Photo-induced nuclear cooperation

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Reactions \( n, \gamma d + A_2^z X \to (n, \pm 1) \gamma + p + A_2^{z+1} X \) and especially the reaction \( d + A_2^z X \to \gamma + p + A_2^{z+1} X \), called photo-induced nuclear cooperation and cooperative spontaneous \( \gamma \) emission with neutron exchange, respectively, are investigated theoretically. In the case of photo-induced nuclear cooperation it is supposed that the energy of \( \gamma \) photons of the beam is less than the binding energy of the deuteron. The cross section and the transition probability per unit time, respectively, are determined with the aid of standard second order perturbation calculation of quantum mechanics. The calculations are extended to photo-induced nuclear cooperation and cooperative spontaneous \( \gamma \) emission with proton exchange as well. With the aid of the results obtained, recent observations of nuclear activity of samples of large deuteron content after irradiation by photon-flux of photon energy smaller than the deuteron binding energy are discussed.

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I. INTRODUCTION

In a recent report \(^1\) observation of nuclear activity in deuterated materials subjected to low-energy photon beam was announced. In the experiments different mixtures of deuterated materials were subjected to a photon beam of photon energy less than the deuteron binding energy. The specimens are made from Er\( D_2 + C_{30} D_{71} + Mo \) and Hf\( D_2 + C_{36} D_{74} + Mo \) mixtures. The gamma activity measurements made after irradiation showed significant presence of \(^{163}\)Er, \(^{171}\)Er, \(^{99}\)Mo and \(^{101}\)Mo radioisotopes of zero natural abundance in specimens of deuterated erbium and \(^{180m}\)Hf, \(^{181}\)Hf, \(^{99}\)Mo and \(^{101}\)Mo radioisotopes also of zero natural abundance in specimens of deuterated hafnium. In both cases presence of \(^{99m}\)Tc and \(^{101}\)Tc isotopes and creation of neutrons were also found. In control experiments, i.e. with specimens made from hydrogenated or non-gas-loaded (without any hydrogen isotope) materials gamma spectra revealed no new isotopes. In this work it is attempted to give a mechanism to expound the observations.

The results of \(^1\) indicate that to induce nuclear activity a joint presence of deuterons and a photon flux is necessary. Thus it is expected that those processes may be responsible for nuclear activity in which electromagnetic radiation virtually breaks up deuterons and that the virtually-free neutron is captured by another nucleus. Namely the reaction

\[
n, \gamma d + A_2^z X \to (n, \pm 1) \gamma + p + A_2^{z+1} X + D_j, \quad (1)
\]

which further on will be called photo-induced nuclear cooperation with neutron exchange, will be investigated. Here \( \gamma \) denotes photon of energy less than the binding energy \( E = 2.225 \) MeV of the deuteron, \( n, \) is the number of \( \gamma \) photons initially present, \( d \) is deuteron which is 'virtually' broken up due to electromagnetic interaction, \( A_2^z X \) stands for the cooperating (target) nucleus which absorbs the 'virtual' free neutron, \( A_2^{z+1} X \) denotes final nucleus (of \( A_3 = A_2 + 1 \)) and \( D_j \) is the energy of the reaction.

A special case of \((1)\) when \( n, \gamma \) = 0, i.e. when initially photons are not present. In this case \((1)\) reads as

\[
d + A_2^z X \to \gamma + p + A_2^{z+1} X + D_j. \quad (2)
\]

This reaction is called cooperative spontaneous \( \gamma \) emission with neutron exchange.

The transition probability per unit time and the cross section of processes \((1)\) and \((2)\) can be determined with the aid of second order standard perturbation calculation of quantum mechanics \(^2\). (The term 'virtual' refers to the intermediate state in standard second order perturbation calculation.)

Accordingly, in photo-induced nuclear cooperative process the electromagnetic field-matter and strong interactions are essential. The interaction Hamiltonian \( H_I \) has two terms:

\[
H_I = V^{em} + V^{st}. \quad (3)
\]

\( V^{em} \) describes the interaction of electromagnetic radiation with matter primarily with deuteron and \( V^{st} \) stands for strong interaction acting between the 'virtual' free neutron and the cooperating (target) nucleus \( A_2^z X \) which absorbs the 'virtual' free neutron. In the case of photo-induced nuclear cooperation 'first' \( V^{em} \) and 'after' \( V^{st} \) acts when determining the transition probability per unit time \( (W_{fi, \gamma n}) \) and the cross section \( (\sigma_{\gamma n}) \) of this second order process in standard manner (e.g. see \(^2\)). (The terminology 'first' and 'after' corresponds to the time ordering of the operators in the calculation.) Since in control experiments \(^1\) nuclear activity did not appear, the observed phenomenon may be attached to 'virtual' photo-disintegration of deuteron. Therefore our investigation is restricted to this case only and in the description one can apply the theoretical results obtained during cross section calculation of real photo-disintegration of the deuteron \(^3\), \(^4\). The momentum of the photon is
neglected compared to the momenta of the proton and the final nucleus $A_{2}^{+1}X$.

The photo-induced nuclear cooperation with neutron exchange is dealt with in Section II, where initial, intermediate and final states and energy relations of the process, the cooperation factor, the transition probability per unit time of spontaneous photo-induced nuclear cooperation and the cross section of photo-induced nuclear cooperation with neutron exchange are given. Section III. is devoted to nuclear cooperation with proton exchange dealing with the Coulomb factor in nuclear cooperation with proton exchange, the transition probability per unit time of spontaneous decay and the cross section of photo-induced nuclear cooperation with proton exchange. In section IV. the explanation of observations is discussed comparing the rate of the process induced by irradiation of a photon flux and the rate of the spontaneous process. As a numerical example the rate of unstable $^{99}Mo$ isotope creation in the spontaneous process is given too. Section V. is devoted to conclusions. In the Appendix the interaction Hamiltonians, the matrix elements of $V_{em}$ and $V_{et}$, which are necessary to second order perturbation calculation, and some details of calculation of $W_{fiγn}$ are given.

II. PHOTO-INDUCED NUCLEAR COOPERATION WITH NEUTRON EXCHANGE

A. Initial, intermediate and final states

Deuterons are somewhere in the sample of volume $V_{s}$. Their initial state describing the motion of the center of mass (CM) of the deuteron is a wave of amplitude $V_{s}^{−1/2}$. This is the most simple choice. The state of the deuteron in the relative (neutron-proton separation) coordinate $(r)$ reads as $\varphi_{d} = (4\pi)^{−1/2}\sqrt{\alpha/(2\pi)} e^{−\alpha r}/r$, where $\alpha = \hbar^{−1}\sqrt{B_{d}}$ and $r = |r|$. [3], [4].

The initial state of target nucleus (of mass number $A_{2}$) is a wave of amplitude $V_{t}^{−1/2}$, i.e. the target nucleus is somewhere in the volume of normalization $V_{t}$.

The motion of center of mass of intermediate neutron and the final proton states are plane waves of wave number vector $k_{n}$, $k_{p}$ and volumes of normalization $V_{n}$, $V_{p}$, respectively. The cooperation (of deuteron) by neutron with an other nucleus is taken into account with the aid of spherical waves the source of which is the deuteron and which behaves far away (at incidence on nucleus $A_{2}^{+1}X$) as a plane wave.

For the final bound neutron states of excitation energy $\varepsilon_{j}$ of nucleus of mass number $A_{3}$, $A_{3} = A_{2} + 1$, where $A_{2}$ is the mass number of target nucleus, we take $\Phi(r_{n}) = \sqrt{3/R_{3}^{3}}\phi_{3}(R_{3}x)Y_{j,m_{j}}(\Omega_{n})$ where $x = r_{n}/R_{3}$, $R_{3} = r_{0}A_{3}^{1/3}$ is the radius of a nucleus of nucleon number $A_{3}$ with $r_{0} = 1.2 \times 10^{-13}$ cm, the $Y_{j,m_{j}}(\Omega_{n})$ is a spherical harmonics and $\int_{0}^{\infty} |\phi_{3}(R_{3}x)|^{2}x^{2}dx = 1/3$. In the Weisskopf-approximation to be used $\phi_{3}^{W}(R_{3}x) = 1$, if $x \leq 1$ and $\phi_{3}^{W}(R_{3}x) = 0$ for $x > 1$. The final state which describes the motion of center of mass of the final nucleus (of mass number $A_{3}$) is also a plane wave of wave number vector $k_{3}$ and of volume of normalization $V_{3}$. It is supposed that $V_{3} = V_{t}$.

The wave number (momentum) of the photon is much less than the wave numbers of the proton and the final nucleus $A_{2}^{+1}X$, therefore it is neglected in the calculation of momentum conservation.

B. Energy relations

$D_{0n} = \Delta_{−} + \Delta_{+}$ is the energy of reaction into the ground state of the final nucleus with $\Delta_{−} = \Delta_{d} - \Delta_{p}$ and $\Delta_{+} = \Delta_{A_{2}} - \Delta_{A_{2}^{+1}}$ ($D_{0n} = \Delta_{d} - \Delta_{p} + \Delta_{A_{2}} - \Delta_{A_{2}^{+1}}$), $\Delta_{A_{2}}$, $\Delta_{A_{2}^{+1}}$, $\Delta_{d}$, $\Delta_{p}$ and $\Delta_{n}$ are the energy excesses of neutral atoms of mass numbers $A_{2}$, $A_{2}^{+1}$, deuteron, proton and neutron, respectively [3]. It is possible (energetically allowed) that the final nucleus is created in an excited state of energy $\varepsilon_{j}(> 0)$ above its ground state. The reaction energy $D_{j} = D_{0n} - \varepsilon_{j}$ belongs to that reaction which has final state of excitation energy $\varepsilon_{j}$. It is useful to introduce the quantity

$$\Delta_{j}^{±} = D_{0n} - \varepsilon_{j} \pm E_{γ}. \tag{4}$$

Here $E_{γ}$ is the energy of the photon. $\Delta_{j}^{±}$ is the energy which is shared between the kinetic energies of final nucleus and proton. The upper + and − signs throughout correspond to absorption and emission of photon.

C. Spontaneous decay by photo-induced nuclear cooperation with neutron exchange

In the case of $n_{γ} = 0$, i.e. initially photons are not present, only the $V_{E_{ki}}$ and $V_{M_{ki}}$ matrix elements with $g^{−} = 1$ give contribution (see Appendix B.). It is the case of cooperative spontaneous $γ$ emission (see (2)). The phase space of the emitted photon is $(2\pi\hbar)^{−3}V_{e}E_{γ}^{2}d^{3}E_{γ}d\Omega_{γ}$ with which the expression of transition probability per unit time must be multiplied too. ($\hbar$ is the reduced Planck constant and $c$ is the velocity of light.)

1. Cooperation factor

Determining the full transition probability per unit time $W_{fiγn}$, the contributions coming from all cooperating nuclei located at a distance $L$ far from each other in the case of every possible $L$ value must be taken into account. The number of cooperating nuclei in a shell of sphere of radius $L$ and width $dL$ reads as $4\pi L^{2}dLn_{A_{2}}$ with $n_{A_{2}}$ the number density of nuclei of nuclear number $A_{2}$. The emitted ‘virtual’ neutron wave has an amplitude $A(L)$ in this shell (see Appendix C). Using
Transition probability per unit time of spontaneous photo-induced nuclear cooperation with neutron exchange

The full transition probability per unit time $W_{fi,\gamma n}^{-}$ of the spontaneous process has the form

$$W_{fi,\gamma n}^{-} = n_d \Lambda n_{z_2} r_{A_2} A_3 C_W \sum_{\epsilon_j} I_{jn}^{-}$$

Here $n_{z_2}$ is the number density of element of charge number $z_2$, $r_{A_2}$ is the natural abundance of isotope of mass number $A_2$ and

$$C_W = \frac{6\alpha_f r_0^3 c}{\pi^2 (1 - \alpha r_l) \alpha^6 (hc)^5 (A_3 + 1)}.$$}

Here $\alpha_f$ is the fine structure constant ($e^2 = \alpha_f (hc)$), $V_0 = 45 \ MeV$, $m_0 c^2 = 0.931, 494 \ MeV$ is the atomic mass unit, $c$ is the elementary charge, $r_l = 1.759 \times 10^{-13} \ cm$, $C_W = 8.02 \times 10^{-45} \ cm^5 s^{-1}$, and

$$I_{jn}^{-} = \int_0^{\delta_j} u^3 \left[ F_{jlj} \left( \xi_j^+ \right) \left[ S^E \left( \xi_j^+ \right) + \eta_0^2 S^M \left( \xi_j^+ \right) \right] \right] du$$

with $u = E_{\gamma} / B$, $\delta_j = (D_{0n} - \epsilon_j) / B$, $\eta_0^2 = 0.3684$ and

$$\xi_j^+ = \frac{\left( \frac{k_{0j}}{\alpha} \right)^2}{(A_3 + 1)} 2(\delta_j + u).$$

Here $\xi_j^+$ is used.

$$S^E \left( \xi_j^+ \right) = \frac{2 \xi_j^+}{1 + \xi_j^+},$$

$$S^M \left( \xi_j^+ \right) = \frac{1}{[1 + \xi_j^+]^2 (1 + \xi_j^+ a^2 a_0^2)},$$

where $\alpha^2 a_0^2 = 29.89$.

$F_{jlj} \left( \xi_j^+ \right) = \frac{(2l_j + 1) H_{jlj}^2 (\alpha R_3 / \sqrt{\xi_j^+})}{[\xi_j^+ + 1 \mp u]^2 \sqrt{\xi_j^+}}$.

For the definition of $H_{jlj}$ see (11), it is given by (15) and (43) in the Weisskopf- and Weisskopf-long wavelength approximations.

**D. Cross section of photo-induced nuclear cooperation with neutron exchange**

The cross section $\sigma_{\gamma n}^\pm$ of photo-induced nuclear cooperation with neutron exchange due to all cooperating nuclei located in the sample can be obtained from the transition probability per unit time $W_{fi,\gamma n}^{-}$ omitting from it the phase space of $\gamma$ and dividing it by the photon flux $c n_{\gamma} / V_s$, where the $n_{\gamma} + 1 \gg n_{\gamma}$ approximation is used.

The photo-induced cross section $\sigma_{\gamma n}^\pm$ has the form

$$\sigma_{\gamma n}^\pm = \sum_{\epsilon_j} K_{\sigma} u F_{jlj} \left( \xi_j^\pm \right) \left[ S^E \left( \xi_j^\pm \right) + \eta_0^2 S^M \left( \xi_j^\pm \right) \right],$$

where $S^E \left( \xi_j^\pm \right)$ and $S^M \left( \xi_j^\pm \right)$ come from contributions due to electric and magnetic parts of deuteron-photon electromagnetic interaction [see (10) and (11)], $F_{jlj} \left( \xi_j^\pm \right)$ is determined by (12), $\xi_j^\pm$ is given by (9), $u = E_{\gamma} / B$ and

$$K_{\sigma} = V_s^{-1} \Lambda n_{z_2} r_{A_2} A_3 C_{\sigma},$$

with

$$C_{\sigma} = \frac{6\alpha_f r_0^3}{(1 - \alpha r_l) \alpha^6 (hc)^2 (A_3 + 1)}.$$ (15)

($C_{\sigma} = 1.840 \times 10^{-87} \ cm^2$).

**III. NUCLEAR COOPERATION WITH PROTON EXCHANGE**

If the deuteron is ‘virtually’ splitted up by a photon then the reaction

$$n_{\gamma} + d + A_2 X \rightarrow (n_{\gamma} \pm 1) \gamma + n + A_2^{+1} Y + D_{jp},$$ (16)

which is nuclear cooperation with proton exchange and the reaction

$$d + A_2 X \rightarrow \gamma \ + \ n + A_2^{+1} Y + D_{jp},$$ (17)

which is cooperative spontaneous $\gamma$ emission with proton exchange, may happen too. Now $D_{0p} = D_{0p} - \epsilon_j$ with $D_{0p} = D_{0p} - \Delta_n + \Delta_{A_2} - \Delta_{A_2^{+1}}$, $(\Delta_j^\pm = D_{0p} - \epsilon_j \pm E_{\gamma})$ and $\delta_j = (D_{0p} - \epsilon_j) / B$. However these reactions are hindered by the Coulomb repulsion between the proton and the nucleus $A_2^\pm X$, which is manifested in the appearance of the Coulomb factor in the matrix element of $V^{st}$. 
A. Coulomb factor in nuclear cooperation with proton exchange

The Coulomb repulsion can be taken into account using an approximate form of Coulomb-solution, which can be obtained from wave function describing relative motion of like charges of charge numbers \( z_j \) and \( z_k \) and reads as \( \varphi(r) = f_{jk}(E) e^{ikr}/\sqrt{V} \) valid in the nuclear volume. Here \( V \) denotes the volume of normalization, \( r \) is the relative coordinate of the two particles and \( k \) is the wave number vector in their relative motion. \( E \) is the energy taken in the center of mass (CM) coordinate system. \( f_{jk} = (2\pi\eta_{jk}/[\exp(2\pi\eta_{jk}) - 1])^{1/2} \) is the Coulomb factor and

\[
\eta_{jk}(E) = z_j z_k \alpha_f \sqrt{\frac{A_j A_k}{A_j + A_k}} \frac{m_0 c^2}{2E(CM)} \tag{18}
\]

is the Sommerfeld parameter, where \( A_j, A_k \) are mass numbers of the Coulomb interacting nuclei.

In the case of reaction (16) momentum conservations (\( k_n = -k_3 \) in the final state and \( k_p = -k_3 \)) during em interaction) furthermore energy conservation \( \hbar^2 k_3^2/(2m_0) + \hbar^2 k_3^2/(2m_0 A_3) = \Delta^\pm (\text{in the final state}) \) determine the (proton) energy of intermediate state \( E_p(lab) \) in the laboratory frame of reference as \( E_p(lab) = [A_3/(1 + A_3)] \Delta^\pm \). Thus the proton energy in the CM system (of proton and mass of number \( A_2 \)) \( E_p(CM) = \Delta^\pm A_2 E_p(lab) = (1 + A_2)/(1 + A_3) \Delta^\pm \) must be substituted in (18) that results

\[
\eta_{p2}(\xi_j^\pm) = z_2 \alpha_f \sqrt{\frac{m_0 c^2}{B \xi_j^\pm}}. \tag{19}
\]

B. Transition probability per unit time of spontaneous decay with proton exchange

The transition probability per unit time \( W_{fi,\gamma_n}^{-} \) of spontaneous decay with proton exchange can be obtained with the aid of the transition probability per unit time \( W_{fi,\gamma_n}^{+} \) of spontaneous decay with neutron exchange modifying \( I_{jn}^\pm \) in it as

\[
I_{jn}^{-} = \int_{0}^{\delta_j} u^3 f_{p2}^2(\xi_j^\pm) \times \left[ F_{jl}(\xi_j^\pm) \left[ S^E(\xi_j^\pm) + \eta_0^2 S^M(\xi_j^\pm) \right] \right] du,
\]

where

\[
f_{p2}^2(\xi_j^\pm) = 2\pi \eta_{p2}(\xi_j^\pm)/[\exp(2\pi \eta_{p2}(\xi_j^\pm)) - 1] \tag{21}
\]

with \( \eta_{p2}(\xi_j^\pm) \) and \( \xi_j^\pm \) given by (19) and (18).

Furthermore, in estimating the cooperation length the \( \Lambda \lesssim s[E_p(CM)] \) choice (an upper estimate) seems to be acceptable where \( s[E_p(CM)] \) is the stopping range of a proton of energy \( E_p(CM) \) determined above.

C. Cross section of photo-induced nuclear cooperation with proton exchange

Similarly to the above, the cross section \( \sigma_{\gamma p}^{\pm} \) of photo-induced nuclear cooperation with proton can be determined from \( \sigma_{\gamma n}^{\pm} \) as

\[
\sigma_{\gamma p}^{\pm} = \sum_{\xi_j} K_\gamma u f_{p2}^2(\xi_j^\pm) F_{jl}(\xi_j^\pm) \left[ S^E(\xi_j^\pm) + \eta_0^2 S^M(\xi_j^\pm) \right]. \tag{22}
\]

In the rate of photo-induced nuclear cooperation with proton exchange the quantity

\[
J_{jn}^\pm = \int_{4\min}^{4\max} u f_{p2}^2(\xi_j^\pm) \left[ S^E(\xi_j^\pm) + \eta_0^2 S^M(\xi_j^\pm) \right] \left( \frac{d\Phi_\gamma}{du} \right) du \tag{23}
\]

must be used instead of \( J_{jn}^{-} \) (see below).

IV. EXPLANATION OF OBSERVATIONS

A. Rate of isotope creation by photo-induced nuclear cooperation with neutron exchange

Now the rate of nuclear cooperation with neutron exchange is determined in a photon-flux. The photon flux \( d\Phi_\gamma \) in an energy interval \( dE_\gamma \) can be written as \( d\Phi_\gamma = (d\Phi_\gamma/dE_\gamma) dE_\gamma \) where \( (d\Phi_\gamma/dE_\gamma) \) is the photon flux per unit photon energy. The rate \( d[dN_n/dt] \) due to \( d\Phi_\gamma \) can be written as \( d[dN_n/dt] = N_d (d\Phi_\gamma/dE_\gamma) dE_\gamma (\sigma_{\gamma n}^{+} + \sigma_{\gamma n}^{-}) \) and the full rate of nuclear events produced by photons in the energy range \( E_{\gamma \min} < E_\gamma < E_{\gamma \max} \) can be written as

\[
\frac{dN_n}{dt} = N_d \int_{E_{\gamma \min}}^{E_{\gamma \max}} \frac{d\Phi_\gamma}{dE_\gamma} \sum_{\xi_j} (\sigma_{\gamma n}^{+} + \sigma_{\gamma n}^{-}) dE_\gamma, \tag{24}
\]

where \( N_d \) is the number of deuterons in the volume \( V_s \) and \( N_d = V_s n_d \) with \( n_d \) the deuteron number density.
production of somewhat smaller number is obtained in the case of \( T \) and it is \( D \) to compare with the full spontaneous rate \( \epsilon \) range about \( 10^{12} \) with \( \Sigma \). The order of magnitude of the second fraction is determined by \( \Lambda = V_s^{1/3} \), and therefore \( \kappa \). The order of magnitude estimation of \( \kappa \) remains valid in the case of cooperative processes with proton exchange. Consequently, one can conclude that \( \gamma \) irradiation causes negligible effect compared to the spontaneous process.

Investigating numerically the full transition probability per unit time (\( W_{\gamma n}^{\nu} \)), the spontaneous process we take for example \( n_d = n_{ee} = a_0^3 \) with \( a_0 = 4 \times 10^{-8} \) cm, \( \Lambda = 1 \) cm and \( A_3 = 100 \) resulting \( n_d A_n z A_2 C_W = 196 \) s\(^{-1}\). As a model process the case of \( d → ^{98}Mo(0^+) \) cooperation is taken in the Weisskopf-long wavelength approximation (calculating \( H_j^2 \) with the aid of [19]). In this case the contributions of levels of \( \ell_j = 0 \) of \( ^{99}Mo \) (1/2\(^{+}\) levels) are taken into account. Their number of \( \varepsilon_j < D_{0n} \) is \( 8 \sum \varepsilon_j I_j = 0.093 \) and \( r_{98}(Mo) = 0.2413 \) resulting \( W_{\gamma n}^{\nu} = 4.4 \) s\(^{-1}\). (A somewhat smaller number is obtained in the case of production of \( ^{100}Mo \). Natural abundances \( (r A_2) \) and \( D_{0n} = \Delta_d - \Delta_p + \Delta A_2 - \Delta A_{3+1} \) values of those initial isotopes, which are thought to be essential to the explanation of \( \kappa \) can be found in Table I.)

The spontaneous process starts up as soon as the sample is made ready. In the experiment of [1] there was \( T = 6 \) h irradiation, which can be considered as a 'waiting time' from the point of view of the spontaneous process. Thus at least \( W_{\gamma n}^{\nu} T \simeq 9.5 \times 10^4 \) nuclear events happened during this time resulting \( ^{99}Mo \). Similarly in the cases of the other initial isotopes (see Table I) the spontaneous process yields instable isotopes, and it is thought that their traces were observed by gamma spectroscopy [1].

### B. Neutron production

In nuclear cooperation processes with proton exchange (see [10]) free neutrons are created. Since the Coulomb factor decreases strongly with the increase of \( z_2 \) it is expected that neutrons are created mainly in reactions with nuclei of small \( z_2 \). Considering the compositions of samples in the experiment of [1], the following reactions may be candidates of source of neutron creation by cooperative spontaneous \( \gamma \) emission with proton exchange:

\[
d + d → \gamma + n + \frac{3}{2} He + D_{0n},
\]

\[
d + \frac{13}{6}C → \gamma + n + \frac{13}{6} N + D_j,
\]

\[
d + \frac{13}{6}C → \gamma + n + \frac{14}{6} N + D_j.
\]

Naturally their counterparts, i.e. the cooperative spontaneous \( \gamma \) emission with neutron exchange reactions

\[
d + d → \gamma + p + t + D_{0n},
\]

\[
d + \frac{13}{6}C → \gamma + p + \frac{13}{6} C + D_j,
\]

\[
d + \frac{13}{6}C → \gamma + p + \frac{14}{6} C + D_j.
\]

are possible too. However the direct observation of creation of \( ^{13}/6 C \) and \( ^{14}/6 C \) is rather hard.

### V. CONCLUSIONS

The cross section of photo-induced nuclear cooperation and the transition probability per unit time of cooperative spontaneous \( \gamma \) emission both with neutron and proton exchange are determined in deuterated materials. It is found that the full rate of cooperative spontaneous \( \gamma \) emission is many fold larger than the rate of proton-induced nuclear cooperation that would produce a \( \gamma \) source of flux available todate. It is found that the observed activity can not be achieved by irradiation of samples by \( \gamma \) flux. Perhaps, cooperative spontaneous \( \gamma \) emission may be responsible for the observed nuclear activity. [1].
VI. APPENDIX

A. Interaction Hamiltonians

The electric and magnetic fields \( \mathbf{E} = e \mathbf{E} \) and \( \mathbf{H} = e_\mathbf{k} \times e \mathbf{E} \) in electromagnetic wave are perpendicular to each other and to the direction of propagation \( \mathbf{e}_\mathbf{k} = \mathbf{k} / |\mathbf{k}| \), where \( \mathbf{E} = e \mathbf{E} \) is the electric field vector of the quantized field with \( E = i \sum_{\mathbf{k}, \mathbf{e}} (2 \pi E_\mathbf{e} / V_\mathbf{e})^{1/2} [a e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} - a^+ e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})}] \). Here \( E_\mathbf{e} / h = \omega \), \( \mathbf{k} \) and \( \mathbf{e} \) are the angular frequency, wave number vector and vector of state of polarization of \( \mathbf{E}, V_\mathbf{e} \) is the volume of normalization, \( a \) and \( a^+ \) are the photon annihilation and creation operators of the quantized field. The energy of a photon of angular frequency is \( E_\gamma = \hbar \omega \).

The electric \( (V_{\mathbf{E}}^{em}) \) and magnetic \( (V_{\mathbf{M}}^{em}) \) dipole interaction with electromagnetic radiation reads in the electric dipole gauge and in the long wavelength (dipole) approximation \( (c / \omega \gg R_A) \) as:

\[
V_{\mathbf{E}}^{em} = -q \mathbf{r} \cdot e \mathbf{E}_0 \quad \text{and} \quad V_{\mathbf{M}}^{em} = -\mathbf{m} \cdot (\mathbf{e}_\mathbf{k} \times \mathbf{e}) E_0
\]

where \( \mathbf{r} \) is the space vector of the particle having electric charge \( q \) and

\[
\mathbf{m} = \frac{e^2 h}{2m_0 c} (\mu_n \sigma_n + \mu_p \sigma_p)
\]

is the magnetic dipole operator with \( \mu_n = -1.91, \mu_p = 2.79 \), \( \sigma_n \) and \( \sigma_p \) are vectors made from Pauli-spinors (the indices \( n \) and \( p \) refer to neutron and proton, respectively,) and \( E_0 = i \sum_{\mathbf{k}, \mathbf{e}} (2 \pi E_\mathbf{e} / V_\mathbf{e})^{1/2} [a e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} - a^+ e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})}] \).

For the strong interaction the interaction potential

\[
V^{st}(x) = -V_0 \quad \text{if} \quad |x| \leq b \quad \text{and} \quad V^{st}(x) = 0 \quad \text{if} \quad |x| > b
\]

is applied, where the choice for \( V_0 = 45 \text{ MeV} \) and \( b = r_0 A_{1/2} / 2 \) with \( r_0 = 1.2 \times 10^{-13} \text{ cm} \) seem to be reasonable in the case of target particle \[4, 8\].

B. Matrix elements of \( V^{em} \)

The matrix element of the interaction potential of electromagnetic radiation with matter between the initial and intermediate states \( V^{em}_{ki} = V^{em}_{E,ki} + V^{em}_{M,ki} \) according to electric \( (V_{\mathbf{E}}^{em}) \) and magnetic \( (V_{\mathbf{M}}^{em}) \) dipole interaction with electromagnetic radiation. The upper indices + and - correspond to absorption (+) and induced emission (-), respectively.

\[
V_{\mathbf{E},ki}^{em\pm} = g_{\mathbf{E},ki0}^{em\pm} \sin \theta_{kp} \frac{(2\pi)^3}{\sqrt{V_{np}}} \delta(|\mathbf{k} + \mathbf{p}|),
\]

where \( \theta_{kp} \) is the angle between \( \mathbf{k} \) of incident photon and \( \mathbf{k}_p, g^+ = \sqrt{n_\gamma + 1} \) and \( g^- = \sqrt{n_\gamma} \).

\[
V_{\mathbf{E},ki0}^{em} = \frac{1}{V_s^{1/2} \sqrt{2}} \frac{e i (2\pi E_\mathbf{e} / V_\mathbf{e})^{1/2}}{2(1 - \alpha r_1)^{1/2}} \frac{i k \sqrt{8\pi \alpha}}{(\alpha^2 + k^2)^{1/2}}.
\]

with \( k = |\mathbf{k}_p - \mathbf{k}| / 2 \). The factor \( (1 - \alpha r_1)^{-1/2} \) comes from range correction of the zero range approximation of nuclear force \[4\] with \( r_1 = 1.759 \times 10^{-13} \text{ cm} \) \[6\].

\[
V_{\mathbf{M},ki0}^{em} = g_{\mathbf{M},ki0}^{em\pm} \frac{(2\pi)^3}{\sqrt{V_{np}}} \delta(|\mathbf{k} + \mathbf{p}|)
\]

with \( \eta = [\frac{2 (\sigma^{MD} / \sigma^{ED})]^{1/2}, \quad \text{where} \quad \sigma^{MD} \quad \text{and} \quad \sigma^{ED} \quad \text{are the magnetic and electric dipole parts of regular photodissociation cross section} \[4, 6\]. \quad \text{Taking} \quad \sigma^{MD} \quad \text{and} \quad \sigma^{ED} \quad \text{from} \[4\] \quad \eta = \eta_0 \chi (k) \quad \text{with} \]

\[
\eta_0 = |\mu_n - \mu_p| \sqrt{B / (6 \mu c^2)}
\]

and

\[
\chi (k) = \left[ 1 + \frac{(\frac{k}{\alpha})^2}{1 + (\frac{k}{\alpha})^2 \alpha^2 a^2} \right]^{1/2}.
\]

Here \( a_s \) is the scattering length in the singlet state. Taking \( a_s = -2.37 \times 10^{-12} \text{ cm} \) \[4\] \( \eta_0 = 0.607 \).

C. Matrix elements of \( V^{st} \) - Cooperation

\( V_{\mathbf{k} f}^{st} \) is the matrix element of the potential of the strong interaction between the intermediate and final states. When calculating \( V_{\mathbf{k} f}^{st} \) it must be taken into account that the cooperating nuclei are located at a distance \( L \) far from each other. The amplitude \( A(L) \) of the emitted neutron spherical wave in the \( kL \rightarrow \infty \) limit varies as \( |A(L)| = \sin (k_3 L) / (k_3 L) \) (e.g. in an s-wave) or \( |A(L)| = \cos (k_3 L) / (k_3 L) \) (e.g. in a p-wave). Since \( L \) is very large compared to nuclear extension the wave appearing at the cooperating (neutron absorbing) nucleus may be considered to be a plane wave of form \( A(L)e^{ikz} \) with an appropriate choice of the frame of reference. With the aid of a state of this type

\[
V_{\mathbf{k} f}^{st} = -V_0 R_0^{3/2} j_{lf} \sqrt{12 \pi (2l_f + 1)} \quad \text{in the case of} \quad A(L) = \sin (k_3 L) / (k_3 L)
\]

\[
\times H_{i f} (k_3 R_3) \frac{2 \pi^3}{\sqrt{V_3 \sqrt{V_3}}} \delta (k_3 - k_3)
\]

In the Weiskopf-approximation

\[
H_{i f} (k_3 R_3) = \int_0^1 j_{if} (k_3 R_3 x) x^2 \, dx.
\]

In the long wavelength-approximation (LWA, \( k_3 R_3 \ll 1 \) case) \( j_{i f} (k_3 R_3 x) = (k_3 R_3 x)^{l_f + 1} / (2l_f + 1) !! \) which gives (in the Weiskopf-approximation)

\[
H_{i f}^{W} (k_3 R_3) = \frac{(k_3 R_3)^{l_f}}{(l_f + 3) (2l_f + 1)!!}.
\]

The case \( l_f = 0 \) gives the leading term with \( H_{i f}^{W} (k_3 R_3) = 1/3 \) in the LWA.
D. Some details of calculation of $W_{fi,\gamma n}$

In the center of mass frame of reference momentum conservation leads to the appearance of wave number vector Dirac-deltas $\delta (k_n+k_p)$ and $\delta (k_n-k_3)$. Integrating first over $k_n$ it results $k_n = -k_p$ substitution in the integrand of $\int V_{fi}^* V_{ki} V_{ki} \delta (k_n + k_p) \delta (k_n - k_3) d^3 k_n (2\pi)^{-3} V_n = T_{fi}(k_3, k_p)$ while the volume of normalization $V_n$ of the neutron disappears. The square of the remaining Dirac delta $\delta^2 (k_p + k_3) = \delta (0) \delta (k_p + k_3) = V_3 (2\pi)^{-3} \delta (k_p + k_3)$. Taking $V_3 = V_{fi}, V_{fi}$ disappears too. The factors $V_3 (2\pi)^{-3}$ and $V_p (2\pi)^{-3}$ coming from phase space factors of proton and $A_{z+1} X$ final nuclei make disappearing $V_3$ and $V_p$ too. Then integrating $|T_{fi}(k_3, k_p)|^2$ over $k_3$ gives $k_3 = -k_p$ substitution in the standard $W_{fi,\gamma n}$ calculation.

The energy denominator $\Delta E_{ki} = E_k - E_i - \Delta_{ik} + E_\gamma$, where $\Delta_{ik} = \Delta_+ - \Delta_0 = -B$ is the difference between the rest energies of the initial and intermediate states. $\Delta_0 = \Delta_0 - E_p$, $E_i$, $E_k$ and $E_f$ are the kinetic energies in the initial, intermediate and final states, respectively, $E_i = 0$ is supposed.

The energy denominator $\Delta E_{ki}$ reads as

$$\Delta E_{ki} = \frac{h^2 k_p^2}{m_0} + B \mp E_\gamma$$

(47)

after the substitution $k_n = -k_p$. Using $k_3 = -k_p$, the final kinetic energy $E_f$ in the argument of energy Dirac-delta $[\delta (E_f - \Delta_j^\pm)]$ is

$$E_f = \frac{(A_3 + 1) h^2 k_p^2}{2A_3 m_0}.$$  

(48)

The energy Dirac-delta is converted into $\delta (k_0^\pm - k_p)$ where $k_p = |k_p|$ and $k_0^\pm = h^{-1} \sqrt{2m_0 [A_3/(A_3 + 1)] \Delta_j^\pm}$.

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