Object oriented and functional programming
for symbolic manipulation

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Abstract

The advantages of mixed approach with using different kind of pro-
gramming techniques for symbolic manipulation are discussed. This
type of programming environment now is in development. The work is
stimulated by the necessity of managing complicated algebraic expres-
sions and formulae of noncommutative geometry related with HENP.

Functional languages are convenient for representation and manip-
ulation of symbolic data. From the other hand, it is convenient to use
special “shell” around the functional language to make programming
more transparent.

The main purpose of approach offered is merge the methods of
object oriented programming that convenient for presentation data
and algorithms for user with advantages of functional languages for
data manipulation, internal presentation, and portability of software.

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1 Introduction

We can consider two different approaches to computer algebra. On the one hand, it may be application of usual programming techniques to specific tasks and it is not essential to determine the particular programming language we are to use. The main problem here is to express the task by usual simple data types and operations. For example, we can express monomials like $x_1^k x_2^{l-k} y_1^m y_2^{n-m}$ those form $(l/2, n/2)$ representation of Lorentz group by using data “register” with four numbers like $[k \mid l \mid m \mid n]$. Multiplication of the monomials is addition of the registers, complex conjugation is swapping of left and right parts, etc. If we work with more complicated structures, the “advanced” properties of programming language can be more useful.

One of such properties is the constructive conception of data types as some preliminary information about data accessible for translator and possibility to construct new data types and data structures in addition to basic types $\mathbb{U}$. Other properties are discussed further. An object oriented Pascal-like language with some functional objects is used for the demonstration of possible synthesis of object oriented and functional programming. It is called here $\psi^{++}$ and is used in examples.

2 Object Oriented Programming (OOP)

The OOP languages are successfully used in modern computer science. Let us consider standard of C++ or some version of Object Pascal.

The object oriented style supposes some principles of decomposition of complicated tasks:

- The basic structure is an object. The object encloses not only data fields, but methods (procedures, functions) for manipulation with these data.

- An object can be defined as descendant of some other object. The object inherits all fields and methods of ancestor.

- Descendant objects can be used anywhere as parameters of functions or right part of assignment operator instead of ancestor.
An analogy of OOP with language of mathematics should be mentioned. It is useful for applications to \textit{computer algebra}. For example we can define object \textit{Group} with two methods:

\begin{verbatim}
Group = Object;
    function infix +(A,B : Group) : Group;
    { C++ version: friend Group operator + (Group,Group) }
    function prefix -(A : Group) : Group;
end: Group;
\end{verbatim}

It is possible to define \textit{Algebra} = \texttt{Object}((\texttt{Group}) with new multiplicative operation \texttt{"*"}, \texttt{Complex} numbers, \texttt{Dirac algebra} etc. Due to properties of OOP it possible to use function \texttt{'+'} for any descendant of \textit{object Group} and both functions \texttt{"*"}, \texttt{'+'} for any descendant of \textit{Algebra}. The methods like \texttt{"*"} can be different for all descendant objects, but expression like \texttt{A*B} invokes necessary algorithm for particular objects \texttt{A} and \texttt{B} due to standard principles of OOP. In the C++ the operator\texttt{*} can be \texttt{overloaded} for any object.

Another useful “algebraic” property of languages like C++ is the possibility of \textit{type conversion}. It is possible to write expression like \texttt{x := 2*(1+2*i)} instead of \texttt{x := Complex(2,0)*Complex(1,2)} because \texttt{real numbers} like \texttt{2} will be converted to \texttt{(Re : 2, Im : 0)} of type \texttt{Complex} by constructor \texttt{Complex(x : Number)}.

\section{Functional Programming (FP)}

The FP languages like LISP were one of the first tools for \textit{symbolic manipulation} and \textit{artificial intelligence} (AI). One of advantages of LISP for such applications is \textit{list} data structure. On the other hand \textit{lists} also can be simple defined in most of modern imperative languages.

Another advantage of FP is the clear functional structure of programs \cite{1}. In FP the whole program is considered as composition of functions. The “pure” functions are transformations of \textit{argument} to \textit{result} and do not depend on some global external variables, unlike the functions and procedures in most of programming languages.

Due to the pure functional structure in most of FP languages there are very powerful possibilities of construction and evaluation of \textit{new functions at run time}. It is possible to construct a new function as some special structure

\footnote{Within examples comments are inclosed in brackets: ‘{’ and ‘}’}
and perform it by using a special operator like `EVAL` in LISP. This property is very useful for AI due to possibility of “learning” by constructing of new algorithms. In symbolic manipulation the building of new functions is necessary for interpretation of symbolic expressions related with some variables and operations. For example, the string like `abc` often means `OP(a,OP(b,c))` there `OP` is some function and it is necessary to have the ability to perform these `OP` operations when values have been assigned to the variables `a`, `b` or `c`.

4 Combination of OOP and FP styles

The FP and OOP languages are mentioned above are universal languages. Any algorithm that has been written by using one of them can be rewritten by using any other universal language. The only problem is complication.

For example, it is possible to define `Polynomial = Object(Algebra)` and overload operators ‘*’ and ‘+’ for working with symbolic expressions for polynomials in OOP languages like C++.

There some synthesis OOP and FP is possible because both these approaches have tendency to unification of data structures and algorithms. In OOP data fields and methods of manipulation with these data are encapsulated in common structure: an object. Basic elements of FP are functions, but the same structure like `(add 1 x)` can be treated either as a data (the list with three elements: `add`, 1, `x`) or, as the body of a function (the addition of 1 to `x`).

From point of view of OOP the functions in FP can be considered as special type of functional objects. It was mentioned above that such “lazy functions” are essential for symbolic manipulation. It can be useful to add this conception to OOP. Internal realization of such function is some FP Object:

```
  FP_function = Object(FP_Object)
  Arg : Arguments;
  function Evaluate : Result;
  end: { FP_function F(Arguments) : Result }
```
4.1 Type checking

We should also to save the clear principles of types in OOP. For example, the expression \( y := 3 \ast (x+1) \) must be valid. It means that type of 3 (integer) is compatible with functional object \((\text{Arg};(1,x),+)\) i.e. any functional object has type compatible with type of result of the function. The variable \( x \) in this expression does not have an assigned value. Such kind of objects are useful for symbolic manipulation and it has used in FP or logical programing, but it lack of in usual imperative OOP languages. We should distinguish such variable and variable with value, but both types must be compatible.

For using of the FP principles in \( \psi^{++} \) are introduced some extensions of types. Usually if we have some variable of type \( T \) we can assign to it only value of the same or descendant type. In \( \psi^{++} \) there are three variant for every type \( T \): \textit{value} of \( T \), \textit{variable} of \( T \), and \textit{functional object} of \( T \). For example:

\[
\text{var} \quad a, b, c, d : \text{integer}; \{ a, b, c, d : \text{integer variable} \}
\]
\[
a := 1; \{ a : \text{integer value} \}
\]
\[
b := c + d; \{ b : \text{integer functional object} \}
\]

4.2 Algebraic example

After such extension we can continue an analogy with language of mathematics. We can describe property of distributivity for object \textit{Algebra}.

\[
\text{function} \quad \text{Algebra.infix}^\ast (A, B : \text{Algebra}) : \text{Algebra};
\]
\[
\text{par} \quad C, D, E, F : \text{Algebra};
\]
\[
\text{begin} \quad \text{if} \ A = C + D \text{ then } \{ \text{i.e. } A \text{ is record } (+, C, D) \}
\]
\[
\text{Return} := C \ast B + D \ast B
\]
\[
\text{else} \quad \text{if} \ B = E + F \text{ then Return } := A \ast E + A \ast F
\]
\[
\text{else Return } := \text{fail}
\quad \text{end};
\]

The \textit{fail} is value compatible with all types. In OOP it is useful for \textit{inheritance}. For example, for object \textit{Complex} is possible to write:
function Complex infix∗ (A, B : Complex) : Complex;
begin
    Return := Algebra.(A ∗ B);  { Inherited method for algebraic expressions }
    if Return = fail  { No – Directly perform the multiplication }
    then Return := (A.Re ∗ B.Re − A.Im ∗ B.Im, A.Re ∗ B.Im + A.Im ∗ B.Re)
    end;

    After such definition we can freely use the same operation for manipulation with algebraic expression and with usual complex numbers. The calling of inherited part of method is managed with algebraic transformations like distributivity ((i + x) ∗ i → i ∗ i + i ∗ x) and another part perform usual complex multiplication (i ∗ i + i ∗ x → −1 + i ∗ x).

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