Towards Efficient Large-Scale Network Slicing: An LP Rounding-and-Refinement Approach

Wei-Kun Chen, Ya-Feng Liu, Fan Liu, Yu-Hong Dai, and Zhi-Quan Luo

Abstract—In this paper, we propose an efficient algorithm for the network slicing problem which attempts to map multiple customized virtual network requests (also called services) to a common shared network infrastructure and allocate network resources to meet diverse service requirements. The problem has been formulated as a mixed integer linear programming (MILP) formulation in the literature. We first propose a novel linear programming (LP) relaxation of the MILP formulation. We show that compared with a natural LP relaxation of the MILP formulation, the novel LP relaxation is much more compact in terms of smaller numbers of variables and constraints, and much stronger in terms of providing a better LP bound, which makes it particularly suitable to be embedded in an LP based algorithm. Then we design an efficient two-stage LP rounding-and-refinement algorithm based on this novel LP relaxation. In the first stage, the proposed algorithm uses an iterative LP rounding procedure to place the virtual network functions of all services into cloud nodes while taking traffic routing of all services into consideration; in the second stage, the proposed algorithm uses an iterative LP refinement procedure to obtain a solution for traffic routing of all services with their end-to-end delay constraints being satisfied. Compared with the existing algorithms which either have an exponential complexity or return a low-quality solution, our proposed algorithm achieves a better trade-off between the solution quality and the computational complexity. In particular, the worst-case complexity of our proposed algorithm is polynomial, which makes it suitable for solving large-scale problems. Numerical results demonstrate the effectiveness and efficiency of our proposed algorithm.

Index Terms—LP Relaxation, Network Slicing, Resource Allocation, Rounding-and-Refinement.

I. INTRODUCTION

Network function virtualization (NFV) plays a crucial role in the fifth generation (5G) and beyond 5G networks [2]. Different from traditional networks where service functions are processed by specialized hardwares in fixed locations, NFV efficiently takes the advantage of cloud technologies to configure some specific nodes (called cloud nodes) in the network to process network service functions on-demand, and then flexibly establishes a customized virtual network for each service request. However, as virtual network functions (VNFs) of all services run over a shared common network infrastructure, it is crucial to allocate network (e.g., cloud and communication) resources to meet the diverse service requirements. The above resource allocation problem in the NFV-enabled network is called network slicing in the literature.

A. Related Works

Various approaches have been proposed to solve the network slicing problem or its variants; see [3]-[23] and the references therein. These approaches can generally be classified into two categories: (i) exact algorithms that solve the problem to global optimality and (ii) heuristic algorithms that aim to quickly find a feasible solution for the problem.

More specifically, references [3]-[6] proposed the link-based mixed integer linear programming (MILP) formulations for the network slicing problem and used standard MILP solvers like Gurobi [24] to solve their problem formulations. Reference [7] used a logic-based Benders decomposition approach [25] to solve the link-based MILP formulation. References [8]-[11] proposed the path-based MILP formulations and used a column generation approach [26] to solve the related problems. Though the above three approaches can solve the network slicing problem to global optimality, they generally suffer from low computational efficiency as their worst-case complexities are exponential.

Various heuristic and meta-heuristic algorithms has also been proposed to quickly obtain a feasible solution of the network slicing problem; see [12]-[23]. For instance, references [12]-[19] simplified the solution approach by decomposing the network slicing problem into a VNF placement subproblem (which maps VNFs into cloud nodes in the network) and a traffic routing subproblem (which finds paths connecting two adjacent VNFs in the network) and solving each subproblem separately. To obtain a binary solution for the VNF placement subproblem, references [12]-[13] first solved the linear programming (LP) relaxation of the network slicing problem and then used a rounding strategy while references [14]-[19] used some greedy heuristics (without solving any LP). Once the VNFs are mapped to the cloud nodes, the traffic routing subproblem is solved by using shortest path, $k$-shortest path, or multicommodity flow algorithms. However, solving the VNF placement subproblem without taking the global information (i.e., traffic routing of all services) into account can lead to infeasibility or low-quality solutions. Reference [20] decomposed the whole problem into several
subproblems by dividing the services into several groups and iteratively solve the subproblem for each group using standard MILP solvers. Similarly, this approach failed to take the global information (i.e., VNF placement and traffic routing of all services) into consideration and hence can also lead to infeasibility or low-quality solutions. References [21] and [22]-[23] proposed the simulated annealing based and Tabu search based meta-heuristic algorithms, respectively, to obtain a feasible solution for the network slicing problem.

To summarize, the aforementioned approaches to solve the network slicing problem either have an exponential complexity (e.g., [3]-[11]) or return a low-quality solution (e.g., [12]-[23]). The motivation of this paper is to fill this research gap, i.e., develop an algorithm that finds a high-quality solution of the network slicing problem while still enjoys a polynomial-time computational complexity. Integer programming and LP relaxation techniques play vital roles in the developed algorithm.

B. Our Contributions

The main contribution of this paper is the proposed algorithm, that achieves a good trade-off between high solution quality and low computational complexity, for solving the network slicing problem. In particular,

- We first propose a novel LP relaxation of the MILP problem formulation. We show that compared with the natural LP relaxation of the problem (which directly relaxes the binary variables into continuous variables in [0,1]), the novel LP relaxation enjoys two key advantages: (i) it is much more compact in terms of smaller numbers of variables and constraints; (ii) it is much stronger in terms of providing a better LP bound, which is crucial to the effectiveness of the proposed algorithm. We also analyze the integrality gap of the proposed LP relaxation.

- We then develop a two-stage LP rounding-and-refinement algorithm (LPRR) based on the above novel LP relaxation. Specifically, in the first stage, we solve the VNF placement subproblem by using an iterative LP rounding procedure, which takes traffic routing into account; in the second stage, we solve the traffic routing subproblem by using an iterative LP refinement procedure to find a solution that satisfies the end-to-end (E2E) delay constraints of all services. The proposed algorithm has a guaranteed polynomial-time worst-case complexity, and thus is particularly suitable for solving large-scale problems.

Simulation results demonstrate the effectiveness and efficiency of our proposed LP relaxation and algorithm. To be more specific, our simulation results demonstrate that (i) the novel LP relaxation significantly outperforms the natural LP relaxation in terms of the solution efficiency and providing a better LP bound; (ii) our proposed LPRR algorithm is more effective than the existing state-of-the-art algorithms in [3], [13], and [14] in terms of both solution efficiency and quality.

The rest of the paper is organized as follows. Section II briefly reviews the network slicing problem and its mathematical formulation. Sections III and IV propose the novel LP relaxation and the LPRR algorithm for the network slicing problem, respectively. Section V reports the computational results. Finally, Section VI draws the conclusion.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

Let $G = \{\mathcal{I}, \mathcal{L}\}$ be the connected directed network, where $\mathcal{I} = \{i\}$ and $\mathcal{L} = \{(i, j)\}$ are the sets of nodes and links, respectively. Each link $(i, j)$ has an expected (communication) delay $d_{ij}$ [16], [19], [20], and a total data rate upper bounded by the capacity $C_{ij}$. The set of cloud nodes is denoted as $\mathcal{V} \subseteq \mathcal{I}$. Each cloud node $v$ has a computational capacity $\mu_v$ and processing one unit of data rate requires one unit of (normalized) computational capacity, as assumed in [12].

A set of flows $\mathcal{K} = \{k\}$ is required to be supported by the network. The source and destination nodes of flow $k$ are denoted as $S(k)$ and $D(k)$, respectively, with $\{S(k), D(k)\} \notin \mathcal{V}$. Each flow $k$ relates to a customized service, which is given by a service function chain (SFC) consisting of $\ell_k$ service functions that have to be processed in sequence by the network: $f^k_1 \rightarrow f^k_2 \rightarrow \cdots \rightarrow f^k_{\ell_k}$ [27], [28], [29]. To minimize the coordination overhead, each function must be processed at exactly one cloud node, as required in [4], [12], [16].

If function $f^k_s$, $s \in \mathcal{F}(k) := \{1, \ldots, \ell_k\}$, is processed by cloud node $v$ in $\mathcal{V}$, the expected NFV delay is assumed to be known as $\delta_{v,s}(k)$, which includes both processing and queuing delays [16], [19]. For flow $k$, the service function rates before receiving any function and after receiving function $f^k_s$ are denoted as $\lambda_0(k)$ and $\lambda_s(k)$, respectively. Each flow $k$ has an E2E delay requirement, denoted as $\Theta_k$.

The network slicing problem is to determine functional instantiation, the routes, and the associated data rates on the corresponding routes of all flows while satisfying the capacity constraints on all cloud nodes and links, the SFC requirements, and the E2E delay requirements of all flows. Next, we shall introduce the problem formulation in details.

B. Problem Formulation

1) VNF Placement: We introduce the binary variable $x_{v,s}(k)$ to indicate whether or not function $f^k_s$ is processed by cloud node $v$. Notice that in practice, cloud node $v$ may not be able to process function $f^k_s$ [3], [12], and in this case, we can simply set $x_{v,s}(k) = 0$. Each function $f^k_s$ must be processed by exactly one cloud node, i.e.,

$$\sum_{v \in \mathcal{V}} x_{v,s}(k) = 1, \quad \forall \ k \in \mathcal{K}, \forall \ s \in \mathcal{F}(k).$$

Let $y_v = 1$ denote that cloud node $v$ is activated and powered on; otherwise $y_v = 0$. Thus

$$x_{v,s}(k) \leq y_v, \quad \forall v \in \mathcal{V}, \forall \ k \in \mathcal{K}, \forall \ s \in \mathcal{F}(k).$$

The node capacity constraints can be written as follows:

$$\sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{F}(k)} \lambda_s(k) x_{v,s}(k) \leq \mu_v y_v, \quad \forall v \in \mathcal{V}. \quad (3)$$

2) Traffic Routing: Let $(k, s)$ denote the flow which is routed between the two cloud nodes hosting two adjacent functions $f^k_s$ and $f^k_{s+1}$. Similar to [3], we suppose that there are at most $P$ paths that can be used to route flow $(k, s)$
and denote $\mathcal{P} = \{1, \ldots, P\}$. The main reason for introducing notation $P$ is to model the delay of flow $(k, s)$ and allow the corresponding traffic flow to be split into multiple paths. In practice, flow $(k, s)$ can be divided into an arbitrary number of paths, meaning that $P$ can be set as a sufficiently large integer. It is worthwhile highlighting that setting $P = |\mathcal{L}|$ is sufficient for flow $(k, s)$ to be divided into an arbitrary number of paths. Thus is because, from the classical network flow theory, any traffic flow between two nodes can be decomposed into the sum of at most $|\mathcal{L}|$ routes on the paths and a circulation; see \cite[Theorem 3.5]{30}.

Let $r(k, s, p)$ be the fraction of data rate $\lambda_s(k)$ on the $p$-th path of flow $(k, s)$. Then, the following constraint enforces that the total data rate between the two nodes hosting functions $f_k$ and $f_{k+1}$ is equal to $\lambda_s(k)$:

$$\sum_{p \in \mathcal{P}} r(k, s, p) = 1, \ \forall k \in \mathcal{K}, \ \forall s \in \mathcal{F}(k) \cup \{0\}. \quad (4)$$

Let $z_{ij}(k, s, p) \in \{0, 1\}$ denote whether or not link $(i, j)$ is on the $p$-th path of flow $(k, s)$ and $r_{ij}(k, s, p)$ be the associated fraction of data rate $\lambda_s(k)$. Then

$$\forall (i, j) \in \mathcal{L}, \ \forall k \in \mathcal{K}, \ \forall s \in \mathcal{F}(k) \cup \{0\}, \ \forall p \in \mathcal{P}. \quad (5)$$

The total data rates on link $(i, j)$ is upper bounded by capacity $C_{ij}$:

$$\sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{F}(k) \cup \{0\}} \sum_{p \in \mathcal{P}} \lambda_s(k) r_{ij}(k, s, p) \leq C_{ij}, \ \forall (i, j) \in \mathcal{L}. \quad (6)$$

3) SFC: To ensure that the functions of each flow $k$ are processed in the prespecified order $f_k^1 \rightarrow f_k^2 \rightarrow \cdots \rightarrow f_k^\ell_k$ and for each $s \in \mathcal{F}(k) \cup \{0\}$ and $p \in \mathcal{P}$, \{(i, j) : $z_{ij}(k, s, p) = 1$\} forms a path, we need the flow conservation constraint (7) in the next page. We remark that together with constraint (5), constraint (7) also ensures that \{(i, j) : $r_{ij}(k, s, p) > 0$\} forms a path. Notice that in (7), the term $x_{i,s+1}(k) - x_{i,s}(k)$ takes values 1, 0, or -1 depending on whether (i) node $i$ hosts function $f_{s+1}$ but does not host function $f_s$; (ii) node $i$ hosts functions $f_s$ and $f_{s+1}$ but does not host any of the two functions; or (iii) node $i$ hosts function $f_s$ but does not host function $f_{s+1}$. In constraint (7), when $s = 0$ and $s = \ell_k + 1$, we let $x_{i,s}(k) = 1$, if $i = S(k); \quad x_{i,s}(k) = 0$, otherwise.

4) E2E Delay: Let $\theta(k, s)$ denote the communication delay due to the traffic flow from the cloud node hosting function $f_k$ to the cloud node hosting function $f_{k+1}$. Then

$$\theta(k, s) = \sum_{(i, j) \in \mathcal{L}} d_{ij} z_{ij}(k, s, p), \quad \forall k \in \mathcal{K}, \ \forall s \in \mathcal{F}(k) \cup \{0\}, \ \forall p \in \mathcal{P}. \quad (8)$$

To ensure that flow $k$’s E2E delay is less than or equal to its threshold $\Theta_k$, we need the following constraint:

$$\theta_N(k) + \theta_L(k) \leq \Theta_k, \ \forall k \in \mathcal{K}, \quad (9)$$

where $\theta_N(k) = \sum_{v \in \mathcal{V}} \sum_{s \in \mathcal{F}(k)} d_{v,s}(k) y_{v,s}(k)$ and $\theta_L(k) = \sum_{s \in \mathcal{F}(k) \cup \{0\}} \theta(k, s)$ are the total NFV delay on the nodes and the total communication delay on the links of flow $k$, respectively.

5) MILP Formulation: The network slicing problem is to minimize a weighted sum of the total power consumption of the whole cloud network (equivalent to the number of activated cloud nodes \cite{31}) and the total delay of all services:

$$\min_{x, y, r, \theta} \sum_{v \in \mathcal{V}} y_v + \sigma \sum_{k \in \mathcal{K}} (\theta_L(k) + \theta_N(k)) \quad \text{s.t.} \quad (1) - (9), \quad x_{v,s}(k), y_v \in \{0, 1\}, \ \forall v \in \mathcal{V}, \ k \in \mathcal{K}, \ s \in \mathcal{F}(k),$$

$$r(k, s, p) \leq 0, \ \forall (i, j) \in \mathcal{L}, \ k \in \mathcal{K}, \ s \in \mathcal{F}(k) \cup \{0\}, \ p \in \mathcal{P},$$

where $\sigma$ is a small positive value, meaning that the total power consumption term is much more important than the total delay term. Indeed, as shown in \cite[Proposition 2]{3}, the total number of activated cloud nodes returned by solving problem (MINLP) with a sufficiently small positive value $\sigma$ is equal to the total number of activated cloud nodes returned by solving problem (MINLP) with $\sigma = 0$. We remark that incorporating the total delay term into the objective function is also important in the following sense. First, the problem of minimizing the total number of activated cloud nodes often has multiple solutions and the total delay term can be regarded as a regularizer to make the problem have a unique solution as observed in our simulation results. Second, it will help to avoid cycles in each path connecting the two adjacent service functions in the solution of the problem.

The above problem is a mixed integer nonlinear programming (MINLP) problem due to the nonlinearity in constraint (5). However, constraint (5) can be equivalently linearized \cite{31}. Indeed, from (4), we have $0 \leq r(k, s, p) \leq 1$, which, together with $z_{ij}(k, s, p) \in \{0, 1\}$, implies that

$$0 \leq r_{ij}(k, s, p) \leq r(k, s, p) \leq 1.$$ 

Then constraint (5) can be equivalently reformulated as:

$$r_{ij}(k, s, p) \geq z_{ij}(k, s, p) + r(k, s, p) - 1, \quad (10)$$

$$r_{ij}(k, s, p) \leq z_{ij}(k, s, p), \quad (11)$$

$$r_{ij}(k, s, p) \leq r(k, s, p). \quad (12)$$

Together with $r_{ij}(k, s, p) \geq 0$, the above three constraints ensure that if $z_{ij}(k, s, p) = 1$, we have $r_{ij}(k, s, p) = r(k, s, p)$; otherwise $r_{ij}(k, s, p) = 0$. Thus we can present an equivalent mixed integer linear programming formulation:

$$\min_{x, y, r, \theta} \sum_{v \in \mathcal{V}} y_v + \sigma \sum_{k \in \mathcal{K}} (\theta_L(k) + \theta_N(k)) \quad \text{s.t.} \quad (1) - (4), (6) - (12),$$

$$x_{v,s}(k), y_v \in \{0, 1\}, \ \forall v \in \mathcal{V}, \ k \in \mathcal{K}, \ s \in \mathcal{F}(k),$$

$$r(k, s, p) \leq 0, \ \forall (i, j) \in \mathcal{L}, \ k \in \mathcal{K}, \ s \in \mathcal{F}(k) \cup \{0\}, \ p \in \mathcal{P},$$

$$\theta(k, s) \geq 0, \ \forall k \in \mathcal{K}, \ s \in \mathcal{F}(k) \cup \{0\}. \quad \text{(MILP)}$$

Problem (MILP) can be solved to global optimality using standard MILP solvers like Gurobi \cite{24}.

It is worthwhile highlighting that formulation (MILP) is
\[
\sum_{j:(i,j)\in L} z_{ji}(k,s,p) - \sum_{j:(i,j)\in L} z_{ij}(k,s,p) = \begin{cases} 
0, & \text{if } i \in T \setminus V; \\
x_{i,s+1}(k) - x_{i,s}(k), & \text{if } i \in V, \forall k \in K, \forall s \in F(k) \cup \{0\}, \forall p \in P. 
\end{cases}
\]

equivalent to our recently proposed formulation (NS-II) in [3]. However, to present the SFC constraints (cf. (7)), reference [3] introduced an equivalent but larger virtual network, and the formulation was built upon the virtual network. In this paper, we do not introduce the virtual network, and the proposed formulation (MILP) is built upon the original network. As a result, the number of variables and constraints in the proposed formulation (MILP) are much smaller than that in [3].

C. Complexity Analysis

The computational complexity of the (related) network slicing problem has been studied in the literature. In particular, references [32] and [33] studied a related problem where each slice (service) can be a general virtual graph (a graph that is not limited to be a chain) and showed that the corresponding problem is strongly NP-hard when \( P = 1 \) (i.e., only a single path is allowed to transmit the data flow of each service). References [12] and [34] showed the strong NP-hardness of problem (MILP) when the nodes’ capacities are limited and there is only a single service (but the number of functions in the services’ SFC is not a constant), respectively. The following theorem, however, shows the (strong) NP-hardness of problem (MILP) in two new very special cases and thus reveals the intrinsic difficulty of solving it. This motivates us to develop efficient algorithms for approximately solving problem (MILP), especially when the problem’s dimension is large.

**Theorem 1.** (i) Problem (MILP) is NP-hard even when there is only a single service and there is only a single function in the SFC of this service. (ii) Problem (MILP) is strongly NP-hard even when each node’s capacity, link’s capacity, and service’s E2E delay threshold are infinite. Moreover, there does not exist a constant-ratio approximation algorithm to solve it in this case.

**Proof.** The proof can be found in Section I of [35]. \( \square \)

III. PROPOSED NOVEL LP RELAXATION

In this section, we shall derive a novel LP relaxation for the network slicing problem (MILP), which will be employed in the proposed LPRR algorithm in Section IV.

A. Natural LP Relaxation

As the network slicing problem can be formulated as the MILP problem (MILP), simply relaxing the binary variables \( \{y_v\}, \{x_{v,s}(k)\}, \) and \( \{z_{ij}(k,s,p)\} \) to continuous variables in \([0,1]\) will give a natural LP relaxation:

\[
\min_{x, y, r} \sum_{v \in V} y_v + \sigma \sum_{k \in K} (\theta_L(k) + \theta_N(k))
\]

\[
\text{s.t. } (1)-(4), (6)-(12),
\]

\[
x_{v,s}(k), y_v \in [0,1], \forall v \in V, k \in K, s \in F(k),
\]

\[
r(k,s,p), r_{ij}(k,s,p) \geq 0, z_{ij}(k,s,p) \in [0,1],
\]

\[
\forall (i,j) \in L, k \in K, s \in F(k) \cup \{0\}, p \in P, \theta(k,s) \geq 0, \forall k \in K, s \in F(k) \cup \{0\}. \quad \text{(LP-I)}
\]

However, the above natural LP relaxation has two disadvantages, which are detailed as follows.

First, the natural LP relaxation (LP-I) is very weak in terms of providing a poor LP bound. Indeed, while it has been shown that the three constraints (10)-(12) and \( r_{ij}(k,s,p) \geq 0 \) are equivalent to the nonlinear constraint (5) when \( z_{ij}(k,s,p) \in [0,1] \), it can only be shown that (10)-(12) is a relaxation of the nonlinear constraint (5) when \( z_{ij}(k,s,p) \in (0,1) \). Therefore, the feasible region of the natural relaxation (LP-I) can be larger than that of the nonlinear relaxation of problem (MINLP):

\[
\min_{x, y, r} \sum_{v \in V} y_v + \sigma \sum_{k \in K} (\theta_L(k) + \theta_N(k))
\]

\[
\text{s.t. } (1)-(9),
\]

\[
x_{v,s}(k), y_v \in [0,1], \forall v \in V, k \in K, s \in F(k),
\]

\[
r(k,s,p), r_{ij}(k,s,p) \geq 0, z_{ij}(k,s,p) \in [0,1],
\]

\[
\forall (i,j) \in L, k \in K, s \in F(k) \cup \{0\}, p \in P, \theta(k,s) \geq 0, \forall k \in K, s \in F(k) \cup \{0\}. \quad \text{(NLP)}
\]

and hence the natural LP relaxation (LP-I) is weaker than the nonlinear relaxation (NLP). In other words, the optimal objective value of relaxation (LP-I) can be smaller than that of relaxation (NLP). Please see the following example for an illustration.

**Example 1.** Consider the toy example in Fig. 1. There are two links from node \( S \) to node \( D \). The communication capacities are both 0.5 and the communication delays are 1 and 2, respectively. Suppose that there is a flow with its source and destination nodes being \( S \) and \( D \), respectively, and its data rate being 1. For simplicity of presentation, we assume that there does not exist any function in the flow’s SFC and the problem is to route the flow to minimize the delay from node \( S \) to node \( D \).
S to node D. Due to the capacity constraints on both links, the traffic flow must use the two links with both data rates being 0.5. Obviously, the E2E delay is equal to 2.

Let \( P = 2 \) in relaxations (LP-I) and (NLP). Solving the nonlinear relaxation (NLP) returns a solution with the E2E delay (i.e., the objective value) being 1.5, whereas solving the natural LP relaxation (LP-I) returns a solution with the E2E delay being 1.25; see Appendix A for more details. This example clearly shows that the natural LP relaxation (LP-I) can be strictly weaker than the nonlinear relaxation (NLP).

We remark that the weakness of problem (LP-I) is due to the split of traffic flows, i.e., multiple paths are needed to carry out the traffic flow between the two nodes hosting two adjacent functions. Indeed, if only one single path is used to carry out the traffic flow between any two nodes hosting two adjacent functions (i.e., \( z_{ij}(k, s, p) \in \{0, 1\} \)), the two relaxations (LP-I) and (NLP) are equivalent.

Second, the dimension of the natural LP relaxation (LP-I) is very large and there is some redundancy in it. To see this, recall that in problem (MILP), in order to model different paths for flow \((k, s)\), we introduce the notation \( \{p : p \in \mathcal{P}\} \) and use \( \{(i, j) : r_{ij}(k, s, p) > 0\} \) to represent the \(p\)-th path of flow \((k, s)\) (cf. (5) and (7)). However, as \( z_{ij}(k, s, p) \in [0, 1]\) in the natural LP relaxation (LP-I), the traffic flow \( \{r_{ij}(k, s, p) > 0\} \) can be split into multiple paths. This reveals that there is some redundancy in the natural LP relaxation (LP-I); i.e., we do not need to introduce the notation \( \{p : p \in \mathcal{P}\} \) to model different paths for flow \((k, s)\).

The weakness and the large problem size of relaxation (LP-I) make it unsuitable to be embedded in an LP rounding algorithm, i.e., it can lead to a bad performance of an LP rounding algorithm; see the result in Section V further ahead.

### B. Novel LP Relaxation

To overcome the above two disadvantages of relaxation (LP-I), we propose a new LP relaxation for problem (MILP) in this subsection. The proposed LP relaxation is based on the observation that for each \( p \in \mathcal{P} \), the traffic flow \( \{(i, j) : r_{ij}(k, s, p) > 0\} \) can be split into multiple paths when binary variables \( \{z_{ij}(k, s, p)\} \) are relaxed to continuous variables in \([0, 1]\). To this end, we simply set \( P = 1 \) and \( \mathcal{P} = \{1\} \) in the nonlinear relaxation (NLP). Then by (4), we have \( r(k, s, 1) = 1 \), and hence constraint (5) reduces to \( r_{ij}(k, s, 1) = z_{ij}(k, s, 1) \). Furthermore, we can remove constraints (4) and (5), and variables \( \{r_{ij}(k, s, p)\} \), and replace constraints (6)–(8) by

\[
\sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{F}(k)} \lambda_s(k)z_{ij}(k, s, 1) \leq C_{ij}, \quad (i, j) \in \mathcal{L}, \quad (6')
\]

and (7') in the next page, and

\[
\theta(k, s) \geq \sum_{(i, j) \in \mathcal{L}} d_{ij}z_{ij}(k, s, 1), \quad \forall k \in \mathcal{K}, \quad \forall s \in \mathcal{F}(k) \cup \{0\}. \quad (8')
\]

Then problem (NLP) reduces to the following LP problem:

\[
\min_{x, y, z, \theta} \sum_{v \in \mathcal{V}} y_v + \sigma \sum_{k \in \mathcal{K}} (\theta_L(k) + \theta_N(k))
\]

s.t. \((1) - (3), \ (6') - (8'), \ (9)\),

\[
x_{v, s}(k), \ y_v \in [0, 1], \ \forall v \in \mathcal{V}, \ k \in \mathcal{K}, \ s \in \mathcal{F}(k),
\]

\[
z_{ij}(k, s, 1) \in [0, 1], \ \theta(k, s) \geq 0,
\]

\[
\forall (i, j) \in \mathcal{L}, \ k \in \mathcal{K}, \ s \in \mathcal{F}(k) \cup \{0\}. \quad \text{(LP-II)}
\]

Next, we provide more insight into the derivation of LP relaxation (LP-II). In fact, there are two sources of the nonconvexity of problem (MINLP): the binary placement constraints and the multi-path routing and E2E delay constraints. (LP-II) can be derived by using convex techniques to deal with these two nonconvex constraints. The way to deal with the first nonconvex constraint is to relax the binary variables \(x \) and \( y \) into continuous variables in \([0, 1]\). As for the second one, one can remove the multi-path routing constraint (which mathematically corresponds to using the continuous variables \( \{z_{ij}(k, s, 1)\} \) and the related constraints (6')–(7')) and replace the largest delay of the \( P \) paths in flow \((k, s)\) with the average delay (which mathematically corresponds to using constraint (8')).

Now we show the connection between problems (LP-II) and (NLP). Notice that problem (LP-II) is derived from relaxation (NLP) with \( P = 1 \) by removing some redundant variables and constraints. Therefore, problem (LP-II) is indeed a special case of problem (NLP) (i.e., the case that \( P = 1 \)). When \( P \geq 2 \), the main difference of problems (NLP) and (LP-II) is their variables and constraints that are used to formulate the traffic routing of flow \((k, s)\) (variables \( \{r(k, s, p)\}, \{r_{ij}(k, s, p)\} \), and \( \{z_{ij}(k, s, 1)\} \) and constraints (4)–(8) in problem (NLP) while variables \( \{z_{ij}(k, s, 1)\} \) and constraints (6')–(8') in problem (LP-II)). The following theorem shows, somewhat surprisingly, that the above LP problem is also equivalent to the nonlinear problem (NLP) (with any parameter \( P \geq 2 \)) in the sense that for any given feasible solution \( (x, y, r, z, \theta) \) of problem (NLP), we can construct a feasible point \( (X, Y, Z, \Theta) \) of problem (LP-II) such that the two problems have the same objective value at the corresponding solutions and vice versa. The main idea to prove this theorem is to set \( x = X, \ y = Y, \) and \( \theta = \Theta, \) and establish a one-to-one correspondence between \( \{r(k, s, p)\}, \{r_{ij}(k, s, p)\}, \) and \( \{z_{ij}(k, s, 1)\} \) of problem (LP-II); see Eqs. (13)–(16) further ahead.

**Theorem 2.** The LP problem (LP-II) is equivalent to the nonlinear relaxation (NLP) with \( P \geq 1 \).

**Proof.** In this proof, in order to differentiate the feasible points of the two problems, we use \( (x, y, r, z, \theta) \) and \( (X, Y, Z, \Theta) \) to denote the feasible points of problems (NLP) and (LP-II), respectively. We shall prove the theorem by showing that, given any feasible solution \( (x, y, r, z, \theta) \) of problem (NLP), we can construct a feasible point \( (X, Y, Z, \Theta) \) of problem (LP-II) such that the two problems have the same objective value at the corresponding solutions and vice versa.

Given any feasible solution \( (x, y, r, z, \theta) \) of problem (NLP), we shall construct a solution \( (X, Y, Z, \Theta) \) of problem...
\[ \sum_{j:(i,j) \in \mathcal{L}} z_{ji}(k,s,1) - \sum_{j:(i,j) \in \mathcal{L}} z_{ij}(k,s,1) = \begin{cases} 0, & \text{if } i \in \mathcal{I} \setminus \mathcal{V}; \\
 x_{i,s+1}(k) - x_{i,s}(k), & \text{if } i \in \mathcal{V}; \end{cases} \quad \forall \ k \in \mathcal{K}, \forall \ s \in \mathcal{F}(k) \cup \{0\}. \quad (7') \]

Next, we prove the other direction. Given any feasible solution \((X, Y, Z, \Theta)\) of problem (LP-II), we construct a solution \((x, y, r, z, \theta)\) by setting \(x = X\), \(y = Y\), \(\theta = \Theta\), and
\[
\begin{align*}
\sum_{p \in \mathcal{P}} r_{ij}(k,s,p) &= 1, \\
\sum_{p \in \mathcal{P}} r_{ij}(k,s,p) &= 0, \\
\sum_{i,j \in \mathcal{L}} r_{ij}(k,s,p) &= 1. 
\end{align*}
\]

Again, it is simple to see that problems (NLP) and (LP-II) have the same objective value at points \((x, y, r, z, \theta)\) and \((X, Y, Z, \Theta)\), respectively. We still need to show that constraints \((1) - (9)\) in problem (NLP) hold true at the above constructed point \((x, y, r, z, \theta)\). Clearly, constraints \((1) - (3)\) and \((9)\) hold at point \((x, y, r, z, \theta)\). Combining the construction of variables \(r \) and \(z\) in (14)–(16), one can show that constraints \((4)\) and \((5)\) are satisfied. From (15), we have (13), which, together with constraint \((6')\), implies that constraint \((6)\) holds at point \((x, y, r, z, \theta)\). Finally, by the definition of variables \(z\) in (16) and constraints \((7') - (8')\), constraints \((7) - (8)\) also hold true. This completes the proof. \(\square\)

The equivalence of problems (LP-II) and (NLP) in Theorem 2 (i) implies that the LP problem (LP-II) can also be seen as a relaxation of problem (MILP) and (ii) characterizes how tight relaxation (LP-II) can possibly be, i.e., relaxation (LP-II) provides a lower bound of the network slicing problem that is as tight as that relaxation (NLP) can provide. However, in sharp contrast to relaxation (NLP) which is a nonconvex problem (due to the bilinearity in constraint (5)), relaxation (LP-II) is a (convex) LP problem and can be (globally and efficiently) solved using the polynomial-time interior-point method in [36]. In addition, when compared with the natural LP relaxation (LP-I), the novel LP relaxation (LP-II) enjoys the following two advantages. First, LP relaxation (LP-II) is also stronger than LP relaxation (LP-I), which follows from the fact that nonlinear relaxation (NLP) is stronger than LP relaxation (LP-I) and the equivalence of (NLP) and (LP-II) in Theorem 2.

**Corollary 1.** The novel LP relaxation (LP-II) is stronger than the natural LP relaxation (LP-I).

Second, LP relaxation (LP-II) is much more compact than LP relaxation (LP-I). Indeed, the numbers of variables and constraints in LP relaxation (LP-II) are \(O(|\mathcal{L}| \sum_{k \in \mathcal{K}} \ell_k)\) and \(O(\min\{|\mathcal{I}| \sum_{k \in \mathcal{K}} \ell_k, |\mathcal{L}|\})\), respectively. These are much smaller than those in LP relaxation (LP-I), which are \(O(|\mathcal{L}| |\mathcal{P}| \sum_{k \in \mathcal{K}} \ell_k)\). The strengthen and compactness of the proposed LP relaxation (LP-II) play a crucial role in the effectiveness and efficiency of the proposed LPRR algorithm in the next section.
C. Integrality Gap of Relaxation (LP-II)

In this subsection, we analyze the integrality gap of relaxation (LP-II) in some special cases. In particular, we show that the integrality gap of relaxation (LP-II) in some special case (i.e., case (i) in the following Theorem 3) can be infinite, which provides more insight into the intrinsic difficulty of the original problem. In addition, we also identify two other special cases (i.e., cases (ii) and (iii) in the following Theorem 3) where we can derive bounded or zero integrality gaps.

Let us first specify the special cases of interest in this subsection: each link’s capacity is infinite, each link’s delay is zero, and the SFC of each service contains only one function in problem (MILP). In this case, the capacity constraint on all links and the E2E delay constraint of all services are automatically satisfied. Let \( \mathcal{V}(k) \subseteq \mathcal{V} \) denote the set of cloud nodes that can process the function \( f^k_i \), which is the (only) function in the SFC of service \( k \). Without loss of generality, we assume that there exists a path between the source node \( S(k) \) and each node in \( \mathcal{V}(k) \) and between each node in \( \mathcal{V}(k) \) and the destination node \( D(k) \) and the processing capacity of node \( v \) is larger than or equal to the data rate \( \lambda_1(k) \) of function \( f^k_i \), i.e., \( \mu_v \geq \lambda_1(k) \) (otherwise, we can set \( x_{v,1}(k) = 0 \)). Then, we must have

\[
x_{v,1}(k) = 0, \quad \forall k \in \mathcal{K}, \quad v \in \mathcal{V} \setminus \mathcal{V}(k).
\]  

(17)

Hence problem (MILP) reduces to

\[
\min_{x,y} \sum_{v \in \mathcal{V}} y_v \\
\text{s.t.} \quad (1) - (3), \quad (17), \\
x_{v,1}(k), \quad y_v \in \{0, 1\}, \quad \forall v \in \mathcal{V}, \quad k \in \mathcal{K},
\]  

(18)

and relaxation (LP-II) reduces to

\[
\min_{x,y} \sum_{v \in \mathcal{V}} y_v \\
\text{s.t.} \quad (1) - (3), \quad (17), \\
x_{v,1}(k), \quad y_v \in [0, 1], \quad \forall v \in \mathcal{V}, \quad k \in \mathcal{K}.
\]  

(19)

**Theorem 3.** Suppose that each link’s capacity is infinite, each link’s delay is zero, and the SFC of each service contains only one function in problem (MILP). Then,

(i) if \( \mu_v = \infty \) for all \( v \in \mathcal{V} \), the integrality gap of relaxation (LP-II) can be infinite;

(ii) if \( \lambda_1(k) \geq \frac{1}{t} \mu_v \) for all \( k \in \mathcal{K} \) and \( v \in \mathcal{V}(k) \) where \( t \) is some constant, the integrality gap of relaxation (LP-II) is at most \( t \); and

(iii) if each node can host (at most) one function, the integrality gap of relaxation (LP-II) is zero and the original problem (MILP) can be solved to global optimality.

*Proof.* We first prove case (i). In this case, constraint (3) can be removed. Constraints (1) and (2) can be replaced by \( \sum_{v \in \mathcal{V}(k)} y_v \geq 1, \quad \forall k \in \mathcal{K} \). Then problems (18) and (19) reduce to

\[
\min_{y} \sum_{v \in \mathcal{V}} y_v \\
\text{s.t.} \quad \sum_{v \in \mathcal{V}(k)} y_v \geq 1, \quad \forall k \in \mathcal{K},
\]  

(20)

and

\[
\min_{y} \sum_{v \in \mathcal{V}} y_v \\
\text{s.t.} \quad \sum_{v \in \mathcal{V}(k)} y_v \geq 1, \quad \forall k \in \mathcal{K},
\]  

(21)

Problem (20) is the well-known set covering problem and problem (21) is its LP relaxation. From [37, Pages 110-112, the integrality gap of the set covering problem is \( \Omega(\log(|\mathcal{K}|)) \), which can go to infinity as \( |\mathcal{K}| \to \infty \).

Next, we prove case (ii). In this case, summing up constraints (3) and using (17) and (1), we have

\[
\sum_{v \in \mathcal{V}} y_v \geq \frac{1}{t} \mu_v \sum_{k \in \mathcal{K}} \lambda_1(k) x_{v,1}(k)
\]  

\[
= \sum_{k \in \mathcal{K}} \frac{1}{t} \lambda_1(k) x_{v,1}(k)
\]  

\[
\geq \frac{1}{t} \sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}(k)} x_{v,1}(k) = \frac{1}{t} |\mathcal{K}|,
\]

which implies that the objective of relaxation (21) is at least \( \frac{1}{t} |\mathcal{K}| \). This, together with the fact that the objective value of problem (18) is at most \( |\mathcal{K}| \) (as each activated node must host at least one function), shows that the integrality gap in this case is at most \( t \).

Finally, we prove case (iii). In this case, problem (19) reduces to

\[
\min_{x,y} \sum_{v \in \mathcal{V}} y_v \\
\text{s.t.} \quad (1), \quad (17), \quad \sum_{k \in \mathcal{K}} x_{v,1}(k) \leq y_v, \quad \forall k \in \mathcal{K},
\]  

(22)

One can check that the constraint matrix in problem (22) is a totally unimodular matrix, showing that the integrality gap of this problem is zero and the original problem (MILP) can be solved to global optimality; see [26, Chapter 4]. \( \Box \)

IV. AN LP ROUNDING-AND-REFINEMENT ALGORITHM

In this section, we will focus on designing an efficient algorithm, based on the proposed LP relaxation (LP-II), to obtain a high-quality solution for problem (MILP). The basic idea of the proposed algorithm is to decompose the hard problem (MILP) into two relatively easy subproblems and solve two subproblems separately while taking their connection into account. Specifically, in the first stage, we find a binary vector \( (\bar{x}, \bar{y}) \) for the VNF placement subproblem (i.e., \( (\bar{x}, \bar{y}) \) satisfying constraints (1)-(3)) using an iterative LP rounding procedure, which takes traffic routing into account.
In the second stage, based on the binary vector \((\hat{x}, \hat{y})\), we use an LP refinement procedure to solve the traffic routing subproblem to obtain a solution that satisfies the E2E delay constraints \((8)-(9)\) of all services.

A. Solving the VNF Placement Subproblem

In this subsection, we solve the VNF placement subproblem by constructing a binary vector \((\hat{x}, \hat{y})\) that satisfies constraints \((1)-(3)\). Since vector \(\hat{y}\) can be uniquely determined by vector \(\hat{x}\), in the following we concentrate on constructing the binary vector \(\hat{x}\). To do this, we first solve the LP relaxation (LP-II), denoted its solution by \((x^*, y^*, z^*, \theta^*)\). If \(x^*\) is a binary vector, we obtain a feasible solution \(\hat{x} := x^*\) for the VNF placement subproblem. Otherwise, we set \(x_{v,s}(k) = 1\) in relaxation (LP-II) if \(x^*_{v,s}(k) = 1\). Then we choose one variable, denoted as \(x_{v,s}(k_0)\), whose value \(x^*_{v,s}(k_0)\) is the largest among the remaining variables, i.e.,

\[
x^*_{v,s}(k_0) = \max \{x^*_{v,s}(k) : 0 < x^*_{v,s}(k) < 1, \quad v \in V, \quad k \in K, \quad s \in F(k)\}. \tag{23}
\]

Next we decide to round variable \(x_{v,s}(k_0)\) to one or zero. In particular, we first set \(x_{v,s}(k_0) = 1\) in problem (LP-II). If the above modified LP is infeasible, we set \(x_{v,s}(k_0) = 0\) and continue to round other variables respect to the values \(\{x^*_{v,s}(k)\}\). Otherwise, the above modified LP is feasible and we repeat the above procedure to the solution of the modified LP until a binary solution is obtained. The details are summarized in the following Algorithm 1.

**Algorithm 1** An iterative LP rounding procedure for solving the VNF placement subproblem

1. Initialize the set \(A = \emptyset\);
2. Solve problem (LP-II) to obtain its solution \((x^*, y^*, z^*, \theta^*)\);
3. **while** (there exists some \(v \in V, k \in K, \) and \(s \in F(k)\) such that \(0 < x^*_{v,s}(k) < 1\)) **do**
   4. For each \(v \in V, k \in K, \) and \(s \in F(k)\) with \(x^*_{v,s}(k) = 1\), if constraint \(x_{v,s}(k) = 1\) is not in set \(A\), add it into set \(A\);
   5. Compute \((v_0, s_0, k_0)\) such that (23) holds and add constraint \(x_{v_0,s_0}(k_0) = 1\) into set \(A\);
   6. Add the constraints in set \(A\) into problem (LP-II) to obtain a modified LP;
   7. **if** (the modified LP problem is feasible) **then**
      8. Set \((x^*, y^*, z^*, \theta^*) \leftarrow \) the optimal solution of the modified LP problem;
   9. **else**
      10. Replace constraint \(x_{v_0,s_0}(k_0) = 1\) by constraint \(x_{v_0,s_0}(k_0) = 0\) in set \(A\) and set \(x^*_{v_0,s_0}(k_0) \leftarrow 0\);
11. **end if**
12. **end while**
13. If vector \((x^*, y^*)\) satisfies constraints \((1)-(3)\), then the binary vector \((\hat{x}, \hat{y}) \leftarrow (x^*, y^*)\) is feasible for the VNF placement subproblem; otherwise declare that the algorithm fails to find a feasible solution.

Two remarks on the above rounding strategy are as follows. First, the above rounding strategy makes sure that we can round one variable, taking a fractional value at the current solution, at a time and more importantly this variable can be rounded to a binary value that is consistent to other already rounded variables. This is in sharp contrast to the algorithm in [13] where the variables are rounded without ensuring the consistency of the current rounding variable with other already rounded variables. Second, our rounding strategy takes traffic routing into consideration as the modified LP contains the information of traffic routing of all services. Indeed, when setting \(x_{v_0,s_0}(k_0) = 1\) in problem (LP-II), there may not exist traffic routing strategies for some services (due to the limited node/link capacities or the E2E latency thresholds). In this case, the modified LP problem is likely to be infeasible, and we exploit this by re-setting \(x_{v_0,s_0}(k_0) = 0\) in modified LP problem (LP-II); see steps 7-11 for more details.

B. Solving the Traffic Routing Subproblem

Once we get a binary vector \((\hat{x}, \hat{y})\), we still need to solve the traffic routing subproblem that finds paths connecting two adjacent functions and satisfying the E2E delay of all services. This can be done by solving problem (MILP) with \(x = \hat{x}\) and \(y = \hat{y}\). In this case, the objective function in problem (MILP) reduces to

\[
g(\theta) = \sum_{k \in K} \theta_L(k).
\]

Similarly, we solve the LP problem (LP-II) with \(x = \hat{x}\) and \(y = \hat{y}\) to obtain a solution \((z^*, \theta^*)\). Due to the (possibly) fractional values of \(z^*_k(s, 1)\), \(\theta^*(k, s)\) can be smaller than the communication delay incurred by the traffic flow from the node hosting function \(f_k^s\) to the node hosting function \(f_{k+1}^s\). To recompute the communication delay based on solution \((z^*, \theta^*)\), we need to solve the NP-hard Min-Max-Delay problem [38]. Fortunately, there exists an efficient polynomial-time \((1 + \epsilon)\)-approximation algorithm for this problem [38]. After recomputing the communication delays between all pairs of nodes hosting two adjacent functions, we can compute the total delay of each service \(k\), denoted as \(\theta(k)\). If \(\theta(k) > \theta^*(k, s)\) for some service \(k\), the current routing strategy is infeasible as it violates the E2E delay constraint of service \(k\). We then use an iterative LP refinement procedure to try to get a solution that satisfies the E2E delay constraints of all services.

The idea of our refinement procedure is to increase the weights of the variables \(\theta_L(k)\) corresponding to the service whose E2E delay constraint is not satisfied at the current solution, in order to refine the solution. In particular, we change the objective function \(g(\theta)\) in problem (LP-II) into

\[
\hat{g}(\theta) = \sum_{k \in K} \omega_k \theta_L(k),
\]

where \(\omega_k \geq 1\) for all \(k \in K\). At each iteration, we solve problem (LP-II) (with the objective function \(\hat{g}(\theta)\), \(x = \hat{x}\), and \(y = \hat{y}\)) to obtain its solution \((z^*, \theta^*)\). If, for some service \(k\), the E2E delay constraint is violated at this solution, we increase \(\omega_k\) by a factor of \(\rho > 1\), and solve problem (LP-II) again. The procedure is repeated until the solution satisfies the E2E delay constraints of all services or the iteration number reaches a predefined parameter IterMax. We summarize the above procedure in Algorithm 2. Notice that as stated in
An iterative LP refinement procedure for solving the traffic routing subproblem

1. Set $\rho > 1$, IterMax $\geq 1$, $t = 0$, and $\omega_k = 1$ for $k \in \mathcal{K}$;
2. while $t < $ IterMax do
3. Solve problem (LP-II) (with the objective function $\hat{g}(\theta)$, $\bar{x} = \hat{x}$, and $\bar{y} = \hat{y}$) to obtain its solution $(\bar{z}^*, \theta^*)$;
4. For each $k \in \mathcal{K}$, compute the total delay $\bar{\theta}(k)$ based on $(\bar{z}^*, \theta^*)$ and $(\bar{x}, \bar{y})$;
5. if $(\bar{\theta}(k) \leq \Theta_k$ for all $k \in \mathcal{K}$ then
6. Stop with the feasible solution $(\bar{z}^*, \theta^*)$;
7. else
8. For each $k \in \mathcal{K}$ with $\bar{\theta}(k) > \Theta_k$, set $\omega_k \leftarrow \rho \omega_k$;
9. end if
10. Set $t \leftarrow t + 1$;
11. end while

Section II-B.2, $P$ can be set as a sufficiently large number, and hence we can always decompose the traffic flow $\{z_{ij}(k, s, 1)\}$ into a suitable number of paths.

**C. Complexity Analysis**

The dominant computational cost of our algorithm is to solve the LP problems in form of (LP-II). Indeed, the number of solving problems (LP-II) in Algorithms 1 and 2 are upper bounded by $|V| \sum_{k \in \mathcal{K}} \ell_k$ and IterMax, respectively. By summing up the upper bound of solved LPs in Algorithms 1 and 2, we obtain the upper bound of solved LPs in the proposed LPRR algorithm, which are $|V| \sum_{k \in \mathcal{K}} \ell_k + $ IterMax. The worst-case complexity of solving an LP is approximately $O((n + m)^{1.5}n^2)$ using the interior-point method in [36, Section 6.6.1], where $n$ and $m$ are the numbers of variables and constraints, respectively. This, together with the fact that the numbers of variables and constraints in problem (LP-II) are $O(|\mathcal{L}| \sum_{k \in \mathcal{K}} \ell_k)$ and $O(\min(|\mathcal{I}| \sum_{k \in \mathcal{K}} \ell_k, |\mathcal{L}|))$, respectively, implies that problem (LP-II) can be solved in

$$O\left(\left(|V| \sum_{k \in \mathcal{K}} \ell_k + \text{IterMax}\right)^3 \cdot \left(|\mathcal{L}| \sum_{k \in \mathcal{K}} \ell_k\right)^{3.5}\right).$$

In sharp contrast, the worst-case complexity of using the standard MILP solvers like Gurobi [24] to solve problem (MILP) is exponential, which makes it unsuitable for solving large-scale problems.

**V. SIMULATION RESULTS**

In this section, we present simulation results to illustrate the effectiveness and efficiency of the proposed LP relaxation (LP-II) and the proposed LPRR algorithm for solving the network slicing problem. Specifically, we first perform numerical experiments to compare the performance of solving the natural LP relaxation (LP-I) and the proposed LP relaxation (LP-II). Then, we present some simulation results to demonstrate the efficiency and effectiveness of our proposed LPRR algorithm over the state-of-the-art approaches in [3], [13], and [14].

In problem (MILP), we choose $\sigma = 0.001$ and $P = 2$, as suggested in [3]. In Algorithm 2, we set $\rho = 5$ and IterMax $= 10$. We use Gurobi 9.0.1 [24] to solve all MILP and LP problems. When solving the MILP problems, the time limit is set to be 1800 seconds, and the relative gap tolerance is set to be 0.1%, i.e., a feasible solution which has an optimality gap of 0.1% is considered to be optimal. All experiments were performed on a server with 2 Intel Xeon E5-2630 processors and 98 GB of RAM, using the Ubuntu GNU/Linux Server 14.04 x86 64 operating system.

We test all algorithms on the fish network topology studied in [12] and the JANOS-US, NOBEL-GERMANY, and POL-SKA network topologies taken from the SNDLIB [39]. Due to the space limit, we only present the simulation results on the fish network topology in this paper and more simulation results on other network topologies can be found in Section III of [35]. The fish network topology contains 112 nodes and 440 links, including 6 cloud nodes. The cloud nodes’ and links’ capacities are randomly generated within $[50, 100]$ and $[5, 55]$, respectively. The NFV and communication delays on the cloud nodes and links are randomly generated within $\{3, 4, 5, 6\}$ and $\{1, 2\}$, respectively. For each service $k$, node $S(k)$ is randomly chosen from the available nodes and node $D(k)$ is set to be the common destination node; SPC $\mathcal{F}(k)$ is a sequence of functions randomly generated from $\{f^1, \ldots, f^4\}$ with $|\mathcal{F}(k)| = 3$; $\lambda_s(k)$’s are the service function rates which are all set to be the same integer value, randomly generated within $[1, 11]$; $\Theta_k$ is set to $20 + (3 \times \text{dist}_k + \alpha)$ where $\text{dist}_k$ is the delay of the shortest path between nodes $S(k)$ and $D(k)$ and $\alpha$ is randomly chosen in $[0, 5]$. The above parameters are carefully chosen to make sure that the constraints in problem (MILP) are neither too tight nor too loose. For each fixed number of services, 100 problem instances are randomly generated and the results presented below are obtained by averaging over these problem instances.

**A. Comparison of LP Relaxation (LP-I) and Proposed LP Relaxation (LP-II)**

In this subsection, we compare the performance of solving the natural LP relaxation (LP-I) and the proposed LP relaxation (LP-II).

We first compare the computational efficiency of solving two relaxations (LP-I) and (LP-II). Fig. 2 plots the CPU time versus the numbers of services. From Fig. 2, it can be clearly seen that it is much more efficient to solve relaxation (LP-II) than relaxation (LP-I). In particular, when the number
of services is equal to 10, the CPU time of solving relaxation (LP-II) is about 1 second while that of solving relaxation (LP-I) is more than 8 seconds. In addition, we can observe from Fig. 2 that, with the increasing number of services, the CPU time of (LP-I) increases much faster, as compared with that of relaxation (LP-II). This is mainly due to the fact that the numbers of variables and constraints in relaxation (LP-I) increase much faster than those in relaxation (LP-II) as the number of services increases.

Next, we compare the optimal values of the two LP relaxations (LP-I) and (LP-II). As it has been shown in Corollary 1, the LP relaxation (LP-II) is stronger than the natural LP relaxation (LP-I), and in Example 1, relaxation (LP-II) will return a solution with a much larger communication delay, as compared to that returned by relaxation (LP-I). Therefore, we compare the total communication delay of the solutions returned by the two relaxations. To do this, we need to eliminate the effect of the number of activated nodes and the total NFV delay in the objective function. This can be done by fixing variables $x$ and $y$, in relaxations (LP-I) and (LP-II), to the solution returned by solving problem (MILP). We compare the relative gap, which is defined as follows:

$$\text{Gap} = \frac{D(\text{MILP}) - D(\text{LP-II})}{D(\text{MILP}) - D(\text{LP-I})},$$

where $D(\text{MILP})$, $D(\text{LP-I})$, and $D(\text{LP-II})$ are the total communication delay returned by solving problems (MILP), (LP-I), and (LP-II), respectively. The relative gap reflects the tightness of relaxation (LP-II) over that of relaxation (LP-I), i.e., the smaller the relative gap, the stronger relaxation (LP-II) (as compared with relaxation (LP-I)). We remark that the relative gap is a widely used performance measure in the integer programming community [40], [41] to show the tightness of an LP relaxation over another LP relaxation.

Fig. 3 plots the average relative gap versus different numbers of services. As observed in Fig. 3, in all cases, the relative gap is smaller than 1.0, showing that the LP relaxation (LP-II) is indeed stronger than the LP relaxation (LP-I). In addition, with the increasing number of services, the relative gap becomes smaller. This can be explained as follows. As the number of services increases, the traffic in the network becomes heavier. This further results in the situation that the traffic flows between the two nodes hosting two adjacent functions are likely to transmit over multiple paths. Consequently, the total communication delay returned by solving LP relaxation (LP-I) becomes smaller and relaxation (LP-I) becomes looser.

Based on the above computational results, we can conclude that the proposed relaxation (LP-II) significantly outperforms the natural relaxation (LP-I) in terms of the solution efficiency and providing a better LP bound. As it will be seen in the next subsection, the solution efficiency and the tightness of the proposed LP relaxation (LP-II) plays a crucial role in the effectiveness and efficiency of the proposed LPRR algorithm.

B. Comparison of Proposed Algorithm and Those in [3], [13], and [14]

In this subsection, we compare the performance of the proposed LPRR algorithm with the exact approach using standard MILP solvers (called EXACT) in [3], the GREEDY algorithm (which is adapted from the one in [14]), and the LP rounding (LPR) algorithm (which is adapted from the one in [13]). GREEDY maps VNFs into cloud nodes using a greedy heuristic while LPR first solves the LP relaxation (LP-II) of the network slicing problem to obtain its solution and then uses a rounding strategy to map VNFs to cloud nodes based on this solution. The two algorithms then find paths connecting two adjacent VNFs in the network by solving a multicommodity flow problem [30, Chapter 17], which can be solved in polynomial time. Due to the space limit, we provide the detailed description of the GREEDY and LPR in Section II of [35]. In addition, to address the advantage of using the proposed LP relaxation (LP-II) in the LPRR algorithm, we compare the LPRR algorithm with LPRR’, which embeds the natural LP relaxation (LP-I) into Algorithm 1 while still embeds the proposed LP relaxation (LP-II) into Algorithm 2. It is worthwhile remarking that the natural LP relaxation (LP-I) cannot be used in Algorithm 2 as we generally cannot find a traffic routing strategy based on the solution of the natural LP relaxation (LP-I); see the example in Appendix A.

Fig. 4 plots the number of feasible problem instances solved by GREEDY, LPR, LPRR, LPRR’, and EXACT.

1) Effectiveness of LPRR: Figs. 4 and 5 plot the number of feasible problem instances and the average number of activated nodes solved by GREEDY, LPR, LPRR, LPRR’, and EXACT, respectively.

First, we can observe from Fig. 4 that compared with LPRR’, LPRR can solve a larger number of problem instances than LPRR’, especially when the number of services is large.
This clearly shows that the advantage of using the proposed LP relaxation (LP-II) in the proposed LPRR algorithm, i.e., it enables the LPRR algorithm to find feasible solutions for much more problem instances. Second, we can see the effectiveness of the proposed algorithm LPRR over GREEDY and LPR in Figs. 4 and 5. In particular, as shown in Fig. 4, using the proposed algorithm LPRR, we can find feasible solutions for much more problem instances, compared with using GREEDY and LPR. Indeed, LPRR finds feasible solutions for almost all (truly) feasible problem instances (as EXACT is able to find feasible solutions for all (truly) feasible problem instances and the difference of the number of feasible problem instances solved by EXACT and LPRR is small in Fig. 4). In addition, we can also observe that using LPRR, the number of activated cloud nodes is much smaller than that of using GREEDY and LPR, respectively.

2) Efficiency of LPRR: The comparison of the solution time of GREEDY, LPR, LPRR, LPRR', and EXACT is plot in Fig. 6. First, as expected, LPRR is much more computationally efficient than LPRR', especially when the number of services is large. This is mainly due to the fact that solving the proposed LP relaxation is much faster than solving the natural LP relaxation, as demonstrated in Fig. 2. Second, compared with EXACT, LPRR is significantly more efficient and the solution efficiency of GREEDY, LPR, and LPRR is comparable. Overall, LPR and LPRR are several times slower than GREEDY. For example, when $|\mathcal{K}| = 10$, the average CPU time of GREEDY is 1.2 seconds, while the average CPU times of LPR and LPRR are 2.7 seconds and 9.0 seconds, respectively. In sharp contrast, for EXACT whose computational complexity is exponential, the average CPU time is 1156.4 seconds.

C. Summary of Simulation Findings

From the above simulation results, we can conclude that

- Compared with the natural LP relaxation (LP-I), our proposed relaxation (LP-II) is much stronger and more compact, which plays a crucial role in the effectiveness and efficiency of our proposed LPRR algorithm.
- When comparing with the existing state-of-the-art algorithms in [3], [13] and [14], our proposed LPRR achieves a better trade-off between the solution quality and the computational efficiency. More specifically, compared with GREEDY in [14] and LPR in [13], it is able to find a much better solution; compared with EXACT in [3], it is much more computationally efficient.

VI. CONCLUSIONS

In this paper, we have proposed an efficient algorithm called LPRR for solving the network slicing problem. The proposed algorithm is a two-stage LP based algorithm that places the virtual network functions of all services into cloud nodes using an iterative LP rounding procedure in the first stage and finds the traffic routing strategies of all services using an iterative LP refinement procedure in the second stage. Three key features of the proposed algorithm, which make it particularly suitable to solve the large-scale network slicing problems with a high-quality solution, are: (i) it is based on
a newly proposed LP relaxation (LP-II), which, compared with the natural LP relaxation (LP-I), is much stronger (in terms of providing a better LP bound) and more compact (in terms of smaller numbers of variables and constraints); and (ii) it takes the global information of the problem (i.e., the traffic routing of all services) into consideration in the first stage (in contrast to [13] which only takes the local information of the problem); (iii) the worst-case complexity of the proposed algorithm is polynomial. Simulation results show that our proposed algorithm is able to achieve a better trade-off between the solution quality and the computational efficiency than the existing state-of-the-art algorithms.

APPENDIX A
DETAILS OF EXAMPLE 1

Here we will show that solving the nonlinear relaxation (NLP) will return a solution with the communication delay being 1.5, while solving the natural LP relaxation (LP-I) will return a solution with the communication delay being 1.25.

To distinguish the two links in Fig. 1, we denote the top one with communication delay 1 and the bottom one with communication delay 2 as $a$ and $b$, respectively. Let $\theta$ denote the delay incurred by the traffic flow from node $S$ to node $D$. For $p = 1, 2$, let $r(p)$ denote the data rate on the $p$-th path, $z_a(p)$ and $z_b(p)$ denote whether or not links $a$ and $b$ are used by the $p$-th path, respectively, and $r_a(p)$ and $r_b(p)$ denote the associated data rate on links $a$ and $b$, respectively.

The nonlinear relaxation (NLP) for this example reduces to

$$\theta_{NLP} := \min_{r, z, \theta} \theta$$

s.t. (25) – (36), where

$$\theta \geq z_a(1) + 2z_b(1), \quad (25) \quad r(1) + r(2) = 1, \quad (31)$$

$$\theta \geq z_a(2) + 2z_b(2), \quad (26) \quad r_a(1) = r(1)z_a(1), \quad (32)$$

$$z_a(1) + z_b(1) = 1, \quad (27) \quad r_b(1) = r(1)z_b(1), \quad (33)$$

$$z_a(2) + z_b(2) = 1, \quad (28) \quad r_a(2) = r(2)z_a(2), \quad (34)$$

$$r_a(1) + r_a(2) \leq 0.5, \quad (29) \quad r_b(2) = r(2)z_b(2), \quad (35)$$

$$r_b(1) + r_a(2) \leq 0.5, \quad (30) \quad 0 \leq r(p), r_a(p), r_b(p), z_a(p), z_b(p) \leq 1, \quad p \in \{1, 2\}. \quad (36)$$

The natural LP relaxation (LP-I) for this example reduces to

$$\theta_{LP-I} := \min_{r, z, \theta} \theta$$

s.t. (25) – (31), (36) – (48).

Below we shall show that $\theta_{NLP} = 1.5$ and $\theta_{LP-I} = 1.25$, separately.

- Proof of $\theta_{NLP} = 1.5$.
- First, we give a point $(\hat{r}, \hat{z}, \hat{\theta})$ as follows:
  $$\hat{\theta} = 1.5, \quad \hat{r}(1) = 1, \quad \hat{r}(2) = 0,$$
  $$\hat{z}_a(1) = \hat{z}_b(1) = 0.5, \quad \hat{r}_a(1) = \hat{r}_b(1) = 0.5,$$
  $$\hat{z}_a(2) = \hat{z}_b(2) = 0.5, \quad \hat{r}_a(2) = \hat{r}_b(2) = 0.$$

  Obviously, $(\hat{r}, \hat{z}, \hat{\theta})$ is feasible for problem (24), and hence we have $\theta_{NLP} \leq 1.5$. It remains to show $\theta_{NLP} \geq 1.5$. Indeed, summing up constraints (32) – (35) and using constraints (27), (28), and (31), we have

$$r_a(1) + r_b(1) + r_a(2) + r_b(2)$$

$$= r(1)(z_a(1) + z_b(1)) + r(2)(z_a(2) + z_b(2))$$

$$= r(1) + r(2) = 1.$$

This implies that constraints (29) and (30) must hold with equalities:

$$r_a(1) + r_a(2) = 0.5,$$

$$r_b(1) + r_b(2) = 0.5.$$

(50)

Substituting (33) and (35) into (50), we have

$$r_0(1) + r_0(2) = \hat{r}(1)z_b(1) + r(2)z_b(2) = 0.5,$$

which, together with constraint (31), immediately shows $\max\{z_a(1), z_a(2)\} \geq 0.5$. Then, it follows from constraints (25) – (28) that

$$\theta_{NLP} \geq \max\{z_a(1) + 2z_b(1), z_a(2) + 2z_b(2)\}$$

$$= \max\{1 + z_b(1), 1 + z_b(2)\} \geq 1.5.$$

- Proof of $\theta_{LP-I} = 1.25$.

Let $(\hat{r}, \hat{z}, \hat{\theta})$ be given as follows:

$$\hat{\theta} = 1.25, \quad \hat{r}(1) = 0.5, \quad \hat{r}(2) = 0,$$

$$\hat{z}_a(1) = 0.75, \quad \hat{z}_b(1) = 0.25, \quad \hat{r}_a(1) = 0.25, \quad \hat{r}_b(1) = 0,$$

$$\hat{z}_a(2) = 0.75, \quad \hat{z}_b(2) = 0.25, \quad \hat{r}_a(2) = 0.25, \quad \hat{r}_b(2) = 0.$$

It is simple to check that $(\hat{r}, \hat{z}, \hat{\theta})$ is feasible for problem (49) (though we cannot find a traffic routing strategy based on this solution). As a result, we have $\theta_{LP-I} \leq 1.25$. To seek a solution with a better objective value for problem (49), we may assume that

$$z_a(1) + 2z_b(1) = 1 + z_b(1) \leq 1.25,$$

$$z_a(2) + 2z_b(2) = 1 + z_b(2) \leq 1.25.$$

Notice that the above equalities follow from (27) and (28). Then, we have $z_b(1) \leq 0.25$ and $z_b(2) \leq 0.25$, and it follows from constraints (27) and (28) that $z_a(1) \geq 0.75$ and $z_a(2) \geq 0.75$. Summing up constraints (45) and (47) yields

$$r_a(1) + r_a(2) \geq r(1) + r(2) + z_a(1) + z_a(2) - 2$$

$$= 1 + z_a(1) + z_a(2) - 2 \text{ (from (31))}$$

$$= z_a(1) + z_a(2) - 1 \geq 0.5. \quad \text{ (from } z_a(1) \geq 0.75 \text{ and } z_a(2) \geq 0.75).$$

By constraint (29), the above inequality must hold with equality, and hence we have $z_a(1) = z_a(2) = 0.75$, or equivalently $z_b(1) = z_b(2) = 0.25$. As a result, $\theta_{LP-I} = 1.25$.

Acknowledgments We would like to thank the two anonymous reviewers for their insightful comments.

REFERENCES

[1] W.-K. Chen, Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, “An efficient linear programming rounding-and-refinement algorithm for large-scale network slicing problem,” in Proceedings of 46th IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Toronto, Canada, June 2021, pp. 4735-4739.

[2] R. Mijumbi, J. Serrat, J.-L. Garrido, N. Bouten, F. De Turck, and R. Boutaba, “Network function virtualization: State-of-the-art and research challenges,” IEEE Communications Surveys & Tutorials, vol. 18, no. 1, pp. 236-262, Firstquarter 2016.

[3] W.-K. Chen, Y.-F. Liu, A. De Domenico, Z.-Q. Luo, and Y.-H. Dai, “Optimal network slicing for service-oriented networks with flexible routing and guaranteed E2E latency,” IEEE Transactions on Network and Service Management, vol. 18, no. 4, pp. 4337-4352, December 2021.
[4] A. De Domenico, Y.-F. Liu, and W. Yu. “Optimal virtual network function deployment for 5G network slicing in a hybrid cloud infrastructure,” IEEE Transactions on Wireless Communications, vol. 19, no. 12, pp. 7942-7956, December 2020.

[5] A. Jarray and A. Karmouch, “Decomposition approaches for virtual network function placement and routing optimization,” in Proceedings of IEEE 4th International Conference on Cloud Networking (CloudNet), Niagara Falls, Canada, October 2015, pp. 171-177.

[6] A. Jarray and A. Karmouch, “The virtual network function placement problem,” in Proceedings of IEEE 20th International Workshop on Quality of Service (IWQoS), Coimbra, Portugal, June, 2012, pp. 1-4.

[7] S. Ayoubi, S. Sebbah, and C. Assi, “A logic-based Benders decomposition approach for the VNF assignment problem,” IEEE Transactions on Cloud Computing, vol. 7, no. 4, pp. 894-906, October-December 2019.

[8] A. Jarray and A. Karmouch, “Decomposition approaches for virtual network embedding with one-shot node and link mapping,” IEEE/ACM Transactions on Networking, vol. 23, no. 3, pp. 1012-1025, June 2015.

[9] M. C. Luizelli, L. R. Bays, L. S. Buriol, M. P. Barcellos, and L. X. Li and C. Qian, “The virtual network function placement problem,” in Proceedings of 12th International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob), Hong Kong, China, May 2015, pp. 69-70.

[10] A. Mohammadkhan, S. Ghapani, G. Liu, W. Zhang, K. K. Ramkrishnan, and T. Wood, “Virtual network function placement and traffic steering in flexible and dynamic software defined networks,” in Proceedings of IEEE International Workshop on Local and Metropolitan Area Networks (LANMAN), Beijing, China, April 2015, pp. 1-6.

[11] X. Li and C. Qian, “The virtual network function placement problem,” in Proceedings of IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS), Hong Kong, China, May 2015, pp. 69-70.

[12] M. Abu-Lebed, D. Nabousi, R. Gliotho, and C. W. Chouati, “On the placement of VNF managers in large-scale and distributed NFV systems,” IEEE Transactions on Network and Service Management, vol. 14, no. 4, pp. 875-889, December 2017.

[13] N. Promwongs, M. Abu-Lebed, S. Kianpisheh, F. Belqasmi, R. H. Gliotho, H. Elbiaze, N. Crespi, and O. Alfandi, “Ensuring reliability and low cost when using a parallel VNF processing approach to embed delay-constrained slices,” IEEE Transactions on Network and Service Management, vol. 17, no. 4, pp. 2226-2241, October 2020.

[14] Gurobi Optimization, “Gurobi optimizer reference manual,” 2019. [Online]. Available: http://www.gurobi.com.

[15] J. H. Hooker, G. Ottosson, “Logic-based Benders decomposition,” Mathematical Programming, vol. 96, pp. 33-60, April 2003.

[16] M. Conforti, G. Cornájols, and G. Zambelli, Integer Programming. Cham, Switzerland: Springer, 2014.

[17] Y. Zhang, N. Beheshti, L. Beliveau, G. Lefebvre, R. Marghirimalani, R. Mishra, R. Patney, M. Sharzinpour, R. Subrahmanian, C. Truchan, and M. Tatipamula, “SEERING: A software-defined networking for inline service chaining,” in Proceedings of 21st IEEE International Conference on Network Protocols (ICNP), Goettingen, Germany, October 2013, pp. 1-10.

[18] J. Halpern and C. Pignataro, “Service function chaining (SFC) architecture,” 2015. [Online]. Available: https://www.rfc-editor.org/rfc/rfc7656.txt.pdf.

[19] G. Mirjalily and Z.-Q. Luo, “Optimal network function virtualization and service function chaining: A survey,” Chinese Journal of Electronics, vol. 27, no. 4, pp. 704-717, July 2018.

[20] R. K. Abuja, T. L. Magnanti, and J. B. Orlin, Network Flows. Upper Saddle River, NJ, USA: Prentice Hall, 1993.

[21] F. Glover. “Improved linear integer programming formulations of non-linear integer problems,” Management Science, vol. 22, no. 4, pp. 455-460, December 1975.

[22] M. Rost and S. Schmid, “On the Hardness and Inapproximability of Virtual Network Embeddings,” IEEE/ACM Transactions on Networking, vol. 28, no. 2, pp. 791-803, April 2020.

[23] G. S. Paschos, M. A. Abdullah, and S. Vassilaras, “Network slicing with splittable flows is hard,” in Proceedings of IEEE 29th Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), Bologna, Italy, September 2018, pp. 1788-1793.

[24] W.-K. Chen, Y.-F. Liu, F. Liu, Y.-H. Dai, and Z.-Q. Luo, “Network slicing for service-oriented networks under resource constraints,” IEEE Journal on Selected Areas in Communications, vol. 35, no. 11, pp. 2512-2521, November 2017.

[25] J. H. Hooker, G. Ottosson, “Logic-based Benders decomposition,” Mathematical Programming, vol. 96, pp. 33-60, April 2003.

[26] M. Conforti, G. Cornájols, and G. Zambelli, Integer Programming. Cham, Switzerland: Springer, 2014.

[27] Y. Zhang, N. Beheshti, L. Beliveau, G. Lefebvre, R. Marghirimalani, R. Mishra, R. Patney, M. Sharzinpour, R. Subrahmanian, C. Truchan, and M. Tatipamula, “SEERING: A software-defined networking for inline service chaining,” in Proceedings of 21st IEEE International Conference on Network Protocols (ICNP), Goettingen, Germany, October 2013, pp. 1-10.

[28] J. Halpern and C. Pignataro, “Service function chaining (SFC) architecture,” 2015. [Online]. Available: https://www.rfc-editor.org/rfc/rfc7656.txt.pdf.

[29] G. Mirjalily and Z.-Q. Luo, “Optimal network function virtualization and service function chaining: A survey,” Chinese Journal of Electronics, vol. 27, no. 4, pp. 704-717, July 2018.

[30] R. K. Abuja, T. L. Magnanti, and J. B. Orlin, Network Flows. Upper Saddle River, NJ, USA: Prentice Hall, 1993.

[31] F. Glover. “Improved linear integer programming formulations of non-linear integer problems,” Management Science, vol. 22, no. 4, pp. 455-460, December 1975.

[32] M. Rost and S. Schmid, “On the Hardness and Inapproximability of Virtual Network Embeddings,” IEEE/ACM Transactions on Networking, vol. 28, no. 2, pp. 791-803, April 2020.

[33] G. S. Paschos, M. A. Abdullah, and S. Vassilaras, “Network slicing with splittable flows is hard,” in Proceedings of IEEE 29th Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), Bologna, Italy, September 2018, pp. 1788-1793.

[34] W.-K. Chen, Y.-F. Liu, F. Liu, Y.-H. Dai, and Z.-Q. Luo, “A companion technical report on ‘Towards efficient large-scale network slicing: An LP rounding-and-refinement approach’,” Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, China, Technical Report, 2022. [Online]. Available: http://home.cc.ac.cn/~yafliu/Report_Network_Slicing.pdf

[35] A. Ben-Tal and A. Nemirovski, Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications. Philadelphia, USA: Society for Industrial and Applied Mathematics, 2001.

[36] V. V. Vazirani, Approximation Algorithms. Berlin, Germany: Springer, 2014.

[37] Q. Liu, L. Deng, H. Zeng, and M. Chen, “On the min-max-delay problem: NP-completeness, algorithm, and integrality gap,” in Proceedings of IEEE Information Theory Workshop (ITW), Kaohsiung, Taiwan, November 2017, pp. 21-25.

[38] S. Orlowski, R. Wessely, M. Pfofo, and A. Tomaszewski, “SNDlib 1.0 – survivable network design library,” Networks, vol. 55, no. 3, pp. 276-286, May 2010.

[39] J. P. Viehman, S. Ahmed, and G. Nemhauser, “Mixed-integer models for nonseparable piecewise-linear optimization: Unifying framework and extensions,” Operations Research, vol. 58, no. 2, pp. 303-315, March-April 2010.

[40] T. Fukasawa and M. Goycoolea, “On the exact separation of mixed integer knapsack cuts,” Mathematical Programming, vol. 128, pp. 19-41, June 2011.