Learning and Sustainability: an approach from the interaction

The attitude that this new man maintains facing nature will be radically different from the attitude he maintained in previous times.

Werner Karl Heisenberg

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Abstract. We present a study of behaviors, belonging to the domain of emotions, inquiry, persuasion and the use of information and that are deployed in time by teams of students immersed in learning processes. These are transformed into time series on which non-linear dynamics techniques are applied, obtaining the connectivity for each team. From this value and the quotient Positivity / Negativity (referred to the emotions) can be characterized learning dynamics and its performance. The Lyapunov coefficients, calculated to the time series, make it possible to obtain the Kolmogorov entropy. From this value, associated to the experimental time series, the complexity of the system and the loss of information are deduced. This allows us to construct, for the learning process, a quantitative estimate of sustainability.

Keywords: time series, connectivity, Lyapunov coefficients, entropy, sustainability

1. Introduction

Several authors ([1, 2, 3, 4, 5, 6] point out that emotion fulfills an important cognitive role: knowledge of life and the universe is not only intellectual, since the subtle nuances of it brings us the emotiveness (emotivity). Emotions are the other way of knowing about the world and about themselves [4]. In learning, value should be given to facts, people and situations (which form the initial contextualized condition), according to their influence on the emotionality of individuals and teams. This value assignment manifests, neurobiologically, in attendance and perceptive selection. In the selection of what is remembered by the long-term memory, and in the perception that dispositions and attitudes are "felt" as more appropriate [4, p.180, 7]. This makes learning sustainable over time. Thus the question which gives the title to this work arises: Can appropriate mathematical indicators be built to be measured that provide information about sustainability in times of a learning process? [8, 9, 10].

In order to elaborate a quantitative answer to this question, observational follow-up is carried out, during a semester with a group of students whose major is engineering in a technological university.
They constitute a segment of students who enter with their completed secondary studies, without a university selection test, and a significant proportion, form a first-generation human group to access to higher education. In total, five Physical Sciences courses are monitored. The reasons to choose this subject point, essentially, to the emotional commitment to it, from which everything becomes: interest or lack of interest, absenteeism, desertion, etc. Which is supported each semester in the percentages of approved and failed. Each course consists of an average of 32 students (approximate) who are divided, for the specific sessions of follow-up, into two of 16 students forming 4 work teams of 4 students each. The "typical" performance matrix per student team is divided into small time intervals to complete the 90-minute class [5, 11, 12]. It is important to observe the behaviors deployed by the students in the process, in which 6 "macro" variables are identified: IND (IND), Persuasion (PER), Positivity (POS), Negativity (NEG), Internal Information External (IE), constructing the normalized proportions: X = IND / PER, Y = POS / NEG, Z = II / IE. Both numerator and denominator of each fraction contain a subset of 13 behaviors, which are assigned a numerical range from -6 to +6, as shown in the example # 1 schema for Positivity and Inquiry [12, 13].

Scheme No. 1. It indicates the dual polarity domain and the coding scale, in its double polarity, of Positivity and Inquiry, with its thirteen associated behaviors.

This hierarchy determines a total of 78 behaviors to observe during the 90-minute session.

Observational monitoring is a complex process that can be addressed in two ways:
On the one hand, through videos of the class sessions from which the information is extracted and on the other hand, is to train observers to record the behaviors deployed by the teams for each variable "macro" (13), PER (13), POS (13), NEG (13), II (13), and IE (13). With the purpose of exploring collaborative work and self-organization, it is suggested to form four-student teams (2 men and 2 women, etc.). These teams are called Experimental Group. An initial condition is applied in the form of an experimental or theoretical activity, with an emphasis on positive emotionality (solidarity, founded criticism, respect, shared humor, etc.) allowing to each team to deploy their own emotionality interactively. This same initial condition gives the purpose of the activity through questions or challenges. The team must be self-sufficient regarding the proposed question or challenge.

In so far as, the process progresses in time, the respective data and tables of values are generated, a “type” table is shown below:

Table 1. Presents the behavior to be followed in time and its coding taxonomy.
With the purpose of studying validity and reliability criteria of the instruments of measurement of the research, control courses, called Control Groups, without initial condition and without emphasis on the emotional interaction and with the classic format of development of the academic activity, followed by the same variables [6, 13 a]. It is convenient that the constructed intervals of time are very fine, of the order of 1 second for each record of observation. Some of the graphs in the time, for the case with initial condition with emphasis on the emotions are presented below:

![Graph 1](image)

Figure 1. It represents the variation in the time of the Time Series of the Positivity / Negativity (= Y) and the Internal Information / External Information (= Z).

These graphs are the Time Series. These can be represented by Fourier Series and so they are usually called Fourier Time Series [14, 15].

Finally, from the Time Series of X (t), Y (t) and Z (t) we construct the discretized column vectors (although vectors with a minimum of 1000 elements allow to make good estimates, ideally they contain over the 5000 components of the analysis of Lyapunov coefficients). Their graphics in the Space of Phases, characteristically acquire forms such as:

![Graph 2](image)

Figure 2. The graphs show the time evolution of the Time Series X, Y, Z - each one of 1000 data - generated from the time tracking of the different teams of students that satisfy fixed (left) and attractor Chaotic (right).

The graphs allow to classify the dynamics, according to the performance of the teams that make up the Experimental Group, in: Low, Medium and High. It contrasts with the performance of Control Groups, traditional courses with no initial condition and emotional deployment in line with a traditional approach. The cross-correlations by group according to the influence exerted by the variety of emotions Y (= Positivity / Negativity) on the variable X (= Inquiry / Persuasion) [5, 12]:

| Group          | Control | Experimental | Comparison: Experimental / Control |
|----------------|---------|--------------|-----------------------------------|
| Cross correlation | 0.3     | 0.5          | 1.7                               |

That is, the experimental group, that is treated with contextualized initial conditions and climate that leads to high emotional connectivity within each team, shows that the balanced presence of positivity / negativity in their relational ties exerts a $1.7 \approx 2$ influence, two times approximately, superior on the variable X, that is the inquiry / persuasion (the most rational part of the work of the team), leading the team in a more effective and secure towards the achievement of high performance. This influence is translated into connectivity and emotional field evolution, which is reflected in the value that the students give to learning and in the achievements of the same that covers from the experience of
collaborative work, each component is determined in the learning process to formal assessment procedures. These corollaries allow to concentrate in the courses with an initial condition and influence of the emotional fields, Experimental Group. It is possible to apply modeling to the study of Time Series through the MatLab Software, either by the Lorenz equations [15, 16], Neuronal Networks [17] and Cellular Automata [18, 19]. For this study the Lorenz equations [15] are applied through the fourth order Runge - Kutta numerical method [14] to the discretized column vectors of X, Y and Z [5]. These equations have a parameter, called control parameter or connectivity [43, 44, 45], showing that it is directly related to the performance and the Pos / Neg quotient of the emotions of each team [6] Table 2:

| PERFORMANCE                          | THEORETICAL | CENTROID | CORRELATIONAL | AVERAGE | POS / NEG |
|--------------------------------------|-------------|----------|---------------|---------|-----------|
| Low (weak attractor)                 | 16.5        | 20.15    | 16            | 17.5    | 0.375     |
| Medium (medium attractor)            | 19.5        | 22.1     | 20            | 20.5    | 1.95      |
| High (strange or chaotic attractor)  | 31          | 26       | 29            | 28.7    | 5.25*     |

Table 2. It shows the performance according to connectivity and the Positivity / Negativity (POS / NEG). (*: POS / NEG = 5 / 1, [11, 46])

Teams with connectivity and high Pos / Neg (emotional flowering [11]) are sustained over time and achieve the objectives of the activity. Looking at Table 2, we can see an increase in connectivity as we approach to chaotic or complex dynamics, as shown in Graphic 3:

What does this entropic connectivity behavior (called entropic connectivity) mean for learning? Is it possible to calculate it? And how does it relate to the complexity of the learning process under study?

To answer these questions, there are several numerical procedures applied to the Time Series (Sprott, 2006), which allow the determination of Lyapunov coefficients [20], Kolmogorov (SK) entropy [21, 22, 23], complexity [22] and finally uncertainty in information [23].

### 2. Theoretical approach [22]

From a point of view of a physical-mathematical theory of collaborative learning, it is possible from the free energy of Helmholtz [24] to obtain a definition characterization of the sustainability of a learning process.

Thus, for every open thermodynamic system, the total entropy, S, can be written as the sum of the internal entropy of the system, S\text{INTERN}, and the external entropy, S\text{EXTERN}, to it:

\[
S = S_{\text{INTERN}} + S_{\text{EXTERN}} = S_I + S_E
\]

The production of entropy expressed as a variation over time.

\[
\frac{dS}{dt} = \frac{dS_I}{dt} + \frac{S_E}{dt}
\]

The second law of thermodynamics states that the internal production of entropy due to irreversible processes occurring within the system: \(\frac{dS_I}{dt} > 0\)
For the special case where constrictions are constant, irreversible classical thermodynamic theory states that eventually the system will reach a steady state thermodynamic state in which all microscopic variables, including total entropy, are stationary in time, so:

\[
\frac{dS}{dt} = 0
\]

then:

\[
\frac{dS_I}{dt} = -\frac{dS_E}{dt} \Rightarrow \frac{dS_E}{dt} < 0
\]

A system of such characteristics, to be maintained in a stable stationary thermodynamic state, requires a continuous negative flow of entropy into the system. This can be visualized from the perspective of game theory and thinking of two teams A and B and assuming that some players of A assume that one strategy, say A, is better than the other, no matter how the other team, B, plays. Clearly the population of A who assumed this position will decrease over time since their adaptability is lower than the average and will eventually disappear. Then the whole population will be with strategy B, so the adaptability of B and the average of adaptability are equal and the production of entropy will be zero reaching the steady state. From the social point of view, this process requires a continuous negative flow of entropy that is not sustainable (Think of the countries that have opted for a bad development strategy and another country for a good one, sooner or later everyone will want to be with the "best" strategy in such a way that the negative flow of entropy derived from the decline of those who opted for the "bad" strategy would prove to be ultimately unsustainably planetary, as we see it today).

Sustainability, understood as the ability of a system to reach states of greater longevity, can be obtained by minimizing Helmholtz free energy [24]:

\[
F = U - TS
\]

where T is the system temperature associated with internal randomness and U is the internal energy associated with the energy due to the interactions.

The free energy of Helmholtz can be minimized in two ways. One is to minimize U and the other is to maximize S. Most of the time, the internal energy U is a conserved quantity or cannot be controlled externally, it remains as the only alternative to maximize the entropy.

If the variation in the time of the Entropy is greater than zero:

\[
\frac{dS}{dt} > 0
\]

indicates that the entropy grows and the system is in a more sustainable configuration if the variation in time of the Entropy is less than zero:

\[
\frac{dS}{dt} < 0
\]

means that the entropy decreases and the system is in a configuration far from sustainability.

A positive entropy production is reached when the average adaptability is greater than the local one and this corresponds to cooperative games (or processes), thus a quantitative indicator for the measurement of the sustainability of a system is obtained.

In the real world interactions are an integral part. As the entropy of the system increases, it becomes more stable over time, more sustainable. This allows to make explicit the relationship between information and entropy since the complexity is measured from the information once all the computational calculations of the system have been realized.

Adaptability is a measure of the Helmholtz free energy minimization capacity.

3. Entropy and complexity

Let the equivalence be between Kolmogorov's entropy and entropy:

\[
S_K \leftrightarrow \frac{dS}{dt}
\]

as indicated before, the production of positive entropy is achieved if the average adaptability is greater than the local one. This observation is also consistent with connectivity for the chaotic case, which is on the order of 30 [12], which would indicate that a high connectivity between components of a computer is a condition of high entropy production and vice versa: means that a j-nth component of
the equipment is increasingly establishing connections with the other components and, in turn, these with each other and with it.

Real-world observations show that interactions are a characteristic feature of their evolutionary relational dynamics. This allows us to visualize, as it happens in nature, that the open systems "propitiate" an increasing order in connectivity, being dynamically sustainable, increasing its complexity in time.

If a sequence of characters, X, composed of values (time series) or symbols, which follow a probability distribution P(x), is constructed for information, I [25]:

\[ I = -\sum P(x) \log P(x) = S \]

This expression has the same functional form of the entropy of the Physical Sciences.

According to Gershenson [26, 27] the complexity can be defined:

\[ C = aI_{out} (1 - I_{out}) \]

a is a normalization constant that makes it possible for quantity C to be a percentage value without units.

Using the entropy, S [22]:

\[ C = aS_{obs} (1 - S_{obs}) = aS (1 - S) \]

If \( S_{MAX} \) is the maximum entropy and accounts for the total information capacity of the system, then \( S_{OBS} \) is the observed entropy and corresponding to the present information content. The constructed formal structure leads "naturally" to a definition of the complexity of a system, which can be estimated through its entropy. Since Kolmogorov's entropy is time-dependent, by its relation to the Lyapunov coefficients of the time series, the complexity remains:

\[ C(t) = aS_{Kobs} (1 - S_{Kobs}) \]

4. Experimental results

From the point of view of the Lyapunov coefficients, the Kolmogorov entropy, the complexity and the sustainability for two learning dynamics, the three (weak attractor, medium attractor, strange or chaotic attractor) were studied. Teams of students in learning processes: medium attractor and strange or chaotic attractor.

Medium or fixed attractor Case

The Lyapunov coefficients \( \lambda_x, \lambda_y \) and \( \lambda_z \) are associated to the time series of the Inquiry / Persuasion (X (t)), Positivity / Negativity (Y (t)) and Internal Information / External Information (Z (t)).

The numerical calculation of the Lyapunov Coefficients [43, 44] for the series yields the values:

\( (\lambda_x, \lambda_y, \lambda_z) < 0 \)

The average loss of information, over time, is equal to the mean sum of positive Lyapunov exponents [28], the Kolmogorov entropy:

\[ S_K = \sum_{\lambda_i > 0} \lambda_i = \lambda_x + \lambda_y + \lambda_z = 0 \]

As it was mentioned before, \( S_K \) \( \leftrightarrow \) \( \frac{dS}{dt} \), then \( S_K \) \( \leftrightarrow \) \( \frac{dS}{dt} = 0 \)

There is no disorder increase, which is consistent with the value of the control or connectivity parametric, low or medium if compared to that of the chaotic dynamics, according to calculations by centroid, cross correlation [12], by the theoretical recursive method according to MatLab [29] of the applied nonlinear model [12, 15, 30, 31, 32], shown in Table 2.
Strange or chaotic attraction case:
The numerical calculation of the Lyapunov coefficients [43, 44] for the time series $X(t)$, $Y(t)$, $Z(t)$ [12], yields the values:

$$\lambda_x \approx 0.004, \quad \lambda_y \approx 0.001, \quad \lambda_z \approx 0.004$$

And the average loss of information over time [28] according to the Kolmogorov entropy:

$$S_K = \sum_{i>0} \lambda_i = \lambda_x + \lambda_y + \lambda_z \approx 0.009 \approx 0.01 > 0$$

Following the reasoning of the previous case $S_K \Leftrightarrow \frac{dS}{dt}$, then: $S_K \Leftrightarrow \frac{dS}{dt} > 0$

5. Calculation of complexity according to fixed and strange attractors

From the previous results it is clear that the production of entropy grows, allowing the system to enter a more sustainable configuration, confirming that the average adaptability is greater than the local one since the character of the activity developed by the teams of students corresponds to processes of collaborative or cooperative learning, generating a quantitative indicator for the measurement of the sustainability of the system.

The value of the entropy production is directly related to the control parameter, $r$, called connectivity, of the chaotic case with high performance computers, with a numerical value of 30 [5]. This shows that high connectivity between components of a computer produces high entropy and vice versa (an $i$-th component of the equipment during the learning process leading to high performance increasingly establishes, in the context of the Positivity / Negativity ratio, increasing connections with other components and thus for the remaining $n$-1 components). In everyday life in the real world connectivity [45] is an integral part. As noted, as the number and order of interactions of the system increases, it is found to be more stable over time, in essence more sustainable.

Using the equivalence between Kolmogorov's entropy and entropy, one can write:

Fixed attractor case:

$$C(t) = a S_{K_{\text{obs}}} (1 - S_{K_{\text{obs}}}) = a 0.0 (1 - 0.0) \approx 0.0.$$

The complexity of the system is zero.

Strange or chaotic attraction case:

$$C(t) = a S_{K_{\text{obs}}} (1 - S_{K_{\text{obs}}}) = a 0.01 (1 - 0.01) \approx 0.0099a \approx 0.01a > 0$$

That is, the complexity of the system grows.

The numbers show that the entropic element of the system for the learning process and that leads to high performance is the connectivity ($r$, control parameter) of averaged values:

$$r_{\text{WEAK ATTRACTOR}} = 18; \quad r_{\text{MEDIUM ATTRACTOR}} = 21 \Rightarrow C(t) = 0; \quad r_{\text{CHAOTIC ATTRACTOR}} = 29 \Rightarrow C(t) > 0$$

By increasing connectivity, the system will become characteristically complex and sustainable. Thus, the test based on the observational monitoring of the behaviors deployed by the teams during the process confirms, in a first approximation, that it becomes sustainable as connectivity grows (for this reason it is the entropic element). At lower values of connectivity, the system is "dumped", turning it rigid, weak or adaptive, which

This result confirms the absolutely relational nature of all the elements that make up the learning process. So what is done to the supposedly most insignificant component of the network that sustains life ends up affecting each other.

6. Study of the horizon of projection

Chaotic Dynamic Case: High Performance
What is the maximum time that the process of Learning based on increasing entropic connectivity can be projected, regarding a given activity?

The learning process in chaotic dynamics, within the framework of this research, is considered to be collaborative, which favors the connectivity network of a team of 4 people, regarding an activity set out as a challenge or entrepreneurship for an academic period which is equivalent to 90 minutes.

The Kolmogorov entropy [28]:\[ S_K = - \sum p_i \log_2 p_i = 0.01 \text{ bits/sec} \] for the academic period, with the \( \lambda \) positive maximum values of Lyapunov coefficients of the Time Series.

This numeric value indicates that 0.01 information bits per second are lost in the 90 minutes. Although there is a loss this is relatively low, which indirectly indicates that there is a high percentage of retention during the session.

The projection time for the loss of the system information is:

\[
\text{Lyapunov time} = \frac{1}{S_K} = \frac{1}{0.01} = 100 \text{ sec} = 1.666 \text{ min} \approx 2.0 \text{ min}.
\]

According to this result, in order to maintain an optimal ratio of meaningful learning (with high connectivity generating high performance), for each activity deployed by the student teams, the "ideal" current information should be in a time of the order of 2.0 minutes. Above this limit it is not possible to make predictions, which can be trusted, about the process characterized by contextual initial conditions and proposed activity as a challenge or undertaking, so any strategy that is applied will be useless.

This result also advises that before making an accurate projection in the long term, it is indispensable to calculate the possibility of doing it in any other type of scenario. It is notable that this time horizon does not depend on the historical past of the process on the basis of which the projection has been made. In chaotic systems, it is more favorable to look at the future, to increase the level of reliability of a projection, than to look back.

Assuming that for every second of the 90 minutes an "average information loss rate" of 0.01 bits/sec is generated, then by dividing the 90 minutes of the academic class by the average information loss rate, we get an approximate time of 6.25 days that could be interpreted as a maximum "optimistic" projection time of prediction.

### 7. Determination of loss information

Considering the relationship between Kolmogorov's entropy and the characteristic parameter of chaos, the Lyapunov exponent, \( \lambda \), it follows that this is proportional to the loss of information, if \( \lambda > 0 \) the movement is chaotic, information is lost, since the temporal prediction is less accurate and, therefore, since the loss of information is greater, no prediction can be made outside of that time horizon [23].

Then, the relationship between Kolmogorov's entropy and the characteristic parameter of chaos is shown, the Lyapunov exponent, \( \lambda \), is proportional to the loss of information [23]:

\[
\langle \Delta I \rangle, \log_2 \lambda = -\lambda (i_0 (t)) \Rightarrow \langle \Delta I \rangle_i = \frac{-\lambda (i_0 (t))}{\log_2}, \quad i = X,Y,Z
\]

Fixed attractor case, \( \lambda < 0 \):

\[
\{ \langle \Delta I \rangle_X; \langle \Delta I \rangle_Y; \langle \Delta I \rangle_Z \} = \{ 0.01827 > 0; 0.003654 > 0; 0.0003321 > 0 \}
\]

A system with negative Lyapunov exponent does not progressively lose the information that had initially and is, from a certain time, predictable (it is the matrix idea of the current educational paradigm).

Strange or chaotic attractor case, \( \lambda > 0 \):

\[
\{ \langle \Delta I \rangle_X; \langle \Delta I \rangle_Y; \langle \Delta I \rangle_Z \} = \{ -0.014616 < 0; -0.0043185 < 0; -0.0132877 < 0 \}
\]

There is loss of information in the time series for X(t), Y(t) and Z(t). For X(t), which corresponds to the time series for the Inquiry / Persuasion ratio, we obtain since it indicates there is information loss.
how do we understand this result? And what do you suggest? It can be interpreted that the algorithmic routines, deployed by the teams of high performance students, lead to certain results (positive feedback process) where there is discovery that makes meaningful learning, but with great efficiency to dispense with superfluous information, which, definitely, decreases the information towards what it is actually necessary or relevant.

Regarding Y (t), associated with the Positivity / Negativity Emotion Ratio, the question is: Is it desirable that a system in the process of learning to behave in this way? It seems prudent, assuming there is more openness in teams working collaboratively, in looking forward than still loaded with stereotypes and emotional fixations (fixed point attractors). Emotions exhibit complications in their classification because they are very remote and complex neurobiological mechanisms (their beginnings can be investigated in unicellular ones attracted by variations of light or oxygen, that are able to "feel", pheromones that incite behaviors in social insects, in the Interactions between predator-prey ...) [4,7]. Thus the chaotic dynamics copes with an acceptable break with respect to behavior patterns that favor certain repetitive behaviors [33] that restrain the creativity demanded in contexts of high connectivity - between people, people with the ecosystem and the ecosystem with himself - characteristic complexity of the world we live in, which reminds us that "a butterfly beats its wings in Bibury and a Tsunami affects the coast of Constitución" [34].

In the quotient between internal and external information the ratio Z (t), the calculations show loss of information, how can this corollary be explained? The ratio manifests the relationship between the behaviors exhibited by the team participants in their process of searching for internal and external information to the team, and which of them prevails. In innovative processes (rupture of archetypes), it seems to be more what is discarded than what it is preserved (try new strategies which involves, relatedly, a disposition or a different emotional tone). Thus, it is aligned with the observed.

A process characterized this way shows how they are influenced, affecting the final result, the three-time series. Connectivity, 30, associated with chaotic dynamics and leading to high performance is not understood without the participation and nonlinear interference between emotions, inquiry, persuasion and information, which also marks distance with the reductionist models.

8. Conclusions

The numerical ranks, in line with the theoretical models analyzed [22], indicate the characteristics that make the learning process sustainable [35]. It is shown, in the first approximation, that these depend on: increasing connectivity, quotient of emotions Positivity / Negativity (P / N), Information and uncertainty [ 21, 25, 36] and Entropy. Parameters that arise from the experimental observation and that allow to characterize the complexity of the process. Then and only then, a “healthy system of learning for human species” will lead to: performance, self-organization, diversity and sustainability. A system with such characteristics will go into deep tension with everything that surrounds it [47]. In general, chaotic systems are characterized by consuming considerable energy and information to maintain and increase their level of complexity, while being very sensitive to initial conditions (which may be environmental fluctuations).

From this different approach, it is possible to reflect with a deeper and more specific learning, on our position and activity on this planet which is, without minimizing our very long-term efforts for interplanetary travel, the only safe place we have to live on. [37, 38, 39, 40, 41, 42].

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