A New Networking Technique—The Beeline Diagramming Method

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Abstract. The Critical Path Method (CPM) has gradually increased in importance in the construction industry since the Arrow Diagramming Method (ADM) and the Precedence Diagramming Method (PDM) were introduced in 1956 and 1961, respectively. In an improvement over the ADM, the PDM is able to represent overlapping relationships between consecutive activities. It is limited to representing overlapping relationships only as four combinations that connect the starting and finishing points of two consecutive activities, however; it cannot express the relationships at the exact point of interrelation if that point is in the middle of the activity’s duration. Further, when two consecutive activities have respective multiple milestones that should be connected independently, the PDM cannot represent the multiple overlapping relationships precisely. This research proposes the Beeline Diagramming Method (BDM) as a new networking technique that can represent all kinds of overlapping relationships between activities. The basic concept, principle, interpretation methods, and schedule computation methods of the BDM are defined in this paper.

Introduction

Network schedules have contributed significantly to the planning, control, and on-time completion of construction projects (Callahan 1992). The Critical Path Method (CPM) has gradually increased in importance in the construction industry over the last several decades. It integrates overall project management functions, such as scheduling, cost control, and resource planning.

By the middle of the 1980s, the CPM was widely applied in the construction industry based on the Arrow Diagramming Method (ADM) first introduced by Du Pont in 1956. The ADM has since been replaced by the Precedence Diagramming Method (PDM) proposed by Fondahl in 1961. Most international project management programs applied to construction projects now utilize the PDM as their scheduling function. The conversion from the ADM to the PDM is due to the PDM’s ability to depict the overlapping relationships of two consecutive activities, which permits more realistic project modeling. Further, overlapping permits the planner to reduce the overall time duration of the project (Harris 1978).

In the PDM, overlapping relationships between activities are represented by four combinations that connect the starting and finishing points of two consecutive activities (Ponce-Campos 1972). Confining overlapping relationships to the starting and finishing points is limiting, however, since the overlapping could happen at any point during the activity’s duration. If overlapping relationships occur at any middle point, the PDM cannot properly show the relationships.

The scheduling technique must represent all kinds of relationships between activities realistically and efficiently. This research thus proposes a new networking technique that can depict overlapping relationships at any middle point of activities and not limit the relationships to the starting and finishing points. This new networking technique will allow project teams to establish project schedule plans more realistically and efficiently by representing all kinds of relationships between activities with improved flexibility.
Research Methods

This research is performed in the following steps. First, the major issues of the PDM, the Logical Diagramming Method (LDM), and the chronographic method with regard to overlapping networks are surveyed and the Beeline Diagramming Method (BDM) is proposed as a new networking technique to overcome these limitations and inefficiencies. Second, the basic concept, principle, and characteristics of the BDM are defined, the organization and interpretation of relationships in the BDM are explained, and the schedule computation methods used in the BDM are described. Third, the applicability and rationality of the BDM is verified by performing schedule computations in the complete BDM network.

The BDM proposed in this research is designed to be applied to the construction industry. However, the results of this research could be applied to other industries that are adopting the CPM as a scheduling technique.

Survey of the PDM, the LDM, and the Chronographic Method

The PDM expresses the overlapping relationships of two consecutive activities by utilizing the four link relationships. The first link relationship, Finish-to-Start (FS), does not show overlapping. It is considered an overlapping relationship, however, because the predecessor’s finish determines the successor’s start. The second link relationship is Start-to-Start (SS); this permits the start of the predecessor to set the start of the successor. The third relationship, Finish-to-Finish (FF), allows the finish of the predecessor to establish the finish of the successor. The fourth, Start-to-Finish (SF), permits the start of the predecessor to determine the finish of the successor.

The PDM represents the overlapping between the predecessor and successor by the four link relationships between their starting and finishing points; it is impossible to depict overlapping at any middle point. For instance, let us assume that there are two consecutive activities—the preceding activity A of 10 days and the succeeding activity B of 12 days. If the two activities are connected from the 70% completion of activity A to the 33% completion of activity B, as shown in Figure 1, the PDM can depict their overlapping relationship as SS, FF, and SF according to their locations and linkage preferences. The first option is to select the SS link relationship with lead time 3 (SS3), shown in Figure 2. The second is to choose the FF linkage with lead time 5 (FF5), illustrated in Figure 3. The third is to pick the SF with lead time 15 (SF15), shown in Figure 4. All three options represent the overlapping relationships between activities A and B identically. However, this circular depiction of the overlapping relationships through combinations of the starting and finishing points is not efficient or convenient; the two activities need to be positioned to align connection points. The overlapping linkage type can then be selected and the lead-time for the selected linkage calculated. Of course, overlapping relationships that depend solely on the starting and finishing points of two consecutive activities exist; it is more realistic and reasonable to expect two consecutive activities to interrelate directly at a middle point, however.

Figure 1. Two Consecutive Activities.
If two consecutive activities need multiple overlapping relationships in the PDM, it is best for them to be connected by the compound relationship that depicts only two overlapping linkages; this means that the start of the predecessor and the start of the successor are connected by SS and the finish of the predecessor and the finish of the successor are linked by FF. If the preceding activity A and the succeeding activity B have more than two milestones, they should be interrelated at more than two points. The PDM cannot represent multiple overlapping relationships properly with only two linkages, however. Since the predecessor and the successor should realistically be linked by multiple overlapping relationships, it is necessary to develop a new networking technique that can depict multiple overlapping linkages properly and correctly.

Ponce de Leon (2008) proposed the LDM that is a technique suitable for collaborative planning that is squarely within the reach of project stakeholders who are not necessarily trained schedulers. He noted that LDM allows effortless visualization of activity sequences and timing while also offering interrelationships between activities that are more intuitive and versatile for use by nonschedulers than those offered by PDM or ADM, and it is a recently-unveiled activity-based networking method that modifies and extends ADM to permit PDM logic. Although the LDM looks like to represent the overlapping relationship on the intermediate points of activities, however, it only shows the finish-to-start (FS) relationships between the segmented activities of ADM’s arrow activity. This is confirmed from his paper, it defines “On the LDM, driving relationships are conveyed by a common node (FS only) or, owing to the time scale, a vertical link.” Therefore, the
LDM is an identical methodology with the ADM. Further, it does not have any distinct schedule computation algorithm and overlapping representation methods.

Francis and Miresco (2006) proposed the chronographic method that is a time-scaled scheduling modeling technique that allows a good visual appearance along with an accurate representation of the project. The chronographic modeling approach classifies entities in three categories: (i) primary entities, which symbolize the production tools as activities, resources, etc.; (ii) secondary entities, which represent relational constraints between the principal entities (these relations could be deterministic or probabilistic); and (iii) general entities, which represent the measuring units for the direction of logical flow. Further, it proposes the schedule computation algorithm for the earliest start and latest finish, and six new types of float: the complete float; the start float and the finish float; and the partial complete, partial start, and partial finish floats. However, this method does not have any distinct representation method of overlapping relationships, making it difficult to schedule and control construction projects in tabular format excluding visual expressions. Further, the many and varied representation symbols proposed for a good visual appearance along with an accurate representation of the project make it more difficult for schedulers or individuals who are not familiar with them to manage project schedules. Therefore, it is not easy to say that the chronographic method is an effective and flexible scheduling method compatible with the traditional ADM and PDM that have been applied for more than 50 years in construction projects.

**Beeline Diagramming Method**

**Basic Concept, Principle, and Characteristics**

This research proposes the Beeline Diagramming Method (BDM) as a new networking technique to overcome the inefficiencies and limitations of the PDM, the LDM, and the chronographic method. The basic concept of the BDM is to represent the overlapping relationship of two consecutive activities by the shortest straight line; this has an arrow to represent the direction of work flow. The BDM connects any point of the predecessor to any point of the successor. This research defines the shortest straight line, which indicates a very direct or quick path or trip, as the “beeline” (Wiktionary 2009). Figure 5 shows the basic concept of the BDM; a beeline connects the middle point of the preceding activity A to the middle point of the succeeding activity B. The BDM has only one principle: The BDM represents the single or multiple overlapping relationships of two consecutive activities in the network by a beeline or beelines in any circumstance. Building on the basic concept and principle of the BDM, its characteristics are as follows.

![Figure 5. Basic Concept of Beeline Diagramming Method.](image)

First, the BDM simplifies the PDM’s overlapping relationships into one beeline. Thus, the complicated process of the PDM, which includes the positioning of activities, the selection of linkage types, and the calculation of the lead-time for the selected linkage, is eliminated.
Second, the BDM permits multiple overlapping relationships by means of multiple beelines between two consecutive activities. It therefore overcomes the limitations of the compound relationships found in the PDM, which has only two overlapping linkages.

**Linkage Representation Types in the BDM**

Linkage relationships between two consecutive activities in the BDM are represented differently from those in the PDM. Linkage relationships in the BDM can be represented at any middle point between two consecutive activities; the PDM, in contrast, represents linkage relationships only by FS, SS, FF, and SF relationships with lead-time between the starting and finishing points.

This research proposes three types of linkage representations in the BDM. The first is the “N-N” type shown in Figure 6. This type represents two consecutive activities that are mutually connected at any point in days after their respective starts.

![Figure 6. Representation Type by the Elapsed Days.](image)

The initial “N” in Figure 6 refers to the days that have elapsed from the start date of the preceding activity; the latter “N” refers to the days that have elapsed from the start date of the succeeding activity; the “-” is the separation indicator between the two Ns. An example of the first representation type is illustrated in Figure 7. In this figure, two consecutive activities in the BDM are connected by a “7-4” type—between a point of 7 days after the start date of the preceding activity A and a point of 4 days after the start date of the succeeding activity B.

![Figure 7. An Example by the Elapsed Days.](image)

The second type is “<N>“, shown in Figure 8, wherein the successor starts some days after the completion of the predecessor.

![Figure 8. Linkage Representation by Second Type.](image)

The “N” in Figure 8 refers to the lead-time to be passed after the completion of the preceding activity. The initial “<” and latter “>” indicate the lead-time space indicators. An example of the second representation type is illustrated in Figure 9.
Two consecutive activities in the BDM are connected by “<4>“, wherein the succeeding activity B starts after the preceding activity A has been completed for 4 days.

The third type represents the multiple linkage relationships between two consecutive activities by the elapsed days or the second linkage type. Schedule computations will continue to be performed independently for each individual linkage. Figure 10 shows an example of the multiple beeline relationships between two activities that have multiple milestones.

**Schedule Computation of the BDM**

**Forward Pass Computation**

Forward pass computation determines both the early start date (ESD) and the early finish date (EFD) for the activities in the BDM network.

Figure 11 illustrates the multiple versus single relationship of the BDM wherein activities I₁, I₂, and I₃ are merged into activity J. Activities I₁ and J have “d₁₁-d₁₁” of the BDM relationship, activities I₂ and J have “d₁₂-d₁₂”, and activities I₃ and J have “d₁₃-d₁₃”. In the multiple versus single BDM relationship, the ESDⱼ of the succeeding activity J is determined by the maximum early start date among the BDM relationships of activities I₁, I₂, I₃, and J. Equation (1) expresses a formula to determine the ESDⱼ of the succeeding activity J through the forward pass computation in the multiple versus single BDM relationship.
\[ ESD_j = \max_{vl} ESD_l + d_l - d_j \]  
(1)

\[ EFD_j = ESD_j + D_j \]  
(2)

The symbol \( \max_{vl} \) in equation (1) means that the maximization is to be over all the beelines \( lJ \) that are merged into activity \( J \). This research verifies equation (1) through the simple example of the multiple versus single BDM relationship. Figure 12 shows the multiple versus single relationship of the BDM wherein activities A, B, and C are merged into activity D. Thus activities A and D have a “7-3” BDM relationship, activities B and D have a “7-1” BDM relationship, and activities C and D have a “8-6” BDM relationship.

The ESD of the succeeding activity D in the BDM relationships with the preceding activities A, B, and C is calculated by applying equation (1) as follows: the first ESD from the “7-3” relationship with activity A is calculated as \( ESD_D = 10 + 7 - 3 = 14 \); the second ESD from the “7-1” relationship with activity B is determined as \( ESD_D = 5 + 7 - 1 = 11 \); and the third ESD from the “8-6” relationship with activity C is computed as \( ESD_D = 13 + 8 - 6 = 15 \). The maximum value of “15” then is selected as the ESD of the succeeding activity D and the EFD is calculated as \( EFD_D = 15 + 12 = 27 \) by applying equation (2).

From the above, the forward pass computation of the BDM relationship proposed in this research is proved to be simple, obvious, and reasonable.

**Backward Pass Computation**

Backward pass computation determines the late start date (LSD) and the late finish date (LFD) of the activities in the BDM network. Backward pass computations in the CPM network calculate the LFD of the preceding activity first, and then determine the LSD by subtracting the duration of the preceding activity from the LFD. Due to the characteristics of the BDM network, the LSD of an activity is calculated first and the LFD is computed by adding its duration to the LSD.
Figure 13 illustrates the single versus multiple relationship of the BDM wherein activity I bursts into activities J1, J2, and J3. Activities I and J1 have “d_{I1}-d_{J1}” of the BDM relationship, activities I and J2 have “d_{I2}-d_{J2}” of the BDM relationship, and activities I and J3 have “d_{I3}-d_{J3}” of the BDM relationship. In the single versus multiple BDM relationship, the LSDI of the preceding activity I is determined by the minimum LSD among the BDM relationships of activities I, J1, J2, and J3. Equation (3) expresses a formula to determine the LSDI of the preceding activity I through the backward pass computation in the single versus multiple BDM relationship.

\[
\text{LSD}_I = \min_{\forall J} \text{LSD}_J + d_J - d_I
\]

Equation (3)

\[
\text{LFD}_I = \text{LSD}_I + D_I
\]

The symbol \( \min_{\forall J} \) in equation (3) means that the minimization is to be over all the beelines IJ that burst from activity I. This research verifies equation (3) through the simple example of the single versus multiple BDM relationship. Figure 14 shows the single versus multiple relationship of the BDM wherein activity A bursts into activities B, C, and D. Activities A and B have a “5-2” BDM relationship, activities A and C have a “10-2” BDM relationship, and activities A and D have a “13-3” BDM relationship.
The LSD_A of the preceding activity A in the BDM relationships with the succeeding activities B, C, and D is calculated by applying equation (3) as follows: the first LSD_A of the preceding activity A from the “5-2” relationship with activity B is calculated as LSD_A = 33+2−5 = 30; the second LSD_A from the “10-2” relationship with activity C is determined as LSD_A = 34+2−10 = 26; and the third LSD_A from the “13-3” relationship with activity D is computed as LSD_A = 38+3−13 = 28. The minimum value of “26” then is selected as the LSD_A of the preceding activity A and the LFD_A is calculated as LFD_A = 26+15 = 41 by applying equation (4).

From the above, the backward pass computation of the BDM relationship proposed in this research is verified as simple, obvious, and reasonable, as was the forward pass computation.

**Computation of Free Float in the BDM**

The free float (FF) is defined as the time span within which the completion of an activity may occur without delaying either the completion of the project or the start of any following activity (Harris 1978). During the forward pass computation, a difference between the early start date of an activity and the early finish date of the preceding activity may occur; this is called a link lag (Harris 1978). The link lag (LAG_IJ) between the preceding activity I and the succeeding activity J in the PDM is defined as equation (5).

\[ \text{LAG}_{IJ} = \text{ESD}_J - \text{EFD}_I \]  

(5)

A link lag (LAG_IJ) in the BDM can be defined as a difference between the connecting points of two successive activities; thus, it is stated as equation (6).

\[ \text{LAG}_{IJ} = (\text{ESD}_J + d_J) - (\text{ESD}_I + d_I) \]  

(6)

When a link lag occurs in the BDM network, a beeline is modified into an offset-screwdriver shape with a horizontal line that matches the extent of the link lag’s duration, as shown in Figure 15. This unique representational method of a link lag in the BDM allows the time span between the early finish of the predecessor and the early start of the successor to be visually recognizable, something that is impossible in the PDM.

\[ \text{FF}_I = \min_{\forall j} \text{LAG}_{IJ} = \min_{\forall j} (\text{ESD}_J + d_J) - (\text{ESD}_I + d_I) \]  

(7)

The symbol \( \min_{\forall j} \) in equation (7) means that the minimization is to be over all the beelines IJ that begin with activity I. Figure 16 shows an example for computing the free float of activities in the BDM by applying equations (6) and (7).
In Figure 16, activities A and C have a beeline relationship of “4-2”, activities A and D have a beeline relationship of “6-2”, and activities B and D have a beeline relationship of “7-3”. If the early start dates of activities A, B, and C already have been derived, then, through the application of equation (6), the $\text{LAG}_{AC}$, a link lag between activities A and C, is calculated as $\text{LAG}_{AC} = (15 + 2) - (10 + 4) = 3$, the $\text{LAG}_{AD}$, a link lag between activities A and D, is found as $\text{LAG}_{AD} = (16 + 2) - (10 + 6) = 2$, and the $\text{LAG}_{BD}$, a link lag between activities B and D, is computed as $\text{LAG}_{BD} = (16 + 3) - (12 + 7) = 0$. Through the application of equation (7), the $\text{FF}_A$, the free float of activity A, is derived as $\text{FF}_A = \text{Min} (\text{LAG}_{AC}, \text{LAG}_{AD}) = \text{Min} (3, 2) = 2$ and the $\text{FF}_B$, the free float of activity B, is derived as $\text{FF}_B = \text{Min} (\text{LAG}_{BD}) = \text{Min} (0) = 0$.

The basic concept for deriving the free float of an activity in the BDM is almost identical with the concept used in the PDM. In the BDM network, however, a free float is calculated based on the beeline connecting points between two consecutive activities.

**Computation of Total Float in the BDM**

The total float (TF) is defined as the time span in which the completion of an activity may occur and not delay the termination of the project (Harris, 1978), and it is the maximum float that an activity could possess. The total float of an activity can be derived from a difference between the forward and backward pass computations. Therefore, $\text{TF}_I$ of activity $I$ can be computed by a difference between $\text{LSD}_I$ and $\text{ESD}_I$, or $\text{LFD}_I$ and $\text{EFD}_I$, as expressed on the equation (8).

$$\text{TF}_I = \text{LSD}_I - \text{ESD}_I = \text{LFD}_I - \text{EFD}_I$$  \[8\]

The concept for deriving the total float of an activity in the BDM is identical with the PDM because the BDM performs the forward and backward pass computations as the PDM does.

**Verification of the BDM**

This section verifies the basic concept, principle, and schedule computation methods of the BDM proposed in this research; further, it determines whether or not they are reasonable when they are applied to the complete BDM network for construction projects. The complete BDM network was constructed with 15 activities with various BDM relationships and the schedule computations were performed. The complete BDM network and its schedule computation results, with a critical path of A-C-G-H-L-O, are illustrated in Figure 17.
The results of the complete schedule computations performed in the BDM network confirm that the basic concept and principle of the BDM have been applied reasonably. The BDM thus has all the key elements to evolve into a new networking technique that could replace the existing ADM and PDM.

**Practical Example of the BDM**

Figure 18 shows a practical example that the schedule of interior works in apartment unit of high-rise residential building project is represented by the BDM network. Interior works begin with plastering work after building structure was completed, and will be completed with cleaning and final Inspection. The most distinctive characteristic of the BDM network in Figure 18 is to express multiple relationships on intermediate milestones between door & window work and glass work, furniture work and flooring work, which is impossible in the PDM. Further, it exactly represents the overlapping relationships between door & window work and ceiling & wall papering work, painting work and ceiling & wall papering work, ceiling & wall papering work and furniture work, furniture work and clean & inspection, on the time-scaled format. This unique feature that the BDM can represent all kinds of relationship between consecutive activities on the time-scaled format confirms that it could overcome the limitations and inefficiencies of the existing ADM and PDM.
Figure 18. Practical Example of BDM Network.

Figure 19 shows a sample chart screen of the BDM network of practical example which is realized by the Beeliner (New Scheduling Software based on the BDM Technique).

Conclusions

More than five decades have passed since the CPM was introduced in 1956. As the CPM has developed from the ADM to the PDM, it has become the most critical tool for schedule management in the construction industry. The PDM has the advantage of expressing overlapping relationships between two consecutive activities; thus it has been adopted as the basic scheduling technique in the most popular project management systems. However, the PDM cannot depict
overlapping relationships when they are interrelated at a middle point of an activity’s duration. The PDM only represents overlapping relationships within four combinations that connect the starting and finishing points of two consecutive activities. Further, when two consecutive activities have respective multiple milestones that should be connected independently, the PDM cannot represent their multiple overlapping relationships precisely.

Construction projects are getting bigger and more complex. They need more flexible and innovative scheduling techniques that can be applied to all kinds of project management environments. This research therefore proposes the Beeline Diagramming Method (BDM) as a new networking technique that can represent all kinds of overlapping relationships between activities. This paper defines the basic concept, principle, interpretation methods, and schedule computation methods that the BDM requires to be an effective scheduling technique. To verify the BDM’s adaptability and validity, the techniques proposed in this study have been applied to the complete BDM network. The verification results confirm that the BDM has all the key elements to evolve into a new networking technique. The BDM thus could overcome the limitations and inefficiencies of the existing ADM and PDM. Project teams using the BDM can establish project schedule plans more realistically and efficiently, since the BDM represents all kinds of relationships between activities with improved flexibility.

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