Optimal parameters of EWMA Control Chart for Seasonal and Non-Seasonal Moving Average Processes

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Abstract. The main goal of this paper is to study optimal parameters of an Exponentially Weighted Moving Average (EWMA) control chart for seasonal and non-seasonal Moving Average (SMA and MA) processes. The characteristic of control chart is Average Run Length (ARL) which is the average number of samples taken before an action signal is given. Ideally, an acceptable ARL of in-control process should be large enough, so-called Average Run Length for in-control process (ARL₀). Otherwise, it should be small when the process is out-of-control, so-called Average Run Length for out-of-control process (ARL₁). We obtain explicit formulas of ARL for EWMA chart for Moving Average (MA) process with exponential white noise. In particular, the explicit analytical formulas for evaluating ARL₀ and ARL₁ are able to obtain a set of optimal parameters which depend on a smoothing parameter (λ) and a width of control limit (b) for designing EWMA chart with a minimum ARL₁ value. In addition, the explicit formulas for the EWMA control chart was applied with the practical data of the unemployment rate of Thailand.

1. Introduction

Statistical Process Control (SPC) plays a vital role in monitoring, detecting changes in a process, and it is used for measuring, controlling and improving quality in many areas, such as industrial statistics and manufacturing, economics and finance, computer sciences and telecommunications, and in other areas of applications (see [1]-[3]). The main tool for SPC is control chart. CUSUM control chart was firstly introduced by Page [4], and EWMA control chart was initially presented by Roberts [5]. In this paper, we discuss the Exponentially Weighted Moving Average (EWMA) chart which is used for detecting small changes of parameters. A basic assumption of control charts is that observations from the process at different times are independently and identically distributed (i.i.d) random variables. However, there are many situations in which the process is serially correlated data, such as a chemical process, the manufacture of food, computer intrusion detection, wind speeds, and a daily water flow of a river. Various authors were studied control charts for monitoring processes with serially dependent data, see [6]-[7]. The observations from the variables or factors in real situations are usually collected from stochastic processes, or time series. In general, normally distributed white noise indicates the errors in a time series model with autocorrelated observations. However, the white noise can be distributed differently in some applications, such as exponentially distributed white noise. For instance, the exponential white noise was also used to analyze the autoregressive model, proposed by Mohamed and Hocine [8].
A common characteristic used for comparing the performance of control charts is Average Run Length (ARL) defined as the expected number of observations taken from an in-control process until the control chart falsely signals out-of-control which is denoted by $ARL_0$. An $ARL_0$ will be regarded as acceptance if it is large enough to keep the level of false alarms at an acceptable level. The second common characteristic is the expected number of observations taken from an out-of-control process until the control chart signals that the process is out-of-control, which is denoted by $ARL_1$. Ideally, the $ARL_1$ should be as small as possible.

In literature, there are many methods for evaluating the $ARL$ such as Markov Chain approach (MCA), Numerical Integral Equation approach (NIE) and Monte Carlo simulation (MC). In 1959, Roberts [5] presented evaluating $ARL$ of the EWMA control chart by using the Monte Carlo simulation technique. Later, Crowder [9] presented a numerical procedure for the $ARL$ of the EWMA control chart under a normal observation assumption by using the Integral Equation approach. Recently, Champ and Ridgon [10] studied CUSUM and EWMA charts using the Markov chain and integral equation approaches to evaluate the $ARL$. As discussed earlier, MC, MCA and NIE are the most popular methods for evaluating the characteristics of control charts. However, these methods are difficult and laborious to find the optimal designs. The limitations of the MC, MCA and IE methods provide the motivation for evaluating explicit analytical formulas of $ARL$. Petcharat et al. [11] derived explicit formula of $ARL$ for an Exponentially Weighted Moving Average (EWMA) chart by using integral equation when observations are described by Moving Average order $q$ (MA($q$)) process with exponential white noise. Later, Suntornwat et al. [12] proposed the explicit formulas of $ARL$ which evaluated the Integral Equation technique on the EWMA control chart for ARFIMA process and also compared the analytical solutions.

Recently, Peerajit [13] evaluated the numerical integral equation method of $ARL$ on CUSUM chart. After that, Sunthornwat and Areepong [14] presented explicit formulas of $ARL$ on CUSUM control chart for seasonal and non-seasonal moving average processes with exogenous variables and compared the results with the NIE method. From the above research, the explicit formulas were derived but the optimal values from the formulas for the $ARL$ value have yet found.

In this paper, we develop explicit formulas of $ARL$ for EWMA control chart for Moving Average (MA) process with exponential white noise. In particular, the explicit analytical formulas for evaluating $ARL_0$ and $ARL_1$ be able to get a set of optimal parameters which depend on smoothing parameter ($\lambda$) and control limit ($b$) for designing EWMA control chart with minimum of $ARL_1$.

2. Methodology
In this paper, we consider SPC chart under an assumption that sequential observations $\xi_1, \xi_2, \ldots$, are independent random variables with a distribution function $F(x, \alpha)$. The parameter $\alpha$ is equal to $\alpha_0$ before a change-point time ($\theta \leq \infty$). For in-control process, $\theta = \infty$ means that there are no change at all. For out-of-control process, $\alpha > \alpha_0$ after the change-point time $\theta$. All popular charts are based on use of stopping time $\tau$. The typical condition on choice of the stopping time $\tau$ is the following:

$$E_{\alpha}(\tau) = T, \quad (1)$$

where $T$ is a constant and $E_{\alpha}(\cdot)$ denotes that the expectation under distribution $F(x, \alpha_0)$. In literature on quality control the quantity, $E_{\alpha}(\tau)$ is called an Average Run Length for in-control process ($ARL_0$).

Then, by definition, $ARL_0 = E_{\alpha}(\tau)$ and the typical practical constraint is

$$ARL_0 = E_{\alpha}(\tau) = T. \quad (2)$$

Another typical constraint consists in minimizing the quantity

$$Q(\alpha) = E_{\alpha}(\tau - \theta + 1 | \tau \geq \theta), \quad (3)$$
where \( E_{\theta}(.) \) is the expectation under distribution \( F(x,\alpha_i) \) (out-of-control) and \( \alpha_i \) is a value of parameter after the change-point. We restrict on the special case, usually \( \theta = 1 \). The quantity \( E_{\theta}(\tau) \) is called as Average Run Length for out-of-control process \( (ARL_c) \) and one could expect that a sequential chart has a near optimal performance if \( ARL_c \) is close to a minimal value.

The EWMA statistics based on \( MA(q) \) process is defined by the following recursion:

\[
Z_t = (1 - \lambda)Z_{t-1} + \lambda X_t ; t = 1, 2, \ldots
\]  

(4)

where \( Z_t \) is the EWMA statistics, \( X_t \) is a sequence of \( MA(q) \) process, the initial value is a constant \( (Z_0 = u) \), and \( \lambda \in (0, 1) \) is a smoothing parameter.

The cumulative sum (CUSUM) control chart can be defined as follows:

\[
Y_t = \max \left( Y_{t-1} + X_t - a, 0 \right) ; t = 1, 2, \ldots
\]  

(5)

where \( Y_t \) is the CUSUM statistic, \( X_t \) is the sequence of an \( MA(q) \) process, the initial value is a constant \( (Z_0 = u) \), and \( a \) is the constant recall reference value for the chart.

The \( MA(q) \) process can be described by the following recursion:

\[
X_t = \mu - \theta_1 \xi_{t-1} - \theta_2 \xi_{t-2} - \ldots - \theta_q \xi_{t-q}
\]  

(6)

where \( \xi_t \) is a white noise process assumed to have exponential distribution, \( \theta \) is a moving average coefficient which \( 0 \leq \theta_i \leq 1 \), and \( \mu \) is a constant. We assume the initial value of \( MA(q) \) process \( \xi_{t-1}, \xi_{t-2}, \ldots, \xi_{t-q} = 1 \) as the process mean.

The Seasonal Moving Average process, denoted by \( SMA(Q)_L \), process can be written as:

\[
X_t = \mu - \theta_{Q-L} \xi_{t-L} - \theta_{2L} \xi_{t-2L} - \ldots - \theta_{QL} \xi_{t-QL}
\]  

(7)

where \( \xi_t \) is to be a white noise process assumed with exponential distribution. A seasonal moving average coefficient \( 0 \leq \theta_i \leq 1 \), and \( \mu \) is a constant. We assume the initial value of \( MA(q) \) process \( \xi_{t-L}, \xi_{t-2L}, \ldots, \xi_{t-QL} = 1 \) as the process mean.

The stopping time for the EWMA control chart can be written as:

\[
\tau_b = \inf \left\{ t > 0 : Z_t > b \right\}, b > u ,
\]  

(8)

where \( b \) is a control limit.

The stopping time for the CUSUM control chart can be written as:

\[
\tau_h = \inf \left\{ t > 0 : Y_t > h \right\}, h > u ,
\]  

(9)

where \( h \) is a control limit.

3. Solution for Evaluating \( ARL_0 \) and \( ARL_1 \) of EWMA Procedure

In this section, we present the explicit formulas of \( ARL \) for \( MA(q) \) process in Petcharat et al. [11]. We obtain the explicit formula for \( ARL \) as follows:

\[
ARL_0 = 1 - \lambda \exp \left( \frac{1 - \lambda u}{\lambda \alpha_0} \right) \exp \left( - \frac{b}{\lambda \alpha_0} \right) - 1
\]  

(10)

On the other hand, since the process is out-of-control, parameter \( \alpha = \alpha_1 \), the explicit formula for \( ARL_1 \) can be written as follows:
Using the explicit formulas, we have been able to provide the tables for the optimal smoothing parameter \((\lambda)\) and width of control limit \((b)\) for designing EWMA chart with minimum of \(ARL_1\). We first describe a procedure for obtaining optimal designs for EWMA control chart. The criterions for choosing optimal values are smoothing parameter \((\lambda)\) and control limit \((b)\) for designing EWMA chart with minimum of \(ARL_1\) for a given in-control parameter value \(\alpha_0 = 1\), \(ARL_0 = T\), and a given out-of-control parameter value \((\alpha = \alpha_1)\). We compute optimal \((\lambda, b)\) values for \(T = 370, 500\) and magnitudes of change. Tables of the optimal parameters values are shown in Tables 1-6.

### 3.1 The numerical procedure for obtaining optimal parameters for MA designs

1. Select an acceptable in-control value of \(ARL_0\) and decide on the change parameter value \((\alpha_i)\) for an out-of-control state.

2. For given values \(\alpha_0\) and \(T\), find optimal values of \(\lambda\) and \(b\) to minimize the \(ARL_1\) value given by Equation 11 subjected to the constraint that \(ARL_0 = T\) in Equation 10. Then \(\lambda\) and \(b\) are solutions of the optimality problem.

In addition, the numerical procedure for obtaining optimal parameters for SMA(\(Q_l\)) designs is the same as MA(\(q\)) procedure by using Equations 12 and 13 for \(ARL_0\) and \(ARL_1\) respectively. The optimal values \((\lambda, b)\) for \(T = 370, 500\) and magnitudes of change are shown in Tables 1-4.

### 4. Numerical Results

In this section, the numerical results for optimal design parameters of optimal width of smoothing parameter \((\lambda)\), optimal of a control limit \((b)\), and minimal \(ARL_1\) for EWMA control chart were calculated from Equations 10-13. The optimal parameter values for the EWMA control chart were chosen by setting the desired \(ARL_0 = 370\) and 500. The value of the in-control parameter \(\alpha_0 = 1\) and the out-of-control parameter \(\alpha_i \subseteq [1.01; 1.20]\). Tables 1-3 show optimal design parameters for EWMA control chart for \(MA(q)\) process. The coefficient parameters of the process \(u = 0.1\) was used for the \(MA(1)\) process, \(u = 0.1, v = 0.2\) were used for the \(MA(2)\) process, and \(u = 0.4, v = 0.2, w = 0.3\) were used for the \(MA(3)\) process. For example, if we want to detect a parameter change of \(MA(l)\) process from \(\alpha_0 = 1\) to \(\alpha_1 = 1.05\) and the \(ARL_0\) value is 370 then the optimality procedure given above will give optimal parameter values \(\lambda = 0.2031\) and \(b = 0.2536\). On substituting the values for \(\alpha_i, \lambda\) and \(b\) into Equation 10, we obtain an optimal \(ARL_1\) value 20.265. In Tables 4-5, the optimal parameter values for EWMA control chart for \(MA(Q_l)\) process are presented. The coefficient parameters of the process
\( \theta_1 = -0.1 \) was used for the \( MA(1)_4 \) process, \( \theta_1 = -0.1, \theta_2 = 0.2 \) were used for the \( MA(2)_4 \) process, and \( \theta_1 = -0.1, \theta_2 = -0.2, \theta_3 = 0.3 \) were used for the \( MA(3)_4 \) process. For example, if we want to detect a parameter change of \( MA(1)_4 \) process from \( \alpha_0 = 1 \) to \( \alpha_4 = 1.10 \) and the \( ARL_0 \) value is 370 then the optimality procedure given above will give optimal parameter values \( \lambda = 0.2347 \) and \( b = 0.2381 \). On substituting the values for \( \alpha_1, \lambda \) and \( b \) into Equation 10, we obtain an optimal \( ARL \) value 9.339.

As shown in Tables 1-5, the use of the suggested \( ARL \) explicit formulas for EWMA control chart can greatly reduce the computational time, and is useful for practitioners especially finding optimal parameters of EWMA control chart.

**Table 1.** Optimal design parameters and \( ARL_1 \) for \( MA(2) \) when give \( \theta_1 = 0.1, \theta_2 = 0.2 \)

| \( \alpha_1 \) | \( \lambda \) | \( b \) | \( ARL_0 = 370 \) | \( \alpha_1 \) | \( \lambda \) | \( b \) | \( ARL_0 = 500 \) |
|----------------|----------|----------|-----------------|----------------|----------|----------|-----------------|
| 1.01           | 0.1690   | 0.2585   | 96.519          | 1.01           | 0.1691   | 0.2587   | 103.442         |
| 1.02           | 0.1689   | 0.2581   | 55.877          | 1.02           | 0.1689   | 0.2583   | 58.109          |
| 1.03           | 0.1686   | 0.2577   | 39.520          | 1.03           | 0.1687   | 0.2579   | 40.604          |
| 1.04           | 0.1684   | 0.2573   | 30.681          | 1.04           | 0.1684   | 0.2575   | 31.322          |
| 1.05           | 0.1682   | 0.2569   | 25.146          | 1.05           | 0.1682   | 0.2571   | 26.001          |
| 1.06           | 0.1679   | 0.2565   | 21.355          | 1.06           | 0.1680   | 0.2567   | 21.656          |
| 1.07           | 0.1677   | 0.2561   | 18.596          | 1.07           | 0.1677   | 0.2562   | 18.821          |
| 1.08           | 0.1675   | 0.2557   | 16.498          | 1.08           | 0.1675   | 0.2559   | 16.673          |
| 1.09           | 0.1672   | 0.2552   | 14.849          | 1.09           | 0.1672   | 0.2554   | 14.989          |
| 1.10           | 0.1670   | 0.2548   | 13.519          | 1.10           | 0.1670   | 0.2550   | 13.633          |
| 1.15           | 0.1658   | 0.2527   | 9.467           | 1.15           | 0.1658   | 0.2529   | 9.520           |
| 1.20           | 0.1645   | 0.2506   | 7.407           | 1.20           | 0.1646   | 0.2507   | 7.438           |

**Table 2.** Optimal design parameters and \( ARL_1 \) for \( MA(3) \) when give \( \theta_1 = 0.1, \theta_2 = 0.2, \theta_3 = 0.3 \)

| \( \alpha_1 \) | \( \lambda \) | \( b \) | \( ARL_0 = 370 \) | \( \alpha_1 \) | \( \lambda \) | \( b \) | \( ARL_0 = 500 \) |
|----------------|----------|----------|-----------------|----------------|----------|----------|-----------------|
| 1.01           | 0.1112   | 0.2280   | 147.452         | 1.01           | 0.1122   | 0.2282   | 164.374         |
| 1.02           | 0.1123   | 0.2285   | 91.920          | 1.02           | 0.1124   | 0.2288   | 98.161          |
| 1.03           | 0.1125   | 0.2290   | 66.708          | 1.03           | 0.1126   | 0.2293   | 69.908          |
| 1.04           | 0.1128   | 0.2295   | 52.318          | 1.04           | 0.1128   | 0.2297   | 54.250          |
| 1.05           | 0.1130   | 0.2300   | 43.018          | 1.05           | 0.1130   | 0.2302   | 44.305          |
| 1.06           | 0.1131   | 0.2303   | 36.518          | 1.06           | 0.1132   | 0.2306   | 37.434          |
| 1.07           | 0.1133   | 0.2308   | 31.721          | 1.07           | 0.1134   | 0.2310   | 32.404          |
| 1.08           | 0.1135   | 0.2312   | 28.037          | 1.08           | 0.1135   | 0.2314   | 28.565          |
| 1.09           | 0.1137   | 0.2315   | 25.121          | 1.09           | 0.1137   | 0.2318   | 25.540          |
| 1.10           | 0.1138   | 0.2319   | 22.756          | 1.10           | 0.1139   | 0.2321   | 23.097          |
| 1.15           | 0.1145   | 0.2334   | 15.499          | 1.15           | 0.1145   | 0.2335   | 15.650          |
| 1.20           | 0.1150   | 0.2348   | 11.796          | 1.20           | 0.1150   | 0.2347   | 11.880          |
### Table 3. Optimal design parameters and $ARL_1$ for $SMA(1)_k$ when given $\theta_i = -0.1$

| $\alpha_i$ | $ARL_0 = 370$ | $ARL_1^*$ | $\alpha_i$ | $ARL_0 = 500$ | $ARL_1^*$ |
|------------|----------------|------------|------------|----------------|------------|
|            | $\lambda$ | $b$ | $ARL_1$ | $\lambda$ | $b$ | $ARL_1$ |
| 1.01       | 0.2404   | 0.2447 | 69.602 | 1.01 | 0.2404   | 0.2449 | 73.117 |
| 1.02       | 0.2398   | 0.2439 | 38.866 | 1.02 | 0.2398   | 0.2441 | 39.920 |
| 1.03       | 0.2391   | 0.2432 | 27.174 | 1.03 | 0.2391   | 0.2433 | 27.676 |
| 1.04       | 0.2385   | 0.2424 | 21.011 | 1.04 | 0.2385   | 0.2426 | 21.306 |
| 1.05       | 0.2378   | 0.2417 | 17.206 | 1.05 | 0.2378   | 0.2419 | 17.399 |
| 1.06       | 0.2372   | 0.2409 | 14.623 | 1.06 | 0.2372   | 0.2411 | 14.760 |
| 1.07       | 0.2365   | 0.2402 | 12.754 | 1.07 | 0.2365   | 0.2404 | 12.857 |
| 1.08       | 0.2359   | 0.2395 | 11.339 | 1.08 | 0.2359   | 0.2397 | 11.419 |
| 1.09       | 0.2353   | 0.2388 | 10.231 | 1.09 | 0.2353   | 0.2390 | 10.295 |
| 1.10       | 0.2347   | 0.2381 | 9.339  | 1.10 | 0.2347   | 0.2382 | 9.392  |
| 1.15       | 0.2316   | 0.2346 | 6.636  | 1.15 | 0.2316   | 0.2348 | 6.660  |
| 1.20       | 0.2287   | 0.2313 | 5.267  | 1.20 | 0.2287   | 0.2314 | 5.281  |

### Table 4. Optimal design parameters and $ARL_1$ for $SMA(2)_k$ when given $\theta_i = -0.1, \theta_j = 0.2$

| $\alpha_i$ | $ARL_0 = 370$ | $ARL_1^*$ | $\alpha_i$ | $ARL_0 = 500$ | $ARL_1^*$ |
|------------|----------------|------------|------------|----------------|------------|
|            | $\lambda$ | $b$ | $ARL_1$ | $\lambda$ | $b$ | $ARL_1$ |
| 1.01       | 0.2049   | 0.2562 | 80.318 | 1.01 | 0.2050   | 0.2564 | 85.043 |
| 1.02       | 0.2045   | 0.2555 | 45.493 | 1.02 | 0.2045   | 0.2557 | 46.948 |
| 1.03       | 0.2040   | 0.2549 | 31.946 | 1.03 | 0.2040   | 0.2551 | 32.646 |
| 1.04       | 0.2036   | 0.2542 | 24.739 | 1.04 | 0.2036   | 0.2544 | 25.150 |
| 1.05       | 0.2031   | 0.2536 | 20.265 | 1.05 | 0.2031   | 0.2538 | 20.336 |
| 1.06       | 0.2026   | 0.2529 | 17.217 | 1.06 | 0.2027   | 0.2531 | 17.410 |
| 1.07       | 0.2022   | 0.2523 | 15.007 | 1.07 | 0.2022   | 0.2525 | 15.151 |
| 1.08       | 0.2017   | 0.2517 | 13.331 | 1.08 | 0.2018   | 0.2518 | 13.443 |
| 1.09       | 0.2013   | 0.2510 | 12.016 | 1.09 | 0.2013   | 0.2512 | 12.106 |
| 1.10       | 0.2008   | 0.2504 | 10.957 | 1.10 | 0.2009   | 0.2506 | 11.031 |
| 1.15       | 0.1986   | 0.2473 | 7.741  | 1.15 | 0.1987   | 0.2475 | 7.775  |
| 1.20       | 0.1965   | 0.2443 | 6.109  | 1.20 | 0.1965   | 0.2444 | 6.129  |

### Table 5. Optimal design parameters and $ARL_1$ for $SMA(3)_k$ when given $\theta_i = -0.1, \theta_j = -0.2, \theta_k = 0.3$

| $\alpha_i$ | $ARL_0 = 370$ | $ARL_1^*$ | $\alpha_i$ | $ARL_0 = 500$ | $ARL_1^*$ |
|------------|----------------|------------|------------|----------------|------------|
|            | $\lambda$ | $b$ | $ARL_1$ | $\lambda$ | $b$ | $ARL_1$ |
| 1.01       | 0.2227   | 0.2513 | 74.481 | 1.01 | 0.2227   | 0.2514 | 78.523 |
| 1.02       | 0.2221   | 0.2505 | 41.860 | 1.02 | 0.2221   | 0.2507 | 43.086 |
| 1.03       | 0.2215   | 0.2498 | 29.324 | 1.03 | 0.2215   | 0.2499 | 29.911 |
| 1.04       | 0.2210   | 0.2491 | 22.690 | 1.04 | 0.2210   | 0.2493 | 23.034 |
| 1.05       | 0.2204   | 0.2484 | 18.583 | 1.05 | 0.2204   | 0.2486 | 18.810 |
| 1.06       | 0.2199   | 0.2477 | 15.791 | 1.06 | 0.2199   | 0.2479 | 15.952 |
| 1.07       | 0.2193   | 0.2470 | 13.769 | 1.07 | 0.2193   | 0.2472 | 13.889 |
| 1.08       | 0.2188   | 0.2463 | 12.236 | 1.08 | 0.2188   | 0.2465 | 12.330 |
| 1.09       | 0.2182   | 0.2456 | 11.036 | 1.09 | 0.2183   | 0.2458 | 11.111 |
| 1.10       | 0.2177   | 0.2449 | 10.069 | 1.10 | 0.2178   | 0.2451 | 10.130 |
| 1.15       | 0.2151   | 0.2416 | 7.135  | 1.15 | 0.2151   | 0.2417 | 7.164  |
| 1.20       | 0.2125   | 0.2383 | 5.649  | 1.20 | 0.2125   | 0.2385 | 5.666  |
5. Real world Application

Application to real-world data was conducted to evaluate the $ARL$ by the explicit formula in Equations 10-11. The unemployment rate of Thailand, was collected monthly from January 2012 to December 2019 as the dataset of real observations. The second-order MA model is suitable for fitting the unemployment rate. Therefore, the second-order MA model was constructed with the process coefficients $\mu = 0.916$, $\theta_1 = -0.587$, $\theta_2 = -0.263$, and the error as exponential white noise with $\alpha_0 = 0.1305$. For the $ARL$ performance comparison, the boundary values $b = 0.0000939$, $0.000127$ for the EWMA control chart and $h = 0.2121, 0.21875$ for the CUSUM control chart were used with conditions of $ARL_0 = 370$ and 500, respectively. The smoothing parameter of the EWMA control chart was set to 0.1. In Table 6, the performance of EWMA control chart with the explicit formula is compared with CUSUM control chart. The results of performance comparison show that the EWMA control chart performed better than the CUSUM control chart for all magnitude of shift sizes. To more clearly, Figures 1 show the $ARL_1$ value of the EWMA is lower than the CUSUM charts for all levels change for $ARL_0 = 370$ and 500 respectively.

| $ARL_0$ | EWMA | CUSUM |
|---------|------|-------|
| 370     | 370.69 | 370.23 |
| 500     | 500.38 | 500.55 |

Table 6. Comparison of $ARL_1$ between EWMA and CUSUM charts for $MA(2)$ under data on the unemployment rate of Thailand for in-control process $\alpha_0 = 0.1305$.

Figure 1. Comparison of $ARL_1$ between EWMA and CUSUM charts for $MA(2)$ given (a) $ARL_0 = 370$ and (b) $ARL_0 = 500$. 
6. Conclusion
This research has been applied the ARL explicit formulas of MA process for determining the optimal parameters on the EWMA control chart. It can be seen that the benefit of the ARL explicit formulas is to obtain the appropriate parameters of EWMA control chart at different levels of change. Additionally, this process can be applied for observing real life situations such as the unemployment rate of Thailand. In future studies, the method for evaluating the ARL could be developed for other models and construct explicit formulas to modern control charts.

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