Electron-electron interaction corrections to the thermal conductivity in disordered conductors

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(Dated: December 23, 2021)

We evaluate the electron-electron interaction corrections to the electronic thermal conductivity in a disordered conductor in the diffusive regime. We use a diagrammatic many-body method analogous to that of Altshuler and Aronov for the electrical conductivity. We derive results in one, two and three dimensions for both the singlet and triplet channels, and in all cases find that the Wiedemann-Franz law is violated.

PACS numbers: 73.50.Lw, 72.10.-d

I. INTRODUCTION

The effect of electron-electron interaction on the electrical conductivity of disordered systems has been extensively investigated both theoretically and experimentally over the past two decades. There are two main types of correction to the Drude electrical conductivity which can have similar magnitudes and temperature dependences: weak localization and interaction effects. Weak localization is due to interference between pairs of time-reversed scattering trajectories of electrons from impurities; this effect can occur in a non-interacting system. Interaction effects are due to the increased effective electron-electron interaction strength due to incomplete screening by diffusively moving electrons. Experimentally the two effects can be distinguished by applying a magnetic field; weak localization is suppressed whilst interaction effects are not.

In comparison there has been relatively little work done on the thermal conductivity, either theoretically or experimentally. This is largely because thermal conductivity is hard to measure in low-dimensional systems at low temperature, and it is difficult to separate the electronic and lattice contributions. It is therefore doubly hard to observe the disorder-driven corrections to the electronic thermal conductivity. Moreover there have been theoretical predictions that the Wiedemann-Franz law holds, which allows one to deduce the thermal conductivity, $\kappa$, directly from the electrical conductivity, $\sigma$,

$$\frac{\kappa}{\sigma} = \frac{\pi^2}{3} \left(\frac{k_B}{e}\right)^2 T = L_0 T,$$

where $k_B$ is Boltzmann’s constant, $e$ is the electronic charge, $T$ is the temperature, and $L_0$ is known as the Lorenz number. Clearly it is important to establish whether the Wiedemann-Franz law is valid in interacting disordered systems; if it is, there is nothing to be gained from measuring the thermal conductivity in addition to the electrical conductivity.

In this paper we evaluate the interaction corrections to the thermal conductivity in one, two and three dimensions in both the singlet and triplet channels. We find that the Wiedemann-Franz law is violated in all cases. Our results are presented in detail in Table 1. We have not included a result for the singlet channel in one dimension since this is just one third of the corresponding triplet channel result with $F$ replaced by an effective singlet channel interaction $F_s = \kappa_2^0 a^2 \ln(D\kappa_2^0/T)$, as explained in the figure caption. Each term consists of two pieces: the first piece leads to violation of the Wiedemann-Franz law; the second piece does not. The exception is the triplet channel in two dimensions which does not possess a violating piece. Note that in each case both terms have the same parametric dependence; it is only the constant prefactors which are different.

Our calculation is the exact analogue of the original Altshuler-Aronov calculation for electrical conductivity. There have been several previous attempts at this calculation; however these are in disagreement with each other, and we believe all of them to be incorrect. The fact that it has taken so long (over twenty years) to get the correct result for the thermal conductivity is due to three inherent difficulties in the problem. First, the heat-current operator is not uniquely defined, and some choices of definition will be renormalized by the electron-electron interaction. Second, the heat-current operator has opposite electron-hole parity to the charge-current operator, which leads to the presence of an extra “heat-drag” diagram for the thermal conductivity which is vanishingly small for the electrical conductivity. (We use the term “heat-drag” by analogy to the Coulomb drag effect between two layers of electrons; this effect is described by the same diagram. Third, spurious ultraviolet divergences occur in the diagrammatic approach to thermal conductivity; these can be understood and evaluated within the framework of divergent series theory (their origin is due to illegal series expansions in the derivation of the perturbation theory).

Before we proceed to the details of our calculation, we present a short history of the field. We will first consider effects which do not require interaction, such as weak localization and the Anderson transition. Chester and Thellung used an exact eigenstates approach to show that the Wiedemann-Franz law should hold in a non-interacting disordered system, independent of the strength of impurity scattering. Strinati and Castellani...
The aim of the present work is therefore to resolve the discrepancy between previous calculations and to present the first consistent calculation of the interaction corrections to thermal conductivity.

The remainder of the paper is organised as follows: in section 2 we derive the Drude contribution to electronic thermal conductivity; in section 3 we present an outline of the calculation of the interaction corrections to thermal conductivity; finally in section 4 we analyse our results and draw conclusions.
The heat current kernel is then given by in the Matsubara frequency representation that we use. The Feynman diagram for the Drude thermal conductiv-
ity is obtained from the imaginary time heat response by deriving the Drude result. The thermal conduc-
tivity to thermal conductivity is obtained directly from the Drude result if the frequencies 

\[ \kappa T = \lim_{\Omega \to 0} \left[ \frac{Q_{hh}(i\Omega_p)}{\Omega_p} \right]_{\Omega_p \to \Omega + i\delta} \]

The Feynman diagram for the Drude thermal conductivity is shown in Fig. 1. The solid lines are disordered electron Green functions

\[ G(k, i\epsilon_l) = \frac{1}{i\epsilon_l - \xi_k + \frac{\pi}{2} \text{sgn}(\epsilon_l)} \]

where \( \epsilon_l = 2\pi T(l + 1/2) \) is a Fermi Matsubara frequency, \( \xi_k = k^2/2m - \mu \) is the electronic excitation spectrum, and \( T \) is the elastic scattering time. The black dots represent heat-current vertices, which are given by

\[ j_h(k, \epsilon_l, \epsilon_l + i\Omega_p) = \frac{k}{2m} i(2\epsilon_l + i\Omega_p). \]

in the Matsubara frequency representation that we use. The heat current kernel is then given by

\[ Q_{hh}^{\alpha\beta}(i\Omega_p) = 2T \sum_{\epsilon_l} \frac{k_\alpha k_\beta}{m^2} \left[ i(\epsilon_l + \Omega_p/2) \right]^2 \times G(k, i\epsilon_l)G(k, i\epsilon_l + i\Omega_p) \]

Performing the \( k \)-integral, we only obtain a non-zero result if the frequencies \( \epsilon_l + \Omega_p \) and \( \epsilon_l \) have opposite sign, which means that the \( \epsilon_l \) is restricted to the range \( -\Omega_p < \epsilon_l < 0 \). Upon changing the sign of \( \epsilon_l \) we obtain

\[ Q_{hh}^{\alpha\beta}(i\Omega_p) = -4\pi N(0) D \delta_{\alpha\beta} T \sum_{0 < \epsilon_l < \Omega_p} (\epsilon_l - \epsilon_l - \Omega_p/2)^2 \]

where \( N(0) \) is the single-spin electronic density of states at the Fermi surface. We see that the response function is isotropic, so we drop the spatial indices. We can then perform the \( \epsilon_l \) sum to obtain

\[ Q_{hh} = -16\pi^3 N(0) T^2 \sum_{l=0}^{p-1} (l + 1/2 - p/2)^2 \]

\[ = -\frac{4\pi^3}{3} N(0) T^3 (p^2 - p), \]

and we finally extract \( \kappa \) using Eq. (2) to get the Drude result

\[ \kappa_0 = \frac{2\pi^2}{3} N(0) DT = \frac{\pi^2 n T}{3} \]

where \( n \) is the electron number density and we have used the Einstein relation \( 2N(0) D = nT/m \). From the corresponding Drude formula for electrical conductivity, \( \sigma_0 = ne^2\tau/m \), we see that the Wiedemann-Franz law is obeyed (note that \( \hbar = 1 \) and \( k_B = 1 \) in our calculations). This diagrammatic technique offers a simple proof that the Wiedemann-Franz law is obeyed for a non-interacting disordered system with arbitrary disorder strength. For any diagrammatic contribution to electrical conductivity, \( \sigma \), there is a corresponding contribution to thermal conductivity, \( \kappa \). Moreover the corresponding expressions differ only in the form of the current vertices, yielding a factor \( k_\alpha k_\beta \epsilon^2/m^2 \) for \( \sigma \) and \( k_\alpha k_\beta [1(\epsilon_l + \Omega_p/2)]^2/m^2 \) for \( \kappa \). The only essential difference between \( \kappa \) and \( \sigma \) then lies in the frequency sums; the ratio of these two sums is independent of disorder and leads to the Wiedemann-Franz ratio. In particular, the weak localization correction to thermal conductivity is obtained directly from the Wiedemann-Franz law. Note that this proof relies on the Sommerfeld expansion since we are linearising our energy integrals about the Fermi surface. Interaction effects can violate the Wiedemann-Franz law since the presence of an interaction line can alter the Matsubara frequencies at the two current vertices.

**III. THE INTERACTION CORRECTIONS**

In this section we calculate the interaction corrections arising from the singlet and triplet interaction channels — we do not evaluate Cooperonic contributions since these are expected to be small for a system with repulsive interactions. In the singlet channel the dominant contribution arises from small energy and momentum transfers between electrons. This is dominated by the bare Coulomb interaction, which takes the form

\[ V_0(q) = \begin{cases} 
4\pi e^2 & d = 3 \\
2\pi e^2 & d = 2 \\
e^2 \ln \left( \frac{1}{q^2 a^2} \right) & d = 1
\end{cases} \]
In the triplet channel the dominant contribution arises from momentum transfers of the order of \(2q\), \(i\omega\) where the polarization operator \(\Pi(q, i\omega_n)\) takes the universal form,

\[
V_s(q, i\omega_n) = \frac{1}{V_0(q)^{-1} + \Pi(q, i\omega_n)},
\]

where the disorder-screened singlet interaction then takes the form

\[
\Pi(q, i\omega_n) = 2N(0)\frac{Dq^2}{Dq^2 + |\omega_n|}.
\]

If any integrals we obtain involving \(V_s(q, i\omega_n)\) are convergent at small momentum, \(q\), we can ignore the \(V_0(q)^{-1}\) term in Eq. (10) in comparison to \(\Pi(q, i\omega_n)\);\(V_s(q, i\omega_n)\) then takes the universal form,

\[
2N(0)V_s(q, i\omega_n) = \frac{Dq^2 + |\omega_n|}{Dq^2}.
\]

In the triplet channel the dominant contribution arises from momentum transfers of the order of \(2k_F\), and the unscreened triplet interaction can be treated as a constant. The disorder screened triplet interaction then takes the form

\[
2N(0)V_t(q, i\omega_n) = F \frac{Dq^2 + |\omega_n|}{(F + 1)Dq^2 + |\omega_n|},
\]

where \(F = 2N(0)V_t^0\) and \(V_t^0\) is the bare interaction in the triplet channel. Note that \(F\) includes Fermi liquid corrections in the absence of disorder (it is only unscreened with respect to the disorder). \(F\) may be determined from the measured paramagnetic spin susceptibility in the experimental system of interest

\[
\chi = \frac{2N(0)}{1 + F}
\]

If we first calculate the contribution from the singlet channel, we can then obtain the corresponding result for the triplet channel by replacing \(V_s\) by \(V_t\) and multiplying by \(3\) – the extra factor of \(3\) arises because there are \(3\) times as many modes in the spin-one channel than in the spin-zero channel.

Applying standard perturbation theory we find that the leading order interaction corrections are given by the Feynman diagrams shown in Fig. 2. These diagrams are identical to those considered by Altshuler and Aronov for the electrical conductivity, with the exception of diagram (F) which we call the “heat-drag” term. The corresponding diagram for electrical conductivity is a factor \((T/E_F)^2\) smaller than the other diagrams due to cancellation of electron and hole charge currents; the heat current has opposite electron-hole parity and thus electron and hole heat currents reinforce each other. Consequently diagram (F) is of the same size as the other diagrams in the case of thermal conductivity. The use of the Matsubara heat current vertex defined in Eq. (11) ensures that no interaction renormalization of vertices is needed, greatly simplifying the calculation with respect to other choices of heat current operator (obviously the final results must be independent of this choice).

![Feynman diagrams](image)

FIG. 2: Feynman diagrams for the interaction correction to the heat-current response function. Diagrams A–E are structurally identical to the diagrams which contribute to electrical conductivity. Diagram F is the additional “heat-drag” diagram which must be included to obtain a consistent result.

Details of the calculation of the contributions from diagrams A–F to the heat-current response function, \(Q_{hh}(i\Omega)\), are presented in the appendix (note that in the following we suppress the subscripts on Matsubara frequencies \(\Omega_p = 2\pi T p\) and \(\omega_n = 2\pi T n\) for convenience). It is shown there that \(Q_{hh}(0)\) vanishes, as it must for internal consistency, and that \(\kappa T\) may be written as the limit of the expression

\[
+4N(0)D \frac{T}{\Omega} \sum_{0<\omega\leq\Omega} \sum_q \omega^3 \left[ 1 - \frac{8}{3d (Dq^2 + \omega)} \right] \frac{V(q, \omega)}{(Dq^2 + \omega)^2}
\]

\[
-\frac{16}{3d} T^2 N(0)D \left[ \frac{T}{\Omega} \sum_{\omega>0} \sum_{0<\omega\leq\Omega} \omega \sum_q Dq^2 V(q, \omega) \right] \frac{1}{(Dq^2 + \omega)^3}
\]

(15)
as $\Omega$ tends to zero. As explained in the appendix, this statement is to be interpreted in the sense of first continuing $i\Omega_p$ to real frequencies, and then taking the limit of real frequency going to zero. The second line of Eq. (15) is just the interaction contribution to the electrical conductivity multiplied by the factor $\pi^2 T^2/3e^2$. In other words, this is the contribution to $\kappa T$ predicted by the Wiedemann-Franz law. The other two terms thus lead to violation of the Wiedemann-Franz law if they yield a non-zero result (as we indeed find that they do). The expressions in Eq. (15) may now be evaluated in one, two and three dimensions for both the singlet and triplet interactions to yield the results listed in Table 1. Note that all three terms in Eq. (15) give results having the same parametric form but with different constant prefactors; we must therefore evaluate all of them to get a correct final result. This statement is not quite true in two dimensions because of the presence of logarithmic terms, and we should keep the most singular terms in this case.

IV. CONCLUSIONS

We have calculated the interaction corrections to thermal conductivity in the diffusive regime of a disordered conductor. Our main result is that the Wiedemann-Franz law is violated in all dimensions; the predicted interaction results have the expected parametric dependences, but different numerical coefficients. For example, in the singlet channel in two dimensions, the actual logarithmic correction is half that predicted by the Wiedemann-Franz law. Unfortunately the experimental work of Bayot et al is the only work we know of on thermal conductivity in disordered conductors, and this only isolates a weak localization correction. In particular the two-dimensional system would seem to be a promising one to investigate. Bayot et al's disordered graphitic system showed weak localization effects whose magnitude was roughly 20% of the Drude term in the electronic thermal conductivity at a temperature of 2.9K. If it were possible to cleanly extract the phonon term, one could look at the electronic term as a function of both temperature and magnetic field, and distinguish weak localization and interaction effects. This would then experimentally settle the question of whether the Wiedemann-Franz law is violated in an interacting disordered system. Another sensible quantity to investigate experimentally would be the thermal Hall conductivity, which arises solely from electrons (although there can be phonon drag effects). In future work we intend to derive the interaction corrections to this quantity.

Acknowledgments

We thank A.M. Finkel’stein, I.V. Lerner and I.V. Yurkevich for helpful discussions. We acknowledge support from the UK EPSRC.

APPENDIX: DIAGRAMMATIC CONTRIBUTIONS

In this appendix we list the complete set of diagrammatic contributions for archival purposes. This should allow interested researchers to reproduce our results in detail. The total contribution to the heat-current response function, $Q_{h\alpha}(i\Omega)$, from all the diagrams in Fig. 2 is given by

\[
+16\pi N(0)^2 DT \sum_{\omega>0} \sum_q \omega^3 \left[ \frac{2Dq^2 V(q,\omega)}{(Dq^2 + \omega)^2} \right. \\
+ \frac{\partial V(q,\omega)}{\partial \omega} \left. \frac{Dq^2 V(q,\omega)}{(Dq^2 + \omega)^4} \right]
\]

\[
−8\pi N(0) D\delta_{\alpha\beta} T \sum_{\omega>0} \sum_{0<\epsilon<\omega} \left[ \epsilon + \Omega \right]^2 \frac{V(q,\omega)}{(Dq^2 + \omega)^2} \]

\[
−8\pi N(0) D\delta_{\alpha\beta} T \sum_{\omega>\Omega} \sum_{0<\epsilon<\omega-\Omega} \left[ \epsilon + \Omega \right]^2 \frac{V(q,\omega)}{(Dq^2 + \omega)^2} \]

\[
+8\pi N(0) D\delta_{\alpha\beta} T \sum_{\omega<\Omega} \sum_{0<\epsilon<\omega} \left[ \epsilon - \Omega \right]^2 \frac{V(q,\omega)}{(Dq^2 + \omega)^2} \]

\[
+8\pi N(0) D\delta_{\alpha\beta} T \sum_{\omega>\Omega} \sum_{0<\epsilon<\omega} \left[ \epsilon - \Omega \right]^2 \frac{V(q,\omega)}{(Dq^2 + \omega)^2} \]

\[
+16\pi N(0) D\delta_{\alpha\beta} T \sum_{\omega>\Omega} \sum_{0<\epsilon<\omega-\Omega} \left[ \epsilon + \Omega \right]^2 \frac{V(q,\omega)}{(Dq^2 + \omega)^2} \]
and checking that to evaluate by setting the external frequency, \( \Omega = 0 \), check that we have chosen a consistent set of diagrams.

Rem to convert the electrical conductivity calculation. At this point we can where in the last step we have used the divergence theorem to convert the \( q \)-integral into a surface integral with

\[
+32\pi N(0)D\delta_{\alpha\beta} T \sum_{\omega > 0} \sum_q \omega^3 \frac{V(q, \omega)}{(Dq^2 + \omega)^2} - 4N(0)D\delta_{\alpha\beta} T \sum_{0 < q \leq \Omega} \sum_q \omega^2 \Omega \frac{V(q, \omega)}{(Dq^2 + \omega)^2}
\]  

\[
+32\pi N(0)D\delta_{\alpha\beta} T \sum_{\omega > \Omega} \sum_q \omega^3 \frac{V(q, \omega)}{(Dq^2 + \omega + \Omega)^2} - 4N(0)D\delta_{\alpha\beta} T \sum_{0 < q \leq \Omega} \sum_q \omega^2 \Omega \frac{V(q, \omega)}{(Dq^2 + \omega + \Omega)^2}
\]  

\[
-32\pi N(0)D\delta_{\alpha\beta} T \sum_{\omega > \Omega} \sum_q \omega^3 \frac{V(q, \omega)}{(Dq^2 + \omega)^2} - 4N(0)D\delta_{\alpha\beta} T \sum_{0 < q \leq \Omega} \sum_q \omega^2 \Omega \frac{V(q, \omega)}{(Dq^2 + \omega)^2}
\]  

\[
-32\pi N(0)D\delta_{\alpha\beta} T \sum_{\omega > \Omega} \sum_q \omega^3 \frac{V(q, \omega)}{(Dq^2 + \omega + \Omega)^2} - 4N(0)D\delta_{\alpha\beta} T \sum_{0 < q \leq \Omega} \sum_q \omega^2 \Omega \frac{V(q, \omega)}{(Dq^2 + \omega + \Omega)^2}
\]  

Note that the contributions from diagrams A, B and C do not cancel each other as they do in the corresponding electrical conductivity calculation. At this point we can check that we have chosen a consistent set of diagrams to evaluate by setting the external frequency, \( \Omega = 0 \), and checking that \( Q_{hh}(0) = 0 \). This is a very powerful test that should not be omitted – it is very dangerous to merely calculate \( [Q_{hh}(\Omega) - Q_{hh}(0)]/\Omega \), especially when a large number of diagrams are involved. One always runs the risk of missing important physical processes; in fact we only became aware of the presence of the heat-drag term of diagram (F) when this check failed in its absence. Setting \( \Omega = 0 \) here yields

\[
Q_{hh}(0) = 4N(0)D\delta_{\alpha\beta} T \sum_{\omega > 0} \sum_q \left[ -\delta_{\alpha\beta} + \frac{4Dq_{\alpha}q_{\beta} V(q, \omega)}{(Dq^2 + \omega)^2} + \frac{4\omega Dq_{\alpha}q_{\beta} N(0)V(q, \omega)}{(Dq^2 + \omega)^2} \right] \omega^3 V(q, \omega) \frac{(Dq^2 + \omega)^2}{(Dq^2 + \omega)^2}
\]

where in the last step we have used the divergence theorem to convert the \( q \)-integral into a surface integral with its bounding surface at infinity. Note that this check
not only gives us confidence that all relevant diagrams have been included, but also that each diagram has been given the correct combinatorial factors. In the above derivation we have treated $V_0(q)^{-1}$ as though it had no $q$-dependence. This is justified as the terms ignored are smaller in magnitude, and would be cancelled by higher order diagrams. In addition to the above proof that $Q_{hh}(0) = 0$, we have also directly evaluated $Q_{hh}(0)$ from Eq. (13). After performing this derivation we have treated $q$-dependence. This is justified as the terms ignored are smaller in magnitude, and would be cancelled by higher powers of $\Omega_p$. This approach is legitimate provided that any operation we perform on $\Omega_p$ would carry over unchanged to the same operation on the corresponding real frequency after analytic continuation. The advantage of this method is simply convenience in calculation – no illegal operations occur, as we have checked by performing the analytic continuation first and then taking the zero-frequency limit. This procedure leads to the expression for $\kappa T$ as the zero-frequency limit of

$$+4N(0)\frac{T}{\Omega} \sum_{0<\omega\leq\Omega} \sum_{q} \omega^3 \left[ 1 - \frac{8}{3d} \frac{Dq^2}{(Dq^2 + \omega)} \right] \frac{V(q, \omega)}{(Dq^2 + \omega)^2}$$

$$-\frac{16}{3d} \pi^2 T^2 N(0) D \left[ T \sum_{\omega>0} \Omega + \frac{T}{\Omega} \sum_{0<\omega\leq\Omega} \omega \right] \sum_{q} Dq^2 V(q, \omega) \frac{(Dq^2 + \omega)^3}{(Dq^2 + \omega)^2}$$

$$+\frac{16}{d} N(0)^2 DT \sum_{\omega>0} \sum_{q} \omega^3 \left[ \frac{2Dq^2 V(q, \omega)}{(Dq^2 + \omega)} + \omega \frac{\partial V(q, \omega)}{\partial \omega} \right] \frac{Dq^2 V(q, \omega)}{(Dq^2 + \omega)^4} \quad (A.4)$$

We can then perform the $q$-integral using the standard identification

$$\sum_{q} = \int \frac{d^dq}{(2\pi)^d} \quad (A.5)$$

and the formulae for the $V(q, \omega)$ given in Eq. (10) and Eq. (13). After performing this $q$-integral we end up with terms which are infinite sums over powers of $\omega$. Provided that the sums are ultraviolet divergent, we can identify them as zeta functions via

$$T \sum_{\omega>0} \omega^k = T(2\pi T)^k \sum_{n>0} n^k = T(2\pi T)^k \zeta(-k). \quad (A.6)$$

Any sum that is infrared divergent must be cut off correctly since such a divergence is physical – such a situation occurs in two-dimensions where we obtain a logarithmic sum which is cut off at $\omega \sim T$ at low frequency and $\omega \sim 1/\tau$ at high frequency. We have used this particular divergent series trick as it allows a very direct evaluation of results. As a check of its legality we have recalculated the various terms using standard analytic continuation methods and obtained the same results – albeit after a lot more algebraic manipulation.

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