Dynamic Kalman filtering to separate low-frequency instabilities from turbulent fluctuations: Application to the Large-Eddy Simulation of unsteady turbulent flows

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Abstract. A dynamic method based on Kalman filtering is presented to isolate low-frequency unsteadiness from turbulent fluctuations in the large-eddy simulation (LES) of unsteady turbulent flows. The method can be viewed as an adaptive exponential smoothing, in which the smoothing factor adapts itself dynamically to the local behavior of the flow. Interestingly, the proposed method does not require any empirical tuning. In practice, it is used to estimate a shear-improved Smagorinsky viscosity, in which the low-frequency component of the velocity field is used to estimate a correction term to the Smagorinsky viscosity. The LES of the flow past a circular cylinder at Reynolds number \(Re_D = 4.7 \times 10^4\) is examined as a challenging test case. Good comparisons are obtained with the experimental results, indicating the relevance of the shear-improved Smagorinsky model and the efficiency of the Kalman filtering. Finally, the adaptive cut-off of the Kalman filter is investigated, and shown to adapt locally and instantaneously to the complex flow around the cylinder.

1. Introduction

Large-eddy simulation (LES) of turbulent flows refers to the numerical integration of fluid dynamical equations on a mesh, whose resolution is reduced. Therefore, only the large-scale motions are resolved. This is physically justifiable since large-sized eddies contain most of the kinetic energy, and their strength make them the efficient carriers of momentum. On the contrary, small-sized eddies are mainly responsible for dissipation and contribute little to momentum transport. Avoiding the numerical integration of small-scale motions is therefore desirable in most situations. The large-scale motions are solutions of the flow equations, e.g. the Navier-Stokes equations, supplemented by an unknown term accounting for the stress exerted by the unresolved subgrid-scale (SGS) excitations on the simulated flow. A common thread is to consider that this stress is essentially responsible for diffusion at grid scale, which in turn calls for the modeling of an additional SGS viscosity in the large-scale-flow equations. Unlike the constant-property molecular diffusion coefficients, the SGS viscosity is here a space-and-time-dependent quantity that is closely related to the nature of turbulent motions. In the context of engineering flows, which may experience unsteady events such as shear-layer mixing, vortex...
shedding or external disturbances, the modeling of the SGS viscosity is recognized to be a difficult problem.

An elementary model for the SGS viscosity has been introduced by Smagorinsky (1963):

$$\mu_{SGS} = \bar{\rho} (C_s \Delta)^2 |\mathbf{S}|,$$

where $C_s = 0.18$ is the Smagorinsky constant, $\Delta$ is the local grid spacing and $|\mathbf{S}| = \sqrt{2 S_{ij} S_{ij}}$ is a norm of the resolved rate-of-strain tensor ($S_{ij} = \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$).

The Smagorinsky model is mostly limited to isotropic turbulence and fails to properly account for flow regions affected by a strong shear (e.g. near-wall flows) or transitional behaviors. The shear-improved Smagorinsky model (SISM) is a simple evolution of the original Smagorinsky model accounting for the effect of a shear, at a moderate computational cost (Lévéque et al., 2007). In the SISM, the SGS viscosity is calculated as

$$\mu_{SGS} = \bar{\rho} (C_s \Delta)^2 \left( |\mathbf{S}| - |\mathbf{\tilde{S}}| \right).$$

This expression arises from a careful analysis of the scale-by-scale energy budget of locally homogeneous shear flow (Lévéque et al., 2007). The expression is fairly simple, and only introduces a correcting term to the original Smagorinsky formulation. This term involves the rate-of-strain of the mean component of the velocity field, i.e. $\mathbf{\tilde{S}}$. The performance of the SISM has been evaluated on a plane-channel-flow configuration (Lévéque et al., 2007), and a backward-facing-step flow by (Toschi et al., 2006). In these configurations, homogeneous directions allow the use of averaging in space to evaluate $\mathbf{\tilde{S}}$. However, practical flow configurations are generally inhomogeneous. They can also involve unsteadiness (vortex shedding, rotor/stator blade rows in a turbine, etc.) that generally occurs at relatively low frequency with respect to the turbulent spectrum range. Consequently, the present study proposes to replace the averaging (overtilde symbol) by a low-pass filtering in time, i.e. temporal smoothing.

2. Adaptive Kalman filtering

In the context of large-eddy simulation, spatial filtering is a classical tool for physical reasons (e.g. scale separation) or numerical purposes (e.g. damping of numerical instabilities). However, time filtering offers an interesting alternative for some applications (cf. Pruett et al. (2006)). Indeed, time is unidimensional and generally discretized with a constant time-step. Consequently, time-filter stencils are generally simpler. A simple way to extract the low frequency component of a time signal is to smooth it out by a weighted moving average (in time). A well-known algorithm is the exponential smoothing, which applies exponentially-decreasing weights to the contribution of previous instants. It formulates very conveniently in a recursive manner: the smoothed variable $\tilde{u}(x)^{(n)}$, at instant $n$, of a given variable $\bar{u}(x)^{(n)}$ satisfies the recursion:

$$\tilde{u}(x)^{(n+1)} = (1 - \alpha) \cdot \tilde{u}(x)^{(n)} + \alpha \cdot u(x)^{(n+1)},$$

at some arbitrary position $x$ in the flow. The smoothing factor $\alpha$ controls the influence of the past iterations on the average; the higher $\alpha$, the faster the observations are discounted. Formally, this smoothing can be regarded as a first-order low-pass filter, whose cut-off frequency expresses as:

$$f_c = \frac{\sqrt{3\alpha}}{2\pi \Delta t},$$

where $\Delta t$ is the time step of the velocity signal (cf. Cahuzac et al. (2010)).
An obvious limitation of the exponential smoothing is the requirement to select an \textit{a priori} physically-relevant cut-off frequency (or a smoothing factor) for the whole flow. In unsteady non-homogeneous turbulent flows, one expects the smoothing factor to vary both in time and space. This motivates the development of an \textit{adaptive exponential smoothing} (our main contribution) in which the smoothing is made according to (3) but with a smoothing factor $\alpha^{(n)}(x, t)$ that is now inferred locally from the flow itself, as a function of time. The starting point is to formulate the relationship between the low-frequency component of the velocity and the (complete) velocity as a crude state-space model, namely a simple random walk model. From that, the inference on $\alpha^{(n)}(x, t)$ relies on Kalman filtering (Harvey, 1989).

In brief, given the postulated state-space model, Kalman filtering yields a sequential prediction of the state, here the smoothed flow. The theory shows that, for a random walk model, the Kalman filters can be written as an exponential smoothing where the Kalman gain $\alpha^{(n)}(x, t)$ plays the role of the smoothing factor. It is given by the ratio between the predicted error variance $P^{(n)}$ for the low-frequency component and the error variance for the complete velocity. More precisely, the recursion for the evolution of these variables writes

$$
\alpha^{(n)}(x) = P^{(n)}(x)/\left( P^{(n)}(x) + \sigma^2_u(x) \right)
$$

and

$$
P^{(n+1)}(x) = \left( 1 - \alpha^{(n)}(x) \right) \cdot P^{(n)}(x) + \sigma^2_u(x).
$$

An estimate of the two variances $\sigma^2_u$ and $\sigma^2_e$ is required at each time and position $x$. Our Kalman filter is designed such that, in a steady state, it reduces to an exponential smoothing with a constant smoothing factor $\sigma_u/\sigma_u$. Accordingly, $\sigma_u^{(n)}(x) = 2\pi f_c \Delta t \cdot u_c/\sqrt{3}$ is kept constant, whereas $\sigma^2_e^{(n)}(x) = u_c \cdot |\tilde{u}(x)^{(n)} - \tilde{u}(x)^{(n)}|/\Delta t$ is updated at each time and position; $f_c$ and $u_c$ are fixed reference frequency and velocity necessary for the calibration of the Kalman filter. An additional constraint is to impose a (non-zero) lower bound on $\sigma^2_u$ so that the Kalman gain does not diverge in laminar regions, where the filtered velocity is equal to the complete velocity: $\min(\sigma^2_u) = 0.1 \cdot u_c^2$ is chosen in practice.

3. Solver and test-case configuration

3.1. Solver: \textit{Turb'Flow}

\textit{Turb'Flow} (cf. Smati \textit{et al.} (1998), Boudet \textit{et al.} (2007)) solves the compressible flow equations for an ideal gas. It is based on a finite-volume discretization for multiblock structured grids. In the present study, a LES approach is used, relying on the SISM model with adaptive Kalman filtering. Convective fluxes are interpolated with a 4-point centered stencil, and 4th order numerical viscosity (cf. Jameson (1982), coefficient: 0.01). Viscous fluxes are interpolated with a 2-point centered stencil. A 3-step Runge-Kutta scheme is used for the unsteady resolution. Boundary conditions are imposed through compatibility relations and supplementary nodes.

3.2. Test-case configuration: flow past a cylinder in subcritical regime

The capabilities of the method are now investigated by considering the LES of the flow past a circular cylinder, at Reynolds number $Re_D = 4.7 \times 10^4$ (calculated from the rod diameter $D = 0.01m$, and the upstream flow velocity $U\infty = 70$m/s). In this regime, the attached boundary layers are laminar, and transition occurs in the separated shear layers. The wake is turbulent, but governed by large vortices shed at a Strouhal number $St = f_{vs} \cdot D/U\infty \sim 0.2$ (where $f_{vs}$ is the vortex shedding frequency). Both cartesian coordinates ($x$: upstream flow direction, $y$: cross-stream, $z$: span-wise) and cylindrical coordinates ($r$: radial, $\theta$: azimuthal, $z$: span-wise) are used, with origin at the center of the cylinder. The computational domain extends over 3 diameters in the span-wise direction ($z$). The structured grid is mostly cylindrical,
with an increased density in the separated region, downstream of the cylinder. In this turbulent region, the cell dimensions at the wall are: $\Delta r^+ \approx 1$, $r^+ \Delta \theta^+ \approx 15$ and $\Delta z^+ \approx 24$ (in wall units). This resolution agrees with standard practice in LES, according to Sagaut (2001). The nondimensional time step is: $\Delta t U_\infty / D = 4 \times 10^{-1}$. Periodic boundary conditions are imposed in the spanwise direction, in order to make the computation representative of higher aspect ratios (span/diameter). A no-slip adiabatic condition is imposed at the wall. The components of velocity and the static temperature are imposed at the inflow, while the static pressure is imposed at the outflow.

The smoothing algorithm requires for its calibration a reference frequency and a reference velocity. These two values are chosen respectively equal to the expected vortex-shedding frequency $f_{vs} = St \cdot U_\infty / D = 1400$ Hz (based on a Strouhal number $St = 0.2$) and the upstream flow velocity $U_\infty$.

4. Flow results and comparisons with experimental data

First, instantaneous snapshots of both the flow field and the smoothed field are presented in Fig. 1, at the same instant. In the LES flow field (left picture), the vorticity shows the diversity of the turbulent scales in the wake, embedded within the large vortices shed from the cylinder. In comparison, the smoothed flow field (right picture) mostly retains the largest scales of the flow, including the vortex shedding. This is exactly the behavior expected for the calculation of the SISM viscosity.

The global characteristics of the flow are summarized in Table 1: mean drag, root-mean-squared drag and lift, and vortex-shedding Strouhal number. They are compared with various experimental data, at different Reynolds numbers in the same regime. The LES forces (mean and fluctuating) fall within the experimental ranges. This is a first indication of the quality of the simulation. The vortex-shedding Strouhal number is calculated with the peak frequency from the Fourier transform of the pressure signal on a shoulder of the cylinder $(\theta = 90 \, \text{deg})$. Physically, the shedding frequency is closely controlled by the size of the recirculation downstream of the cylinder. This quantity is also well predicted by the LES.

The fluctuating pressure distribution around the cylinder is plotted in Fig. 2, in comparison with experimental data. This is a particularly delicate quantity to capture, and a sensitivity to subgrid-scale modeling has been observed by Cahuzac et al. (2010). Here, a rather good
Table 1. Global characteristics of the flow: LES in comparison with various experimental data from the literature (at different Reynolds numbers, in the same regime).

| Characteristic | LES (Re = 4.7 × 10^4) | experimental data in literature |
|----------------|-----------------------|-------------------------------|
| \(\langle C_D \rangle\): mean drag coefficient | 1.230 | 1.35 Szepessy & Bearman (1992) (Re = 4.3 × 10^4) [0.98, 0.8] Cantwell & Coles (1983) (Re = 4.8 × 10^4) [1.0, 1.3] Achenbach (1968) (Re = 4.8 × 10^4) [1.1, 1.3] Zdravkovich (2002) (Re ∈ [10^4, 10^5]) |
| \(C_D'\): rms drag coefficient | 0.065 | 0.16 Szepessy & Bearman (1992) (Re = 4.3 × 10^4) [0.05, 0.1] Gerrard (1961) (Re = 4.8 × 10^4) [0.06, 0.1] Zdravkovich (2002) (Re ∈ [10^4, 10^5]) |
| \(C_L'\): rms lift coefficient | 0.603 | 0.45, 0.55 Szepessy & Bearman (1992) (Re = 4.3 × 10^4) [0.4, 0.8] Gerrard (1961) (Re = 4.8 × 10^4) [0.6, 0.82] Zdravkovich (2002) (Re ∈ [10^4, 10^5]) |
| St: vortex-shedding Strouhal number | 0.204 | 0.18, 0.22 Cantwell & Coles (1983) (Re = 4.8 × 10^4) [0.185, 0.195] Norberg (2003) (Re = 6.1 × 10^4) |

Figure 2. Coefficient of the root-mean-squared pressure fluctuations around the cylinder. — : LES at Re = 4.7 × 10^4; □: experimental data at Re = 6.1 × 10^4 Norberg (2003); ○: experimental data at Re = 6.1 × 10^4 Nishimura & Taniike (2001); △: experimental data at Re = 10^5 Yokuda & Ramaprian (1990); ◊: experimental data at Re = 10^5 Batham (1973).

agreement is observed with the experimental data obtained at lower (and most comparable) Reynolds number. Interestingly, a Reynolds number dependency seems to appear in this figure, from the different results gathered: the lower the Reynolds number, the higher the fluctuating pressure coefficient.

Finally, a spectral analysis of the velocity in the wake is presented in Fig.3. This power spectral density is calculated from the instantaneous component of velocity in the plane perpendicular to the cylinder (as measured by single hot-wire), from a probe at \(x = 1.3D\) and \(y = 0.5D\). As already mentioned, the LES predicts accurately the frequency of the peak associated with the vortex shedding. The amplitude of the peak is also in rather good agreement with the experiment. The broadband level, associated with the turbulent scales, is also well captured by the LES. However, the slope of the decay at the highest frequencies appears under-predicted. This needs to be further investigated, as it is related with subgrid-scale modeling.

5. Analysis of the Kalman filter behavior

The adaptive behavior of the Kalman filter is investigated in Fig.4, where the equivalent cut-off frequency is shown on a cutting plane, at a given instant. The equivalent cut-off frequency is calculated by analogy with the exponential smoothing, substituting the Kalman gain \(\alpha(x, t)\)
Figure 3. Velocity spectrum measured at $x = 1.3D$ and $y = 0.5D$. —— : experiment from Jacob et al. (2005), - - - : present LES.

Figure 4. Instantaneous cut-off frequency of the Kalman filter, normalized by the vortex shedding frequency, on a cutting plane at constant $z$.

within equation (4) from the exponential smoothing. This yields:

$$f_c(x, t) = \frac{\sqrt{3} \alpha(x, t)}{2\pi \Delta t},$$

(7)

In steady state, this would exactly equates the cut-off frequency. In Fig.4, the adaptive cut-off frequency is shown to be highest in the laminar (unperturbed) regions of the flow, where the smoothed flow accompanies closely the instantaneous flow. On the contrary, the cut-off frequency is lowest in the most perturbed region of the flow: the wake. In this region, the cut-off frequency (and thus the Kalman gain) is lowest, and reaches normalized values around 1.5. Consequently, the largest scales (in particular: the vortex shedding, associated with normalized frequency equal to 1) are separated from the high-frequency turbulent fluctuations.

Finally, the instantaneous angle of separation of the boundary layer (associated with null
friction) is presented in Fig.5, at a given spanwise position \((z)\), on the upper shoulder of the cylinder. It is plotted for both the LES flow field and the smoothed flow field. The angle of separation of the instantaneous flow field does not exhibit turbulent fluctuations. It is governed by the vortex shedding frequency, with modulations of the amplitude. Indeed, the upstream boundary layer is laminar, but the large vortex shedding induces potential effects traveling upstream. Consequently, the smoothed angle of separation is nearly identical to the instantaneous one, because the vortex shedding frequency is retained within the smoothed field. However, there is a delay between the instantaneous angle and the smoothed one (about 15% of the vortex shedding period). This is a classical issue with Kalman filtering. Considering the expression of the SISM SGS viscosity (eq. (2)), any phase lag between \(|\vec{S}|\) and \(|\tilde{\vec{S}}|\) can induce a misprediction of the model viscosity. This issue is currently investigated: a posteriori tests already show that introducing the slope of the smoothed velocity variation in the filter formulation reduces the time delay associated, at the price of an increased computational cost (Cahuzac et al., 2011).

6. Conclusion

An innovative adaptive-smoothing algorithm has been introduced in the context of LES. It enables extraction of the low-frequency component of the flow field, in unsteady conditions and complex geometries. It has been used for identifying the mean component of shear, as required for subgrid-scale modeling by the SISM model. The capabilities of this approach have been investigated on a particularly challenging test-case: the flow past a circular cylinder in subcritical regime, combining vortex shedding and turbulence in the wake. Comparisons with experimental data are good, regarding mean and fluctuating forces, vortex shedding frequency, wall pressure fluctuations and a velocity spectrum in the near wake. A discrepancy is observed on the spectral slope at high frequencies, and requires further investigations, in relation with the subgrid-scale modeling and the associate smoothing. Finally, the cut-off frequency of the Kalman filter is shown to adapt to the local and unsteady flow, with higher values in the laminar regions, and lower values (about the vortex shedding frequency) in the near wake. A classical delay associated with filtering is observed on the separation angle, between the unsteady and smoothed flows. This probably induces errors in the calculation of the subgrid-scale viscosity, but evolutions of the Kalman filter are currently investigated to reduce this delay.
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