Projection Bias in Effort Choices*

Marc Kaufmann (Central European University)

April 12, 2021

Abstract

Working becomes harder as we grow tired or bored. I model individuals who underestimate these changes in marginal disutility – as implied by “projection bias” – when deciding whether or not to continue working. This bias causes people’s plans to change: early in the day when they are rested, they plan to work more than late in the day when they are rested. Despite initially overestimating how much they will work, people facing a single task with decreasing returns to effort work optimally. However, when facing multiple tasks, they misprioritize urgent but unimportant over important but non-urgent tasks. And when they face a single task with all-or-nothing rewards (such as being promoted) they start, and repeatedly work on, some overly ambitious tasks that they later abandon. Each day they stop working once they have grown tired, which can lead to large daily welfare losses. Finally, when they have either increasing or decreasing productivity, people work less each day than previously planned. This moves people closer to optimal effort for decreasing, and further away from optimal effort for increasing productivity.

JEL — D03, J22

1 Introduction

Our tastes fluctuate, often rapidly: we grow tired and thirsty from running, and we savor food or crave coffee more the longer we go without. Furthermore, evidence from a variety of domains suggests that our perceptions of our tastes are biased towards our current tastes: we misperceive future tastes as being closer to our current tastes than they will

---

*Central European University, Department of Economics and Business. Email: kaufmannm@ceu.edu. I am grateful to Matthew Rabin, David Laibson, Gautam Rao, and Josh Schwartzstein. I also thank Ben Bushong, Krishna Dasaratha, Anastassia Fedyk, Tristan Gagnon-Bartsch, James Hodson, Botond Köszegi, Annie Liang, Ben Lockwood, Neil Thakral, Linh T. Tô, and Gal Wettstein, as well as seminar participants at CEU, Harvard, Humboldt, the Hungarian Academy of Sciences, Liser, and LMU for helpful comments.
be (Loewenstein, O'Donoghue, and Rabin (2003)). This projection bias can trigger undesirable and unintended habits and behaviors, such as buying too much when shopping on an empty stomach, or becoming addicted due to under-appreciating the future intensity of cravings. In this paper, I study effort choices where the distaste for work changes, such as when students grow bored of studying or employees become tired of working. Due to projection bias, individuals mispredict future disutility of work, which can cause them to mis-prioritize between tasks, waste time on never-to-be-completed tasks, and inefficiently choose when to work on those tasks.

I provide and discuss the formal model in Section 2. The agent works continuously on a single task in one period and stops working the moment she perceives stopping to be optimal given the monetary benefits and the perceived disutility from work. Letting $s$ denote the total time she has worked so far that day, the instantaneous disutility from continuing to work is equal to $D'(S)$. She misperceives her future disutility due to projection bias: she predicts that her marginal disutility after $e$ hours of work lies between her current marginal disutility, $D'(s)$, and her true future marginal disutility, $D'(e)$. So when she is rested and marginal disutility is thus particularly low, she overestimates how easy it will be to work, while when tired and marginal disutility is thus particularly high, she underestimates it. This leads to changing and inconsistent plans which in turn can lead to the time-inconsistent behavior that I study in this paper.

In Section 3, I analyze the implications from underestimating the disutility of future work in single-period, single-task settings. Because I assume that a person grows tired the longer she works, she underestimates how unpleasant the work will be later that day and overestimates how much she will work. Despite this, with decreasing returns to effort, she works optimally. She works until the marginal benefits equal her marginal disutility, and then stops. I then consider all-or-nothing rewards, for which she receives known, fixed benefits if she completes the task by the end of a given day. A projection-biased person starts some overly costly all-or-nothing tasks, but as work becomes more unpleasant, she realizes some of the higher costs and may give up. If she does give up, she is better off than if she had committed herself to finishing the task, because she still overestimates how much she should work.

Even for decreasing returns to effort she only works optimally if she faces a single task. If, as I assume in Section 4, she has to allocate her time across two consecutive tasks, she spends too much time on the first task compared to the second: when she switches from the first to the second task, she overestimates how much she will work on the second task. If she realized how much she will actually work, she would switch earlier. This suggests

---

1Evidence for projection bias has been found for food (Read and Van Leeuwen (1998); Nordgren, Pligt, and Harreveld (2008)), drink (Van Boven and Loewenstein (2003)), sexual arousal (Loewenstein, Nagin, and Paternoster (1997); Ariely and Loewenstein (2006)), effortful tasks (Augenblick and Rabin (2019)), heroin substitute cravings (Badger et al. (2007)), the endowment effect (Loewenstein and Adler (1995)) and for predictions of gym attendance (Acland and Levy (2015)). Projection bias resembles immune neglect (Gilbert et al. (1998)) whereby people overestimate how long they will feel bad about negative events. Bushong and Gagnon-Bartsch (2020) additionally find evidence for interpersonal projection bias for choices over effort.
more generally that, when multi-tasking, projection bias leads people to work too much on early stages of tasks or on urgent but unimportant tasks, compared to later stages or flexible but important tasks.

In Section 5, I extend the basic model to consider multi-day settings with all-or-nothing tasks that are due on some fixed future day. I simplify the analysis by considering a continuous-time setup with a continuum of days. At the start of each day, the person may plan to complete the task efficiently, yet stop working earlier than anticipated once she has grown tired, in which case she plans to drop the task entirely. This repeated starting and stopping – due to naiveté about her bias – leads to one of two outcomes. Either she completes the task despite working too little in the early days, but has to work harder later on. Or she repeatedly wastes time on working on a task that she eventually drops for good. In fact, as long as benefits are insufficient to lead the person to complete the task, increasing benefits makes her worse off, because they lead her to waste even more time across more days for no change in the outcome. Even an almost unbiased person can thus be induced to complete almost all the work, yet fail to complete the task.

In Section 6, I consider a setting in which the main benefit from working consists in having less work in the future, so that the perceived value of the benefit fluctuates with current tiredness. Specifically, the person has to produce a fixed total output by a given day, but her productivity either increases or decreases over time. Then, the benefit of more work today is to have less work in the future. On low-productivity days, the person works little, and therefore perceives the benefits from saving, say, 5 hours of future work as low. On high-productivity days, the person works a lot and therefore perceives the same 5 hours of future work more costly and hence more valuable to save them. Thus the person works too much on high-productivity days and too little on low-productivity days. Moreover, when productivity changes monotonically, whether it is increasing or decreasing, people work less on a given day than they planned at the end of the previous day. This change in plans moves them closer to optimal behavior when productivity is decreasing, but further from optimal behavior when productivity is increasing, which highlights again that committed choices are not always preferable to on-the-spot decisions.

My paper is most closely related to Loewenstein, O'Donoghue, and Rabin (2003) who formalize the model of projection bias and apply it to durable goods consumption, the endowment effect, and habit formation. I focus on the time-inconsistent choices resulting from changes in fatigue in effort choices, an economically important domain. Since agents repeatedly reoptimize as they grow tired or rested, the final success of work depends on combining the many decisions made while having inconsistent plans. The constant re-optimization in the multi-day model is closest to the model of instantaneous gratification of Harris and Laibson (2013), including in using ordinary differential equation in continuous time to approximate the discrete-time optimization of naïve agents. This approach

---

2Herrnstein and Prelec (1991)’s model of melleration under distributed choices is somewhat related, but differs substantially in the sense that plans don’t matter in their model, while they are central to the results in my paper.

3Like Harris and Laibson (2013), I assume naïveté: a person is unaware of her bias, which allows for repeated changes in plans. It is beyond this paper to study how a person could be aware of her
may help integrate projection bias into specific economic settings, in particular to time (mis)management and personnel economics. For instance, Buehler, Griffin, and Ross (1994) find that students believe that they will finish their bachelor’s thesis earlier than they actually do – which they explain by students underestimating the number of hours necessary for the task. Projection bias provides a complementary explanation that people overestimate how much they will work. Regarding my results on multi-tasking, Coviello, Ichino, and Persico (2015) and Bray et al. (2016) find empirical evidence that it decreases productivity. Coviello, Ichino, and Persico (2014) show that workers may engage knowingly in intrinsically inefficient multi-tasking due to lobbying by co-workers and superiors. With projection bias, even if multi-tasking is not intrinsically inefficient, such lobbying will lead workers to multi-task inefficiently. Given the possibly large welfare losses under inconsistent behavior, my results highlight the potential to expand the study of projection bias beyond domains with large swings in taste (Levy (2009); Chaloupka, Levy, and White (2019)) and binding choices (Conlin, O’Donoghue, and Vogelsang (2007); Busse et al. (2015); Buchheim and Kolaska (2017); Michel and Stenzel (2020)).

2 A Model of Projection Bias in Effort Choices

2.1 The Formal Model

Environment Consider a baseline setup with a single period during which a person works for a single block of \( e \) hours. She earns a monetary benefit \( B(e) \) and incurs a disutility \( D(e) \) for this work, where \( D(\cdot) \) is continuously differentiable, with \( D(0) = 0 \), \( D'(e) \geq 0 \), and \( D''(0) \geq 0 \). The marginal disutility \( D'(e) \) is the instantaneous disutility of continuing to work at time \( e \) – at a time when the person has worked for a duration \( e \).

Perceived Disutility Projection bias as defined by Loewenstein, O’Donoghue, and Rabin (2003) leads a person to misperceive her future taste for work as more similar to her current taste for work than it will actually be. As people work, they grow more tired of working which is captured by increasing marginal disutility, and so they perceive future work as more onerous the more tired they currently are. Formally:

**Definition 1 (Projection Bias).** At time \( s \) – when a person has worked for a time \( s \) – the person mispredicts marginal disutility at a future time \( e \) to be

\[
\tilde{D}'(e|s) = (1 - \alpha)D'(e) + \alpha D'(s)
\]

where \( \alpha \in [0, 1] \) is the degree of projection bias. Moreover, she is naive with respect to her projection bias: she does not realize that her perception depends on \( s \) and thus believes that she always perceives the marginal disutility as she does currently.

With this definition the perceived disutility from total effort \( e \) is \( \tilde{D}(e|s) = (1 - \alpha) \cdot D(e) + \alpha \cdot D'(s) \cdot e \), since \( D(0) = 0 \). Note that, since the taste for money (for consuming goods changing plans without being aware of the underlying reason for the changes.

4
that money can buy) does not change substantially nor systematically while working, the benefits \( B(\cdot) \) are being perceived correctly.

**Behavior** I assume that the person stops working once she perceives working as suboptimal, which then determines the total amount of work done. Since the actual and perceived utility from stopping at time \( s \) is \( B(s) \), and the perceived utility of continuing until time \( e > s \) is \( B(e) - (\bar{D}(e|s) - \bar{D}(s|s)) \), we have the following behavior, which I describe as *momentary* work decisions:

**Definition 2** (Momentary Work Decision). A projection-biased person works until \( \bar{e}^* \) given by

\[
\bar{e}^* = \inf \{ s : B(e) - \bar{D}(e|s) < B(s) - \bar{D}(s|s), \forall e > s \}
\]

### 2.2 Discussion of the Model

Projection bias leads to projecting marginal disutility under two assumptions. First, at each instant the person makes a single binary decision of whether or not to work. If the person could choose the level of work intensity at each moment, then the instantaneous disutility would depend on the level of intensity chosen at time \( s \), so that \( D'(s) \) could no longer capture the instantaneous disutility. Second, since projection bias is not about projecting indirect decision utilities, but about projecting immediate hedonic utility, I assume that working at time \( s \) incurs the *instantaneous hedonic* disutility equal to \( D'(s) \).

Now consider a projection-biased person who decides *momentarily* – moment-by-moment – whether or not to work right now, but who cannot commit to how much she will work in total. If, as I assume, the person is naive with respect to their own projection bias and hence does not anticipate that she will deviate from her current plan,\(^4\) then she will only work as long as she perceives it optimal.\(^5\) This determines total effort that period under the assumption that there is no opportunity for resting within a period – which I exclude by assuming that all the work is being done in one block. If resting were possible, the person might decide to resume work after a break. Of course, both resting and intensity of work are important, since a projection-biased person may misoptimize both, but I study them in separate ongoing work.

Finally notice that when \( \alpha = 0 \) the person has no projection bias, and the actual work done \( \bar{e}^* \) equals the optimal work \( e^* \), if it is unique.\(^6\) Thus the setup nests the unbiased case.

---

\(^4\)The evidence from (Read and Van Leeuwen 1998) shows that, despite experience with fluctuations in hunger, adults still display projection bias over hunger. A nice experiment by Le Yaouanq and Schwardmann (2019) in the context of present bias shows however that participants do make less overoptimistic predictions about their own future behavior after making initial predictions and receiving feedback on it. This shows that we need richer models of learning about one’s biases.

\(^5\)In case the agent is indifferent between working and stopping, the model assumes that the agent works, since the person only stops if it is strictly suboptimal to continue working. In the settings that I study, this happens only for non-generic edge-cases.

\(^6\)If \( s < e^* \), then \( B(e^*) - \bar{D}(e^*|s) - (B(s) - \bar{D}(s|s)) = B(e^*) - D(e^*) - (B(s) - D(s)) > 0 \), since \( \bar{D}(e|s) = D(e) \) and since \( e^* \) maximizes \( B(e) - D(e) \). Hence \( e^* \geq s^* \). But similarly, for \( s = e^* \), then \( B(s) - D(s) = B(e^*) - D(e^*) > B(e) - D(e) \) for every \( e > e^* = s \), hence \( e^* \leq s = e^* \). Thus \( \bar{e}^* = e^* \).
Related Literature In terms of modeling projection bias, Loewenstein, O’Donoghue, and Rabin (2003) define and formalize projection bias as the general tendency for people to perceive their future tastes to be more similar to their current tastes than they are, when tastes in any given moment depend on some state $s$. They focus on habit formation, durable goods, and loss aversion. Gagnon-Bartsch and Bushong (2019) develop a model in which people mislearn about their preferences for the good as they learn about it from experience. People enjoy a good or activity more when it turns out better than expected, and less when it turns out worse than expected, but misattribute these feelings of elation or disappointment to the good itself. Mislearning under projection bias instead may lead people to neglect how much their state affected the enjoyment of a good, thus misattributing their current state partially to the consumed good. Haggag et al. (2019) develop a simple model of such state misattribution and find evidence for it in two consumer decisions.

My simplest setup where the person decides when to stop working on a single task is similar to stopping problems under time-inconsistent preferences, such as Quah and Strulovici (2013), Hsiaw (2013), and Huang and Nguyen-Huu (2018). In these models, the time inconsistency stems from present bias or dynamically inconsistent changes in patience, rather than from state changes that depend themselves on earlier decisions. In the multi-tasking and multi-period settings that I study, the agent no longer faces a single stopping decision, but several decisions that jointly determine the final outcome. This setup is thus more closely related to Harris and Laibson (2013) and Ahn, Iijima, and Sarver (2020), both of which allow for repeated decisions to combine over time to a final outcome such as final savings.

Having defined projection bias and the person’s decision problem, the next sections explore how the inconsistent plans of a projection-biased person due to changing tastes affect their work decisions.

3 Single-Task Choices

Let us start with single-day decisions where people maximize their utility in momentary work decisions over a single task as described in section 2: people who have worked for $s$ hours so far keep working if they perceive it optimal at time $s$. When the disutility $D(\cdot)$ is convex and the benefits $B(\cdot)$ are linear or concave, a projection-biased person works optimally – despite (in fact, because of) her changing plans. Nonetheless, such a person has overoptimistic beliefs about how much she will work. I then consider all-or-nothing tasks, where a person receives a known reward if she completes a minimum amount of work. I show that people start overly ambitious tasks, so that they either end up completing the task despite it not having been worthwhile, or they quit the task without receiving any benefit for their effort.
3.1 Optimal Behavior and Optimistic beliefs with Convex Disutility and Linear Benefits

Consider Anna, a projection-biased student with $\alpha = 0.5$, who has an exam tomorrow. The benefits of every additional hour of studying are equal to 3, and studying becomes more unpleasant the longer she studies. Specifically, Anna’s daily disutility is quadratic in total time studied, thus $D(e) = \frac{e^2}{2}$ and $D'(e) = e$. After having studied for $s$ hours, Anna plans to study until her currently perceived marginal disutility is equal to her marginal benefits (which are constant and equal to 3). I denote the time at which she plans to stop by $\tilde{e}^*(s)$, the total hours she plans to work after having worked for $s$ hours. She perceives her marginal disutility after studying for $e$ hours to lie between her current marginal disutility, $D'(s)$, and her actual marginal disutility after $e$ hours of studying, $D'(e)$:

$$\tilde{D}'(e|s) = (1 - \alpha) D'(e) + \alpha D'(s) = \frac{1}{2}(D'(e) + D'(s))$$

At the start of the day, Anna hasn’t studied at all and $s = 0$. So she thinks that her marginal disutility after $e$ hours of studying will be $\tilde{D}'(e|0) = \frac{1}{2}D'(e)$. She plans to work for $\tilde{e}^*(0)$ hours, with $\tilde{D}'(\tilde{e}^*(0)|0) = 3 \implies 1/2 \cdot (D'(\tilde{e}^*(0)) + 0) = \frac{1}{2} \tilde{e}^*(0) = 3 \implies \tilde{e}^*(0) = 6$. Anna plans to study for 6 hours and thus starts studying. After 2 hours of studying, the current marginal disutility is $D'(2) = 2$. Anna now plans to study for $\tilde{e}^*(2)$ hours in total, with $\tilde{D}'(\tilde{e}^*(2)|2) = 3$ – the first order condition as she perceives it now. This leads to $1/2 \cdot (D'(\tilde{e}^*(2)) + D'(2)) = 3 \implies \tilde{e}^*(2) = 4$ hours. Finally, once she has completed 3 hours of studying, the current marginal disutility is $D'(3) = 3$, so that $\tilde{e}^* = 3$ and Anna stops studying.

The same logic applies when the returns to effort are decreasing rather than constant, which leads to Proposition 1. All proofs can be found in the appendix.

**Proposition 1.** Let $D(.)$ be a strictly convex function with $D'(.) \to \infty$, let $\alpha \in [0,1)$, and $B(.)$ be both differentiable and linear or concave. Then a projection-biased person who makes momentary work decisions works optimally. Moreover, letting $e^*$ be the optimal amount of work and $\tilde{e}^*(s)$ be the perceived optimal amount of work after $s$ hours of work, then $\tilde{e}^*(s) > e^* \forall s < e^*$.

Proposition 1 relies on momentary work decisions. If Anna had to make an irreversible (or hard-to-reverse) choice, then she would choose to work too much. This is not likely in the case of studying, but may be the case if Anna is grading exams for a course or working on a common project with a friend. In such situations, due to being overoptimistic, Anna will overcommit to working too much.

Proposition 1 also highlights that Anna constantly overestimates how much she will work. Why? By assumption, the marginal disutility of effort increases, so that Anna – who projects her current marginal disutility – underestimates how high marginal disutility will be later that day when she stops, and therefore overestimates how long she will study.
This strongly limits the scope of the result: while behavior is optimal when the disutility is convex and benefits are linear or concave, the beliefs over future work are overoptimistic. As long as overoptimistic beliefs don’t affect other decisions, everything is well. However, as soon as some decisions rely on predictions of future effort, mistakes will be made. I highlight this major caveat in Proposition 3 in the next section. Now, I consider tasks with all-or-nothing benefits that are received only if the person completes a minimum number of hours.

3.2 All-or-Nothing Tasks

Definition 3 (All-or-Nothing Task). A single-period all-or-nothing task \((E_0, B_0)\) is a task that pays benefits \(B_0\) if the person works at least \(E_0\) hours by the end of the period, and pays 0 otherwise: \(B(e) = B_0 \cdot 1(e \geq E_0)\).

Each instant, the person chooses whether to start or continue the task. She does so if she currently thinks that completing the task is better than quitting the task. Suppose that Alice, a projection-biased high-school student with \(\alpha = 0.5\), has a deadline to finish a college application tonight, which will take her 6 hours. With quadratic disutility \(D(e) = e^2/2\), we have \(D(6) = 18\), and let us suppose that \(B = 12\), so that Alice should not do the application. Thus an unbiased person does not do start nor complete the task.

Does Alice start the application and, if so, does she finish it? She starts if the perceived disutility at \(s = 0\), \(\tilde{D}(6|0)\), is less than \(B\), where \(\tilde{D}(6|0) = (1 - \alpha)D(6) + \alpha D'(0) \cdot 6 = 9 + \alpha D'(0) \cdot 6\). Since \(D(e) = e^2/2\), then \(\tilde{D}(6|0) = 9 < 12 = B\) and Alice starts the application. Now imagine what happened if Alice worked for another two hours – which, as we will see, does not happen. Then the perceived disutility of completing the application would be \(\tilde{D}(6|2) - \tilde{D}(2|2) = 13 > 12 = B\), so she would have stopped working before reaching two hours of work. The reason is that Alice perceives the final 4 hours of work as so much more unpleasant after 2 hours of working than at the start of the day that she perceives the task no longer worth completing – despite having less work left to do. Proposition 2 states formally when this happens.

Proposition 2. [All-or-Nothing] Let \(D(.)\) be a strictly convex function with \(D'(\cdot) \to \infty\), let \(\alpha \in [0, 1)\). Let \(\tilde{e}^*(E_0, B_0)\) be the actual effort exerted by a person making momentary work decisions for a single-period all-or-nothing task. Then there exists a unique \(E_H \geq 0\) such that the following statements hold:

1. \(\forall E_0\), if \(B \in (\tilde{D}(E_0|0), D(E))\) then \(\tilde{e}^*(E_0, B_0) > 0\), yet \(B_0 - D(E_0) < 0\)
2. \(\forall E_0 < E_H\) if \(\tilde{e}^*(E_0, B_0) > 0\) then \(\tilde{e}^*(E_0, B_0) = E_0\).
3. \(\forall E_0 > E_H, \exists B_0(E)\ s.t. 0 < \tilde{e}^*(E_0, B_0) < E_0\).

The proposition states that, first, the person starts tasks that are not worth doing; that if the task requires sufficiently low effort, every task that is started is completed; and if the task requires sufficiently high effort, there is a task-specific payment such that the person starts the task yet fails to finish it, thus wasting effort for no benefit. Note that the first two
points imply that all worthwhile tasks are started and finished. Moreover, when \( D'(0) = 0 \), then \( E_H = 0 \), so that starting and stopping can happen for all tasks. Proposition 2 applies more generally to tasks with sufficiently convex benefits, not just all-or-nothing tasks. Thus it also applies to situations with few discrete outcomes, such as promotions or grades on exams, where the probability of success is S-shaped and hence convex for each outcome.

The result that people start but don’t finish a project that they start superficially resembles the result by O’Donoghue and Rabin (2008). There, however, people procrastinate on a project with no deadline, and therefore never finish it, expecting to do so eventually, whereas here they start and stop because of the close deadline. Thus both the predictions on planned behavior and the welfare implications of the results differ: while commitment would benefit a naive procrastinator, it would hurt a projector.

4 Multi-Tasking with Concave Benefits

Let us revisit the situation with convex disutility and decreasing returns to effort, but with a twist: the person now divides her time between two tasks, each of which has decreasing returns to effort.

4.1 Multi-Tasking Model

Environment A projection-biased person works on \( T \) consecutive tasks in a single period: she works on task 1 for a duration \( e_1 \), then switches to task 2 for a duration \( e_2 \) and so on until working on task \( T \) for a duration \( e_T \). The disutility depends on total effort, so that we can write it as \( D(\sum_{t=1}^{T} e_t) \), while the monetary benefits are task-specific, so that we can write them as \( \sum_{t=1}^{T} B_t(e_t) \).

Behavior Consider a person who is working on the \( i \)th task and let \( E_{i-1} \) be the total amount of effort exerted on the first \( i - 1 \) tasks, which can no longer be changed. Let \( V_i(e_i, E_{i-1}|s) \) denote the perceived value from the remaining tasks \( i \) through \( T \) when planning to put total effort \( e_i \) on task \( i \), perceived when the person’s current tiredness is \( s \) – when the person has exerted total effort \( s \) during this period. Then this is given as follows:

**Definition 4 (Perceived Continuation Value).** Let \( E_{i-1} \) be the total effort exerted on past tasks 1 through \( i - 1 \). Then

\[
V_i(e_i, E_{i-1}|s) = \max_{(e_j)_{j>i}} \sum_{t=i}^{T} B(e_t) - (D(E_{i-1} + \sum_{t=i}^{T} e_t) - D(E_{i-1}))
\]

The person stops working on task \( i \) and switches to task \( i+1 \) once she perceives it as strictly suboptimal to continue working on task \( i \): when \( V_i(e_i, E_{i-1}|s) < V_i(s - E_{i-1}, E_{i-1}|s) \) for all \( e_i > s - E_{i-1} \), since if she stops right away, she will have spent a time \( s - E_{i-1} \) on task \( i \). This leads to the following generalization of Definition 2:
**Definition 5** (Multiple Effort Decisions). A projection-biased person who works on $T$ consecutive tasks in a single period exerts effort $\tilde{e}^*_i$ on task $i$ given by

$$\tilde{e}^*_i = \inf \{ s : \tilde{V}_i(e_i, E_{i-1} | s) < \tilde{V}_i(s - E_{i-1}, E_{i-1} | s), \forall e_i > s - E_{i-1} \}$$

where $E_{i-1} = \sum_{t=1}^{i-1} \tilde{e}^*_t$.

**4.2 Multi-Tasking Results**

To illustrate, suppose that Elaine has two problem sets due the same day, one in economics due at 3pm and one in mathematics due at 8pm. Given these deadlines, she starts working on the economics problem set first. For simplicity, assume that the benefits for each problem set are the same and given by $B(\cdot)$, which has decreasing marginal returns. After working on the first problem set for $s$ hours, she plans to spend $\tilde{e}^*(s)$ hours on each assignment. She thus stops working on the economics assignment when she thinks that she has done half the work. Let's say that this happens after 5 hours, at which point she expects to do another 5 hours on the mathematics assignment. She is of course wrong, and overestimates how long she will keep working. Thus she may stop working after only 3 hours on the mathematics assignment. We know from Proposition 1 that this choice is optimal conditional on her having spent 5 hours on the economics assignment – so the mistake she makes is to spend too much time on the economics assignment, because she overestimates at that time how much she will work on the mathematics assignment.

**Proposition 3.** There are two tasks with strictly concave and continuously differentiable benefits $B_1(\cdot)$ and $B_2(\cdot)$ that have to be completed one after the other in that order. Let $\tilde{e}^*_1$ and $\tilde{e}^*_2$ be the actual effort spent on the two tasks, and $e^*_1$ and $e^*_2$ be the optimal effort levels. Then $B'(\tilde{e}^*_2) > B'(e^*_2) = B'(\tilde{e}^*_1) > B'(e^*_1)$ and $\tilde{e}^*_2 + \tilde{e}^*_1 > e^*_2 + e^*_1$.

The Proposition states that the person works too much in total, working too much on the earlier and too little on the later task.

The same mistake occurs when the person works on a single task consisting of two or more subtasks, as long as each subtask is best done in one block. If the subtasks have a natural sequence, so that one subtask makes the subsequent subtask easier, then Elaine will work too much on the earlier stages than on the later stages. For instance, suppose that Elaine plans to read both the lecture notes and to finish a problem set for the same class today. If she believes that the problem set will be easier after reading the lecture notes, then she reads the lecture notes first and consequently spends too much time on them.

While I assume that the order of the tasks is fixed exogenously in Proposition 3, what is necessary for the result to hold is that each task is done in one go, for which switching cost are a sufficient condition.
5 Multi-Day All-or-Nothing Tasks

In single-day all-or-nothing tasks, a person with increasing marginal disutility always underestimates the costs of finishing the task. In multi-day all-or-nothing tasks, she underestimates these costs at the beginning of each day, but if she works long enough, she overestimates them once her current marginal disutility is higher than the average marginal disutility from completing the task. In this section, I develop a continuous-time model to study how these fluctuations affect effort choices and welfare.\(^7\)

5.1 Model for Multi-Day All-or-Nothing Tasks

Consider a continuous-time setup where every time \(x \in [0,1)\) represents a different period. In each (continuous-time) period \(x\), the person stops working once she perceives it as optimal doing so, which determines the flow effort \(e_x\) at time \(x\). She incurs a flow disutility of effort equal to \(D(e_x)\). The person receives a benefit of \(B\) at time \(x = 1\) if she has completed total effort equal to or exceeding \(E\) and she receives no benefit otherwise.

So when does the projection-biased person stop working at time \(x\)? Let us fix a period \(x\) from the continuum of periods. Then by this definition, the person has a fraction \(1 - x\) of periods left to complete the task, and I denote the amount of work left to do as \(E_x\), i.e. \(E_x = E - \int_0^x e_t dt\). If the projection-biased person plans on completing the task, then she plans completing it efficiently, working the same amount each day, which will incur a cost of \(D\left(\frac{E_x}{1 - x}\right)(1 - x)\) in total. After having worked for \(s\) hours on period \(x\), she perceives the cost of completing the task efficiently as \(G(x, s, E) := \tilde{D}\left(\frac{E_x}{1 - x}\right)\left(1 - x\right)\). If at the start of the period when \(s = 0\) the perceived costs exceed the benefits, then the person doesn’t work at all, so that \(e_x = 0\). If instead even after having worked for \(s = \frac{E_x}{1 - x}\) hours the person still considers it worth doing, then she stops working that period when \(e_x = \frac{E_x}{1 - x}\). Otherwise she stops once the \(G(x, s, E) = B\), since this implies that \(G(x, s + \varepsilon, E) > B\), so she would never work more than \(s\) hours. This shows that the following definition is in line with Definition 2:

**Definition 6** (Continuous-Time All-or-Nothing Task). Consider a projection-biased person working on a multi-period all-or-nothing task \((E, B)\) that requires effort \(E\) to complete and pays monetary benefits \(B\) if the task is completed by time \(x = 1\) and pays 0 otherwise. Let \(E_x\) for \(x \in [0,1)\) be the effort that remains to be done at time \(x\) in order to complete the task. Then \(E_x\) is given by the ODE

\[
\dot{E}_x = -e_x
\]

\[
e_x = \begin{cases} 
0, & \text{if } G(x, 0, E_x) > B \\
\frac{E_x}{1 - x}, & \text{if } G(x, \frac{E_x}{1 - x}, E_x) < B \\
e_x^*, & \text{otherwise, with } G(x, e_x^*, E_x) = B 
\end{cases}
\]

\(^7\)In a working paper – see Kaufmann (2020) – I prove that this continuous-time solution is the limit of the discrete-time solution as \(T \to \infty\), which I take as a given here.
with initial condition \( E_0 = E \). A task is completed if \( E_1 = 0 \).

Notice that because we normalize the time until the deadline to 1, the total benefit \( B \) and effort \( E \) approximate the average “daily” benefit and effort in the discrete-time setting. That is, the limit as \( T \to \infty \) holds the average daily benefits and effort levels constant, not the totals.

### 5.2 Results for Multi-Day All-or-Nothing Tasks

I first give a discrete-time example, the intuition of which carries over to the continuous-time case. Consider Beth, a student who is working on an all-or-nothing task with a deadline in \( T \) days. She has an economics exam in 100 days and knows that she will receive a B in her final if she does nothing but attend the required lectures. Getting an A on the final is worth 1250 more than receiving a grade B. If she studies 5 hours a day on average, Beth is sure to receive an A, if she studies less, she is sure to receive a B.

Suppose that the daily disutility is quadratic: \( D(e) = \frac{e^2}{2} \) so that \( D'(e) = e \). First, note that Beth at every moment either plans to complete the task efficiently, or to not do the task at all. After all, at any given moment she plans to do what an unbiased person would do whose actual disutility was given by \( \tilde{D}(\cdot|s) \). On the first day, Beth therefore studies as long as she perceives it worthwhile to study 5 hours every day. The disutility of studying 5 hours per day is \( 100 \cdot D(5) = 1250 \), so an unbiased student would be indifferent between studying and not studying. But Beth is projection-biased, with \( \alpha = 0.5 \). At the start of the first day she underestimates the disutility of the task and starts studying. After 2.5 hours of studying, her marginal disutility is \( D'(2.5) \), and she perceives the disutility of working 5 hours on every future day correctly: \( \tilde{D}(5|2.5) = (1 - \alpha)D(5) + \alpha D'(2.5) \cdot 5 = D(5) \). She therefore perceives the remaining disutility of studying 5 hours every day almost correctly and soon stops working.\(^8\) When she stops, she believes, mistakenly, that she won’t resume it the next day. Yet, come the next day, she is rested and starts studying again, planning to get an A, only to stop once more when she grows sufficiently tired.

Every day, Beth thus either doesn’t study at all, studies inefficiently given how much work still remains to be done, or studies efficiently. It is not difficult to see that if Beth doesn’t study at all on day \( t \), than she won’t study on day \( t + 1 \) or any later day either, and therefore not get an A. Similarly, if she studies efficiently on day \( t \), then she will study efficiently on all future days and thus get an A. For instance, if after 50 days, Beth had only completed 50 hours of studying, she would have to study 9 hours per day on the remaining days, and she wouldn’t start studying any longer. Alternatively, if after 75 days Beth had completed 300 hours of studying, she would have to work 8 hours a day for the remaining 25 days to receive the full benefits worth 1250. She would work 8 hours a day, since she would perceive this as worthwhile even after 8 hours of work: \( \tilde{D}(8|8) = 16 + 32 = 48 < 50 = \frac{1250}{25} \).

Proposition 4 formally states that for any average daily effort required, each of these two outcomes – wasting effort on a task that won’t be completed and working inefficiently for

\(^8\)She still slightly underestimates it because she underestimates the disutility of the 2.5 hours of work she has to complete on the first day.
a while on a task that is completed – happens for some average daily benefit. To state the result, I now define the time \( \tau_0(E, B) \), which is (roughly) the fraction of days on which the person doesn’t work at all; and \( \tau_F(E, B) \) which is (roughly) the fraction of days on which the person works efficiently on the task. Formally:

**Definition 7.** Let \( \tau_0(E_0, B_0) := \inf\{\tau \in [0, 1] : G(x, 0, E_x(E_0, B_0)) < B_0 \forall x < 1 - \tau\} \). Thus for any \( x < 1 - \tau_0 \), we have \( e_x > 0 \).

Let \( \tau_F(E_0, B_0) := \inf\{\tau \in [0, 1] : G(x, \frac{E_x(E_0, B_0)}{1-x}, E_x(E_0, B_0)) > B_0 \forall x < 1 - \tau\} \). Thus for any \( x < 1 - \tau_F \), the person does not work fully on the task (either not at all or only partially).

We can now formally state the proposition.

**Proposition 4.** The disutility of effort is strictly convex, with \( D''(\cdot) > d \) for some \( d > 0 \) and \( \lim_{e \to \infty} D'(e) = \infty \). Consider a multi-period all-or-nothing task \((E_0, B_0)\). Then there exist \( B_H(E_0) > B_C(E_0) > B_L(E_0) > 0 \) such that:

- if \( B_0 > B_H \), then the task is completed efficiently, i.e. \( \tau_F = 1 \).
- if \( B_H > B_0 > B_C \), then \( \tau_F(E_0, B_0) \in (0, 1) \) and the task is completed.
- if \( B_C > B_0 > B_L \), then \( \tau_0(E_0, B_0) \in (0, 1) \) and the task is not completed.
- if \( B_L > B_0 \), then no effort is spent on the task, i.e. \( \tau_0 = 1 \).

with \( \tau_0 \) continuous and decreasing in \( B_0 \), and \( \tau_F \) continuous and increasing in \( B_0 \).

Moreover \( \lim_{B_0 \to B^-} u_0(E_0, B_0) \leq -D(E_0) \).

The proposition states that for any an all-or-nothing task \((E_0, B_0)\) there exist thresholds \( B_H > B_C > B_L > 0 \) depending on \( E_0 \) such that:

1. Beth never starts the task if \( B_0 < B_L \);
2. Beth spends some days working on the task but eventually gives up if \( B_0 \in (B_L, B_C) \);
3. Beth spends early days working inefficiently on the task but eventually finishes the remainder efficiently if \( B_0 \in (B_C, B_H) \);
4. Beth works on the task efficiently from the start and finishes it if \( B_0 > B_H \).

Moreover, as \( B_0 \in (B_L, B_C) \) increases, Beth works a larger fraction of days on the task and wastes more time on a task she doesn’t finish. As \( B_0 \) approaches \( B_C \) from below, Beth ends up doing almost all the work, yet fails to complete the task, hence she incurs almost the full cost of completing the task. When \( B_0 \) increases on \((B_C, B_H)\), Beth works more earlier, which makes her better off, as she completes the task more efficiently.

Notice that the large cost from repeatedly working and stopping can occur for arbitrarily small biases, although the range of payments for which inefficient work happens becomes
smaller the less biased a person is. Thus the likelihood of mistakes decreases, but the range of possible costs does not.

6 Productivity and Effort Allocation

In previous sections, I assumed that the benefits are not projected, since the value of money does not fluctuate much with tiredness. In this section, I relax this assumption indirectly by considering situations where a total output has to be produced, but the productivity (output per hour of work) is either increasing or decreasing across periods. In such a situation, the benefit of working consists in having less work in the future.

6.1 Productivity and Time Discounting

Let us consider a setting where people have different productivities on different days, and let us start with a warm-up example. Doris has to complete an assignment by tomorrow night that requires \( E \) hours. Her productivity \( p \) on day 1 is twice as high as on day 2, because a friend has offered to give feedback at the end of day 1. Thus every hour of work exerted on day 1 leads to \( p = 2 \) hours worth of output, so she has to choose \( e_1 \) and \( e_2 \) s.t.

\[
2 \cdot e_1 + e_2 = E.
\]

On the first day after having worked for \( s \) hours, she plans to stop after completing \( e_1(s) \) hours today, given by

\[
\frac{D'(e_1(s)|s)}{p} = \frac{D'(e_2(s)|s)}{p} \iff \frac{D'(e_1(s)|s)}{p} - \frac{D'(e_2(s)|s)}{p} = 0
\]

\[
\iff (1 - \alpha) \frac{1}{p} \cdot D'(e_1(s)) + \alpha \frac{1}{p} \cdot D'(s) - (1 - \alpha)D'(e_2(s)) + \alpha D'(s) = 0
\]

\[
\iff \frac{1}{p} \cdot D'(e_1(s)) - D'(e_2(s)) = -\frac{\alpha}{1 - \alpha} \frac{1}{p} \cdot D'(s)(1 - \alpha)
\]

which shows that \( e_1(s) > e_1^* \), since \( \frac{1}{p} = \frac{1}{2} < 1 \) and \( D'(s) > 0 \): the LHS has to increase as \( \alpha \) increases, which requires that either \( e_1(s) \) increases or \( e_2(s) \) decreases. But if \( e_2(s) \) decreases, then \( e_1(s) \) increases and similarly if \( e_1(s) \) increases then \( e_2(s) \) decreases given the constraint on total effort. She stops working when her current perceived plan is equal to (or less than) what she has done, that is when \( e_1(s) = s \). Substituting this into the above equations we get:

\[
D'(\tilde{e}_1) \left( \frac{1}{p} + \frac{\alpha}{1 - \alpha} \left( \frac{1}{p} - 1 \right) \right) = D'(\tilde{e}_2) \iff \frac{D'(\tilde{e}_1)}{p} \frac{1 - p\alpha}{1 - \alpha} = D'(\tilde{e}_2)
\]

and so she acts as if her productivity was \( \frac{1 - \alpha}{1 - p\alpha} > p \), so that she works too much on day 1.
This result is a special case of the proposition 5.

**Proposition 5.** Let $D(.)$ be a strictly convex function with $D'(\cdot) \to \infty$ and $D'(0)$, and let $\alpha \in [0,1)$. Consider a person who works momentary each of $T$ periods under the constraint $E = \sum_{t=1}^{T} p_t \cdot e_t$, where $p_t$ is her (known, exogenously given) productivity on day $t$. Denote by $E^*_t$ the total amount of work done by the beginning of day $t$ and by $E^*_t$ the optimal total amount of work done at the beginning of day $t$.

- If $p_t$ is strictly increasing, then $E^*_t \leq E^*_t \forall t > 1$.
- If $p_t$ is strictly decreasing, then $E^*_t \geq E^*_t \forall t > 1$.

Let $\tilde{e}^*_t \forall t$ denote the amount of work the person plans, at the end of day $t$, to exert on day $t + 1$. Then $\tilde{e}^*_t \leq \tilde{e}^*_t$, that is she works less on day $t + 1$ than predicted at the end of day $t$. When productivity is increasing, this change of plan moves her further away from optimal effort that day; when productivity is decreasing, this change of plan moves her closer to optimal effort that day.

Here is an example of proposition 5 in action. Betsy has to complete an assignment that would take her $E = 18$ hours of work if each day she was as productive as she is today. Fortunately for her, she has lectures tomorrow and office hours the day after that: during lectures and office hours she will learn shortcuts for completing the problems on the assignment. Specifically, every hour of work done tomorrow is worth 1.5 hours of work today, while every hour of work done in two days is worth 1.5 hours of work tomorrow.

Obviously, she should work less today than tomorrow, since she will become more efficient at solving questions. Suppose that her disutility is quadratic. The optimal effort levels should satisfy the first order conditions $D'(e^*_1) = \frac{1}{p} D'(e^*_2) = \frac{1}{p} D'(e^*_3)$, which leads to $e^*_1 \approx 2.2$, $e^*_2 \approx 3.33$, and $e^*_3 \approx 4.95$. On day 1, Betsy instead solves her perceived first order conditions, which we can derive as in the 2-day case to be $D'(\tilde{e}_1) = \frac{1-\alpha}{1-\alpha p} D'(\tilde{e}_2) = \frac{1-\alpha}{1-\alpha p} D'(\tilde{e}_3)$, where $\tilde{e}_i$ indicates that it is the effort Betsy perceives to be optimal at the end of day $i$. These are given by $\tilde{e}_1 \approx 1.52$, $\tilde{e}_2 \approx 3.03$, and $\tilde{e}_3 \approx 5.29$.

Yet, on day 2 she will not do what she thought she would do. She solves her new perceived first order condition, which is now exactly as in the 2-day case, taking into account that she worked roughly 1.52 hours on day 1: $D'(\tilde{e}_2) = \frac{1-\alpha}{1-\alpha p} D'(\tilde{e}_3)$. Solving this, we find that $\tilde{e}_2 \approx 2.75$ and that $\tilde{e}_3 \approx 5.50$. Betsy was already planning to work less than she should, planning to do 3.03 instead of 3.33, yet she ends up doing even less, namely 2.75. Thus, Betsy postpones too much work, and at the end of day 1 she thinks that she will have done more by the end of day 2 than will be the case. The reason is that the more tired she is, the more Betsy wants to do effort on a productive day – which leads to postponing more work in this setting. Betsy correctly understands that doing 1 hour less of work requires her to do 40 minutes more work tomorrow. Thus she saves 20 minutes, which she perceives as more unpleasant the more unpleasant effort is right now. Therefore she is willing to delay more work until tomorrow to take advantage of her higher productivity. Since tomorrow she will work more, she will be more tired at the end of the day when she decides to stop,
and therefore she will want to delay more at the end of day 2 than at the end of day 1 and stops working earlier than anticipated.

Since a person with constant productivity and exponential discounting with discount factor $\delta$ solves an identical problem to a person whose whose productivity increases by $\frac{1}{\delta}$ each period, the next corollary follows immediately.

**Corollary 1.** Let $D(.)$ be a strictly convex function with $D'(\cdot) \to \infty$ and $D'(0)$, and let $\alpha \in [0, 1)$. Consider a person who works momentary each of $T$ periods under the constraint $E = \sum_{t=1}^{T} e_t$ and discounts disutility exponentially with discount factor $\delta$. Then $\tilde{E}^*_t \leq E^*_t$ $\forall t > 1$, and $\tilde{e}^*_{t+1|t} \geq e^*_{t+1}$.

Loewenstein, O’Donoghue, and Rabin (2003) highlighted the potential for projection bias to cause time inconsistent plans under habit formation, that is in a case where the utility from consumption decreases the more one has recently consumed. What my results highlight though is that projection bias can cause time-inconsistent behavior much more generally, in fact whenever there are incentives towards unequal effort over time. Moreover, it shows that the departure from earlier plans can improve choices when effort is decreasing.

It is important to note however that projection-bias leads to time-inconsistency only if the states in which choices are made are different, and that they are not driven by time-inconsistent preferences but by a failure to predict how different future preferences will be from current preferences. This differs from temptation models such as Gul and Pesendorfer (2001) or models of present bias (Laibson (1997); O’Donoghue and Rabin (1999)), both of which assume that the actual preferences over future outcomes depend on the immediacy of the choices. Therefore projection bias does not preferentially lead to present focus (Ericson and Laibson (2019)), although it leads to overestimating how much one will work, if the final decision is made when people are the most tired. Thus committed high-effort choices may be the mistake, rather than the failure to implement them. Nonetheless, through magnifying pre-existing present bias or present focus, projection bias is more likely to magnify such behaviors. Since this magnification is larger the more tired people are at the time of making committed choices, this may bias estimates of time preferences when comparing across populations or across times, with more tired populations appearing as more impatient – although careful laboratory designs can avoid such concerns by keeping (expected) tiredness constant across choice elicitation (see Fedyk (2018), Augenblick and Rabin (2019), Le Yaouanc and Schwardmann (2019)).

### 7 Discussion and Conclusion

Throughout this paper, I highlighted how projection bias causes will turn changing tiredness into changing plans, which leads to inefficient task management. I made two major assumptions on the instantaneous disutility. First I assumed that a person either works or doesn’t work, ruling out intensity of effort. Second I assumed that the instantaneous disutility depends only on total time a person has worked so far, ruling out breaks and rest during a day. The major assumptions that I made about projection bias were that people make momentary decisions and that they are naive so that they never realize that they do
not execute their earlier plans.

Adding intensity and rest to the model will help to integrate projection bias better into applied settings, which is part of ongoing work. Conceptually, more limiting is the assumption that people learn *nothing* from their repeated fluctuations. Without such naiveté, plans will not fluctuate as often nor as much. While I believe that naiveté is more appropriate than is often assumed, it seems clear that people sometimes do display a sense of metasophistication: they realize that they repeatedly fall short of their own expectations, that they behave inconsistently, yet without a clearly articulated cause for this behavior. The more challenging question, and one that is relevant for all types of misperceptions, is thus what people learn or do when we neither assume that people *must* learn nor that they *cannot* learn.
Proof of proposition 1.

Proof. Remember that the biased person stops at $\tilde{e}^* = \inf\{s : B(e) - \tilde{D}(e|s) < B(s) - \tilde{D}(s|s), \forall e > s\}$. It is therefore enough to show that when $s < e^*$, the projection-biased person perceives it as better to continue working a little more, and that if $s > e^*$, she perceives it as strictly optimal to stop right away, since then $\tilde{e}^* = e^*$.

Notice then that for $s < e^*$, we have $\tilde{D}'(s|s) = D'(s) < D'(e^*) = B'(e^*) \leq B'(s)$, and moreover, even the projection-biased person realizes that the marginal disutility will only increase and the marginal benefit decrease. Hence it is strictly better to stop right now rather than work more, so that $\tilde{e}^* \leq e^*$. Together, these imply that $\tilde{e}^* = e^*$.

Proof of proposition 3:

Proof. The person – by assumption – first works on the first task, and then on the second task. After having worked on the first task for a time $s$, she plans on working $\tilde{e}_1^*(s)$ on task 1 and $\tilde{e}_2^*(s)$ on task 2 given by the following first order conditions:

$$\tilde{D}'(\tilde{e}_1^*(s) + \tilde{e}_2^*(s)|s) = B'_1(\tilde{e}_1^*(s)) = B'_2(\tilde{e}_2^*)$$

Then she switches from working on task 1 to working on task 2 at time $\tilde{e}_1^*$, at which time she has to perceive her current marginal benefit from task 1 to be equal to her perceived future marginal disutility at the end of the period:\footnote{The argument is similar to that in proposition 1: for any $s$ before the time where the perceived first order condition holds, the person considers working strictly more as strictly better. For any $s$ after that time, they perceive it as strictly worse. Hence they stop when the first order condition, as perceived in that moment, holds.}

$$\tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*|\tilde{e}_1^*) = B'_1(\tilde{e}_1^*) = B'_2(\tilde{e}_2^*)$$

where $\tilde{e}_{2|1}$ is the amount she plans to work on task 2 at the time when she switches. Note that when $B'_1(\tilde{e}_1^*) = B'_2(\tilde{e}_{2|1}^*)$ then the following are equivalent: $\tilde{e}_1^* > e_1^*$; $\tilde{e}_{2|1}^* > e_2^*$; and $\tilde{e}_1^* + \tilde{e}_{2|1}^* > e_1^* + e_2^*$, since the $B'_i$ are strictly concave and disutilities are strictly convex. That is, if she plans to work more on the first task than is optimal, then she also plans to work more on the second task than is optimal and vice versa; and hence both of these imply, and are implied by, her planning to work more in total than is optimal.
We can now show that $\tilde{e}_1^* > e_1^*$. Suppose not. Then she plans to work less in total than is optimal and we get:

$$
\tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*(s)|s) = B'_2(\tilde{e}_2^*(s))
$$

and as before, she stops once this holds her current $s$ equal to final total effort, $s = \tilde{e}_1^* + \tilde{e}_2^*$:

$$
\tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*(s)|s) = B'_2(\tilde{e}_2^*(s))
\iff D'(\tilde{e}_1^* + \tilde{e}_2^*) = B'(\tilde{e}_2^*)
\iff D'(\tilde{e}_1^* + \tilde{e}_2^*) = 0
\iff D'(\tilde{e}_1^* + x + \tilde{e}_2^*) = 0 \text{ for } x = \tilde{e}_1^* - e_1^* > 0
$$

Since $B'(\tilde{e}_2^*) = D'(\tilde{e}_1^* + e_2^* + x) > D'(\tilde{e}_1^* + e_2^*) = B'(e_2)$, we have that $\tilde{e}_2^* < e_2^*$. Therefore, $D'(\tilde{e}_1^* + \tilde{e}_2^*) = B'(\tilde{e}_2^*) > B'(e_2) = D'(e_1^* + e_2^*)$, so that $\tilde{e}_1^* + \tilde{e}_2^* > e_1^* + e_2^*$ and we are done. 

Here is the proof of proposition 2.

**Proof.** Let $\tilde{R}(E|s) := \tilde{D}(E|s) - \tilde{D}(s|s)$, the perceived remaining disutility of completing the task after $s$ hours of work have already been completed. Before proving the main result, we have to deal with a technicality: the perceived optimal effort $\tilde{e}^*(s)$ at time $s$ is not necessarily unique as assumed in Section 2, since the person is indifferent between completing the task and stopping right away when $\tilde{R}(E|s) = B$. Thus in principle, the final outcome may be ambiguous, depending which way the indifference is broken. However, generically indifference will not matter. Let $s_0$ be the minimum $s$ such that $\tilde{R}(E|s_0) = B$. Such a minimum exists, since $\tilde{R}(E|\cdot)$ is continuous and hence the infimum is a minimum. But then, since $\tilde{R}(E|s)$ is increasing towards $B$, this means that generically the derivative of $\tilde{R}(E|\cdot)$ with respect to $s$ is strictly positive at $s_0$, so that $\tilde{R}(E|s_0 + \varepsilon) > B$, and hence the person never works past $s_0$. Hence, the indifference at $s_0$ does not affect outcomes and the person stops at $s_0$. In what follows, I therefore assume that the person works as long as $\tilde{R}(E|s) < B$ and never works when $\tilde{R}(E|s) > B$, ignoring what happens at the non-generic points where $\tilde{R}(E|s) = B$.

The first part of the proposition claims that for all $E > 0$, if $B \in (\tilde{D}(E|0), D(E))$, then the person starts working on the task even though the task is not worth doing. Notice that
$\hat{D}(E|0) = (1 - \alpha)D(E) + \alpha D'(0) \cdot E < D(E)$, since $D(E) = \int_0^E D'(s)ds > \int_0^E D'(0)ds = D'(0) \cdot E$, as $D(\cdot)$ is strictly convex and $E > 0$. Thus for $B \in (\hat{D}(E|0), D(E))$, since $\hat{R}(E|0) = \hat{D}(E|0)$ and since $\hat{R}(E|s)$ is continuous in $s$, we have that $\hat{R}(E|\epsilon) < B$ for some sufficiently small $\epsilon > 0$, so that the person will work at least for a time $\epsilon$. This proves the first part of the proposition.

Let $\hat{R}_{\text{max}} := \max_s \hat{R}(E|s)$ the worst perceived remaining disutility for the biased person. Of course, for an unbiased person, the worst remaining disutility is always at the start when the most work remains to be done, but this won’t necessarily hold for projection-biased people. The more she works, the more tired she gets, which makes her perceive the remaining work as worse than she perceived it earlier. Notice that if $\hat{R}_{\text{max}}(E) < B$, then the remaining task is always perceived worth doing and therefore is completed. If $\hat{R}_{\text{max}}(E) > B$, then the task is definitely not completed, since at some point the person perceives it not worth doing. Finally, if $\hat{R}(E|0) < B$ and $\hat{R}_{\text{max}}(E) > B$, then the person starts the task, but does not complete it.

Let $\mathcal{E} := \{E \geq 0 : \hat{R}_{\text{max}}(E) > \hat{R}(E|0)\}$, the set of tasks for which the worst perception of the task happens after exerting some effort. I will show that $\mathcal{E} = (E_H, \infty)$ for some finite $E_H > 0$, which proves that if $E > E_H$, then we can pick $B$ in the non-empty interval $(\hat{R}(E|0), \hat{R}_{\text{max}}(E))$ and the person starts the task but fails to complete it. Moreover, I will show that if $E < E_H$, then $\hat{R}(E|0) > \hat{R}(E|s) \forall s \in (0, E]$, which means that, for such tasks, if the person starts the task, she also completes it. These results then establish both the second and third part of the proposition.

First, let us show that $\mathcal{E}$ is not the empty set. Pick some $s > 0$ such that $D'(s) > 0$. Notice that $\hat{R}(E|s) - \hat{R}(E|0) = \hat{D}(E|s) - \hat{D}(E|0) = (1 - \alpha)(D(s) - D(0)) + \alpha E(D'(s) - D'(0)) + \alpha s D'(s) = -D(s)(1 - \alpha) + \alpha E(D'(s) - D'(0)).$

Since $D'(s) - D'(0) > 0$, this expression becomes positive for sufficiently large $E$, say for $E > \bar{E}$, so that $\hat{R}(E|s) - \hat{R}(E|0) > 0$ for all $E > \bar{E}$. Thus $\mathcal{E}$ is not empty.

Further, from $\hat{R}(E|s) - \hat{R}(E|0) = -D(s)(1 - \alpha) + \alpha s D'(s) + \alpha E(D'(s) - D'(0))$, we immediately see that this expression is strictly increasing in $E$. Hence if $\hat{R}_{\text{max}}(E) - \hat{R}(E|0) > 0$ and $E' > E$, then there is some $s$ such that $\hat{R}(E'|s) - \hat{R}(E'|0) > 0$ by definition of $\hat{R}_{\text{max}}(E)$. Thus $\hat{R}(E'|s) - \hat{R}(E'|0) > 0$ and thus $\hat{R}_{\text{max}}(E') - \hat{R}(E'|0) > 0$. Therefore if $E \in \mathcal{E}$, then $E' \in \mathcal{E}$. Let $E_H = \inf \mathcal{E}$. Then if $E > E_H$, by definition of $E_H$, there is some $E' \in (E_H, E)$ s.t. $E' \in \mathcal{E}$. Therefore all $E > E_H$ are in $\mathcal{E}$.

Moreover, $E_H \notin \mathcal{E}$, since either $E_H = 0$ (in which case it is obvious) or $E_H > 0$. If $E_H > 0$ and $E_H \in \mathcal{E}$, then $\hat{R}(E_H|s) > \hat{R}(E_H|0)$ for some $s > 0$, and thus $\hat{R}(E_H - \epsilon|s) > \hat{R}(E_H - \epsilon|0)$ for sufficiently small $\epsilon$, which implies that $E_H - \epsilon \in \mathcal{E}$. This contradicts the definition of $E_H$ as $\inf \mathcal{E}$.

Finally, note that when $E < E_H$, we must have that $0 > \hat{R}(E|s) - \hat{R}(E|0) \forall s > 0$. If not, then $\hat{R}(E|s) - \hat{R}(E|0) = 0$ for some $s$ and we know that the LHS strictly increases in $E$, which would imply that $E_H \in \mathcal{E}$. And thus we are done. \qed
A.2 Proofs for Section 4

Proof of proposition 3:

\textit{Proof.} The person – by assumption – first works on the first task, and then on the second task. Consider the time \( t^* \) that satisfies the following:

\[
\hat{D}'(t^* + \varepsilon_{2|1} | t^*) = B'_1(t^*) = B'_2(\varepsilon_{2|1})
\]

where \( \varepsilon_{2|1} \) is the amount she plans to work on task 2 at time \( t^* \). Then I claim that \( t^* = \varepsilon_1^* = \inf\{t : \hat{V}_1(\varepsilon_1 | s) < \hat{V}_1(s | s), \forall \varepsilon_1 > s\} \) – that \( t^* \) is the switching time. First, when \( s < t^* \), then since \( \hat{D}(\cdot | s) \) is strictly increasing in \( s \), we have that \( \hat{D}'(t^* + \varepsilon_{2|1} | s) < \hat{D}'(t^* + \varepsilon_{2|1} | t^*) = B'_1(t^*) = B'_2(\varepsilon_{2|1}) \). Hence she must plan to work more on total, and hence more on each task, than she does at \( s = t^* \). Therefore she must plan to work more on both tasks (since she wants to equalize marginal benefits), therefore there is some \( e > t^* > s \) such that \( \hat{V}_1(e | s) > \hat{V}_1(s | s) \). Similarly, when \( s > t^* \), we have that \( \hat{D}'(s + \varepsilon_{2|1} | s) > \hat{D}'(t^* + \varepsilon_{2|1} | t^*) = B'_1(t^*) = B'_2(\varepsilon_{2|1}) \). But this means that the person plans less than \( s + \varepsilon_{2|1} \) total work, hence she has to plan to work less on at least one task. But she cannot work less on the first (she already spent time \( s \)), so it must be the second task, in which case the marginal benefit from the second task is strictly larger. Therefore, fixing the total amount of work, she should spend all the remaining time on the second task, and hence stops right now, i.e. \( \hat{V}_1(e | s) < \hat{V}(s | s) \) for all \( e > s \). This shows that \( \varepsilon_1^* \) satisfies:

\[
\hat{D}'(\varepsilon_1^* + \varepsilon_{2|1} | \varepsilon_1^*) = B'_1(\varepsilon_1^*) = B'_2(\varepsilon_{2|1})
\]

and \( \varepsilon_{2|1} \) is the amount she plans to work on task 2 at the time of switching.

Note that when \( B'_1(\varepsilon_1^*) = B'_2(\varepsilon_{2|1}) \) then the following are equivalent: \( \varepsilon_1^* > \varepsilon_1^1; \varepsilon_{2|1} > \varepsilon_2^2; \) and \( \varepsilon_1^1 + \varepsilon_{2|1} > \varepsilon_1^1 + \varepsilon_2^2 \), since the \( B_i^* \) are strictly concave and disutilities are strictly convex. That is, if she plans to work more on the first task than is optimal, then she also plans to work more on the second task than is optimal and vice versa; and hence both of these imply, and are implied by, her planning to work more in total than is optimal.

We can now show that \( \varepsilon_1^* > \varepsilon_1^1 \). Suppose not. Then she plans to work less in total than is optimal and we get:

\[
\hat{D}'(\varepsilon_1^* + \varepsilon_{2|1} | \varepsilon_1^*) < \hat{D}'(\varepsilon_1^* + \varepsilon_{2|1} | \varepsilon_1^1 + \varepsilon_{2|1})
\]

\[
= \hat{D}'(\varepsilon_1^* + \varepsilon_{2|1})
\]

\[
\leq \hat{D}'(\varepsilon_1^* + \varepsilon_2^2), \text{ since total optimal effort is larger then total planned effort}
\]

\[
= B'_i(\varepsilon_1^*),
\]

\[
\leq B'_i(\varepsilon_1^1), \text{ since } \varepsilon_1^1 \leq \varepsilon_1^*
\]

which shows that it does not satisfy the first order condition. Thus \( \varepsilon_1^* > \varepsilon_1^1 \).

Once she switches, she keeps working on the second task. While she wants to reduce \( \varepsilon_1 \), she can no longer do so, and thus takes \( \varepsilon_1^* \) as a given. Thus she now simply solves the first
order condition
\[ \tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*(s)) = \tilde{B}'_2(\tilde{e}_2^*(s)) \]
and as before, she stops once this holds her current \( s \) equal to final total effort, \( s = \tilde{e}_1^* + \tilde{e}_2^* \):

\[
\begin{align*}
\tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*|\tilde{e}_1^* + \tilde{e}_2^*) &= \tilde{B}'_2(\tilde{e}_2^*) \\
\iff \tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*) &= \tilde{B}'(\tilde{e}_2^*) \\
\iff \tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*) - \tilde{B}'(\tilde{e}_2^*) &= 0 \\
\iff \tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*) - \tilde{B}'(\tilde{e}_2^*) &= 0 \text{ for } x = \tilde{e}_1^* - \tilde{e}_2^* > 0
\end{align*}
\]

Since \( \tilde{B}'(\tilde{e}_2^*) = \tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^* + x) > \tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*) = \tilde{B}'(\tilde{e}_2^*) \), we have that \( \tilde{e}_2^* < \tilde{e}_1^* \). Therefore, \( \tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*) = \tilde{D}'(\tilde{e}_2^*) = \tilde{D}'(\tilde{e}_1^* + \tilde{e}_2^*) \), so that \( \tilde{e}_1^* + \tilde{e}_2^* > \tilde{e}_1^* + \tilde{e}_2^* \) and we are done. \( \Box \)

### A.3 Proofs for Section 5

Let me state again the initial value problem (henceforth IVP) determining the behavior of a projection-biased person working on a task requiring total effort \( E \) for a total benefit \( B \):

\[
\begin{align*}
\dot{E}_x &= -e_x \\
e_x &= \begin{cases} 
0, & \text{if } G(x, 0, E_x) > B \\
\frac{E_x}{1-x}, & \text{if } G(x, \frac{E_x}{1-x}, E_x) < B \\
e_x^*, & \text{otherwise, with } G(x, e_x^*, E_x) = B
\end{cases}
\end{align*}
\]

where \( G(x, E, E) = (1 - x) \cdot \tilde{D}(\frac{E}{1-x}) \). When \( G(x, e_x^*, E_x) = B \) holds, we have \( e_x^* = f(x, E, B) := (\tilde{D}')^{-1} \left( \frac{B-(1-x)(1-E)\tilde{D}(E)}{\alpha E} \right) \), with \( f(\cdot) \) existing by strict convexity of \( \tilde{D}(\cdot) \).

As a reminder, we have the following definitions of \( \tau_0 \) and \( \tau_F \):

\[
\begin{align*}
\tau_0(E_0, B_0) &:= \inf \{ \tau \in [0, 1] : G(x, 0, E_x(E_0, B_0)) < B_0 \ \forall x < 1 - \tau \} \\
\tau_F(E_0, B_0) &:= \inf \{ \tau \in [0, 1] : G(x, \frac{E_x(E_0, B_0)}{1-x}, E_x(E_0, B_0)) > B_0 \ \forall x < 1 - \tau \}
\end{align*}
\]

Intuitively (but not quite), \( \tau_0 \) is how close to the deadline the person stops working and doesn’t resume again; and \( \tau_F \) is how close to the deadline the person starts working on the task efficiently.

With this, let us first prove that \( E_x(E_0, B_0) \) for \( x < 1 \) is Lipschitz continuous in a neighborhood of \((E_0, B_0, x)\) and that it is increasing in \( E_0 \). I will use the following theorem (from https://www.math.washington.edu/~burke/crs/555/555_notes/continuity.pdf) to prove continuity.

**Theorem 1.** Consider the initial value problem

\[ x' = f(t, x, \mu), \ x(t_0) = y \]
where \( x' \) is the derivative of \( x(t) \) with respect to time. If \( f \) is continuous in \( t, x, \mu \) and Lipschitz in \( x \) with Lipschitz constant independent of \( t \) and \( \mu \), then \( x(t, \mu, y) \) is continuous in \((t, \mu, y)\) jointly.

Then we get continuity as follows:

**Lemma 1.** Suppose that \( D''(x) > d \) for some \( d > 0 \). The solution \( E_x(E, B) \) to the continuous-time problem restricted to \( x \in [0, 1 - \varepsilon] \) with \( \varepsilon > 0 \) exists and is Lipschitz continuous in \( x, E, \) and \( B \), on \([0, 1 - \varepsilon] \times [E, \bar{E}] \times [0, \infty] \), for some \( \bar{E} > E > 0 \).

**Proof.** It is clear that \( E_x \leq E_0 \), so we can pick \( \bar{E} > E_0 \). We then show that, starting with \( E_0 \in [E, \bar{E}] \), we will not fall below \( E \) before time \( x \). Given that the maximum instantaneous effort is given by \( \frac{E_0}{x} \) it is not hard to see that at most a fraction \( x \) of the total effort will be completed by time \( x \) (the efficient amount, conditional on trying to complete the task).\(^{10}\) Thus if \( E_0 \geq \frac{1}{\varepsilon} \frac{E_0}{x} \), then \( E_x \) will be larger than \( E \) for all \( x \leq 1 - \varepsilon \).

Given theorem 1 and our IVP (3), we only need to show that \( e_x(x, E, B) \) is continuous in \( t \), \( E \), and \( B \), and Lipschitz continuous in \( E \) independent of \( t \) and \( B \). Notice that \( G \) and \( f \) are continuous functions, given that \( x \) is bounded away from 1 and \( E \) is bounded away from 0.

First, notice that when \( G(x, 0, E) = B \), by definition of \( G \) and \( f \), we have that \( f(x, E, B) = (D')^{-1}(D'(0)) = 0 \), and similarly when \( G(x, \frac{E}{x}, E) = B \), we have that \( f(x, E, B) = \frac{E}{x} \).

Thus \( e(x, E, B) \) restricted to \( A := \{ (x, E, B) : G(x, 0, E) \geq B \} \) is the constant 0 function, \( e(x, E, B) \) restricted to \( B := \{ (x, E, B) : G(x, \frac{E}{x}, E) \leq B \} \) is equal to \( \frac{E}{x} \), and \( e(x, E, B) \) restricted to \( C := \{ (x, E, B) : G(x, 0, E) \leq B \) and \( G(x, \frac{E}{x}, E) \geq B \} \) is equal to \( f(x, E, B) \).

If we can show that \( e(x, E, B) \) restricted to \( A, B, \) and \( C \) is Lipschitz in all parameters (which is stronger than what we need), then \( e(x, E, B) \) is Lipschitz continuous in all parameters over the union of \( A, B, \) and \( C \). The reason is that all three regions are closed, and thus contain their limit points: Suppose we have two points \( x = (x, E, B) \) and \( x' = (x', E', B') \) and we want to show that \( |e(x, E, B) - e(x', E', B')| < K(|x - x'| + |E - E'| + |B - B'|) \) for some \( K \). First, if both points are in the same region, then this immediately holds, by the assumption that the function is Lipschitz in that region. Now suppose that the two points are in regions \( A \) and \( C \). These two regions share a common border. Thus there exists some point \( x'' = (x'', E'', B'') \) on the line connecting the two points that belongs to both regions (this is the part that requires both \( A \) and \( C \) to be closed), so that \( |e(x, E, B) - e(x', E', B')| = |e(x, E, B) - e(x'', E'', B'') + e(x'', E'', B'') - e(x', E', B')| \leq |e(x, E, B) - e(x'', E'', B'')| + |e(x'', E'', B'') - e(x', E', B')| \leq K(|x - x''| + |E - E''| + |B - B''|) = K(|x - x'| + |E - E'| + |B - B'|) \) where \( |x - x'| + |x'' - x'| = |x - x'| \) because the point \( x'' \) lies between the two points (is a convex combination of) \( x \) and \( x' \). Thus the function is Lipschitz continuous over the union of \( A \) and \( C \), and by an exactly identical argument over the union of the three regions.

Restricting ourselves to \( E_0 \in [E, \bar{E}] \), it is clear that \( e(x, E, B) \) is Lipschitz on \( A \), where it is constant. It is equally clear that \( e(x, E, B) \) is Lipschitz continuous on \( B \) since (by assumption) we are only considering \( x \leq 1 - \varepsilon \), that is \( 1 - x \geq \varepsilon \).

\(^{10}\)This statement is proved in Lemma 4
Finally, $e(x, E, B)$ is Lipschitz continuous on $C$ if $f(\cdot)$ is. But $f(\cdot)$ is the inverse function of $D'(\cdot)$, so as long as the derivative of $D'(\cdot)$ is strictly bounded away from 0 everywhere, $f(\cdot)$ is Lipschitz. This holds since we assume $D''(x) > d$ for some $d > 0$. Thus we have shown that $e(x, E, B)$ is Lipschitz continuous when $x \leq 1 - \varepsilon$, $E \in [E, \bar{E}]$ and $B \geq 0$, for any $\varepsilon > 0$, $\bar{E} > 0$, $\bar{E} > 0$.

\[ \text{Lemma 2. If } G(x, 0, E_x) > B, \text{ then } G(x', 0, E_{x'}) > B \text{ for all } x' \geq x. \text{ Similarly, if } G(x, E_{\frac{1}{2}x}, E_{x'}) < B, \text{ then } G(x', E_{\frac{1}{2}x'}, E_{x'}) < B \text{ for all } x' \geq x. \]

\[ \text{Proof. Suppose not. Then there exists } 1 > x' > x \text{ such that } G(x', 0, E_{x'}) \leq B. \text{ Note that by lemma 1, } E_x \text{ is continuous on } [0, x + \varepsilon] \text{ for sufficiently small } \varepsilon, \text{ and because } G \text{ is continuous in all its arguments, we know that } G(x + \varepsilon_1, 0, E_{x+\varepsilon_1}) > B \text{ for sufficiently small } \varepsilon_1. \text{ Thus for } x^* := \inf\{x' > x : G(x', 0, E_{x'}) \leq B\}, \text{ we have } x^* > x. \text{ Moreover, } G(y, 0, E_y) > B \text{ for all } x \leq y < x^* \text{ and therefore } e_y = 0. \text{ Thus } E_{x^*} = E_x - \int_{x^*}^{x} e_y dy = E_x. \text{ Hence we have that } G(x^*, 0, E_{x^*}) = G(x^*, 0, E_x) > G(x, 0, E_x) > B, \text{ since } G \text{ is strictly increasing in } x. \] But then by continuity of $E_x$ and $G$, we have that $G(x^* + \varepsilon_2, 0, E_{x^*+\varepsilon_2}) > B$ for sufficiently small $\varepsilon_2$, which contradicts the definition of $x^*$.

A similar argument works for the second part of the lemma. \[ \square \]

\[ \text{Lemma 3. For a fixed } x < 1 \text{ and } B_0 > 0, \text{ } E_x(E_0, B_0) \text{ is strictly increasing in } E_0. \]

\[ \text{Proof. Let } \Delta_x = E_x(E'_0, B_0) - E_x(E_0, B_0) \text{ for some } E'_0 > E_0 > 0. \text{ We need to show that } \Delta_x > 0 \text{ for all } x < 1. \]

Notice that $\Delta_0 = E'_0 - E_0 > 0$ and that $\frac{d\Delta_x}{dx} = -e'_x + e_x$. Since $E_x$ is continuous, we have that $\Delta_x > 0$ for all $x < \varepsilon$ at least. Suppose that the claim is false, so that $x^* := \inf\{x : \Delta_x \leq 0\}$ exists. Then $x^* \geq \varepsilon > 0$ and for all $x < x^*$ we have $\Delta_x > 0$. Thus $E'_x > E_x$.

We then have check that for all possible cases of values for $e_x$, we can limit how large $e_x'$ is. If $G(x, 0, E_x) > B$, then $G(x, 0, E'_x)$, and hence $e_x = e'_x = 0$. If $G(x, e_x, E_x) = B$, then $G(x, e_x, E'_x) > B$, since $G$ is increasing in $E$, and therefore $e'_x < e_x$ because $G$ is increasing in its second argument. Finally, if $G(x, E_{\frac{1}{2}x}, E_x) < B$, then $e_x = E\frac{1}{2}x$ and $e'_x \leq E\frac{1}{2}x$.

Thus we see that in all cases $\frac{d\Delta_x}{dx} = -e'_x + e_x \geq -\frac{E'_x + E_x}{1-x} = -\frac{\Delta_x}{1-x}$, and therefore for $x < x^* \Delta_x' = \Delta_x + \int_{x^*}^{x} \frac{d\Delta_x}{dx} dx \geq \Delta_x - \int_{x^*}^{x} \frac{\Delta_x}{1-y} dy$. Let $\delta < \frac{1}{3}(1 - x^*)$. We know by the definition of $x^*$ that $\Delta_x > 0$ for $x < x^*$. Pick $x \in [x^* - \delta, x^*)$ that achieves the maximum of $\Delta_x$ in this interval, which exists since $\Delta_x$ is continuous. Then we have that $\Delta_x \geq \Delta_x - \int_{x^*}^{x} \frac{\Delta_x}{1-y} dy \geq \Delta_x - \Delta_x \frac{\delta}{1-x^*} > \frac{1}{3} \Delta_x > 0$. Thus $\Delta_x > 0$ and therefore (by continuity) $\Delta_{x^*+\varepsilon} > 0$ for some small $\varepsilon > 0$, which contradicts the definition of $x^*$. Thus the claim is proved. \[ \square \]

\[ G \text{ is strictly increasing in } x \text{ because } (1-x) \cdot D(E/(1-x)) = E \cdot 1 / X \cdot D(X) \text{ where } X = (1-x)/E. \text{ But } D(X)/X \text{ is the average disutility per unit of effort, which strictly increases for a strictly convex function } D(\cdot). \]
Lemma 4. \( E_x(E_0, B_0) \geq E_x'(E_0, B_0) \frac{1}{1-x} \) for \( 1 > x > x' \geq 0 \), and \( E_x(E_0, B_0) \) is decreasing in \( B_0 \).

**Proof.**

\[
\dot{E}_x \geq \frac{E_x}{1-x} \implies \dot{E}_x \geq -\frac{1}{1-x} \\
\implies \frac{d}{dx} \log(E_x) \geq -\frac{1}{1-x} \\
\implies \log(E_{x'}) - \log(E_x) \geq -\int_x^{x'} \frac{1}{1-y} \, dy \\
\implies \log\left(\frac{E_{x'}}{E_x}\right) \geq \int_x^{x'} \frac{d}{dy} \log(1-y) \, dy \\
\implies \log\left(\frac{E_{x'}}{E_x}\right) \geq \log \frac{1-x}{1-x'} \\
\implies \frac{E_{x'}}{E_x} \geq \frac{1-x}{1-x'}
\]

The proof that \( E_x(E_0, B_0) \) is decreasing in \( B_0 \) is similar to the proof of lemma 3, and thus I omit it.

Now let us prove that \( \tau_0(E_0, B_0) \) and \( \tau_F(E_0, B_0) \) are continuous in \( E_0 \).

**Lemma 5.** Suppose \( D'(E) \to \infty \) as \( E \to \infty \) and \( D''(\cdot) > d \) for some \( d > 0 \). Then \( \tau_0(E, B) \) is increasing in \( E \) and decreasing in \( B \), and if \( \tau_0(E, B) \in (0, 1) \) it is continuous in \((E, B)\) in a neighborhood of \((E_0, B_0)\). Similarly, \( \tau_F(E, B) \) is decreasing in \( E \) and increasing in \( B \), and if \( \tau_F(E, B) \in (0, 1) \) it is continuous in \((E, B)\) in a neighborhood of \((E_0, B_0)\).

**Proof.** The proofs are essentially identical for \( \tau_0 \) and \( \tau_F \), so I prove the first. We know

\[
\tau_0(E_0, B_0) = \inf \{ \tau : 1 - \tau \in [0, 1] \text{ and } G(x, 0, E_x(E_0, B_0)) < B_0 \forall x < 1 - \tau \} = \inf \Gamma_0
\]

Notice that \( 1 \in \Gamma_0 \), thus \( \tau_0 \) always exists. Then take \( x > 1 - \tau_0 \). Suppose \( E_0' > E_0 \) and let \( \tau_0' := \tau_0(E_0, B_0) \) and \( \tau_0' := \tau_0'(E_0', B_0) \) and similarly for \( \Gamma_0 \) and \( \Gamma_0' \). Note that if \( G(x, 0, E_x(E_0, B_0)) > B_0 \) then, by lemma 3, \( E_x' \geq E_x \), and thus (since \( G \) is increasing in its third argument) \( G(x, 0, E_x(E_0, B_0)) > B_0 \). Therefore for \( \tau \notin \Gamma_0 \), then there exists some \( x < 1 - \tau \) with \( G(x, 0, E_x) \geq B_0 \) and therefore \( G(x, 0, E_x') > B_0 \) so that \( \tau \notin \Gamma_0' \). Hence \( \Gamma_0' \subset \Gamma_0 \) and thus \( \tau_0 \geq \tau_0' \).

Notice that as we increase \( B_0 \) to \( B_0' \), every \( \tau \) in \( \Gamma_0 \) is necessarily also in \( \Gamma_0' \): if \( G(x, 0, E_x(E_0, B_0)) < B_0 \), then \( G(x, 0, E_x(E_0, B_0')) < B_0' \), since \( E_x \) weakly decreases in \( B_0 \), and thus \( G(\cdot) \) decreases, while the RHS increases. Thus \( \Gamma_0 \subset \Gamma_0' \), hence \( \tau_0 \leq \tau_0' \).
Let us now show continuity for $\tau_0 \in (0,1)$. Fix $(E_0, B_0)$, then $\tau_0$ is s.t. for $x < 1 - \tau_0$ we have $G(x, 0, E_x(E_0, B_0)) < B_0$ and for every $\varepsilon > 0$ there is some $x \in (1 - \tau_0, 1 - \tau_0 + \varepsilon)$ with $G(x, 0, E_x(E_0, B_0)) \geq B_0$. Suppose by contradiction that $\tau_0$ is not continuous. Then there is some $\delta$ s.t. for every $\varepsilon_i$ we have $(E_i, B_i)$ within $\varepsilon_i$ distance from $(E_0, B_0)$ with either some $x_i \leq 1 - \tau_0 - \delta$ and $G(x_i, 0, E_{x_i}(E, B)) \geq B$ so that $\tau_i \geq \tau_0 + \delta$, or we have for all $x < 1 - \tau_0 + \delta$ we have $G(x, 0, E_x(E, B)) < B$, so that $\tau_0 \leq \tau_0 - \delta$.

**Contradiction Case 1:** For $\varepsilon_i \to 0$, there is a sequence $(E_i, B_i)$ s.t. $G(x, 0, E_x(E_i, B_i)) < B_i$ for all $x < 1 - \tau_0 + \delta$.

Since $E_x(E, B)$ is (Lipschitz) continuous in $(E, B)$ and $G(\cdot)$ in its arguments in the range observed, this converges uniformly for all $x < 1 - \tau_0 + \delta$. Applying this to the closed range $x < 1 - \tau_0 + 1/2\delta$, we find that $G(x, 0, E_x(E_0, B_0)) < B_0$ for all $x < 1 - \tau_0 + 1/2\delta$, which contradicts the value of $\tau_0$.

**Contradiction Case 2:** For $\varepsilon_i \to 0$, there is a sequence $(E_i, B_i)$ and some $x_i \leq 1 - \tau_0 - \delta$ s.t. $G(x, 0, E_x(E_i, B_i)) > B_i$.

Given that the ranges are all finite, $(E_i, B_i, x_i)$ converges to $(E_0, B_0, x)$, with $x \leq 1 - \tau_0 - \delta$, with $G(x, 0, E_x(E_0, B_0)) \geq B_0$. But this directly contradicts the definition of $\tau_0$, since this implied that $G(x, 0, E_x(E_0, B_0)) < B$ for all $x < 1 - \tau_0$.

Hence $\tau_0(E, B)$ is continuous in $(E, B)$.

**Lemma 6.** Let $D$ be convex with $D'(e) \to \infty$ as $e \to \infty$. Then $\forall K > 0$, $\exists E$ s.t. $D(e) > K \cdot e$ $\forall e > E$. That is, $D(e)/e \to \infty$ as $e \to \infty$.

**Proof.** Since $D'(e) \to \infty$, pick $E$ s.t. $D'(E/2) > 2 \cdot K$. Then for $e > E$

$$D(e) = \int_0^e D'(s)ds \geq \int_0^E D'(s)ds \geq \int_{E/2}^E 2 \cdot K ds \geq \frac{e - E/2}{2} \cdot K = e \cdot K$$

Proof of proposition 4:

**Proof.** Let $B_L = (1 - \alpha)D(E_0) + \alpha D'(0)E_0$. Then $G(0, 0, E_0) = B_L$ and therefore if $B < B_L$ we have $G(0, 0, E_0) > B_L$ and hence by lemma 2 we know that $G(x, 0, E_x) > B_L$ for all $x \geq 0$. Hence $e_x = 0$ and $\tau_0(E_0, B_0) = 1$. Similarly, if $B_H = (1 - \alpha)D(E_0) + \alpha D'(E_0)E_0$, then $G(0, 0, E_0) = B_H$. Hence if $B > B_H$ we have $G(0, 0, E_0) < B$ and again by lemma 2 this holds for all $x \geq 0$ and thus $\tau_0 = 1$ and $e_x = E_0$ (this last part in effect requires solving the same differential equation as we did in lemma 4, which I omit).

Moreover, note that if $B < B_H$ then we have that $G(0, 0, E_0) > B$ and thus (by continuity of $E_x$ and $G$) we have that $G(x, E_0, E_x) > B$ for all sufficiently small $x$. Therefore, $\tau_0 < 1$. Similarly, if $B > B_L$ we have that $\tau_0 < 1$.  

26
It is clear by lemma 2 that if \( \tau_0 > 0 \) then \( \tau_F = 0 \) and if \( \tau_F > 0 \) then \( \tau_0 = 0 \): if \( \tau_0 > 0 \), then there is some \( x \in [1 - \tau_0, 1 - \tau_0 + \varepsilon] \) such that \( e_x = 0 \), hence by lemma 2 we have \( e_x' = 0 \) for all \( x' > x \), so the person never works efficiently on the task. The other direction is similar. Let 
\[ B_{C,0} = \inf \{ B : \tau_0(E_0, B) = 0 \} \] — roughly the smallest \( B \) for which there is some work done at all times \( x \). Then because \( \tau_0 \) is decreasing in \( B_0 \) by lemma 5, we know that if \( B < B_{C,0} \) then \( \tau_0(E_0, B) > 0 \), since if \( \tau_0(E_0, B) = 0 \), then \( \tau_0(E_0, B') = 0 \) for all \( B' \geq B \), contradicting the definition of \( B_{C,0} \). Similarly we can define \( B_{C,F} = \limsup \{ B : \tau_F(E_0, B) = 0 \} \) and show that if \( B > B_{C,F} \) then \( \tau_F > 0 \).

To finish the proof, we need to show that \( B_{C,F} = B_{C,0} \). Notice that if \( B_0 \in [B_{C,0}, B_{C,F}] \) we have that \( \tau_0 = 0 \) and \( \tau_F = 0 \). Therefore \( G(x, e_x, E_x(E_0, B_0)) = B \) for all \( x < 1 \). Suppose that \( B_{C,0} < B_{C,F} \).\(^{12}\) Let \( e_{x,0} \) be the effort for \( B_{C,0} \) and \( e_{x,F} \) the effort for \( B_{C,F} \). Then, since \( G(x, e_{x,0}, E_{x,0}) = B_{C,0} < B_{C,F} = G(x, e_{x,F}, E_{x,F}) \) for all \( x \), we must have that \( e_{x,F} > e_{x,0} \) or \( E_{x,F} > E_{x,0} \) for every \( x \). Since \( E_{0,C} = E_{0,F} \), by continuity of \( E_x \) in \( x \) and \( G \) in \( E \), we can pick \( \varepsilon > 0 \) such that for all \( x < \varepsilon \), \( G(x, e_{x,0}, E_{x,F}) \) is arbitrarily close to \( G(x, e_{x,0}, E_{x,0}) = B_{C,0} \). Therefore the inequality holds only if \( e_{x,F} > e_{x,0} \) for \( x < \varepsilon \), and thus \( E_{x,F} < E_{x,0} \). We can then show that \( e_{x,F} > e_{x,0} \) for all \( x \). Suppose not, then we must have that \( E_{x,F} > E_{x,0} \) for some \( x \) and therefore there exists a smallest \( x^* > \varepsilon \) such that \( E_{x^*,F} = E_{x^*,0} \). But \( e_{x,F} > e_{x,0} \) for all \( x < x^* \), therefore \( E_{x^*,F} < E_{x^*,0} \), which is a contradiction.

Thus we have shown that \( E_{x,F} < E_{x,0} \) and that \( e_{x,F} > e_{x,0} \) for all \( x > 0 \). Let \( \delta = E_{1,0} - E_{1,F} > 0 \), then \( E_{x,0} - E_{x,F} \geq \delta \) for \( x > \frac{1}{2} \) and therefore \( E_{x,0} \geq \delta > 0 \) for all \( x \). Therefore \( D(\frac{E_{x,0}}{1-x})(1-x) \geq D(\frac{\delta}{1-x})(1-x) \delta \to \infty \) as \( x \to 1 \) by lemma 6. But this means that \( G(x,0, E_{x,0}) \to \infty \) and therefore that \( G(x,0, E_{x,0}) > B \) as \( x \to 1 \), so that \( \tau_0 > 0 \). Therefore, we cannot have that \( B_{C,0} < B_{C,F} \), so that \( B_{C,0} = B_{C,F} = B_C \), and we are done.

Now let us show that the utility is continuous and decreasing on \((B_L, B_C)\) and continuous and increasing on \((B_C, B_H)\).

**Lemma 7.** The utility \( u_0(E_0, B_0) := -\int_0^1 D(e_x)dx \) is continuous and decreasing on \((B_L(E_0), B_C(E_0))\) and the utility \( u_F(E_0, B_0) := B - \int_0^1 D(e_x)dx \) is continuous and increasing on \((B_C(E_0), B_H(E_0))\).

**Proof.** Notice that when \( B \in (B_L, B_C) \) then we know that \( \tau_0 \in (0,1) \) and the task is not completed, hence the definition of the utility as \( u_0 \) is correct. Moreover \( u_0 = \int_0^{1-\tau_0} D(e_x)dx \). We can show that \( \tau_0 \) and \( E_x \) are continuous and decreasing in \( B_0 \). Picking \( B_0 < B_0' \), we therefore have that \( \tau_0' < \tau_0 \) and that for \( x \leq 1 - \tau_0 \) we have \( G(x, e_x, E_x) = B_0 < B_0' = G(x, e_x', E_x') \). Since \( E_x' \leq E_x \) we therefore have that \( e_x' > e_x \) and therefore \( u_0' > \int_0^{1-\tau_0} D(e_x') dx > \int_0^{1-\tau_0} D(e_x) dx = u_0 \). Moreover, if \( B_0' \) is close to \( B_0 \) then \( E_x \) is close to \( E_x' \) by Lipschitz continuity and therefore \( e_x' \) and \( e_x \) are close together, since \( e_x \) is Lipschitz continuous in all the parameters as well (I haven’t shown this in detail, but this is where I use the condition \( D'(0) > 0 \)). Therefore the \( u_0' \) and \( u_0 \) are close.

\(^{12}\)We cannot have \( B_{C,0} > B_{C,F} \) since then \( \tau_F > 0 \) and \( \tau_0 > 0 \) for all \( x \in (B_{C,F}, B_{C,0}) \), but both cannot happen jointly.
Now suppose $B_0, B'_0 \in (B_C, B_H)$ then $\tau_F \in (0, 1)$. Let $B_0 < B'_0$. We can show in a similar way as before that $\tau_F > \tau_F$ and that $e'_x > e_x$ for $x \leq 1 - \tau_0$. Then notice that $\int_0^1 e_x = E_0 = \int_0^1 e'_x$. Let $F(e) := \mathbb{P}(e_x \leq e) = \int_0^1 1(e_x \leq e)dx$ and $G(e) := \mathbb{P}(e'_x \leq e) = \int_0^1 1(e'_x \leq e)dx$, where the interpretation as probabilities is to help intuition, although it can be made formal by drawing $x$ uniformly from $[0, 1)$. We want to show that $\int_0^1 D(e) dF(e) > \int_0^1 D(e) dG(e)$. By strict convexity of $D(\cdot)$, this holds if $F(\cdot)$ is a mean-preserving spread of $G(\cdot)$. Let $\bar{\epsilon}_G = e_{1-\tau_F}$, be the effort the person exerts under $G(\cdot)$ once they work fully, with $B'_0$. Then if $e < \bar{\epsilon}$, $G(e) < F(e)$, since $e_x > e_x$ for all $x \leq 1 - \tau_0$, which are the only $e$ that can below $\bar{\epsilon}$ under $G(\cdot)$, while $F(\cdot)$ can get contributions from $e > \bar{\epsilon}_G$. However $G(\bar{\epsilon}) = 1$ for $e > \bar{\epsilon}_G$, while $F(\bar{\epsilon}) \geq 1$, we have that $F(e) \geq G(e)$ for $e \geq \bar{\epsilon}$. Therefore $F$ is a mean-preserving spread of $G$, moving effort from above $\bar{\epsilon}$ to below, and thus the disutility for $e_x$ is higher than for $e'_x$. Continuity follows again by noting that, until time $1 - \tau_0$, $e_x$ is Lipschitz continuous in all parameters, and thereafter it is constant. Therefore the utility is Lipschitz continuous.

I will need the following lemma to prove the second part of the proposition.

**Lemma 8.** Let $D$ be convex and such that $D'(e) \to \infty$ as $e \to \infty$. Fix $B$ and $\epsilon > 0$. Let $e_\epsilon$ be s.t.

$$D(e_\epsilon) \cdot \epsilon = B \quad (4)$$

Then $e_\epsilon \cdot \epsilon \to 0$ as $\epsilon \to 0$.

**Proof.** First note that as $\epsilon$ goes to $0$, $e_\epsilon$ goes to $\infty$, since if it was bounded, then $D(e_\epsilon) \cdot \epsilon$ would go to $0$. By lemma 6, we know that $\frac{D(e_\epsilon)}{e_\epsilon} \to \infty$. Dividing both sides of equation 4 by $\epsilon \cdot e_\epsilon$ yields

$$\frac{D(e_\epsilon)}{e_\epsilon} = \frac{B}{e_\epsilon \cdot \epsilon} \iff e_\epsilon \cdot \epsilon = \frac{B}{D(e_\epsilon)/e_\epsilon} \to 0$$

which proves the claim. \hfill \square

Here is the proof of the second part of proposition 4.

**Proof.** Since $\tau_0$ is continuous and decreasing on $(B_L(E_0), B_H(E_0))$, and since $\tau_0$ can be $0$ and $1$, we know that for every $\tau \in (0, 1)$ there is some $B_0 \in (B_L(E_0), B_H(E_0))$ such that $\tau_0(E_0, B_0) = \tau$. Notice that at time $\tau_0$ we have that $G(1 - \tau_0, 0, \frac{E_1 - \tau_0}{\tau_0}) = B_0$. Therefore $\bar{D}(\frac{E_1 - \tau_0}{\tau_0})|\tau_0 = B_0$. As $\tau_0 \to 0$, by Lemma 8 we must therefore have that $E_{1-\tau_0} \to 0$. This means that almost all the work gets done before time $1 - \tau_0$ for which the least disutility is $D(E_0 - \epsilon) > D(E_0) - \delta$ for sufficiently small $\epsilon$ (i.e. $\tau_0$ sufficiently close to 1). Therefore the disutility is at least $D(E_0) - \delta$ for arbitrary $\delta$. Hence the result holds. \hfill \square

\[13\]A rigorous proof of this would require to show that $e'_x < \bar{\epsilon}_G$, so that the final effort exerted is the highest effort ever exerted.
Proof of proposition 5.

Proof. The agent solves the following maximization problem:

\[
\max_e B - \sum_{t=1}^{T} D(e_t), \text{ s.t. } \sum_{t=1}^{T} p_t \cdot e_t = E
\]

where \( B \) is a fixed benefit for completing \( E \) total work, and \( p_t \) is the productivity in period \( t \), that is the amount of effective work done for each unit of effort exerted. This means that optimal effort is determined by the following first order conditions:

\[
\frac{D'(e_t^*)}{p_t} \geq \lambda
\]

In period 1, after having worked for a time \( s \), the optimal perceived effort levels are instead given by the following perceived first order conditions:

\[
\frac{\tilde{D}'(\tilde{e}_t^*(s)|s)}{p_t} \geq \lambda(s) \iff \frac{(1 - \alpha) \cdot D'(\tilde{e}_t^*(s))}{p_t} + \frac{\alpha \cdot D'(s)}{p_t} \geq \lambda(s)
\]

\[
\iff \frac{D'(\tilde{e}_t^*(s))}{p_t} \geq \frac{\lambda(s)}{1 - \alpha} - \frac{\alpha}{1 - \alpha} \frac{D'(s)}{p_t}
\]

\[
\iff \frac{D'(e_t^*|1)}{p_t} \geq \frac{\lambda}{1 - \alpha} - \frac{\alpha}{1 - \alpha} \frac{D'(e_1^*)}{p_t} = \lambda_t
\]

where \( \tilde{e}_{t1}^* \) is the amount of work the person plans to do on day \( t \) at the end of day 1, and \( \tilde{e}_1^* \) is the amount of work done on day 1. Note that when the productivities are strictly increasing, then so are the \( \lambda_i \), and when the productivities are strictly decreasing, then so are the \( \lambda_i \). When the productivities are strictly increasing, then \( \lambda_1 < \lambda^* \) : suppose not, so that \( \lambda_1 \geq \lambda^* \). Then since the \( \lambda_i > \lambda_1 \geq \lambda^* \) for all \( i > 1 \), this implies that whenever the unbiased agent exerts strictly positive effort, then the biased agent plans strictly more effort on all those days, and hence strictly more total effort – which violates the output constraint. So as long as the person exerts effort on more than a single day (the final day), this cannot be – hence \( \lambda_1 < \lambda^* \), and the person works less than optimal on the first day (strictly less if \( e_1^* > 0 \) and \( T \geq 2 \)).

Similarly, when productivities are strictly decreasing, we have that \( \lambda_1 > \lambda^* \). If not, then we have that \( \lambda^* \geq \lambda_1 > \lambda_2 > ... > \lambda_T \). Thus the person plans to do strictly less work on every future day on which it is optimal to exert strictly positive effort – but this means that they plan on producing strictly less output than is required, which is not possible. So as long as it is optimal to exert some strictly positive amount of work on some future day, \( \lambda_1 > \lambda^* \) and thus the person works more on the first day (strictly more if it is optimal to work on at least 2 days).

Let us now prove the following two separate statements for increasing and decreasing pro-
ductivity respectively.

**Case 1: Increasing productivity** Suppose that productivity is strictly increasing, so that \( p_1 < p_2 < \ldots < p_T \). Then \( \lambda(s) \) is strictly increasing in \( D'(s) \) and hence in \( s \).

We will show the following in turn:

1. The agent works strictly less on the first day than they should
2. At the start of every day after the first, the biased agent has completed strictly less work in total than the unbiased agent
3. On every day that is not the first or the last, the agent ends up working strictly less than they expected to work on this day at the end of the day before

**Step 1:** We proved this above.

**Step 2:** Let \( E_t = \sum_{i=1}^{t} p_i \cdot e_i^* \) be the total work completed at the end of day \( t \) by the unbiased agent and \( \tilde{E}_t = \sum_{i=1}^{t} p_i \cdot \tilde{e}_i^* \) be the total work completed at the end of day \( t \) by the biased agent.

Then we want to show that \( \tilde{E}_t < E_t \) for all \( t \) from 1 to \( T - 1 \) – since of course at the end of the last day, the person has completed the same amount of work. We know from step 1 that \( \tilde{E}_1 = \tilde{e}_1^* < e_1^* = E_1 \), so the result holds for \( t = 1 \). We will prove it by induction. Suppose that the result holds for all \( \tau < t \). If \( t = T \), then we are done. If \( t < T \), then we will prove that the result also holds for \( \tau = t \) and hence for all \( \tau \leq t \). Thus by induction it holds for all \( \tau < T \), proving our claim.

Why does the result hold for \( \tau = t \)? Suppose by contradiction that it does not hold. Then \( \tilde{E}_{t-1} < E_{t-1} \) and \( \tilde{E}_t \geq E_t \), hence \( \tilde{e}_t^* > e_t^* \). Consider how much the biased agent would work if suddenly on day \( t \) they would become unbiased: that is, they can not change the fact that they worked suboptimally in the past, but moving forward they will work optimally. Let us denote this person’s variables with a hat rather than a tilde, i.e. \( \hat{e}_t^* \) is the amount of effort they will exert on day \( t \). Since day \( t \) is the first day of the rest of their life, by step 1 we know that they would work more now than they did as a biased agent: \( \hat{e}_t^* > \tilde{e}_t^* \geq e_t^* \). But this means that they started day \( t \) having to complete strictly more work on the remaining days than an agent who worked optimally from the start, yet on day \( t + 1 \) they have strictly less work remaining than such an agent, despite both choosing optimally and having convex disutility of effort. This is a contradiction.

Hence step 2 holds.

**Step 3:** The FOCs for planned work on day 1 are given by:

\[
\frac{D'(\hat{e}_{1|1})}{p_t} \geq \frac{\lambda}{1 - \alpha} - \frac{\alpha D'(\tilde{e}_1^*)}{p_t} = \lambda_t
\]
whereas the FOCs for planned work on day 2 are given by:

$$\frac{D'(\tilde{e}_{t2}^*)}{p_t} \geq \frac{\mu}{1 - \alpha} - \frac{\alpha}{1 - \alpha} \frac{D'(\tilde{e}_{t1}^*)}{p_t} = \mu_t$$

Our claim is that \(\lambda_2 > \mu_2\), since then \(\tilde{e}_{21}^* \geq \tilde{e}_{1}^*\), with strict inequality if \(\tilde{e}_{21}^* > 0\). But given that actual effort on day 2 is certainly higher than actual effort on day 1 (note that as long as the person has worked less than they worked on day 1, they are planning more work on day 2 than they planned), final effort on day 2 is higher. Note that both at the end of day 1 and at the beginning of day 2, the person plans to produce identical total output over the remaining \(T - 1\) days. Now suppose that we have \(\mu_2 \geq \lambda_2\). We have:

\[
\lambda_t = \frac{\lambda}{1 - \alpha} - \frac{\alpha}{1 - \alpha} \frac{D'(\tilde{e}_{t1}^*)}{p_t} \\
= \lambda_2 - \frac{\alpha}{1 - \alpha} \frac{D'(\tilde{e}_{1}^*)}{1/p_t - 1/p_2} \\
\leq \mu_2 - \frac{\alpha}{1 - \alpha} \frac{D'(\tilde{e}_{2}^*)}{1/p_t - 1/p_2} < \mu_2 - \frac{\alpha}{1 - \alpha} \frac{D'(\tilde{e}_{2}^*)}{1/p_t - 1/p_2} \\
= \mu_t
\]

where the first inequality holds because we assumed that \(\mu_2 \geq \lambda_2\), and the second holds because \(1/p_t - 1/p_2 < 0\) and \(\tilde{e}_{2}^* \geq \tilde{e}_{1}^*\). But if \(\mu_t > \lambda_t\) for all \(t > 2\) and \(\mu_2 \geq \lambda_2\), then the person plans to do more work on every single day and hence produce more output in total on day 2 than they planned at the end of day 1. As long as they plan to do some positive amount of work on some day (which they do on the last day), they plan strictly more work, hence this is a contradiction, unless the second day is the last day. Thus \(\mu_2 < \lambda_2\) and the person works strictly less on the second day than they planned if they planned on doing a non-zero amount of work.

**Case 2** Let us now assume that the productivity strictly decreases over time. We will show the following in turn:

1. The agent works strictly more on the first day than they should
2. At the start of every day after the first, the biased agent has completed strictly more work in total than the unbiased agent
3. On every day that is not the first or the last, the agent ends up working strictly less than they expected to work on this day at the end of the day before

**Step 1** We showed this before.

**Step 2** This is essentially identical to the argument in the previous step 2.

**Step 3** The argument mirrors step 3 from the increasing productivity case, but somewhat surprising maybe, the person does still overestimate the amount of work they will do. Thus
we again want to show that $\lambda_2 > \mu_2$. In fact, the derivation in the equations in 5 still hold: the first inequality holds if we assume (by contradiction) that $\mu_2 \geq \lambda_2$; the second inequality holds because we now have that $1/p_1 - 1/p_2 > 0$, but we also have that $\tilde{e}_2^* < \tilde{e}_1^*$ (as long as there was any effort on day 1) – after all, productivity is higher on day 1, so the person works more on day 1 than on day 2. Thus the result follows.

\[\square\]

**References**

Acland, Dan, and Matthew R. Levy. 2015. “Naiveté, Projection Bias, and Habit Formation in Gym Attendance.” *Management Science* 61 (1): 146–160.

Ahn, David S, Ryota Iijima, and Todd Sarver. 2020. “Naivete About Temptation and Self-Control: Foundations for Recursive Naive Quasi-Hyperbolic Discounting.” *Journal of Economic Theory* 189. Elsevier: 105087.

Ariely, Dan, and George Loewenstein. 2006. “The Heat of the Moment: The Effect of Sexual Arousal on Sexual Decision Making.” *Journal of Behavioral Decision Making* 19 (2): 87–98.

Augenblick, Ned, and Matthew Rabin. 2019. “An Experiment on Time Preference and Misprediction in Unpleasant Tasks.” *Review of Economic Studies* 86 (3). Oxford University Press: 941–975.

Badger, Gary J., Warren K. Bickel, Louis A. Giordano, Eric A. Jacobs, George Loewenstein, and Lisa Marsch. 2007. “Altered States: The Impact of Immediate Craving on the Valuation of Current and Future Opioids.” *Journal of Health Economics* 26 (5): 865–876.

Bray, Robert L, Decio Coviello, Andrea Ichino, and Nicola Persico. 2016. “Multitasking, Multiarmed Bandits, and the Italian Judiciary.” *Manufacturing & Service Operations Management* 18 (4). Informs: 545–558.

Buchheim, Lukas, and Thomas Kolaska. 2017. “Weather and the Psychology of Purchasing Outdoor Movie Tickets.” *Management Science* 63 (11). Informs: 3718–3738.

Buehler, Roger, Dale Griffin, and Michael Ross. 1994. “Exploring the' Planning Fallacy': Why People Underestimate Their Task Completion Times.” *Journal of Personality and Social Psychology* 67 (3). American Psychological Association: 366.

Bushong, Benjamin, and Tristan Gagnon-Bartsch. 2020. “An Experiment on Interpersonal Projection Bias.”

Busse, Meghan R, Devin G Pope, Jaren C Pope, and Jorge Silva-Risso. 2015. “The Psychological Effect of Weather on Car Purchases.” *The Quarterly Journal of Economics* 130 (1). Oxford University Press: 371–414.

Chaloupka, IV, Frank J, Matthew R Levy, and Justin S White. 2019. *Estimating Biases in Smoking Cessation: Evidence from a Field Experiment*. Working Paper 26522. Working Paper Series. National Bureau of Economic Research. doi:10.3386/w26522.
Conlin, Michael, Ted O’Donoghue, and Timothy J. Vogelsang. 2007. “Projection Bias in Catalog Orders.” *American Economic Review* 97 (4): 1217–1249.

Coviello, Decio, Andrea Ichino, and Nicola Persico. 2014. “Time Allocation and Task Juggling.” *American Economic Review* 104 (2): 609–623. doi:10.1257/aer.104.2.609.

Coviello, Decio, Andrea Ichino, and Nicola Persico. 2015. “The Inefficiency of Worker Time Use.” *Journal of the European Economic Association* 13 (5): 906–947. doi:10.1111/jeea.12129.

Ericson, Keith Marzilli, and David Laibson. 2019. “Intertemporal Choice.” In *Handbook of Behavioral Economics: Applications and Foundations 1*, 2:1–67. Elsevier.

Fedyk, Anastassia. 2018. “Asymmetric Naivete: Beliefs About Self-Control.” Available at SSRN 2727499.

Gagnon-Bartsch, Tristan, and Benjamin Bushong. 2019. *Learning with Misattribution of Reference Dependence*. Technical report, Michigan State University.

Gilbert, Daniel T., Elizabeth C. Pinel, Timothy D. Wilson, Stephen J. Blumberg, and Thalia P. Wheatley. 1998. “Immune Neglect: A Source of Durability Bias in Affective Forecasting.” *Journal of Personality and Social Psychology* 75 (3): 617.

Gul, Faruk, and Wolfgang Pesendorfer. 2001. “Temptation and Self-Control.” *Econometrica* 69 (6). Wiley Online Library: 1403–1435.

Haggag, Kareem, Devin G Pope, Kinsey B Bryant-Lees, and Maarten W Bos. 2019. “Attribution Bias in Consumer Choice.” *The Review of Economic Studies* 86 (5). Oxford University Press: 2136–2183.

Harris, Christopher, and David Laibson. 2013. “Instantaneous Gratification.” *The Quarterly Journal of Economics* 128 (1). MIT Press: 205–248.

Herrnstein, Richard J, and Dražen Prelec. 1991. “Melioration: A Theory of Distributed Choice.” *The Journal of Economic Perspectives*. JSTOR, 137–156.

Hsiaw, Alice. 2013. “Goal-Setting and Self-Control.” *Journal of Economic Theory* 148 (2). Elsevier: 601–626.

Huang, Yu-Jui, and Adrien Nguyen-Huu. 2018. “Time-Consistent Stopping Under Decreasing Impatience.” *Finance and Stochastics* 22 (1). Springer: 69–95.

Kaufmann, Marc. 2020. “Projection Bias in Effort Choices.” Working Paper. https://trichotomy.xyz/publication/projection-bias-in-effort-choices/projection-bias-in-effort-choices.pdf.

Laibson, David. 1997. “Golden Eggs and Hyperbolic Discounting.” *The Quarterly Journal of Economics* 112 (2). MIT Press: 443–478.

Levy, Matthew. 2009. “An Empirical Analysis of Biases in Cigarette Addiction.” Working Paper.
Le Yaouanc, Yves, and Peter Schwardmann. 2019. “Learning About One’s Self.” CEPR Discussion Paper No. DP13510.

Loewenstein, George, and Daniel Adler. 1995. “A Bias in the Prediction of Tastes.” The Economic Journal. JSTOR, 929–937.

Loewenstein, George, Daniel Nagin, and Raymond Paternoster. 1997. “The Effect of Sexual Arousal on Expectations of Sexual Forcefulness.” Journal of Research in Crime and Delinquency 34 (4). Sage Publications: 443–473.

Loewenstein, George, Ted O’Donoghue, and Matthew Rabin. 2003. “Projection Bias in Predicting Future Utility.” The Quarterly Journal of Economics 118 (4): 1209–1248.

Michel, Christian, and André Stenzel. 2020. “Model-Based Evaluation of Cooling-Off Policies.”

Nordgren, Loran F, Joop van der Pligt, and Frenk van Harreveld. 2008. “The Instability of Health Cognitions: Visceral States Influence Self-Efficacy and Related Health Beliefs.” Health Psychology 27 (6). American Psychological Association: 722.

O’Donoghue, Ted, and Matthew Rabin. 1999. “Doing It Now or Later.” The American Economic Review 89 (1): 103–124.

O’Donoghue, Ted, and Matthew Rabin. 2008. “Procrastination on Long-Term Projects.” Journal of Economic Behavior & Organization 66 (2): 161–175.

Quah, John K-H, and Bruno Strulovici. 2013. “Discounting, Values, and Decisions.” Journal of Political Economy 121 (5). University of Chicago Press Chicago, IL: 896–939.

Read, Daniel, and Barbara Van Leeuwen. 1998. “Predicting Hunger: The Effects of Appetite and Delay on Choice.” Organizational Behavior and Human Decision Processes 76 (2): 189–205.

Van Boven, Leaf, and George Loewenstein. 2003. “Social Projection of Transient Drive States.” Personality and Social Psychology Bulletin 29 (9). Sage Publications: 1159–1168.