Abstract—This work investigates fault-resilient federated learning when the data samples are non-uniformly distributed across workers, and the number of faulty workers is unknown to the central server. In the presence of adversarially faulty workers who may strategically corrupt datasets, the local messages exchanged (e.g., local gradients and/or local model parameters) can be unreliable, and thus the vanilla stochastic gradient descent (SGD) algorithm is not guaranteed to converge. Recently developed algorithms improve upon vanilla SGD by providing robustness to faulty workers at the price of slowing down convergence. To remedy this limitation, the present work introduces a fault-resilient proximal gradient (FRPG) algorithm that relies on Nesterov’s acceleration technique. To reduce the communication overhead of FRPG, a local (L) FRPG algorithm is also developed to allow for intermittent server-workers parameter exchanges. For strongly convex loss functions, FRPG and LFRPG have provably faster convergence rates than a benchmark robust stochastic aggregation algorithm. Moreover, LFRPG converges faster than FRPG while using the same communication rounds. Numerical tests performed on various real datasets confirm the accelerated convergence of FRPG and LFRPG over the robust stochastic aggregation benchmark and competing alternatives.

Index Terms—Communication-efficient learning, fault-resilient learning, federated learning.

I. INTRODUCTION

Traditional machine learning algorithms are mostly designed for centralized processing of collected data at a single server or the cloud. However, the copies of data kept in the cloud render cloud-centric learning vulnerable to privacy leakage [1–3]. Leveraging the ever-improving computational capability of network-edge devices such as mobile terminals and Internet of Things (IoT) devices, distributed on-device learning has emerged to alleviate these privacy concerns.

As an implementation of distributed on-device learning, federated learning has attracted growing attention from both industry and academia [4–5]. A popular realization of federated learning employs a parameter server collaborating with multiple workers, which are the network-edge devices in practical systems. Specifically, the parameter server updates and broadcasts the global model parameters using local messages (e.g., local gradients and/or local model parameters) from/to the workers. Based on the received global model parameters and local datasets, the workers compute local messages in parallel. Since the datasets are kept at the workers in federated learning, the risk of privacy leakage is reduced. From the design of learning algorithms at the upper layer [4], recent research on federated learning moves to resource allocation at the physical layer [5]. The two major topics in the design of learning algorithms deal with fault-resilient and communication-efficient federated learning. The present work builds on this research front.

A. Related Works

Fault-resilient federated learning. Faulty workers may strategically corrupt the local datasets in federated learning such that the local messages uploaded to the parameter server are unreliable. When unreliable local messages are used by the parameter server, the convergence of the vanilla gradient descent algorithm is not guaranteed. Indeed, it has been demonstrated in [6] that the vanilla gradient descent algorithm and its stochastic version (Stochastic Gradient Descent, SGD) fail to converge when each faulty worker uploads an unreliable message to the parameter server. Therefore, it is crucial to deal with faulty workers. Recent research has reported fault-resilient learning approaches relying on the full gradient per update [6–9]. For strongly convex loss functions, the geometric median (GeoMed) algorithm [6] converges to a near-optimal solution when less than 50% of the local messages from the workers is unreliable. For (strongly) convex and smooth non-convex loss functions, [7] developed a component-wise median and component-wise trimmed mean algorithms to secure parameter updates at the server over the faulty workers. For non-convex loss functions, the component-wise median and component-wise trimmed mean algorithms may converge to a saddle point that is far away from a real local minimizer. As a remedy, [8] advocated a Byzantine perturbed gradient algorithm for which the obtained solution is an approximate local minimizer of the non-convex loss function.

1For example, the model parameters are employed to provide a mapping between the data samples and labels in classification problems.
For large datasets however, evaluating the full gradient per iteration is computationally prohibitive. Several efforts to improve computational efficiency of fault-resilient stochastic federated learning, include Krum [10], Bulyan [11], Byzantine SGD [12], Zeno [13], and DRACO [14]. When less than 50% of the workers are unreliable, the Kr"urn algorithm can obtain the dissimilarity score of local gradients. Using the local gradient with the smallest dissimilarity score to update the global model parameters, the Krum iteration converges to a neighborhood of stationary points [10]. When less than 25% of the workers are unreliable, the Buylan algorithm [11] can reduce the radius of the neighborhood obtained by the Krum algorithm. Using historical gradients, the Byzantine SGD algorithm [12] allows the parameter server to remove the faulty local gradients before performing gradient aggregation. The Zeno algorithm [13] ranks the reliability of local gradients based on the weighted descent value and magnitude of local gradients. Averaging the top-ranked local gradients, the Zeno algorithm can tolerate up to $Q-1$ faulty workers, where $Q$ is the number of workers. Based on coding theory and sample redundancy (i.e., multiple copies of a data sample across different workers), the DRACO algorithm [14] converges when there is at least one reliable worker.

The aforementioned works deal with homogeneous datasets in which samples from different workers are independent and identically distributed. In several practical settings though, the collected data are heterogeneous. For example, different YouTube subscribers are provided with different categories of advertisements and video clips based on their search history. As a result, developing fault-resilient federated learning algorithms over heterogeneous datasets has emerged as an important research task. For heterogeneous datasets, [15] introduced a robust stochastic aggregation framework to optimize fault-resilient stochastic federated learning. The resultant robust stochastic aggregation (RSA) algorithm can converge to a near-optimal solution with convergence rate $O(\log K/\sqrt{\lambda K})$, where $K$ is the number of iterations; see also [16] where multi-task federated learning is effective for heterogeneous datasets over worker clusters having different model parameters, but not as effective for a single model parameter set.

**Communication-Efficient Federated Learning.** Frequent communications between the server and workers are inevitable in federated learning. Since bandwidth is a scarce resource for the parameter server, the communication overhead becomes the bottleneck [17], [18]. To reduce this overhead, a line of research focuses on skipping the unnecessary communication rounds [19], [20], where the so-called LAG algorithm avoids redundant information exchanges, and can be extended to employ just quantized gradients. Compared with the vanilla gradient descent algorithm, the LAG enjoys comparable convergence at reduced communication overhead; see also [21]–[23] that leverage local SGD to allow intermittent server-worker exchanges. The issue remaining unexplored is whether LAG and local SGD algorithms are resilient to faulty workers.

**B. Contributions**

Motivated by the need for communication-efficient robust learning over heterogeneous datasets, we propose two communication-efficient federated learning algorithms, which are robust to faulty workers. Our contributions to this end, are as follows.

- In the presence of faulty workers, heuristically using Nesterov’s acceleration leads to divergence of the vanilla SGD algorithm. As a remedy, we develop a fault-resilient proximal gradient (FRPG) algorithm by tailoring Nesterov’s acceleration [24], [25] and stochastic approximation for fault-resilient federated learning.
- To further reduce communication overhead, we also develop a local (L)FRPG algorithm where the parameter server periodically communicates with workers, and prove that LFRPG has lower communication overhead than FRPG.
- We establish the convergence rates for the proposed FRPG and LFRPG algorithms, which are challenging to analyze when faulty workers are present. Our theoretical results demonstrate that the proposed FRPG and LFRPG algorithms can converge faster than the existing federated learning schemes.

Numerical tests corroborate our analytical findings. The remaining work is organized as follows. The investigated problem is described in Section II. The FRPG algorithm and its convergence analysis are the subjects of Section III, and the LFRPG algorithm and its convergence analysis are presented in Section IV. Numerical results are shown in Section V, and conclusions are drawn in Section VI.

**Notation.** Vectors are denoted by bold lowercase letters. The $\ell_2$-norm of a vector is denoted by $\|\cdot\|$. The operator $\langle\cdot, \cdot\rangle$ denotes the inner product of two vectors. The operator $\mathbb{E}_x[\cdot]$ denotes the expectation over the random variable $x$. The polynomial of $x$ is denoted by $O(x)$. The proximal operator for a function $f$ is defined as

$$
\text{prox}_{\alpha f}(w) := \arg\min_x \left\{ \alpha f(x) + \frac{1}{2} \|x - w\|^2 \right\}.
$$

The nomenclature of this work is listed in Table I.

| Notations | Definitions |
|-----------|-------------|
| $Q$       | Number of workers |
| $N$       | Number of reliable workers |
| $B$       | Number of faulty workers |
| $G$       | Maximum gradient power of penalty functions |
| $\lambda$ | Weight factor for penalty functions |
| $f_i(w_0)$ | Regularization function at the server at $w_0$ |
| $f_i(w_n)$ | Local loss at the nth worker at $w_n$ |
| $f(w_n; x_n)$ | Loss value at the nth worker w.r.t. random variable $x_n$ |
| $p_n(w_{n-1}, w_n)$ | Penalty function at the nth worker |
| $\beta_k$ | Step size in the $k$th slot |
| $\alpha_n, k$ | Step sizes in the $k$th slot of server and the $n$th worker |
| $u_{n, k}, v_{n, k}$ | Auxiliary sequences at the $k$th slot of server and the $n$th worker |
| $w_{n, k}$ | Model parameters at the $k$th slot of server and the $n$th worker |
| $\Delta_{n, k}$ | Gradient noise at the $k$th slot |

**TABLE I. Nomenclature**
II. PROBLEM STATEMENT

Consider a federated learning setup, comprising a parameter server, $Q$ workers, and overall loss given by

$$\sum_{n=1}^{Q} f_n(w_n) + f_0(w_0)$$

(1)

where $w_0 \in \mathbb{R}^d$ denotes model parameters at the server; $w_n \in \mathbb{R}^d$ are model parameters at the $n$th worker; and $f_0(w_0)$ is a regularization function. The local loss at the $n$th worker is

$$f_n(w_n) = \mathbb{E}_{x_n} [f(w_n; x_n)]$$

where $\mathbb{E}_{x_n} [\cdot]$ denotes the $n$th worker’s specific expectation over the random data vector $x_n$, and $f(w_n; x_n)$ is the corresponding loss with respect to $w_n$ and $x_n$.

The objective of fault-resilient federated learning is to minimize in a distributed fashion the loss in (1) subject to the consensus constraints, expressed as

$$w_0 = w_n, \quad n = 1, \ldots, Q.$$  

(3)

When there are multiple faulty workers, several researchers have demonstrated that obtaining the minimizer of (1) subject to (3) is less meaningful [6], [10], [15]. For this reason, our goal will be to minimize the loss function while avoiding consensus with faulty workers. The server cannot differentiate reliable from faulty workers, and does not even know the number of faulty workers. Our novel algorithms will seek resilience to faulty workers under these challenging conditions. But when analyzing the convergence rate in the presence of faulty workers, we will assume that among $Q$ workers, $N$ are reliable, and for notational convenience we will index the $B = Q - N$ faulty workers by $n = N + 1, \ldots, Q$.

Dropping the losses of faulty workers in (1), the ideal minimization task with $w := \text{vec}([w_0, w_1, \ldots, w_N])$, is

$$\min_{w} \sum_{n=1}^{N} f_n(w_n) + f_0(w_0) \quad \text{s.t. } w_0 = w_n, \quad n = 1, \ldots, N.$$  

(4)

Without information about faulty workers, it is ideal (and thus not meaningful) for the server to seek the solution of (4). Instead, we will adapt the robust stochastic aggregation approach of [15] by adding a penalty term $p_n(w_n - w_0)$ with weight $\lambda > 0$ per local loss $f_n(w_n)$. We will then target to approach the solution of the penalized version of (4), namely

$$\min_{w} F(w) := \sum_{n=1}^{N} (f_n(w_n) + \lambda p_n(w_n - w_0)) + f_0(w_0).$$

(5)

Remark 1: Different from (3), the penalty terms in (5) allow the server parameters and those of faulty workers to differ. This flexibility is in par with the data heterogeneity across workers. We will select convex and differentiable $\{p_n(\cdot)\}$, e.g., of the Huber type. Moreover, the gradients of $\{p_n(\cdot)\}$ for reliable and faulty workers must be similar, so that the undesirable influence of faulty workers is mitigated.

Our communication-efficient solvers of a non-ideal version of (5) will be developed in Sections III and IV, based on the following assumptions about $f_0$, $f_n$, and $p_n$, for $n = 1, \ldots, N$.

Assumption 1 (Lipschitz Continuity [24] eq. (1.2.11)): Regularizer $f_0$ has an $L_0$-Lipschitz continuous gradient, and $f_n$ has an $L_n$-Lipschitz continuous gradient for $n = 1, \ldots, N$.

Assumption 2 (Strong Convexity [24] eq. (2.1.20)): Regularizer $f_0$ is strongly convex with modulus $\delta_0$, and $f_n$ is strongly convex with modulus $\delta_n$ for $n = 1, \ldots, N$.

Assumption 3 (Penalty): Penalty function $p_n(w_n - w_0)$ is convex and differentiable, with $\|\nabla w_n p_n(w_n - w_0)\| \leq G$, and $\|\nabla w_n p_n(w_n - w_0)\| \leq G$ for $n = 1, \ldots, Q$.

Assumptions 1 and 2 are standard when the learning criterion entails smooth and strongly convex local loss functions. The negative effects of faulty workers can be bounded through Assumption 3 which is satisfied by e.g., a Huber-type penalty.

III. FAULT-RESILIENT PROXIMAL GRADIENT

In this section, we develop a novel fault-resilient proximal gradient (FRPG) algorithm for the server to solve the non-ideal version of (5), with $Q$ replacing $N$ since faulty workers can be present. Subsequently, we will analyze the convergence of our iterative FRPG solver.

A. Algorithm

Along the lines of [23], the parameter server in our federated learning approach maintains three sequences per slot $k$, namely $u_{0,k}$, $w_{0,k}$ and $v_{0,k}$. The resultant FRPG algorithm updates these three sequences using the recursions

$$u_{0,k} = (1 - \beta_k)u_{0,k-1} + \beta_k v_{0,k-1}$$

(6a)

$$w_{0,k} = u_{0,k} - \frac{1}{\alpha_{0,k}} \nabla f_0(u_{0,k})$$

(6b)

$$v_{0,k} = v_{0,k-1} - \frac{\delta_0(v_{0,k-1} - u_{0,k}) + \nabla f_0(u_{0,k}) + \sum_{n=1}^{Q} g_{n,k}}{\delta_0 + \alpha_{0,k} \beta_k}$$

(6c)

where $w_{0,k-1}$ are the server parameters on slot $(k - 1)$; and likewise for the auxiliary iterates $v_{0,k-1}$; scalars $\alpha_{0,k}$ and $\beta_k$ are step sizes; and the sum over $Q$ in (6c) accounts for the non-ideal inclusion of faulty workers, where $g_{n,k}$ is given by

$$g_{n,k} := \lambda \nabla w_n p_n(w_{0,k} - w_{n,k}).$$

(7)
Algorithm 1 FRPG Algorithm

1: Initialize: \( w_{n,0} \) and \( v_{n,0} \) for \( n = 0, \ldots, N \), and step sizes as (10) and (17)
2: for \( k = 1, \ldots, K \) do
3: \[ u_{0,k} \text{ and } w_{0,k} \text{ via (6a) and (6b)} \]
4: \[ \text{The server broadcasts the model parameters } w_{0,k} \]
5: \[ \text{parfor } n = 1, \ldots, Q \text{ do } \]
6: \[ \text{if } n = 1, \ldots, N \text{ then } \]
7: \[ \text{The } n\text{th reliable worker updates } w_{n,k} \text{ via (8)} \]
8: \[ \text{end if } \]
9: \[ \text{if } n = N + 1, \ldots, Q \text{ then } \]
10: \[ \text{The } n\text{th faulty worker generates faulty parameters } \]
11: \[ \text{end if } \]
12: \[ \text{end parfor } \]
13: \[ \text{All workers upload } g_{n,k} \text{ to the server } \]
14: \[ \text{The server updates } v_{0,k} \text{ via (6c)} \]
15: end for

Each reliable worker also maintains sequences \( u_{n,k}, w_{n,k} \) and \( v_{n,k} \) per slot \( k \), that are locally updated as

\[
\begin{align*}
\dot{u}_{n,k} &= (1 - \beta_k) \dot{u}_{n,k-1} + \beta_k \dot{v}_{n,k-1} \\
\dot{w}_{n,k} &= \dot{w}_{0,k} - \text{prox}_{\alpha_n \eta_n} \left( \dot{w}_{0,k} - \dot{v}_{n,k} + \frac{\nabla f}(\dot{u}_{n,k}; \dot{x}_{n,k}) \right) \\
\dot{v}_{n,k} &= \dot{v}_{n,k-1} - \frac{\delta_n}{n} (\dot{v}_{n,k-1} - \dot{u}_{n,k}) + \frac{\nabla f}(\dot{u}_{n,k}; \dot{x}_{n,k}) - g_{n,k}
\end{align*}
\]
(8a)
(8b)
(8c)

where subscript \( k = 1 \) indices the previous slot; while \( \alpha_n \) and \( \beta_k \) denote stepsizes as before; and \( x_{n,k} \) is a realization of \( x_n \) at slot \( k \). Without adhering to (8a)-(8c), faulty workers generate parameters \( \{w_{n,k}\}_{n=N+1} \) using an unknown mechanism.

Based on (6) and (8), our novel FRPG solver of (5) is listed under Algorithm 1 with lines 5-12 showing that the workers generate their local model parameters in parallel.

Remark 2: Note that while our algorithm is inspired by (25), the updates in (6) and (8) are distinct in three aspects. The update step in (6a) does not require a proximal operation since \( w_{0,k} \) and \( w_{n,k} \) must be iteratively updated. Since FRPG is a distributed algorithm, the update steps in (6c) and (8c) require server-worker exchanges of \( \{g_{n,k}\}_{Q=1}^{Q} \) that also include exchanges from faulty workers. These three differences render the ensuing convergence analysis of FRPG challenging.

B. Convergence analysis

Our analysis here is for a single realization of \( x_n \) per slot, but can be directly extended to mini-batch realizations of \( x_n \). Let us define the gradient error at worker \( n \) per slot \( k \) as

\[
\Delta_{n,k} := \nabla f(\dot{u}_{n,k}; \dot{x}_{n,k}) - \nabla f_n(\dot{u}_{n,k})
\]
(9)

and adopt the following assumption on its moments that are satisfied, e.g., when stochastic gradients are employed (26).

Assumption 4 (Bounded Stochastic Noise): The gradient error (a.k.a. noise) is zero mean, that is \( \mathbb{E}_{x_n}[\Delta_{n,k}] = 0 \), with bounded variance \( \mathbb{E}_{x_n}[\|\Delta_{n,k}\|^2] \leq \sigma^2_n \), for \( n = 1, \ldots, N \).

\[
\text{Lemma 1: If Assumptions [1][3] hold, (6b) implies that}
\]
\[ f_0(\dot{w}_{0,k}) - f_0(u_0) \leq \left( \sum_{n=1}^{N} \dot{g}_{n,k} + \Delta_{0,k}, u_0 - \dot{w}_{0,k} \right) - \left( \alpha_{0,k} - \frac{L_n}{2} \right) \| \dot{w}_{0,k} - w_{0,k} \|^2 \\
+ \left( - \frac{\delta_n}{2} \right) \| u_0 - u_{0,k} \|^2 \\
- \left( \alpha_0 \left( u_{0,k} - w_{0,k} \right) + \sum_{n=1}^{Q} g_{n,k}, u_0 - u_{0,k} \right) , \forall u_0
\]
(10)

where \( \Delta_{0,k} := \sum_{n=N+1}^{Q} g_{n,k} \).

\[
\text{Lemma 2: If Assumptions [1][3] hold, (8b) implies that}
\]
\[ f_n(\dot{w}_{n,k}) - f_n(u_n) \leq \left( \Delta_{n,k} - g_{n,k}, u_n - w_{n,k} \right) - \left( \alpha_n - \frac{L_n}{2} \right) \| u_{n,k} - w_{n,k}, u_n - u_{0,k} \|^2 \\
- \left( - \frac{\delta_n}{2} \right) \| u_n - u_{0,k} \|^2 , \forall u_n
\]
(11)

As \( p_n(w_0 - w_n) \) is convex and differentiable (cf. Assumption 3), (7) implies that \( \lambda \nabla w_n p_n(w_0 - w_n) = -g_{n,k} \), and thus
\[ \lambda p_n(\dot{w}_{0,k} - w_{0,k}) = - g_{n,k} \]
(12)

Summing up (10)-(12) and using the definition of \( F(w) \) in (5), we obtain
\[ F(\dot{w}_k) - F(u) \leq \left( \sum_{n=1}^{N} \dot{g}_{n,k} + \Delta_{0,k}, u_n - w_{0,k} \right) + \left( - \frac{\alpha_n - \frac{L_n}{2}}{n} \right) \| u_{n,k} - w_{0,k} \|^2 \\
- \left( - \frac{\delta_n}{2} \right) \| u_n - u_{0,k} \|^2 \\
- \left( \alpha_0 \left( u_{0,k} - w_{0,k} \right) + \sum_{n=1}^{Q} g_{n,k}, u_0 - u_{0,k} \right) , \forall u_n
\]
(13)

where \( \dot{w}_k := \text{vec}(w_{0,k}, \ldots, w_{N,k}) \), and \( u := \text{vec}(u_0, \ldots, u_n) \).

Based on the definition of \( \Delta_{0,k} \) and Assumption [3] it follows that \( \| \Delta_{0,k} \| \leq B \| g_{1,k} \| \) and \( \sum_{n=1}^{Q} g_{n,k} \| \leq Q \| g_{1,k} \|. \) Using also that \( \| g_{1,k} \|^2 \leq \lambda^2 G \), \( \| \sum_{n=1}^{Q} g_{n,k} \| \leq Q \| g_{1,k} \| \) and \( \| \Delta_{0,k} \| \leq B \| g_{1,k} \| \), we deduce that
\[ \left( \sum_{n=1}^{Q} g_{n,k} \right) + \| \Delta_{0,k} \|^2 \leq \lambda^2(Q + B)^2 G := \sigma^2_0.
\]
(14)

\[
\text{Lemma 3: Under Assumptions [1][4] the FRPG iterates at the server relative to the optimum } u^* \text{ satisfy}
\]
\[ F(\dot{w}_k) - F(u^*) \leq (1 - \beta_k)(F(\dot{w}_{k-1}) - F(u^*)) + \sum_{n=0}^{N} (\etaS_{s,n,k} + \etaG_{s,n,k}) + 2\frac{\lambda^2 Q^2 G}{\alpha_0} + \frac{\lambda^2 B^2 G}{2\varepsilon} \beta_k
\]
(15)
Fig. 2. LFRPG iteration, where the server broadcasts $w_i^0$ at the beginning of the $i$th frame, and the workers upload $T^{-1} \sum_{k=1}^T g_{n,k}^i$ at the end of the $i$th frame, $n = 1, \ldots, Q$.

where $\epsilon > 0$, while the scalars $\eta_{5,n,k}$ and $\eta_{6,n,k}$ are positive constants.

Using Lemma 3 our FRPG convergence is asserted next.

**Theorem 1 (Convergence of FRPG):** If under Assumptions 1–4 the step sizes are updated as

$$\alpha_{n,k} = \begin{cases} \frac{\sqrt{\bar{N}}}{13}(k+2)^2 + \frac{2}{3} L_0, n = 0 \\ \frac{1}{4}(k+2)^2 + L_m, n = 1, \ldots, N \end{cases}$$

and

$$\beta_k = \frac{2}{k+2}$$

FRPG converges as

$$F(w_k) - F(u^*) \leq \frac{4}{(k+2)^2} \left( F(w_0) - F(u^*) + \sum_{n=0}^N \eta_j n \right) + \frac{2K}{(k+2)^2} \sum_{n=0}^N \eta_j 10n + O \left( \frac{\lambda^2B^2G}{\delta_0} \right)$$

where $K$ is the number of communication rounds, while scalars $\eta_{j,n}$ and $\eta_{j10,n}$ are positive constants.

As confirmed by the last term in (18), FRPG converges to a neighborhood of the optimum with radius on the same order as that of RSA, with rate $O(1/k^2 + 1/k)$, which is faster than $O(\log k/\sqrt{k})$ of RSA. This implies that FRPG is more communication-efficient than RSA. While achieving a faster convergence rate, FRPG still requires that the workers contact the parameter server on each slot. Our LFRPG algorithm developed in the next section reduces this overhead by skipping several communication rounds.

However, two questions remain: i) what is the convergence rate of LFRPG? and, ii) how does the convergence of LFRPG depend on the communication period between the workers and parameter server? We answer these two questions next.

**IV. LOCAL FAULT-RESILIENT PROXIMAL GRADIENT**

To reduce the communication overhead, the model parameters at the server $w_0^i$ and the step sizes $\alpha_{n,k}^i$ and $\beta^i$ are updated at the start of the $i$th frame, $n = 0, 1, \ldots, N$ (as shown in Fig. 2), with each frame consisting of $T$ slots. The model parameters at the workers are updated in every slot. With $u_0^i$, $w_0^i$, and $v_0^i$ denoting the server sequences per frame $i$, the model parameters at the server are updated as

$$u_0^i = (1 - \beta^i)w_0^{i-1} + \beta^i w_0^i$$

$$w_0^i = u_0^i - \frac{1}{\alpha_{0}^i} \nabla f_0(u_0^i)$$

$$v_0^i = v_0^{i-1} - \frac{\delta_0}{\alpha_0^i} (v_0^{i-1} - u_0^i) - \nabla f_0(u_0^i) + \frac{T}{T} \sum_{k=1}^Q g_{n,k}^i$$

where superscripts $i$ and $i-1$ index the corresponding frame in the sequences and step sizes $\alpha_{j,k}^i, \beta^i$; while $g_{n,k}^i$ is defined as

$$g_{n,k}^i := A \nabla w_0 P_n (w_0^i - w_{n,k}^i).$$

Accordingly, sequences at reliable worker $n$, slot $k$, and frame $i$ are updated using stepsizes $\alpha_{n,k}^i, \beta^i$, as

$$u_{n,k}^i = (1 - \beta^i)w_{n,k}^{i-1} + \beta^i v_{n,k}^{i-1}$$

$$w_{n,k}^i = w_0^i - \text{prox}_{\frac{\delta_n^i}{\alpha_n^i}} \left( \frac{\delta_n}{\alpha_n^i} (v_{n,k}^{i-1} - u_{n,k}^i) + \nabla f(u_{n,k}^i; x_{n,k}^i) \right)$$

$$v_{n,k}^i = v_{n,k}^{i-1} - \frac{\delta_n}{\alpha_n^i} (v_{n,k}^{i-1} - u_{n,k}^i) - \nabla f_0(u_{n,k}^i; x_{n,k}^i) - g_{n,k}^i$$

while the resultant gradient noise is given by

$$\Delta_{n,k}^i := \nabla f(u_{n,k}^i; x_{n,k}^i) - \nabla f_0(u_{n,k}^i).$$

Based on (19) and (21), our novel scheme that we abbreviate as LFRPG, is listed in Algorithm 2, where lines 6–13 show that the workers update local model parameters in parallel. To proceed with convergence analysis of LFRPG, we need an assumption on the per-frame gradient noise too.

**Assumption 5 (Bounded Stochastic Noise):** The gradient noise is zero mean; that is, $E_n [\Delta_{n,k}^i] = 0$, with bounded mean-square error: $E_n [\|\Delta_{n,k}^i\|^2] \leq \sigma_{n,k}^2$ for $n = 1, \ldots, N$. 

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**Algorithm 2 LFRPG Algorithm**

1. Initialize: $w_0^i, v_0^i, w^0_{n,0}$ and $w_{n,0}^i$ for $n = 1, \ldots, N$, and step sizes as $\alpha_{0}^i$ and $\beta^i$
2. for $i = 1, \ldots, I$ do
3. The server updates $u_0^i$ and $w_{n,0}^i$ via (19a) and (19b)
4. The server broadcasts the model parameters $w_0^i$
5. for $k = 1, \ldots, T$ do
6. for $n = 1, \ldots, Q$ do
7. if $n = 1, \ldots, Q$ then
8. The $n$th reliable worker updates $w_{n,k}^i$ via (21)
9. end if
10. if $n = N + 1, \ldots, Q$ then
11. The $n$th faulty worker generates faulty parameters
12. end if
13. end for
14. end for
15. All workers upload $\frac{T}{T} \sum_{k=1}^Q g_{n,k}^i$ to the server
16. The server updates $v_0^i$ via (19a)
17. end for
Lemma 4: Under Assumptions 1, 3 and 5, the descent loss at the server implied by the LFRPG iterates in (19b), satisfies
\[ f_0(w^i_0) - f_0(u_0) \leq \frac{N}{\sum_{i=1}^{\Omega} g^i_{n,k}} \|u^i_0 - w^i_0\|^2 + \sum_{i=1}^{\Omega} g^i_{n,k} \|u^i_0 - w^i_0\|^2 \]
and stepsizes are respectively updated as
\[ \alpha^i_0 = \frac{L_0}{2} \]
where \( \Delta^i_{n,k} := \sum_{\alpha=0}^{\Omega} g^i_{n,k} \).
Proof: The proof follows directly from Lemma 1.

Lemma 5: Under Assumptions 1, 3 and 5, the descent loss per worker implied by LFRPG iterates in (21b), obeys
\[ f_n(w^i_{n,k}) - f_n(u_n) \leq \left( \frac{\alpha^i_n - L_n}{2} \right) \|u^i_{n,k} - w^i_{n,k}\|^2 \]
Proof: The proof follows readily from Lemma 2.

Since \( u^i_{n,k} \) and \( w^i_{n,k} \) are updated at the start of frame \( i \), we set \( u^i_{0,k} = u^i_{0,k} \) and \( w^i_{0,k} = w^i_{0,k} \) with \( k = 1, \ldots, T \). Summing (23) and (24), it follows after straightforward algebraic manipulations that the overall loss at \( w^i_{k} := \text{vec}([w^i_{0,k}, w^i_{1,k}, \ldots, w^i_{N,k}]) \), obeys
\[ F(w^i_{k}) - F(u^i) \leq \frac{N}{\sum_{i=1}^{\Omega} g^i_{n,k}} \|u^i_{0,k} - w^i_{0,k}\|^2 + \sum_{i=1}^{\Omega} g^i_{n,k} \|u^i_{0,k} - w^i_{0,k}\|^2 \]
where \( \eta^i_{14,n} \) and \( \eta^i_{15,n} \) are positive constants.

Lemma 6: Under Assumptions 1, 3 and 5, LFRPG iterates incur loss relative to the optimum \( u^* \), that is bounded by
\[ \frac{1}{T} \sum_{k=1}^{T} F(w^i_{k}) - F(u^i) \leq \frac{1 - \beta^i}{T} \left( \frac{1}{T} \sum_{k=1}^{T} F(w^i_{k-1}) - F(u^*_{k}) \right) + \frac{\lambda^2 B^2 G}{2e} \beta^i + \sum_{i=0}^{N} \left( \eta^i_{14,n} + \eta^i_{15,n} \right)^2 \]
where \( \eta^i_{14,n} \) and \( \eta^i_{15,n} \) are positive constants.

Lemma 6 leads to the convergence result for LFRPG.

Theorem 2 (Convergence of LFRPG): If Assumptions 1, 3 and 5 hold, and stepsizes are respectively updated as
\[ \alpha^i_n = \begin{cases} \frac{L_0}{2} (i + 2)^2 + \frac{L_n}{2} (i + 2)^2 + L_n, & n = 0 \\ \frac{L_0}{2} (i + 2)^2 + \frac{L_n}{2} (i + 2)^2 + L_n, & n = 1, \ldots, N \end{cases} \]
then average LFRPG iterates \( \bar{w}^i := \frac{1}{T} \sum_{k=1}^{T} w^i_{k} \) converge
\[ F(\bar{w}^i) - F(u^*) \leq \frac{2\eta_16}{T(I + 2)^2} + \eta_{17} + I_{18} + O \left( \frac{\lambda^2 B^2 G}{\delta_0} \right) \]
where \( \eta_{16}, \eta_{17} \) and \( \eta_{18} \) are positive constants.

The first fraction in (29) reveals that LFRPG outperforms FRPG in communication efficiency; while the last fraction asserts that LFRPG converges to the neighborhood of FRPG.

V. Experiments

To validate our analytical results, we tested the performance of FRPG and LFRPG numerically on real datasets (USPS [27], MNIST [28] and FMNIST [29]). In the USPS set, we used 8,000 data vectors of size 256 \times 1 for training, and 3,000 for testing. In MNIST, we used 60,000 data vectors of size 784 \times 1 for training, and 10,000 for testing. In FMNIST, we used 60,000 data vectors of size 784 \times 1 for training, and 10,000 for testing. The heterogeneity of datasets was manifested as follows. Each pair of workers were assigned data of the same handwritten digits, and 50% of the handwritten digits were removed. For example, the data samples with labels 6, 7, 8 and 9 were removed in half of the tests. We consider the Label-Flipping attack 7, and the Gaussian attack 4, to verify the robustness of FRPG and LFRPG. For the Label-Flipping attack the original label \( y \) was skewed to \( 9-y \); while for the Gaussian attack we set \( w_n = c \times \mathcal{N}(0,1) \) with \( c = 1 \times 10^4 \). The tests were run on MATLAB R2018b with Intel i7-8700 CPU @ 3.20 GHz and 16 Gb RAM.

The multinomial logistic regression was employed as the loss with regularizer \( (\delta_n/2) \|w_n\|^2 \). At the parameter server, we set \( f_0(w_0) = (\delta_n/2) \|w_0\|^2 \). Huber’s cost with smoothing constant \( \mu = 10^{-3} \) was adopted as the penalty function
\[ p_n(w_0 - w_n) = \begin{cases} \frac{1}{2} \|w_0 - w_n\|^2, & \|w_0 - w_n\| \leq \mu \\ \frac{1}{2} \|w_0 - w_n\|^2 - \frac{\mu}{2}, & \text{otherwise} \end{cases} \]
We considered a setting with \( Q = 20 \) workers, \( N = 16 \) reliable ones, and weight \( I = 1.6 \). The training data were evenly distributed across the workers. With faulty workers attacking by flipping labels, the mini-batch size was set to 15; while for those adopting a Gaussian attack, the mini-batch size was set to 10. To obtain a good top-1 accuracy convergence, we set the step sizes for benchmark schemes to \( \frac{3}{4} \). A strongly convex modulus with \( \delta_n = 0.003 \) was chosen for \( n = 0, 1, \ldots, N \); while the Lipschitz constants for the USPS, MNIST and FMNIST datasets were respectively set to 156, 295, and 524. The workers in LFRPG communicated with the server every ten slots.

We also tested communication efficiency in comparison with Krum [17], GeoMed [9], and RSA [15] benchmarks. To demonstrate the negative effects of different attacks, we employed SGD by averaging heuristically the local gradients of workers. Figures 3 and 4 show the convergence of FRPG, LFRPG and RSA under Label-Flipping, and Gaussian attacks, respectively. After 4,000 communication rounds, FRPG and
LFRPG converge faster than RSA, while LFRPG outperforms FRPG for the same number of rounds. To reach the same loss value with the FMNIST dataset, LFRPG takes around 400 communication rounds versus 800 required by FRPG under Label-Flipping attacks.

Figs. 3 and 4 compare the top-1 accuracy with Krum, GeoMed and RSA, under Label-Flipping and Gaussian attacks, respectively. With Label-Flipping, both FRPG and LFRPG converge faster than the benchmarks. FRPG and LFRPG also achieve better top-1 accuracy for the USPS, MNIST and FMNIST datasets, while Krum fails because it is designed for homogeneous datasets. Since Label-Flipping attacks do not change the magnitude of local gradients, their negative effects on SGD are limited when heterogeneous datasets are used. For this reason, we considered the more severe Gaussian attack. Fig. 6 illustrates that SGD fails in the presence of Gaussian attacks. However, both FRPG and LFRPG converge faster and achieve better top-1 accuracy than Krum, GeoMed and RSA. In the FMNIST with Gaussian attacks present, the top-1 accuracy of FRPG and LFRPG is 4.13% better than that of GeoMed, and 9.89% better than that of RSA.

VI. CONCLUSIONS

This work dealt with fault-resilient federated learning. Cross-fertilizing benefits of the robust stochastic aggregation framework and Nesterov’s acceleration technique, two algorithms were developed to reduce the communication overhead involved. Both were proved to attain performance gains relative to the benchmarks in terms of communication efficiency. Numerical tests also confirmed this improved communication efficiency over different real datasets.

REFERENCES

[1] B. Li, M. Ma, and G. B. Giannakis, “On the convergence of SARAH and beyond,” arXiv preprint arXiv:1906.02351, June 2019.
[2] B. Li, L. Wang, and G. B. Giannakis, “Almost tune-free variance reduction,” arXiv preprint arXiv:1908.09345, Aug. 2019.
[3] J. Konečný, B. McMahan, and D. Ramage, “Federated optimization: Distributed optimization beyond the datacenter,” arXiv preprint arXiv:1511.03575, Mar. 2015.
[4] Y. Dong, J. Cheng, M. J. Hossain, and V. C. M. Leung, “Secure distributed on-device learning networks with Byzantine adversaries,” IEEE Netw., to be published, Apr. 2019.
[5] M. Chen, Z. Yang, W. Saad, C. Yin, H. Y. Poor, and S. Cui, “A joint learning and communications framework for federated learning over wireless networks,” arXiv preprint arXiv:1909.07972, Sept. 2019.
[6] Y. Chen, L. Su, and J. Xu, “Distributed statistical machine learning in adversarial settings: Byzantine gradient descent,” Proc. ACM Meas. Anal. Comput. Syst., vol. 1, no. 2, pp. 44:1–44:25, Dec. 2017.
[7] D. Yin, Y. Chen, R. Kannan, and P. Bartlett, “Byzantine-robust distributed learning: Towards optimal statistical rates,” in Proc. International Conference on Machine Learning (ICML), Stockholmsmässan, Stockholm, Sweden, July 2018, pp. 5650–5659.
[8] ——, “Defending against saddle point attack in Byzantine-robust distributed learning,” in Proc. International Conference on Machine Learning (ICML), vol. 97, Long Beach, California, USA, June 2019, pp. 7074–7084.
[9] L. Su and J. Xu, “Securing distributed gradient descent in high dimensional statistical learning,” in Proc. ACM Meas. Anal. Comput. Syst., vol. 3, no. 1, Mar. 2019, pp. 12:1–12:41.
[10] P. Blanchard, E. M. El Mhamdi, R. Guerraoui, and J. Stainer, “Machine learning with adversaries: Byzantine tolerant gradient descent,” in Proc. Advances in Neural Information Processing Systems (NIPS), Long Beach, USA, Dec. 2017, pp. 119–129.
[11] E. M. El Mhamdi, R. Guerraoui, and S. Rouault, “The hidden vulnerability of distributed learning in Byzantium,” in Proc. International Conference on Machine Learning (ICML), Stockholmsmässan, Stockholm, Sweden, July 2018, pp. 3521–3530.
[12] D. Alistarh, Z. Allen-Zhu, and J. Li, “Byzantine stochastic gradient descent,” in Proc. Advances in Neural Information Processing Systems (NIPS), Montreal, CA, Dec. 2018, pp. 4614–4624.
Fig. 5. Top-1 accuracy over the number of communication rounds under Label-Flipping attack and heterogeneous datasets.

Fig. 6. Top-1 accuracy over the number of communication rounds under Gaussian attack and heterogeneous datasets.

[13] C. Xie, S. Koyejo, and I. Gupta, “Zeno: Distributed stochastic gradient descent with suspicion-based fault-tolerance,” in Proc. International Conference on Machine Learning (ICML), Long Beach, California, USA, June 2019, pp. 6893–6901.

[14] L. Chen, H. Wang, Z. Charles, and D. Papailiopoulos, “DRACO: Byzantine-resilient distributed training via redundant gradients,” in Proc. International Conference on Machine Learning (ICML), Stockholm, Sweden, July 2018, pp. 903–912.

[15] L. Li, W. Xu, T. Chen, G. B. Giannakis, and Q. Ling, “RSA: Byzantine-robust stochastic aggregation methods for distributed learning from heterogeneous datasets,” in Proc. AAAI Conference on Artificial Intelligence, vol. 33, no. 01, Jan. 2019, pp. 1544–1551.

[16] A. Ghosh, J. Hong, D. Yin, and K. Ramchandran, “Robust federated learning in a heterogeneous environment,” arXiv preprint arXiv:1906.06629, 2019.

[17] M. Li, D. G. Andersen, A. J. Smola, and K. Yu, “Communication efficient distributed machine learning with the parameter server,” in Proc. Advances in Neural Information Processing Systems (NIPS), Palais des Congrès de Montréal, Montréal, Dec. 2014, pp. 19–27.

[18] M. I. Jordan, J. D. Lee, and Y. Yang, “Communication-efficient distributed statistical inference,” Journal of the American Statistical Association, vol. 114, no. 526, pp. 668–681, 2019.

[19] T. Chen, G. B. Giannakis, T. Sun, and W. Yin, “LAG: Lazily aggregated gradient for communication-efficient distributed learning,” in Proc. Advances in Neural Information Processing Systems (NIPS), Montreal, CA, Dec. 2018, pp. 5050–5060.

[20] J. Sun, T. Chen, G. B. Giannakis, and Z. Yang, “Communication-efficient distributed learning via lazily aggregated quantized gradients,” in Proc. Advances in Neural Information Processing Systems (NIPS), to be published, Sept. 2019.

[21] S. U. Stich, “Local SGD converges fast and communicates little,” in Proc. International Conference on Learning Representations (ICLR), Addis Ababa, Ethiopia, Apr. 2019.

[22] H. Yu, S. Yang, and S. Zhu, “Parallel restarted SGD with faster convergence and less communication: Demystifying why model averaging works for deep learning,” in Proc. AAAI Conference on Artificial Intelligence, vol. 33, no. 01, Jan. 20, 2019, pp. 5693–5700.

[23] H. Yu, R. Jin, and S. Yang, “On the linear speedup analysis of communication efficient momentum SGD for distributed non-convex optimization,” in Proc. International Conference on Machine Learning (ICML), vol. 97, Long Beach, California, USA, June 2019, pp. 7184–7193.

[24] Y. Nesterov, Introductory Lectures on Convex Optimization: A Basic Course, 1st ed. Springer Publishing Company, Incorporated, 2014.

[25] C. Hu, W. Pan, and J. T. Kwok, “Accelerated gradient methods for stochastic optimization and online learning,” in Proc. Advances in Neural Information Processing Systems (NIPS), Vancouver, Canada, Dec. 2009, pp. 781–789.

[26] A. Nemirovski, A. Juditsky, G. Lan, and A. Shapiro, “Robust stochastic approximation approach to stochastic programming,” SIAM J. Optim., vol. 19, no. 4, pp. 1574–1609, 2009.

[27] J. J. Hull, “A database for handwritten text recognition research,” IEEE Trans. Pattern Anal. Mach. Intell., vol. 16, no. 5, pp. 550–554, May 1994. [Online]. Available: [https://cs.nyu.edu/~roweis/data.html](https://cs.nyu.edu/~roweis/data.html)

[28] Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner, “Gradient-based learning applied to document recognition,” Proc. IEEE, vol. 86, no. 11, pp. 2278–2324, Nov. 1998. [Online]. Available: [http://yann.lecun.com/exdb/mnist/](http://yann.lecun.com/exdb/mnist/)

[29] H. Xiao, K. Rasul, and R. Vollgraf, “Fashion-MNIST: a novel image dataset for benchmarking machine learning algorithms,” arXiv preprint arXiv:1708.07747, Sept. 2017. [Online]. Available: [https://www.kaggle.com/kushalkr/fmnist](https://www.kaggle.com/kushalkr/fmnist)