Multi-period data envelopment analysis models and resource allocation: A case study

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Abstract. Data envelopment analysis is a general tool for efficiency and performance measurement of the set of decision making units based on solving linear optimization problems. The paper deals with using this group of models for resource allocation analysis among Czech economic faculties within 7 years period from 2008 until 2014. The resources in the period \( t \) are divided among faculties according to their teaching and research performance in the period \((t-1)\). The teaching performance is measured by the number of students and number of graduated in the given period. The research performance is defined according to the national rules by the number of so called RIV points. The overall performance level in the given period is estimated using data envelopment analysis model with constant returns to scale assumption. The input in the period \( t \) is partly based on the efficiency or super-efficiency scores from the preceding period. Super-efficiency score given by Andersen and Petersen model is used for discrimination among efficient units. The results of the allocation in the last period are compared to the ones calculated to the standard methodology. Numerical experiments are realized using own MS Excel add-ins and LINGO modelling and optimization system.

1. Introduction
Allocation of a limited amount of resources among the set of decision making units (DMUs) is an important task that finds its applications in various areas of human activities. One of the typical problems of this nature is allocation of funds among higher education institutions which is a problem that significantly influences all activities of the units under evaluation. The common practice in the Czech Republic consists in dividing of available resources among universities and then among faculties according to their performance in the past one or several periods. The overall performance of the universities (faculties) is given by their partial performances in various activities carried out. Among the main activities of higher educational institutions belong teaching, research, international relations, co-operation with partners, etc. Of course teaching and research are the most important ones in this context. That is why evaluation of teaching and research performance of the set of educational institutions (let us denote them further units or faculties) is a very important problem and the methodology used for this evaluation must reflect and respect many requirements.

The main aim of this paper is to contribute to the discussion in this field and try applying not so widely used techniques. The common practice in the Czech Republic uses basic multiple criteria decision making approaches, i.e. assigns expert weights to the defined criteria that describe various aspects of overall performance and then applies simple additive weighting. The given performance score is a basis for a future allocation of funds. In the last several tens of years a more sophisticated methodological approaches for efficiency and performance evaluation have been proposed and verified by various researchers. Data Envelopment Analysis (DEA) models introduced by Charnes et
al. [1] belong without any doubts among the most theoretically developed and applied techniques. Applications of DEA models in higher education are numerous. Let us mention at least three of them – Beasley [2], Thanassoulis et al. [3] and Jablonsky [4].

The paper is organized as follows. Section 2 defines shortly basic DEA models and introduces multi-period DEA models which is a class of models that evaluates the performance of the set of units within several consecutive periods. An iterative approach that combines results of the performance evaluation in the period $t$ and the amount of funds allocated to the unit in the period $t$ (based on the evaluation in the period $(t-1)$) as the main input variable in the period $(t+1)$ is introduced in Section 3. The proposed approach is illustrated on the real data set of 19 Czech economic faculties in multiple periods from 2008 until 2014. The given results are summarized in the final section of the paper.

2. Multi-period DEA models

Traditional DEA models analyse relative technical efficiency of the set of $n$ DMUs that are described by $m$ inputs and $r$ outputs in one period. The efficiency score $\theta_q$ of the DMU$_q$ is defined as the weighted sum of outputs divided by the weighted sum of inputs as follows:

$$\theta_q = \sum_{k=1}^{r} u_k y_{kq} \left/ \sum_{i=1}^{m} v_i x_{iq} \right.$$  \hspace{1cm} (1)

where $u_k$, $k = 1, 2, ..., r$ is the positive weight of the $k$-th output, $v_i$, $i = 1, 2, ..., m$ is the positive weight of the $i$-th input, and $x_{ij}$, $i = 1, 2, ..., m$, $j = 1, 2, ..., n$ and $y_{kj}$, $k = 1, 2, ..., r$, $j = 1, 2, ..., n$ are non-negative values for the DMU$_j$ of the $i$-th input and the $k$-th output respectively. Traditional DEA models maximize the efficiency score (1) under the assumption that the efficiency scores of all other DMUs do not exceed 1 (100%). The presented problem is not linear in the objective function but it can be converted into a linear optimization problem and then solved easily. The transformation consists in maximization of the nominator or minimization of the denominator in expression (1). In some cases, it is more convenient working with dual problem to the linearized version of the model described above. This model, often referenced as input oriented envelopment CCR (Charnes, Cooper and Rhodes) model is formulated as follows:

minimize $\theta_q^{CCR}$

subject to $\sum_{j=1}^{n} x_{ij} \lambda_j + s^-_i = \theta_q^{CCR} x_{iq}$, \hspace{1cm} $i = 1, 2, ..., m$, \hspace{1cm} (2)

$\sum_{j=1}^{n} y_{kj} \lambda_j - s^+_k = y_{kj}$, \hspace{1cm} $k = 1, 2, ..., r$

$\lambda_j \geq 0$, \hspace{1cm} $j = 1, 2, ..., n$.

The optimal objective function value of model (2) is lower than 1 for inefficient units and equals 1 for the units weakly or fully efficient. In order to rank efficient units many models based on various principles have been proposed. An important group of these models are so called super-efficiency models. The first model of this nature was introduced in Andersen and Petersen [5]. Its input oriented formulation is very close to model (2). In Andersen and Petersen (AP) model the weight of the DMU$_q$ $\lambda_q$ is set to zero. It causes that the DMU$_q$ is removed from the set of units and the efficient frontier changes its shape after this removal. Super-efficiency score $\theta_q^{AP}$, that is greater than 1 for units CCR fully efficient, measures the distance of the unit DMU$_q$ from the new efficient frontier. Its value expresses how many times the inputs may increase (in input oriented version of the model) in order the evaluated unit remains efficient.
Traditional DEA models (1) and (2) evaluate the efficiency or performance of the set of units in one time period only according to the values of input and output variables but it is clear that the performance in the period $t$ can depend not only on the inputs in this period but on the inputs in one or several preceding periods. That is why various attempts how to solve this problem have been proposed in the past. This group of models is called multi-period DEA models. The first model of this class was introduced by Park and Park [6]. Unfortunately, their model (further as PP model) is not a real multi-period model because it returns just the best efficiency score of the unit under evaluation within all periods considered. It is the model oriented on the best period. It is clear that according to this property the PP model cannot be used as a tool for future allocation of resources. Its mathematical formulation can be found e.g. in Park and Park [6] or Jablonsky [4]. Similarly to the PP model it is possible formulate models that are oriented on the worse period of the unit under evaluation or the model that returns average efficiency score over all periods. Their formulation is given in Jablonsky [4]. In addition, this paper contains an original SBM multi-period model that measures efficiency during all periods using undesirable slacks of inputs and outputs. Nevertheless, this model does not take into account interconnections among particular periods. In order to do it, the appropriate model is not linear and its solution is questionable. The DEA model that uses efficiency score given in the period $(t-1)$ as the only input variable in the period $t$ is as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{q=1}^{n} \sum_{t=2}^{T} \theta_{q}^t \\
\text{subject to} & \quad \sum_{j=1}^{n} \theta_{j}^{t-1} y_{jq}^t \leq \theta_{q}^t \theta_{q}^{t-1}, \quad t = 2,3,…,T, i = 1,2,…,m, q = 1,2,…,n, \\
& \quad \sum_{j=1}^{n} y_{kj}^{t} \lambda_{jq}^t \geq y_{kq}^t, \quad t = 1,2,…,T, k = 1,2,…,r, q = 1,2,…,n, \\
& \quad \theta_{q}^t = 1, \quad q = 1,2,…,n, \\
& \quad \lambda_{jq}^t \geq 0, \quad t = 1,2,…,T, j = 1,2,…,n, \quad q = 1,2,…,n.
\end{align*}
\]

where the DMUs are described by the same set of outputs and one input in $T$ consecutive time periods $t = 1, 2, \ldots, T$, and assume that $y_{ji}, k = 1, 2, \ldots, r, j = 1, 2, \ldots, n$ are the values of the $k$-th output in the $t$-th period of the DMU. The only input in the period $t$ is the efficiency score of the units in the period $(t-1)$ and its values for the first period are set to 1 for all units. $\theta_{q}^t$ is the aggregative efficiency score of DMU in the period $t$. It is clear that model (3) is not linear in the first set of constraints (moreover, the number of variables and constraints can be very high) and that is why it cannot be solved easily. To overcome this problem we propose in the next section of the paper an iterative approach for analysis of multi-period systems.

3. Resource allocation – a case of Czech economic faculties

In this section we will test the hypothesis that the multi-period DEA analysis can generate results usable for resource allocation of the set of homogenous faculties. For this purpose the set of 19 Czech public economic faculties was investigated. Each of the faculties is described by two inputs - number of academic staff and labour costs. Three most important performance characteristics as outputs are taken into account – the number of students, the number of graduated, and the number of RIV points generated by the faculty in particular years (RIV points is an aggregated characteristic that measures the quality of publication outputs of individuals and/or educational units in the Czech Republic). The data set is available since 2007 until 2013. Table 1 contains efficiency scores $\theta_{CR}$ and super-efficiency scores $\theta_{O}$ for all faculties and periods. The best (highest) values are bolded.
Table 1. Efficiency (super-efficiency) scores in all periods.

| Faculty   | 2007   | 2008   | 2009   | 2010   | 2011   | 2012   | 2013   | Avg   |
|-----------|--------|--------|--------|--------|--------|--------|--------|-------|
| FSV UK    | 1.4722 | **1.7058** | 1.1984 | 1.6965 | 1.1766 | 1.5307 | 1.3887 | 1.4527 |
| EkF ČU    | 0.6730 | 0.6825 | **0.9610** | 0.6937 | 0.7666 | 0.6818 | 0.6175 | 0.7252 |
| FSE UJEP  | 0.7713 | 0.6978 | 0.7977 | 0.7678 | 0.9260 | 0.8206 | **0.9849** | 0.8237 |
| ESF MU    | 0.6921 | 0.6466 | 0.9762 | 1.0216 | **1.0444** | 1.0315 | 0.9765 | 0.9127 |
| OPF SU    | 0.6206 | 0.6824 | 0.8831 | **0.9331** | 0.9072 | 0.8495 | 0.8372 | 0.8162 |
| FE ZČU    | 0.9104 | 0.8333 | 0.8441 | 0.9530 | **1.0022** | 0.9381 | 0.8750 | 0.9080 |
| HF TUL    | 0.4978 | 0.5363 | 0.6122 | 0.6490 | 0.5923 | **0.8037** | 0.6573 | 0.6212 |
| HES UP    | 0.8354 | 0.6271 | **0.8623** | 0.8270 | 0.7358 | 0.6568 | 0.7726 | 0.7596 |
| FP VUT    | **1.0290** | 0.9581 | 0.8996 | 0.8446 | 1.0065 | 1.0142 | 0.9721 | 0.9606 |
| EkF VŠB   | **0.9919** | 0.9457 | 0.8883 | 0.8779 | 0.9000 | 0.9682 | 0.8525 | 0.9178 |
| FME Zlíns | 1.3748 | 1.4282 | 1.3081 | 1.3163 | **1.6378** | 1.0406 | 1.4069 | 1.3590 |
| FFU VŠE   | **1.1239** | 1.0924 | 0.8858 | 0.8394 | 0.8629 | 0.8370 | 0.7877 | 0.9184 |
| FMV VŠE   | 0.7464 | 0.6835 | 0.6572 | 0.6340 | 0.7071 | **0.7236** | 0.6744 | 0.6895 |
| FPH VŠE   | 0.9634 | **0.9881** | 0.8743 | 0.8047 | 0.8138 | 0.7390 | 0.6931 | 0.8395 |
| FIS VŠE   | 0.7533 | 0.7983 | 0.8435 | 0.8182 | 0.8443 | **0.9640** | 0.8495 | 0.8387 |
| NH VŠE    | **1.5045** | 1.4318 | 1.0453 | 0.9920 | 0.9135 | 1.0390 | 0.9321 | 1.1226 |
| FM VŠE    | 0.7225 | 0.7049 | 0.8124 | 0.8441 | 0.8253 | **0.9756** | 0.9482 | 0.8333 |
| PEF ČZU   | 0.9038 | 1.1285 | 1.3193 | 1.3149 | 1.3638 | **1.4432** | 1.3190 | 1.2561 |
| PEF MZLU  | 0.8037 | 0.8461 | **0.9368** | 0.7982 | 0.8450 | 0.9348 | 0.8331 | 0.8568 |

4. Resource allocation – a case of Czech economic faculties

The results presented in Table 1 show that only two faculties are efficient in all periods (FSV UK and FME Zlín). Application of the PP multi-period model (orientation on the best period) leads to the following ranking: FSV UK (1.7058), FME Zlín (1.6378), NH VŠE (1.5045), ..., FMV VŠE (0.7236). The multi-period model oriented on the worse period (see Jablonský [4]) generates the same ranking in first three places but the worse faculty is changed in this case, i.e. FSV UK (1.1766), FME Zlín (1.0406), NH VŠE (1.0390), ..., HF TUL (0.4978). The same ranking (first three and last place) is given by average efficiencies over all periods (the last column of Table 1). The efficiency (super-efficiency) scores in the period \( t \) express the level of relative performance of the faculties according to the given model and variables (inputs/outputs) taken into account but they just hardly can be used as indicators for allocation of resources in the future periods because the results do not consider interconnections among the periods. In order to try overcoming this drawback we propose the following simple iterative procedure:

- The set of outputs contains the same three variables as in the previous experiments, i.e. the number of students, the number of graduated and the number of RIV points.
- The set of inputs includes two variables – the number of employees and the labor costs but this second variable is given in a different way comparing to the previous model. In 2007 the values of this variable correspond to the reality. In the other periods it is the result of a simple formula. The amount of 50% of the labor costs in the period \( t \) is calculated according to the labor costs’ share in the previous period and the remaining 50% according to the efficiency scores’ share in this period.
• The efficiency scores are calculated using model (2) and, if necessary, super-efficiency scores using the AP model derived from model (2). Either two mentioned inputs are considered or just labor costs derived using the presented way are taken into account. The results of allocation of resources among faculties according to the given approach are presented in Table 2. The first 8 columns of the table contain differences in allocation of funds given by the model with two inputs and original allocation by government methodology. The positive/negative values indicate that the allocation by the model is higher/lower than the reality. The last column of Table 3 contain the same information but given by the modified model (3). The results show that there are faculties overfunded in all periods (e.g. PEF ČZU and FMV VŠE). In the contrary some of the faculties are underfunded in all or almost all periods (e.g. FME Zlín, NH VŠE, and others).

| Faculty   | 2008  | 2009  | 2010  | 2011  | 2012  | 2013  | 2014  | 2014a |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|
| FSV UK    | 5997  | 17544 | 10269 | 20844 | -7166 | -6503 | -17364| -16567|
| EKF JČU   | 20431 | 14085 | 15480 | 10396 | 7587  | 1016  | -1228 | 1642  |
| FSE UJEP  | 3691  | 4762  | 4864  | 8954  | 8491  | 13237 | 15948 | 10085 |
| ESF MU    | -9655 | -12901| -7325 | -1177 | 2112  | 996   | 5945  | 3112  |
| OPF SU    | -8758 | -9311 | -4260 | -8153 | -6975 | -5020 | 1934  | 6253  |
| FE ZČU    | 7804  | 12856 | 15178 | 13311 | 13740 | 16804 | 16230 | 9831  |
| HF TUL    | -4088 | -4645 | -1970 | -960  | -199  | 5278  | 1094  | 1860  |
| FES UP    | -643  | -2569 | 3251  | 5887  | -4    | 6462  | 6626  | 7111  |
| FP VUT    | 6321  | 8784  | 10445 | 7175  | 6786  | 6426  | 5138  | 5013  |
| EKF VŠB   | -17022| -17904| -19730| -15926| -12632| -14866| -15413| -10018|
| FME Zlín  | 18709 | 20030 | 19368 | 15750 | 23395 | 18230 | 22479 | 17873 |
| FFU VŠE   | 6616  | 8052  | 3143  | -327  | 83    | -487  | -212  | -118  |
| FMV VŠE   | -15821| -18543| -24991| -30903| -26917| -30064| -34109| -29677|
| FPH VŠE   | -852  | -1517 | -2978 | -6524 | -11828| -14165| -8428 | -4523 |
| FIS VŠE   | -1322 | 113   | -1668 | -4084 | -1161 | -1731 | -5411 | -2075 |
| NH VŠE    | 22668 | 25877 | 21707 | 20693 | 17412 | 17566 | 18870 | 11498 |
| FM VŠE    | 7779  | 13082 | 16868 | 17407 | 20597 | 20076 | 19965 | 10512 |
| PEF ČZU   | -37916| -55918| -55652| -49018| -34498| -28167| -27135| -22003|
| PEF MZLU  | -3937 | -1876 | -2001 | -3346 | 1177  | -5088 | -4926 | 194   |

5. Conclusions
Evaluation of efficiency and performance of higher educational institutions is of a high importance and the results of this evaluation can serve as starting points for allocation of limited resources among these institutions. The commonly used practice in the Czech Republic is based on weighting of several criteria among which the most important is the number of students and research performance of the institution in the last five years measured by RIV points. The paper compares this methodology with DEA approach – a traditional CCR input oriented model and its super-efficiency modification with five (or four) variables is applied. The given results show significant differences in funding of the faculties under evaluation according to the official methodology and the DEA models. Of course the proposed models do not cover all criteria considered by the official methodology but the differences in
funding can just partly be explained by this drawback. A future research in this field can be focused on extension of the number of variables in the model and application of other DEA models. SBM models, e.g. Tone’s efficiency and super-efficiency models (see Tone [7]), could be an interesting alternative to the traditional DEA models.

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