Evidence of $A_{CP}(D^0 \to \pi^+\pi^-)$ implies observable $CP$ violation in the $D^0 \to \pi^0\pi^0$ decay

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Abstract

Inspired by the discovery of $CP$ violation in the $D^0 \to \pi^+\pi^-$ decay on LHCb, we study $CP$ asymmetries in the $D \to \pi\pi$ system in the isospin and topological analysis. The formulas of direct $CP$ asymmetries in the $D^0 \to \pi^+\pi^-$ and $D^0 \to \pi^0\pi^0$ decays are derived. We find the ratio between penguin and tree amplitudes $P/(T + C)$ in the $D \to \pi\pi$ system is greater than 2 in most values of the strong phase of penguin. And $D^0 \to \pi^0\pi^0$ decay is a potential mode to reveal the $CP$ violation of the order of $10^{-3}$ and would be observed by Belle II in the foresee future. Besides, we try to explain the large $CP$ asymmetries in the $D \to \pi\pi$ system in the rescattering mechanism.

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1. INTRODUCTION

$CP$ asymmetry in the $D$ meson decay, which is defined as

$$A_{CP}(D \rightarrow f) = \frac{\Gamma(D \rightarrow f) - \Gamma(D \rightarrow \bar{f})}{\Gamma(D \rightarrow f) + \Gamma(D \rightarrow \bar{f})},$$

provides a window to test the Standard Model (SM) and search for new physics (NP) in the up-type quark weak decay in hadrons. In 2019, the LHCb collaboration observed $CP$ asymmetry in charm [1],

$$\Delta A_{CP} \equiv A_{CP}(D^0 \rightarrow K^+K^-) - A_{CP}(D^0 \rightarrow \pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}. \quad (2)$$

After that, many experimental efforts are devoted to the measurement of $CP$ violation and mixing parameters in charm system [2–8]. In theoretical aspect, there are two controversial viewpoints for the observed $CP$ asymmetry difference in literature, regarding it as signal of new physics [9–12], or the non-perturbative QCD enhancements to penguin [13–21]. It attributes to the large ambiguities in evaluating penguin topologies and the absence of enough information given by experiments.

Recently, the LHCb collaboration reported the first evidence of a non-vanishing $CP$ asymmetry in an individual decay of $D^0 \rightarrow \pi^+\pi^-$ by measuring $\Delta A_{CP}$ and $CP$ violation in the $D^0 \rightarrow K^+K^-$ decay [22]. The $CP$ asymmetries in the $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays are given by

$$A_{CP}(D^0 \rightarrow K^+K^-) = (0.77 \pm 0.57) \times 10^{-3}, \quad (3)$$
$$A_{CP}(D^0 \rightarrow \pi^+\pi^-) = (2.32 \pm 0.61) \times 10^{-3}. \quad (4)$$

$CP$ asymmetry in the individual decay mode is more significant compared to the difference between two decay modes because it allows us to extract more knowledge of non-perturbative QCD. Recent theoretical work showed the newest data indicates a very large $U$-spin breaking in the $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ modes, which is beyond the naive expectations of $\varepsilon \sim m_s/\Lambda_{QCD} \sim 30\%$ [23].

In this work, we analyze the implications of the new measurement of $CP$ violation in the $D^0 \rightarrow \pi^+\pi^-$ decay. By applying the isospin and topological analysis, we show the ratio between penguin and tree amplitudes $P/(T + C)$ in the $D \rightarrow \pi\pi$ system is greater than 2 in most values of strong phase. And $CP$ violation in the $D^0 \rightarrow \pi^0\pi^0$ decay could reach to be $O(10^{-3})$, which is available on Belle II in the future. Besides, we try to understand the large $CP$ asymmetries in the $D \rightarrow \pi\pi$ system in the $t$-channel final-state interaction (FSI) which are widely used in heavy meson and baryon weak decays [24–37].
This paper is organized as follows. In Sec. II we study the CP asymmetries in the \( D \to \pi \pi \) system in the isospin and topological analysis. In Sec. III we try to give an explanation of the large CP violation in charm in the final state interaction. And Sec. IV is a short summary.

II. ISOSPIN AND TOPOLOGICAL ANALYSIS

In the \( D \to \pi \pi \) system, \( (D_0, D^+) \) form an isospin doublet, \( (\pi^+, \pi^0, \pi^-) \) form an isospin triplet. Isospin decompositions of the \( D_0 \to \pi^+ \pi^- \), \( D_0 \to \pi^0 \pi^0 \) and \( D^+ \to \pi^+ \pi^0 \) modes are

\[
A(D_0 \to \pi^+ \pi^-) = \frac{1}{2\sqrt{3}} A_{3/2} + \frac{1}{\sqrt{6}} A_{1/2},
\]

(5)

\[
A(D_0 \to \pi^0 \pi^0) = \frac{1}{\sqrt{6}} A_{3/2} - \frac{1}{2\sqrt{3}} A_{1/2},
\]

(6)

\[
A(D^+ \to \pi^+ \pi^0) = \frac{\sqrt{6}}{4} A_{3/2},
\]

(7)

in which \( A_{3/2} \) and \( A_{1/2} \) are the amplitudes with \( \Delta I = 3/2 \) and \( \Delta I = 1/2 \), respectively. Topological decompositions of the \( D \to \pi \pi \) modes can be expressed as

\[
A(D_0 \to \pi^+ \pi^-) = \lambda_d (T + E + P_b) - \lambda_b P,
\]

(8)

\[
A(D_0 \to \pi^0 \pi^0) = \frac{1}{\sqrt{2}} \lambda_d (C - E - P_b) + \frac{1}{\sqrt{2}} \lambda_b P,
\]

(9)

\[
A(D^+ \to \pi^+ \pi^0) = \frac{1}{\sqrt{2}} \lambda_d (T + C),
\]

(10)

where \( \lambda_d = V^*_{cd} V_{ud} \) and \( \lambda_b = V^*_{cb} V_{ub} \). \( T, C \) and \( E \) denote the tree amplitudes and \( P \) and \( P_b \) denote the penguin amplitudes. \( P_b \) is the difference between \( P_d \) and \( P_s \); \( P_b = P_d - P_s \), and \( P = P_s \). \( P_d \) and \( P_s \) are the topologies with \( d \) and \( s \) in the quark loop respectively. In literatures, \( P \) is usually written as penguin plus penguin annihilation diagrams, \( P + 2 PA \). In order to math the isospin amplitudes, the quark compositions of \( D \) and \( \pi \) mesons are defined as \( D^0 = -c\bar{u}, D^+ = c\bar{d}, \pi^+ = u\bar{d}, \pi^0 = \frac{1}{\sqrt{2}} (d\bar{d} - u\bar{u}) \) and \( \pi^- = -d\pi \) in Eqs. (8)∼(10). By comparing Eqs. (8)∼(10) and Eqs. (5)∼(7), the relations between isospin amplitudes and topological amplitudes are found to be

\[
A_{3/2} = \frac{2\sqrt{3}}{3} \lambda_d (T + C),
\]

(11)

\[
A_{1/2} = \sqrt{6} \lambda_d (E + P_b + \frac{2}{3} T - \frac{1}{3} C) - \sqrt{6} \lambda_b P.
\]

(12)

Eqs. (11) and (12) can also be derived from the effective Hamiltonian of charm decay by analyzing the isospin structure of tree and penguin operators, see literature such as Ref. [38] for details.

In the SM, \( \lambda_b \) is much smaller than \( \lambda_d \); \( \lambda_b/\lambda_d \sim \mathcal{O}(10^{-4}) \) [39]. Then the last term in Eq. (12) can be neglected safely in the branching fractions. We define an approximate \( \Delta I = 1/2 \) amplitude
without the $\lambda_b P$ term as

$$A'_{1/2} = \sqrt{6} \lambda_d (E + P_b + \frac{2}{3} T - \frac{1}{3} C).$$

(13)

The strong phase of $A_{3/2}$ is usually set to be zero. And $A'_{1/2}$ is written as

$$A'_{1/2} = A^{s}_{1/2} e^{i \delta_I},$$

(14)

with magnitude $A_{1/2}^{s}$ and relative strong phase $\delta_I = \delta_{1/2} - \delta_{3/2}$. The isospin amplitudes $A_{3/2}$ and $A_{1/2}^{s}$ can be extracted from branching fractions of the $D \to \pi\pi$ decay modes. Branching fractions of the $D^0 \to \pi^+\pi^-$, $D^0 \to \pi^0\pi^0$ and $D^+ \to \pi^+\pi^0$ decays are given by [39]

$$Br(D^0 \to \pi^+\pi^-) = (1.454 \pm 0.024) \times 10^{-3}, \quad Br(D^0 \to \pi^0\pi^0) = (0.826 \pm 0.025) \times 10^{-3},$$

$$Br(D^+ \to \pi^+\pi^0) = (1.247 \pm 0.033) \times 10^{-3}.$$  

(15)

The partial decay width $\Gamma$ is parameterized to be

$$\Gamma(D \to \pi\pi) = \frac{|P_c|}{8\pi m_D} |A(D \to \pi\pi)|^2,$$

(16)

in which $P_c$ is the c.m. momentum in the rest frame of $D$ meson. The hadronic parameters $A_{3/2}$, $A_{1/2}^{s}$ and $\delta_I$ are extracted to be

$$A_{3/2} = (0.447 \pm 0.006) \times 10^{-6} \text{ GeV}, \quad A_{1/2}^{s} = (1.090 \pm 0.009) \times 10^{-6} \text{ GeV},$$

$$\delta_{I}^n = (-86.66 \pm 1.31)^{\circ}, \quad \delta_{I}^p = (86.66 \pm 1.31)^{\circ}.$$  

(17)

There are two solutions for the strong phase $\delta_I$. Superscripts $n$ and $p$ are used to distinguish the negative and positive solutions.

To analyze the $CP$ asymmetries in the $D \to \pi\pi$ modes, we parameterize the penguin amplitude $P$ as

$$\lambda_b P = |\lambda_b| P e^{i (\delta_p - \gamma)},$$

(18)

in which $P$ and $\delta_p$ are the magnitude and strong phase (with respect to $A_{3/2}$) of penguin amplitude respectively. $\gamma$ is phase parameter of the CKM matrix, known as $\phi_3$ in the unitarity triangle. In the SM, $\gamma$ is fitted to be $1.144 \pm 0.027$ [39]. The weak phases of $V_{ud}$ and $V_{cd}$ in the isospin amplitudes $A_{3/2}$ and $A_{1/2}$ are negligible compared to $\gamma$. With isospin and topological amplitudes of the $D \to \pi\pi$ modes, $CP$ asymmetries in the $D^0 \to \pi^+\pi^-$ and $D^0 \to \pi^0\pi^0$ decays are derived to be

$$A_{CP}(D^0 \to \pi^+\pi^-) = 4\sqrt{3} \frac{|\lambda_b|}{\lambda_d} \frac{P \sin \gamma \left( \sqrt{2} A_{1/2}^{s} \sin(\delta_I - \delta_p) - A_{3/2} \sin \delta_p \right)}{2(A_{1/2}^{s})^2 + A_{3/2}^2 + 2\sqrt{2} A_{1/2}^{s} A_{3/2} \cos \delta_I},$$

(19)
\[ A_{CP}(D^0 \to \pi^0\pi^0) = 2\sqrt{3} \frac{|\lambda_b|}{\lambda_d} \frac{P \sin \gamma \left( \sqrt{2} A_{1/2}^{s} \sin(\delta_I - \delta_{p}) + 2 A_{3/2} \sin \delta_{p} \right)}{(A_{1/2}^{s})^2 + 2 A_{3/2}^2 - 2\sqrt{2} A_{1/2}^{s} A_{3/2} \cos \delta_I}. \] (20)

In Eqs. (19) and (20), the first term in numerator is \( CP \) violation in \( A_{1/2} \) and the second term is interference between \( A_{1/2} \) and \( A_{3/2} \).

There are two non-determined parameters in Eqs. (19) and (20), \( P \) and \( \delta_{p} \). If the scenario of no new physical effects is assumed, we can solve \( P \) as a function of \( \delta_{p} \) according to the experiment result of \( A_{CP}(D^0 \to \pi^+\pi^-) \). We plot \( P/(T + C) \) dependent on \( \delta_{p} \) in Fig. 1. One can find \( |P/(T + C)| \) is greater than 2 in most values of \( \delta_{p} \) in both negative and positive \( \delta_{I} \). It suggests the penguin topologies are enhanced by non-perturbative QCD in the \( D \to \pi\pi \) decay modes. More generally, it is possible that the large penguin topologies exist in many singly Cabibbo-suppressed charmed hadron decay modes, leading to observable \( CP \) asymmetries. In addition, topology \( P_b \) is comparable to the tree amplitudes, contributing to a large \( SU(3)_F \) breaking effect in the singly Cabibbo-suppressed charm decays and affecting the branching fractions. Thus \( P_b \) cannot be neglected in the global fit of the \( D \) meson or baryon non-leptonic weak decays.

With the function of \( P(\delta_{p}) \), we get the function of \( A_{CP}(D^0 \to \pi^0\pi^0) \) dependent on \( \delta_{p} \), which is plotted in Fig. 2. \( A_{CP}(D^0 \to \pi^0\pi^0) \) is expected to be \( \mathcal{O}(10^{-3}) \) at most values of \( \delta_{p} \), which is available on Belle II at 50ab\(^{-1} \) data set. At some particular values of \( \delta_{p} \), \( A_{CP}(D^0 \to \pi^0\pi^0) \) could reach to be \( \mathcal{O}(10^{-2}) \). So \( CP \) violation in the \( D^0 \to \pi^0\pi^0 \) decay might be the next observed \( CP \) violation in charm sector. On the other hand, once \( CP \) asymmetries in the \( D^0 \to \pi^+\pi^- \) and \( D^0 \to \pi^0\pi^0 \) decays are well determined by experiments, the magnitude and strong phase of penguin \( P \) can be extracted without model calculations. It will deepen the understanding of non-perturbative QCD and charmed hadron weak decays.
FIG. 2: $CP$ violation in the $D^0 \rightarrow \pi^0 \pi^0$ decay dependent on $\delta_p$ in the cases of negative (left) and positive (right) $\delta_I$. The horizontal pink shadow is $1\sigma$ experimental limitation to date [39, 40]. The blue shadow is the expected statistical uncertainties on Belle II at 50ab$^{-1}$ data set [41].

In experiments, the ratio $R$ is used to test new physics in the $\Delta I = 3/2$ amplitude, which is defined as [43, 44]

$$R = \frac{\left|\mathcal{A}(\pi^+\pi^-)\right|^2 - \left|\mathcal{A}(\pi^0\pi^0)\right|^2 + \left|\mathcal{A}(\pi^0\pi^0)\right|^2 - \left|\mathcal{A}(\pi^0\pi^0)\right|^2 - \frac{2}{3}(\left|\mathcal{A}(\pi^+\pi^-)\right|^2 - \left|\mathcal{A}(\pi^0\pi^0)\right|^2) \left|\mathcal{A}(\pi^+\pi^-)\right|^2 + \frac{2}{3}(\left|\mathcal{A}(\pi^+\pi^-)\right|^2 + \left|\mathcal{A}(\pi^0\pi^0)\right|^2)}{3 \left|\mathcal{A}(\pi^+\pi^-)\right|^2 + \frac{3}{2} \left|\mathcal{A}(\pi^+\pi^-)\right|^2 + \frac{3}{2} \left|\mathcal{A}(\pi^0\pi^0)\right|^2 + \frac{2}{3}(\left|\mathcal{A}(\pi^0\pi^0)\right|^2 + \left|\mathcal{A}(\pi^0\pi^0)\right|^2)} \sin(\delta_I - \delta_p).

With the isospin and topological amplitudes of $D \rightarrow \pi\pi$ modes, ratio $R$ is derived to be

$$R = 6\sqrt{6} \frac{|\lambda_b|}{\lambda_d} \frac{\mathcal{A}_{1/2}^{t}}{3 \mathcal{A}_{1/2}^{t}} \sin\gamma \sin(\delta_I - \delta_p) \sin(\delta_I - \delta_p),$$

in the SM. According to Eq. (22), ratio $R$ is determined by the $CP$ violation in the $\Delta I = 1/2$ amplitude. The dependence of ratio $R$ on $\delta_p$ is plotted in Fig. 3. It is found that $R \sim \mathcal{O}(10^{-3})$ in the most values of $\delta_p$. 

FIG. 3: Ratio $R$ dependent on $\delta_p$ in the cases of negative (left) and positive (right) $\delta_I$, in which the horizontal pink shadow is $1\sigma$ experimental limitation to date taken from HFLAV [42].
III. ESTIMATION IN THE FINAL-STATE INTERACTION

To understand the large $CP$ asymmetries in the $D \to \pi\pi$ system in the Standard Model, one should explain how the penguin amplitude enhanced by non-perturbative QCD effects. In the $D \to \pi\pi$ modes, the penguin contributions include $\lambda_d P_d + \lambda_s P_s$. Considering all the tree amplitudes are proportional to $\lambda_d$ in the $D \to \pi\pi$ modes, we write penguin amplitudes as $\lambda_d P_d + \lambda_s P_s = \lambda_d P_d - (\lambda_d + \lambda_b) P_s = \lambda_d P_b - \lambda_b P_s$ in Eqs. (8) $\sim$ (10). $P_b$ can be included into the tree amplitudes. The $CP$ asymmetries are induced by the interference between $\lambda_b P_s$ with other decay amplitudes. In this section, we try to study how large the penguin amplitude $P_s$ could be in the final-state interaction.

For the two-body heavy meson weak decays, the FSI effects can be modeled as exchanges of one particle between two particles generated from the short-distance tree emitted process. There are $s$-channel and $t$-channel contributions in the final state interaction, which are depicted in Fig. 4. In the $s$-channel contribution, the resonance state in the $D \to \pi\pi$ decay has the quantum number $J^{PC} = 0^{++}$ derived from the final state particles. Ref. [18] suggested that $f_0(1710)$ playing an important role in enhancing the penguin amplitude. However, the $CP$ asymmetry ratio between $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ modes is expected to be $-1.06$. It is attributed to the suppression by the ratio of branching fraction, $Br(f_0(1710) \to \pi\pi)/Br(f_0(1710) \to KK) \approx 0.4$. In this work, we study the $t$-channel FSI contribution to the penguin amplitude.

In Ref. [20], we proposed an approach to relate topological diagram to the re-scattering triangle diagram. The re-scattering contribution in the penguin amplitude can be described by Fig. 5. In the topological diagram $P$ in Fig. 5, the indices $i, j, k$ and $l$ present the light quark $u, d$ or $s$ in the charm decay. $P$ diagram can be obtained by twisting quark lines from a short-distance $T$ diagram, as shown in the second diagram, $L(P)$, in Fig. 5. $L(P)$ forms a triangle diagram at hadron level, which is the third diagram in Fig. 5. In the triangle diagram, the vertex in the left is a weak vertex.
and the other vertices are strong vertices. The quark loop in penguin can be understood as the light quark $q_l$ exited from the weak vertex goes through three propagators then returns to the weak vertex in the triangle diagram.

There are two triangle diagrams contributing to the penguin diagram $P_s$, exchanging a vector meson $K^*$ and pseudoscalar meson $K$ respectively, as shown in Fig. 6. Since the weak vertex of triangle diagram is a short-distance $T$ diagram, the factorization approach is used to estimate the weak decay amplitude, as done in Ref. [28]. It is parameterized as the decay constant of the emitted meson and the transition form factor of another meson. For the amplitude of total triangle diagram, there are several calculational methods, in which the treatment of hadronic loop integration is different [24–30]. In this work, we adopt the optical theorem and Cutkosky cutting rule as in Ref. [28], in which the intermediate states are treated to be on their mass shell. In this way, only the absorptive part is calculated.

Considering the exchanged meson being generally off-shell, a form factor is introduced as

$$F(t) = \frac{\Lambda^2 - m_t^2}{\Lambda^2 - t}$$  \hspace{1cm} (23)

to compensate the off-shell effect [45], where $t$ is momentum square of the exchanged meson. $F(t)$ is normalized to unity at the on-shell situation $t = p_t^2 = m_t^2$. The cutoff $\Lambda$ is parameterized as

$$\Lambda = m_t + \eta \Lambda_{QCD}$$  \hspace{1cm} (24)
FIG. 7: The theoretical prediction for the ratio $|P/(T+C)|$ with $\eta$ varying from $-1.6 \sim 1.6$ (blue) and $t = m_t^2$ (red) in the $F(t)$.

with $\Lambda_{QCD} = 330$MeV for the charm decays. The parameter $\eta$ can not be calculated from the QCD method. In Fig. 7, we display the dependence of the ratio $|P/(T+C)|$ on the parameter $\eta$. It is found that $|P/(T+C)|$ is very sensitive to the value of $\eta$. With $\eta$ varying in the range between $-1.6$ and $1.6$, the ratio $|P/(T+C)|$ varies from zero to four. The large $|P/(T+C)|$ in some area indicates that the large penguin is accessible in the non-perturbative QCD enhancement. If we set $t = m_t^2$, $|P/(T+C)|$ is expected to be 1.96. It is consistent with the penguin amplitude extracted from $CP$ asymmetry in the $D^0 \to \pi^+\pi^-$ mode.

IV. SUMMARY

We studied the $CP$ asymmetries in the $D \to \pi\pi$ system based on the isospin and topological analysis. According to the new measurement of $CP$ violation in the $D^0 \to \pi^+\pi^-$ decay, we concluded that $CP$ violation in the $D^0 \to \pi^0\pi^0$ decay could reach to be $\mathcal{O}(10^{-3})$, which is available on Belle II in the foresee future. Besides, the large $CP$ asymmetries in the $D \to \pi\pi$ system might be understood by the final-state interaction.
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