Formation of Quantum Phase Slip Pairs in Superconducting Nanowires

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Abstract

Macroscopic quantum tunneling (MQT) is a fundamental phenomenon of quantum mechanics, related to an actively debated topic of the quantum-to-classical transition. In addition, implementation of quantum computing schemes, involving qubits, is often dependent on our ability to realize an MQT process, as well as on the protection of the resulting quantum state against decoherence. Decoherence in qubits can be reduced by means of topological protection, e.g., by exploiting various parity effects. In particular, the double-phase-slip effect can provide such protection for superconducting qubits. Here, we report direct observation of quantum double phase slips in thin-wire superconducting loops. We show that, in addition to conventional single phase slips, changing superconducting order parameter phase by $2\pi$, there are quantum transitions changing the phase by $4\pi$. Quantum double phase slips represent a synchronized occurrence of two macroscopic quantum tunneling events, i.e., the cotunneling. We demonstrate the existence of a striking regime in which double phase slips are exponentially more probable than single ones.
Decoherence and quantum noise are primary roadblocks for large-scale implementation of quantum computing, which, in many realizations, is based on the macroscopic quantum tunneling phenomenon\textsuperscript{1-5}. Decoherence is inherently related to a collapse of the macroscopic wave function due to various interactions with the environment. In quantum computers, such an outcome can be avoided by means of quantum error correction schemes as originally proposed by Shor\textsuperscript{6}. An alternative, physical, approach was suggested by Kitaev\textsuperscript{7} who argued that the use of topologically ordered quantum systems can eliminate the necessity of quantum error correction. For such systems distinct qubit states, e.g., realized by braided anyons, are topologically protected and cannot be shifted by local noise. Less exotic schemes of topological protection, namely those based on various parity effects\textsuperscript{8,9}, have also been presented\textsuperscript{10,11}. We focus on topological parity phenomenon represented by quantum double phase slips (DPS)\textsuperscript{12} in homogeneous superconducting nanowires, analogous to those proposed in Refs.\textsuperscript{13,14}. The reason they might occur is that, while gapless environmental modes can suppress quantum tunneling of single phase slips (SPS), DPS remain only weakly affected due to their reduced interaction with the environment.

Historically, the phase slips were proposed by Little\textsuperscript{15} as thermally activated topological transitions in the space of the winding numbers of the superconducting condensate in a one-dimensional wire. At low temperatures such events should proceed through MQT. The unique characteristic of MQT in thin wires (as opposed to tunnel junctions) is that an additional dissipative component is created at the location of the tunneling process; it is formed by normal electrons at the core of the phase slip. One may question if MQT is still possible in these circumstances. So far, demonstrations of quantum phase slips in homogeneous wires have relied on indirect evidences, such as overheating of the wire by a phase slip\textsuperscript{16-19}, or it was related to collective effects of many phase slips\textsuperscript{20-23}. The important advancement was recently made, showing a delocalization of wave-function due to a coherent tunneling of multiple phase slips in strongly disordered InO\textsubscript{x}\textsuperscript{24} and NbN\textsuperscript{25} nanowires.

In the present work we employ a microwave measurement technique, capable of individual phase slip detection\textsuperscript{26}, to study MQT in thin-wire superconducting loops, and report the following results. First, we observe individual phase slips as isolated macroscopic tunneling events. Second, we discover that, in addition to the conventional SPS, there also exist quantum DPS events that change the loop vorticity by 2, and hence preserve the parity of its winding quantum number. Such transitions correspond to simultaneous tunneling
(cotunneling) of two fluxoids in or out the loop. They can be viewed as quantum synchronization, or pairing, of distinct macroscopic events, taking place due to minimization of action, rather than minimization of energy. More importantly, we show that at low bias DPS are exponentially more likely than SPS.

The geometry of our samples is illustrated in Fig. 1. A pair of superconducting nanowires fabricated by molecular templating (see SI-1) is integrated into the center conductor (c.c.) of a superconducting coplanar Fabry-Perot resonator. The wires connected with the c.c. superconducting strips form a closed loop. An external magnetic field, \( B \), applied perpendicular to the resonator’s surface, threads the loop, giving rise to Little-Parks (LP) oscillations of various types\(^{26,27}\). Sufficiently small thickness of the film allows us to neglect its effect on the external field. Below, we present data for Sample A. Data obtained on Sample B show similar behavior and are relegated to the Supplementary Information (SI-3,8).

We detect phase slips by measuring the resonator’s transmission phase shift \( \phi_{S21} \) defined as the difference between phases of the output and input microwave signals. The frequency of the input signal is equal to that of the resonator’s fundamental mode at \( B = 0 \). After

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**FIG. 1:** Schematic representation of the samples (not to scale). Superconducting MoGe film is shown in gray, regions where it is removed – in white. The nanowires are suspended over a trench in the substrate (not shown; see SI-1). The superconducting loop is threaded by an external magnetic field, \( B \). Yellow stripes depict gold coating. DPS is shown as a pair of fluxoids simultaneously crossing both nanowires. Each fluxoid changes the phase accumulated on the loop by \( \Delta \phi = 2\pi \) and generates voltage on each wire \( V = (d\Delta \phi/dt)/2e \). Plus and minus signs indicate the charge distribution during a double phase slip.
cycling the magnetic field generated by current $J$ in solenoid multiple times, we obtain the dependence shown in Fig. 2. The phase shift $\phi_{S21}(J)$ is a multivalued function composed of a periodic set of “parabolas” representing the LP effect\(^{20}\). Each parabola corresponds to a state with a particular number of phase fluxoids trapped in the loop, i.e., a state with a particular vorticity $n$. Every jump up to the nearest parabola represents exactly one SPS, which takes place in one of the wires\(^{15}\). Interestingly, apart from the $n \to (n + 1)$ jumps, we also detect $n \to (n + 2)$ events, changing the vorticity by 2. In the latter case the phase jumps up to the next nearest parabola. This detection method does not rely on the overheating of the nanowire by the phase slips, as was the case in the experiments of Refs.\(^{16,17}\). In the present case, the transitions between the different vorticity states are quantum transitions between the quantized energy levels of the systems.

The switching of the loop vorticity has a stochastic nature, i.e., in every measurement

![FIG. 2: Dependence of the transmission phase on the current in the solenoid. Periodically spaced “parabolas”, each corresponding to a particular vorticity or the quantum winding number, n, are clearly observed. SPS result in jumps of $\phi_{S21}$ to the next parabola (black arrow) and $n \to (n + 1)$ change in the vorticity. DPS result in jumps of $\phi_{S21}$ to the next nearest parabolas (red arrow) and $n \to (n + 2)$ change in the vorticity. The Little-Parks period is marked as $J_{LP}$. Regions where switching events form clusters correspond to magnetic fields and vorticity values at which the current in the wires is near the critical current.](image-url)
it occurs at a slightly different current, $J$. To characterize the switching events, we employ a statistical approach. Specifically, for each current interval where phase slips take place, we extract values of switching currents for SPS and DPS individually, and compute their distributions (see SI-3, Fig. S2). From a series of distributions measured at different temperatures we compute standard deviations of the switching currents, $\sigma_1$ for SPS and $\sigma_2$ for DPS, and plot them as functions of temperature in Fig. 3. When the sample is cooled down, $\sigma_1$ exhibits the expected behavior in accordance with the Kurkijärvi power-law scaling\textsuperscript{28}. The best fit is set by $\sigma_1(T) = 0.136T^{0.54}$, proving the thermally activated nature of the vorticity jumps. At low temperatures we observed a saturation of $\sigma_1$. Temperature $T_q$ is understood as a crossover between the regimes of thermally activated phase slips ($T>T_q$) and quantum tunneling ($T<T_q$). The obtained value of $T_q \approx 0.90$ K is very close to 0.87 K that is expected from the previously determined linear dependence\textsuperscript{18} of $T_q$ and the critical temperature $T_c$ (see SI-4). Moreover, the average SPS switching current continues to grow even when standard deviation does not change (see inset in Fig. 3). These two facts provide strong evidence that the observed crossover is not a result of incomplete thermalization or external noise, but due to MQT of SPS. The dependence of $\sigma_2$ on temperature significantly differs from that of $\sigma_1$. First, DPS are almost never observed in the thermal activation regime ($T>T_q$), suggesting that DPS is an essentially quantum phenomenon. Second, at temperatures where DPS do occur, their probability distributions are broad, $\sigma_2>\sigma_1$, reflecting the fact that DPS rate does not drop as sharply as that of SPS, when the current in the wire is shifted away from the critical value.

According to Kurkijärvi’s theory\textsuperscript{28}, for a given current sweep speed $v_J = dJ/dt$, distribution of switching currents $P(J)$ (see SI-3, Fig. S2a) has a one-to-one correspondence to the switching rate $\Gamma(J)$. In contrast with $P(J)$, $\Gamma(J)$ is independent of $v_J$. Following the method outlined in references\textsuperscript{19,28,29}, we performed a statistical analysis of more than $10^4$ switching events and computed the rates of SPS and DPS denoted as $\Gamma_1$ and $\Gamma_2$, respectively (see SI-5). Fig. 4 shows the rates as functions of the normalized current $j = J/J_{LP}$ carried by the solenoid, where $J_{LP}$ is the LP period (Fig. 2). Such normalization reveals how far the system can be driven away from its equilibrium vorticity before a phase slip takes place. Notice that if the system were completely classical and followed its lowest energy vorticity state, then the first SPS would occur at $j=1/2$ (assuming $n=0$ initially) and DPS would never happen. Experimentally, the phase slips take place at values slightly higher than $j=3$.
(see Fig. 4), which is a metastable state quite far from the equilibrium. Therefore, it is clear that at \( j = 0 \) and \( n = 0 \) and even at \( j = 1/2 \) and \( n = 0 \) the barrier for phase slips is much higher than the scale of thermal or quantum fluctuations.

Remarkably, Fig. 4 shows that at low currents the rate of DPS exceeds that of SPS by approximately one order of magnitude. Demonstration of such regime constitutes the major finding of the present work. If the observed trend continues down to zero current, the rate of SPS is expected to be negligible at \( J = 0 \) compared to the DPS rate. Since DPS preserve the parity of the winding number of the superconducting order parameter, the observed phenomenon could lead to novel topologically protected qubits\(^{7-9,12}\).

To explain our findings, we will employ two simple arguments. The first one is that,
in principle, two distinct types of DPS events can exist, namely co-tunneling (DPS\(_{4\pi}\)) and sequential tunneling (DPS\(_{2\pi+2\pi}\)). DPS\(_{4\pi}\) happens at once and leads to the instantaneous phase change of \(4\pi\). Sequential DPS\(_{2\pi+2\pi}\) consist of two SPS events, the first one of which causes the second one to occur. These two events are separated by a very short time lag, which is on the order of the energy relaxation time, \(\tau_r\). Being at nanosecond time scale\([10]\), this lag is much shorter than the resolution of our setup, thus DPS\(_{2\pi+2\pi}\) are also registered as \(4\pi\)-slip events. Note that if the delay would be much larger, i.e., within the resolution limit of our setup, then we would detect some DPS as split events, but practically such splitting is never observed. The second argument is that energy \(Q\) dissipated during either phase slip is proportional to the current in the wire\([30]\); \(I\), (note that \(I/J/\approx const\), i.e., \(Q_1/\approx\Phi_0\) (for SPS) and \(Q_2/\approx 2I\Phi_0\) (for DPS), where \(\Phi_0\) is the flux quantum.

Let us notice, that the DPS rate \(\Gamma_2(j)\) consists of two regions with different slopes joined in the vicinity of \(j_x\). At the same time, \(\Gamma_1(j)\) does not change its slope at \(j=j_x\). This drastic change of \(\Gamma_2(j)\) slope around \(j_x\) can be explained if we assume that for \(j>j_x\) DPS\(_{2\pi+2\pi}\) can be initiated by SPS with a probability \(P_{2\pi+2\pi}\), which is less than 0.5. The conclusion that

![Graph showing rates of SPS and DPS as functions of normalized current in the solenoid. The bath temperature is 60 mK. Single (black squares) and double (blue ovals) phase slip rates are obtained from the distributions shown in Supplementary Information, Fig. S1a. The solid black line is the Kurkijärvi-Garg fit of \(\Gamma(j)\) for SPS. The dashed red line is the DPS fit, generated according to Korshunov’s theory of DPS\([14]\). Rates of SPS and DPS are equal at the current \(j_x\).](image-url)
\[ P_{2\pi + 2\pi} < 0.5 \] follows from the fact that \( \Gamma_1 > \Gamma_2 \) for \( j > j_x \). Noting that \( \Gamma_2 \) is parallel to \( \Gamma_1 \) for \( j > j_x \), we conclude that mostly sequential \( \text{DPS}_{2\pi + 2\pi} \) processes are observed here, in which case one phase slip sometimes causes the second one to occur within nanoseconds. Indeed, SPS shifts the system from a deep metastable state, \( n=0 \), to the next state, \( n=1 \), which is closer to the equilibrium value, \( n_{eq} = \text{round}(j) = 3 \). Each phase slip is accompanied by the suppression of the order parameter and dissipation of the energy \( Q_1 \) in a nanowire. As a result, the energy barrier is reduced during time \( \sim \tau_r \), helping the second SPS to occur with an elevated probability, and thus producing \( \text{DPS}_{2\pi + 2\pi} \). On the other hand, the first SPS reduces the equilibrium supercurrent in the loop. So, if the second phase slip does not occur within nanoseconds after the first one, it then has to wait till the magnetic field travels to the next critical current region.

The aforementioned arguments cannot explain the fact that \( \Gamma_2 > \Gamma_1 \) at low currents (Fig. 4), \( i.e., \) at \( j < j_x \). Indeed, since the energy dissipated during SPS decreases with the current (because \( Q_1 \approx J \Phi_0 \)), the probability that one SPS causes the second one decreases along with \( I \), \( i.e., \) \( P_{2\pi + 2\pi}(j < j_x) < P_{2\pi + 2\pi}(j > j_x) \). Recalling that \( P_{2\pi + 2\pi}(j > j_x) < 0.5 \), we obtain \( P_{2\pi + 2\pi}(j < j_x) < 0.5 \) (given that one phase slip does occur).

If there were only two types of events, namely SPS and \( \text{DPS}_{2\pi + 2\pi} \), then SPS should be observed more frequently at low currents. This obviously contradicts our observations (Fig. 4). Thus, another type of DPS transitions must exist that are dominant at low currents \( (j < j_x) \). The only events not yet included in our consideration are \( \text{DPS}_{4\pi} \). The main difference is that although \( \text{DPS}_{4\pi} \) involves two phase slips, it is not true that one causes the other. They occur together, at the same time. Since thermal fluctuations are negligible at \( T < T_q \), it is justified to characterize \( \text{DPS}_{4\pi} \) as \( \text{cotunneling of phase slips} \). We have also verified that the observed dominance of \( \text{DPS}_{4\pi} \) over SPS is not caused by external noise, since making a better filtering of the signal lines only leads to a higher percentage of DPS at low currents (see SI-6 and Fig. S3).

To approximately describe short superconducting nanowires, such as ours\textsuperscript{[13]}, one can employ the Stewart-McCumber model\textsuperscript{[10]}, within which the phase difference represents the motion of some effective “phase particle” in a “washboard” potential. The effective mass \( m = \hbar^2 C / (2e)^2 \) of the phase particle is proportional to the shunt capacitance, \( C \), connected in parallel with a superconducting junction. The friction experienced by the particle is described by the effective viscosity \( \eta = \hbar^2 / 4e^2 R_n \), inversely proportional to the shunt
impedance, $R_n$. The character of the particle motion is determined by the ratio between $m$, $\eta$ and the potential barrier height $U = \hbar(I_c^2 - I^2)^{1/2}/2e$, where $I$ is the bias current and $I_c$ is the critical current. In an underdamped regime ($mU \gg \eta^2$), the “heavy” particle moves in a non viscous environment. The viscosity in this case does not play a significant role and thus can be neglected. The main parameter influencing the phase slip rate at a particular bias current is the barrier height $U$.

Let us now show that SPS process corresponds to the underdamped motion of the particle. From the resonance frequency, $f_0 \approx 5 \text{ GHz}$, and the inductance of the sample $L \approx 6 \text{ nH}$, one can estimate the effective electric capacitance (the c.c. electrodes) $C_1 = 1/(4\pi^2 f_0^2 L) \sim 10^{-13} \text{ F}$. The effective shunt is set by the waveguide impedance of the resonator, $R_n \sim 50 \Omega$. Using all of the above values, we arrive at the following estimates: $m \sim 10^{-44} \text{ J-s}^2$, $\eta \sim 10^{-33} \text{ J-s}$, $U \approx \hbar I_c/4e \sim 10^{-21} \text{ J}$ ($I_c$ of the nanowires is around $10 \mu A$). Such parameters indeed correspond to the underdamped regime, $mU \gg \eta^2$. And therefore, SPS can be analyzed in the framework of the Kurkijärvi-Garg (KG) theory. Following Aref et al., we fit the rate of SPS in amorphous MoGe wires using the equation: $\Gamma_1(j) = \Omega \exp[-(2\sqrt{2}hI_c/3ek_B T_q)(1 - j/j_c)^{3/2}]$. Here $\Omega$ is the phase slip attempt frequency, which we assume is a constant, $k_B$ is the Boltzmann constant, $j_c$ is the normalized critical current in the solenoid, defined as $j_c=J_c/J_{LP}=I_c/I_{LP}$. Note that $J_c$ is the current in the solenoid that induces a Meissner current, entering the nanowires, that is equal to their critical current $I_c$. $I_{LP}$ is the Little-Parks period measured in the units of the current in the nanowires. The best fit generated by the KG formula with the use of three fitting parameters is shown in Fig. 4. It corresponds to $\Omega = 0.9 \sim 10^{11} \text{ Hz}$, $I_c = 7.8 \mu A$, $j_c = 4.04$. The best fit values of $\Omega$ and $I_c$ are somewhat different from the previously reported values but they are of the same order of magnitude. The $j_c$ is close to, but higher than the experimentally observed maximum switching current $j_{max} \sim 3.5$ (see Fig. 4), as it should be. These results related to SPS are in good agreement with the indirect SPS observations, achieved with dc current measurements.

We now discuss the DPS effect. A pair of fluxoids moving in opposite directions and crossing both wires simultaneously (see Fig. 1) does not charge the capacitor formed by the c.c. electrodes. Thus, such mechanism of DPS requires less energy and therefore is more probable than the one, in which a fluxoid pair crosses a single wire. Due to the specifics of the fabrication process, our wires are not completely identical. The role
of asymmetry is not clearly understood. Nonetheless, since DPS$_{4\pi}$ have been observed in two samples, we conclude that the asymmetry, if any, does not play a crucial role in our experiment. Because the effective capacitance is much smaller for DPS$_{4\pi}$ than for SPS, the effective mass of DPS$_{4\pi}$ particle is also much smaller, and the rate of DPS$_{4\pi}$ becomes determined mostly by the Caldeira-Leggett dissipation. For the special case of sinusoidal dissipation, it can be computed by considering small fluctuations near Korshunov’s instanton. We generalize that by including an additional parameter $\alpha$ characterizing the peculiarities of the spatiotemporal shape of the phase slips in superconducting wires. Our final expression for the DPS$_{4\pi}$ rate is $\Gamma_2 = B_2 (j/j_c)^{-2/3} \exp[A_2 (j/j_c)^{2/3}]$. Here $A_2$ and $B_2$ are defined through $I_c$, effective viscosity, $\eta_2$, effective capacitance, $C_2$, and the quantum resistance $R_q = \pi \hbar/2e^2$ as: $A_2 = 12\pi (\alpha \eta_2 R_q C_2 I_c / 4\pi \hbar e)^{1/3}$, $B_2 = (12\pi / A_2) (\alpha \eta_2 / \hbar)^{1/2} (I_c / 2e) \exp[-8\pi \{\eta_2 / \hbar + [2/(\alpha \eta_2 / \hbar)^{1/2}] (A_2 / 12\pi)^{3/2}\}]$. The theory is valid in the limit of large viscosity, namely when $\beta = \eta_2 / (\hbar^2 R_q C_2 I_c / 4\pi \hbar e)^{1/2} \gg 1$. This condition is satisfied by choosing $\alpha = 6$, $C_2 = 3.1 \times 10^{-17}$ F, $\eta_2 = 2.8 \times 10^{-34}$ J-s, which deviate minimally from their expected values (see SI-7). From the best fit for the rate of $4\pi$ events (red dashed line in Fig. 4) we determine $A_2 = 91.4$, $B_2 = 1.5 \cdot 10^{-36}$ s$^{-1}$. Value of the normalized critical current is not independent, but is set by the SPS fit, i.e., $j_c = 4.04$. The fitting parameters are in agreement with the assumption that the viscosity is strong ($mU \ll \eta^2$). They also correlate well with the fitting parameters for the sample B (see SI-8).

Qualitatively speaking, the higher rate of DPS$_{4\pi}$ compared to SPS is explained by three facts: (1) The DPS$_{4\pi}$ are effectively “lighter” since the net voltage they generate on each electrode is zero and, thus, the capacitive effect is weak for them. (2) The DPS$_{4\pi}$ are less coupled to the environment, according to the Guinea-Schön-Korshunov action (see SI-9). (3) The electromagnetic emission generated by DPS$_{4\pi}$ into the resonator, which is a dissipative effect slowing down MQT, is virtually zero, again because the pair of phase slips does not produce any net voltage difference between the electrodes and the ground planes.

In conclusion, we report a study of the stability of fluxoid states in superconducting loops using microwave measurements. We provide direct evidence that at low temperatures the change of the loop vorticity is realized by macroscopic quantum tunneling of individual phase slips or pairs of phase slips through nanowires forming the loop. We identify a low bias regime in which cotunneling of two phase slips, i.e., a double phase slip, is exponentially more likely to occur than a single phase slip. The dominance of such parity conserving
transitions, which serve as a macroscopic analogue of electron tunneling, might present nanowire loops as a new platform for building parity-protected quantum qubits.

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1. Leggett, A. J. Macroscopic quantum systems and the quantum theory of measurement. Prog. Theor. Phys. Suppl. 69, 80-100 (1980).
2. Lee, C. W. and Hyunseok J. Quantification of macroscopic quantum superpositions within phase space. Phys. Rev. Lett. 106, 220401 (2011).
3. Clarke, J., Cleland A. N., Devoret, M. H., Esteve D. and Martinis, J. M. Quantum mechanics of a macroscopic variable: the phase difference of a Josephson junction. Science 239, 992-997 (1988).
4. Jackel, L. D. et al. Decay of the zero-voltage state in small-area, high-current-density Josephson junctions. Phys. Rev. Lett. 47, 697-700 (1981).
5. Wernsdorfer, W. et al. Macroscopic quantum tunneling of magnetization of single ferrimagnetic nanoparticles of barium ferrite. Phys. Rev. Lett. 79, 4014-4017 (1997).
6. Shor, P. W. Scheme for reducing decoherence in quantum computer memory. Phys. Rev. A 52, R2493-R2496 (1995).
7. Kitaev, A. Yu. Fault-tolerant quantum computation by anyons. Ann. Phys. 303, 2-30 (2003).
8. Gottesman, D., Kitaev, A. Yu. and Preskill, J. Encoding a qubit in an oscillator. Phys. Rev. A 64, 012310 (2001).
9. Doucot, B., Vidal, J. Pairing of Cooper pairs in a fully frustrated Josephson junction chain. Phys. Rev. Lett. 88, 227005 (2002).
10. Gladchenko, S. et al. Superconducting nanocircuits for topologically protected qubits. Nat. Phys. 5, 48-53 (2009).
11. Bell, M. T., Paramanandam, J., Ioffe, L. B. and Gershenson, M. E. Protected Josephson rhombus chains. Phys. Rev. Lett. 112, 167001 (2014).
12. Doucot, B., Ioffe, L. B. Physical implementation of protected qubits. Rep. Prog. Phys. 75, 072001 (2012).
Guinea, F. and Schön, G. Coherent charge oscillations in tunnel junctions. *Europhys. Lett.* **1** (11), 585-593 (1986).

Korshunov, S. E. Coherent and incoherent tunneling in a Josephson junction with a “periodic” dissipation. *JETP Lett.* **45**, 434 (1987).

Little, W. A. Decay of persistent currents in small superconductors. *Phys. Rev.* **156**, 396-403 (1967).

Sahu, M. et al. Individual topological tunneling events of a quantum field probed through their macroscopic consequences. *Nat. Phys.* **5**, 503-508 (2009).

Li, P. et al. Switching currents limited by single phase slips in one-dimensional superconducting Al nanowires. *Phys. Rev. Lett.* **107**, 137004 (2011).

Aref, T., Levchenko, A., Vakaryuk, V. and Bezryadin, A. Quantitative analysis of quantum phase slips in superconducting Mo$_{76}$Ge$_{24}$ nanowires revealed by switching-current statistics. *Phys. Rev. B* **86**, 024507 (2012).

Bezryadin, A. *Superconductivity in Nanowires*. Wiley-VCH, 1 edition (2012).

Giordano, N. Evidence for macroscopic quantum tunneling in one-dimensional superconductors. *Phys. Rev. Lett.* **61**, 2137-2140 (1988).

Bezryadin, A., Lau, C. N. and Tinkham, M. Quantum suppression of superconductivity in ultrathin nanowires. *Nature* **404**, 971-974 (2000).

Lau, C. N., Markovic, N., Bockrath, M., Bezryadin, A. and Tinkham, M. Quantum phase slips in superconducting nanowires. *Phys. Rev. Lett.* **87**, 217003 (2001).

Arutyunov, K., Golubev, D. S. and Zaikin, A. D. Superconductivity in one dimension. *Phys. Rep.* **464**, 1–70 (2008).

Astafiev, O. V. et al. Coherent quantum phase slip. *Nature* **484**, 355-358 (2012).

Peltonen, J. T. et al. Coherent flux tunneling through NbN nanowires. *Phys. Rev. B* **88**, 220506(R) (2013).

Belkin, A., Brenner, M., Aref, T., Ku, J. and Bezryadin, A. Little–Parks oscillations at low temperatures: gigahertz resonator method. *Appl. Phys. Lett.* **98**, 242504 (2011).

Hopkins, D. S., Pekker, D., Goldbart, P. M. and Bezryadin, A. Quantum interference device made by DNA templating of superconducting nanowires. *Science* **308**, 1762-1765 (2005).

Kurkijärvi, J. Intrinsic fluctuations in a superconducting ring closed with a Josephson junction. *Phys. Rev. B* **6**, 832-835 (1972).
29 Fulton, T. A. and Dunkleberger, L. N. Lifetime of the zero-voltage state in Josephson tunnel junctions. *Phys. Rev. B* **9**, 4760-4768 (1974).

30 Anderson, P. W. and Dayem, A. H. Radio-frequency effects in superconducting thin film bridges. *Phys. Rev. Lett.* **13**, 195-197 (1964).

31 Garg, A. Escape-field distribution for escape from a metastable potential well subject to a steadily increasing bias field. *Phys. Rev. B* **51** (21), 15591-15595 (1995).

32 Caldeira, A. O. and Leggett, A. J. Influence of dissipation on quantum tunneling in macroscopic systems. *Phys. Rev. Lett.* **46**, 211-214 (1981).

33 Averin, A. D. and Nazarov, Y. Virtual electron diffusion during quantum tunneling of the electric charge. *Phys. Rev. Lett.* **65**, 2446-2449 (1990).
Supplementary Information (SI)

1. Molecular templating

A trench is made on a Si – SiO₂ – Si₃N₄ substrate, and a fluorinated carbon nanotube is placed across the trench. The nanotube is then sputter-coated with a superconducting alloy, amorphous Mo₂₉Ge₇₁, thus producing a superconducting nanowire (SNW). Both the resonator and the nanowires are formed during the same sputtering session, allowing to avoid any contact resistance. Sample A has the following parameters: nanowires’ length is 200 nm, their width is 22-25 nm, and thickness is 17 nm. The distance between the nanowires is 13 μm. Sample B has the same nanowire length, the width is 25-30 nm, and the thickness is 20 nm. Nanowire separation is 10 μm. The distance between the input and output “mirrors” of the resonator in both cases is 6 mm, resulting in the resonance frequency of the fundamental mode \( f₀ \sim 5 \text{ GHz} \) (for sample A) and \( f₀ \sim 5.5 \text{ GHz} \) (for sample B). The width of the center conductor of the resonator is \( w = 20 \mu \text{m} \). The input, output and the ground planes are partially covered by 100 nm thick gold film (Fig. 1) to facilitate the connection of external gold wires to them. Gold wires have been used to connect the device to the microwave measurement cables because such Au wires stay normal at sub-Kelvin temperatures and allow a good thermal anchoring of the sample to its Faraday cage.

![Fig. S 1: Schematic of the molecular templating. Fluorinated carbon nanotube is stretched across a trench etched into Si₃N₄/SiO₂ on a Si chip. The nanotube and the banks are coated with superconducting Mo₂₁Ge₇₉.](image)
2. Low-temperature setup: $^3$He and dilution refrigerator

Each cryostat was equipped with two stainless steel semirigid coaxial cables, thermalized by a sequence of attenuators thermally anchored at low temperatures. The sample was mounted inside a brass Faraday cage that screens external RF noise but is transparent for DC magnetic field. The ground planes of the resonator have been connected to parts of the Faraday cage to achieve a superior thermalization of the sample. The signal from the output of the resonator was amplified by a low temperature microwave amplifier LNF-LNC4.8A (Low Noise Factory), which has the input noise temperature of 2.6 K. To prevent this noise from impacting the sample, we placed two thermally anchored isolators between the sample and the amplifier. For simple transmission measurements a network analyzer Agilent PNA5230A was used.

3. Phase slip distributions

Figure S2a shows probability density functions for SPS and DPS events for sample A. This data was used to calculate phase slip rates shown in Fig. 4. As it clearly follows from the graph, probability density curves intersect. The same effect was confirmed on the different sample B (see Fig. S2b). Corresponding rates and theoretical fits are discussed below.

![Fig. S2: Probability density of SPS and DPS as a function of normalized current in solenoid. Black squares indicate SPS, blue ovals – DPS events. (a) Sample A, $T = 60$ mK. (b) Sample B, $T = 350$ mK. Distributions are computed using the bin size of $\Delta j = 0.01$.](image-url)
4. Critical temperature of superconducting nanowires

The critical temperature of MoGe nanowires can be estimated based on the wire cross sectional area, $S$. Using the dependence from Ref.\textsuperscript{34}, we find $T_c \approx 5.3$ K for $S \approx 400$ nm$^2$ (sample A).

Once the critical temperature is obtained, the expected crossover temperature $T_q$ is readily calculated from the linear relationship\textsuperscript{19} ($T_q = 0.164T_c$), giving us $T_q = 0.87$ K.

5. Phase slip rate calculation

We calculate $\Gamma_1$ and $\Gamma_2$ using the following expression: $\Gamma_k = \frac{v_j}{\Delta J} \cdot \frac{N_k(J)}{N_T(J)}$. Here $N_k(J)$ is a number of phase slips of type $k$ ($k = 1$ for SPS, $k = 2$ for DPS) observed in the current interval $\Delta J$ centered at $J$, $N_T$ is the total number of phase slips observed at currents greater than $J$.

6. Influence of external photons on DPS probability

To verify that DPS are not caused by a stray electromagnetic radiation from the environment we covered the inner part of our Faraday cage by radiation-absorbing black coating\textsuperscript{35} that has an absorptivity of 90% over a wide angle in the 0.3-2.5 THz range\textsuperscript{36}. In addition, we varied the input power sent to the resonator by 5 dBm. The impact of these two modifications on the described phenomenon is shown in Fig. S3. There, we plot the difference between probability densities of DPS and SPS events, $(P_2 - P_1)$, as a function of normalized current, $j$.

It is clear that the peak height, describing the predominance of DPS over SPS, has not become bigger after we had increased the power and removed the black coating, which filters out the environmental photons. We also find that in the case of better filtering, the net percentage of DPS increases from 18% to 34%. This makes us confident that DPS are not caused by external perturbations, but vice versa, it is a delicate macroscopic quantum phenomenon, which can be suppressed by the slightest external perturbation. Thus, the probability of observing DPS in “quieter” environments (in terms of lesser numbers of stray photons) is higher.
7. **Expected values of $\alpha$ and $C_2$**

We estimate the rate of DPS$_{4\pi}$ events by considering small fluctuations near Korshunov’s instanton. We encapsulate the difference between a Josephson junction, for which the instanton was originally found, and a nanowire by taking the instanton-antiinstanton interaction to be of the same (dipole) form as in Ref.\textsuperscript{14}, but with a magnitude proportional to a free parameter $\alpha$ (a constant of order unity). In Ref.\textsuperscript{14}, $\alpha$ is approximately 2, while in our case a good fit to the data is achieved with $\alpha = 6$. We believe this happens due to the fact that the spatio-temporal form of the phase slip in thin wire does not coincide with the form of the phase slip in Josephson junction, for which Korshunov has developed his theory.

To estimate the capacitance $C_2$, we use the expression for a cylindrical capacitor: $C = 2\pi \epsilon \epsilon_0 L / \ln(R_{\text{out}}/R_{\text{in}})$. Here $\epsilon$ is the dielectric constant, $\epsilon_0$ is the vacuum permittivity, $L$ is the length of the capacitor, $R_{\text{out}}$ and $R_{\text{in}}$ are the outer and inner radii of the capacitor, respectively. Applying this formula to our system we associate $L$ with the nanowire length (200 nm), $R_{\text{in}}$ – with its radius (10 nm), and $R_{\text{out}}$ – with the distance to an effective ground. The latter quantity could be estimated as the distance from the nanowire center to the c.c. plane, i.e., as $L/2$ (100 nm). If the dielectric constant $\epsilon = 1$, we obtain $C_2 \approx 0.5 \times 10^{-17}$ F.

![Fig. S 3: Comparison of different measurement conditions. Difference between DPS, $P_2$, and SPS, $P_1$, probability densities before (red triangles) and after (green diamonds) the infrared absorbing coating is added and the input power is reduced.](image-url)
8. Analysis of SPS and DPS rates for sample B

Distributions of SPS and DPS were measured in $^3$He system at 350 mK. Corresponding rates are calculated and the results are shown in Fig. S4. Similarly to sample A, $\Gamma_1$ and $\Gamma_2$ intersect, and above the intersection go almost parallel to each other.

We were able to fit the rate of SPS using the following parameters: $\Omega = 2.3 \cdot 10^{11}$ Hz, $I_c = 8.0 \mu$A, $j_c = 3.51$. From the best fit for the DPS rate we find that $A_2 = 93.6$, $B_2 = 7.3 \cdot 10^{-37}$ 1/s, $j_c = 3.51$. As one can see, these fitting parameters are close to those obtained for sample A.

9. Guinea-Schön-Korshunov action for SPS and DPS

Our starting point is the Euclidean action for a Josephson junction in the case when a dissipative component with the single-electron charge periodicity is present. This is given by\textsuperscript{13,14}:

$$S[\phi(t)] = \int dt \left[ \frac{m}{2} \left( \frac{d\phi}{dt} \right)^2 - \frac{I_c}{2e} \cos \phi - \frac{I}{2e} \phi \right] + \frac{4\eta}{\pi} \int \int dtdt' \frac{\sin^2 \{[\phi(t) - \phi(t')] / 4\}}{(t-t')^2}$$  \hspace{1cm} (1)

![Fig. S4: Rates of SPS and DPS as functions of normalized current in the solenoid for sample B. The bath temperature is 350 mK. Single (black squares) and double (blue ovals) phase slip rates are obtained from the distributions shown in Fig. S2b. The solid black line is the Kurkijärvi-Garg fit for SPS, the dashed red line is the Korshunov fit for DPS.](image)
The last term describes the influence of dissipation on tunneling events (either SPS or DPS). Although we do not expect the cosine form of the potential and the sine form of the dissipative term to apply directly to the case of nanowires, the qualitative arguments of this section are based only on the periodicity properties and should apply in that case as well.

A tunneling event can be thought to occur around a certain value of the Euclidean time, when the phase $\phi(t)$ changes by $2\pi$ (for SPS) or $4\pi$ ($\text{DPS}_{4\pi}$). In the former case the last term in (1) logarithmically diverges at large values of $t - t'$, while in the latter case it remains finite\textsuperscript{14}. As a result, at low temperatures and currents, tunneling by $2\pi$ is additionally suppressed. A qualitative explanation of this is as follows. When an SPS happens, the system tunnels from one minimum of the washboard potential to the next one, changing the phase of the condensate by $2\pi$. During this event, a small voltage develops and rotates the phase of the normal electrons, coupled to the condensate, by $\pi$ (assuming that normal electrons in the environment are quasi-independent and can be described by separate single-particle wave functions). Since the environment can be considered infinitely large for all practical purposes, such a change of the normal-electron phase is “too heavy” for the tunneling to occur. On the other hand, if the phase of the condensate rotates by $4\pi$, the corresponding rotation of the phase of the normal electrons is $2\pi$, which is a trivial change since their wavefunction is $2\pi$-periodic. Therefore, cotunneling or tunneling by $4\pi$ in the washboard potential is not suppressed as strongly.

In addition, the effective mass $m$ is strongly dependent on the environment and can be expected to be larger for SPS than for $\text{DPS}_{4\pi}$. Indeed, $\text{DPS}_{4\pi}$ is an event in which two single phase slips are generated on both superconducting nanowires simultaneously (see Fig. 1). In such a case, the voltage generated by the phase slip on one wire would be opposite to the voltage on the other wire. Since the mean voltage produced by the phase slips is near zero, the c.c. electrodes do not charge, and the effective shunt capacitance becomes much smaller for $\text{DPS}_{4\pi}$, than it is for SPS.

Small capacitance corresponds to a light phase particle. Given that heavier particles do not tunnel as easily as lighter ones, the rate of SPS is further suppressed relative to the rate of $\text{DPS}_{4\pi}$.

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34 Kim, H., Jamali, S. and Rogachev, A. Superconductor-insulator transition in long MoGe nanowires. *Phys. Rev. Lett.* **109**, 027002 (2012).

35 Barends, R. *et al.* Minimizing quasiparticle generation from stray infrared light in superconducting quantum circuits. *Appl. Phys. Lett.* **99**, 113507 (2011).

36 Klaassen, T. O. *et al.* Absorbing coatings and diffuse reflectors for the Herschel platform submillimeter spectrometers HIFI and PACS. *2002 IEEE Tenth International Conference on Terahertz Electronics*, 32-35 (2002).