Tailoring Broadband Kerr Soliton Microcombs via Post-Fabrication Tuning of the Geometric Dispersion

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Geometric dispersion in integrated microresonators plays a major role in nonlinear optics applications, especially at short wavelengths, to compensate the natural material normal dispersion. Tailoring of geometric confinement allows for anomalous dispersion, which in particular enables the formation of microcombs which can be tuned into the dissipative Kerr soliton (DKS) regime. Due to processes like soliton-induced dispersive wave generation, broadband DKS combs are particularly sensitive to higher-order dispersion, which in turn is sensitive to the ring dimensions at the nanometer-level. For microrings exhibiting a rectangular cross section, the ring width and thickness are the two main control parameters to achieve the targeted dispersion. The former can be easily varied through parameter variation within the lithography mask, yet the latter is defined by the film thickness during growth of the starting material stack, and can show a significant variation (few percent of the total thickness) over a single wafer. In this letter, we demonstrate that controlled dry-etching allows for fine tuning of the device layer (silicon nitride) thickness at the wafer level, allowing multi-project wafers targeting different wavelength bands, and post-fabrication trimming in air-clad ring devices. We demonstrate that such dry etching does not significantly affect either the silicon nitride surface roughness or the optical quality of the devices, thereby enabling fine tuning of the dispersion and the spectral shape of the resulting DKS states.

Integrated microresonators have been demonstrated in a variety of different materials in which the frequency spacing of a set of optical modes, the free spectral range (FSR), increases with the optical frequency, a behavior termed as anomalous dispersion. This allows for compensation of the intensity-dependent Kerr frequency shift, enabling the fundamental conservation of both energy and momentum - or frequency and angular momentum in the special case of periodic whispering gallery modes structures. This fundamental dispersion property allows for a variety of \( \chi^{(3)} \) nonlinear processes to be effectively realized. In particular, four-wave mixing (FWM) and optical parametric oscillation from a continuous-wave pump can occur, seeding the so-called modulation instability frequency microcomb, which is a first step in obtaining a stable pulse state in the dissipative Kerr soliton (DKS) regime. In photonic platforms where DKS states have been demonstrated, for example in microring, material dispersion (which is normal) is compensated by geometric dispersion, i.e. the confinement of the guided mode within the structure relative to the wavelength, to obtain anomalous dispersion. The amount of dispersion compensation needed is particularly acute at shorter wavelengths as photonic materials, such as \( \text{Si}_3\text{N}_4, \text{SiO}_2, \text{AlN} \) or \( \text{LiNbO}_3 \), exhibit pronounced normal dispersion in the visible range.

However, obtaining anomalous dispersion is usually not sufficient for supporting a DKS state that suits application needs. In particular, for full comb stabilization requiring an octave of bandwidth, the DKS spectrum has to be extended using dispersive waves (DWs), a phenomenon relying on higher-order dispersion. In this case, a DKS comb tooth overlaps with a cavity resonance far away from the pumped mode, exhibiting resonant enhancement at this particular wavelength, and increasing the comb power locally. The spectral positions of such DWs are highly sensitive to the geometric dispersion, which in turn is controlled primarily through the ring width and the ring thickness. Dimensional control down to the few nanometer level is required for precise targeting of DW spectral locations. The ring width, being an in-plane parameter, can be addressed at the design level where different structures on the same chip are given a slight variation in their ring width. However, thickness is defined during growth and requires subsequent post-growth processes if it is to be adjusted.

In this letter, we demonstrate a dry etch trimming approach that enables fine tuning of the thickness of \( \text{Si}_3\text{N}_4 \) microresonator frequency comb devices with a resolution down to the few nanometer level (Fig. 1). One application of this trimming is in chip-to-chip compensation of the natural thickness variation across a wafer, which is typically at the percent level or higher for growth via low pressure chemical vapor deposition. The dry etch trimming can also be used to realize thickness differences of several tens of nanometers, enabling multi-project wafers in which different thicknesses are needed to target, for example, frequency combs operating in different spectral bands or other \( \chi^{(3)} \) nonlinear nanophotonic devices. Working in an air-clad system in which the \( \text{Si}_3\text{N}_4 \) device layer remains accessible after fabrication (see Fig. 1), we
perform optical characterization of the microcomb devices before and after trimming steps. We show that the frequency shift of the resonator modes due to dry etch trimming can be repeatably controlled and does not adversely impact their optical quality factor \( Q \). We further show that DKS microcomb states, and in particular, DW positions, are modified through thickness trimming, with the DW shift predictable based on the impact of device layer thickness on higher-order dispersion.

Most of the reported DKS microcombs in Si\(_3\)N\(_4\) resonators have been demonstrated in devices that are fully clad with SiO\(_2\). This approach has the advantages of making the resonator relatively insensitive to its environment and allows for better heat dissipation. However, post-fabrication steps to modify the geometry based on measurement results are made impossible as the resonator is not directly accessible. Here, we use a system where the ring resonator does not have any material as the top cladding. Such air-clad resonators have been demonstrated to support high-\( Q \) and octave-spanning microcombs\), as well as other wide-band nonlinear processes\). Yet in our system, the facet waveguides are still embedded in SiO\(_2\) (Fig. 2). This selectively-clad chip thus enables direct access to the microresonator section for trimming purposes, while providing more optimized facet coupling regions for improved insertion losses from/to lensed optical fibers (Fig. 2(a)). In particular, the full SiO\(_2\) cladding both provides for a more symmetric mode than what would be possible with an air-clad waveguide and ensures that a guided mode is supported even when inverse tapers are used to expand the mode size to best match that of lensed optical fibers (asymmetrically clad waveguides are not guaranteed to support even a fundamental guided mode if the taper width is too small). Together, this results in insertion losses that are predicted to be as low as 2 dB per facet at 1060 nm (Fig. 2(a)), an improvement on the 6 dB per facet loss that could be reached in air-clad waveguides tapered to a larger width at the chip facets (Fig. 2(b)). In practice, we typically measure insertion losses between 3.5 dB to 4 dB per facet for a nominal inverse taper width of 150 nm (see supplementary material). We note that the Fresnel reflection between the two cladding regions, which could potentially lead to other insertion losses, is negligible and in the 0.1 dB range (see supplementary material). The selective SiO\(_2\) cladding is realized through a photoresist lift-off process (Fig. 2(c)-(d)), where removal of the top SiO\(_2\) cladding over the micror-
while the AFM suggests that RIE trimming results in at most a small increase in the Si$_3$N$_4$ film roughness, it is still important to understand the impact of the trimming on the optical properties of the resonator modes, in particular, the influence on resonator $Q$ and modal frequencies, and the extent to which the trimming can be repeatably applied. To do so, we need to track the same resonator modes before and after trimming, which can be difficult in microring resonators due to the large number of modes present. We use the selective mode splitting technique employed in Ref. [25] where the ring resonator has an inner sidewall modulation with a period set to couple and frequency split the initially degenerate clockwise and counterclockwise traveling wave modes with a specific azimuthal mode number. Here, we target the azimuthal mode number of 237 ($2 \times 237$ periods along the ring inner circumference), resulting in a resonance frequency that is split close to $287.5$ THz (Fig. 4(a)). Using the trim etch recipe presented earlier, we perform a series of seven trims for devices which have been designed with different nominal ring width ($RW$) varying from 920 nm to 1015 nm on the same chip, with each trim step lasting 20 s and at 45 W of RF power. The reduced RF power relative to that used in the AFM measurements in Fig. 3 provided finer trimming resolution, while ensuring that the induced surface roughness was minimized. We observe a nearly uniform frequency shift of $\approx 0.29$ THz with each step for each of the considered $RW$ values (Fig. 4(b)), though there are some small discrepancies from this trend, for example, the first trim step produced a somewhat smaller shift than the subsequent ones. Using finite element method eigenmode simulations, we retrieve the thickness dependence of the resonance frequencies $\partial f_{res}/\partial H = -73.2$ GHz/nm for the $RW$ range under study (from 900 nm to 1100 nm), and from the frequency shifts measured in Fig. 4(b), we estimate that each 20 s trim step reduces the thickness by 4 nm. Even finer trimming steps (e.g., 2 nm for a 10 s etch) should be possible; however, we note that a linear scaling of etched amount with etch time should not necessarily be assumed due to plasma ignition delay (for short etch times) and sample heating (for long etch times). Finally, we consider the intrinsic quality factor ($Q_i$) for 14 resonances in our lin-
ear frequency scan for each trimming step. We find that the average $Q_i$ remains about the same (within the one standard deviation uncertainty values determined by the spread in the $Q_i$ data). This is consistent with the AFM measurements indicating that the top surface roughness is only slightly increased by the trimming, and suggests that the post-fabrication trimming process can provide a method to tune the geometric dispersion of devices without significantly impacting their losses, which is critical for nonlinear applications.

Fine tuning of the thickness of a ring resonator not only allows for fine shifting of the frequency of resonances - which can be of interest for post-fabrication tuning to overlap with atomic transitions or other frequencies of interest - but also impacts the overall dispersion of the resonator, in particular the higher order dispersion coefficients responsible for dispersive wave creation. For such microcombs, it is convenient to introduce the integrated dispersion $D_{\text{int}}(\mu) = \omega_{\text{res}}(\mu) - \omega_{\text{DKS}}$, where $\mu$ is the mode number relative to the pumped mode (i.e. $\mu_{\text{pmp}} = 0$), which effectively measures the discrepancy between the resonator mode frequencies and the supported DKS frequency comb teeth $\omega_{\text{DKS}} \approx \omega_{\text{pmp}} + D_1 \mu$, which are spaced by the resonator free spectral range that is approximately given by $D_1/2\pi$. As the modal confinement is partly set by the resonator thickness, the resonator dispersion (calculated in Fig. 5(a)) and therefore its integrated dispersion (calculated in Fig. 5(b)) can be significantly impacted by modifying its thickness for a fixed ring width. This ultimately results in a variation in the frequency of the DWs (Fig. 5(c)) on both the low and high frequency sides of the spectrum. DWs are crucial elements in realizing octave spanning DKS frequency combs that can be self-referenced and reach atomic transition frequencies and control of their spectral positions is important to optimize the power available for the $f-2f$ technique. Post-fabrication trimming thus provides the option of being able to measure a device, determine its DW positions, and then trim its thickness to move them closer to the targeted values.

To demonstrate the possibility to post-process tune a DW through thickness trimming, we measure a DKS state obtained in a ring resonator with a starting thickness $H = 459$ nm and $RW = 1040$ nm and pumped at 976 nm in the fundamental transverse electric mode, while actively-cooled using a counter-propagative wave cross-polarized at 1060 nm, which allows for adiabatic tuning onto the soliton state. Here, we proceed to monitor the DKS state as we trim the thickness by more than 30 nm, which corresponds to the full variation of Si$_3$N$_4$ thickness over a 100 nm wafer (Fig. 1). We use the same recipe as for the trimming calibration, namely a 20 sec RIE etch at 45 W RF power. By pumping the exact same mode, which is slightly shifted in wavelength for each trim step from approximately 307 THz (976 nm) to 309.5 THz (969 nm), a clear and constant red shift of the DW is observed for each trimming step. Assuming the previously calibrated trimming amount of 4 nm per step, the dispersion at each trimming step can be found theoretically, and the linear approximation of the DW position based on the zero-crossing of the integrated dispersion (Fig. 5(b)-(c)) provides a reasonable estimate of the observed DW shift. It is important to note that for an asymmetric integrated dispersion profile (odd dispersion coefficients dominating), the linear approximation of predicting the DW position based on the zero-crossing of the integrated dispersion will present a discrepancy with respect to the actual DW frequency. To ad-

FIG 4. (a) Linear transmission data for a microring resonator ($RW = 920$ nm) with an initial thickness of $H = 430$ nm, after several different trimming steps, where for each step the trim was for 20 s and with 45 W of RF power. The frequency-split mode indicated in-between the dashed black lines is the result of an inner sidewall modulation whose period was set to target a specific mode of the ring (azimuthal mode number $M = 237$), providing an easy reference by which mode frequencies can be tracked. (b) Frequencies of the tracked microring mode for several different $RW$ values and several different trim steps, showing a nearly constant frequency shift per step. (c) Average intrinsic quality factor ($Q_i$) of 14 different resonances at each trim step and for $RW = 945$ nm and $RW = 1015$ nm. The error bars represent one standard deviation values based on the spread in the $Q_i$ values.
In conclusion, we have demonstrated a dry etch technique that allows for fine control of the thickness of Si$_3$N$_4$ microring resonators with a thickness step as low as 4 nm and over a range of 30 nm. The post-processing trimming does not significantly impact the Si$_3$N$_4$ surface roughness or resonator quality factors. By using an air-clad structure with oxide cladding still present at the chip facets, we are able to measure the resonator performance in-between trimming steps while retaining low insertion losses to lensed optical fibers. We demonstrate the utility of such fine trimming in microcomb engineering by showing how a dissipative Kerr soliton comb pumped in the 980 nm band can have its dispersive wave tuned across a total range of 40 THz in few THz steps. This trimming approach has numerous potential uses in the development of microresonator frequency combs and related Kerr nonlinear optical devices. For example, it can enable multi-project wafer runs in which different thickness are needed across the wafer; recent demonstrations of four-wave mixing Bragg scattering and optical parametric oscillation were realized for Si$_3$N$_4$ thicknesses (480 nm and 500 nm, respectively) that could be combined with the microcomb devices shown in this work. Finally, the trimming technique also makes it possible to compensate for the natural thickness variation of the Si$_3$N$_4$ material across a full wafer, so that the number of devices exhibiting a desired dispersion can be greatly increased.

See the supplementary material for information related to the insertion losses of the dual air/oxide cladding structure.

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DATA AVAILABILITY - The data that support the findings...
of this study are available from the corresponding author upon reasonable request.

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**Supplementary Materials**

S-1. Insertion Losses in Selectively Clad Devices – Experimental Data

![Graph](image1)

**FIG S1.** (a) Experimentally measured per facet insertion losses for the same material thickness as in Fig. 2(a) from the main text and inverse taper widths from 150 nm to 250 nm. (b) Chromatic dispersion of the per facet insertion losses in the 1020 nm to 1070 nm band. The one standard deviation uncertainty in per facet insertion loss is at the ±0.5 dB level and is due to measured variations in coupling for nominally identical devices.

Our system utilizes a selective SiO$_2$ cladding, with the absence of cladding over the microresonator region allowing direct access for post-processing, while keeping an SiO$_2$ cladding at the facets that is useful for reduced insertion losses. Here we present experimental data supporting the simulations presented in the main article, which show that the insertion losses are lower than that in an air-clad system. We perform measurement of the insertion losses with the inverse taper width (Fig. S1(a)). As expected from the simulation, the insertion losses for the first order transverse electric mode increase with the inverse taper width, where the optimum value is at 150 nm width (smaller tapers are limited by the fabrication process; in particular, the ability to realize high-aspect ratios for thick Si$_3$N$_4$ films). Finally, we measured that such inverse tapers present very low chromatic dispersion in their losses across the pump band, and present similar insertion losses (≈ 3.5 dB per facet) across the range between 1020 nm to 1070 nm (Fig. S1(b)).

![Graph](image2)

**FIG S2.** (a) Schematic of silicon nitride waveguide cladding transition between oxide-clad and air-clad regions. (b) Mode profile of $y$-oriented electric field component of the first order transverse electric mode in the oxide clad region for a waveguide cross section of 550 nm width and 420 nm thickness. (c) Mode profile for the same polarization and waveguide cross-section, but with an air top/side cladding. (d) Finite-difference time-domain simulation results for the transmission between the oxide and air-clad regions.

S-2. Losses from the Cladding Transition

In order to estimate the potential Fresnel losses from the oxide-clad to air-clad transition (Fig. S2(a)), we perform finite-difference time-domain simulation of the structure, where a silicon nitride waveguide of 550 nm width and 420 nm thickness is halfway embedded in SiO$_2$ at the left edge, and then transitions to a waveguide that has only a bottom SiO$_2$ cladding and a top and side air cladding. Although the mode profile of the first order transverse electric mode presents some disparity between the two regions (Fig. S2(b)), due to the difference of refractive index gradient in the two regions that modifies the confinement of the mode, the simulated transmission from the oxide-clad region to the air-clad region remains well above 90% (i.e. losses lower than 0.1 dB). This level of losses is negligible in comparison to the facet coupling loss.