Observation of an in-plane magnetic-field-driven phase transition in a quantum Hall system with SU(4) symmetry

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In condensed matter physics, the study of electronic states with SU(N) symmetry has attracted considerable and growing attention in recent years, as systems with such a symmetry can often have a spontaneous symmetry-breaking effect giving rise to a novel ground state. For example, pseudospin quantum Hall ferromagnet of broken SU(2) symmetry has been realized by bringing two Landau levels close to degeneracy in a bilayer quantum Hall system [1]. In the past several years, the exploration of collective states in other multi-component quantum Hall systems has emerged [2, 3, 4, 5]. Here we show the conventional pseudospin quantum Hall ferromagnetic states with broken SU(2) symmetry collapsed rapidly into an unexpected state with broken SU(4) symmetry, by in-plane magnetic field in a two-subband GaAs/AlGaAs two-dimensional electron system at filling factor around \( \nu = 4 \). Within a narrow tilting range angle of 0.5 degrees, the activation energy increases as much as 12 K. While the origin of this puzzling observation remains to be exploited, we discuss the possibility of a long-sought pairing state of electrons with a four-fold degeneracy.

The studies of multi-component quantum Hall systems so far have been limited to systems with a SU(2) symmetry. Very recently, interest has been further extended to systems with a higher order symmetry, such as SU(4), motivated mainly by the surge of research in graphene, where the two-fold spin and two-fold valley degeneracy lead to a four fold degeneracy [6, 7]. The SU(4) symmetry can be readily created in a multi-component quantum Hall system. For a typical two subband semiconductor heterostructure, where both the symmetric and anti-symmetric subbands of the confined quantum well are occupied, two distinct sets of Landau levels are present. Through varying the density or magnetic field, levels with different Landau orbital indices originating from the two subbands can be brought into degeneracy. Due to the very small energy difference of the spin-splitting in GaAs [2, 3, 8, 9, 10], the system can provide us with the desired SU(4) symmetry. The experimental studies presented in this paper are specifically focused on this interesting physical regime.

The sample was grown by molecular-beam epitaxy and consists of a symmetrical modulation-doped 24 nm wide single GaAs quantum well bounded on each side by Si \( \delta \)-doped layers of AlGaAs with doping level \( n_d = 10^{12} \) cm\(^{-2}\). Heavy doping creates a very dense 2DEG, resulting in the filling of two subbands in the well. As determined from the Hall resistance data and Shubnikov-de Haas oscillations in the longitudinal resistance, the total density is \( n = 8.0 \times 10^{11} \) cm\(^{-2}\), where the first and the second subband have a density.
of \( n_1 = 6.1 \times 10^{11} \text{ cm}^{-2} \) and \( n_2 = 1.9 \times 10^{11} \text{ cm}^{-2} \). The sample has a low-temperature mobility \( \mu = 4.1 \times 10^5 \text{ cm}^2/\text{V s} \), which is extremely high for a 2DEG with two filled subbands. The samples are patterned into Hall bars using standard lithography techniques. A NiCr top gate is evaporated on the top of the sample, approximately 350 nm away from the center of the quantum well. By applying a negative gate voltage on the NiCr top gate, the electron density can be varied continuously. Magneto-transport measurements were carried out in an Oxford Top-Loading Dilution Refrigerator with a base temperature of 15 mK and a \textit{in situ} motorized rotating stage with a resolution better than 0.01 degree. To measure the longitudinal and Hall resistance \( R_{xx} \) and \( R_{xy} \), we used a standard ac lock-in technique with electric current ranging from 10 nA to 100 nA at a frequency of 11.3 Hz. Two devices from the same wafer were studied, and they have produced remarkably identical results. For consistency, however, we present the data from only one sample.

In this experiment, we have concentrated our study around \( \nu = 4 \), where four energy levels are filled. In the absence of an in-plane magnetic field (i.e., with zero tilting angle), we have essentially reproduced the results of earlier studies \[3, 10\] on devices from the same wafer. The topology of longitudinal resistance \( R_{xx} \) in the density(\( n \))-magnetic field(\( B \)) plane exhibits a square-like structure around \( \nu = 4 \), as shown in Fig. 1a. Here, point B corresponds to the degeneracy point of \((|S, 1, \downarrow\rangle)\) and \((|A, 0, \uparrow\rangle)\) while point C corresponds to that of \((|S, 1, \uparrow\rangle)\) and \((|A, 0, \downarrow\rangle)\), as illustrated schematically in Fig.(1f). Here we label the single-particle levels \((i, N, \sigma)\), and \(i(=S, A), N, \) and \(\sigma(=\uparrow, \downarrow)\) are the subband, orbital and spin quantum numbers. One prominent feature is the disappearance of the extended states (i.e., bright lines) that complete the two-arms of the square. The disappearance is due to the pseudospin gaps of easy-axis quantum Hall ferromagnetic states where the electrons are suddenly transferred from \((S, 1, \downarrow)\) to \((A, 0, \uparrow)\), and from \((S, 1, \uparrow)\) to \((A, 0, \downarrow)\), or vice versa \[2, 10, 11\].

Now we turn our attention to the behavior of the topology phase diagram in the tilted magnetic field. The density(\( n \))-perpendicular magnetic field(\( B_\perp \)) phase diagrams of \( R_{xx} \) at several tilted angles \( \theta = 0^\circ, 4.62^\circ, 5.82^\circ, 6.02^\circ, 6.32^\circ \) for \( \nu = 4 \) are shown in Fig.(1I). Here the angle \( \theta \) is defined as \( \tan \theta = \frac{B_\parallel}{B_\perp} \). When the tilting angle is increased to \( 4.62^\circ \), Fig.(1b), the size of the square, a measure of the energy separation between the two degeneracy points, shrinks only slightly. However, to increase merely another 1-degree, the two ”arms” of the square structure almost collapse together, Fig.(1c), and become one point at slightly larger angles, Fig.(1d),(1e). The evolution of the positions of the anti-crossing points B and C
(roughly in the middle of each of the "arms" of the square structure at $\nu = 4$, also the local minimum of energy gap in the anti-crossing region), are plotted in Fig. (2) as a function of the tilting angle (See also the movie in the supplement). One can clearly visualize that the two points get close with each other in a narrow range of tilting angle and emerge as one point when $\theta$ is increased, which implies four Landau levels are brought into degeneracy. In other words, the two degeneracy points with a SU(2) symmetry become a single degeneracy point with a SU(4) symmetry.

Since there seems to be a drastic change in the characteristics of the phase diagram, in a very narrow range of the in-plane magnetic field, it is natural to obtain a measurement of the energy scale of the associated energy gaps there. In Fig. (3), we present the $\theta$ dependence of the energy gap $E_a$, deduced from the thermal activation relation $\rho_{xx} \sim \exp(-E_a/2T)$, at two points B, C marked by the dots in Fig. (1). The energies are found to be nearly constant, $\approx 3$ K before $\theta = 5.82^\circ$, while increasing sharply for a small angle region into a rough angle independent region $\approx 15$ K. Without any assist of theoretical analysis, the raw data strongly suggests a phase transition.

Before we speculate on the physical origin of the observations, we would like to compare the current experiment with others in similar 2DEG systems in the presence of an in-plane magnetic field. There are indeed examples of in-plane magnetic field induced phase transitions in multi-component quantum Hall systems. The system studied by Murphy et al. [12], a strongly coupled double quantum well at a filling factor $\nu = 1$, showed a relatively large energy gap change when the tilted angle was increased. It is now commonly believed that this change reflects a commensurate to an incommensurate phase transition in the pseudospin field. However, in this case, only the two lowest Landau levels are involved before and after this quantum phase transition, in contrast to four Landau levels degeneracy in our system. In a recent study of Si-SiGe heterostructures by Lai et al. [4], the $\nu = 3$ and 5 valley gaps rise rapidly when the angle of tilting approaches about 65$^\circ$. In this experiment, the effect is driven by the coincidence of two different Landau levels when the Zeeman splitting becomes comparable to the cyclotron energy, which is not possibly consistent with our small tilting angle. In another recent study of the 2DEG in a AlAs quantum well by Vakili et al. [5], the size of the energy gap of $\nu = 3$ valley changes rapidly through a coincidence angle of about 65$^\circ$. The change is induced also by the crossing of two different Landau levels, made possible again by the large tilting angle.

Changes in the energy gap $E_a$ normally reflect changes in the spectrum of charged excita-
tions and the 2DEG ground state. At first sight, one might expect that the phase transition is induced by the spin and pseudospin flips due to variations in the Zeeman energy $g\mu_B B_{tot}$. The Zeeman energy, however, changes only about 30 mK upon tilting from $\theta = 0^\circ$ to $10^\circ$ as the total magnetic field changes only about 10 percent, which is negligibly small compared to the observed change 10 K in $E_a$. The in-plane magnetic field can also affect the magnetite of the exchange energy since the distributions of both the symmetric and anti-symmetric wavefunctions are expected to vary considerably when the in-plane magnetic field ranges from 0 at $\theta = 0^\circ$ to 0.8 T at $\theta = 8^\circ$. However the sudden change of the activation energy cannot simply be due to the quantitative change of the exchange energy by the in-plane magnetic field. First, the experimental data of the energy gap $E_a$ is nearly constant in a wide range of tilted angle $\theta$ at small in-plane fields, as shown in Fig. (3). We therefore expect that the energy of the pseudospin states is rather insensitive to response relative to the in-plane magnetic field. Second, we found that such a phase transition is totally absent at a filling factor $\nu = 6$.

We speculate the observations involve a quantum phase transition induced by an in-plane magnetic field. There is a competition between two ground states, one of which, at $\theta < \theta_c$, takes advantage of Coulomb interactions by forming pseudospin quantum Hall ferromagnets and the other, which becomes a more favorable many-body configuration when $\theta > \theta_c$. From the energetic degeneracy of Landau levels before and after the critical point, we infer that there is a symmetry change: from SU(2) to SU(4) symmetry in our system. As shown in the standard Landau level fan diagram (Fig. (4)), at $\nu = 4$, there is a SU(2) symmetry at crossing points B and C. Since the two crossing Landau levels have opposite spins, the exchange energy is highly pseudospin dependent, and leads to an easy-axis quantum Hall ferromagnet. As a result, the energy gap here represents an exchange energy penalty for a pseudospin flip. In our system, as the tilted magnetic field increases, the B and C points are close to each other, which means a total of four Landau levels with different subband, orbital, and spin indices are brought close in energy near the phase transition. Thus there is a SU(4) symmetry at the region where B and C overlapped. Coupling of all four-fold levels may give rise to a more complex many-body state. In light of this, the energy gap jump may be due to suppression of the low energy excitations which originates from the formation of a new ground state. One possibility is that, in this region, electrons with different spins and subbands can pair up, condensed in a pseudospin-singlet pairing state of four-fold Landau levels as $|S\rangle = \Pi_k [C^\dagger_{(S,1,\uparrow),k} + e^{i\varphi} C^\dagger_{(A,0,\downarrow),k}] [C^\dagger_{(S,1,\downarrow),k} + e^{i\varphi} C^\dagger_{(A,0,\uparrow),k}] |0\rangle$. In this new state, the
electrons condense in a superposition of states with different spins and subbands sharing the same phase $e^{i\phi}$. Thus the energy gap of 15 K for $\theta > \theta_c$ can be taken as a measurement of the pairing strength of pseudospin coherence.

As far as we know, there are only a few theoretical investigations of two-fold and four-fold Landau level degeneracy in two-subband quantum Hall systems [13, 14]. Nevertheless, there are quite a few theoretical studies of multi-fold degeneracy states in the coupled bilayer system. A soft barrier originating from Coulomb interaction among electrons in the quantum well separates our system into effective strongly coupled bilayers. As a result, the associated wave functions with subband index can be transformed as those with layer index $\Phi_L = [\Phi_{AS} + \Phi_S]/\sqrt{2}$, $\Phi_R = [\Phi_{AS} - \Phi_S]/\sqrt{2}$. Thus we believe that the theoretical description of quantum Hall phenomenon in two-subband single wells only requires slight modifications of the idealized bilayer model while retaining the basic results of the many-body Hartree-Fock formalism. On one hand, for an idealized bilayer model at filling factor $\nu = 4$ in zero in-plane magnetic field, where two aligned Landau levels with different subband and orbital indices are brought into degeneracy, the ground state can be an easy-axis quantum Hall ferromagnet dependent on the details of the system [11]. The quasiparticle energy gap of easy-axis quantum Hall ferromagnet is weakly dependent on the tilted angle [15], which is consistent with our observations for $\theta < \theta_c$. On the other hand, for a bilayer system at $\nu = 2$, where sometimes four Landau levels are close to degeneracy, the ground state is theoretically studied by several groups with the Hartree-Fock method [16, 17]. It was found that there are rich phases including ferromagnetic, spin-singlet, and canted phase and transitions between them. In the presence of strong in-plane magnetic fields and large tunneling, the ground state is expected to be a spin-singlet state resulting from the pairing of two adjacent layers [17, 18]. In the present system, since the energy difference between the symmetry and antisymmetry states, $\Delta_{SAS}$, is as large as $0.75 \frac{e^2}{\varepsilon l}$, measured from Shubnikov-de Haas oscillations, one expects a similar pairing state of pseudo-spins, or sub-bands $|S\rangle$ when four Landau levels are brought into degeneracy. Thus we can construct a phenomenological model as shown in the Fig. (4), where the ground state is an ordinary easy-axis quantum Hall ferromagnet for the case of two degenerate Landau levels and becomes a new state for four degenerate Landau levels, when the electrons with different spins and subband pair up and coherence associated with the phase $e^{i\phi}$ is maintained. We speculate that similar SU(2) to SU(4) phase transition phenomenon can be observed in bilayer systems at some appropriate regions. Actually, the activation energies we measured at $\theta > \theta_c$ are consistent with the experimental values of
energy gap of spin-singlet state in bilayer $\nu = 2$, which is about $15 \text{ K}$\textsuperscript{[19]}. Of course, there are possibly different competing-orders, such as a stripe state with broken translational and spin symmetries, which is commonly believed to occur at very large in-plane magnetic fields \textsuperscript{[14, 20, 21]}. More work is desired to calculate the ground state energy of the proposed SU(4) as a function of the in-plane field.

In summary, we find experimental evidence for intriguing and unexpected quantum phase transition from the typical pseudospin quantum Hall ferromagnetic states with broken SU(2) symmetry into a state with broken SU(4) symmetry driven by the in-plane magnetic field, around a filling factor of $\nu = 4$ in a two-subband GaAs-AlGaAs 2DEG. While the origin of this new state is unclear, we discussed the possibility of the pairing state of electrons with different spins and subbands. More experiments, such as electrically detected nuclear magnet resonance \textsuperscript{[22]}, are needed to further investigate the microscopic aspects of this quantum phase transition.

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FIG. 1: Topology structures of the longitudinal resistance $R_{xx}$ in $\nu - B_{\perp}$ phase diagram at filling factor $\nu = 4$ with the tilted angle $\theta$ from $0^\circ$ to $7^\circ$, which are measured at the base temperature. (a)-(e): phase diagram at tilted angle $\theta = 0^\circ$, $4.62^\circ$, $5.82^\circ$, $6.02^\circ$, $6.32^\circ$. (f): schematic drawing of the anti-crossing between different subband and spin indices Landau levels in two corresponding places marked by B and C in fig. (a)-(e).
FIG. 2: Evolution of the positions of the degenerate points B and C as a function of the tilted angle $\theta$. The corresponding in-plane magnetic field are displayed on the top axis. Insert: indication of the points that are measured.
FIG. 3: Energy gap of two anti-crossing points B and C as a function of in-plane magnetic field $B_\parallel$. 

- $\theta = 0^\circ$
- $\theta = 2.62^\circ$
- $\theta = 5.82^\circ$
- $\theta = 6.62^\circ$
- $\theta = 8.62^\circ$
FIG. 4: Energy profile of the system from two-fold degeneracy to four-fold degeneracy. The solid line represents the energy of the respective ground state stabilized. The left is an ordinary easy-axis quantum Hall ferromagnet (QHF), and the right is a pseudospin-singlet electron pairing state with different subband and spin indices.