Solution of Tasks of Energy Efficiency of the Process of Drying Raw Plant Materials by Methods of Mathematical Modeling

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Abstract. Beet pulp has a high feed value, but after pressing it has a sufficiently high humidity, which leads to rapid spoilage. One of the ways to stabilize the quality of beet pulp is drying with superheated steam of reduced pressure. Mathematical modeling of heat transfer during convective drying of beet pulp is proposed. A feature of the model is the description of the drying process, which ensures its high intensity due to high heat and mass transfer ratios and preserves the quality of the finished product as a result of the process at a reduced pressure of transfer medium. In the period of a constant drying speed, dehydration of macroscopic pores topologically related to the sample surface, which are characterized by a relatively low binding energy of water molecules with the liquid phase, is taken into account. According to experimental data, the proportion of water content in these pores is 60...70% of the total moisture content. In the final stage of this period, an increase in the binding energy occurs due to the removal of water from microcapillaries, which is explained by the formation of structured clusters in thin wetting films of water molecules. The change in steam temperature as a result of passing through the product layer was 8 ... 10 K, and the relative temperature change was 5 ... 10%. The parameters of the models were identified in the following range of parameters: superheated steam temperature 393 ... 453 K; speed of superheated steam 3 ... 5
m/s; the pressure inside the drying chamber is 40 ... 100 kPa; the specific load of the product on the grate is 8 ... 24 kg/m². The interpretation of the simulation results is presented by the dependences of the temperature difference between beet pulp and heat carrier $\Delta T$ on the temperature of superheated steam $T_s$ at various pressures $P$: 1 - 100 kPa; 2 - 80 kPa; 3 - 60 kPa; 4 - 40 kPa; $\nu = 4$ m/s; at various specific loads of beet pulp on the gas distribution grid $q_{sp}$: 1 - 8 kg/m²; 2 - 16 kg/m²; 3 - 24 kg/m²; $\nu = 4$ m/s; $P = 60$ kPa and various speeds of superheated steam $\nu$: 1 - 5 m/s; 2 - 4 m/s; 3 - 3 m/s. A sufficient convergence of the results was ensured, in which the deviation of the calculated data from the experimental data did not exceed 12.5% in absolute value. The obtained results of the simulation can be used in the operational control of the technological parameters of the process of drying beet pulp with superheated steam of reduced pressure with a restriction on the temperature mode due to the receipt of a high quality product.

**Key words**: drying, beet pulp, superheated steam of reduced pressure, kinetics.

### 1. Introduction

The problem of the production and use of non-traditional types of raw materials, such as waste from the sugar industry, in particular beet pulp, which, having high feed value and taking into account the scale of production, can greatly contribute to solving one of the most important issues in the development of the country’s agriculture - creating a solid fodder base for livestock breeding. However, the pulp after pressing has a sufficiently high humidity (72 ... 75%), resulting in its rapid deterioration and souring during storage and, therefore, reduction efficiency of its use as a valuable fodder raw material or as filler in the production of biologically active additives used in the organization of full feeding of different species and age groups of agricultural animals and poultry [1, 2, 3].

It has been established that due to the application of thermal effect, it is possible to ensure high quality of beet pulp, increase shelf life and be effectively used in animal feed technology. One of the ways to stabilize the quality of beet pulp is drying with superheated steam of reduced pressure, which largely provides a high-quality product without reducing the intensity the process [4, 5]. The temperature modes during drying of beet pulp determine the intensity and duration of the process, and the temperature gradients in the product, as a rule, determine its quality. Due to the complexity of the experimental determination of temperature fields in single particles, it is crucial to develop modeling methods to determine the temperature fields in the product, which as a rule, they allow the analysis of many alternative options for hardware and technological design of drying processes and are optimal in terms of heat and power costs and indicators of quality of the product to be dried.

### 2. Materials and methods

In this regard, mathematical modeling of thermal exchange in convective drying of beet pulp due to the transfer of thermal energy of superheated steam of reduced pressure through the surface of the product particle and the endothermic phase transition of water molecules from the bound state to the gaseous state is proposed [6],[7].

### 3. Results and discussion

When formulating the simulation problem, the following simplifying assumptions were made:

1. The particle shape of the product is represented by a thin plate, which is blown with superheated steam at a speed of $\nu$. The plate is located in the plane $(x, y)$ of the coordinate system, and the heat carrier flow is directed along the $z$ axis.

2. The characteristic time of heating of the samples is determined by the ratio, $s$:

$$\tau_f = \frac{l^2}{a}$$

where $l$- thickness of the plate material, m; $a$ - coefficient of thermal diffusivity of the material, m²/sec. Since the drying process time exceeds the sample heating time by almost two orders, the heat flux quantity entering the product particle from the heat carrier is controlled by the thermal conductivity of superheated steam, and the temperature of the product particle is assumed to be the same throughout its volume.
3. The change in temperature of superheated steam in the working volume of the drying unit is taken into account only from one spatial variable \( z \). The \( z \) axis is directed along the steam speed vector, and the coordinate origin coincides with the location of the gas distribution grid. The steam temperature depends on two variables: the \( z \) coordinate and time \( t \). 

4. The studies on the kinetic laws of the beet pulp drying process in an environment of superheated steam of reduced pressure show the presence of two drying periods: a period of constant and decreasing drying speed [11]. In the period of a constant drying speed, dehydration of macroscopic pores topologically related to the sample surface, which are characterized by a relatively low binding energy of water molecules with the liquid phase, is taken into account. According to experimental data, the proportion of water content in these pores is 60...70% of the total moisture content. In the final stage of this period, an increase in the binding energy occurs due to the removal of water from microcapillaries, which is explained by the formation of structured clusters in thin wetting films of water molecules [8]. In the period of decreasing drying speed, water is removed from the pores due to the diffusion of water molecules through the walls of the product tissue.

The general problem of heat exchange during drying of a particle located in a moving heat carrier flow is that after the drying time some thermal equilibrium is established and it is necessary to determine the resulting temperature difference \( (T_s - T) \).

The kinetic equation for changing the particle temperature \( T \) is considered, based on the balance of thermal energy:

\[
Vc \frac{dT}{dt} = aS(T_s - T) - Sef,
\]

where \( V \)- volume of one particle of the product, m\(^3\); \( c \)-specific heat of the product, J/(kg·K); \( T_s \)-temperature of the heat carrier near the product particle, K; \( S \)-area of the product particle, m\(^2\); \( a \)-heat transfer coefficient from the heat carrier to the product particle, W/(m\(^2\)·K); \( e \)-binding energy of the water molecule with the liquid phase; \( j \)-the flux density of water molecules evaporated from the sample; \( f \)-change speed of the sample temperature, K/s; \( aS(T_s - T) \)-heat energy flow from steam to the product particle; \( Sef \)-endothermic process of water evaporation from the surface of a particle.

The left-hand side of equation (1) represents the change speed of the thermal energy of the particle, the first term on the right-hand side determines the heat transfer of the thermal energy of superheated steam to the product particle, the second term - the decrease in the thermal energy of the particle as a result of evaporation [9],[10].

The equilibrium concentration of water molecules in steam at the temperature of the sample is presented in the following form, 1/cm\(^3\):

\[
n_s(T) = n_{atm}\left(e^{\frac{u}{kT_{atm}} - \frac{u}{kT}}\right),
\]

where \( T_{atm} \)- boiling point of water at atmospheric pressure \( (T_{atm} = 373K) \); \( n_{atm} \)-concentration of water molecules under these conditions \( (n_{atm} = 2 \cdot 10^{19} 1/cm^3) \); \( u \)-parameter found from the experimental data.

The temperature dependence of the kinetic coefficient \( k \) is determined by the expression:

\[
k = \frac{R u_n}{4},
\]

where \( u_n \)-thermal speed of the movement of water molecules; \( R \)-condensation coefficient, which represents the probability of “sticking” of a water molecule from the heat carrier to the surface of the condensed state of the substance upon impact on the surface.

The equation for changing the steam temperature based on the balance of the thermal energy of the heat carrier was considered similarly (1):

\[
3n_s kT_s = -3n_s k u \frac{\partial n_s}{\partial z} - \alpha S ef (T_s - T).
\]

On the right side of expression (3), the change speed of the thermal energy density of water steam is recorded. \( f \)-change speed of the steam temperature, K/s; \( n_s \)-the number of product particles per unit volume, 1/m\(^3\); \( v \)-steam speed in the drying chamber, m/s.
The first term in (3) represents the change speed of thermal energy at some point in the volume of the drying apparatus due to the movement of steam at a speed \( v \). The second term takes into account the decrease in thermal energy of steam due to heat transfer to heat the product.

Substituting (2) in (1) taking into account (3) allows us to formulate a model of the drying process of beet pulp in a closed system of differential equations in which the quantities to be found are \( T, T_s, \ldots \)

\[
T = \frac{aS}{Vc} (T_s - T) - \frac{SiK}{Vc} (n_p - n_s), \tag{4}
\]

where \( n_s \) is the concentration of water molecules in the superheated steam in the drying chamber, \( 1/cm^3 \).

\[
T_s = -v \frac{\partial T_s}{\partial z} - \frac{aSn_f}{3Kn_s} (T_s - T). \tag{5}
\]

Under the conditions under consideration, other characteristics of the system, such as the speed of the heat carrier and its concentration, may also depend on the \( z \) coordinate. The change in temperature \( n_s \) with height \( z \) is due to changes in steam pressure and its temperature. The change in temperature of the steam as a result of passing through the product layer was 8 ... 10 K, and the relative change in temperature was 5 ... 10 %. The same quantity is the relative seal of the heat carrier. Therefore, in the system (4), (5), the change in the steam density was neglected.

A similar situation arose in connection with a change in steam pressure before and after the product layer. This meant that a decrease in the density of water molecules in the heat carrier arising from a decrease in pressure can also be neglected. For the same reason, in the system of equations (4), (5), the steam density and its speed are considered constant over the entire thickness of the product layer. These characteristics, together with the initial temperature \( T_0 \) and the mass of the product, were the controlling parameters of the process and their values are set during an experimental study of the kinetics of the drying process.

Based on the above assumptions, the solution of system (4), (5) is determined for the drying process of beet pulp in a stationary mode, i.e. \( \dot{T} = 0, T_s = 0 \). Hence follows:

\[
T_s - T = \frac{uK}{a} (n_p(T) - n_s), \tag{6}
\]

\[
\frac{\partial T_s}{\partial z} = -\frac{aSn_f}{3Kn_s} (T_s - T). \tag{7}
\]

To find a solution to system (6), (7), a function is introduced that represents the temperature difference between the heat carrier and the product:

\[
\theta = T_s - T = \frac{ux}{a} (n_p(T) - n_s), \tag{8}
\]

where \( \chi \) is the kinetic coefficient relating the flux density of evaporating water molecules from the product to the deviation of the concentration of water molecules in superheated steam from equilibrium.

Then the system of equations (6), (7) takes the form:

\[
\frac{\partial T_s}{\partial z} = T + \theta(T); \tag{9}
\]

\[
\frac{\partial T_s}{\partial z} = -\frac{1}{\lambda} \theta(T), \tag{10}
\]

where \( \lambda \) characterizes the linear section along the height of the drying chamber, on which the steam temperature decreases. This quantity is determined by the expression:

\[
\lambda = \frac{3Kn_s u}{aSn_f}. \tag{11}
\]

Equation (9) was differentiated according to the \( z \) coordinate:

\[
\frac{\partial T_s}{\partial z} = \frac{\partial T}{\partial z} + \theta_0 \frac{\partial \theta}{\partial T} \frac{\partial T}{\partial z}, \tag{12}
\]

and after substituting (12) into (10), we obtained:

\[
\frac{\partial T}{\partial z} \left( 1 + \frac{\partial \theta}{\partial T} \right) = -\frac{1}{\lambda} \theta(T). \tag{13}
\]

The solution of equation (13) is presented in the form:

\[
\int \frac{\partial T}{\theta(T)} + \ln \theta(T) = -\frac{z-z_0}{\lambda}, \tag{14}
\]

where \( z_0 \) is the integration constant, which is found from the condition \( T(0) = T_0 \), here \( T_0 \) is the temperature of the product near the gas distribution grid, K.
The drying model of beet pulp (6) - (12) reflects the situation with a low value of the degree of layer expansion, in which the mixing of product particles is poorly expressed. This may occur, including due to the dispersion of product particles in size. Larger particles occupy relatively low positions in the heat carrier flow, and the temperature of the product particles becomes dependent on the height $z$.

Since the numerical estimates of the integrand in (14) take a value significantly less than unity, this integral is neglected. As a result, we get:

$$\theta(T) = \theta(0)e^{-\frac{z}{\lambda}}.$$  

Hence, the dependence $T(z)$ was found by the formula:

$$T = \frac{kT_{\text{in}}}{uk} \ln \left( \frac{n_s}{n_{\text{atm}}} \left( 1 + \frac{\alpha(T_{\text{os}} - T_{\text{os}}) - n_s}{uk} \right) \right),$$  

(15)

here $T_{\text{os}}$ - the temperature of the steam at the input to the drying plant, K; $T_{k}$ - the final temperature of the product, K.

When drying beet pulp in a fluidized bed, the particle temperature $T$ is averaged over the time spent in different parts of the drying chamber and takes a fixed value. Then the solution of the kinetic equation (3) for the steam temperature at each section of the drying chamber in the stationary mode $T_{s}$ = 0 has the form:

$$T_s = \bar{T} + (T_{\text{os}} - \bar{T})e^{-\frac{z}{\lambda}}.$$  

(16)

The average temperature of the product particles $\bar{T}$ is found from the balance condition for the absorbed heat and energy that goes to the water evaporation.

The density of thermal energy per unit time absorbed by the particles of the product at a height $z$ is equal, W/m$^3$:

$$q = \alpha n_f S(T_s - \bar{T}).$$  

(17)

As a result of substituting (16) into (17), we obtain:

$$q(z) = \alpha n_f S(T_{\text{os}} - \bar{T})e^{-\frac{z}{\lambda}}.$$  

Integrating this expression over the height of the product layer $d$, m, the amount of heat energy $Q$, W, transferred by the heat carrier to the product in the layer having a unit area is obtained:

$$Q = \int_0^d q(z)dz = \lambda \alpha n_f S(T_{\text{os}} - \bar{T}) \left( 1 - e^{-\frac{d}{\lambda}} \right).$$  

The number of water molecules $n_{\text{lay}}$ evaporated in this layer:

$$n_{\text{lay}} = kSn_f \left( n_p(T) - n_s \right) d,$$

whered - the height of the product layer, m

Hence we obtain the equation for the definition $\bar{T}$:

$$\alpha k d (n_p(T) - n_s) = \lambda \alpha (T_{\text{os}} - \bar{T}) \left( 1 - e^{-\frac{d}{\lambda}} \right).$$  

(18)

Substituting in (18) $n_p(T)$ from (4), we obtain:

$$\frac{ukd}{\alpha \lambda} \left( n_{\text{atm}} e^{\frac{k}{T_{\text{atm}}}} - 1 \right) - n_s = (T_{\text{os}} - \bar{T}) \left( 1 - e^{-\frac{d}{\lambda}} \right).$$  

(19)

According to the experiment, a linear approximation is established:

$$d = d_0 + d_1 v = (1 + 2v),$$  

(20)

where $d_0$ - начальная высота слоя продукта, м; $d_1$ - высота слоя продукта при скорости $v$, м, а также зависимость коэффициента теплоотдачи от скорости пара:

$$\text{where } c - \text{ the initial height of the product layer, m; } d_f - \text{ the height of the product layer at a speed } v, m,$$

and also the dependence of the heat transfer coefficient on the steam speed:

$$\alpha = (1 + 8v).$$  

(21)

Taking into account (19) and (20), equation (18) is reduced to the form:

$$he k_{\text{atm}} \frac{v}{\kappa f} - 1 = \frac{T_{\text{os}} - \bar{T}}{T_s},$$  

(22)
where $n_{atm} > 1; T_0 = \frac{k\alpha S d n_f}{3k v (1 - e^{-g})}; g = \frac{a S d n_f}{3k n v}
$

Thus, the average temperature of the product particles depends on three control parameters $T_0$, $d$, and $T^*$. Simplifying the product $S d n_f$, we get:

$$ q = \frac{S d n_f}{S_{as}} = \frac{N}{S_{as}}; \quad (23) $$

where $S_{as}$ - the cross-sectional area of the drying chamber, $m^2$; $N$ - the total number of product particles in the drying chamber, pcs.; $S_{as}/S$ - the number of product particles stacked in one layer on the cross-sectional area of the drying chamber, pcs.; $S$ - the number of product layers in the apparatus, pcs.

Substituting (11) in (19) we obtain:

$$ T^* = \frac{kus}{3k v (1 - e^{-g})}, g = \frac{aS}{3knv}. \quad (24) $$

Based on the simulation results, calculation formulas were obtained for determining the temperature of a beet pulp particle during drying under reduced pressure with superheated steam, both in the filter layer along one spatial coordinate in the direction of heat carrier motion (15) and in a fluidized bed for various sections of the drying chamber (20).

The parameters of the models were identified in the following parameter variation range: superheated steam temperature 393 ... 453 K; superheated steam speed 3 ... 5 m/s; pressure inside the drying chamber 40 ... 100 kPa; specific product load on the grate 8 ... 24 kg/m$^2$.

The interpretation of the simulation results is presented by the dependences of the temperature difference between beet pulp and superheated steam $\Delta T$ on the temperature of superheated steam $T_s$ at various pressures $P$: 1 - 100 kPa; 2 - 80 kPa; 3 - 60 kPa; 4 - 40 kPa; $v = 4$ m/s; $q_{sp} = 16$ kg/m$^2$ (Table 1); at various specific loads of beet pulp on the gas distribution grate $q_{sp}$: 1 - 8 kg/m$^2$; 2 - 16 kg/m$^2$; 3 - 24 kg/m$^2$; $v = 4$ m/s; $P = 60$ kPa (Table 2) and various speeds of superheated steam $v$: 1 - 5 m/s; 2 - 4 m/s; 3 - 3 m/s; $q_{sp} = 16$ kg/m$^2$; $P = 60$ kPa (Table 3). A sufficient convergence of the results was ensured, in which the deviation of the calculated data from the experimental data did not exceed 12.5% in absolute value.

### Table 1. The temperature difference between beet pulp and superheated steam at various pressures of superheated steam in the drying chamber

| Temperature $T_s$, K | Superheated steam pressure in the drying chamber $P$, kPa |
|----------------------|----------------------------------------------------------|
|                      | 100 | 80 | 60 | 40 |
| 393                  | 24  | 32 | 40 | 52 |
| 413                  | 35  | 42 | 53 | 64 |
| 433                  | 49  | 58 | 67 | 79 |
| 453                  | 67  | 77 | 86 | 96 |

### Table 2. The temperature difference between beet pulp and superheated steam at different specific product loads on the gas distribution grid

| Temperature $T_s$, K | The specific load of the product on the grate $q_{sp}$, kg/m$^2$ |
|----------------------|---------------------------------------------------------------|
|                      | 8 | 16 | 24 |
| 393                  | 35 | 40 | 46 |
| 413                  | 45 | 53 | 58 |
| 433                  | 60 | 67 | 72 |
| 453                  | 80 | 86 | 91 |

### Table 3. The temperature difference between beet pulp and superheated steam at different speeds of superheated steam in the drying chamber

| Temperature $T_s$, K | The speed of superheated steam in the drying chamber $v_s$, kg/m$^2$ |
|----------------------|---------------------------------------------------------------|
|                      | 3 | 4 | 5 |
| 393                  | 38 | 40 | 44 |
4. Conclusion
The obtained simulation results can be used in the operational control of the technological parameters of the process of drying beet pulp with superheated steam of reduced pressure with a restriction on the temperature mode due to the receipt of a high quality product.

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