Non-reciprocity and heat transfer in far from equilibrium processes

Alejandro Cabo Montes de Oca
Instituto de Cibernetica, Matematica y Fisica, Calle E 309, Vedado, Ciudad Habana, Cuba

A non-reciprocal phonon model for microwave or optical isolators is considered. It gives a simpler framework to further investigate the formerly argued possibility for a heat transfer between black bodies at common temperatures. While the non-dissipative device retains the Detailed Balance property, the presence of dissipation breaks it. This property allows a net transfer of heat between the two black bodies at common temperatures, whenever the absorptive elements are at lower temperatures than the one being common to the bodies.

PACS numbers: 44.10.+i,44.40.+a,44.90.+c
Keywords: Statistical Physics, Second Law, Non-reciprocity

In this work we continue the previously started examination of the physics of the heat transfer among bodies and the fact that such energy flows are expected to always spontaneously occur from higher to lower temperature regions. This property is equivalent to the Law of increasing Entropy. However, in our view its theoretical derivation from statistical physics deserves more close attention. A concrete fact underlining this need is the lack of definition of the entropy for physical processes occurring far from the equilibrium situation. It can be understood that for a very wide class of physical systems having reciprocal dynamics, the heat will always travel from higher to lower temperature regions. However, when the bodies have a non-reciprocal dynamical response, this property is no so naturally evident. Non-reciprocal phenomena occurs in practice within devices such as optical and microwave isolators and can be found more generally in all the physics of the magnetism having so many applications in technology.

The former paper made an analysis of two types of isolators connecting two regions containing black-body radiation at common temperatures. In the present article we consider the realization of the same kind of phenomena within a simple soluble 1D non-reciprocal phonon model. It is defined by a string having a uniform charge density which is placed within a magnetic field orientated along the equilibrium line of the string. The systems wave modes are explicitly determined. They show two independent Faraday waves for each direction of propagation. These are employed for defining a phonon version of the optical and microwave isolators discussed in.

A main new possibility introduced here is the addition of a lateral string (the ”vane”) at one of the ends of the isolator which simulates the dissipative resistance vane in the microwave or optical versions of these devices. It is then shown that when the vane is disconnected, the transmission coefficients through both sides is identical no matter the frequency or the orientations of the slits simulating the optical polarizers. This result in fact represent a clean solution of the so called Wien paradox in optical systems. Also it illustrates the validity of the Detailed Balance principle for the two terminal conservative version of the isolator. After that, the waves modes for the system including the vane were also solved. The calculated spectrum for the transmission coefficients implied the breaking of the Detailed Balance principle for the isolator including the vane. However, it follows that when the three outputs are at the same temperature, the net flux through any of the three terminals exactly vanish. This result excludes the presence of the heat flux unbalance under full thermal equilibrium. However, a net heat flow between Black-Bodies at common temperatures is predicted when the vane is placed in contact with a very low temperature heat reservoir.

Seeking for a simple model for the optical and microwave isolators discussed in let us consider a Lagrangian system formed by a uniformly charged string in presence of an also uniform magnetic field. The Lagrangian of the system and its Euler equations are given by

\[ L = \int dx \left( \frac{\rho_m}{2c_s} (\partial_t \vec{u})^2 - \frac{k_h}{2} (\partial_x \vec{u})^2 + \frac{\rho_q}{2c_e} \vec{u} \times \vec{B} \right), \]

\[ 0 = -\rho_m \partial_t \vec{u} + k_h \partial_x \vec{u} + \frac{\rho_q}{2c_e} (\partial_t \vec{u}) \times \vec{B}. \]

where \( \vec{u}(x,t) \) are the vector displacement of the points of the string with respect to their equilibrium positions and \( \rho_m \) and \( \rho_q \) are the linear mass and charge densities, \( k_h \) is the Hook law constant. The magnetic field is oriented along the string in the x direction \( \vec{B} = (1,0,0) \). It should be mentioned that the self interaction of the charged string will be disregarded in order to simplify the discussion. The coulomb interaction, basically will produce the appearance of a plasmon mode introducing a gap at low frequencies. Searching for the oscillation modes in the usual form

\[ \vec{u}(x,t) = \exp(i.kx - ik_0x_o) \vec{u}(k) \]

in which \( k_o = w/c_s, x_o = c_s t \) and \( c_s = \sqrt{k_h/\rho_m} \) is the sound velocity in the cord, \( c \) is the light velocity. The Euler equation in (1) after written in components takes...
be noticed that for a fixed energy \( \epsilon \), microwave or optical isolators investigated in section changes proportionally with the displacement along the same fixed positive frequency, Faraday waves can be obtained. c) Let us also consider that the frequency lies in the linear part of the dispersion relations, that is, when \( \epsilon = \hbar k_o c_s \)\( k_o = k, k_o = \sigma M + \sqrt{\kappa^2 + M^2}, \sigma = \pm, M = |\omega B|c_s| \). Inversely, the momenta and frequencies obey the relations \( k_o = \sqrt{(\kappa_o - \sigma M)^2 - M^2} \), with \( \sigma = \pm \). It should be noticed that for a fixed energy \( \epsilon = \hbar k_o c_s \) there are two solutions having momenta \( k_+ \) and \( k_- \). After performing linear combinations of two modes associated with the same fixed positive frequency, Faraday waves can be obtained. That is, they show the Faraday effect: their polarizations are linear at any point and its angular orientation changes proportionally with the displacement along the string.

A phonon version of the experimentally existing microwave or optical isolators investigated in Fig. 1 is accomplished by considering an arrangement depicted in Fig. 1.

![Image](66x306 to 287x471)

**FIG. 1: Phonon Isolator**

The device can be described in the following way: a) Two slits forming an angle \( \theta \) among them are situated a distance \( L \) restricting the movement of a long piece of the string described in previous section. They lay within a plane being orthogonal to the string and it is assumed that there is no friction between them and the cord. In other words they form conservative mechanical constraints on the system. b) At a distance \( \delta \ll L \) of one of the them, let us say the right one, a second string is attached to the main one, by mean of a bar sliding without friction inside a cylinder at rest. That is, the point at the junction is constrained to move linearly in a direction orthogonal to the initial string. This fact implies that the newly added string, that we will call from now on, the "vane", is allowed to oscillate only longitudinally. c) Let us also consider that the frequency lies in the linear part of the dispersion relations, that is, when \( M \ll k_o \). Then, the length \( L \) is chosen to assure that a Faraday mode propagating to the right and starting at the slit 1 being polarized along it, after arriving to the slit 2 will be also polarized along it. This property, in turns will assure that the waves incident on the left of slit 1, being polarized along it will have a transmission coefficient equal to one, whenever the coupling with the vane is disconnected. d) Next, the angular orientation of the vane at the fixing point distant a length \( \delta \) from the slit 2 is selected to coincide with the exact polarization of the before analyzed Faraday wave having transmission coefficient equal to one. e) At last, let us consider that outside the two slits, the charge density of the string vanish and also that additional non-dissipative mechanical constraints exist which only allow the transversal polarization of the waves in the directions of the associated slits. The only purpose of this assumption is to reduce the number of independent wave modes of the system in order to simplify the discussion. It should be notice that in the proposed arrangement the longitudinal oscillations do not couple with the transversal ones (within the linear approximation only).

The above conditions, then assure that the system can be described by a three terminal scattering matrix \( S(i,j) = \delta_{ij} \)\( b_i = S_{ij} a_j, i,j = 1,2,3 \), where the pair \( (a_1, b_1) \) contains the incident \( a_1 \) and reflected \( b_1 \) wave amplitudes at the slit one and \( (a_2, b_2), (a_1, b_1) \) are the corresponding quantities at the slit 2 and the vane respectively.

In order to solve for the matrix \( S \), auxiliary Faraday modes propagating in the two internal regions, shown in Fig 1 were defined. Then, the amplitudes for such modes were related among themselves by imposing the continuity of the amplitudes and the components of the forces along the directions of the slits 1 and 2 and the vane’s direction. Further, after defining the scattering modes associated to the input, let say the one with number \( i(i = 1,2,3) \), as such wave modes for which the incident amplitude \( a_i = 1 \) and all the other incident amplitudes \( a_j = 0 \), \( j \neq i \) at the rest of the terminals vanish, the set of linear equations defined by the boundary conditions were solved. More precisely, as one of the propagating modes shows a gap for low frequencies, this wave turns to be spatially damped boundary mode when the frequency is low enough to be in the gap. The effects of these low frequency oscillations were disregarded in this initial discussion.

The above described equations for finding the scattering modes were solved initially for the case in which the vane is disconnected from the central string. This situation corresponds to an optical or microwave isolator in which the "resistive element" absorbing the power in the output 2 is absent. The system is also completely analog to the one related with the so called Wien paradox. The transmission coefficient for the resulting two terminal device \( T = \frac{b_2}{a_1} \)\( a_2 = 0 \), for the wave incident upon the slit one was then evaluated as a function of the wave vector and the angle \( \theta \) between the two slits. The results are shown in the two dimensional plot in Fig. 2. It, should be...
reminded that the transmission coefficient for the wave coming in through the slit 2 can be extracted from the figure by changing the sign of the angle $\theta$. The data correspond to a length $L$ for which the angle $\theta = \pi/4$ produces a value one for the transmission coefficient from left to right at high frequencies. The picture remarkably shows that in the absence of the vane, in spite of the non-reciprocity of the system, the transmission coefficient associated to the two inputs are exactly equal whatever the value of the angle and the frequency. In other words, the fraction of the incident power transmitted from one side to another is exactly the same whatever side is taken as the input. The equality to one of the transmission for the wave associated to the two inputs are exactly equal whatever the value of the angle and the frequency. In other words, the equality to one of the transmission coefficients are illustrated at Fig. 3 as functions of the energy in the range $k_o/M \in (3, 10)$ where the linearity condition of the dispersion relations is obeyed very approximately.

Figure 3 clearly indicates a breaking of the detailed Balance in the three terminal system simulating the optical and microwave isolators. This conclusion follows from the fact that $t_{12}(k_o) \neq t_{21}(k_o) = 1$. The possibility for the existence of physical systems not satisfying this property was elderly advanced by the same Boltzmann long time ago.

However, let us now argue that under the assumption of the thermal contact of the three terminals with heat reservoirs characterized by a common value of the temperature, the net heat flux through any of the inputs is equal to zero. For the slit number one this outcome is a direct consequence of the results in Fig. 3 and the identical formula for the link between the flux of energy and the amplitudes of the waves in each of the outputs. This last property is determined by the assumed common physical parameters (density $\rho_m$ and Hook law constant $k_h$ ) of the strings associated to any of the terminals. The relevant property of Fig. 2 determining the vanishing of the heat flux at the left of the slit 1 is the relation $t_{12}(k_o) + t_{13}(k_o) = 1$, which is clearly satisfied by the data in the picture. This is also the situation for any of the other terminals as a general consequence of the conservative character of the three terminal device (See Ref[1]), which implies that the $S$ matrix is unitary. To see this explicitly, it is useful to consider that the columns of $S$ defined by a fixed value of $j$, give the components $b_i^{(j)}$, $i = 1, 2, 3,$ of the scattering modes normalized by the condition $a_j = 1$ and $a_l = 0$ for $l \neq j$. Therefore, let us consider the power $P^{(i)}_e$ of the waves entering from output $i$ under the assumption that equal powers are entering through the three terminals due to the common temperature conditions. Then it follows $P^{(i)}_e = p_o \sum_{j=1}^{3} b_i^{(j)}(b_i^{(j)})^* = p_o \sum_{j=1}^{3} S_{ij} S_{ij}^* = p_o a_i a_i^*$, where $p_o$ is the constant relating the square of the amplitudes (defined by some convention) with the associated power flow in a travelling wave. The above relation implies that the power of the outgoing waves at any of the terminals is identical to the power of the thermal incident waves entering through the same input, whenever all the heat reservoirs are at a common temperature $T$.

Finally, we will consider that the vane terminal is laying at a very much small temperature $T_v << T$ where $T$ is the temperature of the left and right heat reservoirs in contact with the pieces of the central string arriving at the left and right slits 1 and 2. Also the angle $\theta$ will be selected to furnish a transmission coefficient equal to one for the waves incident through the slit 1. Under this conditions it follows that at least for a very extended region of the energies for which the relation $M \ll k_o$ is valid, the net flux passing from left to right of the isolator is greater than the one circulating in the opposite
FIG. 3: Transmission coefficients determining the outgoing heat flow in the input at the left of the slit 1.

sense. Since this property is independent of the temperature, we will assume that it can be elevated sufficiently enough for to be able for disregarding the low energy region, when deciding about the question of the net heat balance. Under this central assumption, it can be concluded from the data in Fig. 2 that the system will produce a heat flow passing from the Black -Body at the left of the slit 1 to the black body at the right of slit 2. This conclusions motivate to consider in further works the possibility for a reduction of the statistical entropy through concrete mechanisms for the extraction of energy through the vane. In ending, I would like to acknowledge the discussions with Dr. A. Gonzalez, C. Trallero and F. Comas and the general support of Dr. D. Sheehan, the Org. Committee of the Conference and the Associate Scheme of the ASICTP.

* Electronic address: cabo@cidet.icmf.inf.cu

1 M. W. Zemansky, *Heat and Thermodynamics*, McGraw-Hill, New York, 1968.
2 D. Ruelle, in *Statistical Physics, Invited Papers form STATPHYS 20*, Eds. A. Gervois, D. Iagolnitzer, M. Moreau and Y. Pomeau, Elsevier-North Holland, 1999.
3 A. Cabo, *Int. Jour. Mod .Phys.* B15, (2001) 2993.
4 B. Lax, K. J. Button and L. M. Roth, *J. Appl. Phys.* 25 (1954)1413.
5 B. Lax and B. Button, *Microwave Ferrites and Ferrimagnetics*, MacGraw-Hill, New York,1962.
6 K. Chang (Ed.), *Handbook of Microwave and Optical Components*, Vol.1, *Microwave Passive and Antennae Components*, John Wile & Sons, New York.
7 T. Koryu Ishii, *Handbook of Microwave Technology, Vol. 1, Components and Devices*, Academic Press, New York.
8 P. E. Green Jr., *Fiber Optics Networks*, Prentice Hall, New Jersey.
9 I.P. Bazarov, *Thermodynamics, (in Russian)*, Fizmatgiz, Moscow,1961.
10 C. G. Montgomery, R.H. Dicke and E. M. Purcell, *Principles of Microwave Circuits*, McGraw-Hill, New York,1948.