Bi-Large Neutrino Mixing See-Saw Mass Matrix with Texture Zeros and Leptogenesis

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Abstract

We study constraints on neutrino properties for a class of bi-large mixing See-Saw mass matrices with texture zeros and with the related Dirac neutrino mass matrix to be proportional to a diagonal matrix of the form $\text{diag}(\epsilon, 1, 1)$. Texture zeros may occur in the light (class a)) or in the heavy (class b)) neutrino mass matrices. Each of these two classes has 5 different forms which can produce non-trivial three generation mixing with at least one texture zero. We find that two types of texture zero mass matrices in both class a) and class b) can be consistent with present data on neutrino masses and mixing. None of the neutrinos can have zero masses and the lightest of the light neutrinos has a mass larger than about 0.039 eV for class a) and 0.002 eV for class b).

In these models although the CKM CP violating phase vanishes, the non-zero Majorana phases can exist and play an important role in producing the observed baryon asymmetry in our universe through leptogenesis mechanism. The requirement of producing the observed baryon asymmetry can further distinguish different models and also restrict the See-Saw scale to be in the range of $10^{12} \sim 10^{15}$ GeV.

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I. INTRODUCTION

There are abundant experimental data showing that neutrinos have small but non-zero masses and also mix with each other\[1, 2, 3, 4, 5, 6, 7\]. One of the most interesting mechanisms of naturally generating small neutrino masses is the See-Saw mechanism\[8\]. This mechanism requires introduction of right-handed neutrinos $\nu_R$ into the theory. At a phenomenological level, the See-Saw neutrino mass matrix, in the left-handed neutrino $\nu_L$ and the charge conjugated right-handed neutrino $\bar{\nu}_R^c$ basis ($\nu_L$, $\nu_R^c$) with the charged lepton mass matrix already diagonalized, can be written as

$$M = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix},$$

where $M_D = v_\nu Y_\nu$ is the Dirac neutrino mass term which can be generated through the Yukawa couplings of a Higgs doublet $H_\nu$ to the left- and right-handed neutrinos, $\bar{\nu}_R(Y_\nu v_\nu)\nu_L$ with $v_\nu$ being the vacuum expectation value of $H_\nu$. $M_R$ is from the Majorana mass term $(1/2)\bar{\nu}_R M_R \nu_R^c$.

With three generations of left- and right-handed neutrinos, $M_R$ is a $3 \times 3$ symmetric matrices, and $M_D$ is a $3 \times 3$ arbitrary matrix. The elements in $M_D$ can be of the same order of the magnitude as the corresponding charge leptons, and the scale of $M_R$ is a new scale characterizing possible new physics beyond SM which is expected to be much larger than the weak scale. To the leading order, the mass matrices $M_h$ and $M_\nu$ for the heavy and light neutrinos are given by

$$M_h \approx M_R, \quad M_\nu \approx -M_D^T M_R^{-1} M_D = -v_\nu^2 Y_\nu^T M_R^{-1} Y_\nu.$$

The light neutrino masses are suppressed compared with their charged lepton partners by a factor of $M_D/M_R$ resulting in very small neutrino masses compared with the masses of their corresponding charged leptons. The eigen-values and eigen-vectors of $M_\nu$ are the light neutrino masses and mixing measured by low energy experiments. The mixing matrix is the unitary matrix $U$ (the PMNS matrix\[9\]) which diagonalizes the mass matrix, and is defined, in the basis where the charged lepton is already diagonalized, by $D = U^T M_\nu U = \text{diag}(m_1, m_2, m_3)$ with the eigenvalues $m_i$ to be larger or equal to zero. One can always decompose $U$ into a product of a CKM matrix\[10\] like unitary matrix $V$ and a phase matrix
\[ P = \text{diag}(e^{i\rho_1}, e^{i\rho_2}, e^{i\rho_3}) \], \quad U = VP. \] The phase \( \rho_i \) is the Majorana phase. It is sometimes convenient to write \( \tilde{D} = V^T M_\nu V \). In this case the eigenvalues are in general complex which will be indicated by \( \tilde{m}_i = m_i e^{-i2\rho_i} \). The commonly used parametrization for \( V \) is given by

\[
V = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13} e^{i\delta} \\
0 & 1 & 0 \\
-s_{13} e^{-i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{21} & s_{21} & 0 \\
-s_{21} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix},
\] (3)

where \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \). \( \delta \) is a CKM like CP violating phase.

If See-Saw mechanism is responsible for neutrino masses and mixing, the properties of the right-handed neutrinos play a very important role in determining the light neutrino properties. \( M_R \) not only provides a scale for new physics responsible for the mechanism to explain the smallness of neutrino masses, but also affects the low energy mixing, and vice versa. It may also provide important ingredients to explain the baryon asymmetry of our universe (BAU) through lepton number violating decays of the heavy neutrino to light neutrinos and Higgs particles by the Yukawa coupling \( Y_\nu \), the leptogenesis mechanism \[11, 12, 13\]. If one takes leptogenesis as a requirement, important information about the mass matrix and the associated CP violating phases can be obtained \[14, 15\]. The CP violating Majorana phases \( \rho_i \) can play an important role in explaining BAU through leptogenesis mechanism which will be discussed later.

The present neutrino oscillation experimental data on neutrino masses and mixing angles from solar, atmospheric, reactor neutrino oscillation experiments can be summarized as the following. The 3\( \sigma \) allowed ranges for the mass-squared differences are constrained to be \[6, 7\]:

- \( 1.6 \times 10^{-3} \, \text{eV}^2 \leq \Delta m^2_{\text{atm}} = |\Delta m^2_{32}| = |m_3^2 - m_2^2| \leq 3.6 \times 10^{-3} \, \text{eV}^2 \),
- \( 7.3 \times 10^{-5} \, \text{eV}^2 \leq \Delta m^2_{\text{solar}} = \Delta m^2_{21} = m_2^2 - m_1^2 \leq 9.3 \times 10^{-5} \, \text{eV}^2 \), with the best fit values given by \( \Delta m^2_{\text{atm}} = 2.2 \times 10^{-3} \, \text{eV}^2 \), and \( \Delta m^2_{\text{solar}} = 8.2 \times 10^{-5} \, \text{eV}^2 \).

The mixing angles are in the ranges of \( 0.28 \leq \tan^2 \theta_{12} \leq 0.60 \) (best fit value 0.39), \( 0.5 \leq \tan^2 \theta_{23} \leq 2.1 \) (best fit value 1.0), and \( \sin^2 \theta_{13} \leq 0.041 \).

There are many theoretical studies of neutrino masses and mixing \[16\]. The mixing matrix can be nicely represented by the so called bi-large mixing matrix. By an appropriate choice
of sign and phase conventions, the bi-large mixing matrix can be written as

\[
V = \begin{pmatrix}
c & s & 0 \\
-\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
-\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{pmatrix},
\]

where \( c = \cos \theta \) and \( s = \sin \theta \) with \( \tan \theta \approx \tan \theta_{12} \). In this form the eigen-masses \( \tilde{m}_{\nu_i} \) of the light neutrinos, in general, have non-zero Majorana phases \( \rho_i \).

This class of models has \( V_{13} = 0 \) which is allowed by present experimental data and can be tested by future experiments. Several experiments are planned to measure \( V_{13} \) with greater precisions \[17\]. Obviously should a non-zero value for \( V_{13} \) be measured, modifications for the model considered are needed. The bi-large mixing model can be taken as the lowest order approximation. The bi-large mixing model captures many features of the present data and deserves more careful theoretical studies.

The bi-large mixing mass matrix is of the form \[16, 18\]

\[
M_{\nu} = \begin{pmatrix}
M_{11} & M_{12} & M_{12} \\
M_{12} & M_{22} & M_{23} \\
M_{12} & M_{23} & M_{33}
\end{pmatrix}
= \begin{pmatrix}
(c^2 \tilde{m}_{\nu_1} + s^2 \tilde{m}_{\nu_2}) & \frac{cs}{\sqrt{2}} (-\tilde{m}_{\nu_1} + \tilde{m}_{\nu_2}) & \frac{cs}{\sqrt{2}} (-\tilde{m}_{\nu_1} + \tilde{m}_{\nu_2}) \\
\frac{cs}{\sqrt{2}} (-\tilde{m}_{\nu_1} + \tilde{m}_{\nu_2}) & \frac{1}{2} (s^2 \tilde{m}_{\nu_1} + c^2 \tilde{m}_{\nu_2} + \tilde{m}_{\nu_3}) & \frac{1}{2} (s^2 \tilde{m}_{\nu_1} + c^2 \tilde{m}_{\nu_2} - \tilde{m}_{\nu_3}) \\
\frac{cs}{\sqrt{2}} (-\tilde{m}_{\nu_1} + \tilde{m}_{\nu_2}) & \frac{1}{2} (s^2 \tilde{m}_{\nu_1} + c^2 \tilde{m}_{\nu_2} - \tilde{m}_{\nu_3}) & \frac{1}{2} (s^2 \tilde{m}_{\nu_1} + c^2 \tilde{m}_{\nu_2} + \tilde{m}_{\nu_3})
\end{pmatrix}.
\]

The above mass matrix produces bi-large mixing matrix, but the neutrino masses cannot be completely fixed using the two mass different \( \Delta m^2_{\text{sol}} \) and \( \Delta m^2_{\text{atm}} \) measurements even one knows all information about the mixing matrix. Additional inputs are needed to further constrain or to determine the parameters in the neutrino mass matrix \[16, 18, 19, 20, 21, 22\]. Several proposals have been made to reduce the parameters, such as texture zero \[20\], determinant zero requirement \[21\], and traceless requirement \[22\] for the mass matrix. In this paper we impose texture zeros on bi-large mixing mass matrix to constrain neutrino masses and Majorana phases and also study the implications for leptogenesis.

To study leptogenesis, one needs further information on the heavy neutrino mass matrix. Since \( M_{\nu} = -M_D M_R^{-1} M_D^T \), if \( M_D \) is known one can obtain the form of \( M_R \) at higher energy. If \( M_D \) is proportional to a unit matrix, \( M_R \) will also have a similar form as the one given
in eq. (5), and the associated mixing matrix $V_R P_R$ is equal to $V^* P^*$ in the basis where all eigen-masses are real and positive, and the heavy neutrino eigen-masses are proportional to $1/m_i$. $M_\nu$ and $M_R$ are trivially related. We will show later that in this case the Majorana phases will play no role in leptogenesis. We therefore consider a simple, but non-trivial relation between $M_\nu$ and $M_R$ with

$$M_D = v_\nu Y_\nu = v_\nu b \, d i a g(\epsilon, 1, 1).$$

With this form for $M_D$, $M_R$ also has the bi-large mixing mass matrix form, but with $V_R P_R$ not equal to $V^* P^*$. With these forms for $M_\nu$ and $M_R$, there is no CKM like CP violating phase in the charged current interaction with $W$ boson. The Majorana phases can, however, play a non-trivial role in leptogenesis \cite{14, 15, 23} and will be studied in more details later. The heavy neutrino mass matrix can be expressed as

$$M_R = -v_\nu^2 b^2 \left( \begin{array}{ccc}
\epsilon^2 \left( \frac{c^2}{m_{\nu_1}} + \frac{s^2}{m_{\nu_2}} \right) & \epsilon c_s \sqrt{2} \left( -\frac{1}{m_{\nu_1}} + \frac{1}{m_{\nu_2}} \right) & \epsilon c_s \sqrt{2} \left( -\frac{1}{m_{\nu_1}} + \frac{1}{m_{\nu_2}} \right) \\
\epsilon c_s \sqrt{2} \left( -\frac{1}{m_{\nu_1}} + \frac{1}{m_{\nu_2}} \right) & \frac{1}{2} \left( \frac{s^2}{m_{\nu_1}} + \frac{c^2}{m_{\nu_2}} + \frac{1}{m_{\nu_3}} \right) & \frac{1}{2} \left( \frac{s^2}{m_{\nu_1}} + \frac{c^2}{m_{\nu_2}} - \frac{1}{m_{\nu_3}} \right) \\
\epsilon c_s \sqrt{2} \left( -\frac{1}{m_{\nu_1}} + \frac{1}{m_{\nu_2}} \right) & \frac{1}{2} \left( \frac{s^2}{m_{\nu_1}} + \frac{c^2}{m_{\nu_2}} - \frac{1}{m_{\nu_3}} \right) & \frac{1}{2} \left( \frac{s^2}{m_{\nu_1}} + \frac{c^2}{m_{\nu_2}} + \frac{1}{m_{\nu_3}} \right)
\end{array} \right).$$

(6)

We note that none of the neutrino masses can be zero.

We comment that when discussing See-Saw neutrino mass matrices, there are two scales, the light neutrino and the heavy neutrino mass scales. The mass matrices at the two scales may be different due to renormalization group running effects \cite{24}. A mass matrix element is zero at a particular scale may not be zero at another scale unless there are certain symmetries to guarantee this \cite{25}.

An important problem in neutrino physics is to understand the origin of neutrino masses and mixing. There are many attempts have been made to understand this problem, at present the answer is, however, far from satisfaction. We will not attempt to carry out a model building investigation of the mass matrices, instead we will study phenomenological implications here. However, we would like to point out there are models which can produce some models we study. For example, in Ref. \cite{19} it was shown that for three generations of left-handed leptons and three generations of right-handed charged leptons and neutrinos, it is possible to obtain $M_D \sim d i a g(\epsilon, 1, 1)$ and $M_R$ of the form in eq. (6) with $M_{23} = 0$ if there are 3 Higgs doublets and 2 neutral signets which transform, non-trivially, under a discrete group $Z_2^{(r)} \times Z_2^{(tr)} \times Z_2^{(aux)}$. In this model the flavor symmetries of the model dictates that the light neutrino mass matrix structure is essentially determined by the heavy neutrino mass matrix \cite{25}.
We will generalize the discussion to include all possible texture zeros in both the light and heavy bi-large mass matrices. Although the classes of models have simple structures, there are very rich phenomenological implications on the neutrino masses, CP violating phases and on leptogenesis.

II. BI-LARGE NEUTRINO MIXING MASS MATRIX WITH TEXTURE ZEROS

There are two different ways the texture zeros can be imposed: a) The texture zeros are imposed on the light neutrino mass matrix $M_\nu$; And b) The texture zeros are imposed on the heavy neutrino mass matrix $M_R$. There are five different cases for each of class a) and class b) types of models which give non-trivial three generation mixing.

For class a) the five cases indicated by $M_{Li}$ are

$$M_{L1} = \begin{pmatrix} 0 & M_{12} & M_{12} \\ M_{12} & M_{22} & M_{23} \\ M_{12} & M_{23} & M_{22} \end{pmatrix}, \quad M_{L2} = \begin{pmatrix} 0 & M_{12} & M_{12} \\ M_{12} & 0 & M_{23} \\ M_{12} & M_{23} & 0 \end{pmatrix}, \quad M_{L3} = \begin{pmatrix} 0 & M_{12} & M_{12} \\ M_{12} & M_{22} & 0 \\ M_{12} & 0 & M_{22} \end{pmatrix},$$

$$M_{L4} = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & M_{22} & 0 \\ M_{12} & 0 & M_{22} \end{pmatrix}, \quad M_{L5} = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & 0 & M_{23} \\ M_{12} & M_{23} & 0 \end{pmatrix}. \quad (7)$$

For class b) the five cases have the same forms as above. We denote them and their matrix elements by $M_{Ri}$ and $M_{ij}^h$, respectively.

Before discussing the above mass matrices in detail, we make some comments on the neutrino masses. There are three light neutrino masses, and there are two observable mass differences $\Delta m_{sol}^2$ and $\Delta m_{atm}^2$ from neutrino oscillation. If one of the neutrino masses, or a combination of them, is known, the rest of the masses can be determined in terms of $\Delta m_{21,32}^2$. We express $m_2$ and $m_3$ as a function of $m_1$ as,

$$m_{\nu_2} = \sqrt{m_{\nu_1}^2 + \Delta m_{21}^2}, \quad m_{\nu_3} = \sqrt{m_{\nu_1}^2 + \Delta m_{21}^2 + \Delta m_{32}^2}. \quad (8)$$

There are additional constraints on the neutrino masses. One of them comes from WMAP data which limits $m_{sum} = m_1 + m_2 + m_3$ to be less than 0.71 eV at the 95% C.L.. In the future cosmological data can improve the sensitivity and reach 0.03 eV. There are another two constraints, the effective mass for neutrinoless double beta decays, $m_{ee} = |M_{11}|$,
and the effective mass for tritium beta decay, \( \langle m_\nu \rangle = \left( |V_{11}|^2 m_1^2 + |V_{12}|^2 m_2^2 + |V_{13}|^2 m_3^2 \right)^{1/2} \). The current experimental upper bounds for \( m_{ee} \) and \( \langle m_\nu \rangle \) are 1.35 eV\(^2\) and 3 eV\(^{1/2}\), respectively. These bounds are less stringent than the WMAP constraint. However, in the future, the sensitivities for \( m_{ee} \) and \( \langle m_\nu \rangle \) can reach 0.01 eV\(^2\) and 0.12 eV\(^{1/2}\), respectively, by laboratory experiments. These experiments can provide interesting constraints.

If there are additional constraints, such as texture zeros, it may be possible to determine the neutrino masses or relate the masses to the CP violating Majorana phases which are otherwise very difficult to measure.

A. Constraints From Texture Zeros In The Light Neutrino Mass Matrix

In this section we study the consequences of the above texture zeros in class a). For the case L1, we have

\[
M_{11} = c^2 \tilde{m}_1 + s^2 \tilde{m}_2 = 0. \tag{9}
\]

This is a special case studied in Ref.\(^3\)\(^\text{[31]}\) by requiring \( m_{ee} = 0 \).

The phase of \( \tilde{m}_1 \) can be chosen to be zero which results in \( \tilde{m}_2 = -m_1 / \tan^2 \theta \). Using this relation, the value of \( m_2 \) is determined by

\[
m_2^2 = \frac{\Delta m^2_{sol}}{1 - \tan^4 \theta}. \tag{10}
\]

The mass \( m_3 \) can be expressed as

Normal hierarchy : \( m_3^2 = \Delta m^2_{atm} + \frac{\Delta m^2_{sol}}{1 - \tan^4 \theta} \).

Reversed hierarchy : \( m_3^2 = -\Delta m^2_{atm} + \frac{\Delta m^2_{sol}}{1 - \tan^4 \theta} \). \tag{11}

The phase of \( \tilde{m}_3 \) is not determined from the above consideration. The reversed hierarchy is not a physical solution in this case because \( m_3^2 \) is minus numerically when constraints from data are imposed.

From eqs (9) and (10), we obtain the central value of \( m_1 \) for the normal hierarchy to be 0.0038 eV and the 3\( \sigma \) lower bound to be 0.0025 eV. The sum of the masses \( m_{sum} \) is equal to 0.062 \pm 0.010 eV which satisfies the WMAP bound and can be probed by future cosmological data. The effective mass \( m_{ee} \) is identically equal to zero and \( \langle m_\nu \rangle = 0.0062 \pm \)}
0.0011 eV which are safely within the current experimental bounds and is very difficult to be probed by near future experiments.

The case L2 is a special case of the minimal Zee mass matrix [32] and has been shown to be ruled out [33]. The cases L2 and L3 cannot be consistent with data, it is easy to understand from the simultaneous requirements $M_{11} = 0$, and $M_{22} = 0$ ($M_{23} = 0$). Because of this, one has $\tilde{m}_3 = s^2\tilde{m}_1 + c^2\tilde{m}_2$ and $c^2\tilde{m}_1 + s^2\tilde{m}_2 = 0$ which leads to

$$\frac{\Delta m_{\text{atm}}^2}{\Delta m_{\text{sol}}^2} = \frac{\tan^2\theta(2 - \tan^2\theta)}{1 - \tan^2\theta}. \quad (12)$$

With $0.28 < \tan^2\theta < 0.6$, the above ratio is in the range 0.5 to 1.3 which is in conflict with data. This class of models is therefore ruled out.

We now discuss the case L4. The constraint from $M_{22} = m_1 s^2 e^{-i2\rho_1} + m_2 c^2 e^{-i2\rho_2} + m_3 e^{-i2\rho_3} = 0$ implies that $m_1 s^2 + m_2 c^2 \geq m_3$. This implies that only reversed mass hierarchy, $m_2 > m_1 > m_3$, is allowed. We choose the convention where the phase $\rho_2 = 0$. We find that the Majorana phases $\rho_{1,3}$ are determined by the masses and mixing angles as

$$\cos 2\rho_1 = \frac{1}{2m_1m_2s^2c_2}(m_3^2 - m_1^2s^4 - m_2^2c^4),$$

$$\cos 2\rho_3 = \frac{1}{m_3}(m_1 s^2 \cos 2\rho_1 + m_2 c^2),$$

$$\sin 2\rho_3 = \frac{1}{m_3}m_1 s^2 \sin 2\rho_1. \quad (13)$$

There are two solutions, due to the undetermined sign of $\sin 2\rho_1 = \pm\sqrt{1 - \cos^2 2\rho_1}$, even with $\cos 2\rho_1$ fixed. They cannot be distinguished by oscillation and laboratory mass measurement experiments. If leptogenesis is responsible for the baryon asymmetry of our universe, we will show later that the two different solutions give different signs for the baryon asymmetry and the solution with positive $\sin 2\rho_1$ has to be chosen.

In Fig. 1 (a) and (b), we show $\cos 2\rho_1$ and $\cos 2\rho_3$, and $\sin 2\rho_1$ and $\sin 2\rho_3$, respectively, as functions of $\log(m_1)$ with the best fit values for $\Delta m_{\text{sol}}^2$, $\Delta m_{\text{atm}}^2$ and $\tan\theta$. We see that in order to have physical solutions there is a minimal value for $m_1$ which is about 0.05 eV for the input parameters with central values. When errors in the input parameters are included, the lower bound can be reduced to 0.039 eV at 3$\sigma$ level. One can also express the masses as functions of the phase $\rho_1$.

There is no upper bound for the neutrino masses from the above considerations. If one takes the WMAP constraint, $m_1$ is bounded to be less than 0.238 eV. This implies that $\cos 2\rho_1$ is to be smaller than 0.906.
FIG. 1: $\cos 2\rho_1$ and $\cos 2\rho_3$ (Fig. 1(a)), and $\sin 2\rho_1$ and $\sin 2\rho_3$ (Fig. 1(b)), respectively, as functions of $\log(m_1)$ for the central values of $\Delta m^2_{\text{solar}}$, $\Delta m^2_{\text{atm}}$, and $\tan\theta$. Both solutions with $\pm|\sin 2\rho_1|$ are drawn. In (a) the solid line is for $\cos 2\rho_1$ and the dotted line is for $\cos 2\rho_3$, while in (b) the solid line is for $\sin 2\rho_1$ and the dotted line is for $\sin 2\rho_3$ respectively.

The effective masses $m_{ee} = |M_{11}| = (m_1^2 c^4 + m_2^2 s^4 + 2m_1 m_2 s^2 c^2 \cos 2\rho_1)^{1/2}$ and $\langle m_\nu \rangle = (c^2 m_1^2 + s^2 m_2^2)^{1/2}$ are constrained. Compared with current experimental upper bounds for $m_{ee} < 1.35$ eV and $\langle m_\nu \rangle < 3$ eV, we get the upper bound for $m_1$ to be 0.95 eV which is above the WMAP bound. The lower bound for $m_{ee}$ and $\langle m_\nu \rangle$ may be calculated too. They are, at $3\sigma$ level, 0.0217 eV and 0.0392 eV respectively. The lower bound on $m_{ee}$ can be probed by future neutrinoless double beta decays.

The constraints on $L_5$ can be obtained by replacing $m_3$ by $-m_3$ in eq. (13). The net result is to change the signs of $\cos 2\rho_3$ and $\sin 2\rho_3$. Leptogenesis will select the solution with $\sin 2\rho_1$ to be positive again.

B. Constraints From Texture Zeros In The Heavy Neutrino Mass Matrix

We now discuss the situation for the cases in class b). The cases R1, R2 and R3 all require $M_{11}^h = 0$, which implies

$$s^2 \tilde{m}_1 + c^2 \tilde{m}_2 = 0.$$  \hspace{1cm} (14)

Since data requires that $s^2 < c^2$ and $m_2^2 > m_1^2$, it is not possible to satisfy the above...
equation. Cases R1, R2 and R3 are therefore ruled out by data.

A specific realization of R4 was discussed by Grimus and Lavoura in Ref. [18]. We choose the convention with $\rho_2 = 0$. In this case since $M_{23} = s^2/\tilde{m}_1 + c^2/\tilde{m}_2 - 1/\tilde{m}_3 = 0$, when combined with $m_2 > m_1$ from data, one obtains $m_3 > m_1$. Since data show that $|\Delta m^2_{21}|$ is larger than $|\Delta m^2_{32}|$, only normal hierarchy neutrino mass pattern is allowed.

The condition $|M_{23}| = 0$ also leads to

$$
\cos 2\rho_1 = \frac{1}{2m_1m_2c^2s^2}\left(\frac{m^2_2m^2_1}{m^2_3} - m^2_2s^4 - m^2_1c^4\right),
$$

$$
\cos 2\rho_3 = \frac{m_3}{m_1m_2}(m_2s^2\cos(2\rho_1) + m_1c^2),
$$

$$
\sin 2\rho_3 = \frac{m_3}{m_1}s^2\sin(2\rho_1). \tag{15}
$$

Similar to the case for L4, there are two solutions due to the undetermined sign of $\sin 2\rho_1$. Neutrino oscillation and laboratory neutrino mass measurement experiments will not be able to decide which solution to take. However, leptogenesis will select the solution with positive $\sin 2\rho_3$.

In Fig. 2 (a), we show $\cos 2\rho_1$ and $\cos 2\rho_3$ as functions of $m_1$ with the best fit values for $\Delta m^2_{sol}$, $\Delta m^2_{atm}$, $\tan \theta$. In Fig. 2 (b) we show the two solutions for $\sin 2\rho_1$ and $\sin 2\rho_3$ as functions of $\log(m_1)$. There is a region, around $m_1 = 0.01$, not allowed. We see that there is a minimal value for $m_1$ which is about 0.003 eV. With errors in the input parameters, the low bound at $3\sigma$ level is 0.002 eV. One can also express the masses as functions of the phase $\rho_1$.

Similar to the case L4, there is no upper bound for the neutrino masses for R4 from the above considerations. If one takes the WMAP constraint, $\cos 2\rho_1$ is bounded to be smaller than 0.905.

The effective masses $m_{ee} = |M_{11}| = m_1m_2/m_3$ and $\langle m_\nu \rangle = \left(c^2m^2_1 + s^2m^2_2\right)^{1/2}$ are constrained. Compared with current experiment upper bounds for $m_{ee} < 1.35$ eV and $\langle m_\nu \rangle < 3$ eV, we get the upper bound for $m_1$ to be 1.35 eV which is again above the WMAP bound. The lower bound at $3\sigma$ for $m_{ee}$ and $\langle m_\nu \rangle$ are $3.8 \times 10^{-4}$ eV and $4.7 \times 10^{-3}$ eV respectively.

Again the constraints for case R5 can be obtained by simply changing $\tilde{m}_3$ to $-\tilde{m}_3$. 

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FIG. 2: $\cos 2\rho_1$ and $\cos 2\rho_3$ (Fig.2 (a)), and $\sin 2\rho_1$ and $\sin 2\rho_3$ (Fig.2 (b)), respectively, as functions of $\log(m_1)$ for the central values of $\Delta m^2_{\text{solar}}$, $\Delta m^2_{\text{atm}}$, and $\tan\theta$. Both solutions with $\pm |\sin 2\rho_1|$ are drawn. In (a) the solid line is for $\cos 2\rho_1$ and the dotted line is for $\cos 2\rho_3$, while in (b) the solid line is for $\sin 2\rho_1$ and the dotted line is for $\sin 2\rho_3$ respectively.

III. HEAVY NEUTRINO MASSES AND MIXING MATRIX

In our previous discussions we have concentrated only on the light neutrino masses, mixing and phases. We have seen that the mass matrix is completely specified by experimental measurable quantities. In fact once the light neutrino mass matrix is known, the right-handed neutrino mass matrix is almost specified as can be seen from eq. (6).

There are three new parameters $v_\nu$, $b$ and $\epsilon$ in $M_R$. In the cases considered here, only the combination $v_\nu b$ appears in the calculations. We will normalize $v_\nu$ to have the SM values of 174 GeV and let $b$ be a free parameter. It is interesting to note that if one knows $\epsilon$, all information on the mixing matrix $U_R$ is known, and also the ratios of the heavy neutrino masses $M_i/M_j$ are known once the light neutrino masses and mixing angles are fixed.

In the limit $\epsilon = 1$, $U_R = U^*$ and $M_i = v_\nu^2 b^2/m_i$. When $\epsilon$ is not equal to 1, the situation is more complicated. But from eq. (6) it is clear that the unitary matrix $U_R$ which diagonalizes $M_R$ still has the bi-large mixing form. When $\epsilon$ is close to one, the heavy neutrino mass hierarchies are $M_3 > M_1 > M_2$ and $M_1 > M_2 > M_3$ for the reversed and normal light neutrino hierarchies, respectively. When $\epsilon$ deviates from one, the mass hierarchy pattern will change and $U_R$ is non-trivially related to $U$. But $M_3$ is always equal to $v_\nu^2 b^2/\tilde{m}_3$. 

11
In Fig. 3 we show the heavy neutrino masses as functions of $\epsilon$ for several fixed values of $m_1$ for illustration. For case L1, we use the central value 0.0038 eV, and for L4 and R4 we use two typical values 0.055 eV and 0.1 eV for $m_1$, respectively. The cases L4 and L5 have the same eigen-masses, and R4 and R5 also have the same eigen-masses. From Fig. 3 we can clearly see that the mass hierarchy changes with $\epsilon$.

The mixing matrix $U_R$ is more complicated. It has the general form

$$
U_R = \begin{pmatrix} e^{i\delta_1} & 0 & 0 \\ 0 & e^{i\delta_2} & 0 \\ 0 & 0 & e^{i\delta_2} \end{pmatrix} \begin{pmatrix} c & s' & 0 \\ -s'/\sqrt{2} & c'/\sqrt{2} & 1/\sqrt{2} \\ -c'/\sqrt{2} & c'/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma_2} & 0 \\ 0 & 0 & e^{i\gamma_3} \end{pmatrix}.
$$

(16)

In the following we give, as examples, $U_R$ for the cases L1, L4 and R4. In the case L1 with $m_1 = 0.0038$ eV,

$$
U_R = \begin{pmatrix} 0.7175 & 0.6965 & 0 \\ -0.4925 & 0.5074 & 0.707 \\ -0.4925 & 0.5074 & -0.707 \end{pmatrix}.
$$

(17)

For cases L4 and R4, in the basis where all eigen-masses are real and positive, we write $U_R$ for two typical values of $m_1$, 0.055 eV and 0.1 eV. We have

For case L4 with $m_1 = 0.055$ eV,

$$
U_R = \begin{pmatrix} 0.2946 + 0.1841 \ i & -0.4986 + 0.7942 \ i & 0 \\ 0.6516 - 0.1225 \ i & -0.0449 - 0.2415 \ i & -0.6941 + 0.1351 \ i \\ 0.6516 - 0.1225 \ i & -0.0449 - 0.2415 \ i & 0.6941 - 0.1351 \ i \end{pmatrix},
$$

(18)

and with $m_1 = 0.1$ eV,

$$
U_R = \begin{pmatrix} 0.0532 + 0.1285 \ i & -0.9162 + 0.3760 \ i & 0 \\ 0.6929 - 0.1012 \ i & -0.0139 - 0.0973 \ i & -0.6996 + 0.1029 \ i \\ 0.6929 - 0.1012 \ i & -0.0139 - 0.0973 \ i & 0.6996 - 0.1029 \ i \end{pmatrix}.
$$

(19)

For case R4 with $m_1 = 0.055$ eV,

$$
U_R = \begin{pmatrix} 0.1202 + 0.1778 \ i & -0.8122 + 0.5424 \ i & 0 \\ 0.6777 - 0.1332 \ i & -0.0284 - 0.1491 \ i & -0.6938 + 0.1364 \ i \\ 0.6777 - 0.1332 \ i & -0.0284 - 0.1491 \ i & 0.6938 - 0.1364 \ i \end{pmatrix},
$$

(20)
FIG. 3: $M_i(\text{GeV})/v^2b^2$ for cases L1, L4 and R4 as functions of $\epsilon$. For L1, $m_1$ is equal to 0.0038 eV ((a)) (determined from the central values of the mixing angles). For L4 and R4, $m_1$ is not fixed by the mixing angles. We draw figures for $m_1 = 0.055$ eV ((b)) and $m_1 = 0.1$ eV ((c)) for illustrations. The solid, dotted and dashed lines are for $M_1$, $M_2$ and $M_3$, respectively.

and with $m_1 = 0.1$ eV,
\[ U_R = \begin{pmatrix} 0.0431 + 0.1172i & -0.9324 + 0.3393i & 0 \\ 0.6954 - 0.0931i & -0.0114 - 0.0875i & -0.7009 + 0.0938i \\ 0.6954 - 0.0931i & -0.0114 - 0.0875i & 0.7009 - 0.0938i \end{pmatrix}. \]  

(21)

### IV. LEPTOGENESIS

There are extensive discussions on implications of leptogenesis for See-Saw neutrino mass matrix \([11, 12, 13, 14, 15]\). With a general See-Saw mass matrix, it has been shown that there is enough room in parameters space to reproduce the observed BAU \([11, 12, 13]\). There are also more restrictive forms of mass matrix with texture zeros which can also reproduce the observed BAU \([14, 15]\). The mass matrices discussed in the previous sections are a class of very restrictive matrices, in particular that there is no CKM like CP violating phase. It is interesting to see if such models can also produce the observed BAU. We find that although there is no CKM like CP violating phase, the required CP violation can come from the Majorqana phase. There is a large parameter space with which BAU can be reproduced. Taking leptogenesis as a requirement, we show that interesting constraints on the scale of the right-handed neutrino can be obtained. We now proceed to provide more details.

The baryon number asymmetry problem, why our universe is dominated by matter, is one of the most outstanding problems in modern physics. This problem is related to the ratio \( \eta_B = n_B/n_\gamma \). Here \( n_B \) is the baryon number density and \( n_\gamma \) is the photon number density. If the universe contains equal matter and anti-matter initially with baryon number conserved, the expected ratio for \( \eta_B \) is about \( 10^{-20} \). Observations from Big-Bang Nucleosynthesis (BBN) and Cosmic Microwave Background (CMB) radiation determine \( \eta_B \) to be \( 6.5^{+0.4}_{-0.3} \times 10^{-10} \). There is a huge difference between the expected and the observed values. Sakharov showed that if there are \( 35 \): 1) baryon number violation, 2) C and CP violation, and 3) occurrence of non-thermal equilibrium when 1) and 2) are effective, it is possible to create a matter dominated universe from a symmetric one in the early epoch of the universe.

In the Standard Model due to \( SU(2)_L \) anomaly, there are baryon number violating interactions. This interaction becomes strong at high temperatures \( 36 \). This interaction violates \( B + L \), but conserves \( B - L \). Fukugita and Yanagida \([11]\) noticed that if in the early universe there was lepton number asymmetry, this interaction can transfer lepton number asymmetry...
a_i produced by heavy neutrino decays, for example, to baryon number asymmetry.

The surviving baryon asymmetry from lepton number asymmetry due to the “lth” heavy neutrino is given by

$$\eta_B = \frac{s}{n_\gamma} \left| \frac{\omega}{\omega - 1} \right| \frac{a_l \kappa_l}{g_{sl}},$$

(22)

where $s = (2\pi^2/45)g_{s0}T^3|_0$ and $n_\gamma = (2/\pi^2)\zeta(3)T^3|_0$ are the entropy and photon densities of the present universe with $g_{s0} = 43/11$ being the effective relativistic degrees of freedom. The parameter $\omega$ is calculated to be $\omega = (8N_F + 4N_H)/(22N_F + 13N_H)$ depending on the number of $SU(2)_L$ doublet Higgs scalars $N_H$ and fermions $N_F$. $g_{sl}$ is the effective relativistic degrees of freedom at the temperature where the lepton number asymmetry $a_l$ is generated from the “lth” heavy neutrino decay. For the lightest heavy neutrino decay contribution, $g_{sl} = (28 + (7/8)\times 90)_{SM} + 4(N_H - 1) + 2(7/8)$ is of order 100. Here the last term comes from the lightest heavy Majorana neutrino which produces the lepton number asymmetry. The number $N_H$ depends on the details of the specific model. We have checked the sensitivity of $\eta_B$ on $N_H$ and find that there is only about a 10% reduction for $N_H$ varying from 1 to 5. We will assume that there is just one Higgs doublet in our numerical calculations. $\kappa_l$ is a dilute factor which depends on the ratio of heavy Majorana neutrino decay rate and the Hubble parameter at the time of heavy neutrino decay, $K_l = \Gamma_l/H_l$ with $\Gamma_l = (\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)_{ll}M_l/8\pi$ and $H_l = 1.166\sqrt{g_{sl}}M_l^2/M_{planck}$. Here $\tilde{Y}_\nu = V_R^T Y_\nu$ is the Yukawa coupling in the basis where $M_R$ is diagonalized.

The heavy neutrino mass is of order $M_l \sim (v^2/m_3)(\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)_{ll}$, one would obtain $\Gamma_l/H_l \sim 10^4(m_3/eV)(100 GeV/v_\nu)^2$. For $m_3$ within the allowed lower bound discussed earlier and upper bound from WMAP, the factor $K_l$ is within the range of $10 \sim 10^6$. In this range the dilute factor $\kappa_l$ is approximated by $\kappa_l \approx 0.3/K_l(\ln K_l)^{3/5}$. In our numerical calculations we will use this approximate form.

We now study $a_i$ in the models considered. The lepton number asymmetry $a_i$ generated by the “ith” heavy neutrino is given by

$$a_i \approx -\frac{1}{8\pi} \frac{1}{[\tilde{Y}_\nu \tilde{Y}_\nu^\dagger]_{i\bar{i}}} \sum_j \text{Im} \{[\tilde{Y}_\nu \tilde{Y}_\nu^\dagger]_{ij} \} f \left( \frac{M_j^2}{M_i^2} \right),$$

(23)

where

$$f(x) = \sqrt{x} \left( \frac{2}{x - 1} + \ln \frac{1 + x}{x} \right).$$

(24)
Applying the above equation to the models discussed in the previous section, we obtain the lepton number asymmetries due to heavy neutrino decays to be

\[
\begin{align*}
a_i &= -\frac{1}{8\pi}b^2(\epsilon^2 - 1)^2 \sum_j \text{Im}(U_{R1i}U_{R1j}^*)^2 \frac{f(M_j^2/M_i^2)}{1 + (\epsilon^2 - 1)|U_{R1i}|^2}, \\
a_1 &= -\frac{1}{8\pi}b^2(\epsilon^2 - 1)^2 \text{Im}(U_{R11}U_{R12}^*)^2 \frac{f(M_2^2/M_1^2)}{1 + (\epsilon^2 - 1)|U_{R11}|^2}, \\
a_2 &= -\frac{1}{8\pi}b^2(\epsilon^2 - 1)^2 \text{Im}(U_{R12}U_{R11}^*)^2 \frac{f(M_1^2/M_2^2)}{1 + (\epsilon^2 - 1)|U_{R12}|^2}, \\
a_3 &= 0,
\end{align*}
\]

with \( \text{Im}(U_{R11}U_{R12}^*)^2 = -c^2s^2\sin(2\gamma_2) \). In the above we have used the fact that \( U_{R13} = 0 \).

Note that for \( \epsilon = 1 \), no lepton number asymmetry can be generated.

From eq.(22) we see that only solutions which generate negative \( a_i \) can be candidate producing the right sign for baryon asymmetry. This criterion selects out solutions obtained in Section II which are not able to be distinguished by low energy experimental data.

Several studies of leptogenesis with bi-large neutrino mixing matrix have been carried out[15]. Here we follow similar strategy to systematically study the models discussed earlier. To demonstrate that the See-Saw model discussed here can indeed explain the observed baryon number asymmetry, in the following we consider a simple case with large hierarchical structure for the heavy Majorana neutrino mass. In this case the dominant contribution to the surviving baryon asymmetry is from the lightest heavy neutrino decay. The heavy neutrino with mass of \( M_3 \) does not produce a non-zero asymmetry, it cannot be the lightest heavy neutrino since it will washout baryon asymmetries produced by the other two heavier ones in our case. One needs to work in the parameter space where \( M_3 \) is not the smallest. Large \( \epsilon \) tends to make \( M_{1,2} \) bigger, while does not affect \( M_3 \) as can be seen from Fig. 3. Therefore leptogenesis favors small \( \epsilon \). Numerically we find that with \(|\epsilon|\) less than around 0.5 the lightest heavy neutrino mass is \( M_2 \) and the mass squared is at least 10 times smaller than \( M_3^2 \) as can be seen from Fig. 4. In this range, the washout effect of the CP conserving decay of the heavy neutrino of mass \( M_3 \) would be small. We will present our results for the baryon asymmetry produced by the lightest heavy neutrino with \( \epsilon \) satisfying the condition that the lightest heavy neutrino mass squared is at least 10 times smaller than the next lightest heavy neutrino mass squared.

For the case L1, \( U_{R11}U_{R12}^* \) is real. This leads to zero lepton asymmetry \( a_L \). This type of models cannot explain the baryon number asymmetry in the universe.
FIG. 4: The allowed ranges for $\epsilon$ and $b$ for case L4 with $\eta_B$ in the range of $4 \times 10^{-10} \sim 8 \times 10^{-10}$, with (a) $m_1 = 0.055$ eV and (b) $m_1 = 0.1$ eV.

FIG. 5: The allowed ranges for $\epsilon$ and $b$ for case R4 with $\eta_B$ in the range of $4 \times 10^{-10} \sim 8 \times 10^{-10}$, with (a) $m_1 = 0.055$ eV and (b) $m_1 = 0.1$ eV.

In Figs.4 and 5 we show $\eta_B$ as functions of $b$ and $\epsilon$ for $m_1 = 0.055$ eV and 0.1 eV for the cases L4 and R4. Only the cases with negative $a_i$ which produces the right sign for the observed baryon number asymmetry are shown. There are two solutions with different signs for $\sin 2\rho_1$ in the case of L4 which satisfies neutrino mass and oscillation experimental
FIG. 6: $M_i$ for case L4 as functions of $\epsilon$ with (a) $m_1 = 0.055$ eV and (b) $m_1 = 0.1$ eV. The solid, dotted and dashed lines are for $M_1$, $M_2$ and $M_3$ respectively.

FIG. 7: $M_i$ for case R4 as functions of $\epsilon$ with (a) $m_1 = 0.055$ eV and (b) $m_1 = 0.1$ eV. The solid, dotted and dashed lines are for $M_1$, $M_2$ and $M_3$ respectively.

constraints as discussed in section II. If the model is required to produce the baryon number asymmetry, we find that only the solution with the positive $\sin 2\rho_3$ is allowed. Similar situation happens for the case of R4, positive $\sin 2\rho_1$ has to be chosen. For the cases L5 and R5, the solutions with positive $\sin 2\rho_3$ have to be chosen.

We see from Figs. 4 and 5 that the observed baryon number asymmetry can be produced
in the models considered here. We also see that the requirement of generating the correct baryon number asymmetry, the parameters $\epsilon$ and $b$ are constrained. One can use this fact to obtain the allowed mass ranges for the heavy neutrino masses $M_i$. In Figs. 6 and 7 we show $M_i$ as functions of $\epsilon$ for the central value of $\eta_B$. These masses represent possible new physics scale and are constrained to be in the range of $10^{12} \sim 10^{15}$ GeV.

V. CONCLUSIONS

In this paper we have studied constraints from texture zeros in bi-large mixing See-Saw neutrino mass matrices and also from leptogenesis. We have systematically investigated two classes of models with one of them (class a)) to have the texture zeros imposed on the light neutrino mass matrix, and another (class b)) to have the texture zeros imposed on the heavy neutrino mass matrices.

Assuming a simple form proportional to $\text{diag}(\epsilon, 1, 1)$ for the Dirac mass matrix which relates the left- and right- handed neutrinos, both light and heavy neutrinos can simultaneously have the bi-large mixing matrix form. Both classes a) and b) of mass matrices can have 5 different forms which produce non-trivial three generation mixing. We find that only three (L1, L4, L5 ) in class a) and two (R4, R5) in class b), respectively, can be consistent with present data on neutrino masses and mixing constraints. In all the models none of the neutrino masses can be zero. Using present data, the lightest neutrino is bounded to be heavier than, 0.0025 eV, 0.039 eV and 0.002 eV for L1, L4 and L5, and, R4 and R5, respectively. Future experiments can provide further tests and even rule out some of the models.

Because $V_{13} = 0$, there is no CKM type of CP violating phase in the light neutrino mixing matrix. No CP violating effects can be observed in neutrino oscillation experiments. However, there can be non-trivial Majorana phases. These phases can play an important role in explaining the observed baryon number asymmetry in our universe. We have shown that in the models considered there are parameter spaces where the observed baryon number asymmetry can indeed be generated through the leptogenesis mechanism. It is interesting to note that the requirement of producing the observed baryon number asymmetry rules out several models which are, otherwise, impossible to achieve by Laboratory experiments. This requirement also provides a condition to fix the allowed scale for the heavy neutrinos.
We find that the masses are in the range of $10^{12} \sim 10^{15}$ GeV.

In the models we considered $V_{13}$ is zero which is allowed by present experimental data and can be tested by future experiments. Several experiments are planned to measure $V_{13}$ with greater precision\cite{17}. Obviously should a non-zero value for $V_{13}$ be measured, modifications for the model considered are needed. However, the models considered can be taken as the lowest order approximations. How to obtain such mass matrices deserves more future theoretical studies.

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Bi-Large Neutrino Mixing See-Saw Mass Matrix with Texture Zeros and Leptogenesis

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Abstract

We study constraints on neutrino properties for a class of bi-large mixing See-Saw mass matrices with texture zeros and with the related Dirac neutrino mass matrix to be proportional to a diagonal matrix of the form $\text{diag}(\epsilon, 1, 1)$. Texture zeros may occur in the light (class a)) or in the heavy (class b)) neutrino mass matrices. Each of these two classes has 5 different forms which can produce non-trivial three generation mixing with at least one texture zero. We find that two types of texture zero mass matrices in both class a) and class b) can be consistent with present data on neutrino masses and mixing. None of the neutrinos can have zero masses and the lightest of the light neutrinos has a mass larger than about 0.039 eV for class a) and 0.002 eV for class b). In these models although the CKM CP violating phase vanishes, the non-zero Majorana phases can exist and play an important role in producing the observed baryon asymmetry in our universe through leptogenesis mechanism. The requirement of producing the observed baryon asymmetry can further distinguish different models and also restrict the See-Saw scale to be in the range of $10^{12} \sim 10^{15}$ GeV.

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I. INTRODUCTION

There are abundant experimental data showing that neutrinos have small but non-zero masses and also mix with each other \[1, 2, 3, 4, 5, 6, 7\]. One of the most interesting mechanisms of naturally generating small neutrino masses is the See-Saw mechanism \[8\]. This mechanism requires introduction of right-handed neutrinos $\nu_R$ into the theory. At a phenomenological level, the See-Saw neutrino mass matrix, in the left-handed neutrino $\nu_L$ and the charge conjugated right-handed neutrino $\nu^c_R$ basis ($\nu_L$, $\nu^c_R$) with the charged lepton mass matrix already diagonalized, can be written as

$$M = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix}, \quad (1)$$

where $M_D = v_\nu Y_\nu$ is the Dirac neutrino mass term which can be generated through the Yukawa couplings of a Higgs doublet $H_\nu$ to the left- and right-handed neutrinos, $\bar{\nu}_R(Y_\nu v_\nu)\nu_L$ with $v_\nu$ being the vacuum expectation value of $H_\nu$. $M_R$ is from the Majorana mass term $(1/2)\bar{\nu}_R M_R \nu^c_R$.

With three generations of left- and right-handed neutrinos, $M_R$ is a $3 \times 3$ symmetric matrices, and $M_D$ is a $3 \times 3$ arbitrary matrix. The elements in $M_D$ can be of the same order of the magnitude as the corresponding charge leptons, and the scale of $M_R$ is a new scale characterizing possible new physics beyond SM which is expected to be much larger than the weak scale. To the leading order, the mass matrices $M_h$ and $M_\nu$ for the heavy and light neutrinos are given by

$$M_h \approx M_R, \quad M_\nu \approx -M_D^T M_R^{-1} M_D = -v_\nu^2 Y_\nu^T M_R^{-1} Y_\nu. \quad (2)$$

The light neutrino masses are suppressed compared with their charged lepton partners by a factor of $M_D/M_R$ resulting in very small neutrino masses compared with the masses of their corresponding charged leptons. The eigen-values and eigen-vectors of $M_\nu$ are the light neutrino masses and mixing measured by low energy experiments. The mixing matrix is the unitary matrix $U$ (the PMNS matrix \[9\]) which diagonalizes the mass matrix, and is defined, in the basis where the charged lepton is already diagonalized, by $D = U^T M_\nu U = \text{diag}(m_1, m_2, m_3)$ with the eigenvalues $m_i$ to be larger or equal to zero. One can always decompose $U$ into a product of a CKM matrix \[10\] like unitary matrix $V$ and a phase matrix
\( P = \text{diag}(e^{i\rho_1}, e^{i\rho_2}, e^{i\rho_3}), \ U = VP. \) The phase \( \rho_i \) is the Majorana phase. It is sometimes convenient to write \( \tilde{D} = V^T M_\nu V. \) In this case the eigenvalues are in general complex which will be indicated by \( \tilde{m}_i = m_i e^{-i2\rho_i}. \) The commonly used parametrization for \( V \) is given by

\[
V = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{-i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{21} & s_{21} & 0 \\
-s_{21} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

where \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij}. \) \( \delta \) is a CKM like CP violating phase.

If See-Saw mechanism is responsible for neutrino masses and mixing, the properties of the right-handed neutrinos play a very important role in determining the light neutrino properties. \( M_R \) not only provides a scale for new physics responsible for the mechanism to explain the smallness of neutrino masses, but also affects the low energy mixing, and vice versa. It may also provide important ingredients to explain the baryon asymmetry of our universe (BAU) through lepton number violating decays of the heavy neutrino to light neutrinos and Higgs particles by the Yukawa coupling \( Y_\nu, \) the leptogenesis mechanism [11, 12, 13]. If one takes leptogenesis as a requirement, important information about the mass matrix and the associated CP violating phases can be obtained [14, 15]. The CP violating Majorana phases \( \rho_i \) can play an important role in explaining BAU through leptogenesis mechanism which will be discussed later.

The present neutrino oscillation experimental data on neutrino masses and mixing angles from [1, 2, 3, 4, 5] solar, atmospheric, reactor neutrino oscillation experiments can be summarized as the following. The 3\( \sigma \) allowed ranges for the mass-squared differences are constrained to be [6, 7]:

\[ 1.6 \times 10^{-3} \text{ eV}^2 \leq \Delta m^2_{\text{atm}} = |\Delta m^2_{32}| = |m^2_3 - m^2_2| \leq 3.6 \times 10^{-3} \text{ eV}^2, \]

and

\[ 7.3 \times 10^{-5} \text{ eV}^2 \leq \Delta m^2_{\text{solar}} = \Delta m^2_{21} = m^2_2 - m^2_1 \leq 9.3 \times 10^{-5} \text{ eV}^2, \]

with the best fit values given by \( \Delta m^2_{\text{atm}} = 2.2 \times 10^{-3} \text{ eV}^2, \) and \( \Delta m^2_{\text{solar}} = 8.2 \times 10^{-5} \text{ eV}^2. \) The mixing angles are in the ranges of

\[ 0.28 \leq \tan^2 \theta_{12} \leq 0.60 \] (best fit value 0.39),

\[ 0.5 \leq \tan^2 \theta_{23} \leq 2.1 \] (best fit value 1.0),

and

\[ \sin^2 \theta_{13} \leq 0.041. \]

There are many theoretical studies of neutrino masses and mixing [16]. The mixing matrix can be nicely represented by the so called bi-large mixing matrix. By an appropriate choice
of sign and phase conventions, the bi-large mixing matrix can be written as

\[ V = \begin{pmatrix} c & s & 0 \\ -s \sqrt{\frac{1}{2}} & c \sqrt{\frac{1}{2}} & \frac{1}{\sqrt{2}} \\ -s \sqrt{\frac{1}{2}} & c \sqrt{\frac{1}{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \]  

where \( c = \cos \theta \) and \( s = \sin \theta \) with \( \tan \theta \approx \tan \theta_{12} \). In this form the eigen-masses \( \tilde{m}_{\nu_i} \) of the light neutrinos, in general, have non-zero Majorana phases \( \rho_i \).

This class of models has \( V_{13} = 0 \) which is allowed by present experimental data and can be tested by future experiments. Several experiments are planned to measure \( V_{13} \) with greater precisions\(^{17} \). Obviously should a non-zero value for \( V_{13} \) be measured, modifications for the model considered are needed. The bi-large mixing model can be taken as the lowest order approximation. The bi-large mixing model captures many features of the present data and deserves more careful theoretical studies.

The bi-large mixing mass matrix is of the form\(^{16, 18} \)

\[ M_{\nu} = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & M_{22} & M_{23} \\ M_{12} & M_{23} & M_{33} \end{pmatrix} \]

\[ = \begin{pmatrix} (c^2 \tilde{m}_{\nu_1} + s^2 \tilde{m}_{\nu_2}) & \frac{c s}{\sqrt{2}} (-\tilde{m}_{\nu_1} + \tilde{m}_{\nu_2}) & \frac{c s}{\sqrt{2}} (-\tilde{m}_{\nu_1} + \tilde{m}_{\nu_2}) \\ \frac{c s}{\sqrt{2}} (-\tilde{m}_{\nu_1} + \tilde{m}_{\nu_2}) & \frac{1}{2} (s^2 \tilde{m}_{\nu_1} + c^2 \tilde{m}_{\nu_2} + \tilde{m}_{\nu_3}) & \frac{1}{2} (s^2 \tilde{m}_{\nu_1} + c^2 \tilde{m}_{\nu_2} - \tilde{m}_{\nu_3}) \\ \frac{c s}{\sqrt{2}} (-\tilde{m}_{\nu_1} + \tilde{m}_{\nu_2}) & \frac{1}{2} (s^2 \tilde{m}_{\nu_1} + c^2 \tilde{m}_{\nu_2} - \tilde{m}_{\nu_3}) & \frac{1}{2} (s^2 \tilde{m}_{\nu_1} + c^2 \tilde{m}_{\nu_2} + \tilde{m}_{\nu_3}) \end{pmatrix}. \]  

The above mass matrix produces bi-large mixing matrix, but the neutrino masses cannot be completely fixed using the two mass different \( \Delta m^2_{\text{sol}} \) and \( \Delta m^2_{\text{atm}} \) measurements even one knows all information about the mixing matrix. Additional inputs are needed to further constrain or to determine the parameters in the neutrino mass matrix\(^{16, 18, 19, 20, 21, 22} \). Several proposals have been made to reduce the parameters, such as texture zero\(^{20} \), determinant zero requirement\(^{21} \), and traceless requirement\(^{22} \) for the mass matrix. In this paper we impose texture zeros on bi-large mixing mass matrix to constrain neutrino masses and Majorana phases and also study the implications for leptogenesis.

To study leptogenesis, one needs further information on the heavy neutrino mass matrix. Since \( M_{\nu} = -M_D M_R^{-1} M_D^T \), if \( M_D \) is known one can obtain the form of \( M_R \) at higher energy. If \( M_D \) is proportional to a unit matrix, \( M_R \) will also have a similar form as the one given
in eq. (5), and the associated mixing matrix $V_R P_R$ is equal to $V^* P^*$ in the basis where all eigen-masses are real and positive, and the heavy neutrino eigen-masses are proportional to $1/m_i$. $M_\nu$ and $M_R$ are trivially related. We will show later that in this case the Majorana phases will play no role in leptogenesis. We therefore consider a simple, but non-trivial relation between $M_\nu$ and $M_R$ with $M_D = v_\nu Y_\nu = v_\nu b \text{diag}(\epsilon, 1, 1)$. With this form for $M_D$, $M_R$ also has the bi-large mixing mass matrix form, but with $V_R P_R$ not equal to $V^* P^*$. With these forms for $M_\nu$ and $M_R$, there is no CKM like CP violating phase in the charged current interaction with $W$ boson. The Majorana phases can, however, play a non-trivial role in leptogenesis\cite{14,15,23} and will be studied in more details later. The heavy neutrino mass matrix can be expressed as

$$
M_R = -v^2 b^2 \begin{pmatrix}
\epsilon^2 \left( \frac{\epsilon^2}{m_{\nu_1}} + \frac{s^2}{m_{\nu_2}} \right) & \frac{c_s}{\sqrt{2}} \left( -\frac{1}{m_{\nu_1}} + \frac{1}{m_{\nu_2}} \right) & \frac{c_s}{\sqrt{2}} \left( -\frac{1}{m_{\nu_1}} + \frac{1}{m_{\nu_2}} \right) \\
\frac{c_s}{\sqrt{2}} \left( -\frac{1}{m_{\nu_1}} + \frac{1}{m_{\nu_2}} \right) & \frac{1}{2} \left( \frac{\epsilon^2}{m_{\nu_1}} + \frac{s^2}{m_{\nu_2}} + \frac{1}{m_{\nu_3}} \right) & \frac{1}{2} \left( \frac{\epsilon^2}{m_{\nu_1}} + \frac{c^2}{m_{\nu_2}} - \frac{1}{m_{\nu_3}} \right) \\
\frac{c_s}{\sqrt{2}} \left( -\frac{1}{m_{\nu_1}} + \frac{1}{m_{\nu_2}} \right) & \frac{1}{2} \left( \frac{\epsilon^2}{m_{\nu_1}} + \frac{s^2}{m_{\nu_2}} - \frac{1}{m_{\nu_3}} \right) & \frac{1}{2} \left( \frac{\epsilon^2}{m_{\nu_1}} + \frac{c^2}{m_{\nu_2}} + \frac{1}{m_{\nu_3}} \right)
\end{pmatrix}.
$$

(6)

We note that none of the neutrino masses can be zero.

We comment that when discussing See-Saw neutrino mass matrices, there are two scales, the light neutrino and the heavy neutrino mass scales. The mass matrices at the two scales may be different due to renormalization group running effects\cite{24}. A mass matrix element is zero at a particular scale may not be zero at another scale unless there are certain symmetries to guarantee this\cite{25}.

An important problem in neutrino physics is to understand the origin of neutrino masses and mixing. There are many attempts have been made to understand this problem, at present the answer is, however, far from satisfaction. We will not attempt to carry out a model building investigation of the mass matrices, instead we will study phenomenological implications here. However, we would like to point out there are models which can produce some models we study. For example, in Ref.\cite{19} it was shown that for three generations of left-handed leptons and three generations of right-handed charged leptons and neutrinos, it is possible to obtain $M_D \sim \text{diag}(\epsilon, 1, 1)$ and $M_R$ of the form in eq. (6) with $M_{23} = 0$ if there are 3 Higgs doublets and 2 neutral signets which transform, non-trivially, under a discrete group $Z_2^{(r)} \times Z_2^{(tr)} \times Z_2^{(aux)}$. In this model the flavor symmetries of the model dictates that the light neutrino mass matrix structure is essentially determined by the heavy neutrino mass matrix\cite{25}.
We will generalize the discussion to include all possible texture zeros in both the light and heavy bi-large mass matrices. Although the classes of models have simple structures, there are very rich phenomenological implications on the neutrino masses, CP violating phases and on leptogenesis.

II. BI-LARGE NEUTRINO MIXING MASS MATRIX WITH TEXTURE ZEROS

There are two different ways the texture zeros can be imposed: a) The texture zeros are imposed on the light neutrino mass matrix $M_{\nu}$; And b) The texture zeros are imposed on the heavy neutrino mass matrix $M_R$. There are five different cases for each of class a) and class b) types of models which give non-trivial three generation mixing.

For class a) the five cases indicated by $M_{L_i}$ are

$$\begin{align*}
M_{L1} &= \begin{pmatrix} 0 & M_{12} & M_{12} \\ M_{12} & M_{22} & M_{23} \\ M_{12} & M_{23} & M_{22} \end{pmatrix}, \\
M_{L2} &= \begin{pmatrix} 0 & M_{12} & \ast \\ M_{12} & 0 & M_{23} \\ M_{12} & M_{23} & 0 \end{pmatrix}, \\
M_{L3} &= \begin{pmatrix} 0 & M_{12} & M_{12} \\ M_{12} & 0 & \ast \\ M_{12} & M_{22} & \ast \end{pmatrix},
\end{align*}$$

$$M_{L4} = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & M_{22} & 0 \\ M_{12} & 0 & M_{22} \end{pmatrix}, \\
M_{L5} = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & 0 & M_{23} \\ M_{12} & M_{23} & 0 \end{pmatrix}. \quad (7)$$

For class b) the five cases have the same forms as above. We denote them and their matrix elements by $M_{R_i}$ and $M_{ij}^h$, respectively.

Before discussing the above mass matrices in detail, we make some comments on the neutrino masses. There are three light neutrino masses, and there are two observable mass differences $\Delta m^2_{\text{sol}}$ and $\Delta m^2_{\text{atm}}$ from neutrino oscillation. If one of the neutrino masses, or a combination of them, is known, the rest of the masses can be determined in terms of $\Delta m^2_{21,32}$. We express $m_2$ and $m_3$ as a function of $m_1$ as,

$$m_{\nu_2} = \sqrt{m_{\nu_1}^2 + \Delta m^2_{21}}, \quad m_{\nu_3} = \sqrt{m_{\nu_1}^2 + \Delta m^2_{21} + \Delta m^2_{32}}. \quad (8)$$

There are additional constraints on the neutrino masses. One of them comes from WMAP data which limits $m_{\text{sum}} = m_1 + m_2 + m_3$ to be less than $0.071$ eV at the 95% C.L.. In the future cosmological data can improve the sensitivity and reach $0.03$ eV. There are another two constraints, the effective mass for neutrinoless double beta decays, $m_{ee} = |M_{11}|$,
and the effective mass for tritium beta decay, \( \langle m_\nu \rangle = (|V_{11}|^2 m_1^2 + |V_{12}|^2 m_2^2 + |V_{13}|^2 m_3^2)^{1/2} \). The current experimental upper bounds for \( m_{ee} \) and \( \langle m_\nu \rangle \) are 1.35 eV \(^{28}\) and 3 eV \(^7\), respectively. These bounds are less stringent than the WMAP constraint. However, in the future, the sensitivities for \( m_{ee} \) and \( \langle m_\nu \rangle \) can reach 0.01 eV \(^{29}\) and 0.12 eV \(^{30}\), respectively, by laboratory experiments. These experiments can provide interesting constraints.

If there are additional constraints, such as texture zeros, it may be possible to determine the neutrino masses or relate the masses to the CP violating Majorana phases which are otherwise very difficult to measure.

A. Constraints From Texture Zeros In The Light Neutrino Mass Matrix

In this section we study the consequences of the above texture zeros in class a). For the case L1, we have

\[
M_{11} = c^2 \tilde{m}_1 + s^2 \tilde{m}_2 = 0. \tag{9}
\]

This is a special case studied in Ref. \(^{31}\) by requiring \( m_{ee} = 0 \).

The phase of \( \tilde{m}_1 \) can be chosen to be zero which results in \( \tilde{m}_2 = -m_1 / \tan^2 \theta \). Using this relation, the value of \( m_2 \) is determined by

\[
m_2^2 = \frac{\Delta m_{sol}^2}{1 - \tan^4 \theta}. \tag{10}
\]

The mass \( m_3 \) can be expressed as

\[
\text{Normal hierarchy} : \quad m_3^2 = \Delta m_{atm}^2 + \frac{\Delta m_{sol}^2}{1 - \tan^4 \theta}.
\]

\[
\text{Reversed hierarchy} : \quad m_3^2 = -\Delta m_{atm}^2 + \frac{\Delta m_{sol}^2}{1 - \tan^4 \theta}. \tag{11}
\]

The phase of \( \tilde{m}_3 \) is not determined from the above consideration. The reversed hierarchy is not a physical solution in this case because \( m_3^2 \) is minus numerically when constraints from data are imposed.

From eqs (9) and (10), we obtain the central value of \( m_1 \) for the normal hierarchy to be 0.0038 eV and the 3\( \sigma \) lower bound to be 0.0025 eV. The sum of the masses \( m_{sum} \) is equal to 0.062 ± 0.010 eV which satisfies the WMAP bound and can be probed by future cosmological data. The effective mass \( m_{ee} \) is identically equal to zero and \( \langle m_\nu \rangle = 0.0062 \pm \)
0.0011 eV which are safely within the current experimental bounds and is very difficult to be probed by near future experiments.

The case L2 is a special case of the minimal Zee mass matrix and has been shown to be ruled out. The cases L2 and L3 cannot be consistent with data, it is easy to understand from the simultaneous requirements \( M_{11} = 0 \), and \( M_{22} = 0 \) (\( M_{23} = 0 \)). Because of this, one has \( \tilde{m}_3 = s^2 \tilde{m}_1 + c^2 \tilde{m}_2 \) and \( c^2 \tilde{m}_1 + s^2 \tilde{m}_2 = 0 \) which leads to

\[
\frac{\Delta m_{atm}^2}{\Delta m_{sol}^2} = \frac{\tan^2 \theta (2 - \tan^2 \theta)}{1 - \tan^2 \theta}.
\]

(12)

With \( 0.28 < \tan^2 \theta < 0.6 \), the above ratio is in the range 0.5 to 1.3 which is in conflict with data. This class of models is therefore ruled out.

We now discuss the case L4. The constraint from \( M_{22} = m_1 s^2 e^{-i \rho_1} + m_2 c^2 e^{-i \rho_2} + m_3 e^{-i \rho_3} = 0 \) implies that \( m_1 s^2 + m_2 c^2 \geq m_3 \). This implies that only reversed mass hierarchy, \( m_2 > m_1 > m_3 \), is allowed. We choose the convention where the phase \( \rho_2 = 0 \). We find that the Majorana phases \( \rho_{1,3} \) are determined by the masses and mixing angles as

\[
\cos 2 \rho_1 = \frac{1}{2m_1 m_2 s^2 c^2} (m_3^2 - m_1^2 s^4 - m_2^2 c^4),
\]

\[
\cos 2 \rho_3 = \frac{1}{m_3} (m_1 s^2 \cos 2 \rho_1 + m_2 c^2),
\]

\[
\sin 2 \rho_3 = \frac{1}{m_3} m_1 s^2 \sin 2 \rho_1.
\]

(13)

There are two solutions, due to the undetermined sign of \( \sin 2 \rho_1 = \pm \sqrt{1 - \cos^2 2 \rho_1} \), even with \( \cos 2 \rho_1 \) fixed. They cannot be distinguished by oscillation and laboratory mass measurement experiments. If leptogenesis is responsible for the baryon asymmetry of our universe, we will show later that the two different solutions give different signs for the baryon asymmetry and the solution with positive \( \sin 2 \rho_1 \) has to be chosen.

In Fig. (a) and (b), we show \( \cos 2 \rho_1 \) and \( \cos 2 \rho_3 \), and \( \sin 2 \rho_1 \) and \( \sin 2 \rho_3 \), respectively, as functions of Log\( (m_1) \) with the best fit values for \( \Delta m_{sol}^2, \Delta m_{atm}^2 \) and \( \tan \theta \). We see that in order to have physical solutions there is a minimal value for \( m_1 \) which is about 0.05 eV for the input parameters with central values. When errors in the input parameters are included, the lower bound can be reduced to 0.039 eV at 3\( \sigma \) level. One can also express the masses as functions of the phase \( \rho_1 \).

There is no upper bound for the neutrino masses from the above considerations. If one takes the WMAP constraint, \( m_1 \) is bounded to be less than 0.238 eV. This implies that \( \cos 2 \rho_1 \) is to be smaller than 0.906.
FIG. 1: cos 2$\rho$ and cos 2$\rho_3$ (Fig. 1(a)), and sin 2$\rho_1$ and sin 2$\rho_3$ (Fig. 1(b)), respectively, as functions of Log($m_1$) for the central values of $\Delta m_{\text{solar}}^2$, $\Delta m_{\text{atm}}^2$, and tan$\theta$. Both solutions with $\pm |\sin 2\rho_1|$ are drawn. In (a) the solid line is for cos 2$\rho_1$ and the dotted line is for cos 2$\rho_3$, while in (b) the solid line is for sin 2$\rho_1$ and the dotted line is for sin 2$\rho_3$ respectively.

The effective masses $m_{ee} = |M_{11}| = (m_1^2 c^4 + m_2^2 s^4 + 2 m_1 m_2 s^2 c^2 \cos 2\rho_1)^{1/2}$ and $\langle m_\nu \rangle = (c^2 m_1^2 + s^2 m_2^2)^{1/2}$ are constrained. Compared with current experimental upper bounds for $m_{ee} < 1.35$ eV and $\langle m_\nu \rangle < 3$ eV, we get the upper bound for $m_1$ to be 0.95 eV which is above the WMAP bound. The lower bound for $m_{ee}$ and $\langle m_\nu \rangle$ may be calculated too. They are, at 3$\sigma$ level, 0.0217 eV and 0.0392 eV respectively. The lower bound on $m_{ee}$ can be probed by future neutrinoless double beta decays.

The constraints on L5 can be obtained by replacing $m_3$ by $-m_3$ in eq. (13). The net result is to change the signs of cos 2$\rho_3$ and sin 2$\rho_3$. Leptogenesis will select the solution with sin 2$\rho_1$ to be positive again.

B. Constraints From Texture Zeros In The Heavy Neutrino Mass Matrix

We now discuss the situation for the cases in class b). The cases R1, R2 and R3 all require $M_{11}^h = 0$, which implies

$$s^2 \tilde{m}_1 + c^2 \tilde{m}_2 = 0.$$  \hspace{1cm} (14)

Since data requires that $s^2 < c^2$ and $m_2^2 > m_1^2$, it is not possible to satisfy the above
equation. Cases R1, R2 and R3 are therefore ruled out by data.

A specific realization of R4 was discussed by Grimus and Lavoura in Ref. [18]. We choose the convention with $\rho_2 = 0$. In this case since $M_{23} = s^2/\tilde{m}_1 + c^2/\tilde{m}_2 - 1/\tilde{m}_3 = 0$, when combined with $m_2 > m_1$ from data, one obtains $m_3 > m_1$. Since data show that $|\Delta m^2_{32}|$ is larger than $|\Delta m^2_{21}|$, only normal hierarchy neutrino mass pattern is allowed.

The condition $|M_{23}| = 0$ also leads to

$$\cos 2\rho_1 = \frac{1}{2m_1m_2c^2s^2}\left(\frac{m_2m_1^2}{m_3^2} - m_2s^4 - m_1c^4\right),$$

$$\cos 2\rho_3 = \frac{m_3}{m_1m_2}(m_2s^2\cos(2\rho_1) + m_1c^2),$$

$$\sin 2\rho_3 = \frac{m_3}{m_1}s^2\sin(2\rho_1).$$

(15)

Similar to the case for L4, there are two solutions due to the undetermined sign of $\sin 2\rho_1$. Neutrino oscillation and laboratory neutrino mass measurement experiments will not be able to decide which solution to take. However, leptogenesis will select the solution with positive $\sin 2\rho_3$.

In Fig. 2 (a), we show $\cos 2\rho_1$ and $\cos 2\rho_3$ as functions of $m_1$ with the best fit values for $\Delta m^2_{sol}$, $\Delta m^2_{atm}$, $\tan \theta$. In Fig. 2 (b) we show the two solutions for $\sin 2\rho_1$ and $\sin 2\rho_3$ as functions of Log($m_1$). There is a region, around $m_1 = 0.01$, not allowed. We see that there is a minimal value for $m_1$ which is about 0.003 eV. With errors in the input parameters, the low bound at 3σ level is 0.002 eV. One can also express the masses as functions of the phase $\rho_1$.

Similar to the case L4, there is no upper bound for the neutrino masses for R4 from the above considerations. If one takes the WMAP constraint, $\cos 2\rho_1$ is bounded to be smaller than 0.905.

The effective masses $m_{ee} = |M_{11}| = m_1m_2/m_3$ and $\langle m_\nu \rangle = (c^2m_1^2 + s^2m_2^2)^{1/2}$ are constrained. Compared with current experiment upper bounds for $m_{ee} < 1.35$ eV and $\langle m_\nu \rangle < 3$ eV, we get the upper bound for $m_1$ to be 1.35 eV which is again above the WMAP bound. The lower bound at 3σ for $m_{ee}$ and $\langle m_\nu \rangle$ are $3.8 \times 10^{-4}$ eV and $4.7 \times 10^{-3}$ eV respectively.

Again the constraints for case R5 can be obtained by simply changing $\tilde{m}_3$ to $-\tilde{m}_3$. 

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FIG. 2: cos $2\rho_1$ and cos $2\rho_3$ (Fig.2 (a)), and sin $2\rho_1$ and sin $2\rho_3$ (Fig.2 (b)), respectively, as functions of Log($m_1$) for the central values of $\Delta m^2_{\text{solar}}$, $\Delta m^2_{\text{atm}}$, and $\tan\theta$. Both solutions with $\pm |\sin 2\rho_1|$ are drawn. In (a) the solid line is for cos $2\rho_1$ and the dotted line is for cos $2\rho_3$, while in (b) the solid line is for sin $2\rho_1$ and the dotted line is for sin $2\rho_3$ respectively.

III. HEAVY NEUTRINO MASSES AND MIXING MATRIX

In our previous discussions we have concentrated only on the light neutrino masses, mixing and phases. We have seen that the mass matrix is completely specified by experimental measurable quantities. In fact once the light neutrino mass matrix is known, the right-handed neutrino mass matrix is almost specified as can be seen from eq. (6).

There are three new parameters $v_\nu$, $b$ and $\epsilon$ in $M_R$. In the cases considered here, only the combination $v_\nu b$ appears in the calculations. We will normalize $v_\nu$ to have the SM values of 174 GeV and let $b$ be a free parameter. It is interesting to note that if one knows $\epsilon$, all information on the mixing matrix $U_R$ is known, and also the ratios of the heavy neutrino masses $M_i/M_j$ are known once the light neutrino masses and mixing angles are fixed.

In the limit $\epsilon = 1$, $U_R = U^*$ and $M_i = v_\nu^2 b^2/m_i$. When $\epsilon$ is not equal to 1, the situation is more complicated. But from eq. (6) it is clear that the unitary matrix $U_R$ which diagonalizes $M_R$ still has the bi-large mixing form. When $\epsilon$ is close to one, the heavy neutrino mass hierarchies are $M_3 > M_1 > M_2$ and $M_1 > M_2 > M_3$ for the reversed and normal light neutrino hierarchies, respectively. When $\epsilon$ deviates from one, the mass hierarchy pattern will change and $U_R$ is non-trivially related to $U$. But $M_3$ is always equal to $v_\nu^2 b^2/\tilde{m}_3$. 

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In Fig. 3 we show the heavy neutrino masses as functions of $\epsilon$ for several fixed values of $m_1$ for illustration. For case L1, we use the central value 0.0038 eV, and for L4 and R4 we use two typical values 0.055 eV and 0.1 eV for $m_1$, respectively. The cases L4 and L5 have the same eigen-masses, and R4 and R5 also have the same eigen-masses. From Fig. 3 we can clearly see that the mass hierarchy changes with $\epsilon$.

The mixing matrix $U_R$ is more complicated. It has the general form

$$U_R = \begin{pmatrix} e^{i\delta_1} & 0 & 0 \\ 0 & e^{i\delta_2} & 0 \\ 0 & 0 & e^{i\delta_2} \end{pmatrix} \begin{pmatrix} c' & s' & 0 \\ -\frac{s'}{\sqrt{2}} & \frac{c'}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{s'}{\sqrt{2}} & \frac{c'}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma_2} & 0 \\ 0 & 0 & e^{i\gamma_3} \end{pmatrix}.$$  \hspace{0.5cm} (16)

In the following we give, as examples, $U_R$ for the cases L1, L4 and R4. In the case L1 with $m_1 = 0.0038$ eV,

$$U_R = \begin{pmatrix} 0.7175 & 0.6965 & 0 \\ -0.4925 & 0.5074 & 0.707 \\ -0.4925 & 0.5074 & -0.707 \end{pmatrix}.$$ \hspace{0.5cm} (17)

For cases L4 and R4, in the basis where all eigen-masses are real and positive, we write $U_R$ for two typical values of $m_1$, 0.055 eV and 0.1 eV. We have

For case L4 with $m_1 = 0.055$ eV,

$$U_R = \begin{pmatrix} 0.2946 + 0.1841 i & -0.4986 + 0.7942 i & 0 \\ 0.6516 - 0.1225 i & -0.0449 - 0.2415 i & -0.6941 + 0.1351 i \\ 0.6516 - 0.1225 i & -0.0449 - 0.2415 i & 0.6941 - 0.1351 i \end{pmatrix},$$ \hspace{0.5cm} (18)

and with $m_1 = 0.1$ eV,

$$U_R = \begin{pmatrix} 0.0532 + 0.1285 i & -0.9162 + 0.3760 i & 0 \\ 0.6929 - 0.1012 i & -0.0139 - 0.0973 i & -0.6996 + 0.1029 i \\ 0.6929 - 0.1012 i & -0.0139 - 0.0973 i & 0.6996 - 0.1029 i \end{pmatrix}.$$ \hspace{0.5cm} (19)

For case R4 with $m_1 = 0.055$ eV,

$$U_R = \begin{pmatrix} 0.1202 + 0.1778 i & -0.8122 + 0.5424 i & 0 \\ 0.6777 - 0.1332 i & -0.0284 - 0.1491 i & -0.6938 + 0.1364 i \\ 0.6777 - 0.1332 i & -0.0284 - 0.1491 i & 0.6938 - 0.1364 i \end{pmatrix}.$$ \hspace{0.5cm} (20)
FIG. 3: \( M_i(\text{GeV}/v^2b^2 \) for cases L1, L4 and R4 as functions of \( \epsilon \). For L1, \( m_1 \) is equal to 0.0038 eV ((a)) (determined from the central values of the mixing angles). For L4 and R4, \( m_1 \) is not fixed by the mixing angles. We draw figures for \( m_1 = 0.055 \) eV ((b)) and \( m_1 = 0.1 \) eV ((c)) for illustrations. The solid, dotted and dashed lines are for \( M_1 \), \( M_2 \) and \( M_3 \), respectively.

and with \( m_1 = 0.1 \) eV,


\[ U_R = \begin{pmatrix}
0.0431 + 0.1172i & -0.9324 + 0.3393i & 0 \\
0.6954 - 0.0931i & -0.0114 - 0.0875i & -0.7009 + 0.0938i \\
0.6954 - 0.0931i & -0.0114 - 0.0875i & 0.7009 - 0.0938i \\
\end{pmatrix}. \quad (21) \]

IV. LEPTOGENESIS

There are extensive discussions on implications of leptogenesis for See-Saw neutrino mass matrix\[^{11, 12, 13, 14, 15}\]. With a general See-Saw mass matrix, it has been shown that there is enough room in parameters space to reproduce the observed BAU\[^{11, 12, 13}\]. There are also more restrictive forms of mass matrix with texture zeros which can also reproduce the observed BAU\[^{14, 15}\]. The mass matrices discussed in the previous sections are a class of very restrictive matrices, in particular that there is no CKM like CP violating phase. It is interesting to see if such models can also produce the observed BAU. We find that although there is no CKM like CP violating phase, the required CP violation can come from the Majorqana phase. There is a large parameter space with which BAU can be reproduced. Taking leptogenesis as a requirement, we show that interesting constraints on the scale of the right-handed neutrino can be obtained. We now proceed to provide more details.

The baryon number asymmetry problem, why our universe is dominated by matter, is one of the most outstanding problems in modern physics. This problem is related to the ratio \( \eta_B = n_B/n_\gamma \). Here \( n_B \) is the baryon number density and \( n_\gamma \) is the photon number density. If the universe contains equal matter and anti-matter initially with baryon number conserved, the expected ratio for \( \eta_B \) is about \( 10^{-20} \). Observations from Big-Bang Nucleosynthesis (BBN) and Cosmic Macrowave Background (CMB) radiation determine \( \eta_B \) to be \( 6.5^{+0.4}_{-0.3} \times 10^{-10} \). There is a huge difference between the expected and the observed values. Sakharov showed that if there are\[^{35}\]: 1) baryon number violation, 2) C and CP violation, and 3) occurrence of non-thermal equilibrium when 1) and 2) are effective, it is possible to create a matter dominated universe from a symmetric one in the early epoch of the universe.

In the Standard Model due to \( SU(2)_L \) anomaly, there are baryon number violating interactions. This interaction becomes strong at high temperatures\[^{36}\]. This interaction violates \( B + L \), but conserves \( B - L \). Fukugita and Yanagida\[^{11}\] noticed that if in the early universe there was lepton number asymmetry, this interaction can transfer lepton number asymmetry
produced by heavy neutrino decays, for example, to baryon number asymmetry.

The surviving baryon asymmetry from lepton number asymmetry due to the “lth” heavy neutrino is given by \[11, 37\]

\[
\eta_B = \frac{s}{n_\gamma} \left| \frac{\omega}{\omega - 1} \frac{a_l \kappa_l}{g_{*l}} \right|,\tag{22}
\]

where \( s = (2\pi^2/45)g_{*0}T^3|_0 \) and \( n_\gamma = (2/\pi^2)\zeta(3)T^3|_0 \) are the entropy and photon densities of the present universe with \( g_{*0} = 43/11 \) being the effective relativistic degrees of freedom. The parameter \( \omega \) is calculated to be \[37\]

\[
\omega = (8N_F + 4N_H)/(22N_F + 13N_H) \]

depending on the number of \( SU(2)_L \) doublet Higgs scalars \( N_H \) and fermions \( N_F \). \( g_{*l} \) is the effective relativistic degrees of freedom at the temperature where the lepton number asymmetry \( a_l \) is generated from the “lth” heavy neutrino decay. For the lightest heavy neutrino decay contribution, \( g_{*l} = (28 + (7/8)\times 90)_{SM} + 4(N_H - 1) + 2(7/8) \) is of order 100. Here the last term comes from the lightest heavy Majorana neutrino which produces the lepton number asymmetry. The number \( N_H \) depends on the details of the specific model. We have checked the sensitivity of \( \eta_B \) on \( N_H \) and find that there is only about a 10% reduction for \( N_H \) varying from 1 to 5. We will assume that there is just one Higgs doublet in our numerical calculations. \( \kappa_l \) is a dilute factor which depends on the ratio of heavy Majorana neutrino decay rate and the Hubble parameter at the time of heavy neutrino decay, \( K_l = \Gamma_l/H_l \) with \( \Gamma_l = (\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)_{ll} M_l/8\pi \) and \( H_l = 1.166\sqrt{g_{*l} M^2_l/M_{\text{planck}}} \). Here \( \tilde{Y}_\nu = V_R^T Y_\nu \) is the Yukawa coupling in the basis where \( M_R \) is diagonalized.

The heavy neutrino mass is of order \( M_l \sim (v_\nu^2/m_3)(\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)_{ll} \), one would obtain \( \Gamma_l/H_l \sim 10^4(m_3/eV)(100 GeV/v_\nu)^2 \). For \( m_3 \) within the allowed lower bound discussed earlier and upper bound from WMAP, the factor \( K_l \) is within the range of \( 10 \sim 10^6 \). In this range the dilute factor \( \kappa_l \) is approximated by \[13\]

\[
\kappa_l \approx 0.3/K_l(\ln K_l)^{3/5}.\]

In our numerical calculations we will use this approximate form.

We now study \( a_i \) in the models considered. The lepton number asymmetry \( a_i \) generated by the “ith” heavy neutrino is given by \[11, 12\]

\[
a_i \approx \frac{1}{8\pi} \frac{1}{[\tilde{Y}_\nu \tilde{Y}_\nu^\dagger]_{ii}} \sum_j \text{Im}\{[\tilde{Y}_\nu \tilde{Y}_\nu^\dagger]_{ij}\} f \left( \frac{M^2_j}{M^2_i} \right),\tag{23}
\]

where

\[
f(x) = \sqrt{x}(\frac{2}{x - 1} + \ln \frac{1 + x}{x}).\tag{24}
\]
Applying the above equation to the models discussed in the previous section, we obtain the lepton number asymmetries due to heavy neutrino decays to be

\[ a_i = -\frac{1}{8\pi} b^2 (\epsilon^2 - 1)^2 \sum_j \text{Im}(U_{\text{R}1i}U_{\text{R}1j}^*)^2 \frac{f(M_j^2/M_i^2)}{1 + (\epsilon^2 - 1)|U_{\text{R}1i}|^2}, \]

\[ a_1 = -\frac{1}{8\pi} b^2 (\epsilon^2 - 1)^2 \text{Im}(U_{\text{R}11}U_{\text{R}12}^*)^2 \frac{f(M_2^2/M_1^2)}{1 + (\epsilon^2 - 1)|U_{\text{R}11}|^2}, \]

\[ a_2 = -\frac{1}{8\pi} b^2 (\epsilon^2 - 1)^2 \text{Im}(U_{\text{R}12}U_{\text{R}11}^*)^2 \frac{f(M_1^2/M_2^2)}{1 + (\epsilon^2 - 1)|U_{\text{R}12}|^2}, \]

\[ a_3 = 0, \]

(25)

with \( \text{Im}(U_{\text{R}11}U_{\text{R}12}^*)^2 = -c^2 s^2 \sin(2\gamma_2) \). In the above we have used the fact that \( U_{\text{R}13} = 0 \). Note that for \( \epsilon = 1 \), no lepton number asymmetry can be generated.

From eq. (22) we see that only solutions which generate negative \( a_i \) can be candidate producing the right sign for baryon asymmetry. This criterion selects out solutions obtained in Section II which are not able to be distinguished by low energy experimental data.

Several studies of leptogenesis with bi-large neutrino mixing matrix have been carried out\[15\]. Here we follow similar strategy to systematically study the models discussed earlier. To demonstrate that the See-Saw model discussed here can indeed explain the observed baryon number asymmetry, in the following we consider a simple case with large hierarchical structure for the heavy Majorana neutrino mass. In this case the dominant contribution to the surviving baryon asymmetry is from the lightest heavy neutrino decay. The heavy neutrino with mass of \( M_3 \) does not produce a non-zero asymmetry, it cannot be the lightest heavy neutrino since it will washout baryon asymmetries produced by the other two heavier ones in our case. One needs to work in the parameter space where \( M_3 \) is not the smallest.

Large \( \epsilon \) tends to make \( M_{1,2} \) bigger, while does not affect \( M_3 \) as can be seen from Fig. \[3\] Therefore leptogenesis favors small \( \epsilon \). Numerically we find that with \( |\epsilon| \) less than around 0.5 the lightest heavy neutrino mass is \( M_2 \) and the mass squared is at least 10 times smaller than \( M_3^2 \) as can be seen from Fig.\[4\] In this range, the washout effect of the CP conserving decay of the heavy neutrino of mass \( M_3 \) would be small. We will present our results for the baryon asymmetry produced by the lightest heavy neutrino with \( \epsilon \) satisfying the condition that the lightest heavy neutrino mass squared is at least 10 times smaller than the next lightest heavy neutrino mass squared

For the case L1, \( U_{\text{R}11}U_{\text{R}12}^* \) is real. This leads to zero lepton asymmetry \( a_i \). This type of models cannot explain the baryon number asymmetry in the universe.
FIG. 4: The allowed ranges for $\epsilon$ and $b$ for case L4 with $\eta_B$ in the range of $4 \times 10^{-10} \sim 8 \times 10^{-10}$, with (a) $m_1 = 0.055$ eV and (b) $m_1 = 0.1$ eV.

FIG. 5: The allowed ranges for $\epsilon$ and $b$ for case R4 with $\eta_B$ in the range of $4 \times 10^{-10} \sim 8 \times 10^{-10}$, with (a) $m_1 = 0.055$ eV and (b) $m_1 = 0.1$ eV.

In Figs.4 and 5 we show $\eta_B$ as functions of $b$ and $\epsilon$ for $m_1 = 0.055$ eV and 0.1 eV for the cases L4 and R4. Only the cases with negative $a_i$ which produces the right sign for the observed baryon number asymmetry are shown. There are two solutions with different signs for $\sin 2\rho_1$ in the case of L4 which satisfies neutrino mass and oscillation experimental...
FIG. 6: $M_i$ for case L4 as functions of $\epsilon$ with (a) $m_1 = 0.055$ eV and (b) $m_1 = 0.1$ eV. The solid, doted and dashed lines are for $M_1$, $M_2$ and $M_3$ respectively.

FIG. 7: $M_i$ for case R4 as functions of $\epsilon$ with (a) $m_1 = 0.055$ eV and (b) $m_1 = 0.1$ eV. The solid, doted and dashed lines are for $M_1$, $M_2$ and $M_3$ respectively.

constraints as discussed in section II. If the model is required to produce the baryon number asymmetry, we find that only the solution with the positive $\sin^2 \rho_3$ is allowed. Similar situation happens for the case of R4, positive $\sin 2\rho_1$ has to be chosen. For the cases L5 and R5, the solutions with positive $\sin 2\rho_3$ have to be chosen.

We see from Figs. 4 and 5 that the observed baryon number asymmetry can be produced
in the models considered here. We also see that the requirement of generating the correct baryon number asymmetry, the parameters $\epsilon$ and $b$ are constrained. One can use this fact to obtain the allowed mass ranges for the heavy neutrino masses $M_i$. In Figs. 6 and 7 we show $M_i$ as functions of $\epsilon$ for the central value of $\eta_B$. These masses represent possible new physics scale and are constrained to be in the range of $10^{12} \sim 10^{15} \text{GeV}$.

V. CONCLUSIONS

In this paper we have studied constraints from texture zeros in bi-large mixing See-Saw neutrino mass matrices and also from leptogenesis. We have systematically investigated two classes of models with one of them (class a)) to have the texture zeros imposed on the light neutrino mass matrix, and another (class b)) to have the texture zeros imposed on the heavy neutrino mass matrices.

Assuming a simple form proportional to $\text{diag}(\epsilon, 1, 1)$ for the Dirac mass matrix which relates the left- and right- handed neutrinos, both light and heavy neutrinos can simultaneously have the bi-large mixing matrix form. Both classes a) and b) of mass matrices can have 5 different forms which produce non-trivial three generation mixing. We find that only three (L1, L4, L5 ) in class a) and two (R4, R5) in class b), respectively, can be consistent with present data on neutrino masses and mixing constraints. In all the models none of the neutrino masses can be zero. Using present data, the lightest neutrino is bounded to be heavier than, 0.0025 eV, 0.039 eV and 0.002 eV for L1, L4 and L5, and, R4 and R5, respectively. Future experiments can provide further tests and even rule out some of the models.

Because $V_{13} = 0$, there is no CKM type of CP violating phase in the light neutrino mixing matrix. No CP violating effects can be observed in neutrino oscillation experiments. However, there can be non-trivial Majorana phases. These phases can play an important role in explaining the observed baryon number asymmetry in our universe. We have shown that in the models considered there are parameter spaces where the observed baryon number asymmetry can indeed be generated through the leptogenesis mechanism. It is interesting to note that the requirement of producing the observed baryon number asymmetry rules out several models which are, otherwise, impossible to achieve by Laboratory experiments. This requirement also provides a condition to fix the allowed scale for the heavy neutrinos.
We find that the masses are in the range of $10^{12} \sim 10^{15} \text{GeV}$.

In the models we considered $V_{13}$ is zero which is allowed by present experimental data and can be tested by future experiments. Several experiments are planned to measure $V_{13}$ with greater precision\cite{17}. Obviously should a non-zero value for $V_{13}$ be measured, modifications for the model considered are needed. However, the models considered can be taken as the lowest order approximations. How to obtain such mass matrices deserves more future theoretical studies.

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