A novel method for improved DEM deformation detection

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Abstract
The deformation detection method without control points is one of the key techniques for multi-temporal digital elevation model (DEM) analyses, and represents an attractive and difficult research topic. A novel method for improved DEM deformation detection is proposed in this paper, based on the Least Z-Difference algorithm with differential model (DM-LZD). In the new method, an additional parameter is employed to improve the weighting function for the observations in the matching algorithm. Three indexes are designed to give an in-depth and quantitative analysis of the performance, according to the possible two types of errors occurring in the weighting function. The experimental result, based on the simulated dataset, shows that with an appropriate additional parameter it will achieve a better balance between the deformation-detecting ability and the matching accuracy, and so generates better performance.

Keywords: DEM matching, differential model, deformation detection, performance index.

Introduction
Since the 1990s, the modern laser scanning techniques, terrestrial laser scanning (TLS) and aerial laser scanning (ALS), have developed very quickly, allowing a digital surface model of an area of interest to be regularly, or irregularly, and repeatedly acquired [Lichti, 2007; Barbarella and Fiani, 2013; Costantino and Angelini, 2013]. Moreover, its accuracy and the spatial resolution of DEM derived from laser scanning have increased over these years, with the discovery of more potential applications in many fields [Hu et al., 2009; Jaboyedoff et al., 2012; Barbarella and Fiani, 2013; Fanti et al., 2013; Pirotti et al., 2013]. The time-varying information about terrestrial changes contained in these multi-temporal DEMs is critical for hazard research and prevention [Gigli and Casagli, 2011; Lague et al., 2013]. However, the efficiency of data analysis is much lower than that of data acquisition, because data analysis usually requires ground control points (GCP) [Rau et al., 2012]. Constructing and maintaining GCP is both time-consuming and labor-intensive [Huggel et al., 2003; Kaufmann et al., 2005].
The DEM co-registration technique has great potential in multi-temporal DEM analysis, which can spatially align two DEMs without GCP, and then describe the changes between them [Rosenholm and Torelegård, 1988; Besl and McKay, 1992; Akca, 2007]. Finding a good match, which is a precondition for deformation detection, is key to the analysis of multi-temporal DEM, which is a basic job for studying geological hazards.

DEM deformation detection without GCP is usually developed based on the DEM co-registration technique. The Least Z-Difference (LZD) [Rosenholm and Torelegård, 1988; Zhang, 1994] is often employed for its relative higher efficiency, and many deformation detection methods are reported. By adopting the M-estimator, Pilgrim [1996] proposed the M-LZD algorithm, which can detect no more than 25% deformation. Li et al. [2001] obtained the LMS-LZD algorithm by using the least median of squares (LMS) estimator. It can obtain matching with no more than 50% deformation. Zhang et al. [2006] introduced a differential model (DM), and produced a DM-LZD algorithm, whose performance efficiency is higher as a result of using the spatial relationship and magnitude of deformation. In the case of deformation detection, both the matching efficiency and deformation-detecting ability play equivalently important roles, therefore, pursuing a better balance between these two factors is very valuable.

In this paper, the principle of a novel algorithm, DM-LZD with an additional parameter, is introduced. Then three indexes are designed, to measure the performance of the co-registration, according to the two classes of errors, which appear in the weighting function. Using these indexes, an in-depth and comprehensive analysis is performed quantitatively. The proposed algorithm is tested on a simulated dataset, and experimental results show that it obtain a better balance between the deformation-detecting ability and the matching efficiency.

**DM-LZD with additional parameter**

A robust LZD algorithm using the differential model (DM-LZD) proposed by Zhang et al. [2006], showed very good deformation detectability. Associated with the differential model, the magnitude of difference and the spatial relationship between differences can be considered in the DM-LZD algorithm. However, about half of the observations will be discarded because the weight is set to zero if an observation is larger than the median of all observations.

Although the deformation-detection ability is greatly enhanced, the co-registration efficiency is limited by this weight strategy. To overcome this shortcoming, a new weight function with an additional parameter is given, to balance the deformation-detecting ability and the co-registration efficiency.

**DM-LZD with additional parameter algorithm:**

1. Build the differential model;
2. Establish the objective function of the modified algorithm

\[
\sum (w_{DM} \cdot dz^2) = \min \quad [1]
\]

where \(w_{DM} = w_M \cdot w_R\), \(w_M\) is the additional weight, \(w_R\) is the weight given by the robust estimator, and \(dz\) is the height difference between corresponding points located in the
The additional weight \( w_M \) is determined by the following steps:

1. Construct a statistical vector \( D = \{d_{ij}\} \) according to the differential model

\[
d_{ij} = \left\{ \frac{1}{8} \sum_{r=1, s=1}^{M, N} dz_{rs} - dz_{ij} \right\} \quad (1 < i < M, 1 < j < N) \quad [2]
\]

where \( d_{ij} \) refers to the spatial variance ratio, \( dz_{rs} \) is spatially direct adjacent to \( dz_{ij} \), and \( M \) and \( N \) are the number of columns and rows of DEM;

2. The additional weight can be derived from the statistical vector \( D = \{d_{ij}\} \) by the following equation

\[
w_M(i, j) = \begin{cases} 
1 & d_{ij} < d^{(M \times N \times k)}_{ij} \\
0 & \text{Other}
\end{cases} \quad [3]
\]

where \( d^{(M \times N \times k)}_{ij} \) is the \( M \times N \times k \) -th element in the ascending-order statistical vector \( D \). \( \lfloor \cdot \rfloor \) refers to downward rounding, and the \( k \) is the additional parameter (0 < \( k \) ≤ 1). The additional parameter \( k \) determines how many observations will contribute to the co-registration.

The \( d^{(M \times N \times k)}_{ij} \) can be calculated by the divide and conquer technique, with time complexity \( O(\log n) \). The lowest time complexity of any sorting method, based on comparison, is \( O(n \log n) \) [Zhang and Cen, 2008]. The larger the dataset, the larger \( n \), more efficiency improvement, will be achieved.

**Analysis of performance**

**Indexes of performance**

To facilitate analysis of the new method, suppose that the DEM surface contains two parts: one is the unchanged region (\( S \)); and the other the deformed region (\( D \)).

The critical factor of the new method’s performance is whether it can judge if points are located in the deformed region, or in the unchanged region. This judgment is mainly carried out by the weight function (Eq. [4]). The weight function may generate two kinds of errors.

1. The first type of error (\( E_I \)). The observation located in the deformed region is judged to be in the unchanged region. \( E_I \) will cause the observation containing the deformation into an objective function. It is the main reason for failed matching, or poor matching accuracy.

2. The second type of error (\( E_{II} \)). The observation located in the unchanged region is judged to be in the deformed region. \( E_{II} \) will reduce the number of observations in the
objective function, and decrease its efficiency, even resulting in failed matching. These two types of error are directly related to the performance of the new deformation detection algorithm. To analyze their influence, three indexes \((e_I, e_{II}, e_{III})\) are introduced in this paper:

\[
e_I = \frac{\sum E_I(P_i)}{\text{Sum}(D)} \quad [4]
\]

\[
e_{II} = \frac{\sum E_{II}(P_i)}{\text{Sum}(S)} \quad [5]
\]

\[
e_{III} = \frac{\sum E_I(P_i)}{\text{Sum}(S)+\text{Sum}(D)} \quad [6]
\]

where \(E_I(P_i)=1\) when the first type of error \((E_I)\) happens in \(P_i\), otherwise \(E_I(P_i)=0\); \(E_{II}(P_i)=1\) when the second type of error \((E_{II})\) happens in \(P_i\), or \(E_{II}(P_i)=0\); \(\text{Sum}(D)\) is the total number of observations located in the deformed region \((D)\), and \(\text{Sum}(S)\) is the total number of observations located in the unchanged region \((S)\).

**Deformation detection ability**

The deformation detection ability is an important performance index, and can be measured by the ratio \((\eta)\).

\[
\eta = \max \left( \frac{\text{Sum}(D)}{\text{Sum}(S)+\text{Sum}(D)} \right) \quad [7]
\]

Therefore, \(\eta\) is the maximum detectable percentage, in terms of finding a successful match. The deformation detection ability \((\eta)\) of M-LZD and LMS-LZD is nearly equal to the breakdown ratio \((\epsilon^*)\) of the robust estimator adopted.

\[
\eta \approx \epsilon^* \quad [8]
\]

Suppose the least square estimator (LSE) is adopted in the DM-LZD algorithm. For correct matching, the following two conditions should be satisfied, because the breakdown ratio \(\epsilon^*\) of LSE is equal to zero.

\[
e_I = 0, e_{II} < 1 - p/\text{Sum}(S) \quad [9]
\]

where \(p\) is the number of parameters to be solved.
The first condition \((e_I=0)\) means that no observations located in the deformed area are taken into the objective function, and the last condition guarantees that the number of observations in the objective function is larger than the number of parameters to be solved. That is, the weight function used in the new method should identify all observations located in the deformed region, and discard them. Usually, \(e_{II}\) increases while \(e_I\) decreases. In two extreme cases, a) when \(w \equiv 0, e_I=0, e_{II}=1;\) b) and when \(w \equiv 1, e_I=1, e_{II}=0.\) Therefore, ensuring the two conditions in Equation [9] is almost impossible in real applications.

A robust estimator, e.g., an M-estimator or LTS estimator, should be adopted in DM-LZD. To obtain correct transformation parameters, the two following conditions should be met.

\[
e_{III} < e^* , e_{II} < 1 - \frac{p}{\text{Sum}(S)} \quad [10]
\]

That is, the ratio of observations assigned a non-zero weight value should be less than \(e^*\) of the estimator, and the number of observations in the unchanged region should be more than the parameter number. Substituting Equation [4] into Equation [6] leads to

\[
e_{III} = \frac{\sum E_1 (P)}{\text{Sum}(S) + \text{Sum}(D)} = \frac{\text{Sum}(D) \times e_I}{\text{Sum}(S) + \text{Sum}(D)} \quad [11]
\]

Thus,

\[
\frac{\text{Sum}(D)}{\text{Sum}(S) + \text{Sum}(D)} = \frac{e_{III}}{e_I} \quad [12]
\]

According to Equation [7], the above equation can be rewritten as:

\[
\eta_{\text{DM-LZD}} = \max \left( \frac{1}{e_I} e_{III} \right) \quad [13]
\]

Therefore, the relationship between the deformation-detecting ability \((\eta)\) of the new method, and the breakdown ratio \((e^*)\) of the robust estimator, can be described as:

\[
\eta_{\text{DM-LZD}} \approx \frac{1}{e_I} e^* \quad [14]
\]

The worst case of \(\eta_{\text{DM-LZD}}\) is \(e_I=1.\) In this case, according to Equation [4], \(\sum E_1 (P) = \text{Sum}(D).\) All observations in the deformed region are in the objective function; the weight function (Eq.[3]) is invalid. In this case, the \(\eta_{\text{DM-LZD}}\) is nearly equal to that used directly by the robust estimator, such as M-LZD and LTS-LZD.
\[ \eta_{\text{DM-LZD}} \mid \varepsilon_i = 1 \approx \varepsilon^* \quad [15] \]

At the same time, \( e_{\text{II}} \neq 0 \) in this case. Its efficiency is less than that of existing methods using fewer observations.

In the case \( 0 \leq e_i < 1 \), only partial observations in the deformed region will enter the objective function. The relationship between \( \eta_{\text{DM-LZD}} \) and \( \varepsilon^* \) can be written as

\[ \eta_{\text{DM-LZD}} \approx \frac{1}{e_i} \varepsilon^* > \varepsilon^* \left( 0 \leq e_i < 1 \right) \quad [16] \]

This shows that the deformation detectable ratio may overcome the breakdown ratio of the robust estimator.

**Matching efficiency**

The DEM matching efficiency is another important aspect of the new method. To enhance the deformation-detecting ability, the observations in the deformed region are discarded as much as possible. Excessive discards will result in the matching efficiency decreasing greatly.

The observations in the unchanged region will contribute positively to the objective function, and those in the deformed region will have a negative effect. Therefore, we introduce the ratio (\( \gamma \)), to measure the matching efficiency.

\[ \gamma = \frac{\text{Sum}(S) - \sum E_{\text{II}}(P_i)}{\text{Sum}(S)} \quad [17] \]

Substituting Equation [5] into Equation [17] leads to:

\[ \gamma = 1 - e_{\text{II}} \quad [18] \]

It is obvious that \( 0 \leq \gamma \leq 1 \). When \( e_i = 1, \gamma = 0 \). All observations in the unchanged region will be discarded, leading to the failure of the algorithm. When \( e_{\text{II}} = 0, \gamma = 1 \). All observations in the unchanged region will be taken into account. Only in this case is the matching efficiency optimal.

**The balance between deformation-detecting ability and matching efficiency**

Three indexes, \( \varepsilon^* \), \( e_{\text{II}} \) (or \( \gamma \)), and \( e_{\text{III}} \) are introduced to measure the deformation-detecting ability, and the matching efficiency. Actually, these two elements are linked, and both should be considered simultaneously for a comprehensive assessment.

Some specific cases are discussed first. When \( e_i = 1 \) and \( e_{\text{II}} = 0 \), all observations in the deformed region are included in objective functions. Therefore, the performance would be best. When \( e_i = 0, e_{\text{II}} = 1 \), or \( e_i = 1, e_{\text{II}} = 1 \), no observations, or no observations in the unchanged region,
are included in the objective functions. This will result in failure. When \( e_I = 0 \) and \( e_{II} = 0 \), the observations in the objective functions are all in the unchanged region, the matching efficiency is optimal, and the deformation-detecting ability is at a maximum.

Actually, \( e_{II} \) is decreasing while \( e_I \) is increasing, so \( e_I \) and \( e_{II} \) cannot be zero at the same time; that is, the matching efficiency and the deformation-detecting ability cannot reach an optimum simultaneously. Therefore, only an appropriate balance between the two can result in optimal performance, obtained by taking these three indexes, \( e_I \), \( e_{II} \) (or \( \gamma \)), and \( e_{III} \) into account.

**Experiments**

**Dataset**

To facilitate the analysis of the experimental results, the experiments are performed with a simulated dataset (Fig. 1). In a real application of ground surface deformation detection, the DEM obtained in the first epoch is usually employed as the reference ground, and then the terrestrial changes can be given by comparing the DEMs of each following epoch. Therefore, DEM obtained in the first epoch is used as the reference DEM in experiments. In this research, a DEM with 500×500 grids and an interval of 2m is used in tests (Fig. 1). It is extracted from an aerial LiDAR-derived DEM of a typical mountain area located in southwest of China. The height of points in the DEM is between 485m and 708m.

![Figure 1 – DEM.](image)

The deformed DEM is generated by two steps: 1) adding simulated deformation and random errors, subject to normal distribution \( N\left(\mu, \sigma^2\right) \) in the reference DEM; and 2) applying the given transformation parameters to the reference DEM.

For convenience, the surface is equally divided into 3×3=9 sub-blocks, and labeled from
number 1 to number 9. In tests, one sub-block is either added as a deformation to the whole surface, or none is added. The deformed percentage is set by the number of sub-blocks of added deformation, and the tested percentages are \(1/9(\approx 11.1\%)\), \(2/9(\approx 22.2\%)\)… and \(5/9(\approx 55.6\%)\), respectively.

When the percentage of deformation is fixed, several possible deformation distributions are tested.

With one fixed deformation percentage, several different possible deformed distributions are tested to analyze the effect of the deformation distribution. The detailed position of deformations tested is listed in Table 1.

| Deformation percentage | Number of tests | Position of the deformation* |
|------------------------|----------------|-------------------------------|
| 1/9(\approx 11.1\%)   | 9              | [1],[2],[3],[4],[5],[6],[7],[8],[9] |
| 2/9(\approx 22.2\%)   | 9              | [1,2],[2,3],[3,4],[4,5],[5,6],[7,8],[8,9],[2,8],[4,6] |
| 3/9(\approx 33.3\%)   | 9              | [1,2,3],[4,5,6],[7,8,9],[1,4,7],[2,5,8],[3,6,9],[1,5,9],[3,5,7],[2,4,6] |
| 4/9(\approx 44.4\%)   | 6              | [1,2,4,5],[2,3,5,6],[4,5,7,8],[1,3,7,9],[3,4,6,7],[5,6,8,9] |
| 5/9(\approx 55.6\%)   | 1              | [2,4,5,6,8]                   |

* Note. The numbers in the square brackets denote the position of deformation, and a pair of square brackets denotes one test.

It should be noted that the last fine-matching step in DM-LZD is a great help for enhancing the matching accuracy, but it also covers the difference from the algorithm parameters. Therefore, the last fine-matching step is neglected in all tests.

**The effect of the additional parameter**

The three indexes, \(e_I\), \(e_{II}\) and \(e_{III}\), introduced in this paper, are adopted to analyze the performance. They are closely related to the deformation percentage, deformation magnitude, and the additional parameter \(k\). Therefore, the percentage of deformation is fixed firstly in tests, to analyze the effect of the additional parameter in different magnitudes of deformation. The changing trends of three parameters are similar in cases of different deformation percentage, and the experimental results in the case of the deformation percentage \(4/9(\approx 44.4\%)\) are given in Figure 2.

Then the magnitude of deformation is fixed to analyze the effect of the additional parameter in different deformation percentages. The variation trends of three parameters are also similar in the case of different magnitudes of deformation, and the experimental results in the case of magnitude \(5\sigma\) are given in Figure 3.
Figure 2 - Indexes varying with additional parameter $k$ with different deformation magnitude.
Figure 3 - Indexes varying with additional parameter $k$ with different deformation percentage.
Figure 4 - Matching error of the proposed method; the horizontal axis is the additional parameter.
In Figure 2, the deformation magnitude has little influence on $e_I$, $e_{II}$ and $e_{III}$. In Figure 3, the deformation percentage has a clear influence on $e_I$, $e_{II}$ and $e_{III}$. Therefore, the deformation percentage is the main factor, and the deformation magnitude is the minor factor in terms of performance. This finding is in agreement with the results reported in the references.

In Figures 2 and 3, $e_I$, $e_{II}$ and $e_{III}$ change greatly with the additional parameter $k$, and the effect of the additional parameter $k$ on the variation in the value of $e_I$, $e_{II}$ and $e_{III}$ is larger than the effect of the deformation magnitude and percentage on it. Consequently, the additional parameter $k$ is the most important influential factor.

As shown by the change in value of the three indexes, as shown in Figure 2 and Figure 3, when the additional parameter $k$ increases, $e_{II}$ decreases, and at the same time, $e_I$ and $e_{III}$ increase, but with $e_I$ at a higher rate. That is to say, that the matching accuracy decreases when the deformation-detecting ability becomes stronger.

The determination of the additional parameter
When the additional parameter $k \leq 0.8$, $e_I < 0.15$ and $e_{II} > 0.6$, according to the experimental results in Figure 2. That is, over 85% of the observations in the deformed region are removed from the objective function, and less than 40% of observations in the unchanged region are kept in the objective function, so the matching efficiency is low. The proposed method is not obviously superior to the LZD with LTS, in the case of a slight deformation percentage, but is much better in the case of a large deformation percentage.

The matching accuracy of the proposed method is given in Figure 4, when the deformation percentage is $3/9$ ($\approx 33.3\%$), $4/9$ ($\approx 44.4\%$) and $5/9$ ($\approx 55.6\%$), respectively. The matching accuracy includes the error of rotation angle and shift, which comprises the average value of three rotation angle errors and three shift errors.

When $k=1$, the weighting function (Eq. [3]) makes no contribution to the matching process, so the method applies the LZD algorithm, and uses a robust estimator directly.

When the additional parameter $k=0.6~0.8$, and the deformation percentage is $3/9$ ($\approx 33.3\%$) and $4/9$ ($\approx 44.4\%$), the improved method has a higher matching accuracy than LZD with the LTS estimator. When the deformation percentage is $5/9$ ($\approx 55.6\%$), it is higher than 50%, the breakdown ratio of LTS estimator, so the proposed method can also obtain correct matching with high accuracy with the additional parameter $k=0.7$.

Conclusions
An improved DEM deformation detection is proposed in this paper by introducing an additional parameter to the DM-LZD algorithm, obtaining a better performance. The experimental results, based on the simulated dataset, show that its deformation-detecting ability can surpass the breakdown ratio of the robust estimator, and can deal with a higher deformation percentage.

For in-depth analyses of the deformation detection method, three indexes were designed, according to the two classes of potential errors in the weight of observations in the method, and then the deformation-detecting ability, and the matching efficiency could be discussed quantitatively. Deformation percentage has more influence on $e_I$, $e_{II}$ and $e_{III}$ than that of deformation magnitude. When the additional parameter $k$ increases, $e_{II}$ decreases, and at the same time, $e_I$ and $e_{III}$ increase, but $e_I$ does so at a higher rate.
The additional parameter \( k \) has a remarkable effect on performance, since the matching accuracy decreases when the deformation-detecting ability becomes stronger. When \( k=0.6\sim0.8 \), an optimal balance between the deformation-detecting ability and the matching accuracy will be achieved, according to the experimental results.

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References
Akca D. (2007) - *Matching of 3D surfaces and their intensities*. ISPRS Journal of Photogrammetry and Remote Sensing, 62 (2): 112-121. doi: http://dx.doi.org/10.1016/j.isprsjprs.2006.06.001.

Barbarella M., Fiani M. (2013) - *Monitoring of large landslides by terrestrial laser scanning techniques: Field data collection and processing*. European Journal of Remote Sensing, 46: 126-151. doi: http://dx.doi.org/10.5721/EuJRS20134608.

Besl P.J., McKay N.D. (1992) - *A method of registration of 3-D shapes*. IEEE Transactions on Pattern Analysis and Machine Intelligence, 14 (2): 239-256. doi: http://dx.doi.org/10.1109/34.121791.

Costantino D., Angelini M.G. (2013) - *Production of DTM quality by TLS data*. European Journal of Remote Sensing, 46: 80-103. doi: http://dx.doi.org/10.5721/EuJRS20134606.

Fanti R., Gigli G., Lombardi L., Tapete D., Canuti P. (2013) - *Terrestrial laser scanning for rockfall stability analysis in the cultural heritage site of Pitigliano (Italy)*. Landslides, 10 (4): 409-420. doi: http://dx.doi.org/10.1007/s10346-012-0329-5.

Gigli G., Casagli N. (2011) - *Semi-automatic extraction of rock mass structural data from high resolution lidar point clouds*. International Journal of Rock Mechanics and Mining Sciences, 48 (2): 187-198. doi: http://dx.doi.org/10.1016/j.ijrmms.2010.11.009.

Hu P., Liu X., Hu H. (2009) - *Accuracy assessment of digital elevation models based on approximation theory*. Photogrammetric Engineering & Remote Sensing, 75 (1): 49-56. doi: http://dx.doi.org/10.14358/PERS.75.1.49.

Huggel C., Kääb A., Haeberli W., Krummenacher B. (2003) - *Regional-scale gis-models for assessment of hazards from glacier lake outbursts: Evaluation and application in the Swiss alps*. Natural Hazards and Earth System Sciences, 3: 647-662. doi: http://dx.doi.org/10.5194/nhess-3-647-2003.

Jaboyedoff M., Oppikofer T., Abellán A., Derron M.-H., Loye A., Metzger R., Pedrazzini A. (2012) - *Use of lidar in landslide investigations: A review*. Natural Hazards, 61(1): 5-28. doi: http://dx.doi.org/10.1007/s11069-010-9634-2.

Kaufmann V., Ladstädter R., Lieb G.K. (2005) - *Quantitative assessment of the creep process of Weissenkar rock glacier (central alps,Austria)*. 8th International Symposium on High Mountain Remote Sensing Cartography, La Paz, Bolivia.

Lague D., Brodu N., Leroux J. (2013) - *Accurate 3D comparison of complex topography with terrestrial laser scanner: Application to the Rangitikei Canyon (n-z)*. ISPRS Journal of Photogrammetry and Remote Sensing, 82 (1): 10-26. doi: http://dx.doi.org/10.1016/
Li Z., Xu Z., Cen M., Ding X. (2001) - Robust surface matching for automated detection of local deformations using least-median-of-squares estimator. Photogrammetric Engineering & Remote Sensing, 67 (11): 1283-1292.

Lichti D.D. (2007) - Error modelling, calibration and analysis of an AM-CW terrestrial laser scanner system. ISPRS Journal of Photogrammetry and Remote Sensing, 61 (5): 307-324. doi: http://dx.doi.org/10.1016/j.isprsjprs.2006.10.004.

Pilgrim L.J. (1996) - Robust estimation applied to surface matching. ISPRS Journal of Photogrammetry and Remote Sensing, 51: 243-257. doi: http://dx.doi.org/10.1016/0924-2716(96)00010-X.

Pirotti F., Guarnieri A., Vettore A. (2013) - State of the art of ground and aerial laser scanning technologies for high-resolution topography of the earth surface. European Journal of Remote Sensing, 46: 66-78. doi: http://dx.doi.org/10.5721/EuJRS20134605.

Rau J.-Y., Chang K.-T., Shao Y.-C., Lau C.-C. (2012) - Semi-automatic shallow landslide detection by the integration of airborne imagery and laser scanning data. Natural Hazards, 61 (2): 469-480. doi: http://dx.doi.org/10.1007/s11069-011-9929-y.

Rosenholm D., Toreleggård K. (1988) - Three-dimensional absolute orientation of stereo models using digital elevation models. Photogrammetric Engineering & Remote Sensing, 54 (10): 1385-1389.

Zhang T., Cen M. (2008) - Robust DEM co-registration method for terrain changes assessment using least trimmed squares estimator. Advances in Space Research, 41 (11): 1827-1835. doi: http://dx.doi.org/10.1016/j.asr.2007.06.035.

Zhang T., Cen M., Zhou G., Wu X. (2006) - A new method for debris-flow detection using multi-temporal dems without ground control points. International Journal of Remote Sensing, 27 (21): 4911-4921. doi: http://dx.doi.org/10.1080/01431160600794613.

Zhang Z. (1994) - Iterative point matching for registration of free-form curves and surfaces. International Journal of Computer Vision, 13 (2): 119-152. doi: http://dx.doi.org/10.1007/BF01427149.

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