Security-Aware Synthesis Using Delayed-Action Games

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Abstract. Stochastic multiplayer games (SMGs) have gained attention in the field of strategy synthesis for multi-agent reactive systems. However, standard SMGs are limited to modeling systems where all agents have full knowledge of the state of the game. In this paper, we introduce delayed-action games (DAGs) formalism that simulates hidden-information games (HIGs) as SMGs, by eliminating hidden information by delaying a player’s actions. The elimination of hidden information enables the usage of SMG off-the-shelf model checkers to implement HIGs. Furthermore, we demonstrate how a DAG can be decomposed into a number of independent subgames. Since each subgame can be independently explored, parallel computation can be utilized to reduce the model checking time, while alleviating the state space explosion problem that SMGs are notorious for. In addition, we propose a DAG-based framework for strategy synthesis and analysis. Finally, we demonstrate applicability of the DAG-based synthesis framework on a case study of a human-on-the-loop unmanned-aerial vehicle system that may be under stealthy attack, where the proposed framework is used to formally model, analyze and synthesize security-aware strategies for the system.

1 Introduction
Stochastic multiplayer games (SMGs) are used to model reactive systems where nondeterministic decisions are made by more than one entity [4,13,20]. SMGs extend probabilistic automata by assigning a player to each choice to be made in the game. This extension allows for modeling complex systems where the behavior of players is unknown in design time. A player’s strategy is a set of rules that the player follows when making decisions as the game advances. The strategy synthesis problem is concerned with finding a winning strategy, i.e., a strategy that guarantees that a set of objectives (or winning conditions) is satisfied [6,18].

Several algorithms and tools have been developed for this purpose. For instance, value iteration methods are utilized to synthesize strategies for SMGs such that multiple reward-based objectives are satisfied [29,16]. To tackle the state-space explosion problem, [25] presents an assume-guarantee strategy synthesis framework that relies on synthesizing strategies on the component level first, before combining them through an assume-guarantee framework to find a global winning strategy. Mean-payoffs and ratio rewards are further investigated in [8] to synthesize $\varepsilon$-optimal strategies. Formal tools that support strategy synthesis via SMGs include PRISM-games [7] and Uppaal Stratego [10].
SMGs are classified based on the number of players that can make choices at each state. In concurrent games, more than one player are allowed to concurrently make choices at a given state. Conversely, turn-based games assign one player at most to each state. Since they provide a relatively less complex structure, we focus on turn-based games in the rest of this paper.

Games are also classified according to the nature of information available to different players across the game states [23]. Complete-information games (also known as perfect-information games [5]) grant all players complete access to the information within the game — including other players’ choices. Conversely, partially-observable stochastic games, introduced in [14], allow agents to have different belief states by including uncertainty about both the current state and adversarial plans into the game state. Moreover, in symmetric games some of the information is equally hidden from all players. On the contrary, asymmetric games allow some players to have access to more information than the others [23]. Note that the equivalence between turn-based semi-perfect information games and concurrent perfect-information games was shown [5]. Since a player’s strategy mainly rely on full knowledge of the game state [9], using SMGs for synthesis produces strategies that may violate synthesis specifications in cases where required information is hidden from the player.

In this work, we are motivated by security-aware models of systems in which stealthy adversarial actions are potentially hidden from the system, and where the system can probabilistically and intermittently gain full knowledge about or access to its current state — e.g., when intrusion detection mechanisms are executed. To model such systems, hidden-information games (HIGs) can be used, where the hidden information is captured by private variables [5]. However, standard model checkers can only synthesize strategies for (full-information) SMGs. Hence, an alternative representation of HIGs is required.

Consequently, we introduce delayed-action games (DAGs) — a new class of games that can simulate HIGs by utilizing the concept of delaying actions. That is, a DAG hides information from one player by delaying the actions of the others. The omission of private variables enables the use of off-the-shelf tools (e.g., PRISM-games [7]) to implement and analyze DAG-based models. Next, we show how DAGs (under some mild and practical assumptions) can be decomposed into a number of independent subgames. This approach can reduce the time required for synthesis by employing parallel computation to explore each subgame. Moreover, we propose a DAG-based framework for strategy synthesis and analysis of security-aware systems. Finally, we demonstrate the applicability of the proposed framework on a case study focused on security-aware planning for an unmanned-aerial vehicle (UAV) system prone to stealthy cyber attacks; specifically, we develop a DAG-based system model and further utilize the proposed framework to synthesize strategies with strong probabilistic security guarantees.

The paper is organized as follows. Sec. 2 presents SMGs, HIGs, and problem formulation. In Sec. 3 we introduce DAGs and show that they can simulate HIGs. Sec. 4 proposes a DAG-based synthesis framework, which we use for security-aware planning for UAVs in Sec. 5 before concluding the paper in Sec. 6.
2 Stochastic Games

In this section, we present turn-based stochastic games, which assume that all players have full information about the state of the game. We then introduce hidden-information games and their private-variable semantics. We start by presenting the employed notation.

Notation. We employ standard notation with \( \mathbb{N}_0 \) denoting the set of non-negative integers. \( \mathcal{P}(A) \) denotes the powerset of \( A \) (i.e., \( 2^A \)). A variable \( v \) has a set of valuations \( Ev(v) \), where \( \eta(v) \in Ev(v) \) denotes one. We use \( \Sigma^* \) to denote the set of all finite words over alphabet \( \Sigma \), including the empty word \( \epsilon \). The mapping \( \text{Eff}: \Sigma^* \times Ev(v) \rightarrow Ev(v) \) indicates the effect of a finite word on \( \eta(v) \). Finally, for general indexing, we use \( s_i \) or \( s^{(i)} \), for \( i \in \mathbb{N}_0 \), while \( PL_\gamma \) denotes Player \( \gamma \).

2.1 Turn-Based Stochastic Games

Stochastic multiplayer games (SMGs) can be used to model reactive systems that undergo both stochastic and nondeterministic transitions from one state to another. In a turn-based game, actions can be taken at any state by at most one player. Formally, an SMG can be defined as follows [1,24,25].

Definition 1 (Turn-Based Stochastic Game). A turn-based game (SMG) with players \( \Gamma = \{I, II, \emptyset\} \) is a tuple \( G = \langle S, (S_I, S_{II}, S_\emptyset), A, s_0, \delta \rangle \), where

- \( S \) is a finite set of states, partitioned into \( S_I, S_{II} \) and \( S_\emptyset \);
- \( A = A_I \cup A_{II} \cup \{\tau\} \) is a finite set of actions where \( \tau \) is an empty action;
- \( s_0 \in S_{II} \) is the initial state; and
- \( \delta : S \times A \times S \rightarrow [0,1] \) is a transition function, such that \( \delta(s, a, s') \in \{0,1\} \), \( \forall s \in S_I \cup S_{II}, a \in A \) and \( s' \in S \), and \( \delta(s, \tau, s') \in [0,1], \forall s \in S_\emptyset \) and \( s' \in S_I \cup S_{II} \), where \( \sum_{s' \in S_I \cup S_{II}} \delta(s, \tau, s') = 1 \) holds.

For all \( s \in S_I \cup S_{II} \) and \( a \in A_I \cup A_{II} \), we write \( s \xrightarrow{a} s' \) if \( \delta(s, a, s') = 1 \). Similarly, for all \( s \in S_\emptyset \) we write \( s \xrightarrow{p} s' \) if \( s' \) is randomly sampled with probability \( p = \delta(s, \tau, s') \).

2.2 Hidden-Information Games

SMGs assume that all players have full knowledge of the current state, and hence provide perfect-information models [5]. In many applications, however, this assumption may not hold. A great example are security-aware models where stealthy adversarial actions can be hidden from the system; e.g., the system may not even be aware that it is under attack. On the other hand, hidden-information games (HIGs) refer to games where one player does not have complete access to (or knowledge of) the current state. The notion of hidden information can be formalized with the use of private variables (PVs) [5]. Specifically, a game state can be encoded using variables \( v_T \) and \( v_B \), representing the true information, which is only known to \( PL_I \), and \( PL_{II} \) belief, respectively.

1 The term turn-based indicates that at any state only one player can play an action. It does not necessarily imply that players take fair turns.
**Definition 2 (Hidden-Information Game).** A hidden-information stochastic game (HIG) with players $I = \{I, \Pi, \bigcirc\}$ over a set of variables $V = \{v_T, v_B\}$ is a tuple $G_H = \langle S, (S_I, S_{II}, S_{O}), A, s_0, \beta, \delta \rangle$, where

- set of states $S \subseteq Ev(v_T) \times Ev(v_B) \times \mathcal{P}(Ev(v_T)) \times I$, partitioned in $S_I, S_{II}, S_{O}$;
- $A = A_I \cup A_{II} \cup \{\tau, \theta\}$ is a finite set of actions, where $\tau$ denotes an empty action, and $\theta$ is the action capturing PL$_{II}$ attempt to reveal the true value $v_T$;
- $s_0 \in S_{II}$ is the initial state;
- $\beta : A_{II} \rightarrow \mathcal{P}(A_I)$ is a function that defines the set of available PL$_{II}$ actions, based on PL$_{II}$ action; and
- $\delta : S \times A \times S \rightarrow \{0, 1\}$ is a transition function such that $\delta(s_I, a, s_{O}) = \delta(s_{O}, a, s_I) = 0$, and $\delta(s_{II}, \theta, s_{O})$, $\delta(s_{II}, a, s_I)$, $\delta(s_I, a, s_{II}) \in \{0, 1\}$ for all $s_I \in S_I$, $s_{II} \in S_{II}$, $s_{O} \in S_{O}$ and $a \in A$, where $\sum_{s' \in S_{II}} \delta(s_{O}, \tau, s') = 1$.

In the above definition, $\delta$ only allows transitions $s_I$ to $s_{II}$, $s_{II}$ to $s_I$ or $s_{O}$, with $s_{II}$ to $s_{O}$ conditioned by action $\theta$, and probabilistic transitions $s_{O}$ to $s_{II}$. A game state can be written as $s = (t, u, \Omega, \gamma)$, but to simplify notation we use $s_\gamma(t, u, \Omega)$ instead, where $t \in Ev(v_T)$ is the true value of the game, $u \in Ev(v_B)$ is PL$_{II}$ current belief, $\Omega \in \mathcal{P}(Ev(v_T)) \setminus \{\emptyset\}$ is PL$_{II}$ belief space, and $\gamma \in \Gamma$ is the current player’s index. When the truth is hidden from PL$_II$, the belief space $\Omega$ is the information set $\{23\}$, capturing PL$_{II}$ knowledge about the possible true values.

**Example 1 (Belief vs. True Value).** Our motivating example is a system that consists of a UAV and a human operator. For localization, the UAV mainly relies on a GPS sensor that can be compromised to effectively steer the UAV away from its original path. While aggressive attacks can be detected, some may remain stealthy by introducing only bounded errors at each step $\{17, 22, 19, 15\}$. For example, Fig. 1 shows a UAV (PL$_{II}$) occupying zone A and flying north (N). An adversary (PL$_I$) can launch a stealthy attack targeting its GPS, introducing a bounded error (NE, NW) to remain stealthy. The set of stealthy actions available to the attacker depends on the preceding UAV action, which is captured by the function $\beta$, where $\beta(N) = \{NE, N, NW\}$. Being unaware of the attack, the UAV believes that it is entering zone C, while the true new location is D due to the attack (NE). Initially, $\eta(v_T) = \eta(v_B) = z_A$, and $\Omega = \{z_A\}$ as the UAV is certain it is in zone $z_A$. In $s_2$, $\eta(v_B) = z_C$, yet $\eta(v_T) = z_D$. Although $v_T$ is hidden, PL$_{II}$ is aware that $\eta(v_T)$ is in $\Omega = \{z_B, z_C, z_D\}$.

**HIG Semantics.** The semantics of $G_H$ is described using the rules shown in Fig. 2 where H2 and H3 capture PL$_{II}$ and PL$_I$ moves, respectively. If PL$_{II}$ executes $\theta$ to attempt to reveal the true value of the game, H4 shows that this attempt can succeed with probability $p_I$ where PL$_{II}$ belief is updated (i.e., $u' = t$), or remains unchanged otherwise.

**Example 2 (HIG Semantics).** Continuing Example 1, let us assume that $A_I = A_{II} = \{N, S, E, W, NE, NW, SE, SW\}$, and that $\theta = GT$ is a geolocation task that
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H1: \( s_0 = s_{11}(t_0, u_0, \Omega_0) \) if \( t_0 = u_0, \Omega_0 = \{t_0\} \)

H2: \( s_{11}(t, u, \Omega) \xrightarrow{a} s_1(t', u', \Omega') \) if \( a_i \in A_{11}, t' = t, u' = E_{ff}(a_i, u), \Omega' = \{t' \mid t' = E_{ff}(b_i, t) \forall b_i \in \beta(a_i), t \in \Omega\} \)

H3: \( s_1(t, u, \Omega) \xrightarrow{b_i} s_{11}(t', u', \Omega') \) if \( b_i \in \beta(a_i), t' = E_{ff}(b_i, t), u' = u, \Omega' = \Omega \)

H4: \( s_\circ(t, u, \Omega) \xrightarrow{\delta_i} s_{11}(t', u', \Omega') \) if \( t' = t, u' = u, \Omega' = \Omega \)

\( \delta_i \) is the adversary's action, \( \beta \) is the set of possible actions for the adversary, \( E_{ff} \) is the effect function, and \( \Omega \) is the set of states.

\( s_0, s_1, s_2, s_3, s_4, s_5, s_6 \)

Assignment

State

Transition

Hidden information

\( \mathcal{M}_{uav} \)

\( \mathcal{M}_{adv} \)

\( \mathcal{M}_{as} \)

\( \mathcal{M} \)

\( \mathcal{G} \)

\( N \)

\( NE \)

\( W \)

\( NW \)

\( GT \)

\( idle \)

\( fly \)

\( attack \)

\( guard \)

\( channel \ transmit \ (!) \)

\( channel \ receive \ (?) \)

\( task \ activates \)

\( location \)
UAV believes to be current \((v_B)\), and the ground truth location that the UAV actually occupies \((v_T)\). Reasoning about the flight plan using such model becomes problematic since the ground truth \(v_T\) is inherently unknown to the UAV (i.e., PL_{II}), and thus so is \(p(v_T, v_B)\). Furthermore, such representation, where some information is hidden, is not supported by off-the-shelf SMG model checkers. Consequently, for such HIGs, our goal is to find an alternative representation that is suitable for strategy synthesis using off-the-shelf SMG model-checkers.

3 Delayed-Action Games

In this section, we provide an alternative representation of HIGs that eliminates the use of private variables — we introduce Delayed-Action Games (DAGs) that exploit the notion of delayed actions. Furthermore, we show that for any HIG, a DAG that simulates the former can be constructed.

3.1 Delayed Actions

Informally, a DAG reconstructs an HIG such that actions of PL_{I} (the player with access to perfect information) follow the actions of PL_{II}, i.e., PL_{I} actions are delayed. This rearrangement of the players' actions provides a means to hide information from PL_{II} without the use of private variables, since in this case, at PL_{II} states, PL_{I} actions have not occurred yet. In this way, PL_{II} can act as though she has complete information at the moment she makes her decision, as the future state has not yet happened and so cannot be known. In essence, the formalism can be seen as a partial ordering of the players' actions, exploiting the (partial) superposition property that a wide class of physical systems exhibit. To demonstrate this notion, let us consider DAG modeling on our running example.

Example 3 (Delaying Actions). Fig. 5 depicts the (HIG-based) scenario from Fig. 3, but in the corresponding DAG, where the UAV actions are performed first (in \(\hat{s}_0, \hat{s}_1, \hat{s}_2\)), followed by the adversary delayed actions (in \(\hat{s}_3, \hat{s}_4\)). Note that, in the DG model, at the time the UAV executed its actions (\(\hat{s}_0, \hat{s}_1, \hat{s}_2\)) the adversary actions had not occurred (yet). Moreover, \(\hat{s}_0\) and \(\hat{s}_6\) (Fig. 5) share the same belief and true values as \(s_0\) and \(s_6\) (Fig. 3), respectively, though the transient states do not exactly match. This will be used to show the relationship between the games.

The advantage of this approach is twofold. First, the elimination of private variables enables simulation of an HIG using a full-information game. Thus, the formulation of the strategy synthesis problem using off-the-shelf SMG-based tools becomes feasible. In particular, a PL_{II} synthesized strategy becomes dependent on the knowledge of PL_{I} behavior (possible actions), rather than the specific (hidden) actions. We formalize a DAG as follows.

Definition 3 (Delayed-Action Game). A DAG of an HIG \(G_H = \langle S, (S_I, S_{II}, S_{\Box}), A, s_0, \beta, \delta \rangle\), with players \(\Gamma = \{I, II, \Box\}\) over a set of variables \(V = \{v_T, v_B\}\) is a tuple \(G_D = \langle \hat{S}, (\hat{S}_I, \hat{S}_{II}, \hat{S}_{\Box}), A, \hat{s}_0, \beta, \delta \rangle\) where

\[
\hat{S} \subseteq Ev(v_T) \times Ev(v_B) \times A_{II}^* \times \mathbb{N}_0 \times \Gamma
\]

is the set of states, partitioned into \(\hat{S}_I, \hat{S}_{II}\) and \(\hat{S}_{\Box}\);
Fig. 5. The same scenario as in Fig. 3 modeled as a DAG. Solid squares represent UAV belief, while solid diamonds represent the ground truth. The UAV action GT denotes performing a geolocation task.

D1: \( s_0 = s_{11}(\hat{t}_0, \hat{u}, w_0, 0) \) if \( \hat{t}_0 = \hat{u}, w_0 = \epsilon \)

D2: \( s_{11}(\hat{t}, \hat{u}, w, 0) \xrightarrow{a_i} s_{11}(\hat{t}', \hat{u}', w', 0) \) if \( a_i \in A_{11}, \hat{t}' = \hat{t}, \hat{u}' = \text{Eff}(a_i, \hat{u}), w' = w a_i \)

D3: \( s_1(\hat{t}, \hat{u}, w, j) \xrightarrow{b_i} s_1(\hat{t}', \hat{u}', w', j+1) \) if \( b_i \in \beta(w_j), \hat{t}' = \text{Eff}(b_i, \hat{t}), \hat{u}' = \hat{u}, w' = w, j < |w| - 1 \)

D4: \( s_\circ(\hat{t}, \hat{u}, w, j) \xrightarrow{p_i} s_{11}(\hat{t}', \hat{u}', w', 0) \) if \( \hat{t}' = t_0, \hat{u}' = \hat{u}, w' = w, q_i = \delta(s_\circ, s_{11}) \)

DAG Semantics. A DAG state is a tuple \( \hat{s} = (\hat{t}, \hat{u}, w, j, \gamma) \), where for simplicity we shorthand as \( \hat{s} \), \( (\hat{t}, \hat{u}, w, j) \), where \( \hat{t} \in \text{Ev}(v_T) \) is the last known true value, \( \hat{u} \in \text{Ev}(v_B) \) is PL_{11} belief, \( w \in A_{11} \) captures PL_{11} actions taken since the last known true value, \( j \in N_0 \) is an index on \( w \), and \( \gamma \in \Gamma' \) is the current player index. The game transitions are defined using the semantic rules from Fig. 6.

Note that, in contrast to transition function \( \delta \) in HIG \( G_H \), \( \hat{\delta} \) in DAG \( G_D \) only allows transitions \( s_0 \) to \( s_{11} \) or \( s_1 \), as well as \( s_0 \) to \( s_1 \) or \( s_\circ \), and probabilistic transitions \( s_\circ \) to \( s_{11} \). Note also that \( s_\circ \) to \( s_{11} \) is conditioned by the action \( \theta \).

In both \( G_H \) and \( G_D \), we label states where all players have full knowledge of the current state as proper. We also say that two states are similar if they agree on the belief, and equivalent if they agree on both the belief and ground truth.
Definition 4 (States). Let \( s, (t, u, \Omega) \in S \) and \( \hat{s}, (\hat{t}, \hat{u}, w, j) \in \hat{S} \). We say:
- \( s, (t, u, \Omega) \) is proper if \( \Omega = \{ t \} \), denoted by \( s, (t, u, \Omega) \in \text{Prop}(G_H) \).
- \( \hat{s}, (\hat{t}, \hat{u}, w, j) \) is proper if \( w = \epsilon \), denoted by \( \hat{s}, (\hat{t}, \hat{u}, w, j) \in \text{Prop}(G_D) \).
- \( s, (t, u, \Omega) \) and \( \hat{s}, (\hat{t}, \hat{u}, w, j) \) are similar if \( \hat{u} = u, t \in \Omega \), and \( \gamma = \hat{\gamma} \), denoted by \( s, (t, u, \Omega) \sim \hat{s}, (\hat{t}, \hat{u}, w, j) \).
- \( s, (t, u, \Omega) \) and \( \hat{s}, (\hat{t}, \hat{u}, w, j) \) are equivalent if \( t = \hat{t}, u = \hat{u}, w = \epsilon \), and \( \gamma = \hat{\gamma} \), denoted by \( s, (t, u, \Omega) \simeq \hat{s}, (\hat{t}, \hat{u}, w, j) \).

From the above definition, we have that \( s \simeq \hat{s} \implies s \in \text{Prop}(G_H), \hat{s} \in \text{Prop}(G_D) \).
We now define execution fragments, possible progressions from a state to another.

Definition 5 (Execution Fragment). An execution fragment (of either an SMG, DAG or HIG) is a finite sequence of states, actions and probabilities
\[ \varrho = s_0 a_1 p_1 s_1 a_2 p_2 s_2 \ldots a_n p_n s_n \text{ such that } (s, s_{i+1}) \in A_i, \forall i \in [0, n] \]
We use \( \text{first}(\varrho) \) and \( \text{last}(\varrho) \) to refer to the first and last states of \( \varrho \), respectively. If both states are proper, we say that \( \varrho \) is proper as well, denoted by \( \varrho \in \text{Prop}(G_H) \).
Moreover, \( \varrho \) is deterministic if no probabilities appear in the sequence.

Definition 6 (Move). A move \( \gamma \) of an execution \( \varrho \) from state \( s \in G \), denoted by \( \text{move}(s, \varrho) \), is a sequence of actions \( a_1 a_2 \ldots a_i \in A_\gamma^* \) that player \( \gamma \) performs in \( \varrho \) starting from \( s \).

By omitting the player index we refer to the moves of all players. To simplify notation, we use \( \text{move}(\varrho) \) as a short notation for \( \text{move}(\text{first}(\varrho), \varrho) \). We write \( \text{first}(\varrho) = \text{last}(\varrho) \) to denote that the execution of move \( m \) from the \( \text{first}(\varrho) \) leads to the \( \text{last}(\varrho) \). This allows us to now define the delay operator as follows.

Definition 7 (Delay Operator). For an \( G_H \), let \( m = \text{move}(\varrho) = a_1 b_1 \ldots a_n b_n \theta \) be a move for some deterministic \( \varrho \in \text{TS}(G_H) \), where \( a_1 \ldots a_n \in A_{II}^*, b_1 \ldots b_n \in A_{II}^* \).
The delay operator, denoted by \( m \), is defined by the rule \( m = a_1 \ldots a_n \theta b_1 \ldots b_n \).

Intuitively, the delay operator shifts \( PL_I \) actions to the right of \( PL_{II} \) actions up until the next probabilistic state. For example,
\[
\begin{align*}
\varrho &= s_I^{(0)} \xrightarrow{a_1} s_I^{(1)} \xrightarrow{b_2} s_{II}^{(2)} \xrightarrow{\theta} s_{II}^{(3)} \xrightarrow{\tau} s_{II}^{(4)} \xrightarrow{a_4} s_{II}^{(5)} \xrightarrow{b_5} s_{II}^{(6)} \xrightarrow{a_6} s_{II}^{(7)} \xrightarrow{b_7} s_{II}^{(8)} \\
n &= a_1 b_2 \xrightarrow{\theta} b_2 \tau a_4 b_5 a_6 b_7, \\
\overline{m} &= a_1 b_2 \tau a_4 b_5 b_7.
\end{align*}
\]

3.2 Simulation Relation

Given an HIG \( G_H \), we first define the corresponding DAG \( G_D \).

Definition 8 (Correspondence). Given an HIG \( G_H \), a corresponding DAG \( G_D = D[G_H] \) is a DAG that follows the semantic rules displayed in Fig. [7].

For the rest of this section, we consider \( G_D = D[G_H] \), and use \( \varrho \in \text{TS}(G_H) \) and \( \hat{\varrho} \in \text{TS}(G_D) \) to denote two execution fragments of the HIG and DAG, respectively. We say that \( \varrho \) and \( \hat{\varrho} \) are similar, denoted by \( \varrho \simeq \hat{\varrho} \), iff \( \text{first}(\varrho) \simeq \text{first}(\hat{\varrho}) \), \( \text{last}(\varrho) \simeq \text{last}(\hat{\varrho}) \), and \( \text{move}(\varrho) = \text{move}(\hat{\varrho}) \).

\(^3\) For deterministic transitions, \( p = 1 \), hence omitted from \( \varrho \) for readability.
\(^4\) An execution fragment lives in the transition system (TS), i.e., \( \varrho \in \text{Prop}(\text{TS}(G)) \).
We omit TS for readability.
Move to the first simulation. For any \( s \), (Theorem 1 (Probabilistic Simulation).

\[
\text{Definition 9 (Game Proper Simulation). A game } G \text{ properly simulates } G_H, \text{ denoted by } G \models G_H, \text{ iff } \forall \varrho \in \text{Prop}(G_H), \exists \bar{\varrho} \in \text{Prop}(G) \text{ such that } \varrho \sim \bar{\varrho}.
\]

Before proving the existence of the simulation relation, we first show that if a move is executed on two equivalent states, then the terminal states are similar.

**Lemma 1 (Terminal States Similarity).** For any \( s_0 \simeq s_0 \) and a deterministic \( \varrho \in \text{TS}(G_H) \) where \( \text{first}(\varrho) = s_0 \), \( \text{last}(\varrho) \in S_H \), then \( \text{last}(\varrho) \sim \left(\text{move}(\varrho)\right)(s_0) \) holds.

**Proof.** Let \( \text{last}(\gamma_l) = s_{i_l}^{(i_l)}(t_i, u_i, \Omega_i) \) and \( \left(\text{move}(\varrho)\right)(s_0) = s_{i_0}^{(i_0)}(t_0, u_0, \Omega_0) \), where \( \text{move}(\varrho) = a_1 b_1 \ldots a_i b_i \). We then write \( \text{move}(\varrho) = a_1 \ldots a_i \theta b_1 \ldots b_i \). We use induction over \( i \) as follows:

- **Base \((i = 0)\):** \( \varrho_0 = s_0 \implies s^{(0)} \simeq s^{(0)} \) where \( u_0 = \bar{u}_0 \) and \( t_0 = \bar{t}_0 \).
- **Induction \((i > 0)\):** Assume that the claim holds for \( \text{move}(\varrho_{i-1}) = a_1 b_1 \ldots a_{i-1} b_{i-1} \theta \), i.e., \( u_{i-1} = \bar{u}_{i-1} \) and \( t_{i-1} = \bar{t}_{i-1} \). For \( \gamma_l \) we have that \( u_i = \text{Eff}(a_i, u_{i-1}) \) and \( \bar{u}_i = \text{Eff}(a_i, \bar{u}_{i-1}) \). Also, \( t_i = \text{Eff}(b_i, t_{i-1}) \in \Omega_i \) and \( \bar{t}_i = \text{Eff}(b_i, \bar{t}_{i-1}) \). Hence, \( u_i = \bar{u}_i \), \( t_i \in \Omega_i \) and \( \gamma_i = \gamma_i = \bar{\gamma}_i \). Thus, \( s^{(i)} \sim s^{(i)} \) holds. The same can be shown for \( \text{move}(\varrho) = a_1 b_1 \ldots a_i \theta b_i \) where no \( \theta \) occurs. \( \square \)

**Theorem 1 (Probabilistic Simulation).** For any \( s_0 \simeq s_0 \) and \( \varrho \in \text{Prop}(G_H) \) where \( \text{first}(\varrho) = s_0 \), it holds that

\[
\Pr[\text{last}(\varrho) = s'] = \Pr[\left(\text{move}(\varrho)\right)(s_0) = s'] \quad \forall s', s' \text{ s.t. } s' \sim s'.
\]

**Proof.** We can rewrite \( \varrho \) as \( \varrho = \varrho_0 \circ \varrho_1 \cdots \varrho_{n-1} \circ \varrho_n \), where \( \varrho_0, \varrho_1, \ldots, \varrho_{n-1} \) are deterministic. Let \( \text{first}(\varrho_l) = s_l^{(l)}(t_l, u_l, \Omega_l) \), \( \text{last}(\varrho_l) = s_{l'}^{(l')}(t_{l'}, u_{l'}, \Omega_{l'}) \), and \( \left(\text{move}(\varrho)\right)(s_0) = s^{(n)}(t_n, u_n, w_n, j_n) \). We use induction over \( n \) as follows:

- **Base \((n = 0)\):** for \( \varrho \) to be deterministic and proper, \( \varrho = \varrho_0 = s^{(0)} \) holds.
- **Case \((n = 1)\):** \( \varrho_1 = \varrho(t_0, u_0) \). From Lemma 1, \( \bar{u}_1 = u_1 \) and \( \bar{t}_1 = t_1 \). Hence, \( \Pr[\text{last}(\varrho) = s^{(1)}] = \Pr[\left(\text{move}(\varrho)\right)(s_0) = s^{(1)}] = \varrho(t_0, u_0) \) and \( s^{(1)} \simeq \bar{s}_1^{(1)} \).
- **Induction \((n > 1)\):** assume that the previous case holds for \( \varrho_{n-2} \), i.e., \( s^{(n-1)} \simeq \bar{s}_1^{(n-1)} \). It is straightforward to infer that \( \varrho_n = \varrho(t_{n-1}, u_{n-1}) \), and hence \( \Pr[\text{last}(\varrho) = s^{(n)}] = \Pr[\left(\text{move}(\varrho)\right)(s^{(0)}) = s^{(n)}] = P \), and \( s^{(n)} \sim \bar{s}_1^{(n)} \). \( \square \)
Note that in case of multiple $\theta$ attempts, the above probability $P$ satisfies

$$P = \prod_{i=1}^{n} \sum_{j=1}^{m_i} p_i \left( t_{i-1}', u_{i-1}' \right) \left( 1 - p_{i-1} \left( t_{i-1}', u_{i-1}' \right) \right)^{(j-1)},$$

where $m_i$ is the number of $\theta$ attempts at stage $i$.

Finally, since Theorem 1 imposes no constraints on $move(\varrho)$, it holds for all possible moves. That is, a DAG can simulate all proper executions that exist in the corresponding HIG. Therefore, the following result holds.

**Theorem 2 (DAG-HIG Simulation).** For any HIG $\mathcal{G}_H$ there exists a DAG $\mathcal{G}_D \equiv \mathcal{D}[\mathcal{G}_H]$ such that $\mathcal{G}_D \rightsquigarrow \mathcal{G}_H$ (as defined in Def. 7).

4 Properties of DAG and DAG-based Synthesis

In this section, we discuss some of the DAG features and show how a DAG can be decomposed into a set of subgames by restricting the simulation to finite executions. We also examine preservation of DAG safety properties before proposing a DAG-based synthesis framework.

**Transitions.** In DAGs, nondeterministic actions of different players underline different semantics. Specifically, PL$_I$ nondeterminism captures what is known about the adversarial behavior, rather than exact actions, where PL$_I$ actions are constrained by the earlier PL$_{II}$ action. Conversely, PL$_{II}$ nondeterminism abstracts the player’s decisions. This distinction reflects how DAGs can be used for strategy synthesis under hidden information. To illustrate this, suppose that a strategy $\pi_{II}$ is to be obtained based on a worst-case scenario. In that case, the game is explored for all possible adversarial behaviors. Yet, if a strategy $\pi_{I}$ is known about PL$_I$, a counter strategy $\pi_{II}$ can be found by constructing $\mathcal{G}_D^{\pi_I}$.

Probabilistic behaviors in DAGs are captured by a virtual player PL$_{\varnothing}$, and is characterized by the transition function $\hat{\delta}: \hat{S}_\varnothing \times \hat{S}_{II} \rightarrow [0,1]$. The specific definition of $\hat{\delta}$ depends on the modeled system. For instance, if the transition function (i.e., the probability) is state-independent, i.e., $\hat{\delta}(\hat{s}_\varnothing, \hat{s}_{II}) = c, c \in [0,1]$, the obtained model becomes trivial. Yet, with a state-dependent transition function, i.e., $\hat{\delta}(\hat{s}_\varnothing, \hat{s}_{II}) = p(t, \hat{u})$, the probability that PL$_{II}$ successfully reveals the true value depends on both the belief and the true value, and the transition function can then be realized since $\hat{s}_\varnothing$ holds both $\hat{t}$ and $\hat{u}$.

**Decomposition.** Consider an execution $\hat{\varrho}^* = \hat{s}_0a_1\hat{s}_1a_2\hat{s}_2\ldots$ that describes a scenario where PL$_{II}$ performs infinitely many actions with no attempt to reveal the true value. To simulate $\hat{\varrho}^*$, the word $w$ needs to infinitely grow. Since we are interested in finite executions, we impose stopping criteria on the DAG, such that the game is trapped whenever $|w| = h_{max}$ is true, where $h_{max} \in \mathbb{N}$ is an upper horizon. We formalize the stopping criteria as a deterministic finite automaton (DFA) that, when composed with the DAG, traps the game whenever the stopping criteria hold. Note that imposing an upper horizon by itself is not a sufficient criterion for a DAG to be considered a stopping game. Conversely, consider a proper (and hence finite) execution $\hat{\varrho} = \hat{s}_0\ldots\hat{s}'$, where $\hat{s}_0, \hat{s}' \in$
Prop($G_D$). From Definition 9 it follows that a DAG initial state is strictly proper, i.e., $\hat{s}_0 \in \text{Prop}(G_D)$. Hence, when $\hat{s}'$ is reached, the game can be seen as if it is repeated with a new initial state $\hat{s}'$. Consequently, a DAG game (complemented with stopping criteria) can be decomposed into a (possibly infinite) countable set of subgames that have the same structure yet different initial states.

**Definition 10 (DAG Subgames).** The subgames of a $G_D$ are defined by the set \( \hat{G}_i \mid \hat{G}_i = \left\langle \hat{S}^{(i)}, (\hat{S}^{(i)}_I, \hat{S}^{(i)}_H, \hat{S}^{(i)}_O), A, \hat{s}_0^{(i)}, \hat{s}^{(i)} \right\rangle, \ i \in \mathbb{N}_0 \}, \) where $\hat{S} = \bigcup_i \hat{S}^{(i)}$, $\hat{S}_\gamma = \bigcup_i \hat{S}_\gamma^{(i)} \forall \gamma \in \Gamma$; and $\hat{s}_0^{(i)} = \hat{s}_0^{(i)}$ s.t. $\hat{s}_0^{(i)} \in \text{Prop}(G_D^{(i)})$, $\hat{s}_I^{(i)} \neq \hat{s}_H^{(i)} \forall i, j \in \mathbb{N}_0$.

Intuitively, each subgame either reaches a proper state (representing the initial state of another subgame) or terminates by an upper horizon. This decomposition allows for the independent (and parallel) analysis of individual subgames, as well as further utilization of the obtained results for studying the corresponding supergame; drastically reducing both the time required for synthesis and the explored state space, and hence improving scalability. An example of this decompositional approach is provided in Sec. 5.

**Preservation of safety properties.** In DAGs, the action $\theta$ denotes a transition from $PL_H$ to $PL_I$ states and thus the execution of any delayed actions. While this action can simply describe a revealing attempt, it can also serve as a what-if analysis of how the true value may evolve at stage $i$ of a subgame. We refer to an execution of the second type as a hypothetical branch, where Hyp$(\hat{\theta}, h)$ denotes the set of hypothetical branches from $\hat{\theta}$ at stage $h \in \{1, \ldots, n\}$. Let $L_{\text{safe}}(s)$ be a labeling function denoting if a state is safe. The formula $\Phi_{\text{safe}} := [G \text{ safe}]$ is satisfied by an execution $\theta$ in HIG iff all $s(t, u, \Omega) \in \theta$ are safe.

Now, consider $\hat{\theta}$ of the DAG, where $\hat{\theta} \sim \theta$. We identify three cases:

(i) $L_{\text{safe}}(s)$ depends only on belief value $u$, then $\theta \models \Phi_{\text{safe}}$ iff all $\hat{s}_H \in \hat{\theta}$ are safe.

(ii) $L_{\text{safe}}(s)$ depends only on true value $t$, then $\theta \models \Phi_{\text{safe}}$ iff all $\hat{s}_I \in \text{Hyp}(\hat{\theta}, n)$ are safe.

(iii) $L_{\text{safe}}(s)$ depends on both true and belief values $t, u$, then $\theta \models \Phi_{\text{safe}}$ iff last$((\hat{\theta}_h))$ is safe for all $\hat{\theta}_h \in \text{Hyp}(\hat{\theta}, h)$, $h \in \{1, \ldots, n\}$, where $n$ is the number of $PL_H$ actions. These relations are to be taken into account when using DAGs for synthesis.

**Synthesis Framework.** We here propose a framework for strategy synthesis using DAGs, summarized in Fig. 8. We start by formulating the automata $M_I$, $M_H$ and $M_O$, representing $PL_I$, $PL_H$ and $PL_O$ abstract behaviors, respectively. Next, a FIFO memory stack $(m_1, \ldots, m_n)_{i=1}^n \in A^n_H$ is implemented using two automata $M_{\text{mrd}}$ and $M_{\text{mwr}}$ to perform reading and writing operations, respectively. The DAG $G_D$ is constructed by following the procedure summarized in Algorithm 1.

Given a synthesis query $\phi_{\text{syn}}$ and the constructed DAG $G_D$, the procedure described in Algorithm 2 can be used for strategy synthesis. Starting with the initial subgame, the procedure checks for each horizon of size $h$ whether $\phi_{\text{syn}}$ is satisfied (hence a strategy exists) if $\theta$ is performed at stage $h$. These iterations are terminated when reaching the maximum search horizon $h_{\text{max}}$ or failing to satisfy $\phi_{\text{syn}}$. Subgames can then be pruned by discarding strategies that fail.
Model Refinement
Primary Components $\mathcal{M}_I, \mathcal{M}_{II}, \mathcal{M}_{\circ}$
Auxiliary Components $\mathcal{M}_{mrd}, \mathcal{M}_{mwr}$

Fig. 8. Synthesis and analysis framework based on the use of DAGs.

Algorithm 1: Procedure for DAG construction

| Step | Description |
|------|-------------|
| 1    | while ¬(end criterion) do |
| 2    | while $a \neq \emptyset$ do |
| 3    | $\mathcal{M}_{II}.v_B \leftarrow \text{Eff}(a, v_B), \mathcal{M}_{mwr}.\text{write}(a, ++wr)$ |
| 4    | while $wr \leq rd$ do |
| 5    | $\mathcal{M}_{mrd}.\text{read}(a, ++rd), \mathcal{M}_{I}.v_T \leftarrow \text{Eff}(\beta(a), v_T)$ |
| 6    | if $\text{draw } x \sim \text{Brn}(p(v_T, v_B))$ then |
| 7    | $\mathcal{M}_{II}.v_B \leftarrow \mathcal{M}_{I}.v_T, wr \leftarrow 0, rd \leftarrow 0$ |
| 8    | else $rd \leftarrow 0$ |

5 Case Study

In this section, we consider a case study which is an extension of our running example – a system where a human operator supervises a UAV prone to stealthy attacks on its GPS sensor. The UAV mission is to visit a number of targets after being airborne from a known base (initial state), while avoiding hazard zones that are known a priori. Moreover, the presence of adversarial stealthy attacks via GPS spoofing is assumed. We use the DAG framework to synthesize strategies for both the UAV and an operator advisory system (AS) that schedules geolocation tasks for the operator.

Modeling. We model the system as a delayed-action game $G_D$, where $PL_I$ and $PL_{II}$ represent the adversary and the UAV-AS coalition, respectively. Fig. 9 shows the model primary and auxiliary components. In the UAV model $\mathcal{M}_{\text{uav}}$, $x_B = (x_B, y_B)$ encodes the UAV belief, and $A_{\text{uav}} = \{N, S, E, W, NE, NW, SE, SW\}$ is the set of available movements. The AS can trigger the action $\text{activate}$ to initiate
Algorithm 2: Procedure for strategy synthesis

**Input:** Initial location \((x_0, y_0)\), synthesis query \(\phi_{\text{syn}}\)

**Output:** PL₁₁ strategies \(H_{\text{II}}\)

1. \(\ell \leftarrow [(x_0, y_0)], i \leftarrow 0, \text{stop} \leftarrow \bot\)
2. while \(i < |\ell|\) do
   3. \(s_0 \leftarrow (\ell[i], \ell[i], \epsilon, 0, \text{II})\)
   4. while \(h \leq h_{\text{max}} \land \neg \text{stop}\) do
   5. \((\pi_{\text{II}}, \varphi) \leftarrow \text{Synth}\left(G_{\text{so}}^n, \phi_{\text{syn}}\right)\)
   6. if \(\pi_{\text{II}} \neq \emptyset\) then
      7. \(\Pi_t \leftarrow \Pi_t \cup (\pi_{\text{II}}, \pi_h, \varphi)\)
   8. else \(\text{stop} \leftarrow \top\)
9. \(\text{Prune} (\Pi_t)\)
10. \(\ell \leftarrow \ell \cdot (\text{Reachable} (\Pi_t) \setminus \ell), i++\)

\(\text{Explore all reachable subgames}\)
\(\text{Construct initial state}\)
\(\text{Explore subgame till upper horizon}\)
\(\text{Synthesize strategy for horizon h}\)
\(\text{Save synthesized strategy}\)
\(\text{Prune subgame strategies}\)
\(\text{Update reachability}\)

Fig. 9. Primary DAG components: UAV (\(M_{\text{uav}}\)), adversary (\(M_{\text{adv}}\)), and AS (\(M_{\text{as}}\)). Auxiliary DAG components: memory write (\(M_{\text{mwr}}\)) and memory read (\(M_{\text{mrd}}\)) models, capturing DA representation. At stage \(i\), the next memory location to write/read is \(m_i\).

A geolocation task, attempting to confirm the current location. The adversary behavior is abstracted by \(M_{\text{adv}}\), where \(x_T = (x_T, y_T)\) encodes the UAV true location. The adversarial actions are limited to one directional increment at most.\(^7\) If, for example, the UAV is heading \(N\), then the adversary set of actions is \(\beta(N) = \{N, NE, NW\}\). The auxiliary components \(M_{\text{mwr}}\) and \(M_{\text{mrd}}\) manages a FIFO memory stack \((m_i)_{i=0}^{n} \in A_{\text{mwr}}\). The last UAV movement is saved in \(m_i\) by synchronizing \(M_{\text{mwr}}\) with \(M_{\text{nav}}\) via \text{write}, while \(M_{\text{mrd}}\) synchronizes with \(M_{\text{adv}}\) via \text{read} to read the next UAV action from \(m_j\). The subgame terminates whenever action \text{write} is attempted and \(M_{\text{mwr}}\) is at state \(n\) (i.e., out of memory).

Let \((x_0, y_0)\) be the initial UAV location that is known to be true. The goal is to find strategies for the UAV-AS coalition based on the following requirements:

- **Target reachability.** To overcome cases where targets are not reachable due to hazard zones, checkpoints can be selected as intermediate targets to render the objective incrementally feasible. Hence, the label \(\text{reach}\) is assigned to the set of states with acceptable checkpoint locations (including

\(^7\) To detect aggressive attacks, techniques from literature (e.g., [21][22][15]) can be used.
the target), where the objective for all encountered subgames is formalized as $\Pr_{\max}[\text{reach}] \geq p_{\min}$ for some bound $p_{\min}$.

- **Hazard Avoidance.** Similar to target reachability, the label $\text{hazard}$ is assigned to the states corresponding to hazard zones. The objective $\Pr_{\max}[\text{¬hazard}] \geq p_{\min}$ is then specified for all encountered subgames.

By refining the aforementioned objectives, synthesis queries are used for both the subgames and the supergame. Specifically, the query

$$\phi_{\text{syn}}(k) := \langle\langle\text{uav}\rangle\rangle \Pr_{\max=?}[\neg\text{hazard} \cup^{\leq k} (\text{locate} \land \text{reach})]$$  \hspace{1cm} (1)

is specified for each encountered subgame $\hat{G}_i$, where $\text{locate}$ indicates a successful geolocation task. By following Algorithm 2 for a $q$ number of reachable subgames, the supergame is reduced to an MDP $G_{\{\pi_i\}}^{q}_i = 1$ (whose states are the reachable subgames), which is checked against the query

$$\phi_{\text{ana}}(n) := \langle\langle\text{adv}\rangle\rangle \Pr_{\min,\max=?}[\text{F}^{\leq n} \text{target}]$$  \hspace{1cm} (2)

to find the bounds on the probability that the target is reached under a maximum number of geolocation tasks $n$.

**Experimental Results.** Fig. 10(a) shows the map setting used for implementation. The UAV’s ability to actively detect an attack depends on both its belief and the ground truth. Specifically, the probability of success in a geolocation task mainly relies on the disparity between the belief and true locations, captured by $f_{\text{dis}}: Ev(x_B) \times Ev(x_T) \rightarrow [0, 1]$. We also include the map boundaries to the set of hazard zones to prevent the UAV from reaching boundary values. For the UAV to ever arrive to any designated target, a practical assumption is to prohibit the adversary from launching any attack for at least the first step, preventing the UAV model from infinitely bouncing around the target location.

We implemented the model in PRISM-games [7] and performed the experiments on an Intel Core i7 4.0 GHz CPU, with 10GB RAM dedicated to the tool. Fig. 10(b) shows the supergame obtained by following the procedure in Algorithm 2. A vertex $\hat{G}_{xy}$ represents a subgame (composed with its strategy) that starts at location $(x, y)$, while the outgoing edges points to subgames reachable from the current one. Note that each edge represents a probabilistic transition. Subgames with more than one outgoing transition imply nondeterminism that is resolved by the adversary actions. Hence, the directed graph depicts an MDP.

By analyzing the supergame against $\phi_{\text{ana}}$ from (2), we study the effect of the maximum number of geolocation tasks ($n$) on the probability to reach the target. As shown in Fig. 11(b), the minimum number of geolocation tasks to guarantee a non-zero minimum probability of reaching the target (regardless of the adversary

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8 Note that an LTL query can be expressed as a deterministic Rabin automaton (DRA) that accepts only the paths satisfying that query. The DAG and DRA are further composed to perform model checking (see [9]).

9 In this case study, $f_{\text{dis}}$ is obtained by assigning probabilities for each pair of locations according to their features (e.g., presence of landmarks) and further smoothed using a Gaussian 2D filter. A thorough experimental analysis, where probabilities are extracted from experiments with human operators is described in [11].
strategy) is 3, where the probability bounds obtained are (33.7%, 94.4%). By allowing for an extra geolocation task (i.e., $n = 4$), the minimum probability becomes almost twice as before (63.8%), while the upper bound slightly increases (98.9%). The increase in the lower bound slows down until both bounds level off at $n=8$. Note that these results include the geolocation task at the target itself.

For the initial subgame, Fig. 11(a) shows the maximum probability of a successful geolocation task if performed at stage $h$, and the remaining distance to target. The trends show that, assuming the adversary can launch attacks after the stage $h_{adv} = 2$, the maximum probability of detection can be achieved by performing the geolocation task at step 4 (for all possible adversary strategies). For $h_{adv} = 1$, however, a geolocation task at stage $h=3$ has the highest probability of success, which drops to 0 at $h=6$; a stage where no possible flight plan exists without encountering a hazard zone. Conversely, for $h_{adv} = 2$, hazard areas can still be avoided up till $h = 6$. The synthesized strategy for ($h_{adv} = 2, h = 4$) is demonstrated in Fig. 10(c).

The experimental data obtained for this case study are listed in Table 1. Note that, for the same grid size, more complex maps require more time for
Table 1. Results for strategy synthesis using queries $\phi_{\text{syn}}$ and $\phi_{\text{ana}}$.

| Subgame $\hat{G}$ | Model Size | Time (sec) |
|-------------------|-------------|-------------|
|                   | States      | Transitions | Choices |
|                   | $\phi_{\text{syn}}$ | $\phi_{\text{ana}}$ |
| $8 \times 8$ 1 | 11,608      | 17,397      | 15,950  |
|                  | 2.810       | 0.072       | –       |
|                  | 57,129      | 87,865      | 83,267  |
|                  | 14.729      | 0.602       | –       |
|                  | 236,714     | 366,749     | 359,234 |
|                  | 62.582      | 1.293       | –       |
|                  | 876,550     | 1,365,478   | 1,355,932 |
|                  | 231.741     | 6.021       | –       |
| $2 \times 4$ 2 | 6,078       | 9,230       | 8,394   |
|                  | 2.381       | 0.042       | –       |
|                  | 33,904      | 48,545      | 45,354  |
|                  | 10.251      | 0.367       | –       |
|                  | 141,622     | 204,551     | 198,640 |
|                  | 37.192      | 1.839       | –       |
|                  | 524,942     | 763,144     | 754,984 |
|                  | 145.407     | 8.850       | –       |
| Supergame $\phi_0$ | 6,212      | 8,306      | 6,660   |
|                  | 2.216       | –           | 2.490   |

synthesis while the state space size remains unaffected. The number of states grows exponentially with the explored horizon size, i.e., $O((|A_{uav}||A_{adv}|)^h)$, and is typically slowed by, e.g., the presence of hazard areas, since the branches of the game transitions are trimmed upon encountering such areas. Interestingly, for $h=6$ and $h=7$, while the model construction time (size) for $h_{adv}=1$ is almost twice (quadruple) as those for $h_{adv}=2$, the time for checking $\phi_{\text{syn}}$ declines in comparison. This reflects the fact that, in case of $h_{adv}=1$ compared to $h_{adv}=2$, the UAV has higher chances to reach a hazard zone for the same $k$, leading to a shorter time for model checking.

6 Discussion and Conclusion

In this paper, we have introduced Delayed Action Games (DAGs) and have shown how DAGs can simulate Hidden-Information Games (HIGs) by delaying players’ actions. We have also derived a DAG-based framework that allows for strategy synthesis and analysis using off-the-shelf model checkers that support Stochastic Multiplayer Games (SMGs). Under some practical assumptions, we have demonstrated that DAGs can be decomposed into a set of independent subgames. This decomposition provides the advantage of utilizing parallel computation to reduce the time needed for model analysis, as well as the size of the state space. We have further demonstrated the applicability of the proposed framework on a case study focused on synthesis and analysis of active attack detection strategies for UAVs prone to cyber attacks.

The concept of delaying actions implicitly assumes that the adversary knows the UAV actions a priori. This does not present a concern in the presented case study as an abstract (i.e., nondeterministic) adversary model is analogous to synthesizing against the worst-case attacking scenario. That being said, it is clear that strategies synthesized using DAGs (and SMGs in general) are inherently conservative. Depending on the investigated system, this can easily lead to finding no feasible solution. Future work will include more case studies, e.g., scheduling active attack detection for multiple-UAV systems, and how synthesis based on multiple objectives can be utilized in DAGs.

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