Deformation of composite plates with a layer of aging materials

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Abstract. The paper presents a calculation of the stress-strain state of composite plates. The mathematical model obtained by the authors earlier has been applied. In contrast to the previous results, the viscous-elastic properties of the material are taken into account in describing the behavior of the intermediate layer. The developed creep kernel of the layer material contains the expression that serves to describe the properties of the aging material. The compliance of the proposed formulas with actual processes has been proved by comparison with experimental data. In the numerical implementation decomposition of the expressions under integral sign into the Maclaurin series has been used. Based on the experimental data, the parameters of creep kernels have been determined. The parameters have been determined by minimizing the calculated data standard deviation from the experimental ones. The algorithm based on the random search method has been implemented. The results of the calculation according to the developed mathematical model have been given. The analysis of time variation in the stress-strain state of the joints between the layers of the composite structure has also been given.

Keywords: composite plate and shallow shell theory, composite structures, composite plate bending, visco-elastic properties, aging material.

1. Introduction

The article covers the solution to the problem of composite plate bending taking into account the creeping and relaxation processes of the material of the most frequently implemented structures – composite shells, plates and beams [1]. Connections between the layers of composite plates can be provided by adhesive bond or anchors [2, 3]. A record of the relations between stresses and strains for aging material has been proposed. On the basis of the elastic-creeping body theory the possibility of analytical description of the high-rate creep movement has been shown. Using the N. Kh. Arutyunyan and A. A. Zevin method new creep kernels have been built with respect to aging material creeping. A numerical implementation of the kernels has been considered on the example of A. A. Ross’ experimental data of [4]. Error estimates have been given.

The algorithm for solving the problem of composite plate bending during long-term deformation has been developed. The algorithm is based on an introduction of a discrete time scale and creep calculation by areas in accordance with the P. I. Vasilyev stepping-stone method.
2. Main points
We assume that the material of the construction has creep and aging properties. Then, in the elastic-creeping body linear theory the relationship between stresses and strains in such a construction with a uniaxial stress state has the form [5].

Using the method [5], new kernels to describe aging materials creep were determined. The possibility of calculating the obtained functions using power series for a sufficiently wide time interval was shown [6].

To describe the relationship between stresses and strains in accordance with the N. Kh. Arutyunyan and A. A. Zevin method a development of new kernels was implemented taking into account material aging:

\[
K_1(t, \tau) = Q(t, \tau) + B(t - \tau) + \int_\tau^t Q(t, s)B(s - \tau)ds ;
\]

\[
K_2(t, \tau) = Q(t, \tau) + B(t - \tau) + \int_\tau^t Q(s, \tau)B(t - s)ds .
\]

(1)

where \(Q(t, \tau)\) is a regular part of the creep kernel; \(B(t - \tau)\) is a differential part of the creep kernel.

As a differential component, we use one of the most meeting the entire necessary requirements weakly singular kernel [7].

As a regular component, we use a kernel based on the creep measure of the form [5].

For a numerical implementation the exponent in the expression under integral sign [5] is decomposed into a Maclaurin series.

The error of such a transition is determined by the accuracy of the integral count.

The possibility of using the obtained relations to describe the creep of actual materials was tested on the A. A. Ross’ experimental data [4] (Figure 1).

![Figure 1. Experimental and theoretical creep measure curves of concrete samples of different age \(\tau\)](image)

Creep measure was calculated by formula:

\[
C(t, \tau) = \frac{V_r(t, \tau)}{E(t)} - \frac{1}{E(\tau)} + \frac{1}{E(t)},
\]

(2)
where, \(E(t)\) is the modulus of elasticity, \(E(\tau)\) is the stiffness ratio of the concrete at different age.

The kernel parameters were determined according to these data by minimizing the standard deviation from the experiment [6]. The results were used to estimate the accuracy of the transition from the exact relation to approximate [5]. Here, the algorithm was implemented based on the random search method [4]. In the presence of four or more parameters instability in experimental data processing was observed [6]. Therefore, the value of \(\alpha\) (the exponent of the kernel expression degree) varied with a step of 0.25 from 0 to 1, and the remaining parameters were determined at fixed \(\alpha\). The parameter search intervals were set based on the minimum error condition when comparing the numerical implementation of the obtained expressions with the experiment.

The approximation of the experimental curves taking into account the obtained parameters is shown in figure 1 (dashed line). The discrepancy between the theoretical and experimental values was not more than 10%.

The results of the numerical implementation show that the error of the approximate relations is not big with the appropriate set of parameters. These relations can be used both for short times and fairly long ones without using special approximation techniques.

As an example, the problem of bending a three-layer composite plate taking into account the viscoelastic properties of the aging material in the second layer was solved (Figure 2). The dimensions of the plate are: in plan \(a = b = 1200\ mm\), thickness \(h = 120\ mm\). The outer layers are taken as equal: \(E^{(1)} = E^{(3)} = 2.1\times10^5\ MPa;\ \nu^{(1)} = \nu^{(3)} = 0.3;\ h^{(1)} = h^{(3)} = 1\ mm\). The parameters of the intermediate layer are: \(E^{(2)} = 8\times10^3\ MPa;\ \nu^{(2)} = 0.2;\ h^{(2)} = 118\ mm\). The stiffness for connection shear between the layers is described by linear relations. The stiffness ratio of the connections varies within: \(\eta = 0 \div 20\ \text{N/mm}^3\).

Resting the pack edge on a hinge has been selected as boundary conditions. The ends are bound with a tape, absolutely rigid in its plane and absolutely flexible from the plane. On the edges \(X = 0\); and we have the following conditions:

\[
\frac{\partial^2 W}{\partial x^2} = \frac{\partial^2 W}{\partial y^2} = \frac{\partial^2 \phi^i}{\partial x^2} = \frac{\partial^2 \phi^i}{\partial y^2} = 0.
\]

The load is evenly distributed in the center of the plate surface on the pad with dimensions of 200 x 200 mm. In the calculations the maximum load intensity was as \(q = 8\ \text{MPa}\).

![Figure 2. Composite three-layer plate](image-url)
The calculations were performed for linear deformation of the structure.

The problem, taking into account all the factors, was solved with the step-by-step method [8]. In the initial moment of time $t_1$ a calculation of instantaneous bending was carried out. In the second calculation cycle, with an underestimated value of the secant module, the visco-elastic properties of the material in the intermediate layer were taken into account. The creep strain was determined by the moment of time $t_2$. Then the secant modulus and the coefficient of transverse deformation were calculated. The integral of the creep kernel had the following form [5].

Thus, the solution to the problem of bending a three-layer composite plate taking into account the visco-elastic properties of the aging material of the intermediate layer was brought to the required moment of time $t_n$.

The calculation results are reflected in figures 3-5. The following designations are accepted: the solid line is the solution of the linear creep problem; the dashed line is the solution of the problem of time deformation, taking into account the physically nonlinear properties of the material; and the dash-dot line is an additional account of the material destruction in the stretched zone of the intermediate layer. The calculation was carried out at the local load of $q = 5$ MPa and joint stiffness $\eta = 1$ N/mm$^3$. The curves of change of the determined values are built for the loading moments: $\tau = 8$ days, 28 days and 300 days. The description of the intermediate layer material time deformation is used in the form [5].

The analysis of the results shows that the age of the material of the structure significantly depends on its stress-strain state. If the nonlinearity of the material properties (at some load values) leads to insignificant changes in the deformation of the pack (for example, the deflection changes by no more than 5%), then the account of the destruction of the stretched zone of the intermediate layer causes a significant change in the stress-strain state ($W$ increases by 30-40%).
The younger the material of the structure when it is loaded, the more pronounced the creep and relaxation processes are. The displacements and stresses in the composite plate vary by one and a half or more times depending on the age at loading.

The affect associated with the change of stresses in the intermediate layer has been revealed. If the layer material has physically nonlinear properties, the stresses in it are reduced in comparison with the linear version of the calculation. If the material destruction of the stretched zone of the layer occurs, then the stresses in the second layer increase due to the decrease in the stiffness of the pack and the increase in its strain (Figure 4 a).

![Figure 4](image)

**Figure 4.** Time variation of normal stresses in the intermediate layer (a) and shear stresses in the joint (b). See designations in figure 3.

Taking into account material destruction also leads to a significant asymmetry of the stress state of the pack along the height of its section. This can be seen from the comparison of the normal stress values in the outer surfaces of steel sheets (Figure 5). The difference in values varies between 40 and 75 %.

During the time deformation not only the value but also the form of stress distribution changes, even in a linear problem. This is most noticeable in the distribution of shear stresses in the joint.

Loading the plate which has the intermediate layer material of an earlier age results in a significant increase in shear stresses when we compare with calculations performed when loading the pack with the second layer material of other ages. This happens for the reason of that the younger material has minor stiffness. Since the stresses are reduced in this case, and the deflection of the pack increases, the joints and outer layers incur a major part of the load.
For the same reason, during time deformation of the plate, the normal stresses in the outer surface of the first layer $\sigma_x^{(1,1)}$ increase by 140% (8 and 108 days), while the stresses in the upper surface of the second layer $\sigma_x^{(2,1)}$ only decrease by 25% if the intermediate layer material is loaded at an earlier age. I.e. the overlap of the processes of increasing $W$ and decreasing the stiffness of the intermediate layer changes $\sigma_x^{(2,1)}$ insignificantly. Since the stiffness of the outer layers does not vary in time, the growth of deformation of the plate causes a sharp increase in $\sigma_x^{(1,1)}$.

### 3. Conclusion

The calculation of the composite plate in time has showed that the age of the structure material at which it is loaded significantly effects its stress-strain state. In time, there is a redistribution of the strain between the layers. Accordingly, the stress state of the layers changes. Shear stresses in the joints are characterized not only by quantitative but also qualitative changes.

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