{1,p} = \frac{\pi \chi_1^3}{2\rho_2}(\rho_1 l_1, p(\tilde{x}_1))(z_2 + a_1) + \rho_1 (f_1, p(\tilde{x}_1) - \tilde{y}_d)$$

$$- \frac{\chi_1^2}{\zeta_2}\hat{W}_1^Tz_1 + \frac{3\pi^2\rho_1^2}{8\rho_2^2}\tilde{z}_1^2\tilde{g}_{1,p}(\tilde{x}_1)\tilde{g}_1, p(\tilde{x}_1).$$ (15)

By using Young’s inequality, one has

$$\frac{\pi \rho_1^2}{2\rho_2^2}\tilde{g}_{1, l_1, p}(\tilde{x}_1)\tilde{z}_2 \leq \frac{3}{4}\pi^4\frac{1}{4}\tilde{y}_d(1)$$

$$+ \frac{1}{4}\tilde{g}_1, p(\tilde{x}_1)\tilde{z}_2^4.$$ (16)
\[
\frac{3\pi^2 \rho_2^2}{8\rho_2^2} \varepsilon_1^2 g_1^T(\xi_1)g_1(\xi_1) \leq 3\pi^4 \rho_2^4 \frac{1}{16\rho_2^2} \varepsilon_1^2 \|g_1(\xi_1)\|^2 + \frac{3}{16} \varepsilon_1^2.
\]

(17)

Therefore, (15) can be rewritten as

\[
\ell V_{1,p} \leq \frac{\pi \rho_1}{2\rho_2^2} \varepsilon_1^3 f_{1,p}(\xi_1) + \frac{1}{4} l_1 \varepsilon_1^2 + \varepsilon_1^3 \tilde{f}_{1,p}(Z_1)
- \frac{3}{4} \varepsilon_1^4 - \frac{b_{1,m}^i}{l_1} \tilde{W}_i \tilde{W}_i + \frac{3}{16} \varepsilon_1^2.
\]

(18)

where \( \tilde{f}_{1,p}(Z_1) = \rho_1 f_{1,p}(\xi_1) + \frac{3\pi^3 \rho_2^3}{16\rho_2^2} \varepsilon_1^2 \|g_1(\xi_1)\|^4 - \rho_1 \tilde{\nu}_d + \frac{3}{4} l_1 \varepsilon_1^2 \|g_1(\xi_1)\|^4 - \frac{3}{4} l_1 \tilde{\nu}_d. \)

By using fuzzy logic system \( \gamma_{i1}(Z_1) \) approximate \( \tilde{f}_{1,p}(Z_1) \), we have

\[
\tilde{f}_{1,p}(Z_1) = \psi_{i1,p} \gamma_{i1}(Z_1) + \delta_{i1}^p(Z_1)
\]

(19)

where \( |\delta_{i1}^p(Z_1)| \leq \tau_i \) with \( \tau_i > 0 \). According to Young’s inequality, it can be given

\[
\varepsilon_1^3 \tilde{f}_{1,p}(Z_1) \leq \frac{b_{1,m}^i Z_1^2 \tilde{W}_i^T}{2a_i^2} S_1^T(Z_1) S_1(Z_1) + \frac{1}{2} a_i^2
+ \frac{3}{4} \varepsilon_1^4 + \frac{1}{4} \varepsilon_1^4.
\]

(20)

Substituting (10), (12) and (20) into (18) yields

\[
\ell V_{1,p} \leq -\frac{\pi}{2} c_1 b_{1,m}^i \varepsilon_1^4 - \int_{j=2}^i \left( c_j b_{j,m}^i \varepsilon_1^4 + \int_{j=1}^i b_{j,m}^i \tilde{W}_j \tilde{W}_j + \frac{1}{2} l_1 \varepsilon_1^2 + \tilde{T}_1 \right)
\]

(21)

where \( \tilde{T}_1 = \frac{1}{16} a_i^2 + \frac{1}{4} \epsilon_i^4 \).

**Step i** \((2 \leq i \leq n - 1)\): From (3) and (9), one has

\[
d\xi_i = (l_{i,p}(\xi_1)(z_{i+1} + \alpha_i) + f_{i,p}(\tilde{\xi}_i) - \ell \alpha_{i-1}) dt
+ (g_{i,p}(\tilde{\xi}_i) - \int_{j=1}^{i-1} \frac{1}{\varepsilon_1 \varepsilon_j} (g_{j,p}(\tilde{\xi}_j)) \, dw
\]

(22)

where

\[
\ell \alpha_{i-1} = \sum_{j=1}^{i-1} \frac{1}{\varepsilon_1 \varepsilon_j} (l_{j,p}(\tilde{\xi}_j) x_{j+1} + f_{j,p}(\tilde{\xi}_j))
+ \sum_{j=1}^{i-1} \frac{1}{\varepsilon_1 \varepsilon_j} (g_{j,p}(\tilde{\xi}_j) g_{s,p}(\tilde{\xi}_s))
+ \sum_{j=1}^{i-1} \frac{1}{\varepsilon_1 \varepsilon_j} (g_{j,p}(\tilde{\xi}_j) g_{s,p}(\tilde{\xi}_s)).
\]

The following Lyapunov function candidate is defined

\[
V_{i,p} = V_{i-1,p} + \frac{1}{4} \varepsilon_1^4 + \frac{b_{1,m}^i}{2l_1} \tilde{W}_i^2
\]

(23)

and then, we have

\[
\ell V_{i,p} = \ell V_{i-1,p} + \frac{3}{4} l_i \varepsilon_1^2 \varepsilon_1^4 + \int_{j=1}^{i-1} \frac{1}{\varepsilon_1 \varepsilon_j} (g_{j,p}(\tilde{\xi}_j) g_{s,p}(\tilde{\xi}_s)).
\]

(24)

Therefore, (25) can be rewritten as

\[
\frac{3}{2} \tilde{\nu}_d \phi_{i,p} \phi_{i,p} \leq \frac{3}{4} \varepsilon_1^2 - \frac{2}{3} \varepsilon_1^4 \|\phi_{i,p}\|^4 + \frac{3}{4} \varepsilon_1^2.
\]

(26)

Using (25)-(26), (24) can be rewritten as

\[
\ell V_{i,p} \leq \ell V_{i-1,p} + \frac{3}{4} l_i \varepsilon_1^2 \varepsilon_1^4 + \int_{j=1}^{i-1} \frac{1}{\varepsilon_1 \varepsilon_j} (g_{j,p}(\tilde{\xi}_j) g_{s,p}(\tilde{\xi}_s)).
\]

(27)

\[
\ell V_{i,p} \leq \ell V_{i-1,p} + \frac{3}{4} l_i \varepsilon_1^2 \varepsilon_1^4 - \ell \alpha_{i-1} + \frac{3}{4} l_i \varepsilon_1^2 \|\phi_{i,p}\|^4 - \frac{3}{4} l_i \varepsilon_1^2 \varepsilon_1^4 + \frac{3}{4} l_i \varepsilon_1^2 \varepsilon_1^4.
\]

(28)

Substituting (11), (12) and (29) into (27) result in

\[
\ell V_{i,p} \leq -\frac{\pi}{2} c_1 b_{1,m}^i \varepsilon_1^4 - \int_{j=2}^i \left( c_j b_{j,m}^i \varepsilon_1^4 + \int_{j=1}^i b_{j,m}^i \tilde{W}_j \tilde{W}_j + \frac{1}{2} l_i \varepsilon_1^2 + \tilde{T}_j \right)
\]

(30)

where \( \tilde{T}_j = \sum_{j=1}^i \left( \frac{1}{4} a_i^2 + \frac{1}{4} \epsilon_i^4 \right) \).

**Step n**: First, let \( T_{i_k}^a = \tau_i \), \( T_{i+1}^a = \tau_{i+1} \), \( T_1^a, T_2^a, \ldots, T_k^a \) are the switching times on \([t_k, t_{k+1})\). The MDETM is designed as follows

\[
u^p(t) = - (1 + \lambda)(\alpha_n \tan(\frac{3^n \rho_n}{3^n \rho_n} \alpha_n))
+ h_1 \tan(\frac{3^n \rho_n}{3^n \rho_n} \alpha_n h_1)
- \frac{1 + \lambda}{1 - \lambda} \rho_{T_w} \tan(\frac{3^n \rho_n}{3^n \rho_n} \alpha_n T_w),
\]

(31)

\[
u(t) = u^{\sigma t}(t_k), \quad t_k \leq t < t_{k+1}, \quad T_{k+1} = \inf \{r \in R | \|\beta^{\sigma t}(t)\| \geq \lambda |u^{\sigma t}(t_k)| + \epsilon + T_{w'} \}
\]

(33)

\[
T_{w'} = \begin{cases} u^{\sigma t}(T_{i_k}^a) - u^{\sigma t}(T_{i_k}^a), & t \in [T_{i_k}^a, T_{i_k}^a], \\ 0, & otherwise \end{cases}
\]

(34)

where \( \beta^{\sigma t}(t) = u^{\sigma t}(t_k) - u^{\sigma t}(t_k), h_1 > \frac{1}{\sqrt{\lambda}}, \epsilon > 0, \rho_{T_w} > 0 \) and \( 0 < \lambda < 0.5 \) are design parameters.

**Remark 4**: For the studied switched stochastic nonlinear systems, the MDETM that relies on switching signals is cleverly designed, which not only mitigates the communication burden, but also eliminates the impact of asynchronous switching on the system performance.

**Remark 5**: It can be seen that the triggering error of the designed MDETM is discontinuous at the switching moment,
and switching may cause additional continuous triggers, which
may lead to Zeno behavior. The introduction of variable $T_w$
effectively avoids the above problems.

From (3) and (9), one has

$$d z_n = (l, n, \phi_x) u + f_{i, n} (\phi_x) - \ell a_{n-1} dt
+ \left(l, n, \phi_x\right) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{j-1}}{\partial x_j} l_{j, n} (\phi_x) dt$$

(35)

where

$$\ell a_{n-1} = \sum_{j=1}^{n-1} \frac{\partial \alpha_{j-1}}{\partial x_j} l_{j, n} (\phi_x) x_{j+1} + f_{j, n} (\phi_x)$$

$$+ \sum_{j=0}^{n-1} \frac{\partial \alpha_{j-1}}{\partial x_j} \delta_j + \sum_{j=1}^{n} \frac{\partial \alpha_{j-1}}{\partial x_j} \dot{\phi}_{n, j}$$

$$+ \frac{1}{2} \sum_{j=1}^{n-1} \frac{\partial^2 \alpha_{j-1}}{\partial x_j^2} g_{j, n} (\phi_x) g_{s, n, p} (\phi_x).$$

Define the Lyapunov function candidate

$$V_{n, p} = V_{n-1, p} + \frac{1}{4} z_n^4 + \frac{b_{n, p}^2}{2} \sum_{j=1}^{n} \omega_n.$$

(36)

From (35) and (36), we have

$$\ell V_{n, p} = \ell V_{n-1, p} + \frac{3}{4} (l, n, u + f_{n, n} (\phi_x) - \ell a_{n-1})
- \frac{b_{n, p}^2}{l_n} \dot{\omega}_n + \frac{3}{4} z_n^2 f_{n, n} (\phi_n) \phi_{n, n} (\phi_x)$$

(37)

where $\phi_{n, n} (\phi_x) = g_{n, n} (\phi_x) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{j-1}}{\partial x_j} g_{j, n} (\phi_x)$.

By utilizing Young’s inequality, the following inequality holds

$$\frac{3}{2} \frac{2}{3} z_n^2 f_{n, n} (\phi_n) \leq \frac{3}{4} a_n^2 z_n^4 \|\phi_{n, n}\|^4 + \frac{3}{4} a_n^2.$$

(38)

By substituting (38) into (37), it gets

$$\ell V_{n, p} \leq \ell V_{n-1, p} + \frac{3}{4} z_n^2 f_{n, n} (\phi_x) u + \frac{3}{4} z_n^2 f_{n, n} (\phi_x) - \ell a_{n-1}
- \frac{b_{n, p}^2}{l_n} \dot{\omega}_n + \frac{3}{4} z_n^2 f_{n, n} (\phi_x)$$

(39)

where $f_{n, n} (\phi_x) = f_{n, n} (\phi_x) + \frac{3}{4} a_n^2 z_n^2 \|\phi_{n, n}\|^4 - \ell a_{n-1} + \frac{1}{4} l_{n-1} (\phi_x) z_n^4$.

By using Young’s inequality, we have

$$z_n^3 f_{n, n} (\phi_x) \leq \frac{b_{n, p}^2}{2a_n^2} \sum_{j=1}^{n} \omega_n (\phi_x) S_{n} (\phi_x) + \frac{1}{2} a_n^2
+ \frac{3}{4} z_n^4 + \frac{4}{4} l_n^4.$$

(41)

By using (39), (39) can be converted into

$$\ell V_{n, p} \leq \ell V_{n-1, p} + \frac{b_{n, p}^2}{2a_n^2} \sum_{j=1}^{n} \omega_n (\phi_x) S_{n} (\phi_x) + \frac{1}{2} a_n^2
+ \frac{3}{4} z_n^4 + \frac{4}{4} l_n^4.$$

Next, we will divide the system dynamics into two parts for discussion based on whether the $p$th subsystem is syn-
chronized with the candidate controller within the triggering interval $[t_k, t_{k+1})$.

Part 1: synchronous interval.

At this time, $\sigma (t) = \sigma (t) = p, T_w = 0$. From (31)-(33), we have

$$u_p (t) = (1 + \phi_x (t) \lambda) u_p (t) + \phi_x (t) \epsilon, \forall t \in [t_k, t_{k+1}),$$

where $\phi_x (t) \in [-1, 1], \phi_x (t) \in [-1, 1]$. Then, the actual controller can be expressed as

$$u = u_p (t) = \frac{u_p (t)}{1 + \phi_x (t) \lambda} - \frac{\phi_x (t) \epsilon}{1 + \phi_x (t) \lambda}.$$

(43)

Therefore, (42) can be repeated as

$$\ell V_{n, p} \leq \ell V_{n-1, p} + \frac{b_{n, p}^2}{2a_n^2} \sum_{j=1}^{n} \omega_n (\phi_x) S_{n} (\phi_x) + \frac{5}{4} a_n^2
+ \frac{1}{1 + \phi_x (t) \lambda} z_n^3 (\phi_x) u_p (t)
+ \frac{1}{1 + \phi_x (t) \lambda} z_n^3 (\phi_x) r_n (\phi_x)
- \frac{b_{n, p}^2}{l_n} \dot{\omega}_n + \frac{1}{4} \epsilon^4 + \frac{1}{4} l_{n-1} (\phi_x) z_n^4.$$

(44)

Based on $\frac{z_n^3 (\phi_x) u_p (t)}{1 + \phi_x (t) \lambda} \leq \frac{z_n^3 (\phi_x) u_p (t)}{1 + \phi_x (t) \lambda} \leq l_n^2 (\phi_x) \frac{1}{1 + \phi_x (t) \lambda}$, it follows that

$$\ell V_{n, p} \leq \ell V_{n-1, p} + \frac{b_{n, p}^2}{2a_n^2} \sum_{j=1}^{n} \omega_n (\phi_x) S_{n} (\phi_x) + \frac{b_{n, p}^2}{l_n} \dot{\omega}_n
+ \frac{1}{1 + \phi_x (t) \lambda} z_n^3 (\phi_x) (\phi_x)\theta_{n, p}
+ \frac{1}{1 + \phi_x (t) \lambda} z_n^3 (\phi_x) (\phi_x)\theta_{n, p}
+ \frac{5}{4} a_n^2 + \frac{1}{4} \epsilon^4 + \frac{1}{4} l_{n-1} (\phi_x) z_n^4 + 0.557 \rho_p^p
+ z_n^3 (\phi_x) (\phi_x)\theta_{n, p}.$$

(45)

Using a process similar to Step i, it gets

$$\ell V_{n, p} \leq \frac{\pi}{2} c_j b^{p, m} (\phi_x) - \sum_{j=2}^{n} (c_j b^{p, m} (\phi_x)) + \sum_{j=1}^{n} \frac{b_{n, m} (\phi_x)}{2a_n^2} \sum_{j=1}^{n} \omega_n (\phi_x) S_{n} (\phi_x) + \frac{1}{2} a_n^2
+ \frac{3}{4} z_n^4 + \frac{4}{4} l_n^4.$$

(46)

By means of Young’s inequality, we get

$$\sum_{j=1}^{n} b_{n, m} (\phi_x) \dot{\omega}_j \leq \frac{1}{2} \sum_{j=1}^{n} b_{n, m} (\phi_x) + \frac{1}{2} \sum_{j=1}^{n} b_{n, m} (\phi_x).$$

(47)

where $\sum_{j=1}^{n} b_{n, m} (\phi_x) \dot{\omega}_j \leq \frac{1}{2} \sum_{j=1}^{n} b_{n, m} (\phi_x) + \frac{1}{2} \sum_{j=1}^{n} b_{n, m} (\phi_x).$

(48)

$$\ell V_{n, p} \leq \frac{\pi}{2} c_j b^{p, m} (\phi_x) - \sum_{j=2}^{n} (c_j b^{p, m} (\phi_x))
+ \frac{1}{2} \sum_{j=1}^{n} \frac{b_{n, m} (\phi_x)}{2a_n^2} \sum_{j=1}^{n} \omega_n (\phi_x) S_{n} (\phi_x) + \frac{1}{2} a_n^2
+ \frac{3}{4} z_n^4 + \frac{4}{4} l_n^4.$$

(42)
Part 2: asynchronous interval.

1) If \( \sigma(t_k) \neq r, \sigma(t) = p \) with \( t \in [\tau^k_1, \tau^k_2) \). At this moment, the MDET (33) can ensure that
\[
|u^{\sigma(t_k)}(t_k) - u^p(t)| \leq \lambda|u^0(t)(t_k)| + T_w + \epsilon. \tag{49}
\]

Similar to the derivation in Part 1, we have
\[
z^3_{\sigma n}p(\tilde{x}_n)u(t) = z^3_{\sigma n}p(\tilde{x}_n)u^{\sigma(t_k)}(t_k) \leq \frac{z^3_{\sigma n}p(\tilde{x}_n)u^p(t)}{1 + \lambda} \tag{50}
\]
\[
+ \left| z^3_{\sigma n}p(\tilde{x}_n) \frac{\epsilon + T_w}{1 - \lambda} \right|.
\]

Take the same steps as Part 1 to get
\[
\ell V_{n,p} \leq -\pi c_1 b^1_{1,m} z^4 + \frac{n}{2} (c_j b^p_{j,m} z^4) - \frac{n}{2} \sum_{j=1}^n b^p_{j,m} \tilde{W}_j^2 + \Delta_p \tag{51}
\]
\[
+ \left| \frac{z^3_{\sigma n}p(\tilde{x}_n)T_w}{1 - \lambda} - \frac{z^3_{\sigma n}p(\tilde{x}_n)T_w}{1 - \lambda} \text{tanh} \left( \frac{z^3_{\sigma n}p(\tilde{x}_n)}{\rho^p} \right) \right|.
\]
\[
\leq -\pi c_1 b^1_{1,m} z^4 - \frac{n}{2} (c_j b^p_{j,m} z^4) - \frac{n}{2} \sum_{j=1}^n b^p_{j,m} \tilde{W}_j^2 + \Delta_p + 0.557 \rho^p.
\]

2) This interval is nonempty only if \( r > 1 \). At this time, \( \sigma(t_k) \neq r, \sigma(t) = p \) with \( t \in [\tau^k_{\frac{i-1}{k}}, \tau^k_{i+1}) \), \( i = 2, \cdots, r \). The MDET (33) is the same as Part 1 to ensure
\[
|u^{\sigma(t_k)}(t_k) - u^p(t)| \leq \lambda|u^0(t)(t_k)| + \epsilon. \tag{52}
\]

Then, using the similar derivation given in Part 1, we get
\[
z^3_{\sigma n}p(\tilde{x}_n)u(t) = z^3_{\sigma n}p(\tilde{x}_n)u^{\sigma(t_k)}(t_k) \leq \frac{z^3_{\sigma n}p(\tilde{x}_n)u^p(t)}{1 + \lambda} \tag{53}
\]
\[
+ \frac{z^3_{\sigma n}p(\tilde{x}_n)}{1 - \lambda} \frac{\epsilon}{1 - \lambda}.
\]

It can be obtained by using the same procedure as in Part 1
\[
\ell V_{n,p} \leq -\pi c_1 b^1_{1,m} z^4 - \frac{n}{2} (c_j b^p_{j,m} z^4) - \frac{n}{2} \sum_{j=1}^n b^p_{j,m} \tilde{W}_j^2 + \Delta_p. \tag{54}
\]

The synchronous/asynchronous discussion between the subsystem and the candidate controller is completed. Next, by selecting the Lyapunov function candidate \( V_p = V_{n,p} \), we have
\[
\ell V_p \leq -\eta_p \dot{V}_p + \Lambda \tag{55}
\]
where \( \eta_p = \min(2\pi c_1 b^1_{1,m}, 4c_i b^p_{i,m}, l_i, i = 2, 3, \cdots, n) \), \( \Lambda = \max(\Delta_p + 0.557 \rho^p, p \in M) \).

Theorem 1: For the switched stochastic nonlinear systems (3) under Assumptions 1-3. The actual controller (32), the adaptive law (12) and the MDET (31)-(34) are constructed for \( \sigma(t) \) with MDADT \( \tau_{ap} \geq \tau^a_{ap} = \frac{T_l}{\eta_p}, \mu = \max(b^p_{i,m}, i = 1, 2, \cdots, n, k \in M) \), it can ensure the following:

1) All the resulting system signals are SGUUB in probability.
2) The tracking error \( e_1 \) gets into a prescribed boundary no later than a setting time.
3) The designed MDET is Zeno-free.

Proof. First of all, we prove that all signals of the control system are bounded, and the discussion is divided into two cases.

Case 1: When \( \mu_p = 1(p \in M) \), we get \( V_p = V_q, q \in M \). Therefore, the common Lyapunov function \( V = V_p \) for all subsystems satisfies (55), which means
\[
E[V(t)] \leq V(0) + \frac{\Lambda}{\eta_{\min}} \forall t \geq 0 \tag{56}
\]
where \( \eta_{\min} = \min(\eta_p, p \in M) \). Therefore, it can be concluded that all the signals in the control system are SGUUB.

Case 2: When \( \exists \mu_{p,q} > 1(p, q \in M) \). There are functions \( \gamma, \Psi \in K_{\infty} \), such that \( \gamma(\|Y\|) \leq V_p(Y) \leq \Psi(\|Y\|) \), for an arbitrary \( T > 0 \), let \( t_0 = 0 \) and \( t_1, t_2, \cdots, t_1, t_{i+1}, \cdots, t_{N_p}(0, T) \) are the switching times on \( [0, T] \), in which \( N_p(T, 0) = 0 \).

Consider the piecewise continuous function \( H(t) = e^{\mu_1 t} V_{\sigma(t)}(Y(t)) \). From (55), on each interval \( [t_j, t_{j+1}] \), one has
\[
\dot{H}(t) \leq \lambda_{\sigma(t)} e^{\mu_1 t} \dot{V}_{\sigma(t)}(Y(t)) + e^{\mu_1 t} V_{\sigma(t)}(Y(t)) \tag{57}
\]

Invoking the fact \( E[d\dot{V}(t)] = 0 \), we have
\[
E \left\{ \int_{t_j}^{t_{j+1}} \dot{H}(t) dt \right\} = E[H(t_{j+1})] - E[H(t_j)] \leq E \left\{ \int_{t_j}^{t_{j+1}} e^{\mu_1 t} \Lambda dt \right\} \tag{58}
\]

It is shown from \( V_p(Y(t)) \leq \mu_p V_q(Y(t)) \), one has
\[
\dot{E}[H(t_{j+1})] \leq \mu_{\sigma(t_{j+1})} E \left\{ \sum_{i=0}^{j} (\eta_{\sigma(t_{i+1})} - \eta_{\sigma(t_{i})}) H(t_{i+1}) \right\} \tag{59}
\]
\[
\leq \mu_{\sigma(t_{j+1})} E \left\{ \sum_{i=0}^{j} (\eta_{\sigma(t_{i+1})} - \eta_{\sigma(t_{i})}) H(t_{i+1}) \right\} \tag{59}
\]

where \( Q_{1,l} = \sum_{i=0}^{j} (\eta_{\sigma(t_{i+1})} - \eta_{\sigma(t_{i})}) \exp \left\{ \sum_{i=0}^{j} (\eta_{\sigma(t_{i+1})} - \eta_{\sigma(t_{i})}) H(t_{i+1}) \right\} \)

Hence
\[
E[H(T^-)] \leq E\left\{ \sum_{i=0}^{j} (\eta_{\sigma(t_{i+1})} - \eta_{\sigma(t_{i})}) H(t_{i+1}) \right\} \tag{59}
\]
\[
\leq \mu_{\sigma(t_{j+1})} E \left\{ \sum_{i=0}^{j} (\eta_{\sigma(t_{i+1})} - \eta_{\sigma(t_{i})}) H(t_{i+1}) \right\} \tag{59}
\]

where
\[
Q_2 = \prod_{i=0}^{j} \mu_{\sigma(t_{i+1})} \exp \left\{ \sum_{i=0}^{j} (\eta_{\sigma(t_{i+1})} - \eta_{\sigma(t_{i})}) H(t_{i+1}) \right\},
\]
\[
 \prod_{i=0}^{j} \mu_{\sigma(t_{i+1})} \exp \left\{ \sum_{i=0}^{j} (\eta_{\sigma(t_{i+1})} - \eta_{\sigma(t_{i})}) H(t_{i+1}) \right\}
\]
Then get from (60) that
\[
E[V_{\sigma(T^-)}(Y(T))]
\leq E\{Q_3 V_{\sigma(0)}(Y(0))\}
+
E\left\{\sum_{s=0}^{N_e(T,0)-1} \left( Q_{3,l} e^{-e min_{l+1}} \int_{t_l}^{t_{l+1}} e^{e min_l} \Lambda dt \right) \right\}
+
E\left\{e^{-\frac{d}{d} \sum_{p} N_{0p} \mu_p} e^{-e min_{l+1}} \int_{t_l}^{t_{l+1}} e^{e min_l} \Lambda dt \right\}
\leq E\{\tilde{Q}_3 V_{\sigma(0)}(Y(0))\}
+
E\left\{\sum_{l=0}^{N_e(T,0)-1} \left( \tilde{Q}_{3,l} e^{-e min_{l+1}} \int_{t_l}^{t_{l+1}} e^{e min_l} \Lambda dt \right) \right\}
+
E\left\{e^{-e min_{l+1}} \int_{t_l}^{t_{l+1}} e^{e min_l} \Lambda dt \right\}
(61)

where
\[
Q_3 = \prod_{j=0}^{N_e(T,0)-1} \mu_{\sigma(t_{j+1})} \exp\left\{\sum_{j=0}^{N_e(T,0)-1} \left( e^\sigma(t_{j+1}) - e^\sigma(t_{j+1}) T + e^\sigma(t_{j+1}) T \right) \right\},
\]
\[
Q_{3,l} = \prod_{i=0}^{N_e(T,0)-1} \mu_{\sigma(t_{i+1})} \exp\left\{\sum_{i=0}^{N_e(T,0)-1} \left( e^\sigma(t_{i+1}) - e^\sigma(t_{i+1}) T + e^\sigma(t_{i+1}) T \right) \right\},
\]
\[
\tilde{Q}_3 = \prod_{p=1}^{d} \mu_{\sigma(t_{j+1})} \exp\left\{-\sum_{p=1}^{H} [\eta_p \sum_{i \in \phi} (t_{i+1} - t_i)] \right\}
- \eta_p (T_{T} - T_{N_e(T,0)}) \right\},
\]
\[
\tilde{Q}_{3,l} = \prod_{l=0}^{N_e(T,0)-1} \mu_{\sigma(t_{i+1})} \exp\left\{\sum_{i=0}^{N_e(T,0)-1} \left( e^\sigma(t_{i+1}) - e^\sigma(t_{i+1}) T + e^\sigma(t_{i+1}) T \right) \right\},
\]
\[
\tilde{Q}_3 = \prod_{p=1}^{d} \mu_{\sigma(t_{j+1})} \exp\left\{\sum_{p=1}^{H} T \eta_p \sum_{i \in \phi} \left( e^\sigma(t_{i+1}) \right) \right\}
- \eta_p T_{T} \right\},
\]
\[
\tilde{Q}_{3,l} = \prod_{l=0}^{N_e(T,0)-1} \mu_{\sigma(t_{i+1})} \exp\left\{-\sum_{p=1}^{H} T \eta_p T_{T} \right\},
\]
\[
\frac{d}{dt} |\beta^p| = \text{sign}(\beta^p) \dot{\beta}^p \leq |\dot{u}^p(t)|.
\]

\(\phi(p)\) stands for the set of \(l\) satisfying \(\sigma(t_l) = p, t_l \in \{t_0, t_1, \ldots, t_{l+1}, \ldots, t_{N_e-1}\}\), and \(\epsilon_{min} = \min\{e_p, p \in M\} \) with \(e_p \in (0, \eta_p - \ln \mu_p / e_{ap})\).

Then, from \(e_{ap} \geq (\ln \mu_p / \eta_p)\) together with (8) get
\[
E[V_{\sigma(T^-)}(Y(T))]
\leq E\{Q_4 V_{\sigma(0)}(Y(0))\} + E\left\{e^{-e min_{l+1}} \int_{t_l}^{t_{l+1}} e^{e min_l} \Lambda dt \right\}
+
E\left\{\sum_{l=0}^{N_e(T,0)-1} \left( \prod_{p=1}^{d} \mu_{\sigma(t_{j+1})} \sum_{p=1}^{H} \eta_p T_{T} \right) \right\}
- \frac{d}{d} \sum_{p=1}^{d} N_{0p} \ln \mu_p \max\{0, (\ln \mu_p / e_{ap}) - \eta_p\} \int_{t_l}^{t_{l+1}} e^{e min_l} \Lambda dt \right\}
\leq E\{Q_4 V_{\sigma(0)}(X(0))\} + E\left\{e^{-e min_{l+1}} \int_{t_l}^{t_{l+1}} e^{e min_l} \Lambda dt \right\}
+
E\left\{\sum_{l=0}^{N_e(T,0)-1} \left( \prod_{p=1}^{d} \mu_{\sigma(t_{j+1})} \sum_{p=1}^{H} N_{0p} \ln \mu_p \max\{0, (\ln \mu_p / e_{ap}) - \eta_p\} \int_{t_l}^{t_{l+1}} e^{e min_l} \Lambda dt \right) \right\}
\leq E\{Q_4 V_{\sigma(0)}(Y(0))\} + E\left\{\sum_{l=0}^{N_e(T,0)-1} \left( \prod_{p=1}^{d} \mu_{\sigma(t_{j+1})} \sum_{p=1}^{H} N_{0p} \ln \mu_p \max\{0, (\ln \mu_p / e_{ap}) - \eta_p\} \right) \right\}
+ \frac{H}{\mu_p} \sum_{p=1}^{d} N_{0p} \mu_p \Lambda \epsilon_{min}
(62)

where \(Q_4 = \exp\{H \sum_{p=1}^{d} N_{0p} \ln \mu_p \exp\{H \sum_{p=1}^{d} (\frac{\eta_p}{e_{ap}} - \eta_p) T_{p} \} \).

Therefore, all signals in the control system are SGUUB under MDADT method. Furthermore, we need to prove that \(-z(t) < \epsilon_1 < z(t), t \geq T > 0\). The help of the boundedness of \(z(t)\) and \(\tan(\frac{\pi}{2}) = \infty\), it follows that
\[-z(t) < \epsilon_1 < z(t).\]

From (6) and (7), we have
\[-z(t) < \tan(\epsilon_1) < z(t).\]
With the help of (31), it follows that \(u^p\) is differentiable and \(\dot{u}^p\) is bounded. From \(\beta^p(t_k) = 0\) and \(\lim_{t \to t_{k+1}} \beta^p(t) = (\lambda|u^p(t_k)| + \epsilon)\) to get \(t_{k+1} - t_k \geq (\lambda|u^p(t_k)| + \epsilon)/\varrho_1 > 0\), where \(\varrho_1\) is a positive constant satisfying \(|\dot{u}^p| \leq \varrho_1\).

**Case 2 (Triggering Interval With One Switch):** Assume that the switch occurs at \(T_{k}^{1} \in (t_{k}, t_{k+1})\). Noting that, it can be seen from \(|u^\sigma(t_k) - u^\sigma(T_{k}^{1})| < \lambda|u^\sigma(t_k)| + \epsilon + T_w\) that no additional trigger will be generated at \(T_{k}^{1}\); From case 1, \(\frac{\partial \beta^p(t_k)}{\partial t} \leq \varrho_1\) can be obtained in \((t_k, T_{k}^{2})\). As in case 1, it can be guaranteed that \(\frac{1}{2}\beta^p(T_{k}^{1}) \leq \varrho_2 \in (T_{k}^{1}, t_{k+1})\), where \(\varrho_2\) is a positive constant. It is shown from the above analysis that \(t_{k+1} - t_k \geq (\lambda|u^\sigma(t_k)| + \epsilon)/(\max(\varrho_1, \varrho_2)) > 0\).

**Case 3 (Triggering Interval With Multiple Switches):** \(N_k\) is the number of switches on the \(k\)th triggering interval, and obviously \(t_{k+1} - t_k \geq N_k t_d > 0\).

Based on the above analysis, the designed MDETM is Zeno-free. The proof is completed.

**Remark 6:** To prove the stability of the switched system based on the multiple Lyapunov function techniques, it is important to construct the relationship between any two Lyapunov functions. In this paper, the cumulative relationship of two Lyapunov functions is found by using uniform coordinate transformation and common adaptive law for all subsystems.

### IV. Simulation Examples

In this section, the effectiveness of the proposed theoretical results is verified by numerical example and practical example.

**Example 1:** Consider the following numerical example

\[
\begin{align*}
\dot{x}_1 &= (l_{1,1}(t)\tilde{x}_1)x_2 dt, \\
\dot{x}_2 &= (l_{2,1}(t)\tilde{x}_2)u + f_{2,1}(t)(\tilde{x}_2) dt \\
&\quad + g_{2,1}(t)\tilde{x}_2 dw, \\
y &= x_1,
\end{align*}
\]

where \(\sigma(t) : [0, \infty) \rightarrow M = \{1, 2\}, l_{1,1} = 1 + 0.3\cos(x_1), l_{1,2} = 1 + 0.5\cos(x_1), l_{2,1} = 1 + 0.7\sin(x_1 x_2), l_{2,2} = 1 + 0.6\cos(x_1 x_2), f_{2,1} = 0.1\sin(x_1), f_{2,2} = 0.1\cos(x_2^2), g_{2,1} = x_1\sin(x_1), g_{2,2} = \sin(x_1).\) The desired signal \(y_d(t) = 0.7\sin(t)\). The following membership functions are selected:

\[
\begin{align*}
\mu F^1_1 &= e^{-0.5(x_1 + 1.5)^2}, & \mu F^2_1 &= e^{-0.5(x_1 + 1)^2}, \\
\mu F^3_1 &= e^{-0.5(x_1 + 0.5)^2}, & \mu F^4_1 &= e^{-0.5(x_1)^2}, \\
\mu F^5_1 &= e^{-0.5(x_1 - 0.5)^2}, & \mu F^6_1 &= e^{-0.5(x_1 - 1)^2}, \\
\mu F^7_1 &= e^{-0.5(x_1 - 1.5)^2}.
\end{align*}
\]

The initial conditions are \([x_1(0), x_2(0)]^T = [-0.5, 0.4]^T, \dot{\tilde{x}}_1(0) = 15, \dot{\tilde{x}}_2(0) = 4\). The design parameters are chosen as \(a_1 = 10, \alpha_1 = 5, l_1 = 0.8, a_2 = 10, c_1 = 1, l_2 = 0.7, \lambda = 0.3, \tilde{h}_1 = 5, \epsilon = 2, \rho^1 = 1, \rho^2 = 0.8, \tilde{z}_0 = 0.1, \tilde{z}_\infty = 0.01, \kappa_1 = 0.7, s_0 = 5, s_1 = 3, \kappa_2 = 20, N = 10\) and the setting time \(T = 2s\). Furthermore, it is shown from \(\mu_1 = 1.4, \eta_1 = 0.7, \mu_2 = 1.3, \eta_2 = 0.7\) that \(\tau_{a1} \geq 0.4807, \tau_{a2} \geq 0.4110\).

The simulation results are shown in Figs. 2-8. Fig. 2 displays the tracking performance of the output \(y\). Fig. 3 depicts the control effect of tracking error \(e_1\), and \(e_1\) gets into a prescribed boundary no later than a setting time \(T = 2s\) without \(-\tilde{z}_1(0) \leq e_1(0) \leq \tilde{z}_1(0)\). The system state \(x_2\) and the adaptive parameters \(\dot{W}_1, \dot{W}_2\) are described in Figs. 4 and 5, respectively. The trajectory of control input \(u\) is shown in Fig. 6. Fig. 7 gives the time interval of event-triggered. Finally, the responses of the switching signal is illustrated in Fig. 8.

**Example 2:** In order to verify the practicability of the proposed control method, the RLC circuit given in [16] is
considered.

\[
\begin{cases}
\dot{x}_1 = x_2 dt, \\
\dot{x}_2 = (u - \frac{1}{C_1(\tau)}) - \frac{R}{L}x_2 dt + \frac{1}{L}x_2 \sin(x_1) d\omega 
\end{cases}
\]  

(67)

where \( L = 1H, C_1 = 0.5F, C_2 = 0.8F, R = 0.1\Omega \). Define \( x_1 = q_L, x_2 = \phi_L \).

The membership functions are the same as Example 1. The initial conditions are \([x_1(0), x_2(0)]^T = [1, 0.8]^T, \hat{W}_1(0) = 1, \hat{W}_2(0) = 1\). The design parameters are chosen as \( a_1 = 1, c_1 = 25, l_1 = 0.5, a_2 = 5, c_2 = 5, l_2 = 0.1, \lambda = 0.3, h_1 = 0.25, \epsilon = 0.1, \rho^1 = 1, \rho^2 = 0.8, \zeta_0 = 0.1, \zeta_\infty = 0.01, \kappa_1 = 0.7, s_0 = 5, s_1 = 3, \kappa_2 = 20, N = 10 \) and the setting time \( T = 3s \). Especially, we get \( \mu_1 = \mu_2 = 1 \), which means that a common Lyapunov function can be found for system (67), and Theorem 1 holds under arbitrary switching signals.
The simulation results for the RLC circuit are shown in Figs. 9-15. From Figs. 9-10, we can see that $e_1$ can be constrained in performance function no later than a setting time $T = 3s$ without $-\xi_1(0) \leq e_1(0) \leq \xi_1(0)$. Figs. 11-13 display the boundedness of $x_2$, $\hat{W}_1$, $\hat{W}_2$ and $u$. The trajectories of the trigger time interval and the system signal are shown in Figs. 14 and 15. Simulation results verify the practicality of the proposed control algorithm.

V. CONCLUSION

Based on the event-triggered strategy, this paper solves the problem of fuzzy control for stochastic switched nonlinear systems with set-time predefined performance. Combined with MDADT method and Lyapunov function stability analysis, a fuzzy performance algorithm is proposed. The contribution of this study is to introduce the MDETFS into the performance control design of switched stochastic nonlinear systems. The proposed control algorithm can not only ensure that the tracking error enters the predefined region no later than a setting time, but also overcome the adverse impact of asynchronous switching on the system performance. Finally, the theoretical results are verified by two simulation examples. In the future, we will study the set-time PPC design of MIMO systems, large-scale systems and multi-agent systems.

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