N=2 6-dimensional Supersymmetric $E_6$ Breaking

Chao-Shang Huang$^a$, Jing Jiang$^b$, Tianjun Li$^c$ and Wei Liao$^d$

$^a$Institute of Theoretical Physics, Academia Sinica, P. O. Box 2735, Beijing 100080, P. R. China
$^b$HEP Division, Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, IL 60439
$^c$Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104
$^d$The Abdus Salam International Center for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy

Abstract

We study the $N=2$ supersymmetric $E_6$ models on the 6-dimensional space-time where the supersymmetry and gauge symmetry can be broken by the discrete symmetry. On the space-time $M^4 \times S^1 / (Z_2 \times Z'_2) \times S^1 / (Z_2 \times Z'_2)$, for the zero modes, we obtain the 4-dimensional $N=1$ supersymmetric models with gauge groups $SU(3) \times SU(2) \times SU(2) \times U(1)^2$, $SU(4) \times SU(2) \times SU(2) \times U(1)$, and $SU(3) \times SU(2) \times U(1)^3$ with one extra pair of Higgs doublets from the vector multiplet. In addition, considering that the extra space manifold is the annulus $A^2$ and disc $D^2$, we list all the constraints on constructing the 4-dimensional $N=1$ supersymmetric $SU(3) \times SU(2) \times U(1)^3$ models for the zero modes, and give the simplest model with $Z_9$ symmetry. We also comment on the extra gauge symmetry breaking and its generalization.

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$^1$E-mail: tli@bokchoy.hep.upenn.edu, phone: (215) 573-5820, fax: (215) 898-2010.
1 Introduction

Grand Unified Theory (GUT) gives us an simple and elegant understanding of the quantum numbers of quarks and leptons, and the success of gauge coupling unification in the Minimal Supersymmetric Standard Model strongly supports this idea. Although the Grand Unified Theory at high energy scale has been widely accepted now, there are some problems in GUT: the grand unified gauge symmetry breaking mechanism, the doublet-triplet splitting problem, and the proton decay, etc.

Recently, a new scenario proposed to address above questions in GUT has been discussed extensively \[1, 2, 3\]. The key point is that the GUT gauge symmetry exists in 5 or higher dimensions and is broken down to the 4-dimensional $N = 1$ supersymmetric Standard Model like gauge symmetry for the zero modes due to the discrete symmetries in the neighborhoods of the branes or on the extra space manifolds, which become non-trivial constraints on the multiplets and gauge generators in GUT \[3\]. The attractive models have been constructed explicitly, where the supersymmetric 5-dimensional and 6-dimensional GUT models are broken down to the 4-dimensional $N = 1$ supersymmetric $SU(3) \times SU(2) \times U(1)^{n-3}$ model, where $n$ is the rank of GUT group, through the compactification on various orbifolds and manifolds. The GUT gauge symmetry breaking and doublet-triplet splitting problems have been solved neatly by the discrete symmetry projections. Other interesting phenomenology, like $\mu$ problems, gauge coupling unifications, non-supersymmetric GUT, gauge-Higgs unification, proton decay, etc, have also been discussed \[1, 2, 3\].

All of the models \[1, 2, 3\] discussed previously have gauge group $SU(N)$ or $SO(N)$. So, we study the $E_6$ model in the present paper, which is as interesting as $SU(5)$ and $SO(10)$ GUT models. Because $E_6$ is a rank 6 exceptional group, in order to break the gauge symmetry and supersymmetry, we need to consider at least two extra dimensions. In addition, the 6-dimensional $N = 1$ supersymmetric theory is chiral, where the gaugino (and gravitino) has positive chirality and the matters (hypermultiplets) have negative chirality, so, it often has anomaly unless we put the Standard Model fermions on the brane, and add one multiplet in the adjoint representation of the gauge group or some suitable matter contents in the bulk to cancel the gauge anomaly. And the 6-dimensional non-supersymmetric $E_6$ models and $N = 1$ supersymmetric $E_6$ models can be considered as special cases of $N = 2$ supersymmetric $E_6$ models, therefore, we only discuss the 6-dimensional $N = 2$ supersymmetric $E_6$ models. Moreover, because $N = 2$ 6-dimensional supersymmetric theory has 16 real supercharges, which corresponds to $N = 4$ 4-dimensional supersymmetric theory, we can not have hypermultiplets in the bulk. Therefore, we have to put the Standard Model fermions on the brane or brane intersection.

In this paper, we first review the discussions of $E_6$ breaking by Wilson line in our context \[4\]. Then, we study $E_6$ breaking on the space-time $M^4 \times S^1/(Z_2 \times Z_2') \times S^1/(Z_2 \times Z_2')$, where $M^4$ is the 4-dimensional Minkowsky space-time. For the zero modes, we obtain the 4-dimensional $N = 1$ supersymmetric models with gauge groups $SU(3) \times SU(2) \times SU(2) \times U(1)^2$, $SU(4) \times SU(2) \times SU(2) \times U(1)$, and $SU(3) \times SU(2) \times U(1)^3$ with one extra pair of Higgs doublets from the vector multiplet.
In addition, considering that the extra space manifold is an annulus \( A^2 \) and a disc \( D^2 \), we can define \( \mathbb{Z}_n \) symmetry on the extra space manifold. We list all the constraints on constructing the 4-dimensional \( N = 1 \) supersymmetric \( SU(3) \times SU(2) \times U(1)^3 \) model for the zero modes, and give the simplest model with \( \mathbb{Z}_9 \) symmetry. Furthermore, we comment on the extra gauge symmetry breaking and its generalization.

We would like to explain our convention. For simplicity, we define

\[
(\alpha, \beta, \gamma) \equiv \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix}.
\]

(1)

In addition, suppose \( G \) is a Lie group and \( H \) is a subgroup of \( G \). In general, for \( G = SU(N) \) and \( G = SO(N) \), \( H \) can be the subgroup of \( U(N) \) and \( O(N) \), respectively. We denote the commutant of \( H \) in \( G \) as \( G/H \), i.e.,

\[
G/H \equiv \{ g \in G | gh = hg, \ \text{for any} \ h \in H \}.
\]

(2)

And if \( H_1 \) and \( H_2 \) are the subgroups of \( G \), we define

\[
G/\{H_1 \cup H_2\} \equiv \{G/H_1\} \cap \{G/H_2\}.
\]

(3)

2 Background of \( E_6 \) Breaking

In 1985, a lot of work has been done on \( E_6 \) breaking because the \( E_6 \) model can be obtained from the compactification of the weakly coupled heterotic \( E_8 \times E_8 \) string theory on the Calabi-Yau manifold by spin connection embedding \([4, 5]\). We would like to review the discussions of \( E_6 \) breaking by Wilson line in our context \([4]\), which will be used to discuss \( E_6 \) breaking in this paper.

Suppose the space-time manifold is \( M^4 \times K \) where \( K \) is the \( k \)-dimensional extra space manifold, and we can define a discrete symmetry \( \Gamma \) on \( K \). In general, \( \Gamma \) can be the product of discrete groups, and \( \Gamma \) may not act freely on \( K \). And when it does not act freely on \( K \), there exists a brane at each fixed point, line or hypersurface, where the Standard Model fermions can be located.

The gauge fields of \( E_6 \) are in the adjoint representation of \( E_6 \) with dimension 78, and \( E_6 \) contains a maximal subgroup \( SU(3)_C \times SU(3)_L \times SU(3)_R \) where \( SU(3)_C \) is color \( SU(3) \), \( SU(3)_L \) and \( SU(3)_R \) describe the weak interactions of left-handed and right-handed quarks, respectively \([6]\). Under the gauge groups \( SU(3)_C \times SU(3)_L \times SU(3)_R \), the \( E_6 \) gauge fields decompose to \((8, 1, 1), (1, 8, 1), (1, 1, 8), (\overline{3}, \overline{3}, 3), (3, \overline{3}, 3)\) \([3]\).

For simplicity, we assume that \( \Gamma \) is \( \mathbb{Z}_n \) in the following discussions, where \( \mathbb{Z}_n \) is generated by the \( n \)-th roots of unity. Since the discussions for the product of cyclic groups are similar, we do not repeat them here. Let \( \gamma \) be a generator of \( \Gamma \), we choose the following matrix representation for \( \gamma \), which will give us the representations of all the elements in \( \Gamma \),

\[
R_\gamma = (+1, +1, +1) \otimes (\alpha, \alpha, \beta) \otimes (\delta, \rho, \sigma),
\]

(4)
the proton decay, the unbroken subgroups do not contain $SU(1)$, for the zero modes, the gauge group is $SU(2)$ couplings would evolve at low energy to be as strong as that of $\alpha$. For the zero modes of gauge fields, the group $SU(3)$ includes the generator $A$, that: (1) $A$, (2) $A$, (3) $A$.

With the choice of $R_\gamma$, we discussed the breaking of $E_6$ ($E_6/R_\gamma$) might not form a group if $R_\gamma$ were not a subgroup of $E_6$. Because we require $E_6/R_\gamma$ to be a group, we choose that $R_\gamma \subset SU(3)_C \times SU(3)_L \times SU(3)_R$ in the following discussions, i.e., $\alpha^2 \beta = \delta \rho \sigma = 1$. With the choice of $R_\gamma \subset SU(3)_C \times SU(3)_L \times SU(3)_R$, we can embed all the generators back to those of $E_6$. Because $R_\gamma$ is an abelian subgroup of $E_6$, it is easy to prove that $E_6/R_\gamma$ also forms a subgroup of $E_6$ with rank 6. However, there is an exception in our discussions. For $Z_2$ case, in order to break $E_6$ down to $SU(3)_C \times SU(3)_L \times SU(3)_R$, we choose $R = (+1, +1, +1) \otimes (+1, +1, +1) \otimes (-1, -1, -1)$ or $R = (+1, +1, +1) \otimes (-1, -1, -1) \otimes (+1, +1, +1)$.

Now, we discuss the $E_6$ breaking. For simplicity, we just consider $E_6$ gauge field $A_\mu = A_\mu^{B_T}B_T$, where $\mu = 0, 1, 2, 3$ and $B = 1, 2, ..., 78$. Let us denote the $E_6$ gauge fields in $SU(3)_C \times SU(3)_L \times SU(3)_R$ as $A_\mu = A_\mu^{\hat{a} \hat{a}}$, where $T^a$ is the product of three $3 \times 3$ matrices, for example, if $t^a$ was a Lie algebra for $SU(3)_C$, the corresponding $T^a = t^a \otimes (+1, +1, +1) \otimes (+1, +1, +1)$. We also denote the $E_6$ gauge fields in (3, 3, 3) and (3, 3, 3) as $A_\mu = A_\mu^{\hat{a} \hat{a}}$ and $A_\mu = A_\mu^{\hat{a} \hat{a}}$, respectively, where $T^\hat{a}$ and $T^\hat{a}$ are the products of three $3 \times 1$ columns.

Because the extra space manifold has $\Gamma = Z_n$ symmetry, for any $\gamma \in \Gamma$, we have

$$A_\mu^{B}(x^\mu, \gamma_iy^1, \gamma_iy^2, ..., \gamma_iy^k)B = (R_\gamma)^{l_A}A_\mu^{B}(x^\mu, y^1, y^2, ..., y^k)B (R_\gamma^{-1})^{m_A},$$

where $y^i$ for $i = 1, 2, ..., k$ are the coordinates for the extra space manifold, and $(l_A, m_A)$ are equal to $(1, 1), (1, 0)$, and $(0, 1)$ for $B = a, \hat{a}, \bar{a}$, respectively. The $E_6$ gauge fields $A_\mu^{B}$ will have zero modes only if

$$A_\mu^{B}(x^\mu, \gamma_iy^1, \gamma_iy^2, ..., \gamma_iy^k) = A_\mu^{B}(x^\mu, y^1, y^2, ..., y^k).$$

The zero modes of gauge fields form the group $E_6/R_\gamma$ with rank 6.

From the phenomenological point of view, for the zero modes, we require that: (1) $SU(3)_L$ and $SU(3)_R$ cannot be completely unbroken for otherwise their couplings would evolve at low energy to be as strong as that of $SU(3)_C$; (2) To avoid the proton decay, the unbroken subgroups do not contain $SU(5)$, $SU(6)$ and $SO(10)$.

The first requirement implies that $\alpha \neq \beta$, and that $\delta, \rho, \sigma$ are not all equal. The second requirement implies that we can have at most one pair of color triplets.

As an example, if $\delta \neq \rho \neq \sigma$, and $\alpha \delta, \alpha \rho, \alpha \sigma, \beta \delta, \beta \rho, \beta \sigma$ are all not equal to one, for the zero modes, the gauge group is $SU(3)_C \times SU(2)_L \times U(1)^3$. And for $Z_2,$
if we chose \( R = (+1,+1,+1) \otimes (+1,+1,+1) \otimes (-1,-1,-1) \) or \( R = (+1,+1,+1) \otimes (-1,-1,-1) \otimes (+1,+1,+1) \), we break \( E_6 \) down to \( SU(3)_C \times SU(3)_L \times SU(3)_R \).

3 \( E_6 \) Breaking on \( M^4 \times S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2) \)

In this section, we will discuss \( E_6 \) breaking on the space-time \( M^4 \times S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2) \). We consider the 6-dimensional space-time which can be factorized into the product of the ordinary 4-dimensional Minkowski space-time \( M^4 \), and the torus \( T^2 \) which is homeomorphic to \( S^1 \times S^1 \). The corresponding coordinates for the space-time are \( x^\mu, (\mu = 0,1,2,3) \), \( y \equiv x^5 \) and \( z \equiv x^6 \). And the radii for the circles along the \( y \) direction and \( z \) direction are \( R_1 \) and \( R_2 \), respectively. We also define \( y' \) and \( z' \) by \( y' \equiv y - \pi R_1/2 \) and \( z' \equiv z - \pi R_2/2 \). The orbifold \( S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2) \) is obtained from \( S^1 \times S^1 \) by modulating the following equivalent classes:

\[
y \sim -y \quad , \quad z \sim -z \quad , \quad y' \sim -y' \quad , \quad z' \sim -z' \quad .
\]

The corresponding operators for the \( Z_2 \) symmetries \( y \sim -y \), \( z \sim -z \), \( y' \sim -y' \) and \( z' \sim -z' \) are \( P^y, P^z, P^{y'} \) and \( P^{z'} \), respectively. Allowing a little abuse of notation, we also denote the matrix representations of \( P^y, P^z, P^{y'} \) and \( P^{z'} \) as \( P^y, P^z, P^{y'} \) and \( P^{z'} \), as used in the literature \([1, 2, 3]\).

Let us explain the 6-dimensional gauge theory with \( N = 2 \) supersymmetry. \( N = 2 \) supersymmetric theory in 6-dimensions has 16 real supercharges, corresponding to \( N = 4 \) supersymmetry in 4-dimensions. Therefore, only the vector multiplet can be introduced in the bulk, and the Standard Model fermions are confined on the 4-branes, 3-branes or 4-brane intersections. In terms of the 4-dimensional \( N = 1 \) supersymmetry language, the theory contains a vector multiplet \( V(A_\mu, \lambda_1) \) in which \( \lambda_1 \) is the gaugino, and three chiral multiplets \( \Sigma_5, \Sigma_6, \) and \( \Phi \). All of them are in the adjoint representation of the gauge group. In addition, the \( \Sigma_5 \) and \( \Sigma_6 \) chiral multiplets contain the gauge fields \( A_5 \) and \( A_6 \) in their lowest components, respectively.

In the Wess-Zumino gauge and 4-dimensional \( N = 1 \) supersymmetry language, the bulk action is \([7]\):

\[
S = \int d^6x \left\{ \operatorname{Tr} \left[ \int d^2\theta \left( \frac{1}{4kg^2} W^\alpha W_\alpha + \frac{1}{kg^2} \left( \Phi \partial_5 \Sigma_6 - \Phi \partial_6 \Sigma_5 - \frac{1}{\sqrt{2}} \Phi [\Sigma_5, \Sigma_6] \right) \right) \right] + \text{H.C.} \right\} + \int d^4\theta \frac{1}{kg^2} \operatorname{Tr} \left[ \sum_{i=5}^{6} \left( (\sqrt{2}\partial_i + \Sigma_i^\dagger) e^{-V} (-\sqrt{2}\partial_i + \Sigma_i) e^V + \partial_i e^{-V} \partial_i e^V \right) \right] + \Phi^\dagger e^{-V} \Phi e^V \right\} .
\]

And the gauge transformation is given by

\[
e^V \rightarrow e^{\Lambda} e^V e^{\Lambda^\dagger},
\]

\[
\Sigma_i \rightarrow e^{\Lambda} (\Sigma_i - \sqrt{2}\partial_i) e^{-\Lambda},
\]

\[
\Phi \rightarrow e^{\Lambda} \Phi e^{-\Lambda},
\]

(9) - (11)
where \( i = 5, 6 \).

From the action, we obtain the transformations of vector multiplet under the \( Z_2 \) operators \( P^y, P^z \)
\[
V(x^\mu, -y, z) = (P^y)^l V(x^\mu, y, z)((P^y)^{-1})^{m_V},
\]
\[
\Sigma_5(x^\mu, -y, z) = -(P^y)^l \Sigma_5(x^\mu, y, z)((P^y)^{-1})^{m_{\Sigma_5}},
\]
\[
\Sigma_6(x^\mu, -y, z) = (P^y)^l \Sigma_6(x^\mu, y, z)((P^y)^{-1})^{m_{\Sigma_6}},
\]
\[
\Phi(x^\mu, -y, z) = -(P^y)^l \Phi(x^\mu, y, z)((P^y)^{-1})^{m_\Phi},
\]
\[
V(x^\mu, y, -z) = (P^z)^l V(x^\mu, y, z)((P^z)^{-1})^{m_V},
\]
\[
\Sigma_5(x^\mu, y, -z) = (P^z)^l \Sigma_5(x^\mu, y, z)((P^z)^{-1})^{m_{\Sigma_5}},
\]
\[
\Sigma_6(x^\mu, y, -z) = -(P^z)^l \Sigma_6(x^\mu, y, z)((P^z)^{-1})^{m_{\Sigma_6}},
\]
\[
\Phi(x^\mu, y, -z) = -(P^z)^l \Phi(x^\mu, y, z)((P^z)^{-1})^{m_\Phi},
\]
where \((l_V, m_V), (l_{\Sigma_5}, m_{\Sigma_5}), (l_{\Sigma_6}, m_{\Sigma_6}), \) and \((l_\Phi, m_\Phi)\) are equal to \((1,1)\) if the gauge fields were in the representations \((8, 1, 1), (1, 8, 1), (1, 1, 8), \) and \((l_V, m_V), (l_{\Sigma_5}, m_{\Sigma_5}),\)
\((l_{\Sigma_6}, m_{\Sigma_6}), \) and \((l_\Phi, m_\Phi)\) are equal to \((1,0)\) if the gauge fields were in the representation \((3, 3, 3), \) and \((l_V, m_V), (l_{\Sigma_5}, m_{\Sigma_5}), (l_{\Sigma_6}, m_{\Sigma_6}), \) and \((l_\Phi, m_\Phi)\) are equal to \((0,1)\) if the gauge fields were in the representation \((3, 3, 3). \) Moreover, the transformations of vector multiplet under the \( Z_2 \) operators \( P^y' \) and \( P^z' \) are similar.

In the following models, for the zero modes, we will break the 4-dimensional \( N = 4 \) supersymmetry down to \( N = 1 \) supersymmetry, and break the \( E_6 \) gauge group down to \( E_6/\{P^y \cup P^z \cup P^y' \cup P^z' \}. \) Including the KK modes, the intersection 3-branes and boundary 4-branes preserve the 4-dimensional \( N = 1 \) and \( N = 2 \) supersymmetry, respectively. The general 4-dimensional supersymmetry and gauge groups on the intersection 3-branes and boundary 4-branes are given in the Table 1. The KK mode expansions and the detail of this set-up can be found in Ref. [2].

### 3.1 Models without the Zero Modes of \( \Sigma_5, \Sigma_6 \) and \( \Phi \)

We will first discuss the models without the zero modes of \( \Sigma_5, \Sigma_6 \) and \( \Phi \). In order to project out all the zero modes of \( \Sigma_5, \Sigma_6 \) and \( \Phi \), we choose the matrix representations of \( P^z \) and \( P^z \) as product of three \( 3 \times 3 \) unit matrices
\[
P^y = P^z = (+1, +1, +1) \otimes (+1, +1, +1) \otimes (+1, +1, +1).
\]
Table 1: For $E_6$ models on $S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2)$, the number of 4-dimensional supersymmetry and gauge groups on the 3-branes, which are located at the fixed points $(y = 0, z = 0)$, $(y = 0, z = \pi R_2/2)$, $(y = \pi R_1/2, z = 0)$, and $(y = \pi R_1/2, z = \pi R_2/2)$, or on the 4-branes which are located at the fixed lines $y = 0$, $z = 0$, $y = \pi R_1/2$, $z = \pi R_2/2$.

| Brane Position | SUSY | Gauge Symmetry |
|----------------|------|----------------|
| $(0, 0)$       | $N = 1$ | $G/\{P^y \cup P^z\}$ |
| $(0, \pi R_2/2)$ | $N = 1$ | $G/\{P^y \cup P^z'\}$ |
| $(\pi R_1/2, 0)$ | $N = 1$ | $G/\{P^y' \cup P^z\}$ |
| $(\pi R_1/2, \pi R_2/2)$ | $N = 1$ | $G/\{P^y' \cup P^z'\}$ |
| $y = 0$       | $N = 2$ | $G/P^y$ |
| $z = 0$       | $N = 2$ | $G/P^z$ |
| $y = \pi R_1/2$ | $N = 2$ | $G/P^y'$ |
| $z = \pi R_2/2$ | $N = 2$ | $G/P^z'$ |

So, considering the zero modes, under $P^y$ projection, we can break the 4-dimensional $N = 4$ supersymmetry down to the $N = 2$ supersymmetry with $(V, \Sigma_6)$ forming a vector multiplet and $(\Sigma_5, \Phi)$ forming a hypermultiplet, and we can break the 4-dimensional $N = 2$ supersymmetry down to the $N = 1$ supersymmetry further by the $P^z$ projection.

We define 5 matrices which will be used in the following discussions

$$A = (+1, +1, +1) \otimes (-1, -1, +1) \otimes (-1, -1, +1), \quad (21)$$
$$B = (+1, +1, +1) \otimes (+1, +1, +1) \otimes (-1, -1, +1), \quad (22)$$
$$C = (+1, +1, +1) \otimes (-1, -1, +1) \otimes (+1, +1, +1), \quad (23)$$
$$D = (+1, +1, +1) \otimes (+1, +1, +1) \otimes (-1, -1, -1), \quad (24)$$
$$E = (+1, +1, +1) \otimes (-1, -1, -1) \otimes (+1, +1, +1). \quad (25)$$

Because $A, B, C, D, E$ are order 2 elements and the unit element (or indentity) $e$ commutes with all the elements in the group, we define $E_6/A \equiv E_6/\{e, A\}$ for simplicity and similarly for the others. As an example, we will explain how to obtain $E_6/A$. As we know, $E_6$ has three maximal subgroups with rank 6: $SO(10) \times U(1)$, $SU(6) \times SU(2)$ and $SU(3) \times SU(3) \times SU(3)$ [6]. Because $A$ is an order 2 subgroup of $SU(3) \times SU(3) \times SU(3)$, $E_6/A$ forms a maximal subgroup and must be one of the
three $E_6$ maximal subgroups with rank 6. $E_6/A$ has 46 gauge generators by simple counting, therefore, we obtain that $E_6/A$ is $SO(10) \times U(1)$. Similarly, we can calculate the other commutants. In short, we have

\[
E_6/A \approx SO(10) \times U(1) , \tag{26}
\]

\[
E_6/B \approx E_6/C \approx SU(6) \times SU(2) , \tag{27}
\]

\[
E_6/D \approx E_6/E \approx SU(3) \times SU(3) \times SU(3) , \tag{28}
\]

\[
E_6/\{A \cup B\} \approx E_6/\{A \cup C\} \approx E_6/\{B \cup C\} \approx SU(4) \times SU(2) \times SU(2) \times U(1) , \tag{29}
\]

\[
E_6/\{A \cup D\} \approx E_6/\{A \cup E\} \approx SU(3) \times SU(2) \times SU(2) \times U(1)^2 . \tag{30}
\]

**Model I.** We choose the matrix representations for $P^{y'}$ and $P^{z'}$ as

\[
P^{y'} = A , \quad P^{z'} = B . \tag{31}
\]

For the zero modes, the bulk 4-dimensional $N = 4$ supersymmetric $E_6$ model is broken down to the $N = 1$ supersymmetric $SU(4) \times SU(2) \times SU(2) \times U(1)$ model. Including the KK modes, the gauge groups on the intersection 3-branes at $(y = 0, z = 0), (y = \pi R_2 /2, z = \pi R_2 /2)$ are $E_6$, $SU(6) \times SU(2), SO(10) \times U(1)$ and $SU(4) \times SU(2) \times SU(2) \times U(1)$, respectively. And the gauge groups on the 4-branes at $y = 0, z = 0, y = \pi R_1 /2$ and $z = \pi R_2 /2$ are $E_6, E_6, SO(10) \times U(1)$ and $SU(6) \times SU(2)$, respectively. Similarly, one can discuss the model by choosing $P^{y'} = A$ and $P^{z'} = C$.

**Model II.** We choose the matrix representations for $P^{y'}$ and $P^{z'}$ as

\[
P^{y'} = B , \quad P^{z'} = C . \tag{32}
\]

For the zero modes, the bulk 4-dimensional $N = 4$ supersymmetric $E_6$ model is broken down to the $N = 1$ supersymmetric $SU(4) \times SU(2) \times SU(2) \times U(1)$ model. Including the KK modes, the gauge groups on the intersection 3-branes at $(y = 0, z = 0), (y = \pi R_2 /2, z = \pi R_2 /2)$ are $E_6$, $SU(6) \times SU(2), SU(6) \times SU(2)$ and $SU(4) \times SU(2) \times SU(2) \times U(1)$, respectively. And the gauge groups on the 4-branes at $y = 0, z = 0, y = \pi R_1 /2$ and $z = \pi R_2 /2$ are $E_6, E_6, SU(6) \times SU(2)$ and $SU(6) \times SU(2)$, respectively.

**Model III.** We choose the matrix representations for $P^{y'}$ and $P^{z'}$ as

\[
P^{y'} = A , \quad P^{z'} = D . \tag{33}
\]

For the zero modes, the bulk 4-dimensional $N = 4$ supersymmetric $E_6$ model is broken down to the $N = 1$ supersymmetric $SU(3) \times SU(2) \times SU(2) \times U(1)^2$ model. Including the KK modes, the gauge groups on the intersection 3-branes at $(y = 0, z = 0), (y = \pi R_2 /2, z = \pi R_2 /2)$ are $E_6$, $SU(3) \times SU(3) \times SU(3), SO(10) \times U(1)$ and $SU(3) \times SU(2) \times SU(2) \times U(1)^2$, respectively. And the gauge groups on the 4-branes at $y = 0, z = 0, y = \pi R_1 /2$ and $z = \pi R_2 /2$ are $E_6, E_6, SO(10) \times U(1)$ and $SU(3) \times SU(3) \times SU(3)$, respectively. Similarly, one can discuss the model by choosing $P^{y'} = A$ and $P^{z'} = E$.
3.2 Model with Gauge-Higgs Unification

In this subsection, we will present the model with $SU(3) \times SU(2) \times U(1)^3$ gauge symmetry and one pair of $SU(2)_L$ Higgs doublets from $\Phi$. We choose the matrix representations for $P^y$, $P^z$, $P^{y'}$ and $P^{z'}$ as

$$P^y = P^z = A = ( +1, +1, +1 ) \otimes ( -1, -1, +1 ) \otimes ( -1, -1, +1 ), \tag{34}$$
$$P^{y'} = ( +1, +1, +1 ) \otimes ( -1, -1, +1 ) \otimes ( -1, -1, +1 ), \tag{35}$$
$$P^{z'} = ( +1, +1, +1 ) \otimes ( -1, -1, +1 ) \otimes ( +1, -1, -1 ). \tag{36}$$

And we would like to point out the commutant groups

$$E_6/A \approx E_6/P^{y'} \approx E_6/P^{z'} \approx SO(10) \times U(1), \tag{37}$$
$$E_6/\{A \cup P^{y'}\} \approx E_6/\{A \cup P^{z'}\} \approx E_6/\{P^{y'} \cup P^{z'}\} \approx SU(5) \times U(1)^2, \tag{38}$$
$$E_6/\{A \cup P^{y'} \cup P^{z'}\} \approx SU(3) \times SU(2) \times U(1)^3. \tag{39}$$

We project out all the zero modes of $\Sigma_5$ and $\Sigma_6$ by choosing $P^y = P^z$. And we project out all the zero modes of $\Phi$ except one pair of $SU(2)_L$ doublets, which can be viewed as one pair of Higgs doublets. Considering the zero modes, the bulk 4-dimensional $N = 4$ supersymmetric $E_6$ model is broken down to the $N = 1$ supersymmetric $SU(3) \times SU(2) \times U(1)^3$ model. Including the KK modes, the gauge groups on the intersection 3-branes at $(y = 0, z = 0)$, $(y = 0, z = \pi R_2/2)$, $(y = \pi R_1/2, z = 0)$ and $(y = \pi R_1/2, z = \pi R_2/2)$ are $SO(10) \times U(1)$, $SU(5) \times U(1)^2$, $SU(5) \times U(1)^2$, and $SU(5) \times U(1)^2$, respectively. In addition, the gauge groups on the 4-branes at $y = 0$, $z = 0$, $y = \pi R_1/2$ and $z = \pi R_2/2$ are all $SO(10) \times U(1)$.

4 $E_6$ Breaking on $M^4 \times A^2$ and $M^4 \times D^2$

In this section, we would like to discuss $E_6$ breaking on the space-time $M^4 \times A^2$ and $M^4 \times D^2$, where $A^2$ and $D^2$ are the two dimensional annulus and disc, respectively. And we only show the models with $SU(3) \times SU(2) \times U(1)^3$ gauge symmetry and 4-dimensional $N = 1$ supersymmetry for the zero modes. Similarly, one can discuss the models with gauge groups $SU(3) \times SU(2) \times SU(2) \times U(1)^2$, or $SU(4) \times SU(2) \times U(1)^3$, or $SU(4) \times SU(2) \times SU(2) \times U(1)$ and 4-dimensional $N = 1$ supersymmetry for the zero modes.

The convenient coordinates for the annulus $A^2$ is polar coordinates $(r, \theta)$, and it is easy to change them to the complex coordinates by $z = re^{i\theta}$. We call the inner radius of the annulus as $R_1$, and the outer radius of the annulus as $R_2$. When $R_1 = 0$, the annulus becomes the disc $D^2$, which is an special case of $A^2$. We can define the $Z_n$ symmetry on the annulus $A^2$ by the equivalent class

$$z \sim \omega z , \tag{40}$$
where $\omega = e^{i2\pi \over n}$. And we denote the corresponding generator for $Z_n$ as $\Omega$ which satisfies $\Omega^n = 1$. The KK mode expansions and the detail of this set-up can be found in Ref. [3].

The $\mathcal{N} = 2$ supersymmetry in 6-dimension corresponds to the $\mathcal{N} = 4$ supersymmetry in 4-dimension, thus, only the gauge multiplet can be introduced in the bulk. This multiplet can be decomposed under the 4-dimensional $\mathcal{N} = 1$ supersymmetry into a vector multiplet $V$ and three chiral multiplets $\Sigma, \Phi$, and $\Phi^c$ in the adjoint representation, with the fifth and sixth components of the gauge field, $A_5$ and $A_6$, contained in the lowest component of $\Sigma$. The Standard Model fermions are on the boundary 4-brane at $r = R_1$ or $r = R_2$ for the annulus $A^2$ scenario, and on the 3-brane at origin or on the boundary 4-brane at $r = R_2$ for the disc $D^2$ scenario.

In the Wess-Zumino gauge and 4-dimensional $\mathcal{N} = 1$ supersymmetry language, the bulk action is [4]

$$
S = \int d^4x \left\{ \text{Tr} \left[ \int d^2\theta \left( \frac{1}{4kg^2} \mathcal{W}^a \mathcal{W}_a + \frac{1}{kg^2} \left( \Phi^c \partial \Phi - \frac{1}{\sqrt{2}} \Sigma [\Phi, \Phi^c] \right) \right) \right] + \text{h.c.} \right\}
$$

$$
+ \int d^4\theta \frac{1}{kg^2} \text{Tr} \left[ (\sqrt{2}\partial^i + \Sigma^l)e^{-V}(-\sqrt{2}\partial + \Sigma)e^V \right]
$$

$$
+ \int d^4\theta \frac{1}{kg^2} \text{Tr} \left[ +\Phi^i e^{-V} \Phi e^V + \Phi^i e^{-V} \Phi^c e^V \right].
$$

From above action, we obtain the transformations of gauge multiplet under $\Omega$ as

$$
V(\omega z, \omega^{n-1} \bar{z}) = (R_\Omega)^{l_V} V(z, \bar{z})(R_\Omega^{-1})^{m_V},
$$

$$
\Sigma(\omega z, \omega^{n-1} \bar{z}) = \omega^{n-1}(R_\Omega)^{l_\Sigma} \Sigma(z, \bar{z})(R_\Omega^{-1})^{m_\Sigma},
$$

$$
\Phi(\omega z, \omega^{n-1} \bar{z}) = \omega^{n-1}(R_\Omega)^{l_\Phi} \Phi(z, \bar{z})(R_\Omega^{-1})^{m_\Phi},
$$

$$
\Phi^c(\omega z, \omega^{n-1} \bar{z}) = \omega^2(R_\Omega)^{l_{\Phi^c}} \Phi^c(z, \bar{z})(R_\Omega^{-1})^{m_{\Phi^c}},
$$

where $(l_V, m_V)$, $(l_\Sigma, m_\Sigma)$, $(l_\Phi, m_\Phi)$, and $(l_{\Phi^c}, m_{\Phi^c})$ are equal to $(1,1)$ if the gauge fields were in the representations $(8, 1, 1)$, $(1, 8, 1)$, $(1, 1, 8)$, and $(l_V, m_V)$, $(l_\Sigma, m_\Sigma)$, $(l_\Phi, m_\Phi)$, and $(l_{\Phi^c}, m_{\Phi^c})$ are equal to $(1,0)$ if the gauge fields were in the representation $(3, 3, 3)$, and $(l_V, m_V)$, $(l_\Sigma, m_\Sigma)$, $(l_\Phi, m_\Phi)$, and $(l_{\Phi^c}, m_{\Phi^c})$ are equal to $(0,1)$ if the gauge fields were in the representation $(3, 3, 3)$.

Moreover, we choose the following matrix representation for $R_\Omega$

$$
R_\Omega = (+1, +1, +1) \otimes (\omega^{n_1}, \omega^{n_1}, \omega^{n_2}) \otimes (\omega^{n_3}, \omega^{n_4}, \omega^{n_5}).
$$

9
In order to have the models with $SU(3) \times SU(2) \times U(1)^3$ gauge symmetry and 4-dimensional $N = 1$ supersymmetry for the zero modes, we obtain the following constraints on $n_i$

\[(a) \ 2n_1 + n_2 = 0 \mod n , \quad (47)\]
\[(b) \ n_3 + n_4 + n_5 = 0 \mod n , \quad (48)\]
\[(c) \ n_1 \neq n_2 \mod n , \quad (49)\]
\[(d) \ n_3 \neq n_4 \neq n_5 \mod n , \quad (50)\]
\[(e) \ |n_1 - n_2| \neq 1 \text{ and } n - 2 \mod n , \quad (51)\]
\[(f) \ |n_i - n_j| \neq 1 \text{ and } n - 2 \mod n , \text{ for } i, j = 3, 4, 5 \text{ and } i \neq j , \quad (52)\]
\[(g) \ |n_i + n_j| \neq 0, 1 \text{ and } n - 2 \mod n , \text{ for } i = 1, 2 \text{ and } j = 3, 4, 5 . \quad (53)\]

Because $R_\Omega \subset SU(3)_C \times SU(3)_L \times SU(3)_R$, we obtain the constraints (a) and (b). And the constraints (c) and (d) will break the $SU(3)_L$ down to $SU(2)_L \times U(1)$ and $SU(3)_R$ down to $U(1)^2$, respectively. In addition, the constraints (e) and (f) will project out all the zero modes of $\Sigma, \Phi$ and $\Phi^c$ in the representations $(\mathbf{8}, \mathbf{1}, \mathbf{1})$, $(\mathbf{1}, \mathbf{8}, \mathbf{1})$, $(\mathbf{1}, \mathbf{1}, \mathbf{8})$, and the constraint (g) will project out all the zero modes of $V$, $\Sigma$, $\Phi$ and $\Phi^c$ in the representations $(\overline{\mathbf{3}}, \mathbf{3}, \mathbf{3})$ and $(\mathbf{3}, \overline{\mathbf{3}}, \mathbf{3})$.

Let us give the simplest model with $Z_9$ symmetry, the matrix representation for $R_\Omega$ is

$$R_\Omega = (+1, +1, +1) \otimes (\omega^2, \omega^2, \omega^5) \otimes (+1, \omega^3, \omega^6) . \quad (54)$$

It is easy to check that all the constraints are satisfied.

First, we consider that the extra space manifold is the annulus $A^2$. For the zero modes, we have 4-dimensional $N = 1$ supersymmetry and $SU(3) \times SU(2) \times U(1)^3$ gauge symmetry in the bulk and on the 4-branes at $r = R_1$ and $r = R_2$. Including the KK states, we will have the 4-dimensional $N = 4$ supersymmetry and $E_6$ gauge symmetry in the bulk, and on the 4-branes at $r = R_1$ and $r = R_2$.

Second, we consider that the extra space manifold is the disc $D^2$. For the zero modes, we have 4-dimensional $N = 1$ supersymmetry and $SU(3) \times SU(2) \times U(1)^3$ gauge symmetry in the bulk and on the 4-brane at $r = R_2$. Including all the KK states, we will have the 4-dimensional $N = 4$ supersymmetry and $E_6$ gauge symmetry in the bulk, and on the 4-brane at $r = R_2$. In addition, because the origin $(r = 0)$ is the fixed point under the $Z_9$ symmetry, we always have the 4-dimensional $N = 1$ supersymmetry and $SU(3) \times SU(2) \times U(1)^3$ gauge symmetry on the 3-brane at origin in which only the zero modes exist. And if we put the Standard Model fermions on the 3-brane at origin, the extra dimensions can be large and the gauge hierarchy problem can be solved for there does not exist the proton decay problem at all.
5 Discussion and Conclusion

The extra gauge symmetry must be broken around or above the TeV scale. This can be done by Higgs mechanism, for example, the $SU(4)$ can be broken down to $SU(3)_C$ and $SU(2)_R$ can be broken by introducing the Higgs fields in their fundamental representations, and the extra $U(1)$ can be broken by introducing the Standard Model singlets which are charged under the extra $U(1)$. The right-handed neutrinos might also become massive during the extra gauge symmetry breaking. And with the ansatz that there exist discrete symmetries in the neighborhoods of the branes, one can discuss the general $E_6$ breaking on the space-time $M^4 \times M^1 \times M^1$, where the extra dimensions can be large and the KK states can be set arbitrarily heavy [3]. On the outlook, the gauge coupling unification, supersymmetry breaking, the $\mu$ problem, how to forbid the proton decay operators by $R$ symmetry, and how to explain the fermion mass hierarchy and mixing angles in our models deserve further study.

In short, we have studied the $N = 2$ supersymmetric $E_6$ models on the 6-dimensional space-time where the supersymmetry and gauge symmetry can be broken by the discrete symmetry. On the space-time $M^4 \times S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2)$, for the zero modes, we obtain the 4-dimensional $N = 1$ supersymmetric models with gauge groups $SU(3) \times SU(2) \times SU(2) \times U(1)^2$, $SU(4) \times SU(2) \times SU(2) \times U(1)$, and $SU(3) \times SU(2) \times U(1)^3$ with one extra pair of Higgs doublets from the vector multiplet. In addition, considering that the extra space manifold is the annulus $A^2$ and disc $D^2$, we list all the constraints on constructing the 4-dimensional $N = 1$ supersymmetric $SU(3) \times SU(2) \times U(1)^3$ models for the zero modes, and give the simplest model with $Z_9$ symmetry.

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