Global fixed-time synchronization of coupled neutral-type neural network with mixed time-varying delays

Mingwen Zheng1,2, Lixiang Li3,4*, Haipeng Peng3, Jinhua Xiao1,4, Yixian Yang3, Yanping Zhang2, Hui Zhao5

1 School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, China, 2 School of Mathematics and Statistics, Shandong University of Technology, Zibo 255000, China, 3 Information Security Center, State Key Laboratory of Networking and Switching Technology, National Engineering Laboratory for Disaster Backup and Recovery, Beijing University of Posts and Telecommunications, Beijing 100876, China, 4 State Key Laboratory of Information Photonics and Optical Communications, Beijing University of Posts and Telecommunications, Beijing 100876, China, 5 Shandong Provincial Key Laboratory of Network Based Intelligent Computing, School of Information Science and Engineering, University of Jinan, Jinan 250022, China

* These authors contributed equally to this work.

Current address: School of Cyberspace Security, Beijing University of Posts and Telecommunications, Beijing 100876, China

Li_lixiang2006@163.com

Abstract

This paper mainly studies the globally fixed-time synchronization of a class of coupled neutral-type neural networks with mixed time-varying delays via discontinuous feedback controllers. Compared with the traditional neutral-type neural network model, the model in this paper is more general. A class of general discontinuous feedback controllers are designed. With the help of the definition of fixed-time synchronization, the upper right-hand derivative and a defined simple Lyapunov function, some easily verifiable and extensible synchronization criteria are derived to guarantee the fixed-time synchronization between the drive and response systems. Finally, two numerical simulations are given to verify the correctness of the results.

Introduction

The artificial neural network (ANN) is a very active frontier interdisciplinary, which has strong engineering application background and great research potential. It has been widely used in intelligent computing, pattern recognition, signal processing, associative memory, automatic control, and so on [1–5]. In the process of using large-scale integrated circuits to form a neural network, it inevitably leads to the emergence of various time-delays. These time-delays occur not only in the current states of the system, but also in the derivatives of the past states, i.e. there exists a neutral behavior in some systems. Scientific experiments show that the neutral behaviors have a great impact on the system, which has prompted lots of scholars to study the dynamics of neutral-type neural networks [6–11]. Furthermore, the synchronization of the coupled system, such as coupled oscillators, is a basic phenomenon of nonlinear dynamics. A series of papers have already studied the dynamic behaviors of coupled nonlinear oscillators,
such as the dynamic behaviors of coupled Kuramoto oscillators with time delay [12], chimera states of [13, 14], coexistence phenomena of coherence and incoherence [15], etc [16].

Along with mathematical models of various kinds of neural networks have been put forward, the dynamic behavior of their structures and characteristics, such as the existence of the equilibrium points, the stability and boundedness of solutions, have attracted the wide attention and research of many scholars in many fields [17–21]. In the real world, the synchronous discharge of neurons is a universal phenomenon. For example, the synchronization in the visual cortex of conscious monkeys [22], the synchronization of the hippocampus and the cerebral cortex during the maze task [23], the synchronization of local brain regions in patients with Parkinson’s disease [24], the synchronization of neurons in circadian clock [25–28], and so forth [29, 30]. Neural networks have more complex dynamic characteristics and chaos phenomena. This naturally has made a lot of scholars to transform from the study of chaotic synchronization to neural network synchronization. Synchronization problem can be seen as an extension of the stability problem. And synchronization is a behavior of two or more dynamical systems under the action of external driving or mutual coupling, and constantly adjusts their dynamic characteristics to form a certain kind of overall consistency. At present, the main control methods to achieve synchronization have drive-response control [31–36], feedback control [37], adaptive control [38], impulsive control [39], intermittent control [40], sliding control [41] and pinning control [42]. And the synchronization forms mainly include complete synchronization [43, 44], lag synchronization [45–47], generalized synchronization [48, 49], etc [50, 51].

In many practical problems, finite-time synchronization is of interests rather than the synchronization over infinite time. Here, there are two ways of understanding. One is finite-time synchronization that means the system converges within a finite-time interval for given any initial value, and different initial values result in different synchronization time; the other is fixed-time synchronization that means the convergence time has a uniform upper-bounds for all initial values within a defined interval. About the finite-time synchronization of neural networks, there has existed some literatures on this study. Ref. [52] investigated the finite-time synchronization problem of a class of neutral complex dynamical networks with Markovian switching by using pinning control technique. Ref. [53] studied the finite-time synchronization for a class of the complex dynamical network with non-derivative and derivative coupling and proposed a new finite-time synchronization theory. Refs. [54, 55] discussed the finite-time synchronization problem of a class of complex dynamical network with coupled items. Because the initial values of many practical systems are difficult to determine, the final settling time in finite-time synchronization is not easy to be obtained. The fixed-time synchronization theory can overcome this shortcoming. However, the references about the fixed-time synchronization are relatively less. Ref. [56] studied the fixed-time master-slave synchronization of Cohen–Grossberg neural networks, which contains only one kind of time-varying delay \(\tau(t)\) and has not coupling items. Ref. [57] studied the fixed-time synchronization control protocol of multi-agent systems. Refs. [58–62] mainly focused on the fixed-time stability of some simple nonlinear systems. For example, Ref. [61] considered the finite-time and fixed-time stability control problems of linear multi-input system. But, there are few studies on the fixed-time synchronization of coupling neutral-type neural networks with mixed time-varying delays.

Through the above discussions, the motivation of our research is summarized as follows: (1) in the theory aspect, there is little research on the fixed-time synchronization problem of dynamical neural network, especially the fixed-time synchronization of coupled neutral neural networks with mixed delays; (2) in the application aspect, the fixed-time synchronization is more suitable for practical application than the finite-time synchronization or asymptotic synchronization. Because the settling time of the finite-time synchronization depends on the initial value of the system, but the initial value is not easy to obtain. And the settling time of
asymptotic synchronization may be infinite. Motivated by the above discussions, this paper studies the globally fixed-time synchronization control problem for the drive-response coupling neutral-type neural network with mixed time-varying delays, which can achieve synchronization in fixed time independent of the initial values. It has not been studied in the existing references. The proposed coupled neutral-type neural networks is general and many models can be considered as a special case of this model [7, 63–65]. According to the proposed model, we design a general nonlinear and discontinuous control law which involves a variety of different time-delays. During the proof process of the conclusion, a simple Lyapunov function is constructed. With the aid of upper right-hand derivative, the definition of fixed-time synchronization, and related lemmas, we obtain simple synchronization criteria of the drive-response coupled neutral-type neural networks with mixed time-varying delays. The main contributions are outlined as follows:

1. The globally fixed-time synchronization problem for the drive-response coupling neutral-type neural networks with mixed time-varying delays is studied. At present, there are few studies on the globally fixed-time synchronization problem of coupled neutral-type neural networks.

2. The network model we propose has not only neutral-type time-varying delays and discrete time-varying delays, but also distributed time-varying delays. In addition, the coupling term is also included in our model. Therefore, the obtained results in this paper are more general than the aforementioned works.

3. We design a class of new and general discontinuous fixed-time synchronization feedback controllers, define a simple Lyapunov function and derive some easily verifiable and extensible synchronization criteria to achieve the fixed-time synchronization of drive-response systems. The sufficient conditions in our results are more simple and easier to calculate than other methods (such as LMI method).

4. Two simulation examples are given to verify the validity of the main theorem and corollary.

The rest of this paper is organized as follows. Section 2 introduces the coupled neutral-type neural network model, some definitions and lemmas about fixed-time synchronization. Section 3 gives the fixed-time synchronization controller and derives the sufficient conditions to ensure the drive-response system to achieve fixed-time synchronization. Two numerical examples to verify the main results are given in Section 4. Finally, we summarize the paper and put forward the prospect in Section 5.

**Network model and preliminaries**

In this section, we will give the mathematical model of the coupled neutral-type neural network, some assumptions, definitions, and lemmas about the fixed-time synchronization problem.

**Network model**

Inspired by Ref. [66, 67], the mathematical model of coupled neutral-type neural network which contains \( N \) identical networks in this paper is given as follows:

\[
\hat{x}_i(t) = d_i \dot{x}_i(t) - c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f(x_j(t)) + \sum_{j=1}^{n} b_{ij} g(x_j(t)) + \sum_{j=1}^{n} h_{ij} \int_{t-\tau_1(s)}^{t} \dot{q}(x_j(s))ds + \kappa \sum_{j=1}^{N} m_{ij} x_j(t) + I_i, \quad i = 1, 2, \cdots, N, \tag{1}
\]
with initial conditions

\[ x_i(t) = \phi(t), \quad t \in [-\tau, 0], \]

\[ \tau = \max_{i \in \mathbb{N}} \{ \tau_1(t), \tau_2(t), \tau_3(t) \}, \]

where \( x_i(t) = [x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t)]^T \) is the state variable of the \( i \)th neutral-type neural network.

\[ D = \text{diag}(d_1, d_2, \cdots, d_n), \quad A = (a_{ij})_{n \times n}, \quad B = (b_{ij})_{n \times n}, \quad H = (h_{ij})_{n \times n} \]

are state time-varying delays feedback connection weight matrices of the neurons, respectively, and \( \tau_1(t), \tau_2(t), \tau_3(t) \) are time-varying delays.

\[ C = \text{diag}(c_1, c_2, \cdots, c_n) \]

is the state self-feedback diagonal matrix. \( I = [I_1, I_2, \cdots, I_n]^T \)

is the external input vector. \( f(x) = [f(x_{11}(t)), f(x_{12}(t)), \cdots, f(x_{1n}(t))], \quad g(x) = [g(x_{11}(t)), g(x_{12}(t)), \cdots, g(x_{1n}(t))], \quad q(x) = [q(x_{11}(t)), q(x_{12}(t)), \cdots, q(x_{1n}(t))]^T \) are activation functions. \( \kappa \) denotes the coupling strength. \( M = (m_{ij})_{N \times N} \) is the outer coupling matrix defined as follows: if there exist communications between two neural networks via an edge, then \( m_{ii} = 1 \); otherwise, \( m_{ii} = 0, \quad i \neq i, \quad \text{while}, \quad m_{ii} = m_{ii} \).

\( \Gamma = \text{diag}(\gamma_1, \gamma_2, \cdots, \gamma_n) \) is the inner coupling matrix, where \( \gamma_i > 0, \quad i = 1, 2, \cdots, n. \)

Let the network model (1) be the drive system, and the corresponding response system is formulated as

\[
\dot{y}_i(t) = d \dot{y}_i(t - \tau_i(t)) - c_i y_i(t) + \sum_{j=1}^{n} a_{ij} f(y_j(t)) \\
+ \sum_{j=1}^{n} b_{ij} g(y_j(t - \tau_j(t))) + \sum_{j=1}^{n} h_{ij} \int_{t-\tau_j(s)}^{t} q(y_j(s)) ds \\
+ \kappa \sum_{i=1}^{N} m_{ij}(t) \Gamma y_i(t) + I_i + u_i(t),
\]

with initial conditions

\[ y_i(t) = \varphi(y_i(t)), \quad t \in [-\tau, 0], \]

\[ \tau = \max_{i \in \mathbb{N}} \{ \tau_1(t), \tau_2(t), \tau_3(t) \}, \]

where \( y_i(t) = [y_{i1}(t), y_{i2}(t), \cdots, y_{in}(t)]^T \) is the state variable of the \( i \)th neutral-type neural network. \( u_i(t) = [u_{i1}, u_{i2}, \cdots, u_{in}]^T \) is the controller to be designed in main results. The other parameters are the same as the model (1).

For the parameters of drive-response systems (1) and (2) throughout this paper, we introduce the following assumptions.

\textbf{Assumption 1.} The activation functions \( f(x), g(x), q(x) \) are Lipschitz continuous, i.e. there exist Lipschitz constants \( f_i, g_i, q_i \quad i = 1, 2, \cdots, n \) satisfying the following conditions.

\[ |f(y_i) - f(x_i)| \leq f_i |y_i - x_i|, \]

|g(y_i) - g(x_i)| \leq g_i |y_i - x_i|, \]

|q(y_i) - q(x_i)| \leq q_i |y_i - x_i|.

And \( F = \text{diag}(f_1, f_2, \cdots, f_n), \quad G = \text{diag}(g_1, g_2, \cdots, g_n), \quad Q = \text{diag}(q_1, q_2, \cdots, q_n). \)
Denote the error system \( e_i(t) = y_i(t) - x_i(t) \), and the dynamical equation of \( e_i(t) \) can be expressed as

\[
\dot{e}_i(t) = d\dot{e}_i(t - \tau_i(t)) - c_i e_i(t) + \sum_{j=1}^{n} a_{ij} (f(y_j(t)) - f(x_j(t))) + \sum_{j=1}^{n} b_{ij} (g(y_j(t - \tau_j(t))) - g(x_j(t - \tau_j(t)))) + \sum_{j=1}^{n} h_{ij} \int_{t-\tau_j(t)}^{t} (q(y_j(s)) - q(x_j(s))) ds
\]

\[+ \kappa \sum_{l=1}^{N} m_{il} \Gamma e_i(t) + u_i(t),\]

with the initial conditions

\[ e_i(t) = \phi(t) - \phi(t), t \in [-\tau, 0], \]
\[ \tau = \max \{ \tau_1(t), \tau_2(t), \tau_3(t) \}. \]

**Definitions and lemmas**

In this subsection, we will introduce some definitions and lemmas related to the fixed-time synchronization. They are necessary in the process of derivation of the main results.

Suppose the origin be an equilibrium of (3) (if the equilibrium is not origin, we can move the equilibrium point to the origin through the translation transformation.)

**Definition 1.** ([60]) The origin of system (3) is said to be globally uniformly finite-time stable if it is globally uniformly asymptotically stable and there exists a locally bounded function \( T : C^{w}[-\tau, 0] \to \mathbb{R} \cup \{0\} \), such that \( e(t, e_0(t)) = 0 \) for all \( t \geq T(e_0(t)) \), where \( e(t, e_0(t)) \) is an arbitrary solution of the error system (3). The function \( T \) is called the settling-time function.

**Definition 2.** ([59]) The origin of error system (3) is said to be globally fixed-time stable if it is globally uniformly finite-time stable and the settling-time \( T \) is globally bounded, i.e. \( \exists T_{\max} \in \mathbb{R}^{+} \) such that \( T(e_0(t)) \leq T_{\max} \), \( \forall e_0(t) \in \mathbb{R}^{n} \).

**Definition 3.** If \( e(t) \) is Lyapunov stable, then the drive-response systems (1) and (2) are said to achieve globally fixed-time synchronization if there exists \( T(e_0(t)) \) in some finite time such that

\[
\begin{aligned}
\lim_{t \to T(e_0(t))} ||e(t)|| &= 0, \\
e(t) &= 0, \forall t \geq T(e_0(t)) \\
T(e_0(t)) &\leq T_{\max}, \forall e_0(t) \in C^{w}[-\tau, 0].
\end{aligned}
\]

**Remark 1.** In the Definition 1, \( e(t) = 0 \Rightarrow y(t) = x(t), u_i(t) \) to be designed is a function of \( e_i(t) \) and \( u_i(t) = 0 \) when \( e_i(t) = 0 \).

**Remark 2.** According to the Definition 1 and Definition 2, we can see the main difference between finite-time stability and fixed-time stability is whether the settling time is independent to the initial value. The settling-time of fixed-time stability is independent to the initial value.
Remark 3. From the Definition 2 and Definition 3, we can conclude the globally fixed-time stability of system (3) is equivalent to the fixed-time synchronization of systems (1) and (2).

Lemma 1. [59] If there exists a continuous radically unbounded function $V : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ such that (1) $V(x) = 0 \Rightarrow x = 0$, (2) Any solution $e(t)$ of system (3) satisfies

$$
\dot{V}(e(t)) \leq -aV^p(e(t)) - bV^q(e(t)),
$$

for some $a, b > 0, p > 1, 0 < q < 1$, where $\dot{V}$ denotes the derivative of $V$. Then,

$$
V(e(t)) = 0, t \geq T(e_0),
$$

with the settling time bounded by

$$
T(e_0) \leq T_{\text{max}} = \frac{1}{a(p-1)} + \frac{1}{b(1-q)}. \tag{5}
$$

Lemma 2. [68] If there exists a continuous radically unbounded function $V : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ such that 1) $V(x) = 0 \Rightarrow x = 0$, (2) Any solution $e(t)$ of system (3) satisfies

$$
\dot{V}(e(t)) \leq -aV^p(e(t)) - bV^q(e(t)),
$$

for some $a, b > 0, p = 1 - \frac{1}{2\rho}, q = 1 + \frac{1}{2\rho}, \rho > 1$, where $\dot{V}$ denotes the derivative of $V$. Then the origin is globally fixed-time stable for system (3) and more exact estimation of the settling time can be obtained as

$$
T(e_0) \leq T_{\text{max}} = \frac{\pi \rho}{\sqrt{ab}}. \tag{6}
$$

Lemma 3. [69] Let $a_1, a_2, \ldots, a_N \geq 0, 0 < p \leq 1, q > 1$, then the following two inequalities hold

$$
\sum_{i=1}^{N} a_i^p \geq \left( \sum_{i=1}^{N} a_i \right)^p, \quad \sum_{i=1}^{N} a_i^q \geq N^{1-q} \left( \sum_{i=1}^{N} a_i \right)^q.
$$

**Main results**

In this section, we will design the controller $u(t)$ and deduce some sufficient conditions in order to achieve fixed-time synchronization of neutral-type neural networks (1) and (2).

The nonlinear controller in response system (2) is designed as follows:

$$
u_i(t) = -\text{sign}(e_i(t))(\xi_i |e_i(t)| + \omega_{i1} |e_i(t - \tau_1(t))| + \omega_{i2} |e_i(t - \tau_2(t))| + \omega_{i3} \int_{t-[\xi_1(t)]}^{t} |e_i(s)| ds + k_i |e_i(t)|^\alpha + r_i |e_i(t)|^\beta,
$$

where $\text{sign}(\cdot)$ denotes sign function, $\Xi = \text{diag}([\xi_1, \xi_2, \ldots, \xi_n])$, $\Omega_1 = \text{diag}([\omega_{11}, \omega_{12}, \ldots, \omega_{1N}])$, $\Omega_2 = \text{diag}([\omega_{21}, \omega_{22}, \ldots, \omega_{2N}])$, $\Omega_3 = \text{diag}([\omega_{31}, \omega_{32}, \ldots, \omega_{3N}])$, $K = \text{diag}([k_1, k_2, \ldots, k_n])$, $R = \text{diag}([r_1, r_2, \ldots, r_n])$, $\alpha > 1, 0 < \beta < 1$ are constants.
Replacing error system (3) with $u_i(t)$ and according to Assumption 1, we have

$$
\dot{e}_i(t) = d_i \dot{e}_i(t - \tau_i(t)) - c_i e_i(t) + \sum_{j=1}^{n} a_{ij} (f(y_j(t)) - f(x_i(t))) \\
+ \sum_{j=1}^{n} b_{ij} (g(y_j(t - \tau_j(t))) - g(x_i(t - \tau_j(t)))) \\
+ \sum_{j=1}^{n} h_{ij} \int_{t-\tau_j(t)}^{t} (q(y_i(s)) - q(x_i(s))) ds \\
+ \kappa \sum_{j=1}^{N} m_{ji} \Gamma e_j(t) + u_i(t) \\
\leq d_i \dot{e}_i(t - \tau_i(t)) - c_i e_i(t) + \sum_{j=1}^{n} a_{ij} e_j(t) \\
+ \sum_{j=1}^{n} b_{ij} e_j(t - \tau_j(t)) + \sum_{j=1}^{n} h_{ij} \int_{t-\tau_j(t)}^{t} e_j(s) ds \\
+ \kappa \sum_{j=1}^{N} m_{ji} \Gamma e_j(t) - \text{sign}(e_i(t))(|\tilde{\xi}_i| e_i(t)) \\
+ \omega_{ii} |\dot{e}_i(t - \tau_i(t))| + \omega_{ii} |e_i(t - \tau_i(t))| \\
+ \omega_{ii} \int_{t-|\tau_i(t)|}^{t} |e_i(s)| ds + k_i |e_i(t)|^\alpha + r_i |e_i(t)|^\beta.
$$

**Theorem 1.** Suppose Assumption 1 holds, then the drive-response systems (1) and (2) can achieve globally fixed-time synchronization under the controller (8) if the following conditions hold:

$$
\begin{aligned}
\sum_{j=1}^{n} a_{ij} f_j - c_i - \tilde{\xi}_i &< 0, \\
d_i - \omega_{ii} &< 0, \\
\sum_{j=1}^{n} b_{ij} g_j &< 0, \\
\sum_{j=1}^{n} h_{ij} q_j &< 0, \\
i = 1, 2, \ldots, N.
\end{aligned}
$$

Moreover,

$$
\lim_{t \to T_{\text{max}}} \|e(t)\| = 0, \ e(t) = 0, \forall t \geq T_{\text{max}},
$$

and the settling time given as

$$
T_{\text{max}} = \frac{1}{(\alpha - 1) \min_{1 \leq i \leq N} \left\{ k_i N^{1-\alpha} \right\}} \\
+ \frac{1}{(1 - \beta) \min_{1 \leq i \leq N} \left\{ r_i \right\}}.
$$
Proof. Consider a Lyapunov function defined by

$$V(e_i(t)) = \sum_{i=1}^{N} |e_i(t)|. \quad (12)$$

Calculate the upper right-hand derivative of $V(e_i(t))$ along the error system (3) and replace the inequality (9) with $\dot{e}_i(t)$, we can obtain

$$\dot{V}^+(e_i(t)) = \sum_{i=1}^{N} \text{sign}(e_i(t)) \dot{e}_i(t)$$

$$\leq \sum_{i=1}^{N} \text{sign}(e_i(t)) \left( d_i \dot{e}_i(t) - c_i e_i(t) + \sum_{j=1}^{n} a_{ij} e_j(t) \right)$$

$$+ \sum_{i=1}^{n} b_{g_i} e_i(t - \tau_2(t)) + \sum_{j=1}^{n} h_{q_i} \int_{t-\tau_1(t)}^{t} e_j(s) ds + \kappa \sum_{i=1}^{N} m_i \Gamma e_i(t)$$

$$- \text{sign}(e_i(t)) \left( \xi_i |e_i(t)| + \omega_{a_i} |\dot{e}_i(t) - \tau_1(t)| + \omega_{b_i} |e_i(t) - \tau_2(t)| \right)$$

$$+ \omega_{a_i} \int_{t-\tau_1(t)}^{t} |e_i(s)| ds + k_i |e_i(t)|^p + r_i |e_i(t)|^q \right)$$

$$\leq \sum_{i=1}^{N} d_i |\dot{e}_i(t) - \tau_1(t)| - \sum_{i=1}^{N} c_i |e_i(t)|$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{n} a_{ij} |e_j(t)| + \sum_{j=1}^{n} b_{g_i} |e_i(t - \tau_2(t))|$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{n} h_{q_i} \int_{t-\tau_1(t)}^{t} |e_j(s)| ds + \kappa \sum_{i=1}^{N} m_i \Gamma |e_i(t)| - \sum_{i=1}^{N} \xi_i |e_i(t)|$$

$$- \sum_{i=1}^{N} \omega_{a_i} |\dot{e}_i(t) - \tau_1(t)| - \sum_{i=1}^{N} \omega_{b_i} |e_i(t) - \tau_2(t)|$$

$$- \sum_{i=1}^{N} \omega_{a_i} \int_{t-\tau_1(t)}^{t} |e_i(s)| ds - \sum_{i=1}^{N} k_i |e_i(t)|^p - \sum_{i=1}^{N} r_i |e_i(t)|^q$$

$$\leq \sum_{i=1}^{N} \left( \sum_{j=1}^{n} a_{ij} e_j(t) - c_i e_i(t) \right) |e_i(t)|$$

$$+ \sum_{i=1}^{n} (d_i - \omega_{a_i}) |\dot{e}_i(t) - \tau_1(t)| + \sum_{j=1}^{n} (b_{g_i} - \omega_{b_i}) |e(t) - \tau_2(t)|$$

$$+ \sum_{i=1}^{N} \left( \sum_{j=1}^{n} h_{q_i} q_i - \omega_{a_i} \right) \int_{t-\tau_1(t)}^{t} |e_j(s)| ds$$

$$- \sum_{i=1}^{N} k_i |e_i(t)|^p - \sum_{i=1}^{N} r_i |e_i(t)|^q.$$
According to the conditions (10) and Lemma 3, we have

\[ V^+(e(t)) \leq -\sum_{i=1}^{N} k_i |e_i^a(t)| - \sum_{i=1}^{N} r_i |e_i^b(t)| \]

\[ \leq -\min_{1 \leq i \leq N} \{k_i\} \sum_{i=1}^{N} |e_i(t)|^\alpha - \min_{1 \leq i \leq N} \{r_i\} \sum_{i=1}^{N} |e_i|^\beta \]

\[ \leq -\min_{1 \leq i \leq N} \{k_iN^{1-\alpha}\} V^+(e(t)) - \min_{1 \leq i \leq N} \{r_i\} V^\beta(e(t)) \]

Taking \( a = \min_{1 \leq i \leq N} \{k_iN^{1-\alpha}\}, \ b = \min_{1 \leq i \leq N} \{r_i\}, \ p = \alpha, \ q = \beta, \) then \( p > 1, \ q < 1 \) and from Lemma 3, we have \( V(e(t)) = 0, \ t \geq T_{\max} \) and the settling time \( T_{\max} \) is given as follows:

\[ T_{\max} = \frac{1}{(x - 1) \min_{1 \leq i \leq N} \{k_iN^{1-\alpha}\}} \]

\[ + \frac{1}{(1 - \beta) \min_{1 \leq i \leq N} \{r_i\}}. \]

Thus, we complete the proof.

**Corollary 1.** According to Lemma 2, if \( x = 1 - \frac{1}{2p}, \ \beta = 1 + \frac{1}{2p}, \ \rho > 1 \), the setting time in Theorem 1 can be rewritten as the following form:

\[ T_{\max} = \frac{\pi \rho}{\sqrt{\min_{1 \leq i \leq N} \{k_iN^{1-\alpha}\} \min_{1 \leq i \leq N} \{r_i\}}}. \]

**Remark 4.** Our proposed model is more general than the other models in some literatures [52, 56, 70]. When coupling strength \( \kappa = 0 \), model (1) is changed into the common neutral-type neural networks without coupling items. When \( d_i = 0 \), the model becomes the common neural networks without neutral items. For these special circumstances, we have the following corollaries.

**Corollary 2.** Suppose Assumption 1 holds and the coupling strength \( \kappa = 0 \) in the drive-response system (1) and (2), then they can achieve a globally fixed-time synchronization under the controller (8) if the inequality conditions (10) hold.

**Remark 5.** In the proof of Theorem 1, the coupling item is removed according to the dissipativeness of coupling matrix \( M \). Therefore, the information of coupling item is not included in the synchronization conditions of Theorem 1 and Corollary 2.

When \( d_i = 0 \) in the drive-response systems (1) and (2), we define the following controller

\[ u_i(t) = -\text{sign}(e_i(t))(\xi |e_i(t)| + \omega_2 |e_i(t - \tau_2(t))|) \]

\[ + \omega_3 \int_{t-\tau_2(t)}^{t} |e_i(s)| ds + k_i |e_i(t)|^\alpha + r_i |e_i(t)|^\beta. \]  

**Corollary 3.** Suppose the Assumption 1 holds and \( d_i = 0 \) in the drive-response system (1) and (2), then they can achieve globally fixed-time synchronization under the controller (13) if...
the following conditions hold:

\[
\begin{align*}
\sum_{j=1}^{n} a_{ij} f_i - c_i - \xi_i &< 0, \\
\sum_{j=1}^{n} b_{ij} g_i - \omega_{2i} &< 0, \\
\sum_{j=1}^{n} h_{ij} q_i - \omega_{3i} &< 0.
\end{align*}
\] (14)

When \( h_i = 0 \) in the drive-response systems (1) and (2), we modify (8) as the following controller

\[
\begin{align*}
u_i(t) &= -\text{sign}(e_i(t))((\xi_i |e_i(t)| + \omega_3 |\dot{e}_i(t - \tau_i(t))|) \\
&+ \omega_2 |e_i(t - \tau_2(t))| + k_i |\dot{e}_i(t)|^\beta + r_i |e_i(t)|^\beta).
\end{align*}
\] (15)

**Corollary 4.** Suppose Assumption 1 holds and \( h_i = 0 \) in the drive-response system (1) and (2), then they can achieve the globally fixed-time synchronization under the controller (15) if the following conditions hold:

\[
\begin{align*}
\sum_{j=1}^{n} a_{ij} f_i - c_i - \xi_i &< 0, \\
d_i - \omega_{2i} &< 0, \\
\sum_{j=1}^{n} b_{ij} g_i - \omega_{2i} &< 0, \\
i = 1, 2, \ldots, N.
\end{align*}
\] (16)

**Remark 6.** In the Corollaries 1-3, the settling time \( T_{\text{max}} \) is the same as that in Theorem 1 because this two parameters \( \alpha, \beta \) have not changed.

By modifying designed controller (8), we can obtain the fixed-time synchronization of the drive-response systems. The modified controller is given as follows

\[
\begin{align*}
u_i(t) &= -\text{sign}(e_i(t))((\xi_i |e_i(t)| + \omega_3 |\dot{e}_i(t - \tau_i(t))|) \\
&+ \omega_2 |e_i(t - \tau_2(t))| + \omega_3 \int_{t-[\tau_3(t)]}^{t} |e_i(s)| ds + r_i |e_i(t)|^\beta).
\end{align*}
\] (17)

where \( 0 < \beta < 1 \), the other parameters are the same as those in the controller (8).

**Corollary 5.** Suppose Assumption 1 holds and \( h_i = 0 \) in the drive-response systems (1) and (2), then they can achieve the globally finite-time synchronization under the controller (17) if the conditions (10) hold in the Theorem 1. Moreover,

\[
\lim_{t \to T_{\text{max}}} \|e(t)\| = 0, e(t) = 0, \forall t \geq T_{\text{max}},
\]
and the settling time is given as

\[ T_{\text{max}} = \frac{\sum_{i=1}^{N} |x_i(0)|^{1-\beta}}{(1-\beta)\min_{i \leq N\{r_i\}}}. \]

**Remark 7.** Compared with existing research on the fixed-time synchronization [56, 57, 71], although the theoretical framework of fixed-time is derived from the research of Polyakov et al. [59–61], they study different system models. In this paper, we study the globally fixed-time synchronization of the coupling neutral-type neural networks with mixed time-varying delays, which no one seems to have studied such a model.

**Numerical examples**

In order to verify the rightness of the theoretical results, we give two numerical examples.

**Example 1:** Consider the following two-dimensional neutral-type neural network with three identical neurons:

\[
\begin{align*}
\dot{x}_i(t) &= d\dot{x}_i(t - \tau_i(t)) - c x_i(t) + \sum_{j=1}^{2} a_{ij} f(x_j(t)) + \sum_{j=1}^{2} b_{ij} g(x_j(t - \tau_j(t))) \\
&+ \sum_{j=1}^{2} h_{ij} \int_{t-\tau_j(t)}^{t} q(x_j(s))ds + \kappa \sum_{j=1}^{3} m_{ij} \Gamma x_j(t) + I_i, i = 1, 2, 3.
\end{align*}
\]

The corresponding response system is shown below:

\[
\begin{align*}
\dot{y}_i(t) &= d\dot{y}_i(t - \tau_i(t)) - c y_i(t) + \sum_{j=1}^{2} a_{ij} f(y_j(t)) + \sum_{j=1}^{2} b_{ij} g(y_j(t - \tau_j(t))) \\
&+ \sum_{j=1}^{2} h_{ij} \int_{t-\tau_j(t)}^{t} q(y_j(s))ds + \kappa \sum_{j=1}^{3} m_{ij} \Gamma y_j(t) + I_i + u_i(t), i = 1, 2, 3.
\end{align*}
\]

where the time-varying delays are \(\tau_1(t) = \tau_2(t) = 0.5|\sin(t)|, \tau_3(t) = 0.5|\cos(t)|, f(e_i(t)) = g(e_i(t)) = h(e_i(t)) = \tanh(e_i(t)).\) Other parameters in the model (18) are chosen as follows:

\[
D = \text{diag}\{0.6, 0.8\}, C = \text{diag}\{0.5, 0.6\}, A = \begin{bmatrix} 0.2 & 0.3 \\ 0.5 & 1 \end{bmatrix}, B = \begin{bmatrix} 0.5 & 0.4 \\ 0.6 & 0.3 \end{bmatrix},
\]

\[
H = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 1 \end{bmatrix}, I = \begin{bmatrix} 0.1, 0.1 \end{bmatrix}^T, F = \text{diag}\{1, 1\}, G = \text{diag}\{1, 1\}, H = \text{diag}\{1, 1\}
\]

\[
M = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}, \Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \kappa = 2.
\]
According to the conditions (10) in Theorem 1, the parameters of controller (8) are set as follows:

\[ \Xi = \text{diag}(5, 5), \Omega_1 = \text{diag}(2, 2), \Omega_2 = \text{diag}(2, 2), \]
\[ \Omega_3 = \text{diag}(2, 2), K = \text{diag}(4, 3), R = \text{diag}(3, 4), \alpha = 1.25, \beta = 0.25. \]

The initial values of (18) and (19) are

\[ x_1(t_0) = [0.5 + \sin(t), \cos(t) - 0.5]^T, \]
\[ x_2(t_0) = [\sin(t) - 0.5, 0.5 + \cos(t)]^T, \]
\[ x_3(t_0) = [\sin(t) - 0.5, 0.5 + \cos(t)]^T. \]

When no controller \( u_i(t) \) is added into the system, the trajectories of the drive system (18) and the response system (19), and the phase diagram of the first neuron for the drive-response systems are shown in Figs 1, 2, 3 and 4.

When using controller \( u_i(t) \), the trajectories and the error curves of the drive-response systems (18) and (19) are shown in the following Figs 5, 6, 7 and 8.

From the Eq (11), we can calculate the settling-time \( T_{\text{max}} \approx 1.9740 \).

Remark 8. Obviously, from Figs 1–4, we can see that the drive-response systems (18) and (19) can not reach the synchronization state when the controller (8) is not used. However, when the controller (8) is used, the drive-response systems achieve synchronization, which is easy to see from Figs 5–8. The simulation results verify the effectiveness of Theorem 1.

Next, we verify the rightness of Corollary 4. through the following example.

Example 2: Consider the following two-dimensional drive neutral-type neural network:

\[ \dot{x}(t) = D\dot{x}(t - \tau_1(t)) - Cx(t) + Af(x(t)) + Bg(x(t - \tau_2(t))), \]

\[ \dot{y}(t) = D\dot{y}(t - \tau_1(t)) - Cy(t) + Af(y(t)) + Bg(y(t - \tau_2(t))), \]

Fig 1. The first dimensional trajectories of the drive system (18) with initial conditions \( x_1(t_0) = [0.5 + \sin(t), \cos(t) - 0.5]^T, x_2(t_0) = [\sin(t) - 0.5, 0.5 + \cos(t)]^T, x_3(t_0) = [\sin(t) - 0.5, 0.5 + \cos(t)]^T \) and response system (19) with initial conditions \( y_1(t_0) = [0.1 + \sin(t), \cos(t) - 0.1]^T, y_2(t_0) = [\sin(t) - 0.1, 0.1 + \cos(t)]^T, y_3(t_0) = [0.5 + \sin(t), \cos(t) - 0.5]^T \) when no controller (8).
Fig 2. The second dimensional trajectories of the drive system (18) with initial conditions $x_i(t_0) = [0.5 + \sin(t), \cos(t) - 0.5]^T$, $x_i(t_0) = [\sin(t) - 0.5, 0.5 + \cos(t)]^T$, $x_i(t_0) = [\sin(t), \cos(t)]^T$ and response system (19) with initial conditions $y_i(t_0) = [0.1 + \sin(t), \cos(t) - 0.1]^T$, $y_i(t_0) = [\sin(t) - 0.1, 0.1 + \cos(t)]^T$, $y_i(t_0) = [0.5 + \sin(t), \cos(t) - 0.5]^T$ when no controller (8).

https://doi.org/10.1371/journal.pone.0191473.g002

Fig 3. The phase diagram of the first neuron($x_{11} - x_{12}$) of the drive system (19) with initial conditions $x_i(t_0) = [0.5 + \sin(t), \cos(t) - 0.5]^T$, $x_i(t_0) = [\sin(t) - 0.5, 0.5 + \cos(t)]^T$, $x_i(t_0) = [\sin(t), \cos(t)]^T$ when no controller (8).

https://doi.org/10.1371/journal.pone.0191473.g003
Fig 4. The phase diagram of the first neuron \((y_{11} - y_{12})\) of the response system (19) with initial conditions \(y_1(t_0) = [0.1 + \sin(t), \cos(t) - 0.1]^T\), \(y_2(t_0) = [\sin(t) - 0.1, 0.1 + \cos(t)]^T\), \(y_3(t_0) = [0.5 + \sin(t), \cos(t) - 0.5]^T\) when no controller (8).

https://doi.org/10.1371/journal.pone.0191473.g004

Fig 5. The first dimensional trajectories of drive-response systems (18) and (19) with initial conditions \(x_1(t_0) = [0.5 + \sin(t), \cos(t) - 0.5]^T\), \(x_2(t_0) = [\sin(t) - 0.5, 0.5 + \cos(t)]^T\), \(x_3(t_0) = [\sin(t), \cos(t)]^T\) and response system (19) with initial conditions \(y_1(t_0) = [0.1 + \sin(t), \cos(t) - 0.1]^T\), \(y_2(t_0) = [\sin(t) - 0.1, 0.1 + \cos(t)]^T\), \(y_3(t_0) = [0.5 + \sin(t), \cos(t) - 0.5]^T\) under the controller (8).

https://doi.org/10.1371/journal.pone.0191473.g005
Fig 6. The second dimensional trajectories of with initial conditions $x_i(t_0) = [0.5 + \sin(t), \cos(t) - 0.5]^T$, $x_i(t_0) = [\sin(t) - 0.5, 0.5 + \cos(t)]^T$, and response system (19) with initial conditions $y_i(t_0) = [0.1 + \sin(t), \cos(t) - 0.1]^T$, $y_i(t_0) = [\sin(t) - 0.1, 0.1 + \cos(t)]^T$ under the controller (8).

https://doi.org/10.1371/journal.pone.0191473.g006

Fig 7. The first dimensional error curves ($e_i$, $i = 1, 2, 3$) of drive-response systems (18) and (19) with the controller (8).

https://doi.org/10.1371/journal.pone.0191473.g007
and the response system as
\[
\dot{y}(t) = D\dot{y}(t - \tau_1(t)) - C\dot{y}(t) + Af(y(t)) + Bg(y(t - \tau_2(t)))
\] (21)
where \(\tau_1(t) = 0.2 + 0.5|\sin(t)|, \tau_2(t) = 0.3 + 0.6|\cos(t)|, f(x) = \tanh(x), g(x) = 0.5(|x + 1| - |x - 1|).

Other parameters in the model (20) are chosen as follows:
\[
D = \text{diag}\{0.2, 0.2\}, C = \text{diag}\{3.6, 3.6\}, A = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1.2 & 0.1 \\ 0.1 & 1.2 \end{bmatrix},
\]

According to the conditions (16) in Corollary 4, the parameters of controller (15) are set as follows:
\[
\Xi = \text{diag}\{1, 1\}, \Omega_1 = \text{diag}\{1, 1\}, \Omega_2 = \text{diag}\{1, 1\},
\]
\[
K = \text{diag}\{2, 2\}, R = \text{diag}\{3, 3\}, \alpha = 1.5, \beta = 0.5.
\]

The initial conditions of (20) and (21) are \(x(t_0) = [3 + \sin(t), 2 - \cos(t)]^T, y(t_0) = [1 - 2\sin(t), 1 + \cos(t)]^T\).

The corresponding results are shown in Figs 9 and 10.

Similarly, we can calculate the settling-time \(T_{\max} \approx 1.6667\) from the Eq (11).

**Remark 9.** The synchronization of nonlinear systems, including neural networks, has many potential practical applications, such as synchronization-based secure communication, signal transmission, automatic control, pattern recognition, etc. In these applications, it is sometimes necessary to accomplish a certain task within a finite/fixed time. For example, in robotics control, we need to drive the robot to reach a specified position at a given time [72]; in a traffic
Fig 9. The trajectories of drive-response systems (20) and (21) with initial conditions $x(t_0) = [3 + \sin(t), 2 - \cos(t)]^T$, $y(t_0) = [1 - 2\sin(t), 1 + \cos(t)]^T$ under the controller (15).

https://doi.org/10.1371/journal.pone.0191473.g009

Fig 10. The error curves of drive-response systems (20) and (21) under the controller (15).

https://doi.org/10.1371/journal.pone.0191473.g010
dynamics of signalized intersections network, the intersections must be automatically regulated in a fixed time [73]. Therefore, the study of this paper has some practical significance.

Conclusion
In this paper, the globally fixed-time synchronization problem of a class of coupled neutral-type neural networks with mixed time-varying delays is studied. The proposed network model is more general than the model of earlier publications. A general discontinuous feedback controller is designed to guarantee the drive-response systems to achieve fixed-time synchronization. By virtue of the definition of fixed-time synchronization, some lemmas, the upper right-hand derivative of discontinuous function, and a simple Lyapunov function, some fixed-time synchronization criteria are obtained through mathematical derivation. Some corollaries about the fixed-time synchronization and some special cases of proposed model have been also given. Finally, the effectiveness of the proposed theorem and corollaries has been validated by two numerical examples. For future research topics, (1) more simple controllers and more easily validated conditions will be studied to guarantee the fixed-time synchronization of neutral-type neural networks or other complex dynamics networks; (2) based on some existing literatures [74], we will consider the problem of fixed-time synchronization of neural-type neural networks with stochastic factors or Markovian jump; (3) considering that the neuron model studied in this paper is to artificial, we will investigate some classical physical-biological models, as shown in Refs. [75, 76].

Acknowledgments
This paper is supported by the National Key Research and Development Program (Grant Nos.2016YFB0800602), the National Natural Science Foundation of China (Grant Nos.61472045, 61573067, 61771071).

Author Contributions
Conceptualization: Yanping Zhang.
Funding acquisition: Lixiang Li, Haipeng Peng, Yixian Yang.
Investigation: Mingwen Zheng, Haipeng Peng, Yanping Zhang, Hui Zhao.
Methodology: Mingwen Zheng, Jinghua Xiao.
Supervision: Lixiang Li, Jinghua Xiao, Yixian Yang.
Writing – original draft: Mingwen Zheng.

References
1. Schmidhuber J. Deep learning in neural networks: An overview. Neural Networks. 2015; 61:85–117. https://doi.org/10.1016/j.neunet.2014.09.003 PMID: 25462637
2. Wang N, Er MJ, Han M. Dynamic tanker steering control using generalized ellipsoidal-basis-function-based fuzzy neural networks. Fuzzy Systems, IEEE Transactions on. 2015; 23(5):1414–1427. https://doi.org/10.1109/TFUZZ.2014.2362144
3. Nguyen A, Yosinski J, Clune J. Deep neural networks are easily fooled: High confidence predictions for unrecognizable images. In: Computer Vision and Pattern Recognition (CVPR), 2015 IEEE Conference on. IEEE; 2015. p. 427–436.
4. Ali JB, Fnaiech N, Saidi L, Chebel-Morello B, Fnaiech F. Application of empirical mode decomposition and artificial neural network for automatic bearing fault diagnosis based on vibration signals. Applied Acoustics. 2015; 89:16–27. https://doi.org/10.1016/j.apacoust.2014.08.016
5. Zhao L, Jia Y. Neural network-based adaptive consensus tracking control for multi-agent systems under actuator faults. International Journal of Systems Science. 2016; 47(8):1931–1942. https://doi.org/10.1080/00207721.2014.960906

6. Park JH, Park C, Kwon O, Lee SM. A new stability criterion for bidirectional associative memory neural networks of neutral-type. Applied Mathematics and Computation. 2008; 199(2):716–722.

7. Park JH. Synchronization of cellular neural networks of neutral type via dynamic feedback controller. Chaos, Solitons & Fractals. 2009; 42(3):1299–1304. https://doi.org/10.1016/j.chaos.2009.03.024

8. Samli R, Arik S. New results for global stability of a class of neutral-type neural systems with time delays. Applied Mathematics and Computation. 2009; 210(2):564–570. https://doi.org/10.1016/j.amc.2009.01.031

9. Zhou W, Zhu Q, Shi P, Su H, Fang J, Zhou L. Adaptive synchronization for neutral-type neural systems with stochastic perturbation and Markovian switching parameters. Cybernetics, IEEE Transactions on. 2014; 44(12):2848–2860. https://doi.org/10.1109/TCYB.2014.2317236

10. Dharani S, Rakkiyappan R, Cao J. New delay-dependent stability criteria for switched Hopfield neural networks of neutral type with additive time-varying delay components. Neurocomputing. 2015; 151:827–834. https://doi.org/10.1016/j.neucom.2014.10.014

11. Yeung MKS, Strogatz SH. Time Delay in the Kuramoto Model of Coupled Oscillators. Physical Review Letters. 1999; 82(3):648–651. https://doi.org/10.1103/PhysRevLett.82.648

12. Abrams DM, Mirollo R, Strogatz SH, Wiley DA. Solvable model for chimera states of coupled oscillators. Physical Review Letters. 2008; 101(8):084103. https://doi.org/10.1103/PhysRevLett.101.084103 PMID: 18764617

13. Ma R, Wang J, Liu Z. Robust features of chimera states and the implementation of alternating chimera states. Epl. 2010; 91(4):40006–40011(6). https://doi.org/10.1209/0295-5075/91/40006

14. Kuramoto Y, Battogtokh D. Coexistence of Coherence and Incoherence in Nonlocally Coupled Phase Oscillators. Physics. 2002; 5(4):380.

15. Zhang X, Pikovsky A, Liu Z. Dynamics of oscillators globally coupled via two mean fields. Scientific Reports. 2017; 7(1):2104. https://doi.org/10.1038/s41598-017-02283-1 PMID: 28522836

16. Zhu Q, Cao J, Rakkiyappan R. Exponential input-to-state stability of stochastic Cohen–Grossberg neural networks with mixed delays. Nonlinear Dynamics. 2015; 79(2):1085–1098. https://doi.org/10.1007/s11071-014-1725-2

17. Li Y, Yang L, Wu W. Anti-periodic solution for impulsive BAM neural networks with time-varying delays on time scales. Neurocomputing. 2014; 149:536–545. https://doi.org/10.1016/j.neucom.2014.09.020

18. Nie X, Zheng WX. Multistability of neural networks with discontinuous non-monotonic piecewise linear activation functions and time-varying delays. Neural Networks. 2015; 65:65–79. https://doi.org/10.1016/j.neunet.2015.01.007 PMID: 25703511

19. Buzsáki G, Schomburg EW. What does gamma coherence tell us about inter-regional neural communication? Nature neuroscience. 2015; 18(4):484–489. https://doi.org/10.1038/nn.3952 PMID: 25706474

20. Gu C, Xu J, Ros R, Yang H, Liu Z. Noise Induces Oscillation and Synchronization of the Circadian Neurons. Plos One. 2015; 10(12):e0145360. https://doi.org/10.1371/journal.pone.0145360 PMID: 26691765

21. Cagnan H, Duff EP, Brown P. The relative phases of basal ganglia activities dynamically shape effective connectivity in Parkinson’s disease. Brain. 2015; 138(6):1667–1678. https://doi.org/10.1093/brain/awv093 PMID: 25888552
27. Gu C, Rohling JH, Liang X, Yang H. Impact of dispersed coupling strength on the free running periods of circadian rhythms. Physical Review E. 2016; 93(3-1):032414. https://doi.org/10.1103/PhysRevE.93.032414 PMID: 27078397

28. Gu C, Liang X, Yang H, Rohling JHT. Heterogeneity induces rhythms of weakly coupled circadian neurons. Scientific Reports. 2016; 6:21412. https://doi.org/10.1038/srep21412 PMID: 26898574

29. Ciocchi S, Passecker J, Malagon-Vina H, Mikus N, Klausberger T. Selective information routing by ventral hippocampal CA1 projection neurons. Science. 2015; 348(6234):560–563. https://doi.org/10.1126/science.aaa3245 PMID: 25931556

30. Hanslmayr S, Staresina BP, Bowman H. Oscillations and Episodic Memory: Addressing the Synchronization/Desynchronization Conundrum. Trends in neurosciences. 2016; 39(1):16–25. https://doi.org/10.1016/j.tins.2015.11.004 PMID: 26763659

31. Pecora LM, Carroll TL. Synchronization in chaotic systems. Physical review letters. 1990; 64(8):821. https://doi.org/10.1103/PhysRevLett.64.821 PMID: 10042089

32. Gu C, Tang M, Yang H. The synchronization of neuronal oscillators determined by the directed network structure of the suprachiasmatic nucleus under different photo periods. Scientific Reports. 2016; 6:28878. https://doi.org/10.1038/srep28878 PMID: 27358024

33. Gu C, Tang M, Rohling JH, Yang H. The effects of non-self-sustained oscillators on the entrainment ability of the suprachiasmatic nucleus. Scientific Reports. 2016; 6:37661. https://doi.org/10.1038/srep37661 PMID: 27869182

34. Gu C, Yang H, Ruan Z. Entrainment range of the suprachiasmatic nucleus affected by the difference in the neuronal amplitudes between the light-sensitive and light-insensitive regions. Physreeve. 2017; 95(042409).

35. Gu C, Yang H, Rohling JH. Dissociation between two subgroups of the suprachiasmatic nucleus affected by the number of damped oscillated neurons. Physreeve. 2017; 95(032302).

36. Gu C, Yang H. The asymmetry of the entrainment range induced by the difference in intrinsic frequencies between two subgroups within the suprachiasmatic nucleus. Chaos. 2017; 27(6):063115. https://doi.org/10.1063/1.4989385 PMID: 28679229

37. Franklin GF, Powell JD, Emami-Naeini A. Feedback control of dynamics systems. Addison-Wesley, Reading, MA. 1994.

38. Yassen M. Adaptive control and synchronization of a modified Chua’s circuit system. Applied Mathematics and Computation. 2003; 135(1):113–128. https://doi.org/10.1016/S0096-3003(01)00318-6

39. Yang T, Chua LO. Impulsive stabilization for control and synchronization of chaotic systems: theory and application to secure communication. Circuits and Systems I: Fundamental Theory and Applications, IEEE Transactions on. 1997; 44(10):976–988. https://doi.org/10.1109/81.633887

40. Xia W, Cao J. Pinning synchronization of delayed dynamical networks via periodically intermittent control. Chaos. 2008; 19(1):013120. https://doi.org/10.1063/1.3071933

41. Tavazoei MS, Haeri M. Synchronization of chaotic fractional-order systems via active sliding mode controller. Physica A: Statistical Mechanics and Its Applications. 2008; 387(1):57–70. https://doi.org/10.1016/j.physa.2007.08.039

42. Yu W, Chen G, Lü J. On pinning synchronization of complex dynamical networks. Automatica. 2009; 45(2):429–435. https://doi.org/10.1016/j.automatica.2008.07.016

43. Lin W, He Y. Complete synchronization of the noise-perturbed Chua’s circuits. Chaos: An Interdisciplinary Journal of Nonlinear Science. 2005; 15(2):023705. https://doi.org/10.1063/1.1938627

44. Lopes LM, Fernandes S, Grácia C. Complete synchronization and delayed synchronization in couplings. Nonlinear Dynamics. 2015; 79(2):1615–1624. https://doi.org/10.1007/s11071-014-1764-8

45. Gu C, Wang J, Wang J, Liu Z. Mechanism of phase splitting in two coupled groups of suprachiasmatic-nucleus neurons. Physical Review E Statistical Nonlinear & Soft Matter Physics. 2011; 83(2):046224. https://doi.org/10.1103/PhysRevE.83.046224

46. Totz JF, Snari R, Yengi D, Tinsley MR, Engel H, Showalter K. Phase-lag synchronization in networks of coupled chemical oscillators. Physical Review E. 2015; 92(2):022819. https://doi.org/10.1103/PhysRevE.92.022819

47. Al-mahbashi G, Noorani MM, Bakar S, Al-sawalha MM. Robust projective lag synchronization in drive-response dynamical networks via adaptive control. The European Physical Journal Special Topics. 2016; 225(1):51–64. https://doi.org/10.1140/epjst/e2016-02920-1

48. Ouannas A, Odbat Z. Generalized synchronization of different dimensional chaotic dynamical systems in discrete time. Nonlinear Dynamics. 2015; 81(1-2):765–771. https://doi.org/10.1007/s11071-015-2026-0
49. Wang S, Wang X, Han B. Complex Generalized Synchronization and Parameter Identification of Non-identical Nonlinear Complex Systems. PloS one. 2016; 11(3):e0152099. https://doi.org/10.1371/journal.pone.0152099 PMID: 27014879

50. Vaidyanathan S, Azar AT. Anti-synchronization of Identical Chaotic Systems Using Sliding Mode Control and an Application to Vaidyanathan–Madhavan Chaotic Systems. In: Advances and Applications in Sliding Mode Control systems. Springer; 2015. p. 527–547.

51. Wu W, Zhou W, Chen T. Cluster synchronization of linearly coupled complex networks under pinning control. Circuits and Systems I: Regular Papers, IEEE Transactions on. 2009; 56(4):829–839. https://doi.org/10.1109/TCSI.2008.2003373

52. Liu X, Yu X, Xi H. Finite-time synchronization of neutral complex networks with Markovian switching based on pinning controller. Neurocomputing. 2015; 153:148–158. https://doi.org/10.1016/j.neucom.2014.11.042

53. Xu Y, Zhou W, Fang J, Xie C, Tong D. Finite-time synchronization of the complex dynamical network with non-derivative and derivative coupling. Neurocomputing. 2016; 173:1356–1361. https://doi.org/10.1016/j.neucom.2015.09.008

54. He P, Ma SH, Fan T. Finite-time mixed outer synchronization of complex networks with coupling time-varying delay. Chaos: An Interdisciplinary Journal of Nonlinear Science. 2012; 22(4):043151. https://doi.org/10.1063/1.4773005

55. Li D, Cao J. Finite-time synchronization of coupled networks with one single time-varying delay coupling. Neurocomputing. 2015; 166:265–270. https://doi.org/10.1016/j.neucom.2015.04.013

56. Wan Y, Cao J, Wen G, Yu W. Robust fixed-time synchronization of delayed Cohen–Grossberg neural networks. Neural Networks. 2016; 73:86–94. https://doi.org/10.1016/j.neunet.2015.10.009 PMID: 26575975

57. Zhou Y, Sun C. Fixed time synchronization of complex dynamical networks. In: Proceedings of the 2015 Chinese Intelligent Automation Conference. Springer; 2015. p. 163–170.

58. Cruz-Zavala E, Moreno JA, Fridman LM. Uniform robust exact differentiator. Automatic Control, IEEE Transactions on. 2011; 56(11):2727–2733. https://doi.org/10.1109/TAC.2011.2160030

59. Polyakov A. Nonlinear feedback design for fixed-time stabilization of linear control systems. Automatic Control, IEEE Transactions on. 2012; 57(8):2106–2110. https://doi.org/10.1109/TAC.2011.2179869

60. Polyakov A, Efimov D, Perruquet W. Finite-time and fixed-time stabilization: Implicit Lyapunov function approach. Automatica. 2015; 51:332–340. https://doi.org/10.1016/j.automatica.2014.10.082

61. Polyakov A, Efimov D, Perruquet W. Robust stabilization of MIMO systems in finite/fixed time. International Journal of Robust and Nonlinear Control. 2016; 26(1):69–90. https://doi.org/10.1002/rnc.3297

62. Lu W, Liu X, Chen T. A note on finite-time and fixed-time stability. Neural Networks. 2016; 81:11–15. https://doi.org/10.1016/j.neunet.2016.04.011 PMID: 27239892

63. Dai Y, Cai Y, Xu X. Synchronization criteria for complex dynamical networks with neutral-type coupling delay. Physica A: Statistical Mechanics and its Applications. 2008; 387(18):4673–4682. https://doi.org/10.1016/j.physa.2008.03.024

64. Zhu Q, Zhou W, Zhou L, Wu M, Tong D. Mode-dependent projective synchronization for neutral-type neural networks with distributed time-delays. Neurocomputing. 2014; 140:97–103. https://doi.org/10.1016/j.neucom.2014.03.032

65. Xu Y, Xie C, Tong D. Adaptive synchronization for dynamical networks of neutral type with time-delay. Optik-International Journal for Light and Electron Optics. 2014; 125(1):380–385. https://doi.org/10.1016/j.ijleo.2013.08.002

66. Liu Y, Wang Z, Liang J, Liu X. Synchronization of coupled neutral-type neural networks with jumping-mode-dependent discrete and unbounded distributed delays. Cybernetics, IEEE Transactions on. 2013; 43(1):102–114. https://doi.org/10.1109/TSMCB.2012.2199751

67. Wang W, Li L, Peng H, Wang W, Kurths J, Xiao J, et al. Anti-synchronization of coupled memristive neutral-type neural networks with mixed time-varying delays via randomly occurring control. Nonlinear Dynamics. 2016; 83(4):2143–2155. https://doi.org/10.1007/s11071-015-2471-9

68. Parsegov S, Polyakov A, Shcherbakov P. Nonlinear fixed-time control protocol for uniform allocation of agents on a segment. In: Decision and Control (CDC), 2012 IEEE 51st Annual Conference on. IEEE; 2012. p. 7732–7737.

69. Khalil HK, Grizzle J. Nonlinear systems. vol. 3. Prentice hall New Jersey; 1996.

70. Wang L, Shen Y. Finite-time stabilizability and instabilizability of delayed memristive neural networks with nonlinear discontinuous controller. Neural Networks and Learning Systems, IEEE Transactions on. 2015; 26(11):2914–2924. https://doi.org/10.1109/TNNLS.2015.2460239
71. Cao J, Li R. Fixed-time synchronization of delayed memristor-based recurrent neural networks. Science China Information Sciences. 2017; 60(3):032201. https://doi.org/10.1007/s11432-016-0555-2

72. Haimo VT. Finite Time Controllers. Siam Journal on Control & Optimization. 2014; 24(4):760–770. https://doi.org/10.1137/0324047

73. Muralidharan A, Pedarsani R, Varaiya P. Analysis of fixed-time control. Transportation Research Part B. 2015; 73:81–90. https://doi.org/10.1016/j.trb.2014.12.002

74. Zhu Q, Rakkiyappan R, Chandrasekar A. Stochastic stability of Markovian jump BAM neural networks with leakage delays and impulse control. Neurocomputing. 2014; 136:136–151. https://doi.org/10.1016/j.neucom.2014.01.018

75. Rabinovich MI, Varona P, Selverston AI, Abarbanel HDI. Dynamical principles in neuroscience. Reviews of Modern Physics. 2006; 78(4):1213–1265. https://doi.org/10.1103/RevModPhys.78.1213

76. Dhamala M, Jirsa VK, Ding M. Enhancement of Neural Synchrony by Time Delay. Physical Review Letters. 2004; 92(7):074104. https://doi.org/10.1103/PhysRevLett.92.074104 PMID: 14995856