Supersymmetric domain wall $\times G/H$ solutions of $IIB$ supergravity

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Abstract

1-brane nonmaximally supersymmetric solutions of $D = 10$ chiral supergravity are discussed. In the dual frame, their near brane geometry is the product of a 3-dimensional domain wall spacetime and a 7-dimensional homogeneous Einstein space $(G/H)_7$. 

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It has been conjectured some time ago [1, 2, 3] that the AdS/CFT correspondence (for a review see [1] and references therein) could be extended to a DW/QFT correspondence between supergravity on domain wall spacetimes and (nonconformal) quantum field theories living on their boundary. This is a generalization of the gauge/gravity correspondence, well tested in the framework of AdS/CFT, but still in need of explicit support in the nonconformal scenarios.

In the present Letter we study in detail a particular class of 1-brane solutions of $D = 10$ IIB chiral supergravity, and classify their supersymmetry content. These solutions involve nontrivial scalar and two-form fields. In the dual frame of ref. [2] (obtained from the Einstein frame after a Weyl transformation on the metric) the near brane geometry factorizes into the product of a three dimensional domain wall spacetime ($DW$)$_3$ and a compact homogeneous 7-dimensional coset manifold $(G/H)_7$. These solutions preserve a fraction $N/16$ of the original $D = 10$ supersymmetry, where $N$ is the number of Killing spinors of $G/H$. The literature on $p$-brane solutions of supergravity theories is quite extensive, see for ex. the review [5].

Our motivation is to set up a convenient playing ground for testing the DW/QFT correspondence between $d = 3$ nonmaximal supergravities and $d = 2$ nonconformal field theories.

Compact (and noncompact) gaugings of maximal supergravities in $d = 3$ have been recently constructed in [4]. It will be worthwhile to investigate also the nonmaximal cases: some of these should correspond to consistent reductions of the $D = 10$ solutions we are discussing. For a recent review on supergravity gaugings in diverse dimensions and their use in $p$-brane physics see for ex. [7].

Chiral IIB $D = 10$ supergravity contains a complex anti-Weyl gravitino $\psi_M$ and a complex Weyl spinor $\lambda$. The bosonic fields are the graviton $g_{MN}$, a complex antisymmetric tensor $A_{MN}$, a real antisymmetric tensor $A_{MNRS}$ (restricted by a self-duality condition) and a complex scalar $\phi$.

After setting the spinor fields to zero, the field equations read [8, 3, 10]:

\[
2R_{MN} = P_M P^*_N + P^*_M P_N + \frac{1}{6} F^{PQRS}_M F_{PQRSN} + \frac{1}{8} (G^{PQ}_M G^*_{PQN} + G^{* PQ}_M G_{PQN} - \frac{1}{6} g_{MN} G^{PQR} G^*_P Q R) \tag{1}
\]

\[
F_{M_1 \ldots M_5} = \frac{1}{5!} \epsilon_{M_1 \ldots M_5 N_1 \ldots N_5} F^{N_1 \ldots N_5} \tag{2}
\]

\[
(\nabla^S - iQ^S) G_{MNS} = P^S G^*_{MNS} - \frac{2i}{3} F_{MNPQR} G^{PQR} \tag{3}
\]

\[
(\nabla^M - 2iQ^M) P_M = -\frac{1}{24} G^{PQR} G_{PQR} \tag{4}
\]

where $R_{MN} \equiv R^S_{M \ldots S N}$ and the curvature two-form is defined as $R^S_{M \ldots S N} \equiv dB^S_M + B^S_N \wedge B^N_M$; the vectorial quantities $P_M$ (complex) and $Q_M$ (real) are related to the scalar field and its first derivative, $F_{PQRSN}$ and $G_{PQN}$ to the field strengths of the four-form and of the two-form, and $\nabla$ is the Lorenz covariant derivative. Here we have adopted the normalizations and conventions of [8]; note however a
sign correction in \([4]\), already found in \([10]\) and noted also in \([11]\). Moreover the following Bianchi identities hold (a consequence of the field definitions):

\[
\begin{align*}
(\nabla_{[M} - 2iQ_{[M})P_{N]} &= 0, \\
\partial_{[M}Q_{N]} &= -iP_{[M}P_{N]} \quad (5) \\
(\nabla_{[M} - iQ_{[M})G_{NRS}] &= -P_{[M}G_{NRS}^* \quad (6) \\
\partial_m F_{M_1...M_5} &= \frac{1}{8} \text{Im} \, G_{[NM_1M_2}G_{M_3M_4M_5]} \quad (7)
\end{align*}
\]

The supersymmetry variations of the bosonic fields are proportional to Fermi fields, and these vanish in the type of backgrounds we are considering. On the other hand, the supersymmetry variations of the fermionic fields are:

\[
\delta \lambda = i\Gamma^M \varepsilon^* P_M - \frac{1}{24} iG_{MNP}\Gamma^{MNP} \varepsilon \quad (8)
\]
\[
\delta \psi_M = (\nabla_M - \frac{i}{2}Q_M)\varepsilon + \frac{i}{480} F_{N_1...N_5}\Gamma^{N_1...N_5}\Gamma_M \varepsilon + \frac{1}{96}(\Gamma^M_{N_1-N_3}G_{N_1-N_3} - 9\Gamma^{N_1N_2}G_{M_1N_2})\varepsilon^* \quad (9)
\]

(in backgrounds with \(\psi = 0, \lambda = 0\)) \([3]\). A solution is supersymmetric if there exist spinors \(\varepsilon\) for which these variations vanish in the corresponding background.

We search for solutions of the IIB field equations of the type (two-blocks brane Ansatz):

\[
ds^2 = e^{2A(r)}d\sigma^\mu d\sigma^\nu \eta_{\mu\nu} - e^{2B(r)}[dr^2 + r^2\lambda^{-2}ds^2_{G/H}] \quad (10)
\]
\[
A_{\mu\nu} = \varepsilon_{\mu\nu}e^{C(r)} \quad (11)
\]
\[
P_\bullet = -E'(r) \quad (12)
\]
\[
Q_M = F_{M_1...M_5} = 0 \quad (13)
\]

where \(\lambda\) is a constant parameter with dimension of a length, the metric is \(\eta_{\mu\nu} = (+,-,-,...,-)\), and \(A, B, C, E\) are real functions of \(r\). The indices run as follows: \(\mu, \nu ... = 1, 2; \bullet\) labels the radial direction; \(m, n ...\) run on the internal coset manifold \(G/H\) directions; \(M, N ...\) run over 1...10. The 3-form \(G_{\mu\nu\rho}\) is proportional to the curl of the potential \(A_{\mu\nu}\):

\[
G_{\mu\nu\rho} = 3e^{E(r)}\partial_{[\mu}A_{\nu\rho]} \quad (14)
\]

Eq.s \([11]-[13]\) provide the \(G/H\) generalization of the standard (two-blocks) \(p\)-brane Ansatz extensively considered in the literature (see for instance \([3]\)). As such, we know that it satisfies the field eq.s \([1]-[4]\) if the functions \(A, B, C, E\) have a specific form, and the internal coset space is an Einstein manifold \([12,13]\). This we now verify in detail: in fact the explicit expression for the connection will be useful when discussing the supersymmetry content of the solutions.

The vielbein and spin connection corresponding to the Ansatz are given by:
\[ E^{\mu} = e^{A(r)} dx^{\mu}, \quad E_{\nu} = e^{B(r)} dr, \quad E^{m} = e^{B(r)} r \lambda^{-1} E_{m} \]

\[ \omega^{\mu \nu} = 0, \quad \omega^{\mu \nu} = -e^{-B(r)} A'(r) E^{\nu}, \quad \omega^{m m} = 0 \]

\[ \omega^{m \nu} = -e^{-B(r)} (B'(r) + r^{-1}) E^{m}, \quad \omega^{m \nu} = \omega^{m \nu} \]

where flat indices are underlined, and \( E^{m}, \omega^{m \nu} \) are the vielbein and spin connection of the coset manifold \( G/H \).

It is convenient to express the only nonvanishing components of the 3-form field strength using flat indices:

\[ G^{\mu \nu} = C^\prime e^{C + E - 2A - B} \epsilon_{\mu \nu} \quad (15) \]

The Ricci tensor is:

\[ R^{\mu \nu} = \frac{1}{2} \eta_{\mu \nu} e^{-2B} (A'' + 2A'^{2} + 6A'B' + 7r^{-1} A') \quad (16) \]

\[ R_{\nu \nu} = -\frac{1}{2} e^{-2B} (2A'' + 2A'^{2} - 2A'B' + 7B'' + 7r^{-1} B') \quad (17) \]

\[ R_{mm} = \frac{1}{2} \eta_{mm} e^{-2B} (2A'B' + 2r^{-1} A' + 13r^{-1} B' + B'' + 6B'^{2} + 6r^{-2}) + R_{mn} \quad (18) \]

where \( R_{mm} \) is the Ricci tensor of the coset manifold.

Inserting the Ansatz (10)-(13) and (15) into the Einstein field eq. (1) yields

\[ 2R^{\mu \nu} = \frac{3}{8} \eta_{\mu \nu} C'^{2} e^{2(C + E - 2A - B)} \quad (19) \]

\[ 2R_{\nu \nu} = -\frac{3}{8} C'^{2} e^{2(C + E - 2A - B)} + 2E'' \quad (20) \]

\[ 2R_{mm} = -\frac{1}{8} \eta_{mm} C'^{2} e^{2(C + E - 2A - B)} \quad (21) \]

while equations (3) and (4) become:

\[ C'' + C'^{2} - 2A'C' + 6B'C' + 7r^{-1} C' + 2C'E' = 0 \quad (22) \]

\[ E'' + 6E'B' + 2A'E' + 7r^{-1} E' + \frac{1}{4} C'^{2} e^{2(C + E - 2A)} = 0 \quad (23) \]

The Ansatz satisfies the Bianchi identities for \( Q \) and \( F \) trivially. The other two Bianchi identities hold because of the particular form of the spin connection (for ex. use \( \omega_{[\mu \nu]} = 0 = \omega_{[m \nu]} \) in the \( P \) Bianchi identity).

Requiring that the solutions preserve a certain amount of the \( D = 10 \) supersymmetry, i.e. that the variations (8),(9) vanish, places further restrictions on the functions \( A, B, C, E \), which we now deduce. We adopt the following real representation for the \( SO(1,9) \) gamma matrices \((32 \times 32)\):

\[ \Gamma^{M} = (\gamma^{\mu} \otimes 1_{8} \otimes \sigma^{2}, \gamma^{\cdot} \otimes 1_{8} \otimes \sigma^{2}, 1_{2} \otimes \Gamma^{m} \otimes \sigma^{1}) \quad (24) \]
where the $SO(1,2) \gamma$-matrices are $(\gamma^1, \gamma^2, \gamma^3) = (\sigma^2, i\sigma^1, i\sigma^3)$ and $\sigma^i$ are the Pauli matrices. $\Gamma^m$ are the real $SO(7)$ matrices given by the octonionic structure constants (see for ex. [14]). This tensor representation is well adapted to our $(2+1+7)$ Ansatz.

Correspondingly, any anti-Weyl $D = 10$ spinor $\epsilon$ can be decomposed as:

$$
\epsilon = c \xi \otimes \eta(r, y) \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}
$$

where $\xi$ is a constant real $SO(1,2)$ spinor, $\eta$ a real $SO(7)$ spinor, and $c \in \mathbb{C}$.

Requiring that the supersymmetry variation (8) vanishes in the Ansatz background is equivalent to:

$$
\frac{1}{4} \alpha C' e^{C+E-2A} = E'
$$

(26)

$$
(1_2 - \sigma^3) \xi = 0
$$

(27)

where $\alpha = \frac{c}{c^*} = \pm 1$ (so that $\alpha = \alpha^{-1}$); note that eq.(27) projects out half of the components of the $d = 3$ supersymmetry parameter. Using the condition on $\xi$, the supersymmetry variation $\delta \psi_\mu$ of the gravitino vanishes provided

$$
A' + \frac{3}{8} \alpha C' e^{C+E-2A} = 0
$$

(28)

Together with (26), this relation gives $A$ and $C$ in terms of $E$, and fixes $\alpha$:

$$
A = -\frac{3}{2} E, \quad C = -4 E, \quad \alpha = -1
$$

(29)

Using these equalities in the condition $\delta \psi_\bullet = 0$ yields:

$$
\partial_r \eta = -\frac{1}{4} A' \eta
$$

(30)

whose solution is:

$$
\eta(r, y) = \eta_0(y) e^{-\frac{1}{4}A}
$$

(31)

Finally $\delta \psi_m = 0$ leads to:

$$
B' - \frac{1}{8} \alpha C' e^{C+E-2A} = 0
$$

(32)

$$
\left( \nabla_m^{G/H} + \frac{1}{2\alpha} \Gamma_m \right) \eta_0 = 0
$$

(33)

where $\nabla_m^{G/H}$ is the $G/H$-covariant derivative. From the first equation we find $B$ in terms of $E$:

$$
B = \frac{1}{2} E
$$

(34)
while the second, after identifying the Freund-Rubin parameter as $e = \frac{1}{2\lambda}$, reduces to the familiar Killing spinor equation for a $\frac{G}{H}$, found in studying the residual supersymmetry of Freund-Rubin vacua. The solutions of this equation have been exhaustively studied in the eighties, see for ex. [15, 16, 17] for a review.

Now we come back to the equations of motion. After expressing the functions $A, B, C$ in terms of $E$, the four equations (19), (20), (22) and (23) all reduce to:

$$E'' + \frac{7}{r}E' + 4(E')^2 = \nabla^2 E + 4E'^2 = 0 \quad (35)$$

or

$$\nabla^2 e^{4E(r)} = 0 \quad (36)$$

where $\nabla^2$ is the $(G/H)_7$-covariant Laplacian.

This equation is solved by:

$$e^{4E(r)} = H(r) \equiv 1 + \frac{k}{r^6} \quad (37)$$

and we have chosen the integration constant such that $E(\infty) = 0$. With use of (29), (34) we determine the exponentials appearing in the Ansatz:

$$e^{2A} = H(r)^{-\frac{3}{4}}, \quad e^{2B} = H(r)^{\frac{1}{4}}, \quad e^{C} = H(r)^{-1} \quad (38)$$

Finally, the Einstein equation in the $G/H$ directions (21) becomes:

$$R_{mn} = -\frac{3}{\lambda^2} g_{mn} \quad (39)$$

where $g_{mn}$ is the $G$-invariant $G/H$ metric. This equation is solved by any compact 7-dimensional Einstein manifold, and in particular by the homogeneous manifolds $(G/H)_7$ endowed with a $G$-invariant Einstein metric, classified in [15]. A short review on coset space geometry can be found in [17, 18]. For self-consistency we recall here the subset of such manifolds that admit $N \geq 1$ Killing spinors. The notations are as in ref.s [15, 17].

To each of these coset spaces $G/H$ corresponds a supersymmetric, nonconformal 1-brane solution of IIB supergravity, where $N/16$ of the $D = 10$ spacetime supersymmetries are preserved. For example the round seven sphere solution has 1/2 of the original $D = 10$ supersymmetry. This halving of supersymmetry is a familiar phenomenon in brane solutions [5]; technically here it arises because of eq. (27).

These 1-brane solutions of IIB supergravity are a special case of the p-brane classical solutions of the D-dimensional action

$$\int d^D x \sqrt{-g} \left[ 2R[g] - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2(p+2)!} e^{-a\phi} F_{p+2}^2 \right] \quad (40)$$

corresponding to a consistent truncation of some D-dimensional supergravity bosonic action, involving a scalar field and a $p+2$ form $F_{p+2}$. As shown in [12, 13] the field
Table 1: 7-dimensional Einstein coset spaces with Killing spinors

| G/H | G                | H                | N |
|-----|------------------|------------------|---|
| $S^7$ | $SO(8)$ | $SO(7)$ | 8 |
| squashed $S^7$ | $SO(5) \times SO(3)$ | $SO(3) \times SO(3)$ | 1 |
| $M^{ppr}$ | $SU(3) \times SU(2) \times U(1)$ | $SU(2) \times U(1)^2$ | 2 |
| $N^{010}$ | $SU(3) \times SU(2)$ | $SU(2) \times U(1)$ | 3 |
| $N^{prq}$ | $SU(3) \times U(1)$ | $U(1)^2$ | 1 |
| $Q^{ppp}$ | $SU(2)^3$ | $U(1)^3$ | 2 |
| $B^{7}_{irred}$ | $SO(5)$ | $SO(3)_{max}$ | 1 |
| $V_{5,2}$ | $SO(5) \times U(1)$ | $SO(3) \times U(1)$ | 2 |

Equations derived from (40) admit the following elementary (or electric) p-brane solution:

\[ ds^2 = H(r)^{-\frac{4d}{D-2}} dx^\mu \otimes dx^\nu \eta_{\mu\nu} - H(r)^{\frac{4d}{D-2}} (dr^2 + \frac{r^2}{\Delta} ds_X^2) \]  

(41)

\[ F_{p+2} = \frac{2}{\sqrt{\Delta}} \varepsilon_{\mu_1...\mu_{p+1}} d[H(r)^{-1}] \wedge dx^{\mu_1} \wedge ... \wedge dx^{\mu_{p+1}} \]  

(42)

\[ e^{\phi(r)} = H(r)^{-\frac{2a}{\Delta}} \]  

(43)

where $x^\mu, (\mu = 0,...,p)$ are the coordinates on the p-brane worldvolume, and $d \equiv p+1, \tilde{d} \equiv D-d-2$ are the worldvolume dimensions of the p-brane and its magnetic dual. The remaining coordinates of the $D$-dimensional spacetime span a generalized cone with radial coordinate $r$ (radial distance from the brane) and whose basis is a homogeneous $(D-d-1)$-dimensional Einstein space $X$ with metric $ds_X^2$. Moreover

\[ \Delta \equiv a^2 + 2 \frac{dd}{D-2} \]  

(44)

\[ H(r) \equiv 1 + \frac{k}{r^a} \]  

(45)

where $k$ is the electric charge of the brane, and $H(r)$ is a harmonic function ($\nabla^2 H(r) = H'' + (\tilde{d} + 1)r^{-1}H' = 0$) on the space transverse to the brane worldvolume. When $X$ is the usual round sphere, the metric (41) is asymptotic to $D$-dimensional Minkowski spacetime as $r$ goes to infinity; if $X$ is any other Einstein space the same metric is no longer asymptotic to $D$-Minkowski spacetime, although it is still asymptotically flat [12].

The IIB supergravity 1-brane solution we have discussed in detail can be easily cast in the form (43) with the following identifications

\[ E(r) = -\frac{\phi(r)}{2}, \quad G_3 = e^{-\frac{\phi}{2}} F_3, \quad a = 1, \quad d = 2, \quad \tilde{d} = 6, \quad \Delta = 4 \]  

(46)
The discussion of ref. (2) can be applied “in toto”, with a slight generalization: the sphere $S^{D-d-1}$ becomes an arbitrary $G/H$ Einstein manifold, with $N \geq 1$ Killing spinors. Then in the so called “dual” frame the near brane geometry factorizes into $DW_3 \times (G/H)_7$, i.e. a 3-dimensional domain-wall spacetime times the Einstein manifold $(G/H)_7$. The reduction of the action (40) over the $G/H$ part of the near-brane geometry proceeds in the same fashion as discussed in ref.s (2, 19, 3). One then obtains a $D = 3$ action admitting a domain wall spacetime with parameter $\Delta_{DW} = -3$. What is exactly the gauged $D = 3$ supergravity whose truncation yields this $D = 3$ action is still to be elucidated. Only very recently a systematic analysis of maximal (non)compact gaugings of $D = 3$ supergravity has been carried out in [6]. Clearly also the nonmaximal gaugings are of interest for the solutions we have studied here, since these have nonmaximal supersymmetry.

In conclusion, we have given the detailed supersymmetry analysis of 1-brane solutions of IIB supergravity having in mind to find the corresponding noncompact gaugings of nonmaximal $D = 3$ supergravity. This remains still to be done, and to be used to test the $DW_3/QFT_2$ correspondence between these classical supergravities and the 2-dimensional QFT living on the brane worldsheet.

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