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Article Description of Dressed-Photon Dynamics and Extraction Process

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Abstract: Several interesting physical phenomena and industrial applications explained by the dressed photon have been reported in recent years. These require a novel concept in an off-shell science that deviates from the conventional optics, satisfying energy and momentum conservation laws. In this paper, starting from an original model that captures dressed-photon characteristics phenomenologically, the dynamics of the dressed photon in a nanomatter system and the mechanism for extracting internal degrees of freedom of the dressed photon to an external space have been examined by theoretical and numerical approaches. Our proposal is that basis states of the dressed photon can be transformed to the form that reflects the spatial distribution of the dressed-photon steady state in the system, and some of basis states with predetermined spatial distribution can relate to the dissipation components in the external space by means of the renormalization technique. From the results of numerical simulation, it is found that quasi-static states are regarded as the photon with light mass or massless, and the extraction of active states strongly affects the spatial distribution in a new steady state. The concept for extracting dressed-photon energy to an external space will contribute to a detailed understanding of dressed-photon physics and future industrial applications.

Keywords: dressed photon; localization; dissipation; off-shell science; non-equilibrium open system; quantum master equation; quantum density matrix; projection operator; renormalization

1. Introduction

In recent years, some novel and fundamental experimental studies have been reported that originate from the photon localized at a nanometer scale. For example, a Si light emitter with a nanostructure of boron dopants has been demonstrated, where Si is an indirect semiconductor and such an optical transition is forbidden in the conventional optics [1,2]. For microfabrication techniques, size-selective and non-adiabatic photochemical reactions (etching [3] and deposition [4]) have been observed on rough surfaces with nanostructures and under nanometrically tapered optical fiber probes. Furthermore, a giant magneto-optical effect using a ZnO single crystal with a nanostructure of the dopant has been confirmed as a surprising experimental result [5]. To explain these experimental facts, it is necessary to step into an off-shell science [6,7], which is a concept that overcomes the conventional optics limited by energy and momentum conservation laws. The origin of the appearance of strange optical phenomena in the off-shell region is considered to be environmental effects of background materials, such as the electronic excitation field and the phonon field, on the internal photon field, which is called the dressed photon. However, it is a challenging task to build a complete theory, since this would require incorporating an unknown contribution of infinite degrees of freedom. Thus, a simple theoretical expression without losing the essence of the dressed photon is strongly desired.

In this paper, a phenomenological model of the dressed photon is proposed without touching on the specific generation process of the dressed photon. At first glance, such a model may resemble an exciton–polariton picture, but the dressed photon is considered to be a quasi-particle bounded in a finite distance with the help of the surrounding electronic
and phononic excitations, and the energy transfer of the dressed photon via the off-shell region or non-resonant region is allowed. This is the difference between the dressed photon and the exciton polariton. A numerical simulation is also demonstrated for expressing the dressed photon dynamics, and discussing the extraction of energy from the internal dressed-photon system to the external field.

The logical flow in this paper is summarized as follows. The spatial distribution of the dressed photon has been decomposed into plural characteristic basis states reflecting a certain steady state. At this time, the basis states can be distinguished into strong and weak contributions to the system dynamics by referring to the formula for renormalizing the weak interaction into the strong one. The weak-interacting basis states can be regarded as quasi-particle states with a light mass that resemble a photon reservoir system. In this way, the influence of a microscopic system on a macroscopic one can be formulated, and the microscopic system can be controlled from the macroscopic one. It will be a clue to explain the emergence of optical functions via the dressed photon, such as the light emission from indirect semiconductors.

The following sections are constructed to evaluate the above concept as follows. Section 2 describes the formulation of dressed-photon dynamics. Here, in addition to providing the equation of motion in a non-equilibrium system, dressed-photon basis states characterized by the spatial distribution is introduced for the subsequent discussions. In Section 3, a method for dividing a dressed-photon system into the systems with strong and weak contributions is proposed using the renormalization technique. Section 4 gives an insight for connecting the dressed photon with the external free photon, based on the method obtained up to Section 3. In addition, we will discuss how to control the dressed photon from the external degree of freedom. Finally, Section 5 summarizes this paper.

2. Theoretical Model of a Dressed-Photon System

2.1. Quantum Master Equation

From the experimental situation for a nanophotonics system, one can understand that the system always exists under an environment with the external photoexcitation and the dissipation, and the balance of the input and the output is maintained. This is a non-equilibrium open system. For describing the dressed-photon dynamics, the dressed photon is assumed to be a carrier bounded in a nanomatter system, and a part of energy dissipates to the external field as the free photon, where the optical coherence is disappeared. In other words, it is a problem to analyze the internal states of the dressed photon distributed in a non-equilibrium open system.

A nanomatter with an arbitrary shape is expressed as a collection of nodes that bind the dressed photon, and are freely arranged inside a matter. To avoid misunderstanding, note that the node does not mean the atomic site, but a center of mass for the dressed photon with spatial spreading. Therefore, the nodes are not restricted by a periodic array structure representing the translational symmetry of such an electron wave, and can set freely. This paper is not intended to describe a rigorous structure of a matter, but rather to represent adequately the essence of dressed-photon mediating phenomena. From this point of view, this model is equivalent to a quantum walk on a graph. Several studies have also been reported that suggest that dressed-photon phenomena correspond to some stationary solution in a quantum-walk system [8,9].

The dressed photon as a carrier is assumed to be transferred among the nodes by the hopping conduction, such that the coupling strength is expressed as a function of distance between a target node and all the others. Figure 1 illustrates a dressed-photon system, where a nanoscale two-dimensional taper structure and a nanomatter are expressed as just a collection of nodes without distinguishing the two separated parts. In this system, the dressed photon is injected from the upper part of a taper structure, released to the external field radiatively, and returns to the input side non-radiatively. This model is a non-
equilibrium open system. The equation of motion, that is a quantum master equation, in such a system can be described using the quantum density operator $\rho(t)$ as follows [10–12],

$$\frac{\partial \rho^I(t)}{\partial t} = -\frac{i}{\hbar} \left[ H^I_{\text{int}} + H^I_{\text{exc}}, \rho^I(t) \right] + \mathcal{L}^{(nr)} \rho^I(t) + \mathcal{L}^{(r)} \rho^I(t), \quad (1)$$

where the superscript $I$ for each operator represents the interaction picture, and the hopping energy transfer and the coherent excitation are expressed as $H^I_{\text{int}}$ and $H^I_{\text{exc}}$, respectively. The square brackets represent the commutation relation, and $\mathcal{L}^{(r)}$ and $\mathcal{L}^{(nr)}$ mean the Lindblad-type radiative and the non-radiative dissipations. The following devotes explanation of each component in (1), where the superscript $I$ for the interaction picture is omitted to avoid the complexity of the subscript and superscript expressions. In defining the operators that are used in this research, the basis states are assumed to be a one or zero dressed photon. This means an assumption of the weak excitation limit. In the future, the many-body interaction of the dressed photon, i.e., the nonlinear problem, should be considered, and it will be reported somewhere.

![Figure 1](image_url)

**Figure 1.** Schematic illustration of a dressed-photon system that consists of plural arbitrarily arranged nodes and models a taper structure of an optical fiber probe and a nanomatter. It is not necessary to distinguish between the taper and the nanomatter, as it is regarded as just a collection of nodes. In this system, the dressed photon is coherently excited from the upper part of the taper structure, and transfers via the hopping conduction among nodes with the coupling strength according to the distance. Some dressed photon dissipates out of the system as the free photon, and some returns to the input side non-radiatively.

2.1.1. Dressed-Photon Excitation by External Field

The external excitation of the dressed photon is assumed to be given coherently from the upper part in a taper structure in Figure 1. When the creation and annihilation operators $a^\dagger_i$ and $a_i$ of the dressed photon at a node $i$ are predetermined, the excitation is expressed as

$$H_{\text{exc}} = \sum_{i \in \text{edge}} \hbar A (a_i + a^\dagger_i), \quad (2)$$

where $\hbar A$ denotes the strength of the excitation that is related to the amplitude of the external input field. The form of (2) is inspired by the theoretical description of the conventional electric dipole excitation.
2.1.2. Inter-Node Energy Transfer

The hopping energy transfer means mathematically an exchange of the dressed photon between two different nodes, which is given by the following equation,

\[ H_{\text{int}} = \sum_{i \neq j} \hbar V(|r_i - r_j|)(a_i^\dagger a_j + a_i a_j^\dagger), \]  

(3)

where the coupling strength \( \hbar V(r) \) is assumed to have a finite interaction range for expressing the localization nature of the dressed photon. Readers with knowledge of quantum theory may pay attention to a positive sign of the interaction Hamiltonian in comparison with some known models of material systems, such as the Bose–Hubbard model and the tight-binding model [13]. The interaction Hamiltonian is based on the theoretical derivation of the transition probability of the electronic excitation between two nanomatters in our published reports [14–16], where the transition probability was obtained by assuming that the constraint of the energy and momentum conservation laws can be overcome. According to the detailed explanation in [17], the coupling strength \( \hbar V(r) \) with a finite interaction distance and a positive sign is derived as the form so-called Yukawa potential,

\[ V(r) = \frac{V_0 e^{-m_{\text{eff}} r}}{r}, \]  

(4)

where \( V_0 \) and \( m_{\text{eff}} \) are an appropriate constant and an effective mass which determines the interaction range, respectively. The Yukawa function often appears to give a screening effect in a many-body interaction system. In the case of the dressed-photon energy transfer, degrees of freedom of an environment leads to the equivalent effect to the many-body interaction.

2.1.3. Radiative Dissipation

The Lindblad-type radiative dissipation in (1), which shows the emission of the free photon into an external space, is given as the following equation,

\[ \mathcal{L}^{(r)} \rho(t) = \frac{\gamma^{(r)}}{2} \sum_{i,j} \left( 2a_i \rho(t) a_j^\dagger - \left\{ a_j^\dagger a_i, \rho(t) \right\} \right), \]  

(5)

where \( \gamma^{(r)} \) represents the relaxation constant via the free photon in an external space, and the curly brackets are the notation of the anti-commutation relation. It is worth noting the summation of nodes labeled by the indices \( i \) and \( j \). The relaxation involves both allowed and forbidden transitions of the free photon depending the symmetry of the spatial distribution of the total dressed photon excitation.

2.1.4. Non-Radiative Dissipation

As shown in Figure 1, the dressed photon is simultaneously excited and dissipated from the input side of a nanomatter system because it is an open system. Since the actual system of interest should be regarded as a microscopic part of an infinite system, it is difficult to accurately model the whole picture of the matter structure that is continuously connected from microscopic to the macroscopic systems. In our formulation, the Lindblad-type non-radiative dissipation is assumed for simply realizing a non-equilibrium open system. This assumption is approximately inadequate, but deep physical consideration in this topic is beyond the scope of this paper. It is expected a theoretical model will be built that accurately incorporates the macroscopic system hidden in the background. There are several studies for connecting a microscopic system with a macroscopic one [18,19].

Here, the non-radiative dissipation, i.e., the third term in (1), is given qualitatively as a similar manner in (5) as

\[ \mathcal{L}^{(nr)} \rho(t) = \frac{\gamma^{(nr)}}{2} \sum_{i,j \in \text{edge}} \left( 2a_i \rho(t) a_j^\dagger - \left\{ a_j^\dagger a_i, \rho(t) \right\} \right), \]  

(6)
where $\gamma^{(\text{nr})}$ is the non-radiative relaxation constant of the local dressed photon that is obviously faster than the radiative one. This is almost the same as (5), but the region with the dissipation is limited to a part of a taper structure.

2.2. Spatial Mode Expansion

In this subsection, the basis states for expressing the spatial distribution of the dressed photon are discussed using the quantum master equation formulated in Section 2.1. So far, the basis states of the dressed photon are set as the dressed photon exists or not at local nodes in a nanomatter system as an implicit understanding. In Figure 2, a steady-state solution in the case of a two-dimensional taper structure is calculated and mapped with the color gradation that represents the occupancy probability of the dressed photon. Figure 2a–c denote the snapshots of the temporal evolution at the time steps, $t = 5, 50,$ and $5000$, respectively. The simulation parameters used in these calculations are written in the caption in Figure 2, and are commonly used in the following calculations. At each time step, the spatial distribution of the dressed photon reflects the weight coefficient $c_i$ of the basis states in a quantum superposition state $|\psi\rangle = c_0 |0,\ldots,0\rangle + c_1 |1,0,\ldots,0\rangle + c_2 |0,1,0,\ldots,0\rangle + \cdots + c_N |0,\ldots,0,1\rangle$, where, for example, $|0,1,0,\ldots,0\rangle$ represents a state in which the dressed photon exists at the node labeled as the position 2. The temporal evolution can be interpreted as follows; in the early stage, the dressed photon runs down as the ballistic conduction at a part of the taper slopes, and then is reflected at a boundary of the taper tip, i.e., a spatial singular point of the system, leading to a steady state. Finally, it is found that the dressed photon makes spatial localization near around the tip, similar to a standing wave. In addition, there are locations inside the taper where the occupancy probability of the dressed photon is highly established quasi-periodically. The spatial distribution is, of course, determined depending on the shape of a matter system, such as the size of the taper structure and the steepness of the taper slopes. It also depends on the coupling strength $\hbar V(r)$.

As mentioned in the Introduction, the dressed photon should be controlled in a nanomatter system, and observed via the free photon radiated from the system. In the following, a way to extract the characteristics of the spatial distribution of the dressed
photon is discussed from a viewpoint of the basis transformation. Although the Fourier transformation and/or the Bloch’s theorem, which are based on translational symmetry, are used in the cases of the conventional optics and solid-state physics to catch the clear description of a wave nature, they cannot be applied for the description of the dressed photon because of the spatial singularity of the matter boundary and the impurity. Therefore, focusing on the fact that this system converges to a non-equilibrium steady state, the basis transformation which diagonalizes the steady state is proposed. According to the obtained basis states, there is no energy transfer between such basis states at a steady state, and the dressed photon dynamics can be separable depending on the spatial distribution of the basis states which strongly reflects a geometrical nature of a nanomatter.

From the steady-state solution (Figure 2c), the matrix $U$ that diagonalizes the quantum density matrix can be determined numerically and uniquely. As a result of this transformation, (1) is rewritten as

$$\frac{\partial \rho_{st}(t)}{\partial t} = -\frac{i}{\hbar} [H_{\text{int},st} + H_{\text{exc},st}, \rho_{st}(t)] + L^{(nr)}_{st} \rho_{st}(t) + L^{(r)}_{st} \rho_{st}(t), \quad (7)$$

$$O_{st} \equiv U^{-1} O U, \quad (8)$$

$$L^{(nr,r)}_{st} \rho_{st} \equiv \frac{\gamma^{(nr)}_{st}}{2} \left\{ 2a_{st}^{\dagger} \rho_{st}(t) a_{st} + \left\{ a_{st}^{\dagger} a_{st}, \rho_{st}(t) \right\} \right\}, \quad (9)$$

where the subscript “st” means the operators after the basis transformation as the quantum density matrix being diagonalized, and $O$ is an arbitrary operator. To decompose the individual row of the matrix $U$ is intuitive because the elements of a certain row are constructed from the weight coefficients of the linear combination of the basis states in the local-node description, and it is sorted in descending order of the occupancy probability. The basis states can be visualized as shown in Figure 3.

**Figure 3.** Color map images of the basis states reconstructed from the transformation matrix $U$ in (8). Since the state $n = 27$ corresponds to the coherent excitation, there is no meaning in the state over $n = 27$, and almost of those are excluded from visualization.
From the perspective of the spatial distribution, there are several characteristic basis states. The state of $n = 27$ in Figure 3 apparently corresponds to the excitation due to the external field at an input interface, and thus, the states in the region labeled $n > 27$ are no longer excited in this system, and most of these are excluded from the drawing. The states $n = 14, 16, 24$ have quasi-periodic spatial structures that resemble standing waves in a waveguide. In the states of $n = 8, 12$, the dressed photon occupies the taper slopes. Several basis states of $n \leq 9$ show localization of the dressed photon at a taper tip; therefore, it is predicted that these states couple strongly with each other.

The dynamics of the quantum density matrix for the transformed basis states (Figure 3) can be recalculated numerically. In Figure 4, the density matrix elements are depicted as the color map images, where the time steps are similarly set as $t = 5, 50, 5000$, and the colors represent absolute values of the density matrix elements. In an early stage of a time evolution, the occupancy probability (diagonal elements) concentrates in the basis states with a localization nature ($n \leq 7$), and the off-diagonal elements which represent the transition probability between the different basis states also change actively. After some time, the central area of the color map becomes active, in which there are a few characteristic basis states with high occupancy probability. In the final stage, the system goes to the steady state that consists only of the diagonal matrix elements. The following two points from this basis transformation approach are noticeable. One is that the basis states labeled by $n > 27$ are not excited and negligible, and this contributes to decrease the numerical calculation volume and the calculation time. The other is that there are components growing slowly and unidirectionally without exchange of energies among the other basis states. These are reminiscent of the dissipation process for the free photon.

![Figure 4. Color map images of the quantum density matrices at the time steps of (a) $t = 5$, (b) 50, and (c) 5000, respectively. The occupancy probability and the transition matrices of the dressed photon are represented as the diagonal and off-diagonal matrix elements, respectively. All simulation parameters are the same in Figure 2. Meanwhile, in the early state, the dressed photon concentrates in the states with a localization nature and goes and returns aggressively among themselves; the basis states with the intermediate spatial size show slightly calm movement, which is reminiscent of the radiative dissipation to the external field of the free photon.](image)

3. Renormalization of Quasi-Static Basis States

In the previous section, novel basis states inspired by a non-equilibrium steady state are proposed to capture the spatial property that distinguishes the dressed-photon dynamics, and the temporal evolution of the dressed photon is visualized numerically in a space of the quantum density matrix. This seems to suggest the distinction between the matter-like and the free photon-like properties of the dressed photon. Based on this insight, this section is devoted to discussing a way to focus the principal modes of the dressed photon with a localization nature.

First, let us pay attention to the coupling strength between the unitary transformed basis states that can be observed in the interaction Hamiltonian, $H_{\text{int,st}}$. The interaction Hamiltonian before and after the basis transformation is visualized as the color map images in Figure 5. In the case before the transformation, a quasi-periodic structure appears depending on the lattice structure of the nodes as illustrated in Figure 1. The unitary
transformation drastically changes the appearance, which is shown in Figure 5b. The effective transition among the basis states restricts in the several basis states, and many basis states stay in their own modes that are described as the diagonal matrix elements, which represent the energy shift in the system dynamics. In the following, the projection operator method is applied to extract the principal basis states with a localization nature of the dressed photon, and to eliminate the basis states with the weak contribution.

Figure 5. Color map images of matrix elements of the interaction Hamiltonian $H_{\text{int}}$, which is given in (3). (a) The matrix before the unitary transformation has a quasi-periodic structure reflected by the range-dependent coupling strength among a certain node and nearly arranged ones, and all matrix elements in the diagonal part are zero. (b) The matrix after the unitary transformation shows characteristic structure. There are two distinct areas divided at $n = 27$, which corresponds to the mode of the dressed-photon excitation. In the base $n \leq 27$, the diagonal matrix elements have large values, i.e., staying in their own modes, and it is found that several basis states dominantly contribute to the dressed-photon dynamics via off-diagonal matrix elements.

3.1. Projection Operator Method

The projection operator method is a mathematical technique that divides the entire system into a target space ($P$) and a complementary space ($Q$), and inserts the influence of the complementary space into the target space [14,20]. A state vector of the entire system $|\psi_{st}\rangle$ is divided into the two sub-spaces using the projection operators,

$$
|\psi^P_{st}\rangle = P|\psi_{st}\rangle, \quad (10a)
$$

$$
|\psi^Q_{st}\rangle = Q|\psi_{st}\rangle, \quad (10b)
$$

where the projection operators $P$ and $Q$ satisfy the following relations,

$$
P + Q = 1, \quad (11a)
$$

$$
P^2 = P, \quad (11b)
$$

$$
Q^2 = Q. \quad (11c)
$$

Using the Schrödinger equation,

$$
H_{\text{exc,}st}|\psi_{st}\rangle = \Delta E|\psi_{st}\rangle, \quad (12)
$$

the state vector in the $Q$-space can be expressed as the sum of the contributions from the state vector in the $P$-space, i.e.,

$$
|\psi^Q_{st}\rangle = \sum_{n=1}^{\infty} \left( \Delta E^{-1} Q H_{\text{int,}st} \right)^n |\psi^P_{st}\rangle \approx \Delta E^{-1} Q H_{\text{int,}st} |\psi^P_{st}\rangle. \quad (13)
$$

In (12), $\Delta E$ corresponds to the energy shift of the basis states unitary transformed from the basis states expressed by the local nodes. It should be noted that the contribution of the radiative and non-radiative dissipations, and the excitation of the dressed photon
are ignored in (12) and (13) because the dissipation and the excitation originate from the interaction with the external field, but it is qualitatively negligible by assuming that only the hopping conduction of the dressed photon contributes to the transition between the \( P \) and the \( Q \)-spaces. In the last part of (13), the first-order perturbation is applied by assuming that the basis states in the \( Q \)-space weakly affect the \( P \)-space dynamics.

3.2. Modified Quantum Master Equation

Applying the approximate expression given in Section 3.1, the equation of motion for the quantum density operator can be transformed in the \( P \)-space representation, where the influence of the \( Q \)-space is renormalized into the original interaction Hamiltonian, and the creation and annihilation operators in the dissipation terms. Omitting the redundant mathematical transformations, the quantum master equation is modified as follows,

\[
\frac{\partial \rho_{st}(t)}{\partial t} = \frac{\partial \rho_{st}^{P}(t)}{\partial t} = -\frac{i}{\hbar} \left[ H_{\text{int,}st}^{P} + H_{\text{ext,}st}^{P} \rho_{st}^{P}(t) \right] + \mathcal{L}_{st}^{(\text{nr})} \rho_{st}^{P}(t) + \mathcal{L}_{st}^{(\text{r})} \rho_{st}^{P}(t),
\]

where the quantum density matrix operator in the \( P \)-space is \( \rho_{st}^{P}(t) = P \rho_{st}(t) P \), and the modified interaction Hamiltonian reads

\[
H_{\text{int,}st}^{P} = P H_{\text{int,}st} P + \sum_{m \in Q} \frac{P H_{\text{int,}st} \langle \phi_m^Q \rangle \langle \phi_m^Q | H_{\text{int,}st} P \phi_m^Q \rangle}{\langle \phi_m^Q | H_{\text{int,}st} | \phi_m^Q \rangle}.
\]

In (15), the operator \( Q \) is rewritten by the intermediate states \( | \phi_m^Q \rangle \) for clear understanding, i.e.,

\[
Q = \sum_{m \in Q} | \phi_m^Q \rangle \langle \phi_m^Q |,
\]

where the summation is applied to the artificially selected basis states in the \( Q \)-space. (15) means that the interaction Hamiltonian with the coherent dynamics is corrected by the transition between the basis states in the \( Q \)- and \( P \)-spaces.

The dissipation terms in (5) and (6) are similarly rewritten as the following form,

\[
\mathcal{L}_{st}^{(\text{nr})} \rho_{st}^{P}(t) = \frac{\gamma_{(\text{nr})}}{2} \sum_{i,j} \left\{ 2 a_{i}^{P} P a_{j}^{P} - \left\{ a_{i}^{P} a_{j}^{P} \right\} \right\},
\]

where

\[
a_{i}^{P} \equiv P a_{i} P + \sum_{m \in Q} \frac{P a_{i} \langle \phi_m^Q \rangle \langle \phi_m^Q | H_{\text{int,}st} P \phi_m^Q \rangle}{\langle \phi_m^Q | H_{\text{int,}st} | \phi_m^Q \rangle}.
\]

Ideally, the contribution of the dissipation should be renormalized into the relaxation constants \( \gamma_{(\text{nr})} \), and the creation and annihilation operators should be left as the original form of the basis states transformed by a non-equilibrium steady state. However, (17) is only an approximate expression for the operators, since the theoretical formulation has not been completed in this stage. This is a problem to be solved in the future. In (18), the second term means that there are dissipation processes with the energy flow from the \( P \)-space to the \( Q \)-space.

3.3. Numerical Demonstration of Renormalization

Using the above formulation, the concrete temporal evolution of the quantum density matrix is calculated numerically, and the validity of the approximation is evaluated. Figure 6 shows the steady-state solution for the three steps of the coarse graining, which are the original result without approximation, the result simply removing the basis states over \( n = 27 \), and the result after renormalization using (14)–(17). Prior to the renormalization, the \( Q \)-space components are selected as \( n = 17, 19, 20, 22, \) and \( 23 \), by referring the correction term of \( H_{\text{int,}st} / \langle \phi_m^Q | H_{\text{int,}st} | \phi_m^Q \rangle \), that weakly couple to the basis state for the excitation. Despite reducing the number of the basis states, the obtained steady-state solutions are almost
the same in all three cases, and the calculation time has been significantly reduced. In the case of Figure 6, the number of the basis states for obtaining the calculation results has been reduced by less than half against no coarse graining, and thus the number of the differential equations to be solved is 22% less. To confirm the validity of this approach, the spatial distribution of the occupancy probability of the dressed photon is reconstructed from the quantum density matrix by applying renormalization or not, that is shown in Figure 7. Both color map images of the occupancy probability before and after renormalization are in good agreement with each other.

Figure 6. Color map images of steady-state solutions for the quantum density matrix in the cases of (a) no eliminating the extra basis states, eliminating the states of \( n > 27 \), and additional renormalization of the states \( n = 17, 19, 20, 22, \) and 23. The number of the matrix elements decreases from (a–c) as \( 47^2, 27^2, \) and \( 22^2 \). In all three cases, the steady-state solutions converge to only diagonal elements, and the occupancy probability can be reproduced after coarse graining using renormalization of the quasi-static basis states.

Figure 7. Steady-state solutions of occupancy probability for the dressed photon are mapped in the geometrical structure of taper that are calculated using (a) the original basis states defined by nodes, and (b) reconstructed from the basis coarse-grained by eliminating extra base and renormalization. The renormalization condition is the same as that in Figure 6.

So far, a method to distinguish the heavy and the light components of the dressed photon has been proposed using the original basis transformation, and the numerical demonstration shows the potential for reducing the amount of computation. As a similar approach, a method where a macroscopic system is expressed with a small number of basis states using the basis states that are predetermined by the steady-state solutions in a small space step by step has been already published [18,19]. These papers report a large reduction in the amount of the quantum calculation. This method is very similar to our approach, in which basis transformation and renormalization are used for reducing the number of the principal basis states. Meanwhile, our main purpose in this research is to observe and control a behavior of the dressed photon localized in a nanometer space. This point will be considered in the next section.

4. Discussion on Control of Dressed Photon Distribution

This section discusses the physical meaning for renormalizing particular basis states. In Section 3.3, from the characteristics of the basis states defined by a steady-state solution,
the basis states with the weak contribution can be converted to dissipative component in the system by applying the renormalization method in the first-order perturbation approximation. Such a situation is equivalent to a free photon reservoir. According to the intuitive image, the dressed photon can be regarded as stripping off the mass caused by the interaction with the environment and changes into the massless free photon, where the dressed-photon basis states staying in a nanomatter system are responsible for the stripped mass via renormalization.

On the other hand, it is interesting to consider how to affect the spatial distribution characteristics of the dressed photon that stays inside a nanomatter system. As an example, let us consider extracting a certain principal basis state with the strong localization of the dressed photon into the $Q$-space. When the localization basis state is selected as $n = 7$ in Figure 3, where the dressed photon energy concentrates at a tip position, is assigned in the $Q$-space, the quantum density matrix is calculated in the same manner as explained in the previous section, where the approximation of the weak coupling has been already exceeded. Figure 8a is the numerical result of the simulation, and the quantum density matrix cannot converge on the diagonal matrix elements. If the strict quantitativeness is neglected, this corresponds to change of a steady-state solution, i.e., the spatial distribution of the dressed photon can be modified by extracting artificially the principal localization basis state. In Figure 8b, the steady-state solutions are shown as a color map image in the taper geometry reconstructed from Figure 8a. One can observe that the localization of the dressed photon at a tip position disappears after such renormalization. It should be noted that this result represents the characteristic behavior of dressed-photon mediated phenomena. Removing the dressed photon out of a nanomatter system for an experimental observation makes another new internal state of the spatial distribution of the dressed photon in the system. For controlling and optimizing dressed-photon mediated phenomena, the renormalization of the basis states of interest, that is proposed in this paper, will be an extremely important concept.

Figure 8. Numerical calculation result of the quantum density matrix when the basis state of $n = 7$ is additionally extracted as the $Q$-space. (a) Color map image of a steady-state solution for the density matrix elements, and (b) reconstructed color map image on the geometrical structure of taper. The extraction of the state actively contributing to the localization drastically changes convergence property of the quantum density matrix as well as spatial distribution of the dressed photon in a nanomatter.

5. Conclusions

In this paper, a phenomenological model that regards the dressed photon as a particle localized at a certain node in a collection of nodes representing nanomatter has been proposed, and its spatio-temporal dynamics has been formulated using the quantum density operator. This system includes the radiative and non-radiative dissipation processes that are given by the Lindblad-type dissipation based on the first-order Born–Markov approximation, and the external excitation, and thus, the system dynamics converges to a non-equilibrium steady state. For such a model, a mechanism for extracting a part of the dressed-photon energy to the external field, which corresponds to observation instruments, has been considered to access the localized state of the dressed photon, and to explain
interesting experimental facts mediated by the dressed photon. Specifically, the methods to
describe the dressed photon by characteristic basis states inspired by a non-equilibrium
steady state as well as to separate the basis states into the target and the complementary
spaces have been proposed and formulated using the projection operators. Contribution of
the complementary space is renormalized in the target space by means of the lowest-order
perturbation approximation. These theoretical and numerical approaches are a pioneering
study that elucidates the principle of continuously connecting the dressed photon to the
free photon. In this research, the process in which the bound or massive dressed photon
dissociates its mass and is converted to the free photon has been interpreted by considering
the energy transfer among the basis states with different spatial characteristics.

In the last part of this paper, a concept for accessing the principal basis states in a
nanomatter system has been discussed using the same manner of renormalization. This is a
qualitative proposal, but an important finding that the external manipulation of the dressed
photon associated with the concept of renormalization.

The basis transformations using a predetermined steady state are inconsistent for the
purpose of simulating the dressed-photon dynamics in an unknown system. However,
in the experimental systems in which the dressed photon is mediated, the structural
changes of the nanomatter always appears, such as an optimal rearrangement of atoms.
Therefore, our approach to focus on changes from the steady states seems to be effective
for explaining the experimental facts. In that sense, our proposed method is worth enough
aiming at solving a dressed-photon optimization problem.

In the present research stage, the theoretical formulation is somewhat insufficient to
explain the experimental facts quantitatively, but this paper has provided a meaningful
consideration as a challenge by stepping into the essence of the underlying physical
mechanism of the dressed photon and an off-shell science. It is expected to lead to a detailed
understanding of dressed-photon physics and industrial applications in the near future.

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