DEFECT FORMULA FOR NODAL COMPLETE INTERSECTION THREEFOLDS

SLAWOMIR CYNK

Abstract. We generalize Werner’s defect formula for nodal hypersurfaces in \( \mathbb{P}^4 \) to the case of a nodal complete intersection threefold.

1. Introduction

The main goal of this paper is to give a formula for Hodge numbers of a nodal complete intersection threefold satisfying certain non-degeneracy condition. Hodge numbers of a transversal complete intersection in a projective space can be computed from the generating function of \( \chi_y \)-genus [14, Thm. 22.1.1, Thm. 22.1.2]. In the special case of a threedimensional complete intersection \( X \) of hypersurfaces of degrees \( (d_1, d_2, \ldots, d_r) \) in \( \mathbb{P}^{r+3} \) we can use the Hirzebruch–Riemann–Roch theorem for the vector bundle \( \Omega_X^1 \) and the Lefschetz hyperplane theorem to compute

\[
h^{1,2}(X) = \frac{1}{24} c_1 c_2 - \frac{1}{2} c_3 + 1
\]

and then

\[
h^{1,2}(X) = \left( \frac{11}{24} \sigma_1^3 - \frac{5(r+4)}{12} \sigma_1^2 + \left( \frac{(r+4)(9r+25)}{48} - \frac{11}{12} \sigma_2 \right) \sigma_1 + \frac{5(r+4)}{12} \sigma_2 + \frac{1}{3} \sigma_3 - \frac{3(r+4)(r+3)}{48} \right) \sigma_r + 1
\]

where \( \sigma_i \) is the \( i \)-th elementary symmetric function evaluated at \( (d_1, d_2, \ldots, d_r) \). If \( X = \{ F = 0 \} \) is a degree \( d \) hypersurface in \( \mathbb{P}^4 \) there is moreover isomorphism

\[
H^{2,1}(X) \cong (K[X_0, \ldots, X_4]/\text{Jac}(F))_{2d-5}
\]

of the Hodge group with degree \( 2d-5 \) component of the Jacobian algebra of \( X \) (an explicit isomorphism is described in [19]).

First formulae for the Hodge numbers of singular threefolds were given by Clemens [3] for double coverings of \( \mathbb{P}^3 \) branched along a nodal double surface and then by Werner [26] for nodal hypersurfaces in \( \mathbb{P}^4 \). Clemens’ and Werner’s formulae relate the Hodge numbers of a resolution of a nodal double solid and a nodal hypersurface to the defect of certain linear system. These results were reproved with algebraic methods (characteristic free) and generalized to the case of hypersurfaces with A-D-E singularities satisfying certain vanishings. The proofs follow the line of [19], vanishing of a certain cohomology group breaks–up the long cohomology sequence.

Our goal is to generalize Werner’s formula to the case of a nodal complete intersection in projective space, in this case the considered exact sequence does not break, instead of vanishing we explicitly describe the image of one of the maps in

2000 Mathematics Subject Classification. Primary: 14J30; Secondary 14C30, 32S25.
Research partially supported by the National Science Center grant no. 2014/13/B/ST1/00133.
This work was partially supported by the grant 346300 for IMPAN from the Simons Foundation and the matching 2015-2019 Polish MNiSW fund.
the sequence. Three dimensional node admit two types of a special resolution. The first one is the blow–up of the singular locus and is called big resolution. Small resolution replace singular point with a line, in general small resolution need not be projective. In our proofs we consider the big resolution, but the Hodge numbers of any small one follows easily.

Nodal threefolds play important role in several branches of algebraic geometry, first examples of Calabi–Yau threefolds with small absolute value of the Euler characteristic were constructed as small resolutions of nodal hypersurfaces and complete intersection (cf. [12, 14, 27, 24, 18]). A $\mathbb{Q}$–factorial nodal quartic 3–folds and nodal double sextic are non–rational which raised the question of minimal number of nodes on non–$\mathbb{Q}$–factorial nodal threefold of given type (cf. [2, 6, 17, 16, 21]). Special properties of small resolutions of nodal threefolds were used to constructed examples of Calabi–Yau spaces in positive characteristic non–liftable to characteristic zero. Contraction of a class of lines on a Calabi–Yau threefold to nodes followed by a smoothing of the nodal threefolds is the so–called conifold transition which can connect different families of Calabi–Yau threefolds ([23]).

2. Preliminaries

Let $X = H_1 \cap \cdots \cap H_r \subset \mathbb{P}^{r+3}$ be a nodal complete intersection in $\mathbb{P}^{r+3}$ of smooth hypersurfaces of dimensions $d_1, \ldots, d_r$, denote $d := d_1 + \cdots + d_r$. Assume moreover that the intersections $Y = H_1 \cap \cdots \cap H_{r-1}$ is smooth.

We have the following Bott–type vanishing

$$H^i(\Omega^1_Y(kX)) = 0, \quad \text{for } i > 4, \; k > 0.$$ 

Let $\Sigma := \text{Sing } X$ be the singular locus of $X$, $\mu = \# \Sigma$ – the number of nodes of $X$ and let $\sigma: \tilde{Y} \longrightarrow Y$ be the blow–up of $Y$ at the singular locus of $X$. Denote by $\tilde{X}$ the strict transform of $X$, let $E := \sigma^{-1}(\Sigma)$ be the exceptional divisor of $\sigma$. Then $\tilde{X}$ is non–singular and $E$ is a disjoint union of projective 3–spaces.

**Proposition 1.**

$$H^0(\Omega^1_Y(\tilde{X})) \cong H^0(\Omega^1_Y(X)),$$

$$H^i(\Omega^1_Y(\tilde{X})) = 0, \quad \text{for } i > 0,$$

$$H^i(\Omega^1_Y(2\tilde{X})) \cong H^i(\Omega^1_Y(2X) \otimes \mathcal{J}_E), \quad \text{for } i \geq 0.$$ 

**Proof.** We have $\Omega^1_Y(\tilde{X}) \cong \sigma^*\Omega^1_Y(X) \otimes \mathcal{O}_Y(E)$, first two assertions follows now from $\sigma_*\mathcal{O}_Y(E) \cong \mathcal{O}_Y$, $R^i\sigma_*\mathcal{O}_Y(E) = 0$, projection formula and (degenerate case) of Leray spectral sequence. Applying the direct image functor to the exact sequence $0 \longrightarrow \mathcal{O}_Y(-E) \longrightarrow \mathcal{O}_Y \longrightarrow \mathcal{O}_E \longrightarrow 0$ we get $\sigma_*\mathcal{O}_Y(-E) \cong \mathcal{J}_E$ and $R^i\sigma_*\mathcal{O}_Y(-E) = 0$, the last assertion follows now from $\Omega^1_Y(2\tilde{X}) \cong \sigma^*\Omega^1_Y(2X) \otimes \mathcal{O}_Y(-E)$.

**Corollary 2.** We have the following exact sequence

$$H^0\Omega^1_Y(2X) \longrightarrow H^0(\Omega^1_Y(2X) \otimes \mathcal{O}_X) \longrightarrow H^1\Omega^1_X(\tilde{X}) \longrightarrow 0$$

**Proof.** By adjunction formula $\Omega^3_X(\tilde{X}) \cong \Omega^1_Y(2\tilde{X}) \otimes \mathcal{O}_{\tilde{X}}$, assertion follows now from the previous proposition and the long exact sequence associated to

$$0 \longrightarrow \Omega^1_Y(\tilde{X}) \longrightarrow \Omega^1_Y(2\tilde{X}) \longrightarrow \Omega^1_Y(2\tilde{X}) \otimes \mathcal{O}_{\tilde{X}} \longrightarrow 0.$$
Proposition 3.

\[ H^i(\Omega^3_Y(\tilde{X})) \cong H^i(\Omega^3_Y(X)), \quad i \geq 0, \]
\[ H^i(\Omega^3_Y) = 0, \quad i \leq 2, \]

**Proof.** By direct computations in local coordinates we verify

\[ \sigma^*\Omega^3_Y(\log E)(-3E) \]
and so

\[ \sigma^*(\Omega^3_Y(X)) \cong \Omega^3_Y(\log E)(-3E) \otimes \sigma^*O_Y(X). \]

Tensoring the exact sequence

\[ 0 \to \Omega^3_Y(\log E)(-E) \to \Omega^3_Y \to \Omega^2_E \to 0 \]
with \( O_Y(\tilde{X}) \cong O_Y(-2E) \otimes \sigma^*(O_Y(X)) \) we get

\[ 0 \to \sigma^*(\Omega^3_Y(X)) \to \Omega^3_Y(\tilde{X}) \to O_E(-2) \to 0. \]

Now, using the direct image operator and projection formula we get

\[ \sigma_*\Omega^3_Y(\tilde{X}) \cong \Omega^3_Y(X) \quad \text{and} \quad R^i\sigma_*\Omega^3_Y(\tilde{X}) = 0, \]

the assertion follows from the Leray spectral sequence. Second assertion follows in a similar manner from the exact sequence

\[ 0 \to \sigma^*(\Omega^3_Y) \otimes O_Y(2E) \to \Omega^3_Y \to \Omega^2_E \to 0 \]
and the Lefschetz hyperplane theorem \( H^1(\Omega^3_Y) = 0. \)

Lemma 4. The following sequence is exact

\[ 0 \to H^1\Omega^3_Y \to H^1\Omega^3_Y(\log \tilde{X}) \to H^1\Omega^2_X \to 0 \]

**Proof.** We have \( H^0(\Omega^2_X) = H^2(O_X) = 0 \) (Prop. 3) and \( H^2(\Omega^3_Y) = 0 \) (Prop. 3), now the assertion follows by the long cohomology exact sequence derived from

\[ 0 \to \Omega^3_Y \to \Omega^3_Y(\log \tilde{X}) \to \Omega^2_X \to 0 \]

Lemma 5. The following sequence is exact

\[ 0 \to H^0(\Omega^3_Y(X)) \to H^0(\Omega^3_X(\tilde{X})) \to H^1(\Omega^3_Y(\log \tilde{X})) \to H^1(\Omega^3_Y(X)) \to H^1(\Omega^3_X(\tilde{X})) \]

**Proof.** Follows from the short exact sequence

\[ 0 \to \Omega^3_Y(\log \tilde{X}) \to \Omega^3_Y(X) \to \Omega^3_X(\tilde{X}) \to 0 \]
and previous lemmata.
3. Main result

Now, we shall formulate and prove our main result

**Theorem 6.** Let $F_1, \ldots, F_r \in S := k[X_0, \ldots, X_{r+3}]$ be homogeneous polynomials in $r + 4$ variables such that

- varieties $V(F_1, \ldots, F_i)$ are smooth for $i = 1, \ldots, r - 1$,
- variety $X := V(F_1, \ldots, F_r)$ is a threefold with ordinary double points as the only singularities.

Denote by $\Sigma := \text{Sing}(X)$ the set of singular points of $X$, $\mu := |\Sigma|$ number of its elements and $d := d_1 + \cdots + d_r$. Let $V$ be a linear combination of rows of the matrix $\bigwedge^{r-1} \text{Jac}(F_1, \ldots, F_r)$ which does not vanish at any point of $\Sigma$ and let $I$ be the ideal generated by entries of $V$.

Then
\[ h^{1,1}(\tilde{X}) = 1 + \delta, \quad h^{1,2}(\tilde{X}) = h^{1,2}(X_{\text{smooth}}) - \mu + \delta \]
where
\[ \delta := \mu - (\dim_k I^{2d-2r-3} - \dim_k (I \cap J_\Sigma)^{2d-2r-3}) \]
is the defect of the ideal $I$ at the singular locus of $X$.

**Lemma 7.** There exists an epimorphism
\[ \bigoplus_{i=1}^{r-1} S^{d_i - 1} \rightarrow H^1(\Omega_X^3(X)). \]

**Proof.** Let $Z$ be a complete intersection of $r - 2$ hypersurfaces $H_i$. Using Bertini theorem we can assume without lost of generality that $Z := H_1 \cap \cdots \cap H_{r-2}$ is a smooth fivefold. By similar arguments as before we easily get exact sequences
\[ H^1(\Omega_X^3(X)) \rightarrow H^1(\Omega_X^4(X)) \rightarrow 0 \]
\[ H^0(\Omega_X^3(X) \otimes \mathcal{O}_Z(Y)) \rightarrow H^1(\Omega_X^3(X)) \rightarrow H^1(\Omega_X^4(X + Y)) \rightarrow 0 \]
By adjunction and the Bott vanishing we get recursively that $H^0(\Omega_X^3(X) \otimes \mathcal{O}_Z(Y))$ is an image of $S^{d + d_r - 1}$. Now, the lemma follows by induction.

Consider the following commutative diagram
\[
\begin{array}{ccc}
S^{d_i - 1} & \rightarrow & H^1(\Omega_X^3(X)) \\
\bigoplus_{i=1}^{r-1} S^{d_i - 1} & \rightarrow & H^1(\Omega_X^3(X)) \\
H^0(\Omega_X^3(2X)) & \delta & H^0(\Omega_X^3(2X) \otimes \mathcal{O}_Z) \\
\beta & & \gamma \\
S^{d + d_r - 1} & \eta & H^1(\Omega_X^3(X)) \\
\kappa^\mu & \theta & H^1(\Omega_X^3(2X) \otimes J_\Sigma)
\end{array}
\]
All the maps except $\beta$ are determined by the proofs we presented, on the other hand the identification $H^0(\Omega_X^3(2X) \otimes \mathcal{O}_Z) \cong \kappa^\mu$ is not given explicitly.

Denote by $\Omega$ the form $\Omega := \sum_{i=0}^{r+3} X_i dX_0 \wedge \cdots \wedge dX_i \wedge \cdots \wedge dX_{r+3}$. The map $\theta$ to a function $\lambda$ associates Poincare residue of the form $\frac{dF_1}{F_1} \wedge \cdots \wedge \frac{dF_r}{F_r}$ evaluated at points of $\Sigma$. When we want to identify values of
θ with vectors we have to evaluate coefficients of resulting form, which is the same as evaluate quotients of A by \((r - 1) \times (r - 1)\) minors of the jacobian matrix of \(F_1, \ldots, F_r\).

At each point of \(Σ\) the jacobian matrix \(\text{Jac}(F)\) has rank \(r - 1\), so the matrix \(\bigwedge^{r-1} \text{Jac}(F)\) of \((r - 1) \times (r - 1)\) minors has rank 1. By our assumption all the rows of this matrix are non–zero, so at every point of \(Σ\) some columns are zero the other columns have are proportional and have only non–zero entries. It may happen however that each column vanish at some point of \(Σ\). In order to circumvent this problem we take a random linear combination of columns \((V_1, \ldots, V_r)\) which does not vanish at any point.

Composing with \(α, γ, φ\), we see that \(β\) can be identified in the same manner as \(θ\) through remaining \((r - 1) \times (r - 1)\) minors of the jacobian matrix \(\text{Jac}(F)\) of \(F_1, \ldots, F_r\), main difference is that from \(S^{d_i - r - 4}\) we pass through \(H^0(Ω^4_Y(X) \otimes \mathcal{O}_Z(Y))\) instead of \(H^0(Ω^4_Y(2X))\) which means that we have to multiply by \(F_r/F_i\). Evaluating at a singular point we have to pass to the limit equal \(V_i/V_r\). Finally, denoting \(Σ := \{P_1, \ldots, P_r\}\) the value of \(β\) at \(P_i \in S^{d_i - r - 4}\) is \(\mathcal{V}_i(F)\). Denote the ideals \(I = (V_1, \ldots, V_r), J = I \cap \mathcal{J}_Σ\) and by \(I^k\) (resp. \(J^k\)) vector space of degree \(k\) forms in \(I\) resp. \(J\). We have proved the following proposition.

**Proposition 8.**

\[
\dim(\text{Im}(δ) + \text{Im}(β)) = \dim I^{2d - 2r - 3} - \dim J^{2d - 2r - 3}.
\]

**Proof of Thm.** By simple linear algebra we get

\[
h^1(Ω^2_X) = h^0(Ω^4_Y(2X)) - h^0(Ω^4_Y(X)) - h^0(Ω^4_Y(X)) + h^1(Ω^3_Y(X)) - \dim(\text{Im}(β + \text{Im}(δ)).
\]

Repeating the computations for a smooth complete intersection \(X_{\text{smooth}}\) of the same type we get

\[
h^1(Ω^2_{X_{\text{smooth}}}) = h^0(Ω^4_Y(2X)) - h^0(Ω^4_Y(X)) - h^0(Ω^4_Y(X)) + h^1(Ω^3_Y(X))
\]

so by previous Proposition

\[
h^{1,2}(\tilde{X}) = h^{1,2}(X_{\text{smooth}}) - μ + δ.
\]

As \(\tilde{X}\) is the blow–up of \(μ\) lines in any small resolution \(\tilde{X}\) we get formula for \(h^{1,2}(\tilde{X})\), formula for \(h^{1,1}(\tilde{X})\) follows now from an easy Milnor number computation.  

### 4. Examples

Defect formula in main theorem can be easily implemented in a computer algebra system, we use Magma code [11].

**Example.** Denote by \(X(d_1, \ldots, d_r; e_1, \ldots, e_{r+1})\) general complete intersection of hypersurfaces of degrees \(d_1, \ldots, d_r\) in \(\mathbb{P}^{r+3}\) containing general complete intersection surface of degrees \(e_1, \ldots, e_{r+1}\). In [6] these nodal threefolds were studied as candidates for non–factorial nodal complete intersections with minimal number of nodes (cf. [10], [17]). Using our main result we check that the defect equals 1 for the following cases with \(r = 2\) and \(d_1 + d_2 = 6, (\text{Calabi–Yau cases})\),
Example. We use our main result to verify computations of the Hodge numbers of some rigid Calabi–Yau complete intersections.

Complete intersection of four quadrics in projective space $\mathbb{P}^7$

$Y_0^2 = X_0^2 + X_1^2 + X_2^2 + X_3^2$
$Y_1^2 = X_0^2 - X_1^2 + X_2^2 - X_3^2$
$Y_2^2 = X_0^2 + X_1^2 - X_2^2 - X_3^2$
$Y_3^2 = X_0^2 - X_1^2 - X_2^2 + X_3^2$

studied by van Geemen and Nygaard in [11]. Using counting points in characteristic 17 they proved that small resolution of this complete intersection is rigid, i.e. $h^{1,1} = 32, h^{1,2} = 0$. The Hodge numbers of a smooth complete intersection of four quadrics equal

$h^{1,1} = 1, h^{1,2} = 65.$

Using magma code we compute

$\dim_C I^5 = 144, \dim_C (I \cap J) = 79, h^{1,1} = 96, h^{1,2} = 96 - (144 - 79) = 31$

and finally for the Hodge numbers of the van Geemen Nygaard complete intersection equals

$h^{1,1} = 1 + 31 = 32, h^{1,2} = 65 - 96 + 31 = 0.$

as computed in [11].

For the complete intersection of a quadric in quartic in $\mathbb{P}^5$ given by [27]

$x_1^2 + x_2^2 + x_3^2 = x_4^2 + x_5^2 + x_6^2$
$x_1^4 + x_2^4 + x_3^4 = x_4^4 + x_5^4 + x_6^4$

In this case

$\dim_C I^5 = 200, \dim_C (I \cap J) = 111, h^{1,1} = 122, h^{1,2} = 122 - (200 - 111) = 33$

and

$h^{1,1} = 1 + 33 = 34, h^{1,2} = 89 - 122 + 33 = 0.$

Desingularized self fiber product of the Beauville surface $\Gamma(3)$ (constructed by Schoen in [24]) is birational to the complete intersection

$x_1^3 + x_2^3 + x_3^3 = x_4^3 + x_5^3 + x_6^3$
$x_1x_2x_3 = x_4x_5x_6$

with 108 nodes. We get

$\dim_C I^5 = 219, \dim_C (I \cap J) = 146, h^{1,1} = 108, h^{1,2} = 108 - (219 - 146) = 35$

and

$h^{1,1} = 1 + 35 = 36, h^{1,2} = 73 - 108 + 35 = 0.$

| $d_1$ | $d_2$ | $e_1$ | $e_2$ | $e_3$ | $\mu$ | $h^{1,1}$ | $h^{1,2}$ |
|-------|-------|-------|-------|-------|-------|-----------|-----------|
| 4     | 2     | 1     | 1     | 1     | 13    | 2         | 77        |
| 4     | 2     | 2     | 1     | 1     | 18    | 2         | 72        |
| 4     | 2     | 2     | 2     | 1     | 24    | 2         | 66        |
| 4     | 2     | 2     | 2     | 2     | 32    | 2         | 58        |
| 4     | 2     | 3     | 2     | 1     | 18    | 2         | 72        |
| 4     | 2     | 3     | 2     | 2     | 24    | 2         | 66        |
| 4     | 2     | 3     | 3     | 2     | 18    | 2         | 72        |
We have also computed Hodge numbers of nodal complete intersections studied in [18, Ch. 5] confirming Meyer’s results.

References

[1] W. Bosma, J. Cannon, C. Playoust, The Magma algebra system. I. The user language, J. Symbolic Comput., 24 (1997), 235–265.
[2] I. Cheltsov, On factoriality of nodal threefolds, J. Alg. Geom. 14 (2005), 663–690.
[3] C. H. Clemens, Double solids. Adv. in Math. 47 (1983), 107–230.
[4] S. Cynk, Defect of a nodal hypersurface. Manuscripta Math. 104 (2001), 325–331.
[5] Defect via differential forms with logarithmic poles, Math. Nachr. 284 (2011), no. 17–18, 2148–2158.
[6] S. Cynk, S. Rams, Non-factorial nodal complete intersection threefolds. Commun. Contemp. Math. 15 (2013), no. 5, 1250064.
[7] V. Di Gennaro, D. Franco, Factoriality and Néron-Severi groups. Commun. Contemp. Math. 10 (2008), 745–764.
[8] A. Dimca, Betti numbers of hyperplanes and defects of linear systems, Duke Math. Jour. 60 (1990), 285–294.
[9] A. Dimca, Singularities and topology of hypersurfaces. Springer 1992.
[10] H. Esnault, E. Viehweg, Lectures on vanishing theorems. Birkhäuser 1992.
[11] B. van Geemen, N. Nygaard, On the geometry and arithmetic of some Siegel modular threefolds, Journal of Number Theory 53 (1995), 45–87.
[12] B. van Geemen, J. Werner, Nodal quintics in P4. Arithmetic of complex manifolds (Erlangen, 1988), 48–59, Lecture Notes in Math., 1399, Springer, Berlin, 1989.
[13] F. Hirzebruch, Some examples of threefolds with trivial canonical bundle. In: Hirzebruch, F. (ed.). Gesammelte Abhandlungen. Collected papers, vol. II, pp. 757–770. Berlin Heidelberg New York: Springer 1987.
[14] F. Hirzebruch, Topological methods in algebraic geometry. Reprint of the 1978 edition. Classics in Mathematics. Springer-Verlag, Berlin, 1995.
[15] K. Hulek, R. Kloosterman, Calculating the Mordell-Weil rank of elliptic threefolds and the cohomology of singular hypersurfaces. Preprint available at arXiv: math/0806.2025, 2008.
[16] R. Kloosterman, Nodal complete intersection threefold with defect
[17] D. Kosta, Factoriality of complete intersections in P5. Tr. Mat. Inst. Steklova 264, 109–115, 2009.
[18] C. Meyer, Modular Calabi-Yau Threefolds, Fields Institute Monograph 22 (2005), AMS.
[19] C. Peters, J. Steenbrink, Infinitesimal variations of Hodge structure and the generic Torelli problem for projective hypersurfaces. Classification of algebraic and analytic manifolds (Katata, 1982), 399–463, Progr. Math. 39, Birkhäuser 1983.
[20] C. Peters, J. Steenbrink, Mixed Hodge Structures. Springer 2008
[21] V. V. Przhiyalkovskii, I. Cheltsov, K. A. Shramov, Hypervallaric and trigonal Fano threefolds. (Russian) Izv. Ross. Akad. Nauk Ser. Mat. 69 (2005), 145–204.
[22] S. Rams, Defect and Hodge numbers of hypersurfaces. Adv. Geom. 8 (2008), 257–288.
[23] M. Reid, Miles, The moduli space of 3-folds with K=0 may nevertheless be irreducible. Math. Ann. 278 (1987), no. 1-4, 329–334.
[24] C. Schoen, On fiber products of rational elliptic surfaces with section. Math. Z. 197 (1988), no. 2, 177–199.
[25] D. van Straten, A quintic hypersurface in P4 with 130 nodes, Topology 32 (1993), 857-864.
[26] J. Werner, Kleine Auflösungen spezieller dreidimensionaler Varietäten, Bonner Math. Schriften 186 (1987).
[27] J. Werner, B. van Geemen, New examples of threefolds with c1=0. Math. Z. 203 (1990), no. 2, 211–225.

Institute of Mathematics, Jagiellonian University, ul. Łojasiewicza 6, 30-348 Kraków, Poland
E-mail address: slawomir.cynk@uj.edu.pl

Institute of Mathematics of the Polish Academy of Sciences, ul. Śniadeckich 8, 00-956 Warszawa, Poland