Reduction in energy dissipation rate with increased effective applied field

Zdeněk Janů and František Soukup

Institute of Physics of the CAS, Na Slovance 2, CZ-182 21, Prague, Czech Republic

E-mail: janu@fzu.cz

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Abstract

Dynamics of the response of type-II superconductors to a time-varying magnetic field can exhibit a rate-independent or rate-dependent hysteresis. An energy dissipation rate in a superconductor placed in a time-varying magnetic field depends on its wave form and type of hysteresis, which depends on temperature. The same wave form may reduce the energy dissipation rate in the case of true hysteresis, while it may increase the energy dissipation rate in the case of dynamic hysteresis compared with an energy dissipation rate in a pure sinusoidal field. We present experimental data which confirm the energy dissipation rate calculated using the critical state theory for the case of rate-independent hysteresis and limiting behavior in a normal state for the case of rate-dependent hysteresis.

Keywords: critical state, hysteresis, energy dissipation rate, field waveform

1. Introduction

Magnetic hysteresis in type-II superconductors in an external time-varying magnetic field may vary between rate-independent hysteresis and rate-dependent hysteresis, depending on temperature [1]. The rate-dependent hysteresis and the rate-independent hysteresis bring about markedly different energy dissipation rates, depending on a wave form of the external field. When, e.g., a field of the third harmonic frequency in phase with the pure sinusoidal field is added, a peak value of the field is trimmed, the field rate changes, and the effective field increases. We compare the dissipation rates in the case of pure sinusoidal and first-plus-third harmonic or (approximated) square waves. One can reduce the ac losses when the superconductor is in the critical state by adding harmonics, even if the effective applied field is increased. On the contrary, we observe an increase in dissipation with the number of harmonics when the superconductor is in the normal state, as expected.

A growing application potential of thin superconducting films employed, for example, in recent second-generation high-temperature superconducting wires or superconducting electronics makes an exploration of the field wave form-dependent energy dissipation rate in the thin films relevant. Recently, critical-state models for thin circular disks and strips in a transverse magnetic field were developed [2, 3]. A validity of these models was proved later [4, 5]. The models give the complete analytical expressions needed to calculate magnetization loops in a time-varying magnetic field. On the basis of these magnetization loops the energy dissipation rate may be calculated for the case of the rate-independent hysteresis, which occurs at temperatures below the critical depinning temperature. At temperatures above the critical depinning temperature, a superconductor is in a normal state with ohmic conductivity. The magnetization loops manifest the rate-dependent hysteresis. For a low frequency ac field the limiting behavior of the energy dissipation rate may be obtained on the basis of the eddy current model [6]. In a temperature interval between the critical depinning temperature and the critical depairing temperature a hysteresis is a rate-dependent. The shape of the magnetization loops depends both on a peak value and a rate of the field because of screening current damping, of which the time dependence changes from logarithmical to exponential with increasing temperature [7].

Our experimental results support the foregoing hypothesis. We have measured temperature dependence of the magnetic moment of a thin Nb film subjected to an external time-varying magnetic field whose wave form was synthesized to modify a peak value and rate. The energy dissipation
Magnetic properties of superconductors are commonly described via external ac susceptibility $\chi_e$ related to a magnetic moment as $\chi_e/I_0 = M_s/m_{CM}$, where $\chi_e = [(1/V)(dm/dH)]_{H=0}$ is the initial susceptibility, dependent on the shape of the sample and its orientation in a field, and $m_s = \chi_e V H$ is the magnitude of the magnetic moment of the sample in the case of perfect screening [9, 10].

At temperature below the critical depinning temperature $T_c$, vortices are pinned, and flux density profiles created in an applied field are quasistatic with the gradient $|\nabla B| = \mu_0 J_c > 0$, where $J_c$ is the critical depinning current density. Due to a large aspect ratio of thin film samples in the transverse magnetic field configuration, the particular in-plane shape almost does not influence the response to an external magnetic field, and normalized magnetization loops for the critical state differ by less than 1.3% between disks, strips, and squares over the whole range of applied fields [7, 10]. The only difference is that fitting of data measured on squares to the model for disks gives a $J_c$ underestimated by 6%, while fitting to the model for strips gives a $J_c$ overestimated by 16%. However, complete analytical expressions known for an initial magnetization curve and magnetization loops of disks and strips allow fast calculations, unlike the numerical calculations needed for rectangles [2, 3]. The magnetization loops are an analytical function of $H_p/H_d$, where $H_p$ is the peak value of the ac field and $H_d$ is the characteristic field. For the disks $H_d = J_c d/2$ and $\chi_e = 8\pi/3\alpha R$ where $R$ is the disk radius and $d$ is the disk thickness [3].

In the critical state with $J_c > 0$, as the temperature approaches $T_c$, thermal activation of flux lines over a pinning barrier with activation energy $U$ causes the created flux density profiles to relax spontaneously by diffusion. The diffusivity $D = (E_c/\mu_0 J_c)(J/J_c)^{\gamma-1}$, where $E_c/J_c$ is a typical resistivity parameter of the material, is a nonlinear function of the screening current $J$. With increasing temperature the creep exponent $a = U/k_B T$ falls from $a \to \infty$ (Bean critical state) to $a = 1$ (normal state with ohmic conductivity). The critical depinning current density $J_d$ drops to zero at $T_c$. In the normal state, above the critical depairing temperature $T_d$, a superconductor obeys Ohm’s law with a conductivity $\sigma$ and diffusivity $D = 1/\mu_0 \sigma$. Using numerical calculations Brandt has shown that the nonlinear ac susceptibility in a sinusoidal field with an amplitude $H_p$ and frequency $f$ may be written as

$$\chi(f, H_p) = g\left(H_p/f^{1/(\alpha-1)} + \beta H_p^{1-\alpha}\right),$$  

(1)

where $g(f, H_p)$ is a universal function depending only on geometry [7]. In the Bean critical state, $\chi$ depends only on $H_p$, while in the normal state, $\chi$ depends only on $f$. As the temperature increases from $T_c$ to $T_d$, nonlinearity in $D(J)$ decreases, and $\chi_e/I_0 = M_s/m_{CM}$ is reduced and $\chi_e/I_0 = M_s/m_{CM}$ enhanced at all ac amplitudes $H_p$. The maximum of $M_s/m_{CM}$ increases from $0.24$ for the critical state to $\approx 0.4$ for the normal state at $R = \delta$, where $R$ is the dimension of the sample in a direction perpendicular to the field and $\delta = (\mu_0 \pi \sigma)^{-1/2}$ is the skin depth [11]. The range of the temperature interval between $T_c$ and $T_d$ depends on the

![Figure 1. Temperature dependence of the normalized fundamental and third harmonic ac magnetic moment. Symbols represent experimental data measured in the applied field with the amplitude $\mu_0 H = 100 \mu T$ and frequency 1.5625 Hz. The curves represent data calculated on the basis of the model and transformed to a temperature domain using effective temperature.](image)
strength of pinning and an applied field. For weak applied fields these temperatures may coincide.

In the normal state, at temperatures above \( T_\text{c} \), the magnetic moment \( \mathcal{M}_1 \) produced by currents induced in a sample by the changing magnetic field may be calculated on the basis of the eddy current model [11]. The magnetization loops are ellipses, and for linear \( m(H) \) dependence only the fundamental components of the ac magnetic moment are present. In the low-frequency limit of an applied ac field, a sample is somewhat transparent for a field when \( \delta \gg R \), and components of the ac magnetic moment have the limiting behavior \( \mathcal{M}'_1 \propto -(-R/\delta)^2 \) and \( \mathcal{M}''_1 \propto (R/\delta)^3 \propto \sigma \) [6]. The area of the magnetization loop \( W(f) = \pi \mu_0 \mathcal{M}'_1(j_\text{c}) H_1 \approx j_\text{c} \sigma H_1^2 \) increases linearly with the frequency of the external ac field. The mean value of the energy dissipation rate \( P = f_1 W(f) \propto j_\text{c} \sigma H_1^2 \) grows with the square of the frequency.

Figure 1 shows the experimental ac magnetic moment measured as a function of temperature and the theoretical ac magnetic moment of a disk in the critical state. In order to fit experimental data \([T, \mathcal{M}_1]\) and theoretical data \([H_\text{p}, H_\text{d}, \mathcal{M}_1]\) we use the phenomenological scaling form of the temperature dependence of the critical depinning current density

\[
\frac{J_c(T)}{J_c(0)} = \left( 1 - \left( \frac{T}{T_c} \right)^b \right)^{\frac{b}{6}},
\]

with the exponent \( b \), typically ranging from 1–3 in experiments. By analogy with \( J_c(T) \) we can define the effective temperature \( (T/T_c)_\text{eff} \equiv 1 - (-c H_\text{d}/H_\text{p})^{1/b} \) for theoretical data, where \( c \equiv H_\text{p}/H_\text{d}(0) = 2 H_\text{p} J_c(0)/d \) [5]. A relation for the effective temperature maps \( H_\text{p}/H_\text{d} \) from the interval \([H_\text{p}/H_\text{d}(0), \infty) \) to the reduced temperature \( T/T_c \) from the interval \([0, 1) \) and vice versa, using free parameters \( c, T_c, \) and \( b \), which fit experimental data to theoretical data and give \( J_c(0) \) and \( J_c(T) \). A good agreement, including distinctive behavior of the third harmonic, is obtained with \( T_c = 8.925 \) K, \( J_c(0) = 112 \) GA/m², and \( b = 1.33 \). The third harmonic components indicate the critical state with \( J_c \) approaching zero clearly. Starting by low temperatures and fixed \( H_\text{p} \), with increasing temperature, i.e., increasing \( H_\text{p}/H_\text{d} \), the energy dissipated per ac field cycle increases as \( \mathcal{M}'_1/\mathcal{m}_\text{M} \propto (H_\text{p}/H_\text{d})^2 \). The peak in \( \mathcal{M}'_1/\mathcal{m}_\text{M} \) has a value consistent with an expected value \( \approx 0.24 \) and occurs at temperature \( T/T_c \approx 0.988 \) when \( H_\text{p}/H_\text{d} = 1.943 \). A further increase in temperature causes a decrease in \( \mathcal{M}'_1/\mathcal{m}_\text{M} \), because the magnetization loop saturates. For \( H_\text{p}/H_\text{d} \gg 1 \) we have \( \mathcal{M}'_1/\mathcal{m}_\text{M} \propto (H_\text{p}/H_\text{d})^{-1} \). Since \( J_c \) approaches zero as \( T \) approaches the critical depinning temperature \( T_c \), the rate-independent \( \mathcal{M}'_1/\mathcal{m}_\text{M} \propto J_c \) turns to the rate-dependent \( \mathcal{M}'_1/\mathcal{m}_\text{M} \propto \sigma \) with temperature dependence given by the temperature dependence of conductivity in the normal state. The critical state is established by the presence of the third harmonic components \( \mathcal{M}_3 \). Since a peak in experimental \( \mathcal{M}'_3/\mathcal{m}_\text{M} \) is not pronounced at \( T/T_c \approx 0.995 \) while the experimental \( \mathcal{M}'_3/\mathcal{m}_\text{M} \) fits to the predicted curve, a clear conclusion cannot be drawn.

The applied field pinwheels the pure sine wave and its third harmonic, \( H(t) = H_1 \sin(2\pi f_1 t) + H_2 \sin(2\pi 3f_1 t) \). For \( H_2/H_1 \leq 1/9 \) the field \( H(t) \) increases and decreases monotonously between \( -H_\text{p} \) and \( H_\text{p} \). The added third harmonic trims \( H_\text{p} \) so that, in the critical state, the energy dissipated per cycle of the fundamental field \( H_1 \) decreases with increasing \( H_2/H_1 \). At the same time the effective value of the field increases, \( H_\text{m} = 2^{-1/2} \sqrt{H_1^2 + H_2^2} > 2^{-1/2} H_1 \). Because of the nonlinear \( m(H) \) dependence in the critical state, both \( \mathcal{M}(f) \) and \( \mathcal{M}(3f) \) depend on both applied fields \( H_1 \) and \( H_2 \), from which energy is absorbed. The total value of the energy dissipation rate is \( P = f_1 W(f) + 3f_1 W(3f) \).

Figure 2 shows the impact of the added third harmonic component on the total value of the energy dissipation rate \( P \) and normalized \( P/R \) plotted versus \( H_2/H_1 \) for the fixed values \( H_2/H_1 = 0, 1/16, \) and \( 1/9 \). Here, \( P_1 \) is the energy dissipation rate in the pure sine field \( H_1 \). The experimental \( P \) is calculated from the magnetization loops measured at \( \mu_0 H_1 = 100 \) µT, \( f_1 = 1.5625 \) Hz, and varying temperature with rate 0.1 K/min. Temperature dependence is transformed into \( H_2/H_1 \) dependence on the basis of \( J_c(T) \) found using the data shown in figure 1. Since the field increases and decreases monotonously between \( -H_\text{p} \) and \( H_\text{p} \), we can apply the model for disks. For \( H_2/H_1 \ll 1 \) the simulation predicts a decrease in the energy dissipation rate to \( P/R = 0.733 \) and 0.625 for \( H_2/H_1 = 1/16 \) and \( 1/9 \), respectively. While the peak in the field is trimmed to \( H_2/H_1 \approx 0.9375 \) and 0.8889, the effective field \( H_\text{m} \) is increased from 0.707 to 0.708 and 0.711, respectively. An increase in \( H_\text{m} \) by \( \approx 1\% \) causes a decrease in the energy dissipation rate \( P \) by \( \approx 40\% \). For \( H_2 \gg H_\text{d} \) the effect is smaller: \( P/R = 0.936 \) and 0.887 for \( H_2/H_1 = 1/16 \) and \( 1/9 \), respectively. Figure 2 shows that the experimental data are in good agreement with the theoretical predictions,
including a step in the normalized energy dissipation rate at \( H_p/H_d \approx 2 \).

Apparently, the optimum applied field wave form to minimize the total energy dissipation rate in the critical state and maximize the effective field at the same time is the square wave form, because it has the lowest crest factor \( H_p/H_{rms} \) of all waveforms. However, because of experimental limitations, e.g., a detection system slew rate and solenoid charging rate, we would rather synthesize the square wave-form field using a series expansion

\[
H(t) = H_1 \sum_{n=1,3,5,\ldots}^{\infty} \sin\left(\frac{n2\pi f_i}{n}\right). \tag{3}
\]

As \( N \) approaches infinity both \( H_p \) and \( H_{rms} \) approach \( (\pi/4)/H_d \approx 0.785 H_d \), which represents, in comparison with the pure sine field, an increase in the effective field by 11%. The total energy dissipation rate in the critical state is

\[
P = \sum_{n=1,3,5,\ldots}^{N} n f_i W(n f_i). \tag{4}
\]

In this case the model for disks is not applicable, since the field is not monotonously non-decreasing and non-increasing between the peak values for small \( N \). However, a ‘true’ square wave meets this condition, so we can evaluate a difference between an energy dissipation rate in sine and square wave fields, which have the same effective value. For \( H_p/H_d \ll 1 \), \( M_s^2/m_d \propto (H_p/H_d)^2 \), which yields \( P/R = 0.5 \) in favor of the square wave. On the other hand, for \( H_p/H_d \gg 1 \), \( M_s^2/m_d \propto (H_p/H_d)^{-1} \), which yields \( P/P_1 = 1.4 \), an increase in the energy dissipation rate. The peak in \( P_1 \) occurs at \( H_{rms}/H_d = 1.55 \), while the peak in \( P \) occurs at \( H_{rms}/H_d = 2.14 \). Both wave forms bring about the same energy dissipation rate at \( H_{rms}/H_d = 1.7 \).

In our case with the square wave form given by equation (3), the total value of the energy dissipation rate \( P = 0.616 R \) is decreased by 38%, while the effective field is increased by 11%.

On the other hand, in the normal state each added sine wave, independently of its phase, increases the total value of the energy dissipation rate. Since the energy dissipation rate increases as a square of the frequency, while the amplitudes decrease as a reciprocal value of the frequency,

\[
P \propto \sum_{n=1,3,5,\ldots}^{N} (nf_i)^2 \sigma H^2 n^2 \propto \frac{N+1}{2}, \tag{5}
\]

the total value of the energy dissipation rate increases with the number of terms in series expansion, if the conductivity is frequency-independent.

Figure 3 shows temperature dependencies of the total energy dissipation rate \( P \) and normalized total energy dissipation rate \( P/R \) for \( N = 1, 3, 5 \), and 7 measured at \( \mu_0 H = 100 \mu T \) and \( f_0 = 1.5625 \text{ Hz} \). While the normalized total energy dissipation rate in the critical state changes only slightly, \( P/R \approx 0.944, 0.942, \) and 0.941, for \( N = 3, 5 \), and 7, it increases to \( P/R = 2.03, 2.88, \) and 3.58 in the normal state. These values are close to those predicted by equation (5).

3. Conclusions

In conclusion, we have proved experimentally a theoretical prediction that the energy dissipation rate in a quasistatic system with diverging relaxation times excited by a square wave form field is lower than the energy dissipation rate in a pure sine field of the same effective value, unlike a system with diffusion dynamics, where the reverse is true. An analogous affect is observed, for example, when a proper third harmonic of the sinusoidal field that trims the peak value of the field is superposed.

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