Contact calculation is of great importance in predicting the material removal (MR) of flexible grinding process (FGP). The contact is mostly considered approximately constant in the existing MR models, while the situations that contact varies a lot after FGP are ignored. Therefore, a novel model is proposed in this paper to take those situations into consideration. Firstly, the nonconstant-contact situation is introduced. Then, an equivalent method is developed to convert the nonconstant-contact grinding process into the accumulation of several quasi-constant-contact grinding processes. Based on the equivalent method, a MR model is established, and the procedure to obtain the model parameters by the finite element analysis (FEA) is introduced. In the end, the equivalent method and the MR model are tested by a series experiments of different process parameters. Results show that the proposed MR model can predict the material removal effectively for the nonconstant-contact situations.

Keywords Flexible grinding · Nonconstant contact · Material removal · Equivalent method · FEA
Since the grinding tool is much softer than workpiece, the contact is different with that in the conventional grinding, resulting the invalidity of conventional material removal (MR) model. To develop the removal model suitable for FGP, some simplifications are mostly made including rigid workpieces and steady and constant contact. At the same time, the Preston equation or the Archard wear equation is mostly adopted for removal calculation, while it is necessary for the two equations to obtain the contact pressure between the tool and workpiece. For example, Rao et al. treated the grinding tool as a simple Winkle elastic foundation and the workpiece as a rigid base [14]. Zhu et al. assumed that the contact between the tool and workpiece obeyed the Hertz contact theory, and the original profiles of grinding tool and workpiece were chosen for calculation [15]. Considering the specific contact state, the finite element method (FEM) was also selected for contact calculation [16].

With the establishment of removal model, FGP was reported to be applied in the finishing stage of many precision parts such as aero-engine blades, optical lenses, and medical instruments [17–19]. Meanwhile, higher requirements were put forward to FGP for more situations, which made the mentioned simplifications irrational sometimes. For example, the deformation of low-rigidity parts may also influence the calculation of contact [20] and the contact may not be steady due to the grinding vibration [21]. Another typical situation is the correction of local error on the workpiece. Due to the tool wear and other reasons, it is inevitable that local error exceeds the limits of tolerance, such as the local protrusion caused by tool wear and the nonuniform allowance distribution of the precision-forged aero-engine blades, which is shown in Figs. 1 and 2 [22, 23], respectively. In those cases, it is difficult to reprocess since the allowance of most area of workpiece has reached the tolerance limit, and it is also hard for local machining processes to maintain a smoothly transition between the processing area and the other area. Since the ability of FGP to achieve a smoothly transition, it was thus reported for local error correction. However, it can be seen that the profile of local area varies a lot after local processing, so that the assumption that the contact maintains constant is not suitable in this situation, introducing the failure of the exiting FGP removal models. In the above literatures 22 and 23, the removal model was developed through the empirical approach, which was considered unable to reveal the physics of the process and unbeneficial for the control of processing period and cost [15]. As a result, a novel MR model is proposed in this paper to take the nonconstant-contact situations into account.

The rest of this paper is arranged as follows. The nonconstant-contact situations and the equivalent method in FGP are explained in Sect. 2. Then, the MR model is developed in Sect. 3. In Sect. 4, a series experiments are carried out for verification. And in Sect. 5, results are analyzed and discussed. Conclusions are summarized in the last section.

2 An equivalent method for the nonconstant-contact situations

2.1 Nonconstant contact in FGP

In order to explain the change of contact, an experiment was conducted with different feed rates. Figure 3a shows...
the structure of the tool, and the actual tool in this experiment is shown in Fig. 3b and denoted as TOOL_1. The flexible tool used is composed of tool core, flexible substrate, and grinding film, the material of which is steel, nitrile rubber (NBR), and CBN sand belt, respectively. The workpiece is made from steel and pre-processed on a surface grinding machine. The grinding process is shown in Fig. 4a where the direction of feed is perpendicular to that of the grinding speed. A plate workpiece (W_P) is used and the surface after grinding is shown in Fig. 4b, where the grinding mark is changing gradually with feed rate, and the contact profile tends to be the geometric tool-workpiece interference when the total dwelling time increases.

To calculate the MR, a section S is taken for illustration which is perpendicular to the workpiece surface and parallel with the feed direction. CA is an arbitrary point on the intersection line of contact area and section S. The contact is shown in Fig. 5, where MN is the contact line. Before grinding, the contact is symmetric as described in most literatures shown in Fig. 5a. Due to MR, partial material at CA has been removed before M reached there, causing the decrease of actual tool offset within MN. Specifically, the amount of the decrease becomes larger from N to M, causing an unusual and asymmetric contact as shown in Fig. 5b. Suppose the contact pressure distribution of MN before grinding is $P(x)$ for a given tool offset $\delta_0$. With the growing MR, the real contact pressure distribution within MN is not $P(x)$ but $P'(x)$, $P''(x)$, and so on as shown in Fig. 6, which is much different with the $P(x)$ and can hardly be calculated directly.

To solve this problem, an equivalent method is developed to convert the unusual and asymmetric contact process into the accumulation of several quasi-constant-contact processes. The implementation method is to convert a one-pathed grinding with large MR into the accumulation of several repeated one-pathed grindings with small MR. Since the MR of each one-pathed grinding in the repeated grinding processes is small, the contact of each grinding process can be approximately regarded constant. As a result, a large MR can be calculated.

### 2.2 Explanation and deduction of the equivalent method

Let the MR in CA after a one-pathed grinding with feed rate $F_0$ be $h_{F_0}$, and call this process the single-pathed
grinding (SPG). Then, for the several repeated one-pathed grindings, which is called multi-pathed grinding (MPG), the other parameters except for feed rate remain the same. Suppose the number of one-pathed grindings is $N_g$. For the first time of one-pathed grinding in the MPG, let the feed rate be $F_1$, then for the second grinding is $F_2$, and so on for the last grinding is $F_{N_g}$. Let the MR of $i$th one-pathed grinding with feed rate $F_i$ be $h_{F_i}$, where $1 \leq i \leq N_g$, so the total MR is $\sum h_{F_i}$. The equivalent method means that when the total dwelling time spent on the same grinding distance remains the same, the total MR is approximately equal for the SPG and MPG. That is, if

$$1/F_0 = \sum_{i=1}^{N_g} 1/F_i$$

(1)

then

$$h_{F_0} \approx \sum_{i=1}^{N_g} h_{F_i}$$

(2)

A discrete method is utilized to calculate the MR of the two grinding processes. The contact line MN is divided into several equidistant grinding segments, the number of which is $N_c$. The segment is denoted as $S_j$, and the tool offset at one grinding segment is considered equal and denoted as $\delta_j$, where $1 \leq j \leq N_c$. Some assumptions are made in the calculation as follows.

(a) The MR process is relatively slow in FGP, and the material is removed bit by bit. Therefore, a minimum unit dwelling time $\Delta t_m$ for each segment can be defined to calculate the MR, and when the dwelling time of each segment is $\Delta t_m$, the grinding process is denoted as $G^*$ in this paper. It is clear that for any total dwelling time, a multi-pathed grinding can always be found through a superposition of several $G^*$s. The number of $G^*$ is recorded as $N_m$, and can be calculated as

$$N_m = \frac{T}{N_c \Delta t_m}$$

(3)

where $T$ is total dwelling time. As a result, if the MR of the SPG and the MPG consisted of several $G^*$ is approximately equal for any $N_m$, then the equivalent method can be seen correct.

(b) For each segment on the flexible tool, the relation between the contact pressure $P$ and the tool offset $\delta$ is the same, written as $P = K(\delta)$, where $K$ is a certain function.

(c) The MR can be calculated by Preston’s equation.

(d) The MR of any segment during a single $\Delta t_m$ is small compared with the tool offset at that segment, which can be calculated according to Preston’s equation [24], written as

$$\Delta h = k_{pr} K(\delta) \Delta t_m$$

(4)

where $k_{pr}$ is a constant related to the material properties and the unit of each parameter. As a result, $\Delta h/\delta$ is
considered of the same order as $\Delta t_m$. For any two neighboring unit dwelling time of any segment, the MR of the former is a little bit larger than that of the latter, which can be obtained using Taylor expansion as:

the former: $\Delta h_{\text{former}} = k_{pr}K(\delta)\Delta t_m$

the latter: $\Delta h_{\text{latter}} \approx k_{pr}K(\delta - \Delta h_{\text{former}})\Delta t_m$

$= k_{pr}K(\delta)\Delta t_m - k_{pr}K'(\delta)\Delta t_m \Delta h_{\text{former}}$

$= \Delta h_{\text{former}}(1 - k_{pr}K'(\delta)\Delta t_m)$

(5)

Similarly, $k_{pr}K'(\delta)\Delta t_m$ is also considered of the same order as $\Delta t_m$.

(e) The shape of flexible tool is smooth, and the tool offset of each segment is symmetrical with respect to the perpendicular bisector of MN, which is reported in most literatures. Suppose $N_c$ is even, so that the tool offset is:

$\delta_1, \delta_2, ..., \delta_{N_c/2-1}, \delta_{N_c/2}, \delta_{N_c/2}, \delta_{N_c/2-1}, ..., \delta_2, \delta_1$.

The MR can be calculated by accumulating the MR of each segment during each unit dwelling time $\Delta t_m$. Given a total dwelling time, $N_m$ can then be calculated according to Eq. (3), and the total grinding number of all segments is $N_cN_m$. Some symbols are defined for simplification. The ratio of the MR of any segment during a single $\Delta t_m$ to the tool offset written as:

$a_j = k_{pr}K(\delta)\Delta t_m / \delta_j$

(6)

Denote the temporary total grinding number of all segments as $S$, where $1 \leq k \leq N_cN_m$. The MR of $k$th grinding is $\Delta h_k$, and the total MR after $k$th grinding is $h_k$. $F(\delta)$ is defined as:

$F(\delta) = k_{pr}K'(\delta)\Delta t_m$

(7)

For the SPG, the process is that $S_1$ removes the material with $N_m$ times, then $S_2$ with $N_m$ times and so on until $S_{N_c}$ with $N_m$ times. So:

$\Delta h_k^S = k_{pr}K(\delta_1)\Delta t_m = a_1\delta_1$

(8)

where the superscript “$S$” represents the SPG. Since the removal of the first grinding is small, then the removal of the second grinding can be obtained using Taylor expansion as:

$\Delta h_k^S = k_{pr}K(\delta_1 - h_1^S)\Delta t_m \approx k_{pr}K(\delta_1)\Delta t_m - k_{pr}K'(\delta_1)\Delta t_m \Delta h_1^S$

$= a_1\delta_1 - F(\delta_1)a_1\delta_1$

(9)

where the third and the higher orders of $\Delta t_m$ are ignored in this paper. Accordingly, for $1 \leq k \leq N_cN_m$, the removal can be obtained as:

$\Delta h_k^S = \begin{cases} a_1\delta_1, & k = 1 \\ a_1\delta_1 - \sum_{i=2}^{k-1} F(\delta_1) a_1\delta_1 - h_1^S \sum_{i=1}^{k-1} F(\delta_1) a_1\delta_1, & k \geq 2 \end{cases}$

(10)

where $h_1^S$ is defined to be zero.

For the MPG, the process is that all the grinding segments remove the material once successively from $S_1$ to $S_{N_m}$, then for the second time and so on until for the $N_m$ th time. For the first grinding with $S_1$, the removal is:

$\Delta h_1^M = k_{pr}K(\delta_1)\Delta t_m = a_1\delta_1$

(11)

where the superscript “$M$” represents the MPG. For the second grinding with $S_2$, the removal is:

$\Delta h_2^M = k_{pr}K(\delta_2 - h_1^M)\Delta t_m \approx k_{pr}K(\delta_2)\Delta t_m - k_{pr}K'(\delta_2)\Delta t_m h_1^M$

$= a_2\delta_2 - F(\delta_2) a_1\delta_1$

(12)

Then for $1 \leq k \leq N_cN_m$, the removal can be written as:

$\Delta h_k^M = \begin{cases} a_1\delta_1, & k = 1 \\ a_1\delta_1 - \sum_{i=2}^{k-1} F(\delta_1 - h_i^M) a_1\delta_1 - h_1^M, & k \geq 2 \end{cases}$

(13)

where $q = k - ([k/N_c] - 1)N_c$ and $h_1^M$ is defined to be zero. By comparing Eqs. (10) and (13), it can be seen that the total removal after $N_cN_m$ th grinding is consisted of the sum of several terms with different orders of $\Delta t_m$, and the terms of the first order of $\Delta t_m$ are the same for the two grinding processes. As for the terms of the second order of $\Delta t_m$, the sum of those can be written using the last assumption as:

$S_{2nd} = \sum_{j=1}^{N_c/2} B_j a_1\delta_1$

(14)

where $B_j$ is the coefficient and can be described using Eqs. (10) and (13) as:

$B_j = \sum_{j=1}^{N_c/2} \sum_{k=0}^{N_c/2-1} B_{1,j,k} F(\delta_j - h_k)$

(15)

where $B_{1,j,k}$ is the coefficient. It can be verified that for any $1 \leq l \leq N_c/2$, coefficients $B_l^S$ and $B_l^M$ are approximately equal for the two grinding processes. For example, coefficient $B_l^S$ can be calculated as:

$B_l^S = \sum_{j=2}^{N_c} \sum_{k=0}^{N_c-1} F(\delta_1 - h_k^S) + \sum_{j=2}^{N_c} \sum_{k=0}^{N_c-1} (F(\delta_j - h_k^S))$

(16)
and coefficient $B_1^M$ can be calculated as

$$B_1^M = \sum_{i=2}^{N_r} \sum_{k=0}^{w-2} \left( F(\delta_i - h_{N_c}) + F(\delta_i - h_{N_c}) \right)$$

$$+ \sum_{j=2}^{N_r} \sum_{k=0}^{w-2} \left( F(\delta_j - h_{N_c}) + F(\delta_j - h_{N_c}) \right) + N_m \sum_{j=2}^{N_r} F(\delta_j)$$

(17)

According to the fourth assumption, the subtraction of $F(\delta - h_i)$ and $F(\delta - h_j)$ is considered as the second order of $\Delta t_{im}$, where $1 \leq k, k \leq N_mN_c$. Since the number of the terms with $F(\delta - h_i)$ is equal for any $1 \leq l \leq N_c/2$ in Eqs. (16) and (17), the subtraction of $B_1^S$ and $B_1^M$ is also of the second order of $\Delta t_{im}$. As a result, the total MR is approximately equal for the two grinding processes, and the equivalent method can thus be supposed correct.

3 Establishment of material removal model

3.1 Derivation process of the model

Considering the MPG consisted of several $G^*$, the number of $G^*$ is $N_m$. For the $r$th unit grinding, $G^*$ denote the total cutting number of abrasives acting on CA as $n_r$. Let $P_i(j)$ be the contact pressure in the $j$th segment. Since the distribution of abrasives is uniform, the MR can then be written according to Preston equation as

$$h_i^M = k_{pr} \sum_{j=1}^{N_c} P_i(j) \frac{n_1}{N_c}$$

(18)

And $n_1$ can be calculated as

$$n_1 = \frac{W_{FR}v_{PF}^\rho_{PR}}{F_i}$$

(19)

where $W_F$ is the length of MN and $\rho_{PR}$ is the number of abrasives per unit length in the feed direction. If $N_c$ is large enough, Eq. (18) can be rewritten as

$$h_i^M = k_{pr} \sum_{j=1}^{N_c} P_i(j) \frac{W_{FR}v_{PF}^\rho_{PR}}{F_i} \frac{1}{N_c}$$

$$= k_{pr} v_{PF}^\rho_{PR} \sum_{j=1}^{N_c} (P_i(j) \frac{W_{FR}}{N_c}) \frac{1}{F_i}$$

$$= K^* \int_0^{N_c} P_i dx \frac{1}{F_i}$$

(20)

where $K^* = k_{pr} v_{PF}^\rho_{PR}$ depending on the material of workpiece, the tool, and the grinding speed. $\int_0^{N_c} P_i dx$ is the integral of contact pressure to the contact length in MN, denoted as $P^*$ and called equivalent contact pressure in this paper. Considering the equivalent method is found, the MR in the SPG with feed rate $F_0$ can be calculated as

$$h_{F_0} = \sum_{i=1}^{N_c} K^* P_i \frac{1}{F_i} = K^* \int_0^{1/N_c} P^* (\frac{1}{F})$$

(21)

As a result, the general differential form of the relation between MR $h$ in SPG and the reciprocal of feed rate $1/F$ is

$$dh = K^* P^* (\frac{1}{F})$$

(22)

It can be seen from Eq. (22) that the relation between $P^*$ and $1/F$ or that between $P^*$ and $h$ is needed to calculate $h$. The latter relation is chosen in this paper, which only depends on the contact characteristic between the tool and workpiece. For the convenience of measurement, the position of the maximum MR, denoted as $h_0$, is taken for example to explain and verify of the model. The equivalent contact pressure at that position is recorded as $P^*_0$. However, it is not easy to measure $P^*_0$ in practice, so an assumption is made that the larger $P^*_0$ is, the faster it decreases as $h_0$ decreases. This idea comes from some natural phenomena such as the heat conduction becomes faster when the difference in temperature of the two contact bodies enlarges, and can be written as

$$\frac{\Delta P^*_0}{\Delta h_0} = aP^*_0 + b$$

(23)

where $a$ and $b$ are the coefficients. Integrate Eqs. (23) and (24)

$$P^*_0 = c_1 (e^{c_2 \delta - h_0}) - c_3$$

(24)

can be obtained, where $c_1$, $c_2$, and $c_3$ are the coefficients to be determined. The boundary condition is that the equivalent contact pressure approaches zero when the MR is close to initial tool offset, that is

$$\lim_{h_0 \to 0} P^*_0 = 0$$

(25)

And it can thus be determined that $c_3$ equals 1. Coefficients $c_1$ and $c_2$ can be obtained by using the derivative of $P^*_0$ to $h_0$. Denote the derivative as $Ks$ when $h_0$ is close to zero and as $Ke$ when $h_0$ is close to initial tool offset, then they can be written as

$$Ks = \lim_{h_0 \to 0} \frac{\Delta P^*_0}{\Delta h_0} = -c_1 c_2 e^{c_2 \delta}$$

(26)

and
respectively. After \(c_1\) and \(c_2\) are determined, \(h_0\) can be obtained by substituting Eqs. (24) to (22) and integral calculation. Meanwhile, the boundary conditions should be met, which are the initial MR is zero and the final MR is equal to tool offset, written as

\[
\lim_{\frac{1}{F} \to 0} h_0(\frac{1}{F}) = 0
\]  

and

\[
\lim_{\frac{1}{F} \to \infty} h_0(\frac{1}{F}) = \delta
\]  

Consequently, once \(K_s\) and \(K_e\) are obtained, the MR can then be calculated.

### 3.2 Method of determining \(K_s\) and \(K_e\) using FEA

As mentioned above, the equivalent contact pressure is of great importance to determine \(K_s\) and \(K_e\). In this paper, the FEA is adopted for contact calculation for the tool structure is complex and the influence of sand belt is nonnegligible. The shape of workpiece is needed to explain the procedure and for the subsequent verifications. And since the edge processing of aero-engine precision-forged blades is a typical situation of nonconstant contact, a semicircular cylinder whose radius \(R\) equals 0.3 mm is modeled for example. The analysis is conducted on the software ABAQUS 6.14.

#### 3.2.1 Modeling of FEA model

The simulation process is shown in Fig. 7a, where the pellets on the sand belt are ignored for simplification. A quarter model is built owing to the symmetry for analysis efficiency, and to improve the accuracy, the contact region is fine-meshed as shown in Fig. 7b. The rubber substrate and the sand belt are supposed linear elastic for the tool offset is mostly small compared to the size of the tool. The elastic constants of the sand belt are measured on the universal testing machine, and the hardness of the rubber is measured using a Shore hardness tester, which is used to calculate the Young’s modulus. The density of each component is also measured to include the influence of centrifugal force. Properties of the parts are listed in Table 1.

#### 3.2.2 Verification of the FEA model

Since it is not easy to measure the contact pressure precisely, the contact force and marks are used to verify the FEA model. Experiments are conducted on a plate workpiece (W_P) for observation. Results are shown in Tables 2 and 3. It can be seen that the maximum error of the simulated contact force is 0.2 N, which is less than 10% of the measured force. The simulation contact marks are close to the removal marks when the removal is very small. Thus, the FEA model can be considered reliable.

#### 3.2.3 \(K_s\) determination

The calculation of \(K_s\) is shown in Eq. (26), but it is difficult to get the exact value; hence, an approximate way is adopted as \(K_s \approx \frac{\Delta p_0^*}{\Delta h_0}\), where \(\Delta h_0\) is small enough. As a result, the contact of the initial contact and the contact after a small removal \(\Delta h_0\) are needed to be analyzed. The contact profile is simplified to be an arc, and the change of contact arc length \(\Delta W\) is calculated as

\[
\frac{\Delta W}{W_\infty - W_0} \approx \frac{\Delta h_0}{\delta}
\]  

where \(W_0\) is the initial contact arc length and \(W_\infty\) is length of the removed boundary when \(h_0\) equals \(\delta\), as shown in Fig. 8.

### Table 1 Properties of the FEA model parts

| Parts           | Material properties             |
|-----------------|---------------------------------|
| Tool core, workpiece | \(E = 210\) GPa, \(\nu = 0.3, \rho = 7.8 \times 10^3\) kg/m³ |
| Rubber substrate | HA 40°, \(E = 1.69\) MPa, \(\nu = 0.47, \rho = 1.0 \times 10^3\) kg/m³ |
| Sand belt       | \(E = 180\) MPa, \(\nu = 0.4, \rho = 1.0 \times 10^3\) kg/m³ |

### Table 2 Comparison of contact force

| Tool offset (mm) | Measured force (N) | Simulated force (N) |
|-----------------|--------------------|---------------------|
| 0.1             | 1.3                | 1.4                 |
| 0.2             | 2.6                | 2.8                 |
| 0.3             | 4.2                | 4.3                 |
A typical contact pressure distribution is shown in Fig. 9, and the equivalent contact pressure can be calculated as

$$p_0^* = \int_M P \, dx \approx \sum P_j \times L$$

where $P_j$ is the contact pressure in the contact segments, and $L$ is length of the element. Denote the initial equivalent pressure and that after $\Delta h_0$ removed as $P^*_a$ and $P^*_b$, respectively. Then, $K_s$ can be calculated as

$$K_s = \frac{2(P^*_b - P^*_a)}{\Delta h_0}$$

### 3.2.4 Ke determination

$Ke$ appears when the MR is close the given tool offset. Meanwhile, the contact profile is close to that of the tool as well. Thus, the profile of workpiece in the FEA can be modeled as a nearly complementary arc to the tool with the radius slightly larger than that of the tool, and the maximum MR can be firstly set as equal as the tool offset, as illustrated in Fig. 10a. At the same time, the tool should be in full contact with the workpiece instead of partial contact, and there should not appear stress concentration at the contact boundaries for the radius of workpiece profile should remain decreased, as shown in Fig. 10b. Therefore, the tool offset needs then to be readjusted to satisfy the contact conditions mentioned above. Denote the readjusted amount as $\Delta h_0'$, and the equivalent contact pressure here as $P^*_c$, then $Ke$ can be calculated as

$$Ke = \frac{-2P^*_c}{\Delta h_0'}$$

### Table 3 Comparison of contact marks

| Tool offset (mm) | Removal marks | Simulated marks |
|------------------|---------------|-----------------|
| 0.1              | ![Image]      | ![Image]        |
| 0.2              | ![Image]      | ![Image]        |
| 0.3              | ![Image]      | ![Image]        |

Fig. 8 Assumption of contact boundary change
So far, coefficients $c_1$, $c_2$, and $c_3$ can be obtained through the mentioned procedure. Although there is a constant $K^*$ in Eq. (22), it can be determined by the calibration experiments for a certain material, and the whole MR model can then be obtained.

4 Experiments

4.1 Comparison in MR of SPG and MPG

To verify the effectiveness of equivalent method, experiments are conducted under three different conditions. Since the shape of the grinding tool and the workpiece are mainly concerned in practice, another type of grinding tool (TOOL_2) and a cylindrical workpiece (W_C) whose radius is 3 mm are introduced as shown in Fig. 11 besides TOOL_1 and the plate workpiece (W_P). The arrangement of the experiments is listed in Table 4. As for the grinding parameters, they are set the same for each condition. The tool offset and spindle rotation speed is 0.3 mm and 10,000 r/min, respectively. Three groups of feed rates are tested under each condition for the SPG, denoted as $F_A$, $F_B$, and $F_C$, and for each feed rate of $F_A$, $F_B$, or $F_C$, several groups of feed rates in MPG are set to remain the total dwelling time identical. The arrangement of feed rates is listed in Table 5 where the unit is mm/min. Experiments are carried out on a grinding machine QMK 50A, and each experiment is repeated three times (E1, E2, and E3) to reduce the contingency. The maximum MR $h_0$ is collected for comparison which is measured by the Taylor Hobson profilometer.

4.2 Verification of the MR model

Fillets of three sizes $R_{0.1}$, $R_{0.2}$, and $R_{0.3}$ are pre-milled as the workpiece to simulate the edges of the aero-engine blades as shown in Fig. 12a, grinding machine QMK 50A and TOOL_1 are used for the test, and the schematic of grinding process is shown in Fig. 12b. The tool offset for fillets of $R_{0.1}$, $R_{0.2}$, and $R_{0.3}$ is set to 0.1 mm, 0.2 mm, and 0.3 mm, respectively, and the spindle rotation speed is set to 10,000 r/min. Denote the reciprocal of feed rate $1/F$ as 1 when the feed rate is 2000 mm/min, and the scope of $1/F$ is set from 0.125 to 128 by adjusting the feed rate. Each experiment is repeated three times to improve the reliability of the data. The maximum MR $h_0$ is measured on the coordinate measuring machine HRSW PONY.
5 Results and analysis

5.1 Comparison experiments

Material removal of MPG and SPG under three conditions is listed in Table 6, and the average removal is shown in Fig. 13 for comparison. The fluctuation of MR in each repeated experiment is no more than 3 μm and 5% of the average MR, indicating the reliability of the experimental results. For each condition, the average MR comes to 93%, 49%, and 80% of the given tool offset, respectively. The maximum error between the two grinding processes is 5 μm for all the data, and 3.7 μm for the average MR. For the data and the average MR with maximum error, the relative error is 1.7% and 1.3%, respectively. Considering the error caused by tool wear and measurement, the results demonstrate that within a quite large range of MR, the MR of the two grinding processes remains approximately equal independent of the shape of the tool and workpiece. Consequently, the equivalent method is validated.

5.2 Verification experiments

MR results of three group radii are illustrated in Fig. 14. The maximum error for all the data is 5 μm, the relative error of which is 5.5%. Results of the three repeated experiments show good accordance with each other, so that the MR under each \( I/F \) is averaged for representation below. As mentioned above, a group of experiments are needed for calibration. In this paper, the results of \( R_{0.3} \) are selected. Different values of \( K^* \) are selected for

Table 4 Arrangement of different conditions

| Condition | Grinding tool | Workpiece |
|-----------|---------------|-----------|
| Condition1 | TOOL_1        | W_P       |
| Condition2 | TOOL_2        | W_P       |
| Condition3 | TOOL_2        | W_C       |

Table 5 Arrangement of feed rates

| Feed rate | Single-patched grinding | Multi-patched grinding |
|-----------|-------------------------|------------------------|
|           | \( i = 1 \)   | \( i = 2 \)   | \( i = 3 \)   | \( i = 4 \)   |
| \( F_A \) | 1000           | 2000           | 2000           | /             | /             |
| \( F_B \) | 100            | 500            | 333.3          | 200           | /             |
| \( F_C \) | 10             | 100            | 50             | 33.3          | 25            |
comparison as shown in Fig. 15a. It can be seen that when
the MR is small, the relation between $h_0$ and $1/F$ can be
seen nearly linear as described in most existing models,
while when MR becomes large, a strongly nonlinear rela-
tion appears instead. When the value of $K*$ lies in 1 ~ 1.2,
the deviation between the prediction and the measurement
is relatively small, so the value of $K*$ is set to be 1.2 in
rest of experiments, then the MR for $R0.1$ and $R0.2$ can
be calculated and are compared as shown in Fig. 15b.
It should be pointed out that the real tool offset when $R$
equals 0.1 mm is not 0.1 mm but 0.12 mm. This is due to
the error of tool alignment, and it is adjusted when calcu-
lation. Coefficients $c_1$ and $c_2$ are then calculated using the
mentioned method, as listed in Table 7.

Results in Fig. 15 indicate that the calculated MR is in
good agreement with that of the experiments. For each
measured data, the error of the proposed model is shown
in Fig. 16. The maximum error is 15 μm, and the mean
error is 4.6 μm for all measured data as shown in Fig. 16a.
The relative error is mostly less than 10% with the per-
centage of 91%, the maximum relative error is 32.8%, and
the mean relative error is 5% as shown in Fig. 16b. Though
the maximum relative error is large, the real error there
is less than 10 μm. The possible error sources include
the simplification of FGP, the equivalent method, FEA
model, and tool wear. In general, the prediction error of
the MR model lies in the interval of (−10, +15) μm, and
mostly less than 10 μm with the percentage of 85%, which
Fig. 13  Maximum MR comparison of the two grinding processes. a Condition1, TOOL_1 + M_P. b Condition2, TOOL_2 + M_P. c Condition3, TOOL_2 + M_C

![Graphs showing MR comparison](image)

(a) Condition1, TOOL_1 + M_P  
(b) Condition2, TOOL_2 + M_P  
(c) Condition3, TOOL_2 + M_C

Fig. 14  Experimental results of workpiece with different radius. a Results of R0.1. b Results of R0.2. c Results of R0.3

![Graphs showing experimental results](image)

(a) Results of R0.1  
(b) Results of R0.2  
(c) Results of R0.3
is almost of the same precision as the finish milling. As a result, the MR model can meet the manufacturing tolerance of most components such as the aero-engine blades whose manufacturing tolerances are usually 0.1 mm, and the model can thus be validated.

Fig. 15 Results of grinding experiments. a Comparison of different $K^*$. b Results of $R0.1$ and $R0.2$

Table 7 Values of coefficients $c_1$ and $c_2$

| $R$ (mm) | $\delta$ (mm) | $c_1$ | $c_2$ |
|---------|---------------|-------|-------|
| 0.1     | 0.12          | 3.6   | 0.03  |
| 0.2     | 0.2           | 6     | 0.02  |
| 0.3     | 0.3           | 3.6   | 0.014 |

Fig. 16 Error analysis of the proposed MR model. a Error. b Relative error
6 Conclusion

In order to predict the material removal of flexible grinding processes for nonconstant-contact situations, a novel MR model is established in this paper. The equivalent method is explained that the material removal of the single-pathed grinding can be converted to that of the multi-pathed grinding. The material removal model is then deduced based on the equivalent method and developed by using FEA. Experiments show that the error caused by the equivalent method is less than 3% under different grinding conditions, and the material grinding experiments show that the maximum error and the mean error of the proposed model are 15 μm and 4.6 μm for a large range of material removal, respectively, which indicates an acceptable accuracy in practice and the effectiveness of the material model.

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Data availability All data and materials generated or analyzed during this study are included in this manuscript.

Declarations

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