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ABSTRACT

Copper and Iron based high temperature superconductors exhibit d-wave type superconducting gap and order parameter and posses a universal phase diagram. Here a potential is introduced that accounts for high temperature superconductivity, justifies d-wave symmetric behavior, and successfully explains phase diagram’s salient features. This potential is stipulated by principles of special relativity and arises from the difference between the electric potential of moving electrons and the potential of stationary nuclei. In quasi-two-dimensional materials this difference results in an uncompensated angular dependent attraction force in preferred directions of motion and a repulsion force in the perpendicular directions. The attraction force causes d-wave angular dependent superconducting gap and order parameter at high temperatures for d or p orbitals, which are the orbitals involved in Copper and Iron based superconductors. The repulsion force justifies the existence of angular dependent pseudogap and since the attraction and repulsions forces confine electrons to two directions of motion the number of allowed momentum states are reduced resulting in anti-ferromagnetic Mott-insulator behavior. The combination of the attraction and repulsion forces is shown to create charge density waves in these quasi-two-dimensional materials. This potential is able to justify the main features of the universal phase diagram self-consistently.

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INTRODUCTION

The attraction force proposed by conventional quantum mechanical description of superconductivity (BCS) vanishes at cryogenic temperatures above about 30 K. In 1986, however, a new class of superconductors was discovered with critical temperatures much higher than this limit. These new high-Tc superconductors were all layered quasi 2-D material, which shared CuO₂ planes for electron transport. In 2006 another class was found in which Iron plays the role of Copper in their layered planes. Both have atoms with d-orbital chemical bonds and they also share signature characteristics such as antiferromagnetism, pseudogap formation, charge density waves and a universal phase diagram. The are theories attempting to introduce an attraction force, such as resonating valence bond, interlayer tunneling, plasmons, spin-bags, to justify high Tc superconductivity but they fail to account consistently for the accompanying characteristics.

Although the common characteristics of high Tc superconductors such as anisotropic two-dimensionality, d-orbital bonds, d-wave type superconducting band gap, d-wave type superconducting order parameter, pseudogap formation, antiferromagnetic Mott-insulator and charge density waves, seem to add to the complexity of the problem at first glance, they help narrow our search to one particular potential that is the cause of all these behaviors. And in fact such a potential has been speculated by principles of special relativity.

There exists another unconventional class of superconductors in which spin-triplet electrons are paired. This class of superconductors has found much theoretical as well as practical attention because of its potential for very low power electronic devices. These materials are all s-wave superconductors and almost all theoretical investigations have considered s-wave triplet superconductivity. The potential presented here cannot create s-wave superconductivity, therefore, we are only concerned with conventional
zero spin superconductivity for high Tc superconductors, which are d-wave.

The paper is organized as follows: we first introduce the relativistic potentials in quasi-one and two-dimensional materials and show that they are identical to \( d_{z^2} \) and \( d_{x^2-y^2} \) orbitals, respectively. In the subsequent section this potential is used in BCS theory to show d and p orbitals result in a superconducting state at high temperatures. Then the role of repulsion part of the potential in creating pseudogap is explained and following that section the reasoning behind creation of charge density waves is provided. Next the Mott-insulator antiferromagnetic behavior of these materials is justified based on this potential. The role hole doping plays in creating pseudogap is explained and following that section the relationship between critical temperature and doping is treated afterwards. In the last section all the arguments are brought together to justify the distinct features of the universal phase diagram.

The fact that one potential is capable of explaining all these behaviors cannot be incidental and deserves much attention and scrutiny.

**CONSEQUENCES OF THE CHANGE IN ELECTRON’S POTENTIAL**

In general, crystals are charge neutral because equal and opposite electron and nuclei potentials cancel each other. However, special relativity claims that the potential around a moving charged particle is different from a stationary particle. This means that special crystal structures could be envisioned in which charge neutrality does not hold everywhere and a built-in angular dependent uncompensated potential is developed inside them as demonstrated below.

The potential of a stationary positive charge is the well-known Coulomb potential written as:

\[
\Phi(r) = -\frac{q^2}{r},
\]

which is spherically symmetric. If this charge moves, the potential is skewed based on special relativity in the following manner:

\[
\Phi(\theta, r) = -\frac{q^2}{r} \left( 1 - \frac{v^2}{c^2} \right) \left( 1 - \frac{v^2}{c^2 \sin^2 \theta} \right),
\]

where \( v \) is particle speed, \( c \) is the speed of light in the medium in which the particle is traveling, and \( \theta \) is measured from the direction of motion. Fig. 1A illustrates the symmetric potential of a stationary nuclei and the relativistic potential of an electron moving either to the left or the right. These potentials are no longer identical and won’t cancel each other out. Their non-zero superposition potential is obtained as:

\[
\Phi(\theta, r) = -\frac{q^2}{r} \left( 1 - \frac{v^2}{c^2} \right) \left( 1 - \frac{v^2}{c^2 \sin^2 \theta} \right),
\]

and is shown in Fig. 1B. Green lobes in the figure represent the attraction and blue represent repulsion forces for electrons. For an electron speed equal to 0.01c, which is the Fermi speed of electrons in the celebrated YBCO cuprate, the maximum attraction potential in the direction of motion is \( 10^{-4} \times \frac{q^2}{r} \) and the maximum repulsion potential in perpendicular directions is \( 0.5 \times 10^{-4} \times \frac{q^2}{r} \). As seen in the figure resultant potential looks very similar to \( d_z \) \( \frac{1}{\sqrt{2}} \right) (3 \cos^2 \theta - 1) \) orbital, in fact for electron speed of 0.01c they are identical as illustrated in Fig. 2, in which they are drawn alongside for two different multiplication constants.

This potential does not manifest itself in regular isotropic three-dimensional material because electrons have an equal chance of moving in any direction. For such materials the potential of Fig. 1B must be rotated and summed over all possible directions of motion. The resultant potential would be zero and no net attraction or repulsion potential would be detected. However, if a crystal is anisotropic
and quasi-2D dimensional, such as in cuprates and iron based superconductors, a net attraction and repulsion force would be developed. For instance imagine electrons have two preferred lines of motions along $x$ and $y$ directions. Two potentials similar to Fig. 1B but one in $x$ and the other in $y$ direction must be added together. The result is provided in the Fig. 3. The resultant potential is identical to the angular form of $d_{x^2-y^2} = \frac{1}{2} \sqrt{\frac{2}{\pi}} \sin \theta \cos \theta$ in its attraction part and it is also similar to spherical harmonic $Y_2^0$ in its repulsion part. Fig. 4 illustrates that the attraction potential in $xy$ plane is identical to $d_{x^2-y^2}$ orbital.

**SUPERCONDUCTIVITY**

The question is whether this potential can cause superconductivity at high temperatures. Based on BCS theory superconductivity occurs if the contribution of attraction force to energy lowering is larger than the repulsion force contribution to energy increase. To ascertain such contributions, and determine unequivocally whether a potential can cause superconductivity, it is sufficient to calculate the so-called scattering amplitude between initial pair states $(k^\uparrow, -k^\downarrow)$ and final pair states $(k'^\uparrow, -k'^\downarrow)$ as:

$$V_{k,k'} = \langle k'^\uparrow, k'^\downarrow | \Phi_{(0,0)} | k^\uparrow, k^\downarrow \rangle,$$

where $\Phi_{(0,0)}$ is either the potential given in Fig. 1B for one-dimensional case or the one provided in Fig. 3D and E for two-dimensions. Scattering amplitude determines how much energy of the system is lowered if electrons are scattered from one pair state to another pair state when these two states are correlated by the attraction potential. Summation of all such possible scattering events gives us the superconducting gap.

As a general rule a negative value to above integral leads to a superconducting state, meaning a lower state of energy is achieved due to the scattering event. Both the potential and orbitals (pair states) involved in transitions determine the scattering amplitude. It is easily seen that if $s$-orbitals were used in the above integral the result would be zero because the repulsion parts compensate the attraction parts. Hence, $s$-wave superconductivity does not occur with this potential. The situation is different for high $T_c$ materials. Fig. 5 illustrates the orbitals and potentials created in CuO$_2$ planes of cuprates showing two directions of motion for electrons in $x$ and $y$ directions. Cu atoms have $d_{z^2}$ orbitals shown in reddish-brown and Oxygen atoms have $p$ orbitals, which are shown in pink. At all Oxygen sites a potential similar to $d_{z^2}$ orbital is developed but are only shown for four Oxygen sites at the top left corner. The green lobes correspond to attraction and blue to repulsion coming in and going out of the page.

**FIG. 3.** Built-in relativistic potential in quasi-2D material. If electrons are moving in two orthogonal directions of $x$ and $y$, two orthogonal potentials shown in A and B should be added. C. Two potentials superimposed. D. The resulting potential has an attraction part identical to $d_{x^2-y^2}$ orbital. E. Looking from the side at D showing the repulsion part in the $z$ direction, which is almost identical to $Y_2^0$ spherical harmonic.

**FIG. 4.** A. The $d_{x^2-y^2}$ orbital $\frac{1}{2} \sqrt{\frac{2}{\pi}} \sin \theta \cos \theta$ drawn with the potential of Fig. 3 multiplied by 30000 as $30000 \left\{ \left( \frac{1-10^{-4}}{\left(1-10^{-4}\sin^2 \theta\right)^2} \right)^{\frac{1}{2}} \right\}$. B. The multiplication factor has been changed to 25565 and the two figures are overlaid exactly.

**FIG. 5.** Orbitals and relativistic attraction and repulsion potentials in CuO$_2$ planes of cuprates showing two directions of motion for electrons in $x$ and $y$ directions. Cu atoms have $d_{z^2}$ orbitals shown in reddish-brown and Oxygen atoms have $p$ orbitals, which are shown in pink. At all Oxygen sites a potential similar to $d_{z^2}$ orbital is developed but are only shown for four Oxygen sites at the top left corner. The green lobes correspond to attraction and blue to repulsion coming in and going out of the page.
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planes of cuprates. Electrons are confined to x and y directions only, where they create $d_{xy}$ looking potential around Oxygen atoms, and at the intersections, where Cu atoms reside, they add to give $d_{z^2-r^2}$ orbital type attraction force. To obtain the scattering amplitude, $d_{z^2}$ potential should be used with p-orbitals of Oxygen and d$_{z^2-r^2}$ attraction for d-orbitals of Cu in the above integral. For p orbitals of the Oxygen atom the following integral must be evaluated:

$$V_{p_{\theta'}} = \langle \Phi_{\theta'} | p | \Phi_{\theta} \rangle.$$  

Assuming $3\Delta^0$ as an average for $r$, $\frac{\Delta}{\Delta} = 0.33$ mev and $p = \frac{1}{2} \sqrt{\frac{\Delta}{2}} \cos \theta$, we used MATLAB program to solve the integral and obtained $V_{p_{\theta'}} = -1.5$ mev.

For $d_{z^2-r^2}$ orbitals we must calculate:

$$V_{d_{z^2-r^2}} = \langle d_{z^2-r^2} | \Phi_{\theta} \rangle | d_{z^2-r^2} \rangle.$$

where $\Phi_{\theta}$ is:

$$\Phi_{\theta} = -\frac{q^2}{r} \left( \frac{1 - \frac{q^2}{2}}{1 - \frac{q^2}{2} \sin^2 \theta} \right) \frac{q^2}{r} \left( 1 - \frac{q^2}{2} \sin^2 (\theta + \pi/4) \right).$$

$V_{d_{z^2-r^2}}$ turns out to be about -0.83 mev.

Negative values mean that superconducting state is possible and to calculate the band gaps the same steps as outlined in the BCS theory can be followed. The contributions of scattering amplitudes of all possible scattering events must be added. Assuming a constant theory can be followed. The contributions of scattering amplitudes $V_{d_{z^2-r^2}}$ should be used with p-orbitals of Oxygen and orbital type attraction force. To obtain the scattering amplitude, $x$ cal temperature around 93 K. For Iron based superconductors only band gap is then obtained from:

$$\Delta = \frac{\nabla E}{\sinh \left( \frac{1}{N(k)} V(k') \right)},$$

where the energies are written with respect to Fermi energy. The band gap is then obtained from:

$$\Delta = \frac{\nabla E}{\sinh \left( \frac{1}{N(k)} V(k') \right)},$$

If we use the linear density of states, which is given in Ref. 32 and replace it by a constant average, transition between $d_{z^2-r^2}$ orbitals gives a gap of 17.5 mev for an energy interval of 15 mev (using Debye energy). And the gap for p orbital transitions turns out to be around 30 mev. Since there are two Oxygen atoms for each Cu atom the average gap is calculated to be about 25.8 mev, which places critical temperature around 93 K. For Iron based superconductors only $d_{z^2-r^2}$ orbitals are involved and 17.5 mev gap would be obtained, which places the critical temperature around 50 K.

It is therefore seen that this potential causes high critical temperatures and the importance of CuO$_2$ bonds or Iron bonds in creating superconductivity becomes clear. Both $d_{z^2-r^2}$ and p orbitals do not overlap with the repulsion areas of the potential and negative contributions to scattering amplitude is obtained. There is no other orbital that can do this. The scattering amplitude for all other orbitals is less than these two or zero. The reason for d-wave order parameter and superconducting gap of cuprates is clear as well. The d-wave order parameter of cuprates follows the d-orbitals of CuO$_2$ bonds and d-wave superconducting gap follows the attraction force, as it should. What gives this potential more credence as the cause of high Tc superconductivity is its ability to explain accompanying non-superconducting characteristics.

PSEUDOGAP

One inseparable characteristic of high Tc materials is the occurrence of pseudogap,~12 which refers to existence of a spectral gap above superconducting temperature with strong momentum anisotropy. Here, the repulsion portion of this potential can easily be shown to explain this enigmatic phenomenon. Imagine repulsion potential $w$ is in the $z$ direction, free electrons moving in this direction must satisfy following Schrodinger equation:

$$-\frac{\hbar^2}{2m} \psi''(z) + w\psi(z) = E\psi(z)$$

transferring $w$ to the right hand side we obtain:

$$-\frac{\hbar^2}{2m} \psi''(z) = (E-w)\psi(z)$$

depending on the relative value of electron energy with respect to the repulsion potential two answers are obtained:

$$E < w \Rightarrow \psi(z) = A e^{-kz}$$

$$E < w \Rightarrow \psi(z) = e^{akz}$$

For electrons with energy less than repulsion there are no free electron solutions and the answers are evanescent. Only when energy larger than potential is given to electrons the equation provides a free electron solution. This simply means there are no allowed states in $z$ direction when energy is less than repulsion potential or differently put a band gap is created in the directions where repulsion force exists. This gap or pseudogap’s magnitude should be calculated by summing up the contributions of all electrons to the repulsion potential. But it is possible to obtain an estimate by comparing it to superconducting gap, which measures the contributions of all electrons to attraction potential. Since repulsion magnitude in $z$ direction is twice the attractions in $x$ and $y$ directions for a given electron, as given in Fig. 3E, its total contribution can be approximated as twice 17.5 mev and equal to about 35 mev for a regular high Tc material such as YBCO.

We return to examine pseudogap and its response to doping and temperature later. For now lets see how this potential results in charge density waves (CDW) that are seen in high Tc superconductors.

CHARGE DENSITY WAVES

In quasi one and two-dimensional material a periodic distortion of lattice can happen when electron and holes are coupled via phonon scattering, or when electrons and holes affect the nuclei. The periodic distortion in turn modulates either charge or spin of electrons.~37,33,34 Mathematically speaking, free electron gas in a crystal with density $p(r)$, under the periodic potential of crystal $\psi(r)$, is redistributed by a change in its electron density.~34,35 This new distribution of electrons is related to potential by:

$$p(r) = \psi(r)\chi(r),$$
where $\chi(r)$ relates potential to the electrons and is the Lindhard response function, which has the following general form,

$$
\chi(q) = \int \frac{dk}{(2\pi)^d} \frac{f_k - f_{k+q}}{\epsilon_k - \epsilon_{k+q}}
$$

(15)

where $d$ is the number of dimensions, $q$ and $k$ are wave-vectors, $f_k$ is the Fermi-Dirac distribution and $\epsilon$ is the dispersion relation for electron gas.

It can be shown that the Lindhard function diverges at a wave-vector $q=2k_F$ in one-dimensional lattices and therefore has the largest value for quasi one-dimensional structures. This is normally referred to as optimum Fermi nesting.\textsuperscript{34,35} Physically above equations imply that if electrons somehow are causing a potential that affects nuclei, and they can interact with crystal easily via phonons, the material exhibits CDWs. In our case both conditions are met. Optimum nesting by the quasi 2-dimensional structure is provided and the potential, which is caused by electrons, should interact with nuclei through attraction and repulsion forces.

Fig. 3d gives a very good indication of how CDWs should look like in the structures with such potential. The repulsive parts of the potential, which are perpendicular to the plane of the motion, are attraction potentials for positive nuclei. These cause crystal to be distorted in transverse directions while following electron density variations or waves. On the other hand, the attraction parts act as repulsion for nuclei and can cause longitudinal distortions in the planes of motion.

**ANTIFERROMAGNETIC MOTT-INSULATOR**

Another salient characteristic of high Tc superconductors is that they act as Mott-insulators with antiferromagnetic arrangements of electrons,\textsuperscript{36–38} while according to conventional band theory they must be metals. Conventional band theory tells us that there is twice as many allowed states in a band as the number of primitive cells per unit volume. This means that if each primitive cell releases an odd number of electrons to the sea of free electrons the band would be half-filled and the material acts as a metal. The situation is the opposite for many materials that have 3d-orbitals at their outer shell. These materials such as MnO, FeO, CoO and NiO all contribute an odd number of electrons to the lattice but defying the conventional band theory they act as insulators.

To account for this discrepancy Hubbard theory introduces a repulsion force between electrons as the cause for anomalous insulating behavior.\textsuperscript{39–40} The Hamiltonian written is as follows:

$$
H = -\sum_{\langle ij \rangle, \sigma} t_{ij}(c^\dagger_i \sigma c_j \sigma + c^\dagger_j \sigma c_i \sigma) + U \sum_i n_i^\uparrow n_i^\downarrow
$$

(16)

where $i$ and $j$ run over nearest neighbors, $\sigma$ runs over spin, $c$ is the creation operator, $U$ is the repulsion energy, and $n$ counts the spin. The summation is a measure of electron moving from one atom to another and recreates the band diagram while the second term is the repulsion contribution to energy. Electrons of opposite spin occupying a state are considered to repel each other and increase energy by $U$. It is easily shown that the lowest energy state for such material is when electrons arrange themselves in antiferromagnetic configuration, meaning that electrons occupy opposite spins in adjacent atoms. This in itself means that the number of available states has decreased by half and a normally metal turns into an insulator.

By the discovery of high-Tc superconductors situation has become complicated.\textsuperscript{40} The Copper based and Iron based superconductors all exhibit Mott-insulator behavior at low temperatures and low doping regimes. As a result attempts have been made to reconcile Copper based superconductivity with Hubbard model of Mott-insulator behavior,\textsuperscript{40} but further progress in this area has stalled and no prediction or justification of high-Tc behavior has resulted.

There seems to be an inherent irreconcilable difference between superconductivity and Mott-insulator antiferromagnetic behavior because one is caused by attraction and the other is caused by repulsion between electrons. The discrepancy is removed if it were shown that attraction between electrons could also cause antiferromagnetic Mott-insulator in anisotropic two-dimensional material such as high Tc superconductors.

Consider the quasi two-dimensional structure where electrons are forced to move in $x$ and $y$ directions due to attraction potential, which is designated as $w$ and drawn in blue in Fig. 6. The potential is creating lower energy conduits for electrons and as long as thermal energy is much less than $W$, KT<<$W$, electrons are confined to move in these two directions. The solution to Hamiltonian for this quasi two-dimensional lattice is quite easily obtained by two independent Bloch functions of the form:

$$
\psi_{n}(x) = e^{\pm ik_n x} u_{n}(x)
$$

(17)

$$
\psi_{n}(y) = e^{\pm ik_n y} u_{n}(y)
$$

(18)
Where $k_{(n+1)}X$ is momentum in the $n$th row and $k_{(n)}Y$ is momentum in the $n$th column, $u_{(n)}$ denotes the periodicity of atomic sites in that row or column, and $n$ is the index identifying each row or column.

Assuming the parallel paths are far apart such that their orbitals do not overlap electrons in each path behave independently of their neighbors. In such a case the band diagrams are two one-dimensional band diagrams in $x$ and $y$ directions and each has $2N$ states for $N$ atomic sites in each direction. If each atom contributes one or odd number of electrons to each path a half-filled band and a metal is obtained. Now imagine we bring the rows and columns closer to each other such that electrons of neighboring sites overlap as shown in the figure. Overlapping electrons must obey Pauli’s exclusion principle and must be different at least in one of their good quantum numbers. Let’s see what this entails. For a given energy level $E$, only two momentum values of $\pm k$ exist in each path. Electron orbitals are the same for all paths so the only concise or good quantum numbers are energy, momentum and spin.

Normally two electrons with opposite spins can reside on any $\pm k$. But now each electron is sharing its orbital with its nearest neighbors, which have the same momenta of $\pm k$. It means that if its spin is up, its neighbors with the same energy and momentum must acquire a down spin and vice versa.

Therefore the only allowed spin states for these two neighboring electrons of the same energy in the $n$th and $(n+1)$th row or $n$th and $(n+1)$th column would be the following possibilities:

$$\begin{pmatrix} -k_{(n+1)} \uparrow, -k_{(n)} \downarrow \\ +k_{(n+1)} \downarrow, +k_{(n)} \uparrow \end{pmatrix} \cong \begin{pmatrix} -k_{(n+1)} \downarrow, -k_{(n)} \uparrow \\ +k_{(n+1)} \uparrow, +k_{(n)} \downarrow \end{pmatrix}$$

where the up and down arrows denote two spin directions and $|k_{(n+1)}| = |k_{(n)}|$.

In other words Pauli’s exclusion principle has forced each state to accommodate only one spin. Of the four possibilities the two at the top and the two at the bottom are equivalent. The two at the bottom basically mean that all electrons in a row and all electrons in a column have their spins in one direction. The two at the top, however, make it possible for electrons to have opposite spins in a row and in a column. To decide which situation occurs we need to consider the effect their spins have on total energy of the system. Equal spins in adjacent atoms increase the energy while opposite spins decrease it. This means that antiferromagnetic ordering along each path and between paths is what happens in reality. (Classically it amounts to attaching a number of flat magnets alongside and beside each other, which is only possible when their north and south poles are placed alternatively or in antiferromagnetic fashion).

Based on this analysis confinement of electrons to only two preferred paths decreases the states by a factor of two because only one spin can occupy each momentum. This decreases the number of states to $N$ in each band and in a one-dimensional structure one electron per unit cell fills the first band and creates an insulator. If there are two electrons per unit cell the second band would be filled and again an insulator is obtained and so is the case for all even and odd number of electrons per unit cell. In regular crystals there is a constant energy surface of momenta available for electrons to occupy and at each state two spins can reside without violating Pauli’s exclusion principle.

**HOLE DOPING**

The analysis above showed that quasi-2D materials could become Mott-insulators due to the discussed potential and how all the energy levels in the topmost energy band are filled with electrons. For superconductivity to occur there must be empty states into which electrons can scatter. This fact is implicit in equations 5 and 6 where electrons occupying an initial state are scattered into final vacant states. One way to create vacant states is to dope the material with holes. A measure of total energy lowering or critical temperature versus hole doping can be obtained simply by multiplying the number of filled states by the number of empty states. Consider the fraction of empty sates or the number of holes as $p$, which changes between 0 and 1 for when all the states are full and when all are empty, respectively. The fraction of the filled states or number of electrons would then be $(1-p)$. In the first order the product of the number of electrons and holes or $p(1-p)$ would be a measure of critical temperature. But energy lowering is also related to the value of attraction force, which is a function of hole doping itself. As material is hole doped electrons are frozen at the impurity sites and are no longer free to move, which means their contribution to the relativistic potential vanishes and the total attraction potential per material decreases. If we assume a linear relationship between the number of free electrons and total attraction force they create, the formula representing critical temperature becomes $p(1-p)(1-p)$. Furthermore, as the number of filled states decreases so does the average energy and speed of electrons, which in turn decreases the relativistic potential in equation 2 and 3. Again assuming for simplicity a linear relationship between the number of electrons and their contribution to relativistic potential due to their speed we have $p(1-p)(1-p)$ as the relationship between doping and critical temperature. In other words:

$$T_c(p) = Cp(1-p)^3$$

where $T_c(p)$ denotes the critical temperature as a function of doping and $C$ is a constant obtained from equations 5 and 6. For this relationship the maximum critical temperature occurs at an optimum doping of $p_{\text{optimum}} = 24\%$. Above equation provides a graph that resembles the experimental results but still a more accurate equation can be obtained as well. In a real sample there is another mechanism that affects the optimum doping. Doping in general introduces random positive charges that can have detrimental affect on the attraction force by creating random paths for electrons and redistribution of positive charges. So one expects for the optimum doping to decrease below 24%. The optimal doping for high Tc material are around 15% to 20%.

We can try to estimate the contribution of random charges by fitting above equation to experimental data. We therefore include a new term as:

$$T_c(p) = Cp(1-p)^3f(p),$$

where $f(p)$ represents the detrimental affect of impurity dopants. Experimental data suggest that superconductivity is lost at about
30% doping levels and a minimum doping level of about 7% is necessary for superconducting state to start. Replacing $p$ with $(p - 0.07)$ and $f(p)$ with $(1 - 3.5p)$ we arrive at the following equation that closely resembles experiments.

$$T_c(p) = C(p - 0.07)(1 - p)^3(1 - 3.5p).$$

In Fig. 7, which illustrates the universal phase diagram of high $T_c$ superconductors, the upper boundary of the yellow area corresponds to the above equation. Superconductivity starts at 7% doping and vanishes at about 30% doping level. The optimum doping level has shifted to about 16% on its own and the general shape follows the so-called bell shaped superconducting phase of experimental data.

One can create scattering events and superconductivity by doping the material with electrons as well. But electron doping has orders of magnitude more detrimental effect on the value of attraction force compared to hole doping. An extra electron introduces an uncompensated electron’s repulsive potential in the preferred paths of the motion without its compensating proton accompanying it and as a result it only contributes to repulsion and decreases overall attraction potential by about $\frac{e^2}{r^2}$. Recall that each regular electron adds only $10^{-4} \times \frac{e^2}{r^2}$ to the attraction potential. Therefore, one doped electron destroys the attraction force created by $10^4$ regular electrons and if we assume there are $10^{12}$ electrons per cm$^3$, a doping of $10^{18}$ per cm$^3$ is enough to quench superconductivity. In other words a 0.01% electron doping is capable of destroying the attraction force.

The situation, however, can be mitigated to a certain degree if dopant atoms are located exactly along the directions of motion.

**PHASE DIAGRAM**

Fig. 7 is a rough illustration of the universal phase diagram of high $T_c$ superconductors. Except for the superconductivity phase obtained from equation 21, drawings and numbers are not to be taken literally. There are Mott-insulator, pseudogap, charge density wave, superconductivity and metal phases appearing at different doping-temperature regions. We have already discussed how the relativistic potential can cause all these phases. Their position on the diagram and with respect to each other is what we intend to discuss in this section.

The material starts as an antiferromagnetic Mott-insulator at zero doping. When electrons arrange themselves in antiferromagnetic fashion a minimum energy is reached similar to when small magnets are attached north south in rows and columns. To break this arrangement energy must given to the system or a lower energy arrangement must be introduced. Therefore as doping starts the material remains in its antiferromagnetic Mott-insulator phase until the introduction of holes creates a situation where the rearrangement of electrons is energetically more favorable. For example in our case at about 7% doping superconductivity presents a lower state of energy and electrons transition from Mott-insulator to superconducting phase. The transition to a non-superconducting state is also conceivable and the borders of the two regions do not need to meet at one point as given in the figure. As doping is increased we enter superconducting phase, which was discussed in the previous section.

The pseudogap regime above superconductivity simply means that the repulsion force, which is responsible for pseudogap, exists at these temperature levels. This is as expected because as long as electrons move primarily in the preferred directions of motion repulsion force should exist as well. Loss of superconductivity is not because the attraction force has disappeared but because thermal energy is more than the energy lowering of electrons. As long as repulsion force is present the pseudogap it creates is discernible in experiments and if we had other means beside superconductivity to assess the existence of attraction force, we could have observed it in the pseudogap regime as well.

The borderline between the pseudogap and metal phase is quite interesting especially where it intersects the doping axis. It indicates that pseudogap vanishes sooner than superconductivity with doping. An enormous amount of data collected by APRES measurements, corroborate this fact. This peculiar behavior is easily explained by our mechanism by considering that the repulsion force is affected by doping much more than the attraction force. In the previous section the decrease in attraction force due to doping was discussed in detail to obtain equation 21. All those arguments equally apply to repulsion force; however, there is another mechanism that reduces the repulsion force without directly affecting the attraction force. It has to do with the fact that holes are attracted to the repulsion areas and essentially screen and weaken it. Therefore pseudogap disappears before attraction force and superconductivity.

The charge density wave phase appears at the center of the diagram covering parts of pseudogap and superconductivity regions. Since attraction and repulsion forces are crucial for CDW creation...
it is expected for this behavior not to occur in the metal part of the diagram where attraction and repulsion forces have been lost. Also, CDWs cannot occur in Mott-insulator regime because a prerequisite for their creation is for electrons to scatter to empty states as a result of their interactions with lattice vibrations and Mott-insulator is devoid of empty states. So CDW phase placing on the diagram is consistent with our analysis. Charge density wave provides a minimum energy phase for electrons and is expected to vanish at high enough temperatures placing an upper boundary for this region.

The metal phase at high doping levels occurs when the attraction and repulsion forces are destroyed due to random distribution of dopant atoms that redistribute charges. Loss of attraction and repulsion forces means that the material is no longer antiferromagnetic and two electrons with opposite spins are allowed to fill each energy level. The energy band is half filled and a metal is created. In the metal state above the superconducting region the attraction force may not be diminished yet so it is conjectured that the metal above the superconducting region might behave differently compared to the metal to the right of the superconducting state.

CONCLUSIONS

In quasi two-dimensional structures a potential comprised of angular dependent attraction and repulsion portions is created. Based on conventional superconductivity theory the attraction part causes high Tc d-wave superconductivity when physical parameters of cuprate or Iron based materials are used in evaluations. The repulsion part causes the so-called pseudogap, which is inseparable from high Tc superconductivity. The attraction and repulsion forces confine electrons to particular directions of motion and result in Mott-insulator antiferromagnetic behavior when the material is relatively undoped. The same attraction and repulsion forces explain the charge density wave creation in these materials. Main features of the universal phase-diagram are consistently explained by this simple classical potential warranting further evaluation of its properties and ramifications in quasi-one, two or three-dimensional materials.

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