A refined trans-Planckian censorship conjecture

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We propose a refined version of trans-Planckian censorship conjecture (TCC), which could be elaborated from the strong scalar weak gravity conjecture combined with some entropy bounds. In particular, no fine-tuning on the inflation model-building is required in the refined TCC, and it automatically passes the tests from those stringy examples that support the original TCC. Furthermore, our refined TCC could be consistent with hilltop eternal inflation.

inflation, swampland conjectures, weak gravity conjecture

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1 Introduction

Decades of explorations along the string theory as a promising candidate for quantum gravity reveal a landscape \cite{1} of a huge number of low-energy effective theories, beyond which the low-energy effective theories are conjectured mostly in the swampland \cite{2} that admits no consistent completion into quantum gravity in the ultraviolet (UV). The string-inspired swampland criteria (see, for example, a review \cite{3}) have been proposed to discriminate those seemingly innocent theories consistent at low-energy in the infrared (IR).

The difficulties of locating de Sitter (dS) vacua in string theory have inspired the swampland dS conjecture (SdSC) \cite{4} and its refinements \cite{5-10}. The original SdSC \cite{4} simply forbids any dS local extrema, which is in direct tension with the standard model Higgs potential \cite{11-15} (see also ref. \cite{16}) that has a dS local maximum. Therefore, the refined SdSC \cite{5-8,10,17} was proposed to relax the constraint so that those local maxima with large second-derivatives in field space are allowed, while the rest of extrema are forbidden including all dS local minima. In particular, both requirements of no quantum broken dS \cite{5,18} and no eternal inflation \cite{7} disfavour any stable dS local minima. Similarly, the trans-Planckian censorship conjecture (TCC) \cite{19} was recently proposed to further relax the constraint that, all modes that exit the Hubble horizon at the end of inflation should have their physical length larger than the Planck length at the beginning of inflation, namely, $a_i/(a_f H_f) > 1/M_{Pl}$. This strongly constrains the duration of inflation on the elapsed e-folding number,

$$e^N < \frac{M_{Pl}}{H_f}.$$  \hfill (1)

Therefore, TCC further relaxes the swampland dS criteria by forbidding any stable dS local minima but sparing the life of metastable dS local minima. As an immediate result, inflation cannot be eternal both into the past \cite{20} and future directions \cite{5,7,18,21}.

When applied to the inflationary phenomenology \cite{22}, TCC sets an upper bound on the inflationary Hubble scale...
and corresponding tensor-to-scalar ratio by
\[ H < e^{-N_{\text{Pl}}} \approx O(10) \, \text{MeV}, \tag{2} \]
\[ r \equiv \frac{2 \pi^2 \mathcal{P}_R \left( \frac{H}{M_{\text{Pl}}} \right)^2}{r > 6.8 \times 10^{-33}, \tag{3} \]
where \( N = (1/3) \ln(M_{\text{Pl}}/H_0) \approx 46.2 \) for an instantaneous reheating history [22,23] and \( \mathcal{P}_R \approx 2.1 \times 10^{-9} \) from Planck 2018 [24]. This imposes a severe fine-tuning problem on the inflationary model-building with an initial condition on the slow-roll parameter down to \( \epsilon \approx 10^{-33} \), which could be relaxed by invoking a non-instantaneous reheating history [23] at the price of an ultra-low reheating temperature (see ref. [25] for its implications on the mass of primordial black holes). Other trials of model-building within TCC include inflationary dark matter [26], warm inflation [27,28], initial states [29,30], non-standard post-inflationary history [31,32], D-term hybrid inflation [33], multi-stage inflation [34-36], inflection-point inflation [37], and negative running of spectral index [38].

Beyond the original statement of TCC, ref. [39] proposes a generalized TCC to save \( \kappa \)-inflation [40] from the problem that sub-Planckian fluctuations could exit the Hubble horizon without violating the original TCC. Furthermore, ref. [41] casts the doubt on TCC as a universal swampland criterion but proposes a probabilistic interpretation.

Although TCC requires all trans-Planckian quantum modes to maintain their quantum nature by never exiting the Hubble horizon, the apparent physical consequences of having trans-Planckian fluctuations stretched out of the Hubble horizon are unclear (see, however, refs. [42-44] for self-completeness of Einstein gravity), which should be elaborated with other more established arguments. One inspiration comes from some earlier variants of refined SdSC [5,7,18] that also prohibit long-lasting dS vacua. Therefore, although the refined SdSC in ref. [8] is silent about the dS lifetime, the arguments they used to obtain the refined SdSC could shed light on the physical interpretation of TCC. Note that the refined SdSC [8] was found following the swampland distance conjecture (SDC) [45] combined with the Bousso entropy bound [46] applied to an accelerating universe, which could also be used in ref. [21] to derive an entropic quasi-dS instability time. In particular, TCC was recently elaborated from SDC [47] in the large field excursion limit, which could also be derived from some entropy bounds [48]. To elaborate TCC over the whole field space, we use a strong version [49] of scalar weak gravity conjecture (WGC) [50] combined with some entropy bounds, and then obtain a weaker version of TCC to naturally evade the fine-tuning problem of initial conditions without turning to any exotic model-building.

The rest of the paper is organized as follows. In sect. 2, we review the scalar WGC and then derive a bound for the species number from both scalar WGC and its strong version. In sect. 3, using the aforementioned bound on species number, we propose a refined TCC from two separate arguments of UV-IR hierarchy and entropy bounds. In sect. 4, the refined TCC is reformulated for eternal inflation. In sect. 5, we draw the conclusion and discuss the refined TCC implications for inflation without the fine-tuning problem. In sect. 6, we discuss several issues on our refined TCC.

2 \ Scalar weak gravity conjecture

The scalar WGC [50] implements the belief that the net force mediated by scalar fields should be larger than the gravitational force. Specifically, one considers a complex scalar field \( \phi \) with a field-dependent mass term \( m^2(\phi)\phi^2 \), where a canonically normalized scalar field \( \phi \) mediates the scalar force between \( \phi \) particles via a trilinear coupling term \( \partial_\mu m^2(\phi)\partial^\mu\phi^2 \) when expanding \( \phi = \phi_0 + \delta\phi \) around a vacuum expectation value (vev) \( \phi_0 \). By requiring the scalar force larger than the gravity,
\[ F_{\text{scalar}} \equiv \frac{(\partial_\mu m)^2}{4\pi^2} > \frac{m^2}{8\pi M_{\text{Pl}}r^2} \equiv F_{\text{gravity}}, \tag{4} \]
the scalar WGC reads
\[ 2(\partial_\mu m)^2 > \frac{m^2}{M_{\text{Pl}}^2}. \tag{5} \]

Similar form also holds for fermion or massless multi-scalar fields \( \phi_i \) with kinetic term \( g_{ij}\partial^i\phi\partial^j\phi \), that is, there should have a state with mass \( m \) satisfying the bound
\[ 2g^{ij}(\partial_\mu m)(\partial_\nu m) > \frac{m^2}{M_{\text{Pl}}^2}. \tag{6} \]

In particular, as an explicit fulfillment of eq. (5), SDC [45] allows for a large distance replacement (corresponding to weak coupling limit) over the moduli space without a potential (or field space in the effective theory with a potential [51,52]) if an infinite tower of states becomes light with mass scale
\[ m(\phi) \sim m(\phi_0)e^{-\alpha |\Delta\phi|/M_{\text{Pl}}}. \tag{7} \]
descending from the UV with some \( \mathcal{O}(1) \) constant \( \alpha > 0 \). Note that the scalar WGC is formulated for any field value in field space, while the SDC holds most straightforwardly at large distances in field space, \( |\Delta\phi|/M_{\text{Pl}} \gtrsim 1 \). Therefore, our use of scalar WGC in replacement of SDC is a non-trivial generalization of the arguments used in ref. [21]. Nevertheless, we all arrive at the same dS lifetime as shown in eq. (43).
2.1 A bound for species number

Since both scalar WGC and SDC could constrain the mass scale of an infinite tower of states for a scalar theory coupled to gravity, the effective description of the scalar theory might be jeopardized by including a sufficient number of states. Therefore, there exists a bound on the number of states included, above which we are not in any weakly-coupled gravitational regime but a strong coupling regime. Or equivalently, given \( N_s \) particle states, gravity becomes strongly coupled above a dubbed species scale (conjecture) [53-60],

\[
\Lambda_s \equiv \frac{M_{Pl}}{\sqrt{N_s}}.
\]

For example, the number of Kaluza-Klein (KK) modes \( N_s \) we can include in the lower \( d \) dimensional gravity theory before reaching the true quantum gravity cutoff scale \( M_{Pl}^D \) of the higher \( D \) dimensional gravity theory is \( N_s \sim M_{Pl}^D/m_{KK} \). Likewise, \( N_s \sim \Lambda_s/m \) is expected in general. Hence the species number reads

\[
N_s \sim \left( \frac{M_{Pl}}{m(\phi)} \right)^{\frac{2}{3}},
\]

which, after the use of scalar WGC eq. (5), becomes

\[
(\partial_\phi \ln N_s)^2 \gtrsim \frac{2}{9} M_{Pl}^2.
\]

This could be integrated directly to give

\[
\ln N_s \gtrsim \frac{\sqrt{2} |\Delta \phi|}{3 M_{Pl}}.
\]

On the other hand, if \( \phi \) also drives the background expansion with the Hubble rate \( H \), one could define the corresponding Hubble slow-roll parameter as:

\[
\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2M_{Pl}^2 H^2},
\]

where we have used the Friedmann equation \( M_{Pl}^2 H = -\frac{1}{2} \dot{\phi}^2 \) in the last step. The e-folding number during a given field excursion in the non-eternal inflationary regime is well-known,

\[
|\Delta N| = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi = \int_{\phi_i}^{\phi_f} \frac{1}{\sqrt{2\epsilon_H} M_{Pl}} |\Delta \phi|,
\]

\[
\equiv \left( \frac{1}{\sqrt{4\pi}} \right) \frac{|\Delta \phi|}{\sqrt{2} M_{Pl}},
\]

where in the second line we define the averaged value for the inverse square-root of the Hubble slow-roll parameter. Therefore, eq. (11) becomes

\[
N_s \gtrsim e^{\frac{1}{2} |\Delta N|/(\epsilon_H^{1/2})} \equiv e^{\frac{1}{2} N_s},
\]

where the abbreviation \( N_s \equiv |\Delta N|/(\epsilon_H^{1/2}) \) is introduced for convenience. The physical meaning of the bound (14) is illuminating: for given inflationary potential in an effective theory valid below \( \Lambda_s \), the e-folding number is bounded from above by eq. (14), otherwise a larger e-folding number leads to a larger field excursion, and hence a larger number of light particle states would be included to spoil the effectiveness of the original theory unless lowering \( \Lambda_s \) accordingly.

2.2 A strong scalar weak gravity conjecture

However, there are some concerns [49] on the scalar WGC, for example, it does not apply to all the scalar fields (e.g., the massless mediator \( \phi \) itself), and it does not simultaneously accommodate the SDC for both \( \phi \rightarrow \pm \infty \). Furthermore, it is not consistent with axion-like particles and fifth-force constraints, and it might be in the phenomenological tension [61] with the (refined) SdSC [4-10]. As a result, a stronger version of the scalar WGC was proposed in ref. [49] for any canonically normalized real scalar \( \phi \) coupled to the gravity.

The scalar potential \( V(\phi) \) at any field value should meet

\[
2(V''')^2 - V''V''' \gtrsim \frac{(V''^2)}{M_{Pl}^2},
\]

which, after writing \( m^2 = V'' \), becomes

\[
2(\partial_\phi m^2)^2 - m^2(\partial_\phi^2 m^2) \gtrsim \frac{(m^2)^2}{M_{Pl}^2}.
\]

The above bound implies a clear physical meaning [62] that the first and second terms on the left-hand-side denote the attractive and repulsive scalar forces, respectively. Therefore, this strong version of scalar WGC simply states that the net scalar force should surpass the gravity force over the whole field space.

To see whether the bound (14) is still held in the strong version of scalar WGC, we could insert eq. (9) into eq. (16) and find

\[
9(\partial_\phi \ln N_s)^2 + 3(\partial_\phi^2 \ln N_s) \gtrsim M_{Pl}^2,
\]

which could be directly solved as:

\[
N_s \gtrsim \cosh \frac{1}{2} \frac{|\Delta \phi|}{M_{Pl}} \gtrsim e^{\frac{1}{2} N_s}.
\]

Apart from an \( O(1) \) factor difference in the exponent, this is the same as eq. (14), which could also be derived in ref. [47] from applying the SDC on eq. (9) in a specific form of \( m(\phi) \) with \( m(\phi_0) \sim M_{Pl} \). However, our derivation of eq. (18) is quite generic in the sense that it follows for any scalar field at any field value, and does not rely on the slow-roll approximation.
3 A refined trans-Planckian censorship conjecture

Armed with eq. (18), a refined TCC could be targeted with the following two separate arguments.

3.1 UV-IR hierarchy

The Hubble horizon scale as an IR cutoff scale should be bounded by the UV cutoff scale set by the species scale [47],

\[ H < \Lambda_s \equiv \frac{M_{Pl}}{\sqrt{N_s}} \leq M_{Pl} e^{-\frac{S}{N_s}}, \]  

(19)

with eq. (18) used in the last step, from which a refined TCC could be directly read off,

\[ e^{\frac{S}{N_s}} \leq \frac{M_{Pl}}{H}. \]  

(20)

The original TCC (1) serves as a stronger statement than eq. (20) since \( N_e \ll |\Delta N| \) for the slow-roll inflation with \( \epsilon_H \ll 1 \). Therefore, all those stringy examples that support the original TCC also automatically hold for the refined TCC. The physical consequence of the refined TCC could be understood in the following way: for a given potential with the expansion history \( (\epsilon_H^{1/4}) \) fixed, the violation occurs for a large enough \( |\Delta N| \), which leads to a large field excursion \( |\Delta \phi| \). Hence the species number \( N_s \) would be so large that the UV scale set by species scale \( \Lambda_s \) could be even smaller than the IR scale set by Hubble horizon scale \( H \), which is not allowed by the hierarchy of scales.

3.2 Entropy bounds

Consider a spherical region of size \( R \) consisting of \( N_s \) species numbers of particles as radiations at a temperature \( T \), the total energy and entropy scale as:

\[ E \sim N_sR^3T^4, \quad S \sim N_sR^3T^3. \]  

(21)

The absence of black hole formation imposes \( R > R_{BH} \sim \frac{E}{M_{Pl}^2} \sim N_sR^3T^4/M_{Pl}^2 \), which leads to a maximum radius \( R < M_{Pl}/(\sqrt{N_s}T^2) \), or equivalently a maximum entropy \( S \sim N_sR^3T^3 < M_{Pl}^4/(\sqrt{N_s}T^3) \). This maximum entropy should be bounded by the Bekenstein-Hawking entropy, \( S_{BH} \sim M_{Pl}^4/(N_sT^3) \), as:

\[ \frac{M_{Pl}^4}{\sqrt{N_s}T^3} < \frac{M_{Pl}^4}{N_sT^3}, \]  

(22)

which in turn gives rise to a maximum temperature \( T < \frac{M_{Pl}}{\sqrt{N_s}} [59, 63, 64] \) larger than the Hubble temperature,

\[ \frac{H}{2\pi} < \frac{M_{Pl}}{\sqrt{N_s}} \leq M_{Pl} e^{-\frac{S}{N_s}}, \]  

(23)

namely,

\[ e^{\frac{S}{N_s}} \leq \frac{M_{Pl}}{H}. \]  

(24)

The similar bound could also be obtained if the radiation entropy is bounded by the Hubble entropy [65, 66],

\[ N_sR^3T^3 < M_{Pl}^2R^3H. \]  

(25)

Hence there is also a maximum temperature, \( T < (M_{Pl}^2H/N_s)^{1/3} \), which should be larger than the Hubble temperature,

\[ H^3 < \frac{M_{Pl}^2H}{\sqrt{N_s}} \leq M_{Pl}^2He^{-\frac{S}{N_s}}, \]  

(26)

namely,

\[ e^{\frac{S}{N_s}} \leq \frac{M_{Pl}}{H}. \]  

(27)

Note that similar arguments of using entropy bounds are also adopted in ref. [48] to elaborate the SDC.

The above arguments implicitly assume that all \( N_s \) species numbers of particles have their masses spectra below the temperature. However, it is fairly possible that some of the particle masses are heavier than the temperature so that they could have contributions to the total energy \( E \sim k_BN_sT \) (\( k_B \equiv 1 \) hereafter) but no contribution to the total entropy, which should be bounded by the Bekenstein entropy [67-69] for any weakly gravitating matter fields,

\[ S_{matter} \leq 2\pi ER \sim 2\pi N_sTR. \]  

(28)

If the matter inside the sphere is gravitationally stable so that a black hole does not form, the Bekenstein entropy should be further bounded by the Bekenstein-Hawking entropy,

\[ 2\pi N_sTR \leq M_{Pl}^2R^2. \]  

(29)

Therefore, there is a maximum temperature \( T < \frac{M_{Pl}^2R}{2\pi N_s} \), which should be larger than the Hubble temperature,

\[ \frac{H}{2\pi} < \frac{M_{Pl}^2R}{2\pi N_s} \leq \frac{M_{Pl}^2e^{-\frac{S}{N_s}}}{2\pi}, \]  

(30)

namely,

\[ e^{\frac{S}{N_s}} \leq \frac{M_{Pl}}{H}. \]  

(31)

The same bound could also be obtained by bounding the Bekenstein entropy with Hubble entropy,

\[ 2\pi N_sTR \leq M_{Pl}^2R^3H. \]  

(32)
Hence there is also a maximum temperature \( T < \frac{M_{\text{Pl}}^2 H R^2}{2\pi N_s} \), which should be larger than the Hubble temperature,

\[
\frac{H}{2\pi} < \frac{M_{\text{Pl}}^2 H R^2}{2\pi N_s} \lesssim \frac{M_{\text{Pl}}^2}{2\pi H} e^{-\frac{2\pi}{N_s}},
\]

namely,

\[
e^{-\frac{2\pi}{N_s}} \lesssim \frac{M_{\text{Pl}}}{H}.
\]

Note that the entropy bound alone is not sufficient to guarantee the refined TCC (eqs. (24), (27), (31), (34)), since the UV-IR hierarchies \( H < T \) and \( R < H^{-1} \) are still needed for the relevant derivation of inequalities.

In summary, with eq. (18), the refined TCC is always implied by some entropy bounds either from the radiation entropy (21) or from the Bekenstein entropy (28), which are bounded by either the Bekenstein-Hawking entropy (eqs. (22) and (29)) or the Hubble entropy (eqs. (25) and (32)).

4 Eternal inflation

There are three types of eternal inflation frequently discussed in the literature: the old eternal inflation (a trapped dS vacuum) [70, 71], the stochastic eternal inflation [72-75] (a hilltop potential [76]), and the topological eternal inflation [77-79] (an eternally inflating topological defect from a hilltop potential [80]). Whether the eternal inflation is in the swampland is an important question currently under debate [7, 81-87]. The old eternal inflation is generically inconsistent with the original SdSC [81, 82], but the stochastic eternal inflation was argued to be marginally consistent with the refined SdSC [83], which, however, was questioned with either the perturbative [84] or entropic [85] arguments. Nevertheless, ref. [7] speculated a possibility that some variants of SdSC should be regarded as the approximate consequences of the absence of eternal inflation. Furthermore, some topological eternal inflation models could be constructed to evade the swampland criteria for an eternally inflating bubble wall [86] and the domain wall on the brane [87].

Our refined TCC (eq. (20)) is elaborated from the arguments of eqs. (11), (13) and (19). Note that eq. (13) is restricted to the non-eternal inflationary regime where the field excursion is dominated by the classical rolling. For the eternal inflation, the elapsed e-folding number could be arbitrarily large as long as the field excursion is limited in the range where the quantum fluctuations dominate over the classical rolling. In this case, one could use eqs. (11) and (19) directly without eq. (13) to render the general version of our refined TCC,

\[
e^{-\frac{2\pi}{N_s}} \lesssim \frac{M_{\text{Pl}}}{H},
\]

which shares a similar form derived in ref. [88] from SDC on the inflation. To see whether the stochastic eternal inflation could survive the above criterion, we use a quadratic hilltop potential

\[
V(\phi) = V_0 - \frac{1}{2} m^2 \phi^2, \quad m^2 > 0,
\]

where the eternal inflation is allowed in the small field region \( |\Delta \phi| < \phi_c \) when the averaged quantum random walks dominate over the classical rolling during each Hubble time,

\[
\frac{H}{(2\pi)} \equiv \langle \dot{\phi} \rangle_0 > \frac{\phi_c}{H}. \phi_c \text{ is therefore solved from}
\]

\[
\mathcal{P}_c(\phi_c) \equiv \left( \frac{H^2(\phi_c)}{2\pi \phi_c} \right)^2 \approx \frac{V(\phi_c)}{24\pi^2 M_{\text{Pl}}^2 \epsilon_V(\phi_c)} = 1.
\]

Our refined TCC (eq. (35)) is ensured provided that

\[
e^{-\frac{2\pi}{N_s}} \lesssim \frac{M_{\text{Pl}}}{H(\phi_c)},
\]

which could be solved numerically as shown in Figure 1 with the blue solid line. Furthermore, eq. (38) automatically guarantees the slow-roll condition for \( \epsilon_V \) (dashed lines),

\[
\epsilon_V(\phi_c) < \frac{1}{8\pi^2} e^{-\frac{2\pi}{N_s}} < 1.
\]

The other slow-roll condition \( |\eta_V(\phi_c)| \ll 1 \) for \( \eta_V(\phi_c) \) (red dashed lines) also automatically guarantees \( \eta_V > -3 \) [7] so that the exponentially suppressed probability to stay within \( |\Delta \phi| < \phi_c \) beats the exponential expansion of the universe. On the other hand, the perturbativity argument [84] further requires

![Figure 1](Color online) The parameter space for a quadratic hilltop eternal inflation out of swampland by our refined TCC enclosed by the red and blue solid lines. The slow-roll conditions are also checked with the blue and red dashed lines.
\[ P_{\epsilon} < \frac{1}{16(\eta_{\nu} - \epsilon_{\nu}) - \epsilon_{\nu}(\eta_{\nu} - \epsilon_{\nu}) - \epsilon_{\nu} \eta_{\nu}}. \]  

which is shown in Figure 1 with the red solid line. Therefore, there is a parameter space for the quadratic hilltop eternal inflation out of swampland by our refined TCC (eq. (35)).

5 Conclusion

In this paper, we proposed a refined TCC as a consequence of a strong version of the scalar WGC combined with either UV-IR hierarchy or some entropy bounds,

\[ e^{\frac{\Delta N}{O(1)}} \leq O(1) \frac{M_{Pl}}{H}, \]  

which is consistent with the hilltop eternal inflation. For non-eternal inflation, it further reduces to

\[ e^{N/\epsilon H} \leq O(1) \frac{M_{Pl}}{H}. \]  

where the \( O(1) \) factor on the left-hand-side is associated with a certain combination of spacetime dimension, which could be easily worked out from the previous arguments, and the \( O(1) \) factor on the right-hand-side is already mentioned in a footnote in ref. [19], which could be the sound speed in the generalized TCC in ref. [39]. The presence of an averaged Hubble slow-roll parameter in the exponent \( N_{e} \equiv N/\langle \epsilon_{H}^{-1/2} \rangle \) would prolong the dS lifetime by

\[ \Delta a_{s} < \frac{1}{H_{\epsilon}} \int_{a_{t}}^{a_{i}} H dt = \frac{1}{H_{t}} \ln \frac{a_{t}}{a_{i}} \approx \frac{\langle \epsilon_{H}^{-1/2} \rangle}{H_{t}} \ln \frac{M_{Pl}}{H_{t}}, \]  

which resembles the form of the entropic quasi-de Sitter instability time [21] (such a logarithmic correction to the Hubble time also appears in the scrambling time [19, 89]). Now the constraint on the Hubble scale could be greatly relaxed, for instance, \( N_{e} \equiv N/\langle \epsilon_{H}^{-1/2} \rangle \approx N/10 = 6 \) for an illustrative \( \epsilon_{H} \approx 0.01 \) and the e-folding number \( N = 60 \), to

\[ H \leq e^{-N} M_{Pl} \approx 6 \times 10^{15} \text{ GeV}. \]  

The same relaxation could also be achieved even for a near critical expansion with \( \epsilon_{H} \approx 1 \) due to the presence of an \( O(1) \) factor on the left-hand-side of eq. (42), for example, as shown in eq. (20),

\[ H \leq e^{-\frac{4}{5} N} M_{Pl} \approx 2 \times 10^{12} \text{ GeV}, \]  

for the same illustrative \( N = 60 \). Therefore, our refined TCC resolves the fine-tuning problem of inflationary initial conditions without invoking any exotic model-building. Furthermore, since our refined TCC is a weaker version of the original TCC, it would be automatically supported by those stringy examples that also hold for the original TCC.

6 Discussions

Several comments are given in order below regarding some concerns on our refined TCC.

Firstly, although initially motivated to relax the original TCC constraint on the inflationary Hubble scale, we have not directly addressed the trans-Planckian problem yet. Similar to the implication from the entropic quasi-de Sitter instability time [21], the physical length of those modes at the beginning of inflation that freeze out the Hubble horizon at the end of inflation is

\[ k_{\max}^{-1} = \frac{a_{i}}{a_{t} H_{t}} = \frac{1}{e^{N_{\text{sat}} H_{t}}, \]  

which, after replacing the total inflationary e-folding number \( N_{\text{sat}} \) with the saturation of our TCC bound for the non-eternal inflation ref. (42),

\[ e^{N_{\text{sat}}} \approx \left( O(1) \frac{M_{Pl}}{H_{t}} \right)^{O(1) \langle \epsilon_{H}^{-1/2} \rangle}, \]  

allows for the trans-Planckian modes

\[ k_{\max} \approx H_{t} \left( O(1) \frac{M_{Pl}}{H_{t}} \right)^{O(1) \langle \epsilon_{H}^{-1/2} \rangle} > O(1) M_{Pl}. \]  

This is not surprising since the original TCC that exactly forbids the exit of the trans-Planckian modes is refined (weaken) in our formulation. However, the arguments that put forward recently in ref. [90] against the Hubble horizon being a scale of singular significance of the trans-Planckian problem have shown that the classicalization does not necessarily require a trans-Planckian mode to cross the Hubble horizon at all. Therefore, we would not consider the allowance of trans-Planckian modes as a serious problem for our refined TCC. Nevertheless, it also weakens the connection to the original TCC that is closely attached to the trans-Planckian problem. As a result, our refined TCC might be better regarded as another weaker version among other swampland conjectures.

Secondly, as one could elaborate from the derivation of our refined TCC, the relation (9) (or more precisely \( N_{s} \sim \Lambda_{s}/m \)) is a crucial assumption. In fact, \( N_{s} \sim \Lambda_{s}/m \) closely follows the SDC that, on the large distance in the field space, \( |\Delta \phi| > M_{Pl} \), a tower of light states with mass and number

\[ m(\phi) \sim m(\phi_{0}) e^{-\alpha|\Delta \phi|/M_{Pl}}, \]  

\[ N_{s}(\phi) \sim e^{\alpha|\Delta \phi|/M_{Pl}}, \]  

appear descending from the UV if \( m(\phi_{0}) \sim \Lambda_{s} \) is identified. Although this relation has been widely used in the literature (see, for example, ref. [3] around eq. (5.17) and references
Thirdly, the naive combination of the scalar WGC with either UV-IR hierarchy or some entropy bounds leads to eq. (41), which is nothing but a weaker version of SDC,

$$\frac{V}{\Delta N} |\Delta | = O(1) \ln \left( \frac{\Delta M_{\text{Pl}}}{\Delta N^{2}} \right),$$

with an extra logarithmic correction. In fact, this relaxed form of SDC could easily meet the current observational constraints on inflation by expressing the field excursion with the Lyth bound and the Hubble scale with $A_s = 2.1 \times 10^{-9}$,

$$\Delta N \left( \frac{r}{\Delta N} \right)^{z} \lesssim O(1) \ln \left( \frac{\sqrt{2} O(1)}{\pi \sqrt{r A_{s}}} \right),$$

which constrains the tensor-to-scalar ratio as:

$$r \lesssim \frac{8 O(1)^{2}}{\Delta N^{2}} W \left( \frac{O(1) |\Delta N|}{2 \pi O(1) r A_{s}} \right)^{2} \sim O(0.1 - 1)$$

in terms of a typical $|\Delta N| = 60$ and the Lambert function $W(z) e^{W(z)} = z$. To further ensure the validity of effective theory of inflation, the mass scale of the infinite tower of light states due to the field excursion $|\Delta | = |\delta f - f|$ should be larger than the inflationary energy scale $V_{1/4}$ $\approx M_{\text{Pl}}^{1/2} H^{1/2}$,

$$m(\phi) e^{-\alpha |\Delta | / M_{\text{Pl}}} \geq m(\phi) \left( \frac{M_{\text{Pl}}}{H} \right)^{O(1)} \sim O(1),$$

which constrains the mass scale of the infinite tower of states before field excursion by

$$m(\phi) \gtrsim O(1)^{O(1)} \left( \frac{M_{\text{Pl}}}{H} \right)^{O(1)-1} M_{\text{Pl}}.$$
