Study of the Hadronic Decays of $\chi_c$ States

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Abstract

Hadronic decays of the P-wave spin-triplet charmonium states $\chi_{cJ}(J = 0, 1, 2)$ are studied using a sample of $\psi(2S)$ decays collected by the BES detector operating at the BEPC storage ring. Branching fractions for the decays $\chi_{c1} \rightarrow K_s^0 K^+ \pi^-, \chi_{c0} \rightarrow K_s^0 K_s^0, \chi_{c2} \rightarrow K_s^0 K_s^0, \chi_{c0} \rightarrow \phi \phi, \chi_{c2} \rightarrow \phi \phi$ and $\chi_{cJ} \rightarrow K^+ K^- K^+ K^-$ are measured for the first time, and those for $\chi_{cJ} \rightarrow \pi^+ \pi^- \pi^+ \pi^-, \chi_{cJ} \rightarrow \pi^+ \pi^- K^+ K^-, \chi_{cJ} \rightarrow \pi^+ \pi^- p \bar{p}$ and $\chi_{cJ} \rightarrow 3(\pi^+ \pi^-)$ are measured with improved precision. In addition, we determine the masses of the $\chi_{c0}$ and $\eta_c$ to be $M_{\chi_{c0}} = 3414.1 \pm 0.6(stat) \pm 0.8(sys)$ MeV and
$M_{\eta_c} = 2975.8 \pm 3.9(stat) \pm 1.2(sys) \text{ MeV.}$

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I. INTRODUCTION

The P-wave spin-triplet charmonium states were originally observed \[1\] in radiative decays of the $\psi(2S)$ soon after the discovery of the $J/\psi$ and $\psi(2S)$ resonances. A number of decay modes of these states have been observed and branching fractions reported \[2\]. Most of the existing results are from the Mark I experiment, which had a data sample of 0.33 million $\psi(2S)$ decays \[3\]. Because the photon capabilities of the Mark I detector were limited, the detection of the photon from the $\psi(2S) \rightarrow \gamma \chi_{cJ}$ process was not required, and one constraint kinematic fits were used to reconstruct the final states.

Recently there has been a renewed interest in the P-wave charmonium states. Since in lowest-order perturbative QCD the $\chi_{c0}$ and $\chi_{c2}$ decay via the annihilation of their constituent $c\bar{c}$ quarks into two gluons, followed by the hadronization of the gluons into light mesons and baryons, these decays are expected to be similar to those of a bound $gg$ state; a detailed knowledge of the hadronic decays of the $\chi_{c0}$ and $\chi_{c2}$ may provide an understanding of the decay patterns of glueball states that will help in their identification.

The mass differences between the three $\chi_c$ states provide information on the spin-orbit and tensor interactions in non-relativistic potential models and lattice QCD calculations. The masses of the $\chi_{c1}$ and $\chi_{c2}$ have been precisely determined (to a level of $\sim \pm 0.12$ MeV) by Fermilab experiment E760 \[4\] using the line shape measured in the $p\bar{p} \rightarrow \chi_{c1,2}$ formation reaction. In contrast, the $\chi_{c0}$ mass is much more poorly known; the PDG average for $M_{\chi_{c0}}$ has an uncertainty of $\pm 2.8$ MeV \[2\].

In this paper, we report the analyses of all-charged-track final states from $\chi_{cJ}$ decays, including $\pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-K^+K^-$, $\pi^+\pi^-p\bar{p}$, $K^+K^-K^+K^-$, $K_s^0K^+\pi^- + c.c.$ and $3(\pi^+\pi^-)$. The results for $\chi_{cJ}$ decays into $\pi^+\pi^-$, $K^+K^-$ and $p\bar{p}$ have been reported elsewhere \[5\]. We use the combined invariant mass distribution from all of the channels under study to determine the $\chi_{c0}$ mass with improved precision.

A byproduct of this analysis is a determination of the mass of the $\eta_c$. This is of interest
because the $M_{J/\psi} - M_{\eta_c}$ mass difference measures the strength of the hyperfine splitting term in heavy quark interactions. However, in spite of a number of measurements, the current experimental value of $M_{\eta_c}$ remains ambiguous: the PDG [2] average is based on a fit to seven measurements with poor internal consistency [6–8] and the confidence level of the fit is only 0.001. A recent measurement from E760 [6] disagrees with the value reported by the DM2 group [7] by almost four standard deviations. Additional measurements may help clarify the situation.

The data used for the analysis reported here were taken with the BES detector at the BEPC storage ring at a center-of-mass energy corresponding to $M_{\psi(2S)}$. The data sample corresponds to a total of $(3.79 \pm 0.31) \times 10^6 \psi(2S)$ decays, as determined from the observed number of inclusive $\psi(2S) \rightarrow \pi^+ \pi^- J/\psi$ decays [9].

II. THE BES DETECTOR

BES is a conventional solenoidal magnet detector that is described in detail in Ref. [10]. A four-layer central drift chamber (CDC) surrounding the beampipe provides trigger information. A forty-layer cylindrical main drift chamber (MDC), located radially outside the CDC, provides trajectory and energy loss ($dE/dx$) information for charged tracks over 85% of the total solid angle. The momentum resolution is $\sigma_p/p = 0.017\sqrt{1 + p^2}$ ($p$ in GeV/c), and the $dE/dx$ resolution for hadron tracks is $\sim 11\%$. An array of 48 scintillation counters surrounding the MDC measures the time-of-flight (TOF) of charged tracks with a resolution of $\sim 450$ ps for hadrons. Radially outside of the TOF system is a 12 radiation length thick, lead-gas barrel shower counter (BSC) operating in the limited streamer mode. This device covers $\sim 80\%$ of the total solid angle and measures the energies of electrons and photons with an energy resolution of $\sigma_E/E = 22\%/\sqrt{E}$ ($E$ in GeV). Outside the BSC is a solenoid, which provides a 0.4 Tesla magnetic field over the tracking volume. An iron flux return is instrumented with three double layers of counters that identify muons of momentum greater than 0.5 GeV/c.
III. MONTE CARLO

We use Monte Carlo simulated events to determine the detection efficiency ($\varepsilon$) and the mass resolution ($\sigma_{\text{res}}$) for each channel analyzed. The Monte Carlo program (MC) generates events of the type $\psi(2S) \rightarrow \gamma \chi_{cJ}$ under the assumption that these processes are pure $E1$ transitions \cite{3,11}: the photon polar angle distributions are $1+\cos^2 \theta$ ($\chi_{c0}$), $1-\frac{1}{3} \cos^2 \theta$ ($\chi_{c1}$) and $1+\frac{1}{13} \cos^2 \theta$ ($\chi_{c2}$). Multihadronic $\chi_{cJ}$ decays are simulated using phase space distributions. For each channel, either 10000 or 5000 events are generated, depending on the numbers of events for the corresponding mode that are observed in the data sample.

IV. EVENT SELECTION

A. Photon Identification

A neutral cluster is considered to be a photon candidate when the angle in the $xy$ plane between the nearest charged track and the cluster is greater than $15^\circ$, the first hit is in the beginning 6 radiation lengths, and the difference between the angle of the cluster development direction in the BSC and the photon emission direction is less than $37^\circ$. When these selection criteria are applied to kinematically selected samples of $\psi(2S) \rightarrow \pi^+\pi^-\pi^+\pi^-$ and $\psi(2S) \rightarrow \pi^+\pi^-K^+K^-$ events, fewer than 20% of the events have $\gamma$ candidates, which indicates that the fake-photon rejection ability is adequate (see Fig. \(\text{I}\)). The number of photon candidates in an event is limited to four or less. The photon candidate with the largest energy deposit in the BSC is treated as the photon radiated from $\psi(2S)$ and used in a four-constraint kinematic fit to the hypothesis $\psi(2S) \rightarrow \gamma + \text{charged tracks}$. 
B. Charged Particle Identification

Each charged track is required to be well fit to a three-dimensional helix and be in the polar angle region $|\cos \theta_{MDC}| < 0.8$. For each track, the TOF and $dE/dx$ measurements are used to calculate $\chi^2$ values and the corresponding confidence levels to the hypotheses that the particle is a pion, kaon and proton ($Prob_\pi, Prob_K, Prob_p$). The reliability of the confidence level assignments is verified using a sample of $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$, $J/\psi \rightarrow \rho \pi$ and $J/\psi \rightarrow K^+K^-$ events, where the particle identification confidence levels ($ProbID$) of the tracks in different momentum ranges are found to be distributed uniformly between zero and one as expected [12]. Typically the $ProbID$ value of each track for a given decay hypothesis is required to be greater than 1% in our analysis.

C. Event Selection Criteria

For all decay channels, the candidate events are required to satisfy the following selection criteria:

1. The number of charged tracks is required to be four or six with net charge zero.

2. The maximum number of neutral clusters in an event is eight, and the number of photon candidates remaining after the application of the photon selection is required to be four or less.

3. The sum of the momenta of the lowest momentum $\pi^+$ and $\pi^-$ tracks is required to be greater than 550 MeV; this removes contamination from $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ events.

4. The $\chi^2$ probability for a four-constraint kinematic fit to the decay hypothesis is greater than 0.01.

5. The particle identification assignment of each charged track is $ProbID > 0.01$. 
1. $\gamma\pi^+\pi^+\pi^-\pi^-$ and $\gamma\pi^+\pi^-K^+K^-$

A combined probability of the four-constraint kinematic fit and particle identification information is used to separate $\gamma\pi^+\pi^+\pi^-\pi^-$ and the different particle assignments for the $\gamma\pi^+\pi^-K^+K^-$ final states. This combined probability, $Prob_{all}$, is defined as

$$Prob_{all} = Prob(\chi^2_{all}, ndf_{all}),$$

where $\chi^2_{all}$ is the sum of the $\chi^2$ values from the four-constraint kinematic fit and those from each of the four particle identification assignments, and $ndf_{all}$ is the corresponding total number of degrees of freedom used in the $\chi^2$ determinations. The particle assignment with the largest $Prob_{all}$ is selected, and further cuts on the kinematic fit probability and particle identification probability are imposed.

Figure 2 shows a scatterplot of $\pi^+\pi^-$ vs $\pi^+\pi^-$ invariant masses for events with a $\pi^+\pi^-\pi^+\pi^-$ mass between 3.2 and 3.6 GeV. The cluster of events in the lower left-hand corner indicates the presence of a $K_s^0K_s^0$ signal. A fit of a Gaussian function to the $\pi^+\pi^-$ mass distribution gives a peak mass at 499.3±1.2 MeV and a width $\sigma = 11.8\pm1.0$ MeV that is consistent with the MC expectation for the mass resolution. We select $\gamma K_s^0K_s^0$ candidates by requiring the mass of both $\pi^+\pi^-$ combinations in the event to be within $\pm2\sigma$ of the nominal $K_s^0$ mass.

The invariant mass distributions for the $\pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-K^+K^-$ and $K_s^0K_s^0$ events that survive all the selection requirements are shown in Figs. 3 and 4. There are peaks corresponding to the $\chi_{cJ}$ states in each of the plots. (The high mass peaks in Figs. 3 and 4 correspond to the $\psi(2S)$ decays to all charged tracks final states that are kinematically fit with a fake low-energy photon.)

We fit the $\pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-K^+K^-$ or $K_s^0K_s^0$ invariant mass distribution between 3.20 and 3.65 GeV with three Breit-Wigner resonances convoluted with Gaussian mass resolution functions and a linear background shape using an unbinned maximum likelihood method. In
the fit, the mass resolutions are fixed to their MC-determined values and the widths of the \( \chi_{c1} \) and \( \chi_{c2} \) are fixed to the PDG average values of 0.88 and 2.00 MeV \[2\], respectively. The results of the fit are listed in Table I and shown in Figs. 3, 4 and 5. Table I also lists the MC-determined efficiencies and mass resolutions.

2. \( \gamma\pi^+\pi^-p\bar{p} \)

If one of the four tracks is identified as a proton or antiproton, the event is assumed to be \( \gamma\pi^+\pi^-p\bar{p} \). We assign probabilities to the remaining particle assignment using the same technique that was used for \( \pi^+\pi^-K^+K^- \) decays; the combination with the highest probability is selected.

The \( \pi^+\pi^-p\bar{p} \) invariant mass distribution for the selected events is shown in Fig. 6. Here clear signals for all three \( \chi_{cJ} \) states are apparent. We fit the mass spectrum using the same method described in the previous section; the results are listed in Table II and shown as the smooth curve in Fig. 6.

3. \( \gamma K^+K^-K^+K^- \)

For the case where all the tracks are kaons, the contamination from \( \pi^+\pi^-J/\psi \) is not an important background, and the requirement on total momentum of the lowest momentum \( \pi^+ \) and \( \pi^- \) tracks, which is aimed at removing these events, is not used. The \( K^+K^-K^+K^- \) invariant mass distribution is shown in Fig. 4.

Figure 8 shows a scatterplot of \( K^+K^- \) vs \( K^+K^- \) invariant masses for the events with \( K^+K^-K^+K^- \) mass between 3.2 and 3.6 GeV. The concentration of events in the lower left-hand corner of the plot indicates the presence of \( \phi\phi \) final states. A fit to the \( K^+K^- \) mass distribution with a Gaussian function gives a peak mass of 1021.9 ± 0.8 MeV and a width \( \sigma = 5.3 ± 0.6 \) MeV, consistent with MC expectations. Events where the mass of two \( K^+K^- \)
combinations are in the range $0.99 < M_{K^+K^-} < 1.05$ GeV are identified as $\gamma\phi\phi$ candidates. The $\phi\phi$ mass distribution for these events is shown in Fig. 9, where there are clear signals for the $\chi_{c0}$ and $\chi_{c2}$.

The $K^+K^-K^+K^-$ mass and $\phi\phi$ mass plots are fitted with three Breit-Wigner resonances and two Breit-Wigner resonances, respectively, as described previously. The results of the fit are listed in Table III and are shown as smooth curves in Figs. 7 and 8.

Because of the large fraction of $\phi\phi$ intermediate events observed in the $K^+K^-K^+K^-$ mode and the significant difference between the detection efficiency for phase-space events and those coming from $\phi\phi$ decays, the detection efficiency for the $\chi_{c0}$ and $\chi_{c2} \rightarrow K^+K^-K^+K^-$ channels is a weighted average of the phase space and $\phi\phi$ efficiency. The detection efficiencies and mass resolutions are listed in Table III.

4. $\gamma K^0_s K^+\pi^- + c.c.$

The $\chi_{cJ} \rightarrow K^0_s K^+\pi^- + c.c.$ decay channels have serious potential backgrounds from $\gamma\pi^+\pi^-\pi^+\pi^-$ (including $\gamma K^0_s K^0_s$) and $\gamma\pi^+\pi^-K^+K^-$ final states. To eliminate these backgrounds, we exploit the feature that there is one and only one $K^0_s$ with a secondary vertex in real $K^0_s K^+\pi^- + c.c.$ events.

In each event, we determine $NKSHORT$, the number of two charged track combinations with net charge zero and effective mass within $\pm 200$ MeV of $M_{K^0}$, when the tracks are assigned a pion mass. The combination with mass closest to $M_{K^0}$ is considered to be a $K^0_s$ candidate. The $K^0_s$ vertex is defined as the point of closest approach of these two tracks; the primary vertex is defined as the point of closest approach of the other two charged tracks in the event. Two parameters are used to identify the $K^0_s$: the distance between primary vertex and secondary vertex in the $xy$ plane, $L_{xy}$, and the cosine of the angle between the $K^0_s$ momentum vector and its vertex direction $CSKS$, which is expected to be very near unity for a real $K^0_s$ event.
Candidate $\gamma K^0_s K^+\pi^- + c.c.$ events are selected by requiring the mass of the $K^0_s$ candidate determined from the track four-vectors returned by the 4C-fit to be within $\pm 2\sigma$ (i.e. $\pm 28$ MeV) of the nominal $K^0$ mass, $NKSHORT = 1$, $L_{xy} > 5$ mm, and $CSKS > 0.98$. In the invariant mass distribution of the selected events, shown in Fig. 10, only a $\chi_{c1}$ signal is prominent. The MC simulation indicates that the numbers of events in the the $\chi_{c0}$ and $\chi_{c2}$ mass region are consistent with residual backgrounds from $\gamma \pi^+\pi^-\pi^+\pi^-$, $\gamma K^0_s K^0_s$ and $\gamma \pi^+\pi^- K^+K^-$ final states. We set upper limits on the branching fractions of $\chi_{c0}$ and $\chi_{c2}$.

The $K^0_s K^+\pi^- + c.c.$ invariant mass distribution between 3.20 and 3.65 GeV are fitted with the procedure described above. The mass resolutions are fixed at their MC-determined values, the width of the $\chi_{c0}$ is fixed at the recent BES value of 14.3 MeV [5] and those of the $\chi_{c1}$ and $\chi_{c2}$ at their PDG values [2]. The mass of the three $\chi_c$ states are also fixed at their PDG [2] values. The fit results are listed in Table IV and are shown as a smooth curve in Fig. 10.

5. $\gamma 3(\pi^+\pi^-)$

After the selections based on the kinematic fit and particle ID, the main background to the $\chi_{cJ} \rightarrow 3(\pi^+\pi^-)$ decays comes from the decay chain $\psi(2S) \rightarrow \pi^+\pi^- J/\psi, J/\psi \rightarrow \gamma \pi^+\pi^- \pi^+\pi^-$. The requirement on the total momentum of the lowest momentum $\pi^+$ and $\pi^-$ tracks removes one third of the MC-simulated events while rejecting almost all the $\pi^+\pi^- J/\psi$ background.

The $3(\pi^+\pi^-)$ invariant mass distribution for the selected events is shown in Fig. 11, where prominent signals for all three $\chi_{cJ}$ states can be seen. The smooth curve in the figure is the result of the fitting procedure described above. The results of the fit and the MC-determined efficiencies and resolutions are listed in Table V.
V. BRANCHING FRACTION DETERMINATION

We determine branching fractions from the relation

\[ \mathcal{B}(\chi_{cJ} \rightarrow X) = \frac{n_{\text{obs}}/\varepsilon(\chi_{cJ} \rightarrow X)}{N_{\psi(2S)} \mathcal{B}(\psi(2S) \rightarrow \gamma \chi_{cJ})}, \]

where the values for \( \mathcal{B}(\psi(2S) \rightarrow \gamma \chi_{cJ}) \) are taken from the PDG tables [2]. For the \( K_s^0 K_s^0 \) \([\phi\phi]\) channel, a factor of \( \mathcal{B}(K_s^0 \rightarrow \pi^+\pi^-)^2 \mathcal{B}(\phi \rightarrow K^+K^-)^2 \) is included in the denominator.

A. Systematic errors

Systematic errors common to all modes include the uncertainties in the total number of \( \psi(2S) \) events (8.2%) and the \( \psi(2S) \rightarrow \gamma \chi_{cJ} \) branching fractions (8.6%, 9.2% and 10.3% for \( \chi_{c0} \), \( \chi_{c1} \) and \( \chi_{c2} \), respectively). Other sources of systematic errors were considered. The variation of our results for different choices of the selection criteria range from 10% for high statistics channels to 25% for those with low statistics. The systematic errors due to the statistical precision of the MC event samples range from 2% to 5% depending on the detection efficiencies of the channels. Changes in the detection efficiency when the phase space event generator is replaced by one using possible intermediate resonant states indicate that the systematic error on the efficiency due to the unknown dynamics of the decay processes is 15%. The variation of the numbers of observed events due to shifts of the mass resolutions and the total widths of the \( \chi_{cJ} \) states is 7%; that coming from changes in the shape used for the background function is less than 5%. The total systematic error is taken as the quadrature sum of the individual errors and ranges from 25% to 35%, depending on the channel.

B. Branching fraction results

The branching fraction results are listed in Table VI, where all BES results for \( \chi_{cJ} \) branching fractions are given, including those for the two-charged track modes reported
in Ref. [5]. In each case, the first error listed is statistical and the second is systematic. For comparison, we also provide the previous world averages for those channels when they exist [2].

Our branching fractions for $\chi_{c1} \rightarrow K_s^0 K^+ \pi^-, c.c., \chi_{c0} \rightarrow K_s^0 K_s^0, \chi_{c2} \rightarrow K_s^0 K_s^0, \chi_{c0} \rightarrow \phi \phi, \chi_{c2} \rightarrow \phi \phi$ and $\chi_{cJ} \rightarrow K^+ K^- K^+ K^- (J=0,1,2)$ are the first reported measurements for these decays. The results for $\chi_{c0}$ and $\chi_{c2} \rightarrow K_s^0 K_s^0$ are in agreement with the isospin prediction of the $\chi_{cJ}$ decays compared with the corresponding $K^+ K^-$ branching ratios.

For the other decay modes, signals with large statistics are observed and the corresponding branching fractions are determined with precisions that are significantly better than those of existing measurements. Note that our results are consistently lower than the previous measurements, sometimes by as much as a factor of two or more. We can find no obvious explanation for these discrepancies.

VI. DETERMINATION OF $M_{\chi_{c0}}$ AND $M_{\eta_c}$

We determine $M_{\chi_{c0}}$ by fitting the combined invariant mass distribution of all of the channels discussed above to three resolution-broadened Breit-Wigner functions with the resolution fixed at the value of 13.8 MeV, which is determined from fits to the $\chi_{c1}$ and $\chi_{c2}$, and the total widths of the $\chi_{c1}$ and $\chi_{c2}$ fixed at the PDG values [2]. The masses of all three $\chi_{cJ}$ states and the total width of the $\chi_{c0}$ are left as free parameters. The results of the fit for $M_{\chi_{c1}}$ (3509.4 ± 0.9 MeV) and $M_{\chi_{c2}}$ (3556.4 ± 0.7 MeV) agree with the PDG values within errors. The fit value for $M_{\chi_{c0}}$ is 3414.1 ± 0.6 MeV, where the error is statistical. The fit gives a total width for the $\chi_{c0}$ that is in good agreement with the recently reported BES result [2].

Figure 12 shows the combined invariant mass distribution for the $\pi^+ \pi^- \pi^+ \pi^-, \pi^+ \pi^- K^+ K^-, K^+ K^- K^+ K^-$, and $K_s^0 K^+ \pi^- + c.c.$ channels in the region of the $\eta_c$, where an $\eta_c$ signal is evident. Superimposed on the plot is a fit to the spectrum using a resolution-smeared
Breit-Wigner line shape with a mass that is allowed to vary, a total width fixed at the PDG value of $\Gamma_{\eta_c} = 13.2$ MeV [2], and a fourth-order polynomial background function. The fit gives a total of $63.5 \pm 14.4$ events in the peak and has a $\chi^2/dof = 97.4/92$, which corresponds to a confidence level of 27.9%. The mass value from the fit is $M_{\chi_c^0} = 2975.8 \pm 3.9$ MeV, where the error is statistical. (A fit with only the background function and no $\eta_c$ has a confidence level of 0.8%.)

The systematic error on the mass determination includes a possible uncertainty in the overall mass scale ($\pm 0.8$ MeV), which is determined from the rms average of the differences between the fitted values for $M_{\chi_c^1}$ and $M_{\chi_c^2}$ and their PDG values. The systematic errors associated with uncertainties is the particle’s total widths and the experimental resolutions ($\pm 0.95$ MeV for $M_{\eta_c}$ and less than $\pm 0.2$ MeV for $M_{\chi_c^0}$) are added in quadrature. The resulting masses and errors are:

$$M_{\chi_c^0} = 3414.1 \pm 0.6\text{(stat)} \pm 0.8\text{(sys)} \text{ MeV},$$

and

$$M_{\eta_c} = 2975.8 \pm 3.9\text{(stat)} \pm 1.2\text{(sys)} \text{ MeV}.$$  

The precision of our $M_{\chi_c^0}$ measurement represents a substantial improvement on the existing PDG value of $3417.3 \pm 2.8$ MeV [4]. Our result for $M_{\eta_c}$ agrees with the DM2 group’s value of $2974.4 \pm 1.9$ MeV [4] and is 2.4 standard deviations below the E760 group’s result of $2988.3^{+3.3}_{-3.1}$ MeV [5].

**VII. SUMMARY**

Events of the type $\psi(2S) \rightarrow \gamma\chi_{cJ}$ in a $3.79 \times 10^6 \psi(2S)$ event sample are used to determine branching fractions for $\chi_{cJ}$ decays to four and six charged particle final states. Our results for $K^0 K^+\pi^- + c.c., K^0 K^0 s, \phi \phi$, and $K^+ K^- K^+ K^-$ are the first measurements for these decays. The branching fractions for $\chi_{cJ} \rightarrow \pi^+\pi^-\pi^+\pi^-, \pi^+\pi^-K^+ K^-, \pi^+\pi^-p\bar{p}$, and
3(π⁺π⁻) final states are measured with better precision and found to be consistently lower than previous measurements. $M_{χ_c^0}$ and $M_{η_c}$ were determined using the same data sample.

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[12] Using the $\psi(2S) \to \pi^+\pi^-K^+K^-$ events sample and the particle identification procedures described in the text, we determine the branching fraction $\mathcal{B}(J/\psi \to K^+K^-) = (2.35 \pm 0.34 \pm 0.44) \times 10^{-4}$, which is in good agreement with the world average.$^\mathbb{2}$.
FIGURES

FIG. 1. The distribution of the number of photon candidates found in $\psi(2S) \rightarrow \pi^+\pi^-\pi^+\pi^-$ and $\psi(2S) \rightarrow \pi^+\pi^-K^+K^-$ events.

FIG. 2. A scatterplot of $\pi^+\pi^-$ vs $\pi^+\pi^-$ invariant masses for selected $\gamma\pi^+\pi^-\pi^+\pi^-$ events (two entries per event).

FIG. 3. The $\pi^+\pi^-\pi^+\pi^-$ invariant mass distribution. The smooth curve is the result of a fit described in the text.
FIG. 4. The $\pi^+\pi^-K^+K^-$ invariant mass distribution. The smooth curve is the result of a fit described in the text.

FIG. 5. The $K^0_sK^0_s$ invariant mass distribution. The smooth curve is the result of a fit described in the text.

FIG. 6. The $\pi^+\pi^-p\bar{p}$ invariant mass distribution. The smooth curve is the result of a fit described in the text.
FIG. 7. The $K^+K^-K^+K^-$ invariant mass distribution. The smooth curve is the result of a fit described in the text.

FIG. 8. A scatterplot of $K^+K^-$ vs $K^+K^-$ masses from selected $\gamma K^+K^-K^+K^-$ events (two entries per event).

FIG. 9. The $\phi\phi$ invariant mass distribution. The smooth curve is the result of a fit described in the text.
FIG. 10. The $K_s^0 K^+ \pi^- + c.c.$ invariant mass distribution. The smooth curve is the result of a fit described in the text.

FIG. 11. The $3(\pi^+\pi^-)$ invariant mass distribution. The smooth curve is the result of a fit described in the text.

FIG. 12. The four charged track invariant mass distribution for selected events in the $\eta_c$ mass region. The superimposed curve is the result of the fit described in the text.
TABLES

TABLE I. Fit results for $\chi_{cJ} \rightarrow \pi^+ \pi^- \pi^+ \pi^-$, $\pi^+ \pi^- K^+ K^-$ and $K^0_s K^0_s$ decays.

| Channel                  | $n^{obs}$     | $\varepsilon$ (%) | $\sigma_{res}$ (MeV) |
|--------------------------|---------------|-------------------|---------------------|
| $\chi_{c0} \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ | 874 ± 30  | 16.06             | 15.1                |
| $\chi_{c1} \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ | 277 ± 19   | 17.06             | 15.6                |
| $\chi_{c2} \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ | 425 ± 21   | 15.90             | 13.4                |
| $\chi_{c0} \rightarrow K^0_s K^0_s$          | 49.3 ± 7.0  | 15.16             | 10.9                |
| $\chi_{c2} \rightarrow K^0_s K^0_s$          | 11.7 ± 3.2  | 13.92             | 9.4                 |
| $\chi_{c0} \rightarrow \pi^+ \pi^- K^+ K^-$      | 587 ± 27   | 11.32             | 14.4                |
| $\chi_{c1} \rightarrow \pi^+ \pi^- K^+ K^-$      | 192 ± 16   | 12.91             | 15.3                |
| $\chi_{c2} \rightarrow \pi^+ \pi^- K^+ K^-$      | 267 ± 18   | 11.42             | 15.1                |

TABLE II. Fit results for $\chi_{cJ} \rightarrow \pi^+ \pi^- p\bar{p}$ decays.

| Channel | $n^{obs}$     | $\varepsilon$ (%) | $\sigma_{res}$ (MeV) |
|---------|---------------|-------------------|---------------------|
| $\chi_{c0}$ | 81 ± 10    | 14.62             | 13.9                |
| $\chi_{c1}$ | 27.1 ± 6.9 | 16.72             | 14.3                |
| $\chi_{c2}$ | 50.9 ± 8.1  | 13.98             | 13.0                |
TABLE III. Fit results for $\chi_{cJ} \to K^+K^-K^+K^-$ and $\phi\phi$ decays.

| Channel         | $n_{obs}$ | $\varepsilon$ (%) (PS/$\phi\phi$) | $\sigma_{res}$ (MeV) |
|-----------------|-----------|-------------------------------------|----------------------|
| $\chi_{c0} \to K^+K^-K^+K^-$ | 57.8 ± 6.9 | 7.38/10.06                          | 15.4                 |
| $\chi_{c1} \to K^+K^-K^+K^-$   | 11.7 ± 4.2 | 8.52/no                             | 15.2                 |
| $\chi_{c2} \to K^+K^-K^+K^-$   | 36.6 ± 5.9 | 7.64/9.76                           | 14.7                 |
| $\chi_{c0} \to \phi\phi$      | 7.6 ± 2.8  | 9.78                                | 8.9                  |
| $\chi_{c2} \to \phi\phi$      | 13.6 ± 3.7 | 9.54                                | 10.8                 |

TABLE IV. Fit results for $\chi_{cJ} \to K_0^0K^+\pi^- + c.c.$ decays. The upper limits are 90% confidence level values.

| Channel | $n_{obs}$ | $\varepsilon$ (%) | $\sigma_{res}$ (MeV) |
|---------|-----------|-------------------|----------------------|
| $\chi_{c0}$ | < 8.5    | 4.94              | 10.3                 |
| $\chi_{c1}$ | 31.4 ± 5.6 | 5.64              | 14.2                 |
| $\chi_{c2}$ | < 10.6   | 4.93              | 14.7                 |

TABLE V. Fit results for $\chi_{cJ} \to 3(\pi^+\pi^-)$ decays.

| Channel | $n_{obs}$ | $\varepsilon$ (%) | $\sigma_{res}$ (MeV) |
|---------|-----------|-------------------|----------------------|
| $\chi_{c0}$ | 191 ± 16  | 4.62              | 15.8                 |
| $\chi_{c1}$ | 98 ± 12   | 5.20              | 15.0                 |
| $\chi_{c2}$ | 112 ± 12  | 4.23              | 14.7                 |
TABLE VI. The $\chi_{cJ}$ hadronic decay branching fractions, determined using $\mathcal{B}(\psi(2S) \rightarrow \gamma\chi_{c0}) = (9.3 \pm 0.8)\%$, $\mathcal{B}(\psi(2S) \rightarrow \gamma\chi_{c1}) = (8.7 \pm 0.8)\%$ and $\mathcal{B}(\psi(2S) \rightarrow \gamma\chi_{c2}) = (7.8 \pm 0.8)\%$.

| Channel | $n_{\text{obs}}$ | Branching Ratio | World Average |
|---------|------------------|-----------------|---------------|
| $\chi_{c0} \rightarrow \pi^+\pi^-$ | 720 ± 32 | $(4.68 \pm 0.26 \pm 0.65) \times 10^{-3}$ | $(7.5 \pm 2.1) \times 10^{-3}$ |
| $\chi_{c2} \rightarrow \pi^+\pi^-$ | 185 ± 16 | $(1.49 \pm 0.14 \pm 0.22) \times 10^{-3}$ | $(1.9 \pm 1.0) \times 10^{-3}$ |
| $\chi_{c0} \rightarrow K^+K^-$ | 774 ± 38 | $(5.68 \pm 0.35 \pm 0.85) \times 10^{-3}$ | $(7.1 \pm 2.4) \times 10^{-3}$ |
| $\chi_{c2} \rightarrow K^+K^-$ | 115 ± 13 | $(0.79 \pm 0.14 \pm 0.13) \times 10^{-3}$ | $(1.5 \pm 1.1) \times 10^{-3}$ |
| $\chi_{c0} \rightarrow p\bar{p}$ | 15.2 ± 4.1 | $(15.9 \pm 4.3 \pm 5.3) \times 10^{-5}$ | $< 9.0 \times 10^{-4}$ |
| $\chi_{c1} \rightarrow p\bar{p}$ | 4.2 ± 2.2 | $(4.2 \pm 2.2 \pm 2.8) \times 10^{-5}$ | $(8.6 \pm 1.2) \times 10^{-5}$ |
| $\chi_{c2} \rightarrow p\bar{p}$ | 4.7 ± 2.5 | $(5.8 \pm 3.1 \pm 3.2) \times 10^{-5}$ | $(10.0 \pm 1.0) \times 10^{-5}$ |
| $\chi_{c0} \rightarrow \pi^+\pi^-\pi^+\pi^-$ | 874 ± 30 | $(15.4 \pm 0.5 \pm 3.7) \times 10^{-3}$ | $(3.7 \pm 0.7) \times 10^{-2}$ |
| $\chi_{c1} \rightarrow \pi^+\pi^-\pi^+\pi^-$ | 277 ± 19 | $(4.9 \pm 0.4 \pm 1.2) \times 10^{-3}$ | $(1.6 \pm 0.5) \times 10^{-2}$ |
| $\chi_{c2} \rightarrow \pi^+\pi^-\pi^+\pi^-$ | 425 ± 21 | $(9.6 \pm 0.5 \pm 2.4) \times 10^{-3}$ | $(2.2 \pm 0.5) \times 10^{-2}$ |
| $\chi_{c0} \rightarrow K^0_sK^0_s$ | 49.3 ± 7.0 | $(1.96 \pm 0.28 \pm 0.52) \times 10^{-3}$ | |
| $\chi_{c2} \rightarrow K^0_sK^0_s$ | 11.7 ± 3.2 | $(0.61 \pm 0.17 \pm 0.16) \times 10^{-3}$ | |
| $\chi_{c0} \rightarrow \pi^+\pi^-K^+K^-$ | 587 ± 27 | $(14.7 \pm 0.7 \pm 3.8) \times 10^{-3}$ | $(3.0 \pm 0.7) \times 10^{-2}$ |
| $\chi_{c1} \rightarrow \pi^+\pi^-K^+K^-$ | 192 ± 16 | $(4.5 \pm 0.4 \pm 1.1) \times 10^{-3}$ | $(9 \pm 4) \times 10^{-3}$ |
| $\chi_{c2} \rightarrow \pi^+\pi^-K^+K^-$ | 267 ± 18 | $(7.9 \pm 0.6 \pm 2.1) \times 10^{-3}$ | $(1.9 \pm 0.5) \times 10^{-2}$ |
| $\chi_{c0} \rightarrow \pi^+\pi^-p\bar{p}$ | 81 ± 11 | $(1.57 \pm 0.21 \pm 0.54) \times 10^{-3}$ | $(5.0 \pm 2.0) \times 10^{-3}$ |
| $\chi_{c1} \rightarrow \pi^+\pi^-p\bar{p}$ | 27.1 ± 6.9 | $(0.49 \pm 0.13 \pm 0.17) \times 10^{-3}$ | $(1.4 \pm 0.9) \times 10^{-3}$ |
| $\chi_{c2} \rightarrow \pi^+\pi^-p\bar{p}$ | 50.9 ± 8.1 | $(1.23 \pm 0.20 \pm 0.35) \times 10^{-3}$ | $(3.3 \pm 1.3) \times 10^{-3}$ |
| $\chi_{c0} \rightarrow K^+K^-K^+K^-$ | 57.8 ± 6.9 | $(2.14 \pm 0.26 \pm 0.40) \times 10^{-3}$ | |
| $\chi_{c1} \rightarrow K^+K^-K^+K^-$ | 11.7 ± 4.2 | $(0.42 \pm 0.15 \pm 0.12) \times 10^{-3}$ | |
| $\chi_{c2} \rightarrow K^+K^-K^+K^-$ | 36.6 ± 5.9 | $(1.48 \pm 0.26 \pm 0.32) \times 10^{-3}$ | |
| $\chi_{c0} \rightarrow \phi\phi$ | 7.6 ± 2.8 | $(0.92 \pm 0.34 \pm 0.38) \times 10^{-3}$ | |
| \( \chi_{c2} \to \phi\phi \) | 13.6 ± 3.7 | \((2.00 \pm 0.55 \pm 0.61) \times 10^{-3}\) |
|---|---|---|
| \( \chi_{c0} \to K_s^0 K^+ \pi^- + c.c. \) | < 8.5 | \(< 0.71 \times 10^{-3}\) |
| \( \chi_{c1} \to K_s^0 K^+ \pi^- + c.c. \) | 31.4 ± 5.6 | \((2.46 \pm 0.44 \pm 0.65) \times 10^{-3}\) |
| \( \chi_{c2} \to K_s^0 K^+ \pi^- + c.c. \) | < 10.6 | \(< 1.06 \times 10^{-3}\) |
| \( \chi_{c0} \to 3(\pi^+\pi^-) \) | 191 ± 16 | \((11.7 \pm 1.0 \pm 2.3) \times 10^{-3}\) | \((1.5 \pm 0.5) \times 10^{-2}\) |
| \( \chi_{c1} \to 3(\pi^+\pi^-) \) | 98 ± 12 | \((5.8 \pm 0.7 \pm 1.2) \times 10^{-3}\) | \((2.2 \pm 0.8) \times 10^{-2}\) |
| \( \chi_{c2} \to 3(\pi^+\pi^-) \) | 112 ± 12 | \((9.0 \pm 1.0 \pm 2.0) \times 10^{-3}\) | \((1.2 \pm 0.8) \times 10^{-2}\) |