Lateral-Torsional Buckling Behaviour of Triangularly Corrugated Web Beam

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Abstract. Lateral torsional buckling (LTB) is a common failure mode of large span beams. In this phenomenon, the beam becomes unstable along the unbraced length. This instability of beams can be identified by out-of-plane deflection and twisting. In this paper LTB strength of triangularly corrugated web beams under bending condition is investigated. Examined steel beams consist of two flanges and a thin triangularly corrugated web, connected by automatic welding. In the literature, the LTB strength problem of steel beams was dealt with mostly for steel beams with plate, sinusoidal and trapezoidal corrugated webs. Researches of the LTB behaviour of beams with triangularly corrugated webs were found out to be very limited. A parametric study is carried out for various beam spans and corrugation densities. A general-purpose finite element program (ABAQUS) was used. The corrugation densities adopted in this study represent practical geometries, which are common used for such structures in building practice. Plot showing the influence of compressed flange slenderness on value of reduction factor for LTB is presented. It is determined that existing buckling curves poorly describe the change of reduction factor depending on slenderness of compressed flange for triangularly corrugated web beams. Finally, recommendations were proposed for the design of simple supported steel beams with corrugated webs against lateral torsional buckling in accordance with numerical results.

1. Introduction
Corrugated web beams are fabricated structures with a thin-walled corrugated web and flanges made of plate steel. Currently, such beams used as ceiling girders in multi-story apartment houses, large span roof girders in industrial and administrative buildings etc. In the world practice of construction, sections with trapezoidal, sinusoidal and triangular corrugation profiles are used. Beam with different type of web corrugation is shown in figure 1. Triangularly corrugated webs have several advantages over others. For example, they do not require expensive equipment for the production and the webs could be thicker than sinusoidal ones. This is especially important for heavy operating conditions that characterized by aggressive environment and large values of loads on the structures. Due to its profiled form, corrugated web exhibits enhanced shear stability and therefore eliminate the need for additional transverse stiffeners or thicker web plates. The profiling of the web generally avoids failure of the beam before the web ultimate load is reached by web yielding. A distinctive feature of beams with a corrugated web is the almost complete perception of the bending moment by the flanges, and the normal stresses along the height of the web are significant just in narrow areas near the flanges and insignificant for most of the web height.
Generally, lateral-torsional buckling (LTB) is a major design aspect of flexural thin-walled members. When a slender I-girder is subjected to flexure about its strong axis with insufficient lateral bracing, out-of-plane bending and twisting may occur as the applied load approaches its critical value. At this critical value, lateral-torsional buckling occurs.

Research studies on the LTB effect of corrugated web beams were very limited. Aschinger and Lindner [1] and Sayed-Ahmed [2] have studied the LTB of I-girders with trapezoidal corrugated webs. Aschinger and Lindner [1] have proposed the empirical formula of the warping constant of such I-girders on the basis of test results. They have also studied the interaction between local flange buckling and the overall lateral-torsional buckling. Sayed-Ahmed [2] has performed a series of finite element analyses and concluded that resistances to the lateral–torsional buckling of the I-girder with corrugated webs are higher than those of an I-girder with flat webs, and the equations used to calculate the critical lateral–torsional buckling strength of the I-girder with flat webs would underestimate the capacity of the I-girder with corrugated webs. However, the behaviour of the lateral–torsional buckling of the I-girder with corrugated webs has not yet been explained clearly, and it should be investigated. Moon et al. [3], have proposed approximated methods for locating the shear centre and calculating the warping constant of trapezoidal corrugated profile. Lopes et al. [4] are proposed a method to obtain the warping or torsional constant of beams with sinusoidal corrugated webs based on a numerical study for a wide range of different corrugation profiles and lengths.

The present paper deals with the lateral-torsional buckling effect of steel beams with triangularly corrugated webs. In the literature, the LTB problem in steel beams with corrugated webs was dealt with mostly for beams with trapezoidal and in several cases for sinusoidal corrugation. In present work, a numerical parametric study of finite element analysis (FEA) was carried out for simply supported corrugated web beams. Within the parametric study, various cases were considered including span of the beam and the corrugation density expressed as a ratio between height and length of half-wave of corrugation. The software ABAQUS (2017) was used to carry out linear buckling analyses (LBA) and to obtain the elastic critical moments of triangularly corrugated web beams. Results compared with estimations of the currently available design guidance [5-9].

2. Design of triangularly corrugated web steel beam

Limit states considered for the design of a corrugated web beam is, in general, similar to those considered for a steel plate girder with a flat web plate. The destruction of steel beam with a corrugated web may occur for several reasons: due to the exhaustion of the load-bearing capacity of flanges at bending or corrugated web at shear; due to local buckling of compressed flange or at particular areas of the web; due to overall buckling of the entire structure. In addition, according to the
conditions of suitability for normal operation, the maximum vertical deflections of the beam must be limited [5].

![Diagram of bending moment](image)

**Figure 2.** (a) Considered beam design scheme; (b) Cross section of beam with corrugated web; (c) Geometrical parameters of corrugated web with triangular shaped corrugations

Since the corrugated web is not supposed to resist axial forces due to the so-called “accordion effect” [5], corrugated web beams subjected to bending are designed considering that only the flanges resist of the bending moment. In European [7] and Russian [8] building design codes the bending resistance of a corrugated beam is given as flange resistances multiplied by the section height and no contribution of the web is taken into account:

$$M_y = b_f t_f h_f R_y \gamma_c$$

in which $b_f$, $t_f$, $h_f$ are the flange width, thickness and distance between centres of gravity of compressed and tension flanges respectively, $R_y$ is the material yield strength, $\gamma_c = 1.0$ is working condition factor specified by [9].

Elastic critical moment at lateral-torsional buckling according to [8] and [7] can be respectively expressed as

$$M_{cr} = \varphi_f \cdot M_y$$

$$M_{cr} = \chi \cdot M_y$$

in which $\varphi_f$ and $\chi$ are the reduction factors for out of plane buckling according to [8] and [7] respectively.

These reduction factors are computed using a simplified procedure in both design codes. In such procedure reduction factor is defined as for a centrally compressed rod depending on the out-of-plane slenderness of the compressed flange which is defined as follows
\[
\bar{\lambda} = \frac{l_{ef}}{i_f} \sqrt{R_y/E}
\]  
(4)
in which \(\bar{\lambda}\) is slenderness parameter of the compressed flange, \(l_{ef}\) is the distance between lateral restraints of the compressed flange, \(i_f = 0.29b_f\) is the radius of gyration for compressed flange, and \(E = 206000\) MPa is modulus of elasticity for steel.

In European [6] and Russian [9] design codes adopted different curves to obtain reduction factor at lateral-torsional buckling as shown in figure 4. According to Russian design code [9] this curve can be described by following expressions

\[
\varphi_f = \begin{cases} 
1 & \text{for } \bar{\lambda} < 0.6, \\
7.6/\bar{\lambda} & \text{for } \bar{\lambda} > 4.4, \\
0.5\left(\delta - \sqrt{\delta^2 - 39.48\bar{\lambda}^2}\right)/\bar{\lambda} & \text{for other cases},
\end{cases}
\]
(5)

\[
\varphi_f = \begin{cases} 
1 & \text{for } \bar{\lambda}_{LT} \leq 0.2, \\
1 & \Phi + \Phi - \frac{2}{\lambda_{LT}} & \text{for other cases},
\end{cases}
\]
(6)

\[
\varphi_f = 0.5\left[1 + 0.76(\bar{\lambda} - 0.2) + \frac{\bar{\lambda}_{LT}^2}{\Phi}\right], \quad \lambda_{LT} = \bar{\lambda}/\pi.
\]
(7)
in which \(\delta = 9.87(0.96 + 0.09\bar{\lambda}) + \bar{\lambda}^2\).

Lateral-torsional buckling curve for corrugated web beam according to European design code [6] can be described by corresponding expressions

3. Numerical parametric study
Typical finite element models adopted for the corrugated web beams are shown in figure 3a. A type of four-node doubly curved shell element (S4R) which is available in ABAQUS (2017) was employed in the models. A typical model is composed of upper and lower flanges of 10mm x 200mm in size, representing a plate 200 mm wide and 10 mm thick and 4mm x 600mm corrugated web which is a thin sheet of steel plate having 4mm thickness and 600mm height.

For the webs were assumed three different corrugation densities. As schematically explained in figure 2 the corrugation density is defined as the ratio of the magnitude of the wave height \(f\) to the corrugation length \(a\). In the models were used three different corrugation densities: \(f/a = 50\text{mm}/100\text{mm}, f/a = 60\text{mm}/150\text{mm}\) and \(f/a = 70\text{mm}/200\text{mm}\). The corrugation densities adopted in this study represent practical geometries, which are common used for such structures in building practice [5]. Within the parametric study, for each corrugation type seven different beam spans were considered with values of \(l\) is equal to 3m, 4.5m, 5m, 6m, 7.5m, 9m and 12m. All models were loaded by two point loads \(P = 1\text{kN}\) located in thirds of the span on the upper flange at the level of the beam centre line.

Boundary conditions were applied to either ends of the beam models at lower flange surface and at web outermost nodes by restraining appropriate degrees of freedom to simulate the simply supported condition and lateral restraints just only on beam supports. An elastic material model was assumed with a yield strength value of 240MPa, modulus of elasticity \(E = 206000\) MPa and Poisson’s ratio 0.3.
Figure 3. (a) FE model of triangularly corrugated web beam adopted for buckling analysis; (b) Example of the global buckling mode for triangularly corrugated web beam

Table 1. Results obtained from the buckling analysis

| Model          | $\bar{\lambda}$ | $v$ | $M_{cr}$, kN.m | $M_{y}$, kN.m | $M_{cr}/M_{y}$ | $\phi_f$ by Eq. (5)-(7) | $\phi_p$ by Eq. (8)-(9) | $\chi$ by Eq. (11)-(12) | $\varphi$ by Eq. (11)-(12) |
|----------------|-----------------|-----|----------------|----------------|-----------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $f/a = 50/100, l = 3$ m | 1.77  | 602.11 | 602.1 | 2.06 | 0.86 | 0.86 | 1.0 |
| $f/a = 50/100, l = 4.5$ m | 2.65  | 198.87 | 298.3 | 1.02 | 0.71 | 0.70 | 1.0 |
| $f/a = 50/100, l = 5$ m | 2.94  | 147.53 | 245.9 | 0.84 | 0.65 | 0.64 | 0.88 |
| $f/a = 50/100, l = 6$ m | 3.53  | 88.53 | 177.1 | 0.60 | 0.54 | 0.52 | 0.61 |
| $f/a = 50/100, l = 7.5$ m | 4.41  | 48.11 | 120.3 | 0.41 | 0.39 | 0.38 | 0.39 |
| $f/a = 50/100, l = 9$ m | 5.30  | 29.73 | 89.2 | 0.30 | 0.27 | 0.28 | 0.27 |
| $f/a = 50/100, l = 12$ m | 7.06  | 14.5 | 58.0 | 0.20 | 0.15 | 0.17 | 0.15 |
| $f/a = 60/150, l = 3$ m | 1.77  | 626.6 | 626.6 | 2.14 | 0.86 | 0.86 | 1.0 |
| $f/a = 60/150, l = 4.5$ m | 2.65  | 198.8 | 298.3 | 1.02 | 0.71 | 0.70 | 1.0 |
| $f/a = 60/150, l = 5$ m | 2.94  | 147.4 | 245.7 | 0.84 | 0.65 | 0.64 | 0.88 |
| $f/a = 60/150, l = 6$ m | 3.53  | 89.0 | 178.0 | 0.61 | 0.54 | 0.52 | 0.61 |
| $f/a = 60/150, l = 7.5$ m | 4.41  | 48.3 | 121.1 | 0.41 | 0.39 | 0.38 | 0.39 |
| $f/a = 60/150, l = 9$ m | 5.30  | 29.9 | 89.9 | 0.31 | 0.27 | 0.28 | 0.27 |
| $f/a = 60/150, l = 12$ m | 7.06  | 14.6 | 58.5 | 0.20 | 0.15 | 0.17 | 0.15 |

4. Results of the parametric study
In this work, all models were assumed to buckle under perfect conditions, where there is no initial imperfections and eccentric load. Overall view of the global buckling mode for triangularly corrugated web beam was obtained from FE buckling analysis is shown in figure 3b. In order to calculate the critical buckling load, the load factors from the eigenvalues analysis were multiplied by the applied load. For example, $P$ for this study is 1kN, thus the critical buckling load $P_{cr} = 1kN \cdot \nu$, where $\nu$ is eigenvalue received from FE buckling analysis. In the case under consideration critical moment of lateral-torsional buckling can be obtained as
\[ M_{cr} = P_{cr}/3. \] (10)

The values of the reduction factors for lateral-torsional buckling obtained from the results of FE analysis were calculated as ratio \( M_{cr}/M_y \) and given in table 1. Lateral-torsional buckling curves for trianularly corrugated web beams evaluated according to [6] and [9] along with values of the reduction factors obtained from buckling analysis are given in figure 4. As shown in this figure, the curves obtained according different design codes look almost identical and poorly describe the changing of buckling reduction factor depending on slenderness parameter for compressed beam flange. The greatest difference from the FE analysis results is demonstrated at values of \( \bar{\lambda} < 4.4 \). For some cases the difference in values is in twice. Much better FE analysis results are consistent with the existing buckling curves for slenderness parameter \( \bar{\lambda} > 4.4 \). Given the fact that in existing design rules adopted the simplified method to evaluate buckling reduction factor for corrugated web beams it can be allowed that web corrugation has a significant impact on sustainability of all structure and increase lateral-torsional buckling resistance of beam. For this reason, in that work proposed following expressions to calculate buckling reduction factor for triangularly corrugated web beams

\[ \varphi_p = 1 \text{ for } \bar{\lambda} < 2.7, \] (11)
\[ \varphi_p = 7.6/\bar{\lambda}^2 \text{ for } \bar{\lambda} \geq 2.7. \] (12)

Figure 4. Buckling curves for trianularly corrugated web beam and results of FE analysis

5. Conclusions
In present work an investigation into the lateral-torsional buckling behaviour and design of trianularly corrugated steel web beams was carried out. A general-purpose finite element analysis program was used to model steel beams with varying beam length and different corrugation densities. Using this program, a numerical parametric study was carried out for simply supported corrugated
steel web beams loaded by point loads in third of the span. The chosen models represented practical geometries in terms of production and structural application.

According to the results of the performed work, it is shown that corrugated web beam has increased resistance against lateral-torsional buckling while bending in strong plane. In addition, it is shown that existing European [6, 7] and Russian [8, 9] design rules are not perfect in the matter of evaluation buckling resistance for beams with triangularly corrugated webs. For practical design it is possible to calculate buckling reduction factors for triangularly corrugated web beam by expressions (11) and (12) but the issue of buckling resistance for such structures requires further investigation considering initial imperfections, type and location of the applied load, the shape of the bending moment, end support conditions and so on.

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