Quantum charge pumping through Majorana bound states

Krashna Mohan Tripathi¹, Sumathi Rao¹ and Sourin Das²,³

¹Harish-Chandra Research Institute, HBNI, Chhatnaug Road, Jhusi, Allahabad 211 019, India.
²Department of Physical Sciences, Indian Institute of Science Education and Research, Kolkata, West Bengal, 741252, India.
³Department of Physics and Astrophysics, University of Delhi, Delhi - 110 007, India.

We study adiabatic charge pumping through a Majorana bound state tunnel coupled to multiple normal leads. We show that for most of the parameters such a pump does not lead to any net pumped charge between the various leads unless a multiply connected geometry is implemented. We introduce an Aharonov-Bohm ring geometry at the junction to implement such a multiply connected geometry. We further show that the Fourier transform of the pumped charge with respect to flux inserted through the ring shows a clear distinction between the case of an Andreev bound state and the Majorana bound state. Hence such a Fourier analysis can serve as a diagnostic for the detection of Majorana bound states in the proposed geometry.

I. INTRODUCTION

One of the first steps that is required for the application of Majorana modes in quantum computation is its unambiguous identification. This has proved difficult in experiments, since the usual diagnostic of Majorana bound states (MBS), a zero bias peak in the conductance, can have many other origins besides signaling the presence of a Majorana mode. This fact has led to considerable work in recent years, encompassing study of various toy models and promising physical systems and their electrical transport signatures.

However, there has been no definitive experimental confirmation so far, which has proved the existence of Majorana modes in any system and hence, it is still of interest to look for different ways to confirm the existence of Majorana modes. In this context, it is worth exploring the question of charge pumping through a Majorana mode and examining whether there are unique signals which can identify it and differentiate it from pumping through other resonant leads or Andreev bound states. Charge pumping or the phenomenon of obtaining current in the absence of bias by local variations of parameters of the quantum system, has been studied in many contexts, beginning with Thouless who considered the effect of a travelling periodic potential that could drag the electrons along. The analysis performed by Thouless was in the spirit of closed quantum system.

Later the idea of pumping was extended to open quantum system in the Refs. 12 and 13 where the pumping of charge is induced between different electron reservoirs by periodically varying independent parameters of scattering matrix that describes the scattering of electrons between the different electron reservoirs. When the variation of the parameters is much slower than the transport time, then the pumping is adiabatic and the Brouwer formula can be applied. Adiabatic charge pumping has attracted a great deal of interest in the last several years, and different aspects of it have been studied in great detail. There has also been some work on normal metal-superconductor interfaces including Majorana mediated charge pumps.

There has also been recent interest in cases when the pumped charge is quantized, and in particular for topological reasons, so that it is stable to disorder and could be used for metrological applications. As mentioned above, this was first studied by Thouless who showed that the quantised adiabatic charge transport was related to the Chern number of the band, which also counts the number of monopoles or equivalently gapless points enclosed by the pumping contour. In recent work, it has been shown that the presence of a single transmitting channel at the interface between a normal wire and a superconductor enables quantization of the pumped charge by tuning the system through topological phase transitions so that isolated topological trivial regions are surrounded by topological regions. Thus pumping paths can be chosen to make non-contractible loops in the parameter space which could lead to quantized charge pumping.

In an earlier paper, we studied the conductance through a Majorana bound state (MBS) embedded in a Aharonov-Bohm ring geometry and showed that the currents at the two leads tunnel coupled to the Aharonov-Bohm ring were anti-correlated and the degree of anti-correlation could be tuned by the Aharonov-Bohm (AB) flux threading the ring. In this paper, we will explore charge pumping through the MBS, in the same geometry and study the role of the AB ring geometry which exhibits non-trivial topology. Unlike Ref. 47 where the pumping of charge required going through topological phase transitions as one traverses along the pumping contour, here we will show show that it possible to obtain quantized pumped charge even when the superconductor hosting the MBS does not undergo phase transitions.

The ring geometry plays a crucial role here since we will show that there is no pumped charge when the two leads are just connected to the MBS (normal-MBS-normal or simple two-lead geometry without the ring). This is
unlike the earlier study of conductance where the anti-correlation existed even in the two-lead geometry and the \(\mathcal{AB}\) geometry was required only to provide a tuning parameter for tuning the degree of anti-correlation. Here, on the contrary, the \(\mathcal{AB}\) geometry is crucial to get non-zero pumping. In this geometry, we will study the pumped charge at each of the leads using a scattering matrix approach, restricting ourselves to the adiabatic regime, and we will compute the pumped charge using the analog\(^{38}\) of the Brouwer\(^{13}\) formula for a normal-superconductor junction. Finally, we will show that a Fourier analysis of the pumped charge as a function of the flux through the ring, leads to a single frequency domination for the MBS, as opposed to many harmonics for an Andreev bound state (ABS); this can be thought of as a diagnostic for the MBS.

II. SIMPLE CONNECTED GEOMETRY WITH TWO LEADS

The Hamiltonian for two normal leads which are tunnel coupled to an MBS situated at the end of a one-dimensional p-wave superconductor\(^{1}\) is given by

\[
H = \sum_{\alpha} H_{\alpha} + H_{T},
\]

where the form of the lead Hamiltonian for the two \((\alpha = 1,2)\) leads are given by

\[
H_{\alpha} = \int_{-\infty}^{\infty} dx \psi_{\alpha}^\dagger(x)(-i v_F \partial_x) \psi_{\alpha}(x)
\]

and the tunneling Hamiltonian is given by

\[
H_{T} = i \gamma \sum_{\alpha} (u_{\alpha} \psi_{\alpha}(x = 0) + u_{\alpha}^\dagger \psi_{\alpha}^\dagger(x = 0)).
\]

Here \(\gamma\) represents the Majorana fermion operator and \(u_{\alpha}\) represents the amplitude of coupling between the lead \(\alpha\) and the MBS which is complex number in general. The scattering matrix describing the scattering of electrons and holes between the leads via the MBS corresponding to the situation described by the above tunnel Hamiltonian is found by applying the Weidemannformula\(^{51}\) as

\[
S(E) = \left( \begin{array}{cc} S^{ee}(E) & S^{eh}(E) \\ S^{he}(E) & S^{hh}(E) \end{array} \right),
\]

where

\[
S^{ee}(E) = 1 - \frac{2i \pi \nu}{d(E)} \left( |u_{1}|^2 |u_{2}|^2 \right).
\]

\[
S^{he}(E) = \frac{2i \pi \nu}{d(E)} \left( u_{1}^2 u_{1}^\dagger u_{2}^\dagger u_{2} \right),
\]

with \(d(E) = E + 2i \pi \nu(|u_{1}|^2 + |u_{2}|^2)\). Here the scattering matrix is written in a basis where the first two rows and columns correspond to electrons from lead-1 and lead-2 respectively, and the next two rows and columns correspond to holes from lead-1 and lead-2 respectively, and \(\nu\) represents the density of states of the electrons which has been assumed to be same in both leads for simplicity.

Using the extension of Brouwer’s formula for the case of superconducting junction\(^{38}\), the pumped charge at each lead is given by

\[
Q_{\alpha} = -e \int_{A} dX_{1}dX_{2} Im[C_{\alpha,\alpha}]
\]

where the matrix

\[
C = \frac{1}{\pi} \left( \frac{d}{dX_{1}} S^{ee} - \frac{d}{dX_{2}} S^{he} \times \frac{d}{dX_{2}} (S^{he})^\dagger \right).
\]

Here \(X_{1}\) and \(X_{2}\) represent the pumping parameters which are periodic functions of the time \(t\). They trace out a closed loop in the \(X_{1}-X_{2}\) plane over one time period such that the area enclosed by the loop is finite and is given by \(A\). Also note that the pumped charge in each lead can be decomposed into a particle-like process which depends on \(S^{ee}\) alone and a particle-hole conversion process which depends on \(S^{he}\) alone.

We can choose the \(u_{\alpha}\)'s to be the pumping parameters, \(i.e.,\), we can choose \(X_{1} = u_{1}\) and \(X_{2} = u_{2}\). In this case, we find that

\[
Im[C_{11}] = 0 = Im[C_{22}]
\]

\(i.e.,\) the integrand itself vanishes and there is no pumped charge. Note that the \(u_{\alpha}\)'s can be taken to be real since their phase can be gauged away as long as it is not time dependent. Alternatively, if we intend to use the phase of the \(u_{\alpha}\)'s as a pumping parameter \((i.e.,\) make it time dependent), the implementation of such a pumping protocol requires the order parameter of the superconducting hosting the MBS to be varied in time which in turn calls for Josephson junction type setup which is beyond the scope of the present work.

Now, the vanishing of the pumped charge can be attributed to the particle-hole symmetry of the couplings between the MBS and the leads about zero energy due to the fact that the MBS is pinned to zero energy. To contrast the case of MBS to a more regularly encountered bound state in the context of normal-superconducting hybrid structures, the Andreev bound state (ABS), we will show now that the vanishing of the pumped charge is not true in general. When we couple an ABS to leads, we will see that it leads to a finite pumped charge even when the ABS is tuned to zero energy. To evaluate the pumped charge via an ABS, we start by replacing the tunnel Hamiltonian in Eq.[2] by\(^{51}\)

\[
H_{T} = \alpha^{\dagger} \sum_{\alpha,k} (t_{\alpha} c_{\alpha k} + u_{\alpha}^{\dagger} c_{\alpha k}^{\dagger}) + h.c.,
\]
where now $a^\dagger$ denotes the creation operator for the ABS, (which, unlike the MBS does not have to be real) and the tunneling amplitudes to the electron and hole states on the leads are given by $t_\alpha$ and $v_\alpha^\dagger$ respectively. Considering $t_1$ and $t_2$ to be pumping parameters, and parametrising the contour in terms of a scale $R_2$, $(t_1^2 + t_2^2/R_2^2 = 1)$, we can obtain the pumped charge.

As the analytic expression for the pumped charge in this case gets cumbersome, we perform a numerical analysis for some representative values to obtain it using Eq.[6,7]. This is presented in Fig.2. Note that the total charge pumped from the normal metal leads into the superconductor is given by $Q_+ = Q_1 + Q_2$ and the total charge pumped from lead-1 to lead-2 via the MBS is given by $Q_- = Q_1 - Q_2$ over a single pumping cycle. We observe that both these quantities are finite for the chosen pumping contour and they asymptotically reach a steady value as the amplitude of pumping parameter($R$) gets larger and larger. Hence, this fact itself, presents a clear distinction between the ABS and MBS.

III. MULTIPLY CONNECTED GEOMETRY AND THE MBS

From the above analysis it is clear that a simple set up involving two leads tunnel coupled to an MBS does not lead to net charge being pumped either from one lead to another or from the leads to the superconductor over a pumping cycle. Next, we explore the possibility of pumping in a multiply connected geometry where the MBS is embedded in a ring.

**MBS embedded in an AB geometry:-** We now look for a multiply connected geometry and the simplest choice is to consider a ring geometry where a magnetic flux is piercing the ring. The ring geometry is realized by considering an MBS which is tunnel coupled to two leads that are also directly tunnel coupled to one another as shown in Fig.1. The Hamiltonian for the system is the same as that given in Eq.[1] except that we now have an additional direct tunneling term given by

$$H_{direct} = \tau \psi_1(x=0)\psi_2(x=0) + h.c. \ . \ (10)$$

Here $\tau$ denotes the amplitude for the direct tunnel coupling of the two leads to each other. The scattering matrix now involves the direct coupling term as well and is given by

$$S^{re}(E) = \frac{1}{1 + \pi^2 \nu^2 |\tau|^2} \begin{pmatrix} 1 - \pi^2 \nu^2 |\tau|^2 & -2i\pi \nu \tau \\ -2i\pi \nu \tau^* & 1 - \pi^2 \nu^2 |\tau|^2 \end{pmatrix} - \frac{2i\pi \nu}{(1 + \pi^2 \nu^2 |\tau|^2)^2 D(E)} \begin{pmatrix} u_{1+}^* u_{1-} & u_{1+}^* u_{2-} \\ u_{2+} u_{1-} & u_{2+} u_{2-} \end{pmatrix} \ \ (11)$$

and

$$S^{he}(E) = -\frac{2i\pi \nu}{(1 + \pi^2 \nu^2 |\tau|^2)^2 D(E)} \begin{pmatrix} u_{1+} u_{1-} - u_{1+} u_{2-} \\ u_{2+} u_{1-} - u_{2+} u_{2-} \end{pmatrix} \ \ (12)$$

with $D(E) = E + \frac{2i\pi \nu}{(1 + \pi^2 \nu^2 |\tau|^2)}(|u_1|^2 + |u_2|^2)$, $u_{14} = u_1 + si\pi \nu \tau^* u_2$, $u_{25} = u_2 + si\pi \nu \tau u_1$ and $s = +, -$. Once again, the pumped charge can be computed using the analog of Brouwer’s formula. Here, we find that the integrand is given by

$$Im[C_{11}] = \frac{16\pi^2 \nu^2 \tau(1 + \pi^2 \nu^2 \tau^2) \cos(\phi) E^2 (u_1^2 + u_2^2)}{|E^2 (1 + \pi^2 \nu^2 \tau^2)^2 + 4\pi^2 \nu^2 (u_1^2 + u_2^2)^2|^2} = -Im[C_{22}] \ . \ \ (13)$$

As before, the $u_s$ are taken to be real, and the direct tunneling term is taken to be $\tau = \tau_0 e^{i\phi}$ where $\phi$ plays the role of the AB flux. Clearly, this expression is zero when the direct tunneling amplitude is zero and it agrees with...
the earlier result. Using this in Eq.[6], we see that the integrand and consequently, the pumped charge through each lead is zero even in the presence of a direct tunneling term, at $E = 0$.

So although, we have allowed for a finite direct tunneling amplitude ($\tau_0$) for the electrons, leading to a multiply connected geometry, we are still unable to break the particle-hole symmetry of the pumped charge about $E = 0$, which prevents pumping of net charge. The MBS in many aspects is very similar to a resonant level (RL). The MBS allows for resonant injection of a pair of electron into superconductor via resonant Andreev process and the RL allows for a single electron to resonantly transmit across it. Hence to gain insight in the MBS pumping analysis, our next analysis is to consider pumping of charge across a RL embedded into an AB geometry.

**AB geometry and the resonant level:** We can now contrast the above observed behaviour of the MBS to the case where the MBS is replaced by a RL. The only change in the above model is that the first line of the tunneling term in Eq.[10] that represents tunneling through the MBS is now replaced by

$$H_T = (d^\dagger \sum_\alpha u_\alpha \psi_\alpha(x=0) + h.c.) \quad (14)$$

where $d^\dagger$ represents the creation operator of the electron on the resonant level. The direct coupling term between the leads remains the same. The scattering matrix in this case is given by:

$$S(E) = \begin{pmatrix} S_{11}(E) & S_{12}(E) \\ S_{21}(E) & S_{22}(E) \end{pmatrix}$$

$$S_{ij;i=\bar{j}}(E) = -1 + \frac{2(E + i\nu\nu_{ij})}{d(E)}$$

$$\hat{d}(E) = E(1 + \pi^2\nu^2\tau_0^2) + 2\pi^2\nu^2\tau_0 \cos(\phi)u_1u_2 + i\nu\nu(u_1^2 + u_2^2). \quad (15)$$

where $u_{11} = u_1^2$, $u_{22} = u_2^2$ and $\phi_{12} = \phi$, $\phi_{21} = -\phi$. Now the pumped charge can be evaluated from the expressions for $\text{Im}[C_{11}]$ and $\text{Im}[C_{22}]$ as given below,

$$\text{Im}[C_{11}] = \frac{8\pi^2\nu^3}{|d(E)|^4} (u_1^2 + u_2^2)(E^2\tau_0 \cos(\phi)(1 + 2\pi^2\nu^2\tau_0^2) + E\{u_1u_2(1 + 2\pi^2\nu^2\tau_0^2 \cos(2\phi)) - \pi\nu\tau_0 \sin(\phi)(u_1^2 - u_2^2)\}) - \text{Im}[C_{22}]. \quad (16)$$

Note that unlike the MBS case, where the integrand vanishes without direct tunneling between the leads, here the integrand is non-zero even for $\tau_0 = 0$. This clearly indicates that although the direct tunneling term or the ring geometry was absolutely necessary to even get a non-zero integrand for the MBS case, that is not the case for the RL case. However, the pumped charge through each lead continues to be identically zero in both cases at $E = 0$.

**AB geometry and the role of pumping parameters:** We note above that for both the MBS and the RL, the pumped charge is zero at $E = 0$, while it is finite in general for ABS. Hence it is natural is ask whether the choice of pumping parameters can change this fact. For the case of the MBS embedded in a ring we replace the pumping parameter $u_2$, which is one of the hopping amplitudes to the MBS, by $\bar{\tau}_0 = \pi\nu\tau_0^2$ which is the amplitude for direct tunneling between the leads. Note that this explicitly breaks the symmetry between the two leads as far as the pumping contour is concerned. In this case, we find that even at $E = 0$ the integrand is finite and is given by

$$\text{Im}[C_{11}] = \frac{2\cos(\phi)u_2(u_1^2 - \bar{\tau}_0^2u_2^2)}{\pi(u_1^2 + u_2^2)(1 + \bar{\tau}_0^2) \tau_0^2}$$

$$\text{Im}[C_{22}] = -\frac{2\cos(\phi)u_2(u_2^2 - \bar{\tau}_0^2u_1^2)}{\pi(u_1^2 + u_2^2)(1 + \bar{\tau}_0^2) \tau_0^2}. \quad (17)$$

In Fig.3, we show a plot of the integrands $C_{11}$ and $C_{22}$ as a function of the two pumping parameters. The pumped charge at the two leads can now be computed by choosing various contours. We show below the pumped charge for a few representative contours and note how asymptotically, (almost) quantised charge is pumped either to the superconductor or between the two leads.

**Pumped charge for various pumping contours:** As the physically relevant quantities are $Q_+$ and $Q_-$, we first plot the integrands $\text{Im}(C_{11} + C_{22})$ and $\text{Im}(C_{11} - C_{22})$ as a function of the pumping parameters $u_1$ and $\tau_0$ as shown in Fig.3. Note that the maximum value for $Q_+$ and $Q_-$ are concentrated about the $\tau_0 = 0$ axis respectively. Also, note that the sign of $\text{Im}(C_{11} + C_{22})$ and $\text{Im}(C_{11} - C_{22})$ remains the same as we move along the axis about which the maxima of these functions are mostly distributed. On the other hand $\text{Im}(C_{11} + C_{22})$ and $\text{Im}(C_{11} - C_{22})$ do change sign along the axis perpendicular to the axis of the distribution of the maxima.

This fact will strongly influence the asymptotic values of the pumped charge as we go to larger and larger contour sizes. For obtaining large values of pumped charge we need to analyze the symmetries of the distribution of values of $\text{Im}(C_{11} + C_{22})$ and $\text{Im}(C_{11} - C_{22})$ and design pumping contours which will efficiently enclose a large fraction of the maxima of these functions in the parameter space. In principle, appropriately chosen contours can lead to asymptotically quantized value for pumped charge\textsuperscript{17,29,31,36,46}. Keeping this fact in mind we consider elliptical shapes of the contours in the plane of pumping parameters $(u_1, \tau_0)$ given by $u_2^2/R_2^2 + \tau_0^2/R_1^2 = 1$.

We have produced plots for three different kinds of contours which are given by (a) $R_1 = R_2 = R$ where the asymptotic pumped charge is obtained for $R \to \infty$ limit, (b) $R_1/R_2 > 1$ where the asymptotic pumped charge is obtained for $R_1 \to \infty$ limit, and (c) $R_2/R_1 > 1$ where
Parameters under the asymptotic pumped charge is obtained for $R_{\text{blue}}$ and $R_{\text{red}}$ respectively. The corresponding asymptotic pumped charges $Q_+$ and $Q_-$ for the cases $(a)$ discussed above in cyan, blue and red respectively. The asymptotic pumped charge is given in Fig.4 where the color code of $Q_+$ and $Q_-$ shows representative contours for the cases $(a)$, $(b)$ and $(c)$ discussed above in cyan, blue and red respectively. The corresponding asymptotic pumped charge is given in Fig.4 where the color code of the direct hopping $\phi = 0$. The three contours $a,b$ and $c$ (explained in the text) for which we have computed the pumped charges $Q_+$ and $Q_-$ in Fig.(4) are shown in cyan, blue and red respectively.

Let us first discuss the results corresponding to the $(b)$-type contour which is depicted in blue in Figs.3 and 4. We note that $Q_+ \rightarrow 2$, i.e., gets asymptotically quantized while $Q_- \rightarrow 0$ as $R_1 \rightarrow \infty$. This fact is consistent with our observation that the maximum of $\text{Im}(C_{11} + C_{22})$ is distributed around the $u_1 = 0$ axis and the $(b)$-type contour maximally encloses the area around this axis, hence leading to quantization of $Q_+$. On the other hand, $\text{Im}(C_{11} - C_{22})$ changes sign as we move along the $u_1 = 0$ axis and hence $Q_-$ shows a non-monotonic behaviour and finally goes to zero as $R_1 \rightarrow \infty$.

The same logic can be used to understand the fact that $(a)$-type contour always shows a non-monotonic behaviour for the pumped charge which always goes to zero in the asymptotic limit. This is so because the $(a)$-type contour always engulfs areas where $\text{Im}(C_{11} + C_{22})$ and $\text{Im}(C_{11} - C_{22})$ both undergo sign changes, hence canceling to zero in the asymptotic limit. Finally it is clear from the above arguments that the $(c)$-type contour will show a behaviour which is exactly complementary to the $(b)$-type contour since the maximum values of $\text{Im}(C_{11} + C_{22})$ and $\text{Im}(C_{11} - C_{22})$ are distributed around complementary axis (i.e., $u_1 = 0$ and $\tau = 0$ axis respectively). Hence we have shown that by choosing appropriate contours we are in a position to selectively pump charge from the leads to the superconductor ($Q_+ \neq 0$) while keeping the relative transfer of charge between the leads to be zero ($Q_- = 0$) or pump charge between the leads while keeping the superconductor decoupled (i.e., $Q_+ = 0$).

**Fourier analysis of pumped charge:** Finally we would like to point out a crucial difference in the scattering matrix for the MBS in the multiply connected geometry and other forms of bound states like the ABS or the RL in the same geometry. The other form of bound states in general would lead to a $\phi$ dependent denominator which appears due to the Fabry-Perot type interference due to the circulating paths of electrons or holes around the multiply connected geometry. But due to the fine tuned symmetry between an electron and a hole for the MBS, all such phases cancel out to provide a $\phi$ independent denominator. This can be seen clearly by comparing the expression for $D(E)$ in Eq.12 with $d(E)$ in Eq.15. Also note that the $\phi$ dependence for MBS appears in the numerator of the scattering matrix as a pure cosine. The same difference in dependence also persists in the expression for the integrand of the pumped charge, as can be seen from Eqs.16 and 17. This essentially means that the pumped charge for the MBS has a single periodicity with respect to variation of $\phi$ as opposed to the RL or the ABS which will have superperiodics in $\phi$. This can serve as a diagnostic for the MBS. This fact can be seen very clearly from a Fourier analysis for the pumped charge shown in Fig.5.

As expected the MBS case show a clear delta function like peak which is independent of the parameters chosen for the analysis due to the fact that only the first of the harmonics contributes to this case, whereas for the ABS, there are multiple frequencies signifying the presence of higher harmonics which can be traced back to the $\phi$ dependent expression for $d(E)$.

**Discussions and conclusion :**

In this letter, we have discussed the importance of a ring geometry to get non-zero pumped charge through the MBS. We note that asymptotically two units of charge can either be pumped between the leads or from the leads to the superconductor. We do not get quan-
FIG. 5. (color online) Plot of discrete Fourier transform of $Q_\omega$, $|A(\omega)|^2 = \left| \frac{1}{N} \sum_{n} Q[\phi] e^{2i\pi n \omega \phi / N} \right|^2$ as a function of the frequency $\omega$ for pumping in $v_1 = \bar{\tau}_0$ plane for ABS in red and MBS in blue. The contour is chosen to be circular with $R = 25$. The number of points $N$ is chosen to be 100 and the other parameters are given by $u_2 = 1$, $v_1 = 3$, $v_2 = 2$.

We then show that the Fourier analysis of the pumped charge through an $AB$ ring, can be used as a diagnostic to distinguish between MBS from other spurious zero energy states. In particular, the charge pumped through the MBS is different from the charge pumped through either the resonant level or the ABS in that it has no higher harmonics. This is true independent of choice of the contour, and is consequently a strong diagnostic.

Acknowledgments

The research of K.M.T was supported in part by the INFOSYS scholarship for senior students.

---

1. A. Kitaev, Physics-Uspekhi 44, 16 (2001).
2. J. Alicea, Rep. Prog. Phys. 75, 076501 (2012).
3. C. W. J. Beenakker, Ann. Rev. Cond. Matt. Phys. 4, 113 (2013).
4. V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, L. P. Kouwenhoven, Science 336, 1003 (2012).
5. M. T. Deng et al, NanoLett. 12, 6414 (2012); A. Das et al, Nat. Pys. 8, 887 (2012); L. P. Rokhinson, X. Liu and J. K. Furdyna, Nat. Phys. 8, 795 (2012); A. D. K. Finck et al, Phys. Rev. Lett. 110, 126406 (2013); S. Nadji-Perge et al, Science 346, 602 (2014); R. Pawlak et al, cond-mat/1505.0678; M. Ruby et al, cond-mat/1507.0314.
6. D. Sen and S. Vishweswara, EPL 91 660009 (2010); W. DeGottardi, D. Sen and S. Vishweswara, New J. Phys. 13, 065028 (2011); P. W. Brouwer, M. Dukheim, A. Romito and F. von Oppen, Phys. Rev. Lett. 107, 196804 (2011); ibid, Phys. Rev. B84, 144526 (2011); S. Gangadharaiah, B. Braunecker, P. Simon and D. Loss, Phys. Rev. Lett. 107, 036801 (2011); B. H. Wu and J. C. Cao, Phys. Rev. B85, 085415 (2012); M. Leijnse and K. Flensberg, Semi-cond. Sci. Tech. 27, 124003 (2012); H. F. Lu, H. Z. Lu and S. Q. Shen, Phys. Rev. B86, 075318 (2012); B. Zocher and B. Rosenow, Phys. Rev. Lett. 111, 036802 (2013).
7. Y. Oreg, G. Rafael, and F. von Oppen, Phys. Rev. Lett.105 177002 (2010); I. C. Fulga, F. Hassler, A. R. Akhmerov and C. W. J. Beenakker, Phys. Rev. B83, 155429 (2011); A. R. Akhmerov, J. P. Dahlhaus, F. Hassler, M. Wimmer and C. W. J. Beenakker, Phys. Rev. Lett. 106 057001(2011); H. F. Lu, H. Z. Lu and S. Q. ShenPhys. Rev. B86, 075318 (2012); P. Wang, Y. Cao, M. Gong, G. Xiong and X. Q. Li, Europhys. Lett. 103,57016 (2013).
8. L. Fu and C. L. Kane, Phys. Rev. Lett.100, 096407.
9. J. D. Sau, R. M. Lutchyn, S. Tewari, and S. Das Sarma, Phys. Rev. Lett.104, 040502 (2010); T. Stanescu and S. Tewari, J. Phys. Cond-mat. 25, 233201 (2013).
10. K. T. Law, P. A. Lee and T. K. Ng, Phys. Rev. Lett.107, 237001 (2011); A. Golub and B. Horovitz, Phys. Rev. B85, 153415 (2011); A. Haim, F. Berg, F. von Oppen and Y. Oreg, Phys. Rev. Lett.114 166406 (2015); A. Haim, E. Berg F. von Oppen and Y. Oreg, cond-mat/1509.00463.
11. J. D. Thouless, Phys. Rev. B27, 6083 (1983).
12. M. Buttiker, H. Thomas, and A. Pretre, Z. Phys.B94, 133 (1994).
13. P. W. Brouwer, Phys. Rev. B58, R10 135 (1998).
14. L. Aleiner and A. V. Andreev, Phys. Rev. Lett.81, 1286(1998).
15. F. Zhou, B. V. Spivak and B. L. Altschuler, Phys. Rev. Lett.82, 608 (1999).
16. Y. Wei, J. Wang and H. Guo, Phys. Rev. B62, 9947 (2000).
17. Y. Levinson, O. Entin-Wohlman and P. Wolfle, Physica A 302, 335 (2001).
18. F. Renzoni and T. Brandes, Phys. Rev. B64, 245301 (2001).
19. M. Blaauboer and E. J. Heller, Phys. Rev. B64, 241301(R) (2001).
20. M. L. Polianski and P. W. Brouwer, Phys. Rev. B64, 075304 (2001).
21. J. E. Avron, A. Elgart, G. M. Graf, and L. Sadun, Phys. Rev. Lett.87, 236601 (2001).
22. O. Entin-Wohlman, A. Aharony and Y. Levinson, Phys. Rev. B65, 195411 (2002).
23. A. Aharony and O. Entin-Wohlman, Phys. Rev. B65, 241401(R) (2002).
24. J. N. H. J. Cremers and P. W. Brouwer, Phys. Rev. B65, 115333 (2002).
25. M. L. Polianski, M. G. Vavilov, and P. W. Brouwer, Phys. Rev. B65, 245314 (2002).
26. M. Moskalets and M. Buttiker, Phys. Rev. B66, 205320(2002).
27. P. Sharma and C. Chamon, Phys. Rev. B68, 035321(2003).
28. R. Citro, N. Andrei, and Q. Niu, Phys. Rev. B68, 165312(2003).
29. A. Banerjee, S. Das, and S. Rao, arXiv:cond-mat/0307324 (unpublished).
30. M. Moskalets and M. Buttiker, Phys. Rev. B69, 205316(2004).
31. S. Das and S. Rao, Phys. Rev. B70, 155420 (2004).
32. E. Sela and Y. Oreg, Phys. Rev. B\textbf{71}, 075322 (2005).
33. J. Splettstoesser, M. Governale, J. Konig, and R. Fazio, Phys. Rev. Lett.\textbf{95}, 246803 (2005).
34. S. Banerjee, A. Mukherjee, S. Rao and A. Saha, Phys. Rev. B\textbf{75}, 153407 (2006).
35. A. Agarwal and D. Sen, Phys. Rev. B\textbf{76}, 035308 (2007).
36. S. Das and V. Shpitalnik, EPL 83, 17004 (2008).
37. J. Wang, Y. Wei, B. Wang and H. Guo, Appl. Phys. Lett.\textbf{79}, 3977 (2001).
38. M. Blaauboer, Phys. Rev. B\textbf{65}, 235318 (2002).
39. B. Wang and J. Wang, Phys. Rev. B\textbf{65}, 153311 (2002).
40. B. Wang and J. Wang, Phys. Rev. B\textbf{66}, 201305 (2002).
41. F. Taddei, M. Governale, and R. Fazio, Phys. Rev. B\textbf{70}, 052510 (2004).
42. M. Governale, F. Taddei, R. Fazio, and F. W. J. Hekking, Phys. Rev. Lett.\textbf{95}, 256801 (2005).
43. N. Kopnin, A. S. Melnikov, and V. M. Vinokur, Phys. Rev. Lett.\textbf{96}, 146802 (2006).
44. J. Splettstoesser, M. Governale, J. Konig, F. Taddei, and R. Fazio, Phys. Rev. B\textbf{75}, 235302 (2007).
45. S. Russo, J. Tobiska, T. M. Klapwijk, and A. F. Morpurgo, Phys. Rev. Lett.\textbf{99}, 086601 (2007).
46. A. Saha and S. Das, Phys. Rev. B\textbf{78}, 075412 (2008).
47. M. Gibertini, R. Fazio, M. Polini and F. Taddei, Phys. Rev. B\textbf{88}, 140508(R) (2013).
48. M. Alos-Palop, Rakesh P. Tiwari, and M. Blaauboer, Phys. Rev. B\textbf{89}, 045307 (2014).
49. Ganesh C. Paul and Arijit Saha Phys. Rev. B\textbf{95}, 045420 (2017).
50. F. W. J. Hekking and Y. V. Nazarov, Phys. Rev. Lett.\textbf{71}, 1625 (1993).
51. K. M. Tripathi, S. Das, S. Rao, Phys. Rev. Lett.\textbf{116}, 166401 (2016).