Influence of temperature restrictions on the heliocentric motion controlling of a solar-sailing spacecraft

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Abstract. Solar sailing spacecraft uses the light pressure to change the trajectory and are most effective in the immediate Sun vicinity, but the sail surface temperature most significant increases in the same area. Therefore, when designing solar sailing "hot" missions, we should consider the temperature restriction when choosing the control laws. The paper proposes the following approach to the implementation of the temperature restriction. When exceeding a predetermined temperature limit values the sail deviates from the direction of the radiation flux so that the direction of the acceleration projection on the Ecliptic plane is not changed, but the total sail orientation angle increased. The additional acceleration that occurs when using this approach, directed along the Ecliptic normal, can be used to rotate the orbital plane, if this is necessary to fulfill the flight purpose or to form a cylindrical orbit.

1. Introduction

The advantage of a solar sail spacecraft (SSSC) is the lack of fuel that the mission may continue as long as it take. The disadvantages of a solar sail spacecraft are the smallest of the acceleration and the dependence of its acceleration on the distance to the Sun: the farther from the Sun, the less is the light pressure and there by the acceleration. In addition, the solar sail efficiency depends on the sail loading (areal density) which is the total mass spacecraft divided by the sail area. The smaller the ratio, the greater the acceleration from the forces of light pressure and the faster the sail will arrive at the chosen star. Moreover, the front solar sail surface should be made of well reflective material to achieve its full potential.

The spacecraft with a solar sail in the nearest vicinity of the Sun is subjected to forces significantly different from the classical Newtonian bounded two-body problem. When calculating the gravitational forces it is necessary to take into account the relativistic effects of a rotating attracting body. The light pressure should be calculated from the Sun model as a non-point radiation source. The characteristics of the solar sail surface under the action of a significant radiation flux will rapidly degrade. In addition, it is necessary to take into account the design restrictions on the temperature of the surface of the sail.

It is impossible to obtain an explicit and optimal solution for launching a solar sail spacecraft to the target orbit, taking into account all these disturbances or it is associated with SS gigantic computational difficulties. In this work, we propose to use the locally known optimal control laws and the method of their combination developed by the authors to select the ballistic scheme of the mission taking into account all the described disturbances.
2. Statement of solar sail’s mission design

We assume the mission is performed in the following scenario. The spacecraft is removed from the Earth's action sphere due to the upper stage with a zero hyperbolic raisin and after that flight to purposed heliocentric orbit in the minimum duration. It is required to determine the optimal control program to the sail's installation angles, which ensures the fulfillment of the boundary conditions of the interplanetary flight and the temperature restriction to the sail with fixed design parameters:

$$u_{\text{opt}}(t) = \arg\min_{u(t) \in U} \left\{ t_m(u(t), x(t)) \mid x(t_0) = x_0, x(t_f) = x_f, T(t) \leq T_{\text{max}} \right\}. \quad (1)$$

Here $$\lambda_1(t), \lambda_2(t)$$ are the sail’s installation angles: between projection of acceleration and orbital plane and between acceleration and orbital plane respectively, $$x(t_0) = x_0, x(t_f) = x_f$$ are the phase coordinates of solar sail spacecraft in initial and finish time respectively, $$t_m$$ is a total mission duration, $$T(t)$$ is an equilibrium temperature of the sail surface, $$T_{\text{max}}$$ is a maximum possible equilibrium temperature.

Figure 1. The direction acceleration form solar light is defined by two controlled angles.

The geometry of space-time in the vicinity of the slowly rotating Sun is described up to linear order in the angular momentum J of the Sun by the large-distance limit of the Kerr metric sentence. If this, the motion equations can be given in spherical phase coordinates $$x = (r, \varphi, \theta, V_r, V_\varphi, V_\theta)$$ as follows.

$$\dot{r} = V_r,$$

$$\dot{V}_r = \frac{V_\varphi^2 - \sin^2 \varphi}{r \sin^2 \theta} - \frac{GM}{r^2} f + \frac{GM}{r^3} f^2 V_r^2 + \frac{fV_r^2}{r} + \frac{2GJ}{c^2 r^3} fV_\varphi + a_r + f_r,$$

$$\dot{\varphi} = \frac{V_\varphi}{r \sin \theta},$$

$$\dot{V}_\varphi = -\frac{V_r V_\phi \cos \theta}{r \sin \theta} + \frac{2GJ}{c^2 r^3} V_\varphi + a_\varphi + f_\varphi,$$

$$\dot{\theta} = \frac{V_\theta}{r},$$

$$\dot{V}_\theta = \frac{V_\varphi^2 \cos \theta}{r \sin \theta} - \frac{V_r V_\phi}{r^2} + \frac{4GJ V_\varphi}{r^2} \cos \theta + a_\theta + f_\theta.$$
Here \( f = 1 - \frac{2GM}{c^2 r} \), \( \mathbf{v} = (V_x, V_y, V_z)^T \) is a vector of spacecraft velocity, \( \mathbf{a} = (a_x, a_y, a_z)^T \) is an acceleration from Sun radiation pressure, \( \mathbf{f} = (f_x, f_y, f_z)^T \) is a disturbance acceleration, \( t \) is time as measured by a distant static observer, \( G \) is the gravitational constant, \( M = 1.99 \times 10^{30} \) kg is the mass of the Sun, \( c \) is velocity of light, and \( J = (1.9 \pm 0.015) \times 10^{41} \) kg\( \cdot m^2 \) is an angular momentum of a Sun. These equations we used to simulate the motion. All calculations have done in dimensionless values, which allow improving algorithm convergence.

We have considered a sail with an imperfectly reflecting surface and that the acceleration is determined by two components directed along the normal \( a_\perp \) and along \( a_\parallel \) the surface:

\[
\begin{align*}
a_\perp &= 2 \frac{S_r}{cm} S \cos \lambda_1 \cos \lambda_2 \left( a_1 \cos \lambda_1 \cos \lambda_2 + a_2 \right), \\
a_\parallel &= 2 \frac{S_r}{cm} S \cos \lambda_1 \cos \lambda_2 a_1 \sqrt{1 - \left( \cos \lambda_1 \cos \lambda_2 \right)^2}, \\
a_1 &= \frac{1}{2} \left( 1 + \varphi \rho \right), \quad a_2 = \frac{1}{2} \left( 1 - \varphi \rho \right), \quad (3)
\end{align*}
\]

where \( S_r \) is a solar electromagnetic radiation flux density per unit area of a sail located at a heliocentric distance \( r \); \( c \) is the speed of light; \( m \) is the mass of the spacecraft; \( S \) is the sail surface area; \( \rho \) is a reflection coefficient; \( \varphi \) is a spectral reflection factor of the sail’s surface, describing the dependence of the surface reflection coefficient on the wavelength of the light flux; \( \varepsilon_f, \varepsilon_b \) are radiation coefficients of the front and back sail’s surface respectively, \( B_f, B_b \) are non-Lambert coefficients of the front and back surface of the sail, which describe the angular distribution of emitted and diffusely reflected photons [1].

In order to take into account the Sun as an extended source of radiation pressure, we recalculate the light pressure included in the expressions (3) by the formula.

\[
S_r = \frac{I_0}{3\pi R_e^2} \left[ 1 - \left( 1 - \left( \frac{R_e}{r} \right)^2 \right)^{3/2} \right]. \quad (4)
\]

Here \( R_e = 1.392 \times 10^8 \) m is the Sun’s radius. Thus, we take into account only the fall of the value, but do not take into account the change in the direction of the light flux.

When moving near the Sun, the surface temperature of the solar sail can rise to critical values, leading to the destruction of structural elements. Therefore, the temperature limit must be taken into account in such missions. The equilibrium temperature of the sail surface was calculated by a refined model based on the Stefan-Boltzmann law and taking into account the change in the optical characteristics of materials when the surface temperature changes and described:

\[
T = \left( \frac{S_r}{\sigma_{SB} \varepsilon_f + \varepsilon_b} \cos \psi (t) \right)^{1/4} \left( \frac{r_0}{r} \right)^{2/5} \quad (5)
\]

Here \( \sigma_{SB} = 5.67 \times 10^{-8} \) W\( \cdot \)m\( ^2 \)K\(^{-4} \) is the Stefan-Boltzmann constant, \( r_0 \) is a nominal radius (in the paper - the Earth’s orbit radius), \( \psi (t) \) is the angle between the direction of solar light flux and normal of sails.
surface. Equation (5) takes into account that the emission factor linearly increases with increasing surface temperature [2]. In Fig. 2 we calculated temperature variation with distance from the Sun and installation angle for aluminum (Fig. 2a) and beryllium (Fig. 2b) solar sails.

Figure 2. Temperature change for solar sail front surface covered with a) aluminum and b) beryllium.

More detailed information about the effects of optical degradation of the sail surface can be found in paper [3]. We use the same parametric model, which is assumed that the reflection coefficient, the spectral reflection factor, and the emissivity coefficient of the front sail’s surface depend on the dimensionless cumulative dose of solar radiation \( \Sigma(t) \) obtained by the sail. We reckon the emissivity coefficient of the back sail’s surface, and the non-Lambert coefficients of the front and back surface do not change in the flight duration.

\[
\rho(t) = \rho_0 \frac{1 + d_\rho e^{-\delta_\rho \Sigma(t)}}{1 + d_\rho},
\]

\[
\zeta(t) = \zeta_0 \frac{1 + d_\zeta e^{-\delta_\zeta \Sigma(t)}}{1 + d_\zeta},
\]

\[
\varepsilon_f(t) = \varepsilon_{f0} \frac{1 + d_\varepsilon e^{-\delta_\varepsilon \Sigma(t)}}{1 + d_\varepsilon}.
\]

Here \( \delta_\rho \), \( \delta_\zeta \), \( \delta_\varepsilon \) are the degradation coefficients and \( d_\rho \), \( d_\zeta \), \( d_\varepsilon \) are the degradation factors of the corresponding optical parameters. The degradation coefficient is determined based on half the lifetime of the sail under the influence of solar radiation

\[
\delta_{\rho,\zeta,\varepsilon} = \frac{\ln 2}{\Sigma_{\rho,\zeta,\varepsilon}},
\]
where \( \hat{\rho}_{\mu,\nu} \) is the solar radiation dose, which leads to a half deterioration of the corresponding optical characteristics. The degradation factor determines the value of the optical characteristic at which the sail should stop functioning:

\[
\rho_{\mu} = \frac{\rho_{0}}{1 + d_{r}}, \quad \zeta_{\mu} = \frac{\zeta_{0}}{1 + d_{r}}, \quad \epsilon_{\mu} = \epsilon_{0}(1 + d_{r}).
\] (8)

The dimensionless total dose of solar radiation is calculated as the ratio of the total radiation power received by the sail during the flight to the solar radiation power received by the one square meter of an area at a distance of one AU during one year.

\[
\Sigma(t) = \int_{0}^{t} \frac{\cos \psi(t)}{r^2} dt.
\] (9)

The formulae (5) demonstrates that the surface temperature results to not only the Sun’s distance and the optical parameters of the surface, but also to the angle of installation of the sail. Therefore, we can influence to the surface temperature by changing the installation angles.

\[
\cos \psi = \cos \lambda_{1} \cdot \cos \lambda_{2}.
\] (10)

Since the installed angle \( \lambda_{i} \) is determined by the control program to achieve the flights purpose, we can use the change in angle \( \lambda_{2} \) to reduce the surface temperature. The value of angle \( \lambda_{2} \) can be obtained from (5) at a given limit temperature \( T_{\max} \):

\[
\lambda_{2} = \arccos \left( \frac{\sigma_{sh} T_{\max}^{4} \epsilon_{r} + \epsilon_{n}}{(1 - \rho) \cos(\hat{\lambda}_{1})} \left( \frac{r}{r_{0}} \right)^{8/5} \right).
\] (11)

We used the known control laws [4], providing the maximum speed of semi-major axis \( A \) reduction and eccentricity \( e \) reduction to determine the plane installation angle \( \lambda_{1} \). Keplerian elements equations for the ideal reflective and non-degradation solar sail are of the form:

\[
\frac{dA}{dt} = 2a\epsilon \left( \frac{p}{GM} \right) \sqrt{1 - e^{2}} \left( \sin \vartheta \cos^{3} \lambda_{i} + (1 + e \cos \vartheta) \cos^{2} \lambda_{i} \sin \lambda_{i} \right),
\]

\[
\frac{de}{dt} = a \left( \frac{p}{GM} \right) \left( \sin \vartheta \cos^{3} \lambda_{i} + \frac{e \cos^{2} \vartheta + 2 \cos \vartheta + e}{1 + e \cos \vartheta} \cos^{2} \lambda_{i} \sin \lambda_{i} \right).
\] (12)

Here \( \vartheta \) is a true anomaly; \( p = A(1 - e^{2}) \) is an orbital parameter. Generalizing formulas (12) we can write, the law changes of the Keplerian elements \( K \) of the form:

\[
\frac{dK}{dt} = f_{1}(\vartheta) \cos^{3} \lambda_{i} + f_{2}(\vartheta) \cos^{2} \lambda_{i} \sin \lambda_{i}.
\] (13)

The locally optimal control laws [Macdonald et al., 2007; Gorbunova and Starinova, 2017] can provide the most rapid change in the Keplerian elements \( K \):

\[
\lambda_{1} = \frac{1}{2} \arcsin \left( \frac{f_{2}(\vartheta)}{f_{1}(\vartheta) + \sqrt{9f_{1}(\vartheta)^{2} + 8f_{2}(\vartheta)^{2}}} \right).
\] (14)
Here \( f_1, f_2 \) are functions depending on the eccentricity and the true anomaly, sign “+” corresponds to increasing the Keplerian element, and the sign “-” corresponds to decreasing. In particular, to change the eccentricity it is:

\[
f_1 = \sin \theta, \quad f_2 = \frac{e \cos^2 \theta + 2 \cos \theta + e}{1 + e \cos \theta},
\]

and the semi-major axis it is:

\[
f_1 = e \sin \theta, \quad f_2 = 1 + e \cos \theta.
\]

The control laws (14-16) were deduced that planar interplanetary trajectories are consists mainly of two sections: a section changing semi-major axis and a section decreasing of eccentricity. This has been demonstrated when we are calculating the flights to Mercury, Venus, Mars and Jupiter and transferring Solar-sail’s spacecraft to the nearest Sun’s orbit. It is a mission that aims to fly in a circular heliocentric orbit with radius 0.05 AU (7.5 million km or about ten radii of the Sun).

3. Simulation results

We used the author’s software for modeling interplanetary flight with a solar sail. This software allows us using the different ballistic, optical, and design parameters of the SSSC to consider its heliocentric motion. We developed blending technique of locally optimal control laws, which are described in our works [5]. Simulation of the SSSC controlled heliocentric motion performed numerically in a plane polar coordinate system. Trajectories of the motion planet is considered circular; the angular position of the planet was calculated based on the current date. We use the duration of the flight as the optimality criterion.

The spacecraft, the movement of which we will model in this article, will have the following design and optical parameters: \( S=2,000 \text{ m}^2, m=100 \text{ kg}, \rho_0=0.98, \varphi_0 = 0.94, \varphi_{f0}=0.01, \varphi_s=0.55, \beta_j=0.79, \beta_k=0.55 \). These parameters [6] correspond to the sail with a highly reflective aluminum-coated front side and a highly emissive chromium-coated backside. We believe the optical parameters of the sail degrade with parameters \( d_\rho=0.1, \delta_\rho=0.231, d_\delta=0.1, \delta_\delta=0.139, d_{\delta \rho}=0.1, \delta_{\delta \rho}=0.231 \). That corresponds to the deterioration of the reflection coefficient by 50 % for the ten years of the sail in Earth’s orbit.

The results of flight simulation to planets are shown in table 1 and the results to the nearest near-solar neighborhood flight in table 2. Data on the interplanetary flights show the sail surface temperature does not exceed the limit value. This limitation should be taken into account when calculating missions whose target is significantly closer to the Sun than the Mercury’s orbit.

One can notice that in order to maintain required solar sail temperature for the flight towards heliocentric orbit with radius 0.05 AU, we have to implement angle \( \lambda \) control. This control leads to temperature stabilization around 1000 K, but reduces value of the acceleration produced by solar radiation pressure. As result the flight duration increased by 265 days. Moreover, the trajectory when approaching critical temperature deviates from planar motion. As it shown in Fig. 3 the plane of the orbit declines from initial, and that can be used to form a specific polar orbit for the Sun observation. It is also possible to keep initial plane with changing sign of \( \lambda \) periodically.

| Flight parameters                  | Mercury Date of exit from the Earth's action sphere | Venus Date of exit from the Earth's action sphere | Mars Date of exit from the Earth's action sphere | Jupiter Date of exit from the Earth's action sphere |
|-----------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Date of exit from the Earth's action sphere | 6 Jun 2022                                    | 12 Jun 2021                                   | 18 Aug 2022                                   | 1 Sep 2022                                   |
| Duration of the section of semi-major axis reduction, days | 284,75                                       | 165                                          | 127,2                                        | 4608                                         |
Table 2. Results of the near Sun’s vicinity flight simulation.

| Flight parameters                                      | Without constraints on temperature | With constraints on temperature |
|--------------------------------------------------------|------------------------------------|---------------------------------|
| Date of exit from the Earth’s action sphere            | 10.05.2022                         | 10.05.2022                      |
| Duration of the section of semi-major axis reduction, days | 2090                               | 2218.3                          |
| Duration of the section of eccentricity reduction, days | 5                                  | 141.5                           |
| Date of the working orbit formation                    | 30.05.2028                         | 1.02.2029                       |
| The maximum steady-state temperature, °K               | 1310.9                             | 1000                            |
| Flight duration, days                                  | 2095                               | 2360                            |

Figure 3. The heliocentric trajectory towards to the 0.05 AU orbit.

Such trajectories always occur when applying thrust perpendicular to orbital plane. If it necessary to keep planar motion during all flight one can maintain required temperature via controlling only angle $\lambda$. However, for such kind of flight will be implemented different locally optimal control law with restriction on possible control angle. It will drastically increase flight duration.

4. Conclusion
We outlined the most significant processes that should be considered when calculating trajectory and control for the close to the Sun mission with a solar sail. Equations (2) describe motion near massive rotating body and equation (4) shows how the solar radiation pressure is calculated when considering
the Sun as extended body. For such mission optical parameters degradation and sail surface temperature became extremely important characteristics and we calculated them by equations (5) and (6). Suggested locally optimal control laws (14-16) can be effectively used as a first iteration for calculating optimal trajectories and estimation of the mission possibility. The simulation results showed how the temperature limitation changes trajectory, particularly orbital plane orientation. This deviation can be used to form unique cylindrical orbits that hovers above Ecliptic plane or any other heliocentric orbit. Overall, this work demonstrated the importance of considering temperature in solar sail motion and possible control to maintain it at acceptable limit.

References
[1] Forward R L 1999 Grey Solar Sails Journal of the Astronautical Sciences 38(2) 161-185
[2] Kezerashvili R Y 2014 Solar sail: materials and space environmental effects Berlin Heidelberg: In Advances in Solar sailing / Springer 573-592
[3] Dachwald B, Mengali G, Quarta A A and Macdonald M 2006 Parametric model and optimal control of Solar sails with optical degradation Journal of Guidance, Control, and Dynamics 29(5) 1170-1178
[4] Macdonald M, McInnes C and Dachwald B 2007 Heliocentric Solar sail orbit transfers with locally optimal control laws Journal of Spacecraft and Rockets 44(1) 273-276
[5] Starinova O L and Chernyakina I V 2018 The Mission’s Design of a Solar Sail Spacecraft to the Nearest Circumsolar Space Based on Local-optimal Control Laws. Advances in Astronautics Science and Technology 1(1) 81-85
[6] Wright J L 1992 Space sailing Gordon and Breach Science Publishers: Philadelphia