Effect of flexible tube wall on fluid flow based on a modified immersed boundary-lattice Boltzmann method

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Abstract. In this work, a modified immersed boundary-lattice Boltzmann method is introduced to investigate the peristaltic flow in an axial section of a tube. In this scheme, the lattice Boltzmann method is employed to describe the fluid flow, and the modified immersed boundary method is employed to describe the coupling between the moving boundary and the surrounding fluid. We analyze the flow field and investigate the effects of the crucial factors, including the amplitude ratio and the frequency of the traveling wave, kinematic viscosity of the fluid and wavenumber, have on the flow rate of the peristaltic tube. The comparison between the numerical and the published results shows that the method is reasonable and effective.

1. Introduction
Peristaltic flow is generated by the propagation of waves along a flexible wall in a tube or channel. Early studies of peristaltic flows were developed according to physiological requirements [1, 2]. To investigate the transport mechanism of peristaltic motion, a number of studies have been undertaken [3, 4]. However, these analytical works were carried out under the constraints of small Reynolds numbers, long wavelengths and/or small amplitude approximations. These results may not be applicable to situations where inertia cannot be ignored and amplitude is not small relative to the radius. Some traditional numerical methods have been adopted to study peristaltic flows [5, 6], however these conventional numerical methods have some limitations in dealing with the moving boundary, such as difficulty reconstructing the mesh and time-consuming computations. The novel immersed boundary method (IBM) [7] considers a structure immersed in fluid as a kind of force source in the Navier-Stokes equations rather than a real body, which avoids the problem of generating a body-adaptive grid. The IBM has a high computational efficiency when dealing with interactions between the fluid and the complex structures immersed in the fluid, and it has been applied in various application aspects, including studying the deformation of red blood cells [8], aggregation of platelets [9], movement of aquatic organisms in fluid [10], and simulating a deformable fiber in a flow [11]. In the lattice Boltzmann method (LBM) [12], the flow field is represented by a fixed, non-adaptive grid system. This numerical method has proven to be an efficient computational tool, as it avoids the grid reconstruction. The applications of LBM have achieved great success in multiphase fluid flows [13], suspended particles fluid [14], non-Newtonian fluid mechanics [15], micro-fluidics [16], and flows in complex geometries [17]. Fauci et al. [18] and Chrispell and Fauci [19] performed numerical simulations on the transport of macroscopic particles using peristaltic flow in a channel adopting IBM,
and Connington et al. [20] studied the same subject with LBM. However, little work has been done to study peristaltic flows using a combination of IBM and LBM (IB-LBM).

In this study, a modified immersed boundary-lattice Boltzmann method (MIB-LBM) [21] is adopted to investigate the peristaltic flow. Using this method, the fluid dynamics are solved numerically by LBM over the entire fluid domain, and the interaction between the moving boundary and the fluid is implemented using the modified immersed boundary method (MIBM), where the moving boundary is treated as a velocity source, which is different from the traditional immersed boundary method, where the moving boundary is regarded as a force source. Compared to the traditional IB-LBM [8, 11] and other numerical methods, the MIB-LBM has many advantages, including good computational efficiency, easy implementation for fluid-structure interaction and parallel computing capabilities. Moreover, the re-meshing of the fluid domain can be avoided with the magnitudes of wave-amplitude and wavelengths unlimited.

2. Theory and method

2.1. The lattice Boltzmann method

LBM models the fluid as a collection of a limited number of fictive particles with discrete velocities, which propagate and collide over a discrete lattice mesh. The fluid flow can be described by the flowing lattice Boltzmann BGK equation (LBE) [12]:

\[
f_i(x + e_i \Delta t, t + \Delta t) - f_i(x, t) = -\frac{1}{\tau} [f_i(x, t) - f_i^{eq}(\rho, u)]
\]

where the \( f_i(x, t) \) is the density distribution function along the \( i \)-th direction with the velocities \( e_i \), at position \( x \) and time \( t \); \( \tau \) is the relaxation time, its relationship with the kinematic viscosity of the fluid is as follows: \( \nu = (\tau - 0.5)/3 \); and \( f_i^{eq} \) is the equilibrium distribution given by

\[
f_i^{eq}(\rho, u) = \omega_i \rho [1 + 3e_i \cdot u + \frac{9}{2}(e_i \cdot u)^2 - \frac{3}{2}u^2]
\]

where the values of the weighting coefficients are \( \omega_i = 4/9 \) for \( i=0 \), \( \omega_i = 1/9 \) for \( i=1,2,3,4 \) and \( \omega_i = 1/36 \) for \( i=5,6,7,8 \). The discrete lattice velocities in the above equation are defined as: \( e_0 = (0,0) \), \( i=0; e_i = (\cos[(i-1)\pi/2], \sin[(i-1)\pi/2]) \), \( i=1,2,3,4; e_i = (\cos[(2i-1)\pi/4], \sin[(2i-1)\pi/4]) \), \( i=5,6,7,8 \).

The fluid density and the velocity are calculated using:

\[
\rho = \sum_{i=0}^{s} f_i, \quad u = \frac{1}{\rho} \sum_{i=0}^{s} e_i f_i.
\]

2.2. The modified immersed boundary method

The IBM is suitable for simulating boundary motion at a specified speed, and for solving the motion of the flexible boundary coupled with fluid [7].

The MIBM adopted in this work directly introduce the immersed boundary velocity into LBE as a velocity source, which means the velocity at a boundary marker \( X \) induced by boundary motion is imposed on the nearby fluid grid points \( x \) via a Dirac delta function by

\[
\Delta u(x,t) = \int_0^1 U(X,t) \delta(x - X) dX
\]

\[
u(x,t + \Delta t) = u(x,t) + \Delta u(x,t)
\]
Where $U(X,t) = \partial X/\partial t$ is the specified boundary velocity, $u(x,t) = (u_x, u_y)$ is the fluid velocity, $\delta(x)$ is a two-dimensional Dirac delta function, its form is given as

$$\delta(x) = \frac{1}{h^2} \varphi(\frac{x}{h})\varphi(\frac{y}{h})$$

(6)

here $x=(x,y)$ are the Eulerian coordinates, $h$ the grid size. The function $\varphi$ is of the form

$$\varphi(r) = \begin{cases} 
\frac{1}{4} (1 + \cos(\frac{\pi r}{2})), & |r| \leq 2 \\
0, & |r| > 2 
\end{cases}$$

(7)

Equations (4) and (5) state that the fluid velocity $u$ is obtained from the velocities of boundary nodes, which describe the coupling between the moving boundary and the surrounding fluid. The method simplifies the calculation procedure.

3. Formulation of the problem

In this work, a numerical study on the peristaltic flow in an axial section of a tube based on the MIB-LBM is performed. As shown in figure 1, only the traveling wave on one side of the tube wall is considered because of the symmetry of the tube. The tube wall is treated as an immersed boundary between the air and the fluid, the tube is filled with viscous incompressible newtonian fluid in its initial state. During the computations, a $1000 \times 200$ lattice is employed to discrete the computing domain $(L \times H)$. A periodic boundary condition is adopted for the inlet and outlet of the tube, and a symmetrical boundary condition is applied for the symmetry axis $y=0$.

The deformed tube wall is uniformly discretized into a series of Lagrangian points moving with prescribed velocities, horizontal coordinate $X'$ and vertical coordinate $Y'$, the traveling wave propagates on the tube wall in the positive x-direction with velocity $c'$, the movement of the deformable wall is set to [22]

$$Y'(X', t') = R' + B_0' \cos \frac{2\pi}{\lambda'} (X' - c't')$$

(8)

where $R' = H/2$ is the radius of the tube, $B_0'$ is the wave amplitude, $\lambda'$ is the wavelength, the frequency of the traveling wave is $f' = c'/\lambda'$. The corresponding dimensionless form of these parameters are respectively denoted by $R$, $B_0$, $\lambda$ and $f$.

The dimensionless transformation of the variables is carried out according to
\[ Y = \frac{Y'}{R'}, \quad X = \frac{X'}{\lambda'}, \quad t = f't' \]  

Similarly, we introduce several other dimensionless parameters,

\[ \text{Re} = \frac{R'c'}{\nu}, \quad \alpha = \frac{R'}{\lambda'}, \quad \phi = \frac{B_y'}{R'} \]  

where, Re is the Reynolds number, \( \nu \) is the kinematic viscosity of fluid, \( \alpha \) is the normalized wavenumber, and \( \phi \) is amplitude ratio.

Instantaneous volume flow rate at the outlet is

\[ Q(t) = \int_0^Y u_x(L, y, t)\,dy \]

where \( u_x(L, y, t) \) is the component of the velocity in the \( x \)-direction at position \((L, y)\) and time \( t \). The time-mean flow rate is

\[ Q_{\text{avg}} = \frac{1}{t - t_0} \int_{t-\epsilon}^{t_0} Q(t)\,dt. \]

The corresponding dimensionless form are given by,

\[ \Theta = \frac{Q}{R'c'}, \quad \Theta_{\text{avg}} = \frac{Q_{\text{avg}}}{R'c'} \]

4. Results and discussion

4.1. Analysis of the flow field and the instantaneous volume flow rate

In this study the motion results from the moving boundary after five wave cycles are analyzed. Note that the channel is initialized at rest in a straight configuration, the amplitude of the wave increases from zero to its maximum amplitude after a period of time.

Figure 2 shows several snapshots of the velocity and pressure distribution in the tube. In physical units, we set \( R' = 0.1 \) cm, and \( c' = 0.25 \) cm/s, while the kinematic viscosity is adjusted to yield the desired Re after \( \alpha \) is specified. In corresponding lattice units \( R = 100 \) and \( \tau = 1 \). For this simulation \( \alpha = 0.2, \phi = 0.5 \) and \( \text{Re} = 1 \). The colors represent the pressure, and the arrows represent the velocity distribution. As seen in figure2 (a)-(e), the fluid in expanding parts flows along the same direction as the wave, while the fluid in contracted parts flows in the opposite direction of the wave, which is the same as described in the literature [3]. The reason for this is that higher pressure takes place on the side of the boundary that pushes the fluid than the side which pulls the fluid. In other words, a particular point on the boundary is exposed to a higher pressure when the slope of the boundary at that point is positive and a lower pressure when the slope is negative. The slope of the boundary changes with the wave propagation, and the high and low pressure regions are alternately distributed in the direction of wave propagation, forming the differential pressure to drive the fluid flow.
Figure 2. Snapshots of the velocity field at different time for $\phi=0.5$, $\alpha=0.2$, $f=0.6$ and $Re=1$.

Figure 3 shows the instantaneous volume flow rate at the outlet of the channel over time. It is shown that as the amplitude of the wave increases to its maximum, the flow curve becomes steady in a periodic sense, with a net flow in the direction the wave is traveling.

4.2. Analysis of the parameters

Figure 4 shows the relationships between the time-mean flow rate and amplitude ratio $\phi$, the frequency $f$, the fluid viscosity $\nu$ and the wavenumber $\alpha$. For a particular group of simulations, the bulk of the parameters are the same as in figure 2, with the parameter of interest being varied.

Jaffrin and Shapiro [3] derived many results for the case where both $Re$ and $\alpha$ were negligible. If there is no imposed pressure gradient, $\Theta_{avg}$ can be given as a function of amplitude ratio,

$$\Theta_{avg} = \frac{3\phi^2}{2 + \phi^2}$$  \hspace{1cm} (14)
Figure 4. Relationship between the time-mean flow rate and (a) amplitude ratio; (b) frequency of the traveling wave; (c) kinematic viscosity of the fluid; (d) wavenumber.

Figure 4(a) shows the effects of the amplitude ratio on the flow. Using $\phi$ as the independent variable by changing $B_0$. Compared to the theoretical values derived by Jaffrin and Shapiro [3], the results are in good agreement with the analytical solution, conforming the effectiveness of our method.

Figure 4(b) shows that the time-mean flow increases linearly with the frequency of the traveling wave. The reason for this is that as the frequency of the traveling wave increases, the momentum transmitted to the surrounding fluid from the moving boundary increases. Accordingly, the flow rate at the outlet increases. Figure 4(c) shows that the time-mean flow increases nonlinearly as the kinematic viscosity increases, and the rate of growth decreases gradually, when $\nu>50*10^{-6}\text{m}^2/\text{s}$, the flow is almost unchanged. Figure 4(d) shows the effect of the wavenumber $\alpha$ on the time-mean flow rate. The time-mean flow rate decreases nonlinearly with $\alpha$, and the rate of decrease slows as $\alpha$ increases.

5. Conclusions
In conclusion, the MIB-LBM is adopted to investigate the peristaltic flow in an axial section of a tube due to the deformation of the tube wall. In this method, the velocity of the moving boundary is directly distributed to the grid points of the surrounding fluid by the Dirac delta function, and the fluid flow is solved through the lattice Boltzmann equation. Compared with the traditional IB-LBM and other numerical methods, the MIB-LBM avoids the enormous and complicated calculations of the force generated by the boundary deformation, and it is easy to carry out with high efficiency. Correspondingly, numerical simulation is performed to find out the influence of the traveling wave along the tube wall on the contained fluid. The velocity and pressure distribution and the flow rate curves are used to analyze the flow field in the tube. Then we probe the effects of the relevant parameters, such as the amplitude ratio and the frequency of the traveling wave, the viscosity of the fluid and the wavenumber, on the flow rate. It turns out that the average flow rate has a nonlinear increase as the amplitude ratio and the kinematic viscosity increase, and it decreases nonlinearly with the increase of the normalized wavenumber and increases linearly with the increase of the frequency. The results from the MIB-LBM are in good agreement with the results from published literatures. This work demonstrates that the MIB-LBM provides an efficient means for the study of the peristaltic flow.
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