Electric–magnetic duality and renormalization in curved spacetimes

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We point out that the duality symmetry of free electromagnetism does not hold in the quantum theory if an arbitrary classical gravitational background is present. The symmetry breaks in the process of renormalization, as also happens with conformal invariance. We show that a similar duality-anomaly appears for a massless scalar field in 1 + 1 dimensions.

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Introduction and summary. The Maxwell equations in vacuo are highly symmetric. In addition to their relativistic (Poincaré) invariance in Minkowski spacetime, they exhibit two additional symmetries: conformal invariance and electric–magnetic duality. The former is the symmetry under Weyl (or conformal) transformations $g_{\mu\nu} \rightarrow \Omega^2(x) g_{\mu\nu}$. This is a symmetry of the classical theory in arbitrary spacetimes, and it is also an exact symmetry of the quantum theory in Minkowski spacetime. However, as first pointed out in [1], conformal invariance cannot be extended to quantum field theory (QFT) in curved backgrounds. Conformal symmetry implies the tracelessness of the energy-momentum tensor $T_{\mu\nu}$. Since $T_{\mu\nu}$ is quadratic in the field $F_{\mu\nu}$, renormalization is required to compute its expectation values. It turns out that generally covariant methods of renormalization in curved spacetime produce a non-vanishing trace $\langle T^\mu_\mu \rangle$ which breaks the conformal invariance [2]. The value of this trace is independent of the state in which the expectation value is evaluated, and is written in terms of curvature tensors. The breakdown of conformal symmetry is a renormalization effect and therefore it is only manifest when composite operators are considered, such as $T_{\mu\nu}$ (the equations of motion and correlation functions are still conformally invariant). This is the celebrated conformal or trace anomaly, which constitutes a robust prediction of renormalization in curved spacetimes and has important physical consequences [2].

Another important symmetry of electromagnetism in the absence of charges is invariant under duality transformations $F_{\mu\nu} \rightarrow *F_{\mu\nu}$ (see e.g. [3, 4]) where the (Hodge) dual tensor is defined in the standard way, $*F_{\mu\nu} = \mp 1/2 \varepsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$. In terms of the electric and magnetic fields, this discrete transformation reads $E \rightarrow B, B \rightarrow -E$. It can be also viewed as a particular case of the electric–magnetic rotation $E \rightarrow E \cos \theta + B \sin \theta, B \rightarrow -B \cos \theta + E \sin \theta$. Maxwell’s equations, $\nabla_\mu F^{\mu\nu} = 0$, are manifestly invariant. The classical (Maxwell) energy-momentum tensor, which can be written in the symmetric form

$$T^M_{\mu\nu} = -\frac{1}{2} (F_{\mu\alpha} F_{\nu}^\alpha + *F_{\mu\alpha} *F_{\nu}^\alpha) ,$$

is also invariant. This duality can be extended to the QFT in Minkowski spacetime. One can show that the duality transformation is implemented by a unitary operator in the Fock space which leaves the Minkowski vacuum invariant. As a consequence, vacuum-correlation functions are duality-invariant, e.g. $\langle F_{\mu\alpha}(x) F_{\nu\alpha}(x') \rangle = \langle *F_{\mu\alpha}(x) *F_{\nu\alpha}(x') \rangle$ for all $x \neq x'$. Vacuum expectation values of composite (non-linear) operators are also invariant, although renormalization is required to make sense of the otherwise divergent expressions. In Minkowski spacetime normal-order (i.e. subtraction of the vacuum expectation value) does the job. As an example, one trivially obtains $\langle F_{\mu\alpha}(x) F_{\nu\alpha}(x) \rangle = \langle *F_{\mu\alpha}(x) *F_{\nu\alpha}(x) \rangle = 0$.

The goal of this paper is to show that the classical electric–magnetic duality symmetry can not be extended to QFT in curved spacetime. In order to show the influence of the gravitational background in the sharpest way, we will work as closely as possible to the theory in Minkowski spacetime. We will consider free electromagnetism, $\mathcal{L} = -1/4 \sqrt{|g|} F_{\mu\nu} F^{\mu\nu}$, in a spatially flat Friedman–Lemaitre–Robertson–Walker (FLRW) spacetime. This background is conformally Minkowskian and, since the electromagnetic field equations are conformally invariant, the quantum theory shares multiple properties with the Minkowski spacetime formulation. In particular, both theories have the same Hilbert space. This relation allows the definition of a preferred vacuum state in FLRW backgrounds (the so-called conformal vacuum), and also implies the absence of particle (i.e. photon) creation by the expanding spacetime, in sharp contrast with other non-conformally invariant fields [2]. However, the presence of a non-trivial spacetime curvature manifests itself in an important way in the process of renormalization. Although there exist a preferred vacuum, the normal-order prescription is not a satisfactory renormalization prescription in FLRW. This is because that procedure for subtracting the ultraviolet divergences is
neither generally covariant nor local. Therefore, out of Minkowski spacetime, normal-order does not satisfy the axioms on which the theory of renormalization in curved spacetime relies \[3\]. Instead, we will use the adiabatic renormalization method developed by Parker and Fulling \[4, 5\] —which has been proven to be equivalent to DeWitt-Schwinger point-splitting renormalization \[6, 7\]— adapted to the electromagnetic field. (See also \[8, 9\] for the extension to fermionic fields.) We will see that the quantity

\[ \Delta_{\mu\nu} \equiv \langle F_{\mu\alpha}(x) F_{\nu}^{\alpha}(x) \rangle - \langle * F_{\mu\alpha}(x) * F_{\nu}^{\alpha}(x) \rangle \]  

(2)

takes a non-vanishing value given (we use the same geometric conventions as in Refs. \[2, 4\]) by

\[ \Delta_{\mu\nu} = \frac{1}{480\pi^2} \left( -\frac{9}{2} R_{\alpha\beta} R^{\alpha\beta} + \frac{23}{12} R^2 + 2\Box R \right) g_{\mu\nu}, \]

(3)

where \( R_{\alpha\beta} \) is the Ricci tensor and \( R \) its trace. This expression implies that the fluctuations of the electric and magnetic field in the vacuum state—which is duality-invariant in FLRW—are different, i.e. \( \langle \vec{E}^2 \rangle \neq \langle \vec{B}^2 \rangle \), and therefore the duality-symmetry is broken.

We analyze the same issue in the case of a massless, minimally coupled scalar field in an arbitrary \( 1 + 1 \) dimensional space-time. We use the Hadamard renormalization method and reach with similar conclusions: the presence of a classical gravitational background breaks not only conformal invariance, but also the duality symmetry.

**Sketch of the calculation for the electromagnetic field.** The goal of this section is to compute the vacuum expectation values \( \langle F_{\mu\alpha}(x) F_{\nu}^{\alpha}(x) \rangle, \langle * F_{\mu\alpha}(x) * F_{\nu}^{\alpha}(x) \rangle \), and the energy-momentum tensor in a spatially flat FLRW background with line element \( ds^2 = a(\eta)^2 (d\eta^2 - d\vec{x}^2) \), where \( \eta \) is the conformal time. All tensor components in this section will refer to the coordinates \( \eta, \vec{x} \). As pointed out above, the conformal invariance of the equations of motion greatly facilitates the formulation of the theory. The electromagnetic field operator can be written in terms of the vector potential as \( F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \), where the operator \( A_\mu \) can be represented in terms of Fourier modes of the two physical polarizations (we work in the Lorenz gauge \( \nabla^\mu A_\mu = 0 \)) by

\[ A_\mu(x, \eta) = \int \frac{dk}{(2\pi)^3} \sum_{\alpha=1}^2 \hat{a}_k^{(\alpha)}(\eta) \varphi_k(\eta) e^{i\vec{k} \cdot \vec{x} + \text{h.c.}}. \]

(4)

where \( \varphi_k(\eta) = e^{-ik\eta} / \sqrt{2k} \), \( k = |\vec{k}| \) is the length of the comoving mode \( \vec{k} \), and \( \hat{a}_k^{(\alpha)} \) and \( \hat{a}_k^{(\alpha)\dagger} \) are creation and annihilation operators for the polarization \( \alpha \). The polarization vectors \( e_{\mu}^{(\alpha)}(\vec{k}) \) depend on \( \vec{k} \) and are transversal, \( k^\mu e_{\mu}^{(\alpha)}(\vec{k}) = 0 \). It is convenient to choose them to be mutually orthogonal, \( g^{\mu\nu} e_{\mu}^{(\alpha)} e_{\nu}^{(\alpha')} = a^{-2} \delta^{\alpha\alpha'} \).

Direct substitution shows that the quantity \( \langle F_{\mu\alpha} F_{\nu}^{\alpha} \rangle \) is ultraviolet-divergent, and therefore requires renormalization. In adiabatic renormalization the physically relevant, finite expression is obtained by subtracting mode-by-mode, i.e. under the Fourier integral sign, terms that would produce state-independent ultraviolet divergences. The terms to be subtracted are identified by performing a Liouville or WKB-type asymptotic expansion for large values of the physical frequency of the Fourier modes \( \omega(k) \), or, mathematically equivalently, an expansion for small values of the time derivatives of the scale factor \( a(\eta) \) (this is the reason for the name adiabatic, although the method is primarily concerned with ultraviolet issues). See \[2\] for further details.

As in the DeWitt-Schwinger renormalization method, to find the subtraction terms one has to first temporarily introduce a mass in the theory, then take the limit \( m \to 0 \) at the end of the calculation. This mass introduces a third polarization for the adiabatic modes. So the adiabatic expansion of the vector potential \( A_\mu^{(\text{Ad})} \) contains two transverse and one longitudinal polarizations. The transverse ones take the same form as in equation \[4\], but now the modes \( \varphi_k^{(\text{Ad})}(\eta) \) satisfy the equation \( \partial_k^2 \varphi_k^{(\text{Ad})} + \omega(k, \eta)^2 \varphi_k^{(\text{Ad})} = 0 \), with \( \omega(k, \eta) = k^2 + m^2 a(\eta)^2 \). The longitudinal polarization can be chosen as \( e_\mu^{(\text{Ad})}(\eta) \), where \( e_\mu^{(\text{Ad})} \) has components \( \langle f(k, \eta), \vec{k} \rangle \), with \( f(k, \eta) = -i k^\mu / (\omega a) \), and the re-scaled mode \( \psi_k(\eta) = \xi_m \chi_k^{(\text{Ad})}(\eta) \) satisfies the equation

\[ \psi_{k''} + 2 \frac{\omega''}{a} \psi_k' + \left( \frac{\omega''}{\omega} + 2 \frac{\omega'}{a} \omega + \omega^2 \right) \psi_k = 0. \]

(5)

The adiabatic expansion of both the transversal and longitudinal polarizations up to fourth adiabatic order provides the subtraction terms needed to renormalize the expectation values we are looking for. Note that one must include terms of up to fourth adiabatic order in the subtractions because this is the order at which divergences appear for general values of the mass \( m \). In the \( m \to 0 \) limit there are no divergences at fourth order, but one still must apply the general prescription. A lengthy calculation produces

\[ \langle F_{\mu\alpha} F_{\nu}^{\alpha} \rangle = \theta_{\mu\nu} + \frac{1}{4} \gamma(\eta) g_{\mu\nu} + t_{\mu\nu}, \]

(6)

\[ \langle * F_{\mu\alpha} * F_{\nu}^{\alpha} \rangle = \theta_{\mu\nu} - \frac{1}{4} \gamma(\eta) g_{\mu\nu} + *t_{\mu\nu}. \]

(7)

In these expressions \( t_{\mu\nu} \) and \( * t_{\mu\nu} \) are traceless tensors encoding all the information regarding the quantum state, and both vanish for the conformal vacuum. \( \theta_{\mu\nu} \) is a traceless, local geometric tensor given by

\[ \theta_{\mu\nu} = \frac{1}{480\pi^2} \left[ -\frac{16}{3} R_{\mu\alpha} R_{\nu}^{\alpha} + \frac{61}{18} R R_{\mu\nu} + \frac{23}{9} \Box R_{\mu\mu} \right] + \frac{4}{3} R_{\alpha\beta} R^{\alpha\beta} g_{\mu\nu} - \frac{61}{72} R^2 g_{\mu\nu} + \frac{1}{6} \Box R g_{\mu\nu}. \]

(8)
and $\gamma(\eta) = \frac{1}{3} [−9R_{\alpha\beta}R^{\alpha\beta} + 2R^2 + 4\Box R]$. Applying equations (6) and (7) for the vacuum state, one obtains $\Delta_{\mu\nu} = 1/2\gamma(\eta)g_{\mu\nu}$, as anticipated in equations (2) and (3). This quantity is different from zero for a generic scale factor $a(\eta)$. Also note that taking the trace of equation (6) one obtains $\langle F^2 \rangle = \langle F_{\mu\alpha}^* F_{\mu\alpha} \rangle = \langle \bar{E}^2 \rangle - \langle \bar{B}^2 \rangle = \gamma(\eta)$. Since the vacuum state is duality-invariant, these results indicate a breakdown of duality.

As in Minkowski spacetime, in FLRW there also exists a unitary operator implementing the duality transformation in the representation of the (linear) Heisenberg algebra of field operators. However, the previous result indicates that the renormalized expectation values of composite (non-linear) operators do not transform as expected under this unitary operator. The geometric quantities—curvature tensors—involved in the renormalization procedure break the duality symmetry.

We finish this section by providing the expression for the renormalized energy-momentum tensor. The aim is to show that our techniques are consistent with well-known results. From expression (11) the vacuum expectation value of the Maxwell tensor is

$$\langle T_{\mu\nu}^M(x) \rangle = -\frac{1}{2} \left( \langle F_{\mu\alpha}F_{\nu}^{\alpha} \rangle + \langle F_{\mu\alpha}F_{\nu}^{\alpha} \rangle \right) = -\theta_{\mu\nu}. \quad (9)$$

By construction, this tensor is traceless. However, $\langle T_{\mu\nu}^M(x) \rangle$ is not a suitable candidate for the source of the gravitational field, i.e., for the right hand side of the semi-classical Einstein equations $G_{\mu\nu} = -8\pi G \langle T_{\mu\nu} \rangle$, since $\langle T_{\mu\nu}^M(x) \rangle$ is not conserved, $\nabla^{\nu} \langle T_{\mu\nu}^M(x) \rangle \neq 0$. Explicit computations show that

$$\nabla^{\nu} \langle T_{\mu\nu}^M \rangle = -\frac{1}{2(4\pi)^2} [\nabla^{\nu} v_{\mu\nu} - \frac{3}{4} \nabla_{\nu} v_{\mu} + \nabla_{\nu} v] \quad (10)$$

where $v_{\mu\nu}$ and $v$ are objects constructed from curvature tensors:

$$v_{\mu\nu} = \frac{1}{3} R_{\mu\alpha}R^{\alpha\nu} - \frac{3}{10} RR_{\mu\nu} - \frac{1}{45} \nabla_{\nu} \nabla_{\mu} R \quad (11)$$

$$+ \frac{1}{180} R_{\alpha\beta}R^{\alpha\beta}g_{\mu\nu} + \frac{113}{2160} R^2 g_{\mu\nu} - \frac{1}{360} \Box R g_{\mu\nu},$$

$$v = \frac{13}{1080} R^2 + \frac{1}{30} \Box R + \frac{1}{180} R^{\alpha\beta} R_{\alpha\beta}. \quad (12)$$

One can construct a suitable conserved energy-momentum tensor from $\langle T_{\mu\nu}^M(x) \rangle$ in two different ways. The shortest one is to use the procedure commonly employed in Hadamard renormalization $[3, 11, 13]$. It consists in simply adding to $\langle T_{\mu\nu}^M \rangle$ a geometric tensor that makes it conserved. From equation (10) we find that a solution is

$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu}^M \rangle + \langle T_{\nu\mu}^{Ad} \rangle + c_1 H_{\mu\nu}^{(1)}. \quad (13)$$

where $T_{\mu\nu}^{Ad} = \frac{1}{2(4\pi)^2} [\nabla^{\nu} v_{\mu\nu} + (\nabla^{\nu} v_{\mu}^{\nu} + \nabla^{\nu} v) g_{\mu\nu}]$. Of course, this method can only define $\langle T_{\mu\nu} \rangle$ up to a conserved tensor. In FLRW this ambiguity is all encoded in the last term of the previous equation, where $c_1$ is an arbitrary real number and $H_{\mu\nu}^{(1)}$ is the tensor obtained by functional variation of $\sqrt{-g} R^2$ with respect to the metric; therefore it is conserved, $\nabla_{\mu} H_{\mu\nu}^{(1)} = 0$. Note that the freedom in the value of $c_1$ in (13) coincides with the well-understood ambiguity in the renormalized energy-momentum tensor in curved spacetimes $[3, 11]$.

Another way of finding $\langle T_{\mu\nu} \rangle$ in adiabatic renormalization is by direct application of the method. But to follow this route one has to deal carefully with the gauge invariance. A convenient approach in curved backgrounds is provided by the Faddeev–Popov scheme (see e.g. [4]). This method introduces new contributions to the energy-momentum tensor, namely the so-called gauge breaking terms and the contribution of a ghost field. Explicit computations produce results that agree with (13).

From (13) it is easy to check that the trace of the renormalized energy-momentum tensor is nonzero and is given by $\langle T^{\mu}_{\mu} \rangle = \frac{1}{2(4\pi)^2} [-62(\Box R^2 - \frac{1}{2} R^2) - (2 + 6 \times 2880 \pi^2 c_1) \Box R]$. This is the well-known conformal or trace anomaly. Any other renormalization method would provide an expression for $\langle T^{\mu}_{\mu} \rangle$ that would possibly differ from (13) in the value of the coefficient $c_1$. Note that the existence of the anomalous trace does not imply the duality anomaly. The trace arises from the geometric term $\langle T_{\mu\nu}^{Ad} \rangle$ in (13), while the duality anomaly appears already in the expectation values (6) and (7).

Results in de Sitter universe and related comments. For the de Sitter–FLRW solution ($a(t) = e^{-Ht}$ in cosmic time and $a(\eta) = -1/(H\eta)$ in conformal time, with $dt = a d\eta$) the conformal vacuum — also called the Bunch–Davies vacuum — is de Sitter invariant. Evaluation of equation (11) produces $\langle T_{\mu\nu} \rangle = 0$. This result is expected from symmetry arguments, since there are no two-covariant tensors which are simultaneously de Sitter invariant and traceless. If one (incorrectly) assumes, following the standard lore, the validity of electric-magnetic duality (see e.g. [12, 14]), one would conclude that $\langle B^2 \rangle = 0 = \langle E^2 \rangle$ in this spacetime (14). However, particularizing eqns. (7) and (9) to de Sitter space one obtains, instead, $\langle B^2 \rangle = \frac{19}{160} \pi^2 H^4$ — in agreement with (13) — and $\langle E^2 \rangle = -\langle B^2 \rangle$. The negative value of the quadratic quantity $\langle E^2 \rangle$ should not be surprising since that is common for renormalized quantities. The same happens in the usual Casimir effect (see e.g. [16]).

Duality anomaly in a 2D conformal scalar theory. The duality anomaly in curved spacetimes can also be illustrated in a simpler scenario: a minimally coupled, massless scalar field in 1 + 1 dimensions. This theory is very similar to free electromagnetism in the sense that it can be described by an abelian 1-form $F_{\mu\nu}$ [17], and classically it shows both conformal and duality invariance. This framework has been extensively discussed in the context of conformal field theory and string theory. The classical stress-energy tensor can be expressed as $T_{\mu\nu} = \frac{1}{2} (F_{\mu} F_{\nu} + F_{\nu} F_{\mu})$, where $* F_{\mu} = |g|^{1/2} \epsilon_{\mu\nu} F^\nu$ is the dual
of $F_\mu$. The classical field equations are $\nabla^\mu F_\mu = 0$ and $\nabla^\mu \ast F_\mu = 0$, where the scalar field $\phi$ plays the role of the potential of the field $F_\mu$. $F_\mu = \nabla_\mu \phi$. The classical equations are invariant under both conformal and duality transformation $F_\mu \to \ast F_\mu$. In this section we consider an arbitrary spacetime metric (not necessarily homogeneous), which can always be written as $ds^2 = e^{2\rho}dx^+ dx^-$, in term of the null coordinates $x^\pm \equiv t \pm x$. Because the spacetime is not necessarily homogeneous, we cannot use adiabatic regularization. We will use instead the Hadamard point-splitting method [3, 12, 13], which gives us the chance to show the existence of the duality anomaly for a different renormalization prescription. In this theory there is once again a preferred vacuum state, the conformal vacuum. This state is dual-invariant, and so are the vacuum-correlation functions: $\langle F_\pm(x) F_\pm(x') \rangle = \frac{1}{16\pi} (x^\pm - x'^\pm)^{-2} = \langle \ast F_\pm(x) \ast F_\pm(x') \rangle$ and $\langle F_+(x) F_-(x') \rangle = 0 = \langle \ast F_+(x) \ast F_-(x') \rangle$, for $x \neq x'$. However, for $x = x'$ the subtractions required for renormalization are no longer dual-invariant. These subtractions are obtained from the singular part of the Hadamard two-point function, $1/4\pi V(x,x') \ln |\sigma(x,x')|$, where $2\sigma(x,x')$ is the square of the geodesic distance between $x$ and $x'$ and $V$ is a geometric biscalar [13]. We obtain

$$\langle F_\mu(x) F_\nu(x) \rangle = \tilde{\theta}_{\mu\nu} + \frac{1}{4\pi} \tilde{\gamma} g_{\mu\nu},$$

(14)

$$\langle \ast F_\mu(x) \ast F_\nu(x) \rangle = \tilde{\theta}_{\mu\nu} - \frac{1}{4\pi} \tilde{\gamma} g_{\mu\nu},$$

(15)

where $\tilde{\theta}_{\mu\nu}$ is a traceless tensor with components

$$\tilde{\theta}_{\pm\pm} = -1/12\pi[(\partial_{\pm}\rho)^2 - \partial^2_{\pm}\rho], \quad \tilde{\theta}_{+-} = 0,$$

and $\tilde{\gamma} = 5/(12\pi) R$. Therefore, $\tilde{\Delta}_{\mu\nu} \equiv \langle F_\mu(x) F_\nu(x) \rangle - \langle \ast F_\mu(x) \ast F_\nu(x) \rangle = 1/2 \tilde{\gamma} g_{\mu\nu}$. From (14) one can also obtain the vacuum expectation value of the energy-momentum tensor following the procedure summarized for the electromagnetic case. Taking into account that $\nabla^\mu \tilde{\theta}_{\mu\nu} = \frac{1}{4\pi} \nabla_{\nu} \nabla^\rho R$, one obtains $\langle T_{\mu\nu} \rangle = \frac{1}{4\pi} \tilde{\gamma} g_{\mu\nu}$. It is well known that for $x \neq x'$ the correlation function $\langle \partial_+ \phi(x) \partial_- \phi(x') \rangle$ vanishes, as mentioned before, which is commonly referred to as the decoupling of left- and right-moving modes. A consequence of the duality anomaly is that this is no longer true for $x = x'$. Rather, equation (14) provides $\langle \partial_+ \phi \partial_- \phi \rangle = \frac{1}{4\pi} \tilde{\gamma} e^{2\rho} = \frac{5}{12\pi} \partial_{+} \partial_{-} \rho$.

**Conclusions and final comments.** QFT is intrinsically more involved than a quantum-mechanical system having a finite number of degrees of freedom. This difference arose in the early stages of quantum electrodynamics due to the emergence of divergent expressions in physical quantities. It was nicely solved with the renormalization program, which has provided many important and surprising results. In particular, when applied in the presence of a classical gravitational background, renormalization has been shown to break some of the important symmetries of the theory under consideration. The chiral current anomaly for free massless fermions or the conformal anomaly are examples with important physical consequences. In this paper we have proven that the duality symmetry cannot hold in QFT in arbitrarily curved spacetimes. We have shown this explicitly with some of the most common renormalization methods. However, it could still be possible to build a renormalization scheme for which the symmetry is preserved. Even in the case such a method exists, which we believe is unlikely, it would be highly unnatural or fine-tuned.

Phenomenologically, although the duality is an exact physical symmetry of the classical theory only in the absence of charges, it still plays an important role in certain situations in which charge density is negligible. This happens, for instance, during cosmic inflation. At the conceptual level, electric–magnetic duality has been the focus of several theoretical developments, and an important ingredient in different scenarios, like in the Montonen–Olive dualities [18] in non-abelian gauge and supersymmetric theories. Therefore, the duality anomaly presented in this paper may have interesting physical and theoretical consequences which merit further exploration.

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