Resilient Task Allocation in Heterogeneous Multi-Robot Systems

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Abstract—For a multi-robot system equipped with heterogeneous capabilities, this paper presents a mechanism to allocate robots to tasks in a resilient manner when anomalous environmental conditions such as weather events or adversarial attacks affect the performance of robots within the tasks. Our primary objective is to ensure that each task is assigned the requisite level of resources, measured as the aggregated capabilities of the robots allocated to the task. By keeping track of task performance deviations under external perturbations, our framework quantifies the extent to which robot capabilities (e.g., visual sensing or aerial mobility) are affected by environmental conditions. This enables an optimization-based framework to flexibly reallocate robots to tasks based on the most degraded capabilities within each task. In the face of resource limitations and adverse environmental conditions, our algorithm minimally relaxes the resource constraints corresponding to some tasks, thus exhibiting a graceful degradation of performance. Simulated experiments in a multi-robot coverage and target tracking scenario demonstrate the efficacy of the proposed approach.

I. INTRODUCTION

In recent years, heterogeneous multi-robot systems have demonstrated a potential to achieve complex real-world objectives due to their versatility in accomplishing specialized tasks which might require collaboration among different types of robots, e.g. [1], [2], [3]. A crucial step towards achieving such behaviors is multi-robot task allocation (MRTA), which concerns itself with allocating robots to tasks in such a way that the resources required to execute the tasks successfully are made available (see [4], [5], [6] for a taxonomy and survey of the topic). For instance, a possible approach is to classify the robots according to their heterogeneous capabilities (e.g., speed, sensor range, battery life, etc), and then assign aggregated capabilities to each task, based on given specifications [7], [8].

For heterogeneous multi-robot systems operating in dynamic and complex environments, the diversity in the capabilities of the robots presents another advantage—resilience: the ability to continuously operate and recover from failures with limited resources, e.g. [9], [10]. In our context, when a multi-robot system experiences difficulties in executing tasks due to changing environmental conditions or certain types of adversarial attacks, reallocating robots to tasks can significantly improve their performance as a team, e.g., [11]. Such a reallocation can take different forms, based on the type of failure that has occurred. For instance, if a team of ground robots tasked with surveilling an area encounters slippery terrain, a reallocation of aerial robots to the task might be desirable. However, if an adversarial attack were to reduce the effective communication range of the ground robots, supplying additional robots of the same kind to act as intermediate communication links might be a better solution. Note that, in these scenarios, specific capabilities of the robots were affected by disturbances, i.e., ground mobility and communication range, respectively. Hence, a way to facilitate effective and resilient task allocation is by: i) identifying the extent to which the robot capabilities within each task are affected; and ii) performing a suitable reallocation which ensures progress in each of the tasks.

In this paper, we propose a novel heterogeneous multi-robot task allocation framework which explicitly quantifies the extent to which robot capabilities—pertaining to relevant aspects of the robots’ operation such as ground speed or sensor coverage—are degraded by environmental disturbances. The primary objective of our optimization-based formulation is to allocate a team of robots to a set of given tasks in a deterministic manner such that constraints on the minimum aggregate capability requirements for each task are satisfied. Distinct from previous works in the literature [7], [8], [12], we impart resilience to our framework in two ways. First, we explicitly model the fact that, a given task can be accomplished via multiple possible combinations of robot capabilities—one of which can be selected based on the extent to which robot capabilities have been degraded by environmental disturbances. This achieves resilience via reconfiguration—by allowing the algorithm to move robots to tasks where they can contribute the most. Second, in situations where the capabilities of the robots are too degraded to satisfy the requirements for all the tasks, we allow the algorithm to minimally relax the capability requirement constraints for some tasks, to ensure that constraints corresponding to higher priority tasks continue to be met. Such a graceful degradation of performance ensures that infeasible task allocation specifications in the face of significant environmental disturbances are handled effectively.

Leveraging robot heterogeneity in MRTA problems has classically been approached by scoring the ability of each robot to perform different tasks [12], [13], [14], and by explicitly enumerating the various task-related capabilities of the robots [7], [15]. The above discussed features of the proposed task allocation algorithm are owed to a quantifiable understanding of how different robot capabilities are degraded due to changing environmental conditions. Towards
this end, we restrict our framework to tasks whose execution can be encoded as the minimization of a non-negative scalar cost function, such as in distributed coordinated multi-robot tasks [16], [17], [18]. At every point in time, we allow each robot to measure the discrepancy between the expected and measured progress that it makes towards minimizing its portion of the overall cost function. A similar approach is presented in [11], where the real-time performance of robots at tasks is used to modify the suitability of robots towards different tasks. In this paper, we instead leverage the heterogeneity model to identify which capabilities are primarily responsible for the observed performance deviations—thus allowing the algorithm to make more expressive reallocation decisions.

The capability degradation metrics are then leveraged by a centralized mixed-integer quadratic program (MIQP) which i) selects a capability configuration that is best suited for each task, ii) generates the robot-to-task allocations to meet the requirements set by the chosen configuration, and iii) minimally violates the constraints corresponding to the allocation requirements of some tasks, if required. Our framework deploys robots in a resource-aware manner, by minimizing the team size and allowing the mission designer to specify a cost of deployment for each type of robot. Similarly, the mission designer can also specify which tasks are less critical to the mission than others (and hence should be degraded in quality first). Lastly, robots experiencing high performance degradation—based on a user defined threshold—are automatically excluded from the allocation process.

To circumvent the computationally intensive nature of solving MIQPs frequently, we present an event-triggered execution framework, where the MIQP is solved only when the estimated capability degradations change beyond a certain threshold. Figure 1 illustrates the system architecture for the resilient task allocation paradigm presented in this paper. \( V_i \) denotes the task cost associated with robot \( i \), which is used to compute the difference between the measured and predicted performance of the robots. This information is used to compute degradation metrics for the different robot capabilities by the mission evaluation block which decides if a reallocation of robots to tasks is warranted or not.

### II. TASK PERFORMANCE EVALUATION

In this section, we first characterize the heterogeneity within the robot team in terms of the different types of robots available, and the capabilities possessed by each type of robot. This framework is then coupled with a task execution model to quantify the extent to which robot capabilities are affected by environmental disturbances within each of the tasks.

#### A. Robot Heterogeneity

We consider a team of \( N \) heterogeneous robots, indexed by the set \( \mathcal{R} = \{1, \ldots, N\} \). Let \( \mathcal{U} \) denote the total number of unique task-related capabilities available to the robot team, e.g., perception, ground mobility, aerial mobility, object manipulation, etc. Individual robots can exhibit different combinations of capabilities, depending on their size, power, and cost constraints. In the literature, robots with identical sets of capabilities are often said to belong to the same species [19]. Let \( S \) denote the total number of species in the team. Let \( \mathbf{Q} \in \mathbb{R}^{S \times U} \) denote the capability matrix, which specifies the capabilities available to each robot species:

\[
\mathbf{Q} = \begin{bmatrix} \mathbf{q}^{(1)} & \mathbf{q}^{(2)} & \cdots & \mathbf{q}^{(S)} \end{bmatrix}^T,
\]

where \( \mathbf{q}^{(s)} = [q_1^{(s)}, \ldots, q_U^{(s)}]^T \in \mathbb{R}^U \) is a vector describing the capabilities available to species \( s \). Section IV and various examples throughout the paper will demonstrate how physically meaningful values can be assigned to the robot capabilities. Let \( \mathbf{Q} \in \{0, 1\}^{S \times U} \) denote the binary version of \( \mathbf{Q} \), where, \( \mathbf{Q}_{su} = 1 \) if and only if \( q_u^{(s)} > 0 \). Similarly, let \( \mathbf{P} \in \{0, 1\}^{S \times N} \) denote the robot-species mapping matrix, whose binary-valued element \( \mathbf{P}_{si} = 1 \) if and only if robot \( i \) belongs to species \( s \).

#### B. Task Execution

We now introduce a model for the execution of different tasks by the multi-robot team. The following sections will leverage this model to allow each robot to evaluate its performance at a given task. We let \( x_i \in \mathbb{R}^p \) denote the state of robot \( i \in \mathcal{R} \), and \( u_i \in \mathbb{R}^q \) denote the control input, which modifies the state according to the following control-affine dynamics:

\[
\dot{x}_i = f(x_i) + g(x_i)u_i.
\]

Let \( M \) denote the total number of tasks among which the robots must be allocated. Subsequently, let \( \mathcal{T}_m \subseteq \mathcal{R} \) represent the index set of robots that are currently allocated to task \( m \in \{1, \ldots, M\} := \mathcal{M} \). We assume that robots can only contribute to one task at a time, so \( \mathcal{T}_m \cap \mathcal{T}_n = \emptyset, \forall m \neq n \in \mathcal{M} \). Let \( \mathbf{x}_m \in \mathbb{R}^{p|\mathcal{T}_m|} \) represent the stacked ensemble
state of robots allocated to task \( m \), where \( | \cdot | \) denotes the set cardinality operator.

In this paper, we encode the execution of tasks as the minimization of a non-negative scalar cost function. A large class of robotic tasks can be encoded via such a formulation, e.g., when robots modify their states according to the gradient flow of a cost functional [20], which might represent planning, mapping, or target tracking objectives of the robots [21]. To this end, let \( V_m : \mathbb{R}^{[T_m]} \rightarrow \mathbb{R} \) denote the cost function corresponding to task \( m \in \mathcal{M} \). We assume that this cost can be expressed as the sum of robot-wise costs,

\[
V_m(x_m) = \sum_{i \in T_m} V_m^{(i)}(x_m).
\]  

(3)

Note that, the individual robot cost function \( V_m^{(i)} \) can depend on the states of other robots in the task, as is common in coordinated control multi-robot tasks [20].

C. Task Performance Discrepancy

As discussed in Section [3], we would like to endow the robots with an ability to evaluate their performance in the tasks, with the aim of quantifying the extent of degradation of different robot capabilities within each task. Ultimately, these metrics will be leveraged in Section [4] to design a resilient task allocation algorithm. Towards this end, we allow each robot in task \( m, i \in T_m \), to compute a predicted cost function \( \text{pred} V_m^{(i)} \) which represents the value of the cost function at the next time step as predicted by robot \( i \). More specifically, we consider discrete time intervals, indexed by \( t \in \mathbb{N} \), and evenly spaced by a small time interval \( \Delta t \), at which the predicted cost function is computed as,

\[
\text{pred} V_m^{(i)}[t + 1] = V_m^{(i)}(x_m[t]) + \Delta t \frac{dV_m^{(i)}(x_m[t])}{dt}
\]  

(4)

where,

\[
\frac{dV_m^{(i)}(x_m[t])}{dt} = \frac{\partial V_m^{(i)}(x_m[t])}{\partial x_i} \dot{x}_i + \sum_{r \in \mathcal{N}_i} \frac{\partial V_m^{(i)}(x_m[t])}{\partial x_r} \dot{x}_r.
\]  

(5)

Here, \( \mathcal{N}_i \) represents the neighborhood set of robot \( i \), and can be described using a graph embedding—for example, representing physical proximity among the robots [20]. Some examples of multi-robot tasks described in this fashion include coverage control [16], formation control [18], rendezvous [22], and target tracking [17].

At discrete time \( t \), robot \( i \) can then use (2) to compute the predicted cost at time \( t + 1 \). Comparing this against the measured cost function at the next time step allows the robot to evaluate its task performance as discussed next.

Definition 1 (Task Performance Discrepancy). Let \( \Delta V^{(i)}[t + 1] \) denote the discrepancy associated with the task performance of robot \( i \) at time \( t + 1 \), given as,

\[
\Delta V^{(i)}[t + 1] = \min \left( \max \left( 1 - \frac{V_m^{(i)}(x_m[t + 1]) - V_m^{(i)}(x_m[t])}{\text{pred} V_m^{(i)}[t + 1] - V_m^{(i)}(x_m[t])}, 0 \right), 1 \right).
\]  

(6)

For a small time interval \( \Delta t \), the task-performance discrepancy \( \Delta V^{(i)} \) encodes the fractional deviation between how much progress the robot made towards modifying its cost function (encoded in the numerator) and how much progress it expected to make in the same time interval (encoded in the denominator).

As seen in Definition 1 if the robot did not experience any disturbance, the predicted cost \( \text{pred} V_m^{(i)}(x_m[t + 1]) \) and the actual measured cost function \( V_m^{(i)}(x_m[t + 1]) \) would be equal, implying that the task performance discrepancy \( \Delta V^{(i)}[t + 1] \) would be equal to 0. Similarly, a discrepancy value of 1 implies that the robot did not make any progress towards the task execution. As seen in (2), we cap values of \( \Delta V^{(i)} \) which are less than 0 or greater than 1, which correspond to situations where the robot did better than expected, or its actions resulted in an unexpected direction of change of the cost function, respectively. In the following example, we demonstrate how the task performance discrepancy can quantify the real-time disturbances experienced by a multi-robot system.

Example 1. Consider a multi-robot team composed of two robots: a ground “leader” robot \( r_1 \) and an aerial “follower” robot \( r_2 \). The ground robot is tasked with tracking a moving goal and the aerial robot is tasked with maintaining a pre-specified distance with respect to the ground robot. The robot-wise cost functions, whose minimization encodes these objectives, are given as,

\[
V^{(1)} = 0.5||x_1 - g||^2
\]  

(7)

\[
V^{(2)} = 0.5(||x_2 - x_1|| - d)^2,
\]  

(8)

where \( g \) represents the location of the goal and \( d \) represents the desired following distance for the aerial robot. For simplicity, we model the motion of both robots using single integrator dynamics: \( \dot{x} = u \). At time \( t = 0.66s \), gusts of head wind affect the motion of the aerial robot, but not the ground robot. We model this disturbance as a multiplier to the control input applied by the robot. More specifically, for robot \( r_2 \) \( \dot{x}_2 = (1 - w)u_2 \) where \( w \) gradually increases from 0 to 0.3. Figure 2a shows how the task performance discrepancy corresponding to both the robots, evolves. The higher values of discrepancy for robot \( r_2 \) capture the fact that, the robot is making a smaller amount of progress towards minimizing its cost function than it expects, as computed by (6).

D. Capability Degradation Metrics

While (6) gives us the robot-wise task performance discrepancies, it does not tell us which robot capabilities are
affected by environmental disturbances. Towards that end, we assemble the task performance discrepancies of the robots in task \( m \in M \) into a vector denoted as \( \Delta V_m \in [0, 1]^{T_m} \). In the following definition, we use the heterogeneous mappings described in Section II-A to compute a capability degradation metric for the robots in each task.

**Definition 2.** Let \( d_{m}^{*}[t] \in [0, 1]^{U} \) denote the extent to which each capability is degraded within task \( m \) at time \( t \). The higher the score, the more ineffectual the robots having this capability are, at executing task \( m \). We compute this capability degradation metric based on the task performance discrepancy values computed in (9).

\[
d_{m}^{*}[t] = \Theta_m^{T} \Delta V_m[t],
\]

where \( P_{S_m,T_m} \) denotes a submatrix of \( P \) which contains only the rows and columns corresponding to the species and indices of robots currently present in task \( m \), respectively. \( Q_{S_m,-} \) contains the rows corresponding to the species of robots in task \( m \) along with all columns. The rows of \( P_{S_m,T_m} \) and columns of \( Q_{S_m,-} \) are normalized to preserve the value of the disturbances between 0 and 1.

Note that (9) represents the instantaneous capability degradation at time \( t \) based on the task performance discrepancies \( \Delta V_m[t] \). We introduce the following update law to capture a time-averaged version of the capability degradation metrics,

\[
d_{m}[t + 1] = d_{m}[t] + \Delta t \Theta_m[t] \left( d_{m}[t] - d_{m}[t] \right),
\]

where \( d_{m}[t] \) now represents the time-averaged capability degradation at discrete time \( t \). Here, \( \Theta_m[k] \) is a binary diagonal matrix, whose \( u^{th} \) diagonal element indicates whether capability \( u \) is currently available on any robot allocated to task \( m \), defined as,

\[
\Theta_m[k] = \text{diag}(1^{T} Q_{S_m,-})
\]

where for \( g \in \mathbb{R}^n \), \( \text{diag}(g) = G \in \mathbb{R}^{n \times n} \) and the columns of \( Q_{S_m,-} \) are normalized as before. The introduction of \( \Theta_m \) allows us to update only the degradation values for the capabilities which are currently deployed in task \( m \), and keep the other values constant. The following example continues the scenario presented in Example 1 to illustrate how the update law presented in (10) can be leveraged.

**Example 2.** For the heterogeneous multi-robot team presented in Example 1 we first specify the capability matrix \( Q \), which consists of three capabilities—perception (measured in terms of the area that the robot can sense around it), ground mobility, and aerial mobility (both measured in terms of speed):

\[
Q = \begin{bmatrix} 10 \text{ m}^2 & 2 \text{ m/sec} & 0 \text{ m/sec} \\ 10 \text{ m}^2 & 0 \text{ m/sec} & 5 \text{ m/sec} \end{bmatrix}.
\]

The robot-species mapping is simply: \( P = I_2 \), and \( \Theta = I_3 \) since all three capabilities are present in the task. For the same scenario presented in Example 1 Fig. 2a plots each element of the capability degradation metric \( d \) computed according to the update law presented in (10) (note that the task index is hidden). As seen, the degradation metric for aerial mobility increases as the task performance discrepancy for the aerial robot is mapped to the capabilities it possesses using (9). The degradation metric for the perception capability also increases, where the lower magnitude is explained by the fact that it represents the average degradation experienced in this capability by the ground and the aerial robot (the former of which is unaffected by the wind).

III. RESILIENT TASK ALLOCATION

In this section, we develop an optimization-based task allocation framework which meets the resilience objectives described in Section I. Towards this end, we take into account the fact that, tasks can often be accomplished with one of multiple possible capability configurations. For example, a surveillance task over a large region could be accomplished by slow moving ground robots with large perception ranges, or fast moving aerial robots with smaller perception ranges. This notion is formalized in the definition below.
Definition 3 (Task Requirement Matrix). Let $K_m$ denote the number of possible alternative configurations of capabilities which can support the accomplishment of a given task $m \in \mathcal{M}$. We denote $Y_m^*: \mathbb{R}_{\geq 0}^{K_m \times U}$ as the requirement matrix for task $m$, which specifies the aggregated capabilities required to effectively execute the task in each of the different configurations. In other words, each row of $Y_m^*$ specifies a possible combination of minimum aggregated capabilities which need to be assigned to task $m$.

In this paper, we are interested in generating an allocation matrix, $A \in \{0,1\}^{M \times N}$, whose element $A_{ji} = 1$ if and only if robot $i$ is allocated to task $j$. For each task $m \in \mathcal{M}$ and candidate allocation $A$, let $c_m \in \mathbb{R}_+^U$ denote the total aggregated capabilities assigned to the task (computed in a similar manner to [8]), given as,

$$c_m = (A_m,-P^TQ)^T, \quad (13)$$

where $A_m,-$ denotes the $m$th row of $A$. However, as discussed in Section II, the performance of different robot species will be different in the tasks, due to environmental disturbances. To explicitly account for these variations in the allocation process, we introduce the effective total capabilities assigned to a given task, which leverages the capability degradation metrics computed in (10). Thus, the effective aggregated capabilities in task $m$ can be given as,

$$\hat{c}_m = c_m - (d_m \odot c_m), \quad (14)$$

where $\odot$ is the Hadamard product. Using (14), the following definition outlines the conditions which would ensure that a sufficient amount of aggregated capabilities are assigned to the tasks.

Definition 4 (Effective Task Execution). The capability requirements for a given task $m \in \mathcal{M}$ are met, when the effective aggregated capabilities of the robots allocated to it are greater than those specified by one or more of the configurations in the task requirements matrix (see Definition 3). This is encoded by the following two conditions:

$$\hat{c}_m - (\iota_m Y_m^*)^T = \delta_m \quad (15)$$

$$\delta_m \geq 0 \quad (16)$$

where $\geq 0$ is interpreted element-wise, and $\iota_m \in \{0,1\}^{K_m}$ is an indicator matrix specifying which of the $K_m$ possible configurations is selected. In particular, the condition $\iota_m 1 = 1$ ensures that only one configuration is selected at a given point in time. $\delta_m \in \mathbb{R}_+$ then represents the aggregated capability margin, which is the difference between the total available capabilities assigned to the task and the requirements of the task.

However, environmental conditions might force a situation where it is impossible to meet the requirements for every task, i.e. constraint (16) might not be satisfied $\forall m \in \mathcal{M}$. To impart a second layer of resilience to our framework, we introduce a task relaxation matrix $\phi_\varepsilon \in \{0,1\}^M$ which indicates the requirements for each task will be met or not,

$$\phi_m = \begin{cases} 0, & \text{task } m \text{ requirements are being met} \\ 1, & \text{otherwise.} \end{cases} \quad (17)$$

Using $\phi$, we can modify the requirement in (16) as follows:

$$\delta_m \geq -\phi_m \delta_{\max}, \quad (18c)$$

where $\delta_{\max} \in \mathbb{R}$ represents the maximum extent to which the task requirements constraints can be violated for all capabilities.

We now present the mixed-integer quadratic program (MIQP) which can be solved to generate a resilient task allocation for the multi-robot team.

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minimize $1^T(A^T \lambda \cdot T)W_s 1 + w_t^T \phi +$, \quad (18a)

subject to $\hat{c}_m - (\iota_m Y_m^*)^T = \delta_m$, \quad (18b)

$D \geq -\phi^T \delta_{\max}$, \quad (18c)

$1^T(A^T \lambda \cdot T) \leq \lambda^T$, \quad (18d)

$1^T \xi 1 \leq 1 + (\Delta \text{V}_{\text{thresh}} - \Delta \text{V})$, \quad (18e)

$\iota_m 1 = 1$, \quad (18f)

$\forall m \in \mathcal{M}$,

where $D = [\delta_1, \delta_2, \ldots, \delta_M]^T \in \mathbb{R}^{M \times U}$ and the inequality in constraint (18c) holds elementwise. We will now define the symbols and the roles played by various terms in the above defined optimization problem. First, $W_s \in \mathbb{R}_+^{S \times S}$ is a diagonal weight matrix which represents the cost-of-deployment associated with robots of different species (for instance, a robot with an expensive LIDAR might have a higher cost of deployment associated with it). Furthermore, $w_t \in \mathbb{R}_+^M$ represents the relative importance among the various tasks, which is taken into account when considering which task constraint to relax first. For example, when considering the mission objective of defending a perimeter [23], it might be better to relax the constraints of the patrol task (which detects new intruders) than the defense task (which intercepts them) if both cannot be achieved simultaneously.

The third term in the cost function (18a), $\|D1\|_2^2$, scaled by a positive constant $l$, serves two purposes: it penalizes excessive allocation of capabilities to a given task and also ensures that in case the constraints corresponding to a given task are relaxed due to significant environmental disturbances, they are done so to a minimal extent. The final term in the cost function (18a), $1^T T(A - A_p)\|1\|_1$, represents the cost of transitioning robots between the tasks. In this regard, $A_p$ simply represents the current allocation of the multi-robot team to tasks (computed as the solution of the MIQP in the previous iteration, see Algorithm 1). The transition cost matrix, $T \in \mathbb{R}_+^{M \times M}$ is a diagonal matrix, where $|T_{i,i} - T_{j,j}|$ represents the cost incurred by each robot when it transitions from task $i$ to task $j$, or vice versa. Similarly, $T_{i,i}$ simply indicates the cost associated with an
idle (unallocated) robot being assigned to task $i \in \mathcal{M}$. For instance, these costs can be assigned by the mission designer based on the distances that robots have to traverse when transitioning between tasks.

The vector $\lambda \in \mathbb{N}^S$ represents the total number of robots of each species available for allocation and thus, constraint (18) ensures that the resource constraints of the overall team are accounted for by the allocation algorithm. Along a similar vein, constraint (18e) ensures that each robot is allocated to only one task at most. Here, $\Delta V \in \{0, 1\}^N$ represents the stacked task-discrepancies corresponding to the entire team (see Definition 1), and $\Delta V_{\text{thresh}} \in \{0, 1\}$ represents the maximum acceptable task performance discrepancy that a given robot can have, for it to be eligible for allocation by the algorithm. Thus, if the following condition holds for robot $i \in \mathcal{R}$,

$$\Delta V_{\text{thresh}} - \Delta V^{(i)} < 0,$$

(19) then robot $i$ will not be allocated to any task, since it is deemed unfit to perform any task. For instance, a ground robot stuck in a crevice might not be able to perform any task in the environment, and will not be considered in the allocation.

As discussed earlier, the capability degradation metric $d_m$ is updated every $\Delta t$ seconds, which is then incorporated into the resilient task allocation problem (18) via constraint (18a). However, if we assume that environmental disturbances affect the multi-robot team at time scales much larger than $\Delta t$, it is clear that the MIQP described by (18) need not be solved every $\Delta t$ seconds. This idea is further reinforced by the fact that, the MIQP must be solved in a centralized manner, and is not amenable to real-time solutions due to its NP-complete nature [24].

Indeed, a reallocation of tasks to robots is warranted only when there are significant changes in the capability degradation metrics associated with any of the tasks. Let $t_l$ denote the time index when the MIQP was most recently solved. We introduce a binary variable $\beta[t]$, which determines if the MIQP should be solved at time $t$,

$$\beta[t] = \begin{cases} 1, & \text{if } \exists m \in \mathcal{M}, \text{ s.t. } \max (d_m[t] - d_m[t_l]) \geq \Delta, \\ 0, & \text{otherwise} \end{cases}$$

(20)

where $\Delta$ is a user defined threshold on the change in any capability degradation value. Algorithm 1 outlines the operations of the resilient task allocation framework. Step 6 computes the task performance discrepancy values $\Delta V^{(i)}$ based on the environmental disturbances experienced by the robots. Following this, step 7 computes the capability degradation metrics $d_m$ for each task $m \in \mathcal{M}$ and uses this to compute $\beta$ using (20). If $\beta = 1$, the task allocation MIQP presented in (18) is solved to generate the allocation matrix $A$ which subsequently results in a rearrangement of robots among the tasks. The next section illustrates the salient features of the proposed framework in a heterogeneous multi-robot coverage control and target tracking scenario.

### Algorithm 1 Resilient task allocation via online task performance evaluation

**Require:**
- Robot team heterogeneity specifications $Q, S, P, \lambda$
- Task Specifications $Y_{m,r}$, $T$
- Parameters $W_s$, $w_t$, $\Delta V_{\text{thresh}}$, $l$, $\delta_{max}$, $\Delta$

**Parameters** $W_s$, $w_t$, $\Delta V_{\text{thresh}}$, $l$, $\delta_{max}$, $\Delta$

1: Initialize: $t = t_l = 0$, $A_p = \emptyset^{M \times M}$
2: Compute $A$ and transmit to robots. [18]
3: Set $A_p = A$
4: while true do
5: Execute tasks $m \in \mathcal{M}$
6: Compute $\Delta V^{(i)}$, $\forall i \in \mathcal{R}$ [6]
7: Update capability degradation $d_m$, $\forall m \in \mathcal{M}$ [10]
8: Compute reallocation trigger $\beta$ [20]
9: if $\beta = 1$ then
10: Compute $A$ and transmit to robots [18]
11: Set $A_p = A$ and $t_l = t$
12: end if
13: $t = t + 1$
14: end while

### IV. ENVIRONMENT COVERAGE AND TARGET TRACKING: AN APPLICATION

We consider a team of aerial and ground robots which need to be allocated among three tasks: tracking target 1 (task 1), tracking target 2 (task 2), and monitoring of the environment (task 3). In particular, we use the coverage control algorithm [16] to execute the monitoring task, with the importance density function chosen as a zero-centered Gaussian function. The robots performing tracking tasks 1 and 2 are also required to maintain a certain quality of surveillance on the target, which is modeled as a function of both the distance to the target and a scalar state $e_i$ denoting the environmental effects on the sensing. These task objectives are encoded into the following cost functions whose minimization represents the execution of the tasks,

$$V_k = \frac{1}{2} \sum_{i \in T_k} ((x_i - \gamma_k)^2 - d_k)^2 + e_i \|x_i - \gamma_k\|^2 +$$

(21)

$$\left(\sum_{j \in (T_k \setminus i)} \frac{1}{\|x_i - x_j\|^2} - \frac{1}{d_0^2}\right)^2, k = 1, 2,$$

$$V_3 = \frac{1}{2} \sum_{i \in T_3} \|x_i - c_i\|^2,$$

(22)

where $\gamma_k$ denotes the locations of target $k$, $d_k$ denotes the desired distance to be maintained between the robots and target $k$, $d_0$ determines the minimum distance maintained between the robots in the task, and $c_i$ denotes the centroid of the Voronoi cell corresponding to robot $i$ [16].

The simulated experiment considers two species of robots: an aerial and a ground platform. The heterogeneity among the robots is characterized via five capabilities: perception ($\text{m}^2$), sensing resolution (m), air speed (m/sec), ground speed (m/sec), and communication rate (mb/sec). These specifica-
tions are captured by the robot capability matrix:

\[
Q = \begin{bmatrix}
5 \text{ m}^2 & 1 \text{ m} & 3 \text{ m/sec} & 0 \text{ m/sec} & 5 \text{ Mb/sec} \\
2 \text{ m}^2 & 3 \text{ m} & 0 \text{ m/sec} & 1 \text{ m/sec} & 8 \text{ Mb/sec}
\end{bmatrix}. \tag{23}
\]

The task requirements matrix (see Definition 3) is given as,

\[
Y_1^* = Y_2^* = \begin{bmatrix}
7 & 10.5 \\
4 & 2.1 \\
3 & 6.3 \\
1 & 0 \text{.5}
\end{bmatrix}, \tag{24}
\]

\[
Y_3^* = \begin{bmatrix}
20 & 4 \\
12 & 0 \\
15 & \text{}
\end{bmatrix}. \tag{25}
\]

As seen, the tracking tasks can either be accomplished using a team of aerial and ground robots (configuration 1) or only aerial robots (configuration 2). The parameters for the optimization program are chosen as follows: \( w_1 = [100, 100, 100] \), (indicating that the coverage task is less critical to mission success compared to the target tracking tasks), \( W_s = \text{diag}(0.1, 0.1, 0.1) \), where \( \text{diag} \) is the diagonalization operator, \( \Delta V_{\text{thresh}} = 0.7 \), \( l = 1.0 \), \( \delta_{\text{max}} = 1000 \), \( \Delta = 0.3 \), and \( T = \text{diag}(65, 18, 45) \).

Figure 3a shows the initial deployment of robots to tasks as generated by solving the task allocation MIQP (18). In particular, the red and green dots represent aerial and ground robots, respectively. The robots in the middle of the domain are executing the monitoring task using coverage control. Two aerial robots remain idle at their starting locations (denoted by purple circles), as all task requirements are met by the rest of the team. At a certain point in time, target 2 enters a region of low-friction terrain—indicated by the blue colored area—where, as seen in Fig. 3b, the motion of the ground robot allocated to the task is impeded and it cannot track the assigned target anymore. Thanks to the capability degradation computations in Section III, this anomaly is accounted for in constraint (18b) of the MIQP. In particular, Fig. 4 illustrates how the capability margin corresponding to ground mobility for task 2 (\( D_{2,4} \)) decreases after the failure and becomes negative. Consequently, the event triggered MIQP switches task 2 to the second configuration (showcased by \( t_2 \) in the top left corner of Fig. 3b). This ensures that only aerial robots—unaffected by the slippery ground—are deployed to track target 2. Figure 3b shows the two additional aerial robots joining task 2 (highlighted by the green ellipse). Furthermore, constraint (18e) ensures that the stuck ground robot is not assigned to any task.

At time 12 seconds, a weather event affects the ability of the robots in task 2 to maintain an effective tracking quality of the target—signified by an increase in the value of \( c_{t} \). This is depicted in Fig. 3c as a white shaded area around target 2 and a decreasing capability margin \( D_{2,2} \) corresponding to the “resolution” capability in the right tile of Fig. 4. Since there are no more robots available to join task 2, the algorithm relaxes the constraints corresponding to the monitoring task, and reallocates one aerial robot to the tracking task ensuring that the overall capability margin stays above zero, while that for task 3 (depicted by \( D_{3,2} \)) falls below zero. This demonstrates the ability of our algorithm to gracefully degrade performance when necessary.

In order to verify the ability of the proposed allocation algorithm to deal with large robot teams and varied environmental conditions, we ran multiple randomized trails of the coverage and target tracking mission described above with a team of 32 aerial robots and 8 ground robots (with a modified (24) and (25)). The timing corresponding to the target movement, weather events, as well as the initial positions of the robots were randomized in each of the trials. Over 20 independent trials, Fig. 5 depicts the minimum (worst case among all runs) capability margins corresponding to two cases: with and without the event-triggered resilient task allocation algorithm. For the second case, the allocation algorithm was only executed once at the beginning of each trial. As seen, the resilient allocation algorithm (executed on average in 0.04 seconds for \( N = 40 \)) ensures that the capability margins remain close to or at zero, ensuring that the tasks can progress successfully, despite the environmental variations.

V. CONCLUSIONS

For heterogeneous multi-robot systems operating in dynamic conditions, we present a resilient task allocation framework which explicitly leverages information pertaining to the real-time task performance of robots when generating robot-task assignments. We endow the allocation algorithm with the ability to reconfigure robots among tasks to ensure that the detrimental effects of environmental disturbances are mitigated, thus showcasing a degree of resilience. The resulting framework not only degrades performance of the system gradually but presents a flexible mechanism for the mission designer to specify important parameters like robot deployment costs and task priorities.

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corresponding to tasks 2 (shown by the yellow ellipse). The corresponding increase in capability degradation causes the event triggered resilient task allocation algorithm to switch task 2 to an aerial only configuration and deploy two additional robots to join the task (highlighted by the green ellipse), thus mitigating the effects of the failure. In Fig. 3c, a perception failure is depicted near task 2 requiring the presence of additional robots to ensure that the aggregated capabilities sufficiently meet the needs of the task. The allocation algorithm autonomously chooses to relax the constraints corresponding to the monitoring task and redirects a robot from it to join the target tracking task (highlighted by the green ellipse in Fig. 3c). In each subfigure, the top left corner indicates the environment. As seen in Fig. 3b, the ground robot tracking target 2 becomes immobilized after encountering a low friction surface depicted in blue (shown by the yellow ellipse). The corresponding increase in capability degradation causes the event triggered resilient task allocation algorithm to switch to a different configuration, thus mitigating the failure. (Right) Margins for the task 2, respectively). As seen, the capability margin D2,2 drops. The resilient allocation algorithm reallocates a robot from task 3 to task 2, ensuring that D2,2 remains above 0, while intentionally sacrificing performance in task 3.

![Fig. 3](image)

Fig. 3. Aerial and ground robots (denoted by red and green dots, respectively) allocated among three tasks: tracking target 1, target 2, and monitoring. As seen in Fig. 3, the ground robot tracking target 2 becomes immobilized after encountering a low friction surface depicted in blue (shown by the yellow ellipse). The corresponding increase in capability degradation causes the event triggered resilient task allocation algorithm to switch to a different configuration, thus mitigating the failure. In Fig. 3c, a perception failure is depicted near task 2 requiring the presence of additional robots to ensure that the aggregated capabilities sufficiently meet the needs of the task. The allocation algorithm autonomously chooses to relax the constraints corresponding to the monitoring task and redirects a robot from it to join the target tracking task (highlighted by the green ellipse in Fig. 3c). In each subfigure, the top left corner indicates the current capability configuration chosen by the algorithm, with red text indicating that a task constraint has been relaxed.

![Fig. 4](image)

Fig. 4. (Left) Margins for aerial and ground mobility capabilities corresponding to tasks 2 (D2,3 and D2,4, respectively). As seen, the mobility failure of the ground robot in Fig. 3c is captured by a decrease in D2,4. A value below 0 indicates that the requirements for the task are not met anymore. This prompts the allocation algorithm to switch to a different configuration, thus mitigating the failure. (Right) Margins for the sensing resolution capability corresponding to tasks 2 and 3 (D2,2 and D3,2, respectively). Due to the environmental disturbance in task 2 (see Fig. 3c), the capability margin D2,2 drops. The resilient allocation algorithm reallocates a robot from task 3 to task 2, ensuring that D2,2 remains above 0, while intentionally sacrificing performance in task 3.

![Fig. 5](image)

Fig. 5. Capability margin (see Definition 4) results from 20 randomized trials of the coverage and tracking task with a team of 40 robots. The bold lines represent the lowest worst-case capability margins over all the trials for target tracking tasks 1 and 2 corresponding to two cases: with and without the event triggered task allocation algorithm. In the case where resilient reallocations are regularly applied, the capability margins for the tasks remain significantly closer to 0, ensuring they are effectively executed (solid lines). For this case, the shaded regions represent the ±1 standard deviation of the results obtained over the trials.

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