The PSI np data and their effect on the charged $\pi$NN coupling constant

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Abstract

Differential cross sections of elastic neutron-proton scattering have been measured for the energy range from 200 MeV to 580 MeV. The angular interval for the detection of the recoiling protons ranges up to 48° in the laboratory system. This corresponds to an interval of the scattering angle from 80° to 180° in the centre-of-mass system. For absolute normalization the simultaneously measured np $\rightarrow$ d$\pi^+$ reaction was used above 280 MeV. The charged $\pi$NN coupling constant has been determined to $f_{\pi NN}^2 = 0.076 \pm 0.001$.

1 Introduction

Neutron-proton elastic scattering at backward angles in the medium energy regime has been the subject of several experiments [1–6]. The common feature of these investigations is a steep rise of the differential cross section towards the backward scattering angle of 180°. The slope of the sharp backward peak suggests a connection to the one pion exchange (OPE) amplitude. Several suggestions have been made for a theoretical description of the experiments [7–16]. All of these proposals are able to describe the backward spike, but most of them are purely phenomenological and some of them fail to describe other observables.

Precise backward scattering data offer the opportunity to evaluate the pion–nucleon coupling constant $f_{\pi NN}^2$. The medium energy region is particularly suited for this purpose, because the pion pole is not very far from the physical region, so that one could expect a reliable extrapolation to it. However, the values determined so far by this method are not all in accordance with each other and with the value obtained from pion-nucleon scattering [18].

The results presented in this paper have been obtained from four separate experiments of different angular ranges, labeled I–IV [4, 19–21]. Together they span the interval from about 80° to 180° in the centre-of-mass system. The neutron energies range from 200 MeV to 580 MeV in steps of 20 MeV.

2 Experiment

The experimental set-ups and techniques of experiments I–IV have been quite similar but with differences in detail of the accelerator performance, the beam arrangement and the detection equipment. Here we outline the common features only and refer to a forthcoming paper [19] for details.

The experiments have been performed at the Paul Scherrer Institute (PSI), the former Swiss Institute for Nuclear Research (SIN). The proton beam of the ring cyclotron of 589 MeV energy consists of bunches with a width of less than 1 ns at a rate of 50.63 MHz for experiments I and III and 16.88 MHz for II and IV, corresponding to bunch spacings of 19.75 ns and 59.25 ns, respectively. The beam current during the data taking was 60–100 $\mu$A. The neutrons were produced on a thick target of beryllium (I–III) or carbon (IV). Neutrons escaped through a collimator hole in the beam dump at an angle of 60 mrad with respect to the incident protons. They were shaped by two additional collimators to a beam of about 2 x 2 cm$^2$ at a distance of 61 m from the production target. Charged particle contaminations were eliminated by cleaning magnets behind the collimators. A lead filter was inserted in order to reduce the photon component of the beam which mainly originates from the decay of neutral pions in the production target.

The continuous neutron energy spectrum consists of the 40 MeV wide quasielastic peak at about 540 MeV and a broad distribution at lower energies, which can be ascribed to inelastic processes [22]. The detailed shape of the spectrum depends on the target material and its thickness.

After a flight path of 61 meters, kept at rough vacuum of about 100 Pa, the neutron beam hits a liquid hydrogen target. For the analysis of the scattering products a magnet spectrometer was installed on a turn table. The spectrometer was equipped with drift chambers and two scintillation counters, a thin one (1 mm thick) in front of the magnet, and a thicker one (1 cm thick) behind it. They allow to measure both, the time-of-flight of the particle detected by the spectrometer, and that of the incoming neutron with respect to the rf-signal of the accelerator. The angular acceptance is almost 20°. The average momentum resolution is about 3 % FWHM.

3 Data taking and analysis

The data have been taken at different run periods, each extending over several weeks. The contribution of the target surroundings and spectrometer materials was measured...
with an empty target. It was subtracted after normalization to the neutron intensity of the full target measurement. Corrections have been applied for the event rate dependent dead-time losses.

The long periods of data taking required special attention to the long term variations of the whole system, particularly of the neutron time-of-flight measurement. The stability of the electronics has been checked regularly and close control of the drift chamber gas flow and high voltage supply has been maintained.

The neutron intensity was monitored in three different ways. The integrated primary proton beam intensity was provided from the accelerator control centre. This signal, though convenient, was not sufficiently reliable because of variations of the focusing of the primary beam on the production target. The second monitor consisted of a three stage scintillation counter telescope, which recorded charged particles emerging from a thin polyethylene target placed in the beam at about 44 m from the neutron target. A third monitor was installed behind the magnet spectrometer. It recorded elastic np scattering events from a polyethylene block in coincidence.

These monitors were intended to measure the relative neutron intensity. Above 280 MeV the absolute normalization was performed by the simultaneously recorded np → dπ⁰ reaction as reference cross section. This is discussed in sect. 3.2.

3.1 Time-of-flight calibration

As mentioned above, the energy spectrum of the incident neutrons is continuous. Therefore, the neutron time–of-flight measurement is the basis of the incident energy determination, and the control of its stability is crucial. It requires careful calibration of the zero point and the conversion gain of the time–to–digital converters. For the control of the time zero point we used the high energy photons in the beam. The lead filter in the neutron beam was removed for these calibration runs, and the hydrogen target was replaced by a lead slab for a higher conversion rate. With the spectrometer field inverted and reduced appropriately, the converted electrons were detected. From the width of the peak the time resolution has been evaluated to be 0.8 ns FWHM including the bunch width of the primary beam and the contribution from electronics. The corresponding neutron energy resolutions vary from 1.2 MeV at 200 MeV to 7.6 MeV at 580 MeV.

These calibrations have been performed in regular intervals. Deviations from the overall mean have been corrected for each run.

3.2 Data reduction and event selection

The first step in the off-line analysis was a reduction of the data by setting cuts in order to select good events. These cuts included a unique track in the drift chambers, starting in the target volume and emitted within the full acceptance of the spectrometer in vertical and horizontal direction. Next, the mass of the particle in the spectrometer was determined from the measured time-of-flight through the spectrometer and the measured momentum. A mass selection has been applied by rejecting masses \( m < 0.5 m_p \) and \( m > 1.5 m_p \), where \( m_p \) is the proton mass. The upper limit was increased to \( m > 2.5 m_p \) for experiments I and II, in order to keep also deuteron events, which were used for the normalization of the np data (see sect. 3.3). Finally, low momentum particles have been excluded by cuts at \( p = 500 \) MeV/c for protons and \( p = 620 \) MeV/c for deuterons.

3.3 Neutron energy determination

The emission angle of the recoil proton is determined by the set of drift chambers in front of the magnet, and its momentum by the deflection in the magnetic field. The proton energy is obtained after correction of the energy loss in traversing the material of target and spectrometer. The incident neutron energy is determined from the measured neutron time-of-flight \( t_{\ell} \). A complication arises from the fact, that the bunch interval given by the rf-signal (19.75 ns or 59.25 ns) is much shorter than the time-of-flight from the production target to the hydrogen target (more than 200 ns). This introduces ambiguities on the proper multiple of the bunch spacing time which has to be added to the measured value. However, by comparison with the time-of-flight \( t_{\text{calc}} \), calculated from the measured recoil proton momentum vector, the correct number of bunch spacings can be determined.

4 Experimental results

4.1 np differential cross sections

The data were binned in energy intervals of 20 MeV, and angular bins of 0.5 degrees in the centre-of-mass (CM) system. Each bin has been corrected for absorption of the recoil protons in the target and the spectrometer material. This energy dependent correction was below 1% in all cases. Another correction was necessary due to a small polarization component of the neutron beam in vertical direction of \( 0.5 \lesssim |P_n| < 0.8 \). This comes about since the neutrons are produced at an angle of 60 mrad (cf. sect. 2.3). The energy and angle dependent effect of the analyzing power of the np elastic scattering has been taken into account. The correction to the differential cross section was below 2.5%. The errors of these corrections contribute to the errors of the differential cross sections with less than 0.3%.

The four sets of data have been combined in an appropriate way. For the experiments I and II a common angular region exists which has been used to evaluate the multiplying factor for experiment I. The uncertainty introduced by this procedure for the different energies has been determined as 0.6% from the spread of the individual factors from the average. In a similar way the data set from experiment III
Figure 1: Relative differential np cross sections as a function of the CM-scattering angle $\theta_{\text{CM}}$. 

- •: data set I, $150^\circ \leq \theta_{\text{CM}} \leq 180^\circ$; 
- •: data set II, $140^\circ \leq \theta_{\text{CM}} \leq 180^\circ$; 
- ×: data set III, $129^\circ \leq \theta_{\text{CM}} \leq 156^\circ$; 
- ⋆: data set IV, $80^\circ \leq \theta_{\text{CM}} \leq 126^\circ$. The data are normalized to 1 at $\theta_{\text{CM}} = 179.2^\circ$. Errors are not visible within the symbol size.

has been linked to the combined data of set I and set II. Here the uncertainty was 0.7%. This procedure could not be followed for the normalization of experiment IV because no overlap of data points exists. We therefore fitted for each neutron energy the data points of experiment III for CM-angles smaller than $145^\circ$ together with the same number of data points of set IV by Legendre polynomials of 4th order with a free scaling factor. Each data point has been given the same statistical weight of 2% in the fit procedure in order to assign the same weight for the two data sets. The average value of $\chi^2$ for the relative normalization was 1.41, and the mean error of the scaling factor is 2%.

The statistical errors of the differential cross sections range from 1% to 2%. The spread of the data with respect to smooth fits with polynomials of an appropriate order is more like 2–2.5%. This reflects systematic uncertainties due to

- short term changes in the phase of the rf-signal
- imperfections of the drift chambers and other detectors
- remaining effects of gain shifts of detectors and electronics.

Taken together they are in the order of 1–2%, depending on the set-up. Systematic errors of 1.4%, 0.6% and 1.7% have been added in quadrature to the data sets II, III and IV,
respectively. A systematic error of 0.5\% had been added earlier to data set I.  

The relative differential cross sections for the four separate experiments are displayed together in Fig. 1. The angular distributions are normalized to 1.0 at the largest angle of data set II. The numerical values are tabulated in ref. [24].

4.2 Discussion

A common feature of the angular dependence at all energies is the sharp rise towards the backward direction. It is followed by a less steep decrease passing through a wide minimum, which is shifted to smaller angles with decreasing energy. In the transition region of the two different slopes there is an indication of a bump. A similar shape is indicated in the LAMPF data [8] but not as clear.

The energy dependence of the cross section at 180° is shown in Fig. 2. It is roughly constant in our energy range, and is well described by the phase shift predictions. With decreasing angle a deviation of the data from the phase shift predictions develops within the full energy range. This can be clearly seen by a comparison of the cross section ratios \( \sigma(\theta)/\sigma(180°) \) as shown in Fig. 2 for \( \theta = 135° \) and 90°. The deviations occur for both phase shift solutions of Arndt et al. [25] and Bystricky et al. [26]. It reflects the fact that for the extreme backward scattering angles several precise and consistent measurements exist [3] which pin down the phase shift solutions, whereas the angular region around \( \theta = 90° \) has been covered only scarcely at singular energies so far. The cross section ratios, as displayed in Fig. 2, are decreasing with increasing energy, except for the ratio \( \sigma(90°)/\sigma(180°) \) which is almost energy independent above 250 MeV.

5 Absolute normalization

For the absolute normalization of the cross section the incident neutron intensity has to be known as a function of energy. In our case the neutron intensity was obtained for experiment II from a comparison with the simultaneously measured reaction

\[ \text{np} \rightarrow \text{d} \pi^0, \]  

(1)

where the deuterons have been recorded by the spectrometer like the recoiling protons of the elastic scattering.

Isospin independence is used to relate the cross section of the process (1) to the cross section of reaction

\[ \text{pp} \rightarrow \text{d} \pi^+, \]  

(2)

for which precise data exist. However, isospin symmetry, which relates the two reactions by \( \sigma_{\text{d} \pi^+} = \frac{3}{4} \sigma_{\text{p} \pi^+} \), is not exact since the masses in the initial and in the final states are different for (1) and (2). Also, the Coulomb interaction for process (2) must be taken into account. This is discussed in detail in section 5.4.

5.1 Measurement of \( \frac{d \sigma}{d \Omega} (\text{np} \rightarrow \text{d} \pi^0) \)

The angular acceptance of the magnet spectrometer is sufficiently large to cover the full angular range of the deuterons, \( \theta^\text{max}_{\text{d}} \leq 12° \), so that a complete angular distribution of reaction (1) is obtained. The energy of the deuterons emitted in (1) varies strongly with the emission angle. For an incident neutron energy of 400 MeV, as an example, the deuteron energy varies from 130 MeV to 260 MeV. In this energy region the deuteron cross section on nuclei is strongly energy dependent and distorts the angular distribution. Besides the energy loss the absorption by the spectrometer material has to be corrected for. While the energy loss correction is straightforward, the correction for absorption loss is more involved because of the missing knowledge of deuteron cross sections on nuclei.

For the deuteron total cross section on a nucleus with mass number \( A \) we use [27]

\[ \sigma_{\text{d}A} = 0.97(\sigma_{nA} + \sigma_{pA}). \]  

(3)

The material around the target and the spectrometer consists mainly of hydrogen and the self-conjugate nuclei C, N, O and Ar, for which we assume \( \sigma_{nA} = \sigma_{pA} \). The deuteron
cross section eq. (3) reduces then to
\[ \sigma_{dA} = 1.94\sigma_{nA}. \] (4)

Neutron total cross sections on nuclei have been measured in a previous work [25]. For N and Ar, which have not been measured directly, the cross section has been evaluated from a parameterization of the energy and mass number dependence given in [25]. For the deuteron cross section on hydrogen \( \sigma_{dp} \) at the energy \( T \) we used \( \sigma_{np} \) data or the \( \sigma_{np} \) data of [24] at \( T/2 \).

The absorption correction is then obtained by summing up the contributions of the different elements according to their thickness in the spectrometer. The resulting correction varies from 5.2\% at \( T_d = 120 \text{ MeV} \) to 2.5\% at 400 MeV. As in the case of elastic np scattering (cf. sect. 4.1) the effect of a small polarization of the incoming neutrons

The Coulomb correction and apply Coulomb penetration factors for a point proton with an extended proton and for a point pion with an extended deuteron. The weighted sum of the

\[ R_{d\pi^+} = C_{cc}^2 \cdot R_{d\pi^+} \cdot \delta R, \] (6)

Table II shows some relevant information used to transform the cross section \( \sigma_{d\pi^+} \) of (2) to \( \sigma_{d\pi^0} \) of the process (1) by the relation

The uncertainty for \( \delta R/R_{av} \) is according to Ref. [30] about 0.01. Together with the Coulomb corrections which we applied to the pp initial state, this increases to 0.015. This error contribution is included in the errors given in col. 9 of Table II.

5.3 Parameterization of \( \sigma(pp \rightarrow d\pi^+) \)

Cross sections for reaction (2) as well as for the inverse reaction \( \pi^+d \rightarrow pp \) have been measured with sufficient accuracy in the relevant energy range from threshold up to 640 MeV. The data base [31–35] consists of 61 integrated cross sections for the reaction (2) between 288 and 641 MeV incident proton energy, and of 74 cross sections for the inverse process between 1.8 and 174 MeV incident pion energy. The latter ones have been converted to (2) via detailed balance. Statistical and systematic errors of each data point have been added in quadrature (Gaussian). The data set has been fitted by a parameterization similar to Bystricky et al. [31] as a function of \( \eta \). The fit error of \( \sigma_{d\pi^+} \) ranges from 2.1\% at \( \eta = 0.396 \) to 0.7\% at \( \eta = 1.582 \).

5.4 Absolute normalization above 300 MeV

The absolute normalization factor for the elastic np scattering data is given with respect to the highest measured angle \( \theta_{max} = 179.2^\circ \) of data set II

\[ \frac{d\sigma}{d\Omega}(\theta_{max})_{np} = \frac{\tilde{N}_{np}(\theta_{max})}{\int \tilde{N}_{d\pi^0}d\Omega_{d\pi^0}} \cdot \sigma_{d\pi^0}, \] (8)

where \( \tilde{N}_{np} \) and \( \tilde{N}_{d\pi^0} \) are the solid angle corrected event numbers for the two processes in corresponding energy bins. The results are given in col. 10 of Table II. Close to pion threshold and below, this method can not be applied.

6 The \( \pi NN \) coupling constant \( f_{\pi NN}^2 \)

6.1 The Chew method

The value of the \( \pi NN \) coupling constant \( f_{\pi NN}^2 \) has focussed new interest recently in connection with a discussion on a conceivable breaking of charge independence [37–41]. The
### Table I: Absolute normalization of the elastic np cross sections.

All cross section related quantities are given in the CM system.

| $T_n$ (MeV) | $\eta$ | $T_p$ (MeV) | $C^2_{\text{ee}}$ | $\frac{dP}{d\omega_n}$ | $\frac{dP}{d\omega_p}$ | $\sigma_{d \pi^+}$ (mb) | $\sigma_{d \pi^0}$ (mb) | $\Delta \sigma$ (%) | $\frac{\sigma_{d \pi^+}}{\sigma_{d \pi^0}}$ (%) | $\Delta \sigma_{\pi^+}$ (%) |
|-------------|--------|-------------|----------------|-----------------|-----------------|----------------|----------------|-------------|----------------|----------------|
| 300.2       | 0.3960 | 312.0       | 0.9362         | -0.194          | -0.0095         | 0.1099         | 0.1225         | 0.0084      | 0.0684 ± 2.6 | 10.63 ± 3.1 |
| 320.1       | 0.5397 | 321.4       | 0.9471         | 0.0950          | -0.0084         | 0.0866         | 0.2288         | 0.1248      | 0.1248 ± 2.2 | 10.52 ± 2.7 |
| 340.0       | 0.6527 | 350.9       | 0.9525         | 0.0778          | -0.0074         | 0.0704         | 0.3639         | 0.1952      | 0.1952 ± 2.1 | 10.60 ± 2.4 |
| 360.1       | 0.7558 | 370.7       | 0.9559         | 0.0643          | -0.0066         | 0.0577         | 0.5206         | 0.2805      | 0.2805 ± 2.1 | 10.86 ± 2.3 |
| 379.9       | 0.8486 | 390.1       | 0.9584         | 0.0525          | -0.0059         | 0.0462         | 0.7213         | 0.3777      | 0.3777 ± 2.0 | 10.87 ± 2.3 |
| 400.2       | 0.9375 | 410.1       | 0.9606         | 0.0412          | -0.0053         | 0.0359         | 0.9474         | 0.4910      | 0.4910 ± 1.9 | 10.91 ± 2.2 |
| 420.3       | 1.0206 | 429.9       | 0.9622         | 0.0325          | -0.0048         | 0.0277         | 1.200          | 0.6171      | 0.6171 ± 1.9 | 10.71 ± 2.1 |
| 440.5       | 1.1004 | 449.9       | 0.9638         | 0.0254          | -0.0043         | 0.0211         | 1.481          | 0.7563      | 0.7563 ± 1.8 | 10.50 ± 2.1 |
| 460.4       | 1.1760 | 469.6       | 0.9651         | 0.0200          | -0.0039         | 0.0161         | 1.777          | 0.9030      | 0.9030 ± 1.8 | 10.56 ± 2.0 |
| 480.5       | 1.2498 | 489.5       | 0.9665         | 0.0146          | -0.0036         | 0.0110         | 2.085          | 1.054       | 1.054 ± 1.8  | 10.54 ± 2.0 |
| 501.0       | 1.3229 | 509.8       | 0.9678         | 0.0132          | -0.0032         | 0.0100         | 2.392          | 1.208       | 1.208 ± 1.7  | 10.30 ± 1.8 |
| 520.8       | 1.3916 | 529.4       | 0.9690         | 0.0225          | -0.0030         | 0.0195         | 2.664          | 1.358       | 1.358 ± 1.7  | 10.45 ± 1.8 |
| 540.3       | 1.4577 | 548.7       | 0.9700         | 0.0400          | -0.0027         | 0.0373         | 2.889          | 1.499       | 1.499 ± 1.7  | 10.00 ± 1.8 |
| 559.6       | 1.5217 | 576.9       | 0.9711         | 0.0560          | -0.0025         | 0.0535         | 3.054          | 1.611       | 1.611 ± 1.7  | 9.61 ± 1.8  |
| 578.1       | 1.5820 | 586.2       | 0.9719         | 0.0700          | -0.0023         | 0.0677         | 3.143          | 1.682       | 1.682 ± 1.7  | 9.42 ± 1.9  |

6.2 Test with pseudo data

We have tested this extraction method with pseudo data between 240 MeV and 540 MeV neutron laboratory kinetic energy in 100 MeV steps. The data were generated from the regularized OPE model of Gibbs and Loiseau [16] in the angular range from $80^\circ$ to $180^\circ$ in $1^\circ$ steps. The input coupling constant was fixed at $\lambda_e^2 = 0.076$. Uncertainties of the pseudo data were generated with a Gaussian random error distribution. If these uncertainties are $\leq 0.1\%$, the
model coupling constant can be reproduced with an error of \( \approx 0.25\% \). For pseudo data with uncertainties of 2\%, corresponding to the present experiment, the error for \( f_c^2 \) is 2.4\%. Moreover, the extracted coupling constant is systematically smaller than its input value. This is because the extracted \( f_c^2 \) rises with the maximum polynomial order of the fit as shown in Fig. 3 (open circles). Applying an \( F \) test [44] to the fit gives a lower maximum order of the polynomials of eq. 9 for data with higher uncertainties. At \( T_n = 440 \text{ MeV} \), for instance, and for uncertainties \( \leq 0.1\% \), the \( F \) test results in the maximum order \( j_{\text{max}} = 10 \) and \( f_c^2 = 0.0761 \pm 0.2\% \). For uncertainties of 2\% the \( F \) test results in \( j_{\text{max}} = 7 \) and \( f_c^2 = 0.0746 \pm 2.4\% \).

### 6.3 Chew method with conformal mapping

The convergence properties of the polynomial expansion is improved by conformal mapping of the variable \( x \) before extrapolating to the pole [45]. We used this method similar to Dumbrajs et al. [46]. The position of the neutral pion pole and the onset of the two pion (\( \pi^+\pi^- \)) exchange cut are mapped to (+1,0) and (-1,0) in the complex plane, respectively. The other exchange cuts lie on the unit circle. In addition, the data points are symmetrized around zero.

This mapping method was tested as before with pseudo data. Again, the model coupling constant is reproduced for uncertainties \( \leq 0.1\% \). The convergence is faster than without conformal mapping, and the extrapolation error is of the same order or slightly larger. For the example at \( T_n = 440 \text{ MeV} \) given above, the \( F \) test results a value of \( f_{c,\text{map}}^2 = 0.0759 \pm 0.3\% \) with a reduced polynomial order of \( j_{\text{max}} = 8 \) for uncertainties of \( \leq 0.1\% \). With uncertainties of 2\%, the \( F \) test results in the lower value \( j_{\text{max}} = 5 \) and \( f_{c,\text{map}}^2 = 0.0786 \pm 1.6\% \). Thus, the conformal mapping method results in a systematic upward shift for the extracted coupling constant, as shown by the triangles in Fig. 3. Again, this comes about because the extracted \( f_{c,\text{map}}^2 \) depends on the maximum polynomial order, which is lower for less precise pseudo data.

Similar results were obtained with pseudo data from phase shift predictions of Arndt et al. [25]. Again, the input value \( f_{c}^2 = 0.076 \) could be reproduced only with high precision \( (\leq 0.1\%) \) pseudo data by both methods. For uncertainties of 2\% the result was systematically too low without, and too high with conformal mapping. Thus, both methods have systematic offsets of opposite sign. In principle, they can be taken into account by a correction factor

\[
k_{\text{corr}} = \frac{f_{c,\text{input}}^2}{f_{c,\text{extracted}}^2}.
\]

### 6.4 Determination of \( f_c^2 \) from our data

Besides the systematic uncertainties caused by the errors of the data, there is also a dependence on the angular range used in the fit. With decreasing range also the extracted coupling constant is decreasing. This can be seen from the results on \( f_j^2 \) and \( f_{c,\text{map}}^2 \) obtained with the real data at \( T_n = 440 \text{ MeV} \), as shown in Fig. 4. For the fits at this energy all data points were given a statistical error of 2.8\%. With this choice the reduced \( \chi^2 \) of the fit is of the order of 1 for the whole angular range. The error of \( f_j^2 \) is about 15\% for an angular range \( \Delta \theta = 15^\circ \) and drops to about 3\% for \( \Delta \theta = 100^\circ \). According to Fig. 3 an angular range \( \Delta \theta \geq 50^\circ \) is needed. The variation of the extracted \( f_j^2 \) values is minimal for \( 75^\circ \leq \Delta \theta \leq 95^\circ \). For all energies it was found, that the angular range should extend to the region where the differential cross section reaches its minimum. In order to have no different weights from the four data sets, we have chosen for each laboratory kinetic energy a common relative error per data point, which was adjusted to get a reduced \( \chi_{\text{R}}^2 \) of the order of 1 for the whole angular range. The maximum polynomial order \( j_{\text{max}} \) was determined either when a minimum of \( \chi_{\text{R}}^2 \) was reached or when \( F \leq 4 \), which corresponds to a significance of at least 95\% for the term with \( j_{\text{max}} \). The fit error of \( f_j^2 \) is then below 5\%. The results of the fits at energies where an absolute normalization exists are collected in Table I.

Figure 4: Dependence of \( f_j^2 \) fits on the angular range at \( T_n = 440 \text{ MeV} \). Full dots are the results with conformal mapping.

The correction factors \( k_{\text{corr}} \), which were needed to correct for the systematic offset introduced by the Chew extrapolation were determined for both methods by an extended study with OPE pseudo data. For each energy we have generated 1000 sets of pseudo data, with randomly distributed errors \( s \) as given by the experimental data (see Table I). For the Chew method without conformal mapping no unique \( j_{\text{max}} \) was found for a given energy. Therefore we have used the \( k_{\text{corr}} \) obtained for that value of \( j_{\text{max}} \) which was obtained for the real data. This complication does not arise with conformal mapping, where the fit criteria (F-test) lead to the same value of \( j_{\text{max}} \) at all energies. Moreover, the energy variation of \( k_{\text{corr}} \) is only \( \approx 1\% \) as compared with almost 10\% without conformal mapping. Therefore we prefer to extract an energy independent value of \( f_j^2 \) using the conformal mapping method. The maximum polynomial order \( j_{\text{max}} \), the resulting value of \( f_j^2 \), the fit error \( \Delta f_j^2 \) and the correction factor \( k_{\text{corr}} \) is given in Table I for both methods at each energy.
Table II: Fit results for $f^2$, obtained with the standard Chew method ($f^2_c$), and with conformal mapping ($f^2_{c,\text{map}}$) for all energies with absolute normalization. $\Delta \theta$: angular range; $N$: number of data points; $s$: error of individual data point; $j_{\text{max}}$: maximum polynomial order according to $F$ test; $\Delta f^2$: fit error of $f^2_c$ and $f^2_{c,\text{map}}$, respectively; $k_{\text{corr}}$: correction factor for systematic offset.

| $T_n$ (MeV) | $\Delta \theta$ (°) | $N$ | $s$ (%) | $j_{\text{max}}$ | $f^2_{c}$ | $\Delta f^2$ (%) | $k_{\text{corr}}$ | $j_{\text{max}}$ | $f^2_{c,\text{map}}$ | $\Delta f^2$ (%) | $k_{\text{corr}}$ |
|------------|------------------|----|--------|-----------------|----------|----------------|-----------------|----------------|-----------------|----------------|----------------|
| 300.2      | 94.2             | 108| 2.3    | 7               | 0.0766   | 4.7           | 0.980           | 5               | 0.0730          | 3.1           | 0.977         |
| 320.1      | 92.5             | 106| 2.3    | 7               | 0.0778   | 4.2           | 0.983           | 5               | 0.0762          | 2.8           | 0.975         |
| 340.0      | 88.8             | 106| 2.4    | 6               | 0.0713   | 3.4           | 1.040           | 5               | 0.0765          | 3.1           | 0.973         |
| 360.1      | 87.0             | 106| 2.4    | 7               | 0.0794   | 4.4           | 0.985           | 5               | 0.0807          | 3.2           | 0.974         |
| 379.9      | 85.2             | 104| 2.5    | 7               | 0.0791   | 4.5           | 0.982           | 5               | 0.0778          | 3.2           | 0.971         |
| 400.2      | 83.5             | 104| 2.8    | 6               | 0.0713   | 3.9           | 1.049           | 5               | 0.0786          | 3.6           | 0.969         |
| 420.3      | 83.7             | 106| 2.8    | 6               | 0.0700   | 3.8           | 1.056           | 5               | 0.0776          | 3.4           | 0.970         |
| 440.5      | 79.9             | 103| 2.8    | 6               | 0.0742   | 3.5           | 1.049           | 5               | 0.0797          | 3.6           | 0.967         |
| 460.4      | 80.1             | 104| 2.5    | 6               | 0.0725   | 3.1           | 1.060           | 5               | 0.0780          | 3.1           | 0.967         |
| 480.5      | 78.3             | 103| 2.6    | 6               | 0.0734   | 3.2           | 1.060           | 5               | 0.0792          | 3.2           | 0.965         |
| 501.0      | 74.4             | 101| 2.5    | 6               | 0.0757   | 3.0           | 1.053           | 5               | 0.0783          | 3.3           | 0.965         |
| 520.8      | 74.6             | 104| 2.1    | 6               | 0.0769   | 2.3           | 1.060           | 5               | 0.0811          | 2.6           | 0.964         |
| 540.3      | 74.9             | 104| 2.2    | 6               | 0.0746   | 2.4           | 1.063           | 5               | 0.0794          | 2.6           | 0.963         |
| 559.6      | 75.1             | 103| 2.5    | 6               | 0.0741   | 2.6           | 1.077           | 5               | 0.0815          | 2.7           | 0.963         |

A contribution to the error which has not been taken into account so far is due to the uncertainty of $\approx 2\%$ in the relative normalization of data set IV with respect to the other sets. It was investigated systematically and a variation of about 1.5\% of the extracted $f^2$ was observed. This contribution is not contained in $\Delta f^2$ of Table II. It was, however, added in quadrature to each $\Delta f^2$ before the energy averaged weighted mean $\langle f^2_{\text{np}} \rangle = \langle f^2 \cdot k_{\text{corr}} \rangle$ has been calculated.

For the preferred method with conformal mapping we obtain as our final result the weighted mean of the charged coupling constant in the energy range $300 \text{ MeV} \leq T_n \leq 560 \text{ MeV}$:

$$\langle f^2_{\text{np}} \rangle = 0.0760 \pm 0.0008.$$

The given error is the propagated error calculated from the individual $\Delta f^2$. It is slightly larger than the standard deviation of the mean.

It has to be kept in mind, that both, the relative error of the data points as well as the absolute normalization enter and limit the determination of $f^2$. In our case both error types are of the same order. With more precise data, i.e. smaller errors of the relative cross sections, the fitting procedure would be more stable and the systematic effects be smaller. Changes in the absolute normalization factor and its error, on the other hand, propagate to the $f^2$ values only with a factor $1/2$ since the Chew function at the pole depends on $f^4$.

All method tests and the calculation of $k_{\text{corr}}$ have been performed with OPE model data. Therefore, a model contribution should eventually be added to the error margin.

Our value of 0.0760 for the charged coupling constant is remarkably lower than the value obtained by Hohler et al. \cite{37} from pion-nucleon scattering. It is, however, in good agreement with more recent determinations of $f^2$ from both, pion-nucleon partial wave analysis \cite{47} and nucleon-nucleon scattering \cite{37}.

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