Exploring the effects of Delta Baryons in magnetars

Kauan Dalfovo Marquez
marquezkauan@gmail.com

with the collaboration of
D. P. Menezes, V. Dexheimer, D. Chatterjee, M. R. Pelicer and B. C. T. Backes

25/10/2022
- DEXHEIMER, V.; MARQUEZ, K. D.; MENEZES, D. P. Delta baryons in neutron-star matter under strong magnetic fields. EUROPEAN PHYSICAL JOURNAL A 57 216, 2021. [arXiv:2103.09855]

- BACKES, B. C. T.; MARQUEZ, K. D.; MENEZES, D. P. Effects of strong magnetic fields on the hadron-quark deconfinement transition. EUROPEAN PHYSICAL JOURNAL A 57 229, 2021. [arXiv:2103.14733]

- MARQUEZ, K. D.; PELICER, M. R.; GHOSH, S.; PETERSON, J.; CHATTERJEE, D.; DEXHEIMER, V.; MENEZES, D. P. Exploring the effects of Delta Baryons in magnetars. PHYSICAL REVIEW C 106 035801, 2022. [arXiv:2205.09827]
\[ \mathcal{L} = \sum_b \bar{\psi}_b \left[ \gamma_\mu \left( i \partial_\mu - g_{\omega b} \omega_\mu - g_{\phi b} \phi_\mu - \frac{g_{\rho b}}{2} \vec{\tau} \cdot \vec{\rho}_\mu \right) - M \right] \psi_b \]

\[ + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m^2 \sigma^2) - \frac{\lambda_1}{3} \sigma^3 - \frac{\lambda_2}{4} \sigma^4 \]

\[ - \frac{1}{4} \Omega_{\mu \nu} \Omega^{\mu \nu} + \frac{1}{2} m^2 \omega_\mu \omega_\mu - \frac{1}{4} \Phi_{\mu \nu} \Phi^{\mu \nu} + \frac{1}{2} m^2 \phi_\mu \phi^\mu \]

\[ - \frac{1}{4} \vec{R}_{\mu \nu} \cdot \vec{R}^{\mu \nu} + \frac{1}{2} m^2 \vec{\rho}_\mu \cdot \vec{\rho}_\mu + g_{\omega \rho} \omega_\mu \omega^\mu \vec{\rho}_\mu \cdot \vec{\rho}_\mu , \quad (1) \]
Relativistic effective models in compact star description

\[ \mathcal{L} = \sum_b \bar{\psi}_b \left[ \gamma_{\mu} \left( i \partial^\mu - g_{\omega b} \omega^\mu - g_{\phi b} \phi^\mu - \frac{g_{\rho b}}{2} \vec{\tau} \cdot \vec{\rho}^\mu \right) - M \right] \psi_b \]

\[ + \frac{1}{2} \left( \partial_{\mu} \sigma \partial_{\mu} \sigma - m_{\sigma}^2 \sigma^2 \right) - \frac{\lambda_1}{3} \sigma^3 - \frac{\lambda_2}{4} \sigma^4 \]

\[ - \frac{1}{4} \Omega_{\mu \nu} \Omega^{\mu \nu} + \frac{1}{2} m_{\omega}^2 \omega_{\mu} \omega^\mu - \frac{1}{4} \Phi_{\mu \nu} \Phi^{\mu \nu} + \frac{1}{2} m_{\phi}^2 \phi_{\mu} \phi^\mu \]

\[ - \frac{1}{4} \vec{R}_{\mu \nu} \cdot \vec{R}^{\mu \nu} + \frac{1}{2} m_{\rho}^2 \vec{\rho}_{\mu} \cdot \vec{\rho}^\mu + g_{\omega \rho} \omega_{\mu} \omega^\mu \vec{\rho}_{\mu} \cdot \vec{\rho}^\mu, \]

(1)

| Model   | \( n_0 \) | \( B/A \) | \( K \) | \( S \) | \( L \) | \( M/m \) |
|---------|--------|--------|--------|-------|-------|--------|
| GM1     | 0.153  | 16.33  | 300.5  | 32.5  | 94    | 0.70   |
| L3\( \omega \rho \) | 0.156  | 16.20  | 256    | 31.2  | 74    | 0.69   |
| DDME2   | 0.152  | 16.14  | 251    | 32.3  | 51    | 0.57   |
| Constr. | 0.148–0.170 | 15.8–16.5 | 220–260 | 28.6–34.4 | 36.0–86.8 | 0.6–0.8 |

**Table:** symmetric nuclear matter properties at saturation density for the models employed in this work.
\[ \mu_b = \mu_n - q_b \mu_e \]  
\[ \sum_{i=b,l} q_i n_i = 0 \]
\[ \mu_b = \mu_n - q_b \mu_e \] (2)

\[ \sum_{i=b,l} q_i n_i = 0 \] (3)

\[ \varepsilon = \sum_b \frac{1}{\pi^2} \int_0^{p_F} dp \, p^2 \sqrt{p^2 + M_b^2} + \frac{1}{2} m^2 \sigma_0^2 + \frac{\lambda_1}{3} \sigma_3^3 + \frac{\lambda_2}{4} \sigma_4^4 \\
- \frac{1}{2} m^2 \omega_0^2 - \frac{1}{2} m^2 \phi_0^2 - \frac{1}{2} m^2 \rho_0^2 - g_{\omega \rho} \omega_0^2 \rho_0^2 + \varepsilon_{\text{leptons}}, \] (4)

\[ P = -\varepsilon + \sum_b \mu_b n_b \] (5)
\[ \mu_b = \mu_n - q_b \mu_e \]  
\[ \sum_{i=b,l} q_i n_i = 0 \]  
\[ \varepsilon = \sum_b \frac{1}{\pi^2} \int_{p\mu}^{p_{Fb}} dp \, p^2 \sqrt{p^2 + M_b^2} + \frac{1}{2} m^2 \frac{\rho^2}{3} \sigma^2 + \frac{\lambda_1}{3} \sigma^3 + \frac{\lambda_2}{4} \sigma^4 \]  
\[ -\frac{1}{2} m^2 \omega_0^2 - \frac{1}{2} m^2 \phi_0^2 - \frac{1}{2} m^2 \rho_0^2 - g_{\omega \omega} \omega_0^2 \rho_0^2 + \varepsilon_{\text{leptons}}, \]  
\[ P = -\varepsilon + \sum_b \mu_b n_b \]  
\[ \frac{dP}{dr} = - \frac{[\varepsilon(r) + P(r)] [m(r) + 4\pi r^3 P(r)]}{r [r - 2m(r)]}, \]  
\[ m(r) = \int_0^r dr' 4\pi r'^2 \varepsilon(r') \].
### Relativistic effective models in compact star description

|       | $M_b$ (MeV) | $q_b(e)$ | $I_{3b}$ | $S_b$ | $\mu_b/\mu_N$ | $\kappa_b/\mu_N$ |
|-------|-------------|----------|----------|-------|----------------|------------------|
| $p$   | 939         | $+1$     | $+1/2$   | $1/2$ | 2.79           | 1.79             |
| $n$   | 939         | 0        | $-1/2$   | $1/2$ | $-1.91$        | $-1.91$          |
| $\Lambda$ | 1116     | 0        | 0        | $1/2$ | $-0.61$        | $-0.61$          |
| $\Sigma^+$ | 1193    | $+1$     | $+1$     | $1/2$ | 2.46           | 1.67             |
| $\Sigma^0$ | 1193    | 0        | 0        | $1/2$ | 1.61           | 1.61             |
| $\Sigma^-$ | 1193    | $-1$     | $-1$     | $1/2$ | $-1.16$        | $-0.37$          |
| $\Xi^0$  | 1315       | 0        | $+1/2$   | $1/2$ | $-1.25$        | $-1.25$          |
| $\Xi^-$  | 1315       | $-1$     | $-1/2$   | $1/2$ | $-0.65$        | 0.06             |
| $\Delta^{++}$ | 1232  | $+2$     | $+3/2$   | $3/2$ | 4.99           | 3.47             |
| $\Delta^+$ | 1232   | $+1$     | $+1/2$   | $3/2$ | 2.49           | 1.73             |
| $\Delta^0$ | 1232   | 0        | $-1/2$   | $3/2$ | 0.06           | 0.06             |
| $\Delta^-$ | 1232   | $-1$     | $-3/2$   | $3/2$ | $-2.45$        | $-1.69$          |

$$g_{ib} = x_{ib} g_i$$

(8)
Figure 2 The $P - \dot{P}$ diagram illustrating the placement of the different isolated neutron star classes. The blue dots mark pulsars detected both in the radio and X-ray bands, the red ones those observed only at X-ray energies. The lines of constant age and magnetic field are also shown (courtesy R.P. Mignani).
Matter composition under extreme magnetic fields

\begin{equation}
\int d^3 k \rightarrow \frac{|q| B}{(2\pi)^2} \sum_{\nu} \int d k_z, \quad \text{where} \quad \nu = n + \frac{1}{2} - \frac{s}{2} \frac{q_b}{|q_b|}
\end{equation}

\begin{equation}
\nu_{\text{max} b}(s) = \left\lfloor \frac{(E_{Fb}^* + s\kappa B)^2 - M_{b}^{*2}}{2|q_b|B} \right\rfloor
\end{equation}
\[
\int d^3k \rightarrow \frac{|q|B}{(2\pi)^2} \sum_{\nu} \int dk_z, \quad \text{where} \quad \nu = n + \frac{1}{2} - \frac{s}{2} \frac{q_b}{|q_b|}
\]

\[
\nu_{\text{max}b}(s) = \left[ \frac{(E_{Fb}^* + s\kappa B)^2 - M_b^* 2}{2 |q_b|B} \right]
\]

$q_b = 0$:

\[
k_{F,b}^2(s) = E_{Fb}^* 2 - (M_b^* - s\kappa B)^2
\]

\[
n_b = \frac{1}{2\pi^2} \sum_s \left\{ \frac{k_{Fb}^3(s)}{3} - s\kappa B \left[ \frac{1}{2} \left( M_b^* - s\kappa B \right) k_{Fb}(s) E_{Fb}^* 2 \left( \arcsin \left( \frac{M_b^* - s\kappa B}{E_{Fb}^*} \right) - \frac{\pi}{2} \right) \right] \right\}
\]

\[
n_{sb} = \frac{M_b^*}{4\pi^2} \sum_s \left[ E_{Fb}^* k_{Fb}(s) - (M_b^* - s\kappa B)^2 \ln \left| \frac{k_{Fb}(s) + E_{Fb}^*}{M_b^* - s\kappa B} \right| \right]
\]

$q_b \neq 0$:

\[
k_{F,b}^2(\nu, s) = E_{Fb}^* 2 - \left( \sqrt{M_b^* 2 + 2\nu |q_b|B - s\kappa B} \right)^2
\]

\[
n_b = \frac{|q_b|B}{2\pi^2} \sum_{\nu, s} k_{Fb}(\nu, s)
\]

\[
n_{sb} = \frac{|q_b|B M_b^*}{2\pi^2} \sum_{s, \nu} \sqrt{M_b^* 2 + 2\nu |q_b|B - s\kappa B} \ln \left| \frac{k_{Fb}(\nu, s) + E_{Fb}^*}{\sqrt{M_b^* 2 + 2\nu |q_b|B} - s\kappa B} \right|
\]
Figure: Particle composition of neutron-star matter with $\Delta$s, with $B = 0$ (top panels) and magnetic field $B = 3 \times 10^{18}$ G (bottom panels), when considering (solid lines) or disregarding (dashed lines) the effects of the AMMs.
Matter composition under extreme magnetic fields

\[ Y_{\text{spin}} = \frac{\sum_{b,s} s n_b(s)}{\sum_{b,s} n_b(s)} , \]  

(17)

Figure: Spin polarization fraction as a function of baryon number density for neutron-star matter with magnetic field \( B = 3 \times 10^{18} \) G, when considering (solid lines) or disregarding (dashed lines) the effects of the AMMs.
Macroscopic structure effects of magnetic fields

![Graph showing the relationship between stellar mass and equatorial radius for different compositions and interaction strengths.](image)

**Figure:** Stellar mass as a function of equatorial radius for different compositions and interaction strengths, for central magnetic fields $B = 0$ (solid lines), $B = 5 \times 10^{17}$ G (dashed lines), and $B = 10^{18}$ G (dotted lines).

| $B$ (G) | $n_c$ (fm$^{-3}$) | $\varepsilon_c$ (MeV/fm$^3$) |
|---------|------------------|-----------------------------|
|         | N+H             | N+H+\Delta                 | N+H             | N+H+\Delta                 |
| 0       | 0.672            | 0.618 (0.614)              | 742             | 658 (657)                  |
| $5 \times 10^{17}$ | 0.701          | 0.659 (0.653)              | 783             | 712 (708)                  |
| $1 \times 10^{18}$ | 0.747          | 0.714 (0.707)              | 850             | 786 (783)                  |
| 0       | 0.629            | 0.625                       | 678             | 672                        |
| $5 \times 10^{17}$ | 0.680          | 0.677                       | 747             | 741                        |
| $1 \times 10^{18}$ | 0.749          | 0.746                       | 843             | 837                        |

**Table:** Central baryon ($n_c$) and energy ($\varepsilon_c$) densities as a function of magnetic field strength for neutron stars of radius 12 km with L3$\omega$\rho model for $x_{\sigma\Delta} = x_{\omega\Delta} = 1.0(1.2)$ in the top panel and CMF model in the bottom panel.
Figure: Magnetic field distribution inside a neutron star of mass $1.8M_{\odot}$ and central magnetic field of $B = 5 \times 10^{17}$ G. Solid, dashed, dashed-dotted and dotted are, respectively, the first four even multipoles of the magnetic field norm ($l = 0, 2, 4, 6$).

Figure: Magnetic field distribution inside a neutron star of mass $1.8M_{\odot}$ and central magnetic field of $B = 5 \times 10^{17}$ G. Solid, dashed and dotted are the dominant monopolar ($l = 0$) term at the polar ($\theta = 0$), intermediate ($\theta = \pi/4$) and equatorial ($\theta = \pi/2$) orientations.
Magnetic field effects on the deconfinement transition

K. D. MARQUEZ
ICTP-SAIFR, IFT-UNESP, São Paulo, Brazil

Universo Primordial
LHC, RHIC
Ponto Crítico Final
LQCD
Outras colisões de ions pesados
Plasma de quarks e glúons
Fase Quarkiônica?
Matéria de hadrons
Núcleos
REGIÃO DE TRANSIÇÃO
Supercondutor de cores
Estrelas de nêutron
Estrelas de nêutron

Assimetria
Densidade bariônica (fm$^{-3}$)

Temperatura (MeV)
**Figure:** Mass-radius diagram for hybrid EoS with chemical equilibrium in both phases, showing results without magnetic field effects.

**Figure:** Example of equations of state of parameter choices that allow the hadron-quark phase transition to occur at $B = 3 \times 10^{18}$ G.

\[
m_i = m_{i0} + \frac{D}{n_b^{1/3}} + C n_b^{1/3} = m_{i0} + m_l,
\]  

(18)
### Magnetic field effects on the deconfinement transition

Table: Values for $\mu_0$ (in MeV) and $p_0$ (in MeV/fm$^3$) for which the conditions of phase coexistence are satisfied at $T = 0$. The latter column specifies whether or not the Bodmer-Witten conjecture is satisfied.

| $C$      | $\sqrt{D}$ | $B = 0$        | $B = 3 \times 10^{18}$ G | B-W |
|----------|-------------|----------------|---------------------------|-----|
| 0        | 155 MeV     | no crossing    | no crossing               | yes |
| 0.23     | 155 MeV     | $\mu_0 = 1130$| $\mu_0 = 1145$            | no  |
| 0.365    | 142 MeV     | $\mu_0 = 1105$| $\mu_0 = 1109$            | yes |
| 0.5      | 135.75 MeV  | $\mu_0 = 1202$| $\mu_0 = 1242$            | yes |
| 0.68     | 130 MeV     | $\mu_0 = 1440$| $\mu_0 = 1475$            | yes |

**K. D. MARQUEZ**
ICTP-SAIFR, IFT-UNESP, São Paulo, Brazil
- The understanding of the meson-delta coupling parameters can be refined by symmetry group considerations, as it is made for the hyperon coupling schemes.

- The magnetic field effects on $\Delta$-admixed matter can be more robustly understood by having the complete solution of the spin-$3/2$ Rarita-Schwinger equation under a magnetic field.
- The understanding of the meson-delta coupling parameters can be refined by symmetry group considerations, as it is made for the hyperon coupling schemes.

- The magnetic field effects on $\Delta$-admixed matter can be more robustly understood by having the complete solution of the spin-3/2 Rarita-Schwinger equation under a magnetic field.

Muito Obrigado!

Kauan Dalfovo Marquez
marquezkauan@gmail.com
Exploring the effects of Delta Baryons in magnetars

Kauan Dalfovo Marquez
marquezkauan@gmail.com

with the collaboration of
D. P. Menezes, V. Dexheimer, D. Chatterjee, M. R. Pelicer and B. C. T. Backes

25/10/2022