Application of non-equal interval GM (1,1) model in prediction of building settlement

Jiameng Wei, a Huagen Jiang2ab and Jianpeng Diao3, c

1, 2, 3 Surveying Engineering, Southwest Forestry University, kunming, Yunnan Province, 650000, China
a Email: WJM@swfu.edu.cn

Abstract. In order to solve the impact of the uneven settlement of the building on the construction process and safety evaluation of the building, this paper takes the building settlement as the monitoring object, and uses the non-equidistant GM (1, 1) model to construct the model, and applies it to the construction of a building. Settlement analysis. In addition, the prediction accuracy of the model is tested for the posterior error ratio and small probability error to ensure the reliability and accuracy of the model application.

1. Introduction

The settlement phenomenon is common in nature. In construction engineering, as the construction progresses, the building will inevitably settle as the load increases1. In order to make the work proceed smoothly, it is necessary to monitor the deformation of the building, that is, to use instruments to Dynamic buildings carry out static and periodic measurements2. Through the analysis of monitoring data, the safety monitoring and forecasting of buildings can be completed. According to the characteristics of the non-equal time interval of building settlement monitoring, the 1-WAGO sequence is generated with the adjacent observation time interval as the weight, and the gray system non-equal interval GM (11,1) model is established3. The results are tested by the posterior difference ratio, The simulation forecast data analysis shows that the settlement result predicted by the model is correct, reliable and accurate4.

2. Grey model construction

2.1. Gray generation

Let the original data sequence be
\[ x^0 = \{x^0(t_1), x^0(t_2), \ldots, x^0(t_i)\} \]

among them:
\[ \Delta t_i = t_i - t_{(i-1)} \neq \text{const}, \quad k = 2, 3, \ldots, n \]

Perform 1-WGAO generation on the original sequence:
\[ \{x^{(1)}(t_k) = \{x^{(1)}(t_1), x^{(1)}(t_2), \ldots, x^{(1)}(t_n)\} \]

among them:
\[ x^{(1)}(t_k) = \left\{ \begin{array}{l} x^{(0)}(t_1) \quad k = 1 \\ \sum_{i=1}^{k} \Delta t_i x^{(0)}(t_i) \quad k = 2, 3, \ldots, n \end{array} \right. \] (1)

The sequence \( x^{(1)} \) is generated next to the mean value:
Among them, \( Z^{(1)} = \{Z^{(1)}(t_1), Z^{(1)}(t_2), ..., Z^{(1)}(t_n)\} \)

then the gray differential equation model of the non-equidistant GM (1, 1) model is:

\[
\frac{dx^0}{dt} + ax^{(1)} = u
\]

Among them, \( a \) and \( b \) are undetermined parameters.

2.2. Parameter solving

Suppose matrix \( B \) is a gray sequence matrix, and use least squares to solve the undetermined parameters according to the monitoring data\([5]\):

\[
(a, u)^T = \left(B^TB\right)^{-1} B^TY
\]

Among them, \( a \) is the development coefficient, which reflects the development status of the original sequence and the 1-WGAO generated sequence, and \( u \) is the gray effect, which reflects the relationship between data changes. The \( B \) and \( Y \) matrix sequence is as follows:

\[
B = \begin{bmatrix}
-\frac{1}{2}(x^{(1)}(t_1) + x^{(1)}(t_2)) & 1 \\
-\frac{1}{2}(x^{(1)}(t_2) + x^{(1)}(t_3)) & 1 \\
\vdots & \vdots \\
-\frac{1}{2}(x^{(1)}(t_{n-1}) + x^{(1)}(t_n)) & 1 
\end{bmatrix},
Y = \begin{bmatrix}
x^{(0)}(t_2) \\
x^{(0)}(t_3) \\
\vdots \\
x^{(0)}(t_n)
\end{bmatrix}
\]

Solve the above differential equation to get the time response function:

\[
x(k+1) = [x^0(1) - \frac{u}{a}e^{-ak}] + \frac{u}{a}, k = 1, 2, 3, ..., n.
\]

Revert the above equation to a function related to time \( t \) in a non-equidistant series:

\[
x^{(1)}(t) = \left[x^{(0)}(1) - \frac{u}{a}\right]e^{-\frac{u}{a}(t-t_1)} + \frac{u}{a}
\]

\[
x^{(0)}(t) = x^{(1)}(t_k) - x^{(1)}(t_{k-1})
\]

Where \( t \) is the time interval between the monitoring time and the first monitoring time.

2.3. Model checking

Whether the gray non-equidistant model can be applied to deformation monitoring, generally its reliability and accuracy are judged by residual error size test, correlation test, posterior variance test, etc. Here, residual statistical characteristics are used for posterior variance test. Based on the predicted value or the residual error of the fitted value, the variance ratio \( c \) and the small error probability value \( p \) are obtained after the test, and the value of the posterior variance and small probability is used as the evaluation accuracy\([6]\]. The reference standard is shown in Table 2. Proceed as follows:

1. Find the residual

\[
e(t_k) = x^{(0)}(t_k) - \hat{x}^{(0)}(t_k)
\]

2. Mean residual and mean value of measurement:

\[
\bar{e} = \frac{1}{n} \sum_{k=1}^{n} e(t_k), \quad \bar{X} = \frac{1}{n} \sum_{k=1}^{n} x^{(0)}(t_k)
\]

3. The variance of the residual:
3. Variance of original data:

\[ S_1^2 = \frac{1}{n-1} \sum_{k=1}^{n} (e(t_k) - \bar{e})^2 \]  

(4)

\[ S_2^2 = \frac{1}{n} \sum_{k=1}^{n} (x^{(0)}(k) - \bar{x})^2 \]  

(7)

(5) The posterior difference ratio \( c = \frac{S_1}{S_2} \), the probability of small error \( p = p \{ |e(t_k) - \bar{e}| < 0.67445S_2 \} \). Model accuracy standards are shown in Table (1)

| Model accuracy grade | Posterior variance ratio \( c \) | Small error probability value \( P \) |
|----------------------|-----------------|------------------|
| First level (good)   | \( c < 0.35 \)  | \( 0.95 \leq p \) |
| Level 2 (Qualified)  | \( 0.35 \leq c < 0.5 \) | \( 0.8 \leq p < 0.95 \) |
| Level 3 (barely)     | \( 0.5 \leq c < 0.65 \) | \( 0.7 \leq p < 0.8 \) |
| Level 4 (unqualified)| \( c \geq 0.65 \)  | \( p < 0.7 \) |

3. Case Analysis

Taking the observation results of an affordable housing in a certain place in Yunnan as an example, there are eight periods of data in the original data table in Table 2. Among them, the first six periods are modeling and fitting, and the latter two periods are predicted as follows:

| Observation date | 8-10 | 9-1 | 10-8 | 10-18 | 10-24 | 10-29 | 11-8 | 11-20 |
|------------------|------|-----|------|-------|-------|-------|------|-------|
| Observation period \( t \) | 1 | 23 | 60 | 70 | 76 | 81 | 91 | 103 |
| Cumulative settlement (mm) | 1.0510 | 1.5497 | 2.1226 | 2.1575 | 2.2038 | 2.2319 | 2.2653 | 2.2653 |

The modeling method of this article is as follows:

- The second step is to generate an accumulation of the equally spaced sequence \( x_0 \):
  \[ x_1 = [1.051 \ 35.144 \ 113.681 \ 135.256 \ 148.478 \ 159.638] \]
- The second step is to generate the immediate mean value of \( x_1 \):
  \[ x_1^{(1)} = [-18.098 \ -74.413 \ -124.468 \ -141.867 \ -154.058] \]
- The third step is to use the least square method to solve the parameters:
  \( a = -0.0046, \ u = 1.581 \)
- The fifth step, the time response equation is:
  \[ x^{(1)}(t) = 344.747 \times e^{-0.0046(t-1)} - 343.696 \]
- The sixth step, the prediction results are displayed, the simulated prediction diagram is shown in Figure 1; the residual analysis diagram is shown in Figure 2; the simulation prediction results table is shown in Table 3;
Figure 1 Simulation forecast map

Figure 2 Residual analysis diagram

Table 3 Simulation prediction results table

| Observation time t | Observation x | Analog value (mm) | Absolute residual value (mm) | Absolute percentage error % |
|-------------------|---------------|-------------------|------------------------------|-----------------------------|
| Fit data          |               |                   |                              |                             |
| 1                 | 1.0510        | 1.051             | 0                            | 0                           |
| 23                | 1.5497        | 1.6690            | 0.119                        | 7.7                         |
| 60                | 2.1226        | 1.9131            | 0.209                        | 9.9                         |
| 70                | 2.1575        | 2.1291            | 0.028                        | 1.3                         |
| 76                | 2.2038        | 2.2089            | 0.005                        | 0.23                        |
| 81                | 2.2319        | 2.2655            | 0.034                        | 1.5                         |
| Forecast data     |               |                   |                              |                             |
| 91                | 2.2653        | 2.3428            | 0.163                        | 7.2                         |
| 103               | 2.3371        | 2.5380            | 0.201                        | 8.6                         |
According to the model development coefficient, it can be seen that the model development coefficient is -0.0046, which can be applied to mid- and long-term forecasting. It can be seen from Figure 1 that the forecast value has an upward trend in the seventh and eighth periods. Combining its development coefficient, it can be concluded that the model can be used for mid-term forecasts. In addition, the posterior difference ratio c of the model is 0.097, the probability of small error is 1, and the model prediction level is level 1, which fully meets the accuracy requirements. The absolute residual values of the calculated model are all within 0.3 mm, which fully meets the requirements of model prediction.

4. Conclusion
Through data analysis, the prediction accuracy and fitting effect are good, and the model can be applied to the settlement prediction of buildings. In addition, some problems were encountered in the forecasting process, such as failure to calculate the specific forecast time range based on the forecast data parameters, the initial value of the gray model was replaced by the initial value of the original data sequence, the failure to calculate the initial value, and so on. However, with a small amount of raw data, the model can simulate and predict well.

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