Space- and time-like electromagnetic pion form factors in light-cone pQCD

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Abstract

We present a combined analysis of the space- and time-like electromagnetic pion form factors in light-cone perturbative QCD with transverse momentum dependence and Sudakov suppression. Including the non-perturbative “soft” QCD and power suppressed twist-3 corrections to the standard twist-2 perturbative QCD result, the experimental pion data available at moderate energies/momentum transfers can be explained reasonably well. This may help towards resolving the bulk of the existing discrepancy between the space- and time-like experimental data.

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Introduction. The electromagnetic (e.m.) form factor of hadrons are important physical observables that play a key role in understanding the transition from the perturbative to the non-perturbative behavior in particle physics. The space- and time-like e.m. pion form factors $F_\pi$ and $G_\pi$, respectively, are specified through the following matrix elements:

\[
e(P' + P)_\mu F_\pi(Q^2) = \langle \pi^\pm(P') | J^\text{em}_\mu(0) | \pi^\pm(P) \rangle ,
\]

\[
e(P' - P)_\mu G_\pi(Q^2) = \langle \pi^+(P') \pi^-(P) | J^\text{em}_\mu(0) | 0 \rangle ,
\]

where $J^\text{em}_\mu$ is the e.m. current, and $P = (Q/\sqrt{2}, 0, 0_T)$ and $P' = (0, Q/\sqrt{2}, 0_T)$ are, respectively, the initial and final state external light-cone pion 4-momenta in the Breit-frame. For the space-like momentum transfers, $q^2 = (P' - P)^2 = -Q^2 \leq 0$, whereas for the time-like momentum transfers $q^2 = (P' + P)^2 = Q^2 \geq 0$.

Theoretical predictions based on standard “asymptotic” QCD rely on collinear factorization [1] that lead to the celebrated quark counting rule, \{F, G\}_\pi(Q^2) \sim 1/Q^2 [2]. Naively, one may then conclude that at high enough energies/momentum transfers the space- and time-like form factors are essentially of the same magnitude. However, at the present experimentally accessible energies the reported pion form factor results differ significantly, with the time-like results [3–5] being up to a factor of four more than the space-like results [6]. Efforts to explain the above difference with conventional Vector Meson Dominance (VMD) [6] and perturbative QCD (pQCD) lead to the general conclusion that the time-like and the space-like data are inconsistent with each other. The purpose of this letter, is to show a possible “clean” scenario where the above difference could be reconciled with the standard treatment of the parton transverse momentum dependence (TMD), the sub-leading twist-3 contributions and the so-called “soft” QCD corrections. However, in dealing with the parton picture in pQCD, one should naturally be aware of the fact that in reality there are additional difficulties with hadronization and other final state interactions, and resonances. Our approach assumes that these effects for $Q^2 \gg \Lambda_{\text{QCD}}^2$ are comparatively small (as data suggest) and would not come in conflict with our predictions that account for the largeness of the existing discrepancy between the space-and time-like pion form factor data.

The charged pion form factor can be written as [7] $\{F, G\}_\pi(Q^2) = \{F, G\}_\pi^{\text{soft}}(Q^2) + \{F, G\}_\pi^{\text{hard}}(Q^2)$. The factorizable hard part $\{F, G\}_\pi^{\text{hard}}(Q^2)$ is calculated using light-cone pQCD with explicit TMD of the constituent valence partons; whereas, the non-factorizable soft part $\{F, G\}_\pi^{\text{soft}}(Q^2)$ is modeled using QCD sum rules (QCDSR) via local quark-hadron
Parametrically, both the soft and higher twist contributions to the form factor are expected to be small at large momentum transfers compared to the leading hard (twist-2) contributions due to the relative $1/Q^{2n}$ suppression. Despite this, their contributions turn out to be unnaturally large at moderate range of energies. In this paper, for the first time, we show that the twist-3 corrections to the time-like pion form factor are very large and essentially account for the bulk of the observed discrepancy between theory and experimental data. Note that the first attempt to explain both the space- and time-like data in the context of pQCD includes only the twist-2 effects [8]. However, the present consensus is that the twist-2 effects are much too small to explain the form factor data [7,9–12]. Furthermore, one must use appropriate Sudakov factors [13–17] to suppress the kinematic enhancements that may invalidate factorization. The advantage of such a modified ‘$k_T$’-factorization [13,14,18] approach is the elimination of large logarithms in the hard kernel through the TMD of the valence partons. This extends the range of applicability of pQCD down to very moderate range of energies and has been widely applied to inclusive and exclusive processes, and especially, to exclusive $B$-meson decays [9,16,17,19].

**Factorized pQCD.** We now present the essentials of our calculations. The dominant contributions come only from the leading order (LO) Fock state, i.e., a $q\bar{q}$ valence quark configuration with one hard gluon exchange in the scattering kernel sandwiched between 2-particle wavefunctions/distribution amplitudes (DAs). One of four diagrams contributing to each of the Born amplitudes $\pi\gamma^* \rightarrow \pi$ and $\gamma^* \rightarrow \pi^+\pi^-$ is shown in Fig. [1]. The other diagrams correspond to allowing the gluon to interact on the other side of the photon vertex and allowing the photon also to couple to the other valence quark. The higher Fock state contributions are neglected being suppressed by higher powers of $1/Q^2$. Since, we are only concerned with the leading Fock states in the scattering kernel of the pion, we must consider only the 2-particle pion DAs for our analysis neglecting the multi-particle components. Nevertheless, one can show that for the 2-particle twist-3 DAs are not independent of the 3-particle twist-3 DA, being related by QCD equations of motions. To next-to-leading order in conformal twist there is just one 2-particle twist-2 collinear DA $\phi_{2,\pi}(x,\mu)$ with an axial-vector structure, and two 2-particle twist-3 collinear DAs, one with a pseudo-scalar structure $\phi_{3,\pi}^p(x,\mu)$ and the other with a pseudo-tensor structure $\phi_{3,\pi}^\sigma(x,\mu)$. They can be derived from light-cone QCD sum rules (LCSR) and are usually expressed as truncated conformal series expansion over
FIG. 1: LO representative matrix elements contributing to (a) the space-like $F_\pi(Q^2)$, and (b) the time-like $G_\pi(Q^2)$ pion form factors. The blobs represent the pion wavefunctions $\tilde{\psi}_\pi$.

Gegenbauer polynomials \[20–22\]. Their asymptotic forms are given by

$$
\phi_{2,\pi}^{(as)}(x) = \frac{3f_\pi}{\sqrt{2N_c}} x(1-x) ; \quad \phi_{3,\pi}^{(as)}(x) = \frac{f_\pi}{2\sqrt{2N_c}} x(1-x) ,
$$

where $f_\pi \approx 131$ MeV is the pion decay constant and $x$ is the longitudinal momentum fraction of the valence partons. The intrinsic TMD of the total pion wavefunctions is modeled via the Brodsky-Huang-Lepage (BHL) prescription \[23\] having the general impact “$b$”-space representation:

$$
\check{\mathcal{P}}_{t,\pi}(x, b, \mu, M_q) = A_{t,\pi} \phi_{t,\pi}(x, \mu) \exp \left[ -\frac{\beta_{t,\pi}^2 M_q^2}{x(1-x)} \right] \exp \left[ -\frac{b^2 x(1-x)}{4\beta_{t,\pi}^2} \right] ; \quad t = 2, 3 ,
$$

where $\phi_{t,\pi}(x, \mu)$ is one of the twist-2 or twist-3 non-asymptotic collinear DAs at any given scale $\mu$. Such DAs satisfy the Efremov-Radyushkin-Brodsky-Lepage (ER-BL) evolution equation \[1\], e.g., the twist-2 DA is given at the LO by the following non-asymptotic expression, in terms of Gegenbauer polynomials $C_n^{3/2}(2x - 1)$:

$$
\phi_{2,\pi}(x, \mu) = \phi_{2,\pi}^{as}(x) \sum_{n=0,2,4,\cdots} \infty a_n^\pi(\mu_0^2) C_n^{3/2}(2x - 1) \left( \frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)} \right)^{-4\gamma_n(0)/9} + O(\alpha_s) ,
$$

where $\alpha_s$ is the standard (two loop) \overline{MS} QCD coupling with $\Lambda_{QCD} = 0.2$ GeV, $a_n^\pi$’s are the moments of the DA, depicting the genuine non-perturbative inputs, and $\gamma_n(0)$’s are the corresponding standard LO anomalous dimensions. A compilation of the numerical values of the Gegenbauer moments as well as the LO RGE behavior of the various non-perturbative parameters of the twist-2 and twist-3 DAs can be found in \[12, 22\], normalized to the mass scale $\mu_0 = 1$ GeV.
The BHL Gaussian parameters $A_{t;\pi}$ and $\beta_{t;\pi}$ are fixed using phenomenological constraints from $\pi^0 \to \gamma\gamma$ and $\pi \to \mu\nu_\mu$ decays (see, e.g., \cite{12}), and $M_q \approx 0.33$ GeV is the constituent ($q = u, d$) quark mass, introduced to parameterize the QCD vacuum effects. These parameters could be additionally constrained using a combined analysis of data from lattice simulations and from experiments like CLEO, BaBar and FermiLab E791 diffractive dijet production. However, since the analysis \cite{12} showed that the sensitivity to the model DA parameters is less than 5%, while the experimental error bars are much larger, we refrain from doing such an involved analysis at the moment. With the availability of higher quality data in future such a systematic combined analysis may provide important constraints to our results.

Next, using the TMD modified factorization ansatz in the operator convolution form $\{F,G\}_\pi^{hard} \sim \tilde{\mathcal{P}}_\pi \otimes \mathcal{M}_{LO} \otimes \tilde{\mathcal{P}}_\pi$, where the LO matrix elements $\mathcal{M}_{LO}$ are diagrammatically represented in the Fig.1 and $\otimes$ represents the phase space integration, one can obtain the pQCD contribution to the hard form factor in a standard way up to twist-3 corrections, given by $\{F,G\}_\pi^{hard}(Q^2) = \delta\{F,G\}_\pi^{(twist2)}(Q^2) + \delta\{F,G\}_\pi^{(twist3)}(Q^2)$ where,

$$
\delta\{F,G\}_\pi^{(twist2)}(Q^2) = \frac{64\pi}{3} Q^2 \int_0^1 dx dy \int_0^\infty db_1 db_2 \alpha_s(t) \left[ \pm x \mathcal{P}_{2;\pi}(x, b_1) \mathcal{P}_{2;\pi}(y, b_2) \right] \times H_\pm(x, y, Q, b_1, b_2) S_t(x) \exp[-S(x, y, b_1, b_2, Q)] ; \quad (5)
$$

$$
\delta\{F,G\}_\pi^{(twist3)}(Q^2) = \frac{128\pi}{3} \mu_\pi^2 \int_0^1 dx dy \int_0^\infty db_1 db_2 \alpha_s(t) \left[ \bar{x} \mathcal{P}_{3;\pi}^p(x, b_1) \mathcal{P}_{3;\pi}^p(y, b_2) + \frac{1}{6} \partial_x \mathcal{P}_{3;\pi}^{\sigma}(x, b_1) \mathcal{P}_{3;\pi}^{\sigma}(y, b_2) + \frac{1}{2} \mathcal{P}_{3;\pi}^{\sigma}(x, b_1) \mathcal{P}_{3;\pi}^{p}(y, b_2) \right] \times H_\pm(x, y, Q, b_1, b_2) S_t(x) \exp[-S(x, y, b_1, b_2, Q)] . \quad (6)
$$

In the above equations, “+” and “-” correspond to the space-like and time-like cases, respectively, $\mathcal{P}_{t;\pi}(x, b) \equiv \tilde{\mathcal{P}}_{t;\pi}(x, b, 1/b_1, \mathcal{M}_{u,d})$ and $t = \max(\sqrt{x} Q, 1/b_1, 1/b_2)$ is related to the factorization scale. The so-called “chiral” parameter $\mu_\pi$ arises from the standard definitions of the twist-3 collinear DAs defined at a suitable low energy scale, $\mu_\pi(\mu_0 \approx 1$ GeV) = $m_\pi^2/(m_u + m_d) \sim 1.7$ GeV \cite{20}. However, in the context of intermediate energies, $\mu_\pi$ is usually taken to be slightly lower $\approx 1.3 - 1.5$ GeV which is consistent with fits to the $B \to \pi$ transition form factors \cite{9, 16, 17, 19}, $\chi$PT estimates \cite{21, 24} and the moment calculation applying QCDSR \cite{25}. Since, the twist-3 results can be somewhat sensitive to this parameter, here we use $\mu_\pi = 1.5 \pm 0.2$ GeV and indeed show that its variation contributes to a large
uncertainty in the time-like region. The hard kernels $H_{\pm}$ could be expressed in terms of the standard Bessel functions $K_0(\theta)$, $I_0(\theta)$, $H_0^{(1)}(\theta) = J_0(\theta) + iY_0(\theta)$ and $J_0(\theta)$:

$$H_+(x, y, Q, b_1, b_2) = K_0(\sqrt{xy} Q b_2)$$

$$\times \left[ \theta(b_1 - b_2) K_0(\sqrt{x} Q b_1) I_0(\sqrt{x} Q b_2) + \theta(b_2 - b_1) K_0(\sqrt{x} Q b_2) I_0(\sqrt{x} Q b_1) \right] ; \quad (7)$$

$$H_-(x, y, Q, b_1, b_2) = \left( \frac{i\pi}{2} \right)^2 H_0^{(1)}(\sqrt{xy} Q b_2)$$

$$\times \left[ \theta(b_1 - b_2) H_0^{(1)}(\sqrt{x} Q b_1) J_0(\sqrt{x} Q b_2) + \theta(b_2 - b_1) H_0^{(1)}(\sqrt{x} Q b_2) J_0(\sqrt{x} Q b_1) \right] , \quad (8)$$

The Sudakov factor $S(Q)$ and the jet function $S_t(x)$ are introduced to organize to all orders the large double logarithms $\alpha_s \ln^2 k_T$ ($k_T$ is the generic transverse parton momenta) and $\alpha_s \ln^2 x$, respectively, that arise from radiative gluon effects and may otherwise invalidate perturbative factorization. Such resummations result in the natural suppression of possible non-perturbative and kinematic endpoint enhancements of the scattering kernel, thereby, improving convergence and making perturbative evaluation self-consistent. For their explicit expressions, one is referred to [12–17]. A few comments regarding our factorized results (Eqs. 5 and 6) are now in order:

1) Here, we have presented a LO analysis of the hard kernel which is apparently gauge dependent (light-cone gauge), arising from the contribution of the single gluon propagator. However, in [26] it was shown that for the $\pi\gamma^* \rightarrow \gamma$ transition form factor, the gauge invariance of the hard kernel is a consequence of the gauge-dependence cancellation between the quark level diagrams of the full QCD and effective diagrams of the pion wavefunction, order by order in perturbation theory using the principle of mathematical induction. In this way, the hard kernel and the resulting predictions from the $k_T$-factorization turn out to be gauge-invariant to all orders. The above reference also claims that such an approach could be extended to other elastic and transition form factors, at least up to the level of NLO corrections.

2) Our result for the hard form factor depends on the renormalization/factorization scale which is typical of all fixed order calculations. The Sudakov factor that resums a certain class of radiative soft-gluon contributions to all orders in perturbation theory is inherently
factorization scale dependent, while the LO hard kernel that is used to evaluate the hard form factor depends on the renormalization scale through the running of $\alpha_s$. In this case, the scale dependence is minimized by adhering to a fixed prescription with the renormalization/factorization scale set to the momentum transfer $Q^2$ [26, 27]. It is, however, believed that a systematic higher order calculation can eventually absorb this scale dependence.

3) There may be a simple rationale why the TMD factorization is expected to work at the level of $1/Q^2$ power suppressed corrections, although a more rigorous proof is beyond the scope of this paper. Firstly, note that the “active” soft gluons which may arise e.g., from the 3-particle twist-3 DA that probe the hard kernel, bring about additional power corrections. Compared to the 2-particle twist-3 corrections considered in this work, the 3-particle twist-3 corrections is not chirally enhanced (there is a large parametric enhancement from $\mu_\pi$ in the definition of the 2-particle twist-3 DAs, which brings about a sensitivity to the chiral scale), and should be numerically small. Secondly, the rest of the “long-distance” soft gluons that do not interfere with the hard kernel may break the TMD factorization. However, in the large $Q^2$ limit, a hadron tends to have a small “color-dipole” due to the Lorentz contraction and the Sudakov suppression. Such gluons can not probe the small “color-dipole” configurations of $q \bar{q}$ within the hadronic bound state, and their effects cancel each other. This is the so-called “color transparency hypothesis”. With this assumption, one only needs to care about collinear gluon effects and their factorization. Using similar arguments, the authors in [28] have explicitly proven TMD factorization at the twist-2 level and collinear factorization at the twist-3 level. Hence, it is our assumption that the approach presented in the above reference can even be straightforwardly extended to include the twist-3 TMD factorization.

**Soft QCD.** Next, following [4], we include the soft (Feynman mechanism) contribution via Local Duality (LD) for the space-like form factor [29],

$$F^\text{soft}_\pi(Q^2)|_{LD} = 1 - \frac{1 + 6s_0/Q^2}{(1 + 4s_0/Q^2)^{3/2}}, \quad (9)$$

where $s_0 \approx 0.68$ GeV$^2$ is the duality interval for higher excited and continuum thresholds which is very naturally almost the “middle” between pion mass $m_\pi^2 \approx 0$ and that of the $A_1$ resonance $m_{A_1}^2 \approx 1.6$ GeV$^2$. The VMD models and $\chi$PT predictions are not expected to work beyond $\approx 1$ GeV, while standard pQCD with only twist-2 operators completely fails to explain the available experimental data. The soft contribution, on the other hand, is significantly large at moderate energies [7] and so are the twist-3 power corrections [12].
FIG. 2: The relative magnitudes of the soft (double-dot black lines), twist-2 (thin solid blue lines) and twist-3 (thick solid red lines) corrections to the pion form factor. The twist-2 and twist-3 corrections without including the pre-factors are also displayed.

However, both the soft and the twist-3 corrections are expected to fall off rapidly as $\sim 1/Q^4$ for large $Q$, so that asymptotically ($Q \to \infty$) one recovers the rigorous leading twist-2 contributions $\sim 1/Q^2$ which dominate the form factor. This aspect is clearly revealed through our analysis (see, Figs. 2 and 3).

To extend the analysis to the time-like region, one may analytically continue Eq. 9 from the space-like region. Using such a model ansatz, the authors of [30] were able to show for the first time a much larger contribution to the form factor in the time-like region than in the space-like, and hence were partly able to resolve the bulk of the discrepancy for large $Q$. However, this gives rise to a single pole at $4s_0 \approx 2.71$ GeV$^2$ which does not correspond to any of the real physical bound states or resonances (e.g., $\rho, \omega, \ldots$) seen in the time-like data. In fact, the observed spectrum around 2.7 GeV already appears to be rather “smooth” and well above the resonance region (below $\pi\omega$ threshold). Hence, rather than trying to reproduce the actual time-like data, including the various bound states and resonances, we try to explain the continuum contribution with a smooth $G_{\pi}^{\text{soft}}$ which has the same leading $1/Q^4$ dependence as $F_{\pi}^{\text{soft}}$ under analytic continuation. Thus, we choose the same form of the time-like soft factor, i.e., $G_{\pi}^{\text{soft}}(Q^2) = F_{\pi}^{\text{soft}}(Q^2) + O(1/Q^6)$ for large $Q$. Here again we should stress that our analysis is entirely based on the assumption that the physically
observed low-lying resonances would not spoil the continuum contribution which appear as “superposed peaks” on a continuum spectrum.

Results and Discussion. To this end, it may be notable that the rather ad hoc incorporation of the soft part from QCDSR have no a priori correspondence with the hard parts, and therefore, may lead to the possibility of some double-counting between the respective soft and hard contributions at the intermediate regime. However, such double counting could partly be removed by imposing the the vector Ward-identity \( \{F, G\}_\pi(0) = 1 \). Following the argument detailed in \cite{7,12}, we introduce appropriate power correcting pre-factors to restore the Ward-identity, and hence we arrive at our final expression for the space- and time-like pion form factors given by

\[
\{F, G\}_\pi(Q^2) = 1 - \frac{1 + 6s_0/Q^2}{(1 + 4s_0/Q^2)^{3/2}} + \Delta\{F, G\}_\pi^{(\text{twist2})}(Q^2) + \Delta\{F, G\}_\pi^{(\text{twist3})}(Q^2); \\
\Delta\{F, G\}_\pi^{(\text{twist2})}(Q^2) = \left(\frac{Q^2}{2s_0 + Q^2}\right)^2 \delta\{F, G\}_\pi^{(\text{twist2})}(Q^2), \\
\Delta\{F, G\}_\pi^{(\text{twist3})}(Q^2) = \left(\frac{Q^4}{4s_0^2 + Q^4}\right)^2 \delta\{F, G\}_\pi^{(\text{twist3})}(Q^2). \tag{10}
\]

The above pre-factors of \( \delta\{F, G\}_\pi^{(\text{twist2})} \) and \( \delta\{F, G\}_\pi^{(\text{twist3})} \) ensure a “smooth” matching of the different power-law \( Q^2 \) behavior between the soft and the hard parts that preserve the gauge invariance condition \( \{F, G\}_\pi^{\text{hard}}(0) = 0 \). In principle, this vector Ward identity can also be achieved with larger \( n \) values in the \( Q^{2n}/((2s_0)^n + Q^{2n}) \) factors in front of the hard part. However, as \( n \to \infty \), the factor becomes a step function which is not smooth. Thus, we have chosen the minimum \( n \)’s to achieve the maximum smoothness. The individual contributions of the soft \( \{F, |G|\}_\pi^{\text{soft}}, \text{twist-2 } \Delta\{F, |G|\}_\pi^{(\text{twist2})} \text{ and twist-3 } \Delta\{F, |G|\}_\pi^{(\text{twist3})} \) are summarized in Fig. \( \ref{fig:2} \). The soft and the twist-3 terms turn out to give dominant contributions at the low and moderate range of \( Q^2 \)-values with anomalously large twist-3 contributions in the time-like region. Nevertheless, both the corrections exhibit sharp fall-off with increasing \( Q^2 \), such that the plot extended beyond \( Q^2 \approx 50 \text{ GeV}^2 \), will clearly show the twist-2 contributions as the being the only dominant ones, both for the space- and time-like domains. Note that our factorized hard results are calculated using non-asymptotic collinear twist-2 and twist-3 DAs (up to NLO in conformal twist), taken from Ball et al. \cite{22} where these are obtained in the context of LCSR.

The final result for the total space- and time-like form factors (Eq. \ref{eq:10}) is displayed in Fig. \ref{fig:3} along with the existing experimental pion data. The solid (red) curves correspond
FIG. 3: The total space- and time-like e.m. pion form factors calculated using Eq. [10], denoted by the solid (red) lines; the soft from factor \( \{ F, |G| \}_\text{soft} \) is denoted by the double dashed (black) lines; the Ward-identity violating result that does not include the pre-factor modification is also displayed. For comparison, the standard asymptotic pQCD result [2] is displayed. The shaded area is roughly our estimated theoretical error beyond \( Q^2 \approx 5 \text{ GeV}^2 \). The world pion data are taken from [3–6].

to our central result obtained with the chiral parameter \( \mu_\pi = 1.5 \text{ GeV} \), while the shaded area can be regarded as our estimated theoretical error, reliable only beyond the resonance region. Due to the overwhelming time-like twist-3 contributions at intermediate energies, the modulus of the total time-like form factor \( |G_\pi| \) shows a big enhancement, at least by a factor of \( \approx 2 - 4 \) compared to \( F_\pi \), although both clearly show asymptotic trends, numerically approaching the standard pQCD result [2] (beyond \( \sim 50 - 100 \text{ GeV}^2 \)). The above enhancement is largely due to two model independent features:

1. The quark and gluon propagators in the hard kernel can become on-shell at non-zero \( q^2 \) in the time-like but not the space-like region. Thus, in general \( |G_\pi^{\text{hard}}| \) should be bigger than \( F_\pi^{\text{hard}} \). In pQCD, this generic feature is captured only if the \( k_T \) dependence is kept, so that the denominator of the hard kernel has terms proportional to \( q^2 \) and \( k_T^2 \) that cancel each other in the time-like but not the space-like region (see, e.g., Eqs. 27 and 43 of [12] before they are Fourier transformed into Eq. [7] of this paper). Without the \( k_T \) dependence,
this enhancement will be missing.

(2) The twist-3 contributions are more important than the twist-2 ones for intermediate range of $Q^2$ due to two notable reasons: Firstly, the twist-3 terms have the aforementioned parametric enhancement arising from $\mu_\pi$ which is absent in the twist-2 case. Secondly, the finiteness of $\phi_{3,\pi}^F$ and the derivative of $\phi_{3,\pi}$ with respect to $x$ (see, Eq. (6)) at the endpoints $x = 0, 1$. These features together with $H \propto x^{-3/2}$, being oscillatory in the time-like region and exponentially decaying in the space-like region (i.e., the time-like parton propagators in momentum representation develop poles which are absent in the space-like), account for the characteristic relative enhancement of the time-like twist-3 contributions.

With these two rather robust features and typical treatments of the soft and sub-leading twist-3 contributions, it appears that the previously reported discrepancy between the experimental data and theoretical predictions [5] can be ostensibly reconciled. To this end, we also present our results without including the pre-factors to demonstrate their effect. As revealed from Figs. 2 and 3 without the pre-factors the hard contributions tend to grow very rapidly as $Q^2 \to 0$ and become unreliable, while at the same time beyond $Q^2 \approx 5 - 10$ GeV$^2$ the effect of the pre-factors is hardly discernable. Clearly, then our predictions convincingly agrees with most of the space- and time-like experimental pion data, including the recent CLEO result: $Q^2|G_\pi(13.48 \text{ GeV}^2)| = 1.01 \pm 0.11 \text{(stat)} \pm 0.07 \text{(syst)} \text{ GeV}^2$ [5], and also the theoretical prediction $M^2_{J/\psi} \left| G_\pi(M^2_{J/\psi} = 9.6 \text{ GeV}^2) \right| = 0.94 \pm 0.08 \text{ GeV}^2$ [31], fixed from branching ratios of $J/\psi \to \pi\pi$ and $J/\psi \to e^+e^-$ decays.

Finally, to comment on the error estimate of our approach, we first look at the error band in Fig. 3. While the width of the error band is too narrow to be even noticeable in the space-like region, it is anomalously large in the time-like region. Over 90% of this error is essentially due to the variation of the chiral parameter $\mu_\pi$ between 1.3 – 1.7 GeV with increasing contribution to the pion form factor. The remaining difference generously over-estimates the other model (parameter) dependences in the DAs from QCDSR, but it seems to be a reasonable range of theoretical error when the error of the soft part is also included. However, we again stress that the estimate only applies beyond the resonance region. Furthermore, several aspects deserve to be noted: The extent of the theoretical error from our LO analysis is large enough to completely subsume the systematic errors that may arise, e.g., considering NLO effects (in the QCD coupling $\alpha_s$) [15, 32, 33], sub-leading twists (see, e.g., [34]) for the twist-4 and twist-6 contributions to the pion form
factor in the context of QCDSR), and effects due to higher Fock state corrections which are expected to be rather nominal. For example, even without explicit calculations, it is easily understandable that the 2-particle twist-4 power corrections are, in fact, very small being being proportional to $m_π^2 \to 0$. Again, the contribution of the 3-particle twist-3 DA, being proportional to the “tiny” non-perturbative parameter $f_{3π} \approx 0.45 \times 10^{-2}$ GeV$^2$ (to be compared with the 2-particle twist-3 DA parameter $\mu_π \approx 1.5$ GeV), is also strongly suppressed. Thus, the 2-particle twist-3 contributions are indeed very special in this regard. Moreover, it is estimated that the NLO corrections in the case of the $\pi\gamma^* \to \gamma$ transition form factor amount to only about 5% under specific factorization scheme with the factorization scale set to the energy/momentum transfer $Q$. This is not expected to be very different for the pion form factor. To conclude, the unnaturally large twist-3 contribution, especially in the time-like region, is certainly non-intuitive and may constitute an important step toward understanding the large asymmetry seen in the experimental data, unaccountable otherwise.

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