Probing the momentum dependence of medium modifications of the nucleon-nucleon elastic cross sections

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Abstract

The momentum dependence of the medium modifications on nucleon-nucleon elastic cross sections is discussed with microscopic transport theories and numerically investigated with an updated UrQMD microscopic transport model. The semi-peripheral Au+Au reaction at beam energy $E_b = 400A$ MeV is adopted as an example. It is found that the uncertainties of the momentum dependence on medium modifications of cross sections influence the yields of free nucleons and their collective flows as functions of their transverse momentum and rapidity. Among these observables, the elliptic flow is sensitively dependent on detailed forms of the momentum dependence and more attention should be paid. The elliptic flow is hardly influenced by the probable splitting effect of the neutron-neutron and proton-proton cross sections so that one might pin down the mass splitting effect of the mean-field level at high beam energies and high nuclear densities by exploring the elliptic flow of nucleons or light clusters.

Keywords: medium modifications of cross sections, elliptic flow, splitting effects

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It is well-known that the extensive explore of the equation of state (EOS) of the nuclear matter is one of the hottest topics during a long period of time. With continuous improvements in the reduction of the stiffness uncertainty of the EOS endeavored in recent years [1, 2, 3, 4], much more attention have been kept on going to the more consistent treatments of the mean field and the collision term, which both originate from the same effective Lagrangian density based on the QHD theory [3, 4, 7].

In the past, two-body nucleon-nucleon (NN) cross sections adopted in the microscopic transport models are often treated to be in free space partly for simplicity and partly for the lack of the information of medium modifications of cross sections. Recently, various medium modifications on the NN elastic and inelastic cross sections have been investigated with various theories and simulated with transport models by several groups. And vivid effects of these modifications on dynamics of the heavy ion collisions (HICs) have been found in quite a few sensitive observables [7, 8, 9, 10, 11, 12]. Based on QHD-II type effective Lagrangian, in which the interaction between nucleons is described by exchanges of $\sigma$, $\omega$, $\pi$, $\rho$ [13] and $\delta$ [7, 14] mesons, the in-medium neutron-proton, proton-proton and neutron-neutron elastic scattering cross sections ($\sigma_{np}^*$, $\sigma_{pp}^*$, and $\sigma_{nn}^*$) had been systematically studied within the framework of the self-consistent RBUU transport theory [6, 15]. Further, in Ref. [16], such medium modifications of the NN elastic cross section on several observables was investigated within neutron-rich intermediate-energy HICs. At that time, the so-called splitting effect of the effective neutron and proton masses (‘NR’ and ‘Dirac’ modes) was paid more attention and the HICs at lower beam energies ($\sim 100 A$ MeV) were in use. We found that, although the transverse flow as a function of rapidity and the nuclear stopping quantities, such as $Q_{zz}$ as a function of momentum and the ratio of halfwidths of the transverse to that of longitudinal rapidity distribution $R_t/l$, are very sensitive to the medium modifications of the cross sections, the mass-splitting effect on these observables is quite small and deserves more investigations. We notice that some of these findings were also demonstrated independently by the other theoretical group [10].

On the mean field level, the sensitive probes to the controversial mass-splitting effect at intermediate beam energies ($\sim 400 A$ MeV) have been studied [17, 18]. It is found that the elliptic flow, especially the flow difference of free neutrons and protons (or light isobars), is quite sensitive to the mass-splitting indicated by various theories. In view of the same origin of the medium modifications on both the mean field and the collision, one might ask: might
the splitting effect on the two-body NN (elastic) cross sections also be seen from the elliptic flow observable? If so, is the total sensitivity to the elliptic flow reduced or enhanced with the consideration of the splitting effect in the cross sections? Obviously, the answer to these questions is essential to determine the trend of the mass-splitting at high densities.

As well-known from many previous investigations, see, e.g., [7, 8, 9, 10, 19, 20, 21], the density dependence of cross sections is drastically influenced by the relative momentum of the colliding particles in the NN center-of-mass system. In order to consider the momentum dependence of the medium modifications on cross sections in the real transport models, some phenomenological scaling models are adopted in previous calculations [10, 16]. However, we know that different momentum dependent forms of the cross sections can be obtained based on different parameterizations used in RBUU calculations [7, 13]. In this work, we would also like to investigate the influence of various momentum dependence on the observables related to the splitting effect. It is even more valuable than the splitting effect itself and is necessary to be resolved in a timely manner since it is comparable to the density dependence of the cross sections in the nuclear medium and should influence the determination of the stiffness of the EOS when comparing same observables with experimental data.

The new-updated UrQMD model, which is suitable for studies of HICs at SIS energies, is adopted for calculations in this work [16, 22, 23]. It is well-known that the UrQMD microscopic transport model is based analogous principles as the quantum molecular dynamics model (QMD) [24] and the relativistic quantum molecular dynamics model (RQMD) [25]: the mean-field potential applied to hadrons is treated similar to QMD, while the treatment of the collision term is similar to RQMD. And starting from the version 2.0, the PYTHIA code is incorporated into UrQMD in order to investigate the jet production and fragmentation at high SPS and RHIC energies [26]. Hadrons are represented by Gaussian wave packets in phase space and the phase space of the hadron is propagated according to Hamilton’s equation of motion. Besides the cascade mode, and in terms of better description of the experimental data, the effective two-body interaction potential terms are taken into account carefully. In the current version of the UrQMD model [16, 22], the potential energies include the two-body and three-body (which can be approximately written in the form of two-body interaction) Skyrme- (also called as the density-dependent terms), Yukawa-, Coulomb-, Pauli-, density-dependent-symmetry-, and momentum-dependent- terms. With the updates of the UrQMD transport model, some successful theoretical analyses, predic-
tions and comparisons with data have been accomplished.

In the previous work \cite{16} the in-medium NN elastic cross sections \( \sigma^*_{\text{el}} \) are treated to be factorized as the product of a medium correction factor \( (F(u, \alpha, p), u = \rho/\rho_0 \) is the nuclear reduced density and \( \alpha = (\rho_n - \rho_p)/\rho_0 \) the isospin-asymmetry) and the free NN elastic ones \( \sigma^\text{free}_{\text{el}} \). For the inelastic channels \( \sigma_{\text{in}} \), we still use the experimental free-space cross sections \( \sigma^\text{free}_{\text{in}} \). It is believed that this assumption does not have serious influence on our present study at intermediate energies. Therefore, the total two-body scattering cross section of nucleons \( \sigma^*_{\text{tot}} \) will be modified to

\[
\sigma^*_{\text{tot}} = \sigma_{\text{in}} + \sigma^*_{\text{el}} = \sigma^\text{free}_{\text{in}} + F(u, \alpha, p)\sigma^\text{free}_{\text{el}}.
\]

As for the medium correction factor \( F(u, \alpha, p) \), it is proportional to both the isospin-scalar density effect \( F_u \) and the isospin-vector mass-splitting effect \( F_\alpha \), please read \cite{16} for more details. Furthermore, the factors \( F_u \) and \( F_\alpha \) should be constrained by the relative momentum of the two colliding particles in the NN center-of-mass system \( (p_{NN}) \). In \cite{16}, they are formulated as,

\[
F_{\alpha,u}^p = \begin{cases} 
  f_0 & p_{NN} > 1\text{GeV/c} \\
  F_{\alpha,u} - f_0 + \frac{f_0 - f_0}{1 + (p_{NN}/p_0)^\kappa} & p_{NN} \leq 1\text{GeV/c}
\end{cases}
\]  

The parameters \( f_0, p_0 \) and \( \kappa \) in Eq. (1) can be varied in order to obtain various momentum dependence of, for example, \( F_u \). In this work, we select several parameter sets, which are shown in Table 1. The corresponding \( F_{\alpha,u}^p \) functions are illustrated in Fig. 1 at a reduced density \( u = 2 \) (and \( F_{u=2} = 0.35 \)). The FP1 set was used in our previous works when the medium modifications of cross sections were considered. It is also used in this work as a base. The function of FP2 (FP3) gives a rapid increase at smaller (larger) \( p_{NN} \) as compared to the case with FP1. This demonstrates the uncertainty of the momentum dependence to the density dependent cross sections. With a certain set of isospin dependent EOS, the NN elastic cross section might be even enhanced at large momenta \cite{13, 27} (which arises from the differences between the isoscalar and isovector channels), when compared to the cross section at free space. FP4 in Fig. 1 gives one example to show 30% enhancement at large \( p_{NN} \). On the contrary, the dash-dot-dotted line is to show the case without any momentum constraint on \( F_u \).

A soft EOS with momentum dependence (SM-EOS) and with a soft symmetry potential energy (corresponding stiffness factor \( \gamma \) of the energy form \( S_0 u^\gamma \) is set to 0.5. \( S_0 = 32 \text{ MeV} \) is the symmetry energy at the normal density) is adopted in this work. The reaction Au+Au
TABLE I: Four parameter sets FP1 ∼ FP4 used in this work for various momentum dependence of $F_u$. The case without $p_{NN}$ limit is also considered if one sets $f_0$ to be $F(u)$ in Eq. (1).

| Set   | $f_0$ | $p_0$ [GeV c$^{-1}$] | $\kappa$ |
|-------|-------|-----------------------|-----------|
| FP1   | 1     | 0.425                 | 5         |
| FP2   | 1     | 0.225                 | 3         |
| FP3   | 1     | 0.625                 | 8         |
| FP4   | 1.3   | 0.425                 | 4         |
| no $p_{NN}$ limit | $F(u)$ | /                      | /         |

FIG. 1: $F_u^p$ as a function of the relative momentum $p_{NN}$ with four parameter sets FP1 ∼ FP4 and without $p_{NN}$ limit. The reduced density $u = 2$ is chosen.

at a beam energy $E_b = 400 A$ MeV and for impact parameter $b = 7$ fm is chosen. For each case 100 thousand events are calculated and the freeze-out time is taken to be 100fm c$^{-1}$. After freeze-out, a conventional phase-space coalescence model [28] is used to construct clusters, in which the nucleons with relative momenta smaller than $P_0$ and relative distances smaller than $R_0$ are considered to belong to one cluster. In this work, $P_0$ and $R_0$ are chosen to be 0.3GeV c$^{-1}$ and 3.5 fm, respectively. The change of $P_0$ and $R_0$ values certainly alters the yields of clusters, but it shall not change the main conclusions drawn in this paper.
FIG. 2: Rapidity distribution of protons (left plot) and neutrons (right plot) for Au+Au reactions at beam energy $E_b = 400$ A MeV and impact parameter $b = 7$ fm. SM-EoS with soft symmetry potential energy is adopted for calculations with various (from FP1 to FP4) and without momentum dependence (“no $p_{NN}$ limit”) of medium modifications on NN elastic cross sections.

Fig. 2 shows the rapidity (in the nucleus-nucleus center-of-mass system) distribution of unbound protons (left plot) and neutrons (right plot) for Au+Au reactions at beam energy $E_b = 400$ A MeV and impact parameter $b = 7$ fm. It is interesting to see that in each plot the results can be divided into two camps: The results with FP1, FP2, and FP4 are similar to each other and somewhat higher than the results with FP3 and without $p_{NN}$ limit. It implies that at such beam energy the collision dynamics of nucleons is sensitive to medium modifications of elastic cross sections in the $0.3 \lesssim p_{NN} \lesssim 0.6$ GeV $c^{-1}$ region which is understandable. And, it is also easy to understand that stronger reduction of the NN elastic cross sections leads to weaker emission of nucleons. However, in each camp of each plot, it is still hard to distinguish them by taking only the rapidity distribution of the yield of nucleons into account and one needs to go further on.

Fig. 3 shows the directed flows $v_1$ of protons ($v_1 = p_x/p_t$ where $p_t = \sqrt{p_x^2 + p_y^2}$ is the
FIG. 3: Rapidity distribution of the directed flow $v_1$ of protons for Au+Au reactions at beam energy $E_b = 400$A MeV and impact parameter $b = 7$ fm. Results with FP1 $\sim$ FP4 momentum dependence of medium modifications of cross sections is compared to the one without momentum constraint.

transverse momentum of the particle) as a function of the rapidity. The uncertainty of momentum dependence on medium modifications of cross sections is not much obviously seen in the rapidity distribution of $v_1$ although the two camps shown already in Fig. 2 appear again. And, it is clear that larger NN elastic cross sections in the nuclear medium makes bigger positive directed flow at this beam energy, which is due to larger transverse expansion.

Let us further investigate the sensitivity of the momentum limits to the elliptic flow $v_2$ ($v_2 = < (p_x^2 - p_y^2)/p_t^2 >$) of protons as functions of the transverse momentum $p_t$ which is shown in Fig. 4 and of the rapidity which is shown in Fig. 5, respectively. It is clear that at $0.5 \lesssim p_t \lesssim 1.0$ GeV c$^{-1}$ (in Fig. 4) or at mid-rapidity (in Fig. 5), the elliptic flow of protons is sensitive to the treatment of the momentum dependence of medium modifications of cross sections. It is known that stronger two-body collisions lead to larger negative elliptic flow at such beam energy, which can be examined explicitly by relating Fig. 1 to Figs. 4 and 5. It is further found that the order of flows shown in Figs. 4 and 5 follows the momentum
dependent forms in the $0.3 \lesssim p_{NN} \lesssim 0.5$ GeV $c^{-1}$ region shown in Fig. 1. Furthermore, in the $p_t$ distribution of Fig. 4, FP1 and FP4 cases deviate from each other with the increase of $p_t$ which is obviously due to the enhancement of cross sections in the FP4 case. It is interesting to see that the difference of results between with FP3 and without $p_{NN}$ limit can even be also detected from Fig. 4 when $p_t \gtrsim 0.6$ GeV $c^{-1}$. A beam-energy scan of the elliptic flow under various momentum conditions might be useful for giving further constraints on cross sections in medium.

Now that the elliptic flow can be taken as a sensitive probe for the momentum dependence of the medium modifications of cross sections, it is supposed that it might also be a good candidate for detecting the splitting effect probably shown in the NN elastic cross sections. Fig. 6 shows the rapidity dependence of $v_2$ of protons (left plot) and neutrons (right plot) with NR- and Dirac-type mass-splitting. In the NR case, $m_n^* > m_p^*$ so that $\sigma_{nn}^* > \sigma_{pp}^*$, while in the Dirac case, the trend is on the contrary [29]. The detailed forms of the splitting effect $F_\alpha$ on $\sigma_{nn}^*$ and $\sigma_{pp}^*$ had been discussed in Ref. [16]. In Fig. 6 the flows without momentum dependence of $F_\alpha$ are also calculated for comparison with the ones having momentum constraint. First of all, it is seen that the $p_{NN}$ limit plays strong role on the final elliptic flow while the splitting effect does not. For protons, the flow with the NR-typed splitting
is slightly larger than that with the Dirac case at mid-rapidity because of a bit smaller $\sigma_{pp}^*$ than $\sigma_{nn}^*$, while for neutrons, the inverse observation is made which is certainly due to the same reason. Finally, it is also interesting to find that the splitting effect of elastic cross sections on the elliptic flow is too small to affect the sensitivity of the elliptic flow to the mass-splitting effect in the mean field calculations claimed in Refs. [17, 18]. Actually in that work these results were obtained just on the basis of pure mean field effects, i.e. different momentum dependence of the symmetry potentials for neutrons and protons, without changing the elastic cross sections.

To summarize, in order to gain deep insight into the dynamics of particles in the nuclear medium at SIS energies, the momentum dependence of the medium modifications on nucleon-nucleon elastic cross sections is analyzed based on microscopic transport theories and numerically investigated with an updated UrQMD microscopic transport model in which the EOS and the medium modified cross sections had been considered and examined before. The semi-peripheral Au+Au reaction at $E_b = 400A$ MeV is adopted since it produces large negative collective flows. It is found that the uncertainties originating from the momentum dependence on medium modifications of cross sections, such as the slope at moderate relative momenta as well as the possible enhancement of cross sections at high momenta, influence
FIG. 6: Rapidity dependence of $v_2$ of protons (left plot) and neutrons (right plot) with NR- and Dirac-typed splitting. For each typed splitting effect, the flow is calculated with and without the momentum dependence on the density dependent term $F_u$.

The emission of free nucleons as well as their flows. Among these, the elliptic flow is seen to be sensitively dependent on the detailed forms of the momentum constraint on cross sections. However, the flow is still insensitive to the splitting effect of the neutron-neutron and proton-proton cross sections in the isospin-asymmetric nuclear medium. This result can be partially related to the naive $(m^*/m)^2$ scaling of the cross sections. The mass-splitting will be then not affecting the dominant $(n, p)$ collisions.

In order to pin down the mass-splitting effect obtained in the mean field calculation at high beam energies and nuclear densities by exploring the elliptic flow of nucleons or light clusters, it is quite necessary to dig deeper into the momentum dependence on medium modifications of the cross sections. Moreover, the in-medium modification of the angular distributions should also be properly accounted for [32, 33]. A good check would be to test the effect on the stopping, i.e. on the longitudinal and transverse rapidity distributions.

In the next step, using the self-consistent RBUU theory, we plan also to investigate more
systematically the energy dependence of the NN elastic cross sections in the neutron-rich nuclear medium within the reduced density region $\mu \lesssim 3$ and the temperature region $T \lesssim 100$ MeV\cite{34}.

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We note that in a full relativistic approach a connection can be worked out between the Dirac and NR (Non-Relativistic) evaluation. The relation is however strongly affected by the poorly known momentum dependence of the nucleon self-energies, see Sect.6.3.1 of Ref. [30] and the detailed Dirac-Brueckner calculation of Ref. [31]. In this paper we have followed the rough choices described in the text in order to test possible effects on reaction observables.

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