Broken unitarity of the SM and a new theory of EW Interactions

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Abstract

It is shown that in an axial-vector field theory the axial-vector field is always accompanied by a spin-0 field which has negative metric. Therefore, unitarity is broken. The same results are found in a theory of charged vector fields which are coupled to two fermions whose masses are different. These results are applied to the SM. It is found that both Z and W fields contain spin-0 component. Their masses are \( m_{\phi^0} = m_t e^{28.4} = 3.78 \times 10^{14} GeV \) and \( m_{\phi^\pm} = m_t e^{27} = 9.31 \times 10^{13} GeV \) respectively.

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They have negative metric which leads to negative probability. Therefore, the unitarity of the SM is broken at about $\sim 10^{14}$ GeV. The masses of $W$ and $Z$ can be generated by two types of interactions mentioned above. A new Lagrangian has been constructed, which is the same as the one of the SM without Higgs sector and the fermions are massive. The masses of the $W$ and the $Z$ bosons are obtained to be $m_{W}^2 = \frac{1}{2} g^2 m_t^2$ and $m_Z^2 = \rho m_W^2 / \cos^2 \theta_W$ with $\rho \simeq 1$. $G_F = \frac{1}{\sqrt{2} m_t}$. A cut-off which is less than $10^{14}$ GeV has to be introduced to the new theory.

1 Introduction

The Lagrangian of the SM after spontaneous symmetry breaking is

$$\mathcal{L} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{q} \{ i \gamma \cdot \partial - m_q \} q$$

$$+ \bar{q}_L \left\{ \frac{g}{2} \gamma \cdot A^i + g' \frac{Y}{2} \gamma \cdot B \right\} q_L + \bar{q}_R g' \frac{Y}{2} \gamma \cdot B q_R$$

$$+ \bar{l} \{ i \gamma \cdot \partial - m_l \} l + \bar{l}_L \left\{ \frac{g}{2} \gamma \cdot A^i - \frac{g'}{2} \gamma \cdot B \right\} l_L - \bar{l}_R g' \gamma \cdot B l_R$$

$$+ \frac{1}{2} m_Z^2 Z^\mu Z^\mu + m_W^2 W^+ W^- + \mathcal{L}_{Higgs}. \quad (1)$$

Comparing with QED and QCD, there are two new interactions, taking (t,b) generation as an example, $Z$, and $W$ are

$$\mathcal{L} = \frac{g}{4} \bar{t} \gamma_{\mu} \gamma_5 t Z^\mu - \frac{g}{4} \bar{b} \gamma_{\mu} \gamma_5 b Z^\mu, \quad (2)$$
\[
\mathcal{L} = \frac{g}{2\sqrt{2}} \bar{t} \gamma_{\mu} (1 + \gamma_5) b W^{+\mu} + \bar{b} \gamma_{\nu} (1 + \gamma_5) t W^{-\nu}.
\]

\[m_t \neq m_b\]

It has been known for a long time that the unitarity of renormalizability of the SM after spontaneous symmetry breaking has been proved in Ref.\[1\]. However, these two new vertices are not included in ’t Hooft’s paper[1]. We need to study their effects.

## 2 Theory of axial-vector field

The Lagrangian of a model of an axial-vector field and a fermion is constructed as[2]

\[
\mathcal{L} = -\frac{1}{4} (\partial_{\mu} a_{\nu} - \partial_{\nu} a_{\mu})^2 + \bar{\psi} \{ i \gamma \cdot \partial + e \gamma \cdot a \gamma_5 \} \psi - m \bar{\psi} \psi.
\]

(3)

The amplitude of the vacuum polarization of axial-vector field is

\[
\Pi_{\mu\nu}^a = \frac{1}{2} (p_{\mu} p_{\nu} - p^2 g_{\mu\nu}) F_{a1}(z) + F_{a2}(z) p_{\mu} p_{\nu} + \frac{1}{2} m_c^2 g_{\mu\nu},
\]

(4)

where

\[
F_{a1}(z) = 1 + \frac{e^2}{(4\pi)^2} \left\{ \frac{1}{3} D \Gamma(2 - \frac{D}{2}) \left( \frac{\mu^2}{m^2} \right) \right\} - 8 f_1(z) + 8 f_2(z) \}
\]

\[
f_2(z) = \frac{1}{z} \int_0^1 dx \log \{ 1 - x (1 - x) z \},
\]

\[
F_{a2}(z) = -\frac{4e^2}{(4\pi)^2} f_2(z),
\]

3
\[ m_a^2 = \frac{2e^2}{(4\pi^2)}D\Gamma(2 - \frac{D}{2})(\frac{\mu^2}{m^2})^2 m^2. \]

The function \( F_{a1} \) is used to renormalize the \( a_\mu \) field. A mass term is generated by massive fermion loop. \( F_{a2} \) is another new term generated by the axial-vector coupling between axial-vector field and massive fermion. The function \( F_{a2} \) is finite and rewritten as

\[ F_{a2}(z) = \xi + (p^2 - m_{a_\phi}^2)G_{a2}(p^2), \quad (5) \]

where \( m_{a_\phi}^2 \) is the mass of a spin-0 state whose existence will be studied below, \( G_{a2} \) is the radiative correction of the term \((\partial_\mu a_\mu)^2\), and

\[ \xi = F_{a2}(\frac{m_{a_\phi}^2}{m^2}). \quad (6) \]

The new perturbation theory of axial-vector field theory is constructed as

\[ \mathcal{L}_{a0} = -\frac{1}{4}(\partial_\mu a_\nu - \partial_\nu a_\mu)^2 + \xi(\partial_\mu a_\mu)^2 + \frac{1}{2}m_{a_\mu}^2 a_\mu^2, \quad (7) \]
\[ \mathcal{L}_{a\bar{a}} = e\bar{\psi}\gamma_\mu\gamma_5\psi a_\mu + \mathcal{L}_c, \quad (8) \]
\[ \mathcal{L}_c = -\xi(\partial_\mu a_\mu)^2 - \frac{1}{2}m_{a_\mu}^2 a_\mu. \quad (9) \]

\( \mathcal{L}_c \) is a counter term of the Lagrangian. \( \mathcal{L}_{a0} \) is the Stueckelberg’s Lagrangian. The axial-vector field \( a_\mu \) has four independent components.

The equation satisfied by \( \partial_\mu a_\mu \) is derived

\[ \partial^2(\partial_\mu a_\mu) - \frac{m_{a_\mu}^2}{2\xi}(\partial_\mu a_\mu) = 0. \quad (10) \]
\( \partial_\mu a^\mu \) is a pseudoscalar field and we define

\[
\partial_\mu a^\mu = b\phi.
\]  

(11)

The equation of the new field \( \phi \) is found

\[
\partial^2 \phi - \frac{m_a^2}{2\xi} \phi = 0, \quad m_\phi^2 = -\frac{m_a^2}{2\xi}.
\]  

(12)

the mass of the \( \phi \) boson is the solution of the equation

\[
2F_{a2}(\frac{m_\phi^2}{m^2})m_\phi^2 + m_a^2 = 0.
\]  

(13)

In order to show the existence of solution we take

\[
\frac{e^2 m^2}{\pi^2 m_a^2} = 1
\]

as an example. The numerical calculation shows that \( \xi < 0 \) and the solution is found to be

\[
m_\phi = 8.02m.
\]

If

\[
\frac{e^2 m^2}{\pi^2 m_a^2} = 0.5
\]

is taken we obtain

\[
m_\phi = 20.42m.
\]
The value of $m_\phi$ increases while $e^2$ decreases. It is necessary to point out that $F_{\alpha\beta}(z)$ is negative in the region of the mass of $\phi$ boson. We separate $a_\mu$ field into a massive spin-1 field $a'_\mu$ and a pseudoscalar field $\phi$

$$a_\mu = a'_\mu + c\partial_\mu \phi,$$

$$\partial_\mu a'^\mu = 0.$$  \hspace{1cm} (14, 15)

The free Lagrangian is divided into two parts

$$L_{a0} = L_{a'0} + L_{\phi0},$$  \hspace{1cm} (16)

$$L_{a'0} = -\frac{1}{4}(\partial_\mu a'_\nu - \partial_\nu a'_\mu)^2 + \frac{1}{2}m_a^2 a'_\mu a'^\mu,$$  \hspace{1cm} (17)

$$L_{\phi0} = \frac{1}{2m_\phi^2} \partial_\mu \phi (\partial^2 + m_\phi^2) \partial^\mu \phi.$$  \hspace{1cm} (18)

The coefficient $c$ is determined by the normalization of $L_{\phi0}$

$$c = \pm \frac{1}{m_a},$$  \hspace{1cm} (19)

and we obtain

$$b = -cm_\phi^2 = \mp \frac{m_\phi^2}{m_a}.$$  \hspace{1cm} (20)

The signs don’t affect the physical results when $\phi$ appears as virtual particle. The propagator of the $a_\mu$ field is derived

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m_a^2} \{-g_{\mu\nu} + (1 + \frac{1}{2\xi}) \frac{p_\mu p_\nu}{p^2 - m_\phi^2}\}.$$  \hspace{1cm} (21)
It can be rewritten as

$$
\Delta_{\mu\nu} = \frac{1}{p^2 - m_a^2} \left\{ -g_{\mu\nu} \frac{p_\mu p_\nu}{m_a^2} \right\} - \frac{1}{m_a^2} \frac{p_\mu p_\nu}{p^2 - m_a^2}.
$$

(22)

The first part is the propagator of the massive spin-1 field and the second part is the propagator of the pseudoscalar field. The propagator of the pseudoscalar is determined to be

$$
-\frac{1}{p^2 - m_\phi^2}.
$$

(23)

It is different from the propagator of a regular spin-0 field by a minus sign. There are problems of indefinite metric and negative probability when the $\phi$ field is on mass shell. $\phi$ is a ghost. Therefore, in the energy region of $m_\phi$ the unitarity of this theory is broken.

### 3 Theory of charged vector fields

In this section we study a theory of charged vector fields which are coupled to two fermions whose masses are different. The Lagrangian is constructed as[2]

$$
\mathcal{L} = -\frac{1}{2} (\partial_\mu v_\nu^+ - \partial_\nu v_\mu^-)^2 + e (\bar{u} \gamma_\mu d v_\mu^- + \bar{d} \gamma_\mu u v_\mu^+) - m_u \bar{u} u - m_d \bar{d} d.
$$

The amplitude of the vacuum polarization is derived

$$
\Pi_{\mu\nu} = \frac{1}{2} (p_\mu p_\nu - p^2 g_{\mu\nu}) F_{\nu-1}(p^2) + F_{\nu-2}(p^2)p_\mu p_\nu + \frac{1}{2} m_\nu^2 g_{\mu\nu}.
$$
where
\[
m_{v^*}^2 = \frac{4D}{(4\pi)^2} g^2 \Gamma(2 - \frac{D}{2}) \int_0^1 dx (\frac{\mu^2}{L_0})^2 \left( m_- + m_+(2x - 1) \right)
\]
\[
F_{v^*1}(p^2) = \frac{4D}{(4\pi)^2} g^2 \Gamma(2 - \frac{D}{2}) \int_0^1 dx (1 - x) (\frac{\mu^2}{L})^2
\]
\[
F_{v^*2}(p^2) = -\frac{4D}{(4\pi)^2} g^2 \frac{1}{p^2} \int_0^1 dx m_- \left( m_- + m_+ (2x - 1) \right) \log \left\{ 1 - \frac{1}{L_0} x (1 - x) p^2 \right\}
\]
where
\[
m_- = \frac{1}{2} (m_u - m_d), \quad m_+ = \frac{1}{2} (m_u + m_d),
\]
\[
L_0 = x m_u^2 + (1 - x) m_d^2, \quad L = L_0 - x (1 - x) p^2.
\]

The equations show that nonzero \( m_{v^*}^2 \) and \( F_{v^*2} \) are resulted in \( m_u \neq m_d \).

The structure of the vacuum polarization is the same as the case of axial-vector field. Both mass and divergence of the vector fields are generated by massive fermion loop. Therefore, the charged vector fields have four independent components too: three spin-1 components and one scalar whose mass can be determined. The scalar component has negative metric which leads to negative probability. The unitarity of this theory is broken at the mass of the scalar.
4 Weinberg’s $2^n d$ sum rule

Pion, $\rho$ meson, and $a_1$ meson are made of u and d quarks.

Why pion is so light?

Why $\rho$ meson is much heavier than pion?

Why $a_1$ meson is much heavier than $\rho$ meson?

It is well known that pion mass is originated in explicit chiral symmetry breaking, the Gell-Mann, Oakes, and Ranner formula. In 1967 Weinberg published a paper[3] in which the Weinberg’s second sum rule is obtained

$$m_a^2 = 2m_{\rho}^2,$$

with an assumption about the high energy behavior of the propagator of the axial-vector fields.

The question is that based on QCD how can we understand the masses of pion, $\rho$, $a_1$, and Weinberg’s rule. Masses of mesons are associated with some kind of symmetry breaking. In QCD there are only two mechanism of breaking chiral symmetry: explicit and dynamical chiral symmetry breaking.

Where is the third one? In Ref.[4] we have proposed an effective chiral theory of light mesons. Taking two flavors as an example,

$$\mathcal{L} = \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a\gamma_5 - mu(x))\psi(x) - \bar{\psi}(x)M\psi(x)$$
\[ + \frac{1}{2} m_0^2 (\rho^\mu \rho_{\mu i} + \omega^\mu \omega_\mu + a^\mu a_{\mu i} + f^\mu f_\mu) \]  

(24)

where \( a_\mu = \tau_\mu a^\mu_i + f_\mu, \ v_\mu = \tau_\mu \rho^\mu + \omega_\mu, \) and \( u = \exp\{i \gamma_5 (\tau_i \pi_i + \eta)\} \), and \( M \) is the current quark matrix. Since mesons are bound states solutions of \( QCD \) they are not independent degrees of freedom. Therefore, there are no kinetic terms for meson fields. The kinetic terms of meson fields are generated from quark loops. \( m \) is a parameter, the constituent quark mass, which related to quark condensate, dynamical chiral symmetry breaking.

This theory has following features:

1. The theory is chiral symmetric in the limit of \( m_q \to 0 \),

2. The constituent quark mass is introduced as \( m \) and the theory has dynamically chiral symmetry breaking(m),

3. VMD is a natural result

\[ \frac{e}{f_v} \left\{ -\frac{1}{2} F^{\mu \nu} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) + A^\mu j^\mu \right\}. \]

4. Axial-vector currents are bosonized

\[ -\frac{g_W}{4 f_a f_a} \left\{ -\frac{1}{2} F^{i \mu \nu} (\partial_\mu a^i_\nu - \partial_\nu a^i_\mu) + A^{i \mu} j^i_\mu \right\} - \frac{g_W}{4} \Delta m^2 f_a A^i_\mu a^{i \mu} - \frac{g_W}{4} f_\pi A^{i \mu} \partial_\mu \pi^i, \]

5. The Wess-Zumino-Witten anomalous action is the leading term of the imaginary part of the effective Lagrangian,
6. Weinberg’s first sum rule is satisfied analytically,

7. All the 10 coefficients of the ChPT are predicted by this theory. ChPT is the low energy limit of the effective chiral theory of mesons.

8. The theory is phenomenologically successful: theoretical results of the masses and strong, E&M, and weak decay widths of mesons agree well with data,

9. The form factors of pion, $\pi_{l3}$, $K_{l3}$, $\pi \rightarrow e\gamma\nu$, and $K \rightarrow e\gamma\nu$ are obtained and agree with data,

$$<r^2>_{\pi} = 0.445 \text{ fm}^2, \quad \text{Exp.} = 0.44 \pm 0.01 \text{ fm}^2, \quad \rho - \text{pole} = 0.39 \text{ fm}^2.$$  

$\pi\pi$ and $\pi K$ scatterings are studied. Theory agrees with data,

10. The parameters of this theory are: $m$(quark condensate), $g$(universal coupling constant), and three current quark masses,

11. Large $N_C$ expansion is natural in this theory. All loop diagrams of mesons are at higher orders in $N_C$ expansion,

12. A cut-off has been determined to be $1.8 \text{GeV}$. All the masses of mesons are below the cut-off. The theory is self consistent,
To the leading order in quark mass expansion, the masses of the octet pseudoscalar and vector mesons are derived

\[
m^2_\pi = -\frac{2}{f_\pi^2} (m_u + m_d) < 0|\bar{\psi}\psi|0>,
\]

\[
m^2_\rho = 6m^2.
\] (25)

Therefore, in this theory explicit chiral symmetry breaking is responsible for \(m_\pi\) and \(m_\rho\) is revealed from dynamical chiral symmetry breaking. A new mechanism of chiral symmetry breaking is found in this theory, which is obtained from

\[
\bar{\psi}\gamma_\mu\gamma_5\tau^i\psi a^i_\mu.
\]

It is not anomaly, there are two \(\gamma_5\). However, because of the anticommuter property of \(\gamma_5\) an additional mass term is obtained from quark loop

\[
(1 - \frac{1}{2\pi^2g^2})m^2_a = m^2_\rho + m^2_\rho
\]

On the LHS there is a factor

\[
1 - \frac{1}{2\pi^2g^2}
\]

which is obtained from the the behavior of the propagator \(a_1\) field at high energy. Besides the kinetic term of \(a_1\) field

\[
-\frac{1}{4}(\partial_\mu a_\nu - \partial_\nu a_\mu)^2
\]
there is additional divergent term
\[
\frac{1}{4\pi^2 f^2_a} (\partial_\mu a^{i\mu})^2
\]
is obtained. The new chiral symmetry breaking originates in the axial-vector coupling between axial-vector fields and massive fermions \(m\).

5 Spin-0 component of the Z-field

We choose unitary gauge to study the properties of Z and W fields of the SM\([2]\). Of course, in the SM besides the diagrams of vacuum polarization by fermions there are other diagrams in which propagators of the intermediate bosons are involved. However, only after taking the vacuum polarization of fermions into account the propagator of boson can be defined. Therefore, we study the effects of vacuum polarization of fermions first. Using the unitary gauge, after spontaneous symmetry breaking the free Lagrangian of W and Z bosons in the original perturbation theory is defined as

\[
\mathcal{L}_0 = -\frac{1}{4} (\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 + \frac{1}{2} m^2_Z Z_\mu Z^\mu \\
- \frac{1}{2} (\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) (\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) + m^2_W W^+_\mu W^-\mu.
\]

There are other free Lagrangian of photon, fermions and Higgs respectively. Both W and Z are massive spin-1 fields which have three independent degrees of freedom.
We take the t- and b-quark generation as an example

\[ \mathcal{L} = \frac{\bar{g}}{4} \{ (1 - \frac{8}{3} \alpha) \bar{t} \gamma \mu t + \bar{t} \gamma \mu \gamma_5 t \} Z^\mu - \frac{\bar{g}}{4} \{ (1 - \frac{4}{3} \alpha) \bar{b} \gamma \mu b + \bar{b} \gamma \mu \gamma_5 b \} Z^\mu, \quad (26) \]

where \( \alpha = \sin^2 \theta_W \). The S-matrix element of the vacuum polarization of t and b quark generation at the second order is obtained

\[
<Z|s^{(2)}|Z> = i(2\pi)^4 \delta(p - p') e^\mu e^\nu \frac{\bar{g}^2}{8} \frac{N_C}{(4\pi)^2} D \Gamma(2 - \frac{D}{2}) \int_0^1 dx \{ x(1 - x)(p_\mu p_\nu - p^2 g_{\mu \nu})[(\frac{\mu^2}{L_t})^2 [(1 - \frac{8}{3} \alpha)^2 + 1] \\
+ (\frac{\mu^2}{L_b})^2 [(1 - \frac{4}{3} \alpha)^2 + 1]] \\
+ m_t^2(\frac{\mu^2}{L_t})^2 g_{\mu \nu} + m_b^2(\frac{\mu^2}{L_b})^2 g_{\mu \nu}, \quad (27) \]

where \( L_t = m_t^2 - x(1 - x)p^2 \), \( L_b = m_b^2 - x(1 - x)p^2 \) and \( \bar{g}^2 = g^2 + g'^2 \). The kinetic term is generated by both the vector and the axial-vector couplings and the mass terms originate in the axial-vector coupling only.

The interaction Lagrangian between Z-boson and the leptons of e and \( \nu_e \) is obtained

\[ \mathcal{L} = \frac{\bar{g}}{4} \bar{\nu}_e \gamma \mu (1 + \gamma_5) \nu_e Z^\mu - \frac{\bar{g}}{4} \{ (1 - 4\alpha) \bar{e} \gamma \mu e + \bar{\nu}_e \gamma \mu \gamma_5 e \} Z^\mu. \quad (28) \]

We obtain

\[
<Z|s^{(2)}|Z> = i(2\pi)^4 \delta(p - p') \bar{g}^2 e^\mu e^\nu \frac{1}{8} \frac{1}{(4\pi)^2} D \Gamma(2 - \frac{D}{2}) \int_0^1 dx \{ x(1 - x)(p_\mu p_\nu - p^2 g_{\mu \nu})[(\frac{\mu^2}{L_e})^2 [(1 - 4\alpha)^2 + 1] + 2(\frac{\mu^2}{L_\nu})^2] \\
+ m_e^2(\frac{\mu^2}{L_e})^2 g_{\mu \nu} + m_\nu^2(\frac{\mu^2}{L_\nu})^2 g_{\mu \nu}, \quad (29) \]

14
There are other two lepton generations contributing to the vacuum polarization.

The amplitude of the vacuum polarization of fermions is expressed as

\[ \Pi_{\mu\nu}^Z = \frac{1}{2} F_{Z1}(z)(p_\mu p_\nu - p^2 g_{\mu\nu}) + F_{Z2}(z)p_\mu p_\nu + \frac{1}{2} \Delta m_Z^2 g_{\mu\nu}, \quad (30) \]

\[
F_{Z1} = 1 + \frac{g^2}{64\pi^2} \left( \frac{D}{12} \Gamma(2 - \frac{D}{2}) \right) [N_C y_q \sum_q \left( \frac{\mu^2}{m_q^2} \right) z_q + y_l \sum_l \left( \frac{\mu^2}{m_l^2} \right) z_l] \\
- 2[N_C y_q \sum_q f_1(z_q) + y_l \sum_l f_1(z_l)] + 2[\sum_q f_2(z_q) + \sum_{l=e,\mu,\tau} f_2(z_l)],
\]

\[
F_{Z2} = -\frac{g^2}{64\pi^2} \left( \frac{D}{12} \right) \left( N_C \sum_q f_2(z_q) + \sum_{l=e,\mu,\tau} f_2(z_l) \right), \quad (31) \]

\[
\Delta m_Z^2 = \frac{1}{8} \frac{g^2}{(4\pi)^2} \Gamma(2 - \frac{D}{2}) \left( N_C \sum_q m_q^2 \left( \frac{\mu^2}{m_q^2} \right) z_q + \sum_l m_l^2 \left( \frac{\mu^2}{m_l^2} \right) z_l \right).
\]

where \( y_q = 1 + (1 - \frac{8}{3}\alpha)^2 \) for \( q = t, c, u, \) \( y_q = 1 + (1 - \frac{4}{3}\alpha)^2 \) for \( q = b, s, d, \) \( y_l = 1 + (1 - 4\alpha)^2, \) for \( l = \tau, \mu, e, \) \( y_l = 2 \) for \( l = \nu_e, \nu_\mu, \nu_\tau, \) \( z_i = \frac{\mu^2}{m_i^2}. \) In the SM the Z boson gains mass from the spontaneous symmetry breaking and the mass term \( \Delta m_Z^2 \) has been refereed to the renormalization of \( m_Z^2. \) Both the vector and axial-vector couplings contribute to \( F_{Z1}. \) Only the axial-vector coupling contributes to \( F_{Z2}. \) \( F_{Z2} \) is finite.

The function \( F_{Z1} \) is used to renormalize the Z-field. The term \( F_{Z2} \) in Eq.(31) indicates that Z field has four independent degrees of freedom(see section (2)). We rewrite it as

\[ F_{Z2}(z) = \xi_Z + (p^2 - m_{\phi^0}^2) G_{Z2}(p^2), \quad (32) \]

where \( m_{\phi^0}^2 \) is the mass of a new neutral spin-0 field, \( \phi^0, \) which will be studied, \( G_{Z2} \) is the
radiative correction of this term, and

\[ \xi_Z = F_{Z2}|_{p^2=m_{\phi^0}^2}. \]  (33)

In the original free Lagrangian Z field only has three degrees of freedom and the divergence of Z field is zero. Therefore, the divergence term, \( F_{Z2} \), must be included in the new "free Lagrangian" to satisfy unitarity. The new free Lagrangian of the Z-field is constructed as

\[ \mathcal{L}_Z = \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 + \xi_Z (\partial_\mu Z^\mu)^2 + \frac{1}{2}m_{Z}^2 Z_\mu^2. \]  (34)

The equation of \( \partial_\mu Z^\mu \) is derived

\[ \partial^2 (\partial_\mu Z^\mu) - \frac{m_Z^2}{2\xi_Z} (\partial_\mu Z^\mu) = 0. \]  (35)

\( \partial_\mu Z^\mu \) is an independent spin-0 field. Following section(2) we have

\[ Z_\mu = Z'_\mu \pm \frac{1}{m_Z} \partial_\mu \phi^0, \]  (36)

\[ \partial_\mu Z'^\mu = 0, \]  (37)

\[ \phi^0 = \mp \frac{m_Z}{m_{\phi^0}} \partial_\mu Z^\mu, \]  (38)

\[ \partial^2 \phi^0 - \frac{m_Z^2}{2\xi_Z} \phi^0 = 0. \]  (39)

The mass of \( \phi^0 \) is obtained

\[ 2m_{\phi^0}^2 F_{Z2}|_{p^2=m_{\phi^0}^2} + m_Z^2 = 0, \]  (40)

\[ m_{\phi^0}^2 = -\frac{m_Z^2}{2\xi_Z}. \]  (41)

16
\[ 3 \sum_q \frac{m_q^2}{m_Z^2} z_q f_2(z_q) + \sum_l \frac{m_l^2}{m_Z^2} z_l f_2(z_l) = \frac{32\pi^2}{g^2}. \]  

(42)

For \( z > 4 \) it is found that

\[ f_2(z) = -\frac{2}{z} - \frac{1}{z} \left(1 - \frac{4}{z}\right)^{\frac{1}{2}} \log \frac{1 - (1 - \frac{4}{z})^\frac{1}{2}}{1 + (1 - \frac{4}{z})^\frac{1}{2}}. \]  

(43)

Because of the ratios of \( \frac{m_q^2}{m_Z^2} \) and \( \frac{m_l^2}{m_Z^2} \) top quark dominates and the contributions of other fermions can be ignored. The equation has a solution at very large value of \( z \). For very large \( z \) Eq.(42) becomes

\[ \frac{2(4\pi)^2}{g^2} + \frac{6m_t^2}{m_Z^2} = 3 \frac{m_t^2}{m_Z^2} \log \frac{\phi_0}{m_t^2}. \]  

(44)

The mass of the \( \phi^0 \) is determined to be

\[ m_{\phi^0} = m_t e^{\frac{m_t^2}{3932} + 1} = m_t e^{28.4} = 3.78 \times 10^{14} \text{GeV}, \]  

(45)

and

\[ \xi_Z = -1.18 \times 10^{-25}. \]

The neutral spin-0 boson is extremely heavy.

The propagator of Z boson is found

\[ \Delta_{\mu\nu} = \frac{1}{p^2 - m_Z^2} \left\{ -g_{\mu\nu} + \left(1 + \frac{1}{2\xi_Z} \right) \frac{p_\mu p_\nu}{p^2 - m_{\phi^0}^2} \right\}, \]  

(46)

It can be separated into two parts

\[ \Delta_{\mu\nu} = \frac{1}{p^2 - m_Z^2} \left\{ -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_Z^2} \right\} - \frac{1}{m_Z^2} \frac{p_\mu p_\nu}{p^2 - m_{\phi^0}^2}. \]  

(47)
The first part is the propagator of the physical spin-1 Z boson and the second part is the propagator of a new neutral spin-0 meson, $\phi^0$.

The propagator of Z boson takes the same form in the renormalization gauge in original perturbation theory. However, from physical point of view they are different. The SM is gauge invariant before spontaneous symmetry breaking. Therefore, in general, there is a gauge fixing term in the Lagrangian of the SM. In the study presented above the gauge parameter has been chosen to be zero, unitary gauge. The differences between present propagator and the propagator of renormalization gauge in the original perturbation theory of the SM are

1. The $\xi_z$ is determined dynamically. As mentioned above, this result is obtained in unitary gauge. Physics results depend on $\xi_z$. The gauge parameter of the renormalization gauge is determined by choosing gauge and because of unitarity physics results are independent of the gauge parameter.

2. In renormalization gauge ghosts are accompanied. However, there are no additional ghosts.

There is a pole at $p^2 = m_{\phi^0}^2$. On the other hand, the minus sign of Eq.(47) indicates that the Fock space has indefinite metric and there is problem of negative probability when $\phi^0$ is on mass shell. $\phi^0$ is a ghost. Unitarity of the SM is broken at $E = m_{\phi^0}$. 

18
The couplings between $\partial_\mu \phi^0$ and $t, b, e, \nu_e$ fermions are found

$$\mathcal{L} = \pm \frac{1}{m_Z} \frac{\bar{g}}{4} \{ \left( 1 - \frac{8}{3} \alpha \right) \bar{\ell} \gamma_\mu t + \bar{\ell} \gamma_\mu \gamma_5 t \} \partial_\mu \phi^0$$

$$\pm \frac{1}{m_Z} \frac{\bar{g}}{4} \{ -(1 - \frac{4}{3} \alpha) \bar{b} \gamma_\mu b - \bar{b} \gamma_\mu \gamma_5 b \} \partial_\mu \phi^0$$

$$\pm \frac{1}{m_Z} \frac{\bar{g}}{4} \{ \bar{\nu}_e \gamma_\mu (1 + \gamma_5) \nu_e - (1 - 4 \alpha) \bar{e} \gamma_\mu e - \bar{e} \gamma_\mu \gamma_5 e \} \partial_\mu \phi^0. \quad (48)$$

The couplings with other generations of fermions take the same form. Using the equations of fermions, it can be found there are couplings between $\phi^0$ and fermions

$$\pm \frac{1}{m_Z} \frac{\bar{g}}{4} 2i \sum_i m_i \bar{\psi}_i \gamma_5 \gamma_5 \psi_i \phi^0, \quad (49)$$

where $i$ stands for the type of fermion. It is the same as the coupling between Higgs and fermion that the interaction is proportional to the fermion mass. However, here is pseudoscalar coupling.

### 6 Spin-0 components of W-fields

In the SM the fermion-W vertices are

$$\mathcal{L} = \frac{g}{4} \bar{\psi} \gamma_\mu (1 + \gamma_5) \tau^i \psi W^{i\mu}, \quad (50)$$

where $\psi$ is a doublet of fermions and summation over all fermion generations is implicated.
The expression of the vacuum polarization of fermions is obtained[2]

\[ \Pi_{\mu\nu}^W = F_{W1}(p^2)(p_\mu p_\nu - p^2 g_{\mu\nu}) + 2F_{W2}(p^2)p_\mu p_\nu + \Delta m_W^2 g_{\mu\nu}, \]  

(51)

where

\[ F_{W1}(p^2) = 1 + \frac{g^2}{32\pi^2} D\Gamma(2 - \frac{D}{2}) \int_0^1 dx x(1 - x) \left\{ N_C \sum_{iq} \left( \frac{\mu_q^2}{L_q^i} \right)^{\xi} + \sum_{il} \left( \frac{\mu_l^2}{L_l^i} \right)^{\xi} \right\} \]

\[ - \frac{g^2}{16\pi^2} \left\{ N_C \sum_{iq} f_{iq}^i + \sum_{il} f_{il}^i \right\} + \frac{g^2}{16\pi^2} \left\{ N_C \sum_{iq} f_{2iq}^i + \sum_{il} f_{2il}^i \right\}, \]  

(52)

\[ F_{W2}(p^2) = -\frac{g^2}{32\pi^2} \left\{ N_C \sum_{iq} f_{2iq}^i + \sum_{il} f_{2il}^i \right\}, \]  

(53)

\[ \Delta m_W^2 = \frac{g^2}{4} \frac{1}{(4\pi)^2} D\Gamma(2 - \frac{D}{2}) \int_0^1 dx \left\{ N_C \sum_{iq} L_q^i \left( \frac{\mu_q^2}{L_q^i} \right)^{\xi} + \sum_{il} L_l^i \left( \frac{\mu_l^2}{L_l^i} \right)^{\xi} \right\}. \]  

(54)

where

\[ L_q^1 = m_b^2 x + m_t^2 (1 - x), \quad L_q^2 = m_s^2 x + m_c^2 (1 - x), \quad L_q^3 = m_d^2 x + m_u^2 (1 - x), \]  

(55)

\[ L_l^1 = m_c^2 x, \quad L_l^2 = m_{c'}^2 x, \quad L_l^3 = m_{c''}^2 x, \]

\[ f_{1q}^i = \int_0^1 dx x(1 - x) \log[1 - x(1 - x) \frac{p^2}{L_q^i}], \]  

(56)

\[ f_{1l}^i = \int_0^1 dx x(1 - x) \log[1 - x(1 - x) \frac{p^2}{L_l^i}], \]  

(57)

\[ f_{2q}^i = \frac{1}{p^2} \int_0^1 dx L_q^i \log[1 - x(1 - x) \frac{p^2}{L_q^i}], \]  

(58)

\[ f_{2l}^i = \frac{1}{p^2} \int_0^1 dx L_l^i \log[1 - x(1 - x) \frac{p^2}{L_l^i}], \]  

(59)
The function $F_{W1}(p^2)$ is used to renormalize the W-field. In the SM W boson gains mass from spontaneous symmetry breaking and the additional mass term has been treated by renormalization. The divergence $F_{W2}$ leads to the existence of two charged spin-0 states, $\phi^\pm$, in the SM. Now we apply the results obtained in sections 2, 3 to the case of W fields. $F_{W2}$ is rewritten as

$$F_{W2} = \xi_W + (p^2 - m_{\phi_W}^2)G_{W2}(p^2),$$

$$\xi_W = F_{W2}(p^2)|_{p^2=m_{\phi_W}^2},$$

where $G_{W2}$ is the radiative correction of the term $(\partial_\mu W^\mu)^2$ and $m_{\phi_W}^2$ is the mass of the charged spin-0 states, $\phi^\pm$, whose existence will be shown below.

The free part of the Lagrangian of W-field is redefined as

$$\mathcal{L}_{W0} = -\frac{1}{2}(\partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu)(\partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu) + 2\xi_W \partial_\mu W^{\pm\mu} \partial_\nu W^{-\nu} + m_W^2 W^{+\mu} W^{-\mu}.$$  \(62\)

The equation satisfied by the divergence of the W-field is derived

$$\partial^2 (\partial_\mu W^{\pm\mu}) - \frac{m_W^2}{2\xi_W} (\partial_\mu W^{\pm\mu}) = 0.$$ \(63\)

\(\partial_\mu W^{\pm\mu}\) are spin-0 fields. Therefore, the W field of the SM has four independent components. The W-field is decomposed as

$$W^{\pm}_\mu = W^\mu_\mu \pm \frac{1}{m_W} \partial_\mu \phi^\pm,$$ \(64\)
\[ \partial_{\mu}W^{'+\mu} = 0, \quad (65) \]

\[ \phi^{\pm} = \pm \frac{m_W}{m_{\phi W}^2} \partial_{\mu}W^{\pm\mu}, \quad (66) \]

where \( W' \) is a massive spin-1 field and \( \phi^{\pm} \) are spin-0 fields. The equation of \( \phi^{\pm} \) is derived

\[ \partial^2 \phi^{\pm} - \frac{m_W^2}{2\xi_W} \phi^{\pm} = 0. \quad (67) \]

The mass of \( \phi^{\pm} \) is determined by the equation

\[ 2m_{\phi W}^2 F_{W2}(p^2)|_{p^2=m_{\phi W}^2} + m_W^2 = 0 \quad (68) \]

and

\[ m_{\phi W}^2 = -\frac{m_W^2}{2\xi_W}. \quad (69) \]

Numerical calculation shows that top quark is dominant in \( F_{W2} \). Keeping the contribution of top quark only,

\[ \frac{p^2}{m_{\phi W}^2} F_{W2} = -\frac{3g^2}{32\pi^2} \frac{m_t^2}{m_W^2} \left\{ -\frac{3}{4} + \frac{1}{2z} + \frac{1}{2} - \frac{1}{z} + \frac{1}{2z^2} \log(z - 1) \right\}, \quad (70) \]

where \( z = \frac{p^2}{m_t^2} \). It has a solution at very large \( z \). At very large \( z \) Eq.\( (68) \) becomes

\[ \frac{p^2}{m_{\phi W}^2} F_{W2} = -\frac{3g^2}{64\pi^2} \frac{m_t^2}{m_W^2} \log z. \quad (71) \]

The mass of \( \phi^{\pm} \) is determined to be

\[ m_{\phi W} = m_t e^{\frac{m_W^2}{3g^2 m_t^2}} = m_t e^{27} = 9.31 \times 10^{13} \text{GeV}, \quad (72) \]
and

$$\xi_W = -3.73 \times 10^{-25}.$$ 

The charged $\phi^\pm$ are very heavy too.

The propagator of $W$-field is derived

$$\Delta^W_{\mu\nu} = \frac{1}{p^2 - m_W^2} \{-g_{\mu\nu} + (1 + \frac{1}{2\xi_W})\frac{p_\mu p_\nu}{p^2 - m_W^2}\},$$

(73)

and it can be separated into two parts

$$\Delta^W_{\mu\nu} = \frac{1}{p^2 - m_W^2} \{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_W^2}\} - \frac{1}{m_W^2} \frac{p_\mu p_\nu}{p^2 - m_W^2}.$$  

(74)

The first part is the propagator of physical spin-1 $W$-field and the second part is related to the propagator of the $\phi^\pm$ field. The physics of the propagator is different from the one derived by using renormalization gauge in the original perturbation theory of the SM. There are no associated charged ghosts. $\xi_W$ is dynamically generated.

The propagator shows that at very high energy there is a pole at $\sqrt{p^2} = m_{\phi W}$. On the other hand, the minus sign of Eq.(73) indicates that there are problems of indefinite metric and negative probability when $\phi^\pm$ are on mass shell. $\phi^\pm$ are ghosts. Unitarity of the SM is broken at $E \sim m_{\phi W}$.

The Lagrangian of interactions between fermions and $\partial_\mu \phi^\pm$ field is found

$$\mathcal{L}_{q\phi} = \pm \frac{1}{m_W} \frac{g}{4} \sum_j \bar{\psi}_j \gamma_\mu (1 + \gamma_5) \tau^i \psi_j \partial^\mu \phi^i,$$

(75)

23
where \( j \) is the type of the fermion and

\[
\phi^1 = \frac{1}{\sqrt{2}}(\phi^+ + \phi^-) \quad \phi^2 = \frac{1}{\sqrt{2}i}(\phi^+ - \phi^-).
\]  

Using the dynamical equations of fermions of the SM, for \( t \) and \( b \) quark generation we obtain

\[
\mathcal{L}_{q\phi} = \pm \frac{i}{m_W} \frac{g}{4}(m_t + m_b) \{ \bar{\psi}_t \gamma_5 \psi_b \phi^+ + \bar{\psi}_b \gamma_5 \psi_t \phi^- \}.
\]

It is the same as Higgs that the coupling is proportional to the fermion mass. However, it is pseudoscalar coupling.

\section{Effects of vacuum polarization by intermediate bosons}

Besides the vacuum polarization of fermions because of the nonlinear nature the intermediate bosons contribute to the vacuum polarization too. Before we proceed to study the effects of the vacuum polarization by intermediate bosons it is necessary to restate the theoretical approach exploited in this paper. The spin-o states of \( Z \) and \( W \) fields are revealed from the vacuum polarization of fermions. Of course, there are vacuum polarizations of intermediate bosons. However, the propagators of \( Z \) and \( W \) fields can be defined only after the vacuum polarization of fermions are taken into account. This is the reason why the vacuum polarization of fermions has been treated differently from others.
After the propagators of $Z$ and $W$ are defined we can proceed to study the effects of the vacuum polarization by intermediate bosons. The interaction Lagrangian of intermediate bosons is obtained from the SM

$$\mathcal{L}_i = i\bar{g}(\partial^\mu Z_\nu - \partial^\nu Z_\mu)W^{-\mu}W^{+\nu}$$

$$+ i\bar{g}Z^\mu\{(\partial^\mu W^+ - \partial^\nu W^+_{\mu})W^-_{\nu} - (\partial^\mu W^- - \partial^\nu W^-_{\mu})W^+_{\nu}\}$$

$$- \bar{g}^2\{Z_\mu Z^\mu W^+_{\nu}W^{-\nu} - Z_\mu Z^\nu W^{+\mu}W^{-\nu}\}. \tag{78}$$

We calculate the contribution of $W$ bosons to the vacuum polarization of $Z$ boson. In the amplitude of the vacuum polarization of $Z$ boson there are three parts: kinetic term, mass term, and the term proportional to $p_\mu p_\nu, F'_{Z2}p_\mu p_\nu$. We are interested in the last term. The calculation shows that only the second term of the Lagrangian contributes to $F'_{Z2}$. We obtain

$$F'_{Z2} = \frac{2\bar{g}^2}{(4\pi)^2} \left\{ \frac{1}{4} \Gamma\left(\frac{2}{D} - \frac{1}{2}\right)(\frac{\mu^2}{m_W^2})^\frac{\gamma}{2} - \frac{1}{12} \right\}$$

$$+ \frac{3}{2} \int_0^1 dx \left[ \frac{3}{z} x(1 - x) - x(5 - 7x) \right] \log[1 - x(1 - x)z]$$

$$+ \frac{2\bar{g}^2b}{(4\pi)^2} \left\{ \frac{13}{40} + \frac{3}{2} \int_0^1 dx x^3(1 - x) \right\}$$

$$[x(m_{\phi_W}^2 - m_W^2) + m_W^2] [x(m_{\phi_W}^2 - m_W^2) + m_W^2 - x(1 - x)p^2]^{-1}$$

$$- \frac{3}{b m_W^2} \int_0^1 dx [2x^3 - \frac{3}{2}] [m_W^2 - x(1 - x)p^2] \log[1 - x^2(1 - x)p^2]$$

$$[x(m_{\phi_W}^2 - m_W^2) + m_W^2 - x(1 - x)p^2]^{-1} \}$$

25
\[-\frac{9}{2} \frac{1}{p^2} \int_0^1 dx \int_0^x dy \left[ y(m_{\phi W}^2 - m_W^2) + m_W^2 \right] \]

\[ \log \left\{ [y(m_{\phi W}^2 - m_W^2) + m_W^2 - x(1 - x)p^2] \right\} \]

\[ [y(m_{\phi W}^2 - m_W^2) + m_W^2]^{-1} \}, \]

where \( z = \frac{p^2}{m_W^2} \) and \( b = 1 + \frac{1}{2\xi_W} \).

There is divergence, therefore, renormalization of the operator \((\partial_\mu Z^\mu)^2\) is required. \( F'_{Z2} \) is written as

\[ F'_{Z2} = F'_{Z2}(m_{\phi^0}) + (p^2 - m_{\phi^0}^2) G'_{Z2}(p^2). \]  \(79\)

\( F'_{Z2} \) is divergent and \( G'_{Z2} \) is finite and another term of radiative correction of \((\partial_\mu Z^\mu)^2\). We define

\[ \xi_Z + F'_{Z2}(m_{\phi^0}) = \xi_Z Z_Z, \]  \(80\)

\[ Z_Z = 1 + \frac{1}{\xi_Z} F'_{Z2}(m_{\phi^0}). \]  \(81\)

\( Z_Z \) is the renormalization constant of the operator \((\partial_\mu Z^\mu)^2\). The renormalization constant \( Z_Z \) defined guarantees the Z boson the same propagator.

In the same way, the contribution of Z and W bosons to the vacuum polarization of W boson can be calculated and the renormalization constant \( Z_W \) can be defined. After renormalization of \( \partial_\mu W^{+\mu} \partial_\nu W^{-\nu} \) we still have the same propagator of W boson. In the same way, the contribution of Higgs to vacuum polarization can be studied too.
8. \( G_F = \frac{1}{2\sqrt{2} m_t^3} \), \( M_Z \), and \( M_W \) without spontaneous symmetry breaking

The SM works until 10^{14} \text{ GeV}. At this energy level unitarity in the SM is broken. New elements need to be added into the theory.

Let’s revisit the Higgs mechanism and spontaneous symmetry breaking:

1. \( W \) and \( Z \) bosons gain masses

2. Unitarity of the SM. For example, \( ee^+ \rightarrow W^+W^- \)

3. renormalizability. For example, \( WW \rightarrow ZZ \).

However,

1. Unitarity is broken at 10^{14} \text{ GeV} and the behavior of the SM at high energies has a problem which affects the renormalizability

2. \( W \) and \( Z \) can gain masses from fermion masses

We try to get rid of the Higgs’s mechanism and spontaneous symmetry breaking, but keep the success of the SM. We propose to take[5]

\[
\mathcal{L} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{q} \{ i\gamma \cdot \partial - M \} q
\]
\[
\begin{align*}
+ & \bar{q}_L \left\{ \frac{g}{2} \tau_i \gamma \cdot A^i + g' \frac{Y}{2} \gamma \cdot B \right\} q_L + \bar{q}_R g' \frac{Y}{2} \gamma \cdot B q_R \\
+ & \bar{l} \left\{ i \gamma \cdot \partial - M \right\} l + \bar{l}_L \left\{ \frac{g}{2} \tau_i \gamma \cdot A^i - g' \frac{2}{2} \gamma \cdot B \right\} l_L - \bar{l}_R g' \gamma \cdot B l_R
\end{align*}
\] (82)

as the Lagrangian of the electroweak interactions. This is the Lagrangian of the SM after spontaneous symmetry breaking in unitary gauge without Higgs. \( A_\mu^i \) fields are still Yang-Mills fields. The Lagrangian doesn’t have \( SU(2)_L \times U(1) \) symmetry. U(1) symmetry is kept and electric current is conserved. As mentioned above, this L has problem at \( 10^{14} \) GeV. A cutoff has to be introduced under this energy. Therefore, we don’t need to worry about the renormalizability.

As shown in previous sections the masses of W and Z bosons are resulted in fermion masses

1. Z boson gains mass from the axial-vector couplings with massive fermions

2. W bosons gain mass from both the axial-vector couplings with massive fermions and the vector couplings with fermions whose masses are different.

\[
\mathcal{L}_M = \frac{1}{2} \frac{N_C}{(4\pi)^2} \left\{ \frac{D}{4} \Gamma(2 - \frac{D}{2}) \left( 4\pi \frac{\mu^2}{m_1^2} \right)^2 + \frac{1}{2} \ln(1 - ln(1 - x)) \right\}
\]

\[
+ \left\{ \frac{D}{4} \Gamma(2 - \frac{D}{2}) \left( 4\pi \frac{\mu^2}{m_1^2} \right)^2 - \frac{1}{2} \ln(1 - x) \right\}
\]

\[
+ \sqrt{x} \ln \left\{ \frac{1 + \sqrt{x}}{1 - \sqrt{x}} \right\} m_1^2 (g^2 + g'^2) Z_\mu Z^\mu,
\] (83)
where $N_C$ is the number of colors and $x = \left(\frac{m_t^2}{m_1^2}\right)^2$. It is reasonable to redefine the fermion masses by multiplicative renormalization

$$Z_m m_1^2 = m_{1,p}^2,$$

$$Z_m = \frac{N}{(4\pi)^2} \left\{ \frac{N_C}{4} \frac{D}{D} \Gamma\left(2 - \frac{D}{2}\right)(4\pi)^\frac{3}{2} \left(\frac{\mu^2}{m_1^2}\right)^\frac{D}{2} + \frac{1}{2} \left[ 1 - \ln(1 - x) \right] \right\},$$

for each generation of fermions. The index "P" is omitted in the rest of the paper. Now the mass of W boson is obtained

$$m_W^2 = \frac{1}{2} g^2 \left\{ m_t^2 + m_b^2 + m_c^2 + m_s^2 + m_u^2 + m_d^2 + m_e^2 + m_\nu_e + m_\nu_\mu + m_\nu_\tau + m_\tau^2 \right\} \quad (84)$$

Obviously, the top quark mass dominates the $m_W$

$$m_W = \frac{g}{\sqrt{2}} m_t. \quad (85)$$

Using the values $g = 0.642$ and $m_t = 174.3 \pm 5.1 GeV$, it is found

$$m_W = 79.1 \pm 2.3 GeV, \quad (86)$$

which is in excellent agreement with data $80.41 \pm 0.056 GeV$. The Fermi coupling constant is derived

$$G_F = \frac{1}{2\sqrt{2}m_t^2} = 1.024(1 \pm 0.029) \times 10^{-5} m_N^{-2}.$$
The mass formula of the Z boson is written as

\[ m_Z^2 = \rho m_W^2 (1 + \frac{g_2^2}{g_1^2}), \]  

(87)

where

\[ \rho = (1 - \frac{\alpha}{4\pi} f_4)^{-1} \]

\[ f_4 \] is a determined quantity[5]. Ignoring the electromagnetic correction

\[ \rho = 1. \]

Therefore,

\[ m_Z = m_W / \cos\theta_W \]  

(88)

This is the prediction of the Higgs mechanism.

9 conclusions

1. The unitarity of the SM is broken at $10^{14}$ GeV. A cutoff has to be introduced.

2. Based on the success of the SM a new L without Higgs sector is proposed.

3. In this new L W and Z gain masses from massive fermion loops. Correct $m_Z$ and $m_W$ are obtained.
4. The free Lagrangian of Z and W fields are expressed by Eqs.(34,62).

5. In the new theory a cut-off which is less than $10^{14}$ GeV has to be introduced.

6. The propagators of Z and W are

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m_Z^2} \{-g_{\mu\nu} + \left(1 + \frac{1}{2\xi_Z}\right)\frac{p_\mu p_\nu}{p^2 - m^2_{\phi^0}}\}.$$ \hspace{1cm} (89)

$$\Delta^W_{\mu\nu} = \frac{1}{p^2 - m_W^2} \{-g_{\mu\nu} + \left(1 + \frac{1}{2\xi_W}\right)\frac{p_\mu p_\nu}{p^2 - m^2_{\phi^W}}\}.$$ \hspace{1cm} (90)

7. This theory predicts no Higgs. This theory can be tested by precision measurements.

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