Researchament of Mainlobe Anti-jamming Algorithms

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Abstract. Main lobe interferences usually cause the distortion and peak offset of main lobe and the heighten of side lobe level in adaptive beamformers. The block matrix algorithm utilized the direction vector which was determined by both the antenna arrays and the direction of interference to eliminate the main lobe interference. But this algorithms is sensitive to the error occurred in the interference arrive angle and would consume the array of degrees of freedom. In this paper a new algorithm is proposed which projects the receive signal into a space which is orthogonal to the signal subspace to achieve the goal. The space can get by three steps. Frist we obtain eigenvalues by the decomposition of the covariance matrix of the snapshots to evaluate the noise power. Then we get a new matrix by removing the noise from the covariance matrix of the snapshots. At last we get the target space by conducting the singular value decomposition to the new matrix. Performance is mainly assessed in terms of the beamforming gain and the signal to interference and noise (SINR) ratio. The computer simulation shows the effective of this method.

1. Introduction

Adaptive beamforming is a hot issue of array signal processing and is widely used in radar, sonar, seismology and even in artificial intelligence field[1]. Its main idea is to strengthen the signal power in the specific direction, and at the same time form nulls in the interference position to suppress interferences[2, 3].

In recently years, the use of antenna arrays in combination with signal processing algorithms offer a good basis for developing techniques to eliminate interference in the side lobes of the beam pattern, which achieve a good performance[4]. However, the real communication environment is so complex that interferences may fall into the main lobe. In this case some shortcomings appear. For example, high side lobe level and main lobe distortion, which lead to the limitation of the application of adaptive beamformer in practice since the performance of adaptive beamformer will suffer a severe degradation in its output Signal-to-Interference-plus-NoiseRatio(SINR)[5]. Therefore, it is necessary to find an effective method to solve this problem even when interference falls into the main lobe.

In the past, researchers have studied the main lobe interference elimination algorithms, like the method by using spatial polarization characteristics of antennas[6]. This method processes the sample data by using the scanning property of a signal polarized antenna. The orthogonal polarization decomposition and polarization estimation of the receiving signal based on the spatial polarization characteristics of the antenna are realized. Therefore, the interference is diminished. However, this method needs a large amount of calculation. In addition, other researchers concentrate on changing the antenna arrays[5] and this method employ a main antenna array to obtain radar data, which contains target signals as well as interferences and an auxiliary antenna array to estimate the interferences,
which aims to estimate the interferences only. However, these ways require additional hardware of an auxiliary antenna array to estimate the external interferences. Besides, there is another method to solve the main-lobe problems called the block matrix method (BM)\[7, 8\]. This method eliminates main lobe interference by setting the direction vectors of interferences in the main lobe to zero. However, the BM method depends largely on the interference arrival angles and obtains a poor fault tolerance, so it is hard to put into use in real scenes. Therefore, it is necessary to develop a main lobe interference suppression algorithm which is easy to realize.

In this paper, an innovative algorithm is proposed which projects the receive signal into a space which is orthogonal to the signal subspace. The space can be obtained by three steps. Firstly, we obtain eigenvalues by the decomposition of the covariance matrix of the snapshots to evaluate the noise power. Then we get a new matrix by removing the noise from the covariance matrix of the snapshots. At last, we get the target space by conducting the singular value decomposition to the new matrix. Simulation results show that this method has excellent SINR performance.

2. Signal Model
Considering an isotropic uniform linear array which contains \(M\) antenna arrays and spacing for half a wavelength. The noise is Gaussian white noise, and its variance is \(\sigma^2\). Interferences come from the direction of \(\{\theta_1, \theta_2, \ldots, \theta_D\}\) and \(M > D\). Then signals received by antenna arrays can be represented as:

\[
X = a(\theta_0) s_0(k) + A S(k) + n(k)
\]

(1)

Where \(a(\theta_0)\) is the desired signal, \(s_0(k)\) is the signal steering vector. \(X = [x_1(k), x_2(k), \ldots, x_M(k)]^T\) is the signals received by \(M\) antenna arrays. \(A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_D)]\) is the steering vector of interferences. \(S(k) = [s_1(k), s_2(k), \ldots, s_D(k)]^T\) is the envelope of interference signals. \(n(k) = [n_1(k), n_2(k), \ldots, n_M(k)]^T\) is the channel noise of antenna arrays.

In reality, antennas are often used in the environment where the desired signal is weak while the interference signals are strong, such as satellite navigation environment. Therefore, formula (1) can be rewritten as:

\[
X = AS(k) + n(k)
\]

(2)

Then the covariance matrix can be represented as:

\[
R_X = E[X(k)X^H(k)] = AR_SM + \sigma^2 I
\]

(3)

According to the literature\[9\], the best adaptive weights is:

\[
W = \mu R_X^{-1} a_q
\]

(4)

Where \(\mu\) is a constant. \(a_q\) is the steering vector of the desired signal. Using weights, a beam pattern with nulls in the position of interferences is formed to achieve the purpose of anti-jamming.

However, if the interference falls into the main lobe, the main lobe will be distorted and peak offset. To solve this problem, interference should be eliminated before weights calculating.

3. Mainlobe Interference Cancelling Via Block Matrix

3.1. Getting the block matrix \(B\)
Before using this method, directions of interference signals should be known. For the main idea of this paper is to focus on the main lobe, which is relatively narrow. To reduce the amount of calculation, Capon DOA Estimation methods\[8\] is used. Eq. can be represented as:
\[ P(\theta) = \frac{1}{a(\theta)^T R^{-1} a(\theta)} \]  

(5)

Where the scope of \( \theta \) is the main lobe. The angle which make \( P \) the maximum is the arrive angle of interference signals.

Then data received by antenna arrays should be processed to eliminate main lobe interference. In this paper, assume there are two main lobe interference signals. Then assume \( Y \) is the data after processing.

\[ Y = B X = B A G = C_{(M-2)} D G_{D/1} \]  

(6)

Where \( B \) is the block matrix, \( A \) is the direction vector, \( X \) is the received data, \( G \) is the matrix of interferences. \( C \) should have the following form:

\[
C = \begin{bmatrix}
0 & 0 & c_{1L} & c_{1D} \\
0 & 0 & c_{2L} & c_{2D} \\
M & M & M & M \\
0 & 0 & c_{(M-2)L} & c_{(M-2)D}
\end{bmatrix}
\]

Then

\[
B = \begin{bmatrix}
b_{11} & b_{12} & b_{13} & L & 0 & 0 \\
M & M & M & M & M & M \\
0 & 0 & L & L & b_{(M-2)(M-2)} & b_{(M-2)(M-1)} & b_{(M-2)M}
\end{bmatrix}
\]

According to \( B \) and \( C \), the following equation can easily get:

\[
\begin{align*}
b_{(m-2)(m-2)} a_{(m-2)(m-2)} + b_{(m-2)(m-2)} a_{(m-1)(m-1)} + b_{(m-2)m} a_m &= 0 \\
b_{(m-2)(m-2)} a_{(m-2)(m-2)} + b_{(m-2)(m-1)} a_{(m-1)(m-1)} + b_{(m-2)m} a_m &= 0
\end{align*}
\]  

(7)

The solves of equation (7) is:

\[
\begin{align*}
b_{(m-2)(m-2)} &= \frac{a_{(m-2)(m-2)} - a_{(m-2)m} a_{m}}{a_{(m-1)(m-1)} - a_{(m-1)m} a_{m}} b_{(m-2)(m-2)} \\
b_{(m-2)M} &= \frac{a_{(m-2)M} - a_{(m-2)m} a_{m}}{a_{(m-1)M} - a_{(m-1)m} a_{m}} b_{(m-2)(m-2)}
\end{align*}
\]  

(8)

Where \( m = 3, 4L M \). When calculating, \( b_{(m-2)(m-2)} = 1 \). Using the Eq. above and interference arrive angles, we can get block matrix \( B \). Then getting \( Y \) in which interference signals have already been eliminated. However, the degrees of freedom of antenna arrays are consumed.

3.2. Adaptive Beamforming

After getting \( Y \), the covariance matrix can be calculated using the following Eq.:

\[ R_{\gamma} = E [YY^H(k)] = B A A^H B^H + \sigma_n^2 B B^H \]  

(9)

Like Eq. (4), adaptive weights can be got by:

\[ W_B = \mu R_{\gamma}^{-1} a_\gamma \]  

(10)

Weights calculated by Eq. (10) will cause the main beam deflection. For when main lobe interference signals are eliminated, Eq. (10) can be replaced by the follow Eq.:

\[ R_{\gamma} = \sigma_n^2 B B^H \]  

(11)
Then the best adaptive weights is $W_{opt}$:

$$W_{opt} = \mu'R_y^{-1}a_s = \frac{\mu(BB'^{-1})a_s}{\sigma^2_n}$$

It is obvious that the offset of the main beam is caused by $(BB'^{-1})$. At the time of calculating weights a compensation factor can be added before $a_s$, then the problem can be solved.

Using the above method, problems caused by main lobe interference can be solved. However, it depends on the expense of consuming freedom, on the other hand interference arrive angles should be accurate known which is hard to reach in reality. The block matrix method is no longer applicable when there are errors exist in the predicted arrival angle. It can be seen in the back of the simulation.

4. Proposed Method

4.1. Getting the New Block Matrix

About this method, signals are projected into the subspace which is orthogonal to the signal subspace.

$$R_x^{-1} = \sum_{i=1}^{D} \lambda_i e_ie_i^H + \sum_{i=D+1}^{M} \sigma^2_n e_ie_i^H$$

From the Eq. (13), it is obviously that interferences are in the signal subspace $(e_ie_i^H)$, to eliminate interference, it is necessary to construct a space which is orthogonal to the signal subspace, then to project the received signal into the space to achieve the purpose of anti-jamming.

In Eq. (3) the covariance matrix of received signal is got. It is easy to see that every column of $R_s$ is belong to the signal subspace. when the noise power $\sigma^2_n$ is known. Signal subspace can be got by $R - \sigma^2_n I$, while signals are limited number of samples, and the noise power is different from the ideal value. On the other hand the desired signal is very weak, while the interference signals are strong, it is need to estimate the noise power. So here, we conduct the eigenvalue decomposition to $R_x$.

$$R_x = \sum_{i=1}^{D} \lambda_i e_ie_i^H + \sum_{i=D+1}^{M} \sigma^2_n e_ie_i^H$$

By removing the noise, it is obtained

$$\hat{R} = R - \hat{\sigma}^2_n I$$

In Eq. (15), $\hat{R}$ is close to $A^RTA^H$ but it can not keep the renaturation. So here we conduct the singular value decomposition of matrix to $\hat{R}$:

$$\hat{R} = U\Lambda V^H$$

Then it is easy to get the signal subspace, as following described:

$$U_s = [u_1, u_2, \ldots, u_D]$$

Though $U_s$ is not euqal to the signal subspace, it is one with the minimum disturbance. The step after is to get the new block matrix $E$: 
4.2. Adaptive Beamforming

Using the new block matrix $E$ described in Eq. (18) to deal with the data:

$$Y = EX$$

The

$$R_r = E[YY^H] = EAR_A^HE^H + \sigma^2 EE^H$$

As described above, under the condition of limited samples, characteristic vector will appear disturbance, and the bigger the characteristic vector is the smaller the disturbance is. So under the condition of limited snapshots, after main lobe interferences were eliminated, the adaptive pattern is unstable and different from the static pattern which can get under the condition of big snapshots. To solve the problem, we introduce a diagonal loading quantity $\delta$:

$$\hat{R}_r = R_r + \delta I$$

Then like the Eq.(10), we can get weights:

$$W_e = \mu \hat{R}_r^{-1}a_q$$

Here we can see that the proposed method do not need know accurately the mainlobe interferences’ arrive angles and also do not need to consume array degrees of freedom.

5. Experiment Results

In this section a 21-channel uniform linear array which array spacing is half the wavelength is used. The angle of the desire signal is 0° with a SNR(Signal to Noise Ratio) level 3dB, the INR(Interference to Noise Ratio) of two main lobe interferences is 40dB and 45dB. The noise is the zero mean Gaussian white noise.

Three algorithms: the Minimum Variance Distortionless Response(MVDR) algorithm, the Block Matrix algorithm (BM), the proposed algorithm were compared to show the superiority of the proposed algorithm.

5.1. Different arrive angles of mainlobe interferences

Two main lobe interferences were set to evaluate the performance of the proposed method, and a static vector with -30dB homogeneous low sidelobe is set.

Figure 1 shows adaptive pattern of three methods with interferences come from the azimuth angle -1 and 2 in (a) and (b). While in (c) and (d), interference azimuth angles are -2 and 3. It is obvious that interferences are suppressed by using all three method, while the main beam is distorted and the level of sidelobes rise with the MVDR method. In this case, the desired signal is suppressed too. The other two method can form good beam patterns at the same time of suppressing main lobe interferences.

5.2. These Guidelines, Written in the Style of a Submission to IOP Conference Series, Show the Best Layout for Your Paper Using Microsoft Word. If You Don’T Wish to Use the Word Template Provided, Please Use the Following Page Setup Measurements. DOA Estimation with Small Errors

In most cases, we have to estimate arrive angles of mainlobe interferences, and there are always errors between the estimate value and the real value.

Now on the basis of Figure 1, we add a 0.5° error to the interference whose azimuth angle is -1. From Fig. 2(a), it is found that with the BM method, interference at -1degree can not be eliminated, and like the MVDR method, there is a null at the position of -1 degrees. Then add a 0.5° error to another mainlobe interference whose azimuth angle is 2°. From the Fig. 2 (b), it is easy found the BM
method loses its function. However, the proposed method still keep a good performance that form a good beam pattern.

![Figure 1](image1.png)

**Figure 1.** Adaptive pattern of three methods

(a) 0.5° errors for the interference at is -1°

(b) 0.5° for the interference at is 2°

**Figure 2.** Adaptive pattern of three methods with direction errors

From Fig. 2, we can know that the BM method is sensitive to the arrive angles of interferences, while the proposed method is not affected by interference arrive angles.

5.3. Comparison of SINR of Three Algorithms

Fig. 3 shows the convergence characteristics of three algorithms versus snapshots, we can see that the SINR of the BM method and the proposed method is higher than the MVDR method, because in figure 1, two mulls in main lobe lead to the rise of sidelobe and distortion main beam. The maximum gain is not in the direction of the desired signal. On the other hand, the other two algorithms can eliminate main lobe interferences and form good patterns.

When errors exist in interference arrive angles, for example a 0.5° error is added to the interference whose azimuth angle is -1 as it shows in Fig.3, then the SINR will lose about 18 dB as it shows in Fig.3 in the BM method. However the proposed method can keep a high SINR, no matter there are errors in interference arrive angles or not.
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7. Conclusion
In this paper, we propose a method to solve the problems in adaptive beamforming when interferences came into the main lobe, such as high side lobe and main lobe distortion. In the proposed scheme, the block matrix method is introduced for comparison, which could eliminate main lobe interferences, but the block matrix method was sensitive to the interference arrive angle and consumed the degree of freedom of the array. To overcame its limitations we proposed a new method which projected the received signals into the space which is orthogonal to the signal subspace. From the result of the simulation, we can see the new method is effective and practical.

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