LARGE-ANGLE BHABHA SCATTERING
AND LUMINOSITY AT DAΦNE

Carlo Michel Carloni Calame 1,2, Cecilia Lunardini 3, Guido Montagna 1,2, Oreste Nicrosini 2,1, Fulvio Piccinini 2,1

1 Dipartimento di Fisica Nucleare e Teorica, Universita’ di Pavia
2 INFN, Sezione di Pavia
3 SISSA and INFN, Sezione di Trieste

Abstract

The accurate knowledge of luminosity at $e^+e^-$ flavour factories requires the precision calculation of the Bhabha cross section at large scattering angles. In order to achieve a theoretical accuracy at the 0.1% level, the relevant effect of QED radiative corrections is taken into account in the framework of the Parton Shower method, which allows exclusive event generation. On this scheme, a Monte Carlo event generator (BABAYAGA) is developed for data analysis. To test the reliability of the approach, a benchmark calculation, including exact $O(\alpha)$ corrections and higher-order leading logarithmic contributions, is developed as well, implemented in a Monte Carlo integrator (LABSPV) and compared in detail with the BABAYAGA predictions. The effect of initial-state and final-state radiation, $O(\alpha)$ next-to-leading and higher-order leading corrections is investigated and discussed in the presence of realistic event selections. The theoretical precision of BABAYAGA is estimated to be at the 0.5% level.

1 Introduction

The precise determination of the machine luminosity is necessary for the successful accomplishment of the physics program of $e^+e^-$ colliders operating in the region of the low-lying hadronic resonances, such as DAΦNE (Frascati) [1], VEPP-2M (Novosibirsk) [2], as well as for the BABAR [3, 4] and BELLE [5, 6] experiments at PEP-II and KEKB. In particular, the precise measurement of

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the hadronic cross section requires a luminosity determination with a total relative error better than 1% [6]. It is well known that the luminosity of $e^+e^-$ colliders can be precisely derived by the relation $L = N/\sigma_{th}$, where $N$ and $\sigma_{th}$ are the number of events and the theoretical cross section of a given reference reaction. In order to make the total luminosity error as small as possible, the reference process should be characterized by a large cross section and calculable with high theoretical accuracy. At low-energy $e^+e^-$ machines, the best candidate fulfilling the above criteria is the Bhabha process ($e^+e^- \rightarrow e^+e^-$) detected at large scattering angles.

On the theoretical side, precision calculations of the large-angle Bhabha (LABH) cross-section are therefore demanded, with a theoretical accuracy at the $O(10^{-3})$ level. This requires the inclusion in the calculation of all the relevant radiative corrections, in particular the large effects due to photonic radiation. The complete and exclusive simulation of events in generators is also strongly required by the experimental analysis.

### 2 Theoretical approach

The calculation of the Bhabha scattering cross-section, corrected by the effects of photon radiation, and the corresponding event generation is performed according to the master formula [7]

\[
\sigma_{corrected} = \int dx_- dx_+ dy_- dy_+ \int d\Omega_{lab} D(x_-, Q^2) D(x_+, Q^2) \times
\]

\[
D(y_-, Q^2) D(y_+, Q^2) \frac{d\sigma_0}{d\Omega_{cm}} (x_-, x_+, s, \theta_{cm}) J(x_-, x_+, \theta_{lab}) \Theta(cuts). \tag{1}
\]

In the previous equation, the electron Structure Function (SF) $D(x, Q^2)$ is the solution of DGLAP equation in QED. It takes into account soft-photon exponentiation and multiple hard bremsstrahlung emission in the leading log (LL) approximation [8], both for the QED initial-state (ISR) and final-state radiation (FSR). The QED-DGLAP equation can be exactly solved by means of the QED Parton Shower (PS) algorithm [9], which allows also exclusive photon generation in the LL approximation. A more detailed discussion about the implementation of the PS algorithm as adopted in the present analysis will be given elsewhere [10]. In eq. [1], $d\sigma_0/d\Omega$ is the “hard-scattering” differential cross section relevant for centre of mass (c.m.) energy around 1 GeV, including the photonic $s$- and $t$-channel diagrams, their interference and the (small) contribution due to $\Phi$ exchange. In the hard-scattering cross section, the correction due to vacuum polarization is taken into account as well, according to the parameterization and the recipe given in ref. [11]. The effect of the running
coupling constant at $\sqrt{s} \simeq M_\Phi$ is to enhance the cross section by $\approx 2(2.5)\%$ for $20^\circ(50^\circ) \leq \vartheta_\pm \leq 160^\circ(130^\circ)$. The jacobian factor $J(x_-, x_+, \theta_{lab})$ in eq. 1 accounts for the boost from the c.m. to the laboratory frame due to emission by initial state $e^+$ and $e^-$ of unbalanced radiation, while $\Theta(cuts)$ stands for (arbitrary) experimental cuts implementation.

Upon the above-sketched theoretical background, a new Monte Carlo (MC) generator (BABAYAGA) for simulation of the LABH process at $\Phi$-factories has been developed. In the program both ISR and FSR are simulated and the complete kinematics of the generated events is reconstructed in the LL approximation. The possibility of performing an up to $O(\alpha)$ calculation of eq. 1 is included as well, in view of a comparison with the exact $O(\alpha)$ perturbative results.

In order to test the precision and the reliability of the Bhabha generator, an exact $O(\alpha)$ calculation has also been addressed, by computing the up to $O(\alpha)$ corrected cross-section, consisting of soft+virtual [12] and hard photon corrections [13]. Moreover, higher-order LL terms can be summed on top of the exact $O(\alpha)$ cross section within the collinear SF approach, following the algorithm of ref. [14]. This formulation is available in the form of a MC integrator (LABSPV), which is a suitable modification of the SABSPV code described in ref. [15].

3 Numerical results

In the following, the selection criteria adopted for the analysis and the simulations correspond to realistic data taking at DAΦNE and VEPP-2M, at c.m. energy $\sqrt{s} = 1.019$ GeV. The energy cut imposed on the final-state electron and positron is $E_{min}^\pm = 0.4$ GeV, with the angular acceptance of $20^\circ \leq \vartheta_\pm \leq 160^\circ$ or $50^\circ \leq \vartheta_\pm \leq 130^\circ$ and the (maximum) acollinearity cut allowed to vary in the range $\xi_{max} = 5^\circ-25^\circ$.

A sample of simulations obtained by means of BABAYAGA is shown in fig. 1, where the energy, the cosine of the angle, the $p_\perp$ of the most energetic photon of each event and the missing mass of the event are plotted. As expected, the behaviour of photonic radiation (soft and collinear to charged particles) is well reproduced by the PS. The effect of FSR has also been investigated. The corrected cross section including only ISR has been compared with the corrected cross section including both ISR and FSR. We noticed that, as a consequence of the rather severe cuts, the total effects of photon radiation is to reduce the integrated cross section by an $O(10\%)$ amount. As expected, half of the whole effect must be ascribed to FSR when non-calorimetric (“bare”) event selection [14, 15].
The comparison between the exact $O(\alpha)$ calculation and the $O(\alpha)$ predictions of the PS generator allows to evaluate the size of the $O(\alpha)$ next-to-leading-order (NLO) corrections missing in the LL approximation PS predictions. Moreover, this comparison can be a useful guideline to improve the agreement between perturbative and PS results, for example, by properly choosing the virtuality $Q^2$ in the electron SF in such a way that the bulk of $O(\alpha)$ NLO terms is effectively reabsorbed into the LL contributions. The scale choice $Q^2 = st/u$ ($s$, $t$ and $u$ are the usual Mandelstam variables) allows to keep under control the dominant structure due to initial-, final- and initial-final-state interference radiation \cite{17}. As a function of the acollinearity cut, the relative difference between the exact $O(\alpha)$ cross section and the corresponding PS one is shown in fig. 2 for the angular acceptances $20^\circ \leq \vartheta_{\pm} \leq 160^\circ$ and $50^\circ \leq \vartheta_{\pm} \leq 130^\circ$ and for two different choices of the $Q^2$ scale in the PS, i.e. $Q^2 = st/u$ and $Q^2 = 0.75 \cdot st/u$. It can be seen that, with the scale $Q^2 = 0.75 \cdot st/u$, the difference between the exact $O(\alpha)$ calculation and the PS predictions is within 0.5%. This naive ex-
ample illustrates how, for a given selection criterion, the level of agreement can be substantially improved by a simple redefinition of the maximum virtuality of the electromagnetic shower. Going beyond this simple recipe would require a true merging between perturbative calculation and PS scheme, which is beyond the scope of the present analysis.

In addition to the evaluation of the $O(\alpha)$ NLO corrections, it is important, for an assessment of the theoretical precision, to quantify the amount of the higher-order LL contributions with typical experimental cuts. The size of LL $O(\alpha^n L^n)$ ($n \geq 2$) corrections can be derived in the PS scheme by comparing the full all-order predictions with the corresponding up to $O(\alpha)$ truncation, as shown in fig. 3. The comparison shows that the $O(\alpha^n L^n)$ corrections are unavoidable for a theoretical precision better than 1%, being their contribution 0.7% at $\xi_{\text{max}} = 5^\circ$ and 0.3-0.4% for larger acollinearity cuts in the angular acceptance $20^\circ \leq \vartheta_\pm \leq 160^\circ$. The $O(\alpha^n L^n)$ corrections are even more important, of the order of 1.5%, in the narrower angular range $50^\circ \leq \vartheta_\pm \leq 130^\circ$. It is worth noticing that the generators so far used by the experimental groups at Frascati and Novosibirsk include only $O(\alpha)$ corrections, missing the important effect of higher-order contributions.
4 Conclusions and perspectives

In order to provide predictions of interest for the luminosity determination at $e^+e^-$ flavour factories, a precision calculation of the LABH process has been addressed. It is based on a QED PS algorithm (the details of the formulation as adopted in the present paper will be given elsewhere [10]), which accounts for corrections due to ISR and FSR (and interference) in the LL approximation and allows the complete event generation. A new MC event generator (BABAYAGA) has been developed and is available for a full experimental simulation; actually, it is under test at Frascati and Novosibirsk. The overall precision of the PS approach has been checked by means of a benchmark calculation, which includes exact $O(\alpha)$ and higher-order LL corrections and is available as a MC integrator (LABSPV), allowing for precise cross section calculations. Critical comparisons between the exact $O(\alpha)$ and the $O(\alpha)$ PS calculations pointed out that the contribution of the $O(\alpha)$ NLO corrections is important for the required theoretical precision. Moreover, the effect of higher-order $O(\alpha^n L^n)$ LL corrections has been evaluated to be at the 1-2% level.
By virtue of its generality, the PS approach could be employed to simulate and to evaluate radiative corrections to other large-angle QED processes, as for example $e^+e^- \rightarrow \gamma\gamma$ or $e^+e^- \rightarrow \mu^+\mu^-$. An interesting application of PS would be the simulation of processes with tagged photons, e.g. $e^+e^- \rightarrow \text{hadrons} + \gamma$.

In conclusion, our analysis points out that theoretical predictions aiming at a $O(10^{-3})$ precision must include the effects of both $O(\alpha)$ NLO terms and $O(\alpha^n L^n)$ LL contributions. As a consequence of that, we can estimate the present accuracy of our generator BABAYAGA to be at 0.5% level and the accuracy of the integrator LABSPV at 0.1% level. In the future, an improvement of the presented approach is needed by means of an appropriate merging of the exact $O(\alpha)$ matrix element with the exclusive photon exponentiation realized by the PS algorithm.

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