Light Meson decays at BESIII

Benhou Xiang
(for the BESIII Collaboration)
Institute of High Energy Physics, Beijing, China

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Outline

➢ Light meson physics

➢ BESIII: a light meson factory

➢ $\eta/\eta'$ decays at BESIII
  • Decay mechanisms
  • Form factors

➢ Summary
Light Meson Physics

- Light mesons
  - Important roles in particle physics, e.g. strong interactions, Quark Model, CP violation …

- Rich physics
  - Test ChPT predictions
  - EM Form factors
  - Test fundamental symmetries
  - Probe new physics beyond the SM
The BESIII detector records symmetric $e^+e^-$ collisions provided by the BEPCII storage ring.

The facility is used for studies of $\tau$-charm physics.

Collected 10 billion $J/\psi$ Events!

$J/\psi \rightarrow \gamma P, VP, ...$
Decay list of light meson in BESIII

| Decay channel | Physics | Publication |
|---------------|---------|-------------|
| \( \eta' \rightarrow \rho \pi \) | First Observation, BR | PRL118, 012001 (2017) |
| \( \eta' \rightarrow \gamma \gamma \pi^0 \) | BR, B Boson | PRD96, 012005 (2017) |
| \( \eta' \rightarrow \gamma \pi^+ \pi^- \) | BR, Box anomaly | PRL120, 242003 (2018) |
| \( \eta' \rightarrow \pi^+ \pi^- \eta, \eta' \rightarrow \pi^0 \pi^0 \eta \) | Matrix elements, Cusp effect | PRD97, 012003 (2018) |
| \( P \rightarrow \gamma \gamma \) | BRs, Chiral anomaly | PRD97, 072014 (2018) |
| \( \eta' \rightarrow \gamma \gamma \eta \) | UL | PRD100, 052015 (2019) |
| Absolute BR of \( \eta' \) decays | BRs | PRD122, 142002 (2019) |
| \( \eta' \rightarrow 4\pi^0 \) | CP violation, UL | PRD101, 032001 (2020) |
| Absolute BR of \( \eta \) decays | BRs | PRD104, 092004 (2021) |
| \( \eta' \rightarrow \pi^+ \pi^- e^+ e^- \) | BR, CP violation asymmetry | PRD103, 092005 (2021) |
| \( \eta \rightarrow \pi^+ \pi^- \mu^+ \mu^- \) | BR, Decay dynamics | PRD103, 072006 (2021) |
| \( \eta' \rightarrow e^+ e^- e^+ e^- \) | BR | PRD.105.112010(2022) |
| \( \eta' \rightarrow \pi^0 \pi^0 \eta \) | Cusp effect | PRL130, 081901 (2023) |
| \( \eta \rightarrow \pi^+ \pi^- \pi^0, 3\pi^0 \) | Matrix elements, \( m_u - m_d \) | PRD107, 092007 (2023) |
| \( \eta' \rightarrow 4\pi \) | Amplitude analysis | PRD109, 032006 (2024) |
| \( \eta'/\eta \rightarrow \gamma e^+ e^- \) | Form factor | PRD109, 072001 (2024) |
| \( \eta' \rightarrow \pi^+ \pi^- l^+ l^- \) | Form factor, CP violation | JHEP07, 135(2024) |

BESIII: an important role in \( \eta/\eta' \) decays

- Decay mechanisms
- Form factors
Decay mechanisms

- Evidence of the cusp effect in $\eta' \to \pi^0\pi^0\eta$  
  PRL130, 081901 (2023)

- Improved measurement of the decays $\eta' \to \pi^+\pi^-\pi^+(0)\pi^-(0)$ and search for the rare decay $\eta' \to 4\pi^0$  
  PRD109, 032006 (2024)
$\eta' \rightarrow \pi^0 \pi^0 \eta$

high term of $\pi\pi$ rescattering

EPJC 62, 511 (2009)
\eta' \rightarrow \pi^0 \pi^0 \eta

A2 Collaboration
PRD 98, 012001 (2018)

PRD 97, 012003 (2018)
\[ \eta' \rightarrow \pi^0 \pi^0 \eta \]

- Non-relativistic effective field theory
- Evidence of the cusp effect around \(3.5\sigma\).

**With cusp effect**

| Parameters   | Fit I                  | Fit II                | Fit III               | Fit IV                |
|--------------|------------------------|-----------------------|-----------------------|-----------------------|
| \(a\)        | \(-0.075 \pm 0.003 \pm 0.001\) | \(-0.207 \pm 0.013\) | \(-0.143 \pm 0.010\) | \(-0.077 \pm 0.003 \pm 0.001\) |
| \(b\)        | \(-0.073 \pm 0.005 \pm 0.001\) | \(-0.051 \pm 0.014\) | \(-0.038 \pm 0.006\) | \(-0.066 \pm 0.006 \pm 0.001\) |
| \(d\)        | \(-0.066 \pm 0.003 \pm 0.001\) | \(-0.068 \pm 0.004\) | \(-0.067 \pm 0.003\) | \(-0.068 \pm 0.004 \pm 0.001\) |
| \(a_0 - a_2\) | -                      | \(0.174 \pm 0.066\)  | \(0.225 \pm 0.062\)  | \(0.226 \pm 0.060 \pm 0.012\) |
| \(a_0\)      | -                      | \(0.497 \pm 0.094\)  | -                     | -                     |
| \(a_2\)      | -                      | \(0.322 \pm 0.129\)  | -                     | -                     |
| Statistical Significance | -                    | \(3.4\sigma\)        | \(3.7\sigma\)        | \(3.6\sigma\)        |
Chiral anomaly: triangle anomaly, box anomaly, pentagon anomaly

\[ \pi^0 \rightarrow \gamma \gamma \quad \eta' \rightarrow \gamma \pi^+ \pi^- \quad K^+ K^- \rightarrow \pi^+ \pi^- \pi^0 \]

Combination of ChPT and VMD model: (PRD 85, 014014 (2012))
First measurement:
\[ \alpha = 1.22 \pm 0.33 \pm 0.04 \]

If \( \alpha = 1 \), triangle anomaly would be dominated.
\[ \eta' \rightarrow \pi^+ (0) \pi^- (0) \pi^0 \pi^0 \]

\[ B(\eta' \rightarrow \pi^+ \pi^- \pi^0 \pi^0) = (2.12 \pm 0.12 \pm 0.1) \times 10^{-4} \]

\[ B(\eta' \rightarrow \pi^0 \pi^0 \pi^0 \pi^0) < 1.24 \times 10^{-5} \]
Form factors

- Improved measurements of the Dalitz decays $\eta/\eta' \rightarrow \gamma e^+e^-$  
  PRD109, 072001 (2024)

- Measurement of the Electromagnetic Transition Form Factors in the decays $\eta' \rightarrow \pi^+\pi^-l^+l^-$  
  JHEP07, 135(2024)
Form Factor Physics

✓ Describe the complex internal structure or intermediate processes

✓ It determines the size of hadronic quantum corrections in the calculation of the $(g - 2)_{\mu}$

$$a_{\mu} = \frac{1}{2} (g - 2)_{\mu}$$

$$a_{\mu}^{SM} = a_{\mu}^{QED} + a_{\mu}^{EW} + a_{\mu}^{hadron}$$

$$a_{\mu}^{hadron} = a_{\mu}^{HVP} + a_{\mu}^{HLbL}$$

$$a_{\mu}^{HVP} = 6845(40) \times 10^{-11}$$

Hadronic Vacuum Polarization (LO)

$$a_{\mu}^{HLbL} = 92(18) \times 10^{-11}$$

Hadronic Light-by-Light

✓ Experimental input is needed to improve the precision of predictions!
Form Factor Physics

✓ The coupling of $\pi^0$, $\eta$, and $\eta'$ with photon in HLbL can be described using transition form factor (TFF).

✓ TFFs are experimentally accessible in three different processes

TFFs as experimental input!
The decay rate

\[
d\Gamma(P \rightarrow \gamma l^+ l^-) = \frac{2\alpha}{3\pi} \frac{1}{q^2} \sqrt{1 - \frac{4m_l^2}{q^2}} \left(1 + \frac{2m_l^2}{q^2}\right)^3 |F(q^2)|^2
\]

\[
= [\text{QED}(q^2)] \times |F(q^2)|^2
\]

Single-pole: \( F(q^2) = \frac{1}{1 - q^2/\Lambda^2} \)

Multi-pole: \( |F(q^2)|^2 = \frac{\Lambda^2(\Lambda^2 + \gamma^2)}{(\Lambda^2 - q^2)^2 + \Lambda^2 \gamma^2} \)

Slope parameter: \( b_{\eta'} = \frac{a|F(q^2)|}{dq^2} \bigg|_{q^2=0} \)
$\eta/\eta' \rightarrow \gamma e^+e^-$

- **Unbinned** maximum likelihood fit with $M(e^+e^-)$
  - less systematic uncertainties
  - better consideration of resolution

![Graph (a)](image1)

![Graph (b)](image2)
\[ \eta/\eta' \rightarrow \gamma e^+ e^- \]

✧ **Single-pole formula is sufficient for \( \eta \)**

\[
F(q^2) = \frac{1}{1 - q^2/\Lambda^2}
\]

\[ \Lambda_\eta = (0.749 \pm 0.026 \pm 0.008) \text{ GeV}/c^2 \]

✧ **Multi-pole formula for \( \eta' \)**

\[
|F(q^2)|^2 = \frac{\Lambda^2(\Lambda^2 + \gamma^2)}{(\Lambda^2 - q^2)^2 + \Lambda^2\gamma^2}
\]

\[ \Lambda_{\eta'} = (0.802 \pm 0.007 \pm 0.008) \text{ GeV}/c^2 \]

\[ \gamma_{\eta'} = (0.113 \pm 0.009 \pm 0.002) \text{ GeV}/c^2 \]
 Decay amplitude

\[
|A_{\eta'\rightarrow\pi^{+}\pi^{-}l^{+}l^{-}}|^{2}(s_{\pi\pi}, s_{ll}, \theta_{\pi}, \theta_{1}, \phi) = \frac{e^{2}}{8k^{2}} |M(s_{\pi\pi}, s_{ll})|^{2} \times \lambda \left( m_{\eta'}^{2}, s_{\pi\pi}, s_{ll} \right) \times [1 - \beta_{1}^{2} \sin^{2} \theta_{1} \sin^{2} \phi]s_{\pi\pi} \beta_{\pi}^{2} \sin^{2} \theta_{\pi}
\]

\[M(s_{\pi\pi}, s_{ll}) = M_{\text{mix}} \times \text{VMD}(s_{\pi\pi}, s_{ll})\] contains the information of the decaying particle and the form factor.

Within the VMD model, TFF can be parameterized into three separate parts

\[
\text{VMD}(s_{\pi\pi}, s_{ll}) = 1 - \frac{3}{4} \left( c_{1} - c_{2} + c_{3} \right) + \frac{3}{4} \left( c_{1} - c_{2} - c_{3} \right) \frac{m_{V}^{2}}{m_{V} - s_{ll} - im_{V} \Gamma(s_{ll})} + 3 \frac{c_{3}}{2} \frac{m_{V}^{2}}{m_{V}^{2} - s_{ll} - im_{V} \Gamma(s_{ll})} \frac{m_{V,\pi}^{2}}{m_{V,\pi}^{2} - s_{\pi\pi} - im_{V,\pi} \Gamma(s_{\pi\pi})}
\]

Axial anomaly  

VMD contribution  

VMD contribution
By adjusting the values of the $c_i$-parameters, we can switch between the various VMD models.

I. Hidden gauge model: $c_1 - c_2 = c_3 = 1$

II. Full VMD model: $c_1 - c_2 = \frac{1}{3}, c_3 = 1$

III. Modified VMD: $c_1 - c_2 \neq c_3$

For $\eta' \to \pi^+\pi^-e^+e^-$ decay

- $\rho^0$ only can not describe data well.
- $\omega \to \pi^+\pi^-$ decay is necessary!

$$\frac{m^2_{\nu,\pi}}{m^2_{\nu,\pi} - s_{\pi\pi} - im_{\nu,\pi}\Gamma(s_{\pi\pi})} + \frac{\beta e^{i\theta}m^2_\omega}{m^2_\omega - s_{\pi\pi} - im_\omega\Gamma(s_{\pi\pi})}$$
First time to study form factors with $\eta' \rightarrow \pi^+ \pi^- l^+ l^-$:

$$b_{\eta'} = 1.30 \pm 0.19 \text{ (GeV}/c^2)^{-2}$$
Summary

✧ BESIII: a Light Meson Factory!
  ✓ A unique place for light mesons
  ✓ Allow to study light meson decays with high precision

✧ Significant progresses achieved on $\eta/\eta'$ decays
  ✓ $\eta/\eta'$: Decay mechanisms, Form factors…

✧ More results are expected to come soon!
  ✓ $\eta' \rightarrow \pi^+\pi^-\eta$, $\eta' \rightarrow e^+e^-\omega$, ...
  ✓ Rare decays

THANKS
Backup
\( \eta' \rightarrow \pi^0 \pi^0 \eta \)

\[ M = M_{\text{tree}} + M_{\text{one-loop}} + M_{\text{two-loop}} \]

- **Non-relativistic effective field theory**
  
  B. Kubis and S. P. Schneider, EPJC 62, 511 (2009)

- The statistical significance is found to be around \( 3.5\sigma \).
\[ \eta / \eta' \rightarrow \gamma e^+ e^- \]

**Time-like:** \[
\eta' \rightarrow \gamma l^+ l^-
\]

**Space-like:** \[
e^+ e^- \rightarrow e^+ e^- \eta'
\]

**Theory**

![Graph showing data points and theoretical predictions for \( b_\eta \) (GeV/c^2)^2]
**TFF Results**

| \( \eta' \rightarrow \pi^+\pi^-e^+e^- \) | Model I | Model II | Model III |
|--------------------------------------|---------|----------|-----------|
| \( c_1 - c_2 = c_3 = 1 \)          |         | 1/3, \( c_3 = 1 \) |          |
| \( m_V (\text{MeV}/c^2) \)          | 954.3 ± 82.5 ± 36.4 | 857.4 ± 74.3 | 787.5 ± 137.9 |
| \( m_{V^*} (\text{MeV}/c^2) \)      | 765.3 ± 1.1 ± 20.2 | 765.4 ± 1.1 | 764.8 ± 1.3 |
| \( \beta (10^{-3}) \)               | 8.5 ± 1.4 | 8.5 ± 1.4 | 8.1 ± 1.4 |
| \( c_1 - c_2 \)                      | 1.4 ± 0.3 ± 0.1 | 1.4 ± 0.3 | 1.4 ± 0.4 |
| \( \chi^2/\text{ndof}(e^+e^-, \pi^+\pi^-) \) | 65.3/82.0, 44.5/65.0 | 66.1/82.0, 44.3/65.0 | 66.8/82.0, 42.2/65.0 |
| \( b_V (\text{GeV}/c^2)^{-2} \)     | 1.10 ± 0.19 ± 0.07 | 1.36 ± 0.24 | 1.61 ± 0.56 |

| \( \eta' \rightarrow \pi^+\pi^-\mu^+\mu^- \) | Model I | Model II | Model III |
|-----------------------------------------------|---------|----------|-----------|
| \( c_1 - c_2 = c_3 = 1 \)                   |         | 1/3, \( c_3 = 1 \) |          |
| \( m_V (\text{MeV}/c^2) \)                  | 649.4 ± 52.3 ± 35.6 | 601.6 ± 24.0 | 589.6 ± 24.2 |
| \( m_{V^*} (\text{MeV}/c^2) \)              | 757.3 ± 22.6 ± 18.0 | 765.4 ± 17.6 | 774.4 ± 40.7 |
| \( c_1 - c_2 \)                              | 1/3 | 1/3 | 0.01 ± 0.42 |
| \( c_3 \)                                    | 1 | 1 | 0.98 ± 0.38 |
| \( \chi^2/\text{ndof}(\mu^+\mu^-, \pi^+\pi^-) \) | 36.1/34.0, 30.4/46.0 | 36.1/34.0, 30.4/46.0 | 37.4/35.0, 29.9/46.0 |
| \( b_V (\text{GeV}/c^2)^{-2} \)              | 2.37 ± 0.38 ± 0.27 | 2.76 ± 0.22 | 2.88 ± 0.24 |

→ **Large statistical uncertainty of** \( m_V \) **and** \( c_1 - c_2 \)

- A test with \( c_1 - c_2 = c_3 \) gives
  \[
  c_1 - c_2 = c_3 = 1.03 \pm 0.02
  \]
- Provide a weighted average of the slope parameter for \( \eta' \rightarrow \pi^+\pi^-e^+e^- \) and \( \eta' \rightarrow \pi^+\pi^-\mu^+\mu^- \) based on Model I.
  \[
  b_{\eta'} = 1.30 \pm 0.19 \text{ (GeV}/c^2)^{-2}\]