Low latency search for Gravitational waves from BH-NS binaries in coincidence with Short Gamma Ray Bursts

Andrea Maselli and Valeria Ferrari
Dipartimento di Fisica, Università di Roma “La Sapienza” & Sezione, INFN Roma1, P.A. Moro 5, 00185, Roma, Italy

We propose a procedure to be used in the search for gravitational waves from black hole-neutron star coalescing binaries, in coincidence with short gamma-ray bursts. It is based on two recently proposed semi-analytic fits, one reproducing the mass of the remnant disk surrounding the black hole which forms after the merging as a function of some binary parameters, the second relating the neutron star compactness, i.e. the ratio of mass and radius, with its tidal deformability. Using a Fisher matrix analysis and the two fits, we assign a probability that the emitted gravitational signal is associated to the formation of an accreting disk massive enough to supply the energy needed to power a short gamma ray burst. This information can be used in low-latency data analysis to restrict the parameter space searching for gravitational wave signals in coincidence with short gamma-ray bursts, and to gain information on the dynamics of the coalescing system and on the internal structure of the components. In addition, when the binary parameters will be measured with high accuracy, it will be possible to use this information to trigger the search for off-axis gamma-ray bursts afterglows.

Keywords: gravitational waves, neutron stars, short gamma-ray bursts

Introduction.— The advanced gravitational wave detectors LIGO and Virgo (to hereafter AdvLIGO/Virgo) are expected to detect signals emitted by coalescing compact binaries, formed by neutron stars (NS) and/or black holes (BH) \cite{1}. These catastrophic events have an electromagnetic counterpart. For instance, the coalescence of NS-NS and BH-NS binaries has been proposed as a candidate for the central engine of short Gamma Ray Bursts (SGRB), provided the stellar-mass BH which forms after merging is surrounded by a hot and sufficiently massive accreting disk, but this model needs to be validated \cite{2} and references therein. Since the electromagnetic emission is produced at large distance from the central engine, it does not give strong information on the source. In addition, the emission is beamed, and consequently these events may not be detected if one is looking in the wrong direction. Conversely, the gravitational wave (GW) emission is not beamed, and exhibits a characteristic waveform (the chirp) which should allow a non ambiguous identification of the source. GRBs are characterized by a prompt emission, which lasts a few seconds, and an afterglow, whose duration ranges from hours to days.

Thus, gravitational wave detection may be used to trigger the afterglow search of GRBs which have not been detected by the on-axis prompt observation, and to validate the “jet-model” of SGRB. Or, in alternative, the observation of a SGRB may be used as a trigger to search for a coincident GW signal. Indeed, this kind of search has already been done in the data of LIGO and Virgo \cite{3,4}. A large number of numerical studies of BH-NS coalescence, have allowed to derive two interesting fits. The first \cite{7} gives the mass of the accretion disk, \( M_{\text{rem}} \), as a function of the the NS compactness \( C = M_{\text{NS}} / R_{\text{NS}} \), where \( M_{\text{NS}} \) and \( R_{\text{NS}} \) are the NS mass and radius, the dimensionless BH spin, \( \chi_{\text{BH}} \in [-1,1] \), and the mass ratio \( q = M_{\text{BH}} / M_{\text{NS}} \):

\[
\frac{M_{\text{rem}}}{M_{\text{NS}}} = K_1(3q)^{1/3}(1-2C) - K_2q \, C \, R_{\text{ISCO}}.
\]

Here \( M_{\text{NS}}^{\text{b}} \) is the NS baryonic mass which, following \cite{8}, we assume to be 10% larger than the NS gravitational mass; \( R_{\text{ISCO}} \) is the radius of the innermost, stable circular orbit for a Kerr black hole:

\[
\frac{R_{\text{ISCO}}}{M_{\text{BH}}} = \frac{3 + Z_2 - \text{sign}(\chi_{\text{BH}})}{\sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}},
\]

where \( Z_1 = 1 + (1 - \chi_{\text{BH}}^2)^{1/3} \left[ 1 + (1 + \chi_{\text{BH}})^{1/3} + (1 - \chi_{\text{BH}})^{1/3} \right] \) \( Z_2 = (3\chi_{\text{BH}} + Z_1^2)^{1/2} \) \cite{9}. The two coefficients \( K_1 = 0.288 \pm \)
where $Q$ is projected onto the parallel transported tetrad attached to the e.

$\bar{\lambda}$ by class of equations of state (EoS). Hereafter, we shall denote $(0\bar{M})_{\text{rem}}$ is a key parameter in our study. Neutrino-antineutrino annihilation processes extract energy from the disk, and several studies have shown that this process could supply the energy required to ignite a short gamma-ray burst, if $M_{\text{rem}} \in (0.01 \div 0.03)M_\odot$ [15][17]. In the following we shall assume as a threshold for SGRB formation $M_{\text{rem}} = 0.01 M_\odot$. The results we will show do not change if we choose $M_{\text{rem}} = 0.03 M_\odot$.

The second fit [18] is a universal relation between the NS compactness $C$ and the tidal deformability $\chi_{\text{BH}} = C_{ij}\varepsilon_{ij} R_{\alpha\beta\gamma\delta}$ is the tidal tensor, i.e. the Riemann tensor projected onto the parallel transported tetrad attached to the star $e_{ij}$.

$$C = 0.371 - 3.9 \times 10^{-2} \ln \bar{\lambda} + 1.056 \times 10^{-3} (\ln \bar{\lambda})^2,$$  \hspace{1cm} (3)

where $\bar{\lambda} = \lambda_2 / M_{\text{NS}}^3$. This fit is found to reproduce the values of the star compactness with accuracy greater 3%, for a large class of equations of state (EoS). Hereafter, we shall denote by $C_1$ the NS compactness obtained from this fit.

Let us now assume that the gravitational wave signal emitted in a BH-NS coalescence is detected; a suitable data analysis allows to find the values of the symmetric mass-ratio $\nu = (M_{\text{NS}} M_{\text{BH}}) / (M_{\text{NS}} + M_{\text{BH}})^2$ and of the chirp mass $M = \nu^{3/5}(M_{\text{NS}} + M_{\text{BH}})$, from which the mass ratio $q$ can be derived, and of the black hole spin $\chi_{\text{BH}}$, with the corresponding errors. Knowing $q \pm \sigma_q$ and $\chi_{\text{BH}} \pm \sigma_{\chi_{\text{BH}}}$, using the fit [1], we can trace the plot of Fig. 1 in the $q - C$ plane, for an assigned disk mass threshold, say $M_{\text{rem}} = 0.01 M_\odot$. This plot allows to identify the parameter region where a SGRB may occur, i.e. the region $M_{\text{rem}} \geq 0.01 M_\odot$ (below the fit curve in the figure), and the forbidden region above the fit ($M_{\text{rem}} \leq 0.01 M_\odot$). In addition, we identify four points $X_1, \ldots, X_4$, which are the intersection between the contour lines for $\chi_{\text{BH}} \pm \sigma_{\chi_{\text{BH}}}$ and the horizontal lines $q \pm \sigma_q$. Let us indicate as $C_1, \ldots, C_4$ the corresponding values of the neutron star compactness. Since the fit [1] is monotonically decreasing, $C_1 < C_2 < C_3 < C_4$. At this stage we still cannot say whether the detected binary falls in the region allowed for the formation of a SGRB or not.

In order to get this information, we need to evaluate $C$. As discussed in [19][23], Advanced LIGO/Virgo are expected to measure the gravitational wave phase with an accuracy sufficient to estimate the NS tidal deformability $\lambda_2$. Thus, using the fit [3], the NS compactness $C_i$ and the corresponding uncertainty

$$\sigma_{C_i}^2 = \sigma_{\bar{\lambda}}^2 + \sum_{ij} \frac{\partial C_i}{\partial p_i} \frac{\partial C_i}{\partial p_j} \text{Cov}(p_i, p_j)$$  \hspace{1cm} (4)

can be inferred. In [3] $p_i = (\lambda_2, M_{\text{NS}})$, and $\text{Cov}(p_i, p_j)$ is their covariance. As shown in [18], $\sigma_{\bar{\lambda}} = 0.03 C$ is the largest relative discrepancy between the value of $C$ obtained from the fit and the value computed solving the equations of stellar perturbations, for a set of EoS covering a large range of stiffness.

Knowing the parameters and their uncertainties, the probability that a SGRB is associated to the detected coalescence can now be evaluated.

We assume that $(q, C_i, \chi_{\text{BH}})$ are described by a multivariate Gaussian distribution:

$$P(q, C_i, \chi_{\text{BH}}) = \frac{1}{(2\pi)^{3/2} \Sigma^{1/2}} \exp \left[ -\frac{1}{2} \Delta^T \Sigma^{-1} \Delta \right],$$  \hspace{1cm} (5)

where $\Delta = (\vec{x} - \vec{m})$, $\vec{m} = (q, C_i, \chi_{\text{BH}})$, $\Sigma$ is the covariance matrix. Then, we define the maximum and minimum probability that the binary coalescence produces an accretion disk with mass over the threshold, $M_{\text{rem}}$, as

$$P_{\text{MAX}}(M_{\text{rem}} \geq M_{\text{rem}}) = P(C_i \leq C_4),$$  \hspace{1cm} (6)

$$P_{\text{MIN}}(M_{\text{rem}} \geq M_{\text{rem}}) = P(C_i \leq C_1),$$

where $P(C_i \leq C_i)$ is the cumulative distribution of Eq. (5), which gives the probability that the measured compactness $C_i$, estimated through the fit [3], is smaller than an assigned value $C_i$.

As an illustrative example, we now evaluate the probability that a given BH-NS coalescing binary produces a SGRB, assuming a set of equations of state for the NS matter and evaluating the uncertainties on the relevant parameters using a Fisher matrix approach.

**Evaluation of the uncertainties on the binary parameters.**—

The accuracy with which future interferometers will measure a set of binary parameters $\theta$, is estimated by comparing the gravity-wave data-stream with a set of theoretical templates. For strong enough signals, $\theta$ are expected to have a Gaussian distribution centered around the true values, with covariance

![Contour plot of the fit](image-url)
matrix
\[
\text{Cov}^{ab} = (\Gamma^{-1})^{ab}, \quad \Gamma^{ab} = \left( \frac{\partial h}{\partial \theta^a} \right) \left( \frac{\partial h}{\partial \theta^b} \right).
\] (7)

being \( \Gamma \) the Fisher information matrix \([25]\). \((\cdot)\) is the inner product between two GW templates \( \tilde{h}(f) \) and \( \hat{g}(f) \):
\[
(h|g) = \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{\tilde{h}(f)\hat{g}^*(f) + \hat{g}(f)\tilde{h}^*(f)}{S_n(f)} df,
\] (8)

\* denotes complex-conjugation, and \( S_n(f) \) is the noise spectral density of the considered detector. To model the waveform we use the TaylorF2 approximant in the frequency domain, assuming the stationary phase approximation \([26]\):
\[
h(f) = A(f) e^{i\phi(f)} = \sqrt{\frac{5}{24\pi^3/2}} f^{-7/6} M^{5/6}_\text{BH} \sum_{i=0}^{7} e_i (m\pi f)^{i/3},
\] (9)

where \( d \) is the source distance. The post-Newtonian expansion of the phase includes spin-orbit and tidal corrections. It can be written as \( \psi(f) = \psi_{pp} + \psi_T \), where the point-particle term is
\[
\psi_{pp}(f) = 2\pi c_4 - \phi_c - \frac{\pi}{4} \left( M\pi f \right)^{3/2} \sum_{i=0}^{7} e_i (m\pi f)^{i/3},
\] (10)

and \( c_4 \) and \( \phi_c \) are the time and the phase at coalescence. The coefficient \( e_i \) are listed in \([27, 28]\). The tidal contribution \( \psi_T \) is given by \([20, 29]\):
\[
\psi_T(f) = \frac{-117A}{8\sqrt{\pi} m^3} x^{5/3} \left[ 1 + \frac{3115}{1248} x - \frac{\pi x^{3/2}}{16} + \left( \frac{23073805}{3302208} + \frac{20}{81} \right) x^2 - \frac{4283}{1092} \pi x^{5/3} \right],
\] (11)

where \( x = (m\pi f)^{5/3} \) and \( A \) is the averaged tidal deformability, which for BH-NS binaries reads \([30]\): \( A = \Lambda^{(3/12)^2} \).

We consider non rotating NSs, as this is believed to be a reliable approximation of real astrophysical systems \([31, 32]\). The GW waveform is therefore described in terms of the following set of parameters, \( \theta = (\ln A, t_c, \phi_c, \ln M, \ln v, \Lambda, \beta) \), where \( \beta \) is the 2 PN spin-orbit contribution in \( \psi_{pp} \):
\[
\beta = \frac{\chi_{\text{BH}}}{12} \left[ 113 \frac{M_{\text{BH}}^2}{m^2} + 75\nu \right] \mathbf{L} \cdot \mathbf{S}_{\text{BH}},
\] (12)

where \( \mathbf{L}, \mathbf{S}_{\text{BH}} \) are the unit vectors in the direction of the orbital angular momentum and of the spin, respectively. We choose the BH spin aligned with the orbital angular momentum. Moreover, since \( \chi_{\text{BH}} \leq 1 \), \( \beta \leq 9.4 \); therefore we consider the following prior probability distribution on \( \beta \):
\[
p^{\text{prior}}(\beta) \propto \exp \left[ - \frac{1}{2} \left( \beta / (9.4)^2 \right)^2 \right].
\]

Thus, we need to compute a \( 7 \times 7 \) Fisher matrix. However, since \( \ln A \) is uncorrelated with the other variables, we perform derivatives only with respect to the remaining six parameters \( \theta = (t_c, \phi_c, \ln M, \ln v, \Lambda, \beta) \). In our analysis we consider both second (AdvLIGO/Virgo) and third generation (Einstein Telescope, ET, \([33]\)) detectors. For AdvLIGO/Virgo we use the ZERO_DET.high_P noise spectral density of AdvLIGO \([34]\), in the frequency ranges \([20 \ Hz, f_{\text{isco}}]\); for the Einstein Telescope we use the analytic fit of the sensitivity curve provided in \([35]\), in the range \([10 \ Hz, f_{\text{isco}}]\). \( f_{\text{isco}} \) is the frequency of the Kerr ISCO including corrections due to NS self-force \([36]\):
\[
f_{\text{isco}} = \frac{M_{\text{BH}}}{m \pi} \left[ \Omega_{\text{BH}} + 1 + \nu_{\text{GEO}}(\Omega_{\text{BH}}) \right],
\] (13)

with \( \Omega_{\text{BH}} = \Omega(\Omega_{\text{BH}}) \frac{M_{\text{BH}}}{f_{\text{isco}}} \Omega(\Omega_{\text{BH}} + 1). \)

We model the NS structure by means of piecewise polytropes, \([24]\). The core EoS changes with an overall pressure shift \( P_1 \), specified at the density \( \rho_1 = 5.0119 \times 10^{14} \text{ g/cm}^3 \). Choosing \( \Gamma_{\text{core}} = 3 \) and \( P_1 = (10^{13.85}, 10^{13.55}, 10^{13.45}, 10^{13.35}) \text{ g/cm}^3 \) we obtain four EoS, 2H, H, HB and B, which denote very stiff, moderately stiff and soft EoS, respectively. The stellar parameters for \( M_{\text{NS}} = (1.2, 1.35) M_{\odot} \), are shown in Table \[I\].

| EoS | \( M_{\text{NS}}(M_{\odot}) \) | \( C \) (km) | \( \beta \) (km) |
|-----|-----------------|----------|--------|
| 2H  | 1.2             | 0.117    | 75991  |
|     | 1.2             | 0.145    | 21232  |
|     | 1.2             | 0.153    | 15090  |
|     | 1.2             | 0.162    | 10627  |

| EoS | \( M_{\text{NS}}(M_{\odot}) \) | \( C \) (km) | \( \beta \) (km) |
|-----|-----------------|----------|--------|
| 2H  | 1.35            | 0.131    | 72536  |
|     | 1.35            | 0.163    | 18964  |
|     | 1.35            | 0.172    | 13161  |
|     | 1.35            | 0.182    | 8974   |

**Table I.** For each EoS we show the NS mass, the compactness \( C = M_{\text{NS}}/R_{\text{NS}} \), and the tidal deformability \( \beta \).

**Numerical results.**—Following the strategy previously outlined, we compute the minimum and maximum probabilities \([9]\) that the coalescence of a BH-NS system produces a remnant disk with mass above a threshold \( \dot{M}_{\text{rem}} \), for the NS models listed in Table \[I\] and different values of the mass ratio \( q \). The results are given in Table \[I\] for \( q = 3 \) and \( q = 7 \), black hole spin \( \chi_{\text{BH}} = (0.2, 0.5, 0.9) \), \( M_{\text{NS}} = (1.2, 1.35) M_{\odot} \), and disk mass thresholds \( \dot{M}_{\text{rem}} = 0.01 M_{\odot} \).

For AdvLIGO/Virgo we put the source at a distance of 100 Mpc. For ET the binary is at 1 Gpc. In this case the signal must be suitably redshifted \([23, 37]\), and we have assumed that \( z \) is known with a fiducial error of the order of 10% \([38]\).

The first clear result is that as the BH spin approaches the highest value we consider, \( \chi_{\text{BH}} = 0.9 \), and for low mass ratio \( q = 3 \), the probability that a BH-NS coalescence produces a disk with mass above the threshold is insensitive to the NS internal composition, and it approaches unity for all considered configurations. These would be good candidates for GRB production. For the highest mass ratio we consider, \( q = 7 \), the probability to form a sufficiently massive disk depends on the NS mass and EoS, and on the detector. In particular, it decreases as the EoS softens, and as the NS mass increases. This is a general trend, observed also for smaller values of \( \chi_{\text{BH}} \). However, when \( \chi_{\text{BH}} = 0.9 \) the probability that the coalescence is associated to a SGRB is always \( \geq 50\% \).

Let us now consider the results for \( \chi_{\text{BH}} = 0.2 \). If the NS mass is \( 1.2 M_{\odot} \), the probability that a detected GW signal from a BH-NS coalescence is associated to the formation of a black hole with a disk of mass above threshold is \( \geq 50\% \) for both
Expected to reach a sensitivity such that the steady improvement of the efficiency of computational facilities experienced in recent years will continue, reducing the time needed to obtain these parameters from a detected signal. Moreover, the higher sensitivity will allow to detect sources in a much larger volume space, thus increasing the detection rates. In this perspective, the method we envisage in this paper will be useful to trigger the electromagnetic follow-up of a GW detection, searching for the afterglow emission of a SGRBs.

Waiting for the future, the method we propose can be used in the data analysis of advanced detectors as follows:

- Table II indicates the systems which are more likely to produce accretion disks sufficiently massive to generate a SGRB. The table can be enriched including more NS equations of state or more binary parameters; however, it already contains a clear information on which is the range of parameters to be used in the GW data analysis, if the goal is to search for BH-NS signals which may be associated to a GRB. For instance, Table II suggests that searching for mass ratio smaller than, or equal to, 3-4, and values of the black hole angular momentum larger than 0.5-0.6 would allow to save time and computational resources in low latency search. In addition, it would allow to gain sensitivity in externally triggered searches performed in time coincidence with short GRBs observed by gamma-ray satellites.

- If a SGRB is observed sufficiently close to us in the electromagnetic waveband, the parameters of the GW signal detected in coincidence would allow to set a threshold on the mass of the accretion disk. If the GW signal comes, say, from a system with a BH with spin $\chi_{BH} = 0.5$, mass ratio $q = 7$, and neutron star mass $M_{NS} = 1.2 M_\odot$, from Table II equations of state softer than the EOS 2H would be disfavoured. Thus, we would be able to shed light on the dynamics of the binary system, on its parameters and on the internal structure of its components. We would enter into the realm of gravitational wave astronomy.

Finally, it is worth stressing that as soon as the fit [1] will be extended to NS-NS coalescing binaries, this information will be easily implemented in our approach. Being the rate of NS-NS coalescence higher than that of BH-NS, our approach will acquire more significance, and will be a very useful tool to study these systems.

Acknowledgements.—We would like to thank M. Branchesi for her comments on the paper, which helped us to clarify several issues. We would like to thank L. Pagano, R. Amato and M. Muccino for useful discussions and comments. A. M. is supported by a "Virgo EGO Scientific Forum" (VESF) grant. Partial support comes from the CompStar network, COST Action MP1304.

---

Table II. We show the probability range $[P_{MIN}, P_{MAX}]$ that the coalescence of a BH-NS binary produces a disk mass larger than $M_{d \text{rem}} = 0.01 M_\odot$ for AdLIGO/Virgo (AdV) and for the Einstein telescope (ET), for binaries with $q = 3$ and $q = 7$, NS masses $(1.2, 1.35) M_\odot$, and BH spin $\chi_{BH} = (0.2, 0.5, 0.9)$. Sources are assumed to be at $d = 100$ Mpc for advanced detectors, and $d = 1$ Gpc for the Einstein Telescope. The star compactness $C_s$ is estimated throughout the universal relation [3].

| $M_{NS} = 1.2 M_\odot$ | $q = 3$ | $d = 100$ Mpc | $C_s$ | $\chi_{BH}$ | $M_{\text{d,rem}}$ | $q = 3$ | $d = 1$ Gpc | $C_s$ | $\chi_{BH}$ | $M_{\text{d,rem}}$ |
|------------------------|--------|---------------|------|-------------|-----------------|--------|---------------|------|-------------|-----------------|
| $\chi_{BH}$ | 0.5 | 0.9 | 0.2 | 0.5 | 0.9 | 0.2 | 0.5 | 0.9 | 0.2 | 0.5 | 0.9 |
| 2H | 0.118 | 1 | 1 | 1 | 1 | 0.4 | 0.8 | 0.9 | 1 | 0.3 | 0.4 | 0.7 | 0.8 | 0.9 | 1 |
| H | 0.147 | [0.6-0.9] | 1 | 1 | [0.9-1] | 1 | 1 | 0.4 | 0.4 | [0.8-0.9] | 0.3 | [0.3-0.4] | 1 |
| HB | 0.155 | [0.5-0.7] | [0.9-1] | 1 | [0.7-0.8] | 1 | 1 | 0.4 | 0.4 | [0.7-0.8] | 0.3 | 0.3 | 0.9 |
| B | 0.164 | [0.4-0.6] | [0.7-0.8] | 1 | [0.5-0.6] | 1 | 1 | 0.4 | 0.4 | [0.6-0.7] | 0.3 | 0.3 | [0.7-0.8] | 1 |

---

[1] http://www.ligo.caltech.edu  http://www.ego-gw.it
