Non-perturbative non-locality: $V \pm A$ correlators in euclidean position space

Ralf Hofmann

Max-Planck-Institut für Physik
Werner-Heisenberg-Institut
Föhringer Ring 6, 80805 München
Germany

Abstract

Using an OPE modification, which takes non-perturbative non-locality into account, the difference and sum of vector and axial-vector correlators are evaluated in euclidean position space. For distances up to 0.8 fm the calculated behavior is close to the instanton liquid result and the ALEPH data, in contrast to the implications of the conventional OPE.
The determination of the momentum or distance range in which a truncated operator product expansion (OPE) in QCD approximates well is very important for many applications. Based on global quark-hadron duality the low-energy precision measurements of the spectral functions of vector and axial-vector channels (ALEPH experiment $\tau \rightarrow \nu_\tau + \text{hadrons}$ [1]) could be confronted with the associated OPE’s [2] but also with the instanton liquid model [3] and lattice calculations [4].

In this letter we investigate how OPE-based non-perturbative non-locality affects the behavior of the $V \pm A$ correlators in euclidean position space. The results are compared to the instanton liquid data of [3] and the conventional OPE approach.

Working in the chiral limit, in which both $\Pi_{V,A}^{\mu\nu}$ are transverse, and applying naive vacuum saturation at dimension 6, we obtain the following truncated OPE’s [3] at a large normalization scale $Q_0$

\[
R_{V-A}(x)|_{Q_0} \equiv \left. \frac{\Pi^V(x) - \Pi^A(x)}{2\Pi^0(x)} \right|_{Q_0} = \frac{\pi^3}{9} \alpha_s(Q_0) \langle \bar{q}q \rangle_{Q_0} \log[(xQ_0)^2] x^6;
\]
\[
R_{V+A}(x)|_{Q_0} \equiv \left. \frac{\Pi^V(x) + \Pi^A(x)}{2\Pi^0(x)} \right|_{Q_0} = 1 - \frac{\pi^2}{96} \left< \frac{\alpha_s}{\pi} \left( F_{\mu\nu}^a \right)^2 \right>_{Q_0} x^4 - \frac{2\pi^3}{81} \alpha_s(Q_0) \langle \bar{q}q \rangle_{Q_0} \log[(xQ_0)^2] x^6.
\]

Thereby, we have only taken into account the leading-order $\alpha_s$ corrections in each mass dimension. $\Pi^0$ denotes the parton-model result which is equal for $V \pm A$. The $V - A$ correlator is free of purely perturbative contributions and only sensitive to chiral symmetry breaking. On the other hand, the $V + A$ correlator includes pure perturbation theory and moreover should directly reflect non-perturbative gluonic dynamics. Since the running of $\alpha_s$ almost cancels the log-powers at dimension 6, which come from the anomalous dimensions of the contributing operators, we can consider $\alpha_s$ to be fixed at scale $Q_0$ [2].

A modification of the conventional OPE was proposed in [5] and applied to light-quark channels in [6]. Since the idea is explained at length in these two papers we can be short here. For dimension 4 it was argued in [5] that a finite correlation in the non-perturbative piece of the associated gauge invariant correlation function must decrease the relevance of fundamental field operators at low resolution. Coarse graining the vacuum expectation values (VEV’s) of dimension 4 operators, an evolution equation was obtained which has the following solution in euclidean momentum space

\[
A(Q) = A(Q_0) \exp \left[ -\frac{4}{5\lambda} \left( \frac{1}{Q} - \frac{1}{Q_0} \right) \right].
\]

Thereby, $A(Q_0)$ is the VEV of a fundamental, gauge invariant composite and $Q_0$ the “fundamentality” scale down to which the description in terms of fundamental fields is sufficiently accurate. The (resolution independent) correlation length is denoted by $\lambda$.  

1
Figure 1: $R_{V-A}$ as a function of distance $x$ in the chiral limit. The solid line corresponds to the non-perturbatively coarse grained OPE, the dashed line to the conventional OPE. Crosses depict the result of the instanton liquid calculation of [3] which is taken from [4].

Assuming naive vacuum saturation [2] in the sense of 1) in [2], the relevant 2 point functions to evaluate the modified version of eq. (1) are the gauge invariant scalar quark and field strength correlators [7] which have been measured on the lattice [8]. The following parameter values ($q$=quark, $g$=gluon) were obtained for $N_F = 4$ (staggered) and a quark mass $a \cdot m = 0.01$

$$
\lambda_q = 3.1 \text{GeV}^{-1}, \quad A_q(a^{-1} = Q_0 \sim 2 \text{GeV}) = (0.212 \text{GeV})^3; \\
\lambda_g = 1.7 \text{GeV}^{-1}, \quad A_g(a^{-1} = Q_0 \sim 2 \text{GeV}) = 0.015 (\text{GeV})^4.
$$

Using (3) and $\alpha_s(2 \text{ GeV}) = 0.2$ [2] as inputs, appealing to eqs. (1,2) (substituting $Q = 1/x$ in (2)), we are now in a position to calculate $R_{V\pm A}$. Figs. 1 and 2 show the results. For a comparison with the ALEPH data see ref. [3] where a spectral threshold $s_0 = 2.5 \text{GeV}^2$ for the onset of perturbation theory is used in the $x$-space dispersion relation. Due to the moderation by exponential factors the power-like blow up of the non-perturbative corrections in conventional OPE’s is not observed, and the modified OPE seems to converge also at larger distances than the 0.3 fm determined in [3]. Extrapolating the behavior of the lowest dimensions under non-perturbative coarse graining to higher dimensions $d < d_c$, each correction develops a maximum at $x_d = d \times \lambda_d$. Thereby, $d_c$ denotes the critical dimension of the asymptotic expansion which even the modified OPE is believed to be [3]. It is a rather striking observation that our solid lines in Figs. 1 and 2 match the lattice results of ref. [4] almost perfectly. In quenched QCD this lattice calculation is based on the evaluation of the (truncated) overlap quark propagator which subsequently is used to build the correlators.

To summarize we have shown that the concept of a non-perturbative component of OPE-based coarse graining leads in a wider range of distances to more realistic
results for the euclidean $V \pm A$ correlators in position space than the conventional OPE's do.

### Acknowledgements

The author would like to thank Uli Nierste for a stimulating conversation. Financial support from CERN’s theory group during a research stay in June are gratefully acknowledged. The author is indebted to V. I. Zakharov for numerous useful discussions, valuable comments, and the encouragement to write this paper.

### References

[1] R. Barate et al. [ALEPH Collaboration], Z. Phys. C76, 15 (1997); Eur. Phys. J. C4, 409 (1998).

[2] M. Shifman, A. Vainshtein, and V. Zakharov, Nucl. Phys. B147, 385 (1979).

[3] T. Schäfer and E. V. Shuryak, Phys. Rev. Lett. 86, 3973 (2001).

[4] T. deGrand, [hep-lat/0106001](https://arxiv.org/abs/hep-lat/0106001).

[5] R. Hofmann, [hep-ph/0109007](https://arxiv.org/abs/hep-ph/0109007).

[6] R. Hofmann, [hep-ph/0109008](https://arxiv.org/abs/hep-ph/0109008).

[7] H. G. Dosch, Phys. Lett. B 190, 177 (1987).

H. G. Dosch and Yu. A. Simonov, Phys. Lett. B 205, 339 (1988).
[8] M. D’Elia, A. Di Giacomo, and E. Meggiolaro, Phys. Lett. B408, 315 (1997).
A. Di Giacomo, E. Meggiolaro, and H. Panagopoulos, Nucl. Phys. B483, 371 (1997).
M. D’Elia, A. Di Giacomo, E. Meggiolaro, Phys. Rev. D59, (1999) 054503.

[9] A. Bode et al., hep-lat/0105003.