CUMULATIVE PHENOMENA THROUGH
THE QUARK-PARTON DIAGRAM SUMMATION
AT THRESHOLDS

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Abstract

A microscopic treatment of cumulative phenomena based on perturbative QCD
calculations of the corresponding quark diagrams near the threshold is presented.
To sum all diagrams like these the special technique based on the recurrence relations was developed. The $x$-behaviour of the nuclear structure function $F_2(x)$ in the cumulative region $x > 1$ was found to be roughly exponential, governed by an effective coupling constant, which depends on the QCD coupling constant and quark mass. Two mechanisms for cumulative particle production, direct and spectator ones, were analysed. It was shown that due to final state interactions the leading terms of the direct mechanism contribution are cancelled and the spectator mechanism is the dominant one. It leads to a smaller slope of the particle production rates compared to the slope of the nuclear structure function in the cumulative region, in agreement with the recent experimental data. The slope difference is due to additional multiple interactions between nuclear and projectile partons which enter the spectator mechanism for the cumulative production. The different versions of hadronization mechanisms of the produced cumulative quarks into hadrons are also discussed.

1 Introduction

One of the most interesting fragmentation processes is the production of fast hadrons in $hA$ interactions in the nuclear fragmentation region with the value of the Feynman scaling variable: $x > 1$, i.e. in the region in which the production of this fast hadron is kinematically prohibited for the $hN$ interaction. Here $x$ is the longitudinal momentum per nucleon of the fast moving nucleus carried away by the produced particle. It means that a produced particle carries a momentum much greater than the average one of a constituent nucleon. The nuclear fragmentation processes like these usually are referred as the cumulative ones. For the deep inelastic $lA$ scattering the cumulative region is $x > 1$ in terms of Bjorken scaling variable defined for individual $lN$ interactions.

The theoretical investigations of the nuclear fragmentation processes in this cumulative region are of great physical importance as they enable to study multi-nucleon short-range correlations in nuclei (so called fluctons) and to get the information on the high-density nuclear matter clusters which always are being in nucleus.

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From the modern point of view these compact multi-quark dense hadronic matter clusters in nuclei can be also considered as the quark-gluon plasma clusters at zero temperature which investigation is of interest also in the light of recent efforts to detect the indications of high temperature quark-gluon plasma formation in heavy-ion collisions on colliders at super high energies.

An adequate treatment of the cumulative effect requires the quark language \[1, 2\]. A phenomenological treatment of cumulative phenomena based on nuclear quark distributions and QCD evolution equations was proposed in \[2\]. It uses an \textit{ad hoc} relation between the structure functions and particle production rate, based on the nucleonic (or fluctonic) picture of the nucleus rather than the quark one. Starting from the quark picture of hadrons the famous quark counting rules have been proposed \[3, 4\], which, in particular, describe the threshold behaviour of the structure functions. In \[4\] it was noted that these rules also apply to nuclei at the deep cumulative threshold \(x \to A\). In \[5, 6\] it was stressed that the cumulative phenomena involve a particular type of contributions, the so-called intrinsic diagrams, in which several partons of the nucleus cooperate to determine the intrinsic hardness of the nuclear wave function.

However the quark counting rules alone do not allow to find the cumulative probabilities. To calculate these, one needs besides to know the coefficients of the quark counting rules behaviour, which determine the relative weights with which different contributions combine to give the total probability, as well as the overall normalization. Recently we have proposed \[7, 8\] a microscopic model for the theoretical description of the cumulative phenomena based on perturbative QCD calculations of the corresponding quark-parton Feynman diagrams near the thresholds and calculated these coefficients in the framework of the proposed model. By means of the specially developed technique based on the recurrence relations all diagrams of that kind were summed. As a result, we obtain formulas for the nuclear structure function and particle production rate at \(x > 1\), which allow direct comparison with experiment. The proposed method of the summation of intrinsic diagrams at cumulative thresholds based on the recurrence relation technique are also discussed in the present paper.

2 General formulae

2.1 The nuclear structure function at \(x > 1\)

Consider the virtual \(\gamma\)-nucleus scattering cross-section in the system, where the nucleus is moving fast along the \(z\)-axis with the longitudinal momentum per nucleon \(p_z\). As a reasonable first approximation, we may treat the nucleus as a collection of \(N = 3A\) valence quarks, which, on the average, carry each momentum \(x_0p_z\) with \(x_0 = 1/3\). In the following we assume that the quark longitudinal momentum distribution is sharp enough, so that in all places, except in the quark wave function itself, we can safely put the initial quark longitudinal momentum equal to \(x_0p_z\). The probing \(\gamma\) quantum will see a "active" quark with its momentum much greater than \(x_0p_z\) only if this quark has interacted with several other quarks ("donors") and has taken some of their longitudinal momenta. The maximum possible value of the longitudinal momentum thus accumulated is \(Nx_0p_z = Ap_z\). It corresponds to \(N - 1\) interactions with all other quarks, whose
longitudinal momentum has become equal to zero each. It is well-known that interactions which make the longitudinal momentum of one of the quark equal to zero may be treated by perturbation theory [6]. These considerations were used in numerous calculations of hadronic structure functions near $x = 1$, which revealed that the results depend heavily on the quark distribution in the initial hadron (see e.g [9]). In the case of a nucleus the calculations in principle remain the same. With a number of quarks considerably larger, their distribution in $x$ evidently becomes much narrower, so that we may hope that the results would be less sensitive to its form nor to its spin-colour structure. On the other hand the number of relevant diagrams rises tremendously. Their summation presents the main technical difficulty in the nuclear case.

In general form the diagram for the deep inelastic $\gamma A$ scattering is shown in Fig. 1. In the following we use the light-cone variables $p_{\pm} = (p_0 \pm p_z)/\sqrt{2}$. The momentum of the active quark will be denoted as $k$ and its scaling variable is $\xi = k_+/p_+ \simeq k_z/p_z$. We shall study a general case when $n - 1$ quarks acquire their momenta equal to zero as a result of interquark interactions, giving their momentum to the active quark. Evidently with $n - 1$ momentum exchanges the $\xi$ maximal value is $nx_0$. The upper blob in Fig. 1 represents the active quark’s structure function. For simplicity we shall treat quarks as scalar particles. Then the contribution of the diagram of Fig. 1 to the nuclear structure function $F_2^A$ is given by

$$F_2^A(x, q^2) = \int_x^1 (d\xi/\xi) D(\xi) f_2(x/\xi, q^2)$$

where $D(\xi)/\xi$ has the meaning of the probability to find in the nucleus a quark with the longitudinal momentum $k_z = \xi p_z$, $f_2$ is the quark structure function and $x = q^2/2qp$. In the cumulative region $1 < x < \xi < A$.

2.2 Cumulative particle production

We now turn to the production of cumulative particles which have $x > 1$ in the system where the nucleus is moving fast along the $z$-axis. All the contributions can be divided into a direct part, in which the projectile interacts with the created cumulative quark (Fig. 2) and which is a straightforward generalization of Fig. 1, and a spectator part in
which the projectile interacts with other quarks. The direct contribution is evidently given
by the formula analogous to (1). We shall limit ourselves with the inclusive cross-section
integrated over the transverse momentum \( I_A(x) = x d\sigma/dx \). The direct part is then

\[
I_A^{\text{dir}}(x) = \int_x (d\xi/\xi) D(\xi) h(x/\xi)
\]

where again \( D(\xi)/\xi \) is the probability defined in the preceding subsection to find in the
nucleus a cumulative quark with scaling variable \( \xi \), \( h(x/\xi) \) is the inclusive cross-section
with this quark as a target, \( x = k_+/p_+ \approx k_z/p_z \) and \( \xi = k'_+/p_+ \approx k'_z/p_z \). Again in the
cumulative region \( 1 < x < \xi < A \).

The spectator part has a much more complicated structure. We have analysed its
contribution in details in [7, 8]. We have found that near the thresholds significant
cancellations occur in this part, which lead to the domination of the diagrams, with \textit{each}
donor quark interacting with the projectile. As a result the contribution of the spectator
mechanism \textit{has not} such a simple convolutional form (2) as in the direct case. With the
quasi-eikonal parametrization of the partonic amplitude chosen to account for diffraction,
we found [7, 8] for the contribution of the spectator mechanism to the cumulative particle
production in the case when \( n-1 \) donors transfer their momenta to the active quark:

\[
I_A^{\text{sp}}(x) = D_n(x) C_P^{-1} \int d^2B |4\pi m^2 j(B)|^{n-1}
\]

with

\[
j(B) = C_P \nu_P \int d^2b d^2b' \lambda^2(b) \rho_P(b') |a(B + b' - b) - a(B + b')|^2
\]

where

\[
\lambda(b) = K_0(mb)/2\pi
\]

Here \( \nu_P \) is the mean number of partons in the projectile, \( \rho_P \) is the one parton distribution
normalized to unity and \( C_P \) is the quasi-eikonal factor for the projectile. For example, for
NN-interaction \( C_N^2 = 1 + \sigma_{NN}^{\text{dif}}/\sigma_{NN}^{\text{el}} \). The \( a \) is the partonic amplitude of the interaction
between a parton of the projectile and a donor quark from the nucleus.
Figure 3: Formation of the hard parton component of the flucton in the nucleus.

Figure 4: An example of the diagram which contribute to $B_n$.

3 The cumulative quark production probability

We see that both the nuclear structure function (1) and the particle production rate (2,3) in cumulative region are determined by $D(\xi)/\xi$ - function which has the meaning of the probability to find in the nucleus a quark with the scaling variable $\xi > 1$.

Study the general case when $n - 1$ quarks acquire their momenta equal to zero as a result of interquark interactions, giving their momentum to the active quark (Fig. 3). The scaling variables for quarks are defined as $x_i = k_i/p_+$. According to our assumption initially in the nucleus all $x_i \simeq x_0$ and the distribution in $x_i$ is sharp. The momentum of the active quark is denoted as $k = k_1$ and its scaling variable is $\xi = x_1$. Evidently with $n - 1$ momentum exchanges its maximal value is $n x_0$ and when $x_1 = \xi \to n x_0$ then all $x_i \to 0$ for $i = 2, 3, ..., n$. Particularly on this fact the use of the perturbation theory is based [6].

The block $B_n$ describes the formation of the hard parton component of the flucton as a result of $n - 1$ interquark interactions. Due to the quark counting rules [3,4] one has $D_n(\xi) \sim (n x_0 - \xi)^{2n-3}$ when $\xi \to n x_0$. But as we have mentioned in the introduction for to calculate the cumulative quark production probability one needs besides to know the coefficients of the quark counting rules behaviour, which determine the relative weights with which contributions with different $n$ combine to give the total probability, as well as the overall normalization. So unlike [4] we are trying to calculate theoretically the structure function of the flucton at least its hard component.

In the perturbative approach we find Feynman diagrams for the $B_n$ of the type shown in Fig. 4. The shown diagram is only an example. It contains a particular set of interquark
interactions, in which donors successively give their momentum to the active quark and become soft. However other type of interquark interactions are also possible: say, the $n - 1$ donors may give their momenta successively to each other, the last giving all the accumulated momentum to the active quark (see second diagram in Fig. 3). On these diagrams dash vertical lines refer to one gluon exchange. One can choose the Coulomb part depends only on the scaling variables of the participant quarks before $(x_1, x_2)$ and after $(x_1', x_2')$ the interaction (see Fig. 7). If we neglect the colour then

$$V = 4\pi\alpha(x_1 + x_1')(x_2 + x_2')/(x_1 - x_1')^2 = 4\pi\alpha V$$

(6)

with $\alpha$ the interaction constant. To approximately take the colour into account we average over the quark colour variables, which introduces a factor $\sqrt{2}/3$ into (6).

We are using the time ordered perturbation theory in the light cone variables. In the energy denominators of the intermediate states the contributions from soft final quarks dominate as their "energies" $E_i = (m^2 + k_i^2)/x_i$ rise with $x_i \to 0$ for $i = 2, 3, ..., n$. Thus the contribution of the diagram in Fig. 3 will be proportional to $[E_2(E_2 + E_3)...(E_2 + ... + E_n)]^{-1}$ or after symmetrization on $x_2, ..., x_n$ to $[E_2E_3...E_n]^{-1}$.

The advantage of the time ordered perturbation theory is that the energy denominators have the same form for any diagram in the threshold limit and coincied after symmetrization on $x_2, ..., x_n$. To illustrate, we have presented in Fig. 5 one Feynman diagram as the sum of two time ordered diagrams. The Feynman diagram is proportional to $[E_2E_3E_4(E_2 + E_3 + E_4)]^{-1}$ and the time ordered diagrams are proportional to $[E_2(E_2 + E_4)(E_2 + E_4 + E_3)]^{-1}$ and to $[E_4(E_4 + E_2)(E_4 + E_2 + E_3)]^{-1}$ respectively in the $\xi \to 4x_0$ limit. After symmetrization on $x_2, ..., x_n$ the contributions of the last two diagrams both become proportional to $[E_2E_3E_4]^{-1}$. Moreover in this case they are equal which leads to the possibility to take into account only one of them multiplied by the "time ordering" factor 2.

As result the $B_n$ can be presented in the form

$$B_n = X_n \frac{(4\pi\alpha)^{n-1}}{(x_0)^{n-2}} \frac{1}{E_2E_3...E_n}$$

(7)

The denominator leads after integration over $x_2, ..., x_n$ to the quark counting rule behavior for the cumulative quark production rate from the $n$-quark flucton. But our aim is to calculate the coefficient in this behavior, which determines the relative weights with which contributions on different thresholds combine to give the total probability. For to do this we have to calculate the sum of all possible Feynman diagrams of the type shown in Fig. 3. In (7) the $X_n$ denotes this sum after the extraction of the common factor $(4\pi\alpha)^{n-1}(x_0)^{2-n}[E_2E_3...E_n]^{-1}$. 

Figure 5: Decomposition of the Feynman diagram on the time ordered diagrams.
Figure 6: The diagrams which contribute to $X_n$ for $n = 6$ as an example. All $x_i$ are in units $x_0$.

$$x_1 x_2 = \frac{(x_1 + x'_1)(x_2 + x'_2)}{(x_1 - x'_1)^2}; \quad x_i = \frac{1}{x_i}$$

Figure 7: The diagram rules for $X_n$.

After this extraction one can go to the limit $x_1 = \xi = nx_0$ and $x_i = 0$ for $i = 2, ..., n$ in the rest of the diagram, which leads to the diagram rules in Fig. 7 for $X_n$. All $x_i$ in Fig. 6-8 are in units $x_0$. We have also to multiply each diagram on the proper “time ordering” factor for to take into account the different time ordering possibilities for gluon exchanges.

The main problem here is that the number of these diagrams grows rapidly with $n$, as $n!$. For to sum all these diagrams we derive a recurrency relation. The derivation is based on the important observation that only diagrams with all intermediate $x$ greater than zero should be retained. Separating the last exchange, which raises the momentum of the active quark up to its final value we present $X_n$ as a sum over $k = 1, 2, ..., n - 1$ of the diagrams shown in Fig. 8, where it has been assumed that $X_1 = 1$ by definition. Times of $k - 1$ gluon exchanges in the upper blob $X_k$ relative to those in the lower blob $X_{n-k}$ may be arbitrary, which leads to a time ordering factor $C^{n-1}_{n-2}$. Taking into account the diagram rules in Fig. 7 and denoting $f_n \equiv X_n/n!$ we find that Fig. 8 translates into a recurrency relation for $f_n$:

$$f_n = \frac{1}{n(n-1)} \sum_{k=1}^{n-1} \frac{n+k}{n-k} f_k f_{n-k}$$

with the initial condition $f_1 = 1$. The recurrency relation (8) enables easy calculate $f_n$ for an arbitrary $n$ starting from $f_1 = 1$. For large $n$ (8) evidently admits asymptotical

Figure 8: The diagrammatic recurrency relation for $X_n$. 

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solutions of the form

\[ f_n \simeq [(6/5)n + o(n)] \exp(-an) \]  

(9)

where \( a \) is arbitrary. Numerical studies reveal that with \( f_1 = 1 \)

\[ a = 0.24421... \]

and also show that the asymptotical expression (9) approximates the true solution quite well starting from \( n = 3 \), i.e. for all physically interesting values.

So we have calculated the \( X_n = f_n n! \) and consequently the \( B_n \) which describes the formation of the hard parton component of the flucton. Now from diagram in Fig. 3 we can calculate the \( D(\xi)/\xi \) - the probability to find in the nucleus a hard cumulative quark with scaling variable \( \xi \).

The block \( B_n \) is not dependent on the momentum variables of the initial quarks. As the consequence we find that all donor quarks have their coordinates relative to the active one equal to zero.

Now assume that on the average quarks are homogeneously distributed inside nucleons (three in each of them). In this picture the probability \( w_n^{(q)} \) to find \( n-1 \) quarks within the distance \( R_1 \) from the active one, where \( R_1 \) is the nucleon radius, is equal to the probability to find within the same distance \( \pi \) nucleons, where \( \pi = [(n - 1)/3] + 1 \) (i.e. the entire part of \( (n - 1)/3 \) plus one nucleons):

\[ w_n^{(q)} = w_\pi \]  

(10)

If the nucleons, in their turn, are homogeneously distributed inside the nucleus of the finite volume \( V_A = AV_0 \) then the latter probability will be given by

\[ w_\pi^{(N)} = C A^{-1} (V_1/V_A)^{\pi - 1} \]  

(11)

where \( V_1 \) is the nucleon’s volume. Actually, as well-known, this approximation is rather poor due to the rise of the nuclear density towards its center. To account for this fact we introduce a factor \( q_\pi \) to be calculated from a realistic nuclear distribution. In particular for the Saxon-Woods distribution

\[ \rho(r) = \rho_0/(1 + \exp(r - R_A)/r_1) \]  

(12)

with \( R_A = A^{1/3} 1.17 \text{ fm} \) the nuclear radius and \( r_1 = 0.51 \text{ fm} \), for \( A >> 1 \)

\[ q_\pi = 1 - (3r_1/R_A) \sum_{k=1}^{\pi - 1} (1/k) = 1 - 1.31 A^{-1/3} \sum_{k=1}^{\pi - 1} (1/k) \]  

(13)

An explicit calculation gives for the cumulative quark distribution

\[ D_n(\xi) = 3AC A^{-1} X_n^2 \left( \frac{\alpha^2}{9m^3 V_1} \right)^{n-1} q_\pi \left( \frac{V_1}{V_A} \right)^{\pi - 1} \frac{\theta(\Delta_n)\Delta_n^{2n-3}}{(2n-3)!} \]  

(14)

where \( \pi - 1 = [(n - 1)/3] \) is the ”cumulative number” (i.e. the number of nucleons in the flucton \( \pi \) minus one) and \( \Delta_n = (nx_0 - \xi)/x_0 \). All nontrivial \( A \) dependence of the distribution \( D \) is contained in the nuclear factor \( q_\pi \). It rises with \( A \) approaching its
asymptotic value 1. The rise is quicker for higher $\bar{\pi}$. Studies of the cumulative particle production rate in the nucleon picture, where the same $A$-dependence emerges [10], revealed that it agrees rather well with the experimental observations.

With (9) we finally find

$$D_n(\xi) = \frac{216}{25} \frac{A}{(\bar{\pi} - 1)!} \left( \frac{V_1}{V_0} \right)^{(\bar{\pi} - 1)} (n - 1)n^{9/2}\gamma^{n-1}\theta(\Delta_n)\Delta_n^{2n-3}$$

(15)

where,

$$\gamma = \frac{\alpha^2}{36m^3V_1}\exp 2a$$

(16)

and we have assumed $A > 1$. The behavior of $D_n(\xi)$ near the threshold $\Delta_n = 0$ is determined by the factor $\Delta_n^{2n-3}$, which corresponds to the well-known quark counting rules [10]. The magnitude of the coefficient is determined mainly by the factor $\gamma^{n-1}$. As we shall see later, the values of $\gamma$ extracted from the known experimental data on nuclear structure functions in the cumulative region result quite small. Because of this the power factor $\gamma^{n-1}$ ensures that the coefficient in the $\xi$-dependence of (15) drops very fast with the growth of $n$. Down from the threshold the contribution $D_n(\xi)$ rises as a power. This rise has evidently to stop at some distance from the threshold where our approximations become invalid and terms with higher powers of $\Delta_n$ become important. We expect, however, that the overall order of the contribution remains to be governed by the small factor $\gamma^{n-1}$ coming from $n - 1$ gluon exchanges.

As a result, due to the smallness of $\gamma$, the contribution from diagrams with a given $n$ will always dominate over those with greater $n$’s, as soon as we slightly move down from the threshold point $\xi = nx_0$. Therefore we may safely assume that $D_n(\xi)$ given by (14) or (15) represents the true quark distribution for $\xi$ starting from the threshold $\xi = nx_0$ down to the point $\xi = (n - 1)x_0$, which is the threshold for the diagrams with $n - 2$ donors, i.e. in the region

$$(n - 1)x_0 \leq \xi < nx_0, \quad x_0 = 1/3$$

(17)

Alternatively, to avoid discontinuities, we may take the total distribution $D(\xi)$ as a sum of $D_n(\xi)$ over all possible $n$’s with an additional assumption that $\Delta_n = 1$ for $\xi$ smaller than $(n - 1)x_0$, which means freezing the contribution $D_n$ at the threshold for the contribution $D_{n-1}$. As a result we have the roughly exponential behaviour of $D(\xi)$ in the cumulative region, governed by an effective coupling constant, which depends on the QCD coupling constant and quark mass.

4 Numerical calculations, discussion and the comparison to experimental data

Both convolutional formulas (14,2) lead to the same $x$ dependence for the structure function $F_A^2(x)$ and for the direct mechanism production rate $I_A^{dir}(x)$ in the region $x > 1$. It is roughly an exponential in $x$

$$I_A^{dir}(x) \sim F_A^2(x) \sim \exp(-b_0x)$$

(18)
where the slope $b_0$ is determined by the QCD coupling constant and quark mass.

The spectator mechanism \[8\] besides involves interactions between partons of the projectile and target nucleus. It was shown in \[7\] that each donor quark has to interact with the projectile. As a result the spectator contribution $I^{sp}_A(x)$ also behaves in $x$ as an exponential but with a different slope

$$I^{sp}_A(x) \sim \exp(-b_s x)$$ (19)

the slope $b_s$ depending not only on the QCD coupling constant and quark mass but also on the partonic amplitude.

In \[8\] we have found that final state interactions cancel the leading terms in the direct contribution, so that it becomes much less than the spectator one. Then particle production in the cumulative region indeed goes predominantly via the spectator mechanism. To be able to explain the experimental slope we have studied in \[8\] the quasi-eikonal parametrization for the partonic amplitude. The spectator slope $b_s$ results very sensitive to the magnitude of the hadronic diffractive cross-sections. We have chosen the maximum possible value for the diffractive parameter for the case of the nucleon as a projectile $C_P = C_N = 1.5$ (see \[4\]). Then the quasi-eikonal parametrization leads to values of the partonic amplitude $a$ \[4\] considerably larger than without diffraction because of a stronger screening effect introduced by diffractive states.

The results of the calculations are shown in Fig. 9 for the cumulative charged pion production on $^{181}$Ta, respectively. For comparison the corresponding nuclear structure function $F^A_2(x)$ at $x > 1$, is also shown, as well as the available experimental data from \[11\]-\[13\].

One clearly observes that the spectator mechanism, with a quasi-eikonal parametrization of the partonic amplitude chosen to account for diffraction, leads to a considerably smaller slope of the production spectra ($b_s \sim 7 \div 9$) \[19\] compared to the slope of the structure function \[18\] in the region $x > 1$ ($b_0 \sim 16$), in a good agreement with experimental data.

Thus our model correctly predicts two different exponential in the cumulative particle production and in the nuclear structure function at $x > 1$. The difference is due to additional multiple interactions between projectile and target which enter the spectator mechanism for the cumulative production.

A more phenomenological attempt to explain smaller slopes for the cumulative particle production is made in \[14\], where the existence of multiquark clusters in the nucleus and their properties are postulated and the quark-gluon string model formalism of \[15\] is used to calculate production rates.

Note that at a nucleon level of analysis a similar approach to a description of cumulative phenomena was suggested in our paper \[16\] in which to find the cumulative particle production rates we used the fragmentation functions of multi-nucleon fluctons calculated in the framework of Regge approach. In a sense, the calculations of the probabilities "to slow" the quark system with some definite quantum numbers usually discussed in the quark-gluon string model approach \[13, 17, 18\] is close to the idea of counting of the quantum numbers transferring from the flucton fragmentation region to the central one in triple reggeon like approach \[16\]. In the last case these quantum numbers transferring
in $t$-channel determine the number and the type of Regge trajectories which have to be included in the "shoulders" of the triple like reggeon diagram.

To calculate the hadron production rates in the cumulative region we need also to analyze the process of fragmentation of one (or several) fast quark(s) into hadrons of different flavors. In principle the different versions of hadronization mechanisms of the produced fast cumulative quarks into colorless states (hadrons) can be suggested. Apparently that in the case of the production of cumulative protons the hadronization through the coalescence of three cumulative quarks is favorable than the usual hadronization through one cumulative quark fragmentation into proton. This leads to the higher production rates for the protons compared with the pions but approximately with the same slopes of the protons and mesons spectra in agreement with experiment.

The idea that in QCD a quark can hadronize by coalescing with a comoving spectator parton was suggested in the paper \[19\]. It was used later for the description of the fragmentation of protons and pions into charm and beauty hadrons at large $x$ \[20, 21\]. It was shown that the coalescence or recombination of one or both intrinsic charm quarks with spectator valence quarks of the Fock state leads in a natural way to leading charm and beauty production.

The conception of quark fusion for the description of hadronic fragmentation into $\pi^\pm$ and $K^\pm$ mesons was also used in the papers \[22, 23\]. It was shown that the valence quarks of the projectile hadron play a dominating role in meson production in the projectile fragmentation region and while the $x$-distribution for $\pi^+, \pi^-$ and $K^+$ are the convolutions of the $u_v, d_v$ and $u_g$ valence quarks with those of the corresponding antiseaquarks $\bar{d}_s, \bar{u}_s$ and $\bar{s}_s$, the $x$-distribution for $K^-$ is that of the a sea quark $s_s$ and an antiseaquark $\bar{d}_s$. The spin properties of these fragmentation processes have been also studied within framework of this approach \[24\]-\[26\]. In particular it has been shown that the observed left-right asymmetry can be readily described in the framework of a relativistic quark model in which the observed $\pi^+$ and $\pi^-$ are respectively the fusion products of the valence quarks $u_v$ and $d_v$ of $p(\uparrow)$ and antiseaquarks $\bar{d}_s$ and $\bar{u}_s$.

Note that both the intrinsic mechanism of the cumulative quark production when the quarks of several nucleons concentrated in one nuclear flucton transfer their longitudinal momenta to the distinguished quark and the hadronization through the cumulative quarks coalescence break the QCD factorization theorem.

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Figure 9: $I_A = \frac{d\sigma}{dx}(mb)$ is the calculated inclusive cross-section (per nucleon) for cumulative charged pion production on $^{181}$Ta at $\sqrt{s} = 23.5 \text{ GeV}$ (solid curve) and $1800 \text{ GeV}$ (dashed one). $\Delta$ - the experimental data [12] on the cumulative charged pion production on $^{181}$Ta by 400 GeV incident proton beam. $F_2^{A}/A$ is the calculated nuclear structure function for the $^{181}$Ta at $Q^2 = 50 \text{ GeV}^2$ (dashed curve) and $^{12}$C at $Q^2 = 100 \text{ GeV}^2$ (solid curve). $\Box$ and $\times$ - the experimental data [13] on the $^{12}$C structure function at $Q^2 = 61 \text{ GeV}^2$ and $150 \text{ GeV}^2$, respectively.