Production of gravitational waves during preheating with 
nonminimal coupling

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Abstract

We study the preheating and the in-process production of gravitational waves (GWs) after inflation in which the inflaton is nonminimally coupled to the curvature in a self-interacting quartic potential with the method of lattice simulation. We find that the nonminimal coupling enhances the amplitude of the density spectrum of inflaton quanta, and as a result, the peak value of the GW spectrum generated during preheating is enhanced as well and might reach the limit of detection in future GW experiments. The peaks of the GW spectrum not only exhibit distinctive characteristics as compared to those of minimally coupled inflaton potentials but also imprint information on the nonminimal coupling and the parametric resonance, and thus the detection of these peaks in the future will provide us a new avenue to reveal the physics of the early universe.
I. INTRODUCTION

Inflation, an accelerated expansion in the early universe, is an elegant idea proposed to resolve the horizon, flatness, and monopole problems in the big bang standard cosmology [1]. At the same time, the quantum fluctuations of the inflaton field also provide the seed for the formation of cosmic structures [2]. The slow-roll single-field inflationary models predict that the fluctuant spectrum of curvature perturbations is nearly scale-invariant. This prediction is consistent with the observations of the cosmic microwave background (CMB) [3, 4], which limit the spectral index to be $n_s = 0.968 \pm 0.006$ at 68% confidence level (CL) [5].

During inflation, there are also tensor perturbations of the spacetime metric, which lead to the production of a stochastic background of gravitational waves (GWs). Since the amplitude of the power spectrum of GWs depends on the energy scale of inflation in the slow-roll single-field inflationary models, the combination of the ratio of tensor to scalar fluctuations $r$, which is constrained to be $r < 0.09$ by current CMB data [5], and the spectral index $n_s$ is capable of discriminating a host of inflationary models. For example, the simple cubic and quartic potentials are nearly ruled out by the Planck 2015 data, and the simple quadratic potential is also disfavored [5].

After inflation, the universe enters a reheating era in which the potential energy of the inflaton is transferred to a thermal bath of the matter species that are present in our Universe today. The first stage of reheating is preheating [6, 7], in which there exists an explosive particle production of the inflaton quanta or a scalar matter field coupled to the inflaton due to the parametric resonance. Since only a part of momenta are in the resonance bands, the Fourier modes of the inflaton quanta or the scalar matter field with resonant momenta grow exponentially while all other modes do not. This results in that the matter distribution has large and time-dependent density inhomogeneities in the position space, and thus possesses substantial quadruple moments. Therefore, the parametric resonance of preheating can source a significant production of GWs [8, 9]. Different from vacuum fluctuations of tensor perturbations during inflation, the amplitude of the GW spectrum generated during preheating is independent of the energy scale of inflation which only determines the present peak frequency [10, 11]. Recently, the production of GWs during preheating with special inflation potentials, such as asymmetric potential around the minimum and cuspy potential, has been investigated in [12, 13]. It was found that there is a pronounced peak in the GW
spectrum for the asymmetric potential, and the pronounced peak becomes two in the case of the cuspy potential.

Although simple quadratic and quartic potentials are disfavored phenomenologically, they however agree with observations very well after a simple extension which assumes the existence of a nonminimal coupling between the inflation field and the curvature scalar [14, 15]. Moreover, nonminimal couplings can be generated naturally when quantum corrections are considered and are essential for the renormalizability of the scalar field theory in curved space [16]. Therefore, it is of great interest to investigate the production of GWs during preheating in the case of symmetric and simple power law inflationary potentials with nonminimal couplings as opposed to those potentials with rather peculiar shapes and see whether the nonminimal coupling shows as peculiar gravitational wave signatures. To understand the physics of the GW production in details, the process of preheating needs to be investigated thoroughly. However, all current studies (to the best of our knowledge) on the preheating after inflation with nonminimal couplings consider only the linear perturbations of the scalar field and use the Hartree approximation to account for the backreaction of the amplified quantum fluctuations [17–19], and this leaves unfortunately some other interesting physical processes, such as scattering among different modes and the evolution of the energy density spectra of inflaton quanta, unclear. So, in this paper, we first plan to fill this gap by a full investigation of preheating after the nonminimally coupled scalar field inflation with a self-interacting potential by performing a numerical lattice simulation. We find that the nonminimal coupling has an appreciable effect on the amplitude of the density spectrum of inflaton quanta, since, as is revealed by our study, the amplitude grows with the increase of the coupling parameter. Then we study the production of GWs during preheating. The existence of the nonminimal coupling gives rise to two new source terms in the GW equation, which become more and more important as the coupling parameter grows. With the increase of the coupling parameter, the amplitude of GW spectrum also grows constantly and might reach the detection limit of future GW experiments.
II. PREHEATING

In a flat Friedmann-Robertson-Walker (FRW) background, the scalar field $\phi$ nonminimally coupled with gravity in the form $\xi \phi^2 R$ satisfies the equation

$$\ddot{\phi} + 3H \dot{\phi} - \frac{1}{a^2} \nabla^2 \phi + \frac{dV(\phi)}{d\phi} + \xi R \phi = 0, \quad (1)$$

where $\xi$ is the coupling parameter, $R$ is the curvature scalar, a dot denotes the derivative with respect to the cosmic time $t$, $a$ is the cosmic scale factor, $H = \dot{a}/a$ is the Hubble parameter, $V(\phi) = \frac{\lambda}{4} \phi^4$ is the self-interacting potential with $\lambda$ being a constant, and $\nabla^2$ is the Laplacian operator. This inflationary model is consistent with observations since the negative nonminimal coupling yields a slight increase of $n_s$ as well as a significant decrease of $r$ [15, 20]. The joint of Planck and WMAP polarization data gives $\xi < -0.0019$ at 95% CL [14, 15]. After inflation, the scalar field oscillates around $\phi = 0$.

We assume that the inflaton is weakly coupled to other fields and thus only the parametric resonance of inflaton quanta is considered during preheating. For convenience, one can divide the scalar field into the homogeneous and fluctuant parts: $\phi(t, x) = \phi_0(t) + \delta\phi(t, x)$. To the linear order, the perturbations in the momentum space obey the equation of motion

$$\delta\ddot{\phi}_k + 3H \delta\dot{\phi}_k + \left[\frac{k^2}{a^2} + 3\lambda \phi_0^2 + \xi R\right] \delta\phi_k = 0. \quad (2)$$

By defining a conformal field $\varphi_0 \equiv a\phi_0$ and its fluctuation $\delta\varphi_k \equiv a\delta\phi_k$ and introducing the conformal time $\eta \equiv \int a^{-1} dt$, Eq. (2) can be rewritten as

$$\frac{d^2}{d\eta^2} \delta\varphi_k + \omega_k^2 \delta\varphi_k = 0, \quad (3)$$

with

$$\omega_k^2 \equiv k^2 + 3\lambda \varphi_0^2 + \left(\xi - \frac{1}{6}\right) Ra^2. \quad (4)$$

Since the background field is time dependent, Eq. (3) describes an oscillator with a time varying frequency. When $\xi = 0$, due to that the evolution of the scale factor can be approximately expressed as $a \sim \eta$ after using the time averaged relation $\frac{1}{2} \langle \dot{\phi}^2 \rangle = 2 \langle V \rangle$, the time averaged value of $R$ vanishes as a results of $R = \frac{6}{a^3} a''$, where a prime denotes the derivative with respect to the conformal time $\eta$. Eq. (3) belongs to the class of Lamé equation, and thus $\delta\varphi_k$ has exponentially growing modes for certain momenta $k$. But the resonant bands are in narrow ranges since the amplitude of $\varphi_0$ is very small. For the
nonminimal coupling case, the oscillations of the inflaton and its perturbations differ as compared to the minimal coupling case because of the last term in Eqs. (1) and (4), so that the resonant structure is modified and the resonant bands become broad with the increase of $|\xi|$.

To obtain a panoramic view of preheating, the best way is to perform numerical lattice simulations. By modifying the publicly available C++ package LATTICEEASY [21], we investigate the evolution of the nonminimally coupled inflaton field in the configuration space in an expanding universe. The results are simulated with $N^3 = (128)^3$ points and the minimum momentum $k/(\sqrt{\lambda}\phi_i) \sim \mathcal{O}(0.2)$. Here, $\phi_i$ is the value of the scalar field at the beginning of preheating, which is given by [18, 19]

$$\phi_i = \left[ \frac{\sqrt{(1 - 24\xi)(1 - 8\xi)} - 1}{16\pi(1 - 6\xi)|\xi|} \right]^{1/2} m_{\text{pl}},$$

where $m_{\text{pl}}$ is the Planck mass. We use the conformal vacuum as the initial state with the initial conditions of the fluctuations being $\delta \varphi_k(0) = 1/\sqrt{2\omega_k(0)}$ and $\delta \varphi_k'(0) = -i\omega_k(0)\delta \varphi_k(0)$, and stop the simulation when the density spectrum of the inflaton quanta does not change appreciably.

We show in Fig. 1 the evolution of the energy density spectrum $k^3 \rho_k/(\lambda\phi_i^4)$ of the inflaton quanta, where $\rho_k = \frac{1}{2} (\omega_k^2 |\varphi_k|^2 + |\varphi_k'|^2)$ is the comoving energy density of created momentum modes [22], as a function of $k/(\sqrt{\lambda}\phi_i)$ with time $\tau \equiv \eta/(\sqrt{\lambda}\phi_i)$ for different values of $\xi$ ($\xi = 0, -5, -20$ and $-30$). Although the coupling constant is constrained to be $\xi < -1.9 \times 10^{-3}$ [14, 15], we still consider the case of $\xi = 0$ as a comparison. One can see that the nonminimal coupling modifies the resonant structure and affects the amplitude of the density spectrum which increases with the increase of $|\xi|$, but does not change the spectrum shape. At the initial linear stage of preheating, the growth of inflaton quanta takes place mainly in the resonance bands. This leads to the formation of the peak structure in the spectra. When the backreaction effects of created momentum modes are significant, the evolution of the inflaton quanta enters a fully nonlinear stage and then the main peaks of the spectra stop growing and reach their maxima. Due to the scattering among different modes, the peaks created during the early stages are gradually washed out. An interesting feature is that the maximum peak gradually decreases and moves lightly to $k = 0$, but it does not disappear completely.
FIG. 1: The evolutions of the energy density spectra \( k^3 \rho_k/(\sqrt{\lambda} \phi_i^4) \) of the inflaton quanta as a function of \( k/(\sqrt{\lambda} \phi_i) \) with time \( \tau \equiv \eta/(\sqrt{\lambda} \phi_i) \). The spectra from bottom to up are plotted with the time interval \( \Delta \tau = 50 \), with green line corresponding to the final result. The purple and red lines represent the early and late times, respectively.

Now we have seen that the perturbations of the scalar inflaton are amplified by the parametric resonance. But, these perturbations do not cause significant changes on the scalar power spectrum at large length scales and hence do not affect the predictions of inflation for the CMB [7]. On small scales, the growth of fluctuations may lead to copious production of primordial black holes [7]. However, whether the amplitude of the power spectrum of the scalar inflaton perturbations at short length scales is consistent or not with the bounds from ultracompact minihalo objects and primordial black holes [23] remains unclear at present. Nonetheless, this is an interesting issue which we would rather leave for future investigation, since it is beyond the scope of the present paper.
Now, we study the production of GWs during preheating, which corresponds to the transverse-traceless tensor perturbations $h_{ij}$ of the flat FRW metric. In the gravitational theory with a nonminimal coupling between the scalar field and the curvature scalar, $h_{ij}$ obeys, to the first order, the following equation of motion

$$
\ddot{h}_{ij} + \left(3H + \frac{\dot{F}}{F}\right)\dot{h}_{ij} - \frac{1}{a^2} \nabla^2 h_{ij} = \frac{2\kappa^2}{a^2F} \Pi_{ij}^{TT},
$$

(6)

with

$$
\Pi_{ij} \equiv (1 - 2\xi)\partial_i\phi\partial_j\phi - 2\xi\phi\partial_i\partial_j\phi,
$$

(7)

where $F \equiv 1 - \xi\kappa^2\phi^2$, $\kappa^2 = 8\pi G$, and $\Pi_{ij}^{TT}$ is the transverse-traceless part of $\Pi_{ij}$. The term $\Pi_{ij}^{TT}$ associated with the inhomogeneous inflaton field sources the gravitational radiation. In comparison with the minimally coupled model, we find that the source of gravitational waves $\Pi_{ij}^{TT}$ contains two extra $\xi$-dependent terms which predominate for $|\xi| > 1/2$. The energy density associated with GWs is given by [24]

$$
\rho_{gw} = \sum_{i,j} \frac{1}{32\pi G} \langle \dot{h}_{ij}^2 \rangle.
$$

(8)

Here $\langle \cdot \cdot \cdot \rangle$ is the spatial average.

In our numerical calculation, we introduce the transverse-traceless projection operator [25]: $\Lambda_{ij,lm} = P_i P_j - \frac{1}{2} P_{ij} P_{lm}$, where $P_{ij}(k) = \delta_{ij} - \frac{k_i k_j}{k^2}$, to obtain the transverse-traceless part of $\Pi_{ij}$ in the momentum space

$$
\Pi_{ij}^{TT}(k) = \Lambda_{ij,lm} \Pi_{lm}(k).
$$

(9)

Using this projection operator, one can define a new tensor $u_{ij}$ which satisfies the following relation in the momentum space

$$
h_{ij}(k) = \Lambda_{ij,lm} u_{lm}(k).
$$

(10)

Instead of directly investigating the evolution of $h_{ij}$ in the configuration space, we solve numerically the equation of motion of $u_{ij}$,

$$
\ddot{u}_{ij} + \left(3\frac{\dot{a}}{a} + \frac{\dot{F}}{F}\right)\dot{u}_{ij} - \frac{1}{a^2} \nabla^2 u_{ij} = \frac{2\kappa^2}{a^2F} [(1 - 2\xi)\partial_i\phi\partial_j\phi - 2\xi\phi\partial_i\partial_j\phi].
$$

(11)
FIG. 2: The evolutions of the density spectra of GWs as a function of \( k/(\sqrt{\lambda_i}) \) with time \( \tau \). The spectra from bottom to up are plotted with the time interval \( \Delta\tau = 50 \), with green line corresponding to the final result. The purple and red lines represent the early and late times, respectively.

with \( u_{ij} \) and its derivative being initialized as zero. This method is different from both the one of solving Eq. (6) directly [26] and the one based on the Green’s function [27]. Then the energy density (8) can be rewritten as

\[
\rho_{gw} = \frac{1}{8GL^3} \int d\ln k k^3 \int \frac{d\Omega_k}{4\pi} \Delta_{ij,lm} \dot{u}_{ij}(k) \dot{u}^*_{lm}(k),
\]  

with \( L \) the length of one side of the lattice, and the corresponding spectra of GWs, normalized to the critical energy density \( \rho_c \), can be obtained from

\[
\Omega_{gw} \equiv \frac{1}{\rho_c} \frac{d\rho_{gw}}{d\ln k} = \frac{\pi k^3}{3H^2L^3} \int \frac{d\Omega_k}{4\pi} \Delta_{ij,lm} \dot{u}_{ij}(k) \dot{u}^*_{lm}(k).
\]  

The evolutions of the GW density spectra from the lattice simulation are shown in Fig. 2 with \( \xi = 0, -5, -20 \) and \(-30\). A paramount characteristic is that the final density spectra of GWs have several distinct peaks. These peaks are relative to the resonant bands. Furthermore, the late time evolution of GW spectra leads to a minor peak in the low momentum
FIG. 3: Today’s spectra of GWs with $\lambda = 3 \times 10^{-38}$. The black curve is the expected sensitivity curve of the fifth observing run (O5) of the aLIGO-Virgo detector network.

region, which originates from the non-vanishing peak of the energy density spectrum of the inflaton quanta. For the case of a minimal coupling ($\xi = 0$), the peak value of GWs generated in the main resonance band is noticeably larger than those from subordinate bands, which is consistent with the result from the $\phi^4$ chaotic inflation with an interaction between inflaton and a massless scalar field [9, 28]. For the case of $\xi \neq 0$, one can see that there is a pronounced peak of GWs at the initial stage, which disappears completely in the final spectrum. Our numerical check indicates that this peak results from the contribution of the last term on the right hand side of Eq. (7). This term also suppresses the growth of the low-frequency GWs, and enhances the high-frequency part, so that the produced GWs in the subordinate resonance bands become substantial.

With the increase of $|\xi|$, the peak values of the spectra increase since the energy density amplitude of inflaton quanta is strengthened. The maximum peak values of the spectrum are $1.57 \times 10^{-5}$, $8.59 \times 10^{-4}$, $6.32 \times 10^{-3}$ and $1.30 \times 10^{-2}$ for $\xi = 0, -5, -20$ and -30, respectively. The numerical results tell us that the maximum peak value of the spectrum has a larger value for a larger $|\xi|$ and thus the generated GWs will have strong backreactions on the background evolution. The backreaction effect of GWs, however, is beyond the scope of this paper, so we do not consider further bigger values of $|\xi|$.

Since the model parameter $\lambda$ is constrained by the amplitude of the CMB temperature fluctuations to be $\lambda \simeq 4 \times 10^{-10} \xi^2$ [15, 29] (when $|\xi \kappa^2 \phi^2| \gg 1$ during inflation), after projecting the energy spectrum of GWs generated during preheating into today, its peak frequency is beyond $10^7 \text{Hz}$ and blueshifts with the increase of $|\xi|$. This frequency exceeds many orders of magnitude the frequency possibly detectable by current GW experiments.
However, the energy scale of inflation determines the peak frequency scale of GWs rather than the peak amplitude of the energy spectrum [10, 11]. Thus, if we relax the constraint on $\lambda$ from the CMB observations, frequencies lying in the present experiments detection range might be possible. For example, setting $\lambda = 3 \times 10^{-38}$, the peak frequency is about $30\text{Hz}$ when $\xi = -30$. Since the amplitude of GW spectrum does not reduce, its spectrum could lie above the expected sensitivity curve of the fifth observing run (O5) of the aLIGO-Virgo detector network [30]. Fig. 3 represents our predictions for GW spectra today. One can see that there are two peaks for $\xi = -5$ and four peaks for $\xi = -20$ and $-30$ falling within the range of the O5 detection. Our results are different from that obtained in [12, 13] where the pronounced peak is one for the asymmetric potential and two for the cuspy potential. They are also different from that of the hybrid preheating [31] where the amplitude of GWs is significant for the high-scale model of inflation, but has no apparent peaks. Thus, the detection of GWs in the future will provide us a chance to differentiate different inflationary models, to understand underlying physics of preheating since the GW peaks are related with the parametric resonance, as well as to obtain an upper limit on the coupling strength as the GW amplitude is determined by the value of the coupling parameter.

Now a few comments are in order for our choice of $\lambda \sim 10^{-38}$ to get a GW spectrum, which might be within the reach of the aLIGO-Virgo detector network, and our neglect of couplings of the inflaton to other fields during the preheating. First, we want to point out that a value of $\lambda$ as tiny as $\sim 10^{-38}$ may not be as unrealistic as it appears to be at the first sight. For example, it is well known that in the hybrid inflationary scenario [32] $\lambda$ is essentially a free parameter, and $\lambda \sim 10^{-38}$ corresponds to the inflationary energy scale of about $10^{7}\text{Gev}$, which is within the allowed region of a successful model of inflation. Second, in this paper we only examined the $\delta\phi$—particle production. In addition to this process, the $\phi$ field can also decay to other particles $\chi$ through the interaction, i.e. $\frac{1}{2}g^2\phi^2\chi^2$, but the $\delta\phi$—particle production appears to be the leading process for $g^2 \ll \lambda$ in the minimally coupled case [33]. So, if the coupling of the inflation field with matter is very weak, it is reasonable to assume these additional couplings can be neglected safely for the model considered here. However, in this case, our universe might not be thermalized to a high enough temperature by the production of $\delta\phi$—particles. This is because that the relation between the reheating temperature $T$ and the coupling constant $\lambda$ has been shown to be $T \propto \sqrt{\lambda}$ in the case of nonminimal coupling [34], and $T \sim 0.1\text{Gev}$ when $\lambda \sim 10^{-38}$. Therefore,
the $g^2 \phi^2 \chi^2$-like couplings are needed to thermalize our universe. The periodic oscillation of the field will lead to the resonant production of $\chi$ particles inevitably, which can amplify the reheating temperature. If the universe can be thermalized to a high enough temperature, the standard model thermal plasma can be generated by the self-interaction of $\delta \phi$-particles along with the decay of inflaton field through the $g^2 \phi^2 \chi^2$-like interactions. But the reheating temperature may not be able to reach a high enough value to allow for big bang baryogenesis when $g^2/\lambda$ is very small. For example, in the minimally coupled case, it has been found that the reheating temperature can expressed as $\frac{1}{10} m_{pl} \lambda^{1/4} \left( \frac{g^2}{\lambda} \right)^2 \log^{-1}(1/\lambda)$ when the parametric resonance is considered [35] and therefore it can not reach the electroweak energy scale when $g^2/\lambda < 0.01$ and $\lambda \sim 10^{-38}$. As the reheating temperature depends on the magnitude of the couplings between the inflaton field and other matter, we also need to consider the resonant productions of both inflaton quanta and $\chi$ particles at the same time. Moreover, the reheating also depends on whether the $\chi$ field couples with gravity or not. If the $\chi$ field is coupled nonminimally with gravity ($\xi' R \chi^2$), our numerical simulations indicate that the tachyonic preheating will occur because the effective mass of the $\chi$ field becomes negative when $\phi$ drops below a critical value. Since an systematic analysis of preheating with the couplings of the inflaton field with other matter fields not neglected is very complicated and the tachyonic preheating is different from the preheating considered in this paper, we will not go any further on them but leave to a future paper.

IV. CONCLUSION

We have investigated the preheating and the in-process production of GWs after inflation in which the inflaton is nonminimally coupled to the curvature in a self-interacting quartic potential. We find that the amplitude of the density spectrum of inflaton quanta is enhanced by the nonminimal coupling and it increases with the increase of the coupling parameter. As a result, the peak values of the GW spectrum generated during preheating also increase with the increase of the coupling parameter and might reach the detection limit of future GW experiments. We also find that the peaks of GW spectrum generated during preheating in which the inflaton is nonminimally coupled to the curvature exhibit distinctive characteristics as compared to those of minimally coupled inflaton potentials and reflect the story of parametric resonance. Thus, the detection of these peaks in the future will provide us a
new avenue to reveal the physics of the early universe.

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