Development of non-stationary buoyancy-induced jets and conjugate heat transfer at the jet inleakage on an obstacle of finite thermal conductivity

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Abstract. The evolution of a buoyancy induced jet in a high-viscosity liquid with $Pr = 2700$ over a suddenly switched-on linear heat source is numerically investigated. The dependence of the velocity, temperature, and local heat fluxes on time has been investigated at a layer height of 50 mm and the Grashof number $Gr = 172$. Numerical simulation is performed using the finite element method. The complete system of equations of nonstationary thermogravitational convection for a two-dimensional flow is solved. The dependence of the spatial forms of flow and velocity fields on the power of the heat source is studied experimentally. Digital video shooting and computer processing of video films are used to determine the velocity fields. The results obtained can potentially be useful in the creation of adequate models of thermal plumes.

1. Introduction

With the development of geodynamics, new tasks arise that require an interdisciplinary approach to their solution. One of such problems is the processes of the origin and development of mantle plumes. Mantle plumes are the streams of a heated or molten substance rising to the surface of the Earth. The theory of mantle plumes originated in the 1960s and today is one of the central theories of global disasters in the history of the Earth, the emergence of volcanic islands and deposits of minerals [1-5].

An integrated approach requires clarifying the nature of the origin of plumes and the depth of the source, the determination of the power of the heat sources necessary to create and move the ascending flow to the base of the earth's crust. According to some hypotheses, molten matter rises from the boundary between the mantle and the core, lying at a depth of 2900 km from the Earth's surface [3]. The study of mantle plumes in the natural environment is complicated by the scale of the processes and the low rate of their development. Currently, the only method to study the state of deep layers of the Earth and observe geodynamic processes is seismic tomography, based on measuring the parameters of seismic waves [6]. Deciphering the processes of propagation of longitudinal and transverse seismic waves allowed to restore the layered structure of the earth's interior and even to obtain information about local inhomogeneities of the density of deep-lying layers. Studies based on modern methods of processing multi-angle seismology data allow one to propose hypotheses about the depth of plume origin [6], as well as the global circulation of substances in the outer core and mantle of the Earth within the framework of the superplume theory [1-4, 7]. It is practically impossible to observe slow processes with a characteristic time of 1 million years, so the problem of physical and numerical simulation of mantle processes and their influence on the temperature fields in the earth's
crust and on its surface is relevant. The statement of simulation problems is complicated by the fact that a number of thermophysical properties of substances and many geophysical parameters of the Earth's depths are known only on the basis of indirect data [7]. Analogous problems arise in the analysis of processes of interaction of the head part of plumes with the earth’s crust. The thermophysical properties of substances in deep layers, at least in order of magnitude, are determined in experiments with rock melts in high-pressure installations. As shown by experimental studies, they have a very high viscosity [7]. Therefore, a liquid with a large Prandtl number of PES-5 was chosen for experimental modelling [8, 9]. To model the processes of thermal interaction of plumes with the Earth's crust, Plexiglas was chosen as a model of the crust. Plexiglas thermal conductivity value is close to PES-5.

It is practically impossible to experimentally obtain data on nonstationary temperature fields inside solid walls when carrying out investigations on physical models. Measurements of temperature fields in non-stationary modes of convection are also a laborious process. Therefore, the application of mathematical simulation is relevant. Hydrodynamics in the development of plumes over a linear heat source has been experimentally studied. This part is a continuation of [8, 9]. The development of an ascending natural convective jet over a local heat source and the associated convective heat exchange of its head part with a horizontal obstacle of finite thermal conductivity are numerically studied.

Numerical simulation of free convection was carried out by the finite element method on the basis of the dimensionless system of Navier-Stokes equations in the Boussinesq approximation written in terms of temperature, vorticity and the stream function:

$$\frac{\partial T}{\partial t} + \left( \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) = \frac{1}{\text{Pr}} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

$$\frac{\partial \omega}{\partial t} + \left( \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} \right) - \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) = \text{Gr} \frac{\partial T}{\partial x}$$

Here, Gr = $g \beta \Delta T H^3/\nu^2$ is the Grashof number, where $g$ is the gravitational acceleration, $\beta$ is the thermal expansion coefficient of the liquid, Pr = $\nu/\alpha_t$ is the Prandtl number, $\alpha_t$ is the thermal diffusivity

2. Problem definition
The problem of conjugate convective heat transfer of an ascending buoyancy-induced jet with an upper heat-conducting wall is solved in a two-dimensional formulation in Cartesian coordinates. The computational domain, as shown in figure 1, is a two-dimensional rectangular cavity $\Omega_1$ filled with fluid. Above, the liquid layer is closed by a rigid wall $\Omega_2$ of a specified thickness of 10 mm. The heat source HS is located on the lower rigid adiabatic wall $S_3$ of the cavity and has the shape of a flat strip of a predetermined width $W_{HS}$. The upper boundary of the source is at the level of the adiabatic wall. The lateral vertical walls $S_2$ and $S_4$ of the cavity are rigid and adiabatic.

As the scale of the geometric dimensions, the thickness of the liquid layer $H$ is chosen. The temperature scale is $T_1 - T_2$, where $T_1$ and $T_2$ are the temperatures at the heat source and on the outer side of the upper wall, respectively. The velocity scale is $v/H$, where $v$ is the kinematic viscosity of the liquid, the time scale is $H^2/\nu$. Convective heat transfer in a fluid is described by the dimensionless system of Navier-Stokes equations in the Boussinesq approximation, written in terms of temperature, vorticity and the stream function:
of the liquid, $T$ is the dimensionless temperature, $\omega$ is dimensionless vorticity, $\psi$ is the dimensionless stream function. Numerical experiments were carried out with polyethylsiloxane liquid PES-5 with the Prandtl number $Pr = 2728$ (at 20 °C).

The conductive heat transfer in the upper wall is described by the equation of thermal conductivity:

$$\frac{\partial T}{\partial t} + \alpha_s \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0,$$

where $\alpha_s$ is the coefficient of thermal diffusivity of the material of the upper wall. For the upper wall, the properties of plexiglas (plexiglass SOL) are taken: the coefficient of thermal conductivity $\lambda = 0.184 \text{ W/m•K}$ and the coefficient of thermal diffusivity $\alpha_s = 1.21 \times 10^{-7} \text{ m}^2/\text{s}$. Below are the results of calculations performed with a heat source in the form of an isothermal strip, at $H = 50 \text{ mm}$, $T_1 = 20 \degree\text{C}$ and $T_2 = 21.45 \degree\text{C}$. The horizontal cavity size is $L = 300 \text{ mm}$. The dimensioned width of the heat source is $20 \text{ mm}$, the dimensionless width is 0.4.

The system of equations is discretized using the finite element method on an inhomogeneous triangular mesh. The mesh of the calculated area consists of 6498 nodes and 12667 elements (triangles). At the liquid - upper boundary and liquid - surface of the heat source interfaces the condition of ideal contact, that is, the continuity of temperature and heat flux, is set.

3. Results and discussion

At the initial time, the system shown in figure 1 is in the isothermal state. The temperature of the system is equal to the temperature on the outside of the boundary $S5 T_1 = 20 \degree\text{C}$. At the instant $t = 0$, the heat source HS with a predetermined temperature $T_2 = 21.45 \degree\text{C}$ is suddenly switched on. The properties of

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Snapshots of the evolution of thermal field in the liquid with $Pr = 2728$, $Gr = 172$.}
\end{figure}
liquid PES-5 are taken at a temperature of 20 °C. At a layer height $H = 50 \text{ mm}$ and a given temperature difference, this corresponds to the Grashof number $\text{Gr} = 172$. At 20 °C the Prandtl number $\Pr = 2728$. The gravity vector is directed along the normal to the S3 boundary (figure 1).

The evolution of the temperature field after the switching of a heat source is shown in figure 2. At the initial stage after the heat source is switching on, an incubation period is observed when the liquid is heated near the source in the thermal conductivity regime. The duration of this period in the dimensionless form is $0 \leq t \leq 3.5$. The time scale for translation into the dimensional form is 9.24 s. When the critical mass of the heated liquid is reached, it begins to float ($t \geq 4.5$) under the action of buoyancy forces. A compact ascending jet is formed. There is a transition to the mode of convective heat transfer from the surface of the heat source.

One of the main characteristics of a non-stationary floating jet is the distribution of instantaneous local heat fluxes. The heat transfer from the rising plume to the environment determines the buoyant force reserve and the ascent velocity of the plume. The final stage of the plume development process is the interaction of its head part with the upper boundary. The amount of heat brought by the plume to the upper boundary is determined by the power of the heat source and the losses during the plume evolution. Energy losses are determined by overcoming the forces of viscous friction when generating convective currents and heating the environment. Distributions of local heat fluxes during the rise of the plume are shown in figure 3. Heat fluxes are indicated by arrows, the length of which characterizes their absolute values. A special feature is the formation of the heated mushroom head of the plume and its detachment from the lower part at some stage of the development of the plume. When the head part is separated from the rising jet of the heated liquid, an ejection of the cold liquid into the paraxial region is observed. This explains the reverse direction of the local heat flux in figure 3. Inside the plume, heat flows can be directed not only in the lateral directions, but also downwards. This shows that at some stage of the plume development, a jumper appears where the heat flux along the axis of the plume is directed towards each other and the heat transfer in the lateral directions takes place. Such a paradoxical distribution of heat fluxes can be explained by temperature distributions along the plume axis during the separation of its head.

**Figure 3.** The distribution of the local heat fluxes in a fluid with $Pr = 2728$ at $Gr = 172$.  

| $t$ | Heat Flux Distribution |
|-----|------------------------|
| 6.0 | ![Heat Flux at t=6.0](image1) |
| 7.0 | ![Heat Flux at t=7.0](image2) |
| 7.5 | ![Heat Flux at t=7.5](image3) |
| 8.0 | ![Heat Flux at t=8.0](image4) |
The distributions of the vertical component of velocity along the horizontal coordinate at different distances from the surface of the heat source are shown in figure 4. The velocity profiles are presented in a dimensional form. This shows their characteristic values and allows us to compare the data of the numerical simulation with the results of the laboratory experiment.

The heating of the upper wall begins when the head of the plume infiltrates its lower boundary. Distributions of local heat fluxes at the liquid-wall interface are shown in figure 5 at different times. The time evolution of the temperature field in the upper wall is shown in figure 6. This process demonstrates a possible scenario of thermal interaction of a plume with the Earth's crust. It is rather difficult to obtain experimentally the same detailed information on nonstationary temperature fields in a solid wall.

The information obtained will help to estimate the characteristic times of the development of the process of conjugate heat exchange and the time of the thermal trace exit to the upper boundary of the wall. These data will be used for thermal imaging studies on a physical model.

The development of the floating jet is the result of heat transfer to the liquid from the surface of the heat source. From the point of view of geophysics it is important to have estimates of the amount of

Figure 4. Profiles of velocity at the levels of the layer height: 1 - \( y = 5 \) mm, 2 - 10 mm, 3 - 15 mm, 4 - 20 mm, 5 - 25 mm, 6 - 30 mm, 7 - 45 mm.

Figure 5. Local heat fluxes at the liquid-top interface at different times: 1 - \( t = 8.0 \), 2 - \( t = 9.0 \), 3 - \( t = 10.0 \), 4 - \( t = 11.0 \).

Figure 6. The evolution of the temperature field in the upper wall.
source’s energy of the delivered heat amount to the bottom of the earth's crust converted into kinetic energy of the fluid. The distribution of local heat fluxes on the surface of the source at different times is shown in figure 7. The integral amount of heat pumped into the ascending stream in different times is shown in figure 8. A special feature of the heat transfer from the source is observed. The maximum heat flux is observed after the sudden heating of the source. Then the heat flux monotonically drops when the hot surface is blocked by the hot liquid. Further the heat flux monotonically grows and approaches to a constant value during the jet development for the almost steady flow of the source.

To obtain experimentally the same detailed information on nonstationary temperature fields not only at the wall, but also in the liquid is rather difficult [8, 9, 11]. Therefore, it is more appropriate to compare the results of calculations with the data of experimental studies of hydrodynamics. The experimental data have been obtained using the experimental apparatus, described in detail in references [8, 9]. The methods of measurement and control of the boundary conditions are described in detail in these references as well. In hydrodynamic studies, we used digital video recording of visualized fluid flow and computer processing of video films. To visualize the transparent liquid, flat particles of 10 ÷ 15 μm in size and a flat laser beam for illumination were used.

In this series of experiments, the liquid layer had a height of 50 mm, a length of 300 mm and a width of 60 mm. Experimental data has been obtained on the development of the flow spatial shape (figure 9) and on the velocity fields (figures 10, 11) in different time moments. The processing of video films allows to obtain the temporal dependence of the kinetic energy of the fluid for a given power of the heat source. The velocity fields in a 1 mm thick layer in the L × H plane were determined. The velocities of the motion of liquid volumes of 1 mm³ are used to estimate the integral values of the kinetic energy (figure 12). The regularities of the change in the velocity amplitude along the jet axis at the moment of its touching the upper boundary in figure 4 and in figure 10 coincide. For close values of the power of the sources, the absolute values of the calculated and experimental

**Figure 7.** Distributions of local heat fluxes on the surface of the heat source as a function of time: 1 - 1 = 9.24 s, 2 - 18.48 s, 3 - 27.72 s, 4 - 36.96 s, 5 - 46.2 s, 6 - 55.44 s.

**Figure 8.** The time dependence of the integral heat flux from the source surface per second.

**Figure 9.** Evolution of the jet in the PES - 5 layer at H = 50 mm and at the specific power of the linear heat source P = 7.1 W/m.
amplitudes of the velocity are similar. In figure 11, the velocity distribution on the jet axis is represented in a form normalized to the maximum value $V_m$. The maximum velocity values are indicated for a discrete set of powers of the heat source. The data in figure 11 show that scenarios of the jet development in high-viscosity liquid at low (curve 1) and high (curves 2-5) power of the heat source are different. It is possible that at low flow velocities the development of jets cannot be described by the Boussinesq approximation. A comparison with the data of [8, 9] shows that this feature does not depend on the height of the liquid layer.

4. Conclusion
The process of development of a floating jet in a liquid of high viscosity over a heat source in the form of a flat narrow strip located on the adiabatic bottom of the cavity has been numerically studied. Investigations were carried out for the Prandtl number Pr = 2728. Temporal evolutions of velocity, temperature, and local heat flux have been studied for a layer height of 50 mm and the Grashoff number $Gr = 172$. The process of thermal interaction of the jet with the solid wall bounding the area occupied by the liquid has been studied. Numerical simulation has been carried out using the finite element method. The complete system of equations for nonstationary thermogravitational convection

Figure 10. Vertical velocity profile in the PES-5 layer at $H = 50$ mm and specific power of the linear heat source $q = 7.1$ W/m at the moment of the rising of the jet’s head to the surface: 1 – $y = 17$ mm, 2 – 20 mm, 3 – 23 mm, 4 – 27 mm, 5 – 30 mm, 6 – 33 mm.

Figure 11. Distribution of the normalized amplitude of the vertical velocity along the height of the layer, depending on the power of the heat source: 1 – $P = 7.1$ W/m, $V_m = 0.28$ mm/s; 2 – $P = 28$ W/m, $V_m = 0.83$ mm/s; 3 – $P = 64$ W/m, $V_m = 1.34$ mm/s; 4 – $P = 110$ W/m, $V_m = 1.82$ mm/s; 5 – $P = 180$ W/m, $V_m = 2.31$ mm/s.

Figure 12. Dependence of the kinetic energy of a fluid on the power of a heat source.
is solved in the two-dimensional formulation of the problem. The dependence of the spatial forms of flow and velocity fields on the heat source power has been studied experimentally. To determine the velocity fields, digital video shooting and computer processing of video films have been used. The results obtained can potentially be useful for elaboration of adequate models of thermal plumes.

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