Renormalized Quarkonium

J. Segovia, D.R. Entem, F. Fernández, and E. Ruiz Arriola

1Grupo de Física Nuclear and IUFFyM, Universidad de Salamanca, E-37008 Salamanca, Spain
2Departamento de Física Atómica, Molecular y Nuclear, Universidad de Granada, E-18071 Granada, Spain.
3Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, E-18071 Granada, Spain.

(Dated: May 3, 2014)

We extend our previous study of the vector charmonium states within a renormalization approach with boundary conditions to the full spectrum of charmonium and bottomonium. On the light of the predicted spectrum we comply to assignments suggested in the literature. A comparison with the regularized quark model is also included.

PACS numbers: 14.40.Pq, 12.39.Pn, 11.10.Gh
Keywords: Heavy quarkonia, potential models, renormalization in field theory

I. INTRODUCTION

The discovery of charmonium-like states by the B-factories named XYZ mesons, which could not simply be described by the naive quark model, have increased the interest for new and more accurate charmonium potentials trying to accommodate these states.

The static charmonium potential can be parametrized phenomenologically as a coulomb plus linear part together with spin-spin, spin-tensor and spin-orbit terms as leading spin dependent corrections [1, 2]. From a more fundamental point of view, this potential can be derived from first principles of QCD in the lattice with relativistic corrections classified in powers of the inverse quark mass [3]. Recently these potentials have been improved either by new approaches in lattice QCD [4] or by matching the long range part calculated by lattice simulations of the full QCD with results from perturbative QCD at short range [5]. Despite these improvements the short range part of all these potentials still suffers from singularities which can be only handled by using ad hoc regulators. This triggers an unpleasant short distance sensitivity.

In Ref. 6 we have developed an approach which reduces the effects of the ad hoc regulators, treating exactly the singular contributions of the potential following renormalization ideas. Instead of fitting the regulators to reproduce the ground state mass we use this mass as input parameter of our calculation. The rest of the spectrum is predicted from the orthogonality with the ground state. The short range uncertainties are encoded in this input and the rest of the spectrum only depends on the well established pieces of the potential.

We have shown in Ref. 6 that this procedure accurately reproduces the same numerical results as the standard constituent quark model with extra regulators for the particular case of the \(J^{PC}=1^{--}\) states. In this paper we extend the calculation to the full charmonium and bottomonium spectrum showing that for all purposes the regulators only account for the ground states of \(c\bar{c}\) and \(b\bar{b}\) systems with \(J^{PC}\) quantum numbers. Therefore the approach constitutes a useful tool to complement more fundamental potentials. For completeness, we will also compare the results with those of the original model (including regulators), where generally an almost perfect agreement is found. Once these short range uncertainties have been resolved we try to analyze which of the currently existing XYZ states could be identified as purely quarkonium states.

The plan of the paper is as follows. Section II explains briefly the features of our constituent quark model and describes the renormalization approach extended to all possible channels of quantum numbers. Sections III and IV are devoted to comment the spectrum of charmonium and bottomonium sectors within the renormalization approach. Section V includes a comparison with the original model. We finish summarizing the work and giving some conclusions in Section VI.

II. RENORMALIZED QUARKONIUM MODEL

The renormalization procedure we use exploits the local character of the potentials and uses a radial regulator in terms of boundary condition at a given cut-off radius \(r_c\) which is made smaller than any other length scale of the problem (typically \(r_c \sim 0.01\) is enough). Within the present context the procedure has been explained in detail in [6] for which we refer for further aspects and motivation. Here, we will only present its generalization for any channel different than the \(J^{PC}=1^{--}\).

The inter-quark potential we use is based on the one developed by Vijande et al. [7] which is able to describe...
meson phenomenology from the light to the heavy quark sector. It also successfully describes hadron phenomenology and hadronic reactions \[8\,11\]. The quark model includes a screened linear confinement potential based on unquenched lattice calculations together with spin-dependent terms determined by perturbative one-gluon-exchange as a Fermi-Breit type interaction. Although a dependent terms determined by perturbative one-gluon-

The OGE potentials are

\[ V_{\text{OGE}}^C(\vec{r}) = -\frac{4a_s}{3} \frac{1}{r}, \quad (1) \]

\[ V_{\text{OGE}}^T(\vec{r}) = \frac{1}{3} \frac{a_s}{m^2 r^3} S_{12}, \quad (2) \]

\[ V_{\text{OGE}}^{SO}(\vec{r}) = \frac{2a_s}{m^2 r^3} \vec{L} \cdot \vec{S}. \quad (3) \]

As we see, the tensor and spin-orbit potentials present \(1/r^3\) singularities at short distances. Note that we have discarded the Dirac delta function (which has the same dimensions as \(1/r^3\)) which is traditionally included. While this may seem weird, these are distributions around the origin which are not seen by the compact support test functions implied by the boundary condition regularization below the radial cut-off radius \[6\]. This also applies to any derivatives of the Dirac delta function. The result was suggested \[13\] and explicitly checked by using a momentum space regularization with so-called counterterms \[14\].

Once the constituent quark model we use has been presented, we will apply the renormalization with boundary conditions to eliminate the regulators of the model and treat exactly the singular contribution of the potential.

This scheme has been explained in detail in \[6\] and we will provide here only the most relevant additional aspects to deal with all \(J^{PC}\) quarkonium states.

In Ref. \[6\] it is shown that for unique and finite normalizable solutions the number of free independent parameters of the regularized theory depends on the behavior of the solution at the origin and can be established by a simple analysis of the potentials. The different cases that we can find are

- Uncoupled channels with a singular attractive potential. In this case, besides the normalization condition which eliminates one constant, one more parameter is needed to renormalize the solution. If the potential is repulsive at short range all the constants which determine the wave function are defined and the bound state is predicted as it usually happens for the standard non singular quantum mechanics problem.

- In the case of coupled channels the number of free parameters depends of the values of the potential near the origin, and more specifically on the corresponding eigenvalues of the coupled channel potential matrix. For two attractive potential eigenvalues we need three observables to fix the wave function. If there is one attractive eigenvalue only one parameter is needed. Finally, in the case of two repulsive eigenvalues the wave function is completely determined without any additional input parameter.

Focusing on quarkonium systems, we will work within the nonrelativistic framework, so the dynamics of the system is given by the Schrödinger equation. For a tensor and spin-orbit interaction we find the following situations

- Singlet channel (\(s = 0, l = J\))

\[ -\frac{1}{2\mu} u''_n(r) + \left[ V_{J,J}^{\alpha}(r) + \frac{J(J+1)}{2\mu r^2} \right] u_n(r) = E_n u_n(r) \quad (4) \]

- Triplet uncoupled channel (\(s = 1, l = J\))

\[ -\frac{1}{2\mu} u''_n(r) + \left[ V_{J,J}^{\alpha}(r) + \frac{J(J+1)}{2\mu r^2} \right] u_n(r) = E_n u_n(r) \quad (5) \]

- Triplet coupled channel (\(s = 1, l = J \pm 1\))

\[ -\frac{1}{2\mu} u''_n(r) + \left[ V_{J,J}^{\alpha}(r) + \frac{(J-1)J}{2\mu r^2} \right] u_n(r) + V_{J+1,J+1}^{\alpha}(r) w_n(r) = E_n u_n(r), \]

\[ -\frac{1}{2\mu} w''_n(r) + V_{J+1,J+1}^{\alpha}(r) u_n(r) + \left[ V_{J+1,J+1}^{\alpha}(r) + \frac{(J+1)(J+2)}{2\mu r^2} \right] w_n(r) = E_n w_n(r), \quad (6) \]

where the energy is defined with respect to the \(Q - \bar{Q}\) threshold, \(E_n = M_n - m_Q - m_{\bar{Q}} = M_n - 2m\).
As mentioned, in order to determine the number of independent constants we have to study the potential at short distances for different channels. In the \( r \to 0 \) limit the dominant contributions are the tensor and the spin-orbit terms of the OGE potential. We need to know their character for the different cases:

- Singlet channel \((s = 0, l = J)\). We have \(\langle S_{12}\rangle = 0\) and \(\langle \mathbf{L} \cdot \mathbf{S} \rangle = 0\), so the potential is regular and there is no observable free parameter. For the \(^1S_0\) channel we have to take into account that in this case the potential has an attractive \(\delta\)-function and so is singular, therefore we have to set an observable for a regularized solution.

- Triplet uncoupled channel \((s = 1, l = J)\). We have \(\langle S_{12}\rangle = +2\) and \(\langle \mathbf{L} \cdot \mathbf{S} \rangle = -1\), so the potential is singular attractive and an observable must be fixed for a regularized solution.

- Triplet coupled channel \((s = 1, l = J \pm 1)\). If we denote \(l = J - 1\) and \(l' = J + 1\), we will have

\[
\begin{align*}
- (\langle J_l | S_{12} | J'_{l'} \rangle) &= 6\sqrt{J(J+1)} J_{2J+1}, \\
- (\langle J_l | S_{12} | J_l \rangle) &= -2(J-1) J_{2J+1}, \\
- (\langle J'_{l'} | S_{12} | J'_{l'} \rangle) &= -2(J+2) J_{2J+1}, \\
- (\langle J_l | \mathbf{L} \cdot \mathbf{S} | J_l \rangle) &= J - 1, \\
- (\langle J'_{l'} | \mathbf{L} \cdot \mathbf{S} | J'_{l'} \rangle) &= -(J + 2),
\end{align*}
\]

and diagonalizing the potential matrix the eigenvalues are

\[
E(J) = -10 \pm 6 \sqrt{1 + J + J^2}
\]

with \(J \geq 1\), so we always have one negative eigenvalue and it requires one parameter.

In order to describe a bound state we seek normalizable solutions

\[
\int_0^\infty \left[ u(r)^2 + w(r)^2 \right] = 1,
\]

where \(w(r) = 0\) for the uncoupled channels. This imposes conditions on the wave functions both at infinity as well as at the origin.

Moreover, the set of equations \((4)\) \((5)\) \((6)\) must be accompanied by asymptotic conditions at infinity. Once we have discard the irregular function at long distances, the wave functions at infinity have the following behavior

\[
\begin{align*}
u(r) &\to A_{J-1} e^{-\eta r}, \\
w(r) &\to A_{J+1} e^{-\eta r},
\end{align*}
\]

where \(A_{J-1}\) is the normalization factor and the asymptotic \(J + 1 / J - 1\) ratio parameter is defined by \(\eta = A_{J+1} / A_{J-1}\). Ideally, one would integrate the Schrödinger equation taking its solutions at infinity, Eq. \((9)\), which depend on the binding energy and \(\eta\). The singular structure of the problem at short distances requires a specific analysis of the coupled equations as it has been done extensively elsewhere \((13)\) and we adapt here for our particular situation. The result amounts to integrate from infinity for the physical value of \(M_0\) and \(\eta\) (or \(M_0\) in the case of singlet channels). Generally, the solutions diverge strongly at the origin, so that the normalization of the state is precluded. However, there is a particular value of \(\eta\) which guarantees that the wave function becomes normalizable \(^1\). The rest of the spectrum is then built by imposing orthogonality of states in coupled channels (see \((3)\) for details in the \(J/\psi\) case).

### III. PHENOMENOLOGY OF CHARMONIUM STATES

The charmonium spectrum consists of eight narrow states below the open-charm threshold (3.73 GeV) and several tens of states above the threshold, some of them wide, because they decay into charmed mesons, some of them still narrow, because their decay to open-charm is forbidden by some conservation rule. Below the threshold all states are well established. Above threshold, however, there are new charmonium-like states that are very difficult to accommodate theoretically.

In Tables 4 and 9 of Ref. \((10)\) the updated new conventional and unconventional states in the \(c\bar{c}\), \(b\bar{b}\) and \(b\bar{b}\) sectors are given. If we focus on the \(c\bar{c}\) region, there are three new states which have been recognized as \(q\bar{q}\) pairs in the PDG \((17)\) during the last years. They are the \(h_c\) which is the \(^1P_1\) state of charmonium, singlet partner of the long-known \(\chi_{cJ}\) triplet \(^3P_J\) states, the \(\eta_c(2S)\) which is the first radial excitation of the pseudoscalar ground state \(\eta_c(1S)\) and the \(Z(3940)\) whose assignment as the \(^2P_2\) state, \(\chi_{c2}(2P)\), seems widely accepted. The rest of the resonances of this sector, namely the \(X(3872)\), \(X(3915), Y(3940), X(3940)\) still lack for a clear assignment.

The results of the study of the full charmonium spectrum up to total spin \(J = 2\) and for the first radial excitations within the renormalization with boundary conditions scheme are shown in Table I. In the following we will discuss our predictions for the different channels except the \(J_{PC} = 1^{--}\) which has been extensively discussed in Ref. \((4)\) already.

In columns four and five we compare the calculated masses within the renormalization approach with the experimental data. As we see, once the experimental value

\(^1\) Thus, if one imposes the regularity condition at the origin one will determine \(\eta\) and therefore the wave function of the bound state. In practice, however, the converging solution is rather elusive since integrated-in solutions quickly run into diverging solution due to the round-off errors and dominate over the converging solution.
of the ground state mass is taken as input in the calculation, the rest of the spectrum is reproduced fairly well.

In order to size the relevance of form factors in the excited spectrum we also show in columns six and seven the comparison with the original model including form factors \cite{3} and taking the predicted mass as a parameter of the renormalization approach. The observed tiny deviations are below a few MeV corresponding to a marginal influence of the form factors on the excited states. These results actually point to the idea that the gluonic regulators are fitted just to provide the ground state energies. Once we have eliminated possibles bias due to the use of regulators, we are in a favorable position to discuss possible \( q\bar{q} \) assignments for the old and new states.

### A. \( \eta_c \) and \( \eta_{c2} \) states

An \( \eta_c(1S) \) candidate has been observed thirty years ago by CBAL \cite{21} and MARK II \cite{22} Collaborations with a mass measurement very close to the updated world average 2980.3 \( \pm \) 1.2. In the renormalization scheme this state is taken as a parameter and in essence fixes the spin-spin contact term interaction.

The search for a reproducible \( \eta_c(2S) \) signal has a long history. Recently, Belle \cite{22} found a signal in \( B \to K\eta_c(2S) \) in the exclusive \( \eta_c(2S) \to K_S^0 K^- \pi^+ \) decay mode (a favorite all-charged final state for \( \eta_c(1S) \)), at 3654 \( \pm \) 6 \( \pm \) 8 MeV. Since then measurements of \( \eta_c(2S) \) in that mass region have been reported by BaBar \cite{24}, CLEO \cite{25}, and Belle \cite{26} in \( \gamma\gamma \)-fusion to \( KK\pi \) final states and by BaBar \cite{27} and Belle \cite{28} in double charmonium production.

Our predicted mass for the \( \eta_c(2S) \) is 3634 MeV in very good agreement with the updated world average reported in Ref. \cite{17}. Nothing is known of the next excitation, \( \eta_c(3S) \). Our prediction is around 4.05 GeV.

The potential at short distances is regular for the \( \eta_c \) states. Therefore the mass of the ground state for the \( \eta_{c2} \) meson is not a parameter and we predict 3812, 4166 and 4437 MeV for the ground state and the first two radial excitations.

We can clearly identify a second state \( \eta_{c2}(2S) \) with mass \( M = 4166 \) MeV and width \( \Gamma = 122.9 \) MeV with the resonance recently reported by Belle at \( M = 4156.2_{-20}^{+25} \) MeV with a width \( \Gamma = 139.1_{-61}^{+111} \) MeV \cite{29} in the \( e^+e^- \to D^+\bar{D}^*J/\psi \). The decay of \( \eta_{c2} \) to \( D\bar{D} \) is forbidden being the \( X(4160) \to D^*\bar{D}^* \) the most favored decay channel as shown by the data.

### B. \( h_c \) and \( \chi_{cJ} \) states

Two experiments reported the observation of the \( h_c(1P) \) in 2005. CLEO \cite{29, 31} obtained a 6\( \sigma \) statistical significance in the isospin-forbidden decay \( e^+e^- \to \psi(2S) \to \pi^0h_c, h_c \to \gamma(1S) \). E835 \cite{32} found a 3\( \sigma \) evidence in \( p\bar{p} \to h_c, h_c \to \eta(1S), \eta_c(1S) \to \gamma\gamma \).

The precision measurement of its mass was reported by CLEO in 2008 \cite{33} as 3525.28 \( \pm \) 0.19 \( \pm \) 0.12 MeV. Later BES III \cite{34} has confirmed this with a mass of 3525.40 \( \pm \) 0.13 \( \pm \) 0.18 MeV. It was important to measure the mass of this state because the spin-averaged centroid of the triplet states

\[
\langle m(1^3P_J) \rangle = \frac{m_{\chi_{c0}} + 3m_{\chi_{c1}} + 5m_{\chi_{c2}}}{9},
\]

\( (10) \)
TABLE II: The theoretical masses, in MeV, of the first two radial excitations of $h_c$ compared with the spin-averaged centroid, in MeV, of the triplet states. We compare with the experimental data [17].

|       | CQM  | REN. |
|-------|------|------|
| $nL$  | $M(h_c)$ $C_{the}$ | $M(h_c)$ $C_{the}$ | $C_{exp}$ |
| $1P$  | 3515 | 3513 | 3516 | 3513 | 3525.30 ± 0.20 |
| $2P$  | 3905 | 3955 | 3957 | 3955 |

is expected to be near the $h_c(1P)$ mass. The lattice data show a vanishing long-range component of the spin-spin potential. Thus, the potential appears to be entirely dominated by its short-range, delta-like, part, suggesting that the $1P_1$ should be close to the center-of-gravity of the $3P_J$ system. So it makes the hyperfine mass splitting, $\Delta m_{P_J}[h_c(1P)] = (m(1^3P_J)) - m_1[h_c(1P)]$ an important measurement of the spin-spin interaction.

The centroid of the $1^3P_J$ states ($\chi_{0,1,2}$) is known to be [17] 3525.30 ± 0.04 MeV and then the hyperfine splittings are +0.02 ± 0.23 MeV, from CLEO and −0.10 ± 0.22 MeV from BES III.

Table II shows the masses for three radial excitations of the singlet $1P_1$ and the triplet $3P_J$ mesons. In Table II we show the comparison between the centroid of $3P_J$ states and the corresponding $h_c$ mass for the ground state and the first radial excitation, showing that our spin-spin interaction is negligible for these channels and it is in perfect agreement with the lattice expectations and the experimental measurements for the ground state.

The mean $2P$ multiplet mass is predicted to be near 3.95 GeV. Although no $2P$ $c\bar{c}$ states have been clearly seen experimentally, there are a lot of states reported from the different Collaborations which claim enhancements in that energy region. Concerning the rest of the new states, none of them appears in our calculation as a $c\bar{c}$ state except the $Z(3930)$. This fact agrees with the results of a recent calculation which shows that these states can be interpreted as molecular resonances [35].

The $Z(3930)$ was reported by Belle in $\gamma\gamma \rightarrow DD$ with a mass and width $M = 3929 ± 5 ± 2$ MeV and $\Gamma = 29.9±10±2$ MeV [18]. The two photon width is measured to be $\Gamma_{\gamma\gamma}B(Z(3930) \rightarrow DD) = 0.18 ± 0.05 ± 0.03$ keV. Moreover the $DD$ angular distribution is consistent with $J = 2$. The $\chi_{c2}(2P)$ state is a good candidate for the $Z(3930)$. We obtained a mass of 3974 MeV and the total width $\Gamma = 49.1$ MeV is comparable with the experimental data. Finally the experimental two photon width compares nicely with our result $\Gamma_{\gamma\gamma}B(Z(3930) \rightarrow DD) = 0.15$ keV.

| Particle $J^{PC}$ | $nL$ | REN1 | EXP. |
|-------------------|-----|------|------|
| $\eta_c$          | 0–+ | 1S input | 9390.9 ± 2.8 |
|                   | 2S  | 9957 | 9999.0 ± 3.5 ± 2.9 [36] |
|                   | 3S  | 10306 |
| $\chi_{c0}$       | 0++)| 1P input | 9859.44 ± 0.42 ± 0.31 |
|                   | 2P  | 10226 | 10232.5 ± 0.4 ± 0.5 |
|                   | 3P  | 10505 |
| $h_b$             | 1++ | 1P | 9879 | 9898.25 ± 1.06 ± 1.03 |
|                   | 2P  | 10241 | 10259.76 ± 0.64 ± 1.43 |
|                   | 3P  | 10516 |
| $\Upsilon$        | 1−− | 1S input | 9460.30 ± 0.26 |
|                   | 2S  | 9992 | 10023.26 ± 0.31 |
|                   | 1D  | 10117 |
|                   | 3S  | 10331 | 10355.2 ± 0.5 |
|                   | 2D  | 10414 |
|                   | 4S  | 10592 | 10579.4 ± 1.2 |
|                   | 3D  | 10653 |
|                   | 5S  | 10805 | 10865 ± 8 |
|                   | 4D  | 10853 |
|                   | 6S  | 10984 | 11019 ± 8 |
|                   | 5D  | 11023 |
| $\chi_{b1}$       | 1++ | 1P input | 9892.78 ± 0.26 ± 0.31 |
|                   | 2P  | 10254 | 10255.46 ± 0.22 ± 0.50 |
|                   | 3P  | 10527 |
| $\eta_{b2}$       | 2−+ | 1D | 10123 |
|                   | 2D  | 10419 |
|                   | 3D  | 10658 |
| $\chi_{b2}$       | 2++ | 1P input | 9912.21 ± 0.26 ± 0.31 |
|                   | 2P  | 10248 | 10268.65 ± 0.22 ± 0.50 |
|                   | 1F  | 10315 |
| $\Upsilon_2$      | 2−− | 1D input | 10163.7 ± 1.4 |
|                   | 2D  | 10418 |
|                   | 3D  | 10657 |

TABLE III: Masses in MeV of bottomonium states. The notation for the quantum numbers is the same as in Table II. We compare with the well established states in Ref. [17].

IV. BOTTOMONIUM

In this Section we extend the renormalization with boundary conditions scheme to the study of the bottomonium states. Although the $B$-factories are not usually considered as ideal facilities for the study of the bottomonium spectrum since their energy is tuned to the peak of the $\Upsilon(4S)$ resonance, which decays in almost 100% of cases to a $B\bar{B}$ pair, BaBar and Belle have reported data samples at various energies in the bottomonium region that made possible discoveries like the $\eta_c$ [35], $h_b(1P)$ and $h_b(2P)$ [38]. Nevertheless the number of states is...
As in the previous Section, column four of Table III shows the calculated masses within the renormalization approach up to total spin \( J = 2 \) and for the first radial excitations. As in the case of the charmonium spectrum, when applicable, we indicate the fact that the ground state mass is used as an input parameter.

In the \( J^P_C = 1^- \) channel there are less data for the bottomonium spectrum than for the charmonium. The \( S \) and \( D \) wave states are almost degenerate for the highest excited states. In the charmonium these doublets has been measured and we can assign them to \( S \) and \( D \) waves. In the bottomonium the splittings are smaller and experimentally they have not been resolved. We follow the assignment of the Particle Data Group up to 4\( S \) states and assumed that the others are \( S \) wave. This is the reason why we assigned the experimentally measured state at \( M = 11019 \) MeV to our state 6\( S \) and not to our 5\( D \).

As a general trend the experimental data are well reproduced by the calculation and a number of excited states are given which can be tested at LHCb and other B-factories. The \( \eta_b(2S) \) has been very recently measured by the Belle Collaboration [36] with a mass \( M = 9999.0 \pm 3.5^{+2.8}_{-1.9} \) which is in reasonable agreement with our \( M = 9957 \) MeV result.

The hyperfine mass-splitting of singlet-triplet states, \( \Delta m_{hf}[\eta_b(1S)] = m(T(1S)) - m(\eta_b(1S)) \), probes the spin-dependence of bound-state energy levels, and, once measured, imposes constraints on theoretical descriptions. It is given experimentally by

\[
\Delta m_{hf}[\eta_b(1S)] = 69.6 \pm 2.9 \text{ MeV} \tag{11}
\]

In the renormalization scheme this splitting is fixed and we can only predict the splittings for radial excitations which has not yet been measured.

In the case of the centroid of the \( \chi_{bJ}(nP) \) states with \( n = 1, 2 \) is known to be [17] 9899.87 \( \pm 0.27 \) MeV and 10260.24 \( \pm 0.36 \) MeV, respectively. The hyperfine splittings measured by the Belle Collaboration [36] are \( \Delta m_{hf}[h_b(1P)] = +1.6 \pm 1.5 \) MeV and \( \Delta m_{hf}[h_b(2P)] = +0.5^{+1.4}_{-1.2} \) MeV which are compatible with zero.

Table IV shows the masses for three radial excitations of the singlet \( h_b \) and the triplet \( \chi_{bJ} \) mesons. They are in reasonable agreement with the experimental data. In Table IV we show the comparison between the centroid of \( \chi_{bJ} \) states and the corresponding \( h_b \) mass for the ground state and the first radial excitation, showing that our spin-spin interaction is negligible.

\( \chi_{bJ}(1P) \) and \( \chi_{bJ}(2P) \) with \( J = 2, 1, 0 \) were discovered earlier in 1982 [39] and 1983 [41] respectively. Their masses have not changed much since then and our theoretical prediction through both schemes are very close to the experimental values.

### V. Conclusions

Based on an earlier work, we have reanalyzed the calculation of the charmonium spectrum in a constituent quark model using a renormalization scheme with boundary conditions. This approach avoids explicitly the introduction of phenomenological form factors taking as a parameter the mass of the ground state. Thus, the only relevant physical information on the form factors is to tune the value of the ground state energy. Once this fact has been established we have applied the renormalization framework to provide some basis to “bare” \( \bar{q}q \) assignments of mesons.

We obtain a spectrum in reasonable agreement with the experimentally well established data. For instance, we obtain \( \Delta m_{hf}[h_c(nP)] \) compatible with zero due to the fact that the hyperfine contact term has been included in the renormalization conditions. We also assign certain \( XYZ \) mesons according to our model.

For the phenomenologically successful model of Ref. [7] where \( \text{ad hoc} \) form factors are introduced as regulators, we find an almost perfect agreement with the renormalization approach. This result provides confidence on the way the original model took into account the unknown short distance dynamics. In addition, we have extended this study to the bottomonium sector obtaining similar conclusions as in the charmonium sector.

**Acknowledgments**

This work has been partially funded by Ministerio de Ciencia y Tecnología under Contract No. FPA2010-21750-C02-02, by the European Community-Research Infrastructure Integrating Activity ‘Study of Strongly Interacting Matter’ (HadronPhysics3 Grant No. 283286), the Spanish Ingenio-Consolider 2010 Program CPAN (CSD2007-00042), the Spanish DGI and FEDER funds with grant no. FIS2011-24149, and by Junta de Andalucía grants no. FQM225.
[1] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D 17, 3090 (1978).
[2] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D 21, 203 (1980).
[3] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005).
[4] T. Kawanai and S. Sasaki, Phys. Rev. Lett. 107, 091601 (2011).
[5] A. Laschka, N. Kaiser, and W. Weise, Phys.Lett. B715, 190 (2012).
[6] J. Segovia, D. Entem, F. Fernandez, and E. Ruiz Arriola, Phys.Rev. D85, 074001 (2012).
[7] J. Vijande, F. Fernandez, and A. Valcarce, Journal of Physics G: Nuclear and Particle Physics 31, 481 (2005).
[8] F. Fernandez, A. Valcarce, P. Gonzalez, and V. Vento, Physics Letters B 287, 35 (1992), ISSN 0370-2693.
[9] H. Garcilazo, A. Valcarce, and F. Fernandez, Phys. Rev. C 63, 035207 (2001).
[10] H. Garcilazo, A. Valcarce, and F. Fernandez, Phys. Rev. C 64, 058201 (2001).
[11] J. Vijande, H. Garcilazo, A. Valcarce, and F. Fernandez, Phys. Rev. D 70, 054022 (2004).
[12] J. Segovia, A. M. Yasser, D. R. Entem, and F. Fernandez, Phys. Rev. D 78, 114033 (2008).
[13] M. Pavon Valderrama and E. R. Arriola, Phys. Rev. C74, 054001 (2006), nucl-th/0506047.
[14] D. R. Entem, E. Ruiz Arriola, M. Pavon Valderrama, and R. Machleidt, Phys. Rev. C77, 044006 (2008), 0709.2770.
[15] A. Calle Cordon and E. Ruiz Arriola, Phys. Rev. C81, 044002 (2010), 0905.4933.
[16] N. Brambilla et al., The European Physical Journal C - Particles and Fields 71, 1 (2011).
[17] K. Nakamura et al., Journal of Physics G: Nuclear and Particle Physics 37, 075021 (2010).
[18] S. Uehara et al. (Belle Collaboration), Phys. Rev. Lett. 96, 082003 (2006).
[19] X. L. Wang et al. (Belle Collaboration), Phys. Rev. Lett. 99, 142002 (2007).
[20] G. Pakhlova et al. (The Belle Collaboration), Phys. Rev. Lett. 101, 172001 (2008).
[21] R. Partridge et al., Phys. Rev. Lett. 45, 1150 (1980).
[22] T. M. Himel et al., Phys. Rev. Lett. 45, 1146 (1980).
[23] S.-K. Choi et al. (The Belle Collaboration), Phys. Rev. Lett. 89, 102001 (2002).
[24] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 92, 142002 (2004).
[25] D. M. Asner et al. (CLEO Collaboration), Phys. Rev. Lett. 92, 142001 (2004).
[26] H. Nakazawa (BELLE), Nucl. Phys. Proc. Suppl. 184, 220 (2008).
[27] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 72, 031101 (2005).
[28] K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. 98, 082001 (2007).
[29] P. Pakhlov et al. (Belle Collaboration), Phys. Rev. Lett. 100, 202001 (2008).
[30] P. Rubin et al. (CLEO Collaboration), Phys. Rev. D 72, 092004 (2005).
[31] J. L. Rosner et al. (CLEO Collaboration), Phys. Rev. Lett. 95, 102003 (2005).
[32] M. Andreotti et al. (Fermilab E835 Collaboration), Phys. Rev. D 72, 032001 (2005).
[33] S. Dobbs et al. (CLEO Collaboration), Phys. Rev. Lett. 101, 182003 (2008).
[34] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 104, 132002 (2010).
[35] P. Ortega, D. Entem, and F. Fernandez, arXiv:hep-ph/1205.1699 (2012).
[36] R. Mizuk et al. (Belle Collaboration), arXiv:hep-ex/1205.6351 (2012).
[37] B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 101, 071801 (2008).
[38] I. Adachi et al. (Belle) (2011), 1103.3419.
[39] K. Han et al., Phys. Rev. Lett. 49, 1612 (1982).
[40] G. Eigen et al., Phys. Rev. Lett. 49, 1616 (1982).
[41] C. Klopfenstein et al., Phys. Rev. Lett. 51, 160 (1983).
[42] F. Pauss et al., Physics Letters B 130, 439 (1983).