How can one detect the rotation of the Earth “around the Moon”?
Part 3: With a simple gravity pendulum

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Abstract  The attraction of the Moon on objects at the surface of the Earth gives rise to a so-called tidal force which is of the order of \(1/10,000,000\) times the gravitational force of the Earth. For instance, when the Moon is located between the Earth and the Sun (new moon) the distance from a given terrestrial location to the Moon is shorter at noon than at midnight. This reduces the gravitational acceleration and therefore increases the period of a simple pendulum by a small amount. Although the change is of the order of \(0.1\mu s\) it appears that it can be detected. We give some preliminary results and discuss how the accuracy can be further improved. It is hoped that the present paper will encourage new experiments in this direction.

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Introduction

The methods presented in the first two parts of this series of papers were both based on detecting the *angular velocity vector* of the rotation under consideration. This method of detection is independent of the size, attraction or nature of the celestial body which provokes the rotation. The only variable which matters is the magnitude of the angular velocity. In this framework the movement around the Moon is 13 times easier to detect than the rotation around the Sun in spite of the fact that the mass of the Sun is $2.7 \times 10^7$ times larger.

There is another class of detection methods which rather focus on gravitational attraction. Obviously any movement that takes place on the Earth is affected by the attraction of other celestial bodies (of which the Moon and the Sun are the most important) but this influence may be more or less easy to detect.

The simplest way to detect the gravitational influence of the Moon and Sun is to observe the sea tides. Unfortunately, tides have a fairly complicated connection with the gravitational attraction because they are also dependent on the size and depth of the lakes, seas or oceans under consideration. Offshore, in the deep ocean, the difference in tides is of the order of 50 cm to one meter, but in bays and estuaries it can reach several meters. Is it possible to find a physical system for which the connection would be simpler to analyze?

In 1913-1914 Albert A. Michelson set up an experiment which allowed him to observe tidal waves in the laboratory (Michelson 1914). A pipe, half-filled with water, 166 meter long and 15 centimeter in diameter was laid in a 2-meter deep trench cut in a East-West direction. At both end the pipe was sealed with a glass wall. The level of the water was read with the help of a micrometer microscope. Level changes of about 20 micrometer were observed. Because the pipe was 2 meter deep under ground dilatation due to temperature changes was negligible. The level changes were observed over a period of several months and were well in agreement which what was expected.

In the mid-19th century direct observation of tidal forces was attempted with the help of an “horizontal” pendulum. Such a pendulum is similar to a door whose axis of rotation would not be exactly vertical and would be able to rotate with very small friction. Such a device is very sensitive to small changes in the direction of the axis of rotation which is why it is used up to the present day to set up seismometers. Because the arm of such a pendulum is subject to a tiny gravity $g \sin \epsilon$ (where $\epsilon$ is the angle of the axis of rotation with the vertical) it is sensitive to very small forces such

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1Except for the fact that the magnitude of the changes was only 70% of what would have been observed if the Earth had been perfectly rigid. As a matter of fact, the main purpose of the experiment was to estimate the rigidity of the Earth.
as tidal forces. Needless to say, such a device is very sensitive to external vibrations or thermal dilatation effects.

An obvious candidate for the detection of small gravity changes induced by tidal forces is the simple gravity pendulum. In contrast to the Foucault pendulum which has two rotational degrees of freedom, the simple pendulum moves around a fixed axis of rotation and has therefore only one degree of freedom. Back in the 18th and 19th century the simple pendulum has played an essential role in the study of many physical effects. Can it be used to detect the effect of the gravitational field of the Moon?

At the center of the Earth the attraction of the Moon $F_M(r)$ is exactly balanced by the centrifugal force due to the rotation of the Earth “around the Moon” (in fact around the center of mass of the Earth-Moon system) but at points located at the surface of the Earth the two forces do not cancel one another exactly. The difference is called the tidal force due to the Moon. The order of magnitude of the corresponding acceleration is:

$$a_t = 2GMr/R^3$$

where:
- $G$: constant of universal attraction, $G = 6.6 \times 10^{-11}$ (SI units)
- $M$: mass of the Moon, $M = 7.3 \times 10^{22}$ kg
- $r$: radius of the Earth, $r = 6,400$ km
- $R$: distance from the Moon to the Earth, $R = 384,000$ km

Numerical computation gives: $a_t = 1.1 \times 10^{-7}$ g. This small difference in gravity will change the period $T_0 = 2\pi \sqrt{L/g}$ of a pendulum by the following amount:

$$T = 2\pi \sqrt{\frac{L}{g + a_t}} = T_0 \left(1 - 0.5 \frac{a_t}{g}\right)$$

As a result, the period of a seconds pendulum for which $T_0 = 2$ s will be changed by about $0.2 \mu$s. The question is whether such a small difference can be measured.

We will see that the answer is “yes”. In the following sections we give some preliminary results and explain what can be done in order to improve the accuracy of the measurement.

We considered the “new moon” situation for the sake of simplicity but it is clear that a similar experiment can be done also in other configurations. For instance once the

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2In this connection it can be recalled that the definition of the meter was chosen to be almost equal to the length $L$ of the so-called “seconds pendulum” which is a pendulum whose period is precisely two seconds, one second for a swing in one direction and one second for the return swing: $L = 0.994$ m. As a matter of fact, before being defined as $10^{-7}$ times the distance from the Equator to the North Pole, the meter had been defined as the length of a seconds pendulum.

3There is a similar tidal force due to the Sun.
Moon has achieved one fourth of a revolution it will be in the so-called “first quarter” position. In this situation the distance between the Moon and the surface of the Earth will be shortest for the locations where the (solar) time is 6pm and it will be farthest for the locations where the time is 6am.

![Diagram](image)

**Fig. 1: Tidal force on a pendulum at noon and midnight.** At the center of the Earth the attraction of the Moon (in magenta) is exactly balanced by the centrifugal force (in green) due to the rotation of the Earth in the Earth-Moon system. At the surface of the Earth the cancellation is no longer an exact one however. The (small) difference is the so-called tidal force. Its order of magnitude is about $10^{-7}$ times the Earth gravity acceleration \( g = 9.81 \text{m/s}^2 \). As a result, the periods of a pendulum at noon and midnight will differ by a small amount which is of the order of 0.2\( \mu \text{s} \) for a pendulum whose period is 2 s. For the sake of simplicity the diagram shows two specific configurations where the Moon, the Sun and the axis of rotation of the Earth are in the same plane. Of course, the same effect also exists in other configurations although the exact amount of the difference between the longest and shortest periods may be slightly different.

**Implementation of the experiment**

The description of the tidal force (Fig. 1) suggests a procedure for carrying out the experiment.

**Noon-midnight difference**

In a situation where the Moon is located between the Earth and the Sun (which is called “new moon” or more appropriately “dark moon”) a person located on the Earth will be closest to the Moon at noon and farthest at midnight. As the Moon’s attraction reduces the gravity, \( g \) will be smaller at noon and the period of a pendulum should therefore be larger. The question is how to organize the measurements so that the best accuracy can be achieved.

**How many periods should one measure?**

An important uncertainty comes from the time measuring process. A quartz clock may have a short-term accuracy of about 1\( \mu \text{s} \). However the device which starts and stops the clock (usually an infrared detector) may be less accurate. In order to limit the number of starts and stops it is best to measure the time for a large number of swings. Thus, if the interval during which the measurement is performed covers one hour at noon and one hour at midnight, one may in each case perform for instance
6 measurements, each of which will include 600 swings of 1s (i.e. 300 periods of 2s). The $0.1\mu s$ accuracy needed on one swing will translate into a $60\mu s$ requirement for the series of 600 swings. The 6 successive measurements will allow to check the stability of the pendulum’s swings and the reliability of the measurement process.

**The damping issue**

At this level of precision, the amplitude plays an important role even for small angles. In the small angle approximation the relationship between the period and the amplitude $\theta_1$ is the following (Cabannes 1966 p. 203):

$$\frac{T - T_0}{T_0} \sim \frac{\theta_1^2}{16} \quad (D1)$$

For an angle of 1 degree and $T_0 = 2$ s one gets $T - T_0 = 38 \mu s$. In words, this means that for an amplitude of one degree the period is 38 microseconds longer than the ideal period $2\pi \sqrt{L/g}$ of a simple pendulum. In order to make the pendulum move during one hour without too much damping taking place there are two possible options.

- The first option is to reduce the friction as much as possible.
- The second option is to maintain the amplitude of the movement through a propulsion device which gives back to the pendulum the energy that it loses through friction.

Two pendulums designed by Mr. Marcel Bétrisey meet both requirements. They have a mass of 4 kg and a period of 2 s and are contained in a glass tube in which a low air pressure of 1000 Pa is maintained. An initial swing with an amplitude of 2 cm (which corresponds to an angle of 1.15 degree) can keep them in movement for several hours: 6 hours for “Chronolithe” and 24 hours for “Conti”. Moreover, the amplitude can be maintained by using a “light engine” based on the same principle as Crookes’s radiometers.

**Preliminary results**

In this section we give the results already obtained. They should be considered as preliminary and open to improvement for the following reasons:

- The trigger device which starts and stops the clock was an infrared device which is known to be less accurate as the clock itself.
- Instead of measuring the time for 600 swings (as suggested above) it was measured for subsets of only 10 swings. This required the trigger device to work much more often and therefore amplified the inaccuracy due to this device.

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4On [http://www.betrisey.ch/econti.html](http://www.betrisey.ch/econti.html) this pendulum can be found in the section “Radiometric clocks”; the names of these specific clocks are “Chronolithe” and “Conti”.
Table 1  Half period -1 of a seconds pendulum at noon and midnight (10-13 April 2002)

| Trial | Noon m [µs] | Noon  σ [µs] | Midnight m [µs] | Midnight σ [µs] | Noon-midnight m [µs] | Noon-midnight σ [µs] |
|-------|-------------|--------------|-----------------|-----------------|----------------------|----------------------|
| 1     | -1.49       | 11.1         | -1.57           | 12.0            | 0.07                 | 1.22                 |
| 2     | -3.71       | 17.1         | -2.88           | 9.9             | -0.82                | 1.42                 |
| 3     | -0.25       | 12.4         | -3.85           | 9.2             | 3.61                 | 1.14                 |
| 4     | -0.12       | 16.9         | -2.05           | 10.9            | 1.94                 | 1.47                 |
| Average | 14           | 10           | 1.2 ± 0.7       |                 |                      |                      |

Notes: Each swing was detected by an infrared sensor and the computer printed the average half-period every 10 swings that is to say about every 10 seconds. The two time intervals were one-hour intervals centered at noon and midnight, that is to say 11:30-12:30 and 23:30-00:30. In each of these intervals there were 360 measurements: m denotes the average of these measurements and σ their standard deviation. As this experiment was done in time of new moon (new moon was on 12 April 2002) one expects g to be smaller at noon which means that the period should be longer than at midnight. The expected difference for the half-period is $T'_{12} - T'_{24} = 0.09 \mu s$ which decomposes into 0.066µs for the Moon and 0.028µs for the Sun. The overall average and standard deviation can be computed by line or by column. For the average the two computations give of course the same result. For the standard deviation one gets two slightly different estimates, namely $\sqrt{\frac{1}{4}} = 0.98 \mu s$ for the σ of the averages and 0.58µs for the average of the σ. Because the error-bar which was used here is ±σ. it means that the probabilistic level of confidence is 0.68 and not 0.95 as would be the case for ±2σ.

Source: [http://www.betrisey.ch/eindex.htm](http://www.betrisey.ch/eindex.htm)

The results obtained for 5 noon-midnight comparisons are summarized in Table 1. Although the overall average (last line of the table) shows, that, as expected, the period is longer at noon than at midnight the error bar is still too large to make this result really conclusive. As suggested in the discussion, the accuracy can certainly be improved in forthcoming experiments.

In addition to the improvements already suggested, it is clear that the accuracy of the experiment can be improved by increasing the number of trials. If instead of 5 trials, one gets results for 20 trials, the standard deviation of the average will be divided by $\sqrt{20/5} = 2$.

Incidentally, Table 1 shows that the standard deviation is almost always larger at noon than at midnight. For the 5 trials the average of the standard deviation is 16µs at noon and 12µs at midnight.

Conclusions

The three experiments considered in the present series of papers rely on two different approaches. The approach of Parts 1 and 2 consists in observing the vectors of angular velocity associated with the movements of rotation whereas in the approach of Part 3 one measures gravitational forces. How do these approaches compare?
The second approach depends upon several parameters: the mass and distance of the body $A$ which produces the attraction, the diameter of the body $B$ (i.e. the Earth in our case) on which the measurement is performed, the distance between the centers of $A$ and $B$. As an illustration, let us for a moment assume that the Earth has the size of Jupiter. Its orbit around the Sun would be almost the same but the tidal force due to the Sun would be much larger. On the contrary if the Earth were of the size of an asteroid such as Ceres (480 km) the tidal force would be very small.

In addition, the tidal effect depends upon internal properties of the Earth such as the rigidity or mobility of its structural layers. This is the origin of the so-called gravific factor, an empirical parameter of the order of 1.5, which has to be applied to the tidal potential to get actual gravity changes.

In contrast, the angular velocity approach does not depend on the diameter of $B$ nor does it depend (directly) upon the distance $AB$. The only parameter which really matters is the period of rotation. Moreover the first approach allows the detection of rotations (such as the rotation of the Earth around its axis or its precessional axial rotation) which are not rotations around another astronomical body. In short, the first approach seems to provide a broader and “cleaner” view.

Of the three measurement methods that we have considered which one is the easiest to implement? At the present moment it is still difficult to answer this question because the most meaningful test, namely the detection of the rotation of the Earth around the Moon, has not yet been performed by more than one method.

The only comparison which can be made concerns the rotation of the Earth around its axis. This rotation can of course be detected with a Foucault pendulum. It can also be detected with a simple pendulum provided one compares measurements of the period performed at different locations. Such an observation was made for the first time in 1672 by the astronomer Jean Richer when he discovered that a pendulum which has a period of 2 s in Paris (at a latitude of 49 degrees) has a period shorter by 3.4 ms in the town of Cayenne in French Guyana at a latitude of 4 degrees. The fact that the Richer experiment took place almost two centuries before the Foucault experiment suggests that somehow it was easier to do. However, it required a long

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5 Even its precise shape will play a role for instance the fact that the Earth is not exactly a sphere; in contrast such details are completely irrelevant in the angular velocity approach.

6 The angular velocity is given by Kepler’s third law which says that if the major axis of the orbit is the same then the orbital period will also be the same.

7 Such movements can also be detected with a simple pendulum although in such cases the detection relies on observing the centrifugal force (rather than the tidal force) at two places located at different latitudes; see below for more details.

8 The formula is the following:

\[ T \simeq T_0 \left(1 - \frac{\Delta g}{2g}\right), \quad \text{where:} \quad \Delta g = \Omega^2 R \cos \lambda, \quad \Omega = \frac{2\pi}{T}, \quad T = 24 \times 3600 \text{ s}, \quad \lambda = \text{latitude} \]
journey from Paris to the Equator.

Appendix A: Formulas for tidal forces

This appendix gives theoretical results for the magnitude of the tidal effect. We are only concerned in the change of $g$. As the theory of tidal forces can be found in many textbooks we focus mainly on a number of key-points.

As in all problems of classical mechanics there are two crucial preliminary steps (i) What simplifying assumptions do we make? (ii) In which frame of reference do we work?

Simplifications

We make the following simplifying hypotheses.

1. In a first step we concentrate on the Earth-Moon system and forget the influence of the Sun.
2. We assume that the orbit of the Moon is a circle so that the Earth-Moon distance does not change in the course of time.
3. We assume that the Earth is a perfect sphere so that its radius does not change with the latitude.

The most drastic simplification is the first one. However, once we have been able to get the answer for the Earth-Moon system we will observe that the argument is basically the same for the Earth-Sun system.

Frame of reference

The investigation of any phenomenon in classical mechanics requires that one defines two frames of reference: one ($F_1$) in which one will work and one ($F_0$) that is considered as absolute and one ($F_1$) in which one will work. For many problems (for instance the acceleration of a car) the frame of reference in which one works can be considered as absolute even though it is not. In the example of the the car the corrective forces due to the fact that the frame of reference rotates with the Earth are small with respect to the forces which act on the car.

For the tidal effect the frames $F_0$ and $F_1$ are given in Fig. A1. Because $F_1$ rotates around the center of mass of the Earth-Moon system, a correction must be introduced in the form of a centrifugal force. Fig. A2 confirms that, as expected, this force is the same for all points of the Earth.

Formulas

9 An absolute (or inertial) frame of reference is one in which one can apply Newton’s law $\vec{F} = m \vec{a}$ without introducing any additional corrective force such as the centrifugal or Coriolis forces.
10 For a pendulum the Coriolis force will be perpendicular to the plane of oscillation of the pendulum. As this pendulum has only one degree of freedom (instead of two for a Foucault pendulum) the Coriolis force will play no role.
Fig. A1: Frames of reference for the Earth-Moon system. For every problem in classical mechanics one must indicate in which frame of reference one will work and which frame of reference is considered as absolute that is to say steady except for a translation.

The frame of reference in red ($F_0$) is considered as absolute. Its origin is at center of mass of the Earth-Moon system and its axis have fixed directions with respect to distant stars.

The frame of reference in green ($F_1$) is the one in which we will work. Its origin is at the center of the Earth and the directions of its axis are also fixed with respect to distant stars. In this frame the coordinates of the North Pole (for instance) will remain unchanged while the Earth moves around the center of mass which is what one wants for the sake of simplicity.

As this frame of reference has a movement of rotation, in order to be able to apply Newton’s law one must introduce two correcting forces: the centrifugal force and the Coriolis force. Because we will focus on the static aspect of the tidal effect, the Coriolis force will play no role and we can forget it.

The three formulas which are needed for computing the tidal effect on the beat of a pendulum are the following:\footnote{Formula (A1) comes from Leroy (2004, p. 12).}

\[ p = \frac{a_v}{g} = \frac{M_L}{M_T} \left( \frac{R}{d} \right)^3 [3 \cos^2 \theta_L - 1] \]  \hspace{1cm} (A1)

where:
\[ a_v: \text{ Vertical component of the tidal force} \]
\[ g: \text{ Acceleration of gravity} \]
\[ M_L: \text{ Mass of the Moon} \]
\[ M_T: \text{ Mass of the Earth} \]
\[ R: \text{ Radius of the Earth} \]
\[ d: \text{ Distance from the Earth to the Moon} \]
\[ \theta_L: \text{ Angle defined in Fig. A3a} \]

The angle $\theta_L$ is defined by:

\[ \cos \theta_L = \cos \mu \sin \phi \cos \theta + \sin \mu \cos \alpha \sin \phi \sin \theta + \sin \mu \sin \alpha \cos \phi \]  \hspace{1cm} (A.2)
Fig.A2: Circular movement of the Earth around the center-of-mass of the Earth-Moon system. The purpose of the figure is to describe the movement of the Earth around the center-of-mass of the Earth-Moon system when one assumes that there is no rotation of the Earth itself around its center. In this case all the points of the surface of the Earth move on circles which are identical to the trajectory of the center of the Earth (blue dotted circle) except for an appropriate translation. Thus, the North Pole (points $A_1, A_2, A_3$) moves on the green circle centered on $a$.

For the sake of clarity of the figure the center of mass (CM) has been represented between the Earth and the Moon whereas in fact it is contained inside of the Earth at a distance of 4,600 km of its center.

Fig.A3a: Definition of the angle $\theta_L = \widehat{LCP}$. The angle $\theta_L$ between the vectors $\overrightarrow{CP}$ and $\overrightarrow{CL}$ plays a key role in the formula of the tidal effect. Fig. A3b shows how it can be expressed as a function of several other angles which characterize the position of the observation point $P$ and the location of the Moon on its orbit around the Earth.

where:

$\theta_L$: Angle defined in Fig. A3a

$\mu$: Angle between $Ox$ and the Moon in the plane which contains both $Ox$ and the
**Fig. A3b:** Definition of the frame of reference for computing the angle $\theta_L$. The origin $O$ of the frame of reference $Oxyz$ is at the center of the Earth, $Oz$ is on its axis of rotation, $Oy$ is in the direction of the Sun. The location $P$ is defined by the two spherical angles $\theta, \phi$. The figure shows the Moon in “new moon” position; when the Moon moves on its orbit it will be described by an angle $\mu$ with respect to $Ox$ in the plane which contains $Ox$ and the Moon.

**Fig. A4:** Theoretical variations of the beat of a pendulum (new moon, April). The horizontal scale gives the time starting from 6 am to 6 am on the next day which on the graph corresponds to time 6+24=30:00. The latitude of the place is 46 degree. In this configuration the difference between noon and midnight is $\Delta T' = 0.094\mu s$; incidentally, the difference between noon and 8 pm is slightly larger, namely. $0.106\mu s$. The shape of the curve would be almost the same in an other month (provided that one is a new moon configuration) but the horizontal time scale would be different. In June for instance one gets $\Delta T' = 0.098\mu s$. On the contrary, in a first (or last) quarter configuration the difference is much smaller; thus, for a (first quarter, June) configuration one gets: $\Delta T' = 0.028\mu s$. Some rules providing quick estimates of the tidal force are given in the text.

**Moon**

$\theta, \phi$: Spherical angles as defined in Fig. A3b; $\phi$ is the co-latitude $\pi/2 - \lambda$ where $\lambda$ is the latitude of $P$; $\theta$ increases as the Earth turns around its axis. With the frame of reference of Fig. A3b, $\theta = \pi/2$ at noon and consequently $\theta = 0$ at 6 am.
\( \alpha \): Angle between the normal to the plane which contains the orbit of the Moon and \( OZ \); on 21 June (situation shown in Fig. A3b) \( \alpha = \epsilon + m = 23 + 5 = 28 \) degrees. On 21 December, \( \alpha = \epsilon - m = 18 \) degrees. On other dates \( \alpha \) is comprised between these limits and given by the formula:

\[
\cos \alpha = [\sin \epsilon \sin \epsilon + (1/m) \cos \epsilon](1 + 1/m^2)^{-1/2}
\]

where \( \epsilon \) is the angle that determines the position of the Earth on its orbit with respect to its position on 21 March.

Finally, the change in the duration of a beat \( T' = T/2 \) of a pendulum is given by:

\[
T' = \pi \sqrt{L/g'} = \pi \sqrt{L/g(1+p)} = T_0'(1 - p/2)
\]

(A.3)

The graph in Fig. A.4 illustrates the use of the above formulas in a specific case.

**Qualitative rules**

In order to get quick estimates for the tidal effect at a location \( P \) one can use the following rules.

1. As the tidal effect is nonexistent at the center (\( C' \)) of the Earth, it is the distance \( CP' \) which matters, where \( P' \) is the projection of \( P \) on the ecliptic.

2. For places whose “latitude” \( \lambda_e \) with respect to the ecliptic (instead of equator) is near 90 degree the tidal force is mainly downward that is to say \( \Delta g > 0 \); for places for which \( \lambda_e \sim 60 \) degree, the tidal force is almost horizontal. which makes its vertical projection (the only one which matters for a pendulum) fairly small.

3. The difference in the period for noon and midnight is largest for new moon or full moon.

4. For almost all latitudes the effect of the Sun is about one half the effect of the Moon. However, this is not true for places where the two forces are fairly horizontal because in such cases even a small difference in direction can produce a substantial change for the vertical component.

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