Thermal masses in leptogenesis

Clemens P. Kießig and Michael Plümacher

Max-Planck-Institut für Physik, Föhringer Ring 6, 80805 München, Germany

Abstract. We investigate the validity of using thermal masses in the kinematics of final states in the decay rate of heavy neutrinos in leptogenesis calculations. We find that using thermal masses this way is a reasonable approximation, but corrections arise through quantum statistical distribution functions and leptonic quasiparticles.

Keywords: leptogenesis, thermal bath, thermal masses, thermal field theory, hard thermal loop resummation, dispersion relation, plasmino

PACS: 98.80.Cq, 11.10.Wx

1. INTRODUCTION

Leptogenesis [1, 2] is an extremely successful theory in explaining the baryon asymmetry of the universe by adding three heavy right-handed neutrinos $N_i$ to the standard model,

$$\delta \mathcal{L} = \bar{N}_i i \partial_\mu \gamma^\mu N_i - \lambda_{i\alpha} \bar{N}_i \phi^\alpha \ell_\alpha - \frac{1}{2} M_i \bar{N}_i N_i + h.c.,$$

(1)

with masses $M_i$ at the GUT-scale and Yukawa-couplings $\lambda_{i\alpha}$ similar to the other fermions. This also solves the problem of the light neutrino masses via the see-saw mechanism without fine-tuning.

The heavy neutrinos decay into lepton and Higgs after inflation, the decay is out of equilibrium since there are no gauge couplings to the SM. If the CP asymmetry in the Yukawa-couplings is large enough, a lepton asymmetry is created by the decays which is then partially converted into a baryon asymmetry by sphaleron processes. As temperatures are extremely high, interaction rates need to be calculated using thermal field theory (TFT) [3] rather than vacuum quantum field theory.

2. THERMAL CORRECTIONS

2.1. Thermal masses

When using bare thermal propagators in TFT [4], one can encounter IR singularities and gauge dependent results. In order to cure this problem, the hard thermal loop (HTL) resummation technique has been invented [5, 6], where for soft momenta $K \ll T$, resummed propagators are used. For a scalar field,

$$i\Delta^* = i\Delta + i\Delta(-i\Sigma)i\Delta + \ldots = \frac{i}{\Delta^{-1} - \Sigma} = \frac{i}{K^2 - m^2 - \Sigma}.$$  

(2)
The self energy $\Sigma$ then acts as a thermal mass $m_{th}^2(T) := m^2 + \Sigma$.

2.2. May you put in thermal masses “by hand”?

Consider the decay rate $\Gamma(N_1 \to LH)$. The decay density which appears in the Boltzmann equations is

$$\gamma_D = \int d\hat{p}_N d\hat{p}_L d\hat{p}_H (2\pi)^4 \delta^4(P_N - P_L - P_H) \left| M \right|^2 f_N(1 + f_H)(1 - f_L),$$

(3)

where $d\hat{p}_i = \frac{p_i^3}{(2\pi)^4 2E_i}$ and we are using Fermi-Dirac and Bose-Einstein statistics with Fermi blocking $(1 - f)$ and Bose enhancement factors $(1 + f)$. Thermal masses are put in “by hand” and appear in the dispersion relation of Higgs and lepton doublet $E_{H,L} = \sqrt{m_{H,L}^2(T) + p_{H,L}^2}$. The decay is only allowed if $M_N > m_L(T) + m_H(T)$.

Ref. [3] included thermal masses this way, but chose to approximate the quantum statistical distribution functions with Maxwell-Boltzmann distributions, where blocking and enhancement factors disappear. The deviation of interaction rates from the quantum statistical case is typically around 20–30%. This ad-hoc treatment of thermal masses seems intuitive and plausible, but requires a closer look since TFT describes Green’s functions rather than external states and their kinematics.

The consistent way [4,7,8] to deal with TFT effects on external states is by looking at the discontinuities of the corresponding loop diagrams, where all propagators are treated thermally and the external states like vacuum states. We consider the self-energy of the heavy neutrino (Fig. 1) and resum the Higgs and the lepton propagator using the HTL resummation technique [4] in order to account for thermal masses. In the imaginary time formalism, the self-energy is

$$\Sigma(N_1) = -\left(\lambda'\lambda\right)_{11} T \sum_{P_L=2\pi inT} \frac{d^3P_L}{(2\pi)^3} S^*(P_L)D^*(P_H).$$

(4)

The resummed Higgs propagator is $D^*(P_H) = \frac{1}{P_H^2 - m_H^2(T)}$, where $m_H^2(T) = \left(\frac{3}{16}g_2^2 + \frac{1}{16}g_Y^2 + \frac{1}{4}Y^2 + \frac{1}{2}\lambda\right)T^2$. The resummed lepton propagator is

$$S^*(K) = \frac{1}{2D^+}(\gamma_0 - \hat{k} \cdot \gamma) + \frac{1}{2D^-}(\gamma_0 + \hat{k} \cdot \gamma),$$

(5)
where
\[
D_\pm(K) = -k_0 \pm k + \frac{m_L(T)^2}{k} \left( \pm 1 - \frac{k_0 - k}{2k} \ln \frac{k_0 + k}{k_0 - k} \right) \tag{6}
\]
and
\[
m_L^2(T) = \left( \frac{g_2^2}{\pi^2} + \frac{1}{32}\right) T^2.
\]
For simplicity, we approximate the lepton propagator with
\[
S^*(k) = \frac{k}{k^2 - m_L^2(T)} \tag{4}
\]
and come back to the full HTL-result later.

According to finite-temperature cutting rules \[4, 7, 8\], the discontinuity of the self-energy is
\[
\text{tr}(\bar{p}_N \text{Im} \Sigma) = -\int d\bar{p}_L d\bar{p}_H (2\pi)^4 \delta^4(p_N - p_H(T) - p_L(T)) \times |\mathcal{M}|^2 (1 - f_L + f_H), \tag{7}
\]
where the factor \(1 - f_L + f_H = (1 - f_L)(1 + f_H) + f_L f_H\) accounts for both decays \(N \to LH\) and inverse decays \(LH \to N\). The interaction rate \(\Gamma = -\frac{1}{2E_N} \text{tr}(\bar{p}_N \text{Im} \Sigma)\) gives the rates for decays and inverse decays, \(\Gamma_D = (1 - f_N^{eq}(E_N)) \Gamma\) and \(\Gamma_{ID} = f_N^{eq}(E_N) \Gamma\), where \(f_N^{eq}\) denotes the equilibrium distribution function. The (inverse) decay density in the Boltzmann equation is
\[
\gamma_{D,ID} = \int \frac{d^3p_N}{(2\pi)^3} f_N(E_N) \Gamma_{D,ID}, \tag{8}
\]
where \(f_N\) needs to be out of equilibrium \[1\].

The result equals the intuitive “per hand” treatment with two caveats: The correct quantum statistical distribution functions always appear in TFT even without including thermal masses, therefore it seems inconsistent to use thermal masses and Maxwell-Boltzmann approximation at the same time, as has been done by ref. \[3\]. Moreover, the HTL lepton propagator (eq. 5) has a different structure than the approximate propagator.

### 2.3. Two leptonic quasiparticles

The full HTL resummed lepton propagator (eq. 5) has two poles at \(D_\pm(\omega, k) = 0\), which correspond to two different dispersion relations \(\omega_\pm(k)\) (Fig. 2) \[9, 10, 11, 12\].

For very large momenta, \(\omega_+ \to \sqrt{k^2 + 2m_L^2(T)}\) and \(\omega_- \to k\). In this limit, \(\omega_+\) corresponds to a thermal mass of \(\sqrt{2m_L(T)}\) and \(\omega_-\) to a massless mode. The decay rate for the heavy mode \(\omega_+\) will thus be reduced and the threshold for this decay will be at \(M_N = m_H(T) + \sqrt{2m_L(T)}\), whereas the decay in the light mode will be possible up to \(M_N = m_H(T)\). The detailed shape of the decay rate including the two leptonic modes and its influence on the Boltzmann equations will be examined in a later work \[13\].

### 3. CONCLUSIONS

Using HTL resummation and the optical theorem, we confirm that putting in thermal masses “by hand” as has been done by ref. \[3\] is a reasonable approximation. However,
since quantum statistical distribution functions always appear in the TFT treatment, it is inconsistent to use thermal masses and Maxwell-Boltzmann distributions at the same time. Moreover, the full HTL lepton propagator gives two leptonic quasiparticles and will have an effect on the decay rate which has to be examined in a future work [13].

Looking further ahead it also seems desirable to look at thermal corrections to scattering processes and the CP asymmetry in a consistent thermal field theoretic treatment.

ACKNOWLEDGMENTS

We would like to thank Georg Raffelt, Markus Thoma, Dietrich Bödeker, Florian Hahn-Wörnle, Steve Blanchet and Denis Besak for fruitful and inspiring discussions.

REFERENCES

1. A. D. Sakharov, *Pisma Zh. Eksp. Teor. Fiz.* 5, 32–35 (1967).
2. M. Fukugita, and T. Yanagida, *Phys. Lett.* B174, 45 (1986).
3. G. F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, *Nucl. Phys.* B685, 89–149 (2004), [hep-ph/0310123](https://arxiv.org/abs/hep-ph/0310123).
4. M. Le Bellac, *Thermal field theory*, Cambridge University Press, Cambridge, UK, 1996.
5. E. Braaten, and R. D. Pisarski, *Nucl. Phys.* B337, 569 (1990).
6. E. Braaten, and R. D. Pisarski, *Nucl. Phys.* B339, 310–324 (1990).
7. H. A. Weldon, *Phys. Rev.* D28, 2007 (1983).
8. R. L. Kobes, and G. W. Semenoff, *Nucl. Phys.* B272, 329–364 (1986).
9. V. V. Klimov, *Sov. J. Nucl. Phys.* 33, 934–935 (1981).
10. H. A. Weldon, *Phys. Rev.* D26, 1394 (1982).
11. H. A. Weldon, *Phys. Rev.* D26, 2789 (1982).
12. H. A. Weldon, *Phys. Rev.* D40, 2410 (1989).
13. C. P. Kiessig, and M. Plumacher (2009), in preparation.