Signaling Design for MIMO-NOMA with Different Security Requirements
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Abstract—Signaling design for secure transmission in two-user multiple-input multiple-output (MIMO) non-orthogonal multiple access (NOMA) networks is investigated in this paper. The base station broadcasts multicast data to all users and also integrates additional services, unicast data targeted to certain users, and confidential data protected against eavesdroppers. We categorize the above MIMO-NOMA with different security requirements into several communication scenarios. The associated problem in each scenario is nonconvex. We propose a unified approach, called the power splitting scheme, for optimizing the rate equations corresponding to the scenarios. The proposed method decomposes the optimization of the secure MIMO-NOMA channel into a set of simpler problems, including multicast, point-to-point, and wiretap MIMO problems, corresponding to the three basic messages: multicast, private/unicast, and confidential messages. We then leverage existing solutions to design signaling (covariance matrix) for the above problems such that the messages are transmitted with high security and reliability. Numerical results illustrate the efficacy of the proposed covariance matrix (linear precoding and power allocation) design in secure MIMO-NOMA transmission. The proposed method also outperforms existing solutions, when applicable.

In the case of no multicast messages, we also reformulate the nonconvex problem into weighted sum rate (WSR) maximization problems by applying the block successive maximization method and generalizing the zero duality gap. The two methods have their advantages and limitations. Power splitting is a general tool that can be applied to the MIMO-NOMA with any combination of the three messages (multicast, private, and confidential) whereas WSR maximization shows greater potential for secure MIMO-NOMA communication without multicasting. In such cases, WSR maximization provides a slightly better rate than the power splitting method.

Index Terms—MIMO-NOMA, physical layer security, power splitting, weighted sum rate, wiretap, multicast, unicast.

I. INTRODUCTION

The unprecedented wave of emerging devices has dramatically increased the requirements and challenges of resource allocation and spectrum utilization. To fulfill the demands, nonorthogonal multiple access (NOMA) at the physical (PHY) layer is a promising technique [2], [3], and has attracted remarkable attention both in academia and industry. For example, NOMA has also been proposed for inclusion in the Third Generation Partnership Project (3GPP) Long-Term Evolution (LTE-A) standard [4]. Therein, NOMA is referred to as multi-user superposition transmission (MUST) [5] built on LTE resource blocks.

While several code-domain uplink NOMA schemes are developed in the literature [3], [6], downlink NOMA is based on well-known information-theoretic techniques for the broadcast channel (BC) [7]. Then, in single-input single-output (SISO) NOMA networks, the superposition coding (SC) at the transmitter and successive interference cancellation (SIC) at the receiver is the optimal strategy. Hence, a large body of work assumes NOMA to be equivalent to SC-SIC, and applies it to multiple-input multiple-output (MIMO) channels [8]–[11]. However, it is known that SC-SIC is not capacity-achieving in the MIMO-BC and dirty-paper coding (DPC) is the only optimal strategy [12]–[14]. Similarly, in MIMO-NOMA with PHY layer security, SC-SIC cannot achieve secure capacity, and secret DPC (S-DPC) is the optimal solution [15], [16]. In this paper, MIMO-NOMA is defined broadly and refers to any technique that allows simultaneous transmission over the same resources, i.e., concurrent non-orthogonal transmission. That is, it is equivalent to the MIMO-BC.

A. MIMO-NOMA with Secrecy

Today, there is a trend to merge multiple services in one transmission. This is referred to as PHY layer service integration [17]. Integrated services usually include three fundamental services: multicast, unicast, and confidential services, which can be realized by common, private/individual, and confidential messages, respectively. Especially, secure transmission of confidential messages requires PHY layer security which has been introduced as additional protection for secure transmission [18].

This work is concerned with different security requirements for the two-user MIMO-NOMA networks, in which three different types of messages can be transmitted:

- **Common message** $M_0$ [19]: a common message is transmitted in such a way that all users can decode it. For example, the base station (BS) broadcasts daily news or amber alerts to all online users.
- **Private message** $M_p$ [7]: a private or unicast/individual message is a message intended for a specific user, since each user is only interested in its message. For instance, the BS provides targeted advertisements and recommended videos that are available only to interested users. This message is not encoded securely, and as such, it can be decoded by other users.
- **Confidential message** $M_c$ [20]: a confidential message is similar to a private message but is to be kept secret from other users. For example, personal email accounts access and online banking transactions. Here, encoding is such that the message cannot be decoded by others.

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Early information-theoretic works \cite{12, 16, 21, 22} and have established the capacity region of two-user MIMO-BC with different security requirements. This includes the MIMO-BC with one common and two independent private messages \cite{12, 21, 22}, the MIMO-BC with private, confidential, and common messages \cite{23, 24}, and the MIMO-BC with one common and two confidential messages \cite{16, 25}. However, their primary purpose is to derive capacity regions or to construct coding strategies that characterize certain rate regions. The solutions are based on DPC or S-DPC and usually are given as a union over all possible transmit covariance matrices satisfying certain power constraints. Implementation of DPC requires sophisticated random coding \cite{26}, and finding practical dirty paper codes close to the capacity is not easy \cite{27}. Linear precoding is a popular alternative to simplify the transmission design \cite{14, 27}.

The two-user MIMO-NOMA can be classified into three communication scenarios with different security requirements as shown in Fig. 1. The classification is mainly based on the well-established information-theoretic results:

- **Scenario A (no security):** two independent private messages $M_{1p}$ and $M_{2p}$ (one for each user) and a common message $M_0$ for both users are ordered \cite{12, 21, 22}. In this case, we have a MIMO-NOMA with common and two private messages $(M_0, M_{1p}, M_{2p})$;
- **Scenario B (security for one user):** a confidential message $M_{1c}$ for user 1, a private message $M_{2p}$ for user 2, and one common message $M_0$ for both users are ordered \cite{24}. Then, a MIMO-NOMA with common, private, and confidential messages $(M_0, M_{1c}, M_{2p})$ is formed;
- **Scenario C (security for both users):** two confidential messages $M_{1c}$ and $M_{2c}$ (one for each user), and a common message $M_0$ for both users are needed \cite{16, 25}. In this case, a MIMO-NOMA with common and confidential messages $(M_0, M_{1c}, M_{2c})$ is obtained.

The three scenarios overall cover nine problems, or communication scenarios (see Table 1). The combinations of different types of messages are also named integrated services \cite{17}.

### B. Motivation and Related Problems

While the capacity regions of the three different messages are characterized, it is still unknown how to identify optimal or implementation-efficient solutions. Thus, this paper is motivated by the following question: How can we maximize the secrecy rate for the MIMO-NOMA with different security requirements in an acceptable computational complexity?

The state-of-the-art includes solutions only for some special combinations of the messages and the orthogonal multiple access (OMA) case in which only one message, out of the three messages we mentioned earlier, is transmitted. These are summarized in Table 1 and some are highlighted below.

- **Two private messages** \cite{7, 33, 34}: When there is no common message in Scenario A, the problem reduces to the MIMO-BC and DPC gives the capacity region. Alternatively, multiple access channel (MAC) to BC duality \cite{33} can be applied to iteratively achieve the capacity \cite{35}. An analytic linear precoding scheme based on generalized singular value decomposition (GSVD)—which decomposes the MIMO channel into parallel SISO channels—is also designed for the special case where the two users are equipped with the same number of antennas \cite{16}.
- **One confidential message** \cite{20}: When there is neither a common nor private message in Scenario B, the system reduces to a MIMO wiretap channel \cite{20}. Various linear precoding schemes, including GSVD \cite{30}, alternating optimization and water filling (AOWF) algorithm \cite{31}, and rotation modeling (RM) \cite{32} are known for this problem.
- **Two confidential messages** \cite{15}: When $M_0$ is empty in Scenario C, the problem reduces to the MIMO-BC with two confidential messages. It is proven in \cite[Theorem 1]{} that both users can reach their respective maximum secrecy rates simultaneously by S-DPC under input covariance constraints. Low-complexity approaches, such as GSVD \cite{37}, weighted sum rate (WSR) maximization with block successive maximization method (BSMM) \cite{38}, and power splitting (PS) method \cite{39} are developed. We generalize the PS into a more general case.
- **Common and confidential messages** \cite{40}: Different linear precoding schemes, including a GSVD-based precoding \cite{41} and RM with random search \cite{42} are proposed in this case.
- **MIMO with only a common message** \cite{17, 28}: If there is only a common message and no individual messages to be transmitted, the system becomes a multicast channel. Heuristic precoding with iterations is investigated in \cite{28}, analytical solutions with a convex tool are in \cite{14}.

However, these problems are all special cases of the three general scenarios mentioned earlier. Signaling designs (solutions) for the general cases are still unknown in general.

### C. Contributions and Organization

As illustrated, the problems listed in Table 1 are all related to the three scenarios shown in Fig. 1. Nonetheless, there are
no solutions for the general cases. In this paper, we propose a new solution, named the power splitting method, which applies to all of those problems. This method decomposes the secure MIMO-NOMA channel into point-to-point (P2P), wiretap, and multicasting MIMO channels. Then, we design one algorithm that can be used in all problems in Table I to approach their corresponding capacity regions. The main contributions can be summarized as follows:

- We first split the total power among the three messages and then reformulate the secrecy capacity optimization problems into three sub-problems. Particularly, Scenario A (only private messages) is decomposed into two P2P MIMO problems; Scenario B (private and confidential messages) is decomposed into one P2P MIMO problem and one wiretap channel; and, Scenario C (only confidential messages) is decomposed into two wiretap channels.

- Linear precoder and power allocation matrices are designed for private and confidential messages by extending the analytical solution of the P2P MIMO problem and the numerical solution of wiretap channels to the MIMO-NOMA with different secrecy scenarios. For multicasting, we use a combination of analytical solutions and numerical solution based on a convex tool. Finally, we propose an algorithm for all different secrecy scenarios.

- When there is no common message, a WSR maximization is formulated in all scenarios. We prove that the WSR problem has zero duality gap in all scenarios, and the KKT conditions are necessary for the optimal solutions. Besides, we derive and generalize an iterative algorithm for all scenarios by applying the BSMM [38], [43]. Especially, in Scenario A, we provide an alternative solution that directly optimizes the WSR of the DPC region with BSMM instead of applying MAC-BC duality.

One main benefit of the proposed signaling design (power splitting, linear precoding, and power allocation) is its ability to be generalized to more complicated scenarios, e.g., when there are more than two users.

The remainder of this paper is organized as follows. In the next section, we discuss the channel model and formulate the problems for the three scenarios. We introduce a power splitting method to all scenarios in Section III-A and a signaling design for each in Section III-B. For the subcases without common messages, we generalize a WSR based on BSMM for all scenarios in Section IV. We then present numerical results in Section V and conclude the paper in Section VI.

**Notations:** \( \text{tr} (\cdot) \) and \( (\cdot)^T \) denote trace and transpose of matrices. \( E\{\cdot\} \) denotes expectation. \( \text{diag}(\lambda_1, \ldots, \lambda_n) \) represents diagonal matrix with elements \( \lambda_1, \ldots, \lambda_n \). \( Q \succeq 0 \) represents that \( Q \) is a positive semidefinite matrix. \( [x]^+ \) gives the max value of \( x \) and \( 0 \). \( I \) is the identity matrix.

## II. System Model

Considering a two-user MIMO-NOMA network, the BS equipped with \( n_t \) antennas simultaneously serves two users, in which user 1 and user 2 are equipped with \( n_1 \) and \( n_2 \) antennas, respectively. The transmitted signal to user 1 and user 2 share the same time slot and frequency. The received signals at user 1 and user 2 are given by

\[
\begin{align*}
y_1 &= H_1 x + w_1, \quad (1a) \\
y_2 &= H_2 x + w_2, \quad (1b)
\end{align*}
\]

where \( H_1 \in \mathbb{R}^{n_1 \times n_t} \) and \( H_2 \in \mathbb{R}^{n_2 \times n_t} \) are the channel matrices for user 1 and user 2, respectively. \( w_1 \in \mathbb{R}^{n_1 \times 1} \) and \( w_2 \in \mathbb{R}^{n_2 \times 1} \) are independent identically distributed (i.i.d.) Gaussian noise vectors whose elements are zero mean and unit variance. The input \( x \in \mathbb{R}^{n_t \times 1} \) is a vector consisting of three components

\[
x = x_0 + x_1 + x_2, \quad (2)
\]

where \( x_k \sim \mathcal{N}(0, Q_k), k = 0, 1, 2 \), is the input corresponding to two kinds of services: the multicast message \( M_0 \) and secure messages (private \( M_p \) and/or confidential \( M_c \)) of user 1 and user 2, in which \( Q_k \succeq 0 \) is the covariance matrix corresponding to \( x_k \). The channel input is subject to an average total power constraint

\[
\text{tr}(E\{xx^T\}) = \text{tr}(Q_0 + Q_1 + Q_2) \leq P. \quad (3)
\]

We denote \( R_0, R_{jp}, \) and \( R_{jc}, j = 1, 2 \), as the achievable rates associated with the multicast, private, and confidential messages transmitted by the corresponding user \( j \), respectively.

In the following, we provide the achievable rate region for each scenario.

### A. Scenario A (one common and two private messages)

The DPC rate region of the MIMO-NOMA with common and two private messages is realized by [12], [21].

\[
R_A(P) = \text{conv} \left\{ R_{12}^{\text{DPC}} \cup R_{21}^{\text{DPC}} \right\} \quad (4)
\]
in which \( \text{conv} \) is the convex hull operator. \( R_{12}^{\text{DPC}} \) consists of all triplets \((R_{1p}, R_{2p}, R_0)\) satisfying

\[
R_0 \leq \min(R_{01}, R_{02}), \quad (5a)
\]
\[
R_{1p} \leq \frac{1}{2} \log \left| I + H_1 Q_1 H_1^T \right|, \quad (5b)
\]
\[
R_{2p} \leq \frac{1}{2} \log \left| I + (I + H_2 Q_2 H_2^T)^{-1} H_2 Q_2 H_2^T \right| \quad (5c)
\]

where

\[
R_{0j} = \frac{1}{2} \log \left| I + \frac{H_j Q_j H_j^T}{(1 + H_j (Q_j + Q_2) H_j^T)} \right|, \quad j = 1, 2 \quad (6)
\]

with the total power constraint \( (3) \). \( R_{12}^{\text{DPC}} \) is obtained from \( R_{12}^{\text{DPC}} \) by swapping the subscripts 1 and 2 corresponding to different DPC encoding orders. Specifically, when each of the users has a single antenna, the problem can be transferred to a linear semi-definite convex optimization \[12, \text{Section III}\], but the MIMO case is in general still unknown. Without the common message, the capacity of the MIMO BC is given in \[33], \[34]\.

### B. Scenario B (common, private, and confidential messages)

In this scenario, only user 1 requires a confidential message. The secrecy capacity region \( R_B(P) \) under a total power constraint \( (3) \) is given by a set of rate triplets \((R_{1c}, R_{2p}, R_0)\) satisfying \[24, \text{Theorem 2}\]

\[
R_0 \leq \min(R_{01}, R_{02}), \quad (7a)
\]
\[
R_{1c} \leq \frac{1}{2} \log \left| I + H_1 Q_1 H_1^T \right| - \frac{1}{2} \log \left| I + H_2 Q_1 H_2^T \right|, \quad (7b)
\]
\[
R_{2p} \leq \frac{1}{2} \log \left| I + (I + H_2 Q_2 H_2^T)^{-1} H_2 Q_2 H_2^T \right|. \quad (7c)
\]

The entire secrecy capacity region is achieved using DPC to cancel out the signal of private \( M_{2p} \) at user 2, the other variant, i.e., DPC against \( M_{1c} \) at user 1, is unnecessary. This is different from Scenario A for which the capacity region is exhausted by taking the convex hull of both variants \((R_{12}^{\text{DPC}} \text{ and } R_{12}^{\text{DPC}})\).

### C. Scenario C (one common and two confidential messages)

The secrecy capacity region \( R_{1c}(P) \) of the MIMO-BC with one common and two confidential messages under the average total power constraint \( (3) \) can be expressed as \[16, \text{Theorem 2}\]

\[
R_0 \leq \min(R_{01}, R_{02}), \quad (8a)
\]
\[
R_{1c} \leq \frac{1}{2} \log \left| I + H_1 Q_1 H_1^T \right| - \frac{1}{2} \log \left| I + H_2 Q_1 H_2^T \right|, \quad (8b)
\]
\[
R_{2c} \leq \frac{1}{2} \log \left| I + (I + H_2 Q_2 H_2^T)^{-1} H_2 Q_2 H_2^T \right|
- \frac{1}{2} \log \left| I + (I + H_1 Q_1 H_1^T)^{-1} H_1 Q_2 H_1^T \right|, \quad (8c)
\]

The secrecy capacity region is characterized by S-DPC \[16, \[25\]. In this scenario, both of the users are legitimate and secret from each other. User 1 with confidential messages \( M_{1c} \) treats user 2 as an eavesdropper, and vice versa.

The secrecy capacity regions in \[5], \[7], \[8\] are obtained via DPC or S-DPC which can be obtained by an exhaustive search over the set of all possible input covariance matrices. However, the complexity of such methods for practical implementations is prohibitive, which motivates us to develop a simpler signaling scheme. The covariance matrices achieving the capacity regions are not known in general due to the non-convexity.

### III. POWER SPLITTING METHOD FOR MIMO-NOMA IN ALL SCENARIOS

In order to introduce a new simpler and faster solution, we split the total power for three messages in each scenario. Then, we decouple the MIMO-NOMA channel of all secrecy scenarios into three different problems and solve them separately.

#### A. Decomposing Secure MIMO-NOMA into Simpler Channels

We decompose MIMO-NOMA into different problems in this section. Due to some overlapping, such as the privacy part in Scenario A and Scenario B, the confidentiality part in Scenario B and Scenario C, we start with \textit{Step 1} to split the power between user 1 and user 2 for different usages; Then, \textit{Step 2} is for user 1 with private messages in Scenario A and confidentiality in Scenario B, respectively; \textit{Step 3} is for user 2 with private messages in Scenario A, and confidentiality in Scenario C, correspondingly; Last, \textit{Step 4} is for common message in all scenarios.

#### Step 1: Introducing power splitting factor \( \alpha_k \in [0, 1] \), and \( \sum_k \alpha_k = 1, k = 0, 1, 2 \). We dedicate a fraction \( \alpha_k \) of the total power to user 1 for the private message \( M_{1p} \) or the confidential message \( M_{1c} \) \((P_1 = \alpha_1 P)\), and fraction \( \alpha_2 \) to user 2 for the secure message \( M_{2p} \) or \( M_{2c} \) \((P_2 = \alpha_2 P)\). Last, we allocate the remaining power to the common message \( M_0 \) for both users \((P_0 = \alpha_0 P = (1 - \alpha_1 - \alpha_2) P)\).

#### Step 2a: We design the secure precoding for user 1 with private message \( M_{1p} \) in \[56\] for Scenario A. Since the rate \( R_{1p}(\alpha_1) \) of user 1 is only controlled by the covariance matrix \( Q_1 \) under the power constraint \( P_1 \), the interference-free link can be seen as a P2P MIMO with power \( P_1 \), which is

\[
R_{1p}(\alpha_1) = \max_{Q_1 \succeq \text{diag}(1/\alpha_1)} 1 - \frac{1}{2} \log \left| I + H_1 Q_1 H_1^T \right| \quad (9a)
\]

\[
s.t. \quad \text{tr}(Q_1) \leq P_1 = \alpha_1 P. \quad (9b)
\]

The solution for \( Q_1^* \) has been well-developed analytically through singular value decomposition (SVD) and water filling (WF) \[29\].

\footnote{The S-DPC can assure security between the two users because a precoding matrix is selected such that it satisfies two goals \[15, \text{Remark 5}\]. First, it helps to cancel the precoding signal representing message \( M_{2c} \), so that \( M_{1c} \) can be served with an interference-free legitimate user channel. Second, it boosts the secrecy for message \( M_{2c} \), by causing interference (artificial noise) to user 1. In other words, user 1 can remove the interference of user 2 but is not able to decode the message of user 2.}

\footnote{The optimal solution using total power throughout the paper.
Step 2b: In Scenario B and Scenario C, we design secure precoding for user 1 with confidential messages $M_{1c}$ while treating the second user as an eavesdropper. Because covariance matrix $Q_1$ is the only variable in (7b) and (8b), the problem can be seen as a wiretap channel under a transmit power $P_1$, which is

$$R_{1c}(\alpha_1) = \max_{Q_1 \succeq 0} \frac{1}{2} \log \frac{|I + H_1 Q_1 H_1^T|}{|I + H_2 Q_1 H_2^T|},$$

s.t. $\text{tr}(Q_1) \leq P_1 = \alpha_1 P$. \hspace{1cm} (10a)

This problem is now the well-known MIMO wiretap channel [32], and standard MIMO wiretap solutions can be applied to obtain $Q_1^*$.

Step 3a: We design secure precoding for user 2 to maximize the secrecy rate $R_{2p}(\alpha_2)$ of user 2 with private message $M_{2p}$, we apply $Q_1^*$ obtained in Step 2a and Step 2b to (5c) and (7c) for Scenario A and Scenario B, respectively. The interference channel can be transformed to an interference-free link over modified channels. Thus, (5c) or (7c) can be represented as

$$R_{2p}(\alpha_2) = \max_{Q_2 \succeq 0} \frac{1}{2} \log \left| I + (I + H_2 Q_1^* H_2^T)^{-1} H_2 Q_2 H_2^T \right|,$$

s.t. $\text{tr}(Q_2) \leq P_2 = \alpha_2 P$. \hspace{1cm} (11a)

Since $Q_1^*$ is given after solving decomposing MIMO-NOMA into different problems, in the following we show that the above problem can be seen as a P2P MIMO problem under power $P_2$.

Theorem 1. The optimization problem in (11) with interference from user 1 can be converted to the optimization of a standard P2P MIMO channel

$$\tilde{H}_2 \triangleq B^{-\frac{1}{2}} C^T H_2$$

for user 2, in which $B$ and $C$ are the eigenvalue and eigenvector of $I + H_2 Q_1^* H_2^T$.

Proof. Define

$$\Sigma \triangleq I + H_2 Q_1^* H_2^T = CBC^T.$$ \hspace{1cm} (13)

Then, the secrecy rate for user 2 with private $M_{2p}$ can be written as

$$R_{2p}(\alpha_2) = \max_{Q_2 \succeq 0} \frac{1}{2} \log \left| I + \Sigma^{-1} H_2 Q_2 H_2^T \right| = \max_{Q_2 \succeq 0} \frac{1}{2} \log \left| I + CB^{-1} C^T H_2 Q_2 H_2^T \right| \overset{(a)}{=} \max_{Q_2 \succeq 0} \frac{1}{2} \log \left| I + B^{-\frac{1}{2}} C^T H_2 Q_2 H_2^T CB^{-\frac{1}{2}} \right| = \max_{Q_2 \succeq 0} \frac{1}{2} \log \left| I + H_2 Q_2 H_2^T \right|, \hspace{1cm} (14)$$

in which (a) holds because of Sylvester’s determinant theorem, i.e., $\det(I + XY) = \det(I + YX)$, $C$ is orthogonal, i.e., $C^{-1} = C^T$, and $B$ is a diagonal matrix.

In view of (14), the problem in (11) becomes the standard P2P MIMO without interference over a modified channel. The solution $Q_2^*$ can be obtained as the same as Step 2a.

Step 3b: We design secure precoding for user 2 to maximize the secrecy rate $R_{2e}(\alpha_2)$ of user 2 with confidential message $M_{2e}$, we apply $Q_1^*$ obtained in Step 2b to (6c) for Scenario C. Thus, (6c) can be represented as

$$R_{2e}(\alpha_2) = \max_{Q_2 \succeq 0} \left\{ \frac{1}{2} \log \left| I + \frac{H_2 Q_2 H_2^T}{I + H_1 Q_1^* H_2^T} \right| - \frac{1}{2} \log \left| I + \frac{H_2 Q_2 H_2^T}{I + H_1 Q_1^* H_1^T} \right| \right\},$$

s.t. $\text{tr}(Q_2) \leq P_2 = (1 - \alpha) P$. \hspace{1cm} (15a)

Since $Q_1^*$ is given after solving (10), next we show that the problem (15) can be seen as a wiretap channel where users 2 and 1 are the legitimate user and eavesdropper, respectively.

Theorem 2. The above channel can be converted to a standard MIMO wiretap channel with

$$\tilde{H}_1 \triangleq D_a^{-\frac{1}{2}} E_a^T H_1,$$

$$\tilde{H}_2 \triangleq D_b^{-\frac{1}{2}} E_b^T H_2,$$

in which $D_a$ and $E_a$ are the eigenvalues and eigenvectors of $I + H_1 Q_1^* H_1^T$, and $D_b$ and $E_b$ are the eigenvalues and eigenvectors of $I + H_2 Q_1^* H_2^T$.

Then, the rate for user 2 can be written as

$$R_{2e}(\alpha_2) = \max_{Q_2 \succeq 0} \frac{1}{2} \log \left| I + \tilde{H}_2 Q_2 \tilde{H}_2^T \right|,$$ \hspace{1cm} (17)

In view of (17), it is seen that similar to (10a), (15a) is the rate for a MIMO wiretap channel with channels $\tilde{H}_2$ for the legitimate user and $\tilde{H}_1$ for the eavesdropper. This problem now transfers to a MIMO wiretap channel, and we can obtain $Q_2^*$ using any standard MIMO wiretap solutions.

Step 4: After distributing the power to both users for secrecy messages, we allocate the remaining power $P_0 = \alpha_0 P, \alpha_0 = 1 - \alpha_1 - \alpha_2$ to the common message $M_0$ for both users. The equation (5c), (7a), and (8a) becomes

$$R_0(\alpha_0) = \max_{Q_0 \succeq 0} \min\{R_{0j}, j = 1, 2\} \hspace{1cm} \text{s.t.} \hspace{0.5cm} \text{tr}(Q_0) \leq P_0 = \alpha_0 P, \hspace{1cm} (18a)$$

Since $Q_1^*$ and $Q_2^*$ are given, we can show that the above problem becomes MIMO multicasting [19] by applying the same approach as Theorem 1 again into (6). Specifically, let us define the denominator of (6) as

$$K_j \triangleq I + H_j (Q_1^* + Q_2^*) H_j^T \triangleq F_j G_j F_j^T,$$ \hspace{1cm} (19)

for $j = 1, 2$, where the second equality is given by eigenvalue decomposition. Then, $R_{0j}$ can be rewritten as

$$R_{0j} = \frac{1}{2} \log \left| I + K_j^{-1} H_j Q_0 H_j^T \right| = \frac{1}{2} \log \left| I + G_j^{-\frac{1}{2}} F_j^T H_j Q_0 H_j^T F_j G_j^{-\frac{1}{2}} \right| = \frac{1}{2} \log \left| I + H_j Q_0 H_j^T \right| \hspace{1cm} (20)$$
Then, we can generate $Q$ optimally.

Step 2a: The solutions of (9) in Step 2 are exhaustively searched over all power fractions $\alpha_1$, $\alpha_2$, and $\alpha_0$, to solve precoding matrices $Q_1^*$, $Q_2^*$, and $Q_3^*$ (and thus $R_{1p}(\alpha_1)$, $R_{2p}(\alpha_2)$, and $R_0(\alpha_0)$). Alternatively, $R_{21}^{\text{DPC}}$ is obtained by encoding the private messages for user 2 and user 1, then the common message for both. We can solve $Q_2^*$ followed by $Q_1^*$ and $Q_3^*$ to obtain $R_{1p}(\alpha_1)$, $R_{2p}(\alpha_2)$, and $R_0(\alpha_0)$, respectively.

Corollary 1. The achievable DPC rate region for secure MIMO-NOMA Scenario A under the total power is the convex hull of all rate triplets

$$R_A(P) = \text{conv} \left\{ \left( \bigcup_{k=1}^{K} R_{12}^{\text{DPC}}(\alpha_k) \right) \bigcup \left( \bigcup_{k=1}^{K} R_{21}^{\text{DPC}}(\alpha_k) \right) \right\},$$

where $R_{12}^{\text{DPC}}(\alpha_k) = (R_{1p}(\alpha_k), R_{2p}(\alpha_k), R_0(\alpha_k))$, $k = 0, 1, 2$, and is obtained by encoding the private messages for first user 1 then user 2 followed by the common message for both, whereas $R_{21}^{\text{DPC}}(\alpha_k) = (R_{1p}(\alpha_k), R_{2p}(\alpha_k), R_0(\alpha_k))$ is obtained in the reverse order of private messages (first user 2 then user 1).

Scenario B: (Step 1 $\rightarrow$ Step 2b $\rightarrow$ Step 3a $\rightarrow$ Step 4) In light of our decomposition in the previous section, Scenario B is characterized as one MIMO wiretap channel in Step 2b and one P2P MIMO problem in Step 3a. Standard MIMO wiretap solutions can be applied to design covariance matrix $Q_1$. One fast approach is the rotation-based linear precoding [32]. In this method, the covariance matrix $Q_1$ is eigendecomposed into one rotation matrix $V_1$ and one power allocation matrix $\Lambda_1$ [32, 42] as

$$Q_1 = V_1 \Lambda_1 V_1^T.$$

Consequently, the secrecy capacity of user 1 is

$$R_{1c}(\alpha_1) = \max_{\lambda > 0} \frac{1}{2} \log \left| I + H_1 V_1 \Lambda_1 V_1^T H_1^T \right|,$$

subject to

$$\sum_{n=1}^{n_1} \lambda_{1n} \leq P_1 - \alpha P,$$

in which $\lambda_{1n}, n = \{1, \ldots, n_1\},$ is a diagonal element of matrix $\Lambda_1 = \text{diag}(\lambda_{11}, \ldots, \lambda_{n_1})$. The rotation matrix $V_1$ can be obtained by

$$V_1 = \prod_{p=1}^{n_1} \prod_{q=p+1}^{n_1} V_{pq},$$

in which the basic rotation matrix $V_{pq}$ is a Givens matrix which is an identity matrix except that its elements in the $p$th row and $q$th column, i.e., $v_{pp}$, $v_{pq}$, $v_{qp}$, and $v_{qq}$ are replaced by

$$\begin{bmatrix} v_{pp} & v_{pq} \\ v_{qp} & v_{qq} \end{bmatrix} = \begin{bmatrix} \cos \theta_{1pq} & -\sin \theta_{1pq} \\ \sin \theta_{1pq} & \cos \theta_{1pq} \end{bmatrix},$$

in which $\theta_{1pq}$ is rotation angle corresponding to the rotation matrix $V_{pq}$. Then, we will optimize the parameterized problem

3 This paper is focused on the two-user case. For $K$-user MIMO wiretap channel with an external eavesdropper, $K$ covariance matrices should be constructed independently. When applying the rotation method for user $k$, we will have independent rotation parameters for each user to optimize. For multiple external eavesdroppers, a large-scale analysis may be needed to evaluate the security performance [44].
by applying numerical approaches such as Broyden-Fletcher-Goldfarb-Shanno (BFGS) method \[45\] to obtain the solution $Q_1^*$ (thus $R_{1c}^*(\alpha_1)$) with respect to rotation angles and power allocation parameters. Then, to obtain $Q_2^*$ and $R_{2p}^*(\alpha_2)$ in Step 3a, $Q_1^*$ above is applied in Theorem 1 and we solve the modified P2P MIMO problem using Lemma 1. The precoding approach for Step 4 is the same as Scenario A. The achievable secrecy rate for Scenario B is given by the following corollary.

**Corollary 2.** The achievable rate region for secure MIMO-NOMA Scenario B under the total power is the convex hull of all rate triplets

$$R_B(P) = \bigcup_{\alpha_k} (R_{1c}^*(\alpha_1), R_{2p}^*(\alpha_2), R_0^*(\alpha_0)). \quad (26)$$

**Scenario C:** (Step 1 $\rightarrow$ Step 2b $\rightarrow$ Step 3b $\rightarrow$ Step 4)

In Scenario C, the steps are the same as Scenario B except for Step 3b which can be seen as a wiretap channel instead of P2P MIMO. Then, we apply Theorem 2 and solve \[17\] instead. Similar to the precoding in Step 2b of Scenario B, the covariance matrix $Q_2$ can be written by rotation method as $Q_2 = \mathbf{V}_2 \mathbf{A}_2 \mathbf{V}_2^T$, where the rotation matrix $\mathbf{V}_2$ is defined similarly to $\mathbf{V}_1$ in \[24\] with rotation angles $\theta_{2p}$. Therefore, the optimization problem for $R_{2c}^*(\alpha_2)$ becomes

$$R_{2c}^*(\alpha_2) = \max_{Q_2 \succeq 0} \frac{1}{2} \log \frac{|I + H_2 \mathbf{V}_2 \mathbf{A}_2 \mathbf{V}_2^T H_2^T|}{|I + H_1 \mathbf{V}_2 \mathbf{A}_2 \mathbf{V}_2^T H_1^T|} \quad (27a)$$

s.t. \( \sum_{n=1}^{n_c} \lambda_{2n} \leq P_2 = (1-\alpha)P \) \quad (27b)

in which $\mathbf{A}_2 = \text{diag}(\lambda_{21}, \ldots, \lambda_{2n_r})$. This problem is again similar to \[23\].

In the power splitting scheme, we solve $Q_1^*$, $Q_2^*$, and $Q_0^*$ to obtain $R_{1c}^*(\alpha_1)$, $R_{2c}^*(\alpha_2)$, and $R_0^*(\alpha_0)$ with respect to power splitting parameters pair $(\alpha_1, \alpha_2, \alpha_0)$. Alternatively, we can first solve for $Q_2^*$ followed by $Q_1^*$ last $Q_0^*$ (i.e., first $R_{1c}^*(\alpha_1)$, $R_{2c}^*(\alpha_2)$, then $R_0^*(\alpha_0)$). In general, changing the order of optimization will result in a different rate region. The convex hull of the two solutions with different orders enlarges the achievable rate region. The achievable secrecy rate for Scenario C is given by Corollary 3.

**Corollary 3.** The achievable DPC rate region for the secure MIMO-NOMA Scenario C under the total power is the convex hull of all rate triplets

$$R_C(P) = \text{conv} \left\{ \bigcup_{\alpha_k} R_{12}(\alpha_k) \bigcup_{\alpha_k} R_{21}(\alpha_k) \right\}, \quad (28)$$

in which $R_{12}(\alpha_k) = (R_{1c}^*(\alpha_1), R_{2c}^*(\alpha_2), R_0^*(\alpha_0))$, $k = 0, 1, 2$, is obtained by encoding the confidential messages for user 1 first, then user 2, and lastly the common message for both, whereas $R_{21}(\alpha_k) = (R_{1c}^*(\alpha), R_{2c}^*(\alpha), R_0^*(\alpha_0))$ is obtained in the reverse order of confidential messages (first user 2 then user 1).

Algorithm 1 summarizes the power splitting method for all of the scenarios. $\epsilon_1$ is the searching step for the power allocation factor. If $\alpha_1 = \alpha_2 = 0$ and $\alpha_0 \neq 0$, then the system reduces to multiantenna transmission. If no power is allocated to the common message, it is the private transmission cases in the next section. If only the power of one of the secrecy messages is zero ($\alpha_k = 0, k = 1$ or 2), the problem is integrated service with confidential and common messages.

The precoding order for secrecy messages at different scenarios is not the same. Corollary 1 and Corollary 3 in Scenario A and Scenario C require an exchange of subscripts. For Scenario A, this is because the encoding order affects the achievable rate region. For Scenario C, although encoding order is irrelevant to the achievable rate in S-DPC, the order of optimization (solve the covariance matrix) will affect the solution \[39\]. This is because the power splitting method splits the power among the messages and solves them one by one. This simplifies the problem but is sub-optimal in general. Then, changing the precoding order may enlarge the achievable rate region. For scenario B, as proved in \[24\] Remark 4, it is always better to cancel the private message $M_p$ at user 1 and treat the confidential message $M_{1c}$ at user 2 as noise. Thus, there is no need to exchange the precoding order.

---

**Algorithm 1** Power splitting for all three scenarios

1. inputs: secrecy scenario $L \in \{A, B, C\}$, and $\epsilon_1$;
2. for $\alpha_1 = 0 : \epsilon_1 : 1$ do
3. for $\alpha_2 = 0 : \epsilon_1 : 1 - \alpha_1$ do
4. $\alpha_0 = 1 - \alpha_1 - \alpha_2$;
5. switch $L$
6. case $A$:
7. Obtain $Q_1^*$ using Lemma 1 in problem (9);
8. Compute $R_{1p}$ in (9);
9. case $B$ or $C$:
10. Obtain $Q_2^*$ by solving (23) using BFGS;
11. Compute $R_{1c}$ in (10);
12. end switch
13. switch $L$
14. case $A$ or $B$:
15. Obtain $Q_2^*$ using Theorem 2, the $Q_1^*$ in Line 7 or Line 10 with respect to A or B, and Lemma 1 in problem (11);
16. Compute $R_{2p}$ in (11);
17. case $C$:
18. Obtain $Q_2^*$ using Theorem 2, the $Q_1^*$ in Line 10, and BFGS by solving (27);
19. Compute $R_{2c}$ in (15);
20. end switch
21. Compute $R_0$ as described in Step 4;
22. end for
23. end for
24. if $L = A$ or $L = C$ then
25. swap all subscripts of 1 and 2 in (5) or (8);
26. repeat switch and obtain $R_{2p}(\alpha_1)$ or $R_{2p}(\alpha_1)$ in Corollary 1 and Corollary 3
27. end if
28. outputs: $R_L(P)$. 
KKT conditions are necessary for the optimal solution. Lemma 2. and the dual problem is given by

\[ \text{minimize } \sum_{i} R_i \quad \text{subject to } Q_i \geq 0, \quad Q_i \text{ being a positive definite matrix.} \]

Thus, the complexity of Scenario A is \( O(m^3 + n_i^3) \). For Scenario B and Scenario C, the complexity of solving wiretap channels in Step 2b and Step 3b has \( O(m^3 + n_i^4) \). Ignore the coefficient, the overall complexity of Algorithm 1 is \( O(m^3 + n_i^4) \).

Thus, the complexity of Scenario A is \( O(m^3 + n_i^4) \) \( m = \max(n_1, n_2) \) and Scenario B in this paper.

Remark 1 (Complexity). For Scenario A, Step 2a, Step 3a, Case 1, and Case 2 in Step 4 are analytical, which only requires the computation of matrix multiplications and matrix inverse. The computation of matrix multiplications and matrix inverse has the complexity of \( O(m^3) \) in which \( m = \max(n_1, n_2) \).

Case 3 in Step 4 uses \texttt{fmincon} which is achieved mainly by BFGS. The BFGS algorithm yields the complexity \( O(n^2) \) \( n = \frac{(m+1)n}{2} \) is rotation parameters \( n \) = \( \frac{(m+1)n}{2} \). Thus, the complexity of Scenario A is \( O(m^3 + n_i^4) \). For Scenario B and Scenario C, the complexity of solving wiretap channels in Step 2b and Step 3b has \( O(m^3 + n_i^4) \). Ignore the coefficient, the overall complexity of Algorithm 1 is \( O(m^3 + n_i^4) \).

The BFGS algorithm yields the complexity \( O(n^2) \) \( n = \frac{(m+1)n}{2} \) is rotation parameters \( n \) = \( \frac{(m+1)n}{2} \). Thus, the complexity of Scenario A is \( O(m^3 + n_i^4) \). For Scenario B and Scenario C, the complexity of solving wiretap channels in Step 2b and Step 3b has \( O(m^3 + n_i^4) \). Ignore the coefficient, the overall complexity of Algorithm 1 is \( O(m^3 + n_i^4) \).

Thus, the complexity of Scenario A is \( O(m^3 + n_i^4) \) \( m = \max(n_1, n_2) \) and Scenario B in this paper.

IV. WEIGHTED SUM RATE FORMULATION FOR SECRECY

We consider the subcases of the three scenarios of the MIMO-NOMA without a common message (\( M_0 = \emptyset \)) in this section. A weighted sum rate maximization based on BSMM \( \text{(38), (43)} \) is generalized to all scenarios. The WSR maximization for the MIMO-NOMA with private and confidential messages under a total power constraint is formulated as

\[ \varphi(P) = \max_{Q_k \geq 0} \sum_k w_k R_k, \quad k = 1, 2 \]

\[ \text{s.t. } \text{tr}(Q_k) \leq P, \]

where \( R_k := R_{kp} \) in Scenario A, \( R_1 := R_{1e} \) and \( R_2 := R_{2p} \) in Scenario B, and \( R_k := R_{ke} \) in Scenario C. \( w_k \geq 0 \) is a weight. The Lagrangian of the problem \( \text{(29)} \) is

\[ L(Q_1, Q_2, \lambda) = w_1 R_1 + w_2 R_2 - \lambda (\text{tr}(Q_1 + Q_2) - P), \]

where \( \lambda \) is the Lagrange multiplier related to the total power constraint. The dual function is a maximization of the Lagrangian

\[ g(\lambda) = \max_{Q_k \geq 0} L(Q_1, Q_2, \lambda), \]

and the dual problem is given by

\[ \min_{\lambda \geq 0} g(\lambda). \]

Lemma 2. The problem in \( \text{(29)} \) has zero duality gap and the KKT conditions are necessary for the optimal solution.

Proof: See the proof in Appendix A.

Since the problem in \( \text{(31)} \) is a nonconvex problem in any secrecy transmission, the BSMM \( \text{(38), (43)} \) can be considered which alternatively updates covariance matrix by maximizing a set of strictly convex local approximations. Specifically, Scenario C has been studied in \( \text{(38)} \). We discuss Scenario A and Scenario B in this paper.

A. Scenario A

It is worth noting that the MAC-BC duality \( \text{(33)} \) is applied to WSR maximization in \( \text{(35)} \) where the WSR on MAC rate region can be transformed to BC rate region by an iterative algorithm. Then, the WSR can be solved using convex optimization. Once the optimum uplink covariance matrices are determined by any standard convex optimization tool, the equivalent downlink covariance matrices can be obtained through the duality transformation \( \text{(33)} \). The optimization in MAC requires a descent algorithm over a line search with a tolerance. It also mentions that the DPC rate region is difficult to compute without employing duality \( \text{(33)} \). Yet in this paper, we provide an alternative solution without applying the MAC-BC duality. We form a WSR for the DPC rate region directly and solve the maximization by using BSMM.

Since \( \text{(32)} \) is nonconvex, we can apply BSMM which updates covariance matrices by successively optimizing the lower bound of local approximation of \( f(Q_1, Q_2) = L(Q_1, Q_2, \lambda) \)

and solve the maximization by using BSMM. Then, we optimize the right-hand side of \( \text{(37)} \) by omitting the constant terms, which is equivalent as

\[ \sum_{k=1}^{m} \frac{w_k}{2} \log |I + H_k Q_k H_k^H| - \text{tr}(\lambda I + A) Q_k. \]
Next, we optimize $f(Q_1^{(i)}, Q_2)$ by fixing $Q_1^{(i)}$, which is equivalent as
\[
Q_2^{(i)} = \arg \max_{Q_2} w_2 \log |I + (I + H_2 Q_1^{(i)} H_2^T)^{-1} H_2 Q_2 H_2^T| - \lambda \text{tr}(Q_2). \tag{39}
\]

The optimal solution for $Q_2$ can be achieved by the following lemma [38].

**Lemma 3.** [38] For some $S > 0$, the optimal solution of the problem
\[
\max_{Q > 0} w \log |I + R^{-1} HQHT| - \text{tr}(SQ) \tag{40}
\]
is given by
\[
Q^* = S^{-1/2} V A V^T S^{-1/2}. \tag{41}
\]

To use Lemma 3, we set $w = \frac{w_1}{T}$, $S = \lambda I + A$, and $R = I$ for (38); and $w = \frac{w_2}{T}$, $S = \lambda I$, and $R = I + H_2 Q_1^{(i)} H_2^T$ for (39). $V$, $U$, and $A$ are obtained by eigenvalue decomposition of $R^{-1/2} HS^{1/2} = U \text{diag}(\sigma_1, \sigma_2, \cdots, \sigma_m) V^T$, $\sigma_i > 0$, $\forall i$, $A = \text{diag}((w - 1/\sigma_1^2)^+, \cdots, (w - 1/\sigma_m^2)^+)$, and $(x)^+ = \max(x, 0)$.

**B. Scenario B**

In Scenario B, what makes it different from Scenario A is the formulation of convex and concave functions, which can be written as
\[
f_1(Q_1) = \frac{w_1}{2} \log |I + H_1 Q_1 H_1^T| - \lambda \text{tr}(Q_1) \tag{42a}
\]
\[
f_2(Q_1, Q_2) = -\frac{w_2}{2} \log |I + H_2 Q_1 H_2^T| + \frac{w_2}{2} \log |I + (I + H_2 Q_1 H_2^T)^{-1} H_2 Q_2 H_2^T| - \lambda \text{tr}(Q_2). \tag{42b}
\]

$f_1(Q_1)$ is also a concave function of $Q_1$, while $f_2(Q_1, Q_2)$ is convex by fixing $Q_2$, because the second term in (42b) is convex over $Q_1$. For the $i$th iteration, the function for $f_2(Q_1, Q_1^{(i-1)})$ is lower-bounded by its first-order Taylor approximation as the expression in (35), in which the power price matrix
\[
A = -\nabla Q_1 f_2(Q_1^{(i-1)}, Q_1^{(i-1)})
= \frac{w_1 + w_2}{2} H_2^T (I + H_2 Q_1^{(i-1)} H_2^T)^{-1} H_2
- \frac{w_2}{2} H_2^T (I + H_2 Q_1^{(i-1)} + Q_2^{(i-1)} H_2^T)^{-1} H_2. \tag{43}
\]

Finally, we optimize the right-hand side of (37) with the power price matrix in (43), which is equivalent as
\[
Q_1^{(i)} = \arg \max_{Q_1} \frac{w_1}{2} \log |I + H_1 Q_1 H_1^T| - \text{tr}((\lambda I - A) Q_1). \tag{44}
\]

Next, we optimize $L(Q_1^{(i)}, Q_2)$ by fixing $Q_1^{(i)}$, which is equivalent as
\[
Q_2^{(i)} = \arg \max_{Q_2} \frac{w_2}{2} \log |I + (I + H_2 Q_1^{(i)} H_2^T)^{-1} H_2 Q_2 H_2^T| - \lambda \text{tr}(Q_2). \tag{45}
\]

**Algorithm 2** WSR maximization for all three scenarios without a common message

1: inputs: $\lambda_{\text{max}}, \lambda_{\text{min}}, \epsilon_2, \epsilon_3$, secrecy scenario $L \in \{A, B, C\}$
2: while $\lambda_{\text{max}} - \lambda_{\text{min}} > \epsilon_2$ do
3: $\lambda := (\lambda_{\text{max}} + \lambda_{\text{min}})/2$
4: $Q_1^{(0)} := Q_2^{(0)} := \frac{L}{2m} I$
5: $R^{(0)} := 0$
6: $i = 0$
7: while $1$ do
8: $i = i + 1$
9: switch $L$
10: case A:
11: Solve $Q_1^{(i)}$ and $Q_2^{(i)}$ in (38)-(39) using Lemma 3
12: Compute $R_1$ and $R_2$ in (5)
13: case B:
14: Solve $Q_1^{(i)}$ and $Q_2^{(i)}$ in (44)-(45) using Lemma 3
15: Compute $R_1$ and $R_2$ in (7)
16: case C:
17: Solve $Q_1^{(i)}$ and $Q_2^{(i)}$ in (38) Algorithm 1, lines 5-13
18: Obtain $R_1$ and $R_2$ in (8)
19: end switch
20: $R^{(i)} := w_1 R_1 + w_2 R_2$
21: if abs($R^{(i)} - R^{(i-1)} < \epsilon_3$ then
22: break;
23: end if
24: if $\text{tr}(Q_1^{(i)} + Q_2^{(i)}) < P$ then
25: $\lambda_{\text{max}} := \lambda$
26: else
27: $\lambda_{\text{min}} := \lambda$
28: end if
29: end while
30: end while
31: outputs: $\lambda^* := \lambda$, $R_k := R_k$, and $Q_k^* = Q_k^{(i)}, k \in \{1, 2\}$.

The WSR maximization for all scenarios without common message is summarized in Algorithm 2. $\epsilon_2$ and $\epsilon_3$ are the bisection search accuracy and convergence tolerance of BSMM, respectively. If $w_1 = 0$ and $w_2 = 1$, the problem reduces to a P2P MIMO with an analytical solution. Algorithm 2 becomes a WF regime. If $w_1 = 1$ and $w_2 = 0$, then problem in (29) is the secrecy rate maximization over MIMO wiretap channel. Then, Algorithm 2 is nothing but AOWF [31]. WSR maximization using BSMM can solve the special cases of the secrecy capacity regions in all three scenarios. However, it is hard to extend directly to the general cases of the three scenarios, because the max-min problem of multicasting is not derivable in BSMM although the multicasting problem owns convexity. Thus, we propose a power splitting method for the general cases in Section III.

Encoding order for secrecy messages in different scenarios is distinguished. In Scenario A, the weight determines the optimal encoding order. For example, if $w_1 > w_2$, the optimal
encoding order is encoding for user 1 of private message $M_{1p}$ first and then private message $M_{2p}$ for user 2 is encoded last. The capacity region is taken over all permutations of the users’ order [34]. In Scenario B, the entire capacity region is using DPC to cancel the signal of the private message $M_{2p}$ intended for user 2 at user 1 only. The other variant which treats the private message $M_{2p}$ of user 2 as interference for user 1 is unnecessary [24, Remark 4]. In Scenario C, the S-DPC owns the invariant property that the achievable rate region is irrelevant with respect to the encoding order [16].

The three scenarios without common messages differentiate the security requirements. The achievable region is limited with a higher secrecy requirement. For comparison, we show an example in Fig. 3 with the same channel settings as [15], [24]. First, when the secrecy message of user 2 is zero, i.e., $M_{2s} = 0$ in Scenario B and $M_{2c} = 0$ in Scenario C, the maximal achieving rates for user 1 in the two cases are the same, and the two problems drop to the Gaussian wiretap channel. Second, when the secrecy message of user 1 is zero, i.e., $M_{1p} = 0$ in Scenario A and $M_{1c} = 0$ in Scenario B, the achieving rates for user 2 in the two cases become the same P2P MIMO problem. Third, imposing a secrecy constraint on two users in Scenario C strictly shrinks the capacity region compared with Scenario A.

Remark 2 (Complexity). The number of iterations of the BSMM is $O(1/\epsilon_3)$, and the bisection search requires $O(\log(1/\epsilon_2))$. The weighted sum rate of Algorithm 2 has the complexity of $O(\frac{n \log(1/\epsilon_2)}{\epsilon_1})$ with a search step $\sigma$ over the weight [38], [39]. On the other hand, the computation complexity of Algorithm 1 without common messages is $O(\frac{n^3 \log(n)}{\epsilon_1})$ with only one layer of search loop over $\alpha_1$.

V. NUMERICAL RESULTS

In this section, we perform numerical results to illustrate the achievable secrecy rate region of the three scenarios under the average power constraint and then verify Algorithm 1 and Algorithm 2.

A. Secrecy Rate Regions for Three Scenarios

First, we verify the transmission rates for all of the scenarios. In this simulation, the channels for user 1 and user 2 are chosen to be

$$H_1 = \begin{bmatrix} 0.3861 & 0.6355 \\ 0.9995 & 0.6259 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.4977 & 0.9658 \\ 0.9245 & 0.6116 \end{bmatrix},$$

where the channel coefficients are generated independently according to Gaussian distribution, and the total power is 10. The search steps for $\alpha_1$ in Algorithm 1 is 0.05. Fig. 4 depicts the secrecy rate regions of the three scenarios. The PS scheme is compared with TDMA based scheme which is realized by transmitting messages in three orthogonal time slots with equal length. Also, the upper bounds are achieved by DPC [21], [22] for Scenario A, capacity rate regions [24] and [16] for Scenario B and C, respectively, which are realized by exhaustive search over all possible covariance matrices. It is shown that the proposed precoding and power allocation method significantly outperforms the TDMA strategy, and it is close to that of the capacity rate regions. The projection of the secrecy capacity region onto the $(R_1, R_2)$ or $(R_0, R_k)$, $k = 1, 2$, plane is the capacity region with two secrecy messages or only one secrecy message, which is going to appear in the next subsections.

It is worth mentioning that in Scenario A, given a set of power allocation parameters, we can analytically obtain the rate triplets, i.e., SVD and WF in Step 2a and Step 3a. The complexity of the algorithm for finding one point on the region only comes from matrix operations, and no search is needed. In [12, Section III] where each user is equipped with one antenna, the rate maximization optimization is transferred to the power minimization problem, and thus a linear semi-definite convex optimization is obtained, but it needs a binomial search of one parameter and then apply one numerical method using standard semi-definite programming methods, e.g., CVX [48].

B. Secrecy Rate Regions without Common Messages

Consider the MIMO-NOMA case without common messages, the achievable rate region is realized by Algorithm 1 with $M_0 = \emptyset$ and $\alpha_0 = 0$. Also, WSR with BSMM in Algorithm 2 is compared. The capacity regions are achieved by the parameters including the search step 0.01, the total power $P = 2, 4, 10$, respectively, and the channels denote as

$$H_1 = \begin{bmatrix} 0.1560 & -0.6372 & -0.4055 \\ -1.1450 & -0.1417 & 0.0708 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} -1.5032 & 0.5503 & -0.0334 \end{bmatrix}.$$

Figure 5 compares the rate regions of the proposed power splitting scheme with the capacity region achieved by DPC for Scenario A [7], [34] generated using the iterative algorithm with MAC-BC duality presented [35], and Scenario B [24], respectively, and S-DPC [15] for Scenario C. In general, the proposed algorithms can outperform OMA and achieve capacity.

![Fig. 3: Secrecy capacity regions of three scenarios under an average total power constraint without common message over the channel $H_1 = [0.3 2.5; 2.2 1.8]$ and $H_2 = [1.3 1.2; 1.5 3.9]$, and $P = 12$.](image-url)
In Scenario A, we consider another case when the numbers of receivers’ antennas are limited to be the same, i.e., $n_1 = n_2$. The channels are

\[
\mathbf{H}_1 = \begin{bmatrix}
-1.3784 & 0.2593 & -0.2040 \\
-1.0689 & -2.4811 & -1.2978
\end{bmatrix},
\]

\[
\mathbf{H}_2 = \begin{bmatrix}
-0.3403 & 0.1358 & -1.9706 \\
-2.2982 & -1.8135 & 0.2904
\end{bmatrix},
\]

and $P = 10$. From Fig. 6, the proposed algorithms can achieve a larger rate region than GSVD [36] and OMA. In Scenario C, the GSVD [37] and BSMM proposed by [38] are compared. The secrecy capacity region, i.e., DPC-based rate region, is obtained by [33]. The proposed method can reach the secrecy rate region. Besides, the PS method in Algorithm 1 is more general for all scenarios, while the WSR method in Algorithm 2 is specifically for the case without any broadcasting requirements.

We provide one case with the same settings in [33, Fig. 3], where the channels are:

\[
\mathbf{h}_1 = \begin{bmatrix} 1 & 0.4 \end{bmatrix}, \quad \mathbf{h}_2 = \begin{bmatrix} 0.4 & 1 \end{bmatrix},
\]

and $P = 10$. The results are shown in Fig. 7. The iteration tolerance $t$ in [35] is set as $10^{-3}$, and a bisection search is applied to find the optimal $t$. We set our iteration accuracy $\epsilon_2$ and convergence tolerance $\epsilon_3$ in Algorithm 2 as $10^{-3}$. The complexity is the same because both methods require finding the covariance matrices iteratively. The tolerance in [35] and the Lagrange multiplier in Algorithm 2 are both optimized through bisection search. Algorithm 1 is very fast without any search for one power allocation factor but is sub-optimal.

### C. Multicast and One Confidential Messages

If we set $\alpha_2 = 0$ in Scenario C, then the general problem is reduced to the integrated services with one confidential and one common messages\(^4\), i.e., $(R_0, R_{1c})$. As shown in Fig. 8, one can also set $\alpha_1 = 0$ and change the order of channels for $(R_0, R_{2c})$ which finally will resort to the same results due to duality.

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\(^4\)One can also set $\alpha_1 = 0$ and change the order of channels for $(R_0, R_{2c})$ which finally will resort to the same results due to duality.
the proposed method substantially outperforms the GSVD-based orthogonal subchannel precoding method in [41], in which the turning point is a switch of subchannel selection schemes. Compared with the GSVD-based orthogonal subchannel decomposition, Algorithm 1 can make a better use of channel without decomposing the channel into many orthogonal subchannels. Also, our method is very close to the use of channel without decomposing the channel into many subchannel decomposition, Algorithm 1 can make a better

decomposes the MIMO BC into the P2P MIMO, the wiretap, and the multicasting channels. Then, existing solutions can be applied to obtain the precoding and power allocation matrices. The proposed PS can achieve near-capacity rate regions which are significantly higher compared to the existing orthogonal methods. On the other hand, in the case of the MIMO-NOMA

We notice that GSVD has been applied to many subcases. Examples are two private messages in Scenario A [36], two confidential messages in Scenario C [37], and one confidential message and one common message [49]. Thus, it also has the potential to become an efficient and general tool for all scenarios. But, it should be noted that the performance of GSVD is affected by the number of antennas at the transmitter and users [36]. [46]. Algorithm 2 outperforms GSVD and sometimes Algorithm 1 but it is not easy to extend it to common messages. Algorithm 1 balances the two methods. We summarize the benefits and properties of the precoding schemes in Table II. Specifically, three linear precoding families in the MIMO-NOMA with different secrecy requirements are:

- GSVD: is the fastest general tool but has poor performance in some antenna settings.
- PS: Algorithm 1 is a general suboptimal tool. It balances time and performance.
- WSR: Algorithm 2 is locally optimal with KKT as the optimal necessary conditions. It has relatively high time complexity.

VI. CONCLUSIONS

In this paper, we investigate a two-user MIMO-NOMA network with different security requirements. Specifically, three scenarios are differentiated according to the required services: multicast, private, and/or confidential services. Although the capacity rate regions are known from information theory, the precoding and power allocation are still unclear for all three scenarios in general. A PS scheme is proposed which decomposes the MIMO BC into the P2P MIMO, the wiretap, and the multicasting channels. Then, existing solutions can be applied to obtain the precoding and power allocation matrices. The proposed PS can achieve near-capacity rate regions which are significantly higher compared to the existing orthogonal methods. On the other hand, in the case of the MIMO-NOMA
networks without multicasting, a WSR maximization based on BSMM is formulated for all three scenarios. We generalize and prove that the duality holds for the WSR maximization, and the KKT conditions are necessary for the optimality. Numerical results demonstrate the performance of the WSR maximization. The two methods have their advantages. PS is a general tool for the MIMO-NOMA with different scenarios of security, while WSR maximization provides a great potential for the secure MIMO-NOMA without multicasting, and both methods are computationally efficient compared with the DPC or S-DPC.

### APPENDIX A

**PROOF OF LEMMA 2**

The duality gap is zero in Scenario A because the problem can be transformed as a convex problem satisfying Slater’s condition [47]. Scenario C has zero duality gap and satisfies Lemma 1 [38] (Theorem 1). Now, we only need to prove that Scenario B also has zero duality gap.

The time-sharing property holds if the optimal value of the problem \( \varphi(P) \) in (29) is concave of total power \( P \), which implies a zero duality gap, i.e., the primal problem \( \varphi(P) \) and the dual problem in (32) have the same optimal value [50] (Theorem 1). The problem \( \varphi(P) \) meets the time-sharing property because the secrecy capacity for Scenario B without common message under total power constraint is a union of all the possible rates under covariance constraint [24] Corollary 1. The convex hull operation such as the time-sharing scheme will not enlarge the capacity region. Therefore, the problem \( \varphi(P) \) has zero duality gap and KKT conditions are necessary [50].

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### REFERENCES

[1] Y. Qi and M. Vaezi, “Power splitting based precoding for the MIMO-BC with multicast and confidential messages,” in Proc. IEEE Global Communications Conference, pp. 1–6, 2020.

[2] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, “Non-orthogonal multiple access (NOMA) for cellular future radio access,” in Proc. IEEE Vehicular Technology Conference (VTC Spring), pp. 1–5, 2013.

[3] M. Vaezi, Z. Ding, and H. V. Poor, "Multiple access techniques for 5G wireless networks and beyond," Cham, Switzerland: Springer, 2019.

[4] 3rd Generation Partnership Project (3GPP), “Study on downlink multiuser superposition transmission for LTE,” 2015.

[5] H. Lee, S. Kim, and J.-H. Lim, “Multituser superposition transmission (MUST) for LTE-A systems,” in Proc. IEEE International Conference on Communications, pp. 1–6, 2016.

[6] Y. Cao, H. Sun, J. Soriaga, and T. Ji, “Resource spread multiple access—A novel transmission scheme for 5G uplink,” in Proc. IEEE Vehicular Technology Conference (VTC-Fall), pp. 1–5, 2017.

[7] H. Weingarten, Y. Steinberg, and S. S. Shamai, “The capacity region of the Gaussian multiple-input multiple-output broadcast channel,” IEEE Transactions on Information Theory, vol. 52, no. 9, pp. 3936–3964, 2006.

[8] Z. Ding, F. Adachi, and H. V. Poor, “The application of MIMO to non-orthogonal multiple access,” IEEE Transactions on Wireless Communications, vol. 15, no. 1, pp. 537–552, 2015.

[9] M. Zeng, A. Yadav, O. A. Dobre, G. Tsiropoulos, and H. V. Poor, “On the sum rate of MIMO-NOMA and MIMO-OMA systems,” IEEE Wireless Communications Letters, vol. 6, no. 4, pp. 534–537, 2017.

[10] V.-D. Nguyen, H. D. Tuan, T. Q. Duong, H. V. Poor, and O.-S. Shin, “Precoder design for signal superposition in MIMO-NOMA multilcell networks,” IEEE Journal on Selected Areas in Communications, vol. 35, no. 12, pp. 2681–2695, 2017.

[11] Y. Huang, C. Zhang, J. Feng, Y. Jing, L. Yang, and X. You, “Signal processing for MIMO-NOMA: Present and future challenges,” IEEE Wireless Communications, vol. 25, no. 2, pp. 32–38, 2018.

[12] H. Weingarten, Y. Steinberg, and S. Shamai, “On the capacity region of the multi-antenna broadcast channel with common messages,” in Proc. IEEE International Symposium on Information Theory, pp. 2195–2199, 2006.

[13] M. Vaezi and H. V. Poor, “NOMA: An information-theoretic perspective,” in Multiple Access Techniques for 5G Wireless Networks and Beyond, pp. 167–193, Cham, Switzerland: Springer, 2019.

[14] B. Clerckx, Y. Mao, R. Schober, E. Jorswieck, D. J. Love, J. Yuan, L. Hanzo, G. Y. Li, E. G. Larsson, and G. Caire, “Is NOMA efficient in multi-antenna networks? A critical look at next generation multiple access techniques,” IEEE Open Journal of the Communications Society, 2021.

[15] R. Liu, T. Liu, H. V. Poor, and S. Shamai, “Multiple-input-multiple-output Gaussian broadcast channels with confidential messages,” IEEE Transactions on Information Theory, vol. 56, no. 9, pp. 4215–4227, 2010.

[16] R. Liu, T. Liu, H. V. Poor, and S. Shamai, “New results on multiple-input multiple-output broadcast channels with confidential messages,” IEEE Transactions on Information Theory, vol. 59, no. 3, pp. 1346–1359, 2013.

[17] R. F. Schaefer and H. Boche, “Physical layer service integration in wireless networks: Signal processing challenges,” IEEE Signal Processing Magazine, vol. 31, no. 3, pp. 147–156, 2014.

[18] A. D. Wyner, “The wire-tap channel,” Bell system technical journal, vol. 54, no. 8, pp. 1355–1387, 1975.

[19] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, “Transmit beamforming for physical-layer multicasting,” IEEE Transactions on Signal Processing, vol. 54, no. 6-1, pp. 2272–2281, 2006.

[20] A. Khisti and G. W. Wornell, “Secure transmission with multiple antennas—Part II: The MIMOME wiretap channel,” IEEE Transactions on Information Theory, vol. 11, no. 56, pp. 5515–5532, 2010.

[21] E. Ekrem and S. Ulukus, “On Gaussian MIMO broadcast channels with common and private messages,” in Proc. IEEE International Symposium on Information Theory, pp. 565–569, 2010.

[22] Y. Geng and C. Nair, “The capacity region of the two-receiver Gaussian vector broadcast channel with private and common messages,” IEEE Transactions on Information Theory, vol. 60, no. 4, pp. 2087–2104, 2014.

### TABLE II: Comparison among different precoding schemes for the MIMO-NOMA with different communication scenarios.

| Performance | DPC/S-DPC | PS (proposed for all scenarios) | GSVD | OMA |
|-------------|-----------|---------------------------------|-------|-----|
|             | optimal   | suboptimal (but close to optimal) | suboptimal (but close to optimal) | suboptimal | highly suboptimal |
| Speed       | generally slow | acceptable for a small \( m \) \((m = \max(n_t, n_1, n_2))\) | fast for a small \( n_t \) | fast | very fast |
| Complexity  | generally high | acceptable | acceptable | low | low |
| Generality  | ✓ | not easy to generalize for common message | ✓ | ✓ | ✓ |
S. A. A. Fakoorian and A. L. Swindlehurst, “On the optimality of linear precoding for GSVD-based beamforming in the MIMO Gaussian wiretap channel,” in Proc. IEEE International Symposium on Information Theory, pp. 2321–2325, 2012.

Q. Li, M. Hong, H.-T. Wai, Y.-F. Liu, W.-K. Ma, and Z.-Q. Luo, “Transmit solutions for MIMO wiretap channels using alternating optimization,” IEEE Journal on Selected Areas in Communications, vol. 31, no. 9, pp. 1714–1727, 2013.

M. Vaezi, W. Shin, and H. V. Poor, “Optimal beamforming for Gaussian MIMO wiretap channels with two transmit antennas,” IEEE Transactions on Wireless Communications, vol. 16, no. 10, pp. 6726–6735, 2017.

S. Vishwanath, N. Jindal, and A. Goldsmith, “Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels,” IEEE Transactions on Information Theory, vol. 49, no. 10, pp. 2658–2668, 2003.

H. Weingarten, Y. Steinberg, and S. Shamai, “The capacity region of the Gaussian MIMO broadcast channel,” in Proc. IEEE International Symposium on Information Theory, p. 174, 2004.

H. Vishwanathan, S. Venkatesan, and H. Huang, “Downlink capacity evaluation of cellular networks with known-interference cancellation,” IEEE Journal on Selected Areas in Communications, vol. 21, no. 5, pp. 802–811, 2003.

Z. Chen, Z. Ding, X. Dai, and R. Schober, “Asymptotic performance analysis of GSVD-NOMA systems with a large-scale antenna array,” IEEE Transactions on Wireless Communications, vol. 18, no. 1, pp. 575–590, 2019.

S. A. A. Fakoorian and A. L. Swindlehurst, “On the optimality of linear precoding for secrecy in the MIMO broadcast channel,” IEEE Journal on Selected Areas in Communications, vol. 31, no. 9, pp. 1701–1713, 2013.

D. Park, “Weighted sum rate maximization of MIMO broadcast and interference channels with confidential messages,” IEEE Transactions on Wireless Communications, vol. 15, no. 3, pp. 1742–1753, 2015.

Y. Qi and M. Vaezi, “Secure transmission in MIMO-NOMA networks,” IEEE Communications Letters, vol. 24, no. 12, pp. 2696–2700, 2020.

H. Ly, T. Liu, and Y. Liang, “Multiple-input multiple-output Gaussian broadcast channels with common and confidential messages,” IEEE Transactions on Information Theory, vol. 56, no. 11, pp. 5477–5487, 2010.

W. Mei, Z. Chen, and J. Fang, “GSVD-based precoding in MIMO systems with integrated services,” IEEE Signal Processing Letters, vol. 23, no. 11, pp. 1528–1532, 2016.

M. Vaezi, Y. Qi, and X. Zhang, “A rotation-based precoding for MIMO broadcast channels with integrated services,” IEEE Signal Processing Letters, vol. 26, no. 11, pp. 1708–1712, 2019.

M. Razaviyayn, M. Hong, and Z.-Q. Luo, “A unified convergence analysis of block successive minimization methods for nonsmooth optimization,” SIAM Journal on Optimization, vol. 23, no. 2, pp. 1126–1153, 2013.

Y. Liu, Z. Qin, M. Elkashlan, Y. Gao, and L. Hanzo, “Enhancing the physical layer security of non-orthogonal multiple access in large-scale networks,” IEEE Transactions on Wireless Communications, vol. 16, no. 3, pp. 1656–1672, 2017.

J. Nocedal and S. Wright, Numerical optimization. New York, NY, USA: Springer, 2006.