Role of interference in MM-wave driven DC transport in two dimensional electron gas

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In this paper we point out that in addition to the density of states effect proposed in Ref.\textsuperscript{[3, 4]} one should consider the effect of constructive interference between the multi-MM-wave-photon processes shown in Fig.2. This process enhances the dark value of the conductivity. When the sample is very pure, i.e., when the transport life time is very long, this interference effect quickly diminishes as the MM-wave frequency deviates from the cyclotron frequency. In this paper we also present the linear response theory in the presence of strong harmonic time-dependent perturbation.

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The recent observation of strong suppression of the longitudinal resistivity of a two-dimensional electron gas (2DEG) by a millimeter (MM) radiation source\textsuperscript{[1, 2]} has stimulated considerable interests in the condensed matter community\textsuperscript{[3, 4, 5, 6, 7, 8, 9, 10, 11, 12]}. In Ref.\textsuperscript{[3]} and Ref.\textsuperscript{[4]} it is pointed out that the combined effect of photo-excitation by the MM wave photon and scattering by impurities can lead to a sinusoidal modulation of the conductivity as a function of $\omega_0/\omega_c$ (the MM wave photon frequency over the cyclotron frequency). When the amplitude of this modulation becomes big, the conductivity becomes negative and the system becomes unstable. In Ref.\textsuperscript{[3]} and Ref.\textsuperscript{[5]} it is postulated that this leads the system to self-organize into a state with zero conductivity.

In this work we propose another mechanism that may also be of importance for the observed phenomenon. In essence our mechanism associate the above experimental results with a phenomenon called “electromagnetically induced transparency”\textsuperscript{[13]}. This phenomenon occurs when an optical transition can take place through many alternative processes. When these processes destructively interfere with one another, the net optical transition amplitude vanishes. This has been observed when two of the levels of a three-level system is resonantly coupled together by a strong coupling laser. As a consequence, the resonant peak due to the absorption from the third level to one of these strongly mixed states is suppressed to zero.

In the present case we assume that the MM wave is sufficiently strong to couple many electronic states together. When an additional low frequency probing source is turned on, the transition that involves the absorption/emission of a single probing photon can occur via many virtual processes with varying number of MM wave photons absorbed and subsequently emitted. The interference between these processes can lead to electromagnetically induced transparency. The effect is illustrated in Fig.1 where the vertical arrows indicate the emission/absorption of the MM wave photons, and the slanted horizontal arrows indicate the absorption of the probing photons. If the matrix elements associated with the slanted horizontal arrows at different vertical levels are in phase, this leads to constructive interference. Otherwise destructive interference will be resulted. In the DC limit (vanishing probing photon frequency) destructive interference implies a strong suppression of the longitudinal conductivity. This is equivalent to a suppression of the longitudinal resistivity when $\sigma_{xy} > > \sigma_{xx}$. In the following we shall argue that the ratio between the MM wave frequency and the cyclotron frequency determines whether the interference is constructive or destructive.

Our picture of this $\omega_0/\omega_c$ dependence is the following. When $\omega_0$ is an integral multiple of $\omega_c$, the MM wave couples together states that have (essentially) the same guiding center orbit but different Landau level indices. Due to energy conservation we argue that the absorption/emission of a (low frequency) probing photon can only change the guiding center orbits not the Landau level index. More explicitly we assume that, with the initial and final guiding center orbits of the processes depicted in Fig.1 being all the same, the matrix elements corresponding to the slanted horizontal arrows are approximately equal. The processes shown in Fig.1 tend therefore to interfere constructively. However, when $\omega_0$ is not close to an integer multiple of $\omega_c$, the initial and final guiding center orbits of the different slanted horizontal arrows are different. This will generally give rise to matrix elements with different phases and lead to destructive interference.
The mechanism discussed here (like in Ref. [4]) the electron-electron interaction does not play a central role. However it is important to keep in mind that electron-electron interaction affects the self-consistent potential each electron sees hence affect the eigenstates $|i>$ in Eq. (1). In particular in the mechanism proposed in Ref. [4] where significant redistribution of charges can be resulted by the action of the MM wave radiation, the steady-state self-consistent potential can differ significantly from that in the absence of MM wave radiation. In the rest of the paper we shall ignore the electron-electron interaction with the understanding that the potential the electrons see is the steady-state self-consistent one.

In the following we first develop the formalism for describing this problem. We obtain a Kubo-like formula (Eq. (29)) that allows one to compute the AC/DC conductivity in the presence of a strong harmonic time-dependent driving field. We would like to stress that method developed here is not restricted to the specific problem considered. Rather it is a general formulation of linear response in the presence of a strong time-dependent driving field. We emphasize that the mechanism described here is not restricted to the specific method proposed in Ref. [5] where significant redistribution of charges can be

In the rest of the paper we shall use the term coupling field to denote the MM radiation field, and probing field to denote the low frequency field associated the linear response measurement.

I. THE GENERAL FORMALISM

A. The Floquet eigenvalue problem

Let us assume $\{ |i> \}$ are the exact single particle eigenstates (in the presence of disorder) in the absence of the coupling field

$$ H_0 |i> = \epsilon_i |i> . $$

(1)

With the coupling field turned on, the Hamiltonian becomes

$$ H(t) = H_0 + g_c \left[ e^{-i\omega_0 t} \sum_{i<j} P_{ij} |i><j| + e^{i\omega_0 t} \sum_{i<j} P^*_{ij} |i><j| \right] . $$

(2)

In the above $\omega_0$ is the frequency of the coupling field, and $P_{ij}$ is the matrix element of the current operator.

The solution of the time-dependent Schrödinger equation

$$ i\partial_t |\psi(t)> = H(t) |\psi(t)> $$

(3)

can be written as

$$ |\psi(t)> = \sum_{i} \phi_i(t) |i> , $$

(4)

where $\phi_i(t)$ satisfies

$$ i\partial_t \phi_i(t) = \epsilon_i \phi_i(t) + g_c e^{-i\omega_0 t} \sum_{j} P_{ij} \phi_j(t) + g_c e^{i\omega_0 t} \sum_{j} P^*_{ij} \phi_j(t) . $$

(5)

Due to the symmetry of Eq. (5) upon time translation

$$ t \rightarrow t + \frac{2\pi}{\omega_0} $$

(6)

the solutions of Eq. (5) $\phi_i^\alpha(t)$ can be written in the form

$$ \phi_i^\alpha(t) = e^{-iE_\alpha t} \sum_{n} \phi_n^\alpha e^{-in\omega_0 t} , $$

(7)

where the index $\alpha$ labels the different solutions. In the above

$$ -\frac{\omega_0}{2} \leq E_\alpha < \frac{\omega_0}{2} $$

(8)

is the “Brillouin zone” in frequency. This time version of the Bloch theorem is called the Floquet theorem [14]. By substituting Eq. (7) into Eq. (5) and equate coefficients of $e^{-i(E_\alpha + n\omega_0)t}$ we obtain the time independent equation

$$ E_\alpha \phi_n^\alpha = (\epsilon_i - n\omega_0)\phi_n^\alpha + g_c \sum_{j} \left[ P_{ij} \phi_{n-1j}^\alpha + P^*_{ji} \phi_{n+1j}^\alpha \right] . $$

(9)

Eq. (9) is an algebraic eigenvalue problem which can be solved to obtain eigenvalue $E_\alpha$ and corresponding eigenvector $\phi_n^\alpha$. Since $\phi_n^\alpha$ and $\phi_{n+1j}^\alpha$ are eigenvectors of Eq. (9), they satisfy (when properly normalized) the orthogonality relation

$$ \sum_{n,j} (\phi_{ni}^\beta)^* \phi_{nj}^\alpha = \delta_{\alpha\beta} . $$

(10)

It is straightforward to prove the following fact concerning the solutions of Eq. (9). If $\phi_n^\alpha$ is the eigenvector of Eq. (9) with eigenvalue $E_\alpha$, then

$$ \phi_{ni}^\alpha = \phi_{n+1i}^\alpha $$

(11)

is also an eigenvector of Eq. (9). The eigenvalue associated with the latter is

$$ E_{\alpha'} = E_\alpha + \omega_0 . $$

(12)
By repeated application of Eq. (11) we can generated a family of solutions $\phi_{n,i}^{\alpha}$, $\phi_{n,i}^{\prime \alpha}$, $\phi_{n,i}^{\prime \prime \alpha}$, ... of Eq. (11). It is simple to
to show that all these solutions lead to the same time-
dependent solution

$$|\tilde{\alpha}(t)\rangle = e^{-iE_{\tilde{\alpha}}t} \sum_{n,i} \phi_{n,i}^{\alpha} e^{-i\omega_n t} |i\rangle .$$  \hspace{1cm} (13)

In the above $\tilde{\alpha}$ denotes the entire class of Floquet eigen-
vectors related by Eq. (11) and Eq. (12).

It is simple to prove that

$$<\beta(0)|\tilde{\alpha}(0)> = \delta_{\beta\tilde{\alpha}} .$$  \hspace{1cm} (14)

deepth solutions

Thus

$$|\tilde{\alpha} > \equiv |\tilde{\alpha}(0) > = \sum_{n,i} \phi_{n,i}^{\alpha} |i\rangle$$  \hspace{1cm} (15)

can be used as an orthonormal basis just as $\{|i\rangle\}$.

We also note that because the time evolution is unitary,
Eq. (14) ensures that

$$<\beta(t)|\tilde{\alpha}(t)> = \delta_{\beta\tilde{\alpha}}$$  \hspace{1cm} (16)

for any $t > 0$.

Now we have obtained the an orthonormal set of solution
of the time-dependent Schrödinger equation

$$\{ |\tilde{\alpha}(t) > \} .$$  \hspace{1cm} (17)

Of course, any linear combination of these solutions

$$\sum_{\alpha} A_{\alpha} |\tilde{\alpha}(t) >$$  \hspace{1cm} (18)

is itself a solution of the Schrödinger equation. The coeffi-
cient $A_{\alpha}$ in Eq. (18) are determined by the initial
condition. For example by properly choosing $A_{\alpha}$ we

$$|i(0) > = |i\rangle .$$  \hspace{1cm} (19)

Using Eq. (14) and Eq. (15) it is simple to show that

$$|i(t) > = \sum_{\alpha} <\tilde{\alpha} |i\rangle |\tilde{\alpha}(t) > .$$  \hspace{1cm} (20)

To gain some intuitive feeling for what Eq. (9) rep-

tresents we consider the following model. For each

$$H = \sum_n \sum_i (\epsilon_i - n\omega_0)c_{n,i}^{\dagger}c_{n,i} + g_{\alpha} \sum_{n,i} [P_{ij}c_{n,i}^{\dagger}c_{n-1,j} + h.c.] .$$  \hspace{1cm} (21)

In the above $c_{n,i}^{\dagger}$ create an electron in the $i$th states of the
nth replica. The physical interpretation of the $n$ variable

is the photon number of the coupling field. The single
particle states in the $n$th layer are $|n, i\rangle$, and the
associated eigen energies are $\epsilon_i$. The replicas are coupled
together by hopping (the terms proportional to $g_{\alpha}$), and
an “electric field” is turned on so that the potential energy
of the $n$th replica is $-n\omega_0$. The replica model is
constructed so that Eq. (11) is its time-independent eigen
equation. Each solution $\phi_{n,i}^{\alpha}$ of Eq. (11) uniquely defines
an eigen state

$$\sum_{n,i} \phi_{n,i}^{\alpha} |n, i\rangle$$  \hspace{1cm} (22)

of the replica model. Eq. (11) and Eq. (12) link a whole
family of replica states together.

B. The linear response theory

In this section we derive the formula for the AC conduc-
tivity in the presence of the coupling field. The formal-
ism developed in this section is rather general. The
only restriction is that the electron-electron interaction
is neglected.

The AC conductivity is the response of the system to
a time-dependent probing field. The Hamiltonian in the
presence of such probing field is given by

$$H'(t) = H(t) + H_p(t)$$

$$H_p(t) = g_p e^{i\omega_p t} \sum_{ij} D_{ij} |i\rangle <j| + h.c.,$$  \hspace{1cm} (23)

where $D_{ij}$ is the current matrix element coupling to the
probing field. Unlike the coupling field, the probing field
is very weak. Therefore we will treat its effect perturba-
tively to the lower order in $g_p$.

If we use $|i(t) >$ as basis, the probability amplitude
that the probing field will induce a transition from $|i\rangle$
at time zero to $|j\rangle$ at time $t$ is given by

$$A_{ji} = \delta_{ji} - i \int_0^t d\tau <j(\tau)|H_p(\tau)|i(\tau) > + ... .$$  \hspace{1cm} (24)

If the we model the decoherence by a single decoherence

time $\tau = 1/\Gamma$ the the averaged transition rate between

$$W_{ji} = \frac{2\Gamma}{\int_0^\infty dt e^{-\Gamma t}} |j(\tau)|H_p(\tau)|i(\tau) >|^2 .$$  \hspace{1cm} (25)

Straightforward calculation shows that

$$W_{ji} = W_{ji}^{\alpha} + W_{ji}^{\prime \alpha} .$$  \hspace{1cm} (26)

("a" stands for absorption and "e" stands for emission),

where

$$W^{\alpha}_{ji} = 2\Gamma \left| \sum_{l} \sum_{\alpha\beta} \frac{<j|\vec{\alpha} > D_{l\alpha\beta}^{\ast} <\beta|i >}{E_{\alpha} - E_{\beta} - \omega_p + i\omega_0 + i\Gamma} \right|^2 .$$  \hspace{1cm} (27)
In Eq. (27)

\[ D_{\alpha\beta} = \sum_n \sum_{ij} (\phi_{n+i}^{\alpha})^* D_{ij} \phi_{n+j}^{\beta} \]  

(28)

For the \( \alpha \) and \( \beta \) in the summation we choose one \( \alpha \) and one \( \beta \) for each family of solutions \( \alpha \) and \( \beta \). We note that if these are chosen so that \( E_{\alpha} - E_{\beta} \) is small compared to \( \omega_0 \) (we shall assume this choice in the rest of the paper), the main contribution in the sum over \( l \) comes from \( l = 0 \), since \( \omega_p \) is small.

The total absorption and emission rates and the AC conductivity are given by

\[ A = \sum_{ij} f_i (1 - f_j) W_{ij}^a \]

\[ E = \sum_{ij} f_i (1 - f_j) W_{ij}^e \]

\[ \sigma_{xx}(\omega_p) \sim \frac{1}{\Omega} \lim_{\omega_p \to 0} \frac{1}{\omega_p} (A - E), \]

(29)

where \( f_i = f(\epsilon_i) \) is the Fermi function and \( \Omega \) is the volume (or area for 2D) of the sample.

In the limit of \( \Gamma \to 0 \) only the diagonal terms in the expansion of \( \ldots \pi \) in Eq. (27) survive. In that case \( E_{\alpha} - E_{\beta} \), is not strictly fixed by energy conservation, but can take value within a range \( \Gamma \) around \( \pm \omega_p \). In this case \( A \) and \( E \) do not only get contributions from the diagonal terms in the expansion of (27) but also from crossing terms. When the phase coherence length is much smaller than the sample dimension, the above formula should also be averaged over the configurations of the random potential.

Eq. (27), Eq. (28) and Eq. (29), which express the linear response coefficients of a strongly driven system in terms of the eigenenergies and eigenfunctions of a time-independent Hamiltonian (\( H_r \) in Eq. (21)), is a main result of this paper.

C. A simple model

In order to illustrate the application of the above formalism we now consider a simple model where the “dark” energy levels are labeled by two parameters, \( i = (\nu, a) \), and the energies are given by

\[ \epsilon_{\nu a} = \nu \omega_0 + V_a \]

(30)

where \( -\omega_0/2 \leq V_a < \omega_0/2 \). In the following we shall refer to \( \nu \) as the “vertical” and \( a \) as the “horizontal” indices. We shall assume resonant coupling, i.e., the absorption/emission of a coupling photon results in vertical transition \( |\nu, a \rangle \to |\nu \pm 1, a \rangle \), and the absorption/emission of a probing photon results in horizontal transition \( |\nu, a \rangle \to |\nu, b \rangle \). More specifically we consider the following Hamiltonian

\[ H(t) = H_0 + H_c(t) + H_p(t) \]

\[ H_c(t) = g_c \left[ e^{-i\omega_0 t} \sum_{\nu a} D_{\nu a}^\nu |\nu + 1, a \rangle < \nu, a \rangle + h.c. \right] \]

\[ H_p(t) = g_p \left[ e^{-i\omega_p t} \sum_{\nu a b} D_{\nu a b}^\nu |\nu + 1 < \nu, a, b \rangle + h.c. \right]. \]  

(31)

To simplify the Floquet eigen equation we further assume

\[ p_a^\nu = \bar{P}. \]  

(32)

Given Eq. (31) and Eq. (32) the Floquet eigenvectors are given by

\[ \phi_{\nu a}^\alpha = \phi_{\nu a}^\alpha \delta_{\nu, \nu_0} + g_{\nu} (P \phi_{\nu a}^\alpha - 1 + \bar{p}^\nu \phi_{\nu a}^{\alpha+1}) \]  

(34)

This equation is invariant under translations \( n \to n + 1 \) and has, according to Bloch’s theorem, solutions of the form

\[ \phi_{\nu a}^\alpha = \frac{1}{\sqrt{N}} e^{i n \theta_\alpha} \]  

(35)

For simplicity we assume the variable \( n \) to take a finite number of values \( N \), with \( \phi_{\nu a}^\alpha \) as a periodic function. This implies

\[ \theta_{\alpha} = \frac{2\pi}{N} n_{\alpha}, \quad n_{\alpha} = 0, 1, \ldots, N - 1. \]  

(36)

with \( n_{\alpha} \) as a new discrete parameter that characterizes the state \( \alpha \). The energy of the Floquet state \( \alpha \) now is given by

\[ E_{\alpha} = \nu_{\alpha} \omega_0 + \lambda \cos \left( 2\pi \frac{n_{\alpha}}{N} + \delta \right) \]  

(37)

with the parameters \( \lambda \) and \( \delta \) defined by

\[ \lambda e^{-i\delta} = g_{\nu} \bar{P}. \]  

(38)

With the expressions for \( \phi_{\nu a}^\alpha \) and \( E_{\alpha} \) inserted in Eq. (27), the absorption/emission rate gets the form

\[ W_{\nu a} = 2\Gamma \frac{1}{N^2} \times \]

\[ \left| \sum_{p q} \frac{1}{N} \sum_{n} e^{i 2\pi \frac{n}{N} n} D_{p q}^{\nu + n} \overline{V_{p q}} + 2\lambda \sin(2\pi \frac{n}{N}) \sin(2\pi \frac{n}{N} + 2\delta) \pm \omega_p \right|^2. \]  

(39)

with \( V_{\nu a} = V_{\nu} - V_{\nu a} \). This expression is illustrated by the diagram of Fig.1, referred to in the introduction. Without the coupling field turned on the transition induced by the probing field is restricted to one value of \( \nu \) for each pair \( (a, b) \). However, with the coupling field on, the
coupling field introduced we do not see enhancement of the amplitudes
our idea. We note however, that with the simplifications
ations, in particular only including resonant couplings.
i.e. reduces to its “dark value”, i.e. its value without the coupling field
\[ W_{ji}^{a/c} = 2\Gamma \frac{D_{ab}}{V_{ab} \pm \omega_p + i\Gamma}^2. \] (40)
However, if the phase of \( D_{ab}^{\nu+n} \) varies randomly with \( n \)
and the phase generically varies randomly with \( p \). In the
limit \( \Gamma \to 0 \) only the diagonal terms in the expansion of
\( \cdots \) in Eq. (39) contribute and, owing to Eq. (41), \( W_{ji}^{a/c} \sim N^2/N^3 = 1/N \). Hence we have destructive interference.
When \( \Gamma \neq 0 \) but much smaller compared to \( \lambda \), the non-
diagonal terms become important. In that case simple estimate leads to
\[ W_{ji}^{a/c} \approx K \left[ c_1 \Gamma N + c_2 \Gamma \frac{\Gamma}{\lambda} + c_3 \left( \frac{\Gamma}{\lambda} \right)^2 \right]. \] (41)
where \( K, c_1, c_2, c_3 \) are \( n \)-independent constants. From
Eq. (42) we see that the suppression by large \( N \) depends on \( \Gamma \ll \lambda \). It is important to note that in
the simple model considered here the suppression of the
DC/AC conductivity is complete only when \( N \to \infty \) and
\( \Gamma/\lambda \to 0 \).
In the model we have introduced several simplifications,
in particular only including resonant couplings. However, we believe this model convey the essence of
our idea. We note however, that with the simplifications introduced we do not see enhancement of the amplitudes
for constructive interference, but rather a return to the
dark value.

II. APPLICATION TO THE 2-DIMENSIONAL ELECTRON GAS.

When discussing the application of the general formalism to the 2DEG, there are two regimes of interest to
discuss separately. The first one is the weak coupling limit, where the Floquet state is well localized with re-
spect to layer index \( n \) and where the coupling field as well, as the probing field, can be treated perturbatively.
The other case is the strong coupling regime, where the state is extended through many \( n \)'s. We first treat the
weak coupling case and discuss this in a general way with focus especially on the effect of the density of states.
The strong coupling case we discuss more qualitatively, with specific reference to the simple model discussed in the
previous section.

A. Weak coupling. The density of states effect.

In this limit the eigenstates of \( H_r \) are qualitatively sim-
ilar to those in the absence of the hopping between the
replica’s. In this limit the our calculation produces result
similar that obtained in Ref. [2] and Ref. [3]. For a com-
parison with these references we restrict ourself to the
case of total coherence, i.e. \( \Gamma \to 0 \).
Assuming \( \phi_{ni} \) leaks weakly to the adjacent layers we obtain (upon using Eq. (15))
\[ |\tilde{\alpha} \rangle \approx |j > + \sum_{j'} \eta_{jj'} |j' > + \sum_{j''} \chi_{jj''} |j'' > \]
\[ |\tilde{\beta} \rangle \approx |i > + \sum_{i'} \eta_{ii'} |i' > + \sum_{i''} \chi_{ii''} |i'' >. \] (43)
In the above \( |j' >, |j'' > \) are states with energy \( \omega_0 \) above
those of \( |i >, |j > \) and \( |i'' >, |j'' > \) has energy \( \omega_0 \)
below those of \( |i >, |j > \). In the rest of this section we shall treat \( \eta \) and \( \chi \) as first order in the coupling constant
\( g_c \) and obtain \( A \) and \( E \) to \( O(g^2) \) by substituting Eq. (15)
into Eq. (29) and take the \( \Gamma \to 0 \) limit. The result for \( A \) is
\[ A = 2\pi g_p^2 \left\{ \sum_{ij} \delta(E_{ji} - \omega_p) f_i (1 - f_j)|D_{ji} + \delta D_{ji}|^2 \right. \\
+ \sum_{ij} \delta(E_{ji} - \omega_p - \omega_0)|f_i (1 - f_j)|\delta D_{ji}^*|^2 \right. \\
+ \sum_{ij} \delta(E_{ji} - \omega_p)|f_i \left( \sum_{j'} (1 - f_{j'}) |\eta_{jj'}|^2 \right)|D_{ji}|^2 \right. \\
+ \sum_{ij} \delta(E_{ji} - \omega_p)|\sum_{ii'} |\chi_{ii'}|^2 f_{ii'}| (1 - f_j)|D_{ji}|^2 \right\}. \] (44)
In the above
\[ E_{ji} = E_j - E_i, \] (45)
and \( \delta D_{ji}, \delta D_{ji}' \) are of order \( O(\epsilon) \) respectively.

The first term of Eq. (44) is the value in the absence of
coupling field (the matrix elements are slighted modified).
distribution function due to the coupling laser. If we
assume that the matrix elements in Eq. (14) are smooth
function of energy we can replace this equation by
\[ A = 2\pi g_p^2 \left[ \int dE f(E)(1 - f(E + \omega_p))N(E) \right. \\
\times N(E + \omega_p)|M_1|^2 + \int dE N(E)N(E + \omega_0 + \omega_p) \right. \\
\times f(E)(1 - f(E + \omega_0 + \omega_p))|M_2|^2], \] (46)
where \( M_1 \) and \( M_2 \) are \( E \)-dependent functions that we
do not write out explicite. The corresponding emission
From Eq. (46) and Eq. (47) we can compute the DC

eigenstates among many replicas. For the the transitions
in a randomly varying background potential.

As previously discussed, with $D_{ab}^\nu$ independent of $\nu$, the sum over $n$ in Eq. (39) will include terms that interfere destructively. Although one should note that in the expression found there was no enhancement relative to the dark value. Such an enhancement is clearly present in the observed effect. We believe such an enhancement is due to an increase in the magnitude of $D_{ab}^\nu$ with $\nu$ and/or to the effect of including non-resonant terms both of which are ignored in our simple model.

Let us next turn to the case where the coupling frequency is not close to $n\omega_c$. In this case different $|\nu, a >$ and $|\nu + 1, a >$ label two distant guiding center orbits with the potential energy of the second orbit $\omega_0$ higher than that of the first.

Since the matrix element $D_{ab}^\nu$, for different $\nu$, refers to transitions between distant pairs of guiding center orbits, they are no longer strongly correlated and may change substantially from one value of $\nu$ to the next. In an adiabatic approximation the matrix element can be related to variations in the drift velocity along the guiding center orbit, and in a randomly varying background potential we therefore expect a corresponding random variation in the matrix element. In this case the $n$ sum in Eq. (39) add destructively.

To recap, the difference between the case $\omega_0 = m\omega_c$ and $\omega_0 \neq m\omega_c$ lies in the fact that for the former the $n$ sum in the absorption/emission amplitude tend to add non-destructively, whereas for the latter the sum tends to add destructively. Thus, we propose that in addition to the density of state mechanism discussed in Ref. [3, 4], and seen in our discussion of the weak coupling limit, the above interference mechanism will contribute, for $\omega_0$ between integer multiples of $\omega_c$, to suppress the magnitude of the longitudinal resistivity. This can reduce the
resistivity or even suppress it to zero when the effect is very strong. However, since our discussion at this point is qualitative, we cannot estimate the real strength of the effect.

III. SUMMARY

In this paper we present a general formalism for describing the DC/AC transport in the presence of an oscillating coupling field. The oscillating field couples together states in the form of a Floquet state, and by use of the expression for this state we derive a general form for the absorption and emission probabilities of an additional low frequency probing field. This gives a generalized Kubo formula for the conductivity in the presence of an oscillating field.

We further discuss in a general way the application of this to the 2D electron gas in a magnetic field radiated by a MM wave. For a weak coupling field we show the presence of oscillations due to variations in the density of states. For strong coupling we discuss in a qualitative way the difference between constructive interference when the frequency matches the energy difference between two Landau levels and destructive interference for intermediate values of the frequency. The discussion of the interference effects is illustrated by a simplified model.

Quantum interference between different virtual processes described in this paper should in general exist. However, complete destructive interference requires a large number of interfering processes and small decoherence. We are currently uncertain about the values of these parameters for the experimental system. The mechanism presented here raises the question of whether an independent study of the strength of coherence effects can be performed. Clearly, if the coherent MM source is replaced by an incoherent source at the same frequency the coherence effects will be destroyed. As a simpler experiment we suggest that the importance of coherence for the observed effect can be studied by use of two independent coupling fields. If one of these is tuned to one of the peak values (say $2\omega_c$) and the other to a suppressed value (say, around $1/2\omega_c$), we predict that the effect of the second source will be to suppress the peak of the first source, even to destroy the peak completely if the coherent effect is sufficiently strong.

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