GEOMETRIZATION OF VACUUM CONDENSATE EFFECTS IN QUARKONIUM
POTENTIAL MODEL

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It is suggested the modification of traditional potential model, in which nontrivial structure
of inside-hadron vacuum condensate is simulated by geometric properties of inside-hadron space.
Confinement of quarks is ensured by closed effective (Riemannian) space. Interquark potential
represents itself Coulomb’s low in the effective space (Poisson equation). In the framework of such
approach the quarks dynamics completely is defined only by metric of the effective space, which in
turn in conformally-Euclidean case (we consider) is defined by sole phenomenological function. Also
it is supposed, that inside-hadron vacuum is essential nonperturbative one both on large and on
small distances and this is taken into account by special parameter in the metric of effective space.
For final Schrödinger equation the exact analytical solutions for nonrelativistic energy spectrum
and wave functions of quarkonium are obtained. From the basic principles of geometrized potential
model and under the perturbation theory the spin-dependent relativistic corrections are calculated.
Charmonium and bottomonium spectra are simulated. Suggested model gives a very good fit to
experimental data: accuracy of spectra fitting makes $4.06 \times 10^{-2}$ for charmonium and $3.28 \times 10^{-2}$
for bottomonium and many predictions are made. On the basis of charmonium and bottomonium
analysis conclusions about role of vacuum condensate in hadron structure are done.

I. INTRODUCTION.

Quarkonia are bound states of heavy quarks and antiquarks $c\bar{c}, b\bar{b}, t\bar{t}$ form the special class of objects in the
hadrons physics. Experimentally observable quarkonia properties are used for clearing up of static and dynamic
quark properties. The quarkonia spectroscopy is studied in detail experimentally and gives in to effective theoretical
modeling. The nonrelativistic character of heavy quarks moving within quarkonia allows to apply for calculation of
their spectra nonrelativistic Schrödinger equation, in which effective potential of quarks interaction
$U(r)$ and quarks reduced mass $\mu(r)$ as functions of coordinates are selected phenomenologically. Criterion of functions
$U(r)$ and $\mu(r)$ choice is a good fit of potential model outcomes with experimental data. Smallness of relativistic effects allows to
specialize nonrelativistic potential models by inserting to Hamiltonian effects of spin-orbit and spin-spin interactions
as small corrections computed under perturbations theory. Modern condition of potential models is reflected in the
review paper and for more detailed information see lectures.

Central object in realization of the nonrelativistic potential program is the potential of interquark interaction $U(r)$,
which should take into account effect of absolute quarks confinement within hadron. Its choice is ambiguous in view
of unsufficient methods of the interquark interactions analysis on large distances. The potential is selected usually
from some physical reasons in combination with the aesthetic requirements of simplicity. The existing theoretical
indications concerning the form of potential most explicitly are reflected in so-called Cornell potential

$$U(r) = -\frac{g^2}{r} + \sigma r,$$

(1)

where the first term is motivated by perturbative QCD with $r \rightarrow 0$ with an accuracy to logarithmic corrections, taking
into account the perturbative vacuum polarization effects; second term is predicted by nonperturbative lattice QCD
with $r \rightarrow \infty$. Majority of used potentials differ among themselves by character of interpolation on intermediate
distances, between two limiting conditions reflected in the formula (1).

Concerning of reduced mass, as function of coordinates, only its asymptotic on small distances, found in perturbative
QCD, is known. For potential model, however the dependence of reduced masses from coordinates on average and
large distances is necessary, which unfortunately theoretically is not found. For this reason the reduced mass is
simulated, as a rule, by constant parameter, which value is determined from the requirement of best agreement of
theoretical outcomes with experiment.

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The use of the above-stated procedures allows in whole to achieve a good quantitative agreement of the theory with experiment, however, a problem of physical completeness and justification of the potential approach remains open. In the framework of the fundamental QCD hadron is an area of reconstructed vacuum – is a cavity inside nonperturbative quark-gluon condensate stabilized by valent quarks, residing within it. Moreover, the inside-hadron reconstructed vacuum exert influence on quarks dynamics, exactly on reduced mass and interquark potential. Thus, quarkonium is complicated self-consistent object, in which quarks condition depends from condition of reconstructed nonperturbative vacuum and condition of vacuum inside quarkonium depends from quarks condition. In this situation it is worth to suggest the new phenomenological approaches in the theory of quarkonia. The more general potential models, at first should contain additional gang of functional and numerical parameters in the interquark potential, supposing the interpretation in the terms of nonperturbative vacuum physics; secondly, in these models the problem of reduced mass of quarks within nonperturbative inside-hadron vacuum and problem of interquark potential should be considered from uniform positions.

In present work one of the variants of new potential models, possessing by the above-stated properties, is suggested. Our approach is based on the consideration of inside-hadron space as manifold with the closed topology. The quarks reduced mass and the interquark potential completely are defined by geometric properties of the effective Riemannian space; metric of this space represents itself as universal phenomenological functional parameter. We also assume, that vacuum inside quarkonium is essential nonperturbative one both on large and on small distances and take this into account by specially selected parameter in the metric of effective space, which then occurs in quarks reduced mass and interquark potential functional dependencies from coordinates of real (Euclidean) space.

The paper is constructed as follows. In section II the basic notions about vacuum element in hadron are stated; in section III the geometrization of potential model is carried out and its mathematical structure is stated; in section IV the generalized Cornell potential, parametric taking into account nonperturbative structure of vacuum on all distances is introduced; here the exact analytical solutions for nonrelativistic Schrödinger equation are obtained; in section V the relativistic effects are considered; section VI is devoted to the results and conclusions.

II. THE VACUUM ELEMENT IN THE HADRON MASS.

In the framework of the qualitative script of the hadron formation the reconstruction of vacuum condensate, described quite determined energy, leading to vacuum constituting element in hadron mass, as a necessary condition is contained. This vacuum constituting element reads

\[ E_{\text{vac}} = \int \varepsilon_{\text{vac}}(\vec{r}) dV \]

where

\[ \varepsilon_{\text{vac}}(\vec{r}) = \varepsilon_{\text{in}}(\vec{r}) - \varepsilon_{\text{out}} > 0, \]

\( \varepsilon_{\text{out}} \) is the energy density of outside-hadron vacuum condensate (nonperturbative quark-gluon condensate), \( \varepsilon_{\text{in}}(\vec{r}) \) is the energy density of inside-hadron (reconstructed) vacuum condensate, here the coordinate dependence takes into account the spatial heterogeneity of condensate. The mass of hadron is represented as follows:

\[ M = E_{\text{vac}} + E_q, \]  (2)

where \( E_q \) is a sum of rest energy, kinetic energy and energy of quarks interactions with each other and with vacuum condensate. The birth threshold of hadron agrees (4) is defined not only by quark masses, but also energy \( E_{\text{vac}} \), expended to quite defined reconstruction, after which inside a new vacuum the processes of quark excitations births with masses \( m_q \) become possible.

III. THE MODEL.

Let us to start from quarkonium Lagrangian, which in the framework of potential approach reads

\[ L = \frac{1}{2} \mu(\vec{r}) \left( \frac{dE}{dt} \right)^2 - U(\vec{r}), \quad dE^2 = dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \]  (3)

where \( \mu(\vec{r}) \) is the quarks reduced mass and \( U(\vec{r}) \) is the quark-antiquark potential.
Having limited by a class of isotropic phenomenological functions $\mu_r = m f^2(r)/2, U(r)$, let’s rewrite Lagrangian \( L \) as

$$ L = \frac{m}{4} \left( \frac{dl}{dt} \right)^2 - U(r), \quad (4) $$

\( dl^2 = f^2(r)(dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)) \equiv \gamma_{\alpha\beta} dx^\alpha dx^\beta. \quad (5) $$

The first term in \( L \) formal coincides with kinetic energy of particle with mass \( m/2 = const \), moving in curved Riemannian space with metric \( (5) \). It means the condensate influence to the quark inert properties can be mathematically taken into account by transition from real Euclidean space to effective Riemannian space. Function \( U(r) \), appearing in \( (4) \), within the framework of accepted hypothesis is identified with Coulomb’s law in Riemannian space with metric \( (5) \):

$$ \frac{1}{\sqrt{\gamma}} \partial_x \gamma \partial_x \equiv \frac{1}{r^2 f^3(r)} \frac{d}{dr} r^2 f(r) \frac{dU}{dr} = g^2 \delta(r). \quad (6) $$

From the equation \( (6) \) follows

$$ \mu_r = \frac{m}{2} \left( \frac{r^2}{g^2} \frac{dU}{dr} \right)^{-2}, \quad U(r) = g^2 \int_{r_0}^r \frac{dr}{r^2 f(r)}. \quad (7) $$

The effective potential in \( (7) \) is defined with an accuracy to additive constant, which value is parameterized by a limit inferior in quadrature. Such model’s ambiguity, however, is insignificant, since this additive constant is swallowed by vacuum energy \( (2) \), assigning count beginning of energy spectrum.

For geometrized potential model the hypothesis about additional quark-condensate interaction, which energy proportional to curvature of the effective Riemannian space is mathematically natural. The insertion this interaction to model lead up to Lagrangian

$$ L = \frac{m}{4} \gamma_{\alpha\beta} \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} - U(r) + \kappa R(r), \quad (8) $$

where \( \kappa \) - phenomenological quark-condensate coupling constant;

$$ R(r) = 2 \left[ \frac{1}{f^4} \left( \frac{df}{dr} \right)^2 - \frac{2}{f^3} \frac{d^2 f}{dr^2} - \frac{4}{rf^3} \frac{df}{dr} \right] \quad (9) $$

is scalar curvature, evaluated in terms of coordinates \( (3) \).

For Lagrangian \( (8) \) the corresponding Schrödinger equation is

$$ \left( -\frac{1}{m} \gamma^{\alpha\beta} \nabla_\alpha \nabla_\beta + U(r) - \kappa R(r) \right) \Psi = E \Psi \quad (10) $$

where \( \nabla_\alpha \) is covariant derivation operator in space with metric \( (3) \). After variable separation

$$ H \Psi_{nLM} = E_{nL} \Psi_{nLM}, \quad \Psi_{nLM}(r, \theta, \phi) = R_{nL}(r)Y_{LM}(\theta, \phi), \quad (11) $$

equation \( (10) \) is reduced to equation for the radial part of wave function

$$ - \frac{1}{m} \frac{1}{r^2 f^3} \frac{d}{dr} r^2 f \frac{dR_{nL}}{dr} + \left[ \frac{1}{m} \frac{L(L+1)}{r^2 f^2} + g^2 \int_{r_0}^r \frac{dr}{r^2 f} - 2\kappa \left( \frac{1}{f^4} \left( \frac{df}{dr} \right)^2 - \frac{2}{f^3} \frac{d^2 f}{dr^2} - \frac{4}{rf^2} \frac{df}{dr} \right) \right] R_{nL} = E_{nL} R_{nL} \quad (12) $$

with invariant normalization

$$ \int_0^\infty \left| \Psi_{nLM} \right|^2 \sqrt{\gamma} d^3x = \int_0^\infty R_{nL}^2(r) f^3(r) r^2 dr = 1. \quad (13) $$
From the viewpoint of physical representations, explained above, the choice of non-Euclidean measure of integration in (13) is finishing mathematical operation under the account of condensate influence on spatial localization of quarkonium state.

The radial coordinate \( r \), used in (4) – (13), is common both for the real Euclidean space and for the effective Riemannian space. Therefore, arguing of problem about radial dependence of quark-antiquark potential is carried out in terms of this coordinate. However, in mathematical research of the equation (12) it is more convenient to use dimensionless coordinate \( \rho \) and phenomenological function \( \chi(\rho) \):

\[
\rho = \frac{1}{a} \int_{r_0}^{r} f(r)dr, \quad \chi(\rho) = \frac{1}{a^2} \int_{r_0}^{r} f(r) dr, \quad \rho_{max} = \rho(\infty)
\]  

The metric (5), curvature (9) and Coulomb’s law (7) become:

\[
dl^2 = \frac{a^2}{\rho^2} \left[ d\rho^2 + \chi^2(\rho) \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right] = \gamma_{\alpha\beta} dx^\alpha dx^\beta \\
R(\rho) = 2 \frac{g^2}{a^2} \left[ \frac{1}{\chi^2} - 2 \frac{\chi''}{\chi} - \left( \frac{\chi'}{\chi} \right)^2 \right], \quad U(\rho) = \frac{g^2}{\rho} \int_{\rho_0}^{\rho} \frac{d\rho}{\chi^2(\rho)}
\]  

In (13) and in the further primes derivative are designated on \( \rho \). The physical and the geometrical meaning of dimensional parameter \( a \) become clear after giving representations for potential, satisfying to condition of quarks confinement within hadron.

Further, it is convenient to turn to new radial function with simple normalization:

\[
R_{nL} = \frac{1}{a^{3/2} \chi(\rho)} \phi_{nL}(\rho), \quad \int_0^{\rho_{max}} \phi_{nL}^2(\rho) d\rho = 1
\]

and to dimensionless quantities:

\[
\varepsilon_{nL} = \frac{ma^2}{\hbar^2} E_{nL}, \quad 2\gamma = \frac{g^2 ma}{\hbar^2}, \quad \lambda = \frac{2k m}{\hbar^2}.
\]

Now equation (12) corresponds as follows:

\[
\phi''_{nL} + \left[ \varepsilon_{nL} - 2\gamma \int_{\rho_0}^{\rho} \frac{d\rho}{\chi^2} \frac{L(L+1)}{\chi^2} - \lambda \left( \frac{1}{\chi^2} - 2 \frac{\chi''}{\chi} - \left( \frac{\chi'}{\chi} \right)^2 \right) \right] \phi_{nL} = 0
\]

IV. THE GENERALIZED CORNELL POTENTIAL AND EXACT SOLUTIONS OF THE SCHRÖDINGER EQUATION.

In quarkonium geometrized potential model the radial dependence of the effective potential is set simultaneously with the metric and the curvature of effective Riemannian space by sole phenomenological function:

\[
f(r) = \left( \frac{r^2 dU}{g^2 dr} \right)^{-1}.
\]

Among of potentials satisfying to condition of quarks confinement within hadron the potentials of whirlpool form stands out of the simplicity:

\[
U(r) = \frac{g^2}{2a} \left[ - \left( \frac{2a}{r} \right)^{1/k} + \left( \frac{r}{2a} \right)^{1/k} \right],
\]

Where \( 2a \) is characteristic quarkonium size ("diameter"); \( k \) is positively defined numerical parameter. With \( k = 1 \) expression (20) passes to Cornell potential (4), joining known asymptotics. Euclidean-Coulomb asymptotic of Cornell
potential with \( r \to 0 \) corresponds to supposition that properties of inside-hadron vacuum in a neighborhood of point \( r = 0 \) limiting are closed to properties of perturbative vacuum. It is possible to expect, with \( k \neq 1 \) generalized Cornell potential (20) simulates situation with nonperturbative inside-hadron vacuum condensate. For potential (20) the evaluations under the formulas (19), (14), (15) give:

\[
f(r) = \frac{k}{\left(\left(\frac{2a}{r}\right)^{k-1} + \left(\frac{r}{2a}\right)^{k+1}\right)}, \quad r = 2a \tan^k \frac{\rho}{2k^2}, \quad 0 \leq \rho < \pi k^2
\]

\[
\chi(\rho) = k \sin \frac{\rho}{k^2}, \quad U(\rho) = -\frac{g^2}{a} \cot \frac{\rho}{k^2}, \quad R(\rho) = \frac{2}{k^4a^2} \left(3 - \frac{1 - k^2}{\sin^2 \frac{\rho}{k^2}}\right).
\] (21)

The equation (18) and normalization (16) become:

\[
\frac{d^2 \phi_{nL}}{d\eta^2} + \left(k^4 \varepsilon_{nL} + 3\lambda + 1 + 2\gamma k^4 \cot \eta - \frac{1}{\sin^2 \eta} \left[k^2 L(L+1) + \nu(\nu+1)\right]\right) \phi_{nL} = 0,
\] (22)

\[
k^2 \int_0^\pi \phi_{nL}^2(\eta) d\eta = 1,
\] (23)

where

\[
\eta = \rho/k^2, \quad \nu(\nu+1) = \lambda(1-k^2).
\]

The solutions of the equation (22) satisfy to condition of absolute quarks confinement within hadron, if:

\[
\lambda(1-k^2)\nu = -\frac{1}{2} + \sqrt{\frac{1}{4} + \lambda(1-k^2)} \geq 0.
\] (24)

The inequality (24) is the sole restriction on the phenomenological parameters of geometrized model appropriating to generalized Cornell potential (20). With \( k = 1 \) the effective geometry, as it is visible from (21), represents a closed homogeneous and isotropic space with constant positive curvature \( R = 6/a^2 \). The equation (22) in this case describes hydrogen-like system in such space:

\[
\phi''_{nL} + \left[\varepsilon_{nL} + 3\lambda + 1 + 2\gamma k^4 \cot \rho - \frac{L(L+1)}{\sin^2 \rho}\right] \phi_{nL} = 0.
\] (25)

From symmetry reasons is beforehand obvious, that spectrum of such system depends only from the sum of radial and orbital numbers \( N = n + L \), i.e. degenerates on orbital number.

With \( k \neq 1 \) the degeneracy is removed, but mathematical structure of the equation for wave function on comparison with (25) practically does not vary. Really, after insertion variable \( \eta \) and intermediate auxiliary parameter \( l \):

\[
l(l+1) = \nu(\nu+1) + k^2 L(L+1), \quad l = -\frac{1}{2} + \sqrt{\left(\nu + \frac{1}{2}\right)^2 + k^2 L(L+1)},
\]

the equation (22) corresponds as follows

\[
\frac{d^2 \phi_{nL}}{d\eta^2} + \left[k^4 \varepsilon_{nL} + 3\lambda + 1 + a\gamma k^4 \cot \eta - \frac{l(l+1)}{\sin^2 \eta}\right] \phi_{nL} = 0,
\] (26)

and differs from (25) only by overdetermination of dimensionless quantities \( \gamma \to \gamma k^4, \varepsilon_{nL} \to \varepsilon_{nL} k^4, \quad L \to l \). The mathematical uniformity of the equations (25) and (26) does not cancel, however, their physical distinctions: in (24) \( l \) is not orbital quantum number, accepting only integer value. In model with potential (20) and with additional quark-condensate interaction, proportional to curvature, with \( k \neq 1 \), parameter \( l \) with realization of the condition (24) can accept any positive values even for states with real orbital moment \( L = 0 \). The solutions of the equation (26) are energy spectrum:
\[ \varepsilon_{nL} = -\frac{1}{k^4}(1 + 3\lambda) + \frac{1}{k^4}(n + l)^2 - \frac{\gamma^2 k^4}{(n + l)^2}, \]  
and exact eigenfunctions

\[ \phi_{nL} = \left[ \prod_{s=1}^{n} \left( -\frac{\partial}{\partial \eta} + \frac{(l + s) \cos \eta}{\sin \eta} - \frac{\gamma k^4}{l + s} \right) \right] \phi_{nL}. \]  

In (28) line-ups of operators is ordered in such a way, that operators, corresponding to smaller values \( s \), stand to the left of operators, corresponding to large \( s \). The auxiliary functions \( \phi_{nL} \) satisfy to equations

\[ \left(-\frac{\partial}{\partial \eta} - \frac{(l + n) \cos \eta}{\sin \eta} + \frac{\gamma k^4}{l + n}\right) \phi_{nL} = 0, \]

having exact solutions,

\[ \phi_{nL} = C_{nL} \exp(-\frac{\gamma n}{l + n}) \sin^{n+l} \eta, \]

where \( C_{nL} \) - normalizing constants, computed numerically from normalization (23).

Turning to dimensional quantities under the formulas (17) and choosing additive constant in the correspondence with (2), we shall receive from (27) the expression for nonrelativistic quarkonium spectrum:

\[ M_{nL} = m_0 + m_1 \left( n - \frac{1}{2} + \sqrt{\left( \nu + \frac{1}{2} \right)^2 + k^2 L(L + 1)} \right)^2 - \frac{m_2}{\left( n - \frac{1}{2} + \sqrt{\left( \nu + \frac{1}{2} \right)^2 + k^2 L(L + 1)} \right)^2}, \]

where parameters are defined as follows:

\[ m_0 = 2m + 2\pi^2 a^3 k^4 \varepsilon_0, \quad m_1 = -\frac{1}{m a^2 k^4}, \quad m_2 = \frac{m}{4} \gamma^2 k^4. \]

Energy, expended to reconstruction of nonperturbative vacuum condensate inside quarkonium, here is presented as \( E_{vac} = 2\pi^2 a^3 k^4 \varepsilon_0 \), where \( V = 2\pi^2 a^3 k^4 \) is quarkonium volume, calculated under the metric of effective Riemannian space; \( \varepsilon_0 \) - parameter quantitatively describing vacuum reconstruction.

\[ V. \text{ FINE AND HYPERFINE SPLITTING.} \]

For the analysis of the relativistic effects it is necessary beforehand to agree about interpretation of nonrelativistic levels (23). There are two possible interpretation. According to the first of them the eigenvalues of the nonrelativistic Hamiltonian are centers of gravity of the spin triplets i.e. ortho\((S = 1)\)quarkonium levels, according to the second the nonrelativistic Hamiltonian gives the centers of gravity of the full spin multiplets. For the states with \( L \neq 0 \), this interpretations are equivalent and the difference occurs only for the \( S \)-states. In the framework of potential ideology there is not justification for one interpretation rather than the other. From the practical reasons that the masses of the spin singlets for bottomonium are not known we interpret nonrelativistic levels as the centers of gravity of orthoquarkonium.

In present work we shall be limited by consideration only spin-dependent interactions – \( H_{SD} \). Following to standard procedure, based on Breit-Fermi Hamiltonian \( \frac{3}{2} \frac{i}{m} \varepsilon_{\alpha \beta \gamma}(\sigma^\alpha_Q + s^\alpha_Q \gamma) (\nabla^\beta U) (\nabla^\gamma) - \frac{1}{m^2} (s^\beta_Q \nabla \alpha) (\sigma^\beta_Q \nabla \beta) U + \frac{1}{m^2} [\nabla^\alpha \nabla \beta U] (\gamma_{\alpha \beta} s^\alpha_Q s^\beta_Q) \),

where \( \nabla \alpha \) is the covariant derivation operator; \( s^\alpha_Q = \frac{1}{2} \sigma^\alpha_Q \) is (anti)quark spin operator. The matrixes \( \sigma^\alpha_Q \) are defined in the effective Riemannian space by condition

\[ \sigma^\alpha_Q \sigma^\beta_Q + \sigma^\beta_Q \sigma^\alpha_Q = 2 \gamma^{\alpha \beta}, \]
\[ \nabla_\beta \sigma^\alpha_Q = 0 \] (32)

Representing the metric (33) as follows
\[ dt^2 = f^2(x, y, z) \left[ dx^2 + dy^2 + dz^2 \right] = \gamma_{\alpha\beta} dx^\alpha dx^\beta \]
and further taking into account the spherical symmetry of potential and ratio (39) Hamiltonian (40) is reduced to
\[ H_{SD} = H_{LS} + H_T + H_{hyp}, \]
where
\[ H_{LS} = \frac{1}{2m^2} \left( \frac{3g^2}{r^3} \right) L_\alpha S^\alpha, \]
\[ H_T = \frac{1}{2m^2} \left( \frac{3g^2}{r^3} \right) \left[ 1 + \frac{1}{3} \frac{df}{dr} \right] \left\{ \frac{S^\beta x_\beta (S^\beta x_\beta)}{r^2} - \frac{1}{3} S^\alpha S_\alpha \right\}, \]
\[ H_{hyp} = \frac{2}{m^2} \left[ \nabla_\beta \nabla_\alpha \right] \left( \frac{S^\alpha S_\alpha}{2} - \frac{3}{4} - \frac{1}{4} \right) = \frac{2g^2}{3m^2} \delta(r^2) \left( \frac{S^\alpha S_\alpha}{2} - 1 \right), \]
here \( S^\alpha = s^\alpha_Q + s^\alpha_Q \) and \( H_{LS}, H_T, H_{hyp} \) are accordingly the spin-orbit and the tensor interactions, responsible for the fine structure of orthoquarkonium levels, \( H_{hyp} \) is the hyperfine interaction, splitting the states ortho and para\((S = 0)\)quarkonium. The number \(-\frac{1}{4}\) in (36) corresponds to our assumption that the nonrelativistic Hamiltonian gives the orthoquarkonium levels.

The numerical values for the fine and hyperfine splitting of nonrelativistic levels are found by calculation of matrix elements for the operators \( H_{LS}, H_T, H_{hyp} \) on wave functions of corresponding states and by virtue of factorization (34) the determination of the matrix elements of spin and spatial operators can be carried out separately.

Let us consider at first the fine splitting. The matrix elements for the spin operators in (34) and (35) are
\[ K_{JL}^{(LS)} = \langle J, L, 1 | LS | J, L, 1 \rangle = \frac{1}{2} \left[ J^2 - L^2 - 2 \right] \]
\[ K_{JL}^{(T)} = \langle J, L, 1 | \left( n_\alpha n_\beta - \frac{1}{3} \gamma_{\alpha\beta} \right) S^\alpha S^\beta | J, L, 1 \rangle = -\frac{1}{3} \left[ \frac{6 (LS)^2 + 3 (LS) - 4L^2}{4L^2 - 3} \right] \]
where \( n_\alpha = x_\alpha / r \), \( J = L + S \). After transition in (34) and (35) to coordinate \( \eta \), we can write the final expressions for numerical corrections as follows
\[ \delta E_{nLJ}^{(LS)} = A_{nL} K_{JL}^{(LS)} \]
\[ A_{nL} = \frac{g^2 k}{8m^2 \alpha^3} \int_0^\pi \frac{1}{\sin \eta \tan 2k} \frac{\phi^2_{nL}}{2} d\eta \]
\[ \delta E_{nLJ}^{(T)} = B_{nL} K_{JL}^{(T)} \]
\[ B_{nL} = \frac{g^2 k}{8m^2 \alpha^3} \int_0^\pi \frac{1}{\sin \eta \tan 2k} \left( \frac{2k + \cos \eta}{3k} \right) \phi^2_{nL} d\eta \]
Let us note that expressions (39) and (40) do not contain any free parameters.

When analyzing hyperfine interaction, we suppose additional quark-condensate correlation, conditioned by influence of spin-spin states of quarks on the state of nonperturbative vacuum inside quarkonium. We understand that is

\[ 1 \text{ Expressions (34) and (35) are nonrelativistic limit of fundamental spatial-spin connection } \gamma^i \gamma^k + \gamma^k \gamma^i = 2g^{ik} \text{ and } \nabla_i \gamma_k = 0. \]
reflected in the form of interquark potential, formed with active participation of this vacuum. For account with supposition we replace phenomenological parameter \( k \) (appropriating to orthoquarkonium) in the potential (20) by a new phenomenological parameter \( k_0 \) appropriating to paraquarkonium, which value is fixed from the adoption of the experimental data. Since there is not experimental data for parabottomonium we shall not consider the spin-spin interactions in this quarkonium.

Further we shall notes, that the \( \delta \) function in (36) comes from the unnatural nonrelativistic reduction and will become a smooth function with a finite range if this is calculated correctly. The standard way to calculate \( H_{hyp} \) is to replace \( \delta(\vec{r}) \) by a smeared function \([5]\), which we assume

\[
S(r) = \left[ \frac{1}{4 \pi f^4 r^2} \right] \frac{\sin \{\mu r\}}{r}, \quad \lim_{\mu \to \infty} S(r) = \delta(\vec{r})
\]

where \( \mu \) is the range of smearing, which is a free phenomenological parameter, we choose \( \mu = \frac{1}{3 \sqrt{a}} \).

In terms of coordinate \( \eta \) the numerical values of hyperfine corrections are

\[
\delta E_{nL}^{(hyp)} = \frac{2 g^2}{3 m^2 a^3 k} \int_0^\pi \frac{\sin \left\{ \frac{\mu}{2} \tan^k \left( \frac{\eta}{2} \right) \right\}}{\sin^3 \eta} \phi_{nL}^2 d\eta,
\]

(41)

The criterion of adaptability of such mode calculation \( H_{hyp} \) can be following. For a P state, the center of gravity \((COG)\) of triplet \( ^13P \) must coincide with singlet \( ^11P \) if \( H_{hyp} \) is pointlike interaction, since \( |\Psi(0)|^2 = 0 \). From experimental data for charmonium, one finds

\[
^13P_{(COG)} - ^11P_1 = 1 \text{ Mev}
\]

Our calculation under the formula (41) give

\[
^13P_{(COG)} - ^11P_1 = 5 \text{ Mev}
\]

This is a reasonable accuracy and this means the range of \( H_{hyp} \) is reasonable.

Finally, the paraquarkonium levels are determined in two stage: on the first stage the nonrelativistic levels (29) with parameter \( k_0 \) are found and on the second the corrections (41) are calculated.

Wave functions \( \phi_{nL} \) used in expressions (39), (40) and (41) in correspondence with (28) have a following form

\[
\phi_{1L} = C_{1L} \exp \left( -\frac{\gamma k^4 \eta}{l+1} \right) \sin^{l+1} \eta;
\]

\[
\phi_{2L} = C_{2L} \left( \frac{2^{-2} \gamma k^4 \eta}{l+1} \right) \sin^{l+2} \eta \left[ (2l + 3) \gamma k^4 \sin \eta - (2l^3 + 9l^2 + 13l + 6) \cos \eta \right] ;
\]

\[
\phi_{3L} = C_{3L} \left( \frac{2^{-3} \gamma k^4 \eta}{l+1} \right) \sin^{l+3} \eta \left[ \gamma^2 k^8 l + 10 \gamma^2 k^8 45 - 93l - 65l^2 - 2l^4 - 19l^3 \right] \sin^2 \eta
\]

\[
+ (4l^5 + 44l^4 + 187l^3 - 369l + 381l^2 + 135) \cos^2 \eta - (8 \gamma k^4 l^3 + 56 \gamma k^4 l^2 + 126 \gamma k^4 + 90 \gamma k^4) \cos \eta \sin \eta \right] ;
\]

and are normalized according to (23).

VI. RESULTS AND CONCLUSION.

In Table 1 the values of parameters, appearing in expression for nonrelativistic spectra (29) are presented, which were used at all stages of charmonium and bottomonium spectra numerical modeling. In Table 2 the values of quarkonia physical parameters are presented. Table 3 contain outcomes of charmonium and bottomonium spectra modeling for experimentally observable levels. On Fig. 1, 2 in standard representation the complete charmonium and
bottomonium spectra, including experimentally known levels and our theoretical prediction for not yet opened levels are presented. The levels have spectroscopic designation $^{2S+1}L_J$ and are located in corresponding to experimentally fixed quantum numbers $J^{PC}$.

Accuracy of spectra theoretical modeling was estimated under the formula

$$\delta = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{M_{i(th)} - M_{i(ex)}}{M_{i(ex)} - M_{1S}} \right)^2} \tag{42}$$

where $N$ - number of quarkonium levels, on which is conducted comparison theories $M_{i(th)}$ and experiment $M_{i(ex)}$ behind elimination of ground state of orthocharmonium $M_{1S} = 3097$ MeV and orthobottomonium $M_{1S} = 9460$ MeV, accepted for a count beginning. For all experimentally known levels of charmonium $N = 11$; in this case the calculation under the formula (42) give $\delta^{(c)}_{11} = 4.06 \cdot 10^{-2}$. Levels below charm threshold is estimated under the same formula (42) with $N = 7$; here we have $\delta^{(c)}_{7} = 5.05 \cdot 10^{-2}$. For all experimentally known levels of bottomonium $N = 11$, here $\delta^{(b)}_{11} = 3.28 \cdot 10^{-2}$. For levels below bottom threshold $N = 8$, $\delta^{(b)}_{8} = 0.96 \cdot 10^{-2}$. In Table 4 the accuracy of spectra modeling of the several best models known for us, calculated under the formula (42), are presented.

From numerical values of charmonium and bottomonium parameters, presented in Tables 1 and 2, follows the quarkonia are essentially nonperturbative objects. Foundations for this conclusion serve, at first large value of a coupling constant $g^2$ on quarkonium scale; secondly, essential difference of parameter $k$ from one, showing the absence of Coulomb’s asymptotics with $r \to 0$, that, in turn, means essential nonperturbative properties of vacuum inside quarkonium on all scales. The last conclusion is confirmed by comparison of parameter $\varepsilon_0$, describing a power of vacuum condensate partial reconstruction inside quarkonium, with the module of outside-vacuum condensate energy density.

$$|\varepsilon_{out}| = 0.3 \cdot \langle 0 \left| \frac{\alpha}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right| 0 \rangle \approx (275 MeV)^4$$

As we see, the deconfinement of quarks within quarkonium take place already with $\varepsilon_0/|\varepsilon_{out}| \approx 0.03$ for charmonium and with $\varepsilon_0/|\varepsilon_{out}| \approx 0.05$ for bottomonium. Conclusion about more force reconstruction of nonperturbative vacuum inside bottomonium in comparison with charmonium is qualitatively coordinated with values of nonperturbative coupling constant $g^2$: in bottomonium it is less, than in charmonium.

In the end let us to make conclusion: in the framework of our model we can quantitatively see, that nonperturbative vacuum play key role in hadron structure, it is necessary and defining ingredient of quarks dynamics and, therefore observable properties of quarkonia. By justification of this conclusion the good fit to experimental data serve.

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TABLE I. Parameters of model.

| Parameter | Charmonium | Bottomonium |
|-----------|------------|-------------|
| $\nu$     | 0.158      | 0.139       |
| $2k_0^2$  | 1.291      | --          |
| $2k^2$    | 1.179      | 1.360       |
| $m_0$     | 3628.209 MeV | 9963.116 MeV |
| $m_1$     | 49.348 MeV | 48.136 MeV |
| $m_2$     | 800.825 MeV | 734.050 MeV |

TABLE II. Parameters of quarkonia.

| Parameter | Charmonium | Bottomonium |
|-----------|------------|-------------|
| $m$       | 1700 MeV   | 4905 MeV    |
| $g^2$     | 2.328      | 1.138       |
| $a$       | 5.856 $\times$ 10$^{-3}$ MeV$^{-1}$ | 3.028 $\times$ 10$^{-3}$ MeV$^{-1}$ |
| $\varepsilon_0$ | (113.442 MeV)$^4$ | (129.298 MeV)$^4$ |
| $\Lambda$ | 57.243 MeV | 68.278 MeV |

TABLE III. Fitted levels of quarkonia (MeV).

| State | Data [1] | This work | State | Data [1] | This work |
|-------|----------|-----------|-------|----------|-----------|
| $1^1S_0$ | 2980     | 2974      | $1^1S_1$ | 9460     | 9460      |
| $1^3S_1$ | 3097     | 3097      | $1^3P_0$ | 9860     | 9860      |
| $1^1P_1$ | 3526     | 3512      | $1^3P_1$ | 9892     | 9901      |
| $1^3P_0$ | 3415     | 3413      | $1^3P_2$ | 9913     | 9916      |
| $1^3P_1$ | 3511     | 3517      | $2^3S_1$ | 10023    | 10023     |
| $1^3P_2$ | 3556     | 3556      | $2^3P_0$ | 10232    | 10222     |
| $2^1S_0$ | 3594     | 3647      | $2^3P_1$ | 10255    | 10254     |
| $2^1P_1$ | 3686     | 3686      | $2^3P_2$ | 10269    | 10266     |
| $1^3D_1$ | 3770     | 3772      | $3^3S_1$ | 10355    | 10363     |
| $3^3S_1$ | 4040     | 4040      | $2^3D_1$ | 10580    | 10522     |
| $2^3D_1$ | 4159 ± 20 | 4127      | $4^3S_1$ | 10865    | 10745     |
| $4^3S_1$ | 4415 ± 6  | 4435      | $3^3D_1$ | 11020    | 10936     |

TABLE IV. Accuracy of spectra modeling (%).

| Accuracy | EQ [8] | GJ [9] | MZ [10] | ZOR [11] | BBZ [12] | This work |
|----------|--------|--------|---------|----------|----------|-----------|
| $\delta^{(c)}$ | 6.36   | 1.9    | 2.37    | 9.83     | 5.34     | 5.05      |
| $\delta^{(b)}$ | --     | --     | --      | 9.83     | --       | 4.06      |
| $\delta_0^{(b)}$ | 5.15   | --     | 0.14    | 3.25     | 2.04     | 0.96      |
| $\delta_1^{(b)}$ | --     | --     | --      | 4.85     | --       | 3.28      |
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