Recently, the CDF collaboration has reported a measurement of the CP asymmetry in the $B \to \psi K_S$ decay: $a_{\psi K_S} = 0.79^{+0.41}_{-0.44}$. We analyze the constraints that follow from this measurement on the size and the phase of contributions from new physics to $B - \overline{B}$ mixing. Defining the relative phase between the full $M_{12}$ amplitude and the Standard Model contribution to be $2\theta_d$, we find a new bound: $\sin 2\theta_d \gtrsim -0.6$ ($-0.87$) at one sigma (95% CL). Further implications for the CP asymmetry in semileptonic $B$ decays are discussed.
Recently, the CDF collaboration has reported a measurement of the CP asymmetry in the $B \to \psi K_S$ decay \cite{1}:

$$a_{\psi K_S} = 0.79^{+0.41}_{-0.44},$$

(1)

where

$$\frac{\Gamma(B^0_{\text{phys}}(t) \to \psi K_S) - \Gamma(B^{0*}_{\text{phys}}(t) \to \psi K_S)}{\Gamma(B^0_{\text{phys}}(t) \to \psi K_S) + \Gamma(B^{0*}_{\text{phys}}(t) \to \psi K_S)} = a_{\psi K_S} \sin(\Delta m_B t),$$

(2)

(Previous searches have been reported by OPAL \cite{2} and by CDF \cite{3}.) Within the Standard Model, the value of $a_{\psi K_S}$ can be cleanly interpreted in terms of the angle $\beta$ of the unitarity triangle, $a_{\psi K_S} = \sin 2\beta$. The resulting constraint is still weak, however, compared to the indirect bounds from measurements of $|V_{ub}/V_{cb}|$, $\Delta m_B$ and $\varepsilon_K$ \cite{4}:

$$\sin 2\beta \in [+0.4, +0.8].$$

(3)

Yet, the CDF measurement is quite powerful in constraining contributions from new physics to the $B - \bar{B}$ mixing amplitude. It is the purpose of this work to investigate this constraint.

We focus our analysis on a large class of models of new physics with the following features:

(i) The $3 \times 3$ CKM matrix is unitary. In particular, the following unitarity relation is satisfied:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$ 

(4)

(ii) Tree-level decays are dominated by the Standard Model contributions. In particular, the phase of the $\bar{B} \to \psi K_S$ decay amplitude is given by the Standard Model CKM phase, $\arg(V_{cb}V_{cs}^*)$, and the following bound, which is based on measurements of Standard Model tree level processes only, is satisfied:

$$R_u \equiv \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right| \lesssim 0.45.$$ 

(5)

The first assumption is satisfied by all models with only three quark generations (that is, neither fourth generation quarks nor quarks in vector-like representations of the Standard Model). The second assumption is satisfied in many extensions of the Standard Model,
such as most models of supersymmetry with $R$-parity and left-right symmetric (LRS) models. There exist, however, viable models where this assumption may fail, such as supersymmetry without $R$-parity (see, for example, the discussion in [5,6] or specific multi-scalar models [7]). Within the class of models that satisfies (i) and (ii), our analysis is model-independent.

The effect of new physics that we are interested in is the contribution to the $B - \bar{B}$ mixing amplitude, $M_{12} - i\frac{1}{2}\Gamma_{12}$. Our second assumption implies that

$$\Gamma_{12} \approx \Gamma_{12}^{SM}. \quad (6)$$

The modification of $M_{12}$ can be parameterized as follows (see, for example, [8,4]):

$$M_{12} = r_d^2 e^{2i\theta_d} M_{12}^{SM}. \quad (7)$$

The experimental measurement of $\Delta m_B$ provides bounds on $r_d^2$ while the new CDF measurement of $a_{\psi K_S}$ gives the first constraint on $2\theta_d$.

The implications for CP violation in $B$ decays of models with the above features has been discussed in refs. [9-17]. Analyses that are similar to ours have also appeared, prior to the CDF measurement, in refs. [18-23,8].

To derive bounds on $r_d^2$ and $2\theta_d$ we need to know the allowed range for the relevant CKM parameters. Assuming CKM unitarity (4) and Standard Model dominance in tree decays (5), we get:

$$0.005 \lesssim |V_{td}V_{tb}^*| \lesssim 0.013, \quad (8)$$

$$0 \lesssim \beta \lesssim \pi/6 \text{ or } 5\pi/6 \lesssim \beta \lesssim 2\pi. \quad (9)$$

Note that these ranges are much larger than the Standard Model ranges. The reason for that is that we do not use here the $\Delta m_B$ and $\varepsilon_K$ constraints. These are loop processes and, in our framework, could receive large contributions from new physics.

Let us first update the constraint on $r_d^2$. To do so, we write the Standard Model contribution to $\Delta m_B$ in the following way (see [24,4] for definitions and numerical values of the relevant parameters):

$$\left[ \frac{2M_{12}^{SM}}{0.471 \text{ ps}^{-1}} \right] = \left[ \frac{\eta_B}{0.55} \right] \left[ \frac{S_6(x_t)}{2.36} \right] \left[ \frac{f_{B_d}\sqrt{B_{B_d}}}{0.2 \text{ GeV}} \right]^2 \left[ \frac{V_{td}V_{tb}^*}{8.6 \times 10^{-3}} \right]^2. \quad (10)$$
The main uncertainties in this calculation come from eq. (8) and from
\[ f_{B_d} \sqrt{B_{B_d}} = 160 - 240 \text{ MeV}. \] (11)

Using
\[ \Delta m_B = r_d^2 |2M_{12}^{\text{SM}}|, \] (12)
we find:
\[ 0.3 \lesssim r_d^2 \lesssim 5. \] (13)

Next we derive the new constraint on \( 2\theta_d \). With the parameterization (7), we have
\[ a_{\psi K_S} = \sin 2(\beta + \theta_d). \] (14)

Defining
\[ \beta_{\text{max}} \equiv \arcsin[(R_u)_{\text{max}}], \]
\[ 2\bar{\beta}_{\text{min}} \equiv \arcsin[(a_{\psi K_S})_{\text{min}}], \] (15)
where both \( \beta_{\text{max}} \) and \( \bar{\beta}_{\text{min}} \) are defined to lie in the first quadrant, we find that the following range for \( 2\theta_d \) is allowed:
\[ 2(\bar{\beta}_{\text{min}} - \beta_{\text{max}}) \leq 2\theta_d \leq \pi + 2(\beta_{\text{max}} - \bar{\beta}_{\text{min}}). \] (16)

The constraint (16) can be written simply as
\[ \sin 2\theta_d \geq -\sin 2(\beta_{\text{max}} - \bar{\beta}_{\text{min}}). \] (17)

Within our framework, the allowed range for \( \beta \) is given in (9), that is \( 2\beta_{\text{max}} \approx \pi/3 \). Taking the CDF measurement (11) to imply, at the one sigma level,
\[ a_{\psi K_S} \gtrsim 0.35, \] (18)
or, equivalently,
\[ 2\bar{\beta}_{\text{min}} \approx \pi/9, \] (19)
we find \( 2(\beta_{\text{max}} - \bar{\beta}_{\text{min}}) \approx 2\pi/9 \) and, consequently,
\[ \sin 2\theta_d \gtrsim -0.6. \] (20)
If we take a more conservative approach and consider the 95% CL lower bound,

\[ a_{\psi K_S} \geq 0, \]  

(21)

or, equivalently,

\[ 2\bar{\beta}_{\text{min}} \approx 0, \]  

(22)

we find \( 2(\beta_{\text{max}} - \bar{\beta}_{\text{min}}) \approx \pi/3 \) and, consequently,

\[ \sin 2\theta_d > -0.87. \]  

(23)

Eq. (20) (or the milder constraint (23)), being the first constraint on \( \theta_d \), is our main result.

There are two main ingredients in the derivation of the bounds (20) and (23). The validity of one of them, that is the bound on \( \sin 2(\beta + \theta_d) \) from the value of \( a_{\psi K_S} \), depends on the size of contributions to the \( b \to c\bar{c}s \) decay that carry a phase that is different from \( \arg(V_{cb}V_{cs}^*) \). To understand the effects of such new contributions, we define

\[ \theta_A = \arg(\bar{A}_{\psi K_S}/\bar{A}_{\psi K_S}^{\text{SM}}), \]  

(24)

where \( \bar{A}_{\psi K_S} \) is the \( B \to \psi K_S \) decay amplitude. For \( \theta_A \neq 0 \), eq. (14) is modified into

\[ a_{\psi K_S} = \sin 2(\beta + \theta_d + \theta_A). \]  

(25)

The bounds (20) and (23) apply now to the combination of new phases \( \theta_d + \theta_A \). Since, however, \( |\sin \theta_A| \leq |\bar{A}_{\psi K_S}^{\text{NP}}/\bar{A}_{\psi K_S}^{\text{SM}}| \), we expect \( \theta_A \) to be small. Then, we can still use (20) and (23), with the right hand side relaxed by \( \mathcal{O}(\theta_A) \), as lower bounds on \( \sin 2\theta_d \). Examining the actual numerical values of the bounds (20) and (23), we learn that for \( |\bar{A}_{\psi K_S}^{\text{NP}}/\bar{A}_{\psi K_S}^{\text{SM}}| \lesssim 0.01 \), the effect is clearly unimportant. It takes a very large new contribution, \( |\bar{A}_{\psi K_S}^{\text{NP}}/\bar{A}_{\psi K_S}^{\text{SM}}| \gtrsim 0.4(0.25) \), to completely wash away our one sigma (95% CL) bounds. We are not familiar with any reasonable extension of the Standard Model where the new contribution is that large. For example, in the framework of supersymmetry with \( R_p \), a model independent analysis of supersymmetric contributions to the \( b \to c\bar{c}s \) decay [25] finds an upper bound, \( |\bar{A}_{\psi K_S}^{\text{SUSY}}/\bar{A}_{\psi K_S}^{\text{SM}}| \lesssim 0.1 \). The bound can be saturated only with light supersymmetric spectrum and maximal flavor changing gluino couplings. In
most supersymmetric flavor models, however, the relevant coupling is of order \( |V_{cb}| \) and \( |\bar{A}_{\psi K_S}^{\text{SUSY}}/\bar{A}_{\psi K_S}^{\text{SM}}| \) is well below the percent level. This is the case, for example, in models of universal squark masses, of alignment and of non-Abelian horizontal symmetries (see e.g. ref. [8]). In LRS models, with \( m(W_R) \gtrsim 1 \text{ TeV} \) and \( |V_{cb}^R| \sim |V_{cb}^S| \), we have \( |\bar{A}_{\psi K_S}^{\text{LRS}}/\bar{A}_{\psi K_S}^{\text{SM}}| \ll 0.01 \).

The other ingredient of our analysis, that is the bound on \( \sin \beta \) from \( R_u \), suffers from hadronic uncertainties in the determination of the allowed range for \( R_u \). We have used \( |V_{ub}/V_{cb}| \lesssim 0.10 \). We emphasize, however, that uncontrolled theoretical errors, that is the hadronic modelling of charmless \( B \) decays, are the main source of uncertainty in determining the range for \( |V_{ub}/V_{cb}| \). It would be misleading then to assign a confidence level to our bound on \( \sin 2\theta_d \). (See a detailed discussion in ref. [4].) All we can say is that if indeed \( |V_{ub}/V_{cb}| \leq 0.10 \) holds, as suggested by various hadronic models, then \( \sin 2\theta_d \geq -0.6(-0.87) \) at one sigma (95% CL). The measurement of \( a_{\psi K_S} \) would not provide any bound on \( \sin 2\theta_d \) at one sigma (95% CL) if \( |V_{ub}/V_{cb}| \) were as large as 0.17 (0.15).

When investigating specific models of new physics, it is often convenient to use a different parameterization of the new contributions to \( M_{12} \). Instead of (7), one uses (see, for example, [27] in the supersymmetric framework and [26] in the left-right symmetric framework):

\[
M_{12}^{\text{NP}} = he^{i\sigma} M_{12}^{\text{SM}},
\]

(26)

where \( M_{12}^{\text{NP}} \) is the new physics contribution. The relation between the two parametrizations is given by

\[
r_d^2 e^{2i\theta_d} = 1 + he^{i\sigma}.
\]

(27)

To derive the CDF constraints in the \((h, \sigma)\) plane, the following relations are useful:

\[
r_d^2 = \sqrt{1 + 2h \cos \sigma + h^2}.
\]

(28)

\[
\sin 2\theta_d = \frac{h \sin \sigma}{\sqrt{1 + 2h \cos \sigma + h^2}}.
\]

(29)

The bound of eq. (13) corresponds to the allowed region in the \((h, \sigma)\) plane presented in Figure 1.
The situation is particularly interesting for values of $\sigma$ close to $\pi$. Here, the Standard Model and the new physics contributions add destructively. Consequently, large values of $h$ up to

$$h_{\text{max}} = (r_d^2)_{\text{max}} + 1 \approx 6$$

are allowed; this means that new physics may still be dominant in $B - \bar{B}$ mixing. On the other hand, values of $h$ close to 1 are forbidden since the new physics contribution cancels the Standard Model amplitude, yielding values of $\Delta m_B$ that are too small.

The bounds of eqs. (20) and (23) are presented in Figure 2.
Figure 2. The $a_{\psi K_S}$ constraint. The allowed region corresponding to the one sigma (95% CL) bound, $a_{\psi K_S} \geq 0.35 \ (0)$, is given by the light (light plus dark) grey area.

We would like to emphasize some features of the excluded region:

1. Since only negative $\sin 2\theta_d$ values are excluded, only negative $\sin \sigma$ values are excluded.

2. For very large $h$, the Standard Model contribution is negligible and, consequently, $\sin \sigma \approx \sin 2\theta_d$. Therefore, for large $h$ values, $\sigma$-values in the range $[\pi + 2(\beta_{\text{max}} - \bar{\beta}_{\text{min}}), 2\pi - 2(\beta_{\text{max}} - \bar{\beta}_{\text{min}})]$ are excluded.

3. For $\sigma$ arbitrarily close to $\pi$ (from above), there is always an excluded region corresponding to $h$ similarly close to 1.

Finally, in Figure 3 we show the combination of the $\Delta m_B$ and $a_{\psi K_S}$ bounds. It is obvious that the latter adds a significant exclusion region in the $(h, \sigma)$ plane.
The parameters that we have constrained here are related to other physical observables. The ratio between the difference in decay width and the mass difference between the two neutral $B$ mesons, $\Delta \Gamma_B / \Delta m_B$, and the CP asymmetry in semileptonic decays, $a_{SL}$, are given by

\[
\frac{\Delta \Gamma_B}{\Delta m_B} = \Re \frac{\Gamma_{12}}{M_{12}}, \\
ar_{SL} = \Im \frac{\Gamma_{12}}{M_{12}}. 
\]

The Standard Model value of $\Gamma_{12}/M_{12}$ has been estimated [27-29,23]:

\[
\left( \frac{\Gamma_{12}}{M_{12}} \right)^{\text{SM}} \approx -(0.8 \pm 0.2) \times 10^{-2}, 
\]

\[
\arg \left( \frac{\Gamma_{12}}{M_{12}} \right)^{\text{SM}} = \mathcal{O} \left( \frac{m_c^2}{m_b} \right). 
\]
We emphasize that there is a large hadronic uncertainty in this estimate, related to the assumption of quark-hadron duality. Eq. (32) leads to the following estimates:

\[
\left| \frac{\Delta \Gamma_B / \Delta m_B}{\Delta m_B} \right|_{\text{SM}} \sim 10^{-2}, \\
|a_{\text{SL}}|_{\text{SM}} \lesssim 10^{-3}.
\]

(34)

The (possible) measurement of \(a_{\text{SL}}\) can be used to constrain the Standard Model CKM parameters \(^{23}\).

Since \((\Gamma_{12}/M_{12})_{\text{SM}}\) is real to a good approximation, the effects of new physics, within our framework, can be written as follows:

\[
\frac{\Delta \Gamma_B}{\Delta m_B} = \left( \frac{\Gamma_{12}}{M_{12}} \right)^{\text{SM}} \frac{\cos 2\theta_d}{r_d^2}, \\
a_{\text{SL}} = -\left( \frac{\Gamma_{12}}{M_{12}} \right)^{\text{SM}} \frac{\sin 2\theta_d}{r_d^2}.
\]

(35)

Note the following relation between the two observables:

\[
\sqrt{(\Delta \Gamma_B / \Delta m_B)^2 + (a_{\text{SL}})^2} = \left| \frac{\Gamma_{12}}{M_{12}} \right|_{\text{SM}} \frac{1}{r_d^2}.
\]

(36)

The lower bound on \(r_d^2\) in eq. (13) implies then that neither \(\Delta \Gamma_B / \Delta m_B\) nor \(a_{\text{SL}}\) can be enhanced compared to \((\Gamma_{12}/M_{12})_{\text{SM}}\) by more than a factor of about 3, that is a value of approximately \(3 \times 10^{-2}\). Moreover, if one of them is very close to this upper bound, the other is suppressed. (This is actually the situation within the Standard Model: \(\Delta \Gamma_B / \Delta m_B\) saturates the upper bound with \(r_d = 1\), and \(a_{\text{SL}}\) is highly suppressed.)

The new bound on \(\sin 2\theta_d\) that we found, eq. (20) (or the milder bound (23)), do not affect the allowed region for \(\Delta \Gamma_B / \Delta m_B\). The reason is that \(\cos 2\theta_d\) is not constrained and could take any value in the range \([-1, +1]\). On the other hand, the range for \(a_{\text{SL}}\) is affected. Taking into account also the lower bound on \(r_d^2\) in (13), we find

\[
-3.3 \lesssim \frac{a_{\text{SL}}}{(\Gamma_{12}/M_{12})_{\text{SM}}} \lesssim 2.0.
\]

(37)

The reduction in the upper bound from 3.3 to 2.0 is due to the \(a_{\psi K_S}\) bound. Note that \((\Gamma_{12}/M_{12})_{\text{SM}}\) is negative, so that the \(a_{\psi K_S}\) constraint is a restriction on negative \(a_{\text{SL}}\) values.
Similar analyses will be possible in the future for the $B_s$ system. At present, there is only a lower bound on $\Delta m_{B_s}$,

$$\Delta m_{B_s} \geq 12.4 \text{ ps}^{-1}. \quad (38)$$

The main hadronic uncertainty comes from the matrix element,

$$f_{B_s} \sqrt{|B_{B_s}|} = 200 - 280 \text{ MeV}. \quad (39)$$

We find

$$r_s^{2} \sim 0.6. \quad (40)$$

Consequently, $|a_{SL}(B_s)|$ is constrained to be smaller than 1.6 times the Standard Model value of $|\Gamma_{12}(B_s)/M_{12}(B_s)|$.

Once an upper bound on a CP asymmetry in $B_s$ decay into a final CP eigenstate is established, we will be able to constrain $2\theta_s$. It will be particularly useful to use $b \rightarrow c\bar{c}s$ decays, such as $B_s \rightarrow D_s^+ D_s^-$. The Standard Model value, $a_{B_s \rightarrow D_s^+ D_s^-} \approx \sin 2\beta_s$, is very small, $\beta_s \equiv \arg[-(V_{ts} V_{tb}^*)/(V_{cs} V_{cb}^*)] = O(10^{-2})$. Therefore, the Standard Model contribution can be neglected when the bounds on $a_{B_s \rightarrow D_s^+ D_s^-}$ are well above the percent level. The approximate relation, $a_{B_s \rightarrow D_s^+ D_s^-} \approx -\sin 2\theta_s$, will make the extraction of a constraint on $\sin 2\theta_s$ particularly clean and powerful.

To summarize our main results: the CDF measurement of the CP asymmetry in $B \rightarrow \psi K_S$ constrains the size and the phase of new physics contributions to $B - \bar{B}$ mixing. The constraints are depicted in Figures 2 and 3. They can be written as a lower bound, $\sin 2\theta_d \gtrsim -0.6 (-0.87)$ at one sigma (95% CL), where $2\theta_d = \arg(M_{12}/M_{12}^{SM})$. This, together with constraints from $\Delta m_B$, gives the one sigma bounds on the CP asymmetry in semileptonic $B$ decays, $-2 \times 10^{-2} \lesssim a_{SL} \lesssim 3 \times 10^{-2}$.

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