LOCAL HUBBLE EXPANSION:
CURRENT STATE OF THE PROBLEM

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Abstract: We present a brief qualitative overview of the current state of the
problem of Hubble expansion at the sufficiently small scales (e.g., in planetary
systems or local intergalactic volume). The crucial drawbacks of the avail-
able theoretical treatments are emphasized, and the possible ways to avoid
them are outlined. Attention is drawn to a number of observable astronomical
phenomena that could be naturally explained by the local Hubble expansion.

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1. Introduction: theoretical approaches to the problem of local Hubble
expansion

The problem of small-scale cosmological effects has a long history: the question
if planetary systems are affected by the universal Hubble expansion was posed by
McVittie as early as 1933 [35], i.e., approximately at the same time when the concept
of Hubble expansion became the dominant paradigm in cosmology. Although this
question never was a hot topic, the corresponding papers occasionally appeared in
the astronomical literature in the subsequent eight decades [1, 7, 9, 10, 12, 19, 21,
27, 33, 37, 41]. Using quite different physical models and mathematical approaches,
most of these authors arrived at the negative conclusions. As a result, it is commonly
believed now[1] that Hubble expansion should be strongly suppressed or absent at all
at the sufficiently small scales, for example, in planetary systems or inside galaxies.

1 One of a few exceptions is a short review by Bonnor [3], which appealed for a critical recon-
consideration of the available studies.
However, a surprising thing is that the commonly-used arguments not only prohibit the local Hubble expansion but also strongly contradict each other. For example, the most popular criterion for the suppression of Hubble expansion (especially, among the observational astronomers) is just a gravitational binding of the system, e.g., determined by the virial theorem of classical mechanics \cite{32}. Namely, if mass of the particles concentrated in the system becomes so large that the corresponding energy of gravitational interaction approaches by absolute value the double kinetic energy, then orbits of the particles should be bounded, i.e., no overall expansion of the system is possible. In other words, just the classical forces of gravitational attraction break the global Hubble flow in the regions of local mass enhancement.

On the other hand, yet another well-known theoretical argument against the local Hubble expansion, based on the self-consistent theoretical analysis in the framework of General Relativity (GR), is the so-called Einstein–Strauss theorem \cite{19}, illustrated in Figure 1. Let us consider a uniform distribution of the background matter with density $\rho$ and then assume that substance in a spherical volume with radius $R$ is cut off and concentrated in its center, thereby forming the point-like mass $M = (4\pi/3)R^3\rho$. Then, according to the this theorem, there will be no Hubble expansion inside the empty cavity, but the Hubble flow is restored again beyond its boundary with the background matter distribution (and this boundary itself moves exactly with Hubble velocity).

It is important to emphasize that, as distinct from the first criterion, there is no any excessive mass in the above-mentioned sphere and, moreover, the Hubble expansion is absent just in the empty space rather than in the region of mass enhancement. In principle, this fact is quite natural: according to the standard GR formula,
Hubble constant $H$ is related to the local energy density $\rho$ in the spatially-flat Universe as
\[ H = \sqrt{\frac{8\pi G}{3}} \rho, \tag{1} \]
where $G$ is the gravitational constant. So, from the relativistic point of view, it is not surprising that Hubble constant tends to zero when the energy density disappears.

Therefore, the above discussion demonstrates that the attempts to treat the problem of local Hubble expansion in terms of the classical gravitational forces can be very misleading. Indeed, the global Hubble expansion exists even in the perfectly-uniform Universe, where there are no any “classical” gravitational forces at all (since such forces can be produced only by nonhomogeneity of mass distribution). In other words, it should be kept in mind that Hubble expansion corresponds to another “degree of freedom” of the relativistic gravitational field as compared to the degrees of freedom reduced to the classical gravitational forces.

Unfortunately, a lot of textbooks tried to estimate the local Hubble expansion in terms of the “classical” gravity or just postulated its absence in the small-scale systems. A typical example is the famous textbook [36], where the behavior of small-scale systems (galaxies) in the globally-expanding Universe was pictorially described as a set of coins pinned to the surface of an inflating ball (see Figure 27.2 in the above-cited book), but no justification for such a picture was given.

2. Hubble expansion in the dark-energy-dominated cosmology

Because of the oversimplified geometry of the Einstein–Straus model (particularly, a presence of the void, which can hardly have a reasonable astrophysical interpretation), it is desirable to consider not so idealized situations. Unfortunately, a serious obstacle in this way is the problem of separation between the peculiar and Hubble flows of matter in a spatially inhomogeneous system. Namely, if there is no empty cavity, and boundary with the background matter distribution is not perfectly sharp, as in Figure 1, then substance in the vicinity of the central mass will experience a quite complex radial motion in the course of time, depending on the initial conditions. In general, we do not have any universal criterion to answer the question: what part of this motion should be attributed to the Hubble flow?

Fortunately, the situation is simplified very much in the case of idealized dark-energy-dominated Universe, where the entire cosmological contribution to the energy–momentum tensor of GR equations is produced by the $\Lambda$-term (cosmological constant). The $\Lambda$-term is distributed, by definition, perfectly uniform in space and, therefore, does not experience any back reaction from the additional (e.g., point-like)

\[ \text{We use everywhere the system of units where the speed of light is equal to unity (} c \equiv 1) \text{ and, therefore, there is no difference between the mass- and energy-density.} \]

\[ \text{Of course, strictly speaking, formula (1) is applicable only to the totally uniform Universe.} \]
As a result, it becomes not so difficult to consider the “restricted cosmological two-body problem”, i.e., motion of a test particle in the local gravitational field of the central mass embedded into the cosmological background formed by the Λ-term.

From our point of view, just this model enables one to get the simplest but reliable estimate for the magnitude of the local Hubble expansion, which can be used as a benchmark in more sophisticated studies.

The above-mentioned problem can be separated into two steps: Firstly, using GR equations, one needs to find a space–time metric of the point-like mass $M$ against the Λ-background. Secondly, using the standard geodesic equations, we should calculate the trajectories of test particles in this metric.

The first task was actually solved long time ago, in 1918, by Kottler [28]. The required metric reads as:

$$d{s^2} = - \left(1 - \frac{2GM}{r} - \frac{Λr^2}{3}\right)dt^2 + \left(1 - \frac{2GM}{r} - \frac{Λr^2}{3}\right)^{-1}dr^2 + r^2(dθ^2 + \sin^2θdϕ^2),$$

for more general discussion, see also [29].

Just this metric was widely used starting from the early 2000’s — when the importance of Λ-term in cosmology was clearly recognized — to study the motion of test particles. The quite sophisticated mathematical treatments can be found, for example, in papers [2, 23]; and the respective formulas were used for the analysis of observational data on planetary dynamics in the Solar system [5, 24, 25]. Unfortunately, the original Kottler metric (2) does not possess the correct cosmological asymptotics at infinity (which is not surprising, since it was derived well before a birth of the modern cosmology). The above-cited works, of course, reveal some features of particle dynamics in the dark-energy-dominated Universe, but they are unrelated (or, probably, partially related) to the Hubble expansion by itself.

So, to study effects of the Hubble expansion per se, it is necessary to transform metric (2) to the standard Robertson–Walker coordinates, commonly used in the cosmological calculations. Such a procedure was performed in our paper [15]; and the resulting expressions for the “cosmological” Kottler metric can be found there. Next, this metric should be used to solve the geodesic equations for a test particle moving in the field of the central mass [17]:

4 We do not discuss here the models with “dynamical” dark energy (where Λ-term is replaced by a new field), because they are not so necessary to explain the available observational data.

5 According to the standard terminology of celestial mechanics, the term “restricted” implies that one of the bodies (test particle) has infinitely small mass.

6 It is often called in the modern literature the Schwarzschild–de Sitter metric; although, from our point of view, this term is not sufficiently correct.
Figure 2: Orbits of a test body in the field of the central mass at $r_g = 10^{-2}$ and various values of $r_\Lambda$, assuming that $R_0 = 1$.

Here, as distinct from formula (2), $t$ and $r$ are the Robertson–Walker coordinates; and dot denotes a derivative with respect to the proper time of the moving particle.

An important feature of these equations is that they involve three characteristic spatial scales — Schwarzschild radius $r_g = 2GM$, de Sitter radius $r_\Lambda = \sqrt{3/\Lambda}$, and the initial radius of orbit of the test body (e.g., a planet) $R_0$ — which differ from each other by many orders of magnitude. For example, in the case of the Earth–Moon system (where Earth is the central mass; and Moon, the test body), we
have: \( r_g \sim 10^{-2} \text{ m}, \ R_0 \sim 10^0 \text{ m}, \) and \( r_\Lambda \sim 10^{27} \text{ m}. \) This makes the problem of accurate numerical integration very hard.

However, for simplicity — just to reveal the possibility of local Hubble expansion in the gravitationally-bound system — we can consider a toy model, where these parameters differ from each other not so much, e.g., by only two or three orders of magnitude. For example, let us take the initial orbital radius as the unit of length (i.e., \( R_0 \equiv 1 \)); and let the Schwarzschild radius be \( r_g = 10^{-2} \), and de Sitter radius \( r_\Lambda = 10^9 \text{ or } 2 \cdot 10^9 \). The corresponding numerical orbits are presented in Figure 2. As is seen, when \( \Lambda \) (i.e., the dark-energy density) increases and, respectively, \( r_\Lambda \) decreases, the orbits become more and more spiral. In other words, \textit{a test particle orbiting about the central mass can really experience the local Hubble expansion}. This quantitative analysis argues against the commonly-accepted intuitive point of view that the remote cosmological action would result just in a partial compensation of the gravitational attraction to the center, i.e., the orbit will be slightly disturbed but remain stationary \cite{33}. According to our calculations, the secular (time-dependent) effects are really possible.

Unfortunately, it is not so easy to get the reliable numerical values of such an effect in the realistic planetary systems, because of the above-mentioned huge difference in the characteristic scales and the need for integration over a very long time interval. Besides, since the set of equations is strongly nonlinear, it is difficult to predict how the other kinds of celestial perturbations (e.g., by the additional planets) will interfere with the secular Hubble-type effects. Moreover, it is unclear in advance if the local Hubble expansion will follow the standard linear relation:

\[
\dot{r} = \frac{H_0^{(\text{loc})}}{r},
\]

where \( H_0^{(\text{loc})} \) is the local Hubble constant (which, generally speaking, can be different from the global one). In principle, the corresponding relation in the vicinity of the central massive body might be substantially nonlinear. So, all these questions are still to be answered.

### 3. Observable footprints of the local Hubble expansion

A crucial factor supporting the interest to a probable manifestation of Hubble expansion at the small scales is that there is a number of observable phenomena — both in the Solar system and local intergalactic volume — that could be naturally explained by the local Hubble expansion. A detailed list of such effects in the Solar system can be found in papers \cite{30, 31}. Particularly, they are:

- the so-called faint young Sun problem (i.e., the insufficient luminosity of the young Sun to support development of the geological and biological evolution on the Earth),
the problem of liquid water on Mars (which actually has the same origin as the above-mentioned one),

• the anomalous rate of recession of the Moon from the Earth (called also the lunar tidal catastrophe),

• the long-term dynamics of the so-called fast satellites of Mars, Jupiter, Uranus, and Neptune,

• the efficiency of formation of Neptune and comets in the Kuiper belt from the protoplanetary disk.

3.1. The lunar tidal catastrophe

From our point of view, the most appealing example for the existence of the local Hubble expansion is the anomalous Earth–Moon recession rate. Namely, it was known for a long time that tidal interaction results in the deceleration of proper rotation of the Earth $\Omega_E$ and acceleration of orbital rotation of the Moon $\Omega_{ME}$ [26]. This is pictorially explained in Figure 3, since $\Omega_E > \Omega_{ME}$, a tidal bulge on the Earth’s surface is slightly shifted forward (in the direction of Earth’s rotation) because of the finite-time relaxation effects. Such a shifted bulge pulls the Moon forward, thereby accelerating it; and simultaneously, due to the back reaction, the proper rotation of the Earth decelerates.
Method Measurement by the lunar laser ranging Estimate from the Earth’s tidal deceleration

| Effects involved | (1) geophysical tides | (1) geophysical tides |
|------------------|-----------------------|----------------------|
| (2) local Hubble expansion |                       |                      |
| Numerical value  | 3.8±0.1 cm/yr         | 1.6±0.2 cm/yr        |

Table 1: Relative contributions of various processes to the recession rate of the Moon from the Earth.

The increasing orbital momentum of the Moon results in the increase of its distance from the Earth with the following rate:

\[
\dot{r} = k \dot{T}_E, \tag{7}
\]

where \(T_E\) is the Earth’s diurnal period, and \(k = 1.81 \cdot 10^5 \text{ cm/s} \) \cite{14}. So, if secular variation in the length of day is known from astrometric observations, relation (7) can be used to derive the rate of secular increase in the lunar orbit \(\dot{r}\).

On the other hand, the same quantity can be measured immediately by the lunar laser ranging (LLR). This became possible since the early 1970’s, when a few optical retroreflectors were installed on the lunar surface. The accuracy of LLR quickly improved in the subsequent two decades, and its errors were reduced to 2–3 cm, which enabled ones to measure immediately the secular expansion of the lunar orbit \cite{11}. Surprisingly, the measured value of \(\dot{r}\) turned out to be substantially greater than the value obtained from formula (7), as summarized in Table 1 \cite{16}.

Then, a lot of attempts were undertaken to reduce this discrepancy. Namely, the value presented in the last column of the table corresponds to

\[
\dot{T}_E = (8.77±1.04) \cdot 10^{-6} \text{ s/yr}. \tag{8}
\]

It was derived from a series of astronomical observations accumulated since the middle of the 17th century, when telescopic data became available (they are compiled, for example, in monograph \cite{42}). In principle, the period of three centuries might be insufficiently long, because the length of day \(T_E\) experience also some quasi-periodic variations on the longer time scales, which can affect the linear trend (5).

One of the ways to get around this obstacle is to employ the ancient data on eclipses, which cover the period over two millennia. Such an approach was pursued by a number of researchers (e.g., review \cite{43}), and the obtained values of \(\dot{T}_E\) sometimes enabled them to get a reasonable agreement with LLR data. However, the various sets of ancient observations give the results different from each other by almost two times, and it is not clear \textit{a priori} which of them are more reliable.
Yet another idea to avoid the discrepancy presented in Table 1 is to take into account a secular variation in the Earth’s moment of inertia, which is commonly characterized by the second gravitational harmonic coefficient $J_2$. Its decreasing trend at the present time is assumed to be caused by the so-called viscous rebound of the solid Earth from the decrease in load due to the last deglaciation. (Namely, the Earth was compressed by the ice caps in polar regions during the glacial period and now restores its shape.) The first determination of the above-mentioned parameter by Lageos satellite \cite{1} led to the value $J_2 = -3 \cdot 10^{-11}/\text{yr}$, which seemed to be consistent with LLR data. However, as was established later, such a determination may be very unreliable \cite{8} and even can give the opposite sign of $J_2$.

In view of the above difficulties, a promising explanation of the discrepancy 2.2±0.3 cm/yr in Table 1 can be based just on the presence of local Hubble expansion. Assuming validity of the standard relation (6), this corresponds to the value of the local Hubble constant

$$H_0^{(\text{loc})} = 56 \pm 8 \text{ (km/s)/Mpc},$$

which is quite close to its “global” value $H_0$. So, such an interpretation is not meaningless.

Unfortunately, as was mentioned in Section 2 by now we cannot reliably explain this quantity in terms of parameters of the Earth–Moon system because of the problems in the numerical integration of the equations of motion. Instead, we shall present here a more crude but universal estimate of the relation between the local and global Hubble rates, which is actually applicable to any “small-scale” system.

It is reasonable to assume that the local Hubble expansion is formed only by the uniformly-distributed dark energy ($\Lambda$-term), while the irregularly distributed (clumped) forms of matter affect the rate of cosmological expansion only at the sufficiently large distances, where they can be characterized by their average values. (At smaller distances, the clumped forms of matter manifest themselves by the “classical” gravitational forces.) So, if the Universe is spatially flat and filled only with dark energy and the dust-like matter with densities $\rho_{\Lambda 0}$ and $\rho_{D 0}$, respectively, then general expression (11) can be rewritten as

$$H_0 = \sqrt{\frac{8\pi G}{3}} \sqrt{\rho_{\Lambda 0} + \rho_{D 0}},$$

$$H_0^{(\text{loc})} = \sqrt{\frac{8\pi G}{3}} \sqrt{\rho_{\Lambda 0}}.$$

Therefore, a ratio of the local to global Hubble constants will be

$$\frac{H_0^{(\text{loc})}}{H_0} = \left[1 + \frac{\Omega_{D 0}}{\Omega_{\Lambda 0}}\right]^{-1/2},$$

where $\Omega_{\Lambda 0} = \rho_{\Lambda 0}/\rho_{\text{cr}}$ and $\Omega_{D 0} = \rho_{D 0}/\rho_{\text{cr}}$ are the corresponding relative densities.

\footnote{Subscripts “0” denote here the values of the corresponding quantities at the present time.}
Taking for a crude estimate $\Omega_\Lambda_0 = 0.75$ and $\Omega_D_0 = 0.25$, we arrive at

$$H_0 / H_0^{(\text{loc})} \approx 1.15.$$  (13)

Consequently, the local value (9) corresponds to the global value

$$H_0 = 65 \pm 9 \text{ (km/s)/Mpc},$$  (14)

which is in a good agreement with the modern cosmological data (especially, based on studies of type Ia supernovae).

Let us emphasize that the performed analysis crucially depends on the accepted value of secular increase in the length of day $\dot{T}_E$. The qualitative idea of such analysis was put forward in our work [13], and in the first quantitative study of this subject [14] we used the value corrected for the ancient eclipses, $\dot{T}_E = 1.4 \times 10^{-5}$ s/yr, which was considered by some researchers as the best option [13]. As a result, we arrived at the substantially reduced magnitude of the local Hubble constant, $H_0^{(\text{loc})} = 33 \pm 5$ (km/s)/Mpc, which had no reasonable interpretation. On the other hand, when in the later work [16] we employed $\dot{T}_E$ derived purely from the set of astrometric observations in the telescopic era [42] without any further corrections, the resulting value of $H_0^{(\text{loc})}$ was found to be in accordance with the large-scale cosmological data.

### 3.2. The faint young Sun paradox

Yet another appealing example for the existence of local Hubble expansion is the problem of insufficient flux of energy from Sun to the Earth in the past; e.g., review [22]. Namely, according to the modern models of stellar evolution, the solar luminosity increases by approximately 30% during the period after its birth (about $5 \times 10^9$ yr). This means that the energy input to the Earth’s climate system, e.g., 2–4 billion years ago was appreciably less than now and, therefore, the most part of water must be in a frozen state. This would preclude the geological and biological evolution of the Earth and contradicts a number of well-established facts on the existence of considerable volumes of liquid water in that period of time. Although a lot of attempts were undertaken to resolve this problem by the inclusion of additional influences (first of all, the atmospheric greenhouse effect), no definitive solution is available by now.

An interesting option was suggested recently by Krížek and Somer [30, 31], who proposed to take into consideration the local Hubble expansion of the Earth’s orbit. As a result, the Sun–Earth distance in the past would be appreciably less than now and, consequently, the solar irradiation of the Earth’s surface increased. In particular, the quantitative analysis performed in the above-cited papers have shown that at $H_0^{(\text{loc})} \approx 0.5 H_0$ expansion of the Earth’s orbit compensates the increasing solar luminosity with very good accuracy; so that the Earth’s surface received almost the
same flux of energy in the past $3.5 \times 10^9$ yr and will continue to do so for a considerable period in future.

From our point of view, the above-mentioned idea is very promising. Unfortunately, the value of local Hubble constant used in these papers is poorly consistent with the one derived from our analysis of the Earth–Moon system in Section 3.1, $H_0^{(\text{loc})} \approx 0.85 H_0$. So, it is interesting to check if the same mechanism will work at other rates of the local Hubble expansion? Such analysis was performed in our recent work [18].

Namely, let solar luminosity increase linearly with time:

$$L(t) = L_0 + \left(\frac{\Delta L}{\Delta T}\right) t,$$

(15)

where $L_0$ is its present-day value (at $t = 0$), and $\Delta L$ is the variation of luminosity over the time interval $\Delta T = 5 \times 10^9$ yr (for the sake of estimate, we shall use here the rounded values). Then, assuming validity of the standard relation (6), a temporal variation in the irradiation of the Earth’s surface can be found from a simple geometric consideration. The resulting curves for a number of hypothetical solar models with $\Delta L/L_0 = 0.3, 0.4, 0.5, 0.6$ and various rates of the local Hubble expansion $H_0^{(\text{loc})} = 0.5, 0.6, 0.7, 0.8, 0.9 H_0$ are presented in Figure 4.

It is seen that the Krížek–Somer case ($\Delta L/L_0 = 0.3, H_0^{(\text{loc})}/H_0 = 0.5$) really provides a very stable energy input to the Earth for a few billion years both in the past and future. At the higher rates of the local Hubble expansion (which would be more consistent with our analysis of the Earth–Moon dynamics), a quite favorable situation exists, for example, at $\Delta L/L_0 = 0.5$ and $H_0^{(\text{loc})}/H_0 = 0.8$: the solar irradiation at $t < 0$ is almost as stable as in the Krížek–Somer case, and more appreciable variation at $t > 0$ is not so important because we actually do not know the Earth’s evolution in the future.

Is it reasonable to consider the solar model with $\Delta L/L_0 = 0.5$? In fact, such enhanced variations $\Delta L$ were typical for the first quantitative models of the Sun [40]. However, the subsequent investigations resulted in the progressively less values of $\Delta L$; and it is commonly accepted now that the increase in luminosity amounts to about 7% per Gyr over the past evolution of 4.57 Gyr. Nevertheless, we may imagine processes like mixing in the solar interiors to change this value. This would imply the star with a small convective core. The problem is that the Sun is just at the limit of mass where convective cores appear [34].

Of course, one should keep in mind that the above calculations of solar irradiation cannot be immediately confronted with the relevant data from paleoclimatology, because it is necessary to take into account a lot of additional geophysical and geochemical processes, first of all, the greenhouse effect. From this point of view, the Earth–Moon system discussed in Section 3.1 represents a more “clean” case, where the probable local Hubble expansion is less obscured by other phenomena.
Figure 4: Temporal variations in solar irradiation of the Earth’s surface $F/F_0$ for different models of solar evolution (characterized by $\Delta L/L_0$) and various rates of the local Hubble expansion (numbers near the curves denote the ratio $H_{0}^{(loc)}/H_0$). The straight lines marked by 0.0 correspond to the case when the local Hubble expansion is absent at all.
3.3. Other systems

A number of other effects in the Solar system that might be associated with local Hubble expansion have been already listed in the beginning of Section 3. Unfortunately, they are much less studied than the lunar tidal catastrophe and the faint young Sun paradox. So, we shall not discuss them in the present article; for more details, see papers [30, 31].

Besides, a few researchers studied dynamics of all solar-system bodies, including the major asteroids, on the basis of data by optical and radio astrometry collected in the last decades [38, 39]. Their conclusion was that, in general, a self-consistent picture of planetary motion (the high-precision ephemerides) can be obtained without taking into account any local cosmological influences. However, it should be kept in mind that such analyses involved a lot of fitting parameters, which were attributed, e.g., to the unknown masses of asteroids, solar oblateness, effects of the solar wind on radio wave propagation, etc. On the other hand, the probable Hubble expansion was never included into their equations in explicit form. So, the small resulting residuals might be merely a mathematical fact: it is well known from statistics that any empirical data can be fitted as accurately as desirable if the number of free parameters becomes sufficiently large.

If the local Hubble expansion is present in the Solar system, it should be naturally expected also in galaxies. Unfortunately, the entire pattern of galaxy evolution is very complicated by the formation of stars and their proper motions. So, as far as we know, the problem of cosmological effects at the scale of galaxies remains completely unexplored by now.

A much more elaborated subject is Hubble expansion in the local intergalactic volume. It was believed for a long time that the standard Hubble flow can be traced only at the distances starting from 5–10 Mpc, where it becomes possible to introduce the average cosmological matter density. Nevertheless, by the end of the 20th century, the Hubble flow was detected also at the considerably less scales, down to 1–2 Mpc. At the same time, the concept of dark-energy-dominated Universe became the main paradigm in cosmology. So, it was natural to explain both the presence of the Hubble flow at the sufficiently small scales and its regularity (“quiescence”) just by the perfectly-uniform dark energy (or Λ-term) [6, 20]. Unfortunately, it remains unclear by now if the effective value of Hubble constant in the Local Group is smaller or larger than at the global scales and, therefore, if the relation (12) between \( H_0^{(loc)} \) and \( H_0 \) is applicable in this situation?

Let us mention also that the most of available theoretical works on the dynamics of galaxies in the Local Group are based on the effective gravitational forces derived from Kottler metric [2]:

\[
F_{\text{eff}}(r) = M_1 \left( -\frac{GM_2}{r^2} + \frac{\Lambda r}{3} \right); \tag{16}
\]
the last term often being called the “antigravity” force. Unfortunately, such treat-
ment has a limited scope of applicability: Firstly, as was already mentioned in Sec-
tion 2, the static metric (2) does not possess a correct cosmological asymptotics at
infinity and, therefore, the corresponding force (16) is unable to describe the entire
Hubble flow, including the large distances. Secondly, strictly speaking, the above-
written effective force is adequate only for the restricted two-body problem (where
$M_1$ is the mass of a test particle, and $M_2$ is the mass of the central gravitating
body). This is evident, in particular, from the fact that masses $M_1$ and $M_2$ appear
in expression (16) by different ways. So, there is no reason to assume validity of this
formula when $M_1$ and $M_2$ are comparable to each other or, especially, to apply it to
the many-body problem.

4. Concluding remarks

1. Despite a lot of theoretical works rejecting the possibility of local Hubble ex-
pansion, we believe that this problem is still unresolved: Firstly, the available
arguments often contradict each other. Secondly, the most of them become in-
applicable to the case when the Universe is dominated by the perfectly-uniform
dark energy (or Λ-term). Moreover, a self-consistent theoretical treatment of
the simplest models (such as the restricted two-body problem against the Λ-
background) demonstrates a principal possibility of the local cosmological in-
fluences: the Hubble expansion is not suppressed completely in the vicinity of
a massive body.

2. A few long-standing problems in planetology, geophysics, and celestial mechan-
ics can be well resolved by the assumption of local Hubble expansion whose
rate is comparable to that at the global scales. It is quite surprising that many
theorists believe that the possibility of local cosmological influences is strictly
prohibited just by the available observational data, while a lot of observers
believe that there are irrefutable theoretical proofs that Hubble expansion is
absent at small scales.

3. However, the important conceptual question still persists: What is the spa-
tial scale from which the cosmological expansion no longer takes place? This
is of crucial importance since otherwise, as pictorially explained by Misner et
al. [36, p. 719], the “meter stick” will also expand and, therefore, it will be
meaningless to speak about any expansion at all... We cannot give a definitive
numerical answer to this question. However, we believe that the systems domi-
nated by non-gravitational interactions should not experience the cosmological
expansion (e.g., the meter stick, the solid Earth, etc. do not expand).
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References

[1] Anderson, J. L.: Multiparticle dynamics in an expanding Universe. Phys. Rev. Lett. 75 (1995), 3602.

[2] Balaguera-Antolínez, A., Böhmer, C. G., and Nowakowski, M.: Scales set by the cosmological constant. Class. Quant. Grav. 23 (2006), 485.

[3] Bonnor, W. B.: Local dynamics and the expansion of the Universe. Gen. Rel. Grav. 32 (2000), 1005.

[4] Bourda, G. and Capitaine, N.: Precession, nutation, and space geodetic determination of the Earth’s variable gravity field. Astron. Astrophys. 428 (2004), 691.

[5] Cardona, J. F. and Tejeiro, J. M.: Can interplanetary measures bound the cosmological constant? Astrophys. J. 493 (1998), 52.

[6] Chernin, A., Teerikorpi, P., and Baryshev, Y.: Why is the Hubble flow so quiet? Adv. Space Res. 31 (2003), 459.

[7] Cooperstock, F. I., Faraoni, V., and Vollick, D. N.: The influence of the cosmological expansion on local systems. Astrophys. J. 503 (1998), 61.

[8] Cox, C. M. and Chao, B. F.: Detection of a large-scale mass redistribution in the terrestrial system since 1998. Science 297 (2002), 831.

[9] Davis, T. M., Lineweaver, C. H., and Webb, J. K.: Solutions to the tethered galaxy problem in an expanding Universe and the observation of receding blueshifted objects. Amer. J. Phys. 71 (2003), 358.

[10] Dicke, R. H. and Peebles, P. J. E.: Evolution of the Solar system and the expansion of the Universe. Phys. Rev. Lett. 12 (1964), 435.

[11] Dicke, J. O. et al.: Lunar laser ranging: A continuing legacy of the Apollo program. Science 265 (1994), 482.

[12] Domínguez, A. and Gaite, J.: Influence of the cosmological expansion on small systems. Europhys. Lett. 55 (2001), 458.
[13] Dumin, Y. V.: Using the lunar laser ranging technique to measure the local value of Hubble constant. Geophys. Res. Abstr. 3 (2001), 1965.

[14] Dumin, Y. V.: A new application of the lunar laser retroreflectors: Searching for the ‘local’ Hubble expansion. Adv. Space Res. 31 (2003), 2461.

[15] Dumin, Y. V.: Comment on ‘Progress in lunar laser ranging tests of relativistic gravity’. Phys. Rev. Lett. 98 (2007), 059001.

[16] Dumin, Y. V.: Testing the dark-energy-dominated cosmology by the solar-system experiments. In: H. Kleinert, R. Jantzen, and R. Ruffini (Eds.), Proc. 11th Marcel Grossmann meeting on General Relativity, p. 1752. World Sci., Singapore, 2008.

[17] Dumin, Y. V.: Perturbation of a planetary orbit by the Lambda-term (dark energy) in Einstein equations. In: N. Capitaine (Ed.), Proc. Journées 2010 Systèmes de référence spatio-temporels: New challenges for reference systems and numerical standards in astronomy, p. 276. Observ. Paris, 2011.

[18] Dumin, Y. V.: The faint young Sun paradox in the context of modern cosmology. Astronomicheskii Tsirkulyar (Astron. Circular) 1623 (2015), 1. http://comet.sai.msu.ru/~gmr/AC/AC1623.pdf.

[19] Einstein, A. and Straus, E.G.: The influence of the expansion of space on the gravitation fields surrounding the individual stars. Rev. Mod. Phys. 17 (1945), 120.

[20] Ekholm, T., Baryshev, Y., Teerikorpi, P., Hanski, M. O., and Paturel, G.: On the quiescence of the Hubble flow in the vicinity of the Local Group: A study using galaxies with distances from the Cepheid PL-relation. Astron. Astrophys. 368 (2001), L17.

[21] Faraoni, V. and Jacques, A.: Cosmological expansion and local physics. Phys. Rev. D 76 (2007), 063510.

[22] Feulner, G.: The faint young Sun problem. Rev. Geophys. 50 (2012), RG2006.

[23] Hackmann, E. and Lämmerzahl, C.: Geodesic equation in Schwarzschild-(anti-) de Sitter space-times: Analytical solutions and applications. Phys. Rev. D 78 (2008), 024035.

[24] Iorio, L.: Can solar system observations tell us something about the cosmological constant? Int. J. Mod. Phys. D 15 (2006), 473.

[25] Kagramanova, V., Kunz, J., and Lämmerzahl, C.: Solar system effects in Schwarzschild–de Sitter space–time. Phys. Lett. B 634 (2006), 465.
[26] Kaula, W.: *An introduction to planetary physics: The terrestrial planets*. J. Wiley & Sons, New York, 1968.

[27] Klioner, S. A. and Soffel, M. H.: Refining the relativistic model for Gaia: Cosmological effects in the BCRS. In: C. Turon, K. S. O’Flaherty, and M. A. C. Perryman (Eds.), *Proc. Symp. The three-dimensional Universe with Gaia (ESA SP-576)*, p. 305. ESA Publ. Division, Noordwijk, Netherlands, 2005.

[28] Kottler, F.: Über die physikalischen Grundlagen der Einsteinschen Gravitationstheorie. Ann. Phys. (Leipzig) **56** (1918), 401.

[29] Kramer, D., Stephani, H., MacCallum, M., and Herlt, E.: *Exact solutions of Einstein’s field equations*. Cambridge University Press, Cambridge, 1980.

[30] Krížek, M.: Dark energy and the anthropic principle. New Astron. **17** (2012), 1.

[31] Krížek, M. and Somer, L.: Manifestations of dark energy in the Solar system. Grav. Cosmol. **21** (2015), 59.

[32] Landau, L. D. and Lifshitz, E.M.: *Mechanics*. Pergamon Press, Oxford, 1976, 3rd edn.

[33] Lineweaver, C. H. and Davis, T. M.: Misconceptions about the Big Bang. Sci. American **292**, no. 3 (2005), 36.

[34] Maeder, A.: Private communication (2016).

[35] McVittie, G. C.: The mass-particle in an expanding Universe. Mon. Not. Royal Astron. Soc. **93** (1933), 325.

[36] Misner, C. W., Thorne, K. S., and Wheeler, J. A.: *Gravitation*. W. H. Freeman & Co., San Francisco, 1973.

[37] Noerdlinger, P. D. and Petrosian, V.: The effect of cosmological expansion on self-gravitating ensembles of particles. Astrophys. J. **168** (1971), 1.

[38] Pitjeva, E. V.: High-precision ephemerides of planets–EPM and determination of some astronomical constants. Solar System Res. **39** (2005), 176.

[39] Pitjeva, E. V.: Relativistic effects and solar oblateness from radar observations of planets and spacecraft. Astron. Lett. **31** (2005), 340.

[40] Schwarzschild, M.: *Structure and evolution of the stars*. Princeton Univ. Press, Princeton, N.J., 1958.

[41] Sereno, M. and Jetzer, P.: Evolution of gravitational orbits in the expanding Universe. Phys. Rev. D **75** (2007), 064031.
[42] Sidorenkov, N. S.: Physics of the Earth’s rotation instabilities. Nauka-Fizmatlit, Moscow, 2002. In Russian.

[43] Stephenson, F. R. and Morrison, L. V.: Long-term changes in the rotation of the Earth: 700 B.C. to A.D. 1980. Phil. Trans. Royal Soc. Lond. A 313 (1984), 47.

[44] Yoder, C. F. et al.: Secular variation of Earth’s gravitational harmonic $J_2$ coefficient from Lageos and nontidal acceleration of Earth rotation. Nature 303 (1983), 757.