Distributed Compute-and-Forward Based Relaying Strategies in Multi-User Multi-Relay Networks

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Abstract

The compute-and-forward (CMF) relaying technique, proposed by Nazer et al., provides a substantially higher network coding throughput in multiuser cooperative networks, compared to the other conventional relaying techniques. The improvement is due to exploiting the multiuser interference rather than avoiding it. However, a major drawback of this strategy is that the equations decoded by different relays are not necessarily linearly independent, and hence a rank failure may occur at the destination; which results in a substantial reduction of the system rate as well as the diversity order. In this paper, we propose three practical distributed schemes to deal with the problem in a multi-user multi-relay network without central coordinator, in which the relays have not any prior information about each other. In first scheme, a new relaying strategy based on CMF, named incremental compute-and-forward (ICMF), is proposed that requires cooperation among relays. Second, a novel simple strategy, called amplify-forward and compute (AFC), is introduced in which the equations are recovered in the destination rather than in the relays. Finally, the two schemes ICMF and AFC strategies are combined to present hybrid compute-amplify and forward (HCAF) relaying scheme, which improves the performance of the ICMF considerably. We evaluate the performance of our proposed strategies and compare the results with those of the conventional CMF strategy and also the well known Decode and Forward (DF) strategy. The results indicate the substantial superiority of the proposed schemes, specially at the higher number of users and also relays.

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I. INTRODUCTION

In a multiuser relay network, the users desire to transfer their messages to a common destination or to different destinations with the help of some relays in an efficient and reliable way. To date, most proposed relaying schemes like amplify-and-forward (AF) and decode-and-forward (DF) perform quite well when there is not any multiuser interference [1-2], e.g., the users transmit in orthogonal channels (for instance by using TDMA) at the cost of low network throughput. On the other hand, if the users transmit simultaneously, the performance will be degraded due to the multiuser interference or noise amplification. By utilizing network coding along with DF or AF relaying scheme, a combination of the users’ messages can be constructed in each relay in a way to improve the system throughput [3-4].

The novel relaying technique, known as compute-and-forward (CMF) [5], has been designed for multiuser applications with the aim of increasing the network throughput. In this scheme, each relay, based on a noisy received combination of simultaneously transmitted signals of the users, attempts to recover an equation (a linear integer-combination) of users’ messages, instead of recovering each individual message separately. To enable the relay to recover the equation, the CMF scheme is usually implemented based on using a proper lattice code [6]. Since the equation coefficients are selected according to the channel coefficients, this method is also called physical layer network coding [7]. An attractive characteristic of this scheme is that the channel state information is not needed in the transmitters, which makes it be practical for most applications. The recovered equations by the relays are then forwarded to the common destination that attempts to solve and recover the users’ messages. In fact, the CMF method exploits rather than combats the interference, towards a better network performance.

Though CMF method has been considered and studied in different applications and scenarios in the literature, such as in multi antenna systems [8], two way relaying systems [9-10], cooperative distributed antenna systems [11], multi-access relay channels [12], generalized multi-way relay channels [13], and in two transmitter multi-relay systems [14], its substantial problem in a distributed MIMO systems has not received enough attentions. That is since each relay independently selects its equation coefficients with the aim of maximizing its own rate, the equations that are received from different relays at the destinations can be linearly dependent. On the other word, the coefficient matrix of the equations received by the destinations can
encounter rank failure. In this case, the destination cannot recover the users’ messages and the system performance deteriorates considerably, for instance see [14]. Until now, this problem in general has been considered only when there exists a central coordinator with global knowledge to compute and allocate independent equations with a maximum computation rate, i.e. the equation detecting rate, to the relays [11,15], which is not practical in many applications. On the other hand, the most strategies based on AF and DF schemes, which do not encounter the rank failure problem, have only a simple timer on each relay to coordinate multiple relays in their strategies [16-17]. In this paper, using the simple timer on each relay, we propose novel strategies to handle the rank failure problem for a general multi-user multi-relay wireless network that does not have a center to coordinate the relays’ equation computation, which can compete with the conventional AF and DF schemes practically.

First, we propose a relaying strategy based on CMF, named "incremental compute-and-forward" (ICMF), in which through cooperation among the relays, the linearly independent equations are recovered one by one, only by using a simple timer in each relay. We then introduce a new strategy called "amplify-forward-and-compute" (AFC), based on using the conventional amplify-and-forward relaying method and the integer-forcing linear receiver (IFLR) introduced by Zhan et al [8]. In AFC, each relay simply amplifies and forwards the received combination of the users’ noisy signals to the destination, and then the destination recovers all the required equations. Hence, the relay structure in AFC is much simpler than the ones in CMF and ICMF. In fact, in AFC method, the destination performs as a computation center, while in CMF and ICMF the computation (recovering the equations) is done by the relays in a distributed manner.

We, finally, propose "hybrid compute-amplify and forward" (HCAF) scheme, based on the combination of ICMF and AFC schemes described above. In this strategy, first the linearly independent equations are recovered one by one by the relays based on ICMF till the one for which the maximum rate derived by the relay is less than the target rate. Then for the rest of the equations the AFC technique is used and the related equations are recovered by the destination. We evaluate the performance of our proposed schemes and compare the results with the CMF and the commonly used DF relaying schemes. Our numerical results show that the ICMF and AFC strategies perform significantly better than the CMF method and provide a higher diversity order, in term of the system outage probability. Although the AFC strategy performs worse than the ICMF, especially when the channels between the relays are strong, it has much less
complexity. HCAF can improve the performance of the ICMF at the expense of a more complex receiver structure at the destination. Our proposed schemes substantially perform better than the conventional DF scheme. Moreover, this improvement becomes more significant by the increase of the number of users and also relays.

The reminder of the paper is organized as follows. In Section II, the system model and the related CMF strategy are described. Our new proposed methods, namely ICMF, AFC, and HCAF, are presented in Section III, Section IV, and Section V, respectively. Numerical results are given in Section VI. Finally, Section VII concludes the paper.

Notations: The superscripts $(.)^*$ and $||.||$ stand for conjugate transposition and frobenius norm of a matrix, respectively. The symbol $|x|$ is the absolute value of the scalar $x$, while $\log^+(x)$ denotes $\max\{\log(x), 0\}$. $\mathbb{E}\{\cdot\}$ is the expectation of a random variable. $I$ denotes identity matrix.

II. SYSTEM MODEL AND RELATED WORK

A. System Model

We consider a multi-user multi-relay cooperative network, shown in Fig. 1, consisting of $L$ users, $M$ relays and one common destination. Each user $i$ exploits a lattice encoder, with power constraint $P_T$, to map its message $w_i$ to a complex-valued codeword $x_i$ of length $n$ with $||x_i||^2 \leq nP_T$. We denote the received signals at relay $m$ by $y_m^r$ and at the destination from the relay $m$ by $y_m$. The power constraint on each relay is $P_R$. The element $h_{im}$ of the channel matrix $H$ represents the channel coefficient from user $i$ to relay $m$, $f_m$ indicates the channel coefficient from relay $m$ to the destination, and $g_{ab}$ denotes the channel coefficient from relay $a$ to relay $b$. The channel coefficient $h_{im}$, $f_m$, and $g_{ab}$ are assumed to be independent complex Gaussian distributed random variables with the variances $\sigma_{im}^2$, $\sigma_m^2$, and $\sigma_{r,ab}^2$, respectively. Moreover, block fading is assumed in which the channels are considered to be constant during the transmission periods required for message exchanges. We assume that each relay has only information about its own channels and is not aware of the other relays’ channel states.

At the first time slot, all $L$ users transmit their code words simultaneously to the relays. We assume that there are not direct links between the users and the destination. In the following $L$ or $M$ time slots, depending on the schemes used, $L$ or $M$ signals are transmitted by the relays, each in its own dedicated slot. The received signals $y_m^r$ and $y_m$ at the relay $m$ and at the
destination can respectively be written as,

\[ y_r^m = \sum_{i=1}^{L} h_{im} x_i + z_m \]  

(1)

\[ y_m = f_m x_r^m + \eta_m \]  

(2)

where \( z_m \) and \( \eta_m \) are independent additive white Gaussian noises with the same variance \( N_0 \), and \( x_r^m \) denotes the signal transmitted by the relay \( m \).

**B. Related Compute-and-Forward Strategy**

In this subsection, we briefly describe the conventional compute-and-forward (CMF) method [5,14]. In the first time slot, all the \( L \) users transmit their own codewords \( x_i, i = 1, \ldots, L \), simultaneously to the relays. Based on its received signal \( y_r^m \), each relay \( (m) \) attempts to detect an equation \( s_m \), a linear combination of user codewords, with complex integer equation coefficients vector (ECV) \( \mathbf{a}_m = [a_{1m}, \ldots, a_{Lm}] \in (\mathbb{Z} + i\mathbb{Z})^L \), i.e., \( s_m = \sum_{i=1}^{L} a_{im} x_i = \mathbf{a}_m^* \mathbf{X} \), where the vector \( \mathbf{X} = [x_1, \ldots, x_L]^* \) includes the codewords of all users. The coefficient vector in each relay is selected based on maximizing the relay’s computation rate, i.e., the rate of detecting the equation \( s_m \), as follows [5]:

\[ \mathbf{a}_m = \arg\min_{\mathbf{a}_l \in (\mathbb{Z} + i\mathbb{Z})^L, \mathbf{a}_l \neq \mathbf{0}} (\mathbf{a}_l^* \mathbf{H}_m \mathbf{a}_l) \]  

(3)

where \( \text{SNR}_T = P_T / N_0 \). The vector \( \mathbf{h}_m \) and the matrix \( \mathbf{H}_m \) are defined as,

\[ \mathbf{h}_m = [h_{1m}, \ldots, h_{Lm}]^* \]  

(4)
\[ H_m \triangleq I - \frac{\text{SNRT}}{1 + \text{SNRT}||h_m||^2} h_m h_m^* \]  

(5)

An efficient algorithm for solving the above integer optimization problem has been proposed in [15]. To detect the equation \( s_m \), the relay \( m \) quantizes the scaled received signal \( \alpha_m y_m^r \) to its nearest lattice point \( s_m = Q(\alpha_m y_m^r) \), where [5]

\[ \alpha_m = \frac{\text{SNRT}\ h_m^* a_m}{1 + \text{SNRT}||h_m||^2} \]  

(6)

and the \( Q(.) \) denotes the lattice quantizer function. The achievable computation rate of \( s_m \) is equal to [5]:

\[ r_m = \log^+((a_m^* H_m a_m)^{-1}) \]  

(7)

The \( M \) equations, \( s_m, m = 1, \ldots, M \), each independently detected by one of the relays, are then orthogonally transmitted with power \( P_r \) to the destination in the next \( M \) consecutive time slots. Since the channel from the relay \( m \) to the destination is a simple point to point channel, according to the (2), the transmission rate over this channel is,

\[ \tilde{r}_m = \log \left(1 + \text{SNRR}|f_m|^2\right) \]  

(8)

where \( \text{SNRR} = P_R/N_0 \). This rate is achievable by using the CMF strategy in one user case [5]. Hence, the overall rate of equation \( s_m \) at the destination is determined by,

\[ R_m = \min (r_m, \tilde{r}_m) \]  

(9)

The destination receives \( M \) equations from the \( M \) relays. To recover the users’ messages, the destination should select \( L \) equations from these \( M \) equations. This can be done in \( \binom{M}{L} \) different ways. Let \( S_u \) denotes the set of selected equations for \( u \)-th way, i.e.

\[ S_u = \{u_1, \ldots, u_L\} ; u \in \left\{1, 2, \ldots, \binom{M}{L}\right\} \]  

(10)

where \( u_i \) indicates the \( i \)-th equation in the set \( S_u \), and the matrix \( A_u \) denotes the ECVs corresponding to the equations in \( S_u \). For each set \( S_u \), its symmetric achievable rate \( R_{su} \), i.e. the rate of recovering all the messages, is

\[ R_{su} = \begin{cases} \min(R_{u_1}, \ldots, R_{u_L}) & , \text{rank}(A_u) = L \\ 0 & , \text{O.W.} \end{cases} \]  

(11)
It is noteworthy that if the ranks of all possible sets of equations $S_u$ are less than $L$, a rank failure is occurred and the destination cannot recover the messages; which leads to an outage event. Therefore, the achieved rate of the CMF method can be written as

$$R_{CMF} = \frac{L}{M+1} \max \left( R_{s_1}, \ldots, R_{s_{\binom{M}{L}}} \right)$$

(12)

where the coefficient $\frac{L}{M+1}$ is due to the fact that, in CMF method, the transmission and the recovering of the $L$ messages at the destination take place in $M + 1$ time slots.

III. INCREMENTAL COMPUTE-AND-FORWARD (ICMF)

In this method, $L$ independent equations with highest computation rates are recovered and sent to the destination through the cooperation among the relays in a distributed manner. This algorithm works as follows. First, each relay calculates its own overall computation rate (using (9)). Then the relay with the highest computation rate transmits its recovered equation to the destination, which is received by the other relays as well. For the second equation, each relay again computes another equation independent from the first one, which has the maximum computation rate. Among them, the relay with the highest second computation rate transmits its derived equation, which is again received by the other users as well. This process is repeated until all $L$ equations are derived and transmitted to the destination. That is, to recover a new equation, each relay finds an equation with the maximum rate, which is linearly independent of the previously derived and transmitted equations, and then the relay with the highest rate at that stage is selected to transmit the new recovered equation, as will be described later. In each relay, in each stage, the previous equations are exploited to increase the rate of recovering the new equation, similar to the successive compute-and-forward described in [18].

Specifically, the ICMF can be described as follows. At stage $k$, the $k$-th best equation is recovered and transmitted to the destination in the corresponding time slot, as follows. Each relay knows the $k-1$ best equations, $s_{\text{max}}^1, \ldots, s_{\text{max}}^{k-1}$, that are transmitted in the previous $k-1$ time slots simply by listening and detecting the signals transmitted in the earlier slots, as discussed above. In our performance evaluation, we consider the possible failure at the relay in detecting
the \( k - 1 \) equations. Let’s define the matrices

\[
E_k = \begin{bmatrix} e_1^* \\ \vdots \\ e_{k-1}^* \end{bmatrix}, \quad S_k = \begin{bmatrix} s_{\text{max}}^1 \\ \vdots \\ s_{\text{max}}^{k-1} \end{bmatrix}
\]

where \( e_i, i = 1, \ldots, k - 1 \), is the ECV of the \( i \)-th transmitted equation. Therefore, we can write,

\[
S_k = E_k X
\]

where \( X = [x_1, \ldots, x_L]^* \). Assume that the equation \( s_{\text{max}}^j, j = 1, \ldots, k - 1 \), is computed and transmitted by relay \( n_j \). The rate of receiving this equation at relay \( m \) is,

\[
r^e_m = \min \left( r^j_{n_j}, r_{mn_j} \right)
\]

where \( r_{mn_j} \) is the rate of the point-to-point channel between relays \( m \) and \( n_j \) as follows:

\[
r_{mn_j} = \log \left( 1 + SNR_R |g_{mn_j}|^2 \right)
\]

\( r^j_{n_j} \) is the computation rate of recovering this equation in relay \( n_j \), will be given in (34). Hence, the overall achievable rate of all of the \( k - 1 \) previously transmitted equations at relay \( m \) is,

\[
r^{e,k}_m = \min_{j=1,\ldots,k-1} (r^e_j)
\]

Now, each relay \( (m) \) attempts to recover its new equation based on the \( k - 1 \) equations received in the previous time slots and its received signal \( y^e_m \). First, the effect of previous equations is removed from the received signal \( y^e_m \) using projection space method [19] as:

\[
\hat{y}^k_m = y^e_m - h^*_m E_k^* (E_k E_k^*)^{-1} S_k
\]

This makes the optimum ECV derivation simpler, which will be shown later in Theorem 1. Then, a controlled and desired linear combination of this signal and the previously derived equations is made as follows:

\[
\tilde{y}^k_m = \beta^k_m \hat{y}^k_m + c^k_m S_k
\]

The coefficients of this linear combination are selected in a way to maximum the computation rate of the relay; the rate of recovering an equation, from signal \( \tilde{y}^k_m \), that is independent from the \( k - 1 \) previously transmitted equations. To this end, we define the vector \( g^k_m \) as,

\[
g^k_m \triangleq h^*_m \left( I - E_k^* (E_k E_k^*)^{-1} E_k \right)
\]
From (18-20), to recover the equation \( a_i^* X \) from \( \tilde{y}_m^k \), we rewrite (19) as,

\[
\tilde{y}_m^k = a_i^* X + \left( \beta_m^k g_m^k + c_m^k E_k^* - a_i^* \right) X + \beta_m^k z_m
\]

(21)

The effective noise variance for this equation is,

\[
N_{eq} = \mathbb{E} \left\{ \| \tilde{y}_m^k - a_i^* X \|_2^2 \right\} = \| \beta_m^k \|^2 + \text{SNR}_T \| \beta_m^k g_m^k + E_k^* c_m^k - a_i \|_2^2
\]

(22)

where \( \text{SNR}_T = P_T / N_0 \). Hence, the computation rate of this equation according to the above effective noise variance is given by

\[
r_m^k = \log^+ \left( \frac{\text{SNR}_T}{\| \beta_m^k \|^2 + \text{SNR}_T \| \beta_m^k g_m^k + E_k^* c_m^k - a_i \|_2^2} \right)
\]

(23)

Note that an equation with message transmission power \( P \) and effective recovery noise \( N_{eq} \) has computation rate \( \log^+ \left( \frac{P}{N_{eq}} \right) \) [5].

From (23), to obtain the maximum computation rate for recovering the equation with ECV \( a_i \), we should solve the following optimization problem:

\[
\max_{\beta_m^k, c_m^k} \log^+ \left( \frac{\text{SNR}_T}{\| \beta_m^k \|^2 + \text{SNR}_T \| \beta_m^k g_m^k + E_k^* c_m^k - a_i \|_2^2} \right)
\]

(24)

**Theorem 1:** In stage \( k \), the optimum values of the \( \beta_m^k \) and vector \( c_m^k \) for recovering the equation with coefficient vector \( a_i \) at relay \( m \) are

\[
\beta_m^{k, \text{opt}} = \frac{g_m^k a_i}{\text{SNR}_T + \| g_m^k \|^2}
\]

(25)

\[
c_m^{k, \text{opt}} = a_i^* E_k^*(E_k E_k^*)^{-1}
\]

(26)

**Proof:** From (24), the optimum coefficient vectors are obtained by minimizing the following function:

\[
f(\beta_m^k, c_m^k) = \frac{1}{\text{SNR}_T} \| \beta_m^k \|^2 + \| \beta_m^k g_m^k + E_k^* c_m^k - a_i \|_2^2
\]

\[
= \frac{1}{\text{SNR}_T} \| \beta_m^k g_m^k + E_k^* c_m^k - a_i \|^2 + \| \beta_m^k g_m^k + E_k^* c_m^k - a_i \|_2^2
\]

\[
= \frac{1}{\text{SNR}_T} \| \beta_m^k g_m^k + E_k^* c_m^k - a_i \|_2^2 + 2 \beta_m^k g_m^k E_k^* c_m^k - 2 \beta_m^k g_m^k a_i
\]

\[
+ c_m^k E_k^* c_m^k - 2 c_m^k E_k a_i + a_i^* a_i
\]

(27)
From the definition of $g_m^k$ in (20), we have,

$$g_m^k E_k^* c_m^k = 0$$  \hspace{1cm} (28)

Hence, we can write

$$f = \beta_m^k \beta_k^* \left( \frac{1}{SNR_T} + ||g_m^k||^2 \right) - 2\beta_m^k g_m^k a_t + c_m^k E_k^* c_m^k - 2c_m^k E_k a_t + a_t^* a_t$$  \hspace{1cm} (29)

The optimum value for $\beta_m^k$ is obtained by setting the derivative of $f$ with respect to $\beta_m^k$ equal to zero

$$\frac{\partial f (\beta_m^k, c_m^k)}{\partial \beta_m^k} = 2\beta_m^k \left( \frac{1}{SNR_T} + ||g_m^k||^2 \right) - 2g_m^k a_t = 0$$  \hspace{1cm} (30)

which leads to:

$$\beta_m^{k, opt} = \frac{g_m^k a_t}{\frac{1}{SNR_T} + ||g_m^k||^2}$$  \hspace{1cm} (31)

In a similar way, to obtain the optimum value for $c_m^k$, we set:

$$\frac{\partial f (\beta_m^k, c_m^k)}{\partial c_m^k} = 2E_k^* E_k c_m^k - 2E_k a_t = 0$$  \hspace{1cm} (32)

which leads to:

$$c_m^{k, opt} = a_t^* E_k^* (E_k E_k^*)^{-1}$$  \hspace{1cm} (33)

Thus, the theorem is proved.

By substituting (25) and (26) in (23), the computation rate of the equation with the coefficient vector $a_t$, at relay $m$ and at stage $k$, is easily computed as

$$r_m^k = \min \{ \log^+ ((a_t^* V_m^k a_t)^{-1}, r_{e,k}^m) \}$$  \hspace{1cm} (34)

where $r_{e,k}^m$ is given in (17) and

$$V_m^k \triangleq I - \frac{g_m^k g_m^k *}{\frac{1}{SNR_T} + ||g_m^k||^2} - E_k^* (E_k E_k^*)^{-1} E_k$$  \hspace{1cm} (35)

The relay $m$, at stage $k$, has to find the equation with the highest possible rate in (34) that is linearly independent from the previous $k - 1$ equations, i.e., $e_1, \ldots, e_{k-1}$. Hence, from (34), the relay $m$ finds its optimum ECV based on the following optimization problem

$$a_m^k = \min_{a_t \in (\mathbb{Z} + i\mathbb{Z})^L} (a_t^* V_m^k a_t)$$
subject to
\[
\text{rank} \left( [a_l, e_1, \ldots, e_{k-1}] \right) = k
\]  \hspace{1cm} (36)

**Lemma 1:** To find the optimum ECV \( a^k_m \) in problem (36), it is sufficient to check the space of all integer vectors \( a_l \) with norm satisfying
\[
||a_l||^2 \leq \frac{1}{SNR_T} - \frac{1}{SNR_T + ||g^k_m||^2} - \left| \left| E_k^* (E_k E_k^*)^{-1} E_k \right| \right|
\]  \hspace{1cm} (37)

**Proof:** From (34), in stage \( k \), the computation rate of relay \( m \) is zero for all \( a_l \) satisfying
\[
a^*_l V^k_m a_l \geq 1
\]  \hspace{1cm} (38)

From (35), we can rewrite the left side of (38) as
\[
a^*_l V^k_m a_l = \frac{||a_l||^2 - \frac{1}{SNR_T} - \left| \left| g^k_m \right| \right|^2}{SNR_T + \frac{1}{||g^k_m||^2}} - \left| \left| a_l \right| \right|^2 \left| \left| E_k^* (E_k E_k^*)^{-1} E_k \right| \right|
\]  \hspace{1cm} (39)

Using Cauchy-Schwarz inequality, \( \left| \left| g^k_m \right| \right|^2 \leq \left| \left| a_l \right| \right|^2 \left| \left| g^k_m \right| \right|^2 \) and \( a^*_l E_k^* (E_k E_k^*)^{-1} E_k a_l \leq \left| \left| a_l \right| \right|^2 \left| \left| E_k^* (E_k E_k^*)^{-1} E_k \right| \right| \), this can be lower bounded by
\[
a^*_l V^k_m a_l \leq \left| \left| a_l \right| \right|^2 - \left| \left| a_l \right| \right|^2 \left| \left| g^k_m \right| \right|^2 \left| \left| \frac{1}{SNR_T} + \frac{1}{||g^k_m||^2} \right| \right| - \left| \left| a_l \right| \right|^2 \left| \left| E_k^* (E_k E_k^*)^{-1} E_k \right| \right|
\]  \hspace{1cm} (40)

Hence,
\[
\frac{1}{SNR_T} - \frac{1}{SNR_T + ||g^k_m||^2} - ||E_k^* (E_k E_k^*)^{-1} E_k|| \geq a^*_l V^k_m a_l \geq 1
\]  \hspace{1cm} (41)

The equation corresponding to this ECV is recovered by quantization of \( \tilde{y}^k_m \) in (19) as
\[
s^k_m = Q \left( \tilde{y}^k_m \right)
\]  \hspace{1cm} (42)

\( \beta^k_m \) and \( c^k_m \) in (19) are substituted from (25) and (26). The overall rate of receiving this equation at the destination is
\[
R^k_m = \min \left( r^k_m, \tilde{r}_m \right)
\]  \hspace{1cm} (43)
TABLE I

ALGORITHM 1: ICMF Procedure

for stage $k = 1, \ldots, L$

I. for relay $m = 1, \ldots, M$
   1. Recovering equation transmitted in the time slot $k - 1$
   2. Finding ECV $a_{k}^{m}$ by solving of (36)
   3. Recovering equation with ECV $a_{k}^{m}$ by quantization of (42)
   end

II. Best relay selection by timer setting

III. Best relay transmission

end

where $\tilde{r}_{m}$ and $r_{m}^{k}$ are given in (8) and (34), respectively. Now, the relay with the highest rate $R_{m}^{k}$ sends its equation to the destination, by using the technique similar to the one presented in [16], as follows. Each relay $m$ sets a timer with the value $T_{m}$ proportional to the inverse of its rate $R_{m}^{k}$, which counts down to zero simultaneously. The relay that its timer reaches zero first has the highest rate and broadcasts a flag to inform the other relays, and then transmits its equation in the $k$-th time slot. In the stage $k$, we denote $s_{\text{max}}^{k} = e_{k}^{*}X$ as the transmitted equation by the best relay $n_{k}$.

After $L$ stages, $L$ independent equations are recovered and sent to the destination, and based on them, all users’ messages are decoded at the destination. Assume that for the equation $L$, relay $n_{L}$ is selected as the best relay. It can be easily observed that the achievable rate of the proposed scheme, ICMF, (for recovering all users’ messages at the destination) is,

$$R_{ICMF} = \frac{L}{L+1} R_{n_{L}}^{L}$$

(44)

$R_{n_{L}}^{L}$, which denotes the rate of relay $n_{L}$, is obtained from (43). Please note that $L + 1$ time slot is required to transmit $L$ complex equations, in contrast to the CMF that requires $M + 1$ time slots. The mentioned ICMF procedure is summerized in algorithm 1.

In this algorithm, when the cooperation among the relays is not possible because of the bad quality of the inter-relays channels, only the best relay with the maximum computation rate is selected using the technique described above, and that relay transmits its $L$ best independent equations to the receiver.
IV. AMPLIFY-FORWARD-AND-COMPUTE (AFC)

This technique is based on using the conventional amplify-and-forward relaying method and integer-forcing linear receiver (IFLR) introduced by Zhan, et al [8], which is developed for the point-to-point MIMO channels. In AFC strategy, in the first time slot, like CMF, all the $L$ users transmit their own codewords $x_i$s simultaneously to the relays. Each relay ($m$) amplifies its received signal $y^r_m$ by a gain $\gamma_m$ and then forwards the amplified signal to the destination at its dedicated time slot. Hence, similar to CMF, AFC requires $M+1$ time slots to transmit the $L$ messages. According to the power constraint $P_R$ at each relay $m$, $\gamma_m$ is easily computed as,

$$\gamma_m = \sqrt{\frac{SNR_R}{SNR_T||h_m||^2 + 1}}$$  \hspace{1cm} (45)$$

where $SNR_R = P_R/N_0$ and $SNR_T = P_T/N_0$. From (1) and (2), the received signal from each relay $m$, at the destination is given by,

$$y_m = f_m \gamma_m y^r_m + \eta_m = f_m \gamma_m \sum_{i=1}^{L} h_{im} x_i + f_m \gamma_m z_m + \eta_m, m = 1, \ldots, M$$  \hspace{1cm} (46)$$

By defining the following vectors,

$$Y = [y_1, \ldots, y_M]^*; X = [x_1, \ldots, x_L]^*; \gamma = [\gamma_1, \ldots, \gamma_M]^*; f = [f_1, \ldots, f_M]^*; Z = [\eta_1, \ldots, \eta_m]^*; Z^r = [z_1, \ldots, z_M]^*$$  \hspace{1cm} (47)$$

And also,

$$F \triangleq diag (f) \times diag (\gamma)$$  \hspace{1cm} (48)$$

The set of equations (46) from all relays can be rewritten in the following matrix form

$$Y = FHX + FZ^r + Z$$  \hspace{1cm} (49)$$

The receiver structure at the destination is shown in Fig. 2. The receiver exploits a projection matrix $B_{L \times M}$ to recover $L$ independent equations with the complex integer coefficient matrix $A_{L \times L}$, as

$$B = \begin{bmatrix} b_1^* \\ \vdots \\ b_L^* \end{bmatrix}, A = \begin{bmatrix} a_1^* \\ \vdots \\ a_L^* \end{bmatrix}$$  \hspace{1cm} (50)$$
where \( a_l \) and \( b_l \) are the ECV and the projection vector corresponding to the \( l \)-th equation, respectively. From Fig. 2 and (49), for recovering the equations \( AX \), the projected received vector is written as:

\[
\tilde{Y} = BY = BFHX + BFZ' + BZ = AX + (BFH - A)X + BFZ' + BZ
\]  

(51)

The \( l \)-th row of the vector \( \tilde{Y} \) indicates the \( l \)-th projected signal for recovering \( l \)-th equation, i.e. \( a_l^*X \), which is calculated as

\[
\tilde{y}_l = a_l^*X + (b_l^*FH - a_l^*)X + b_l^*FZ' + b_l^*Z
\]  

(52)

From (51-52), \( \tilde{y}_l \) is in fact a linear combination (with coefficients vector \( b_l^* \)) of received signals from \( M \) relays, i.e., \( y_1, ..., y_M \), that will be used to recover the equation \( a_l^*X \). From (52), the variance of the effective noise in recovering equation \( a_l^*X \) is easily computed as,

\[
N_{eq} = \mathbb{E} \left\{ |\tilde{y}_l - a_l^*X|^2 \right\} = ||b_l||^2 + ||F^*b_l||^2 + SNR_T||H^*F^*b_l - a_l||^2
\]  

(53)

As a result, from (52) and (53), the computation rate of recovering an equation with ECV \( a_l \) using the projection vector \( b_l \) is given by,

\[
R_l = \log^+ \left( \frac{SNR_T}{||b_l||^2 + ||F^*b_l||^2 + SNR_T||H^*F^*b_l - a_l||^2} \right)
\]  

(54)

For a maximum computation rate, we should solve the following optimization problem:

\[
\max_{b_l} \log^+ \left( \frac{SNR_T}{||b_l||^2 + ||F^*b_l||^2 + SNR_T||H^*F^*b_l - a_l||^2} \right)
\]  

(55)

**Theorem 2**: The optimum projection vector \( b_l^* \) for recovering an equation with ECV \( a_l \) is,

\[
b_l^{opt} = a_l^*H^*F^* \left( \frac{1}{SNR_T} (I + FF^*) + FHH^*F^* \right)^{-1}
\]  

(56)

And hence, the optimum projection matrix \( B \) can be written as

\[
B_{opt} = AH^*F^* \left( \frac{1}{SNR_T} (I + FF^*) + FHH^*F^* \right)^{-1}
\]  

(57)

**Proof**: To maximize (55), the following function should be minimized:

\[
f (b_l) = \frac{1}{SNR_T} ||b_l||^2 + \frac{1}{SNR_T} ||F^*b_l||^2 + ||H^*F^*b_l - a_l||^2
\]

\[
= \frac{1}{SNR_T} b_l^* b_l + \frac{1}{SNR_T} b_l^* FF^*b_l + (H^*F^*b_l - a_l)^*(H^*F^*b_l - a_l)
\]

\[
= b_l^* \left( \frac{1}{SNR_T} (I + FF^*) + FHH^*F^* \right) b_l - 2b_l^* FHa_l + a_l^*a_l
\]  

(58)
The optimum projection vector $b_l$ is found by solving $\frac{df(b_l^{opt})}{db_l} = 0$, which results to:

$$2 \left( \frac{1}{SNR_T} (I + FF^*) + FHH^*F^* \right) b_l^{opt} - 2FHa_l = 0$$  \hspace{1cm} (59)$$

Hence,

$$b_l^{opt} = a_l^*H^*F^* \left( \frac{1}{SNR_T} (I + FF^*) + FHH^*F^* \right)^{-1}$$  \hspace{1cm} (60)$$

Thus, the theorem is proved.

By substituting (56) in (54), the optimum computation rate for recovering an equation with ECV $a_l$ is easily obtained as

$$R_l = \log^+ \left( (a_l^*Va_l)^{-1} \right)$$  \hspace{1cm} (61)$$

where,

$$V = I - H^*F^* \left( \frac{1}{SNR_T} (I + FF^*) + FHH^*F^* \right)^{-1}$$  \hspace{1cm} (62)$$

If $L$ independent ECVs $a_1, ..., a_L$, forming the coefficient matrix $A$, are used, the AFC rate for recovering all users’ messages is given by,

$$R_{AFC} = \frac{L}{M + 1} \min (R_1, ..., R_L)$$  \hspace{1cm} (63)$$

where $R_l$ is the computation rate of $a_l$ given by (61). Note that due to linear independency of ECVs $a_1, ..., a_L$, the rank failure problem is solved. To maximize the rate $R_{AFC}$, from (61) and (63), the optimum coefficient matrix $A^{opt}$ is calculated as,

$$A^{opt} = \arg \max_{A \in \mathbb{Z}^{+L \times L}} \min_{l=1,...,L} \log((a_l^*Va_l)^{-1})$$

subject to,

$$\begin{cases} 
A = \begin{bmatrix} 
a_1^* \\
\vdots \\
a_L^* 
\end{bmatrix} \\
\det (A) \neq 0 \\
a_l \in \mathbb{Z}^L + i\mathbb{Z}^L, l = 1, ..., L 
\end{cases}$$  \hspace{1cm} (64)$$
Fig. 2. Receiver structure of destination for AFC method (D shows lattice decoder)

**Lemma 2:** To find the optimum ECVs in problem (64), it is sufficient to check the space of all integer vectors $a_l$ with norm satisfying

$$||a_l||^2 \leq \frac{1}{1 - \|H^*F^*\left(\frac{1}{SNR_t} (I + FF^*) + FHH^*F^*\right)^{-1}\|}$$

(65)

**Proof:** The proof is similar to the one given for lemma 1.

Then, based on the matrix $A^{opt}$, the receiver calculates the optimum projection matrix $B^{opt}$ from (57). The receiver quantizes the projected signals, i.e., the rows of $\tilde{Y}$ given in (51), to the nearest lattice points to find the $L$ independent equations. Finally, by solving the set of the equations, the users’ messages are recovered.

It is noteworthy that while in the CMF method each relay has to find the optimum ECV and perform a lattice decoding, in the AFC strategy, the relays simply amplify and forward the received signals and the destination finds all the required optimum ECVs which are guaranteed to be independent. Therefore, in AFC, the relay complexity is much less than those of CMF and ICMF; which makes AFC be more desirable in practice.

**V. Hybrid Compute-Amplify-and-Forward (HCAF)**

This strategy can improve the performance of the ICMF method by combining it with AFC. In ICMF, when the computation rate of the best relay in a stage $k$ is lower than the target rate $R_t$, the system encounters an outage event. In this case, recovering and sending an equation by the best relay cannot help the destination. As an alternative method, when in some stage $k$ the highest computation rate is less than the target rate, the remaining $L - k + 1$ best relays, with the
highest rates (though all less than $R_t$), can use the AFC strategy to simply amplify and forward their received signals to the destination in the remaining $L - k + 1$ time slots. These signals can then be exploited by the destination with the help of the previously $k - 1$ received equations to recover the remaining equations, which can lead higher rates compared to the ICMF strategy. In this strategy, the best relays are selected by utilizing count-down timers described in Section III for the ICMF strategy.

Suppose that the relays $n_1, ..., n_{k-1}$, named as computing relays, have recovered and transmitted equations $d_1^*X = s_{CF}^{1}$, $d_{k-1}^*X = s_{CF}^{k-1}$, or in the matrix form of $S^{CF} = DX$, where

$$D \triangleq \begin{bmatrix} d_1^* \\ \vdots \\ d_{(k-1)}^* \end{bmatrix}$$

and $S^{CF} = \begin{bmatrix} s_1^{CF} \\ \vdots \\ s_{(k-1)}^{CF} \end{bmatrix}$, in the first $k - 1$ slots to the destination, and at the stage $k$, the highest rate is less than the target rate. Suppose at this stage, the relays $n_k, ..., n_L$ are selected as amplifying relays, based on their computation rates (which are higher than the rates of the other relays). For amplifying relays, we define the following vectors

$$Y^{AF} = [y_{n_k}, ..., y_{n_L}]^*, \gamma^{AF} = [\gamma_{n_k}, ..., \gamma_{n_L}]^*, f^{AF} = [f_{n_k}, ..., f_{n_L}]^*,$$

$$Z^{AF} = [\eta_{n_k}, ..., \eta_{n_L}]^*, Z^r^{AF} = [z_{n_k}, ..., z_{n_L}]^*$$

(66)

And matrices,

$$H^{AF} = \begin{bmatrix} h_{n_k}^* \\ \vdots \\ h_{n_L}^* \end{bmatrix}$$

(67)

$$F^{AF} \triangleq diag(f^{AF}) \times diag(\gamma^{AF})$$

(68)

All the components of these vectors and matrices are as defined in Section IV for the AFC. By the above definitions, the received signals from amplifying relays at the destination can be simply written as

$$Y^{AF} = F^{AF}H^{AF}X + F^{AF}Z^r^{AF} + Z^{AF}$$

(69)

The block diagram of the receiver at the destination is shown in Fig. 3. As shown in this Fig., at first, the effects of the received equations from the computing relays, i.e., relays $n_1, ..., n_{k-1}$,
are removed from the signals received by the amplifying relays using projection space method [19],

\[
\hat{Y}^{AF} = Y^{AF} - F^{AF}H^{AF}D^*(DD^*)^{-1}S^{CF}
\]  

(70)

As mentioned previously for the ICMF, this makes the later derivations simpler, as will be shown in proof of Theorem 3. The destination exploits two projection matrices \(B_{(L-k+1)\times (L-k+1)}\) and \(C_{(k-L+1)\times (k-1)}\) for the signals received from the amplifying relays and the equations received from the computing relays, respectively. After the projections, the results are simply added as

\[
\tilde{Y}^{AF} = B\hat{Y}^{AF} + CS^{CF}
\]  

(71)

where \(B = \begin{bmatrix} b_1^* & \vdots & b_{L-k+1}^* \end{bmatrix}\) and \(C = \begin{bmatrix} c_1^* & \vdots & c_{L-k+1}^* \end{bmatrix}\). From \(\tilde{Y}^{AF}\), the remaining \(L-k+1\) linearly independent equations are recovered with the complex integer coefficient matrix \(A_{AF}^{(L-k+1)\times L}\), as follows.

\[
A^{AF} = \begin{bmatrix} a_1^* & \vdots & a_{L-k+1}^* \end{bmatrix}
\]

We define the matrix \(G\) as

\[
G \Delta F^{AF}H^{AF}(I - D^*(DD^*)^{-1}D)
\]  

(72)

From this definition, the \(l\)-th row of the vector \(\tilde{Y}^{AF}\) in (71) is given by

\[
\tilde{y}_l^{AF} = b_l^*Y^{AF} + c_l^*S^{CF} = b_l^*GX + c_l^*DX + b_l^*F^{AF}Z^{rAF} + b_l^*Z^{AF}
\]  

(73)

The equation \(a_l^*X\) is recovered from \(\tilde{y}_l\) as

\[
\tilde{y}_l^{AF} = a_l^*X + (b_l^*G + c_l^*D - a_l^*)X + b_l^*F^{AF}Z^{rAF} + b_l^*Z^{AF}
\]  

(74)

The effective noise variance in this computation is equal to

\[
N_{eq} = \mathbb{E}\left\{\left|\tilde{y}_l^{AF} - a_l^*X\right|^2\right\} = ||b_l||^2 + ||F^{AF^*}b_l||^2 + SNR_T||G^*b_l + D^*c_l - a_l||^2
\]  

(75)

Hence, from (74) and (75), the computation rate of the equation with coefficient of \(a_l\) is given by

\[
R_l = \log^+ \left( \frac{SNR_T}{||b_l||^2 + ||F^{AF^*}b_l||^2 + SNR_T||G^*b_l + D^*c_l - a_l||^2} \right)
\]  

(76)
For a maximum computation rate, we should solve the following optimization problem:

$$\max_{b_l, c_l} \log^+ \left( \frac{SNR_T}{||b_l||^2 + ||F^{AF*}b_l||^2 + SNR_T||G^*b_l + D^*c_l - a_l||^2} \right)$$  \hspace{1cm} (77)

**Theorem 3**: The optimum values of vectors $b_l$ and $c_l$ for recovering an equation with coefficient vector $a_l$ are given by

$$b_{opt,l}^* = a_l^* G^* \left( \frac{1}{SNR_T} (I + F^{AF} F^{AF*}) + G G^* \right)^{-1}$$  \hspace{1cm} (78)

and

$$c_{opt,l}^* = a_l^* D^* (D D^*)^{-1}$$  \hspace{1cm} (79)

Therefore, the matrices $B$ and $C$ can be written as

$$B = A^{AF} G^* \left( \frac{1}{SNR_T} (I + F^{AF} F^{AF*}) + G G^* \right)^{-1}$$  \hspace{1cm} (80)

and

$$C = A^{AF} D^* (D D^*)^{-1}$$  \hspace{1cm} (81)

**Proof**: From (77), the optimum values are obtained by minimizing the following function:

$$f(b_l, c_l) = \frac{1}{SNR_T} ||b_l||^2 + \frac{1}{SNR_T} ||F^{AF*}b_l||^2 + ||G^*b_l + D^*c_l - a_l||^2$$

$$= \frac{1}{SNR_T} b_l^* b_l + \frac{1}{SNR_T} b_l^* F^{AF*} F^{AF} b_l + (G^* b_l + D^* c_l - a_l)^* (G^* b_l + D^* c_l - a_l)$$

$$= \frac{1}{SNR_T} b_l^* \left( I + F^{AF} F^{AF*} \right) b_l + b_l^* G G^* b_l + 2 b_l^* G D^* c_l - 2 b_l^* G a_l$$

$$+ c_l^* D D^* c_l - 2 c_l^* D a_l + a_l^* a_l$$  \hspace{1cm} (82)

From definition of $G$ in (72), we easily obtain:

$$b_l^* G D^* c_l = 0$$  \hspace{1cm} (83)

Hence, we have

$$f = b_l^* \left( \frac{1}{SNR_T} (I + F^{AF} F^{AF*}) + G G^* \right) b_l - 2 b_l^* G a_l + c_l^* D D^* c_l - 2 c_l^* D a_l + a_l^* a_l$$  \hspace{1cm} (84)

The optimum value of $b_l$ is the solution of

$$\frac{\partial f(b_l, c_l)}{\partial b_l} = 2 \left( \frac{1}{SNR_T} (I + F^{AF} F^{AF*}) + G G^* \right) b_l - 2 G a_l = 0$$  \hspace{1cm} (85)
Hence,

\[ b_{opt,l}^* = a_l^* G^* \left( \frac{1}{SNR_T} (I + F^{AF} F^{AF*}) + G G^* \right)^{-1} \]  \hspace{1cm} (86)  

In a similar way, the optimum value of \( c_l \) is found from the solution of

\[ \frac{\partial f(b_l, c_l)}{\partial c_l} = 2 D D^* c_l - 2 D a_l = 0 \]  \hspace{1cm} (87)  

which leads to

\[ c_l^* = a_l^* D^*(DD^*)^{-1} \]  \hspace{1cm} (88)  

Thus, the theorem is proved.

By substituting (78) and (79) in (76), the computation rate of an equation with ECV \( a_l \) can be written as

\[ R_l = \min \left( \log \left( (a_l^* U a_l)^{-1} \right), R_{n_{k-1}}^{k-1} \right) \]  \hspace{1cm} (89)  

where,

\[ U \triangleq I - G^* \left( \frac{1}{SNR_T} (I + F^{AF} F^{AF*}) + G G^* \right)^{-1} G - D^*(DD^*)^{-1} D \]  \hspace{1cm} (90)  

and \( R_{n_{k-1}}^{k-1} \) denotes the equation rate of relay \( n_{k-1} \) as the best relay at stage \( k-1 \). It is clear that the rate of the recovered remaining equations in (89) is lower than the rate of the computing relays. Hence, the rate of this strategy can be easily written as

\[ R_{HCAF} = \frac{L}{L+1} \min \left( R_1, \ldots, R_{(L-k+1)} \right) \]  \hspace{1cm} (91)  

Please note that the HCAF, similar to the ICMF, needs \( L + 1 \) time slots to transmit the \( L \) messages. To maximize the computation rate (91), from (89), \( L - k + 1 \) linearly independent equations, which are also independent from the computing equations, must be found from the following optimization problem like as in AFC,

\[ A_{opt,AF} = \arg \min_{A^{AF} \in (Z+iZ)^{(L-k+1) \times L}} \max_{l=1, \ldots, L-k+1} \left( a_l^* U a_l \right) \]  

subject to

\[ \begin{cases} A^{AF} = \begin{bmatrix} a_1^* \\ \vdots \\ a_{L-k+1}^* \end{bmatrix} \\ \det \left( [A^{AF}; D] \right) \neq 0 \\ a_l \in (Z+iZ)^L, l = 1, \ldots, L - k + 1 \end{cases} \]  \hspace{1cm} (92)
Lemma 3: To find the optimum ECVs in problem (92), it is sufficient to check the space of all integer vectors \( \mathbf{a}_l \) with norm satisfying
\[
\|
\mathbf{a}_l
\|^2 \leq \frac{1}{1 - \left\| G^* \left( \frac{1}{SNR_t} (\mathbf{I} + \mathbf{F}^{AF} \mathbf{F}^{AF*}) + \mathbf{GG}^* \right)^{-1} \mathbf{G} \right\| - \left\| \mathbf{D}^* (\mathbf{DD}^*)^{-1} \mathbf{D} \right\|}
\]
(93)

Proof: The proof is similar to the proof of lemma 1.

The coefficient matrix corresponding to all equations at the destination can be written as
\[
\mathbf{A}^{opt} = \begin{bmatrix} \mathbf{A}^{opt,AF} \\ \mathbf{D} \end{bmatrix}
\]
(94)
The projection matrices \( \mathbf{B} \) and \( \mathbf{C} \) are calculated by substituting the matrices \( \mathbf{A}^{opt,AF} \) and \( \mathbf{D} \) in (80) and (81). The remaining equations, i.e. the rows of \( \mathbf{S}^{AF} \), are recovered by quantization of \( \hat{\mathbf{Y}}^{AF} \) as
\[
\mathbf{S}^{AF} = Q \left( \hat{\mathbf{Y}}^{AF} \right) = \mathbf{A}^{AF, opt} \mathbf{X}
\]
(95)
where,
\[
\hat{\mathbf{Y}}^{AF} = \mathbf{B} \hat{\mathbf{Y}}^{AF} + \mathbf{CS}^{CF}
\]
(96)
Finally, by solving the \( L \) independent equations, obtained from the \( k - 1 \) computing relays and the \( L - k + 1 \) amplifying relays, the destination can recover all of the users’ messages.

VI. Simulation Results

In this section, we evaluate and compare the performance of our proposed methods through computer simulations. In simulations, \( L \) users and \( M \) relays are considered. We assume the special case in which all of the nods, i.e., the users and the relays, have equal transmission powers, and \( \sigma_{im}^2 = \sigma_h^2, \forall i, m, \sigma_{r,ab}^2 = \sigma_g^2, \forall a, b, \) and \( \sigma_m^2 = \sigma_f^2, \forall m \). Threshold rate is set equal to one \( (R_t = 1) \).

Fig. 4 and Fig. 5 shows the plots of the outage probabilities versus SNR for three proposed schemes along with those of the conventional CMF and DF relaying schemes for \( L = 2, M = 3, \sigma_h^2 = 1, \sigma_f^2 = 10, \) and \( \sigma_g^2 = 1 \) and 0.1, respectively. For the DF strategy, the best relay with maximum rate jointly decodes the users’ messages utilizing the successive interference cancellation method ([4] and [20]) and then transmits them to the destination. From this figures,
the ICMF and AFC methods perform significantly better than the CMF and DF methods, especially at high SNR. For example for $\sigma^2_g = 1$ and at outage probability of 0.01, the proposed schemes perform approximately 10$dB$ and 3$dB$ better than the CMF and the DF strategies, respectively. Moreover, both the ICMF and AFC methods achieve a significantly higher diversity order than CMF in which due to the rank failure problem at the destination, the diversity order is low. As realized from the figures, the HCAF always shows better performance than ICMF; the amount of the improvement decreases by the increase of the channel qualities among the relays, i.e., higher $\sigma^2_g$. For example for $\sigma^2_g = 0.1$ and 1, HCAF performs approximately 1.5$dB$ and 0.5$dB$ better at outage probability of 0.01, respectively. Furthermore, ICMF can perform better than the AFC for $\sigma^2_g$ higher than a certain threshold, due to the fact that the ICMF requires each relay to correctly decode the other relays transmissions. For example at outage 0.01, while at $\sigma^2_g = 0.1$, ICMF performs approximately 2$dB$ worse than the AFC, at $\sigma^2_g = 1$ it performs 1$dB$ better.

In Fig. 6, we consider another case in which $L = 2$, $M = 3$, $\sigma^2_h = 1$, $\sigma^2_f = 1$, and $\sigma^2_g = 1$. By comparison of Figs. 5 and 6, it can be realized that the performance of the proposed schemes is better when the channels from the relays to the destination have higher SNR, i.e., higher $\sigma^2_f$. As can be observed and expected, the effect of $\sigma^2_f$ on the performance of AFC is more substantial than the other schemes, and the amount of the improvement of ICMF over AFC decreases for higher $\sigma^2_f$. For example at $\sigma^2_f = 1$ and 10, and at outage 0.01, ICMF performs approximately
$4dB$ and $1dB$ better than AFC, respectively.

In Fig. 7, the effect of the number of relays on the performance has been studied and compared. The values of the parameters are: $L = 2$, $\sigma_n^2 = 1$, $\sigma_f^2 = 10$, $\sigma_g^2 = 1$, and $M = 2$ and $3$. By the increase of the number of relays, the performance and also the diversity order are significantly improved. For example at outage 0.02, the proposed schemes with $M = 3$ performs approximately $5.5dB$ better than the ones with $M = 2$.

In Fig. 8, we consider three users and three relays, i.e. $L = 3$ and $M = 3$, and we set $\sigma_n^2 = 1$, $\sigma_f^2 = 10$, and $\sigma_g^2 = 1$. At outage probability of 0.01, it can be observed that the proposed schemes have approximately $8dB$ better performance than the DF method, and they provide significant improvement in comparison with the CMF scheme. From Fig. 4 and 8, by the increase of the number of users, the superiority of the proposed schemes is more substantial.

VII. Conclusion

In this paper, we proposed three novel relaying strategies for multi-user multi-relay networks, named as ICMF, AFC, and HCAF. In these strategies, new ideas are exploited to overcome the drawbacks of the conventional CMF strategy and to provide efficient and reliable frameworks for practical applications in multiuser cooperative networks. In ICMF, in a distributed and cooperative manner, each relay exploits the previously transmitted equations to extract a new independent equation with highest computation rates. In AFC, based on the concept of computation, the relays just simply amplify and forward their received signals and the destination, as a center of computation, recovers all required equations. In HCAF, a combination of ICMF and AFC approaches are used in which whenever the highest computation rate of the relays is lower than the target rate, the relays switch from computing nodes to amplifying nodes. Numerical results indicate that the outage performance and diversity order of the proposed strategies are considerably better than those of the conventional CMF and DF strategies specially at high number of users and also relays. Moreover, numerical results show that ICMF performs better than AFC only when the links among the relays have good enough qualities. It is notable that the complexity of AFC is much lower than that of the ICMF. HCAF strategy provides a better performance than ICMF, at the cost of more complicated receiver. It is noteworthy to mention that the ICMF and HCAF schemes, independent of the number of relays ($M$), need $L + 1$ time slots to transmit the $L$ users’ messages, in contrast to AFC and CMF that require $M + 1$ time slots.
slots.
Fig. 4. Outage probabilities of the proposed schemes along with CMF and DF versus SNR for $L=2$ and $M=3$, $R_t = 1$, $\sigma_h^2 = 1$, $\sigma_f^2 = 10$, $\sigma_g^2 = 1$.

Fig. 5. Outage probabilities of the proposed schemes along with CMF and DF versus SNR for $L=2$ and $M=3$, $R_t = 1$, $\sigma_h^2 = 1$, $\sigma_f^2 = 10$, $\sigma_g^2 = 0.1$. 
Fig. 6. Outage probabilities of the proposed schemes along with CMF versus SNR for $L=2$ and $M=3$, $R_i = 1$, $\sigma_h^2 = 1$, $\sigma_f^2 = 1$, $\sigma_g^2 = 1$.

Fig. 7. Outage probabilities of the proposed schemes and CMF versus SNR for $L=2$, and $M=2$ and 3, $R_i = 1$, $\sigma_h^2 = 1$, $\sigma_f^2 = 10$, $\sigma_g^2 = 1$. 

Fig. 8. Outage probabilities of the proposed schemes along with CMF and DF versus SNR for \( L=3 \) and \( M=3 \), \( R_t = 1 \), \( \sigma_h^2 = 1 \), \( \sigma_f^2 = 10 \), \( \sigma_g^2 = 1 \).

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