Lifting the bandwidth limit of optical homodyne measurement with broadband parametric amplification

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Homodyne measurement is a cornerstone method of quantum optics that measures the quadratures of light—the quantum optical analog of the canonical position and momentum. Standard homodyne, however, suffers from a severe bandwidth limitation: while the bandwidth of optical states can span many THz, standard homodyne is inherently limited to the electronically accessible MHz-to-GHz range, leaving a dramatic gap between relevant optical phenomena and the measurement capability. We demonstrate a fully parallel optical homodyne measurement across an arbitrary optical bandwidth, effectively lifting this bandwidth limitation completely. Using optical parametric amplification, which amplifies one quadrature while attenuating the other, we measure quadrature squeezing of 1.7 dB simultaneously across 55 THz, using the pump as the only local oscillator. As opposed to standard homodyne, our measurement is robust to detection inefficiency, and was obtained with >50% detection loss. Broadband parametric homodyne opens a wide window for parallel processing of quantum information.

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The standard representation of a nearly monochromatic field is either as a complex amplitude \( a = |a|e^{i\phi} \) to reflect the amplitude and phase of the field oscillation \( E(t) = ae^{-i\phi} + a^*e^{i\phi} = |a|(\cos(\Omega t + \phi)) \) (\( \Omega \) the carrier frequency), or as a superposition of two quadrature oscillations \( E(t) = x\cos\Omega t + y\sin\Omega t, \) where \( x = a + a^* \) and \( y = (a - a^*)/2i \) are the real quadrature amplitudes of the cosine-wave and sine-wave components. While the quadrature representation may be just a mathematical convenience in classical electromagnetism, it is of fundamental importance in quantum optics. The two quadrature operators \( x = a + a^* \) and \( y = (a - a^*)/2i \) form a conjugate pair of non-commuting observables \( [x, y] = 2i \) analogous to position and momentum in mechanics, indicating that their fluctuations are related by quantum uncertainty \( \Delta x \Delta y \geq 1 \). This conjugation is most emphasized with quantum squeezed light, where the quantum uncertainty of one quadrature amplitude is reduced (squeezed), while the uncertainty of the other is inevitably increased (stretched), that is, \( \Delta x < 1 < \Delta y \).

Homodyne measurement, which extracts the quadrature information of the field, forms the backbone of coherent detection in physics and engineering, and plays a central role in quantum information processing, from measuring non-classical squeezing, through quantum state tomography, generation of non-classical states, quantum teleportation, quantum key distribution, and quantum computing. To measure the field quadratures, homodyne detection compares the optical signal to a strong and coherent quadrature reference (local oscillator—LO), where the specific quadrature axis to be measured is selected by tuning the phase of the LO. Hence, the heart of a homodyne detector encompasses an external LO and a field multiplier. This is most evident for homodyne measurement in the radio-frequency (RF) domain, where the input radio-wave and the LO are directly multiplied using an RF frequency mixer. In optics, however, direct frequency mixers do not exist. Instead, standard optical homodyne relies on a beam splitter to superpose the optical input and the LO (see Fig. 1a) and on the nonlinear electrical response of square-law photo-d Detectors as the field multipliers that generate an electronic signal proportional to the measured \( x \) or \( y \) quadrature. Thus, measuring quadratures with standard homodyne is inherently limited to the electronic bandwidth of the photo-detectors (MHz-to-GHz). In addition, homodyne detection is highly sensitive to the noise level and quantum efficiency of the detectors, which leads to decoherence due to the addition of vacuum noise.

Yet, optical states of light can easily span optical bandwidths of 10–100 THz and more, where the quadratures \( x(t), y(t) \) vary rapidly on a time scale comparable to the optical cycle (\( E(t) = x(t)\cos(\Omega t + \phi(t)) \)). Thus, the detection method enforces an inherent distinction between nearly monochromatic and broadband fields. In the near monochromatic case, the instantaneous quadrature amplitudes vary slowly over millions of optical cycles, and can be directly observed from the time-dependent electronic signal of the homodyne output. For broadband light, however, photo-detectors are too slow to follow the quadrature variations, demanding an inherently different measurement approach.

Two examples can illuminate both the potential utility of broad bandwidth in quantum information and the difficulty of standard methods to exploit it. One example is one-way quantum computation with a quantum computer, which forms the most promising realization of scalable quantum information to date. This approach exploits the large bandwidth of frequency mode pairs from a single parametric oscillator (two-mode squeezed vacuum) as a set of quantum modes (Q-modes), where coupling among near Q-modes demonstrated the largest entangled cluster states to date along with a complete set of quantum gate operations. The number of parallel Q-modes is dictated by the squeezing bandwidth of the parametric oscillator, which can extend up to a full optical octave by rather simple means (limited only by phase matching of the nonlinear interaction).

Assuming a squeezing bandwidth of 10–100 THz, the number of simultaneous Q-modes can easily exceed 10^5. The limitation of this approach to quantum computation is the bandwidth of the measurement, where each Q-mode requires a separate homodyne detection using a precise pair of phase-correlated LOs. A broad bandwidth of Q-modes requires a dense set of correlated LOs and multiple homodyne measurements, quickly multiplying the complexity to impracticality. In our experiment, we simultaneously measure the entire bandwidth of a broadband two-mode squeezed vacuum with only one LO—the pump field that generates the squeezed light to begin with.

Another example is in quantum key distribution, where enhanced bandwidth was employed to increase the data rate by increasing the number of bits per photon. The concept here is to divide the photon readout time, which is limited by photodetectors, into multiple short time-bins, which act as an additional time stamp for each photon (or pair). The time stamp (bin), which is usually detected using a Pranson interferometer, enhances the number of bits per photon to \( \log_2 N \), where \( N \) is the number of time-bins. Theoretically, if the bandwidth limit of the detector could be lifted, all time (or frequency) bins could be detected independently, and a \( \times N \) higher flux of photons could be used, allowing full parallelization of the communication across the available bandwidth and enhancement of the data throughput by a larger factor \( N \) (compared to \( \log_2 N \)).

Here we present a different approach to optical homodyne, which resorts to a broadband optical nonlinearity—parametric amplification, as the field multiplier. Using this method we measure the entire bandwidth simultaneously with a single homodyne device and a single LO. Specifically, since parametric gain only amplifies one quadrature of the input signal but attenuates the other, analysis of the output spectrum enables evaluation of the input quadratures. Due to the parametric amplification of the quadrature of interest, our measurement is insensitive to detection inefficiency (and to the added vacuum noise it introduces). Indeed, our observation of broadband squeezing was easily obtained with >50% loss in the detection channel. With sufficient parametric gain, any given x quadrature can be amplified to overwhelm the attenuated orthogonal y quadrature, even if it was originally squeezed, such that the resulting output signal is practically proportional only to the input x quadrature. Even if the parametric gain in the measurement is not high enough to completely diminish the y quadrature, measurement is simple, once the desired x quadrature is sufficiently enhanced above the vacuum level. Specifically, two orthogonal measurements, one for each quadrature, enable extraction of both quadratures (average) over the entire optical bandwidth, as detailed hereon.

**Results**

**Experiment.** The basic concept of our method for broadband homodyne detection is illustrated in Fig. 1, showing in Fig. 1a the standard homodyne method and in Fig. 1b the parametric homodyne detection as realized by a broadband parametric amplifier acting on the quadratures of the light. To describe the effect of the parametric amplifier in Fig. 1b, we use the common expression for the optical field at the output of a parametric amplifier (based on either three-wave or four-wave mixing (FWM)) that generates a quadrature reference signal: assuming \( a_{in} = a_{in}\cos(g) + a_{in}\sinh(g) = x_{in}e^{i\phi} + y_{in}e^{-i\phi} \), where \( a_{in} \) are the input field operators, and \( x_{in}, y_{in} \) are the input quadratures, and \( g \) is the parametric gain. Hence, the parametric amplification amplifies one input quadrature (\( x_{in}e^{i\phi} \))...
while attenuating the other \( \langle y_\mu e^{-i\delta} \rangle \), indicating that for sufficient amplification, the output field reflects one quadrature of the input primarily without adding noise to the measured quadrature, thus offering a quadrature selective quantum measurement. The amplification process responds instantaneously to time variations of the quadrature amplitudes \( x(t) \), \( y(t) \) and the amplification bandwidth is limited only by the phase matching conditions in the nonlinear medium, which can easily span an optical bandwidth of 10–100 THz (implications of the time dependence are deferred to a later discussion). In our experiment, we measured the spectrally resolved intensity of the chosen input quadrature \( x^\dagger(\omega)x(\omega) \) simultaneously across the entire bandwidth by detecting the output spectrum of a parametric amplifier with an input of broadband squeezed vacuum.

We note that the parametric amplifier used in the measurement need not be ideal. Specifically, since the attenuated quadrature is not measured, it is not necessarily required to be squeezed below vacuum, only to be sufficiently suppressed compared to the amplified quadrature. Consequently, restrictions on the measurement amplifier are considerably relaxed compared to sources of squeezed light, allowing it to operate with much higher gain.

The common source for squeezed light or squeezed vacuum, in our experiment, is also a parametric amplifier. If the amplification is spontaneous (vacuum input), the amplifier attenuates one of the quadratures of the vacuum input state, squeezing its quantum uncertainty. For measuring the squeezing, we exploit the same nonlinearity and the same pump that generates the squeezed state—spontaneous (vacuum input), the amplifier attenuates one of the quadratures of the vacuum input state, squeezing its quantum uncertainty. For measuring the squeezing, we exploit the same nonlinearity and the same pump that generates the squeezed state. Hence, the quadrature information over a broad frequency range is obtained simultaneously by measuring the spectrum of the light at the output of the detection parametric amplifier. With a single LO—the pump, each individual frequency component is measured independently, and the number of accessible Q-modes (or Q-bits) that could be utilized simultaneously would be multiplied by \( N \).
Fig. 2 Experimental schematic of the parametric homodyne. The experiment consists of two parts: (1) generation of broadband squeezed light and (2) homodyne measurement of the generated squeezing. Broadband two-mode squeezed light is generated via spontaneous four-wave mixing (FWM) in a photonic crystal fiber (PCF) pumped by 12 ps laser pulses (786 nm). After generation, the pump is replaced by an appropriately delayed copy of the original pump light, via a narrowband filter, which allows independent intensity and phase control, to tune the parametric gain and to select the specific quadrature to be measured. Then, the new pump and the FWM light enter the second PCF for the homodyne measurement. After this second (measurement) pass through the amplifier, the pump is separated from the FWM light by a narrowband filter and the FWM light is measured by a spectrometer.

The observed squeezing in our experiment is far from ideal, primarily due to the fact that the pump is pulsed, which induces an undesirable time dependence of both the magnitude and phase of the parametric gain in the squeezing process, as well as in the parametric homodyne detection via self-phase and cross-phase modulation—SPM and XPM. Since our pump pulses are relatively long, their time dependence can be regarded as adiabatic, indicating that the instantaneous squeezing (source) and parametric amplification (measurement) are ideal, but the quadrature axis, squeezing level, and gain of the two amplifiers vary with time, not necessarily at the same rate. Thus, the measured spectrum, which represents a temporal average of the light intensity over the entire pulse, diminishes somewhat the expected squeezing (see illustration in the Methods).

Even with a pulsed pump, however, the various homodyne and calibration measurements are consistent and unequivocal for weak enough pump intensity (see Methods for further details on the pulse-averaging effects). With a pure CW pump, as is generally used in squeezing applications, this pulse-averaging limitation would not exist. Another limitation in our measurements is the need to re-couple the FWM back into the PCF, which introduces an inevitable loss of 30% and reduces the observed squeezing. This “known” loss can either be avoided completely in other experimental configurations or can be calibrated out to estimate the “bare” squeezing level of the measured light source (see Methods).

We verified the properties of the parametric homodyne detection in several ways. We measured the squeezed quadrature $(\langle x^2 \rangle,\langle y^2 \rangle)$, and the uncertainty area, $(\langle x^2 \rangle \times \langle y^2 \rangle)$, of the squeezed state. Ideally, the generated squeezed light should be a minimum uncertainly state of $(\langle x^2 \rangle \times \langle y^2 \rangle) = 1$, independent of the generation gain; and the average intensity of the squeezed quadrature should exponentially decrease with the gain. The results are presented in the Methods, showing a clear reduction of the normalized squeezed quadrature intensity down to $(\langle x^2 \rangle) \approx 0.68$ (32% below the vacuum level), and the uncertainty area remains nearly ideal at $(\langle x^2 \rangle \times \langle y^2 \rangle) < 1.3$, up to a pump power of...
60 mW. Further increase of the pump does not improve the measured squeezing due to pulse effects, and the minimum uncertainty property deteriorates. Based on the measured squeezing, the instantaneous squeezed quadrature at the peak of the pulse was estimated to be >3 dB (see Methods). Additional verification measurements of the broadband squeezing are presented and illustrated in the Methods.

**Two-mode quadratures.** The fundamental quadrature oscillation—a single-frequency component of a quadrature amplitude $x(\omega)$, $y(\omega)$, is a two-mode combination of frequencies $\omega_s = \Omega + \omega$ and $\omega_i = \Omega - \omega$—the signal and idler. Using the field operators of the signal $a_s = a(\omega)$ and the idler $a_i = a(-\omega)$, the quantum operators of the quadratures $x(\omega)$, $y(\omega)$ are (see Methods for an intuitive reasoning)

$$
\begin{align*}
  x(\omega) &= a_s + a_i^\dagger \\
  y(\omega) &= i(a_i^\dagger - a_s)
\end{align*}
$$

This definition preserves the commutation relation $[x, y] = 2i$ and reduces in the monochromatic case to the single-mode quadratures $x = a + a^\dagger$, $y = i(a^\dagger - a)$.

The generalization of the standard quadratures to two-mode quadratures requires some attention. As opposed to the standard quadrature operators, which are hermitian and represent the time-independent real amplitude of the cosine (sine) oscillation, the two-mode quadrature operators of Eq. (1) are non-hermitian $x^\dagger(\omega) \neq x(\omega)$ and represent a time-dependent beat between the signal and idler modes with an envelope frequency $\omega_s$ carried by a cosine (sine) wave at frequency $\Omega$ (see Methods for some intuitive reasoning). The quadrature operators $x(\omega)$, $x^\dagger(\omega)$ represent the beat envelope, which has an amplitude and phase, in some similarity to the field operators $a, a^\dagger$ that represent the amplitude and phase of the carrier oscillation. Yet, the two-mode quadrature $x$ is an observable quantity (in contrast to the field operator $a$). Since $x$ commutes with its conjugate $[x, x^\dagger] = 0$ (as opposed to $[a, a^\dagger] = 1$), it is possible to simultaneously measure both the real and imaginary part of the quadrature envelope, and thereby obtain complete information on both amplitude and phase of the single quadrature:

$$
\begin{align*}
  \text{Re}[x] &= x + x^\dagger = X_s + X_i \\
  \text{Im}[x] &= i(x - x^\dagger) = Y_s - Y_i
\end{align*}
$$

where $X_s, X_i, Y_s, Y_i$ are the standard single-mode quadratures of the signal and idler modes. Our experiment measured $x^\dagger(\omega)x(\omega)$.

Since the phase of the two-mode quadrature relates to commutating observables (as opposed to the carrier phase), it does not reflect a non-classical property of the quantum light field, but rather defines the classical temporal mode in which the field is measured. Specifically, the temporal mode of measurement is the two-frequency beat pattern of frequency $\omega_s$ (see Methods for an illustration), where the envelope phase defines the temporal offset of the beat. This offset, along with other mode parameters, such as polarization, spatial mode, carrier frequency, and so on define the mode of the LO. Of course, quantum entanglement is possible between the two envelope modes (cosine or sine) in direct equivalence to entanglement of a single photon (or photon pair, or cat state) between polarization modes, which is widely used for quantum information. However, this “quantumness” between modes is different and additional to the intra-mode quantum state, which is described by the quadratures $x, y$.

Due to the bandwidth limitation of standard homodyne measurement, the commonly used expression to interpret two-mode quadratures of optical frequency separation $\omega$ does not rely

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![Figure 3](image URL) The procedure of parametric homodyne. Measurement of the quadratures includes three stages: **a** raw output measurement, **b** calibration, and **c** quadrature extraction. **a** Raw output measurements: In the most general case of arbitrary parametric gain, two measurements are needed to extract the quadrature information: (1) amplifying one quadrature (black); and (2) amplifying the orthogonal quadrature (purple). The specific quadrature to be amplified is defined by tuning the pump phase. The reduction of the raw output beneath the vacuum input level (dashed green) directly indicates squeezing. The inset shows the effect of loss at the input FWM light on the parametrically amplified output. As the loss is increased, the squeezing is reduced and the observed fringe minima rise towards the vacuum level (vertical arrows) even though the total input intensity is considerably decreased (a non-classical signature). **b** Calibration: To calibrate the parametric amplifier, the output response is measured for a set of three known inputs: (1) idler-input only (blocked signal, $I_s$—solid blue), (2) vacuum input (blocked entire FWM—both signal and idler, $I_v$—dashed green), and (3) zero amplification (blocked pump, $I_p$—dotted red). **c** Extracted quadratures (black and purple in accordance with **a**) with the analysis detailed in the Methods, quadrature information is extracted. Quadrature squeezing is evident across the entire 55 THz spectrum down to $\langle x^\dagger x \rangle \approx 0.68$, 32% below the vacuum level (~1.7 dB).
on Eq. (1), but rather on Eq. (2). Two independent homodyne measurements of the signal and idler quadratures, $x_{s,i}$, $y_{s,i}$ need to be made relative to two correlated LOs at their respective frequencies $\omega_{s}, \omega_{i}$ so that the two output homodyne signals are within the electrical bandwidth. Thus, the standard procedure to measure just a single-frequency component of the two-mode quadrature $x(\omega)$ (and its squeezing) requires two separate homodyne measurements of the independent quadratures of both the signal and the idler using a pair of phase-correlated LOs$^{17,28}$. For a broadband spectrum, standard two-mode homodyne requires a dense set of correlated pairs of LOs for each frequency component of the measurement. As we have shown, however, in our experiment above, a single LO is sufficient to simultaneously extract a specific quadrature across the entire optical bandwidth, just as a single pump laser can simultaneously generate the entire bandwidth of quadrature squeezed mode pairs.

Quantum derivation of the parametric amplified output. To model quantum mechanically the parametric homodyne process, we derive an expression for the parametric output intensity (photon-number) operator of the signal (or idler) mode, $N_s(\omega) = a_s^\dagger(a_s + a_s^\dagger)$ ($g$ is the parametric gain) in terms of the input complex quadratures $x(\omega), y(\omega)$. Mathematically, our method relies on the similarity between the quadrature operators of interest (Eq. (1)) $x(\omega) = a_s + a_i, y(\omega) = a_s - a_i$ and the field operator at the output of a parametric amplifier:

$$a_s(\omega) = a_s \cosh(g) + e^{i\theta}a_i \sinh(g) \equiv C a_s + D a_i,$$

where the coefficients $C$ and $D$ are generally complex. Since field operators must fulfill $[a_s^\dagger(a_s + a_i)] = 1$, the two coefficients $C$ and $D$ must obey $|C|^2 - |D|^2 = 1$, which leads to the common description of $C= \cosh(g)$ and $D = e^{i\theta}\sinh(g)$. However, the attributed phase of the parametric process $\theta$, which is determined by the pump phase and the phase matching conditions in the nonlinear medium, can also be expressed explicitly, leaving the two coefficients $C, D$ real and positive (rather than complex), using

$$a_s(\omega, \theta) = (Ca_s e^{i\theta} + Da_i e^{-i\theta}) e^{i\theta},$$

Since the overall phase $\theta$ does not affect the photon-number calculations, we may discard it as $\theta = 0$. In this expression we account for the phase of the pump as a rotation of the input quadrature axis---$a_{s,i} \rightarrow a_{s,i} e^{i\theta}$. Accordingly, the rotated complex quadrature operators (Eq. (1)) become

$$x(\omega) = a_s e^{i\theta} + a_i e^{-i\theta} \text{ and } y(\omega) = i(a_s e^{-i\theta} - a_i e^{i\theta}).$$

Parametric amplification directly amplifies one quadrature of the input and attenuates the other, as evident by expressing the field operators $a_s(\omega)$ at the output using the quadrature operators $x,y$ of the input:

$$a_s(\omega) = \frac{C + D}{2} x + i \frac{C - D}{2} y = e^{i\theta} x + e^{-i\theta} y,$$

where the index $\theta$ was dropped for brevity. Finally, the parametric photon-number operator at the output is

$$N_s(\omega) = a_s^\dagger(a_s + a_s^\dagger) = \frac{N_s - N_i - 1}{2} + \left(\frac{C+1}{2}\right)^2 x^2 + \left(\frac{C-1}{2}\right)^2 y^2,$$

$$= \frac{N_s - N_i - 1}{2} + e^{2\theta} x^2 + e^{-2\theta} y^2,$$

where $N_s = a_s^\dagger a_s$ represent the input photon numbers (intensities) of the signal and idler. When access is available simultaneously to the intensities of both the signal and the idler, their sum of intensities provides the cleanest measurement of the quadrature intensities

$$N_s(\omega) + N_i(\omega) = \frac{e^{2\theta}}{2} x^2 + \frac{e^{-2\theta}}{2} y^2 - 1.$$  

Note that $N_s(\omega) - N_i(\omega) = N_s - N_i$ is a constant of the amplification, independent of the parametric gain.

With sufficient parametric gain, any given $x$ quadrature at the input can be amplified above the vacuum noise to a "classical" level, even if it was originally squeezed, which allows complete freedom in measurement since vacuum fluctuations are no longer the limiting noise. If the measurement gain considerably exceeds the generation gain, such that $e^{2\theta} x^2 \gg e^{-2\theta} y^2$, the amplified quadrature will dominate the intensity of the output light allowing to neglect the intensity of the attenuated orthogonal $y$ quadrature, and the measurement of the light intensity spectrum at the output will directly reflect (after calibration, see Methods) the single-shot value of the input quadrature intensity $x^2(\omega)x(\omega)$, just like the standard measurement of the electrical spectrum at the output of standard homodyne.

Although the concept of parametric homodyne is conveniently understood in the limit of large gain, where the quadrature of interest dominates the output light field, parametric homodyne is equally effective with almost any finite gain. When the measurement gain is not large enough and the attenuated quadrature cannot be neglected, the two quadrature intensities can be easily extracted using a pair of measurements; setting the pump phase to amplify one quadrature ($\theta = 0$) and then to amplify the other ($\theta = \pi/2$), as illustrated in Fig. 3. Indeed, the output intensity in this case will not directly reflect the quadrature intensity, but it still provides equivalent information about the quadrature at any finite gain, since two light intensity measurements along orthogonal axes uniquely infer the two quadrature intensities at any finite gain, indicating that the information content of a measurement of the output intensity is the same as that of the quadrature intensity. An analytic derivation of this equivalence is provided in the Methods.

Applicability to quantum tomography. Quantum state tomography is a major application of homodyne measurement. It allows reconstruction of an arbitrary quantum state (or its density matrix or Wigner function) from a set of quadrature measurements along varying quadrature axes$^7$. Unique reconstruction requires a complete measurement of the quadrature distribution function, which necessitates single-shot measurements of the instantaneous quadrature value, not just its average. Although both standard two-mode homodyne and parametric homodyne provide complete quadrature information in a single shot (in somewhat different ways), they still allow reconstruction of the quantum state under some assumptions. Hereon we review the different limitations of both methods and their implications to quantum tomography, leading to a conclusion that a combination of parametric homodyne followed by standard homodyne alleviates all the limitations and allows unambiguous reconstruction of arbitrary states.

Standard two-mode homodyne cannot provide a complete measurement of $x(\omega)$ in a single shot since standard homodyne is a destructive measurement. Specifically, observation of $\text{Re}[x(\omega)] = x_s + x_i$ requires a standard homodyne measurement of both frequency modes, which inevitably destroys the quantum state by photo-detection and prevents a consecutive measurement of $\text{Im}[x(\omega)] = Y_s - Y_i$. Splitting the state into two measurement channels is impossible since such a splitting will inevitably introduce additional vacuum noise. Thus, although $\text{Re}[x(\omega)]$ and $\text{Im}[x(\omega)]$ commute, standard two-mode homodyne can evaluate...
only one of them in a single shot. In analogy to light polarization, standard homodyne acts as an absorptive polarizer that detects one polarization but absorbs the other, preventing complete analysis of the polarization state.

Our current realization of parametric homodyne suffers from a different ambiguity in a single shot (envelope phase). Since parametric homodyne measures only the instantaneous intensity of the quadrature $x^\dagger x$ (across a wide spectrum), but not its phase, only the probability distribution of the intensity $P(x^\dagger x)$ can be measured.

Let us analyze the ambiguity that is introduced to the reconstruction of a two-mode quantum state by the incomplete measurement, for both standard homodyne (only real part) and parametric homodyne (only intensity). For standard homodyne, the interpretation of a null result is ambiguous: a zero value of the quadrature $x^\dagger x$ can only be measured for a null value of the envelope phase, fixed and known a priori.

For parametric homodyne, where the quadrature intensity is measured, null (or any intensity) is unambiguously interpreted for any envelope phase, but the sign of the measured quadrature is ambiguous. Thus, complete reconstruction is possible (for any envelope phase) only if the symmetry of the quadratures is known, which is relevant to a large set of important quantum contexts: already the seminal paper of Caves from 1981 that introduced squeezed vacuum to the unused port of an interferometer for sub-shot noise interferometric measurement,5 suggested to include a parametric amplifier in the detection arm to overcome the quantum inefficiency of photo-detectors.29 Leonhard and Paul30 later suggested a similar use of parametric amplification for quantum tomography that is insensitive to loss, Ralph31 suggested it for teleportation and Davis et al.32 for the analysis of atomic spin-squeezing. Most recently, this concept was experimentally implemented for atomic spin measurements in enabling phase detection down to 20 dB below the standard quantum limit with inefficient detectors.

Comparison to standard homodyne. It is illuminating to examine on equal footing standard homodyne measurement and the parametric homodyne method. After all, the balanced detection in standard homodyne produces a down-converted RF field at the difference-frequency of the two optical inputs (LO and signal), similar to optical down-conversion, which is the core of parametric amplification. In that view, the well-known homodyne gain of balanced detection (proportional to the LO field) produces an amplified electronic version of the input quantum quadrature, directly analogous to the parametric gain (proportional to the pump amplitude), which optically amplifies a single input quadrature. Thus, both the standard homodyne gain and the optical parametric gain serve the same homodyne purpose—to amplify the quantum input of interest (the optical quadrature) to a classically detectable output level, which is sufficiently above the measurement noise (the electronic noise for standard homodyne or the optical vacuum noise for parametric homodyne). Consequently, standard and parametric homodyne are two faces of the same concept.

The difference between the two schemes is both technical and conceptual. On the technical level, the gain of standard homodyne is generally very large, allowing to a priori neglect
any effect of the unmeasured quadrature on the electrical output, whereas the optical parametric gain may not be sufficient to justify such an a priori assumption and may require more careful analysis of the output with finite gain, as we described earlier. On the conceptual level, parametric homodyne provides an optical output, as opposed to standard homodyne that destroys the optical fields. Since the optical parametric output can be sufficiently “classical” (amplified above the vacuum level), it is far less sensitive to additional vacuum noise from optical loss or detector inefficiency. Consequently, parametric homodyne does not only preserve the optical bandwidth across the quantum-classical transition (see Fig. 1), but can also allow complete reconstruction of the two-mode quadrature in a single shot, as was explained in the previous sub-section. Hence, adding a layer of optical parametric gain before the electronic photo-detection, be it intensity detection or homodyne provides an important freedom to quantum measurement beyond the ability to preserve the optical bandwidth.

Beyond the pure two-mode field. Last, let us briefly consider broadband time-dependent states of light beyond the single-frequency two-mode state. Any classical wave packet with spectral envelope \( f(\omega) = |f(\omega)|e^{i\theta(\omega)} \) around the carrier frequency \( \Omega \) (normalized to \( |f(\omega)|^2 = 1 \)) can be regarded as an electromagnetic mode with associated quantum field operators

\[
\begin{align*}
  a(t) &= \int d\omega f(\omega)a(\omega)e^{-i\omega t}, \\
  a^\dagger(t) &= \int d\omega f^*(\omega)a^\dagger(\omega)e^{i\omega t},
\end{align*}
\]

and associated temporal quadratures

\[
\begin{align*}
  x_f(t) &= \int d\omega e^{-i\omega t}[f(\omega)a(\omega) + f^*(\omega)a^\dagger(\omega)], \\
  y_f(t) &= i\int d\omega e^{i\omega t}[f^*(\omega)a^\dagger(\omega) - f(\omega)a(\omega)],
\end{align*}
\]

which is just the Fourier transform of Eq. (1) (see also Eq. (11) in the Methods).

We can express the temporal quadrature \( x_f(t) \) in terms of the two-mode quadratures \( x(\omega), y(\omega) \) as

\[
\begin{align*}
  x_f(t) &= \int d\omega e^{-i\omega t}\left[\frac{f(\omega)+f^*(\omega)}{2}\right]x(\omega) + \frac{i(f(\omega)-f^*(\omega))}{2}y(\omega), \\
  y_f(t) &= \int d\omega e^{i\omega t}\left[\frac{f(\omega)+f^*(\omega)}{2}\right]y(\omega) - \frac{i(f(\omega)+f^*(\omega))}{2}x(\omega),
\end{align*}
\]

where the symmetric and antisymmetric parts of the wave packet \( f(\omega)+f^*(\omega)/2 \) and \( f(\omega)-f^*(\omega)/2 \) are the Fourier transforms of \( \text{Re}(t), \text{Im}(t) \) the real and imaginary parts of the field envelope in time.

Equation (9) can be simplified considerably when the spectrum of the wave packet is symmetric \( |f(\omega)| = |f(-\omega)| \), which is the major situation to employ a quadrature representation to begin with. The temporal quadrature \( x_f(t) \) is then simply a superposition of many two-mode components \( x(\omega) \) with a spectrally varying axis \( \theta(\omega) \) and envelope phase \( \delta(\omega) \)

\[
\begin{align*}
  x_f(t) &= \int d\omega e^{-i\omega t}|f(\omega)|e^{i\theta(\omega)}x(\omega) \\
  &= \int d\omega e^{-i\omega t}|f(\omega)|e^{i\theta(\omega)}\left[\frac{x(\omega)\cos(\theta(\omega)) + y(\omega)\sin(\theta(\omega))}{2}\right] \\
  y_f(t) &= \int d\omega e^{i\omega t}|f(\omega)|e^{i\theta(\omega)}y(\omega) \\
  &= \int d\omega e^{i\omega t}|f(\omega)|e^{i\theta(\omega)}\left[\frac{x(\omega)\cos(\theta(\omega)) - y(\omega)\sin(\theta(\omega))}{2}\right].
\end{align*}
\]

The quadrature axis of each two-mode component is dictated by its carrier phase \( \theta(\omega) = \frac{\varphi(\omega) + \varphi^*(-\omega)}{2} \) — the symmetric part of the spectral phase of the wave packet \( \varphi(\omega) \); and the two-mode envelope phase \( \delta(\omega) = \frac{\varphi(\omega) - \varphi^*(-\omega)}{2} \) relates to the antisymmetric part of \( \varphi(\omega) \). Thus, for a transform limited mode, where \( \varphi(\omega) = 0 \), both the envelope phase and the quadrature axis are constant across the spectrum \( \delta(\omega) = 0, \theta(\omega) = 0 \). An antisymmetric phase variation, \( \varphi(\omega) = -\varphi(-\omega) \), will affect only the envelope phase, but keep the quadrature axis constant \( \theta(\omega) = 0 \), as is the case for down-converted light. A purely symmetric phase \( \varphi(\omega) = \varphi(-\omega) \), as due to material dispersion, will affect only the quadrature axis, but keep the envelope constant \( \delta(\omega) = 0 \).

Therefore, measurement of an arbitrary generalized quadrature of broadband light requires measurement (or knowledge) of two spectral degrees of freedom—the quadrature axis \( \theta(\omega) \) and the envelope phase \( \delta(\omega) \). Parametric homodyne with intensity measurement provides complete information of the quadrature axis \( \theta(\omega) \) (by measuring the output spectrum for varying pump phase), but is insensitive to \( \delta(\omega) \). It therefore allows measurement if \( \delta(\omega) \) is either unimportant (down-conversion) or known a priori (transform limit or well-defined pulse), which is relevant to all current sources of broadband quantum light in spite of the limitations. The combination of parametric gain followed by standard homodyne allows complete arbitrary measurement, as explained above.

Discussion

It is interesting to note that the effect of two parametric amplifiers in series was deeply explored previously in the context of quantum interference\(^{20, 25}\). In such a series configuration, interference occurs between two possibilities for generating bi-photons, either in the first amplifier or in the second, depending on the pump phase. The interference contrast can reach unity when the parametric gain of the two amplifiers is identical (assuming no loss), which testifies to the quantum nature of the light in both the single-photon regime\(^{21}\) and high power\(^{20, 35}\). Here, however, we consider the second amplifier as a measurement device, independent of the source of light to be measured. This light source can be, but is certainly not limited to, a squeezing parametric amplifier. Clearly, any other source of quantum light is relevant when homodyne measurement is of interest, such as single photons, Fock states, NOON states, Schrödinger cat states, and so on.

A different optical measurement of quantum light was recently reported in ref. \(^{36}\), where vacuum fluctuations of THz radiation were observed in time. There too an optical nonlinearity (of several THz bandwidth) was utilized for a direct measurement, where the large bandwidth of the nonlinearity was key to enable time sampling of the vacuum fluctuations, well within a single optical cycle of the measured THz mode.

To conclude, we presented an approach to optical homodyne measurement with practically unlimited bandwidth, which adds a layer of optical parametric amplification before the photo-detection, and enables simultaneous quadrature measurement across the entire spectrum with a single LO. This measurement removes major limitations of optical homodyne and opens a wide window for efficient utilization of the bandwidth resource for parallel quantum information processing. An interesting expansion of this concept would be where the pump itself includes more than one mode, for measurement of “hyper” entanglement between different frequency pairs of the frequency comb with a multi-mode pump\(^{13, 37}\).

Methods

Two-mode quadratures: time and frequency representation. The direct mathematical relation of the time varying field to broadband quadrature amplitudes is simple and illuminating in both time and frequency, and yet, it is rarely used outside the context of near monochromatic light. For a classical time-dependent field \( E(t) = a(t) \exp \Delta t + c.c., the two quadratures in time are the real
and imaginary parts of the field amplitude $a(t)$

\[
\begin{align*}
    x(t) &= a(t) + a^*(t) = 2\text{Re}[a(t)], \\
    y(t) &= i(a^*(t) - a(t)) = 2\text{Im}[a(t)].
\end{align*}
\]  

In frequency, therefore, the quadrature amplitudes $x(\omega)$, $y(\omega)$, represent the symmetric and antisymmetric parts of the field spectral amplitude $a(\omega)$

\[
\begin{align*}
    x(\omega) &= a(\omega) + a(-\omega), \\
    y(\omega) &= i(a^*(-\omega) - a(\omega)),
\end{align*}
\]  

where $\omega$ is the offset from the carrier frequency $\Omega$, possibly of optical separation, and $a(\omega) = \int_{-\infty}^{\infty} a(t)e^{-i\omega t}dt$. Strictly speaking, Eqs. (1) and (12) define the quadratures of the nonlinear dipole within the medium, not of the emitted light field. Specifically, they do not include the frequency dependence of the optical field operator $E(\omega)$, a(\omega)\sqrt{\Omega} / \sqrt{\Omega}$, which is different for the signal and idler modes. Yet, to avoid cumbersome nomenclature we simply refer to these as the “two-mode quadratures,” since they correctly represent the quantum correlation and squeezing of a two-mode field.

Figure 5 illustrates the temporal field of a single two-mode component of a pure quadrature oscillation, which represents a beat pattern: slow sinusoidal envelope of quadrature oscillations, showing the two-mode beat envelope at frequency $\omega$ modulating the carrier frequency oscillation. The quadrature axis is defined by the phase $\phi_{\text{fi}}$ of the carrier of the two-mode oscillation relative to a reference LO (black) $\pm \chi$ for the $x(\omega)$ quadrature (cosine) and $\pm \frac{\chi}{2}$ for $y(\omega)$. The phase of the quadrature amplitude $\phi_{\text{fi}}$ reflects the temporal offset of the beat envelope with respect to a time reference.

For calibration we use measurements that are independent of phase-coherent terms $\langle a|a\rangle = \langle a^*a\rangle = 0$ or $D = 0$, allowing us to write

\[
I_s = n_s^2|C|^2 + D^2 + (N_s + 1),
\]

where $n_s^2 = n_s^2|D|^2$, and

\[
I_i = n_i^2|D|^2 + (N_i + 1),
\]

indicating that the ratio between these two measurements yields the idler average photon-number $N_i = \frac{I_i}{I_s} - 1$. Note that these two measurements act as a simple method for acquiring the input number of photons independent of the parametric gain. The signal photon-number $N_s = \frac{I_s}{I_s^2}$ can be acquired by measuring the output number intensities in the same way (or be assumed equal to $N_i$, if appropriate).

Next, we use the knowledge of the input photon numbers for calibrating the overall detector response $n^2_s$. We measure: (3) $I_{p\nu}$ blocking the pump (zero amplification, $|C| = 1$, $I_{m} = 0$, letting the signal and idler through), and we obtain, from Eq. (14) we find $n^2_s = \frac{I_s}{I_{p\nu}}$.

Once the detector response is obtained, we can obtain the parametric gain coefficients $G_s, D$ with the $I_{m} = 0$ measurement, since $|D|^2 = \frac{I_{pm}}{I_{p\nu}}$, and $G_s^2 = \frac{I_{pm}}{I_{p\nu}^2} (\frac{I_s}{I_{pm}} + 1)$ (or may be assumed ideal $|G|^2 = |D|^2 + 1$, if appropriate). Note that the ratio is needed only once, as long as the parametric measurement gain is constant, and the average photon-number difference at the input $N_s = N_i$, does not change (typically for squeezed input, this difference is simply zero).

**Extraction of the average quadratures.** The two quadratures cannot be measured simultaneously, but their average intensities can be both extracted from two measurements of the parametric output intensity, amplifying one quadrature first ($I_s$) and then the other ($I_i$), according to

\[
\begin{align*}
    \langle x^2 \rangle &= \frac{1}{2} \left[ r(I_s/n_s^2 - p) - q(I_s/n_i^2 - p) \right], \\
    \langle y^2 \rangle &= \frac{1}{2} \left[ r(I_s/n_i^2 - p) - q(I_s/n_s^2 - p) \right],
\end{align*}
\]

where $n_i^2$ is the detector response per single photon and the coefficients $p, q, r$ are:

\[
\begin{align*}
    p &= \frac{1}{2} (N_s - N_i - 1) \\
    q &= \frac{1}{2} |G|^2 |D|^2, \\
    r &= \frac{1}{2} |D|^2.
\end{align*}
\]

**Parametric homodyne with finite gain.** To consider more formally the equivalence of parametric amplification to extraction of quadrature information at any finite gain, let us examine the relation between the field operators at the output of the amplifier and the quadratures of the input. (Eq. (3)) $a_0(\theta, g) = a_0e^{i\theta}\cos g + a_0^*e^{i\theta}\sin g = e^{-i\theta}a_0^* + e^{i\theta}a_0$. As mentioned, the field operator converges in the limit of large gain to an amplified single quadrature operator $a_0(\theta, g) = e^{i\theta}a_0$, but this convergence can never be exact since the commutation relation of field
operators $|a, a\rangle = 1$ is inherently different than that of quadrature operators $|x, x\rangle = 0$. To illuminate the smooth transition from a field operator to a quadrature, let us express the field operator for any finite parametric gain in the form of a generalized quadrature operator along an axis of a complex angle $\theta = \theta + iy$,

$$a(g) = M(\sqrt{x} \cos \theta + y \sqrt{\sin \theta}) = Mx_{\gamma},$$

(17)

where the imaginary part of the quadrature axis and the normalization factor $M$ relate to the gain $g$ by $\tan \gamma = e^{-3}$, $M^2 = \frac{2}{\sinh 2\gamma}$.

Thus, the single-shot measurement of the output light intensity with any parametric gain reflects the intensity of the “generalized” quadrature at this gain value, and not the standard (real) quadrature. The commutation relation of these generalized quadratures is

$$[x_{\gamma}, x_{\gamma}'] = 1/M^2 \approx e^{-3},$$

(18)

where the approximation is valid already for moderate gain of $g \geq 1$. Consequently, the commutator of the measured generalized quadratures, converges very quickly to that of the real quadratures.

**Details of the experimental setup.** In our experiment (Figs. 2 and 6), we generate an ultra-broadband two-mode squeezed vacuum via collinear FWM in a PCF, which is pumped by narrowband 12 ps pulses at 786 nm with up to 100 mW average power. The broad bandwidth is obtained by closely matching the pump wavelength to the zero dispersion of the fiber at 784 nm, resulting in a signal and idler bandwidth of ~55 THz each, with ~90 THz mean frequency separation between the mode centers (700 nm—signal center, and 900 nm—idler center). After generation, the pump is separated from the FWM field by a narrowband filter (NBF1—Semrock FF776-Dio1), allowing independent control of the relative pump phase. The pump phase is actively locked to the FWM and pump frequencies by rotating its polarization before the first pass with a half-wave plate. Thus, rotating the pump polarization before the first pass with a half-wave plate we could transfer part of the pump power through the fiber without affecting the FWM. This power could later be used in the second pass by rotating its polarization back to the PCF axis with a quarter-wave plate in the pump beam path. This extra pump power accumulated almost the same as the FWM, but without affecting the squeezing generation.

The various calibration measurements were performed by manipulating the FWM light between the passes either by physically blocking the FWM beam (vacuum input) or pump beam (zero amplification) or with a high-efficiency optical long pass filter (after input only) (Semrock FF776-Dio1). The two orthogonal homodyne measurements (amplifying the squeezed quadrature or the stretched quadrature) were acquired by tuning the offset of the active feedback loop that locked the pump phase.

**Effects of the pulsed pump.** In our experiment, the pump for both generation of the squeezed light and for the parametric homodyne measurement (second pass) is a pulsed laser of 12 ps duration. Since the bandwidth of the generated FWM (55
Fig. 8 Expanded homodyne results. a Measured squeezed quadrature as a function of squeezing strength (696 nm)—(solid line for guidance only). As the squeezing strength in the first pass is increased, the measured squeezed quadrature decreases, down to \(\langle \xi_x^2 \rangle \sim 0.68\) at a pump power of ~60 mW. Further increase of the pump degrades the observed squeezing due to temporal effects of the pulsed pump. b Minimum uncertainty conservation (696 nm)—(solid line for guidance only). Ideal squeezed vacuum is a minimum uncertainty state of \(\langle \xi_x^2 \rangle \langle \xi_y^2 \rangle = 1\), independently of the squeezing strength. Up to a pump power of 60 mW, the uncertainty area is indeed nearly conserved \(\langle \xi_x^2 \rangle \langle \xi_y^2 \rangle < 1.3\). Beyond this limit the pulse-averaging effect washes out the minimum uncertainty property. c, d The effect of loss on the squeezed state. c We apply a series of loss values (30–66%) to a given squeezed state and observe the influence on the squeezed/stretch quadratures. d The reconstructed “bare” squeezed/stretch quadratures that calibrated out the loss from all the curves of c, demonstrating collapse of all the curves to nearly the same value, as expected.

However, since the integration time of the photo-detectors in the CCD spectrograph is much longer (~10 ms), the measured homodyne data is averaged over the entire shape of many pulses.

The effect of the pulse on the parametric gain alone changes the generated squeezing and the measurement gain with time, measuring weak squeezing with weak parametric gain at the edges of the pulse, and strong squeezing with strong parametric gain at the peak. The phase modulation (SPM, XPM) of the FWM process has a more severe effect, since it modulates in time the quadrature axis to be amplified. As a result, due to the pump pulse shape, the amplified quadrature axis of the FWM field rotates with time. Luckily, when the pump itself experiences nearly the SPM it can still act as a near-perfect LO (phase regarding) for measuring the FWM, even after passage through the fiber. The small residual difference between the pump SPM and the FWM XPM causes the amplified FWM quadrature to rotate with time, mixing different quadrature axes together in the same measurement, smearing out some of the squeezing.

Ideally, we would like to extract the maximum squeezing that occurs at the peak of the pulse from the time-averaged measurements. To estimate this peak squeezing, we numerically simulated the entire FWM generation and parametric amplification along the pump pulse with 50 fs temporal resolution (corresponding to the coherence time of the FWM). The simulation incorporated the measured pump pulse energy, the measured loss and fiber coupling efficiencies, and an assumed hyperbolic-secant temporal shape of the pump pulse (12 ps). Using the simulation, we could calculate both the average and the peak outputs of the process, allowing us to estimate the squeezing at the peak of the pulse from the measured averaged homodyne output. Figure 7 demonstrates the relation between the peak homodyne output and the average homodyne output, as the parametric measurement gain is varied. As long as the generation pump power does not exceed a specific limit (~60 mW in our experiment), the pulse-averaging only affects the absolute measured squeezing values (which can be roughly estimated) but not the expected trends of the experiment (increasing the loss, the squeezing power, or the parametric power).

**Expanded results.** To verify the properties of the parametric homodyne, we measured the quadrature squeezing \(\langle \xi_x^2 \rangle\), and the uncertainty area, \(\langle \xi_x^2 \rangle \times \langle \xi_y^2 \rangle\) of the squeezed state as described in the main text.

Another important verification of our squeezing measurement is to observe the effect of loss on the quadrature squeezing and stretching. We measured the quadrature intensities after applying a set of known attenuations (30–66% loss), and reconstructed the “bare” quadratures before loss, which indeed collapsed to the same value, as shown in Fig. 8c, d. The effect of loss on the quadrature intensity can be regarded as propagation through a beam splitter with one open port. The relations between the operators of the two inputs \((a_1, a_2)\) and two outputs \((b_3, b_4)\) of the beam splitter can be defined as \(b_3 = b_1 + b_2\) and \(b_4 = b_1 - b_2\), where \(a\) and \(r\) are the transmission and reflection (loss) amplitudes. In these terms, the quadrature operator at output port 3 is: \(x_3 = x_1 + r x_2\), and the expectation value of the quadrature intensity is

\[
\langle x_3^2 \rangle = |r|^2 \langle x_1^2 \rangle + |r|^4 \langle x_2^2 \rangle + 2 |r|^2 \langle x_1 x_2 \rangle / \langle x_1 \rangle.
\]

(19)

Assuming a vacuum state at the open input port 2, the final expression becomes:

\[
\langle x_3^2 \rangle = |r|^2 \langle x_1^2 \rangle + |r|^4.
\]

(20)

Hence, the “bare” quadratures, before the loss, can be reconstructed using

\[
\langle x_3^2 \rangle_{\text{bare}} = \left( \langle x_1^2 \rangle_{\text{measured}} - |r|^2 \right) / |r|^2.
\]

(21)

As a complementary evaluation, we studied the parametric measurement-amplifier output as a function of its own gain, while maintaining the squeezing generation gain constant. For this, we gradually increased the pump power in the second pass up to 5.5 times the pump power that generated the squeezing in the first pass. When the parametric gain is strong enough, the output intensity relative to the vacuum level (without input) is directly proportional to the input quadrature. Hence, we expect the relative output to stabilize as the parametric gain is increased, and indeed the observed reduction below the vacuum level stabilized at 5%. Figure 7 shows the measured results and addresses the pulse effects on this measurement.

**Data availability.** All relevant data are available from the authors.

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Author contributions

Y.S., A.P., and M.R. designed the experiment, Y.S. and A.P. developed the theory, Y.S., Y.M., and R.Z.V. performed the experiment, L.B. participated in the analysis of results and continuous discussions. All authors contributed to finalizing the manuscript.

Additional information

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