Clustering of vector nulls in homogeneous isotropic turbulence

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We analyze the vector nulls of velocity, Lagrangian acceleration, and vorticity, coming from direct numerical simulations of forced homogeneous isotropic turbulence at $Re_\lambda \in [40 - 610]$. We show that the clustering of velocity nulls is much stronger than those of acceleration and vorticity nulls. These acceleration and vorticity nulls, however, are denser than the velocity nulls. We study the scaling of clusters of these null points with $Re_\lambda$ and with characteristic turbulence length scales. We also analyze datasets of point inertial particles with Stokes numbers $St = 0.5, 3,$ and $6$, at $Re_\lambda = 240$. Inertial particles display preferential concentration with a degree of clustering that resembles some properties of the clustering of the Lagrangian acceleration nulls, in agreement with the proposed sweep-stick mechanism of clustering formation.

I. INTRODUCTION

Single-phase and particle-laden turbulent flows are of interest in many industrial, natural and environmental situations. But despite their relevance, there are still many open questions that severely limit our understanding of these flows. For instance, the study of geometrical properties of the velocity, the Lagrangian acceleration, and the vorticity fields in turbulent flows has received considerable attention in the last decades. The geometrical properties of these fields can be useful to model important phenomena in turbulent flows such as superdiffusivity, preferential concentration of particles, vortex reconnection, among many others. Some works, focused on characterizing simply connected regions of vorticity, have found that such regions tend to cluster [26, 27, 32, 51]. Other studies have focused on the vector field nulls [12, 20, 30], the points where the modulus of vectorial quantities is equal to zero (i.e., the set of points $X_p = \{ x_n = (x_n, y_n, z_n) \in \mathbb{R}^3 | p(x_n, y_n, z_n) = 0 \}$, where $p$ is some vector field as, e.g., the vorticity). It is nevertheless unclear how these quantities relate to each other and, furthermore, their dependence on different parameters such as the Reynolds number or the homogeneity and isotropy of the underlying flow.

In spite of the limited knowledge on their spatial distribution and scaling properties, the geometry of these vector nulls, also known in some cases as stationary or fixed points, has successfully been related to the underlying physics of turbulence and to turbulence-particle interactions. For instance, their statistics and scaling properties have been associated with the fractal nature of turbulence. Scaling laws for the distribution of vector nulls have been derived from fractal dimensions of the velocity field compatible with Kolmogorov scalings [8, 12, 20]. Also, it has been noted that the velocity nulls carry information about the turbulent kinetic energy dissipation rate $\varepsilon$ (see [12, 22, 29, 48]) and can be used to model relative pair dispersion [15]. Furthermore, the Lagrangian acceleration nulls have been related to the degree of preferential concentration found when inertial particles are added to a turbulent flow [10, 20, 53].

Some specific results for three choices of these null points are worth mentioning. On the one hand, it has been shown that properties of the set of velocity nulls (satisfying $v(x_n) = 0$ where $v$ is the fluid velocity field, and referred in the following as “stagnation points” or STPS) from a one-dimensional (1D) measurement, can be related to the Taylor microscale $\lambda$ of homogeneous isotropic turbulence (HIT) via the Rice theorem [29, 45]. More recently, Goto and Vassilicos extended these results to three-dimensional (3D) fields [22]. On the other hand, the set of Lagrangian acceleration nulls (with $a(x_n) = 0$, and referred in the following as ZAPS for “zero acceleration points”), are at the core of the proposed sweep-stick mechanism [10], that models preferential concentration in inertial-particle-laden flows. This model suggests that the inertial particles mimic the spatial distribution of ZAPS, for particles with a Stokes number $St$ above unity [40, 49, 53]. Finally, the vorticity nulls ($\omega(x_n) = 0$, or WZERO in the following), have been related to vortex reconnection events and the turbulent cascading process [30, 59]. Furthermore, low vorticity regions are also expected to control the centrifugal expulsion of inertial particles (and their clustering) for $St < 1$. Within this mechanism, dense particles are expected to be expelled from the core of eddies and to accumulate in regions of high strain and low vorticity [56].

Despite the interest and ongoing research in the topic, to the authors best knowledge no joint systematic study on the global properties and $Re_\lambda$-dependence of the clustering of these points has been carried out (where $Re_\lambda$ is the Reynolds number based on the flow Taylor microscale), nor of their relation with inertial particle concentration fields. Vassilicos and collaborators have conducted the most extensive studies on the geometry of STPS and ZAPS.
using pair distribution functions [8, 10, 12, 16, 21]. These works have provided analytical predictions on these points’ statistical properties, and some recent studies [2, 40, 53] have retrieved some evidence that partially validates some of these predictions. Considering that Vassilicos and collaborators used pair distribution functions to examine the vector nulls clustering, they could not examine the local vicinity around a null point so that conditioned statistics could be computed and related to surrounding turbulent phenomena (e.g., to preferential concentration). In this respect, the study of Obligado and collaborators [40] found that the 2D spatial distribution of inertial particles with $St = 2$ and 4 (characterised via Voronoi tessellations) presents similarities with the regions of low Lagrangian acceleration.

In this work we analyze velocity, Lagrangian acceleration, and vorticity nulls through Voronoi tessellations [18]. The fields examined come from forced direct numerical simulations (DNSs) of HIT. We study seven different DNS datasets, exploring a wide range of Reynolds numbers ($Re_x \in [40 - 610]$) and different forcing schemes. Voronoi tessellations, contrary to pair correlation functions, allow the examination of the nulls local “concentration” maps and cluster size distributions, similar to previous studies of clustering of inertial particles [17, 34]. Our results show that the degree of clustering of STPS is much larger than the respective ones for ZAPS or WZERO (while the concentration follows an inverse trend), and display a clear scaling with $Re_x$. The results also confirm that the concentration of STPS is at least one order of magnitude smaller than those of ZAPS or WZERO, in agreement with the scalings of Chen et al. [8]. Also, while the properties of ZAPS and WZERO have a similar trend with $Re_x$, STPS spatial structure presents a different dependence with this parameter.

Finally, for one of our DNS (with $Re_x = 240$) we also studied the behaviour of dense, point-like inertial particles. The objective is to generalise the results from the previous work of Obligado and collaborators [40]. In this aspect, the novelty of the present work compared to previous ones is twofold: we generalise the study to 3D Voronoi tessellations (thus eliminating any bias caused by projecting fields to 2D) and instead of studying regions with low vorticity or acceleration, we use an interpolation method that allows to actually detect nulls as points (as detailed in Sec. II B). We then compare the clustering properties of different nulls datasets with the clustering of inertial particles with Stokes numbers $St$ of 0.5, 3 and 6. We find that, in agreement with previous works, the spatial segregation of particles with $St > 1$ presents larger similarities with the ZAPS than with the other vector nulls, consistently with the behavior expected from the sweep-stick mechanism.

## II. METHODOLOGY

### A. Numerical simulations

Our numerical datasets of the Eulerian velocity, Lagrangian acceleration, and Eulerian vorticity came from DNSs. These simulations follow standard practices regarding their temporal integration, de-aliasing procedures, and have an adequate spatial resolution of the smallest scales, i.e., $\kappa Np \gtrsim 1$ [42]. Here $\eta$ is the Kolmogorov lengthscale, $\eta = (\nu^3/\varepsilon)^{1/4}$ (where $\varepsilon$ is the kinetic energy dissipation rate, and $\nu$ the kinematic viscosity of the fluid), and $\kappa = N/3$ the maximum resolved wavenumber in Fourier space (with $N$ the linear spatial resolution). Fully dealiased pseudospectral methods with second-order Runge-Kutta methods for the time stepping are used. The 3D simulation domain for all datasets has dimensions of $2\pi \times 2\pi \times 2\pi$. All relevant simulation parameters can be found in Table I.

Numerical simulations solve the incompressible Navier-Stokes equations for the velocity $v$ with a random solenoidal forcing $f$,

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla p' + \nu \nabla^2 v + f, \quad (1)$$

where $p' = p/\rho$ (with $p$ is the pressure and $\rho$ a uniform mass density), which is obtained from the incompressibility condition $\nabla \cdot v = 0$. In Eq. (1), $Dv/Dt = a$ is the Lagrangian acceleration of the fluid elements, while the vorticity field is given by $\omega = \nabla \times v$. We define the r.m.s. velocity as $u' = (\langle |v_i|^2 \rangle)^{1/2}$ (where $v_i$ is a Cartesian component of the velocity), the Taylor scale as $\lambda = (15\nu u'^2/\varepsilon)^{1/2}$, and the integral scale as $\mathcal{L} = \pi/(2u'^2) \int E(k)/k \, dk$ (where $E(k)$ is the isotropic energy spectrum).

We use a total of seven numerical datasets. The first five datasets (labeled in the following as “DNS-N”), where $N$ is the linear resolution of each dataset) were obtained using the GHOST code (see [31, 46] for further details of the code). In this case, the solenoidal forcing $f$ is given by a superposition of Fourier modes with random phases in the shell with wavenumber $k = 1$. A new random forcing was generated every 0.5 large-scale turnover times, and the forcing was linearly evolved from its previous state to the next state along this period of time. This results in a continuous and slowly evolving random forcing with correlation time of 0.5 turnover times, which at the largest resolution considered has an integral scale $\mathcal{L}/(2\pi) \approx 0.309$, and which will be useful for simulations with inertial particles as discussed below. These simulations also use the largest Reynolds number attainable at their given spatial resolution, with
Particles are integrated following the equations 
\[ \kappa \eta \]
of the dependence of nulls on 
also have different forcing schemes, which can also affect the distribution and number of null points, a detailed study of statistical moments, with intermittency increasing as 
lower-order moments of the vector fields, but much more stringent conditions are needed to capture higher-order statistical moments of the vector fields. Comparing simulations with different values of 
that as the vector fields become more intermittent, the number of zeros in the fields increases. Previous studies 
in the simulations with 
will be shown below, we obtain consistent results in all simulations, but with the simulations with larger values of 
indicate that as the vector fields become more intermittent, the number of zeros in the fields increases. Previous studies comparing simulations with different values of 
also indicate that this could be the case: the simplest criterion that the Kolmogorov scale should be resolved yields simulations with accurate estimations of the statistical lower-order moments of the vector fields, but much more stringent conditions are needed to capture higher-order statistical moments, with intermittency increasing as \( \kappa \eta \) increases. However, as the DNS and JHU simulations also have different forcing schemes, which can also affect the distribution and number of null points, a detailed study of the dependence of nulls on \( \kappa \eta \) using a unique forcing scheme is left for the future.

For DNS-512 we also have data of \( 10^6 \) inertial point particles without gravity, which will be considered in Sec. IV. Particles are integrated following the equations
\[ \frac{dx_p}{dt} = v_p, \quad \frac{dv_p}{dt} = \frac{1}{\tau_p} [v(x_p) - v_p], \]
where \( x_p \) is the particle position, \( v_p \) the particle velocity, \( v(x_p) \) the fluid velocity at the particle position, and \( \tau_p \) the Stokes time. The Stokes number of the particles is then defined as \( St = \tau_p / \tau_\eta \), where \( \tau_\eta \) is the eddy turnover time at the Kolmogorov scale. These equations are integrated with a high-order Runge-Kutta method to evolve in time, and a high-order three-dimensional spatial spline interpolation to estimate the fluid velocity \( v(x_p) \) at the particle position (see [1, 61] for details).

The Taylor-based Reynolds number, \( Re_\lambda = u' \lambda / \nu \), spans one and a half decades. We have \( Re_\lambda \in [40, 610] \) for spatial resolutions of \( 64^3, 128^3, 256^3, 512^3, 1024^3 \), and \( 4096^3 \) grids points. We took enough snapshots of the vector fields to have adequate global statistics. The JHU datasets were post-processed using the Sciserver platform [50], and we used the Python library FREUD [44] to compute the 3D Voronoi diagrams for all datasets.

### B. Nulls calculation

We applied the method proposed by Haynes and collaborators [23, 24, 39] to compute our data vector nulls. Although they developed this method for magnetic fields, recent studies have used the same method to compute nulls in the simulations.

| Dataset    | \( N \) | \( L/(2\pi) \) | \( 10^3 \eta \) | \( Re_\lambda \) | \# snapshots |
|------------|--------|---------------|----------------|-----------------|--------------|
| DNS-64     | 64     | 0.304         | 50             | 40              | 50           |
| DNS-128    | 128    | 0.291         | 24             | 70              | 50           |
| DNS-256    | 256    | 0.291         | 12             | 120             | 50           |
| DNS-512    | 512    | 0.238         | 6              | 240             | 20           |
| DNS-1024   | 1024   | 0.309         | 3              | 520             | 9            |
| JHU-1024   | 1024   | 0.217         | 2.8            | 430             | 15           |
| JHU-4096   | 4096   | 0.221         | 1.4            | 610             | 1            |

**TABLE I:** DNS parameters. \( N \) is the number of points in each direction, such that \( N^3 \) is the total number of grid points in the simulation domain. \( L/(2\pi) \) is the integral lengthscale in units of the domain linear size of length \( 2\pi \). \( \eta \) is the Kolmogorov dissipation scale, multiplied by \( 10^3 \) for convenience. \( Re_\lambda \) is the Reynolds number based on the Taylor microscale \( \lambda \), and \# snapshots is the number of snapshots of the vector fields used for the analysis.
of vorticity fields [30]. We briefly describe this algorithm main steps: First, for each cell in the domain, we survey its vector values at the cell’s corners (i.e., at the grid points) to determine if there is a change of sign in all components of the target vector field (i.e., we survey the 8 corners of the target cubic cell). For a properly resolved and dealiased DNS, if any of the $x$, $y$, or $z$ components of the field do not change in sign within the cell, there cannot be a zero inside it. If, on the other hand, there are changes of sign in every vector component inside the cell (i.e., there is a change of sign in any of the 8 corners of the cubic cell, and for each vector component $x$, $y$, and $z$), we use the 8 corner component values to feed a trilinear interpolation algorithm, and thereby, we build a local vector interpolation function. Then, we proceed by feeding this function into a Newton-Raphson method [43] to verify if there is a zero within the cell. This algorithm is somewhat similar to those proposed by Vassilicos and collaborators [10, 11], and can be easily parallelized.

However, considering the non-linearity and resulting spatial complexity of the turbulent fields here studied, there is a caveat: some of the cells’ zeros can be located outside the target cell. Haynes and Parnell [24] propose that these zeros could be accepted if the zeros’ locations are not very far from the local cell. Although these “satellite” zeros increase the nulls density (and improve the statistics), we opted for a more conservative approach and considered as valid nulls only those zeros found inside the target cell. We took this decision based on a benchmark calculation that showed that including these “satellite” nulls may lead to pathological behaviors of the global parameters coming from the Voronoï tessellation analyses.

### III. NULLS ANALYSIS

We computed the nulls of velocity, Lagrangian acceleration, and vorticity following the Haynes algorithm for all the data in table I. Table II summarizes the total number of zeros found by this algorithm, as well as the average number of zeros per field snapshot in each simulation. We then applied the 3D Voronoï tessellation analysis on these nulls positions. This analysis followed the same protocol of studies focusing on inertial particle clustering [34, 40]. Thus, we quantified their degree of clustering via the standard deviation $\sigma_V$ of the normalized Voronoï cells volume $V = V/V_\text{avg}$ (where $V$ denotes the average volume), which ultimately quantifies the effects of the “voids” (i.e., of low density regions [49]) present in the nulls spatial distribution. We consider that clustering is present when $\sigma_V > \sigma_{\text{RPP}}$ [34], where $\sigma_{\text{RPP}} \approx 0.42$ is the standard deviation of a 3D random Poisson process (RPP), which has no correlations at any scale [52].

Analogously to inertial particle studies, we also computed the clusters volume probability distribution function (PDF) via the algorithm proposed by Monchaux et al. [34]. We selected from the volume cells collection those cells that are below a threshold $V_{\text{th}}$, and considered as clusters the groups of two or more of those cells sharing a boundary (face). We picked $V_{\text{th}}$ as the location of the first crossing (i.e., for $V < 1$) between an RPP PDF and the Voronoï cells’ PDF. In our analysis we took $V_{\text{th}} \approx 0.5$ (and $V_{\text{th}} \approx 0.56$ for the inertial particles in Sec. II B). Interestingly, this threshold did not depend strongly on $Re_{\lambda}$ or on the dataset we analyzed. We note, however, that there were not sufficient clusters in single snapshots of the DNS-64 dataset to reach adequate statistics.

Nevertheless, there is a discrepancy in the values of $V_{\text{th}}$ between our simulations and the JHU datasets, as will be shown later. This is partially related to the fact that the JHU datasets have a larger number of nulls. Although it could be expected both JHU and our DNS datasets should strictly follow the same power-law fitting exponents, it is worth pointing out that due to their different large-scale forcing methods, this may not be the case. This argument is supported by the simulations and theoretical predictions of Goto and Vassilicos [22] that show that the stagnation point structure changes depending on the forcing method, and the forcing wave number. In other words,

| Dataset  | total # of STPS | $\langle \text{STPS} \rangle_{\text{snap}}$ | total # of ZAPS | $\langle \text{ZAPS} \rangle_{\text{snap}}$ | total # of WZERO | $\langle \text{WZERO} \rangle_{\text{snap}}$ |
|----------|----------------|-------------------------------------------|----------------|-------------------------------------------|----------------|-------------------------------------------|
| DNS-64   | 1000           | 20                                        | 16000          | 320                                        | 87000          | 1740                                       |
| DNS-128  | 6000           | 120                                       | 145000         | 2900                                       | 800000         | 16000                                      |
| DNS-256  | 22200          | 440                                       | 750000         | 15000                                      | 4600000        | 920000                                     |
| DNS-512  | 71000          | 3550                                      | 4800000        | 240000                                     | 26000000       | 13000000                                   |
| DNS-1024 | 57000          | 6333                                      | 5800000        | 644444                                     | 38000000       | 60000000                                   |
| JHU-1024 | 220000         | 14667                                     | --             | --                                         | --             | --                                         |
| JHU-4096 | 100000         | 10000                                     | --             | --                                         | --             | --                                         |

| Dataset | total # of STPS | $\langle \text{STPS} \rangle_{\text{snap}}$ | total # of ZAPS | $\langle \text{ZAPS} \rangle_{\text{snap}}$ | total # of WZERO | $\langle \text{WZERO} \rangle_{\text{snap}}$ |
|---------|----------------|-------------------------------------------|----------------|-------------------------------------------|----------------|-------------------------------------------|
| DNS-64  | 1000           | 20                                        | 16000          | 320                                        | 87000          | 1740                                       |
| DNS-128 | 6000           | 120                                       | 145000         | 2900                                       | 8000000        | 160000                                      |
| DNS-256 | 22200          | 440                                       | 750000         | 15000                                      | 46000000       | 9200000                                    |
| DNS-512 | 71000          | 3550                                      | 4800000        | 240000                                     | 260000000       | 130000000                                   |
| DNS-1024| 57000          | 6333                                      | 5800000        | 644444                                     | 380000000       | 600000000                                   |
| JHU-1024| 220000         | 14667                                     | --             | --                                         | --             | --                                         |
| JHU-4096| 100000         | 10000                                     | --             | --                                         | --             | --                                         |

TABLE II: Number of zero nulls found for each vector field (STPS, ZAPS and WZERO) and simulation. Both total numbers of zeros (i.e., aggregated for all times available) and averages are given. The averages, indicated by angular brackets, refer to the average number of the respective zeros per field snapshot.
even simulations with similar $Re_\lambda$ but different forcing may exhibit different topologies (besides the effect of spatial resolution already discussed in Sec. II A). These authors further argued that differences in STPS topology may impact the turbulent cascading process via changes in the normalized dissipation rate $C_\varepsilon$. Weiss et al. [57] also reported that the degree of particle clustering depends on the forcing used to sustain the turbulence. Taking into account that Coleman and Vassilicos have linked particle clustering to properties of the ZAPS, the effect of spatial resolution, and the results of Weiss and collaborators that highlight the non-negligible influence of the large scales on the turbulent field topology, we consider that such a discrepancy is to be expected. Moreover, as will be shown next, once $V_{ih}$ is defined as described above, other results from all the datasets are compatible between themselves.

### A. Scaling of averaged quantities for all vector field nulls

Vassilicos and collaborators [8, 12, 20] report that the number density ($n_s \sim \langle V \rangle^{-1}$, i.e., the inverse of the average Voronoi cell volume) of 3D STPS and of 3D ZAPS scale as $n_s \sim (\mathcal{L}/\eta)^{3}$, where $\delta$ (a fractal dimension) takes the values of 2 and 3 respectively for each set of nulls. For STPS, this fractal dimension can be seen as a consequence of viewing turbulence as a self-similar process. Under such assumption, the energy spectrum exponent (i.e., the $-5/3$ power law) can be related to the fractal exponent $\delta$ via Orey’s theorem leading to $\delta = 2$ for the 3D STPS (see [8, 12]). Likewise, Moisy and Jimenez [32] report a box dimension for the number density of vortical structures (resp., WZERO points) close to $-3$. Our results for the average Voronoi volume size ($\langle V \rangle \sim n_s^{-1}$) of the different null points are consistent with the mentioned scalings and observations (see Fig. 1a). However, we may not have enough scale separation in the inertial range (only a decade in terms of $\mathcal{L}/\eta$) to ascertain without doubt their exact numerical values. Moreover, our results do reveal that the STPS are indeed very scarce compared to ZAPS or WZERO, and thereby, their larger average Voronoi cell volume, which as already mentioned is inversely proportional to the number density of the respective nulls: in other words, note from Fig. 1a that $\langle V \rangle|_{\lambda=0} > \langle V \rangle|_{a=0} > \langle V \rangle|_{\omega=0}$ for all $Re_\lambda$ considered.

The Voronoi volume standard deviation for the nulls, except the for smallest value of $Re_\lambda$, roughly satisfies a similar ordering as the mean (see Fig. 1b), i.e., $\sigma_V|_{\lambda=0} > \sigma_V|_{a=0} > \sigma_V|_{\omega=0}$. This is consistent with the findings of Chen et al. [8], who used pair distribution functions to characterize the geometry of STPS and ZAPS. At increasing $Re_\lambda$, the velocity nulls (STPS) cluster more strongly than the acceleration and vorticity nulls, with $\sigma_V|_{\lambda=0}$ growing with $Re_\lambda$ (a power law is indicated in the figure as a reference). The increased complexity (at all scales) of the STPS topological structure is reflected in its respective PDF (see Fig. 2a and the discussion in Sec. III B), which shows that at increasing $Re_\lambda$ a power law close to $-5/3$ emerges in the PDF of the Voronoi volumes of these nulls. In addition, the standard deviations of the cell volumes of vorticity and acceleration nulls depend weakly on $Re_\lambda$, if at all. As will be shown later, this is a consequence of the behavior of their respective PDFs (see Figs. 3a and 4a). These PDFs, for different values of $Re_\lambda$, roughly collapse for $\langle V \rangle > 1$, which correspond to the cells that contribute the most to $\sigma_V$.

Interestingly, at our smallest values of $Re_\lambda$, the ZAPS and WZERO exhibit similar numerical magnitudes of the standard deviation and of the normalized average cluster size $\langle V_c \rangle/\langle V \rangle$ (see Fig. 1c, where $V_c$ denotes the volume of the clusters). In other words, under the Voronoi analysis criteria, both fields display a similar degree of clustering. This observation is consistent with studies of Coleman and Vassilicos [10] and of Bragg et al. [6], who advanced that regions where $a = 0$ are associated with regions of low vorticity. Moreover, the average cluster size ($\langle V_c/\langle V \rangle \rangle$) of clusters for ZAPS and WZERO in Fig. 1c also seems independent, or at least weakly dependent, on $Re_\lambda$.

On the other hand, Fig. 1d shows the mean linear size of clusters for all nulls, $\langle V_c \rangle^{1/3}$, compared against the Kolmogorov and integral length scales. For ZAPS we retrieve $\langle V_c \rangle^{1/3}/\eta \approx 30 - 50$ in lieu of $\langle V_c \rangle^{1/3}/\eta \approx 10 - 20$ in [40]. This mild discrepancy is due to our definition of a cluster: at least two cells ($N_{PC} \geq 2$) below the threshold $\mathcal{V}_{ih}$, and that share a face (resp. edge in 2D) to define a cluster. Using this definition yields an average cluster size 2 to 4 times larger than when a condition $N_{PC} \geq 1$ is used. Hence, it is thus unsurprising that for ZAPS we obtain a slightly larger value of $\langle V_c \rangle^{1/3}/\eta$ than in other studies. Interestingly, average cluster sizes seem to be related to the existence of certain fluid structures within the turbulent flow. For instance, by means of a 2D numerical study, Faber and Vassilicos [16] argue that vortical structures centered around ZAPS scale with some scale larger than $\eta$: in fact, a length scale between $\eta$ and $\mathcal{L}$. If clusters of ZAPS scale similarly to these vortical structures identified by Faber and Vassilicos, we find that such lengthscale should be close to $0.1\mathcal{L}$. In addition, the behavior of this quantity is also in agreement with the study of Sunbekeva et al. [49] for inertial particles, which suggests that the average cluster size is an increasing function of the Reynolds number.

In the following we continue our analysis characterizing the different shapes of the PDFs of the Voronoi cell volumes. As it will be detailed below, not only the global parameters present important differences for STPS, ZAPS, and WZERO, but each set of nulls also has different PDFs and clusters with very different geometrical properties.
FIG. 1: Global Voronoï statistics for the different field nulls. a) Average Voronoï volume sizes of STPS, ZAPS, and WZERO. STPS for the JHU datasets (“STPS-JHU”) are indicated by a different marker here an in the following panels. b) Standard deviation of the Voronoï cell volumes with respect to the one from a Poisson distribution (RPP). c) Average cluster volume normalized by the average cell volume. d) Average cluster size over Kolmogorov ($\eta$, left vertical axis with closed markers) and integral length scales ($\mathcal{L}$, right vertical axis with open markers). Markers shapes are the same for all panels. Power laws and some reference values are indicated by straight lines.

B. Probability density functions of velocity nulls

The Voronoï cell volume PDF for velocity nulls exhibits an increasingly wider power-law behavior with an exponent close to $-5/3$ (see Fig. 2a) at increasing values of $Re_\lambda$. Mora and Obligado [38] have also recovered this trend in laboratory experiments downstream of an active grid via 1D Voronoï tessellations. This power-law can be a consequence of the power-law behavior of the velocity autocorrelation function. From [47], successive zero crossings (nulls in 1D) of a Gaussian process have power-law behavior with exponent $-2 + \beta$ if its autocorrelation function is of the form $\rho(r) = 1 - O(r^{2\beta})$ for $r \ll 1$ (see also [12, 41]). Our volume PDFs also hint that the degree of clustering for STPS scales with $Re_\lambda$, in agreement with the results in the previous section.

The PDF of the cluster volumes of velocity nulls exhibits an even clearer power-law (see Fig. 2b), also with an exponent close to $-5/3$. The power-law widens over several decades as $Re_\lambda$ increases. However, some previous studies have proposed that this behavior may be trivial or spurious. For instance, Uhlmann and collaborators [53, 54] have shown that the cluster detection algorithm applied to synthetic random (RPP) data can also yield power-laws. The latter prompts the question of how to differentiate random structures from turbulence driven ones. Mora et al. [37] have addressed this problem using a PDF mixture model [19] (see also Sec. III D). After analyzing the histograms of the number of points inside a cluster ($N_{PC}$), they suggested that this power-law behavior in turbulent flows follows
from the functional dependence of these histograms. They then argue that if the probability \( P \) of finding a cluster with \( N_{PC} \) points goes as \( P(N_{PC}) \sim N_{PC}^{\gamma} \), the respective cluster volumes PDF will have a power-law with an exponent close to \( \gamma \). Although in 3D the cluster volumes in a RPP may also exhibit such behavior for certain values of the \( \mathcal{V}_{th} \) threshold, Mora et al. [37] found this behavior is of much wider extent for turbulence-driven clusters.

C. Zero acceleration points and vorticity nulls

Contrary to the STPS, the zero Lagrangian acceleration points PDFs of Voronoi cell volumes do not exhibit a power-law behavior, and interestingly, when \( \mathcal{V} = O(1) \), they display an almost exponential decay (see Fig. 3a). Their clusters PDFs exhibit a strong power-law with an exponent close to \(-2\) (see Fig. 3b). Obligado et al. [40] report a similar algebraic exponent and argue it is a signature of the ZAPS clusters’ fractal nature. Note that for ZAPS, and for WZERO in Sec. III C, we only report data from the “DNS” datasets, as a result of data availability.

Likewise, the vorticity nulls Voronoi cell PDFs display (see Fig. 4a) a similar behavior as the one found for the ZAPS volume cell PDF. However, the PDFs of cluster volumes of vorticity nulls have slightly broader power-law behavior than the ZAPS ones (see Fig. 4b). In these figures, and for the sake of comparison, the same power laws as those shown in Figs. 3a and 3b are indicated as references.

D. Probability density functions of clusters

We now examine the PDFs of ZAPS and WZERO clusters volumes using the approach of Mora et al. [37]. These authors claim that clusters PDFs, obtained by the clustering detection algorithm in Monchaux et al. [34] (see also Sec. III), can be analytically described by a mixture PDF model [19]. PDF mixture models are based on PDFs linear superpositions; individual PDFs \( f_i \) are multiplied by weights \( \alpha_i \), i.e., \( f_{mix} = \sum_i^{N_P} \alpha_i f_i \), where \( N_P \) is the number of PDFs to combine. For instance, \( N_P \) can be associated with the number of points (resp. particles) in the clusters, and the PDFs of clusters of two, three, four, and up to \( N_P \) points can be combined to construct a PDF which represents the statistics of \( V_C / \langle V \rangle \). These \( N_P \)-points cluster PDFs are computed via convolutions (i.e., assuming statistical independence and strong-mixing conditions [5, 25]) using a limited 3D random Poisson distribution as base function. For more details, see [37].

Mora et al. [37] suggest computing the weights \( \alpha_i \) as \( \alpha_i = \text{number of clusters with } N_{PC} \text{ points divided by the total number of clusters} \). Thus, we estimated these weights by computing histograms \( (S_N) \) of the number of
clusters conditioned on the number of null points (resp. particles) in a cluster $N_{PC}$ (see Fig. 6a). These histograms have a power-law behavior with an exponent close to $-16/9$ or to $-2$ (maybe slightly dependent on the field nulls considered), and similar to the exponent proposed by Yoshimoto and Goto [62] for inertial particles. The observation of this scaling cannot be overlooked, as it gives credence to some aspects proposed by the sweep-stick mechanism: particle clustering is a multi-scale process that resembles the clustering of acceleration nulls. Indeed, it is remarkable that previous studies with inertial particles [21] found a compatible self-similar structure than that followed by these pairs, triplets, quartets, and so on of vector nulls.

We feed the mixture PDF model with $\alpha_{PC} \sim N_{PC}^{-2}$, and compute it up to $N_{PC} = 5000; \max(V_C/(V)) \approx 5000\nu_{ih} \approx 2500$ for $\nu_{ih} = 0.5$. We find good agreement, recovering the power-law behavior of the PDFs of ZAPS and WZEROS for several decades (see Fig. 6b). This result is remarkable considering that the PDF mixture model uses convolutions of limited RPP distributions, each of them with no correlation at any scale [18]. However, their superposition can have correlations given by the coefficients in the expansion. Therefore, the broader power-law behavior seen in the clusters PDFs (Figs. 2b, 3b, and 4b) can only have a turbulent origin. Indeed, its origin resides in the power law scaling of the $\alpha_i$ weights. The resulting behavior, although it may depend on the threshold $\nu_{ih}$ used, is therefore
IV. INERTIAL PARTICLES IN HIT

As previously mentioned, Coleman and Vassilicos [10] have related the geometry of ZAPS to inertial particle clustering (also known as preferential concentration). To examine this phenomenon, we tracked point inertial particles in DNS-512 with Stokes numbers $St$ equal to 0.5, 3, and 6 respectively. Each dataset of particles (for each value of $St$) comprised 20 snapshots containing instantaneous 3D positions of $10^6$ inertial particles.

We found evidence of clustering of particles for all Stokes numbers considered (Fig. 7a, which also shows a comparison with the PDFs of cell volumes of STPS, ZAPS and WZERO). Interestingly, the particles Voronoï volumes PDF have better agreement with the WZERO PDF for $\nu = 5/3$, and on the contrary, they exhibit better agreement with the not a spurious artifact of the 3D Voronoï tessellations (see also Fig. 13 in [9]); turbulence increases the probability –through the weights $\alpha_i$– of having very large structures as those found in STPS, ZAPS, or WZERO.
FIG. 7: Voronoi tessellation analysis of inertial particles’ instantaneous positions. a) PDF of the normalized Voronoi cells volumes $V = V / \langle V \rangle$. Markers follow the legend in panel b). b) PDF of the clusters volumes normalized by the average volume ($V_C / \langle V \rangle$) using Monchaux et al. criteria. The inset shows a detail of the peak of the PDFs in linear scale. Open markers refer to STPS, ZAPS and WZERO in DNS-512, while closed markers are for particles with different Stokes numbers. c) Histogram of number of clusters conditioned on the number of points in the cluster, for STPS, ZAPS, WZERO, and particles with different $St$. d) Linear cluster size (normalized by the Komogorov and integral lengths) against the number of particles inside the cluster. In these panels, RPP distributions and power laws are shown as references.

ZAPS PDF for $\mathcal{V} \gtrsim 1$. Thus, it is unsurprising that the inertial particles degree of clustering shows an overall better agreement with the ZAPS field (as later shown in Fig. 8b), as larger cells, also known as voids, are the main contributors to the standard deviation of the cell volumes $\sigma_V$ [49]. This observations suggest that the sweep-stick mechanism may only be an approximate representation of the phenomenon underlying the physics at all scales, as discussed by Bragg et al. [6].

The behavior for $\mathcal{V} \ll 1$ has not been observed by the Obligado et al. study [40], which used experimental data for inertial particles (using 2D high-speed imaging) and patches of low acceleration obtained via DNS. These patches were also averaged into 2D planes with several pixels thickness (to mimic the finite thickness of a laser sheet). Therefore, both experiments (due to finite spatial resolution) and DNS (due to the averaging) did not resolve the range $\mathcal{V} \ll 1$ corresponding to very small cells. For larger cells, a statistical agreement between particles clustering and ZAPS was reported, in agreement with the present results.

Interestingly, we retrieve a power law (see Fig. 7b) in the PDF of cluster volumes analogous to the one found for ZAPS and WZERO. The cluster volume PDFs peak at $V_C / \langle V \rangle \approx 1$ can be explained from the histograms of number of clusters conditioned on the number of particles inside a cluster $N_{PC}$ (Fig. 7c); clusters with $N_{PC} = 2$ have a larger
probability, and therefore, $V_C/(V) \approx 2V_{th} \approx 1$ is the most likely value. Considering that the normalized cells $V$ close to the threshold $V_{th}$ have the highest probability, our observation is insensitive to the reported linear behavior between $N_{PC}$ and cluster size (see Fig. 7d and [33]). Closer inspection of this histogram reveals good agreement between the different particles sets and ZAPS or WZEROs for $St = 0.5$ and 3, whereas for $St = 6$ discrepancies arise at $N_{PC} > 10$. The latter discrepancy agrees with the expectation that at larger Stokes numbers the particles filter out certain flow scales [1, 4]. In particular, for large values of $St$, particles are expected to be less sensitive to fast changing (and small scale) motions in the fluid. Also, the probability of small clusters $V_C/(V) \ll 1$ (see Fig. 7b) is higher for the vector nulls than for the particles. Although dispelling why this is the case is very interesting, we refrain from analyzing these high concentration regions, as the assumption of the one-way coupling approximation used in our particle-laden simulations may present limitations on the proper modeling of particles in those regions [3, 14, 36].

Interestingly, inertial particle clusters volumes are (on average) smaller than the average STPS clusters, of the same order of ZAPS clusters, and larger than WZERO clusters (see Fig. 8a, which compares the ratios to these three nulls). The ratios seem to grow with $St$ until saturating. It is again remarkable the similarities between the clustering of ZAPS and of inertial particles, as both quantities have average cluster volumes of the same order of magnitude, i.e., $\langle V_C\rangle_{ZAPS}/\langle V_C\rangle_{STPS} \in [0.5 - 2]$. Obligado et al. [40] reported a similar trend from experimental measurements taken via 2D imaging. The ratio of the standard deviations (which quantifies the strength of the clustering [34]) in Fig. 8b is in agreement with the results of Obligado et al. [40]. This observation further supports the observation that the degree of clustering inertial particles for $St \geq 1$ has indeed a close resemblance to self-similar clustering of ZAPS.

We therefore find that, while the 3D nature and more detailed spatial resolution of this study shows that no set of nulls perfectly mimics the inertial particles spatial distribution, for $St \geq 1$ the results from [40] remain valid: the spatial segregation of particles is consistent with that of ZAPS (specially for larger Voronoi volumes), supporting an agreement with the sweep-stick mechanism. On the other hand, very small particles’ Voronoi cells deviate from this behavior, and particles with $St = 0.5$ do not present complete similarities with any of the set of nulls discussed here. It is nevertheless expected that for lower values of the $St$ number, inertial particles will eventually agglomerate within low vorticity regions of the flow.

As a closing comment, note other scaling relations can be inferred or confirmed from these results. Previous studies [35, 37] indicate that the linear cluster size of inertial particles $L_C = \langle V_C\rangle_{St}^{1/3}$ is of $O(\ell/\eta)$, Our results also indicate $L_C/\ell \in [0.1 - 0.15]$, with similar values for linear cluster sizes of ZAPS for $Re_{\lambda} > 200$. In a related observation, Wittmeier and Shirnpton [58] recently reported that when the product between the particle number density and the Kolmogorov length scale is held constant, i.e., $n_{p}^{1/3}\eta = \langle V \rangle_{\text{particle}}^{1/3}$, some measures used to quantify preferential concentration become independent of the Reynolds number for $Re_{\lambda} \geq 200$. We can test this claim in the following way: first, we assume that the particles follow the sweep-stick mechanism, and that the number density of ZAPS and of inertial particles are similar such that we can write $n_{p}^{1/3}\eta \sim n_{ZAPS}^{1/3}\eta$. Then, using Fig. 1a (see also [8]) we can advance $n_{ZAPS} \sim \langle \ell/\eta \rangle^3$ and thus $n_{p}^{1/3}\eta \sim n_{ZAPS}^{1/3}\eta \sim \ell$. Our DNS results supports this proposal, and the degree of clustering of ZAPS appears to saturate for $Re_{\lambda} \in [250, 610]$, i.e., $\langle V \rangle_{ZAPS}/\sigma_{RPP} \approx 4$. 

FIG. 8: Particle clusters global parameters. a) Inertial particles average cluster size over the different average cluster sizes coming from the nulls of fields of the DNS-512 dataset. b) Ratio of the standard deviation of the inertial particle cluster volumes over the standard deviation for the different nulls fields, also for the DNS-512 dataset.
V. CONCLUDING REMARKS

We have analyzed the velocity, Lagrangian acceleration, and vorticity nulls in datasets coming from high fidelity numerical simulations, in a wide range of Taylor-based Reynolds numbers. Mean values and standard deviations of Voronoi cells volumes for these fields nulls display scaling dependence with $Re_{\lambda}$. The number density of the velocity and acceleration nulls roughly follow the scalings proposed by Vassilicos and collaborators [8, 12, 20]. Vorticity nulls (the densest of all fields) also exhibit a scaling similar to the acceleration nulls, as reported by Moisy and Jimenez[32]. The velocity nulls are scarce, but they are the most strongly clustered field at increasing $Re_{\lambda}$, as indicated by the standard deviation of Voronoi cells volumes. On the contrary, clustering of vorticity and Lagrangian acceleration nulls (again as indicated by the standard deviations) barely changes with $Re_{\lambda}$, with their normalized cluster size depending weakly on $Re_{\lambda}$.

Our results confirm the presence of a power-law with an exponent close to $-5/3$ in the Voronoi volume cell PDF for velocity nulls (or stagnation points) at increasing values of $Re_{\lambda}$. This behavior is absent for acceleration and vorticity nulls. Moreover, when considering the PDFs of cluster volumes, the PDFs for all null fields show a power-law behavior with an algebraic exponent close to $-5/3$ for velocity nulls, and to $-2$ for Lagrangian acceleration and vorticity nulls. We showed evidence that this behavior is not an artifact of the 3D Voronoi tessellation, and that the extent of the scaling stems from the underlying dynamics of the turbulent flow.

When considering the clustering (or preferential concentration) of point inertial particles, our results show that for Voronoi cells with normalized volume $V > 1$ (i.e., for volumes larger than the mean), the Voronoi cell PDF of inertial particle clustering better matches the ZAPS Voronoi cell PDF. Likewise, the average cluster volume of both inertial particles and ZAPS have the same order of magnitude. These observations give credence to the observation that on the average, the preferential concentration mimics the topology of the zero acceleration points, as reported elsewhere [10, 40]. However, for very small particles’ Voronoi cells, deviations from this behavior are observed, indicating that the sweep-stick mechanism may be only an an approximate representation of a more complex physical process underlying the preferential concentration of particles. Finally, we find evidence that the cluster linear size scales with the integral length scale, $L_C = \mathcal{O}(L/10)$, in agreement with previous studies by Mora et al. [37].

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