Entropia: A Family of Entropy-Based Conformance Checking Measures for Process Mining

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Abstract—This paper presents a command-line tool, called Entropia, that implements a family of conformance checking measures for process mining founded on the notion of entropy from information theory. The measures allow quantifying classical non-deterministic and stochastic precision and recall quality criteria for process models automatically discovered from traces executed by IT-systems and recorded in their event logs. A process model has “good” precision with respect to the log it was discovered from if it does not encode many traces that are not part of the log, and has “good” recall if it encodes most of the traces from the log. By definition, the measures possess useful properties and can often be computed fast.

I. INTRODUCTION

Process mining is a research field concerned with extracting knowledge from event sequence data that is stored in event logs. Conceptually, process mining techniques assume that events have at least three attributes: a timestamp, a case identifier and an activity type [1]. Process mining techniques support various process analysis tasks including automatic process discovery, conformance checking, and variant analysis [2].

Conformance checking refers to those process mining techniques that compare the behavior captured in an event log with a normative process model [3]. A key challenge for research on conformance checking is the definition of appropriate measures that quantify the extent of correspondence between the log and the model. A rich spectrum of measures have been proposed, though many of them in an ad hoc way [4]. The recent stream of work on entropy-based measures provides a solid theoretical foundation for conformance checking measures with sound properties [5], [6], [7], [8], [9], but tool support has been limited in the past.

In the paper at hand, we address this gap. Specifically, we present a command-line tool, called Entropia, that implements entropy-based conformance checking techniques. The tool is publicly available[1] and aims to support process analysts in several scenarios for measuring commonalities and discrepancies between process models and event logs. Finally, the reader can take a look at a screencast[2] that demonstrates the tool and check the user guide[3] that contains a comprehensive collection of examples and tutorials on using Entropia.

The paper proceeds as follows. Section II gives an overview of the theoretical foundations of conformance checking. Section III introduces the Entropia tool using a practical use case highlighting its analysis features. Section IV discusses the maturity of the work. Section V provides illustrative examples. Section VI discusses computational performance and current limitations, before Section VII concludes.

II. CONFORMANCE CHECKING

The assessment of the model quality with respect to an event log is paramount for process mining [1]. Buijs et al. [10] introduce four main quality dimensions, namely fitness, precision, generalization, and simplicity, which are currently considered the de facto standard. Fitness captures the degree to which the traces recorded in the event log can be replayed on the process model. Precision penalizes the extra behavior introduced by the model that is not recorded in the event log. Conversely, generalization indicates how well the model can support unforeseen traces. Finally, simplicity denotes the capability of the model to express the behavior of the event log while keeping the model easy to understand.

Conformance checking techniques provide a number of approaches for assessing the four quality dimensions. One can broadly classify them into two categories: descriptive and quantitative. Descriptive techniques construct comprehensive artifacts that aim to explain various aspects of the studied

[1] https://github.com/jbpt/codebase/tree/master/jbpt-pm
[2] https://youtu.be/RZVEFMuH684
[3] https://github.com/jbpt/codebase/tree/master/jbpt-pm/entropia/guide.pdf
criterion, e.g., a description of all the commonalities and discrepancies between a trace and a process model. Quantitative techniques measure the quantity of the studied phenomenon, e.g., as a number between zero and one. Orthogonal to this classification is the classification of conformance checking techniques into non-stochastic and stochastic ones. Stochastic conformance checking techniques study relations between some stochastic aspects of the compared model and log, e.g., distributions of traces recorded in the log and described in the model. In contrast, non-stochastic techniques, even though they may rely on the probabilistic aspects of the individual compared artifacts, do not analyze the relations between them.

III. ENTROPIA

This section presents Entropia by specifying the use cases it supports (Section III-A), the core principle behind the entropy-based measuring of precision and recall (Section III-B), and the command-line interface (CLI) of the tool (Section III-C).

A. Use Cases

Entropia implements the techniques for quantifying the precision and recall quality criteria in conformance checking presented in [5], [6], [7], and [9]. Two techniques [5], [9] can be used to measure aspects that relate to stochastic precision and recall quality criteria.

B. Entropy-Based Conformance Checking

The key idea for quantifying precision and recall between a model that describes “relevant” behavior and a model that captures “retrieved” behavior is to measure the magnitude of the behavior the two models share in relation to the magnitude of the behavior of one of the models.

Specific to the process mining context, one can think of an event log as a model that specifies the relevant behavior, i.e., the behavior that provides information about the true behavior it was sampled from. On the other hand, a process model discovered from an event log specifies the “retrieved” behavior, i.e., the behavior the applied discovery algorithm constructed from the input event log.

Then, by following the principle for defining the corresponding quality criteria in information retrieval [5], one can define precision as the ratio of the magnitude of the shared behavior specified by the models of relevant and retrieved behaviors to the magnitude of the retrieved behavior. In turn, recall amounts to the ratio of the magnitude of the shared behavior to the magnitude of the relevant behavior. Figure 1 visualizes the presented idea schematically. Note that \( \text{rel} \cap \text{ret} \) refers to the behavior shared by the relevant and retrieved behaviors of the compared models.

In the figure, \( \text{rel} \) and \( \text{ret} \) stand for the relevant behavior and retrieved behavior, respectively. Function \( m \) is used to measure the magnitude of the corresponding (part) of the behavior. The conformance checking approaches implemented in Entropia interpret the behaviors of the compared models as collections of the traces that these models describe, where a trace is a sequence of process actions. Function \( m \) is implemented as the measure of the entropy of a collection of traces; note that the implemented conformance checking techniques use different notions of entropy, also in different ways, refer to Section IV.

The benefit from using the entropy to measure the magnitudes of collections of traces when calculating precision and recall is twofold. First, one can measure entropy of an arbitrary (potentially infinite) collection of traces. Second, the entropy-based precision and recall measures can achieve a range of desired properties [4], [11], [5], [8].

C. Interface

By August 2020, the Entropia tool is in version 1.5. One can invoke the tool by executing this command:

```
java -jar jbpt-pm-entropia-1.5.jar <options>.
```

The core CLI options of Entropia are listed in Table I.

| Option (full) | Option | Parameter | Description |
|--------------|--------|-----------|-------------|
| --help       | -h     |           | print help message |
| --relevant   | -rel   | <path>    | model that describes relevant traces |
| --retrieved  | -ret   | <path>    | model that describes retrieved traces |
| --silent     | -s     |           | run tool in the silent mode |
| --version    | -v     |           | get version of this tool |

The --h and --v options print the help message and tool version, respectively, while options --rel and --ret are used to specify the models of relevant and retrieved traces, respectively. To refer to a model, the user specifies its file path. The user can use option --s to run the tool in the silent mode. In this mode, the tool only prints the result of the invocation omitting the debug information and execution details. The tool accepts input models specified in one of the following formats: Extensible Event Stream (XES) [12], Petri Net Markup Language (PNML) [13], Stochastic Petri Net Markup Language (sPNML), Directly-Follows Graph (DFG), Stochastic Deterministic Finite Automaton (SDFA). The latter three formats are specific to our tool.

Further tool options allow selecting a conformance measure to be applied on the input data and configuring it. These CLI options are detailed in the next section.

IV. MATURITY

The work on the code base of the tool started in August 2017, together with the start of the work on the entropy-based approach for measuring precision and recall presented in [5]. The tool is integrated into the jBPT library [14], a compendium of open-source business process technologies, the work on which commenced in January 2009.

The approach presented in [5] suggests measuring precision and recall by interpreting the compared models, e.g., process

![Figure 1](image-url)
model and event log, as collections of traces that they describe. The models are said to specify shared behavior if and only if they describe identical traces. The magnitude of the behavior captured by each of the compared models, and of the behavior shared by the models, is determined as topological entropy [15] of the corresponding collection of traces.

In [6], the authors generalize the approach from [5] by replacing every collection of traces involved in the calculations of the precision and recall measures with the collection of all subtraces of all the traces it contains. Consequently, the shared behavior of two collections of traces is identified as a collection of all sequences of actions that are subtraces of some traces in both compared collections. Therefore, this approach considers all the shared subsequences of actions in the compared models of traces for the measurements.

While the measures described in [5] and [6] can be seen as extremes of the spectrum, either no or all subtraces are considered when determining the magnitudes of the collections of traces, the approach presented in [7] allows for a flexible analysis. In particular, based on the knowledge about the compared models, the user can specify the maximal number of allowed skipped actions in a trace described by each of the models of traces for determining the shared subtraces. This way, the user can tune the measures towards the desired sensitivity to the discrepancies in the compared behaviors.

In [8], entropy is used to extend conformance checking to stochastic process mining. An event log and a stochastic process model can be compared on whether they exhibit the same control flow, but also whether the frequency of behavior in the event log matches the probabilities of behavior in the model. To this end, both log and model are translated into stochastic deterministic finite automata, the conjunction of these automata is constructed, and the entropy of these three automata yields two measures: stochastic recall and stochastic precision. In [8], an evaluation shows the practical applicability by searching for pairwise similar process models in a 4000-model repository.

Finally, the entropic relevance measure presented in [9] is a stochastic conformance measure computed as the average number of bits required to compress (i.e., to perform the entropy coding of) a trace from the log using the information on the relative likelihood of traces encoded in the model. Table II lists the tool options to select and configure the supported conformance measures.

| Option   | Parameter | Description                  | Publ.   |
|----------|-----------|------------------------------|---------|
| -emp     |           | exact matching precision     | [6]     |
| -emr     |           | exact matching recall        | [6]     |
| -pmp     |           | partial matching precision   | [7]     |
| -cpmp    |           | partial matching recall      | [7]     |
| -cpmr    |           | controlled partial matching precision | [7]     |
| -cpmr    |           | controlled partial matching recall | [7]     |
| -srel    | <num>    | number of allowed skips in relevant traces | [9]     |
| -sret    | <num>    | number of allowed skips in retrieved traces | [9]     |
| -sp      |           | stochastic precision         | [9]     |
| -sr      |           | stochastic recall            | [9]     |
| -r       |           | entropic relevance           | [9]     |

Table III summarizes the characteristics of the conformance checking approaches implemented in Entropia by specifying the input models of retrieved and relevant traces (L–event log, M–process model) supported by the approach presented in the corresponding publication (Publ.), the ability to address the stochastic aspect of the input models of traces (Stoch.), and event log and process model formats the tool supports (Log–event log, and Model–process model).

The approaches listed in Table III except the technique presented in [9], can be used to quantify precision and recall conformance criteria between two (possibly infinite) collections of traces. The approach in [9] measures the relevance of a stochastic process model to an event log. Relevance reflects a compromise between the precision and recall criteria and has meaningful units.

V. Examples

In this section, we provide some examples of using Entropia. To this end, we use the Petri net N in Fig. 2 SDFA A in Fig. 3 and event log $E = \{abcdce, ace, bce^2, abcdce, abdcb, aaacbe\}$; $E$ contains two instances of trace bce.

One can compute the entropy-based exact matching precision between $N$ and $E$ presented in [5] using CLI options:

```
-emp -rel=E.xes -ret=N.pnml
```

If one wants to allow up to two skips in traces of the Petri net and up to one skip in the traces of the log, as described in [7], when identifying similar traces in the computation of precision, one should use CLI options:

```
-cpmp -rel=E.xes -ret=N.pnml -srel=1 -sret=2
```

To compute entropic relevance of $A$ to $E$, use options:

```
-r -rel=E.xes -ret=A.sdfa
```
The computed values of exact matching precision, controlled partial matching precision, and entropic relevance are 0.776, 0.833, and 11.368 bits, respectively. For more examples of using Entropia and the serialized models and log used in the examples discussed above, refer to the user guide.

VI. Discussion

All the techniques implemented in Entropia ver. 1.5 support process models that describe arbitrary (potentially infinite) collections of traces and impose no limitations on input logs provided that they are explicitly recorded and, thus, are finite. However, process models must be bounded, i.e., they must induce finite reachability graphs. Various notions of semantic correctness of process models require process models to be bounded. Nevertheless, process models used in practice can be incorrect, thus potentially unbounded. Hence, each process model provided as input to Entropia, which is not guaranteed by definition to be bounded, is tested by default for boundedness using LoLA ver. 2.0 [16]. One can check if a process model is bounded using option \(-b\) of the tool. If the boundedness of process models is established, one can invoke Entropia with option \(-t\) to skip the model correctness tests.

Entropia is implemented in Java and integrates with the LoLA tool compiled for Windows. To use Entropia on another platform, one needs to recompile LoLA for that platform.

Different conformance techniques implemented in Entropia have different performance characteristics. The computation time of entropic relevance [9] is linear in the size of the event log (number of traces times average length of a trace). The computation of entropy-based precision and recall [5], [6], [7] is low polynomial in the size of the reachability graphs of the compared models of traces. However, in practice, a reachability graph can be large, e.g., exponential in the size of the original model due to the state explosion problem. Empirical evidence suggests that the approach grounded in the exact matching of traces [5] runs in the order of seconds on real-world datasets, as the state explosion does not manifest often. The approach grounded in the partial matching of traces [6], on the other hand, induces large reachability graphs. Thus, it is recommended for small inputs, e.g., when calibrating a new automated process discovery technique. Hence the controlled partial matching technique [7], which can be configured by the user towards the desired performance, i.e., the number of allowed mismatches between similar traces and, thus, runtime \(-c\) the less the number of allowed mismatches, the faster the computation, with the techniques reported in [5], [6] constituting the two extremes of the spectrum. Finally, the computation of the stochastic measures presented in [8] relies on an iterative procedure which converges deterministically, and often fast, to the correct values, but can also lead to prolonged computation times on real-world datasets.

Future work on Entropia will aim to further improve the runtime performance of the supported techniques and implement new state-of-the-art information theoretic approaches to conformance checking, including those that assess the quality criteria that go beyond precision and recall. In [17], the authors discuss initial ideas on using entropy for measuring the simplicity of a model automatically discovered from a log.

VII. Conclusion

This paper presents Entropia, an open-source command-line tool for quantifying precision and recall conformance quality criteria in process mining. The current version of the tool implements several measures, all grounded in the notion of entropy of a collection of traces described by a process model or event log. The supported measures can be used to assess both classical and stochastic precision and recall, and fulfill a wide range of desired properties suggested by the process mining community. The development of the tool’s code base commenced in 2017 and is maintained by the authors of the implemented techniques.

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