Suppression of the shear viscosity near the critical temperature in hot QCD

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Abstract

We consider QCD near but above critical temperature \( T_c \). The pressure, susceptibilities and the renormalized Polyakov loop — which is an order parameter for the deconfining phase transition — dramatically change up to temperatures a few times \( T_c \). We refer to this region as a "semi"-QGP, where partial confinement plays important role. We show that the shear viscosity \( \eta \) is suppressed by two powers of the Polyakov loop. This suggests that \( \eta/T^3 \) decreases markedly as QCD cools down to temperatures near \( T_c \). We also show a ratio of the viscosity to the entropy becomes small near \( T_c \) \cite{1}.

1. Introduction: Semi-QGP

To understand nature of the deconfining phase transition in QCD at finite temperature and/or density is one of current topics of great interest in hadron physics. The experimental results on heavy-ion collisions at relativistic heavy ion collider (RHIC), which may have probed a quark-gluon plasma (QGP), have brought remarkable results. One of them is elliptic flow suggesting a small ratio of the shear viscosity to the entropy. This result cannot be reproduced by an ordinary weakly coupled plasma, so it has been described, rather, as a strongly coupled quark-gluon plasma. In this talk, we discuss how the shear viscosity can be small even in perturbation theory, for moderate values of the coupling near the phase transition.

At the deconfining phase transition, the physical degrees of freedom change from mesons and baryons, for which the pressure is of order one, to quarks and gluons, with pressures of order \( N_f N_c \) and \( N_c^2 \), where \( N_c \) and \( N_f \) are the number of colors and flavors. It is useful to view deconfinement as the ionization of color charge. In the confined phase, there is no ionization of color. Conversely, far into the deconfined phase, color is completely ionized. The ionization in non Abelian gauge theory is characterized by the renormalized Polyakov loop, which is an order parameter for a global \( Z(N_c) \) sym-
While the Polyakov loop is a strict order parameter in pure Yang-Mills theory, in QCD it is only an approximate order parameter. Lattice simulations, however, find that the renormalized Polyakov loop is numerically close to a good order parameter, e.g., the susceptibility of the Polyakov loop has a sharp peak at $T_c$ [2]. At extremely high temperature, quarks and gluons are almost freely moving, and one has a complete QGP, where the renormalized Polyakov loop is near one, and is insensitive to temperature. In the hadronic phase, the value of Polyakov loop is almost zero up to near $T_c$.

In the intermediate region, it is expected that the Polyakov loop changes strongly with temperature, and represents the partial ionization of color. We refer to this region as a "semi"-QGP. Obvious questions are: how wide is the semi-QGP, and how do observables change in it? Lattice simulations provide partial answers to these questions: for three colors, they find that a semi-QGP window is $T_c$ to $4T_c$ in pure Yang-Mills theory and $0.8T_c$ to $2 - 3T_c$ with $2 + 1$ flavors of dynamical quarks [2,3]. The RHIC experiment could probe a semi-QGP. We expect that not only static observables, such as the pressure, but also dynamical quantities, such as transport coefficients, change strongly in the semi-QGP.

2. Shear viscosity in the semi-QGP

The transport coefficients are parameters in hydrodynamics which cannot be determined by itself. They are calculated in several methods such as Kubo formula and kinetic theory. Here, we consider how the shear viscosity changes in the semi-QGP. We do not give the explicit calculation but a simple explanation for how the shear viscosity can be small even with a small coupling constant in the semi-QGP. In classical transport theory, the shear viscosity $\eta$ is given by

$$\eta \approx \frac{1}{3} n \bar{p} \lambda,$$

where $n$ is the number density of quarks and gluons, $\bar{p}$ is the mean momentum, and $\lambda$ is the mean free path. There are three possibilities to get small viscosity: the number of particles, mean momentum, or mean free path become small. In a relativistic plasma, $\bar{p} \sim T$ for light particles. There are three possibilities to get small viscosity: the number of particles, mean momentum, or mean free path become small. In a relativistic plasma, $\bar{p} \sim T$ for light particles. One may expect that the mean momentum does not get small, so that we treat it as of order $T$. In general, the mean free path depends on the number of particles. One may expect that the more dilute the system is, the longer the mean free path is. In fact, in classical transport theory, the mean free path is $\lambda \sim 1/(n\sigma)$, where $\sigma$ is the transport cross section; thus, $\eta \sim T/\sigma$. This is true in a QGP at very high temperatures, where $n \sim N_c^2 T^4$, $\sigma \sim (g^4 \ln 1/g)/T^2$, and $g$ is the gauge coupling constant. The logarithmic term, $\ln 1/g$, is known as the Coulomb logarithm, and is caused by an infrared singularity at the forward scattering region. As a result, $\eta \sim T^3/(g^4 \ln 1/g)$, which gives the same result of the Boltzmann equation, up to overall constant [4].

In the semi-QGP, the situation is different. The interaction and the number density of the particles may depend on their color structures because of the partial ionization of color. Then, the average mean free path is

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1 The expectation value of the renormalized Polyakov loop has an ambiguity associated with the renormalization scheme. Here we assume a scheme in which zero point energy vanishes.
Fig. 1. The function $R(L)$ of Eq. (3), versus $\ell$. “Step” and “GW” denote eigenvalue distributions of the Wilson line with a simple step-function and that in the Gross-Witten matrix model [5], respectively (left). The ratio of our viscosity to the entropy calculated on the lattice (right).

$$\lambda^{-1} \sim \frac{\sum_{a,b} n_a n_b \sigma_{a,b}}{\sum_a n_a}.$$  \hspace{1cm} (2)

where $\sigma_{a,b}$ and $n_a$ is the transport cross section and number density for color indices $a, b$. If the transport cross section is independent of color, the mean free path reduces to $\lambda \sim 1/(n\sigma)$.

We found $\sum_a n_a \sim N_c^2 \ell^2$, and $\sum_{a,b} n_a n_b \sigma_{a,b} \sim N_c^4 \ell^2\sigma$ in the semi-QGP [1], where $\ell$ is the expectation value of the Polyakov loop. Therefore the mean free path $\lambda \sim N_c^2/\sigma$ does not become small as the density decreases; the shear viscosity becomes small, $\eta \sim T\ell^2/\sigma$, by two powers of the Polyakov loop. This implies that the interactions canceling color are more favored, i.e., the correlation of colored particles is stronger even for small values of the coupling constant. Consequently, the shear viscosity can be small in the semi-QGP.

3. Numerical Results

In the previous section, we discussed how the shear viscosity is suppressed for small values of the Polyakov loop. In this section, we show numerical results. We make numerous drastic assumptions to characterize the semi-QGP. We assume that the coupling constant is small even near $T_c$. For simplicity, we take the number of colors and flavors to be large. We work in a semiclassical approximation, so that we can treat the Polyakov loop as a constant classical background field about which we expand. The background field corresponds to an eigenvalue distribution of the Polyakov loop. Since this distribution is so far unknown, we need to make assumptions about it. We use two forms to check the sensitivity of our results. The first is to take a simple step function, of width $\beta$, about the origin. The other is to take a distribution as in the Gross-Witten matrix model [5]. We then calculate the shear viscosity by employing a Boltzmann equation to leading order in $g$ and $\log(1/g)$ [1]. We take the following parameterization to compare with previous work [4]:

$$R(L) = \frac{\eta(L)}{\eta(0)}, \quad \eta(0) = \frac{c_\eta}{g^4 \ln(\kappa/g)},$$  \hspace{1cm} (3)

where $\eta(0)$ is the viscosity in absence of the background. The coefficient $c_\eta$ is the numerical constant depending on the number of colors and flavors. $\kappa$ is higher order correction
beyond the leading-log order; here we treat it as a parameter.

In Fig. 1, we show numerical results of $R(L)$ with $N_f/N_c = 0, 1$. The differences between eigenvalue distributions are very small, most a few percent over the entire range of $\ell$. We find that the shear viscosity is suppressed near $T_c$ or at a small $\ell$, as we discussed in the previous section. A bump is observed near $\ell \approx 0.9$. The value is 25% larger than $\eta(0)$, which is analytically estimated $\sim 1 + 1.47\sqrt{1-\ell}$. We expect that this non-analytic behavior will be washed out by corrections to higher order.

For hydrodynamics, a more important quantity than $\eta$ is the dimensionless ratio, $\eta/s$, where $s$ is the entropy density. This is related to a diffusion constant, $D = \eta/(sT)$. In Fig. 1, we compare the shear viscosity to the lattice entropy $s_{\text{lat}}$. The coupling constant used is a one-loop running coupling with $\alpha_s(T_c) = 1/3$; an unknown parameter beyond the leading-log order is taken $\kappa = 8, 32$ and 64. The result strongly depends on the choice of $\kappa$; however, in any case, $\eta/s_{\text{lat}}$ is small near $T_c$, and becomes larger as temperature increases.

4. Summary

We have shown that the shear viscosity is suppressed by two powers of the Polyakov loop in the semi-QGP region near $T_c$. It is natural to suspect that heavy ion collisions at RHIC have probed some region in the semi-QGP. Since one needs a small value of the shear viscosity to fit the experimental data, perhaps one is near $T_c$. Heavy ion collisions at the LHC may probe temperatures which are significantly higher, possibly well into the complete QGP. If so, then at small times collisions at the LHC creates a system with large shear viscosity; as the system cools through $T_c$, the shear viscosity then drops. Thus the semi-QGP predicts that at short times, the hydrodynamic behavior of heavy ion collisions at the LHC is qualitatively different from that at RHIC.

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