Large N gauge theories – Numerical results

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Some physical results in four dimensional large N gauge theories on a periodic torus are summarized.

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1. Introduction

The large N limit of four dimensional non-abelian gauge theories is interesting from the view point of QCD phenomenology and string theory. Lattice QCD is a useful technique for extracting fundamental results in the large N limit of QCD. Fermions are naturally quenched in the 't Hooft limit of large N QCD and this significantly reduces the computational cost in a lattice calculation. In addition, there is a concept of continuum reduction, namely, physics does not depend on the size of box \( l^4 \) for \( l > l_c \) and \( l_c \) is a physical critical size. These two observations have been used to extract physical results in the large N limit of QCD using numerical techniques on the lattice.

2. Phases of large N QCD

Large N QCD on a continuum torus \( l^4 \) has several phases depending upon the size of the torus as shown in Fig. 1. The continuum action has \( U^4(1) \) symmetries associated with the Polyakov loops in the four directions and the various phases correspond to the number of directions in which this
symmetry is broken. The continuum limit is obtained by going to the top-right corner of Fig. 1 and different approaches to this corner will result in one of the five continuum phases.

The 0h-phase present for $b < 0.36$ for all $L$ is an unphysical phase that does not survive the continuum limit. The 0h to 0c transition is associated with the single plaquette operator opening up a gap around $\pi$ in its eigenvalue distribution. Gauge fields come in disconnected pieces in all the Xc-phases due to the presence of the gap in the single plaquette operator.\(^5\)

The 0c-phase is the confined phase of large N QCD and the 1c-phase is the deconfined phase and $l_c = 1/t_c$. An immediate consequence of continuum reduction is that large N QCD does not feel temperatures below $t_c$.\(^6\) Numerical analysis has shown that lattice spacing effects are small in the 0c-phase and it is sufficient to work at $L \sim 9$ and $N \sim 30$ to extract continuum results. Therefore, numerical computations can be performed on a serial computer and a cluster of computers can be efficiently used to generate statistics in a Monte-Carlo calculation.

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Fig. 1. The various phases of four dimensional large N QCD as viewed from the lattice.
3. Chiral symmetry breaking in finite volume

Since physics does not depend on the box size in \( 0c \)-phase, one should show that chiral symmetry is spontaneously broken in finite volume in order to properly reproduce physics in this phase \(^7\). The order of limits are important and one has to take the large \( N \) limit before taking the quark mass to zero at finite physical volume. The low lying spectrum of the massless Dirac operator shows evidence for spontaneous chiral breaking since the eigenvalues, \( i\lambda \), scales like \( z = \lambda \Sigma N L^4 \) with \( z \) obeying a universal distribution. The chiral condensate, \( \Sigma \), is independent of \( l \). Results of a calculation of the chiral condensate on the lattice is shown in Fig. 2. Results from chiral random matrix theory \(^8\) were used to extract the chiral condensate at a fixed \( N, L \) and lattice coupling. Two lowest non-zero eigenvalues in the \( Q = 0 \) and \( Q = 1 \) topological sectors were used to show consistency. The plot shows that there is a limit as \( N \to \infty \) and this limit is independent of \( L \). Results obtained at different lattice couplings yield \( \frac{\lambda^3}{N^2} \langle \bar{\psi} \psi \rangle \approx (0.65)^3 \).
4. Pions in large N QCD

Since chiral symmetry is broken in large N QCD even in finite volume, one should be able to observe massless pions in finite volume. This result emerges in the following manner. Properties of a single quark in a background gauge field $A_\mu(x)$ cannot depend on a shift of $A_\mu(x) \rightarrow A_\mu(x) + p_\mu$ for arbitrary $p_\mu$ since the $U_A(1)$ symmetries associated with the Polyakov loops are not broken in the $0\mathrm{c}$-phase. But the propagator of a non-singlet meson will depend on $p_\mu$, if one quark sees $A_\mu(x) \rightarrow A_\mu(x) + p_\mu$ and the other sees $A_\mu(x) \rightarrow A_\mu(x) - \frac{p_\mu}{2}$ as their respective gauge fields. This is referred to as the quenched momentum prescription for the computation of meson propagators in the large N limit of QCD. One can use the results for the chiral condensate, $\Sigma(b)$, and critical lattice size, $L_c(b)$, to plot the pion mass as a function of $m < \bar{\psi} \psi >$. The results fall on a single universal
curve as shown in fig. 3 and \( f_\pi l_c = \frac{1}{\sqrt{2} \Lambda_\pi} \approx 0.269 \).

5. Chiral symmetry restoration

The 0c to 1c phase transition is the confinement-deconfinement phase transition since the \( U(1) \) symmetry associated with one of the Polyakov loops is broken. This transition is first order since there is a latent heat associated with the single plaquette \(^{11}\). The fermion determinant does matter in the 1c-phase and it picks the correct boundary conditions for fermions in the broken direction, namely, anti-periodic with respect to the Polyakov loop \(^{12}\). The lowest eigenvalue of the Dirac operator with the correct boundary conditions can be used to study the gap in the 1c-phase and one finds that chiral symmetry is restored in the 1c-phase \(^{12}\). Furthermore, the chiral transition is first order as shown in Fig. 4. If one were to super-cool the 1c-phase into the 0c-phase, a second order transition with a square root singularity would be observed at 0.93/\( l_c \).
6. Phase transition in the Wilson loop operator

Non-abelian gauge theories in the confined phase are strongly interacting at large distances and weakly interacting at short distances. If this is seen as a phase transition in some observable, one could use the universal behavior of this transition to connect the low energy physics of QCD to the high energy physics of QCD. The Wilson loop operator as a function of its size is the most likely candidate to study this transition within the $0c$-phase of large $N$ QCD\textsuperscript{13}. Such a transition exists in two dimensional large $N$ QCD\textsuperscript{14} and it is claimed that the transition in four dimensional QCD is in the same universality class. This transition is referred to as the Durhuus-Olesen phase transition in Fig 1.

![Phase transition in the Wilson loop operator](image)

Fig. 5. The phase transition in the expectation value of the Wilson loop operator as a function of its size in large $N$ QCD. The distribution of the eigenvalues of the smeared Wilson loop operator are compared to the Durhuus-Olesen distributions. Results are shown for loops of sizes $l/l_c = 0.740, 0.660, 0.560, 0.503$. They match with $k = 4.03, 2.30, 1.41, 1.15$ where $k$ is the dimensionless area in two dimension large $N$ QCD. $k = 2$ is the critical area.

Wilson loop operators suffer from a perimeter divergence in four dimensions and one has to eliminate it when defining this operator if one were to
study its eigenvalue distribution. One possible way to suppress the perimeter divergence is to use smeared operators on the lattice. Numerical studies of the eigenvalue distribution of the smeared Wilson loop operator on the lattice shows clear evidence for a phase transition as a function of the size of the Wilson loop \(^{13}\). The universality class is the same as the one found in two dimensional large \(N\) QCD as shown in Fig. 5 and the critical loop size is roughly \(0.6l_c\).

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