Decay Processes in the Presence of Thin Superconducting Films

Per K. Rekdal a,∗ and Bo-Sture K. Skagerstam b,†

a Institut für Theoretische Physik, Karl-Franzens-Universität Graz, Universitätsplatz 5, A-8010 Graz, Austria
b Complex Systems and Soft Materials Research Group, Department of Physics, The Norwegian University of Science and Technology, N-7491 Trondheim, Norway

In a recent paper [Phys. Rev. Lett. 97, 070401 (2006)] the transition rate of magnetic spin-flip of a neutral two-level atom trapped in the vicinity of a thick superconducting body was studied. In the present paper we will extend these considerations to a situation with an atom at various distances from a dielectric film. Rates for the corresponding electric dipole-flip transition will also be considered. The rates for these atomic flip transitions can be reduced or enhanced, and in some situations they can even be completely suppressed. For a superconducting film or a thin film of a perfect conducting material various analytical expressions are derived that reveals the dependence of the physical parameters at hand.

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I. INTRODUCTION

Harnessing the interactions of electromagnetic field and matter is one of the ultimate goals of atom optics. One promising approach towards control of matter waves on small scales is to trap and manipulate cold neutral atoms in microtraps near structures microfabricated on a surface, known as atom chips [1]. Magnetic traps on such atom chips are commonly generated either by microfabricated current-carrying wire [1] or by poled ferromagnetic films [2, 3] attached to some dielectric or metallic body. However, the proximity of the atoms to the surface threatens to decohere the quantum state of the atoms through electromagnetic field fluctuations. This effect arises because the resistivity of the surface is always accompanied by field fluctuations as a consequence of the fluctuation-dissipation theorem. For an atom close to the surface of a dielectric body these fluctuating fields can be strong enough to drive magnetic and electric dipole transitions in the atom, as e.g. shown in recent experimental studies [4, 5, 6]. If the atom is in a magnetic or electric trap, these flip transitions may lead to atom loss. Such transitions are therefore most often undesirable, and we want to reduce them or even suppress them completely.

In the present paper we intend to explicitly write down the flip rate for both of these types of transitions for the dielectric slab as shown in Fig. 1. We will e.g. consider a normal conducting slab as well as a superconducting slab, as described in Ref. [7]. To the best of our knowledge, this is not done for a general spin or dipole orientation, despite the fact that it in principle has been known for a long time (see e.g. Ref. [8]).

II. THEORY

A. Magnetic Spin Transition

We begin by considering an atom in an initial state |i⟩ and trapped at position \( \mathbf{r}_A = (0, 0, z) \) in vacuum, near a dielectric body. The rate \( \Gamma_B \) of spontaneous and thermally stimulated magnetic spin-flip transition into a final state |f⟩ has e.g. been derived in Ref. [9],

\[
\Gamma_B = \frac{\mu_0}{\hbar} \frac{2(\mu_B g_S)^2}{\hbar} \sum_{j,k=1}^{3} S_j S_k^* \times \text{Im} \left[ \nabla \times \nabla \times G(\mathbf{r}_A, \mathbf{r},\omega) \right]_{jk} (\pi_{th} + 1),
\]

\( j = 1,2,3 \) and \( S_j \) is the vector of the nuclear spin corresponding to the atomic state |i⟩.
where we have introduced the dimensionless components $S_j \equiv \langle f | S_j | h | i \rangle$ of the electron spin operators $S_j$ corresponding to the transition $| i \rangle \rightarrow | f \rangle$, with $j = x, y, z$. Here $\mu_B$ is the Bohr magneton, $g_S \approx 2$ is the electron spin $g$ factor, and $G(r_A, r_A, \omega)$ is the dyadic Green tensor of Maxwell’s theory. Eq. 1 follows from a consistent quantum-mechanical treatment of electromagnetic radiation in the presence of absorbing bodies [10, 11]. Thermal excitations of the electromagnetic field modes are accounted for by the factor $(\pi T_{\text{th}} + 1)$, where $\pi T_{\text{th}} = 1/(e^{\hbar/2k_B T_{\text{th}}}-1)$ is the Planck distribution giving the mean number of thermal photons per mode at frequency $\omega$ of the spin-flip transition. Here $T$ is the temperature of the dielectric body, which is assumed to be in thermal equilibrium with its surroundings, and $k_B$ is Boltzmann’s constant. The dyadic Green tensor is the unique solution to the Helmholtz equation

$$\nabla \times \nabla \times \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) - k^2 \epsilon(\mathbf{r}, \omega) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}') \mathbf{1},$$

with appropriate boundary conditions. Here $k = \omega/c$ is the wavenumber in vacuum, $c$ is the speed of light and $\mathbf{1}$ the unit dyad. The tensor $\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega)$ contains all relevant information about the geometry of the material and, through the electric permittivity $\epsilon(\mathbf{r}, \omega)$, its dielectric properties. Due to causality, any complex dielectric function must in general obey the Kramers-Kronig relations. Since we only consider non-zero frequencies in a suitable finite range, such dispersion relations will be of no concern in the present paper.

The current density in a superconducting media is commonly described by the Mattis-Bardeen theory [12]. Following Ref. [7], assuming non-zero frequencies $0 < \omega \ll \omega_D = 2\Delta(0)/\hbar$, where $\omega$ is the angular frequency and $\Delta(0)$ is the energy gap of the superconductor at zero temperature, the current density is well described by means of a two-fluid model [13, 14]. The dielectric function is in this case given by

$$\epsilon(\omega) = 1 - \frac{1}{k^2 \lambda_L^2(T)} + i \frac{2}{k^2 \delta^2(T)},$$

where $\lambda_L(T) = e\sqrt{m/\mu_0 n_s(T)}$ is the London penetration length and where $\delta(T) \equiv \sqrt{2/\omega_0 \mu_0 \sigma_n(T)}$ is the skin depth associated with the normal conducting electrons. As usual, $\mu_0$ is the permeability of vacuum and $e$ is the elementary charge. The total electron density $n_0(T)$ is constant and given by $n_0 = n_s(T) + n_n(T)$, where $n_s(T)$ and $n_n(T)$ are the electron densities in the superconducting and normal state, respectively, at a given temperature $T$. The optical conductivity corresponding to Eq. 3 is

$$\sigma(T) = 2/\omega_0 \delta^2(T) + i/\omega_0 \mu_0 \lambda_L^2(T).$$

Above the transition temperature $T_c$, the dielectric function in Eq. 3 reduces to the well known Drude form. We also stress that the theory in this paper is particular to non-magnetic media.

The rate $\Gamma_B$ of a magnetic spin-flip transition for an atom in the unbounded free-space is well known (see e.g. Refs. [11]), with the result

$$\Gamma_B = \Gamma_B S^2,$$

with

$$\Gamma_B = \mu_0 (\mu_B g_S)^2 \frac{\hbar^3}{3\pi}. \tag{5}$$

Here we have introduced the dimensionless spin factor $S^2 \equiv S_x^2 + S_y^2 + S_z^2$. The unbounded free-space lifetime corresponding to this magnetic spin-flip rate is $\tau_B^0 \equiv 1/\Gamma_B$.

In the following we apply our model to the geometry shown in Fig. 4, where an atom is located in vacuum at a distance $z$ away from a dielectric slab with thickness $H$. This slab is described by dielectric function as given by Eq. 6. The total magnetic transition rate

$$\Gamma_B = (\Gamma_B^0 + \Gamma_B^{\text{slab}})(\pi T_{\text{th}} + 1), \tag{6}$$

can then be decomposed into a free part and a part purely due to the presence of the slab. The latter contribution for an arbitrary spin orientation is given by

$$\Gamma_B^{\text{slab}} = 2\Gamma_B^0 \left( (S_x^2 + S_y^2) I_\| + S_z^2 I_\perp \right), \tag{7}$$

with the atom-spin orientation dependent integrals

$$I_\| = \frac{3}{8} \text{Re} \left( \int_0^\infty \frac{dq}{\eta_0} e^{i2\eta_0 kqz} \left[ C_N(q) - q^2 C_M(q) \right] \right), \tag{8}$$

and

$$I_\perp = \frac{3}{4} \text{Re} \left( \int_0^\infty \frac{dq}{\eta_0} q^3 e^{i2\eta_0 kqz} C_M(q) \right). \tag{9}$$

The scattering coefficients $C_N(q)$ and $C_M(q)$ are given by

$$C_N(q) = r_p(q) \frac{1 - e^{i2\eta(q) kH}}{1 - r_p(q) e^{i2\eta(q) kH}}, \tag{10}$$

and

$$C_M(q) = r_s(q) \frac{1 - e^{i2\eta(q) kH}}{1 - r_s(q) e^{i2\eta(q) kH}}, \tag{11}$$

with the electromagnetic field polarization dependent Fresnel coefficients

$$r_s(q) = \frac{\eta_0 - \eta(q)}{\eta_0 + \eta(q)}, \quad r_p(q) = \frac{\epsilon(q) - \eta(q)}{\epsilon(q) + \eta(q)}. \tag{12}$$

Here we have defined $\eta(q) = \sqrt{\epsilon(q) - q^2}$ and $\eta_0 = \sqrt{1 - q^2}$. For a thick slab with $H = \infty$, the above equations are reduced to the results in Ref. [15].
B. Electric Dipole Transition

The previous section concerns magnetic field fluctuations. In this section we will consider electric field fluctuations. For an electrical dipole transition, the rate spontaneous and thermally stimulated decay is given by (see e.g. Refs. [11]):

$$\Gamma_E = \frac{\mu_0}{\hbar} \frac{2\omega^2}{h} \sum_{j,k=1}^{3} d_j d_k^*$$

$$\times \Im \left[ G(r_A, r_A, \omega) \right]_{jk}(\pi_{th} + 1), \quad (13)$$

where $d_j \equiv \langle f|\hat{d}_j|i\rangle$, with $j = x, y, z$, is the matrix element of the atomic dipole operator $\hat{d}_j$ in the direction $j$, corresponding to the transition $|i\rangle \rightarrow |f\rangle$.

Let us apply this model to the geometry shown in Fig. 4 where an atom is located in vacuum at a distance $z$ away from a dielectric slab with thickness $H$. The total electric transition rate

$$\Gamma_E = (\Gamma_E^0 + \Gamma_{E,lab}) (\pi_{th} + 1), \quad (14)$$

can then be decomposed into a free part and a part purely due to the presence of the slab. The latter contribution for an arbitrary dipole orientation is given by

$$\Gamma_{E,lab} = 2\bar{\Gamma}_E^0 \left( (d_x^2 + d_y^2) J_\parallel + d_z^2 J_\perp \right), \quad (15)$$

where we have introduced the dipole factor $d^2 \equiv d_x^2 + d_y^2 + d_z^2$. The dipole orientation dependent integrals are

$$J_\parallel = \frac{3}{8} \Re \left( \int_0^\infty dq \, \frac{q}{\eta_0} \, e^{i2\eta_0 k_z} \left[ C_M(q) - \eta_0^2 C_N(q) \right] \right), \quad (16)$$

and

$$J_\perp = \frac{3}{4} \Re \left( \int_0^\infty dq \, \frac{q^3}{\eta_0} \, e^{i2\eta_0 k_z} C_N(q) \right). \quad (17)$$

Here we have defined

$$\bar{\Gamma}_E^0 = \bar{\Gamma}_E d^2, \quad (18)$$

with

$$\bar{\Gamma}_E = \mu_0 \frac{c^2}{3\pi \hbar} k^3, \quad (19)$$

i.e. the dipole-flip rate in unbounded vacuum for the electric dipole transition $|i\rangle \rightarrow |f\rangle$, with the corresponding free-space lifetime $\tau_{th}^0 \equiv 1/\bar{\Gamma}_E^0$. We mention that Eq. (18) is consistent with the definition in Refs. [11].

III. TWO LIMITING CASES

A. Magnetic Spin Transition

1. The Limit $\lambda_L(T) \ll \delta(T), H, \lambda$

Let us now consider a special case of the dielectric function in Eq. (8). The superconducting term dominates over the normal conducting term provided that $\lambda_L(T) \ll \delta(T)$. If, in addition, $\lambda_L(T) \ll \lambda$, which holds true in practically all cases of interest, then we can neglect the unit term in Eq. (9). Here $\lambda = 2\pi/k$ is the wavelength associated to the magnetic spin-flip transition. The dominant factor in the dielectric function is in this case real, and the main contribution to the integrals in Eqs. (5) and (8) occurs for values of $q$ such that $0 \leq q \leq 1$. For a slab with a thickness such that $\lambda_L(T) \ll H$, which also holds true in practically all cases of interest, the exponential functions in the scattering coefficients Eqs. (10) and (11) can be neglected. The scattering coefficients are then reduced to $C_N(q) \approx r_\| (q)$ and $C_M(q) \approx r_\perp (q)$ for all relevant values of $q$. Furthermore, with the above mentioned assumptions, the Fresnel coefficients are reduced to $r_\| (q) \approx 1$ and $r_\perp (q) \approx -1$. The integrals in Eqs. (5) and (8) can then be solved analytically. The total magnetic spin-flip rate for an atom above a slab is then

$$\Gamma_B^{pc} \approx \bar{\Gamma}_B^0 (\pi_{th} + 1)$$

$$\times \left[ S^2 + \frac{3}{2} f_\parallel (kz) (S_x^2 + S_y^2) + 3 f_\perp (kz) S_z^2 \right], \quad (20)$$

where we have defined

$$f_\parallel (kz) \equiv \frac{\sin(2kz)}{2kz} + f_\perp (kz), \quad (21)$$

$$f_\perp (kz) \equiv \frac{2kz \cos(2kz) - \sin(2kz)}{(2kz)^3}. \quad (22)$$

Note that Eq. (20) is not valid for an arbitrary small thickness $H$ of the slab. In the limit $\lambda_L(T) \rightarrow 0$, the magnetic spin-flip rate in Eq. (20) is exact. This result is consistent with Ref. [12].

Let us now consider the near-field case $kz = 2\pi z \ll 1$, which holds true in practically all cases of interest. The magnetic spin-flip rate is then reduced to

$$\Gamma_B^{pc, \perp} \approx \frac{(2kz)^2}{10} \bar{\Gamma}_B^0 (\pi_{th} + 1), \quad (23)$$

provided the atomic spin is oriented perpendicular to the slab, i.e. provided that $\langle f|\hat{S}_z|i\rangle = \langle f|\hat{S}_y|i\rangle = 0$. This result implies that, for an atom at the surface of the slab, i.e. $z = 0$, there is no magnetic spin-flip at all (see lower graph in Fig. 2). The particular atomic spin orientation under consideration is the only one that can give zero
spin-flip rate despite the presence magnetic field fluctuations. Furthermore, when the atomic spin is oriented parallel to the slab, the magnetic spin-flip rate is

$$\Gamma_{B}^{\parallel} \approx 2 \Gamma_{B}^{0} (\pi_{th} + 1),$$

(24)

for the near-field case $kz \ll 1$. This result shows that, in the near-field regime, the magnetic dipole-flip rate is twice the rate as compared to an atom in unbounded free-space, as e.g. pointed out in Ref. [18]. In current atomic chip design (see e.g. Ref. [4]) the typical atomic frequency is $\omega/2\pi = 560$ kHz and a typical atom-surface distance is $z = 50 \mu$m. Hence, we have $kz \sim 10^{-7}$, i.e. far within the near-field condition $kz \ll 1$.

In passing we also give the small $H$ expansion. For sufficiently small thickness of the slab, i.e. $H \ll \delta^2(T)/\lambda_L(T), z, \lambda_L(T)$, the magnetic spin-flip rate is

$$\Gamma_{B} \approx \Gamma_{B}^{0} (\pi_{th} + 1) \times \left[ S^2 + (S_x^2 + S_y^2) \frac{3}{64} \left( \frac{k \delta(T)}{k \lambda_L(T)} \right)^2 \left( \frac{H}{z} \right)^2 \right] \times \left[ S^2 + \lambda_L(T) \frac{3}{64} \left( \frac{k \delta(T)}{k \lambda_L(T)} \right)^2 \left( \frac{kH}{kz^2} \right)^2 \right].$$

(25)

This expression for the magnetic spin-flip rate as it is only valid for thicknesses of the slab smaller than the London penetration length.

2. The Limit $\delta(T) \ll \lambda_L(T), H, \lambda, z$

Let us now consider the case when the dielectric function as given by Eq. [3] is dominated by the normal conducting term, i.e. when $\delta(T) \ll \lambda_L(T)$ and $\delta(T) \ll \lambda$. The dominant factor in the dielectric function is in this case imaginary, corresponding to the well known Drude form, and the main contribution to the integrals in Eqs. [8] and [9] is for values of $q$ such that $q \lesssim 1/kz$. If, in addition, $\delta(T) \ll z$ then the exponential functions in the scattering coefficients in Eqs. [10] and [11] are negligible for all values $q \lesssim 1/kz$ provided that $\delta(T) \ll H$. The scattering coefficients are then reduced to $C_N(q) \approx 1$ and $C_M(q) \approx 1$, i.e. the same result as in the last subsection. Hence, the conditions $\delta(T) \ll \lambda_L(T), H, \lambda, z$ and $\lambda_L(T) \ll \delta(T), H, \lambda$ give the same results. In particular, for the perfect normal conducting limit, i.e. $\delta(T) \to 0$, the magnetic spin-flip rate as given by Eq. [20] is exact. Note that the perfect normal conducting limit is only valid for the case $\delta(T) \ll \lambda_L(T), H, \lambda, z$, which e.g. means that the slab can not be arbitrarily thin. It also means that, in contrast to the case as described in last subsection, the atom-surface distance $z$ can not be chosen arbitrary small.

We close this section by mentioning that, following the two-fluid model [13, 14] and using the Gorter-Casimir temperature dependence [13], the limit $\delta(T) \to 0$ is obtained for $T \to 0$, as e.g. described in Ref. [7].

![Graph](image-url)

**FIG. 2:** The lifetime $\Gamma_{B}^{\parallel}/\Gamma_{B}^{0} (\pi_{th} + 1)$ according Eq. (24). This fraction only depends on the product $kz$ and the atomic spin orientation. *Upper figure:* The spin-orientation is the same as in Refs. [2, 17], i.e. $|\langle f|S_y|i\rangle|^2 = |\langle f|S_x|i\rangle|^2$ and $\langle f|\hat{S}_z|i\rangle = 0$. *Lower figure:* The atomic spin is oriented perpendicular to the slab, i.e. $\langle f|\hat{S}_x|i\rangle = \langle f|\hat{S}_y|i\rangle = 0$. The spin-flip rate is completely suppressed for $kz = 0$ in this case.

**B. Electric Dipole Transition**

The only difference between the integrals in Eqs. [16, 17] and Eqs. [8, 9] is the position of the scattering coefficients $C_N(q)$ and $C_M(q)$. Hence, the correction to the vacuum dipole-flip rate corresponding to electric field fluctuations for the two limits as mentioned above is in general opposite in sign as compared to that of the magnetic spin-flip case. This was also pointed out in Ref. [8]. It can be understood in physical terms, since the electric and magnetic field are perpendicular. Hence, if $\delta(T) \ll \lambda_L(T), H, \lambda, z$ or $\lambda_L(T) \ll \delta(T), H, \lambda$ then the total electric dipole-flip rate for an atom above a slab is given by
\[ \Gamma_{E}^{pc} \approx \Gamma_{E} (\pi_{th} + 1) \]
\[ \times \left[ 2d^2 - \frac{3}{2} f_{||}(kz)(d_x^2 + d_y^2) - 3 f_{\perp}(kz)d_z^2 \right]. \]

This equation is consistent with the results in Ref. \[18\].

Let us again consider the near-field case \( kz \ll 1 \). The electric dipole-flip rate is then reduced to

\[ \Gamma_{E}^{pc, ||} \approx \frac{(2kz)^2}{5} \frac{\Gamma_{E}^0}{\pi_{th} + 1}, \]

provided that the atomic dipole is oriented parallel to the slab, i.e. provided that \( \langle f|d_{\parallel}|i \rangle = 0 \). This result implies that, for an atom at the surface of the slab, i.e. \( z = 0 \), there is no electric dipole-flip at all. The particular atomic dipole orientation under consideration is the only one that can give zero dipole-flip rate despite the presence of electric field fluctuations. Furthermore, when the atomic spin is oriented perpendicular to the slab, the electric dipole-flip rate is

\[ \Gamma_{E}^{pc, \perp} \approx 2 \frac{\Gamma_{E}^0}{\pi_{th} + 1}, \]

for the near-field case \( kz \ll 1 \). This result was pointed out by Babiker in Ref. \[18\].

To summarize, in the present paper we have reported results on the magnetic as well as electric decay properties of a neutral two-level atom trapped in the vicinity of a dielectric body. For a slab with vacuum on both sides (see Fig. \[1\]), we have obtained the flip rate for both of these types of transitions, for any spin or dipole orientation. The expression for the electric and magnetic transition rate can be solved exactly in two limiting cases, i.e. in the small skin depth limit for normal conducting metals and in the small London length limit for superconductors. In these limits, the correction to the vacuum rate for an electric dipole transition is opposite in sign as compared to that of a magnetic spin transition. These results are consistent with well known results, e.g. Refs. \[12\].

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[1] R. Folman, P. Krueger, J. Schmiedmayer, J. Denschlag, and C. Henkel, At. Mol. Opt. Phys. 48, 236 (2002).
[2] E.A. Hinds and I.A. Hughes, J. Phys. D: Appl. Phys 32, R119 (1999).
[3] S. Eriksson, F. Ramirez-Martinez, E.A. Curtis, B.E. Sauer, P.W. Nutter, E.W. Hill and E.A. Hinds, Appl. Phys. B 79, 811 (2004).
[4] M.P.A. Jones, C.J. Vale, D. Sahagun, B.V. Hall, and E.A. Hinds, Phys. Rev. Lett. 91, 080401 (2003).
[5] Y.J. Lin, I. Teper, C. Chin, and V. Vuletic, Phys. Rev. Lett. 92, 050404 (2004).
[6] D.M. Harber, J.M. McGuirk, J.M. Obrecht, and E.A. Cornell, J. Low. Temp. Phys. 133, 229 (2003).
[7] B.-S. Skagertam, U. Hohenester, A. Eiguren, and P.K. Rekdal, Phys. Rev. Lett. 97, 070401 (2006).
[8] P.W. Milonni and P.L. Knight, Opt. Commun. 9, 119 (1973).
[9] P.K. Rekdal, S. Scheel, P.L. Knight, and E.A. Hinds, Phys. Rev. A 70, 013811 (2004).
[10] C. Henry and R. Kazarinov, Rev. Mod. Phys. 68, 801 (1996).
[11] L. Knöll, S. Scheel, and D.-G. Welsch, *QED in dispersing and absorbing media*, in Coherence and Statistics of Photons and Atoms, ed. J. Peřina (Wiley, New York, 2001); T.D. Ho, L. Knöll and D.-G. Welsch, Phys. Rev. A 62, 053804 (2000); S. Scheel, L. Knöll and D.-G. Welsch, Phys. Rev. A 60, 4094 (1999); S. Scheel, L. Knöll and D.-G. Welsch, Phys. Rev. A 60, 1590 (1999); H. T. Dung, S. Y. Buhmann, L. Knöll, D.-G. Welsch, S. Scheel, and Jürgen Kästel, Phys. Rev. A 68, 043816 (2003);
[12] D.C. Mattis and J. Bardeen, Phys. Rev. 111, 412 (1958).
[13] H. London, Nature (London) 133, 497 (1944); H. London, Proc. R. Soc. London, Ser. A. 176, 522 (1940).
[14] C.S. Gorter and H. Casimir, Z. Phys. 35, 963 (1934); Z. Tech. Phys. 15, 539 (1934); C.J. Gorter, in *Progress in low Temperature Physics*, (North-Holland, Amsterdam, 1955).
[15] C. Henkel, S. Pötting and M. Wilkens, Appl. Phys. B 69, 379 (1999); C. Henkel and M. Wilkens, Europhys. Lett. 47, 414 (1999); B. Zhang, C. Henkel, E. Haller, S. Wildermuth, S. Hofferberth, P. Krüger, and J. Schmiedmayer, Eur. Phys. J. D 35, 97-104 (2005).
[16] P.B. Miller, Phys. Rev. 113, 1209 (1959).
[17] S. Scheel, P.K. Rekdal, P.L. Knight, and E.A. Hinds, Phys. Rev. A 72, 042901 (2005).
[18] M. Al-Amri and M. Babiker, Phys. Rev. A 67, 043820 (2003); M. Babiker, J. Phys. A: Math. Gen. 9 799 (1976).