We present results on lepton energy and recoil hadronic mass moments in semileptonic $B$ decay using a total of 9.4 fb$^{-1}$ of data taken with the CLEO detector at the $\Upsilon (4S)$. These results are discussed in the context of Heavy Quark Effective Theory and compared to theory predictions as a function of the minimum lepton energy requirement. We also measure the $B$ semileptonic branching fraction, $\mathcal{B}(B \to X e^+ \nu_e)$, as $(10.91 \pm 0.09 \pm 0.24)\%$.

1 Inclusive Semileptonic $B$ Decay

Experimentalists have long used semileptonic $B$ decay as a place to measure the magnitude of the elements $V_{ub}$ and $V_{cb}$ of the CKM quark-mixing matrix. In the case of inclusive semileptonic measurements, where only the lepton and neutrino are studied, a significant source of error in the measurements of CKM matrix elements arises from the lack of understanding of the dynamics of the $b$ quark inside the $B$ meson. The non-perturbative interaction of the $b$ quark with the light degrees of freedom of the meson leaves its imprint on the spectra of kinematical variables measured in inclusive semileptonic decay.

Heavy Quark Effective Theory (HQET) combined with QCD implemented in an Operator Product Expansion (OPE) has provided a framework to understand how the $b$ quark interacts with the light degrees of freedom of the meson. The expansion combines QCD with HQET to handle short and long range interactions within the meson. The result is that inclusive observables can be expressed in powers of $m_B^{-1}$, where $m_B$ is $B$ meson mass. This expansion has one free parameter at $\mathcal{O}(m_B^{-1})$, $\bar{\Lambda}$, where

$$\bar{\Lambda} = m_B - m_b|_{m_b \to \infty}$$

and represents the energy of the light cloud. At $\mathcal{O}(m_B^{-2})$, two more parameters, $\lambda_1$ and $\lambda_2$ appear. The parameter $\lambda_1$ is related to the average momentum squared of the $b$ quark in the
rest frame of the meson, and $\lambda_2$ describes the spin-dependent chromo-magnetic interaction of the $b$ quark with the light cloud which can be precisely determined from $B/B^*$ and $D/D^*$ mass splittings. We should note that these two parameters are related to the frequently used parameters $\mu_2^{\pi}$ and $\mu_2^g$ by $\lambda_1 \equiv -\mu_2^{\pi}$ and $\lambda_2 \equiv \mu_2^g/3$.

Expressions for the moments of kinematical variables in inclusive semileptonic $B$ decay at $\mathcal{O}(m_B^2)$ explicitly contain these parameters. Therefore, measuring many such moments constrains the parameters and provides a mechanism to validate the theory.

2 Recoil Hadronic Mass Moments

The hermetic CLEO detector and clean initial state allows one to “reconstruct” neutrinos from the missing energy and momentum. By reconstructing the lepton and neutrino from a semileptonic $B$ decay we measure the full triply-differential decay rate in terms of the kinematical variables $q^2$, $M_X^2$, and $\cos \theta_{Wl}$, which are the mass squared of the virtual $W$, the mass squared of the recoil hadronic system, and the the angle between the lepton in $W$ rest frame and $W$ in the $B$ rest frame (assumed to be the lab frame).

A three dimensional fit is performed to the measured spectrum. Components of the fit consist of Monte Carlo modeled signal $B$ semileptonic decay modes along with estimated background contributions from sources such as fake or secondary leptons. The $M_X^2$ projection of the fit is shown in Figure 1 and the agreement with data is excellent.

The analysis is repeated with a variety of values for the minimum lepton energy. Below we shows results for the first and second moments of the $M^2_X$ distribution for lepton energy cuts of 1.0 and 1.5 GeV. The errors are statistical, experimental systematic, and model dependence uncertainties. In Figure 1 we also show the constraints placed on the HQET parameters $\lambda_1$ and $\bar{\Lambda}$ using the pole mass scheme of Bauer et al.\textsuperscript{2}.

| Moment | $E_l > 1.0$ GeV | $E_l > 1.5$ GeV |
|------------------|-----------------|-----------------|
| $\langle M_X^2 - M_{D*}^2 \rangle$ (GeV$^2$/c$^4$) | 0.456 ± 0.014 ± 0.045 ± 0.109 | 0.293 ± 0.012 ± 0.033 ± 0.048 |
| $\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle$ (GeV$^4$/c$^8$) | 1.266 ± 0.065 ± 0.222 ± 0.631 | 0.629 ± 0.031 ± 0.088 ± 0.113 |
\[
\frac{dN}{dp}(l^\pm e^\mp) = N_l \eta(p) \epsilon(p) \left[ (1 - \chi) \frac{d\mathcal{B}(b \to c \to e)}{dp} + \chi \frac{d\mathcal{B}(b \to c \to e)}{dp} \right] \\
\frac{dN}{dp}(l^\pm e^\pm) = N_l \eta(p) \left[ \chi \frac{d\mathcal{B}(b \to e)}{dp} + (1 - \chi) \frac{d\mathcal{B}(b \to c \to e)}{dp} \right]
\]

where \( \eta(p) \) and \( \epsilon(p) \) are the efficiencies for the signal electron and the cut to remove same-\( B \) secondary leptons. \( N_l \) is the number of tag leptons and \( \chi \) is the \( B \) mixing parameter times the fraction of \( \Upsilon(4S) \) events that produce neutral \( B \) mesons. While not depicted in the equations above, we also allow for the possibility that the secondary spectra differ in charged and neutral \( B \) decays.

Using equations 2 and 3 and the measured like- and unlike-sign spectra we can extract the primary electron spectrum for semileptonic \( B \) decays, shown in Figure 2. The zeroth moment
of the primary spectrum gives a measurement of $B(B \to Xe^+\nu_e)$, as $(10.91 \pm 0.09 \pm 0.24)$%. The first and second moments of the spectrum can be computed with various lepton energy momentum cuts and are shown below. In Figure 2 we also show the constraints placed on the HQET parameters $\lambda_1$ and $\bar{\Lambda}$ by these moments using the pole mass scheme of Bauer et al.\cite{2}.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Moment & $E_l > 0.7$ GeV & $E_l > 1.5$ GeV \\
\hline
$\langle E_l \rangle$ & $1.4509 \pm 0.0035 \pm 0.0079$ & $1.7792 \pm 0.0021 \pm 0.0027$ \\
$\langle E_l^2 \rangle - \langle E_l \rangle^2$ & $0.1374 \pm 0.0015 \pm 0.0018$ & $0.0316 \pm 0.0008 \pm 0.0010$ \\
\hline
\end{tabular}
\end{table}

4 Comparison With HQET Predictions

We can use the data with a 1.5 GeV minimum lepton energy cut along with the CLEO $b \to s\gamma$ measurement\cite{4} to constrain the values of the HQET parameters. Using these values in the Bauer pole mass scheme\cite{2}, we can predict the moments at lower lepton energy cut. Figure 3 shows this prediction along with the measured moments at various lepton energy cuts. A discrepancy emerges in the first moment of the $E_l$ spectrum as the cut is lowered.

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