How can gravitational-wave standard sirens and 21 cm intensity mapping jointly provide a precise late-universe cosmological probe?

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In the next decades, the gravitational-wave (GW) standard siren observations and the neutral hydrogen (HI) 21 cm intensity mapping (IM) surveys, as two promising non-optical cosmological probes, will play an important role in precisely measuring cosmological parameters. In this work, we make a forecast for cosmological parameter estimation with the synergy between the GW standard siren observations and the 21 cm IM surveys. We choose the Taiji observatory and the Einstein Telescope (ET) as the representatives of the GW detection projects and choose the Square Kilometre Array (SKA) phase I mid-frequency array as the representative of the 21 cm IM experiments. We find that the synergy of the GW standard siren observations and the 21 cm IM surveys could break the cosmological parameter degeneracies. The joint Taiji+ET+SKA data give $\sigma(H_0) = 0.23 \text{ km s}^{-1} \text{ Mpc}^{-1}$ in the $\Lambda$CDM model, $\sigma(w) = 0.025$ in the $w$CDM model, and $\sigma(w_a) = 0.057$ and $\sigma(w) = 0.275$ in the CPL model, which are better than the results of Planck 2018 TT,TE,EE+lowE+lensing+SNe+BAO. In the $\Lambda$CDM model, the constraint accuracies of $H_0$ and $\Omega_m$ are less than or rather close to 1%, indicating that the magnificent prospects for non-optical precision cosmology are worth expecting.

I. INTRODUCTION

The precise measurements of the cosmic microwave background (CMB) anisotropies initiated the era of precision cosmology [1, 2]. The $\Lambda$CDM model as the standard model of cosmology can fit the CMB data with breathtaking precision. Nevertheless, extra cosmological parameters in the extended cosmological models cannot be tightly constrained by solely using the CMB data due to the strong cosmological parameter degeneracies. Thus, the measurements of the late universe are needed as the supplements of the CMB data to break the cosmological parameter degeneracies. However, there are inconsistencies between the early and late universe. For example, the tension between the values of the Hubble constant inferred from the CMB observation [3] and the Cepheid-supernova distance ladder measurement [4] has now reached $4.2\sigma$ [4]. The Hubble tension has been intensively discussed in the literature [5–23]. It is now commonly believed that the Hubble tension is a severe crisis for cosmology [24, 25]. To solve the current cosmological tensions, one crucial way is to develop new powerful late-universe cosmological probes, besides conceiving novel cosmological models. Since the current measurements of the late universe are mainly based on optical observations, it is important to develop non-optical cosmological probes. Obviously, the gravitational-wave (GW) standard siren observations and the neutral hydrogen (HI) 21 cm radio observations are two promising non-optical cosmological probes.

The standard siren method could be applied in measuring cosmological parameters by establishing the relation between luminosity distance and redshift, which was first proposed by Schutz [26]. The absolute luminosity distance to the GW source could be directly obtained from the analysis of the GW waveform. If the source’s redshift can also be obtained by identifying its electromagnetic (EM) counterpart, then this GW-EM event could be treated as a standard siren for exploring the expansion history of the universe [27]. The first actual application of standard siren is using GW170817 [28] and its EM counterpart (GRB 170817A) [29, 30] to measure the Hubble constant, which gives a result with around 15% precision [31]. A further forecast analysis shows that the measurement accuracy of the Hubble constant could achieve about 2% using 50 similar standard siren events [32]. It can be anticipated that GWs could help resolve the Hubble tension with the accumulation of observed standard siren events. Recently, the GW standard sirens have been widely discussed [33–65].

The observations of GWs will be greatly developed in the next decades. The third-generation ground-based GW detectors, i.e., the Cosmic Explorer [66, 67] and the Einstein Telescope (ET) [68, 69], aimed at high frequency-band (a few hundred hertz) GW detections, will begin observing in the 2030s. At the same time, the space-based GW detectors, i.e., LISA [70–75], Tian-Qin [76–80], and Taiji [81–84], will open the window of detecting milli-hertz frequency-band GWs. These GW standard siren observations could play an important role in breaking the cosmological parameter degeneracies generated by the EM cosmological probes [53–62]. Since the ground-based and space-based GW detectors focus on different frequency bands, the multi-band observations of GWs are also important for the study of cosmology. Recently, the multi-band observations of GWs have been detailedly discussed. For example, Grimm and Harms [85] used the multi-band observations of GWs to estimate the GW sources’ parameters. Ng et al. [86] used the multi-band observations of GWs to search for ultra-
light bosons. Valiante et al. [87] used multi-band observations of GWs to unveil early black hole (BH) growth. In this work, we also simulate the standard sirens based on the multi-band GW observations.

Moreover, the HI 21 cm radio observation is another promising cosmological probe. In the post-reionization epoch of the universe, the HI is thought to reside in dense gas clouds embedded in galaxies, so it is essentially a tracer of the galaxy distribution. Actually, it is difficult to detect enough HI-emitting galaxies to make accurate cosmological analysis. However, we can simply measure the total HI intensity over comparatively large angular scales to study the large-scale structure of the universe, of which the method is called 21 cm intensity mapping (IM). Using the 21 cm IM technique, one could measure the scale of baryon acoustic oscillations (BAO) that is a cosmological standard ruler, thus accurately measuring the late-time expansion history of the universe. Many 21 cm IM experiments have been proposed to measure the HI power spectrum and other features of the large-scale structure, e.g., the Baryon acoustic oscillations from Integrated Neutral Gas Observations [88–91], the Five-hundred-meter Aperture Spherical radio Telescope [92–97], the Square Kilometre Array (SKA) [98–101], and the Tianlai cylinder array [102–104]. A series of forecasts indicate that 21 cm IM could play an important role in the cosmological parameter estimation [100, 105–107] (see Ref. [108] for a brief review).

Actually, as two promising non-optical cosmological probes, standard sirens and 21 cm IM have different advantages. The standard siren allows the direct measurement of $d_L(z)$ that is inversely proportional to $H_0$, so a large number of standard sirens could constrain $H_0$ well. The 21 cm IM survey, compared to the optical survey, has some advantages in such as larger survey volumes, deeper redshifts, higher survey efficiency, and so forth. In addition, the BAO measurements by 21 cm IM can provide the information of $H(z)$ that is related to $w(z)$ by only one integral, therefore, compared with the distance–redshift relation that is related to $w(z)$ by two integrals, radial BAO may provide better constraints on $w(z)$. This implies that the combination of standard sirens and 21 cm IM may constrain both $H_0$ and $w(z)$ well. Hence, we wish to investigate the capability of estimating cosmological parameters using the combination of these two non-optical cosmological probes.

Based on the motivations described above, in this work we focus on the synergy of the GW standard siren observations and the 21 cm IM surveys in cosmological parameter estimation. For the simulations of standard sirens, we choose Taiji and ET as the representatives of the GW detection projects. For the simulations of 21 cm IM observations, we choose SKA as the representative of the 21 cm IM experiments. Since the SKA phase I mid-frequency (SKA1-MID) array focuses on exploring the evolution of the late universe [99], we only consider SKA1-MID in this work. For the cosmological models, we take the ΛCDM, wCDM, and CPL models as typical examples. The flat ΛCDM model is taken as the fiducial model to generate mock data, with the fiducial values of cosmological parameters being set to the constraint results from Planck 2018 TT,TE,EE+lowlE [3].

This work is organized as follows. In Sec. II, we introduce the methods of simulating GW standard sirens. In Sec. III, we briefly describe the methods of simulating 21 cm IM data based on SKA. In Sec. IV, we give the constraint results and make some relevant discussions. The conclusion is given in Sec. V. Unless otherwise stated, we adopt the system of units in which $G = c = 1$ throughout this paper.

II. GRAVITATIONAL WAVE STANDARD SIREN OBSERVATION

A. Simulation of GW standard sirens from ET

The frequency band detected by the ground-based GW detectors corresponds to the coalescences of binary stellar-mass black holes, binary neutron stars (BNSs), or neutron star-black hole binaries. In this work, we assume that all the GW standard siren events detected by ET are produced by the coalescences of BNSs. For the redshift distribution of BNSs, we adopt the form in Ref. [47]. The Fourier transform $\tilde{h}(f)$ of the time domain waveform is given by

$$\tilde{h}(f) = A f^{-7/6} e^{i \Psi(f)} ,$$

where the Fourier amplitude $A$ is given by

$$A = \frac{1}{d_L} \sqrt{F_x^2 (1 + \cos^2(\iota))^2 + 4 F_y^2 \cos^2(\iota) \times \sqrt{5 \pi/96 \pi^{-7/6} \mathcal{M}_c^{5/6}}} ,$$

where $d_L$ is the luminosity distance to the GW source; $F_{x,y}$ are antenna pattern functions; $\iota$ is the inclination angle between the binary’s orbital angular momentum and the line of sight; $\mathcal{M}_c = (1 + z) m_1 m_2 / (m_1 + m_2) / (m_1 + m_2)$ is the observed chirp mass with component masses $m_1$ and $m_2$; $\eta = m_1 m_2 / (m_1 + m_2)^2$ is the symmetric mass ratio. The definition of the function $\Psi(f)$ can be found in Ref. [109]. The antenna pattern functions of ET are [47, 49]

$$F_x^{(1)}(\theta, \phi, \psi) = \sqrt{3} \left[ \frac{1}{2} (1 + \cos^2(\theta)) \cos(2\psi) \cos(2\psi) \right] ,$$

$$F_y^{(1)}(\theta, \phi, \psi) = \sqrt{3} \left[ \frac{1}{2} (1 + \cos^2(\theta)) \cos(2\phi) \sin(2\psi) + \cos(\theta) \sin(2\phi) \cos(2\psi) \right] ,$$

where $(\theta, \phi)$ are angles describing the location of the source in the sky, and $\psi$ is the polarization angle. Since
ET has three interferometers with 60° inclined angles between each other, the other two pattern functions are
\( F_{+,x}^{(2)}(\theta, \phi, \psi) = F_{+,x}^{(1)}(\theta, \phi + 2\pi/3, \psi) \) and \( F_{+,x}^{(3)}(\theta, \phi, \psi) = F_{+,x}^{(1)}(\theta, \phi + 4\pi/3, \psi) \).

Then we need to select the GW event based on its signal-to-noise ratio (SNR) and we set the SNR threshold to 8 in our simulation. The combined SNR for the network of \( N \) independent interferometers is given by
\[
\rho = \sqrt{\sum_{i=1}^{N} (\rho^{(i)})^2},
\]
where \( \rho^{(i)} = \sqrt{\langle \hat{h}^{(i)}, \hat{h}^{(i)} \rangle} \). The inner product is defined as
\[
\langle a, b \rangle = 4 \int_{f_{\text{lower}}}^{f_{\text{upper}}} \frac{\tilde{a}(f) \tilde{b}^*(f) + \tilde{a}^*(f) \tilde{b}(f)}{2} \frac{df}{S_n(f)},
\]
where \( \sim \) above the function denotes the Fourier transform of the function. Here, \( f_{\text{lower}} = 1 \) is the lower cutoff frequency; \( f_{\text{upper}} = 2/(63/2\pi m_{\text{obs}}) \) is the frequency at the last stable orbit with \( m_{\text{obs}} \) being \( m_{\text{obs}} = (m_1 + m_2)/(1+z) \); \( S_n(f) \) is the one-side noise power spectral density (PSD) and we adopt the PSD of ET from Ref. [47].

A few \( \times 10^5 \) BNS mergers per year could be observed by ET and only about 0.1% of them could have \( \gamma \)-ray bursts toward us [65], which means that a few \( \times 10^2 \) GW events’ redshifts could be obtained per year. In this work, we simulate 1000 GW standard siren events generated by BNS mergers corresponding to the 10-year operation time of ET. For each simulated source, the sky location \( (\theta, \phi) \), the mass of NS, and the polarization angle are evenly sampled in the ranges of \([0, \pi]\), \([0, 2\pi]\), \([1, 2]M_\odot\), and \([0, 2\pi]\), respectively, where \( M_\odot \) is the solar mass. Here the fiducial value of the inclination angle \( \iota \) is simply set to \( \iota = 0 \) [110]. As for the effect of \( \iota \) on the practical measurement uncertainty of \( d_L \), we shall discuss below.

For ET, we consider three measurement errors of \( d_L \), consisting of the instrumental error, the weak-lensing error, and the peculiar velocity error. The total error of \( d_L \) is
\[
(\sigma_{d_L})^2 = (\sigma_{d_L}^{\text{inst}})^2 + (\sigma_{d_L}^{\text{len}})^2 + (\sigma_{d_L}^{\text{pv}})^2.
\]

Using the Fisher information matrix, we can get the instrumental error of \( d_L \). It can be found that \( \sigma_{d_L}^{\text{inst}} \approx d_L/\rho \). Note that we have set \( \iota = 0 \) when we simulate the GW source, which is an ideal situation. Therefore, we add a factor of 2 to \( d_L/\rho \) to take into account the correlation between \( d_L \) and \( \iota \) [110].
\[
\sigma_{d_L}^{\text{inst}} = \frac{2d_L}{\rho}.
\]

In addition, the measurement of luminosity distance is affected by the weak lensing and we adopt the form in Refs. [111, 112]
\[
\sigma_{d_L}^{\text{len}}(z) = d_L(z) \times 0.066 \left[ 1 - (1 + z)^{-0.25} \right]^{1.8}.
\]
The error caused by the peculiar velocity of the source is given by [113]
\[
\sigma_{d_L}^{\text{pv}}(z) = d_L(z) \times \left[ 1 + \frac{c(1+z)^2}{H(z)d_L(z)} \right] \sqrt{\frac{\langle v^2 \rangle}{c}},
\]
where \( H(z) \) is the Hubble parameter, \( \sqrt{\langle v^2 \rangle} \) is the peculiar velocity of the source and we roughly set \( \sqrt{\langle v^2 \rangle} = 500 \text{ km s}^{-1} \).

### B. Simulation of GW standard sirens from Taiji

The frequency band detected by the space-based GW detectors corresponds to the massive black hole binary (MBHB) mergers. The unknown birth mechanisms of MBHB lead to the uncertainties in predicting the event rate of MBHB. Based on a semi-analytical galaxy formation model, three population models of MBHBs, i.e., the pop III, Q3d, and Q3nod models are proposed, based on the various combinations of the mechanisms of seeding and delay [114]. In Refs. [59, 60, 112], it is found that the Q3nod model gives the best constraints since the Q3nod model yields the most data points among these three models. In this paper, we only simulate standard siren events based on the Q3nod model.

The response functions of Taiji are given by
\[
F_+(t; \theta, \phi, \psi) = \frac{1}{2} \left( \cos(2\psi)D_+(t; \theta, \phi) - \sin(2\psi)D_\times(t; \theta, \phi) \right),
\]
\[
F_\times(t; \theta, \phi, \psi) = \frac{1}{2} \left( \sin(2\psi)D_+(t; \theta, \phi) + \cos(2\psi)D_\times(t; \theta, \phi) \right).
\]

Based on the low-frequency approximation, the forms of \( D_+, \times \) are given by [115]
\[
D_+(t; \theta, \phi) = \sqrt{\frac{3}{64}} \left[ -36\sin^2 \theta \sin(2\alpha(t) - 2\beta) + (3 + \cos(2\theta)) \right.
\]
\[
\left. \times \left( \cos(2\phi) \left( 9 \sin(2\beta) - \sin(4\alpha(t) - 2\beta) \right) \right.ight.
\]
\[
\left. + \sin(2\phi) \left( \cos(4\alpha(t) - 2\beta) - 9 \cos(2\beta) \right) \right]
\]
\[
- 4\sqrt{3} \sin(2\theta) \left( \sin(3\alpha(t) - 2\beta - \phi) - 3 \sin(\alpha(t) - 2\beta + \phi) \right) \right],
\]
\[
D_\times(t; \theta, \phi) = \frac{1}{16} \left[ \sqrt{3} \cos \theta \left( 9 \cos(2\phi - 2\beta) - \cos(4\alpha(t) - 2\beta - 2\phi) \right.ight.
\]
\[
\left. - 6 \sin \theta \left( \cos(3\alpha(t) - 2\beta - \phi) + 3 \cos(\alpha(t) - 2\beta + \phi) \right) \right].
\]
where $\alpha = 2\pi f_{\text{min}} t + \kappa$ is the orbital phase of the guiding center, and $\beta = 0$ is the relative phase of three spacecrafts. Here $\kappa = 0$ is the initial ecliptic longitude of the guiding center and $f_{\text{min}} = 1/\text{yr}$. Following Ref. [116], we equivalently consider Taiji as a combination of two independent interferometers with an azimuthal difference of $\pi/4$. Another equivalent antenna pattern function is $F_{\phi}^{(2)}(t; \theta, \phi, \psi) = F_{\phi}^{(1)}(t; \theta, \phi - \pi/4, \psi).

In order to study the signal in the Fourier space, we replace the observation time $t$ by \[ t(f) = t_c - \frac{5}{256} M_c^{-5/3}(\pi f)^{-8/3}, \tag{13} \]

where $t_c$ is the coalescence time of MBHB. In our analysis, we set $t_c = 0$.

We calculate SNR of each GW event using Eqs. (4)–(5), and we choose the SNR threshold of 8 for Taiji. In Eq. (5), Taiji’s PSD is taken from Ref. [82]. $f_{\text{lower}} = 10^{-4}$ is the lower frequency cutoff, and $f_{\text{upper}} = c^3/6\pi GM (1 + z)$ is the innermost stable circular orbit [119]. Following Ref. [60], we assume that Taiji’s detection rates of MBHBs are identical to those of LISA. We adopt the redshift distribution given in Ref. [120] and simulate 41 standard siren events based on 5-year operation time of Taiji. For each GW source, the sky position $(\theta, \phi)$, the mass of BH, the inclination angle, and the polarization angle are evenly sampled in the ranges of $[0, \pi], [0, 2\pi], [10^4, 10^7] M_\odot, [0, \pi]$, and $[0, \pi]$.

It is expected that MBHBs can produce EM signals since they are believed to merge in a gas-rich environment that may power EM emissions through jets, disk winds, or accretions. These EM signals can be applied in measuring their corresponding redshifts. If the redshift is measured spectroscopically, the errors produced by measuring the redshifts of the EM counterparts could be ignored. While if the redshift is measured photometrically, then the redshift error should be taken into account. Since the spectroscopic redshift in the range of $z > 2$ is almost unavailable [75, 121], we assume that the redshifts of GW events satisfying $z > 2$ are measured by photometry, while those satisfying $z < 2$ are measured spectroscopically. Hence, for the GW events satisfying $z > 2$, we take into account the additional redshift error $\sigma_{\text{red}}$ [75] in Eq. (6). We estimate the error on the redshift measurement as $\sigma_{\text{red}} = \frac{\partial d_L}{\partial z}(\Delta z)_n$ [122] and propagate it to the error on $d_L$

\[ \sigma_{\text{red}} = \frac{\partial d_L}{\partial z}(\Delta z)_n. \tag{14} \]

III. 21 CM INTENSITY MAPPING SURVEY

The 21 cm IM surveys will also be developed into a powerful non-optical cosmological probe. In this paper, we consider the SKA1-MID array that has 133 15-m SKA dishes and 64 13.5-m MeerKAT dishes as the representative of the 21 cm IM experiments. Note that we only consider the Wide Band 1 Survey of the SKA1-MID array (with the redshift range of $0.35 < z < 3$) with perfect foreground removal and calibration. For simplicity, we consider SKA1-MID as an array with 197 15-m dishes. In the following, we briefly introduce the signal power spectrum and the noise power spectrum, and construct the Fisher matrix.

The mean 21 cm brightness temperature is given by

\[ T_b(z) = 180 \Omega_{\text{HI}}(z) h (1 + z)^2 \frac{H(z)}{H_0} \text{ mK}, \tag{15} \]

where $h$ is the dimensionless Hubble constant. $\Omega_{\text{HI}}(z)$ is derived from a simulated HI halo mass function that can be written as

\[ \Omega_{\text{HI}}(z) \equiv (1 + z)^{-3} \rho_{\text{HI}}(z)/\rho_{c,0}, \tag{16} \]

where $\rho_{c,0}$ is the critical density today. $\rho_{\text{HI}}(z)$ is the proper HI density that can be calculated by

\[ \rho_{\text{HI}}(z) = \int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn}{dM} M_{\text{HI}}(M, z), \tag{17} \]

where $M$ is the mass of the dark matter halo, $dn/dM$ is the proper halo mass function, and $M_{\text{HI}}(M, z)$ is the HI mass in a halo of mass $M$ at the redshift $z$. For the detailed calculations, see Ref. [123].

Considering the effect of redshift space distortions (RSDs) [124] caused by the peculiar velocities of the HI clouds and the galaxies in which they reside, the signal power spectrum can be written as [123, 125]

\[ P_S(k, \mu, z) = T_b^2(z) \frac{D_A^2(z) H(z)}{D_A(z) H(z) f} b^2_{\text{HI}}(z)[1 + \beta_{\text{HI}}(z) \mu^2]^2 \times \exp(-k^2 \mu^2 \sigma_{\text{NL}}^2) P(k, z), \tag{18} \]

where the subscript “f” denotes the quantities calculated in the fiducial cosmology and $D_A(z)$ is the angular diameter distance. $b_{\text{HI}}(z)$ is the HI bias that can be calculated by

\[ b_{\text{HI}}(z) = \rho_{\text{HI}}^{-1}(z) \int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn}{dM} M_{\text{HI}}(M, z) b(M, z), \tag{19} \]

where $b(M, z)$ is the halo bias (for the detailed calculation, see Ref. [104]). $\mu = k \cdot \hat{z}$, and $\beta_{\text{HI}} \equiv f/b_{\text{HI}}$ is the RSD parameter, where $f \equiv d\ln D/d\ln a$ is the linear growth rate [with $a = 1/(1 + z)$ being the scale factor]. The exponential term accounts for the “Fingers of God” effect and $\sigma_{\text{NL}} = 7 \text{ Mpc}$ is the non-linear dispersion scale [126]. $P(k, z) = D^2(z) P(k, z = 0)$, with $D(z)$ being the growth factor and $P(k, z = 0)$ being the matter power spectrum at $z = 0$ that can be generated by CAMB [127].

Next, we consider the thermal noise and effective beams. The frequency resolution of IM survey performs very well due to the narrow channel bandwidths of SKA’s
receivers, so we ignore the instrumental response function in the radial direction and only consider the response due to the finite angular resolution:

$$W^2(k) = \exp \left[-k_\perp^2 r^2(z) \left( \frac{\theta_B}{\sqrt{2\pi}} \right)^2 \right], \quad (20)$$

where $k_\perp$ is the transverse wave vector, $r(z)$ is the comoving radial distance at redshift $z$, and $\theta_B$ is the full width at half-maximum of the beam of an individual dish.

The survey volume of a redshift bin between $z_1$ and $z_2$ can be written as

$$V_{\text{sur}} = \Omega_{\text{tot}} \int_{z_1}^{z_2} d\frac{r^2(z)}{H(z)}, \quad (21)$$

where $\Omega_{\text{tot}} = S_{\text{area}}$ is the solid angle of the survey area. The pixel volume $V_{\text{pix}}$ is also calculated with the similar formula with $\Omega_{\text{tot}}$ substituted by $\Omega_{\text{pix}} \approx 1.139 \theta_B^2$.

For the SKA1-MID array, the pixel noise is given by [123]

$$\sigma_{\text{pix}} = \sqrt{\frac{T_{\text{sys}}}{\Delta\nu}} \frac{\lambda^2}{A_e \theta_B^2} \frac{1}{\sqrt{N_{\text{dish}} N_{\text{beam}}}}, \quad (22)$$

where $T_{\text{sys}}$ is the system temperature; $N_{\text{dish}} = 197$ is the number of dishes; $N_{\text{beam}} = 1$ is the number of beam; $t_{\text{tot}} = 10000$ h is the total integration time; $A_e \equiv \eta I (D_{\text{dish}}/2)^2$ is the effective collecting area of each element; $\theta_B \approx \lambda / D_{\text{dish}}$; $D_{\text{dish}} = 15$ m is the diameter of the dish; $\eta$ is an efficiency factor (we adopt 0.7 in this work) and $S_{\text{area}} = 20000$ deg$^2$ is the survey area.

The system temperature of the SKA1-MID array can be divided into four parts [100],

$$T_{\text{sys}} = T_{\text{rec}} + T_{\text{spl}} + T_{\text{CMB}} + T_{\text{gal}}, \quad (23)$$

where $T_{\text{spl}} \approx 3$ K is the contribution from spill-over, $T_{\text{CMB}} \approx 2.73$ K is the CMB temperature, $T_{\text{gal}} \approx 25$ K $\times$ (408 MHz/$\nu$)$^{2.75}$ is the contribution from the Milky Way for a given frequency $\nu$, and $T_{\text{rec}}$ is the receiver temperature which is assumed to be [100]

$$T_{\text{rec}} = 15 \text{ K} + 30 \text{ K} \left( \frac{\nu}{\text{GHz}} - 0.75 \right)^2. \quad (24)$$

Finally, the noise power spectrum is given by

$$P_N(k) = \sigma_{\text{pix}}^2 V_{\text{pix}} W^{-2}(k), \quad (25)$$

and the Fisher matrix for a set of parameters $\{p\}$ is given by [128]

$$F_{ij} = \frac{1}{8\pi^2} \int_{-1}^{1} d\mu \int_{k_{\text{min}}}^{k_{\text{max}}} k^2 dk \frac{\partial \ln P^S}{\partial p_i} \frac{\partial \ln P^S}{\partial p_j} V_{\text{eff}}, \quad (26)$$

where the “effective volume” is defined as [123, 129]

$$V_{\text{eff}} = V_{\text{sur}} \left( \frac{P^S}{P^S + P_N} \right)^2. \quad (27)$$

In this work, we assume that $b_{\text{HH}}$ only depends on the redshift $z$. This assumption is appropriate only for large scales, so we impose a non-linear cutoff at $k_{\text{max}} \approx 0.14(1 + z)^2/3$ Mpc$^{-1}$ [130]. In addition, the largest scale the survey can probe corresponds to a wave vector $k_{\text{min}} \approx 2\pi / V_{\text{sur}}^{1/3}$ [130]. We choose the parameter set $\{p\}$ as $\{D_A(z), H(z), [f\sigma_8(z)], [b_{\text{HH}}\sigma_8(z)], \sigma_{\text{NL}}\}$, and only use the forecasted observable parameters $\{D_A(z), H(z), [f\sigma_8(z)]\}$ to constrain cosmological models.

The method of making forecast for cosmological parameter estimation using 21 cm IM surveys has been described in detail in Refs. [123, 131], and we follow the methods described in Refs. [123, 131] to perform the forecast for SKA1-MID. First, we measure the full anisotropic power spectrum to obtain the constraints on the angular diameter distance $D_A(z)$, the Hubble parameter $H(z)$, and the RSD observable $[f\sigma_8(z)]$, which are considered to be independent in each redshift bin. Then, we invert the Fisher matrix to obtain covariance matrices for $\{D_A(z_j), H(z_j), [f\sigma_8(z_j)]; j = 1...N\}$ in a series of $N$ redshift bins $\{z_j\}$. Finally, we use these covariance matrices and the fiducial cosmology to generate the mock data of SKA1-MID.

IV. COSMOLOGICAL PARAMETER ESTIMATION

![FIG. 1. Constraints (68.3% and 95.4% confidence level) on the $\Lambda$CDM model by using the data combinations of Taiji+ET, Ska, and Taiji+ET+SKA.](image-url)

In this section, we shall report the constraint results of cosmological parameters. Here we consider three typical dark energy models, the $\Lambda$CDM model [$w(z) = -1$], the $w$CDM model [$w(z) = \text{constant}$], and the CPL model [$w(z) = w_0 + w_a z/(1+z)$]. We use the simulated standard siren data and the 21 cm IM data to perform the Markov-chain Monte Carlo analysis [132] to constrain these three cosmological models. The constraint results are shown in Figures 1–3 and summarized in Table I. Note here that we use SKA to denote the 21 cm IM surveys of the SKA1-
In Figure 1, we show the constraints on the ΛCDM model in the $\Omega_m - H_0$ plane from Taiji+ET, SKA, and the combination of them. The contours of SKA and Taiji+ET show different degeneracy orientations and thus the combination of them could break the parameter degeneracies. We also see that Taiji+ET could provide a tight constraint on $H_0$, with $\sigma(H_0) = 0.35 \text{ km s}^{-1} \text{ Mpc}^{-1}$, better than the result of $\sigma(H_0) = 0.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$ with the Planck 2018 TT, TE, EE+lowE+lensing+BAO data [3]. SKA gives $\sigma(\Omega_m) = 0.006$ that is comparable with the Planck 2018 TT, TE, EE+lowE+lensing+BAO result of $\sigma(\Omega_m) \approx 0.006$ [3]. The combination of Taiji+ET and SKA gives tighter constraints on both $\Omega_m$ and $H_0$. The joint data give the results of $\sigma(H_0) = 0.23 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\sigma(\Omega_m) = 0.005$, which is better than the results of Planck 2018 TT, TE, EE+lowE+lensing+SNe+BAO with $\sigma(H_0) = 0.40 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\sigma(\Omega_m) = 0.0054$ [64]. What’s more, with the joint data, the constraint accuracy of $H_0$ is 0.3%, and the constraint accuracy of $\Omega_m$ is 1.4% (rather close to 1%), indicating that standard sirens and 21 cm IM could jointly provide a precise late-universe cosmological probe.

In Figure 2, we show the constraint results for the $w$CDM model in the $w - \Omega_m$ and $w - H_0$ planes. We clearly see that the parameter degeneracy orientations of SKA and Taiji+ET are almost orthogonal in the $w - H_0$ plane and thus the combination of them could significantly break the parameter degeneracies. Also, we see that SKA could tightly constrain $\Omega_m$ and $w$, while Taiji+ET could tightly constrain $H_0$, and thus the combination of them could tightly constrain all of these three parameters. Concretely, with the Taiji+ET data, the constraint accuracies of $\Omega_m$, $H_0$, and $w$ are 7.2%, 0.6%, and 10.0%, respectively. With the SKA data, the constraint accuracies of $\Omega_m$, $H_0$, and $w$ are 2.1%, 1.0%, and 3.3%, respectively. The joint constraint gives $\sigma(w) = 0.025$, which is better than the result of Planck 2018 TT, TE, EE+lowE+lensing+SNe+BAO with $\sigma(w) = 0.032$ [3]. With the joint data, the constraint accuracies of $\Omega_m$, $H_0$, and $w$ are 1.6%, 0.4%, and 2.5%, respectively.

In Figure 3, we show the case for the CPL model in the $w_0 - w_a$ plane. We find that Taiji+ET cannot constrain these two dark-energy parameters well, but could constrain $H_0$ well, with the constraint accuracy of 0.7%. On the contrary, SKA gives better constraints on $\Omega_m$, $w_0$, and $w_a$, but cannot constrain $H_0$ well. Therefore, the joint data could give tight constraints on all these four parameters. Concretely, the joint constraints give the results of $\sigma(H_0) = 0.36 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\sigma(w_0) = 0.057$, and $\sigma(w_a) = 0.275$, which are all better than the results

### Table I

| Model   | Error  | Taiji+ET | SKA    | Taiji+ET+SKA |
|---------|--------|----------|--------|--------------|
| ACMD    | $\sigma(\Omega_m)$ | 0.010    | 0.006  | 0.005        |
|         | $\sigma(H_0)$       | 0.35     | 0.51   | 0.23         |
|         | $\varepsilon(\Omega_m)$ | 0.032  | 0.020  | 0.014        |
|         | $\varepsilon(H_0)$  | 0.005    | 0.008  | 0.003        |
| wCDM    | $\sigma(\Omega_m)$ | 0.023    | 0.007  | 0.005        |
|         | $\sigma(H_0)$       | 0.38     | 0.67   | 0.30         |
|         | $\varepsilon(\omega)$ | 0.101  | 0.033  | 0.025        |
|         | $\varepsilon(\Omega_m)$ | 0.072  | 0.021  | 0.016        |
|         | $\varepsilon(H_0)$  | 0.006    | 0.010  | 0.004        |
|         | $\varepsilon(\omega)$ | 0.100  | 0.033  | 0.025        |
| CPL     | $\sigma(\Omega_m)$ | 0.168    | 0.016  | 0.008        |
|         | $\sigma(H_0)$       | 0.44     | 1.04   | 0.36         |
|         | $\sigma(w_0)$       | 0.214    | 0.119  | 0.057        |
|         | $\sigma(w_a)$       | 1.479    | 0.445  | 0.275        |
|         | $\varepsilon(\Omega_m)$ | 0.529  | 0.050  | 0.025        |
|         | $\varepsilon(H_0)$  | 0.007    | 0.015  | 0.005        |
|         | $\varepsilon(w_0)$  | 0.215    | 0.119  | 0.056        |
As two promising non-optical cosmological probes, standard sirens and 21 cm IM could play a crucial role in the cosmological parameter estimation. Hence, we wish to investigate the capability of estimating cosmological parameters using the combination of these two non-optical cosmological probes. In this work, we simulate the standard siren data based on the 5-year operation time of Taiji and the 10-year operation time of ET, and simulate the 21 cm IM data based on the 10000-h total integration time of SKA. By comparing the results of Taiji+ET+SKA with those of Taiji+ET and SKA, we find that standard sirens and 21 cm IM could jointly provide a precise late-universe cosmological probe. In the $\Lambda$CDM model, using the joint data, the constraint accuracy of $H_0$ is 0.3% (less than 1%), and the constraint accuracy of $\Omega_m$ is 1.4% (around 1%), indicating that the forthcoming non-optical precision cosmology era is worth expecting.

In addition, we find that these two cosmological probes could effectively break the parameter degeneracies. Taking the $w$CDM model as an example, the parameter degeneracy orientations of SKA and Taiji+ET are almost orthogonal in the $w - H_0$ plane and thus the combination of them could significantly break the parameter degeneracies. This implies that standard sirens and 21 cm IM could complement each other. Actually, the standard siren could directly measure $d_L(z)$, thus providing a powerful constraint on $H_0$, while the measurement of BAO by 21 cm IM with a large survey volume could provide the information of $H(z)$, which could constrain $w(z)$ well. Hence, the combination of standard sirens and 21 cm IM could tightly constrain both the Hubble constant and EoS of dark energy. The joint data of Taiji+ET+SKA could give the constraint $\sigma(w) = 0.025$, which is better than the result of Planck 2018 TT,TE,EE+lowE+lensing+SNe+BAO, providing a powerful late-universe cosmological probe.

The improvements of cosmological constraints due to the synergy between standard sirens and 21 cm IM could also be seen in the CPL model. The Taiji+ET data alone could provide a tight constraint on $H_0$, but cannot constrain $w_0$ and $w_a$ well. However, the joint data of Taiji+ET+SKA could give tight constraints on $H_0$, $w_0$, and $w_a$ at the same time, with $\sigma(H_0) = 0.36$ km s$^{-1}$ Mpc$^{-1}$, $\sigma(w_0) = 0.057$, and $\sigma(w_a) = 0.275$, which are all better than the results of Planck 2018 TT,TE,EE+lowE+lensing+SNe+BAO. Therefore, we can conclude that standard sirens and 21 cm IM could jointly provide a precise late-universe cosmological probe.

In the next decades, the forth-generation dark-energy programs such as LSST [133], Euclid [134], and WFRST [135] will be implemented, and the cosmological probes based on the optical observations will be greatly developed. In addition, the lower frequency (nano-Hz) GWs produced by the inspiralling of supermassive black hole binaries could be detected by the global network of pul-

**V. CONCLUSION**
The multi-band GW observations combined with the optical, near-infrared, and radio observations will usher in a new era of cosmology.

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