Transmittance spectra in one-dimensional dielectric photonic crystals with defects

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Abstract. We have calculated the dependence of transmittance spectra with wavelength for TE wave at normal incidence in one-dimensional dielectric photonic crystals (1DPC) using the transfer matrix method. We demonstrate that when consider two defects layers spatial breaking periodicity in the 1DPC, the presence of defect modes located within the photonic band gap can be found. Additionally we have calculated the effect on the transmittance spectrum by varying the thickness of the defects in the 1DPC.

1. Introduction
In 1987 two works were published independently, which marked the birth of what today is known as photonic crystals (PC) [1, 2]. Such crystals are characterized by having a periodic modulation in space of its dielectric constant and are essentially act as light semiconductors, in them the light always find some direction from which it can propagate. The light waves that are allowed to propagate in the crystal are known as modes, modes groups form the bands. An element of interest for the design and subsequent use of the PC is to determine the regions of existence of forbidden photonic bands (PBG), in which no mode with energy found in this region will propagate [3]. Among the numerical methods allowing to obtain information from the band structure, the plane wave expansion method, the finite difference method in the time domain and of the dispersion matrix method [4, 5]. The existence of PBG in PC gives rise to different optical phenomena as the inhibition of spontaneous emission, waveguides with low losses and Fabry Perot resonators.

The introduction of defects that broken the spatial periodicity of the PC, may be point, linear or planar, and produce electromagnetic modes locating or the radiation guidance within the PBG. The defects are introduced in the PC by changing the thickness of the layers, inserting another dielectric structure, or removing a layer of PC. It is in this field where PCs are attractive, planar microcavities using with such systems with high quality factors [6]. Photonic crystals can be formally described with Maxwell's electromagnetic theory, which makes them systems of scalable features, the electromagnetic properties of a microscopic crystal whose work is in the visible spectral range are maintained if the crystal is scaled to macroscopic size in the range of microwaves. The purpose of this work is to investigate the transmittance spectrum using the method of the transfer matrix when two defects are introduced in the one-dimensional dielectric photonic crystals (1DPC).
2. Transfer matrix method
Consider first a light beam incident with an angle \( \theta \) from the left of a 1DPC, \( z \) direction, as shown in figure 1. In 1DPC, \( N \) is the number of periods of the bilayers, \( H \) and \( L \) are layers of refractive index, \( n_H \) high and \( n_L \) low, respectively. The medium surrounding the dielectric 1DPC heterostructure is air.

![Figure 1. Schematic diagram of a one-dimensional photonic crystal](image)

The electric field for TE mode satisfies the Helmholtz equation:

\[
\frac{\partial^2}{\partial z^2} E(z) + \frac{w^2}{c^2} \epsilon(z) E(z) = 0, \tag{1}
\]

where \( \epsilon(z) \) is the dielectric constant related to the refractive index \( \epsilon(z) = n^2(z) \). The solution of equation (1) for the \( j \)-th layer is given by

\[
\vec{E}_j(x, z) = (A_j e^{ik_j z} + B_j e^{-ik_j z}) e^{-ik_x x} \hat{y}, \tag{2}
\]

where the TE mode is defined as the component of electric field parallel to interface 1DPC, \( k_{j,z} = \sqrt{(w/c)^2 \epsilon_j - k_x^2} \) and \( \epsilon_j \), are the \( z \) component of the wave vector and the dielectric constant in the \( j \)-th layer, respectively. The transverse component of the wave vector is \( k_x = k_0 \sin \theta \), with \( k_0 \) the wave vector in the incident medium.

The \( A_j \) and \( B_j \) parameters are calculated using the continuity conditions of the tangential components of electromagnetic fields at each interface of 1DPC. However, requires solving a substantial number of algebraic equations, therefore it is necessary to use the transfer matrix method (TMM) [7]. According to the TMM, each layer may be represented by a matrix.

\[
M_j = D_j P_j D_j^{-1} ; j = H, L. \tag{3}
\]

In equation (3), propagation matrix is defined as

\[
P_j = \begin{pmatrix} e^{i\phi_j} & 0 \\ 0 & e^{-i\phi_j} \end{pmatrix}, \tag{4}
\]

where the \( \phi_j \) phase is

\[
\phi_j = k_{j,x} l_j = \frac{2\pi d_j}{\lambda} n_j \cos \theta_j. \tag{5}
\]

In equation (5), \( d_j, n_j \) and \( \theta_j \) are the thickness, the refractive index and the angle at the \( j \)-th layer, respectively. The dynamical matrix is given by

\[
D_j^{TE} = \begin{pmatrix} 1 & 1 \\ n_j \cos \theta_j & -n_j \cos \theta_j \end{pmatrix}, \tag{6}
\]

for TE waves, and...
For the structure, \( \text{Air}/(HL)^N/\text{Air} \), the total transfer matrix is
\[
M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = D_0^{-1} [M_H M_L]^N D_0,
\tag{7}
\]
where \( D_0 \) is the dynamical matrix for air.

The reflectance \( R \) and transmittance \( T \) in the case of normal incidence on the structure is given by
\[
R = \left| \frac{M_{21}}{M_{11}} \right|^2 \quad \text{and} \quad T = \left| \frac{1}{M_{11}} \right|^2,
\tag{8}
\]
where \( M_{11} \) and \( M_{21} \) represent the matrix elements of the transfer matrix, equation (7).

3. Results and discussion

In the following calculations, we consider a 1DPC structure with a PBG in the visible region. Here, \( H \) is taken as \( TiO_2 \) with a refractive index \( n_{TiO_2} = 2.2 \) \( y \) \( SiO_2 \) with \( n_{SiO_2} = 1.45 \) for layer \( L \), the period is \( N = 6 \), [8]. In addition, both layers are a quarter wavelength, this is,
\[
n_{TiO_2} d_{TiO_2} = n_{SiO_2} d_{SiO_2} = \frac{\lambda_0}{4},
\tag{9}
\]
where \( \lambda_0 \) is the designed wavelength which is assumed to be 500 nm. Our study is restricted to the case of normal incidence to TE waves, where the left band edge \( \lambda_L \) and right band edge \( \lambda_R \) can be calculated by the theory of Bragg reflector, using the following equations [9],
\[
\lambda_L = \frac{\pi (n_{TiO_2} d_{TiO_2} + n_{SiO_2} d_{SiO_2})}{\cos^{-1}(-\rho)}, \quad \lambda_R = \frac{\pi (n_{TiO_2} d_{TiO_2} + n_{SiO_2} d_{SiO_2})}{\cos^{-1}(\rho)},
\tag{10}
\]
where \( \rho = (n_{TiO_2} - n_{SiO_2})/(n_{TiO_2} + n_{SiO_2}) \) is the coefficient of Fresnel. Based on equation (10), together with parameters of the material, we find that for a 1DPC, \( \text{Air}/(HL)^N/\text{Air} \), the edges of the band are \( \lambda_L = 441.79 \) nm and \( \lambda_R = 575.87 \) nm, as shown on figure 2.

If we introduce a defect located in the center of the 1DPC, the total transfer matrix for the structure \( \text{Air}/(HL)^N/D/(HL)^N/\text{Air} \), is
\[
M = D_0^{-1} [M_H M_L]^N M_D [M_H M_L]^N D_0,
\tag{11}
\]
in our calculations we consider that the defect \( D \) is the layer of low refractive index \( L \), \( SiO_2 \). The transmittance of this structure is shown on figure 3, a transmission peak is observed with a transmittance of 87.2% located in \( \lambda_0 \) within the PBG called defect mode, [8].
In the following calculations, we will investigate the introduction of two defects of SiO\textsubscript{2} to 1DPC in the transmittance spectrum, as shown in figure 4.

It can be seen from figure 5, two transmission peaks located around \( \lambda_0 \) within the PBG with a transmittance of 87.4\% and 85.1\%, at wavelengths of 503 nm and 507.5 nm, respectively. The TE mode with a wavelength value mentioned above, will propagate through the 1DPC structure and will not be fully reflected in the PBG. The appearance of these peaks is used to function as narrowband transmission filters multilayers which are also known as Fabry Perot multilayers resonators.
Let us finally examine the effect of thickness on the defect modes in 1DPC, \((HL)^N/mD/(HL)^N/mD/(HL)^N\), where \(m\) represents the repeated number of defects with the condition \(m < 6\). For an odd number of \(m\), \(m = 3\) see figure 6 and \(m = 5\) see figure 7, the two defect modes are fixed around the \(\lambda_0\) design wavelength. However, for \(m\) even, pronounced transmission peaks that are located within the PBG have transmittance values of 97.8% at wavelengths of 449.3 nm and 562.4 nm, as shown in figure 8. By increasing \(m\), the separation of defect modes decreases, figure 9 shows the presence within the PBG of four defect modes with a transmittance of 95% for wavelengths of 444.5 nm, 457.4 nm, 551.9 nm and 574.4 nm.

**Figure 5.** Transmittance spectrum for 1DPC, \(Air/(HL)^6/D/(HL)^6/D/(HL)^6/Air\)

**Figure 6.** Transmittance spectrum for 1DPC, \(Air/(HL)^6/mL/(HL)^6/mL/(HL)^6/Air\), with \(m=3\)

**Figure 7.** Transmittance spectrum for 1DPC, \(Air/(HL)^6/mL/(HL)^6/mL/(HL)^6/Air\), with \(m=5\)
Figure 8. Transmittance spectrum for 1DPC, $\text{Air}/(\text{HL})^6/\text{mL}/(\text{HL})^6/\text{Air}$, with $m = 2$

Figure 9. Transmittance spectrum for 1DPC, $\text{Air}/(\text{HL})^6/\text{mL}/(\text{HL})^6/\text{Air}$, with $m = 4$

4. Conclusions

The unusual property of photonic crystals to allow propagation of light for certain ranges of wavelengths, is demonstrated using the transfer matrix method when calculating the transmission spectrum for normal incidence TE waves in 1DPC. Introducing defects in 1DPC, causes the appearance of transmittance peaks which are located in the PBG, called defect modes. Unlike a 1DPC structure with a defect, where only a defect mode appears located at the design wavelength $\lambda_0$, Introducing two defects in the 1DPC causes two transmittance peaks around $\lambda_0$. The transmittance spectrum depends on the number of times that the defects are repeated. For odd values of the thickness, the defect modes maintain their position around $\lambda_0$. While for even of values of thicknesses, one can find the presence of additional defect modes in the PBG, whose separation decreases as the thickness of such modes increases.

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6. References

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