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Abstract. We investigate strong mechanical feedback for a single electron tunneling (SET) device coupled to an underdamped harmonic oscillator in the high-frequency case, when the mechanical energy of the oscillator exceeds the tunnel rate, and for weak coupling. In the strong feedback regime, the mechanical oscillations generated by the telegraph signal from the SET device in their turn modify the electric current. In contrast to the previously published results for the low frequencies, the current noise is not enhanced above the Poisson value.

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1. Introduction

One of the key questions in the field of nanoelectromechanical systems (NEMS) [1] is the effect of mechanical motion on the electric properties of the systems. It is equally important for understanding of fundamental questions such as behavior of non-equilibrium, dissipative, driven systems and for the prospects of NEMS practical applications, such as switches, relays and actuators. Electron motion in both external ac electromagnetic fields (see e.g. [2]) and subject to time-dependent noise [3] have been extensively studied in the past. The new feature brought

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by NEMS is that not only electrons move in the field created by the mechanical vibrations (for brevity, phonons), but also non-equilibrium phonons are created by the electron motion through the NEMS device.

In this paper, we concentrate on NEMS operating in the single electron tunneling (SET) regime. These include experimental realizations based on single molecules (for a review, see [4]), semiconductor beams [5]–[7], and suspended carbon nanotubes [8]–[12]. In the SET regime, a NEMS can be represented as a SET device coupled to a mechanical (harmonic) oscillator. The coupling is provided by a force (typically of electrostatic origin, see e.g. [13]) acting on the oscillator. The value of the force depends on the charge state of the SET device, providing the feedback. This feedback can be strong even if the electron–phonon coupling is weak: indeed, if one considers a SET device next to the Coulomb blockade threshold, only two charge states, say, \( n = 0 \) and 1, with \( n \) being the number of electrons at SET, are important. The occupation of a SET device above the Coulomb blockade threshold fluctuates between zero and one, providing a force which is a random telegraph signal. The signal swings an underdamped oscillator to large amplitudes, which in turn affect the transport properties of the SET device by modifying the tunnel matrix elements and the energy difference between the two charge states.

The transport properties of NEMS devices crucially depend on a number of parameters (figure 1). One is the coupling strength \( g \), defined as the squared ratio of the displacement of the oscillator center under the action of the force and the amplitude of the zero-point motion, \( g = F^2/(\hbar M \omega^3) \), where \( M \) and \( \omega \) are the mass and the frequency of the oscillator, and \( F \) is the difference of the forces acting on the oscillator in the charge states \( n = 1 \) and 0. It is important that the Planck constant is in the denominator, and thus strong coupling actually means quantum regime. Other parameters determining the behavior are given by ratios of various energy scales characterizing the NEMS device. The most relevant one is the ratio between the typical tunnel rate \( \Gamma \) and the oscillator frequency \( \omega \).

Surprisingly, the strong coupling (quantum regime) is more extensively studied in the literature. Assuming the oscillator is underdamped, for \( g \geq 1 \) and \( \omega \gg \Gamma \), the behavior of the system is dominated by Franck–Condon physics. The tunnel rates are modified due to emission of phonons in the course of electron tunneling [3]. For the coupling to a single mode, this leads to the steps in the current as the function of the applied bias voltage, the height of a step being determined by the coupling constant [14]–[16]. For \( \omega \ll \Gamma \), the electron level width becomes bigger than the distance between the steps, and the Franck–Condon structure disappears. In this regime, the motion of the oscillator is much slower than the electron tunneling, and one can use the Born–Oppenheimer approximation, considering electrons in the quasi-stationary field potential provided by the oscillator. In this situation, the oscillator may become bistable, and the electron tunneling is dominated by switching events between the two states of the oscillator [17]–[21]. All these phenomena are the manifestation of strong mechanical feedback. Similar effects have been studied in [22] in the context of superconducting NEMS.

It is less obvious that strong feedback is also possible at weak coupling \( g \ll 1 \). This is the classical regime, where the Boltzmann equation serves as the starting point [23, 24]. Blanter et al [25] studied the low-frequency case \( \omega \ll \Gamma \). The oscillator motion in this case is described by the Fokker–Planck equation with effective diffusion and effective damping (originating from the electron tunneling out of the SET device), both determined by the energy dependence of the tunnel rates. It turns out that the strong feedback regime is feasible, but the behavior of the current strongly depends on the energy dependence of the tunnel rates. For instance, in the two most
commonly investigated cases—electron tunneling through a single level and electron tunneling through a continuum of levels with constant density, as in a single electron transistor—strong feedback does not appear. Four distinct regimes have been identified: (i) no phonons generated; (ii) the oscillations are generated with fixed finite amplitude; (iii) the oscillator is bistable, one state has oscillations with zero amplitude, and the other oscillations with a finite amplitude and (iv) the system is bistable, with the two states representing oscillations with two different amplitudes. In the regimes (ii), (iii) and (iv), current is strongly modified with respect to the case when the phonons are not generated. A quantity even more sensitive to the oscillations is the current noise. The natural measure of the current noise in SET devices is the Poisson value of the zero-frequency spectral density $S$, $S_P = 2eI$, $I$ being the average current [26]. It turns out that the noise is strongly enhanced above the Poisson value in the regimes (ii), (iii) and (iv), and may even become super-Poissonian in the regime (i), when the current is not renormalized.

In this paper, we consider the only regime not addressed so far: weak coupling $g \ll 1$ and high frequency $\omega \gg \Gamma$. We show that it shares many features with the low-frequency classical regime described above. Even though we have not been able to identify any bistable regime, we still find the two regimes of zero and finite amplitude, the latter demonstrating strong mechanical feedback. We find that the noise, in contrast to the low-frequency case, is always sub-Poissonian.

2. Boltzmann equation

At weak coupling, the motion of the oscillator is classical, and the behavior of the system is characterized by the joint distribution function, $P_n(x, v, t)$. Here, $n$ is the charge state of the SET. We assume that the SET is biased close to the edge of one of the Coulomb diamonds, so that only two charge states are important for transport, for definiteness $n = 0$ and 1. Furthermore, $x$ and $v$ are the coordinate and the velocity of the oscillator. The starting point of our classical
approach is the Boltzmann (master) equation for the distribution function [23, 25],
\[
\frac{\partial P_n}{\partial t} + \left\{ v \frac{\partial}{\partial x} + \frac{\partial}{\partial v} \mathcal{F} \right\} P_n - \text{St}[P] = 0, \tag{1}
\]
\[
\mathcal{F} = -M\omega^2 x - \frac{M\omega v}{Q} + F_n, \tag{2}
\]
\[
\text{St}[P] = (2n - 1) \left( \Gamma^+(x) P_0 - \Gamma^-(x) P_1 \right), \tag{3}
\]
which holds for an arbitrary relation between \( \omega \) and \( \Gamma \). Here, the total force \( \mathcal{F} \) acting on the oscillator is the sum of the elastic force friction force and charge-dependent coupling force, respective to the order of terms in equation (2), \( Q \gg 1 \) is the quality factor. We count the position of the oscillator from its equilibrium position in the \( n = 0 \) state. In this case, \( F_n = nF \). The ‘collision integral’ \( \text{St}[P] \) in the right-hand side represents SET. There are four tunnel rates, \( \Gamma_{L,R}^\pm \), where the subscripts L and R denote tunneling through the left or right junction, and the superscripts + and − correspond to the tunneling to and from the island, respectively; \( \Gamma^\pm = \Gamma_L^\pm + \Gamma_R^\pm \). Each rate is a function of the corresponding energy cost \( \Delta E_{L,R}^\pm \) associated with the addition/removal of an electron to/from the island in the state \( n = 0/1 \) via left or right junction (\( \Delta E_{L,R}^+ = -\Delta E_{L,R}^- \)). Two independent energy differences are determined by electrostatics and depend linearly on the voltages. There is also a contribution to each energy from the shift of the oscillator, \( \Delta E_L^+ = \Delta E_L^{(0)} - F_x \) and \( \Delta E_R^- = \Delta E_R^{(0)} + F_x \), where \( \Delta E^0 \) are the corresponding energy differences in the absence of mechanical motion.

The condition \( \omega \gg \Gamma \equiv \Gamma^+ + \Gamma^- \) means that the motion of the oscillator is fast on the scale of the typical time an electron spends in the SET device. In this situation, the probabilities \( P_0 \) and \( P_1 \) average over the fast oscillator motion. In the leading order, if we parameterize \( x = \epsilon \sin \varphi, \ v = \omega \epsilon \cos \varphi \) and \( \epsilon = \sqrt{2E/M\omega^2} \), these probabilities do not depend on \( \varphi \). Consequently, we expand the probabilities in the following way,
\[
P_n(x, v, t) \approx P_n^a(E, t) + \cos \varphi P_n^b(E, t) + \sin \varphi P_n^c(E, t),
\]
with \( P_n^{b,c} \ll P_n^a \). Here and below, we disregard the terms proportional to \( \sin m\varphi, \cos m\varphi \), with \( m \geq 2 \). This procedure is similar to the transformation of the Boltzmann equation into the diffusion equation in the semi-classical theory of electron transport in metals.

We can obtain a closed set of equations for \( P_n^{a,b,c} \) by multiplying equation (1) with 1, \( \cos \varphi \) and \( \sin \varphi \) and subsequently averaging over the phase, throwing out \( \sin 2\varphi \) and \( \cos 2\varphi \) terms. It is important that the tunnel rates only depend on the coordinate and not on the velocity of the oscillator, and thus are functions of \( \sin \varphi \) and not \( \cos \varphi \). After some algebra, we obtain
\[
P_{0,1}^b = P_0^c = 0,
\]
\[
P_0^a(E, t) = \frac{\langle \Gamma^- \rangle}{\langle \Gamma^+ \rangle} P(E, t), \quad P_1^a(E, t) = \frac{\langle \Gamma^+ \rangle}{\langle \Gamma^- \rangle} P(E, t),
\]
\[
P_1^c(E, t) = \frac{\langle \sin \varphi \Gamma^+ \rangle - \langle \sin \varphi \Gamma^- \rangle}{\langle \Gamma^+ \rangle (\sin^2 \varphi \Gamma^-)} P(E, t),
\]
where the angular brackets denote averaging over the phase \( \varphi \), and the function \( P(E) \) obeys the equation
\[
\frac{\partial P}{\partial t} = \mathcal{L} P \equiv \sqrt{\frac{g\hbar \omega^3 E}{2}} \langle \Gamma^+ \rangle \left( -\frac{\partial P}{\partial E} + \sqrt{\frac{1}{2g\hbar \omega E}} \frac{\langle \sin \varphi \Gamma^+ \rangle}{\langle \Gamma^+ \rangle} P - \frac{\langle \sin \varphi \Gamma^- \rangle}{\langle \Gamma^- \rangle} \langle \sin^2 \varphi \Gamma^- \rangle \right) \tag{4}
\]
and we have disregarded the mechanical damping \( Q^{-1} \).
Figure 2. The stability regions in the gate–bias voltage plane. Bold solid lines indicate the edge of the Coulomb diamonds. Insets show the sketch of $P(E)$ in each region. The horizontal lines indicate bias voltages used for current scans in figure 3.

3. Distribution and current

The stationary solution of equation (4) is easily found as

$$P(E) = P(0) \exp \left( - \int_0^E \gamma(E') dE' \right),$$

(5)

$$\gamma(E) = \sqrt{\frac{1}{2g\hbar E}} \left( \langle \sin \varphi \Gamma^+ \rangle \langle \Gamma^- \rangle - \langle \sin \varphi \Gamma^- \rangle \langle \Gamma^+ \rangle \right).$$

Note that, similarly to the low-frequency case $\omega \ll \Gamma$, the distribution (5) is very sharp. Indeed, the typical energy $E$ is of the order of the applied voltage $eV$, and all energies which occur in equation (5) are in our classical consideration much bigger than the quantum energy $\hbar \omega$ of the oscillator. The function $\gamma$ can have positive as well as negative sign. If it becomes negative at some values of energy (amplification instead of dissipation), there is a possibility that the strong feedback regime emerges.

To illustrate the energy dependence of the probability, we have chosen exponential energy dependence used previously for low frequencies in [25],

$$\Gamma_{L,R}^+ = 2 \Gamma_{L,R}^0 e^{-a_{L,R} \Delta E_{L,R}} \left( 1 - f_F \left( -\Delta E_{L,R}^+ \right) \right), \quad \Gamma_{L,R}^- = \Gamma_{L,R}^0 e^{a_{L,R} \Delta E_{L,R}} f_F \left( \Delta E_{L,R}^- \right),$$

(6)

the factor of 2 accounting for the spin degeneracy of the state $n = 1$. For concrete illustration, we choose $a_L = 0.3$, $a_R = 0.75$ and $\Gamma_{L,R}^0 = \Gamma_{L,R}^0$. In the figures, $W = eV_g$, the gate $V_g$ and bias $V$ voltages are measured in units of $W$, the parameter associated with the energy dependence of the tunneling rates, with a value smaller than the charging energy.

Figure 2 presents the regions in the gate–bias voltage plane corresponding to the two regimes. In regime (i), the probability distribution has a sharp peak around zero energy. Since the energy represents the mechanical motion of the oscillator, it corresponds to positive damping at any energy and the absence of mechanical feedback—phonons are not generated by the electron tunneling events. In regime (ii), the distribution function sharply peaks around a finite value of
the oscillator energy: mechanical oscillations are generated. We were not able to detect the existence of bistable regimes, similar to (iii) and (iv) described in the introduction. Regime (ii) of phonon generation only emerges outside the Coulomb diamonds.

For (i), the distribution function can be approximated as

\[ P(E) = \gamma(0) \exp(-\gamma(0)E), \]  

where we have normalized the solution. For (ii), we have the Gaussian centered around the most probable value \( E_m \),

\[ P(E) = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{\partial \gamma(E_m)}{\partial E}} \exp \left( -\frac{\partial \gamma(E_m)}{\partial E} \frac{(E - E_m)^2}{2} \right). \]

The current is found as

\[ I = \int I(E) P(E) \, dE, \quad I(E) \equiv e^{\frac{\langle \Gamma_{L} \rangle \langle \Gamma_{R} \rangle - \langle \Gamma_{L}^{-} \rangle \langle \Gamma_{R}^{-} \rangle}{\langle \Gamma_{L} \rangle}}. \]

Figure 3 shows the results for the voltage dependence of the current. Trace (a) is taken in regime (i), and the current is not modified by mechanical motion. Traces (b), (c) and (d) cross region (ii), and the current dependence in this regime deviates from the one without mechanical motion. The deviations are stronger for higher bias voltages; additional peaks in the current develop.
4. Current noise

The current noise spectral power at zero frequency is found from the expression [25]

\[ S = -4 \int_0^\infty \delta I(E) u(E) \, dE, \quad \delta I(E) \equiv I(E) - \int_0^\infty I(E) \, P(E) \, dE \]  

and \( u \) solves the equation \( Lu = \delta I(E) \, P(E) \). In contrast to the low-frequency case, \( L \) is a first-order differential operator. This fact simplifies the calculations and readily provides analytical estimates for noise in the two regimes. We parameterize \( u(E) = v(E) \, P(E) \), and the equation for \( v \) reads

\[ \sqrt{\frac{g\hbar \omega}{2}} \frac{\langle \Gamma^+ \rangle}{\langle \Gamma \rangle} \frac{\partial v}{\partial E} = \delta I(E). \]

Solving it and substituting the result into the expression for noise, we obtain

\[ S = 4 \sqrt{\frac{2}{g\hbar \omega}} \int_0^\infty \delta I(E) \, P(E) \int_0^E \frac{\langle \Gamma_i \rangle}{\langle \Gamma^+ \rangle} \delta I(E') \, dE'. \]  

This expression can be evaluated with the use of approximations (7) and (8).

In regime (i), we write \( \delta I(E) \approx (g\hbar \omega/2e^2) \partial^2 I/\partial V_g^2 (E - \gamma^{-1}(0)) \), where the second derivative is evaluated at \( E = 0 \). Substituting this into the expression for noise, we obtain

\[ S = \sqrt{\frac{2\pi}{\omega}} (g\hbar \omega)_0^{3/2} \left( \frac{\partial^2 I}{e^2 \partial V_g^2} \right)^2 \left| \frac{\langle \Gamma_i \rangle}{\langle \Gamma^+ \rangle} \right|_{E=0} \left| \frac{\langle \Gamma \rangle}{\langle \Gamma^+ \rangle} \right|^{5/2}. \]  

The comparison with the Poisson value of noise \( S_p = 2eI \) gives the estimate \( S/S_p \sim (\Gamma^2 / \omega^2) (g\hbar \omega/eV_g)^3/2 \). In the high-frequency regime \( \Gamma \ll \omega \), both factors are small, and thus in regime (i) the mechanically induced noise is always sub-Poissonian.

In regime (ii), expanding \( \delta I = (\partial I(E)/\partial E)(E - E_m) \), we obtain

\[ S = \frac{16}{3\omega} \sqrt{\frac{2E_m}{g\hbar \omega}} \left( \frac{\partial I}{e\partial V_g} \right)_{E=E_m} \left( \frac{\partial \gamma}{\partial E} \right)^{-1} \left( \frac{\langle \Gamma \rangle}{\langle \Gamma^+ \rangle} \right). \]  

The estimate for the noise power is \( S/S_p \sim (\Gamma^2 / \omega^2) (g\hbar \omega/eV_g)^1/2 \). It is not surprising that the noise is relatively higher than in regime (i), where there is no mechanical motion induced. However, the noise is still below the Poisson value, which means that the behavior of the current noise is dominated by the shot noise.

The plots of the current noise at two different bias voltages are shown in figure 4.

5. Conclusions

In this paper, we studied current and current noise for an SET device coupled to an underdamped harmonic oscillator in the only regime not considered so far: weak coupling \( g \ll 1 \) and high frequency \( \omega \gg \Gamma \). We find that, similarly to other regimes, coupling to mechanical modes of the oscillator—phonons—excited by the tunneling of electrons through the SET device, may have a strong effect on the transport properties of the SET device. This is the phenomenon of strong mechanical feedback.
However, we also find that the feedback effects are the weakest of all the regimes. It was indeed expected that the effect of phonons on electron transport is stronger for stronger coupling. But, we also find that for high frequencies the effects are less pronounced than for low frequencies. Indeed, no bistability regimes have been discovered, the strong feedback only manifests in oscillations with a fixed amplitude. The mechanical contribution to the current noise is small compared with the shot noise contribution. It can still be separated from the white shot noise due to its frequency dependence, however, the noise behavior is less spectacular than for low oscillator frequencies, where it is sometimes exponentially enhanced in comparison with the Poisson value.

In this paper, we assume that the energy dependence of the tunnel rates is not too strong. All our results are expressed in terms of the tunnel rates averaged over the period of mechanical oscillations. One can imagine an opposite situation—strongly energy-dependent rates (but at each energy still below the frequency of the oscillator). Since the oscillator moves quickly on the scale of the typical time the electron spends in one of the SET charge states, the instant tunnel rate performs fast oscillations with a big amplitude. In this situation, it is the easiest for an electron to tunnel when the tunnel rate is the highest, which typically would correspond to the maximum displacement of the oscillator. In this synchronization regime, the electron jumps are synchronized with the oscillator period. The condition for the appearance of the synchronization regime is $(d\Gamma/dE)\delta E \gg \Gamma$, where $\delta E \sim (eV)^2/(\hbar \omega)$ is the shift of the mechanical energy if the oscillator is displaced between the two extreme positions. Presumably, in the synchronization regime the transport properties are determined by the maximum (rather than the averaged) value of the tunnel rate over the oscillation period. Detailed analysis of the synchronization regime lies outside the scope of this paper, but we expect that the current is enhanced as compared with the ‘regular’ strong-feedback regime considered above, whereas the current noise is suppressed since the electron stream becomes more regular.

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