A quantum-mechanical Maxwell’s demon†

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Abstract: A Maxwell’s demon is a device that gets information and trades it in for thermodynamic advantage, in apparent (but not actual) contradiction to the second law of thermodynamics. Quantum-mechanical versions of Maxwell’s demon exhibit features that classical versions do not: in particular, a device that gets information about a quantum system disturbs it in the process. In addition, the information produced by quantum measurement acts as an additional source of thermodynamic inefficiency. This paper investigates the properties of quantum-mechanical Maxwell’s demons, and proposes experimentally realizable models of such devices.

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Introduction

In 1871, Maxwell noted that a being that could measure the velocities of individual molecules in a gas could shunt fast molecules into one container and slow molecules into another, thereby creating a difference in temperature between the two containers, in apparent violation of the second law of thermodynamics.\(^1\) Kelvin called this being a ‘demon’: by getting information and being clever how it uses it, such a demon can in principle perform useful work. Maxwell’s demon has been the subject of considerable discussion over the last century.\(^2\) The contemporary view of the demon, spelled out in the last decade,\(^3\) is that a demon could indeed perform useful work \(k_B \ln 2 T\) for each bit obtained, but must increase entropy by at least \(k_B \ln 2\) for each bit erased (a result known as ‘Landauer’s principle’\(^4\)). As a result, a demon that operates in cyclic fashion, erasing bits after it exploits them, cannot violate the second law of thermodynamics.

Up to now, Maxwell’s demon has functioned primarily as a gedankenexperiment, a thought experiment that allows the exploration of theoretical issues. This paper, in contrast, proposes a model of a Maxwell’s demon that could be realized experimentally using magnetic or optical resonance techniques—all that is required of a device to function as a quantum ‘demon’ is the ability to perform simple quantum logic operations.

Any experimentally realizable model of a ‘demon,’ like the molecules of Maxwell’s original example, must be intrinsically quantum-mechanical. The classic reference on quantum demons is Zurek’s treatment of the quantum Szilard engine\(^5\) (see also references 6-7). There are compelling reasons to investigate quantum-mechanical models of Maxwell’s demon. First, as Zurek emphasized, a quantum-mechanical treatment of the demon is necessary for making sense of the underlying thermodynamics. Second, the demon operates by obtaining information, i.e., by making measurements, and measurements on quantum systems tend to disturb the system measured. Third, measurement of quantum systems introduces ‘new’ bits of information into the world. As noted by Haus\(^8\), Landauer’s principle implies that these new bits come with a thermodynamic cost: quantum demons suffer from peculiarly quantum sources of inefficiency.

This paper presents a simple quantum-mechanical model of a Maxwell’s demon that allows these three issues to be addressed. The results are in accordance with Zurek’s treatment of the quantum Szilard engine: quantum mechanics allows the consistent treatment of thermodynamics, and introduces additional complications because of the nature of quantum measurement. The mechanisms by which infor-
Information is obtained, exploited, and erased are explored in detail. Naturally, such a device cannot violate the second law of thermodynamics, but if supplied with heat reservoirs at different temperatures it can undergo a Carnot cycle and function as a heat engine. A thorough investigation of the thermodynamics of the engine shows that Landauer’s principle is validated, and that each bit of information introduced by quantum measurement and decoherence functions as an extra bit of entropy, decreasing the engine’s efficiency. A quantum-mechanical engine that processes information is limited not by the Carnot efficiency, but by the potentially lower quantum efficiency $\varepsilon_Q$ defined below.

1. **A magnetic resonance model of a quantum ‘demon.’**

   To see how a quantum system can function as a heat engine, consider a spin in a magnetic field. If the spin points in the same direction as the field it has energy $-\mu B$, where $\mu$ is the spin’s magnetic dipole moment, and $B$ is the field strength. If it points in the opposite direction it has energy $+\mu B$. The state of the spin can be controlled by conventional magnetic resonance techniques: for example, the spin can be flipped from one energy state to the other by applying a $\pi$ pulse at the spin’s Larmor precession frequency $\omega = 2\mu B/h$.\(^{9-10}\)

   When the spin flips, it exchanges energy with the oscillatory field. If the spin flips from the lower energy state to the higher energy state it coherently absorbs one photon with energy $\hbar\omega$ from the field; if it flips from the higher energy state to the lower, it coherently emits one photon with energy $\hbar\omega$ to the field. The energy exchange involves no entropy increase or loss of quantum coherence:\(^{11}\) either the field does work on the spin, or the spin does work on the field.

   It is clear how a device that acquires information about such a spin could use the information to make the spin do work. Suppose that some device can measure whether the spin is in the low-energy quantum state $|\downarrow\rangle$ that points in the same direction as the field, or in the high-energy quantum state $|\uparrow\rangle$ that points in the opposite direction to the field, and if the spin is in the high-energy state, send in a $\pi$-pulse to extract its energy. The device can then wait for the spin to come to equilibrium at temperature $T_1 \gg 2\mu B/k_B$ and repeat the operation. Each time it does so, it converts an average of $\mu B$ of heat into work. The device gets information and uses that information to convert heat into work. The amount of work done by such a device operating on a single spin is negligible; but many such devices operating in parallel could function as a ‘demon’ maser, coherently amplifying the pulse that flips the spins.
Landauer’s principle prevents such a device from violating the second law of thermodynamics. To operate in a cyclic fashion, the device must erase the information that it has gained about the state of the spin. When this information is erased, entropy $S_{\text{out}} \geq k_B \ln 2$ is pumped into the device’s environment, compensating for the entropy $S_{\text{in}} \approx k_B \ln 2$ of entropy in the spin originally. If the environment is a heat bath at temperature $T_2$, heat $k_B T_2 \ln 2$ flows to the bath along with the entropy, decreasing the energy available to convert into work.

The overall accounting of energy and entropy in the course of the cycle is as follows: heat in $Q_{\text{in}} = T_1 S_{\text{in}}$, heat out $Q_{\text{out}} = T_2 S_{\text{out}}$, work out $W_{\text{out}} = Q_{\text{in}} - Q_{\text{out}}$, efficiency

$$\varepsilon = \frac{W_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{T_2 S_{\text{out}}}{T_1 S_{\text{in}}} \leq 1 - \frac{T_2}{T_1} \equiv \varepsilon_{\text{C}},$$

where $\varepsilon_{\text{C}}$ is the Carnot efficiency. Since $S_{\text{out}} \geq S_{\text{in}}$, $W_{\text{out}}$ can be greater than zero only if $T_1 > T_2$. Landauer’s principle implies that instead of violating the second law of thermodynamics, the device operates as a heat engine, pumping heat from a high-temperature reservoir to a low-temperature reservoir and doing work in the process.

Why quantum measurement introduces added inefficiency into the operation of such a device can be readily understood. Suppose that the spin is originally in the state $|\to\rangle = 1/\sqrt{2}(|\uparrow\rangle + |\downarrow\rangle)$. One way to extract energy from such a spin is to apply a $\pi/2$ pulse to rotate the spin to the state $|\downarrow\rangle$, extracting work $\mu B$ in the process. A second way to extract energy is to repeat the process described above: measure the spin to see if it is in the state $|\uparrow\rangle$, and if it is, apply a $\pi$ pulse to extract work $2\mu B$. This process also generates work $\mu B$ on average, but in addition generates a ‘waste’ bit of information that costs energy $k_B T_2 \ln 2$ to erase. Quantum measurement introduces added inefficiency to the process of getting information about a quantum system and exploiting that information to perform work.

More generally, suppose the spin is initially described by a density matrix $\rho$. The device makes a measurement on the spin that takes

$$\rho \to \rho' = \sum_i P_i \rho P_i,$$

where $P_i$ are projection operators onto the eigenspaces of the operator corresponding to the measurement.\(^\text{12}\) The extra information generated by quantum measurement is

$$\Delta S_Q / k_B \ln 2 = -\text{tr} \rho' \log_2 \rho' - (\text{tr} \rho \log_2 \rho) \geq 0$$

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The efficiency of the device in converting heat to work is limited not by the Carnot efficiency $\varepsilon_C$ but by the quantum efficiency $\varepsilon_Q = 1 - T_2(S_{\text{in}} + \Delta S_Q)/T_1 S_{\text{in}} = \varepsilon_C - T_2 \Delta S_Q/T_1 S_{\text{in}}$. (4)

Equation (4) quantifies the inefficiency due to any process, such as decoherence, that destroys off-diagonal terms of the density matrix $\rho$.\textsuperscript{12-14}

To investigate more thoroughly how quantum measurement and decoherence introduce thermodynamic inefficiency in a quantum information-processing ‘demon’ requires a more detailed model of how such a device gets and gets rid of information. One of the simplest quantum systems that can function as a measuring device is another spin. Magnetic resonance affords a variety of techniques, called spin-coherence double resonance, whereby one spin can coherently acquire information about another spin with which it interacts.\textsuperscript{9-10} The basic idea is to apply a sequence of pulses that makes spin 2 flip if and only if spin 1 is in the excited state $|\uparrow\rangle_1$, while leaving spin 1 unchanged. If spin 2 is originally in the ground state $|\downarrow\rangle_2$, then after the conditional spin-flipping operation, the two spins will either be in the state $|\uparrow\rangle_1 |\uparrow\rangle_2$ if spin 1 was originally in the state $|\uparrow\rangle_1$, or in the state $|\downarrow\rangle_1 |\downarrow\rangle_2$ if spin 1 was originally in the state $|\downarrow\rangle_1$. Spin 2 has acquired information about spin 1. A variety of spin coherence double resonance techniques (going under acronyms such as \textit{INEPT} and \textit{INADEQUATE}) can be used to perform this conditional flipping operation,\textsuperscript{9-10} which can be thought of as an experimentally realizable version of Zurek’s treatment of the measurement process in reference (5). Readers familiar with quantum quantum computation will recognize the conditional spin flip as the quantum logic operation ‘controlled–\textit{NOT}.’\textsuperscript{15-17}

How can this information be used to extract energy from spin 1? Simply apply a second pulse sequence to flip spin 1 if and only if spin 2 is in the state $|\uparrow\rangle_2$, while leaving spin 2 unchanged. The energy transfer from spins to field is as follows. If spin 1 was originally in the state $|\downarrow\rangle_1$, then spin 1 and spin 2 remain in the state $|\downarrow\rangle$ through both pulses and no energy is transferred to the field. If spin 1 was originally in the state $|\uparrow\rangle$, first spin 2 flips, then spin 1, yielding a transfer of energy from spins to field of $\hbar(\omega_1 - \omega_2) = 2(\mu_1 - \mu_2)B$, which is $> 0$ as long as $\mu_1 > \mu_2$. The average energy extracted is half this value. As long as the conditional spin flips are performed coherently, the amount of energy extracted depends only on overall conservation of energy, and is independent of the particular double resonance technique used. Note that the entire process maintains quantum coherence and can be reversed simply by repeating the conditional spin flips in reverse order.
2. A quantum heat engine

To complete the treatment of the thermodynamics of this device and to understand the role of decoherence and quantum measurement in its functioning, we must investigate how the ‘demon’ interacts with its thermal environment to take in heat and erase information. This section will show that a quantum device that interacts with a thermal environment can indeed get information and ‘cash it in’ to do useful work, but not by violating the second law of thermodynamics: a detailed model of the erasure process confirms Landauer’s principle (one bit of information ‘costs’ entropy $k_B \ln 2$). As a result, instead of functioning as a perpetual motion machine, the device operates as a heat engine that undergoes a Carnot cycle.

The environment for our spins will be taken to consist of two sets of modes of the electromagnetic field, the first a set of modes at temperature $T_1$ with average frequency $\omega_1$ and with frequency spread greater than $|\gamma|$ but less than $\omega_1 - \omega_2$, and the second a set of modes at temperature $T_2$ with average frequency $\omega_1$ and the same frequency spread. Such an environment can be obtained, for example, by bathing the spins in incoherent radiation with the given frequencies and temperatures. The purpose of such an environment is to provide effectively separate heat reservoirs for spin 1 and spin 2: spin 1 interacts strongly with the on-resonance radiation at frequency $\omega_1$, and weakly with the off-resonance radiation at frequency $\omega_2$ — *vice versa* for spin 2. Over short times, to a good approximation spin 1 can be regarded as interacting only with mode 1, and spin 2 as interacting only with mode 2. A spin can be put in and out of ‘contact’ with its reservoir by isentropically altering the frequency of the reservoir mode to put the spin in and out of resonance.

With this approximation, the initial probabilities for the state of the $j$-th spin are (ignoring for the moment the coupling between the spins)

$$p_j(\uparrow) = e^{-\mu_j B/k_B T_j}/Z_j, \quad p_j(\downarrow) = e^{\mu_j B/k_B T_j}/Z_j,$$

yielding energy

$$E_j = -\mu_i B \tanh(\mu_j B/k_B T_j),$$

and entropy

$$S_j = -k_B \sum_{i=\uparrow,\downarrow} p_j(i) \ln p_j(i) = E_j/T_j + k_B \ln Z_j,$$

where $Z_j = e^{-\mu_j B/k_B T_j} + e^{\mu_j B/k_B T_j} = 2 \cosh(\mu_j B/k_B T_j)$. Even though it does not start out in a definite state, spin 2 can still acquire information about spin 1, and this information can be exploited to do electromagnetic work. The spins can function as a heat engine by going through the following cycle:
(1) Using spin coherence double resonance, flip spin 2 iff spin 1 is in the state $|\uparrow\rangle_1$. This causes spin 2 to gain information $(\hat{S}_2 - S_2)/k_B\ln 2$ about spin 1 at the expense of work $W_1 = p_1(\uparrow)2\mu_2B\tanh(\mu_2B/k_BT_2)$ supplied by the oscillating field. Here $\hat{S}_2 = -k_B \sum_{i=\uparrow,\downarrow} \tilde{p}_2(i) \ln \tilde{p}_2(i)$, where $\tilde{p}_2(\uparrow) = p_1(\uparrow)p_2(\downarrow) + p_1(\downarrow)p_2(\uparrow)$ and $\tilde{p}_2(\downarrow) = p_1(\downarrow)p_2(\downarrow) + p_1(\uparrow)p_2(\uparrow)$ are the probabilities for the states of spin 2 after the conditional spin flip.

(2) Flip spin 1 iff spin 2 is in the state $|\uparrow\rangle_1$. This step allows spin 2 to ‘cash in’ $(S_2 - S_1)/k_B\ln 2$ of the information it has acquired, thereby performing work $-\mu_1B(\tanh(\mu_1B/k_BT_1) - \tanh(\mu_2B/k_BT_2))$ on the field.

(3) Spin 2 still possesses information $(\hat{S}_2 - S_1)/k_B\ln 2$ about spin 1, which can be converted into work by flipping spin 2 iff spin 1 is in the state $|\uparrow\rangle_1$, thereby performing work $p_2(\uparrow)2\mu_2B\tanh(\mu_1B/k_BT_2)$ on the field.

It is straightforward to verify that after these three conditional spin flips, spin 1 has probabilities $p'_1(i) = p_2(i)$ while spin 2 has probabilities $p'_2(i) = p_1(i)$. That is, the sequence of pulses has ‘swapped’ the information in spin 1 with the information in spin 2.¹² As a result, $S'_1 = S_2$, $S'_2 = S_1$, and the new energies of the spins are $E'_1 = -\mu_1B\tanh(\mu_2B/k_BT_2)$ and $E'_2 = -\mu_2B\tanh(\mu_1B/k_BT_1)$. The total amount of work done by the spins on the electromagnetic field is

$$W = -(E'_1 + E'_2 - E_1 - E_2) = -(\mu_1 - \mu_2)B(\tanh(\mu_1B/k_BT_1) - \tanh(\mu_2B/k_BT_2)).$$

(6)

This formula for the work done depends only on conservation of energy and does not depend on the specific set of coherent pulses that are used to ‘swap’ the spins.

Equation (6) shows that $W > 0$ iff either $\mu_1 > \mu_2$, $\mu_1/T_1 < \mu_2/T_2$ or $\mu_1 < \mu_2$, $\mu_1/T_1 > \mu_2/T_2$. If $T_1 = T_2$, $W$ is zero or negative: no work can be extracted from the spins at equilibrium. The device cannot function as a perpetuum mobile of the second kind. The cycle can be completed by letting the spins re-equilibrate with their respective reservoirs. Each time steps (1-3) are repeated, heat $Q_{\text{in}} = E_1 - E'_1$ flows from reservoir 1 to spin 1 and heat $Q_{\text{out}} = E'_2 - E_2$ flows from spin 2 into reservoir 2. The efficiency of this cycle is $W/Q_{\text{in}} = 1 - \mu_2/\mu_1 < 1 - T_2/T_1 = \varepsilon_C$: when the spins equilibrate with their respective reservoirs, heat flows but no work is done.

The following steps can be added to the cycle to allow the spins to re-equilibrate isentropically:

(5) Return spin 1 to its original state:
(i) Take the spin out of ‘contact’ with its reservoir by varying the frequency of the reservoir modes as above.

(ii) Alter the quasi-static field from \( B \rightarrow B_1 = BT_1/T_2 \) adiabatically, with no heat flowing between spin and reservoir.

(iii) Gradually change the field from \( B_1 \rightarrow B \) keeping the spin in ‘contact’ with the reservoir at temperature \( T_1 \) so that heat flows isentropically between the spin and the reservoir.

During this process, entropy \( S_2 - S_1 \) flows from the spin to the reservoir, while the spin does work \( E_1 - E'_1 - T_1(S_2 - S_1) \) on the field.

(6) Return spin 2 to its original state by the analogous set of steps.

The total work done by the spins on the electromagnetic field throughout the cycle is

\[
W_C = (T_1 - T_2)(S_1 - S_2) \quad .
\]

With the added steps (5-6) performs a Carnot cycle and in principle operates at the Carnot efficiency \( 1 - T_2/T_1 \). In practice, of course, the steps that go into operation of the such an engine will be neither adiabatic nor isentropic, leading to an actual efficiency below the Carnot efficiency.

3. Thermodynamic cost of quantum measurement and decoherence

So far, although the demon has functioned within the laws of quantum mechanics, quantum measurement and quantum information have not entered in any fundamental way. Now that the thermodynamics of the demon have been elucidated, however, it is possible to quantify precisely the effects both of measurement and of the introduction of ‘new’ quantum information on the demon’s thermodynamic efficiency. The simple quantum information-processing engine of the previous section can in principle be operated at the Carnot efficiency: as will now be shown, when the engine introduces ‘new’ information by a process of measurement or decoherence, it cannot be operated even in principle (let alone in practice) above the lower, quantum efficiency \( \varepsilon_Q \) of equation (4).

To isolate the effects of quantum information, first consider the simple model of section 1 above, in which spin 1 is initially in the state \( |\rightarrow_1 = 1/\sqrt{2}(|\uparrow_1 + |\downarrow_1) \). This state has non-minimum free energy which is available for immediate conversion into work: simply apply a \( \pi/2 \) pulse to rotate spin 1 into the state \( |\downarrow_1 \), adding energy \( \hbar \omega_2/2 = \mu_1B \) to the oscillating field in the process. Suppose, however,
that instead of extracting this energy directly, the demon operates in information-gathering mode as above, using magnetic resonance techniques to correlate the state of spin 2 with the state of spin 1 (cf. reference 5). Suppose spin 2 is initially in the state $| \downarrow \rangle_2$.\textsuperscript{18} In this case, coherently flipping spin 2 iff spin 1 is in the state $| \uparrow \rangle_1$ results in the state, $1/\sqrt{2}(| \uparrow \rangle_1 | \uparrow \rangle_2 + | \downarrow \rangle_1 | \downarrow \rangle_2)$, a quantum ‘entangled’ state in which the state of spin 2 is perfectly correlated with the state of spin 1. Continuing the energy extraction process by flipping spin 1 iff spin 2 is in the state $| \uparrow \rangle_2$ as before allows on average an amount of energy $(\mu_1 - \mu_2)B$ to be extracted from the spin. The resulting state of the spins is $1/\sqrt{2} | \downarrow \rangle_1 (| \uparrow \rangle_2 + | \downarrow \rangle_2) = 1/\sqrt{2} | \downarrow \rangle_1 \rightarrow \rangle_2$. Up until this point, no extra thermodynamic cost has been incurred. Indeed, since the conditional spin flipping occurs coherently, the process can be reversed by repeating the steps in reverse order to return to the original state $| \rightarrow \rangle_1$, with a net energy and entropy change of zero.

When is the cost of quantum measurement realized? When decoherence occurs. In the original cycle, decoherence takes place when spin 2 is put in contact with the reservoir to ‘erase’ it:\textsuperscript{13–14} the exchange of energy between the spin and the reservoir is an incoherent process during which the pure state $| \rightarrow \rangle_2 = 1/\sqrt{2}(| \uparrow \rangle_2 + | \downarrow \rangle_2)$ goes to the mixed state described by the density matrix $(1/2)(| \uparrow \rangle_2 \langle \uparrow_{2} + | \downarrow \rangle_2 \langle \downarrow_{2})$. The time scale for this process of decoherence is equal to the spin dephasing time $T_2^*$ and is typically much faster than the time scale for the transfer of energy.\textsuperscript{14} In effect, interaction with the reservoir turns the process by which spin 2 coherently acquires quantum information about spin 1 into a decoherent process of measurement, during which a bit of ‘new’ information is created. This bit corresponds to an increase in entropy $k_B \ln 2$. During the process of erasure, this entropy is transferred from spin 2 to the low-temperature reservoir, in accordance with Landauer’s principle.

By decohering and effectively measuring the spin, the demon has increased entropy and introduced thermodynamic inefficiency. The amount of inefficiency can be quantified precisely by going to the Carnot cycle model of the demon above. In general, the state of spin 1 is described initially by a density matrix

$$
\rho'_1 = p_1(|\uparrow'\rangle \langle \uparrow'|) + p_1(|\downarrow'\rangle \langle \downarrow'|),
$$

where $|\uparrow'\rangle$ and $|\downarrow'\rangle$ are spin states along an axis at some angle $\theta$ from the $z$-axis. Without loss of generality, $T_1$ and $B$ can be taken to be such that $p_1(|\uparrow'\rangle) = e^{-\mu_1 B/k_B T_1} / Z_1$, $p_1(|\downarrow\rangle) = e^{\mu_1 B/k_B T_1} / Z_1$. This state is not at equilibrium, and possesses free energy that can be extracted by applying a tipping pulse that rotates
the spin by $\theta$ and takes $|\uparrow\rangle \rightarrow |\uparrow\rangle$ and $|\downarrow\rangle \rightarrow |\downarrow\rangle$. The amount of work extracted is

$$W^* = E_1^* - E_1$$

where

$$E_1 = \mu_1 B(p_1(\uparrow) - p_1(\downarrow)), E_1^* = \mu_1 B(p_1^* (\uparrow) - p_1^* (\downarrow))$$

and

$$p_1^* (\uparrow) = p_1 (\uparrow) \cos^2 \theta + p_1 (\downarrow) \sin^2 \theta, \quad p_1^* (\downarrow) = p_1 (\downarrow) \cos^2 \theta + p_1 (\uparrow) \sin^2 \theta$$

Running the engine through a Carnot cycle by steps (1-5) above then extracts work $(T_1 - T_2)(S_1 - S_2)$. This process extracts the free energy of the spin isentropically, without increasing entropy.

Inefficiency due to measurement arises when instead of first applying the tipping pulse to extract spin 1’s free energy, one simply operates the engine cyclically as before. The steps are as above: three conditional flips ‘swap’ the states of spin 1 and spin 2, so that spin 1 is in the state $\rho_2$ and spin 2 is in the state $\rho_2' = \rho_1'$. The interaction with the heat reservoir then decoheres spin 2, destroying the off-diagonal terms in the density matrix so that $\rho_2' \rightarrow p_1^*(\uparrow)|\uparrow\rangle\langle \uparrow | + p_1^*(\downarrow)|\downarrow\rangle\langle \downarrow |$ with entropy $S_1^* = -k_B \sum_{i=\uparrow, \downarrow} p_1^*(i) \ln p_1^*(i)$. $(S_1^* - S_1)/k_B \ln 2$ is the ‘extra’ information introduced by decoherence. The entropy $S_1^* - S_2$ that flows out to reservoir 2 is greater than the entropy $S_1 - S_2$ that flowed in from reservoir 1. The total amount of work done is $k_B T_1 (S_1 - S_2) - T_2 (S_1^* - S_2) + W^*$, $T_2 (S_1^* - S_1)$ less than the work $(T_1 - T_2)(S_1 - S_2) + W^*$ done by simply undoing the tipping pulse and operating the engine as before. The overall efficiency with decoherence and measurement included is

$$\varepsilon_Q = 1 - T_2 (S_1^* - S_2)/T_1 (S_1 - S_2) \leq \varepsilon_C,$$

in accordance with equation (4) above. The extra information introduced by quantum measurement and decoherence has decreased the efficiency of the demon. The quantum efficiency $\varepsilon_Q$ rather than the Carnot efficiency $\varepsilon_C$ provides the upper limit to the maximum efficiency of such an engine.

4. Conclusion

This paper presented general arguments and specific models that show how quantum measurement and decoherence decrease the efficiency of heat engines. The systems discussed are experimentally realizable examples of Zurek’s quantum Szilard engine *gedankenexperiment*. The techniques given here for exploiting quantum
information to perform work can be extended in a variety of ways. If operated in
the regime where $\mu_1/T_1 > \mu_2/T_2$, the demon functions as a refrigerator, using the
information gained with the help of work from the electromagnetic field to pump
heat from the reservoir at temperature $T_2$ to the reservoir at temperature $T_1$.\textsuperscript{19–20}
Once again, quantum measurement and decoherence introduce inefficiency in the
operation of such a ‘Maxwell’s fridge.’ The use of magnetic resonance techniques to
describe a quantum Maxwell’s demon was for the sake of convenience of exposition
and potential experimental realizability: many other quantum systems could be
suitable for performing the heat–information–energy conversion described above.
At bottom, a quantum ‘demon’ consists of nothing more than an interaction be-
tween two quantum systems that allows the controlled transfer of information from
one to the other. In particular, any system that can provide the coherent quan-
tum logic operation controlled-$\text{NOT}$ that flips one quantum bit conditioned on the
state of another could form the basis for an information-processing quantum heat
engine.\textsuperscript{21–22}

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(i) Apply a $\pi/2$ pulse with frequency $\omega_2 = 2\mu_2 B/\hbar$ and width $\ll 2|\mu_1 - \mu_2|$ and $> 2\gamma$.

(ii) Wait for time $\pi/2\gamma$.

(iii) Apply a second $\pi/2$ pulse with a phase delay of $3\pi/2$ from the first pulse.

Step (i) rotates spin 2 by $\pi/2$, step (ii) allows spin 2 to acquire a phase of $\pm \pi/2$ conditioned on whether spin 1 is in the state $|\uparrow\rangle$ or $|\downarrow\rangle$, and step (iii) either rotates spin 2 back to its original state if spin 1 = $|\downarrow\rangle$ or rotates spin 2 to an angle of $\pi$ from its original state is spin 1 = $|\uparrow\rangle$. Another way to flip spin 2 iff spin 1 = $|\uparrow\rangle$ is to apply a highly selective $\pi$ pulse with frequency $\omega_2$ and width $\ll 2\gamma$. Spin 1
is off-resonance and does nothing, while spin 2 is on-resonance and flips iff spin 1 = | ↑⟩.

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18. In principle, a slight extension of the cycle allows spin 2 to be placed in the state | ↓⟩₂ at the start of the Carnot cycle. Starting with spin 2 at equilibrium with reservoir 2 at temperature T₂ as above, gradually raise the external field B → B’ >> k_B T₂/μ₂ while keeping spin 2 in contact with the reservoir, allowing heat T₂(k_B ln2 – S₂) to flow isentropically from the spin to the reservoir: spin 2 is now in the state | ↓⟩₂ with high probability. Now adiabatically lower the field from B’ → B. (Spin 1 should be kept out of contact with its reservoir throughout this process.) Spin 2 now begins the cycle in the state | ↓⟩₂. In practice, of course, magnetic fields high enough to perform this extension are hard to come by.

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