Effective optical constants in stratified metal-dielectric metamaterial

Masanobu Iwanaga
Department of Physics, Graduate School of Science, Tohoku University, Sendai 980-8578, Japan
(Dated: March 31, 2022)

We present effective optical constants of stratified metal-dielectric metamaterial. The effective constants are determined by two complex reflectivity method (TCRM). TCRM reveals full components of effective permittivity and permeability tensors and indicates the remarkable anisotropy of metallic and dielectric components below effective plasma frequency. On the other hand, above the plasma frequency, one of the effective refractive indexes takes a positive value less than unity and is associated with small loss. The photonic states are confirmed by the distribution of electromagnetic fields.

Photonic metamaterials attract great interest as a new type of materials including magnetic resonance at optical frequency. A famous example exploiting magnetic resonance is negative refraction.\(^1\)

Evaluation of effective optical constants at optical frequency has been performed for metamaterials of thin layers.\(^2\) In analyzing bulk metamaterials, reflective polarimetry is only way to obtain effective optical constants. It is necessary in the analysis to satisfy equation of dispersion. In this Letter we present a way of analysis, two complex reflectivity method (TCRM), to determine full components of effective tensors of permittivity \(\varepsilon\) and permeability \(\mu\). Moreover, the photonic states implied by the effective optical constants are examined.

Figure 1 shows stratified metal-dielectric metamaterial (SMDM) and coordinate system. The composite material is obviously uniaxial and the \(z\) axis is set parallel to the optical axis. We set here metal to be Ag and dielectrics MgF\(_2\). The pair of constituents is selected to reduce loss in materials and the interfaces.\(^3\)

We assume that effective tensors \(\varepsilon\) and \(\mu\) are diagonal and describe local response in media. Principal axes of both tensors are set to be \(x, y,\) and \(z\) axes in Fig. 1(a). In the configuration shown in Fig. 1(b), incident light is \(s\) polarization (that is, \(E_{in}||y\)), and the following relation is derived from Maxwell boundary conditions concerning bulk material in vacuum \((n_1=1, \varepsilon_1=1,\) and \(\mu_1=1)\):

\[
\frac{k_z(\theta)}{\mu_x} = \frac{n_1 \cos \theta}{\mu_1} \cdot \frac{1 - r_s(\theta)}{1 + r_s(\theta)}.
\] (1)

We set \(E(r, t) \propto \exp(\text{i}(k \cdot r - \omega t))\) and \(k = k/k_0\) where \(k_0\) is wavenumber of light in vacuum. The component \(k_z\) represents the refraction; in particular, \(k_z(0)\) is refractive index along \(z\) axis. Complex reflectivity \(r_s\) was obtained by the numerical calculation based on scattering-matrix method\(^4\) improved in numerical convergence.\(^5\) Optical constants of silver were taken from literature\(^6\) and the refractive index of MgF\(_2\) was set 1.38.

Equation of dispersion under \(s\) polarization is

\[
\varepsilon_y = \frac{k_z(\theta)^2}{\mu_x} + \frac{k_x(\theta)^2}{\mu_z}
\] (2)

where \(k_x(\theta) = n_1 \sin \theta\).

After substituting Eq. (1) for Eq. (2), two different angles enable to evaluate \(\varepsilon_y/\mu_x\) and \(\mu_x\mu_z\) uniquely. The products of \(\varepsilon_x/\mu_y\) and \(\mu_y\mu_z\) are obtained by permuting configuration \((x, y, z) \rightarrow (y, z, x)\). In uniaxial media of \(\varepsilon_x = \varepsilon_y\) and \(\mu_x = \mu_y\), only the two configurations are enough to determine all the values of tensors of \(\varepsilon\) and \(\mu\) except for the sign. We call this procedure TCRM. This method is generalization of two reflectance method (TRM),\(^7,8\) which is valid for materials of \(\mu = 1\). In TRM, conformal mapping between reflectance at two incident angles plays a crucial role and retrieves the phase of complex reflectivity. On the other hand, if it is assumed that \(\mu 
(\neq 1)\), we need two complex reflectivity to evaluate \(\varepsilon\) and \(\mu\) tensors. Thus, though TCRM stems from TRM, the formalism is quite different.

It is nontrivial how to determine the sign of \(\mu_x\). Actually we take the sign to satisfy \(\mu_x \approx 1\) at off-resonance frequency.\(^9\)

FIG. 1: (a) Schematic drawing of SMDM and coordinate configuration. Gray indicates metal (silver) layers and write is dielectric (MgF\(_2\)) layers. (b) Optical configuration of TCRM. Incident light is \(s\) polarization, that is, \(E_{in}||y\).
The incidence on

FIG. 2: (a) Reflectance spectra. Solid line: under normal

incidence on $\mu$ in a wide energy

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behavior is discussed later in more details.

The resonance deviates from simple Lorentzian
ds and is associated with resonant behavior of

The effective plasma frequency $\omega_{p,eff}$ is located at $h\omega_{p,eff} = 2.6$ eV. On the other hand, $\varepsilon_z$ implies that SMDM is dielectrics for the light of $E[z]. Below 3.5 eV there exist no prominent magnetic resonance in SMDM, while magnetic resonance appears at 3.7 eV. The resonance deviates from simple Lorentzian dispersion and is associated with resonant behavior of permittivity. The 3.7 eV resonance is thus mixed one of electromagnetic (EM) components and shows complex response due to anisotropy of SMDM. This resonant behavior is discussed later in more details.

Figure (d) presents effective refractive indexes. SMDM shows a metallic color similar to gold in seeing from $xy$ plane, while it is transparent dielectric material of refractive index 1.8 in visible range for the light of $E[z]$ in $yz$ or $xz$ planes. In particular, loss in SMDM is quite suppressed in the wide range of 2.6-3.5 eV. This is a feature of SMDM. Although the permittivity of bulk silver implies large loss in the energy range, the composite of silver and MgF$_2$ is nearly free from loss. This result

energy of 1.5 eV and to connect $\mu_x$ at resonance without discontinuous jump. Once the sign of $\mu_x$ is identified, other components of $\mu_z$, $\varepsilon_x$, and $\varepsilon_z$ are evaluated uniquely; besides, the refractive component $k_z$ is also evaluated uniquely by Eq. (1). As shown in Fig. 2 this way determines effective tensors $\varepsilon$ and $\mu$ in a wide energy range.

Figure (a) shows reflectance spectra in the two configuration for TCRM: solid line indicates reflectance spectra under normal incidence on $xy$ plane and dotted line under normal and $E_{in}\parallel z$ incidence on $yz$ plane. The periodic-
FIG. 4: Profile of electric field $E_z$ at 3.74 eV (solid line) under normal incidence on $xy$ plane. Dashed line is the profile calculated from the effective refractive index $n_z$ at 3.74 eV in Fig. 2(d). Weak reflection is not included in vacuum for simplicity.

suggests that EM fields in metallic layers help transmission highly efficient; that is, it suggests that there exists EM-field enhancement. Indeed, above $\omega_{p,\text{eff}}$, several layers of metal and dielectrics shows high transmittance. Figure 2(d) indicates that high-efficient transmission in SMDM is associated with effective refractive index $n_z$ of $0 < \text{Re}(n_z) < 1$. The effective index thus provides a concise description for the photonic state. Next, we examine EM-field distribution in SMDM to clarify whether the state is actually realized.

Figure 3 displays the $E_y$ profile along $z$ axis at 3.0 eV. Incident light travels along $z$ axis and illuminates on $yz$ plane with $E_{\text{in}}||y$. Solid line shows incident light in vacuum and the refracted component in SMDM. In this configuration the effective refractive index is $n_z = 0.67 + 0.04i$ from Fig. 2(d). The $E_y$ reproduced by $n_z$ is drawn with dashed line. The wavelength in SMDM is obviously close to $\lambda/\text{Re}(n_z)$ and longer than $\lambda$ (wavelength in vacuum). Consequently, it is definitely confirmed that the effective description for SMDM works well. In particular, the effective description is excellent agreement within a $\lambda/|4\text{Re}(n_z)|$ scale from the interface, and suggests that TCRM mainly analyzes the EM responses within the $\lambda/|4\text{Re}(n_z)|$ depth. The index $n_z$ represents the low-loss photonic state in a wide range of 2.6-3.5 eV, where the phase velocity exceeds the velocity of light $c$ in vacuum. Figure 3 also shows that the EM wave in SMDM is not ideal plane wave and is affected by periodic structure. It is one of the inevitable properties in mesoscopic metamaterial optics.

At the end of discussion, we refer to the unusual behavior of $\text{Im}(n_z)$ at 3.7 eV in Fig. 2(d). At the resonance, $\text{Im}(n_z)$ takes negative value and seems to imply exponential growing wave. However, as shown in Fig. 4, the EM distribution at 3.7 eV (solid line) does not show any such growing wave, and indicates EM-field enhancement in metallic layers. The index $n_z$ implies the profile drawn with dashed line in Fig. 4 and roughly reproduces the EM fields in SMDM within a half wavelength scale from the interface. Why does such unusual behavior appear? In Fig. 4, the wavelength in vacuum is $\lambda = 331$ nm and the periodicity of SMDM is 75 nm. It therefore seems still possible to use the effective description; however, the other component is $\text{Re}(n_x) = 2.0$ and $\lambda/\text{Re}(n_x) = 166$ nm which is about twice the periodicity. The condition in Fig. 4 is likely close to limits of effective description. Additionally we note that the limits of effective description were eagerly debated in GHz range; in the limits of the index is at present understood to be $\lambda/|\text{Re}(n_{\text{eff}})| \sim \text{periodicity}$.

From numerical results and discussion, we can extract a few lessons in TCRM analysis: (i) as for wavelength $\lambda$ in vacuum and periodicity $a$ in metamaterial, it is at least necessary that $\lambda/|\text{Re}(n_{\text{eff}})| > 2a$, in order to obtain usual effective refractive index, $\text{Im}(n_{\text{eff}}) \geq 0$. We state that this condition is satisfied below 3.5 eV in Fig. 4 and that both $\varepsilon$ and $\mu$ are determined successfully: (ii) effective optical constants are determined by the optical response mainly within a $\lambda/|4\text{Re}(n_{\text{eff}})|$ scale from the interface.

In conclusion, we have presented the scheme of TCRM, applied TCRM to SMDM, and revealed the full components of $\varepsilon$ and $\mu$. Analysis of EM distribution has confirmed that the effective refractive index well describes the photonic states in SMDM below 3.5 eV. The effective index indicates that the low-loss photonic states with phase velocity larger than $c$ exists in a wide energy range above $\omega_{p,\text{eff}}$.

The author thanks T. Ishihara for discussion and comments on manuscript. This study is partially supported by Research Foundation for Opto-Science and Technology, and by Information Synergy Center, Tohoku University in numerical implementation.

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