Current evolution and plasma density space
distribution in the reflex discharge with ring cathodes

A A Samokhin, G D Liziakin, A V Gavrikov, R A Usmanov and
V P Smirnov
Joint Institute for High Temperatures of the Russian Academy of Sciences, Izhorskaya 13
Bldg 2, Moscow 125412, Russia
E-mail: glizyakin@gmail.com

Abstract. In this paper the numerical model of direct current gas discharge in drift-diffusion
approximation is considered. For two-component plasma the processes of the gas discharge
development in the reflex geometry with ring cathodes at a helium pressure of 35 mTorr are
studied. We investigate the influence of: (a) the boundary conditions on the dielectric, (b) the
electron temperature and (c) the coefficient of the secondary ion–electron emission on the
$I-U$ curve of the discharge. In a magnetic field of 50 Gauss the impact of the discharge voltage
$U = 300–700$ V on the evolutionary process of the discharge is examined. The effect of diffusion
on maintaining steady state discharge is researched. The parameters of the existence of a
high-current (tens of $\mu$A) and low voltage (tens of mA) discharge modes are defined.

1. Introduction
In the method of plasma processing of spent nuclear fuel [1,2] radial electrostatic field is created
by electrodes placed at the ends of the cylindrical chamber. Buffer plasma generated by rf
inductor is adjacent to the electrodes. Its potential is distributed in the plasma volume along a
magnetic field lines. In the absence of the buffer plasma the electrodes in combination with a
grounded vacuum chamber form the geometry of the reflex discharge [3]. Following the principle
“from simple to complex”, we examine this system first without the buffer plasma.

By the calculations of the discharge evolution without a magnetic field all obtained solutions
are corresponded to a steady state. Signs of the steady state are an establishment of constant
value of (a) a total current on the electrodes, (b) the maximum and average volume densities of
the particles. By the calculations of the discharge evolution with a magnetic field the discharge
current and the maximum density of the ions are considerably increased, which limits the time
step in the numerical integration procedure of the finite-difference equations system of particles
balance in an electric field. In this regard, in most cases with non-zero magnetic field the
stationary solution is not attained.

Calculations are made in helium for a pressure of $P = 35$ mTorr assuming that the diffusion
coefficient $D_e$ is associated with the mobility $\mu_e$ by the Einstein relation at constant temperature $T_e$. The temperature is corresponded to an isotropic distribution function of electrons in the
velocity space.
2. The equation system in diffusion-drift approximation

Two component plasma of electrons and singly charged ions in a magnetic field was investigated numerically in the framework of the diffusion-drift approximation [4]. It was solved the Poisson’s equation for the potential $\phi$:

$$ -\Delta \phi = 4\pi e(n_i - n_e), $$

where $e$ is the electron charge, $n_i$, $n_e$ concentration of ions and electrons respectively. As well as the system of equations for concentration of electrons $n_e$ and ions $n_i$:

$$\frac{\partial n_e}{\partial t} + \text{div} \vec{\Gamma}_e = \dot{q}_e, \quad \vec{\Gamma}_e = -((\mu_e \vec{E} + D_e \vec{\Delta})n_e),$$

$$\frac{\partial n_i}{\partial t} + \text{div} \vec{\Gamma}_i = \dot{q}_i, \quad \vec{\Gamma}_i = -((\mu_i \vec{E} + D_i \vec{\Delta})n_i),$$

where $\dot{q}_e, \dot{q}_i$ is the rate of electron and ion appearance per unit volume, $\vec{E}$ is the vector of electric field strength, $D_e, D_i$ is the diffusion coefficients of electrons and ions, respectively. In turn, the rate of appearance of charged particles is given by:

$$\dot{q}_e = \dot{q}_i = \alpha v_e n_e,$$

$$\alpha(E/P) = 5.4 P \exp(-14 \sqrt{P/E} - 0.0017E/P),$$

where $v_e$ is the electron velocity, $\alpha$ is the first Townsend coefficient [5], $P$ is the gas pressure. The mobility $\mu_e, \mu_i$ writes by collision frequency $\nu_{ea}, \nu_{ia}$ and the Hall’s parameter $\chi_e, \chi_i$:

$$\mu_e = \frac{e}{m_e \nu_{ea}} M(\chi_e), \quad \mu_i = \frac{e}{A m_i \nu_{ia}} M(\chi_i),$$

$$M(\chi) = \begin{pmatrix}
\frac{1}{\chi + 1} & \frac{\chi}{\chi + 1} & 0 \\
\frac{\chi}{\chi + 1} & \frac{1}{\chi + 1} & 0 \\
0 & 0 & 1
\end{pmatrix},$$

$$\chi_e = \frac{\omega_{Be}}{\nu_{ea}}, \quad \omega_{Be}[1/c] = 1.7 \times 10^7 \text{H[G]},$$

$$\chi_i = \frac{\omega_{Bi}}{\nu_{ia}}, \quad \omega_{Bi}[1/c] = 0.96 \times 10^4 A^{-1} \text{H[G]}.$$  

Here $A = 4$ is the atomic mass of the helium atoms.

3. Computational domain and boundary conditions

Computational domain of the cylindrical chamber ($r < R = 45$ cm, $0 < z < L = 200$ cm) is limited by conductive cylinder 1 (anode) in $r$ direction and dielectric wall 2 with ring $(8 < R < 12$ cm) flat electrodes 3 (cathodes) in $Z$ direction figure 1.

On the chamber axis in the central plane ($z_c = 0$ cm) the symmetry condition is:

$$(\partial n_k/\partial z)|_{r=0}, \quad (\partial n_k/\partial r)|_{z_c = 0}, \quad k = e, i,$$

where $n_k[\text{cm}^{-3}]$ is the density of ions ($i$) and electrons ($e$).

On the cathode surface condition of ion–electron emission is:

$$\Gamma_e = \gamma \Gamma_i, \quad \Gamma_k = \mu_k E_z n_k - D_k \Delta_z n_k, \quad k = e, i,$$

where $\gamma$ is the coefficient of ion–electron emission, $D_k$ is the diffusion coefficient, $\mu_k$ is mobility, $E_z$ is electric field component directed along the axis $Z$. On the dielectric surface of the chamber
end \((z = 100 \text{ cm})\) from the condition of the potential symmetry and lack of stream of particles on the surface follow the symmetry condition on the concentration of particles:

\[
(\partial n_k / \partial z)|_{z=100} = 0, \quad k = e, i. \tag{11}
\]

At the anode the flux density of ions from the surface equals to 0. Neglecting the reflected particles, we impose the condition on electron density:

\[
(\partial n_e / \partial r)|_{r=R} = 0. \tag{12}
\]

In initial time the entire space is filled with a plasma with density \(n_i = n_e = 5 \times 10^4 \text{ cm}^{-3}\).

4. Discharge current–voltage characteristic without a magnetic field

Before discussing the results of the calculations note that the experimentally current–voltage \((I–U)\) characteristic has been obtained of the glow discharge in the configuration that matches the configuration of the computational domain (figure 1) [6]. The cathodes were made of stainless steel, the dielectric ends were made of fluoroplastic. The results are presented in figure 2, curve number 5 (blue dots).

Consider how (a) the boundary conditions on the dielectric, (b) the coefficient of the secondary ion–electron emission \(\gamma\) and (c) the diffusion coefficient \(D_e, D_i\) influence on \(I–U\) characteristics of the discharge.

From figure 2 it is seen that the change in the boundary conditions at the cathodes with \(n_e/n_i = \gamma \mu_e/\mu_i\) on \(j_e = \gamma j_i\) does not lead to any noticeable change of the discharge current (figure 2 (1)). The condition change on the dielectric surface (accounting for leakage on the surface) leads to a decrease of the derivative \(dI/dU\) of 1.2 times (figure 2 (2)). Without the diffusion of electrons \((T_e = 0)\), the discharge current is decreased 2 times (figure 2 (3)). The discharge parameters strongly depends on emission coefficient value—the change in \(\gamma\) of 2.5 times from 1 to 0.4 leads to a decrease of the current on the order (figure 2 (4)). When the value of \(\gamma = 0.2\) the current drops to \(\sim 0.1 \text{ mA}\).

The influence of the electron diffusion coefficient and coefficient \(\gamma\) on the plasma distribution is presented in figure 3. From figure 3 it can be seen that both parameters affect the discharge current and the spatial distribution of particles. However, the influence of the diffusion coefficient on the spatial distribution pronounced more clearly. So when \(T_e = 0\) the region
Figure 2. The discharge current–voltage characteristics: (1) \( \left( \frac{\partial n_k}{\partial z} \right)_{z=100} = 0, k = e, i, \gamma = 1, T_e = 4 \text{ eV}; \) (2) \( n_k|_{z=100} = 0; \) (3) \( T_e = D_e = 0; \) (4) \( \gamma = 0.4; \) (5) experimental data.

Figure 3. Spatial distribution of ion and electron density: \( H = 0, U = 800 \text{ V}, \gamma = 1, T_e = 4 \text{ eV}, \) \( I = 6.9 \text{ mA} \) (a, d), \( \gamma = 0.4, T_e = 4 \text{ eV}, I = 0.82 \text{ mA} \) (b, e), \( \gamma = 1, T_e = 0, I = 4.1 \text{ mA} \) (c, f). Maximum density: (a) \( n_e = 0.93 \times 10^6 \text{ cm}^{-3}; \) (b) \( n_e = 0.35 \times 10^5 \text{ cm}^{-3}; \) (c) \( n_e = 3.6 \times 10^7 \text{ cm}^{-3}; \) (d) \( n_i = 3.0 \times 10^6 \text{ cm}^{-3}; \) (e) \( n_i = 4.8 \times 10^5 \text{ cm}^{-3}; \) (f) \( n_i = 3.7 \times 10^7 \text{ cm}^{-3}. \)

with \( n_e \geq 0.3 \) \( n_{e,\text{max}} \) is an order of magnitude less than in case when \( T_e = 4 \text{ eV}. \) At the same time the effect of the diffusion coefficient on the discharge current is significantly less than the impact of \( \gamma. \) In the presented example, by lowering the electron temperature of 4 eV to 0, the discharge current decreased only 2 times. Note that for \( T_e = 4 \text{ eV}, \) the electron density and the
The influence of the discharge voltage and the boundary conditions at the cathode on the evolution of the current and the plasma density at $H = 50$ Gauss, $T_e = 4$ eV; (a), (b) $U = 600$ V, $\gamma = 1$, $j_e = \gamma j_i$; (c) discharge current at $U = 300$ V; (1) $\gamma = 1$, $j_e = \gamma j_i$; (2) $\gamma = 1$, $n_e = \gamma n_i \mu_i / \mu_e$; (3) $\gamma = 0.02$, $n_e = \gamma n_i \mu_i / \mu_e$.

5. The discharge current evolution in a magnetic field of 50 Gauss

The magnetic field affects both the possibility of breakdown and the current value. When the value of $H = 50$ Gauss, the Hall’s parameter are $\chi_e = 9.7$ for electrons and $\chi_i = 0.08$ for ions. When the coefficient $\gamma = 1$, the electron temperature of 4 eV and a voltage of 600 V in 150 $\mu$s the current reaches 75 mA. With the continuing growth of the maximum and average ion concentration occurs at an increasing rate (figure 4). At a voltage of 300 V the current grows slower in 10 times and with increasing speed during 400 $\mu$s (figure 4, curve 1). The change in boundary conditions at the cathode with $j_e = \gamma j_i$ on $n_e = \gamma n_i \mu_i / \mu_e$ decreases the growth rate of current (figure 4, curve 2), but it does not allow to achieve a steady state. After 0.42 ms, we have reduced the coefficient $\gamma$ to a value of 0.02, but it also does not allow to achieve a steady state.

With the same initial conditions at $\gamma = 0.02$ it is observed steady state with current of $\sim 10$ $\mu$A (low current level) and non-steady solutions with current of $\sim 10$ mA (high-current level) depending on voltage (figure 5). The transition boundary from low current to high current takes place in the region of 500 - 550 V. The pronounced maximum in the figure 5b shows the competition between the processes of death and reproduction of particles. The decrease of the diffusion coefficient by reducing the temperature from 3 to 1 eV leads to discharge extinction (figure 6). In this case $T_e = 4$ eV is a critical value that define the transition into high current mode.

Low-current discharge is characterized by negative space charge in the chamber–the total number of electrons are greater than the number of ions in 2.7 times (figure 7). The figure shows that near the cathode the ion layer is formed with thickness of $\sim 30$ cm. In addition, there is the potential drop of the order of magnitude of the applied voltage (figure 8a) near the cathode. The potential drop in the radial direction is $\sim 20...40$ V (figure 8b).

High-current discharge is characterized by the formation a quasi-neutral plasma with the relatively narrow 8 cm cathode potential drop $\sim 400$ V (figure 9). Figures 9 and 10 shows the spatial distribution of such discharge characteristics as the electric field strength, the concentration of ions and electrons, and ionization rate. An important property of the high-current discharge is to form the anode potential drop $\sim 200$ V with wide of $\sim 20$ cm (figure 10). The Increasing of the ionization near the anode at the anode drop reducing and the growth of the field strength leads to an increasing of discharge current.
Figure 5. The influence of the discharge voltage on the current evolution: $T_e = 4 \text{ eV}$, $U[kV] = 0.3$ (1), 0.4 (2), 0.5 (3), low-current mode (a); $U[kV] = 0.55$ (1), 0.6 (2), 0.7 (3), high-current mode (b).

Figure 6. The influence of the diffusion coefficient on (a) discharge current and (b) plasma density: $U = 700 \text{ V}$, $T_e[\text{eV}] = 1$ (1), 2 (2), 3 (3), 4 (4).

Figure 7. Spatial distribution of ion and electron density along the Z-axis in the radius of 10 cm (a) and in the radial direction at a distance of 50 cm from the end (b) voltage $U[\text{V}] = 300$ (1), 400 (2), 500 (3), $T_e = 4 \text{ eV}$.
Figure 8. The potential along the Z-axis at a radius of 10 cm (a) and in the radial direction at a distance of 50 cm from the end (b), $T_e = 4$ eV.

Figure 9. The electric field strength (1), the density of ions (2) and electrons (3) and the ionization rate (4) along the Z axis at a distance of 10 cm from the axis of the chamber for the voltage of 500 V, the current of 34 $\mu$A (a) and 700 V, the current of 12 mA (b).

Figure 10. The electric field strength (1), the density of ions (2) and electrons (3) and the frequency of ionization (4) in the radial direction at a distance of 50 cm from the end for the voltage 500 V, the current of 34 $\mu$A (a) and 700 V, the current of 12 mA (b).
6. Conclusion
We have studied the model of the gas discharge in the drift-diffusion approximation. It has been shown that taking into consideration leakage on dielectric end surface leads to derivative increase \( dI/dU \) in 1.2 times. It has been demonstrated that the influence of the electron diffusion on the discharge current is limited by a factor of order unity. At the same time, the variation of the secondary ion–electron emission \( \gamma \) from 1 to 0.4 leads to a decrease of the current on the order. We have found a high current (tens of mA) and low current (tens of \( \mu \)A) discharge modes in a magnetic field of 50 Gauss. Transition to high current mode occurs when the voltage across the discharge gap more than 550 V. The effect of electron temperature on maintaining the discharge in a magnetic field has been demonstrated. It has been shown that the temperature of 4 eV is the minimum acceptable value for the existence of a high-current mode. The spatial distribution of the electric field strength, the electron and ion density, the ionization rate in the high and low current modes has been investigated. It has been noted that in the low-current mode the formation of the cathode potential drop with wide of \( \sim 30 \) cm and anode potential drop in the high-current mode \( \sim 200 \) V with wide of \( \sim 20 \) cm took place.

Acknowledgments
The study was supported by the Russian Science Foundation (project No. 14-29-00231).

References
[1] Smirnov V P, Samokhin A A, Vorona N A and Gavrikov A V 2013 Plasma Phys. Rep. 39 456–466
[2] Vorona N, Gavrikov A, Samokhin A, Smirnov V and Khomyakov Yu 2015 Phys. At. Nucl. 78 1624–1630
[3] Hooper Jr E B 1970 A Review of Reflex and Penning Discharges vol 27
[4] Mitchner M and Kruger C 1973 Partially ionized gases (Wiley)
[5] Tkachev A and Yakovlenko S 2003 Tech. Phys. Lett. 29 683
[6] Liziakin G and Usmanov R 2015 Phys. Procedia 71 138