Effect of PAPR reduction techniques on BER Performance of OFDM System

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Abstract— In this work, several companding techniques such as Tangent Rooting Companding (tanhR), Logarithmic Rooting Companding , Exponential Companding (EXP), µ Companding etc., for PAPR reduction in orthogonal frequency division multiplexing (OFDM) systems is studied through simulations in the case of higher order QAM. Also the Performance of OFDM system is evaluated under different channels using several PAPR reduction techniques in terms of CCDF(Complementary Cumulative Distribution Function) & BER(Bit Error Rate) of orthogonal frequency division multiplexing system using MATLAB. It is observed that the tangent rooting companding provides better performance in terms of PAPR reduction than the remaining companding techniques and exponential companding technique showed better performance in terms of BER as the order of QAM increases.

Keywords— Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), PAPR reduction techniques, companding techniques

I. INTRODUCTION

OFDM has several properties which make it an attractive modulation scheme for digital TV broadcasting [1,2], such as immunity to impulse noise and intersymbol interference (ISI), low complexity and high spectral efficiency. One major difficulty, however, is its large peak-to-average power (PAPR) ratio, which reduces the resolution of the digital-to-analog (D/A) and analog-to-digital (A/D) converters in the transmitter and receiver. To reduce the PAPR ratio, several techniques have been proposed. Among these techniques, nonlinear companding transforms are the most attractive schemes due to their good system performance, the simplicity of implementation, without restriction on the number of subcarriers, the type of constellation and any bandwidth expansion [5]–[8].

In this work, several companding techniques such as Tangent Rooting Companding (tanhR), Logarithmic Rooting Companding , Exponential Companding (EXP), µ-law Companding etc., for PAPR reduction in orthogonal frequency division multiplexing (OFDM) systems is studied through simulations in the case of higher order QAM.

Also the Performance of OFDM system is evaluated under different channels using several PAPR reduction techniques in terms of CCDF(Complementary Cumulative Distribution Function) & BER(Bit Error Rate) of orthogonal frequency division multiplexing system using MATLAB. The prime focus of carrying out this work is that for a set of PAPR reduction techniques we can predict which one yields the least BER performance under different channels. From the simulations, it is found that the tangent rooting companding provides better performance in terms of PAPR reduction than the remaining companding techniques as the order of QAM increases .

II. OFDM SYSTEM MODEL

The block diagram of the OFDM system is as shown in Fig.1. An OFDM carrier signal is the sum of a number of orthogonal sub-carriers, with baseband data on each sub-carrier being independently modulated using QAM. This composite baseband signal is typically used to modulate a main RF
carrier. Fig. 1 shows the block diagram of an OFDM system with companding transform. The discrete-time transmitted OFDM signal is given by

$$x_n = \frac{1}{\sqrt{NL}} + \sum_{k=0}^{N-1} x_k e^{j2\pi kn/L} \quad 0 \leq n \leq N \quad \cdots (1)$$

$$X = X_1, X_1, X_1, \ldots, X_N, 0, \ldots, 0, X_N, \ldots, X_N$$

Here, \(X\) is the input signal vector with each data symbol modulated by QAM. \(N\) is the number of subcarriers and \(L\) is the oversampling factor. Based on the central limit theory, \(x_n\) can be approximated as a complex Gaussian process when \(N\) is large enough.

**III. PEAK-TO-AVERAGE POWER RATIO**

High PAPR of transmitted signals is one of the major issues of the OFDM system\[3\]. A large dynamic range of input data symbols is the main cause of getting high PAPR. An OFDM signal consists of \(N\) independent data symbols modulated on \(N\) orthogonal subcarriers, and when these \(N\) signals are added to the same phase, higher peak amplitude is observed. The value of this peak may be times of the average amplitude. The PAPR of the discrete time OFDM signal is given by

$$PAPR[x(n)] = \frac{\text{MAX}_{0 \leq n \leq N} ||x(n)||^2}{E[||x(n)||^2]} \quad \cdots (3.1)$$

PAPR of an OFDM symbol is defined as the ratio of peak power to average power within symbol duration

$$PAPR[dB] = \frac{\text{MAX}_{0 \leq n \leq N} ||x(n)||^2}{NL ||x(n)||^2} \quad \cdots (3.2)$$

**3.1μ-Law Companding Technique:**

In the μ-law companding \[2\], the compressor characteristic is piecewise, made up of a linear segment for low level inputs and a logarithmic segment for high level inputs. The signal by utilized μ-Law compression characteristic is defined as:
Where \( V \) is the peak amplitude of the signal, and \( x \) is the instantaneous amplitude of the input signal. Decompression is simply the inverse of equation (4.1). The performance of \( \mu \)-law companding in reduction of PAPR in terms of CCDF for different values of \( M \) in \( M \)-ary QAM are shown in fig. 2. Also the BER performance of OFDM system using \( \mu \)-law companding technique is plotted varying SNR from 0 to 30 using Rayleigh and Rician channels and is shown in fig. 3 and fig. 4 respectively. The SNR required for BER of \( 10^{-2} \) using \( \mu \)-law companding over a Rayleigh and Rician channels for different \( M \)-values are tabulated and shown in table 1 and table 2 respectively. From fig. 2 it is observed that

\[
y(x) = V \frac{\log(1 + \mu \frac{|x|}{V})}{\log(1 + \mu)} sgn(x) \quad (4.1)
\]

the PAPR reduces with increasing \( M \) using \( \mu \)-law technique. From fig. 3 and fig. 4 it is observed that to achieve BER of \( 10^{-2} \) with \( M=64 \) the SNR required is 22dB and 35dB using Rayleigh and Rician channels respectively.

**Table 1: the BER performance of OFDM using \( \mu \)-law companding over a Rayleigh channel**

| M-ary QAM | BER   | SNR |
|-----------|-------|-----|
| M=16      | \( 10^{-2} \) | 16  |
| M=32      | \( 10^{-2} \) | 19  |
| M=64      | \( 10^{-2} \) | 22  |

**Table 2: the BER performance of OFDM using \( \mu \)-law companding over a Rician channel**

| M-ary QAM | BER   | SNR |
|-----------|-------|-----|
| M=16      | \( 10^{-2} \) | 26  |
| M=32      | \( 10^{-2} \) | 30  |
| M=64      | \( 10^{-2} \) | 35  |
3.2 Exponential Companding (EXP):

The signal by utilized exponential compression characteristic is defined as:

\[ h(x) = \text{sgn}(x) \sqrt[\alpha]{1 - \exp\left(-\frac{x^2}{\sigma^2}\right)} \]  \hspace{1cm} (4.2)

The positive constant \( \alpha \) determines the average power output signals. In order to keep the input and output signals at the same average power level, the inverse function of \( h(x) \) is used in the De-companding operation at the receiver side.

\[ \alpha = \left( \frac{E[|s_n|^2]}{\sqrt{1 - \exp\left(-\frac{E[|s_n|^2]}{\sigma^2}\right)^2}} \right)^{\frac{1}{\alpha}} \]  \hspace{1cm} (4.3)
At the receiver side, the inverse function $h(x)$ of is used in the De-companding operation.

$$h^{-1}(x) = \text{sgn}(x)\sqrt{\frac{\sigma^2}{2\alpha}}\log\left(1 - \frac{x^d}{\alpha}\right)$$

(4.4)

The performance of exponential companding in reduction of PAPR in terms of CCDF for different values of $M$ in M-ary QAM are shown in fig.5. Also the BER performance of OFDM system using exponential companding technique is plotted varying SNR from 0 to 30 using Rayleigh and Rician channels and is shown in fig. 6 and fig.7 respectively. From fig.5 it is observed that the PAPR remains same with increasing $M$ using exponential companding technique. From fig.6 and fig.7 it is observed that using exponential companding technique BER is very high compared to $\mu$-law companding technique using Rayleigh and Rician channels respectively.

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**Fig.5** PAPR reduction performance of OFDM system using exponential companding transforms for different M-ary QAM

**Fig.6** Bit error rate performance of OFDM system using exponential companding schemes for different $M$ values over a Rayleigh channel
Tangent Rooting Companding (tanhR):

The hyperbolic tangent (tanh) [6] companding equation will be as follows

\[ f(x) = \tanh (|x| \times k) \times \text{sgn}(x) \quad (4.5) \]

Where \( k \) is positive numbers controlling the companding level applied to the envelope \( x \), \( |x| \) and \( \text{sgn}(x) \) was used to maintain the phases of the OFDM signal. Decompanding equation will be as follows:

\[ f^{-1}(x) = \left( \tanh \left( \frac{|x|}{k} \right) \right)^{\frac{1}{2}} \times \text{sgn}(x) \quad (4.6) \]

The performance of tangent Rooting companding (tanhR) in reduction of PAPR in terms of CCDF for different values of \( M \) in \( M \)-ary QAM are shown in fig.8. Also the BER performance of OFDM system using tangent Rooting companding (tanhR) technique is plotted varying SNR from 0 to 30 using Rayleigh and Rician channels and is shown in fig. 9 and fig.10 respectively. From fig.8 it is observed that the PAPR performance using tangent Rooting companding (tanhR) has improved compared to \( \mu \) -law and exponential techniques with increasing \( M \). From fig.9 and fig.10 it is observed that using tangent Rooting companding (tanhR) technique BER is very high similar to exponential companding technique using Rayleigh and Rician channels respectively.
Fig:8  PAPR reduction performance of OFDM system using tangent Rooting companding (tanhR) transforms for different M-ary QAM

Fig:9 Bit error rate performance of OFDM system using tangent Rooting companding (tanhR) transforms for different M-ary QAM over a Rayleigh channel
Logarithmic Companding (logR):
The logarithm companding [6] equation will be as follows

$$f(x) = \log((|x| + k_2)^y + 1) * \text{sgn}(x) \quad (4.7)$$

Decomping equation of logarithm companding is

$$f(x)^{-1} = \left(\exp\left(\frac{|x|}{k_2}\right) - 1\right)^{\frac{1}{y}} \text{sgn}(x) \quad (4.8)$$
Where $k$ is a positive number controlling the amount of companding. $K$ is used to control the companding level applied to the envelope $x$, $|x|$ and sign($x$) was used to maintain the phases of the OFDM signal.

The performance of Logarithmic Companding (logR) technique in reduction of PAPR in terms of CCDF for different values of $M$ in $M$-ary QAM are shown in fig.11. Also the BER performance of OFDM system using Logarithmic Companding (logR) technique is plotted varying SNR from 0 to 30 using Rayleigh and Rician channels and is shown in fig. 12 and fig.13 respectively. From fig.11 it is observed that the PAPR performance using Logarithmic Companding (logR) has improved compared to other PAPR reduction techniques with increasing $M$. From fig.12 and fig.13 it is observed that using Logarithmic Companding (logR) technique BER is very high similar to exponential companding technique and tangent rooting companding technique using Rayleigh and Rician channels respectively.

![Fig:12 Bit error rate performance of OFDM system using Logarithmic Companding (logR) technique transforms for different $M$-ary QAM over a Rayleigh channel](image)

**IV. RESULTS AND DISCUSSION**

The performance comparison of various PAPR reduction techniques in terms of CCDF for $M=64$ QAM is done and shown in fig.14. Also the BER performance of OFDM system using various PAPR reduction techniques for $M=64$ is plotted varying SNR from 0 to 30 using Rayleigh and Rician channels and is shown in fig. 15 and fig.16 respectively. From fig.14 it is observed that tangent rooting Companding showed better PAPR reduction performance than the other companding techniques with $M=64$. From fig.15 and fig.16 it is observed that exponential companding technique showed better BER performance compared to other techniques using Rayleigh and Rician channels respectively.
**Fig: 14** Performance comparison of OFDM system using various PAPR reduction techniques in terms of CCDF with 64-QAM

**Fig: 15** BER Performance comparison of OFDM with 64-QAM using various PAPR reduction techniques in terms of CCDF using Rayleigh channel
Though tangent rooting companding technique showed better PAPR reduction performance but BER performance is very poor. Exponential companding technique has showed better performance in both PAPR reduction as well as BER performance. The effect of exponential companding technique on PAPR reduction besides giving better BER performance can be explained as follows: In exponential companding [4] technique the PAPR of the transmitted (companded) OFDM signals is effectively reduced by transforming the statistics of the amplitudes of these signals into uniform distribution. In its operation it maintains a constant average power level. Exponential companding technique adjusts both large and small signals and can keep the average power at the same level [4].

V. CONCLUSION

The effect of various PAPR reduction techniques on the BER performance of OFDM is observed through MATLAB simulations. Based on the simulations we can predict which PAPR reduction technique yields the better BER performance under different channels besides PAPR reduction. From the simulations, it is found that the tangent rooting companding provides better performance in terms of PAPR reduction than the remaining companding techniques and exponential companding technique showed better performance in terms of BER as the order of QAM increases.

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