Spin relaxation in the presence of electron-electron interactions.

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The D’yakonov-Perel’ spin relaxation induced by the spin-orbit interaction is examined in disordered two-dimensional electron gas. It is shown that, because of the electron-electron interactions different spin relaxation rates can be obtained depending on the techniques used to extract them. It is demonstrated that the relaxation rate of a spin population is proportional to the spin-diffusion constant \( D_s \), while the spin-orbit scattering rate controlling the weak-localization corrections is proportional to the diffusion constant \( D \), i.e., the conductivity. The two diffusion constants get strongly renormalized by the electron-electron interactions, but in different ways. As a result, the corresponding relaxation rates are different, with the difference between the two being especially strong near a magnetic instability or near the metal-insulator transition.

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part of the interference processes, leading to the well-known anti-localization effect \[17\]. In the case of the DP mechanism, the SO rate is:
\[
\tau_{so}^{-1} = \Delta_{so}^2 \tau / 2 = D(\Delta_{so}/v_F)^2 .
\] (4)

Note that, \( \tau_{so}^{-1} \) is proportional to \( D \), and not \( D_s \) as in Eq. (3).

Since the two rates \( \tau_s^{-1} \) and \( \tau_{so}^{-1} \) are directly related to the behavior of the different transport coefficients, let us clarify the relationship between the diffusion constants in a disordered electron liquid \[13, 14\]. The charge-diffusion constant \( D_c \) is related to the conductivity \( \sigma \) and the diffusion constant \( D \) by the Einstein relation:
\[
\sigma / e^2 = \frac{\partial n}{\partial \mu} D_c = 2 \nu D .
\] (5)

The spin-diffusion constant \( D_s \) is in turn related to \( D \) by the “Einstein” relation for the spin density as:
\[
\chi_s D_s = \chi_s^0 D .
\] (6)

The compressibility \( \partial n / \partial \mu \) controls the static limit of the polarization operator just like \( \chi_s \) controls the static limit of the spin-spin correlation function. Eqs. (6) and (7) taken together reflect the fact that both the charge and the spin are carried by the same particles. Because of the rather different renormalization of the respective charge and spin-diffusion constants, a strong deviation in the rates \( \tau_s^{-1} \) and \( \tau_{so}^{-1} \) may occur. The difference between the two rates can be especially strong near a magnetic instability or near the metal-insulator transition \[18\] where \( \chi_s \to \infty \) whereby \( D_s \to 0 \) and hence \( \tau_s^{-1} \to 0 \), while both \( D \) and \( \tau_{so}^{-1} \) remain finite.

In disordered conductors in 2d the parameters of the electron liquid, in particular the diffusion constants \( D \) and \( D_s \), acquire logarithmically divergent corrections as a function of temperature due to the combined action of the e-e interaction and disorder \[18\]. The question arises how the different spin relaxation rates are renormalized as a result of these corrections. In this paper we show that for the practically important case of the Bychkov-Rashba SO interaction the expressions observed in Eqs. (3) and (4) at the Fermi-liquid level are preserved in the course of the logarithmic renormalizations.

In 2d systems with structure inversion asymmetry, typical to heterostructures, the SO interaction has the form of the Bychkov-Rashba term \[19\]:
\[
H_{so} = \frac{p^2}{2m} + \alpha_{so} \hat{d} \cdot (\hat{z} \times \hat{p}) .
\] (7)

Naively, the SO interaction can be looked upon as a momentum dependent “Zeeman” interaction. While the Zeeman splitting induced by a magnetic field is strongly renormalized in the presence of disorder and e-e interactions \[12, 14\], we find that the momentum dependence of the SO interaction radically changes the situation. The momentum dependence allows \( H_{so} \) to be rewritten (up to a constant) in the gauge form:
\[
H_{so} = \frac{1}{2m} (\vec{p} + p_{so} \bar{\sigma} )^2 ,
\] (8)

where, the SO interaction appears as a spin dependent vector potential, \( \bar{\sigma} = \frac{1}{2}(\hat{d} \times \hat{z}) \), with effective charge \( p_{so} = 2m \alpha_{so} \). (In terms of \( p_{so} \), the spin splitting at the Fermi-surface \( \Delta_{so} = p_{so} v_F \).) We show below that \( p_{so} \), unlike the Zeeman term, is not renormalized.

The renormalization of the parameters of the disordered electron gas is best described by the matrix nonlinear sigma model \[20, 21\]. The SO interaction can also be succinctly described within this model. The disorder-averaged N-replica partition function of the interacting problem reads:
\[
\langle Z_N \rangle = \int DQ e^{-S[Q]} ,
\] (9)

\[
S[Q] = \int d^2 r \frac{\pi \nu}{4 \tau} \text{Tr} Q^2 - \text{Tr} \ln G^{-1} + Q \hat{\Gamma} Q ,
\] (10)

where the \( Q \)-field is an auxiliary matrix field describing the impurity scattering. The Green’s function in Eq. (10) is easily deduced from Eq. (3):
\[
G^{-1} = i \epsilon + \mu - \frac{1}{2m} (-i \nabla + p_{so} \bar{\sigma})^2 + \frac{i}{2 \tau} Q ,
\] (11)

with the last term appearing because of the impurity scattering. On expanding the action \( S[Q] \) about its saddle point solution the problem of electron diffusion in the field of impurities, e-e interactions, and SO interaction \[22\] reduces to the non-linear sigma model:
\[
S[Q] = \frac{\pi \nu}{4} \int \text{Tr} D \left( \nabla Q + i p_{so} [\bar{\sigma}, Q] \right)^2 - 4 \bar{\z} \text{Tr} (\bar{\epsilon} Q) + Q \hat{\Gamma} Q .
\] (12)

The matrix \( Q \) satisfies the constraints: \( Q^2 = 1 \), \( Q = Q^\dagger \), and \( \text{Tr} Q = 0 \). The components of \( Q \) are defined
as $Q_{n_1,n_2}^{ij,\alpha\beta}$, where $n_1, n_2$ are the fermionic energy indices with $\epsilon_n = (2n + 1)\pi T$; $i,j$ are the replica indices and $\alpha, \beta$ are the spin indices. The trace is taken over all these variables. The energy matrix $\hat{\epsilon} = \epsilon_n \delta_{nm}\delta_{ij}\delta_{\alpha\beta}$. The factor $z$ is the frequency renormalization factor that determines the relative scaling of the frequency with respect to the length scale; $z = 1$ for free electrons.

The fluctuations of the $Q$-field in the particle-hole channel (diffusons) determine the propagators $D(q,\omega)$, which in the absence of the SO interaction ($p_{so} = 0$) have a diffusion-like singularity $D(q,\omega) = 1/(Dq^2 + z\omega)$. These propagators describe the diffusive evolution of the charge and spin density fluctuations at large scales. The fluctuations responsible for the interference corrections are in the particle-particle channel (cooperons). Since the Bychkov-Rashba SO interaction is invariant under time-reversal, the particle-hole diffusion and the particle-particle cooperon propagators are related by charge conjugation. It therefore suffices to study the renormalization of the spin-orbit scattering in the diffuson channel only.

The spin fluctuations can be classified in terms of the total spin, $S$, of the particle-hole pairs. The interaction matrix $\Gamma = \Gamma_s$ and $\Gamma_t$ represent the multiple scattering induced by the $e-e$ interactions in the singlet $S = 0$ and the triplet $S = 1$ diffusion channels, respectively. They are related to the static Fermi-liquid parameters $\Gamma_1$ and $\Gamma_2$ as $\Gamma_1 = -\Gamma_2/2$ and $\Gamma_t = -\Gamma_{2}/2$. The presence of the quadratic term $Dp^2_{so}(Q\overrightarrow{Q}\overrightarrow{Q})$ in Eq. (12) introduces in the triplet $S = 1$ diffusion and cooperon propagators a cutoff, i.e., a gap, proportional to the SO scattering rate $\tau^{-1} = Dp^2_{so}$. Note that the linear term proportional to $Dp^2_{so}(\overrightarrow{Q}\overrightarrow{Q})$ in Eq. (12) may be gauged away by the transformation $\overrightarrow{Q}(\overrightarrow{r}) \to e^{ip_{so}\overrightarrow{Q}(\overrightarrow{r})}$. This will, however, generate higher order terms in $p_{so}$ since Pauli matrices do not commute with each other. The question of the renormalization of the quadratic term therefore still remains.

The quadratic term leads to inserting two $\overrightarrow{Q}$-vertices into the particle-hole propagator $D$. These vertices are marked by the solid triangles (▲) in Fig. 1. As shown in the figure, there are two possible ways indicated by the kernels $K_h$ and $K_v$ in which this can be done. To determine the scaling behavior of the cutoff $\tau_{so}^{-1}$ in the diffuson channel we study how both these kernels are renormalized when the $e-e$ interaction corrections are included in the presence of disorder.

![FIG. 2: $\Gamma_1$ corrections to the $K_v$ kernel in Fig. 1 involving only one momentum integration.](image2)

Since each momentum integration involving the diffusion propagators leads to one power of $\rho = 1/(2\pi)^2 \nu D$ in 2d, the corrections to first order in the disorder strength $\rho$ are evaluated by taking into account diagrams with only one momentum. In the following, we demonstrate how the scaling equations for $\tau_{so}^{-1}$ are obtained by considering the renormalization of the kernel $K_v$ as an example.

Corrections to $K_v$ limited to only one momentum integration involving the interactions $\Gamma_1$ and $\Gamma_2$ are shown in Figs. 2 and 3 respectively. (Similar corrections exist for the $K_h$ kernel as well.) With the exception of the first diagram in Fig. 2, these corrections originate from the linear SO term $iDp_{so}(\overrightarrow{Q}\overrightarrow{Q})$ applied twice. They lead to the renormalization of the cutoff $1/\tau_{so}$. The renormalized cutoff $1/\tau'_{so}$ can be expressed in terms of the renormalized parameters $D \to D'$ and $p_{so} \to p'_{so}$:

$$1/\tau'_{so} = D'p'^2_{so} = Dp^2_{so} \left(1 + \delta D/D + 2\delta p_{so}/p_{so} \right), \quad (13)$$

where $\delta D$ and $\delta p_{so}$ are the corrections to $D$ and $p_{so}$, respectively. Given $\delta D$, one can extract $\delta p_{so}$ from the calculated $1/\tau_{so}$. $\delta D$ corresponds to the well known Altshuler-Aronov corrections to the conductivity $\delta D = -4\nu \int d^2 q (2\pi)^2 \int d\omega 2\pi D^3(q,\omega) Dq^2(\Gamma_1 - 2\Gamma_2) \quad (14)$

Then for $\delta p_{so}$, we get:
Note that $\delta p_{so}$ depends only on the triplet amplitude.

The corrections given in Eqs. (13) and (15) have so far employed the static amplitudes $\Gamma_1$ and $\Gamma_2$. To include the effects of dynamical screening, these amplitudes are extended via the ladder summation [21]:

$$
U_{s,t}(q, \omega) = \Gamma_s(t(Dq^2 + z\omega)/(Dq^2 + (z - 2\Gamma_{s,t})\omega)).
$$

The dynamical resummation allows the evaluation of the corrections to infinite order in the interaction amplitudes so that $\rho$ remains as the only expansion parameter. Remarkably, $\delta p_{so}$ vanishes after this replacement of the $\Gamma_2$ amplitude in Eq. (15) by $U_t(q, \omega)$:

$$
\delta p_{so} = 0.
$$

(16)

Similar analysis when extended to the kernel $K_h$ and to the linear term $Dp_{so}(\nabla Q \mp Q)$ in Eq. (12) leads to the result that the gauge form $D(\nabla Q + ip_{so}\hat{Q})^2$ of the action in Eq. (12) with the bare value of $p_{so}$ is preserved under renormalization. This implies that the cutoff $\tau^{-1} = Dp_{so}^2$ retains the same form as in Eq. (4) with $D$ as the renormalized diffusion constant and with $p_{so}$ unrenormalized.

Note that the diagrams presented in Figs. 2 and 3 resemble those that are used for the calculation of the spin susceptibility [13, 14] with the triangle at the top as the starting point and ending at the bottom triangle. This analogy is, however, misleading since the SO interaction $Dp_{so}(\nabla Q \mp Q)$ in addition to the Pauli matrices is proportional to momentum. It is this dependence on momentum that makes the calculation of the renormalization of $p_{so}$ different from the renormalization of the Zeeman term.

We now study the renormalization effects on the DP spin relaxation rate $\tau^{-1}$. To this end, we analyze the form of the renormalized dynamic spin susceptibility that follows from the action defined in Eq. (12):

$$
\chi^{xx}(q, \omega_n) = \chi^{xx}_{s}(z + \Gamma_2) = \frac{Dq^2 + Dp_{so}^2}{(z + \Gamma_2) \omega_n + Dq^2 + Dp_{so}^2},
$$

(17)

where, the renormalized spin-diffusion constant:

$$
D_s = D/(z + \Gamma_2).
$$

(18)

Note that Eq. (17) has the same structure as the expression for $\chi^{xx}(q, \omega)$ in the disordered Fermi-liquid with the parameter $(z + \Gamma_2)$ substituted for $(1 + \Gamma_2)$ in $\chi_s$ and $D_s$. It follows from Eq. (17) that the form of the DP spin relaxation rate, $\tau^{-1} = D_s p_{so}^2$, given in Eq. (6) is also preserved under renormalization.

To conclude, we have shown that the form of the functional $S(Q)$ in Eq. (12) is preserved under renormalization by the combined action of the $e-e$ interaction and disorder. It follows from this observation that the relaxation rate of the spin density and that of the cutoff in the $S = 1$ cooperon/diffuson propagators are given by Eqs. (3) and (4), respectively, where the parameters $D_s$ and $D$ are the renormalized diffusion constants [26], and the spin splitting $p_{so} = \Delta_{so}/v_F$ is unchanged. As a result, under the circumstances when $\chi_s$ diverges, either near a magnetic instability ($\Gamma_2 \to \infty$), or near the metal-insulator transition ($z \to \infty$), the spin-diffusion constant $D_s$ and $\tau^{-1}$ remain finite.

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