Simulation of Quantum Field Theory and Gravity in Superfluid He-3.

G.E. Volovik *

Helsinki University of Technology, Low Temperature Laboratory, P.O.Box 2200, FIN-02015 HUT, Finland
*permanent address: Landau Institute for Theoretical Physics, Moscow, Russia

Superfluid phases of $^3$He are quantum liquids with the interacting fermionic and bosonic fields. In many respects they can simulate the interacting quantum fields in the physical vacuum. One can observe analogs of such phenomena as axial anomaly, vacuum polarization, zero-charge effect, fermionic charge of the vacuum, baryogenesis, ergoregion, vacuum instability, etc. We discuss some topics on example of several linear defects in $^3$He-A: (1) disgyration, which simulates the extremely massive cosmic string; (2) singular vortex, which is analogous to the spinning cosmic string; and (3) continuous vortex, which motion causes the "momentogenesis" which is the analog of baryogenesis in early Universe. The production of the fermionic momentum by the vortex motion (the counterpart of the electroweak baryogenesis) has been recently measured in Manchester experiments on rotating superfluid $^3$He-A and $^3$He-B. To simulate the other phenomena one needs rather low temperature and high homogeneity, which probably can be reached under microgravity conditions.

1. Introduction. Relativistic Fermions in $^3$He-A.

Here we demonstrate several of numerous examples of analogy between high energy physics and superfluid $^3$He. We consider 3 types of topologically stable linear defects in $^3$He-A, which exhibit the properties of different types of cosmic strings. The "gravity" and the axial anomaly in the presence of such defects are discussed.

The quasiparticles in $^3$He-A are chiral and massless fermions. Close to the gap nodes, ie at $k \approx \pm p_F \hat{l}$ ( $\hat{l}$ is unit vector in the direction of the gap
G.E. Volovik

nodes in the momentum space) the energy spectrum $E(k)$ of the gapless A-phase fermions are "relativistic":

$$E^2(k) = g^{ik}(k_i - eA_i)(k_k - eA_k) .$$ (1.1)

Here the "vector potential" $A$ is dynamical: $A = kF\hat{l}$; $e = \pm 1$; and the metric tensor is

$$g^{ik} = c^2(\delta^{ik} - \hat{l}^i\hat{l}^k) .$$ (1.2)

where $c_\perp = \Delta/p_F$ and $c_\parallel = v_F$ (with $c_\perp \ll c_\parallel$) are "speeds of light" propagating transverse to $\hat{l}$ and along $\hat{l}$ correspondingly; $k_F$ is the Fermi momentum; $v_F = k_F/m_3$ is the Fermi velocity; $m_3$ is the mass of $^3$He atom; $\Delta \ll k_Fv_F$ is the gap amplitude in $^3$He-A.

In the presence of superflow with the superfluid velocity $v_s$ the following term is added to the energy $E(k)$:

$$k \cdot v_s \equiv (k - eA) \cdot v_s + eA_0 , \quad A_0 = kF\hat{l} \cdot v_s .$$ (1.3)

The second term corresponds to the scalar potential $A_0$ of the "electromagnetic field", while the first one leads to the nonzero element $g_{0i} = v_s^i$ of the metric tensor. As a result the Eq.(1.1) transforms to

$$g^{\mu\nu}(k_\mu - eA_\mu)(k_\nu - eA_\nu) = 0 ,$$ (1.4)

with $k_\mu = (k, E)$, $A_\mu = (A, A_0)$ and the metric tensor

$$g^{00} = -1, \quad g^{0i} = v_s^i, \quad g^{ik} = c^2(\delta^{ik} - \hat{l}^i\hat{l}^k) + c^2i\hat{l}^i\hat{l}^k - v_s^iv_s^k ,$$ (1.5)

2. Disgyration as cosmic string. Conical singularity.

The so called radial disgyration is one of the topologically stable linear defects in $^3$He-A. This is an axisymmetric distribution of the $\hat{l}$ vector

$$\hat{l}(r, \phi) = \hat{r} ,$$ (2.1)

where $\hat{z}, \hat{r}, \hat{\phi}$ are unit vectors of the cylindrical coordinate system with $\hat{z}$ along the axis of the defect line. The vector potential $A = p_F\hat{r}$ can be removed by gauge transformation since the "magnetic" field is zero: $B = \nabla \times A = 0$. Thus the radial disgyration provides only the "gravity" field, acting on the $^3$He-A fermions, with the metric tensor

$$g^{00} = -1, \quad g^{0i} = 0, \quad g^{ik} = c^2(\hat{z}^i\hat{z}^k + \hat{\phi}^i\hat{\phi}^k) + c^2\hat{r}^i\hat{r}^k .$$ (2.2)

The interval corresponding to this metric

$$ds^2 = -dt^2 + \frac{1}{c^2_\perp}dz^2 + \frac{1}{c^2_\parallel}(dr^2 + \frac{c^2_\parallel}{c^2_\perp}r^2d\phi^2) ,$$ (2.3)
Simulation of Quantum Field Theory and Gravity.

has a conical singularity (if \( c_\perp \neq c_\parallel \)) with curvature being concentrated at the axis of disgyration \((r = 0)\). The space is flat everywhere outside the axis but the length of circumference of radius \( r \) around the axis is \( 2\pi r (c_\parallel / c_\perp) \). Since \( c_\parallel \gg c_\perp \), this is analogous to rather unusual cosmic string with a very big positive or negative mass \( M \) per unit length:

\[
4GM = 1 \pm \frac{c_\parallel}{c_\perp} , \quad 4G|M| \gg 1 .
\] (2.4)

Since the linear mass \( M = 1/4G \) corresponds to the chain of the point masses \( m \) with the distance between the neighbouring masses equal twice the Schwarzschild radius \((r_g = 2Gm): M = m/2r_g = 1/4G \) the case \( |M| > 1/4G \) corresponds to the chain of the overlapping black holes.

3. Symmetric vortex and spinning string.

The most symmetric vortex in \(^3\text{He}-A\) can be realized in thin films where the vector \( \hat{1} \) is fixed along the normal to the film, while the superfluid velocity is circulating around the vortex axis:

\[
\hat{1} = \hat{z} , \quad v_s = \frac{\hbar}{2m_3} \phi .
\] (3.1)

The "magnetic" field and the scalar potential are absent \((B = k_F \nabla \times \hat{1} = 0, A_0 = 0)\). One has again only the gravity field, now with the metric tensor

\[
g^{00} = -1 , \quad g^{0i} = \frac{\hbar}{2m_3} \phi^i , \quad g^{ik} = c_\parallel^2 z^i z^k + c_\perp^2 (x^i x^k + y^i y^k) - v_s^i v_s^k .
\] (3.2)

The corresponding interval is

\[
ds^2 = -\left(1 - \frac{v_s^2(r)}{c_\perp^2}\right) \left(dt + \frac{\hbar \phi}{2m_3 (c_\perp^2 - v_s^2(r))}\right)^2 + \frac{dz^2}{c_\parallel^2} + \frac{dr^2}{c_\perp^2} + \frac{r^2 d\phi^2}{c_\perp^2 - v_s^2(r)} .
\] (3.3)

There are two important properties of this interval:

(1) There is a region, where the velocity field \( v_s(r) \) exceeds the transverse speed of light, \( c_\perp \). This is the ‘ergoregion’: the vacuum in the ergoregion is unstable towards creation of pairs of particles.

(2) Far from the vortex axis, where \( v_s(r) \) is small and can be neglected:

\[
ds^2 = -\left(dt + \frac{d\phi}{\omega}\right)^2 + \frac{1}{c_\parallel^2} dz^2 + \frac{1}{c_\perp^2} (dr^2 + r^2 d\phi^2) ,
\] (3.4)

where the angular velocity

\[
\omega = \frac{2m_3 c_\parallel^2}{\hbar} .
\] (3.5)
This metric corresponds to that outside the so called spinning cosmic string, which has the angular momentum in the core. In our case this is unusual spinning string with the angular momentum \( J = \hbar / (8m_3G) \) per unit length (\( G \) is the gravitational constant), but with zero mass.

The connection between the time and the angle in Eq.(3.4) suggests that the energy \( E \) and angular momentum \( J \) of fermions on the background of this spinning string are related as

\[
E = J \omega .
\] (3.6)

The spectrum of bound states in the core of this vortex was calculated by Kopnin. He found that the factor \( \omega \) in Eq.(3.6) is of the same order, though is not equal to that in Eq.(3.5). What is more important, the calculated \( \omega \) appeared to be independent of the momentum \( k_z \) along the vortex axis in a complete agreement with Eq.(3.5). This is distinct from the spectrum of bound states in the core of vortices in other systems: In \( s \)-wave superconductors and in \( ^3\text{He-B} \) there is an essential dependence of \( \omega(k_z) \) on \( k_z \).

4. ATC vortex and baryogenesis.

The continuous vortex, discussed by Chechetkin and Anderson and Toulouse (ATC vortex), has the following distribution of \( \hat{l} \) and \( v_s \) fields:

\[
\hat{l}(r, \phi) = \hat{z} \cos \eta(r) + \hat{r} \sin \eta(r) , \quad v_s(r, \phi) = -\frac{\hbar}{2m_3r}[1 + \cos \eta(r)]\hat{\phi} , \quad (4.1)
\]

where \( \eta(r) \) changes from \( \eta(0) = \pi \) to \( \eta(\infty) = 0 \) in the so called soft core of the vortex. The stationary vortex generates the “magnetic” field and when the vortex moves with a constant velocity \( \mathbf{v}_L \) it also generates the “electric” field, since \( \mathbf{A} \) depends on \( \mathbf{r} - \mathbf{v}_Lt \):

\[
\mathbf{B} = k_F \hat{\nabla} \times \hat{l} , \quad \mathbf{E} = \partial_t \mathbf{A} = -k_F (\mathbf{v}_L \cdot \hat{\nabla})\hat{l} . \quad (4.2)
\]

The quantum vacuum with massless chiral fermions exhibits the axial anomaly: the presence of the electric and magnetic fields leads to the production of left particles from the vacuum at the rate:

\[
\dot{n} = \partial_\mu j^\mu = \frac{e^2}{4\pi^2} \mathbf{E} \cdot \mathbf{B} . \quad (4.3)
\]

In \( ^3\text{He-A} \) there are two species of fermions, left and right, with \( e = \pm 1 \). The left quasiparticle carries the linear momentum \( p_F \hat{l} \), while the righthanded quasiparticle carries the opposite momentum \(-p_F \hat{l}\). This asymmetry between left and right gives the net product of the fermionic linear
Simulation of Quantum Field Theory and Gravity.

momentum $\mathbf{P}$ in the time-dependent texture:

$$
\partial_t \mathbf{P} = 2 \int d^3r \, \mathbf{\hat{n}} \cdot \mathbf{p_F} \mathbf{\hat{l}} = \hbar \frac{k_F^2}{2\pi^2} \int d^3r \, \mathbf{\hat{l}} \cdot (\mathbf{\nabla} \times \mathbf{\hat{l}}) .
$$

Integration of the anomalous momentum transfer in Eq.(4.4) over the cross-section of the soft core of the moving ATC vortex gives the rate of the momentum transfer between the condensate (vacuum) and the heat bath (matter), mediated by the moving vortex:

$$
\partial_t \mathbf{P} = -\pi \hbar N_v C_0 \mathbf{\hat{z}} \times \mathbf{v}_L .
$$

Here $N_v$ is the winding number of the vortex ($N_v = -2$ in the case of Eq.(4.1)) and $C_0 = k_F^2/3\pi^2$. This "momentogenesis" from the vacuum gives an extra nondissipative force acting on the moving continuous vortex. This result, derived for the ATC vortex from the axial anomaly equation (4.3), was confirmed in the microscopic theory, which took into account the discreteness of the quasiparticle spectrum in the soft core. This was also confirmed in experiments on vortex dynamics in $^3$He-A where it was found that the extra force on the vortex nearly cancels the conventional Magnus force.

In the Weinberg-Salam model the similar asymmetry between left and right leads to the "baryogenesis" – production of the baryonic charge in the presence of the $SU(2)$ and $U(1)$ fields. Such fields can be generated in the core of the topological defects (monopoles, domain walls, spherons and electroweak cosmic strings) evolving in the expanding Universe.

Experiments in $^3$He-A and also in $^3$He-B, where the "momentogenesis" due to the axial anomaly in the singular core has been measured in a broad temperature range, support ideas on electroweak baryogenesis in early Universe.

5. Conclusion. $^3$He droplets in microgravity.

We discussed here only a very small fraction of the analogies which can simulate numerous phenomena in particle physics and gravitation.

In superfluid $^3$He it is very often that the container walls prevent the conducting of the "pure" experiments. Such nonsuperfluid environment should be removed since the Universe, according to our present knowledge, has no external environment. One example: in the process of nucleation of quantized vortices (the phenomenon which is now believed to be important in the cosmological models) the surface roughness of container dominates over the intrinsic nucleation. But only the latter is interesting for the cosmological community. The absence of gravity gives us a chance to produce the free droplet without the container boundaries and thus to deal with the
pure intrinsic nucleation. The rotation of a free droplet $^3$He-A liquid can be made by utilizing the unique magnetic properties of superfluid $^3$He: one can apply the rotating magnetic field.

Another interesting problem related to the droplets of $^3$He is the dependence of its superfluid properties on the size of droplet. The extreme case – the cluster with a small number $N$ of $^3$He atoms – corresponds to the other finite system – atomic nucleus which also represents the cluster of fermions (protons and neutrons). These clusters have very similar properties, determined by their fermionic quantum statistics and interaction: shell structure, magic numbers, single and collective excitations, rotational degrees of freedom, fission and fusion, deformed and superdeformed states of clusters with large angular momentum, the superfluid (pair-correlated) properties of nuclei which evolve with increasing $N$, etc. The advantage of $^3$He clusters, that it is relatively easy to study them using the NMR technique.

I thank T. Jacobson, P. Mazur and K. Rama for discussions.

REFERENCES

1. G.E. Volovik and T. Vachaspati, *Int. J. Mod. Phys. B* 10, 471 (1996).
2. D.D. Sokolov and A.A. Starobinsky, *Doklady AN SSSR* 234, 1043 (1977).
3. M. Banados, C. Teitelboim and J. Zanelli, *Phys. Rev. Lett.* 69, 1849 (1992).
4. G.E. Volovik, *Pisma ZhETF* 62, 58 (1995); [JETP Lett. 62, 65 (1995)].
5. P.O. Mazur, *Phys. Rev. Lett.* 57, 929 (1986); *Phys. Rev. D* 34, 1925 (1986); P.O. Mazur, [hep-th/9611206].
6. N.B. Kopnin, *Physica B* 210, 267 (1995).
7. C.Caroli, *et al., Phys. Lett.* 9, 307 (1964).
8. V.R. Chechetkin, *JETP* 44, 706 (1976).
9. P.W. Anderson and G. Toulouse, *Phys. Rev. Lett.* 38, 508 (1977).
10. S. Adler, *Phys. Rev. Lett.* 177, 2496 (1969).
11. J.S. Bell, R. Jackiw, *Nuovo Cim. A* 60, 47 (1969).
12. G.E. Volovik, *JETP* 75, 990 (1992).
13. N.B. Kopnin, *Phys. Rev. B* 47, 14354 (1993).
14. T.D.C. Bevan, A.J. Manninen, J.B. Cook, *et al., Nature* 386, 689 (1997)
15. A.D. Dolgov, *Phys. Rep.* 222, 310 (1992).
16. A.Vilenkin, E.P.S.Shellard, *Cosmic Strings and Other Topological Defects*, (Cambridge University Press,1994).
17. M.B. Hindmarsh and T.W.B. Kibble, *Phys. Rep.* 58, 477 (1995).
18. N. Turok, in *Formation and Interactions of the Topological Defects*, eds. A.C. Davis and R. Brandenberger, Plenum Press, New York and London, 1995.
19. T. Vachaspati, G.B. Field, *Phys. Rev. Lett.* 73, 373 (1994); 74, *Errata* (1995).
20. J. Garriga and T. Vachaspati, *Nucl. Phys.* B438, 161 (1995).
21. V.M.H.Ruutu, V.B.Eltsov, A.J.Gill, T.W.B Kibble, M.Krusius, Yu.G.Makhlin, B.Placais, G.E.Volovik, W.Xu, *Nature* 382, 334 (1996).