Analysis of spectral efficiency for OFDM cooperative cognitive networks with non-linear relay

Samira Hadavi1 | Seyyed Saleh Hosseini2 | Siamak Talebi1

1 Department of Electrical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran
2 Department of Electrical and Computer Engineering, McGill University, Montreal, QC, Canada

Correspondence
Samira Hadavi, Department of Electrical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran.
Email: samirahadavi@yahoo.com

Abstract
This paper analyzes the downlink achievable spectral efficiency of an orthogonal frequency-division multiplexing (OFDM) cooperative cognitive network with a non-linear relay. The analysis is carried out subject to the power amplifier’s (PA) non-linear effect on the relay node which operates in the amplify-and-forward (AF) mode. Specifically, an analytical expression for the power spectral density (PSD) of relay output in terms of its source power is derived. By using the obtained PSD, the power of relay’s PA output is derived for each subcarrier and the 1 dB compression point is determined. Then, the adjacent channel power (ACP) of each subcarrier is analytically derived in terms of secondary user (SU) source input power. Next, the signal to interference and noise ratio (SINR) of each subcarrier at destination (SU receiver) is calculated by considering the non-linear effect of PA at the relay. Having the SINRs of all subcarriers, a constrained optimization problem on the source’s input power is formulated in which the achievable spectral efficiency is the objective function and all ACPs being less than the interference temperature limit are its constraints. Finally, we perform some simulations and show that the numerical results are consistent with the analytical findings.

1 | INTRODUCTION

Due to the ever increasing demand for more transmission bandwidth in the next generation of wireless communications systems, the optimal management of scarce spectrum resources among users has immensely attracted researchers’ attention in recent years [1]. For example, the authors in [2] develop a spectrum management approach for massive multiple-input multiple output (MIMO) technology which is recognised as a promising candidate of 5G [3–6]. So far, various spectrum management schemes such as command-and-control-based and cognitive radio (CR) allocation methods have been developed and applied in the literature [7]. Among these methods, the CR-based techniques have been recognized as the first and popular candidate since they take the advantage of cognitive capability and reconfigurability [8].

The key idea behind the CR-based methods is that an opportunistic user, defined as secondary user (SU), could potentially enjoy the intact licensed spectrum of another user, defined as primary user (PU), provided that the resultant adjacent channel interference (ACI) is less than a given threshold which is called the interference temperature limit [1, 9]. The crucial factor which gives rise to the ACI is the operation of SUs’ power amplifiers (PAs) at their saturation points so as to achieve the maximum power efficiency [10]. This leads to the spectral regrowth of PA’s output signal which in turn, imposes a severe impact on the system performance by amplitude to amplitude (AM/AM) and amplitude to phase (AM/PM) distortion [11, 12]. Hence, the non-linear analyses in CR communications networks are of paramount importance which have recently attracted researchers’ attention [12–14]. The PA analysis can be generally classified into two categories. In the first group, the PA’s signal is linearized by a linearization technique such as pre-distortion and feed-forward for PA’s input or feedback for PA’s output, and then, the particular analysis of CR network will be conducted [15, 16]. However, the linearization methods are far too expensive and their feasibility at relays are limited to the bandwidth which, in turn, reduces the efficiency of PAs [10]. To overcome this difficulty, in the second group, the CR network analyses are carried out under the assumption
of PA’s non-linear effect. For example, in [12] the power allocation of a single carrier CR network is analyzed by considering the non-linear effect of PA under both peak and average ACI constraints. Specifically, it is demonstrated that the power allocation scenario under peak ACI constraints has more normalized spectral efficiency degradation than the one under average ACI constraints.

For the frequency selective scenario, the orthogonal frequency division multiplexing (OFDM) can be efficiently utilized in combination with CR networks [17]. The efficacy of such systems is evaluated in [13, 18] where the adjacent channel power (ACP) is first extracted by using the calculated power spectral density (PSD) of the OFDM system and then, the problem of spectral efficiency optimization is assessed according to the obtained signal-to-interference-and-noise ratio (SINR). However, one common challenge faced by OFDM systems is high peak to average power ratio (PAPR) of output signal at the receiver which significantly reduces the power efficiency in the RF amplifier [19]. In order to combat the effect of PAPR, there exist several PAPR reduction techniques such as clipping and filtering, coding, selective mapping, and partial transmission which are developed and examined in the literature [20–25]. Although these methods might mitigate the PAPR effect to some extent, this impairment cannot be completely eliminated and is inevitable for OFDM-based settings [13]. Moreover, this issue is more pronounced in uplink communications due to the size and power limitation at user terminals which necessitates taking account of PA’s non-linear effects in uplink analysis of OFDM systems [26]. However, the PAPR problem is not very critical in downlink scenarios because linearization methods can be utilized to diminish significantly the non-linearity effect of PA’s in base stations (BS) [27, 28].

One natural and adequate way to enhance the performance of the CR networks is utilizing the cooperative diversity technique which is capable of increasing the achievable spectral efficiency without the need of multiple antennas at the terminal side [29]. The key idea is that the single-antenna users send the transmitted signals both directly and indirectly through a relay which provides independent copies of signals at the receiver side and leads to achieving the diversity gain [30]. Based upon the type of relaying protocol, cooperative techniques could be categorized into three different groups: (i) amplify-and-forward (AF), (ii) decode-and-forward (DF), and (iii) selective decode-and-forward (SDF) [31]. Among these methods, the AF relaying is the most popular and applied scenario in CR settings due to the simpler implementation and lower latency in comparison with other types [32]. The optimization problem of power allocations for the cognitive multi-node relay networks systems is considered in [33] where authors propose a low complexity algorithm to derive a suboptimal solution. Furthermore, a power allocation problem which maximizes the spectral efficiency of relay-assisted OFDM-based CR networks is formulated and solved in [34]. In order to solve this problem with low computational complexity, an efficient suboptimal algorithm which significantly reduces computational costs is proposed in [35]. However, the crucial non-linear effect of PA’s on the relays is not considered.

In an attempt to develop the non-linear analysis of AF cooperative OFDM systems, a closed-form expression for a tight lower-bound of the outage probability is obtained in [36] where non-linearity of PA’s is contemplated for the relays. In [37], authors extract the outage probability of OFDM-based non-linear relay systems in which the AF relays are capable of two-way communications. In [27], the outage probability and average symbol error rate analyses of OFDM-based non-linear relay systems are extended to independently but not necessarily identically distributed frequency selective channels with Nakagami fading. So far, there have been carried out considerable research efforts directed towards analysis of cooperative and cognitive OFDM-based settings in the literature [38]. In [39] the non-linear effect is considered in the cooperative CR system only for single carrier modulation. However, the analysis of achievable spectral efficiency for a cooperative cognitive system which uses a multi-carrier modulation scheme and a non-linear relay has not been addressed yet. This fact motivates us to consider the problem of achievable downlink spectral efficiency for a cognitive relay network which employs the OFDM as its multi-carrier modulation scheme and a relay with non-linear PA. Specifically, the contribution of this paper is as follows:

- We first formulate the time-domain expression of the signal at the output of the relay’s non-linear PA as a function of the linear and non-linear impulse response of the pulse shifting filter.
- By using the obtained time-domain expression, we then derive the PSD of the output signal and derive rigorous analytical expressions for the ACP and 1 dB compression point. This point refers to the value of input power at which the output power of the non-linear PA is 1 dB less than that of the linear PA. We also present clearly the details of these derivations, which are cumbersome tasks.
- Next, the SINR of the received signal at the destination (SU receiver) is derived, as a function of the transmitted subcarriers, by finding the Fourier transform of signal, noise, and interference terms.
- Having the SINRs of all subcarriers, we formulate a constrained optimization problem in which the achievable spectral efficiency of the cognitive cooperative system is the objective function and the imposed constraints are (i) all ACPs should be equal or less than the interference temperature limit and (ii) the maximum transmission power by the relay must be equal or less than the value of 1 dB compression point. Due to the non-convexity of the objective function, finding a global optimal solution is not an easy task and we, hence, resort to finding a local solution with the aid of interior point method (IPM).
- We perform Monte Carlo simulations to validate the accuracy of our analytical results. We also compare the spectral efficiency of the CR system for the cases where it is equipped with non-linear and linear PAs. This comparison is done via solving our formulated optimization problem and that corresponds to the CR system equipped with the linear PA. The results reveal that the effect of non-linear PA can be considerable as interference temperature limit grows. Finally, we
investigate the impact of interference temperature limit upon the spectral efficiency by solving our formulated optimization problem for different cases where the interference temperature limit of left and right primary users are not equal.

The rest of the paper is organized as follows: The detailed system model is presented in Section 2. In the next section, we analyze the relay PSD and relay output power for the OFDM cognitive relay network with the assumption of non-linear PA at relay. Moreover, the ACP and 1 dB compression point expressions of non-linear PA relay are derived in Section 3. In Section 4, we obtain the SINR value for each subcarrier and solve the maximum achievable spectral efficiency problem subject to both the ACI and the maximum transmission power constraints. In Section 5, some simulations are conducted in order to show that the theoretical analysis is valid and accurate. Finally, a general conclusion is drawn in Section 6.

**Notations:** The operators *(∗)* and *(⊙)* represent convolution and circular convolution, respectively. We denote modulo-\(N\) integer number \(n\) by \((n)_{N}\). The operator \(\Re\{\}\) represents the real part of a complex number. The operator \(E[\cdot]\) denotes the expectation value of a random variable. The rectangular pulse function is denoted by \(\Pi(\cdot)\). Finally, the operator \(\mathcal{F}[\cdot]\) denotes the Fourier transform of a time-domain signal.

## 2 SYSTEM MODEL

We consider an OFDM cognitive network consisting of four different nodes during the data transmission process: (i) one BS (or SU source) which transmits data to the SU relay and SU destination, (ii) one SU relay: the user which amplifies the received signal from the BS and then, transmits it to the SU destination, (iii) one SU destination: the user which receives transmitted data from the BS and SU relay and, (iv) two PUs: two neighboring users which are located on the right and left sides of the SU destination. Each node is equipped with a single transmit and/or receive antenna. We assume that there is a direct path between the SU source and the SU destination. The channels between all nodes are assumed to be slow frequency selective fading and are statistically independent of each other. At the receiver side, the maximum ratio combing (MRC) is utilized to detect the received signal. Figure 1 depicts the underlying system model.

According to Figure 1, the impulse response between nodes \(a \in \{s,r\}\) and \(b \in \{r,d\}\) can be written as follows:

\[
h_{ab}(t) = \sum_{m=0}^{L-1} F_{ab}^{(m)} \delta(t - mT_s),
\]

where \(L\) is the number of channel taps, \(T_s\) is the subcarrier symbol duration, and \(F_{ab}^{(m)}\) is the \(m\)th path gain between nodes \(a\) and \(b\). The parameters \(F_{ab}^{(m)}\) are independent identical distribution (i.i.d) complex Gaussian random variables distributed as \(CN(0, \sigma_m^2)\) where \(\sigma_m^2\) is the variance of \(m\)th path. The complex envelope of the OFDM signal at the input of BS can be expressed as [40]

\[
x(t) = \sum_{n=-\infty}^{+\infty} g(t - nT_s) \sum_{c=1}^{N} B_{c,s} e^{j2\pi f_{c,s} t},
\]

where \(g(t)\) is the real impulse response of the pulse shaping filter with length \(T_s\), \(N\) is the number of subcarriers, and \(\Delta f_s = (2c - N - 1)/2T_s\) is the \(c\)th subcarrier frequency position relative to the central frequency. The parameter \(B_{c,s} \triangleq U_{c,s} e^{j\phi_{c,s}}\) is the \(n\)th symbol on the \(c\)th subcarrier where \(U_{c,s}\) and \(\phi_{c,s}\) are the modulus and phase of \(B_{c,s}\), respectively. Similarly to [13], we assume that a phase modulation, such as BPSK or QPSK, is utilized such that \(|B_{c,s}| = \sqrt{P}\) where \(P\) is the signal power at the input of BS.

The transmission of the OFDM signal \(x(t)\) is performed in two successive time slots. In the first time slot, the BS amplifies \(x(t)\) and sends it to both SU relay and SU destination. The received signals at the SU relay and destination can be written as

\[
\begin{align*}
J_{sr}(t) &= \sqrt{P_r} x(t) \ast h_{sr}(t) + n_{sr}(t), \\
J_{sd}(t) &= \sqrt{P_d} x(t) \ast h_{sd}(t) + n_{sd}(t),
\end{align*}
\]

where \(P_r\) is the transmission power, \(h_{sr}(t)\) and \(h_{sd}(t)\) are the impulse response of channels from BS to SU relay and destination, respectively. The parameters \(n_{sr}(t)\) and \(n_{sd}(t)\) refer to the circularly symmetric additive white Gaussian noise (AWGN) with variance \(N_0\) at the SU relay and destination, respectively. By inserting (1) and (2) into (3), we have

\[
J_{sr}(t) = \sqrt{P_r} \sum_{n=-\infty}^{+\infty} \sum_{c=1}^{N} \sum_{m=0}^{L-1} F_{sr}^{(m)} B_{c,s} g(t - nT_s - mT_s) x(t) e^{j2\pi f_{c,s}(t-nT_s)} + n_{sr}(t).
\]

In the second time slot, the SU relay amplifies the received signal \(J_{sr}(t)\) and transmits it to the SU destination. The output signal...
of relay with non-linear amplifier can be expressed as [41, 42]

\[ y_r(t) = \sum_{k=0}^{K} a_{2k+1} y_{n}(t)|y_{r}(t)|^{2k}, \]

(6)

where \( a_{2k+1} \) is the \((2k + 1)\)th complex coefficient of the non-linear polynomial and \((2K + 1)\) is the order of non-linearity. It should be noted that we neglect the small time delay between the first and second time slots in (6). This reasonable assumption is widely considered in several works such as [26, 27, 33]. Also, we consider only the odd-order terms in (6) since the even-order terms cause a further decrease in the spectral regrowth [43]. Similar to [12, 13, 44], we assume that the order of non-linearity is three, that is \( K = 1 \) and neglect the effect of high order terms. By plugging (5) into (6), we get

\[ y_r(t) = a_1 y_{n}(t) + a_3 y_{r}(t)|y_{r}(t)|^2 = \sum_{d=1}^{8} \lambda_d(t), \]

(7)

where the values of \( \lambda_d(t) \) are listed in Table 1. In this table, \( Q_{n,r,r'}(t) \) is defined as

\[ Q_{n,r,r'}(t) = \sum_{n_1 + \cdots + n_N = w} \frac{B_{n_1}^{w} B_{n_2}^{w} \cdots B_{n_N}^{w}}{r_{1}! r_{2}! \cdots r_{N}!}, \]

(8)

where \( \delta q = r_q - r_{q'} \), \( r_q \) and \( r_{q'} \) are non-negative integer values for \( q = 1, 2, \ldots, N \).

As mentioned earlier, PUs performances are affected by ACP which in turn is due to the non-linear behavior of relay’s PA. To get more insight into this effect, Figure 2 depicts the PSDs of the input and output signals of relay’s PA where the order of non-linearity is three. As seen from the figure, the PSD bandwidth of output signal is three times wider than that of input signal. Hence, the transmitted signal for the destination can interfere with those intended to the PUs. This effect is called ACI.

### TABLE 1 Different values of \( \lambda_d(t) \) in (7)

| \( d \) | \( \lambda_d(t) \) |
|---|---|
| 1 | \( a_1 \sqrt{T} \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} F_{n,m}^{(d)} (t - nT_m - mT_i) Q_{n,m}(t) \) |
| 2 | \( a_1 y_{n}(t) \) |
| 3 | \( 2a_1 y_{n}(t) \) |
| 4 | \( 2a_3 P_{n,m}^*(t) (t - nT_m - mT_i) Q_{n,m}(t) \) |
| 5 | \( 2a_3 P_{n,m}^*(t) \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} F_{n,m}^{(d)} (t - nT_m - mT_i) Q_{n,m}(t) \) |
| 6 | \( 2a_3 P_{n,m}^*(t) \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} F_{n,m}^{(d)} (t - nT_m - mT_i) Q_{n,m}(t) \) |
| 7 | \( a_3 y_{n}(t) |y_{r}(t)|^2 \) |
| 8 | \( a_3 y_{n}(t) |y_{r}(t)|^2 \) |

Figure 2: Power spectral densities of the input and output signals of relay’s non-linear PA with \( K = 3 \)

### 3 | SPECTRUM ANALYSIS OF THE OUTPUT SIGNAL FOR RELAY’S PA

In this section, we first perform a spectral analysis of the relay’s PA output so as to compute its PSD. Then, we derive analytical expressions for the ACP and 1 dB compression point by using the obtained PSD.

#### 3.1 | PSD derivation

The PSD computation of relay’s PA can be divided into three cases based on the type of resulting term from the multiplication of \( y_r(t) y_r^*(t) \): \( \sum_{d=1}^{8} \sum_{d'=1}^{8} \lambda_d(t) \lambda_{d'}^*(t) \):

1. If the resulting term, that is \( \lambda_d(t) \lambda_{d'}^*(t) \) includes only single terms, we apply the methodology used in [45, pp. 36-41], [13, 40] which consists of two steps:
   - A two-dimensional Fourier transform, defined as \( \Gamma(f_1, f_2) = E[\lambda_d(t) \lambda_{d'}^*(t)] \) corresponding to the term \( \lambda_d(t) \lambda_{d'}^*(t) \) is first calculated where \( \lambda_d(t) = F[\lambda_d(t)] \). Note that \( \Gamma(f_1, f_2) \) is also the Fourier transform of the autocorrelation function \( R(t_1, t_2) = E[\lambda_d(t_1) \lambda_{d'}^{*}(t_2)] \).
   - It is shown that \( \Gamma(f_1, f_2) \) can be written as follows:

\[ \Gamma(f_1, f_2) = \Gamma(f_1) \delta(f_1 - f_2) + \Delta(f_1, f_2), \]

(9)

where \( \Delta(f_1, f_2) = 0 \) for \( f_1 = f_2 \) and hence, our desired term is equal to \( \Gamma(f_1) \).
2. If it includes only the noise terms, we find the Fourier transform of autocorrelation function \( R(t_1, t_2) = E[\delta_j(t_1)\delta_j^*(t_2)] = R(\tau) \), where \( \tau = t_1 - t_2 \).

3. If it includes both the signal and noise terms, we combine methods \((i)\) and \((ii)\). Specifically, since the noise and signal terms are independent of each other, the autocorrelation function of the mixed (multiply) term is equal to the multiplication of autocorrelation functions of signal and noise terms. Hence, we convolve the Fourier transform of the signal term (obtained as in \((i)\)) with the Fourier transform of autocorrelation function of noise term (obtained as in \((ii)\)).

By applying the above methodology to \((7)\), the PSD of \( y(t) \) can be obtained as follows:

\[
\Gamma(f) = \sum_i \Gamma_i(f),
\]

where \( \Gamma_i(f) \)'s are presented in Table 2. Note that we do not present the details of calculations here since they are lengthy and straightforward. In this table, we have

\[
S_{(a,a',\rho)} = \frac{1}{T} \sum \frac{1}{N} v_i \cdots + v_N = \omega \prod_{j=1}^N v_i v_j
\]

where \( \omega \) is the Fourier transform of \( \delta^k(t) \). In order to obtain an expression for ACP, we first need to simplify the values of PSD given in Table 3. To this end, we find for each possible pair of \((k,k')\) the terms including \( G_k(\cdot)G_{k'}(\cdot) \) where \( k, k' = 1, 2, 3 \). These terms correspond to the signal components of PSD. Since the right hand side of \((11)\) satisfy for different values of \( v, v', \alpha \), and \( \alpha' \), multiple terms can be extracted for each pair of \((k,k')\), with an extra condition \( \delta_{v \alpha} = \delta_{\alpha v} \). The extracted terms are presented in Table 3 where \( \mu = \sum_{j=0}^{N-1} \sigma_j^2 \), \( \Omega = |a|^2 \), \( \beta = a^*a \), \( \omega = \frac{1}{T} \), and \( 1 \leq i, j, t \leq N \).

Thus, the terms without \( G_k(\cdot)G_{k'}(\cdot) \) are considered as the PSD of noise and can be written as \( f_j^1(\omega) + f_j^2N_0^2 + f_jN_3 \), where \( f_j \)'s are tabulated in Table 4. In this table, \( P_{g_s} \) and \( P_{g_o} \) are the power of \( g_s(t) \) and \( g_o(t) \) \((g_s(t))^* \), respectively. In the next section, we use the PSD of signal and noise in order to derive the ACP.

### 3.2 ACP derivation

In this section, we assess the ACP value at \( \Delta f \) with the aid of extracted PSD components in previous section. To this end, we need to evaluate the integrals of out-of-band PSD components by imposing condition \( |\Delta f| > F_{N-1} \) upon frequency shifts of \( G_k(f - \Delta f) \)'s in PSD components of Table 2. Due to the symmetry of PSD spectrum, we only compute the ACP for the right hand side of spectrum and the same results hold for the left hand side. Moreover, we may divide the ACP calculations...
TABLE 3  Signal components of Table 2

| $k, k'$ | $v_i, v'_i, \alpha_i, \alpha'_i$ | Simplified PSD terms including $G_k(\cdot)G_{k'}(\cdot)$ |
|-------|-----------------|------------------|
| $k = k' = 1$ | $v_i = \alpha_i = 1, v'_i = \alpha'_i = 0$ | $P_s\mu \re[\varepsilon]G_1(f - \Delta f_i)^2 + \Omega \Re \Delta t \Re G_1(f + \Delta f_i)^2P$ |
| $k = 1, k' = 3$ | $v_i = 1, \alpha_i = 2, v'_i = 0, \alpha'_i = 1$ | $4P_s^2 \mu^2 \Re \varepsilon G_1(f - \Delta f_i)G_1(f + \Delta f_i)P^2$ |
| $k = k' = 3$ | $v_i = \alpha_i = 1, v'_i = \alpha'_i = 0, i \neq j$ | $8(N - 1)P_s^2 \mu^2 \Re \varepsilon G_1(f - \Delta f_i)G_1(f - \Delta f_j)P^2$ |

into two parts based on the number of frequency shifts included in arguments $G_k(f - \Delta f_i)$'s: (i) terms with multiple shifts, (ii) terms with only one shift.

According to Table 3, there exist two PSD components with multiple shifts, that is the last two rows. After imposing the out-of-band condition $|\Delta f_i| > \frac{N - 1}{2T_s}$, the sum of PSD components related to these two terms can be written as follows:

$$S_{sl}(f) = \sum_{\Delta f_{iN}} \Delta f_N = \Delta f_i + \Delta f_j - \Delta f_i \leq \Delta f_{2N}$$

$$1 \leq i, j \leq N$$

$$24P_s^3 \mu^3 \re \varepsilon G_1(f - \Delta f_i - \Delta f_j + \Delta f_i)^2P^3$$

$$+ \sum_{\Delta f_{iN}} \Delta f_N \leq 2\Delta f_i - \Delta f_j \leq \Delta f_{2N}$$

$$1 \leq i, j \leq N$$

$$6P_s^3 \mu^2 \re \varepsilon G_1(f - 2\Delta f_i + \Delta f_j)^2P^3. \tag{12}$$

The inequalities in (12) can be readily translated into $N < i + j - t \leq 2N, N < 2i - j \leq 2N$, and $i \neq j$ by using the definition of $\Delta f_i$ in (2). In order to evaluate $S_{sl}(f)$, we define $u_i$ and $u'_i$ as the number of integer solutions for equations $i + j - t = \varepsilon$ and $2i - j = \varepsilon$, respectively and rewrite (12) as follows:

$$S_{sl}(f) = \sum_{\epsilon = N + 1}^{2N} 6(4u_i + u'_i)P_s^3 \mu^3 \re \varepsilon G_1(f - \Delta f_i)^2P^3, \tag{13}$$

where the $u_i$ and $u'_i$ can be derived using the combinatorics techniques in [13, 46]. Now, by taking the integral of $S_{sl}(f)$ over $[\Delta f_i - \frac{1}{2T_s}, \Delta f_i + \frac{1}{2T_s}]$, the out-of-band power is obtained as

$$P_{sl} \left( \Delta f_i \right) = 6P_s^3 \mu^3 \re \varepsilon \Re \varepsilon (4u_i + u'_i)P_{s, G_i, G_j}P^3, \tag{14}$$

where $P_{s, G_i, G_j}$ is defined as follows:

$$P_{s, G_i, G_j} = \int_{\Delta f_i - \frac{1}{2T_s}}^{\Delta f_i + \frac{1}{2T_s}} G_k(f)G_{k'}(f)df, \quad \epsilon = 1, \ldots, N. \tag{15}$$

As given in Tables 3 and 4, the out-of-band power corresponding to one shift terms and the noise power can be readily obtained by taking the integrals of these terms over the interval $[\Delta f_i - \frac{1}{2T_s}, \Delta f_i + \frac{1}{2T_s}]$. By doing so, we get

$$P_{sl} \left( \Delta f_i \right) = \Re \varepsilon \Re \varepsilon (J_0N_0^3 + J_2N_0^2 + J_1N_0)$$

$$+ \sum_{j = -N}^{N} [P\mu(\varepsilon + 4\Re \varepsilon \Re \varepsilon N_0^2 \Re \varepsilon)P_{j, G_i, G_j}]^2P$$

$$+ P_s^2 \mu^2 \Re \varepsilon G_1(f - \Delta f_i - \Delta f_j + \Delta f_j)^2P^3$$

$$+ \Re \varepsilon (2N^2)P_{j, G_i, G_j}P^3. \tag{16}$$

Using (14) and (16), the ACP value of right adjacent channel at $\Delta f_i$ can be expressed as follows:

$$\text{ACP}_{r}\left( \Delta f_i \right) = P_{sl} \left( \Delta f_i \right) + P_{sl} \left( \Delta f_i \right)$$

$$= D_0 + D_1P + D_2P^2 + D_3P^3. \tag{17}$$

where

$$D_0 = \Re \varepsilon (J_0N_0^3 + J_2N_0^2 + J_1N_0)$$

$$D_1 = \sum_{j = -N}^{N} \left[ P\mu(\varepsilon + 4\Re \varepsilon \Re \varepsilon N_0^2 \Re \varepsilon)P_{j, G_i, G_j} \right]^2P$$

$$D_2 = \sum_{j = -N}^{N} \left[ P_s^2 \mu^2 \Re \varepsilon G_1(f - \Delta f_i - \Delta f_j + \Delta f_j)^2P^3 \right]$$

$$D_3 = \sum_{j = -N}^{N} \left[ P_s^3 \mu^3 \Re \varepsilon (2N^2)P_{j, G_i, G_j} \right]^3$$

$$+ 6P_s^3 \mu^3 \Re \varepsilon (4u_i + u'_i)P_{s, G_i, G_j}.$$
3.3 1 dB compression point

As previously mentioned, 1 dB compression point is a point where the output power level of a non-linear PA is 1 dB lower than that of the linear PA. Hence, two steps are required to obtain the 1 dB compression point: (i) deriving output power of the non-linear PA’s relay, denoted by \(P_{NL} \), and then, (ii) solving the equation \(10\log_{10}P_{NL} = 10\log_{10}P_{L} + 1 \) where \(P_{L} \) is the output power of linear PA’s relay. The value of \(P_{NL} \) can be readily calculated by taking the integral of all PSD components over the whole frequency range. By so doing, we have

\[
P_{NL} = A_0 + A_1 P + A_2 P^2 + A_3 P^3, \tag{19}
\]

where

\[
A_0 = J_3 N_0^3 + J_2 N_0^2 + J_1 N_0
\]

\[
A_1 = N[P_{\mu}(\mu + 4\Omega N^2 N_0^2 \omega) P_{G_1, G_1}]
\]

\[
A_2 = N[P_{\mu}^2 32 N \Re(\beta) P_{G_1, G_1}]
\]

\[
A_3 = N[P_{\mu}^3 6 \Re(24 N^2 - 48 N + 30) P_{G_1, G_1}].
\]

Now, one can simply derive \(P_{L} \) from (19) by putting the value of \(\Omega \) and \(\beta \) into zero. Note that \(\Omega \) and \(\beta \) are a function of \(a_1 \) which is the non-linear coefficient in (7).

Now, we need to solve the 1 dB compression point equation \(10\log_{10}P_{NL} = 10\log_{10}N P_{\mu}P_{G_1, G_1} - 1 \) for parameter \(P \). Rewriting this equation, we yield

\[
A_3 P^3 + A_2 P^2 + A_1 P + A_0 - 10^{-0.1} N P_{\mu} P_{G_1, G_1} = 0. \tag{21}
\]

Hence, \(P_{\text{dB}} \) is the smallest positive root of this equation which can be obtained using a numerical method.

4 SPECTRAL EFFICIENCY OPTIMIZATION OF SU DESTINATION

In this section, we formulate the optimization problem of achievable spectral efficiency (bits/s/Hz) by SU destination under two constraints: (i) maximum tolerable ACI (due to the relay and source) by PUs, and (ii) the relay received power is limited to \(P_{\text{dB}} \). To this end, we first write the equivalent discrete-time signal models given in (3) and (6) as follows:

\[
y_t[n] = a_1 y_{\alpha}[n] + a_3 y_{\alpha}[n] y_t[n], \tag{22}
\]

where \(y_t[n] = \sqrt{\beta} y_{\alpha}[n] \oplus x[n] + n_t[n], \quad n = 1, \ldots, N\)

\[
y_{\alpha}[n] = \frac{1}{N} \sum_{\tau=1}^{N} R e^{2\pi i \tau / N}. \tag{23}
\]

where \(x[n] = \frac{1}{N} \sum_{\tau=1}^{N} R e^{2\pi i \tau / N}. \tag{24}
\]

By taking the fast Fourier transform (FFT) of (22), we get

\[
Y_t(c) = a_1 \sqrt{\beta} Z(c) + a_3 P \sqrt{\beta} Z(c) \oplus Z(c) \oplus \* (−c) N
\]

\[
+ 2 a_3 \sqrt{\beta} Z(c) \oplus N_{\alpha}(c) \oplus N_{\alpha}^* (−c) N
\]

\[
= Y_{r1}(c) + Y_{r2}(c) + Y_{r3}(c) + Y_{r4}(c)
\]

\[
= Y_{r1}(c) + Y_{r2}(c) + Y_{r3}(c) + Y_{r6}(c), \quad (23)
\]

where \(Z(c) = H_{\alpha}(c)B \) and \(N_{\alpha}(c) \) is the FFT of \(n_{\alpha}[n] \). The details of derivation for \(Y_{r2}(c)-Y_{r7}(c) \) are given in Appendix. By plugging the obtained values of \(Y_{r2}(c)-Y_{r7}(c) \) into (23) and separating the desired signal, interference and noise terms, \(Y_t(c) \) can be rewritten as follows:

\[
Y_t(c) = \sqrt{\beta} a_1 Z(c) + \sqrt{\beta} P \bigg[ 2 a_1 \sum_{l=0}^{N-1} |\bar{Z}(l)|^2
\]

\[
+ 2 a_3 \sum_{l=0}^{N-1} |\bar{N}_{\alpha}(l)|^2
\]

\[
+ 4 a_3 \sqrt{\beta} \Re\left( \sum_{l=0}^{N-1} \bar{Z}(l) \, \bar{N}_{\alpha}^*(l) \right) \, \bar{Z}(c) W(c)
\]

where \(\bar{Z}(c) \) and \(\bar{N}(c) \) are the periodic forms of \(Z(c) \) and \(N(c) \), respectively, \(W(c) = \left\{ \begin{array}{ll} 1 & 0 \leq c < N - 1 \\ 0 & \text{otherwise} \end{array} \right. \) is the rectangular window, and the interference and noise terms are listed in Table 5 for ease of presentation.

By taking expectations over the sum of square of desired, interference, and noise terms, the signal, interference, and noise power of the in-band subcarrier \(c \) can be obtained as:

\[
P_{t1}(c) = K_{t1} P + K_{t2} P^2 + K_{t3} P^3, \tag{25}
\]

and

\[
P_{t2}(c) = I_{t0} P + I_{t1} P + I_{t2} P^2 + I_{t3} P^3, \tag{26}
\]

respectively, where

\[
K_{t1} = P_t |H_{\alpha}(c)|^2 |a_1|^2 + 4 \Re(a_1 a_3^*) P_t N N_0
\]

\[
+ 4 |a_3|^2 + 2(N^2 + N) N_0^2,
\]

\[
K_{t2} = 4 P_t^2 |H_{\alpha}(c)|^2 \left( \sum_{l=0}^{N-1} |H_{\alpha}(l)|^2 |\Re(a_1 a_3^*)| \right)
\]
TABLE 5 The noise and interferences terms in (24)

|   |                                                                                       |
|---|----------------------------------------------------------------------------------------------------------------------------------|
| 1 | $a_1 P_s \sqrt{|P|} |Z(c)|^2$                                                                 |
| 2 | $a_1 P_s \sum_{\substack{m=0 \atop \not \in \text{spec}}}^{N-1} \sum_{\substack{n=0 \atop \not \in \text{spec}}}^{N-1} |Z(m)|^2 |Z(n)|^2 |Z(l)|^2$ |
| 3 | $2a_1 P_s \sum_{\substack{m=0 \atop \not \in \text{spec}}}^{N-1} \sum_{\substack{n=0 \atop \not \in \text{spec}}}^{N-1} |Z(m)|^2 |N_{m,n}(l)^2 N_{n,m}(l)^2$ |
| 4 | $a_1 P_s \bar{Z}(c) N_{m}(c)^2 W(c)$                                                                                     |
| 5 | $a_1 P_s \sum_{\substack{m=0 \atop \not \in \text{spec}}}^{N-1} \sum_{\substack{n=0 \atop \not \in \text{spec}}}^{N-1} |Z(m)|^2 |Z(n)|^2 |Z(l)|^2$ |
| 6 | $2a_1 P_s |\bar{Z}(c)|^2 N_{m}(c)^2 W(c)$                                                                                   |
| 7 | $2a_1 P_s \sum_{\substack{m=0 \atop \not \in \text{spec}}}^{N-1} \sum_{\substack{n=0 \atop \not \in \text{spec}}}^{N-1} |Z(m)|^2 |N_{m,n}(l)^2 N_{n,m}(l)^2 |Z(l)|^2$ |
| 8 | $a_1 N_{m}(c)^2$                                                                                                          |

where $\text{SNR}_{sd}^{\text{rd}} = \frac{P_r |H_{\text{sd}}(\omega)|^2}{N_0}$ is the received SNR from the source to the destination.

In the CR systems, the resultant ACI caused by the relay and source must be less than the PUs interference temperature limit. Moreover, the maximum transmission power of the relay must be less than $P_{1\text{dB}}$ to avoid the non-linear region of the relay PA. By considering these limitations and (25)–(29), the maximum achievable spectral efficiency problem can be formulated as follows:

$$
\max_{\mathbf{P}} \frac{1}{2} \sum_{c=1}^{C} \log \left(1 + \frac{P_c(\omega) |H_{\text{rd}}(\omega)|^2}{P_{\text{N,tP}(\omega)} |H_{\text{rd}}(\omega)|^2 + N_0} + \frac{P_c(\omega) |H_{\text{rd}}(\omega)|^2}{N_0}\right),
$$

subject to:

$$
\sum_{\omega=0}^{N-1} \{D_{1\text{dB}}^2 + D_{2\text{dB}}^2 + D_{3\text{dB}}^2\} |H_{r_{\text{P}P}(\omega)}|^2 < I_R,
$$

$$
\sum_{\omega=0}^{N-1} \{D_{1\text{dB}}^2 + D_{2\text{dB}}^2 + D_{3\text{dB}}^2\} |H_{r_{\text{P}L}(\omega)}|^2 < I_L,
$$

$$
\sum_{c=1}^{C} \sum_{\omega=0}^{N-1} P_c |H_{r_{\text{P}P}(\omega)}|^2 < I_R
$$

$$
\sum_{c=1}^{C} \sum_{\omega=0}^{N-1} P_c |H_{r_{\text{P}L}(\omega)}|^2 < I_L,
$$

where $I_R$ and $I_L$ are the interference temperature limits for the right and left PUs, respectively. Also, $H_{r_{\text{P}P}(\omega)}$, $H_{r_{\text{P}L}(\omega)}$, $H_{r_{\text{P}P}(\omega)}$, and $H_{r_{\text{P}L}(\omega)}$ are the right and left adjacent channels from the source and relay to the left and right primary users, respectively. Since the obtained objective function in (30) is non-convex in terms of $\mathbf{P}$ [33], it is not feasible to find the global optimal solution. Hence, we resort to a local optimal solution by using the IPM in the next section.

### 5 | SIMULATION RESULTS

In this section, we present some simulation results to validate the accuracy of the derived analytical expressions. We compare the expressions of total ACP, $P_{\text{N,ACP}}$, and $P_{1\text{dB}}$ in (17), (19), and (21), respectively with those obtained via Monte Carlo simulations.

For the simulations, an AF relaying OFDM-based system consisting of non-linear (at the SU relay) and linear PAs (at the SU source) is considered. We assume the symbol rate $\frac{1}{T_s}$ of 5MHz and simulate a number of 1000 OFDM symbols generated by the BPSK signaling with 64 subcarriers for each. To avoid inter symbol interference (ISI), a cyclic prefix (CP) of length 16 is embedded into each OFDM symbol. Furthermore, the channel state information (CSI) is known at both SU relay and destination. The coefficients of non-linear relay PA in (6) are set to $a_1 = 15.0008 + j0.0098$ and $a_2 = -23.0826 + j0.0098$. Note that we assume, similar to [13], the distribution of an OFDM signal tends towards Gaussian when the number of subcarriers is the order of 64 or higher [47]. By using (25)–(27), the received SINR of each in-band subcarrier can be derived as

$$
\text{SINR}_{\text{rd}} = \text{SINR}_{\text{rd}}^{\text{rd}} + \text{SINR}_{\text{rd}}^{\text{rd}},
$$

for the indirect path $s \rightarrow r \rightarrow d$ (see Figure 1). Moreover, since there exists the direct path from the source to the destination and the MRC is used at the receiver, the total received SINR of the $\text{th}$ subcarrier can be written as

$$
\text{SINR}_i = \text{SINR}_{\text{rd}}^{\text{rd}} + \text{SINR}_{\text{rd}}^{\text{rd}},
$$

where $SINR_{sd}^{rd} = \frac{P_r |H_{sd}(\omega)|^2}{N_0}$ is the received SNR from the source to the destination.
TABLE 6  The power of each of the terms in Table 5

\[ j \quad L_{ji} \]

\[ 0 \quad |a_j|^2(2N^2 + 6N + 4)N_0^2 + 8\Re(a_1a_j^*)N_0N_0^2 + |a_j|^2N_0^2 \]

\[ 1 \quad 2|a_j|^2P_2N_0^3 \sum_{m=0}^{N-1} \sum_{l \neq j} |H_{al}(l - \epsilon + w)N_0|^2 \]

\[ 2 \quad + 4|a_j|^2P_2N_0^3 \sum_{m=0}^{N-1} |H_{al}(l)|^2 + \frac{N_0}{2} \sum_{m=0}^{N-1} |H_{al}(l)|^2 + 2 \frac{N_0}{P_2} \sum_{m=0}^{N-1} |H_{al}(l)|^2 + 4N_0 \sum_{m=0}^{N-1} |H_{al}(m)|^4 \]

\[ 3 \quad 2|a_j|^2P_2^3(3|H_{al}(c)|)^4 + \left\{ \frac{1}{2} \sum_{m=0}^{N-1} |H_{al}(m)|^2 |H_{al}(l)|^2 |H_{al}((l - \epsilon + w)N_0)|^2 \right\} + \sum_{m=0}^{N-1} |H_{al}(l)|^4 |H_{al}((2l - \epsilon)N_0)|^2 \]

**FIGURE 3**  Relay PA output power versus input signal power of SU source \( P \)

**FIGURE 4**  The theoretical and estimated ACP, versus input signal power of SU source \( P \)

3.3133 [48]. In the CR relay network, it is assumed that the direct and adjacent channels are frequency selective with \( L = 10 \) taps and have an exponentially decaying power profile with factor \( \epsilon = 2 \) for each tap. The variances of complex Gaussian random variables \( F_{ab}^{(m)} \) in (1) are equal to \( \frac{\exp(-\frac{m}{\epsilon})}{\sum_{m=0}^{N-1} \exp(-\frac{m}{\epsilon})} \) and the AWGN variance \( N_0 \) is normalized to one.

Figure 3 compares the derived analytical expression of output power \( P_{NL} \) (dB) versus the input power of SU source \( P \) (dB) with its simulation counterpart. The output power of a system with the linear PA is also depicted as the reference. As seen from the figure, there exists an excellent match between the analytical and experimental results which validates the derived analytical expression in (19). Moreover, the output power of the non-linear PA system starts to deviate from that of the linear when the input power is around –45 dB and observes a reduction of one dB at point \( P_{nlb} = -40 \).

The derived analytical expression of ACP in (17) versus \( P \) (dB) is compared with its simulation counterpart in Figure 4. The simulation curve is obtained with the aid of Welch method to estimate the PSD of PA output signal [49]. As this figure shows, the derived analytical expression completely conforms to the simulation results and verifies its correctness.

In order to obtain the system performance, the constraint optimization problem, given in (30), is solved by using the IPM in two different cases: (i) the interference temperature limit is same for both PUs, that is \( I_{IL} = I_{IR} \) and, (ii) the right interference-temperature limit is fixed for three different values. For the first case, Figure 5 depicts the optimized spectral efficiency (bits/sec/Hz) of non-linear and linear (ideal) systems versus
interference temperature limit (dB). From this figure, one can clearly observe the non-linearity effect of PA on the system spectral efficiency. For example, there is an approximate loss of 1.6 (bits/s/Hz) when $I_L = I_R = -5$ dB. Note that the ACP is not zero even for the linear case since the considered pulse shape is time limited [13].

Figure 6 displays the optimal spectral efficiency (bits/s/Hz) of a non-linear system versus $I_L$ (dB) for three different values of $I_R \in \{-12, -18, -24\}$ dB. It can be observed that the system spectral efficiency increases as $I_R$ grows since the suboptimal value of $P$ can be found over a larger feasible set. Regarding the effect of $I_L$, the system spectral efficiency, up to the point $I_L = I_R$, monotonically increases with respect to $I_L$ and then, is saturated. Since the spectrum symmetry leads to the same results if $I_R$ and $I_L$ are interchanged, it can be concluded that the system spectral efficiency is limited by $\min(I_R, I_L)$ and the values of $I_L$ which are greater than $I_R$ does not affect the system performance and vice versa. Finally, Figure 7 shows the normalized spectral efficiency of secondary destination for the linear and two different non-linear PA’s relay versus the interference threshold on the primary users. It is assumed that the coefficients of the non-linear PA’s relay models are $\{a_1, a_3\}$ and $\{\alpha_1, 4\alpha_3\}$, respectively. From Figure 7, we observe that by increasing the non-linearity the normalized spectral efficiency of secondary destination significantly decreases.

6 | CONCLUSION

This paper considered and analyzed the downlink achievable spectral efficiency of an OFDM cognitive relay network. The analysis was carried out subject to the PA’s non-linear effect on the relay node which operates in the AF mode. The PSD was first derived as a function of source input power and then, we obtained the ACP by use of the calculated PSD. Also, we derived the relay output power and $P_{1dB}$ in terms of source input power. Furthermore, we formulated the achievable spectral efficiency under the existence of the adjacent channel interference and the maximum transmission power constraint. Due to the non-convexity of the problem, the suboptimal achievable spectral efficiency was obtained via the IPM. Finally, we compare the analytical expressions with those obtained through the Monte Carlo simulations and verify the accuracy of analytical results. It should be noted that the analysis provided in this work can be applicable for cognitive systems where it is difficult to design a linear relay’s PA for large bandwidth. As a possible future work, a more complicated optimization problem might be solved by increasing the degree of non-linearity from 3 to a higher value to investigate how higher harmonics of relay’s non-linear PA can
affect the spectral efficiency of the OFDM cooperative cognitive network.

REFERENCES

1. Haykin, S.: Cognitive radio: brain-empowered wireless communications. IEEE J. Sel. Areas Commun. 23(2), 201–220 (2005)
2. Xie, H.: A full-space spectrum-sharing strategy for massive MIMO cognitive radio systems. IEEE J. Sel. Areas Commun. 34(10), 2537–2549 (2016)
3. Hosseini, S., Champagne, B., Chang, X.-W.: A green downlink power allocation scheme for cell-free massive MIMO systems. IEEE Access 9, 6498–6512 (2021)
4. Morsali, A.: Design criteria for omnidirectional STBC in massive MIMO systems. IEEE Wireless Commun. Lett. 8(5), 143–145 (2019)
5. Hosseini S.S., Abouei J., Uysal M.: Fast-decodable MIMO HARQ systems. IEEE Trans. Wireless Commun. 14(5), 2827–2840 (2015)
6. Kaur, N., Hosseini, S., Champagne, B.: 2020 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC), pp. 76–81, Auckland, New Zealand (2020)
7. Cave, M., Doyle, C., Webb, W.: Essentials of Modern Spectrum Management. Cambridge University Press, New York (2012)
8. Alyildiz, I.F., et al.: A survey on spectrum management in cognitive radio networks. IEEE Commun. Mag. 46(4), 40–48 (2008)
9. Goldsmith, A., et al.: Breaking spectrum gridlock with cognitive radios: an information theoretic perspective. Proc. IEEE 97(5), 894–914 (2009)
10. Mohammadi, A., Ghannouchi, F.M.: RF Transceiver Design for MIMO Wireless Communications, vol. 145. Springer, New York (2012)
11. Cripps, S.: RF Power Amplifiers for Wireless Communications, vol. 250, 2nd ed. Artech House, Inc., Norwood (2006)
12. Majidi, M., Mohammadi, A., Abdipour, A.: Analysis of the power amplifier nonlinearity on the power allocation in cognitive radio networks. IEEE Trans. Commun. 62(2), 467–477 (2014)
13. Baghari, M., et al.: Analysis and rate optimization of OFDM-based cognitive radio networks under power amplifier nonlinearity. IEEE Trans. Commun. 62(10), 3410–3419 (2014)
14. Baghari, M., Mohammadi, A., Majidi, M.: An accurate analysis of the nonlinear power amplifier effects on SC-FDMA signals. Wireless Netw. 25, 533–543 (2019)
15. Kang, X., et al.: Optimal power allocation for fading channels in cognitive radio networks: ergodic capacity and outage capacity. IEEE Trans. Wireless Commun. 8(2), 940–950 (2009)
16. Qian, L., et al.: Power control for cognitive radio Ad hoc networks. In: Proceedings of IEEE Workshop on Local and Metropolitan Area Networks, New York (2007)
17. Saltzberg, B.R.: Comparison of single-carrier and multitone digital modulation for ADSL applications. IEEE Commun. Mag. 36(11), 114–121 (1998)
18. Baghari, M., Mohammadi, A., Majidi, M.: Optimum power allocation in OFDM systems under power amplifier nonlinearity. Analog Integr. Circuits Signal Process. 99, 33–38 (2018)
19. Han, S., Lee, J.: An overview of peak-to-average power ratio reduction techniques for multicarrier transmission. IEEE Wireless Commun. 12(2), 56–65 (2005)
20. Zhe, X., et al.: Simplified approach to optimized iterative clipping and filtering for PAPR reduction of OFDM signals. IEEE Trans. Commun. 61(5), 1891–1901 (2013)
21. Deng, S.K., Lin, M.C.: Recursive clipping and filtering with bounded distortion for PAPR reduction. IEEE Trans. Commun. 55(1), 227–230 (2007)
22. Jiang, T., Zhu, G., Zheng, J.: Block coding scheme for reducing PAPR in OFDM systems with large number of subcarriers. J. Electron. 21(6), 482–489 (2004)
23. Sharma, P., Verma, S.: PAPR reduction of OFDM signals using selective mapping with turbo codes. Int. J. Wireless Mobile Netw. 3(4), 217–223 (2011)
24. Chen, J.C., et al.: A suboptimal tone reservation algorithm based on cross-entropy method for PAPR reduction in OFDM systems. IEEE Trans. Broadcasting. 57(3), 752–756 (2011)
25. Yang, L., et al.: PAPR reduction of an OFDM signal by use of PTS with low computational complexity. IEEE Trans. Broadcasting. 52(1), 83–86 (2006)
26. Fernandes, C.A.R., da Costa, D.B., de Almeida, A.L.F.: Performance analysis of cooperative amplify-and-forward orthogonal frequency division multiplexing systems with power amplifier non-linearity. IET Commun. 8(18), 3223–3233 (2014)
27. Kumar, N., Singya, P.K., Bhatia, V.: Performance analysis of orthogonal frequency division multiplexing-based cooperative amplify-and-forward networks with non-linear power amplifier over independently but not necessarily identically distributed Nakagami-m fading channels. IET Commun. 11(7), 1008–1020 (2017)
28. Singya, P.K., Kumar, N., Bhatia, V.: Performance analysis of AF OFDM system using multiple relay in presence of nonlinear-PA over inid Nakagami-m fading. Int. J. Commun. Syst. 31(1) (2018)
29. Laneman, J.N., Tse, D.N.C., Wornell, G.W.: Cooperative diversity in wireless networks: efficient protocols and outage behavior. IEEE Trans. Inf. Theory 50(12), 3062–3080 (2004)
30. Nosratinia, A., Hunter, T.E., Hedayat, A.: Cooperative communication in wireless networks. IEEE Commun. Mag. 42(10), 74–80 (2004)
31. Chu, Y., Champagne, B., Zhu, W.-P.: NOMA-based cooperative relaying for secondary transmission in cognitive radio networks. IET Commun. 13(12), 1840–1851 (2019)
32. Zhang, C., et al.: Optimal relay power allocation for amplify-and-forward OFDM relay networks with deliberate clipping. In: Proceedings of IEEE Wireless Communications and Networking Conference (WCNC), Paris (2012)
33. Choi, M., Park, J., Choi, S.: Simplified power allocation scheme for cognitive multi-node relay networks. IEEE Trans. Wireless Commun. 11(6), 2008–2012 (2012)
34. Kaiser, M.S., Ahmed, K.M., Shah, R.A.: Power allocation in OFDM-based cognitive radio networks. In: IEEE International Conference on Wireless Communications, Networking and Information Security, Beijing (2010)
35. Sidhu, G.A.S., et al.: Resource allocation in relay-sided OFDM cognitive radio networks. IEEE Trans. Veh. Technol. 62(8), 3700–3710 (2013)
36. Silva, S.L., Fernandes, C.A.R.: Outage analysis of AF OFDM relaying systems with power amplifier nonlinearity. In: Proceedings of International Telecommunications Symposium (ITS), Sao Paulo (2014)
37. Simmons, D.E., Coon, J.P.: Two-way OFDM-based non-linear amplify-and-forward relay systems. IEEE Trans. Veh. Technol. 65(5), 3808–3812 (2016)
38. Rajkumar, S., Thiruvengadam, J.S.: Outage analysis of OFDM-based cognitive radio network with full duplex relay selection. IET Signal Proc. 10(8), 865–872 (2016)
39. Majidi, M., et al.: Characterization and performance improvement of cooperative wireless networks with nonlinear power amplifier at relay. IEEE Trans. Veh. Technol. 69(3), 3244–3255 (2020)
40. Cottais, E., Wang, Y., Toutain, S.: Spectral regrowth analysis at the output of a memoryless power amplifier with multicarrier signals. IEEE Trans. Commun. 56(7), 1111–1118 (2008)
41. Zhou, G.T., Kenney, J.S.: Predicting spectral regrowth of nonlinear power amplifier driven by cyclostationary modulated signals. Analog Integr. Circuits Signal Process. 74(2), 425–437 (2013)
42. Teles, L.C.S., Fetnes, C.A.R., Magalhaes, S.R.C.: Power allocation methods for OFDM systems with nonlinear power amplifier. In: Proceedings of IEEE Symposium on Computers and Communications (ISCC), Natal (2018)
43. Jeon, W.G., Chang, K.H., Cho, Y.S.: An adaptive data predistorter for compensation of nonlinear distortion in OFDM systems. IEEE Trans. Commun. 45(10), 1167–1171 (1997)
44. Majidi, M., Mohammadi, A., Abdipour, A.: Accurate analysis of spectral regrowth of nonlinear power amplifier driven by cyclostationary modulated signals. Analog Integr. Circuits Signal Process. 74(2), 425–437 (2013)
45. Benedetto, S., Biglieri, E.: Principles of Digital Transmission with Wireless Applications, 2nd ed. Kluwer Academic/Plenum Publishers, New York (2002)

AORCID

Samira Hadavi 10 https://orcid.org/0000-0003-2776-5030
Seyed Saleh Hosseini 10 https://orcid.org/0000-0002-6498-9423
46. Tucker, A.: Applied Combinatorics, 3rd ed. Wiley, Hoboken (1994)
47. Dimitrov, S., Sinanovic, S., Haas, H.: Clipping noise in OFDM based
optical wireless communication systems. IEEE Trans. Wireless Commun.
60(4), 1072–1081 (2012)
48. Zhou, G.T., Raich, R.: Spectral analysis for bandpass nonlinearity with
cyclostationary input. In: Proceedings of IEEE International Conference
on Acoustics, Speech, and Signal Processing, Montreal (2004)
49. Hayes, M.H.: Statistical Digital Signal Processing and Modeling. Wiley,
New York (1996)

How to cite this article: Hadavi S, Hosseini SS, Talebi S. Analysis of spectral efficiency for OFDM cooperative cognitive networks with non-linear relay. IET Commun., 2021;15:1174–1186.
https://doi.org/10.1049/cmu2.12151

APPENDIX: DERIVATION FOR $\gamma_{r2}(\epsilon) - \gamma_{r7}(\epsilon)$ IN (23)
We start with $\gamma_{r2}(\epsilon)$ and rewrite it as follows:

$$Y_{r2}(\epsilon) = a_3 P_1 \sqrt{P} \bigg[ \sum_{l=1}^{N-1} \bigg[ \sum_{m=0}^{N-1} Z(l) \overline{Z}(m-l) \overline{Z}^*(\epsilon - m) \bigg] \bigg] W(\epsilon)$$

where $\overline{Z}(\epsilon)$ is the periodic form of $Z(\epsilon)$ and $W(\epsilon) = \frac{1}{\sqrt{N}}$ is the rectangular window. Then, we have

$$Y_{r2}(\epsilon) = a_3 P_1 \sqrt{P} \bigg[ \sum_{l=0}^{N-1} \bigg[ \sum_{m=0}^{N-1} Z(m) \overline{Z}(l) \overline{Z}^*(\epsilon - m) \bigg] \bigg] W(\epsilon)$$

Since

$$\sum_{l=0}^{N-1} \sum_{m=0}^{N-1} Z(m') Z(l) \overline{Z}^*(\epsilon - m')$$

we can rewrite (A.2) as follows:

$$Y_{r2}(\epsilon) = a_3 P_1 \sqrt{P} \bigg[ \sum_{l=0}^{N-1} \bigg[ \sum_{m=0}^{N-1} Z(m') \overline{Z}(l) \overline{Z}^*(\epsilon - m') \bigg] \bigg] W(\epsilon)$$

Similarly, $Y_{r3}(\epsilon) - Y_{r7}(\epsilon)$ can be obtained as follows:

$$Y_{r3}(\epsilon) = 2a_3 \sqrt{P} \bigg[ \sum_{l=0}^{N-1} \bigg[ \sum_{m=0}^{N-1} Z(m') \overline{N_{r5}}(l) \overline{N_{r5}}^*(\epsilon - m') \bigg] \bigg] W(\epsilon)$$

$$Y_{r4}(\epsilon) = a_3 P \bigg[ \sum_{l=0}^{N-1} \bigg[ \sum_{m=0}^{N-1} Z(m') Z(l) \overline{N_{r5}}^*(\epsilon - m') \bigg] \bigg] W(\epsilon)$$

$$Y_{r5}(\epsilon) = 2a_3 \sqrt{P} \bigg[ \sum_{l=0}^{N-1} \bigg[ \sum_{m=0}^{N-1} Z(l) \overline{N_{r5}}^*(\epsilon - m') \bigg] \bigg] W(\epsilon)$$

$$Y_{r6}(\epsilon) = a_3 P \bigg[ \sum_{l=0}^{N-1} \bigg[ \sum_{m=0}^{N-1} Z(m') Z(l) \overline{N_{r5}}^*(\epsilon - m') \bigg] \bigg] W(\epsilon)$$

$$Y_{r7}(\epsilon) = a_3 P \bigg[ \sum_{l=0}^{N-1} \bigg[ \sum_{m=0}^{N-1} Z(m') Z(l) \overline{N_{r5}}^*(\epsilon - m') \bigg] \bigg] W(\epsilon)$$
\[ Y_{r5}(c) = 2a_3 P Z(c) \otimes N_{st}(c) \otimes Z^* \left( (-c)_N \right) \]
\[ = 2a_3 P \sum_{l=0}^{N-1} \left[ \sum_{m' \neq c} Z(l) \tilde{N}_{st}(l) Z^* (l - c + m') \right] W(c) \]
\[ = 2a_3 P \sum_{l=0}^{N-1} \left[ \sum_{m' \neq c} \tilde{N}_{st}(l) Z^* (l) Z(c) + |Z(c)|^2 \tilde{N}_{st}(c) \right] W(c), \quad (A.7) \]

\[ Y_{r6}(c) = a_3 \sqrt{P} N_{st}(c) \otimes N_{st}(c) \otimes Z^* \left( (-c)_N \right) \]
\[ = a_3 \sqrt{P} \sum_{l=0}^{N-1} \left[ \sum_{m' \neq c} \tilde{N}_{st}(m') \tilde{N}_{st}(l) Z^* (l - c + m') \right] W(c), \quad (A.8) \]

\[ Y_{r7}(c) = a_3 N_{st}(c) \otimes N_{st}(c) \otimes N_{st}^* \left( (-c)_N \right) \]
\[ = a_3 \sum_{l=0}^{N-1} \left[ \sum_{m' \neq c} \tilde{N}_{st}(m') \tilde{N}_{st}(l) \tilde{N}_{st}^* (l - c + m') \right] W(c). \quad (A.9) \]