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THE ELECTROWEAK PHASE TRANSITION IN THE
MINIMAL SUPERSYMMETRIC STANDARD MODEL

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Table of Contents

Abstract ........................................................................................................ ii
Acknowledgements ...................................................................................... iii
List of Figures .............................................................................................. vi

1. Introduction ............................................................................................ 1

2. Overview of Baryon Number Violation and the Electroweak Phase Transition .................................................................................. 3

3. Finite Temperature Field Theory ............................................................ 4

4. High Temperature Dimensional Reduction ............................................. 5

5. The Electroweak Phase Transition in the MSSM ........................................ 6
   5.1. Introduction ..................................................................................... 6
   5.2. Discussion of Numerical Results ...................................................... 7
      5.2.1. Dependence on tan $\beta$ and $M_A$ ........................................... 8
      5.2.2. Dependence on other parameters ............................................. 10
      5.2.3. Validity of approximations ...................................................... 12
   5.3. Conclusions .................................................................................... 13

Appendix A. MSSM in Four Dimensions ..................................................... 14

Appendix B. Explicit Relationships between Parameters ............................ 15

Appendix C. 2HDM and NMSSM .............................................................. 16

Appendix D. Appendix of Finite Temperature Formulae .............................. 17

Appendix E. Relation to Physical Parameters ............................................. 18

Appendix F. Figures ................................................................................... 23
List of Figures

F.1. Plot of $x_c$ vs. $M_A$ for several different values of tan $\beta$. The solid line corresponds to tan $\beta = 13.3$, the dashed line to tan $\beta = 1.75$, the dashed-dot line to tan $\beta = 1.5$, and the dotted line to tan $\beta = 1.25$. .......................... 23

F.2. Plot of $x_c$ vs. $M_A$, for tan $\beta = 1.25, 1.75, 13.3$. The solid line corresponds to $M_{SU3Y} = 10^{12}$ GeV, the dotted line to $M_{SU3Y} = 10^9$ GeV. .......................... 24

F.3. Plot of $x_c$ vs. $M_A$ for tan $\beta = 1.25, 1.75$. The solid line corresponds to $m_o = 50$ GeV, the dotted line to $m_o = 150$ GeV. .......................... 24

F.4. Plot of $x_c$ vs. $M_A$ for tan $\beta = 1.25, 1.75$. The solid line corresponds to the case with no squark mixing, dashed line to $\mu = 200$ GeV, dotted line to $A = 150$ GeV. .......................... 25

F.5. Plot of $x_c$ vs. $M_A$ for tan $\beta = 1.5, 1.75$. The dashed line corresponds to $m_t = 165$ GeV, the solid line to $m_t = 175$ GeV, the dotted line to $m_t = 190$ GeV. .......................... 25

F.6. Plot of $x_c$ vs. $M_A$ for tan $\beta = 1.25, 1.75$. The solid line corresponds to $m_{U_3} = 200$ GeV, the dashed line to $m_{U_3} = 100$ GeV, the dotted line to $m_{U_3} = 50$ GeV. .......................... 26

F.7. Plot of $x_c$ vs. $M_A$ including (neglecting) gluino contributions correspond to the solid (dashed) lines, for four different values of tan $\beta$. The stop soft supersymmetry breaking masses vary as functions of tan $\beta$ and $M_A$. .......................... 26

F.8. Plot of $x_c$ vs. $M_A$ including (neglecting) gluino contributions correspond to the solid (dashed) lines, for four different values of tan $\beta$. We have fixed the values of the stop soft supersymmetry breaking masses. .......................... 27

F.9. Plot of $x_c$ vs. $M_A$ including (neglecting) gluino contributions solid (dashed) lines, and including contributions arising only from third generation squarks dotted lines, for two different values of tan $\beta$. We take fixed values of the stop soft supersymmetry breaking masses in all three cases. .......................... 27

F.10. Plot of $\theta$ vs. $T$ for $M_A = 40$ GeV. The solid line corresponds to tan $\beta = 1.25, T_c = 63.3$ GeV, the dashed line is for tan $\beta = 13.3, T_c = 75.6$ GeV. .......................... 28
F.11. Plot of $\theta$ vs. $T$ for $M_A = 300$ GeV, $T_c = 60, 75.6$ GeV. The solid line corresponds to $\tan \beta = 1.25$, the dashed line to $\tan \beta = 13.3$.  

F.12. Plot of $x_c$ vs. $T$ for $\tan \beta = 1.25$, $M_A = 40$ GeV, $T_c = 60.8$ GeV.  

Chapter 1
Introduction

The main goal of the research presented in this thesis is to analyze the electroweak phase transition in the context of the Minimal Supersymmetric Standard Model (MSSM). The principal motivation resides in the implications of the phase transition on the study of the baryon asymmetry of the universe. It has been shown, that unless the phase transition is sufficiently first order, electroweak scale physics cannot account for the observed asymmetry [1].

In chapter two we present an overview of baryon number violation and its relation to the electroweak phase transition. In chapter three we introduce the formulation of finite temperature field theory. The contents of these two chapters are well known. The purpose is to point out to the reader the main issues related to the baryon asymmetry which inspire the study of the phase transition.

The Standard Model has been fully investigated in relation to these issues. The analysis of this model depends on only one unknown parameter, the Higgs mass. A complete study of the phase transition must address the problem posed by the infrared divergences which are a characteristic of gauge theories at finite temperature. In order to do this non-perturbative methods must be implemented. The most accurate calculations have ruled out the Standard Model as the responsible for the baryon asymmetry for any value of the Higgs mass [2].

Extensions of the Standard Model may provide desirable features that affect the conclusions about the electroweak phase transition. In particular, the MSSM contains additional particles which could significantly change certain aspects of the analysis of the strength of the phase transition. The number of unknown parameters is increased, and a detailed study of their effects must be performed.

In chapter 4 we construct an effective theory of the MSSM requiring the presence of a single light Higgs at the phase transition. This allows us to use the constraints, obtained by lattice calculations, on the first order phase transition for the effective 3D theory of the Standard Model. The fifth chapter explores the MSSM parameter space to determine the regions for which the phase transition is sufficiently first order.
Chapters 4 and 5, as well as the results presented in Appendices B, C and F are original contributions to this problem. The contents of chapter 4 and appendices A-D have appeared in reference [3].
Chapter 2
Overview of Baryon Number Violation and the Electroweak Phase Transition
Chapter 3
Finite Temperature Field Theory
Chapter 4
High Temperature Dimensional Reduction

See hep-ph/9605266.
Chapter 5

The Electroweak Phase Transition in the MSSM

5.1 Introduction

Many different authors have studied the order of the electroweak phase transition in the Minimal Supersymmetric Standard Model (MSSM). Most of these studies relied on a one- and two-loop finite temperature effective potential analysis of the phase transition \[4, 5, 6, 7, 8\] in which the stops were expected to make the most significant contribution from supersymmetric particles. The authors of these studies, in the limit of a large pseudoscalar Higgs mass, \( m_A \to \infty \), have identified a region of parameter space for which the transition is strong enough. This corresponds to low values of \( \tan \beta \), and values of the soft supersymmetry breaking right stop mass, \( m_{U_3}^2 \), which are small or even negative \(^1\).

A different approach consists of separating the perturbative and non-perturbative aspects of the phase transition. This is performed through the perturbative construction of effective three dimensional theories, and a subsequent lattice analysis of the reduced theory \[9, 10, 2, 11\]. In addition, this also provides a check to perturbative results for the phase transition. One may construct effective 3D theories for different models at finite temperature to study the electroweak phase transition. As shown in chapter 4, for the case in which the reduced theory contains a single light Higgs field, characterized by a Higgs self-coupling, \( \lambda_3 \), and an effective 3D gauge coupling, \( g_3 \), the condition for a sufficiently strong first order phase transition becomes \[2\]

\[
x_c = \frac{\lambda_3}{g_3^2} \lesssim 0.04. \tag{5.1}
\]

The quantity \( x_c \) is a function of the different parameters appearing in the original 4D model.

The analysis of parameter space for the reduced theory of the Standard Model was performed in \[2, 11\]. The conclusion was that for no value of the Higgs mass is

---

\(^1\)In reference \[6\] the analysis was extended for the full range of allowed values of \( m_A \). It was found that larger values of \( m_A \) are favored.
electroweak baryogenesis possible. Although inconsistent with current experimental constraints, purely perturbative studies had allowed electroweak baryogenesis for small enough values of the Higgs mass.

In chapter 4 we constructed a 3D theory for the MSSM including Standard Model particles and additional corrections arising from gauginos, higgsinos and all squarks and sleptons. Additionally, one-loop corrections from dimensional reduction to all couplings in the model were calculated. We use these results to explore the MSSM parameter space in order to determine the regions for which electroweak baryogenesis may occur. An important element in this analysis is to establish the relation between the running parameters in the original 4D theory and physical parameters. This is done in appendix E. In section 5.2 we discuss our numerical results from scanning parameter space. In section 5.3 we conclude.

In references [12, 13] 3D theories for the MSSM have recently been constructed, and analyzed to determine the regions of parameter space where the criteria given by equation (5.1) is fulfilled. In these papers only the contribution from gauge bosons, higgses and third generation quarks and squarks to the 3D reduction was included. Nor did they incorporate one-loop corrections to all of the parameters in the theory. In our work all one-loop corrections have been included, as well as contributions from all SUSY particles. This allows us to investigate the effect of extra supersymmetric particles, in addition to third generation squarks, on the strength of the phase transition. Furthermore the results of [12, 13] are not totally in agreement. In reference [13] the results agreed basically with those found in the perturbative effective potential analysis. The most favorable region of parameter space was found to be \( m_h \lesssim m_W \) (low \( \tan \beta \)), small stop mixing, \( m_{U_3} \lesssim 50 \) GeV and \( m_A \gtrsim 200 \) GeV. In addition to this region, reference [12] found another region of parameter space in which arbitrary values of \( \tan \beta \) and a range of values for the pseudoscalar Higgs mass, \( 40 \lesssim m_A \lesssim 80 \) GeV, give a sufficiently strong phase transition.

### 5.2 Discussion of Numerical Results

As mentioned above, the quantity \( x_c \) becomes a function of the parameters in the model: \( x_c = x_c(M_A, m_o, \mu, m_{\tilde{g}}, A, \tan \beta, T_c) \), as well as of the gauge couplings. \( A \) is the trilinear soft SUSY breaking parameter, taken to be universal. \( \mu \) is the supersymmetric mass parameter. \( m_o, m_{\tilde{g}}, m_{\tilde{g}} \) denote the common squark/slepton mass at the SUSY breaking scale, the \( SU(2) \) gaugino and gluino mass respectively. We take \( M_A \) to be the physical pole mass of the pseudoscalar Higgs, and \( \tan \beta \) is the ratio of
the vacuum expectation values of the Higgs fields in the renormalized zero temperature theory. The ratio in $x_c$ also depends indirectly on the scale $M_{SUSY}$. $M_{SUSY}$ is the scale at which we have assumed a universal mass parameter for squarks and sleptons, as well as the scale at which the SUSY boundary conditions on the quartic Higgs couplings appearing in the Higgs potential are imposed [4]. All of the above mentioned quantities are our input parameters chosen in such a way that experimental and theoretical constraints are satisfied. In addition, in order to study the effect of the masses of third generation squarks we keep the stop soft supersymmetry breaking masses $m_{Q_3}, m_{U_3}$, as independent parameters.

We define the critical temperature $T_c$, from the requirement of the existence of a direction in field space at the origin of the Higgs potential for which the transition to the minimum of the potential in the broken phase can occur classically. In the 3D lattice calculations [10, 2] the critical temperature is defined by the temperature at which phase coexistence disappears. In general, these two values of temperature are close. In fact, the actual value of the critical temperature lies between these two values. We will remark later on the circumstances under which there can be a significant difference arising from this distinction.

The procedure to evaluate $x_c$ is the following:
- restrict initial parameter space by experimental constraints on the masses of the particles,
- calculate the critical temperature for a fixed set of parameters,
- check the validity of the high temperature expansion,
- check the adequate suppression of non-renormalizable terms and the validity of perturbation theory,
- determine the value of $x_c$.

Throughout our analysis we will concentrate on the regions of parameter space which describe an effective theory in which there is a single light scalar and thus the bound given by equation (5.1) is valid. However, we mention that another possibility is the scenario in which two scalars, e.g. one Higgs and additionally a right stop, are both nearly massless at $T_c$ [7].

5.2.1 Dependence on $\tan \beta$ and $M_A$

As is well known and is shown in appendix E, we can parametrize the Higgs sector in terms of two quantities: $\tan \beta$ and the pole mass of the pseudoscalar Higgs boson, $M_A$. The range of variation explored for these input parameters is taken as follows:
- we allow $M_A$ to vary between 40-300 GeV. The lower bound is taken from experimental considerations, the upper limit to insist on the validity of the high-temperature expansion.

- we vary $\tan \beta$ between 1.25 and 13.3. For values of $\tan \beta$ outside of this range the results have qualitatively the same behaviour.

In general, the masses of all particles are taken such that the high temperature expansion is valid. The experimental constraints we impose on the masses are: for stop masses $m_{\tilde{t}_2} \gtrsim 50$ GeV, $m_{\tilde{t}_1} \gtrsim m_t$, for first and second generation squarks $m_{\tilde{q}} \gtrsim 200$ GeV, sleptons $m_{\tilde{e}} \gtrsim 50$ GeV, the gluino mass either $\lesssim 1$ GeV or $\gtrsim 150$ GeV [15,16]. In addition, the value of the left soft supersymmetry breaking stop mass $m_{Q_3}$ must be such that the contribution from stops and sbottoms to the $\rho$ parameter is not too large.

The critical temperature $T_c$, is evaluated from the temperature dependent Higgs mass matrix as explained in chapter 4. The requirement of a zero eigenvalue of this mass matrix will define the direction in field space for which the curvature of the potential at the origin is zero. We shall make a few general remarks of the dependence of the value of the critical temperature with respect to the input parameters in the model. As the value of $\tan \beta$ decreases, the critical temperature also decreases. The dependence on the pseudoscalar Higgs mass is not very strong, but as $M_A$ increases the critical temperature decreases. For a given value of $\tan \beta$, the value of $T_c$ varies at most on the order of 5 GeV as $M_A$ takes on values in the range mentioned above. The only other parameters which significantly affect the critical temperature are the masses of the squarks/sleptons. With respect to the dependence of critical temperature on the squark masses, we note in particular that a lower value of the right stop supersymmetric breaking mass increases the value of the critical temperature. We mention that we have checked that the difference in the critical temperature from the diagonalization of the Higgs mass matrix, equation (10) in [8] and from equation (7.9) in [13] is extremely small ($\lesssim 0.1$ GeV) and for our purposes negligible.

We have placed all of the plots of the results of the analysis of the strength of the phase transition into appendix F. Figure F.1 shows the value of $x_c$, for the case of no squark mixing, as a function of the pseudoscalar Higgs mass for values of $\tan \beta$ ranging from 1.25 to 13.3. We have fixed the other parameters to be $m_o = 50$ GeV, $m_1 = 50$

\footnote{In the dimensional reduction procedure, the explicit dependence on all Yukawa couplings was kept. For our numerical analysis, except for the top Yukawa coupling, we will take the value of these couplings to be zero. It might be thought that for large values of $\tan \beta$ the bottom Yukawa coupling can also be relevant. We have explicitly checked that this is not the case.}
GeV, \( m_{\tilde{g}} = \frac{\alpha_\text{em}}{\alpha_\text{W}} m_1 \), \( M_{\text{weak}} = m_t \), \( M_{\text{SUSY}} = 10^{12} \text{ GeV} \). For large values of \( \tan \beta \), there is no value of \( M_A \) for which \( x_c \) fulfills the condition given by equation (5.1), and \( x_c \) varies very little as you vary \( M_A \). However, for low values of \( \tan \beta \) and large enough values of \( M_A \), \( x_c \) can be small enough for the phase transition to be sufficiently first order. The strong dependence of \( x_c \) on the value of the pseudoscalar Higgs mass, for low values of the ratio of the vacuum expectation values, arises basically through the dependence of the quantity \( \tilde{\lambda}_3 \) in equation (5.1) on the mixing angle, see equation (17) in [3]. It is easy to see to lowest order the same dependence on \( M_A \) arising in finite temperature effective potential analysis [6].

5.2.2 Dependence on other parameters

We now discuss the consequences of the variation of the other parameters in the model on the strength of the phase transition. Figures F.2 show \( x_c \) vs. \( M_A \), for \( \tan \beta = 1.25, 1.75, 13.3 \), and for two different values of the SUSY scale. The value of the SUSY scale makes an important change in \( x_c \). This is expected because the weak scale values for the masses and couplings change when we change the SUSY scale (see appendix E). In particular, the value of the quartic Higgs couplings, which largely determine the value of \( \tilde{\lambda}_3 \) in equation (5.1), is changed very much as we vary the SUSY scale.

We plot in figure F.3 the dependence of \( x_c \) on the soft-supersymmetry breaking mass \( m_o \), for \( \tan \beta = 1.25, 1.75 \). We show the results for two values of the soft supersymmetry breaking mass for the squarks/sleptons, the solid line corresponds to \( m_o = 50 \) GeV, the dotted line to \( m_o = 150 \) GeV. We see that, keeping the gluino and \( SU(2) \) gaugino mass fixed, the strength of the phase transition becomes weaker for a larger value of \( m_o \). However, we cannot lower the value of \( m_o \) much due to experimental constraints on the squark/slepton masses. The dependence of the critical temperature and of \( x_c \) on the mass of the \( SU(2) \) gaugino is very small. When we increase the value of the gluino mass by an amount of order of 50 GeV, it produces a small variation in the value of the critical temperature, and changes the value of \( x_c \) by an extremely small amount.

We plot in figure F.4 the variations of \( x_c \) for different values of \( \mu \) and the \( A \) parameter. The solid line corresponds to no mixing, the dashed line to \( \mu = 200 \) GeV, and the dotted line to \( A = 150 \) GeV. A small value of \( \mu \) is favored in most of parameter space

\footnote{Note that in our approximation the values of the first and second generation squarks and the slepton masses are fixed by \( m_o, m_{\tilde{g}}, m_1 \), through the renormalization group running, and are constant as we vary \( \tan \beta \) and \( M_A \). The left and right stop masses change as we move on the curves plotted in figure F.1.}
while changing $A$ makes a negligible effect.

In figure F.5, we plot the influence of the mass of the top, which implies a change in the top Yukawa coupling, on the strength of the phase transition. We show the effect for two values of $\tan \beta = 1.5, 1.75$, where the dashed line corresponds to $m_t = 165$ GeV, the solid line to $m_t = 175$ GeV, and the dotted line to $m_t = 190$ GeV. The same dependence on top mass was observed in [13]. The results of varying the right stop soft supersymmetry breaking mass, $m_{U_3}$, are shown in figure F.6. We plot the values of $x_c$ as a function of the pseudoscalar Higgs mass for $\tan \beta = 1.25, 1.5, 1.75$, and $m_{U_3} = 50, 100, 200$ GeV. We see that we recover the expected dependence on $m_{U_3}$ [6, 7, 13]: lowering $m_{U_3}$ causes $x_c$ to decrease.

We have also compared the results of our general analysis, to simplifying cases which did not include the effects from all supersymmetric particles. In all cases we have kept the full one-loop corrections to the 3D couplings. In figures F.7 and F.8, we plot $x_c$ vs. $M_A$ for $\tan \beta = 1.25, 1.5, 1.75, 13.3$. The solid line corresponds to the general case including all supersymmetric particle contributions to the dimensionally reduced theory. The dotted line represents the case in which gluino and $SU(2)$ gaugino contributions are neglected. In figure F.7, we allow the running stop masses to vary as functions of $\tan \beta$ and $M_A$. In figure F.8, the values of the running stop masses are fixed. Figure F.9 shows the variation of $x_c$ with $M_A$ for three different cases. The solid line corresponds to our general analysis, as given above. The dashed line is for the case in which only the effect of third generation squarks is included. The gluino/gaugino thermal screening contribution to the 3D masses of the squarks is also excluded. The dotted line corresponds to the case in which we include all of other squarks and sleptons, ignoring all gluino and gaugino contributions to the three dimensional theory. In order to compare the approximations, the masses of the squarks and sleptons have been fixed to be the same in all three cases. As expected, the dependence on the values of $\tan \beta$ and $M_A$ is very similar in each case. We can see that as a result of including the contributions of all scalars the strength of the first order phase transition is enhanced. We have checked for all cases that the dependence on the value of the right stop soft supersymmetric breaking mass has the same effect of decreasing the value of $x_c$. However, we see that for the case in which the effect of all squarks and sleptons is included the change induced in the strength of the phase transition is not so large as for the case in which only the contribution from third generation squarks [13] was considered. We have also compared the two cases in which only third generation squarks were included with and without thermal screening arising from the gluino and gaugino. The differences in the values of $x_c$ for this case are negligible.
5.2.3 Validity of approximations

Previous treatments of this problem \[5, 6, 8, 13\] were content with the result of the dependence of the strength of the phase transition on \(\tan \beta\) and \(M_A\). We wish to emphasize however that for some regions of parameter space it may not be correct to conclude from this analysis that the phase transition is not sufficiently strongly first order. In particular, for some regions of parameter space \(x_c\) is an extremely strong function of temperature near the critical temperature. This occurs when the mixing angle \(\theta\), which diagonalizes the 3D Higgs mass matrix at finite temperature, varies rapidly in certain regions of temperature. In fact, what is happening is that the diagonal elements of the 3D Higgs mass matrix are becoming equal for a certain value of the temperature. Nevertheless, the two eigenvalues of the mass matrix differ substantially close to \(T_c\). That is, one the eigenvalues is much larger than the other, the latter becoming equal to zero at the critical temperature. This indicates that our procedure for integrating out the heavy Higgs doublet is correct. If the critical temperature for the phase transition is close to the value of the temperature where this rapid variation occurs, then the value of \(\theta\) and consequently of \(x_c\) will vary exceedingly close to \(T_c\).

The value of the temperature at which this rapid variation occurs depends strongly on the value of the pseudoscalar Higgs mass. As \(M_A\) increases this temperature also increases. In addition, for larger values of \(\tan \beta\) the mixing angle dependence on the temperature is less strong than for low values of \(\tan \beta\). We show in figures F.10, F.11 the dependence of \(\theta\) on the temperature. In figure F.10 we have fixed the value of \(M_A = 40\) GeV, where the solid line corresponds to \(\tan \beta = 1.25\) and the dashed line to \(\tan \beta = 13.3\), keeping all other parameters fixed. Figure F.11 is the same thing for \(M_A = 300\) GeV. In figure F.12, we plot the value of \(x_c\) as a function of temperature close to \(T_c\) for \(M_A = 40\) GeV. We see that a variation on the order of 5 GeV in the temperature induces a change in the value of \(x_c\), \(\Delta x_c \sim .13\). For the case in which \(M_A = 300\) GeV, the variation around \(T_c\) implies a variation of \(\Delta x_c \sim 0.005\). We conclude that possibility of large uncertainty in the mixing angle is relevant only for low values of \(\tan \beta\) and \(M_A\).

Low values of \(\tan \beta\) imply a larger value of the top Yukawa coupling. This in turn, decreases the value of \(m_{\tilde{U}_3}^2\), which could even take on negative values. However, we cannot analyze this situation as it implies a breakdown of our approximations. A very light right stop cannot be integrated out in the construction of the effective theory. The effect of the gluino mass is to increase the values of the running soft supersymmetry breaking mass parameters for squarks as the energy scale decreases. For a given value
of $m_o$, the masses of the running squarks and sleptons will be much smaller as the gluino mass decreases. Consequently, a light gluino case, in which the masses of the gluino and $SU(2)$ gaugino are taken to be zero, can easily accommodate the scenario originally proposed in [7] in which there are a light Higgs and a light right stop at the phase transition.

5.3 Conclusions

To summarize, we conclude that a sufficiently strong electroweak phase transition can be fulfilled in the MSSM for values of $\tan \beta < 1.75$. The value of $x_c$ decreases as the pseudoscalar mass increases. The pseudoscalar Higgs mass can be as low as $M_A = 100$ GeV, depending on the value of $\tan \beta$, and still give the required $x_c \leq 0.04$. For low values of $\tan \beta$ and $M_A$ the rapid variation of the mixing angle of the 3D Higgs mass matrix, as a function of the temperature, does not allow us to conclude definitely whether it is possible to have a sufficiently strong phase transition. A precise determination of the critical temperature is needed in this case which requires consideration of tunneling. The inclusion of all supersymmetric scalars has the effect on enhancing the strength of the phase transition. In particular, it is not necessary to have a very light right stop for equation (5.1) to be fulfilled. That is, in contrast to purely pertubative analysis, having universal soft supersymmetry breaking masses at the SUSY breaking does not impede the phase transition from being sufficiently first order. The effect of the parameters $A, \mu, m\tilde{g}, m\tilde{\chi}, m_t$ is either small or increases the value of $x_c$. Using the results presented in appendix E we can obtain the values of the lightest Higgs boson mass. For the allowed region of parameter space, we observe that the corresponding upper bound on the lightest physical Higgs mass, $m_h \leq 70$ GeV, is very close to being experimentally accessible.
Appendix A
MSSM in Four Dimensions

See hep-ph/9605266.
Appendix B

Explicit Relationships between Parameters

See hep-ph/9605266.
Appendix C

2HDM and NMSSM

See hep-ph/9605266.
Appendix D

Appendix of Finite Temperature Formulae

See hep-ph/9605266.
Appendix E

Relation to Physical Parameters

The one-loop relations of the running parameters in the $\overline{MS}$-scheme to the parameters of the effective 3D theory are given in appendix B. Additionally, the running parameters in the $\overline{MS}$-scheme should be given with the same accuracy in terms of physical parameters. This requires one-loop renormalization of the 4D zero temperature theory. In the construction of the effective 3D theory we started out with a 4D Lagrangian with running parameters at the scale $\mu_4$. Generically, we can express any coupling or mass parameter of the dimensionally reduced 3D theory in terms of the running parameters by

$$F_{3D} = f(\mu_4) - \beta^b_f L_b - \beta^f_f L_f,$$

(E.1)

(plus additional temperature squared dependent terms for the three dimensional mass terms), where $L_b$ and $L_f$ are defined in appendix D. The superscripts $b, f$ denote the bosonic and fermionic contributions respectively, to the one-loop beta function of the corresponding parameter. All scale dependence, implicit in the quantities $L_b$ and $L_f$, of the 3D parameters will drop out when we relate the running parameters to physical variables.

For the masses and couplings we use the zero temperature one-loop renormalization group equations to relate the values of the parameters at the scale $\mu_4$ to their values at the weak scale. We approximate the beta function coefficients to be scale independent. Ignoring particle decoupling we can then write,

$$f(M_{weak}) = f(\mu_4) - (\beta^b_f + \beta^f_f) \log(\frac{\mu_4}{M_{weak}}),$$

(E.2)

Substituting equation (E.2) into equation (E.1) the explicit dependence on $\mu_4$ is eliminated and the relevant logarithmic ratio is $M_{weak}/T$.

We now discuss the relevant couplings and masses which we are concerned with and which must be treated separately.

- The strong gauge coupling, $g_s$, only enters through the expressions for the beta-function coefficients. We use $\alpha_s(m_t) = 0.12$. 
- The parameters $\mu$ and $A$ are not universal at one-loop. We do not try to fix them in terms of physical parameters. However, we do require they satisfy experimental and theoretical constraints. $A$ must not be too large in order to avoid colour symmetry breaking. $\mu$ is the higgsino mass in the unbroken phase and must satisfy the high temperature expansion criterion.

- Even though in principle we could fix exactly the top Yukawa coupling in terms of pole masses, since the value of the top mass is not known exactly we will not include the finite corrections. The top Yukawa coupling is taken to be

$$f_t(M_{\text{weak}}) = m_t(M_{\text{weak}})\sqrt{2}/(\sin \beta v).$$  \hspace{1cm} (E.3)

- We obtain the value of the quartic Higgs couplings at the weak scale by running the $SU(2)$ gauge coupling from its measured value at the weak scale, $g(M_{\text{weak}}) = \frac{2}{3}$, up to the SUSY breaking scale $M_{\text{SUSY}}$. The Higgs self coupling constants $\lambda_i$ at $M_{\text{SUSY}}$ are obtained imposing the boundary conditions given in [14]. Finally, we obtain the values of $\lambda_i(M_{\text{weak}})$ by integrating the renormalization group equations from the SUSY breaking scale to the weak scale. With this procedure we incorporate the one-loop leading logarithmic corrections to the weak gauge and quartic couplings.

- We have assumed a common soft supersymmetry breaking mass term $m_o$, for squarks and sleptons at the SUSY breaking scale. Using the renormalization group equations which have the form of equation (E.2), we obtain $m_{Q_i}$, $m_{U_i}$, $m_{D_i}$ at the weak scale. Here $m_{Q_i}$, is the soft supersymmetry breaking left squark/slepton mass, and $i$ denotes a family index. $m_{U_i}$ is the right handed soft supersymmetry breaking up-type squark/slepton mass and $m_{D_i}$ is the right handed soft supersymmetry breaking down-type squark/slepton mass. We then require that the physical squark/slepton masses $m_{\tilde{q}_i,1}$, $m_{\tilde{q}_i,2}$ satisfy the experimental constraints mentioned in section 5.2.

The physical squark/slepton masses $m_{\tilde{q}_i,1}$, $m_{\tilde{q}_i,2}$ are the eigenvalues of the squared mass matrices

$$M_{\tilde{q}}^2 = \begin{pmatrix} m_{\tilde{q}_L}^2 & m_{X_q}^2 \\ m_{X_q}^2 & m_{\tilde{q}_R}^2 \end{pmatrix},$$  \hspace{1cm} (E.4)

where $m_{\tilde{q}_L}^2$, $m_{\tilde{q}_R}^2$ are the mass terms for the left-handed and right-handed squarks in the broken phase, while $m_{X_q}^2$ denotes the left-right squark mixing parameter. For the stops these mass parameters are given by

$$m_{\tilde{t}_L}^2 = m_{\tilde{Q}_3}^2 + f_t^2 v_2^2 + \frac{1}{4}(g^2 - \frac{1}{3}g^{'2}) (v_1^2 - v_2^2)$$  \hspace{1cm} (E.5)
\[ m^2_{t_R} = m^2_{U_3} + f^2_i v^2_1 + \frac{1}{3} g^2 (v^2_1 - v^2_2) \]  
(E.6)

\[ m^2_{\tilde{X}_i} = f_i (A v_2 - \mu v_1). \]  
(E.7)

Using coupling and vacuum expectation values at the scale \( M_{\text{weak}} \), \( m_{\tilde{q}_{i,1}} \), \( m_{\tilde{q}_{i,2}} \) obtained from (E.4) are not pole masses for the squarks and sleptons. However, we cannot do better than this since actual values of the squark and slepton masses are not known.

- In order to express the mass terms in the Higgs potential in terms of physical parameters we follow the procedure of references [17, 18, 14] to obtain the renormalization group improved tree level potential. That is, in equation (22) of [3] we use the quartic Higgs couplings at the weak scale, obtained by integrating the renormalization group equations. Minimizing the potential gives

\[ m^2_1 = -m^2_3 \tan \beta - v^2 \cos^2 \beta (2 \lambda_1 + \lambda_3 \tan^2 \beta + \lambda_4 \frac{\tan^2 \beta}{2}) \]  
(E.8)

\[ m^2_2 = -m^2_3 \cot \beta - v^2 \sin^2 \beta (2 \lambda_2 + \lambda_3 \cot^2 \beta + \lambda_4 \frac{\cot^2 \beta}{2}), \]  
(E.9)

where all quantities are evaluated at the weak scale.

The ratio of the vacuum expectation values of the neutral components of the Higgs doublets defines \( \tan \beta = \frac{v_2}{v_1} \).

As is well known, the model contains five physical Higgs bosons: a charged pair \( m_{H^\pm} \), two neutral \( CP \)-even scalars \( m_{h,H} \) , and a neutral \( CP \)-odd scalar \( m_A \) [14, 19, 20]. The running masses of the physical Higgs particles at the weak scale are given by [14],

\[ m^2_A = -m^2_3 (M_{\text{weak}}) / \sin \beta \cos \beta \]  
(E.10)

\[ m^2_{H^\pm} = -m^2_3 (M_{\text{weak}}) / \sin \beta \cos \beta - \frac{1}{2} \lambda_4 v^2. \]  
(E.11)

The \( CP \)-even Higgs mass matrix elements are

\[ M^2_{11} = -m^2_3 \tan \beta + v^2 \lambda_1 \cos^2 \beta \]  
(E.12)

\[ M^2_{12} = M^2_{21} = m^2_3 + (\lambda_3 + \lambda_4) v^2 \sin \beta \cos \beta \]  
(E.13)
\[ M_{\text{22}}^2 = -m_3^2 \tan \beta + v^2 \lambda_2 \sin^2 \beta. \]  

(E.14)

with corresponding eigenvalues given by

\[ m_{h,H}^2 = \frac{1}{2} \left( \text{tr} M^2 \mp \left( (\text{tr} M^2)^2 - 4 \det M^2 \right)^{\frac{1}{2}} \right), \]

(E.15)

where \( \text{tr} M^2 = M_{11}^2 + M_{22}^2 \) and \( \det M^2 = M_{11}^2 M_{22}^2 - (M_{12}^2)^2 \). The mixing angle \( \alpha \) is determined from

\[
\sin 2\alpha = \frac{2M_{12}^2}{(\text{tr} M^2)^2 - 4 \det M^2}^\frac{1}{2}, \\
\cos 2\alpha = \frac{M_{11}^2 - M_{22}^2}{(\text{tr} M^2)^2 - 4 \det M^2}^\frac{1}{2}.
\]

(E.16)

To correct for the fact that the effective potential is defined at zero external momentum, the pole masses are obtained from the expression for the running masses by [17, 18]

\[ M_\phi^2 = m_\phi^2 + \text{Re} \Delta \Pi_\phi(M_\phi^2) \]  

(E.17)

for \( \phi = h, H, A, H^\pm \), and the self-energy is defined by

\[ \Delta \Pi_\phi(M_\phi^2) = \Pi(M_\phi^2) - \Pi(0). \]

(E.18)

We calculate the Higgs self-energies including corrections from top and stop loops. These results can be found for example in [18].

For the pseudoscalar Higgs we have

\[
\Delta \Pi_A(M_A^2) = -\frac{3}{16\pi^2} f_t^2 \cos^2 \beta M_A^2 F(m_t^2, m_t^2, M_A^2) \\
+ \frac{3}{16\pi^2} f_t^2 (A \cos \beta + \mu \sin \beta)^2 F(m_t^2, m_t^2, M_A^2)
\]

(E.19)

and for the lightest scalar Higgs

\[
\Delta \Pi_h(M_h^2) = -\frac{3}{8\pi^2} f_t^2 \cos^2 \alpha (-2m_t^2 + \frac{1}{2} M_h^2) F(m_t^2, m_t^2, M_h^2) \\
+ \sum_{i,j} \frac{3}{16\pi^2} C_{hij}^2 F(m_{h_i}^2, m_{h_j}^2, M_h^2)
\]

(E.20)

where
\[ F(m_1^2, m_2^2, p^2) = \int_0^1 dx \log \frac{m_1^2(1-x) + m_2^2x - p^2(1-x)}{m_1^2(1-x) + m_2^2x} \]

\[ = -1 + \frac{1}{2} \left( \frac{m_1^2 + m_2^2 - \delta}{m_1^2 - m_2^2} \right) \log \frac{m_2^2}{m_1^2} \]

\[ + \frac{1}{2} r \log \left[ \frac{(1+r)^2 - \delta^2}{(1-r)^2 - \delta^2} \right] \]  

(E.21)

with

\[ \delta = \frac{m_1^2 - m_2^2}{p^2} \]  

(E.22)

\[ r = \left[ (1 + \delta)^2 - \frac{4m_1^2}{p^2} \right]^{\frac{1}{2}} \]  

(E.23)

Additionally,

\[ C_{h_{ij}} = \frac{4 \sin^2 \theta_W m_2^2 Z}{3 v} \sin(\beta + \alpha) \left[ \delta_{ij} + \frac{3 - 8 \sin^2 \theta_W}{4 \sin^2 \theta_W} r^{1i} r^{1j} \right] \]

\[ - f_i^2 v \sin \beta \cos \alpha \delta_{ij} - \frac{1}{\sqrt{2}} f_i (A \cos \alpha - \mu \sin \alpha)(r^{1i} r^{2j} + r^{1j} r^{2i}) \]  

(E.24)

where \( r^{11} = r^{22} = \cos \tau \), and \( r^{12} = -r^{21} = \sin \tau \). The mixing angle \( \tau \) diagonalizes the stop mass matrix (E.4). These relations and the expressions given in appendix B give us an expression for \( x_c \) as a function of physical parameters.
Figure F.1: Plot of $x_c$ vs. $M_A$ for several different values of $\tan\beta$. The solid line corresponds to $\tan\beta = 13.3$, the dashed line to $\tan\beta = 1.75$, the dashed-dot line to $\tan\beta = 1.5$, and the dotted line to $\tan\beta = 1.25$. 

Appendix F

Figures
Figure F.2: Plot of $x_c$ vs. $M_A$, for $\tan\beta = 1.25, 1.75, 13.3$. The solid line corresponds to $M_{SUSY} = 10^{12}$ GeV, the dotted line to $M_{SUSY} = 10^3$ GeV.

Figure F.3: Plot of $x_c$ vs. $M_A$ for $\tan\beta = 1.25, 1.75$. The solid line corresponds to $m_o = 50$ GeV, the dotted line to $m_o = 150$ GeV.
Figure F.4: Plot of $x_c$ vs. $M_A$ for $\tan\beta = 1.25, 1.75$. The solid line corresponds to the case with no squark mixing, dashed line to $\mu = 200$ GeV, dotted line to $A = 150$ GeV.

Figure F.5: Plot of $x_c$ vs. $M_A$ for $\tan\beta = 1.5, 1.75$. The dashed line corresponds to $m_t = 165$ GeV, the solid line to $m_t = 175$ GeV, the dotted line to $m_t = 190$ GeV.
Figure F.6: Plot of $x_c$ vs. $M_A$ for $\tan \beta = 1.25, 1.75$. The solid line corresponds to $m_{U_3} = 200$ GeV, the dashed line to $m_{U_3} = 100$ GeV, the dotted line to $m_{U_3} = 50$ GeV.

Figure F.7: Plot of $x_c$ vs. $M_A$ including (neglecting) gluino contributions correspond to the solid (dashed) lines, for four different values of $\tan \beta$. The stop soft supersymmetry breaking masses vary as functions of $\tan \beta$ and $M_A$. 
Figure F.8: Plot of $x_c$ vs. $M_A$ including (neglecting) gluino contributions correspond to the solid (dashed) lines, for four different values of $\tan \beta$. We have fixed the values of the stop soft supersymmetry breaking masses.

Figure F.9: Plot of $x_c$ vs. $M_A$ including (neglecting) gluino contributions solid (dashed) lines, and including contributions arising only from third generation squarks dotted lines, for two different values of $\tan \beta$. We take fixed values of the stop soft supersymmetry breaking masses in all three cases.
Figure F.10: Plot of $\theta$ vs. $T$ for $M_A = 40$ GeV. The solid line corresponds to $\tan \beta = 1.25, T_c = 63.3$ GeV, the dashed line is for $\tan \beta = 13.3, T_c = 75.6$ GeV.

Figure F.11: Plot of $\theta$ vs. $T$ for $M_A = 300$ GeV, $T_c = 60, 75.6$ GeV. The solid line corresponds to $\tan \beta = 1.25$, the dashed line to $\tan \beta = 13.3$, 
Figure F.12: Plot of $x_c$ vs. $T$ for $\tan \beta = 1.25$, $M_A = 40$ GeV, $T_c = 60.8$ GeV.
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