A novel mass generation scheme for an Abelian vector field

Constantin Bizdadea∗, Solange-Odile Saliu†
Department of Physics, University of Craiova
13 Al. I. Cuza Str., Craiova 200585, Romania

Abstract

A novel mass generation procedure for an Abelian vector field is proposed. This procedure is based on the construction of a class of gauge theories whose free limit describes a free massless vector field and a set of massless real scalar fields by means of the antifield-BRST deformation technique. The relationship between our results and those arising from the Higgs mechanism based on the spontaneous symmetry breaking of an Abelian gauge symmetry is emphasized. Some examples with one, two, and three scalars are given.

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1 Introduction

It is commonly believed that the only possible way to generate vector field masses is the Higgs mechanism based on spontaneous symmetry breaking [1–4]. The starting point of the simplest Abelian version of the Higgs mechanism is the Lagrangian action (expressed in terms of two real scalars)

\[ W_0[A^\mu, \varphi_1, \varphi_2] = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \varphi_1) D^\mu \varphi_1 \\
+ \frac{1}{2} (D_\mu \varphi_2) D^\mu \varphi_2 - V(\varphi_1, \varphi_2) \right], \]

(1)

∗E-mail: bizdadea@central.ucv.ro
†E-mail: osaliu@central.ucv.ro
where

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \]  
\[ D_\mu \varphi_1 = \partial_\mu \varphi_1 - q \varphi_2 A_\mu, \quad D_\mu \varphi_2 = \partial_\mu \varphi_2 + q \varphi_1 A_\mu, \]  
\[ V(\varphi_1, \varphi_2) = \frac{1}{2} \mu^2 (\varphi_1^2 + \varphi_2^2) + \frac{1}{16} \lambda (\varphi_1^2 + \varphi_2^2)^2 \]

and the real constants \( \mu^2 \) and \( \lambda \) are taken such that \( \mu^2 < 0 \) and \( \lambda > 0 \).

Action (1) is invariant under the gauge transformations

\[ \delta_\epsilon A^\mu = \partial^\mu \epsilon, \quad \delta_\epsilon \varphi_1 = q \varphi_2 \epsilon, \quad \delta_\epsilon \varphi_2 = -q \varphi_1 \epsilon. \]  

In this setting we find that the potential of the form (1) has an absolute minimum for \( \sqrt{\varphi_1^2 + \varphi_2^2} = \sqrt{-4\mu^2/\lambda} \equiv v_0 \). Introducing some new fields defined by \( \tilde{\varphi}_1 = \varphi_1 - v_0 \) and \( \tilde{\varphi}_2 = \varphi_2 \) (whose associated field operators display zero vacuum expectation values) and reformulating relations (1) and (5) accordingly, we find the action

\[ W_0[A^\mu, \tilde{\varphi}_1, \tilde{\varphi}_2] = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} q^2 v_0^2 A_\mu A^\mu + \frac{1}{2} \mu^2 (\tilde{\varphi}_1^2 + \tilde{\varphi}_2^2) + \frac{1}{16} \lambda (\tilde{\varphi}_1^2 + \tilde{\varphi}_2^2)^2 \right. 
\]

\[ \left. + \frac{1}{2} (\partial_\mu \tilde{\varphi}_1) \partial^\mu \tilde{\varphi}_1 + \frac{1}{2} (\partial_\mu \tilde{\varphi}_2) \partial^\mu \tilde{\varphi}_2 \right. 
\]

\[ \left. - \frac{1}{16} \lambda (\tilde{\varphi}_1^2 + \tilde{\varphi}_2^2) (\tilde{\varphi}_1^2 + \tilde{\varphi}_2^2 + 4v_0 \tilde{\varphi}_1) \right) 
\]

\[ + q A_\mu (\tilde{\varphi}_1 \partial^\mu \tilde{\varphi}_2 - \tilde{\varphi}_2 \partial^\mu \tilde{\varphi}_1) + q v_0 A_\mu \partial^\mu \tilde{\varphi}_2 \]

\[ + \frac{1}{2} q^2 (\tilde{\varphi}_1^2 + \tilde{\varphi}_2^2 + 2v_0 \tilde{\varphi}_1) A_\mu A^\mu \right], \]

invariant under the gauge transformations

\[ \delta_\epsilon A^\mu = \partial^\mu \epsilon, \quad \delta_\epsilon \tilde{\varphi}_1 = q \varphi_2 \epsilon, \quad \delta_\epsilon \tilde{\varphi}_2 = -q (\tilde{\varphi}_1 + v_0) \epsilon. \]  

Formula (6) may be synthesized into: (a) the vector field \( A^\mu \) acquires the mass \( \sqrt{q^2 v_0^2} \); (b) the scalar field \( \tilde{\varphi}_1 \) (the Higgs boson) becomes massive, with the mass equal to \( \sqrt{-2\mu^2} \); (c) the scalar field \( \tilde{\varphi}_2 \) (the Goldstone boson) is massless. The previous conclusions are involved by the existence of the solution \( v_0 \) that minimizes the potential (1).

Let us consider now the case where \( \mu^2 \) is arbitrary and \( \lambda > 0 \) in formula (1). If we add to action (1) the functional

\[ \tilde{W}_0[A^\mu, \varphi_1, \varphi_2] = \int d^4x \left[ q v A_\mu \partial^\mu \varphi_2 + \frac{1}{2} q^2 v^2 A_\mu A^\mu + q^2 v \varphi_1 A_\mu A^\mu \right] \]
where $v$ is an arbitrary, nonvanishing real constant, then we find that action

$$\bar{W}_0[A^\mu, \varphi_1, \varphi_2] = W_0[A^\mu, \varphi_1, \varphi_2] + \tilde{W}_0[A^\mu, \varphi_1, \varphi_2],$$

is invariant under the gauge transformations

$$\delta_\epsilon A^\mu = \partial^\mu \epsilon, \quad \delta_\epsilon \varphi_1 = q \varphi_2 \epsilon, \quad \delta_\epsilon \varphi_2 = -(\varphi_1 + v) \epsilon.$$

We mention that in (9) and (10) there is no a priori relation among the constants $\mu^2$, $\lambda$, and $v$. Thus, expression (9) emphasizes that the mass of the vector field $A^\mu$, $\sqrt{q^2v^2}$, is independent both of $\mu^2$ and $\lambda$. In fact, the constants $\mu^2$ and $\lambda$ are involved in (9) in the self-interactions and (possibly) some mass terms of the scalar fields. Let us take a fixed value of $v$, say $\bar{v}$. If $\mu^2 \geq 0$, then the scalars $\varphi_1$ and $\varphi_2$ are massive, their masses being $\sqrt{\mu^2 + (3/4)\lambda \bar{v}^2}$ and $\sqrt{\mu^2 + (1/4)\lambda \bar{v}^2}$, respectively. Let us analyze now the case $\mu^2 < 0$. If $\mu^2$ and $\lambda$ satisfy the inequality $(-4\mu^2/\lambda) < \bar{v}^2$, then the two scalars remain massive and their masses are precisely those from the previous situation. If $(-4\mu^2/\lambda) > \bar{v}^2 > (-4\mu^2/3\lambda)$, then the scalar $\varphi_1$ remains massive (the value of its mass being the same from the above situations) whereas the quantity $(-1/2)(\mu^2 + (1/4)\lambda \bar{v}^2)$ (that multiplies $\varphi_2^2$) should be regarded as a parameter, and so on. Moreover, it is simple to see that if $\mu^2 + (1/4)\lambda \bar{v}^2 = 0 \Leftrightarrow \bar{v} = v_0$, then the gauge theory described by (9) and (10) reduces to that governed by relations (6) and (7) modulo the identifications $\varphi_1 \leftrightarrow \tilde{\varphi}_1$, $\varphi_2 \leftrightarrow \tilde{\varphi}_2$. These considerations argue that relations (9) and (10) underlie a more general class of gauge theories than that corresponding to formulas (6) and (7).

The previous discussion raises the following problem: is there a procedure, different from the Higgs mechanism, by which one may generate mass for a vector field in the context of its interactions to an arbitrary set of real scalar fields? The aim of this paper is to investigate the above problem. In view of this, we implement the following steps: (i) we start from a free theory in $D = 4$ whose Lagrangian action is expressed like the sum between the Maxwell action for a single vector field and that for a (finite) collection of massless real scalar fields; (ii) we construct a general class of gauge theories whose free limit is that from step (i) by means of the deformation of the solution to the master equation [5, 6] with the help of local BRST cohomology [7–9]. On the one hand, the procedure described so far does not account in
any way for the Higgs mechanism. On the other hand, it will be proved to produce the next results: (iii) the vector field acquires mass irrespective of the number of scalar fields from the collection; (iv) the gauge transformations are deformed with respect to those from the free limit, but the associated gauge algebra remains Abelian; (v) the propagator of the massive vector field emerging from the gauge-fixed action behaves, in the limit of large Euclidean momenta, like that from the massless case. In this way, the answer to the investigated problem is affirmative. In the meantime, the method based on steps (i) and (ii) enables a proper comparison with the Higgs mechanism. In this context we show that our approach: (vi) is a cohomological extension of the Abelian Higgs mechanism; (vii) reveals an appropriate interpretation of the Higgs mechanism in the framework of the BRST symmetry. Outcomes (iii)–(vii) stand for the main results of our paper.

We stress that, although the antifield-BRST deformation method is well known [5, 6], its application to a free theory with an Abelian vector field and a set of massless real scalar fields with the aim of generating mass for the vector field in mind has not been approached so far. This represents the core novelty of our scheme.

The paper is organized into nine sections. In Section 2 we construct the antifield-BRST symmetry of the free theory. Section 3 briefly reviews the antifield-BRST deformation procedure. In Section 4 we compute the deformed solution to the master equation for the theory under consideration in the presence of some standard hypotheses from field theory. The identification of the class of interacting gauge theories is developed in Section 5. In Section 6 we focus on the comparison between the Abelian Higgs mechanism and our procedure, while in Section 7 we give an interpretation of the Abelian Higgs mechanism in the light of the antifield-BRST symmetry. Section 8 is devoted to the exemplification of our general results to three particular cases. Section 9 closes the paper with the main conclusions.

2 BRST symmetry of the free theory

We start with a Lagrangian action written as the sum between the action of an Abelian vector field $A^\mu$ and that describing a finite set of massless real scalar fields $\{\varphi^A\}_{A=1,N_0}$

$$S_L^0[A^\mu, \varphi^A] = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} k_{AB} (\partial_{\mu} \varphi^A) \partial_{\mu} \varphi^B \right]$$
\[ S_{0}^{L,\text{Maxwell}} [A_{\mu}] + S_{0}^{L,\text{scalar}} [\varphi^{A}], \]  

where the Abelian field strength is like in (2). We work with a mostly negative metric in a Minkowski spacetime of dimension \( D = 4 \), \( \sigma^{\mu\nu} = \sigma_{\mu\nu} = (+-\ -\ -) \) and a metric tensor \( k_{AB} \) with respect to the matter field indices (constant, symmetric, invertible, and positively defined), \( \varphi_{A} = k_{AB} \varphi^{B} \). In this context, the elements of its inverse will be symbolized by \( k^{AB} \). It is easy to see that the number of physical degrees of freedom of the starting theory is equal to \( N_{0} + 2 \).

Action (11) is invariant under the gauge transformations

\[ \delta_{\epsilon} A^{\mu} = \partial^{\mu} \epsilon, \quad \delta_{\epsilon} \varphi^{A} = 0, \quad A = \overline{1, N_{0}}, \]  

that are Abelian and irreducible (independent). The previous properties combined with the linearity of the field equations following from action (11) in all fields allow us to conclude that the overall free model under consideration is a linear gauge theory with a definite Cauchy order, equal to two.

The construction of the antifield-BRST symmetry [10–19] for this free theory starts with the identification of the algebra on which the BRST differential \( s \) acts. The generators of the BRST algebra are of two kinds: fields/ghosts and antifields. The ghost spectrum for the model under study reduces to the fermionic ghost \( \eta \) associated with the gauge parameter \( \epsilon \) from (12). The antifield spectrum is organized into the antifields \( \{ A^{\ast}_{\mu}, \varphi^{A\ast} \} \) of the original fields together with the antifield of the ghost, \( \eta^{\ast} \). The Grassmann parity (\( \varepsilon \)) of the BRST generators reads

\[ \varepsilon(A^{\mu}) = \varepsilon(\varphi^{A}) = 0, \quad \varepsilon(\eta) = 1, \]  

(13)

\[ \varepsilon(A^{\ast}_{\mu}) = \varepsilon(\varphi^{A\ast}) = 1, \quad \varepsilon(\eta^{\ast}) = 0. \]  

(14)

Since the gauge generators from (12) are field-independent, it follows that the BRST differential \( s \) simply reduces to

\[ s = \delta + \gamma, \]  

(15)

where \( \delta \) signifies the Koszul–Tate differential, graded by the antifield number \( \text{agh}(\delta) = -1 \) and \( \gamma \) stands for the longitudinal exterior derivative (in this case a true differential), whose degree is named pure ghost number \( \gamma \) (\( \text{pgh}(\gamma) = 1 \)). These two degrees do not interfere \( \text{agh}(\gamma) = 0, \text{pgh}(\delta) = 0 \). The overall degree that grades the BRST algebra is known as the ghost
number (gh) and is defined like the difference between the pure ghost number
and the antifield number, such that $\text{gh}(s) = \text{gh}(\delta) = \text{gh}(\gamma) = 1$. According
to the standard rules of the BRST method, the corresponding degrees of the
generators from the BRST algebra are valued like

$$\text{agh}(A_\mu) = 0, \quad \text{agh}(\varphi^A) = 0, \quad \text{agh}(\eta) = 0, \quad (16)$$
$$\text{agh}(A^*_\mu) = 1, \quad \text{agh}(\varphi^*_A) = 1, \quad \text{agh}(\eta^*) = 2, \quad (17)$$
$$\text{pgh}(A_\mu) = 0, \quad \text{pgh}(\varphi^A) = 0, \quad \text{pgh}(\eta) = 1, \quad (18)$$
$$\text{pgh}(A^*_\mu) = 0, \quad \text{pgh}(\varphi^*_A) = 0, \quad \text{pgh}(\eta^*) = 0. \quad (19)$$

The actions of $\delta$ and $\gamma$ on the BRST generators that enforce the fundamental
cohomological requirements of the antifield BRST theory $^{[14,19]}$ are given
by

$$\delta A_\mu = 0, \quad \delta \varphi_A = 0, \quad \delta \eta = 0, \quad (20)$$
$$\delta A^*_\mu = \partial^\nu F_{\nu\mu}, \quad \delta \varphi^*_A = k_{AB} \partial_\mu \varphi^B, \quad \delta \eta^* = - \partial^\mu A^*_\mu, \quad (21)$$
$$\gamma A_\mu = \partial^\mu \eta, \quad \gamma \varphi_A = 0, \quad \gamma \eta = 0, \quad (22)$$
$$\gamma A^*_\mu = 0, \quad \gamma \varphi^*_A = 0, \quad \gamma \eta^* = 0. \quad (23)$$

where both operators were taken to act like right derivations.

The Lagrangian BRST differential admits a canonical action in a structure
named antibracket and defined by decreeing the fields/ghosts conjugated with
the corresponding antifields, $s \cdot (\cdot, S)$, where $(,)$ signifies the antibracket
and $S$ denotes the canonical generator of the BRST symmetry. It is a bosonic
functional of ghost number zero, involving both field/ghost and antifield
spectra, that obeys the master equation

$$(S, S) = 0. \quad (24)$$

The master equation is equivalent to the second-order nilpotency of $s$ and
its solution, $S$, encodes the entire gauge structure of the associated theory.
The solution to the master equation for the free model under study takes the
simple form

$$S = S_0^L [A^\mu, \varphi^A] + \int d^4 x A^*_\mu \partial^\mu \eta. \quad (25)$$

### 3 Deformation procedure: a brief review

Now, we consider the problem of consistent interactions that can be intro-
duced in gauge field theories in such a way that the couplings preserve the
original number of independent gauge symmetries. This matter is addressed by means of reformulating the problem of constructing consistent interactions as a deformation problem of the solution to the master equation corresponding to a given “free” theory \([5, 6]\) in the framework of the local BRST cohomology \([7–9]\). Such a reformulation is possible due to the fact that the solution to the master equation contains all the information on the gauge structure of the theory. If a consistent interacting gauge theory can be constructed, then the solution \(S\) to the master equation associated with the “free” theory can be deformed into a solution \(\bar{S}\)

\[
S \rightarrow \bar{S} = S + gS_1 + g^2S_2 + g^3S_3 + g^4S_4 + \cdots, \quad \varepsilon(\bar{S}) = 0, \quad gh(\bar{S}) = 0 \quad (26)
\]

of the master equation for the deformed theory that displays the same ghost and antifield spectra, namely,

\[
(\bar{S}, \bar{S}) = 0. \quad (27)
\]

According to the deformation parameter \(g\), equation (27) splits into:

\[
g^0 : (S, S) = 0, \quad (28)
\]

\[
g^1 : sS_1 = 0, \quad (29)
\]

\[
g^2 : \frac{1}{2}(S_1, S_1) + sS_2 = 0, \quad (30)
\]

\[
g^3 : (S_1, S_2) + sS_3 = 0, \quad (31)
\]

\[
g^4 : \frac{1}{2}(S_2, S_2) + (S_1, S_3) + sS_4 = 0, \quad (32)
\]

The first equation is satisfied by hypothesis. The remaining ones are to be solved recursively, from lower to higher orders, such that each equation corresponding to a given order of perturbation theory, say \(i (i \geq 1)\), contains a single unknown functional, namely, the deformation of order \(i\), \(S_i\). Once the deformation equations (29)–(32), etc., have been solved by means of specific cohomological techniques, from the consistent nontrivial deformed solution to the master equation one can extract all the information on the gauge structure of the resulting interacting theory. It is important to mention that the antifield-BRST deformation method briefly exposed in the above has been successfully applied to various models \([20–33]\).
4 Consistent interactions between a collection of scalar fields and one vector field: deformed solution to the master equation

In the sequel we apply the deformation procedure exposed previously with the purpose of generating consistent interacting gauge theories in $D = 4$ whose free limit is precisely the gauge theory described by relations (11) and (12). We are interested only in (nontrivial) deformations that comply with the standard hypotheses from field theory: analyticity in the coupling constant, Lorentz covariance, spacetime locality, and Poincaré invariance. Moreover, we require that the maximum number of derivatives allowed within the interaction vertices is equal to two, i.e. the maximum number of derivatives from the free Lagrangian (derivative-order assumption).

If we make the notation $S_1 = \int d^4 x \, a$, with $a$ a local function, then equation (29), which we have seen that controls the first-order deformation, takes the local form

$$sa = \partial_\mu j^\mu, \quad gh(a) = 0, \quad \varepsilon(a) = 0,$$  \hspace{1cm} (33)

for some local $j^\mu$. Its solution is unique up to addition of trivial quantities $a \rightarrow a' = a + s\tilde{a} + \partial_\mu \tilde{j}^\mu$, $j^\mu \rightarrow j'^\mu = j^\mu + s\tilde{j}^\mu + \partial_\nu k^{\nu\mu}$ (with $k^{\nu\mu} = -k^{\mu\nu}$), in the sense that $sa - \partial_\mu j^\mu \equiv sa' - \partial_\mu j'^\mu = 0$. At the same time, if the general solution to (33) is found to be completely trivial, $a = s\tilde{a} + \partial_\mu \tilde{j}^\mu$, then it can be made to vanish, $a = 0$. In other words, $a$ is constrained to pertain to a nontrivial class of the local BRST cohomology (cohomology of $s$ modulo $d$ — with $d$ the exterior differential in spacetime) in $gh = 0$ computed in the algebra of (local) nonintegrated densities, $H^0(s|d)$. In addition, all such solutions for $a$ will be selected such as to comply with the working hypotheses mentioned in the above. The nonintegrated density of the first-order deformation splits naturally into three components

$$a = a^{(A)} + a^{(\varphi)} + a^{\text{int}},$$  \hspace{1cm} (34)

where $a^{(A)}$ and $a^{(\varphi)}$ describe the self-interactions of the vector field $A^\mu$ and respectively of the scalar fields $\{\varphi^A\}$, whereas $a^{\text{int}}$ governs the couplings among them. The three components display different contents of BRST generators ($a^{(A)}$ involves only $\{A^\mu, \eta, A^*_\mu, \eta^*_\}$, $a^{(\varphi)}$ only $\{\varphi^A, \varphi^*_A\}$, and $a^{\text{int}}$ mixes both sectors), such that equation (33) becomes equivalent to three
independent equations, one for each piece,

\[ \begin{align*}
  sa^{(A)} &= \partial_\mu j^{\mu}_{(A)}, \\
  sa^{(\varphi)} &= \partial_\mu j^{\mu}_{(\varphi)}, \\
  sa^{\text{int}} &= \partial_\mu j^{\mu}_{\text{int}}.
\end{align*} \tag{35} \]

The solution to the first equation from (35) is completely trivial \[20\], \( a^{(A)} = 0 \), while the solution to the second equation reduces to its component of antifield number 0

\[ a^{(\varphi)} = \frac{1}{2} \mu_{AB}(\varphi) (\partial_\mu \varphi^A) \partial^\mu \varphi^B - \mathcal{V}(\varphi), \tag{36} \]

where \( \mu_{AB} \) and \( \mathcal{V} \) are some arbitrary, smooth real functions depending only on the undifferentiated scalar fields, with

\[ \mu_{AB}(\varphi) = \mu_{BA}(\varphi), \quad \mu_{AB}(\varphi) \neq \partial_\mu \varphi^A + \partial_\nu \varphi^B. \tag{37} \]

Conditions (37) ensure the nontriviality of \( a^{(\varphi)} \) in \( H^0(s|d) \).

In order to analyze the solutions to the last equation from (35) we decompose \( a^{\text{int}} \) along the antifield number. Since the starting free theory is linear and its Cauchy order is equal to 2 (see (11) and (12)), it follows that we can stop the previously mentioned decomposition in antifield number 2, \( a^{\text{int}} = a^{\text{int}}_0 + a^{\text{int}}_1 + a^{\text{int}}_2 \), with \( \text{agh}(a^{\text{int}}_k) = k \). Relying on the requirement \( \text{gh}(a) = 0 \), it results that \( \text{pgh}(a_k) = k \), and hence we have that \( a^{\text{int}}_2 = \bar{\alpha}_2 \eta^2 \equiv 0 \) due to the fermionic behaviour of the ghost \( \eta \). In consequence, \( a^{\text{int}} \) reduces to the sum between its first two components only, \( a^{\text{int}} = a^{\text{int}}_0 + a^{\text{int}}_1 \). Inserting this decomposition of \( a^{\text{int}} \) together with splitting (15) of \( s \) into the last equation from (35), we arrive at

\[ \begin{align*}
  \gamma a^{\text{int}}_1 &= 0, \\
  \delta a^{\text{int}}_1 + \gamma a^{\text{int}}_0 &= \partial_\mu j^{\mu}_{\text{int},0}.
\end{align*} \tag{38, 39} \]

Strictly speaking, equation (38) should have been written like \( \gamma a^{\text{int}}_1 = \partial_\mu j^{\mu}_{\text{int},1} \). Since the antifield number of both hand sides of this equation is strictly positive (equal to 1), it can be safely replaced by its homogeneous version without loss of nontrivial terms, namely, one can always take \( j^{\mu}_{\text{int},1} = 0 \). The proof of this result is done in a standard manner (for instance, see \[8, 22, 28, 34, 37\]). Equation (38) shows that \( a^{\text{int}}_1 \) can be taken as a \( \gamma \)-closed object of pure ghost number one. By means of formulas (16)–(19), (22), and (23), we find that

\[ a^{\text{int}}_1 = (A^*_\mu h^\mu([\varphi],[F_{\mu\nu}]) + \varphi_A^* h^A([\varphi],[F_{\mu\nu}])) \eta, \tag{40} \]
where the notation \( h([y]) \) means that \( h \) depends on \( y \) and its spacetime derivatives up to a finite order. The existence of the solution \( a_{0}^{\text{int}} \) to equation (39) requires that \( \alpha_{1} = A_{\mu}^{\nu} h^{\nu}([\varphi], [F_{\mu\nu}]) + \varphi^{*} A^{A}([\varphi], [F_{\mu\nu}]) \) should be a nontrivial element of the local homology of the Koszul–Tate differential in \( a_{h} = 1 \), \( H_{1}(\delta|d) \) (meaning that \( \delta \alpha_{1} = \partial_{\mu} l_{\mu} \), with \( \alpha_{1} \neq \delta \beta_{2} + \partial_{\mu} k^{\mu} \)). Taking into account the working hypotheses (including the derivative-order assumption), after some computation we infer that the most general nontrivial representative of \( H_{1}(\delta|d) \) corresponds to

\[
\begin{align*}
    h^{\mu}([\varphi], [F_{\mu\nu}]) &= 0, \\
    h^{A}([\varphi], [F_{\mu\nu}]) &= T^{AB} k_{BC} \varphi^{C} + n^{A},
\end{align*}
\]  
(41)

where \( T^{AB} \) and \( n^{A} \) are some arbitrary, real constants, with

\[
T^{AB} = -T^{BA}.
\]  
(42)

Then, from (40) and (41) we obtain that

\[
a_{1}^{\text{int}} = \varphi^{*} A_{A}^{A} (T^{AB} k_{BC} \varphi^{C} + n^{A}) \eta. 
\]  
(43)

Substituting (43) in (39) we deduce the component of antifield number 0

\[
a_{0}^{\text{int}} = -k_{AB} (T^{A}_{C} \varphi^{C} + n^{A}) A_{\mu} \partial^{\mu} \varphi^{B} + \frac{1}{2} F_{\mu\nu} (U(\varphi) F^{\mu\nu} + \varepsilon_{\mu
u\rho\lambda} \tilde{U}(\varphi) F_{\rho\lambda}), \]

(44)

with \( T^{A}_{C} = T^{AB} k_{BC} \). In formula (44) the objects \( U \) and \( \tilde{U} \) denote some arbitrary, smooth real functions depending on the undifferentiated scalar fields and \( \varepsilon_{\mu
u\rho\lambda} \) stand for the components of the Levi-Civita symbol in \( D = 4 \). In order to avoid trivial couplings the two functions \( U \) and \( \tilde{U} \) should contain no additive constants. In consequence, the first-order deformation of the solution to the master equation reads

\[
S_{1} = \int d^{4}x (a^{(\varphi)} + a_{1}^{\text{int}} + a_{0}^{\text{int}}),
\]  
(45)

with \( a^{(\varphi)} \), \( a_{1}^{\text{int}} \), and \( a_{0}^{\text{int}} \) governed by relations (36)–(37), (43), and (44), respectively.

Next, we investigate equation (30). By direct computation, we arrive at

\[
\frac{1}{2} (S_{1}, S_{1}) = s \left\{ \int d^{4}x \left[ \mu_{AB}(\varphi) \partial^{\mu} \varphi^{A} - \frac{1}{2} k_{AB} A_{\mu} (T^{A}_{C} \varphi^{C} + n^{A}) \right] \right\} (T^{B}_{D} \varphi^{D}) \\
+ n^{B} A_{\mu} \right\} + \int d^{4}x \left\{ - \frac{\partial V(\varphi)}{\partial \varphi^{A}} (T^{A}_{C} \varphi^{C} + n^{A}) \right\} \eta
\]
\[ + \frac{1}{2} \frac{\partial U(\varphi)}{\partial \varphi^A} \left(T^A_C \varphi^C + n^A\right) F_{\mu\nu} F^{\mu\nu} \eta \]
\[ + \frac{1}{2} \frac{\partial \tilde{U}(\varphi)}{\partial \varphi^A} \left(T^A_C \varphi^C + n^A\right) \varepsilon^{\mu\rho\lambda} F_{\mu\nu} F_{\rho\lambda} \eta \]
\[ + \frac{1}{2} \left[ \mu_{AC}(\varphi) T^C_B + \mu_{BC}(\varphi) T^C_A \right. \]
\[ + \left. \frac{\partial \mu_{AB}(\varphi)}{\partial \varphi^C} \left(T^C_D \varphi^D + n^C\right) \right] \left( \partial^\mu \varphi^A \left( \partial^\nu \varphi^B \right) \right) \eta. \quad (46) \]

Formulas (30) and (46) show that the first-order deformation is consistent at order \( g^2 \) if and only if the following relations are fulfilled:

\[ \frac{\partial V(\varphi)}{\partial \varphi^A} \left(T^A_B \varphi^B + n^A\right) = 0, \quad (47) \]
\[ \frac{\partial U(\varphi)}{\partial \varphi^A} \left(T^A_B \varphi^B + n^A\right) = 0, \quad (48) \]
\[ \frac{\partial \tilde{U}(\varphi)}{\partial \varphi^A} \left(T^A_B \varphi^B + n^A\right) = 0, \quad (49) \]
\[ \mu_{AC}(\varphi) T^C_B + \mu_{BC}(\varphi) T^C_A + \frac{\partial \mu_{AB}(\varphi)}{\partial \varphi^C} \left(T^C_D \varphi^D + n^C\right) = 0. \quad (50) \]

In what follows we call (47)–(50) consistency equations. Under these circumstances, from (46) we find that

\[ S_2 = \int d^4 x \left[ - \mu_{AB}(\varphi) \partial^\mu \varphi^A + \frac{1}{2} k_{AB} A^\mu \left(T^A_C \varphi^C + n^A\right) \right] \left(T^B_D \varphi^D + n^B\right) A_\mu. \quad (51) \]

With the help of relations (45) and (51) we compute the antibracket \((S_1, S_2)\) and then, by means of equation (31), we deduce the third-order deformation

\[ S_3 = \int d^4 x \left[ \frac{1}{2} \mu_{AB}(\varphi) \left(T^A_C \varphi^C + n^A\right) \left(T^B_D \varphi^D + n^B\right) A_\mu A^\mu \right]. \quad (52) \]

Simple computation provides \((S_2, S_2) = 0\) and \((S_1, S_3) = 0\), so the solution to equation (62) can be taken as \( S_4 = 0 \). Then, all the remaining higher-order deformations can be chosen to vanish: \( S_k = 0, k > 4 \).

In consequence, we can state that the complete deformed solution to the master equation for the model under study, which is consistent to all orders in the coupling constant, reads

\[ \bar{S} = S + g S_1 + g^2 S_2 + g^3 S_3, \quad (53) \]
where $S$, $S_1$, $S_2$, and $S_3$ are given by formulas (25), (45), (51), and (52), respectively. The fully deformed solution to the master equations depends on two kinds of real constants ($T^{AB} = -T^{BA}$ and $n^A$) and four types of smooth, real functions of the undifferentiated scalar fields ($V$, $U$, $\bar{U}$, and $\mu_{AB} = \mu_{BA}$). In addition, the above constants and functions are subject to the consistency equations (47)–(50). Thus, our procedure is consistent provided these equations possess solutions.

Everywhere in the sequel we assume that

\[
\text{rank}(T^{AB}) \neq 0, \quad A, B = 1, N_0, \quad N_0 > 1. \tag{54}
\]

For the sake of generality, we consider that the matrix $T^{AB}$ may possess some nontrivial null vectors

\[
T_{AB}^A r^B_i = 0, \quad i = 1, N_0 - \text{rank}(T^{AB}). \tag{55}
\]

It is understood that if $\text{rank}(T^{AB}) = N_0$, then relations (55) are absent. Introducing the quantities

\[
\Omega_i = k_{AB} \varphi^A r^B_i, \quad q_i = k_{AB} n^A r^B_i, \tag{56}
\]

we find that a class of solutions to equations (47)–(50) is given by

\[
V(\varphi) = V(r, \bar{\Omega}_i, r_\alpha), \quad U(\varphi) = \psi(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha), \tag{57}
\]

\[
\bar{U}(\varphi) = \chi(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha), \quad \mu_{AB}(\varphi) = k_{AB} \omega(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha), \tag{58}
\]

where $r$, $\bar{r}$, and $\bar{\Omega}_i$ read

\[
r = \frac{1}{2} k_{AB} \left( T^A C \varphi^C + n^A \right) \left( T^B D \varphi^D + n^B \right), \tag{59}
\]

\[
\bar{r} = k_{AB} T^A C \varphi^C \left( \frac{1}{2} T^B D \varphi^D + n^B \right), \tag{60}
\]

\[
\bar{\Omega}_i = \delta_{bi}, \Omega_i \quad \text{(no summation over } i), \tag{61}
\]

while $r_\alpha(\varphi)$ are other solutions to the equations

\[
\frac{\partial r_\alpha}{\partial \varphi^A} \left( T^A B \varphi^B + n^A \right) = 0, \quad \alpha = 1, \ldots \tag{62}
\]

(if any) and $\bar{r}_\alpha(\varphi)$ are given by

\[
\bar{r}_\alpha = r_\alpha - r_\alpha(\varphi^A = 0). \tag{63}
\]
In (57) and (58) $V(r, \bar{\Omega}_i, \bar{r}_\alpha)$, $\vartheta(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha)$, $\kappa(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha)$, and $\omega(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha)$ are some arbitrary, smooth real functions of their arguments and, in addition, $\vartheta(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha)$, $\kappa(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha)$, and $\omega(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha)$ are constrained to satisfy the conditions

$$\vartheta(0, 0, 0) = \kappa(0, 0, 0) = \omega(0, 0, 0) = 0. \quad (64)$$

The above conditions ensure that the three functions denoted by $\vartheta(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha)$, $\kappa(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha)$, and $\omega(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha)$ contain no additive constants and, as a consequence, none of the functions $U$, $\tilde{U}$, or $\mu_{AB}$ may exhibit trivial components.

We believe that relations (57)–(64) provide the most general class of solutions to equations (47)–(50), but we do not insist on this matter. We remark that in the context of the above solutions the constants $n^A$ remain arbitrary.

In view of this, we choose them such that

$$k_{AB}n^A n^B \neq 0. \quad (65)$$

The first formula from (56) and relation (61) show that the dependence on $\bar{\Omega}_i$'s in (57) and (58) may appear only in the presence of some nontrivial vectors $\tau^B_i$ that obey relations (55). However, for a given set of constants $n^A$ that fulfills (65), the presence of the Kronecker delta $\delta_{0q_i}$ in (61) signalizes that the dependence on $\bar{\Omega}_i$'s in (57) and (58) is nontrivial if and only if the null vectors $\tau^B_i$ satisfy the conditions

$$q_i \equiv k_{AB} n^A \tau^B_i = 0 \quad (66)$$

for at least one $i \in 1, N_0 - \text{rank}(T^{AB})$.

Inserting relations (57) and (58) into (53), we obtain the final form of the deformed solution to the master equation that is consistent to all orders in the coupling constant,

$$\bar{S} = \int d^4 x \left[ -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} g^2 k_{AB} n^A n^B A_\mu A^\mu - g V(r, \bar{\Omega}_i, \bar{r}_\alpha) 
+ \frac{1}{2} k_{AB} (1 + g \omega(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha)) (D_\mu \varphi^A - 2 gn^A A_\mu) D^\mu \varphi^B 
+ \frac{1}{2} g F_{\mu \nu} (\vartheta(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha) F^{\mu \nu} + \epsilon^{\mu \nu \rho \lambda} \kappa(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha) F_{\rho \lambda}) 
+ \frac{1}{2} g^3 k_{AB} \omega(\bar{r}, \bar{\Omega}_i, \bar{r}_\alpha) n^A n^B A_\mu A^\mu 
+ A_\mu^* \partial^\mu \eta + g \varphi^*_A (T^{AB} k_{BC} \varphi^C + n^A) \eta \right], \quad (67)$$

with the covariant derivative of the matter fields defined by

$$D_\mu \varphi^A = \partial_\mu \varphi^A - g T^{AB} \varphi^B A_\mu. \quad (68)$$
The functional $\bar{S}$ satisfies by construction the equation

$$(\bar{S}, \bar{S}) = 0.$$  \hfill (69)

Formulas (57)–(65), (67), and (69) stand for the general results of the deformation procedure under the current working hypotheses.

5 Lagrangian formulation of emerging interacting gauge theories. Gauge-fixed action

Under these circumstances, from (67) and (57)–(65) we extract all the ingredients correlated with the Lagrangian formulation of the resulting interacting gauge theory. The antifield number 0 piece in the deformed solution (67) is nothing but the Lagrangian action of the emerging class of interacting gauge theories

$$\bar{S}_L^0[A^\mu, \varphi^A] = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 k_{AB} n^A n^B A_\mu A^\mu - g V(r, \bar{\Omega}_i, r_\alpha) 
+ \frac{1}{2} k_{AB} (1 + g \omega(r, \bar{\Omega}_i, \bar{r}_\alpha))(D_\mu \varphi^A - 2 g n^A A_\mu) D^\mu \varphi^B 
+ \frac{1}{2} g F_{\mu\nu}(\partial(r, \bar{\Omega}_i, \bar{r}_\alpha) F^{\mu\nu} + \varepsilon^{\mu\nu\rho\lambda}(r, \bar{\Omega}_i, \bar{r}_\alpha) F_{\rho\lambda}) 
+ \frac{1}{2} g^3 k_{AB} \omega(r, \bar{\Omega}_i, \bar{r}_\alpha) n^A n^B A_\mu A^\mu \right].$$  \hfill (70)

From the terms of antifield number 1 present in (67) we read the deformed gauge transformations (which leave invariant action (70)), namely,

$$\delta_\epsilon A^\mu = \partial^\mu \epsilon, \quad \delta_\epsilon \varphi^A = g(T^A_B \varphi^B + n^A) \epsilon, \quad A = 1, \ldots, N_0.$$  \hfill (71)

The previous gauge transformations are Abelian and irreducible. Relations (70) and (71) serve as the general output of steps (i) and (ii) discussed in the introductory section. Now, we are in the position to emphasize and detail the main results announced in introduction.

It is well known that the deformation procedure does not change the number of physical degrees of freedom of the starting theory \cite{5, 6}. Due to the fact that the matrix of elements $k_{AB}$ was taken by assumption to be positively defined and the constants $n^A$ satisfy condition (65), we find that $k_{AB} n^A n^B > 0$. In consequence, the object

$$\frac{1}{2} g^2 k_{AB} n^A n^B A_\mu A^\mu \equiv \frac{1}{2} M_0^2 A_\mu A^\mu$$  \hfill (72)
from (70) is precisely a mass term for the vector field $A_\mu$. It is clear that the quantity $(1/2)g^3k_{AB}\omega(\bar{r},\bar{\Omega}_i,\bar{r}_a)n^A n^B A_\mu A^\mu$ cannot generate mass for $A_\mu$ due to the fact that $\omega(\bar{r},\bar{\Omega}_i,\bar{r}_a)$ contains no additive constants (see requirement (64)). Then, the vector field present in (70) possesses precisely three physical degrees of freedom. It is easy to see that the mass term (72) exists irrespective of the number of scalar fields from the collection. Meanwhile, we remark that the term $-gk_{AB}A_\mu n^A \omega B$ from (70) is non-propagating. As a result, the linear combination of scalar fields $\phi \equiv k_{AB}n^A \varphi^B$ represents an unphysical degree of freedom, so there are $(N_0 - 1)$ scalar physical degrees of freedom in (70). Therefore, the deformed action (70) describes a system with $(N_0 + 2)$ physical degrees of freedom, like its free limit (11). We observe that the mass term (72) is generated by the nonvanishing arbitrary constants $n^A$. On the one hand, the existence of the constants $n^A$ in (70) is a consequence of the existence of the one-parameter global symmetry $\Delta_\theta \varphi^A = (T^A B_{BC} \varphi^C + n^A) \theta$ of (the free) action (11). Thus, the appearance of the mass term (72) is a direct consequence of the deformation method employed here in the context of the free limit described by action (11). At the same time, the constants $n^A$ are involved also in the deformed gauge transformations of the scalar fields from (71), which are nothing but the gauge versions of the one-parameter global transformations mentioned previously. On the other hand, none of the functions $\mathcal{V}(r,\bar{\Omega}_i,\bar{r}_a), \omega(\bar{r},\bar{\Omega}_i,\bar{r}_a), \vartheta(\bar{r},\bar{\Omega}_i,\bar{r}_a)$, or $\kappa(\bar{r},\bar{\Omega}_i,\bar{r}_a)$ that parameterize action (70) may contribute to the mass of the vector field. Actually, these functions are involved in (70) as follows: (A) $\mathcal{V}(r,\bar{\Omega}_i,\bar{r}_a)$ describes the derivative-free self-interactions and possibly some mass terms of the scalar fields; (B) $\omega(\bar{r},\bar{\Omega}_i,\bar{r}_a)$ controls the self-interactions of the form $\omega(\bar{r},\bar{\Omega}_i,\bar{r}_a)k_{AB}(\partial_\mu \varphi^A)\partial_\mu \varphi^B$ among the scalar fields as well as some cross-couplings between the vector field and the matter sector; (C) $\vartheta(\bar{r},\bar{\Omega}_i,\bar{r}_a)$ and $\kappa(\bar{r},\bar{\Omega}_i,\bar{r}_a)$ are responsible solely for some cross-couplings between the Abelian gauge field and the matter scalars. Until now we proved that the procedure based on steps (i) and (ii) leads to results (iii) and (iv) announced in Section 1.

In order to argue that result (v) from Section 1 also holds, we need to construct the gauge-fixed action corresponding to the deformed solution of the master equation given in (67). In view of this, we introduce the cohomologically trivial pairs $\{B, B^*\}$ and $\{\bar{\eta}, \bar{\eta}^*\}$, with $gh(B) = 0 = gh(\bar{\eta}^*)$, $gh(B^*) = -1 = gh(\bar{\eta})$, and $\varepsilon(B) = 0 = \varepsilon(\bar{\eta}^*)$, $\varepsilon(B^*) = 1 = \varepsilon(\bar{\eta})$. Consequently, the non-minimal solution to the master equation corresponding to (67) is given by $\bar{S}_{nm} = \bar{S} + \int d^4x \bar{\eta}^* B$. Since we have already identified the
unphysical scalar degree of freedom, it is no longer necessary to enforce the unitary gauge. Instead, we work with the \( R_\xi \) gauge implemented via the gauge-fixing fermion

\[
K = - \int d^4x \bar{\eta} \left( \partial_\mu A^\mu + \xi g k_{AB} \varphi^A n^B - \frac{1}{2} \xi B \right),
\]

where \( \xi \) is an arbitrary real constant. As a result, the gauge-fixed action becomes \( \tilde{S}_K = \tilde{S}_\text{nm} \left[ \Phi^\alpha_0, \Phi^{*\alpha}_0 = \frac{\delta K}{\delta \Phi^\alpha_0} \right] \), where \( \Phi^\alpha_0 \) is a collective notation for all the fields/ghosts. If we eliminate the auxiliary field \( B \) from \( \tilde{S}_K \) according to its field equation, we infer that

\[
\tilde{S}_K = \int d^4x \left[ - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 k_{AB} n^A n^B A_\mu A^\mu - \frac{1}{2 \xi} (\partial_\mu A^\mu)^2 
+ \frac{1}{2} k_{AB} (D_\mu \varphi^A) D^\mu \varphi^B - g \mathcal{V}(r, \bar{\Omega}_i, r_\alpha) 
+ \frac{1}{2} g F_{\mu\nu} (\partial(r, \bar{\Omega}_i, r_\alpha) F^{\mu\nu} + \varepsilon^{\mu\nu\rho\lambda} \mathcal{X}(r, \bar{\Omega}_i, r_\alpha) F_{\rho\lambda}) 
- \frac{1}{2} \xi g^2 k_{AC} k_{BD} n^C n^D \varphi^A \varphi^B + g^2 k_{AB} n^A T^B C \varphi^C A_\mu A^\mu 
+ (\partial_\mu \bar{\eta}) \partial^\mu \eta - \xi g^2 \bar{\eta} k_{AB} n^B (T^A C \varphi^C + n^A) \eta \right].
\]

(74)

The gauge-fixed action (74) is invariant under the gauge-fixed BRST transformations

\[
\bar{s}_K A^\mu = \partial^\mu \eta, \quad \bar{s}_K \varphi^A = g (T^A C \varphi^C + n^A) \eta, \quad \bar{s}_K \eta = 0, \quad \bar{s}_K \bar{\eta} = \frac{1}{\xi} (\partial_\mu A^\mu + \xi g k_{AB} \varphi^A n^B).
\]

(75)

Formula (74) emphasizes the following features: (I) the massive vector field propagator behaves like \( \tilde{\Delta}_{F_{\mu\nu}}(\bar{p}) \sim |\bar{p}|^{-2} \) for large Euclidean momenta \( |\bar{p}| \to \infty \), just like in the massless case; (II) the unphysical degrees of freedom \( \phi \equiv k_{AB} n^A \varphi^B \) and \( \{\bar{\eta}, \eta\} \) acquire mass; (III) the scalar fields may be coupled nontrivially to the ghosts. Conclusion (I) is nothing but result (v). Clearly, the last conclusion highlights a propagator behaviour that is different from the purely Proca case exposed in Ref. [38].

In agreement with the previous discussion regarding the properties of the coupled gauge model (see statements (B) and (C) from the previous paragraph), we notice that the functions \( \omega(r, \bar{\Omega}_i, r_\alpha), \partial(r, \bar{\Omega}_i, r_\alpha), \) and \( \mathcal{X}(r, \bar{\Omega}_i, r_\alpha) \)
are less relevant. For the sake of simplicity, we will set them equal to zero in what follows

$$\omega(\vec{r}, \vec{\Omega}_i, \vec{r}_\alpha) = 0, \quad \vartheta(\vec{r}, \vec{\Omega}_i, \vec{r}_\alpha) = 0, \quad \kappa(\vec{r}, \vec{\Omega}_i, \vec{r}_\alpha) = 0.$$  (77)

All the above results remain valid in the presence of (77) since the functions \(\omega(\vec{r}, \vec{\Omega}_i, \vec{r}_\alpha), \vartheta(\vec{r}, \vec{\Omega}_i, \vec{r}_\alpha),\) and \(\kappa(\vec{r}, \vec{\Omega}_i, \vec{r}_\alpha)\) were so far arbitrary.

We remark that all the outcomes obtained until now are entirely independent of the Higgs mechanism.

6 Comparison with the Abelian Higgs mechanism

Initially, we briefly address the Abelian Higgs mechanism in the presence of a collection of \(N_0\) real scalar fields. In this situation the starting point is given by the action (we recall that the covariant derivative \(D_\mu \varphi^A\) is introduced in (68))

$$\bar{S}^L_{0,T,\text{Higgs}}[A^\mu, \varphi^A] = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} k_{AB} \left( D_\mu \varphi^A \right) D^\mu \varphi^B - V_{1,\text{Higgs}}^{\varphi^A} \right],$$  (78)

which is assumed to be invariant under the Abelian and irreducible gauge transformations

$$\delta' A^\mu = \partial^\mu \epsilon, \quad \delta' \varphi^A = g T^A_B \varphi^B \epsilon.$$  (79)

Formulas (78) and (79) are nothing but a generalization of relations (11) and (61) for an arbitrary \(N_0\). The gauge invariance of (78) under (79) is equivalent to the fact that the function \(V_{1,\text{Higgs}}^{\varphi^A}\) is gauge-invariant, i.e.,

$$\frac{\partial V_{1,\text{Higgs}}^{\varphi^A}}{\partial \varphi^A} T^A_B \varphi^B = 0.$$  (80)

In addition, we presume that the function \(V_{1,\text{Higgs}}^{\varphi^A}\) possesses an absolute minimum for a (nonvanishing) constant scalar field configuration

$$\varphi^A = v_0^A.$$  (81)
but make no other supplementary presumptions on $V_{1}^{\text{Higgs}}$. Defining some new fields by
\[ \tilde{\phi}^A = \phi^A - v_0^A, \] (82)
whose associated field operators display zero vacuum expectation values, and rewriting formulas (78) and (79) in terms of (82), we arrive at the action
\[ \bar{S}_{0_{\text{Higgs}}}^T[A^\mu, \tilde{\phi}^A] = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 k_{AB} T^A_c T^B_D v_0^C v_0^D A_\mu A^\mu 
+ \frac{1}{2} k_{AB} (D_\mu \tilde{\phi}^A - 2 g T^A_c v_0^C A_\mu) D^\mu \tilde{\phi}^B - V_{1}^{\text{Higgs}}(\tilde{\phi}^A + v_0^A) \right], \] (83)
invariant under the gauge transformations
\[ \delta'_\epsilon A^\mu = \partial^\mu \epsilon, \quad \delta'_\epsilon \tilde{\phi}^A = g T^A_B (\tilde{\phi}^B + v_0^B) \epsilon. \] (84)
Relations (83) and (84) stand for the final output of the Abelian Higgs mechanism in the presence of a collection of $N_0$ real scalar fields and show that the vector field acquires the square mass $g^2 k_{AB} T^A_c T^B_D v_0^C v_0^D$. Formula (80) written in terms of the transformed scalar fields (82) is equivalent to the gauge-invariance of the function $V_{1}^{\text{Higgs}}(\tilde{\phi}^A + v_0^A)$ under transformations (84)
\[ \frac{\partial V_{1}^{\text{Higgs}}(\tilde{\phi}^A + v_0^A)}{\partial \tilde{\phi}^A} T^A_B (\tilde{\phi}^B + v_0^B) = 0. \] (85)
The square masses of the scalar fields are the eigenvalues of the mass matrix
\[ m_{AB} = \frac{\partial^2 V_{1}^{\text{Higgs}}(\tilde{\phi}^A + v_0^A)}{\partial \phi^A \partial \tilde{\phi}^B} \bigg|_{\tilde{\phi}^A = 0}. \] By differentiating (85) with respect to $\tilde{\phi}^B$, particularizing the resulting formula to $\tilde{\phi}^A = 0$, and taking into account that
\[ \frac{\partial V_{1}^{\text{Higgs}}(\tilde{\phi}^A + v_0^A)}{\partial \tilde{\phi}^A} \bigg|_{\tilde{\phi}^A = 0} = 0 \] (\iff \frac{\partial V_{1}^{\text{Higgs}}(\phi^A)}{\partial \phi^A} \bigg|_{\phi^A = v_0^A} = 0), we obtain the relations
\[ m_{AB} T^A_c v_0^C = 0, \] which show that the maximum possible rank of the scalar mass matrix is equal to $(N_0 - 1)$. This means that at least one scalar field (Goldstone mode) is massless. The masses of the remaining scalars depend on the concrete form of $V_{1}^{\text{Higgs}}(\phi^A)$.

The above discussion allows us to conclude that: (a) the Higgs mechanism is applicable if the gauge-invariant function $V_{1}^{\text{Higgs}}(\phi^A)$ that appears in (78) possesses an absolute minimum for a nonvanishing scalar field configuration.

The starting point of our method is represented by the free limit given by relations (11) and (12). At the same time, the starting point of the Higgs mechanism is provided by an interacting theory (see formulas (78) and (79)).
Thus, in order to correctly compare our procedure with the Higgs mechanism, it is necessary to consider an appropriate starting point. In view of this, we begin with an interacting Lagrangian action

\[ \bar{S}_L^T \left[ A^\mu, \varphi^A \right] = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} k_{AB} (D_\mu \varphi^A) D^\mu \varphi^B - V_1(\varphi^A) \right], \tag{86} \]

where \( V_1(\varphi^A) \) is an arbitrary, smooth real function of its arguments. Action (86) is assumed to be invariant under the gauge transformations

\[ \delta'_\epsilon A^\mu = \partial^\mu \epsilon, \quad \delta'_\epsilon \varphi^A = g T^A_B \varphi^B \epsilon. \tag{87} \]

This implies that the function \( V_1(\varphi^A) \) is gauge-invariant, \( \delta'_\epsilon V_1(\varphi^A) = 0 \). We make no further assumption on the function \( V_1(\varphi^A) \), so formulas (86) and (87) can be regarded like a more general starting point than relations (78) and (79).

Now, we prove that starting from action (86) and gauge transformations (87) we can derive a gauge theory with a massive vector field. The main results of the deformation procedure exposed in the above, more precisely formulas (67) and (69), offer a general manner of finding such a theory. The strategy goes as follows. Initially, we construct the solution to the master equation associated with the theory governed by (86) and (87). It reads

\[ \bar{S}_T = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 k_{AB} n^A n^B A_\mu A^\mu - g k_{AB} n^A A_\mu D^\mu \varphi^B - V_1(\varphi^A) \right. \]

\[ + \left. A^*_\mu \partial^\mu \eta + g \varphi^*_A T^A_B \varphi^B \eta \right]. \tag{88} \]

Now, we introduce a (local) functional of fields, ghosts, and antifields, defined by

\[ \bar{S}_n = \int d^4x \left( \frac{1}{2} g^2 k_{AB} n^A n^B A_\mu A^\mu - g k_{AB} n^A A_\mu D^\mu \varphi^B \right. \]

\[ + \left. V_1(\varphi^A) - g \mathcal{V}(r, \bar{\Omega}_i, r_\alpha) + g \varphi^*_A n^A \eta \right), \tag{89} \]

where the arbitrary constants \( n^A \) still satisfy condition (65) and the quantities \( r, \bar{\Omega}_i, \) and \( r_\alpha \) are specified in formulas (59), (61), and (62), respectively. Next, we construct the functional

\[ \bar{S}' = \bar{S}_T + \bar{S}_n = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 k_{AB} n^A n^B A_\mu A^\mu - g \mathcal{V}(r, \bar{\Omega}_i, r_\alpha) \right. \]

\[ \left. + A^*_\mu \partial^\mu \eta + g \varphi^*_A T^A_B \varphi^B \eta \right]. \]
We observe that (90) is nothing but our deformed solution (67) where we implement (77). Consequently, equation (69) ensures that \((S', S') = 0\). Then, the pieces of antifield number 0 and respectively 1 from (90) lead precisely to the Lagrangian action (70) and gauge transformations (71) obtained in the previous section with the particular choice (77)

\[
S^0_{L}[A^\mu, \varphi^A] = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 k_{AB} n^A n^B A_\mu A^\mu - g \mathcal{V}(r, \Omega_i, r_\alpha) \right. \\
\left. + \frac{1}{2} k_{AB} \left( D_\mu \varphi^A - 2 gn^A A_\mu \right) D^\mu \varphi^B \right],
\]  
(91)

which indeed emphasize a gauge theory with a massive vector field.

The last arguments enable us to state the following conclusions: (b) our method in the presence of the starting point (86) and (87) is conceptually different from the Higgs mechanism; (c) it is applicable to an arbitrary gauge-invariant function \(V_1(\varphi^A)\), which is no longer constrained to display an absolute minimum.

At this stage, we remark that the final outputs of the Abelian Higgs mechanism ((83) and (84)) and those of our procedure ((91) and (92)) are different in general. In the sequel we investigate whether our method is capable of rendering the results of the Abelian Higgs mechanism. In view of this, we take \(n^A\) of the form

\[
n^A = T^A_B v^B_0.
\]  
(93)

In this situation, equations (62) become

\[
\frac{\partial r_\alpha}{\partial \varphi^A} T^A_B (\varphi^B + v^B_0) = 0
\]  
(94)

and, by virtue of (85), obviously admit at least the solution

\[
r_\alpha \rightarrow r_1 = V_1^{\text{Higgs}}(\varphi^A + v^A_0),
\]  
(95)

which allows us to choose \(\mathcal{V}(r, \Omega_i, r_\alpha)\) like

\[
\mathcal{V}(r, \Omega_i, r_\alpha) \rightarrow \mathcal{V}(r_1) = \frac{1}{g} r_1 = \frac{1}{g} V_1^{\text{Higgs}}(\varphi^A + v^A_0).
\]  
(96)
Now, we particularize the procedure developed between formulas (88) and (92) to the case where
\[ V_1(\varphi^A) = V_1^{\text{Higgs}}(\varphi^A) \] (97)
and \( n^A \) together with \( V \) are expressed by (93) and (96). The ansatz described by formula (97) leads to the fact that relations (86) and (87) precisely reduce to (78) and (79). Therefore, the solution to the master equation corresponding to the gauge theory described by formulas (78) and (79) is given by
\[
\bar{S}_{T,\text{Higgs}} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} k_{AB} (D_\mu \varphi^A) D^\mu \varphi^B 
- V_1^{\text{Higgs}}(\varphi^A) + A_\mu^* \partial^\mu \eta + g \varphi_A^* T^A_B \varphi^B \eta \right] \] (98)
and satisfies by construction the equation
\[
(\bar{S}_{T,\text{Higgs}}, \bar{S}_{T,\text{Higgs}}) = 0. \] (99)
The role of the functional (89) is played here by
\[
\bar{S}_{v_0} = \int d^4x \left( \frac{1}{4} g^2 k_{AB} T^A_C T^B_D v_0^C T^D_A A_\mu A^\mu 
+ \frac{1}{2} k_{AB} (D_\mu \varphi^A - 2g T^A_C v_0^C A_\mu) D^\mu \varphi^B \right. 
\left. - V_1^{\text{Higgs}}(\varphi^A + v_0^A) + g \varphi_A^* T^A_C v_0^C \eta \right). \] (100)
Functional (100) follows from (89) where we set (93), (96), and (97). By direct computation, we infer that
\[
(\bar{S}_{v_0}, \bar{S}_{v_0}) \neq 0. \] (101)
Finally, we construct the functional (that results from (90) where we use choices (93) and (96))
\[
\bar{S}_{\text{Higgs}}' = \bar{S}_{T,\text{Higgs}} + \bar{S}_{v_0}
= \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 k_{AB} T^A_C T^B_D v_0^C T^D_A A_\mu A^\mu 
+ \frac{1}{2} k_{AB} (D_\mu \varphi^A - 2g T^A_C v_0^C A_\mu) D^\mu \varphi^B \right. 
\left. - V_1^{\text{Higgs}}(\varphi^A + v_0^A) + A_\mu^* \partial^\mu \eta + g \varphi_A^* T^A_B (\varphi^B + v_0^B) \eta \right], \] (102)
which obviously verifies the master equation
\[
(\bar{S}_{\text{Higgs}}', \bar{S}_{\text{Higgs}}') = 0. \] (103)
The projection of \( (102) \) on antifield number 0 provides the Lagrangian action

\[
\bar{S}_{\text{Higgs}}^{\text{L}}[A^\mu, \varphi^A] = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 k_{AB} T^A_C T^B_D v_0^C v_0^D A_\mu A^\mu \\
+ \frac{1}{2} k_{AB} \left( D_\mu \varphi^A - 2 g T^A_C v_0^C A_\mu \right) D^\mu \varphi^B - V_1^{\text{Higgs}}(\varphi^A + v_0^A) \right],
\]

while from the terms of antifield number 1 we read the gauge transformations of \( (104) \) like

\[
\bar{\delta}_\epsilon A^\mu = \partial^\mu \epsilon, \quad \bar{\delta}_\epsilon \varphi^A = g T^A_B (\varphi^B + v_0^B) \epsilon. \tag{105}
\]

It is obvious that \( (104) \) and \( (105) \) are nothing but \( (83) \) and \( (84) \) modulo the identification

\[
\varphi^A \longleftrightarrow \tilde{\varphi}^A. \tag{106}
\]

Formulas \( (104) - (106) \) argue that: (d) in the case described by relations \( (93) \), \( (96) \), and \( (97) \), the results of our procedure do indeed coincide with those the Abelian Higgs mechanism.

In this way, conclusions (a)–(d) obtained in this section prove that our procedure may be regarded as a \textit{cohomological extension} of the Abelian Higgs mechanism, which is precisely result (vi) announced in the introduction.

### 7 BRST interpretation of the Higgs mechanism

Formulas \( (83) \) together with \( (84) \) and \( (104) \) accompanied by \( (105) \) respectively — in the presence of \( (106) \) — emphasize that the final output of the Abelian Higgs mechanism can be obtained in two different manners: either by performing the shift transformations \( (82) \) or by means of the procedure exposed between formulas \( (98) \) and \( (105) \). Therefore, we can view the Abelian Higgs mechanism like the passage from formulas \( (78) \) and \( (79) \) to \( (83) \) and \( (84) \), or, equivalently, from relations \( (78) \) and \( (79) \) to \( (104) \) and \( (105) \).

Now, we are in the position to give an interpretation of the Abelian Higgs mechanism in terms of the antifield-BRST symmetry. In view of this, we adopt the second manner exposed in the above. Actually, the passage from \( (78) \) and \( (79) \) to \( (104) \) and \( (105) \) means, at the level of the BRST formalism, the transit from \( (98) - (99) \) to \( (102) - (103) \). On the one hand, formulas \( (98) \) and \( (99) \) define a differential of ghost number equal to 1 that acts like

\[
\bar{s}_{\text{Higgs}}^T F = (F, \bar{s}_{\text{Higgs}}^T), \quad \bar{s}_{\text{Higgs}}^2 = 0. \tag{107}
\]
The operator $\bar{s}_{T,Higgs}$ signifies the BRST differential associated with the theory governed by (78) and (79). On the other hand, relations (102) and (103) define also a differential of ghost number equal to 1, via

$$\bar{s}^\prime_{Higgs} F = (F, \bar{S}^\prime_{Higgs}), \quad \bar{s}^2_{Higgs} = 0,$$

which is nothing but the BRST differential corresponding to the gauge theory pictured by (104) and (105). At the same time, formulas (100) and (101) induce an odd derivation of ghost number 1

$$\bar{s}_{v_0} F = (F, \bar{S}_{v_0}), \quad \bar{s}^2_{v_0} \neq 0.$$

By means of definitions (107)–(109), relation (102) connects these three operators through

$$\bar{s}^\prime_{Higgs} = \bar{s}_{T,Higgs} + \bar{s}_{v_0}.$$

Obviously, the operators from (110) act on the same BRST algebra, such that we find that

$$H^k(\bar{s}^\prime_{Higgs}) \neq H^k(\bar{s}_{T,Higgs}), \quad k \geq 0,$$

where $H^k(\bar{s}^\prime_{Higgs})$ and $H^k(\bar{s}_{T,Higgs})$ represent the cohomologies of $\bar{s}^\prime_{Higgs}$ and $\bar{s}_{T,Higgs}$ in ghost number $k$ computed in the space of local functionals. In particular, (111) leads to

$$H^0(\bar{s}^\prime_{Higgs}) \neq H^0(\bar{s}_{T,Higgs}),$$

which further implies that the classical observables associated with the theories described by relations (78) and (79) and respectively (104) and (105) are different. We recall that the classical observables of a given gauge theory are gauge-invariant local functionals modulo the field equations. In conclusion, the passage from (78) and (79) to (104) and (105), which we have seen that represents a proper description of the Higgs mechanism, means the transit from the BRST differential $\bar{s}_{T,Higgs}$ to the BRST differential $\bar{s}^\prime_{Higgs}$ (using relation (110)), which implies that the classical observables of these two theories are different. The last statement stays at the core of the interpretation of the Abelian Higgs mechanism in the light of the BRST symmetry and meanwhile proves result (vii) from Section 1. In this context we remark that the role of the (scalar) shift transformations (82) from the traditional approach to the Higgs mechanism is played in the framework of the BRST symmetry by the odd derivation (109). Thus, all the main objectives of this paper have been accomplished.
8 Examples

In this section we will exemplify the general results obtained in Section 5 to the case of the interactions between a vector field and one, two, and three real scalar fields. In view of this, from now on we work with

\[ k_{AB} = \delta_{AB}, \quad \mathcal{V}(r, \bar{\Omega}_i, r_\alpha) \to \mathcal{V}(r, \bar{\Omega}_i) = c_1 r + c_2 \left( r + \frac{1}{2} b_i \bar{\Omega}_i^2 \right)^2 + \frac{1}{2} d_i \bar{\Omega}_i^2, \]  

(113)

where \( c_1, c_2, b_i, \) and \( d_i \) represent some arbitrary real constants. On account of the first choice from (113) all scalar indices \( A, B, C, \) and so on will be set in lower positions. Although \( \mathcal{V}(r, \bar{\Omega}_i) \) can be taken of a more general form, here we work with a polynomial expression, of the form (113), in order to emphasize other interesting aspects of our procedure.

8.1 The case of one scalar field

First, we consider the case \( N_0 = 1, \) which corresponds to the interactions between a vector field and a single real scalar field, to be denoted by \( \varphi_1 \equiv \varphi. \) Due to the antisymmetry property of the arbitrary constants \( T_{AB} \to T_{11}, \) the only possible choice is \( T_{11} = 0. \) The fact that \( \text{rank}(T_{11}) = 0 \) does not contradict condition (54) since here \( N_0 = 1. \) Obviously, the null vectors \( \tau_B^i \) are absent. Then, by means of (59) and of the notation \( n_A \to n_1 \equiv n \neq 0, \) formula (113) leads to the fact that

\[ \mathcal{V}(r, \bar{\Omega}_i) \to \mathcal{V}(r) = (1/2) n^2 (c_1 + \frac{1}{2} c_2 n^2), \]

so it reduces in this situation to an irrelevant constant and will therefore be omitted. Consequently, formulas (70) and (71) in the presence of choice (77) become

\[ \bar{S}_0^H[A^\mu, \varphi] = \int d^4 x \left[ - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} (gnA_\mu - \partial_\mu \varphi)(gnA^\mu - \partial^\mu \varphi) \right], \]  

(114)

\[ \bar{\delta}_c A^\mu = \partial^\mu \epsilon, \quad \bar{\delta}_c \varphi = gn \epsilon, \]  

(115)

whereas the gauge-fixed action (74) with the same choice takes the particular form

\[ \bar{S}_K = \int d^4 x \left[ - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} g^2 n^2 A_\mu A^\mu - \frac{1}{2 \xi} (\partial_\mu A^\mu)^2 
\]

\[ + \frac{1}{2} (\partial_\mu \varphi) \partial^\mu \varphi - \frac{1}{2} \xi g^2 n^2 \varphi^2 + (\partial_\mu \bar{\eta}) \partial^\mu \eta - \xi g^2 n^2 \bar{\eta} \eta \right]. \]  

(116)

Analyzing relations (114) and (115), we observe that they provide nothing but the Stueckelberg coupling between a vector field and a scalar field \( \varphi. \) We
emphasized in the general context from Section 5 that the unphysical scalar degree of freedom is $n_A \varphi_A \rightarrow n \varphi$, so the only scalar field from the present context, $\varphi$, describes no physical degree of freedom. Therefore, the gauge-fixed action (116) comprises three physical degrees of freedom associated with the massive vector field $A^\mu$, as well as the unphysical degrees of freedom corresponding to $\{\varphi, \bar{\eta}, \eta\}$. In this particular situation the ghosts are not coupled to the Stueckelberg scalar (since $T_{11} = 0$).

8.2 The case of two scalar fields

Second, we analyze the case $N_0 = 2$, i.e., the interactions among a vector field and two real scalar fields. We take the elements $T_{AB}$ and the constants $n_A$ of the form

$$T_{AB} = \beta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad n_1 = 0, \quad n_2 \equiv -n,$$

(117)

with both $\beta$ and $n$ nonvanishing. It is easy to see that in this case there are also no nontrivial null vectors $\tau_B^i$, so the dependence of $\mathcal{V}$ on $\bar{\Omega}_i$ is trivial. Then, (113) reduces to $\mathcal{V}(r) = c_1 r + c_2 r^2$. As a consequence, from expressions (70) and (71) where we set (77) we generate the interacting Lagrangian action and accompanying gauge transformations in this case like

$$\bar{S}_0^L[A^\mu, \varphi_1, \varphi_2] = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 n^2 A_\mu A^\mu \\
+ \frac{1}{2} (\partial_\mu \varphi_1) \partial^\mu \varphi_1 - \frac{1}{2} g \beta^2 (c_1 + 3 n^2 c_2) \varphi_1^2 \\
+ \frac{1}{2} (\partial_\mu \varphi_2) \partial^\mu \varphi_2 - \frac{1}{2} g \beta^2 (c_1 + n^2 c_2) \varphi_2^2 \\
- \frac{1}{2} g c_2 \beta^2 (\varphi_1^2 + \varphi_2^2) \left[ \beta (\varphi_1^2 + \varphi_2^2) + 4 n \varphi_1 \right] \\
+ g \beta A_\mu (\varphi_1 \partial^\mu \varphi_2 - \varphi_2 \partial^\mu \varphi_1) + g n A_\mu \partial^\mu \varphi_2 \\
- g n (c_1 + n^2 c_2) \varphi_1 \\
+ \frac{1}{2} g^2 \beta \left[ (\varphi_1^2 + \varphi_2^2) + 2 n \varphi_2 \right] A_\mu A^\mu \right\},$$

(118)

$$\bar{\delta}_\epsilon A^\mu = \partial^\mu \epsilon, \quad \bar{\delta}_\epsilon \varphi_1 = g \beta \varphi_2 \epsilon, \quad \bar{\delta}_\epsilon \varphi_2 = -g (\beta \varphi_1 + n) \epsilon.$$  

(119)

We remark that the real constants $c_1$, $c_2$, $\beta$, and $n$ appearing in (118) and (119) are arbitrary (with $\beta$ and $n$ nonvanishing), such that the mass of the vector field does not depend either on $c_1$, $c_2$, or $\beta$. Relations (118) and (119) represent the most general expressions (taking (17) into consideration) that
describe an interacting gauge theory with one massive vector field and two scalars. We remark that formulas (9) and (10) follow from (118) and (119) in the particular case

\[ g = q, \quad gc_1 = \mu^2, \quad gc_2 = \frac{1}{4}\lambda > 0, \quad \beta = 1, \quad n = v. \tag{120} \]

Obviously, the results emerging from the Abelian Higgs mechanism are obtained from (118)–(120) in the more particular situation

\[ \mu^2 < 0, \quad v = \sqrt{-\frac{4\mu^2}{\lambda}}. \tag{121} \]

The gauge-fixed action (74) where we put (77) takes (for this example) the concrete expression

\[
\bar{S}_K = \int d^4x \left\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2n^2A_\mu A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \\
+ \frac{1}{2}(\partial_\mu \varphi_1)\partial^\mu \varphi_1 - \frac{1}{2}g\beta^2(c_1 + 3n^2c_2)\varphi_1^2 \\
+ \frac{1}{2}(\partial_\mu \varphi_2)\partial^\mu \varphi_2 - \frac{1}{2}g[\beta^2c_1 + n^2(\beta^2c_2 + \xi g)]\varphi_2^2 \\
- \frac{1}{4}gc_2\beta^2(\varphi_1^2 + \varphi_2^2)[\beta(\varphi_1^2 + \varphi_2^2) + 4n\varphi_1] \\
+ g\beta A_\mu(\varphi_1\partial_\mu \varphi_2 - \varphi_2\partial_\mu \varphi_1) - g\beta n(c_1 + n^2c_2)\varphi_1 \\
+ \frac{1}{2}g^2\beta[\beta(\varphi_1^2 + \varphi_2^2) + 2n\varphi_1]A_\mu A^\mu + (\partial_\mu \bar{\eta})\partial^\mu \eta \\
- \xi g^2n^2\bar{\eta}\eta - \xi g^2\beta n\varphi_1\bar{\eta}\eta \right\}. \tag{122} \]

Here, the unphysical scalar degree of freedom is \( n_A \varphi_A \rightarrow -n \varphi_2 \) and hence it reduces precisely to the scalar field \( \varphi_2 \). Accordingly, this model exhibits four physical degrees of freedom (three corresponding to the massive vector field \( A^\mu \) and one associated with \( \varphi_1 \)) and the unphysical degrees of freedom \( \{ \varphi_2, \bar{\eta}, \eta \} \). Moreover, the vertex \((\cdot)\xi g^2\beta n\varphi_1\bar{\eta}\eta\) from (122) signalizes that the physical scalar \( \varphi_1 \) is coupled to the ghosts. This vertex, omitted in QFT textbooks, should be present also in the gauge-fixed action resulting from the Abelian Higgs mechanism, which follows from (122) with the choices (120) and (121). Its presence is important since it ensures the invariance of the gauge-fixed action (122) under the gauge-fixed BRST transformations (73) and (76) particularized to this example, which is otherwise lost.
8.3 The case of three scalar fields

Third, we investigate the case $N_0 = 3$, which provides the interactions among a vector field and three real scalar fields. In this situation we take

$$T_{AB} = \beta \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

with $\beta$ nonvanishing. We remark that matrix (123) possesses the null vector

$$\tau_1 = 0, \quad \tau_2 = 0, \quad \tau_3 = 1.$$  \hfill (124)

First, we choose $n_A$’s of the form

$$n_1 = 0, \quad n_2 = 0, \quad n_3 \equiv n,$$

with $n$ nonvanishing. With the help of (124) and (125) we observe that (66) is not satisfied, so the dependence of $V$ on $\bar{\Omega}_i$ is again trivial, such that (113) reduces to $V(r) = c_1 r + c_2 r^2$. In this context formulas (70) and (71) take the particular form (being understood that we implement (77))

$$\bar{S}_K[A^\mu, \varphi_1, \varphi_2, \varphi_3] = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 n^2 A_\mu A^\mu \\
+ \frac{1}{2} (\partial_\mu \varphi_1) (\partial^\mu \varphi_1) + \frac{1}{2} (\partial_\mu \varphi_2) (\partial^\mu \varphi_2) \\
+ \frac{1}{2} (\partial_\mu \varphi_3) (\partial^\mu \varphi_3) - \frac{1}{2} g \beta^2 (c_1 + n^2 c_2) (\varphi_1^2 + \varphi_2^2) \\
- \frac{1}{4} g c_2 \beta^4 (\varphi_1^2 + \varphi_2^2)^2 + g \beta A_\mu (\varphi_1 \partial^\mu \varphi_2 - \varphi_2 \partial^\mu \varphi_1) \\
- g n A_\mu \partial^\mu \varphi_3 + \frac{1}{2} g^2 \beta^2 (\varphi_1^2 + \varphi_2^2) A_\mu A^\mu \right],$$

\hfill (126)

$$\bar{\delta}_\epsilon A^\mu = \partial^\mu \epsilon, \quad \bar{\delta}_\epsilon \varphi_1 = g \beta \varphi_2 \epsilon, \quad \bar{\delta}_\epsilon \varphi_2 = -g \beta \varphi_1 \epsilon, \quad \bar{\delta}_\epsilon \varphi_3 = g n \epsilon.$$ \hfill (127)

Again, the mass of the vector field does not depend either on $c_1$, $c_2$, or $\beta$. If we choose the constants $c_1$ and $c_2$ such that $g (c_1 + n^2 c_2) > 0$ and set $\beta = 1$, then action (126) describes precisely an $U(1)$-type coupling between the massive physical scalars $\{\varphi_1, \varphi_2\}$ and a massive vector field in the presence of the (unphysical) Stueckelberg scalar field $\varphi_3$. Meanwhile, (126) contains also interactions involving the two physical scalars. The gauge-fixed action (74) corresponding to (126) is given by

$$\bar{S}_K = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 n^2 A_\mu A^\mu - \frac{1}{2s^2} (\partial_\mu A^\mu)^2 \right]$$

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\begin{align*}
&+ \frac{1}{2} (\partial_{\mu} \varphi_1) \partial^{\mu} \varphi_1 + \frac{1}{2} (\partial_{\mu} \varphi_2) \partial^{\mu} \varphi_2 + \frac{1}{2} (\partial_{\mu} \varphi_3) \partial^{\mu} \varphi_3 \\
&- \frac{1}{2} g \beta^2 (c_1 + n^2 c_2) (\varphi_1^2 + \varphi_2^2) - \frac{1}{2} g c_2 \beta^4 (\varphi_1^2 + \varphi_2^2)^2 \\
&+ g \beta A_\mu (\varphi_1 \partial^\mu \varphi_2 - \varphi_2 \partial^\mu \varphi_1) + \frac{1}{2} g^2 \beta^2 (\varphi_1^2 + \varphi_2^2) A_\mu A^\mu \\
&- \frac{1}{2} \xi g^2 n^2 \varphi_3^2 + (\partial_\mu \eta) \partial^\mu \eta - \xi g^2 n^2 \eta \eta \right]. \tag{128}
\end{align*}

This example underlies five physical degrees of freedom (three corresponding to $A^\mu$ and one for each of the scalars $\varphi_1$ and respectively $\varphi_2$), whereas $\{\varphi_3, \eta, \eta\}$ are unphysical. We notice that the scalar fields are no longer coupled to the ghosts, like in the first example.

Second, we take $\eta_{A_1}$’s of the form

$$n_1 = 0, \quad n_2 = -n, \quad n_3 = 0, \tag{129}$$

with $n$ nonvanishing. By means of (1124) and (1129) we obtain that (100) is satisfied, so $\mathcal{V}$ depends nontrivially on $r$ and $\Omega_1 \equiv \varphi_3$. Denoting the corresponding constants $b_1$ and $d_1$ from (1113) with $b$ and respectively $d$, relations (110) and (111) (in the presence of (77)) become

\begin{align*}
\bar{S}_0^L [A^\mu, \varphi_1, \varphi_2, \varphi_3] &= \int d^4 x \left\{ - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 n^2 A_\mu A^\mu \\
&+ \frac{1}{2} (\partial_{\nu} \varphi_1) \partial^{\nu} \varphi_1 - \frac{1}{2} g \beta^2 (c_1 + 3 n^2 c_2) \varphi_1^2 \\
&+ \frac{1}{2} (\partial_{\nu} \varphi_2) \partial^{\nu} \varphi_2 - \frac{1}{2} g \beta^2 (c_1 + n^2 c_2) \varphi_2^2 \\
&+ \frac{1}{2} (\partial_{\nu} \varphi_3) \partial^{\nu} \varphi_3 - \frac{1}{2} g (d + n^2 c_2) \varphi_3^2 \\
&- \frac{1}{2} g c_2 [\beta^2 (\varphi_1^2 + \varphi_2^2) + b \varphi_3^2] \times \\
&\times [\beta^2 (\varphi_1^2 + \varphi_2^2) + b \varphi_3^2 + 4 \beta n \varphi_1] \\
&+ g \beta A_\mu (\varphi_1 \partial^\mu \varphi_2 - \varphi_2 \partial^\mu \varphi_1) + g n A_\mu \partial^\mu \varphi_2 \\
&- g \beta n (c_1 + n^2 c_2) \varphi_1 \\
&+ \frac{1}{2} g^2 \beta [\beta (\varphi_1^2 + \varphi_2^2) + 2 n \varphi_1] A_\mu A^\mu \right\}. \tag{130}
\end{align*}

Like in the previous cases, the mass of the vector field does not depend on the arbitrary constants $c_1$, $c_2$, $\beta$, $b$, and $d$. In this situation the unphysical scalar degree of freedom is $\varphi_2$. Among the five physical degrees of freedom of this model, three correspond to the massive vector field and two to the scalar fields $\varphi_1$ and $\varphi_3$ (the last is gauge-invariant). It is interesting to notice
that if we take the constants $c_1, c_2, b, d$ such that $g(c_1 + 3n^2c_2) > 0$ and $g(d + n^2bc_2) > 0$, then the two physical scalars possess in general different masses. At the same time, in (130) there are present interactions involving all the scalar fields. Although gauge-invariant, the scalar field $\phi_3$ is not coupled to the vector field. The gauge-fixed action (74) associated with (130) takes in the second case the form

\[
\tilde{S}_K = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 n^2 A_\mu A^\mu - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 
+ \frac{1}{2} (\partial_\mu \varphi_1) \partial^\mu \varphi_1 - \frac{1}{2} g \beta^2 (c_1 + 3n^2c_2) \varphi_1^2 
+ \frac{1}{2} (\partial_\mu \varphi_2) \partial^\mu \varphi_2 - \frac{1}{2} g \left[ \beta^2 c_1 + n^2 (\beta^2 c_2 + \xi g) \right] \varphi_2^2 
+ \frac{1}{2} (\partial_\mu \varphi_3) \partial^\mu \varphi_3 - \frac{1}{2} g \left( d + n^2bc_2 \right) \varphi_3^2 
- \frac{1}{2} g c_2 \left[ \beta^2 (\varphi_1^2 + \varphi_2^2) + b \varphi_3^2 \right] \times 
\times \left[ \beta^2 (\varphi_1^2 + \varphi_2^2) + b \varphi_3^2 + 4\xi \beta \right] \varphi_1 
+ g \beta A_\mu \left( \varphi_1 \partial^\mu \varphi_2 - \varphi_2 \partial^\mu \varphi_1 \right) - g \beta n \left( c_1 + n^2 c_2 \right) \varphi_1 
+ \frac{1}{2} g^2 \beta \left[ \beta (\varphi_1^2 + \varphi_2^2) + 2n \varphi_1 \right] A_\mu A^\mu 
+ (\partial_\mu \bar{\eta}) \partial^\mu \eta - \xi g^2 n^2 \bar{\eta} \eta - \xi g^2 \beta n \varphi_1 \bar{\eta} \eta \right\}. \tag{132}
\]

At the level of the gauge-fixed action the fields $\{\varphi_2, \bar{\eta}, \eta\}$ obviously describe unphysical degrees of freedom. Like in the second example, in (132) there appears a vertex that couples the physical scalar $\varphi_1$ to the ghosts.

## 9 Conclusion

To conclude with, in this paper we developed a novel mass generation mechanism for an Abelian vector field. This mechanism is based on the construction of a class of gauge theories whose free limit describes one massless vector field and a set of massless real scalar fields by means of the antifield-BRST deformation method. In this setting it was proved that:

1. The vector field gains mass irrespective of the number of scalar fields from the collection;
2. The gauge transformations are deformed with respect to the free limit, but their gauge algebra remains Abelian;
3. The massive vector field propagator behaves like that from the massless case in the limit of large Euclidean momenta;

4. Our procedure represents a cohomological extension of the Higgs mechanism;

5. Our scheme reveals an appropriate interpretation of the Higgs mechanism in the framework of the BRST symmetry.

Our main results were exemplified to the cases where the number of scalar fields is equal to one, two, and three. The particular situation of interactions in the presence of two scalar fields strengthens that our results include those emerging from the Higgs mechanism and, in addition, lead to a vertex that is omitted in the literature. The same kind of vertex has also been shown to appear for three scalar fields. The procedure exposed in this paper opens the perspective towards its generalization to the case of interactions among a collection of vector fields and a set of real scalar fields. This problem is under consideration [39].

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