Decoy State Quantum Key Distribution With Modified Coherent State

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To beat PNS attack, decoy state quantum key distribution (QKD) based on coherent state has been studied widely. We present a decoy state QKD protocol with modified coherent state (MCS). By destruction quantum interference, MCS with fewer multi-photon events can be get, which may improve key bit rate and security distance of QKD. Through numerical simulation, we show about 2-dB increment on security distance for BB84 protocol.

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I. INTRODUCTION

Quantum Key Distribution (QKD) [1, 2, 3], combining quantum mechanics and conventional cryptography, allows two distant peers (Alice and Bob) share secret string of bits, called key. Any eavesdropping attempt to QKD process will introduce high bit error rate of the key. By comparing part of the key, Alice and Bob can catch any eavesdropping attempt. However, most of QKD protocols, such as BB84, needs single photon source which is not practical for present technology. Usually, real-file QKD set-ups [4, 5, 6, 7, 8] use attenuated laser sources which have photon-number statistic distribution with less probability of multi-photon events, a laser sources which have photon-number statistic distribution. From derivation of formulas for estimating $S_1^f$ and $e_1^f$ [12, 13], we know that the difference between the real value of $S_1^f$ and $e_1^f$ originates from the negligence of multi-photon counts events. Given some new laser sources which have photon-number statistic distribution with less probability of multi-photon events, a more precision estimation of $S_1^f$ and $e_1^f$ should be obtained.

In fact, it’s proven that modified coherent state (MCS) with less probability of multi-photon events could improve the security of QKD by [28]. The scheme of MCS generation relies on quantum interference to depress multi-photon events from the coherent state. We can write the MCS by [28]:

$$|\Psi\rangle_{MCS} = \hat{U}(\alpha) = \sum_{n=0}^{\infty} C_n |n\rangle$$

with

$$\hat{U} = \exp \frac{1}{2} (\zeta^* \hat{a}^2 - \zeta \hat{a}^\dagger)$$

$$C_n = \frac{1}{\sqrt{n! \mu}} \left( \frac{\nu}{2\mu} \right)^n \exp \left( \frac{\nu^2 \alpha^2 - |\alpha|^2}{2} \right) H_n \left( \frac{\alpha}{\sqrt{2\mu\nu}} \right)$$

and

$$\mu = \cosh(|\zeta|), \quad \nu = \frac{\zeta}{|\zeta|} \sinh(|\zeta|), \quad \text{or} \quad \mu^2 = 1 + |\nu|^2.$$
Here, we can simply take $\alpha$, $\mu$, and $\nu$ as real number because the value of $P_n$ only concerns with the module of them. From equation (5), we can conclude that the MCS source is a source that emits $n$-photon state $|n\rangle$ with probability $P_n$.

II. DERIVATION

And now we can deduce formulas for $3$-intensity MCS decoy QKD and $2$-intensity MCS one. Through adjusting the intensities of input coherent states $|\alpha\rangle$, we can get sources of different $\nu$ corresponding to different $\alpha$. Two different sources of density matrices $\rho_\nu$ and $\rho_{\nu'}$ could be get by this way. The counting rates for the two sources ($\nu < \nu'$) are given by:

$$S_\nu = \sum_{n=0}^{\infty} P_n(\nu)S_n$$

$$S_{\nu'} = \sum_{n=0}^{\infty} P_n(\nu')S_n$$

where, $S_n$ represents counting rate for photon number state $|n\rangle$. And quantum bit error rate (QBER) for $\nu'$ is:

$$E_{\nu'}S_{\nu'} = \sum_{n=0}^{\infty} e_n P_n(\nu')S_n$$

In which, $e_n$ is QBER for the key bits generated by photon number state $|n\rangle$. To derive formulas for $S_{\nu}$ and $e_{1\nu'}$, it’s necessary to prove that $\frac{P_2(\nu')}{P_2(\nu)} P_n(\nu) \leq P_n(\nu')$ for all of $n \geq 2$.

$$\frac{P_2(\nu')}{P_2(\nu)} P_n(\nu) \leq P_n(\nu')$$

$$= 2^{n-2} n |H_2(\frac{1}{\sqrt{\nu'}})|^2 \left((1 + \frac{1}{\nu'^2})^{\frac{n-2}{2}} - (1 + \frac{1}{\nu^2})^{\frac{n-2}{2}}\right) \leq 0$$

From equation (9), we have proven $\frac{P_2(\nu')}{P_2(\nu)} P_n(\nu) \leq P_n(\nu')$. Now we can deduce the formulas for calculating $S_{1\nu}$:

$$S(\nu') = P_0(\nu')S_0 + P_1(\nu')S_1 + P_2(\nu')S_2 + P_3(\nu')S_3 + \cdots$$

$$\geq P_0(\nu')S_0 + P_1(\nu')S_1 + \frac{P_2(\nu')}{P_2(\nu)} \sum_{n=2}^{\infty} P_n(\nu)S_n$$

Combining with equation (6), we have

$$S_{1\nu} = \frac{(P_2(\nu)P_0(\nu') - P_2(\nu')P_0(\nu))S_0 + P_2(\nu')S(\nu) - S(\nu')}{P_2(\nu')P_1(\nu) - P_2(\nu)P_1(\nu')}$$

According to equation (8), estimation of $e_{1\nu'}$ is given by:

$$e_{1\nu'} = \frac{(E_{\nu'}S_{\nu'} - \rho_\nu\rho_{\nu'}(e_{1\nu'})}{P_1(\nu'S_1'_{\nu})}$$

Now we have get the formulas for calculating $S_{1\nu}$ and $e_{1\nu'}$ for three-intensity case. In this case Alice randomly emits laser pulses from source $\rho_\nu$, $\rho_{\nu'}$, or doesn’t emit anything, then Bob can get counting rates for the three case: $S_\nu$, $S_{\nu'}$ and $S_0$. Then Alice and Bob perform error correction and private amplification by $S_{1\nu}$ and $e_{1\nu'}$ calculated through equation (11) and (12). The lower bound of security key rate is given by [13]:

$$R^L = q\{-S_{\nu'}f(E_{\nu'})H_2(E_{\nu'}) + P_1(\nu'S_1'_{\nu})[1 - H_2(e_{1\nu'})]\}$$

with $q = \frac{1}{2}$ for BB84, $f(E_{\nu'})$ is he bidirectional error correction efficiency (typically, $f(E_{\nu'}) = 1.2$), and $H_2$ is the binary Shannon information function.

For two-intensity case, Alice randomly emits laser pulses from source $\rho_\nu$ and $\rho_{\nu'}$, then Bob can get counting rates for the two cases: $S_\nu$ and $S_{\nu'}$. Now, Alice and Bob can get $S_0$ firstly, then calculates $S_{1\nu}$ by equation (14) with taking $S_0$ as $S_0$. The formula for calculating $S_0$ can be derived from equation (8) simply, it’s:

$$S_0 = \frac{2E_{\nu'}S_{\nu'}}{P_0(\nu')}$$

So the formula for two-intensity case is given by:

$$S_{1\nu} = \frac{2(P_2(\nu)P_0(\nu') - P_2(\nu')P_0(\nu))E_{\nu'}S_{\nu'} + P_2(\nu')S(\nu) - S(\nu')}{(P_2(\nu')P_1(\nu) - P_2(\nu)P_1(\nu'))P_0(\nu')}$$

To get $e_{1\nu'}$, we can assume $S_0 = 0$ and let $S_0 = S_{1\nu}$, then from equation (12) $e_{1\nu'}$ could be get by:

$$e_{1\nu'} = \frac{E_{\nu'}S_{\nu'}}{P_1(\nu'S_{1\nu})}$$

Equation (11) and (12) are formulas for three-intensity protocol, while equation (15) and (16) are for two-intensity protocol. These are main results of our derivation.

III. IMPROVEMENT FOR DECOY STATE QKD

In this section, our purpose is to show MCS’s improvement for decoy state QKD by numerical simulation. We consider the case when there is no Eve. And from [13]:

$$e_n = \frac{S_{1\nu} + \epsilon_{a+\eta} \eta_n}{S_n}, \eta_n = 1 - (1 - \eta)^n, \eta = 10^{-kl/10} \eta_{Bob},$$
S_n = S_0 + \eta_n$, where, $\epsilon_{\text{det}}$ is the probability that the survived photon hits a wrong detector, $\eta$ is overall yield and $\eta_1$ is yield for photon number state $|n\rangle$, $k$ is transmision fiber loss constance, $L$ is fiber length and $\eta_{\text{Bob}}$ is the transmittance loss in Bob’s security zone. According to $S$, we set $\epsilon_{\text{det}} = 0.0135$, $S_0 = 8 \times 10^{-7}$, $k = 0.2\,\text{dB/Km}$ for numerical simulation. We simply set $\eta_{\text{Bob}} = 1$, because our purpose is a comparison not absolute distance. These are our parameters and formulas for numerical simulation.

A. To Cancel Two-photon Events

Here, we set $c = 1$ to cancel all two-photon events. We cannot use equation (11) and (12) immediately for $P_2 = 0$. But, it’s easy to see that we can replace $P_2$ as $P_3$, and now the equations are available for this case. Firstly, we will show the increment of precision for estimating $S^L_{1}$. Typically, we set $\alpha = \sqrt{0.2}$, $\alpha' = \sqrt{0.6}$ as the two inputs for the MCS generator. With these inputs, one can get two kinds of MCS with $\nu = 0.196$ and $\nu' = 0.53$. Fig1 shows that real $S_1$, $S^L_{1}$ calculated by ordinary decoy state QKD based on coherent state ($\alpha = \sqrt{0.2}$ for decoy state and $\alpha' = \sqrt{0.6}$ for signal state) and $S^L_{1}$ calculated by MCS decoy QKD ($\nu=0.196$ for the decoy state and $\nu'=0.53$ for the signal state). From Fig1, we can conclude that for two-intensity case MCS decoy state QKD is indeed more effective to calculate $S^L_{1}$ than traditional decoy state QKD based on coherent state. We found that in two-intensity protocol the longest length still capable of estimating $S^L_{1}$ precision increases by 20KM.

Secondly, we compare the key bit rate $R$ of MCS decoy QKD and QKD based on coherent state. To compare the two decoy QKD process more fairly, we draw Fig2 in which each point has optimal value of $\alpha$ and $\alpha'$ or $\nu$ and $\nu'$ for two-intensity case. But for three-intensity case, we set the average photon-number of decoy pulses as 0.1 and $\nu'$ or $\alpha'$ has optimal value for each point.

From Fig2, we see that in two-intensity case about 3KM increment on security distance could be get by using MCS and in three-intensity case 2KM increment is given.

B. To Cancel Three-photon Events

Here, we set $c = 3$ to cancel all two-photon events. And equation (11) and (12) can be used immediately. Though MCS without three-photon events has more multi-photon events than the one without two-photon events, former has higher total counting rates and lower QBER, which may increase $R$. The results are drawn in Fig3. In Fig3, each point has optimal value of $\alpha$ and $\alpha'$ or $\nu$ and $\nu'$ for two-intensity case. But for three-intensity case, we set the average photon-number of decoy pulses is 0.1 and $\nu'$ or alpha' has optimal value for each point.

From Fig3, we see nearly 2-dB increment is given on security length both for two states protocol and three states protocol. This result is better than $c = 1$ MCS. We found MCS without three-photon events has higher counting rates and lower QBER than MCS ($c=1$). This is the reason why $c = 3$ MCS has better performance.

In above discussion, we set $c = 1$ to cancel two-photon events or $c = 3$ to cancel three-photon events. However, we can also set $c$ as some arbitrary positive value, provided this value make $R$ rise. And we draw Fig4 in which the relation of increment of security distance between $c$ is given. From Fig4, we see optimal $c$ is different for two-intensity and three intensity cases. For two-intensity case, the optimal value is 3.3 and for three-intensity it’s 2.8.

FIG. 1: Counting rate for single-photon laser pulses ($S_1$) verse fiber length $L$. Solid curve: real value of counting rate for single-photon laser pulses for no eavesdropping case. Dashed curve: $S^L_{1}$ calculated by traditional decoy state QKD based on coherent state. Dotted-dashed curve: $S^L_{1}$ calculated by MCS decoy QKD.

FIG. 2: security key rate ($R^L$) verse fiber length $L$. Solid curves: $R^L$ with 2-intensity and 3-intensity decoy QKD based on coherent state. Dashed curves: $R^L$ with 2-intensity and 3-intensity MCS decoy QKD.

FIG. 3: security key rate ($R^L$) verse fiber length $L$. Solid curves: $R^L$ with 2-intensity and 3-intensity decoy QKD based on coherent state. Dashed curves: $R^L$ with 2-intensity and 3-intensity MCS decoy QKD.

FIG. 4: security key rate ($R^L$) verse fiber length $L$. Solid curves: $R^L$ with 2-intensity and 3-intensity decoy QKD based on coherent state. Dashed curves: $R^L$ with 2-intensity and 3-intensity MCS decoy QKD.
IV. CONCLUSION

According to above discussion, we see that: thanked to MCS’s fewer multi-photon events probability, decoy state with MCS source can indeed provide QKD service of higher key bit rate and longer distance than before. We found about 2-dB increment of security distance is acquired. Generating this kind of MCS laser pulses isn’t difficult for today’s Lab. We expect that our MCS decoy QKD scheme could be implemented earlier.

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FIG. 3: security key rate ($R^L$) verse fiber length $L$. Solid curves: $R^L$ with 2-intensity and 3-intensity decoy QKD based on coherent state. Dashed curves: $R^L$ with 2-intensity and 3-intensity MCS decoy QKD.

FIG. 4: increment of security distance ($\Delta L$) verse $c$. Solid curve: for the 2-intensity case. Dotted curve: for the 3-intensity case. And each point has optimal $\nu$ and $\nu'$. 