Study of electromagnetic decay of $J/\psi$ and $\psi'$ to vector and pseudoscalar

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The electromagnetic decay contributions to $J/\psi(\psi') \rightarrow VP$, where $V$ and $P$ stand for vector and pseudoscalar meson, respectively, are investigated in a vector meson dominance (VMD) model. We show that $J/\psi(\psi') \rightarrow \gamma \rightarrow VP$ can be constrained well with the available experimental information. We find that this process has significant contributions in $\psi' \rightarrow VP$ and may play a key role in understanding the deviations from the so-called “12% rule” for the branching ratio fractions between $\psi' \rightarrow VP$ and $J/\psi \rightarrow VP$. We also address that the “12% rule” becomes very empirical in exclusive hadronic decay channels.

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The decay of charmonia into light hadrons is rich of information about QCD strong interactions between quarks and gluons. Due to the flavor change in the $c\bar{c}$ annihilation, it is also ideal for the study of light hadron production mechanisms, and useful for probing their flavor and gluon contents, such as the search for experimental evidence for glueball and hybrid. In the past decade, data for $J/\psi$ decays have experienced a drastic improvement. We now not only have access to small branching ratio at order of $10^{-6}$, but also have much precise measurements of most of those old channels from BES, DM2 and Mark-III. Such a significant improvement will allow a systematic analysis of correlated channels, from which we expect that dynamical information about the light hadron production mechanisms can be extracted.

In this work, we will study the electromagnetic (EM) decay of vector charmonia ($J/\psi$ and $\psi'$) into light vector and pseudoscalar. From an empirical viewpoint, one can separate the decays of $J/\psi(\psi') \rightarrow VP$ into two classes: i) Isospin conserved channels such as $J/\psi(\psi') \rightarrow \rho\pi$, $K^*K$, $\omega\eta$, $\phi\pi$, etc. These are decays via both strong and EM transitions; ii) Isospin violated channels such as $J/\psi(\psi') \rightarrow \rho\eta$, $\rho\eta'$, $\omega\pi^0$, and $\phi\pi$, of which the leading decay amplitudes are from EM transitions. In association with the above separation is the observation that branching ratios for some of those isospin violated channels, such as $J/\psi(\psi') \rightarrow \rho\eta$, $\rho\eta'$, $\omega\pi^0$, are compatible with the isospin conserved ones such as $\omega\eta'$ and $\phi\eta'$ in $J/\psi$ decays, and $\rho\pi$, $\omega\eta$, $\omega\eta'$, $\phi\pi$, $\phi\eta'$ in $\psi'$ decays. This observation shows that the EM transition may not be as small as we thought in comparison with the strong one. Therefore, its roles played in $J/\psi(\psi') \rightarrow VP$ should be closely investigated.

On a more general ground, the decay channel $J/\psi(\psi') \rightarrow VP$ has attracted a lot of attention in the literature due to its property that the characteristic pQCD helicity conservation rule is violated here\textsuperscript{[2]}. As a consequence, the pQCD power suppression occurs in this channel and leads to a relation for the ratios between $J/\psi$ and $\psi'$ annihilating into three gluons and a single direct photon:

$$R \equiv \frac{BR(\psi' \rightarrow hadrons)}{BR(J/\psi \rightarrow hadrons)} \simeq \frac{BR(\psi' \rightarrow e^+e^-)}{BR(J/\psi \rightarrow e^+e^-)} \simeq 12\%,$$

(1)

which is empirically called “12% rule”. However, much stronger suppressions are found in $\rho\pi$ channel, i.e. $BR(\psi' \rightarrow \rho\pi)/BR(J/\psi \rightarrow \rho\pi) \simeq (2.0 \pm 0.9) \times 10^{-5}$, which gives rise to the so-called “$\rho\pi$ puzzle”. Namely, there exist large deviations from the above empirical “12% rule” in exclusive channels such as $\rho\pi$ and $K^*K + c.c.$

As we know that the “12% rule” is based on the expectation that the charmonium $3g$ strong decays are the dominant ones in exclusive decay channels, the large deviations from the “12% rule” in $\rho\pi$ and $K^*K + c.c.$ naturally imply some underlying mechanisms which can interfere with the $3g$ decays and change the branching ratio fractions. Theoretical explanations for the “12% rule” deviations have been
proposed in the literature [3-8], but so far none of those solutions has been indisputably agreed [18, 19]. This makes it necessary to provide a detailed study of the charmonium EM decays. If compatible strength of the EM transition occurs in some of those exclusive decay channels, one can imagine that large interferences between the EM and strong transitions are possible, and they may be one of the important sources which produce large deviations from the “12% rule” in $V \gamma P$ decay channels. Relevant studies can also be found in the literature for understanding the role played by the EM transitions in $V \gamma P$ decays. Parametrization schemes are proposed to estimate the EM decay contributions to $J/\psi \to V P$ in Refs. [20, 21], but a coherent study of $J/\psi$ and $\psi'$ is still unavailable.

Apart from the above interests, the EM decay of $J/\psi \to V P$ is also rich of dynamical information about the Okubo-Zweig-Iizuka (OZI) rule [22]. The decay of $J/\psi \to \phi \pi^0$ involves both isospin and OZI-rule violations. Although only the upper limit, $B R_{\exp}(J/\psi \to \phi \pi^0) < 6.4 \times 10^{-6}$, is given, this will be an interesting place to test dynamical prescriptions for $J/\psi \to \gamma^* \to V P$. In Refs. [3, 8], apart from the EM process, the OZI doubly disconnected processes are also investigated, which however possesses large uncertainties. In particular, the separation of these two correlated processes is strongly model-dependent.

In this work, we will introduce an effective Lagrangian for $V \gamma P$ couplings, and apply the vector meson dominance (VMD) model to $V \gamma^*$ couplings. By studying the $J/\psi(\psi') \to \gamma^* \to V P$ at tree level, we shall examine the “12% rule” for those exclusive $V \gamma P$ decay channels. Deviations from this empirical rule in the exclusive decays can thus be highlighted.

For $J/\psi(\psi') \to \omega \eta, \omega \eta', \phi \eta, \phi \eta', \rho \pi$, and $K^* K + c.c.$, the strong and EM decay process are mixed, and the former generally plays a dominant role. For $J/\psi(\psi') \to \rho \eta, \rho \eta', \omega \pi$, and $\phi \pi$, the transitions are via EM processes, of which the isospin conservation is violated. Typical transitions for $V_1 \to V_2 P$ are illustrated by Fig. 1, which consists of three contributions: (a) the process that the pseudoscalar meson is produced in association with the virtual photon via $V_1$ annihilation; (b) the pseudoscalar produced at the final state vector meson $V_2$ vertex; and (c) the pseudoscalar produced via the axial current anomaly. Note that isospin conservation can be violated in both Fig. 1(a) and (b), with the observation of non-zero branching ratios for $J/\psi \to \gamma^* \pi^0$ [1]. At hadronic level, these are independent processes where all the vertices can be determined by other experimental measurements. This treatment is different from that of Refs. 3, 8. Although the OZI disconnected diagram considered in Refs. 3, 8 is similar to our Fig. 1(b), our consideration of the $V \gamma^* P$ coupling will allow us to include both OZI and isospin violation effects which can be constrained by experimental data.

We introduce a typical effective Lagrangian for the $V \gamma P$ coupling:

$$\mathcal{L}_{V \gamma P} = \frac{g_{V \gamma P}}{M_V} \epsilon_{\mu \nu \alpha \beta} \partial^\mu V^\nu \partial^\alpha A^\beta P$$

where $V^\nu(\equiv \rho, \omega, \phi, J/\psi, \psi' \ldots)$ and $A^\beta$ are the vector meson and EM field, respectively; $M_V$ is the vector meson mass; $\epsilon_{\mu \nu \alpha \beta}$ is the anti-symmetric Levi-Civita tensor.

The $V \gamma^*$ coupling is described by the VMD model,

$$\mathcal{L}_{V \gamma} = \sum_V \frac{e M_V^2}{f_V} V_\mu A^\mu ,$$

where $e M_V^2/f_V$ is a direct photon-vector-meson coupling in Feynman diagram language, and the isospin 1 and 0 component of the EM field are both included.

The invariant transition amplitude for $V_1 \to \gamma^* \to V_2 P$ can thus be expressed as:

$$\mathcal{M} \equiv \mathcal{M}_A + \mathcal{M}_B + \mathcal{M}_C$$

$$= \left( e \frac{g_{V_1 V_2 P}}{M_{V_1}} \mathcal{F}_a + \frac{e^2}{M_{V_1} M_{V_2}} g_{P \gamma P} \mathcal{F}_b + \frac{g_{P \gamma^* P}}{M_P} \mathcal{F}_c \right) \epsilon_{\mu \nu \alpha \beta} \partial^\mu V_\nu^\prime \partial^\alpha V_2^\beta P$$

where $g_{P \gamma^*}$ is the coupling for the neutral pseudoscalar meson decay to two photons; $\mathcal{F}_a$ and $\mathcal{F}_b$ denote the form factor corrections to the $V \gamma^* P$ vertices in comparison with the real photon transition for $V \to \gamma P$; and $\mathcal{F}_c$ is the form factor for $P \to \gamma^* \gamma^*$.

The partial decay width can thus be expressed as

$$\Gamma(V_1 \to V_2 P) = \frac{|\mathcal{P}_{P,2}|}{8 \pi M_{V_1}^2} \frac{1}{2S + 1} \sum |\mathcal{M}|^2$$
The vector meson radiative decay partial width, and available in experiment.

\[ \Gamma_{\text{exp}}(V \rightarrow \gamma P) = \left[ \frac{12\pi M_V^2 \Gamma_{\text{exp}}(V \rightarrow \gamma P)}{|p_{\gamma'}|^3} \right]^{1/2}, \]

where \( p_{\gamma'} \) is the three-momentum of the photon in the initial vector meson rest frame; \( \gamma \) is the three-momentum of the final state vector meson in the initial-vector-meson-rest frame.

Those three typical coupling constants are determined as follows:

(I) For \( V \rightarrow \gamma P \) decay, following the effective Lagrangian of Eq. (2), we derive the coupling constant:

\[ g_{V\gamma P} = \left[ \frac{12\pi M_V^2 \Gamma_{\text{exp}}(V \rightarrow \gamma P)}{|p_{\gamma'}|^3} \right]^{1/2}, \]

where \( |p_{\gamma'}| \) is the three-momentum of the photon in the initial vector meson rest frame; \( \Gamma_{\text{exp}}(V \rightarrow \gamma P) \) is the vector meson radiative decay partial width, and available in experiment.

For \( \rho \gamma \eta' \) and \( \omega \gamma \eta' \) couplings, we determine the coupling constants in \( \eta' \rightarrow \gamma \rho \) and \( \gamma \omega \):

\[ g_{V\gamma \eta'} = \left[ \frac{4\pi M_V^2 \Gamma_{\text{exp}}(\eta' \rightarrow \gamma V)}{|p'_{\eta'}|^3} \right]^{1/2}, \]

where \( |p'_{\eta'}| \) is the three-momentum of the photon in the \( \eta' \) rest frame.

(II) The \( V^* \) coupling is determined in \( V \rightarrow e^+e^- \) channel. With the partial decay width \( \Gamma_{V \rightarrow e^+e^-} \), the coupling constant \( e/f_V \) can be derived:

\[ \frac{e}{f_V} = \left[ \frac{3\Gamma_{V \rightarrow e^+e^-}}{4\alpha_e |p_e|^3} \right]^{1/2}, \]

where we have neglected the mass of the electron and \( |p_e| \) is the electron three-momentum in the vector meson rest frame; \( \alpha_e = 1/137 \) is the fine-structure constant.

(III) For \( P \rightarrow \gamma \gamma \), we adopt the following form of effective Lagrangian:

\[ L_{P\gamma\gamma} = \frac{g_{P\gamma\gamma}}{M_P} \epsilon_{\mu\nu\alpha\beta} \partial^\mu A^\nu \partial^\alpha A^\beta P, \]

where the coupling constant is normalized to the pseudoscalar meson mass \( M_P \). With the partial decay width \( \Gamma_{\text{exp}}(P \rightarrow \gamma \gamma) \) the coupling constant for real photon in the final state can be derived:

\[ g_{P\gamma\gamma} = \left[ \frac{32\pi \Gamma_{\text{exp}}(P \rightarrow \gamma \gamma)}{M_P} \right]^{1/2}. \]

It is encouraging that for all the decay channels of \( J/\psi(\psi') \rightarrow \gamma^* \rightarrow VP \), the experimental data are available for determining the above coupling constants: \( g_{V\gamma P}, e/f_V, \) and \( g_{P\gamma\gamma} \). We are then left with the only uncertainty from the form factors due to the exchange of off-shell photons.

We find that without form factors, i.e. \( F_a = F_b = F_c = 1 \), the calculated branching ratios for the isospin violated channels will be significantly overestimated. This is expected due to the large virtualities of the off-shell photons and the consequent power suppressions from the pQCD hadronic helicity-conservation. Since we think that the non-perturbative QCD effects might have played a role in the transitions at \( J/\psi \) energy, e.g. in Fig. (a) and (b) a pair of quarks may be created from vacuum as described by \( 3P_0 \) model, the pQCD hadronic helicity-conservation due to the vector nature of gluon is violated quite strongly, thus alternatively, we would like to suggest a monopole-like (MP) form factor dedicated to the suppression effects:

\[ F(q^2) = \frac{1}{1 - q^2/\Lambda^2}, \]

1 For instance such as in the inclusive decay \( J/\psi \rightarrow q\bar{q} \), the ‘virtualness’ of one quark in the created quark pair is less than chiral broken energy scale \( \Lambda_\chi \sim 1.0 \text{ GeV} \), thus, the non-perturbative QCD effects must be sizeable in the concerned decays here.
where \( \Lambda \) can be regarded as an effective mass accounting for the overall effects from possible resonance poles and scattering terms in the time-like kinematic region, and will be determined by fitting the data \(^1\) for \( J/\psi(\psi') \rightarrow \rho \eta, \rho \pi^0 \), and \( \omega \pi^0 \).

It should be noted that this MP form factor can only partly depict the pQCD power suppression due to violations of the hadronic helicity conservation when \( q^2 \gg \Lambda^2 \) \(^2\), but it is quite consistent with the VMD framework.

By adopting the MP empirical form factor, we have already assumed that non-perturbative effects might have played a substantial role in the transitions. In principle it should be tested experimentally via measuring the coupling the processes \( J/\psi(\psi') \rightarrow Pe^+e^- \) and \( e^+e^- \rightarrow Pe^+e^- \), respectively, when the integrated luminosity at \( J/\psi \) and the suitable energies for \( e^+e^- \) colliders is accumulated enough.

The form factor \( F_c \) appearing in Eqs. \(^4\) and \(^5\) can be determined in \( \gamma^* \gamma^* \) scatterings. A commonly adopted form factor is

\[
F_c(q_1^2, q_2^2) = \frac{1}{(1 - q_1^2/\Lambda^2)(1 - q_2^2/\Lambda^2)},
\]

where \( q_1^2 = M_{V_1}^2 \) and \( q_2^2 = M_{V_2}^2 \) are the squared four-momenta carried by the time-like photons. We assume that the \( \Lambda \) is the same as in Eq. \(^1\), thus, \( F_c = F_a F_b \).

Proceeding to the numerical calculations, we first determine the coupling constants, \( e/f \), \( g_{V\gamma P} \) and \( g_{P\gamma \gamma} \) in the corresponding decays, and the results are listed in Tables \(^I\)\(^-II\)\(^-III\)\(^-I\)\(^I\). It shows that the \( e/f \) coupling is the largest one while all the others are compatible. For the \( g_{V\gamma P} \), it is sizeable for light vector mesons and much smaller for \( J/\psi \) and \( \psi' \). Note that there is no datum for \( \psi' \rightarrow \gamma \pi \) available. So we simply put \( g_{\psi'\gamma \pi} = 0 \) in the calculations. The \( P\gamma\gamma \) couplings can be well determined due to the good shape of the data \(^1\).

To examine the role played by the form factors, we first calculate the EM decay branching ratios without form factors, i.e. \( F(q^2) = 1 \). It shows that all the data are significantly overestimated by the theoretical predictions as shown by Tables \(^I\)\(^V\) and \(^V\)\(^-I\)\(^I\). Nonetheless, it shows that without form factors process (b) is the only dominant transition.

To determine the effective mass \( \Lambda \) in the MP form factor, we consider two possible relative phases between process (a) and (b) in fitting the data for the isospin violated channels, \( J/\psi(\psi') \rightarrow \rho \eta, \rho \pi^0 \), and \( \omega \pi^0 \). We mention in advance that the contributions from process (c) will bring only few percent corrections to the results. Since the corrections are within the datum uncertainties, we are not bothered to consider its relative phase to process (a) and (b). In Tables \(^I\)\(^V\) and \(^V\)\(^-I\)\(^I\) the results for process (a) and (b) in a constructive phase (MP-C) with \( \Lambda = 0.616 \pm 0.008 \) GeV, and in a destructive phase (MP-D) with \( \Lambda = 0.65 \pm 0.01 \) GeV are listed. The reduced \( \chi^2 \) values are \( \chi^2 = 4.1 \) in MP-C and 14.2 in MP-D, respectively.

With the above fitted values for \( \Lambda \) (MP-C and MP-D), predictions for those isospin conserved channels in \( J/\psi \rightarrow \gamma^* \rightarrow VP \) and \( \psi' \rightarrow \gamma^* \rightarrow VP \) are listed in Table \(^I\)\(^V\) and \(^V\)\(^-I\)\(^I\) to compare with the experimental data \(^1\).

There arise some basic issues from the theoretical results.

(I) We find that even though with the form factors, process (b) in Fig. \(^I\) is still the dominant one in most channels except for \( \rho \pi^0 \). For most channels the couplings \( g_{J/\psi \gamma P} \) and \( g_{\psi' \gamma P} \) in process (a) are generally small, and similarly are \( e^2/(f_{V_1} f_{V_2}) \) and \( g_{P\gamma \gamma} \) in process (c). However, we find that the amplitudes of process (a) and (b) are compatible in \( J/\psi \rightarrow \rho \eta \). As shown in Table \(^V\)\(^-I\)\(^I\), large cancellations appear in the branching ratio when (a) and (b) are in a destructive phase (Column MP-D). This is due to the relatively large branching ratios for \( J/\psi \rightarrow \gamma \rho \) \(^3\). Such a large difference between these two phases makes the \( \rho \pi^0 \) channel extremely interesting. The branching ratio fraction will be useful for distinguishing the relative phases between (a) and (b) in the isospin violated channels. It also highlights the empirical aspect of the pQCD “12% rule” in exclusive hadronic decays.

We also note that process (a) and (c) do not contribute to \( K^* K^+c.c. \) and \( \rho^+ \pi^- + c.c. \). This turns to be an advantage for understanding the decay mechanism of \( J/\psi(\psi') \rightarrow \gamma^* \rightarrow \rho^+ \pi^- + c.c. \) and \( K^* K^+ c.c. \), and should be also an ideal place to test the “12% rule” in exclusive decays.

To be more specific, we analyze first those four isospin violation decays: \( J/\psi(\psi') \rightarrow \gamma^* \rightarrow \rho \eta, \rho \pi^0, \omega \pi^0 \) and \( \phi \pi^0 \). These decays to leading order are through EM transitions. Transitions of Fig. \(^I\) have shown how the kinematic and form-factor corrections can correlate with the naive pQCD expected ratio: \( \Gamma(\psi' \rightarrow e^+e^-)/\Gamma(J/\psi \rightarrow e^+e^-) \), i.e. the “12% rule”, and makes it very empirical.
As an example, for those channels dominated by process (b), the exclusive decays are still approximately proportional to the charmonium wavefunction at its origin, i.e. $|\psi(0)|^2$, by neglecting the contributions from process (a) and (c). The branching ratio fraction can be expressed as:

$$R^{VP} = \frac{BR(\psi' \rightarrow \gamma^* \rightarrow VP)}{BR(J/\psi \rightarrow \gamma^* \rightarrow VP)} \approx \frac{BR(\psi' \rightarrow e^+e^- | p_e| |p'_{e}|^3}{BR(J/\psi \rightarrow e^+e^- | p_{e}| |p'_{e}|^3}_{F_2^e(M^2_{\psi'})},$$

where $|p_e|$ and $|p'_{e}|$ are three-momenta of the electron in $J/\psi \rightarrow e^+e^-$ and $\psi' \rightarrow e^+e^-$ in the vector meson rest frame, respectively; while $|p_{e}|$, and $|p'_{e}|$ are three-momenta of the final state vector mesons in $J/\psi \rightarrow \gamma^* P$ and $\psi' \rightarrow \gamma^* P$, respectively. It shows that the respect of the “12% rule” requires that the kinematic and form factor corrections cancel each other for all those channels, which however, is not a necessary consequence of the physics at all. Including the contributions from process (a) and (c) will worsen the situation.

To see this more clearly, we list the branching ratio fractions for the choice of MP-C ($R_{1/2}^{VP}$), MP-D ($R_{1/2}^{VP}$) and without form factors ($R_{1}^{VP}$) in Table (V) to compare with the data. It shows that without the form factor corrections, ratio $R_{1/2}^{VP}$ has a values in a range of (19 $\sim$ 21)% for those four channels, which are larger than the expectation of the “12% rule”.

With the form factors, it shows that $R_{1/2}^{VP}$ has a stable range of (7 $\sim$ 9)%, while more drastic changes occur to $R_{1/2}^{VP}$. For instance, we obtain $R_{1/2}^{vp} = 52\%$ which strongly violates the “12% rule”. Unfortunately, the data still have large uncertainties. We expect that an improved branching ratio fraction for this channel will be able to determine the relative phase between process (a) and (b) in our model, and highlight the underlying mechanism.

The branching ratios for $\phi\pi^0$ channel are much smaller than others due to the small $\phi\gamma\pi$ coupling. This is in a good agreement with the OZI rule suppressions expected in $\phi\pi^0$ channel.

(III) For the isospin conserved channels, the EM decay contributions in $J/\psi$ decays turn out to be rather small in both MP-D and MP-C phases. This is consistent with studies in the literature that $J/\psi \rightarrow VP$ is dominated by the $3g$ transitions. Thus, the deviation of the “12% rule” could be more likely due to the suppression of the amplitudes in $\psi' \rightarrow VP$ (see the review of Ref. [18]).

If we simply apply the relations between the strong and EM transitions parametrized by Ref. [21], the ratio between charged and neutral channels can be expressed as:

$$Q = \frac{BR(\psi' \rightarrow K^{+}K^{-} + c.c.)}{BR(\psi' \rightarrow K^{*0}K^{0} + c.c.)} \approx \frac{[g(1-s) + e]^2}{[g(1-s) - 2e]^2},$$

(14)

where $g$ and $e$ denote the strong and EM decay strengths, respectively, and $s \approx 0.1$ is a parameter for the flavor SU(3) breaking. One can see that for $e = (-1/3 \sim -1/2) \times g(1-s)$, we have $Q \approx 0.06 \sim 0.16$, which is in a good agreement with the data, 0.08 $\sim$ 0.28 [1].

We can also check the other two correlated relations:

$$\frac{BR(\psi' \rightarrow K^{*+}K^{-} + c.c.)}{BR(\psi' \rightarrow K^{*0}K^{0} + c.c.)} \approx \frac{e^2}{g(1-s) - 2e} \approx 0.25 \sim 1,$$

(15)

corresponding to $e = (-1/3 \sim -1/2) \times g(1-s)$. This is consistent with the range of $BR_{MP}/BR_{exp} \approx 0.22 \sim 0.56$ and $BR_{MP}/BR_{exp} \approx 0.28 \sim 0.70$.

Similarly, for $\psi' \rightarrow K^{0}\bar{K}^0 + c.c.$, we have

$$\frac{BR(\psi' \rightarrow K^{*+}K^{-} + c.c.)}{BR(\psi' \rightarrow K^{*0}K^{0} + c.c.)} \approx \frac{4e^2}{g(1-s) - 2e} \approx 0.16 \sim 0.44,$$

(16)

corresponding to the same range for $e = (-1/3 \sim -1/2) \times g(1-s)$. It also turns to be compatible with $BR_{MP}/BR_{exp} \approx 0.10 \sim 0.15$ and $BR_{MP}/BR_{exp} \approx 0.12 \sim 0.18$.

For $\rho\pi$ channel, the above relative phase between the strong and EM transitions can explain the relatively small branching ratios for $\psi' \rightarrow \rho\pi$, i.e. the EM amplitude will destructively interfere will the strong one. With $e = (-1/3 \sim -1/2)g(1-s) \approx (-1/3 \sim -1/2)g$ [21], we have a relation:

$$\frac{BR(\psi' \rightarrow K^{*0} + c.c.)}{BR(\psi' \rightarrow \rho\pi)} \approx \frac{e^2}{g(e)} = 0.25 \sim 1,$$

(17)
which is also consistent with $BR_{\psi'\to VP}^{MP}/BR_{\psi'\to VP}^{exp} = 0.18 \sim 0.39$ and $BR_{\psi'\to VP}^{MP}/BR_{\psi'\to VP}^{exp} = 0.22 \sim 0.49$ \cite{1}.

The above analysis suggests that the relative phase between the EM and strong transition amplitudes in $\psi' \to \rho\pi$ favors $180^\circ$. But due to the large uncertainties with the data, other phases may be possible \cite{24}.

Evidently, for those channels of which the branching ratio fractions between $\psi'$ and $J/\psi$ are observed to deviate from the “12% rule” (see Table \text{VI}), such as $\rho\pi$ and $K^*\bar{K} + c.c.$, they turn to have sizeable EM contributions as shown by Tables \text{VII} and \text{V}. Since the $J/\psi$ decays are still dominated by the strong transitions, the measured branching ratio fractions mostly reflect the interfered effects in $\psi'$ decays over the strong transitions in $J/\psi \to VP$.

We also analyze EM decay contributions to $\psi(3770) \to VP$, and find they becomes negligibly small due to the small partial width for $\psi(3770) \to e^+e^-$ and form factor suppressions. In most channels, the EM transitions contribute to the branching ratios at $O(10^{-8})$ or even smaller, which is beyond the access of the present experimental facilities. Therefore, we can conclude that for those measured $\psi(3770) \to VP$ channels the isospin violations must be negligibly small.

This approach can also be applied to the study of the EM decay contributions to $\phi \to \rho\pi$ and $\omega\pi$. For the isospin violated $\omega\pi^0$ channel, with the same form factor (MP-C), we find $BR(\phi \to \omega\pi^0) = 1.8 \times 10^{-6}$ and $BR(\phi \to \rho\pi) = 3.2 \times 10^{-6}$ in contrast with the the experimental data, $BR_{\exp}(\phi' \to \omega\pi^0) = \left(5.2^{+1.3 \, \wedge -1.1}_{-1.1}\right) \times 10^{-5}$ and $BR_{\exp}(\phi \to \rho\pi + \pi^+\pi^-\pi^0) = (15.3 \pm 0.4)\%$ \cite{1}. Note that $\phi$ and $\omega$ are quite close in mass, the results should be sensitive to the form factors, and more elaborate treatment for them are generally required. If we slightly adjust $\Lambda$, we can reproduce the data for $\omega\pi^0$ well. For $\rho\pi$ channel the EM contributions are found much smaller than the data which suggests the dominance of strong decays in $\phi \to \rho\pi$.

In summary, we make a systematic analysis of the EM decay contributions to $J/\psi(\psi') \to \gamma^* \to VP$ essentially in a VMD model. With the constraint from the available independent experimental data, the EM contributions in all the $VP$ hadronic channels can be consistently investigated. Our results are in a good agreement with the data for those isospin violated channels such as $\rho\eta$, $\rho\eta'$, $\omega\pi$, and $\phi\pi$, especially for the ‘MP-C’ model (see Tables \text{VIII} and \text{V}). For most of those isospin conserved channels, we find that the EM decay contributions bring sizeable corrections to the $\psi' \to VP$, which could be a major source accounting for the branching ratio fraction deviations between the $VP$ exclusive decays of $J/\psi$ and $\psi'$. Large difference between the data for $\psi' \to K^{*0}\bar{K}^0 + c.c.$ and $K^{*+}K^- + c.c.$ \cite{1} has shown such a possibility. An improved measurement of these two channels will be able to clarify the predominant role played by the EM transitions.

For the $\rho\pi$ channel, we find that the EM decay contributions in $\psi'$ decays is compatible with the experimental data. This highlights that the interferences from the EM transitions can play essential role in the decay transitions, especially, in the understanding of the abnormally small ratio of $BR(\psi' \to \rho\pi)/BR(J/\psi \to \rho\pi)$. However, it should be pointed out that the compatible strength of the EM decay contributions with the experimental data in $\psi' \to \rho\pi$ may reflect the suppressed strong decay strength. In this sense, a full understanding of the small branching ratio fraction in $\rho\pi$ channel will require a coherent study of the strong decay mechanisms \cite{10,15,16}. This is beyond the focus of this work and should be pursued in further studies.

Finally, we note that a full constraint on this model can be reached by accommodating experimental measurements of $J/\psi(\psi') \to Pe^+e^-$ and $e^+e^- \to Pe^+e^-$ to derive the couplings for $g_{\psi'\rho\pi}$ and $g_{\psi\gamma\rho\pi}$ from which the form factor information can be derived. Ambiguities from the empirical form factors can thus be minimized. Experimental analysis of these processes at BES is thus strongly recommended.

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[1] W. M. Yao et al. [Particle Data Group], J. Phys. G 33 (2006) 1.
\begin{align*}
V_1 & \xrightarrow{\gamma^*} V_2 \\
& \quad \text{g}_{V_1} \gamma \nonumber \\
& \quad \text{P} \\
& \quad \text{f}_{V_2} \\
\end{align*}
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V_1 & \xrightarrow{\gamma^*} V_2 \\
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\begin{align*}
V_1 & \xrightarrow{\gamma^*} V_2 \\
& \quad \text{g}_{V_1} \gamma \nonumber \\
& \quad \text{P} \\
& \quad \text{f}_{V_2} \\
\end{align*}

(a) \hspace{2cm} (b) \hspace{2cm} (c)

FIG. 1: Schematic diagrams for $J/\psi (\psi') \rightarrow \gamma^* \rightarrow VP$.

| Coupling const. $e/f_V$ | Values ($\times 10^{-2}$) | Total width of $V$ | $BR(V \rightarrow e^+e^-$) |
|-------------------------|--------------------------|--------------------|---------------------------|
| $e/f_\rho$              | 4.28                     | 146.4 MeV          | $(4.70 \pm 0.08) \times 10^{-5}$ |
| $e/f_\omega$            | 1.26                     | 8.49 MeV           | $(7.18 \pm 0.12) \times 10^{-5}$ |
| $e/f_\phi$              | 1.60                     | 4.26 MeV           | $(2.97 \pm 0.04) \times 10^{-4}$ |
| $e/f_{J/\psi}$          | 1.92                     | 93.4 keV           | $(5.94 \pm 0.06)$%         |
| $e/f_{e'/e}$            | 1.17                     | 337 keV            | $(7.35 \pm 0.18) \times 10^{-3}$ |

TABLE I: The coupling constant $e/f_V$ determined in $V \rightarrow e^+e^-$. The data for branching ratios are from PDG2006 [1].
| Coupling const. $g_{V\gamma P}$ | Values | Branching ratios |
|-------------------------------|--------|-----------------|
| $g_{\rho\gamma\eta}$         | 0.372  | $(2.95 \pm 0.30) \times 10^{-4}$ |
| $g_{\rho\gamma\eta'}$       | 0.302  | $(29.4 \pm 0.9)\%$ |
| $g_{\rho^0\gamma\eta}$      | 0.197  | $(6.0 \pm 0.8) \times 10^{-4}$ |
| $g_{\rho^0\gamma\eta'}$     | 0.170  | $(4.5 \pm 0.5) \times 10^{-4}$ |
| $g_{\omega\gamma\eta}$      | 0.110  | $(4.9 \pm 0.5) \times 10^{-4}$ |
| $g_{\omega\gamma\eta'}$     | 0.107  | $(3.03 \pm 0.31)\%$ |
| $g_{\omega\gamma\pi}$       | 0.565  | $(8.90 \pm 0.27)\%$ |
| $g_{\phi\gamma\eta}$        | 0.213  | $(1.301 \pm 0.024)\%$ |
| $g_{\phi\gamma\eta'}$       | 0.218  | $(6.2 \pm 0.7) \times 10^{-5}$ |
| $g_{\phi\gamma\pi}$         | 0.041  | $(1.25 \pm 0.07) \times 10^{-3}$ |
| $g_{K^+\gamma\pi}$          | 0.225  | $(9.9 \pm 0.9) \times 10^{-4}$ |
| $g_{K^0\gamma\eta}$         | 0.340  | $(2.31 \pm 0.20) \times 10^{-3}$ |
| $g_{J/\psi\gamma\eta}$      | $3.13 \times 10^{-3}$ | $(9.8 \pm 1.0) \times 10^{-4}$ |
| $g_{J/\psi\gamma\eta'}$     | $7.61 \times 10^{-3}$ | $(4.71 \pm 0.27) \times 10^{-3}$ |
| $g_{\psi'\gamma\eta}$       | $5.49 \times 10^{-4}$ | $(3.3 \pm 0.6) \times 10^{-5}$ |
| $g_{\psi'\gamma\eta'}$      | $1.63 \times 10^{-3}$ | $< 9 \times 10^{-5}$ |

TABLE II: The coupling constant $g_{V\gamma P}$ determined in $V \to \gamma P$ or $P \to \gamma V$. The data for branching ratios are from PDG2006 [1].

| Coupling const. $g_{P\gamma\gamma}$ | Values | Branching ratios |
|-----------------------------------|--------|-----------------|
| $g_{\phi\gamma\gamma}$           | $2.40 \times 10^{-3}$ | $(98.798 \pm 0.032)\%$ |
| $g_{\phi\gamma\gamma}$           | $9.70 \times 10^{-3}$ | $(39.38 \pm 0.26)\%$ |
| $g_{\phi'\gamma\gamma}$          | $2.12 \times 10^{-2}$ | $(2.12 \pm 0.14)\%$ |

TABLE III: The coupling constant $g_{P\gamma\gamma}$ determined in $P \to \gamma \gamma$. The data for branching ratios are from PDG2006 [1].
| Decay channels | $F(q^2) = 1$ | MP-D | MP-C | Exp. data |
|---------------|--------------|------|------|-----------|
| $\rho\eta$    | $6.8 \times 10^{-5}$ | $8.0 \times 10^{-5}$ | $1.6 \times 10^{-4}$ | $(1.93 \pm 0.23) \times 10^{-4}$ |
| $\rho\eta'$   | $3.5 \times 10^{-3}$ | $4.4 \times 10^{-4}$ | $1.5 \times 10^{-4}$ | $(1.05 \pm 0.18) \times 10^{-4}$ |
| $\omega\pi$   | 0.16         | $3.4 \times 10^{-4}$ | $2.8 \times 10^{-4}$ | $(4.5 \pm 0.5) \times 10^{-4}$ |
| $\phi\pi$     | $4.4 \times 10^{-4}$ | $8.1 \times 10^{-7}$ | $8.3 \times 10^{-7}$ | $< 6.4 \times 10^{-6}$ |
| $\rho^0\pi^0$ | $2.0 \times 10^{-2}$ | $3.6 \times 10^{-2}$ | $3.9 \times 10^{-2}$ | $(5.6 \pm 0.7) \times 10^{-3}$ |
| $\rho\pi$     | $5.2 \times 10^{-2}$ | $1.0 \times 10^{-2}$ | $9.2 \times 10^{-3}$ | $(1.69 \pm 0.15) \times 10^{-2}$ |
| $\omega\eta$  | $5.7 \times 10^{-3}$ | $7.0 \times 10^{-6}$ | $1.3 \times 10^{-5}$ | $(1.74 \pm 0.20) \times 10^{-3}$ |
| $\omega\eta'$ | $4.2 \times 10^{-3}$ | $1.6 \times 10^{-6}$ | $1.5 \times 10^{-5}$ | $(1.82 \pm 0.21) \times 10^{-4}$ |
| $\phi\eta$    | $1.1 \times 10^{-2}$ | $2.0 \times 10^{-5}$ | $2.0 \times 10^{-5}$ | $(7.4 \pm 0.8) \times 10^{-4}$ |
| $\phi\eta'$   | $8.3 \times 10^{-3}$ | $1.3 \times 10^{-5}$ | $1.8 \times 10^{-5}$ | $(4.0 \pm 0.7) \times 10^{-4}$ |
| $K^{*+}K^-$ + c.c. | $3.4 \times 10^{-2}$ | $7.4 \times 10^{-5}$ | $5.9 \times 10^{-5}$ | $(5.0 \pm 0.4) \times 10^{-3}$ |
| $K^{*0}\bar{K}^0$ + c.c. | $7.8 \times 10^{-2}$ | $1.6 \times 10^{-4}$ | $1.3 \times 10^{-4}$ | $(4.2 \pm 0.4) \times 10^{-3}$ |

**TABLE IV:** Branching ratios for $J/\psi \rightarrow \gamma^* \rightarrow VP$ without the form factor ($F(q^2) = 1$) and with the monopole (MP) form factor. Column MP-C corresponds to process (a) and (b) in a constructive phase with an effective mass $\Lambda = 0.616$ GeV. Column MP-D corresponds to (a) and (b) in a destructive phase with $\Lambda = 0.65$ GeV. The data for branching ratios are from PDG2006 [1].

| Decay channels | $F(q^2) = 1$ | MP-D | MP-C | Exp. data |
|---------------|--------------|------|------|-----------|
| $\rho\eta$    | $1.3 \times 10^{-3}$ | $7.3 \times 10^{-6}$ | $1.5 \times 10^{-5}$ | $(2.2 \pm 0.6) \times 10^{-5}$ |
| $\rho\eta'$   | $7.3 \times 10^{-3}$ | $2.3 \times 10^{-6}$ | $1.1 \times 10^{-5}$ | $(1.9 \pm 1.7) \times 10^{-5}$ |
| $\omega\pi$   | $3.1 \times 10^{-3}$ | $3.2 \times 10^{-5}$ | $2.6 \times 10^{-5}$ | $(2.1 \pm 0.6) \times 10^{-5}$ |
| $\phi\pi$     | $8.7 \times 10^{-5}$ | $8.8 \times 10^{-8}$ | $7.0 \times 10^{-8}$ | $< 4 \times 10^{-6}$ |
| $\rho^0\pi^0$ | $3.8 \times 10^{-3}$ | $3.9 \times 10^{-6}$ | $3.1 \times 10^{-6}$ | *** |
| $\rho\pi$     | $9.6 \times 10^{-3}$ | $9.8 \times 10^{-6}$ | $7.9 \times 10^{-6}$ | $(3.2 \pm 1.2) \times 10^{-5}$ |
| $\omega\eta$  | $1.1 \times 10^{-3}$ | $6.5 \times 10^{-7}$ | $1.3 \times 10^{-6}$ | $< 1.1 \times 10^{-5}$ |
| $\omega\eta'$ | $8.9 \times 10^{-4}$ | $4.0 \times 10^{-7}$ | $1.2 \times 10^{-6}$ | $(3.2 \pm 2.5) \times 10^{-5}$ |
| $\phi\eta$    | $2.2 \times 10^{-3}$ | $1.9 \times 10^{-6}$ | $2.0 \times 10^{-6}$ | $(2.8 \pm 1.0) \times 10^{-5}$ |
| $\phi\eta'$   | $1.9 \times 10^{-3}$ | $1.6 \times 10^{-6}$ | $1.8 \times 10^{-6}$ | $(3.1 \pm 1.6) \times 10^{-5}$ |
| $K^{*+}K^-$ + c.c. | $6.7 \times 10^{-2}$ | $7.0 \times 10^{-6}$ | $5.6 \times 10^{-6}$ | $(1.7 \pm 0.8) \times 10^{-5}$ |
| $K^{*0}\bar{K}^0$ + c.c. | $1.5 \times 10^{-2}$ | $1.6 \times 10^{-5}$ | $1.3 \times 10^{-5}$ | $(1.09 \pm 0.20) \times 10^{-4}$ |

**TABLE V:** Branching ratios for $\phi' \rightarrow \gamma^* \rightarrow VP$ without the form factor ($F(q^2) = 1$) and with the monopole (MP) form factor. The notations are the same as Table IV. The stars "***" in $\rho^0\pi^0$ channel denotes the unavailability of the data.
TABLE VI: Branching ratio fractions for $\psi' \rightarrow \gamma^* \rightarrow VP$ over $J/\psi \rightarrow \gamma^* \rightarrow VP$. $R_1^{VP}$ refers to $F(q^2) = 1$, while $R_2^{VP}$ and $R_3^{VP}$ correspond to calculations with MP-D and MP-C form factors, respectively. The last column is extracted from the experimental date [1]. The stars "***" in $\rho^0\pi^0$ channel denotes the unavailability of the data.

| Decay channels | $R_1^{VP}$ (%) | $R_2^{VP}$ (%) | $R_3^{VP}$ (%) | Exp. data (%) |
|----------------|----------------|----------------|----------------|--------------|
| $\rho\eta$     | 19             | 9              | 9              | 11.5 ± 5.0   |
| $\rho\eta'$    | 21             | 52             | 7              | 23.5 ± 17.8  |
| $\omega\pi$    | 19             | 9              | 9              | 5.0 ± 1.8    |
| $\phi\pi$      | 19             | 11             | 8              | < 62.5       |
| $\rho^0\pi^0$  | 19             | 11             | 8              | ***          |
| $\rho\pi$      | 19             | 11             | 8              | 0.2 ± 0.1    |
| $\omega\eta$   | 19             | 10             | 8              | < 0.6 ± 0.1  |
| $\omega\eta'$  | 21             | 25             | 8              | 18.5 ± 13.2  |
| $\phi\eta$     | 20             | 10             | 10             | 4.1 ± 1.6    |
| $\phi\eta'$    | 23             | 13             | 10             | 8.7 ± 5.5    |
| $K^{*+}K^- + c.c.$ | 20         | 9              | 9              | 0.4 ± 0.2    |
| $K^{*0}K^{0} + c.c.$ | 20         | 9              | 9              | 2.7 ± 0.7    |