A New Quantile Estimation Method of Weibull-Rayleigh Distribution

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Authors’ contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

In this paper, we compared different Parameter Estimation method of the two parameter Weibull-Rayleigh Distribution (W-RD) namely; Maximum Likelihood Estimation (MLE), Least Square Estimation method (LSE) and three methods of Quartile Estimators. Two of the quartile methods have been applied in literature, while the third method (Q1-M) is introduced in this work. The methods have been applied to simulate data. These methods of estimation were compared using Error, Mean Square Error and Total Deviation (TD) which is also known as Sum Absolute Error Estimate (SAEE). The analytical results show that the performances of all the parameter estimation methods were satisfactory with data set of Weibull-Rayleigh distribution while degree of accuracy is determined by the sample size. The proposed quartile (Q1-M) method has the least Total Deviation and MSE. In addition, the quartile methods perform better than MLE for the simulated data. In particular, the proposed quartile methods (Q1-M) have an added advantage of simplicity in usage than MLE methods.

Keywords: Weibull-Rayleigh distribution; quartile estimator; least square estimator; maximum likelihood estimator; total deviation and bias.

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1 Introduction

Alzaatreh, et al. [1] introduced a new method for generating families of continuous random variable known as the transformed-transformer or Y-X families, in which a random variable X (the transformer) is used to transform another random variable Y (the transformed) defined on [0, ∞). This newly developed method was used to define the Weibull-X family of distributions by making Y the Weibull random variable Bourguignon et al. [2], hence two-parameter Weibull-Rayleigh distribution (W-RD) was introduced and studied by Akarawak et al. [3] without the method of Estimating the parameters of the distribution. In contrast, Merovci and Elbatal [4] developed and applied a three-parameter Weibull-Rayleigh Distribution by using the Weibull generator on an odd ratio of the form \( \frac{G(x)}{1-G(x)} \). The authors applied method of maximum likelihood in their parameter estimation. Luguterah [5] introduced generalized Exponential Raleigh distribution using log logistics quantile function. Ogunsanya et al. [6] studied Odd Lomax Exponential distribution and its properties.

The probability density function (pdf) of the two-parameter Weibull-Rayleigh distribution with shape parameter \( a \) and scale parameter \( k \) is given by:

\[
g(x) = \frac{2a\left(\frac{x^2}{k}\right)^a \exp\left\{-\left(\frac{x^2}{k}\right)^a\right\}}{x} ; x \geq 0, a, k > 0. \tag{1.1}
\]

And cumulative distribution function (cdf) given by:

\[
F(x; a, k) = 1 - e^{-\left(\frac{x^2}{k}\right)^a} \tag{1.2}
\]

It has been proved that if \( X \) follows W-RD with parameters \( a \) and \( k \), then \( X^2 \) follows the Weibull distribution with parameters \( a \) and \( k \). A simulation study also shows that the shape of the distribution is similar to that of the Weibull distribution.

W-RD has potential of possible application to various real life data. This has been illustrated by Akarawak et al. [3], who also applied the distribution to two data sets on marriage survival and insurance claims using maximum likelihood estimation of the parameters. However, obtaining closed form estimators for this distribution is still a major setback to its applicability. Several parameter estimation methods have been studied and used by many researchers, Nwobi [7].

In this paper, We use the maximum likelihood estimators (MLE), Least Square Estimation Method, Quartile methods (Q3 & Median and Q1 & Q3) used by Entisar and Nadia [8] to Exponential distribution and Dorota [9] to Cauchy distribution, to estimate the two parameters of the Weibull-Rayleigh distribution. Teimouri [10] compared some estimation methods without the quantile method of estimation. The present paper introduces third method of quartile using first quartile and median (Q1 & Median).

The remainder of the article is organized as follows. In section 2, the methodology is discussed; section 3 presents the simulation studies and results while section 4 concludes the paper.

2 Materials and Methods

This section examines five different methods of estimation for Weibull-Rayleigh distribution:

2.1 Maximum Likelihood Estimator (MLE)

Let \( X \) be a random variable and \( X_1, X_2, \ldots X_n \) are random sample from a distribution with pdf \( f(x) \) with a joint density
Likelihood function is given as

\[ l(x_1, x_2, \ldots x_n, \theta) = \prod_{i=1}^{n} f(x_i, \theta) \]

And log likelihood is given

\[ L = \log l(x_1, x_2, \ldots x_n, \theta) = \log \left( \prod_{i=1}^{n} f(x_i, \theta) \right) \]

The MLE is the most common estimation method according to Nelson [11] and Johnson et al. [12]. MLE is proved to be asymptotically unbiased.

Let a random sample from a two parameters Weibull-Rayleigh distribution as defined in (1.1), the likelihood function is given by:

\[ L(x; a, k) = \prod_{i=1}^{n} \left\{ \frac{2a(x_i^2)}{k} \exp\left\{ -\left(\frac{x_i^2}{k}\right)^a \right\} \right\} \]

\[ L(x; a, k) = (2a)^n k^{-na} \prod_{i=1}^{n} x_i^{2a-1} \exp\left\{ -\left(\frac{x_i^2}{k}\right)^a \right\} \]

And the log-likelihood function of equation (2.2) is given by;

\[ \log L = n \ln 2a - na \ln k + (2a - 1) \sum x_i - \frac{\sum x_i^{2a}}{k} \]

Taking a partial differentiation of equation (2.3) w.r.t \( a \) and \( k \) respectively and equating to zero, we obtain the estimating equations as follows:

\[ \frac{\partial \ln L}{\partial a} = \frac{n}{a} - n \ln k + 2 \sum \ln x_i - \frac{\sum (x_i^2)^a}{k^a} \ln \left( \frac{\sum x_i^2}{k} \right) \]

(2.4)

\[ \frac{\partial \ln L}{\partial k} = -\frac{na}{k} + \frac{a \sum (x_i^2)}{k^{a+1}} = 0 \]

(2.5)

Solving the above system of two non-linear equations (2.4) and (2.5) with respect to \( a \) and \( k \) simultaneously. We obtain an estimator of \( k \),

\[ \hat{k} = \left( \frac{\sum x_i^{2a}}{n} \right)^{\frac{1}{a}} \]

(2.6)

On substituting estimates of \( k \) in (2.4), we have

\[ \frac{n}{a} - n \ln \left( \frac{\sum x_i^{2a}}{n} \right)^{\frac{1}{a}} + 2 \sum \ln x_i - \frac{\sum (x_i^2)^a}{\left( \frac{\sum x_i^{2a}}{n} \right)^{\frac{1}{a}}} \ln \left( \frac{\sum x_i^2}{\left( \frac{\sum x_i^{2a}}{n} \right)^{\frac{1}{a}}} \right) = 0 \]

(2.7)
The solution of this system of equations (2.6) and (2.7) are not possible because the equations are not in closed form therefore, we use numerical computation in R programming version 3.4.0 to estimate the two parameters of W-RD assuming the initial estimates of \(a\) and \(k\) are given by \(a_0 = 1\) and \(k_0 = 1\) for MLE.

2.2 Least Squares Estimation Method (LSE)

For the estimation of probability distribution parameters, the least squares estimation method (LSE) is extensively used in reliability engineering and mathematics problems.

Let \(t_{1:n}, t_{2:n}, t_{3:n}, \ldots, t_{n:n}\) be ordered statistics

The cumulative distribution function of (1.1) is given by

\[
F(t; \alpha, k) = 1 - e^{-\left(\frac{t}{\alpha}\right)^a} \tag{2.8}
\]

\[
1 - F(t; \alpha, k) = e^{-\left(\frac{t}{\alpha}\right)^a} \tag{2.9}
\]

To get a linear relation between the two parameters, take the natural logarithm of above equation (2.9) twice and further simplify it we get:

\[
\ln t_{i:n} = \frac{\ln k}{2} + \frac{\ln(-\ln[1-F(t_i)])}{2a} \tag{2.10}
\]

This can be represented using a simple linear model of a regression: \(y = \beta x + c\)

Where

\[y = \ln t_{i:n}, \ x = \ln[-\ln(1 - F(t_i))], \ \beta = \frac{1}{2a}, \ c = \frac{\ln k}{2}, \ \text{n is the sample size and } i = 1, 2, \ldots, n.\]

\(F(t_i)\) is estimated and replaced by Median rank method \(F(t_i) = \frac{i-0.3}{n+0.4}\).

To compute \(a\) and \(k\) by simple linear regression, we obtain the least square estimates (LSE) of \(\beta\) and \(c\) as:

\[
\hat{\beta} = \frac{n \sum \left(\ln(t) \ln[-\ln(1 - F(t_i))]\right) - \left[\sum (\ln t) \right] \sum \left[\ln[-\ln(1 - F(t_i))]\right]}{n \sum [\ln(-\ln(1 - F(t_i)))]^2 - \left[\sum \ln[-\ln(1 - F(t_i))]\right]^2}
\]

\[\hat{c} = \bar{y} - \hat{\beta} \bar{x}\]

2.3 Quartile method

Quartile method is a special case of quantile function where \(Q\) (Quantile functions) are inverse of CDF, \(F\) of a Weibull-Rayleigh distribution. Let \(a, k\) be the parameters to be estimated and \(t_{1:n}, t_{2:n}, t_{3:n}, \ldots, t_{n:n}\) be the order statistics obtained from a random sample from \(F(t_{1:n}; a)\), if the function \(F(t_{1:n}; a, k)\) is continuous and strictly monotonically increasing then one can write

\[F(t_{1:n}; a) = P_{1:n; a} = Q_{1:n; a, k}\] \tag{2.11}

Where \(P_{1:n; a}\) and \(Q_{1:n; a, k}\) are \(i\)th percentile and quartile respectively with \(a, k > 0\) are parameters of the above distribution.
Then CDF of a two-parameter Weibull-Rayleigh Distribution is given
\[
F(t_{1:n}; \alpha) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\alpha}}
\]

Hence,
\[
P_{1:n; \alpha} = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\alpha}}, \text{ simplifying the expression}
\]
\[
e^{-\left(\frac{t}{\alpha}\right)^{\alpha}} = 1 - P_{1:n; \alpha} \quad (2.12)
\]

taking logarithm of both sides twice, we have
\[
2\ln t_{1:n} - \alpha \ln k = \ln[-\ln(1 - P_{1:n; \alpha})]
\]
\[
2\alpha \ln t_{1:n} = \alpha \ln k + \ln[-\ln(1 - P_{1:n; \alpha})]
\]
\[
\ln t_{1:n} = \frac{\ln k}{2} + \frac{\ln[-\ln(1 - P_{1:n; \alpha})]}{2\alpha} \quad (2.13)
\]
Equation (2.13) is the generalized quartile (quantile) expression of the W-RD.

**Method 1: (Q1-Q3)**

Matching first quartile and third quartile of Weibull Rayleigh distribution together
\[
\ln t_{3/4:n} = \ln k/2 + \ln \left[-\ln \left(1 - P_{3/4:n}\right)\right]/2a
\]
\[
\ln t_{1:n} = \frac{\ln k}{2} + \frac{\ln[-\ln\left(1 - P_{1:n; \alpha}\right)]}{2a} \quad (2.14)
\]
\[
\ln t_{3:n} = \frac{\ln k}{2} + \frac{\ln[-\ln\left(1 - P_{3:n; \alpha}\right)]}{2a} \quad (2.15)
\]
\[
2\ln t_{3/4:n} - \frac{\ln[-\ln(\frac{3}{4})]}{a} = \ln k \quad (2.16)
\]
\[
\ln t_{1/4:n} = \ln k/2 + \ln \left[-\ln \left(1 - P_{1/4:n}\right)\right]/2a
\]
Solving (2.3) and (2.4) simultaneously for the parameters \( a, k \)
\[
a = \frac{\ln \left[-\ln\left(\frac{3}{4}\right)\right] - \ln \left[-\ln\left(\frac{1}{4}\right)\right]}{2(\ln t_{1/4:n} - \ln t_{3/4:n})} \quad (2.17)
\]
\[
k = \exp \left\{2\ln t_{3/4:n} - \frac{\ln[-\ln(\frac{1}{4})]}{a}\right\} \quad (2.18)
\]

**Method 2: (Q3 – median)**

This method adopts the matching of third quartile and median of Weibull Rayleigh distribution together and solve it simultaneously
\[ \ln t_{3/4} = \ln k/2 + \ln \left(-\ln \left(1 - \frac{1}{3} \right)\right)/2a \]  
\[ \ln t_{1/2} = \ln k/2 + \ln \left(-\ln \left(1 - \frac{1}{2} \right)\right)/2a \]  
\[ \ln t_{3/4} = \ln k/2 + \ln \left(-\ln \left(1 - \frac{1}{4} \right)\right)/2a \]  
\[ S = \frac{\ln \left(-\ln \left(1 - \frac{1}{2} \right)\right) - \ln \left(-\ln \left(1 - \frac{1}{4} \right)\right)}{\ln t_{3/4} - \ln t_{1/2}} \]  
\[ k = \exp \left(2\ln t_{3/4} - \frac{\ln \left(-\ln \left(1 - \frac{1}{4} \right)\right)}{a}\right) \]  

**Method 3: (Q1-Median)**

This article introduced and proposed the method Q1-Median which is matching the first/lower quartile and median expression of the W-RD.

\[ \ln t_{3/4} = \ln k/2 + \ln \left(-\ln \left(1 - \frac{1}{4} \right)\right)/2a \]  
\[ 2\ln t_{3/4} - \frac{\ln \left(-\ln \left(1 - \frac{1}{4} \right)\right)}{a} = \ln k \]  
\[ \ln t_{1/2} = \ln k/2 + \ln \left(-\ln \left(1 - \frac{1}{2} \right)\right)/2a \]  

Hence, substitute equation (2.24) in equation (2.25) we have;

\[ a = \frac{\ln \left(-\ln \left(1 - \frac{1}{2} \right)\right) - \ln \left(-\ln \left(1 - \frac{1}{4} \right)\right)}{\ln t_{3/4} - \ln t_{1/2}} \]  
\[ k = \exp \left(2\ln t_{3/4} - \frac{\ln \left(-\ln \left(1 - \frac{1}{4} \right)\right)}{a}\right) \]  

### 2.4 Goodness of fit analysis

Some methods of goodness of fit analysis are employed in this work; Bias, Mean square error, MSE and Total deviation, TD are three measurements that give an indication of the accuracy of parameter estimation. Nwobi [4], Ahmad [13] and Muhammad and Ahmad [14] referred to the use of the procedure of MSE and TD.

a. Bias

The bias of an estimator \( H \) is the expected value of the estimator less the value \( \theta \) being estimated: \( \text{Bias} = E(H) - \theta \)

If an estimator has a zero bias, we say it is unbiased. Otherwise, it is biased. Let’s calculate the bias of the sample mean estimator

b. Mean Square Errors (MSE) The MSE can be calculated as below

\[ \text{MSE} = \text{Var}(H) + \text{(Bias)}^2 \]

c. Total Deviation is calculated for each method as follows
TOTAL DEVIATION (TD) = \( \left| \frac{a - \bar{\alpha}}{a} \right| + \left| \frac{k - \bar{k}}{k} \right| \)

Where \( a \) and \( k \) are the known parameters, and \( \bar{\alpha} \) and \( \bar{k} \) are the estimated parameters by these methods.

### 3 Simulation Studies, Results and Discussion

A simulation study is conducted to evaluate the MLE and Least Square Estimator and Quartile Estimators in terms of their Parameter estimates, bias, mean square errors and Total Deviation for various parameter combinations and different sample sizes. The following values 0.05, 0.5, 1, 2 are considered for the parameter \( \alpha \), and 0.5, 1 for the parameter \( k \). The process is repeated 1000 times. Four different sample sizes \( n = 100, 250, 500 \) and 1000 are considered. The estimates are presented in Tables 1-4.

All computations and simulations in this study were performed in R version 3.4.0. We relied on the functions \( \text{optim(optimization of MLE)} \) and \( \text{mse} \) from R packages \( \text{Optimr.function and Metrices respectively.} \)

We ranked the performance of the methods based on the least TD and MSE criteria. In Table 1, though the quartiles method show less MSE but the bias is large, this mean that the Weibull-Rayleigh distribution does not converge when parameter \( \alpha \lessless 0.09 \) for all values of \( k \). The Tables 1-4 show that when \( \alpha=[0.2,0.5] \), the distribution is highly skewed to the right and when \( \alpha=[1.4,1.9] \), the distribution shows a symmetric property. Also from the density plot when \( \alpha=[0,0.2) \) the distribution does not perform well for all values of \( k \).

In Tables 1-4, It is observed that the performance of all the parameter estimation methods were satisfactory with data set of Weibull-Rayleigh distribution while level of accuracy is determined by the sample size.

Table 1 showed that when the sample size is less than 100, Least Square Estimator (LSE) work best irrespective of the values of the parameters. In Tables 2 & 4, the proposed method Q1-M performed best with W-RD of sample sizes 250 and 1000. From Table 3, Q3-Q1 method looked most accurate when sample size is 500 but less than 1000 with this distribution.

**Table 1. Summary of estimates, bias, MSE and TD for Weibull-Rayleigh distribution when \( n=100 \)**

| \( \alpha \) | \( k \) | MLE Estimates | LSE Estimates | Q1-M Estimates | Q3-Q1 Estimates | Ranking |
|-----|-----|-------------|-------------|--------------|--------------|--------|
| 0.05 | 0.5 | 0.052 | 0.386 | 0.049 | 0.541 | 0.052 | 23.173 | 0.051 | 4.236 |
|     |     | BIAS | 0.002 | 0.114 | 0.001 | 0.041 | 0.000 | 22.673 | 0.001 | 3.736 |
|     |     | MSE | 0.052 | 0.386 | 0.000 | 0.002 | 0.000 | 50.416 | 0.000 | 0.015 |
|     |     | TD | 0.259 | 0.101 | 0.045 | 45.386 | 0.000 | 0.015 |
|     | 0.5 | 0.515 | 0.487 | 0.490 | 0.504 | 0.518 | 0.504 | 0.507 | 0.492 |
|     |     | BIAS | 0.015 | 0.013 | 0.010 | 0.004 | 0.018 | 0.004 | 0.007 | 0.008 |
|     |     | MSE | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.025 | 0.000 |
|     |     | TD | 0.056 | 0.027 | 0.044 | 0.045 | 0.000 | 0.000 |
| 0.5 | 1 | 0.515 | 0.974 | 0.490 | 1.004 | 1.000 | 0.507 | 0.492 |
|     |     | BIAS | 0.015 | 0.026 | 0.010 | 0.008 | 0.018 | 0.009 | 0.007 | 0.015 |
|     |     | MSE | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.098 | 0.003 | 0.001 |
|     |     | TD | 0.056 | 0.027 | 0.045 | 0.029 | 0.000 | 0.000 |
| 1 | 1 | 1.030 | 0.987 | 0.981 | 1.004 | 1.035 | 0.991 | 1.014 | 0.986 |
|     |     | BIAS | 0.030 | 0.013 | 0.019 | 0.004 | 0.035 | 0.009 | 0.014 | 0.014 |
|     |     | MSE | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.021 | 0.013 | 0.000 |
|     |     | TD | 0.043 | 0.023 | 0.044 | 0.028 | 0.000 | 0.000 |
| 2 | 1 | 2.061 | 0.994 | 0.981 | 2.008 | 2.070 | 0.992 | 2.028 | 0.991 |
|     |     | BIAS | 0.061 | 0.006 | 1.019 | 1.008 | 0.070 | 0.008 | 0.028 | 0.009 |
|     |     | MSE | 0.000 | 0.000 | 0.002 | 0.030 | 0.001 | 0.005 | 0.053 | 0.000 |
|     |     | TD | 0.037 | 1.518 | 0.043 | 0.023 | 0.000 | 0.000 |

| 0.05 | 1.0 | 0.515 | 0.974 | 0.981 | 0.981 | 0.981 | 0.981 | 0.981 | 0.981 | 0.981 |
|     |     | BIAS | 0.015 | 0.026 | 0.010 | 0.008 | 0.018 | 0.009 | 0.007 | 0.015 |
|     |     | MSE | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
|     |     | TD | 0.056 | 0.027 | 0.045 | 0.029 | 0.000 | 0.000 |

**Table 1 continued**
Table 2. Summary of estimates, bias, MSE and TD for Weibull-Rayleigh distribution when \( n=250 \)

| \( \alpha \) | \( K \) | MLE | LSE | Q1-M | Q3-Q1 | Ranking |
|---|---|---|---|---|---|---|
| Estimates | Estimates | Estimates | Estimates | Estimates | Estimates | Estimates |
| 0.05 | 0.5 | 0.055 | 0.794 | 0.056 | 0.732 | 0.050 | 4.534 | 0.050 | 1.141 |
| BIAS | 0.005 | 0.294 | 0.006 | 0.232 | 0.000 | 4.034 | 0.000 | 0.641 |
| MSE | 0.000 | 0.086 | 0.000 | 0.054 | 0.000 | 0.087 | 0.000 | 0.098 |
| TD | 0.681 | 0.587 | 8.068 | 1.282 | LSE |
| 0.5 | 0.5 | 0.547 | 0.524 | 0.561 | 0.519 | 0.501 | 0.508 | 0.502 | 0.492 |
| BIAS | 0.047 | 0.024 | 0.061 | 0.019 | 0.001 | 0.008 | 0.002 | 0.008 |
| MSE | 0.002 | 0.001 | 0.003 | 0.003 | 0.010 | 0.002 | 0.004 | 0.001 |
| TD | 0.141 | 0.161 | 0.018 | 0.020 | Q1-M |
| 1 | 1 | 0.547 | 1.047 | 0.561 | 1.039 | 0.501 | 1.017 | 0.502 | 0.984 |
| BIAS | 0.047 | 0.047 | 0.061 | 0.039 | 0.001 | 0.017 | 0.002 | 0.016 |
| MSE | 0.002 | 0.002 | 0.004 | 0.002 | 0.010 | 0.007 | 0.004 | 0.002 |
| TD | 0.141 | 0.161 | 0.019 | 0.020 | Q1-M |
| 2 | 1 | 2.188 | 1.012 | 2.245 | 1.010 | 2.006 | 1.000 | 2.008 | 0.994 |
| BIAS | 0.188 | 0.012 | 0.245 | 0.010 | 0.006 | 0.000 | 0.008 | 0.006 |
| MSE | 0.035 | 0.000 | 0.600 | 0.000 | 0.165 | 0.001 | 0.066 | 0.000 |
| TD | 0.106 | 0.132 | 0.003 | 0.010 | Q1-M |

Table 3. Summary of estimates, bias, MSE and TD for Weibull-Rayleigh distribution when \( n=500 \)

| \( \alpha \) | \( K \) | MLE | LSE | Q1-M | Q3-Q1 | Ranking |
|---|---|---|---|---|---|---|
| Estimates | Estimates | Estimates | Estimates | Estimates | Estimates | Estimates |
| 0.05 | 0.5 | 0.050 | 0.468 | 0.052 | 0.431 | 0.050 | 1.570 | 0.050 | 0.805 |
| BIAS | 0.000 | 0.032 | 0.002 | 0.069 | 0.000 | 1.070 | 0.000 | 0.305 |
| MSE | 0.000 | 0.001 | 0.000 | 0.006 | 0.000 | 0.005 | 0.000 | 0.117 |
| TD | 0.069 | 0.172 | 2.140 | 0.610 | MLE |
| 0.5 | 0.5 | 0.503 | 0.497 | 0.517 | 0.493 | 0.503 | 0.502 | 0.501 | 0.499 |
| BIAS | 0.003 | 0.003 | 0.017 | 0.007 | 0.003 | 0.002 | 0.001 | 0.001 |
| MSE | 0.000 | 0.000 | 0.000 | 0.000 | 0.006 | 0.000 | 0.004 | 0.003 |
| TD | 0.012 | 0.048 | 0.010 | 0.004 | Q3-Q1 |
| 0.5 | 1 | 0.503 | 0.993 | 0.517 | 0.985 | 0.503 | 1.005 | 0.501 | 0.998 |
| BIAS | 0.003 | 0.007 | 0.017 | 0.015 | 0.003 | 0.005 | 0.001 | 0.002 |
| MSE | 0.000 | 0.000 | 0.000 | 0.000 | 0.006 | 0.000 | 0.004 | 0.011 |
| TD | 0.012 | 0.048 | 0.011 | 0.004 | Q3-Q1 |
| 1 | 1 | 1.005 | 0.997 | 1.033 | 0.993 | 1.006 | 0.999 | 1.003 | 0.997 |
| BIAS | 0.005 | 0.003 | 0.033 | 0.007 | 0.006 | 0.001 | 0.003 | 0.003 |
| MSE | 0.000 | 0.000 | 0.001 | 0.000 | 0.022 | 0.000 | 0.014 | 0.003 |
| TD | 0.008 | 0.040 | 0.007 | 0.006 | Q3-Q1 |
| 2 | 1 | 2.010 | 0.998 | 2.066 | 0.996 | 2.012 | 0.999 | 2.005 | 0.998 |
| BIAS | 0.010 | 0.002 | 0.066 | 0.004 | 0.012 | 0.001 | 0.005 | 0.002 |
| MSE | 0.000 | 0.000 | 0.004 | 0.000 | 0.088 | 0.000 | 0.056 | 0.001 |
| TD | 0.007 | 0.037 | 0.007 | 0.004 | Q3-Q1 |
Table 4. Summary of estimates, bias, MSE and TD for Weibull-Rayleigh distribution when n=1000

| α  | K | MLE Estimates | LSE Estimates | Q1-M Estimates | Q3-Q1 Estimates | TD Estimates | Ranking |
|----|---|---------------|---------------|----------------|----------------|--------------|---------|
| 0.05 | 0.5 | 0.050 | 0.377 | 0.050 | 0.425 | 0.050 | 0.841 | 0.050 | 0.612 |
|       | BIAS | 0.000 | 0.123 | 0.000 | 0.075 | 0.000 | 0.341 | 0.000 | 0.112 |
|       | MSE | 0.000 | 0.015 | 0.000 | 0.006 | 0.000 | 0.001 | 0.000 | 0.099 |
|       | TD | **0.246** | **0.156** | **0.682** | **0.224** | LSE |
| 0.5 | 0.5 | 0.502 | 0.488 | 0.496 | 0.492 | 0.500 | 0.500 | 0.500 | 0.497 |
|       | BIAS | 0.002 | 0.012 | 0.004 | 0.008 | 0.000 | 0.000 | 0.000 | 0.000 |
|       | MSE | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 |
|       | TD | **0.028** | **0.023** | **0.006** | **0.002** | Q1-M |
| 0.5 | 1 | 1.004 | 0.988 | 0.993 | 0.992 | 1.000 | 0.998 | 1.000 | 0.997 |
|       | BIAS | 0.004 | 0.012 | 0.007 | 0.008 | 0.001 | 0.002 | 0.000 | 0.003 |
|       | MSE | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 | 0.001 |
|       | TD | **0.016** | **0.015** | **0.003** | **0.003** | Q1-M |
| 2 | 1 | 2.008 | 0.994 | 1.993 | 1.000 | 2.002 | 0.999 | 1.999 | 0.998 |
|       | BIAS | 0.008 | 0.006 | 0.007 | 0.000 | 0.002 | 0.001 | 0.001 | 0.002 |
|       | MSE | 0.000 | 0.000 | 0.000 | 0.000 | 0.003 | 0.000 | 0.000 | 0.000 |
|       | TD | **0.010** | **0.003** | **0.002** | **0.000** | Q1-M |

4 Conclusion

Five different estimation methods for both scale and shape parameters of two-parameter Weibull-Rayleigh distribution were used in this article but only four were reported. A simulation study was conducted to compare the five methods based on bias, mean square error and total deviation (TD) of estimates. From simulation results, it was observed that Quartile methods and Least Square Method are better estimation methods of Weibull-Rayleigh distribution. Quartile Estimators performed better with data with outlier. Based on total deviation, the proposed Q1-M method is the best estimator for estimating WRD parameters. Based on the criteria adopted in this study, we therefore recommend the proposed method of matching of first quarter and median method (Q1-M) for the estimation of parameters of the Weibull-Rayleigh distributions.

Competing Interests

Authors have declared that no competing interests exist.

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