Energy-Delay-Distortion Problem

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Abstract—An energy-limited source trying to transmit multiple packets to a destination with possibly different sizes is considered. With limited energy, the source cannot potentially transmit all bits of all packets. In addition, there is a delay cost associated with each packet. Thus, the source has to choose, how many bits to transmit for each packet, and the order in which to transmit these bits, to minimize the cost of distortion (introduced by transmitting lower number of bits) and queueing plus transmission delay, across all packets. Assuming an exponential metric for distortion loss and linear delay cost, we show that the optimal order of transmission is the increasing order of packet sizes and optimization problem is jointly convex. Hence, the problem can be exactly solved using convex solvers, however, because of the complicated expression derived from the KKT conditions, no closed form solution can be found even with the simplest cost function choice made in the paper. To facilitate a more structured solution, a discretized version of the problem is also considered, where time and energy are divided in discrete amounts. In any time slot (fixed length), bits belonging to any one packet can be transmitted, while any discrete number of energy quanta can be used in any slot corresponding to any one packet, such that the total energy constraint is satisfied. The discretized problem is a special case of a multi-partitioning problem, where each packet’s utility is super-modular and the proposed greedy solution is shown to incur cost that is at most 2-times of the optimal cost.

1. Introduction

Rate-distortion problem is a classical problem, where the objective is to find minimum transmission rate to support a given distortion constraint under a specific distortion metric. Typically, the problem is considered for single source-destination pair, with average power constraints, and optimal results on the rate-distortion problem are derived when infinitely large blocklengths are allowed [1]. Rate-distortion with finite blocklengths has been considered in [2] and [3].

Real-time communication, communication under quality-of-service (QoS) constraint, energy harvest-

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or erased otherwise, and the decision variable is to
decide which packet to transmit in any slot among
the ones that have not been transmitted successfully
by then, to maximize throughput.

An online version of the considered problem,
which is the part of ongoing work, considers that \( n \)
packets (each with possibly distinct sizes, \( B_i \) bits)
arrive at the source at distinct times and under the cost
function described above, the problem is to find how
many bits to send for each packet, when to begin its
transmission, and for how long to transmit its bits. A
moment’s thought will reflect that the online version
is a general case of the age of information problem
[11], where \( B_i = 1 \) and \( B_i \in \{0, 1\} \), i.e., the problem
is to minimize the delay between the time at which
the packet with one bit arrives at the source and the
time at which the receiver knows about it, if at all.

Thus, the problem formulation introduced in this
paper is quite general, and addresses two important
problems: rate-distortion problem with finite delays
(where rate constraint is an artefact of limited energy),
and the generalized age of information problem,
where the generalization includes the dependence of
the identity of packets and their sizes on the cost
function.

Our contributions are as follows:

- We show that the optimal order of packet
  transmission is the same as the increasing
  order of the packet sizes, and energy-delay-
  distortion problem given this optimal order is
  jointly convex, and hence can be solved using
  any convex solver. Unfortunately, however,
even for the simplest choice of reasonable
  cost function, the KKT conditions (though
  sufficient for optimality) cannot be used to
  find a structured solution. This is in contrast
  to [4], where closed form solution is found
  when the delay cost is not included. Thus,
  including the delay cost, not only makes the
  problem more practically relevant, but is also
  fundamentally different analytically than [4].

- To get a structured solution that does not
  require a brute force search over all possible
  orders of packet transmissions, we also con-
  sider a discretized version of the problem.
  In the discretized version, time is divided into
  discrete slots, and any one slot can be used
  only to transmit bits belonging to the same
  packet. In addition, we also discretize the
  energy into small units of \( e \) each, that is the
  least amount of energy that will be used in one
  slot. Thus, the equivalent problem is to find
  an allocation of resource blocks (rectangles
  of height (energy) \( e \) and width (slot time \( t \))
  to packets, under the total energy constraint,
such that the objective function is minimized.

- This discretized version is a discrete optimiza-
tion problem, which in general is harder to
solve compared to a continuous (and convex
problem in this case) one. The structure of
the problem, however, comes to the rescue by
noting the fact that the discrete problem is a
special case of the multi-partitioning problem,
where the objective is to partition a given set
of resources among multiple agents to min-
imize an overall objective function. For the
discretized problem, we show that a greedy
algorithm achieves at most twice the cost of
the optimal solution, via exploiting the super-
modularity of the cost function for each of the
packets. Thus, the discretized model allows
the use of a simple structured solution that is
guaranteed to be close to the optimal.

2. Problem Formulation

Consider a source that has \( n \) packets with \( B_i, i = 1, \ldots, n \) bits each, that it wants to communicate to its
destination. The total energy available with the source is \( E \), that can be used to transmit any bits of the \( n \)
packets. Finite \( E \) limits the number of bits that can be
sent from the source to its destination, and thus
the source has to judiciously choose how many bits
of each packet can be sent, and the order in which
the packets should be sent since that also impacts the
QoS.

To make this precise, let the source send \( \hat{B}_i \) out of
\( B_i \) bits of packet \( i \) using energy \( E_i \) and time \( t_i \).
The actual method to compress \( B_i \) bits to \( \hat{B}_i \) bits is out
of scope of this paper, and can be found in quantization
literature. Let \( \pi \) be any permutation over \([1 : n]\). Let
the \( i^{th} \) packet be sent at the \( \pi(i) \)th location, then the
cost for packet \( i \) is defined as
\[
U_i = 2^{B_i - \hat{B}_i} + \sum_{k \in \pi^{-1}(\pi(i))} t_k + t_{\pi(i)},
\]
where the second term is the queuing delay. The overall
cost of the source is
\[
U = \sum_{i=1}^{n} U_i. \tag{1}
\]

Then the optimization problem is
\[
\min_{\pi, E_i, t_i, \sum_i E_i \leq E} \sum_i U_i, \tag{2}
\]
i.e., we want to find the optimal order of transmission
\( \pi \), and energy and time dedicated for each packet \( i \)
under \( \pi \). The choice of \( U_i \) is motivated by the fact that
any natural distortion cost function has a diminishing
returns property such that its incremental decrease
reduces as \( \hat{B}_i \) increases. The delay component counts
the delay of packet \( i \) as well as the queuing delay
that it experiences because of transmission of packets
transmitted before it.

Problem 2 has connections with the rate-distortion
theory in finite time and energy, which to the best
of our knowledge is unsolved. To be specific, the
first term of \( U_i \), \( 2^{B_i - \hat{B}_i} \) measures the distortion for
packet \( i \), and the rate restriction follows because of
finite energy \( E \) and the presence of other packets. The
linear delay term weights the rate at which packets
are being delivered to the destination. One can keep
\( U_i \) a general function of \( B_i, \hat{B}_i \), however, the specific
choice made here is quite natural, that has diminishing
returns property, and is convex, without making the
problem trivial.

We use the Shannon rate formula to relate the \( \hat{B}_i, E_i \)
and \( t_i \) for packet \( i \), that is given by
\[
\hat{B}_i = t_i \log \left( 1 + \frac{E_i}{t_i} \right).
\]
Using this rate formula, we can write Problem 2, as
a function of \( E_i \) or \( t_i \) alone. Even under this ’simple’
rate formula, Problem 2 is challenging, where finding the optimal order in which packets should be sent is non-trivial.

3. Optimal Solution For Problem 2

In the following, we first show that the optimal order \( \pi^* \) in which packets should be transmitted is in fact the increasing order of packet sizes \( B_i \), and then prove that Problem 2 is jointly convex problem under the Shannon-rate formula for any order \( \pi \) and in particular \( \pi^* \). Establishing that Problem 2 is convex in both \( E_i \) or \( t_i \) individually is rather easy.

**Theorem 1.** The optimal order \( \pi^* \) in which packets should be transmitted is the increasing order of packet sizes \( B_i \).

**Proof.** With optimal order \( \pi^* \), \( (\pi^*)^{-1}(j) \) is the index of the packet that is sent at the \( j^{th} \) position with \( \pi^* \). Let \( t(j) = (\pi^*)^{-1}(j) \).

First we establish that \( t(j) \leq t(j+1) \) \( \forall j = 1, \ldots, n-1 \). Assume to the contrary, i.e., \( \exists j' \) such that \( t(j') > t(j'+1) \). Then we can swap the order of packets \( j'+1 \) and \( j', \) which does not change the distortion cost for any packet, but reduces the sum of the queuing plus delay cost for the two packets \( \pi(j') \) and \( \pi(j'+1) \) without affecting the queuing plus delay cost for any other packet. Thus, we get a contradiction.

Next, we prove that \( \hat{B}_j \leq \hat{B}_{j+1} \) \( \forall j = 1, \ldots, n-1 \). We already know that \( t(j) \leq t(j+1) \) \( \forall j = 1, \ldots, n-1 \). If we suppose \( \hat{B}_j > \hat{B}_{j+1} \), then we show that we can transmit \( \hat{B}_j \) in longer time slot \( t_j \) and \( \hat{B}_{j+1} \) in shorter time slot \( t_{j+1} \) while consuming lower total energy without changing the distortion cost or the delay cost as follows. This will contradict the claim that \( \pi^* \) is optimal.

Using Shannon capacity formula \( E_i = t_i(2^{\rho_i} - 1) \), the sum of the energies for the two considered packets \( \rho(i) \) and \( i+1 \) is

\[
E = E_i + E_{i+1},
\]

\[
= t_i 2^{\rho_i} - t_i + t_{i+1} 2^{\rho_{i+1}},
\]

or

\[
t_i 2^{\rho_i} - t_i = t_{i+1} 2^{\rho_{i+1}} - t_{i+1} 2^{\rho_{i+1}},
\]

which is the same as the required condition (3) to prove the claim.

Since bits are indistinguishable and \( \hat{B}_j \leq \hat{B}_{j+1} \), we can associate the optimal order \( \pi(j) \) with the increasing order of number of bits of packets, to get a lower distortion cost than any other order, since (1) increases as \( B_i \) increases.

**Theorem 2.** For a fixed order of packet transmission \( \pi \), Problem 2 is jointly convex problem in \( E_i \) and \( t_i \).

Proof can be found in Appendix A. Using the joint convexity, Problem 2 can be solved efficiently by any of the convex solvers. Typically, for jointly convex problems KKT conditions can be used to find closed form expressions for optimal solutions. In this case, however, KKT conditions lead to exponential functions in the variables of interest that cannot be solved in closed form. Thus, one has to rely on the convex solvers to solve this problem for a fixed \( \pi \), and optimize over \( \pi \) thereafter. In the next section, we present a structural solution to the problem that obviates the need for using convex solvers.

4. Discretized Variant of Problem 2

In this section, we work towards finding a more structured solution for a discretized variant of Problem 2 for which we can find theoretical guarantees on the performance. To facilitate this, we consider a discretized version of Problem 2, where time is divided into discrete slots of short fixed length \( \ell \). In each slot, bits from at most one packet can be sent, however, bits from the same packet can be sent in multiple non-contiguous slots. Let set \( \mathcal{P}_i \) be the set of slots assigned to packet \( i \). We also discretize the energy into small units of \( e \) each, that is the least amount of energy that will be used in one slot. We define a resource block as a rectangle of height \( e \) and width \( \ell \) (slot time). For slot \( j \), the number of resource blocks is defined as \( R_j \). Since any one slot is reserved for bits from any one packet, \( eR_j \) is the amount of energy used for transmission of bits for packet \( i \) if \( i \in \mathcal{P}_j \).

For slots \( j \in \mathcal{P}_i \), the cumulative bits sent using resource blocks \( R_j \) is \( \hat{B}_j \), where

\[
\hat{B}_j = \ell \log \left( 1 + \frac{eR_j}{\ell} \right).
\]

Then the cost \( D_i \) for packet \( i \) is

\[
D_i = \sum_{j=1}^{J} eR_j, \quad \text{where } i_{\max} = \max\{j : j \in \mathcal{P}_i\}
\]

is the last slot where any bits of packet \( i \) are sent.

The total energy consumption under this setup is

\[
\sum_{j=1}^{J} eR_j, \quad \text{where } J \text{ is the total number of slots used for transmission. Then the optimization problem is}
\]

\[
\min_{\mathcal{P}_i, R_i, eR_i, e} \sum_{i=1}^{\mathcal{P}_i} D_i.
\]

Problem 5 is a discrete optimization problem, since \( \mathcal{P}_i \) and \( R_j \) are discrete sets and only one packet can
be assigned to any one slot. Thus, it is not evident that Problem 5 is any easier than Problem 2. To understand how to efficiently solve Problem 5, we need the following preliminaries.

Definition 1. Let \( V \) be a finite set, and let \( 2^V \) be the power set of \( V \). A real-valued set function \( f : 2^V \rightarrow \mathbb{R} \) is said to be monotone if \( f(S) \geq f(T) \) for \( S \subseteq T \subseteq V \), and super-modular if
\[
f(S) + f(T) \leq f(S \cap T) + f(S \cup T), \quad \forall S, T \in 2^V.
\]

An equivalent definition of super-modularity is
\[
f(S \cup \{i\}) + f(S \cup \{j\}) \leq f(S) + f(S \cup \{i, j\}) \tag{6}
\]
for every \( S \subseteq V \) and every \( i, j \in V \setminus S \) with \( i \neq j \).

Let \( S \) be a finite set and let \( f_j, 1 \leq i \leq k \), be set functions from \( 2^S \) to the real numbers. The multi-partitioning problem is defined as follows.

Definition 2. Multi-partitioning problem: Partition a given (resource) set \( S \) into \( k \) subsets \( S_1, \ldots, S_k \), \( S_i \cap S_j = \phi \) if \( i \neq j \), such that
\[
\sum_{i=1}^{k} f_j(S_i) \text{ is minimized.}
\]

Theorem 3 ([12], [13]). If all functions \( f_j \) in the multi-partitioning problem are non-negative, monotone, and super-modular, then the GREEDY algorithm outputs a partition whose cost is at most twice that of the optimal partition.

Problem 5 is a multi-partitioning problem, where the resource set \( S \) is the set of resource blocks \( R_i \) for \( P_i \) that needs to be partitioned among the \( n \) packets, such that the total energy constraint is satisfied \( \sum_{j=1}^{n} e_{R_j} \leq E \), where slot \( j \in P_i \) for some \( i \) (packet). One major difference is in the definition of resource blocks that are dynamic for Problem 5 rather than being fixed ahead of time. To be clear, given a set of existing resource blocks, \( R_{ij}, j \in P_i \), \( R_{ij} \rightarrow R_{ij} + 1 \) is allowed only when if additional bits from packet \( i \) are sent using extra energy \( e \) in that slot. Moreover, for any \( R_{ij1}, R_{ij2}, j_1, j_2 \in P_i \), a new resource block is added to \( R_{ij1} \) or \( R_{ij2} \), depending on which one reduces the cost for packet \( i \) more. Only fixed constraint is that the total number of resource blocks is at most \( E/e \), i.e., \( \sum_{j=1}^{n} e_{R_j} \leq E \).

To solve Problem 5, consider the GREEDY algorithm in Fig. 2, which allocates a new resource block to the packet that reduces the incremental cost the most. The novelty in this greedy algorithm is that both the packet index set \( P_i \) (which slot to assign for packet \( i \)) and which packet to assign to a new resource block (if created at a previously un-allotted slot) is being found out greedily, since given the existing set of resource blocks \( R_{ij}, j \in P_i \), a new resource block can be created at any of the existing slots where \( R_{ij} > 0 \) i.e., \( j \in P_i \) for some \( i = 1, \ldots, n \) or at a new slot where \( R_{ij} = 0, j \notin P_i \) for any \( i = 1, \ldots, n \).

We illustrate the functioning of the GREEDY algorithm for solving Problem 5 in Fig. 4. In the considered iteration of the GREEDY algorithm, the distinct colored (other than cyan) rectangles of Fig. 4 are resource blocks that have been already assigned to different packets, where the same color represents blocks that are assigned to the same packet. The new resource block that is to be assigned is among the candidate resource blocks (denoted A) that could either be allocated to one of the slots \( (j^*) \) that is already occupied by some packet or a completely new slot, depending on which choice makes the largest decrease in cost, given the earlier allocation. In case, a resource block is assigned to a previously empty slot, the algorithm also describes which packet \( (i^*) \) should be assigned to that block.

Theorem 4. If all functions \( D_i \) in Problem 5 are non-negative, monotone, and super-modular, then the GREEDY algorithm outputs a partition (allocation of resource blocks) within a factor of 2 of the optimal partition.

Proof is similar to Theorem 3.

To make use of Theorem 4, we now show that the cost functions \( D_i \) are non-negative, monotone and super-modular as follows.

Lemma 1. The cost function \( D_i \) is non-negative, monotone and super-modular.

Proof. The non-negativity of \( D_i \) is obvious, since the first term is an exponential function, while the second term is linear. To show monotonicity, we need to show that \( D_i(\bigcup_{j \in P_i} R_j \cup \{r\}) \leq D_i(\bigcup_{j \in P_i} R_j) \) for any packet \( i \), where \( r \) is a new resource block that is not part of \( \bigcup_{j \in P_i} R_j \). By the definition of the resource block as described earlier, a new resource block corresponding to packet \( i \) is either added to the slots that are already allotted to that packet, i.e., \( P_i \) or to an un-allotted slot depending on which ever one gives larger decrease in cost. Adding a new resource block to the existing time slot \( P_i \) clearly increases the number of bits \( B_i \) for packet \( i \) while not increasing the delay, thereby decreasing the cost \( D_i \). Thus, adding a new resource block to either \( P_i \) or an un-allotted slot cannot increase the cost \( D_i \), thus proving monotonicity. The super-modularity of \( D_i \) is also easy to see since function \( 2^{B_i-B_j} \) is convex and the delay term is linear.

Thus, we have the following Theorem for Problem 5.

Theorem 5. The resource block allocation output by the GREEDY algorithm 2 for Problem 5 has cost that is at most 2 times the optimal cost.

5. Simulation Results

In this section, we present some simulation results for the optimal solution output by convex solvers for Problem 2. In Fig. 4, we consider two packets, and total energy \( E = 50 \) Joules. We plot two curves for the overall cost \( U \) in Fig. 4, where in each, the size
GREEDY algorithm

1. Initialize $P_i = \emptyset$, $R_j = 0$, $R_j \in P_i$, $\forall i$.
2. Let last = $\max\{j : j \in P_i\}$ for some i
3. For slot j = 1 : last
4. If energy is not exhausted : $\sum_{j=1}^{\text{last}} eR_j \leq E$.
5. Find the slot j (or packet i : j $\in P_i$) that benefits the user i most by allocating
   a new resource block with $R_j = R_j + 1$, i.e.,
   $j^* = \arg\min_{j=1,\ldots,\text{last}, j \in P_i} D_i(R_{j-1} \cup R_j) + 1$ where $R_{j-1} = \cup_{k \in P_i} R_k \setminus R_j$
6. Find the packet i, i = 1, . . . , n that benefits most by allocating first resource block at slot last + 1, i.e.,
   $i^* = \arg\min_{i=1,\ldots,n, j \in P_i} D_i(R_j \cup \text{last}+1)$
7. Make $R_j = R_{j^*} + 1$ if $\min_{i=1,\ldots,n, j \in P_i} D_i(R_{j-1} \cup R_j) + 1 < \min_{i=1,\ldots,n, R_i \in P_i} D_i(R_j \cup \text{last}+1)$
   Otherwise create a new slot and assign it to $i^*$, i.e., slot last + 1 $\in P_i^*$ and $R_{\text{last}+1} = 1$
8. Update last = $\max\{j : j \in P_i\}$, go to step 3
End
9. Return $R_j$, $j \in P_i$.
Appendix A.
Proof of Theorem 2

Here we give a proof about joint convexity of cost function $U$, where $U = \sum_{i=1}^{n} 2B_i - B_i + a_i \ell_i$, for some constant $a_i > 0$ that depends on the order of transmission of packets $\pi$.

There are $2n$ unknown variables ($E_s$ and $\ell_s$ for each packet). So Hessian matrix is $2n \times 2n$ sized matrix given by

$$H = \begin{pmatrix}
\frac{d^2U}{dE_1^2} & \frac{d^2U}{dE_1dE_2} & \frac{d^2U}{dE_1dE_n} & \frac{d^2U}{dE_2dE_1} & \frac{d^2U}{dE_2dE_2} & \frac{d^2U}{dE_2dE_n} & \cdots & \frac{d^2U}{dE_n dE_1} & \frac{d^2U}{dE_n dE_2} & \cdots & \frac{d^2U}{dE_n dE_n}
\end{pmatrix}$$

Clearly, the Hessian matrix is block diagonal, and since eigen-values of block diagonal matrix are eigen-values of each block, to prove the joint convexity, we need to prove that the eigen-values of each block are positive, to prove that the Hessian matrix is positive semi-definite.

Thus, it is sufficient to show that any of the block, say the first block $H_1 = \begin{pmatrix}
\frac{d^2U}{dE_1^2} & \frac{d^2U}{dE_1dE_2} & \cdots & \frac{d^2U}{dE_1dE_n}
\end{pmatrix}$ is positive-definite to prove the joint-convexity of $H$.

Writing derivative terms

$$\frac{d^2U}{dE_1^2} = P \frac{\ell_1 + 1}{\ell_1 + E_1},$$

$$\frac{d^2U}{dE_1dE_2} = -P \left( \frac{\ell_1 + 1}{\ell_1 + E_1} + \frac{\ell_1}{\ell_1 + E_2} \right) \log_2 \left( \frac{\ell_1}{\ell_1 + E_1} \right),$$

$$\frac{d^2U}{dE_1dE_n} = P \left( \log_2 \left( \frac{\ell_1}{\ell_1 + E_1} + \frac{E_1}{\ell_1 + E_{1 + 1}} \right) \right) + \frac{E_1^2}{(\ell_1 + E_{1 + 1}) \ell_1},$$

where $P = 2B_1 \frac{E_1 + \ell_1}{\ell_1}$.

Next, we show that $H_1$ has a positive determinant.

$$\det(H_1) = P^2 \left( \frac{\ell_1 + 1}{\ell_1 + E_1} \right)^2 \left( \log_2 \left( \frac{\ell_1}{\ell_1 + E_1} \right) + 2 \log_2 \left( \frac{\ell_1}{\ell_1 + E_1} \right) \right) + \frac{\ell_1}{\ell_1 + E_1} \frac{E_1^2}{(\ell_1 + E_1)^2 \ell_1}$$

$$- P^2 \left( \frac{\ell_1}{\ell_1 + E_1} \right)^2 \log_2 \left( \frac{\ell_1}{\ell_1 + E_1} \right) + \frac{\ell_1}{\ell_1 + E_1} \frac{E_1^2}{(\ell_1 + E_1)^2 \ell_1} + 2 \log_2 \left( \frac{\ell_1}{\ell_1 + E_1} \right) \frac{\ell_1}{\ell_1 + E_1} \frac{E_1^2}{(\ell_1 + E_1)^3 \ell_1},$$

which is clearly positive, hence joint convexity of our objective function is proved.

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