Partially conserved axial vector current and applications

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Lattice 2016, Southampton, UK, July 24 - 30, 2016
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2 Axial Ward identity

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   - Ward identity masses
   - Axial charge $g_A$

4 Short summary and outlook
In 2014 Horsley et al. (2015) we started to investigate the point split axial vector current.

Its divergence exactly satisfies a lattice Ward identity, involving the pseudoscalar density and a number of irrelevant operators.

Such operators naturally appear in derivation of lattice Ward identities, see e.g. Bochicchio, Maiani, Martinelli, Rossi, Testa NPB262 (1985), also Reisz, Rothe PRD62 (2000), Bhattacharya et al. PRD92 (2015).

We proved such an axial Ward identity for clover fermions and check it both perturbatively and nonperturbatively (Schiller Lattice 2015).
Axial Ward identity

Point split axial vector current

\[ A^{ps}_{\mu}(x) = \frac{1}{2} \left[ \bar{\psi}_x \gamma_\mu \gamma_5 U_\mu(x) \psi_x + a_{\hat{\mu}} + \bar{\psi}_{x+a_{\hat{\mu}}} \gamma_\mu \gamma_5 U^\dagger_\mu(x) \psi_x \right] \]

Axial Ward identity

\[ \langle \partial_\mu A^{ps}_\mu \rangle = 2M_0 \langle P \rangle + X \]

with

\[ M_0 = \frac{1}{2\kappa_I} - 4 \]

\[ X = 2\langle O_C \rangle + \langle O_W \rangle \]

\[ P = \bar{\psi}_x \gamma_5 \psi_x, \quad aO_C = \bar{\psi}_x \gamma_5 C_{xx} \psi_x \]

\[ aO_W = 8P - \frac{1}{2} \sum_\mu \left[ \bar{\psi}_x \gamma_5 U_\mu(x) \psi_x + a_{\hat{\mu}} + \bar{\psi}_{x+a_{\hat{\mu}}} \gamma_\mu \gamma_5 U^\dagger_\mu(x) \psi_x + (x \to x - a_{\hat{\mu}}) \right] \]
RI'-MOM scheme with subsequent trafo into RGI and $\overline{MS}$

$$\Gamma_{B}^{ps} = \gamma_{\mu} \gamma_{5} \cos \left( p_{\mu} \frac{2\pi}{L_{\mu}} \right)$$

Lattice setup:
- Gluon action: tree-level Symanzik improved
- Fermion action: $n_f = 2 + 1$ Wilson fermions with clover term
- Analytic stout smeared links in the Dirac kinetic and mass terms, no smearing in the clover term
- $32^3 \times 64$, $c_{sw} = 2.65$, $\omega = 0.1$, $\beta = 5.5$ [$a = 0.074(2)$ fm]

$(\kappa_l, \kappa_s)$ choices: Flavor symmetric line $(\kappa_l = \kappa_s)$ corresponding to pion masses

$$M_{\pi} = 470, 438, 402, 342, 290 \text{ MeV}$$
Transformation from RI’-MOM into RGI scheme (chiral limit)

\[ Z^0_A + b \cdot (a \rho)^2 \]
\[ Z^0_A = 1.0212(12) \]
Result:

\[ Z_{A^{\text{ps}}}^{\text{RGI}} = Z_{A^{\text{ps}}}^{\text{MS}} = 1.0212(12) \]

compare with

- \textit{local} axial vector current (Constantinou et al.):

\[ Z_{A^{\text{loc}}}^{\text{RGI}} = Z_{A^{\text{loc}}}^{\text{MS}} = 0.8728(27) \]

-one-loop PT (Horsley et al.) \textit{(Symanzik gauge action \(\rightarrow\) plaquette gauge action)}:

\[ Z_{1-\text{loop},A^{\text{ps}}}^{\text{MS}} = 0.996 \]
WI masses

- Remember AWI

\[ \langle \partial_\mu A^\text{PS}_\mu \rangle = 2M_0\langle P \rangle + 2\langle O_C \rangle + \langle O_W \rangle \]

⇒ under renormalization

\[ \bar{X} = 2\langle O_C \rangle + \langle O_W \rangle + 2\bar{M}\langle P \rangle + (Z_A - 1)\langle \partial_\mu A^\text{PS}_\mu \rangle \]

- Is it possible to find \(\bar{M}\) to make \(\bar{X}\) vanish for all quark masses?

\[ \Rightarrow Z_A\langle \partial_\mu A^\text{PS}_\mu \rangle = 2(M_0 - \bar{M})\langle P \rangle \equiv 2m^W_I\langle P \rangle \]

In the following we set \(Z_A = 1\)
WI masses

- Lattice setup \((32^3 \times 64, \beta = 5.50)\):

| \(\kappa_l\)  | \(\kappa_s\)  |
|-------------|-------------|
| 0.120900    | 0.120900    |
| 0.121040    | 0.120620    |
| 0.121095    | 0.120512    |
| 0.120990    | 0.120990    |

- \(\bar{m} = 1/3(m_u + m_d + m_s) = \text{const. line}\)

\[
\bar{M} = M_0 - \frac{c}{1 + \alpha Z} \left( \frac{1}{2\kappa_l} - \frac{1}{2\kappa_c} \right), \quad \alpha Z = \frac{Z_m^S}{Z_m^{NS}} - 1 \approx 0.767
\]

- \(SU(3)\) symmetric line

\[
\bar{M} = M_0 - c \left( \frac{1}{2\kappa_l} - \frac{1}{2\kappa_{0,c}} \right)
\]
We find $c = 1.55(4)$ to give $\bar{X} \approx 0$
\( \bar{m} = \text{const. line:} \)

- \( m_{WI}^l = \frac{c}{1 + \alpha_Z} \bar{m}_l \approx 0.86(m_l + \alpha_Z \bar{m}) \)
- The renormalized vector and WI quark masses should be equal

\[
\begin{align*}
m_{V,R}^l &= Z_m^{NS} \Delta Z_m^{\text{sea}} \bar{m}_l \\
m_{WI,R}^l &= \frac{1}{Z_P} \ 0.86 \bar{m}_l
\end{align*}
\]

\[
R = \frac{m_{V,R}^l}{m_{WI,R}^l} = Z_m^{NS} \Delta Z_m^{\text{sea}} Z_P \frac{1}{0.86} \approx 1.01
\]

- The same result is obtained for the \( SU(3) \) symmetric case
$g_A$ and the axial vector current $A_\mu$

Relation between $A_\mu$ and $g_A$ given by the forward matrix element

\[ \langle p, s | A_{\mu}^{u-d} | p, s \rangle = 2g_A \, s_\mu , \]

with

\[ A_{\mu}^{u-d} = A_{\mu}^{s,u} - A_{\mu}^{s,d} \]

and

$| p, s \rangle$: proton state with momentum $p$ and spin $s$.

On the lattice $g_A$ is computed via the ratio of the 3-pt to the 2-pt functions

\[ g_A = R(t_i, t_f, \tau) = \frac{G_3(t_i, t_f, \tau)}{G_2(t_i, t_f)} \]

with $(t_f - t_i) - $ source-sink distance, $\tau$ - source-operator insertion distance.
Figure: Connected and disconnected diagrams contributing to the 3-pt-function.
Lattice setup

- Gluon action: tree-level Symanzik improved
- Fermion action: $n_f = 2 + 1$ Wilson fermions with clover term
- analytic stout smeared links in the Dirac kinetic and mass terms, no smearing in the clover term
- $32^3 \times 64, c_{sw} = 2.65, \omega = 0.1, \beta = 5.5 \ [a = 0.074(2) \text{ fm}]$
- $48^3 \times 96, c_{sw} = 2.34, \omega = 0.1, \beta = 5.8 \ [a = 0.059(3) \text{ fm}]$

- $(\kappa_l, \kappa_s)$ choices: Flavor symmetric line corresponding to pion masses
  \[
  \beta = 5.5: \ M_\pi = 470, 360, 310 \text{ MeV} \\
  \beta = 5.8: \ M_\pi = 427 \text{ MeV}
  \]
Plateau method I

$\beta = 5.5$ : three different pion masses as function of operator insertion time $\tau$

Example: $\kappa_I = 0.121095$ ($M_\pi = 310$ MeV)

$g_A = 1.128(26), \kappa_I = 0.121095$
Plateau method II

$\beta = 5.8$: four different source-sink distances as function of operator insertion time $\tau$

$\hat{\tau} = \tau + \frac{1}{2}(23 - t_{sep})$
Plateau method - a dependence

Comparison for $\beta = 5.5$ and $\beta = 5.8$ at comparable physical source-sink distances

We recognize a very weak dependence on $\beta$, or $a$ resp.
It is obvious that excited states contaminate the measured result. There are several methods which try to handle this issue, e.g.

- **Exponential fit** (Capitani et al., Bali et al., Bhattacharyya et al., Dragos et al.)

\[
C_{2pt}(t_i, t_f) = \sum_{i=0}^{k} A_i^2 e^{-M_i (t_f - t_i)}
\]

\[
C_{3pt}(t_i, \tau, t_f) = A_0^2 g_A e^{-M_0 (t_f - t_i)} + \sum_{i=1}^{k} A_i^2 C_i e^{-M_i (t_f - t_i)} + \text{mixed terms containing functions of } (\tau, t_i, t_f)
\]

- **Summation method** (Capitani et al., Dragos et al.)

  Sum of the ratio \( R(t_i, t_f, \tau) \)

- **Variational method** (Owen et al., Dragos et al., Bhattacharyya et al.)

  Basis of states that couple differently to different energy levels
Some numbers,…

- At the moment we can realistically compare with computation based on the same action, lattice and at the same $\beta$ and $\kappa$ values ($32^3 \times 64$, $\beta = 5.5$, $M_\pi = 470$ MeV)

\[
A^{\text{loc}} : g^{\text{var}}_A = 1.1203(96) \quad \text{(Dragos et al., arXiv: 1606.03195)}
\]

\[
A^{\text{ps}} : g^{\text{plat}}_A = 1.131(13)
\]

- $g_A$ summary
We have proposed to use an point-split axial vector present in a lattice axial Ward identity.

We found the corresponding nonperturbative $Z_A$ factor very near to one.

We found AWI masses which lead to a continuum-like form of the axial Ward identity.

We have used $A_{\nu}^{ps}$ to compute the axial charge $g_A$ inside the proton for two $\beta$ values. The resulting $g_A(\beta = 5.5)$ do not differ within the errors from the corresponding result for the same action using the local axial vector current.

$g_A(\beta = 5.8)$ is somewhat larger if we use methods like 2-exp or summation.
Outlook ($g_A$)

- More detailed investigation of the influence of the excited states (variational method)
- Investigation of volume dependence
- Computation of $g_A$ for lighter pion masses
- Continuum limit