The calculation and analysis of the trailing vortex influence in the flight of aircraft

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Abstract. In the flight, trailing vortex will be produced behind the aircraft. Because it is invisible and intensity is bigger, it would produce certain influence to flight safety. In this article, the composition of trailing vortex and the trailing vortex strength would be calculated, and the dynamic characteristics would be analyzed about entering the trailing vortex.

Introduction

According to statistics, the light aircraft accident due to enter the large aircraft wake is half in landing phase, 30% occurred in the take-off stage, and 20% occurred in the air. The risk of entering trailing vortex, lies in the invisible of trailing vortex. But its air velocity is very big, this will lead the plane to severe changes state. The momentum in Aircraft flight will transfer to surrounding air, and after the plane flow over, the air caused intense turbulence. The air turbulence after the end plane is aircraft wake\cite{1} \cite{2} \cite{3}.

The composition of trailing vortex

In flight, the trailing vortex after the plane drafting is consists of three parts: one is engine jet, the second is turbulent flow of the plane, three is namely wingtip vortex. The jet flow and turbulent flow usually dissipate in a short time. Practice has proved, the engine jet from the nozzle basic dissipate after 15 ~ 20 nozzle diameter, and the turbulent flow of the plane in the 20 ~ 40 meters leaving the plane also disappear completely. Therefore, great influence for flight status and safety is trailing vortex. The trailing vortex has been a long time , for large wing load aircraft, the trailing vortex can persist 1 ~ 2 minutes.

When the wing of limited wingspan produces positive lift, air pressure under the surface is bigger, and the upper surface air pressure is smaller. So the air on the surface will flow on the surface bypass wing tip. In the process of airflow through the wing, on the lower surface the streamline flows from the plane wing symmetry to the wing tips, while on the upper surface the streamline from the wingtip deflection the plane symmetry\cite{4}\cite{5}. Because of the fluctuation on the wing surface and different flow direction when the airflow through the trailing edge, the result is to form spiral. In addition, due to the air viscosity effect and the interaction of vortex, two large vortex rope far from wing is wingtip vortex.

Look from behind the wing, left wing tip vortex is clockwise, and the right wing tip vortex is counterclockwise. The emergence of wingtip vortex is the basic characteristics of three dimensional wing flow. Speed of the aircraft or the lift coefficient are larger, the pressure difference on the surface are greater, and the wing wake vortex strength is stronger. They are often referred to as eddy current.

Trailing vortex motion and the vortex intensity

A little air mass going around a point is called vortex, its center is called vorticity. The vorticity lines along the vortex vector direction are called vortex line\cite{6} \cite{7}. The direction of rotation of vortex can be judged according to the right-hand rule, and the vortex line is expressed as a vector.
Vortex intensity is the product of the circumferential velocity and its circumference as a point outside the vortex core, with $\Gamma$ express. That is:

$$\Gamma = 2\pi RV_T$$  \hspace{1cm} (1)$$

The circumferential velocity distribution through measurement is shown in the Figure 1. The center of the vortex is the vortex core. And the vortex core is axis area of the vortex circumferential velocity from zero increase to the maximum. Leaving the vortex core outward, the circumferential speed will continue to decrease, but the intensity of vortex is unchanged[8].

The trailing vortex core radius of flat wing aircraft is about 1$\sim$2% wingspan length, that is $r_{\text{core}}=0.01\sim0.02L$. The trailing vortex core radius of sweepback wing aircraft is about 1.5$\sim$2.5% wingspan length, that is $r_{\text{core}}=0.04\sim0.06L$.

![Fig.1 The circumferential velocity distribution](image)

**Trailing vortex strength**

The lift formula is often used to calculate the lift size. If the concept of vortex intensity is introduced, the lift is divided the elliptic distribution wing along the wingspan[9]. So it can be recognized as a rectangular wing. The lift calculation formula can be written as:

$$Y = \rho VTT_{\text{vortex}}$$  \hspace{1cm} (2)$$

$\Gamma$ is the vortex intensity of the plane symmetry, $L_{\text{vortex}}$ is a distance of both the vortex centers, i.e. the span of rectangular wing. That is

$$L_{\text{vortex}} = \frac{\pi}{4} L$$  \hspace{1cm} (3)$$

According to lift formula:

$$\Gamma = \frac{2C_yV_b}{\pi}$$  \hspace{1cm} (4)$$

As long as the data of aircraft's $C_y$ and $C_x$ can be achieved, the $\Gamma$ values can be calculated. Because the existence of the trailing vortex, the air flow behind the wing under the action of the trailing vortex induced downwash velocity $V$ and formed washing angle $\varepsilon$ and wash the area outside the wingspan.

According to the vortex shedding circumferential velocity formula:

$$V_T = \frac{\Gamma}{2\pi r}$$  \hspace{1cm} (5)$$

When $r=r_{\text{core}}$, the maximum circumferential speed of the vortex core radius can be calculated.

Although the circumferential velocity in the vortex core radius is the largest, but the vortex core radius is very small. Leaving the vortex core outwards, trailing vortex circumferential velocity will decrease rapidly. Within the scope of the vortex beam between the wings around, the the circular
velocity direction of the trailing vortex is down. So the downwash velocity is formed, its size is about the sum of two bundles of vortex induced velocity[10]. According to the calculation, just below the plane vertical axis, the trailing vortex downwash velocity is only about 1/10 of the maximum vortex core radius. It is can be seen that the circumferential velocity distribution of trailing vortex along the exhibition numerical has a big range.

The air velocity circumference of the trailing vortex o is an important symbol of trailing vortex strength[11]. Because of the existence of circumferential velocity, when the rear plane entering the tail vortex range, the angle of attack, sideslip angle, aerodynamic force and torque will go to change. Thus the aircraft makes roll, yaw, jitter, decelerate and sink. After the air velocity circumference of the trailing vortex is bigger, the emergence into the trailing vortex of the phenomenon is more serious. Because trailing vortex intensity is felt by the pilot, is after a certain time. Therefore, considering the influence of the trailing vortex to the rear plane, will consider the two aspects, one is the circumferential velocity just before the formation of tail vortex, the other is the attenuation surplus value of trailing vortex circumference velocity after a certain time [12].

The formation of the trailing vortex is not rolled into the vortex rope from the start. In fact, Because there is negative pressure behind the wingspan and turbulent flow, the vortex layer behind the wingspan is very thin, and is not stable. Therefore, trailing vortex is only after a long distance "rolled into" two large scale vortex rope. Generally have the following empirical formula:

\[
T_{\text{Rolled up}} = 0.28 \frac{\lambda L}{C_y V} \\
S_{\text{Rolled up}} = 0.28 \frac{\lambda L}{C_y}
\]

Among them:
- \(\lambda\) —— wing aspect ratio;
- \(L\) —— wingspan (m);
- \(T_{\text{Rolled up}}\), \(S_{\text{Rolled up}}\) —— time and distance of trailing vortex.

Because the air has viscosity, the rotation of the trailing vortex is fading. Eventually trailing vortex would be disappear completely. In generally, it has the following empirical formula:

\[
T_{\text{attenuation}} = k_{\lambda} \frac{\pi^2 L^2}{16 \Gamma}
\]

Which factor \(k_{\lambda}\) desirable values as follows:

\[
k_{\lambda} = \frac{0.4}{16\pi \times 0.0002} = 39.81
\]

Subsidence is refers to the vertical height for the trailing vortex center of the plane below the horizontal line, it is an important data to determine the position of trailing vortex. It is the basis of height difference in follow up flight. Subsidence is induced by the wing vortex system. According to the lift for elliptic distribution, the epsilon downwash angle is equal to \(\varepsilon = C_y / \pi \lambda\). So the subsidence can press type calculation:

\[
\Delta H = S \times \tan \left( C_y / \pi \left( \frac{1}{\lambda},[0] + 0.025 \right) \right)
\]

Conclusions

After the rear plane entering the tail vortex, the plane dynamic will change because the trailing vortex aerodynamic force and moment will vary. The change of the aerodynamic force and moment is at the same time and influence each other. So the dynamics of the aircraft has its complicated side,
but from the perspective of pilots judgment and disposal, the dynamic change and regularity of mainly should be paid attention to.

When the dynamic change of the plane enter the trailing vortex is rolling on the most prominent, thus it will cause roll torque, which a form of roll torque is the difference in wing lift. Firstly it is caused by local angle difference between the two wings. The angle of attack change is proportional to the rotational speed of trailing vortex, and the speed is inversely proportional. On the second the vertical tail plane lateral force change also can produce roll torque.

After the rear in the trailing vortex position is different, the rolling characteristics of the aircraft is also different. When the vertical axis is consistent between the front plane and the rear plane, the rear plane wing is affected by the tail vortex. All are under the tail vortex before washing speed scope, lift force is reduced to the plane down. But the left wing lift is equal to the right wing lift, the plane is not rolling. When the rear plane is on the tail vortex center, the half wing within machine after the trailing vortex lift function washing speed decreases, and half a wing baptized on speed of function lift increases, then the plane to the roll torque is the largest. When after the rear plane is in the tail vortex vorticity line, on the air flow effect besides the rear from the tail wing vortex, the outer roll torque will be produced.

Enter the trailing vortex, the aircraft pitch changes occur because the tail and wings upward force changes. Especially large sweepback of the wing tips at the center of gravity of the shuttle, wingtip baptized under the trailing vortex lift lower speed, will appear on the moment. On the contrary, baptized on speed, wingtip lift increases, and will produce down moment. According to this feature, when the rear plane entering the tail vortex, rolled in strengthening, the aircraft's nose trend also enhanced accordingly; When the plane into the outer roll, the plane down trend is enhanced, or make up on the original trend.

After the rear plane entering the tail vortex, the reason for a sinking plane is the total lift slope after the slope, the vertical component of lift is less than the plane gravity. The second reason is when a plane from above or below after enter the trailing vortex, affected by the downwash velocity, the angle of attack decreases, and the lift is reduced causing the plane sank. While shaking because the wings of local angle of attack and sideslip angle change is bigger after entering trailing vortex, the local plane surface emerge flow separation caused the frames.

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