A Study on the Dynamic Problems of Cylindrical Shells under the Coupling of Tensile and Torsional Forces

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Abstract. In this paper, the critical buckling load and dynamic buckling mode are reduced to symplectic eigenvalues and eigensolutions in symplectic space according to the non synchronous propagation of axial stress wave and torsional stress wave in cylindrical shell. Moreover, the eigensolutions are numerically solved by using the bifurcation theory of dynamic buckling. In the numerical computation, the influence factors of critical buckling load and the occurrence and development of buckling modes are discussed.

1. Introduction

Since 1960s, people have begun to study the dynamic buckling of structures. With the in-depth study of the dynamic buckling of long rod and cylindrical shell under the axial impact load, the buckling localization of the impact end or non impact end of the specimen has attracted people's attention. A large number of experiments have observed different degrees of plastic deformation localization or buckling localization phenomenon [1, 2]. The localization phenomenon is that the plastic deformation or buckling wave is not evenly distributed in the whole structure, and the buckling wave appears in some parts of the structure with certain regularity.

The conventional dynamic buckling analysis method ignores the stress wave in the process of dynamic buckling, so it can not explain the phenomenon of plastic deformation localization or buckling localization observed in the experiment in theory [3, 4]. Compared with the general dynamic buckling, the influence of stress wave on the dynamic buckling of structure is shown as follows:

(1) Buckling generally occurs at the local position of the structure. The critical buckling occurs when the stress wave propagates from the shock end and the propagation area increases to the critical length.

(2) In the existing theoretical analysis, most of them take the semi infinite rod and shell as the research object, and regard the wave front as the fixed end constraint. In fact, both the rod and the shell are finite in length. The stress wave will reflect at the non impact end of the rod and shell. The size of the reflected wave is determined by the constraint conditions of the non impact end.

(3) Most of the existing theoretical analysis only involves the propagation of a single elastic wave. However, in engineering practice, most of the structures are in the condition of various stress wave coupling.

In the theoretical analysis of the dynamic buckling problem caused by stress wave, especially the dynamic post buckling problem, the influence of inertia effect on the structural buckling deformation...
should be taken into account, otherwise, the correct results will not be obtained [5-7]. Considering the influence of stress wave in the dynamic buckling problem makes the problem very complicated. At present, this work is initially carried out, but its significance is very obvious. In this respect, special attention should be paid to the experimental research work. Unfortunately, it is very difficult to capture the characteristic parameters at the bifurcation point in the experiment because of the influence of stress wave propagation, reflection and transmission.

2. Dynamic equation of Timoshenko beam

The deformation potential energy density, bending potential energy density and kinetic energy density of elastic cylindrical shell subjected to axial and torsional coupling impact load can be expressed as

$$
\Pi_e = \frac{1}{2}K(e_x^2 + e_{\theta}^2 + 2\nu e_x e_{\theta} + \frac{1-\nu}{2} e_{\theta\theta}^2) + \frac{N_x}{2}(\frac{\partial}{\partial x} w)^2 + \frac{N_{\theta\theta}}{r}(\frac{\partial}{\partial \theta} w)^2
$$

(1)

$$
\Pi_k = \frac{1}{2}D(k_x^2 + k_{\theta}^2 + 2\nu k_x k_{\theta} + 2(1-\nu)k_{\theta\theta}^2)
$$

(2)

and

$$
\Pi_t = \frac{1}{2}\rho h(\frac{\partial}{\partial t} u)^2 + \frac{1}{2}\rho h(\frac{\partial}{\partial t} v)^2 + \frac{1}{2}\rho h(\frac{\partial}{\partial t} w)^2
$$

(3)

respectively. The Lagrangian function is

$$
L = \int \left( \frac{\rho h}{2}(\frac{\partial}{\partial t} u)^2 + \frac{\rho h}{2}(\frac{\partial}{\partial t} v)^2 - \frac{Eh}{2}(\frac{\partial}{\partial t} w)^2 - \frac{1-\nu}{4}(\frac{\partial}{\partial t} v)^2\right) - \frac{Eh}{2r^2}w^2 - \frac{D}{2}(\frac{\partial}{\partial t} w)^2 + \frac{1}{r^2}(\frac{\partial}{\partial \theta} w)^2 - \frac{2(1-\nu)}{r^2}(\frac{\partial}{\partial \theta} w)^2 - \frac{N_{\theta\theta}}{2}(\frac{\partial}{\partial \theta} w)^2 - \frac{N_{x\theta}}{r}\frac{\partial}{\partial \theta} w - \frac{N_{x\theta}}{r}\frac{\partial}{\partial \theta} w \right) r dl dx
$$

(4)

Through variation, we can get the wave equations of torsional stress

$$
\frac{\partial^2}{\partial t^2} u - c_t^2 \frac{\partial^2}{\partial x^2} u = 0, \quad \frac{\partial^2}{\partial t^2} v - c_t^2 \frac{\partial^2}{\partial x^2} v = 0
$$

(5)

and

$$
D\frac{\partial}{\partial x} w + \frac{2D}{r^2}\frac{\partial}{\partial \theta} \frac{\partial}{\partial x} w + \frac{D}{r^2}\frac{\partial}{\partial \theta} w + \frac{Eh}{r^2}w - \frac{2N_{x\theta}}{r}\frac{\partial}{\partial \theta} w = 0
$$

(6)

where

$$
c_t^2 = \frac{k(1-\nu)}{2\rho h} = \frac{E}{2(1+\nu)\rho}
$$

(7)

In this case, the dynamic boundary conditions can be described as follows

$$
\left. w \cdot [D\frac{\partial}{\partial x} w + D\frac{1-\nu}{r^2}\frac{\partial}{\partial \theta} \frac{\partial}{\partial x} w + N_x\frac{\partial}{\partial x} w - \frac{N_{x\theta}}{r}\frac{\partial}{\partial \theta} w] \right|_{x = x_0} = -\left. w \cdot Q_x \right|_{x = x_0} = 0
$$

(8)

$$
\left. \frac{\partial}{\partial x} w \cdot D\left( \frac{\partial}{\partial x} w + \frac{\nu}{r^2}\frac{\partial}{\partial \theta} w \right) \right|_{x = x_0} = -\left. \phi_x \cdot M_x \right|_{x = x_0} = 0
$$

(9a)

For fixed end constraints, the boundary conditions are

$$
Q_x = D\left( \frac{\partial}{\partial x} w + \frac{1-\nu}{r^2}\frac{\partial}{\partial \theta} \frac{\partial}{\partial x} w + N_x\frac{\partial}{\partial x} w - \frac{N_{x\theta}}{r}\frac{\partial}{\partial \theta} w \right) = 0
$$

(9b)

$$
M_x = D\left( \frac{\partial}{\partial x} w + \frac{\nu}{r^2}\frac{\partial}{\partial \theta} w \right) = 0
$$

(9c)
According to Hamiltonian variational principle, we get

\begin{equation}
L(W) = \frac{Eh}{2r^2} w^2 + \frac{D}{2} \left[ (w')^2 + (\ddot{w})^2 + 2\nu w' \dddot{w} + 2(1-\nu)(\dddot{w})^2 \right] + \frac{N_e}{2} (w')^2 - N_{e\omega} w' \ddot{w}
\end{equation}

(10)

\[ \delta \int \left( \frac{Eh}{2r^2} w^2 + \frac{D}{2} \left[ (w')^2 + (\ddot{w})^2 + 2(w')^2 \dddot{w} \right] + \frac{N_e}{2} (w')^2 - N_{e\omega} w' \ddot{w} \right) rd\theta dx = 0 \]

(11)

and

\[ \delta \int \left( \frac{Eh}{2r^2} w^2 + \frac{1}{2D} \rho_2^2 - (\varphi + \dot{w}) \rho_1 + \frac{N_e}{2} (w')^2 + N_{e\omega} w' \varphi \right) rd\theta dx = 0 \]

(12)

3. Numerical example

Figure 1. The critical buckling loads with time of wave propagation (n=1).

Figure 2. The critical buckling loads with time of wave propagation (n=3).
Fig. 1 is the critical buckling load surface of the axial torsional coupling impact load of order n = 1, and Fig. 2 is the critical buckling load surface of order n = 3. It can be seen from the figure that the existence of torsional impact load reduces the axial critical buckling load. When the torsional load is large enough, the axial critical buckling load appears negative value, and the cylindrical shell will still buckle.

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