Basal-Plane Magnetic Anisotropies of High-\(\kappa\) \(d\)-Wave Superconductors in a Mixed State: A Quasiclassical Approach

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We study the basal-plane anisotropies of reversible magnetization and torque in a mixed state of layered \(d\)-wave superconductors based on the quasiclassical version of the BCS-Gor'kov theory. Both the longitudinal magnetization \((M_L)\) and torque \((\tau)\) show fourfold oscillations as a function of the field angle \(\chi\). The relationship between the node position and the oscillatory patterns shown by \(M_L\) and \(\tau\) is clarified. It is also shown that the sign of the \(\tau(\chi)\)-oscillation does not change between \(H_{c1}\) and \(H_{c2}\), while the sign of the \(M_L(\chi)\)-oscillation changes. The newly obtained result for \(\tau\) indicates that the torque experiment can allow us to detect the in-plane anisotropies of \(H_{c2}\) even in a material with strong fluctuations such as cuprate or organic superconductors, where the \(H_{c2}\) itself cannot be determined experimentally.

KEYWORDS: \(d\)-wave superconductor, mixed state, magnetization, torque, in-plane anisotropy

1. Introduction

It has been widely known that the effects of nonlocal electrodynamics in a clean type-II superconductor cause several macroscopic anisotropy phenomena.\textsuperscript{1} For the uniaxial anisotropy in a layered superconductor, or the basal-plane anisotropy in a multicomponent superconductor, the local London or the basal-plane anisotropy phenomenon. Typical examples are the field-angle-dependent oscillations of the specific heat\textsuperscript{13,14} and thermal conductivity.\textsuperscript{15} These phenomena can be used to clarify the node position of the gap function, through the fact that the Doppler-shifted quasiparticles cause the variation of these quantities depending on the angle between the applied field direction and the nodal direction. For more details, we refer the reader to the seminal theoretical work by Vekhter et al.\textsuperscript{16} and a more quantitative calculation by Miranovi\textsuperscript{17} et al.

Magnetization is another fundamental quantity in the mixed state of layered \(d\)-wave superconductors. Thanks to the newly developed “shaking” technique\textsuperscript{18} and the arrival of high-quality low-pinning samples, we can now obtain reversible magnetizations in a broad region of the field-temperature phase diagram. Indeed there exist precise reversible magnetization measurements\textsuperscript{19,20} for borocarbides, showing clear fourfold basal-plane anisotropies. In our previous work,\textsuperscript{21} we studied such a basal-plane anisotropy of the longitudinal magnetization \(M_L\). We demonstrated that the experimental data of \(M_L\) for borocarbides can be explained by considering both gap and Fermi surface anisotropies. However, results on the transverse component \(M_T\) of the reversible magnetization, or torque \(\tau\), have not been discussed.

Experimentally, the basal-plane magnetic torque in a mixed state was measured by Ishida et al.\textsuperscript{22} for an untwinned single crystal, \(YBa_2Cu_3O_7-\delta\) (YBCO). They observed a fourfold oscillation, in addition to a rather large twofold oscillation presumably due to the Cu-O chains specific to YBCO. Referring to the result based on a nonlocal GL theory,\textsuperscript{9} they concluded that the observed fourfold anisotropy is consistent with the simple \(d_{x^2-y^2}\)-wave model. However, their interpretation strongly relies on the conjecture that the sign of the \(\tau\)-oscillation does not change between \(H_{c1}\) and \(H_{c2}\).\textsuperscript{23} In this connection, it is worth emphasizing that the sign of the \(M_L\)-oscillation does change between \(H_{c1}\) and \(H_{c2}\).\textsuperscript{21,24} Namely, the anisotropy of \(M_L\) in low fields is opposite to what is expected from the \(H_{c2}\)-anisotropy. Thus, it is of importance to clarify the \(\tau\)-oscillation behavior in the whole field range from \(H_{c1}\) to \(H_{c2}\).

In this work we study the basal-plane longitudinal magnetization \(M_L\) and torque \(\tau\) in a mixed state of layered \(d\)-wave superconductors within the quasiclassical Eilenberger formalism. The present paper is an extension of our previous work\textsuperscript{21} on the longitudinal magnetization to the transverse component, and clarifies that the sign of \(\tau\)-oscillation does not change while the sign of \(M_L\)-oscillation changes. Then, it is shown that a simple interpretation connecting the presence or absence of sign changes to the anisotropy dependences of \(H_{c1}\) and \(H_{c2}\) is available. In addition, we provide the details of our anal-

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ysis that were not discussed in our previous work. We use a simple \( d_{x^2-y^2} \)-wave model with the Fermi surface being isotropic within the basal plane. Since no detailed theoretical study on the basal-plane magnetic torque has been reported, it is meaningful to study the simple \( d \)-wave model. Of course, in an actual material, another source of complication may be skipped by those who are not so interested in the theoretical study presented in this paper gives a good result, at least the approximate solution in our previous work\(^{21} \) or Pesch’s solution\(^{26} \) the solution obtained by this method is called a “full solution” in this paper.

We use the Cartesian coordinates \((x_1, x_2, x_3)\) constituting the right-handed system, and set the magnetic induction \( B \) parallel to \( \hat{x}_3 \). We divide the microscopic field \( b(r) \) into a spatially uniform part \( B \) (i.e., induction), and a periodic part \( \delta b(r) \) with zero mean within the vortex lattice cell:

\[
b(r) = B + \delta b(r).
\]

Since the latter part is known to be of the order of \( O(1/\kappa^2) \ll 1 \) and negligible in high-\( \kappa \) superconductors except for \( H \approx H_c \), we neglect \( \delta b \) from now on. To be consistent with this assumption, we neglect the demagnetization effect. The set of primitive translation vectors \( m, n \) of the vortex lattice in this paper is given as

\[
m = (2\pi/\nu)\hat{x}_1, \quad n = (2\pi\zeta/\nu)\hat{x}_1 + \nu\hat{x}_2,
\]

where lengths are measured in units of \( r_B = (2|e|B)^{-1/2} \) above and in the following. The real constants \( \zeta \) and \( \nu \) specify the form of vortex lattice. For a triangular vortex lattice, we set \( \zeta = 1/2 \) and \( \nu = (3\pi^2)^{1/4} \), while for a square vortex lattice, we set \( \zeta = 1/2 \) and \( \nu = \sqrt{2} \). In any case, the difference in vortex configurations has a negligible influence on the thermodynamics at the level of the mean field approximation.

We fix the gauge of the vector potential as

\[
A(r) = -Bx_2\hat{x}_1.
\]

The boundary condition for the pair potential \( \Delta(r) \) is gauge dependent, and with the present gauge, \( \Delta(r) \) can be expressed as the superposition of each Landau level function \( \psi_N \):

\[
\Delta(r) = \Delta_0 \sum_{N=0}^{N_{\max}} d_N \psi_N,
\]

where

\[
\langle \cdots \rangle = \frac{\int_{FS} dS(\cdots)/|v|}{\int_{FS} dS/|v|}
\]

denotes the average over the Fermi surface with \( dS \) being the area element of the Fermi surface.

### 2.1 Full solution

In this subsection, we propose a new method of solving the Eilenberger equations for the vortex lattice states in extreme type-II superconductors with large GL parameters, \( \kappa \gg 1 \). Since this method is beyond either the approximate solution in our previous work\(^{21} \) or Pesch’s solution\(^{26} \) the solution obtained by this method is called a “full solution” in this paper.

We use the Cartesian coordinates \((x_1, x_2, x_3)\) constituting the right-handed system, and set the magnetic induction \( B \) parallel to \( \hat{x}_3 \). We divide the microscopic field \( b(r) \) into a spatially uniform part \( B \) (i.e., induction), and a periodic part \( \delta b(r) \) with zero mean within the vortex lattice cell:

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where

\[
\langle \cdots \rangle = \frac{\int_{FS} dS(\cdots)/|v|}{\int_{FS} dS/|v|}
\]

denotes the average over the Fermi surface with \( dS \) being the area element of the Fermi surface.
$g(\varepsilon_n, p, r)$ is a periodic quantity of $r$, and it can be expanded in the lattice sums by Fourier transformation:

$$g(\varepsilon_n, p, r) = \sum_Q g_Q(\varepsilon_n, p) \exp(iQ \cdot r), \quad (12)$$

$$g_Q(\varepsilon_n, p) = \langle g(\varepsilon_n, p, r) \exp(-iQ \cdot r) \rangle_s, \quad (13)$$

where $\langle \cdots \rangle_s$ denotes the spatial average. The reciprocal lattice vector $Q$ is given by

$$Q = m\nu \hat{x}_1 + (n\zeta + n)(2\pi/\nu)\hat{x}_2, \quad (14)$$

with $m$ and $n$ being integers. Substituting eqs. (10) and (12) into eq. (1), we have an expression of $f$,

$$f = 2w_p \sum_{Q,N} g_Q d_N \beta_{Q,N}, \quad (15)$$

$$\beta_{Q,N} = \Delta_0 (2\varepsilon_n + iv \cdot \Pi)^{-1} \langle e^{iQ \cdot r} \psi_N \rangle. \quad (16)$$

The remaining task is to derive the expression of $\beta_{Q,N}$. Using the parameter representation $(2\varepsilon_n + iv \cdot \Pi)^{-1} = \int_0^\infty d\rho e^{-2\varepsilon_n \rho} e^{-i\nu v_2 \rho^2/\rho^2_0} \times e^{-i\nu_1 \Pi_1} e^{-i\nu_2 \Pi_2}$, we obtain

$$(2\varepsilon_n + iv \cdot \Pi)^{-1} = \int_0^\infty d\rho e^{-2\varepsilon_n \rho} e^{-i\nu_2 \rho^2/\rho_0^2 \cdot v_2}$$

$$\times e^{-i\nu_1 \Pi_1} e^{-i\nu_2 \Pi_2}, \quad (17)$$

where the operator identity $e^{A+B} = e^{-\frac{i}{2} [A,B]} e^{A} e^{B}$ was used, which holds if $[[A, B], A] = [[A, B], B] = 0$. Then, by applying the identity $e^{A+B} f(x) = f(x + a)$ for a nonsingular function $f$, and introducing the notation $s = (\rho/\rho_0)(v_1 \hat{x}_1 + v_2 \hat{x}_2)$, we have

$$\beta_{Q,N} = \Delta_0 \int_0^\infty d\rho e^{-2\varepsilon_n \rho}$$

$$\times e^{-\frac{i}{2} \nu_2 \rho^2/\rho_0^2} e^{iQ \cdot (r-s)} \psi_N (r-s), \quad (18)$$

and we can finally solve the Eilenberger equation (1).

To numerically obtain the self-consistent solutions for $f$, $f^\dagger$, and $g$, we use the following procedure. First we give initial values for $\{d_N\}$ and $\{g_Q\}$. Next we use eqs. (16) and (18) to obtain the new $f$, and eqs. (2), (3) to obtain $f^\dagger$ and $g$. From $g(\varepsilon_n, p, r)$, $g_Q(\varepsilon_n, p)$ is obtained using eq. (13). Each $\{d_N\}$ is determined by the following gap equation projected onto each Landau level:

$$\left( \ln \left( \frac{T}{T_c} \right) + 2\pi T \sum_{\varepsilon_n>0} \frac{1}{\varepsilon_n} \right) d_N = \frac{2\pi T}{\Delta_0} \sum_{\varepsilon_n>0} \left[ \psi_N^* (w_p^* f) \right]_s. \quad (19)$$

Finally, we return to eq. (1) and repeat the same procedure until self-consistency is achieved.

Once we correctly determine $f$, $f^\dagger$, and $g$, the physical quantities can be calculated. In particular, the thermodynamic quantities can be obtained from the free energy $F/V$ through the thermodynamic relation. The expression of $F/V = F/V - B^2/8\pi$ is given as

$$\frac{F}{V} = N(0) \left[ \frac{1}{|V|} |\Delta|^2 - 2\pi T \sum_{\varepsilon_n>0} \langle \Delta^+ f + \Delta f^\dagger \rangle \right]$$

$$+ \frac{f(2\varepsilon_n + iv \cdot \Pi) f^\dagger + f^\dagger(2\varepsilon_n + iv \cdot \Pi^*) f}{2(1 + g)} \right]_s \quad (20)$$

$$= \pi TN(0) \sum_{\varepsilon_n>0} \left[ \frac{(g-1) \langle f(w_p \Delta^*) + f^\dagger(w_p \Delta) \rangle}{(g+1)} \right]_s. \quad (21)$$

Here, $|V|^{-1} = \ln(T/T_c) + 2\pi T \sum_{\varepsilon_n>0} \varepsilon_n^{-1}$ is the strength of the attractive interaction. To proceed to the last line, we assumed that $f$, $f^\dagger$, and $\Delta$ satisfy the Eilenberger equations (eqs. (1), (2)) and the self-consistent equation (eq. (4)).

### 2.2 Approximate solution

To solve eq. (1) in a more numerically tractable manner, we introduce the approximate solution that was used in our previous work.\textsuperscript{21} Hereafter, the solution obtained by the method presented in this subsection is called the “approximate solution”.

Let us introduce an approximation which is the generalization of the one used by Pesch:\textsuperscript{26}

$$f(\varepsilon_n, p, r) \approx 2g(\varepsilon_n, p, r) w_p \left( (2\varepsilon_n + iv \cdot \Pi)^{-1} \Delta(r) \right). \quad (22)$$

This approximation relies on the fact that the normal component of the quasiclassical Green’s function $g$ has a weaker spatial dependence than the anomalous component $f$, as pointed out by Brandt et al.\textsuperscript{27} We emphasize here that in contrast to Pesch’s solution, we do not replace $g(\varepsilon_n, p, r)$ in eq. (22) by its spatial average $\langle g(\varepsilon_n, p, r) \rangle_s$, therefore, we can reproduce the first non-Gaussian term of the nonlocal GL free energy given in ref. 28. As we show later, Pesch’s solution can be obtained from the approximate solution by completely neglecting the spatial dependence of the $g$-function and by restricting $\Delta$ to the lowest ($N = 0$) Landau level. Recently, Pesch’s solution has been widely used in slightly different contexts.\textsuperscript{29-33}

To proceed with our approximation, it is convenient to define a new function $\Phi$ in a similar manner as described in ref. 34 (\Phi-parameterization):

$$\Phi(\varepsilon_n, p, r) = 2w_p \left( (2\varepsilon_n + iv \cdot \Pi)^{-1} \Delta(r) \right). \quad (23)$$

This function offers simple representations of $f$, $f^\dagger$, and $g$:

$$f = g\Phi, \quad (24)$$

$$f^\dagger = g^\dagger \Phi, \quad (25)$$

$$g = \frac{1}{\sqrt{1 + \Phi^* \Phi}}. \quad (26)$$

where $\Phi^\dagger(\varepsilon_n, p, r) = \Phi^\star(\varepsilon_n, -p, r)$. Repeating essentially the same argument as described in the previous subsection, we obtain

$$\Phi = 2w_p \sum_{N} \beta_{Q=0,N} d_N. \quad (27)$$
After some calculations, $\beta_{Q=0,N}$ in eq. (27) can be transformed into the form used in ref. 21,

$$\beta_{Q=0,N} = \Delta_0 \int_0^\infty dp e^{-2\epsilon_n p} \alpha_N(p), \quad (28)$$

$$\alpha_N(p) = \sum_{m=-\infty}^\infty \frac{C_m H_N(x_2 + \nu m - \Re \lambda)}{\sqrt{2N \pi N!}} \times e^{-((\lambda^2 - \lambda^2)/4 \epsilon_n)^2} \times (\epsilon_n - x_2 - \nu m - \lambda)^2/2 - \nu m x(29)$$

where $\alpha_N$ depends on $p$ through $\lambda = (x_2 + \nu m)/\rho / r_B$. The numerical procedure for the above approximate method was explained in our previous work.21

Now we show that the approximate solutions of eqs. (24)-(26) can reproduce the nonlocal GL free energy derived in ref. 28 up to the quartic term. We start from eqs. (24)-(26). Expanding $f$, $f^\dagger$, and $g$ in terms of $\Delta$, we have

$$f = \Phi - \frac{1}{2} \Phi^2 \Phi^\dagger + \ldots,$$

$$f^\dagger = \Phi^\dagger - \frac{1}{2} (\Phi^\dagger)^2 \Phi + \ldots, \quad (30)$$

$$g = 1 - \frac{1}{2} \Phi \Phi^\dagger + \ldots.$$ 

Then, we substitute these equations into eq. (21) and obtain

$$\tilde{F}/V = N(0) \left[ \Delta \left( \frac{1}{2|V|} \right) \Delta + \pi T \sum_{\epsilon_n > 0} \left( - \Phi + \frac{1}{4} \Phi^\dagger (\Phi^\dagger)^2 \right) \right] + \text{c.c.} \quad \text{up to } O(|\Delta|^4).$$

This expression of $\tilde{F}/V$ is equivalent to that derived in ref. 28 using diagrammatic perturbative calculation. This fact justifies the approximate solution near $H_c2$.

2.3 Pesch's solution

It is instructive to see how the approximate solution presented above is connected with Pesch's solution. Pesch's solution can be derived from the approximate solution by imposing two additional conditions: (i) completely neglect the spatial dependence of the $g$-function, and (ii) restrict $\Delta$ to the lowest ($N = 0$) Landau level. Below, we briefly explain how to derive Pesch's solution from the approximate solution in a two-dimensional case. Following Pesch (see eq. (7) of ref. 26), we rewrite eq. (26) in the form $g = (1 + \Phi^\dagger)^2 = 1$. Keeping the above-mentioned two conditions and taking the spatial average of both sides of the equation, we obtain (cf. eq. (27) of ref. 30)

$$[g(\epsilon_n, p, r)]_s = \left( 1 + P(\epsilon_n, p) \right)^{-1/2}, \quad (32)$$

$$P(\epsilon_n, p) = 4 |w_p d_0|^2 \Delta_0^2 \left[ \left( \int_0^\infty d\rho_1 e^{-2\epsilon_n \rho_1} \alpha_0(p) \right) \right]^2 \times \left( \int_0^\infty d\rho_2 e^{-2\epsilon_n \rho_2} \alpha_0(-p) \right)_s \left[ (\Delta_0 + \rho \rho_1) \right]^2 \left[ (\Delta_0 + \rho \rho_2) \right]_s \left[ \epsilon_n - (\rho_1 + \rho_2) \right] \left[ \epsilon_n - (\rho_1 + \rho_2) \right]_s.$$ 

After some algebra, this quantity is transformed into

$$P(\epsilon_n, p) = 4 |w_p d_0|^2 \Delta_0^2 \times \int_0^\infty d\rho_1 \int_0^\infty d\rho_2 e^{-2\epsilon_n (\rho_1 + \rho_2) - (\rho_1 + \rho_2)^2}/4\tau_B^2$$

$$= 4 |w_p d_0|^2 (\Delta_0 \tau_B)^2 \int_0^\infty d\rho e^{-\rho^2/4 - 2\epsilon_n \tau_B \rho}. \quad (34)$$

Here, $\rho = (\rho_1 + \rho_2)/\tau_B$ with $\tau_B = r_B/v_F$. Using Dawson's integral35 $W(\xi) = e^{\xi^2} \text{erfc}(\xi)$ with $\xi = 2\epsilon_n \tau_B$, we finally have

$$P(\epsilon_n, p) = \left( \frac{2 |w_p \Delta_0 d_0|^2}{\epsilon_n^2} \right) \xi^2 \left( 1 - \sqrt{\pi} W(\xi) \right). \quad (35)$$

The above expression is equivalent to the result30 reformulated for a two-dimensional superconductor. The method of obtaining the self-consistent solution using Pesch's approximation is described in ref. 30.

2.4 Validity of approximate solution

In this subsection, we compare the approximate solution in § 2.2 with the full solution in § 2.1 and Pesch's solution in § 2.3. Calculations are performed in two-dimensional cases with isotropic Fermi surfaces, where the area element of the Fermi surface is given by $dS = \rho dr d\varphi$ with $\varphi$ being the azimuthal angle of $\mathbf{r}$. First, we briefly explain our numerics. To perform the $\rho$-integral in the calculation of $\beta_{Q,N}$, we use the simple trapezoidal rule. In our calculation, we use the cutoff for reciprocal lattice vector $Q$ in eq. (14) as $-I_{Q_{\text{max}}} \leq m, n \leq I_{Q_{\text{max}}}$; thus the total number of reciprocal lattice vectors equals $(2I_{Q_{\text{max}}} + 1)^2$. For the Fermi surface average, we use Simpson's rule with an equally spaced mesh. For the spatial average, we use the simple trapezoidal rule, and we set $N_{\text{max}} = 12$.

Now we show the result for the free energy density $F/V = F/V - B^2/8\pi$. The magnetization can be derived from $F/V$ through the thermodynamic relation $M = -\nabla_F (F/V)$. Figure 1 shows the field dependence of $F/V$ for an $s$-wave superconductor and a $d$-wave superconductor. Here, the field $B$ is measured in units of the two-dimensional orbital limiting field $B_{\text{orb}}^{2D} = 0.561 (|e|/\pi) / 2\pi^2 \xi_B^2$. A triangular vortex lattice was assumed in the $s$-wave case, while a square vortex lattice was used in the $d$-wave case. We plot the results of the different methods discussed above: the approximate solution; the full solutions with $I_{Q_{\text{max}}} = 1$ and 2; and Pesch's solution.

As was discussed by Brandt et al.,27 the convergence by changing the cutoff $I_{Q_{\text{max}}}$ for the reciprocal lattice vectors is quite excellent when we calculate the full solution of the Eilenberger equations. That is, the higher Fourier components of the $g$-function, $g_{Q \neq 0}$, do not give significant contributions to the thermodynamic quantities. Indeed, we can see in Fig. 1 that data with $I_{Q_{\text{max}}} = 1$ and 2 are almost the same. Moreover, data corresponding to the approximate solution and the full solution with $I_{Q_{\text{max}}} = 2$ cannot be distinguished on this scale. This means that the approximate solution provides a fairly
good result, at least with respect to the thermodynamics. On the other hand, Pesch’s solution seems to overestimate the free energy. This is because Pesch’s solution integrates out the vortex degrees of freedom in advance, so that the system tends to approach the Meissner state.

Next, we study the spatially resolved quantities. Figure 2(a) shows the field dependence of the approximate solution, the full solution with $I_{Q_{\text{max}}}=1$, the full solution with $I_{Q_{\text{max}}}=2$, and Pesch’s solution, respectively. Here, $H^{2D}_{\text{orb}} = 0.561(|c|/\pi)/2\pi\xi_0^2$ ($\xi_0 = v_F/2\pi T_c$) is the two-dimensional orbital limiting field.

The results in this section can be summarized as follows. The approximate solution in § 2.2 gives relatively good results as long as we consider the thermodynamics in the mixed state. Therefore we can use this approximation for the analysis of the basal-plane magnetization and torque, which we present in the next section.

3. Reversible Magnetization and Torque

3.1 Model for layered superconductor

In this subsection, we introduce the basal-plane anisotropy of a layered superconductor into our treatment. First of all, we discuss the anisotropic model that takes into account a layered structure of the materials,
such as cuprate superconductors. An example of such a procedure\textsuperscript{41, 42} is the parameterization of the Fermi surface as a distorted cylinder. We consider a sample arrangement as shown in Fig. 3. We start with the uniaxial anisotropy parameter by \( \gamma = v_F/t_s \), where \( v_F = p_F/m \). Assuming \( \gamma \gg 1 \) and \( p_F s \gg 1 \), the Fermi surface is parameterized as

\[
\hat{p} = p_F \hat{\rho} + p_z \hat{\varepsilon},
\]

where \( \hat{\rho} = \cos(\varphi) \hat{e} + \sin(\varphi) \hat{g} \), \( -\pi \leq \varphi < \pi \) is the azimuthal angle, and \( -\pi/s \leq p_z < \pi/s \). Then, the Fermi velocity \( \mathbf{v} = \nabla_{\mathbf{p}} p_F \) can be expressed as

\[
\mathbf{v} = v_F \left( \hat{\rho} + \gamma^{-1} \sin(p_z s) \hat{\varepsilon} \right),
\]

and the area element \( d\mathbf{S} \) divided by \( |\mathbf{v}| \) is given by

\[
d\mathbf{S}/|\mathbf{v}| = m d\varphi dp_z.
\]

In this paper, we treat a simple \( d_{x^2-y^2} \)-wave model, and we set the pairing function

\[
w_p = \sqrt{2} \cos(2(\varphi + \chi)),
\]

where \( \chi \) is the field angle, as shown in Fig. 3.

Next we define the anisotropic Landau levels suitable for the description of the in-plane vortex state. To connect the formulation in the previous section with the present sample arrangement (see Fig. 3), we should set \( x_1 = z, x_2 = x \) and \( x_3 = y \). In the following argument, we still use the notation \( (x_1, x_2, x_3) \). We define the raising and lowering operators \( \hat{a}_\pm \) as

\[
\hat{a}_\pm = \frac{-1}{\sqrt{2\eta}} \left( \Pi_1 \pm i\eta \Pi_2 \right),
\]

where the real constant \( \eta \) characterizes the anisotropy. If we choose \( \eta > 1 \), then the vortex state defined by the operators is compressed along the \( x_1 \)-direction and stretched along the \( x_2 \)-direction (see Fig. 4(a)). Of course, the isotropic Landau levels are defined by setting \( \eta = 1 \). Now it will be convenient to introduce a new coordinate, \( \mathbf{r}' = (x_1', x_2') \), as

\[
\mathbf{r}' = \hat{T} \mathbf{r} = (\eta^{1/2} x_1, \eta^{-1/2} x_2).
\]

Then the raising-lowering operators \( \hat{a}'_\pm \) and the resultant vortex state, expressed in the distorted \( \mathbf{r}' \)-frame, appear isotropic (see Fig. 4(b)). Therefore, a natural way to define the anisotropic Landau levels \( \tilde{\psi}(\mathbf{r}) \) in the \( \mathbf{r} \)-frame is by

\[
\tilde{\psi}_N(\mathbf{r}) = \psi_N(\hat{T} \mathbf{r}) = \psi_N(\eta^{1/2} x_1, \eta^{-1/2} x_2),
\]

where \( \psi_N \) is the isotropic Landau levels defined in eq. (11). This definition is the extension of the idea used to describe the in-plane vortex state within the anisotropic lowest Landau level.\textsuperscript{43–45} We note here that our treatment contains higher Landau level components. The important point in the above definition of \( \tilde{\psi}_N(\mathbf{r}) \) is that if we work with the \( \mathbf{r}' \)-system in our calculation, the total effect of the uniaxial anisotropy can be absorbed into the anisotropy of the Fermi velocity represented in the distorted \( \mathbf{r}' \)-frame:

\[
v_1' = \eta^{1/2} v_1, \quad v_2' = \eta^{-1/2} v_2.
\]

Thus, all the formulations presented in the previous section can be valid, as long as we set \( x_1 = z \) and \( x_2 = x \) in the following numerical calculation. Although the parameter \( \eta \) can be chosen arbitrarily in principle, the natural choice for \( \eta \) is to set \( \eta = \gamma \). This is because if we construct a linearized anisotropic GL equation\textsuperscript{46} from the model eq. (38), then the Landau levels eq. (42) with \( \eta = \gamma \) are the solutions of it under the field parallel to the \( a \)- or \( b \)-axes.

### 3.2 Longitudinal magnetization

In general, magnetizations can be derived from \( \tilde{F}/V = F/V - B^2/8\pi \) through the thermodynamic relation \( \mathbf{M} = -\nabla_B (\tilde{F}/V) \). Then the longitudinal magnetization \( M_L(\| \mathbf{H} ) \) is obtained by \( M_L = -\frac{dH}{dH} (\tilde{F}/V) \), since the direction of the applied field \( \mathbf{H} \) can be approximated by that of induction \( \mathbf{B} \) in high-\( \kappa \) superconductors. For this component, however, Klein and Pöttling\textsuperscript{47} obtained a more convenient formula that is the extension of the virial theorem derived by Doria et al.\textsuperscript{48}

\[
-4\pi M_L = \frac{4\pi^2 N(0)}{B} \sum_{\varepsilon_n > 0} \left[ \left\langle \left| \Phi_1(w_{\mathbf{p}} \Delta) + f(w_{\mathbf{p}} \Delta)^* \right| \left( \frac{1 + g}{1 + g} \right)^s \right\rangle \right]_n.
\]
In the following, we calculate the longitudinal magnetization $M_L$ in a $d$-wave superconductor. We set $T/T_c = 0.5$ and $\gamma = 3$. Here, $H_{\text{orb}} = 1.037(|c|/\pi)/2\pi \xi_0^2$ is the three-dimensional orbital limiting field in the isotropic case. The inset shows the linear field dependence of $M_L^0 = (M_L^\parallel_{\text{node}} - M_L^\parallel_{\text{antinode}})/2$.

![Figure 5](image)

**Fig. 5.** Logarithmic field dependence of longitudinal magnetization $M_L$ in a $d$-wave superconductor. We set $T/T_c = 0.5$ and $\gamma = 3$. Here, $H_{\text{orb}} = 1.037(|c|/\pi)/2\pi \xi_0^2$ is the three-dimensional orbital limiting field in the isotropic case. The inset shows the linear field dependence of $M_L^0 = (M_L^\parallel_{\text{node}} - M_L^\parallel_{\text{antinode}})/2$.

![Figure 6](image)

**Fig. 6.** Field-angle dependences of $M_L$ in a $d$-wave superconductor. The parameters used are the same as those in Fig. 5. The data at different $B$ are vertically shifted.

In the following, we calculate the longitudinal magnetization $M_L$ in a layered $d$-wave superconductor under an in-plane field. We use the above formula for $M_L$ and assume a triangular vortex lattice in the $r^*$-frame. Since the calculation for an in-plane vortex state consumes much numerical resources, we use Simpson’s rule for the spatial average as well as the Fermi surface average, and we set $N_{\text{max}} = 6$.

Figure 5 shows the field dependence of $M_L$ in a $d$-wave superconductor. Here, $H_{\text{orb}} = 1.037(|c|/\pi)/2\pi \xi_0^2$ is the three-dimensional orbital limiting field in the isotropic case. We fix the temperature at $T/T_c = 0.5$ and set $\gamma = 3$ for the uniaxial anisotropy parameter. From the figure, we can see the London behavior $M_L \propto \ln(B_{c2}/B)$ in a relatively wide range of field, as well as the GL behavior $M_L \propto H_{c2} - B$ just below $H_{c2}$. This is because the GL theory is derived from the quasiclassical Eilenberger formalism through an expansion about $\Delta$, whereas the London theory is derived using a phase-only (London) approximation. Thus we confirm that our analysis captures the essential behavior in the mixed state.

Now we focus on the anisotropic properties. In the main panel of Fig. 5, $M_L$ for the $B \parallel$ node (open circles) and the $B \parallel$ antinode (filled circles) are shown. A small but clear difference in $M_L$ between the two field orientations is seen. As we examine the data more carefully, we find that the sign of the anisotropy changes between $H_{c1}$ and $H_{c2}$. Namely, $|M_L|$ for the $B \parallel$ node is smaller near $H_{c2}$, while the tendency is reversed below field $B^*$. This is more evident in Fig. 6, where the field-angle dependences of $M_L$ are shown. Due to the inherent fourfold anisotropy of $d$-wave pairing, $M_L$ shows a fourfold oscillation as a function of the field angle $\chi$. Clearly, the sign of the $M_L$-oscillation changes at field $B^*/\gamma H_{\text{orb}} \approx 0.3$ between $H_{c1}$ and $H_{c2}$. Thus, the sign change of $M_L$-oscillation can occur even in a simple $d$-wave superconductor.

### 3.3 Torque

Next we discuss the transverse component of the magnetization, or the torque. The transverse magnetization $M_T$ is given by $M_T = -B^{-1} \frac{d}{d\chi}(F/V)$, where $\chi$ is the field angle shown in Fig. 3. In experiments, this quantity is mostly measured as a torque density, $\tau/V = M \times B$ or $\tau/V = -\frac{d}{d\chi}(F/V)$. To obtain our numerical data of $\tau$, we first calculate the $\chi$-dependence of $\tilde{F}/V$, then perform a polynomial interpolation, and calculate $\tau$ using the above relation.

Figure 7 shows the field-angle dependences of torque $\tau$ in a $d$-wave superconductor at $T/T_c = 0.5$. As in the case of longitudinal magnetization, $\tau$ shows a fourfold oscillation as a function of $\chi$. When the field is lowered from $H_{c2}$ ($\approx 0.475\gamma H_{\text{orb}}$ in this case), the oscillation amplitude first increases up to $B/\gamma H_{\text{orb}} \approx 0.3$, as seen in Fig. 7(a), whereas it starts to decrease at lower fields, as seen in Fig. 7(b).

To see the field dependence of the oscillation amplitude, we plot, in Fig. 8(a), the field dependence of $\tau$ at a fixed angle $\chi = \pi/8$. In the figure, we can see that the sign of the oscillation does not change between $H_{c1}$ and $H_{c2}$. Although the accuracy of our numerical solution breaks down near $B = 0$ or $H = H_{c1}$, we believe that this behavior of $\tau$ is not changed qualitatively by including the screening effect. Later, we will give a theoretical interpretation of the phenomena. It is worth noting that the behavior is in contrast to that in the case of longitudinal magnetization $M_L$, where the sign of the oscillation amplitude is reversed at field $B^*$ between $H_{c1}$ and $H_{c2}$.

For comparison, we show, in Fig. 8(b), the corresponding field dependence of $\delta M_L \equiv M_L(\chi = \pi/4) - M_L(\chi = 0)$. It is interesting that near field $B^*$ where the sign of $\delta M_L$ is changed, the field dependence of the torque is also changed from a decreasing one to an increasing one. This coincidence is explained in the next subsection.
3.4 Interpretation of results

We discuss the interpretation of the results that the sign of $M_L$-oscillation changes, while the sign of $\tau$-oscillation does not. For this purpose, it is useful to consider the anisotropy of the free energy $\tilde{F}(\chi)$. On the basis of our numerical results, we schematically plot, in Fig. 9, the field dependences of $\tilde{F}(\chi)$ in a $d$-wave superconductor, corresponding to the two different field orientations ($B || \text{node}$ and $B || \text{antinode}$). Here, the magnitude of anisotropy is enlarged for clarity. It is important to note that the two curves do not cross, and $\tilde{F}/V$ is a convex function of $B$. This can be understood as follows.

Firstly, it is important to note the relationship between the anisotropies of coherence length and penetration depth. Extending this knowledge, we obtain the anisotropy relation between $H_{c1}$ and $H_{c2}$:

$$\frac{H_{c1}(\chi = 0)}{H_{c1}(\chi = \pi/4)} = \frac{H_{c2}(\chi = \pi/4)}{H_{c2}(\chi = 0)}. \quad (45)$$

This implies that if $H_{c2}$ in a certain direction is larger than in other directions, then $H_{c1}$ in that direction is smaller. This can be explained in an alternative way. Recall that $H_{c1} = (\ln \kappa/\sqrt{2}\kappa)H_c$ and $H_{c2} = (\sqrt{2}\kappa)H_c$, where $H_c$ is the thermodynamic critical field. By definition, $H_c$ does not depend on the field orientation, and we can ascribe the anisotropy to $\kappa$. From these expressions, it is obvious that $H_{c1}$ and $H_{c2}$ have opposite tendencies of anisotropies in the high-$\kappa$ case.

Next we consider the anisotropy of $\tilde{F}/V$ using the analytical results.

$$\tilde{F}/V \approx \begin{cases} \frac{(H_{c2} - B)^2}{16\pi\kappa^2} & (H \approx H_{c2}) \\ \frac{F(B = 0)/V + BH_{c1}/4\pi}{(H \approx H_{c1})} & \end{cases} \quad (46)$$

Near $H_{c1}$, the anisotropy of $\tilde{F}/V$ is determined by that of $H_{c1}$, while near $H_{c2}$, the anisotropy of $\tilde{F}/V$ is governed by that of $H_{c2}$. Using eqs. (45) and (46), the free energy anisotropy shown in Fig. 9 can be confirmed analytically near $H_{c1}$ and $H_{c2}$. In the intermediate field region, it is very difficult to imagine that the two curves $\tilde{F}(\chi = 0)$ and $\tilde{F}(\chi = \pi/4)$ cross at some field, i.e., the free energy becomes $\chi$-independent. Otherwise, the $d$-wave superconductor appears effectively isotropic at this particular field. In this way, the free energy anisotropy shown in Fig. 9 can be understood. Moreover, we can show that $\tilde{F}/V$ is a convex function of $B$ if we correctly include the vortex interaction energy.

Now let us first check the sign reversal of $M_L$-oscillation using Fig. 9. $M_L$ is given by $M_L = -\frac{\partial}{\partial \chi}(\tilde{F}/V)$, i.e., the negative slope of the field dependence of $\tilde{F}/V$. Then it is obvious from Fig. 9 that $M_L$ near $H_{c2}$ is larger for the $B || \text{node}$, while near $H_{c1}$, this...
tendency is reversed. This explains the sign change of $M_L$-oscillation. On the other hand, the in-plane torque $\tau = -\frac{d\chi}{d\chi} F$ in the simple d-wave case is roughly estimated by comparing $F(\chi)$ in the two characteristic directions, $\chi = 0 (B \parallel \text{antinode})$ and $\chi = \pi/4 (B \parallel \text{node})$. Namely, we have the simple relation $\tau \sim -\delta F = \tilde{F}(B \parallel \text{node}) - \tilde{F}(B \parallel \text{antinode})$. As is seen in Fig. 9, the sign of this quantity is not changed between $H_{c1}$ and $H_{c2}$. Thus we immediately conclude that the sign of $\tau$-oscillation is not changed. In the same way, we can see that $-\tau \sim \delta F$ is maximum near field $B^*$ where the difference in the slope of the two curves in Fig. 9, i.e., $-\delta M_L = \frac{\partial F}{\partial B}(B \parallel \text{node}) - \frac{\partial \tilde{F}}{\partial B}(B \parallel \text{antinode})$, changes its sign. In this way, we can understand why near $B^*$, where the sign of $\delta M_L$ is changed, the field dependence of the torque is also changed from a decreasing one to an increasing one.

We end this section with some comments on the effect of Fermi surface anisotropies. This effect can be studied by introducing parameter $\beta$, that describes the in-plane anisotropy of the Fermi surfaces, in a similar manner as in ref. 21. We performed a calculation for when this Fermi surface anisotropy essentially competes with the gap anisotropy, and confirmed that the observed tendency of anisotropy is reversed at around $|\beta| = 1.0$ (similar to the difference in Figs. 3(a) and 3(b) of ref. 21). As the reader may note, however, the above argument for the sign change of $M_L$-oscillation and the sign preservation of $\tau$-oscillation is independent of the source of anisotropy. The key is the use of eq. (45), i.e., the fact that $H_{c1}$ and $H_{c2}$ have the opposite tendencies of anisotropy, and this relation holds irrespective of the source of anisotropy. Thus, even if the system is strongly affected by the Fermi surface anisotropy, the statement that the direction with maximal $H_{c2}$ is always energetically stable in the whole field range from $H_{c1}$ to $H_{c2}$ is valid. The importance of this fact is discussed in the next section. Repeating the same argument as in the previous paragraph, we can say that the sign change of $M_L$-oscillation and the sign preservation of $\tau$-oscillation are universal.

4. Conclusion

In this paper, we proposed a new method of obtaining the “full solution” of the quasiclassical Eilenberger equations for high-$\kappa$ superconductors, in addition to introducing a reasonable “approximate solution” to examine thermodynamic quantities. The approximate solution discussed here is a natural extension of the one developed by Pesch and which has been used recently by several authors. Compared the approximate solution with the full solution or with Pesch’s solution, and showed that our approximate method gives an excellent result, at least with respect to the thermodynamic quantities. Recently, we have extended the approximate solution to the case including Pauli paramagnetism.

We then applied our approach to the description of the in-plane magnetic anisotropies in a layered d-wave superconductor such as high-$T_c$ cuprate. We clarified that the sign of the longitudinal magnetization oscillation changes between $H_{c1}$ and $H_{c2}$, while the sign of the torque oscillation does not. Based on the analytical expressions of the free energy and the anisotropy relation between $H_{c1}$ and $H_{c2}$, we showed that there is a close relationship between $M_L$-oscillation and $\tau$-oscillation.

Furthermore, we revealed that the basal-plane torque data always indicate the maximal $H_{c2}$-direction. The implication of the present result is that the basal-plane $H_{c2}$ anisotropies can be studied through torque measurement at low fields, even in materials with strong fluctuations, such as high-$T_c$ cuprates and organic superconductors, where the true $H_{c2}$ cannot be determined experimentally. Moreover, we can detect the node position if we limit ourselves to a d-wave superconductor possessing a less anisotropic Fermi surface. For example, in light of our results, the torque data for YBCO are consistent with $d_{x^2-y^2}$-pairing. In this respect, it will be interesting to measure the basal-plane torque in the d-wave superconductor $\kappa$-(ET)$_2$Cu(NCS)$_2$, since the node position concluded from the earlier theoretical analysis and the recent experiments are controversial. Also of interest is the measurement of the basal-plane magnetic anisotropies in the d-wave superconductor CeCoIn$_5$, since there again is a controversy concerning the node position.

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