RADIATIVE CORRECTIONS TO
P-LEVELS IN THE TWO-BODY QED
PROBLEM

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Abstract

The physical origin of the $m\alpha^5$ radiative corrections to $P$-levels in the two-body QED problem is elucidated. Then we demonstrate that the next order, $m\alpha^6$, corrections to those levels are due to the anomalous magnetic moment only.
1. The problem of \( m \alpha^6 \) corrections to \( 1s \ 2p \ 3P \) helium levels was treated numerically many years ago \([1, 2, 3, 4]\). Corrections of the same order to positronium \( P \)-levels were recently calculated analytically \([5]\). Those corrections are strongly dominated by the relativistic effects generated by the \((v/c)^4\) expansion terms. The true radiative corrections are numerically small, being suppressed by the "geometrical" factor \(1/\pi^2\). It was assumed in Refs.\([2, 5]\) that such corrections are due only to the anomalous magnetic moment contributions.

However, quite recently radiative corrections of a different type were discussed in relation to the same problems \([6]\). Though those contributions are small numerically, the problem is certainly interesting from a general point of view. Moreover, in helium such a contribution would be essential for the comparison with experiment.

In the present note we demonstrate accurately that it is indeed only the anomalous magnetic moment contributions that generate true radiative corrections of the order \( m \alpha^6 \) to \( P \)-states. Unfortunately, paper \([6]\) is too concise and lacks calculational details; therefore we cannot point out the exact cause of disagreement.

2. Let us start with the discussion of the \( m \alpha^5 \) radiative corrections to \( P \)-levels. There are three sources of terms. First, the infrared divergence in the scattering amplitude, i.e. in the electron charge radius (see Fig.1; here and below the dashed line refers to the Coulomb field, the wavy one to a transverse photon). Being cut off at atomic energies, the would be divergence generates both the \( \log \alpha \) term and the Bethe logarithm \( L_{n0} \) in the \( S \)-state Lamb-shift. For \( P \)-states the same divergence leads only to the corresponding Bethe logarithm \( L_{n1} \).

The second kind of correction in the two-body problem originates from the double exchange diagrams presented in Figs. 2-3. Diagrams in Fig.2 with one Coulomb and one magnetic exchange are also infrared divergent, generating again \( \log \alpha \) and \( L_{n0} \) in \( S \)-states, and \( L_{n1} \) in \( P \)-states (see, e.g., Ref.\([7]\)). On the other hand, those diagrams are effectively cut off from above at atomic momenta \( q \). Then, diagrams in Fig.3 with double magnetic exchange are cut off from below at \( q \). The \( \log q \) contributions from diagrams in Figs.2-3, being transformed into the coordinate representation, generate a potential \( \sim r^{-3} \) which has nonvanishing (and convergent) expectation value in \( P \)-states.
And finally, $ma^5$ radiative corrections to $P$-levels are generated by the $\alpha/2\pi$ contribution to the anomalous magnetic moment.

3. Consider now all potential sources of order $ma^6$ corrections to the energy shifts of $P$-levels in the same order as we have done above for the corrections of order $ma^5$.

First of all, the two-loop contribution to the electron charge radius is infrared finite. This can be most easily demonstrated using the Fried-Yennie (FY) gauge for radiative quanta. Therefore there is no analog of the Bethe logarithm $L_{nl}$ either for $S$- or for $P$-states.

As to the double infrared divergence in the scattering amplitude connected with emission of two brehmsstrahlung quanta (see Fig.1 for the case of one brehmsstrahlung quantum), it contains additional power of momentum transfer squared in comparison with the case of one brehmsstrahlung photon and is thus capable of producing corrections no larger than $ma^8$.

Let us pass over now to the two-photon exchange generating the $ma^5$ approximation log $q$ contribution to the scattering amplitude and correspondingly $r^{-3}$ to the interaction potential in the coordinate representation. We are going to consider radiative corrections to these graphs and to prove that respective diagrams do not produce corrections of order $ma^6$ to the energy levels. Note first that insertion of a one-loop polarization in any of the exchanged photons in Figs. 2-3 provides an additional factor $q^2$ in the respective integral for the energy shift. This gets an additional factor $\alpha^2$ after integration. Hence, polarization insertions are irrelevant in order $ma^6$ in the case of exchanged photons with atomic ($\sim ma^3$) momenta.

The case of one-loop radiative insertions in the electron lines seems to be more involved. Once again, we confine for the moment our consideration to the case of small momenta of exchanged quanta; large ($\sim ma$) exchanged momenta will be considered below separately. First of all in the case of two exchanged magnetic quanta the sum of radiative corrections to the Compton scattering amplitudes entering diagrams in Fig.3 vanish. This is simply a direct consequence of the well-known low-energy theorem [8] for the Compton scattering.

Consider next radiative photon insertions in the diagrams with one magnetic and one Coulomb quantum in Fig. 2. FY gauge for the radiative photons is again most suitable for our goals since it provides the most smooth low frequency behavior for all graphs. Let us start with insertion of the
self-energy operator in the electron line in Figs.2. Explicit expression for the renormalized self-energy operator in the FY gauge \(^3\) (see also \(^1\)) is proportional to the squared Dirac operator \((\hat{p} - m)^2\). Then it is easy to estimate the contribution of the graphs in Figs.2 with an inserted self-energy operator. The Coulomb line may be swallowed up by the Schrödinger-Coulomb wave function, one of the Dirac operators in the numerator of the expression for the self-energy is canceled by the remaining electron propagator and we are left with the product of the magnetic exchange and the Dirac operator between the Schrödinger-Coulomb wave functions. It is evident that the free Dirac operator \(\hat{p} - m\) applied to the Schrödinger-Coulomb wave function then produces a factor \(\alpha^2\). Therefore, self-energy insertion in the electron line suppresses the previous order effect not by a factor of \(\alpha\) but by \(\alpha^3\).

Consider now radiative photon insertion in one of the vertices in Fig.2. Respective anomalous magnetic moment contribution produces correction of order \(m\alpha^6\) as was mentioned above. All other terms in the one-loop vertex correction in the FY gauge contain at least one additional suppression factor (see \(^1\)) which after loop integration leads to contribution to the energy shift of order \(m\alpha^7\). We also have to consider insertion of a spanning radiative photon. Once again, as was shown in \(^1\), respective diagrams contain in the FY gauge an additional suppression factor which turns into an additional factor \(\alpha^2\), leading to the contribution of order \(m\alpha^7\) which is too small to be of interest for us now.

As to the diagrams with multiple exchange by soft \((q \sim m\alpha)\) Coulomb quanta, insertion of a radiative spanning photon leads only to the \(m\alpha^5\) Bethe logarithm \(L_{m1}\).

Consider finally graphs with two exchanged photons of high \((\sim m)\) momenta. It is easy to see that in these diagrams all one-loop radiative insertions either in the electron, or in the exchange photon line produce corrections of order \(m\alpha^6\) only to \(S\)-states.

Up to this moment we deliberately omitted graphs with the anomalous magnetic moment corrections. These graphs produce indeed the \(m\alpha^6\) corrections to the \(P\)-levels. There are no other sources of corrections of this order.

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Figure Captions

Fig.1. Diagrams with infrared divergent one-loop charge radius insertion.

Fig.2 Diagrams with one Coulomb and one magnetic exchange.

Fig.3 Diagrams with double magnetic exchange.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9407335v1