NATURAL FLAVOUR MIXING IN THE MSSM AND $\mu \to e, \gamma$

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In the absence of any additional assumption it is natural to conjecture that sizeable flavour-mixing mass entries, $\Delta m^2$, may appear in the mass matrices of the scalars of the MSSM, i.e. $\Delta m^2 \sim O(m^2)$. This flavour violation can still be reconciled with the experiment if the gaugino mass, $M_{1/2}$, is large enough to yield (through the renormalization group running) a sufficiently small $\Delta m^2/m^2$ at low energy. This leads to a gaugino dominance framework (i.e. $M_{1/2} \gg m^2$), which permits a remarkably model-independent analysis. We study this possibility focussing our attention on the $\mu \to e, \gamma$ decay. In this way we obtain very strong and general constraints, in particular $M_{1/2}^2/\Delta m^2 > 34$ TeV.

It is well-known that FCNC processes are very sensitive tests to physics beyond the standard model (SM) and, in particular, to supersymmetric extensions of the SM (SSM)\cite{1}. Furthermore supersymmetry provides new direct sources of flavour violation, namely the possible (and even natural as we will see) presence of off-diagonal terms (say generically $\Delta m^2$) in the squark and slepton mass matrices\cite{2,3,4,5,6}. In the present talk we will focus all our attention on the constraints on $\Delta m^2$ from the $\mu \to e, \gamma$ process because they are very strong and, as we will see, their evaluation is remarkably model-independent.

The minimal supersymmetric standard model (MSSM) is defined by the superpotential, $W$ (from which the supersymmetric part of the Lagrangian is readily obtained), and the soft supersymmetry breaking terms coming from the (unknown) supersymmetry breaking mechanism

$$-\mathcal{L}_{\text{soft}} = \frac{1}{2} M_a \lambda_a \lambda_a + (m_L^2)_{ij} L_i L_j + (m_{e_R}^2)_{ij} e_{Ri} e_{Rj} + (m_Q^2)_{ij} Q_i Q_j + (m_{u_R}^2)_{ij} u_{Ri} u_{Rj} + (m_{d_R}^2)_{ij} d_{Ri} d_{Rj}$$

$$+ \left[ A_{ij}^u h_{ij}^u Q_i H_2 u_{Rj} + A_{ij}^d h_{ij}^d Q_i H_1 d_{Rj} + A_{ij}^e h_{ij}^e L_i H_1 e_{Rj} + B_{\mu} H_1 H_2 + \text{h.c.} \right] ,$$

\begin{equation}
\end{equation}
where \(i, j\) are generation (gauge group) indices, \(\lambda_a\) are the gauginos, and the remaining fields in the formula denote just their corresponding scalar components in a standard notation. In the simplest version of the MSSM the soft breaking parameters are taken as universal (at the unification scale \(M_X\)). Then, the independent parameters of the theory are

\[
\mu, m, M_{1/2}, A, B
\]

(the rest of the parameters can be worked out demanding a correct unification of the gauge coupling constants and correct masses for all the observed particles). However this simplification is not at all a general principle. In particular there is no theoretical argument against non-vanishing off-diagonal \((m_{ij}^2)_{i \neq j} = \Delta m_{ij}^2\) terms. From the previous arguments, it is natural to assume that these off-diagonal entries can be sizeable, or even of the same order as the diagonal terms

\[
\Delta m^2 \sim O(m^2) .
\]

Certainly, there are proposed mechanisms to avoid this, for example the above-mentioned assumption of universality. However one should wonder whether, in the absence of any additional assumption, the perfectly possible and even natural situation of eq. (3) could still be compatible with the experimental data and, more precisely, with the present experimental bound on \(\mu \rightarrow e, \gamma\)

\[
BR(\mu \rightarrow e, \gamma) \leq 5 \times 10^{-11} ,
\]

The expression of \(BR(\mu \rightarrow e, \gamma)\) in the MSSM depends on several low-energy quantities, namely \(\tan \beta \equiv \langle H_2 \rangle / \langle H_1 \rangle\), \(\mu\), \(A\), and the spectrum of masses of sleptons and gauginos. These can be obtained from the initial parameters of the theory (see eq. (3)) through the corresponding RGEs (see ref. 11). On the other hand the ratio \(\Delta m^2/m^2\) will in general be small at low energies (even if it is \(O(1)\) at \(M_X\)), provided that gaugino masses are bigger than scalar masses, \(M_{1/2}^2 \gg m^2\), because of the contribution of the former in the RGEs of the diagonal parts of the latter, which is not the case for the off-diagonal entries. This is the reason why the RGEs have the potential to "cure" initial sizeable values of \(\Delta m\). Therefore, the assumption of naturally large flavour mixing at \(M_X\) leads us necessarily to a \(M_{1/2}^2 \gg m^2\) ("gaugino dominance") scenario, where all the soft breaking parameters are essentially determined at low energies by the value of \(M_{1/2}\) at \(M_X\) independently of their initial values.

Note that all this does not apply to the \(\mu\) parameter, as it renormalises proportional to itself. However, the further requirement of a correct electroweak breaking fixes the value of \(\mu\), giving us the whole spectrum and other relevant low-energy quantities (such as \(A\) and \(\tan \beta\)) in terms of a unique parameter.
In particular, the values of tan $\beta$ obtained in this framework tend to be rather large (ranging from 11 to 26 as $M_{1/2}$ increases from 150 GeV to 10 TeV. The gaugino dominance is a very interesting fact that makes the subsequent analysis rather accurate and model–independent.

At lowest order, the $\mu \rightarrow e, \gamma$ process is induced by one–loop diagrams that involve a flip of the leptonic flavour triggered by the slepton mixing, besides the propagation of a neutralino or chargino (see ref. 11 for details). Since the electron and muon Yukawa couplings are very suppressed, only the gauge part of the couplings of the charginos and neutralinos will play a role in the diagrams. In fact, one important consequence of the gaugino dominance framework is that for large enough values of $M_{1/2}$ the neutralinos (charginos) are almost pure neutral (charged) gaugino and higgsino. Then, the relevant diagrams correspond to bino ($\tilde{B}$) and wino ($W^0, W^-$) exchange. We have evaluated all of them. The expressions given in the previous literature are either incomplete or not directly applicable to our case.

Although in principle all the diagrams can have a similar magnitude (e.g. if we assume $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu} \sim \Delta m^2_{\tilde{\nu}_L\tilde{\mu}_L} \sim \Delta m^2_{\tilde{\nu}_R\tilde{\mu}_R}$, in practice the bino diagram is the dominant one. This comes from the coefficient of proportionality $(A + \mu \tan \beta)$, that appears in its evaluation and turns out to be very important in the gaugino dominance framework due to the large tan $\beta$ value.

The theoretical $BR(\mu \rightarrow e, \gamma)$ depends on two different sets of parameters. First, the different masses involved in the game $(m^2_\psi, m^2_{\tilde{l}_L}, m^2_{\tilde{l}_R}, M_{\tilde{B}}, M_W)$ and certain relevant low-energy quantities $(A, \mu, \tan \beta)$. Second, the three independent flavour-mixing mass entries: $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu}$, $\Delta m^2_{\tilde{\nu}_L\tilde{\mu}_L}$, $\Delta m^2_{\tilde{\nu}_R\tilde{\mu}_R}$. As explained in before, once we are working in the framework of gaugino dominance, $M^2_{1/2} \gg m^2$, the first set is completely determined in terms of the initial gaugino mass, $M_{1/2}$. Recall that we were led to this framework by the mere assumption of naturally large flavour mixing at $M_X$ (see eq. (8)). The three flavour-mixing mass parameters, however, remain independent.

The constraints on the MSSM from $BR(\mu \rightarrow e, \gamma)$ arise by evaluating the previous diagrams and comparing them with the present experimental bound, eq. (8). We have illustrated this in Fig. 1, where an overall mass-mixing parameter $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu} = \Delta m^2_{\tilde{\nu}_L\tilde{\mu}_L} = \Delta m^2_{\tilde{\nu}_R\tilde{\mu}_R} \equiv \Delta m^2$ has been taken for simplicity. Then we have plotted $BR(\mu \rightarrow e, \gamma)$ vs $M_{1/2}$ for different values of $\Delta m$. From this figure we can derive the maximum allowed value of $\Delta m$ (or, equivalently, the minimum allowed value of $M_{1/2}/\Delta m$) for each value of $M_{1/2}$. This is represented in Fig. 2 for four different cases: a) $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu} = \Delta m^2_{\tilde{\nu}_L\tilde{\mu}_L} = \Delta m^2_{\tilde{\nu}_R\tilde{\mu}_R} \equiv \Delta m^2$; b) only $\Delta m^2_{\tilde{\nu}_R\tilde{\mu}_R} \neq 0$; c) only $\Delta m^2_{\tilde{\nu}_L\tilde{\mu}_L} \neq 0$ and d) only $\Delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu} \neq 0$, which gives a complete picture of the results. Notice that the
(d) case is the less restrictive one.

The constraints are in general extremely strong. For case (a), which is the most representative one, the corresponding curve can be approximately fitted by the simple constraint

\[
\frac{M_{1/2}^2}{\Delta m} \gtrsim 34 \text{ TeV}
\]

(similar equations can be written for the other curves). Under the assumption of eq. (3), i.e. \( \Delta m = O(m) \), the results of Fig. 2 or eq. (3) imply that, indeed, a very large hierarchy between the scalar and gaugino masses is needed in order to reconcile the theoretical and experimental results. This gives full justification to our assumption of a gaugino dominance framework once eq. (3) has been conjectured. For example, for \( M_{1/2} \sim 500 \) GeV the assumption \( \Delta m \sim m \) demands \( M_{1/2}/\Delta m > 65 \). Actually, it is hard to think of a scenario where such a dramatical hierarchy can naturally arise. Consequently, we can conclude at this point that a naturally large flavour mixing, as that conjectured in eq. (3), can hardly be reconciled with the experiment in a natural way.

We can now summarize our work. In the absence of any additional assumption it is natural to conjecture that sizeable flavour-mixing mass entries, \( \Delta m^2 \), may appear in the mass matrices of the scalars of the MSSM, i.e. \( \Delta m^2 \sim O(m^2) \). This flavour violation can still be reconciled with the experiment if the gaugino mass, \( M_{1/2} \), is large enough to yield (through the renormalization group running) a sufficiently small \( \Delta m^2/m^2 \) at low energy. We have analyzed this possibility, focusing our attention on the leptonic sector, particularly on the \( \mu \to e, \gamma \) decay, which is by far the FCNC process with higher potential to restrict the value of the off-diagonal terms, \( \Delta m^2 \). The results are the following:

1. The \( \Delta m^2 \sim O(m^2) \) conjecture automatically leads to a gaugino dominance framework (i.e. \( M_{1/2}^2 \gg m^2 \)), where, apart from \( \Delta m^2 \) itself, all the relevant low-energy quantities (mass spectrum, \( A, \mu, \tan \beta \)) are determined in terms of a unique parameter, \( M_{1/2} \). This makes the subsequent analysis and results remarkably model-independent.

2. The resulting constraints in the MSSM, obtained by comparing the calculated \( BR(\mu \to e, \gamma) \) with the experimental bound, are very strong (see Figs. 1, 2 and eq. (3)). This makes, in our opinion, the natural flavour mixing conjecture \( \Delta m^2 \sim O(m^2) \) extremely hard to be reconciled with the experiment in a natural way. Hence, \( \Delta m/m \) should be small already at the unification scale.
Finally, let us comment that the need of starting with small $\Delta m/m$ can be satisfied in some theoretically well-founded scenarios, which become favoured from this point of view. In particular, we would like to stress that many string constructions can be consistent with that requirement. Other scenarios, however, can produce a larger non-universality of the scalar masses, with potentially dangerous contributions to FCNC processes. In any case, these non-universality effects are to produce non-vanishing off-diagonal terms in the scalar mass matrices once the usual rotation of fields to get diagonal fermionic mass matrices is carried out. The phenomenological viability of these physically relevant scenarios undoubtedly deserves further investigation.

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Figure 1: Plot of $BR(\mu \rightarrow e, \gamma)$ vs. $M_{1/2}$, taking for simplicity $\Delta m^2_{\tilde{\nu}_e \tilde{\nu}_\mu} = \Delta m^2_{\tilde{e}_L \tilde{\mu}_L} = \Delta m^2_{\tilde{e}_R \tilde{\mu}_R} \equiv \Delta m^2$. The different curves correspond to $\Delta m = 50, 100, 200, 300, 400, 500$ GeV respectively.

Figure 2: Plot of the minimum allowed value of $M_{1/2}/\Delta m$ vs. $M_{1/2}$ in four different cases: a) $\Delta m^2_{\tilde{e}_L \tilde{\nu}_e} = \Delta m^2_{\tilde{e}_L \tilde{\mu}_L} = \Delta m^2_{\tilde{e}_R \tilde{\mu}_R} \equiv \Delta m^2$; b) only $\Delta m^2_{\tilde{e}_R \tilde{\mu}_R} \neq 0$; c) only $\Delta m^2_{\tilde{e}_L \mu_L} \neq 0$ and d) only $\Delta m^2_{\tilde{\nu}_\mu \tilde{\mu}_\mu} \neq 0$. 

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