Cepheids in open clusters: an 8D all-sky census

Richard I. Anderson,† Laurent Eyer and Nami Mowlavi
Observatoire de Genève, Université de Genève, 51 Ch. des Maillettes, CH-1290 Versoix, Switzerland

ABSTRACT
Cepheids in Galactic open clusters (CCs) are of great importance as zero-point calibrators of the Galactic Cepheid period–luminosity relationship (PLR).

We perform an eight-dimensional all-sky census that aims to identify new bona fide CCs and provides a ranking of membership confidence for known CC candidates according to membership probabilities. The probabilities are computed for combinations of known Galactic open clusters and classical Cepheid candidates, based on spatial, kinematic and population-specific membership constraints. Data employed in this analysis are taken largely from published literature and supplemented by a year-round observing programme on both hemispheres dedicated to determining systemic radial velocities of Cepheids.

In total, we find 23 bona fide CCs, 5 of which are candidates identified for the first time, including an overtone-Cepheid member in NGC 129. We discuss a subset of CC candidates in detail, some of which have been previously mentioned in the literature. Our results indicate unlikely membership for seven Cepheids that have been previously discussed in terms of cluster membership.

We furthermore revisit the Galactic PLR using our bona fide CC sample and obtain a result consistent with the recent calibration by Turner. However, our calibration remains limited mainly by cluster uncertainties and the small number of long-period calibrators.

In the near future, Gaia will enable our study to be carried out in much greater detail and accuracy, thanks to data homogeneity and greater levels of completeness.

Key words: methods: data analysis – astronomical data bases: miscellaneous – catalogues – stars: variables: Cepheids – open clusters and associations: general – distance scale.

1 INTRODUCTION
The search for Cepheids in Galactic open clusters (CCs) has been a topic of interest in astronomy for the past 60 years, owing largely to their importance as calibrators of the Cepheid period–luminosity relation (PLR), discovered a century ago among 25 periodic variable stars in the Small Magellanic Cloud by Leavitt & Pickering (1912).

The proportionality between the logarithm of Cepheid pulsation periods and their absolute magnitudes, i.e. their (logarithmic) luminosities, gives access to distance determinations and has established PLRs as cornerstones of the astronomical distance scale (e.g. Freedman et al. 2001; Sandage et al. 2006). For reviews on Cepheids as distance indicators, see Feast (1999) and Sandage & Tammann (2006), for instance.

The existence of the Cepheid PLR is most obvious among Cepheids in the Magellanic Clouds (e.g. Udalski et al. 1999; Soszynski et al. 2008, 2010), due to common distances (small dispersion), large statistics (thousands) and relative proximity (detectability). However, knowledge of the zero-point(s) of such relations is also required; in this case, the distances to the Magellanic Clouds. For such zero-point calibrations, PLR-independent distance estimates are required, e.g. from trigonometric parallaxes (Feast & Catchpole 1997; Benedict et al. 2007), Baade–Wesselink-type methods (Gieren, Fouqué & Gomez 1997; Storm et al. 2011), or objects located at comparable distance, e.g. water masers (Macri et al. 2006) or open clusters (Turner et al. 2010).

For open clusters, distances can be determined via zero-age main sequence (ZAMS) or isochrone fitting. If membership can be

* Based on observations collected at the European Southern Observatory La Silla Observatory with the CORALIE echelle spectrograph mounted to the Swiss 1.2 m Euler telescope.
† Based on observations obtained with the HERMES spectrograph, which is supported by the Fund for Scientific Research of Flanders (FWO), Belgium, the Research Council of K.U. Leuven, Belgium, the Fonds National Recherches Scientifiques (FNRS), Belgium, the Royal Observatory of Belgium, the Observatoire de Genève, Switzerland and the Thüringer Landessternwarte Tautenburg, Germany. HERMES is mounted to the Mercator Telescope, operated on the island of La Palma by the Flemish Community, at the Spanish Observatorio del Roque de los Muchachos of the Instituto de Astrofísica de Canarias.
‡ E-mail: richard.anderson@unige.ch
assumed at high confidence, the cluster provides the independent estimation of the Cepheid’s distance. Confidence in cluster membership is thus critical for such calibrations.

Since the first discovery of CCs by Irwin (1955, identified S Nor in NGC 6087 and U Sgr in M 25) and Feast (1957, established membership via radial velocities, RVs), many researchers have contributed to this field (e.g. Kholopov 1956; van den Bergh 1957; Efremov 1964; Tsarevsky, Ureche & Efremov 1966; Turner 1986; Turner et al. 1993; Baumgardt, Dettbarn & Wielen 2000; Hoyle, Shanks & Tanvir 2003; An, Ternrup & Pinsoneault 2007; Majaess, Turner & Lane 2008; Turner 2010). Nevertheless, relatively few bona fide CCs (<30) have thus far been discovered.

We therefore carry out an all-sky census of classical Cepheids in Galactic open clusters that aims to increase the number of bona fide CCs and allows us to rank confidence in membership according to membership probabilities. Our approach is eight dimensional in the sense that three spatial, three kinematic and two population parameters (iron abundance and age) are used as membership constraints. Both data inhomogeneity and incompleteness are critical limitations to this work, and are acknowledged in the relevant sections. We describe our analysis in Section 2.

For the first time, we systematically search for cluster members among Cepheid candidates from surveys such as the All Sky Automated Survey (ASAS; e.g. Pojmanski et al. 2005), the Northern Sky Variability Survey (NSVS; Wozniak et al. 2004), the ROTSE All Sky Survey for Variable Stars (Akerlof et al. 2000), and also from the suspected variables in the General Catalogue of Variable Stars. Most data employed to do so are taken from published catalogues or other literature. However, we also perform RV observations of Cepheids on both hemispheres and determine systemic velocities, v_R. To improve sensitivity to binarity, the literature RVs are added to the new observations. The data compilation is described in Section 3.1.

The results of our census are presented in Section 4, starting with cluster-Cepheid combinations (Combos) that were previously studied with respect to membership, see Table 1 in Section 4.1, and followed by Combos highlighted by our work, see Table 2 in Section 4.2. The full table containing all Combos investigated in this work is provided in digital form in the online appendix and via the CDS. Combos that deserve observational follow-up are identified in the text. Particular attention is given to Combos previously discussed in the literature. Discussions of additional Combos can be found in the online appendix. In Section 4.3, we employ our bona fide CC sample in a calibration of the Galactic Cepheid PLR. The method and results are discussed in Section 5, which is followed by the conclusion in Section 6.

2 MEMBERSHIP ANALYSIS

Our all-sky census is structured as shown in Fig. 1. First, lists of known open clusters and known Cepheid candidates are compiled, see Section 3.1 for details. Secondly, the two lists are cross-matched positionally in a many-to-many relationship so that we investigate a given Cepheid’s membership in multiple different open clusters, and a given open cluster can potentially host multiple Cepheids. The correct classification of cross-matched Cepheid candidates is verified by considering light curves and spectra. Misclassified objects are removed from the Cepheids sample. Thirdly, membership probabilities are calculated based on all available membership constraints. These last two points are described in the present section.

Membership probabilities are calculated following Bayes’ theorem that can be formulated as (Jaynes 2003, Section 4):

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)} \propto P(B|A) \times P(A).$$

The posterior probability P(A|B) (membership probability) is proportional to the product of likelihood, P(B|A), and prior, P(A). P(B|A) represents the conditional probability of observing the data under the hypothesis of membership, and P(A) quantifies the degree of initial belief in membership. The normalization term P(B), of which we possess no knowledge, is the probability to observe the data. We define P(A) in equations (3) and (4) and P(B|A) in equation (7).

2.1 Prior estimation and positional cross-match

2.1.1 Positional cross-match

On-sky proximity is a necessary, but insufficient criterion for membership. Intuitively, if no other information is available, one might tentatively assume membership for a Cepheid that falls within the core radius of a potential host cluster.

Therefore, our census starts with a positional cross-match that aims to identify all combinations of cluster-Cepheid pairs that lie sufficiently close on the sky to warrant a membership probability

\[ P(A) \times P(B|A) \sim P(A|B) \]

---

1 http://cds.u-strasbg.fr
calculation (Combos). The cross-match itself is straightforward: if the separation between a cluster’s centre coordinates and the Cepheid’s coordinates is smaller than 2.5 (to avoid unnecessary contamination), and less or equal to 5 limiting cluster radii,2 we include the Combo in our analysis. Using this proximity criterion, we cross-match 990 different open clusters (of 2168 in Dias et al. 2002a) with 1021 Cepheids (of 1821 initially compiled) and obtain 3974 Combos that we investigate for membership.

The initial cross-match is purely positional, and the majority of Combos studied are non-members. Our analysis intends to weed out this majority and to indicate to us the good candidates through a high membership probability.

2.1.2 The prior

We define the prior, \( P(A) \), using the on-sky separation\(^3 \) between cluster centre and Cepheid, weighted by the cluster radius, i.e. its apparent size on the sky.

The radius of an open cluster is typically determined by fitting an exponential radial density profile to a stellar overdensity on the sky, an approach originally developed for globular clusters by King (1962). The method relies on the assumption that two separate distributions are seen: a constant field distribution and one that is attributed to the cluster.

Various ways to define cluster radii can be found in the literature. Among these are the ‘core radius’ (most stars belong to cluster), \( r_c \), and the ‘limiting radius’ for the cluster halo (strong field star contamination), \( r_{\text{lim}} \), see Kharchenko et al. (2005a,b) and Bukowiecki et al. (2011).

Intuitively, the probability of membership is related to separation and cluster radius (cf. Sánchez, Vicente & Alfaro 2010). Let us therefore define the quantity \( x \) as

\[
x = \frac{r - r_c}{2r_{\text{lim}} - r_c},
\]

where \( r \) denotes separation, \( x \) is negative, if the Cepheid lies within the cluster’s (projected) core and becomes unity at a separation equal to twice the limiting radius. We define our prior, \( P(A) \), so that (no other constraints considered) membership is assumed when the Cepheid lies within the cluster’s core, i.e. \( x < 0 \). Outside \( r_c \), inspired by radial density profiles of star clusters, we let the prior fall off exponentially and define it to reach 0.1 percent = \( 10^{-3} \) at \( x = 1 \). Hence,

\[
P(A)(x < 0) \equiv 1
\]

\[
P(A)(x \geq 0) = 10^{-3x}.
\]

Fig. 2 serves to illustrate this definition. The prior thus carries the two-dimensional information of separation and cluster radius, and thereby takes into account how concentrated a cluster is on the sky assuming circularly distributed member stars.

2.2 The likelihood \( P(B|A) \)

The likelihood, \( P(B|A) \), is computed as a hypothesis test. It estimates the probability that the observed data is consistent with the null hypothesis of (true) membership. This approach was inspired by the Hipparcos astrometry-based studies by Robin et al. (1999) and Baumgardt et al. (2000). We extend it here to take into account up to six dimensions using parallax, \( \sigma \), RV, proper motion, \( \mu_\alpha^* \) and \( \mu_\delta \), iron abundance, [Fe/H], and age (open clusters assumed to be co-eval), weighting all constraints equally.

Assuming that a given Cepheid was not used to determine a cluster’s (mean) parameters, we can calculate the quantity

\[
c = x^T \Sigma^{-1} x,
\]

where \( x \) denotes the vector containing as elements the differences between the (mean) cluster and Cepheid quantities:

\[
x = (\mu_{\alpha,\text{Cl}} - \mu_{\alpha,\text{Cep}}; \mu_{\delta,\text{Cl}} - \mu_{\delta,\text{Cep}}; \ldots).
\]

Let \( \Sigma_{\text{Cl}} \) be the covariance matrix of the cluster and \( \Sigma_{\text{Cep}} \) that of the Cepheid. Let \( \Sigma \) then denote the sum of the two and \( \Sigma^{-1} \) its inverse.

Since the data employed in this calculation come from many different sources, no knowledge of correlations between the different parameters is available. We thus make the assumption of independent measurements, which results in diagonal covariance matrices containing only parameter variances. Possible correlations between Cepheid and Cluster parameters are thus assumed to be negligible.

We consider this justified, since we possess no knowledge of the extent of such correlations and assume that Cepheids were not used in the determination of cluster mean values. This formulation furthermore implicitly assumes normally (Gaussian) distributed errors.

Under these assumptions \( c \) is \( \chi^2 \) distributed, i.e. \( c \sim \chi^2_{N_{\text{ dof}}} \), where \( N_{\text{ dof}} \) is the number of degrees of freedom equal to the length of vector \( x \), ranging from 1 to 6. \( c \) thus depends on the number of membership constraints considered (the on-sky position is used in the prior). In cases where no membership constraints are available, i.e. \( N_{\text{ dof}} = 0 \), we set \( P(B|A) \equiv 1 \).

\( P(B|A) \) is obtained by calculating unity minus the \( p \)-value of \( c \), \( p(c) \):\n
\[
P(B|A) = 1 - p(c).
\]

Since the \( \chi^2 \) distribution (and therefore the \( p \)-value computed) is very sensitive to \( N_{\text{ dof}} \) for small \( N_{\text{ dof}} \), \( P(B|A) \) naturally contains information on the number of membership constraints employed.

Of course, we cannot prove the null hypothesis, only exclude it. However, by including the greatest number of the most stringent membership constraints possible, this method very effectively...
filters out non-members. The remaining candidates can therefore be considered bona fide members, provided the constraints taken into account are sufficiently strong.

The filtering effectiveness of the likelihood strongly depends on the uncertainties adopted for the constraining quantities: the larger the error, the weaker the constraint. Conversely, the smaller the error, the more important become systematic differences between quantities measured or inferred through different techniques. Obtaining reasonable estimates of the external uncertainties is of paramount importance to the success of this work, since the data considered are inhomogeneous and listed uncertainties typically provide formal errors or estimates of precision.

For certain quantities, we therefore adopt increased error budgets that we motivate and detail in the following sections. Care is taken to avoid too large or too small error budgets, and to ensure that likelihood remains an effective membership criterion.

3 DATA USED TO COMPUTE LIKELIHOODS

In this section, we describe how we compile the data used for our analysis. The constraints employed are: on-sky separation, parallax, proper motion, RV, and the population parameters iron abundance (as a proxy for metallicity) and age. Most data considered originate from published literature and catalogues. However, we also include RV data from an extensive, year-round observation programme carried out on both hemispheres. Some details on this programme are provided in Section 3.1.2. Further choices made regarding cluster data are presented in the subsections concerning parallax (Section 3.1.2), proper motion (Section 3.1.3), mean RV (Section 3.1.4), iron abundance (Section 3.1.5) and age (Section 3.1.6).

Fig. 3 shows the distribution of clusters (black open circles, scale with limiting radius) and Cepheids (light star symbols) in Galactic coordinates. Clusters closely trace the disc, and no obvious gaps are present in our all-sky census.

3.1.1 Cluster radii

In order to choose which literature radii to adopt, we start by investigating to what degree cluster radii are reliable quantities. To this end, we search the literature for extensive catalogues that provide both core and limiting radii. Three such studies are identified: Kharchenko et al. (2005a,b, from hereon, we refer to the combined catalogue from both studies as K05), Bukowiecki et al. (2011, from hereon: B11) and Kharchenko et al. (2012, from hereon: K12).

We do not include Froebrich, Scholz & Raftery (2007) here, since we notice a suspicious correlation between $r_c$ and $r_{lim}$. Note, however, that some clusters listed in K12 were originally identified by Froebrich et al. (2007).

Since many clusters in K12 were also studied by K05, we compare the three radii defined in K12 with the core and limiting radii in K05 and notice that the limiting radius in K05, $R_{\text{cl}}$, corresponds well to $r_2$ in K12 (though $r_2$ tends to be smaller), while $r_1$ in K12 is rather similar to the core radius in K05, $R_{\text{co}}$. Nevertheless, a fair amount

4 Version V3.3, 2013 January 16.

5 Maintained by E. Paunzen and C. Stütz in Vienna, cf. http://www.univie.ac.at/webda/
of scatter between both studies, see Fig. 4. We consider K12 an update (and extension) of K05 and therefore prefer the newer cluster parameters over the older ones.

We previously compared radii given in K05 and B11 for the clusters common to both works in Anderson, Eyer & Mowlavi (2012). Rather large scatter is present (more than a factor of 2 for an appreciable fraction) and illustrates that cluster radii are subject to significant uncertainty. However, the radii from both studies follow the same trend and we therefore consider them comparable for our purpose, although K05 and K12 are based on optical and B11 on near-infrared (NIR) Two Micron All-Sky Survey (2MASS; Cutri et al. 2003) photometry.

Given the sometimes rather large difference between cluster radii mentioned in the literature, we adopt a 'permissive' scheme that gives preference to the study giving the largest limiting radius for the cluster and thereby bias ourselves towards higher cluster parameters. For example, in K12, the typical mean proper motion error is smaller than the uncertainty on an individual cluster star’s measurement. This is done by multiplying the uncertainty of an individual clust

Figure 5. Radii based on apparent diameters from D02 compared to the core (left-hand panel) and limiting radii (right-hand panel) compiled from K12, K05 and B11, see the text. Median ratios printed on the graph.

Figure 4. Left-hand panel: comparison between $r_1$ in K12 and the core radius, $R_{co}$, in K05; right-hand panel: same for $r_2$ in K12 and the limiting radius, $R_{lim}$, in K05. The radii are comparable.

3.1.3 Mean proper motion

Mean cluster proper motions, $\bar{\mu}_{\alpha,cl}$ and $\bar{\mu}_{\delta,cl}$, are provided in D02. The uncertainties on mean proper motion listed in these references are typically calculated either as intrinsic dispersions (e.g. for clusters closer than approximately 400 pc originally studied in K12), or as standard mean errors, i.e. the error decreases as $\sqrt{N_{cl} - 1}$, where $N_{cl}$ is the number of stars considered members, cf. D02. The quoted uncertainties on the cluster mean are thus much smaller than the uncertainty on an individual cluster star’s measurement. For example, in K12, the typical mean proper motion error is 0.4 mas yr$^{-1}$.

For the majority of Cepheids, however, the uncertainties on proper motion are much larger, and many have been obtained from different data sets, using different techniques. Therefore, to ensure comparability of inhomogeneous data and to reduce our sensitivity to offsets in zero-points due to data-related specificities such as reduction techniques, we adopt a more generous error budget for $\bar{\mu}_{\alpha,cl}$ and $\bar{\mu}_{\delta,cl}$ that resembles the uncertainty of an individual cluster star’s proper motion. This is done by multiplying the uncertainty listed by the factor $\sqrt{N_{cl} - 1}$ and thus slightly reduces the weight of proper motion as a membership constraint. Empirically, we are confident that this is justified, since proper motions of Cepheids typically barely exceed their uncertainties, and care should be taken not to overinterpret their accuracy.

3.1.4 Mean radial velocity

Average cluster RVs and associated errors are listed in D02. However, qualitative differences can exist in the uncertainties listed. For some well-studied clusters, the uncertainty given is an estimate of the intrinsic RV dispersion. For the majority of cluster RVs, however, only a few stars were used to determine the mean cluster RV (about half on two stars or less, cf. Fig. 6). These cases are therefore subject to systematic uncertainties due to implicit membership assumptions, for instance. In addition, unseen binary companions and instrumental zero-point offsets can introduce systematic uncertainties at the level of a few km s$^{-1}$.

We therefore adopt 2 km s$^{-1}$ as a minimum uncertainty of the mean cluster velocity. If no uncertainty estimate is given, we adopt $\sigma(RV_{Cl}) = 10$ km s$^{-1}/\sqrt{N_{RV}}$ as a typical uncertainty on the mean cluster RV, where $N_{RV}$ is the number of stars used to determine the mean cluster RV.

6 See under ‘version 2.3 (25/abr/2005)’ in file: http://www.astro.iag.usp.br/~wilton/whatsnew.txt
3.15 Iron abundance

We adopt iron abundances compiled in D02, including the uncertainties given. The mean uncertainty among the clusters compiled is 0.08 dex.

3.1.6 Cluster age

Ages were available for most clusters, since they are often determined simultaneously with the distance via isochrone fitting. Although for a model-dependent parameter, age does provide a valid constraint for membership, reflecting evolutionary considerations that are empirically validated. Quantifying an uncertainty for age as a parameter, however, is rather difficult.

Younger clusters exhibit a main-sequence turn-off at higher stellar masses than older clusters. As a consequence of the initial mass function (IMF), a younger cluster’s turn-off point tends to be less populated than that of an older cluster. It therefore follows that age estimates tend to become more accurate with age, since the cluster’s turn-off point tends to be defined more clearly against the field and therefore better constrains an isochrone fit.

Fig. 7 corroborates the above reasoning by showing cluster ages against their uncertainties as given in K12. We thus estimate an upper limit on the uncertainty of cluster age as the dashed line in Fig. 7, which is

$$\sigma(\log a_{\text{Cl}}) \leq 0.3 - 0.067(\log a_{\text{Cl}} - 7.0).$$

(8)

3.2 Cepheid data

Cepheid candidates were compiled from the 2012 January version of the General Catalogue of Variable Stars (from hereon: GCVS; Samus et al. 2012) and the 2012 May version of the AAVSO Variable Star Index (VSX).\(^7\) From GCVS and VSX, we import the variability types CEP, CEP(B), DCEP, DCEPS; from VSX, we include the ASAS (Pojmanski 1997; Pojmanski 2002; Pojmanski, Pilecki & Szczygiel 2005) Cepheid candidates classified as DCEP-FU or DCEP-FO. This list also contains Cepheid candidates found by ROTSE (Akerlof et al. 2000) or NSVS (Woźniak et al. 2004), as well as the ones in the suspected variables catalogue (Kukarkin & Kholopov 1982).

This starting point contains an unknown, but probably high, fraction of non-Cepheids. Type-II Cepheids (halo objects) and Cepheids belonging to the Magellanic Clouds are mostly removed from the sample by cross-matching with the clusters (trace the disc, see Fig. 3). To further reduce contamination, we visually inspect all ASAS-3 V-band light curves of Cepheid candidates with ASAS identifiers.

Radial pulsation and colour variations during the pulsation are defining characteristics of Cepheids. We thus use the spectra obtained for RV observations described in Section 3.2.3.1 to verify classification. A total of 151 ASAS Cepheid candidates and 32 others are thus rejected from the Cepheid sample, resulting in a final list of 1821 Cepheid candidates, 1021 of which are cross-matched with open clusters.

The cleaned sample of Cepheids cross-matched with clusters was appended with literature data from many sources, and references are given in the text. Among the most relevant references are:

(i) The Fernie et al. (1995) DDO Cepheid data base.\(^8\)
(ii) The Klagiyivik & Szabados (2009, KS09) Cepheid data base.
(iii) The ASAS Catalogue of Variable Stars (Pojmanski et al. 2005, ACVS) and associated photometry.
(iv) The new Hipparcos reduction (van Leeuwen 2007).
(v) The extended Hipparcos compilation (Anderson & Francis 2012, XHIP).
(vi) The ASCC-2.5 catalogue (Kharchenko 2001) updated by Kharchenko et al. (2007).
(vii) The PPMXL catalogue (Roeser, Demleitner & Schilbach 2010).
(viii) The 2MASS catalogue (Cutri et al. 2003).
(ix) The Cepheid photometry obtained by Berdnikov, Dambis & Vozyakova (2000) and Berdnikov (2008).
(x) The McMaster Cepheid photometry and RV data archive maintained by Doug Welch.\(^9\)
(xi) The RV data, see Section 3.2.3.

3.2.1 Cepheid parallaxes

Parallax, $\sigma$, is a key membership constraint, since cluster membership is virtually guaranteed if a Cepheid occupies the same space volume as a cluster. We combine parallax estimations from different sources, favouring PLR-independent determinations.

Parallax in (mas) is given preference over distance in (pc) here, since the uncertainty, $\sigma_{\pi}$, is normally distributed, in contrast to the error in distance. This is important, since the computation of likelihoods by equation (7) assumes Gaussian uncertainties.

We compile parallaxes from Benedict et al. (2007, 8 Cepheids), Storm et al. (2011, 65 Cepheids), and the new Hipparcos reduction by van Leeuwen (2007, so long as $\sigma_{\pi}/\sigma \leq 0.1$ and $\sigma > 0$, 5 Cepheids). We then calculate PLR-based parallaxes for 622 additional Cepheids, see below.

PLR-based parallaxes of fundamental-mode Cepheids are calculated from distances computed following Turner et al. (2010). Our choice of PLR was motivated mainly by the considerations that (i) V-band magnitudes can be obtained for the largest number of Cepheids; (ii) the above formulation is calibrated for the Galaxy using the most recent observational results, including the Hubble

\(^7\) http://www.aavso.org/vsx/

\(^8\) http://www.astro.utoronto.ca/DDO/research/Cepheids/

\(^9\) http://crocus.physics.mcmaster.ca/Cepheid/
First, we adopt \( (m_V) \) values from KS09 with a fixed error budget of 0.03 mag, since the study carefully investigates amplitudes with a special focus on binarity.

Secondly, we adopt the magnitude-based means from the Fernie data base with uncertainties calculated as the difference between intensity and magnitude mean magnitudes, with a minimum error of 0.03 mag.

Thirdly, we include Berdnikov et al. (2000) mean magnitudes. As seen in Fig. 8, these \( (m_V) \) values are systematically smaller (brighter) by approximately 0.03 mag than KS09. This discrepancy is most likely due to different ways of determining the mean. We remove this offset from the Berdnikov et al. (2000) values for internal consistency and adopt 0.03 mag as error budget for these values, identical to KS09.

Fourthly, we employ median \( V \)-band magnitudes from the Hipparcos catalogue (Perryman & ESA 1997, obtained via XHIP) for eight Cepheids. The median \( V \)-band magnitudes derived from Hipparcos magnitudes can differ significantly from mean magnitudes listed in other references, cf. Fig. 9. Usually, this is due to contamination due to a nearby star within the instantaneous field of view. As error budget for the Hipparcos median \( V \) magnitudes, we adopt \( \sigma (\Delta_{\text{HIP,KS09}}) = 0.110 \text{ mag} \), see Fig. 9. We note that we could find no dependence on period or number of transits for this dispersion.

Fifthly, we adopt average apparent \( V \)-band magnitudes that we determine from ASAS-3 light curves. To this end, we fit Fourier series (same procedure as described for RVs in Section 3.2.3.3) to the phased light curves and use the constant term as the average, \( (m_v) \). Fig. 10 shows a histogram of \( \Delta_{\text{ASAS}} \), the differences between the computed ASAS-based \( (m_V) \) and Fernie or KS09. We remove the offset of \(-0.01 \text{ mag} \) from the ASAS mean magnitudes and adopt the dispersion of 0.10 mag computed as the error budget.

The large dispersion, \( \sigma (\Delta_{\text{ASAS}}) \), in Fig. 10 probably originates from contamination due to nearby stars. To illustrate this, Fig. 11 shows phase-folded ASAS-3 \( V \)-band light curves of two Cepheids, CY Car (left) and BM Pup (right). Our mean magnitude agrees well with the literature value for CY Car. For BM Pup, however, a systematic difference of approximately 0.144 mag is evident, although the light curve appears to be clean otherwise. Inspection of a Digitized Sky Survey image, however, reveals that contamination from a nearby companion is likely. Out of 154 Cepheids for which the ASAS light \( V \)-band curves were inspected, 20 differed by more than 0.1 mag from the reference value and 28 agreed to within 0.01 mag.

\[
\log d = (m_V) - (M_V) - A_V + 5,
\]

where \( (m_V) \) is the apparent mean \( V \)-band magnitude and the average absolute \( V \)-band magnitude, \( \langle M_V \rangle \), is obtained from the pulsation period \( P \) via

\[
\langle M_V \rangle = -(1.304 \pm 0.065) - (2.786 \pm 0.075) \log P.
\]

Equation (10) is valid only for fundamental-mode pulsators, no dispersion of \( \sigma \) dependence on period or number of transits for this dispersion.

\[
\sigma = \frac{1000}{d^2} \text{ (mas),}
\]

with \( d \) in (pc). The parallax uncertainty, \( \sigma_d \), is obtained considering the error budget on the distance, \( \sigma_P \):

\[
\sigma_d = \frac{1000}{d^2} \cdot \sigma_P \text{ (mas).}
\]

Thus, to estimate a Cepheid’s parallax, knowledge of the PLR, \( P \), \( (m_V) \) and \( A_V \) is required. Periods are usually available in the GCVS (\( V \)). Usually, this is due to contamination due to a nearby star within the instantaneous field of view. As error budget for the Hipparcos median \( V \) magnitudes, we adopt \( \sigma (\Delta_{\text{HIP,KS09}}) = 0.110 \text{ mag} \), see Fig. 9. We note that we could find no dependence on period or number of transits for this dispersion.

Fifthly, we adopt average apparent \( V \)-band magnitudes that we determine from ASAS-3 light curves. To this end, we fit Fourier series (same procedure as described for RVs in Section 3.2.3.3) to the phased light curves and use the constant term as the average, \( (m_v) \). Fig. 10 shows a histogram of \( \Delta_{\text{ASAS}} \), the differences between the computed ASAS-based \( (m_V) \) and Fernie or KS09. We remove the offset of \(-0.01 \text{ mag} \) from the ASAS mean magnitudes and adopt the dispersion of 0.10 mag computed as the error budget.

The large dispersion, \( \sigma (\Delta_{\text{ASAS}}) \), in Fig. 10 probably originates from contamination due to nearby stars. To illustrate this, Fig. 11 shows phase-folded ASAS-3 \( V \)-band light curves of two Cepheids, CY Car (left) and BM Pup (right). Our mean magnitude agrees well with the literature value for CY Car. For BM Pup, however, a systematic difference of approximately 0.144 mag is evident, although the light curve appears to be clean otherwise. Inspection of a Digitized Sky Survey image, however, reveals that contamination from a nearby companion is likely. Out of 154 Cepheids for which the ASAS light \( V \)-band curves were inspected, 20 differed by more than 0.1 mag from the reference value and 28 agreed to within 0.01 mag.
If no mean magnitude is obtained from any of the above sources, we perform a (rough) estimate of \( \langle m_V \rangle \) based on the information provided in the GCVS and the VSX, using the magnitude at maximum brightness, \( \min_V \), and the amplitude, \( \amp_V \), of the V-band light curve. \( \amp_V \) is either provided directly by the catalogues, or calculated as the difference between minimum and maximum brightness, \( \amp_V = \max_V - \min_V \).

Since Cepheid light curves are skewed, their mean magnitudes do not necessarily lie at half the amplitude. We therefore estimate the typical fractional amplitude at mean brightness, \( \langle f_a \rangle \), to compute \( \langle m_V \rangle = \min_V + \langle f_a \rangle \amp_V \). Fig. 12 shows a histogram of \( f_a \) computed using mean magnitudes listed in the Fernie data base, \( \langle m_V,J \rangle \), and amplitudes, \( \amp_V \), from the catalogues. We find

\[
\langle f_a \rangle = \text{median} \left( \frac{\langle m_V,J \rangle - \max_V}{\min_V - \max_V} \right) = 0.54 \pm 0.079.
\] (14)

We derive an uncertainty on \( \langle m_V \rangle \) thus obtained using the uncertainty on \( \langle f_a \rangle \), an estimated error on the amplitude, and a prescribed error on the magnitude at maximum brightness (we adopt 0.1 mag for 12th magnitude and below and increase linearly to 0.5 mag at 20th magnitude). The resulting mean error on \( \langle m_V \rangle \) is 0.27 mag.

This estimation works reasonably well, although there exist obvious limitations, such as inhomogeneity of passbands, accuracy of the upper and lower limits, the applicability of the above ratio for a given Cepheid. Nevertheless, it does provide access to rough estimates of \( \langle m_V \rangle \) for Cepheids with little available information.
Wherever possible, $m_V(\phi_J)$ was obtained from the ASAS light curve. If this is impossible, we assume a sinusoidal light curve with the given mean magnitude and (semi-)amplitude. Uncertainties or changes in pulsation period can significantly impact the phase calculated for the single-epoch 2MASS measurement, $\phi_J$. We therefore optimize Cepheid ephemerides for which ASAS data were available. To do so, we compute a grid (at fixed periods) of Fourier series fits around the period provided in the ACVS and retain the solution with the minimum root mean square (rms). Epochs are optimized by simply shifting the phase-folded curve. Fig. 13 illustrates this step for the overtone Cepheid QZ Nor. We then take care to employ the most recently determined pulsation ephemerides available and estimate reddening uncertainties using error propagation for the quantities involved.

We note that this approach may be subject to multiple issues such as: (i) the unknown shape of the light curve; (ii) the applicability of equation (16); (iii) period changes that impact $m_V(\phi_J)$; and (iii) the approximate form of the relationship in equation (15). We therefore compare the 2MASS-based colour excesses to the reference values, see Fig. 14, where the result of a weighted least-squares fit is indicated by a straight line and does not differ much from the diagonal indicated by a dashed line. Despite considerable dispersion, the correspondence is clear and the results are promising (rms of 0.1 mag).

3.2.2 Proper motions

Cepheid proper motions are taken from the following sources in order of preference:

(i) Hipparcos proper motions from the new reduction by van Leeuwen (2007) and

(ii) The PPMXL catalogue by Roeser et al. (2010).

3.2.3 Systemic radial velocities

3.2.3.1 New observations. In order to extend the number of Cepheids with known systemic RVs, $v_\gamma$, we carried out observations between 2010 November and 2012 July using the fibre-fed high-resolution echelle spectrographs CORALIE (Queloz et al. 2001, see also the instrumental upgrades described in Ségransan et al. 2010, $R \sim 60000$) at the 1.2 m Euler telescope at La Silla, Chile and HERMES (Raskin et al. 2011, $R \sim 80000$) at the identically built Mercator telescope on La Palma. In total, we observed 103 Cepheids with CORALIE and 63 with HERMES. 18 Cepheids were observed with both instruments, i.e. from both hemispheres. For 85 of these Cepheids, no RV data are available in the literature.

Efficient reduction pipelines exist for both instruments that include pre- and overscan bias correction, cosmic removal, as well as flat-fielding using Halogen lamps and background modelization. ThAr lamps are used for the wavelength calibration.

The RVs are computed via the cross-correlation technique described in Baranne et al. (1996). We use numerical masks designed for solar-like stars (optimized for spectral type G2) for all cross-correlations. Both instruments are very stable and yield very high precision RVs of $\sim 10$ m s$^{-1}$ (Queloz et al. 2000; Raskin et al. 2011). The measurement uncertainty is therefore not limited by the instrumental precision, but by line asymmetries due to pulsation. A detailed investigation of these effects is out of scope for this paper and will be presented in a future publication. The typical uncertainty on individual measurements is thus at the 100–300 m s$^{-1}$ level, depending on the star and pulsation phase.

The authors are grateful to the observers who contributed in this effort; names are given in the acknowledgments.
3.2.3.2. Literature data. In addition to the previously unpublished RVs described in Section 3.2.3.1, we employ literature data from many references to determine systemic velocities, \( v_\gamma \), see Section 3.2.3.3. The addition of the literature RVs extends the baseline of our otherwise relatively short (1.5 yr) observing programme, thereby enhancing our sensitivity to binarity. For binary Cepheids\(^{11}\) with published orbital solutions, we adopt the literature \( v_\gamma \).

Aside from the systemic RVs in the Fernie data base and KS09, we compile RV time series from the following sources: Lloyd Evans (1980), Gieren (1985), Evans (1983), Coulson, Caldwell & Gieren (1985), Coulson & Caldwell (1985), Barnes, Moffett & Slovak (1987, 1988), Gieren et al. (1989), Wilson et al. (1989), Metzger et al. (1991), Metzger, Caldwell & Schechter (1992), Metzger, Caldwell & Schechter (1998), Gorynya et al. (1992), Gorynya et al. (1996), Gorynya et al. (1998), Gorynya et al. (2002), Evans & Welch (1993), Pont, Burki & Mayor (1994), Pont et al. (1997), Bersier et al. (1994), Bersier (1998), Imbert (1999), Storm et al. (2004), Barnes et al. (2005), Petterson et al. (2005) and Baranowski et al. (2009). The data for most of these sources dated earlier than 1986 are extracted from the McMaster Cepheids data base. Newer data are obtained through VizieR.\(^{12}\)

3.2.3.3 Systemic radial velocities, \( v_\gamma \). The systemic RV, \( v_\gamma \), is obtained by fitting a Fourier series to the RVs. We use pulsation period, \( P \), as a fixed parameter, since it is known for all Cepheids we observed.

The basic analytical form applied was a Fourier series with \( n \) harmonics and phase \( \phi \) is

\[
FS_n = v_\gamma + \sum_{n=1,2,\ldots} a_n \sin \left(2n\pi\phi\right) + b_n \cos \left(2n\pi\phi\right). \tag{17}
\]

Since the number of data points available varies for each star, we do not fix the number of harmonics in this fit. Instead, we iteratively increase the degree of the Fourier series until an \( F \)-test indicates an overly complex representation, i.e. when spurious fit improvement is more likely than 0.27 per cent. For some stars, we therefore use only a simple sine function, whereas stars with many measurements are fitted using up to five harmonics. We show two examples of newly observed RV curves in Figs 15 and 16. A full description and publication of the new RV data will follow in the near future.

We adopt a fixed error budget of 3 km s\(^{-1}\) on \( v_\gamma \). Although this may overestimate the uncertainties for some very good cases, it is intended to account for a range of systematic errors, such as unseen binarity, instrumental zero-point differences and insufficient phase coverage. We are confident that this error budget is sufficiently large to prevent the exclusion of good member candidates, while being sufficiently small to provide a stringent constraint. If insufficient data points render the Fourier fit unsatisfactory, we determine a rough estimate of \( v_\gamma \) and its error budget by eye.

---

\(^{11}\)cf. Szabados’ data base of binary Cepheids available at [http://www.konkoly.hu/CEP/nagytab3.html](http://www.konkoly.hu/CEP/nagytab3.html)

\(^{12}\)[http://vizier.u-strasbg.fr/viz-bin/VizieR](http://vizier.u-strasbg.fr/viz-bin/VizieR)
3.2.4 Iron abundance

We rely mostly on iron abundances by Luck & Lambert (2011) and complement these with the compilation in KS09 that made the enormous effort of homogenizing iron abundances from the literature available, namely from Giridhar (1983), Fry & Carney (1997), Andrievsky et al. (2002a,b,c, 2004), Luck et al. (2003), Groenewegen et al. (2004), Andrievsky, Luck & Kovtyukh (2005), Kovtyukh, Wallerstein & Andrievsky (2005a), Kovtyukh et al. (2005b), Romaniello et al. (2005), Mottini (2006), Yong et al. (2006), Lemasle et al. (2007), as well as the work by Sziládi et al. (2007), Lemasle et al. (2008) and Romaniello et al. (2008).

Unfortunately, the adopted standard value of solar iron abundance can vary among references, possibly introducing systematic offsets between different authors’ studies. Furthermore, an estimation of the iron abundance in a Cepheid is more complex than in a non-pulsating star, since the stellar parameters (e.g. temperature and turbulence) vary during the pulsation cycle, and since the atmosphere is not static. We therefore adopt generous error budgets of 0.1 dex for the values from Luck & Lambert (2011) and 0.15 dex in [Fe/H] for the others.

3.2.5 Age

Cepheid ages can be calculated for first overtone and fundamental-mode pulsators using the period–age (PA) relations given in Bono et al. (2005). For fundamental-mode Cepheids, we use log $t = (8.31 \pm 0.08) - (0.67 \pm 0.01) \log P$. For overtone pulsators, the relation used is log $t = (8.08 \pm 0.04) - (0.39 \pm 0.04) \log P$. Age error budgets are calculated from the uncertainties stated for slope and intercept.

4 RESULTS

This section presents the results from our census. As mentioned in Section 3.1, some host Cepheid clusters known in the literature (e.g. in Turner & Burke 2002, or Turner 2010) are not present in our cluster sample. Such cases are briefly mentioned in Section 4.1.1. We furthermore note that stellar associations are not considered here.

In addition to the membership probabilities computed, we consider the quality of the data employed to constrain membership, and compare our results to the published literature. We then flag Combos as bona fide, inconclusive, unlikely or non-members. These flags are attributed according to the following reasoning:

(i) *bona fide* is attributed to Combos with typically high priors and high likelihoods constrained by multiple parameters, in particular parallax. Closer inspection of the individual membership employed or the literature builds confidence in membership. Some Combos studied in detail in the literature prove to be strong candidates, despite low probabilities computed here, pointing to limitations of the data used as input in our analysis. We consider these Combos bona fide members.

(ii) *inconclusive* CCs are candidates for which the membership constraints available are insufficient to consider them bona fide, e.g. if $P(A) > 0.5$ with no additional membership constraints. We flag newly identified Combos as inconclusive, if the prior vanishes and $0.1 < P(B|A) < 0.8$ has been computed from at least three membership constraints that exceed the combined error budgets. These candidates warrant follow-up.

(iii) *unlikely* CCs have low likelihoods (<10 per cent) due to discrepant membership constraints (more than one constraint off by
approximately 2σ), although evidence supporting membership may exist in the literature. Membership cannot be ruled out altogether for these candidates that may benefit from additional follow-up.

(iv) non-members form the majority of Combos cross-matched. They are clearly inconsistent with membership.

It should be kept in mind that our analysis is of a statistical nature and may not provide the final answer for every Combo. While the benefit of our analysis is a consistent and transparent approach to determining membership, the correctness of our membership probabilities relies entirely on the accuracy of the input data; this is particularly true for reddening and distances, or pulsation modes. Therefore, we caution that Combos previously discussed in the literature that are found to be unlikely or non-members by our analysis should not be discarded fully without additional consideration or follow-up.

We start the presentation of our results with Combos known from the literature (Section 4.1). While we take care to include relevant references, it is almost inevitable that some works are overlooked in a field with this much history. The literature CCs are followed by new candidates and other newly identified Combos of interest (Section 4.2). For brevity of the main body, we defer presentation of inconclusive, unlikely and inconsistent Combos to Appendix A.

In Section 4.3, we then revisit the Galactic Cepheid PLR using our bona fide CC sample.

4.1 Literature Combos

The main references considered for CCs are Feast (1999), Turner & Burke 2002 and Turner (2010, from hereon: T10). Additional Cepheids whose cluster membership was considered in the literature are mentioned where appropriate, cf. also the references given in the caption of Table 1 and Section 4.1.1.

Table 1 lists the CCs previously discussed in the literature that are recovered by our analysis. A horizontal line divides cases that we find to be consistent with membership according to the data compiled (above), and those that tend to be inconsistent with membership in our analysis (below). Two essentially unconstrained

| Cluster | Cepheid | μ | υ | Constraints | [Fe/H] | age | Rc | P(A) | P(B|A) | P(A|B) | CC | Ref. |
|---------|---------|---|---|------------|-------|-----|-----|------|------|------|----|-----|
| IC 4725 | U Sgr   | * | * | * | * | * | 0.3 | 1.0 | 0.984 | 0.984 | y | a, b, c, m |
| NGC 7790| C F Cas  | * | * | * | * | o | 0.9 | 0.955 | 0.975 | 0.931 | y | d, j |
| NGC 129 | D L Cas  | * | * | * | * | o | 0.2 | 1.0 | 0.857 | 0.857 | y | b, i |
| Turner 9 | SU Cyg  | * | * | * | * | o | 1.2σ | 0.0 | 1.0 | 0.807 | 0.807 | y | k, n |
| NGC 7790| C E Cas A | * | * | * | * | o | 1.5 | 0.71 | 0.975 | 0.693 | y | d, j |
| NGC 7790| C E Cas B | * | * | * | * | o | 1.6 | 0.697 | 0.956 | 0.666 | y | d, j |
| NGC 6649| V367 Sco | * | * | * | * | o | 1.3σ | 1.0 | 0.884* | 0.65 | 0.574 | y | o, p |
| NGC 6067| V340 Nor | * | * | * | * | o | 1.9σ | 0.6 | 1.0 | 0.573 | 0.573 | y | g, m, n, l |
| Lyring 6 | T W Nor  | * | * | * | * | o | 1.3σ | 0.6 | 1.0* | 0.453 | 0.453 | y | n, m, q, C |
| vDbergh 1| C VMon   | * | 2.2σ | * | * | o | 0.6 | 1.0 | 0.318 | 0.318 | y | r |
| NGC 6079| S Nor    | * | * | * | * | o | 1.2σ | 2.0σ | 1.2σ | 1.3σ | 0.6 | 1.0 | 0.192 | 0.192 | y | a, b, c, h |
| Trumpler 35| R U Sct  | * | * | * | * | o | 5.1 | 0.194* | 0.840 | 0.840 | 0.163 | y | n, l, t, u |
| Collinder 394| B B Sgr | * | * | * | * | o | 3.7 | 0.208 | 0.637 | 0.133 | y | n, s |
| Turner 2 | W Z Sgr  | * | * | * | * | o | 5.3 | 0.337* | 0.287 | 0.097 | y | A |
| Trumpler 18| G H Car  | * | * | * | * | o | 2.0σ | 2.9 | 0.194 | 0.143 | 0.028 | y | D, E |
| NGC 6067| QZ Nor   | * | * | * | * | o | 7.4 | 0.029 | 0.963 | 0.027 | y | e, g |
| Berkeley 58| C G Cas  | * | * | * | * | o | 5.0 | 0.308 | 0.027 | 0.008 | y | F |
| NGC 5662| V Cen    | * | * | * | * | o | 5.8 | 0.006 | 0.958 | 0.006 | y | f, m, n |
| NGC 6664| E V Sco  | * | * | * | * | o | 7.5 | 0.0 | 0.866 | 0.0 | y | w, x |
| Ruprecht 173| X Cyg   | * | * | * | * | o | 0.878* | 1.0 | 0.878 | 1.0 | 0.17 | i | n, v, w |
| Dolatine 45| V1334 Cyg| * | * | * | * | o | 0.0 | 0.0 | 0.0 | 0.0 | i | n |
| Ruprecht 79| C S Vel  | * | * | * | * | o | 2.3σ | 1.5 | 1.0 | 0.007 | 0.007 | i | n, y |
| Platais 1| V1726 Cyg| * | * | * | * | o | 3.2σ | 1.0 | 1.0 | 1.4 | 0.98 | 0.006 | i | z |
| NGC 1647| S H Tau  | * | * | * | * | o | 1.8σ | 1.4 | 0.0 | 0.0 | 0.047 | 0.0 | u | B |
| NGC 3409| V442 Car  | * | * | * | * | o | 2.6σ | 1.0 | 0.625 | 0.039 | 0.024 | n | G |
| King 4 | U P Cyg | * | * | * | * | o | 12.6 | 0.02 | 0.019 | 0.0 | n | H |
| Turner 5 | T Ant    | * | * | * | * | o | 3.2σ | 0.0 | 1.0 | 0.0 | 0.0 | n | w |
| NGC 4349| R Cru    | * | * | * | * | o | 2.1σ | 1.1σ | 2.1σ | 2.3σ | 0.048 | 0.0 | n | w |
| NGC 4349| T Cru    | * | * | * | * | o | 2.2σ | 2.5σ | 19.9 | 0.0 | 0.0 | n | w |
| NGC 2345| T CMa    | * | * | * | * | o | 5.6σ | 25.0 | 0.001 | 0.0 | 0.0 | n | x, w |
Combos known in the literature are included above the horizontal.

tal line. For each deviant membership constraint, \( i \), we list the level of disagreement between cluster and Cepheid value, i.e. \( |\chi_i| \)
from equation (6), in units of the square-summed uncertainties

\[
\sigma_i^2 = \sigma_{\text{CC},i}^2 + \sigma_{\text{Cep},i}^2
\]

4.1.1 Missed Combons

Our analysis is limited to open clusters listed in D02. The following CCs reside in nearby sparse clusters that are not included in D02 and could thus not be studied in our analysis: α Uma (Turner et al. 2013, but see also van Leeuwen 2013); δ Cep (e.g. Majaess, Turner & Gieren 2012b); ζ Gem (Majaess et al. 2012c); and SU Cas (Majaess, Turner & Gieren 2012a; Majaess et al. 2012d; Turner et al. 2012).

4.1.2 Bona fide CCs

Based on the available data and literature, we flag the following literature CCs as bona fide CCs, cf. Table 1: U Sgr in IC 4725; CF Cas, CE Cas A and B in NGC 7790; DL Cas in NGC 129; SU Cyg in Turner 9; V367 Sct in NGC 6649; V340 Nor and QZ Nor in NGC 6067; TW Nor in Lynág 6; CV Mon in vdBergh 1; S Nor in NGC 6087; BB Sgr in Collinder 394; RU Sct in Collinder 394; CG Cas in Berkeley 58; V Cen in NGC 5662. For more information on those Combs with high priors and high likelihoods, i.e. the more or less obvious members, we refer to the original references listed in Table 1, as well as to the data table supplied in electronic form.

As mentioned in Section 4, some Combs flagged as bona fide CCs require inspection of the available data and literature in addition to the membership probabilities in order to conclude on membership. We discuss these combs in the sections below.

4.1.2.1 V340 Nor and QZ Nor in NGC 6067. We find two Cepheids that appear to belong to NGC 6067, namely V340 Nor, which lies within its core radius, and QZ Nor, an overtone pulsator (K5909) that lies outside \( r_{\text{lim}} \). Cluster membership for QZ Nor was first considered by Eggen (1983), and by Walker (1985b) for V340 Nor. All membership constraints were employed for both Cepheids, and both are consistent with membership for the open cluster data listed in D02. The only discrepant constraint is age for V340 Nor.

If both Cepheids belong to the same cluster, then their respective membership constraints should agree. Interestingly, the distance estimate of QZ Nor by Storm et al. (2011) is much closer to the cluster’s, while there is nearly 400pc difference between the estimates for both Cepheids. In terms of parallax, QZ Nor \([0.74 \pm 0.07 \text{ (mas)}] \) is consistent with NGC 6067 \([0.71 \pm 0.14 \text{ (mas)} ] \), but slightly off from V340 Nor \([0.58 \pm 0.09 \text{ (mas)} ] \). However, the difference in \( v_r \) between the two Cepheids is minimal \( (0.73 \text{ km s}^{-1}) \). Given that V340 Nor is a visual binary, the small offset in proper motion between the two Cepheids is not alarming, and \([\text{Fe/H}] \) is indistinguishable. In terms of age, QZ Nor seems to be slightly older than V340 Nor \( (7.85 \pm 0.07 \text{ versus } 7.60 \pm 0.08) \), and better matches to the cluster’s age \( (8.08 \pm 0.23; \text{D02}) \).

In summary, the two Cepheids have differing parallax and age. The cluster values from D02 happen to lie between the two, oddly slightly off from V340 Nor \([0.58 \pm 0.09 \text{ (mas)} ] \) which is closer to cluster centre. We therefore note that there are some issues with the membership constraints employed here, and detailed follow-up of the cluster is required. Until then, the constraints compiled are consistent with membership for both Cepheids, and we consider both to be bona fide cluster members.

4.1.2.2 CV Mon and van den Bergh 1. CV Mon lies right in the centre of cluster van den Bergh 1 and was studied in detail by Turner et al. (1998). The constraints employed are parallax, RV, proper motion and age, and yield a likelihood of 32 per cent, which is low due to the discrepant \(\mu_\alpha^*\). The average cluster RV determined by Rastorguev et al. (1999) is identical to that of CV Mon and no information on the number of stars involved in its determination is given; it should thus be discarded as membership constraint (this would lower the likelihood to 20 per cent). Aside from this, the cluster data from D02 is largely consistent with the data for the Cepheid. Parallax (also reddening), \(\mu_\alpha^*\), and age agree well between cluster and Cepheid. Thus, the Cepheid likely lies inside the volume occupied by the Cluster, and therefore should be considered to be a bona fide CC. Observational follow-up of cluster proper motion and RV would be beneficial.

4.1.2.3 S Nor and NGC 6087. S Nor’s membership in NGC 6087 was among the first to ever be suggested (Irwin 1955) and confirmed using RVs (Feast 1957), as well as the detailed study by Turner (1986) based on reddening and distance.

The values for the cluster’s mean RV differ greatly among D02 \( (6 \text{ km s}^{-1}) \), K05 \( (9.7 \text{ km s}^{-1}) \) and Feast (1957, 2.0 \text{ km s}^{-1} ), which is important considering the Cepheid’s \( v_r = 2.53 \text{ km s}^{-1} \) (Groenewegen 2008). We note that the value adopted by D02 is measured on the Cepheid itself (Mermilliod, Mayor & Udry 2008), although without taking orbital motion into account, and is therefore not suitable as a membership constraint. Since Feast (1957) investigated the largest number of stars and specifically targeted this cluster, we trust that his 2.0 \text{ km s}^{-1} is the best available estimate for the mean velocity of the cluster.

We note that proper motion, reddening, metallicity and age are slightly discrepant between cluster and Cepheid, resulting in a low likelihood. However, these differences barely exceed the combined uncertainties and may not be significant.

4.1.2.4 RU Sct and Trumpler 35. Based on age and distance from D02, this Combo would appear nearly inconsistent with membership. Unfortunately, the average cluster RV appears to have been measured on the Cepheid (Rastorguev et al. 1999) and can therefore not be considered a valid membership constraint. Membership of RU Sct in Trumpler 35 was studied in detail by Turner (1980), Hoyle et al. (2003) and T10. Closer inspection of these references reveals an underestimated cluster distance in D02 (compared also to Yilmaz 1966). The region around the Cepheid contains multiple associations, which may explain the confusion in D02. Using the cluster parallax and age from Turner (1980), we compute a likelihood of 84 per cent, and consider RU Sct a bona fide member of Trumpler 35.

4.1.2.5 BB Sgr and Collinder 394. BB Sgr lies at 21 arcmin separation from Collinder 394’s centre, i.e. at a distance of 3.7 pc assuming membership. This Combo was first studied in detail by Turner & Pedreros (1985) after having been originally suggested by Tsarevsky et al. (1966). Most membership constraints could be employed and there are only small discrepancies in parallax and \(\mu_\alpha^*\). The low prior may be misleading in this case, since the high likelihood indicates membership. We thus consider BB Sgr a bona fide member of Collinder 394.
4.1.2.6 WZ Sgr and Turner 2. This Combo was first discussed by Turner et al. (1993) when the cluster was first discovered. Most membership constraints compiled from D02, i.e. parallax, μ_r, and age, differ between Cepheid and cluster, which would result in a likelihood of <1 per cent. The cluster parameters listed in D02 were taken from the automated, 2MASS-based study by Tadross (2008). However, the much more detailed study by Turner et al. (1993) should be given higher weight, especially for its thorough treatment of reddening, and the more precise photometry used. Hence, we compute the likelihood using the cluster parameters for parallax and age from Turner et al. (1993) and find a combined membership probability of 10 per cent. The sole discrepant membership constraint remains proper motion. However, this discrepancy alone is not sufficiently strong to indicate non-membership.

4.1.2.7 CG Cas–Berkeley 58. CG Cas lies at a separation of 5.7 arcmin (outside r_{c}) from Berkeley 58’s centre, at roughly half the limiting radius. While the Cepheid’s PLR-based parallax is close to that of the cluster and reddening is in agreement, μ_r is discrepant by 3.3" and does not suggest a common point of origin. However, Turner et al. (2008) conclude in favour of membership based on a detailed study involving age, reddening, distance and RV. Since the cluster is located in the Perseus spiral arm, the proper motion estimate in K12 may well be dominated by Galactic motion. Hence, the likelihood computed here is likely underestimated, and we should trust the result by Turner et al. (2008).

4.1.2.8 V Cen and NGC 5662. Despite the low prior, all membership constraints indicate V Cen’s membership in NGC 5662, yielding a very high likelihood of 92 per cent. At NGC 5662’s distance of 666 pc, the Cepheid lies 5.8 pc from cluster centre. Hence, the prior may be misleading in this case, perhaps due to underestimated radii. We therefore consider V Cen a bona fide CC of NGC 5662.

4.1.2.9 EV Sct and NGC 6664. This Combo, mentioned previously in Turner (1976) and Turner & Burke (2002), would be nearly inconsistent with membership if the cluster values listed in D02 are employed in the calculation: the highly discrepant age (2.5σ) and parallax (0.57 ± 0.09 versus 0.86 ± 0.17) would result in a likelihood of 13 per cent. However, a literature study reveals that the distance and age listed in D02 may be wrong. The distance by Arp (1958) and Schmidt (1982) are both much greater than the 1.1 kpc in D02. We thus adopt the distance and age by Schmidt (1982, 1.4 kpc) and obtain a very high likelihood of 87 per cent, using also proper motion and RV. As already noted by Laney & Caldwell (2007), a modern follow-up campaign is warranted for this cluster.

4.2 New Combos of interest

Our results suggest the following new bona fide CCs: SX Car in ASCC 61, ASAS J182714−1507.1 in Kharchenko 3, S Mus in ASCC 69, UW Car in Collinder 220 and V379 Cas in NGC 129, see Table 2. We discuss these Combos in some detail in the sections below, followed by the identification of some unconstrained high-prior Combos recovered by our work. Some Combos flagged as inconclusive or unlikely members are discussed in Appendix A.

13 According to Turner (private communication), NGC 5662 is actually a double cluster and V Cen belongs to NGC 5662b.

4.2.1 New candidate CCs

4.2.1.1 SX Car and ASCC 61. The 4.86 d Cepheid SX Car is seen to be comoving with ASCC 61 in proper motion. Unfortunately, no average cluster RV is known. Parallax and age, however, agree very well between cluster and Cepheid, lending support to the hypothesis of membership with P(B/A) = 92 per cent. An in-depth analysis of the cluster, including its mean RV, is of the essence, since the cluster is located in a crowded field (K05). We tentatively consider SX Car as a member of ASCC 61.

4.2.1.2 ASAS J182714−1507.1 and Kharchenko 3. The 5.5 d fundamental-mode pulsator ASAS J182714−1507.1 = TYC 6266−797-1 lies at a large separation of 71 arcmin from cluster centre, which translates to approximately 44 pc at the cluster’s estimated heliocentric distance of 2.1 kpc and casts some doubt on possible membership. Reddening for cluster and Cepheid (estimated from 2MASS photometry, see Section 3.2.1.2) are in excellent agreement, while parallax and age are also consistent with membership. Proper motion does not exceed its error bars and is thus of limited constraining power in this case. Detailed observational follow-up is warranted for both Cepheid and cluster.

4.2.1.3 S Mus and ASCC 69. The membership constraints considered (all but [Fe/H]) for the binary Cepheid S Mus (e.g. Pettersen et al, 2005) and ASCC 69 yield a very high likelihood of 88 per cent. ASCC 69 is a sparsely populated cluster for which K05 list merely 12 1st members. Cluster radius and centre coordinates may therefore be rather imprecise. Furthermore, RVs are only of limited value as membership constraints, since the average cluster RV is based on only two stars. However, proper motion clearly indicates that cluster and Cepheid are comoving. We tentatively accept S Mus as a cluster member and stress the need for a detailed study of its candidate host cluster ASCC 69.

4.2.1.4 UW Car and Collinder 220. This new Combo yields a likelihood of 84 per cent from all membership constraints but [Fe/H]. The parallax of the Cepheid is estimated by the PLR, cf. Section 3.1.2. The most compelling evidence of membership comes from proper motion, while the large on-sky separation translates into a distance of 47 pc from cluster centre assuming the cluster’s distance. A detailed review of the cluster’s parameters, as well as a better parallax estimate of the Cepheid would help to conclude on this Combo.

4.2.1.5 V379 Cas and NGC 129. This high likelihood pair at large separation (44 arcmin or 20 pc at the estimated distance to NGC 129) has nearly vanishing proper motion, while the both RVs are in excellent agreement and the ages are consistent. We obtain a likelihood of 91 per cent. No parallax is computed, since V379 Cas is an overtone pulsator. Since NGC 129 has another known member, DL Cas, we can compare the pulsational ages of the two Cepheids and find both to be consistent within the uncertainties (7.70 ± 0.08 for DL Cas and 7.83 ± 0.07 for V379 Cas). Furthermore, the iron abundances of both Cepheids are close, as are their RVs (to within less than 1 km s^{−1}). We further note that V379 Cas and DL Cas have similar reddening values, though E(B − V) of V379 Cas is slightly (0.1 mag) higher (Kovtyukh et al, 2008, both). We therefore tentatively consider V379 Cas a member of NGC 129’s halo, pending a better distance estimate and additional membership constraints. NGC 129
Table 2. New Combos of interest. Columns are described in Table 1. We visually separate (a) Combos with membership constraints that lie inside the core of a cluster, (b) Combos with high likelihood for which \( \sigma \) was available, (c) Combos without \( \sigma \) that yield high likelihoods, and (d) Combos for which no likelihoods could be computed, but that lie close to the core of their potential host clusters. Combos judged unlikely (‘u’) or bona fide (‘y’) are discussed separately in the text. Inconclusive Combos (‘i’) require additional data or stronger membership constraints.

| Cluster | Cepheid | Constraints | [Fe/H] | age | \( R_\text{cl} \) [pc] | \( P(A) \) | \( P(B(A)) \) | \( P(A) | B(A) \) | CC |
|---------|---------|------------|-------|-----|----------------|-------|----------|----------------|-----|
| ASCC 60 | Y Car   | o          | •     | o   | 0.3               | 1.0   | 0.786    | 0.786          | i   |
| ASCC 61 | SX Car  | o          | •     | o   | 20.4              | 0.001 | 0.919    | 0.001          | y   |
| Kharchenko 3 | ASAS J182714—1507.1 | o | o | o | 43.9 | 0.004* | 0.905 | 0.004 | y |
| ASCC 69 | S Mus   | o          | •     | o   | 1.0\( \sigma \)  | 11.4  | 0.004    | 0.879          | 0.004 | y |
| Collinder 220 | UW Car | o | o | o | 1.0 \( \sigma \)  | 47.2  | 0.001*   | 0.838          | 0.001 | y |
| IC 4725 | Y Sgr   | 1.2\( \sigma \) | o     | o | 26.8  | 0.0 | 0.781    | 0.0   | u   |
| NGC 6705 | ASAS J184741—0654.4 | o | o | 1.3\( \sigma \) | 34.6 | 0.0 | 0.778 | 0.0 | i |
| King 4 | GO Cas  | o          | o     | o   | 0.3            | 21.7  | 0.0 | 0.684 | 0.0 | i |
| Berkeley 60 | BF Cas | o          | o     | o   | 1.0\( \sigma \)  | 33.9  | 0.0 | 0.672 | 0.0 | i |
| Tropper 1 | GH Cyg | 1.0\( \sigma \) | o     | o | 1.0\( \sigma \)  | 18.7  | 0.0 | 0.568 | 0.0 | i |
| Feinstein 1 | U Car | 1.6\( \sigma \) | o | o | o | 20.8 | 0.0 | 0.52 | 0.0 | i |
| ASCC 61 | VY Car  | o          | o     | o   | 2.0\( \sigma \)  | 21.9  | 0.0 | 0.385 | 0.0 | i |
| NGC 129 | V379 Cas | o          | o     | o   | 0.3            | 20.3  | 0.0 | 0.896 | 0.0 | y |
| Ruprecht 18 | VZ CMa | o          | o     | o   | 1.2\( \sigma \)  | 9.2   | 0.0 | 0.592 | 0.0 | i |
| Berkeley 82 | ASAS J190929+1232.8 | o | o | o | 0.3\( \sigma \)  | 10.9  | 0.0 | 0.549 | 0.0 | i |
| Ruprecht 118 | ASAS J162811—5111.9 | o | o | o | 1.0\( \sigma \)  | 22.2  | 0.0 | 0.536 | 0.0 | i |
| Hogg 12 | GH Car  | o          | o     | o   | 1.0\( \sigma \)  | 9.8   | 0.0 | 0.503 | 0.0 | i |
| NGC 6649 | ASAS J183652—0907.1 | o | o | o | 1.0\( \sigma \)  | 36.7  | 0.0 | 0.469 | 0.0 | i |
| Trumpler 9 | ASAS J075503—2614.3 | o | o | o | 1.0\( \sigma \)  | 15.2  | 0.0 | 0.463 | 0.0 | i |
| NGC 2345 | ASAS J070911—1217.2 | o | o | o | 1.0\( \sigma \)  | 36.6  | 0.0 | 0.317 | 0.0 | i |
| Ruprecht 100 | NSV 19202 | o | o | o | o | o | o | 0.837 | 1.0 | 0.837 | i |
| FSR 1595 | NSV 18905 | o | o | o | o | o | o | 0.821 | 1.0 | 0.821 | i |
| Dolidze 53 | V415 Vul | o | o | o | o | o | o | 0.759 | 1.0 | 0.759 | i |
| Ruprecht 100 | TY Cru | o | o | o | o | o | o | 0.671 | 1.0 | 0.671 | i |
| SAI 116 | NSV 18942 | o | o | o | o | o | o | 0.621| 1.0 | 0.621 | i |
| Dolidze 34 | TY Sct | o | o | o | o | o | o | 0.547 | 1.0 | 0.547 | i |

is thus particularly interesting, containing both a fundamental-mode and an overtone pulsator, just as NGC 6067.

4.2.2 Unconstrained high-prior Combos

In Table 2, we highlight six Combos with \( P(A) > 50 \) per cent that have thus far not been studied for membership, and for which no membership constraints were available. Hence, no likelihoods could be computed for these cases. We therefore suggest the following Combos for observational follow-up: V415 Vul – Dolidze 34; NSV 18905 – FSR 1595.

4.3 The Galactic Cepheid PLR revisited

Let us now employ our bona fide CC sample to revisit the calibration of the Galactic Cepheid PLR. It represents an ideal sample to this end, due to the high confidence we can have in cluster membership, though small statistics and a lack of long-period calibrators will limit the precision attainable.

A Cepheid’s absolute \( V \)-band magnitude, \( M_V \), is determined using the true distance modulus of the cluster, \( V_0 – M_V | \text{Cl} \), the Cepheid’s mean magnitude, \( m_V \), the ratio of total-to-selective extinction towards the cluster, \( R_V | \text{Cl} \), and the Cepheid’s colour excess, \( E(B – V) \), as

\[
M_V = (m_V) – (V_0 – M_V | \text{Cl}) – R_V | \text{Cl} E(B – V),
\]

where quantities refer to the Cepheid, unless subscripted by ‘Cl’.

As described in Section 3.1, we compile cluster data from D02 for our membership analysis. These data, however, do not take into account line-of-sight dependences of reddening or non-canonical values of \( R_V \). In the following, we refer to the set of data compiled for our membership analysis as the ‘standard’ set.

In an attempt to improve accuracy, we compile more accurate data from detailed studies of the host clusters. True distance moduli based on ZAMS fitting were taken from T10 and An et al. (2007), giving preference to the estimates with the smallest uncertainties. For the two ASCC clusters, we use the dereddened values by Kharchenko et al. (2005b), and for Collinder 220 we rely on the data in D02. For reddening, An et al. (2007) provide a convenient way to calculate \( R_V \) for some lines of sight in our sample as \( R_V = R_{V,0} + 0.22 (B – V)_0 \), with \( R_{V,0} \) tabulated for the cluster and taking into account the intrinsic colour of the Cepheids. For the two ASCC clusters and Collinder 220, we employ the canonical \( R_V = 3.1 \pm 0.2 \) since no other estimate is available. For Turner 9, Berkeley 58 and van den Bergh 1, we use the distance moduli and \( R_V \) values from Turner et al. (1997, 2008, 1998), respectively. For the long-period Cepheid hosts, Trumpler 35 and Turner 2, we employ the data published in Turner (1980) and Turner et al. (1993). Cepheid mean \( V \) magnitudes and \( E(B – V) \) were compiled as described in Section 3.1. Note that ASAS J182714 – 1507.1 was not included in this calibration, since the data compiled were not of sufficient quality. The values thus compiled are listed in Table 3, and we refer to this data set as the ‘optimal’ one.

Fig. 17 shows the fits to both the ‘standard’ (left-hand panel) and the ‘optimal’ (right-hand panel) data sets. In both figures, four
Table 3. Parameters adopted for the ‘optimal’ set used in equation (18) and the right panel of Fig. 17. Newly-identified bona fide Combos employed in the fit are marked with an asterisk next to the Cepheids identifier. True distance moduli of clusters, \((V_0 - M_V)\), and absorption-relevant parameters were adopted according to the criteria specified in the text. We adopt 0.04 mag as the uncertainty on \(\langle M_V \rangle\).

| Cepheid | Cluster | \(\langle m_i \rangle\) | \(E(B-V)\) | \((V_0 - M_V)\) | \(R_{V,Cl}\) | \(\log P\) | \(M_V\) |
|---------|---------|----------------|-------------|----------------|-----------|-----------|--------|
| SU Cyg  | Turner 9| 6.89 ± 0.02  | 0.07 ± 0.02 | 9.33 ± 0.05  | 2.94 ± 0.38| 0.585     | -2.65 ± 0.08|
| CG Cas  | Berkeley 58| 11.37 ± 0.01 | 0.69 ± 0.01 | 12.40 ± 0.12 | 2.95 ± 0.20| 0.640     | -3.07 ± 0.19|
| CE Cas B| NGC 7790| 11.09 ± 0.01 | 0.48 ± 0.01 | 12.46 ± 0.01 | 3.31 ± 0.26| 0.651     | -2.98 ± 0.21|
| SX Car* | ASC 61   | 9.12 ± 0.03  | 0.33 ± 0.03 | 11.14 ± 0.20 | 3.10 ± 0.51| 0.687     | -3.04 ± 0.28|
| CF Cas  | NGC 7790| 11.15 ± 0.03 | 0.48 ± 0.03 | 12.46 ± 0.01 | 3.33 ± 0.26| 0.688     | -2.91 ± 0.16|
| CE Cas A| NGC 7790| 10.94 ± 0.05 | 0.48 ± 0.05 | 12.46 ± 0.01 | 3.33 ± 0.26| 0.711     | -3.12 ± 0.21|
| UW Car* | Collinder 220| 9.46 ± 0.01 | 0.46 ± 0.01 | 11.63 ± 0.20 | 3.10 ± 0.51| 0.728     | -3.60 ± 0.31|
| CV Mon  | vdBergh 1| 10.33 ± 0.05 | 0.68 ± 0.05 | 11.08 ± 0.07 | 3.20 ± 0.04| 0.731     | -2.93 ± 0.18|
| V Cen   | NGC 5662| 6.87 ± 0.05  | 0.25 ± 0.05 | 9.31 ± 0.02  | 3.47 ± 0.38| 0.740     | -3.32 ± 0.20|
| BB Sgr  | Collinder 394| 6.91 ± 0.05 | 0.29 ± 0.05 | 9.38 ± 0.10  | 3.10 ± 0.51| 0.822     | -3.37 ± 0.24|
| U Sgr   | IC 4725  | 6.72 ± 0.02  | 0.39 ± 0.02 | 8.93 ± 0.02  | 3.32 ± 0.21| 0.829     | -3.52 ± 0.11|
| DL Cas  | NGC 129  | 8.98 ± 0.02  | 0.46 ± 0.02 | 11.11 ± 0.02 | 3.46 ± 0.22| 0.903     | -3.73 ± 0.13|
| S Mus*  | ASC 69   | 6.13 ± 0.02  | 0.21 ± 0.02 | 10.0 ± 0.20  | 3.10 ± 0.51| 0.985     | -4.52 ± 0.24|
| S Nor   | NGC 6087 | 6.41 ± 0.05  | 0.12 ± 0.05 | 9.65 ± 0.03  | 3.74 ± 0.85| 0.989     | -3.70 ± 0.22|
| TW Nor  | Lyngå 6  | 11.66 ± 0.03 | 1.24 ± 0.03 | 11.51 ± 0.08 | 3.33 ± 0.10| 1.033     | -3.96 ± 0.18|
| V340 Nor| NGC 6067 | 8.41 ± 0.02  | 0.32 ± 0.02 | 11.03 ± 0.01 | 3.36 ± 0.29| 1.053     | -3.68 ± 0.11|
| RU Sco  | Trumpler 35 | 9.53 ± 0.03 | 0.92 ± 0.03 | 11.58 ± 0.18 | 3.10 ± 0.20| 1.295     | -4.91 ± 0.27|
| WZ Sgr  | Turner 2  | 8.09 ± 0.02  | 0.62 ± 0.02 | 11.26 ± 0.10 | 3.00 ± 0.20| 1.339     | -5.02 ± 0.17|

Figure 17. Cepheid PLRs fitted to the ‘standard’ (left-hand panel) and ‘optimal’ (right-hand panel) data sets. The solid circles indicate previously known CCs, the open circles highlight the three Cepheids in NGC 7790. The solid red large diamonds indicate the new bona fide fundamental-mode CCs S Mus, SX Car and UW Car included in the fit. The dotted line shows the PLR by Benedict et al. (2007), and the dash-dotted line represents Tammann, Sandage & Reindl (2003). The solid and dashed lines indicate weighted and non-weighted least-squares fits. An accurate PLR calibration critically depends on accurate distance estimates from detailed studies that include line-of-sight variations of extinction.

Despite the reasonable formal uncertainties of our ‘optimal’ fit, our solution should not be considered definitive. The result of the fit is very sensitive to the absolute magnitude estimates of the extreme points at short and long periods, and the \(M_V\) estimates of the new candidates are clearly too crude at this point. Furthermore, despite our preference for cluster literature with the smallest uncertainties, there is no guarantee that the most accurate cluster data were employed; there appears to exist too little consensus on some clusters in the literature, e.g. for Lyngå 6, \((V_0 - M_V)_{Cl}\) differs by 0.39 mag between T10 and An et al. (2007), exceeding the published combined uncertainties by a factor greater than 3. Binarity of the Cepheids was not accounted for, since the fit is dominated mainly by the cluster parameters. Yet, we note that our result is consistent with the calibration by T10 \((\langle M_V \rangle = -(2.88 ± 0.18) \log P - (1.02 ± 0.16))\), which is to be expected due to the significant overlap in the sample of CCs used for this calibration.

straight lines indicate the PLR calibrations by Tammann et al. (2003, red dash-dotted), Turner et al. (2010, green dotted, lower zero-point), as well as our non-weighted (dashed cyan) and weighted (solid blue) least-squares fits to the data. The large scatter (rms = 0.61) in the ‘standard’ data set is striking. Contrastingly, the ‘optimal’ set is much better indeed, with an rms of 0.24 mag. Hence, it is evident that the cluster data compiled for the membership analysis is rather limited in precision and sometimes also accuracy, a fact we already encountered when computing membership probabilities, see for instance the case of WZ Sgr in Section 4.1.2.6. Thus, there remains a need for detailed and deep photometric studies of open cluster parameters, and in particular reddening.

We determine the uncertainties on the ‘optimal’ set by linear regression and obtain

\[
\langle M_V \rangle = -(2.88 ± 0.18) \log P - (1.02 ± 0.16).
\]
5 DISCUSSION

5.1 Constraining power and limitations of membership constraints

5.1.1 The prior \( P(A) \)

The form of the prior was motivated by radial density profiles of star clusters, see Section 2.1. However, the degree with which the distribution of stars in the cluster is known varies greatly between clusters, and deviations from circular cluster shapes were ignored for internal consistency. Furthermore, crowding, great distances (~kpc) to host cluster candidates, differential reddening, sparsity, etc. all conspire to complicate the definition of cluster radii. It is thus not surprising that the prior does not perform extremely well as a membership constraint if taken at face value. Nevertheless, it does help to separate the interesting cases from the majority of null matches, since it reduces the question of proximity on the projected sky to a single number that contains information on the density of the cluster, since both cluster radii are used in its definition.

5.1.1.1 Chance alignment. Among our (bona fide or inconclusive) CCs, 7–9 are found to lie within \( r_c \), and 8–17 at \( r_c < R < r_{lim} \). Within the original cross-match, 25 Combos are matched within \( r_c \) and 520 within \( r_c < R < r_{lim} \). Thus, we can estimate the rate of chance alignment within the core radius to be approximately 3:1. At separations inferior to \( r_{lim} \), this ratio increases to between 20:1 and 35:1, depending on whether inconclusive cases are counted, or not.

Note that eight bona fide CCs lie outside \( r_{lim} \) of which are located within two \( r_{lim} \); EV Scotti’s separation from NGC 6664’s centre is 2.6 \( r_{lim} \), and that of V379 Cas from NGC 129 is 2.7 \( r_{lim} \).

5.1.2 The likelihood \( P(B|A) \)

Intuitively, the greatest set of constraints used provides the tightest constraints on membership for any Combo. Fig. 18 illustrates this. It shows a logarithmic normalized histogram of likelihoods for two cases: more than three constraints used to calculate \( P(B|A) \) (light grey distribution); all constraints used to calculate \( P(B|A) \) (dark grey slim bars). However, the constraining power of a given set of constraints is not merely a function of its size. Here we discuss the membership constraining power of the different constraints used to calculate likelihoods.

Proper motions can be very effective at ruling out membership if the motion clearly exceeds the uncertainties on the measurement. This is the case for a cluster cross-matched with a background Cepheid, for instance. However, for a significant fraction of Combos, the proper motion vector’s magnitude was smaller than the uncertainty of the measurement, thus effectively not constraining membership, see Fig. 19. If the magnitude surpasses the uncertainties by at least a factor of 3, proper motion serves as a reliable constraint. For the majority of Combos that fulfil this criterion, membership tends to be excluded.

Distance is a potentially very strong membership constraint, since intuitively, a Cepheid that occupies the same space volume as a cluster should be a member. However, cluster distances can be subject to large systematic uncertainties due to parameter degeneracy (distance, age, reddening), model dependence (rotation, etc.), or previous distance estimates to, e.g., the Pleiades. Furthermore, implicit assumptions on cluster membership can significantly impact the distance determined, especially for relatively young clusters that harbour few stars around the main-sequence turn-off and few or no red giants. Very detailed studies of open clusters, and in particular of the line-of-sight extinction, are required for improvement in this domain, as is shown in Section 4.3, or demonstrated by the discussion of CV Mon’s membership in van den Bergh 1 (cf. Section 4.1.2.2).

RVs have the potential to provide very tight membership constraints, since the RV dispersion within open clusters can be significantly below 1 km s\(^{-1}\) (Lovis & Mayor 2007), approaching the measurement precision on \( v_p \) for non-binary Cepheids. However, estimates of average cluster RVs are usually based on only a few stars, see Fig. 6. In fact, approximately half of all clusters with ‘known’ average RVs are based on measurements of two stars or less, and strong selection effects (e.g. towards late-type stars) can severely impact the estimate. Since Cepheids are very bright and of late spectral type, it is rather likely that a cluster RV is in part based on measurements that include a Cepheid. We therefore highlight the need to observe more RVs of upper main-sequence stars in clusters in order to ensure the most accurate estimates of average cluster RV. Alternatively, larger telescopes may observe the much fainter lower main sequence. The Gaia-ESO public spectroscopic survey Gilmore et al. (2012) will soon provide precise RVs for a large number of clusters and thus can improve the reliability of a future study similar to the present one.

Figure 18. Log-normalized histogram of likelihoods. Light grey, broad bars: combinations with three or more parameters used for \( P(B|A) \). The dark grey, slim bars: combinations with all membership constraints. The more membership constraints are employed, the more separated are high and low likelihoods, i.e. the better constraint is membership.

Figure 19. Likelihoods computed as a function of proper motion. Abscissa: proper motion of Cepheid divided by the squared-summed uncertainties, i.e. \( \mu = \sqrt{(\mu_x^0)^2 + \mu_y^0}; \sigma = \sqrt{\sigma^2 (\mu_x^0)^2 + \sigma (\mu_y^0)^2} \). Ordinate: same for cluster.
The iron abundances compiled here were, arguably, of limited use as membership constraints, since data inhomogeneity, the limited number of cluster stars used for determining the cluster average, and differences in the solar reference values used are at the same order of magnitude as the range of iron abundances found in the sample considered (that lies within the young metal-rich Galactic disc). For a few cases, however, the iron abundance did further strengthen the interpretation of excluded membership.

Age as a membership constraint quantifies valid evolutionary considerations that are established empirically. It is a particularly useful membership constraint, since it is readily available for clusters as well as for Cepheids (from PA relations), especially when few other constraints are available. However, ages for both kinds of objects are subject to model dependence, and the accuracy of the values inferred is difficult to quantify.

All of the above quantities have their own peculiarities, and thus no single one can be named the ‘best’ membership constraint. Instead, the greatest constraining power resides in the combination of all the data, as is seen in the bona fide CCs identified in this work, as well as in Fig. 18.

5.1.3 Membership probabilities \( P(\text{A} | \text{B}) \)

We computed \( P(\text{A} | \text{B}) \) simply as the product of the prior and the likelihood, leaving out the normalization term \( P(\text{B}) \) that would in principle be required, see equation (1). However, in order to make full use of the \( P(\text{A} | \text{B}) \) values computed, \( P(\text{B}) \) should not be neglected. Given that the incompleteness for open clusters is quite significant at heliocentric distances greater than, say, 1 kpc, we did not currently see this as feasible, cf. Fig. 20.

5.2 Incompleteness

Despite our aim to maximize the number of clusters and Cepheids considered, only 23 bona fide CCs were identified. This is a result of the apparent difference in sample completeness for clusters and Cepheids. Fig. 20 shows the histograms of heliocentric open cluster distances for clusters within the appropriate age range \( (\log_{10} \text{age(yr)} = [7.0, 8.5]) \), taken from D02 as the grey filled distribution (peak at smaller d), and the Cepheid distances compiled (cf. Section 3.2.1, black step histogram). It is evident that the detection rate of open clusters stalls at distances greater than 1 kpc, probably since their identification against the field becomes increasingly difficult. Cepheids, on the other hand, are detectable at much greater distances, thanks to their high luminosity and characteristic brightness variations.

However, a cluster’s probability of hosting a Cepheid is governed by stellar evolution and star formation. For a given distribution of stellar masses, only a small fraction will become seen as Cepheids during their lifetime (stars with masses between \( \approx 4 \) and 11 \( M_\odot \)). Due to the nature of the IMF, these intermediate-mass stars constitute a small fraction of the total number of cluster stars, and few such intermediate-mass stars are present in typical (small) open clusters. Furthermore, only a fraction (perhaps 10–20 per cent) of a suitable star’s lifetime during the core helium burning phase is spent on the blue loops, and even less within the instability strip. As a result, few CCs are known. The inverse problem of finding host clusters around known Cepheids (e.g. Turner et al. 1993; Turner 1998; Majaess et al. 2012a,c), however, can be interesting, e.g. to constrain the survival rate of open clusters. Such studies can be very successful, see e.g. the ‘missing Combus’ in Section 4.1.1 and the Cepheids belonging to OB associations listed in T10.

5.3 Implications for the distance scale

We are aware of the fact that our calibration, although performed on an ‘optimal’ set, remains inhomogeneous in the true distance moduli employed and the treatment of extinction. It is furthermore based on V-band data obtained in multiple different passbands with varying post-processing techniques. These are the main limitations of the data employed. Furthermore, the number of CCs employed in the fit (18) is not very large, and the distance to NGC 7790 enters the fit three times, since its three CCs are included in the fit. Finally, the fit obtained is very sensitive to outliers, due to the lack of long-period calibrators.

We advocate that the ‘optimal’ sample presented, although incomplete (missing host clusters), forms an ideal subset for PLR calibrations, since all cluster memberships were self-consistently evaluated. However, to improve upon the calibration of the cluster-based PLR, two things would be particularly useful: a detailed homogeneous deep photometric study of the host clusters that includes careful treatment of extinction, and observational follow-up of the inconclusive CC candidates identified. Given the large discrepancies that are found between recent Galactic PLR calibrations (and their zero-points), such an observational campaign would be very desirable.

6 CONCLUSION

Focusing on Cepheids, we have performed an all-sky cluster membership census. Our analysis considers an up to eight-dimensional membership space that includes spatial and kinematic information, as well as parent population parameters (age, iron abundance). Although in some ways limited by data inhomogeneity and incompleteness, we identify 23 bona fide cluster Cepheids, including most canonical CCs accessible within our sample, as well as five new ones, and multiple additional CC candidates of interest.

The newly identified CCs are: SX Car in ASCC 61, S Mus in ASCC 69, UW Car in Collinder 220, ASAS J182714−1507.1 in Kharchenko 3 (fundamental-mode pulsators) and V379 Cas in NGC 129 (overtone pulsator). The cluster membership of these candidates must have escaped previous discovery, since most of the host clusters are not very well studied, and the Cepheids are located outside the cluster cores, cf. Section 4.2.
Since we can rank candidates according to membership probabilities, we consider our bona fide CC sample ideal for calibrating the Galactic Cepheid PLR. However, data inhomogeneity, large uncertainties on cluster parameters, and a lack of long-period calibrators unfortunately limit the precision of the calibration we perform. We therefore highlight the need for observational campaigns dedicated to the host clusters of our bona fide CC sample, as well as other interesting CC candidates identified.

The limitations that our work suffers due to inhomogeneous and incomplete data will be significantly reduced in the near future, thanks to the Gaia space mission. Specifically, Gaia will improve our study in the following ways:

(i) Thousands of new Cepheids (Eyer & Cuypers 2000; Windmark, Lindegren & Hobbs 2011) will be discovered.

(ii) Accurate absolute trigonometric parallaxes of Cepheids will become available up to distances of 6–12 kpc, depending on extinction (Eyer et al. 2012), thereby enabling a direct calibration of the Galactic PLR similar to the one performed in the seminal paper by Feast & Catchpole (1997). This will remove our partial dependence on existing PLR calibrations when determining cluster membership. Accurate parallaxes to longer period Cepheids will also be obtained, thereby significantly improving the distance estimates to extragalactic Cepheids.

(iii) The accuracy of proper motions will be improved by orders of magnitude, moving from mas yr\(^{-1}\) to tens of μas yr\(^{-1}\), see Lindegren (2010) and the Gaia Science Performance website.\(^{14}\)

(iv) Homogeneous RVs (via the RVS instrument) and metallicity estimates (via the spectrophotometric instruments) will be available as membership constraints for a great number of Cepheids.

(v) Homogeneous metallicity estimates can be obtained through spectrophotometry and photometry (Liu et al. 2012).

(vi) Thousands of new open clusters (ESA 2000) will be discovered, forming a more or less complete census of open clusters to distances up to 5 kpc. As was shown in Fig. 20, the distribution of Cepheids is still increasing at these distances. Hence, one may expect to find many more CCs in these parts of the Galaxy.

(vii) Down to magnitude 20, all-sky homogeneous multi-epoch photometry and colours will be obtained that will include all the bona fide cluster Cepheids mentioned in this work.

(viii) Known clusters will be mapped in unprecedented detail, and intracluster dynamics will be accessible to determine membership.

Gaia’s data homogeneity will significantly improve error budgets, since no offsets in instrumental zero-points (e.g. in RV) will have to be taken into account. The constraining power in terms of membership will thus be augmented considerably. Correlations between parameters, e.g. proper motion, parallax and RV, can be determined self-consistently and accounted for (cf. van Leeuwen 2007). Such factors will make the Gaia era particularly exciting for work such as this.

Acknowledgements

Many heartfelt thanks to the observers who contributed to the Cepheid RV campaigns, in particular to: Lovro Palaversa, Mihaly Váradi and Pierre Dubath. We gratefully acknowledge useful discussions with and comments received from Maria Sú toolbox, Michael W. Feast, and the referee, C. David Laney; all of these helped to improve the manuscript.

This research has made use of: NASA’s Astrophysics Data System Bibliographic Services; the SIMBAD data base and VizieR catalogue access tool (cf. A&AS 143, 23), operated at CDS, Strasbourg, France; the International Variable Star Index (VSX) data base, operated at AAVSO, Cambridge, Massachusetts, USA; the V0-tool TOPCAT,\(^{15}\) see Taylor (2005); the WEBDA data base, operated at the Institute for Astronomy of the University of Vienna; other online data bases that provide Cepheid data, see the main text.

References

Akerlof C. et al., 2000, AJ, 119, 1901
An D., Terndrup D. M., Pinsonneault M. H., 2007, ApJ, 671, 1640
Anderson E., Francis C., 2012, Astron. Lett., 38, 331 (XHIP)
Anderson R. I., Eyer L., Mowlavi N., 2012, in Griffin E., Hanisch R., 
Seaman R., eds. Proc. IAU Symp. 285, New Horizons in Time-Domain 
Astronomy. Cambridge Univ. Press, Cambridge, p. 275
Andrievsky S. M. et al., 2002a, A&A, 381, 32
Andrievsky S. M., Bersier D., Kovtyukh V. V., Luck R. E., Maciel W. J., 
Lépine J. R. D., Beletsky Y. V., 2002b, A&A, 384, 140
Andrievsky S. M., Kovtyukh V. V., Luck R. E., Lépine J. R. D., Maciel W. J., 
Beletsky Y. V., 2002c, A&A, 392, 491
Andrievsky S. M., Luck R. E., Martin P., Lépine J. R. D., 2004, A&A, 413, 159
Andrievsky S. M., Luck R. E., Kovtyukh V. V., 2005, AJ, 130, 1880
Arp H. C., 1958, ApJ, 128, 166
Bakos G. Á., Lázár J., Papp I., Sári P., Green E. M., 2002, PASP, 114, 974
Balona L. A., Laney C. D., 1995, MNARS, 277, 250
Baranne A. et al., 1996, A&AS, 119, 373
Baranowski R. et al., 2009, MNARS, 396, 2194
Barnes T. G., III, Moffett T. J., Slovak M. H., 1987, ApJS, 65, 307
Barnes T. G., III, Moffett T. J., Slovak M. H., 1988, ApJS, 66, 43
Barnes T. G., III, Jeffery E. J., Monteneyar T. J., Skillen I., 2005, ApJS, 156, 227
Baumgardt H., Dettbarn C., Wielen R., 2000, A&AS, 146, 251
Benedict G. F. et al., 2007, AJ, 133, 1810
Berdnikov L. N., 2008, VizieR Online Catalog, II/285
Berdnikov L. N., Dambis A. K., Voyzakova O. V., 2000, A&AS, 143, 211
Bersier D., 2002, ApJS, 140, 465
Bersier D., Burki G., Mayor M., Duquennoy A., 1994, A&AS, 108, 25
Bono G., Marconi M., Cassisi S., Caputo F., Gieren W. P., Pietrzynski G., 
2005, ApJ, 621, 966
Bukowiecki L., Maciejewski G., Konorski P., Strobel A., 2011, Acta Astron., 
61, 231 (B11)
Coulson I. M., Caldwell J. A. R., 1985, South Afr. Astron. Obser. Circ., 9, 5
Coulson I. M., Caldwell J. A. R., Gieren W. P., 1985, ApJS, 57, 595
Cunti R. M. et al., 2003, 2MASS All Sky Catalog of Point Sources. 
NASA/IPAC Infrared Science Archive
Dias W. S., Lepine J. R. D., Alessi B. S., Moitinho A., Lepine J. R. D., 2002a, A&A, 389, 
871 (D02)
Dias W. S., Lepine J. R. D., Alessi B. S., 2002b, A&A, 388, 168
Efremov Y. N., 1964, Perem. Zvezdy, 15, 242
Eggen O. J., 1980, Inf. Bull. Var. Stars, 1853, 1
Eggen O. J., 1983, AJ, 88, 379
ESA, 2000, Gaia: Composition, Formation and Evolution of the Galaxy. 
Technical Report, ESA-SCI(2000)4
Evans N. R., 1992, ApJ, 385, 680
Evans N. R., Welch D. L., 1993, PASP, 105, 836

\(^{14}\) http://www.rssd.esa.int/index.php?project=GAIA&page=Science_
Performance#chapter1

\(^{15}\) http://www.star.bris.ac.uk/~mbt/topcat/
APPENDIX A: DETAILS ON INDIVIDUAL CLUSTER CEPHEID COMBINATIONS

A1 Literature Combos

A1.1 Inconclusive Combos

A1.1.1 CS Vel and Ruprecht 79. Membership of CS Vel in Ruprecht 79 was thoroughly discussed by Harris & van den Bergh (1976) who credited Tsarevsky et al. (1966) with first suggesting this particular combination. It has since been studied multiple times, e.g. by Walker (1987) and T10. Due to the sparse nature of the cluster, its reality as such was doubted by McSwain & Gies (2005), who conclude that Ruprecht 79 rather be a hole in the dust of the Sagittarius–Carina spiral arm than a physical open cluster.

None of the data from D02 are fully consistent with cluster membership for CS Vel, see Table 1, and we calculate a likelihood of not even 1 per cent, which contrasts the Cepheid’s location inside rC. The Cepheid’s colour excess from Laney & Caldwell (2007) agrees with the cluster value, however, and RV is not very far off. The cluster data from D02 and the uncertain existence of the cluster would suggest unlikely membership. However, the difficult parameter determination for this sparse cluster (candidate) means that only a very detailed study of this region can reliably conclude on membership.

A1.1.2 V1726 Cyg and Platais 1. V1726 Cyg’s membership results for Platais 1 are based on separation, proper motion, RV and age. The star was first considered for cluster membership by Platais (1979) and Turner et al. (1994), and is still often considered a bona fide member of Platais 1 (Turner, Billings & Berdnikov 2001; Turner, Usenko & Kovtyukh 2006). However, Platais (1986) concludes that the existence of this cluster is uncertain.

Of the membership constraints compiled, only proper motion does not significantly differ between cluster and Cepheid, although the magnitude of the Cepheid’s motion is a bit larger than that of the cluster. RV differs significantly, with the cluster receding more than 10 km s\(^{-1}\) faster than the Cepheid (Frinchaboy & Majewski 2008). In addition, the cluster’s age is significantly higher than the Cepheid’s. We do not calculate a PLR-based parallax, since V1726 Cyg may or may not be an overtone pulsator; the Fernie data base places V1726 Cyg at approximately 2 kpc, while D02 list 1.3 kpc for Platais1.

Unfortunately, there are great differences in cluster parameters to be found in the literature. For instance, K12 list 3.5 kpc as the cluster’s distance, which would exclude membership and be consistent with the higher proper motion of a foreground Cepheid. Interestingly, however, K12’s average cluster RV (\(-15.4\) km s\(^{-1}\)) would agree with the Cepheid’s, although it is based on 21 stars in a poorly populated and supposedly distant (3.5 kpc in K12) cluster, rendering this estimate suspicious.

In summary, the constraints compiled here would tend to indicate non-membership. However, the significantly discrepant cluster distances and RVs from different references leave considerable doubt as to whether the cluster values adopted here are accurate. In light, these issues and the possible non-existence of the cluster, we cannot conclude on membership of this Combo.

A1.2 Unlikely literature Combos

A1.2.1 GH Car in Trumpler 18 or Hogg 12. Membership of the spectroscopic binary overtone pulsator GH Car (Szabados et al. 2013) in cluster Trumpler 18 was first proposed by Vazquez & Feinstein (1990) and then called into question by Baumgardt et al. (2000) who recommended RV follow-up to draw a firmer conclusion. We compute a low likelihood of 14 per cent based on proper motion, age and RV. No parallax was calculated, since GH Car is an overtone pulsator. However, the Cepheid’s distance listed in the Fernie data base (2.2 kpc) is significantly larger than Trumpler 18’s (1.4 kpc). Furthermore, the Cepheid is reddened by 0.1 mag more...
than the cluster, which is consistent with a greater distance to the Cepheid. The average cluster RV is based on the measurements of a single star, and does not agree with that of the Cepheid. In short, membership of GH Car in Trumpler 18 is unlikely.

However, our analysis identifies Hogg 12 as an alternative host cluster for GH Car. Based on proper motion and age, we compute a likelihood of 50 per cent for this Combo. Proper motion is a very good match, and clearly detected. Finally, reddening for the cluster and the Cepheid are nearly identical, and the Cepheid’s literature distance from the Fernie data base matches the cluster’s distance very well. Follow-up is required to confirm this option.

A1.2.2 SZ Tau and NGC 1647. The membership of SZ Tau in the halo of NGC 1647 was first considered by Efremov (1964) and later studied in more detail by Turner (1992) who concluded that SZ Tau is a ‘coronal’ member, based on star counts, reddening, RV, proper motion from Francic (1989) and assuming overtone pulsation for SZ Tau.

From parallax (Storm et al. 2011), RV, proper motion and age, we compute a likelihood of membership of 5 per cent, the main discrepant constraints being RV and proper motion from Tycho2 and Hipparcos (Dias, Lépine & Alessi 2001; van Leeuwen 2007). The vanishing prior could, of course, be consistent with coronal membership. However, cluster membership based on proper motion was excluded by Geffert et al. (1996) using 2220 stars measured on photographic plates, and by Baumgardt et al. (2000) using Hipparcos proper motions. We therefore consider SZ Tau an unlikely member of NGC 1647, although an ejection cannot be excluded (Turner, private communication).

A1.3 Non-member Combos discussed in the literature

A1.3.1 V442 Car and NGC 3496. V442 Car was previously considered for membership in NGC3496 by Balona & Laney (1995) who concluded it to be a background star, based on age and reddening. From proper motion and separation, we come to the same conclusion. Furthermore, a rough distance estimate for a 14th magnitude 5.5 d Cepheid excludes membership in a cluster located approximately 1 kpc from the Sun.

A1.3.2 UY Per and Czernik 8 or King 4. Turner (1977) suggested that UY Per could be a member of Czernik 8 or King 4. Turner et al. (2010) again mention the latter combination. Our results, however, are inconsistent with membership in either cluster, based on the constraints parallax, proper motion and age.

The ‘likerlier’ of the two Combos is King 4, for which parallax and age are in relatively good agreement; the Cepheid is slightly farther away and has larger reddening. Kinematically, however, the cluster’s vanishing proper motion is inconsistent with membership of the rather fast moving Cepheid ($\mu_\alpha = -6.15 \pm 2.8 \text{ mas yr}^{-1}$, $\mu_\delta = 12.89 \pm 2.9 \text{ mas yr}^{-1}$, PPMXL).

A1.3.3 R Cru, T Cru and NGC 4349. The two Cepheids R Cru and T Cru have previously been considered as members of the open cluster NGC 4349 (Turner & Burke 2002), although they are no longer listed in Turner (2010). Our results are very clearly inconsistent with either Cepheid’s membership in this cluster. However, both Cepheids have very similar parallaxes and proper motions, and lie close to the open cluster Loden 624. Little information is available for this cluster and observational follow-up is warranted.

A1.3.4 TV CMa and NGC 2345. The membership constraints parallax, proper motion (vanishes) and age agree within their respective uncertainties. The Cepheid’s separation of 40 arcmin from cluster centre results in a very low prior. RV differs by $\sim 20 \text{ km s}^{-1}$ between cluster and Cepheid, resulting in a low likelihood. RV is the prime excluding constraint for this combo, and appears to be robust.

A2 New Combos

In the following subsections, we discuss possible membership for selected Combos listed in Table 2. Observational follow-up is warranted for all the inconclusive Combos in the table, even if they are not discussed here in detail. Further information can be found in the data compiled that are available online, cf. Table A1. We furthermore remark that the ASAS targets in Table 2 are particularly worthy of follow-up, since they have not yet received much attention.

A2.1 Inconclusive Combos

A2.1.1 Y Car and ASCC 60. Y Car is a double-mode Cepheid in a triple system with a B9.0V companion (Evans 1992) on a known orbit (see the Szabados 2003 binary Cepheids data base). The Cepheid lies well-inside ASCC 60’s projected core. Since the RV of the cluster in K05 was measured on a single star and is identical to Y Car’s $v_p$, we cannot consider this a valid membership constraint. Proper motion, on the other hand, is clearly measurable and consistent with membership. Reddening is slightly larger (by 0.07 mag) for the Cepheid than for the cluster, and the absolute magnitude of Y Car estimated by Evans (1992) indicates a distance modulus incompatible with that of the cluster, though we note that due to the sparsity of this cluster, a revised distance estimate would be useful. A detailed photometric and RV study of ASCC60 is required to conclude on the possible membership of Y Car. We furthermore note the presence of another Cepheid, CR Car inside ASCC 60’s core radius. This Combo, however, is clearly inconsistent with membership. CR Car lies in the background of the cluster.

A2.1.2 VZ CMa and Ruprecht 18. This combination yields a likelihood of 59 per cent, since metallicity and age are in excellent agreement between the cluster and the Cepheid. Proper motion is better discernible for the Cepheid than for the cluster, it seems, and colour excess is 0.14 mag less strong for the Cepheid than for the cluster. According to D02, the cluster (1.1 kpc) lies bit closer than the Cepheid (1.3 kpc, from the Fernie data base). In summary, both Cepheid and cluster require detailed follow-up for reddening and parallax, before we can conclude on the question of membership.

A2.1.3 WZ Car and ASCC 63. Likelihood and prior both tend to clearly exclude WZ Car’s membership in ASCC 63; the large projected distance from ASCC 63’s core (33 pc assuming membership) lends further support to this interpretation. However, the likelihood computed is completely dominated by the extreme mismatch in line-of-sight velocities ($\delta v_{\text{rad}} = 120 \text{ km s}^{-1}$). Looking at the other constraints, however, we find that age, reddening, parallax (Cepheid parallax from Storm et al. 2011) and proper motion strongly suggest membership. Suspiciously, the cluster RV is based on only two stars, and may therefore not be reliable, or point towards an ejection.
event. We therefore judge this Combo inconclusive and stress the need for follow-up of the cluster.

A2.2 Unlikely Combos

A2.2.1 Y Sgr and IC 4725 (M25). The parallaxes employed [Cepheid \( \sigma_{\text{Cep}} = 2.13 \pm 0.29 \) from Benedict et al. (2007) and Cluster \( \sigma_{\text{Cl}} = 1.61 \pm 0.32 \)] in the calculation nearly agree within their respective error budgets. However, colour excess is 0.3 mag lower for the Cepheid, which indicates that Y Sgr lies in the foreground of M 25.

Using the well-established cluster member U Sgr as a point of reference, we remark that proper motion, age and metallicity are in excellent agreement between both Cepheids. \( v_\gamma \) differs slightly between the two, which could be explained by the known binarity of U Sgr and Y Sgr. The only significant discrepancy is in distance, which might be explained by uncertainties in extinction, since M 25 lies in the Orion arm (XHIP). However, we calculate a very low prior for this Combo, and if Y Sgr were a cluster member, it would lie at a large distance of 27 pc from cluster centre. From these considerations, it appears that Y Sgr is an unlikely cluster member candidate.

### Table A1

An excerpt of the data provided in the machine-readable online table. For each Combo we provide the parameters separation (R), prior, likelihood and combined membership probability, as well as the differences between the individual membership constraints used (defined as cluster value minus Cepheid value), the combined error budgets adopted and relevant references. For RefR, the cluster radius reference is provided. In column Refs, references are listed in the following order: ‘a’ – values based on ASAS photometry, ‘b’ – values based on Berdnikov photometry, ‘d’ – Dias et al. (2002a) catalogue, ‘f’ – Fernie data base, ‘fo’ – Fouqué et al. (2007), ‘g’ – GCVS, ‘h’ – (van Leeuwen 2007), ‘K’ – Kovtyukh et al. (2008), ‘ks’ – Klagyivik & Szabados (2009), ‘l’ – Luck & Lambert (2011), ‘p’ – parallaxes computed from PLR-based distances, ‘P’ – PPMXL (Roeser et al. 2010), ‘s’ – Storm et al. (2011), ‘t’ – Dias et al. (2001), Dias, Lépine & Asselli (2002b), ‘v’ – VSX, ‘∗’ – newly determined \( v_\gamma \) used, ‘**’ – period improved using RV data, ‘B11’ – Bukowiecki et al. (2011), ‘FO’ – first overtone ages from Bono et al. (2005), ‘FU’ – fundamental-mode ages from Bono et al. (2005), ‘K05’ – Kharchenko et al. (2005a, b), ‘K07’ – Kharchenko et al. (2007), ‘K12’ – Kharchenko et al. (2012), ‘MMB, MMU, BDW, KHA’ – see the references list in Dias et al. (2002a). The complete list of 3974 combinations can be retrieved from the online appendix to the paper.

| Cluster | Cepheid | R     | P(A)     | P(B|A)    | Δσ/Δν/Δμ/Δϖ/Δ[Fe/H]/Δ log a |
|---------|---------|-------|----------|----------|---------------------------|
|         |         | RefR  | Refs     | Refs     | Refs                      |
| IC 4725 | U Sgr   | 1.570 | 1.000 00 | 0.983 73 | -0.114 0.328 3.606 0.780 0.210 |
|         |         |       | K05      | d, s, k, V | MMB, t, d, ks            |
| NGC 7790| CF Cas  | 1.054 | 0.954 54 | 0.975 32 | 0.056 0.070 0.360 0.319 0.263 |
|         |         |       | B11      | d, p, k, G | MMU, t, BDW              |
| NGC 129 | DL Cas  | 0.421 | 1.000 00 | 0.857 00 | 0.063 0.127 0.360 0.210 0.181 |
| Turner 9 | SU Cyg | 0.067 | 1.000 00 | 0.807 43 | 0.074 0.250 6.888 2.108 0.234 |
|         |         |       | K12      | d, s, k, G | MMU, t, K05              |
| NGC 1647| SZ Tau  | 127.84| 0.000 00 | 0.046 91 | 0.060 0.376 3.606 3.910 0.272 |
| NGC 2345| TV CMa  | 38.208| 0.000 69 | 0.000 01 | 0.043 0.092 3.606 1.290 0.009 |
| Turner 9 | SU Cyg | 0.067 | 1.000 00 | 0.807 43 | 0.074 0.250 6.888 2.108 0.234 |
| NGC 4349| R Cru   | 14.985| 0.047 71 | 0.000 00 | -0.717 0.106 3.606 2.510 0.518 |
|         |         |       | B11      | d, p, f, A | MMU, K05, BDW, d          |
| ASCC 61 | SX Car  | 41.253| 0.000 56 | 0.919 17 | 0.042 0.122 2.985 0.950 0.110 |
|         |         |       | K05      | d, p, f, A | K05, K05, d               |
| ASCC 69 | S Mus   | 39.173| 0.004 46 | 0.879 29 | -0.166 0.215 12.266 1.414 0.253 |
|         |         |       | K05      | d, s, f, k | K05, d                   |
| NGC 129 | V379 Cas| 42.900| 0.000 00 | 0.895 94 | 0.040 0.440 2.960 1.320 0.053 |
| Turner 9 | SU Cyg | 0.067 | 1.000 00 | 0.807 43 | 0.074 0.250 6.888 2.108 0.234 |
| ASCC 60 | Y Car   | 1.228 | 1.000 00 | 0.786 42 | -2.100 7.985 2.033 1.291 0.251 |
|         |         |       | K05      | K07, K05 | d                       |

More data online
and age are the only available membership constraints available in the literature compiled. Cluster and Cepheid appear to be comoving in proper motion. Furthermore, the pulsational age of the Cepheid is spot-on with the cluster. However, a very rough distance estimate using the V-band magnitude (10.47) found in the Guide Star Catalog V. 2.3.2 (Lasker et al. 2008) yields a distance of 2.5 kpc for the Cepheid, which is five times the cluster distance. Since both objects are thus far not very well studied, we highlight the need for follow-up of both cluster and Cepheid.

### A2.3 Non-members of interest

Non-member Combos of interest are listed in Table A2. We here present Combos with $P(A) = 1$, as well as others with high priors and information on parallax. Some of the Cepheids listed here belong to other open clusters, e.g. V Cen or X Cyg or OB associations, e.g. S Vul (T10). Below, we discuss one of these cases, EY Car, since the membership probability computed is high.

### A2.3.1 EY Car and Alessi 5

The beat Cepheid EY Car lies within the core radius of Alessi 5. The available (kinematic only) membership constraints are consistent with, but not very close to, each other. However, the literature distance from the Fernie data base places the Cepheid nearly 2 kpc farther than the cluster, and the larger magnitude of proper motion is consistent with a foreground cluster. EY Car is thus not a cluster member, mentioned here only due to the high membership probability computed.

### SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article:

#### Table A1

For each Combo we provide the parameters separation (R), prior, likelihood, and combined membership probability, as well as the differences between the individual membership constraints used (defined as cluster value minus Cepheid value), the combined error budgets adopted and relevant references (http://mnras.oxfordjournals.org/lookup/suppl/doi:10.1093/mnras/stt1160/-/DC1).

Please note: Oxford University Press are not responsible for the content or functionality of any supporting materials supplied by the authors. Any queries (other than missing material) should be directed to the corresponding author for the paper.

#### Table A2

| Cluster  | Cepheid  | $\sigma$ | $v_r$ | $\mu_\alpha$ | $\mu_\delta$ | [Fe/H] | age  | $R_\odot$ | $P(A)$ | $P(B|A)$ | $P(A|B)$ |
|----------|----------|---------|------|-------------|-------------|--------|------|-----------|--------|----------|----------|
| Alessi 5 | EY Car   | o      | *    | *           | *           |        |      | 0.6      | 1.0    | 0.725    | 0.725    |
| Koposov 12 | CO Aur  | o      | o    | 2.0$\sigma$ | 1.9$\sigma$ | o      |      | 3.5      | 1.0    | 0.023    | 0.023    |
| Turner 1 | S Vul    | 2.8$\sigma$ | o    | *           | 1.6$\sigma$ | o      |      | 1.3$\sigma$ | 0.2    | 1.0*     | 0.015    |
| NGC 5045 | NSV 19655 | o      | o    | 2.2$\sigma$ | 2.2$\sigma$ | o      |      | 1.8      | 1.0    | 0.008    | 0.008    |
| Ruprecht 18 | AO CMa  | 3.8$\sigma$ | o    | *           | 1.4$\sigma$ | *      |      | *        | 0.5    | 1.0      | 0.005    |
| Collinder 173 | AH Vel  | o      | o    | o          | 1.1$\sigma$ | o      |      | 2.3$\sigma$ | 11.8   | 10*      | 0.036    |
| Collinder 173 | CR Car  | 4.4$\sigma$ | 5.0$\sigma$ | 1.3$\sigma$ | *          | o      |      | 3.2$\sigma$ | 0.7    | 1.0      | 0.0      |
| Collinder 173 | ASAS J083130$-$4429.3 | 3.9$\sigma$ | o    | *           | o          | o      |      | 3.1$\sigma$ | 5.3    | 1.0      | 0.0      |
| Collinder 173 | ASC 60   | 4.4$\sigma$ | 5.0$\sigma$ | 1.3$\sigma$ | *          | o      |      | 3.2$\sigma$ | 0.7    | 1.0      | 0.0      |
| Collinder 173 | V1256 Tau | 4.7$\sigma$ | o    | *           | 1.3$\sigma$ | o      |      | 4.7$\sigma$ | 3.4    | 1.0      | 0.0      |
| Melotte 25 | NSVS 9444700 | o      | o    | o          | o          | o      |      | 7.7$\sigma$ | 1.2    | 1.0*     | 0.0      |
| Stock 2 | GL Cas  | 4.7$\sigma$ | 5.3$\sigma$ | 7.6$\sigma$ | *          | o      |      | 1.4$\sigma$ | 2.9    | 1.0      | 0.0      |
| Turner 11 | X Cyg   | 3.4$\sigma$ | o    | o          | o          | o      |      | 5.2$\sigma$ | 0.0    | 1.0*     | 0.0      |
| Turner 7 | V Cen   | 5.0$\sigma$ | o    | o          | o          | o      |      | 15.5$\sigma$ | 0.0    | 1.0*     | 0.0      |
| King 7 | V933 Per | o      | o    | *           | 2.0$\sigma$ | o      |      | 5.1$\sigma$ | 3.4    | 1.0      | 0.0      |
| NGC 6639 | X Scet  | 2.9$\sigma$ | 1.5$\sigma$ | 1.7$\sigma$ | o          | 5.0$\sigma$ | 1.2 | 0.853    | 0.0    | 0.0      |
| Teutsch 14a | ASAS J180342$-$2211.0 | 1.8$\sigma$ | o    | o          | o          | o      |      | 3.1$\sigma$ | 2.2    | 0.814*   | 0.001    |
| NGC 6873 | ASAS J200829+2105.5 | 4.5$\sigma$ | o    | o          | o          | o      |      | 3.8      | 0.776*| 0.0      | 0.0      |
| SAI 94 | SX Vel  | 3.8$\sigma$ | o    | o          | o          | o      |      | 8.0$\sigma$ | 4.2    | 0.761*   | 0.0      |
| Collinder 240 | FR Car  | 2.8$\sigma$ | 2.2$\sigma$ | *          | o          | 1.5$\sigma$ | 11.6 | 0.733*   | 0.005  | 0.004    |
| NGC 6847 | EZ Cyg  | 2.4$\sigma$ | o    | 1.2$\sigma$ | *          | 5.4$\sigma$ | 8.7  | 0.732*   | 0.0    | 0.0      |
| BH 23 | AT Pup  | 3.5$\sigma$ | 2.6$\sigma$ | 1.2$\sigma$ | *          | 2.9$\sigma$ | 5.8  | 0.616*   | 0.0    | 0.0      |
| Alessi-Teutsch 7 | ASAS J082710$-$3825.9 | 3.9$\sigma$ | o    | 2.6$\sigma$ | *          | o      |      | 17.8     | 0.59* | 0.0      | 0.0      |
| BH 164 | AV Cir  | 1.3$\sigma$ | 1.3$\sigma$ | 2.6$\sigma$ | 5.0$\sigma$ | o      |      | 9.1      | 0.572*| 0.0      | 0.0      |
| Czernik 43 | PW Cas  | 2.8$\sigma$ | o    | *           | o          | o      |      | 1.1$\sigma$ | 2.4    | 0.565    | 0.044    |

This paper has been typeset from a T\textsc{x}/\textsc{t}\textsc{x} file prepared by the author.