Fluctuation Theorem in Spintronics

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Abstract. Microscopic reversibility is a key in deriving the Onsager relation. It even leads a new exact relationship that would be valid far from equilibrium, called fluctuation theorem (FT). The FT provides a precise statement for the second law of thermodynamics; and remarkably, reproduces the linear response theory. We consider the FT in the spin-dependent transport and derive universal relations among nonlinear spin and charge transport coefficients. We apply the relations to a quantum dot embedded in a two-terminal Aharonov-Bohm interferometer and check that the relations are satisfied.

1. Introduction

According to the second law of thermodynamics, the entropy of a macroscopic system driven out of equilibrium increases with time until the equilibrium is reached. Thus the dynamics of such a system is irreversible. In contrast, in a mesoscopic system and in short time $\tau$, the entropy production of the system $\Delta S$ may either increase or decrease. The probability distribution $P_\tau(\Delta S)$, obeys a simple relation, known as the ‘fluctuation theorem’ (FT) [1]:

$$\lim_{\tau \to \infty} \frac{1}{\tau} \ln \frac{P_\tau(\Delta S)}{P_\tau(-\Delta S)} = \lim_{\tau \to \infty} \frac{\Delta S}{\tau}.$$  

FT quantitatively characterizes the probability of reversal processes induced by the thermal fluctuations. It relays only on the microreversibility of the underlying equation of motion, thus remarkably, it remains valid even far from equilibrium.

The FT is fundamentally important in mesoscopic physics. The Jarzynski equality [2], a derivative of FT, leads one form of the second law of thermodynamics. The FT reproduces the fluctuation-dissipation theorem and the Onsager symmetry relations [3]. Further, the FT is powerful to extend the Onsager relations to the nonlinear transport regime [4, 5, 6, 7, 8, 9, 10]. By now the FT and the Jarzynski equality have been verified experimentally in chemical physics and biophysics [11, 12].

In quantum transport problem, the Joule heat $IV$, where $I$ is the current and $V$ is the voltage drop across a mesoscopic conductor, is responsible for the entropy production (we will set $e = k_B = h = 1$). The FT is formulated for the probability $P_\tau(q)$ of transmitted charge $q$ ($I = \lim_{\tau \to \infty} q/\tau$) through a mesoscopic conductor,

$$\lim_{\tau \to \infty} \frac{1}{\tau} \ln \frac{P_\tau(q)}{P_\tau(-q)} = \frac{IV}{T},$$  

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where $T$ is the temperature. Though several attempt have been done to apply the FT to mesoscopic quantum transport [4, 5, 6, 7, 8, 9, 10], much remains unknown for the role of the spin-dependent transport.

2. Universal relations in nonlinear spin-dependent transport

In the last few years, the full counting statistics (FCS) [13, 14, 15] has been recognized as a suitable framework for the FT in quantum transport [4, 6, 7, 8, 10]. FCS addresses the cumulant generating function (CGF):

$$\mathcal{F}(\lambda) = \lim_{\tau \to \infty} \ln Z(\lambda)/\tau, \quad Z(\lambda) = \sum_q P(q) e^{i\lambda q},$$

(3)

where $\lambda$ is called the counting field [13].

Recently, the FT was generalized to the quantum transport regime in the presence of interaction and a magnetic field $B$ [6]. For two-terminal systems, the FT reads,

$$\mathcal{F}(\lambda; B) = \mathcal{F}(-\lambda + iV/T; -B), \quad P(q; B) = P(-q; -B) e^{qV/T},$$

(4)

One important consequence of Eq. (4) is the universal relations among transport coefficients [6, 9]. The transport coefficient $L$ is introduced by expanding the current cumulant with respect to $V$:

$$\langle I^n \rangle = \frac{\partial^n \mathcal{F}(\lambda; B)}{\partial (i\lambda)^n} \bigg|_{\lambda=0} = \sum_{m=0}^{\infty} L_m^n(B) \frac{(V/T)^m}{m!}.$$

(5)

The FT (4) leads to the Kubo formula $L_1^1 = L_0^2/2$ and the Onsager relation $L_{1-} = 0$, where $L_{0-}^m = L_0^m(B) \pm L_m^0(-B)$ is the symmetrized/anti-symmetrized transport coefficient. Furthermore, nontrivial relations among higher-order coefficients are obtained [6]:

$$L_{2-}^2 = \frac{1}{3} L_{1-}^3, \quad L_{2+}^1 = L_{1+}^2, \quad L_{0+}^3 = 0.$$

(6)

This is significant in that the skewness $L_{0-}^3$ can be finite even in equilibrium and proportional to the asymmetric component of nonlinear conductance $L_{1-}^1$ as well as the linear response of noise $L_{2-}^2$.

In the presence of the spin-degrees of freedom, we have to begin with the FT for multi-terminal systems [6]. For a simple two terminal setup, we have actually 4 terminals including spin degrees of freedoms. Then the symmetry reads,

$$\mathcal{F}(\lambda, \lambda_L^\uparrow, \lambda_R^\downarrow; B) = \mathcal{F}(-\lambda + iV/T, -\lambda_L^\uparrow + i\mu_L/T, -\lambda_R^\downarrow + i\mu_R/T; -B),$$

$$P(q, q_L^\uparrow, q_R^\downarrow; B) = P(-q, -q_L^\uparrow, -q_R^\downarrow; -B) e^{qV + q_L^\uparrow V_L^\uparrow + q_R^\downarrow V_R^\downarrow}/T,$$

(7)

(8)

where the voltage drop $V$ is related with the average chemical potential $\mu_r = (\mu_r^\uparrow + \mu_r^\downarrow)/2$ as $V = \mu_R - \mu_L$. The spin current flowing out of the lead $r$ is given by $I_r^s = \lim_{\tau \to \infty} q_r^s/\tau$ and $V_r^s = \mu_r^\uparrow - \mu_r^\downarrow$ is the spin accumulation inside the lead $r$. The results appears to be reasonable: The exponent is related with the Joule heat $Q = qV + q_L^\uparrow V_L^\uparrow + q_R^\downarrow V_R^\downarrow$ caused by the charge transport between two leads and the spin relaxation in each lead. Further, we extend the transport coefficient Eq. (5) for the spin and charge transport as,

$$L_{m_1m_2m_3}^{n_1n_2n_3}(B) = T^{m_1+m_2+m_3} \frac{\partial^{n_1+n_2+n_3}}{\partial V^{m_1} \partial V_L^{m_2} \partial V_R^{m_3} \partial (i\lambda)^n_1 \partial (i\lambda_L^\uparrow)^n_2 \partial (i\lambda_R^\downarrow)^n_3} \mathcal{F}(\lambda, \lambda_L^\uparrow, \lambda_R^\downarrow; B) \bigg|_{\lambda=\lambda_L^\uparrow=\lambda_R^\downarrow=0} \cdot \bigg|_{V=V_L^\uparrow=V_R^\downarrow=0}$$

(9)
Then from Eq. (7), we immediately obtain, for example, the following relations,

\[ L_{100}^{00}(B) = \frac{L_{000}^{00}(B)}{2}, \quad L_{100}^{01}(B) = \frac{L_{000}^{01}(B)}{2}, \quad L_{010}^{00}(B) = L_{000}^{00}(-B), \quad L_{010}^{01}(B) = L_{001}^{01}(-B). \]  

(10)

They are the fluctuation-dissipation theorem and the Onsager-Casimir relations. Further, for higher order nonlinear transport coefficients, we obtain, for example,

\[
\begin{align*}
L_{000}^{03} &= L_{000}^{00} = L_{000}^{03} = L_{000}^{21} = L_{000}^{02} = L_{000}^{02} = L_{000}^{02} = 0, \\
L_{100}^{10} &= L_{100}^{10}, \\
L_{010}^{02} &= L_{010}^{02} = L_{010}^{02} = L_{010}^{02} = L_{010}^{02} = L_{010}^{02} = L_{010}^{02} = L_{010}^{02}, \\
L_{020}^{01} &= L_{020}^{01}, \\
L_{011}^{01} &= L_{011}^{01} = 2 L_{110}^{02} + L_{020}^{01}, \\
L_{010}^{01} &= L_{010}^{01} = 2 L_{110}^{02} + L_{020}^{01} = 2 L_{110}^{02} + L_{020}^{01}, \\
L_{020}^{01} &= L_{020}^{01} = 2 L_{110}^{02} + L_{020}^{01}, \\
L_{020}^{01} &= L_{020}^{01} = 2 L_{110}^{02} + L_{020}^{01} = 2 L_{110}^{02} + L_{020}^{01}, \\
L_{020}^{01} &= L_{020}^{01} = 2 L_{110}^{02} + L_{020}^{01}.
\end{align*}
\]

(11)-(17)

In this way, we obtain infinite number of exact relations. They are universal for the spin and charge transport. Though they appear nontrivial, the micro-reversibility implies only general restriction for the transport coefficients. Therefore, the physical meaning is still not so clear. Even it is not clear if such nonlinear transport coefficients actually remain finite or not. Next, we will apply the relations to a specific system.

3. Two-terminal Aharonov-Bohm interferometer

Here, we will apply the relation to the simplest example, a two-terminal Aharonov-Bohm interferometer embedded with a quantum dot in one arm [see Fig. 1 (a)]. The dot part is described by the single-site Anderson model and within the Hartree approximation out of equilibrium [7], we obtain the nonlinear transport coefficients satisfying the FT (6) as,

\[
L_{2-}^{1} = \frac{L_{1-}^{1}}{3} = \frac{L_{0-}^{3}}{6} = 4 U_{\text{eff}} \chi_{NI} - \chi_{I,IN} + \chi_{I,IN} + L_{2-}^{1} = 4 U_{\text{eff}} \chi_{NI} + \chi_{I,IN} + L_{0-}^{3} = 0.
\]

(18)

where, \( V \) is fixed as 0. \( U_{\text{eff}} = U/(1 - U_{\chi_{NN}}) \) is the screened Coulomb interaction written with the charge susceptibility \( \chi_{NN} = \partial n_{\sigma}/\partial \epsilon_{D} \), where \( \epsilon_{D} \) is the dot-level and \( n_{\sigma} \) is the number of spin \( \sigma \) in the dot. Other susceptibilities are \( \chi_{NI} = T \partial n_{\sigma}/\partial V \), \( \chi_{I,IN} = \partial S_{I}/\partial \epsilon_{D} \), where \( S_{I} = 2T \partial I/\partial V \) is the equilibrium current noise. The subscript \( \pm \) specify the symmetrized/anti-symmetrized component of the susceptibility, e.g. \( \chi_{NI} = [\chi_{NI}(B) \pm \chi_{NI}(-B)]/2 \). Figure 1 shows the Aharonov-Bohm phase dependence of the nonlinear transport coefficients. As we can see they are not zero only when the Coulomb interaction inside the dot \( U \) is finite. This is because of the absence of the centro-symmetry of the system, the electrical magneto-chiral effect [16]. In our case, the mirror symmetry along the horizontal axis [dotted line in Fig. 1 (a)] is absent and thus generally \( \mathcal{F}(B) \neq \mathcal{F}(-B) \). In this way, in addition to the microreversibility, the spacial symmetry can provide restrictions for the nonlinear transport coefficients.

4. Conclusion

Based on the fluctuation theorem, we discuss the universal relations among nonlinear transport coefficients for the charge and spin transport. We obtained universal relations among nonlinear...
charge and spin transport coefficients. We checked the relations by applying them to the two-terminal quantum-dot Aharonov-Bohm interferometer. As a future problem, it would be interesting to also account for the heat transport [10] as well as the spin transport, which would be crucial to analyze, for example, the universal aspects of the spin-Seebeck effect far from equilibrium [17].

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References
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[1] D. J. Evans, E. D. G. Cohen and G. P. Morris, Phys. Rev. Lett 71, 2401 (1993).
[2] C. Jarzynski, Phys. Rev. Lett. 78, 2690 (1997).
[3] G. Gallavotti, Phys. Rev. Lett. 77, 4334 (1996).
[4] J. Tobiska and Yu. V. Nazarov, Phys. Rev. B 72, 235328 (2005).
[5] M. Esposito, U. Harbola and S. Mukamel, Phys. Rev. B 75, 155316 (2007).
[6] K. Saito and Y. Utsumi, Phys. Rev. B 78, 115429 (2008).
[7] Y. Utsumi and K. Saito, Phys. Rev. B 79, 235311 (2009).
[8] H. Förster, and M. Büttiker, Phys. Rev. Lett. 101, 136805 (2008).
[9] D. Andrieux, P. Gaspard, T. Monnai, and S. Tasaki, New J. Phys. 11, 043014 (2009).
[10] E. Iyoda, Y. Utsumi, T. Kato, and K. Saito, arXiv:0903.0985.
[11] G. M. Wang, E. M. Sevick, E. Mittag, D. J. Searles, D. Evans, Phys. Rev. Lett. 89, 050601 (2002).
[12] J. Liphardt, S. Dumont, S. B. Smith, I. Ticono Jr., and C. Bustamante, Science 296, 1832 (2002).
[13] L. S. Levitov and G. B. Lesovik, JETP Lett. 58, 230 (1993); L. S. Levitov, H.-W. Lee, and G. B. Lesovik, J. Math. Phys., 37, 4845 (1996).
[14] Quantum Noise in Mesoscopic Physics, Vol. 97 of NATO Science Series II: Mathematics, Physics and Chemistry edited by Yu. V. Nazarov (Kluwer Academic Publishers, Dordrecht/Boston/London, 2003).
[15] D. A. Bagrets, Y. Utsumi, D. S. Golubev, G. Schön, Fortschr. der Physik 54, 917-938 (2006).
[16] D. Sánchez and M. Büttiker, Phys. Rev. Lett. 93, 106802 (2004); G. L. J. A. Rikken, J. Folling, and P. Wyder, Phys. Rev. Lett. 87, 236602 (2001).
[17] K. Uchida, S. Takahashi, K. Harii, J. Ieda, W. Koshibae, K. Ando, S. Maekawa and E. Saitoh, Nature 455, 778-781 (2008).