Dynamic stress state of underground pipelines at junctions

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Abstract. A one-dimensional statement of unsteady wave problem of a longitudinal monochromatic wave propagation and reflection from a rigid stationary barrier to which an underground pipeline abuts is given. The linear viscoelastic Eyring model, which describes limited creep and limited relaxation, is taken as the pipeline strain law. Eyring model allows us to describe the behavior of underground steel and polymer pipelines under dynamic loading. The problem is solved numerically using the theory of characteristics, followed by the finite difference method in an implicit scheme. Numerical solutions obtained in the form of dependences of plane wave parameters: longitudinal stress, velocity and strain for fixed sections of the pipeline are analyzed in the paper. An analysis of changes in these wave parameters shows that at high frequencies of dynamic load generating the wave, the stress amplitude in the pipeline increases by two or more times compared to the load amplitude. This is due to the superposition of incident and reflected waves in the pipeline and to a high loading rate of the pipeline. At low frequencies of dynamic loading, such an increase is not observed due to the low loading rate. The obtained numerical solutions allow choosing the statement of problems on dynamic stress state of an underground pipeline depending on the frequency of incident wave. It is shown that the greatest longitudinal stress in an underground pipeline arises in points (sections) close to its connection with a rigid stationary solid body.

1. Introduction

Underground pipelines are widely used nowadays as a mean of conveying natural gas, oil and oil products and other liquid and gas substances. Ensuring their reliability, especially during earthquakes, is an urgent problem [1-11]. According to [1-11], the main factors determining the reliability of underground pipelines during earthquakes are:

• external influences and the changes in their parameters [1, 2];
• strain characteristics of soil and its interaction with the pipeline [3-5];
• design features of underground pipelines [6-11].

A detailed analysis of underground steel pipelines safety during earthquakes was given in [10]. An analysis of earthquake aftermath data [10] indicates that damage to underground pipelines conveying natural gas can lead to unpredictable high socio-economic losses.

Therefore, a thorough study of each issue of safety and reliability of underground pipelines is
relevant. Based on this, the stress state of an underground pipeline section that abuts against a massive stationary body are examined in this paper.

Such cases are observed at the jun

ction of underground pipeline with pumping stations, underground storage tanks, etc. In these cases, when the wave propagates through the pipe body, longitudinal and transverse stresses arise.

The aim of the paper is to determine longitudinal seismic stresses in the material of an underground pipeline rigidly resting on a massive stationary body under the influence of seismic loads.

2. Methods

The underground pipeline is considered as a rod. Following the theory of earthquake resistance of underground pipelines by Shunzo Okamoto [1], we assume that the underground pipeline is deformed under the influence of seismic waves together with surrounding soil. In this case, interaction forces (the friction force) do not arise on the pipeline-soil contact surface and the soil medium is actually neglected.

With these assumptions, a one-dimensional motion of the pipeline sections is described by the equations

\[ \rho_v \frac{\partial v}{\partial t} - \rho \frac{\partial \sigma}{\partial x} = 0, \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ (1) \]

where \( v \) is the particle velocity (mass velocity) of the pipeline; \( \sigma \) is the longitudinal strain (along the pipe axis); \( \sigma \) is the longitudinal stress; \( \rho_0 \) is the initial density of the pipe material; \( x \) is the spatial coordinate coinciding with the pipeline axis; \( t \) - time.

The system of equations of motion (1) is closed by the equation of state of the pipe material (a standard viscoelastic body), also known as the generalized Eyring model

\[ \frac{d\sigma}{dt} + \mu \varepsilon = \frac{d\sigma}{E_\sigma dt} + \mu \frac{\sigma}{E_s}, \]

where \( E_\sigma \) is the module of dynamic strain of the pipe material at \( \dot{\varepsilon} = \frac{d\varepsilon}{dt} \to \infty \); \( E_s \) is the module of static strain, at \( \dot{\varepsilon} \to 0 \); \( \mu \) is the parameter of volume viscosity of the pipe material.

The initial section of the pipeline is affected by a seismic load, which varies according to a sinusoidal law

\[ \sigma = \sigma_m \sin(\pi t/T) \text{ at } 0 \leq t \leq \theta \]

\[ \sigma = 0 \text{ at } t > \theta \]

(3)

where \( T \) is the half-period of the load; \( \theta \) is the load duration; \( \pi \) is the Pythagoras number.

The initial conditions of the problem are zero, i.e. before the load application (3) the pipeline is at rest.

The systems of partial differential equations (1), (2) are of hyperbolic type. They have real characteristic lines and characteristic relations on them, which have the form

\[ \frac{d\sigma - C_0^2 \rho_0 \rho v = -C_0^2 \rho_0 g(\sigma, \varepsilon) dt}{dx} \]

\[ \frac{dx}{dr} = +C_0 \]

\[ \frac{d\sigma + C_0^2 \rho_0 \rho v = -C_0^2 \rho_0 g(\sigma, \varepsilon) dt}{dx} \]

\[ \frac{dx}{dr} = -C_0 \]

\[ \frac{d\sigma - C_0^2 \rho_0 \rho \varepsilon = -C_0^2 \rho_0 g(\sigma, \varepsilon) dt}{dx} = 0 \]

\[ g(\sigma, \varepsilon) = \sigma / \eta - E_\sigma E_s (\varepsilon - \sigma / E_\sigma )/(E_\sigma - E_s) \eta \]
where $C_0$ is the longitudinal wave velocity in the pipeline material; $\eta$ is the coefficient of volume viscosity of the pipeline material, related to the viscosity parameter by the ratio [12]

$$
\eta = \frac{E_D E_s}{(E_D - E_s) \mu} 
$$

The transition from partial differential equations (1), (2) to ordinary differential equations (4) increases the accuracy of numerical solution to the wave problem [12].

Equation (2) is a linear equation for a viscoelastic body strain. It takes into account limited relaxation and limited creep of the body. The universality of equation (2) lies in the fact that, at $E_D \to E_s$, it turns into a linearly elastic Hooke’s model. Application of the law of strain (2) allows us to apply the problem solution considered here to metal and polymer pipelines.

A more generalized model of a hereditary viscoelastic body was considered in [13–15]. Nonlinear models of solids strain with variable mechanical characteristics were given in [16–19]. For the problem under consideration in the case of steel and polymer pipelines, equation (2) is fully consistent with the objectives of the task. Methods for solving wave problems in elastic and viscoelastic bodies and media were considered in detail in [20–23].

Equations (1), (2), and, accordingly, equations (4) with boundary conditions in the initial section (3) and at the wave front

$$
\sigma = 0; \quad \varepsilon = 0; \quad v = 0 \quad \text{at} \quad x = C_0 t
$$

cannot be solved analytically. Therefore, it is solved numerically by the finite difference method in an implicit scheme using equations (4) with boundary conditions (3) and (6) and with zero initial conditions.

It is known that when obtaining numerical solutions, the transition to dimensionless variables enhances the accuracy of results, as in this case the accumulation of errors in calculations with large numbers is excluded.

The transition to dimensionless variables and dimensionless parameters is carried out through the relations

$$
x' = \mu x / C_0; \quad t' = \mu t; \quad \sigma' = \sigma / \sigma_{\text{max}}; \\
v' = v / v_{\text{max}}; \quad \varepsilon' = \varepsilon / \varepsilon_{\text{max}}; \\
v_{\text{max}} = -\sigma_{\text{max}} / C_0 \rho_0; \quad \varepsilon_{\text{max}} = \sigma_{\text{max}} / E_D; \\
A = C_0 \rho_0
$$

where $A$ is the acoustic (wave) resistance (impedance) of the pipe material. In these variables, the main equations (1), (2) take the form

$$
\frac{\partial \varepsilon'}{\partial t'} + \frac{\partial \sigma'}{\partial x'} = 0, \quad \frac{\partial \varepsilon'}{\partial x'} + \frac{\partial \varepsilon'}{\partial t'} = 0
$$

where $\gamma = E_D / E_s > 1$.

Characteristic relations and characteristic lines (5) in dimensionless variables (7) have the form

$$
d\sigma' \pm d\varepsilon' = \left(\varepsilon' - \gamma \sigma'\right) dr', \quad dx'/dt' = \pm 1
\begin{align*}
\frac{d\sigma'}{d\varepsilon'} &= \left(\varepsilon' - \gamma \sigma'\right) dr', \quad dx'/dt' = 0
\end{align*}
$$
Boundary conditions (3) and (6) in dimensionless form are:

\[
\sigma' = \sin(\pi t/\mu T), \quad 0 \leq t' \leq \mu \theta \\
\sigma' = 0, \quad t' > \mu \theta \\
\sigma' = \varepsilon' = v' = 0 \quad \text{at} \quad x' = t' 
\]

(10)

The transition to dimensionless variables, in addition, allows applying the numerical solutions obtained in dimensionless parameters to different values of seismic loads and mechanical characteristics of the pipeline material. Equations of state (2) are applicable to both steel and polymer pipelines at correspondingly different values of \( \gamma \).

The solution algorithm was compiled using the calculation scheme described in [22]. In the characteristic plane \( xt \), the propagation of a plane wave generated by the load (3) at \( x = 0 \) is considered. An underground pipeline of length \( L \) is taken. At the section of the pipeline \( x = L \), the following condition is met

\[
v = 0 \quad \text{at} \quad x = L 
\]

(11)
in dimensionless form

\[
v' = 0 \quad \text{at} \quad x' = x' 
\]

(12)

In [22], a similar wave problem based on the theory of characteristics and the subsequent application of the finite difference method using an implicit calculation scheme was solved numerically. The problem of wave propagation in an elastic-viscoplastic soil and its interaction with a rigid barrier in soil is considered. The problem considered in this paper differs from [22] by the law of material strain and the field of application. The methods for solving problems are the same. The reliability of the numerical solution method used here is shown in [22].

3. Results

After transition to dimensionless variables, the initial data for the numerical calculation are the parameters \( \mu T \), \( \gamma \) and \( x' \). In calculations, the values of \( \mu T \) ranged from 5 to 5000, and \( \mu \theta = 20 \mu T \). The values of \( \gamma \) were taken as: 1.02 (an elastic pipeline); 1.1 and 4 (a visco-elastic polymer pipeline), \( x' \) - from 5 to 1000. At values of \( \mu = 10000 \text{s}^1 \), \( C_0 = 5000 \text{m/s} \), dimensionless \( \mu T \) and \( x' \) relate to dimensional values of \( T \), varying from \( 5 \cdot 10^{-4} \text{s} \) to 0.5 s and \( L \) varying from 2.5 m to 500 m. The loads are considered in the frequency range from 2 Hz to 2000 Hz.

Numerical solutions are obtained in the form of a change in wave parameters over time for fixed sections of the pipeline in the form of functions \( \sigma'(t') \), \( v'(t') \), \( \varepsilon'(t') \).

Figure 1 shows the change in dimensionless longitudinal stresses \( \sigma \) in soil over dimensionless time \( t \) (hereinafter, for the simplifications the upper indices are omitted) for fixed pipeline sections \( x = 0, 2, 4, 6, 8 \) and 10 (curves 0-5, respectively) for the calculation option \( \mu T = 10 \); \( \gamma = 2 \) and \( x' = 10 \). At \( x = x' = 10 \) there is a massive stationary rigid body. In this option, the wave frequency is 500 Hz, i.e. it is quite high frequency.

In figure 1, curve 0 refers to a given load at the initial section of the pipeline. As seen from figure 1, the load in the initial section with amplitude \( \sigma = 1 \) according to equation (3) monotonically varies with time.

In pipeline sections \( x = 2; 4 \) and 6 (curves 1, 2, 3) the amplitude of the first stress vibrations damps with distance. The amplitudes of subsequent vibrations increase, with the exception of curve 3, which relates to the cross section of the pipeline \( x = 6 \). Curve 3 at \( x = 6 \) refers to approximately the middle of the pipeline at \( x = 8 \) and 10 (curves 4 and 5); here the amplitudes of the first vibrations are greater
than the load amplitude $\sigma = 1$ at $x = 0$. Such an increase in stress amplitude for curves 4 and 5 and for curves 1-3 during subsequent vibrations occurs due to the waves reflected from the end section of the pipeline at $x = L$.

Figure 1. Change in dimensionless longitudinal stress in different fixed sections of the pipeline over time for a high-frequency wave.

As a result of superposition of the waves propagated and reflected from the massive body, a complex pattern of change in longitudinal stress is obtained in the pipeline body. The stresses in the middle section of the pipeline (curve 3) vary according to a special law. The reflection coefficients in the amplitudes of the first wave vibrations in all sections are less than 2. This is the expected result, since in the case of $\gamma = 1.02$ (an elastic pipeline), the reflection coefficient for the first wave arrivals is 2. Consideration of viscous properties of the pipeline material ($\gamma = 2$) increases the wave dissipation and the amplitude (the maximum values) of stresses along the first vibrations for the sections $x = 2, 4, 6$ decreases.

Continuous influence of load (3) on the initial section of the pipeline $x = 0$ leads to an increase in amplitude of longitudinal stresses at the end section of the pipeline $x = 10$ and the section close to it $x = 8$ and subsequent vibrations at sections $x = 2, 4$ and 6. Moreover, the reflection coefficient in these cases is more than 2 and reaches 2.4 in section $x = 10$.

Such a phenomenon was not previously observed in elastic media. Due to the increase in deformability of the pipeline material, considering the loading rate, there is an increase in longitudinal stresses in the equation of state (2), when incident and reflected waves in the pipeline body are superimposed.

Further, after 6-8 vibrations for high-frequency waves, 10-12 oscillations for low-frequency waves, the stress value tends to a steady one. Here, the reflection coefficient (the ratio of stress amplitude in the considered section to the amplitude of initial load, which generates waves in section $x = 0$) is less than 2 in all sections.

This process is clearly seen in figure 2. Here are the envelopes of the curves of maximum stresses positive values in time in fixed sections of the pipeline at $\mu T = 50$; $\gamma = 2$; $x^* = 10$. In this option, the wave frequency is 100 Hz. Note that the compressive stresses in the pipeline are taken as positive and tensile stresses as negative ones.
Curves 0–7 in figure 2 relate to the following sections of the pipeline $x=0,1,3,4,5,7$ and 10. The points on the envelopes relate to the values of the stress amplitude reached over time and determined along the abscissa axis in figure 2.

Figure 2. Envelopes of maximum longitudinal stresses for positive amplitudes in the pipeline sections.

As seen from figure 2, at $x=0$, the amplitudes of longitudinal stresses over time are constant (the straight line 1).

At the next sections of the pipeline (curves 1–7), the amplitudes of the first and second vibrations increase as a result of wave superposition (incident wave from the initial section and reflected wave from the end of the pipeline). Over time, the intensity of stress amplitude growth decreases and after 10-15 oscillations it becomes steady (curves 1-7).

In figure 2 at $\mu T = 50$, i.e. when the wave frequency is five times less than in the case of figure 1, the reflection coefficient at all vibrations does not exceed 2. An increase in the reflection coefficient to more than 2 is observed for very high-frequency waves. For low-frequency waves at $\mu T = 1000$ and 5000, the reflection coefficient decreases and tends to 1. For low-frequency waves, a change in the wave parameters in the pipeline practically occurs in a quasistatic mode [22].

Similar results are obtained for changes in sections velocities and longitudinal strains of the pipeline.

4. Discussion

Numerical solution obtained for the considered problem of wave process in an underground pipeline resting on a rigid stationary solid body shows that the stress amplitudes depending on the frequency of the incident wave increase by two or more times. The increase in amplitude occurs due to the
superposition of incident and reflected waves with high loading rates. At high wave frequencies observed under explosive loads, the loading rate is also high. So, a stress increase by more than two times is observed for high-frequency waves. A stress increase up to two or more times occurs gradually, after several oscillations. Then the stress amplitude decreases and tends to a steady state at a level less than double the amplitude of the given wave.

For the low-frequency waves observed during earthquakes, the increase in stress amplitude is small. This is explained by low loading rates at low wave frequencies. In these cases, the loading of the pipeline is carried out almost in a quasistatic mode, i.e. at loading rates or strain rates close to zero. In these cases, as well as in statics, the stress in the pipeline body does not exceed a stated load.

These results facilitate the choice of the method for determining stresses in underground pipelines, depending on the dynamics of stated or actual load. Under high-frequency dynamic loads, longitudinal stresses in underground pipelines should be determined from the solution of wave problems, as is done in this paper. Under low-frequency dynamic (seismic) loads, longitudinal stresses can be determined from the solution of wave problems, as well as is done here, or from the solution of problems of stationary oscillations of an underground pipeline, as recommended in [1].

5. Conclusions
1. To study the process of the stress state formation in an underground pipeline under dynamic loads of varying frequencies, a one-dimensional unsteady wave problem is formulated.
2. A pipeline resting on a massive stationary rigid body is selected as a design scheme, since in this case the largest increase in stress is expected in the end section and close to end section of the pipeline.
3. The dependence of the amplitude of longitudinal stresses in the body of the underground pipeline on the frequency of incident wave on the pipeline is shown by numerical solution to the problem. At high frequencies of dynamic load, the amplitude of longitudinal stresses increases up to two or more times in comparison with the amplitude of incident wave. And at low frequencies of the dynamic load, they increase slightly, almost remaining at the level of the amplitude of incident wave.

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