Transformative $A_4$ Mixing of Neutrinos with CP Violation

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Abstract

Given any real $3 \times 3$ Majorana neutrino mass matrix, the application of a familiar $A_4$ transformation turns it into a well-known form, predicting $\theta_{23} = \pi/4$ and $\delta_{\text{CP}} = \pm \pi/2$ with $\theta_{13} \neq 0$ for the neutrino mixing matrix. The natural implementation of this phenomenologically successful scenario leads to a specific radiative seesaw model of neutrino mass with softly broken $A_4$ dark matter.
In recent years, many theoretical studies have been made regarding the pattern of the 3 × 3 neutrino mixing matrix. In particular, the use of non-Abelian discrete symmetries is widespread. This came about from the specific example of $A_4$ \[1, 2, 3\], where it was shown for the first time how the three very different charged-lepton masses may be incorporated into a symmetry for neutrino mixing, which can explain $\sin^2 \theta_{23} = 1/2$. Subsequently, motivated by empirical observation, it was conjectured \[4\] that the pattern could be tribimaximal, with $\sin^2 \theta_{12} = 1/3$ and $\theta_{13} = 0$. It was then shown \[5\] that $A_4$ is indeed suitable for obtaining this result. Since 2005, there have been many papers written regarding this possibility.

In 2011 \[6\] and then more decisively in 2012 \[7, 8\], $\theta_{13}$ was measured to be significantly different from zero, thus falsifying the tribimaximal ansatz. The 2014 Particle Data Group values \[9\] of neutrino parameters are:

\[
\begin{align*}
\sin^2(2\theta_{12}) &= 0.846 \pm 0.021, \quad \Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2, \\
\sin^2(2\theta_{23}) &= 0.999 \begin{pmatrix} +0.001 \\ -0.018 \end{pmatrix}, \quad \Delta m_{32}^2 = (2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2 \text{ (normal)}, \\
\sin^2(2\theta_{23}) &= 1.000 \begin{pmatrix} +0.000 \\ -0.017 \end{pmatrix}, \quad \Delta m_{32}^2 = (2.52 \pm 0.07) \times 10^{-3} \text{ eV}^2 \text{ (inverted)}, \\
\sin^2(2\theta_{13}) &= (9.3 \pm 0.8) \times 10^{-2},
\end{align*}
\]

where (normal) refers to the ordering $m_1 < m_2 < m_3$ of neutrino masses, and (inverted) refers to $m_3 < m_1 < m_2$.

More recently \[10\], combining reactor data, there appears to be a preference for $\delta_{CP} = -\pi/2$ in long-baseline neutrino oscillation data. These new developments are in fact consistent with a special form of the Majorana neutrino mass matrix which first appeared in 2002 \[3, 11\], i.e.

\[
\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},
\]

where $A, B$ are real. This allows $\theta_{13} \neq 0$ and yet $\theta_{23} = \pi/4$ is maintained, together with the
prediction that $\delta_{CP} = \pm \pi/2$. Subsequently, this pattern was shown \[\text{[12]}\] to be protected by a symmetry, i.e. $e \rightarrow e$ and $\mu \leftrightarrow \tau$ exchange with $CP$ conjugation. With the knowledge that $\theta_{13} \neq 0$, this extended symmetry is now the subject of many studies, which began with generalized $S_4$ \[\text{[13]}\].

In this paper, I show how Eq. (5) may be obtained in a very general way, using the familiar unitary $3 \times 3$ transformation

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, which is derivable from $A_4$ as shown in Ref. \[\text{[1]}\]. The idea is very simple. The $3 \times 3$ Majorana neutrino mass matrix is in general complex, but suppose for some reason it is purely real, and its connection to the diagonal charged-lepton mass matrix is through $U_\omega$, then in the ($e, \mu, \tau$) basis, it is given by

$$\mathcal{M}_\nu = U_\omega \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix} U_\omega^T = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},$$

where

$$A = (a + 2b + 2c + d + 2e + f)/3,$$

$$B = (a - b - c + d - e + f)/3,$$

$$C = (a - b - \omega^2 c + \omega d - \omega e + \omega^2 f)/3,$$

$$D = (a + 2b + 2\omega^2 c + \omega d + 2\omega e + \omega^2 f)/3.$$

In other words, the form of Eq. (5) is automatically obtained.

In the context of $A_4$, efforts prior to 2011 were concentrated on how to achieve $c = e$ and $d = f$ for tribimaximal mixing, i.e. a residual $Z_2$ symmetry in the neutrino sector which coexists with the residual $Z_3$ symmetry implied by $U_\omega$ in the charged-lepton sector. This
clash or misalignment of residual symmetries is the origin of a basic theoretical problem which has no simple solution. In hindsight, it is a powerful argument against the naive expectation of an exact tribimaximal form of the neutrino mixing matrix. Here $A_4$ serves simply as a link for a neutrino mass matrix without any symmetry to the charged-lepton sector. The new question is how an arbitrary complex neutrino mass matrix can be guaranteed to be purely real, without imposing explicit $CP$ conservation. In the following, I will show that it may be achieved naturally together with the appearance of $U_\omega$ in a radiative implementation \cite{14,15} of neutrino and charged-lepton masses through dark matter (scotogenic), using only the one Higgs doublet of the standard model (SM), as suggested by the observation \cite{16,17} of the 125 GeV particle at the Large Hadron Collider (LHC).

Under $A_4$, let the three families of leptons transform as

$$ (\nu_i, l_i)_L \sim 3, \quad (l_i)_R \sim \frac{1}{2}, \frac{1}{2}', \frac{1}{2}''. $$

Add the following new particles, all assumed odd under an exactly conserved discrete $Z_2$ (dark) symmetry, whereas all SM particles are even:

$$ (E^0_0, E^-)_L, (N_L, N_R) \sim \frac{1}{2}, s_i \sim \frac{3}{2}, $$

where $(E^0, E^-)$ is a fermion doublet, $N$ a neutral fermion singlet, and $s_{1,2,3}$ are real neutral scalar singlets. Together with the one Higgs doublet $(\phi^+, \phi^0)$ of the SM, one-loop radiative inverse seesaw neutrino masses are generated \cite{18,19} as shown in Fig. 1.

The mass matrix linking $(\bar{N}_L, \bar{E}^0_L)$ to $(N_R, E^0_R)$ is given by

$$ M_{N,E} = \begin{pmatrix} m_N & m_D \\ m_F & m_E \end{pmatrix}, $$

where $m_N, m_E$ are invariant mass terms, and $m_D, m_F$ come from the Higgs vacuum expectation value $\langle \phi^0 \rangle = v/\sqrt{2}$. As a result, $N$ and $E^0$ mix to form two Dirac fermions of masses
\( m_{1,2} \), with mixing angles

\[
m_D m_E + m_F m_N = \sin \theta_L \cos \theta_L (m_1^2 - m_2^2),
\]

\( m_D m_N + m_F m_E = \sin \theta_R \cos \theta_R (m_1^2 - m_2^2). \)

To connect the loop, Majorana mass terms \((m_L/2) N_L N_L\) and \((m_R/2) N_R N_R\) are assumed. Since both \(E\) and \(N\) may be defined to carry lepton number, these new terms violate lepton number softly and may be naturally small, thus realizing the mechanism of inverse seesaw \cite{20, 21, 22}. Using the Yukawa interaction \( f s E_R^0 \nu_L \), the one-loop Majorana neutrino mass is given by

\[
m_\nu = \frac{f^2 m_R}{2} \sin^2 \theta_R \cos^2 \theta_R (m_1^2 - m_2^2) \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_s^2)(k^2 - m_s^2)} \left( \frac{1}{k^2 - m_1^2} \frac{1}{k^2 - m_2^2} \right)
+ \frac{f^2 m_L m_1^2}{2} \sin^2 \theta_L \cos^2 \theta_R \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)(k^2 - m_s^2)} \left( \frac{1}{k^2 - m_1^2} \frac{1}{k^2 - m_2^2} \right)
+ \frac{f^2 m_L m_2^2}{2} \sin^2 \theta_R \cos^2 \theta_L \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)(k^2 - m_s^2)} \left( \frac{1}{k^2 - m_1^2} \frac{1}{k^2 - m_2^2} \right)
- \frac{2f^2 m_L m_1 m_2}{2} \sin \theta_L \sin \theta_R \cos \theta_L \cos \theta_R \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)(k^2 - m_s^2)} \left( \frac{1}{k^2 - m_1^2} \frac{1}{k^2 - m_2^2} \right).
\]

This formula holds for s as a mass eigenstate. If \( A_4 \) is unbroken, then \( s_{1,2,3} \) all have the same mass and \( \mathcal{M}_\nu \) is proportional to the identity matrix. However, if \( A_4 \) is softly broken by the necessarily real \( s_is_j \) mass terms, then the neutrino mass matrix is given by

\[
\mathcal{M}_\nu = \mathcal{O} \begin{pmatrix}
 m_{\nu 1} & 0 & 0 \\
 0 & m_{\nu 2} & 0 \\
 0 & 0 & m_{\nu 3}
\end{pmatrix} \mathcal{O}^T,
\]

5
where $\mathcal{O}$ is an orthogonal matrix. Now each $m_{\nu_i}$ may be complex because $f$, $m_L$, $m_R$ may be complex in Eq. (17), but a common unphysical phase, say for $\nu_1$, may be rotated away, leaving just two relative Majorana phases for $\nu_2$ and $\nu_3$, owing to the relative phase between $m_L$ and $m_R$ with different $s_{1,2,3}$ masses in Eq. (17). Hence $\mathcal{M}_\nu$ is diagonalized by $\mathcal{O}$, which is all that is required to obtain $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm \pi/2$, once $U_\omega$ is applied.

To derive $U_\omega$, the simplest way is to copy Ref. [1] and add three Higgs doublets $\Phi_i \sim \overline{3}$. This leads to the charged-lepton mass matrix

$$M_l = \begin{pmatrix} f_e v_1^* & f_\mu v_1^* & f_\tau v_1^* \\ f_\mu v_2^* & f_\mu \omega v_2^* & f_\tau \omega v_2^* \\ f_\mu v_3^* & f_\mu \omega v_3^* & f_\tau \omega^2 v_3^* \end{pmatrix} = \begin{pmatrix} v_1^* & 0 & 0 \\ 0 & v_2^* & 0 \\ 0 & 0 & v_3^* \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix}. \quad (19)$$

For $v_1 = v_2 = v_3$, a residual $Z_3$ symmetry exists with $m_e = \sqrt{3} f_e v$, etc. and $U_\omega$ becomes the transformation linking $M_l$ to $M_\nu$. However, this scenario requires four Higgs doublets. It is thus somewhat problematic in the face of present data regarding the observed [16, 17] 125 GeV particle, which is entirely consistent with being the one Higgs boson of the SM.

To obtain charged-lepton masses in the context of $A_4$ with just the SM Higgs doublet, the general radiative framework of Ref. [15] is adopted. The specific scenario here requires the addition of two sets of charged scalars odd under dark $Z_2$:

$$x_i \sim \overline{3}, \quad y_i \sim \overline{1}, \overline{1}', \overline{1}''. \quad (20)$$

The one-loop diagram is given in Fig. 2. To connect $x$ with $y$, $A_4$ must be broken, either softly so that the link is again $U_\omega$ to obtain the desired residual $Z_3$ symmetry, or spontaneously using three singlet scalar fields $\chi_i \sim \overline{3}$ with equal vacuum expectation values. In this way, the three Higgs doublets of the original $A_4$ model are replaced in a renormalizable theory for obtaining charged-lepton masses. Note that the latter may be considered as the
ultraviolet completion of the common practice of using the nonrenormalizable dimension-five
term $\bar{l}_Ll_R\phi^0\chi$ for such a purpose.

As a result, the charged-lepton mass matrix is given by

$$M_l = U^\dag_\omega \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix},$$ (21)

with

$$m_e = f f_e\mu_e u \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2_{1e})(k^2 - m^2_{2e})} \left[ \frac{m_1 \cos \theta_L \sin \theta_R}{k^2 - m^2_1} - \frac{m_2 \cos \theta_R \sin \theta_L}{k^2 - m^2_2} \right],$$ (22)

where $f_e$ is the $N_R e_R y^*_1$ Yukawa coupling, $\mu_e$ is the scalar trilinear $xy^*_1\chi$ coupling, $u$ is the vacuum expectation value of $\chi$, and $m_{1e,2e}$ are the mass eigenvalues of the $2 \times 2$ mass-squared matrix

$$M^2_{xy} = \begin{pmatrix} m^2_\chi & \mu_e u \\ \mu_e u & m^2_{y_1} \end{pmatrix},$$ (23)

with mixing angles $\theta_L$ and $\theta_R$, and similarly for $m_\mu$ and $m_\tau$. One immediate consequence of a radiative charged-lepton mass is that the Higgs Yukawa coupling $h\bar{l}l$ is no longer exactly $m_l/v$ as in the SM. Its deviation is not suppressed by the usual one-loop factor of $16\pi^2$ and may be large enough to be observable [23].

As for dark matter, there is a one-to-one correlation of the neutrino mass eigenstates to the $s_{1,2,3}$ mass eigenstates, the lightest of which is dark matter [24, 25]. It is also clear from
Eq. (17) that all three neutrino masses are expected to be of the same order of magnitude, and their mass-squared differences are related to the scalar mass differences. The most recent cosmological data \[26\] imply

\[
\sum m_\nu < 0.23 \text{ eV}. \tag{24}
\]

This would mean that the effective neutrino mass \(m_{ee}\) in neutrinoless double beta decay is bounded below 0.07 eV for normal ordering and 0.08 eV for inverted ordering.

Due to the presence of the \(A_4\) symmetry, the dark matter parity of this model is also derivable from lepton parity \[27\]. Under lepton parity, let the new particles \((E^0, E^-), N\) be even and \(s, x, y\) be odd, then the same Lagrangian is obtained. As a result, dark parity is simply given by \((-1)^{L+2j}\), which is odd for all the new particles and even for all the SM particles. Note that the tree-level Yukawa coupling \(\bar{l}_L l_R \phi^0\) would be allowed by lepton parity alone, but is forbidden here because of the \(A_4\) symmetry.

In conclusion, it has been pointed out that the phenomenologically successful form of the neutrino mass matrix given by Eq. (5) is derivable from the familiar \(A_4\) transformation of Eq. (6) if the Majorana neutrino mass matrix is purely real. To obtain the latter naturally, a specific scotogenic one-loop radiative model of neutrino and charged-lepton masses is proposed, where the particles appearing in the loop have odd dark matter parity. These predicted new particles should have masses at the scale of weakly interacting dark matter, i.e. 1 TeV or less, and be potentially observable at the LHC, which has just resumed operation at CERN.

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