Magnetic Spin-Up of Line-Driven Stellar Winds

Stan Owocki\textsuperscript{1,2} and Asif ud-Doula\textsuperscript{3}

\textsuperscript{1} Bartol Research Institute, University of Delaware, Newark, DE 19350 USA
\textsuperscript{2} Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ UK
\textsuperscript{3} Department of Physics, North Carolina State University, Raleigh, NC 27695-8202 USA

Abstract. We summarize recent 2D MHD simulations of line-driven stellar winds from rotating hot-stars with a dipole magnetic field aligned to the star’s rotation axis. For moderate to strong fields, much wind outflow is initially along closed magnetic loops that nearly corotate as a solid body with the underlying star, thus providing a torque that results in an effective angular momentum spin-up of the outflowing material. But instead of forming the “magnetically torqued disk” (MTD) postulated in previous phenomenological analyses, the dynamical simulations here show that material trapped near the tops of such closed loops tends either to fall back or break out, depending on whether it is below or above the Keplerian corotation radius. Overall the results raise serious questions about whether magnetic torquing of a wind outflow could naturally result in a Keplerian circumstellar disk. However, for very strong fields, it does still seem possible to form a centrifugally supported, “magnetically rigid disk” (MRD), in which the field not only forces material to maintain a rigid-body rotation, but for some extended period also holds it down against the outward centrifugal force at the loop tops. We argue that such rigid-body disks seem ill-suited to explain the disk emission from Be stars, but could provide a quite attractive paradigm for circumstellar emission from the magnetically strong Bp and Ap stars.

1. Background and Context

Within the broad theme of this conference on “Magnetic Fields in O, B, and A Stars”, a key issue is the role of magnetic fields in channeling the line-driven stellar wind outflows from such hot, luminous stars. Initial results of full MHD simulations for models without stellar rotation (ud-Doula and Owocki 2002) are reviewed in these proceedings through the write-up for the talk presented by A. ud-Doula. Here we summarize further simulation results for rotating hot-stars with a magnetic dipole aligned to the stellar rotation axis. These simulations
provide a timely dynamical test of the “Magnetically Torqued Disk” (MTD) paradigm, which, as reviewed in the talk by J. Brown in these proceedings, has recently been promoted as a central mechanism for producing the disks inferred in Be stars (Cassinelli et al. 2002).

To allow greater emphasis on our MHD results, the broader discussion of alternative Be-disk models given in the oral version of the present paper will not be reproduced here, as most of this has already been included in the write-up for a related talk presented at the recent IAU Symposium 215 on Stellar Rotation (Owocki 2003). A key general result of these MHD simulations regards the tendency for magnetic torquing of wind material to result in centrifugal mass ejection rather a circumstellar disk.

2. Alfven Radius vs. Keplerian Rotation Radius

Recent MHD simulations (ud-Doula 2002; ud-Doula and Owocki 2002) indicate that the effectiveness of magnetic fields in channeling a stellar wind outflow can be characterized by the ratio of the magnetic to wind energy densities

$$\eta(r) \equiv \frac{B^2}{2 \rho v^2} = \eta_\ast \frac{(r/R_\ast)^{2-2q}}{(1 - R_\ast/r)^\beta}.$$ (1)

Here $\eta_\ast \equiv B_\ast^2 R_\ast^2/(\dot{M} v_\infty)$ defines an overall “magnetic confinement parameter” in terms of the strength of the equatorial field $B_\ast$ at the stellar surface radius $R_\ast$, and the wind terminal momentum $\dot{M} v_\infty$. The latter equality thus isolates the radial variation in terms of a magnetic power-law index $q$ (= 3 for a dipole) and a velocity index $\beta$ (≈ 1 for a standard CAK wind). If, for simplicity, we ignore the wind velocity variation (i.e. by taking $\beta = 0$), we can easily solve for an “Alfven radius” $\eta(R_A) \equiv 1$ at which the magnetic and wind energy densities are equal

$$R_A = \eta_\ast^{1/4} R_\ast.$$ (2)

As shown by simulation results summarized below, this Alfven radius provides a reasonable estimate for the maximum radius of closed loops in a wind outflow. Moreover, since in rotating models such closed loops tend to keep the outflow in rigid-body rotation with the underlying star, it also defines the radius of maximum rotational spin-up of the wind azimuthal speed.

To characterize such rotational effects, let us next define a Keplerian corotation radius $R_K$ at which rigid-body rotation would yield an equatorial centrifugal acceleration that just balances the local gravitational acceleration from the underlying star,

$$\frac{GM}{R_K^2} = \frac{v_\phi^2}{R_K} = \frac{V_{eq}^2 R_K}{R_\ast^2},$$ (3)

where $V_{eq}$ is the stellar surface rotation speed at the equator. This can be solved to yield

$$R_K = W^{-2/3} R_\ast,$$ (4)

where $W \equiv V_{eq}/V_{crit}$, with $V_{crit} \equiv \sqrt{GM/R_\ast}$ the critical rotation speed.
Finally, it is also worth noting here that corotation out to an only slightly higher “escape radius”,

$$R_E = 2^{1/3} R_K = 2^{1/3} W^{-2/3} R_*,$$

would imply an azimuthal speed that equals the local escape speed from the star’s gravitational field.

### 3. MHD Models with Parameters Optimized for Keplerian Spin-up

The above scalings suggest that a likely necessary condition for propelling outflowing material into a Keplerian disk is to choose a combination of parameters for magnetic confinement vs. stellar rotation such that $R_K < R_A < R_E$. In the parameter plane of $\sqrt{\eta_*} \sim B_*$ vs. $W$ defined in figure 1, the required combination is represented by the relatively narrow gray band. The dark region below this represents parameter combinations for which the azimuthal speed at the Alfven radius is expected to be sub-Keplerian, while the white region above represents cases for which the rotation speed at the Alfven radius should exceed the local gravitational escape speed.

As a sample test case, we focus here on the specific combination $\eta_* = 10$ and $W = 1/2$, which as shown by the dot in figure 1, lies in the middle of the gray domain, and thus should represent an optimal case for magnetic spin-up into Keplerian orbit.

Figures 2 and 3 illustrate results of 2D MHD simulations for this case, using the approach and general model assumptions described in ud-Doula and Owocki (2002), but now extended to include field-aligned rotation. Figure 2 shows that conditions at a time 90 ksec after introduction of the field do superficially resemble a magnetically torqued disk. In the upper panel the darkest regions – which represent a near rigid-body rotation with $V_{\phi}/V_{\text{rigid}} \approx 1$ – include essentially the entire closed magnetic loop. The log-density grayscale in the lower panel shows moreover that magnetic channeling of the outflowing stellar wind leads to formation of a dense equatorial compression that might be interpreted as the expected circumstellar disk.
Figure 2. Results of a 2D MHD simulation for a model with $\eta_* = 10$ and $W = 1/2$, shown at time snapshot 90 ksec after a dipole field is introduced into an initially steady-state, unmagnetized, line-driven wind. In the lower panel the grayscale represents the log of the mass density, while in the upper panel this represents the ratio of the azimuthal speed $V_\phi$ to the local rigid-body rotation speed, $V_{\text{rigid}} = V_{\text{rot}} \sin \theta (r/R_*)$, with white to black corresponding to the range 0.7 to 1. The curves denote magnetic field lines, and the vertical dashed lines indicate the equatorial location of the Keplerian, Alfvén, and Escape radii defined in eqns. (2), (4), and (5) of the text.
Figure 3. Same as lower panel of figure 2, except for time $t=390$ ksec, with arrows illustrating the upward and downward flow direction of dense material above and below the Keplerian radius.
Closer examination shows, however, that most of this equatorial compression does not have the appropriate velocity for a stable, stationary, Keplerian orbit. Thus in just a few ksec of subsequent evolution, this putative “disk” becomes completely disrupted, characterized generally by infall of the material in the inner region, i.e. below the Keplerian radius $R_K$, and by outflow in the outer region above this Keplerian radius. Figure 3 illustrates the irregular form of the dense compression at an arbitrarily chosen later time (390 ksec from the initial start). The arrows emphasize the flow divergence of the dense material both downward and upward from the Keplerian radius. This evolution is most vividly illustrated through animations, which can be viewed on the web at: www.bartol.udel.edu/~owocki/animations/den4wp5eta10.avi

We have carried out similar MHD simulations for a moderately extensive set of combinations for the rotation and magnetic confinement parameters. In all cases we find that any equatorial compressions are dominated by radial inflows and/or outflows, with no apparent tendency to form a steady, Keplerian disk. In weak magnetic spin-up cases with $R_A < R_K$, material trapped on closed loops generally falls back on the star, much as in the non-rotating models discussed in the talk by A. ud-Doula (see also ud-Doula 2002 and ud-Doula and Owocki 2002). The intermediate magnetic spin-up cases with $R_K < R_A < R_E$ (lying in the gray area of figure 1) generally show a combination of infall and outflow, much as described above. Finally, for the strong magnetic spin-up models with $R_A > R_E$ the material density tends to build up at the tops the highest loops before breaking out in semi-regular episodes of discrete “mass ejections”.

Unfortunately technical issues make it difficult to carry out simulations with very high magnetic fields and/or very rapid stellar rotation. For rotations near the critical speed, the star becomes distorted into an oblate spheroid, thus making it difficult to formulate a static atmosphere lower boundary condition within our spherically symmetric version of the ZEUS MHD code. For very strong magnetic fields (i.e. $\eta^* > 100$), the large associated Alfvén speed implies a small Courant time-step within the explicit time-stepping of the Zeus MHD code, and so such cases become very computationally expensive to evolve through the required few characteristic flow evolution times. Further work is thus needed to explore such cases of near-critical rotation and/or very strong magnetic confinement.

4. Magnetically Confined “Rigid-Body” Disks

In lieu of detailed simulations, it is however possible to infer some likely attributes of magnetically torqued outflows in the strong field limit, for which the magnetic lines-of-force can be idealized as “rigid pipes” that channel the wind outflow toward a collision near fixed loop tops. For the case of a rotation aligned dipole, this effectively concentrates material at the combined magnetic/rotational equator. Below the Keplerian radius, such material will again tend to fall back to the star, but above this radius, it will be centrifugally pushed against the loop top, gradually building in mass until it finally has a sufficient rotational energy to break open the field. By comparing the gravitational-rotational energy of this building disk to the magnetic field energy, one can derive an estimate for this “break-out” time. For typical stellar
parameters, the ratio of this to a characteristic wind flow time $\tau_{\text{flow}} = R_*/V_\infty$ scales roughly as

$$\frac{\tau_{\text{break}}}{\tau_{\text{flow}}} \approx \frac{0.01\eta_* W^{-4/3}}{(r/R_K)^4 - (r/R_K)}.$$  \hspace{1cm} (6)

Such a disk would thus have a roughly constant inner edge at the Keplerian radius $R_K$, but a highly variable outer edge. Indeed, having any sort of disk of accumulated material that extends much beyond $R_K$ requires that the numerator in eqn. (6) exceed unity. In that case, we can estimate a maximum outer cutoff radius to be where $\tau_{\text{break}} \approx \tau_{\text{flow}}$, which solves to

$$R_{\text{out, max}} \approx \frac{R_A}{\sqrt{100}}.$$  \hspace{1cm} (7)

A key point here, however, is that such disks would be quite distinct from a Keplerian disk, both physically and in terms of likely observational signatures. In particular, they seem likely to be much more variable, characterized by intervals with substantial radial flow speeds in the outer disk. Perhaps even more notably, their rotation would follow a rigid-body law, $v_\phi \sim r$, instead of the Keplerian form, $v_\phi \sim 1/\sqrt{r}$.

In terms of a proposed application to Be-star disks, both properties seem to run counter to traditional interpretations of Be-disk emission line-profiles (Hanuschik 1995), which in many Be stars exhibit features (e.g. “central-quasi emissions”; Rivinius et al. 1999) that suggest a quite low upper limit (ca. 10-20 km/s) to radial outflow speeds, and moreover seem generally consistent with an azimuthal speed that follows a Keplerian law. The problems becomes particularly acute for extended rigid-body disks, since these imply more rapid rotation speeds than are typically inferred from Be emission-line-widths. For disks with a limited radial extent, the differences from Keplerian rotation are less dramatic, and as explored in the poster contribution by D. Telfer et al., the resulting profiles could indeed be consistent with existing observational analyses.

However, the long-term V/R variations seen in a substantial fraction of Be stars do still seem best explained by a Keplerian disk undergoing a precession of elliptical orbits that characterize an one-arm disk oscillation (Savonije and Heemskerk 1993; Telting et al. 1994; Savonije 1998). In a rigid-body disk in which the individual fluid elements of the disk are tied to the rotation period of the star, it is difficult to see how any processes could reproduce the year-long timescales of V/R asymmetries that require an associated long-term distinction in the physical properties of emitting material in a specific fixed-frame direction.

On the other hand, such rigid-body magnetic disks may represent a good paradigm for the observed circumstellar emission from Bp stars (see, e.g. talk write-up by Groote and references therein). The observed ca. 10 kG fields of these stars combined with their generally modest mass loss rates $\dot{M} \sim 10^{-9} M_\odot/yr$ imply a huge magnetic confinement parameter, of order $\eta_* \sim 10^6$. With the associated Alfvén radii of $R_A \approx 30R_*$, even the generally modest rotations of stars with, say, $W \approx 0.1$, would allow centrifugal disk support above $R_K \approx 10^{2/3}R_* \approx 4.6R_*$, extending perhaps out to $R_{\text{out, max}} \approx 100R_*$. Such low-density winds are moreover subject to ion runaway instability (Krticka and Kubat 2000, 2001; Owocki and Puls 2002), and this likely plays an important role in the
abundance variations often associated with the circumstellar emission from Bp stars. Future efforts should thus explore further such a wind-fed “magnetically rigid disk” (MRD) paradigm for Bp stars.

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