Cosmological constant and late transient acceleration of the universe in the Horava-Witten Heterotic M-Theory on $S^1/Z_2$

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Orbifold branes are studied in the framework of the 11-dimensional Horava-Witten heterotic M-Theory. It is found that the effective cosmological constant can be easily lowered to its current observational value by the mechanism of large extra dimensions. The domination of this constant over the evolution of the universe is only temporarily. Due to the interaction of the bulk and the branes, the universe will be in its decelerating expansion phase again in the future, whereby all problems connected with a far future de Sitter universe are resolved.

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I. INTRODUCTION

Recent observations of supernova (SN) Ia reveal the striking discovery that our universe has lately been in its accelerated expansion phase $^{11}$. Cross checks from the cosmic microwave background radiation and large scale structure all confirm this unexpected result $^2$. Such an expansion was predicted neither by the standard model of particle physics nor by the standard model of cosmology. In fact, in order to have an accelerated expansion, the latter requires the introduction of either a tiny positive cosmological constant $\Lambda$ or an exotic component of matter that has a very large negative pressure and interacts with other components of matter weakly, if there is any. This invisible component is usually dubbed as dark energy.

A tiny $\Lambda$ is well consistent with all observations carried out so far $^3$, and the recent Hubble Space Telescope observations of the nearby galaxy groups Cen A/M83, M81/M82, and their vicinities, are even in favor of it $^4$. Although the introduction of $\Lambda$ may be the simplest resolution of the crisis, considerations of its origin lead to other severe problems: (a) Its theoretical expectation values exceed observational limits by 120 orders of magnitude $^5$. (b) Its corresponding energy density is comparable with that of matter only recently. Otherwise, galaxies would have not been formed. Considering the fact that the energy density of matter depends on time, one has to explain why only now the two are in the same order. (c) Once $\Lambda$ dominates the evolution of the universe, it dominates forever. An eternally accelerating universe seems not consistent with string/M-Theory, because it is endowed with a cosmological event horizon that prevents the construction of a conventional S-matrix describing particle interaction $^6$. Other problems with an asymptotical de Sitter universe in the future were explored in $^7$.

In view of all the above, dramatically different models have been proposed, including quintessence $^8$, DGP branes $^9$, and the $f(R)$ models $^{10}$. For details, see $^{11}$ and references therein. It is fair to say that so far no convincing model has been constructed.

In this Letter, we study the cosmological constant problem and the late transient acceleration of the universe in the framework of the Horava-Witten heterotic M-Theory on $S^1/Z_2$ $^{12}$. In particular, using the Arkani-Hamed-Dimopoulos-Dvali (ADD) mechanism of large extra dimensions $^{13}$, we show that the effective $\Lambda$ on each of the two branes can be easily lowered to its current observational value. The domination of this term is only temporarily. Due to the interaction of the bulk and the brane, the universe will be in its decelerating expansion phase again in the future, whereby all problems connected with a far future de Sitter universe are resolved.

Before showing our above claims, we note that recently some attempts were made to derive a late time accelerating universe from string/M-Theory. In particular, Townsend and Wohlfarth $^{14}$ invoked a time-dependent compactification of pure gravity in higher dimensions with hyperbolic internal space to circumvent Gibbons’ non-go theorem $^{15}$. Their exact solution exhibits a short period of acceleration. The solution is the zero-flux limit of spacelike branes $^{16}$. If non-zero flux or forms are turned on, a transient acceleration exists for both compact internal hyperbolic and flat spaces $^{17}$. Other accelerating solutions by compactifying more complicated time-dependent internal spaces can be found in $^{18}$. In addition, in the string landscape $^{19}$, it is expected there are many different vacua with different local cosmological constants $^{20}$. Using the anthropic principle, one may select the low energy vacuum in which we can exist. However, many theorists still hope to explain the problem without invoking the existence of ourselves in the universe.

In addition, in 4D spacetime there exists Weinberg’s no-go theorem for the adjustment of the CC $^3$. However, in higher dimensional spacetimes, the 4D vacuum energy on the brane does not necessarily give rise to an effective 4D CC. Instead, it may only curve the bulk, while leaving the brane still flat $^{21}$, whereby Weinberg’s no-go theorem is evaded.
II. MODEL IN THE HORAVA-WITTEN HETEROTIC M-THEORY

The 11D spacetime of the Horava-Witten M-Theory is described by the metric \[ ds_{11}^2 = V^{-2/3} \gamma_{ab} dx^a dx^b - V^{1/3} \Omega_{nm} dx^n dx^m, \]
where \( ds_{CY,6}^2 \equiv \Omega_{nm} dx^n dx^m \) denotes the Calabi-Yau 3-fold, and \( V \) the Calabi-Yau volume modulus that measures the deformation of the Calabi-Yau space, and depends only on \( x^a \), where \( a = 0, 1, \ldots, 4 \). Then, the 5D effective action of the Horava-Witten theory is given by \[ S_5 = \frac{1}{2 \kappa_5^2} \int_{M_5} \sqrt{\gamma} \left( R[\gamma] - \frac{1}{2} (\nabla \phi)^2 + 6 \alpha^2 e^{-2\phi} \right) + \sum_{i=1}^2 \epsilon_i \frac{6 \alpha}{\kappa_5^2} \int_{M_4^{(i)}} \sqrt{-g^{(i)}} e^{-\phi}, \]
with \( \epsilon_1 = -\epsilon_2 = 1 \), and
\[ \phi \equiv \ln(V), \quad \kappa_5^2 \equiv \frac{\kappa_{11}^2}{v_{CY,6}}, \]
with \( v_{CY,6} \) being the volume of the Calabi-Yau space,
\[ v_{CY,6} \equiv \int_X \sqrt{\Omega}. \]
The constant \( \alpha \) is related to the internal four-form that has to be included in the dimensional reduction. This four-form results from the source terms in the 11D Bianchi identity, which are usually non-zero. \( g^{(i)} \)’s are the reduced metrics on the two boundaries \( M_4^{(i)} \) (\( i = 1, 2 \)). It should be noted that in general the dimensional reduction of the graviton and the four-form flux generates a large number of fields. However, it is consistent to set all the fields zero except for the 5D graviton and the volume modulus. This setup implies that all components of the four-form now point in the Calabi-Yau directions. In addition, it can be shown that the above action is indeed the bosonic sector of a minimal \( N = 1 \) gauged supergravity theory in 5D spacetimes coupled to chiral boundary theories.

To study cosmological models in the above setup, we add matter fields on each of the two branes,
\[ S_{m}^{(i)} = - \int_{M_4^{(i)}} \sqrt{-g^{(i)}} \mathcal{L}_m^{(i)} (\phi, \chi), \]
with respect to \( \gamma_{ab} \) yields the field equations,
\[ (5) G_{ab} = \kappa_5^{2(5)} T_{ab}^\phi + \kappa_5^2 \sum_{i=1}^2 T_{\mu\nu}^{(i)} e_\alpha^{(i)} e_\beta^{(i)} \sqrt{-g^{(i)}} \gamma \delta (\Phi_i), \]
where \( T_{ab}^\phi \) and \( S_{\mu\nu}^{(i)} \)’s are the energy-momentum tensors of the bulk and branes, respectively, and are given by
\[ \kappa_5^{2(5)} T_{ab}^\phi = \frac{1}{2} \left( \nabla_a \phi \right) \left( \nabla_b \phi \right), \]
\[ T_{\mu\nu}^{(i)} = \frac{6 \alpha \epsilon_i}{\kappa_5^2} e^{-\phi} g^{(i)}_{\mu\nu} + S_{\mu\nu}^{(i)}, \]
\[ S_{\mu\nu}^{(i)} = 2 \frac{\delta L_m^{(i)}}{\delta g^{(i)}_{\mu\nu}} - g^{(i)}_{\mu\nu} \mathcal{L}_m^{(i)}, \]
where \( \xi^\mu (\mu = 0, 1, 2, 3) \) are the intrinsic coordinates on the orbifold branes. \( \delta (\Phi_i) \) denotes the Dirac delta function, and the two orbifold branes are located on the hypersurfaces,
\[ \Phi_i (x^a) = 0, \quad (i = 1, 2). \]
It is interesting to note that the contribution of the modulus field to the branes acts as a varying cosmological constant, as can be seen clearly from Eq. (7).

Variation of the total action (6) with respect to \( \phi \), on the other hand, yields the generalized Klein-Gordon equation,
\[ \Box \phi = 12 \alpha^2 e^{-2\phi} + \sum_{i=1}^2 \left( 12 \alpha \epsilon_i e^{-\phi} + \sigma_{\phi}^{(i)} \right) \times \sqrt{-g^{(i)}} \frac{\delta (\Phi_i)}{\gamma}, \]
where \( \Box \equiv \gamma^{ab} \nabla_a \nabla_b \), and
\[ \sigma_{\phi}^{(i)} = 2 \kappa_5^2 \frac{\delta L_m^{(i)}}{\delta \phi} \frac{\delta e_m^{(i)}}{\delta \phi}. \]

A. Spacetime in the Bulk

Then, it can be shown that the 5D field equations in between the two orbifold branes admit the solution,
\[ ds_5^2 = \gamma_{ab} dx^a dx^b = dt^2 - \left( \frac{6}{5} \right) t^2 \left( dy^2 + \sinh^2 y d\Omega_3^2 \right), \]
where $d\Omega^2_3$ is the metric on the unit 3-sphere. The corresponding volume modulus is given by

$$\phi = \ln (2\alpha t).$$

It should be noted that without the introduction of the matter fields \([5]\), this solution does not satisfy the reduced field equations on the two branes \([24]\). From the expression of $ds_5^2$ we can see that the 5D spacetime is singular at $t = 0$, which divides the whole manifold into two disconnected branches, $t > 0$ and $t < 0$. The branch $t < 0$ represents a collapsing spacetime starting from $t = -\infty$, while the one $t > 0$ represents an expanding universe starting from a big bang singularity at $t = 0$. Since in this Letter we are mainly interested in cosmological model, from now on we shall work only with the branch $t > 0$. Lifting the above solution to 11 dimensions, from Eqs. (3), (14) and (15) we find that the metric \([1]\) can be cast in the form,

$$ds_{11}^2 = \frac{1}{(2\alpha t)^{2/3}} \left( dt^2 - \frac{8}{15} t^2 (dy^2 + \sin^2 y d\Omega^2_3) \right)$$

$$- (2\alpha)^{1/3} \left( \frac{2t}{3} \right)^{1/2} ds_{CY,6},$$

where $t = (2t/3)^{3/2}$. Clearly, the 11D spacetime is also singular at $t = 0$, where the length of each of the 10 spatial dimensions shrinks to zero. Such a singularity may be removed using the ideas from the resolution of curvature singularities in Loop Quantum Gravity \([25]\). However, since in this Letter we are mainly interested in the evolution of the universe in the late times, we shall not consider this possibility here.

### B. The $S^3/Z_2$ Compactification

To write down the field equations on the branes, one can first express the delta function parts of \((5)G_{ab}\) in terms of the discontinuities of the first derivatives of the metric coefficients, and then equal the delta function parts of the two sides of Eq. (17), as shown systematically in \([26]\). The other way is to use the Gauss-Codacci equations to write the 4-dimensional Einstein tensor as \([27]\),

$$(^{(4)}G_{\mu\nu} = \frac{2}{3} (^{(5)}G_{ab} e^a_{(\mu)} e^b_{(\nu)} + (5)E_{\mu\nu}$$

$$- \frac{2}{3} \left( (^{(5)}G_{ab} n^a n^b + \frac{1}{4} (^{(5)}G) g_{\mu\nu}$$

$$+ (K_{\mu\lambda} K_{\nu}^\lambda - KK_{\mu\nu})$$

$$- \frac{1}{2} g_{\mu\nu} (K_{\alpha\beta} K^{\alpha\beta} - K^2) \right),$$

where $(^5)E_{\mu\nu}$ is the projection of the 5D Weyl tensor of the bulk onto the brane, defined as $(^5)E_{\mu\nu} = (^{(5)}C_{abcd} n^a e^b_{(\mu)} n^b e^d_{(\nu)})$, with $n^a$ being the normal vector to the brane. The extrinsic curvature $K_{\mu\nu}$ is defined as $K_{\mu\nu} \equiv e^a_{(\mu)} e^b_{(\nu)} \nabla_n n^b$, and $(^5)G \equiv (^{(5)}G_{ab} \gamma^{ab})$.

Assuming that the two branes are located on the surfaces, $t = t_i(\tau_i)$ and $y = y_i(\tau_i)$, we find that the normal vectors to the two branes are given by $n^{(i)}_{\lambda} = e_i \sqrt{6/5 t_i} \left(-y_i \delta^t_{\lambda} + t_i \delta^{y}_{\lambda}\right)$, where $e_i = \pm 1$, and $\tau_i$'s denote the proper times of the branes, defined by $d\tau_i = \sqrt{1 - \frac{\delta}{2} y^2 (y_i/\sqrt{t_i})^2 dt_i}$. An overdot denotes the ordinary differentiation with respect to $\tau_i$. When $e_i = 1$, the normal vector points in the $y$-increasing direction, and when $e_i = -1$ it points in the $y$-decreasing direction. The reduced metrics on the two branes take the form

$$ds_{11}^{(i)} = g_{\mu\nu}^{(i)} ds_{(i)} = dt^2 - a^2(\tau_i) d\Omega^2_3,$$

where $a_i = \sqrt{6/5 t_i} \sinh(y_i)$. Assume that on each of the two branes there is a perfect fluid, $S_{\mu\nu}^{(i)} = \tau_{\mu\nu}^{(i)} + \lambda^{(i)} g_{\mu\nu}$, where $\lambda^{(i)}$ is the cosmological constant on the ith brane, and $\tau_{\mu\nu}^{(i)} = (\rho^{(i)} + p^{(i)}) u^{(i)}_{\mu} u^{(i)}_{\nu} - p^{(i)} g_{\mu\nu}$, with $u^{(i)}_{\mu} = \delta^{\mu}_{\tau}$. Then, from the Lanzos equations \([28]\),

$$[K_{\mu\nu}] - g_{\mu\nu} [K] = -\kappa_5^2 T_{\mu\nu},$$

where $[K_{\mu\nu}] = K_{\mu\nu} - K_{\nu\mu}$, $[K] \equiv g^{\mu\nu} [K_{\mu\nu}]$, we obtain \([29]\),

$$H^2 + \frac{1}{a^2} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} + \frac{k_3}{36} \rho^2 + \frac{\sinh^2 y}{5a^2}$$

$$+ 2e_i \left( \frac{\pi G}{5\rho_\Lambda} \right)^{1/2} \left( \rho + 2\rho_\Lambda \sinh y \sqrt{a} \right),$$

\(\rho_\Lambda \equiv \Lambda/8\pi G, H \equiv \dot{a}/a, \) and $G$ and $\Lambda$ are, respectively, the 4D Newtonian and effective cosmological constant, given by

$$8\pi G = \frac{1}{6} \kappa_5^4 \Lambda, \quad \Lambda \equiv \frac{1}{12} \left( \frac{48\pi G}{\kappa_5^2} \right)^2.$$

In writing the above expressions we had used the $Z_2$ symmetry, $K_{\mu\nu} = -K_{\nu\mu}$. For the sake of simplicity, we also dropped the indices "i", without causing any confusions. It is interesting to note that $(^4)G_{\mu\nu}$ given by Eq. (17) depends on $K_{\mu\nu}$ quadratically, so that it does not depend on the signs of $e_i$, nor on the choice whether $K_{\mu\nu}^+$ or $K_{\mu\nu}^-$ is going to be used.

The third term in the right-hand side of Eq. (20) represents the brane corrections, which is important in the
early epoch of the evolution of the universe. The fourth term is the projection of the energy-momentum tensor of the scalar field onto the brane, while the last term represents the interaction between the brane and the bulk. It can be very important in the late evolution of the universe, as to be shown below. The first two terms are those that also appear in Einstein’s theory of gravity, although their physical origins are completely different. In particular, in Einstein’s theory \( \lambda \) is an arbitrary constant, while here it is completely fixed by the Newtonian constant \( G \). In other words, it is now a fundamental constant and plays the same role as \( G \) does. On the other hand, assuming that the typical size of the Calabi-Yau space is \( R \), we find \( v_{CY,6} \sim R^6 \). Then, from Eqs. (3), (22) and the relation \( \kappa_D^2 = M_D^{-2-D} \), we obtain

\[
\rho_\Lambda = 3 \left( \frac{R}{l_{pl}} \right)^{12} \left( \frac{M_{11}}{M_{pl}} \right)^{18} M_{pl}^4 ,
\]

where \( M_{pl} \sim 10^{19} \text{ GeV} \) and \( l_{pl} \sim 10^{-35} \text{ m} \) denote, respectively, the Planck mass and length. From the above expression we can see that, if the theory is in the TeV scale \( [30] \), to have \( \rho_\Lambda \simeq \rho_\text{ob} \), the typical size of the Calabi-Yau space \( R \) needs to be only at the scale \( 10^{-22} \text{ m} \); which is by far below the current observational constraints \( [31] \). If \( M_{11} \sim 100 \text{ TeV} \) it needs to be at the scale \( 10^{-24} \text{ m} \); and if \( M_{11} \sim 10^{12} \text{ GeV} \), it is at the Planck scale. Therefore, the Horava-Witten theory on \( S^1/Z_2 \) provides a very viable mechanism to get \( \rho_\Lambda \) down to its current observational value. Hence, the ADD mechanism that was initially designed to solve the hierarchy problem \( [13] \) also solves the cosmological constant problem in the Horava-Witten M-theory on \( S^1/Z_2 \).

Note that to close the system of Eqs. (20) and (21), two additional equations are needed. One of them can be the equation of the state of the perfect fluid, and the other is the equation that describes the motion of the branes, given by

\[
\dot{y} = \frac{5}{5 \coth^2 y - 6} \left( H \coth y \right) + \epsilon \left( \frac{6}{5} \left( \frac{H^2 + \frac{1}{a^2} - \frac{\sinh^2 y}{5a^2}}{a} \right) \right)^{1/2} ,
\]

where \( \epsilon = \pm 1 \). From these equations it can be seen that by properly choosing the initial positions of the branes \( y_i(0) \), the last two terms in the right-hand side of Eq. (20) are neglected until very recently. As a result, the evolution of the universe follows almost the same trajectory as that described by the standard \( \Lambda \)CDM model, in which the \( \Lambda \) term dominates currently. However, the interaction between the bulk and the branes can be very important in the future, so that it leads to a late transient acceleration of the universe.

To show that this is exactly the case, we first fit our model with observational data, and then use these best fitting data as our initial conditions to study the future evolution of the universe. Let us first parameterize the current density of each component as \( \Omega_m \sim \rho_\Lambda / \rho_{cr} \), \( \Omega_\Lambda = \rho_\Lambda / \rho_{cr} \), and \( \Omega_k = 3/(8\pi G a^2 \rho_{cr}) \), where \( \rho_{cr} = 3H_0^2/(8\pi G) \). We use the 182 gold Sn Ia data \([32]\) combined with Baryon Acoustic Oscillation (BAO) parameter from SDSS data \([33]\). We find that the results depend on the choice of \( \epsilon \) and \( \epsilon_i \), where for the positive (negative) tension brane we have \( \epsilon_i = 1 \) (\( \epsilon_i = -1 \)), as can be seen from Eq. (2). In particular, for \( \epsilon = 1 = \epsilon_i \), we find that the minimum of the function \( \chi^2 = \chi^2 = 156.20 \) and \( \Omega_m = 0.28^{+0.04}_{-0.04} \), \( \Omega_\Lambda = 0.87^{+0.05}_{-0.05} \), and \( \Omega_k = 0.16^{+0.06}_{-0.06} \). For \( \epsilon = 1 = -\epsilon_i \), we find \( \chi^2 = 156.22 \), \( \Omega_m = 0.28^{+0.04}_{-0.04} \), \( \Omega_\Lambda = 0.74^{+0.23}_{-0.05} \), and \( \Omega_k = 0.08^{+0.08}_{-0.08} \). For \( \epsilon = -1 = \epsilon_i \), we find \( \chi^2 = 156.20 \), \( \Omega_m = 0.28^{+0.04}_{-0.04} \), \( \Omega_\Lambda = 0.87^{+0.41}_{-0.04} \), and \( \Omega_k = 0.16^{+0.00}_{-0.16} \). And for \( \epsilon = -1 = -\epsilon_i \), we find \( \chi^2 = 155.77 \), \( \Omega_m = 0.16^{+0.15}_{-0.07} \), \( \Omega_\Lambda = 0.71^{+1.12}_{-0.00} \), and \( \Omega_k = 0.09^{+0.00}_{-0.08} \). Taking these best fitting data as initial conditions, the future evolution of the acceleration of the universe is shown in Fig. 1, from which we can see that for the cases where \( \epsilon = \epsilon_i = 1 \) and \( \epsilon = -\epsilon_i = 1 \), after a finite time, the universe will become decelerating again. It should be noted that in all of these four cases \( \rho_m \) always approaches to a very small value. We set up a lowest limit for it, and once it reaches that limit, it will be set to zero for the rest of its evolution.

III. MAIN RESULTS AND CONCLUDING REMARKS

In this Letter, we have studied the evolution of the universe in the the framework of the 11-dimensional Horava-Witten M-theory on \( S^1/Z_2 \). We have shown explicitly that the effective cosmological constant can be easily lowered to its current observational value using the large extra dimensions. The domination of this constant over the evolution of the universe is only currently. Due to the interaction of the bulk and the branes, the universe will be in its decelerating expansion phase again in the
future, whereby all problems connected with a far future de Sitter universe are resolved.

It should be noted that the expression of Eq. (23) is quite general, and does not depend on the specific model considered in this Letter.

It is also interesting to note that the mechanism used in this Letter is different from the so-called “self-tuning” mechanism, proposed in both 5-dimensional spacetimes [34] and in 6-dimensional spacetimes [35]. In the 5D case, it was shown that hidden fine-tunings are required [36], while in the 6D case it is still not clear whether loop corrections can be as small as required by solving the CC problem [37].

Two important problems have not been addressed in this Letter. One is the stability of radion and the other is the constraints from observations. The former has been extensively studied in 5-dimensional spacetimes [38]. The generalization of such studies to our model is straightforward, and is currently under our considerations. We are also investigating the constraints coming from the solar system tests [39].

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