Equivalence between the Weyl-tensor and gauge-invariant graviton two-point functions in Minkowski and de Sitter spaces

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The two-point Wightman function of the free photon field defined in a gauge-invariant manner is known to be equivalent to the field-strength two-point function in any spacetime that is topologically trivial. We show that the gauge-invariant graviton two-point function defined in a similar manner is equivalent to the Weyl-tensor two-point function in Minkowski space and in the Poincaré patch of de Sitter space. This implies that in the Poincaré patch of de Sitter space the gauge-invariant graviton two-point function decays like (distance)$^{-4}$ as a function of coordinate distance for spacelike separation.

I. INTRODUCTION

The inflationary cosmology [1–6], which assumes an era of exponential expansion in very early universe, has been the leading candidate in addressing various conceptual issues in the standard big-bang cosmology such as the flatness and causality problems. The exponentially expanding universe in inflationary cosmology is approximately the expanding half of de Sitter space. For this reason quantum field theory in de Sitter space has been studied recently by many authors. In particular, the infrared (IR) properties of the graviton two-point function in this spacetime has been a subject of lively debate over the past three decades [7–14].

The two-point function in the transverse-traceless-synchronous gauge in the Poincaré patch, i.e. the spatially-flat expanding half of de Sitter space, is IR divergent in the sense that it is not well-defined unless the IR cutoff is inserted [6]. This fact and other observations have led some authors to claim that there is no de Sitter-invariant vacuum state for linearized gravity [12,14] in spite of the fact that IR-finite two-point functions have been constructed in other gauges [15,16].

The graviton two-point function also diverges or tends to a constant as the separation of the two points tends to infinity both in the spacelike and timelike directions [9]. No gauge conditions are known that make the graviton two-point function tend to zero as the separation of the two points becomes infinite. If linearized gravity were not a gauge theory, this behavior of the two-point function would imply that the graviton field had strong long-distance correlation. However, since linearized gravity is a gauge theory, one needs to characterize the long-distance correlation in a gauge-invariant manner. For example, some cosmologists argue that the logarithmic growth of the two-point function for graviton as well as for minimally-coupled massless scalar field cannot be observed [19,24]. However, there is no consensus in the scientific community about the physical significance of the long-distance behavior of the graviton two-point function (see, e.g., Refs. [25,26]).

Recently a gauge-invariant graviton two-point function has been formulated [27] following a similar construction for electromagnetic field [28,29]. The main purpose of this paper is to show that this graviton two-point function reduces to that of the Weyl tensor in the Poincaré patch of de Sitter space, which decays at large distances.

The rest of this paper is organized as follows. In Sec. III we motivate and introduce the gauge-invariant graviton two-point function. In Secs. III and IV we show the equivalence of the Weyl-tensor and gauge-invariant graviton two-point functions in Minkowski and de Sitter spaces. Then, we discuss our result in Sec. V. In the Appendix we present a technical result used in Sec. III.

II. GAUGE-INARIANT GRAVITON TWO-POINT FUNCTION

Let us quickly introduce the gauge-invariant graviton two-point function proposed in Ref. [27] in a heuristic manner. To this end we first briefly discuss quantization of the free graviton field. We linearize the gravitational field about a globally-hyperbolic spacetime satisfying the vacuum Einstein equations with or without a cosmological constant. We write the Lagrangian density for the linearized gravitational field, $h_{ab}$ as

$$\mathcal{L} = \frac{\sqrt{-g}}{2} \left[ K^{abca}b'c' \nabla_a h_{bc} \nabla_c h_{b'd'} + S^{abc'b'} h_{ab} h_{a'b'} \right],$$

where $K^{abca}b'c' = K^{a'b'c'abc} = K^{abc'a'b'}$ and $S^{abc'b'} = S^{abc'a'b'}$. We define the conjugate momentum current as

$$p^{abc} := K^{abca}b'c' \nabla_a h_{b'c'},$$

then the Euler-Lagrange field equation for $h_{bc}$ is

$$-\nabla_a p^{abc} + S^{abc'b'} h_{b'c'} = 0.$$
For any two solutions, \( h_{bc}^{(n)} \) and \( h_{bc}^{(m)} \), and their conjugate momentum currents, \( p^{(n)abc} \) and \( p^{(m)abc} \), we define the symplectic product as

\[
(h^{(n)}, h^{(m)})_{\text{symp}} := \int_{\Sigma} d\Sigma n_{a} (h_{bc}^{(n)} p^{(m)abc} - p^{(n)abc} h_{bc}^{(m)}),
\]

where \( \Sigma \) is any Cauchy surface and where \( n^{a} \) is the past-directed normal to \( \Sigma \). One can readily see that this product is conserved, i.e., independent of the Cauchy surface \( \Sigma \) \[33\], \[34\]. The symplectic product is degenerate because any tensor of the form \( h_{bc}^{(n)} = \nabla_{a} A_{b}^{(n)} + \nabla_{b} A_{a}^{(n)} \) is a solution to Eq. \[3\] and \((h^{(n)}, h^{(m)})_{\text{symp}} = 0 \) for all solutions \( h_{bc}^{(m)} \). This means that the symplectic product is gauge invariant, i.e., if \( h_{ab}^{(n)} \) and \( h_{ab}^{(m)} \) are two solutions and if \( h_{ab}^{(n)} = h_{ab}^{(n)} + \nabla_{a} \Lambda_{b}^{(n)} + \nabla_{b} \Lambda_{a}^{(n)} \) for some \( \Lambda_{b}^{(n)} \) and similarly for \( h_{ab}^{(m)} \), then \((h^{(n)}, h^{(m)})_{\text{symp}} = (h^{(n')}, h^{(m')})_{\text{symp}} \).

In the spacetimes we are interested in, it is possible to impose gauge conditions such that the symplectic product is nondegenerate on the space of solutions satisfying them \[27\]. We expand the quantum field \( h_{ab}(x) \) in terms of a complete set of solutions \( h_{ab}^{(n)}(x) \) satisfying these gauge conditions: \( h_{ab}(x) = \sum n_{a} h_{ab}^{(n)}(x) \). The field \( h_{ab}(x) \) is quantized through the commutation relations \([\delta n_{a}, \delta n_{b}] = (\Omega^{-1})_{nm} \delta n_{m} \), where \( \Omega^{-1} \) is the inverse of the matrix \( \Omega^{nm} := (h^{(n)}, h^{(m)})_{\text{symp}} \).

The field operator \( h_{ab}(x) \) changes under gauge transformations, but the operators \( \delta n_{a} \) are invariant since they can be expressed using the gauge-invariant symplectic product as \( \delta n_{a} = (\Omega^{-1})_{nm} (h_{m})_{\text{symp}} \). Thus, all gauge-invariant content of the field \( h_{ab}(x) \) can be extracted if \((h^{(n)}, h_{\text{symp}}) \) are known for all solutions \( h_{ab}^{(n)}(x) \). In fact it is sufficient for this symplectic product to be known for all solutions with compact support on a Cauchy surface \( \Sigma \).

If the solution \( h_{ab}^{(n)}(x) \) is compactly supported on \( \Sigma \), then one can find a transverse (or divergence-free) tensor \( f^{(n)ab}(x) \) compactly supported in spacetime such that \( (h^{(n)}, h_{\text{symp}}) = (f^{(n)}, h_{\text{st}}) \), where

\[
(f^{(n)}, h_{\text{st}}) := \int dD^{x} \sqrt{-g(x)} f^{(n)ab}(x) \hat{h}_{ab}(x).
\]

(Here, \( D \) is the spacetime dimension.) This can be shown as follows by generalizing the scalar case \[32\], \[33\]. We define \( h_{ab}^{(n,\chi)}(x) := \chi(x) h_{ab}^{(n)}(x) \), where \( \chi(x) = 1 \) in the future of \( \Sigma \) and \( \chi(x) = 0 \) in the past of another Cauchy surface \( \Sigma' \). The function \( \chi \) changes its value smoothly from 1 to 0 between \( \Sigma \) and \( \Sigma' \). Let \( p^{(n,\chi)abc} \) be the conjugate momentum current of \( h_{ab}^{(n,\chi)} \) as given by Eq. \[2\] and define

\[
f^{(n)bc} := -\nabla_{a} p^{(n,\chi)abc} + S^{abc} h_{bc}^{(n,\chi)}. \tag{6}
\]

Since the right-hand side is the linearized Einstein tensor, the tensor \( f^{(n)bc} \) is transverse by the Bianchi identity. Moreover \( f^{(n)bc} \) is compactly supported between \( \Sigma \) and \( \Sigma' \) because \( h_{ab}^{(n,\chi)} \) satisfies the linearized Einstein equations in the future of \( \Sigma \) and vanishes in the past of \( \Sigma' \).

Now,

\[
(f^{(n)}, \hat{h}_{st}) = \int \sqrt{-g} \nabla_{a} p^{(n,\chi)abc} + g^{bcb'} \hat{h}_{bc}^{(n,\chi)} = \int \sqrt{-g} \nabla_{a} p^{(n,\chi)abc} - p^{(n,\chi)abc} \hat{h}_{bc}. \tag{7}
\]

Then by the generalized Stokes theorem we find

\[
(f^{(n)}, h_{\text{st}}) = (h^{(n)}, h_{\text{symp}}).
\]

This observation implies that one does not lose any gauge-invariant information contained in the Wightman two-point function on a state \( \omega \),

\[
\Delta_{abc} \langle x, x' \rangle = \langle \omega | \hat{h}_{ab}(x) \hat{h}_{c'}(x') | \omega \rangle, \tag{8}
\]

by considering only its smeared version, which is gauge invariant:

\[
G_{g}(f^{(1)}, f^{(2)}) = \int d^{D} x \sqrt{-g(x)} \int d^{D} x' \sqrt{-g(x')} \times f^{(1)ab}(x) f^{(2)a'b'}(x') \Delta_{abc} \langle x, x' \rangle, \tag{9}
\]

where the tensors \( f^{(1)ab} \) and \( f^{(2)ab} \) are compactly supported and transverse.

Now, for the gauge-invariant two-point function for non-interacting electromagnetic field \( A_{a} \) analogous to Eq. \[9\], the two-point function \( \langle \omega | A_{a}(x) A_{a'}(x') | \omega \rangle \) is smeared by smooth and compactly-supported transverse vectors \( f^{(1)ab}(x) \) and \( f^{(2)ab}(x') \). If the spacetime is topologically trivial, or contractible, then one can find smooth and compactly-supported antisymmetric tensors \( u^{(1)ab} \) and \( u^{(2)ab} \) such that \( f^{(1)a} = 2 \nabla_{b} u^{(1)ab} \) and \( f^{(2)a} = 2 \nabla_{b} u^{(2)ab} \). (It can be shown that the difference between the supports of \( u^{(1)ab} \) and \( f^{(1)a} \) can be made arbitrarily small.) Then, by integration by parts, the smeared two-point function is shown to be equal to the two-point function of the field-strength tensor, \( \langle \omega | F_{ab}(x) F_{a'b'}(x') | \omega \rangle \),

\[
F_{ab} := \nabla_{a} A_{b} - \nabla_{b} A_{a}, \text{ smeared with } u^{(1)ab}(x) \text{ and } u^{(2)a'b'}(x'). \tag{10}
\]

In the next two sections we show that in Minkowski space and in the Poincaré patch of de Sitter space the gauge-invariant graviton two-point function \( G_{g}(f^{(1)}, f^{(2)}) \) is equal to the two-point function for the linearized Weyl tensor smeared with compactly-supported tensors.

### III. EQUIVALENCE IN MINKOWSKI SPACE

In Minkowski space the linearized Weyl tensor, which equals the linearized Riemann tensor, is given in terms of the graviton field \( \hat{h}_{ac} \) as

\[
C^{dab}_{abcd} = \kappa (\partial_{a} \partial_{c} \hat{h}_{db} - \partial_{a} \partial_{d} \hat{h}_{cb}), \tag{10}
\]

where \( \kappa < 0 \) is a constant. Here we use \( \partial_{a} \) instead of \( \nabla_{a} \) to emphasize that \([\partial_{a}, \partial_{b}]V^{c} = 0\). Now, suppose that for
any smooth and compactly-supported symmetric tensor $f^{ac}$ one can find a smooth and compactly-supported tensor $v^{abcd}$ antisymmetric under $a \leftrightarrow b$ and $c \leftrightarrow d$ and symmetric under $[ab] \leftrightarrow [cd]$ such that $f^{ac} = \partial_{b} v^{abcd}$, and integrating by parts, one finds that $\int f(v^{abcd})\, d\nu = 0$.

We show that any smooth and compactly-supported symmetric tensor $f^{ac}$ can indeed be expressed as $f^{ac} = \partial_{b} v^{abcd}$, where $v^{abcd}$ has the properties mentioned above. (In this and next sections all tensors are smooth and compactly supported unless otherwise stated.) It is sufficient to find a tensor $u^{abc}$, antisymmetric under $a \leftrightarrow b$ and satisfying $\partial_{b} u^{abc} = 0$ such that $f^{ac} = \partial_{b} v^{abcd}$, antisymmetric under $c \leftrightarrow d$ and that the tensor $\partial_{b} v^{abcd} = (\partial_{b} u^{abc}) + (\partial_{b} u^{abc})/2$ will be a tensor with the desired symmetries satisfying $f^{ac} = \partial_{b} \partial_{d} v^{abcd}$.

The metric signature plays no rôle in finding the tensor $u^{abc}$ with the properties described above. We work in $D$-dimensional Euclidean space. We first solve the following equation for $\gamma^{ab}$:

$$\partial_{b} \partial_{d} v^{abcd} = f^{ac},$$

where

$$v^{abcd} := -\delta_{a[c} \gamma^{db]} + \delta_{b[c} \gamma^{da]} + \delta_{a[c} \delta_{b]} \gamma.$$

Here, the tensor $\delta_{ab}$ is the metric on the Euclidean space and $\gamma := \partial_{[a} \gamma_{b]}$. (The tensor $\gamma_{ab}$ is not compactly supported.) Eq. (11) is basically the linearized Euclidean Einstein equations with $f^{ac}$ as the source tensor. We define

$$U^{abc} := \partial^{d} v^{abcd} = \partial_{[a} \gamma_{b]} - \delta_{[a} \partial_{b]} (\partial_{d} \gamma_{[c]} + \partial_{d} \gamma_{b]}).$$

Then we have $\partial_{b} U^{abc} = f^{ac}$ and $\partial_{b} U^{abc} = 0$. Although the tensor $U^{abc}$ is not compactly supported, we show below that a compactly supported tensor $u^{abc}$ with the same symmetries and satisfying the same differential equations as $U^{abc}$ can be constructed.

We work in polar coordinates with $r^{2} := \delta_{ab} x^{a} x^{b}$. Since $f^{ab}$ is compactly supported, we have $f^{ab}(x) = 0$ if $r > R$ for some $R > 0$. Then, one can choose a gauge condition such that $\gamma^{ab}$ is transverse, traceless and with vanishing radial components, $\gamma^{ab} x^{b} = 0$, for $r > R$. This result is demonstrated in the Appendix. Then $U^{abc} = \partial_{[a} \gamma_{b]}$. We define the following differential and integral operators on any (non-compactly-supported) tensor $\Phi$ (with the indices omitted) decaying faster than $r^{-N}$ for large $r$:

$$D_{N} \Phi = -(x^{d} \partial_{d} + N) \Phi,$$

$$G_{N} \Phi := r^{-N} \int_{r}^{\infty} \rho^{N-1} \Phi \, d\rho.$$
where $C_{abcd}(\hat{h})$ is the linearized Weyl tensor in the de Sitter background.

First we show that any symmetric tensor $f_{ac}$ in de Sitter space can be written as

$$f_{ac} = f_{ac}^{(t)} + E_{ac}^{(1)}(\gamma^{(t)}),$$

where the tensor $\gamma^{(t)}_{ab}$ and the transverse-traceless tensor $f_{ac}^{(t)}$ are symmetric. To see this we let

$$\gamma^{(t)}_{ab} := \frac{2}{(D-1)(D-2)\eta^2} [(D-1) f_{ab} - g_{ab} f],$$

where $f := g_{ac} f^{ac}$. Then

$$E_{ac}^{(1)}(\gamma^{(t)}) = \frac{1}{\eta^2} \left( f_{ac} - \frac{1}{\sqrt{\eta}} (g_{ac} \Box f - \nabla_a \nabla_c f) \right) - \frac{1}{\sqrt{\eta}} (2 f_{ac} - g_{ac} f).$$

It can readily be verified that $f_{ac}^{(t)} = f_{ac} - E_{ac}^{(1)}(\gamma^{(t)})$ is traceless. Note also

$$\int d^D x \sqrt{-\eta} f_{ac} \hat{h}_{ac} = \int d^D x \sqrt{-\eta} f_{ac}^{(t)} \hat{h}_{ac}$$

because

$$\int d^D x \sqrt{-\eta} E_{ac}^{(1)}(\gamma^{(t)}) \hat{h}_{ac} = \int d^D x \sqrt{-\eta} \gamma^{(t)}_{ac} E_{ac}^{(1)}(\hat{h})$$

$$= 0.$$  

Next we observe that for any symmetric tensor $\gamma_{ac}$ we have

$$E_{ac}^{(1)}(\gamma) = \nabla_b \nabla^d \Gamma_{abcd} + H^2 g^{bd} \Gamma_{abcd},$$

where

$$\Gamma_{abcd} := -g_{ab} \gamma_{cd} + g_{bc} \gamma_{ad} + (g_{ac} g_{db} - g_{ad} g_{cb}) \gamma$$

with $\gamma := g^{ac} \gamma_{ac}$. Using Eq. (29) in Eq. (21), we find

$$f_{ac} = f_{ac}^{(t)} + \nabla_b \nabla_d \Gamma^{(t)}_{abcd} + H^2 g_{bd} \Gamma^{(t)}_{abcd},$$

where $\Gamma^{(t)}_{abcd}$ is given by Eq. (21) with $\gamma_{ac} = \gamma^{(t)}_{ac}$. Thus, if

$$f_{ac}^{(t)} = \nabla_b \nabla_d V^{(t)}_{abcd}$$

with a traceless tensor $V^{(t)}_{abcd}$ with the symmetry (21), then Eq. (20) will be satisfied with

$$V_{abcd} = V^{(t)}_{abcd} + \Gamma^{(t)}_{abcd}.$$  

We show below that Eq. (32) holds.

First we note that the equation $\nabla_c f_{ac}^{(t)} = 0$ can be written $\partial_a (D^{D+2} f_{ac}^{(t)}) = 0$ in any conformally-flat spacetime for a symmetric traceless tensor $f_{ac}^{(t)}$ [34]. By our result in Minkowski space in the previous section this implies that we can write

$$\Omega^{D+2} f_{ac}^{(t)} = \partial_a \partial_d V_{abcd},$$

where the tensor $V_{abcd}$ has the symmetry $V_{abcd} = V_{[ab][cd]} = V_{[cd][ab]}$. Denoting the traceless part of $V_{abcd}$ by $\tilde{V}_{abcd}$ and defining $\gamma := \eta_{bd} V_{abcd}$ and $\nu := \eta_{ac} V_{ac}$, where $\eta_{ac}$ is the standard metric tensor on Minkowski space, we find

$$V_{abcd}^{(t)} = \eta_{bd} \nu^{(t)}_{abcd} + \frac{2}{D^2} \left( \eta^{acb} \eta_{bd} - \eta^{ac} \eta_{bd} \right) \left( \eta_{ab} \eta_{cd} - \eta_{ac} \eta_{bd} \eta_{ab} \right).$$

(35)

Substitution of this expression into Eq. (34) yields

$$\Omega^{D+2} f_{ac}^{(t)} = \partial_a \partial_d \nu^{(t)}_{abcd} + \frac{1}{D-2} \eta_{ac} \eta_{bd}.$$  

(36)

where

$$\eta_{ac} \eta_{bd} = \nabla^c \nabla^d \nu^{(t)}_{abcd} - \nabla^c \nabla^d \nu^{(t)}_{abcd} - \nabla^c \nabla^d \nu^{(t)}_{abcd}.$$  

(37)

Now, since $\partial_d \partial_a \nu^{(t)}_{abcd}$ is traceless, the tensor $\eta_{bd} V_{abcd}$ satisfies $\partial_a \partial_d \nu^{(t)}_{abcd} = 0$. That is, $\partial_a V_{abcd}$ is transverse. This means that $\partial_a V_{abcd} = \partial_a q_{ac}^{bd}$, i.e., $\partial_d (\nu^{(t)}_{ac} - \eta_{ac}^{bd}) = 0$, where $q_{ac}$ is an antisymmetric tensor. This in turn implies that $\nu^{(t)}_{ac} = \eta_{ac}^{bd} = 2 \partial_a w_{abc}$, where $w_{abc} = w_{a[bc]}$. By symmetrizing this equation we obtain

$$\nu^{(t)}_{ac} = \partial_b w_{abc} + \partial_b w_{cha}.$$  

(38)

We define

$$W_{abcd} := \partial^c w_{dab} - \partial^d w_{cab} + \partial^a w_{bcd} - \partial^b w_{acd}.$$  

(39)

Then

$$\eta_{bd} W_{abcd} = \eta_{bd} (\partial^c w_{dab} + \partial^a w_{bcd} + \nu_{ac}) = 2 \eta_{ac}.$$  

(40)

Let $W^{(t)}_{abcd}$ be the traceless part of $W_{abcd}$. Then, after some algebra we find

$$\partial_a \partial_d W^{(t)}_{abcd} = \frac{D-3}{D-2} \nu_{ac}.$$  

(42)

Hence by defining

$$\tilde{V}^{(t)}_{abcd} := W^{(t)}_{abcd} + \frac{1}{D-3} \nu_{ac}$$  

(43)

we have

$$\Omega^{D+2} f^{(t)}_{ac} = \partial_a \partial_d \tilde{V}^{(t)}_{abcd},$$  

(44)

where $\tilde{V}^{(t)}_{abcd} = \tilde{V}^{(t)}_{[ab][cd]} = \tilde{V}^{(t)}_{[cd][ab]}$. Now, it can readily be shown that

$$\partial_a \partial_d \tilde{V}^{(t)}_{abcd} = \Omega^{D+2} \nabla_b \nabla_d (\Omega^{-D} \tilde{V}^{(t)}_{abcd}).$$  

(45)

Using $\Omega = (H/|H|)^{-1}$ and the fact that $\tilde{V}^{(t)}_{abcd}$ is traceless. Hence Eq. (42) is satisfied with

$$\tilde{V}^{(t)}_{abcd} = \Omega^{-D} \tilde{V}^{(t)}_{abcd}.$$  

(46)
This establishes Eq. (20). Then Eq. (23) implies that the gauge-invariant graviton two-point function in de Sitter space can be expressed as

\[
G_g(f^{(1)}, f^{(2)}) = \kappa^{-2} \int d^D x \sqrt{-g(x)} \int d^D x' \sqrt{-g(x')} \times V^{(1)abcd}(x)V^{(2)ab'd'}(x') \times \langle \omega | \hat{C}_{abcd}(\hat{h}(x))\hat{C}_{ab'd'}(\hat{h}(x')) | \omega \rangle
\]

(47)

if each of \(f^{(1)}_{ac}\) and \(f^{(2)}_{ac}\) has support in a Poincaré patch. This result was used recently in proving a “cosmic no hair theorem” for linearized quantum gravity [33]. From our construction of \(V^{abcd}\) from \(f^{ab}\), it is clear that the difference between the supports of \(V^{abcd}\) and \(f^{ab}\) can be made arbitrarily small.

**V. DISCUSSION**

In this paper, we showed that the gauge-invariant graviton two-point function of Ref. [27] is equivalent to the two-point function of the linearized Weyl tensor in Minkowski space and in the Poincaré patch of de Sitter space. This result is analogous to the following result for the non-interacting electromagnetic field: the gauge-invariant graviton two-point function in de Sitter space can be expressed as

\[
G^{\text{grav}}(\gamma) = 
\]

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In this Appendix we prove that the gauge conditions 

\[
x^a \gamma_{ab} = 0
\]

can be imposed as well as the transverse-traceless conditions on the solutions to linearized Euclidean Einstein equations in the vacuum.

We thank Dave Hunt, Don Marolf, Ian Morrison and Richard Woodard for useful discussions.

**APPENDIX: GAUGE FIXING IN LINEARIZED EUCLIDEAN GRAVITY**

In this Appendix we prove that the gauge conditions 

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