The Effect of Shock-wave Duration on Star Formation and the Initial Condition of Massive Cluster Formation

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Abstract

Stars are born in dense molecular filaments irrespective of their mass. Compression of the interstellar medium by shocks causes filament formation in molecular clouds. Observations show that a massive star cluster formation occurs where the peak of gas column density in a cloud exceeds 10^{23} cm^{-2}. In this study, we investigate the effect of the shock-compressed layer duration on filament/star formation and how the initial conditions of massive star formation are realized by performing three-dimensional isothermal magnetohydrodynamics simulations with gas inflow duration from the boundaries (i.e., shock-wave duration) as a controlling parameter. Filaments formed behind the shock expand after the duration time for short-shock-duration models, whereas long-duration models lead to star formation by forming massive supercritical filaments. Moreover, when the shock duration is longer than two postshock freefall times, the peak column density of the compressed layer exceeds 10^{23} cm^{-2}, and the gravitational collapse of the layer causes the number of OB stars expected to be formed in the shock-compressed layer to reach the order of 10 (i.e., massive cluster formation).

Unified Astronomy Thesaurus concepts: Star formation (1569); Interstellar clouds (834); Magnetohydrodynamics (1964); Star clusters (1567)

1. Introduction

Revealing the triggering mechanism of massive star/cluster formation is a critical target because the massive stars strongly affect the dynamics and evolution of galaxies. Observations show that stars are born in dense molecular filaments irrespective of their mass (André et al. 2010; Fukui et al. 2015, 2019; Shimajiri et al. 2019). The formation of filaments has been extensively studied (e.g., Tomisaka & Ikeuchi 1983; Nagai et al. 1998; Padoan & Nordlund 1999; Hennebelle 2013; Inoue & Fukui 2013; Pudritz & Kevlahan 2013; Chen & Ostriker 2014; Balfour et al. 2015, 2017; Federrath 2016; Abe et al. 2021). Recently, Abe et al. (2021) found that the major filament formation mechanism changes with the shock velocity triggering the filament formation. One of the major mechanisms is “Type O” (Inoue & Fukui 2013; Vaidya et al. 2013; Abe et al. 2021), the details of which is as follows: When the shock wave sweeps a dense clump in the cloud, the shock surface deforms due to large inertia of the clump. Because the deformed shock wave bends the streamlines across the shock, the gas flows toward the convex part of the deformed shock wave, where only the component of flows parallel to the postshock magnetic field lines converge and lead to the formation of a dense filament. A high Mach number shock is required to induce massive star formation (e.g., Dobbs et al. 2020; Liow & Dobbs 2020; Fukui et al. 2021), and a massive filament is formed in the shock-compressed layer by the Type O mechanism (Abe et al. 2021).

Several theoretical studies used numerical simulations of shock compression of molecular clouds via cloud collision. Those results were compared with observations (Habe & Ohta 1992; Takahira et al. 2014; Bishay et al. 2017a, 2017b; Wu et al. 2017a, 2017b; Dobbs et al. 2020; Liow & Dobbs 2020; Sakre et al. 2021). Previous studies have demonstrated that strong magnetohydrodynamics (MHD) shock compression allows for the formation of massive cores (Inoue & Fukui 2013; Inoue et al. 2018; Sakre et al. 2021). However, these studies implicitly assumed a long duration of a shock wave (corresponding to collisions between large clouds), whereas the realistic duration of a shock wave depends on the situation, e.g., the shock created by cloud collision cannot be kept over a crossing time. On the other hand, simulations of unmagnetized cloud collisions by Takahira et al. (2014) demonstrated that gravitationally bound cores do not form in the case of short duration. Therefore, a systematic study into the effect of shock duration on the resulting filament/core formation is required.

Enokiya et al. (2021) demonstrated that the peak column density of a star-forming region correlates with the number of OB stars in the system, and massive star clusters with more than 10 OB-type stars are associated with massive clouds whose peak column density exceeds 10^{23} cm^{-2}. Thus, the physical origin of the threshold peak column density of 10^{23} cm^{-2} must be clarified. It should be noted that the mean column density \( N_H \) is related to the shock duration \( t_{dur} \), because \( N_H = nL \) and \( t_{dur} = L/v_{sh} \) lead to \( N_H \propto t_{dur} \). Here, \( n, L, \) and \( v_{sh} \) are the mean number density of a cloud, cloud size, and shock velocity, respectively. Therefore, we can suppose that shock duration is one of the important parameters in determining peak column density. Previous simulations with a high column-density core/clump as an initial condition successfully demonstrated massive star/cluster formation (Bonnell et al. 2004; Krumholz & McKee 2008; Krumholz et al. 2009, 2012), indicating that the high column-density initial core/clump leads to the massive star/cluster formation. In Krumholz et al. (2009), their initial condition, a dense core of 100 \( M_\odot \), within 0.1 pc, was justified based on observed values of a protostellar
object IRAS 05358+3543. This is an extremely dense core and is not usually found. A recent molecular study of IRAS 05358+3543 by Yamada et al. (2022) found evidence for two colliding molecular clouds at several parsec scales, which formed a dense filament including the IRAS 05358+3543 core. This suggests that the assumed initial conditions in the previous works require advanced strong compression by an external trigger. This suggests that the collision is an essential initial process in high-mass star formation. It is crucial that the collision duration must be longer than the freefall time defined using the mean density in the shock-compressed layer, i.e., the time for realizing the high column density.

In this paper, we study the correlation between the shock duration and resulting star formation by controlling the amount of gas that flows into the numerical domain to examine the effect of shock duration and the origin of threshold peak column density of massive cluster formation. In previous related studies, there have been discrepancies in conclusions. Inoue et al. (2018) and Abe et al. (2021) demonstrated that fast MHD shock compression allows for the formation of massive cores/stars by numerically setting long-lasting shock compression. These studies assumed a long/infinite duration of a shock wave. On the other hand, Takahira et al. (2014) and Sakre et al. (2022) considered collisions of relatively small clouds and concluded that the fast shock compression prevent the formation of gravitationally bound cores. To clarify this contradictory situation, we examine the effect of shock duration on the resulting massive core/star formation. This paper is organized as follows: In Section 2, we provide the setup of our simulations, and we demonstrated and interpreted the results in Section 3. In Section 4, we discussed the peak column density in the shock-compressed layer by developing a simple theoretical model. Finally, we summarize the results in Section 5.

2. Setup for Simulations

We perform three-dimensional (3D) isothermal MHD simulations including self-gravity using SFUMATO code (Matsumoto 2007). The initial condition is the same as that of Abe et al. (2021) except that we additionally introduce a finite duration of shock compression. In this section, the numerical setup is briefly stated. We use a cubic numerical domain with a box size of \( L_{\text{box}} = 6 \) pc, which is filled with a nonuniform gas, a mean density \( n_0 \), and fluctuations with a power spectrum of \( \rho(t) \propto k^{-4} \) due to supersonic turbulence (Larson 1981; Elmegreen & Scalo 2004; Heyer & Brunt 2004; Scalo & Elmegreen 2004; Beresnyak et al. 2005). We set the gas temperature to 10 K (corresponding sound speed \( c_s \sim 0.2 \text{ km s}^{-1} \)). We divide the numerical domain into uniform 512\(^3\) cells, resulting in a spatial resolution of \( \Delta x = 6 \) pc/512 = 1.2 \times 10^{-2} \) pc. The initial velocity field is set to \( v(x, y, z) = v_{\text{turb}}(x, y, z) \) (\( -v_{\text{col}}/2 \) tanh\([z - L_{\text{box}}/2]/0.1] \), where \( v_{\text{turb}} \) is the turbulent velocity, having a dispersion of 1.0 km s\(^{-1}\) with a power spectrum of \( v_{\text{turb}}^2 \propto k^{-4} \), following Larson’s law (Larson 1981). This initial velocity leads to a gas collision at \( z = L_{\text{box}}/2 \) plane. We select an initial magnetic field strength \( B_0 = 10 \mu \text{G} \) (Heiles & Crutcher 2005; Crutcher 2012) and an angle \( \theta_0 \) to the y-z plane 45\(^\circ\), or 60\(^\circ\).

The velocity fields at z-boundaries are given by

\[
v_{\text{boundary}}(x, y) = \left[ \pm \frac{v_{\text{col}}}{2} + v_{\text{turb}}(x, y, z) \right] \times \exp[-(t-t_{\text{stop}})/0.1 \text{ Myr}],
\]

where \( v_{\text{boundary}}^+ \) and \( v_{\text{boundary}}^- \) are the velocity field at \( z = 0 \) and 6 pc, respectively, and \( v_{\text{col}} \) is the relative velocity of two flows colliding at \( z = 3 \) pc. These boundary conditions realize the cessation of gas inflows around \( t = t_{\text{stop}} \) and their implementation differs significantly from that of Abe et al. (2021). The density/turbulence field that is inflow from the boundary is the same density/turbulence spatial distribution as the initial condition. We impose free boundary conditions on the magnetic field at \( z = 0 \), \( L_{\text{box}} \) boundaries. We use periodic boundary conditions for all physical variables for \( x = 0 \), \( L_{\text{box}} \) and \( y = 0 \), \( L_{\text{box}} \) boundary planes. We show the schematic of the initial condition in Figure 1.

We simulate 18 different models. Each model has a unique name, starting with “v” (for “velocity of collision”), followed by the collision velocity (“3”,”5”,”14,” and “18,””24” km s\(^{-1}\)) and, the shock duration (“t”), followed by the timescale (“0.23–1.9” Myr). Models with a different initial magnetic field angle and initial mean density are additionally denoted as “ang60” and “d300” corresponding to \( \theta_0 = 60^\circ \) and \( n_0 = 300 \text{ cm}^{-3} \), respectively. The set of parameters used in our simulations is listed in Table 1.

In the regions where gravitational collapse occurs, we introduce the sink particle (Matsumoto et al. 2015; Inoue et al. 2018). The conditions for generating the sink particles depend on the resolution. The threshold density of sink particle generation in the current simulations with 512\(^3\) cells is \( 5.6 \times 10^4 \text{ cm}^{-3} \), which is lower than that in previous studies. Thus, we perform an adaptive mesh refinement (AMR) simulation to demonstrate whether the results depend on the resolution and threshold density of the sink creation. The Jeans criterion is used for the refinement (Truelove et al. 1997): \( \Delta x \leq \lambda_J/8 \) (where \( \lambda_J = \pi^{2/3} c_s^2/nG \rho \) is the Jeans length), and the minimum cell size for the AMR run is \( \Delta x = 6 \) pc/1024 = 5.9 \times 10^{-3} \) pc. The threshold density of sink particles is 2.2 \times 10^5 \text{ cm}^{-3}. The results are almost the same as those in the non-AMR run, as shown in Section 3.

3. Results

3.1. Column-density Maps

3.1.1. Short-duration Case

Figure 2 shows snapshots of the column-density map of model v24t0.23 at \( t = 0.10, 0.20, 0.30, \) and 0.60 Myr. Panels (a)–(d) and (a′)–(d′) show snapshots in the y-z and x–y plane slices, respectively. The formation of dense filaments is shown in panels (a′)–(d′). Recently, Abe et al. (2021) classified the filament formation mechanisms into several categories. According to their classification, the type of filament formation seen in the results of model v24t0.23 is Type O (the oblique MHD shock compression mechanism). Note that Type O filamentation works behind a strong shock of \( v_{\text{sh}} \gtrsim 5 \text{ km s}^{-1} \), and the shock waves created in this model have \( v_{\text{sh}} \) of approximately 13 km s\(^{-1}\). The evolution of the mass of the dense gas
in the numerical domain is shown in panel (a) of Figure 3. The blue, red, and green points represent the gas masses of the regions with densities greater than $10^3$, $10^4$, and $10^5$ cm$^{-3}$, respectively. Panels (b′)–(d′) in Figure 2 and panel (a) in Figure 3 show that the mass of the dense gas decreases 0.2–0.3 Myr. This evolution is due to the expansion of the shock-compressed layer and the resulting pressure reduction (see Figure 2(d)), which deconfines dense filaments. The time at which the compression layer starts to expand $t_{\text{dur}}$ can be written using $t_{\text{stop}}$ as

$$t_{\text{dur}} \simeq t_{\text{stop}} + \frac{L_{\text{box}}/2 - v_1 t_{\text{dur}}}{v_{\text{col}}/2}$$

(2)

where $v_1 \simeq v_{\text{Alfv}}/\sqrt{2}$ is the shock velocity at the rest frame of the compression layer, and $v_{\text{Alfv}} = B_0 \cos(\theta_B)/\sqrt{4\pi \rho_0}$ is the mean Alfvén velocity. $B_0 \perp = B_0 \cos(\theta_B)$ is the initial magnetic field strength perpendicular to the shock normal. The second term on the right-hand side of Equation (2) represents a retarded time for the gas to reach the compression layer after $t_{\text{stop}}$. In the case of model v24t0.23, $t_{\text{dur}} \simeq 0.23$ Myr corresponds to the time when the dense gas mass in Figure 3 reaches its peak.

The freefall time in the postshock layer, which gives the timescale for self-gravitating sheet fragmentation (Nagai et al. 1998),

$$t_{\text{stop}} + \frac{L_{\text{box}}/2 - v_1 t_{\text{dur}}}{v_{\text{col}}/2} \approx t_{\text{dur}}$$

(3)

Table 1

| Model Name  | $v_{\text{coll}}$ (km s$^{-1}$) | $v_{\text{sh}}$ (km s$^{-1}$) | $t_{\text{stop}}$ (Myr) | $t_{\text{dur}}$ (Myr) | $\theta_B$ | $n_0$ (cm$^{-3}$) | $\Delta x$ (pc) |
|-------------|---------------------------------|-------------------------------|-------------------------|-------------------------|-----------|-----------------|---------------|
| v3t1.4      | 3                               | 2.2                           | 0.0                     | 1.4                     | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v50.94      | 5                               | 3.2                           | 0.0                     | 0.94                    | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v5t1.5      | 5                               | 3.2                           | 0.7                     | 1.5                     | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v14t0.39    | 14                              | 7.7                           | 0.0                     | 0.39                    | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v14t0.40ang60 | 14                             | 7.5                           | 0.0                     | 0.40                    | 60°       | 100             | $1.2 \times 10^{-2}$ |
| v14t0.66    | 14                              | 7.7                           | 0.3                     | 0.66                    | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v14t0.84    | 14                              | 7.7                           | 0.5                     | 0.84                    | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v14t1.0     | 14                              | 7.7                           | 0.7                     | 1.0                     | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v14t1.1d300 | 14                              | 7.4                           | 0.7                     | 1.1                     | 45°       | 300             | $1.2 \times 10^{-2}$ |
| v14t1.5     | 14                              | 7.7                           | 1.2                     | 1.5                     | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v14t1.7ang60| 14                              | 7.5                           | 1.4                     | 1.7                     | 60°       | 100             | $1.2 \times 10^{-2}$ |
| v14t1.9     | 14                              | 7.7                           | 1.7                     | 1.9                     | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v18t0.68    | 18                              | 9.7                           | 0.4                     | 0.68                    | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v24t0.23    | 24                              | 13                             | 0.2                     | 0.23                    | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v24t0.43    | 24                              | 13                             | 0.2                     | 0.43                    | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v24t0.61    | 24                              | 13                             | 0.4                     | 0.61                    | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v24t1.0     | 24                              | 13                             | 1.4                     | 1.6                     | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v24t1.6     | 24                              | 13                             | 1.4                     | 1.6                     | 45°       | 100             | $1.2 \times 10^{-2}$ |
| v24t1.6AMR  | 24                              | 13                             | 1.4                     | 1.6                     | 45°       | 100             | $5.9 \times 10^{-3}$ |
| v24t1.8     | 24                              | 13                             | 1.7                     | 1.8                     | 45°       | 100             | $1.2 \times 10^{-2}$ |
Figure 2. Column-density maps of the result of model v24t0.23 at time $t = 0.3$, 0.8, 1.6, and 1.8 Myr (from top to bottom). Left row (panels (a), (b), (c), and (d)): $y$–$z$ plane column densities. Right row (panels (a$'$), (b$'$), (c$'$), and (d$'$)): $x$–$y$ plane column densities.
can be estimated as

\[
\tau_{\text{ff}} = \left( \frac{1}{2 \pi G \bar{\rho}_1} \right) \frac{\sqrt{2 \pi G \rho_{\text{sh}}}}{\sqrt{4/2 \pi^3/2 G_{\text{coll}}^3/2 (v_{\text{coll}}^2 + B_0 \sqrt{8 \pi \bar{\rho}_0})}} \\
\approx 1.0 \text{ Myr} \frac{B_{0,\perp}}{10 \mu G} \left( \frac{v_{\text{coll}}}{100 \text{ km s}^{-1}} \right)^{1/2} \left( \frac{\bar{n}_0}{100 \text{ cm}^{-3}} \right)^{-3/4} \\
\times \left[ \left( \frac{v_{\text{coll}}}{12 \text{ km s}^{-1}} \right) + 0.17 \frac{B_{0,\perp}}{10 \mu G} \left( \frac{\bar{n}_0}{100 \text{ cm}^{-3}} \right)^{-1/2} \right]^{-1/2} 
\] 

(4)

where \( \bar{\rho}_1 \approx \sqrt{2} \bar{M}_A \bar{\rho}_0 \) is the mean density of the shocked layer (e.g., Inoue & Fukui 2013), and \( \bar{\rho}_{\text{sh}} = v_{\text{coll}} / 2 + \bar{v}_1 \approx v_{\text{coll}} / 2 + B_0 / \sqrt{8 \pi \bar{\rho}_0} \) represents the mean shock velocity. Because \( \tau_{\text{dur}} < \tau_{\text{ff}} \approx 0.68 \text{ Myr} \) in model v240t23, the shock compression layer expands before gravitational fragmentation and no sink particles are created.

### 3.1.2. Long-duration Case

Similar to Figure 2, Figure 4 shows snapshots of the column-density map of model v24t1.6 at \( t = 0.30, 0.80, 1.60, \) and 1.80 Myr. Because the shock velocity is the same as in the previous model (v240t23), dense filamentary structures are created via the Type O mechanism. The “+” symbols in Figure 4 indicate the positions of the sink particles. The evolution of the dense gas mass in the numerical domain is shown in panel (b) of Figure 3. The blue, red, and green points represent the masses of the regions with densities greater than \( 10^3, 10^4, \) and \( 10^5 \text{ cm}^{-3} \), respectively, as in panel (a). It should be noted that the range of the vertical axis is different from that of the panel (a). In this model also, the shock-compressed layer expands after \( t = \tau_{\text{dur}} \approx 1.6 \text{ Myr} \), as in model v240t23. However, the duration of the compression is longer than the freefall time calculated using the mean density in the layer \( \tau_{\text{ff}} \approx 0.58 \text{ Myr} \) (Equation (4)), allowing the filaments to coalesce because of the gravitational contraction of the compression layer. Therefore, in contrast to model v240t23, the dense gas mass continues to increase even after \( \tau_{\text{dur}} \) (panel (a), but for model v24t1.6.

### 3.2. Peak Column Density versus Dense Gas Mass

Observations show that the peak column density of a star-forming cloud correlates with the number of OB stars in the system (Enokiya et al. 2021). Here, we compare the results of our simulations with the observed results.

We consider the following two types of mass in the numerical domain, as a proxy of star formation: the total mass of sink particle, \( M_{\text{sink, tot}}(t) \), and total gas mass in the region with \( A_V > 8 \text{ mag} \). \( (N_{H_2} \approx 7.8 \times 10^{21} \text{ cm}^{-2}) \), \( M_{\text{Av8}}(t) \) (Lada et al. 2010). The sum of these masses \( M_{\text{dense}}(t) \equiv M_{\text{sink, tot}}(t) + M_{\text{Av8}}(t) \), is proportional to the total mass of stars in the system. Figure 5 shows the evolution of \( M_{\text{dense}}(t) \). Panels (a) and (b) show the results for models v240t23 and v24t1.6, respectively. In our simulation, the \( M_{\text{dense}}(t) \) saturates in late time, and we can define a mass associated with star formation. By fitting the \( M_{\text{dense}}(t) \) using a trial function \( \alpha \tanh [2 \pi (t - t_0)/t_w] + \beta \), as shown in the red lines in Figure 5, we obtain the time \( t_{\text{sat}} \equiv t_0 + t_w / 4 \) (dashed lines in Figure 5) at which we measure total dense gas mass representing the star formation activity in the system, where \( \alpha, t_0, t_w, \) and \( \beta \) are fitting parameters. Note that \( t_{\text{sat}} \) corresponds to the time when the fitting curve reaches approximately \( \alpha \tanh [2 \pi (t_{\text{sat}} - t_0)/t_w] + \beta = \tanh (\pi/2) + \beta \approx 0.92 \alpha + \beta \), i.e., this value means mass corresponding to \( (0.92 + 1.0) \times 100/2 = 96 \% \) of the increment in the trial function.

Generally, a peak column density highly depends on a spatial grid (or in other words, spatial resolution) used for the derivation. Thus, the observed peak column densities of cloud–cloud collision (CCC) candidates in the plots of Figure 9 in Enokiya et al. (2021) have potentially independent spatial resolutions. Nevertheless, the correlation between the peak column density and the number of O- and B-type stars is apparent in the plots. This suggests that the spatial resolutions are similar among the observed CCC candidates. We have checked the spatial resolution among the CCC candidates and found it roughly the size of the smaller cloud/10.
Figure 4. Column-density maps in the result of model v24t1.6 at time $t = 0.3, 0.8, 1.6, \text{ and } 1.8 \text{ Myr}$ (from top to bottom). Left row (panels (a), (b), (c), and (d)): $y$-$z$ plane column densities. Right row (panels (a$'$), (b$'$), (c$'$), and (d$'$)): $x$-$y$ plane column densities.
Assuming 5 pc for the size of the smaller cloud (see Figure 9 in Fukui et al. 2021), we estimate the typical spatial resolution of the observed CCC candidates to be 0.5 pc. Thus, before computing the peak column density of the system, we take a smoothing of the column density using the Gaussian kernel function of a width of 0.5 pc, corresponding to the typical beamwidth for massive star-forming regions. The color of the points represents \( t_{\text{dur}}/t_{\text{ff}} \) indicating the influence of self-gravity. Panel (b) of Figure 6 is compiled from panel (a) using Equation (5). The black line is the fitting line in Figure 9(b) of Enokiya et al. (2021).

\[
N_{\text{OB}} \approx M_{\text{dense}}(t_{\text{sat}}) \times \text{SFE}_{\text{dense}} \\
\times \left( \int_{10M_{\odot}}^{150M_{\odot}} dM \frac{MF_{\text{Kroupa}}}{dM F_{\text{Kroupa}}} \right)^{-1},
\]

where \( \int_{10M_{\odot}}^{150M_{\odot}} dM \frac{MF_{\text{Kroupa}}}{dM F_{\text{Kroupa}}} \) on the right-hand side is the typical mass of OB stars. Panel (b) in Figure 6 is compiled from panel (a) using Equation (5). The result shows a correlation similar to that of Enokiya et al. (2021), although our simulations cover only a limited range of peak column density than the observations. The power-law index obtained by fitting data in panel (b) is 0.75, which agrees well with that of observations 0.73 ± 0.11 (Enokiya et al. 2021). The black line is the fitting line in Figure 9(b) of Enokiya et al. (2021).
has been known observationally that massive star cluster formation occurs in the clouds with a peak column density greater than $N_{\text{peak}} \geq 10^{23} \text{ cm}^{-2}$ (Fukui et al. 2016). Our simulations agree with this threshold peak column density (see the region with $N_{\text{peak}} \sim 10^{23} \text{ cm}^{-2}$ in panel (b) of Figure 6). The stability parameter, ratio of the shock duration and the freefall time of the compressed layer $t_{\text{dur}}/t_{\text{ff}}$, indicates that massive stars are expected to be formed when the dense shock-compressed layer is kept over the freefall time. We find that when the $N_{\text{peak}}$ exceeds $10^{23} \text{ cm}^{-2}$ the stability parameter $\geq 2$. Coincidentally, the threshold peak column density takes a similar value to the threshold column density for fragmentation suppression $1 \text{ g cm}^{-2}$ proposed by Krumholz & McKee (2008).

3.3. Sink Mass Histogram

Figure 7 shows the sink mass histograms at $t = t_{\text{sat}}$ for the models v14t0.38, v14t0.84, and v14t1.9. In our simulations, the sink particles correspond to gravitationally collapsing cores. Note that Figure 7 is not a core mass function because low-mass cores are not well resolved in our simulations. A massive sink particle of 355 $M_{\odot}$ and OB-type stars can be formed in the model v14t1.9, which has a long flow duration of $t_{\text{dur}}/t_{\text{ff}} \simeq 2.6$. The mass/number of sink particles increases with $t_{\text{dur}}$, as expected, but only the long-duration model v14t1.9 has $t_{\text{dur}}/t_{\text{ff}} \geq 2$ and exhibits active massive star formation.

Figure 8 shows a sink mass histogram at $t = t_{\text{sat}}$ in the models v24t1.6 and v24t1.6AMR, comparing the results with and without AMR. The number of sink particles with masses $\sim 1 M_{\odot}$ increased because AMR calculation can capture self-gravitational fragmentation of short-wavelength modes in dense filaments. However, the total masses of massive sink particles ($>10 M_{\odot}$) are almost the same between v24t1.6 ($M_{\text{sink}} = 1803 M_{\odot}$) and v24t1.6AMR ($M_{\text{sink}} = 2048 M_{\odot}$), indicating that the results exhibited in Section 3.2 do not change significantly, even if the spatial resolution is improved.

### 4. Discussion

In Section 3.2, we stated that self-gravitational collapse must achieve a peak column density of $10^{23} \text{ cm}^{-2}$. Given that the column-density map obtained from observations approximately $\sim 500 \times 500$ pixels, the peak column density approximately corresponds to $5 \sigma$ away from the mean of the probability density function (pdf) obtained from observations. Here, we estimate the peak column density using a theory of the gas density pdf in a turbulent medium, which shows that gravitational collapse in shock-compressed layers is required to achieve a peak column density of $10^{23} \text{ cm}^{-2}$. The pdf of the gas density in a turbulent medium shows a lognormal pdf of the form (Passot & Vázquez-Semadeni 2003; Padoan et al. 2014)

$$
\rho_*(s) = \frac{1}{\sqrt{2\pi\sigma_*^2}} \exp \left( -\frac{(s - s_1)^2}{2\sigma_*^2} \right)
$$

where $s = \ln(\rho/\rho_1)$, and $\rho_1$ and $s_1$ represent the mean density and mean logarithmic density in the shocked layer, respectively. The latter is related to the standard deviation $\sigma_*$ as $s_1 = -\sigma_*^2/2$. Molina et al. (2012) find that the standard deviation $\sigma_*$ is determined using the turbulence Mach number $\mathcal{M}$, plasma beta $\beta$, and the ratio of solenoidal mode to compressive mode $b$ (pure solenoidal forcing gives $b = 1/3$ and pure compressive forcing gives $b = 1$) as

$$
\sigma_*^2 = \ln \left( 1 + b^2 \mathcal{M}^2 \frac{\beta}{\beta + 1} \right).
$$

Here, we estimate the maximum possible column density of a cloud formed by the gas collision. The density pdf can have maximum width if we substitute the collision velocity for the turbulence velocity dispersion

$$
\mathcal{M}_{\text{max}} \equiv v_{\text{col}}/(2c_s).
$$
The beta of the shock-compressed layer is given by the shock jump condition of the isothermal MHD as

$$\beta_{sh} = \frac{8\pi \rho_1 c_s^2}{B_1^2} \approx \frac{8\pi \rho_0 c_s^2}{\sqrt{2} M_{\lambda,\text{max}} B_0^2}, \quad (9)$$

where $\rho_1 \approx \sqrt{2} M_{\lambda,\text{max}} \rho_0$, $B_1 \approx \sqrt{2} M_{\lambda,\text{max}} B_0$, and $M_{\lambda,\text{max}} = v_{\text{col}}/(2v_{\Lambda,0})$ are the density, the magnetic field in the compression layer, and the Alfvén Mach number of the shock. If we use the parameter $b = 0.4$, which is expected in the shock-compressed layer (Kobayashi et al. 2022), then we can determine $\sigma_{\eta,\text{max}}$ and density pdf.

Numerical experiments of supersonic isothermal turbulence and observations (Goodman et al. 2009) show that the pdf of column density is also close to lognormal (Federrath et al. 2010)

$$p_\eta(\eta) = \frac{1}{\sqrt{2\pi \sigma_\eta^2}} \exp\left(-\frac{(\eta - \eta_1)^2}{2\sigma_\eta^2}\right), \quad (10)$$

where $\eta = \ln(N/N_1)$. $N$ and $N_1$ are column density and the mean column density, respectively. By comparing $s_1$ and $\eta_1$ in numerical simulations by Federrath et al. (2010), $\eta_1$ is approximately expressed as $\eta_1 \approx 0.5 s_1$. The standard deviation is also approximated by $\sigma_{\eta,\text{max}} \approx 0.5 s_\eta$. These relationships can be used to convert density pdf to column-density pdf.

The peak column density is defined as $5\sigma$ away from the mean

$$N_{5\sigma} \equiv \exp(\eta_1 + 5\sigma_{\eta,\text{max}}) N_1, \quad (11)$$

where $\sigma_{0,\text{max}} \approx 0.5\sigma_{\eta,\text{max}}$. By substituting the parameters of the strong collision case ($v_{\text{col}} = 10 \text{ km s}^{-1}$, $n_0 = 100 \text{ cm}^{-3}$, and $B_0 = 10 \mu\text{G}$), we obtain the peak column density $N_{5\sigma} = 8.7 \times 10^{22} \text{ cm}^{-2} < 10^{23} \text{ cm}^{-2}$. Therefore, even if we use an over-estimated velocity dispersion of the turbulence in the shock-compressed layer, the estimated peak column density of the structure created only by turbulence does not exceed $10^{23} \text{ cm}^{-2}$, indicating that gravitational collapse of the shock layer must achieve $10^{23} \text{ cm}^{-2}$. In other words, the shock-compressed layer must be maintained until the gravitational collapse to form a massive star cluster, which enhances the peak column density, starts to activate.

Recently Sakre et al. (2022) have pointed out that, for a given initial cloud, there is a maximum collision speed for triggering of star formation by a cloud collision. This is consistent with our results because high collision speed leads to a short shock duration. However, this does not mean that high shock velocity is negative for induced star formation. As we discuss below, the high shock velocity is generally positive for star formation: the duration of the shock is estimated by $t_{\text{dur}} = L/v_{\text{sh}}$, where $L$ is the spatial scale and $v_{\text{sh}}$ is the typical velocity of the flows. If we suppose supersonic turbulent gas flow collision as an origin of the shock, Larson’s law gives $v_{\text{sh}} \propto L^{0.5}$ or $L \propto v_{\text{sh}}^{-1}$, and then, $t_{\text{dur}} = L/v_{\text{sh}} \propto v_{\text{sh}}$.

Since the average freefall time of the shocked layer is $t_{\text{ff}} \propto v_{\text{sh}}^{-1/2}$ from Equation (4), the ratio of the shock duration and the postshock freefall time is written as $t_{\text{dur}}/t_{\text{ff}} \propto v_{\text{sh}}^{1/2}$. This indicates that a faster shock leads to a longer duration in units of the freefall time, i.e., larger clouds naturally lead to more active star formation. In contrast, if we fix the scale of the flow $L$ and use a different scaling $t_{\text{dur}} = L/v_{\text{sh}} \propto v_{\text{sh}}^{-1}$, we obtain an opposite result $t_{\text{dur}}/t_{\text{ff}} \propto v_{\text{sh}}^{1/2}$. Thus, under the fixed cloud scale, a faster shock leads to a negative effect on star formation, but we should bear in mind that this is due to artificially fixed $L$ and not a general trend.

5. Summary

We performed the shock compression simulations of molecular clouds using 3D isothermal MHD code with self-gravity (SFUMATO). The effect of the duration of the shock-compressed layer on filament and star formation was investigated by treating shock duration as a controlling parameter. We examined the relationship between peak column density and the estimated number of OB-type stars to understand the initial conditions of
massive star formation and compared it with the observation by Enokiya et al. (2021). Our main conclusions are as follows.

1. In the short-shock-duration model, filaments formed behind the shock start to expand/evaporate after the duration timescale of the shock, whereas the long-duration model leads to star formation by forming massive filaments.

2. The number of OB stars expected to be formed in the shock-compressed layer reaches the order of 10 (i.e., massive cluster formation) when the observed peak column density exceeds $10^{23}$ cm$^{-2}$, which is consistent with that of Enokiya et al. (2021; see the region with $N_{\text{peak}} \sim 10^{23}$ cm$^{-2}$ in panel (b) of Figure 6). According to a simple theoretical model, such a high peak column density can be achieved only when the shock-compressed layer undergoes gravitational collapse.

3. The massive star formation can be activated if shock compression is maintained for more than two freefall times in the compressed layer ($t_{\text{ff}}/t_{\text{sh}} \gtrsim 2$). This conclusion is not significantly affected by the spatial resolution.

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