Hot Neutron Stars as a Source for Gamma Ray Bursts at Cosmological Distance Scales

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Abstract

We discuss the possibility that the sources for gamma ray bursts are hot neutron stars at cosmological distance scales. The temperature of such stars would be $T \sim 1\text{ MeV}$. Such hot stars can produce an electromagnetic blast wave provided that the ratio of baryon and photon numbers $N_B/N_\gamma \leq 10^{-6}$. The typical time scale for such blasts, the total luminosity, and correlation of gamma ray energy with time of arrival are shown to be roughly consistent with observation. The spectrum of photons also appears to be consistent with known data.
1 Introduction

Gamma ray bursts (GRBs) have been the subject of much theoretical and experimental study\[1, 2, 3\]. The source of these bursts is the subject of much dispute. If the source is near our galaxy, then such objects might be neutron stars\[4\]. If on the other hand, such objects are at cosmological distance scales, then the total energy released in photons is typical of supernova explosions. In this paper, we will assume that gamma ray bursts are at cosmological distance scales.

The typical energy and number of such energetic photons are so large that a conventional supernova explosion will not describe the bursts. In a supernova, most photons are soft. The neutrinos carry energies typical of one MeV scale, but the photon energy is degraded. Any initially energetic photons are produced in an optically thick region. The photons one sees are produced much later than the time at which the energetic neutrinos and photons are produced, and are emitted after the supernova remnants have cooled.

At first sight, it would seem impossible that photons produced from a hot source in the center of some violent explosive event could maintain their energy. In the seminal work by Paczynski\[5\] and by Goodman\[6\], it was shown that under certain circumstances this is indeed possible. In this work, it was suggested that energetic photons emitted from some hot source would expand hydrodynamically. It is assumed that to a good first approximation, one has an expanding gas of photons and perhaps initially (depending upon the initial temperature) electron-positron pairs in thermal equilibrium. The ratio of baryons to photons, \(N_B/N_\gamma\), is assumed to be very small.

In this hydrodynamical expansion, both the energy and entropy are conserved. The energy per photon is thus roughly constant. (There may be an order one change due to the annihilation of electron-positron pairs as the hot plasma cools.) Near the hot source, the photon gas is moving outward slowly and the energy per photon is large, since the initial temperature is assumed to be large. Far away from the source, the fluid is relativistic and has a large Lorentz \(\gamma\) factor. The local rest frame
temperature is however small, of order $T_s/\gamma$, where $T_s$ is the surface temperature. The local temperature has been traded for the relativistic gamma factor of the fluid, and this allows for the constant energy per photon.

In order that the energy per photon be roughly conserved, it is necessary that the baryon to photon ratio, $N_B/N_\gamma$, be small. We will see that this typically requires $N_B/N_\gamma \leq 10^{-6}$. This small ratio at first sight seems unnatural since any mechanism which produces the photons must almost certainly come from a baryon rich region. If the ratio is larger, the energy of the gammas at the source is significantly degraded at decoupling from the moving fluid.

This picture of Paczynski and of Goodman has led to a blast wave description of the later stages of such an explosive process\textsuperscript{7}. It is claimed that it has the correct properties to explain the afterglow\textsuperscript{8} which has been recently observed in the visible spectrum\textsuperscript{9}. If the observations of this afterglow are correct, then some fraction of gamma ray bursts must occur at cosmological distance scales.

In this conventional picture\textsuperscript{8}, a GRB is produced in two stages. First, a yet unspecified event generates optically thick plasma by radiating a huge amount of heat in a few tens of seconds. Eventually, the energy of this relativistically expanding fireball is all converted to the proton kinetic energy. In the next stage, the proton kinetic energy dissipates producing GRBs either by internal collision and/or by the collisions with interstellar medium.

Our picture here is somewhat different from the above conventional one. In our scenario, the initial photons are the direct source of GRBs. Hence, we require that the protons do not absorb a large portion of the initial photons.

Regardless of the difference, the conventional picture and ours share a common theme – the blast wave. In this paper, we address the issue of how such a blast wave is generated. Since the total energy in such a blast wave is so large, a source such as a neutron star or the collision of neutron stars would appear to be a likely candidate. We will accept this hypothesis here and then see whether the gross properties of
gamma ray bursts can be explained in such a picture.

There have been a multitude of suggestions for neutron-star mechanisms for generating gamma ray bursts at cosmological distance scale. The observed energy per photon and the total luminosity require an energy release per proton of about $E/N_B \sim 1 - 10\text{ MeV}$. The classes of models which could generate such an energy release might be the following:

1. **Neutron (Strange) Star Collisions or Collapsing Neutron Star Binaries**

   In neutron star collisions or the collapse of neutron star binaries, some of the gravitational binding energy is released as heat\[10\]. If the stars collapse to a black hole, the gravitational binding energy of a particle is its rest mass. Only a small fraction of this energy needs be converted into radiation. Whether or not the time scales for such collapse are short enough\[11\] is not certain, and whether some hot emitting region can live long enough outside the event horizon is not clear.

   Another possibility is a collision of two small neutron stars producing not a black hole but a bigger neutron star. The time scale of this may be just about what we need for a GRB.

   Yet another possibility is collisions or a collapse involving strange stars. A strange star as a GRB source may have an additional benefit of possessing less volatile surface than that of a neutron star.

2. **Nucleation of a Black Hole Inside a Neutron Star**

   It might be possible that a black hole is spontaneously nucleated inside a neutron star\[12\]. If so, heat would be released in the core of the star which would heat the surface layers and serve as an emitting region. Whether this emitting region could live long enough to describe gamma ray bursters is not clear\[13\].

3. **Phase Transition of Matter in the Core of a Neutron Star**
A phase transition between different phases of matter might take place in the core of a neutron star [14, 15, 16]. We will not be concerned with the type of phase transition here except to note that the natural energy scale in the core is of order $E/N_B \sim (10 - 100 \text{ MeV})$ per nucleon which when spread out over the entire star should be large enough to describe gamma ray bursts. Moreover, the star will maintain its integrity after the phase transition since the energy release is small compared to the gravitational binding energies of nucleons in the star.

The list above may not be complete. Certainly some or perhaps all of the mechanisms above may not describe gamma ray bursts and more detailed computations within detailed models would be necessary to obtain better resolution of the correctness of the various hypothesis.

We will in this paper make a few simple generic assumptions about the nature of the source of gamma ray bursts and explore the generic consequences of these assumptions. We specifically assume that the source region is of size and baryon density typical of a neutron star. We assume that somehow this region maintains its integrity over the lifetime of the gamma ray burst. We assume the typical energy per nucleon is typically of order $1 - 10 \text{ MeV}$ per nucleon.

With these assumptions, we will make a hot neutron star model of the gamma ray burst. For this model to work, we must show that

1. The time scale of the evolution of the burst is $\sim 1 \text{ sec}$.

2. The surface of the neutron star heats up quickly after the initial generation of heat.

3. The surface temperature ($\sim 1 \text{ MeV}$), when it is established, is not too much smaller than the core temperature.

4. The baryon contamination of the gamma-sphere is small enough so that at the decoupling, a large number of energetic photons originating from the surface
survive.

We will find that the natural time scales for the gamma ray burst work out correctly as a consequence of this model. We also crudely compute the time evolution of the luminosity and energy per photon and argue that this is generically correct. The differential energy spectrum of emitted photons also appears to be qualitatively correct and does not fall as rapidly with decreasing energy as the fireball model of Goodman [6].

The surface temperature of a hot neutron star is estimated by assuming that the hot core generates thermal neutrinos via modified Urca process, and the surface radiates heat by the neutrino pair bremsstrahlung. We first show that the typical neutrino mean free path is so long that a neutrino interacts at most once with an electron inside the star. Hence the neutrinos from the hot core transport heat to the surface with the speed of light. Assuming that the dominant cooling process near the surface is the neutrino pair bremsstrahlung, we will find that the surface temperature is not much different from the core temperature.

The issue of $N_B/N_\gamma$ is more problematic. We have several suggestions which may allow for a sufficiently small $N_B/N_\gamma$ so that our model works: (i) a strong magnetic field near the neutron star surface, (ii) the formation of super-heated Coulomb crystal near the surface, or (iii) the nucleon binding energy of a strange star. Certainly, the Coulomb crystal should exist in a cold neutron star, and strong magnetic field is believed to exist at pulsars.

The physical picture we generate as a consequence of this model seems to be consistent. The region from which the photons decouple from the hot expanding electromagnetic plasma is large compared to a typical neutron star size. Therefore the details of the shape of the emitting region which generates the electromagnetic plasma is not important. It need only be some region of the right dimensions, temperature and density. The calculation of the baryon to photon ratio will involve understanding the local emission from a hot surface. Again, the details of the shape and dynamics
of the emitting region will not be important.

2 The Hot Neutron Star

We assume that a large amount of heat is deposited into the core of a neutron star, which gives an energy of about 10 MeV per nucleon. We assume the time scale for this deposition is much less than the characteristic time scale for a gamma ray burst which we take to be of order $t_{\text{burst}} \sim 1 \text{ sec}$.

The first question we have to ask is how disruptive is this deposition process, and the thermal emission which follows the deposition, to the star. In an ordinary star, such energy deposition would be catastrophic. There would be an explosion and a large fraction of the star would be blown away. This is not the case for a neutron star however. The typical scale in such a star is set by Fermi energies which are typically hundreds of MeV. The gravitational binding energy of a nucleon is also hundreds of MeV. The energy deposited is small compared to these scales, and we therefore do not expect a catastrophic disruption of the star.

On the other hand, near the surface of the star, there will be emission of some of the baryons from the star. We will return to this problem later.

The initial energy deposited into the star can be spread throughout the star by several mechanisms. Shock fronts and burning fronts can be generated. These fronts typically have relativistic velocities in a neutron star, and since the size is small $R \sim 10 \text{ km}$, the time scale for spread of this energy ($\sim 10^{-4} \text{ sec}$) is also much less than a second.

In addition to collective mechanism for transfer of heat, neutrinos also carry and deposit heat throughout the neutron star. The scattering mean free path for a neutrino is

$$\lambda_{\text{scat}} = \frac{1}{\sigma_{\nu e} \rho_e (E_{\nu}/\mu_e)},$$

(1)

where $\sigma_{\nu e}$ is the neutrino-electron scattering cross section, $\rho_e$ is the electron num-
ber density, and $\mu_e$ is the electron chemical potential. The factor $(E_\nu/\mu_e)$ accounts for the Pauli blocking. The neutrino-electron total cross section is given by $\sigma_{\nu e} \sim (E_\nu^2/1\text{MeV}^2)10^{-19}\text{fm}^2[17]$. Since $E_\nu \sim 3T_c$, where $T_c$ is the core temperature, this gives a typical mean free path

$$\lambda_{\text{scat}} \sim 10^3\text{km} \left(\frac{1\text{MeV}}{T_c}\right)^3 \left(\frac{\rho_{\text{nuc}}}{Y_e\rho_B}\right)^{2/3}, \quad (2)$$

where $\rho_{\text{nuc}} = 0.17\text{fm}^{-3}$ is the nuclear matter density and $Y_e \equiv \rho_e/\rho_B$ is the electron-baryon ratio. For most neutrinos, the mean free path is much longer than the size of the neutron star.

One might suspect that since the neutrino mean free path is so long, the surface temperature could not be maintained after the initial rise to $\sim 1\text{MeV}$. This is not so. The amount of energy flowing out of the core can be enormous since the rate is proportional to $T_c^8$ (modified Urca process). Hence, $T_c \sim 5\text{MeV}$ can easily maintain the surface temperature of 1 MeV even if the fraction of energy deposited is small.

To see that this deposit is big enough to maintain the surface temperature consider a steady state maintained by the modified Urca process near the core and direct Urca process near the surface. The hot core puts out neutrino energy with the rate per volume [17]

$$\frac{d^2E}{dt dV} = \kappa_{\text{Urca}}T_c^8, \quad (3)$$

where

$$\kappa_{\text{Urca}} \sim 10^{-4}\text{MeV}^{-7}\text{fm}^{-3}\text{sec}^{-1} \left(\frac{\rho_e}{\rho_{\text{nuc}}}\right)^{2/3}. \quad (4)$$

Assuming uniform temperature within a core radius $R_c$, the total energy output per unit time is

$$\left.\frac{dE}{dt}\right|_{R_c} = \kappa_{\text{Urca}}T_c^8 \frac{4\pi}{3}R_c^3 \quad (5)$$

Since the neutrino mean free path is typically much longer than the neutron star radius $R$, most of neutrinos originating from this hot core reaches $R$ unscathed. The
survival probability of these neutrinos crossing a radial distance $dR$ near $R$ is given by

$$P = 1 - \frac{dR}{\lambda_{\nu e}}, \quad (6)$$

where the neutrino mean free path is

$$\lambda_{\nu e} = \frac{1}{\sigma_{\nu e} \rho_e}. \quad (7)$$

This differs from Eq. (2) because the Pauli blocking is irrelevant near the surface where the density is small. The conversion rate of the neutrino energy into heat is then

$$\frac{dE_{\text{in}}}{dt} = \kappa_{\text{Urca}} T_c^8 \frac{4\pi}{3} R_c^3 dR \lambda_{\nu e}, \quad (8)$$

to the surface. For a steady state to be maintained, this incoming energy must be balanced by the energy going out of the volume $dV = 4\pi R^2 dR$. We take the neutrino pair bremsstrahlung to be the dominant cooling process of the surface due to the fact that the neutron star surface is mainly composed of stable nuclei such as $^{56}_{26}\text{Fe}$. Then heat is radiated with the rate given by [17]

$$\frac{dE_{\text{out}}}{dt} = \kappa_{\text{Brem}} T_s^6 \frac{4\pi}{3} R^2 dR, \quad (9)$$

where $T_s$ is the temperature at $R$ and

$$\kappa_{\text{Brem}} \sim 10^{-6}\text{MeV}^{-5}\text{fm}^{-3}\text{sec}^{-1}\left(\frac{\rho_s}{\rho_{\text{nuc}}}\right). \quad (10)$$

Equating the two, we get

$$T_s \sim 0.1\text{MeV} \left(\frac{T_c}{1\text{MeV}}\right)^{5/3} Y_e^{1/6} \left(\frac{\rho_e}{\rho_{\text{nuc}}}\right)^{1/9} \left(\frac{R_c}{1\text{km}}\right)^{1/2} \left(\frac{10\text{km}}{R}\right)^{1/3}, \quad (11)$$

where $Y_e \equiv \rho_e/\rho_s$ is about 0.5 if the Coulomb lattice is made of $^{56}_{26}\text{Fe}$. With $T_c \sim 5\text{MeV}$ and $R_c \sim 1\text{km}$, the surface temperature of 1 MeV is easily maintained.\footnote{Neutrino pair annihilation process also deposit energy in the form of lepton pairs. However, the mean free path for this process may be orders of magnitude larger than that of the Urca process because the available transverse energy decreases as the radius increases.}
3 The Gamma-sphere and Its Interface with the Hot Neutron Star

Outside the baryonic matter which makes up the neutron star, there are high energy gammas which are escaping the surface. Unlike the situation for stars such as the sun, these gammas are interacting with one another on a time scale which is small compared to the typical expansion time for the photons, that is, the time it takes for the local photon energy density to dilute by a factor of two. Paczynski and Goodman have written down hydrodynamic solutions for the evolution of matter in this region. We will shortly give an argument that the baryon to photon ratio $N_B/N_\gamma$ is small enough for the hydrodynamic picture to be valid. But for now, let us assume that the baryon to photon ratio, $N_B/N_\gamma$, is so small that one can treat the system outside the star as an electroweak plasma. In this case, the pressure is

$$P = N_{\text{dof}} \frac{\pi^2}{90} T^4,$$

(12)

where the number of degrees of freedom $N_{\text{dof}}$ certainly includes photons, may include electron-positron pairs if the temperature is high enough, and might also include neutrinos if they were not yet decoupled from the system.

We will assume that the time evolution of the system occurs on time scales long compared to the time it takes light to propagate the characteristic size scale of the system. If this is the case, we look for a static solution to the hydrodynamic equations,

$$\partial_\nu T^{\nu\mu} = 0.$$

(13)

We will ignore the effects of gravity in the gamma-sphere. (In the interior of the star this is not a good approximation and one must modify the right hand side of the above equation.)

These equations were solved by Paczynski far from the star. They are equivalent to the algebraic equations

$$T_\gamma = T_\gamma,$$

(14)
and
\[ 4\pi r^2 (\rho + P) v \gamma^2 = L. \] (15)

Here the energy density of the electroweak plasma is \( \rho \) and its pressure is \( P \). This electromagnetic plasma is moving with a velocity \( v \) and Lorentz gamma factor \( \gamma \). The temperature is \( T \). The luminosity of the star is \( L \). \( T_s \) is the surface temperature of the star and \( \gamma_s \) is the Lorentz gamma factor of the fluid just outside the surface. A little algebra shows that the minimum velocity for the fluid which composed the gamma-sphere occurs at some minimum radius and is \( v^2 = 1/3 \). See the appendix. At larger distances, the velocity of the fluid increases. At large distances, \( \gamma \sim r/R \) and \( T \sim T_s R/r \), so the fluid rapidly become ultrarelativistic.

It is plausible that the minimum \( v^2 = 1/3 \) occurs at the surface between the gamma-sphere and the neutron star. If the neutron star is not evaporating too rapidly, this must happen, since the rarefaction front associated with the electromagnetic plasma will propagate inward until it is slowed by the surface of the star. Moreover, if one emits free photons from a surface at rest, the average outward velocity of the emitted photons will be \( v^2 = 1/3 \). We therefore take the minimum radius where \( v^2 = 1/3 \) to be the surface of the star.

The first problem which we must address in this picture is how small the ratio \( N_B/N_\gamma \) should be. In order that one can ignore the effect of the baryons on the collective expansion of the fluid, the total energy of photons at decoupling must be larger than the total energy of the baryons

\[ N_B/N_\gamma < T_{\text{decoupling}}/m_{\text{proton}}, \] (16)

where \( T_{\text{decoupling}} \) is the rest frame temperature when the photons start decouple.

This decoupling temperature is determined by the requirement that the photon interaction probability beyond the decoupling radius (and hence the decoupling temperature) is negligible. The decoupling radius is in turn determined by the condition

\[ r_{\text{decoupling}} = \lambda_\gamma, \] (17)
where $\lambda_\gamma$ is the mean free path for photon scattering. We expect decoupling at a temperature less than that for which there are abundant electron positron pairs, so the decoupling temperature is determined by photon-electron Compton scattering

$$\lambda_\gamma = \frac{1}{\sigma_{\text{Compton}} \rho_{\text{electron}}} \sim \frac{m_{\text{electron}}^2}{\alpha^2} \frac{N_\gamma}{N_B} T^{-3},$$

(18)

where we used the fact that $\rho_{\text{electron}} = \rho_{\text{proton}} = (N_B/N_\gamma) \rho_\gamma$. This gives for the decoupling distance

$$R_{\gamma,\text{decoupling}} \sim R_{\text{decoupling}} \sim 10^7 \text{ km} \sqrt{\frac{N_B}{N_\gamma}} \left(\frac{T_s}{1 \text{ MeV}}\right)^{3/2} \left(\frac{R}{10 \text{ km}}\right)^{3/2}.$$  

(19)

The criterion that the baryons are not important (16) now becomes

$$\frac{N_B}{N_\gamma} < 10^{-6} \left(\frac{1 \text{ MeV}}{T_s}\right)^{1/3} \left(\frac{10 \text{ km}}{R}\right)^{1/3}.$$  

(20)

Hence for surface temperature of 1 MeV, the baryon contamination must be less than one part in a million.

This small baryon contamination is hard to achieve by conventional means. For such an estimate for a neutron star heated from inside, we note that on the average, an outgoing photon of energy $T$ transfers a momentum of order $T$ to an electron-proton pair. Assuming a biased random walk, we then have too big $N_B/N_\gamma \sim 10^{-3}$ for $T \sim 1 \text{ MeV}$.

For an estimate of the ratio for a proto-neutron star, assume a radiation dominated atmosphere [18]. When the radiation energy density $T^4$ exceeds that of the gravitational energy density $G M m_{\text{nucleon}} \rho/r$, energy outflow in the form of baryon ejection is expected. This happens when

$$\rho_s \lesssim \rho_0 \equiv \frac{T^4 R}{G M m_{\text{nucleon}}}. $$

(21)

If the matter and radiation flows out with the same velocity, then the ratio $N_B/N_\gamma$ equals the ratio of the baryon density and the photon density. Using the estimate [21] yields again too big $N_B/N_\gamma$:

$$\frac{N_B}{N_\gamma} \sim \frac{\rho_0}{T^3} \sim 10^{-2} \left(\frac{R}{10 \text{ km}}\right) \left(\frac{M_\odot}{M}\right) \left(\frac{T}{1 \text{ MeV}}\right).$$

(22)

11
Another estimate of the mass expulsion was given by Meszaros & Rees [7]. They assumed that the surface of the initially cold neutron star with densities lower than $\rho_0$ is expelled. The outer density profile of a cold neutron star is calculated from the hydrostatic equation

$$\frac{1}{\rho} \frac{dP}{dr} = -\frac{GM}{r^2},$$

with the degenerate electron pressure $P \propto \rho^{4/3}$. They estimate a lower limit of the ejected mass ($\rho(r_0) = \rho_0$)

$$\Delta M \simeq 4\pi R^2 \int_{r_0}^{\infty} \rho(r) dr = 10^{-12} M_\odot \left( \frac{R}{10 km} \right)^{10/3} \left( \frac{T}{1 MeV} \right)^{16/3} \left( \frac{M}{M_\odot} \right)^{1/3}.$$  

Here they assumed a temperature of $T = 63 MeV$ based on an estimate of frictional heating in mergers and consequently obtained $\Delta M \sim 10^{-3} M_\odot$, which is too big. Detailed numerical calculations of mergers have later found lower temperatures [13]. With $T \sim 1 MeV$ the amount of baryon ejected is sufficiently small according to (24) that baryon contamination can be ignored. However, it is only a lower estimate as can be seen by comparing to (22).

Compared to these conventional estimates, one part in a million may seem unnatural. However there may be numerous ways to get around the above naive estimates. We list some of the possibilities here:

**Strong magnetic field**

A surface temperature of $\sim 1 MeV$ can be maintained without ejecting baryons by a magnetic field of $B \sim 3 \times 10^{13} G$ [19]. A magnetic field of such strength is perhaps not unreasonable for a pulsar-like neutron star where differential rotation or dynamo effect can generate a strong magnetic field. This strong magnetic field need not cover the whole surface of the neutron star. A strong magnetic field creates a relatively baryon-free region near the magnetic poles because charged particles follow the diverging magnetic field lines. These baryon-free regions near the magnetic poles would be effective in producing a GRB [20].
Super heated Coulomb lattice

Another possibility is the super-heated Coulomb lattice. Even though the Coulomb lattice near the surface of a neutron star should melt at temperature below 1 MeV, a meta-stable state might exist for some time before melting during which a GRB would be produced. It might also be stable in a strong magnetic field. Whether the super-heated lattice can exist at $T \sim 1\text{ MeV}$ requires more careful study.

Strange star

When the density of matter becomes much higher than the nuclear matter density, a strange matter may be the ground state.

A typical strange star has a thin crust of baryons at the surface. The fireball originating from a strange star, therefore, will be contaminated by baryons from the crust. Fortunately, the crust is very thin ($M_{\text{crust}} \sim 10^{-5} M_\odot$) and the fireball may be relatively free of baryons. Materials inside the crust will not be disturbed by 1 MeV temperature since the nucleon binding energy of a strange star is a few tens of MeV.

Note that our formula for the surface temperature would not work for a strange star since the chemical composition of a strange star is very much different from that of a neutron star. However, the surface temperature and the core temperature should still be related by a power law. Hence, qualitative descriptions in this and the next section should still apply.

4 Cooling of the Star

As the star emits energy it cools. At the temperature of order 1 MeV, the Urca process is the dominant source of the energy loss. Consider a hot core of radius
\( R_c < R \) at \( T_c \). Then as in Eq. (5),

\[
\frac{dE}{dt} = -\kappa T_c^8 \frac{4\pi}{3} R_c^3,
\]

(25)

where the minus sign indicates energy loss.

On the other hand, the total thermal energy inside the core is related to the temperature by

\[
E \sim N_c T_c (T_c/\mu_c) \sim R_c^3 \rho_c T_c (T_c/\mu_c),
\]

(26)

where \( \rho_c \) is the baryon density of the core and \( \mu_c \sim \rho_c^{2/3} / m_{\text{proton}} \) is the Fermi energy associated with \( \rho_c \). We see therefore that

\[
\frac{dT_c}{dt} \sim -\kappa T_c^7 \frac{\mu_c}{\rho_c}.
\]

(27)

This has as a solution

\[
T_c = T_c^{\text{init}} \left(1 + t/t_{\text{scale}}\right)^{-1/6},
\]

(28)

where

\[
t_{\text{scale}} \sim \frac{\rho_c}{3 \mu_c \kappa (T_c^{\text{init}})^{6/5}} \sim 10^4 \text{ sec} \left(\frac{1 \text{ MeV}}{T_c^{\text{init}}}\right)^{6/5} \left(\frac{\rho_{\text{nuc}}}{\rho_c}\right)^{1/3}
\]

(29)

is the time scale of the initial time evolution. For an initial temperature \( T_c^{\text{init}} \sim 10 \text{ MeV}, t_{\text{scale}} \sim 10^{-2} \text{ sec} \). Hence, at large times \( t > 1 \text{ sec} \),

\[
T_c \sim \left(\frac{\rho_c}{\kappa \mu_c t}\right)^{1/6}.
\]

(30)

Notice that this is independent of the initial temperature. Since

\[
\kappa \sim 10^{-4} \text{ MeV}^{-7} \text{ sec}^{-1} \text{ fm}^{-3} \left(\frac{\rho_c}{\rho_{\text{nuc}}}\right)^{2/3},
\]

(31)

we then have

\[
T_c \sim 5 \text{ MeV} \left(\frac{1 \text{ sec}}{t}\right)^{1/6} \left(\frac{\rho_{\text{nuc}}}{\rho_c}\right)^{1/18}.
\]

(32)
The natural time scale of a second for the time evolution of the burst is a simple consequence of neutrino radiation from a neutron star size object at nuclear matter scale density.

We can now use Eq. (11) to calculate the evolution of the surface temperature. The result is

\[
\left( \frac{T_s}{1 \text{ MeV}} \right) \sim \left( \frac{1 \text{ sec}}{t} \right)^{5/18} Y_e^{1/6} \left( \frac{\rho_e}{\rho_{\text{nuc}}} \right)^{1/54} \left( \frac{R_e}{1 \text{ km}} \right)^{1/2} \left( \frac{10 \text{ km}}{R} \right)^{1/3}.
\] (33)

This evolution of the temperature has many consequences. First there should be a correlation between the arrival time and the energy of photons from the gamma ray burst. Second, the luminosity should go like the typical energy to the power of \( \sim 1.1 \) at any time. It seems the data is in rough agreement with such a prediction.

Another consequence is a hardening of the spectrum of emitted photons at lower energy. If the star had only a fixed temperature, then the differential photon spectrum would be

\[
\frac{dN}{d\omega} \sim \frac{\omega^2}{e^{\omega/T_s} - 1},
\] (34)

which for \( \omega \ll T_s \) scales like \( \omega \). If on the other hand, we emit from a range of temperatures, the distribution will be hardened at the low frequency end so long as \( \omega > T_{\text{min}} \), where \( T_{\text{min}} \) is the minimum temperature for which there is thermal emission. We estimate the rate as \( dN \sim E^3 dt \sim T_s^3 dt \) or

\[
\frac{dN}{d\omega} \sim \omega^{-8/5},
\] (35)

since \( T_s \sim 1/t^{5/18} \). The spectrum is therefore a factor of \( \omega^{-13/5} \) harder at low energies than is predicted by the black body law. This seems to work in the correct general direction as the data. A representative of a data parameterized by \( \exp(-\omega/T_0)/\omega \) with \( T_0 = 0.505 \text{ MeV} \) [13] and our result \( \omega^{-8/5} \) is plotted in Fig. 1.
Figure 1: Photon energy spectrum. The solid line represents a parametrization of an observed spectrum, $dN/d\omega \sim \exp(-\omega/T_0)/\omega$ with $T_0 = 0.505\text{MeV}$, and the broken line represents our result $dN/d\omega \sim \omega^{-8/5}$.

5 Summary and Conclusions

We have attempted to model gamma ray bursts as emission from hot neutron stars with a surface temperature $T_s \sim 1\text{MeV}$. In the crude order of magnitude estimates we have made, we see that we can describe the gross features of such emission. We get a qualitatively valid description of the time evolution. We also argued that a small ratio of $N_B/N_\gamma$ in the hot radiating gamma-sphere of the star can be achieved via strong magnetic field or a super-heated Coulomb lattice. Whether or not this model will work in detail needs further analysis.

In this paper we have tried to ignore the detailed dynamics which will describe the region which we refer to as the hot neutron star. If the description above satisfies experimental constraints, then one can proceed with confidence to a more detailed description.
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A A solution to Eqs. (14) and (15)

Using \( P = aT^4/3 \) and the ultra-relativistic approximation \( \rho = 3P \), one can easily obtain the following solution for \( v \)

\[
v(r) = \frac{2}{\sqrt{3}} \cos\left(\frac{\theta(r)}{3}\right)
\]

(36)

where

\[
\theta(r) = \pi - \tan^{-1}\left(\sqrt{\frac{r^4}{r_{\text{min}}^4} - 1}\right)
\]

(37)

Here, the minimum radius is defined to be

\[
r_{\text{min}} \equiv \sqrt[3]{\frac{9\sqrt{3}L}{32aT^4\pi}}
\]

(38)

When \( r = r_{\text{min}} \), \( v^2(r_{\text{min}}) = 1/3 \). Note that this \( r_{\text{min}} \) is different from \( r_0 \) defined in \[5\]. In Ref.\[5\], \( r_0 \) defines the radius where \( \gamma = 1 \) in the large \( r \) approximation. However, this is a definition of convenience. As is clear from our solution, the minimum speed is that of the sound speed of a photon gas. Hence, \( \gamma \) never goes to 1.

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