Non-equilibrium Josephson effect through helical edge states

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(Dated: February 18, 2022)

We study Josephson junctions between superconductors connected through the helical edge states of a two-dimensional topological insulator in the presence of a magnetic barrier. As the equilibrium Andreev bound states of the junction are 4π-periodic in the superconducting phase difference, it was speculated that, at finite dc bias voltage, the junction exhibits a fractional Josephson effect with half the Josephson frequency. Using the scattering matrix formalism, we show that signatures of this effect can be seen in the finite-frequency current noise. Furthermore, we discuss other manifestations of the Majorana bound states forming at the edges of the superconductors.

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Unlike ordinary insulators, topological insulators (TI) admit robust conducting states at their boundaries. These states display unique properties. For instance, a two-dimensional quantum spin-Hall insulator has helical edge states with up spins propagating in one direction and down spins propagating in the other direction [1, 2]. Signatures of these helical edge states have been revealed in transport measurements on HgTe/CdTe [3] and InAs/GaSb [4] quantum well structures.

A conventional superconductor (S) attached to such edge states induces topological superconductivity by the proximity effect. The resulting topological superconductor has been predicted to support zero-energy Majorana bound state (MBS) at an interface with a topologically trivial region [5]. Majorana fermions have attracted a lot of attention because they are promising for topologically protected quantum computation [6]. Indeed, a pair of spatially separated Majorana fermions form a Dirac fermion that could be used as a quantum bit. As the Majorana bound states forming at the edges of the superconductors.

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lead can be written in the form

\[ \textit{perfectly transmitting conventional Josephson junction.} \]

\[ \begin{aligned}
\alpha &\equiv \epsilon
\end{aligned} \]

\[ \begin{aligned}
\epsilon &\equiv \text{states correspond to a superposition of states with energy}
\end{aligned} \]

\[ \begin{aligned}
e &\equiv \text{an incoming electron (e)}
\end{aligned} \]

\[ \begin{aligned}
I &\equiv \text{dc current}
\end{aligned} \]

\[ \begin{aligned}
\text{In Fig. 1 we plot the numerical results at } T = 0 \text{ for the}
\end{aligned} \]

\[ \begin{aligned}
dc current } I_0(V) \text{ as well as the real and imaginary part of the first}
\end{aligned} \]

\[ \begin{aligned}
\text{harmonic of the ac current as a function of applied bias for various}
\end{aligned} \]

\[ \begin{aligned}
\text{transparencies. Here } I_\Delta = G_N \Delta / e \text{ with } G_N = Dc^2 / h.
\end{aligned} \]

\[ \begin{aligned}
\text{where } s = \pm \text{ and } \nu = \{ \epsilon, i, \alpha \} \text{ labels an incoming state}
\end{aligned} \]

\[ \begin{aligned}
\text{with positive energy } \epsilon, \text{ from the lead } i = l, r, \text{ and of the type}
\end{aligned} \]

\[ \begin{aligned}
\text{type } \alpha = e, h.
\end{aligned} \]

\[ \begin{aligned}
\text{Due to multiple Andreev reflections } \text{[9], scattering}
\end{aligned} \]

\[ \begin{aligned}
\text{states correspond to a superposition of states with energy}
\end{aligned} \]

\[ \begin{aligned}
\epsilon &\equiv +2neV \text{ (n integer). For instance, the wave function of an incoming}
\end{aligned} \]

\[ \begin{aligned}
electron (e) \text{ with energy } \epsilon \text{ from the left (l) lead can be written in the form}
\end{aligned} \]

\[ \begin{aligned}
\Phi_\epsilon^l(0, t) = \sum_n \left( \begin{array}{c}
\delta_{n0} + \alpha_{2n} A_n \\
A_n \\
\alpha_{2n} B_n
\end{array} \right) e^{-i(\epsilon + 2neV)t} \quad (2a)
\end{aligned} \]

\[ \begin{aligned}
\text{and}
\end{aligned} \]

\[ \begin{aligned}
\Phi_\epsilon^l(L, t) = \sum_n \left( \begin{array}{c}
C_n \\
\alpha_{2n+1} C_n \\
\alpha_{2n+1} D_n
\end{array} \right) e^{-i(\epsilon + (2n+1)eV)t} \quad (2b)
\end{aligned} \]

\[ \begin{aligned}
I_n = \frac{e}{h} \left\{ \text{DeV} \delta_{n0} - \int \text{dc tanh} \frac{\epsilon}{2T} J^2 \left[ a_{2n} A_n^* + a_{-2n} A_{-n} + \sum_m (1 + a_{2(m+n)} a_{2m}) (A_{m+n}^* A_m - B_{m+n}^* B_m) \right] \right\}. \quad (7)
\end{aligned} \]

\[ \begin{aligned}
\text{This is not surprising as the BdG Hamiltonians for both}
\end{aligned} \]

\[ \begin{aligned}
systems are identical in the absence of backscattering – however, in conventional junctions, there are two copies of that Hamiltonian. As } V \to 0, \text{ the dissipative current}
\end{aligned} \]

\[ \begin{aligned}
\text{reaches the same value (2/\pi)I_c, where } I_c = (e/2h)\Delta \text{ is the}
\end{aligned} \]

\[ \begin{aligned}
\text{critical current of the junction.}
\end{aligned} \]

\[ \begin{aligned}
\text{At finite backscattering (} D < 1, \text{ we find that } I_0 \text{ vanishes in the limit } V \to 0. \text{ This current suppression can}
\end{aligned} \]
be understood due to the energy gap $E_g = (1 - \sqrt{D})\Delta$ between the bound states and the continuum. In addition, singularities appear at voltages $eV = \Delta/q$, where $q$ is an integer. This is to be contrasted with conventional Josephson junctions, where singularities appear at voltages $eV = 2\Delta/q$. These singularities are associated with the energy gap $2\Delta$ between occupied and empty states that quasiparticles have to overcome. Namely, new channels for charge transfer open at the specific voltages $eV = 2\Delta/q$ when quasiparticles can overcome the energy gap by performing $q - 1$ Andreev reflections. In S/TI/S junctions, the presence of the MBS reduces the energy gap between occupied and empty states to $\Delta$.

This manifests itself most clearly in the tunneling regime ($D \ll 1$), where one sees a current onset at $eV = \Delta$, cf. Fig. 1. The analytic expression for the dc current in the tunneling regime reads

$$I_{0\text{un}}(V) = (eD/h) \int dc\nu(e)[f(e) - f(e - V)]$$

where $f(e)$ is the Fermi distribution, and the normalized density of states on either side of the junction, $\nu(e) = \pi|\Delta^2|e^2 - \Delta^2/|e|$, shows the contributions of MBS and continuum. At $T = 0$, the current $I_{0\text{un}}$ is the sum of two terms. The first term corresponds to the transfer from the continuum to the MBS for voltages $eV \geq \Delta$, whereas the second term corresponds to the (conventional) transfer from continuum to continuum for voltages $eV \geq 2\Delta$. The suppression of the square-root singularity at the gap edge in $\nu(e)$ explains that the singular behavior of $I_0(V)$ at $eV = 2\Delta$ is smooth.

For completeness, we provide the analytical expression for the excess current, $I_{\text{exc}} = I_0 - (e^2/h)DV$ at $eV \gg \Delta$,

$$I_{\text{exc}} = (2e^2/h)(D^2\Delta/R)(1 - (D/\sqrt{R})\arctan\sqrt{R})$$

where $R = 1 - D$, which varies between 0 in the tunneling limit and $(8/3\pi)I_e$ at perfect transmission.

We now turn to the ac components of the current. Multiple Andreev reflections yield ac components at multiples of the Josephson frequency $\omega_J$. In particular, concentrating on the first harmonic $I_1$, we notice that, just as the dc current, both its real and imaginary part show MAR features at $eV = \Delta/q$. More strikingly, $I_1$ vanishes in the limit $V \to 0$ (except for perfectly transparent junctions), and in the tunneling limit, $I_1$ is strongly suppressed even at finite voltages.

At first sight, these results seem somewhat paradoxical: Does the vanishing of $I_1$ indicate the absence of an ac Josephson effect? In fact, the fractional Josephson effect with frequency $\omega_J/2$ is absent in the formalism from the outset, cf. Eq. (6), whereas the regular Josephson effect with frequency $\omega_J$ vanishes at small voltage. The latter is in stark contrast with the case of conventional Josephson junctions, where the stationary value of the dc current is recovered as $V \to 0$.\[15\]

To resolve this puzzle, we study the current-current correlations. In particular, we consider the (symmetrized) frequency-dependent current noise,

$$S(\omega) = \int d\tau e^{i\omega\tau} \langle \delta I(t)\delta I(t+\tau) \rangle$$

where $\delta I(t) = I(t) - I(t)$, and the bar denotes a time averaging. In terms of the coefficients $A_n, B_n, C_n, D_n$, the noise can be expressed in the following form,

$$S(\omega) = \frac{e^2}{2h} \sum_{n} \int dc \delta(\epsilon - \epsilon') J(\epsilon) J(\epsilon') \left\{ (f(\epsilon)[1 - f(\epsilon')] + f(\epsilon')[1 - f(\epsilon)]) \right\} \times$$

$$\times \left[ \delta(\epsilon - \epsilon' - \omega + 2peV) \left( \sum_n \left[ A_{n+p}^* A_n + a_{2(n+p)} a_{2n}^* B_n - (1 + a_{2(n+p)} a_{2n}) B_n^* B_n' \right]^2 + \sum_n (1 + a_{2(n+p)}^* a_{2n+1}) \left( C_n^* C_n - D_n^* D_n' \right)^2 \right) \right.$$
Here, $\bar{A}_n = A_n + \delta_{n0}/a$, and all unprimed (primed) quantities are functions of $e$ ($e'$).

We first analyze the zero-frequency noise which is shown in Fig. 2. Taking the ratio of the noise and the dc current, one can define an effective charge $q^* = S/(2I_0)$. As shown in the inset of Fig. 2, the effective charge is $q^* \sim \Delta/V$, which has to be compared to $q^* \sim 2\Delta/V$ in a conventional Josephson junction [16, 17]. This result is directly related to the modified positions of the MAR features in the dc current discussed above.

More interesting is the finite-frequency noise which is shown in Fig. 3 for various values of the applied bias. The most striking feature is a peak at $\omega = eV$. The peak is revealed in the analytical formula for the noise $S^\text{sin}(\omega) = e \sum_n I_n^\text{sin}(V \pm \omega/e) \coth[(eV \pm \omega)/2T]$ in the tunneling regime ($D \ll 1$); it is absent at $D = 1$. In a conventional Josephson junction, the peak is absent at any transmission. Here it is a consequence of the $4\pi$-periodicity of the Andreev bound states. This can be understood in the following way. The two Andreev bound states carry a current $I_n = \pm (e/2h)\sqrt{D\Delta} \sin(\varphi/2)$. If the system would remain in one of the two states indefinitely, one would indeed observe a fractional Josephson effect. However, due to the coupling between the bound state and the continuum in the presence of an applied bias, the switching probability is non-zero. As a consequence, the fractional Josephson effect disappears in the average current as the applied bias introduces relaxation processes. However, its signatures can be seen in the finite-frequency noise which displays peaks at $\omega = eV$. We believe that our results are directly applicable to the S/TI/S junction studied in Ref. [19], where the absence of a supercurrent signature in the differential resistance was attributed to the lack of coherence in the junction. Frequency-dependent noise experiments [20] could reveal whether coherent effects do exist in this sample.

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