A novel ISLE-ESE method for mixed-variable experiment design

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Abstract: Discrete-continuous mixed-variable problem is an inevitable problem in engineering design. However, few researches pay attention to discrete variables. In this paper, a novel method is proposed for solving mixed-variable problem by combing the Enhanced Stochastic Evolution (ESE) method and the improved Successive Local Enumeration (SLE) method. The continuous part of design variables is generated first. And then the improved SLE method is utilized to generate the discrete part. Finally, an overall optimization is conducted utilizing ESE method taking the continuous part of design variables as input. Results of numerical tests show that the proposed method performs better in both space-filling property and orthogonality. Besides, the computation effort is also saved compared to the ESE method.

1. Introduction
Modern computer technology grows rapidly in recent years and enables the wide use of mathematical models for scientific discovery, technology innovation, and quality improvement. However, although the process is much cheaper than a real physical experiment, traversal of every scheme is still impractical. Thus, surrogate model has been widely used in order to reduce the computational effort[1,2]. Design of Experiment (DOE), which is one of the procedures of surrogate based optimization, has a considerable effect on the accuracy of surrogate model. An efficient experiment design can get as much information as possible during limited number of runs, so that the influence of each experimental factor to the results is revealed and computation resource is simultaneously saved[3].

Properly employing of the experimental design techniques can effectively improve the precision and reliability of experiment conclusions[4]. The goal of DOE is to choose optimal input parameters so that the maximum information can be obtained by surrogate model with a limited number of the calls of true model[5]. Generally, two types of designs are considered, i.e., the input based space-filling design and output based design[6]. Space-filling design means the design points should be uniformly spread out over the whole feasible region[7,10]. Latin Hypercube Design is the most well-known method of space-filling design to proceed experiment design of sophisticated physical systems. Several criteria are proved efficient for LHD, such as Maximum entropy criterion[11],
IMSE(Integrated Mean Square Error) Minimax criterion[12], ϕ p criterion[13], Center L2-discrepancy criterion[14], among which Maximum entropy or criteria based on distance usually perform better[7]. Combining one or several criteria as objective, an Optimal Latin Hypercube Design (OLHD) is then constructed through utilizing evolutionary algorithms to search for optimum. Simulated Annealing(SA)[11-13], Swarm Optimization Algorithms[17,18], Enhanced Stochastic Evolution(ESE)[19], Genetic Algorithm(GA)[20,21], Column-wise-pairwise(CP)[22] are popular algorithms utilized in OLHD. Successive Local Enumeration(SLE), proposed by Zhu in 2011, has become the most widely used method because samples generated by SLE possess both space-filling property and projection property[23]. Li proposed a nested maximin design method combining SLE and a modified novel global harmony search (NGHS) algorithm for conducting computer experiment designs when dealing with simulations involving practical engineering design problems[24]. Wang proposed a constrained LHS based successive local enumeration (SLE-CLHS) approach to improve the performance both in space-filling and projection[25]. Long proposed a sequential-successive local enumeration (S-SLE) method to sequentially produce samples in regions of interest considering the space-filling and projective properties[26].

Most literatures focus on continuous input merely, yet engineering problem can involve both continuous and discrete variables. Simply round off the continuous factor to the nearest discrete value may lead to suboptimal design. Only a few numbers of researches about DOE involves discrete factors. The propose of Sliced Latin Hypercube Design (SLHD) solve the problem when the input parameters contain nominal variable[27-29]. Joseph proposed a Maxpro criterion aiming at four different variables—continuous, discrete numeric, nominal and ordinal[27]. Researches on mix-variable experiment design problem are still immature.

To solve discrete-continuous mixed-variable experiment design problem, this paper proposes an improved SLE-ESE(ISLE-ESE) method. Combining the SLE and the ESE method, the ISLE-ESE method possesses both reliability and computing efficiency. The rest part is organized as follows. In Section 2, the ESE method and the SLE method are introduced. Improved SLE-ESE(ISLE-ESE) is proposed for mixed-variable experiment design problem in Section 3. Section 4 contains the verification of proposed algorithm utilizing several cases. Section 5 is the summary of the paper.

2.Introduction of SLE and ESE method

2.1 Construction of normal OLHD and SLE method

A sample with d independent design variables can be represented as $x=[x_1, x_2, \ldots, x_d] \in \mathbb{R}^d$, while the lower and the upper boundary of the design space is $X^L \leq x \leq X^U$, where $X^L=[X^L_1, X^L_2, \ldots, X^L_d]$ and $X^U=[X^U_1, X^U_2, \ldots, X^U_d]$. The main purpose of the experiment design is to collect a set of excellent sample points $x = \{x_1, x_2, \ldots, x_N\}$, where N is the sample size. The design region of the $i$th variable in sample $x_i$ is divided into $N$ bins uniformly and an initial design matrix is subsequently constructed as

$$L = \begin{bmatrix}
    x_{11} & x_{12} & \ldots & x_{1d} \\
    x_{21} & x_{22} & \ldots & x_{2d} \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{N1} & x_{N2} & \ldots & x_{Nd}
\end{bmatrix} \quad (1.1)$$

One or two space-filling criteria can be employed as the objective function, and OLHD can be considered as a permutation optimization problem. Permutation optimization algorithms like SA, ESE are popular choices.

Recently, SLE has become the most popular experiment design method among researches because of computational efficiency and excellent performance. The concept of normal SLE is presented first. As to a n-dimension problem, specific procedures of SLE are shown as follows:

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STEP 1: Divide the design space into \( m^n \) unit, while \( m \) is the number of samples. The first point \( P_1 \) is randomly selected in unit 1, and \( P_1 \) can be defined as \( P_1(i_1, j_1, ..., h_1, 1) \) (\( i_1, j_1, ..., h_1 \in \{1, 2, ..., m\} \)). The sample set \( P \) is now \( P = \{P_1\} \).

STEP 2: The second point is generated in unit 2. Since the position \( (i_1, j_1, ..., h_1, 1) \) is taken up by \( P_1 \), \( P_2 \) should be generated from the rest positions of unit 2. Using \( d \) to represent the Euclidean distance between \( P_2 \) and \( P_1 \), \( P_2 \) should ensure a maximum value of \( d \) among all positions in unit 2. Then \( P_2 \) can be defined as \( P_2(i_2, j_2, ..., h_2, 2) \) (\( i_2, j_2, ..., h_2 \in \{1, 2, ..., m\}, i_2 \neq i_1, j_2 \neq j_1, h_2 \neq h_1 \)), and the sample set is \( P = \{P_1, P_2\} \).

STEP 3: The \( k \)th sample point should be generated from the rest positions of unit \( k \) where the former sample points haven’t taken up. Calculate the minimum distance \( d \) between the possible position and existing sample points \( P = \{P_1, P_2, ..., P_{k-1}\} \), \( P_k \) should be selected when the \( d \) reach a maximum value and the sample set is updated as \( P = \{P_1, P_2, ..., P_k\} \).

STEP 4: Repeat the step 3 until the algorithm complete the selection of the \( (m-1) \)th sample point, and put the \( m \)th point in the last position.

Consider a 2-dimension problem, and the required sample size is four. The design space is divided into \( 4 \times 4 \) unit as shown in Figure 2.1. The first point \( P_1 \) is randomly chosen in unit \( (2, 1) \). \( P_2 \) is placed in unit \( (4, 2) \) to ensure a maximum distance of 3.6056. One of the remaining units \( (1, 3) \) or \( (3, 3) \) need to be chosen to place \( P_3 \). The distance between these two units and \( P_1 \) is the same. However, unit \( (1, 3) \) ensure a further distance with \( P_2 \). Therefore \( P_3 \) turn out to be in unit \( (1, 3) \) and \( P_4 \) is placed in the last unit \( (3, 4) \).

![Figure 2.1. Process of SLE method](image)

### 2.2 Introduction of ESE method
Morris and Mitchell\[13\] proposed an OLHD method on the basis of SA. However, SA is time-consuming when solving optimization with more experiment factors, and suboptimal becomes an inevitable problem. ESE method, proposed by Jin\[19\], has been proved efficient under years of research. ESE method exchange column elements at random and is consisted by an inner loop and an outer loop, as shown in Figure 2.2.
As is illustrated in the figure, the inner loop has $M$ iterations. The algorithm randomly chooses $J$ distinct element-exchanges as different designs in $i \mod m$ column. The best design $X_{try}$ in each iteration will be accepted to replace $X$ if it satisfies criterion (1.2). So a temporarily worse design could be accepted to help jump out the suboptimal.

$$f(X_{try}) - f(X) \leq T_h \cdot \text{random}(0,1)$$

While the outer loop updates $T_h$ according to whether any improvement is made in a run of inner loop. The update process can be divided into the improving process and the exploration process. Once the criterion is improved, i.e., $\text{flag}_{\text{imp}}=1$, the search process will be turned to the improving process to rapidly find the optimal in the way of decreasing the value of threshold $T_h$. In that case, only better designs could be accepted. Otherwise, $T_h$ is increased and decreased in a relatively large range in the exploration process tending to escape from a locally optimal design.

### 3. Improved SLE-ESE for mixed-variable problem

#### 3.1 SLE-ESE method

When it comes to a mixed-variable problem, SLE method is not suitable mostly. Consider an $n$-dimension problem, where $f/n$ dimensions are continuous and the others are discrete. To different engineering problems, the number of possible values of each discrete variable is probably different, which brings difficulty to mixed-variable experiment design. For example, for the $j$th discrete variable, the required sample size is 7, while the number of possible values of the variable is only 5. Here comes a problem that when proceeding an experiment design, engineers need to choose which of these 5 values should be used twice. However, in normal LHD or SLE, each point has a different parameter value in each dimension. That’s the reason why mixed-variable problems can’t apply LHD.
or SLE directly.

The inspiration comes from the SLE method. To manage the mixed-variable problems, improvement method is conducted on SLE. ISLE-ESE method divides the variables into continuous part and discrete part. Firstly, ESE is utilized to optimize the continuous part and an initial continuous sample set is given. Secondly, improved SLE is applied to optimize the discrete part, and a set of sample is obtained. Finally, employ ESE to optimize the whole sample set, taking the continuous sample set as inputs. As for a \( m^n \) problem, while \( m \) is the number of sample points and \( n \) is the dimensions of design vector where \( f \) of them are continuous and the others are discrete, the specific procedures are presented as follows:

STEP 1: If \( f = 1 \), the continuous variable is divided into \( m \) bins; if \( f > 1 \), the ESE method is utilized to optimize the continuous part of the design vector. And finally a design matrix of continuous part \( x_f \) is obtained.

STEP 2: For the \( kth \) discrete variable, the relationship between number of its possible values \( e \) and sample size \( m \) can be represented as \( m = oe + t \). If \( o = 0 \), utilizing normal SLE method to generate all needed \( t \) values of this discrete variable; if \( o \geq 1 \), conduct \( o \) times of SLE, and the last \( t \) points are managed the same as the case of \( o = 0 \). Specially, the first point of the next time of SLE shouldn’t be chosen at random, which should also ensure a maximum distance between this point and the existing points.

STEP 3: Repeat STEP 2 until all discrete variables are managed, and an entire design matrix \( x_n \) is obtained.

STEP 4: Take \( x_f \) as input matrix and the criteria value of the whole \( x_n \) as output, ESE method is utilized to give an overall optimization, and a last \( x_n \) obtained.

![Figure 3.1. Flowchart of the ISLE-ESE algorithm](image)

The entire process is shown in Figure 3.1. To intuitively describe the ISLE method, two cases are conducted. The first case is a 2-dimension problem with one discrete variable of 6 possible values, yet the required number of samples is 4. As Figure 3.2 illustrates, the design space is divided into 4×6 unit. The first point \( P_1 \) is randomly placed in unit (3,1) of row 3, line 1. As normal SLE method, the distance between rest positions and \( P_1 \) is calculated, and unit (6,2) is selected to place \( P_2 \), which ensure a maximum distance of 4.1231. In line 3, four units are left, and the distance
between these units and \( \{ P_1, P_2 \} \) is calculated, among which the minimum distance 4.2426 of unit (1,3) is the maximal. Thus, \( P_3 \) is placed in (1,3). \( P_4 \) has three possible units (3,4), (4,4), (5,4). The distance is calculated respectively, of which the results are 3.1623, 4.1231 and 3.1623. Therefore, \( P_4 \) is placed in unit (4,4).

![Figure 3.2. ISLE- 4 Samples with 6 discrete values](image)

The second case is also a 2-dimensional problem. One is a discrete variable with 4 possible values, while the required number of samples is 6. Since \( 6 = 4 \times 1 + 2 \), the former four points need to be managed utilizing the same way as normal SLE method. Results of \( P_1, P_2, P_3 \) and \( P_4 \) are shown in Figure 3.3. As to the fifth point, which can be considered as the start of another SLE, shouldn’t be selected at random. \( P_5 \) should also ensure a maximum of the minimum distance with the former points. The minimum distance of four units (1,5), (2,5), (3,5), (4,5) are 2, 1.4142, 1 and 1.4142. Therefore \( P_5 \) is placed in unit (1,5). The last point \( P_6 \) has three units left and is finally placed in unit (4,6) with a maximum distance of 3.6056.

![Figure 3.3. ISLE- 6 Samples with 4 discrete value](image)

3.2 Multi-objective measurement criterion

Traditional LHDs are optimized by taking one single criterion as the objective. For example, \( \Phi_p \) criterion, which is one of the most popular criterion of uniformity, is illustrated by equation (1.3)

\[
\Phi_p = \left[ \sum_{i=1}^{s} d_{ij}^{-p} \right]^{1/p}
\]

where \( d_{ij} = (x_i - x_j)^2 \) represents the Euclidean distance of two points, \( s \) represents the numbers of pairs of different distance values within all samples, \( p \) is a constant which mostly values 2.

Through \( \Phi_p \) criterion, an OLHD design with excellent uniformity is obtained. However, a superior OLHD also pay attention to orthogonality, which \( \Phi_p \) criterion can’t guarantee. Correlation coefficient of experimental factors is a popular criterion of orthogonality, and it is shown by equation (1.4)
\[
\rho^2 = \frac{\sum_{i=1}^{k} \sum_{j=1}^{i-1} \rho_{ij}^2}{k(k-1)/2} \tag{1.4}
\]

where \( \rho_{ij} \) represents the linear correlation coefficient of column vector \( v = [v_1, v_2, ..., v_k] \) and \( u = [u_1, u_2, ..., u_k] \) of design matrix, and is shown by equation (1.5)

\[
\rho_{ij} = \frac{\sum_{i=1}^{n} [(v_i - \bar{v})(u_i - \bar{u})]}{\sqrt{\sum_{i=1}^{n} (v_i - \bar{v})^2 \sum_{i=1}^{n} (u_i - \bar{u})^2}} \tag{1.5}
\]

To proceed an OLHD of both orthogonality and uniformity, the algorithm have to optimize two criteria simultaneously. However, \( 0 \leq \rho^2 \leq 1 \) while \( \Phi_p \in (0, \infty) \), thus these two criteria need to be normalized. According to Jin’s research\[19\], \( \Phi_{p,L} \leq \Phi_p \leq \Phi_{p,U} \), where \( \Phi_{p,L} \) and \( \Phi_{p,U} \) is shown as

\[
\Phi_{p,L} = C_p^2 \left[ \frac{\bar{d} - \bar{d}}{\bar{d} - \bar{d}} + \frac{\bar{d} - \bar{d}}{\bar{d} - \bar{d}} \right]^{1/p} \tag{1.6}
\]

\[
\Phi_{p,U} = \left\{ \sum_{i=1}^{n} (n-i) \right\}^{1/p} \tag{1.7}
\]

where \( \lceil \cdot \rceil \) means round up, \( \lfloor \cdot \rfloor \) represents round down. It is easy to find that

\[
0 \leq \Phi_p = \frac{\Phi_{p,U} - \Phi_{p,L}}{(\Phi_{p,U} - \Phi_{p,L})} \leq 1 \tag{1.8}
\]

Thus the optimization criterion selected in this paper is

\[
C_p = w\rho^2 + (1-w)\Phi_p \tag{1.9}
\]

4. Numerical analysis and discussion

4.1 Numerical experiment

In order to verify the validity of the proposed ISLE-ESE method. Three numerical case, 5×3, 16×5 and 25×4, are conducted to compare with other different algorithm. Since the discrete variables is in the minority in real project, assumption is made that among these cases above, one dimension of 5×3 design is discrete with seven different values, and two of the others are discrete with ten different values. The required number of samples are 5, 16 and 25. Results from other literatures are collapsed to the nearest discrete value.

To eliminate the interference of different dimensions, normalization processing is conducted before the experiment. And \( w \) in equation (1.9) is set to be 0.2.

4.2 Results and discussion

Result of 5×3 design matrix is shown in Table 4-1, which is made comparison with normal Maximin Latin Hypercube Design(MLHD)\[12\]. MLHD is a popular choice of OLHD method based on distance after years of research and can ensure excellent space-filling property. Results show that ISLE-ESE method performs better than MLHD method in both space-filling property and orthogonality.
The $16 \times 5$ design matrix optimized by ISLE-ESE method is shown in Table 4-2, and is compared with normal ESE method presented by Jin.[19] As is illustrated in the picture, the $\phi_p$ value of ISLE-ESE method is 11.998, which is similar to 11.811 of ESE method. But the $\rho$ value is superior, compared to ESE method. It means that ISLE-ESE method could ensure both uniformity and orthogonality when dealing with mixed-variable experiment design, which is comparable to ESE method.

Table 4-1. Experiment comparison with MLHD method

| Design matrix | MLHD     | ISLE-ESE |
|---------------|----------|----------|
|               | 0.2      | 0.2      | 0.33     | 0.5      | 0.1      | 0.67     |
|               | 0.4      | 1.0      | 0.67     | 0.7      | 0.7      | 0.0      |
|               | 0.6      | 0.4      | 1.0      | 0.1      | 0.3      | 0.17     |
|               | 0.8      | 0.6      | 0.17     | 0.9      | 0.5      | 1.0      |
|               | 1.0      | 0.8      | 0.83     | 0.3      | 0.9      | 0.83     |
| $\phi_p$      | 4.196    |          | 3.840    |          |
| $\rho$        | 0.5079   |          | 0.1777   |          |

The result of $25 \times 4$ design is compared with Orthogonal-Maximin Latin Hypercube Designs(OMLHD) proposed by Joseph.[16] Normalization is also conducted. Details are shown in the following Table 4-3. Compare the design matrix optimized by these two methods, ISLE-ESE method is apparently superior to OMLHD in both space-filling property and orthogonality. Compared to an improved ESE method proposed by Liu,[31] our method shows better performance in space-filling property for the $\phi_p$ value is 23.711. However, Liu’s method ensures better orthogonality. Orthogonality of ISLE-ESE method can be improved by adjusting weight $w$ in equation (1.9). Besides, when solving an experiment design problem of $25 \times 4$, among which two are discrete

Table 4-2. Experiment comparison with ESE method

| Design matrix | ESE       | ISLE-ESE |
|---------------|-----------|----------|
|               | 1.00      | 0.93     | 0.40     | 0.56     | 0.67     | 0.84     | 0.78     | 0.09     | 1.00     | 1.00     |
|               | 0.80      | 0.13     | 0.60     | 0.89     | 0.89     | 0.41     | 0.47     | 0.03     | 0.00     | 0.00     |
|               | 0.93      | 0.33     | 1.00     | 0.44     | 0.44     | 0.72     | 0.41     | 0.22     | 0.56     | 0.44     |
|               | 0.87      | 0.60     | 0.07     | 0.11     | 0.22     | 0.03     | 0.34     | 0.66     | 0.89     | 0.89     |
|               | 0.40      | 1.00     | 0.53     | 0.33     | 0.11     | 0.28     | 0.03     | 0.72     | 0.11     | 0.67     |
|               | 0.27      | 0.20     | 0.47     | 0.00     | 0.33     | 0.09     | 0.72     | 0.34     | 0.44     | 0.56     |
|               | 0.47      | 0.27     | 0.00     | 0.67     | 0.78     | 0.78     | 0.27     | 0.91     | 0.78     | 0.11     |
|               | 0.33      | 0.53     | 0.93     | 0.33     | 0.56     | 0.59     | 0.97     | 0.41     | 0.22     | 0.22     |
|               | 0.53      | 0.80     | 0.87     | 0.11     | 0.89     | 0.16     | 0.28     | 0.16     | 0.67     | 0.78     |
|               | 0.67      | 0.00     | 0.20     | 0.09     | 0.56     | 0.97     | 0.59     | 0.59     | 0.33     | 0.33     |
|               | 0.60      | 0.40     | 0.33     | 0.89     | 0.00     | 0.34     | 0.66     | 0.53     | 0.00     | 1.00     |
|               | 0.73      | 0.73     | 0.73     | 0.78     | 0.22     | 0.47     | 0.09     | 0.28     | 1.00     | 0.00     |
|               | 0.07      | 0.87     | 0.13     | 0.78     | 0.44     | 0.66     | 0.84     | 0.78     | 0.89     | 0.67     |
|               | 0.13      | 0.067    | 0.80     | 0.56     | 0.11     | 0.22     | 0.91     | 0.84     | 0.56     | 0.11     |
|               | 0.00      | 0.47     | 0.67     | 0.22     | 0.78     | 0.91     | 0.16     | 0.47     | 0.11     | 0.89     |
|               | 0.20      | 0.67     | 0.27     | 0.44     | 1.00     | 0.53     | 0.53     | 0.97     | 0.33     | 0.78     |
| $\phi_p$      | 11.838    |          | 11.998    |          |
| $\rho$        | 0.03319   |          | 0.01450   |          |
variables, ISLE-ESE method actually optimizes a $25 \times 2$ design for the design of discrete variables are left to ISLE method. Thus the computation efforts is drastically reduced.

Table 4-3. Experiment comparison with OMLHD

| Design matrix | OMLHD | ISLE-ESE |
|---------------|-------|----------|
|               | 0.8   | 0.6      | 0.33     | 0.0  | 0.3 | 0.8 | 0.6 |
|               | 0.4   | 0.9      | 1        | 0.33  | 0.9 | 0.6 | 0.0 |
|               | 0.5   | 0.4      | 1        | 0.11  | 0.6 | 0.0 | 0.4 |
|               | 0.8   | 0.2      | 0.56     | 0.78  | 0.1 | 0.8 | 0.2 |
|               | 0.6   | 0.7      | 0.22     | 0.33  | 0.5 | 0.2 | 1.0 |
|               | 0.2   | 0.0      | 0.89     | 0.44  | 0.2 | 0.1 | 0.4 |
|               | 0.0   | 0.9      | 0.11     | 0.22  | 0.7 | 0.4 | 0.6 |
|               | 0.2   | 0.3      | 0.67     | 1     | 0.1 | 0.5 | 0.5 |
|               | 0.3   | 0.2      | 0.33     | 0.11  | 0.8 | 0.9 | 0.7 |
|               | 0.4   | 0.8      | 0.78     | 0     | 0.6 | 0.9 | 0.3 |
|               | 0.1   | 0.2      | 0.78     | 0.67  | 0.7 | 0.2 | 0.0 |
|               | 0.7   | 0.6      | 0.22     | 0.89  | 0.5 | 0.4 | 0.3 |
|               | 0.5   | 0.0      | 0.56     | 0.33  | 0.3 | 0.5 | 0.8 |
|               | 1.0   | 0.4      | 0.44     | 0.56  | 0.8 | 0.1 | 0.7 |
|               | 0.8   | 0.5      | 0.11     | 0.22  | 0.0 | 0.7 | 1.0 |
|               | 0.7   | 1.0      | 0.33     | 0.67  | 0.1 | 0.3 | 0.4 |
|               | 0.3   | 0.1      | 0.33     | 0.89  | 0.4 | 0.1 | 0.6 |
|               | 0.9   | 0.3      | 0.56     | 0.11  | 0.7 | 0.7 | 0.5 |
|               | 0.6   | 0.4      | 0.89     | 0.78  | 0.2 | 0.7 | 0.1 |
|               | 0.6   | 0.1      | 0.11     | 0.56  | 0.9 | 0.6 | 0.2 |
|               | 0.0   | 0.2      | 0.44     | 0.56  | 0.4 | 0.8 | 0.6 |
|               | 0.9   | 0.7      | 0.67     | 0.89  | 0.9 | 0.5 | 1.0 |
|               | 0.4   | 0.8      | 0.67     | 1     | 0.5 | 0.0 | 0.1 |
|               | 0.2   | 0.8      | 0.440    | 0.44  | 0.3 | 0.3 | 0.0 |
|               | 0.1   | 0.5      | 0.67     | 0.3   | 0.9 | 0.8 | 0.7 |

$\phi$ 24.085 23.104
$\rho$ 0.079559 0.02666

5. Conclusion

Most engineering design problems contain mixed variables. However, the common used experiment design researches pay little attention to discrete variables. The ISLE-ESE method proposed in this paper provides an effective solution for mixed-variable problems. Combining the SLE method and the ESE method, the proposed ISLE-ESE method is superior in both reliability and computational efficiency. Three numerical analysis of are conducted and the results show that multi-objective ISLE-ESE method performs better in both space-filling property and orthogonality compared to MLHD and OMLHD. The computation effort is also saved compared to ESE method.

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