Scalar mesons nonet in a scaled effective Lagrangian

M. Jaminon and B. Van den Bossche

Université de Liège, Institut de physique B5, Sart Tilman, B-4000 Liège 1, Belgium

Abstract

A scaled SU(3) Nambu - Jona-Lasinio Lagrangian is used to compute the mass of the nine scalar mesons in the vacuum and the mass of the gluball. It is shown that a suitable choice of the vacuum gluon condensate allows to reproduce the experimental masses of the scalar mesons except for the $K^*_0(1430)$. This choice corresponds to a weak coupling between the gluon and quark condensates, giving a $f_0(1500)$ or a $f_J(1710)$ which is nearly a pure glueball.

Key words: NJL, Scalar mesons, Glueball
PACS: : 12.39.Fe, 12.39.Mk, 14.40.-n

1 Introduction

Solving the scalar mesons puzzle is one of the most important problems in meson spectroscopy. The identification of the scalar nonet is all but clear and literature is full of contradictory statements. The problem is still complicated by the presence of glueballs. Following Montanet [1], an attractive scenario would be to include the $a_0(1450)$, $K^*_0(1430)$, $f_0(1300)$ and $f_0(1525)$ or $f_0(1590)$ in the nonet. This choice would leave out the $a_0(980)$ and the $f_0(980)$ which could be identified with $K\bar{K}$ molecule states. The glueball candidate could be speculated to be the $f_0(1500)$. Palano [2] offers another scenario according to which the nonet would be the $a_0(980)$, $K^*_0(1430)$,$f_0(980)$ and the $f_0(1400)$. The scalar glueball could be identified with the $f_0(1520)$. Surprisingly, its low mass meson $f_0(980)$ seems to be a $s\bar{s}$ state while the high mass one $f_0(1400)$ would be a $u\bar{u} + d\bar{d}$ state. In the same idea, Törnqvist [3] presents the two mesons $f_0(980)$ and $f_0(1300)$ as two manifestations of the same $s\bar{s}$ state. The chiral partner of the pion (the $u\bar{u} + d\bar{d}$ state) should be identified with a very broad resonance ($\Gamma = 880$ MeV) centered at 860 MeV. According to Lindenbaum and Longacre [4], the $0^{++}$ data can be reproduced with the four
mesons $f_0(980), f_0(1300), f_0(1400)$ and $f_0(1710)$. They then suppose that the $	heta(1710)$ (here $f_0(1710)$) has spin 0 and not spin 2 that is far but evident [2,5,6]. These examples show that the situation is quite confused from an experimental point of view.

Things are not much better from the theoretical one. Klemp et al. [7] calculate the masses and some of the decay properties of the scalar nonet, using a relativistic quark model with linear confinement and an instanton-induced interaction. Their results suggest to identify the nonet with $a_0(1450), K^*_0(1430), f_0(980)$ and $f_0(1500)$, the latter being a $s\bar{s}$ state. No glueball is included in their calculations. Lattice QCD calculations provide an idea of the value of the glueball mass. However, there is still some dissent on this point. Initially, the predictions for the mass of the lightest glueball seemed to favour its identification with the $f_0(1500)$ [8,9]. Recently, the valence (quenched) approximation developed by Sexton et al. [10] has predicted the glueball as being the $f_J(1710)$. Moreover, this $f_J(1710)$ should be nearly a pure glueball, the mixing with $q\bar{q}$ states increasing its mass by around 60 MeV. The fact that the pure glueball state disperses over three resonances is also confirmed by the recent analysis of Anisovitch et al. [11]. However, this analysis seems to indicate that the mixing with $q\bar{q}$ decreases the mass of the glueball.

The scaled Nambu - Jona-Lasinio model introduced in Ref. [12] in the SU(2) case and its extended version to the SU(3) case [13] allows for such a mixing between the quark and the glue sectors. In the present paper, we compute the masses of the scalar nonet and of the glueball. Our model contains two free parameters. One of them, the constituent up quark mass, is chosen to reproduce the $a_0(980)$. We show that for a relatively large value of the vacuum gluon condensate ($\chi_0 = 350$ MeV), the scalar nonet assumed to be $a_0(980), K_0^*(1430), f_0(980)$ and $f_0(1370)$ [14] can be well reproduced (except the $K^*_0(1430)$). This large value of $\chi_0$ amounts to a weak coupling between the quark and the gluon condensates. The quark contents of the glueball increases its mass by less than 20 MeV.

The paper is organized as follow. Sec. 2 recalls the useful tools of the scaled NJL model. Sec. 3 specifies our prescription used to avoid the threshold problem when the meson masses lie above the quark-antiquark pair creation threshold. Our results concerning the masses of the mesons and of the glueball are given in Sec. 4. Finally, Sec. 5 draws our conclusions.

2 The model

The model used in this paper is extensively described in Ref. [13,15] under the name of "A-scaling model". We just recall some of the useful tools for the
understanding of the present work. We start from the vacuum SU(3) effective Euclidean action:

\[ I_{\text{eff}}(\varphi, \chi) = -\text{Tr}_{\Lambda}(\gamma_{\mu}i\partial_{\mu} + m + \Gamma_{a}\varphi_{a}) + \int d^{4}x \frac{g^{2}}{2}x^{2}\varphi_{a}\varphi_{a} + \int d^{4}x L_{\chi} \] (1)

which is written in its bosonized version for which the quark degrees of freedom have been integrated out. The meson fields write:

\[ \varphi_{a} = (\sigma_{a}, \pi_{a}), \quad \Gamma_{a} = (\lambda_{a}, i\gamma_{5}\lambda_{a}), \quad a = 0, \ldots, 8 \] (2)

where the \( \lambda_{a} \) are the usual Gell-mann matrices with \( \lambda_{0} = \frac{\sqrt{3}}{3} \). We choose to work in the isospin symmetry limit and the quantity \( m \) stands for the diagonal matrix \( \text{diag}(m_{u}, m_{u}, m_{s}) \). The trace anomaly of QCD is modelized using a scalar dilaton field \( \chi \) that is intimately related to the gluon condensate \( \chi \propto \langle G_{\mu\nu}^{2} \rangle^{1/2} \) [16]:

\[ L_{\chi} = \frac{1}{2}(\partial_{\mu}\chi)^{2} + \frac{1}{16}b^{2}(\chi^{4}\ln\frac{\chi^{4}}{\chi_{G}^{4}}) - (\chi^{4} - \chi_{G}^{4}). \] (3)

Since we are only interested in the scalar sector, the axial anomaly which would give the \( \eta-\eta' \) mass difference is not considered here. The model contains six parameters: the current quark masses \( (m_{u}, m_{s}) \), the strengths \( (a^{2}, b^{2}) \), the gluon parameter \( \chi_{G} \) and the cut-off \( \Lambda \) introduced to regularised the diverging quark loop. Four of these parameters are adjusted to reproduce the pion mass \( (m_{\pi}) \), the weak pion decay constant \( (f_{\pi}) \), the kaon mass \( (m_{K}) \) and the mass of the glueball \( (m_{GL}) \) which is sometimes identified with the \( f_{0}(1500) \) and othertimes with the \( f_{J}(1710) \). We will perform calculations with these two different values. We are then left with two free parameters that we choose to be the constituent up quark mass \( M_{u}^{0} \) (related to \( a^{2} \)) and the vacuum gluon condensate \( \chi_{0} \) (related to \( \chi_{G} \)). These free vacuum parameters as well as the vacuum constituent strange quark mass \( M_{s}^{0} \) correspond to the stationary point of the effective action. They then satisfy three coupled equations that can be found in Ref. [13]. We do not repeat them here.

The mass of the various mesons are obtained by expanding the effective action up to second order in the fluctuating parts of the meson fields \( (\hat{\sigma}_{a}, \hat{\pi}_{a}) \) and of the dilaton field \( \hat{\chi} \):

\[ I^{(2)}(\varphi, \chi) = \frac{1}{2\beta_{0}} \sum_{q} \left( \hat{\phi}_{aq} \Sigma_{ab}^{-1} \hat{\phi}_{b-q} + \hat{\pi}_{aq} \Pi_{ab}^{-1} \hat{\pi}_{b-q} \right), \] (4)

where \( \hat{\phi}_{a} = (\hat{\sigma}_{a}, \hat{\chi}) \). The matrix \( \Pi^{-1} \) is a 9 X 9 matrix while the corresponding scalar matrix \( \Sigma^{-1} \) we are interested in is 10 X 10. It only mixes the three fields
(\tilde{\sigma}_0, \tilde{\sigma}_8) and \tilde{\chi} so that one can write:

\[
\frac{1}{2\beta \Omega} \sum_q \tilde{\phi}_{aq} \Sigma^{-1}_{ab} \tilde{\phi}_{bq} = \frac{1}{2\beta \Omega} \sum_q (\tilde{\sigma}_{0q}, \tilde{\sigma}_{8q}, \tilde{\chi}_q) S^{-1} \begin{pmatrix}
\tilde{\sigma}_{0q} \\
\tilde{\sigma}_{8q} \\
\tilde{\chi}_q
\end{pmatrix}
+ \frac{1}{2\beta \Omega} \sum_q \sum_{i=1}^7 \left( \tilde{\sigma}_{iq} K_{ii}^{-1} \sigma_{i-q} \right).
\] (5)

The inverse propagators \( K_{ii}^{-1} \) take the simple forms:

\[
K_{ii}^{-1} = 4N_c F(M^0_{u}, M^0_{u}) \left( q^2 + 4M^2_u \right) + a^2 \chi^2 \frac{m_u}{M^2_u}, \quad i = 1, 2, 3, \quad (6)
\]

\[
K_{ii}^{-1} = 4N_c F(M^0_{u}, M^0_{s}) \left( q^2 + (M^0_{u} + M^0_{s})^2 \right) + \frac{1}{2} a^2 \chi^2 \left( \frac{m_u}{M^2_u} + \frac{m_s}{M^2_s} \right), \quad i = 4, 5, 6, 7, \quad (7)
\]

where

\[
F(M^0_i, M^0_j) = \int \frac{d^4k}{(k^2 + M^2_i)} \frac{1}{((k - q)^2 + M^2_j)}.
\] (8)

The zeroes of (6) and (7) give the mass of the \( uu \) and the \((u\bar{s} + s\bar{u})\) excitations, assumed to be the \( a_0(980) \) and the \( K^*_0(1430) \), respectively. In order to get the mass of the remaining scalar mesons and of the glueball, we will diagonalize the matrix \( S^{-1} \)

\[
S^{-1} = \begin{pmatrix}
S^{00} & S^{08} & S^{0\chi} \\
S^{80} & S^{88} & S^{8\chi} \\
S^{\chi0} & S^{\chi8} & S^{\chi\chi}
\end{pmatrix}
\] (9)

and will search for the zeroes of its three eigenvalues. The corresponding physical meson fields will be denoted \( \Phi = (\Phi_1, \Phi_2, \Phi_3) \). We shall see that \( \Phi_1 \) is compatible with \( f_0(980) \), while \( \Phi_2 \) reproduces the \( f_0(1300) \) \[1\] \( f_0(1400) \) \[2\], \( f_0(1370) \) \[14\]). In the same way, \( \Phi_3 \) can be associated with the \( f_0(1500) \) \[1,2,9\] or with the \( f_J(1710) \) \[10\].

Before performing the total diagonalization, an intermediate step can help to get some physical insight in the quark contents of the various fields. It amounts
to diagonalize the matrix

\[
\begin{pmatrix}
S^{00} & S^{08} \\
S^{80} & S^{88}
\end{pmatrix}
\]  

(10)

corresponding to the case of a NJL model without glue. One then has:

\[
\frac{1}{2\beta\Omega} \sum_q (\tilde{\sigma}_{0q}, \tilde{\sigma}_{8q}, \tilde{\chi}_q) S^{-1} \begin{pmatrix}
\tilde{\sigma}_{0-q} \\
\tilde{\sigma}_{8-q} \\
\tilde{\chi}_{-q}
\end{pmatrix} = \frac{1}{2\beta\Omega} \sum_q \sum_{i,j=I,II,III} \tilde{\sigma}_{iq}^T (S_R^{-1})_{ij} \tilde{\sigma}_{j-q} 
\]  

(11)

with

\[
S_R^{-1} = \begin{pmatrix}
S^{I,I} & 0 & S^{I,\chi} \\
0 & S^{II,II} & S^{II,\chi} \\
S^{\chi,I} & S^{\chi,II} & S^{\chi,\chi}
\end{pmatrix}
\]  

(12)

where the definition of \(\tilde{\sigma}_I\) and \(\tilde{\sigma}_{II}\) in terms of \(\tilde{\sigma}_0\) and \(\tilde{\sigma}_8\) can be found in Ref. [13] and where \(\tilde{\sigma}_{III} = \tilde{\chi}\). It is helpfull to recall that

\[
S^{I,I} = 4N_c F(M_u^0, M_u^0) \left( q^2 + 4M_u^0 \right) + a^2 \chi^2 \frac{m_u}{M_u^0}
\]  

(13)

and

\[
S^{II,II} = 4N_c F(M_s^0, M_s^0) \left( q^2 + 4M_s^0 \right) + a^2 \chi^2 \frac{m_s}{M_s^0}
\]  

(14)

These expressions show that \(\tilde{\sigma}_I\) and \(\tilde{\sigma}_{II}\) correspond to pure \(u\bar{u}\) and \(s\bar{s}\) excitations respectively. Due to the coupling with the dilaton field the physical fields \(\tilde{\Phi}_I\) and \(\tilde{\Phi}_{II}\) cease to be pure \(u\bar{u}\) and \(s\bar{s}\) excitations and \(\tilde{\Phi}_{III}\) is not a pure glueball anymore. Our model then provides a way of giving some \(q\bar{q}\) contents to the glueball. Depending on the value of the vacuum gluon condensate \(\chi_0\), the mixing between the fields can be large or not and so is this contents in \(q\bar{q}\) of the glueball.
Due to the nonconfining disease of the NJL model, the various scalar mesons lie above the quark-antiquark pair creation threshold whatever is the choice for the free parameters $M_0^u$ and $\chi_0$. Indeed, one can easily see from (6) and (7) that, due to the nonvanishing current quark masses, one always has:

$$m_{a_0} > 2M_0^u$$
$$m_{K_0^*} > M_0^u + M_0^s.$$  \tag{15}

Due to the mixing between the $\tilde{\sigma}_j$, $j = I, II, III$, the associated threshold is $2M_0^u$. Above these thresholds, the poles of the propagators become complex and the mesons acquire some width. For instance, the mass of the $a_0(980)$ and its width should verify:

$$4N_c F((-m_{a_0} + i\epsilon - i\Gamma_{a_0})^2) \left(-m_{a_0} + i\Gamma_{a_0}\right)^2 + 4M_0^{u^2}) + a^2 \chi^2 m_u M_0^u = 0.$$  \tag{16}

where we have dropped the arguments $M_0^u$ in the fonction $F$ (see Eq. (8)), for simplicity. Eq. (16) corresponds to two coupled equations which cannot be decoupled without a suitable and model dependent prescription. In Refs. [17–19], the prescription amounted to replace Eq. (16) by:

$$4N_c F((-m_{a_0} + i\epsilon)^2) \left(-m_{a_0} + i\Gamma_{a_0}\right)^2 + 4M_0^{u^2}) + a^2 \chi^2 m_u M_0^u = 0.$$  \tag{17}

Here, we perform calculations with the prescription which amounts to replace the complex function $F$ by its modulus, yielding a vanishing width and having the merit of simplicity. The mass of the $a_0(980)$ therefore satisfies:

$$4N_c |F(-(m_{a_0} + i\epsilon)^2)\left(-m_{a_0}^2 + 4M_0^{u^2}) + a^2 \chi^2 m_u M_0^u = 0.$$  \tag{18}

Another prescription introduced recently [20] that could have been tempting to follow consists in introducing an infrared cut-off that eliminates all the divergent processes. If there were no coupling between quarks and glueball, this method should have been surely the simplest one. However, due to this mixing, the infrared cut-off has to be taken very large so that one should cut nearly all the momenta of the quark loop!
Table 1
Masses (MeV) of the scalar nonet for \( m_{\Phi_{III}} = m_{f_0} = 1500 \) MeV

| \( m_{\Phi, [f_0(980)]} \) | \( \chi_0 = 350 \) MeV | \( \chi_0 = 200 \) MeV | \( \chi_0 = 125 \) MeV |
|-------------------------|-----------------|-----------------|-----------------|
| \( a_0(980) \)          | 980             | 980             | 980             |
| \( K_0^*(1430) \)       | 1183            | 1183            | 1183            |
| \( m_{\Phi_{II}, [f_0(1370)]} \) | 973 | 949 | 798 |
| \( m_{\Phi_{III}, [f_0(1370)]} \) | 1367 | 1331 | 1229 |
| \( m_{\sigma_I} \)      | 980             | 980             | 980             |
| \( m_{\sigma_{II}} \)   | 1383            | 1383            | 1383            |
| \( m_{\sigma_{III}} \)  | 1482            | 1445            | 1355            |
| \( \Delta_I \)          | 7.2 \( 10^{-3} \) | 3.3 \( 10^{-2} \) | 2.3 \( 10^{-1} \) |
| \( \Delta_{II} \)       | 1.2 \( 10^{-2} \) | 3.9 \( 10^{-2} \) | 1.3 \( 10^{-1} \) |
| \( \Delta_{III} \)      | 1.2 \( 10^{-2} \) | 3.7 \( 10^{-2} \) | 9.7 \( 10^{-2} \) |
| \( \langle \bar{u}u \rangle \) | -208 | -208 | -208 |
| \( \langle \bar{s}s \rangle \) | -207 | -207 | -207 |

4 Results

We choose for the free parameter \( M_u^0 \) the relatively large value \( M_u^0 = 489 \) MeV that allows to reproduce the mass of the \( a_0(980) \). Results for the masses of the scalar nonet are given in tables 1 and 2 for three specific values of the gluon condensate: \( \chi_0 = 350 \) MeV, 200 MeV and 125 MeV. The values of the quark condensates \( \langle \bar{u}u \rangle \) and \( \langle \bar{s}s \rangle \) are also indicated. The "experimental" values of the scalar mesons masses are taken from Ref. [14]. It is tempting to quantify the contents of the physical fields in the \( u\bar{u}, s\bar{s} \), and in the glueball channels. One possible way is just to compare the exact masses with the approximated ones resulting from a vanishing coupling. The latter approximation defines the masses \( m_{\sigma_I}, m_{\sigma_{II}} \) and \( m_{\sigma_{III}} \). Tables 1 and 2 give the values of

\[
\Delta_i = \frac{|m_{\Phi_i} - m_{\sigma_i}|}{m_{\Phi_i}} \quad i = I, II, III.
\]

When \( m_{\Phi_{III}} = m_{f_0} = 1500 \) MeV, the small values of \( \Delta_i \) for \( \chi_0 = 350 \) MeV show that the physical mesons are nearly pure \( u\bar{u}, s\bar{s} \) or glueball excitations while the mixing is large for \( \chi_0 = 125 \) MeV. For \( \chi_0 = 200 \) MeV, one finds that the nonet scalar is rather well reproduced except the mass of the \( K_0^*(1430) \). According to [10], a mass difference of 60 MeV between the \( f_0(1500) \) and the
Table 2
Masses (MeV) of the scalar nonet for $m_{\Phi_{III}} = m_{f_0} = 1710$ MeV

|               | $\chi_0 = 350$ MeV | $\chi_0 = 200$ MeV | $\chi_0 = 125$ MeV |
|---------------|---------------------|---------------------|---------------------|
| $a_0(980)$    | 980                 | 980                 | 980                 |
| $K_0^*(1430)$ | 1183                | 1183                | 1183                |
| $m_{\Phi_I} [f_0(980)]$ | 976               | 964                 | 917                 |
| $m_{\Phi_{III}} [f_0(1370)]$ | 1377               | 1367                | 1343                |
| $m_{\sigma_I}$ | 980                 | 980                 | 980                 |
| $m_{\sigma_{III}}$ | 1383               | 1383                | 1383                |
| $m_{\sigma_{III}}$ | 1706               | 1697                | 1675                |
| $\Delta_I$    | $4.1 \times 10^{-3}$ | $1.7 \times 10^{-2}$ | $6.9 \times 10^{-2}$ |
| $\Delta_{II}$ | $4.4 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $3.0 \times 10^{-2}$ |
| $\Delta_{III}$ | $2.3 \times 10^{-3}$ | $7.6 \times 10^{-3}$ | $2.0 \times 10^{-2}$ |
| $\langle \bar{u}u \rangle_{\frac{1}{2}}$ | -208 | -208 | -208 |
| $\langle \bar{s}s \rangle_{\frac{1}{2}}$ | -207 | -207 | -207 |

pure glueball is also well reproduced. Whatever $\chi_0$, this mass difference is always positive at variance with results of [11]. When $m_{\Phi_{III}} = m_{f_0} = 1710$ MeV, the coupling is weak whatever the value of $\chi_0$. In order to get the mass difference $(m_{\Phi_{III}} - m_{\sigma_{III}}) \approx 60$ MeV, one should consider $\chi_0 \approx 80$ MeV, yielding a much too small value for the mass of the $f_0(980)$ ($m_{f_0(980)} = 573$ MeV).

Another possibility would consist in giving the contents of each physical field $\tilde{\Phi}_j$ in the three components $\tilde{\sigma}_j$. This would amount to calculate the Euler mixing angles of the mesons. However, the definition of these angles assumes that the physical states are orthogonal to each other and consequently that the mixing is energy independent. The latter assumption is not fulfilled here. Exceptional care has then to be stressed when diagonalization is carried out together with the on-mass shell definitions of associated quantities.

After diagonalization, the right-hand side of (11) reduces to:

\[
\frac{1}{2\beta \Omega} \sum_q \sum_i \tilde{\Psi}_{iq} \chi_{ii}(q^2) \tilde{\Psi}_{i-q}
\] (20)
where
\[ \tilde{\Psi}_{iq} = \left( V^{-1}(q^2)\tilde{\sigma}_q \right)_i \quad (21) \]

and where the \( q^2 \) dependent eigenvalues \( \lambda_{ii}(q^2) \) are given by
\[ \lambda_{ii}(q^2) = \left( V^{-1}(q^2)S_R^{-1}V(q^2) \right)_{ii}, \quad (22) \]

the matrix \( V \) being the orthogonal eigenvector matrix. Let us introduce the matrix
\[ G(q^2) = \partial_{q^2} \left[ \text{diag} \left( \lambda_{ii}(q^2) \right) \right] = \left[ \partial_{q^2} \left( V^{-1}(q^2) \right) V(q^2), \text{diag} \left( \lambda_{ii}(q^2) \right) \right] + V^{-1}(q^2) \left( \partial_{q^2}S_R^{-1} \right) V(q^2) \quad (23) \]

whose diagonal elements calculated on shell can be simplified into:
\[ g_{\Phi_i q q}^{-2} = G_{ii}(-m_{\Phi_i}^2) = \left[ V^T(-m_{\Phi_i}^2) \left( \partial_{q^2}S_R^{-1} \right) V(-m_{\Phi_i}^2) \right]_{ii}. \quad (24) \]

\( V(-m_{\Phi_i}^2) \) is the eigenvector matrix whose first column is evaluated at \(-m_{\Phi_i}^2\), the second one at \(-m_{\Phi_{II}}^2\), and its last one at \(-m_{\Phi_{III}}^2\). Had we no meson-meson mixing, Eq. (24) would define the coupling constants of the mesons to the quarks. Note that we lose the orthogonality of the matrix \( V^{-1} \neq V^T \). If the diagonal elements of the matrix \( S_R^{-1} \) had the form
\[ (S_R^{-1})_{ii} = A_i q^2 + B_i \quad (25) \]

with \( A_i \) and \( B_i \) momentum independent as well as its nondiagonal elements, one could show that the expression (20) can be \textit{identically} written:
\[ \frac{1}{2\beta \Omega} \sum_q \sum_i \tilde{\Phi}_{iq}^T \left( q^2 + m_{\Phi_i}^2 \right) \tilde{\Phi}_{q-i} \quad (26) \]

with
\[ \tilde{\Phi}_{iq} = g_{\Phi_i q q}^{-1} \left( V(-m_{\Phi_i}^2) \right)_{iq} \tilde{\sigma}_q \equiv \sum_{j=I,II,III} d_{ij}(-m_{\Phi_i}^2) \left( \partial_{q^2}S_R^{-1} \right)_{jj} \tilde{\sigma}_{qj}. \quad (27) \]

The demonstration of this result is quite tedious. We do not repeat it here. Moreover, the assumptions above are not totally valid. One must be aware of the fact that neither the equality nor the identity of (27) are valid because the...
Table 3
$uu, ss$ and glue contents of the $f_0(980)$, $f_0(1300)$ and $f_0(1500)$

|                | $\chi_0 = 350$ MeV | $\chi_0 = 200$ MeV | $\chi_0 = 125$ MeV |
|----------------|---------------------|---------------------|---------------------|
| $d_{f_0(980)u}$ | 0.995               | 0.981               | 0.924               |
| $d_{f_0(980)s}$ | -0.013              | -0.041              | -0.117              |
| $d_{f_0(980)\chi}$ | 0.098               | 0.188               | 0.664               |
| $d_{f_0(1370)u}$ | 0.046               | 0.136               | 0.337               |
| $d_{f_0(1370)s}$ | 0.939               | 0.842               | 0.698               |
| $d_{f_0(1370)\chi}$ | -0.342              | -0.522              | -0.632              |
| $d_{f_J(1500)u}$ | 0.088               | 0.137               | 0.180               |
| $d_{f_J(1500)s}$ | -0.345              | -0.537              | -0.707              |
| $d_{f_J(1500)\chi}$ | -0.935              | -0.832              | -0.684              |

Table 4
$uu, ss$ and glue contents of the $f_0(980)$, $f_0(1300)$ and $f_J(1710)$

|                | $\chi_0 = 350$ MeV | $\chi_0 = 200$ MeV | $\chi_0 = 125$ MeV |
|----------------|---------------------|---------------------|---------------------|
| $d_{f_0(980)u}$ | 0.998               | 0.993               | 0.979               |
| $d_{f_0(980)s}$ | -0.008              | -0.025              | -0.066              |
| $d_{f_0(980)\chi}$ | 0.063               | 0.113               | 0.192               |
| $d_{f_0(1370)u}$ | 0.016               | 0.048               | 0.123               |
| $d_{f_0(1370)s}$ | 0.993               | 0.978               | 0.945               |
| $d_{f_0(1370)\chi}$ | -0.121              | -0.205              | -0.304              |
| $d_{f_J(1710)u}$ | 0.061               | 0.105               | 0.162               |
| $d_{f_J(1710)s}$ | -0.122              | -0.209              | -0.321              |
| $d_{f_J(1710)\chi}$ | -0.991              | -0.972              | -0.933              |

The quantity $\partial q^2 S_R^{-1}$ is now $q^2$ dependent. However, since the $q^2$ dependence of the coefficients $A_i$ and $B_i$ is small in the investigated region, we still consider that Eq. (27) defines the physical fields of the scalars $f_0(980)$ and $f_0(1370)$ and of the glueball. We choose to evaluate the quantity $\partial q^2 (S_R^{-1})_{jj}$ at the respective zeroes ($-m_{\Phi}^2$) of the diagonal elements of $S_R^{-1}$. With our prescription, the quantity $d_{12}(-m_{\Phi}^2)$, for instance, provides the contents in $ss$ excitations of the $f_0(980)$. We then denote it $d_{f_0(980)s}$ in tables (3) and (4). These tables confirm the results of tables (1) and (2) in the sense that the physical mesons
are nearly pure $u\bar{u}$, $s\bar{s}$ or gluonic excitations for large $\chi_0$ while significant mixing appears for smaller $\chi_0$. Note also that the mixing is larger for the $f_0(1500)$ than for the $f_0(1710)$ due to the fact that larger the mass of the glueball, larger its decoupling. According to us, the numbers given here are however less transparent than the $\Delta_i$ to quantify the glue contents.

5 Conclusion

The SU(3) scaled effective model developed in Ref. [13] provides some glue contents to two mesons of the scalar nonet as well as some $q\bar{q}$ contents to the glueball. Since the model contains two free parameters, one of them (here, the vacuum constituent up quark mass) can always be chosen to reproduce one of the scalars (we chose $a_0(980)$). The other one (the gluon condensate $\chi_0$) can remain free. As briefly reviewed in the introduction, the identification of the scalar nonet as well as the one of the glueball candidate is not clear. Here we have chosen the values of Ref. [14] for the nonet while we have considered two possible candidates for the glueball: the $f_0(1500)$ [8,9] and the $f_J(1710)$ [10]. The best result for the nonet is obtained with a large gluon condensate ($\chi_0 \approx 350 \text{ MeV}$). Note however that the mass of the $K^*_0(1430)$ is always too small. In that case, the $f_0(1500)$ or the $f_J(1710)$ can be said to be pure glueballs, their $q\bar{q}$ contents increasing their mass of 18 MeV and 4 MeV respectively. If one wants to reproduce the contribution of $\approx 60 \text{ MeV}$ of the $q\bar{q}$ excitations to the glueball [10], one has to use $\chi_0 \approx 200 \text{ MeV}$ for $f_0(1500)$ and $\chi_0 \approx 80 \text{ MeV}$ for $f_J(1710)$. In the former case, the nonet is still not badly reproduced but in the latter one, the agreement is completely destroyed. In conclusion, the condition for which our model reproduces the scalar nonet (except the $K^*_0(1430)$) together with the glueball is that only a tiny mixing exists between them.

References

[1] L. Montanet, *Nucl.Phys.* 39B (1995) 281

[2] A. Palano, *Nucl.Phys.* 39B (1995) 287

[3] N.A. Törnqvist, *Proceeding of "Quark Confinement and the Hadron Spectrum II"* (1996) Como

[4] S.J. Lindenbaum and R.S. Longacre, *Phys.Lett. B* 274 (1992) 492

[5] L.P. Chen *et al.*, *Nucl.Phys.* 21B (1991) 80

[6] Z. Zhipeng, *XIV Inter. Symp. on Lepton-Photon Interactions*, Cornell, (1993)
[7] E. Klempt, B.C. Metsch, C.R. Münz and H.R. Petry, *Phys.Lett. B* 361 (1995) 160

[8] C. Amsler *et al.*, *Phys.Lett. B* 355 (1995) 425

[9] C. Amsler and F. Close, *Phys.Lett. B* 353 (1995) 385

[10] J. Sexton, A. Vaccarino and D. Weingarten, *Phys.Rev.Lett.* 75 (1995) 4563

[11] A.V. Anisovich, V.V. Anisovich and A.V. Sarantsev, *hep-ph/9611333* 15 Nov 96

[12] G. Ripka and M. Jaminon, *Ann. Phys.* 218 (1992) 51

[13] M. Jaminon and B. Van den Bossche, *Nucl.Phys. A582* (1995) 517

[14] Particle Data 1996 *Phys. Rev. D54* (1996) 1

[15] B. Van den Bossche, *Ph. D. Thesis* (1996)

[16] H.Gomm, P. Jain, R. Johnson and J. Schechter, *Phys.Rev. D33* (1986) 801

[17] S.P. Klevansky, *Nucl.Phys. A575* (1994) 605

[18] P. Zhuang, J. Hüfner and S.P. Klevansky, *Nucl.Phys. A576* (1994) 525

[19] D. Blaschke, M. Jaminon, Yu.L. Kalinovsky, P. Petrow S. Schmidt and B. Van den Bossche, *Nucl.Phys. A592* (1995) 561

[20] D. Ebert, T. Feldmann and H. Reinhardt, *Phys. Lett. B388* (1996) 154