Renormalizability of non-anticommutative $\mathcal{N} = (1, 1)$ theories with singlet deformation

I.L. Buchbinder $^+$, E.A. Ivanov $^\dagger$, O. Lechtenfeld $^\ddagger$, I.B. Samsonov $^*$, B.M. Zupnik $^\dagger$

$^+$ Department of Theoretical Physics, Tomsk State Pedagogical University, Tomsk 634041, Russia
$^\dagger$ Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, 141980 Moscow Region, Russia
$^\ddagger$ Institut für Theoretische Physik, Universität Hannover, Appelstraße 2, D-30167 Hannover, Germany
$^*$ Laboratory of Mathematical Physics, Tomsk Polytechnic University, 30 Lenin Ave, Tomsk 634050, Russia

Abstract

We study the quantum properties of two theories with a non-anticommutative (or nilpotent) chiral singlet deformation of $\mathcal{N} = (1, 1)$ supersymmetry: the abelian model of a vector gauge multiplet and the model of a gauge multiplet interacting with a neutral hypermultiplet. In spite of the presence of a negative-mass-dimension coupling constant (deformation parameter), both theories are shown to be finite in the sense that the full effective action is one-loop exact and contains finitely many divergent terms, which vanish on-shell. The $\beta$-function for the coupling constant is equal to zero. The divergencies can all be removed off shell by a redefinition of one of the two scalar fields of the gauge multiplet. These notable quantum properties are tightly related to the existence of a Seiberg-Witten-type transformation in both models.
1 Introduction and summary

Recently, there was a string-theory motivated surge of interest in non-anticommutative deformations of Euclidean superspaces and the corresponding deformed supersymmetric field theories [2] – [21]. The non-anticommutative (or “nilpotently deformed”) field theories, introduced and studied in [1, 2], provide an effective description of the low-energy dynamics of D-branes in superstring theory in the presence of constant graviphoton flux. Their possible physical applications, as established in [1], are related, e.g., to modifications of the glueball superpotential and the expectation value for the glueball field in $\mathcal{N} = 1$ supersymmetric field theories. The non-anticommutativity is also of interest from the phenomenological point of view, since it provides a new mechanism for the explicit partial breaking of supersymmetry. When such a deformation is turned on, only a part (generically, half) of the original supersymmetry remains realized in a standard way and can still be regarded as a symmetry of the deformed field-theoretical model. The study of field theories in non-anticommutative $\mathcal{N} = (1/2, 1/2)$ superspace, initiated by ref. [2], was soon extended to the case of non-anticommutative $\mathcal{N} = (1, 1)$ superspace [3, 4].

Non-anticommutative field theories require new geometrical structures for their concise formulation, which is one more source of interest in such theories. The non-anticommutativity brings additional parameters which lead to the deformation of the anticommutation relations for the Grassmann coordinates of superspace, e.g. $\{\theta^\alpha, \theta^\beta\}_\star = C^{\alpha\beta}$ in $\mathcal{N} = (1/2, 1/2)$ superspace [2] or $\{\theta^\alpha_i, \theta^\beta_j\}_\star = C^{\alpha\beta}_{ij}$ for $\mathcal{N} = (1, 1)$ superspace [3], with $C^{\alpha\beta}$ and $C^{\alpha\beta}_{ij}$ being constant matrices. The deformed models are described in superspace by introducing the $\star$-multiplication of superfields which is associative but non(anti)commutative [5]. This means that the deformed models are obtained from the conventional supersymmetric ones just by replacing the usual multiplication of superfields by the $\star$-product. In such a way, the Wess-Zumino and super Yang-Mills (SYM) models in non-anticommutative $\mathcal{N} = (1/2, 1/2)$ Euclidean superspace were constructed in [2]. The non-anticommutative versions of the $\mathcal{N} = (1, 1)$ vector multiplet and hypermultiplet in the $\mathcal{N} = (1, 1)$ harmonic superspace approach were pioneered in [3, 4]. A more detailed treatment of these $\mathcal{N} = (1, 1)$ models, including the analysis of the component structure of the actions respecting unbroken $\mathcal{N} = (1, 0)$ supersymmetry, was undertaken in [6] – [11].

The non-anticommutative field theories reveal surprising quantum properties. An amazing feature is that, for all currently studied examples, the renormalizability properties of deformed theories are not spoiled as compared to the case without deformation. More specifically, non-anticommutative $\mathcal{N} = (1/2, 0)$ supersymmetric Wess-Zumino and Yang-Mills models were proved to be renormalizable [12] – [13]. From the field-theoretical point of view this property looks rather mysterious since such models contain the parameters of non-anticommutativity $C^{\alpha\beta}$ with mass dimension $-1$ and, by naive power-counting arguments, should be divergent at any order of perturbation theory. However, a key feature of non-anticommutativity is that all such models are consistent only in Euclidean space where the reality properties radically differ from those in Minkowski space. An important manifestation of this difference in the quantum computations is that the new
vertices appearing with the parameters $C^{\alpha\beta}$ are not accompanied by their conjugates and, for this reason, only finitely many divergent Feynman graphs with new divergencies appear. For example, for the non-anticommutative Wess-Zumino model it was shown that only a single new divergent term should be added to the classical action, and the model where such an extra term is added from the very beginning is renormalizable in the usual sense [12, 13, 14, 15]. The non-anticommutative super Yang-Mills model was also proved to be renormalizable using component field formulations [13, 16, 17, 18] and superspace techniques [19]. Note that, in $\mathcal{N} = (1/2, 0)$ gauge theories, quantum computations in components also produce new field structures which are not present in the classical action but, in contrast to the Wess-Zumino model, can be removed from the effective action by a simple shift of the gaugino field [18]. Thus, $\mathcal{N} = (1/2, 0)$ non-anticommutative theories provide interesting examples of renormalizable field theories with dimensionful coupling constants, i.e. the deformation parameters.

The study of theories with non-anticommutative deformations of $\mathcal{N} = (1, 1)$ supersymmetry is more involved. In particular, there are different types of chiral deformation of $\mathcal{N} = (1, 1)$ superspace related to different choices of constants $C^{\alpha\beta}_{ij} = \{\theta^\alpha_i, \theta^\beta_j\}$. The simplest case $C^{\alpha\beta}_{ij} = 2\varepsilon^\alpha\varepsilon_{ij}$ is called the singlet deformation [3, 4]. The $\mathcal{N} = (1, 1)$ classical models with singlet non-anticommutative deformation have been constructed in [6, 7, 8]. Note that the singlet non-anticommutative deformations of $\mathcal{N} = (1, 1)$ superspace also emerge from string theory considered on the axionic background [6]. The case of non-singlet deformations was considered in [10, 11], where the structure of the classical action of the super Yang-Mills model was studied in some detail. In this paper we will analyze only models with singlet deformation, which can be described using the following $\mathcal{N} = (1, 1)$ superfield $\star$-product [3],

$$A \star B = A \exp \left( -I \varepsilon^{\alpha\beta} \varepsilon_{ij} \overleftarrow{Q}_\alpha^i \overrightarrow{Q}_\beta^j \right) B, \quad (1.1)$$

where $Q^i_\alpha$ are the $\mathcal{N} = (1, 1)$ supercharges and $I$ is the parameter of singlet non-anticommutativity. The multiplication (1.1) breaks $\mathcal{N} = (1, 1)$ supersymmetry by half, i.e. down to $\mathcal{N} = (1, 0)$. Nevertheless, this deformation preserves chirality and Grassmann harmonic analyticity [3] and therefore suits well for the use of $\mathcal{N} = 2$ harmonic superspace techniques. The classical superfield actions for non-anticommutative $\mathcal{N} = (1, 1)$ models of the vector multiplet and the hypermultiplet were constructed in harmonic superspace in [3]. The component structure of these actions was studied extensively [6, 7, 8]. However, the quantum aspects of such models have never been studied. In particular, the problem of the renormalizability of these models and the problem of constructing the effective actions have not been addressed so far. The study of these issues is of substantial interest since the renormalizability of deformed $\mathcal{N} = (1, 1)$ models is not evident and we can expect that the effective action in the case under consideration will possess a number of very nontrivial properties.

By this paper we begin a systematic study of quantum aspects of the non-anticommutative models with deformed $\mathcal{N} = (1, 1)$ supersymmetry. Here we answer affirmatively the question of renormalizability for non-anticommutative models of the abelian $\mathcal{N} = (1, 1)$
vector multiplet \(^1\), with and without adding a neutral hypermultiplet, in the case of a singlet deformation of the form \(\mathcal{N} = (1/2, 0)\). The renormalizability of these models can be formally established by applying the Wilsonian renormalization group arguments, developed previously for \(\mathcal{N} = (1/2, 0)\) supersymmetric models in [15]. To find the structure of divergent terms in the effective action we perform the one-loop quantum computations and observe several specific properties of the two models. First, the divergent terms appear only in the vector loop in the SYM model or in the scalar loop in the hypermultiplet model. The external lines must carry only the scalar \(\bar{\phi}\) belonging to the vector multiplet. The fermionic contributions are trivial. Second, all divergent contributions combine in two gauge invariant and \(\mathcal{N} = (1, 0)\) supersymmetric expressions depending on the single scalar field \(\bar{\phi}\) only and vanishing at the classical equations of motion. These two divergent terms represent new interactions which were not present in the classical actions of the considered models. However, they do not spoil the renormalizability since they can be completely absorbed into a redefinition of the second scalar field \(\phi\). This situation is very similar to the \(\mathcal{N} = (1/2, 0)\) gauge model studied in [18], where it was shown that the non-anticommutative interaction also generates new terms in the effective action which can be eliminated from the theory by a shift of the gaugino field. Third, we demonstrate that the appropriate change of fields in the classical actions (a Seiberg-Witten-like map)\(^6\) allows one to completely avoid any divergence in the effective action. This fact emphasizes that the divergencies are unphysical from the standard point of view.

Despite the disadvantage that the considered theories have to be regarded as a sort of toy models, they are relevant for the quantum structure of non-abelian non-anticommutative SYM theory, since they appear as a U(1) sector in non-anticommutative U(\(N\)) gauge theories. Hence, the proof of renormalizability of the abelian vector multiplet and hypermultiplet models is a prerequisite for the analogous study of the deformed U(\(N\)) \(\mathcal{N} = (1, 1)\) gauge theories. Moreover, the results obtained support the hypotheses that the non-anticommutative deformations of supersymmetry cannot spoil the renormalizability of supersymmetric theories and provide a way of construction of renormalizable supersymmetric models with partially broken supersymmetry.

The paper is organized as follows. In Section 2 we compute the one-loop effective action in the non-anticommutative \(\mathcal{N} = (1, 1)\) abelian gauge theory formulated in terms of component fields and prove the renormalizability of this model. In Section 3 we prove, in the same way as for the pure gauge theory, the renormalizability of the non-anticommutative deformation of a coupled system of neutral hypermultiplet and abelian gauge multiplet. In Section 4 we study the interplay between the Seiberg-Witten map and the problem of renormalizability of the considered models. Section 5 contains the conclusions and some discussion of the results obtained. In the Appendix we collect the formulas for the regularization of divergent momentum integrals which are met in the calculations of the effective action, and we also list the corresponding Feynman diagrams.

We follow the conventions and notation of refs. \(^6\) [6, 7].

\(^1\)Although the model under consideration has U(1) gauge symmetry, it is interacting due to non-anticommutativity.
2 Renormalizability of non-anticommutative abelian $\mathcal{N} = (1, 0)$ supergauge theory

The classical superfield action of $\mathcal{N} = (1, 1)$ non-anticommutative U(1) gauge model was given in [3, 4, 6]

$$S_{SYM} = \frac{1}{4} \int d^8 z_c d\bar{\theta} \mathcal{W}^2. \quad (2.1)$$

Here $\mathcal{W}$ is the $\mathcal{N} = (1, 1)$ superfield strength which is covariantly chiral, $\nabla^i_{\alpha} \ast \mathcal{W} = (\bar{D}^i_{\alpha} + \bar{A}^i_{\alpha}) \mathcal{W} = 0$, $d^8 z_c = d^4 x_c d^4 \theta$ is the integration measure of the chiral superspace and $d\bar{\theta}$ stands for the integration over the harmonic variables. The action (2.1) is invariant under the deformed U(1) gauge transformations

$$\delta \mathcal{W} = [\mathcal{W}, \Lambda]_\ast = \mathcal{W} \ast \Lambda - \Lambda \ast \mathcal{W}, \quad (2.2)$$

where $\Lambda$ is an arbitrary analytic superfield and the $\ast$-product was defined in (1.1).

The covariantly chiral superfield $\mathcal{W}$ can be decomposed into usual chiral superfields

$$\mathcal{W} = \mathcal{A} + \bar{\theta}^+ \tau - \bar{\theta}^- \tau, \quad (2.3)$$

with $\bar{D}^i\mathcal{A} = \bar{D}^i\tau - \bar{D}^i\tau = 0$. It is easy to demonstrate that only the superfield $\mathcal{A}$ contributes to the action (2.1)

$$S_{SYM} = \frac{1}{4} \int d^8 z_c d\bar{\theta} \mathcal{A}^2. \quad (2.4)$$

The component structure of $\mathcal{A}$ was found in [6]

$$\mathcal{A}(z_c, u) = [\phi + \frac{4IA_mA_m}{1 + 4I\phi} + \frac{16I^3(\partial_m\bar{\phi})^2}{1 + 4I\phi}]$$

$$+ 2\theta^+ [\Psi + \frac{4I(\sigma\bar{\Psi})A_m}{1 + 4I\phi}] - \frac{2\theta^-}{1 + 4I\phi} [\Psi^+ + \frac{4I(\sigma\bar{\Psi})A_m}{1 + 4I\phi}]$$

$$+(\theta^+)^2 \frac{8I(\Psi^-)^2}{1 + 4I\phi} + D^- + \frac{(\theta^-)^2}{1 + 4I\phi} + D^+]$$

$$- \frac{2(\theta^+\theta^-)}{1 + 4I\phi} \frac{8I(\Psi^+\bar{\Psi})}{1 + 4I\phi} + D^+] + (\theta^+\sigma_m\theta^-)(F_{mn} - \frac{8I\partial_m\bar{\phi}A_n}{1 + 4I\phi})$$

$$+ 2i(\theta^+\theta^-)^2 \frac{\Psi^+}{1 + 4I\phi} + 2i(\theta^+)^2 (1 + 4I\phi)\theta^- \sigma_m \partial_m \bar{\Psi}$$

$$- (\theta^+)^2 (\theta^-)^2 \Box \phi. \quad (2.5)$$

Here $D^+ = D^{kl}u^+_ku^+_l$, $D^- = D^{kl}u^-_ku^-_l$, $\phi$, $\bar{\phi}$ are the scalar fields, $\Psi^\pm = \Psi^\pm_i u^+_i$, $\bar{\Psi}^\pm = \bar{\Psi}^\pm_\alpha u^+_\alpha$ are the spinors, $A_m$ is the vector field, $D^{kl}$ is the auxiliary field. Substituting the
expression (2.5) into the action (2.4) we find
\begin{equation}
S_{\text{SYM}} = S_{\phi} + S_{\Psi} + S_{A},
\end{equation}
\begin{equation}
S_{\phi} = -\frac{1}{2} \int d^4x \Box \phi - \frac{4IAmA_{m}}{1 + 4I\phi} + \frac{16I^2\partial_{m}\phi\partial_{m}\phi}{1 + 4I\phi},
\end{equation}
\begin{equation}
S_{\Psi} = i \int d^4x \left( \phi^{ia} + \frac{4IAm\sigma_{m}}{1 + 4I\phi} \right) (\sigma_{\alpha})_{\alpha\beta} \partial_{\beta} \left( \frac{\tilde{\Psi}_{i}^{\beta}}{1 + 4I\phi} \right)
+ \frac{1}{4} \int d^4x \frac{1}{(1 + 4I\phi)^2} \left( 8I\Psi_{\alpha} \tilde{\Psi}_{\beta}^{j} + D^{ij} \right) \left( \frac{8I\tilde{\Psi}_{\alpha} \Psi_{j} + D_{ij}}{1 + 4I\phi} \right),
\end{equation}
\begin{equation}
S_{A} = \int d^4x \left[ -\frac{1}{2} A_{m} \Box A_{n} - \frac{1}{2} \partial_{m} A_{m} \partial_{n} A_{n} + \frac{1}{2} A_{m} A_{n} \Box \ln(1 + 4I\phi)
- \varepsilon_{mnr} \partial_{r} A_{s} A_{m} \partial_{n} \ln(1 + 4I\phi) + \frac{1}{2} A_{m} A_{n} \partial_{m} \ln(1 + 4I\phi) \partial_{n} \ln(1 + 4I\phi)
- \frac{1}{2} A_{m} A_{n} \partial_{m} \ln(1 + 4I\phi) \partial_{n} \ln(1 + 4I\phi) + \partial_{m} A_{m} A_{n} \partial_{m} \ln(1 + 4I\phi) \right].
\end{equation}

At this step it is convenient to eliminate the auxiliary field $D^{ij}$ using its classical equation of motion $D^{ij} = -8I\tilde{\Psi}_{\alpha} \Psi_{j}^{i} / (1 + 4I\phi)$, then the terms in the second line of (2.8) disappear and the action $S_{\Psi}$ simplifies to
\begin{equation}
S_{\Psi} = i \int d^4x \left( \phi^{ia} + \frac{4IAm\sigma_{m}^{a}}{1 + 4I\phi} \right) (\sigma_{\alpha})_{\alpha\beta} \partial_{\beta} \left( \frac{\tilde{\Psi}_{i}^{\beta}}{1 + 4I\phi} \right).
\end{equation}

Note that model (2.6) is formulated in the Euclidean rather than Minkowski space. This means that the fields $\phi, \tilde{\phi}$ and $\Psi_{i}, \tilde{\Psi}_{i}$ are not conjugated to each other.

Before continuing with the quantization of the model (2.6) we would like to adduce here some intuitive arguments in favour of the renormalizability of this theory. Following the work [15], we will use the Wilsonian approach [22] which ensures the renormalizability of lagrangians containing a finite number of local operators with scaling dimensions not greater than four. The lagrangian (2.6) contains a finite number of local operators (interaction terms), but the dimensions of some of these operators are greater than four. For example, the interaction terms
\begin{equation}
\Box \phi A_{m} A_{m} \quad \text{and} \quad \Box \phi \partial_{m} \phi \partial_{m} \phi
\end{equation}
in (2.7) have mass dimensions 5 and 7, respectively. Therefore, the arguments of Wilsonian approach cannot be directly applied to the action (2.6). However, in [15] it was proposed to ascribe non-standard scaling dimensions to the fields involved. Following this way, let us suppose that the dimensions of the coordinates of $N = 2$ superspace are
\begin{equation}
[\theta_{i}^{\alpha}] = 0, \quad [\tilde{\theta}_{i}^{\alpha}] = -1, \quad [x^{m}] = -1.
\end{equation}
Then it is easy to see that the dimensions of physical component fields should be as follows,

\[
[\phi] = 2, \quad [\bar{\phi}] = 0, \quad [A_m] = 1, \quad [\Psi^{i\alpha}] = 0, \quad [\bar{\Psi}^{i\dot{\alpha}}] = 1
\]  

(2.13)

and the parameter of non-anticommutativity is dimensionless, \([I] = 0\). When the asymmetrical scaling dimensions \((2.13)\) are assumed, the operators \((2.11)\), as well as other interaction terms in \((2.6)\), acquire the desired mass dimension 4. Therefore, such a model has to be renormalizable on formal grounds by applying the Wilsonian argument. Note that one can choose the asymmetrical scaling dimensions \((2.13)\) just because the fields \(\phi\) and \(\bar{\phi}\) as well as \(\Psi^{i\alpha}\) and \(\bar{\Psi}^{i\dot{\alpha}}\) are not related to each other by conjugation. The above argument based on the relations \((2.13)\) is useful only for establishing the very fact of renormalizability and does not provide any specific rules for performing the renormalization and/or clarifying its details. Having established the renormalizability of the model in principle, we henceforth adopt the standard canonical dimensions for all fields.

Now we are going to consider the quantization of the model \((2.6)\) in order to study the structure of divergent terms and the details of renormalization procedure. The action \((2.6)\) is invariant under the following residual gauge transformations

\[
\delta \phi = -8IA_m \partial_m \lambda, \quad \delta \bar{\phi} = 0, \\
\delta \Psi^k_\alpha = -4I(\sigma_m \bar{\Psi}^k)_{\alpha} \partial_m \lambda, \quad \delta \bar{\Psi}^k_{\dot{\alpha}} = 0, \\
\delta A_m = (1 + 4I\bar{\phi}) \partial_m \lambda, \quad \delta D_{kl} = 0, 
\]  

(2.14)

with \(\lambda\) being the gauge parameter. Note that the gauge field \(A_m\) has a non-standard transformation law. However, after the redefinition \(A_m \rightarrow a_m = A_m / (1 + 4I\bar{\phi})\) the new field \(a_m\) has the standard gauge transformation law \(\delta a_m = \partial_m \lambda\). Therefore, the standard Lorentz gauge fixing condition reads \(\partial_m a_m = 0\), or

\[
\partial_m \frac{A_m}{1 + 4I\phi} = 0. 
\]  

(2.15)

Further we follow the routine Faddeev-Popov procedure to fix the gauge freedom in the functional integral. Let us introduce the corresponding gauge-fixing function

\[
\chi = \partial_m \frac{A_m}{1 + 4I\phi} = \frac{\partial_m A_m - A_m G_m}{1 + 4I\phi}, 
\]  

(2.16)

where

\[
G_m(x) = \partial_m \ln[1 + 4I\bar{\phi}(x)]. 
\]  

(2.17)

The function \((2.16)\) transforms under the gauge transformations \((2.14)\) as

\[
\delta \chi = \partial_m \frac{\delta A_m}{1 + 4I\phi} = \Box \lambda. 
\]  

(2.18)

Therefore the action for the ghost fields is just the action of free scalars

\[
S_{FP} = \int d^4x \, b\Box c. 
\]  

(2.19)
The generating functional for the Green’s functions is now defined as

\[ Z[J] = \int \mathcal{D}(\phi, \bar{\phi}, \Psi, \bar{\Psi}, A_m, b, c) \delta(\chi - \frac{\partial_m A_m - A_m G_m}{1 + 4I\bar{\phi}}) e^{-\frac{1}{2}(S_{SYM} + S_{FP} + S_J)}, \tag{2.20} \]

where

\[ S_J = \int d^4x [\frac{\phi J_{\phi} + \bar{\phi} J_{\bar{\phi}} + \Psi_i^{\alpha}(J_{\Psi})_{i}^{\alpha} + \bar{\Psi}_{i\dot{\alpha}}(J_{\bar{\Psi}})_{i\dot{\alpha}} + A_m (J_A)_m] \tag{2.21} \]

and \( J_{\phi}, J_{\bar{\phi}}, (J_{\Psi})_{i}^{\alpha}, (J_{\bar{\Psi}})_{i\dot{\alpha}}, (J_A)_m \) being sources associated with the fields \( \phi, \bar{\phi}, \Psi_i^{\alpha}, \bar{\Psi}_{i\dot{\alpha}}, A_m \). We have inserted into (2.20) the functional delta-function which fixes the gauge degrees of freedom in the functional integral over the gauge fields. This delta-function can be easily written in the gaussian form by averaging (2.20) with the factor

\[ 1 = \int \mathcal{D}\chi e^{-\frac{\alpha}{2} \int d^4x (1 + 4I\bar{\phi})^2} = \text{Det}^{-1/2}[\delta(\chi(x) - \chi(x'))(1 + 4I\bar{\phi})^2]. \tag{2.22} \]

The functional integral (2.22) generates the following gauge-fixing action

\[ S_{gf} = \frac{\alpha}{2} \int d^4x (\partial_m A_m - A_m G_m)^2 \]

\[ = \frac{\alpha}{2} \int d^4x [(\partial_m A_m)^2 - 2\partial_m A_m A_n G_m + A_m A_n G_m G_n]. \tag{2.23} \]

Here \( \alpha \) is an arbitrary parameter. For simplicity, in sequel we set \( \alpha = 1 \). As a result, the generating functional (2.20) can be represented in the following form

\[ Z[J] = \int \mathcal{D}(\phi, \bar{\phi}, \Psi, \bar{\Psi}, A_m, b, c) e^{-\frac{1}{2}(S_{tot} + S_{FP} + S_J)}, \tag{2.24} \]

where

\[ S_{tot} = S_{SYM} + S_{gf} \]

\[ = -\frac{1}{2} \int d^4x \Box(\phi + 4I^2 \partial_n \bar{\phi} G_m) \]

\[ + i \int d^4x \left( \Psi_{i\alpha} + \frac{4IA_m \sigma_m^{\alpha \dot{\alpha}} \bar{\Psi}^{i\dot{\alpha}}}{1 + 4I\phi} \right) (\sigma_n)_{\alpha\beta} \partial_n \left( \frac{\bar{\Psi}^{i\dot{\alpha}}}{1 + 4I\phi} \right) \]

\[ - \int d^4x \left[ \frac{1}{2} A_n \Box A_n - A_n G_m \partial_n A_m + A_n G_n \partial_m A_m + \varepsilon_{mnr} G_m A_n \partial_r A_s \right]. \tag{2.25} \]

The functional integral (2.24) with the action (2.25) requires several comments.

1. The ghost fields \( b, c \) enter the action only through their kinetic term. Hence, they fully decouple and can be integrated out.

\[ ^2 \text{Note that the functional integral in the Euclidean space is defined as } \int \mathcal{D}\Phi e^{-\frac{i}{2}S[\Phi]} \text{ as compared with the Minkowski space definition } \int \mathcal{D}\Phi e^{+\frac{i}{2}S[\Phi]}. \]
2. The action (2.25) defines the propagators and vertices, all what we need for performing quantum computations in the model. Upon a careful examination of the Feynman rules, one can prove the following statements (see Appendix A.2):

i. The one-loop effective action in the model is exact since it is impossible to construct any higher loop diagrams;

ii. The fermionic fields $\Psi_i^\alpha$, $\bar{\Psi}_i^{\dot{\alpha}}$ do not produce any quantum corrections to the effective action (excepting for tree diagrams);

iii. Only the field $\bar{\phi}$ can appear at the external legs;

iv. The only contribution to the effective action comes from the vector loops with arbitrary numbers of external $\bar{\phi}$ legs.

3. Note that the field $\bar{\phi}$ enters the action (2.25) only in the dimensionless combination $(I\bar{\phi})$. Then, by the dimensionality reasoning, the most general form of the effective action depending on $(I\bar{\phi})$ should be of the following form

$$\Gamma = \int d^4x [f_1(I\bar{\phi})I^2\Box\bar{\phi}\Box\bar{\phi} + f_2(I\bar{\phi})I^3\Box\bar{\phi}\partial_m\bar{\phi}\partial_m\bar{\phi} + f_3(I\bar{\phi})I^4(\partial_m\bar{\phi}\partial_m\bar{\phi})^2],$$

(2.26)

where $f_1$, $f_2$, $f_3$ are some functions. The Feynman graph computations should specify the unknown functions in (2.26).

The property iv implies that the effective action can be written as

$$\Gamma^{SYM} = \frac{1}{2}\text{Tr} \ln \frac{\delta^2 \tilde{S}}{\delta A_p(x)\delta A_q(x')} ,$$

(2.27)

where the action $\tilde{S}$ is the last line in (2.25)

$$\tilde{S} = \int d^4x \left[ -\frac{1}{2}A_n\Box A_n + A_n G_m\partial_n A_m - A_n G_n\partial_m A_m - \epsilon_{mnrs}G_mA_n\partial_r A_s \right].$$

(2.28)

The second functional derivative in (2.27) can be easily calculated

$$\frac{\delta^2 \tilde{S}}{\delta A_p(x)\delta A_q(x')} = -\delta_{pq} \delta^4(x-x') + 4G_{[q}\partial_{p]} \delta^4(x-x') + 2\epsilon_{pqmn}G^m\partial^n \delta^4(x-x').$$

(2.29)

Substituting the expression (2.29) into (2.27) we have

$$\Gamma^{SYM} = \frac{1}{2}\text{Tr} \ln \left[ \delta^4(x-x') + 4G_{[p}\partial_{q]} \frac{1}{\Box} \delta^4(x-x') - 2\epsilon_{pqmn}G_m\partial_n \delta^4(x-x') \right]$$

$$= \frac{1}{2}\text{Tr} \sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j} \left[ 4G_{[p}\partial_{q]} \frac{1}{\Box} \delta^4(x-x') - 2\epsilon_{pqmn}G_m\partial_n \delta^4(x-x') \right]^j .$$

(2.30)

Note that the one-loop effective action in the Euclidean space is given by $\Gamma = \frac{1}{2}\text{Tr} \ln S''[\Phi]$ rather than the Minkowski space expression $\Gamma = \frac{i}{2}\text{Tr} \ln S''[\Phi]$. Here $S''[\Phi]$ is the second functional derivative of the classical action.
The sum in (2.30) is taken over the external legs $G_m$, where the field $G_m$ depends on $\bar{\phi}$ according to the definition (2.17). Note that the expression (2.30) can be equivalently rewritten in the form

$$\Gamma^{SYM} = \frac{1}{2} \text{Tr} \sum_{j=1}^{\infty} \frac{(-2)^{j+1}}{j} [X_{mnp}(x)\partial_{\mu} \frac{1}{\Delta}(x-x')]^j ,$$  \hspace{1cm} (2.31)$$

where we have introduced the superfield

$$X_{mnp} = G_m \delta_{np} - G_p \delta_{mn} - G_q \varepsilon_{qpmn} .$$  \hspace{1cm} (2.32)$$
The representation of the action (2.31) in terms of Feynman diagrams is given in the Appendix A.2 (Fig. 1a).

The propagators in (2.30) appear in the combination $\partial_m \Box^{-1} \delta^4(x-x')$. On the dimensionality grounds, only the expressions like

$$\left[\frac{\partial_m}{\Box} \delta^4(x-x')\right]^2, \hspace{1cm} \left[\frac{\partial_m}{\Box} \delta^4(x-x')\right]^3, \hspace{1cm} \left[\frac{\partial_m}{\Box} \delta^4(x-x')\right]^4$$

are divergent, all higher powers of these expressions produce finite contributions to the effective action. Therefore, only two-, three- and four-point diagrams make the divergent contributions to the effective action (note that the external line is that of the field $G_m$). We are interested solely in the divergent contributions to the effective action, so we consider the calculations of two-, three- and four-point functions separately.

According to eq. (2.30), the two-point function is defined by

$$\Gamma_2^{SYM} = - \int d^4x_1 d^4x_2 [2G_m(x_1)\partial_{\mu} \frac{1}{\Delta}(x_1-x_2) + \varepsilon_{pqmn} G_m(x_1)\partial_{\mu} \frac{1}{\Delta}(x_1-x_2)]$$

$$\times [2G_m(x_2)\partial_{\nu} \frac{1}{\Delta}(x_1-x_2) + \varepsilon_{pqrs} G_m(x_2)\partial_{\nu} \frac{1}{\Delta}(x_1-x_2)].$$  \hspace{1cm} (2.34)$$

Unfolding the product of square brackets in (2.34) and passing to the momentum space

$$G_m(x) = \int \frac{d^4p}{(2\pi)^4} e^{ip x_n} \tilde{G}_m(p) , \hspace{1cm} \delta^4(x-x') = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')_n} ,$$  \hspace{1cm} (2.35)$$
we find

$$\Gamma_2^{SYM} = -2 \int \frac{d^4p}{(2\pi)^4} \tilde{G}_m(p) \tilde{G}_m(-p) [2\delta_{ms} \delta_{nr} - 2\delta_{mr} \delta_{ns}] \int d^4k \frac{k_n(p+k)_s}{k^2(p+k)^2}.$$  \hspace{1cm} (2.36)$$
The divergent momentum integrals are calculated in the Appendix A.1 (eqs. (A.10), (A.11)). Further we shall consider only the divergent part of the effective action (2.36) \footnote{In the dimensional regularization scheme all divergencies of the effective action appear with the gamma-function factor $\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma + O(\varepsilon)$. Here $\varepsilon = 2 - d/2$, $d$ is the dimension of space-time, $\gamma$ is the Euler constant. Therefore, all divergent contributions to the effective action enter with the factor $1/\varepsilon$, $\varepsilon \to 0$.}

$$\Gamma_2^{SYM}_{\text{div}} = \frac{2\pi^2}{\varepsilon} \int \frac{d^4p}{(2\pi)^8} [\tilde{G}_m(p)] \frac{1}{3} (p_m p_n + \frac{p^2 \delta_{mn}}{2}) \tilde{G}_n(-p) - \tilde{G}_m(p) p^2 \tilde{G}_m(-p)].$$  \hspace{1cm} (2.37)$$
Switching back to the coordinate space and applying the relations (2.17) we obtain:

\[
\Gamma_{2,\text{div}}^{\text{SYM}} = \frac{1}{16\pi^2\varepsilon} \int d^4x \ln(1 + 4I\phi) \Box^2 \ln(1 + 4I\phi) .
\] (2.38)

Consider now the computation of the divergent part of the three-point function

\[
\Gamma_{3}^{\text{SYM}} = \frac{4}{3} \text{Tr}[(2G_{[r}(x)\partial_s] - \varepsilon_{rsmn}G_m(x)\partial_n) \frac{1}{4}\delta^4(x - x')]^3
\]

\[
= \frac{4}{3} \int d^4x_1d^4x_2d^4x_3[(G_1(x_1)\partial_n - G_u(x_1)\partial_l - \varepsilon_{tmn}G_m(x_1)\partial_n) \frac{1}{4}\delta^4(x_1 - x_2)]
\times[(G_u(x_2)\partial_w - G_w(x_2)\partial_u - \varepsilon_{wur}G_r(x_2)\partial_s) \frac{1}{4}\delta^4(x_2 - x_3)]
\times[(G_w(x_3)\partial_l - G_l(x_3)\partial_w - \varepsilon_{wpq}G_p(x_3)\partial_q) \frac{1}{4}\delta^4(x_3 - x_1)].
\] (2.39)

As in the previous case, we do the products of all brackets, pass to the momentum space and regularize the divergent integrals according to eq. (A.12). As a result, we arrive at the following expression for the divergent part of the three-point Green function

\[
\Gamma_{3,\text{div}}^{\text{SYM}} = -\frac{1}{4\pi^2\varepsilon} \int d^4xG_mG_m\partial_nG_n
\]

\[
= -\frac{1}{4\pi^2\varepsilon} \int d^4x\partial_m \ln(1 + 4I\phi)\partial_m \ln(1 + 4I\phi) \Box \ln(1 + 4I\phi) .
\] (2.40)

Here we made use of the definition (2.17).

The same machinery can be applied for computing the four-point Green function

\[
\Gamma_{4}^{\text{SYM}} = -2\text{Tr}[(2G_{[r}(x)\partial_s] - \varepsilon_{rsmn}G_m(x)\partial_n) \frac{1}{4}\delta^4(x - x')]^4
\]

\[
= -2 \int d^4x_1d^4x_2d^4x_3d^4x_4[(G_p\partial_q - G_q\partial_p - \varepsilon_{pqrs}G_r\partial_s) \frac{1}{4}\delta^4(x_1 - x_2)]
\times[(G_q\partial_m - G_m\partial_q - \varepsilon_{mnqs}G_s\partial_n) \frac{1}{4}\delta^4(x_2 - x_3)]
\times[(G_m\partial_n - G_n\partial_m - \varepsilon_{nms}G_s\partial_n) \frac{1}{4}\delta^4(x_3 - x_4)]
\times[(G_n\partial_p - G_p\partial_n - \varepsilon_{tpun}G_t\partial_u) \frac{1}{4}\delta^4(x_4 - x_1)].
\] (2.41)

The divergent part of the action (2.41) is given by (after regularization of momentum integrals in accord with (A.13) and careful counting of the coefficients)

\[
\Gamma_{4,\text{div}}^{\text{SYM}} = -\frac{5}{16\pi^2\varepsilon} \int d^4x(G_mG_m)^2 = -\frac{5}{16\pi^2\varepsilon} \int d^4x[\partial_m \ln(1 + 4I\phi)\partial_m \ln(1 + 4I\phi)]^2 .
\] (2.42)

Finally, we should put together the divergent contributions from two-, three- and four-point functions given by (2.38), (2.40) and (2.42). The result is the total one-loop
divergent contribution to the effective action in the deformed $\mathcal{N} = (1, 1)$ SYM model

$$\Gamma_{\text{div}}^{\text{SYM}} = \frac{1}{16\pi^2\varepsilon} \int d^4x \ln(1 + 4I\bar{\phi})\Box^2 \ln(1 + 4I\bar{\phi})$$

$$- \frac{1}{4\pi^2\varepsilon} \int d^4x \partial_m \ln(1 + 4I\bar{\phi})\partial_m \ln(1 + 4I\bar{\phi})$$

$$- \frac{5}{16\pi^2\varepsilon} \int d^4x [\partial_m \ln(1 + 4I\bar{\phi})\partial_m \ln(1 + 4I\bar{\phi})]^2. \quad (2.43)$$

The expression (2.43), modulo a total derivative under the integral, can be equivalently rewritten as

$$\Gamma_{\text{div}}^{\text{SYM}} = \frac{1}{\pi^2\varepsilon} \int d^4x \frac{I^2\Box\bar{\phi}\Box\bar{\phi}}{(1 + 4I\phi)^2} - \frac{6}{\pi^2\varepsilon} \int d^4x \frac{4I^3\Box\bar{\phi}\partial_m \bar{\phi}\partial_m \bar{\phi}}{(1 + 4I\phi)^3}. \quad (2.44)$$

The action (2.44) is the complete divergent part of the effective action in the deformed abelian $\mathcal{N} = (1, 1)$ gauge model. It matches with the previously guessed structure (2.26).

At first sight, the model looks non-renormalizable, since the quantum computations produce the terms (2.44) which are absent in the classical action (2.6). Therefore, in order to make the model renormalizable we are led to extend the classical action (2.6) by the two extra terms

$$c_1 \int d^4x \frac{I^2\Box\bar{\phi}\Box\bar{\phi}}{(1 + 4I\phi)^2} + c_2 \int d^4x \frac{4I^3\Box\bar{\phi}\partial_m \bar{\phi}\partial_m \bar{\phi}}{(1 + 4I\phi)^3} \quad (2.45)$$

with some coupling constants $c_1, c_2$. However, both these terms can be removed by shifting the scalar field in the classical action

$$\phi \rightarrow \phi - 2c_1 \frac{I^2\Box\bar{\phi}}{(1 + 4I\phi)^2} - 2c_2 \frac{4I^3\partial_m \bar{\phi}\partial_m \bar{\phi}}{(1 + 4I\phi)^3}, \quad \text{while} \quad \bar{\phi} \rightarrow \bar{\phi}. \quad (2.46)$$

Therefore, the $\mathcal{N} = (1, 0)$ gauge model is renormalizable in the sense that all divergencies can be removed by the redefinition of the scalar field $\phi$. Note that the redefinition of fields of the form (2.40) can be made in the functional integral (2.24). Since the Jacobian of such a change of functional variables equals unity, the terms (2.45), being added to the classical action (2.6), do not make new contributions to the effective action. In the language of Feynman diagrams this means that the terms (2.45) generate new vertices for the scalar field. But due to lacking of the propagator $\langle \bar{\phi}\phi \rangle$, no loops with such vertices can be constructed.

This situation is completely analogous to the $\mathcal{N} = (1/2, 0)$ SYM model considered in [18] where it was demonstrated that the quantum computations in this model generate the divergent terms which are not present in the classical action, but these extra divergencies can be removed by a simple shift of the gaugino field (the lowest component in $\mathcal{N} = (1, 1)$ gauge multiplet). In our case the divergencies can also be removed by the shift of lowest component of $\mathcal{N} = (1, 1)$ gauge multiplet (scalar field).

To summarize, we have calculated the full divergent contribution to the effective action in the deformed abelian $\mathcal{N} = (1, 1)$ gauge model. It can be written in the form of two
terms (2.44). Both these terms can be removed by the redefinition of classical field $\phi$ of the form (2.46). Therefore the abelian deformed $\mathcal{N} = (1, 1)$ gauge model with the action (2.6) is renormalizable. It is easy to see that the divergent terms (2.44) vanish on the classical equations of motion, therefore the S-matrix in this model is divergence-free and in this sense one can say that the model under consideration is finite.

3 Renormalizability of non-anticommutative neutral hypermultiplet

In this Section we prove the renormalizability (finiteness) of the non-anticommutative model of a neutral hypermultiplet interacting with an abelian gauge superfield. Firstly we consider the case when the gauge superfield is treated as an external background and then the case of general $\mathcal{N} = (1, 0)$ non-anticommutative model, with both gauge and hypermultiplet superfields on equal footing.

Let us extend the non-anticommutative $U(1)$ gauge model (2.6) by adding the hypermultiplet fields interacting with the vector multiplet. As pointed out in [7], it is possible to consider here the adjoint and fundamental representations of non-anticommutative $U(1)$ group. These theories are called the neutral and charged hypermultiplet models, respectively. We will study further only the neutral hypermultiplet model since it becomes free in the undeformed limit $I \to 0$ similarly to the deformed abelian supersymmetric gauge model considered in Sect. 2. The model of charged hypermultiplet is essentially different since it retains a non-vanishing interaction in the limit $I \to 0$ and the considerations of quantum aspects of this model within the component field formulation is a much more complicated problem. The problem of computing the effective action in the charged hypermultiplet model will be treated elsewhere.

The classical action of the neutral hypermultiplet model in the harmonic superspace [23] is given by [7]

$$S_{hyp} = \int d\zeta du \tilde{q}^{+}(D^{++}q^{+} + V^{++} \ast q^{+} - q^{+} \ast V^{++}).$$

(3.1)

Here $q^{+}, \tilde{q}^{+}$ are hypermultiplet superfields, $V^{++}$ is the vector multiplet field, $D^{++}$ is the harmonic covariant derivative, $d\zeta du$ is the integration measure of the analytic harmonic superspace. For details of the harmonic superspace approach see, e.g., book [24]. The action (3.1) is invariant under the following gauge transformations

$$\delta \tilde{q}^{+} = [\tilde{q}^{+}, \Lambda]_{\ast}, \quad \delta q^{+} = [q^{+}, \Lambda]_{\ast}, \quad \delta V^{++} = D^{++}\Lambda + [V^{++}, \Lambda]_{\ast}.$$  

(3.2)

with gauge parameter $\Lambda$ being analytic superfield. It is obvious from (3.1) that the model under consideration becomes free in the limit $I \to 0$.

The component form of the action (3.1) (with the auxiliary fields eliminated) was given in [7]:

$$S_{hyp} = \int d^4x \left[ \frac{1}{2}(1 + 4I\tilde{\phi})\partial_m f^{ak}\partial_m f_{ak} + \frac{i}{2}(1 + 4I\tilde{\phi})\rho^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\chi_{\dot{\alpha}} \right]$$
Here $f^{ak}$, $\rho^{\alpha a}$, $\chi^{\dot{\alpha} a}$ are physical scalar and spinor fields of the hypermultiplet, $\bar{\phi}$, $\bar{\Psi}^{\dot{\alpha} k}$, $A_m$ are physical scalar, spinor and vector fields of the vector multiplet. The indices $a, k$ running over 1, 2 are doublet indices of two independent internal symmetry SU(2) groups.

Like in the case of the non-anticommutative gauge theory, we can give a formal proof of renormalizability of the model (3.3) by applying the Wilsonian criterion [22]. According to eq. (2.12), we should ascribe the following asymmetrical scaling dimensions to the component fields of the hypermultiplet,

$$[f^{ak}] = 1, \quad [\rho^{\alpha a}] = 1, \quad [\chi^{\dot{\alpha} a}] = 2. \quad (3.4)$$

Using eqs. (2.13), (3.4), it is easy to check that the dimensions of all operators in the action (3.3) are just 4:

$$[(1 + 4I\bar{\phi})^2 \partial_i f^{ak} \partial_m f^{ak}] = 4, \quad [(1 + 4I\bar{\phi})\rho^{\alpha a} \partial_{\alpha a} \chi^{\dot{\alpha} a}] = 4,$$
$$[\bar{\Psi}^{\dot{\alpha} k} \rho^{\alpha a} \partial_{\alpha a} f^{ak}] = 4, \quad [\rho^{\alpha a} A_m \partial_m \rho_{\alpha a}] = 4, \quad [\rho^{\beta a} \rho^{\alpha a} \partial_{\alpha a} A^a_{\dot{\alpha} \beta}] = 4. \quad (3.5)$$

Therefore, the action (3.3) satisfies the conditions of the Wilsonian approach, and the abelian neutral hypermultiplet model is renormalizable. However the details of the renormalization procedure require further analysis.

Now we are going to compute directly the divergent contributions to the effective action of neutral hypermultiplet model. As a prelude, let us comment on the structure of eq. (3.3).

1. We consider the fields of the gauge multiplet ($\bar{\phi}$, $\bar{\Psi}^{\dot{\alpha} k}$, $A_m$) as the external fields which are not quantized. The quantum fields are physical fields of the hypermultiplet. Note that the hypermultiplet fields enter the action (3.3) only quadratically, therefore the effective action in this model is automatically one-loop exact. This is also clear from the form of the superfield action (3.1).

2. As proved in Appendix A.3, the terms in the second line of (3.3) do not make any contribution to the quantum effective action since the corresponding vertices appear without their conjugates, and so the Feynman rules do not allow to compose any loop from such vertices. Therefore for quantum calculations only first two terms in the action (3.3) are really essential, and in what follows we can limit our consideration just to these terms.

3. The first two terms in (3.3) depend only on the background field $\bar{\phi}$. Therefore, the whole effective action is a functional of the form (2.26) containing only $\bar{\phi}$-dependence.

4. It is easy to prove that the term $\frac{i}{2}(1 + 4I\bar{\phi})\rho^{\alpha a} \partial_{\alpha a} \chi^{\dot{\alpha} a}$, which is responsible for the fermionic loop, does not make any non-trivial contribution to the effective action.
Indeed, let us consider a part of the one-loop effective action which is produced by
this fermionic loop

\[ \Gamma_{\text{ferm}} = -\text{Tr} \ln \frac{\delta^2 S_{\text{hyp}}}{\delta \rho^\alpha(x) \delta \chi^\alpha(x')} = -\text{Tr} \left[ \frac{i}{2} (1 + 4I\bar{\phi}) \partial_{\alpha \dot{\alpha}} \delta^4(x - x') \delta_a^b \right] \]

\[ = -2\text{Tr} \ln \left[ \frac{i}{2} (1 + 4I\bar{\phi}) \delta^4(x - x') \right] - 2\text{Tr} \ln [\partial_{\alpha \dot{\alpha}} \delta^4(x - x')] \simeq 0. \quad (3.6) \]

Both terms in the second line of (3.6) make only trivial contributions to the effective
action and so can be discarded.

Taking into account these remarks, the effective action in the hypermultiplet model
(3.3) is defined by

\[ \Gamma_{\text{hyp}} = \frac{1}{2} \text{Tr} \ln \left[ \frac{\delta^2}{\delta f^a(x) \delta f^{a'}(x')} \left[ \frac{1}{2} \int d^4x (1 + 4I\bar{\phi})^2 \partial_m f^{ak} \partial_m f_k \right] \right]. \quad (3.7) \]

Calculating the functional derivative in (3.7) and performing some further manipulations,
we find

\[ \Gamma_{\text{hyp}} = 2\text{Tr} \ln \left[ (1 + 4I\bar{\phi})^2 \Box \delta^4(x - x') + \partial_m (1 + 4I\bar{\phi})^2 \partial_m \delta^4(x - x') \right] \]

\[ = 2\text{Tr} \ln \left[ (1 + 4I\bar{\phi})^2 \delta^4(x - x') \right] + 2\text{Tr} \ln \left[ \delta^4(x - x') + 2 \frac{1}{\Box} G_m(x) \partial_m \delta^4(x - x') \right]. \quad (3.8) \]

The second line of (3.8) does not contribute to the effective action since it is proportional
to \( \delta^4(0) \), which is zero in the dimensional regularization scheme. Therefore, making a
series expansion of the expression in the last line of (3.8), we obtain the following formal
answer for the effective action,

\[ \Gamma_{\text{hyp}} = 2\text{Tr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left[ \frac{2}{\Box} G_m(x) \partial_m \delta^4(x - x') \right]^n. \quad (3.9) \]

Eq. (3.9) is the starting point of the perturbative calculation of the one-loop effective
action in the neutral hypermultiplet model. Resorting to the dimensional arguments, like
in the gauge model considered in Sect. 2, it is easy to show that only two-, three-, and four-
point functions are divergent since they contain the momentum integrals corresponding
to the expressions (2.33). As far as we are interested only in the divergent part of the
effective action, we will consider the computation of two-, three-, and four-point diagrams
separately.

According to eq. (3.9), the two-point function is defined by the expression

\[ \Gamma^2_{\text{hyp}} = -4 \int d^4x_1 d^4x_2 G_m(x_1) G_n(x_2) \frac{\partial_m \delta^4(x_1 - x_2)}{\Box} \partial_n \delta^4(x_2 - x_1). \quad (3.10) \]
Passing to the momentum space by the standard rules (2.35), we obtain
\[ \Gamma_{2}^{\text{hyp}} = -4 \int \frac{d^{4}p}{(2\pi)^{8}} \tilde{G}_{m}(p)\tilde{G}_{n}(-p) \int d^{4}k \frac{k_{m}(p + k)_{n}}{k^{2}(p + k)^{2}}. \] (3.11)

The divergent momentum integral was calculated in the Appendix A.1 (eq. (A.10)). Here we need only the divergent part of this integral which reads
\[ \Gamma_{2,\text{div}}^{\text{hyp}} = \frac{2\pi^{2}}{3\varepsilon} \int \frac{d^{4}p}{(2\pi)^{8}} \tilde{G}_{m}(p)\tilde{G}_{n}(-p) \left( p_{m}p_{n} + \frac{\delta_{mn}}{2}p^{2} \right). \] (3.12)

Switching back to the configuration space, we obtain the divergent two-point contribution to the effective action
\[ \Gamma_{2,\text{div}}^{\text{hyp}} = \frac{2}{3\varepsilon \cdot 16\pi^{2}} \int d^{4}x [G_{m}(x)\partial_{m}G_{n}(x) + \frac{1}{2}G_{m}(x)\Box G_{m}(x)] \]
\[ = -\frac{1}{16\pi^{2}\varepsilon} \int d^{4}x \ln(1 + 4I\tilde{\phi}(x)) \Box^{2} \ln(1 + 4I\tilde{\phi}(x)). \] (3.13)

Up to the sign, the expression (3.13) is equal to the divergence of two-point function (2.38) in the gauge model.

Consider now the three-point function
\[ \Gamma_{3}^{\text{hyp}} = \frac{16}{3} \int d^{4}x_{1}d^{4}x_{2}d^{4}x_{3}G_{m}(x_{1})G_{n}(x_{2})G_{p}(x_{3}) \]
\[ \times \frac{\partial_{m}}{\Box} \delta^{4}(x_{1} - x_{2}) \frac{\partial_{n}}{\Box} \delta^{4}(x_{2} - x_{3}) \frac{\partial_{p}}{\Box} \delta^{4}(x_{3} - x_{1}). \] (3.14)

Passing to the momentum space, we obtain
\[ \Gamma_{3}^{\text{hyp}} = \frac{16}{3} \int \frac{d^{4}p_{1}d^{4}p_{2}d^{4}p_{3}}{(2\pi)^{12}} \tilde{G}_{m}(p_{1})\tilde{G}_{n}(p_{2})\tilde{G}_{p}(p_{3})\delta^{4}(p_{1} + p_{2} + p_{3}) \]
\[ \times \int d^{4}k \frac{k_{m}(k - p_{2})_{n}(k + p_{1})_{p}}{k^{2}(k - p_{2})^{2}(k + p_{1})^{2}}. \] (3.15)

The divergent part of the momentum integral was calculated in the Appendix A.1, eq. (A.12). Using this result, we find
\[ \Gamma_{3,\text{div}}^{\text{hyp}} = \frac{4\pi^{2}}{9\varepsilon} \int \frac{d^{4}p_{1}d^{4}p_{2}d^{4}p_{3}}{(2\pi)^{12}} \delta^{4}(p_{1} + p_{2} + p_{3})[\tilde{G}_{m}(p_{1})\tilde{G}_{m}(p_{2})\tilde{G}_{p}(p_{3})(2p_{1} + p_{2})_{p} \]
\[ - \tilde{G}_{n}(p_{1})\tilde{G}_{n}(p_{2})\tilde{G}_{m}(p_{3})(p_{1} + 2p_{2})_{n} - \tilde{G}_{m}(p_{1})\tilde{G}_{n}(p_{2})\tilde{G}_{n}(p_{3})(p_{1} - p_{2})_{m}] \]
\[ = -\frac{1}{8\pi^{2}\varepsilon} \int d^{4}x (\partial_{m}G_{m})G_{n}G_{n}. \] (3.16)

Thus the contribution to the divergent part of the effective action from this term reads
\[ \Gamma_{3,\text{div}}^{\text{hyp}} = -\frac{1}{8\pi^{2}\varepsilon} \int d^{4}x \ln(1 + 4I\tilde{\phi})\partial_{n} \ln(1 + 4I\tilde{\phi})\partial_{n} \ln(1 + 4I\tilde{\phi}). \] (3.17)
Finally, let us consider the computation of four-point function

\[
\Gamma_{4}^{hyp} = -8 \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 G_m(x_1) G_n(x_2) G_p(x_3) G_r(x_4)
\]
\[
\times \partial_m \delta^4(x_1 - x_2) \partial_n \delta^4(x_2 - x_3) \partial_p \delta^4(x_3 - x_4) \partial_r \delta^4(x_4 - x_1)
\]
\[
= -8 \int \frac{d^4p_1 \ldots d^4p_4}{(2\pi)^{16}} \tilde{G}_m(p_1) \tilde{G}_n(p_2) \tilde{G}_p(p_3) \tilde{G}_r(p_4) \delta^4(p_1 + p_2 + p_3 + p_4)
\]
\[
\times \int d^4k \frac{k_m(k - p_2)_n(k + p_1 + p_4)(p_1 + k)}{k^2(k - p_2)^2(k + p_1 + p_4)^2(p_1 + k)^2}.
\]
\[
(3.18)
\]

Substituting the expression (A.13) for the divergent momentum integral, we obtain the following expression for the divergent part of four-point function

\[
\Gamma_{4,div}^{hyp} = -\frac{\pi^2}{3\varepsilon} \int \frac{d^4p_1 \ldots d^4p_4}{(2\pi)^{16}} \delta^4(p_1 + p_2 + p_3 + p_4) 3 \tilde{G}_m(p_1) \tilde{G}_m(p_2) \tilde{G}_n(p_3) \tilde{G}_n(p_4)
\]
\[
= -\frac{1}{16\pi^2\varepsilon} \int d^4x [G_m(x) G_m(x)]^2.
\]
\[
(3.19)
\]

As a result, the corresponding contribution to the effective action is given by

\[
\Gamma_{4,div}^{hyp} = -\frac{1}{16\pi^2\varepsilon} \int d^4x [\partial_m \ln(1 + 4I\bar{\phi}) \partial_m \ln(1 + 4I\bar{\phi})]^2.
\]
\[
(3.20)
\]

Now we sum up all the divergent contributions to the effective action found above, i.e. (3.13), (3.17) and (3.20)

\[
\Gamma_{div}^{hyp} = -\frac{1}{16\pi^2\varepsilon} \int d^4x \ln(1 + 4I\bar{\phi}) \Box \ln(1 + 4I\bar{\phi})
\]
\[
-\frac{1}{8\pi^2\varepsilon} \int d^4x \Box \ln(1 + 4I\bar{\phi}) \partial_n \ln(1 + 4I\bar{\phi}) \partial_n \ln(1 + 4I\bar{\phi})
\]
\[
-\frac{1}{16\pi^2\varepsilon} \int d^4x [\partial_m \ln(1 + 4I\bar{\phi}) \partial_m \ln(1 + 4I\bar{\phi})]^2.
\]
\[
(3.21)
\]

After some work all three terms in the effective action (3.21) can be shown to reduce to the following simple expression

\[
\Gamma_{div}^{hyp} = -\frac{1}{\pi^2\varepsilon} \int d^4x \frac{I^2\Box \bar{\phi} \Box \bar{\phi}}{(1 + 4I\bar{\phi})^2}.
\]
\[
(3.22)
\]

The expression (3.22) represents the complete divergent contribution to the effective action in the deformed model of hypermultiplet interacting with the abelian gauge multiplet. Once again, eq. (3.22) matches with the general ansatz (2.26).

As in the deformed gauge model, in order to cancel the divergent term (3.22) one should add the corresponding counterterm to the classical action of the gauge model.
In other words, to make the model renormalizable we should add to the classical action \( (2.6) \) the expression

\[
c_1 \int d^4x \frac{I^2 \Box \phi \Box \bar{\phi}}{(1 + 4I\phi)^2},
\]

where \( c_1 \) is some constant. Remarkably, in a close similarity to the consideration in the gauge model, the expression \( (3.23) \) can be completely absorbed into a redefinition of another scalar field of the gauge multiplet in the classical action of the gauge model:

\[
\phi \rightarrow \phi - 2c_1 \frac{I^2 \Box \bar{\phi}}{(1 + 4I\phi)^2}, \quad \bar{\phi} \rightarrow \bar{\phi}.
\]

Therefore, the appearance of such a divergent term does not spoil the renormalizability of the theory in the sense that it can be removed by redefining the scalar field \( \phi \). On the quantum level, the term \( (3.23) \) does not make any contribution to the effective action of the model since we can perform the change of functional variables \( (3.24) \) in the functional integral.

Let us now consider the general abelian \( \mathcal{N} = (1,0) \) non-anticommutative model of gauge superfield field interacting with the hypermultiplet matter. It is described by the classical action

\[
S = S_{SYM} + S_{hyp},
\]

where \( S_{SYM} \) and \( S_{hyp} \) are given by \( (2.6) \) and \( (3.3) \). Using the Feynman rules developed in the Appendices A.2, A.3, it is easy to demonstrate that the total divergent contribution in the model \( (3.25) \) is a sum of divergencies of each model \( (2.44) \) and \( (3.22) \)

\[
\Gamma_{div} = \Gamma_{div}^{SYM} + \Gamma_{div}^{hyp} = -\frac{6}{\pi^2 \varepsilon} \int d^4x \frac{4I^3 \phi \partial_m \bar{\phi} \partial_m \bar{\phi}}{(1 + 4I\phi)^3}.
\]

The divergent term \( (3.26) \) can also be removed by a shift of the scalar field \( \phi \)

\[
\phi \rightarrow \phi - 2c_2 \frac{4I^3 \partial_m \bar{\phi} \partial_m \bar{\phi}}{(1 + 4I\phi)^3},
\]

where \( c_2 = -6/(\pi^2 \varepsilon) \) in this case. After the field redefinition \( (3.27) \) the effective action of general abelian \( \mathcal{N} = (1,0) \) non-anticommutative model is divergence-free. Since the divergent term \( (3.26) \) vanishes on the classical equations of motion, the S-matrix in this models is finite.

## 4 Seiberg-Witten map and renormalizability

The Seiberg-Witten map for \( \mathcal{N} = (1,0) \) gauge model \( (2.6) \) was found in \[6\]. After the redefinition of fields

\[
\varphi = \phi + 4I(1 + 4I\bar{\phi})^{-1}[A_m A_m + 4I^2(\partial_m \bar{\phi})^2],
\]

\[
a_m = (1 + 4I\bar{\phi})^{-1}A_m, \quad \bar{\psi}_a^k = (1 + 4I\bar{\phi})^{-1}\bar{\Psi}_a^k,
\]

\[
\psi_\alpha^k = \Psi_\alpha^k + 4I(1 + 4I\bar{\phi})^{-1}A_{\alpha\dot{\alpha}}\bar{\Psi}_{\dot{\alpha}}^k,
\]

\[
d^{kl} = (1 + 4I\bar{\phi})^{-1}[D^{kl} + 8I(1 + 4I\bar{\phi})^{-1}\bar{\Psi}_a^k \bar{\Psi}_\alpha^l]
\]
the action (2.6) simplifies drastically to
\[ S_{SYM} = \int d^4x \left( \frac{1}{2} \varphi \Box \tilde{\phi} - i \psi^\alpha \partial_\alpha \tilde{\psi}^{\dot{\alpha}} k + \frac{1}{4} d^{kl} d_{kl} \right) + \frac{1}{4} \int d^4x (1 + 4 I \bar{\phi})^2 (f_{mn} f_{mn} + \frac{1}{2} \varepsilon_{mnpq} f_{mn} f_{pq}) \right) . \] (4.2)

Here \( f_{mn} = \partial_m a_n - \partial_n a_m \). Note that the spinor and auxiliary fields are free, while the interaction between the vector field and the scalar \( \tilde{\phi} \) in the second line of (4.2) is still essential.

The action (4.2) is invariant under the abelian gauge transformations
\[ \delta a_m = \partial_m \lambda \] (4.3)
with \( \lambda \) being the gauge parameter. Therefore we use standard Lorentz gauge fixing
\[ \partial_m a_m = 0 . \] (4.4)

Following the Faddeev-Popov procedure for constructing the functional integral, we introduce the gauge fixing function
\[ \chi = \partial_m a_m , \] (4.5)
which transforms as \( \delta \chi = \Box \lambda \). Therefore the ghost fields do not interact with other fields and completely decouple. The ghost action is given again by eq. (2.19). The generating functional for Green’s functions is now given by \(^5\)
\[ Z[J] = \int \mathcal{D}(\varphi, \tilde{\phi}, \psi, \tilde{\psi}, a_m, b, c) \delta(\chi - \partial_m a_m) e^{-\frac{1}{2}(S_{SYM} + S_{FP} + S_J)} , \] (4.6)
where
\[ S_J = \int d^4x \left[ \varphi J_\varphi + \tilde{\phi} J_{\tilde{\phi}} + \psi^\alpha (J_\psi)^\alpha + \tilde{\psi}^{\dot{\alpha}} (J_{\tilde{\psi}})^{\dot{\alpha}} + a_m (J_a)_m \right] . \] (4.7)

To represent the delta-function in the Gaussian form, we average the equation (4.6) with the functional factor (2.22). As a result we obtain the gauge fixing action in the form
\[ S_{gf} = \frac{\alpha}{2} \int d^4x (1 + 4 I \tilde{\phi})^2 \partial_m a_m \partial_n a_n . \] (4.8)

For simplicity we choose the gauge fixing parameter \( \alpha \) to be unity, \( \alpha = 1 \). As a result, the generating functional (4.6) reads
\[ Z[J] = \int \mathcal{D}(\varphi, \tilde{\phi}, \psi, \tilde{\psi}, a_m, b, c) e^{-\frac{1}{2}(S_{tot} + S_{FP} + S_J)} , \] (4.9)

\(^5\)Note that the Jacobian of the change of functional variables (4.1) is unity since this redefinition of fields is local.
where
\[ S_{\text{tot}} = S_{\text{SYM}} + S_{gf} = \int d^4x \left( -\frac{1}{2} \partial^2 \phi - i\bar{\psi}_k \partial_a \tilde{\psi}^{\dot{a}k} + \frac{1}{4} d^{kl} d_{kl} \right) + S_a, \tag{4.10} \]
and
\[ S_a = \frac{1}{2} \int d^4x (1 + 4i\bar{\psi}\partial^2 \phi - i\bar{\psi}_k \partial_a \tilde{\psi}^{\dot{a}k} + 1) \delta\phi \]
\[ \text{expression (2.38) obtained by} \]
\[ \delta \]
\[ \text{The absence of divergencies here is owing to the term} \]
\[ \int \text{eqs. (A.10), (A.11). As a result we find that the two-point function is given by} \]
\[ G \]
\[ \text{It is evident that the scalar and spinor fields as well as the ghosts do not contribute to the effective action. The only contribution comes from the part} \]
\[ \text{part (4.11), namely} \]
\[ \Gamma^{\text{SYM}} = \frac{1}{2} \text{Tr} \ln \frac{\delta^2 S_a}{\delta a_p(x) \delta a_q(x')} \]
\[ \Gamma^{\text{SYM}} = \frac{1}{2} \text{Tr} \ln \left[ \delta_{pq} \partial^4 (x - x') + 2\delta_{pq} G_m \partial_m \delta^4 (x - x') \right] \]
\[ \text{The field} G_m(x) \text{was defined in eq. (2.17). The expression} \]
\[ \text{starting point for perturbative calculations of one-loop effective action in the} \]
\[ \text{sector} \]
\[ \text{two-, three- and four-point diagrams are divergent. The two-point function is given by} \]
\[ \Gamma_{\text{SYM}} = -\int d^4x_1 d^4x_2 \left( \delta_{pq} G_m(x_1) \partial_m \frac{1}{\partial^4 (x_1 - x_2)} + 2G_{[p}(x_1) \partial_{q]} \frac{1}{\partial^4 (x_1 - x_2)} \right) \]
\[ \text{To proceed, we pass to the momentum space and compute the divergent momentum integrals according to eqs. (A.10), (A.11). As a result we find that the two-point function} \]
\[ \text{has no divergent contributions, i.e.} \]
\[ \Gamma_{\text{SYM}}^{\text{div}} = 0. \tag{4.14} \]
\[ \text{The absence of divergencies here is owing to the term} \]
\[ \text{term} \]
\[ \text{similar calculations without this term.} \]
\[ \text{it resembles the first line of eq.(2.30), except for the term} \]
\[ \text{the further computations are very similar to ones given in Sect. 2. As usual, only two-, three- and four-point diagrams are divergent. The two-point function is given by} \]
\[ \Gamma_{\text{SYM}}^2 = -\int d^4x_1 d^4x_2 \left( \delta_{pq} G_m(x_1) \partial_m \frac{1}{\partial^4 (x_1 - x_2)} + 2G_{[p}(x_1) \partial_{q]} \frac{1}{\partial^4 (x_1 - x_2)} \right) \]
\[ \text{The three- and four-point functions are defined by the following formal expressions:} \]
\[ \Gamma_{\text{SYM}}^3 = \frac{4}{3} \text{Tr} \left[ (\delta_{pq} G_m(x) \partial_m + 2G_{[p}(x) \partial_{q]} - \epsilon_{pqmn} G_m(x) \partial_n) \frac{1}{\partial^4 (x - x')} \right]^3, \tag{4.15} \]
\[ \Gamma_{\text{SYM}}^4 = -2 \text{Tr} \left[ (\delta_{pq} G_m(x) \partial_m + 2G_{[p}(x) \partial_{q]} - \epsilon_{pqmn} G_m(x) \partial_n) \frac{1}{\partial^4 (x - x')} \right]^4. \tag{4.16} \]
As a result, we conclude that the abelian \( N = (1,0) \) non-anticommutative gauge model (4.2) is completely finite, thus
\begin{align}
\Gamma^{SYM}_{3,\text{div}} &= 0, \\
\Gamma^{SYM}_{4,\text{div}} &= 0.
\end{align}
(4.17)

As a result, we conclude that the abelian \( N = (1,0) \) non-anticommutative gauge model (4.2) is completely finite, thus
\begin{align}
\Gamma^{SYM}_{\text{div}} &= 0
\end{align}
(4.18)

without the necessity to perform any field redefinition such as (2.46).

The absence of divergencies in the abelian \( N = (1,0) \) non-anticommutative gauge model confirms the results of Sect. 2, where these calculations were performed without the use of Seiberg-Witten map (4.1). This is a consequence of the fact that the considered model has a very specific interaction due to the non-anticommutativity.

One more important comment to be added is as follows. The abelian \( N = (1,0) \) non-anticommutative gauge model is described by the classical actions (2.6) or (4.2) which are related to each other through the Seiberg-Witten map (4.1). It is obvious that the Jacobian of such a change of functional variables (4.1) is unity (in the sense of dimensional regularization). Therefore the effective actions in these two models should also be related by the Seiberg-Witten map. As for the divergent part, we observe that it is trivial for both models (2.6) and (4.2), since it can be removed by the shift (2.46) of the scalar field \( \phi \).

Note that this explains the appearance of only two out of three possible divergent terms (2.20). Indeed, if the third term proportional to \( I^4 \int d^4 x f_3(I \bar{\phi})(\partial_m \phi \partial_m \phi)^2 \) appeared in the divergent part of the effective action, it could not be removed by any shift of the scalar field \( \phi \), which would mean the presence of a nontrivial divergence in the model. However, we have seen in this Section that the effective action in \( N = (1,0) \) non-anticommutative gauge theory is finite.

Let us consider also the general model of an abelian \( N = (1,0) \) non-anticommutative gauge superfield interacting with a neutral hypermultiplet. It is described by the sum of the classical actions (2.3) and (3.3). In [7] it was shown that, after the appropriate redefinition of fields (Seiberg-Witten map), the action of this model is given by
\begin{align}
S &= S_0 + S_1, \\
S_0 &= \int d^4 x \left[ -\frac{1}{2} \bar{\phi} \Box \phi + \frac{1}{2} \partial_m \hat{f}^{ak} \partial_m \hat{f}_{ak} - i \bar{\psi}^a \partial_a \bar{\psi}^\alpha \bar{\chi}_a^\alpha + \frac{i}{2} \rho^{a\alpha} \partial_a \bar{\chi}^\alpha + \frac{1}{4} d_k d_l \delta_{kl} \right], (4.20) \\
S_1 &= \int d^4 x \left[ \frac{1}{4} (1 + 4 I \bar{\phi})^2 (f_{mn} f_{mn} + \frac{1}{2} \varepsilon_{mnrs} f_{mn} f_{rs}) + I (1 + 4 I \bar{\phi})^{-1} \rho^{\beta a} \bar{\rho}_a f_{\alpha \beta} \right], (4.21)
\end{align}

where \( f_{\alpha \beta} = i (\partial_{a\alpha} \bar{a}_a^\beta + \partial_{\beta \alpha} a_a^\alpha) \) is one of two self-dual parts of the Maxwell field strength \( f_{mn} \). The corresponding Seiberg-Witten map reads
\begin{align}
\hat{f}^{ak} &= (1 + 4 I \bar{\phi}) f^{ak}, \\
\rho^{a\alpha} &= (1 + 4 I \bar{\phi}) \rho^{a\alpha}, \\
\bar{\chi}_a^\alpha &= \chi^{\alpha a} + 4 I a_a^\alpha \rho_a^\alpha - 8 I \bar{\psi}_a^\alpha \hat{f}_k^a, \\
\hat{\varphi} &= \varphi - 4 I (1 + 4 I \bar{\phi}) (f^{ak} f_{ak}), \\
\bar{\psi}_k^\alpha &= \psi_k^\alpha - 4 I (1 + 4 I \bar{\phi}) (\rho^{a\alpha} f_{ak}).
\end{align}
(4.22)
Note that the action \( S_0 \) (4.20) is free and it does not contribute to the effective action. It is easy to demonstrate that the last term in (4.21) also does not give rise to any quantum correction since it is impossible to form any loop with such interactions. The only non-trivial contribution to the effective action comes from the first term in (4.21),

\[
\int d^4x \frac{1}{4} (1 + 4I\bar{\phi})^2 (f_{mn}f_{mn} + \frac{1}{2}\varepsilon_{mnrsl}f_{mn}f_{rs}) .
\]

(4.23)

This expression just coincides with the one present in the gauge theory action (4.2). Thus the quantum computations tell us once again that the general abelian \( N = (1,0) \) non-anticommutative model is finite

\[
\Gamma_{\text{div}} = 0 .
\]

(4.24)

This result agrees with the one of Sect.3, modulo some divergent redefinition (3.27) of the scalar field \( \phi \).

To summarize, the use of the Seiberg-Witten map in the models under consideration makes it possible to avoid the divergent expressions in the effective action from the very beginning. Otherwise, such expressions appear but they are removable by some divergent redefinition of the scalar field \( \phi \).

5 Concluding remarks

In this paper we addressed the problem of renormalizability of two supersymmetric models with the nilpotent singlet deformation \( \mathcal{N} = (1,1) \rightarrow \mathcal{N} = (1,0) \): the model of abelian \( \mathcal{N} = (1,1) \) gauge vector multiplet, as well as the model of abelian vector multiplet interacting with a neutral hypermultiplet. Our main conclusion is that both these models are finite.

The consideration is based on component field computations of all divergent Feynman graphs and their regularization. We observe the following common features peculiar to both considered models.

1. The renormalizability of these models can be established by ascribing non-standard scaling dimensions to the component fields and then resorting to the general Wilsonian argument.

2. The effective action is defined only by one-loop contributions. The vertices corresponding to the new interaction induced by the non-anticommutativity have a very specific structure that ensures the absence of higher-loop contributions to the effective action.

3. The analysis of the Feynman rules in the models shows that the effective action depends only on the field \( \bar{\phi} \) (but not on \( \phi \)). In other words, only the field \( \bar{\phi} \) can appear as the external legs while other fields can propagate only inside the loop.
4. The diagrams with fermionic fields inside the loop do not contribute to the effective action (more precisely, these diagrams give an infinite contribution which is commonly discarded within the dimensional regularization scheme). There are only two types of non-trivial diagrams: with the vector field inside the loop in the gauge model and with the scalar fields inside the loop in the hypermultiplet model.

5. The divergent diagrams carry only two, three or four external legs. Any diagrams with more external legs are convergent.

6. The total divergent contribution to the effective action can be written in the form of two terms (2.45) or one term (3.23), which are absent in the original classical actions of the gauge model or the gauge-hypermultiplet model, respectively. However, these divergences can be eliminated by simple redefinitions of the scalar field $\phi$ as in (2.46) or (3.24). Since such a change of fields can be performed in the functional integral defining the effective action (the Jacobian of such a change is unity), we conclude that all divergencies can be eliminated by such redefinitions. An important property is that the coupling constant (deformation parameter) $I$ is not renormalized, so its $\beta$-function is equal to zero. Note also that the divergent terms (2.45, 3.23) vanish on the classical equations of motion, therefore the S-matrix is divergence-free. In this sense the considered models are finite.

7. In the $\mathcal{N} = (1, 0)$ non-anticommutative gauge models, both with and without the hypermultiplet, there exists a Seiberg-Witten map which essentially simplifies the classical actions of these theories. It is an amazing feature of the considered models that in terms of the new fields (after performing the Seiberg-Witten map) the quantum effective action is completely free of divergencies. This emphasizes the “unphysical” nature of the divergent terms which appear when using the original fields (before performing the Seiberg-Witten map).

All these properties look rather strange since they are not featured by conventional field models. However, these peculiarities are explained by the fact that the considered models become free when the non-anticommutativity is turned off. In this connection, it seems important to study the renormalizability and the problem of effective action in various $\mathcal{N} = (1, 0)$ non-anticommutative models which remain interacting in the undeformed limit. One of the simplest theories of this kind is a charged hypermultiplet interacting with an external abelian gauge superfield. As is well known (see e.g. [25, 26]), the low-energy effective action of the undeformed charged hypermultiplet model is described by the holomorphic potential. Therefore, it is very interesting to find the analogous contributions to the effective action in the corresponding non-anticommutative model. Note that the similar problems for $\mathcal{N} = (1/2, 0)$ Wess-Zumino and gauge models were successfully solved in the works [20, 21]. Also it would be useful to investigate the next-to-leading corrections. In conventional (undeformed) $\mathcal{N} = 2, 4$ gauge models such corrections form the non-holomorphic effective potential having rather universal form [27]. It would be interesting to clarify the structure of next-to-leading corrections in the non-anticommutative theories.
Apart from the feature that the considered field theories look like the “toy” models since they become free in the undeformed limit, the proof of their renormalizability is an important first step in attacking the issue of renormalizability of deformed general non-abelian $\mathcal{N} = (1, 0)$ gauge theories. Indeed, these models appear as a $U(1)$ part of general non-abelian $\mathcal{N} = (1, 0)$ gauge theories. The renormalizability in the $U(1)$ sector is necessary (but of course not sufficient) for the whole non-abelian theory to be renormalizable (see, e.g., the analysis of renormalizability of the $\mathcal{N} = (1/2, 0)$ SYM model in [18, 19]). However, the non-abelian generalization of our results is a very non-trivial task, since for the time being the non-abelian deformed models are insufficiently studied even at the classical level [6, 7].

Another possible direction of extending our results is related to the issue of renormalizability of non-anticommutative $\mathcal{N} = (1, 1)$ models with non-singlet deformations considered e.g. in [10, 11].

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Appendices

A.1 Divergent momentum integrals

All divergent momentum (loop) integrals in quantum field theory can be calculated using the dimensional regularization. For example, there is the list of standard formulas [28]

$$\int \frac{d^d k}{(k^2 + 2kQ + M^2)^\alpha} = \frac{\pi^{d/2}}{\Gamma(\alpha)(M^2 - Q^2)^{\alpha - d/2}} \Gamma\left(\alpha - \frac{d}{2}\right),$$  \hspace{1cm} (A.1)

Note that we work in the Euclidean space, therefore our expressions differ from those given in [28] by the imaginary unit factor.
\[
\int \frac{d^d k \, k_m}{(k^2 + 2kQ + M^2)^\alpha} = \frac{-Q_m \pi^{d/2}}{\Gamma(\alpha)(M^2 - Q^2)^{\alpha - d/2}} \Gamma\left(\alpha - \frac{d}{2}\right), \quad (A.2)
\]
\[
\int \frac{d^d k \, k_m k_n}{(k^2 + 2kQ + M^2)^\alpha} = \frac{\pi^{d/2}}{\Gamma(\alpha)(M^2 - Q^2)^{\alpha - d/2}} \left[ Q_m Q_n \Gamma\left(\alpha - \frac{d}{2}\right) + \frac{1}{2} \delta_{mn}(M^2 - Q^2) \Gamma\left(\alpha - 1 - \frac{d}{2}\right) \right], \quad (A.3)
\]
\[
\int \frac{d^d k \, k_m k_n k_p}{(k^2 + 2kQ + M^2)^\alpha} = \frac{\pi^{d/2}}{\Gamma(\alpha)(M^2 - Q^2)^{\alpha - d/2}} \left[ -Q_m Q_n Q_p \Gamma\left(\alpha - \frac{d}{2}\right) - \frac{1}{2} (\delta_{mn} Q_p + \delta_{np} Q_m + \delta_{pm} Q_n) \times (M^2 - Q^2) \Gamma\left(\alpha - 1 - \frac{d}{2}\right) \right], \quad (A.4)
\]
\[
\int \frac{d^d k \, k_m k_n k_p k_r}{(k^2 + 2kQ + M^2)^\alpha} = \frac{\pi^{d/2}}{\Gamma(\alpha)(M^2 - Q^2)^{\alpha - d/2}} \left[ Q_m Q_n Q_p Q_r \Gamma\left(\alpha - \frac{d}{2}\right) + \frac{1}{2} (\delta_{mn} Q_p Q_r + \text{permutations})(M^2 - Q^2) \Gamma\left(\alpha - 1 - \frac{d}{2}\right) + \frac{1}{4} (\delta_{mn} \delta_{pr} + \text{permutations}) \times (M^2 - Q^2)^2 \Gamma\left(\alpha - 2 - \frac{d}{2}\right) \right]. \quad (A.5)
\]

Each of the expressions (A.2)-(A.5) has the corresponding representation in the momentum space:

\[
\left[ \frac{\partial_m \delta^4(x - x')}{\Box} \right]^2 \rightarrow \int d^4 k \, k_m (p + k)_n \frac{k_2}{k^2(p + k)^2}, \quad (A.6)
\]
\[
\left[ \frac{\partial_m \delta^4(x - x')}{\Box} \right]^3 \rightarrow \int d^4 k \, k_m (k - p_2)_n (k + p_1)_p \frac{k_2}{k^2(k - p_2)^2(k + p_1)^2}, \quad (A.7)
\]
\[
\left[ \frac{\partial_m \delta^4(x - x')}{\Box} \right]^4 \rightarrow \int d^4 k \, k_m (k - p_2)_n (p_1 + p_4 + k)_p (p_1 + k)_r \frac{k_2}{k^2(k - p_2)^2(k + p_1 + p_4)^2(p_1 + k)^2}. \quad (A.8)
\]

Using eqs. (A.2), (A.3), one can calculate the integral (A.6):

\[
\int d^4 k \, k_m (p + k)_n \frac{k_2}{k^2(p + k)^2} = -\pi^2 (p^2)^{-\varepsilon} \frac{\Gamma(\varepsilon) \Gamma^2(2 - \varepsilon)}{\Gamma(4 - 2\varepsilon)} \left[ p_m p_n + p^2 \frac{\delta_{mn}}{2(1 - \varepsilon)} \right]. \quad (A.9)
\]

In our calculations we are interested only in the divergent part of the effective action. Therefore we should consider only the divergent part of the expression (A.6) in the limit \( \varepsilon \rightarrow 0 \)

\[
\left[ \int d^4 k \, k_m (p + k)_n \frac{k_2}{k^2(p + k)^2} \right]_{\text{div}} = -\pi^2 \frac{6\varepsilon}{\varepsilon^2} \left[ p_m p_n + p^2 \frac{\delta_{mn}}{2} \right]. \quad (A.10)
\]
In particular,
\[
\left[ \int d^4k \frac{k_n(p + k)_n}{k^2(p + k)^2} \right]_{\text{div}} = -\frac{\pi^2}{2\varepsilon}p^2. \tag{A.11}
\]

The pole factor $1/\varepsilon$ appears here from the asymptotics of the gamma-function $\Gamma(\varepsilon)|_{\varepsilon \to 0} = \frac{1}{\varepsilon} - \gamma + O(\varepsilon)$, where $\gamma$ is Euler constant.

Similarly, using eqs. (A.4), (A.5), we find the divergent parts of the remaining integrals (A.7), (A.8):
\[
\left[ \int d^4k \frac{k_m(k - p_2)_m(p_1 + p_1)_p}{k^2(k - p_2)^2(k + p_1)^2} \right]_{\text{div}} = \frac{\pi^2}{12\varepsilon}[\delta_{mn}(2p_1 + p_2)_p - \delta_{pm}(p_1 + 2p_2)_n - \delta_{np}(p_1 - p_2)_m], \tag{A.12}
\]
\[
\left[ \int d^4k \frac{k_m(k - p_2)_m(p_1 + p_4 + k)_p(p_1 + k)_r}{k^2(k - p_2)^2(k + p_1 + p_4)^2(p_1 + k)^2} \right]_{\text{div}} = \frac{\pi^2}{24\varepsilon}(\delta_{mn}\delta_{pr} + \delta_{mp}\delta_{nr} + \delta_{mr}\delta_{np}). \tag{A.13}
\]

A.2 Feynman graphs in SYM model

The action (2.25) defines the Feynman rules in the deformed gauge model. The propagators have the standard form in the quantum field theory listed in the following table:

| Propagator | Line |
|------------|------|
| $-\frac{1}{2} \int d^4x \phi \Box \phi \rightarrow \langle \phi(x) \bar{\phi}(x') \rangle = -\frac{\Box}{4\delta^4(x - x')}$ | $\phi \quad \bar{\phi}$ |
| $i \int d^4x \Psi^\alpha \partial^\alpha \bar{\Psi}^\dagger \rightarrow \langle \Psi^\alpha(x) \bar{\Psi}^\dagger(x') \rangle = -\frac{2i}{4\delta^4(x - x') \delta^i_j}$ | $\Psi \quad \bar{\Psi}$ |
| $-\frac{1}{2} \int d^4x A_m \Box A_n \rightarrow \langle A_m(x) A_n(x') \rangle = -\frac{2}{4\delta^4(x - x') \delta_{mn}}$ | $A_m \quad A_n$ |

The vertices defined by the action (2.25) have quite complicated form. Schematically, they can be depicted as
Analyzing the propagators and vertices given above, one can observe that there are only two types of nontrivial loop diagrams shown at Fig. 1. Both these diagrams have arbitrary numbers of external lines. The effective action corresponding to the diagram a) is calculated in Sect. 2. The sum of diagrams b) makes the trivial contribution to the effective action. Indeed, it corresponds to the following one-loop effective action

\[ \Gamma_\Psi = \text{Tr} \ln \frac{\delta^2 S_\Psi}{\delta \Psi^\alpha_i(x) \delta \Psi^\beta_i} = \text{Tr} \ln \left[ -i \frac{\delta^3 (\sigma_n)_{\alpha \dot{\alpha}} \partial_n \delta^4 (x - x')} {1 + 4I \Phi} \right] \]

\[ = 2 \text{Tr} \ln [(\sigma_n)_{\alpha \dot{\alpha}} \partial_n \delta^4 (x - x')] + 2 \text{Tr} \ln \left[ \frac{\delta^4 (x - x')}{1 + 4I \Phi} \right] \approx 0 . \]
Both terms in the second line of eq. (A.14) are trivial.

### A.3 Feynman graphs in hypermultiplet model

Feynman rules in the deformed hypermultiplet model are defined by the action (3.3). The propagators have the standard form

| Propagator | Line |
|------------|------|
| $-\frac{1}{2} \int d^4x f^{ak} \Box f_{ak} \rightarrow \langle f^{ak}(x)f_{ak'}(x') \rangle = -2\delta^4(x-x')\delta_a^a \delta_k^k$ | $f^{ak} \quad f_{ak'}$ |
| $\frac{i}{2} \int d^4x \rho^{\alpha a} \partial_{\alpha a} \chi^\dagger_a \rightarrow \langle \rho^{\alpha a}(x)\chi^\dagger_{a'}(x') \rangle = -i\delta^{\alpha a} \delta^4(x-x')\delta_a^a$ | $\rho^{\alpha a} \quad \chi^\dagger_{a'}$ |

The vertices defined by the action (3.3) are given in the following table
Like in the gauge model, one can observe that there are only two types of nontrivial diagrams shown in Fig. 2. The computation of these diagrams is considered in Sect. 3. Note that the diagram with the fermionic loop give only trivial contribution to the effective action, see eq. (3.6).

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