Samade Probability Distribution: Its Properties and Application to Real Lifetime Data

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Author’s contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

A new two-parameter lifetime distribution has been proposed in this study. The distribution is called Samade distribution. The model is motivated by the wide use of the lifetime models derived from the mixture of gamma and exponential distributions. Its mathematical properties which include the first four moments, variance as well as coefficient of variation, reliability function, hazard function, survival function, Renyi entropy measure and distribution of order statistics have been successfully derived. The maximum likelihood estimation of its parameters and application to real life data have been discussed. Application of this model to three real datasets shown that the proposed model yields a satisfactorily better fit than other existing lifetime distributions. The comparism of goodness-of-fits were established using -2Loglikelihood, AIC and BIC.

Keywords: Samade distribution; moments; reliability measure; probability of order statistics; maximum likelihood estimation.

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1 Introduction

In the recent time, many authors have proposed different distributions for modeling real lifetime data, which are paramount in many applied sciences like biomedical, environmental, insurance and finance, engineering, to mention a few. These distributions, such as generalized exponential and gamma, are quite flexible in nature and are appropriate for modeling a wide range of data showing constant, monotonic or non-monotonic failure rate property [1].

Recently, a one-parameter distribution called Pranav distribution was proposed by Shukla [2]. The distribution is a mixture of exponential distribution with scale parameter \( \theta \) and gamma distribution with shape parameter 4 and scale parameter \( \theta \) while their mixing proportions are \( \frac{\theta^4}{\theta^4 + 6} \) and \( \frac{6}{\theta^4 + 6} \).

\[
 f_p(x; \theta) = pg_1(x; \theta) + (1 - p)g_2(x; 4, \theta),
\]

where

\[
 p = \frac{\theta^4}{\theta^4 + 6}, \quad g_1(x; \theta) = \theta e^{-\theta x} \quad \text{and} \quad g_2(x; 4, \theta) = \frac{\theta^4 x^3 e^{-\theta x}}{6}
\]

Therefore, the probability density function (pdf) of the Pranav distribution, \( f_p(x|\theta) \), was defined as:

\[
 f_p(x, \theta) = \frac{\theta^4}{\theta^4 + 6}(\theta + x^3)e^{-\theta x}, \quad x > 0
\]

where \( \theta > 0 \).

Several other work have been done on mixture and generalization of Pranav distribution by many authors including Uwaeme and Akpan [3], Umeh and Ibenegbu [4]. The Sujatha distribution was proposed by Shanker [5]. The author studied the properties of the model and compared its goodness of fit with some existing models to model biomedical sciences data. Similarly, much work have been done on the generalization and mixture of Sujatha distribution by different researchers including Shanker and Hagos [6], Aderoju [7], Tesfay and Shanker [8].

Lindley [9] used a mixture of exponential and length-biased exponential distributions to illustrate the difference between fiducial and posterior distributions [10]. This mixture is called the Lindley distribution (LD). More studies have been done on the properties, generalization and mixture of the Lindley distribution by Ghitany et al. [11], Nadarajah et al. [12], Ghitany et al. [13], Shanker and Mishra [14], Zakerzadeh and Dolati [15], Aderoju et al. [16], Olayide and Yang [10].

The aim of this paper is to propose, study the properties and application of a new two-parameter distribution called Samade distribution, which is a mixture of exponential and gamma distributions at a specific proportion.

The proposed probability distribution is presented in Section 2 and study its mathematical properties in section 3. The corresponding Maximum Likelihood estimates are discussed in Section 4. In Section 5, we discuss its application to real life examples and we present concluding remarks in Section 6.

2 Materials and Methods

In this section, the probability density function (pdf) of a new exponential-gamma mixture distribution called Samade distribution (SD) is derived, which is given by

\[
 f(x|\alpha, \theta) = \begin{cases} 
 \frac{\theta^4}{\theta^4 + 6\alpha} (\theta + ax^3)e^{-\theta x}, & \text{if } x > 0, \\
 0, & \text{elsewhere}
\end{cases}
\] (1)

where $\theta > 0$ is a scale parameter and $\alpha > 0$ is a shape parameter. The two-parameter lifetime distribution is a mixture of exponential ($\theta$) and gamma ($4, \theta$) distributions, we have

$$f(x|\alpha, \theta) = pg_1(x; \theta) + (1 - p)g_2(x; 4, \theta),$$

where

$$p = \frac{\theta^4}{\theta^4 + 6\alpha}, \quad g_1(x; \theta) = \theta e^{-\theta x} \quad \text{and} \quad g_2(x; 4, \theta) = \frac{\theta^4 x^3 e^{-\theta x}}{6}$$

Hence, (1) is defined as:

$$f(x|\alpha, \theta) = \frac{\theta^4}{\theta^4 + 6\alpha}(\theta + \alpha x^2)e^{-\theta x}, \quad x > 0$$

where $\theta > 0$ and $\alpha > 0$.

Note that when $\alpha = 1$ and $\alpha = 0$, $f(x|\alpha, \theta)$ reduces to the one-parameter Pranav distribution and exponential distribution, respectively.

Moreover, the corresponding cumulative distribution function (CDF) of (1) is:

$$F(x) = 1 - \left(\frac{\theta^4 + \alpha \left(6 + x\theta(6 + x\theta(3 + x\theta))\right)}{6\alpha + \theta^4}\right)e^{-\theta x}, \quad x > 0$$

where $\theta > 0$ and $\alpha > 0$.

Graphs of (1) and (2) are shown in Figs. 1 and 2 for varying values of parameters $\theta$ and $\alpha$ respectively.
3 Mathematical Properties

We present some of the mathematical characteristics of the Samade distribution in this section. The properties include the moments, reliability analysis, entropy and the order statistics.

3.1 Moments of the Samade distribution

The $r^{th}$ moment of a random variable $X$ with Samade distribution is given by

$$E(X^r) = \mu_r = \int_0^\infty x^r f(x|\alpha, \theta) dx,$$

where $r$ is a positive integer.

$$= \int_0^\infty x^r \frac{\theta^4}{\theta^4 + 6\alpha}(\theta + ax^3)e^{-\theta x} dx = \frac{r!\theta^4 + (r+3)\alpha}{\theta^r(\theta^4 + 6\alpha)}$$  \hspace{1cm} (3)

From (3), we can express the first four moment as $(r = 1, 2, 3, 4)$:
\[\mu_1 = \frac{24\alpha + \theta^4}{6\alpha \theta + \theta^5}\]
\[\mu_2 = \frac{2(60\alpha + \theta^4)}{6\alpha \theta^2 + \theta^6}\]
\[\mu_3 = \frac{6(120\alpha + \theta^4)}{6\alpha \theta^3 + \theta^7}\]
\[\mu_4 = \frac{840}{\theta^4} - \frac{816}{6\alpha + \theta^4}\]

Note that, the variance \((\sigma^2)\) of the random variable \(X\) can be obtained as:

\[\sigma^2 = E(X^2) - [E(X)]^2 = \mu_2 - [\mu_1]^2 = \frac{144\alpha^2 + 84\alpha\theta^4 + \theta^8}{(6\alpha \theta + \theta^5)^2}\]

The coefficient of variation (CV) and the index of dispersion \((\gamma)\) of SD are obtained as:

\[CV = \frac{\sigma}{\mu_1} = \sqrt{\frac{144\alpha^2 + 84\alpha\theta^4 + \theta^8}{6\alpha \theta + \theta^5}} - \sqrt{\frac{(24\alpha + \theta^4)(6\alpha \theta + \theta^5)}{24\alpha + \theta^4}}\]
\[\gamma = \frac{\sigma^2}{\mu_1} = \frac{144\alpha^2 + 84\alpha\theta^4 + \theta^8}{144\alpha^2 \theta + 30\alpha \theta^5 + \theta^9}\]

3.2 Renyi’s entropy

The Renyi’s entropy of a random variable \(X\) with density function, \(f(x|\alpha, \theta)\), as defined by Renyi [17] is given by

\[R_\mu(x|\alpha, \theta) = \frac{1}{1-p} \log \int_0^{\infty} [f(x|\alpha, \theta)]^p dx\]  
\[= \frac{1}{1-p} \log \left[ \frac{\theta^4}{\theta^4 + 6\alpha} \left(\theta + \alpha x^3 \right) e^{-\theta x} \right]^p dx\]
\[= \frac{1}{1-p} \log \left[ \int_0^{\infty} \frac{\theta^4}{\theta^4 + 6\alpha} \left(\theta + \alpha x^3 \right) e^{-\theta x} dx \right]^p\]
\[= \frac{1}{1-p} \log \left[ \int_0^{\infty} \frac{\theta^4}{\theta^4 + 6\alpha} \left(1 + \frac{\alpha}{\theta} x^3 \right) e^{-\theta x} dx \right]^p\]
\[= \frac{1}{1-p} \log \left[ \frac{\theta^4}{\theta^4 + 6\alpha} \sum_{j=0}^{\infty} \frac{\theta^j}{\theta^j + 6\alpha} \int_0^{\infty} x^{3j+1-1} e^{-\theta x} dx \right]^p\]
\[= \frac{1}{1-p} \log \left[ \frac{\theta^4}{\theta^4 + 6\alpha} \sum_{j=0}^{\infty} \frac{\theta^j}{\theta^j + 6\alpha} \frac{\Gamma(3j + 1)}{(\theta)^{3j+1}} \right]^p\]
where

\[ p \geq 0 \quad \text{with} \quad p \neq 0. \]

### 3.3 Order statistics

Order statistics gives one of the popular fundamental tools for obtaining inference related to reliability data. For independent copies \( X_1, X_2, \ldots, X_n \) of a random variable \( X \) with Samade distribution, we have that the largest and smallest orders are denoted and defined by \( X_n = \max(X_1, X_2, \ldots, X_n) \) and \( X_1 = \min(X_1, X_2, \ldots, X_n) \) respectively. Here, \( n \) is the sample size. Observe that \( X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)} \) are order observations of \( X_1, X_2, \ldots, X_n \). Moreover, the pdf of \( X_{(k)} \), \( k^{th} \) order statistic can be expressed as

\[
 f_{X(k)}(x|\alpha, \theta) = \frac{n!}{(k-1)!(n-k)!} f(x) [F(x)]^{k-1} [1 - F(x)]^{n-k}
\]

(5)

From (5) we can write

\[
 f_{X(k)}(x|\alpha, \theta) = \frac{n! \theta^4}{(k-1)!(n-k)!} \left( \frac{\theta^4 + 6\alpha}{6\alpha + \theta^4} \right)^{(k-1)} \left( \frac{\theta^4 + \alpha \left( 6 + x\theta(3 + x\theta) \right)}{6\alpha + \theta^4} \right)^{n-k}
\]

where

\[ p \geq 0 \quad \text{with} \quad p \neq 0. \]

The functions of the first and \( n^{th} \) order statistics are:

\[
 f_{X(1)}(x|\alpha, \theta) = \frac{n \theta^4}{(n-1)!(\theta^4 + 6\alpha)} \left( \frac{\theta^4 + 6\alpha}{6\alpha + \theta^4} \right)^{n-1} \left( \frac{\theta^4 + \alpha \left( 6 + x\theta(3 + x\theta) \right)}{6\alpha + \theta^4} \right)^{x\theta}
\]

and

\[
 f_{X(n)}(x|\alpha, \theta) = \frac{n \theta^4}{(\theta^4 + 6\alpha)} \left( \frac{\theta^4 + \alpha \left( 6 + x\theta(3 + x\theta) \right)}{6\alpha + \theta^4} \right)^{n-1} \left( 1 - \frac{\theta^4 + \alpha \left( 6 + x\theta(3 + x\theta) \right)}{6\alpha + \theta^4} \right)^{x\theta}
\]

Respectively, where \( p \geq 0 \quad \text{with} \quad p \neq 0. \)

### 3.4 Reliability analysis

The reliability analysis of any given pdf is always considered based on the survival function, \( S \), and the hazard rate function, \( H \), which were derived as shown below.
3.4.1 Survival function

The survival function is generally defined as the probability that an item does not fail prior to some time. It is expressed as

\[ S(x|\alpha, \theta) = 1 - F(x|\alpha, \theta) = \frac{\left(\theta^4 + \alpha(6 + 6x\theta + 3x^2\theta^2 + x^3\theta^3)\right)}{6\alpha + \theta^4}e^{-x\theta}, \]

3.4.2 Hazard rate function

The hazard rate function can be expressed as the conditional probability of failure, given that it has survived to the time. It is given by

\[ h(x|\alpha, \theta) = \frac{f(x|\alpha, \theta)}{S(x|\alpha, \theta)} = \frac{\theta^4(\theta + ax^3)e^{-\theta x}}{(\theta^4 + \alpha(6 + 6x\theta + 3x^2\theta^2 + x^3\theta^3))\ e^{-\theta x}} = \frac{\theta^4(x^3\alpha + \theta)}{\theta^4 + \alpha(6 + 6x\theta + 3x^2\theta^2 + x^3\theta^3)} \]

where \( \theta > 0 \) and \( \alpha > 0 \).

Figs. 3 and 4 represent the graph of the survival function and hazard rate function of the Samade distribution for varying values of the parameters \( \alpha \) and \( \theta \). The hazard rate function graph shows a monotonically increasing shape.

![Fig. 3. Survival function plots of Samade distribution for varying values of the parameters \( \theta \) and \( \alpha \)](image)

![Fig. 4. Hazard rate function plot of Samade distribution for varying values of the parameters \( \theta \) and \( \alpha \)](image)

4 Maximum Likelihood Estimation

Suppose \( X_1, X_2, X_3, \ldots, X_n \) is a random sample of size \( n \) from the Samade distribution, the maximum likelihood function of parameters can be written as
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\[ L(\alpha, \theta) = \prod_{i=1}^{n} \frac{\theta^4}{\theta^4 + 6\alpha} (\theta + \alpha x_i^3) e^{-\theta x_i}, \]

and the log-likelihood function is

\[ \ell(\alpha, \theta) = \log(L(\alpha, \theta)) = n \log(\theta) - n 4 \log(\theta) - n \log(6\alpha) + \sum_{i=1}^{n} \log(\theta + \alpha x_i^3) - \theta \sum_{i=1}^{n} x_i \]

Differentiating the log \((L(\alpha, \theta))\) partially with respect to associated parameters we have

\[ \frac{\partial \log(L(\alpha, \theta))}{\partial \alpha} = -\frac{n}{\alpha} + \sum_{i=1}^{n} x_i^3 (\theta + \alpha x_i^3)^{-1} = 0 \quad (6) \]

\[ \frac{\partial \log(L(\alpha, \theta))}{\partial \theta} = \frac{n}{\theta} - \frac{4n}{\theta} + \sum_{i=1}^{n} (\theta + \alpha x_i^3)^{-1} - \sum_{i=1}^{n} x_i = 0 \quad (7) \]

where \(\theta > 0\) and \(\alpha > 0\).

The Maximum Likelihood Estimates (MLEs), \(\hat{\alpha}\) and \(\hat{\theta}\), of \(\alpha\) and \(\theta\) are solutions of equation (6) and (7). Obviously, analytical expressions for \(\hat{\alpha}\) and \(\hat{\theta}\) are not available. Hence, we computed the MLEs numerically using the \textit{nloptr} package and \textit{bobyqa} function in R software [18].

5 Application to Real Dataset

In this section we used three datasets as detailed below to evaluate the application of the Samade distribution (SD) and compared its goodness of fit with Exponential distribution (ED), quasi Lindly distribution (QLD) by Shanker and Mishra [14], quasi-Sujatha distribution (QSD) by Shanker [19] and Pranav distribution (PD). To evaluate and compare the goodness of fit of the distribution with the newly introduced SD, we obtain the MLEs’ of the parameters, -2lnL, Akaike Information Criteria (AIC) and Bayesian information criteria (BIC).

Dataset 1: This dataset is the strength data of glass of the aircraft window reported by Fuller et al. [20].

18.83 20.80 21.657 23.03 24.05 24.321 25.5 25.52 25.8 26.69
26.77 26.78 27.05 27.67 29.9 31.11 33.20 33.73 33.76 33.89 34.76
35.75 35.91 36.98 37.08 37.09 39.58 44.045 45.29 45.381

Dataset 2: The following dataset represents the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20 mm taken from Bader and Priest [21].

1.312, 1.314, 1.479, 1.552, 1.700, 1.803, 1.861, 1.865, 1.944, 1.958, 1.966, 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.140, 2.179, 2.224, 2.240, 2.253, 2.270, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.426, 2.434, 2.435, 2.478, 2.490, 2.511, 2.514, 2.535, 2.554, 2.566, 2.570, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697, 2.726, 2.770, 2.773, 2.800, 2.809, 2.818, 2.821, 2.848, 2.880, 2.954, 3.012, 3.067, 3.084, 3.090, 3.096, 3.128, 3.233, 3.433, 3.585, 3.585

Dataset 3: The dataset represents the strength of 1.5cm glass fibers measured at the National Physical Laboratory, England; they are taken from Smith and Naylor [22].

0.55, 0.93, 1.25, 1.36, 1.49, 1.52, 1.58, 1.61, 1.64, 1.68, 1.73, 1.81, 2.00, 0.74, 1.04, 1.27, 1.39, 1.49, 1.53, 1.59, 1.61, 1.66, 1.68, 1.76, 1.82, 2.01, 0.77, 1.11, 1.28, 1.42, 1.50, 1.54, 1.60, 1.62, 1.66, 1.69, 1.76, 1.84, 2.24, 0.81, 1.13, 1.29, 1.48, 1.50, 1.55, 1.61, 1.62, 1.66, 1.70, 1.77, 1.84, 0.84, 1.06, 1.30, 1.48, 1.51, 1.55, 1.51, 1.63, 1.67, 1.70, 1.78, 1.89
Table 1. MLEs’, -2lnL, AIC and BIC of the fitted models of the three datasets

| Dataset | Model | Expected Mean (Observed mean) | MLEs | -2lnL | AIC | BIC |
|---------|-------|-------------------------------|------|-------|-----|-----|
| Data 1 | SD    | 30.8114 (30.8114)             | \( \hat{\alpha} = 40.1552 \) \( \hat{\theta} = 0.1298 \) | 232.7728 | 236.7727 | 239.6407 |
|         | PD    | 30.8117 (30.8114)             | \( \hat{\alpha} = 0.0050 \) \( \hat{\theta} = 0.0959 \) | 241.3166 | 245.3165 | 248.1845 |
|         | QSD   | 30.8086 (30.8114)             | \( \hat{\alpha} = 0.0003 \) \( \hat{\theta} = 0.0649 \) | 252.3182 | 256.3182 | 259.1862 |
|         | QLD   | 30.7873 (30.8114)             | \( \hat{\alpha} = 0.0325 \) \( \hat{\theta} = 0.0649 \) | 274.5288 | 276.5289 | 281.3969 |
|         | ED    | 30.8017 (30.8114)             | \( \hat{\alpha} = 9998.9 \) \( \hat{\theta} = 1.6316 \) | 166.6939 | 170.6939 | 175.1621 |
|         | PD    | 2.4514 (2.4513)               | \( \hat{\alpha} = 1.6251 \) \( \hat{\theta} = 1.2251 \) | 217.1224 | 219.1224 | 225.5906 |
|         | QSD   | 2.4511 (2.4513)               | \( \hat{\alpha} = 0.0010 \) \( \hat{\theta} = 1.0806 \) | 198.4925 | 202.4925 | 206.9607 |
|         | QLD   | 2.4507 (2.4513)               | \( \hat{\alpha} = 0.0017 \) \( \hat{\theta} = 0.8157 \) | 211.4644 | 215.4644 | 219.9326 |
|         | ED    | 2.4513 (2.4513)               | \( \hat{\alpha} = 0.4079 \) \( \hat{\theta} = 0.8157 \) | 261.7352 | 263.7352 | 270.2034 |
| Data 3  | SD    | 1.5071 (1.5068)               | \( \hat{\alpha} = 7499.1 \) \( \hat{\theta} = 2.6518 \) | 93.7870  | 97.7870  | 102.0733 |
|         | PD    | 1.6072 (1.5068)               | \( \hat{\alpha} = 1.5607 \) \( \hat{\theta} = 1.5607 \) | 180.9627 | 182.9627 | 189.249 |
|         | QSD   | 1.5087 (1.5068)               | \( \hat{\alpha} = 0.003 \) \( \hat{\theta} = 1.6872 \) | 124.3834 | 128.3834 | 132.6696 |
|         | QLD   | 1.5087 (1.5068)               | \( \hat{\alpha} = 0.0003 \) \( \hat{\theta} = 1.3272 \) | 132.6521 | 136.6521 | 140.9384 |
|         | ED    | 1.5068 (1.5068)               | \( \hat{\alpha} = 0.6636 \) \( \hat{\theta} = 0.6636 \) | 177.6606 | 179.6606 | 185.9469 |

In Table 1 above, we present the observed mean with the corresponding expected mean, values of MLEs’ of the parameter of the models, -2lnL, AIC and BIC. We find that the proposed Samade distribution fit the three datasets better than the Pranav distribution, QSD, QLD and Exponential distribution.

6 Concluding Remarks

The Samade distribution, which is a mixture of exponential and gamma distributions, has been successfully developed; its mathematical properties which include the first four moments, variance, CV, reliability function, hazard function, survival function, quantile, Renyi entropy measure and distribution of order statistics have been successfully derived. The plots of its pdf, CDF, hazard function and survival function are presented at varying values of the parameters. Application of this model were made to three real life datasets and the results shown that the proposed model gives a satisfactorily better fit consistently than exponential, Pranav, quasi-Lindley and quasi-Sujatha distributions for all the three datasets.

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Competing Interests

Author has declared that no competing interests exist.

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