Matter-wave coherence limit owing to cosmic gravitational wave background

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Abstract

We study matter-wave interferometry in the presence of a stochastic background of gravitational waves. It is shown that if the background has a scale-invariant spectrum over a wide bandwidth (which is expected in a class of inflationary models of Big Bang cosmology), then separated-path interference cannot be observed for a lump of matter of size above a limit which is very insensitive to the strength and bandwidth of the fluctuations, unless the interferometer is servo-controlled or otherwise protected. For ordinary solid matter this limit is of order 1–10 mm. A servo-controlled or cross-correlated device would also exhibit limits to the observation of macroscopic interference, which we estimate for ordinary matter moving at speeds small compared to $c$.

Keywords: Gravitational wave, matter wave, interferometry, cosmic inflation, decoherence

1. Introduction

In a series of papers, Lamine et al. [1, 2], have investigated the effect of a stochastic background of weak gravitational waves on matter-wave interferometers. We use their treatment to obtain the following remarkable observation. If the early universe generated gravitational waves with a scale-invariant spectrum over a wide bandwidth (which is expected in a class of inflationary models of Big Bang cosmology), we show that there is a near-universal cut-off distance scale for the observation of matter wave interference. That is, unless the interferometer (as in figure 1a) is shielded or otherwise protected from these primordial waves, the fringe visibility will go rapidly to zero when the radius $R$ of the interfering object exceeds a value which turns out to be of the order of a few millimetres for ordinary solid matter. This is not an absolutely unavoidable decoherence, only a technological difficulty, so it is not possible to settle the quantum measurement problem by this route. However, we also find that cross-correlation or servo-control will not suffice to avoid the decoherence altogether, but merely...

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extend the size or mass limit by an amount which depends weakly on the integration time. To avoid this source of decoherence altogether, it would suffice to shield the interferometer from the fluctuating gravitational background, but this is very hard to do.

2. Calculation

We are interested in a generic interferometer, but for the sake of making precise calculations we consider a Mach-Zehnder interferometer in which de Broglie waves associated with a sphere of proper radius $R$ interfere, and in which the separation of the arms is $2R$ (see figure 1). For such an interferometer, both the interfering object and the separation of the interfering paths becomes of macroscopic size when $R$ is large enough.

Our treatment follows that of Lamine et al. \cite{1,2,3}. In the presence of a stochastic gravitational background with a spectral density of strain fluctuations $S_h(\omega)$, the variance of the phase of a matter-wave interferometer of the type shown in figure 1 is

$$\Delta \phi^2 = \int_0^\infty \frac{d\omega}{2\pi} S_h(\omega) A(\omega) f(\omega)$$

where $A(\omega)$ is the response of the interferometer to a wave of given frequency, and $f(\omega)$ is a high pass filter function. For the case where the sphere’s group velocity $v \ll c$, and the wavelength of the gravitational wave is large compared to the interferometer, $kR \ll 1$, one finds \cite{2}

$$A(\omega) = \left( \frac{4mv^2}{\hbar \omega} \right)^2 \sin^2(2\alpha) \sin^4 \left( \frac{\omega \tau}{2} \right)$$

where $\tau = R/(v \sin \alpha)$ is the time taken to traverse half of one arm of the interferometer.

The filter function represents the fact that low frequency ‘noise’ is not noise but signal—if the fringes move slowly enough then their movement can be tracked by the interference experiment. We adopt the filter function $f(\omega) = \ldots$
Figure 2: Gravitational wave sources. The upper shaded regions are ruled out by various studies\cite{4, 5, 6, 7, 8, 9, 10, 11}; the lower shaded region shows, approximately, the range and distribution of primordial cosmological waves predicted by one type of inflationary scenario. The curves show the spectra of the main ‘ordinary’ sources predicted by standard physics and astronomical surveys.

\begin{equation}
\omega^2/(\omega^2 + \gamma^2)
\end{equation}

where \( \gamma \) is the bandwidth. We will discuss this bandwidth after obtaining an expression for \( \Delta \phi^2 \) in the presence of stochastic cosmological gravity waves.

Sources of gravitational waves include various ‘ordinary’ processes such as neutron star binaries, black hole binaries, and rotating neutron stars, and ‘exotic’ processes that are not yet well established but which are postulated in theoretical descriptions of early universe physics (figure 2)\cite{6, 5, 9, 7, 8}. We suppose the waves from ordinary sources represent a signal which can be discriminated by its temporal and other signatures, and therefore the noise is only given by the exotic sources. This is outside the present range of confident knowledge, but inflationary models of Big Bang cosmology suggest that there is now, throughout the universe, a low-level gravitational noise of very wide bandwidth. Observations of the cosmic microwave background can in principle detect this gravitational noise at very low frequencies, through the Sachs-Wolfe effect\cite{12, 13}. This is not the frequency range we are interested in, but it is useful because it is by far the most sensitive existing experimental observation. The WMAP and Planck measurements currently place an upper bound on the low-frequency gravitational noise close to the level at which inflationary models suggest it is present, thus it remains undetected but the models remain viable\cite{8}.

The cosmological gravitational wave spectrum is usually described in terms of the measure \( \Omega_{GW} = \nu \tilde{\rho}/(\rho_0 c^2) \), where \( \rho_0 \) is the mass density that would close the universe and \( \tilde{\rho} \) is the spectral energy density of the gravitational radiation (that is, the energy density per unit frequency range \( \Delta \nu \)). \( \Omega_{GW} \) is dimensionless, and is related to \( S_h(\nu) \) by \( S_h = 3H_0^2\Omega_{GW}/\omega^3 \) where \( H_0 \approx 2.4 \times 10^{-18} \text{ s}^{-1} \) is
the Hubble parameter \[ H_0 \]. The important point for our discussion is that inflationary models suggest that \( \Omega_{GW} \) is independent of \( \omega \) over a wide bandwidth—a property called scale invariance. The bandwidth is normally reported in the range \( 10^7 \rightarrow 10^9 \) Hz. We will model this by adopting a simple cut-off at a frequency \( \omega_c \). Hence Eq. (1) reads

\[
\Delta \phi^2 = \int_{0}^{\omega_c} \frac{d\omega}{2\pi} \frac{3H_0^2\Omega_{GW}}{\omega^3} A(\omega) \frac{\omega^2}{\omega^2 + \gamma^2}
\]

\[ = \frac{24H_0^2\Omega_{GW}}{\pi \hbar^2} \frac{m^2(\nu \tau)^4}{3} \int_{0}^{\omega_c} \frac{\sin^4(x/2)}{x^3(x^2 + \gamma^2 \tau^2)} \text{d}x \quad (3)
\]

The integrand in (3) falls quickly once \( x > 2\pi \), with the result that the integral is almost independent of \( \omega_c \) for \( \omega_c \tau > 2\pi \). It is then well approximated (except at very low \( \gamma \tau \)) by the expression

\[
\Delta \phi^2 \approx \frac{24\sqrt{3}H_0^2\Omega_{GW}}{\pi \hbar^2} \frac{m^2(\nu \tau)^4}{1 + 20(\gamma \tau)^{1/4} + 10(\gamma \tau)^n} \quad (4)
\]

where \( n = 2 \) (see figure 3). The parameter \( n \) allows us to consider the effect of different filter functions. For example, a perfect high-pass filter with a sharp cut-off at \( \omega = \gamma \) results in a response given approximately by Eq. (4) with \( n = 4 \).

The above calculation has assumed a point-like model of the interfering sphere, and treats the case where the gravitational wavelength is large compared to the interferometer, i.e. \( \omega_c R \ll c \) which implies \( R \ll 0.05 \text{ m} \) when \( \omega_c = 2\pi \times 10^9 \text{ s}^{-1} \). In order to allow for the extended nature of the interfering sphere, we must consider the fact that, for any given arm of the interferometer, different parts of the sphere may experience different amounts of proper time. Consequently the coordinate associated with the finally measured position of
the sphere may become entangled with internal degrees of freedom, leading to decoherence when the latter are averaged over 19. A symmetric ‘bow-tie’ design for the interferometer can cancel the static part of this effect, but for a randomly fluctuating background, as here, the result is a loss of coherence, and it is one that cannot be avoided by acquiring data quickly so as to track the fluctuations. Consequently, a reasonable rough model of this is to extend the validity of Eq. (4) to all values of $R$, but insist that the bandwidth $\gamma$ of the interferometer must satisfy $\gamma \ll \pi c/R$, so that high-frequency fluctuations always contribute to $\Delta \phi^2$. This is a reasonable approach when the arm separation of the interferometer is similar to the size of the interfering object, as here.

The result (4) has some interesting properties. We have $v \tau = R/\sin \alpha$ so for any given value of $\gamma \tau$, (4) gives $\Delta \phi \propto mR^2/\sin^2 \alpha$—the standard deviation of the phase is proportional to a quadrupole moment associated with $m$ and the size of the interferometer. The interference fringe visibility falls rapidly to zero once $\Delta \phi > \pi$. For a uniform sphere of density $\rho$ one has $m = (4/3)\pi R^3 \rho$, so the critical radius, above which any interference pattern is washed out, is given by

$$R_c = \left( \frac{\sqrt{3} \pi \hbar^2}{512 H_0^2} \right)^{1/10} \left( 1 + 20(\gamma \tau)^{1/4} + 10(\gamma \tau)^n \right)^{1/10} \rho^{1/5} / \Omega_{GW}^{1/10}$$

(taking $\alpha = \pi/4$). As a result of this one tenth power, $R_c$ is very insensitive to $\Omega_{GW}$.

3. Discussion

First consider a single interferometer. The maximum rate at which data points can be acquired is $\nu / 2R = (2\tau \sin \alpha)^{-1}$; this gives a rough estimate of the bandwidth $\gamma$. Hence for this case $\gamma \tau \lesssim 1$, and in this region $R_c$ is almost independent of $\gamma \tau$. Hence we can make a simple statement of near-universal validity: there is a size of a lump of ordinary matter above which interference of the type under discussion is not observable, unless the interferometer is somehow shielded from, or servo-controlled to adjust for, the primordial cosmological gravity waves (assuming they are there). We find that for a silica sphere, for example, the critical radius is 3 mm for $\Omega_{GW} = 10^{-15}$, and 8 mm for $\Omega_{GW} = 10^{-19}$ (the sphere’s mass is then $m = 0.3$ grams and $m = 4.6$ grams, respectively). The values for solid spheres of other common materials are of this same order. This rules out an observation of interference of the type under discussion, unless the interferometer is shielded from such gravity waves, or the local spacetime curvature is measured rapidly enough to allow the interference pattern to be stabilised by servo-control.

By using many interferometers together, the data rate can be made arbitrarily high. Then the effective bandwidth is given by the condition that the interferometers must be spaced by $\geq 2R$ and therefore they are only correlated for fluctuations with correlation length $> 2R$. This implies that the maximum possible bandwidth made available by such cross-correlation is $\gamma \simeq \pi c/R$, and
we have already asserted that in order to allow for the finite extension of the sphere, the value of $\gamma$ must be taken small compared to this.

We consider the case $\gamma = \pi c/10R$. Then $\gamma \tau = (\pi/10 \sin \alpha) c/v$, which is $\gg 1$ since the whole discussion has assumed $v \ll c$. The result is that the previous values of $R_c$ have to be multiplied by $(\pi c/10v)^n/10$, so now the critical radius depends on $v$, though not strongly. At $v = 1$ m/s one finds $R_c = 0.12$ m (4.7 m) for a silica sphere when $\Omega_{GW} = 10^{-15}$ and $n = 2$ (4) respectively.

At small $v$ the interferometer is less sensitive to the noise from gravity waves, but it then becomes more sensitive to other sources of decoherence such as mechanical instability, black body radiation, magnetic field noise and collisions with background gas molecules [16].

4. Conclusion

To sum up, the fact that a fluctuating background of gravitational waves will cause decoherence is not in itself the main result of this paper; both this and the mathematical tools to calculate such effects were already established by previous authors. The main observation of the present work is contained in Eqs (4) and (5). It is that when such a stochastic background has a scale-invariant spectrum over a wide frequency range, then matter-wave interference becomes unobservable, in an unprotected interferometer, for ordinary solid objects above a size which is independent of their velocity (for $v \ll c$) and which depends very little on the strength of the stochastic background. This size is of the order millimetres to centimetres.

In the case of larger objects, one could in principle still detect interference through the use of cross-correlation or servo control methods. However, the present study suggests that this only extends the size range up to a few metres, for ordinary matter traversing the interferometer on a timescale of order seconds. We say ‘suggests’ rather than shows since we have only presented a rough estimate of this case. Nevertheless, these calculations suffice to show that it is questionable whether it makes sense to assert that large objects can exhibit interference effects over large path separations, in the context of the universe as it is, if indeed the universe has a broad stochastic gravitational wave background. This issue has already been raised by Lamine et al. [1,2]; our contribution has been to focus attention on some aspects which are remarkable in their simplicity and generality.

We have used the density of silica (2329 kg/m$^3$) for the numerical examples. The critical distance scale varies as the minus-one-fifth power of this. Very low-density objects of somewhat larger size may therefore exhibit interference. Applied to a sphere having the density of the degenerate matter of a neutron star, the calculation gives $R_c \simeq 10 \mu$m. It would be interesting to explore the issue, what limits might there be for interference of the matter waves that are, presumably, associated with a black hole; this is outside the range of applicability of the methods employed here.

Lamine et al. [2] claim that the decoherence associated with this mechanism constitutes ‘the quantum–classical transition’ and that ‘interferometers
cannot be shielded' against this type of noise. Such claims are not formally justified, because one can in principle propose a thought-experiment which employed gravitational shielding. Passive shielding might, for example, employ vast amounts of viscous matter which absorbed the energy of gravitational waves. Active shielding might employ a massive distortable membrane plus servo mechanisms, designed to generate a canceling gravitational wave signal. An interesting avenue to explore is, whether or not there is any fundamental reason why such schemes could not, in principle, succeed in restoring the coherence of a matter-wave interference experiment.

Our treatment employed ordinary quantum theory on a classical spacetime described by general relativity. It therefore makes no proposal regarding new physics beyond that model. However, if a quantum theory of gravity led to effects that were phenomenologically similar to those of a broadband random fluctuation in spacetime, one which was unavoidable, then we would have a possible route to settling the quantum–classical divide. Calculations such as those in this paper and cited works would then indicate how the resulting limit to interference phenomena relates to the stochastic background.

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