Candidates for HyperCharge Axion in Extensions of the Standard Model

Ram Brustein, David H. Oaknin

Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel
email: ramyb, doaknin @bgumail.bgu.ac.il

Abstract

Many theoretically well-motivated extensions of the Standard Model contain heavy pseudoscalars that couple to hypercharge topological density. The cosmological dynamics of such hypercharge axions could, under certain conditions, lead to generation of a net baryon number in a sufficient amount to explain the observed baryon asymmetry in the universe. We examine the Minimal Supersymmetric Standard Model and string/M-theory models and determine specific conditions which heavy axion-like pseudoscalars must satisfy to successfully drive baryogenesis. We find that all candidates in the Minimal Supersymmetric Standard Model fail to obey some of the constraints, and that only in special string/M-theory models some axions may be adequate.
Topologically non-trivial configurations of hypercharge gauge fields can play a relevant role in the electroweak (EW) scenario for baryogenesis [1–5]. A hypothetical heavy pseudoscalar field that couples to hypercharge topological number density, the hypercharge axion (HCA), can exponentially amplify primordial hypermagnetic fields in the unbroken phase of the EW plasma, while coherently rolling or oscillating. The coherent motion provides the three Sakharov’s conditions [6] and can lead, under certain conditions, to generation of a net hypercharge topological number that can survive until the phase transition and then be converted into a net baryon number in a sufficient amount to explain the origin of the baryon asymmetry in the universe [7,8].

In [7,8] we have focused on a simple model with an extra singlet HCA, \(a\), whose only coupling to Standard Model (SM) fields is through the following effective operator

\[
\mathcal{L}_{aY} = \frac{1}{4M_Y} a Y_{\mu\nu} \tilde{Y}^{\mu\nu},
\]

(1)

where \(Y_{\mu\nu}\) is the \(U(1)_Y\) hypercharge field strength and \(\tilde{Y}^{\mu\nu}\) is its dual. The constant \(M_Y\) has units of mass. The HCA is massless at very high energies and gets a mass \(m \sim \Lambda^2/M_A\) from a generic potential

\[
\tilde{V}(a) = \Lambda^4 V(a/M_A),
\]

(2)
generated at an energy scale \(\Lambda\). The potential \(V\) is a bounded periodic function and \(M_A\) is a normalization mass scale.

Unless some fine-tuning mechanism is effective when the axion potential is generated, the pseudoscalar is trapped far from the minimum of its potential and then starts to coherently roll or oscillate around this minimum until the condensate finally decays. Typically, the cosmological misalignment of \(a\), \(\langle a \rangle_c\), is of the same order of magnitude as \(M_A\). The axion rolls if the Hubble time at the scale of potential generation \(t_H \sim M_p/\Lambda^2\) (\(M_p\) being the Planck mass) is shorter or comparable to the characteristic time \(t_s \sim m^{-1}\) for a coherent
oscillation. Otherwise, the axion will oscillate a few times before the topological condensate decays. While the axion rolls it generates a net topological number which is stored in very long wavelength modes, \( k \ll \Lambda \), that remains frozen in the plasma until the EW phase transition. Oscillations should happen just before or during the EW phase transition so that the generated topological number, stored in shorter wavelength modes \( k \sim \Lambda \), does not diffuse in the highly conducting plasma once the oscillations have stopped [7,8].

A crucial element in the scenario of HCA driven baryogenesis is the requirement that the misalignment \( \langle a \rangle_c \sim M_A \) is larger than the decay constant of the HCA into two hypercharge photons, \( M_Y \) [7,8]. If the axion oscillates, it is enough that \( \langle a \rangle_c \) is somewhat larger than \( M_Y \),

\[
\langle a \rangle_c / M_Y \sim M_A / M_Y > 1, \tag{3}
\]

but if it rolls, it is necessary that

\[
\langle a \rangle_c / M_Y \sim M_A / M_Y > \sqrt{M_P / \Lambda}. \tag{4}
\]

The heavy HCA differs significantly from the original axion proposed by Peccei and Quinn (PQ) as an elegant solution to the strong CP problem [9–11]. The PQ axion couples to the nonabelian \( SU(3) \) topological density

\[
\mathcal{L}_{a_{PQ}GG} = \frac{1}{4M_g} a_{PQ} G_{\mu\nu} \tilde{G}^{\mu\nu}, \tag{5}
\]

which generates a potential

\[
\tilde{V}(a_{PQ}) = \Lambda_g^4 V(4\pi^2 a_{PQ} / g_s^2 M_g)
\]

at the QCD scale \( \Lambda_g \sim 200 \text{ MeV} \) through instanton effects. The QCD axion should get its mass mainly from potential (5) in order to fix \( \theta_{QCD} = 0 \). In this case, the normalization scale for the axion potential \( M_A \), is determined by the axion coupling constant \( 1/M_g \) such that \( M_A \sim M_g \). This is not the case for the HCA. The axion coupling (1) to the abelian hypercharge topological density does not generate any potential for the HCA, which gets
its mass from some additional sector. In general the scales $M_Y$ and $M_A$ in the hypercharge sector (1) and in the mass generation sector (2) are not related.

In this paper we try to identify HCA candidates in theoretically well motivated extensions of the SM. Heavy pseudoscalars that couple to topological gauge densities appear in low-energy supersymmetric models with an extended higgs sector. We analyze here the Minimal Supersymmetric SM (MSSM) as a representative of this class of models. Numerous HCA candidates appear also in string/M-theory models. We analyze here 4 dimensional models of heterotic $E_8 \times E_8$ and Horava-Witten theory as representatives of this class of models.

The physical pseudoscalar higgs of the MSSM gets a mass in the TeV range from soft SUSY breaking terms or supersymmetric $\mu$-term and couples to hypercharge topological number density through anomalous quantum effects when chiral symmetry is spontaneously broken by the higgs mechanism. We expect that the scale $M_A$ of the pseudoscalar potential is about that of SUSY breaking, $M_A \sim M_{SUSY}$, while the pseudoscalar inverse coupling to hypercharge topological density is about the EW scale, $M_Y \sim M_{EW}$. Typically $M_A$ is several orders of magnitude larger than $M_Y$, as required by conditions (3) and (4). As we show this promising picture is drastically altered at high temperatures in the unbroken phase of the EW theory. Although the higgs pseudoscalars get heavy masses from SUSY breaking terms or $\mu$-term, their axion-like effective couplings to hypercharge topological density vanish due to chiral symmetry restoration$^1$. So the pseudoscalar higgs particles of the MSSM cannot serve as HCA.

In string/M-theory heavy axions are common. They typically couple with coupling $1/M_O$ to the topological density of an “observable” gauge group which is supposed to contain the

$^1$At this point we would like to stress that although higgsinos and gauginos remain massive in the symmetric phase of the plasma (contrary to leptons and quarks that become massless), the mass terms they get from SUSY breaking or $\mu$-term are invariant under chiral rotations of interaction eigenstates.
SM group, and with coupling \(1/M_H\) to the topological density of a “hidden” gauge group which interacts only through gravitational strength interactions with the observable sector:

\[
\mathcal{L}_S = \frac{1}{4M_O} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{4M_H} a G_{\mu\nu} \tilde{G}^{\mu\nu}.
\]  

(7)

Axions usually get their potentials mainly from instanton effects in the nonabelian hidden sector whose field strength is \(G\), while the abelian hypercharge group \(U(1)_Y\) is contained in the observable sector whose field strength is \(F\). In this case, the scale \(M_A\) is given by the inverse axion coupling to the hidden gauge topological density, \(M_H\), (see eq. (2) and (4)), while \(M_Y\) is given by \(M_O\). Conditions (3) and (4), using \(M_A/M_Y = M_H/M_O\), require that axions couple to the observable sector more strongly than they do to the hidden sector. In 4D models the ratio \(M_H/M_O\) is determined by the compactification scheme. As we will show, standard compactifications restrict parameter space of string candidates for HCA to regions where it is difficult to satisfy in a natural way all the necessary conditions that allow them to successfully drive baryogenesis.

II. HCA IN THE MINIMAL SUPERSYMMETRIC STANDARD MODEL.

The higgs sector of the MSSM contains two complex \(SU(2)_L\) doublets that give masses to the up and down components of the three families of SM fermions. The coupling structure of the MSSM of relevance to us here appears in each one of the two higgs Yukawa sectors, and for each one of the fermion families separately. Therefore, instead of discussing the full and complicated MSSM we first discuss, for the sake of simplicity, a linear \(\sigma\)-model with a \(U(1)_Y\) abelian gauge group, one fermion flavour \(\psi\), and a singlet complex higgs field \(\phi\),

\[
\mathcal{L}_0 = (-\frac{1}{4}) Y_{\mu\nu} Y^{\mu\nu} + i \bar{\psi} (\partial_\mu + ig Y_\mu) \gamma^\mu \psi + (\partial_\mu \phi^*)(\partial^\mu \phi) \\
- (\lambda \bar{\phi} \psi (\frac{1+\gamma^5}{2}) \psi + h.c.) - \mu^2 (\phi^* \phi - f_P Q)^2, 
\]

(8)

which will allow us to analyze relevant issues. We then apply the results to the full MSSM.

The \(\sigma\)-model (8) is invariant, up to anomalies, under a global PQ chiral rotation
FIG. 1. HCA decay into two hypercharge photons through a fermion loop.

\[ \psi \rightarrow e^{i\frac{\pi}{2} \gamma^5} \psi \]

which is spontaneously broken by the vacuum expectation value of the complex scalar field \( <\phi> = f_{PQ} \).

We are interested in the pseudoscalar component \( a \) of the complex higgs that couples axially to the fermion through the Yukawa sector,

\[
L = \left(-\frac{1}{4}\right)Y_{\mu\nu}Y^{\mu\nu} + i\bar{\psi}(\partial_\mu + ig'Y_\mu)\gamma^\mu\psi - m\bar{\psi}\psi \\
+ (\partial^\mu a)^2 + m_a^2 a^2 - i\lambda a \bar{\psi}\gamma^5\psi.
\]  

Here \( m = \lambda f_{PQ} \) is the chiral mass of the fermion, and \( m_a \) is the pseudoscalar mass. In the linear \( \sigma \)-model (8) the chiral symmetry (9) protects the pseudoscalar from getting a perturbative mass, but in the MSSM it gets a mass in the TeV range from soft SUSY-breaking terms or supersymmetric \( \mu \)-term, and therefore we have introduced it in this simple model as an extra free parameter.

The anomalous coupling of the massive axion to the hypercharge topological density (1) can be obtained directly in a standard way from the invariant amplitude for the decay process of \( a \) into two hypercharge photons \( a \rightarrow 2\gamma_Y \) shown in Fig. 1.

\[
- i\mathcal{M} = -2i\lambda g'^2 \int \frac{d^4q}{(2\pi)^4} Tr \left[ \gamma^5 \frac{i}{(q-m)^2} \gamma^\mu \frac{i}{(q-k_1)^2} \gamma^\nu \frac{i}{(q-k_2)^2} \right] \epsilon_\mu \epsilon_\nu \]

\[
= 8\lambda g'^2 m \epsilon^{\mu\nu\rho\sigma} k_\mu k_\nu \epsilon_\rho \epsilon_\sigma I(k_1, k_2) = \frac{1}{M_Y} \epsilon^{\mu\nu\rho\sigma} k_\mu k_\nu \epsilon_\rho \epsilon_\sigma,
\]  

and therefore
where \( k^1, k^2 \) are the 4-momenta of the two outgoing hypercharge photons in the center of mass system, and \( \epsilon^1, \epsilon^2 \) are their polarizations vectors. The scalar function \( I \) is given by

\[
I = \int \frac{d^4q}{(2\pi)^4} \frac{i}{[(q - k^1)^2 - m^2][q^2 - m^2][(q + k^2)^2 - m^2]}.
\]

The integral in eq. (13) is finite and can be computed using standard techniques:

If \( 0 < \xi = \frac{m_a}{m} \leq 2 \),

\[
I(\xi) = -\frac{1}{8\pi^2m^2} \left[ \frac{\text{Arcsin}(\xi/2)}{\xi} \right]^2.
\]

If \( 2 < \xi = \frac{m_a}{m} < \infty \),

\[
I(\xi) = I|_{\xi=2} - \frac{1}{16\pi^2m^2} \frac{1}{\xi^2} \left( -\frac{1}{2} \left[ \ln \left( \frac{1}{1 + \tau} \right) \right]^2 + i\pi\ln(1 - \tau^2) \right),
\]

where \( \tau = (1 - \frac{4}{\xi^2})^{1/2} \).

The function \( M_Y(\xi) \) reaches its global minimum at \( \xi = 2 \), when the fermion circulating in the loop is on-shell,

\[
M_Y(\xi = 2) = \frac{16}{\lambda g^2} m = \frac{16}{g^2} f_{PQ}.
\]

The function decreases slowly from its value at \( \xi = 0 \) (massless axion)

\[
M_Y(\xi = 0) = \frac{4\pi^2}{g^2} f_{PQ},
\]

until it reaches its minimum and then grows fast towards infinity when \( \xi \gg 2 \).

The smaller \( M_Y \) is, the stronger the axion coupling (1) to hypercharge topological density. If we assume the axion mass \( m_a \) is in the TeV range, we can conclude that fermions with masses around a few hundreds GeV or larger contribute significantly to the anomalous coupling, but much lighter fermions do not.

At this point we would like to make two comments:

1) The anomalous axion coupling to gauge topological densities is closely associated with
the spontaneous or explicit breaking of chiral symmetry in the fermionic mass sector. In
the case of a massless fermion the trace of $\gamma$ matrices in (11) vanishes, and therefore
the coupling $M$ vanishes.

2) The scale $M_Y$ is fixed by the scale $f_{PQ}$ at which chiral symmetry is broken.

The effects of high temperature and chiral symmetry restoration on the anomalous
vertex have been studied in the simplest case of a perturbatively massless axion, $\xi \rightarrow 0$ [12–15].

We report the result obtained there for the invariant amplitude:

$$-i\mathcal{M} = 8\lambda g'^2 m(T) \epsilon^{\mu\nu\rho\sigma} k^1_{\mu} k^2_{\nu} \epsilon^1_{\rho} \epsilon^2_{\sigma} I_T,$$

(18)

where $m(T)$ is the chiral fermion mass that now depends on the temperature $T$. At high
temperature the scalar function $I_T$ is proportional to $\frac{1}{x(T) T}$, where $x(T)$ is the infrared cut-off.

Far below the phase transition, the infrared cut-off is provided by the mass of the fermion
$x(T) \sim m(T)$; therefore the invariant amplitude depends on $T$ as $\frac{1}{T}$, and its dependence on
the fermion mass cancels. At temperatures close to chiral symmetry restoration, $m(T) \rightarrow 0$,
hard-thermal corrections are important and provide the effective cut-off, $x(T) \sim g'T$. As
a consequence, the anomalous invariant amplitude goes like $\frac{m(T)}{g'T}$ and strictly vanishes in
the symmetric phase of the plasma when $m(T) = 0$. This conclusion can be generalized
to the case of a massive axion, following the same argument: the axion mass provides an
additional infrared cut-off, $x(T) \sim m_a$, but the anomalous coupling still vanishes because of
chiral symmetry restoration.

The analysis we have presented can be immediately extended to the specific setup of the
MSSM. The MSSM contains two higgs pseudoscalars that couple independently to hyper-
charge topological density when the EW symmetry is spontaneously broken by the non-zero
higgs vacuum expectation values, $f_{PQ}^1, f_{PQ}^2 \sim M_{EW} = 100$ GeV. In the broken phase of
the plasma one of the two pseudoscalar mass eigenstates remains massless, up to anomalies: it
is the pseudogoldstone boson that is absorbed as the longitudinal component of the mas-
sive Z. The second mass eigenstate gets a mass typically in the TeV range from soft SUSY
breaking terms or $\mu$-term and, therefore, it could be a possible candidate for HCA. The
largest contribution from SM fermions to the heavy pseudoscalar coupling to hypercharge
topological density comes from the heavy top quark \([16]\),

\[
M_Y \sim \frac{1}{3} \times \frac{16 M_{EW}}{g'^2} \sim 50 \text{ TeV.}
\] (19)
The factor \(\frac{1}{3}\) takes into account three different colors for the quark.

The heavy pseudoscalar higgs has additional axial coupling to charginos and neutralinos,
through three-point vertices higgs-higgsino-gaugino. These are supersymmetric vertices that
are not generated through the spontaneous breaking of the EW symmetry. The dimension-
less coupling \(\lambda\) in each sector is fixed by SUSY: \(\lambda = g/\sqrt{2}\) for W-inos and \(\lambda = g'/\sqrt{2}\) for
Hyper-inos. These vertices generate at 1-loop an additional contribution to the anomalous
pseudoscalar coupling \([11]\), \(M_Y \sim \frac{16 m_f}{\lambda g'^2}\), of the order of \((19)\), if charginos or neutralinos
masses \(m_f\) are not much heavier than the top mass.

The picture described above changes drastically at temperatures above the EW phase
transition. Both pseudoscalar mass eigenstates are massive but, as we discussed above
eq. \((18)\), the contribution from SM fermions to their anomalous coupling to topological
densities vanishes because the fermion chiral masses vanish.

Charginos and neutralinos, on the other hand, remain massive in the symmetric phase
and their contributions could be nonvanishing. We will describe here the chargino sector
\([16]\), but the situation is very similar for neutralinos. There are two charginos which contain
four Weyl spinors, \(\lambda^+, \lambda^-\) (W-inos), \(\bar{H}_1^-, \bar{H}_2^+\) (charged higgsinos). The mass term in this
sector in the symmetric phase of the MSSM is:

\[
\left(\begin{array}{c}
\lambda^+ \\
\bar{H}_2^+
\end{array}\right)^t \left(\begin{array}{cc}
M_2 & 0 \\
0 & \mu
\end{array}\right) \left(\begin{array}{c}
\lambda^- \\
\bar{H}_1^-
\end{array}\right) + h.c. = \mu \bar{\chi}_1 \chi_1 + M_2 \bar{\chi}_2 \chi_2,
\] (20)

where we have defined mass eigenstates Dirac spinors, \(\chi_1 = \left(\begin{array}{c}\bar{H}_1^- \\
\bar{H}_2^+
\end{array}\right)\), and \(\chi_2 = \left(\begin{array}{c}\lambda^- \\
\bar{\lambda}^+
\end{array}\right)\).

The parameters \(\mu\) and \(M_2\) are a higgs-higgs coupling constant and a soft SUSY breaking
parameter, respectively. The mass term \((20)\) is invariant under a chiral rotation with oppo-
site charges of the Dirac interactions eigenstates, $\psi_1 = \begin{pmatrix} \widetilde{H}_1^- \\ \lambda^+ \end{pmatrix}$, and $\psi_2 = \begin{pmatrix} \lambda^- \\ H_2^- \end{pmatrix}$, and we can guess that the anomalous coupling will vanish because of chiral symmetry restoration. In fact, this is what happens: if we write the pseudoscalar-higgsino-Wino axial vertex,

$$\mathcal{L}_{\text{axial}} = \frac{ig}{\sqrt{2}} \left( \widetilde{H}_2^+ \lambda^- a_2 - \lambda^+ \widetilde{H}_1^- a_1 \right),$$

(21)

where $a_1,a_2$ are the two massive pseudoscalars, in terms of chargino chiral mass eigenstates in the symmetric phase of the plasma $\chi_1,\chi_2$

$$\mathcal{L}_{\text{axial}} = \frac{ig}{\sqrt{2}} \left( \bar{\chi}_1 \frac{1 + \gamma^5}{2} \chi_2 a_2 - \bar{\chi}_2 \frac{1 + \gamma^5}{2} \chi_1 a_1 \right) + h.c.,$$

(22)

we obtain only non-diagonal $\chi_1 - \chi_2$ vertices, while the hyperphoton-chargino-chargino only vertex, $Y_\mu \bar{\chi}_1 \gamma^\mu \chi_1$, is diagonal. The loop shown in Fig. 1 cannot be closed. Therefore, although charginos and neutralinos are massive in the symmetric phase of the plasma, their contributions to the anomalous 1-loop coupling of pseudoscalars to hypercharge topological density also vanish because of chiral symmetry restoration.

The mass matrix (20) gets thermal contributions that we have not written explicitly. It is not clear if non-diagonal thermal terms appear in the mass matrix but, in any case, they do not break the restored chiral symmetry and our conclusion is not expected to be modified: the MSSM extended higgs sector does not contain an HCA capable to drive baryogenesis while coherently rolling or oscillating.

The mechanism described in [7,8] for HCA-driven baryogenesis through coherent amplification of hypercharge fields in the symmetric phase of the EW plasma could possibly serve to amplify primordial ordinary electromagnetic fields in the broken phase of the plasma [17]. From our previous analysis, it may seem that the heavy pseudoscalar of the MSSM could have a relevant role: if, as it is generally assumed, the SUSY breaking terms that give the pseudoscalar its mass are generated at an intermediate scale $\sim 10^8 GeV$, the coherent motion of the pseudoscalar when its potential is generated could have a typical misalignment $\langle a \rangle_c$, which is several orders of magnitude larger than the inverse coupling $M_Y \sim 50 \text{ TeV}$.
(see (19)), as required by conditions (3) and (4) for coherent amplification of the magnetic modes.

But, if the potential is generated at the intermediate SUSY breaking scale and gives the pseudoscalar a mass in the TeV range, the typical time for a coherent oscillation is much shorter than the Hubble time at the epoch of potential generation. In [7,8] we have considered a singlet HCA that couples only to hypercharge photons through the operator (H). The heavy pseudoscalar of the MSSM, on the contrary, couples to many other fields so that the topological condensate decays incoherently after a few oscillations at temperatures much above the electroweak phase transition and does not survive until the EW phase transition when the coupling (H) is generated. So our conclusion is that the pseudoscalar higgs of the MSSM cannot amplify ordinary electromagnetic fields.

The MSSM contains additional pseudoscalar neutral fields, i.e. pseudoscalar components of sneutrinos, which could, in principle, play a similar role to HCA in amplification of hypercharge topological number. But for sneutrinos, a dimension five operator similar to (H) is forbidden by R-parity symmetry. Extensions of the MSSM without R-parity in which sneutrinos couple to topological gauge densities have been discussed in [18] but, as in the case of the pseudoscalar higgs, the coupling to hypercharge topological number is generated through chiral symmetry breaking and therefore it also vanishes in the unbroken phase of the plasma.

III. HCA IN STRING/M-THEORY MODELS

In this section we consider possible candidates for HCA in low energy 4D effective field theory of weakly coupled heterotic $E_8 \times E_8$ string theory (HE) and Horava-Witten (HW) theory [19,20]. Axions in such theories were studied in detail in [21,22] and we recall here the necessary results to be able to discuss their relevance and application to the subject at hand: HCA’s.

Compactified string/M-theory models invariably contain a model-independent axion $a_{MI}$. 
In string theory compactifications $a_{MI}$ corresponds to a perturbatively massless pseudoscalar mode of the antisymmetric 2-index tensor $B_{\mu\nu}$, while in HW theory compactifications $a_{MI}$ corresponds to a perturbatively massless mode of the 3-index antisymmetric tensor $C_{\mu\nu\rho}$. In compactifications preserving at least $N = 1$ 4D supergravity, such as Calabi-Yau compactifications of HE string theory or HW theory, additional model-dependent axions $a_{MD}^i$ exist: $a_{MI}$ is the pseudoscalar component of the dilaton superfield and $a_{MD}^i$ are the pseudoscalar component of moduli superfields.

String/M-theory compactifications contain the SM fields as part of the so-called “observable” sector, and additional matter and gauge fields which couple to the observable sector only through gravitational strength interactions in the so-called “hidden” sector. Stringy axions generically couple to gauge topological density of both sectors. We will be interested, of course, in the linear combination of all the axions which couples to observable hypercharge topological density. Orthogonal linear combinations couple only to the hidden sector density or do not couple to gauge topological densities at all. The axion that is of interest to us couples with coupling $1/M_O$ to the density of the observable group, whose field strength is $F$, and with coupling $1/M_H$ to the density of the hidden group whose field strength is $G$. This perturbatively massless axion usually gets its potential mainly from instanton effects in the nonabelian hidden sector, while the abelian hypercharge group $U(1)_Y$ is contained in the observable sector. In this case, the scale $M_A$ is given by $M_H$, as in eq. (3) and (6), while $M_Y$ is given by $M_O$. Conditions (3) and (4), using $M_A/M_Y = M_H/M_O$, require that the axion couples to the observable sector more strongly than it does to the hidden sector.

In 4D models the ratio $M_H/M_O$ is determined by the compactification scheme.

The model-independent axion $a_{MI}$ coupling to topological gauge density in HE theory

$$\mathcal{L}_{MI} = \frac{1}{4M_1} a_{MI} (F_{\mu\nu} \tilde{F}^{\mu\nu} + G_{\mu\nu} \tilde{G}^{\mu\nu}),$$

(23)

can be obtained directly from the Bianchi identity of the 3-index antisymmetric tensor, $dH = -tr F^2 - tr G^2 + tr R^2$. The normalization mass scale $M_1$ is determined by the ratio between the Yang-Mills coupling constant and the gravitational constant, and it does not depend on
the details of the compactification scheme. In weakly coupled HE theory $M_1 \sim 7 \times 10^{15} GeV$. Similarly, in HW theory the coupling of $a_{MI}$ can be obtained from the Bianchi identity of the 4-index antisymmetric field strength tensor. The mass scale $M_1$ is not determined as well as it is in weakly coupled HE theory since it also depends on the length of the eleventh dimension interval, and on some additional theoretical input, but it is typically somewhat higher, $\sim 10^{17} GeV$.

The model-dependent axions $a_{MD}^i$ coupling to topological gauge density in HE theory is contained in the 10D Wess-Zumino term $S_{WZ} = c \int d^{10}x (Btr F^4 + Btr G^4 + ...)$ necessary to cancel the anomalies of the theory,

$$L_{MD} = \sum_i \frac{1}{4M_2^2} a_{MD}^i (F_{\mu\nu} \tilde{F}^{\mu\nu} - l^i G_{\mu\nu} \tilde{G}^{\mu\nu}).$$

(24)

We will focus on the linear combination of model-dependent axions, the overall model-dependent axion $a_{MD} = M_2 (\sum_i a^i / M_i^2)$, where $M_2 = (\sum_i (1/M_i^2)^2)^{-1/2}$, that couples to the observable gauge density. Then,

$$L_{MD} = \frac{1}{4M_2} a_{MD} (F_{\mu\nu} \tilde{F}^{\mu\nu} - l G_{\mu\nu} \tilde{G}^{\mu\nu}).$$

(25)

The normalization mass scale $M_2$ is determined by the detailed properties of the compactification scheme. In HW compactifications the coupling of $a_{MD}$ can be obtained by treating HW theory as a strongly coupled HE theory, and varying the coupling continuously from weak to strong coupling. The argument is that the axion coupling is determined by some topological invariants and does not change in the process. Typically, $M_2 \lesssim M_1$. In HE models we are considering $l^i = +1$, and therefore $l = +1$, is required by the fact that the three-index antisymmetric tensor $H$ is globally defined, $\int dH = 0$. In HW models $l = +1$ is also required for similar reasons [24], but we keep $l$ here as a free parameter for the sake of generality.

The mass generation scale $\Lambda$ in string/M-theory models depends on hidden sector matter content and interactions in addition to its dependence on compactification details. It is therefore not so well determined, and can be anywhere below $M_\rho$. But in many models,
such as gaugino condensation models it is expected to be of order $\Lambda^2 \sim m_{3/2} M_p$, where $m_{3/2}$ is the gravitino mass, so $\Lambda \sim 10^{11}$GeV. However, in [24] it is argued that in some cases $\Lambda$ can be much smaller. Our attitude here is to take $\Lambda$ as a free parameter, and find out the constraints on it.

We have collected all relevant information about string/M-theory axions and turn to discuss implications and consequences for HCA. The HCA, $a$, is the linear combination of $a_{MI}$ and $a_{MD}$ that couples to the gauge observable sector topological density:

$$a = M_O \left[ \frac{1}{M_1} a_{MI} + \frac{1}{M_2} a_{MD} \right], \quad (26)$$

where

$$M_O = [M_{-1}^2 + M_{-2}^2]^{-1/2}. \quad (27)$$

The HCA coupling to the hidden sector topological density is given by $1/M_H$, where

$$M_H = M_O \frac{M_1^2 + M_2^2}{|M_2^2 - lM_1^2|}. \quad (28)$$

Condition (4) for successful baryogenesis driven by $a$ while it is rolling requires that $M_H/M_O > \sqrt{\frac{M_2^2}{\Lambda}}$. Using (28), (27), condition (4) then becomes a condition on mass scales $M_1$, $M_2$ and $\Lambda$,

$$|M_2^2 - lM_1^2| \lesssim \sqrt{\frac{\Lambda}{M_P}} (M_1^2 + M_2^2), \quad (29)$$

that cannot be naturally satisfied in standard compactification schemes, in which $\Lambda \ll M_p$ and $l = 1$, unless scales $M_1$ and $M_2$ are appropriately tuned by some mechanism. The condition could be more easily realized in special models in which either $l \neq 1$, or $\Lambda \sim M_p$.

Condition (3) for baryogenesis induced by an oscillating axion, on the other hand, is naturally satisfied since it requires that $M_H/M_O \gtrsim 1$. For models in which $l = 1$ this is obviously satisfied since $\frac{M_2^2 + M_1^2}{|M_2^2 - M_1^2|} > 1$. But, as explained in the introduction, to successfully drive baryogenesis, HCA oscillations have to occur just before the EW phase transition. Therefore, the HCA potential has to be generated at temperatures close to the EW phase
transition: $\Lambda \sim M_{EW}$. Additionally, the HCA mass $m_a \sim \frac{\Lambda^2}{M_H}$, has to be about the EW scale, $m_a \sim M_{EW}$. It follows that the HCA coupling to the hidden sector has to satisfy $M_H \sim M_{EW}$, which can be fulfilled only if at least one of the mass scales $M_1, M_2$ is about the EW scale. This is not expected in typical string/M-theory models, but may perhaps be viable in models with a very low string scale $[25]$, or in models in which HCA is protected by some symmetry. In this range of parameters, $M_Y \lesssim M_{EW}$ and the HCA would be directly detectable in future colliders through its decay into 2 photons $[26]$.

IV. CONCLUSIONS.

We have studied the MSSM and 4D string/M-theory low energy effective field theories, looking for suitable candidates for HCA.

The natural candidate in the MSSM is the heavy higgs pseudoscalar that couples in the broken phase of EW theory to topological gauge densities through 1-loop triangle diagrams. But, as we have shown in section II, this coupling vanishes in the symmetric phase of the EW theory due to chiral symmetry restoration in the fermionic mass sector of the theory. The coupling of other possible candidates, i.e. pseudoscalar components of sneutrinos, to topological gauge densities is forbidden by R-parity symmetry. We conclude that the MSSM does not contain an HCA that can successfully drive baryogenesis. In some models with broken R-parity, sneutrinos do couple to topological gauge densities through triangle diagrams but also fail to serve as HCA’s due to chiral symmetry restoration in the unbroken phase of the EW theory.

Stringy axions couple directly to hypercharge topological density at the compactification scale, which is typically much higher than the EW scale. But, we have concluded in section III that in generic compactifications the specific conditions for successful baryogenesis are violated in one way or another. We have outlined requirements for more elaborate models which may lead to HCA’s which obey all the conditions. Such models may be realized in some special compactification schemes, or in scenarios of very low string scale.
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