Phenomenological Implications of the $m_t$ RGE Fixed Point for SUSY Higgs Boson Searches

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Abstract

In minimal SUSY-GUT models with $M_{SUSY} \lesssim 1$ TeV, the renormalization group equations have a solution dominated by the infrared fixed point of the top Yukawa coupling. This fixed point predicts $m_t \simeq (200 \text{ GeV}) \sin \beta$; combined with the LEP results it excludes $m_t \lesssim 130$ GeV. For $m_t$ in the range $130–160$ GeV, it predicts that the lightest scalar $h$ has mass $60–85$ GeV (detectable at LEP II). At SSC/LHC, each of the five scalars $h, H, A, H^\pm$ may be detectable, but not all of them together; in one parameter region none would be detectable.

For a large top-quark mass $m_t > M_W$, the corresponding Yukawa coupling $\lambda_t$ is plausibly large at the GUT scale $M_G$, in which case $\lambda_t$ evolves rapidly toward an infrared fixed point at low mass scales[1, 2, 3, 4, 5, 6]. The evolution of the top quark Yukawa coupling $\lambda_t$ is governed by the one-loop renormalization group equation

$$\frac{d\lambda_t}{dt} = \frac{\lambda_t}{16\pi^2} \left[ -\sum c_i g_i^2 + 6\lambda_t^2 + \lambda_b^2 \right], \quad (1)$$

with $c_1 = 13/15, c_2 = 3, c_3 = 16/3$; the couplings evolve toward a fixed point close to where the quantity in square brackets in Eq. (1) vanishes. Then the known gauge couplings determine the running mass $m_t(m_t) = \lambda_t(m_t)v \sin \beta/\sqrt{2}$ and hence the pole mass

$$m_t(\text{pole}) = m_t(m_t) \left[ 1 + \frac{1}{3\pi}\alpha_s(m_t) \right],$$

two-loop evaluations[3] give

$$m_t(\text{pole}) \simeq (200 \text{ GeV}) \sin \beta, \quad (2)$$

where $\tan \beta = v_2/v_1$ is the usual ratio of two Higgs vevs. If $\lambda_t(M_G)$ is below the fixed point, its convergence to the fixed point is more gradual and Eq. (2) does

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not necessarily apply. But in practice large $\lambda_t(M_G)$ is favored in many SUSY-GUT solutions; large $\lambda_t(M_G)$ facilitates $\lambda_b(M_G) = \lambda_\tau(M_G)$ Yukawa unification and allows intricate relationships between fermion masses and mixings. It is therefore interesting to pursue the phenomenological implication of Eq. (2).

Figure 1 shows how $\lambda_t(M_G)$ and $\lambda_b(M_G)$ values relate to $m_t$(pole) and $\tan \beta$ in SUSY-GUT solutions with Yukawa unification; the lower (upper) shaded branches contains the $m_t$ ($m_b$) fixed-point solutions. There is a small region at the upper right where both fixed point solutions are simultaneously satisfied. Figure 2 shows that the $m_t$ fixed-point behavior is insensitive to GUT threshold corrections in the $\lambda_b/\lambda_\tau$ ratio, at least for threshold corrections $< 10\%$. The sensitivity of the fixed point to threshold corrections is decreased for larger values of $\alpha_s(M_Z)$ where the solutions tend to have a stronger fixed point character, as indicated by Eq. (1). The perturbative limits of the Yukawa couplings near their Landau poles are shown in Fig. 2(a) as the dashed lines $\lambda_t^G = 3.3$ and $\lambda_b^G = 3.1$.

The minimal SUSY Higgs spectrum contains two CP-even scalars $h$ and $H$ ($m_h < m_H$), a CP-odd pseudoscalar $A$ and two charged scalars $H^{\pm}$. At tree level their properties are controlled by two parameters $m_A, \tan \beta$. Radiative corrections depend principally on $m_t$ (constrained now by Eq. (2)) and logarithmically on $m_t$, that we here set at $m_t = 1$ TeV. Assuming $m_t \lesssim 160$ GeV, Eq. (2) constrains $\tan \beta$ to values near 1, where $h$ is relatively light (recall the tree-level relation $m_h < M_Z|\cos 2\beta|$) and the couplings of $h$ are close to those of a Standard Model Higgs boson. LEP Higgs searches
 exclude a region of the $(m_A, \tan \beta)$ plane shown in Fig. 3(a): this translates to forbidden regions in $(m_h, \tan \beta)$ in Fig. 3(b). We see that the fixed-point condition predicts $m_t \gtrsim 130$ GeV, $m_h \gtrsim 60$ GeV, $m_A \gtrsim 70$ GeV; correspondingly $m_{H^{\pm}} \gtrsim 105$ GeV, $m_H \gtrsim 140$ GeV. If in fact $m_t \lesssim 160$ GeV, then $m_h \lesssim 85$ GeV as shown in Fig. 4, and $h$ will be discoverable at LEP II (but none of the other Higgs bosons will). The discovery limits at SSC/LHC (taken here from Ref. [11]) are also
shown in Fig. 4; we see that each of the five Higgs bosons might be discoverable there, but not all at once, and possibly none of them at all.

\[ \alpha_s(M_Z) = 0.118 \quad m_b(m_b) = 4.25 \quad M_{\text{SUSY}} = m_{\text{pole}} \]

Fig. 2: RGE results for \( \alpha_s(M_Z) = 0.118 \) with the boundary condition \( m_b(m_b) = 4.25 \) GeV. (a) GUT threshold corrections to Yukawa coupling unification. The solutions strongly exhibit a fixed point corrections, for threshold corrections \( \lesssim 10\% \). Taking a larger supersymmetric threshold \( M_{\text{SUSY}} \) or increasing \( \alpha_s(M_Z) \) moves the curves to the right, so that the fixed point condition becomes stronger. (b) Evolution of the top quark Yukawa coupling for \( \tan \beta = 1 \). The dashed line indicates \( \frac{d\lambda}{dt} = 0 \) which gives an approximation to the electroweak scale value of \( m_t \) with accuracy of order 10%.

Fig. 3: \( m_t \) fixed-point solution regions allowed by the LEP I data: (a) in the \( (m_A, \tan \beta) \) plane, (b) in the \( (m_h, \tan \beta) \) plane. The top quark masses are \( m_t(\text{pole}) \), correlated to \( \tan \beta \) by Eq. (2).
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References

[1] B. Pendleton and G. G. Ross, Phys. Lett. B98, 291 (1981); C. T. Hill, Phys. Rev. D24, 691 (1981).

[2] H. Arason, et al., Phys. Rev. Lett. 67, 2933 (1991); Phys. Rev. D47, 232 (1993).

[3] S. Dimopoulos, L. J. Hall and S. Raby, Phys. Rev. Lett. 68, 1984 (1992); Phys. Rev. D45, 4192 (1992); G. F. Giudice, Mod. Phys. Lett. A7, 2429 (1992).

[4] V. Barger, M. S. Berger, and P. Ohmann, Phys. Rev. D47, 1093 (1993); V. Barger, M. S. Berger, T. Han and M. Zralek, Phys. Rev. Lett. 68, 3394 (1992).

[5] C. D. Froggatt, I. G. Knowles and R. G. Moorhouse, Phys. Lett. B249, 273 (1990); Phys. Lett. B298, 356 (1993).

[6] M. Carena, S. Pokorski, and C. E. M. Wagner, Munich preprint MPI-Ph/93-10.

[7] M. Chanowitz, J. Ellis and M. Gaillard, Nucl. Phys. B128, 506 (1977).

[8] For details see V. Barger et al., University of Wisconsin-Madison preprint MAD/PH/755.

[9] ALEPH Collaboration: D. Decamp et al, Phys. Lett. B246, 623, (1990), B265, 475 (1991); DELPHI Collaboration: P. Abreu et al, ibid B245, 276 (1990), Nucl. Phys. B373, 3 (1992); L3 Collaboration: B. Adeva et al, Phys. Lett. B294, 457 (1992); OPAL Collaboration: M. Z. Akrawy et al, Z. Phys. C49, 1 (1991).

[10] T. Mori, report to Dallas Conference 1992; E. Gross and P. Yepes, CERN-PPE/92-153.

[11] V. Barger et al., Phys. Rev. D46, 4914 (1992).
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