Internal heating and thermal emission from old neutron stars

Constraints on dense-matter and gravitational physics

1 Introduction

Neutron star matter is composed of degenerate fermions of various kinds: neutrons \( n \), protons \( p \), electrons \( e \), probably muons \( \mu \) and possibly other, more exotic particles. (We refer to electrons and muons collectively as leptons, \( l \).) Neutrons are stabilized by the presence of other, stable fermions that block (through the Pauli exclusion principle) most of the final states of the beta decay reaction $n \rightarrow p + l + \bar{\nu}$. The large chemical potentials $\mu_i (\approx$ Fermi energies) for all particle species $i$ also make inverse beta decays, $p + l \rightarrow n + \nu$, possible. The neutrinos ($\nu$) and antineutrinos ($\bar{\nu}$) leave the star without further interactions, contributing to its cooling (e.g., Shapiro & Teukolsky 1983; Yakovlev & Pethick 2004). The two reactions mentioned tend to drive the matter into a chemical equilibrium state, defined by $\eta_{npl} = \mu_n - \mu_p - \mu_l = 0$.

If a matter element is in some way driven away from chemical equilibrium ($\eta_{npl} \neq 0$), free energy is stored, which is released by an excess rate of one reaction over the other. This energy is partly lost to neutrinos and antineutrinos (undetectable at present), and partly used to heat the matter. The heat is eventually lost as thermal (ultraviolet) photons emitted from the stellar surface.

The chemical imbalance can be caused by a change in the density of the stellar matter. This can in turn be produced in different ways. The first to be considered (by Finzi 1963, Finzi & Wolf 1968) was stellar pulsation; however, so far no clear evidence for this process has been seen. Gravitational collapse (Haensel 1992, Gourgoulhon & Haensel 1993) and mass accretion (Haensel 1992) are other possible mechanisms, but in these contexts the non-equilibrium heating is probably overwhelmed by the energy released through other channels.

Here we review our work on neutron star heating through beta processes in two other contexts, which we consider to be the most promising in revealing information about the physics of dense matter and gravitation. One is rotochemical heating (Reisenegger 1995).
thermal balance equation, case as well. The evolution of the internal temperature, tendency and chemical imbalances for the case of rotochemical heating is described in § 2 of FR05. Here, we just outline the fundamental equations and the modifications required in order to treat the gravitochemical case as well. The evolution of the internal temperature, $T$, taken to be uniform inside the star, is given by the thermal balance equation,

$$
\frac{dT}{dt} = \frac{1}{C(T)} \left[ L_H(\eta_{npl}, T) - L_{\nu}(\eta_{npl}, T) - L_\gamma(T) \right],
$$

(1)

where $C$ is the total heat capacity of the star, $L_H$ is the total power released by the heating mechanism, $L_{\nu}$ the total power emitted as neutrinos, and $L_\gamma$ the power released as thermal photons. Here and in what follows (including all figures), all temperatures, chemical imbalances, stellar radii, and luminosities are “redshifted” to the reference frame of a distant observer at rest with respect to the star.

The evolution of the chemical imbalances is given by

$$
\dot{\eta}_{npl} = -[A_D,l(\eta_{npe}, T) + A_M,l(\eta_{npe}, T)] - [B_D,l(\eta_{npM}, T) + B_M,l(\eta_{npM}, T)] - R_{npl}\Omega + C_{npl}G.
$$

(2)

The functions $A$ and $B$ quantify the effect of reactions towards restoring chemical equilibrium, and thus have the same sign of $\eta_{npl}$ (FR05). The subscripts $D$ and $M$ refer to direct Urca reactions,

$$
n \rightarrow p + l + \overline{\nu},
$$

$$
p + l \rightarrow n + \nu,
$$

(3)

which are possibly forbidden by momentum conservation, and modified Urca,

$$
n + N \rightarrow p + N + l + \overline{\nu},
$$

$$
p + l + N \rightarrow n + N + \nu,
$$

(4)

where an additional nucleon $N$ must participate in order to conserve momentum (e. g., Shapiro & Teukolsky 1983). The scalars $R_{npl}$ and $C_{npl}$ quantify the departure from equilibrium due to the changes in the centrifugal force ($\propto \Omega^2$) and the gravitational constant ($G$), being positive and depending on the stellar model and equation of state (FR05, Reisenegger et al. 2006a, JRF06).

Figure 1 shows the solution of the coupled differential equations 1 and 2 for the evolution of a classical pulsar with a moderate magnetic field and different assumed initial rotation periods under pure rotochemical heating ($\Omega\dot{\Omega} < 0, \dot{G} = 0$). It can be seen that, for very fast initial rotation, the pulsar can be kept warm beyond the standard cooling time of $\sim 10^7$ yr, at a level that is close to current observational constraints.

The case of a “millisecond pulsar” (a neutron star with fast rotation and a weak magnetic dipole field) is illustrated in Figure 2. It first cools down from its high birth temperature, while the chemical potential imbalances $\eta_{npl}$ slowly increase due to the decreasing rotation rate, until rotochemical heating increases the temperature again, and the reactions stop the rise of the chemical potential imbalances.

### 3 Stationary state

If the relevant forcing ($\Omega\dot{\Omega}$ or $\dot{G}$) changes slowly with time, the star eventually arrives at a stationary state, where the rate at which the equilibrium concentrations...
are modified by this forcing is the same as that at which the reactions drive the system toward the new equilibrium configuration, with heating and cooling balancing each other (Reisenegger 1995). The evolution to this state for pure rotochemical heating is illustrated in Figure 2. Figures 3 and 4 show that the state reached is independent on the assumed initial conditions.

The properties of this stationary state can be obtained by the simultaneous solution of equations (1) and (2) with $\dot{T} = \dot{n}_{\text{ppl}} = 0$. The existence of the stationary state makes it unnecessary to model the full evolution of the temperature and chemical imbalances of the star in order to calculate the final temperature, since the stationary state is independent of the initial conditions (see FR05 for a detailed analysis of the rotochemical heating case). For given values of $\Omega \dot{\Omega}$ and $\dot{G}$, it is thus possible to calculate the temperature of an old pulsar that has reached the stationary state, without knowing its exact age.

When only modified Urca reactions operate, it is possible to solve analytically for the stationary values of the photon luminosity $L_{\gamma}^{st}$ and chemical imbalances $n_{\text{ppl}}^{st}$, as a function of stellar model and current value of $\Omega \dot{\Omega}$ and $\dot{G}$. The reason for this is that the longer equilibration timescale given by the slower modified Urca reaction timescales yields stationary chemical imbalances satisfying $n_{\text{ppl}} \gg kT$. In this limit, the term $L_H - L_{\nu}$ in the thermal balance equation can be written as $K_{Lc} n_{\gamma}^{\text{type}} + K_{L\nu} n_{\text{ppl}}^{\text{type}}$, where $K_{Lc}$ and $K_{L\nu}$ are positive constants that depend only on stellar mass and equation of state (FR05; JRF06). For typical equations of state, the photon luminosity in the stationary state is

$$L_{\gamma}^{st} \approx 10^{30-31} \left( \frac{\dot{P}}{\dot{P}_{\text{ms}}} \right)^{10^{-20}} + \frac{\dot{G}/G}{3 \times 10^{-11}} \text{ yr}^{-1} \text{ erg s}^{-1}, \quad (5)$$

where $\dot{P}_{\text{ms}}$ is the rotation period in milliseconds, and $P_{\text{ms}}$ is its time derivative in units of $10^{-20}$ (dimensionless), and the effective surface temperature of the star in the stationary state is

$$T_{s}^{st} \approx (2 - 3) \times 10^5 \left( \frac{\dot{P}}{\dot{P}_{\text{ms}}} \right)^{10^{-20}} + \frac{\dot{G}/G}{3 \times 10^{-11}} \text{ yr}^{-1} \text{ K}. \quad (6)$$

Fig. 2 (taken from FR05) Evolution of the internal temperature and chemical imbalances under the rotochemical heating effect for a $1.4M_\odot$ star calculated with the $\Lambda 18 + \delta v + \text{UIX}^*$ equation of state (Akmal et al 1998), with initial temperature $T = 10^9$ K, null initial chemical imbalances, and magnetic dipole spin-down with field strength $B = 10^8$ G and initial period $P_0 = 1$ ms.

Fig. 3 (from FR05) Evolution of the internal temperature under rotochemical heating for different initial temperatures. We set $n_{\text{ppl}} = \dot{n}_{\text{ppl}} = 0$. The short-dashed line is the quasi-equilibrium solution, obtained by solving $\dot{T} = 0$ and $n_{\text{ppl}} = \dot{n}_{\text{ppl}} = 0$. The stellar model and spin-down parameters are the same as in Figure 2.

Fig. 4 (from FR05) Evolution of the internal temperature under rotochemical heating for different initial chemical imbalances $n_{\text{ppl}} = \dot{n}_{\text{ppl}} = 0$ and the same initial temperature $T = 10^9$ K at $t = 0$. The line styles, the stellar model, and the spin-down parameters are the same as in Figure 9.
Finally, the timescale for the system to reach the stationary state is

$$\tau_{st} \simeq 2 \times 10^7 \left( \frac{P - 20}{P_{3\text{ms}}} \right)^3 \left( \frac{\dot{G}/G}{3 \times 10^{-11} \text{ yr}^{-1}} \right)^{-6/7} \text{ yr.} \quad (7)$$

4 Comparison to observations

In order to verify this model and constrain the value of $|\dot{G}/G|$, we need a neutron star that (1) has a measured surface temperature (or at least a good enough upper limit on the latter), and (2) is confidently known to be older than the timescale to reach the stationary state. So far, the only object satisfying both conditions is the millisecond pulsar closest to the Solar System, PSR J0437-4715 (hereafter J0437), whose surface temperature was inferred from an HST-STIS ultraviolet observation by Kargaltsev et al. (2004). Its spin-down age, $\tau_{\text{sd}} \approx 5 \times 10^9$ yr (e.g., van Straten et al. 2001), and the cooling age of its white dwarf companion, $\tau_{\text{WD}} \approx (2.5 - 5.3) \times 10^9$ yr (Hansen & Phinney 1998), are much longer than the time required to reach the steady state for both rotochemical and gravitochemical heating, in the latter case under the condition that $|\dot{G}/G| \geq 10^{-13}$ yr$^{-1}$.

Consequently, we consider stellar models constructed from different equations of state, with masses satisfying the constraint obtained for J0437 by van Straten et al. (2001), $M_{\text{psr}} = 1.58 \pm 0.18 M_\odot$, and calculate the stationary temperature for each. Figure 5 compares the predictions for the case of pure rotochemical heating with the measured spin-down parameters of J0437 (for various equations of state and neutron star masses) to the temperature inferred from the observation of Kargaltsev et al. (2004).

In Figure 5, we compare the same observational constraints on the temperature of this pulsar to the theoretical predictions for pure gravitochemical heating, assuming $|\dot{G}/G| = 2 \times 10^{-10}$ yr$^{-1}$. As can be seen, this value is such that the stationary temperatures of all stellar models lie just above the 90% confidence contour, and therefore represents a rather safe and general upper limit.

When the stellar mass becomes large enough for the central pressure to cross the threshold for direct Urca reactions, $T_\star$ drops abruptly, due to the faster relaxation towards chemical equilibrium. This occurs in two steps, as electron and muon direct Urca processes have different threshold densities (see, e.g., FR05). Conventional neutron star cooling models reproduce observed temperatures better when only modified Urca reactions are considered (e.g., Yakovlev & Pethick 2004, Page 2006). Restricting our sample to the equations of state that allow only modified Urca reactions in the mass range considered here, namely A18 + $\delta e$, A18 + $\delta e$ + UX, BPAL21, and BPAL31, we obtain a more restrictive upper limit on $|\dot{G}|$, as shown in Figure 7, yielding $|\dot{G}/G| < 4 \times 10^{-12}$ yr$^{-1}$.

5 Discussion and Conclusions

5.1 Rotochemical heating

Using the equations of state that allow only modified Urca reactions within the allowed mass for PSR J0437-
4715, rotochemical heating predicts an effective temperature in the narrow range $T_{\text{eq}} = (6.9 - 7.9) \times 10^4$ K, about 20\% lower than the blackbody fit of Kargaltsev et al. (2004). There are three possible reasons why the prediction does not quite match the observation:

1. We are not taking superfluidity into account. This would reduce Urca reaction rates, lengthening the equilibration timescale and raising the stationary-state temperature (Reisenegger 1997).

2. We are neglecting other heating mechanisms (some of them directly related to superfluidity), which could further raise the temperature at any stage in the thermal evolution. Nonetheless, in millisecond pulsars, all proposed mechanisms appear to be less important than rotochemical heating (Schaab et al. 1999, Kargaltsev et al. 2004).

3. The thermal spectrum could deviate from a blackbody, as for the isolated neutron star RX J1856-3754, which has a well-determined blackbody X-ray spectrum that underpredicts the optical flux (Walter & Matthews 1997), indicating a more complex spectral shape of its thermal emission.

Kargaltsev et al. (2004) stress that PSR J0437-4715 has a higher surface temperature than the upper limit for the younger, “classical” pulsar J0108-1431, $T_s < 8.8 \times 10^8$ K, inferred from the optical non-detection by Mignani et al. (2003) and shown in our Figure 5. In the rotochemical heating model, these two pulsars are in very different regimes: J0437 is in the stationary state in which its temperature can be predicted from its spin-down parameters, whereas J0108 has a 680 times smaller spin-down power ($\propto \Omega^2$), and will therefore not reach a detectable stationary state. Its equilibration timescale, according to equation (7), is $2 \times 10^{11}$ yr, longer than the age of the Universe and certainly much longer than the spin-down age of the pulsar. Thus, its heat content (if any) is due to its earlier, faster rotation, which may have built up a significant chemical imbalance that is currently being decreased by ongoing reactions in its interior (see Fig. 1). Depending on its initial rotation period, its surface temperature may be substantially smaller than both J0437’s observed temperature and its own current upper limit.

5.2 Gravitochemical heating

Table 1 lists some of the many experiments performed so far to test the constancy of $G$ (see Uzan 2003 and Will 2006 for recent reviews). The second column contains the upper limits on its time variation, most usefully expressed as $|\dot{G}/G|$, and the third is a rough time scale over which each experiment is averaging this variation. Based on the latter, the experiments can be separated into three classes. The first two experiments on the list measure the variation of $G$ from the early Universe to the present time, and the constraint on the present-day value of $|\dot{G}/G|$ is based on assuming a time dependence $G(t) \propto t^{-\alpha}$, where $t$ is the time since the Big Bang, and $\alpha$ is a constant constrained by these experiments. The next four are sensitive to variations over long timescales, $10^9 - 10^{10}$ yr, but without reaching into the very early Universe. The last four experiments measure the change of $G$ directly over short, “human” timescales of years or few decades. Even though results from the first category are nominally the most restrictive on a long-term variation of $G$, they depend crucially on the assumed form of the variation of $G$ near the Big Bang. Thus, it is still useful to consider measurements of the second and third categories, which could directly detect variations of $G$ in more recent times.

The new method advocated here, namely gravitochemical heating of neutron stars, falls closest to the second category, as its timescale is much longer than human, but does not reach into the early Universe. However, it probes somewhat shorter timescales than the other methods in this category. In the most general case, when Urca reactions are allowed to operate, we obtain an upper limit $|\dot{G}/G| < 2 \times 10^{-10}$ yr$^{-1}$. Restricting the sample of equations of state to those that allow only modified Urca reactions, we obtain a much more restrictive upper limit, $|\dot{G}/G| < 4 \times 10^{-12}$ yr$^{-1}$ on a time scale $\sim 10^8$ yr (the time for the neutron star to reach its quasi-stationary state), competitive with constraints obtained from the other methods probing similar timescales. However, since the composition of matter above nuclear densities is uncertain and millisecond pulsars are generally expected to be more massive than classical pulsars, we cannot rule out the result for the direct Urca regime.

Further progress in our knowledge of neutron star matter will allow this method to become more effective at constraining variations in $G$. The method can also be improved with an increased sample of objects with measured thermal emission or good upper limits.
Table 1: Previous upper bounds on $|\dot{G}/G|$.

| Method                                | $|G/G|_{\text{max}} \times 10^{-12}$ yr$^{-1}$ | Time scale [yr] | Reference                           |
|----------------------------------------|----------------------------------------------|-----------------|-------------------------------------|
| Big Bang Nucleosynthesis                | 0.4                                          | $1.4 \times 10^{10}$ | Copi et al. (2004)                  |
| Microwave background                    | 0.7                                          | $1.4 \times 10^{10}$ | Nagata et al. (2004)                |
| Globular cluster isochrones             | 35                                           | 10$^{10}$       | Deye et al. (1996)                  |
| Binary neutron star masses              | 2.6                                          | 10$^{10}$       | Thorsett (1996)                     |
| Helioseismology                         | 1.6                                          | $4 \times 10^9$  | Guenther et al. (1998)              |
| Paleontology                            | 20                                           | $4 \times 10^9$  | Eichendorf & Reinhart (1977)       |
| Lunar laser ranging                     | 1.3                                          | 24              | Williams et al. (2004)              |
| Binary pulsar orbits                    | 9                                            | 8               | Kaspi et al. (1994)                 |
| White dwarf oscillations                | 250                                          | 25              | Benvenuto et al. (2004)             |

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References

Akmal, A., Pandharipande, V. R., & Ravenhall, D. G. 1998, Phys. Rev. C, 58, 1804
Benvenuto, O., García-Berro, E., & Isern, J. 2004, Phys. Rev. D, 69, 2002
Brans, C., & Dicke, R. H. 1961, Phys. Rev., 124, 925
Copi, C., Davis, A., & Krauss, L. 2004, Phys. Rev. Lett., 92, 17
Degl’Innocenti, S. et al. 1996, Astron. Astrophys., 312, 345
Dirac, P. 1937, Nature, 139, 323
Eichendorf, W. & Reinhard, M. 1977, Mitteilungen der Astronomischen Gesellschaft Hamburg, 42, 89 (result cited in Uzan 2003)
Fernández, R., & Reisenegger, A. 2005, Astrophys. J., 625, 291: FR05
Finzi, A. 1965, Phys. Rev. Lett., 15, 599
Finzi, A., & Wolf, R. A. 1968, Astrophys. J., 153, 835
Flores-Tulián, S., & Reisenegger, A. 2006, Mon. Not. R. Astron. Soc., 372, 276
Gourgoulhon, E., & Haensel, P. 1993, Astron. Astrophys., 271, 187
Guenther, D. B., Krauss, L. M., & Demarque, P. 1998, Astrophys. J., 498, 871
Haensel, P. 1992, Astron. Astrophys., 262, 131
Jofré, P., Reisenegger, A., & Fernández, R. 2006, Phys. Rev. Lett., 97, 131102: JRF06
Hansen, B. M. S., & Phinney, E. S. 1998, Mon. Not. R. Astron. Soc., 294, 569
Kargaltsev, O., Pavlov, G. G., & Romani, R. 2004, Astrophys. J., 602, 327
Kaspi, V. M., Taylor, J. H., & Rybka, M. F. 1994, Astrophys. J., 428, 713
Mignani, R. P., Manchester, R. N., & Pavlov, G. G. 2003, Astrophys. J., 582, 978
Nagata, R., Chiba, T., & Sugiyama, N. 2004, Phys. Rev. D, 69, 3512
Page, D. 2006, these Proceedings
Prakash, M., Ainsworth, T. L., & Lattimer, J. M. 1988, Phys. Rev. Lett., 61, 2518
Reisenegger, A. 1995, Astrophys. J., 442, 749
Reisenegger, A. 1997, Astrophys. J., 485, 313
Reisenegger, A., Jofré, P., Fernández, R., & Kantor, E., Astrophys. J., in press [astro-ph/0606322]
Schaab, Ch., Sedrakian, A., Weiner, F., & Weigel, M. K. 1999, Astron. Astrophys., 346, 465
Shapiro, S. L., & Teukolsky, S. A. 1983, Black Holes, White Dwarfs, and Neutron Stars (New York: Wiley)
Thorsett, S. E. 1996, Phys. Rev. Lett., 77, 1432
Uzan, J. 2003, Rev. Mod. Phys., 75, 403
van Straten, W., et al. 2001, Nature, 412, 158
Walter, F. M., & Matthews, L. D. 1997, Nature, 389, 358
Will, C. 2006, Living Reviews in Relativity, 9, 3
Williams, J. G., Torslev, S. G., & Boggs, D. H. 2004, Phys. Rev. Lett., 93, 261101-1-4
Yakovlev, D. G., & Pethick, C. J. 2004, Ann. Rev. Astron. Astrophys., 42, 169
Zavlin, V. E., & Pavlov, G. G. 2004, Astrophys. J., 616, 452