Backbending in Dy isotopes within the Projected Shell Model

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Abstract

A systematic study of the yrast band in $^{154-164}$Dy isotopes using the Projected Shell Model is presented. It is shown that, in the context of the present model, enlarging the mean field deformation by about 20\% allows a very good description of the spectrum of yrast band in these isotopes. The dependence of the $B(E2)$ values on angular momentum is also better described when larger deformations are used. The observed oscillation of g-factors at low spin states remains an open question for this model.

PACS numbers: 21.60.Cs, 21.10.Re, 21.10.Ky, 27.70.+q
Keywords: Dy isotopes, projected shell model, yrast band, energies, $B(E2)$ values, quadrupole moments, g-factors.
I. INTRODUCTION

Backbending in the moment of inertia \(I_{2}\), which is a common phenomenon for many heavy and deformed nuclei, is understood as a consequence of crossing between two rotational bands, one being the ground band and the other having a pair of aligned high-\(j\) intruder particles \(3\). Its magnitude is related to the crossing angle between the crossing bands \(4\). A large crossing angle implies that the bands interact over a narrow angular momentum region, and a sharp backbending usually takes place. A small crossing angle implies an interaction which spreads its influence along a wide angular momentum region, therefore producing a smooth backbending; in some cases, only an upbending instead of a backbending is seen.

The above physical picture of the band crossing can be clearly seen in the Projected Shell Model (PSM) \(5\), which has been applied successfully to the description of the energy spectra and electromagnetic transitions in deformed nuclei.

High spin properties along the yrast line in the Dy isotope chain were studied by various cranked mean-field theories, with and without particle number projection \(6\). It was concluded in Ref. \(6\) that, while the average behavior of the experimental data can be reproduced by cranking models, a microscopic interpretation of some observations, in particular those related to band crossings, requires investigations going beyond the mean field approximation. More recent theoretical work \(7\) used the PSM to go beyond the mean field, and the results were compared with the spectrum, the \(B(E2)\) and the gyromagnetic factor (g-factor) data existing at that time. In Ref. \(7\), an improvement in overall description of the Dy isotopes was found but, in many cases, the PSM seemed to exaggerate the backbending.

The main goal of the present article is to demonstrate that, by increasing the mean field deformation by 20\% on average, both the backbending and the angular momentum dependence of the quadrupole moments in the Dy chain can be quantitatively described within the PSM. It will be shown that the increase in deformation is reflected in a rearrangement of single particle states around the Fermi level that smooths backbending curves, in agreement with the experimental observation. Nevertheless, for lighter isotopes, the oscillation in the recent g-factor data around angular momentum \(I = 6\hbar\) and the reduction in \(B(E2)\) values at high angular momenta cannot be reproduced by the PSM within the current model space.

The paper is arranged as follows: In section II, we outline the PSM. Interested readers are kindly referred to the review article \(8\) and the corresponding computer codes \(8\) in the literature. In section III, we introduce the mean field where our shell model space is truncated and discuss how it is related to our final results. Discussions on the spectra in terms of the backbending plots, the \(B(E2)\)'s and the g-factors for the \(^{154-164}\text{Dy}\) isotopes are presented in sections IV, V and VI, respectively, where a large body of the experimental high spin data is compared systematically with the theory. Finally, a summary is given in section VII.

II. THE MODEL

The PSM allows a many body quantum mechanical description of atomic nuclei, while avoiding the use of an extremely large Hilbert spaces commonly required in spherical shell model calculations \(9\). Assuming an axial symmetry in the single particle potential the PSM uses a multi-quasiparticle (qp) basis \(|\phi_k\rangle\) built on the Nilsson + BCS mean field. Its
elegance and efficiency lies in the use of angular momentum projection through the angular momentum projection operator $\hat{P}_{KK'}^I$ to carry the multi-qp states from the intrinsic to the laboratory system $\hat{\mathcal{P}}_{IKK'\kappa}$.

The present type of PSM Hamiltonian is schematic, with three classes of residual interactions: quadrupole-quadrupole, monopole pairing and quadrupole pairing. This is basically the quadrupole plus pairing Hamiltonian and has been widely used in nuclear structure studies [9]. The Hamiltonian can be expressed as

$$\hat{H} = \hat{H}_0 - \frac{\chi}{2} \sum_\mu \hat{Q}_\mu^+ \hat{Q}_\mu - G_M \hat{P}^+ \hat{P} - G_Q \hat{P}_\mu^+ \hat{P}_\mu,$$

where $\hat{H}_0$ is the spherical single particle Hamiltonian. The Hamiltonian (1) is diagonalized in the angular momentum projected multi-qp basis $\{ \hat{P}_{MK}\kappa|\phi_\kappa\}$. The eigenvalue equation is:

$$\sum_{\kappa'K'} (H^I_{\kappaKK'} - EN^I_{\kappaKK'} \hat{P}_{MK'}^I) F^I_{\kappaK'} = 0,$$

with the normalization condition

$$\sum_{\kappa KK'} F^I_{\kappa K'} N^I_{\kappa K'K''} F^I_{\kappa' K''} = \delta_{EE'},$$

where

$$H^I_{\kappa KK'} = \langle \phi_\kappa | \hat{H} \hat{P}_{KK'}^I | \phi_{\kappa'} \rangle,$$

$$N^I_{\kappa K'K''} = \langle \phi_\kappa | \hat{P}_{KK'}^I | \phi_{\kappa'} \rangle.$$

After solving equation (2), we obtain the normalized eigenstates with energy $E$:

$$|\Phi^E_{IM}\rangle = \sum_{\kappa K} F^I_{\kappa K'} \hat{P}_{MK'}^I |\phi_\kappa\rangle.$$ 

Electromagnetic properties of the nuclei are then computed by using the wave functions obtained above.

III. THE MEAN FIELD AND THE RESIDUAL INTERACTION STRENGTHS

In the PSM, the multi-qp basis $|\phi_\kappa\rangle$ is built on qp excitations from the Nilsson + BCS vacuum. The shell model basis (the angular momentum projected multi-qp states) is then truncated according to energy excitation from the qp vacuum. Because for most quantities of interest in spectroscopy only states relatively near the Fermi surface are important, the usual dimension where diagonalization is carried out is about 100 in realistic calculations. Since this is a truncated theory, the quality of a calculation is related to the quality of the mean field, particularly the qp states around the Fermi level if the interest in question is the yrast line. Given the mean field is often used in describing heavy and deformed nuclei considerable knowledge exists concerning the qp structure near the Fermi surface [9].

In the PSM, the quadrupole-quadrupole strength in Eq. (1) is obtained self-consistently from the mean field quadrupole deformation $\epsilon_2$, while the pairing and quadrupole pairing
strengths in Eq. (1) are taken from systematics [5]. In a previous study of \(^{160}\)Dy, it has been shown that low energy rotational bands can only be reproduced if these self-consistent values are used [10].

The monopole and quadrupole-pairing interaction strengths \(G_M\) and \(G_Q\) used in this work are

\[
G_M = (20.12 \mp 13.13 \frac{N-Z}{A})A^{-1}
\]

\[
G_Q = 0.18 G_M,
\]

where the minus(plus) sign is applied to neutrons (protons). These are the same values used in Refs. [4,7].

The deformation \(\epsilon_2\) can be taken from tables [11] which are extracted from experimental \(B(E2, 2_1^+ \rightarrow 0^+_g)\) values by using a geometrical model. These values are very similar to those calculated from mean field models [12,13]. This is expected because in mean field models the relation between the deformation and the electric quadrupole moment is constructed so that the experimental \(B(E2)\) should be reproduced. However, it should be kept in mind that deformation is a model-dependent concept, while the \(B(E2)\) is directly related to measured quantities.

We found that the use of an enlarged deformation \(\epsilon_2'\) strongly improves the description of the spectra and \(B(E2)\) values within the PSM for the Dy chain. The same effect is operative in other rare earth isotope chains [14]. A detailed analysis of the changes in the mean field energies and quasiparticle occupations introduced by this modified deformation, as well as its effects in the backbending plot, are presented in the following sections.

Table 1 shows the deformations used in this article for the different Dy isotopes, which are listed in the first column. In the second and third columns, we list the deformations reported in the literature [11] and the enlarged ones used in our calculation. The latter ones were obtained by looking for the best available description of the energy spectra and back-bending plots for each isotope. Once the deformation was fixed, the quadrupole-quadrupole interaction strength was obtained selfconsistently [5,10]. An overall deformation increase of around 20\% was found necessary to best reproduce the experimental data od Dy isotopes.

IV. THE BACKBENDING PLOTS

The angular frequency \(\omega\), defined as \(\hbar \omega(I) = \frac{1}{2}[E(I+2) - E(I)]\), is plotted as a function of the spin \(I\) for the yrast states in Fig. 1. Results for \(^{154,156,158,160,162,164}\)Dy are presented in figures (a,b,c,d,e,f) respectively. In each plot the full line corresponds to standard deformation \(\epsilon_2\) and the dotted line to the enlarged deformation \(\epsilon_2'\). Diamonds represent the experimental data, taken from Ref. [15,16].

It can be seen that in each case a larger deformation is associated with a smoother curve, thus better reproducing the experimental data. For the first four Dy isotopes, the PSM calculation with standard deformation predicts a second backbending [7], which is visualized as a second minimum seen in Fig. 1 and has no experimental counterpart. This prediction disappears in the calculation with enlarged deformation. Only in \(^{154}\)Dy does the experimental yrast band exhibit a very sharp second backbending at \(I = 30\hbar\), which is not predicted by either the standard or the enlarged deformation calculations.
The same conclusion is emphasized when presented in a plot with twice the moment of inertia $2\Theta$ vs. the square of the angular frequency (the backbending plots), as it is shown in Fig. 2. In these plots the backbending features are emphasized and the improvement obtained in the description of the experimental data using the enlarged deformation is clear. While for the heavier Dy isotopes the moment of inertia changes very slowly as a function of the angular frequency, a strong stretching effect is apparent for the lighter ones. This implies that, in the case of $^{154,156}\text{Dy}$ at small rotational frequencies, the moment of inertia has drastic changes which would require for its proper description the inclusion of a richer mean field basis, rather than a geometrically fixed deformation with axial symmetry. Due to the absence of this ingredient in the present calculation, the moment of inertia at low angular momenta in these two isotopes is not well described.

In order to obtain a deeper understanding of the results seen in the backbending plots with different deformations, in Fig. 3 we present the unperturbed band diagram of $^{158}\text{Dy}$. In this figure the band energy, defined as 

$$E^I_\kappa = \frac{H^I_{\kappa\kappa\kappa}}{N^I_{\kappa\kappa\kappa}} = \frac{\langle \phi_\kappa | \hat{H}^I_{KK} | \phi_\kappa \rangle}{\langle \phi_\kappa | \hat{P}^I_{KK} | \phi_\kappa \rangle},$$  

(8)

is plotted as a function of angular momentum $I$. For the standard deformation (left figure in Fig. 3) the 2-qp band (labeled by number 2) that is the first one to cross the ground state band corresponds to a configuration with particles from smaller $K$-orbits ($K = 1/2$ and $3/2$). In the case of the enlarged deformation (right figure in Fig. 3) the relevant 2-qp band (again labeled as 2) has particles from larger $K$-orbits ($K = 3/2$ and $5/2$). In the first case the crossing angle between the unperturbed ground and that 2-qp band at $I = 12\hbar$, which we loosely define as the angle between the tangents of both curves at the crossing point \[ ] is large, and is reflected in Fig. 2c as an exaggerated zig-zag effect in the full line. On the other hand, a smaller crossing angle at $I = 16\hbar$ for the enlarged deformation case is behind the smooth behavior of the dotted curve, which precisely reproduces the experimental information.

At the standard deformation there are three neutron single particle levels that contribute to the two quasiparticle bands active in the backbending region. They are the Nilsson states with important spherical components $N = 6$, $l=\bar{i}$, $j=13/2$, $m= 5/2$, $-3/2$, $1/2$, denoted $[6 \ i13/2 \ 5/2]n$, $[6 \ i13/2 \ -3/2]n$ and $[6 \ i13/2 \ 1/2]n$ respectively. When the deformation is increased, only the first two levels are relevant for backbending. In Fig. 4, some neutron Nilsson single particle levels close to the Fermi level (indicated in each figure as a black dot) are plotted. The rearrangements of the relative position of these levels explains the displacement to higher energies of many two-qp bands for the enlarged deformation, with the consequent changes in the band crossings and the backbending plots.

Notice that, for $\epsilon_2 \approx 0.3$ and $N = 92$, there is a large gap and a sparse number of single particle states for $92 \leq N \leq 96$. For this reason, the excitation energy and backbending plots for $^{160,162,164}\text{Dy}$ shown in Fig. 1 d,e,f and Fig. 2 d,e,f are normally behaved, with no sudden changes in their slopes.

Until now, we have argued the necessity of changing the single particle distributions by shifting the Fermi level to that corresponding to a larger deformation in the standard Nilsson diagram. It should be noted that the parameters used to generate the Nilsson diagram in the present paper were fitted nearly 30 years ago [17], when not many accurate
and systematic high-spin data were available. An alternative of improving the single particle distribution is to modify the standard Nilsson parameters, i.e. changing the local single particle distribution in the standard Nilsson diagram. It has been noticed recently [18] that the standard Nilsson parameters for the proton $N = 5$ shell need to be modified to reproduce the newest high spin data.

In an early PSM work [4], the modified set of Nilsson parameters [19] for lighter Er and Yb isotopes was used, while the standard Nilsson parameters [17] were employed for heavier isotopes. Namely, if the authors of [4] insisted on assuming the deformations commonly found in the literature, they had to use different sets of the Nilsson parameters for lighter and heavier isotopes. We have tested our prescription of increasing deformation in Er and Yb nuclei, and found that one is able to use one unified (standard) set of Nilsson parameters to span a deformed basis for the PSM, with the lighter isotopes requiring a larger deformation increase.

V. REDUCED QUADRUPOLE MOMENT AND B(E2) TRANSITIONS

The $B(E2; I_i \rightarrow I_f)$ transition probabilities from initial state $(\sigma_i, I_i)$ to final state $(\sigma_f, I_f)$ are given by [3]

$$B(E2; I_i \rightarrow I_f) = \frac{2I' + 1}{2I + 1} |\langle \Phi_I || \hat{Q}_2 || \Phi_I \rangle|^2,$$

where [3]

$$\langle \Phi_I || \hat{Q}_2 || \Phi_I \rangle = \sum_{\nu} \left\{ \sum_{K'K} (IK' - \nu', \lambda \nu | I'K') \langle \phi_{K'} \hat{P}_{K'K} \phi_{K} | F_{I'} F_{I} \rangle \right\}. \quad (10)$$

Here $F_{I'}$ and $F_{I}$ are, respectively, the PSM eigenvectors for spins $I$ and $I'$, as calculated in Eq. (2), and the electric quadrupole operator $Q_{2\nu}$ is defined as [4]

$$Q_{2\nu} = e_p Q_{2\nu}^p + e_n Q_{2\nu}^n, \quad Q_{2\nu}^{p(n)} = e \sum_{i=1}^{Z(Z)} r_i^2 Y_{2\nu}(\theta_i, \phi_i). \quad (11)$$

The effective charges $e_p$ and $e_n$ for protons and neutrons were taken as $e_p = 1.5$ and $e_n = 0.5$ in previous works [3,7]. We have used the same values in the calculations involving the standard deformation in this paper. However, when using the enlarged deformation, it proved necessary to use a slightly smaller effective charges, as prescribed for example in Ref. [20]:

$$e_p = e(1 + \frac{Z}{A}), \quad e_n = e(\frac{Z}{A}) \quad (12)$$

For $^{158}$Dy their numerical values are $e_p = 1.41$ and $e_n = 0.41$.

Fig. 5 exhibits the $B(E2; I_i \rightarrow I_f)$ values for the six Dy isotopes under study. Again the full lines represent the results obtained with the standard deformation and the dotted lines those obtained with the enlarged one. Experimental data are presented with their error bars. There are strong reductions in the $B(E2)$ values predicted around $I = 14\hbar$ using the
standard deformation that clearly contradict the experimental data. This contradiction is now removed in the calculation with the enlarged deformation.

Another quantity of interest is the reduced transition quadrupole moment. It is defined as

\[ Q_t(I \rightarrow I') = \frac{1}{(I_0, 20|I'0)} \sqrt{\frac{2I + 1}{2I' + 1}} B(E2; II'). \]  

(13)

For a rigid rotor with axial symmetry, it is equivalent, up to a sign, to the static quadrupole moment. In this special case

\[ Q_t(I \rightarrow I - 2)/Q_t(2 \rightarrow 0) = 1. \]

An analysis of this quotient shows the extent to which the yrast band \( B(E2) \) values correspond to a symmetric axial rotor. Cescato et al. showed that the cranking models give a rigid rotor behavior for any spin and can not explain the experimental fluctuations in the quadrupole moment as function of angular momentum.

The Fig. 6(a,b,c,d,e,f) shows the behavior of the ratio \( Q_t(I \rightarrow I - 2)/Q_t(2 \rightarrow 0) \) as function of the total angular momentum. As seen in the figure, the theoretical predictions exhibit a sudden change in magnitude at those angular momenta where backbending takes place. As was mentioned in the discussion of \( B(E2) \) values, at standard deformations the PSM predicts more variations than observed, while with the enlarged deformation the agreement with the experimental data is improved.

The origin of the improved agreement in the transition quadrupole matrix elements is similar to that of the improved backbending behavior. A smoothing of the crossing interaction will generally tend to suppress sharp drops of the transition matrix elements at the first backbending.

The reduction of the \( B(E2) \) values observed in \( ^{156,158}\text{Dy} \) for \( I > 20\hbar \) and the corresponding reduction in quadrupole moments can not be reproduced in any of the present calculations which have included only a few states near the Fermi surface. Qualitatively, this would require a more extensive PSM description including in the Hilbert space more 2- and 4-qp states, and possibly even higher orders of qp states, to account for this phenomenon of gradual loss of collectivity.

VI. THE GYROMAGNETIC FACTORS

Gyromagnetic factors are very sensitive to the particle alignment processes, thus allowing us to disentangle the origin of the crossing band at spin around \( 16 - 18\hbar \). The magnetic moment \( \mu \) of a state \((\sigma, I)\) is defined by

\[ \mu(\sigma, I) = \sqrt{\frac{4\pi}{3}} < \sigma, II | M_{10} | \sigma, II > = \frac{[4\pi I]^{1/2}}{3(I + 1)(2I + 1)^{1/2}} < \sigma, II | M_1 | \sigma, I >, \]  

(14)

the operator \( M_{10} \) is given by

\[ M_{10} = \mu_N \sqrt{\frac{3}{4\pi}} \sum_{i=1}^{A} g^{(i)} l^{(i)}_{z} + g^{(i)}_{s} s^{(i)}_{z} = \mu_N \sqrt{\frac{3}{4\pi}} \sum_{\tau=\mu,n} g^{\tau}_l L^{\tau}_{z} + g^{\tau}_s S^{\tau}_{z} \]  

(15)
with $\mu_N$, the nuclear magneton, and $g_l$ and $g_s$ the orbital and the spin gyromagnetic factors, respectively.

The gyromagnetic factors $g(\sigma, I)$, $g_p(\sigma, I)$ and $g_n(\sigma, I)$ are defined by

$$g(\sigma, I) = \frac{\mu(\sigma, I)}{\mu_N I} = g_p(\sigma, I) + g_n(\sigma, I),$$

with $g_\tau(\sigma, I)$, $\tau = p, n$, given by

$$g_\tau(\sigma, I) = \frac{1}{[I(I+1)(2I+1)]^{1/2}} \left( g_\tau^p < \sigma, I||J^\tau||\sigma, I > + (g_\tau^s - g_\tau^l) < \sigma, I||S^\tau||\sigma, I > \right).$$

In the calculations we use for $g_l$ the free values and for $g_s$ the free values damped by the usual 0.75 factor.

$$g_l^p = 1 \quad g_l^n = 0 \quad g_s^p = 5.586 \times 0.75 \quad g_s^n = -3.826 \times 0.75. \quad (18)$$

We emphasize that, unlike many other models, the g-factor is directly computed by using the many-body wave function (Eq. (17)). In particular, there is no need to introduce any core contribution, which is a model-dependent concept.

In general, for proton alignment the contribution $g_l^p < \sigma, I||J^p||\sigma, I >$ is large and positive and we expect an increasing $g$ factor. For neutron alignment $g_s^n < \sigma, I||S^n||\sigma, I >$ is negative and we therefore expect a decreasing $g$ factor.

In Fig. 7 we present the gyromagnetic factor for the six Dy isotopes along the yrast band, again with the full lines representing the results obtained with the standard deformation and the dotted lines those obtained with the enlarged one. Experimental data are presented with their error bars. Our results show a slight increase at low spins, followed by a clear reduction at the band crossing region and then a recovery. The decrease of the total $g$-factor at the band crossing confirms the character of the crossing band as a neutron aligned band. The smoothness of $g$ at high angular momentum indicates a proton alignment. The prediction of this trend is supported by the $^{154}$Dy g-factor data which extended the measurement to high spins. For the low spin part, the PSM predictions are also confirmed by later experiments.

In a recent experiment, Alfer et al. measured g-factors for $^{158,160,162}$Dy at low spins. In contrast to the behavior of the Er g-factors at low spins, Alfer et al. found a clear drop in $^{158,160}$Dy at spin $I = 6\hbar$, which was not predicted by the PSM calculation. The purpose of our present g-factor calculation is to find a possible explanation for the Alfer’s data when enlarged deformation is studied in this paper. However, the result is negative: the basic features of the theoretical g-factor do not seem to change much in our new calculations. In the calculation with enlarged deformation, one sees only a delayed and smaller decrease of the values at the band crossing, but nothing essential has changed for the low spins.

Bengtsson and Åberg suggested an increase in g-factor at low spins due to changes in deformation and pairing. Our microscopic calculations in this paper seem to have the same trend. The admixture from the $i_{3/2}$ single neutrons to the ground band is found to be negligibly small around spin $I = 6\hbar$ in our calculation, thus can not be the reason causing the g-factor drop, as suggested by the authors of Ref. (If this argument is right, one would expect a larger drop of g-factor for the state $I = 8\hbar$). To our knowledge, there have
been no microscopic calculations that can reproduce this variation of the g-factor around spin \(6\hbar\). A similar situation is found in \(^{50}\text{Cr}\) \[^{26}\], when studied using the spherical shell model (complete \(fp\) shell) and the HFB method. Both predictions are nearly identical, but they do not agree with the data \[^{27}\], noticeably for \(I = 4\hbar\), where the g-factor drops in a way similar to that found in Dy isotopes \[^{24}\]. We notice that recent measurements do not find this g-factor dropping at low spins in \(^{50}\text{Cr}\) \[^{28}\].

VII. CONCLUSIONS

We have presented a study of the yrast band in the \(^{154−164}\text{Dy}\) isotopes using the Projected Shell Model (PSM). We have shown that the use of an input deformation 20% larger than the standard value, coupled with a slight reduction in effective charge, leads to an improved description of the yrast band energies, and of \(B(E2)\) values and transition quadrupole moments in these isotopes. The dependence of the \(B(E2)\) values on the angular momentum is also better described when the larger deformations are used.

We have discussed the changes in the distribution of single-particle occupation implied by increased deformations for the unperturbed rotational bands. The different Nilsson single particle energies at enlarged deformations and the associated changes in the Fermi level were shown to be the main source of changes in the yrast spectra and the wave functions. Appropriate modifications of the Nilsson parameters would have had a similar effect.

While general rotational features and the physics related to the band crossings are well described by the present study, limitations of the model are seen from the discussions. The lighter isotopes \(^{154}\text{Dy}\) and \(^{156}\text{Dy}\) exhibit softness against rotation not present in the calculations. A correct description of these nuclei would require a study with a richer Hilbert space than is contained in this simple PSM. The observed reduction in g-factor around \(I = 6\hbar\) can not be explained by the PSM.

After this manuscript was finished, we become aware of a recent extension of the PSM by Sheikh and Hara \[^{29}\] that includes \(\gamma\)–deformation in the basis states and performs 3-dimensional angular momentum projection. Although their preliminary code works in a very limited model space, great improvement for the description of the moment of inertia at low spins in rare earth nuclei with neutron number around 90 is obtained. This could remove the discrepancies found in our present paper for \(^{154}\text{Dy}\) and \(^{156}\text{Dy}\) and strongly extend the predictive power of the PSM.

We finally emphasize that a systematic study for a chain of nuclei is always a serious test for microscopic models, before they can be used to explain or predict an isolated event in specific isotopes.

ACKNOWLEDGMENTS

This work was supported in part by Conacyt (Mexico) and the National Science Foundation. Yang Sun acknowledge the hospitality of the Instituto de Ciencias Nucleares, UNAM, where the final version of this article was completed.
REFERENCES

[1] A. Johnson, H. Ryde and J. Sztarkier, Phys. Lett. 34B (1971) 605.
[2] A. Johnson, H. Ryde and S. A. Hjorth, Nucl. Phys. A179 (1972) 753.
[3] F.S. Stephens and R.S. Simon, Nucl. Phys. A183 (1972) 257.
[4] K. Hara and Y. Sun, Nucl. Phys. A529 (1991) 445.
[5] K. Hara and Y. Sun Int. Jour. Mod. Phys. E4 (1995) 637.
[6] M.L. Crescato, Y. Sun and P. Ring, Nucl. Phys. A533 (1991) 455.
[7] Y. Sun and J.L. Egido, Nucl. Phys. A580 (1994) 1.
[8] Y. Sun and K. Hara, Computer Phys. Comm. 104 (1997) 245.
[9] P. Ring and P. Schuck, The Nuclear Many-Body Problem (Springer-Verlag, New York, 1980).
[10] V. Velázquez, J.G. Hirsch and Y. Sun, Nucl. Phys. A 643 (1998) 39.
[11] P. Möller, At. Data Nucl. Data Tables 59 (1995) 185.
[12] R. Bengtsson, S. Frauendorf, and F.-R. May, At. Data Nucl. Data Tables, Vol.35 (1986) 15.
[13] L.L. Lamm, Nucl. Phys. A125 (1969) 504.
[14] V. Velázquez, J.G. Hirsch and Y. Sun, in preparation.
[15] H. Emling, E. Grosse, R. Kulessa, D. Schwalm and R.S. Simon, Nucl. Phys. A419 (1984) 187.
[16] F. Azgui, H. Emling, E. Grosse, C. Michel, R.S. Simon, W. Spreng, Nucl. Phys. A439 (1985) 573.
[17] S.G. Nilsson, C.F. Tsang, A. Sobiczewski, Z. Szymanski, Z. Wycech, C. Gustafson, I.-L. Lamm, P. Möller, B. Nilsson, Nucl. Phys. A131 (1969) 1.
[18] Y. Sun and K. Hara, Phys. Rev. C57 (1998) 3079.
[19] T. Bengtsson, Nucl. Phys. A512 (1990) 124.
[20] K.H. Bhatt, C.W. Nestor Jr. and S. Raman, Phys. Rev. C46 (1992) 164; S.G. Nilsson and I. Ragnarsson, Shapes and Shells in Nuclear Structure, Cambridge University Press, U.K. 1995.
[21] A. Bohr and B.R. Mottelson, Nuclear Structure Vol I (W.A. Benjamin INC., New York, 1969).
[22] U. Birkenfeld, A.P. Byrne, S. Heppner, H. Hubel, W. Schmitz, P. Fallon, P.D. Forsyth, J.W. Roberts, H. Kluge, E. Lubkiewicz and G. Goldring, Nucl. Phys. A555 (1993) 643.
[23] F. Brandolini, C. Cattaneo, R.V. Ribas, D. Bazzacco, M. De Poli, M. Ionescu-Bujor, P. Pavan, and C. Rossi-Alvarez, Nucl. Phys. A600 (1996) 272.
[24] I. Alfter, E. Bodenstedt, W. Knieloch, and J. Schüth, Z. Phys. A357 (1997) 13.
[25] R. Bengtsson and S. Åberg, Phys. Lett. 172B (1986) 277.
[26] G. Martínez-Pinedo, A. Poves, L.M. Robledo, E. Caurier, F. Nowacki, J. Retamosa and A.P. Zuker, Phys. Rev. C54 (1996) 2150.
[27] A.A. Pakou, J. Billowes, A.A.W. Mountford and D.D. Warner, Phys. Rev. C50 (1994) 2608.
[28] N. Benczer-Koller, XXII Symposium on Nuclear Physics. Oaxtepec, Mexico, January 5-8 1999.
[29] J.A. Sheikh, K. Hara, [nucl-th/9812051].
Figure Captions

Figure 1: Angular frequency $\omega$ vs. angular momentum $I$ for $^{154,156,158,160,162,164}$Dy are shown in inserts (a,b,c,d,e,f) respectively. The PSM results using standard deformations are presented as solid lines; those with enlarged deformations as dotted lines. Experimental data are represented by diamonds.

Figure 2: Twice the inertia moment $\Theta$ vs. the square of the angular velocity $\omega$. The same convention as Fig. 1 is used.

Figure 3: Unperturbed rotational bands for $^{158}$Dy at standard (a) and enlarged (b) deformation. For the 2-qp bands the notation $[N \, \ell \, j \, m]$ is used for each qp component. 4qp means a four quasiparticle state.

Figure 4: Nilsson neutron single particle energies around the Fermi level (represented with a diamond) for standard (a) and enlarged (b) deformation.

Figure 5: B(E2) values, in $e^2b^2$, as a function of the angular momentum $I$. The same convention as Fig. 1 is used.

Figure 6: Reduced transition quadrupole moments $Q_t(I \rightarrow I-2)$ normalized with respect to $Q_t(2 \rightarrow 0)$. The same convention as Fig. 1 is used.

Figure 7: G-factors vs. angular momentum $I$. The same convention as Fig. 1 is used.
TABLES

TABLE I. Standard ($\epsilon$) and enlarged ($\epsilon'$) deformations used in this work for the six Dy isotopes listed in the first column.

| Isotope | $\epsilon_2$ | $\epsilon'_2$ |
|---------|---------------|----------------|
| $^{154}$Dy | 0.192         | 0.250          |
| $^{156}$Dy | 0.217         | 0.270          |
| $^{158}$Dy | 0.242         | 0.286          |
| $^{160}$Dy | 0.250         | 0.290          |
| $^{162}$Dy | 0.258         | 0.308          |
| $^{164}$Dy | 0.267         | 0.280          |