Aspects of ABJ Theory

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December 11, 2013

Abstract

In this paper we will analyse the deformation of a ABJ theory in harmonic superspace. So, we will first discuss deformations of the harmonic superspace by a graviphoton background and then study the ABJ theory in this deformed harmonic superspace. This deformed ABJ theory will be shown to possess $\mathcal{N} = 6$ supersymmetry. We also discuss the BRST symmetry of this theory.

1 Introduction

Harmonic superspace has been studied in four dimensions [1, 2]. It has also been studied in three dimensions [3, 4, 5]. In three dimensions it has $\mathcal{N} = 3$ supersymmetry. Harmonic superspace has been used for analysing the multiple M2-branes [6]. The action for these multiple M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$ orbifold is given by a theory called the ABJM theory [7, 8, 11, 12]. The ABJM theory has $\mathcal{N} = 6$ supersymmetry. However, the ABJM theory coincides with the BLG theory for the only known example of a Lie 3-algebra [13, 14, 15, 16]. Thus, the supersymmetry of the ABJM theory is expected to be enhanced to full $\mathcal{N} = 8$ supersymmetry [17, 18].

The Chern-Simons actions with levels, $k = 1, 2$ forms the gauge part of the ABJM theory. Furthermore, the matter fields in the ABJM theory live in the bifundamental representation of the gauge group $U(N) \times U(N)$. A generalization of the ABJM theory to the ABJ theory has been made [19, 20, 21, 22, 23]. In the ABJ theory the matter fields live in the bifundamental representation of gauge group $U(M) \times U(N)$ with $M \neq N$ has been made. The ABJ theory also has $\mathcal{N} = 6$ supersymmetry.

The presence of NS background causes a noncommutative deformation between the spacetime coordinates [24, 27, 28, 29], and the presence of a graviphoton background causes a noncommutative deformation between the spacetime and Grassmann coordinates [30, 31, 33, 34]. Both these deformations do not break any supersymmetry. However, the presence of a RR background causes a deformation between the Grassmann coordinates, and thus, breaks the a certain amount of the supersymmetry [35, 36, 37, 38, 39, 40]. As M-theory is dual to type II string theory, a deformation of the string theory side will also generate a deformation on the M-theory side. In fact, a noncommutative superspace deformation of the ABJM theory in $\mathcal{N} = 1$ superspace have been already studied.
In this paper we study deformation of the ABJ theory by a graviphoton background in harmonic superspace. It may be noted that the BRST symmetry of the ABJM theory has been analysed in deformed $N = 1$ superspace [11, 12, 42]. So, in this paper we will also analyse the BRST symmetry of deformed ABJ theory in harmonic superspace.

2 Harmonic superspace

In order to construct harmonic superspace, first harmonic variables, $u^\pm$, parameterizing the coset $SU(2)/U(1)$ are constructed. These harmonic variable are subjected to the constraints $u^+u^- = 1$, $u^+u^+_i = u^-u^-_i = 0$. Thus, harmonic superspace coordinates are given by

$$z = (x^{ab}, \theta^+_a, \theta^-_a, \theta^0_a, u^\pm_i),$$

(1)

where $\theta^\pm_a = \theta^+_a u^\pm_i$ and $\theta^0_a = \theta^+_a u^+_i u^-_i$. In order to construct the harmonic superspace the following derivatives are constructed

$$D^{++} = \partial^{++} + 2i\theta^{++}b \theta^b \partial^{++}_{ab} + \theta^{++} \frac{\partial}{\partial \theta^a} + 2\theta^0 \frac{\partial}{\partial \theta^{--}},$$

$$D^{--} = \partial^{--} - 2i\theta^{--}b \theta^b \partial^{--}_{ab} + \theta^{--} \frac{\partial}{\partial \theta^a} + 2\theta^0 \frac{\partial}{\partial \theta^{++}},$$

$$D^0 = \partial^0 + 2\theta^{++}b \partial^0_{ab} - 2\theta^{--}b \partial^0_{ab},$$

(2)

and

$$D^{a--} = \frac{\partial}{\partial \theta^{++} - b} + 2i\theta^{--}b \partial^{a--}_{ab}, \quad D^0 = -\frac{1}{2} \frac{\partial}{\partial \theta^0} + i \theta^0 b \partial^0_{ab},$$

$$D^{a++} = \frac{\partial}{\partial \theta^{--} - b}.$$  

(3)

Here $\partial^{++}, \partial^{--}$ and $\partial^0$ are given by

$$\partial^{++} = u^+_i \frac{\partial}{\partial u^+_i}, \quad \partial^{--} = u^-_i \frac{\partial}{\partial u^-_i},$$

$$\partial^0 = u^+_i \frac{\partial}{\partial u^+_i} - u^-_i \frac{\partial}{\partial u^-_i}.$$  

(4)

Now these derivatives satisfy the following superalgebra,

$$\{D^{++}, D^{--}\} = 2i \partial^{ab}_{ab}, \quad \{D^0, D^0\} = -i \partial_{ab},$$

$$[D^{++}, D^{a--}] = 2D^{a--}, \quad [D^0, D^{a--}] = \pm 2D^{a--},$$

$$\partial^0 = [\partial^{++}, \partial^{--}], \quad [D^{++}, D^{--}] = D^0.$$  

$$\{D^{a\pm}, D^0\} = 0, \quad [D^{a\pm}, D^0] = D^{a\pm}.$$  

(5)

In harmonic superspace the fields which satisfy, $D^{a++} \Phi_A = 0$, are called analytic superfields, $\Phi_A = \Phi_A(\zeta_A)$. Thus, the analytical superfields are independent of the $\theta^-_a$. So, the coordinates for the analytic subspace are given by

$$\zeta_A = (x^{ab}, \theta^+_a, \theta^0_a, u^-_i).$$  

(6)

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where

\[ x^a_{m} = (\gamma_m)^{ab} x^m_b = x^{ab} + i(\theta^{++a} \theta^{--b} + \theta^{++b} \theta^{--a}). \]  

(7)

The harmonic superspace has \( \mathcal{N} = 3 \) supersymmetry, which is generated by

\[ Q^{++}_a = u^{++}_i u^{++}_j Q^{ij}_a, \quad Q^{--}_a = u^{--}_i u^{--}_j Q^{ij}_a, \]

\[ Q^0_a = u^+_i u^-_j Q^{ij}_a, \]

(8)

where

\[ Q^{ij}_a = \frac{\partial}{\partial \theta^{ij}} - \theta^{ij} \partial_{ab}. \]

(9)

The measures in the harmonic superspace are given by

\[ d^3z = -\frac{1}{16} d^3x (D^{++})^2 (D^{--})^2 (D^0)^2, \]

\[ d\zeta^{(-4)} = \frac{1}{4} d^3x d^3 u (D^{++})^2 (D^{--})^2 (D^0)^2. \]

(10)

A conjugation in the harmonic superspace is defined by

\[ \tilde{(u^\pm_i)} = u^\pm_i, \quad (x^m_A) = x^m_A, \]

\[ \tilde{(\theta^{\pm\pm}_a)} = \theta^{\pm\pm}_a, \quad (\theta^0_a) = \theta^0_a. \]

(11)

It is squared to \(-1\) on the harmonics and to 1 on \( x^m_A \) and Grassmann coordinates. So, the analytic superspace measure is real \( d\zeta^{(-4)} = d\zeta^{(-4)} \) and the full superspace measure is imaginary \( d^3z = -d^3z \).

3 Deformed ABJ Theory

In this section we will construct a deformed ABJ theory in the harmonic superspace. The deformation will be caused by a graviphoton background. We expect a noncommutative deformation of the M-theory to occur in curved backgrounds due to the three form field \( C_{\mu\nu\tau} \) and a graviphoton \( \psi^\mu_\nu \). If \( H_{\mu\nu\tau\rho} \) the field strength of this three form field, then we expect a noncommutative deformation proportional to \( (\gamma^\mu \gamma^\nu \gamma^\tau \gamma^\rho H_{\mu\nu\tau\rho})^{ab} \psi^\mu_\nu \) to occur. We thus deform the harmonic superspace by imposing the following relations,

\[ \{ \hat{y}_\mu, \hat{\theta}^{++}_a \} = C^{++}_{\mu a}. \]

(12)

Now we start by defining a vector field \( V^{++} \) in the harmonic superspace. We can express this field on the deformed superspace as

\[ \hat{V}^{++}(\hat{z}) = \int dp \exp(ip \hat{z}) V^{++}(p). \]

(13)

Now we can similarly express the product of two fields on this deformed superspace. We can thus obtain an expression for the product of these fields on ordinary superspace. This product is given by

\[ V^{++}(z) \ast V^{++}(z) = \exp\left\{-\frac{1}{2} \left(C^{+++a\mu}[\partial^{+++2}_a \partial^0_\mu + \partial^{+++1}_a \partial^2_\mu] \right) \right\} \times V^{++}(z_1) V^{++}(z_2) \bigg|_{z_1 = z_2 = z}. \]

(14)
It is also possible to write $V^{--}$ as

$$V^{--}(z,u) = \sum_{n=1}^{\infty} (-1)^n \int du_1 \ldots du_n E^{++},$$  \hspace{1cm} (15)$$

where

$$E^{++} = \frac{V^{++}(z,u_1) \cdot V^{++}(z,u_2) \ldots \cdot V^{++}(z,u_n)}{(u^+_1 u^+_2) \ldots (u^+_n u^+_n)}.$$ \hspace{1cm} (16)$$

The action for the deformed ABJ theory is invariant under the gauge group $U(N) \times U(M)$. Let the the gauge fields corresponding to $U(M)$ and $U(N)$ be denoted by $(V^+_{L})^\alpha_{\mathcal{A}}$ and $(V^+_{R})^A_{\mathcal{B}}$, respectively. Thus, indices corresponding to the gauge group $U(M)$ would be denoted by the underlined indices. Now the matter fields are given by $(q^+)^B_{\mathcal{A}}$ and $(\bar{q}^+)^\bar{A}_{\bar{\mathcal{A}}}$. We can define covariant derivatives for the matter fields in this deformed theory as

$$\nabla^{++} q^+ = D^{++} q^+ + V_L^{++} \cdot q^+ - q^+ \star V_R^{++},$$

$$\nabla^{++} \bar{q}^+ = D^{++} \bar{q}^+ - q^+ \star V_L^{++} + V_R^{++} \star \bar{q}^+, \hspace{1cm} (17)$$

The action for the deformed ABJ theory can now be written as

$$S = S_{CS,k}[V_L^{++}] + S_{CS,-k}[V_R^{++}] + S_M[q^+, \bar{q}^+],$$ \hspace{1cm} (18)$$

where

$$S_{CS,k}[V_L^{++}] = \frac{ik}{4\pi} \int \sum_{n=2}^{\infty} \frac{(-1)^n}{n} \int d^3 x d \theta d u_1 \ldots d u_n H_L^{++},$$

$$S_{CS,-k}[V_R^{++}] = -\frac{ik}{4\pi} \int \sum_{n=2}^{\infty} \frac{(-1)^n}{n} \int d^3 x d \theta d u_1 \ldots d u_n H_R^{++},$$

$$S_M[q^+, \bar{q}^+] = \text{tr} \int d^3 x d \zeta (-4) \bar{q}^+ \star \nabla^{++} \star q^+, \hspace{1cm} (19)$$

and

$$H_L^{++} = \frac{V^{++}(z,u_1)_L \cdot V^{++}(z,u_2)_L \ldots \cdot V^{++}(z,u_n)_L}{(u^+_1 u^+_2) \ldots (u^+_n u^+_n)},$$

$$H_R^{++} = \frac{V^{++}(z,u_1)_R \cdot V^{++}(z,u_2)_R \ldots \cdot V^{++}(z,u_n)_R}{(u^+_1 u^+_2) \ldots (u^+_n u^+_n)}. \hspace{1cm} (20)$$

The harmonic superspace used to write the deformed ABJ theory has manifest $\mathcal{N} = 3$ supersymmetry generated by to the supercharges $Q_a^{++}, Q_a^{-}$ and $Q_a^{0}$. However, the ABJ theory has $\mathcal{N} = 6$ supersymmetry. Thus, we need additional $\mathcal{N} = 3$ supersymmetry. Thus, this deformed ABJ theory is invariant under the following supersymmetric transformations,

$$\delta \bar{q}^+ = ie^a \bar{\nabla}_a^0 \star \bar{q}^+, \hspace{1cm} (21)$$

$$\delta q^+ = ie^a \nabla_a^0 \star q^+, \hspace{1cm} (21)$$

$$\delta V_L^{++} = \frac{8\pi}{k} e^a \theta_a^0 \star q^+ \star \bar{q}^+, \hspace{1cm} (21)$$

$$\delta V_R^{++} = \frac{8\pi}{k} e^a \theta_a^0 \star \bar{q}^+ \star q^+, \hspace{1cm} (21)$$

$$\delta V_L^{++} = \frac{8\pi}{k} e^a \theta_a^0 \star q^+ \star \bar{q}^+, \hspace{1cm} (21)$$

$$\delta V_R^{++} = \frac{8\pi}{k} e^a \theta_a^0 \star \bar{q}^+ \star q^+, \hspace{1cm} (21)$$
where
\[
\hat{\nabla}_a \star q^+ = \nabla_0 a \star q^+ + \theta_a^- (W^+_L \star q^+ - q^+ \star W^+_R),
\]
\[
\nabla_0 a \star q^+ = D_0 q^+ + V_0^L a \star q^+ - q^+ \star V_0^R a,
\]
\[
V_0^L a = -\frac{1}{2} D^{++} V^L_-,
\]
\[
V_0^R a = -\frac{1}{2} D^{++} V^R_-.
\]
(22)
The covariant derivatives $\hat{\nabla}_a \bar{q}^+$ and $\nabla_0 \bar{q}^+$ are obtained via conjugation. Furthermore, the field strengths $W^{++}_L$ and $W^{++}_R$ are given by
\[
W^{++}_L = -\frac{1}{4} D^{++} D^{++} V^L_- - [V^{++}_L, \Lambda_L] \star,
\]
\[
W^{++}_R = -\frac{1}{4} D^{++} D^{++} V^R_- - [\Lambda_R, V^{++}_R] \star.
\]
(23)
Thus, we get
\[
\delta \epsilon S = 0,
\]
(25)
because using Fierz rearrangement, we have $-\delta S_M[q^+, \bar{q}^+], = \delta \epsilon S_{CS,+}[V^{++}_L] \star + \delta \epsilon S_{CS,-}[V^{++}_R] \star$. So, this ABJ theory in the deformed harmonic superspace has $\mathcal{N} = 6$ supersymmetry.

4 BRST Symmetry

Now all the degrees of this deformed ABJ are physical. This is because this theory is invariant under the following infinitesimal gauge transformations
\[
\delta q^+ = \Lambda_L \star q^+ - q^+ \star \Lambda_R,
\]
\[
\delta \bar{q}^+ = \Lambda_R \star \bar{q}^+ - \bar{q}^+ \star \Lambda_L,
\]
\[
\delta V^{++}_L = -D^{++} \Lambda_L - [V^{++}_L, \Lambda_L] \star,
\]
\[
\delta V^{++}_R = -D^{++} \Lambda_R - [\Lambda_R, V^{++}_R] \star.
\]
(26)
As the deformed ABJ theory is invariant under these gauge transformations, it cannot be quantized without fixing a gauge. We incorporate the gauge fixing conditions $D^{++} V^{++}_L = 0$, $D^{++} V^{++}_R = 0$, at a quantum level by adding gauge fixing and ghost terms to the original action. The gauge fixing term is given by
\[
S_{gf} = \int d^3 x d\zeta (-4) \text{tr} \left[ b_L \star (D^{++} V^{++}_L) + \frac{\alpha}{2} b_L \star b_L 
\right. 
- b_R \star (D^{++} V^{++}_R) + \frac{\alpha}{2} b_R \star b_R \right],
\]
(27)
and the ghost term is given by
\[
S_{gh} = \int d^3 x d\zeta (-4) \text{tr} \left[ \bar{c}_L \star D^{++} \nabla^{++} \star c_L - \bar{c}_R \star D^{++} \nabla_a \star c_R \right].
\]
(28)
The sum of the original Lagrangian density with the gauge fixing and ghost terms is invariant under the following BRST transformations

\[
s V_L^{++} = \nabla^{++} \star c_L, \quad s V_R^{++} = \nabla_R^{++} \star c_R, \\
s c_L = -[c_L, c_L], \quad s c_R = -[c_R, c_R], \\
s \bar{c}_L = b_L, \quad s \bar{c}_R = -[b_R, \bar{c}_R], \\
s b_L = 0, \quad s b_R = -[b_R, c_R], \\
s q^+ = c_L \star q^+ - q^+ \star c_L, \quad s \bar{q}^+ = c_R \star \bar{q}^+ - \bar{q}^+ \star c_R.
\]  

(29)

In fact, the sum of the gauge fixing term and the ghost term, is a total BRST variation,

\[
S_{gh} + S_{gf} = \int d^3 x d\zeta \langle (-4) s \text{tr} [\Phi] \rangle 
\]

(30)

where

\[
\Phi = c_L \left( D^{++} V_L^{++} - \frac{i \alpha}{2} b_L \right) - c_R \left( D^{++} V_R^{++} - \frac{i \alpha}{2} b_R \right),
\]

(31)

As sum of the ghost term and the gauge fixing term can be expressed as a total BRST, it is invariant under the BRST transformations, because of the nilpotency of these transformations, \( s^2 = 0 \). The BRST variation of the original classical Lagrangian density is its gauge variation with the gauge parameter replaced by ghosts. As the original classical Lagrangian density was invariant under the gauge transformations, it is also invariant under the BRST transformations. So, the effective Lagrangian density which is defined to be a sum of the original classical Lagrangian density, the gauge fixing term and the ghost term is invariant under the BRST transformations.

As this total Lagrangian density is invariant under the BRST transformations, so we can obtain the Norther’s charge \( Q \) corresponding to the BRST transformations and use it to project out the physical states. To do that we first calculate the conserved current corresponding to this symmetry

\[
J^\mu = \frac{1}{2} \int d^3 x d\zeta^{(-4)} \left[ \frac{\partial L_{\text{eff}}}{\partial \nabla^{++}_L} \star s V_L^{++} + \frac{\partial L_{\text{eff}}}{\partial b_L} \star s c_L + \frac{\partial L_{\text{eff}}}{\partial \nabla^{++}_R} \star s V_R^{++} + \frac{\partial L_{\text{eff}}}{\partial b_R} \star c_R \\
+ \frac{\partial L_{\text{eff}}}{\partial q^+} \star s q^+ + \frac{\partial L_{\text{eff}}}{\partial \bar{q}^+} \star s \bar{q}^+ \right],
\]

(32)

where

\[
\int d^3 x d\zeta^{(-4)} [L_{\text{eff}}] = S_* + S_{gh*} + S_{gf*}.
\]

(33)

Now the conserved BRST charge will be given by

\[
Q = \int d^3 x J^0.
\]

(34)
As the BRST transformations are nilpotent, so for any state $|\phi\rangle$ we have

$$Q^2|\phi\rangle = 0.$$  \hfill (35)

The physical states $|\phi_p\rangle$ can now be defined as states that are annihilated by $Q$

$$Q|\phi_p\rangle = 0.$$  \hfill (36)

All the states that are obtained by the action of $Q$ on unphysical states are physical. However, they are orthogonal to all physical states. Thus, two states differing from each other by addition of such a state will be indistinguishable. Furthermore, if the asymptotic physical states are given by

$$|\phi_{pa, out}\rangle = |\phi_{pa}, t \to \infty\rangle,$$
$$|\phi_{pb, in}\rangle = |\phi_{pb}, t \to -\infty\rangle,$$  \hfill (37)

then a typical $S$-matrix element can be written as

$$\langle \phi_{pa, out}|S|\phi_{pb}\rangle = \langle \phi_{pa}|S\dagger S|\phi_{pb}\rangle.$$  \hfill (38)

Now as the BRST are conserved charges, so they commute with the Hamiltonian and thus the time evolution of any physical state will also be annihilated by $Q$,

$$QS|\phi_p\rangle = 0.$$  \hfill (39)

This implies that the states $S|\phi_p\rangle$ must be a linear combination of physical states denoted by $|\phi_{p,i}\rangle$. So we can write,

$$\langle \phi_{pa}|S\dagger S|\phi_{pb}\rangle = \sum_i \langle \phi_{pa}|S\dagger|\phi_{p,i}\rangle\langle \phi_{p,i}|S|\phi_{pb}\rangle.$$  \hfill (40)

Since the full $S$-matrix is unitary this relation implies that the $S$-matrix restricted to physical sub-space is also unitarity.

## 5 Conclusion

In this paper we analysed the non-anticommutative deformation of the ABJ theory in harmonic superspace. This deformation was caused by a graviphoton backgrounds. We also discuss the BRST symmetry of this theory and used it to show the unitarity of the $S$-matrix. There are other type of deformations that can be studied. These deformations can occur because of non-vanishing values of anti-commutators between Grassmann coordinates and physically correspond to a deformation generated by a $RR$ background in string theory. As M-theory is dual to string theory, these deformations would also generate non-commutative deformations on the M-theory side. The interesting thing about these deformations is that they break some amount of supersymmetry. Thus, the some amount of supersymmetry of the ABJ theory will be broken by non-anticommutative deformation of the harmonic superspace. One such deformation has been recently studied [43]. It may be noted that the addition of the mass term breaks the superconformal invariance without breaking any supersymmetry [44]. It will be interesting to analyse these aspects further in harmonic superspace. Furthermore, by using a novel Higgsing mechanism, the gauge group of the ABJM theory can be spontaneously broken down to its diagonal subgroup [45, 46, 47, 48].
Thus, by using this novel Higgsing mechanism the action for multiple M2-branes can be reduced to the action for multiple D2-branes. This analysis has also been performed in $\mathcal{N} = 1$ superspace deformed by a graviphoton background [49]. It will interesting to analyse this novel Higgsing mechanism in harmonic superspace deformed by a graviphoton background.

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