Lorentz Violation and Superpartner Masses

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Abstract

We consider Lorentz violation in supersymmetric extensions of the standard model. We perform a spurion analysis to show that, in the simplest natural constructions, the resulting supersymmetry-breaking masses are tiny. In the process, we argue that one of the strongest bounds on Lorentz violation in the photon Lagrangian, which comes from the absence of birefringence from distant astrophysical sources, does not apply when Lorentz violation is parametrized by a single vector.

1 Introduction

Of the various ideas for physics beyond the standard model, supersymmetry is especially appealing because it is the unique extension of the Poincare group in four dimensions. Given the central role of Poincare symmetry in the standard model, and the wealth of relevant experimental data at a wide range of energies, it is hard to imagine that it can be broken. But precisely because of its central role, it is important to test Poincare invariance and to understand the consequences of its possible breaking [1–4]. In this letter, we will explore the implications of this breaking for supersymmetry breaking. Consider for example a chiral superfield with a scalar field $s$, a fermion $\psi$, and an auxiliary component $F$. Under a supersymmetry variation, the fermion transforms as

$$\delta_\xi \psi \sim \sigma^\mu \bar{\xi} \partial_\mu s + \xi F,$$

where $\xi$ is the parameter of the variation. If the Lorentz-scalar $F$ is non-zero, supersymmetry is broken with Poincare symmetry intact. Virtually all studies of supersymmetry breaking rely on such scalar $F$ term, or $D$-term, VEVs. Here he will consider instead the possibility that $\partial_\mu s$ is non-zero, so that supersymmetry is broken together with Poincare invariance. Can this breaking generate weak-scale soft masses? In the “gauged ghost condensation” models of [5], the Lorentz breaking scale can be as high as $10^{15}$GeV [5, 6], so it is intriguing to ask whether it can lead to soft masses that are sufficiently large.

To answer this question, we will use a spurion analysis in a supersymmetric extension of the standard model. Namely, we will assume that Poincare invariance is spontaneously broken in some hidden sector, with some field obtaining a Poincare
violating VEV. We will then treat this field as a spurion, and analyze the minimal supersymmetric standard model (MSSM) Lagrangian in the presence of this spurion, in order to estimate the size of the resulting soft masses relative to the scale of Lorentz violation. We emphasize that we do not consider here any additional source of supersymmetry breaking apart from the Lorentz violating spurion. It is well known of course how to generate soft masses for the MSSM by non-zero $F$-terms or $D$-terms. Here we are interested in whether the dominant source of soft masses in the MSSM can be associated with Lorentz violation, so we assume that no other supersymmetry-breaking spurions, namely, nonzero $F$-terms and $D$-terms, exist.

In section 2 we will consider a spurion which resides in a chiral superfield as in eqn. (1). Terms involving only regular derivatives of the spurion superfield will reproduce the results of [7], which imposed only supersymmetry on the MSSM fields, and studied the Lorentz structure of the Lagrangian. However, terms with superspace derivatives acting on the spurion will generate soft terms, as well as explicit supersymmetry breaking terms for the MSSM fields. As we will show, the suppression of scalar masses is identical to the suppression of Lorentz violation in the fermion kinetic terms. Bounds on these Lorentz violating terms then imply that the soft masses generated are tiny. Interestingly, all other Lorentz violating operators are consistent with weak scale (or higher) soft masses.

In the simple model we will construct, CPT is automatically conserved. It is interesting to note that supersymmetry breaking from Lorentz violation is similar to $D$-term breaking, in that it preserves $R$ symmetry. As mentioned above, the soft masses will be constrained by Lorentz violation in the fermion sector. Lorentz violating operators in the photon sector are not problematic. As we will show in section 2.3 the most stringent bound on these operators, which comes from birefringence, does not apply for models in which Lorentz violation can be parametrized by a single vector field vacuum expectation value (VEV), as is the case here.

In section 3 we will discuss the VEV of a Goldstone boson field [8]. Such constructions seem promising candidates for supersymmetric generalizations of gauged ghost condensation models [9].

For completeness we list the relevant spurion couplings to the MSSM fields in Appendix B.

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2With a slight abuse of notation, we will use the term “Lorentz violation” in the following since this is the term commonly used in the literature. We note however that in the supersymmetric setting we will consider, translations symmetry is almost always broken by the background.

3Because of the spurion we use, our analysis will only reproduce those terms of [7] that can be generated with a single Lorentz violating vector.
2 Chiral superfield spurion

We first consider Lorentz violation from a coordinate-dependent scalar field VEV. If this scalar field is the lowest component of a chiral superfield $S$ then,

$$S = s(x^\mu) - i\theta\sigma^\mu\bar{\theta}\partial_\mu s + \cdots$$

where the dots stand for terms involving the fermion and auxiliary components. We take

$$\langle \partial_\mu s \rangle = e^{i\phi}E^2N_\mu,$$

where $N_\mu$ is a timelike unit vector, $E$ is some energy scale, and $\phi$ is a constant phase. There is then a special "ether" frame in which $N_\mu = (1, 0, 0, 0)$, and the rotation group is unbroken.

Since the $s$ VEV is coordinate dependent, direct couplings of $S$ to the MSSM fields would lead to explicit coordinate dependence. To avoid this we impose a shift symmetry following [3],

$$S \rightarrow S + A$$

where $A$ is a constant. The Lagrangian therefore contains only regular derivatives and superspace derivatives of $S$, which give two Lorentz breaking spurions,

$$\langle \partial_\mu S \rangle = e^{i\phi}E^2N_\mu,$$

$$\langle D_\alpha S \rangle = -2ie^{i\phi}\bar{\sigma}^\mu_{\alpha\dot{\alpha}}\bar{\theta}\dot{\theta} E^2N_\mu .$$

The VEVs of higher derivatives of $S$ either vanish, or can be written in terms of (5). In the following, we will examine the possible couplings of $S$ to the MSSM fields. These will give rise to soft supersymmetry-breaking terms, as well as to Lorentz violating terms. Note that, as in $D$-term breaking, supersymmetry can be broken without $R$ symmetry breaking, since we can choose $S$ to have zero $R$-charge. Therefore, gaugino masses are either zero or suppressed compared to squark masses.

2.1 Superpotential

Since the field $DS$ is not chiral it can not appear in the superpotential. The superpotential therefore involves only regular derivatives of $S$

$$W = W(\partial_\mu S) .$$

Consider the MSSM chiral field $Q$. Its regular derivative $\partial_\mu Q$ is chiral, but not gauge invariant. The covariant derivative,

$$D_\mu \equiv -\frac{i}{4}\bar{\sigma}_\mu^{\dot{\alpha}\alpha}D_\alpha D_{\dot{\alpha}}$$

with

$$D_\alpha Q \equiv e^{-V}D_\alpha(e^VQ) ,$$

$$\frac{d\sigma}{4}\bar{\sigma}^\mu_{\alpha\dot{\alpha}}\bar{\theta}\dot{\theta} E^2N_\mu .$$
is not chiral however, and therefore cannot appear in the superpotential. This means that we can not contract $\partial_\mu S$ with derivatives of the MSSM superfields in the superpotential.

Couplings of the form
\[ W \sim \partial^\alpha S \partial_\mu W_0 , \]  
where $W_0$ is some gauge invariant combination of the MSSM fields, are allowed, but would give total derivative terms. The only relevant couplings therefore involve self-contractions of $\partial_\mu S$,
\[ W \sim (\partial_\mu S \partial^\mu S)^n W_0 , \]  
where $W_0$ depends on the MSSM superfields. Such terms merely give Lorentz-preserving renormalizations of the MSSM superpotential. Therefore, the superpotential does not give rise to either soft terms or to Lorentz violation. In the following we will impose an additional global $U(1)$ symmetry on $S$ in order to forbid dangerous Lorentz violating terms in the Kähler potential. Since $S^\dagger$ cannot appear in the superpotential, the couplings (9) and (10) are then forbidden, and the superpotential does not contain any $S$ couplings to the MSSM.

2.2 Kähler potential: the minimal model

The Kähler potential is a lot less restricted. It is easy to see that the lowest order term that leads to scalar masses is
\[ K \sim \frac{1}{M^6} D^\alpha S D_\alpha S \bar{D}^\dagger \bar{D}^{\dagger} S^{\dagger} Q Q , \]  
where $Q$ denotes an MSSM matter superfield, and $M$ is the high scale at which Lorentz violation is communicated to the MSSM. We will usually take this scale to be the Planck scale. The resulting scalar masses are
\[ \tilde{m}_q^2 \sim \left( \frac{E}{M} \right)^8 M^2 . \]  
Gaugino masses require explicit $R$-symmetry breaking, and are generated at lowest order by,
\[ K \sim \frac{1}{M^7} D^\alpha S D_\alpha S \bar{D}^\dagger \bar{D}^{\dagger} S^{\dagger} W^\alpha W_\alpha + \text{c.c.} , \]  
giving
\[ m_{\tilde{g}} \sim \left( \frac{E}{M} \right)^8 M . \]  
The full list of $S$ couplings to the MSSM (to lowest order) appears in Appendix B. We can forbid many dangerous Lorentz-violating terms in this list by imposing a global $U(1)$ symmetry under which $S$ is charged, with all MSSM fields neutral. The only surviving terms then contain pairs of $S$ and $S^\dagger$, so that the vector $N_\mu$ only
appears an even number of times. As a result, CPT is preserved. In fact, if we also require $R$-invariance, the only other surviving terms are

\[ \frac{1}{M^4} \partial_\mu \bar{S} D_a S^{\dagger} Q^\dagger e^V \bar{\sigma}^{\mu\dot{a}\alpha} D_\alpha Q \],

\[ \frac{1}{M^4} D_a S D_{\dot{a}} S^{\dagger} \bar{D}^{\dot{a}} Q^\dagger e^V D^a Q \].

For any Abelian gauge field we can also write down the following $R$-invariant operator:

\[ \frac{1}{M^4} (D^a S W_a)(\bar{D}^{\dot{a}} S^{\dagger} W_{\dot{a}}) \].

Despite its strange form, the operator (15) does not mediate Lorentz violation to the visible sector. Its explicit contribution to the Lagrangian is:

\[ \mathcal{L} \sim 4 \left( \frac{E}{M} \right)^4 \bar{\psi} \sigma^\mu D_\mu \psi - 4i \left( \frac{E}{M} \right)^4 (D_\mu q^* D^\mu q + |F|^2) \],

where $D$ denotes the usual covariant derivative with respect to the standard model gauge group. Thus, this operator just gives harmless renormalizations of the kinetic terms for both bosons and fermions.

The operator (16) mediates both Lorentz and supersymmetry breaking to the visible sector. It gives rise to the Lagrangian

\[ \mathcal{L} = -8iC \left( \frac{E}{M} \right)^4 \psi \sigma^\mu D_\mu \bar{\psi} - 8iC \left( \frac{E}{M} \right)^4 N_\mu N_\nu \psi \sigma^\mu D^\nu \bar{\psi} - 16C \left( \frac{E}{M} \right)^4 N_\mu N_\nu D^\mu \bar{q}^* D^\nu \bar{q} \]

where we introduced the numerical coefficient $C$, which is actually a matrix in flavor space. This term manifestly violates supersymmetry. While its contribution to the fermion kinetic term contains both Lorentz violating and Lorentz preserving terms, its contribution to the scalar kinetic term is completely Lorentz violating.

The suppression of these terms is equal to the suppression of squark masses (see eqn. [12]). The numerical coefficients $C$ would be different generically for left-handed and right-handed fields of the standard model. We therefore denote them by $C_L$ and $C_R$. In terms of 4d Dirac spinors, using the notations of [1], the second term of eqn. (19) then leads to the following Lorentz-violating modification of the fermion kinetic terms,

\[ \mathcal{L} = ic_\mu \bar{\Psi} \gamma^\mu D^\nu \Psi + id_\mu \bar{\Psi} \gamma^{\mu\gamma 5} D^\nu \Psi \]

with

\[ c_{\mu\nu} = 4 \left( \frac{E}{M} \right)^4 N_\mu N_\nu (C_L + C_R) \]

\[ d_{\mu\nu} = 4 \left( \frac{E}{M} \right)^4 N_\mu N_\nu (C_L - C_R) \].
Note that the standard model gauge group structure imposes the condition $C^{(u)}_i = C^{(d)}_i$. In principle, the matrices $c_{\mu\nu}$ and $d_{\mu\nu}$ have non-diagonal elements in flavor space and cause flavor-changing Lorentz violation which is severely constrained [2]. In the following we will concentrate on the diagonal terms, since, as we will see, even these are very problematic.

Combining these results with (12) and assuming no accidental cancellation between the left-handed and right-handed parameters we conclude that

$$
\begin{align*}
  c_{00} & \sim d_{00} \sim \frac{\tilde{m}_\tilde{q}}{M} \\
  c_{0J} & \sim d_{0J} \sim \beta \frac{\tilde{m}_\tilde{q}}{M} \\
  c_{JK} & \sim d_{JK} \sim \beta^2 \frac{\tilde{m}_\tilde{q}}{M}.
\end{align*}
$$

Here $\beta$ is the velocity of the earth (where the relevant measurements are performed) relative to the ether frame, and $J,K$ are the spatial directions. The clock comparison experiment [10] gives $|c_{JK}| \leq 10^{-27}$ for the neutron. A similar bound should hold for $c_{JK}$ at the quark level. Since the Earth’s speed relative to the ether is presumably at least as large as its speed relative to the CMBR, we take $\beta \sim 10^{-3}$, so that the squark mass is bounded by

$$
\tilde{m}_\tilde{q} \lesssim 10^{-21} M. \tag{24}
$$

Even if $M$ is the Planck scale, this is four or five orders of magnitude too low!

The experimental bounds on $d_{\mu\nu}$ are even stronger. For the electron [11],

$$
|\tilde{b}(e^-)_{X,Y}| \leq 10^{-29} \text{GeV}, \tag{25}
$$

which translates into (see ref. [11] for the definition of $\tilde{b}(e^-)_{X,Y}$)

$$
|d_{0J}| \leq 10^{-29} \quad \text{and} \quad \tilde{m} \lesssim 10^{-26} M. \tag{26}
$$

However, this bound is not necessarily applicable in our theory. If the hidden and visible sectors only couple through gravitational loops [5], the coefficients $C_{ij}$ of eqn. (19) would be identical for all the MSSM fields, and $d_{\mu\nu}$, which is proportional to $C_L - C_R$, would vanish.

Our spurion analysis reproduces many of the operators of [7], which wrote down a Lagrangian for the standard model fields (taken to transform in the usual representations of the full Poincare plus supersymmetry algebra) imposing only supersymmetry and translation invariance. Lorentz invariance then emerges as an approximate symmetry of the low-energy theory, with Lorentz violating terms appearing at dimension-5 and higher. This Lagrangian is simply reproduced in our analysis by terms involving different powers of $\partial_\mu S$ (we can only reproduce of course terms that can be generated with a single vector spurion). For example the Kähler potential operator

$$
\frac{1}{M} Q^\dagger e^V \mathcal{D}_\mu Q \tag{27}
$$

6
of [7] originates from
\[
\frac{1}{M^3} \partial^\mu SQ^1 e^V D_\mu Q .
\] (28)

2.3 Lorentz violation in the photon Lagrangian

As we mentioned above, once we impose the global U(1) symmetry, the lowest order Lorentz violating operator in the gauge sector is the photon operator (17). Interestingly however, the Lorentz violating terms arising from this operator alone do not constrain significantly the scale of Lorentz violation, and are in fact consistent with weak-scale scalar masses. The main reason for this is that the most stringent bound on Lorentz violation in the photon Lagrangian, from birefringence of light from distant astrophysical sources [12], simply does not apply here, or more generally, in models where Lorentz violation can be parametrized by a single vector. As shown in [12], the Lorentz violating photon Lagrangian

\[
\Delta \mathcal{L} = (k_F)^{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}
\] (29)

with arbitrary \( k_F \), leads to different dispersion relations for the two independent photon polarizations

\[
E_\pm = (1 + \rho \pm \sigma)|\vec{p}| ,
\] (30)

where

\[
\rho = -\frac{1}{2} \tilde{k}_\alpha , \quad \text{and} \quad \sigma = \sqrt{\frac{1}{2} (\tilde{k}_{\alpha\beta})^2 - \rho^2 ,}
\] (31)

with

\[
\tilde{k}_{\alpha\beta} = (k_F)^{\alpha\mu\beta\nu} \frac{p_\mu p_\nu}{|\vec{p}|^2} .
\] (32)

The polarization of light from a celestial object is then wavelength-dependent. Since the effect is also proportional to the distance from the light source, refs. [12] use measurements of polarized light coming from distant objects to bound some components of \( k_F \) at the level of \( 10^{-32} \).

However, in our case, and whenever Lorentz violation is due to a single vector,

\[
(k_F)^{\mu\nu\rho\sigma} \sim \left( \frac{E}{M} \right)^4 N_{[\mu_1 \eta_1] [\rho_1 N_{\sigma]} .
\] (33)

It is easy to see that for \( k_F \) of this form \( \sigma \) vanishes identically. Therefore, whenever Lorentz violation can be parametrized by a single vector VEV, it is not constrained by birefringence.

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4While we cannot write the analogues of (17) for non-Abelian gauge fields, Lorentz violating terms such as (29) will be generated radiatively, from “vacuum polarization” amplitudes with Lorentz violation and supersymmetry breaking insertions.

5To leading order in \( k_F \).
The next relevant bound on $k_F$ comes from cavity resonator experiments [12, 13] which give $k_F \lesssim 10^{-15} - 10^{-16}$. This gives $\beta^2(E/M)^4 \lesssim 10^{-16}$, and therefore $m_\tilde{q} \lesssim 10^{-10} M$, which can be very high. We note however that the analysis of [13] assumes that some components of the tensor $k_F$ are zero, motivated by the birefringence bound discussed above.

### 3 A Goldstone–or vector field–spurion

The Lorentz violating operators we considered in the previous section were suppressed by large powers of the high scale $M$, because they had to involve derivatives of the spurion. Another way to implement the shift symmetry is to consider a coordinate-dependent VEV of some Goldstone boson [8]. The simplest example is a chiral superfield $G$, charged under a global U(1), whose lowest component has a coordinate-dependent VEV of the form

$$\langle g \rangle = \Lambda e^{i\phi(x)}, \quad (34)$$

with

$$\partial_\mu \phi = E N_\mu. \quad (35)$$

The spurion chiral superfield is then

$$G = \Lambda e^{i\phi} + E \Lambda N_\mu \theta \sigma^\mu \bar{\theta} e^{i\phi} - \frac{1}{4} E^2 \Lambda e^{i\phi} \theta \bar{\theta} \bar{\theta}. \quad (36)$$

Because of the U(1) symmetry, only the combination $G^\dagger G$, which is coordinate-independent, is allowed. In fact, this choice seems like a good starting point for supersymmetrizing the “gauged ghost condensation” construction, since we can simply gauge the U(1) symmetry. The basic invariant to consider is then

$$G^\dagger e^U G, \quad (37)$$

where $U$ is the U(1) gauge superfield. Clearly, from the point of view of our spurion analysis, there is no difference between a vector field spurion, $U = \ldots + \theta \sigma^\mu \bar{\theta} u_\mu + \ldots$, (38)

with

$$\langle u_\mu \rangle = E N_\mu, \quad (39)$$

and the combination $G^\dagger G$, so for concreteness we will consider the latter in the following.

Since we must have both $G$ and $G^\dagger$, no Lorentz violating operator can appear in the superpotential. The lowest-dimension Kähler potential operator coupling the spurion to the MSSM fields is

$$K \sim \frac{GG^\dagger Q^\dagger Q}{M^2}. \quad (40)$$
This operator gives supersymmetry-breaking scalar masses of order,
\[ \tilde{m}_q \sim \frac{E \Lambda}{M^2} M . \tag{41} \]

However, it also induces Lorentz violating fermion “mass” terms
\[ \mathcal{L} \sim \frac{\Lambda^2 E}{M^2} \psi^\alpha (N \cdot \sigma)_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} . \tag{42} \]

Unlike in the previous section, this term contains a single power of \(N_\mu\) and is CPT odd. It is therefore severely constrained, both by neutral-meson oscillations [14], and by terrestrial clock experiments [11]. The latter give
\[ \frac{\Lambda^2 E}{M^2} \lesssim 10^{-29} \text{GeV} . \tag{43} \]

For the scalar mass (41) to be of order a TeV, we then need \(E\) much higher than the Planck scale.

### 4 Conclusions

In this letter, we studied the implications of Lorentz violation in some hidden sector for supersymmetry breaking in the MSSM. It is easy to see that Lorentz violating VEVs can simply mimic \(F\)-term or \(D\)-term supersymmetry breaking. The Lorentz violating spurion typically involves \(\theta \sigma^\mu \bar{\theta}\), so its square gives the usual \(\theta^2 \bar{\theta}^2\) contribution. This depends of course on the form of Lorentz violation. For example, for a lightlike vector \(N_\mu\), the soft masses we found vanish, because they are proportional to \(N^2\), whereas supersymmetry breaking terms such as eqn. (19) remain. As in \(D\)-term breaking, Lorentz violation can lead to supersymmetry breaking without \(R\)-symmetry breaking.

Even though the combination of supersymmetry and an additional \(U(1)\) symmetry can forbid the most dangerous Lorentz violating terms for the standard model fields, and in particular, CPT-odd terms, bounds on Lorentz violating fermion terms imply that the resulting contributions to scalar and gaugino masses are tiny. One can also imagine models in which direct couplings of the MSSM to the Lorentz violating spurion are forbidden, and supersymmetry breaking is transmitted to the MSSM through messenger fields as in gauge-mediation models [15]. Lorentz violating terms for the MSSM fields would then be generated radiatively, and again, the bounds on these would probably imply that the allowed supersymmetry-breaking terms are very small. Still, such a setup would have the advantage that all supersymmetry breaking and Lorentz violating terms in the MSSM would be flavor blind. As we saw, this avoids many bounds on Lorentz violation.

We did not study here the origin and dynamics of Lorentz violation. It would be interesting to extend the analyses of [3, 5] to the supersymmetric case, and we pointed out one possible starting point for such an analysis.
Note added: After this work was completed, ref. [16] appeared, which extends Very Special Relativity to supersymmetry, with “half” the supersymmetry preserved.

Acknowledgements
We thank Nima Arkani-Hamed, Michael Dine, Yuval Grossman, Yossi Nir, Lisa Randall, Yuri Shirman and especially Ari Laor for useful discussions. Research supported by the United States-Israel Science Foundation (BSF) under grant 2002020 and by the Israel Science Foundation (ISF) under grant 29/03.

A Different Lorentz Violating Spurions

We are interested in spontaneous Poincare breaking in 4d theories with $\mathcal{N} = 1$ global supersymmetry. The relevant Poincare-breaking VEVs can therefore appear either in the chiral supermultiplet, or in the vector supermultiplet. Consider first the chiral supermultiplet. The only possible Poincare breaking it can contain is from a nonzero VEV of $\partial_\mu s$, where $s$ is the scalar field. The variation of the fermion is then given by (1) and is nonzero, so that supersymmetry is broken. (This type of spurion is considered in Section 2.)

In the vector multiplet, there are a few possibilities. First, the field strength $F_{\mu\nu}$ could obtain a VEV. Then, the variation of the gaugino is non-zero, since

$$\delta_\xi \lambda = i \xi D + \sigma^{\mu\nu} \xi F_{\mu\nu},$$

(44)

where $\lambda$ is the gaugino and $D$ is the auxiliary field.

Second, the gauge boson $A_\mu$ could get a VEV. If this vector is associated with an unbroken gauge symmetry, this VEV is unphysical and can be rotated away. If however the vector is associated with a spontaneously broken gauge symmetry, then the $A_\mu$ VEV can be gauge rotated into a coordinate-dependent scalar field VEV. (This type of spurion is considered in Section 3.)

In principle, one could also consider a massive vector field. Then one cannot go into the Wess-Zumino gauge, and the fermion which is the theta component of the vector is physical. The supersymmetry variation of this fermion contains the term

$$\xi \sigma^\mu A_\mu,$$

(45)

which is non-vanishing if $A_\mu$ is non-zero.

Note that in all these examples, half the supersymmetry may be preserved, depending on the choice of VEV. For example, in eqn. (1), variations “orthogonal” to $\sigma^\mu \partial_\mu s$ vanish, as appropriate for 3d supersymmetry.
B The list of operators

We now list the leading Kähler potential terms that couple the MSSM fields to the spurion of section 2. We omit operators that merely renormalize the usual MSSM Kähler terms.

Single $S$, chiral MSSM superfields:

\[(1/M^3) \partial_\mu S Q^\dagger e^V \mathcal{D}^\mu Q + \text{c.c.} \]  
\[(1/M^2) D_\alpha S Q^\dagger e^V \mathcal{D}^\alpha Q + \text{c.c.} \]

Single $S$, MSSM gauge superfields:

\[(1/M^3) \partial_\mu S W^\alpha \sigma_\alpha^\mu \bar{W}^\dot{\alpha} + \text{c.c.} \]  
\[(1/M^3) D^\alpha S W^\beta \mathcal{D}_\alpha W_\beta + \text{c.c.} \]  
\[(1/M^4) \partial^\mu S W^\alpha \partial_\mu W_\alpha + \text{c.c.} \]  
\[(1/M^4) \partial^\mu S \sigma_{\mu \dot{\alpha}} W^\beta \mathcal{D}_\alpha W_\beta + \text{c.c.} \]

$S^2$, MSSM chiral superfields:

\[(1/M^3) D^\alpha S D_\alpha S Q^\dagger e^V Q + \text{c.c.} \]  
\[(1/M^4) D_\alpha \bar{D}_\alpha S^\dagger \bar{D}^\alpha Q^\dagger e^V \mathcal{D}^\alpha Q + \text{c.c.} \]  
\[(1/M^4) \partial_\mu S \bar{D}_\alpha S^\dagger Q^\dagger e^V \sigma_{\mu \dot{\alpha}} \mathcal{D}_\alpha Q + \text{c.c.} \]  
\[(1/M^5) \partial_\mu S D_\alpha S D_\mu Q^\dagger e^V \mathcal{D}^\alpha Q + \text{c.c.} \]

$S^2$, MSSM gauge fields:

\[(1/M^6) \partial_\mu S \partial_\nu S^\dagger \partial^\mu W^\alpha \sigma_\alpha^\nu W^\dot{\alpha} + \text{c.c.} \]  
\[(1/M^4) \partial_\mu S \partial_\nu S^\dagger W^\alpha \sigma_{\mu \nu} W_\beta + \text{c.c.} \]  
\[(1/M^4) D^\alpha S D_\alpha S W^\beta W_\beta + \text{c.c.} \]  
\[(1/M^4) D^\alpha S W_\alpha \bar{D}^\alpha S^\dagger W_\dot{\alpha} + \text{c.c.} \]  
\[(1/M^5) D^\beta S \sigma^\mu S^\dagger \sigma_{\mu \dot{\alpha}} \bar{W}^\dot{\alpha} \mathcal{D}_\beta W^\alpha + \text{c.c.} \]  
\[(1/M^6) D^\beta S \sigma^\mu S^\dagger W^\alpha \partial_\mu \mathcal{D}_\beta W_\dot{\alpha} + \text{c.c.} \]

$S^3$, MSSM chiral superfields:

\[(1/M^5) D^\alpha S \sigma_{\alpha \dot{\alpha}} D^\dot{\alpha} S^\dagger \partial_\mu_S Q^\dagger e^V Q + \text{c.c.} \]  
\[(1/M^6) D^\alpha S D_\alpha S + \bar{D}_\alpha S^\dagger \bar{D}^\alpha S^\dagger \partial_\mu (S + S^\dagger) Q^\dagger e^V \mathcal{D}_\mu Q \]  
\[(1/M^5) D^\alpha S D_\alpha S D_\beta S^\dagger \mathcal{D}^\beta Q^\dagger e^V Q + \text{c.c.} \]  
\[(1/M^6) D S \sigma^\mu \bar{D} S^\dagger D_\beta S \mathcal{D}_\mu Q^\dagger e^V \mathcal{D}^\beta Q \]  
\[(1/M^7) D^\alpha S D^\dot{\alpha} S^\dagger \partial_\mu S \mathcal{D}_\alpha Q^\dagger e^V \mathcal{D}^\alpha \mathcal{D}_\alpha Q \]
S³, MSSM gauge superfields:

\begin{align}
(1/M^6) & \quad D S^\sigma \partial_{\mu} S W^\alpha \bar{W}_\alpha + \text{c.c.} \\
(1/M^6) & \quad D^\alpha S D_{\alpha} S \partial_{\mu} S W^\sigma \bar{W} + \text{c.c.} \\
(1/M^6) & \quad \bar{D}_\alpha S^\dagger \bar{D}_\alpha S \partial_{\mu} S W^\beta \bar{D}_\alpha W_\beta \\
(1/M^7) & \quad D^\alpha S D_{\alpha} S \partial_{\mu} S W^\beta \partial_{\mu} W_\beta + \text{c.c.} \\
(1/M^7) & \quad D^\alpha S D_{\alpha} \bar{D}_\beta S^\dagger \bar{W}^\beta W_\gamma W_\gamma
\end{align}

S⁴:

\begin{align}
(1/M^6) & \quad D^\alpha S D_{\alpha} S \bar{D}_\alpha S^\dagger \bar{D}_\alpha S^\dagger W^\beta \bar{W}_\beta + \text{c.c.} \\
(1/M^6) & \quad D^\alpha S D_{\alpha} S \bar{D}_\alpha S^\dagger \bar{D}_\alpha S^\dagger Q^\dagger e^V Q \\
(1/M^8) & \quad \partial_{\mu} S D_{\alpha} S \bar{D}_\alpha S^\dagger \bar{D}_\alpha S^\dagger \bar{D}^\mu Q^\dagger e^V D^\alpha Q + \text{c.c.} \\
(1/M^8) & \quad \bar{D}_\alpha S^\dagger \bar{D}_\alpha S^\dagger D^\alpha S \partial_{\mu} S \sigma_{\mu \gamma \beta} \bar{W}^\gamma \bar{D}_\gamma W_\alpha + \text{c.c.}
\end{align}

If we introduce a U(1) under which S has charge +1, with all MSSM fields neutral, only operators (53-57, 59-61, 72-75) are allowed. Taking the R-charge of S to be zero, only operators (53, 54, 56, 59, 73, 74) preserve both the U(1) and the R-symmetry. Note that scalar masses are generated by (73), gaugino masses are generated by (72) and A-terms are generated by (52).

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