Multiplicity Distributions in QCD and $\lambda\phi^3$-model

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Abstract

It is shown that QCD with vector gluons predicts drastically different multiplicity distributions compared to the scalar $\lambda\phi^3$-model (in 6 dimensions). In particular, the QCD predicted minimum of cumulant moments of the distributions at ranks about $5 \div 7$ does not reveal itself in the scalar model. The oscillations at higher ranks survive only. Thus, the experimental fact of that minimum at ranks about $5 \div 7$ supports the vector nature of gluons and singular kernels in QCD and gluodynamics.

1 Introduction

The multiplicity distribution of particles (partons) in QCD (or gluodynamics) has been a long-standing problem. The solutions of equations for generating functions of multiplicity distributions have been obtained in the lowest double logarithmic approximation \cite{1}. The shape of the distribution appears to be completely different from that observed in multiparticle production experiments for secondary hadrons, even though the energy dependence is rather reasonable and, moreover, the shape fulfils the so-called KNO-scaling \cite{2}. The theoretical shape is much wider. In terms of the moments of the distribution it implies much faster increase of them with rank increasing, compared to experimental data.

Meantime, the theoretical attempts to take into account the conservation laws more precisely were published \cite{3, 4, 5}. The multiplicity distribution became rather narrow\cite{5}. However, in view of some additional assumptions it is difficult to estimate how rigorous the results are. Moreover, the theoretical moments of the distributions differ from experimental ones.

The problem was solved when the solution of the equations for generating functions was found at higher (than double-logarithmic one) approximations in gluodynamics \cite{6, 7} and QCD \cite{8} in case of the running coupling constant. Later, the exact solution of these equations at fixed coupling constant was obtained\cite{9}. The most remarkable prediction \cite{6} of these findings is the minimum of the cumulant moments of the multiplicity distribution at the values of ranks close to 5 and subsequent oscillations of those moments at higher values of the rank \cite{7} which have been confirmed in experiment at analysis of hadron multiplicity distributions in multiparticle production at high energies for various colliding particles \cite{10} (see also the review paper \cite{11}).
Let us note also that no phenomenological distributions well known and widely used in physics (i.e. those distributions of probability theory as Poisson, negative binomial, geometric, fixed multiplicity) can reproduce such a behaviour. In particular, the negative binomial distribution gives rise to the positive and monotonically decreasing ratio of cumulant to factorial moments, while QCD predicts the negative values of this ratio at the first minimum and its subsequent oscillations about the abscissa (where the ranks of the moments are plotted). Even the so-called modified negative binomial distribution, specially designed empirically \cite{12} to get the best fit of experimental multiplicity distributions has failed in some cases \cite{13} to produce the relevant minima and oscillations of the moments. Thus the analysis of multiplicity distributions in terms of cumulant and factorial moments as well as of their ratio is at present the most sensitive method of revealing the specific typical details of these distributions.

The very existence of the minimum in QCD and its location are closely defined by the singularity of the gluon kernel (the vector nature of gluons) and by the value of the coupling constant in QCD. In that connection it is of interest to understand how important the both factors are. According to the formulae of the paper \cite{6}, the main contribution to the minimum location is inverse proportional to the QCD anomalous dimension (i.e. to the coupling constant) and therefore varies in QCD in the limits of its "running" property i.e. comparatively slow and hard to observe if, especially, one takes into account that the ranks of the moments can be integer only. The more important property is the vector nature of massless gluons giving rise to the singularity of the kernel of the equation for the generating function. That is why it would be desirable to confront the QCD (or gluodynamics) predictions to those of the scalar fields model. Fortunately, there is such a model\cite{14} possessing, besides others, the property of the asymptotical freedom what helps to reduce the impact (even if it is not very essential) of the problem of the value of the coupling constant. This is the $\lambda \phi^3$-model in the 6-dimensional space-time.

Therefore the purpose of the present paper is to get the knowledge of the behaviour of the multiplicity distribution moments for the $\lambda \phi^3$-model in the 6-dimensional space-time and to compare the obtained results with those of QCD and of the simplest phenomenological distributions of the probability theory. The main conclusion which we have got from it is that the vector nature of gluons is very crucial for the shape of the multiplicity distribution and qualitatively changes its moments behaviour so that QCD confirms its predictive power once again even at purely partonic level while the scalar model has been unable to reproduce the qualitative features of experimental data.
2 Equations and their solutions

The theoretical problem, we are addressing at, is as follows. Let a scalar, strongly virtual particle (with high time-like 4-momentum squared) has been produced in some collision. During its evolution the virtuality decreases due to emission of other scalar particles. What is the multiplicity distribution of secondary particles if the emission is controled by the $\lambda \phi^3$-interaction in the 6-dimensional space-time? This problem is analogous to the production and evolution of the pair of strongly virtual quark and antiquark in $e^+e^-$-annihilation. The increased dimensionality of the space-time, as we have mentioned, is necessary here just to get the asymptotical freedom in that theory and to make it closer to the real situation in QCD (or in gluodynamics).

The evolution of such a "scalar jet" is described by the equation for generating functions (or functionals) analogous to the commonly used "birth and death equation" and different from QCD equations by its kernel, describing the interaction vertex in the theory. However, just this distinction strongly influences the multiplicity distribution and its moments. The principal difference is that the QCD kernel is singular while in the scalar model it is regular. Physics corollary is an approximate equipartition of the "parent" energy among its "children" in the scalar model while in QCD the energy is shared in unequal parts i.e. one of the produced partons used to get much higher energy than another one.

Let us turn to our problem and reminde some general notations. The generating function is

$$G(y, z) = \sum_{n=0}^{\infty} (1 + z)^n P_n(y),$$

where $P_n(y)$ is the multiplicity distribution, $y = \ln(Q/Q_0)$, $Q$ is the jet virtuality, $Q_0 = \text{const.}$

The normalized factorial moments $(F_q)$ and cumulants $(K_q)$ are defined by the generating function as

$$G(z) = \sum_{q=0}^{\infty} \frac{z^q}{q!} \langle n^q \rangle F_q, \quad \ln G(z) = \sum_{q=0}^{\infty} \frac{z^q}{q!} \langle n^q \rangle K_q,$$

where $\langle n \rangle$ is the average multiplicity of particles (partons).

The generating function satisfies the non-linear integro-differential equation [1] (the prime denotes the $y$-derivative):

$$G'(y) = \int_0^1 d\xi K(\xi) \gamma_0^2 [G(y + \ln \xi)G(y + \ln(1 - \xi)) - G(y)],$$

where $K(\xi)$ is the kernel of the equation and in our cases is written as:

1) for gluodynamics [1]:

$$K(\xi) = 1/\xi - (1 - \xi)[2 - \xi(1 - \xi)],$$

where

$$G(y, z) = \sum_{n=0}^{\infty} (1 + z)^n P_n(y),$$

is the generating function.
2) for $\lambda \varphi^3$-model in 6 dimensions \cite{14}:

$$K(\xi) = 6\xi(1 - \xi).$$

(5)

$\gamma_0^2 = 2N_c\alpha_s/\pi, \; \alpha_s = 2\pi/11y$ in QCD. In $\lambda \varphi^3$-theory one does not have any physics normalization and has to rely on QCD $\gamma_0^2$ in choosing the numerical value of $\lambda$.

Now we discuss $\lambda \varphi^3$-model directly and consider first the running coupling case, following the method of the approximate solution used in \cite{6},\cite{7} for gluodynamics. After the Taylor series expansion of the generating function at point $y$ has been done, one gets

$$G'(y) = \gamma_0^2\{G(y)(G(y)-1)+G(y)\sum_{n=1}^{\infty}(-1)^n h_n G^{(n)}(y) + \sum_{n,m=1}^{\infty}(-1)^{n+m}h_{nm} G^{(n)}(y) G^{(m)}(y)\},$$

(6)

where

$$h_n = \frac{12}{n!}(-1)^n \int_0^1 d\xi \xi(1 - \xi) \ln^n(1 - \xi) = 12\left(\frac{1}{2n+1} - \frac{1}{3n+1}\right),$$

$$h_{nm} = \frac{6}{n!m!}(-1)^{n+m} \int_0^1 d\xi \xi(1 - \xi) \ln^n(\xi) \ln^m(1 - \xi)$$

or within the approximation used in \cite{7}, one gets

$$\langle n \rangle = \exp(\int^y \gamma(y')dy')$$

and we substitute (2) to (7). The coefficients in front of $z^q$-terms should be equal on both sides wherefrom the moments satisfy the equation

$$k_q = \frac{1}{1-H_0 q} \sum_{l=1}^{q-1} k_{q-l} f_l \left\{ \frac{H_0^q}{l} + \gamma_0^2 x \left[ \frac{h_2}{l} + \frac{h_{11}}{q}\right] \right\},$$

(8)

$$x \equiv q\gamma, \; H_0^q \equiv \gamma_0^2[1/x - 5/3 + h_2\gamma'/\gamma], \; k_q \equiv K_q/(q - 1)!, \; f_q \equiv F_q/(q - 1)! \text{ and we have used the relation}$$

$$f_q = k_q + \sum_{l=1}^{q-1} \frac{k_{q-l} f_l}{l}.$$
The recurrent formula (8) has been used to calculate \( k_q \) and to get \( f_q \) with the help of eq.(9) as well as their ratio \( H_q \) if \( \gamma \) is known.

The equation for \( \gamma \) obtained from (8) in the similar way to (7) but for \( q = 1 \) is written as

\[
\gamma - \gamma_0^2 [1 - \frac{5}{3} \gamma + h_2(\gamma' + \gamma^2)] = 0,
\]

what represents \( \gamma \) in terms of \( \gamma_0 \) as

\[
\gamma = \gamma_0^2 [1 - \frac{5}{3} \gamma_0^2] + O(\gamma_0^6).
\]

The results of computing \( H_q \) according to (8) for \( \gamma_0 = 0.48 \) are shown in Fig.1. In distinction to QCD results obtained within the same approximation in \( q\gamma \), there are still no oscillations and \( H_q \) tends to constant asymptotics (it reminds of QCD in the lower order in \( q\gamma \) – see [6]).

Varying \( \gamma_0 \) within the wide limits, one does not observe any essential qualitative changes. The minimum is slightly shifted to larger values of \( q \) at smaller \( \gamma_0 \).

Thus we notice that the absence of the singular term in the kernel (3) in the \( \lambda \varphi^3 \)-model compared to the gluodynamics (the formula (4)) gives rise to qualitatively different behaviour of the moments.

In the case of the fixed coupling constant, the equation (3) can be solved exactly [9]. Assuming that the \( y \)-dependence is contained completely in the average multiplicity \( \langle n \rangle \) and is of the kind \( \langle n \rangle = \exp(\gamma y) \), one gets the recurrent formula for the factorial moments

\[
\bar{f}_q = \frac{6\gamma_0^2}{x(2 + x)(3 + x) + \gamma_0^2(x - 1)(x + 6)} \sum_{l=1}^{q-1} \bar{f}_{q-l} \bar{f}_l,
\]

where \( \bar{f}_q = f_q \Gamma(x + 2)/q = F_q \Gamma(x + 2)/q! \) and

\[
H_q = 1 - \sum_{l=1}^{q-1} \frac{H_{q-l} f_{q-l} f_l}{f_q}.
\]

The analogous formula in gluodynamics looks like (it can be easily obtained from the formulae of [3] if the quark degrees of freedom are neglected)

\[
\bar{f}_q = \frac{\gamma_0^2}{x - \gamma_0^2 M_q} \sum_{l=1}^{q-1} N_{q,l} \bar{f}_{q-l} \bar{f}_l,
\]

where

\[
M_q = \frac{1}{x} + \Psi(1) - \Psi(1 + x) + \frac{11}{12} \frac{2}{1 + x} + \frac{1}{(2 + x)(3 + x)};
\]

\[
N_{q,l} = \left( \frac{1 + x}{l\gamma} - 1 \right) \frac{1}{(1 + l\gamma)(1 + (q - l)\gamma)} + \frac{1}{2(2 + x)(3 + x)}.
\]
In Figs. 2 and 3 we show the results of computing according to formulae (11) and (12) correspondingly. One can conclude from them that the influence of the singularity in the kernel $K(\xi)$ on the location of the minimum consists in its shift to smaller values of ranks $q$ (from $q_1 \approx 14$ to $q_2 \approx q_1/2 = 7$) with the accompanying diminishing of the “period” of the oscillations.

3 Conclusion

The main conclusion one gets from above consideration is that the scalar $\lambda\varphi^3$-model in 6 dimensions with approximate equipartition of energy among the produced particles gives rise to the qualitatively different predictions about the behaviour of the moments of multiplicity distributions compared to the gluodynamics or/and QCD, where the vector nature of massless gluons implies the singularity of the kernel of the equation and, therefore, the drastically unequal shares of energy for produced particles. This fact provides the minimum in the ratio of cumulants to factorial moments at ranks near $5 \div 7$ in gluodynamics and QCD, while in the scalar model such a minimum does not appear and the oscillations at higher values of ranks survive only.

From the purely theoretical approach, this conclusion is hardly extremely important. However, it becomes essential when one confronts it to the experimentally known facts about hadron multiplicity distributions in real events [10, 11]. It happened that the minimum of $H_q$ at $q \sim 5$ was observed [10] in experimental data just after its prediction [6] in gluodynamics. Even though this prediction was done about partons and not about final hadrons, the very fact of the presence of the minimum with the same location as predicted looks very impressive. In combination with the inability of the phenomenological models as well as of the theory field model considered above to reproduce the existence of the minimum and its location, it confirms once again our belief in the predictive power of QCD. Other numerous facts (the hump-backed plateau of the rapidity distribution, the heavy quark effects etc. – see in detail in [1] and in later papers) support the conclusion that the qualitative effects of QCD predicted at the parton level find out their confirmation by the experimental data, and Monte-Carlo simulations provide the quantitative estimates of the influence of hadronization.

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