A note on the Georgi Vector Limit at finite temperature/density

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The mechanism of chiral symmetry restoration in the context of the Georgi Vector Limit (GVL) at finite temperature/density is discussed. It is suggested that, whereas the system may be driven to the GVL, the fact of reaching the point of restoration does not necessarily mean the complete realization of the GVL since the other mechanisms of restoration may contribute at lower temperatures/densities.

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The statement that chiral symmetry is spontaneously broken in QCD is confirmed by the experimental data and there can be no doubt about it. The QCD ground state does not completely share the symmetries of the QCD Lagrangian so that Goldstone bosons and a number of nonzero order parameters appear. However, the experience borrowed from the condensed matter physics tells us that under some circumstances the continuous broken symmetry (in our case chiral symmetry) can be restored. The restoration may occur either in the academic limit of large $N_f$ or in the hadronic system at finite (and high enough) temperature/density. In the limit of the restored chiral symmetry the corresponding order parameters must vanish. One of the possible order parameters which we will focus on is the difference between the correlators of the chiral partners. Namely we will consider the difference $\Pi_V - \Pi_A$ between the vector and axial correlators. There have been proposed several different scenarios of how a vanishing $\Pi_V - \Pi_A$ difference might occur at finite temperature/density. In the standard picture one of the possibilities is that the masses of the chiral partners $\rho$ and $a_1$ become degenerate at the critical point thus providing the vanishing order parameter. A somewhat different variant of the standard pattern assumes that both the vector and axial correlators exhibit peaks of the same strength at both the mass of the $\rho$ and $a_1$ so that their difference should also vanish at the point of restoration. In the other scenario suggested in and called the Georgi Vector Limit (GVL) the $\rho$ meson is treated as the gauge boson of the Hidden Local Symmetry (HLS) which gets its mass through the Higgs mechanism. When the system is driven closer to the critical point the $\rho$ meson gets lighter and eventually massless. Accordingly, the $\rho$ meson (its longitudinal component) becomes the chiral partner of pion. So the longitudinal component of the $\rho$ meson can be treated as an effective massless scalar. Thus, the GVL implies the existence of the relations

$$< 0 | V_\mu | \rho_L(p) > = i p_\mu f_s, \quad (1)$$

and $f_s = f_\pi$, where $f_\pi = 93$ MeV is the the standard weak decay constant. It is worth mentioning that these couplings are not necessarily vanishing in the unbroken $SU(3) \times SU(3)$
phase. This scenario is called Vector Realization (VR) of the symmetry restoration. The other closely related mechanism called “Vector Manifestation” (VM) was suggested in [3]. In the VM the relation \( f_{\pi} = 0 \) holds at the point of restoration. The VM also requires GVL to be realized in the chirally restored phase. The major difference between the VR and VM mechanisms is that in the VR the Goldstone modes exist, whereas chiral symmetry is not broken. In the VM mechanism the Goldstone bosons disappear since \( f_{\pi} = 0 \).

One notes that in the standard scenario of chiral symmetry restoration in some academic limit in vacuum the mass of the \( \rho \) meson can either be massless or massive but should always be degenerated with the mass of the chiral partner. This is what happens in the limit of large \( N_f \). However, the situation becomes more complicated at finite temperature/density. In this case the general expression for the hadron correlator, satisfying the chiral symmetry requirements, can be written as follows

\[
\Pi_{\mu\nu} = \sum_{a,b} \int \frac{d^3 k}{2\omega_k} n_{\pi}(T/\rho, k) \int d^4 x e^{ip \cdot x} \langle A \pi^a(k) | T \{ J(x) J(0) \} | A \pi^b(k') \rangle.
\]

Here \( n_{\pi}(T/\rho, k) \) is the in-medium pion distribution which is the thermal phase factor in the finite temperature case and the average number of pions in the case of nuclear matter. We denoted \( J(x) \) the corresponding interpolating current. The states labeled as \( | A \rangle \) can either represent the Fermi-gas of noninteracting nucleons or the hadron system in the thermal bath with pionic contributions subtracted. At low temperature the hadron medium is primarily formed by the thermal pions so the state \( | A \rangle \) is just the QCD vacuum so that \( | A \pi \rangle \rightarrow | \pi \rangle \). One notes that in the case of finite baryon density pions are virtual so that all time directions must be taken into account. Using the soft pion theorem and current algebra commutation relations one can get the chiral mixing between axial and vector correlators

\[
\Pi_V = \hat{\Pi}_V + \xi (\hat{\Pi}_A - \hat{\Pi}_V)
\]

and similar expression for the axial vector correlator \( \Pi_A \)

\[
\Pi_A = \hat{\Pi}_A + \xi (\hat{\Pi}_V - \hat{\Pi}_A)
\]

Here we defined (for the finite baryon density case)
Here $\sigma_{\pi N}$ is the pionic piece of the pion-nucleon sigma term and $\rho$ is nuclear density. The corresponding thermal factor is

$$\xi(T) = \frac{T^2}{6f_\pi^2}$$

The mixing of the correlators is due to the in-medium pions interacting directly with the vector/axial interpolating currents. We denoted $\Pi_V(\Pi_A)$ the correlator of the vector (axial) currents, calculated without taking into account the soft pion effects, leading to the chiral mixing. The other finite temperature/density effects, like in-medium changes of masses or widths not related to the chiral mixing effect, may, in principle, be included in the “pionless” correlators $\Pi_V(\Pi_A)$. The finite temperature mixing relation was derived in [6] and the finite baryon density case was considered in [7]. Restoration of chiral symmetry corresponds to the complete mixing, which means the equality of the axial and vector correlators so that $\Pi_V - \Pi_A \rightarrow 0$. One notes in passing that Wigner realization of the chiral symmetry does not require the pion and its chiral partner to be massless at the restoration point. The complete mixing and thus chiral symmetry restoration occurs at $T \simeq 0.16$ GeV and $\rho \simeq 3\rho_0$ where $\rho_0 = 0.17 fm^3$ is the nuclear matter density. It is important to mention that, although formally the expressions above are valid at not very high temperature/density the critical points as given by the mixing relations are surprisingly close to those following from more sophisticated models and lattice simulations. It means that the mixing relations reflect the general model independent chiral structure of the in-medium correlators and approximately hold at densities/temperatures close to the critical. It could qualitatively be understood as follows. The critical temperature $T_c \simeq 0.16$ GeV is still significantly lower than the chiral scale $\Lambda \simeq 0.8 GeV$ so that the chiral symmetry based arguments concerning the general structure of the in-medium correlators may still be qualitatively applicable even at the temperatures not far from the critical. Similar arguments hold for the finite density case. In a sense, the mixing relations can be interpreted as the leading order terms in the
chiral expansion of the pertinent correlators where all nongoldstone degrees of freedom are collected in the “pionless” correlators $\hat{\Pi}_V(\hat{\Pi}_A)$.

Considering the mixing relations in the context of the GVL one first notes that the GVL can naturally be accommodated in the chiral mixing relations. Indeed, the longitudinal part of the axial correlator includes the contribution from the pion pole. We assume that the in-medium pion remains massless and focus basically on the finite temperature case. The generalization to the finite density system is straightforward. At low enough temperature the system is just lukewarm pion gas and $\hat{\Pi}_V(\hat{\Pi}_A)$ are the vacuum vector(axial) correlators. The axial correlator can be decomposed in the following way

$$\hat{\Pi}_{A,\mu\nu} = -\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] \hat{\Pi}_1(Q^2) - \frac{q_\mu q_\nu}{q^2} \left[\hat{\Pi}_1(Q^2) + Q^2 \hat{\Pi}_2(Q^2)\right]$$

(7)

The longitudinal part of the axial correlator contains the contribution from the pion pole

$$\hat{\Pi}_A^L \approx \frac{f_\rho^2}{Q^2}$$

(8)

We assume that the mixing relation approximately holds even at temperatures close to the critical one. The vacuum pion decay constant $f^2_\pi$ should, in principle, be replaced with its finite temperature analog $f^2_\pi(T)$. If the system is driven toward the GVL than

$$\hat{\Pi}_V \approx \frac{f_\rho^2(T)}{Q^2}$$

(9)

Where we assumed that the decay constant $f_\rho$ also acquires some temperature dependence $f_\rho \to f_\rho^2(T)$. At some critical temperature $f^2_\pi(T_c) = f_\rho^2(T_c)$ so that GVL is reached. We note however that there is a competing possibility of reaching the chiral symmetry restoration point which is related to the correlator mixing. Taking the difference $\Pi_V - \Pi_A$ one obtains

$$\Pi_V - \Pi_A \approx \frac{1}{Q^2} \left[f_\rho^2(T) - f_\pi^2(T)\right] (1 - 2\xi(T)),$$

(10)

so that the point $\xi(T) = 1/2$ could, in principle, be reached at the temperature lower than that needed for reaching GVL. The physical consequence of this fact is that, even if
the system shows a tendency towards the GVL with increasing temperature, the critical
temperature needed for chiral symmetry restoration does not neceserily coincides with that
at which the GVL is achieved and VM \((f_\rho^2(T) = f_\pi^2(T) = 0)\) is implemented. Therefore, in
the chirally restored phase one still may have massless pions and light (but massive) vector
mesons. In a sense, the chiral mixing and VM are just two competing scenarios, equally
participating in the process of symmetry restoration. The other possible mode is the already
mentioned VR mechanism. Figuring out which mechanism will take over eventually seems to
be a quite difficult problem which may depend on the model used. Neither one can a’priori
be excluded. We stress that the above arguments are admittedly suggestive and aim only at
indicating the possibility that in the GVL scenario at finite temperature/density the fact of
reaching the GVL point implies the chiral symmetry restoration whereas the opposite may
not be true. One notes that at \(T \rightarrow 0\) chiral symmetry restoration implies the mandatory
realization of the GVL.

It is worth mentioning that the existence of a few possible scenarios within one general
pattern of restoration is also the case when the standard mechanism (mixing of \(\rho\) and its
chiral partner \(a_1\)) is assumed. At the point of restoration (the point of complete mixing) the
masses of the chiral partners may either be degenerate, different or become comparable with
its widths. Any of these possibilities is possible. One needs to develop a sophisticated models
to understand which way is prefered by nature. Chiral symmetry alone cannot provide the
answer. Similar thing may take place in the GVL. The symmetry can be restored either in the
VM or VR mode. It is possible that VR \((f_\rho^2(T) = f_\pi^2(T) \neq 0)\) is reached at the temperature
lower than that needed for the complete chiral mixing to occur. Again, only a sophisticated
enough model where the correlator mixing and the possibilities of VM and/or VR modes
are incorporated can provide a reliable understanding of the chiral symmetry restoration
pattern in GVL. The naive, leading order estimates, based on the chiral perturbation theory
\[8\] give

\[f_\pi^2(T) = f_\pi^2(1 - \frac{T^2}{12 f_\pi^2} + O(T^4))\] (11)
indicate that the difference between axial and vector correlators approaches zero at approximately the same temperatures as \( f_\pi(T) \) does. The higher order \( O(T^4) \) corrections do not change this conclusion in any qualitative way. These corrections were calculated in Ref. [9]. The part of them, which is relevant in the context of this paper, amounts to the replacement \( \xi(T) \rightarrow \xi(T) - \xi^2(T)/2 \) in the mixing relations. It gives somewhat higher (\( T_c \sim 200 \) MeV) but still reasonable critical temperature. The \( f_\pi^2(T) \) shows the same tendency when the \( O(T^4) \) corrections are included. This behavior should, of course, be expected since both the difference \( \Pi_V - \Pi_A \) and decay constant \( f_\pi(T) \) are the order parameters. However, to understand the mechanism of chiral symmetry restoration in the GVL one should also know the temperature dependence of the \( f_\rho^2(T) \). In Ref. [9] this constant was parameterized as \( f_\rho = a f_\pi \). The successful hadron phenomenology can be reproduced if the choice \( a = 2 \) is made. The GVL requires \( a(T_{GV\!L}) = 1 \) at some critical temperature \( T_{GV\!L} \). The temperature dependence of the parameter \( a \) was studied in [10] in the framework of the HLS model. It was shown in [10] that up to \( T \sim 250 \) MeV this parameter exhibits very weak dependence on temperature and almost coincides with the value \( a = 2 \) extracted from the vacuum phenomenology so at the temperatures corresponding to the complete chiral mixing the situation is still quite far from the GVL. Our independent one-loop estimates resulted in the practically the same conclusion. We note, however, that the one-loop calculations may not be completely sufficient at the temperatures close to the critical. The other important aspect of the above discussion is the concept of vector dominance (VD) at finite temperature/density. As demonstrated in [10] VD is simply a consequence of a specific parameter choice \( a = 2 \) in the HLS model. Therefore, if GVL mechanism is indeed realized it implies that VD is badly violated at the point of restoration.

One possible way of studying the mechanism of restoration in the GVL is to use lattice methods at finite temperature. Since the pion effects can be included explicitly via chiral mixing and since pions remains almost massless even at the critical point, so that soft pion approximation may still be qualitatively correct, the temperature dependence of the axial and vector correlators (with pion effects subtracted) could be calculated in the quenched or
partially quenched approximation. However, the lattice calculations are not really possible at finite density so in this latter case some analytical models are required.

One needs to mention again that the arguments given in this letter are at best qualitative. A number of effects was not included. Besides the previously mentioned higher order terms ($O(T^4)$ etc) the effects of the heavier mesons should be included at temperatures near critical. However, in our opinion it seems unlikely that these corrections can qualitatively change the results for at least $T \simeq 100$ MeV. The appearance of baryons at some finite temperature may further complicate the situation. Baryons can roughly be considered as “drops” of chirally restored phase providing thus the other competing mechanism of chiral symmetry restoration. However, at $T \simeq 100$ MeV the baryonic gas is still quite dilute so the influence of baryonic degrees of freedom on the mechanism of restoration seems to be rather moderate. The constraints on the baryonic properties in the case of finite nuclear density, following from chiral symmetry, were considered in [12].

Let’s summarize the main points made in this letter. As was noticed in Ref. [5] in the limit of large $N_f$ restoration of chiral symmetry does imply the complete realization of the GVL and VM mechanism. In the VM mechanism the massless $\rho$ meson (longitudinal part) becomes the chiral partner of pion. So in this limit restoration of chiral symmetry and reaching GVL mean the same thing. Such an equivalence between GVL and chiral symmetry restoration may not be the case at finite temperature/density. There are some additional “medium” mechanisms of chiral symmetry restoration which should be taken into account. At finite temperature/density the hadronic system may also be driven toward the GVL so that vector mesons become light. At some critical temperature the GVL is reached and vector mesons turn out massless and degenerated with pions. At this temperature chiral symmetry is no longer broken. So reaching of the GVL does mean the restoration of chiral symmetry. The converse is not true at finite temperature/density. Even if the system shows the tendency toward the GVL, the restoration of chiral symmetry does not mean that the GVL is reached. The correlators of chiral partners get mixed in medium and may, in principle, reach the point of complete mixing (where their difference is zero) at the tempera-
tures (or densities) lower than that needed for GVL to be realized. Our crude estimates show that indeed the symmetry is restored at temperatures/densities lower than that needed for GVL to be reached. Therefore, unlike the large $N_f$ case, the restoration of chiral symmetry and reaching GVL may occur at different temperatures/densities. Moreover, the restored symmetry in the GVL at finite temperature/density can manifest itself in the both VM and VR modes. These two modes may in turn correspond to different critical temperatures and to figure out the actual mechanism of chiral symmetry restoration in the GVL the elaborated models including all these complications are required.

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