Supersums for all supersymmetric amplitudes

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Abstract

We present an on-shell graphical framework for superamplitudes in super Yang-Mills theory with arbitrary supersymmetry. Our diagrammatic procedure is derived through manipulations of the full $\mathcal{N} = 4$ superamplitude and illustrated by a number of explicit examples.
I. INTRODUCTION

Multiloop scattering amplitudes in maximally supersymmetric ($\mathcal{N} = 4$) Yang-Mills theory have been studied extensively over the years [1–12] in connection with for instance the famous AdS/CFT correspondence and possible finiteness of supergravity theories. Remarkable results have been uncovered including new favorable evaluation methods applicable to both tree- and loop-level amplitudes.

An essential part of this progress is the on-shell superspace formalism, which organizes on-shell states and scattering amplitudes in maximally supersymmetric super Yang-Mills theory very elegantly [1, 2, 13–22]. The principle is to arrange the entire supermultiplet as a convenient expansion labeled by $R$-symmetry indices and particle number into $n$ superfields, one for each external leg. All possible scattering combinations are realized by formation of superamplitudes, defined as generating functions with the superfields as input, having all supersymmetric Ward identities automatically satisfied. Individual scattering amplitudes are available from the generating function using appropriate combinations of Grassmann differential operators. Using either the Maximally Helicity Violating (MHV) vertex expansion [23, 24] or the Britto-Cachazo-Feng-Witten (BCFW) on-shell recursion relations [25, 26] superamplitudes for general particle and helicity configurations may be constructed.

In a recent paper [2], overlapping with [27], the maximally supersymmetric superspace setup was generalized to super Yang-Mills theory with reduced supersymmetry, i.e. with $\mathcal{N} < 4$ generators of supersymmetry. All necessary steps towards developing both a holomorphic and a non-holomorphic approach were taken. In particular, it was shown that the most general MHV generating function valid for generic supersymmetry can be derived by simple combinations of truncations and Fourier transforms of the full $\mathcal{N} = 4$ superamplitude.

The introduction of $\mathcal{N} = 4$ superamplitudes spawned important developments including a very convenient diagrammatic representation [1]. For brevity, all possible contractions between external states are tracked, yielding a one-to-one correspondence between diagrams and individual scattering amplitudes. Motivated by [2] we will extend this graphical framework to $\mathcal{N} < 4$ superamplitudes.
II. \( \mathcal{N} = 4 \) SUPERAMPLITUDES

Before developing the superspace formalism and discussing the MHV and \( \overline{\text{MHV}} \) generating functions for super Yang-Mills theory with less than maximal supersymmetry, we briefly remind ourselves about the \( \mathcal{N} = 4 \) setup.

The \( \mathcal{N} = 4 \) vector multiplet is uniquely CPT self-conjugate allowing all on-shell states to be incorporated into a single holomorphic superfield \( \Phi(p, \eta) \) written as an expansion in Grassmann variables \( \eta_a \) with \( a = 1, \ldots, 4 \) being \( R \)-symmetry indices. Within this framework the sixteen physical states in the \( \mathcal{N} = 4 \) supermultiplet are two gluons \( g_+ \) and \( g_{abcd}^- \), four gluino pairs \( f^+_{a} \) and \( f^{abc}^- \), plus six real scalars \( s^{ab} \), all completely antisymmetric. The superfield then takes the form

\[
\Phi(p, \eta) = g_+ + \eta_a f^+_a + \frac{1}{2!} \eta_a \eta_b s^{ab} + \frac{1}{3!} \eta_a \eta_b \eta_c f^{abc} + \frac{1}{4!} \eta_a \eta_b \eta_c \eta_d g^{abcd}_-. \tag{1}
\]

Grassmann Fourier transformation yields an antiholomorphic superfield,

\[
\bar{\Phi}(p, \bar{\eta}) = g^- + \bar{\eta}^a f^-_a + \frac{1}{2!} \bar{\eta}^a \bar{\eta}^b s_{ab} + \frac{1}{3!} \bar{\eta}^a \bar{\eta}^b \bar{\eta}^c f^{abc}_+ + \frac{1}{4!} \bar{\eta}^a \bar{\eta}^b \bar{\eta}^c \bar{\eta}^d g^{abcd}^+, \tag{2}
\]

but with the exact same particle content encoded. Because of this equivalence either representation may be preferred.

In order to shed light on how proliferation of amplitudes in \( \mathcal{N} = 4 \) super Yang-Mills theory is handled, the concept of superamplitudes is introduced. With \( n \) copies of the superfield \( \Phi_i \) arranged, we organize the full \( n \)-point tree-level superamplitude ascendingly according to Grassmann degree in steps of four,

\[
\mathcal{A}_n(p, \eta) = \mathcal{A}(\Phi_1 \cdots \Phi_n) = \mathcal{A}_n^{\text{MHV}} + \mathcal{A}_n^{\text{NMHV}} + \cdots + \mathcal{A}_n^{\overline{\text{MHV}}}, \tag{3}
\]

ranging from eight \( \eta \)'s to \( 4n - 8 \). Explicit formulas for all \( \mathcal{N}^k \text{MHV} \) amplitudes relying on BCFW shifts \([16, 19, 25, 26]\) exist in the literature \([20]\), but our focus is on MHV and \( \overline{\text{MHV}} \) amplitudes. It follows that all MHV amplitudes may be collected into a generating function, which we will call the MHV superamplitude, such that each term corresponds to a regular scattering amplitude involving gluons, fermions and scalars. The MHV superamplitude is defined as \([1]\)

\[
\mathcal{A}_n^{\text{MHV}}(1, 2, \ldots, n) = i \frac{(2\pi)^4 \delta^{(4)}(\sum_{j=1}^{n} p_j)}{\prod_{m=1}^{n}(m(m + 1))} \delta^{(8)} \left( \sum_{j=1}^{n} \lambda^\alpha_{\eta_j} \right), \tag{4}
\]
and contains in addition to the well-known overall momentum conservation, an eightfold Grassmann delta function, which conserves supermomentum \( Q_\alpha^a \equiv \sum_{j=1}^n \lambda^a_j \eta_j^a \). It proves advantageous to expand the superamplitude as a sum of monomials in the \( \eta \)'s. Factorization in the group index and \( \delta(\eta) = \eta \) for Grassmann variables imply that

\[
\delta^{(8)} \left( \sum_{j=1}^n \lambda^a_j \eta_j^a \right) = 4 \prod_{a=1}^4 \delta^{(2)} \left( \sum_{j=1}^n \lambda^a_j \eta_j^a \right) = 4 \prod_{a=1}^4 \sum_{i<j} \langle ij \rangle \eta_i^a \eta_j^a .
\]  

Consequently, the MHV superamplitude can be recast as

\[
A^{\text{MHV}}_n (1, 2, \ldots, n) = i \prod_{a=1}^4 \sum_{i<j} \langle ij \rangle \eta_i^a \eta_j^a \prod_{m=1}^n \left[ \langle m \rangle \langle m+1 \rangle \right] ,
\]

with four-momentum conservation stripped.

Notation of component amplitudes is streamlined in terms of spinor products of supermomenta of the individual legs defined by

\[
\langle q_i a q_j a \rangle \equiv \eta_i^a \langle ij \rangle \eta_j^a , \quad \left[ \tilde{q}_i^a \tilde{q}_j^a \right] \equiv \tilde{\eta}_i^a [ij] \tilde{\eta}_j^a .
\]

For four external legs, some simple examples of component amplitudes are

\[
A^{\text{tree}}_4 (1^- g_{1234}, 2^- g_{1234}, 3^+ g_{34}, 4^+ g_y) = i \prod_{a=1}^4 \langle q_1 a q_2 a \rangle \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle ,
\]

\[
A^{\text{tree}}_4 (1^- g_{abcd}, 2^- g_{abcd}, 3^+ g_{34}, 4^+ g_y) = i \prod_{a=1}^4 \langle q_1 a q_2 a \rangle \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle ,
\]

\[
A^{\text{tree}}_4 (1^- g_{abcd}, 2^- g_{abcd}, 3^+ g_{34}, 4^+ g_y) = i \prod_{a=1}^4 \langle q_1 a q_2 a \rangle \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle .
\]

Analogous to the MHV superamplitude one can define the \( \tilde{\text{MHV}} \) superamplitude [1] by

\[
A^{\tilde{\text{MHV}}}_n (1, 2, \ldots, n) = i (-1)^n \prod_{m=1}^n \left[ \langle m \rangle \langle m+1 \rangle \right] = i (-1)^n \prod_{a=1}^4 \sum_{i<j} \langle \tilde{q}_i^a \tilde{q}_j^a \rangle \prod_{m=1}^n \left[ \langle m \rangle \langle m+1 \rangle \right] ,
\]

built entirely from antiholomorphic superfields [2]. It is mapped from the \( \tilde{\eta} \) coordinates to the untilded superspace using the Grassmann Fourier transform.

### III. ALL MHV SUPERAMPLITUDES

It is desirable to extend the neat \( N = 4 \) superspace formulation to super Yang-Mills theory with nonmaximal supersymmetry. Two different approaches exist [2], neither of which can be described in terms of a single superfield, due to the fact that \( N < 4 \) super
Yang-Mills theory is not CPT self-conjugate. Instead the physical states of the $\mathcal{N} = 1, 2, 3$ supermultiplets can be assembled into either two conjugate superfields $\Phi^N$ and $\tilde{\Phi}^N$, which are related to (1) and (2) by the truncations $\eta_{N+1, \ldots, 4} \to 0$ and $\tilde{\eta}^{N+1, \ldots, 4} \to 0$ respectively, or two holomorphic superfields $\Phi^N$ and $\Psi^N$ obtained from the $\mathcal{N} = 4$ superfield by suitable combinations of truncations and Grassmann integrations.

Here we focus on the $\Phi - \Psi$ formalism, since we would like to avoid the unfortunate mixing of the $\eta$ and $\tilde{\eta}$ variables in the $\Phi - \Phi^\dagger$ picture, which does not lead to any obvious graphical interpretation.

Consider, say, the $\mathcal{N} = 1$ conjugate superfields obtained by letting $\eta_{2,3,4} \to 0$ and $\tilde{\eta}^{2,3,4} \to 0$ in (1) and (2). It follows that $\Phi^{N=1}(p, \eta) = g_+ + \eta_a f_+^a$ and $\tilde{\Phi}^{N=1}(p, \tilde{\eta}) = g^- + \tilde{\eta}^a f_+^a$.

The $n$-point MHV configuration then has two states from the $\tilde{\Phi}^{N=1}$ superfield and $n - 2$ from the other. The essence of the $\Phi - \Psi$ approach is to keep the truncated holomorphic superfields, but discard the ones of negative overall helicity. To achieve this we observe that full description of the particle content can be maintained by supplementing $\Phi_i$ by a new holomorphic superfield introduced in [2],

$$\Psi_i^N(p, \eta) \equiv \int \left( \prod_{a=N+1}^4 d\eta_a \right) \Phi_i^{N=4}(p, \eta),$$

where for instance $\Psi^{N=1} = f_- + \eta_a g_+^a$ with $a = 1$ fixed but kept for notational uniformity.

For clarity we list here all pairs of superfields $[\Phi^N, \Psi^N]$ for $\mathcal{N} = 1, 2, 3$. States of the $\Psi$-sector are hatted to be distinguished from those originating from the non-manipulated superfields. In this notation the indices take values $a = 1, \ldots, \mathcal{N}$.

$$\Phi^{N=1} = g_+ + \eta_a f_+^a, \quad \Psi^{N=1} = \hat{f}_- + \eta_a \hat{g}_-^a,$$

$$\Phi^{N=2} = g_+ + \eta_a f_+^a + \frac{1}{2!} \eta_a \eta_b s^{ab}, \quad \Psi^{N=2} = \hat{s} + \eta_a \hat{f}_-^a + \frac{1}{2!} \eta_a \eta_b \hat{g}_-^{ab},$$

$$\Phi^{N=3} = g_+ + \eta_a f_+^a + \frac{1}{2!} \eta_a \eta_b s^{ab} + \frac{1}{3!} \eta_a \eta_b \eta_c f_-^{abc}, \quad \Psi^{N=3} = \hat{f}_+ + \eta_a \hat{s}^a + \frac{1}{2!} \eta_a \eta_b \hat{f}_-^{ab} + \frac{1}{3!} \eta_a \eta_b \eta_c \hat{g}_-^{abc}.$$  

Let us define what is understood by a MHV amplitude in $\mathcal{N} < 4$ super Yang-Mills theory in this notation. First of all, a MHV amplitude must have $2\mathcal{N}$ paired indices as
a consequence of truncation of the superfields. Moreover, to be MHV requires two states from the Ψ sector and \( n - 2 \) from the Φ superfield. With this in mind we are now ready to derive the MHV superamplitude for \( \mathcal{N} < 4 \). Recall that the maximally supersymmetric superamplitude is a function of \( \Phi_i, i = 1, \ldots, n \). Suppose legs \( i \) and \( j \) represent states of the Ψ-sector. We can then convert \( \Phi_i \) and \( \Phi_j \) to \( \Psi_i \) and \( \Psi_j \) by integrating out \((4 - \mathcal{N})\) \( \eta_i \)'s and \( \eta_j \)'s according to (12). Afterwards we truncate the remaining \( n - 2 \) legs to reduce the content of the Φ superfields. The only obstacle is to rearrange the integration measure appropriately, but actually the overall sign is not very important to us.

\[
A_{\mathcal{N},\text{MHV}}^{n,ij} = i \int \left( \prod_{b=N+1}^{4} d\eta_{ib} \prod_{c=N+1}^{4} d\eta_{jc} \right) \left[ \prod_{a=1}^{n} \sum_{k<l} \langle kl \rangle_{\eta_{ka}\eta_{la}} \right] \left[ \prod_{m=1}^{\mathcal{N}} \langle m(m+1) \rangle \right] \bigg|_{\text{truncate}}
\]

\[
= i (-1)^{\frac{1}{2} \mathcal{N}(\mathcal{N}-1)} \left( \prod_{m=1}^{n} \langle m(m+1) \rangle \right) \int \left( \prod_{b=N+1}^{4} d\eta_{ib} d\eta_{jb} \right) \left( \prod_{a=1}^{n} \sum_{k<l} \langle kl \rangle_{\eta_{ka}\eta_{la}} \right) \bigg|_{\text{truncate}}
\]

\[
= i (-1)^{\frac{1}{2} \mathcal{N}(\mathcal{N}-1)} \frac{\langle ij \rangle^{4-N} \prod_{a=1}^{\mathcal{N}} \sum_{k<l} \langle kl \rangle_{\eta_{ka}\eta_{la}}}{\prod_{m=1}^{n} \langle m(m+1) \rangle}.
\]

(16)

It is noticed that this generic MHV superamplitude, which is equivalent to the one derived in [1], as expected reduces to the original \( \mathcal{N} = 4 \) MHV superamplitude for maximal supersymmetry, and to the Parke-Taylor formula for pure Yang-Mills theory (\( \mathcal{N} = 0 \)).

IV. DIAGRAMMATIC REPRESENTATION

It would be useful to have a simple visualization of the rather abstract expression for the \( \mathcal{N} \leq 4 \) superamplitude. Similar to Feynman graphs, such a scheme should yield a one-to-one correspondence between diagrams and component amplitudes.

The basic ingredients of the \( \mathcal{N} = 4 \) MHV superamplitude appear in (6) as a cyclic spinor string in the denominator and spinor products of supermomenta in the numerator. The pictorial representation for these components was developed in [1] and the transition to \( \mathcal{N} < 4 \) was sketched. Taking (16) into account we see that the only structural difference between the maximally supersymmetric and the \( \mathcal{N} < 4 \) generating function is the number of spinor products of supermomenta. Contrary to our \( \mathcal{N} < 4 \) MHV superamplitude, the \( 4-\mathcal{N} \) integrated index lines responsible for the \( \langle ij \rangle^{4-\mathcal{N}} \) factor still carry Grassmann variables with fixed \( R \)-symmetry indices in [1]. We circumvent these by introducing a new sector line between the two states of the Ψ-sector to catch the \( \langle ij \rangle^{4-\mathcal{N}} \) factor of (16).
Figure 1. The individual terms in the spinor string in the denominator are diagrammatically represented by a curved line without endpoints. The blue index line with endpoints translates into a spinor product of supermomenta of the corresponding individual legs. For $\mathcal{N} < 4$, states of the $\Psi$-sector must be connected with a solid green sector line without endpoints. Identical graphs exist in the $\overline{\text{MHV}}$ picture with obvious continued expressions.

In order to construct an index diagram for a given MHV amplitude with $n$ external legs in $\mathcal{N} \leq 4$ super Yang-Mills theory having $2\mathcal{N}$ paired $R$-symmetry indices, simply follow this prescription:

1. Draw a polygon with $n$ sides of solid, black lines curving inwards, leaving space at each corner for external legs.
2. Distribute the external legs at the gaps and label them with appropriate momentum, helicity and $R$-symmetry indices.
3. Connect paired $R$-symmetry indices with blue index lines with endpoints.
4. For $\mathcal{N} < 4$ insert a single solid green sector line without endpoints between the two states of the $\Psi$-sector.
5. Indicate holomorphicity with $\oplus$ for MHV and $\ominus$ for $\overline{\text{MHV}}$ in the center of the diagram.

The diagram rules are summarized in Fig. 1.

V. TREE-LEVEL EXAMPLES

Let us now see how the diagrammatic representation scheme works out in practice for a number of MHV amplitudes at tree-level in super Yang-Mills theory with $\mathcal{N} = 4$ and fewer supersymmetries. We start with the amplitudes expressions (8)-(10), whose corresponding diagrams are drawn Fig. 2. These amplitudes are viewed for maximal supersymmetry without sector lines, but actually they exist for all $\mathcal{N}$, $\mathcal{N} \geq 1$ and $\mathcal{N} \geq 2$ respectively. In Fig. 3 the corresponding $\mathcal{N} = 2$ graphs are given. Thanks to the $\langle ij \rangle^2$ factor from the sector line the diagrams of Figs. 2 and 3 have pairwise identical expressions.
Figure 2. The analytic expressions for the $\mathcal{N} = 4$ amplitudes (8)-(10) are neatly captured by these three simple index diagrams. If reinterpreted in the MHV picture the first diagram would have four index lines between the two positive helicity gluons for instance.

Figure 3. Many amplitudes exist for several values of $\mathcal{N}$. Here (8)-(10) are shown for $\mathcal{N} = 2$. In general, if a diagram has at most $\Lambda$ grouped index lines in $\mathcal{N} = 4$, then it can be non-zero for reduced supersymmetry only provided $\Lambda \geq 4 - \mathcal{N}$.

We have now shown how easy it is to draw tree-level diagrams for MHV configurations, but it still remains to demonstrate the translation of diagrams into the matching analytic expressions. Consider, say, the six-point $\mathcal{N} = 3$ and a seven-point $\mathcal{N} = 2$ MHV tree-level amplitudes of Fig. 4. These diagrams were constructed simply by selecting two states from the $\Psi$ superfield and then patching up using $\Phi$ respecting that the total number of indices should be $2\mathcal{N}$. Referring to Fig. 1 we almost effortlessly find

\[
A_6^\text{tree}(\hat{1}_g^{abc} - \hat{2}_g^{abc} + 3_g^{abc} - \hat{4}_g^{abc} + 5_g^{abc} - \hat{6}_g^{abc}) = i\frac{(12)}{\prod_{i=1}^6 i(i + 1)} \langle q_{1a}q_{2a} \rangle \langle q_{1b}q_{3b} \rangle \langle q_{1c}q_{3c} \rangle,
\]

(17)

\[
A_7^\text{tree}(\hat{1}_f a^{f_4} - \hat{2}_f a^{f_4} + 3_f a^{f_4} - \hat{4}_f a^{f_4} + 5_f a^{f_4} - \hat{6}_f a^{f_4} - \hat{7}_f a^{f_4}) = i\frac{(13)^2}{\prod_{i=1}^7 i(i + 1)} \langle q_{1a}q_{2a} \rangle \langle q_{3b}q_{4b} \rangle \langle q_{3b}q_{4b} \rangle.
\]

(18)
VI. SUPERSYMMETRIC SUMS IN UNITARITY CUTS

Despite being formulated for tree-level amplitudes, the idea of generating functions and their diagrammatic representation fits excellently with evaluation of loop amplitudes using the generalized unitarity cut method \cite{1}. Indeed, the required supersymmetric sum over all possible on-shell states propagating in the intermediate channels may be realized as Grassmann integration of superamplitudes, while the flow of $R$-symmetry charges is captured by index diagrams.

As an example we study a one-loop unitarity cut of a four-point amplitude with external gluons, and carry out the supersum for the $N = 2$ case. For a more thorough description of unitarity cuts see \cite{28, 29} and later developments \cite{8, 30–33}. Evaluation of supersums has been discussed in \cite{1, 2, 14–17}. Here we follow the strategy of \cite{1}.

The supersum in question receives in total eight contributions, two with internal gluons, four with a fermion loop and two having scalars. Fig. 5 provides two of these diagrams, and using the rules given in Fig. 1, we find that their numerator part translate into

\[
\langle q_{2a} q_{\ell_1 a} \rangle [\bar{q}_{l_1}^a \bar{q}_1^a] \langle q_{2b} q_{\ell_2 b} \rangle [\bar{q}_{l_2}^b \bar{q}_1^b] (2|\ell_1|4)^2 \quad \text{and} \quad \langle q_{2a} q_{\ell_1 a} \rangle [\bar{q}_{l_1}^a \bar{q}_1^a] \langle q_{2b} q_{\ell_2 b} \rangle [\bar{q}_{l_2}^b \bar{q}_1^b] (2|\ell_1|4)^2,
\]

where $\langle ij \rangle [jk] = \langle i|j|k \rangle$ for shorthand.

By inspection all eight diagrams have the same relative sign because the corresponding Grassmann expressions are ordered equivalently. Therefore the $\eta$’s may just be suppressed,
Figure 5. The left and right index diagrams should respectively illustrate internal gluon and fermion contributions in a unitarity cut of the four-point one-loop amplitude. The cut marked by the dashed red line splits the amplitude into MHV and $\bar{\text{MHV}}$ parts. Again the green sector line accounts for reduced supersymmetry. Horizontal flips of these diagrams and two additional diagrams representing internal scalars are very easy to draw, but are left out here.

and the numerator of the supersum thus becomes

$$\langle 2|\ell_1|4\rangle^4 + \langle 2|\ell_2|4\rangle^4 + 2\langle 2|\ell_1|4\rangle^3\langle 2|\ell_2|4\rangle + 2\langle 2|\ell_1|4\rangle\langle 2|\ell_2|4\rangle^3 + 2\langle 2|\ell_1|4\rangle^2\langle 2|\ell_2|4\rangle^2$$

$$= \left(\langle 2|\ell_1|4\rangle + \langle 2|\ell_2|4\rangle\right)^2 \times \left(\langle 2|\ell_1|4\rangle^2 + \langle 2|\ell_2|4\rangle^2\right). \quad (20)$$

An interesting pattern emerges when comparing the supersums for $\mathcal{N} = 1, 2, 3, 4$. Adding the diagrams in the right combinations dictated by the different supermultiplets we see that the displayed resummation property is a common feature. Indeed, for maximal supersymmetry the supersum takes the very compact form $\left(\langle 2|\ell_1|4\rangle + \langle 2|\ell_2|4\rangle\right)^4$, while for $\mathcal{N} < 4$

$$\left(\langle 2|\ell_1|4\rangle + \langle 2|\ell_2|4\rangle\right)^\mathcal{N} \times \left(\langle 2|\ell_1|4\rangle^{4-\mathcal{N}} + \langle 2|\ell_2|4\rangle^{4-\mathcal{N}}\right), \quad (21)$$

in agreement with [1].

Cuts and supersums are by no means limited to MHV amplitudes. Non-MHV amplitudes may be generated from MHV ones using the MHV vertex construction [23, 24] as addressed in [1, 14, 15, 21], and the MHV techniques thus apply.

VII. CONCLUSION

In this paper we have investigated the superspace formalism of $\mathcal{N} = 4$ super Yang-Mills theory and its extension to situations with less than maximal supersymmetry. We have
written a general $\mathcal{N}$-dependent form of the MHV generating function at tree-level. More importantly, an extension to $\mathcal{N}$-fold supersymmetry of a recent scheme for representing $\mathcal{N} = 4$ superamplitudes diagrammatically was presented. Although simple, it is nice to see that the technique carries over from $\mathcal{N} = 4$. With this diagrammatic prescription it is extremely easy to memorize any super Yang-Mills scattering amplitude and translate it into analytic expressions, as we illustrated through several examples at both tree-level and one-loop.

We will leave the exploration of multiloop unitarity cuts of non-MHV amplitudes utilizing the full content of the various $\mathcal{N} < 4$ multiplets for external states for future work. Another direction could be to study diagrams and supersums in super Yang-Mills theory coupled to matter.

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