Density perturbations in Kaluza–Klein theories during a de Sitter phase

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Abstract

In the context of Kaluza-Klein theories, we consider a model in which the universe is filled with a perfect fluid described by a barotropic equation of state. An analysis of density perturbations employing the synchronous gauge shows that there are cases where these perturbations have an exponential growth during a de Sitter phase evolution in the external space.

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1 Introduction

One of the main still open questions within the realm of modern cosmology is the origin of the primordial density fluctuations which played the role of seeds for structure formation. Within the framework of gravitational instability, the two alternative scenarios which could lead to the generation of perturbations are either the amplification of quantum fluctuations during an inflationary era [1], or fluctuations seeded by topological defects [2]. Both inflation and topological defect models lead to approximately scale invariant Harrison-Zel’dovich spectra on large angular scales, thus both are consistent with the COBE-DMR results [3]. The characteristic features of the Doppler peaks in the angular power spectrum of the cosmic microwave background, might be an important discriminating tool between inflationary fluctuations and those produced by topological defects [4]. The dynamics of the initial density fluctuations has been studied by a large number of authors, either in the context of general relativity, or in multi-dimensional theories.

In a generic inflationary model, the inflaton field \( \phi \) experiences quantum fluctuations on all scales smaller than the effective particle horizon at the onset of inflation. During the era of exponential expansion, the scales inflate outside the horizon and therefore quantum fluctuations freeze in as classical fluctuations of \( \phi \), leading to generation of energy density perturbations. The amplitude of the density perturbations
of a given scale $\lambda$ when they cross back the horizon at the end of inflation is [5]

$$\left(\frac{\delta \rho}{\rho}\right)_{\text{hor}} \sim \left(\frac{H^2}{\dot{\phi}}\right)_\lambda.$$  

(1)

Thus, through a linear mechanism one obtains a spectrum of density perturbations of constant amplitude, which under certain choices lead to the required amplitude of initial fluctuations, sufficient to generate structure formation without distorting the isotropy of the cosmic microwave background.

For a number of authors, inflation is plugged with difficulties (e.g., potential fine tuning of coupling constants in order to achieve the required amplitude of density perturbations) and therefore we believe that it is important to investigate alternative scenarios. As such, one can study topological defect models (which will not be addressed here) or the growth of density perturbations in multi-dimensional theories, in particular, during a de Sitter phase. One may wonder whether in order to get non-zero density perturbations during a de Sitter phase while remaining within the framework of general relativity, one should either consider theories where the energy momentum is not conserved, or to remove the condition of a perfect fluid. We will sketch why none of these attempts work and therefore motivate our choice to work with multi-dimensional theories.

A scalar-tensor theory of gravity which involved a non-conserved energy momentum tensor was proposed by Jordan [6], but it was criticized [7], basically because it fails to successfully incorporate non-relativistic matter. A study of the evolution of scalar perturbations in a universe filled with a viscous fluid, in the framework of general relativity, shows that the density perturbations decay (see, appendix A). So, to investigate whether scalar perturbations can grow during a de Sitter phase, we will consider a universe filled with a perfect fluid described by a barotropic equation of state, in the context of Kaluza-Klein [8] theories, where energy momentum conservation allows density perturbations provided the de Sitter phase characterizes only the four-dimensional subspace.

In this work we are only dealing with the evolution of scalar perturbations, without examining their origin. In the context of inflation, for example, initial perturbations are generated by quantum fluctuations of the inflaton field. We will perform our analysis employing the synchronous gauge, while keeping in mind the residual coordinate freedom in order to finally keep only the physical modes.

Our presentation is organized as follows: In section 2 we first set up the problem, specify the notation, write down the background equations and the energy momentum tensor. In section 3 we introduce linear perturbations and derive the set of equations that describe the evolution of scalar perturbations in the synchronous gauge. In section 4 we solve the set of equations for two particular cases, where the external space is in a de Sitter phase. While our choices for these cases may be rather special, our aim here is only to show that it is possible to obtain growth of perturbations; the issue of how generic this situation is, is left for future work. We will show that, within the framework of multi-dimensional theories, we can indeed find at least one example where there is growth of scalar perturbations, in particular with an exponential behaviour, during a de Sitter phase. We summarize our results in section 5. We close with two appendices; in the first one we discuss density perturbations in a viscous universe, while in the second one we analyze the problem of the residual coordinate freedom.
2 The model

The Lagrangian density in a $n$-dimensional spacetime, coupling gravity to ordinary matter, is

$$L = \frac{1}{16\pi G} \sqrt{-g} R - L_m ,$$  \hspace{1cm} (2)

where all quantities are defined in the multi-dimensional spacetime. From this expression, we can deduce the field equations in higher dimensions

$$R_{AB} - \frac{1}{2} g_{AB} R = 8\pi G T_{AB} ,$$  \hspace{1cm} (3)

$$T^{AB} = 0 .$$  \hspace{1cm} (4)

We consider that each spatial section is divided into two spaces, the external one of dimension $d_1$ and the internal one of dimension $d_2$, such that $n = 1 + d_1 + d_2$. We assume the metric to be of the form

$$ds^2 = -dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j + b(t)^2 \gamma_{ab} dx^a dx^b ,$$  \hspace{1cm} (5)

where $\gamma_{ij}$ is the external metric and $\gamma_{ab}$ the internal one, and both spatial sections have a constant curvature, not necessarily zero. Normalizing the curvature, we write $k_1 = -1, 0$ or $1$ and $k_2 = -1, 0$ or $1$ for the curvatures of the spaces with dimensions $d_1$ and $d_2$ respectively.

We consider an anisotropic energy momentum tensor, with different pressures in each subspace. Thus, it takes the form

$$T_{AB} = (\rho + \bar{p}) u^A u^B - p_1 u^i u^{(A} \delta_i^{B)} - p_2 u^a u^{(A} \delta_a^{B)} - \bar{p} g^{AB} + \frac{p_1}{2} g^{i(A} \delta_i^{B)} + \frac{p_2}{2} g^{a(A} \delta_a^{B)} ,$$  \hspace{1cm} (6)

where

$$\bar{p} = \frac{1}{2}(p_1 + p_2) ; \hspace{0.5cm} u^{(A} u^{B)} = \frac{1}{2}(u^A u^B + u^B u^A) ,$$  \hspace{1cm} (7)

with $i = 1, \ldots, d_1$; $a = 1, \ldots, d_2$.

We consider that both pressures have a barotropic equation of state:

$$p_1 = \alpha_1 \rho \hspace{0.5cm} ; \hspace{0.5cm} p_2 = \alpha_2 \rho .$$  \hspace{1cm} (8)

The differential equations we obtain from Eqs. (3) and (4), relating scale factors $a(t), b(t)$ and $\rho$ are

$$d_1 \ddot{a} + d_2 \ddot{b} =$$

$$- \frac{8\pi G}{d_1 + d_2 - 1} \left[ d_1 (1 + \alpha_1) + d_2 (1 + \alpha_2) - 2 \right] \rho ;$$  \hspace{1cm} (9)

$$\frac{\ddot{a}}{a} + (d_1 - 1)(\dot{a})^2 + d_2 \frac{\ddot{b}}{b} + (d_1 - 1) \frac{k_1}{a^2} =$$

$$\frac{8\pi G}{d_1 + d_2 - 1} \left[ 1 - \alpha_1 + d_2 (\alpha_1 - \alpha_2) \right] \rho ;$$  \hspace{1cm} (10)

$$\frac{\ddot{b}}{b} + (d_2 - 1)(\dot{b})^2 + d_1 \frac{\ddot{a}}{a} + (d_2 - 1) \frac{k_2}{b^2} =$$

with $i = 1, \ldots, d_1$; $a = 1, \ldots, d_2$. 


\[
\frac{8\pi G}{d_1 + d_2 - 1} \left[ 1 - \alpha_2 + d_1(\alpha_2 - \alpha_1) \right] \rho \quad ; \quad (11)
\]
\[
\dot{\rho} + d_1(1 + \alpha_1) \dot{\alpha} \rho + d_2(1 + \alpha_2) \dot{\beta} \rho = 0 \quad . \quad (12)
\]

Some special solutions for these equations were found by Sahdev [9].

3 Perturbation equations

We now proceed with the perturbative level. We introduce in Eqs. (3) and (4) the quantities
\[
\tilde{g}_{AB} = \bar{g}_{AB} + h_{AB} \quad , \quad \tilde{\rho} = \bar{\rho} + \delta \rho \quad , \quad \tilde{\rho} = \bar{\rho} + \delta \rho \quad , \quad (13)
\]
where \( \bar{g}_{AB} \), \( \bar{\rho} \) and \( \bar{p} \) represent the background solutions while \( h_{AB} \), \( \delta \rho \) and \( \delta p \) are small perturbations around them. We will also impose that the perturbations behave spatially like plane waves. Due to the anisotropy of the space, the equations take a tractable form only if the wave is defined in just one space. We consider that all perturbed functions depend only on the coordinate of the external space, \( i.e., \delta(x_i, t) = \delta(t) \exp(i\vec{q} \cdot \vec{x}) \), where \( x_i \) denote the coordinates in the space of dimension \( d_1 \).

The equations are invariant under an infinitesimal coordinate transformation \( x_A \rightarrow x_A + \chi_A \). So, we have the freedom of imposing a coordinate condition. We will impose the synchronous coordinate condition \( h_{A0} = 0 \) ; (14)

As it is well known, there is yet a residual coordinate freedom [12], represented by a coordinate transformation that preserves the coordinate condition. This can lead to non-physical modes in the final solutions. We will discuss later this problem, establishing the criterium to eliminate them (see, appendix B).

We define \( h = h_k^k/a^2 \), \( H = h_a^a/b^2 \), \( \Delta = \delta \rho/\rho \). After a long but straightforward calculation, we obtain from Eqs. (3) – (12) the following system of differential equations
\[
\ddot{h} + 2\dot{a} \dot{h} + \dot{H} + 2\frac{b}{b} \dot{H} = 2 \left( d_1 \dot{a} + d_2 \dot{b} \right) \Delta \quad ; \quad (15)
\]
\[
\dot{H} + (d_1 \dot{a} + 2d_2 \dot{b} \frac{b}{b} ) \dot{H} + \left[ \frac{q^2}{a^2} - 2(d_2 - 1) \frac{k_2}{b^2} \right] H = -d_2 \dot{b} \dot{h} + 2d_2 \left( \frac{b}{b} + (d_2 - 1) \frac{b}{b} \right)^2 + d_1 \dot{a} \dot{b} \frac{b}{b} + (d_2 - 1) \frac{k_2}{b^2} \Delta \quad ; \quad (16)
\]
\[
\dot{\Delta} + (1 + \alpha_1) \delta u^i, i + \frac{1}{2}(1 + \alpha_1) h + \frac{1}{2}(1 + \alpha_2) \dot{H} = 0 \quad ; \quad (17)
\]
\[
\dot{\Delta} + (1 + \alpha_1) \delta u^i + (1 + \alpha_1) \left[ \frac{2 - d_1 \alpha_1}{a} - d_2 \alpha_2 \frac{b}{b} \right] \delta u^i = -\frac{1}{2} (\alpha_1 - \alpha_2) H^i - \alpha_1 \Delta^i \quad . \quad (18)
\]

These equations describing the evolution of scalar perturbations, are too complicated to be solved in the general case. In the next section, we will solve them in two particular cases, where the equation of state in the external space is \( p_1 = -\rho \).
4 Specific cases

Here, we will solve the system of differential equations describing the evolution of density perturbations in the framework of multi-dimensional theories with $\alpha = -1$, for the following particular cases:

(i) Both the external and the internal spaces have scale factors with a power-law behaviour, $a \propto t^r \ (r > 0)$ and $b \propto t^s$, respectively. The internal space is also flat. To get a non-vanishing $\delta \rho/\rho$, the set of equations describing the evolution of density perturbations requires $\alpha_2 \neq -1$. On the other hand, the background equations and the energy conservation equation imply

\[ s = \frac{2}{d_2(1 + \alpha_2)}; \]
\[ r = \frac{d_2s(s - 1)}{d_2s + d_1 - 1}. \]  

One can easily check that Eqs. (15) – (18) imply

\[ \Delta = -\frac{1}{2}(1 + \alpha_2)H + \text{const} \]  

and lead to the following third order equation for $H$:

\[ H''' + H'' \frac{1}{\eta(1 - r)} \left[ d_1r + d_2s - r + 1 \right] + H' \left[ q^2 + \frac{1}{\eta(1 - r)} \left\{ d_1(r^2 + 2r) + d_2s(2 + 3r - 2s) - r^2 - 2 \right\} \right] + H \left[ \frac{q^2}{\eta(1 - r)} + \frac{2}{\eta(1 - r)^3} \left\{ d_1r^2 + d_2s(2r - s) - 2r + 1 \right\} \right] = 0, \]  

where primes denote partial derivatives w.r.t. conformal time $\eta$, defined by $a d\eta = dt$. After some manipulations, we obtain that, as far as time evolution is concerned, Eq. (22) reduces to the second order equation

\[ \frac{d^2g}{d\tau^2} + \frac{1}{\tau} \frac{dg}{d\tau} + g \left[ 1 - \frac{1}{\tau^2} \frac{1}{4(1 - r)^2} \left\{ 1 + r^2(d_1^2 - 4d_1 + 4) \right\} + s^2d_2(d_2 + 8) - r(6d_1 - 4) - 6d_2s + 2r sd_2(d_1 - 6) \right] = 0, \]  

where

\[ g = \left[ H\eta^{1/(1-r)} \right]' \tau^\gamma; \]
\[ \gamma = \frac{3 - d_1r - d_2s}{2(r - 1)}; \]
\[ \tau^2 = q^2\eta^2. \]  

Equation (23) is a Bessel equation, whose solutions are Bessel functions of order

\[ \nu = \frac{d_1^2s^4 + 2d_2^2s^3(s + 2) + d_2^2s^3(s + 4) + 8d_2^2s^2 + 4d_2s(s + 2) + 4}{4[d_2s(2 - s) + 2]^2}. \]
This expression simplifies a lot if \( d_1 = 3 \) and \( d_2 s = -1 \), which means that the external space grows like \( t^r \) with \( r > 1 \), while the internal one goes like \( t^s \) where \( s < 0 \). In that case,

\[
\nu = \frac{r}{2(1 - r)} = -\frac{1 + d_2}{2} .
\]

The general solution of Eq. (23) is

\[
g(\tau) = uJ_\nu(\tau) + vJ_{-\nu}(\tau) ,
\]

where \( u \) and \( v \) are arbitrary constants. For the sign in the argument of the Bessel functions we choose \( \tau = +q\eta \).

In terms of the density contrast, we find

\[
\Delta = -\frac{1}{2}(1 + \alpha_2)\eta^{-\frac{1}{2+2d_2}}q^\gamma \int \eta^{-\gamma}(uJ_\nu(q\eta) + vJ_{-\nu}(q\eta))d\eta.
\]

For the particular case we are considering here, and re-expressing the solutions in terms of the cosmic time \( t \), we find the asymptotic behaviour for \( \Delta \), for \( t \to 0 \) \((\eta \to \infty)\) and \( t \to \infty \) \((\eta \to 0)\), namely

\[
t \to 0 \implies \Delta \to t^{-\frac{d_2+2}{2d_2}}(c_1 \cos t^{-\frac{1}{d_2}} + c_2 \sin t^{-\frac{1}{d_2}}),
\]

\[
t \to \infty \implies \Delta \to t^{-\frac{d_2-5+2(d_2+1)}{2d_2}}.
\]

So, initially the density contrast has an oscillatory behaviour with decreasing amplitude, while asymptotically it tends only to decreasing modes.

(ii) The external space is a de Sitter flat space filled with a perfect fluid described by \( p_1 = -\rho \), and its scale factor goes like \( a \propto e^{rt} \), where \( r \) is a constant. The internal space has dimensions \( d_2 > 3 \), constant non-zero curvature and constant scale factor. The energy conservation equation implies \( \rho = \text{const} \) and the background equations lead to \( d_2 < 2/(1 + \alpha^2) \). To get a non-zero \( \delta\rho/\rho \), the perturbation equations require \( \alpha^2 \neq -1 \). The system of equations describing the evolution of the scalar perturbations simplifies to:

\[
\ddot{h} + 2r \dot{h} + \ddot{H} = 6r^2 \Delta ;
\]

\[
\dot{H} + d_1 r \dot{H} - \left[2(d_2 - 1)\frac{k^2}{b^2} - \frac{q^2}{e^{2rt}}\right]H = (d_2 - 1)\frac{k^2}{b^2} \Delta d_2 ;
\]

\[
\Delta + \frac{1}{2}(1 + \alpha_2)\dot{H} = 0 ;
\]

\[
\Delta_{,i} + \frac{1}{2}(1 + \alpha_2)H_{,i} = 0 .
\]

Clearly, Eqs. (34) – (36) again imply

\[
\Delta = -\frac{1}{2}(1 + \alpha_2)H + \text{const} .
\]

To get a growing solution of Eq. (34) we find the same requirement as the one imposed by the energy conservation equation, namely

\[
d_2 < \frac{2}{1 + \alpha_2} .
\]
Then, the solution for the density perturbation is

\[ \Delta = \tau^{d_1/2} \left( c_1 J_{\nu}(q \tilde{\tau}) + c_2 J_{-\nu}(q \tilde{\tau}) \right) \] (39)

where \( \tilde{\tau} = -\frac{e^{-rt}}{a_0 r} \) and \( \nu = \sqrt{\frac{d_1^2}{2} + d_1 + 1 + \alpha_2 (d_1 - 1)}. \) (40)

The asymptotic behaviour is easily obtained. For small \( t, \) large \( \tilde{\tau}, \)

\[ \Delta = e^{-\frac{d_1}{2} t} \cos(\beta e^{-rt} + \delta), \] (41)

where \( \beta \) and \( \delta \) are constants. For large \( t, \) small \( \tilde{\tau}, \) we obtain a mode which exhibits exponential growth,

\[ \Delta \sim e^{r(-\frac{d_1}{2} + \nu)t}. \] (42)

So, the density perturbations exhibit initially an oscillation with decreasing amplitude and then evolve towards a behaviour characterized by an exponential growth.

5 Conclusions

One of the most interesting and important issues within modern cosmology, is undoubtedly the origin of large scale structure. Within the framework of gravitational instability, there are two classes of theories attempting to answer the question of structure formation: inflationary models and topological defect scenarios. The cosmic microwave background anisotropies, attempting to bridge between theoretical models and observational data, might support or rule out one of the two families of structure formation. In the context of inflation, quantum fluctuations of the scalar field give rise to density perturbations only once inflation ended and the perturbations re-entered the horizon. In this work, we are mainly interested in a model where density perturbations can grow during the inflationary era. We are working in the context of Kaluza-Klein theories, where the external space inflates.

We employ some special solutions of multi-dimensional theories, and study more closely two particular cases, one with power-law behaviour of the scale factor, and one with exponential-type behaviour. The first one leads only to asymptotical decaying modes, while the second one, in spite of an oscillatory decreasing initial behaviour, gives asymptotically an exponential growth of density perturbations. This indicates that such multi-dimensional theories may furnish sufficiently large perturbations in the beginning of the radiation dominated era, thus they may provide an alternative picture. However, one should keep in mind that a further analysis, for example the determination of the amplitude of quantum fluctuations, which may allow a confrontation with observational data, is required before a final statement is reached.

Appendices

A Perturbations in a viscous universe

Another way of obtaining a growth of density perturbations during an inflationary phase, is to use an imperfect fluid in the energy-momentum tensor. This can be
achieved, for example, by introducing viscosity. Here we consider bulk viscosity in the context of the four-dimensional general relativity. The bulk viscosity effects can be deduced just by replacing $p \mapsto p - \chi(\rho)\Theta$ in the energy-momentum tensor, Eq. (4), where $\Theta = u^\mu \gamma_{\mu}$. We consider a barotropic equation of state, $p = \alpha \rho$.

The field equations are

\begin{align}
3(\dot{a}^2) &= 8\pi G \rho ;
2\ddot{a} + (\dot{a}^2) &= -8\pi G \alpha \rho + 24\pi G \chi(\rho) \dot{a} \rho ;
\dot{\rho} + 3(1 + \alpha) \rho &= 9 \chi(\dot{a}^2) .
\end{align}

If $\chi(\rho) = \chi_0 = \text{const}$, we have the simple solution $a \propto e^{Ht}$, where $H = (1 + \alpha) \rho / 3 \chi_0$ and $\rho = \text{const}$. If on the other hand, $\chi(\rho) = \chi_0 \rho$, we have the solutions found by Murphy [12], which reduce either to the previous one in the limit $t \to -\infty$, or to the flat perfect fluid solution in the limit $t \to \infty$.

If we perturb Einstein’s equations in the presence of viscosity, we find the following coupled differential equations

\begin{align}
\ddot{h} + 2 \dot{a} \dot{h} - 12\pi G \chi(\dot{h} - \dot{\Psi}) &= 0 ;
\dot{\Delta} + 9 \left(\frac{\dot{a}}{a}\right)^2 (\frac{X}{\rho} - \chi') \Delta + (1 + \alpha - 6 \frac{X \dot{a}}{\rho a}) (\Psi - \dot{h}) &= 0 ;
(1 + \alpha - 3 \frac{\dot{a}}{a} \frac{X}{\rho}) \Psi + \left[ (1 + \alpha)(2 - 3\alpha) \frac{\dot{a}}{a} - 3 \left(\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 \right) + 9\alpha \left(\frac{\dot{a}}{a}\right)^2 \frac{X}{\rho} + 9\chi' \left(\frac{\dot{a}}{a}\right)^2 \left[ (1 + \alpha) - 3 \frac{\dot{a}}{a} \frac{X}{\rho} \right] \Delta - 3 \frac{\dot{a}}{a} \frac{X}{\rho} \Delta - \frac{X}{\rho} (\Psi - \dot{h}) \right] &= 0 ,
\end{align}

where $\Delta = \delta \rho / \rho$, $\Psi = \delta^i i$, and $\chi' = d\chi / d\rho$.

We will now determine the perturbed solutions for the density contrast. If $\chi = \chi_0$, $\chi' = 0$, then using the background relation

$$1 + \alpha = 3(\chi / \rho)(\dot{a} / a) ,$$

we obtain

$$\Delta = e^{-3Ht} .$$

This solution cannot be eliminated by a residual coordinate transformation. So, if the viscosity coefficient is constant, we have a de Sitter phase, during which the density perturbations do not vanish; however they decrease exponentially with time.

Let us now consider the case $\chi = \chi_0 \rho$. In the asymptotic limit $t \to -\infty$, the density contrast decays exponentially as the inverse of the cubic power of the scale factor at that stage. We find again a de Sitter phase, with non-vanishing density perturbations, which however decrease exponentially with time. On the other hand, in the limit $t \to \infty$, the perturbed solutions approach the ones found in the case of a perfect fluid, like the background solutions do.
B  Residual gauge freedom

We consider the infinitesimal coordinate transformation \( \tilde{x}^A \rightarrow x^A + \chi^A \), under which the components of the metric tensor transform to

\[
\tilde{g}_{AB} = g_{AB} + \chi_{(A;B)} .
\]  

(51)

Imposing that the synchronous coordinate condition be preserved, we obtain the following solutions for the time- and space-components (in each of the two subspaces) of \( \chi^A \):

\[
\chi^0 = \Psi(x) ,
\]

(52)

\[
\chi^i = \Psi(x)^i \int \frac{dt}{a^2} + \zeta(x)^i ,
\]

(53)

\[
\chi^a = \Psi(x)^a \int \frac{dt}{b^2} + \Xi(x)^a ,
\]

(54)

where \( \Psi, \zeta \) and \( \Xi \) are arbitrary functions. Since the traces of the perturbed metric tensor, in each subspace, are

\[
\tilde{h}^k_k = h^k_k - a^2 \left[ 2\Psi_{,k,k} \int \frac{dt}{a^2} + 2\zeta_{,k,k} - 2d_1 \frac{\dot{a}}{a} \Psi \right] ,
\]

(55)

\[
\tilde{h}^a_a = h^a_a - b^2 \left[ 2\Psi_{,a,a} \int \frac{dt}{b^2} + 2\Xi_{,a,a} - 2d_2 \frac{\dot{b}}{b} \Psi \right] ,
\]

(56)

one can show, using the definitions \( h = h^k_k/a^2, H = h^a_a/b^2 \) and the equations for the perturbations, that the density contrast transforms under this residual coordinate transformation as

\[
\tilde{\Delta} = \Delta - \left[ (1 + \alpha_1) d_1 \frac{\dot{a}}{a} + (1 + \alpha_2) d_2 \frac{\dot{b}}{b} \right] \Psi(x) .
\]

(57)

Thus, any perturbed solution with a time behaviour \((1+\alpha_1)d_1(\dot{a}/a) + (1+\alpha_2)d_2(\dot{b}/b)\) can in principle be eliminated by a coordinate transformation and has no physical meaning.

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