Dissipative Future Universe without Big Rip

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Abstract

The present study deals with dissipative future universe without big rip in context of Eckart formalism. The generalized chaplygin gas, characterized by equation of state \( p = \frac{-A}{\rho^\alpha} \), has been considered as a model for dark energy due to its dark-energy-like evolution at late time. It is demonstrated that, if the cosmic dark energy behaves like a fluid with equation of state \( p = \omega \rho; \omega < -1 \), as well as chaplygin gas simultaneously then the big rip problem does not arises and the scale factor is found to be regular for all time.

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Recent observations like CMB anisotropy, supernova and galaxy clustering strongly indicate that our universe is spatially flat and there exists an exotic cosmic fluid called dark energy with negative pressure, which constitutes about 70 percent of the total energy of universe. The dark energy is usually described by an equation of state (EoS) parameter \( \omega \equiv \frac{p}{\rho} \), the ratio of spatially homogeneous dark-energy pressure \( p \) to its energy density \( \rho \). A value \( \omega < -\frac{1}{3} \) is required for cosmic acceleration. The simplest explanation for dark energy is cosmological constant, for which \( \omega = -1 \). The increasing evidence from observational data indicates that \( \omega \) lies in a narrow strip around \( \omega = -1 \) quite likely being less than this value \([1]-[3]\). The region where EoS parameter \( \omega < -1 \), is typically referred to as a phantom dark energy universe. The existence of the region with \( \omega < -1 \) opens up a number of fundamental questions. For instance, the entropy of such universe is negative. The dominant energy condition (DEC) for phantom fluid is violated, as a rule. The phantom dominated universe end up with a finite time future singularity called big rip or cosmic doomsday \([4],[5]\). The last property attracted much attention and brought the number of speculations upto the explicit calculation of the rest of the life-time of our universe.

Soon after Caldwell \([4]\) proposed phantom dark energy model with cosmic doomsday of future universe, cosmologists started making efforts to avoid this problem using \( \omega < -1 \) \([6]-[8]\). In the braneworld scenario, Sahni and Shtanov has obtained well-behaved expansion for the future universe without big rip problem with \( \omega < -1 \). They have shown that acceleration is a transient phenomenon in the current universe and the future universe will re-enter matter dominated decelerated phase \([9]\). It is found that general relativity (GR) based phantom model encounters “sudden future singularity” leading a divergent scale factor, energy density and pressure at finite time \( t = t_s \). Thus the classical approach to phantom model exhibits big rip problem. For future singularity model, curvature invariant becomes very strong and energy density is very high near \( t = t_s \) \([10]\). So, quantum effects should be dominated for \(|t = t_s| < \) one unit of time (Early universe) \([11]-[13]\) and it is shown that the an escape from the big rip is possible on making quantum corrections to the energy density and pressure in Friedmann equations.

In the framework of Robertson-Walker cosmology, Chaplygin gas (CG) is also considered as a good source of dark energy for having negative pressure, given as

\[
p = \frac{-A}{\rho}
\]
where \( p \) and \( \rho \) are, respectively, pressure and energy density in a co-moving reference frame, with \( \rho > 0 \); \( A \) is a positive constant.

Moreover, it is only gas having super-symmetry generalization \([14]-[16]\). Bertolami et al \([17]\) have found that generalized Chaplaincy gas (CG) is better fit for latest Supernova data. In case of CG, equation (2) is modified as

\[
p = -\frac{A}{\rho^{1+\alpha}}
\]

where \( 1 \leq \alpha < \infty \). For \( \alpha = 1 \), equation (2) corresponds to equation (1).

Other approaches have considered dissipative effects in CG models, using the framework of Elkhart theory \([18]\). Thai et al \([19]\), have investigated a viscous GCG, assuming that there is a bulk viscosity in a linear borotropic fluid and GCG. It is found that the equation of state of GCG can cross the boundary \( \omega = -1 \). Also in Ref. \([20]\), it is found that a dissipative chaplygin gas can give rise to structurally stable evaluational scenarios. It is interesting to note that the GCG itself can behave like a fluid with viscosity in the context of Eckart formalism \([18]\). Fabris et al \([21]\), have investigated an equivalence GCG and dust like fluid. Recently Cruz et al \([22]\), have studied dissipative generalized chaplygin gas as phantom dark energy and found the cosmological solutions for GCG with bulk viscosity.

The FRW metric for an homogeneous and isotropic flat universe is given by

\[
ds^2 = -dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)
\]

where \( a(t) \) is the scale factor and \( t \) represents the cosmic time.

The field equations in the presence bulk viscous stresses are

\[
a_4^2 \frac{a^2}{a^2} = H^2 = \frac{\rho}{3}
\]

\[
a_4 \frac{a_4}{a} = -\frac{1}{6} (\rho + 3\bar{p})
\]

where \( \bar{p} \) is the effective pressure given by

\[
\bar{p} = p - 3H\xi
\]

Here \( p, \xi \) are the isotropic pressure and bulk viscous coefficient respectively.

The energy conservation equation is given by

\[
\rho_4 + 3H (\rho + \bar{p}) = 0
\]

Here, and in what follows the sub indices 4 on \( a, \rho \) and elsewhere denote differentiation with respect to \( t \).

Using equations (2), (4) and (6), equation (7) leads to

\[
\rho_4 + 3a_4 \frac{a_4}{a} \left(\rho - \frac{A}{\rho^{1+\alpha}} - 3H\xi\right) = 0
\]

In order to obtain solution of equation (5), we will assume that the viscosity has a power-law dependence upon the density

\[
\xi = \xi_0 \rho^n
\]

where \( \xi_0 \) and \( n \) are constant.

On using equation (9) in equation (8), we obtain

\[
\frac{d\rho}{dt} + 3 \frac{da}{a} \frac{\left(\rho^{1+\alpha} - A\right)}{\rho^{1+\alpha}} = 3\rho^{n+1}\xi_0
\]

To solve equation (10), we use the transformation \( \rho_4 = f(\rho) = \rho^{n+1} \), accordingly equation (10) leads to

\[
\rho^{\frac{1+\alpha}{\alpha+1}}(t) = A + \left(\rho_0^{\frac{1+\alpha}{\alpha+1}} - A\right) \left[\frac{a_0}{a(t)}\right]^{\frac{1}{\alpha+1}}
\]
where \( \rho_0 = \rho(t_0) \) and \( a_0 = a(t_0) \); \( t_0 \) is the present time.

In the present model, it is assumed that the dark energy behaves like GCG, obeying equation (2) as well as fluid with equation of state

\[
p = \omega \rho
\]

with \( \omega < -1 \) simultaneously.

From equations (2) and (12), we obtain

\[
\omega(t) = -\frac{A}{\rho(t)^{\frac{1+\alpha}{1+\alpha}}}
\]

So, evolution of equation (13) at \( t = t_0 \) leads to

\[
A = -\omega_0 \rho_0^{\frac{1+\alpha}{1+\alpha}}
\]

with \( \omega_0 = \omega(t_0) \).

Using equation (14), equation (11) leads to

\[
\rho = \rho_0 \left[ -\omega_0 + (1 + \omega_0) \left( \frac{a_0}{a(t)} \right)^{\frac{3(1+\alpha)}{3(1-3\xi_0)}} \right]^{\frac{1+\alpha}{1+\alpha}}
\]

In the homogeneous model of universe, a scalar field \( \phi(t) \) with potential \( V(\phi) \) has energy density

\[
\rho_\phi = \frac{1}{2} \phi_\phi^2 + V(\phi)
\]

and pressure

\[
p_\phi = \frac{1}{2} \phi_\phi^2 - V(\phi)
\]

Equation (16) and (17) lead to

\[
\phi_\phi^2 = \rho_\phi + p_\phi
\]

Using equations (2), (12) and (14), equation (18) reduces to

\[
\phi_\phi^2 = \frac{\rho^{\frac{1+\alpha}{\alpha}} + \rho_0^{\frac{1+\alpha}{\alpha}} \omega_0}{\rho^{\frac{1+\alpha}{\alpha}}} \rho_0^{\frac{1+\alpha}{\alpha}}
\]

Equation (15) and (19) lead to

\[
\phi_\phi^2 = \frac{(1 + \omega_0) \rho_0 \left( \frac{a_0}{a(t)} \right)^{\frac{3(1+\alpha)}{3(1-3\xi_0)}}}{\left[ -\omega_0 + (1 + \omega_0) \left( \frac{a_0}{a(t)} \right)^{\frac{3(1+\alpha)}{3(1-3\xi_0)}} \right]^{\frac{1+\alpha}{1+\alpha}}}
\]

From equation (20), it is clear that for \( 1 + \omega_0 > 0 \), \( \phi_\phi^2 > 0 \), giving positive kinetic energy and for \( 1 + \omega_0 < 0 \), \( \phi_\phi^2 < 0 \), giving negative kinetic energy. \( 1 + \omega_0 > 0 \) and \( 1 + \omega_0 < 0 \) are representing the case of quintessence and phantom fluid dominated universe respectively. Similar result are obtained by Hoyle and Narlikar in C-field with negative kinetic energy for steady state theory of universe [23].

Now, from equation (21) and (14), we obtain

\[
\frac{a_4}{a} = \Omega_0 H_0^2 \left[ |\omega_0| + (1 - |\omega_0|) \left( \frac{a_0}{a(t)} \right)^{\frac{3(1+\alpha)}{3(1-3\xi_0)}} \right]^{\frac{1+\alpha}{1+\alpha}}
\]

where \( |\omega_0| = -\omega \), \( H_0 = 100\text{km/s Mpc} \), present value of the Hubble’s parameter and \( \Omega_0 = \frac{\rho_0}{\rho_{cr,0}} \) with \( \rho_{cr,0} = \frac{3H_0^2}{8\pi G} \).

Equation (21) may be written as

\[
\frac{a_4}{a} = \sqrt{\Omega_0 H_0 |\omega_0|^{\frac{\alpha}{1+\alpha}}} \left[ 1 + (1 - |\omega_0|) \left( \frac{a_0}{a(t)} \right)^{\frac{3(1+\alpha)}{3(1-3\xi_0)}} \right]^{\frac{1+\alpha}{1+\alpha}}
\]
Neglecting the higher powers of \( \frac{(1 - |\omega_0|)}{|\omega_0|} \left( \frac{a_0}{a(t)} \right)^{\frac{3(1+\alpha)}{\alpha(1-3\xi_0)}} \), equation (22) leads to
\[
\frac{a_4}{a} = \sqrt{\Omega_0 H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}}} \left[ 1 + \frac{\alpha(1 - |\omega_0|)}{2(1 + \alpha)|\omega_0|} \left( \frac{a_0}{a(t)} \right)^{\frac{3(1+\alpha)}{\alpha(1-3\xi_0)}} \right]
\]  
(23)

Integrating equation (23), we obtain
\[
a(t) = \left( \frac{a_0}{2(1 + \alpha)|\omega_0|} \right)^{\frac{a(1-3\xi_0)}{2(1+\alpha)}} \times \left[ (\alpha + 2(1 + \alpha)|\omega_0|) e^{6H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}} \sqrt{\Omega_0(t-t_0)}} - \alpha(1 - |\omega_0|) \right]^{\frac{a(1-3\xi_0)}{2(1+\alpha)}}
\]  
(24)

From equation (24), it is clear that as \( t \to \infty \), \( a(t) \to \infty \) which is supported by recent observation of Supernova Ia [24, 25] and WMAP [26, 27]. Therefore the present model is free from finite time future singularity.

Now the horizon distance is obtained as
\[
d_H(t) = \frac{3(1 + \alpha)a(t)}{\alpha(1 - 3\xi_0)a_0} \left( \frac{2(1 + \alpha)|\omega_0|}{\alpha + (\alpha + 2)|\omega_0|} \right)^{\frac{a(1-3\xi_0)}{\alpha(1-3\xi_0)}} \times \left[ e^{6H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}} \sqrt{\Omega_0(t-t_0)}} - \alpha(1 - |\omega_0|) \right]^{\frac{a(1-3\xi_0)}{2(1+\alpha)}}
\]  
(25)

**Fig. 1** depicts the variation of scale factor \( a \) and horizon distance \( d_H \) versus cosmic time, as a representative case with appropriate choice of constants and other physical parameters. From equation (24) and (25) it is clear that \( d_H(t) > a(t) \) i.e. horizon grows more rapidly than scale factor which is clearly shown in **Fig. 1**.

In this case, the Hubble distance is given by
\[
H^{-1} = \frac{1}{\sqrt{\Omega_0 H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}}}} \left[ 1 - \frac{\alpha(1 - |\omega_0|)}{2(1 + \alpha)|\omega_0|} \left( \frac{a_0}{a(t)} \right)^{\frac{3(1+\alpha)}{\alpha(1-3\xi_0)}} \right]
\]  
(26)
Equation (26) is showing the growth of Hubble distance ($H^{-1}$) with time such that $H^{-1} \to \frac{1}{H_0 \sqrt{\Omega_0 |\omega_0|^{2(1+\alpha)}}} \neq 0$ as $t \to \infty$. This behaviour of $H^{-1}$ is clearly depicted in Fig. 2. Thus in present case, the galaxies will not disappear when $t \to \infty$, avoiding big rip singularity. Therefore, one can conclude that if phantom fluid behaves like GCG and fluid with $p = \omega \rho$ simultaneously then the future accelerated expansion of universe will free from catastrophic situation like big rip. Equation (15) may be written as

$$\rho = \rho_0 \left[ |\omega_0| + (1 - |\omega_0|) \left( \frac{a_0}{a(t)} \right)^{\frac{3(1+\alpha)}{a(1-3\alpha)}} \right] \rho_\infty$$

(27)

From equation (27), it is clear that as $t \to \infty$, $\rho \to \rho_0 |\omega_0|^{\frac{1}{1-3\alpha}} > \rho_0$ (since $t \to \infty$, $a(t) \to \infty$). Thus one can conclude that energy density increases with time, contrary to other phantom models having future singularity at $t = t_s$ [10] [11]. Based on Ia Supernova data, Singh et al [28] have estimated $\omega_0$ for model in the range $-2.4 < \omega_0 < -1.74$ up to 95 percent confidence level. Taking this estimate as an example, with $\alpha = 1$, $\rho_\infty$ is found in the range $1.31 \rho_0 < \rho_\infty < 1.54 \rho_0$ and with $\alpha = 2$, $\rho_\infty$ is found in the range $1.44 \rho_0 < \rho_\infty < 1.78 \rho_0$ and so on. This does not yields much increase in energy density as $t \to \infty$ but if the future experiments supports large value of $|\omega_0|$ then $\rho_\infty$ will be high.

It is interesting to see here that present model is derived by Bulk viscosity in the context of Eckart formalism [18]. We see that big rip problem does not arises in the present model. In Refs [10]–[13], for models with future singularity escape from big rip, is demonstrated using quantum corrections in the field equations near $t = t_s$. However the present study deals phantom cosmology with accelerated expansion without catastrophic situations using classical approach. It is also seen that the model of future universe presented by Srivastava [29] is a particular case of the model presented in this letter.

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