Semileptonic $D \to \pi/K$ and $B \to \pi/D$ decays in 2+1 flavor lattice QCD

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We present results for form factors of semileptonic decays of $D$ and $B$ mesons in 2+1 flavor lattice QCD using the MILC gauge configurations. With an improved staggered action for light quarks, we successfully reduce the systematic error from the chiral extrapolation. The results for $D$ decays are in agreement with experimental ones. The results for $B$ decays are preliminary. Combining our results with experimental branching ratios, we then obtain the CKM matrix elements |$V_{cd}$|, |$V_{cs}$|, |$V_{cb}$| and |$V_{ub}$|. We also check CKM unitarity, for the first time, using only lattice QCD as the theoretical input.

1. INTRODUCTION

Semileptonic decays of $B$ and $D$ mesons play crucial roles in CKM phenomenology. The $B$ decays such as $B \to \pi l\nu$ and $B \to D l\nu$ determine |$V_{ub}$| and |$V_{cb}$|, which are essential to constrain the CKM unitarity triangle. On the other hand, the $D$ decays such as $D \to \pi l\nu$ and $D \to K l\nu$ provide a good test of lattice calculations because corresponding CKM matrix elements |$V_{cd}$| and |$V_{cs}$| are relatively well determined. In this paper, we report lattice calculations of semileptonic decays in unquenched ($n_f = 2+1$) QCD. By using a staggered-type fermion, which is fast to simulate,

\[
\begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
|V_{cd}| & |V_{cs}| & |V_{cb}|
\end{pmatrix}^{0.24(3)(2)}
\begin{pmatrix}
0.97(10)(2) & 3.8(1)(6) \times 10^{-2}
\end{pmatrix}
\]

Figure 1. Result for CKM matrix. The first errors are theoretical, and the second experimental.

for light quarks, we are able to reduce uncertainties from the “chiral” ($m_l \to m_{ud}$) extrapolation. We calculate form factors for the above 4 different decays, from which the 4 CKM matrix elements are determined, as summarized in Fig. 1. The results for $D$ decays are published in Ref. 1. The results for $D$ decays are published in Ref. 1.
2. SIMULATION DETAILS

We use $n_f = 2 + 1$ dynamical gauge configurations obtained with an improved staggered (“Asqtad”) quark action on a lattice with $a^{-1} \approx 1.6$ GeV, generated by the MILC collaboration \[2\]. For the valence light quarks we use the same staggered quark action, with the valence light quark $(u, d)$ mass $m_{l}^{val}$ equal to the dynamical light quark mass $m_{l}^{st}$. The light quark masses we simulate range $\frac{m_q}{4} \leq m_l \leq \frac{m_q}{2}$, where $m_q$ is the strange quark mass. For the valence charm $(c)$ and bottom $(b)$ quarks we use a tadpole-improved clover action with the Fermilab interpretation \[3\].

To form the heavy-light bilinears from the staggered-type light quark and the Wilson-type heavy quark, we convert the staggered-type quark to the naive-type quark, as in Refs. \[4,5\]. Relevant 3-point functions are then computed in the initial state meson rest frame using local sources and local sinks. We typically accumulate about 500 configurations, and results at 2-4 source times are averaged to increase the statistics.

For the matching factor of vector current $Z_{V_{ub}}$, we follow the method in Refs. \[6,7\], writing $Z_{V_{ub}} = \rho_{V_{ub}}(Z_{V}^{ao}Z_{V}^{bo})^{1/2}$. The flavor-conserving renormalization factors $Z_{V}^{ao(bb)}$ are determined nonperturbatively from charge normalization conditions. For the remaining factor $\rho_{V_{ub}}$ we use results in one-loop perturbation theory \[8\].

3. RESULTS

3.1. $D \rightarrow \pi(K)$ and $B \rightarrow \pi$

The heavy-to-light decay amplitudes are parameterized as

$$|V^\mu| |H| = f_{+}(q^2) |p_H + p_P - \Delta|^\mu + f_0(q^2) \Delta^\mu = \sqrt{2m_H} \left[v^\mu f_\parallel(E) + p_\perp^\mu f_\perp(E)\right]$$

with $q = p_H - p_P$, $\Delta^\mu = (m_H^2 - m_P^2) q^\mu/q^2$, $v = p_H/m_H$, $p_\perp = p_P - E v$, and $E = E_V$. The differential decay rate $d\Gamma/dq^2$ is proportional to $|V_{CKM}|^2 |f_\perp(q^2)|^2$. Below we briefly describe our analysis procedure; see Ref. \[9\] for details.

We first extract the form factors $f_\parallel$ and $f_\perp$, as in Ref. \[10\], and carry out the chiral extrapolation

$$f_+(q^2) = \frac{f_{+}}{(1-q^2)(1-\alpha q^2)}, \quad f_0(q^2) = \frac{f_{+}}{1-q^2/\beta}$$

where $q^2 = q^2/m_H^2$. We obtain

$$f_{+}^{B\pi} = 0.23(2), \quad \alpha^{B\pi} = 0.63(5), \quad \beta^{B\pi} = 1.18(5),$$

for the $B \rightarrow \pi$ decay, and

$$f_{+}^{DK} = 0.64(3), \quad \alpha^{DK} = 0.44(4), \quad \beta^{DK} = 1.41(6),$$

$$f_{+}^{DK} = 0.73(3), \quad \alpha^{DK} = 0.50(4), \quad \beta^{DK} = 1.31(7),$$

for the $D$ decays, where the errors are statistical only. To estimate the error from BK parameterization, we also make an alternative analysis, where we perform a 2-dimensional polynomial fit in $(m_l, E)$. A comparison between the two analyses are shown in Fig. \[11\].

Finally we determine the CKM matrix elements (Fig. \[12\]) by integrating $|f_{+}(q^2)|^2$ over $q^2$ and using experimental branching ratios \[13,14\]. For $|V_{ub}|$ we use the branching ratio for $q^2 \geq 16$ GeV$^2$ in Ref. \[14\]. The systematic errors are summarized in Table \[11\]. The results for $D$ decays agree with experimental results \[11\].

Figure 2. $m_l$-dependence and chiral fits for $f_{+}^{B\rightarrow \pi}$. in $m_l$ for them at fixed $E$. To this end, we interpolate and extrapolate the results for $f_{\perp}$ and $f_{\parallel}$ to common values of $E$ using the parametrization of Becirevic and Kaidalov (BK) \[11\]. We perform the chiral extrapolation using the NLO correction in staggered chiral perturbation theory (SKPT) \[10\]. We try various fit forms \[9\], as shown in Fig. \[2\] and the differences between the fits are taken as associated systematic errors.
3.2. $B \to D$

The $B \to D$ amplitude is parameterized as

$$\langle D | V^\mu | B \rangle = \sqrt{m_B m_D} \times [h_+(w)(v + v')^\mu + h_-(w)(v - v')^\mu],$$

where $v = p_B/m_B$, $v' = p_D/m_D$ and $w = v \cdot v'$. The differential decay rate of $B \to D \nu$ is proportional to the square of $F(w)$, which is a linear combination of $h_+(w)$ and $h_-(w)$. We calculate the form factors at $w = 1$ by employing the double ratio method [13].

Extrapolating the result linearly to $m_l \to 0$, we obtain

$$F_{B \to D}(1) = 1.074(18)(16),$$

where the first error is statistical, and the second is systematic summarized in Table 1. The light quark mass dependence for $F_{B \to D}(1)$ is shown in Fig. 4. Extrapolating the result linearly to $m_l \to 0$, we obtain

$$F_{n_f}(N_f=2+1) = 1.00(10)(2).$$

Table 1

| decay          | $D \to \pi(K)$ | $B \to \pi$ | $B \to D$ |
|----------------|----------------|-------------|-----------|
| 3-pt function  | 3%             | 3%          | 1%        |
| BK fit         | 2%             | 4%          |           |
| $m_l$ extrap   | 3%(2%)         | 4%          | 1%        |
| matching       | <1%            | 1%          | 1%        |
| $a$ uncertainty| 1%             | 1%          |           |
| finite $a$ error| 9%            | 9%          | <1%       |
| total          | 10%            | 11%         | 2%        |

Since we have all 3 elements of the second row of CKM matrix, we are able to check a CKM unitarity using only our results as theoretical inputs: $(|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(10)(2).

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