High-precision online prediction model of plate crown based on multi-factor interaction mechanism

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Abstract: The crown is a key quality index of strip and plate. Up to now, many plate crown prediction models have been established, however, the calculation accuracy of these traditional models is not very high. This is because these models only consider the effect of various influencing factors on the outlet plate crown under the basic process parameters, but these factors are not independent of each other. Therefore, in order to improve the accuracy of the plate crown prediction model, a high-precision prediction model of plate crown is established by introducing the evaluation coefficient that represents the influence of single factor on plate crown and the correction coefficient that represents the effect of factor $a$ on the evaluation coefficient of factor $b$. This paper takes the steel SPHC as an example, calculates the plate crown without considering the correction coefficient and the plate crown considering the correction coefficient. Then, the two calculation results are compared with the calculation results of the coupled model and the accuracy of the new model is improved after considering the correction coefficient. In general, compared with the traditional crown prediction model, the accuracy of the new model is greatly improved.

Keywords: plate shape; preset; plate crown prediction; evaluation coefficient; correction coefficient

1 Introduction

Plate shape is one of the most important parameters that evaluate the geometrical quality of rolling strip [1, 2] and the plate shape control technology has been rapidly developed based on various shape control devices and simulation strategies [3]. Plate shape control system is one of the core control technologies involved in strip rolling [4, 5]. The shape control system mainly consists of a preset calculation module, an adaptive calculation module, a roll thermal crown calculation module [6, 7], a calculation module of roll wear [8, 9] and a flatness dynamic control module [10].

The preset calculation module is the core of the shape control system [11] and it is a
predictive control module [12]. The shape control system is a complex, multivariable and nonlinear system. The plate shape is influenced by many device parameters [13, 14]. According to the process parameter values (strip thickness, rolling temperature, rolling speed, rolling force, etc.) set for finishing rolling, roll thermal crown, wear crown and self-adaptive value, the roll shift position, the reasonable preset value of the bending force, the AGC compensation coefficient and the parameters required by the flatness dynamic control module for each pass is calculated by the preset calculation module. Then, these set values are transferred to the basic automation module as the basic value of the shape control.

The calculation model of plate crown belongs to the preset calculation module [15] and the calculation model of loaded roll gap crown is the basis of the preset calculation module. Gong and Xu [16-19] has established models of strip crown influencing rate and developed calculation software.

The crown of loaded roll gap during the rolling process determines the crown of the plate [20, 21] and its calculation accuracy is an important condition for obtaining good plate crown and shape [22]. This is because the elastic recovery and thermal shrinkage of rolled piece can be ignored [23]. After the presetting of the outlet crown of each pass, mainly based on the calculation model of the loaded roll gap crown, inversely calculate the CVC lateral movement and bending force of each pass [24].

The plate crown obtained by using the influence function method [25] called the basic plate crown. Then the influence of various factors (unit rolling force, bending roll force, work roll diameter, backup roll diameter, work roll crown, backup roll crown, load distribution, etc.) are analyzed for the plate crown. That is, taking a certain factor variable and other factors unchanged, and obtain a new plate crown value called regenerative plate crown [26]. The final plate crown is composed of the basic plate crown, the crown of the regenerated plate produced by various factors, and the correction value.

Many plate crown prediction models have been established, but the calculation accuracy of these traditional models is not very high. This paper establish a high-precision prediction model to improve the accuracy of the traditional model.
2 Establishment of a high-precision online plate crown prediction model

2.1 Traditional plate crown prediction model and its limitations

Based on the basic plate crown, the overall calculation model of plate crown considering the influence of various factors [27] can be expressed as:

\[ C = C_0 + \sum_{i=1}^{n} K_i \times (X_i - X_0) \]  

(1)

Where \( C \) is the calculation value of the plate crown. \( n \) is the number of influencing parameters (unit rolling force, bending roll force, work roll diameter, backup roll diameter, work roll crown, backup roll crown, load distribution, etc.). \( K_i \) is the influencing rate of plate crown. \( X_i \) is the actual value of the influencing factors. \( X_0 \) is the base value of the influencing factors. \( C_0 \) is the regenerated plate crown when the influencing parameters is the actual value. \( C_0 \) is the basic plate crown when the influencing parameters is the reference value.

The Eq.(1) gives the individual effects of various factors on the outlet plate crown under the basic process parameters, but these factors are not independent of each other. For example, for different reductions, the influence of roll shifting will inevitably change and the effect of bending force will be different for different roll diameters [28]. Suppose the final outlet crown can be calculated by the following function:

\[ C = f(x_1 + \Delta x_1, x_2 + \Delta x_2, L, x_n + \Delta x_n) \]  

(2)

Where \( x_i \) is the factor affecting the plate crown. \( \Delta x_i \) is the amount of change in influencing factors.

Assuming that the outlet plate crown function has derivatives for each influencing factor. When the above formula expand with Taylor series, it can be expressed as:
The zero-order term in the equation is the basic value of plate crown. The primary term in the formula is the correction amount under the effect of each influencing factor alone. The quadratic term in the formula is the correction amount of the mutual influence between the two factors. The third term in the formula is the correction amount of two factors to the third factor, and so on. It can be seen that there is only primary term in the traditional model. In order to improve the accuracy of the calculation and ensure the calculation efficiency, this paper proposes the calculation method of the quadratic term, and ignores the third-order and higher-order terms.

2.2 Evaluation coefficient and correction coefficient of the effect of influencing factors

In the case of neglecting the thermal crown and the wear of the roll, the main process parameters affecting the outlet plate crown includes: plate width, reduction, roll shifting, bending force, inlet plate crown, work roll diameter and backup roll diameter [29]. In order to analyze the influence of these factors on the plate crown, this paper defines the evaluation coefficient $N_p$. The $N_p$ represents the ratio of the change in the outlet plate crown to the outlet plate thickness when the specific factor changes from the minimum to the maximum within the allowable interval under a certain plate width. Since the outlet plate crown is very small, the $N_p$ is expressed by 100 times of the value for convenience of comparison:

$$N_p = 100 \frac{\Delta C_{\text{max}}}{h}$$

Where $\Delta C_{\text{max}}$ is the maximum change in the outlet plate crown when a certain factor changes. $h$ is the outlet plate thickness.
Taking the effect of the roll shifting value on the plate crown as an example to analysis and calculation. Figure 1a) shows the variation of the outlet plate crown with the width of the plate when the roll shifting value is taken as two limit values. Figure 1b) shows the evaluation coefficient of the roll shifting value at different plate widths.

![Outlet plate crown](image1) ![Evaluation coefficient](image2)

**Fig.1 Impact effect analysis of strip crown for roll shift position**

As can be seen from Figure 1, the effect of the roll shifting value on the plate crown varies under different plate widths. When the plate width is 1850mm, the roll shifting value has a great influence. The plate crown changes about 400μm, and the \(N_p\) is about 1.9. When the plate width is 450mm, the roll shifting value has little effect. The plate crown change is only about 30μm, and the \(N_p\) is less than 0.2. Therefore, \(N_p\) can be used to quantitatively analyze and evaluate the effect of the single influencing factor on the outlet plate crown.

In order to quantitatively analyze the influence of a certain factor \(a\) on the evaluation coefficient of another \(b\), this paper introduces a correction coefficient, as shown in the following:

\[
m_{a,b} = \frac{N_p^{a,b}}{N_p^0}
\]

(5)

Where \(m_{a,b}\) is the correction coefficient for the influence of the factor \(b\) when factor \(a\) changes. \(N_p^{a,b}\) is the \(N_p\) of the influencing factor \(b\) when the factor \(a\) changes. \(N_p^0\) is the \(N_p\) of the influencing factor \(b\) when the factor \(a\) takes the base value.

The influence of the factor \(a\) on the evaluation coefficient of factor \(b\) basically fluctuates around the base value \(N_p^0\).

For example, in order to reflect the effect of different work roll diameters on the bending force [30], this paper calculates the \(N_p\) under different work roll diameters, as shown in Figure 2:
As can be seen from Figure 2, when the diameter of the work roll is small, the values of the $N_p$ of bending force is relatively large. The results show that the bending force has an obvious effect on the outlet plate crown. When the diameter of the work roll is large, the flexural rigidity of the work roll is enhanced, the effect of the bending force is reduced, and the $N_p$ of bending force is small.

Taking the plate width of 1250 mm as an example, the effect of the change of the work roll diameter on the bending force is as follows:

When the work roll diameter is 800mm (the base value state), the $N_p$ is 0.248795, the $m_{a,b}$ is 1; When the work roll diameter is 850mm, the $N_p$ is 0.213060, the $m_{a,b}$ is 0.856368; When the work roll diameter is 750mm, the $N_p$ is 0.293625, the $m_{a,b}$ is 1.180189.

It can be seen that when the correction coefficient $m_{a,b}$ is less than 1, the change of the factor $a$ makes the effect of the factor $b$ smaller. When the correction coefficient is more than 1, the change of the factor $a$ makes the effect of the factor $b$ larger.

**2.3 Establishment of modified model**

Based on the above theory, the correction coefficient $m_{a,b}$ can reflect the magnification or reduction multiple of the effect of factor $b$ on the outlet plate crown when the factor $a$ changes. Therefore, the product of the influence coefficient and the correction coefficient can be used to comprehensively reflect the actual influence of the factor $b$ on the outlet plate crown. For example, the effect of the change of bending force on the outlet plate crown can be expressed as:

$$
\Delta C_{FW} = m_{DH,FW} \cdot m_{SP,FW} \cdot m_{CL,FW} \cdot m_{DW,FW} \cdot m_{DB,FW} \cdot K_{FW} \cdot (FW - FW_0)
$$

Where $\Delta C_{FW}$ is the reconstruction plate crown caused by bending force. $m_{DH,FW}$ is the correction coefficient of the effect of the reduction on the bending force. $m_{SP,FW}$ is the correction coefficient of the effect of the roll shifting value on the bending force.
$m_{CI,FW}$ is the correction coefficient of the effect of the inlet plate crown on the bending force. $m_{PW,FW}$ is the correction coefficient of the effect of the work roll diameter on the bending force. $m_{DB,FW}$ is the correction coefficient of the effect of the backup roll diameter on the bending force. $K_{FW}$ is the effect coefficient of the bending force. $FW_j$ is the actual bending force. $FW_0$ is the base value of bending force.

Similarly, the reduction, the roll shifting quantity, the inlet plate crown, the work roll diameter, and the backup roll diameter can all be corrected by all other influencing factors. The final outlet plate crown can be expressed as:

$$C = C_0 + \sum_{i=1}^{6} \left( \prod_{j=1}^{5} m_{j,i} \times K_i \times (X_j - X_0) \right) + \Delta C_{\text{crt}} + \Delta C_{\text{wear}}$$

(7)

Where $m_{j,i}$ is the correction coefficient for the effect of the factor $j$ on the factor $i$. $\Delta C_{\text{crt}}$ is the influence of thermal crown on the outlet plate crown. $\Delta C_{\text{wear}}$ is the influence of wear crown of roll on the outlet plate crown.

Compared with Eq. (1), the Eq. (7) not only reflects the influence of a single factor on the outlet plate crown, but also reflects the interaction between the two influencing factors through the introduction of the correction coefficient $m_{j,i}$. In the Eq.(7), the influence of thermal crown $\Delta C_{\text{crt}}$ and roll wear crown $\Delta C_{\text{wear}}$ on the outlet plate crown is also considered.

3 Verification of the high-precision online plate crown prediction model

This paper uses the improved 3-D coupled model [31] involved in RPFEM (rigid-plastic finite element method) to calculate the plate crown. The coupled model has high accuracy for the calculation of the plate crown. The coupled model is directly calculated offline and it is the basis for the calculation of the entire online prediction model parameters.

The flowchart of coupled model as shown in Figure 3. The deformation diagram of
roll system as shown in Figure 4.

Fig.3 Flowchart of coupled model
Fig.4 Deformation diagram of roll system

In order to verify the calculation accuracy, this paper combines the actual situation of Baosteel's 2050 hot rolling line. Takes the steel SPHC as an example, the plate width is 1150mm, and the outlet thickness is 20mm. The calculation example is calculated using a predictive model that considers the interaction between the influencing factors and a predictive model that does not consider the interaction between the influencing factors. The main process parameters are shown in Table 1 below:

Table 1 Process parameters of the example

| Name                              | Example value | Base value |
|-----------------------------------|---------------|------------|
| Work roll diameter/mm            | 850           | 800        |
| Backup roll diameter/mm          | 1500          | 1500       |
| Roll shifting value/mm           | 0             | 0          |
| Bending force/kN                  | 1000          | 500        |
| Inlet crown/μm                    | 350           | 350        |
| Outlet plate thickness/mm         | 20            | 20         |
| Reduction/mm                      | 15            | 15         |

The finishing mills of Baosteel’s 2050 hot strip continuous rolling mill consists of 7-stands four-high CVC mills. In order to calculate the plate crown influencing rate $K_i$ and the correction coefficient $m_{i,a}$ of each factor, this paper takes the F1 stand as an example, the outlet thickness of the first pass is 20mm, the basic process parameters used in the calculation are shown in Table 2:
Table 2 Basic process parameters

| name                                   | value  |
|----------------------------------------|--------|
| Work roll diameter /mm                 | 800    |
| Length of work roll /mm                | 2250   |
| Distance of work roll bending cylinders /mm | 3150  |
| Backup roll diameter /mm               | 1500   |
| Length of backup roll /mm              | 2050   |
| Center distance of backup roll reduction /mm | 3150  |
| Roll shifting value /mm                | 0      |
| Bending force /kN                      | 500    |
| Inlet crown /μm                        | 350    |
| Plate width /mm                        | 450-1850 |
| Outlet plate thickness /mm             | 20     |
| Reduction /mm                          | 15     |
| Equivalent unit width rolling force /(kN·mm-1) | 20.6   |

3.1 Calculation of the basic plate crown

\[ C_0 = \sum_{i=0}^{5} \left[ a(i) \times B^i \right] \]  

(8)

Where \( a(i) \) is the fifth-degree polynomial fitting coefficient of the basic plate crown. \( B \) is the plate width.

The curve of the basic plate crown with the width of the plate is shown in Figure 5

Fig.5 The curve of the basic plate crown with the plate width

The fifth-degree polynomial fitting coefficient \( a(i) \) of basic plate crown are shown in Table 3:

Table 3 Fit coefficients of the basic center crown

| \( a(0) \) | \( a(1) \) | \( a(2) \) | \( a(3) \) | \( a(4) \) | \( a(5) \) |
|-----------|-----------|-----------|-----------|-----------|-----------|
| -44.49    | 0.4473    | -4.145 \times 10^{-4} | 5.515 \times 10^{-7} | -2.886 \times 10^{-10} | 4.375 \times 10^{-14} |
3.2 Calculation of the influencing rate of plate crown

The influencing rate of the plate crown of each influencing factor varies with the plate width as shown in the Figure 6

a) Influencing rate of reduction b) Influencing rate of roll shifting value

c) Influencing rate of bending force d) Influencing rate of inlet crown

e) Influencing rate of work roll diameter f) Influencing rate of backup roll diameter

Fig.6 The influencing rate of the plate crown of each influencing factor

The relationship between each influencing factor and the plate width can be fitted with a fifth degree polynomial, as shown in the following

\[ K_i = \sum_{i=0}^{5} a(i) \times B^i \]  \hspace{1cm} (9)

Where \( a(i) \) is the fitting coefficient. \( B \) is the plate width.

The fitting coefficients of each influencing factors are in the Table 4

|   | \( a(0) \) | \( a(1) \) | \( a(2) \) | \( a(3) \) | \( a(4) \) | \( a(5) \) |
|---|---|---|---|---|---|---|
| a(i) of \( K_{DH} \) | 1.506 | -3.3242 \times 10^{-3} | 1.3403 \times 10^{-5} | 2.7368 \times 10^{-9} | -5.9015 \times 10^{-12} | 1.0774 \times 10^{-15} |
| a(i) of \( K_{SP} \) | -0.1643 | 8.23 \times 10^{-4} | -1.132 \times 10^{-6} | 2.061 \times 10^{-9} | -1.041 \times 10^{-12} | 1.65 \times 10^{-16} |
| a(i) of \( K_{FW} \) | 0.002046 | -1.09 \times 10^{-5} | -1.677 \times 10^{-9} | -3.38 \times 10^{-13} | 1.707 \times 10^{-14} | -2.284 \times 10^{-18} |
| a(i) of \( K_{CI} \) | 0.01084 | 3.403 \times 10^{-5} | -8.402 \times 10^{-6} | 1.527 \times 10^{-10} | -6.968 \times 10^{-14} | 1.021 \times 10^{-17} |
| a(i) of \( K_{DW} \) | 0.4368 | -2.561 \times 10^{-3} | 6.024 \times 10^{-4} | -7.113 \times 10^{-8} | 3.642 \times 10^{-12} | -6.564 \times 10^{-16} |
| a(i) of \( K_{DB} \) | -0.01486 | 7.46 \times 10^{-5} | -1.226 \times 10^{-7} | 4.958 \times 10^{-11} | -4.289 \times 10^{-14} | 1.456 \times 10^{-17} |

When the plate width is 1150mm, \( K_{pW} = -0.0388508 \), \( K_{dW} = -0.309968 \), \( K_{pW} = 11.4162 \), \( K_{SP} = 0.930762 \), \( K_{CI} = 0.0697609 \), \( K_{DB} = -0.061533 \)
3.3 Calculation of the correction coefficient

3.3.1 Correction coefficient of bending force

When the other influencing factors take the base value, the evaluation coefficient of the bending force is \( N_p^0 \). When the work roll diameter is 850mm, the evaluation coefficient of the bending force is \( N_p^{DW,FW} \).

The relationship between the evaluation coefficient of the bending force \( N_p \) and the plate width is shown in the Figure 7

![Figure 7 Evaluation coefficient of the bending force](image)

The relationship between the evaluation coefficient of the bending force \( N_p \) and the plate width can be fitted with a fifth degree polynomial, as show in the following

\[
N_p = \sum_{i=0}^{5} a(i) \times B^i
\]  

(10)

The fitting coefficients of \( N_p^0 \) and \( N_p^{DW,FW} \) is shown in the Table 5

|        | \( a(0) \) | \( a(1) \) | \( a(2) \) | \( a(3) \) | \( a(4) \) | \( a(5) \) |
|--------|------------|------------|------------|------------|------------|------------|
| a(i) of \( N_p^0 \) | -0.04334   | 0.00023    | -3.577\times10^{-7} | 5.691\times10^{-10} | -2.852\times10^{-13} | 4.867\times10^{-17} |
| a(i) of \( N_p^{DW,FW} \) | -0.04671   | 0.000269   | -5.008\times10^{-7} | 7.192\times10^{-10} | -3.692\times10^{-13} | 6.647\times10^{-17} |

When the plate width is 1150mm, \( N_p^0 = 0.212708 \), \( N_p^{DW,FW} = 0.182107 \).

According to Eq. (5), \( m_{DW,FW} = 0.856136 \).

3.3.2 Correction coefficient of work roll diameter

When the other influencing factors take the base value, the evaluation coefficient of the work roll diameter is \( N_p^0 \). When the bending force is 1000kN, the evaluation coefficient of the work roll diameter is \( N_p^{FW,WD} \).

The relationship between the evaluation coefficient of the work roll diameter \( N_p \) and the plate width is shown in the Figure 8
The relationship between the evaluation coefficient of the work roll diameter $N_p$ and the plate width can be fitted with a fifth degree polynomial, as shown in the following

$$N_p = \sum_{i=0}^{5} a(i) \times B^i$$

(11)

The fitting coefficients of $N_p^0$ and $N_p^{FW, DW}$ is shown in the Table 6

| $a(i)$ of $N_p^0$ | $a(0)$ | $a(1)$ | $a(2)$ | $a(3)$ | $a(4)$ | $a(5)$ |
|-------------------|--------|--------|--------|--------|--------|--------|
|                   | 0.09582| -4.381×10^{-4} | 5.125×10^{-7} | 8.864×10^{-11} | -2.178×10^{-13} | 4.824×10^{-17} |
| $a(i)$ of $N_p^{FW, DW}$ | 0.2453 | -1.115×10^{-3} | 1.677×10^{-6} | -9.212×10^{-10} | 1.935×10^{-13} | -1.543×10^{-17} |

When the plate width is 1150mm, $N_p^0 = 0.120691$, $N_p^{FW, DW} = 0.087250$.

According to Eq. (5), $m_{FW, DW} = 0.722921$.

3.3.3 Correction coefficient of reduction

When the other influencing factors take the base value, the evaluation coefficient of the reduction is $N_p^0$. When the bending force is 1000kN and , the evaluation coefficient of the reduction is $N_p^{FW, DH}$. When the work roll diameter is 850mm, the evaluation coefficient of the reduction is $N_p^{DW, DH}$.

The relationship between the evaluation coefficient of the reduction $N_p$ and the plate width is shown in the Figure 9

Fig.9 Evaluation coefficient of the reduction

The bending force has very little effect on the reduction, because the curve of $N_p^0$ and the curve of $N_p^{FW, DH}$ almost coincide.

The relationship between the evaluation coefficient of the reduction $N_p$ and the plate width can be fitted with a fifth degree polynomial, as shown in the following
The fitting coefficients of $N_p^0$, $N_p^{FW, DH}$ and $N_p^{DW, DH}$ is shown in the Table 7

| $a(i)$ of $N_p^0$ | $a(0)$ | $a(1)$ | $a(2)$ | $a(3)$ | $a(4)$ | $a(5)$ |
|-------------------|--------|--------|--------|--------|--------|--------|
| 0.2075            | -9.997×10^{-4} | 2.575×10^{-6} | -1.876×10^{-9} | 6.943×10^{-13} | -1.277×10^{-16} |
| $a(i)$ of $N_p^{FW, DH}$ | 0.07022 | -2.142×10^{-4} | 9.175×10^{-7} | -2.502×10^{-10} | -6.267×10^{-14} | 7.048×10^{-18} |
| $a(i)$ of $N_p^{DW, DH}$ | 0.1324 | -5.912×10^{-4} | 1.781×10^{-6} | -1.21×10^{-9} | 4.181×10^{-13} | -8.077×10^{-17} |

When the plate width is 1150mm, $N_p^0=0.567606$, $N_p^{FW, DH}=0.561327$, $N_p^{DW, DH}=0.536436$

According to Eq. (5), $m_{FW, DH}=0.988938$, $m_{DW, DH}=0.945085$

3.3.4 Correction coefficient of roll shifting value

When the other influencing factors take the base value, the evaluation coefficient of the roll shifting value is $N_p^0$. When the bending force is 1000kN and , the evaluation coefficient of the roll shifting value is $N_p^{FW, SP}$. When the work roll diameter is 850mm, the evaluation coefficient of the roll shifting value is $N_p^{DW, SP}$.

The relationship between the evaluation coefficient of the roll shifting value $N_p$ and the plate width is shown in the Figure 10

Fig.10 Evaluation coefficient of the roll shifting value

The bending force and work roll diameter has very little effect on the roll shifting value, because the three curves almost coincide.

The relationship between the evaluation coefficient of the roll shifting value $N_p$ and the plate width can be fitted with a fifth degree polynomial, as show in the following:

$$N_p = \sum_{i=0}^{5} \left[ a(i) \times B^i \right]$$ (13)
The fitting coefficients of $N_p^0$, $N_p^{FW,SP}$ and $N_p^{DW,SP}$ is shown in the Table 8

|               | $a(0)$   | $a(1)$   | $a(2)$   | $a(3)$   | $a(4)$   | $a(5)$   |
|---------------|----------|----------|----------|----------|----------|----------|
| $a(i)$ of $N_p^0$ | 0.01281  | -8.231×10^{-5} | 5.287×10^{-7} | 7.065×10^{-10} | -5.249×10^{-13} | 9.084×10^{-17} |
| $a(i)$ of $N_p^{FW,SP}$ | 0.177    | -1.023×10^{-3} | 2.568×10^{-6} | -1.397×10^{-9} | 5.078×10^{-13} | -1.025×10^{-16} |
| $a(i)$ of $N_p^{DW,SP}$ | 0.1706   | -9.882×10^{-4} | 2.437×10^{-6} | -1.202×10^{-9} | 3.473×10^{-13} | -5.537×10^{-17} |

When the plate width is 1150mm, $N_p^0=0.956516$, $N_p^{FW,SP}=0.954049$, $N_p^{DW,SP}=0.925072$

According to Eq. (5), $m_{FW,SP}=0.997421$, $m_{DW,SP}=0.967127$

3.3.5 Correction coefficient of inlet crown

When the other influencing factors take the base value, the evaluation coefficient of the inlet crown is $N_p^0$. When the bending force is 1000kN and , the evaluation coefficient of the inlet crown is $N_p^{FW,CI}$. When the work roll diameter is 850mm, the evaluation coefficient of the inlet crown is $N_p^{DW,CI}$.

The relationship between the evaluation coefficient of the inlet crown $N_p$ and the plate width is shown in the Figure 11

![Fig.11 The evaluation coefficient of the inlet crown](image-url)

The bending force and work roll diameter has very little effect on the inlet crown, because the three curves almost coincide.

The relationship between the evaluation coefficient of the inlet crown $N_p$ and the plate width can be fitted with a fifth degree polynomial, as show in the following:

$$N_p = \sum_{i=0}^{5} a(i) \times B^i$$

(14)

The fitting coefficients of $N_p^0$, $N_p^{FW,CI}$ and $N_p^{DW,CI}$ is shown in the Table 9
When the plate width is 1150mm, \(N_p^0=0.243576\), \(N_{p,FW}^0=0.248697\),
\(N_{p,DW}^0=0.242905\)

According to Eq. (5), \(m_{FW,CI}=1.021024\), \(m_{DW,CI}=0.997245\)

### 3.3.6 Correction coefficient of backup roll diameter

When the other influencing factors take the base value, the evaluation coefficient of the backup roll diameter is \(N_p^0\). When the bending force is 1000kN and , the evaluation coefficient of the backup roll diameter is \(N_{p,FW}^{DB}\). When the work roll diameter is 850mm, the evaluation coefficient of the backup roll diameter is \(N_{p,DW}^{DB}\).

The relationship between the evaluation coefficient of the backup roll diameter \(N_p\) and the plate width is shown in the Figure 12

![Fig.12 Evaluation coefficient of the backup roll diameter](image)

The bending force has very little effect on the inlet crown, because the curve of \(N_{p,FW}^{DB}\) and curve of \(N_p^0\) almost coincide.

The relationship between the evaluation coefficient of the backup roll diameter \(N_p\) and the plate width can be fitted with a fifth degree polynomial, as show in the following:

\[
N_p = \sum_{i=0}^{5} a(i) \times B^i
\]  

(15)

The fitting coefficients of \(N_p^0\), \(N_{p,FW}^{DB}\) and \(N_{p,DW}^{DB}\) is shown in the Table 10

| Table 9 | Fitting coefficients of \(N_p^0\), \(N_{p,FW}^{CI}\) and \(N_{p,DW}^{CI}\) |
|---------|---------------------------------------------------------------|
|         | \(a(0)\) | \(a(1)\) | \(a(2)\) | \(a(3)\) | \(a(4)\) | \(a(5)\) |
| \(a(i)\) of \(N_p^0\) | 0.2976 | -1.688×10^{-3} | 4.009×10^{-5} | -3.954×10^{-9} | 1.864×10^{-12} | -3.288×10^{-16} |
| \(a(i)\) of \(N_{p,FW}^{CI}\) | 0.4087 | -2.267×10^{-3} | 5.093×10^{-5} | -4.87×10^{-9} | 2.221×10^{-12} | -3.81×10^{-16} |
| \(a(i)\) of \(N_{p,DW}^{CI}\) | 0.0098 | -2.525×10^{-6} | 3.23×10^{-7} | -2.187×10^{-10} | 9.799×10^{-14} | -1.488×10^{-17} |
Table 10 Fitting coefficients of $N_p^0$, $N_{p,FW}^{FW, DB}$ and $N_{p, DW}^{DW, DB}$

|       | $a(0)$ | $a(1)$ | $a(2)$ | $a(3)$ | $a(4)$ | $a(5)$ |
|-------|--------|--------|--------|--------|--------|--------|
| a(i) of $N_p^0$ | -0.01763 | 1.079×10^{-4} | -2.574×10^{-7} | 3.352×10^{-10} | -1.409×10^{-13} | 1.914×10^{-17} |
| a(i) of $N_{p,FW}^{FW, DB}$ | 0.04993 | -2.886×10^{-4} | 6.12×10^{-7} | -5.528×10^{-10} | 2.872×10^{-13} | -5.924×10^{-17} |
| a(i) of $N_{p, DW}^{DW, DB}$ | -0.01844 | 1.034×10^{-4} | -2.211×10^{-7} | 2.594×10^{-10} | -8.951×10^{-14} | 7.417×10^{-18} |

When the plate width is 1150mm, $N_p^0 = 0.0679032$, $N_{p,FW}^{FW, DB} = 0.0698321$, $N_{p, DW}^{DW, DB} = 0.0609449$.

According to Eq. (5), $m_{FW, DB} = 1.028407$, $m_{DW, DB} = 0.897526$.

3.4 Comparison of plate crown calculation accuracy

According to Eq. (6), this paper calculates the change in plate crown caused by each influencing factor, as shown in Table 11. The basic plate crown is 347µm and when the bending force and the work roll diameter is changed, the plate crown calculated by coupled model is 320µm.

Table 11  Influence effect analysis of each factor

|       | Without considering correction | Considering correction | Correction coefficient |
|-------|--------------------------------|------------------------|-----------------------|
| $\Delta C_{DH}$/µm | 0 | 0 | 0.934630 |
| $\Delta C_{SP}$/µm | 0 | 0 | 0.964633 |
| $\Delta C_{FW}$/µm | -19 | -16 | 0.856136 |
| $\Delta C_{CI}$/µm | 0 | 0 | 1.018211 |
| $\Delta C_{DW}$/µm | -15 | -11 | 0.722921 |
| $\Delta C_{DB}$/µm | 0 | 0 | 0.923022 |
| $\Delta C$/µm | -34 | -27 | - |
| $C$/µm | 313 | 320 | - |
It can be seen from Table 11 that when the interaction between the influencing factors is not considered, the predicted crown is 313μm and when the correction coefficient of the influencing factors is considered, the predicted crown is 320μm, which is closer to the 320μm calculated by the coupled model. So, the accuracy is improved after considering the correction coefficient. Since only the bending force and the work roll diameter are changed, other factors use the base value, so the other factors cause the crown change amount to be zero. It can also be seen from the table that the bending force correction coefficient and the work roll diameter correction coefficient are both less than 1, which indicates that the increase of the work roll diameter reduces the effect of the bending force. At the same time, the increase of the bending force also reduces the effect of the work roll diameter, thereby reducing the total influence of each influencing factor.

To further verify the accuracy of the modified model, Table 12 below gives a comparison of the calculation results for five sets of identical materials (SPHC) and different process parameters.
| Study number | Plate width/mm | Outlet thickness/mm | Reduction /mm | Roll shifting quantity /mm | Bending force /kN | Inlet crown /μm | Work roll diameter /mm | Backup roll diameter /mm | Without considering the correction | Considering the correction | Coupling model |
|--------------|----------------|---------------------|---------------|---------------------------|-------------------|-----------------|----------------------|--------------------------|--------------------------------|--------------------------|----------------|
| 1            | 920            | 18                  | 13            | -80                      | 0                 | 300             | 780                  | 1400                     | 325                           | 324                      | 322            |
| 2            | 1050           | 19                  | 14            | -40                      | 200               | 350             | 800                  | 1450                     | 355                           | 361                      | 359            |
| 3            | 1230           | 20                  | 15            | 0                        | 400               | 300             | 810                  | 1500                     | 339                           | 334                      | 335            |
| 4            | 1500           | 21                  | 16            | 40                       | 600               | 350             | 820                  | 1550                     | 352                           | 356                      | 355            |
| 5            | 1800           | 22                  | 17            | 80                       | 800               | 300             | 850                  | 1600                     | 274                           | 287                      | 285            |

It can be seen from Table 12 that the prediction model considering the correction coefficient has higher calculation accuracy under different process parameters, which is most obvious when the board width is 1800mm. The error of calculation without the correction coefficient is 11 μm, but only 2 μm after considering the correction coefficient. Because the process parameters in this example are all deviated from the base value, and the variation of the evaluation coefficient generally increases with the increase of the width.

4 Conclusions

(1) The traditional plate crown prediction model only gives the individual effects of various factors on the outlet plate crown under the base value process parameters, but the various factors are not independent of each other. It can be seen from the Taylor series expansion that the quadratic term represents the amount of correction between the two factors and should be considered.

(2) By introducing the evaluation coefficient $N_p$, the influence of various factors on
the plate crown can be analyzed quantitatively. By introducing the correction coefficient $m_{a,b}$, the influence of a certain factor $a$ on another factor $b$ can be quantitatively analyzed. When the correction coefficient $m_{a,b}$ is less than 1, it indicates the change of the factor $a$ makes the influence effect of the factor $b$ become smaller; when the correction factor $m_{a,b}$ is more than 1, it means that the change of the factor $a$ makes the influence effect of the factor $b$ become larger.

(3) Considering the interaction between any two factors, a new prediction model of plate crown is established. The accuracy of the model is verified by comparison.
5 DECLARATION

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Availability of data and materials
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Authors’ contributions
The author’s contributions are as follows: Tao Wang was in charge of the whole trial; Zhi-Qiang Li wrote the manuscript; Yuan-Ming Liu, Ignatov Aleksei Vladimirovih, and Qing-Xue Huang assisted with sampling and laboratory analyses.

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