Abstract—We present complex-valued Convolutional Neural Networks (CNNs) for RF fingerprinting that go beyond translation invariance and appropriately account for the inductive bias with respect to multipath propagation channels, a phenomenon that is specific to the fields of wireless signal processing and communications. We focus on the problem of fingerprinting wireless IoT devices in-the-wild using Deep Learning (DL) techniques. Under these real-world conditions, the multipath environments represented in the train and test sets will be different. These differences are due to the physics governing the propagation of wireless signals, as well as the limitations of practical data collection campaigns.

Our approach follows a group-theoretic framework, leverages prior work on DL on manifold-valued data, and extends this prior work to the wireless signal processing domain. We introduce the Lie group of transformations that a signal experiences under the multipath propagation model and define operations that are equivariant and invariant to the frequency response of a Finite Impulse Response (FIR) filter to build a ChaRRNet. We present results using synthetic and real-world datasets, and we benchmark against a strong baseline model, that show the efficacy of our approach. Our results provide evidence of the benefits of incorporating appropriate wireless domain biases into DL models. We hope to spur new work in the area of robust RF fingerprinting as well as security mechanisms.

Index Terms—Deep Learning, Equivariant Neural Networks, RF Fingerprinting, Specific Emitter Indentification, Deep Learning on Manifolds

I. INTRODUCTION

FEEEDFORWARD neural networks are not inherently comprised of operations robust to transformations of the network’s input. A popular way to alleviate this problem is to increase the effective size of the training set by presenting transformed examples to the network during training, a process known as data augmentation. An alternate solution is to design networks with inherent robustness to known transformations, e.g., translation. In classification tasks, such as image recognition and segmentation, extracting features that are translation equivariant, i.e. translating the input results in a translated version of the latent output, is of great importance as it produces models with better parameter efficiency. Another important property in classification tasks is that of translation invariance, i.e., the output features remain the same regardless of translations of the input. Translation invariant models are both parameter and data efficient. The combination of convolution and max pooling make CNNs approximately translation invariant (convolution is translation equivariant, while max pooling is approximately invariant to small translations). Designing neural networks that exhibit the desired invariances leads to faster learning and better generalization under different train/test data distributions—a principal tenet of System 2 processing, as outlined by Goyal and Bengio [1].

Since the conception of CNNs [2], subsequent work has continued to seek models that are equivariant and invariant to other groups of transformations. Recently, CNNs that are equivariant to different symmetry groups have been introduced by Cohen et al. [3]–[5]. These networks require that the convolution is performed jointly over both the space and the group, increasing the computational burden. Cheng et al. [6] proposed a rotation equivariant CNN using decomposed steerable filters. Under this framework, only the filter expansion coefficients are learned, which significantly reduces the computational burden involved in computing the group convolution. In a similar fashion, Zhu et al. [7] proposed a scale equivariant CNN with guaranteed representation stability under input deformations.

The theory of equivariant CNNs has thus far been primarily focused on real-valued data with applications to computer vision. Wireless signals, such as those used in communications and RADAR, however, are complex-valued. Real-valued networks that treat complex-valued signals as two independent real-valued channels fail to exploit the relationship between the I and Q components of the signal. Furthermore, we seek CNNs that are equivariant/invariant to different transformations than traditional CNN requirements due to the physics of the problem. Chakraborty et al. [8] introduced CNNs for complex-valued data that are invariant to non-zero scaling and planar rotations, which is the group that acts transitively on the complex plane. This CNN uses the polar form of the complex numbers and identifies the non-zero complex plane with the
product space of the positive real numbers and the rotation Lie group, $SO(2)$. The standard Euclidean convolution is replaced by a weighted Frechet Mean (wFM) [9] of points in this product space, which can be shown to be equivariant to complex scaling. To make the CNN invariant, they introduced a second convolutional layer, which operates on the wFM convolution output, with the desired invariance property. This model was tested on the tasks of synthetic aperture radar (SAR) target recognition and modulation recognition of communications signals. The results showed significant improvements in parameter and data efficiency on both the MSTAR and RadioML datasets [10], [11]. Note that a complex scaling corresponds to a 1-tap channel. Although 1-tap channels model Line-of-Sight (LOS) propagation, robustness to them is not sufficient for the space of all multipath propagation environments.

In this paper we propose ChaRRNets as a kind of CNN architecture for robust RF fingerprinting under multipath propagation channels. We empirically show increased representation stability, relative to a strong baseline model, when the channel realizations represented in the train and test sets are drawn from different statistical models, i.e. ChaRRNets produce better out-of-distribution generalization. Our main contributions are as follows:

- We extend the work presented in [8], by formulating the problem in the frequency domain, and we introduce convolutional NN layers that are equivariant and invariant to a frequency response acting upon the transmitted signal’s spectrum.
- We introduce ChaRRNets: a kind of CNN architecture for multipath-robust RF fingerprinting. Up to a boundary effect that is a function of the length of the channel’s impulse response, our network architecture exhibits the desired stability. That is, the network’s output features with and without multipath remain approximately the same.

II. METHODS AND PROCEDURES

A. RF Fingerprints

In a typical RF transmit chain, the I and Q components go through independent low-pass filters or Digital-to-Analog Converters (DACs) and are then quadrature modulated and amplified before being transmitted. Each of these blocks imparts a signature on the transmitted signal that is specific to the device. For example, a DAC’s input-output characteristics impose a nonlinear relationship known as the Integral Nonlinearity (INL), which measures the deviation between the ideal and measured output values for a given input. Furthermore, the poles of the low-pass filters can deviate slightly from their nominal location due to component manufacturing tolerances. This deviation also introduces a device-specific imperfection. Additionally, power amplifiers exhibit nonlinear behavior as input levels increase and may introduce in-band noise or interference in adjacent frequency channels. Lastly, the oscillators can introduce a phase imbalance in the data since there will always be a small phase offset between them causing the resulting I and Q channels to deviate slightly from perfect quadrature. All of these effects could be exploited for the purposes of device identification.

A simplified model, in complex exponential form, for an ideal signal of bandwidth $W$, transmitted by a wireless device is,

$$x(t) = \left[ h_{tx} \ast \frac{1}{2\pi} \int_{-W/2}^{W/2} X(w)e^{jw t} dw \right] e^{j\omega t}$$

where $h_{tx}$ is the complex-valued impulse response for in-phase and quadrature components of the low-pass filters, $X(w)$ is the spectrum of the modulated signal, $w_c$ is the carrier angular frequency, and $\ast$ denotes convolution. By the time the signal reaches the receiver, it would have propagated through a wireless multipath channel. Assuming that the channel remains stationary over the transmission time, then it can be modeled by a Linear Time-Invariant (LTI) system. Thus, the received signal can be modeled as,

$$r(t) = h_c \ast x(t) + n(t),$$

where $h_c$ is a complex-valued impulse response of the FIR propagation channel, and $n(t)$ is a circularly symmetric Gaussian noise term.

For our fingerprinting problem, the training data contains isolated signal bursts (each burst associated with a single device) in which the propagation channels represented in train and test sets are given by $h_{c\text{train}}$ and $h_{c\text{test}}$ respectively. We are interested in the case when $h_{c\text{train}} \neq h_{c\text{test}}$ (e.g. $h_{c\text{train}}$ may follow a Rayleigh model while $h_{c\text{test}}$ follows a Ricean model). As stated previously, the performance of traditional CNNs will degrade significantly in these scenarios due to overfitting on the channel in the training set. Our proposed CNN architecture is resilient to these distribution changes, however, which results in improved generalization.

B. Group Theoretic Framework

Recall the received signal model, $r(t) = x(t) \ast h_c + n(t)$. We wish to extract features from $r(t)$ which are invariant to the channel. Let $f : C^N \rightarrow C^M$ be a learned feature extractor. For $f$ to be invariant to the channel we must have $f(r(t)) = f(r'(t))$, where $r'(t) = x(t) + n(t)$. We call $r(t)$ a transformed version of $r'(t)$. Of particular interest are sets of transformations that produce groups. Consider $SO(2)$, the set of all rotations about the origin in $\mathbb{R}^2$. Any combination of 2D rotations is also a 2D rotation, the order of rotations does not matter, and any rotation can be undone by the inverse rotation. These properties make $SO(2)$ an Abelian group, with matrix multiplication as the group operation. Another class of groups is Lie groups, which are continuous and smooth groups that form a manifold. We shall explore the properties of a Lie group by example with $SO(2)$. Note that for any $g, h \in SO(2)$, the properties of an Abelian group are satisfied under the operation of matrix multiplication. Furthermore, for any angle $\theta$, there is an exponential map from it to an element of the group. In this case, with $A = \begin{pmatrix} 0 & -\theta \\ \theta & 0 \end{pmatrix}$,
In each component is isomorphic to SO. Under this simplification, the transmitted signal, the signal model (2), reduces the wireless channel to an LTI system. Scalar attenuation, i.e. gain in the transmission chain, and multipath model and note that all operations can be defined simply by the Lie algebra of the group.

RF signals are complex-valued, so each element of the vector can be written in polar coordinates as \( re^{i\theta}, r \in \mathbb{R}^+, \theta \in \mathbb{R} \). Note that the radial and angular components each form independent Lie groups: the radial component is the Abelian Lie group \( \mathbb{R}^+ \) under scalar multiplication, and the polar component is the Abelian Lie group \( U(1) = \{ e^{i \theta} | \theta \in \mathbb{R}/2\pi \mathbb{Z} \} \), which is isomorphic to \( SO(2) \). The logarithmic map for each component is \( \ln \). This Lie group \( \mathbb{R}^+ \times U(1) \) captures scalar attenuation, i.e. gain in the transmission chain, and angular rotations, i.e. a phase offset. The simplified received signal model \( \tilde{x} \), reduces the wireless channel to an LTI system. Under this simplification, the transmitted signal, \( x(t) \), is convolved with the channel impulse response, \( h_c \). By the convolution theorem, \( \mathcal{F}(h_c(t) \ast x(t)) = \mathcal{F}(h_c) \cdot \mathcal{F}(x) \), where \( \mathcal{F} \) is the Fourier Transform operator. Hence, each frequency component lies on the above manifold and perturbations from the channel are two points that contain different windows in the frequency domain.

We now define equivariant and invariant operations on this manifold. We perform these network operations by first mapping inputs to the tangent space, applying algebraic operations, then mapping back to the manifold. First, note that for any two points \( r_1 e^{i\theta_1}, r_2 e^{i\theta_2} \in \mathbb{R}^+ \times U(1) \), we can compute means on the manifold as follows. For the radial component, we map to the tangent space, compute a mean, and map to the manifold producing \( \exp \frac{\ln r_1 + \ln r_2}{2} = \sqrt{r_1 \cdot r_2} \), i.e. the geometric mean. For the angular component, we obtain \( \exp i \frac{\theta_1 + \theta_2}{2} \), which we compute via the angular component of the directional mean, \( \arctan \frac{\sin \theta_1}{\sin \theta_2} \), as shown in Figure 1. These means are equivariant to a scaling and a rotation, respectively. If both points are scaled by \( \rho \) and rotated by \( \phi \), then the means become \( \sqrt{\rho \cdot r_1 \cdot r_2} \) and \( \exp i \frac{(\phi + \theta_1) + (\phi + \theta_2)}{2} = \exp i \phi \cdot \exp i \frac{\theta_1 + \theta_2}{2} \). Note that this generalizes to collections of \( N \) points and weighted means.

Furthermore, we can calculate distances on the manifold by mapping distances in the tangent space with the logarithmic mapping. Then, for any two points \( r_1 e^{i\theta_1}, r_2 e^{i\theta_2} \in \mathbb{R}^+ \times U(1) \), we shall compute their manifold distances. For the radial component, \( d_r(r_1, r_2) = |\ln r_2 - \ln r_1| = |\ln r_2/r_1| \). For the angular component, \( d_\theta(\exp i\theta_1, \exp i\theta_2) = |\iota(\theta_2)_{2\pi} - (\theta_1)_{2\pi}| = |\theta_2 - \theta_1|_{2\pi} \). These distances are invariant to a scaling and a rotation, respectively. If both points are scaled by \( \rho \) and rotated by \( \phi \), then the distances become \( d_r(\rho \cdot r_1, \rho \cdot r_2) = |\ln (\rho \cdot r_2)/(\rho \cdot r_1)| = |\ln r_2/r_1| \) and \( d_\theta(\exp i\phi \cdot \exp i\theta_1, \exp i\phi \cdot \exp i\theta_2) = |(\phi + \theta_2)_{2\pi} - (\phi + \theta_1)_{2\pi}| = |\theta_2 - \theta_1|_{2\pi} \). If a line element is described by \( ds^2 = dr^2 + d\theta^2 \), then we obtain a total distance of \( d(r_1 e^{i\theta_1}, r_2 e^{i\theta_2}) = \sqrt{\ln^2(r_2/r_1) + (|\theta_2|_{2\pi} - |\theta_1|_{2\pi})^2} \).

D. Network Architecture

Leveraging the Abelian Lie groups defined above, we can design neural network layers that are equivariant or invariant to multipath channels.

1) Equivariant Layers: Given that means taken with respect to a manifold are equivariant per frequency bin, suppose that we have \( N \) input windows of signals that have all gone through the same channel. Then, if we compute the above means per frequency bin, they are equivariant to the channel’s frequency response for each bin. So, equivariant layers are generalized means that perform convolution by shifting the mean for the Abelian Lie group with learned weights along a tensor’s axis that contains different windows in the frequency domain.
2) **Invariant Layers:** Similarly, leveraging the fact that distances on the manifold are invariant per frequency bin, we can design an invariant layer by computing the distance from the equivariant means to their respective inputs. Since the inputs all experienced the same frequency response and the mean was equivariant, the same frequency response is present for both the input and the mean, and the distance produces a value invariant to the frequency response. Thus, invariant layers first perform a learned convolution as done in the equivariant layer before computing an elementwise distance from each input window of the mean to the mean per frequency bin.

3) **Complete Model:** With equivariant and invariant layers, now we can stack them to design a model that is inherently robust to channel effects. A general, high-level, description of this model is depicted in Figure 1. We compute the Short Time Fourier Transform (STFT) of the input signal burst using a Kaiser window. Then, we apply an equivariant layer by computing learned weighted sums with the Lie algebra’s mean. Finally, we apply an invariant layer, which outputs real-valued latent representations. The invariant vectors are input to a real-valued backbone 1D CNN for classification.

Once we compute the STFT, we require each window to have been convolved with the channel independently of each other. In reality, the first \( L - 1 \) samples of each window have contribution from the previous window, and the last \( L - 1 \) samples where the convolution dies past the edge of the samples in a given window are not present. These boundary effects break the theoretical equivariance and invariance of the the algebraic layers. However, under the assumption that the length of the channel impulse response is small relative to the length of a short-time window, this boundary effect can be mitigated by applying a window function such as a Kaiser window when computing the STFT. The difference between ChaRRNet’s equivariance and invariance to the mathematical ideal of the transformation and the physical reality along with the Kaiser window’s effect are shown in Figure 2.

### III. **RESULTS**

#### A. **Simulated Transmitter Fingerprinting**

We now evaluate ChaRRNet against 802.11g signals generated using GNURadio, where the device fingerprints are imparted using an IIR filter, a standard model for WiFi signals. We set the population size to 65 devices. Each burst passes through a simulated channel with a specified number of reflectors. The attenuation of each reflector is imparted using an IIR filter, a standard model for WiFi signals and center frequency offset (CFO) data augmentation. The Baseline model has 5M trainable parameters, and the ChaRRNet model has 4M trainable parameters. The results are presented in Table I.

We find that the ChaRRNet model obtains stable generalization and higher accuracy across a wide range of simulated multipath channels. Also note that training with the simulated channel improves generalization to other channels, a result that motivates the use of channel data augmentation during training.

#### B. **Real Transmitter Fingerprinting**

Next, we examine model performance on real signals collected in the wild. We use two datasets for evaluating the model performance:

1) The training and testing sets are comprised of outdoor-collected WiFi signals from separate days. Thus, we have unique channel conditions during training that are separate from those seen during testing.

2) The training consists of two days of outdoor-collected WiFi signals while the test-set still contains a single separate day. We expect this to be an easier test case, as the train data contains a diversity of channels across days.

Each day ostensibly has a different set of channel conditions. There are 50 devices in the first dataset, while there are 65 devices in the second dataset. For all experiments, we train with data augmentation using simulated AWGN and CFO. We also augment the training set by applying multipath channels drawn randomly from Rayleigh and Ricean models. In both cases, we see that the ChaRRNet vastly outperforms the baseline complex-valued convolutional architecture, substantiating the claim that invariances lead to greater generalization across unseen domains. Furthermore, we note that the case of training on two separate days in Table II provides a significant boost in performance, as it further helps the model to correctly calibrate its weights to the existence of channel artifacts.

Finally, we compare ChaRRNet to the Baseline model across a broad range of signal fingerprinting tasks in Table III that contain mixes of WiFi and ADS-B signals.

| Test Set | Baseline Trained on: | ChaRRNet Trained on: |
|----------|----------------------|----------------------|
|          | Pristine | LOS200 | Pristine | LOS200 |
| Pristine | 80.4     | 23.5   | 85.5     | 24.1   |
| nLOS 10  | 11.7     | 76.7   | 20.9     | 83.9   |
| LOS 10   | 5.5      | 76.5   | 23.6     | 84.0   |
| nLOS 30  | 11.8     | 78.4   | 26.8     | 84.5   |
| LOS 100  | 5.4      | 70.5   | 25.6     | 83.0   |
| nLOS 400 | 10.8     | 78.6   | 25.4     | 85.1   |
| LOS 400  | 11.5     | 79.3   | 25.4     | 85.3   |
IV. CONCLUSION

In the ChaRRNet model, we leverage Lie groups to define novel convolutional layers that are robust to multipath degradation of signals in the physical model of wireless signal propagation. We have shown that including this domain bias in the model’s operations drastically increases model generalization to multipath channel environments not present at train time. Furthermore, we have shown the windowing processing necessary to bring physical wireless signals with boundary effects closer to the idealized conditions assumed by the model. We hope that this new set of convolutional layers opens a new space of optimizing NNs for operational tasks in the RF domain while easing the burden of data collection.

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Fig. 2. Output of different CNN layers. Here, we test the layer’s equivariance and invariance properties. The first row shows the results of the Baseline complex-valued convolutional layer. The left column tests for layer equivariance, the right column tests for layer invariance. The ideal action applies the channel independently to each STFT window by element-wise multiplication in the frequency domain. Our layers are mathematically equivariant and invariant to this ideal action. The physical action applies the channel by convolving the input signal with the channel’s impulse response, while the physical action + windowing applies the physical action and helps to mitigate the boundary effects discussed in Section II-D. These results show that our layers compute more stable representations relative to a traditional complex-valued convolutional layer under a multipath channel.