We study the electromagnetic Bethe-Heitler type contribution to neutrino-induced deeply virtual meson production ($\nu$DVMP). Such $O(\alpha_{em})$-corrections decrease with $Q^2$ in the Bjorken regime less steeply than the standard $\nu$DVMP handbag contribution. Therefore, they are relatively enhanced at high $Q^2$. The Bethe-Heitler terms give rise to an angular correlation between the lepton and hadron scattering planes with harmonics sensitive to the real and imaginary parts of the DVMP amplitude.

These corrections constitute a few percent effect in the kinematics of the forthcoming MINERVA experiment at Fermilab and should be taken into account in precision tests of GPD parametrizations. For virtualities $Q^2 \sim 100$ GeV$^2$ these corrections become on a par with DVMP handbag contributions. A computational code, which can be used for the evaluation of these corrections employing various GPD models is provided.

PACS numbers: 13.15.+g,13.85.-t

Keywords: Single pion production, generalized parton distributions, Bethe-Heitler contributions

I. INTRODUCTION

Generalized parton distributions (GPD) allow evaluation of cross-sections for a wide class of processes, where the collinear factorization is applicable \cite{1,2}. The main source of experimental information on GPDs has been so far the electron(positron)-proton measurements performed at JLAB and HERA, in particular deeply virtual Compton scattering (DVCS) and deeply virtual meson production (DVMP) \cite{3-10}. The 12 GeV upgrade at Jefferson lab will open new opportunities for further improvement of our knowledge of the GPDs \cite{10}.

However, the practical realization of this program suffers from large uncertainties. For instance, the results at moderately high $Q^2$ can be affected by poorly known higher-twist components of GPDs and distribution amplitudes (DA) of the produced mesons \cite{17-20}.

Neutrino experiments provide a powerful tool for consistency checks for the extraction of GPD from JLAB data, especially of their flavor structure. The study of various processes in the Bjorken regime may be done with the high-intensity NUMI beam at Fermilab, which will switch soon to the so-called middle-energy (ME) regime with an average neutrino energy of about 6 GeV, and potentially may reach energies up to 20 GeV, without essential loss of luminosity. In this setup the MINERVA experiment \cite{21} should be able to probe the quark flavor structure of the targets. Even higher luminosities in multi-GeV regime can be achieved at the planned Muon Collider/Neutrino Factory \cite{22-24}.

Certain information on the GPD flavor structure can be extracted from comparison of analogous processes in neutrino- and electro-induced processes employing the difference of flavor structures of electromagnetic and weak neutral currents. An example is the weak DVCS \cite{25}, which alone, however, is not sufficient to constrain the flavor structure.

Recently we discussed the possibility of GPD extraction from deeply virtual neutrino-production of the pseudo-Goldstone mesons ($\pi, K, \eta$) \cite{26}. The $\nu$DVMP measurements with neutrino and antineutrino beams are complementary to the electromagnetic DVMP. The octet of pseudo-Goldstone bosons, originating from the chiral symmetry breaking, acts in the axial current as a natural probe for the flavor content. Due to the $V-A$ structure of the charged current, in $\nu$DVMP one can access simultaneously the unpolarized GPDs, $H$ and $E$, and the helicity flip GPDs, $\tilde{H}$ and $\tilde{E}$. We expect the contributions of the GPDs $H_T$, $E_T$, $\tilde{H}_T$ and $\tilde{E}_T$, which are controlled by the poorly known twist-3 pion DA $\phi_p$, to be negligible. Besides, important information on the flavor structure can be obtained by studying the transitional GPDs in the processes with nucleon to hyperon transitions. As was discussed in \cite{27}, assuming $SU(3)$ flavor symmetry one can
relate these GPDs to the ordinary diagonal ones in the proton. In this paper we study the Bethe-Heitler (BH) type radiative corrections to the diffractive neutrino-production of charged pseudo-Golstone mesons with the target remained intact, related to meson emission from lepton line with subsequent electromagnetic interaction with the target. Although such processes are formally suppressed by $\alpha_{\text{em}}$, at large $Q^2$ they fall off less steeply than the DVMP cross-section. While at virtualities $Q^2 \lesssim 10$ GeV$^2$, relevant for modern neutrino experiments, this is a few percent correction, already at $Q^2 \sim 100$ GeV$^2$ it becomes comparable with the DVMP cross section. Such corrections are relevant only in case of the $\nu$DVMP: for the electron-induced DVMP $ep \to epM$ they are suppressed by factor $\sim (G_F Q^2)^2$, where $G_F$ is the Fermi coupling, and are negligible unless we consider extremely high $Q^2 \approx M_W^2$. Existence of such diagrams opens a possibility to probe separately the real and imaginary parts of the DVMP amplitude (not only the total cross-sections), in close analogy to DVCS studies [13, 28, 29].

The paper is organized as follows. In Section II we present the results for the DVMP and BH contributions to the experimentally measurable total $\nu$DVMP cross-section (technical details of evaluation may be found in Appendix A). Also, at the end of Section II we construct two asymmetries which have particular sensitivity to the real and imaginary parts of the DVMP amplitude. In Section III for the sake of completeness we discuss the features of the GPD parametrization used in calculations. In Section IV we present the numerical results and make conclusions.

II. CROSS-SECTION OF THE $\nu$DVMP AND BH PROCESSES

Exclusive neutrino-production of pions is presented by the diagram (a) in the Figure 1. Production of pions by the vector current was studied in [30, 31], and was recently extended to neutrino interactions in [26].

![Diagrams](image)

FIG. 1: Diagrams contributing to the neutrino-production of mesons. (a) DVMP process (b,c) BH contributions.

As was shown in [30, 31], at large $Q^2$ where the collinear factorization is applicable, the amplitude of this process is suppressed due to hard gluon exchange in the coefficient function, and is small. This raises a question, how important could be the $O(\alpha_{\text{em}})$ corrections? While a systematic study of radiative corrections is beyond the scope of the present paper, we would like to consider in detail the contributions which decrease with $Q^2$ less rapidly than the diagram (a). In the leading order in $\alpha_{\text{em}}$ and $\alpha_W$ there are two such diagrams (b,c) shown in the Figure 1. These diagrams are enhanced by a factor $\sim Q^2/t$, where $-Q^2 \equiv q^2$ and $t$ are the lepton and proton 4-momentum transfers squared, respectively. This factor is large in the Bjorken regime of $t \ll Q^2$, and despite the formal suppression by $\alpha_{\text{em}}$, the diagrams (b,c) are numerically comparable to the diagram (a). Notice that such corrections are sizable only in the case of neutrino-production of mesons. In case of electroproduction the corresponding corrections are suppressed by $\sim G_F^2 Q^4$, where $G_F$ is the Fermi constant, and are negligible, unless we go to extremely large $Q^2 \sim M_W^2$. In what follows we evaluate the contribution of each diagram in Figure 1.

The cross-section of pion production can be presented as a sum of contributions of DVMP (diagram (a)), BH mechanism
(diagrams (b,c)) and their interference,
\[
\frac{d^4\sigma}{dt\,d\ln x_{Bj}\,dQ^2\,d\phi} = \frac{d^4\sigma^{(DVMP)}}{dt\,d\ln x_{Bj}\,dQ^2\,d\phi} + \frac{d^4\sigma^{(BH)}}{dt\,d\ln x_{Bj}\,dQ^2\,d\phi} + \frac{d^4\sigma^{(int)}}{dt\,d\ln x_{Bj}\,dQ^2\,d\phi},
\]
where \( x_{Bj} = Q^2/(2P\cdot q) \), and \( \phi \) is the angle between the lepton and hadron scattering planes.

Evaluation of the diagram (a) is straightforward and yields
\[
\frac{d^4\sigma^{(DVMP)}}{dt\,d\ln x_{Bj}\,dQ^2\,d\phi} = \frac{G_F^2 f_M^2 x_{Bj}^2}{64\pi^2 Q^2 (1 + Q^2/M_W^2)^2} \left( 1 - y - \frac{m_N^2 x_{Bj}^2 y^2}{Q^2} \right) (1 + 4m_N^2 x_{Bj}^2) \frac{\alpha_s^2}{2\pi} (2\beta_0 - 4) [T_M]^2,
\]
where \( y \) is the fractional loss of lepton energy defined as \( y = P\cdot q/P\cdot k = \nu/E_\nu \). Notice that the DVMP cross section turns out to be independent of the angle \( \phi \) between lepton and hadron planes. This happens because the momentum \( q \) does not have transverse components in the Bjorken reference frame.

For unpolarized target, the matrix element squared \( |T_M|^2 \) in Eqn. (2) can be simplified as,
\[
|T_M|_{unp}^2 = \frac{64\pi^2}{81} \frac{\alpha_s^2}{Q^2 (2 - x_{Bj})} \phi_{-1}^2 4 \left[ 4 (1 - x_{Bj}) \left( \mathcal{H}_M \mathcal{H}_M^* + \mathcal{H}_M \mathcal{H}_M^* \right) - \frac{x_{Bj} t}{4m_N^2} \mathcal{E}_M \mathcal{E}_M^* \right]
\]
\[
- x_{Bj}^2 \left( \mathcal{H}_M \mathcal{E}_M^* + \mathcal{E}_M \mathcal{H}_M^* + \mathcal{H}_M \mathcal{E}_M^* + \mathcal{E}_M \mathcal{H}_M^* \right) - \left( x_{Bj}^2 + (2 - x_{Bj})^2 \frac{t}{4m_N^2} \right) \mathcal{E}_M \mathcal{E}_M^* \right],
\]
where we introduced a shorthand notation,
\[
\phi_{-1} = \int_0^1 dz \frac{\phi_M(z)}{z^2} = \frac{1}{2} \int_0^1 dz \frac{\phi_M(z)}{z^2},
\]
and the script letters \( \mathcal{H}, \mathcal{E}, \mathcal{H}, \mathcal{E} \) signify convolution of the GPDs \( H, E, \hat{H}, \hat{E} \) with corresponding coefficient functions given in Table I.

**TABLE I**: List of the DVMP amplitudes \( \mathcal{H}_M, \mathcal{E}_M, \mathcal{H}_M, \mathcal{E}_M \) for different final states. For a neutron target, in the r.h.s. we flipped \( H_{u/n} \rightarrow H_{d/p} \), \( H_{d/n} \rightarrow H_{u/p} \), so all the GPDs are given for a proton target. To get \( \mathcal{E}, \mathcal{H}, \mathcal{E}, \mathcal{H} \) one should replace \( H \) with \( E, \hat{H}, \hat{E} \) respectively. \( V_{ij} \) are the CKM matrix elements. \( c_{\pm} \) is a shorthand notation \( c_{\pm}(x, \xi) = 1/(x \pm \xi \mp i0) \) for the leading order coefficient function. The NLO corrections to the coefficient functions may be found in [32, 33]. For the sake of brevity we do not show the arguments \( (x, \xi, t, Q) \) for all GPDs and omitted the integral over the quark light-cone fraction \( \int dx \) everywhere.

| Process | type | \( \mathcal{H}_M \) |
|---------|------|------------------|
| \( \nu_p \rightarrow \mu^+ \pi^+ p \) | CC | \( V_{ud}(H_{d\mu c} + H_{d\mu c}) \) |
| \( \bar{\nu}_p \rightarrow \mu^- \pi^- p \) | CC | \( V_{ud}(H_{d\mu c} + H_{d\mu c}) \) |
| \( \nu_p \rightarrow \mu^- K^+ p \) | CC | \( V_{us}(c_+ H_{\bar{d} c} + c_+ H_{\bar{d} c}) \) |
| \( \bar{\nu}_p \rightarrow \mu^+ K^- p \) | CC | \( V_{us}(c_+ H_{\bar{d} c} + c_+ H_{\bar{d} c}) \) |
| \( \nu_p \rightarrow \mu^- K^+ p \) | CC | \( V_{us}(c_+ H_{\bar{d} c} + c_+ H_{\bar{d} c}) \) |
| \( \bar{\nu}_p \rightarrow \mu^+ K^- p \) | CC | \( V_{us}(c_+ H_{\bar{d} c} + c_+ H_{\bar{d} c}) \) |

Comparing different elements in Table I one gets relations,
\[
\frac{d\sigma_{\bar{\nu}_p \rightarrow \mu^+ \pi^- - p}}{d\sigma_{\nu_p \rightarrow \mu^- \pi^+ n}} = \frac{d\sigma^{DVMP}_{\bar{\nu}_p \rightarrow \mu^+ \pi^- - p}}{d\sigma^{DVMP}_{\nu_p \rightarrow \mu^- \pi^+ n}},
\]
(5)
\[
\frac{d\sigma_{\bar{\nu}_p \rightarrow \mu^- \pi^+ p}}{d\sigma_{\nu_p \rightarrow \mu^+ \pi^- n}} = \frac{d\sigma^{DVMP}_{\bar{\nu}_p \rightarrow \mu^- \pi^+ p}}{d\sigma^{DVMP}_{\nu_p \rightarrow \mu^+ \pi^- n}},
\]
(6)
which are just a manifestation of the isospin symmetry. As shown below, these relations are broken by the BH corrections.

In the leading order in $Q^2$ both BH diagrams, Fig. 3(b) and (c), acquire dominant contribution from longitudinally polarized photons. However, as discussed below, certain angular harmonics are suppressed by $\sim \Delta_1/Q$ and get similar contributions from transverse and longitudinal photons. For this reason, for the form factors we include both the longitudinal and transverse photons. However, as discussed below, certain angular harmonics are suppressed by $Q^2$.

The details of the calculation are moved to Appendix A, and we present here the final result. The details of the calculations are moved to Appendix A and we present here the final result, which reads,

$$A_{\mu\nu}^{ab}(q, \Delta) = \frac{1}{f} \int d^4x e^{-iq\cdot x} \langle 0 \left| (V_\mu^{a}(x) - A_\mu^{a}(x)) J_{\nu}^{\alpha}(0) \right| \pi^b(q - \Delta) \rangle,$$

where $V_\mu^{a}(x)$ and $A_\mu^{a}(x)$ are the vector and axial vector isovector currents. The correlator (7) can be evaluated in pQCD in the collinear approximation, because $Q^2$ is large, and we assume that the dominant contribution comes from the leading twist-2 pion DA. Notice that the amplitude $A_{\mu\nu}$ cannot be interpreted as the pion form factor because (i) the virtuality of $W$ is large, (ii) the insertion of the pion state between $A_\mu^{a}$ and $J_{\nu}^{\alpha}$ leads to $A_{\mu\nu}^{ab} \sim q_\mu$, which gives zero acting on the transverse on-shell lepton current. The details of the calculations are moved to Appendix A and we present here the final result, which reads,

$$\frac{d^4\sigma^{(BH)}}{dt d\ln x_B dQ^2 d\phi} = \frac{f^2 G_F^2 \alpha_s^2 \sum_{n=0}^2 C_n^{BH} \cos(n\phi)}{16 \pi^2 t^2 \left(1 + \frac{4m_N^2 x_B^2}{Q^2}\right)^{3/2}}.$$

where

$$C_0^{BH} = C_2^{BH} + \frac{m_N^2}{9Q^2} \left[4 \left((2y^2 + y - 1) (\phi_{-1} - 1) x_B^3 - \left(4(\phi_{-1} - 1)^2 + \frac{t}{2m_N^2} (4\phi_{-1} - 8\phi_{-1} - 5)\right) y^2 - 4 \left((\phi_{-1} - 1)^2 + \frac{t}{4m_N^2} (4\phi_{-1}^2 - 13\phi_{-1} + 10)\right) y + \frac{5t}{2m_N^2} (\phi_{-1} - 2)^2 + (\phi_{-1} - 1)^2\right) x_B^2 + (2y - 1) \frac{t}{m_N^2} (\phi_{-1} - 1) (-\phi_{-1} + y (2\phi_{-1} + 2) x_B - 4 (1 - 2y)^2 \frac{t}{4m_N^2} (\phi_{-1} - 1)^2) F_1(t) + 2F_1(t) F_2(t) x_B \left(x_B (y + 1)^2 - \frac{m_N^2}{4} (y + 1)^2\right) - \frac{t}{m_N^2} \left((8\phi_{-1}^2 - 24\phi_{-1} + 17) y^2 - 2 (8\phi_{-1}^2 - 24\phi_{-1} + 19) y + 10\phi_{-1} - 36\phi_{-1} + 35)\right) x_B + F_2(t) \left((y + 1)^2 \left(\frac{t}{4m_N^2} + 1\right) x_B^4 - (y + 1) \frac{t}{m_N} \phi_{-1} + y \left(\frac{t}{4m_N^2} + 2\phi_{-1} + 1\right) + 2\right) x_B + \frac{t}{m_N} \left((4\phi_{-1}^2 - 16\phi_{-1} + 1) (2\phi_{-1} - 1)^2\right) y^2 + 2 \left(6\phi_{-1}^2 - 20\phi_{-1} + \frac{t}{m_N^2} (2\phi_{-1} - 1) + 17\right) y - 9\phi_{-1}^2 + \frac{t}{4m_N^2} (5 - 4\phi_{-1} + 34\phi_{-1} - 34) - (2y - 1) \left(\frac{t}{m_N^2} \frac{t}{4m_N^2} (\phi_{-1} - 1) (-\phi_{-1} + y (2\phi_{-1} + 2) x_B + (1 - 2y)^2 \left(\frac{t}{4m_N^2}\right)^2 (\phi_{-1} - 1)^2)\right)\right] + \mathcal{O}\left(\frac{m_N^2}{Q^2}, \frac{t^2}{Q^2}\right),$$
\[
c_{1}^{BH} = \frac{K m_{\pi}^2}{9 Q^2} \left[ 4 \left(3 (-4y + 3(y - 2) \phi_{-1} + 9) x_B^3 ight.ight.
\left.\vphantom{\frac{4}{3}} - 2(\phi_{-1} - 1) \left(-\frac{5t}{2m_N^2} + 9\right) \phi_{-1} + 2y \left(\frac{t \phi_{-1}}{m_N^2} - \frac{3t}{4m_N^2} + 3 \phi_{-1} - 6\right) + 18 \right)x_B^2 
+ \frac{3t}{m_N^2} (\phi_{-1} - 1)(-6 \phi_{-1} + y(4 \phi_{-1} - 3) + 3) x_B - (2y - 3) \frac{6t}{m_N^2} (\phi_{-1} - 1)^2 \right] F_1(t) 
+ 4F_1(t)F_2(t)x_B^2 \left(3(y - 3) x_B^2 - 12(y - 3) \frac{t}{4m_N^2} x_B - 8 \frac{t}{4m_N^2} (4y(\phi_{-1} - 3) - 5 \phi_{-1} + 18) (\phi_{-1} - 1) \right) 
- F_2^2(t) (-6(y - 3) x_B^4 
+ \frac{t}{4m_N^2} (-6(y - 3) x_B^2 + 12(-2y + 3(y - 2) \phi_{-1} + 3) x_B + 8(2y(\phi_{-1} - 6) - \phi_{-1} + 18) (\phi_{-1} - 1)) x_B^2 
+ 24 \left(\frac{t}{4m_N^2}\right)^2 (x_B (x_B - 2 \phi_{-1} + 2) + 2(\phi_{-1} - 1)) (x_B (y - 3) - 2(2y - 3)(\phi_{-1} - 1)) \right] 
+ O \left(\frac{m_N^4}{Q^4}, \frac{t^2}{Q^2}\right) 
\]

\[
c_2^{BH} = -\frac{4 K^2}{9} \left[ (5x_B - 4 \phi_{-1} + 4)(\phi_{-1} - 1) F_1^2(t) + 2x_B F_1(t) F_2(t) \right] 
+ \left( \left(1 + \frac{t}{4m_N^2}\right)x_B - \frac{5t \phi_{-1}}{4m_N^2} \left(\phi_{-1} - 1\right) + \frac{t}{m_N^2} (\phi_{-1} - 1)^2 \right) \right] F_2^2(t) \right] 
+ O \left(\frac{m_N^2}{Q^2}, \frac{t}{Q^2}\right) . 
\]

Here, following Refs. [13, 28, 29] we introduced the notations,

\[
K^2 = \frac{\Delta_1^2}{Q^2} \left(1 - y - \frac{y^2 \epsilon^2}{4}\right) 
= -\frac{t}{Q^2} (1 - x_B) \left(1 - y - \frac{t^2 \epsilon^2}{4}\right) \left(1 - \frac{t_{min}}{t}\right) \left\{ \sqrt{1 + \epsilon^2 + \frac{4x_B (1 - x_B) + \epsilon^2 t - t_{min}}{4(1 - x_B)}} \right\}, 
\]
\[
\epsilon^2 = \frac{4m_N^2 x_B^2}{Q^2}, \quad t_{min} = -m_N^2 x_B \frac{t}{1 - x_B} + O \left(\frac{m_N^2}{Q^2}, \frac{t}{Q^2}\right) . 
\]

Notice that in the leading order in $Q^2$ the contributions of diagrams (b) and (c) to the terms $c_0^{BH}$ and $c_1^{BH}$ exactly cancel each other, so that the coefficients $c_0^{BH}$ and $c_1^{BH}$ acquire an extra suppression factor $m_N^2/Q^2$. This can be understood as a screening of the opposite charges of the pion and muon, when both of them move in the forward direction in the limit of massless leptons. As we can see, the BH cross-section is symmetric under the $\phi \rightarrow -\phi$ transformation. For asymptotically large $Q^2$, the $c_1^{BH}$ harmonic is suppressed by $\Delta_1/Q$, whereas $c_0^{BH} \sim c_2^{BH}$, so the distribution is symmetric relative to the replacement $\phi \rightarrow \pi - \phi$. Note also that $K \sim \Delta_1^2$ and vanishes when $\Delta_1 \rightarrow 0$.

The interference term in (11) has a form (see details in Appendix A).
where

\[
C_0^{\text{int}} = -\frac{m_N^2 (1 - y)}{Q^2} \left( -4 (1 - x_B) \Re \mathcal{H} + x_B^2 \Re \mathcal{E} \right) F_1 + \Re \mathcal{E} F_2 \frac{t}{4m_N^2} (x_B - 2)^2 + \left( \Re \mathcal{H} + \Re \mathcal{E} \right) F_2 x_B^2 \right) \right) \right) + \mathcal{O} \left( \frac{m_N^2}{Q^2} \frac{t}{Q^2} \right),
\]

\[
C_1^{\text{int}} = \frac{K}{3} \left[ F_1 \left( 2 \Re \mathcal{E} (y - 3) x_B^2 + \Re \mathcal{H} \left( 4 (x_B - 2) (2y - 3) \phi_{-1} - 4 ((x_B - 4) y + 6) \right) \right) + \frac{F_2}{4m_N^2} (y - 2) \left( 2x_B^2 (y - 3) - (x_B - 2) \left[ 4 (2y - 3) (\phi_{-1} - 1) - 2x_B (y - 3) \right) \right) \right) + \mathcal{O} \left( \frac{m_N^2}{Q^2} \frac{t}{Q^2} \right)
\]

\[
S_1^{\text{int}} = \frac{K (2 - x_B)}{6} \left[ F_1 \left( -2 \Im \mathcal{E} (y + 1) x_B^2 - 2 \Im \mathcal{H} \left( x_B (4\phi_{-1}y - 2y - 2\phi_{-1} + 4) - 4 (2y - 1) (\phi_{-1} - 1) \right) \right) + \frac{F_2}{4m_N^2} (y + 1) \left( 1 + \frac{t}{2m_N^2} \right) x_B^2 + \frac{t}{2m_N^2} (2\phi_{-1}y + y - \phi_{-1} - 2) x_B + (2y - 1) \left[ \phi_{-1} - 1 \right) \right) \right) + \mathcal{O} \left( \frac{m_N^2}{Q^2} \frac{t}{Q^2} \right)
\]

As one can see from (14), the angular dependence of the interference term has a \(\sin \phi\) term which is absent both in the BH and DVMP taken alone. This term stems from the interference of the vector and axial vector current in lepton part of the diagram. It has the same sign for neutrino and antineutrino beams (in contrast, all the other terms in (14) change sign). Such terms are absent in case of DVMP and BH since the interference is asymmetric w.r.t. polarization vectors of the emitted boson. Due to presence of \(\sin \phi\) harmonics, the antisymmetrized cross-section directly probes the imaginary part of the DVMP amplitude as

\[
\frac{d^4 \sigma_{\text{asym}}(\phi)}{dt \, d\ln x_B \, dQ^2 \, d\phi} = \frac{d^4 \sigma(\phi)}{dt \, d\ln x_B \, dQ^2 \, d\phi} - \frac{d^4 \sigma(-\phi)}{dt \, d\ln x_B \, dQ^2 \, d\phi} \sim S_1^{\text{int}} \sin \phi \sim \Im \left( C_1^{\text{int}} \right).
\]

Another way to access the interference term is based on the isospin symmetry for the pure DVMP cross-sections [5, 6]. Since the BH correction nearly vanishes on a neutron target, and that BH cross-sections may be easily calculated, one may directly probe the interference term and extract the real and imaginary parts of the DVMP amplitude as,

\[
\frac{d^4 \sigma^{(\text{int})}_{\nu p \to \mu^+ \pi^-}}{dt \, d\ln x_B \, dQ^2 \, d\phi} \approx \frac{d^4 \sigma^{(\text{int})}_{\bar{\nu} p \to \mu^+ \pi^-}}{dt \, d\ln x_B \, dQ^2 \, d\phi} \approx \frac{d^4 \sigma^{(\text{int})}_{\nu p \to \mu^- \pi^+}}{dt \, d\ln x_B \, dQ^2 \, d\phi} \approx \frac{d^4 \sigma^{(\text{int})}_{\bar{\nu} p \to \mu^- \pi^+}}{dt \, d\ln x_B \, dQ^2 \, d\phi},
\]

\[
\frac{d^4 \sigma^{(\text{int})}_{\nu n \to \mu^- \pi^+}}{dt \, d\ln x_B \, dQ^2 \, d\phi} \approx \frac{d^4 \sigma^{(\text{int})}_{\bar{\nu} n \to \mu^- \pi^+}}{dt \, d\ln x_B \, dQ^2 \, d\phi}
\]

(19)
III. GPD AND DA PARAMETRIZATIONS

As was mentioned in the introduction, an essential uncertainty in the calculations of DVMP originate from the poorly known DAs of the produced mesons. Only the DAs of pions and $\eta$-mesons have been tested experimentally, and even in this case the situation remains rather controversial. The early experiments CELLO and CLEO [34], which studied the small-$Q^2$ behavior of the form factor $F_{M\gamma\gamma}$, found it to be consistent with the asymptotic form, $\phi_{as}(z) = 6z(1-z)$. Later the BABAR collaboration [35] found a steep rise with $Q^2$ of the form factor $Q^2 F_{\pi\gamma\gamma}(Q^2)^2$ in the large-$Q^2$ regime. This observation gave rise to speculations that the pion DA might have a $z$-dependence quite different from the asymptotic form [36] (see also the recent review by Brodsky et. al. in [37, 38]). However, the most recent measurements by the BELLE collaboration [39] did not confirm the rapid growth with $Q^2$ observed in the BABAR experiment. As was found in [40, 41] based on the fits to BELLE, CLEO and CELLO data, the Gegenbauer expansion coefficients of the pion DA are small and give at most 10% correction for the minus-first momentum $\phi_{-1}$ defined in [40]. Although there are no direct measurements of the kaon DAs, it is expected that their deviations from the pion DAs are parametrically suppressed by the quark mass $m_q/GeV$. Numerically this corresponds to a 10-20% deviation.

For this reason in what follows we assume all the Goldstone DAs to have the asymptotic form,

$$\phi_{2,\pi,K,\eta}(z) \approx \phi_{as}(z) = 6z(1-z). \quad (21)$$

For the decay constants we use the standard values $f_{\pi} \approx 93$ MeV, $f_K \approx 113$ MeV, and $f_{\eta} \approx f_{K^*}$. While we neither endorse nor refute any of them, for the sake of concreteness we select the parametrization [42, 43, 44, 48], which succeeded to describe HERA [51] and JLAB [42, 49, 50] data on electro- and photoproduction of different mesons, so it might provide a reasonable description of $\nu$DVMP. This parametrization is based on the Radyushkin’s double distribution ansatz. It assumes additivity of the valence and sea parts of the GPDs, and $q(\beta)$ are discussed in [42, 49]. Notice that in this model the sea is flavor symmetric for asymptotically large $Q^2$.

$$H(x, \xi, t) = H_{val}(x, \xi, t) + H_{sea}(x, \xi, t), \quad (22)$$

which are defined as

$$H_{val}^q = \int_{|\alpha|+|\beta| \leq 1} d\beta d\alpha \delta(\beta - x + \alpha \xi) \frac{3\theta(\beta)(1-|\beta|^2 - \alpha^2)}{4(1-|\beta|^2)^2} q_{val}(\beta) e^{(b_i - \alpha_i \ln|\beta|)t}, \quad (23)$$

$$H_{sea}^q = \int_{|\alpha|+|\beta| \leq 1} d\beta d\alpha \delta(\beta - x + \alpha \xi) \frac{3 sgn(\beta)(1-|\beta|^2 - \alpha^2)^2}{8(1-|\beta|^2)^5} q_{sea}(\beta) e^{(b_i - \alpha_i \ln|\beta|)t}, \quad (24)$$

and $q_{val}$ and $q_{sea}$ are the ordinary valence and sea components of PDFs. The coefficients $b_i, \alpha_i$, as well as the parametrization of the input PDFs $q(x)$, $\Delta q(x)$ and pseudo-PDFs $\epsilon(x), \tilde{\epsilon}(x)$ (corresponding to the forward limit of the GPDs $E, \bar{E}$), are discussed in [42, 49, 50]. The unpolarized PDFs $q(x)$ within the range of $Q^2 \lesssim 40 \text{ GeV}^2$ roughly coincide with the CTEQ PDFs. Notice that in this model the sea is flavor symmetric for asymptotically large $Q^2$,

$$H_{sea}^u = H_{sea}^d = \kappa(Q^2) H_{sea}^s, \quad (25)$$

where

$$\kappa(Q^2) = 1 + \frac{0.68}{1 + 0.52 \ln(Q^2/Q_0^2)}, \quad Q_0^2 = 4 \text{ GeV}^2. \quad (26)$$

The equality of the sea components for $u$ and $d$ quarks in (25) should be considered as a rough approximation, since in the forward limit $\bar{d} \neq \bar{u}$ was firmly established by the E866/NuSea experiment [52]. For this reason the predictions made
with this parametrization of GPDs for the \( p \mapsto n \) transitions in the region \( x_{Bj} \in (0.1...0.3) \) might slightly underestimate the data.

The Dirac and Pauli form factors \( F_1(t) \), \( F_2(t) \) are extracted from GPDs in the standard way,

\[
F_1^{cm}(t) = \sum_q e_q \int_{-1}^{1} dx \, H_q(x, \xi, t),
\]

\[
F_2^{cm}(t) = \sum_q e_q \int_{-1}^{1} dx \, E_q(x, \xi, t).
\]

### IV. NUMERICAL RESULTS AND DISCUSSION

In this section we perform numerical analysis of the electromagnetic corrections to the processes listed in Table I, relying on the GPDs described in the previous section. At small \( Q^2 \) the angular harmonics are small and the cross-section is dominated by the angular-independent DVMP contribution. Therefore, it is convenient to normalize all the coefficients to DVMP cross-section,

\[
\frac{d^4\sigma}{dt \, d\ln x_B \, dQ^2 \, d\phi} = \frac{d^4\sigma^{(DVMP)}}{dt \, d\ln x_B \, dQ^2 \, d\phi} \left( \sum_{n=0}^{2} c_n \cos n\phi + s_1 \sin \phi \right).
\]

Notice that in the limit \( \alpha_{em} \to 0 \), no BH corrections are possible, the coefficient \( c_0 = 1 \), and all other coefficients vanish.

The results for the \( Q^2 \)-dependence of the relative BH corrections to the neutrino-DVMP cross section for pions and kaons are presented in Figure 2.

We see that the isotropic part of the BH correction \( 1 - c_0 \) steeply rises in all channels from few percent or less at \( Q^2 \lesssim 10 \text{ GeV}^2 \) up to few tens of percent at \( Q^2 \sim 100 \text{ GeV}^2 \). It behaves like \( 1 - c_0 \propto Q^2 \) modulo logarithmic corrections. As a result, the cross section is reduced about twice compared to the DVMP contribution. The asymmetry \( s_1 \) also rises with \( Q^2 \) and reaches about \( 15\% \) at \( Q^2 = 100 \text{ GeV}^2 \). Notice that some of the coefficients (e.g. \( c_0 - 1 \), \( c_1 \)) have nodes for \( \pi^+/K^+ \) production, while they are absent for \( \pi^-/K^- \). The reason is purely algebraic: as one can see from the Table II for \( \pi^+/K^+ \) the large \( s \)-channel coefficient function \( c_- \) is convoluted with the small \( d/s \)-quark GPD, whereas the small \( u \)-channel coefficient function \( c_+ \) is convoluted with the large \( u \)-quark GPD. This produces a node in the real part of the DVMP amplitude, because the real parts of the two contributions have opposite signs. Such a node is absent for negatively charged mesons, because the “large” \( u \)-quark GPD is convoluted with the “large” \( c_- \). The full DVMP cross section has no nodes, because it gets a large contribution from the imaginary part, which homogeneously depends on \( Q^2 \) (the coefficient \( s_1 \), which probes the imaginary part, has no nodes). The difference between the Cabibbo suppressed and allowed processes comes from the sensitivity to different flavor combinations of GPDs in the corresponding DVMP amplitude.

The terms \( c_0 - 1 \) and \( c_1 \) in Eqn. (23) are dominated by the interference of the DVMP and BH amplitudes, therefore they have different signs for \( \pi^+ \) and \( \pi^- \) (and \( K^+ \) and \( K^- \)). The term \( c_2 \) gets contribution only from BH process, so it always has the same sign. The \( s_1 \)-term does not change its sign under the \( C \)-conjugation in the lepton part, because it originates from the \( P \)-odd interference between the vector and axial vector currents.

The results for \( \Delta_\perp \)-dependence of the BH corrections are depicted in Figure 3.

It shows that there is a qualitative difference between \( c_0 \) and other angular harmonics. The coefficient \( c_0 \) reaches its maximum at \( \Delta_\perp = 0 \) due to the \( 1/t \) behavior of the BH cross section. In contrast, the angular harmonics \( c_1 \), \( c_2 \), \( s_1 \) vanish at small \( \Delta_\perp \) due the \( K \)-factors in front of them. As a consequence, the harmonics reach their maxima at \( \Delta_\perp \sim 0.1 \text{ GeV} \).

The results for the angular harmonic coefficients vs the elasticity parameter \( y = P \cdot q/P \cdot k = \nu/E_\nu \) are presented in Figure 4.

The coefficient \( c_2 \) does not depend on \( y \) at all due to exact cancellation of the pre-factors \( 1 - y - y^2/4 \) in \( K^2 \) and the pre-factor in the DVMP cross-section Eqn. (2). The harmonics \( c_0 \) and \( c_1 \) have a mild dependence on \( y \), except in the
region $y \sim 1$, where they blow up, because the DVMP cross-section \(2\) is suppressed there by the factor \(\sim 1 - y - y^2\epsilon^2/4\), whereas the harmonics in \(8, 14\) are suppressed at most as \(K \propto \sqrt{1 - y - y^2\epsilon^2/4}\). The harmonic $s_1$ in accordance with Eqn. \(17\) in the kinematics $x_B \ll 1$, $|t| \ll m_N^2$ is proportional to \(\sim (2y - 1)F_1(t)\text{Im} \mathcal{H}\) and has a node near $y \approx 0.5$.

V. SUMMARY

In this paper we studied the electromagnetic Bethe-Heitler corrections to neutrino-induced deeply virtual meson production. We found these corrections to fall with $Q^2$ less steeply compared with the $\nu$DVMP cross section, so they tend to become a dominant mechanism in the Bjorken limit of $Q^2 \to \infty$. Besides, they are enhanced at small-$t$ due to the
t-channel Coulomb pole \( \propto \frac{1}{t} \). Remarkably, these corrections generate an angular correlation between the lepton and hadron scattering planes. Similar to the BH corrections in DVCS, some angular harmonics are sensitive to the real or imaginary parts of the DVMP amplitude (see (15-17)). Notice that the appearance of such angular dependence was also predicted in [53], however there it appears due to interference of the longitudinally and transversely polarized charged bosons, which is a twist-three effect. In contrast, our result is a twist-two effect, and as one can see from Figure 2, it is not suppressed for asymptotically large \( Q^2 \).

Numerically, the BH corrections are subject to the interplay between the suppression factor \( \sim \alpha_{em} \) and the relative enhancement, which is as large as \( \sim Q^2/t \) for some harmonics. In the kinematics of the Minerva experiment, the BH contribution for the proton target represents a few percent correction and thus is important for precision tests of the GPD parametrizations. At \( Q^2 \sim 100 \text{ GeV}^2 \), which can be accessed in future neutrino experiments, these corrections

FIG. 3: (color online) \( \Delta_\perp \)-dependence of the BH correction to the \( \nu \)DVMP process.
are expected to become on par with the DVMP contribution. For a neutron target, these corrections are two orders of magnitude smaller than for a proton and can be neglected up to very high \( Q^2 \). Combining this fact with isospin symmetry of the DVMP amplitude, we construct combinations of the cross-sections Eqs. (19,20), which are sensitive only to the interference term.

The electromagnetic corrections discussed in this paper are important only in neutrino-induced DVMP: in the case of electron-induced processes \( ep \rightarrow epM \) the BH corrections are suppressed by the factor \( (G_F^2 Q^4) \), and are negligibly small. We provide a computational code, which can be used for evaluation of the cross-sections relying on different GPD models.
Appendix A: Evaluation of the $\nu$DVMP and BH cross-sections

In this section we present some technical details of the evaluation of diagrams (a-c) in the Figure 1. The calculation of the diagram (a) in Figure 1 is rather straightforward and yields for the amplitude of the process \cite{26,31,32}

$$T_a = \frac{\alpha_s}{Q} \frac{\phi_{-\gamma}}{\Gamma} \int d\zeta \left[ \tilde{H}_N^\gamma (p_2) \Gamma N (p_1) \right],$$  \hspace{1cm} (A1)

where $\tilde{\mu}(k)$ and $\nu(k)$ are the spinors of the final muon and initial neutrino; $e(k)$ is the polarization vector of the photon; $N(p)$, $\tilde{N}(p)$ are the spinors of the initial/final state baryons; $\phi_M(z)$ is the DA of the produced meson; $f_M$ is the decay constant of the meson $M$; subscript index for each momentum in Eqn. (A1) and in what follows shows to which particle it corresponds; $\sum_f H_f^\gamma (p_2) \Gamma N (p_1)$ is a symbolic notation for summation of all the leading twist GPDs contributions (defined below); and $\tilde{H}_N^\gamma$ are the convolutions of the GPDs $H_f^\gamma$ of the target with the proper coefficient function. Currently, the amplitude of the DVMP is known up to the NLO accuracy \cite{32,33}. Extension of the analysis of \cite{30,31} to neutrinos is straightforward. In contrast to electro-production, due to the $V-A$ structure, the amplitudes acquire contributions from both the unpolarized and helicity flip GPDs.

Four GPDs, $H$, $E$, $H$ and $E$ contribute to this process in the leading twist. They are defined as

$$\tilde{P}^+ \int d\zeta e^{i\zeta P^+ z} \left\langle A (p_2) \left| n_{\zeta} (z) \right| A (p_1) \right\rangle = \left( H_N (x, \xi, t) \tilde{N} (p_2) \gamma_+ N (p_1) + \frac{\Delta_\perp}{2 m_N} E_{\perp} (x, \xi, t) \tilde{N} (p_2) i \sigma_+ k N (p_1) \right),$$  \hspace{1cm} (A2)

where $\tilde{P} = p_1 + p_2$, $\Delta = p_2 - p_1$ and $\xi = -\Delta^+ / 2 \tilde{P}^+ \approx x_B J / (2 - x_B J)$ (see e.g. \cite{11} for the details of kinematics). In what follows we assume that the target is either a proton or a neutron. Since in neutrino experiments the target cannot be polarized due to its large size, it makes no sense to discuss the transversity GPDs $H_T, E_T, H_T, E_T$. We also ignore the contributions of gluons in this paper because in the current neutrino experiments the region of small $x_B \ll 1$ but very high $Q^2$, is hardly accessible, so the amplitude (A1) simplifies to

$$T_a = \frac{8 \pi i G_F}{9} \frac{\tilde{\mu}(k_\mu)}{\sqrt{2} Q} \frac{\phi_M(z)}{(1 + Q^2/M_\Psi^2)} \left[ \tilde{H}_N^\gamma (p_2) \gamma_+ \gamma_5 N (p_1) + \frac{\Delta_\perp}{2 m_N} \tilde{E}_M^\perp (p_2) i \sigma_+ k N (p_1) \right],$$  \hspace{1cm} (A4)

In Table I the corresponding amplitudes are listed for each final state $M$. The DVMP part of the corresponding neutrino cross-section for charged currents is given by (2).

In the leading order in $Q^2$ both BH diagrams Figure 1 (b,c) are dominated by longitudinally polarized photons. Nevertheless, as was mentioned in Section II we evaluate the BH contribution exactly, because various angular harmonics, suppressed by $\sim Q_\perp / Q$, get contribution from the transverse components, which is of the same order as the longitudinal result. Only after that we make expansion in $1/Q^2$.

The dipole scattering amplitude, which contributes to the diagram Figure 1 (b), has the form

$$A^{ab}_{\mu \nu} (q, \Delta) = \frac{1}{F_m} \int d^4 x e^{-i q x} \left\langle \left| V^{a}_{\mu} (x) - A^{a}_{\mu} (x) \right| J^{\nu}_{\nu} (0) \right\rangle \pi^b (q - \Delta),$$  \hspace{1cm} (A5)
where $V^a_\mu(x)$ and $A^a_\mu(x)$ are the vector and axial-vector isovector currents. Notice that the amplitude $A_{\mu\nu}$ should not be interpreted as a pion form factor, because: (i) the virtuality is large; (ii) the insertion of the pion state between $A^a_\mu$ and $J_{\mu}^{em}$ leads to $A_{\mu\nu}^{ab} \sim q_\mu$, which vanishes when multiplied by an on-shell lepton current.

We evaluated $[A]$ in pQCD in the collinear approximation. This is justified in Bjorken kinematics by the high virtuality of the charged boson, so we assume that the dominant contribution comes from the leading twist-2 pion DA. The result reads,

$$A_{\mu\nu}^{ab}(q, \Delta) = \frac{1}{4} (g_{\mu\nu} f_0 + (q_\mu n_\nu + q_\nu n_\mu) f_1 + (\Delta_\mu n_\nu + \Delta_\nu n_\mu) f_2 - i\epsilon_{\mu\nu\beta\gamma} n_\beta q_\gamma g_1 - i\epsilon_{\mu\nu\beta\gamma} n_\beta \Delta_\gamma g_2);$$

$$f_0 = \int dz \phi_M(z) \frac{-2z\bar{z} + (1 - 2z\bar{z}) t/Q^2}{x_B (z - \bar{z} t/Q^2) (\bar{z} - z t/Q^2)} \approx 2\phi_{-1} + O\left(\frac{m_N^2}{Q^2}, \frac{t}{Q^2}\right);$$

$$f_1 = \int dz \phi_M(z) \frac{2z\bar{z} - (1 - 2z\bar{z}) t/Q^2}{x_B (z - \bar{z} t/Q^2) (\bar{z} - z t/Q^2)} \approx \frac{2}{x_B} + O\left(\frac{m_N^2}{Q^2}, \frac{t}{Q^2}\right);$$

$$f_2 = \int dz \phi_M(z) \frac{(1 - 2z\bar{z}) - 2z t/Q^2}{x_B (z - \bar{z} t/Q^2) (\bar{z} - z t/Q^2)} \approx \frac{2}{3x_B} (\phi_{-1} - 1) + O\left(\frac{m_N^2}{Q^2}, \frac{t}{Q^2}\right);$$

$$g_1 = \frac{1}{3 q \cdot n} \int dz \phi_M(z) \frac{-2z\bar{z} + (1 - 2z\bar{z}) t/Q^2}{x_B (z - \bar{z} t/Q^2) (\bar{z} - z t/Q^2)} \approx -\frac{2}{3x_B} + O\left(\frac{m_N^2}{Q^2}, \frac{t}{Q^2}\right);$$

$$g_2 = -\frac{1}{3 q \cdot n} \int dz \phi_M(z) \frac{1 - 2z\bar{z} - 2z t/Q^2}{x_B (z - \bar{z} t/Q^2) (\bar{z} - z t/Q^2)} \approx \frac{2}{3x_B} (1 - \phi_{-1}) + O\left(\frac{m_N^2}{Q^2}, \frac{t}{Q^2}\right).$$

Here $p_\mu$ and $n_\mu$ are the positive and the negative direction light-cone vectors respectively. The plus-components of $q$ and $\Delta$ have the form,

$$n \cdot q = \frac{Q^2 (1 - \sqrt{1 + \epsilon^2})}{2m_N x_B} \approx -x_B + O\left(\epsilon^2\right);$$

$$n \cdot \Delta = \frac{t m_N^2}{x_B (1 - \sqrt{1 + \epsilon^2}) - x_B (2x_B + 1 - \sqrt{1 + \epsilon^2}) t/Q^2};$$

$$\approx -x_B + O\left(\epsilon^2, \frac{t}{Q^2}\right),$$

where $\epsilon = 2m_N x_B/Q$. In what follows we encounter the combination $4 - f_0 - x_B (f_1 - f_2)$, for which we need to make expansion up to $O(Q^{-2})$. While separately the series expansion coefficients for each factor $f_i$ have non-integrable singularities $\sim z^{-2}\bar{z}^{-2}$, which signal a sensitivity to the transverse degrees of freedom and presence of the non-analytic terms $\sim \ln(Q^2/t^2)/Q^2$, in the above-mentioned combination, these terms cancel each other resulting in

$$f_0 + x_B (f_1 - f_2) \approx 4 + 2\phi_{-1} \frac{(1 + x_B) - 2m_N^2 x_B^2}{Q^2} - \frac{2t (1 + x_B) - 6m_N^2 x_B^2}{Q^2}. $$

The amplitude of the axial current transition into an on-shell pion in the diagram Figure[1] (c) according to PCAC has the form,

$$\langle 0 | J_{\mu}^{b \nu}(0) | \pi^a (q - \Delta) \rangle = if_\pi \sqrt{2} q_\mu. $$

In order to simplify the calculation of the leptonic part of the diagram (c), we employ the chain of identities

$$k_\pi (1 - \gamma_5) \nu (k_\nu) = (1 + \gamma_5) \hat{k}_\pi \nu (k_\nu) = (1 + \gamma_5) (\hat{k}_\pi - \hat{k}_\nu) \nu (k_\nu),$$

(A16)
\[ S(k_\nu - k_\pi)(1 + \gamma_5) \left( \tilde{k}_\pi - \tilde{k}_\nu \right) = S(k_\nu - k_\pi) \left( \tilde{k}_\pi - \tilde{k}_\nu \right) (1 - \gamma_5) \]
\[ = -1 + \gamma_5 + m_\mu S(k_\nu - k_\pi) \approx -(1 - \gamma_5), \tag{A17} \]

\[ \mathcal{A}_l \sim e\bar{\mu}(k_\mu) \bar{\epsilon}(k_\gamma) S(k_\nu - k_\pi) \tilde{k}_\pi (1 - \gamma_5) \nu(k_\nu) \]
\[ = -e\bar{\mu}(k_\mu) \bar{\epsilon}(k_\gamma) \nu(k_\nu) + \mathcal{O}(m_\mu), \tag{A18} \]

where in (A16) we make use of the fact that the initial state neutrino is on-shell, \( \tilde{k}_\nu \nu(k) = 0 \). Actually, the simplification of (A18) is a manifestation of the Ward-Takahashi-Slavnov-Taylor identity for the charged lepton current,

\[ i \partial_\alpha (\bar{\mu} \gamma_\alpha (1 - \gamma_5) \nu) = -\frac{g}{\sqrt{2}} \bar{\nu} W^+ (1 - \gamma_5) \nu + \frac{g}{\sqrt{2}} \bar{\mu} A (1 - \gamma_5) \nu - \frac{g}{2} \left( g_{V}^{(\mu)} + g_{A}^{(\mu)} - g_{V}^{(\nu)} - g_{A}^{(\nu)} \right) \bar{\mu} Z (1 - \gamma_5) \nu \neq 0. \tag{A19} \]

This simplification explains why there is no harmonics in the denominator of the BH and interference terms, and it is valid in the limit of massless leptons. The diagrams (b, c) yield for the amplitude (sign corresponds to \( \pi^+ \))

\[ \sim e^2 \frac{\bar{\mu}(k - q) \gamma_\mu (1 - \gamma_5) \nu(k)}{t} i f_x \left( -g_{\mu\nu} + \frac{A_{\mu\nu}}{1 + Q^2/M_W^2} \right) U(P + \Delta) \left( F_1(t) \gamma_\nu + \frac{i\sigma_\nu\Delta_\nu}{2M} F_2(t) \right) U(P) \tag{A20} \]

Further evaluation of Bethe-Heitler and interference terms requires some trivial but tedious Dirac algebra, which was done with FeynCalc [54].

Acknowledgments

This work was supported in part by Fondecyt (Chile) grants No. 1090291, 1100287 and 1120920.

[1] X. D. Ji and J. Osborne, Phys. Rev. D 58 (1998) 094018 [arXiv:hep-ph/9801260].
[2] J. C. Collins and A. Freund, Phys. Rev. D 59, 074009 (1999).
[3] D. Mueller, D. Robaschik, B. Geyer, F. M. Dittes and J. Horajs, Fortsch. Phys. 42, 101 (1994) [arXiv:hep-ph/9812448].
[4] X. D. Ji, Phys. Rev. D 55, 7114 (1997).
[5] X. D. Ji, J. Phys. G 24, 1181 (1998) [arXiv:hep-ph/9807358].
[6] A. V. Radyushkin, Phys. Lett. B 380, 417 (1996) [arXiv:hep-ph/9604317].
[7] A. V. Radyushkin, Phys. Rev. D 56, 5524 (1997).
[8] A. V. Radyushkin, In *Shifman, M. (ed.): At the frontier of particle physics, vol. 2* 1037-1099 [arXiv:hep-ph/0101225].
[9] J. C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. D 56, 2982 (1997).
[10] S. J. Brodsky, L. Frankfurt, J. F. Gunion, A. H. Mueller and M. Strikman, Phys. Rev. D 50, 3134 (1994).
[11] K. Goeke, M. V. Polyakov and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47, 401 (2001) [arXiv:hep-ph/0106012].
[12] M. Diehl, T. Feldmann, R. Jakob and P. Kroll, Nucl. Phys. B 596, 33 (2001) [Erratum-ibid. B 605, 647 (2001)] [arXiv:hep-ph/0009255].
[13] A. V. Belitsky, D. Mueller and A. Kirchner, Nucl. Phys. B 629, 323 (2002) [arXiv:hep-ph/0112108].
[14] M. Diehl, Phys. Rept. 388, 41 (2003) [arXiv:hep-ph/0307382].
[15] A. V. Belitsky and A. V. Radyushkin, Phys. Rept. 418, 1 (2005) [arXiv:hep-ph/0504030].
