Learning Representation over Dynamic Graph using Aggregation-Diffusion Mechanism

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Representation learning on graphs that evolve has recently received significant attention due to its wide application scenarios, such as bioinformatics, knowledge graphs, and social networks. The propagation of information in graphs is important in learning dynamic graph representations, and most of the existing methods achieve this by aggregation. However, relying only on aggregation to propagate information in dynamic graphs can result in delays in information propagation and thus affect the performance of the method. To alleviate this problem, we propose an aggregation-diffusion (AD) mechanism that actively propagates information to its neighbor by diffusion after the node updates its embedding through the aggregation mechanism. In experiments on two real-world datasets in the dynamic link prediction task, the AD mechanism outperforms the baseline models that only use aggregation to propagate information. We further conduct extensive experiments to discuss the influence of different factors in the AD mechanism.

Index Terms—Dynamic Graph, Representation Learning, Aggregation, Diffusion.

I. INTRODUCTION

Representation learning on graph structured data has recently received significant attention due to its wide application scenarios in various domains such as social networks, knowledge graphs, and bioinformatics. Recently, graph neural networks (GNNs)⁹, ¹², ¹³ have been applied to learn high-dimensional and non-Euclidean graph information efficiently. However, most of the existing GNNs are designed for static graphs. Graphs tend to evolve in real-world application scenarios. For example, a new friendship will be established between people in social networks, and human interaction in social networks changes during an epidemic.

Dynamic graphs can be represented in discrete or continuous form⁴, and in this paper, we focus on the continuous form representation, where no temporal aggregation is applied on the graph. Temporal point process based models⁵, ¹⁰, ¹⁷, ¹⁸ are emerging to address representation learning for dynamic networks with continuous representation. The temporal point process (TPP) is modeled by events $o^t = (u, v, t, k)$ where $u$ and $v$ are the interacting nodes, $t$ is the time of the event, and $k$ is the category of this event.

The TPP is parameterized by an RNN. When an event occurs, the previous embeddings of neighbors of the interacting node are aggregated and fed into this RNN to update the interacting node. This node update mechanism assumes that long-distance structural information and timely temporal information can be propagated to interacting nodes by aggregation mechanisms. However, this assumption does not always hold. Fig. 1 shows an example of problems with information propagation based on aggregation mechanisms:

- The information of $a$ at time $t_1^+$ is propagated to $b$ through aggregation when the event occurs at time $t_2$. By serving node $c$ as a bridge, node $e$ can also access the information of node $a$ at time $t_1^+$. However, before $t_6$, the information of $a$ is updated twice at $t_4$ and $t_5$, so $c$ and $e$ receive very laggered information of $a$.
- $b$ is hub-node for $\{a, f, g\}$ and $\{c, d, e\}$, which means that no information can be exchanged between $\{a, f, g\}$ and $\{c, d, e\}$ unless some event happens on $b$, or some event happens between $\{a, f, g\}$ and $\{c, d, e\}$.

To alleviate this limitation, we introduce the aggregation-diffusion (AD) mechanism into TPP-based dynamic network representation learning. The core operation of the AD mechanism is that the interacting nodes actively diffuse the changes to their neighbors after their information is updated by aggregating information about their neighbors. The main intuition behind this mechanism is twofold: 1) aggregation: a node’s information is affected by its neighbors; 2) diffusion: changes in the node itself should be sensed by its neighbors in time, even if there is no explicit event occurring.

We use two representative TPP-based models, DyRep⁵ and LDG⁶, as backbone model to demonstrate the effectiveness of the AD mechanism.
and scalability of the AD mechanism. On two dynamic graph datasets, Social Evolution[9] and Github[4], the model with AD mechanism produce significant performance improvement on both Mean Average Rank (MAR) and HITS@10 compared to the original DyRep(LDG) model. We further conduct extensive experiments to discuss the influence of different factors in the aggregation-diffusion mechanism.

The remainder of this paper is organized as follows: In Section II we introduce related work. In Section III we describe relevant details of the DyRep and LDG. In Section IV we explain AD mechanism in detail. In Section V we give the details of the experiment settings. In Section VI we present the experiment results and discuss different impact factors of the AD mechanism. In the final section, we present the conclusion.

II. RELATED WORK

A. Discrete Dynamic Embedding Approaches

Discrete dynamic embedding approaches treat dynamic graph as a sequence of graph snapshots. Most of discrete dynamic embedding approaches[10], [11], [12], [13], [14] focus on the learning representations of entire dynamic graphs rather than node representations. Some approaches[15], [16], [17], [18] are starting to focus on the dynamic representation at node level, they encode each graph snapshot using static embedding approaches[19], [20], [21], [22], [23], [2] to embed each node, and then combines some time-series models (e.g. LSTM[24], RNN[25]) for per node to model the discrete dynamic.

B. Continuous Dynamic Embedding Approaches

Currently, continuous dynamic embedding approaches are divided into two main categories: RNN based approaches and temporal point processes based approaches[4].

In RNN based approaches, the embedding of interacting nodes is updated by the a RNN based architecture according to the historical information of itself. Representative works of this type of approach are JODIE[26], TGN[27] and Streaming graph neural network[28]. JODIE[26] are designed for user-item interaction networks, which uses two RNN to maintain the embedding of each node. With on RNN for users and another one for items. Instead of keeping the embedding of node directly, TGN[27] calculates the embedding of node at different time by introducing message and memory mechanisms. The architecture of Streaming graph neural network[28] is consist of two components: 1) update component; and 2) propagation component. The update component is used to update the embedding of nodes involved in an event and the propagation component propagates the event to the involved nodes neighbors. The process of our proposed mechanism is similar to[28], but there are several significant differences:

- The aggregated information is different, [28] aggregate the node’s own historical information, while we focus more on aggregating neighbors information, which is more effective in TPP based models[5], [6], [7].

1 https://www.gharchive.org/

- The propagation information, propagation objects, and propagation methods are different.
- [28] focus more on the architecture of the neural network, while we focus on a mechanism that can be adapted to existing TPP based approaches.
- We discuss the influence of different factors on the aggregated-diffusion mechanism in more detail.

Know-Evolve[8] is the pioneer in bringing the temporal point processes[29] to dynamic graph representation learning, which models temporal knowledge graph as multi-relational timestamped edges by parameterizing a TPP by a deep recurrent architecture. DyRep[5] is the successor of Know-Evolve. DyRep extends Know-Evolve by using TPP to model long-term events (topological evolution) and short-term events (node communication) and introducing aggregation mechanisms. LDG[6] argues long-term events are often specific by humans, and can be suboptimal and expensive to obtain. LDG use Neural Relational Inference (NRI) model[30] to infer the type of events on the graph and replaces the self-attention originally used in DyRep by generating a temporal attention matrix to better aggregate neighbor information. GHNN[7] is another TPP based approach, which uses an adapted continuous-time LSTM for Hawkes process[31]. Similar to Know-Evolve, GHNN is specifically designed for knowledge graphs.

In this paper, we choose DyRep and LDG as our backbone model for the following reasons:

- DyRep and LDG are universal and not specifically designed for knowledge graphs or user-item graphs.
- DyRep and LDG can model realistic long-term events and short-term events, which can not be provided by other models.
- DyRep and LDG have an obvious aggregation mechanism when interacting nodes are updating.

It is important to note that theoretically any model that updates node embeddings in continuous dynamic graphs using the aggregation mechanism can be extended using the aggregation-diffusion mechanism.

III. BACKGROUND: DyRep & LDG

In this section, we describe relevant details of the DyRep and LDG model. We strongly recommend readers to read the original article[5], [6] for more details to better understand the details of how DyRep and LDG models work. LDG and DyRep represent the evolution of dynamic graph as two distinct processes:

- Long-term association ($k = 1$). This is also called “dynamic of graph”, in which new nodes or edges are added resulting in a change in the topology of the graph.
- Short-term communication ($k = 0$). This is also called “dynamic on graph”, in which interaction between nodes leads to temporary information flow between these nodes[32], [33]. And this process does not change the topology of the graph.

A. Node update

The node update mechanism is same for both DyRep and LDG. When an event $o = (u, v, t, k)$ occurs between
interacting nodes \( u \) and \( v \) will cause their node embeddings \( z^u, z^v \in \mathbb{R}^d \) to be updated and subsequently update the temporal attention \( S \in \mathbb{R}^{N \times N} \). The update process are shown in Fig. 2.

In particular, when an event occurs, the embedding of participating node \( u \) is updated based on the three terms of Self-propagation, Exogenous Drive and Attention-based Aggregation. Specially, for an event of node \( u \) at time \( t \), updating \( z^u \) as:

\[
z^u(t) = \sigma(\mathbf{W}^s h^u_s(\bar{t}^u)) + \mathbf{W}^r z^u(\bar{t}^u) + \mathbf{W}^t (t - \bar{t}^u) \tag{1}
\]

where \( \mathbf{W}^s \in \mathbb{R}^{d \times d}, \mathbf{W}^r \in \mathbb{R}^{d \times d} \) and \( \mathbf{W}^t \in \mathbb{R}^d \) are learned parameters used to control the effect of above-mentioned three terms on the computation of node embedding, respectively. \( \sigma(\cdot) \) is a nonlinear function. \( z^u(\bar{t}^u) \) is the previous representation of node \( u \). \( \bar{t} \) denotes the time point just before current event \( t \) and \( \bar{t}^u \) represent the time point of last event involved \( u \). \( h^u_s(\bar{t}^u) \in \mathbb{R}^d \) is the output representation obtained from the aggregation of node \( u \)'s neighbors \( \mathcal{N}_u \):

\[
h^u_s(\bar{t}^u) = \text{Agg}(\text{softmax}(S_u(\bar{t})), \mathbf{W}^r z^u(\bar{t}^u)), \forall r \in \mathcal{N}_u) \tag{2}
\]

where \( \text{Agg}(\cdot) \) is an aggregation function and \( \mathbf{W}^r \in \mathbb{R}^{d \times d} \) are learned parameters. The amount of information propagated from node \( u \)'s neighbors is controlled by temporal attention \( S_u(\bar{t}^u) \), which is updated by a hard-coded algorithm in DyRep and learned in LDG. It should be noted the update of \( S \) is affected by nodes’ embedding. In DyRep, temporal attention \( S(t) \) relies on adjacency matrix \( A(\bar{t}) \) and conditional intensity \( \lambda^u_v(t) \) of event record \( o = (u, v, t, k) \):

\[
S(t) = f_S(A(\bar{t}), S(\bar{t}), \lambda^u_v(t)) \tag{3}
\]

where \( f_S \) is the attention update function in DyRep. Conditional intensity \( \lambda^u_v(t) \) models the occurrence of event \( o = (u, v, t, k) \) between \( u \) and \( v \) at time \( t \):

\[
\lambda^u_v(t) = \psi_k \log \left( 1 + \exp \left( \frac{\omega_k^T [z^u(\bar{t}); z^v(\bar{t})]}{\psi_k} \right) \right) \tag{4}
\]

where \( \psi_k \) is trainable scalar parameter, which denotes the rate of events arising from a corresponding process, and \( \omega_k \in \mathbb{R}^{2d} \) is designed to learn time-scale specific compatibility. \( \cdot \) denotes concatenation.

In LDG, they replace the hard-coded node update algorithm \( f_S \) with a learnable bilinear encoder \( f_S^{nc} \), which is a two pass progress to ensure temporal attention \( S(t) \) depends on node embeddings at previous time step:

\[
S(t) = f_S^{nc}(z(t-1)) \tag{5}
\]

IV. AGGREGATION-DIFFUSION MECHANISM

In this paper, we extend DyRep and LDG by using a node embedding update algorithm with aggregation-diffusion (AD) mechanism. As mentioned in Section [III][III] the main intuition behind AD mechanism is straightforward: first, the node’s information is affected by its neighbors, which is aggregation part; second, changes in the node itself should be propagated to its neighbors proactively and in a timely manner, which is diffusion part.

Algorithm [III][III] gives the pseudo-code for node embedding update with AD mechanism. The algorithm consists of two steps: 1) an aggregation step, which is used to update the embeddings of the nodes directly involved in an event and has been explained in Section [III][III] 2) a diffusion step, which is used to update the embedding of other nodes that may be affected and will be discussed in this section.

The diffusion step mainly consists of diffusion message generation, diffusion node selection and update of diffused nodes.
Algorithm 1: Update Node Embedding with Aggregation-Diffusion Mechanism

Input: Event record $o = (u, v, t, k)$; all node embeddings of previous time $z(t)$; most recently updated $A(t)$ and $S(t)$; trainable parameters $W^s$, $W^r$, $W^t$, $W^h$, and $W^d$.

Output: Updated node embeddings $z(t)$;

```plaintext
/* Aggregation step */
for each $j \in \{u, v\}$ do
  $A_j(t) \leftarrow \{r : A_{r,j}(t) > 0\}$
  /* Aggregate information from all one-hop neighbors. Discussed in Section VI-D */
  $h^u_j(t) \leftarrow \text{Agg}(\text{softmax}(S_j(t))_r(W^h z^u(\bar{t})), \forall r \in N_u)$
  /* Update interacting node's embedding */
  $z^u(t) \leftarrow \sigma(W^u h^u_j(t) + W^u z^u(\bar{t}) + W^u(t \bar{t}))$
end
/* Diffusion step */
for each $j \in \{u, v\}$ do
  /* Generating diffusion message. Discussed in Section VI-E */
  $m^u(t) \leftarrow \text{Generator}(z(t), z(\bar{t}), o)$
  /* Selecting candidate diffusion nodes. Discussed in Section VI-D and Section VI-C */
  $N^d_j \leftarrow \text{SelectCandidate}(A(t)) // S(t)$ for LDG
  for each $r \in N^d_j$ do
    /* Update diffused node's embedding. */
    $z^u(t) \leftarrow \sigma(z^u(\bar{t}) + q_{u,r}(\bar{t})W^d m^u(t))$
  end
end
return $z(t)$
```

**Diffusion Message Generation.** It determines what kind of message the interacting node of an event will propagate to its neighbors. We model the message in three ways. The most straightforward way is interacting node $u$ diffuses its updated embedding $z^u(t)$ outward and the formulation is as follows:

$$m^u(t) = z^u(t)$$

An interacting node can also diffuse outward changes in itself rather than just current state:

$$\text{delta: } m^u(t) = z^u(t) - z^u(\bar{t})$$

There is also a way to diffuse the impact of the event outward:

$$\text{edge: } m^u(t) = \sigma(W^1 z^u(t) + W^2 z^v(t))$$

where $v$ is another interacting node in the event, $W^1$, $W^2 \in \mathbb{R}^{d \times d}$ are trainable parameters.

These three different approaches to diffusion message have their own advantages and disadvantages, which will be discussed in detail in Section VI-B.

**Diffusion Node Selection.** This is designed to select the nodes that will receive the diffusion information.

In this paper, we choose the 1-hop neighbors of the interacting node as the diffusion nodes. We do not diffuse more hops because more hops will result in a significant decrease in training speed but not a significant performance improvement or even a decrease in performance due to the introduction of noise. We will discuss the impact of diffusion hops in detail in Section VI-E.

Specially, when diffusing the message $m^u(t)$ generated by node $u$, we will avoid involving another interacting node $v$ in the event, because $v$ has already obtained information about $u$ through aggregation step, and repeatedly obtaining information through diffusion will lead to a negative effect, which will be discussed in Section VI-D. Therefore, the formulation for diffusion node selection is as follows:

$$N^d_u = \{r : A_{r,u}(\bar{t}) > 0 \text{ and } r \neq v\}$$

It should be noted, for LDG, we use $S(t)$ to replace $A(t)$, because $A$ is not maintained in LDG.

In addition, we also tried to mask the aggregation/diffusion nodes randomly and temporally, which are also discussed in Section VI-D.

**Update of Diffusion nodes.** The diffusion nodes will update their embeddings based on their previous embedding and the diffusion message:

$$z^u(t) = \sigma(z^u(\bar{t}) + q_{u,r}(\bar{t})W^d m^u(t)), \forall r \in N^d_u$$

where $W^d \in \mathbb{R}^{d \times d}$ is a trainable parameters, and $q_{u,r}(\bar{t})$ is used to control the strength of diffusion from $u$ to $r$. In this paper we discuss two methods of calculating $q_{u,r}(\bar{t})$. One is **uniform**, where all values are equal to 1:

uniform: $q_{u,r}(\bar{t}) = 1$, $\forall r \in N^d_u$

Another one is **attention**, which uses temporal attention $S(t)$ to calculate $q_{u,r}(\bar{t})$:

$$\text{attn: } q_{u,r}(\bar{t}) = \frac{\exp(S_{u,r}(\bar{t}))}{\sum_{r' \in N^d_u} \exp(S_{u,r'}(\bar{t}))}, \forall r \in N^d_u$$

This will be discussed in Section VI-E.

V. EXPERIMENT SETTINGS

**A. Datasets & Metrics**

| TABLE I | DATASET STATISTICS FOR SOCIAL EVOLUTION AND GITHUB. |
|---------|-----------------------------------------------|
|         | SOCIAL EVOLUTION                   | GITHUB |
| #Nodes  | 83                               | 284    |
| #Initial Associations | 575                        | 149    |
| #Final Associations    | 708                             | 710    |
| #Train Event           | 43,834                          | 11,644 |
| #Test Event            | 10,535                          | 9,082  |

We evaluate the AD mechanism on two real world dynamic graph datasets, Social Evolution[3] and Github, which are also the dataset used in DyRep[5] and LDG[6]. The statistical
results for Social Evolution and Github are presented in Table I.

**Social Evolution**[9]. This dataset is released by MIT Human Dynamics Lab, which consists of over 2M events \( o = (u, v, t, k) \). Follow [5], we treat Proximity, Calls and SMS records between users as communication events (short-term events, \( k = 1 \) ) and all Close Friendship records between users are treated as association events (long-term events, \( k = 0 \)). Follow [6], Proximity records are filtered by the probability that record occurred, because the number of Proximity records is too large and contains a lot of noise. The Social Evolution data is collected from Jan 2008 to June 2009. Similar to [5] and [6], we use the association events between users from Jan 2008 to Sep 10, 2008 to initialize the graph, and events from Sep 11, 2008 to April 2009 is used as training set, and events after May 2009 is used as test set. After pre-processing, the dataset contains 83 nodes, the training set contains 43K events, and the test set contains 10K events.

**Github**. This dataset is released by Github Archive. The original dataset contains over 12K nodes and 600K events and compared to Social Evolution is a large graph with sparse events. Since LDG requires sufficient interactions between nodes to train temporal attention \( S \), we extract a dense subgraph following the processing in [6]. Follow [5], we treat Follow records between users as association events and other records are treated as communication records. We use the association events in 2011-2012 to initialize the graph, and events from Jan 1, 2013 to Sep 30, 2013 is used as training set, and events from Oct 1, 2013 to Dec 31, 2013 is used as test set. After pre-processing, the dataset contains 284 nodes, the training set contains 11K events, and the test set contains 9K events.

During the test, for a given event \((u, ?, t, k)\) or \((?, v, t, k)\), we compute the conditional density of known node with all other nodes and rank them. Same as [5] and [6], we report Mean Average Ranking (MAR) and HIT@10.

**B. Implementation Details**

For LDG and DyRep we directly use the code[3] provide by [6]. We only modify the node update function `update_node_embed` in the code to add the AD mechanism into LDG and DyRep.

The hyper parameter setting of the experiments are also consistent with those of LDG. We use the Adam optimizer[34] with the learning rate set to 0.0002. The hidden units \( d \) per layer is set to 32. Gradient clipping is used to avoid gradient explosion, and the clipping value is set to 100. We do not use dropout and batch size is set to 200. We train for 5 epochs. We run each experiment 10 times and report the average results.

### VI. Results & Discussion

**A. Overview**

In this section we give an overview performance comparison between models using the AD mechanism and not using. The AD mechanism used in this section is '*-AD-base', which follows the principle of simplicity, specifically, using Eq. 6 in generating diffusion message, considering only 1-hop neighbors, and using a uniform strength \( q \) (Eq. 11). We also conduct experiments on models that only use diffusion step ('*-D-base') as well as neither diffusion nor aggregation step ('*-self'). Table I shows the performance comparison.

On Social Evolution, compared with the baseline DyRep, **DyRep-AD-base** reduced MAR from 13.88 to 6.28 and improved HIT@10 from 0.468 to 0.897; compared with baseline LDG, **LDG-AD-base** reduced MAR from 13.06 to 6.94 and improved HIT@10 from 0.276 to 0.564. We find that the performance improvement from the diffusion step is significantly higher than that from the aggregation step, which may be due to the fact that both aggregation and diffusion are essentially for information delivery, and the number of nodes that acquire new information in the aggregation process (1 node) is less than that in the diffusion process (node’s neighbors).

**B. Implementation Details**

For LDG and DyRep we directly use the code[3] provide by [6]. We only modify the node update function `update_node_embed` in the code to add the AD mechanism into LDG and DyRep.

| MODEL       | SOCIAL EVOLUTION | GITHUB       |
|-------------|------------------|--------------|
|             | MAR   | HIT@10 | SPEED | Epoch | MAR   | HIT@10 | SPEED | Epoch |
| DyRep       | 13.88 | 0.486  | 1x    | 5     | 117.83 | 0.165  | 1x    | 5     |
| DyRep-self  | 20.61 | 0.141  | 0.9x  | 5     | 130.99 | 0.160  | 0.9x  | 5     |
| DyRep-D-base| 6.74  | 0.897  | 4x    | 1     | 85.51  | 0.287  | 4x    | 2     |
| DyRep-AD-base| 6.28 | 0.907  | 4x    | 1     | 81.25  | 0.262  | 4x    | 1     |

The hyper parameter setting of the experiments are also consistent with those of LDG. We use the Adam optimizer[34] with the learning rate set to 0.0002. The hidden units \( d \) per layer is set to 32. Gradient clipping is used to avoid gradient explosion, and the clipping value is set to 100. We do not use dropout and batch size is set to 200. We train for 5 epochs. We run each experiment 10 times and report the average results.
number of epochs required for convergence is that one of the purpose of the repeated epochs in the training process is to propagate the delayed information through sample repetition. For example, in Fig. 1 in the \( i \)-th training epoch, the information updated at \( t_4 \), \( t_5 \) in the previous epoch to be propagated to node \( b \) and thus to \( e \) when \( o^6 \) occurs in the \( i \)-th training epoch.

**B. Impact of Diffusion Message**

In this section, we discuss the impact of diffusion message. We replace the diffusion message generation method in \( \ast \)-AD-base with Eq. 7 (\( \ast \)-AD-delta) and Eq. 8 (\( \ast \)-AD-edge). Fig. 3 shows the dynamic link prediction performance comparison with different diffusion messages.

We find that \( \ast \)-AD-delta have a significant performance decline on the Social Evolution dataset and a significant improvement on Github dataset (especially \( \text{LDG-AD-delta} \)), while \( \text{LDG-AD-edge} \) performs better on the Social Evolution dataset.

In the Social Evolution dataset, the average interval for a node to appear in an event is 35.43 much lower than 107.83 in the Github dataset. The frequent interaction of nodes makes the diffusion information \( \mathbf{m} \) generated using Eq. 7 in Social Evolution close is very small, so it cannot diffuse enough information, resulting in poor performance in the Social Evolution dataset. While in Github dataset, the longer interval allows nodes to accumulate enough difference information for diffusion.

The generation of edge diffusion message (Eq. 8) involves both interacting nodes in the event, therefore, during diffusion, if the number of common neighbors between two nodes is high, it will cause duplicate overlap of diffusion message and thus bring negative impact to the performance. In Social Evolution dataset, the average number of common neighbors of interacting nodes in an event is 2.08, while in the Github dataset is 6.02.

**C. Impact of Diffusion Hops**

Fig. 4 shows the dynamic link prediction performance comparison with different diffusion hops. Diffusing more hops on the Social Evolution and Github datasets doesn’t result in improved performance, but instead has a negative effect. There are two main reasons for the negative effect:

- The more hops of diffusion, the greater the risk and amount of noise introduced in the process of diffusion.
- The AD mechanism actually reduces the distance of information propagation between nodes. Compared to using only aggregation, the \( i \)-hop diffusion in the AD mechanism reduces the distance of information propagation by at least \( i \). The average path length between nodes in the Social Evolution and Github datasets is 2.239 and 2.899, respectively. Therefore, 2-hop diffusion will make the information propagation distance less than 1 (0.239 and 0.899) which will result in the aggregated and diffused messages mixed together, thus producing a negative effect.

In addition, we find that as the number of diffusion hops increases, the training time consuming also increase dramatically, so we suggest that diffusion of 1-hop in practice can be a better balance between performance and training cost.
D. Impact of Aggregation and Diffusion Nodes

In this section, we discuss the performance impact of different strategies for selecting aggregation and diffusion nodes. We consider the following selection strategies and the results are shown in Fig. 5:

- ***-AD-v**: This strategy does not remove another interacting node $v$ when selecting diffusion nodes $N^d_u$ of interacting node $u$: $N^d_u = \{ r : A_{r,u}(t) > 0 \}$. We find a significant performance drop in Fig. 5. This is due to $v$ has already obtained information about $u$ through aggregation step, and repeatedly obtaining information through diffusion will lead to negative effect.

- ***-AD-α**: This strategy randomly mask 20% of the neighboring nodes of interacting node at the aggregation step but diffusion is still based on Eq. 9. We find that LDG-AD-α performs slightly better than LDG-AD-base on both Social Evolution and Github datasets, while DyRep-AD-α performs slightly worse than DyRep-AD-base.

- ***-AD-β**: This strategy randomly mask 20% of the neighboring nodes of interacting node at the diffusion step but still aggregate all neighbors. We find that DyRep-AD-β and LDG-AD-β slightly improved the MAR metrics on both Social Evolution and Github datasets, while LDG-AD-β significantly decreased the HIT@10 metric on Github.

- ***-AD-γ**: This strategy randomly mask 20% of the neighboring nodes of interacting node at both aggregation and diffusion step. We observe no significant difference between *-AD-γ and *-AD-base. Compared with LDG-AD-base, LDG-AD-γ slightly improved the HIT@10 metrics on Social Evolution, while there is a significant gap on Github.

- ***-AD-ω**: This strategy mask 20% of the earliest neighboring nodes of interacting node at both aggregation and diffusion step. We observe a significant drop in the performance of both MAR and HIT@10 on both Social Evolution and Github datasets.

E. Impact of Diffusion Attention

| DATASET | MODEL         | MAR  | HIT@10 |
|---------|---------------|------|--------|
| SOCIAL  | DyRep-AD-base | 6.28 | 0.907  |
|         | DyRep-AD-attn | 6.20 | 0.885  |
|         | LDG-AD-base   | 6.40 | 0.918  |
|         | LDG-AD-attn   | 6.63 | 0.907  |
| GITHUB  | DyRep-AD-base | 81.25| 0.262  |
|         | DyRep-AD-attn | 84.78| 0.261  |
|         | LDG-AD-base   | 51.49| 0.480  |
|         | LDG-AD-attn   | 44.96| 0.427  |

In the work of GAT [35], it has been demonstrated that attention mechanisms have an important role in the aggregation process. And in this section, we explore whether existing attention has a positive effect on the diffusion process. Table III shows the performance comparison of whether to use attention during diffusion, where *-AD-attn is the variation of
*-AD-base with replacing Eq. [11] by Eq. [12]. We observed that *-AD-base and *-AD-attn each have strengths in different datasets and different evaluation metrics. Therefore, we believe that the attention mechanism is still meaningful in the diffusion process, and we will explore a more suitable attention mechanism for the diffusion process in our future work.

F. Summary & Suggestions

Based on the above discussion, in this section, we summarize the impact factors in the AD mechanism and give some suggestions for using the AD mechanism in practice:

- In generating the diffusion message, competitive performance can be obtained based on the most concise Eq. [6]. Event (edge) message (Eq. [8]) is a good choice in case of few common neighbors between interacting nodes when focusing on HIT@10 metric.
- In practice, a remarkable performance can be obtained by diffusing 1-hop. When the average path length in the graph is too long, an appropriate increase in the number of hops can be considered.
- There is no need to filter the neighbors during aggregation, but there is a necessity to avoid propagate the diffusion message to another interacting node during the diffusion.

VII. CONCLUSION

We introduce a novel aggregation-diffusion mechanism into the update of node embedding to extend the existing models with TPP-based DyRep and LDG as example. By using the AD mechanism, we get a huge improvement in all evaluation metrics on both Social Evolution and Github dataset compared to the original models. We also construct extensive experiments to explore the effects of different factors on the AD mechanism and give some suggestions for selecting suitable strategies of the aggregation and propagation process based on the graph properties.

In this paper, we have validated the effectiveness of the aggregation-diffusion mechanism mainly from experiments. In the further work, we will try to explain how the aggregation-diffusion mechanism works from the theoretical level.

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Active local structure

Information change