Spindown of magnetars: quantum vacuum friction?

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Received 2015 April 8; accepted 2015 June 20

Abstract Magnetars are proposed to be peculiar neutron stars which could power their X-ray radiation by super-strong magnetic fields as high as ≥ 10^{14} G. However, no direct evidence for such strong fields has been obtained till now, and the recent discovery of low magnetic field magnetars even indicates that some more efficient radiation mechanism than magnetic dipole radiation should be included. In this paper, quantum vacuum friction (QVF) is suggested to be a direct consequence of super-strong surface fields, therefore the magnetar model could then be tested further through QVF braking. The high surface magnetic field of a pulsar interacting with the quantum vacuum results in a significantly high spindown rate (티). It is found that a QVF dominates the energy loss of pulsars when the pulsar’s rotation period and its first derivative satisfy the relationship \( P^3 \dot{P} > 0.63 \times 10^{-16} \xi^{-4} \) s^{-2}, where \( \xi \) is the ratio of the surface magnetic field over the dipole magnetic field. In the “QVF + magnetodipole” joint braking scenario, the spindown behavior of magnetars should be quite different from that in the pure magnetodipole model. We are expecting these results could be tested by magnetar candidates, especially low magnetic field cases, in the future.

Key words: pulsars: general — radiation: dynamics — stars: magnetars — stars: neutron

1 INTRODUCTION

Kinematic rotation was generally thought to be the only energy source for pulsar emission soon after the discovery of radio pulsars until the discovery of accretion-powered pulsars in X-ray binaries. However, anomalous X-ray pulsars/soft gamma-ray repeaters (AXPs/SGRs, magnetar candidates) have long spin periods (thus low spindown power) and no binary companions, which rules out spin and accretion in a binary system as the power sources. The first SGR-giant flare was even observed in 1979 (Mazets et al. 1979), and Paczynski (1992) then pointed out that the super-strong magnetic field may explain the super-Eddington luminosity. AXPs and SGRs were thereafter supposed to be magnetars, peculiar neutron stars with surface/multipole magnetic fields (10^{14} G \sim 10^{15} G) as the energy source, while the initially proposed strong dipole fields could be unnecessary (e.g., Tong et al. 2013). Moreover, the discovery of low magnetic field magnetars (Zhou et al. 2014; Rea et al. 2010, 2012; Scholz et al. 2012) in recent years indicates that some more efficient radiation mechanism than magnetic dipole radiation should be included. Besides failed predictions and challenges in the magnetar model (Xu 2007; Tong & Xu 2011), one of the key points is: can one obtain direct evidence of the strong surface fields?

Here we suggest that quantum vacuum friction (QVF) is a direct consequence of the surface fields, and calculate the spindown of magnetar candidates with the inclusion of the QVF effect.

Magnetodipole radiation could dominate the kinematic energy loss of isolated pulsars (e.g., Manchester & Taylor 1977; Dai & Lu 1998; Lyubarsky & Kirk 2001; Morozova et al. 2008). The derived braking index \( n = \Omega \dot{\Omega} / \dot{\Omega}^2 \) (\( \Omega \) is the angular velocity of rotation) of a pulsar is expected to be 3 for pure magnetodipole radiation. As a result of observational difficulties, only braking indices \( n \) of a few rotation-powered pulsars have been obtained with some certainty (Yue et al. 2007; Livingstone et al. 2007, http://www.atnf.csiro.au/research/pulsar/psrcat). They are PSR J1846–0258 (\( n = 2.65 \pm 0.01 \)), PSR B1509–58 (\( n = 2.839 \pm 0.001 \)), PSR J1119–6127 (\( n = 2.91 \pm 0.05 \)), PSR B0531+21 (the Crab pulsar, \( n = 2.51 \pm 0.01 \)), PSR B0540–69 (\( n = 2.140 \pm 0.009 \)) and PSR B0833–45 (the Vela pulsar, \( n = 1.4 \pm 0.2 \)). These observed breaking indices are all remarkably smaller than the value of \( n = 3 \), which may suggest that other spindown torques do work besides the energy loss via dipole radiation (Xu & Qiao 2001; Beskin et al. 1984; Ahmedov et al. 2012; Menou et al. 2001; Contopoulos & Spitkovsky 2006; Alpar et al. 2001; Chen & Li 2006; Ruderman 2005; Allen & Horvath 1997; Lin & Zhang 2004; Tong & Xu 2014; Tong 2015).

Recently, the research of Davies et al. showed that the QVF effect could be a basic electromagnetic phenomenon (Davies 2005; Lambrecht et al. 1996; Pendry 1997; Feigel 2004; Van Tiggelen et al. 2006; Manjavacas & Garcia de Abajo 2010). If QVF exists, the dissipative energy by QVF would certainly be from rotational kinetic energy of the pulsar. The loss of rotational kinetic energy of the pulsar by QVF may also be transformed into the pulsar’s ther-
mal energy or the energy of the pulsar’s radiating photons which might not be isotropic. This is the same argument as in the work of Manjavacas & García de Abajo (2010), in which the authors argue that at zero temperature, the friction produced on rotating neutral particles by interaction with the vacuum electromagnetic fields transforms mechanical energy into light emission and produces particle heating. A pulsar may transfer its angular momentum to the vacuum when pulsars rub against a quantum vacuum since the angular momentum is conserved. In this case, the vacuum may work as a standard medium (Dupays et al. 2008) of the quantum vacuum, like in the work of Dupays et al. (2008, 2012) even calculated the energy loss due to the pulsars’ interaction with the quantum vacuum by taking account the effect of quantum electrodynamics (QED) in a high magnetic field. The calculations indicate that when the pulsars’ magnetic field is high, QVF would also play an important role in causing a loss of rotational energy in pulsars. Thus, it is necessary to take QVF into account when considering the rotational energy loss of pulsars, especially on the surface of highly magnetized pulsars, like magnetars.

In this paper we assume that a pulsar interacts with a quantum vacuum like in the work of Dupays et al. (2008) and consider the difference between the surface/toroidal magnetic field and dipole/poloidal magnetic field. The braking indices for pure QVF radiation and the surface magnetic field and dipole/poloidal magnetic field. The calculations results and analysis are presented in Section 3. The calculated results and analysis are presented in Section 3. Finally, conclusions and discussions are presented in Section 4.

2 SPINDOWN AND BRAKING INDEX OF MAGNETARS

A pulsar radiates power as a magnetic dipole of

$$
\dot{E}_{\text{dip}} = -\frac{2}{3}c^{-3}\mu^2\Omega^4,
$$

(1)

where

$$
\mu = \frac{1}{2}B_{\text{dip}}R^3\sin\theta
$$

(2)

is magnetic dipole moment and $c$ is the speed of light in a vacuum, $B_{\text{dip}}$ is the dipole magnetic field, $R$ is the radius of the pulsar, and $\theta$ is the inclination angle. For general pulsars, the surface magnetic field is approximately equal to that of a dipole magnetic field because the multipole magnetic field is greatly attenuated. However, for magnetars there is a non-negligible multipole magnetic field due to its extraordinarily strong surface magnetic field. So, a magnetar’s surface magnetic field $B_{\text{surf}}$ is described by a dipole magnetic field $B_{\text{dip}}$ and a multipole magnetic field. We suppose that the ratio of the surface magnetic field to the dipole magnetic field

$$
\xi = \frac{B_{\text{surf}}}{B_{\text{dip}}}
$$

(3)

is a constant. The pulsar rubs against the quantum vacuum and then loses its rotational kinetic energy (Dupays et al. 2008) of

$$
\dot{E}_{\text{QVF}} \simeq -c\frac{3\alpha}{16}\frac{R^4\sin^2\theta}{cB_{\text{surf}}^2}\frac{B_{\text{surf}}^4}{P^2},
$$

(4)

where $\alpha = e^2/\hbar c \simeq 1/137$ is the coupling constant of electromagnetic interaction, $B_c = 4.4 \times 10^{13}G$ is the QED critical field and $P = 2\pi/\Omega$ is the spin period.

The typical radius of a pulsar, $R = 10^6$ cm, is adopted. We also set the inclination angle $\theta = 90^\circ$ for the sake of simplicity. Considering Equation (2) between the magnetic moment of pulsars and magnetic field in the polar region of pulsars, we can obtain the ratio of the energy loss due to QVF over that due to magnetodipole radiation

$$
\frac{\dot{E}_{\text{QVF}}}{\dot{E}_{\text{dip}}} = 7.69 \times 10^{-24}\beta_{\text{dip}}^2P^2\xi^4.
$$

(5)

Assuming the rotational energy loss of a pulsar comes from both magnetodipole radiation and QVF, i.e. $\dot{E} = \dot{E}_{\text{dip}} + \dot{E}_{\text{QVF}}$, the total energy loss of a pulsar is given by

$$
\dot{E} \simeq -\frac{2\mu^2\Omega^4}{3c^3} - c\frac{3\alpha}{16}\frac{R^4\sin^2\theta}{cB_{\text{surf}}^2}\frac{B_{\text{surf}}^4}{P^2}\xi^4.
$$

(6)

From the pulsar’s rotational energy loss $\dot{E} = I\ddot{\Omega}/I$, where $I$ is the moment of inertia with typical value $I = 10^{45}$ g cm², we can obtain a relationship between a pulsar’s period and its the period derivative with respect to time

$$
\dot{P} = \frac{2\pi^2R^6\sin^2\theta}{3c^6I} - \frac{3\alpha R^4\sin^2\theta}{64\pi B_c^2Ic} B_{\text{dip}}^4P^2\xi^4.
$$

(7)
Using the relation between $\Omega$ and $P$, the braking index can be obtained

$$n = \frac{1}{\dot{P}} \left( \frac{2\pi^2 R^6 \sin^2 \theta B_{\text{dip}}^2}{Ic^3} \frac{\dot{P}}{P} + \frac{3\alpha}{64\pi} \frac{R^4 \sin^2 \theta B_{\text{dip}}^2}{B_c^2 Ic} \dot{B}_{\text{dip}} P \xi^4 \right). \quad (8)$$

Numerically, the braking index can be written as

$$n = \frac{7.31 + f(B_{\text{dip}}, P, \xi)}{2.44 + f(B_{\text{dip}}, P, \xi)}, \quad (9)$$

where

$$f(B_{\text{dip}}, P, \xi) = 18.75 B_{\text{dip},12}^2 P^2 \xi^4 \quad (10)$$

with $B_{\text{dip},12} = 10^{-12} B_{\text{dip}}$. We can also express the ratio of the energy loss due to QVF over that due to magnetodipole radiation in terms of a pulsar’s period ($P$) and period derivative ($\dot{P}$) from Equations (5) and (7)

$$\frac{\dot{E}_{\text{QVF}}}{\dot{E}_{\text{dip}}} = 7.69 \times 10^{-24} \left( \frac{-8768}{9c^2} \pi^3 R^2 B_c^2 \sqrt{\left(\frac{8768}{9c^2} \pi^3 R^2 B_c^2\right)^2 + \frac{8768}{3R^8} \pi Ic B_0^2 \xi^4 P^3 \dot{P}} \right). \quad (11)$$

Numerically, the above equation can be written as

$$\frac{\dot{E}_{\text{QVF}}}{\dot{E}_{\text{dip}}} = -\frac{1}{2} + \sqrt{\frac{1}{4} + 3.16 \times 10^{16} \xi^4 P^3 \dot{P}}. \quad (12)$$

3 THE NUMERICAL RESULTS

The periods of observed pulsars are mainly distributed in the range from 0.1 s to 5 s (The ATNF Pulsar Catalogue: http://www.atnf.csiro.au/research/pulsar/psrcat/). Using Equation (5), we plot the ratio of $\dot{E}_{\text{QVF}}/\dot{E}_{\text{dip}}$ as a function of the period $P$ in Figure 1 for $\xi = 10$ and in Figure 2 for $\xi = 100$. From Figure 1 we can see that the QVF may play an important role when the dipole magnetic field is higher than $\sim 10^{10}\text{ G}$ for pulsars whose periods are between 0.1 s and 1 s. The magnetic field of most observed pulsars, derived from pure magnetodipole radiation, is in the range $10^{11} - 10^{13}\text{ G}$, however, if QVF is included in the associated energy loss, the derived magnetic field could be lower. Thus it is necessary to independently measure the magnetic field of pulsars so that we can judge whether QVF has an important contribution to the rotational energy loss of pulsars.

From Figure 2 we can see that QVF may play an important role in pulsar braking when its dipole magnetic field $B_{\text{dip}} > 10^{10}\text{ G}$. For millisecond pulsars, the derived magnetic field from magnetodipole radiation is already so low ($B_{\text{dip}} < 10^{10}\text{ G}$) that we can ignore the QVF’s contribution to its rotational kinetic energy loss, but for magnetars the derived magnetic field from QVF is already so high ($B_{\text{dip}} > 10^{12}\text{ G}$) that we have to consider the QVF’s contribution. We can also express the ratio of the energy loss due to QVF over that due to magnetodipole radiation by a pulsar’s period ($P$) and period derivative ($\dot{P}$) as shown in Equation (12). From this equation we can obtain that QVF dominates the energy loss of a pulsar when a pulsar’s rotation period and its first derivative satisfy the relationship $P^3 \dot{P} > 0.63 \times 10^{-16} \xi^{-4} \text{ s}^2$, where $\xi$ is the ratio of the surface magnetic field over the dipole magnetic field. According to the above relationship and current observed data for confirmed magnetars (see Table 1), QVF will dominate the rotational energy loss in the spindown of all magnetars.

Substituting the observed value of $\dot{P}$ and $P$ into Equation (7), the magnetic field of pulsars can be calculated. We compute the currently confirmed magnetic field of magnetars and list the results in the last column $B_{\text{dip}}^\text{inf}$ of Table 1. The fourth column $B_{\text{surf}}$ is derived from pure magnetodipole radiation. The calculated results demonstrate that the derived dipole magnetic field $B_{\text{dip}}$ from pure magnetodipole radiation is about $10^6 (\xi = 10)$ and $10^{4}(\xi = 100)$ times larger than $B_{\text{dip}}^\text{inf}$ obtained by combining QVF and magnetodipole radiation. Also, the derived surface magnetic field $B_{\text{surf}}$ from pure magnetodipole radiation is about 100 times larger than $B_{\text{dip}}^\text{inf}$ inferred by combining QVF and magnetodipole radiation for both $\xi = 10$ and $\xi = 100$.

If $\dot{E} = \dot{E}_{\text{QVF}}$, from Equation (4) we can obtain $\dot{\Omega} \propto \Omega^3$, and therefore the braking index $n = 1$ for a pulsar’s spindown by pure QVF. Equation (9) shows that a pulsar’s braking index is between 1 $\sim$ 3 in the “QVF + magnetodipole” joint braking scenario. Magnetars have a strong surface magnetic field, longer rotation period and bigger $\xi$, so magnetars have a larger value of the function $f(B_{\text{dip}}, P, \xi)$ (see Eq. (10)), which results in QVF dominating magnetar braking and its braking indices being about 3. However, for some low magnetic field millisecond pulsars, a small value of $f(B_{\text{dip}}, P, \xi)$ leads to magnetodipole radiation becoming the main means of energy loss in its spindown and its braking index is about 3. Considering a pulsar’s spindown by both QVF and mag-
netodipole radiation, we use Equation (9) to calculate the braking indices of magnetars. The results show that all
the braking indices of all magnetars are around 1 for both
$\xi = 10$ and $\xi = 100$. In the future, the model could
be tested by comparing the calculated results to observed
braking indices. This comparison can also provide further
information to understand QVF.

4 CONCLUSIONS AND DISCUSSIONS

We investigate the rotational energy loss of a pulsar from
QVF and compare it with that from magnetodipole radi-
ation in different ranges of magnetic field and periods.
We find that if the ratio $\xi$ of the surface magnetic field
over dipole magnetic field is fixed to 10(100), QVF could
play a critical role in braking for pulsars when $B_{\text{surf}}P > 10^{11}(10^{10})$ G s, but it can be ignored when $B_{\text{surf}} P < 10^{10}(10^9)$ G s. Magnetars may have a high surface mag-
etic field and long period ($B_{\text{surf}}P \gg 10^{12}$ G s) if the
value of magnetic field is inferred by pure classical mag-
etodipole radiation. Therefore, it is necessary to consider
rotational energy loss of magnetars by both magnetodipole
radiation and QVF.

We consider the difference between the surface mag-
etic field and dipole magnetic field of pulsars and com-
pare the energy loss rate of pulsars due to magnetodipole
radiation to that due to QVF. The results show that when
a pulsar has a strong magnetic field or a long period
($B_{\text{surf}}P > 10^{11}$ G s for $\xi = 10$, $B_{\text{surf}}P > 10^{10}$ G s
for $\xi = 100$), compared to QVF, the energy loss by mag-
etodipole radiation can be ignored, but when pulsars have
a weak magnetic field or short period ($B_{\text{surf}}P < 10^{10}$ G s
for $\xi = 10$, $B_{\text{surf}}P < 10^9$ G s for $\xi = 10$) the QVF can
be negligible. We consider that rotational energy loss of
magnetars is the sum of the energy loss due to QVF and

Fig. 1 The ratio of a pulsar’s energy loss rate from QVF over that from magnetodipole radiation, as a function of period, where $\xi = 10$.

Fig. 2 The ratio of a pulsar’s energy loss rate from QVF over that from magnetodipole radiation, as a function of period, where $\xi = 100$. 
Table 1: The parameters and the inferred magnetic field of magnetars. The data on the period of a magnetar ($P$), the period derivative ($\dot{P}$), and dipole magnetic field ($B_{\text{dip}}$) are from the McGill SGR/AXP Online Catalog [http://www.physics.mcgill.ca/pulsar/magnetar/main.html]. The last column of the table, $B_{\text{dip}}^{\text{inf}}$, is the inferred magnetic field from our model based on both magnetodipole radiation and QVF.

| Name             | $P$ (s) | $\dot{P}$ ($10^{-11}$ s$^{-1}$) | $B_{\text{dip}}$ (10$^{13}$ G) | $B_{\text{dip}}^{\text{inf}}$ (10$^{13}$ G, $\xi = 10$) | $B_{\text{dip}}^{\text{inf}}$ (10$^{10}$ G, $\xi = 100$) |
|------------------|---------|---------------------------------|-------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| CXOU J010043.1−721134 | 8.020392(9) | 1.88(8) | 3.9 | 5.946 | 5.946 |
| 4U 0142+61       | 8.68832877(2) | 0.203323(7) | 1.3 | 3.342 | 3.342 |
| SGR 0418+5729    | 9.07838827 | $< 0.0006$ | $< 0.075$ | $< 0.770$ | $< 0.770$ |
| SGR 0501+4516    | 5.76209653 | 0.582(3) | 1.9 | 4.817 | 4.817 |
| SGR 0526−66      | 8.0544(2) | 3.8(1) | 5.6 | 7.082 | 7.082 |
| 1E 1048.1−5937   | 6.4578754(25) | 2.25 | 3.9 | 6.565 | 6.565 |
| 1E 1547.0−5408   | 2.06983302(4) | 2.318(5) | 2.2 | 8.791 | 8.791 |
| PSR J1622−4950   | 4.3261(1) | 1.7(1) | 2.7 | 6.766 | 6.766 |
| SGR 1627−41      | 2.594578(6) | 1.9(4) | 2.2 | 7.905 | 7.905 |
| CXO J164710.2−455216 | 10.6106563(1) | 0.083(2) | 0.95 | 2.541 | 2.541 |
| 1RXS J170849.0−400910 | 11.03027(1) | 1.91(4) | 4.6 | 5.516 | 5.516 |
| CXOU J171405.7−381031 | 3.825352(4) | 6.40(5) | 5.0 | 9.718 | 9.718 |
| PSR J1745−2900   | 3.76363824(13) | 1.385(15) | 2.3 | 6.655 | 6.655 |
| SGR 1806−20      | 7.6022(7) | 75(4) | 24 | 15.145 | 15.145 |
| XTE J1810−197    | 5.5403537(2) | 0.777(3) | 2.1 | 5.229 | 5.229 |
| Swift J1823.2−1606 | 8.43772106(6) | 0.00214(21) | 0.14 | 1.078 | 1.078 |
| SGR 1833−0832    | 7.56504918(8) | 0.439(43) | 1.8 | 4.194 | 4.194 |
| Swift J1834.9−0846 | 2.4823018(1) | 0.796(12) | 1.4 | 6.430 | 6.430 |
| 1E 1841−045      | 11.7828977(10) | 3.93(1) | 6.9 | 6.494 | 6.494 |
| 3XMM J185246.6+003317 | 11.55871346(6) | $< 0.014$ | $< 0.41$ | $< 1.594$ | $< 1.594$ |
| SGR 1900+14      | 5.19987(7) | 9.2(4) | 7.0 | 9.856 | 9.856 |
| 1E 2259+586      | 6.9789484460(39) | 0.048430(8) | 0.59 | 2.466 | 2.466 |
| PSR J1846−0258   | 0.32657128834(4) | 0.7107450(2) | 0.49 | 10.379 | 10.379 |

that due to magnetodipole radiation. Based on this joint mechanism of energy loss, the surface magnetic field of magnetars and braking indices are calculated. Our work indicates that when QVF is included in the process of rotational energy loss, the surface magnetic field of magnetars is $10^{-1}$ times lower than that in a pure magnetodipole radiation model. In this joint braking model, QVF dominates the energy loss of a pulsar when its rotation period and its first derivative satisfy the relationship $P^3 \dot{P} > 0.63 \times 10^{-16} \xi^{-4} \text{s}^2$, where $\xi$ is the ratio of the surface magnetic field over the dipole magnetic field. Also, we find that the braking index of magnetars is around 1 in the joint braking model. The efficiency of rotational energy losses generated by QVF in magnetars is very high compared to magnetodipole radiation. A smaller magnetic field can generate a greater rotation energy loss by QVF compared to magnetodipole radiation. This may explain why some magnetars have high X-ray luminosity and a lower magnetic field (Zhou et al. 2014; Rea et al. 2010, 2012; Scholz et al. 2012).

We expect that the results presented could be tested by X-ray observations of magnetar candidates, especially for cases with lower magnetic field. X-ray data accumulated in advanced facilities in space could show both timing and luminosity features for magnetars, and research using resulting data would be interesting. In summary, further observations for magnetars in the future would test our joint braking model as well as help us understand the effects of QVF in more detail.

Acknowledgements The authors thank Yue You-ling, Feng Shu-hua, Liu Xiong-wei and Yu Meng for helpful discussions. This work is supported by the National Natural Science Foundation of China (11225314), XTP XDA04060604, and SinoProbe-09-03 (201311194-03).

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