D-wave Charmonium Production in $e^+e^-$ Annihilation at $\sqrt{s} = 10.6$ GeV

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Abstract

We calculate the D-wave charmonium production in $e^+e^-$ annihilation at BELLE and BABAR at $\sqrt{s} = 10.6$ GeV using the non-relativistic QCD factorization formalism, including color singlet and color octet contributions. We analyze the contributions of various processes for $\delta J (J = 1, 2)$ production. We find that both the color singlet and color octet channels may give substantial contributions, and the production rates are estimated to be $\sigma(\delta_1) \simeq 0.043 - 0.16 pb$ and $\sigma(\delta_2) \simeq 0.094 - 0.29 pb$, which, however, are very sensitive to the choice of the color octet matrix elements of the D-wave charmonium states. The measurement of D-wave charmonium production at BABAR and BELLE in the future will be very helpful to test the color octet mechanism and to determine the color octet matrix elements.

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Studies of heavy quarkonium production at high energies are hot topics in recent years. In the conventional picture, the heavy quarkonium production is described in the color-singlet model. In this model, it is assumed that the heavy quark pair must be produced in a color-singlet state at short distance with the same angular-momentum quantum number as the charmonium which is eventually observed. However, with the recent Tevatron data on high $p_T J/\psi$ production, this color-singlet picture for heavy quarkonium production has become questionable. The observed cross sections are larger than the theoretical prediction of the color-singlet model by a factor of about $30 \sim 50$. This is called the $J/\psi$ ($\psi'$) surplus problem. On the theoretical side, the naive color-singlet model may be supplanted by the nonrelativistic QCD (NRQCD) factorization formalism, which allows the infrared safe calculation of inclusive charmonium production and decay rates. In this
approach, the production process is factorized into short and long distance parts, while the latter is associated with the nonperturbative matrix elements of four-fermion operators. So, for heavy quarkonium production, the quark-antiquark pair does not need to be in the color-singlet state in the short distance production stage, which is at the scale of $1/m_Q$ ($m_Q$ is the heavy quark mass). At this stage, the color configuration other than the singlet, i.e. the color-octet is allowed for the heavy quark pair. The later situation for heavy quarkonium production is called the color-octet mechanism. In this production mechanism, heavy quark-antiquark pair is produced at short distances in a color-octet state, and then hadronizes into a final state quarkonium (physical state) nonperturbatively. With this color-octet mechanism, one might explain the Tevatron data on the surplus of $J/\psi$ and $\psi'$ production\[4, 5\].

Aside from the $J/\psi$ production at Tevatron, various heavy quarkonium production processes have been studied to test the color octet mechanism. Among them, the charmonium production in $e^+e^-$ annihilation is particularly interesting, since in this process, the parton structure is simpler, and there is no higher twist effect, so the theoretical uncertainty is smaller and it can even be used to extract the color-octet matrix elements. $J/\psi$ production in $e^+e^-$ annihilation process has been investigated within the color-singlet model\[3, 4, 5, 10, 11\] and color-octet model\[12, 13, 14\]. Recently, BABAR\[15\] and BELLE\[16\] have measured the direct $J/\psi$ production in $e^+e^-$ annihilation at $\sqrt{s} = 10.6GeV$. The total cross section and the angular distribution seem to favor the NRQCD calculation over the color-singlet model\[15\], but some issues still remain. The P-wave charmonium $\chi_{cJ}$ production in $e^+e^-$ annihilation has been discussed in \[17\]. The total $\chi_{c1,2}$ cross sections are dominated by color-octet process, because the C-parity suppresses the process $e^+e^- \to c\bar{c}gg$, which dominates the $J/\psi$ production in the color singlet sector. In order to further test the color octet mechanism, in this paper we study the D-wave charmonium production in $e^+e^-$ annihilation. As pointed out by the authors in \[19\] and \[20\], due to the color octet mechanism the D-wave charmonium production rates could be as large as $J/\psi$ or $\psi'$ at the Tevatron, and could be enhanced by two orders of magnitude at the fixed target experiments. Therefore, it would also be interesting to study the D-wave charmonium production in $e^+e^-$ annihilation.

We would like to mention that among the three spin-triplet $D$ wave charmonium states the $^3D_2$ is the most promising candidate to discover. Its mass is predicted to be in the range of $3.81 - 3.84GeV$ in the potential model calcu-
lation [18], which is above the $D\bar{D}$ threshold but below the $D\bar{D}^*$ threshold. The state $^3D_2(J^{PC} = 2^{--})$ is forbidden to decay into $D\bar{D}$ because of the parity conservation. So this state is predicted to have a narrow decay width of $300 - 400 keV$, and we may easily tag this state through the dominant decay channels, e.g. the $E1$ transitions into $\chi_{cJ=1,2}$ states, and the hadronic transition into $J/\psi + \pi\pi$ (see [19] and [20] for detailed discussions). For the $^3D_1$ state $\psi''(3770)$, it is just above the $D\bar{D}$ threshold and thus has a quite narrow width of about $23 MeV$ and may be tagged via $\psi'' \rightarrow D\bar{D}$. As for the $^3D_3(J^{PC} = 3^{--})$ state, its mass is far above the allowed decay channel $D\bar{D}$ threshold and it has more decay modes, therefore it is expected to be a wide resonance and difficult to observe. So, phenomenologically we are only interested in the $^3D_2$ as well as $^3D_1$ states, which will be discussed below.

The power of the NRQCD formalism stems from the fact that factorization formulae for observables are expansions in the small parameter $v$, where $v$ is the average relative velocity of the heavy quark and antiquark in quarkonium bound state. For charmonium $v^2 \sim 0.3$, and for bottomonium $v^2 \sim 0.1$. NRQCD velocity-scaling rules[21] allow us to estimate the relative size of various NRQCD matrix elements. This information, along with the dependence of the short-distance coefficients on $\alpha_s$, permits us to decide which terms must be retained in expressions for observables to reach a given level of accuracy. At low orders, factorization formulas involve only a few matrix elements, so several observables can be related by a small set of parameters.

In NRQCD the Fock state expansion for the physical $D$-wave charmonium (denoted by $\delta_J$) is

$$|\delta_J\rangle = O(1)|cc(3D_J, \underline{1})\rangle + O(v)|cc(3P_J, \underline{8})g\rangle + O(v^2)|cc(3S_{1,8} or \underline{1})gg\rangle + \cdots. \tag{1}$$

The striking feature of this expansion is that although the amplitudes of these Fock states in the expansion are different in $v$, the contributions to the production rates of all these terms are essentially of the same order of $v^7$. This is very different from the case of $J/\psi$ and $\psi'$.

According to the NRQCD factorization formalism[3], the production process $e^+e^- \rightarrow \delta_J + X$ can be expressed as the following form,

$$d\sigma(e^+e^- \rightarrow \delta_J + X) = \sum_{n} F(e^+e^- \rightarrow n + X)(O_{\delta_J}^n). \tag{2}$$
Here, $n$ denotes the $c\bar{c}$ pair configuration in the expansion terms of Eq. (1) (including angular momentum $^{2S+1}L_J$ and color index 1 or 8). $F(e^+e^- \to n + X)$ is the short distance coefficient for the subprocess $e^+e^- \to n + X$. $\langle O_n^f \rangle$ is the long distance non-perturbative matrix element which represents the probability of the $c\bar{c}$ pair in $n$ configuration evolving into the physical state $\delta_J$. The short distance coefficient $F$ can be calculated by using perturbative QCD in powers of $\alpha_s$. The long distance matrix elements are still not available from the first principles at present. However, the relative importance of the contributions from different terms in Eq. (2) can be estimated by using the NRQCD velocity scaling rules.

The main Feynman diagrams for the production of $\delta_J$ in $e^+e^-$ annihilation are shown in Fig.1, where (a) and (b) are the color singlet processes, and (c) is the color octet process.

For the color-singlet process in Fig.1(a),

$$e^+e^- \to \gamma^* \to c\bar{c}[3D_J, \underline{1}] + g + g,$$

(3)

With the spin-triplet case where $J = 1, 2, 3$, we use explicit Clebsch-Gordan coefficients, and get the following relations for the three cases.

$$\sum_{sm}\langle 1J_z|1s2m\rangle \epsilon^{(m)}_{\alpha\beta} \epsilon^{(s)}_{\rho} = -\left[\frac{3}{20}\right]^{1/2}(g_{\alpha\rho} - \frac{p_{\alpha}p_{\rho}}{4M_c^2})\epsilon^{(J_z)}_{\beta} + (g_{\beta\rho} - \frac{p_{\beta}p_{\rho}}{4M_c^2})\epsilon^{(J_z)}_{\alpha} - \frac{2}{3}(g_{\alpha\beta} - \frac{p_{\alpha}p_{\beta}}{4M_c^2})\epsilon^{(J_z)}_{\rho},$$

(4)

$$\sum_{sm}\langle 2J_z|1s2m\rangle \epsilon^{(m)}_{\alpha\beta} \epsilon^{(s)}_{\rho} = \frac{i}{2\sqrt{6}M_c}\epsilon^{(J_z)}_{\alpha\beta\rho}g^{\sigma\sigma'} + \epsilon^{(J_z)}_{\beta\sigma} \epsilon^{(J_z)}_{\alpha\sigma'}p^\tau g^{\sigma\sigma'},$$

(5)

$$\sum_{sm}\langle 3J_z|1s2m\rangle \epsilon^{(m)}_{\alpha\beta} \epsilon^{(s)}_{\rho} = \epsilon^{(J_z)}_{\alpha\beta\rho}.$$  

(6)

Here, $\epsilon_{\alpha}, \epsilon_{\alpha\beta}, \epsilon_{\alpha\beta\rho}$ are $J = 1, 2, 3$ polarization tensors which obey the projection relations

$$\sum_{m}\epsilon^{(m)}_{\alpha} \epsilon^{(m)}_{\beta} = (-g_{\alpha\beta} + \frac{p_{\alpha}p_{\beta}}{4M_c^2}) \equiv \mathcal{P}_{\alpha\beta},$$

(7)

$$\sum_{m}\epsilon^{(m)}_{\alpha\beta} \epsilon^{(m)}_{\alpha'\beta'} = \frac{1}{2}[\mathcal{P}_{\alpha\alpha'}\mathcal{P}_{\beta\beta'} + \mathcal{P}_{\alpha\beta'}\mathcal{P}_{\beta\alpha'}] - \frac{1}{3}\mathcal{P}_{\alpha\beta}\mathcal{P}_{\alpha'\beta'},$$

(8)

$$\sum_{m}\epsilon^{(m)}_{\alpha\beta\rho} \epsilon^{(m)}_{\alpha'\beta'\rho'} = \frac{1}{6}(\mathcal{P}_{\alpha\alpha'}\mathcal{P}_{\beta\beta'}\mathcal{P}_{\rho\rho'} + \mathcal{P}_{\alpha\beta'}\mathcal{P}_{\beta\alpha'}\mathcal{P}_{\rho\rho'} + \mathcal{P}_{\alpha\beta}\mathcal{P}_{\beta\alpha'}\mathcal{P}_{\rho\rho'} + \mathcal{P}_{\alpha\rho}\mathcal{P}_{\beta\beta'}\mathcal{P}_{\rho\rho'} + \mathcal{P}_{\alpha\rho}\mathcal{P}_{\beta\alpha'}\mathcal{P}_{\rho\rho'} + \mathcal{P}_{\alpha\rho}\mathcal{P}_{\beta\beta'}\mathcal{P}_{\rho\rho'})$$
Figure 1: The main Feynman diagrams for the production of $\delta_J$ in $e^+e^-$ annihilation
\[-\frac{1}{15}(P_{\alpha\beta}P_{\rho\alpha'}P_{\beta'\rho'} + P_{\alpha\beta}P_{\rho\beta'}P_{\alpha'\rho'} + P_{\alpha\beta'}P_{\rho\alpha}P_{\beta'\rho'} \\
+ P_{\alpha\rho}P_{\beta\alpha'}P_{\beta'\rho'} + P_{\alpha\rho}P_{\beta\beta'}P_{\alpha'\rho'} + P_{\alpha\rho}P_{\beta'\alpha}P_{\beta'\rho'} + P_{\alpha\rho}P_{\beta'\beta}P_{\alpha'\rho'} + P_{\alpha\rho}P_{\beta\beta'}P_{\alpha'\rho'} + P_{\alpha\rho}P_{\beta'\beta}P_{\alpha'\rho'}). \tag{9}\]

We choose the parameters:

\[m_c = 1.5 GeV, \alpha_s(2m_c) = 0.26, \quad |R''_D(0)|^2 = 0.015 GeV^7 \tag{10}\]

For the color-singlet matrix element, we use

\[\langle O_8^{\delta J}(3D_J) \rangle = \frac{15(2J + 1)N_c}{4\pi}|R''_D(0)|^2. \tag{11}\]

Because these color-singlet processes have the infrared divergence involved, we introduce an infrared cutoff, which can be set to \(m_c\) on the energy of the outgoing gluons in the quarkonium rest frame. For these color-singlet contributions, we finally obtain

\[\sigma_{\text{singlet}}(\delta_1) = 0.027 pb \]
\[\sigma_{\text{singlet}}(\delta_2) = 0.067 pb \tag{12}\]

For the color singlet process in Fig.1(b), we have estimated it in the fragmentation limit, and found that its contribution to the \(\delta_1\) production cross section is smaller by a factor of about 60 than that for the \(J/\psi\) production cross section which is about 0.1 ~ 0.2 pb. This is consistent with the result given in [22]. So the contribution of Fig.1(b) to the \(\delta_1\) production cross section is only 0.002 ~ 0.004 pb, to \(\delta_2\) is 0.003 ~ 0.006 pb, therefore is negligible.

We now discuss the color octet processes in Fig.1(c)

\[e^+ e^- \to \gamma^* \to g + \bar{c} c[2S+1L_J, \delta], \tag{13}\]

we readily have

\[\sigma(e^+ e^- \to \delta J + g) = C_S\langle O_8^{\delta J}(1S_0) \rangle + C_P\langle O_8^{\delta J}(3P_0) \rangle, \tag{14}\]

with

\[C_S = \frac{64\pi^2 \alpha_s^2}{3s^2 m} 1 - r, \tag{15}\]
\[C_P = \frac{256\pi^2 \alpha_s^2}{9s^2 m^3} \left[ \frac{(1 - 3r)^2}{1 - r} + \frac{6(1 + 3)}{1 - r} + \frac{2(1 + 3r + 6r^2)}{1 - r} \right], \tag{16}\]
where \( r = m^2/s \), \( m \) is the mass of \( \delta_J \), and \( s \) is the \( e^+e^- \) c.m. energy squared. Here we have used the approximate heavy quark spin symmetry relations

\[
\langle O^{\delta_J}_{8}({\bar{3}}P_J) \rangle \approx (2J + 1)\langle O^{\delta_J}_{8}({\bar{3}}P_0) \rangle
\]  

Then we calculated the cross sections of \( \delta_J \) production. The cross section is

\[
\sigma(e^+e^- \rightarrow \delta_J + X) = C_0\langle O^{\delta_J}_{1,8}({\bar{3}}S_1) \rangle + C_1\langle O^{\delta_J}_{8}({\bar{3}}P_0) \rangle + C_2\langle O^{\delta_J}_{1}({\bar{3}}D_J) \rangle
\]  

Using the NRQCD velocity scaling rules, we roughly estimate the \( \delta_J \) color octet matrix elements by relating them to the \( \psi' \) color octet matrix elements:

\[
\langle O^{\delta_J}_{8}({\bar{3}}S_1) \rangle \approx \frac{(2J + 1)}{5}\langle O^{\psi'}_{8}({\bar{3}}S_1) \rangle \approx \frac{(2J + 1)}{5} \times 4.6 \times 10^{-3} \text{GeV}^3
\]

\[
\langle O^{\delta_J}_{8}({\bar{3}}P_0) \rangle \approx \frac{(2J + 1)}{5}\langle O^{\psi'}_{8}({\bar{3}}P_0) \rangle \approx \frac{(2J + 1)}{5} \times 1.3 \times 10^{-3} \text{GeV}^5
\]

Using those color-octet matrix elements, we get

\[
\sigma_{\text{octet}}(\delta_1) \approx 0.016 \text{pb}
\]

\[
\sigma_{\text{octet}}(\delta_2) \approx 0.027 \text{pb}
\]

Then we get the total cross section:

\[
\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \delta_1 + X) \approx 0.043 \text{pb}
\]

\[
\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \delta_2 + X) \approx 0.094 \text{pb}
\]

However, if we use a more radical choice of the \( \delta_J \) color octet matrix elements by relating them to the \( J/\psi \) color octet matrix elements, we would get much larger values for the \( \delta_J \) production cross sections, \( \sigma(\delta_1) \approx 0.16 \text{pb} \) and \( \sigma(\delta_2) \approx 0.29 \text{pb} \), because the \( J/\psi \) color octet matrix elements are much larger than the \( \psi' \) color octet matrix elements\((\langle O^{J/\psi}_{8}({\bar{3}}P_0) \rangle \approx 1.1 \times 10^{-2} \text{GeV}^5\), elements\(\langle O^{\psi'}_{8}({\bar{3}}P_0) \rangle \approx 1.3 \times 10^{-3} \text{GeV}^5\)). Therefore the measurement of \( \delta_J \) production rates in the future will be very helpful to determine the values of the \( \delta_J \) color octet matrix elements.

In summary, we have estimated the D-wave charmonium \( \delta_J(J = 1, 2) \) production rates in \( e^+e^- \) annihilation at \( \sqrt{s} = 10.6 \text{ GeV} \). We find that both color singlet and color octet may give substantial contributions to the production cross sections, of which the values are sensitive to the choice of the \( \delta_J \) color octet matrix elements. We hope that these results can be tested
with higher statistic $e^+e^-$ annihilation data in the future at BABAR and BELLE. For $\delta_2$, its branching fraction of decay mode $J/\psi \pi^+\pi^-$ is estimated to be $B(\delta_2 \to J/\psi \pi^+\pi^-) \simeq 0.12^{+2}_{-3}$, which is only smaller than that of $B(\psi' \to J/\psi \pi^+\pi^-) = 0.324 \pm 0.026$ by a factor of 3. We compare the predicted production rate of $2^{-+}$ D-wave charmonium with that of $\psi'$. With the integrated luminosities of about $30 fb^{-1}$ at $\sqrt{s} = 10.6$ GeV, BELLE gives $\sigma(e^+e^- \to \psi' + X) \simeq 0.67 \pm 0.09 pb$ with $143 \pm 19 \psi'$ events decaying to $J/\psi \pi^+\pi^-$. As a rough estimate, if we choose $\sigma(\delta_2) = 0.094 pb$, about 7 events of $2^{-+}$ state decaying to $J/\psi \pi^+\pi^-$ will be detected, and for $\sigma(\delta_2) = 0.29 pb$ (with larger values for the color-octet matrix elements), it is about 23 events. With more data available at the $B$ Factories in the future, it will be possible to detect this $2^{-+}$ D-wave charmonium state (especially in the latter case). The $^3D_1$ $c\bar{c}$ state $\psi''(3770)$ could also be detected via $\psi'' \to D\bar{D}$ decay.

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