Systematics of $q\bar{q}$ states, scalar mesons and glueball

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Abstract

Basing on the latest results of the PNPI (Gatchina) and QM&W College (London) groups, I discuss systematics of the $IJPC_q\bar{q}$ states in terms of trajectories on the $(n, M^2)$ plane, where $n$ is the radial quantum number and $M$ is its mass. In the scalar sector, which is the most interesting because of the presence of extra states with respect to the $q\bar{q}$ systematics, I discuss: 1) the results of the $K$-matrix analysis of the spectra $\pi\pi$, $\pi\pi\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta'/\pi\eta$ and characteristics of the resonances in the scalar sector, 2) $q\bar{q}$-nonet classification of scalar bare states, 3) accumulation of widths of the $q\bar{q}$ states by the glueball due to the overlapping of $f_0$-resonances at 1200–1700 MeV, 4) systematics of scalar $q\bar{q}$ states, both bare states and resonances, on the $(n, M^2)$-plots, 5) constraints on the quark-gluonium content of the resonances $f_0(980)$, $f_0(1300)$, $f_0(1500)$, $f_0(1750)$, and the broad state $f_0(1420^{+150}_{-70})$ from hadronic decays, 6) radiative decays of the $P$-wave $q\bar{q}$-resonances: scalars $f_0(980)$, $a_0(980)$, and tensor mesons $a_2(1320)$, $f_2(1270)$, $f_2(1525)$. The analysis proves that in the scalar sector we face two exotic mesons: the light $\sigma$-meson, $f_0(450)$, and the broad state $f_0(1420^{+150}_{-70})$, which is the descendant of the glueball.

1. Systematics on the $(n, M^2)$-plots.

An important role for the unambiguous interpretation of the data is played by the $q\bar{q}$ systematization of the discovered meson states: this may be a guide for the search for new resonances as well as for establishing signatures of the existing states.

Here, following [1], the systematics of $q\bar{q}$ states is presented in terms of the $(n, M^2)$ trajectories where $n$ is the radial quantum number of the $q\bar{q}$ state and $M$ is its mass. The trajectories on the $(n, M^2)$ planes are drawn for the $(IJPC)$-states with the positive charge parity ($C = +$): $\pi(10^{-+})$, $\pi_2(12^{+-})$, $\pi_4(14^{--})$, $\eta(00^{-+})$, $\eta_2(02^{+-})$, $a_0(10^{++})$, $a_1(11^{++})$, $a_2(12^{++})$, $a_3(13^{++})$, $a_4(14^{++})$, $f_0(00^{++})$, $f_2(02^{++})$, and negative one ($C = -$): $b_1(11^{+-})$, $b_3(13^{+-})$, $h_1(01^{++})$, $\rho(11^{--})$, $\rho_3(13^{--})$, $\omega/\phi(11^{--})$, $\omega_3(13^{--})$, see Figs. 1 and 2. Open points stand for the predicted states.

The main bulk of information about the mass region 2000–2400 MeV, which is crucial for drawing the trajectories, came from the analysis of Crystal Barrel data for the $p\bar{p}$ annihilation in flight [2].

The trajectories on the $(n, M^2)$-plots with a good accuracy are linear:

$$M^2 = M_0^2 + (n - 1)\mu^2.$$  

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*Talk given on "HADRON 2001"
$M_0$ is the mass of basic meson and $\mu^2$ is the trajectory slope parameter: $\mu^2$ is nearly the same for all trajectories: $\mu^2 \simeq 1.2 - 1.3 \text{ GeV}^2$.

Trajectories with the same $IJ^{PC}$ can be created by the states with different orbital momenta, with $J = L \pm 1$; in this way they are doubled: these are the trajectories $(I1^-), (I2^+)\text{,}$ and so on. Isoscalar states are formed by two light flavour components, $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $ss$. Likewise, this also results in doubling isoscalar trajectories.

The representation of $(C = -)$-trajectories is thus determined. The trajectories are nearly linear with the slope $\mu^2 \simeq 1.3 \text{ GeV}^2$, with an exception of the $b_J$ sector where the slope $\mu^2 \simeq 1.1 - 1.2 \text{ GeV}^2$.

For the $C = +$ states, the $\pi_J$-sector is decisively fixed with the slope $\mu^2 \simeq 1.2 \text{ GeV}^2$. The only state which breaks linearity of the trajectory is the pion, that is not surprising because of its specific role in the low-energy physics.

The trajectories in the $\eta_J$-sector are not unambiguously fixed; in Fig. 1b we show the variant with $\mu^2 = 1.3 \text{ GeV}^2$. The uncertainties are mainly due to the region 1700–2000 MeV in the wave $00^{-+}$: it is the region where one may expect the existence of pseudoscalar glueball. Indeed, a strong production of the $00^{-+}$ wave is observed in the radiative $J/\Psi$ decay $[8]$ that may be a glueball signature (although one should note that lattice calculations provide us with a higher value, $\sim 2300 \text{ MeV} [4]$). For sure, this mass region needs an intensive study.

The sector of $a_{1J}$ states, $J = 0, 1, 2, 3, 4$, demonstrates clearly a set of linear trajectories with $\mu^2 \simeq 1.15 - 1.20 \text{ GeV}^2$, Figs. 1c, 1e. The same slope is observed for $f_2$ and $f_4$ mesons, Fig. 1d.

For $f_0$ mesons we have $\mu^2 \simeq 1.3 \text{ GeV}^2$. A superfluous state for $q\bar{q}$-trajectories are the light $\sigma$-meson $[5, 8, 10]$ and the broad resonance $f_0(1420^{+150}_{-70})$ observed in the K-matrix analysis $[11]$: one should consider these states as candidates for the exotics.

In the recently performed study of the reaction $p\bar{p} \rightarrow \eta\eta \pi^0\pi^0$ in flight, the resonance with mass $1880 \pm 20$ in the $(12^{-+})$-wave has been declared $[12]$: this state is also beyond the $\pi_2$-trajectory and should be considered as a hybrid.

2. Scalars.

The existence of superfluous, with respect to $q\bar{q}$ systematics, states is a motivation to perform intensive studies on scalar-isoscalar sector.

1) $K$-matrix analysis and resonances in the scalar-isoscalar sector. In the paper $[8]$, on the basis of experimental data of GAMS group, Crystal Barrel Collaboration and BNL group, the $K$-matrix solution has been found for the waves $00^{++}, 10^{++}, 02^{++}, 12^{++}$ over the range 450–1900 MeV. Also the masses and total widths of resonances have been determined for these waves. The following states have been seen in the scalar-isoscalar sector: $f_0(980), f_0(1300), f_0(1500), f_0(1420^{+150}_{-70}), f_0(1750)$. For the scalar-isovector sector, the analysis $[8]$ points to the presence of the following resonances in the spectra: $a_0(980), a_0(1520)$.

The $K$-matrix amplitude takes correctly into account the threshold singularities of the $00^{++}$ amplitude related to the channels $\pi\pi, \pi\pi\pi, K\bar{K}, \eta\eta, \eta\eta'$. This circumstance allowed us to reconstruct the analytical amplitude in the complex-mass region shown in Fig. 3 by dashed line. In this area, with correctly restored analytical structure of the amplitude $00^{++}$, we find out the resonance characteristics: the amplitude poles
and decay coupling constants. Besides, we know the $K$-matrix characteristics such as the $K$-matrix poles. The $K$-matrix poles are not the amplitude poles, these latter being connected with physical resonances, but when the decays are switched off, the resonance poles turn into the $K$-matrix ones. In the states related to the $K$-matrix poles there is no cloud of real mesons, that is due to the decay processes. This was the reason to call them as "bare states".

Below the mass scale of the $K$-matrix analysis there is a pole related to the light $\sigma$-meson (or $f_0(450)$); its position is shown in Fig. 3 following the results of the dispersion relation $N/D$-analysis (the mass region validated by this analysis is also shown in Fig. 3). Above the mass region of the $K$-matrix analysis there are resonances $f_0(2030)$, $f_0(2100)$, $f_0(2340)$.

2) Classification of scalar bare states. The quark-gluonium systematics of scalar particles, in terms of bare states, has been suggested in [8]. A bare state being a member of the $q\bar{q}$ nonet imposes rigid restrictions upon the $K$-matrix parameters. The $q\bar{q}$ nonet of scalars consists of two scalar-isoscalar states, $f_0^{\text{bare}}(1)$ and $f_0^{\text{bare}}(2)$, scalar-isovector meson $a_0^{\text{bare}}$ and scalar kaon $K_0^{\text{bare}}$. In the leading order of the $1/N_c$-expansion the decays of these four states into two pseudoscalars are determined by three parameters only, which are the common constant $g$, suppression parameter $\lambda$ for strange quark production (in the limit of a precise $SU(3)$ flavour symmetry $\lambda = 1$) and mixing angle $\varphi$ for the $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ components in $f_0^{\text{bare}}$: $n\bar{n} \cos \varphi + s\bar{s} \sin \varphi$. The mixing angle defines scalar–isoscalar nonet partners $f_0^{\text{bare}}(1)$ and $f_0^{\text{bare}}(2)$: $\varphi(1) - \varphi(2) = 90^\circ$. Restrictions imposed on coupling constants allow one to fix unambiguously basic scalar nonet [8]:

$$1^3 P_0 q\bar{q} : f_0^{\text{bare}}(720 \pm 100), \quad a_0^{\text{bare}}(960 \pm 30), \quad K_0^{\text{bare}}(1220^{+50}_{-150}), \quad f_0^{\text{bare}}(1260 \pm 30), \quad (2)$$

as well as mixing angle for $f_0^{\text{bare}}(720)$ and $f_0^{\text{bare}}(1260)$: $\varphi(720) = -70^\circ \pm 5^\circ$.

To establish the nonet of first radial excitations, $2^3 P_0 q\bar{q}$, appeared to be a more difficult task. The $K$-matrix analysis gives us two scalar-isoscalar states at 1200–1650 MeV, $f_0^{\text{bare}}(1230^{+150}_{-30})$ and $f_0^{\text{bare}}(1600 \pm 50)$; the decay couplings for both of them satisfy the requirements imposed for the glueball. To resolve this dilemma, we have performed the systematization of the $q\bar{q}$ states on the $(n, M^2)$ plot. Such a systematization definitely proves that $f_0^{\text{bare}}(1600 \pm 50)$ is an extra state for the $q\bar{q}$ trajectory. In this way, $f_0^{\text{bare}}(1230^{+150}_{-30})$ and $f_0^{\text{bare}}(1810 \pm 30)$ must be the $q\bar{q}$ states.

Then the nonet $2^3 P_0 q\bar{q}$ looks as follows:

$$2^3 P_0 q\bar{q} : f_0^{\text{bare}}(1230^{+150}_{-30}), \quad f_0^{\text{bare}}(1810 \pm 30), \quad a_0^{\text{bare}}(1650 \pm 50), \quad K_0^{\text{bare}}(1885^{+50}_{-100}). \quad (3)$$

The decay couplings of $f_0^{\text{bare}}(1600)$ to channels $\pi\pi, K\bar{K}, \eta\eta, \eta\eta'$ obey the requirements for the glueball decay. This gives us the reason to consider this state as the lightest scalar glueball:

$$0^{++} \text{ glueball} : \quad f_0^{\text{bare}}(1600 \pm 50). \quad (4)$$

The lattice calculations are in reasonable agreement with such a value of the lightest glueball mass.

After the onset of decay channels, the bare states have transformed into real resonances. For scalar–isoscalar sector we observe the following transitions after switching-on the decay channels: $f_0^{\text{bare}}(720) \pm 100 \rightarrow f_0(980), f_0^{\text{bare}}(1260 \pm 30) \rightarrow f_0(1300), f_0^{\text{bare}}(1810 \pm 30) \rightarrow f_0(1885^{+50}_{-100}).$
\[ f_0^{\text{bare}}(1230^{+150}_{-30}) \rightarrow f_0(1500), \quad f_0^{\text{bare}}(1600 \pm 50) \rightarrow f_0(1420^{+150}_{-70}), \quad f_0^{\text{bare}}(1810 \pm 30) \rightarrow f_0(1750). \]

The evolution of bare states into real resonances is illustrated by Fig. 4: the shifts of amplitude poles on the complex-\( M \) plane correspond to a gradual onset of the decay channels. Technically it is done by replacing the phase space \( \rho_a \) for \( a = \pi\pi, \pi\pi\pi\pi, KK, \eta\eta, \eta'\eta' \) in the \( K \)-matrix amplitude as follows: \( \rho_a \rightarrow \xi\rho_a \), the parameter \( \xi \) running in the interval \( 0 \leq \xi \leq 1 \). At \( \xi \rightarrow 0 \) one has pure bare states, while the limit \( \xi \rightarrow 1 \) gives us the position of real resonance.

Note that the broad state is denoted in \[8\] as \( f_0(1530^{+90}_{-250}) \) that is the averaged value for three solutions found in \[8\]; the value of the mass given in Figs. 3 and 4, \( M = (1420^{+150}_{-70}) - i(540 \pm 80) \) MeV, corresponds to the solution for which the scalar glueball is located near 1600 MeV.

3) The overlapping of \( f_0 \)-resonances at 1200–1700 MeV: accumulation of widths of \( q\bar{q} \) states by the glueball. The appearance of broad resonance is not at all an occasional phenomenon. It has originated as a result of a mixing of states which are due to the decay processes, namely, transitions \( f_0(m_1) \rightarrow \text{real mesons} \rightarrow f_0(m_2) \). These transitions result in a specific phenomenon, that is, when several resonances overlap, one of them accumulates the widths of neighbouring resonances and transforms into a broad state.

This phenomenon has been observed in \[8\] for the scalar-isoscalar states, and the following scheme has been suggested in \[11\]: the broad state \( f_0(1420^{+150}_{-70}) \) is the descendant of a pure glueball, which being in the neighbourhood of \( q\bar{q} \) states accumulated their widths and transformed into a part of the gluonium and \( q\bar{q} \) states. In \[11\] this idea has been applied for four resonances \( f_0(1300), f_0(1500), f_0(1420^{+150}_{-70}) \) and \( f_0(1750) \), by using the language of the \( q\bar{q} \) and \( gg \) states for consideration of the decays \( f_0 \rightarrow q\bar{q}, gg \) and mixing processes \( f_0(m_1) \rightarrow q\bar{q}, gg \rightarrow f_0(m_2) \). According to \[11\], the gluonium component is mainly shared between three resonances, \( f_0(1300), f_0(1500), f_0(1420^{+150}_{-70}) \), so every state is a mixture of \( q\bar{q} \) and \( gg \) components, with roughly equal percentage of the gluonium (about 30-40%).

The accumulation of widths of overlapping resonances by one of them is a well-known effect in nuclear physics. In meson physics this phenomenon can play an important role, in particular for exotic states which are beyond the \( q\bar{q} \) systematics. Indeed, being among the \( q\bar{q} \) resonances, the exotic state creates a group of overlapping resonances. The exotic state, which is not orthogonal to its neighbours, after having accumulated the "excess" of width turns into a broad state. This broad resonance should be accompanied by narrow states which are the descendants of states from which the widths have been taken off. In this way, the existence of a broad resonance accompanied by narrow ones may be a signature of exotics. This possibility, in context of searching for exotic states, has been discussed in \[12\].

The broad state may be one of the components which form the confinement barrier: the broad states after accumulating the widths of neighbouring resonances play for these latter the role of locking states. Evaluation of the mean radii squared of the broad state \( f_0(1420^{+150}_{-70}) \) and its neighbours-resonances, performed in \[12\] on the basis of the GAMS data, argues in favour of this idea, for the radius of \( f_0(1420^{+150}_{-70}) \) is significantly larger than that of \( f_0(980) \) and \( f_0(1300) \) thus making it possible for \( f_0(1420^{+150}_{-70}) \) to be the
-locking state.

4) Systematics of the \( q\bar{q} \) scalar states on the \( (n, M^2) \) plot. As is stressed above, the systematics of \( q\bar{q} \) states on the \( (n, M^2) \) plot argues that the broad state \( f_0(1420^{+150}_{-70}) \) and its predecessor \( f_0^{bare}(1600 \pm 50) \) are beyond the \( q\bar{q} \) classification. We plot in Fig. 5a the \( (n, M^2) \)-trajectories for \( f_0, a_0 \) and \( K_0 \) states. All trajectories are roughly linear, and they clearly represent the states with dominant \( q\bar{q} \) component. It is seen that one of the states, either \( f_0(1420^{+150}_{-70}) \) or \( f_0(1500) \), is superfluous for the \( q\bar{q} \) systematics. Looking at the \( (n, M^2) \)-trajectories of bare states, Fig. 5b, one can see that just \( f_0^{bare}(1600) \) does not fall onto any linear \( q\bar{q} \) trajectory. So it would be natural to conclude that the state \( f_0^{bare}(1600) \) is an exotic one, i.e. the glueball.

For resonances belonging to linear trajectories (Fig. 5a) the \( q\bar{q} \) component is dominant. The scalar-isoscalar resonances \( f_0(1300), f_0(1500) \) contain a considerable gluonium component, and certain gluonium admixture exists in \( f_0(1750) \). The location of the \( f_0(980) \) pole near \( K\bar{K} \) threshold allows one to suspect the existence of an admixture of the \( K\bar{K} \)-component in this resonance. To investigate this admixture the precise measurements of the \( K\bar{K} \) spectra in the interval 1000—1150 MeV are necessary: only these spectra could shed the light on the role of the long-range \( K\bar{K} \) component in \( f_0(980) \).

5) Quark-gluonium content of resonances \( f_0(980), f_0(1300), f_0(1500), f_0(1750) \) and the broad state \( f_0(1420^{+150}_{-70}) \) from hadronic decays. The \( K \)-matrix analysis does not supply us with coupling constants of the resonance decay in a direct way. To find them out, additional calculations are needed to know the residues of amplitude poles related to resonances. Such calculations have been carried out in [13] for the channels \( f_0 \rightarrow \pi\pi, \pi\pi\pi, K\bar{K}, \eta\eta, \eta\eta' \). The conclusion is as follows [14]: the decays couplings to the channels \( \pi\pi, K\bar{K}, \eta\eta, \eta\eta' \) do not provide us with a unique solution for absolute weight of the \( n\bar{n}, s\bar{s} \) and gluonium components but give us relative weights only. The mixing angle \( \varphi \) which enters the quark wave function \( q\bar{q} = n\bar{n} \cos \varphi + s\bar{s} \sin \varphi \) can be evaluated as a fraction of the decay couplings for gluonium and quarkonium components \( G(\bar{g}g \rightarrow hadrons)/g(q\bar{q} \rightarrow hadrons) \). The ratio of the couplings squared was conventionally called in [14] as probability for the gluonium component in the \( f_0 \)-meson: \( W \equiv G^2/g^2 \). The following relations for \( \varphi \) versus \( W \) have been found [14]:

\[
\begin{align*}
\varphi[f_0(980)] & \approx -67^\circ \pm 57^\circ \sqrt{W(980)}, \\
\varphi[f_0(1300)] & \approx -5^\circ \pm 28^\circ \sqrt{W(1300)}, \\
\varphi[f_0(1500)] & \approx 8^\circ \pm 16^\circ \sqrt{W(1500)}, \\
\varphi[f_0(1750)] & \approx -27^\circ \pm 42^\circ \sqrt{W(1750)}.
\end{align*}
\]

(5)

A large admixture of the gluonium, \( W \leq 0.4 \), may be expected for \( f_0(1300), f_0(1500), f_0(1750) \), but it should be considerably less in \( f_0(980), W(980) \leq 0.20 \).

The analysis [14] proves that \( f_0(1420^{+150}_{-70}) \) contains the \( q\bar{q} \) in the flavour singlet state only:

\[
\varphi[f_0(1420^{+150}_{-70})] \approx 37^\circ,
\]

(6)

that perfectly agrees with its gluonium origin: This value of mixing angle practically does not depend on the percentage of the \( (q\bar{q})_{\text{singlet}} \) and gluonium components in the broad state.

6) Radiative decays of the \( P \)-wave \( q\bar{q} \)-mesons. The investigation of radiative decays is a powerful tool for establishing the quark structure of hadrons. At the
early stage of the quark model, the radiative decays of vector mesons provided strong
arguments in favour of the idea of constituent quark, a universal object for mesons and
baryons [17]. The radiative decays of the $1^3P_0q\bar{q}$ mesons are equally important for the
verification of the $P$-wave multiplet.

In Ref. [14], partial widths of the decays $f_0(980) \to \gamma\gamma$ and $a_0(980) \to \gamma\gamma$ have been
calculated assuming $f_0(980)$ and $a_0(980)$ to be dominantly $q\bar{q}$ states, that is, $1^3P_0q\bar{q}$
mesons. The results of the calculation agree well with experimental data. On the basis of
experimental data for the decays $\phi(1020) \to \gamma f_0(980)$ and $f_0(980) \to \gamma\gamma$ the $n\bar{n}/s\bar{s}$
content of $f_0(980)$ has been found. Assuming the flavour wave function in the form
$n\bar{n}\cos\varphi + s\bar{s}\sin\varphi$, the experimental data has been described with two possible values
of mixing angle: either $\varphi[f_0(980)] = -48^\circ \pm 6^\circ$ or $\varphi[f_0(980)] = 85^\circ \pm 4^\circ$ (negative value
is more preferable), see Fig. 6a where the allowed region of $\varphi$ versus $R^2_{f_0(980)}$ is shown.

The dominance of the quark-antiquark state does not exclude the existence of other
components in $f_0(980)$ on the level 10% - 20%, the glueball or long-range $KK$ com-
ponent. The existence of the long-range $KK$ component or that of gluonium in the
$f_0(980)$ results in a decrease of the $s\bar{s}$ fraction in the $q\bar{q}$ component: for example, if
the long-range $KK$ (or gluonium) admixture is of the order of 15%, the data require
either $\varphi = -45^\circ \pm 6^\circ$ or $\varphi = 83^\circ \pm 4^\circ$.

There is no problem with the description of the decay $a_0(980) \to \gamma\gamma$ within the
hypothesis about $q\bar{q}$ origin of the $a_0(980)$: the data are in a good agreement with the
results of the calculation by using $R^2_{a_0(980)} \sim 10 - 17$ GeV$^{-2}$.

Although direct calculations of widths of radiative decays agree well with the hypo-
thesis that the $q\bar{q}$ component dominates $f_0(980)$ and $a_0(980)$, to determine reliably
these mesons as members of the $1^3P_0q\bar{q}$ multiplet one more step is necessary. We have
to prove that radiative decays of tensor mesons $a_2(1320)$, $f_2(1270)$, $f_2(1525)$ can be
calculated within the same approach and the same technique as it was carried out for
$f_0(980)$ and $a_0(980)$. Tensor mesons $a_2(1320)$, $f_2(1270)$, $f_2(1525)$ are the basic members
of the $P$-wave $q\bar{q}$ multiplet, and the existence of tensor mesons had been used to suggest
quark–antiquark classification for four $P$-wave nonets [17]. Under this motivation, par-
tial widths of the tensor $q\bar{q}$ states $a_2(1320) \to \gamma\gamma$, $f_2(1270) \to \gamma\gamma$ and $f_2(1525) \to \gamma\gamma$
have been calculated [14]: the agreement with data has been reached for all calculated
partial widths, with similar radial wave functions, that indicates definitely that both scalar
($f_0(980)$, $a_0(980)$) and tensor ($a_0(1320)$, $f_2(1270)$, $f_2(1525)$) mesons belong
to the same $P$-wave $q\bar{q}$ multiplet. In Fig. 6b one can see the region of magnitudes
($\varphi_T, R^2_T$) allowed by data on the decays $f_2(1270) \to \gamma\gamma$ and $f_2(1525) \to \gamma\gamma$; here $\varphi_T$
is mixing angle for $\psi_{f_2(1270)} = \cos\varphi_T n\bar{n} + \sin\varphi_T s\bar{s}$ and $\psi_{f_2(1525)} = -\sin\varphi_T n\bar{n} + \cos\varphi_T s\bar{s}$
and $R_T$ is the tensor-meson radius.

7) Exotics in scalar-isoscalar sector. The established $q\bar{q}$ systematics of scalar
mesons in terms of bare states fixes two nonets: $1^1P_0q\bar{q}$ and $2^1P_0q\bar{q}$. The resonances
which are the descendants of pure $q\bar{q}$ states are located on linear trajectories in the
$(n, M^2)$-plane. The $q\bar{q}$ systematics reveals two extra states which are the light $\sigma$-meson,
with mass $\sim 450$ MeV, or $f_0(450)$, and broad state $f_0(1420^{+150}_{-70})$. The broad state is
the descendant of a pure glueball state which accumulated the widths of neighbouring
$q\bar{q}$ resonances. The origin of the $\sigma$-meson is questionable.

In the paper [18], a hypothesis is discussed that the $\sigma$-meson owes its origin to strong
singularity in the confinement amplitude: the large-$r$ behaviour of the confinement scalar potential $V(r) \sim r$ evokes a strong $t$-channel singularity, $1/t^2$. It was assumed in \[18\] that the singularity of this kind exists at every colour state of the confinement $q\bar{q}$ ladder; then, in the white state related to the $\pi\pi$ channel, the unitarization of singular block might reduce the singularity strenth, thus providing the pole near the $\pi\pi$ threshold, at $\text{Re } s \sim 4\mu^2$, that corresponds to the $\sigma$-meson.

The analysis of data on the decays $D^+ \to \pi^+\pi^+\pi^-$ and $D^+_s \to \pi^+\pi^+\pi^-$ \[7\] agrees with such an idea. The $s\bar{s}$ component at $f_0(980)$ has been evaluated in \[19\] by comparing the branching ratios $D^+_s \to \pi^+\phi(1020)$ and $D^+_s \to \pi^+f_0(980)$ being about 50% (this estimate agrees with hadronic and radiative decays of $f_0(980)$ for the solution with negative mixing angle $\varphi$). The ratio of yields of $f_0(450)$ and $f_0(980)$ in the reaction $D^+ \to \pi^+\pi^+\pi^-$ tells us that $f_0(450)$ is dominantly the $n\bar{n}$ system. The confinement ladder should be formed by the light quarks, $(u,d)$, see for example \[21\]: in this sense, the structure of $f_0(450)$ is just as it was expected, if it originates from the confinement ladder, as it was supposed in \[18\].

Acknowledgement. Thanks are due to A.V. Anisovich, D.V. Bugg, L.G. Dakhno, D.I. Melikhov, V.A. Nikonov, A.V. Sarantsev for numerous discussions. The work is suppored by the RFFI grant N 01-02-17861.

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Figure 1: Trajectories of the \((C = -)\)-states on the \((n, M^2)\) plane.
Figure 2: Trajectories of the \((C = -)\)-states on the \((n, M^2)\) plane.
Figure 3: Complex $M$-plane in the $(IJ^{PC} = 00^{++})$ sector. Dashed line encircle the part of the plane where the $K$-matrix analysis [8] reconstructs the analytic $K$-matrix amplitude: in this area the poles corresponding to resonances $f_0(980)$, $f_0(1300)$, $f_0(1500)$, $f_0(1750)$ and the broad state $f_0(1420 \pm 150)$ are located. Beyond this area the light $\sigma$-meson is located (the position of pole found in the $N/D$ method [6] is shown) as well as resonances $f_0(2030)$, $f_0(2100)$, $f_0(2340)$ [2]. Solid lines stand for the cuts related to the thresholds $\pi\pi, \pi\pi\pi\pi, KK, \eta\eta, \eta'\eta'$. 

Figure 4: Complex $M$-plane: trajectories of the poles for $f_0(980)$, $f_0(1300)$, $f_0(1500)$, $f_0(1750)$, $f_0(1420 \pm 150)$ during gradual onset of the decay processes.
Figure 5: Linear trajectories in $(n, M^2)$-plane for scalar resonances (a) and scalar bare states (b). Open points stand for predicted states.

Figure 6: a) The $(\varphi, R_{f_0(980)}^2)$-plot: the shaded areas are the allowed ones for the reactions $\phi(1020) \rightarrow \gamma f_0(980)$ and $f_0(980) \rightarrow \gamma\gamma$. b) The $(\varphi_T, R_T^2)$-plot for the reactions $f_2(1270) \rightarrow \gamma\gamma$ and $f_2(1525) \rightarrow \gamma\gamma$; the allowed areas are shaded.