An Organizing Principle for the Water-Energy-Food Nexus

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Abstract: The nexus between water, energy, and food has recently evolved as a resource-management concept to deal with this intimately interwoven set of resources, their complex interactions, and the growing and continuously changing internal and external set of influencing factors, including climate change, population growth, habits and lifestyles alternations, and the dynamic prices of water, energy, and food. While an intriguing concept, the global research community is yet to identify a unifying conceptual and mathematical framework capable of adapting to integrate gathered knowledge and ensuring inclusivity by accounting for all significant interactions and feedbacks (including natural processes and anthropogenic inputs) within all nexus domains. We present an organizing roadmap for a conceptual and mathematical representation of the nexus. Our hope is that this representation will organize the nexus research and formalize a way for a generalizable framework that can be used to advance our understanding of those complex interactions, with hope that such an approach will lead to a more resilient future with sustained resources for the future generations.

Keywords: nexus; water-energy-food; resource management

1. Introduction

Agriculture, energy, and water are coupled so strongly that actions to achieve sustainability in any individual domain will directly impact the others. With growing resource demands and shrinking supplies, this interwoven nature creates potential opportunities or threats to our resource supply chains depending on how actions in one domain amplify or dampen benefits (or harms) across all domains.

The Water-Energy-Food (WEF) Nexus [1] has emerged as a widely accepted concept that can replace the classic approaches centered around enhancing individual sectors’ productivities with a more collective philosophy focused on maximizing the nexus-system’s overall efficiencies. Its elegant, yet deceiving, simplicity attracted a wide range of endorsements from researchers and policy-makers, embracing nexus-thinking as the new means to establish tradeoffs and boost synergies across nexus pillars. Early nexus efforts seemed to balance multiple objectives whilst also seeking to balance an annual budget of resources of differing units. Those efforts focused on the seemingly manageable exercise of multi-resource management framework where demands were met with supplies, albeit each uniquely dispersed in space, time, quality, and price (only to name the most significant attributes).

However, as the nexus continued to become unpacked, it revealed new dimensions of interconnectivities that are highly multi-disciplinary at the intersection between the physical sciences, the biogeochemical cycles (carbon, nitrogen, phosphorus, among others), the anthroposphere, technology, economy, and the social sciences. Such findings added layers of complexity to an already complicated problem. Here we define complex as the nonlinear linkages and feedbacks that create
unexpected behaviors, and complicated is defined as increasing number of operations that must be performed. As the number of elements and as nexus users commenced to unpack the nexus concept, this elegant simplicity began to reveal its intrinsic complexities leading to the proliferation of tens of unique nexus topological frameworks (ex. [2–17]).

All of a sudden, this healthy expansion landed an open debate on the scope and boundaries of the nexus and put forwards the following question: what makes a resource available for distribution to satisfy a given demand? One simple answer can be physical availability and thus resource demands can be satisfied from all the resource stock available at any cost (energy, economic, technological among others), but this can be unsustainable, and allow for resource depletion. Alternatively, supplies can draw from an optimum hierarchical pool of options that utilize renewable resources and restricts the depletion of others. We firmly believe that the nexus-thinking should bring supplies and demands to a sustainable balance. Thus, nexus-thinking must have the breadth to cover such a massive scope of resources in space, time, quality, and many other attributes, but also the depth to allow for tracking all “utilizable” resources all the way from their raw, natural source. As such, the amount of drinking water available in location X at time T is impacted by the water cycle (thus the connection with the natural science including biophysics, climate, and ecosystems), infrastructure and technology (thus the connection with policy, management, and the anthroposphere), and allocations (thus connection with economy, populations, and social sciences).

This grew into a debate in search for an agreement upon a unified nexus definition and framework, which remains a challenge. Here, we present a functional nexus framework that is easily transferrable into a mathematical form and leads to a succinct rhetorical nexus definition which we conceptualize as a concept that describes an interwoven system that transforms and transports resources from natural systems to anthropogenic consumption. We reviewed a wide range of unique nexus topological frameworks (ex. [2–17]) and identified four essential framework ingredients: (1) an organizing principle, (2) a protocol for component-component interactions, (3) modifying factors (endogenous or exogenous) that influence system interactions, and (4) a directory of system components. All current frameworks had one or more of those ingredients, but none (to our interpretation) contained all four. Furthermore, many had some of those ingredients implicitly contained, making it hard to trace back the origin of some nexus resources, or to consider a wider range of resource-management options beyond current practices. For example, many current nexus topologies consider water as a final resource ready for allocations to different demand routes, including domestic use as well as food and energy production. It is hard with such topology to trace those resources in a spatiotemporally distributed way and consider the wide range of available management options that can be available. It is also hard to account for pollution and different losses and inefficiencies across steps except through lump sum factors or problem-specific life cycle assessment exercises. Another example is food, being mostly considered as a resource (either imported or farmed locally) rather than seeing it across the continuum starting from land, weather, nutrients, and water through land-management and agriculture, then food-to-crop conversion, transport, retail, storage, and distribution.

The proposed approach outlined below explicitly defines those ingredients, assembles them into a framework, and renders a mathematical description. This work is not meant to be definitive, but rather a point for community discussion that, we hope, will lead to a community consensus on the WEF nexus system definition, framework, and a community model [3].

2. Methodology

This section outlines the four essential nexus framework ingredients briefly discussed in the previous section and lays the grounds for establishing a nexus mathematical framework.

2.1. Four Essential Nexus Framework Ingredients

Here, we briefly describe the four essential nexus ingredients and provide brief examples of their form and use.
2.1.1. Organizing Principles and Protocol for Interaction

Organizing principles are often chosen based on the functionality of the system or its components. That is, the functional properties of the system components are used to classify them. This functionality, expressed in general terms, becomes the basis for an organizing principle. The WEF nexus is a resource-hierarchical process that meets the population’s water, energy, and food demands. Resources are extracted, refined, and redistributed through four generic stages. The WEF organizing principle is the relative allocation of stocks and flows of resources through these four stages. See Figure 1 for the organizing principle’s application to water, energy, and food, respectively.

Stage 0 encompasses the spatiotemporal distribution of resource stocks at a planetary or regional scale, in their different in situ phases, chemical compositions, and thermodynamic state, all at the beginning of the time-step of nexus analysis. It also includes the main elements’ cycles (water vapor, CO$_2$, N, P, K, and S) that are key for natural redistribution of Earth’s resources. Stage 1 is the spatiotemporal redistribution of water, energy, and land/biome (not food yet at this stage) resources in space and time during the nexus simulation or analysis due to natural interactions and energy redistribution, including rain, groundwater recharge, photosynthesis and biomass buildup, and river flow. Stage 2 refines and redistribute resources by adding a combination of work, energy, and technology to improve the quality, transport, or store resources in forms that are ready for anthropogenic use. Stage 3 holds resources ready at the points of consumption for distribution to satisfy the spatiotemporal demands for water, energy, and food (Figure 1a).

![Figure 1](https://example.com/figure1.png)

**Figure 1.** (a) Generic resource management. This provides the organizing principle for a WEF nexus framework. Biophysics, Natural Resource, improved quality and availability, and end use are the resource stages. Movement from a lower stage to a higher stage requires inputs. Each stage is influenced by the modifying factors of material cycles, climate and ecosystem, policy and management, and the population and economy, respectively. (b) Organizing principle applied to the food pillar. (c) Organizing principle applied to the energy pillar. (d) Organizing principle applied to the water pillar. Note that stage 0 can be at a higher level than later stages. This represents the capacity of some natural transitions to happen spontaneously.

Each stage is associated with a level. A resource may move spontaneously from a higher level to a lower level (decomposing food waste) in reaction to Earth’s natural biogeochemical cycles (water,
energy, and elements) and bound to the laws of physics and thermodynamics (not necessarily associated with anthropogenic action; for example, all transitions from level 0 to 1 are natural, while all other transitions have an anthropogenic element). However, a resource requires external inputs to move from a lower level to a higher level (water purification like reverse osmosis or wastewater treatment). Knowledge-based inputs include research, data, or experience which manifest through innovation into technology. The consequence of this input is a change in the material and energetic costs (the efficiency) of a transition. Each stage transition generates byproducts which may be classified as a pollutant or repurposed as a resource.

Furthermore, the collective set of stages of water, energy, and food comprise the system components. The succession of stages for a single resource forms a nexus pillar (Figure 1b–d). A pillar can contain multiple parallel resource threads that allow for attribute variety. For example, the energy pillar can contain a stock or portfolio of coal, oil, wind, solar, and nuclear threads; the food pillar can contain threads for each food variety, and the water thread can contain sources of varying quality.

2.1.2. Modifying Factors

Modifying factors influence resource availability in space and time, as well as their value, quality, and quantity at each resource stage. Those factors can alter the required inputs for stage transitions. An exhaustive list of all possible modifying factors is not presented, rather they were categorized into four types and each associated with a corresponding single stage where they had the highest influence (Figure 1a): materials cycles (e.g., C, N, P, and S cycles) influence the biophysical compounds (stage 0); climate and ecosystem functions influence natural resource (stage 1) (for example, climate change accelerates the hydrologic cycle); policies and management strategies influence the way in which we refine and concentrate resources (stage 2) (for example, the endangered species act in the US restricts resource extraction where it may harm endangered species); while people and economies influence consumption patterns and demands (stage 3) (for example, rates of resource consumption of a nation, per capita, are often correlated with gross domestic product (GDP) of the nation).

The food pillar starts at stage 0 with C, N, P, K, and S and interacts with water, energy, and soil to form the biome (stage 1). Society complements nature’s capacity (rangelands and rain-fed agriculture) by transforming parts of the biome into harvested biomass (stage 2) through stewardship that requires energy and/or water. Harvested biomass is processed into food and transported to the demand nodes (people) for consumption (Figure 1b).

The energy pillar starts with the lowest energy form of each resource (CO₂ for fossil fuels, raw materials for nuclear, and sunlight). Stage 1 encompasses the spatiotemporal distribution of energy resources (potential energy) and their natural interaction with the surrounding ecosystem and climate. Society utilizes technology to concentrate the raw energy resources: sun, wind, fossil fuels, nuclear, water flow, and biomass into energy capacity (available energy, stage 2) which is delivered to demand nodes (people) for consumption at stage 3 (Figure 1c).

The water pillar starts with water stocks divided into three pools: non-liquid form (this is the dynamic water typically in vapor and snow which redistributes in stage 1 as impacted by climate and the energy cycle to renewable groundwater and surface water resources, as well as into green water); saltwater (this is water’s biggest stock and can be utilized by adding energy and knowledge/technology to purify it into usable water using technologies like reverse osmosis); and static stocks (these are existing groundwater and surface water stocks which can be tapped into during years of drought, but should be differentiated from the earlier stock of safe yield). Weather events, topography, land-cover, and geology redistribute moisture spatiotemporally in soil, rivers, lakes, and in groundwater, forming the potential water resource (stage 1). Energy or work is used to extract and purify this water to make it available through Policy and Management in stage 2, and is then delivered to the demand nodes (people) for use (Figure 1d).
2.1.3. Directory of System Components

System connections are the dynamic inter- and intra-resource fluxes needed to lift a resource through the suggested nexus resource hierarchy stages. Mechanistic, connection-based definitions of the major resource fluxes are congruent with the nexus organizing principle and compatible with the established research lexicon (Box 1).

Water used as a material input for stage transitions in the food and energy pillars has been studied extensively [18] through virtual water [19], water footprints [19–21], and integrated water resource management [22,23]. In these research contexts, water is assigned a descriptive color (green, blue, grey) based on its origin and use. These “color definitions” are congruent with the proposed nexus framework and simply redefined in terms of the origination and destination stage. One major water flow within the nexus topology remained unnamed in the water color context, so it was assigned. Red water (subset of blue water) is the water used for energy production, commonly to dispose of waste heat, and may be consumed in the process through evaporation.

The food pillar has the fewest outgoing connections and the largest number of inputs. Agricultural products used for energy production are designated as red food (to be consistent with the water pillar). Other connections include food waste and livestock feed.

**Box 1.** A directory of nexus components.

| **Blue water:** | water which required energy or other resources to arrive at its end use in industry, residential or agriculture applications. |
| **Green water:** | water which did not require energy or other resources to arrive at its end use in industry, residential or agriculture applications. |
| **Grey water:** | repurposed non-consumptive water that has been diminished in quality or availability after industrial or agricultural use. |
| **Red water:** | water used in energy production. |
| **Consumptive water use:** | Water that is returned to the lowest stage. |
| **Non-consumptive water use:** | Water that is returned to a lower stage, but not to the lowest. |
| **Red food:** | agricultural products used in energy production. |
| **Food waste:** | food that is moved from the highest food stage to any lower stage. |
| **Livestock feed:** | available agricultural feed that is returned to the biome stage. |

Energy is a required input for resource stage changes and reallocations. Anthropogenic energy/work devoted to the biome is named stewardship and energy required to extract and refine water is labeled treatment.

2.2. A Nexus Framework

The organizing principle, system components, interactions, and modifying factors are combined to create a simplified representation of the WEF nexus (Figure 2). Figure 2 is complex at first glance, but a few comments will reveal its intrinsic simplicity. The circles, shades, and arrows represent the stage hierarchy, modifying factors, and system connections, respectively. Black arrows represent carbon flows. Arrows going from lighter to darker gray generally represent anthropogenic connections; while those going from darker to lighter gray generally represent natural connections. Crossing arrows represent the presence of a first order feedback. The topological representation in Figure 2 can be expanded to contain more detail by including additional stages, threads, and connections.

The system suggested in Figure 2 can be rendered into a mathematical representation of the nexus by performing mass or energy balance for each resource at each stage. The total number of balance equations is equal to the number of stages \(N_s\) times number of resource threads \(N_r\). Additional mathematical relationships exist that relate each material flow associated stage transitions...
to the system connections. Together, those relationships create a set of equations that can describe the *nexus*. A complete derivation of the balance equations is presented in the next section and supplemental materials.

Figure 2. Organizing principle, modifying factors, components of each pillar, and the connections between those components. Blue elements of the diagram represent water, red represents energy, green represents food, black represents carbon and yellow represents nutrients. NPP is the net primary production or the net change in carbon stocks, and GHG is greenhouse gases. Evapotranspiration is the total transpiration through plant processes and evaporation from soil and open water.

2.3. Governing Equations

The proposed organizing principle and topological framework can be readily translated into a set of balance equations. The balance equations for a four-stage resource management process (material cycles, climate and ecosystem, policy and management, and population and economy) containing three interwoven resources (water, energy, food) can be written by tracking the inputs and outputs to each individual resource stage. Those balance equations represent resource flows and transformations, and are organized into two major categories:

1. **Intra-resource flows (M):** those flows account for resource flow from one stage to another within the same resource pillar. For example, groundwater pumped for drinking or domestic use. We impose the rule that resource flows that proceed from a lower stage and terminate at a higher stage cannot skip stages, and that resource usage flows (M) can only initiate from the final, highest resource stage (population and economy). Resource flows that move stage-wise upward cannot occur spontaneously, whilst resource flows that move stage-wise downwards may, but do not necessarily, occur spontaneously.

2. **Inter-resource flows (G or C):** those flows can be transformational (G) or transitional (C) with inputs from one resource pillar to another. Transformational flows (G) are needed when resources of one pillar are the base resource in the production of a resource from a different pillar (for example:
water for hydropower, agricultural products for bioenergy). Transitional flows (C) represent resources required as inputs for stage transitions across all pillars (Ex.: water to transition agricultural land to crops, water needed for thermoelectric cooling, water needed to process and wash crops into food). Transformational and transitional flows are drawn from a resource’s highest stage and may partition and return to lower stages within the pillar of origin through the so-called pollution flows (P).

Efficiencies in flows (M), transformations (C), and transitions (G) are controlled through matrices \( \alpha, \beta, \text{ and } \gamma \), respectively. The flows to an arbitrary stage \( W_i \) in a water pillar (as an example) are summarized in Figure 3.

The proposed framework presents an elegant balance between simplicity in the organizing principles and topological rules, yet inclusivity in its ability to cover a wide and comprehensive set of natural and anthropogenic processes and controls. Table 1 (stocks) and Table 2 (fluxes, rates of transformation, processes) demonstrates how these proposed concepts can be applied to Figures 1 and 2 and cover almost any process or control at the water-energy-food nexus scale.

**Table 1.** The dynamics and organization of the major resource-stocks for the water-energy-food nexus under the proposed organizing principles and topological rules.

| Named Resource Flow | Designation | General Description |
|---------------------|-------------|---------------------|
| Overall Water Stock | \( W_0 \)   | This includes the entire water stock in the region of study and can be categorized into static and dynamic stocks. Static stocks are like seawater, permafrost, surface, or ground-water (in different qualities and salinity levels). Dynamic water including non-liquid water that eventually precipitates (rain or snow). |
| Potential Water Resource | \( W_1 \) | This is the potentially usable water from the \( W_0 \) stock. It can be the water that precipitates (rain or snow) and become green (soil moisture) and blue water (rivers, lakes, or groundwater recharge). It can also be any part of the static water stock that is potentially extractable including groundwater reserves, “old” water, or even part of seawater if water desalination is considered. |
| Available Water | \( W_2 \) | This is the water that is made available by anthropogenic intervention including treatment, transportation, pumping or any form of purification or extraction. |
| Water Use | \( W_3 \) | This is the part of the available water that is distributed and used to satisfy the water demands for drinking, cooking, and other domestic uses, as well as the water needed for farming (irrigation), as well as food and energy production. |
| Overall Energy Stock | \( E_0 \) | This includes the carbon stock (fossil fuel, coal, forest, \( \text{CO}_2 \), solar radiation (sun), wind, heavy metals, sea waves, or any available sources of potential, kinetic, thermal, or nuclear energy. |
| Potential Energy Resource | \( E_1 \) | This is the potentially usable portion of the \( E_0 \) stock based on location, cost of extraction, and availability of materials, technology, and skills. |
| Available Energy | \( E_2 \) | This is the energy that is made available in usable form (for example: heat, kWh, gas, fuel) by anthropogenic intervention and technology (for example: photovoltaic panels, water or wind turbines, stoves, refineries). |
| Energy Use | \( E_3 \) | This is the part of the available energy that is distributed and used to satisfy domestic and industrial energy demands, as well as energy needed for water extraction, treatment, and distribution, and for agriculture and food processing, transportation, storage, and cooking. |
| Biophysics and Nutrient Cycles | \( F_0 \) | This includes the stocks and distributions of natural and industrial nutrients in the area of study and covers the biogeochemical cycles of basic nutrients (C, N, P, K, S) and the biophysical dynamics of organic matter and microbial communities. It also includes soils, water bodies, and any media that support the growth of organic life. |
| Land and Biome | \( F_1 \) | This is the distribution of nutrients and the quality of medium controls the potential for biomass growth. This is where land is divided into fertile lands or rangelands vs. deserts, for example. |
| Available Biomass | \( F_2 \) | This is where the potentially fertile lands are utilized for food production by anthropogenic intervention and land management. Grazing and agriculture generate crops and livestock biomass. |
| Food Use | \( F_3 \) | This is where crop-to-food conversion and meat is processed, stored, and distributed (including import/export dynamics) to satisfy the population demands. |

**Table 2.** The dynamics and organization of the major processes and fluxes for the water-energy-food nexus under the proposed organizing principles and topological rules.

| Named Resource Flow | Designation | General Description |
|---------------------|-------------|---------------------|
| Rain | \( M_{W_i,W_j} \) | It can also be any other form of precipitation like snow or fog, as well as lateral groundwater movement or any natural movement within the water stock in space and time. |
| Red Water | \( C_{W_i,M_{W_j}} \) | The water required to produce available energy, for possible use, from the larger energy resource pool. |
Table 2. Cont.

| Named Resource Flow | Designation | General Description |
|---------------------|-------------|---------------------|
| Blue Water          | $C_{W_i,M_i,W_i}$ | The water that is used to produce food through agriculture and can be irrigation water or processing water. |
| Grey Water 1        | $M_{W_i,W_i}$  | Wastewater that will be reused at the point of use. This required that black water and grey water are separated. For example, the reuse of shower water for flushing toilets. |
| Grey Water 2        | $M_{W_i,W_i}$  | Wastewater that is returned to surface water, streams, or ground water, typically after treatment. This flow is often associated with the energetic costs of wastewater treatment and water quality change prior to discharge. |
| Non-Consumptive Water Use | $P_{W_i,W_i,W_i} \times C_{W_i,M_i,W_i}$ | The fraction of the red water that is returned to a surface water body, stream, or groundwater (any potential water source). |
| Consumptive Water Use | $P_{W_i,W_i,W_i} \times C_{W_i,M_i,W_i}$ | The fraction of the red water that is evaporated in the creation of available energy. |
| Runoff               | $P_{W_i,W_i,W_i} \times C_{W_i,M_i,W_i}$ | The non-consumptive fraction of irrigation water that ends in a surface water body or stream or deep percolation into the groundwater. |
| Evaporation          | $P_{W_i,W_i,W_i} \times C_{W_i,M_i,W_i}$ | The consumptive fraction of irrigation water that evaporates and transpires. This is the fraction of the total blue water that is used for irrigation returning to the atmosphere. |
| Treatment            | $C_{E_i,M_i,W_i}$ | The energy required to treat a potential water resource to sufficient quality that it becomes available water for use. Drinking water treatment, chlorination for swimming pools, or pumping costs for irrigation are examples. |
| Stewardship          | $C_{E_i,M_i,W_i}$ | Any anthropogenic energetic input that is directed to promote the production of food within the biome. This include fuel for tillage planting and harvest, fertilizer application, and weed control. |
| Red Food             | $G_{F_i,F_i}$  | Harvested biomass that is converted into biofuel. This can be from corn, sugar, switchgrass, seed crops, and more. |
| Feed                 | $M_{F_i,F_i}$  | Harvested biomass that is redirected to feed livestock. This includes feed for aquaculture, ungulates, and poultry. |
| Food Waste           | $M_{F_i,F_i}$  | The fraction of usable food that is not used for any purpose before spoilage. This can include spoilage in the distribution chain or within the home. This definition of waste does not consider elements of non-anthropogenically consumed food that is reused as feed (see above). |
| NPP                  | $G_{F_i,F_i}$  | Net primary production, the carbon that is absorbed into the biome through its natural and anthropogenic processes. |
| GHG                  | $M_{E_i,F_i}$  | Greenhouse gases, typically CO$_2$, that is a biproduct of energy production and consumption. |

Figure 3. Schematic diagram for Water-Energy-Food (WEF) representation at an arbitrary stage in the water pillar. Nodes $W_i$ is preceded by node $W_{i-1}$ and is followed by node $W_{i+1}$, representing the water stock in their corresponding stage.
Nodes are represented as follows: Resources\(i\) with resources referred to as W, E, and F and stages with 1, 2, 3, and 4. Intra-resource flows (M) are represented as \(M_{X_iX_j}\) where the indices \(i\) and \(j\) can have a value of 1, 2, or 3 and the index X can represent F, E, or W.

Furthermore, resource flows that are transformational and exchange resources between pillar stages are represented as \(G_{X_iY_j}\) (e.g., \(G_{F_3E_2}\) is red food) where again the indices \(i\) and \(j\) can be 0, 1, 2, or 3 while indices X and Y can represent F, E, or W, but not all index permutations appear or need to have values. Transformational flows are accompanied by a coefficient \(A_{X_iY_j}\) that is a function of the modifying factors. The demand for resource \(X\) by the population, \(D_{X_i}\), is also a function of modifying factors. Transformational flows are denoted by \(C_{X_iM_{Y_{j+1}Y_{j+1}}}\) and represent the flow of a resource \(X\) (restricted to only originate from stage 3) used in resource \(Y\) as an input during a stage transition \((M_{Y_{j+1}Y_{j+1}})\). The aggregate pollution flows are by \(P_{X_iM_{Y_{j+1}Y_{j+1}}}\) for transformational flows and \(P_{X_iM_{Y_{j+1}Y_{j+1}}}\) for transitional flows. The transformational, transitional, and pollution flows interweave the nexus together across resource pillars.

The general form of the mass balance for each node \((X_i)\) can be represented by the following general equation:

\[
\begin{align*}
\frac{dX_i}{dt} &= \begin{cases} 
(1 - \delta_{i,0})(1 - \alpha_{X_{i-1}X_i})M_{X_{i-1}X_i} \\
-(1 - \alpha_{X_iX_{i+1}})M_{X_iX_{i+1}} \\
+ (1 - \delta_{i,3})M_{X_iX_i} 
\end{cases} \text{Intra - resource flows} \\
&+ \begin{cases} 
A_{Y_{i+1}Y_i}G_{Y_{i+1}Y_i} \\
+ A_{Z_{i+1}X_i}G_{Z_{i+1}X_i} 
\end{cases} \\
- \delta_{i,3} \sum_{j=0}^{2} G_{Y_jX_i} \quad \text{Transformational} \\
- \delta_{i,3} \sum_{j=0}^{2} G_{Z_jX_i} \quad \text{Transformational} \\
- \delta_{i,3} \sum_{j=0}^{2} C_{X_iM_{Y_{j+1}Y_{j+1}}} \quad \text{Inter - resource flows} \\
- \delta_{i,3} \sum_{j=0}^{2} C_{X_iM_{Z_{j+1}Z_{j+1}}} \quad \text{Inter - resource flows} \\
\end{align*}
\]

\[
\begin{align*}
&+ \begin{cases} 
2 \sum_{j=0}^{2} (1 - \gamma_{X_iY_j})P_{X_iG_{X_jY_j}G_{X_jY_j}} \\
+ \sum_{j=0}^{2} (1 - \gamma_{X_iZ_j})P_{X_iG_{X_jZ_j}G_{X_jZ_j}} 
\end{cases} \text{Transformational} \\
&+ \begin{cases} 
2 \sum_{j=0}^{2} (1 - \beta_{X_iM_{Y_{j+1}Y_{j+1}}}P_{X_iM_{Y_{j+1}Y_{j+1}}} \\
+ \sum_{j=0}^{2} (1 - \beta_{X_iM_{Z_{j+1}Z_{j+1}}}P_{X_iM_{Z_{j+1}Z_{j+1}}} \\
+ \sum_{j=0}^{2} (1 - \beta_{X_iM_{Y_{j+1}Y_{j+1}}}P_{X_iM_{Y_{j+1}Y_{j+1}}} \\
+ \sum_{j=0}^{2} (1 - \beta_{X_iM_{Z_{j+1}Z_{j+1}}}P_{X_iM_{Z_{j+1}Z_{j+1}}} 
\end{cases} \text{Transformational} \\
\end{align*}
\]

where \(i = 0, 1, 2, 3\). Table 3 describes the terms of Equation (1).
Note that many of these flows may be zero in targeted applications, but all possible flows are kept for completeness. The Supplement Information document contains the derived equations (from this general form) of all four stages within each of the three nexus pillars. It also contains a derivation of a simplified mathematical form of the nexus equation which we hope can set the stage and initiate the dialogue for advancing a standardized nexus research.

3. Examples

To maintain simplicity, not all possible resource flows or actions are explicitly referenced within the framework as presented in Figure 2. However, the proposed framework was tested and is capable of explicitly representing a wide and comprehensive set of possible flows, processes, and actions. The following examples are included to illustrate the encompassing nature of the framework and how it can be used to determine the conceptual boundary of the WEF nexus.

**Heating water for a cup of tea:** This would consume energy from a home appliance (gas, wood, or electric stove) to heat water used (from the tap or bottled) to the appropriate temperature for its intended consumption: \( G_{E_3 W_2} \). The energy used (function of heater’s efficiency, water volume, and heat capacity of water) to heat the same cup of water is also associated with a carbon cost: \( M_{E_3 E_0} \).

**Hunting and gathering:** Simulating hunting and gathering requires thinking of food calories as the only available energy source. While food calories are typically considered a macro-nutrient and would usually fall under the food pillar \( F_3 \) in modern life, it was the main source of energy for hunters and gatherers and would be considered as energy source \( E_3 \). As such, this can be considered within the stewardship flow as follows: extra calories needed to perform the work of hunting and gathering are burned (energy) to harvest something from the biome and make food available: \( C_{E_3 M_{F_1}} \).

**Hydropower:** A reservoir built for energy production is an available water resource that can be transformed into energy \( G_{W_3 E_2} \). Note that this is not described as \( G_{W_3 E_2} \) because a dam had to be built to make the water available, and the water must go through a turbine (be used) to generate electricity, consistent with the ruleset outlined above. This reservoir also adds a consumptive water loss due to the additional evaporation from the reservoir’s surface: \( P_{W_3 G_{W_3 E_2}} \). A naturally occurring landslide that blocks a stream to create a reservoir would not be part of the nexus framework as there is no anthropogenic action. If, however that same reservoir was outfitted with turbines and

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**Table 3. The Terms of Equation (1).**

| Term | Description |
|------|-------------|
| \( (1 - \alpha_0) (1 - \alpha_{i,i} X_{i}) M_{x_{i}} X_{i} \) | Net incoming resource \( X \) moving from \( X_{i-1} \) to \( X_{i} \). When \( i = 0 \) this term is 0. Example: Rainfall. |
| \( (1 - \alpha_{x,i} X_{i}) M_{x} X_{i} \) | Net outgoing resource \( X \) moving from \( X_{i} \) to \( X_{i+1} \). Example: Surface water pumped to treatment plant. |
| \( (1 - \alpha_3) M_{x} X_{3} \) | Net incoming resource \( X \) moving from \( X_{3} \) to \( X_{6} \). Example: Food waste. |
| \( X_{i} G_{x} Y_{3} X_{i} A_{x} Z_{x} G_{x} Y_{3} X_{i} \) | Incoming transformational (G) flow to \( X_{i} \) coming from \( Y_{3} \), or \( Z_{3} \), respectively. Example: Red food. |
| \( \sum_{j=0}^{2} (1 - \gamma_{x,j} Y_{j}) P_{X_{i} M_{x} Y_{j}} G_{x} Y_{j} \) | Incoming pollution to \( X_{i} \) generated from transformational (G) flow leaving \( X_{j} \) into resource \( Y_{j} \), or \( Z_{j} \), where \( j = 0, 1, 2 \). Example: Waste from biofuel production, or evaporation from surface of hydropower reservoirs. |
| \( \sum_{i=0}^{2} (1 - \gamma_{x,i} Z_{i}) P_{X_{i} M_{x} Z_{i}} G_{x} Z_{i} \) | Outgoing transitional (C) flow leaving \( X_{3} \) into \( M_{Y_{3} Y_{3}} Y_{3} X_{3} \) or \( M_{Z_{3} Z_{3}} Z_{3} X_{3} \). Example: Red water. |
| \( \sum_{i=0}^{2} (1 - \beta_{x,i} M_{x} Y_{i} Y_{i}) P_{X_{i} M_{x} Y_{i} Y_{i}} Z_{i} X_{3} \) | Outgoing transitional (C) flow leaving \( X_{3} \) into \( M_{Y_{3} Y_{3}} Y_{3} X_{3} \) or \( M_{Z_{3} Z_{3}} Z_{3} X_{3} \). Example: The energy required for wastewater treatment. |
| \( \sum_{i=0}^{2} (1 - \beta_{x,i} M_{x} Z_{i} Z_{i}) P_{X_{i} M_{x} Z_{i} Z_{i}} M_{Y_{3} Y_{3}} Y_{3} X_{3} \) | Incoming pollution to \( X_{3} \) generated from transitional (C) flow leaving \( X_{3} \) into \( M_{Y_{3} Y_{3}} Y_{3} X_{3} \) or \( M_{Z_{3} Z_{3}} Z_{3} X_{3} \). Example: Non-consumptive water used for thermoelectric generation that is returned to surface water. |
| \( \sum_{i=0}^{2} (1 - \beta_{x,i} M_{x} Z_{i} Y_{i}) P_{X_{i} M_{x} Z_{i} Y_{i}} M_{Y_{3} Y_{3}} Y_{3} X_{3} \) | Incoming pollution to \( X_{3} \) generated from transitional (C) flow leaving \( X_{3} \) into \( M_{Y_{3} Y_{3}} Y_{3} X_{3} \) or \( M_{Z_{3} Z_{3}} Z_{3} X_{3} \). Example: Energy for grey water treatment before it is discharged into a river. |

\( X \): A reservoir built for energy production is an available water resource that can be transformed into energy \( G_{W_3 E_2} \). Note that this is not described as \( G_{W_3 E_2} \) because a dam had to be built to make the water available, and the water must go through a turbine (be used) to generate electricity, consistent with the ruleset outlined above. This reservoir also adds a consumptive water loss due to the additional evaporation from the reservoir’s surface: \( P_{W_3 G_{W_3 E_2}} \). A naturally occurring landslide that blocks a stream to create a reservoir would not be part of the nexus framework as there is no anthropogenic action. If, however that same reservoir was outfitted with turbines and
other infrastructure, at an energetic and material cost, it would become the transformative flow discussed above.

**Crop-to-Food conversion and virtual water**: Water used to process and wash crops before delivery would be $C_{W_3M_2F_3}$. Virtual water is the time integral $\int (C_{W_3M_2F_2} + C_{W_3M_1F_2}) dt$ which sums water consumption over the crop growth period ($C_{W_3M_1F_2}$) plus the water consumption for the crop-to-food conversion of a particular food item. This concept can be extracted to virtual energy and virtual food if desired.

Economics come into play by determining the distribution of actions across all the resource flows in all the pillars. The economic constraint assigns a value to each stage on the process from the raw resource pool to a resource’s use. What seems to be optimal from a natural resource perspective may not be economically viable. The same applies for technology, which can reduce or change efficiencies within the model.

**Forests and natural resources**: Humanity defines the bounds of the WEF nexus, which continue to enlarge. An untouched wilderness is not a part of the WEF nexus until there is an anthropogenic intervention to exploit it.

4. Discussion and Case Study

The framework as described can be used in multiple ways. The detailed equations in the Supplementary Materials, in conjunction with appropriate input data, can quantify numerically resource allocations and flows. While this is a useful function, it can often be limited by data availability, and the emergence of new technologies is often associated with data paucity. Perhaps a greater utility can be extracted from the framework by seeking answer to simpler questions. What feedbacks loops are impacted when a new technology or practice is implemented? Will this new approach or technology create positive or negative unintended consequences across the nexus? An example of a negative unintended consequence is the increase in irrigation water demand on farmland adjacent to wind turbines [24]. Here, boundary-layer mixing contributes to altered surface gradients of water vapor, thus changing the flux by ~10%. A more exhaustive case study demonstrating positive feedbacks is outlined below.

The emergent technology considers the dual use of land for food and energy production. Agrivoltaics unfolded as disruptive technology that recognizes part of the solar radiation on agricultural lands as needed for crop transpiration (thus food production), while another part that only adds to crop-stress and thus results in increased water requirements at no additional benefit to crop production. Partial shading of agricultural and rangelands with networks of photovoltaic panels reduces total radiation on crops, thus reducing their water demands and provide a steady harvest of kilowatt-hours that can significantly help in building resilience to the agricultural sector, as impacted by climate change and droughts. Agrivoltaics are the emergent technology that transforms the inefficiencies of unneeded solar radiation over agricultural lands into beneficial electrical energy by trying to find the optimal balance of harvesting the most efficient crops (in terms of yield and water requirements) and energy. Images of agrivoltaic systems are presented in Figure 4.

Here, we demonstrate how the nexus system changes if a disruptive technology is added. We consider the emerging case of agrivoltaics within the proposed framework. Agrivoltaic systems combine agricultural activities and solar energy production on the same land area. This has shown to be mutually beneficial in a nascent literature on the subject [25–29]. Figure 5 considers all the nexus connections, identified in Figure 2 demonstrating the general nexus framework, and highlights those connections and resources that would incur major disruption or alteration.
reduces total radiation on crops, thus reducing their water demands and provide a steady harvest of kilowatt-hours that can significantly help in building resilience to the agricultural sector, as impacted by climate change and droughts. Agrivoltaics are the emergent technology that transforms the inefficiencies of unneeded solar radiation over agricultural lands into beneficial electrical energy by trying to find the optimal balance of harvesting the most efficient crops (in terms of yield and water requirements) and energy. Images of agrivoltaic systems are presented in Figure 4.

Figure 4. Images of an agrivoltaic systems. Top: In this implementation, sheep are raised on pastureland below a solar panel array. Bottom: bush beans are grown in the area between and below solar panels.

Those connections represent the flow of resources from all the nexus pillars needed to produce the needed quantities of water, energy, and food in a format (quality, location, and time) that is ready to disseminate to satisfy the nexus demands. Thus, it is generally desirable to have those connectors and resource flows reduced when flows move to a higher order, and increased when flows move to a lower order, as this indicates less resources needed, and thus more efficient production of water, energy, and food. Those resource flows that will be diminished are shaded white, whilst those resource flows that would be increased are shaded black. The addition of agrivoltaic within the WEF nexus would add a new inter resource flow $M_{E1,E2}$ where the land resource is simultaneously considered an energy resource (black arrows in Figure 5). This concept is consistent with the agrivoltaic’s literature.
concept of “land equivalent” whereby the land is mirrored partially to satisfy the land requirements of dual use (food and energy production). E2 is the destination as the product of agrivoltaic systems is electricity. The supplemental energy provided to E2 lessens the burden on alternate energy sources within E1 (by adding additional solar capacity on agricultural lands). The additional energy potential is propagated forward through E2 to E3 and made available for further consumption. This change potentially frees additional water resources previously utilized as red water. That is, both Red Food \( (G_{F3,E1}) \) and red water \( (C_{W3,M_{E1,E2}}) \) are reduced. Mass balance requires that the return flows associated with red water are simultaneously reduced. The reduction of red water also initiates a positive feedback loop whereby the energy needs to support \( C_{W3,M_{E1,E2}} \) are reduced. This in turn reduces the need for red water. The overall reduction in water demand at W3 as a result of the reduced red water flows initiates a reduction of the inter-pillar water flows \( M_{E1,E2} \) and \( M_{E2,E3} \). In the food pillar, the reduction in the flow of red food reduces the inter-pillar food flows and their associated inputs. This further reduces blue water demand, and the associated energetic needs which initiate additional positively reinforced feedbacks that further reduce the withdrawal of resources from the natural system for all pillars. The important takeaway is that the feedback mechanisms that are triggered by the addition of agrivoltaics are all positively reinforcing. That is, agrivoltaic systems reduce the withdrawal of energetic resources, and this creates feedbacks that further reduces resource withdrawals across all pillars.

![Diagram](image_url)

**Figure 5.** Organizing principle, modifying factors, components of each pillar, and the connections between those components as impacted by the agrivoltaic example. Effected flows are refined and colored as white for reduced flows and black for increased flows.
5. Conclusions

The WEF nexus is a concept that describes an interwoven system that transforms and transports resources. This simplified nexus definition results from applying an organizing principle to the system, enumerating the system components, and translating the resulting topology into mathematical form. The topological framework proposed in Figure 2 is consistent with the terminology and approaches of other integrative disciplines, and new definitions for substantial resource flows (e.g., red water or food) extend prior concepts to the WEF nexus. The proposed mathematical representation identifies and organizes the nexus stages, threads, and connections which we hope can form a point of departure for a coordinated WEF research.

Supplementary Materials: The following are available online at http://www.mdpi.com/2071-1050/12/19/8135/s1, Figure S1: Schematic diagram for WEF representation at an arbitrary stage in the water pillar. Nodes $W_i$ is preceded by node $W_{i-1}$ and is followed by node $W_{i+1}$, representing the water stock in their corresponding stage.

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