$SU(4)_L \otimes U(1)_N$ model for the electroweak interactions

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Abstract

Assuming the existence of right-handed neutrinos, we consider an electroweak model based on the gauge symmetry $SU(4)_L \otimes U(1)_N$. We study the neutral currents coupled to all neutral vector bosons present in the theory. There are no flavor changing neutral currents at tree level, coupled with the lightest neutral vector boson.
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Symmetry principles have been used in elementary particle physics at least since the
discovery of the neutron. A symmetry is useful to both issues: the classification of particles
and the dynamics of the interactions among them. The point is that there must be a part
of the particle spectrum in which the symmetry manifests itself at least in an approximate
way. This is the case for quarks \( u \) and \( d \) and in the leptonic sector for the electron-neutrino
and electron. For instance, the \( SU(2) \) appears as an approximate symmetry in the doublets
\((\nu_e, e)^T\). If one assumes this symmetry among these particles and in the sequential families
as well, almost all the model’s predictions are determined.

The full symmetry of the so called Standard Model is the gauge group \( SU(3)_c \otimes SU(2)_L \otimes
U(1)_Y \). This model spectacularly explains all the available experimental data \[1\]. Usually
it is considered that this symmetry emerges at low energies as a result of the breaking of
higher symmetries. Probably, these huge symmetries are an effect of grand unified scenarios
and/or their supersymmetric extensions.

Considering the lightest particles of the model as the sector in which a symmetry is
manifested, it is interesting that the lepton sector could be the part of the model determining
new approximate symmetries. For instance, \( \nu, e \) and \( e^c \) could be in the same triplet of an
\( SU(3)_L \otimes U(1)_N \) symmetry. This sort of model has been proposed recently \[2\]. In this case
neutrinos can remain massless in arbitrary order in perturbation theory, or they get a mass
in some modifications of the models \[3\]. If we admit that right-handed neutrinos do exist,
it is possible to build a model in which \( \nu^c, \nu \) and \( e \) are in the same multiplet of \( SU(3) \) \[4\].
In fact, if right-handed neutrinos are introduced it is a more interesting possibility to have
\( \nu, e, \nu^c \) and \( e^c \) in the same multiplet of a \( SU(4)_L \otimes U(1)_N \) electroweak theory.

Notice that using the lightest leptons as the particles which determine the approximate
symmetry, if each generation is treated separately, \( SU(4) \) is the highest symmetry group to
be considered in the electroweak sector. A model with the \( SU(4) \otimes U(1) \) symmetry in the
lepton sector was suggested some years ago in Ref. \[5\]. However, quarks were not considered
there. This symmetry in both, quarks and leptons, was pointed out recently \[6\] and here we
will consider the details of such a model.
Hence, our model has the full symmetry $SU(3)_c \otimes SU(4)_L \otimes U(1)_N$. These sort of models are anomaly free only if there are equal number of $4$ and $4^*$ (considering the color degrees of freedom), and furthermore requiring the sum of all fermion charges to vanish. Two of the three quark generations transform identically and one generation, it does not matter which one, transforms in a different representation of $SU(4)_L \otimes U(1)_N$. This means that in these models as in the $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ ones, in order to cancel anomalies, the number of families ($N_f$) must be divisible by the number of color degrees of freedom ($n$). Hence the simplest alternative is $n = N_f = 3$. On the other hand, at low energies these models are indistinguishable from the Standard Model.

The electric charge operator is defined as

$$Q = \frac{1}{2}(\lambda_3 - \frac{1}{\sqrt{3}}\lambda_8 - \frac{2}{3}\sqrt{6}\lambda_{15}) + N,$$

where the $\lambda$-matrices are a slightly modified version of the usual ones.

$$\lambda_3 = \text{diag}(1, -1, 0, 0), \quad \lambda_8 = (\frac{1}{\sqrt{3}})\text{diag}(1, 1, -2, 0), \quad \lambda_{15} = (\frac{1}{\sqrt{6}})\text{diag}(1, 1, 1, -3).$$

Leptons transform as $(1, 4, 0)$, one generation, say $Q_{1L}$, transforms as $(3, 4, +2/3)$ and the other two quark families, say $Q_{\alpha L}$, $\alpha = 2, 3$, transform as $(3, 4^*, -1/3)$,

$$f_{\alpha L} = \begin{pmatrix} \nu_a \\ l_a \\ \nu^c_a \\ l^c_a \end{pmatrix}_L, \quad Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ u' \\ J \end{pmatrix}_L, \quad Q_{\alpha L} = \begin{pmatrix} j_i \\ d'_i \\ u_\alpha \\ d_\alpha \end{pmatrix}_L,$$

where $a = e, \mu, \tau$; $u'$ and $J$ are new quarks with charge $+2/3$ and $+5/3$ respectively; $j_i$ and $d'_i$, $i = 1, 2$ are new quarks with charge $-4/3$ and $-1/3$ respectively. We remind that in Eq. (2) all fields are still symmetry eigenstates. Right-handed quarks transform as singlets under $SU(4)_L \otimes U(1)_N$.

Quark masses are generated by introducing the following Higgs $SU(3)_c \otimes SU(4)_L \otimes U(1)_N$ multiplets: $\eta \sim (1, 4, 0)$, $\rho \sim (1, 4, +1/3)$ and $\chi \sim (1, 4, +1)$.
In order to obtain massive charged leptons it is necessary to introduce a \((1,10^*,0)\) Higgs multiplet, because the lepton mass term transforms as \(\bar{f}_L^c f_L \sim (6_A \oplus 10_S)\). The \(6_A\) will leave some leptons massless and some others degenerate. Therefore we will choose the \(H = 10_S\). Explicitly

\[
H = \begin{pmatrix}
H_0^0 & H_1^+ & H_2^0 & H_2^- \\
H_1^+ & H_1^{++} & H_3^+ & H_3^0 \\
H_2^0 & H_3^+ & H_4^0 & H_4^- \\
H_2^- & H_3^0 & H_4^- & H_2^{--}
\end{pmatrix}.
\] (4)

If \(\langle H_3^0 \rangle \neq 0, \langle H_{1,2,4}^0 \rangle = 0\) the charged leptons get a mass but neutrinos remain massless, at least at tree level. In order to avoid mixing among primed and unprimed quarks we can introduce another multiplet \(\eta'\) transforming as \(\eta\) but with different vacuum expectation value (VEV). The corresponding VEVs are the following \(\langle \eta \rangle = (v,0,0,0), \langle \rho \rangle = (0,u,0,0), \langle \eta' \rangle = (0,0,v',0), \langle \chi \rangle = (0,0,0,w),\) and \(\langle H \rangle_{24} = \langle H_3^0 \rangle = \nu''\) for the decuplet. In this way we have that the symmetry breaking of the \(SU(4)_L \otimes U(1)_N\) group down to \(SU(3)_L \otimes U(1)_{N'}\) is induced by the \(\chi\) Higgs. The \(SU(3)_L \otimes U(1)_{N'}\) symmetry is broken down into \(U(1)_{em}\) by the \(\rho, \eta, \eta', H\) Higgs.

The Yukawa interactions are

\[
-L_Y = \frac{1}{2} G_{ab} f_{aL}^c f_{bL} H + F_{1k} \bar{Q}_{1L} u_{kR} \eta + G_{ak} \bar{Q}_{aL} u_{kR} \rho^* + F_{1k} \bar{Q}_{1L} d_{kR} \rho + F_{ak} \bar{Q}_{aL} d_{kR} \eta^* + h_1 \bar{Q}_{1L} u_{kR} \eta' + h_{a\alpha} \bar{Q}_{aL} d_{kR} \eta'^* + \Gamma_{1L} \bar{Q}_{1L} J_{RX} + \Gamma_{ai} \bar{Q}_{aL} j_{iL} \chi^* + H.c.,
\] (5)

where \(a = e, \mu, \tau; k = 1, 2, 3; i = 1, 2\) and \(\alpha, \beta = 2, 3\). We recall that up to now all fields are weak eigenstates.
The electroweak gauge bosons of this theory consist of a 15 $W^i_\mu$, $i = 1, ..., 15$ associated with $SU(4)_L$ and a singlet $B_\mu$ associated with $U(1)_N$.

The gauge bosons $-\sqrt{2}W^+ = W^1 - iW^2$, $-\sqrt{2}V_1^- = W^6 - iW^7$, $-\sqrt{2}V_2^- = W^9 - iW^{10}$, $-\sqrt{2}V_3^- = W^{13} - iW^{14}$, $-\sqrt{2}U^- = W^{11} - iW^{12}$ and $\sqrt{2}X^0 = W^4 + iW^5$ have masses

$$M^2_W = \frac{g^2}{4}(v^2 + u^2 + 2v''^2), \quad M^2_{V_1} = \frac{g^2}{4}(v^2 + u^2 + 2v''^2), \quad M^2_{V_2} = \frac{g^2}{4}(v^2 + u^2 + 2v''^2), \quad M^2_{V_3} = \frac{g^2}{4}(v^2 + u^2 + 2v''^2), \quad M^2_X = \frac{g^2}{8}(v^2 + v'^2), \quad M^2_U = \frac{g^2}{4}(v^2 + u^2 + 4v''^2). \quad (6a)$$

(6b) The matrix in (7) has determinant equal to zero as it must be in order to have a massless photon. There are four neutral bosons: a massless $\gamma$ and three massive ones: $Z, Z', Z''$ such that $M_Z < M_{Z'} < M_{Z''}$. The lightest one, say $Z$, corresponds to the neutral boson of the Standard Model.

The photon field is

$$A_\mu = \frac{1}{(1 + 4t^2)^{\frac{1}{2}}} \left( tW^3_\mu - \frac{t}{\sqrt{3}} W^8_\mu - \frac{2\sqrt{6}}{3} tW^{15}_\mu + B_\mu \right), \quad (8)$$

with the electric charge defined as

$$|e| = \frac{gt}{(1 + 4t^2)^{\frac{1}{2}}} = \frac{g'}{(1 + 4t^2)^{\frac{1}{2}}}. \quad (9)$$
In the following, we will use the approximation \( v = u = v'' \equiv v_1 \ll v' = w \equiv v_2 \). In this approximation the three nonzero masses are given by

\[
M_n^2 \approx \frac{g^2}{4} (4\lambda_n) v_2^2, \quad n = 0, 1, 2;
\]

where

\[
\lambda_n = \frac{1}{3} \left[ A + 2 \left( A^2 + 3B \right)^{1/2} \cos \left( \frac{2n\pi + \Theta}{3} \right) \right],
\]

\[
A = \frac{3}{4} + t^2 + \left( \frac{5}{4} + t^2 \right) a^2, \quad B = -\frac{1}{8} (1 + 3t^2) - \frac{1}{4} (3 + 7t^2) a^2,
\]

\[
C = \frac{3}{32} (1 + 4t^2) a^2, \quad \Theta = \arccos \left[ \frac{2A^3 + 9AB + 27C}{2(2a^2 + 3B)^{1/2}} \right],
\]

and we have defined \( a \equiv v_1/v_2 \). The respective eigenvectors are

\[
Z_{n\mu} \approx x_n W^3_\mu + y_n W^8_\mu + z_n W^{15}_\mu + w_n B,
\]

with

\[
x_n = -\frac{a^2}{2t} \cdot \frac{4\lambda_n - [2(a^2 + 3)t^2 + a^2 + 1]}{(a^2 - 1)(2\lambda_n - a^2)} w_n,
\]

\[
y_n = \frac{\sqrt{3}}{6t(a^2 - 1)(2\lambda_n - a^2)} \left[ 32\lambda_n^2 - 4[8a^2 + 3 + 8(a^2 + 1)t^2] \lambda_n \\
+ [5a^2 + 9 + 2(11a^2 + 17)t^2] a^2 \right] w_n,
\]

\[
z_n = -\frac{2}{\sqrt{6}t(a^2 - 1)} \cdot [2\lambda_n - 2(a^2 + 1)t^2 - a^2] w_n,
\]

\[
w_n^2 = \frac{1}{1 + x_n^2/w_n^2 + y_n^2/w_n^2 + z_n^2/w_n^2}.
\]

The hierarchy of the masses is \( M_0 > M_2 > M_1 \). Hence we can identify the eigenvector with \( n = 1 \) as being the neutral vector boson of the Standard Model.

However, we have checked numerically if the respective eigenvalue satisfies the relation

\[
M_Z^2/M_W^2 = 1/\cos^2 \theta_W,
\]

with \( \theta_W \) the weak mixing angle. Using \( a = 0.01 \) and \( t = 1.79 \).
we obtain $M_Z^2/M_W^2 \approx 0.97$ which does not agree with the value of the Standard Model $M_Z^2/M_W^2 \approx 1.30$ with $\sin^2 \theta_W = 0.2325$. In fact, $M_Z/M_W \approx 0.99$. This suggests that, in this model, $Z$ and $W$ are mass degenerate at tree level. Hence, the right value for the ratio $M_Z/M_W$ must arise only through radiative corrections. We recall that in models with $SU(3)_L \otimes U(1)_N$ symmetry $M_Z/M_W$ is bounded from above and the weak mixing angle has an upper bound.

For these values of $a$ and $t$ the other two neutral bosons have $M_Z'/M_W \approx 60$ and $M_{Z''}/M_W \approx 189$. If $a < 0.1$, all these results depend very weakly on the value chosen for $a$.

The weak neutral currents have been, up to now, an important test of the Standard Model. In particular it has been possible to determine the fermion couplings, so far all experimental data are in agreement with the model. In the present model, the neutral currents couple to the $Z_n$ neutral boson as follows

$$L_{n}^{NC} = -\frac{g}{2c_W}[(\bar{\psi}_L\gamma^\mu \psi_L)_{L} + (\bar{\psi}_R\gamma^\mu \psi_R)_{R}]Z_{n\mu}$$

where $c_W \equiv \cos \theta_W$ and

$$L_{u_1}^n = -c_W \left( x_n + \frac{1}{\sqrt{3}}y_n + \frac{1}{\sqrt{6}}z_n + \frac{4}{3}w_n t \right),$$

(15a)

$$L_{u_\alpha}^n = -c_W \left( x_n - \frac{1}{\sqrt{3}}y_n - \frac{1}{\sqrt{6}}z_n - \frac{2}{3}w_n t \right),$$

(15b)

$$L_{u'}^n = -c_W \left( -\frac{2}{\sqrt{3}}y_n + \frac{1}{\sqrt{6}}z_n + \frac{4}{3}w_n t \right),$$

(15c)

$$R_{u_1}^n = R_{u_\alpha}^n = R_{u'}^n = -\frac{4}{3}c_W w_n t$$

(16)

for the charge $3/2$ quarks, and

$$L_{d_1}^n = -c_W \left( -x_n + \frac{1}{\sqrt{3}}y_n + \frac{1}{\sqrt{6}}z_n + \frac{4}{3}w_n t \right),$$

(17a)

$$L_{d_\alpha}^n = -c_W \left( -x_n - \frac{1}{\sqrt{3}}y_n - \frac{1}{\sqrt{6}}z_n - \frac{2}{3}w_n t \right),$$

(17b)
\[ L_{d_1}^n = -c_W \left(\frac{2}{\sqrt{3}}y_n - \frac{1}{\sqrt{6}}z_n - \frac{2}{3}w_n t\right), \quad (17c) \]

\[ R_{d_1}^n = R_{d_2}^n = R_{d_3}^n = \frac{2}{3}c_W w_n t, \quad (18) \]

for the charge $-1/3$ quarks. We have checked numerically that only for $n = 1$ we have (for a given value of $t$ and $a < 0.1$)

\[ L_{u_1}^1 = L_{u_2}^1 = L_{u_3}^1 \neq L_{u'}^1, \quad (19a) \]

and

\[ L_{d_1}^1 = L_{d_2}^1 = L_{d_3}^1 \neq L_{d_1'}^1 = L_{d_2'}^1. \quad (19b) \]

Hence, we can introduce a discrete symmetry, as in Model I of Ref. [4], in order to obtain a mass matrix which does not mix $u_k$ with $u'$ and $d_k$ with $d_i'$, $k = 1, 2, 3$ and $i = 1, 2$. That is, the mass matrices have a tensor product form, next they can be diagonalized with unitary matrices which are themselves tensor products of unitary matrices. We see that in this case the GIM mechanism [11] is implemented, at tree level, in the $Z_1(\equiv Z^0)$ couplings. We must stress that, if the new quarks $u'$ and $d_i'$ are very heavy, the requirements for natural (independent of mixing angles) flavor conservation in the neutral currents to order $\alpha G_F$ [12] break down, and it should be necessary to impose the restriction that the mixing angles between ordinary and the new heavy quarks must be very small [13].

It is useful to define the coefficient $V = (L + R)/2$ and $A = (L - R)/2$. In the Standard Model, at tree level, we have $V^SM_\psi = t_{3L_\psi} - 2Q_\psi \sin^2 \theta_W$ and $A^SM_\psi = t_{3L_\psi}$, where $t_{3L_\psi}$ is the weak isospin of the fermion $\psi$. Hence, we have $V^SM_u \approx 0.19$ and $A^SM_u = 0.5$ for the charge $2/3$ sector, and $V^SM_\psi \approx -0.345$ and $A^SM_\psi = -0.5$ for the charge $-1/3$ sector. In our model, also at tree level, using $a = 0.01$ and $t = 1.79$ we obtain $V^SM_d \approx 0.19$, $A^SM_d \approx 0.5$ for $u_k$; and $V^SM_d \approx -0.345$, $A^SM_d \approx -0.5$ for $d_k$. We see that the values are in agreement with the values of the Standard Model. On the other hand $V^SM_\psi \approx -0.310$, $A^SM_\psi \approx 0$ and $V^SM_d \approx 0.15$, $A^SM_d \approx 0$.

For leptons we have
\begin{align}
    L_\nu^n &= L_{u_1}^n + \frac{4}{3} c_W w_n t, \quad R_\nu^n = -L_{u_1}^n - \frac{4}{3} c_W w_n t, \\
    L_\mu^n &= L_{d_1}^n + \frac{4}{3} c_W w_n t, \quad R_\mu^n = -\frac{3}{\sqrt{6}} z_n. \quad (20)
\end{align}

For \( a = 0.01 \) and \( t = 1.79 \) we obtain \( V_\nu \approx 0.5, A_\nu \approx 0.5 \) and \( V_\mu \approx -0.036, A_\mu \approx -0.5 \) which are also in agreement with the values of the Standard Model, \( V_{\nu}^{SM} = A_{\nu}^{SM} = 0.5 \) and \( V_{\mu}^{SM} \approx -0.035 A_{\mu}^{SM} = -0.5 \). Notice also that at tree level neutrinos are still massless but they will get a calculable mass through radiative corrections. In this kind of model it is possible to implement the Voloshin’s mechanism i.e., in the limit of exact symmetry, a magnetic moment for the neutrino is allowed, and a mass is forbidden \[5\]. We will not discuss this issue here.

Finally, we write down the charged current interactions in terms of the symmetry eigenstates. In the leptonic sector they are

\begin{equation}
    \mathcal{L}^{CC}_L = -\frac{g}{2} \left[ \bar{\nu}_L \gamma^\mu l_L W^+_\mu + \bar{\nu}_L \gamma^\mu l_L V^+_\mu + \bar{\nu}_L \gamma^\mu \nu^c_L V^+_\mu + \bar{\nu}_L \gamma^\mu l_L U^{++}_\mu \right] + H.c. \quad (21)
\end{equation}

We have also the interaction \((g/2)\bar{\nu}_L \gamma^\mu \nu_L X^0\).

In the quark sector we have

\begin{align}
    \mathcal{L}^{CC}_Q &= -\frac{g}{\sqrt{2}} \left[ \bar{u}_{kL} \gamma^\mu d_{kL} W^+_\mu + (\bar{u}_{L}^\prime \gamma^\mu d_{1L} + \bar{u}_{aL} \gamma^\mu d_{iL}^\prime) V^+_\mu \\
    &\quad + (\bar{J}_L \gamma^\mu u_{1L} + \bar{d}_{1L} \gamma^\mu j_{1L}) V^+_\mu + (\bar{J}_L \gamma^\mu u_{1L}^\prime) \\
    &\quad + (\bar{d}_{iL} \gamma^\mu j_{1L}) V^+_3 + (\bar{J}_L \gamma^\mu d_{1L} - \bar{u}_{iL} \gamma^\mu j_{1L}) U^{++}_\mu \right] + H.c., \quad (22)
\end{align}

where \( k = 1, 2, 3; \alpha = 2.3; i = 1, 2 \). We have also interaction via \( X^0 \) among primed and unprimed quarks of the same charge as \( \bar{u}_{L}^\prime \gamma^\mu u_L \) and so on.

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REFERENCES

[1] Particle Data Group, Phys. Rev. D45, Part II, (1992).

[2] F. Pisano and V. Pleitez, Phys. Rev. D46, 410(1992); P. H. Frampton, Phys. Rev. Lett. 69, 2889(1992); R. Foot, O. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D47, 4158(1993).

[3] F. Pisano, V. Pleitez and M.D. Tonasse, “Radiatively induced electron and electron-neutrino masses” preprint IFT-P.059/93.

[4] J.C. Montero, F. Pisano and V. Pleitez, Phys. Rev. D 47, 2918(1993).

[5] M.B. Voloshin, Sov. J. Nucl. Phys. 48, 512(1988).

[6] V. Pleitez, “Possibilities beyond 3-3-1 models” preprint IFT-P.010/93 (extended version).

[7] The possibility that flavor changing neutral currents processes can discriminate which generation transforms differently in 3-3-1 models it was recently considered by D. Dumm, F. Pisano and V. Pleitez, Flavor changing neutral currents in \( SU(3) \otimes U(1) \) models, Preprint IFT-P.026/93 (extended version), and J.T. Liu, Generation non-universality and flavor changing neutral currents in the 331 model, hep-ph/9312312.

[8] D. Amati et al., Nuovo Cimento. 34, 1732(1964). See the Appendix of Ref. [4].

[9] See for example F. Cajori, An Introduction to the Theory of Equations, Dover Publications, 1969; p.68-72.

[10] This value for \( t \) is chosen in order to obtain the right value for the neutral coupling for quarks in (L3)-(L8).

[11] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2, 1285(1970).

[12] S.L. Glashow and S. Weinberg, Phys. Rev. D 15, 1958(1977).
[13] E.C. Poggio and H.J. Schnitzer, Phys. Rev. D15, 1973(1977).