Parton Distributions, $d/u$, and Higher Twists Effects at High $X$

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A re-analysis of the NMC and SLAC data leads to a great improvement in our knowledge of the valence $d$ and $u$ parton distribution functions (PDF’s) at high $x$. Standard parton distributions with our modifications are in good agreement with QCD predictions for $d/u$ at $x = 1$, and with the CDHSW $\nu p$ and $\bar{\nu} p$ data, the HERA charged current cross section data, the collider high-$P_t$ jet data, and the CDF $W$ asymmetry data. With the inclusion of target mass and higher twist corrections, the modified PDF’s also describe all DIS data up to $x = 0.98$ and down to $Q^2 = 1$ GeV$^2$.

Recent work on parton distributions functions (PDF’s) in the nucleon has focussed on probing the sea and gluon distribution at small $x$. The valence quarks distribution has been thought to be relatively well understood. However, the precise knowledge of the $u$ and $d$ quark distribution at high $x$ is very important at collider energies in searches for signals for new physics at high $Q^2$. In addition, the value of $d/u$ as $x \to 1$ is of theoretical interest. Recently, a proposed CTEQ toy model \cite{1} included the possibility of an additional contribution to the quark distribution at small $x$, beyond the initial HERA high $Q^2$ anomaly \cite{2}, and for the jet excess at high-$P_t$ at CDF \cite{3}. In this Letter we conclude that a re-analysis of data from NMC and SLAC leads to a great improvement in our knowledge of PDF’s at high $x$, and rules out such toy models.

Information about valence quarks originates from the proton and neutron structure function data. The $u$ valence quark distribution at high $x$ is relatively well constrained by the proton structure function $F_2^u$. However, the neutron structure function $F_2^n$, which is sensitive to the $d$ valence quark at high $x$, is actually extracted from deuterium data. Therefore, there is an uncertainty in the $d$ valence quark distribution from the corrections for nuclear binding effects in the deuteron. In past extractions of $F_2^n$ from deuteron data, only Fermi motion corrections were considered, and other binding effects were assumed to be negligible. Recently, the corrections for nuclear binding effects in the deuteron, $F_2^d/F_2^{u+p}$, have been extracted empirically from fits to the nuclear dependence of electron scattering data from SLAC experiments E139/140 \cite{4}. The empirical extraction uses a model proposed by Frankfurt and Strikman \cite{5}, in which all binding effects in the deuteron and heavy nuclear targets are assumed to scale with the nuclear density.

The total correction for nuclear binding effects in the deuteron (shown in Fig. \ref{fig1}(a)), is in a direction which is opposite to what is expected from the previous models which included only the Fermi motion effects. The surprisingly large correction extracted in this empirical way maybe controversial, but is smaller than the recent theoretical prediction \cite{6} (dashed line in Fig. \ref{fig1}(a)).

The ratio $F_2^d/F_2^p$ is directly related to $d/u$. In leading order QCD, $2F_2^d/F_2^p - 1 \simeq (1 + 4d/u)/(4 + d/u)$ at high $x$. We perform a next-to-leading order (NLO) analysis on the precise NMC $F_2^d/F_2^p$ data \cite{7} to extract $d/u$ as a function of $x$. We extract the ratio $F_2^{u+n}/F_2^n$ by applying the nuclear binding correction $F_2^{d}/F_2^{u+p}$ to the $F_2^{d}/F_2^{p}$ data. As shown in Fig. \ref{fig1}(b), the standard PDF’s \cite{8} do not describe the extracted $F_2^{u+n}/F_2^n$. Since the $u$ distribution is relatively well constrained, we find a correction term to $d/u$ in the standard PDF’s (as a function of $x$), by varying only the $d$ distribution to fit the data. The correction term is parametrized as a simple quadratic form, $\delta(d/u) = (0.1 \pm 0.01)(x + 1) x$ for the MRS(R2) PDF, where the corrected $d/u$ ratio is $(d/u)^{\prime} = (d/u) + \delta(d/u)$. Based on this correction, we obtain a MRS(R2)-modified PDF as shown in Fig.\ref{fig1}(a). The correction to other PDF’s such as CTEQ3M/4M is similar. Note that since the $d$ quark level is small at large $x$, all the sum rules are easily satisfied with a very minute change at low $x$. The NMC data, when corrected for nuclear binding effects in

\begin{figure}[h]
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\includegraphics[width=\textwidth]{fig1.png}
\caption{(a) The total correction for nuclear effects (binding and Fermi motion) in the deuteron, $F_2^d/F_2^{u+p}$, as a function of $x$, extracted from fits to the nuclear dependence of SLAC $F_2$ electron scattering data (compared to theoretical model \cite{6}). (b) Comparison of NMC $F_2^{u+n}/F_2^n$ (corrected for nuclear effects) and the prediction in NLO using the MRS(R2) PDF with and without our proposed modification to the $d/u$ ratio.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{The ratio $F_2^d/F_2^p$ (corrected for nuclear effects) as a function of $x$.}
\end{figure}
the deuteron, clearly indicate that \( d/u \) in the standard PDF’s is significantly underestimated at high \( x \) as shown in Fig. 3(a). It also shows that the modified \( d/u \) ratio approaches 0.2\( \pm \)0.02 as \( x \rightarrow 1 \), in agreement with a QCD prediction \cite{11}. In contrast, if the deuteron data is only corrected for Fermi motion effects (as was mistakenly done in the past) both the \( d/u \) from data and the \( d/u \) in the standard PDF’s fits approach 0 as \( x \rightarrow 1 \). Figure 2(a) shows that \( d/u \) values extracted from CDHSW \cite{14} \( \nu p/\overline{\nu}p \) data (which are free from nuclear effects) also favor the modified PDF’s at high \( x \).

Information (which is not affected by the corrections for nuclear effects in the deuteron) on \( d/u \) can be also extracted from W production data in hadron colliders. Figure 2(b) shows that the predicted W asymmetry calculated with the DYRAD NLO QCD program using our modified PDF is in much better agreement with recent CDF data \cite{12} at large rapidity than standard PDF’s. When the modified PDF at \( Q^2=16 \text{ GeV}^2 \) is evolved to \( Q^2=10^4 \text{ GeV}^2 \) using the NLO QCD evolution, we find that the modified \( d \) distribution at \( x = 0.5 \) is increased by about 40\% in comparison to the standard \( d \) distribution. The modified PDF’s have a significant impact on the charged current cross sections \cite{13} in the HERA high \( Q^2 \) region, shown in Fig. 3(a), because the charged current scattering with positrons is on \( d \) quark only. Figure 3(b) shows that the modified PDF’s also lead to an increase of 10\% in the production rate of very high \( p_T \) jets \cite{14} in hadron colliders.

Since all the standard PDF’s, including our modified versions, are fit to data with \( x \) less than 0.75, we now investigate the validity of the modified MRS(R2) at very high \( x \) by comparing to \( F_2^p \) data at SLAC. Although the SLAC data at very high \( x \) are at reasonable values of \( Q^2 (7 < Q^2 < 31 \text{ GeV}^2) \), they are in a region in which non-perturbative effects such as target mass and higher twist are very large. We use the Georgi-Politzer calculation \cite{11} for the target mass corrections (TM). These involve using the scaling variable \( \xi = 2x/(1+\sqrt{1+4M^2x^2/Q^2}) \) instead of \( x \). Since a complete calculation of higher twist effects is not available, the very low \( Q^2 \) data are used to obtain information on the size of these terms.

We use two approaches in our study of the higher twist effects: an empirical method, and the renormalon model. In the empirical approach, the higher twist contribution is evaluated by adding a term \( h(x)/Q^2 \) to the perturbative QCD (pQCD) prediction of the structure function (including target mass effects). The \( x \) dependence of the higher twist coefficients \( h(x) \) is fitted to the global deep-inelastic scattering (DIS) \( F_2 \) (SLAC, BCDMS, and NMC) data \cite{15} in the kinematic region \((0.1 < x < 0.75, 1.25 < Q^2 < 260 \text{ GeV}^2) \) with the following form; \( F_2 = F_2^{pQCD+TM}(1+h(x)/Q^2)f(x) \). Here \( f(x) \) is a floating factor to investigate possible \( x \) dependent corrections to our modified PDF. A functional form, \( a(x^{-1/c} - c) \) for \( h(x) \) is used in the higher twist fit to estimate the size of the higher twist terms above \( x = 0.75 \).

The SLAC and BCDMS data are normalized to the NMC data. In the case of the BCDMS data, a systematic error shift \( \lambda \) (in standard deviation units) is allowed to account for the correlated point-to-point systematic er-

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**FIG. 2.** (a) The \( d/u \) distributions at \( Q^2=16 \text{ GeV}^2 \) as a function of \( x \) for the standard and modified MRS(R2) PDF compared to the CDHSW data. (b) Comparison of the CDF \( W \) asymmetry data with NLO standard CTEQ3M, MRS(R2), and modified MRS(R2) as a function of the lepton rapidity. The standard CTEQ3M with a resummation calculation is also shown for comparison.

**FIG. 3.** (a) The HERA charged current cross section data and (b) the CDF and D0 inclusive jet cross section data are compared with both standard and modified PDF’s.
rors. The empirical higher twist fits with the modified NLO MRS(R2) pQCD prediction with TM have been performed simultaneously on the proton and deuteron $F_2$ data with 11 free parameters (two relative normalizations and three parameters for $h(x)$ per target and the BCDMS $\lambda$). We find that empirical higher twist fit describes the data well ($\chi^2/d.o.f. = 843/805$), and the higher twist contributions in the proton and deuteron are similar. The magnitude is almost half of the size from a previous analysis of SLAC/BCDMS data [19], because that analysis was based on $\alpha_s(M_Z^2) = 0.113$, while $\alpha_s(M_Z^2) = 0.120$ in the MRS(R2) PDF, which is close to the current world average.

In the renormalon model approach [20], the model predicts the complete $x$ dependence of the higher twist contributions to $F_2$, $2xF_1$, and $xF_3$, with only two unknown parameters $A_2$ and $A_4$. We extract the $A_2$ and $A_4$ parameters, which determine the overall level of the $1/Q^2$ and $1/Q^4$ terms by fitting to the global data set for $F_2$ and $R[=F_3(1+4M^2/Q^2)/2xF_1-1]$. The values of $A_2$ and $A_4$ for the proton and deuteron are same in this model. The $x$ dependence of $2xF_1$ differs from that of $F_2$ but is the same as that of $xF_3$ within a power correction of $1/Q^2$. Our fits can also be used to estimate the size of the higher twist effects in $xF_3$ [e.g. the Gross-Llewellyn Smith (GLS) sum rule].

The higher twist fit in this approach has employed the same procedure as the empirical method. Figure 4 shows that the model yields description of the $x$ dependence of higher twist terms in both $F_2$ and $R$ with just the two free parameters ($\chi^2/d.o.f. = 1470/928$). The CCFR neutrino data [21] is shown for comparison though it is not used in the fit. The extracted values of $A_2$ are $-0.093 \pm 0.005$ and $-0.101 \pm 0.005$, for proton and deuteron, respectively. The contribution of $A_4$ is found to be negligible. We find that the floating factor $f(x)$ for the deuteron deviates from 1 and is also bigger than that for the proton, unless the modified MRS(R2) PDF is used. This reflects our earlier conclusion that the standard $d$ distribution is underestimated at high $x$ region. As in the empirical fit, the extracted $A_2$ value is half of the previous estimated value [20] of $A_2$ based on SLAC/BCDMS [$\alpha_s(M_Z^2) = 0.113$] analysis [22]. Since both of these approaches yield a reasonable description for the higher twist effects, we proceed to compare the predictions of the modified PDFs (including target mass and renormalon higher twist corrections) to the SLAC proton $F_2$ data at very high $x$ ($0.7 < x < 1$).

There is a wealth of SLAC data [23] in the region up to $x = 0.98$ and intermediate $Q^2$ ($7 < Q^2 < 31$ GeV$^2$). Previous PDF fits have not used these data. We use the estimate of the higher twist effects from the models, based on the data (below $x < 0.75$) described above. Note that the data for $x > 0.75$ is in the DIS region, and the data for $x > 0.9$ is the resonance region. It is worthwhile to investigate the resonance region also because from duality

![Figure 4](https://example.com/figure4.png)
arguments. It is expected that the average behavior of the resonances and elastic peak should follow the DIS scaling limit curve. Figure 5 shows the ratio of the SLAC data to the predictions of the modified MRS(R2) at relatively large $Q^2$ ($21 < Q^2 < 30$ GeV$^2$) where the elastic contribution is negligible. With the inclusion of target mass and the renormalon higher twist effects, the very high $x$ data from SLAC is remarkably well described by the modified MRS(R2) up to $x = 0.98$. The good description of the data by the modified MRS(R2) is also achieved using the empirical estimate $h(x)/Q^2$ of higher twist effects as shown in Fig. 5(c). Figure 5 also shows that the CTEQ Toy model (with an additional 0.5% component of $u$ quarks beyond $x > 0.75$) overestimates the SLAC data by a factor of three at $x = 0.9$ (DIS region). From these comparisons, we find that the SLAC $F_2$ data do not support the CTEQ Toy model which proposed an additional $u$ quark contribution at high $x$ as an explanation of the initial HERA high $Q^2$ anomaly and the CDF high-$p_T$ jet excess. As indicated in Fig. 5(c), the uncertainties in the PDF’s at high $x$ are small. The difference between CTEQ4M and MRS(R2) (with our $d/u$ modifications) is an estimate of the errors.

In conclusion, we find that nuclear binding effects in the deuteron play a significant role in our understanding of $d/u$ at high $x$. With the inclusion of target mass and higher twist corrections, the modified PDF’s also describe all DIS data up to $x = 0.98$ and down to $Q^2 = 1$ GeV$^2$. The modified PDF’s with our $d/u$ correction are in good agreement with the prediction of QCD at $x = 1$, and with the CDHSW $\nu p$ and $\nu p$ data, the HERA CC cross section data, the collider high-$p_T$ jet data, and with the CDF $W$ asymmetry data. A next-to-next leading order (NNLO) analysis of $R$ indicates that the higher twist effects extracted in the NLO fit at low $Q^2$ may originate from the missing NNLO terms.

**FIG. 5.** Comparison of SLAC $F_2$ data with the predictions of the modified MRS(R2), CTEQ4M and the CTEQ toy model at high $x$ and higher $Q^2$ ($20 < Q^2 < 31$ GeV$^2$). (a) Ratio to pQCD, (b) ratio to pQCD with TM effects, and (c) ratio to pQCD with TM and higher twist effects.

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