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Published in:
Monthly Notices of the Royal Astronomical Society

DOI:
10.1093/mnras/staa1541

Publication date:
2020

Document version:
Accepted manuscript

Citation for published version (APA):
Petersen, J., & Frandsen, M. T. (2020). A Method for Discriminating Between Dark Matter Models and MOND Modified Inertia via Galactic Rotation Curves. Monthly Notices of the Royal Astronomical Society, 496(2), 1077–1091. https://doi.org/10.1093/mnras/staa1541

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Download date: 19. Oct. 2023
A Method for Discriminating Between Dark Matter Models and MOND Modified Inertia via Galactic Rotation Curves

Jonas Petersen\textsuperscript{1} and Mads T. Frandsen\textsuperscript{1,}\textsuperscript{†}

\textsuperscript{1}CP\textsuperscript{3}-Origins, University of Southern Denmark, Campusvej 55, DK-5230 Odense M, Denmark

(Dated: March 8, 2018)

Dark Matter (DM) and Modified Newtonian Dynamics (MOND) models of rotationally supported galaxies lead to curves, \( C \), with different geometries in \((g_{\text{bar}}, g_{\text{tot}})\)-space (g\textsuperscript{2}-space). Here \( g_{\text{tot}} \) is the total centripetal acceleration of matter in a rotationally supported galaxy and \( g_{\text{bar}} \) is that from the baryonic (visible) matter distribution assuming Newtonian gravity. Specifically, in models of the baryonic matter where \( g_{\text{bar}} \) is zero at the galactic origin, the MOND modified inertia curves in g\textsuperscript{2}-space are closed with zero area \( A(C_{\text{MOND}}) = 0 \). In DM models with cored density profiles where \( g_{\text{bar}} \) is also zero at the galactic origin, the curves are again closed, but the area of the closed curves are in general non-zero, \( A(C_{\text{DM}}) \neq 0 \). The geometry of galactic rotation curve data from the SPARC database is investigated in order to discriminate between different models.

Introduction: There is a significant amount of astrophysical indications in favour of dark matter on a range of scales including the apparent dark halos in galaxies \([1]\). However, rotationally supported galaxies also appear to be well described by a modification of Newtonian dynamics (MOND) for accelerations below a characteristic acceleration scale close to the value \( g_0 \sim cH_0 \), where \( c \) is the speed of light and \( H_0 \) is the value of the Hubble constant today \([2]\). In particular MOND provides an explanation \([3]\) of the baryonic Tully-Fisher relation \([4]\). On larger scales the lensing of galaxy clusters \([5]\), observations of cluster mergers \([6]\) and large scale structure surveys \([7]\) all appear consistent with MOND modified inertia \([11,12]\) and SPARC database \([10]\) in \((\text{g2}-\text{space})\). Here \( g_{\text{tot}} \) is the total observed centripetal acceleration of matter in a rotationally supported galaxy as function of radial distance \( r \) from the center. Similarly \( g_{\text{bar}}(r) \) is the centripetal acceleration arising from the baryonic (visible) matter distribution assuming Newtonian gravity. The differences in geometry are also apparent from e.g. \([9]\), but in the present study it is explored how this difference may be used to discriminate between MOND modified inertia and DM models.

Previous analyses of rotation curve data from the SPARC data base \([10]\) in g\textsuperscript{2}-space has found it to be both consistent with MOND modified inertia \([11,12]\) and with DM \([9,13,15]\). See also \([16]\) for a detailed statistical analysis of the SPARC data. This is a priori not at odds with MOND and DM yielding different geometries in g\textsuperscript{2}-space since the differences manifests themselves in each galaxy mainly at small radii \( r \lesssim r_{\text{peak}} \), where \( r_{\text{peak}} \) is the radius of the maximum Newtonian baryonic acceleration \( g_{\text{bar}} \). Most rotation curve data points in the g\textsuperscript{2}-space: The centripetal baryonic acceleration in a galaxy assuming Newtonian gravity, \( g_{\text{bar}} \), is given by

\[
g_{\text{bar}}(r) = \frac{\sum_{i \in \text{bar}} v_i(r)^2}{r},
\]

where bar=gas, disk, bulge refers to the different baryonic components of the galaxies. Galaxy models with exponential mass densities for the baryonic components of the spiral disk and bulge \([22,24]\) yield baryonic accelerations which tend to zero for small \( r \), \( g_{\text{bar}}(0) = g_{\text{bar}}(\infty) = 0 \), see e.g. Fig.2 in \([25]\). However it should be noted that in other baryonic models, e.g when including gas components, or using other bulge models MONDian behaviour is often discussed at the largest radii of galaxies where accelerations are small \( g < g_0 \) because of the flat rotation velocity curve as a function of \( r \) \([17,20]\). But MONDian behaviour has also previously been investigated at small radii \( r \) in spherical systems where accelerations again are small, \( g < g_0 \) \([21]\). In this study, \( r_{\text{peak}} \) simply provides a radius below which the geometry of different models of rotation curves will differ in g\textsuperscript{2}-space, even if they agree at large radii. For some galaxies the entire rotation curve data sample, both at \( r \geq r_{\text{peak}} \) and \( r < r_{\text{peak}} \) may lie in the MONDian regime as e.g. the Galaxy NGC3109 displayed below in Fig.2.

To examine MOND modified inertia and DM models a subset of galaxies in the SPARC data base with data at radii \( r \lesssim r_{\text{peak}} \) are therefore investigated. In fact it proves relevant to include rotation curve data points at somewhat larger radii \( r < r_{\star} \), where \( r_{\star} \geq r_{\text{peak}} \) may be defined as the radius of maximum total acceleration for the galaxy or chosen on a galaxy by galaxy basis. Of course data at the smallest radii are in general subject to the largest uncertainties. Nevertheless they hold important information on the underlying galaxy model.
with e.g de Vaucouleurs density profiles it is no longer true that \( g_{\text{bar}}(0) = 0 \).

**MOND Modified inertia:** In MOND modified inertia models the total acceleration, \( g_{\text{tot},M} \), on a test mass is related to the Newtonian one, \( g_{\text{bar}} \), via

\[
g_{\text{bar}}(g_{\text{tot},M}) = \mu(x)g_{\text{tot},M} \tag{2}
\]

or equivalently

\[
g_{\text{tot},M}(g_{\text{bar}}) = \nu(y)g_{\text{bar}} \tag{3}
\]

where \( x \equiv \frac{g_{\text{tot},M}}{g_0} \), \( y \equiv \frac{g_{\text{bar}}}{g_0} = I(x) \) and \( \nu(y) = \mu(I^{-1}(y))^{-1} \) with \( g_0 \approx 1.2 \cdot 10^{-10} \text{m/s}^2 \) the characteristic acceleration scale of MOND. The function \( \mu(x) \) smoothly interpolates between \( \mu(x) = x \) for \( x \ll 1 \) and \( \mu(x) = 1 \) for \( x \gg 1 \).

A number of interpolating functions have been considered in the literature. The interpolation function

\[
\nu(y) = \frac{1}{1 - e^{-\sqrt{y}}} \tag{4}
\]

introduced in \cite{28,29} has been used to fit the SPARC galaxy data in \cite{11,12}. The interpolation functions of MOND modified inertia all yield single valued functions \( g_{\text{tot},M}(g_{\text{bar}}) \) in \( g \)-space as illustrated in Fig. 1 using the interpolation function in Eq. 4 (blue line). Since \( g_{\text{bar}}(r = 0) = g_{\text{bar}}(r = \infty) = 0 \), the curves, as parametrized by \( r \), are closed curves, \( C_{\text{MOND}} \), with zero area, \( \mathcal{A}(C_{\text{MOND}}) = 0 \).

**Dark Matter:** In DM models the total centripetal acceleration \( g_{\text{tot},DM}(r) = g_{\text{bar}}(r) + g_{\text{halo}}(r) \) is a sum of \( g_{\text{bar}}(r) \) and the acceleration from the DM halo \( g_{\text{halo}}(r) \). Examples of DM density profiles are the Navarro-Frenk-White (NFW) \cite{30} and pseudo-isothermal profiles

\[
\rho_{\text{NFW}}(r) = \frac{\rho_0}{\frac{r}{r_s}(1 + \frac{r}{r_s})^2}, \quad \rho_{\text{iso}}(r) = \frac{\rho_0}{1 + (\frac{r}{r_c})^2} \tag{5}
\]

with characteristic scale heights \( r_s, r_c \) respectively. The Navarro-Frenk-White profile \( \rho_{\text{NFW}}(r) \) is motivated at large radii by fits to simulations of cold collisionless dark matter \cite{30}. The isothermal DM density profile \( \rho_{\text{iso}}(r) \) is motivated (at small radii) by models with sizeable DM self interactions \cite{31,32}. It has recently been proposed that the diversity of galactic rotation curves \cite{33} can be accommodated in models of self interacting DM where the DM density follows the quasi-isothermal profile below a characteristic radius proportional to the self-interaction cross-section and reduces to the NFW profile at large radii \cite{31,32}. For the pseudo-isothermal profile it follows that \( g_{\text{halo}}(r = 0) = g_{\text{halo}}(r = \infty) = 0 \) but \( g_{\text{halo}} \) is not a function of \( g_{\text{bar}} \) so \( g_{\text{tot},DM} \) is not single valued as a function of \( g_{\text{bar}} \). The DM model curves in \( g \)-space are therefore closed, assuming still \( g_{\text{bar}}(r = 0) = g_{\text{bar}}(r = \infty) = 0 \), with \( \mathcal{A}(C_{\text{DM}}) \neq 0 \). This is illustrated in Fig. 1 in which the red solid curve is the DM model prediction, using an exponential disk for \( g_{\text{bar}}(r) \) and the pseudo-isothermal DM halo for \( g_{\text{halo}}(r) \). For NFW DM models \( g_{\text{tot},DM} \) is again not single valued as a function of \( g_{\text{bar}} \) but NFW models do not lead to closed curves in \( g \)-space, due to the divergence of the density profile at \( r = 0 \), see Fig. 1 (black line) or e.g. Fig. 2 in \cite{25}. Moreover DM density profiles may in principle be (extremely) tuned to yield single valued functions in \( g \)-space, e.g. \cite{34}. When discussing the area of the DM curves in this study, a cored DM profile with \( g_{\text{halo}}(r = 0) = 0 \) is assumed such that the curve is closed. Specifically the pseudo-isothermal profile is considered. However, one may of course also assign an area to the NFW curves by simply including the relevant line segment on the \( g_{\text{tot}} \) axis. Note that the model assumptions above do not affect the quantitative analysis below, where we employ
the full baryonic distribution from the SPARC database and do not rely on any DM distribution.

Theoretical Differences: Fig. 1 clearly illustrates the difference in geometry between the considered MOND modified inertia (blue) and pseudo-isothermal DM (red) model curves in g2-space. This difference is succinctly encoded in the fact that $A(C_{\text{MOND}}) = 0$ whereas $A(C_{\text{DM}}) \neq 0$. We also show a DM model curve using the NFW profile and the radius $r_{\text{peak}}$ at which $g_{\text{bar}}(r)$ has a maximum is indicated on each curve with a solid dot. Since viable MOND modified inertia and DM model curves must yield similar rotation curves at large radii, highlighted on the insert of Fig. 1, $r_{\text{peak}}$ defines a radii below which the DM and MOND modified inertia curves differ markedly. It is also clear from the curves in Fig. 1 that they already begin to differ markedly at radii $r > r_{\text{peak}}$ and so a radius $r_\ast \geq r_{\text{peak}}$ is introduced, as mentioned already, below which the DM and MOND modified inertia curves differ appreciably. The radius $r_\ast$ may be chosen as the point of maximum total acceleration (shown on the red curve) in the figure or determined on a case by case basis for different galaxies. The usefulness of $r_\ast$ is more apparent in Fig. 2 where the g2-space data of some individual galaxies are seen to deviate from the MOND modified inertia prediction at radii also larger than $r_{\text{peak}}$.

With these definitions in hand it is a priori expected that both MOND modified inertia and DM models fit data reasonably at radii above $r_{\text{peak}}$ or $r_\ast$ but yield different predictions below. A galaxy example where data does appear to systematically deviate from the MOND modified inertia curve (in blue) below $r < r_{\text{peak}}(r_\ast)$ and instead traces a DM curve (in red), is the galaxy NGC3109 shown in Fig. 2. In order to discriminate between DM and MOND modified inertia an area test is applied to data in what follows.

Data: The SPARC database consists of rotation curve data from 175 rotationally supported galaxies [10, 12]. The database provides observed total rotational velocities, along with the associated uncertainties, as well as the rotational velocity due to each baryonic component. Following [12] 22 galaxies are discarded from the analysis based on the inclination angle of the galaxy and a further quality criteria defined in [12]. For convenience, a further 23 galaxies are discarded in this analysis for having $\leq 7$ data points and lastly 1 galaxy is discarded because of imaginary values of $v_{\text{bar}}$ at the innermost radii. This leaves a group of 129 galaxies.

Following [12] the baryonic velocity ($v_{\text{bar}}$) is computed from ($v_{\text{gas}} \equiv v_d, v_{\text{bul}} \equiv v_b$ and $v_{\text{disk}} \equiv v_d$)

$$v_{\text{bar}} = \sqrt{|v_g|v_g + \Upsilon_d|v_d|v_d + \Upsilon_b|v_b|v_b.} \quad (6)$$

Where $\Upsilon_d \approx 0.5 \frac{M_d}{L_d}$ and $\Upsilon_b \approx 0.7 \frac{M_b}{L_b}$ denotes the mass to light ratio of the different components and the absolute value allows for negative contributions to $v_{\text{bar}}$ from individual components.

Uncertainties: The uncertainties in $g_{\text{obs}}$ are taken from the SPARC database [10] and include uncertainties in the observed rotational velocity, the galaxy distance and the galaxy inclination angle. The dominant uncertainties in $v_{\text{bar}}$ are caused by uncertainties in $v_{\text{gas}}$ and the mass-to-light ratios ($\Upsilon_d$ and $\Upsilon_b$) [10]. Following [10, 12] a 10% uncertainty in $v_{\text{gas}}$ and 25% uncertainties in $\Upsilon_d$ and $\Upsilon_b$ is adopted. A further 20% uncertainty in $g_{\text{bar}}$ from geometrical effects is included at small radii ($r < r_{\text{peak}}(r_\ast)$), as discussed (but not included) in the analysis of [10, 12]. These uncertainties are treated as random but are presumably systematic. However, it is not expected that this will change the global analysis below significantly. The data points at the smallest radii $r \lesssim r_{\text{peak}}(r_\ast)$ are in general the most uncertain. However these are also the ones that most clearly discriminate models in g2-space. By contrast to e.g. [12] data points with more than 10% uncertainty on $v_{\text{obs}}$ are therefore not discarded.

Results: In order to test the g2-space geometry of data and whether $A(C_{\text{data}}) \neq 0$ or not - rather than to test the value of the acceleration scale - the MOND

$^1$ The authors thank F. Lelli and S. McGaugh for clarifying this.

$^2$ Of course real data does not trace the entire closed curve.
modified inertia model of equation \( g_{\text{baryon}}(r = 0) = g_{\text{halo}}(r = \infty) = 0 \) is fit, with \( g_0 \) as a fit parameter, to data at \( r \geq r_{\text{peak}}(r_*) \). At least two data points at \( r \geq r_{\text{peak}}(r_*) \) are required. The fit is then compared to data at \( r < r_{\text{peak}}(r_*) \), also requiring at least two data points. The effective variance method, taking into account uncertainties in both \( g_{\text{obs}} \) and \( g_{\text{bar}} \) is used to compute the statistical significance \([35]\). Using the exact maximum likelihood method reduces the individual significances slightly but does not change the conclusions. There are 47 galaxies with at least two data points at \( r < r_{\text{peak}} \). Two of these deviate with more than 3\( \sigma \) from the MOND modified inertia fit, these are NGC3109 with 3.1\( \sigma \) and D631-7 with 3.7\( \sigma \). The case of NGC3109 is plotted in Fig. 2 including data with error bars (black crosses) along with the MOND modified inertia model fit (blue curve) and a DM model fit (red curve). The black line is included to illustrate how the data points are grouped in radii with the green dot in the lower left corner indicating the data point at the smallest radius. Despite \( A(C_{\text{NGC3109}}) \neq 0 \) and \( A(C_{\text{D631-7}}) \neq 0 \), at more than 3\( \sigma \), there is no global deviation of the 47 galaxies from the MOND modified inertia fit (the deviation is at 0.3 \( \sigma \)), so \( A(C_{\text{data}}) \neq 0 \) is not detected in the data using \( r_{\text{peak}} \). Similarly there are 69 galaxies with at least two data points at \( r < r_* \) and out of these 5 galaxies deviate with more than 3\( \sigma \) from the MOND modified inertia fit (F571-8 with 4.0\( \sigma \), D631-7 with 6.1\( \sigma \), NGC3109 with 3.1\( \sigma \), NGC2915 with 3.9\( \sigma \) and ESO563-G021 with 3.3\( \sigma \)). The global deviation of the 69 galaxies from the MOND modified inertia fit is 4.4\( \sigma \), so \( A(C_{\text{data}}) \neq 0 \) is detected with high significance using \( r_* \).

A region of the \( g_{2}\)-space data points - without uncertainties for visual clarity - of all galaxies are shown in Fig. 3. The 5 galaxies which deviate from the MOND modified inertia fit with more than 3\( \sigma \) are drawn up to illustrate that the deviating galaxies are spread out over a larger region of accelerations. The blue line illustrates the fit function of Eq. \( 4 \) with \( g_0 = 1.2 \times 10^{-10} \text{m/s}^2 \).

Summary and Discussion: It is shown that DM and MOND modified inertia models lead to curves with different geometries in \( g_{2}\)-space, i.e. \((g_{\text{bar}}, g_{\text{halo}})\)-space. These differences are apparent, at least, at radii \( r \leq r_{\text{peak}}(r_*) \) where \( r_{\text{peak}} \) is the radius at which the baryonic acceleration has a maximum value and \( r_* \) can be defined as the radius of maximum total acceleration or chosen in another systematic way on a galaxy by galaxy basis.

For models of the baryonic matter which obey \( g_{\text{baryon}}(r = 0) = g_{\text{bar}}(r = \infty) = 0 \) the geometric difference between MOND modified inertia and DM models, which additionally satisfy \( g_{\text{halo}}(r = 0) = g_{\text{halo}}(r = \infty) = 0 \), may be succinctly summarized in a single quantity, the area \( A(C_{\text{MOND,DM}}) \) of the closed curve \( C_{\text{MOND,DM}} \) predicted by the models in \( g_{2}\)-space. In particular MOND modified inertia models predict closed curves with \( A(C_{\text{MOND}}) = 0 \) under these assumptions. Instead DM models in general predict closed curves with \( A(C_{\text{DM}}) \neq 0 \).

A subset of 129 galaxies of the SPARC database \([10]\) is analyzed. 47 (69) galaxies have at least 2 data points at \( r < r_{\text{peak}}(r_*) \). Fitting the function in Eq. \( 4 \) to the data of these galaxies at \( r \geq r_{\text{peak}}(r_*) \) and comparing the fit to data points at \( r < r_{\text{peak}}(r_*) \) it is found that 2(5) galaxies deviate with more than 3\( \sigma \) from the fit. The galaxies are NGC3109 with 3.1\( \sigma \) and D631-7 with 3.7\( \sigma \) (F571-8 with 4.0\( \sigma \), D631-7 with 6.1\( \sigma \), NGC3109 with 3.1\( \sigma \), NGC2915 with 3.9\( \sigma \) and ESO563-G021 with 3.3\( \sigma \)). The global deviation of the 47(69) galaxies from the MOND modified inertia fit is 0.3\( \sigma \)(4.4\( \sigma \)). The global deviation of the 69 galaxies is a result of a systematic deviation in \( g_{\text{halo}} \) between points at \( r \geq r_* \) and points at \( r < r_* \). This systematic deviation is present above 3\( \sigma \) in 5 out of 69 galaxies, however, when combining data globally the deviation is significant at 4.4\( \sigma \).

The analysis presented here shows that the \( g_{2}\)-space geometry of individual galaxies with data at small radii \( r < r_{\text{peak}}(r_*) \) seem to deviate in a systematic way from the MOND modified inertia relation, exemplified by the global fit function Eq. \( 4 \). The trend in this deviation follows the expectation from a simple DM halo model with a quasi-isothermal DM density profile as illustrated in Fig. 2 and Fig. 3. Such a density profile is e.g. motivated by self-interacting DM \([31, 32]\).

Improved measurements of the mass-to-light ratio
of individual galaxies would eliminate a substantial amount of the uncertainty in \( g_{\text{bar}} \) and improve the ability to determine \( A(C_{\text{data}}) \neq 0 \) in \( g_2 \)-space. It would also be relevant to examine the systematic uncertainties at small radii in greater detail.

The approximate description of the centripetal acceleration in MOND modified gravity models in \([36, 37] \) shows that these curves are also double valued in \( g_2 \)-space. Investigating further the \( g_2 \)-space geometry of galactic rotation curves will be useful to discriminate between different DM density profiles, MOND modified inertia and MOND modified gravity models, in the future.

Acknowledgments: The authors thank S. McGaugh and F. Lelli for clarification regarding the uncertainties in \( g_{\text{bar}} \) and S. Sarkar and the referees for comments that have improved the text. The authors acknowledge partial funding from The Council For Independent Research, grant number DFF 6108-00623. The CP3-Origins center is partially funded by the Danish National Research Foundation, grant number DNRF90.

\* Electronic address: petersen@cp3.sdu.dk

\* Electronic address: frandsen@cp3.sdu.dk

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