Numerical study of radial force and equilibrium position of microscale spherical particle under the effect of inertial focusing in 2D microchannel

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Abstract. Particle inertial focusing is a passive phenomenon at which a particle is acted upon by interaction with fluid and a channel wall, causing it to align at a certain distance from the channel wall called the equilibrium position as all transverse forces balance. In this work, numerical simulations using the finite element method were conducted in order to evaluate the forces acting on a spherical particle under Poiseuille flow in a 2D microchannel. Various fluid and particle parameters investigated include fluid density, viscosity, mean flow velocity, particle and channel dimensions. The calculated particle’s equilibrium position in relation to each individual parameter is found to be proportional to the square of mean fluid velocity and fluid density, and the cube of particle size while inversely proportional to channel half-width and the square of fluid viscosity.

1. Introduction
The phenomenon at which particles with random spatial distribution at the channel inlet align at a certain distance from a channel’s center in an influence of Poiseuille flow in a straight channel is called particle inertial focusing [1]. Although this phenomenon has been applied in various applications [2], the hydrodynamic focusing mechanism has not fully been investigated. In this work, finite element analysis was used for evaluating forces due to pressure and shear stress acting upon a particle flowing under Poiseuille flow in a 2D microchannel under various fluid flow, particle and microchannel properties.

2. Theory
Flow velocity profile, pressure distribution and shear stress that act on flowing particle can be determined by solving Navier-Stokes equation as

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 \mathbf{u} + \mathbf{f},
\]

where \(\mathbf{u}\) is the fluid velocity, \(p\) is the pressure, \(\mu\) is the fluid viscosity, \(\rho\) is the fluid density, and \(\mathbf{f}\) is the external forces per unit volume. Note that gravitational and buoyancy forces were neglected in this study.
as their magnitudes were much smaller than the lift force. Vertical or lift force acting on a particle is calculated from

\[ F = \oint_S (-p\mathbf{1} + \tau)\mathbf{n} \, ds, \]

where \( F \) is the lift force acting on the particle, \( \mathbf{1} \) is the unit tensor, \( \tau \) is the shear stress, and \( s \) is the particle’s surface area. In the study, the particle Reynolds numbers (\( Re_p \)) in all setup conditions is greater than or equal to one since the inertia of the fluid must be significant on this scale [2], as such, \( Re_p \) is defined as

\[ Re_p = Re \left( \frac{a}{R} \right)^2 = \frac{2\rho U a^2}{\mu R}, \]

where \( Re \) is the Reynolds number, \( a \) is the particle radius, \( R \) is the channel half-width, and \( U \) is the mean flow velocity.

3. Materials and Methods
The Navier-Stokes equation was solved numerically using COMSOL Multiphysics laminar flow module in order to derive the forces acting on the particle. Note that a single spherical particle was placed at various radial positions in a microchannel. The simulation assumed steady state laminar flow of an incompressible fluid with no slip on the channel walls. The simulation was divided into two parts, fluid flow simulation without suspended particle and evaluation of the lift force on the particle. For the second part, the particle’s center was set as a reference frame for the ease of force calculation [1,3], where the particle was at rest while the channel walls moved with the speed at which the particle would be moving but in the opposite direction. By ignoring drag force, the speed of fluid as the same position as particle’s center is zero. The accuracy of the simulated results were verified with the first reported observational findings by Segré and Silberberg [4], in which the particle radius is 0.8 mm, channel half-width is 5.8 mm, fluid density is 1.8 g/cm\(^3\), fluid viscosity is 0.4 Pa·s and mean fluid velocity is 0.46 m/s. The ranges of fluid, particle and channel parameters examined in the study are illustrated in table 1. Note that the investigated parameter was varied from 2.5 µm spherical particle in radius flow with fluid that has mean velocity 1.78 m/s, density 0.997 g/cm\(^3\) and viscosity 0.89 mPa·s with the channel that its half-width is 25 µm which the Reynolds number of particle equals one as the minimum value of flow setup parameter.

| Table 1. The values of investigated parameters varying from the minimum flow setup parameter. |
|---------------------------------|------------------|
| Parameter                      | Range            |
| Fluid density (g/cm\(^3\))     | 0.997 - 1.80     |
| Fluid viscosity (mPa·s)        | 0.466 - 0.890    |
| Fluid mean velocity (m/s)      | 1.780 - 4.462    |
| Half-width of channel (µm)     | 7.5 - 25         |
| Radius of particle (µm)        | 2.50 - 12.5      |

4. Results and Discussion
The COMSOL simulation calculated fluid velocity distribution in the laboratory frame without the particle and in the particle reference frame where the inlet flow speed is 1.78 m/s as shown in figure 1 (a) and (b), respectively. Notice that the fluid velocity profile in the particle frame is close to zero in the area surrounding particle while the speed of the wall is equal but in the opposite direction to the speed of the particle in the laboratory frame. To verify the simulated results, Segré’s and Silberberg’s parameters
and results were used for comparison. The particle was placed at various positions from the center of the channel to close to the wall as shown in figure 2 (a). It was found that particle’s equilibrium position (r), where lateral force vanishes, is 0.5685R. This is in agreement with Segré’s and Silberberg’s particle equilibrium position of approximately 0.6R. An example of force calculation investigated of numerous channel sizes is shown in figure 2 (b). Both graphs from figure 2 illustrate that if the particle is close to the channel’s center, the lateral force is directed towards the wall (shear-gradient lift force) whereas when a particle is near the wall, an even stronger lateral force is induced in the direction towards the channel’s center (wall lift force). Figure 3 shows the linear plot of the particle’s equilibrium position versus various fluid flow parameters, particle and channel parameters are listed in table 1. It can be seen that the particle’s equilibrium position approaches the center of the channel with higher fluid density, mean fluid velocity and particle size, while a higher fluid viscosity and larger channel size result in shifting the particle’s equilibrium position away from the center of the channel. The results of the study agrees with previous experimental studies by Toner’s [2] and Di Carlo’s [5] groups.

![Figure 1](image1.png)

**Figure 1.** Example of simulated fluid velocity profiles (a) in a laboratory frame without a particle and (b) in a particle frame with a particle.

![Figure 2](image2.png)

**Figure 2.** Total lift force on the particle’s surface versus the particle position using (a) Segré’s and Silberberg’s parameters and (b) a 2.5 µm particle in a radius flow with fluid that has mean velocity of 1.78 m/s, density of 0.997 g/cm³ and viscosity of 0.89 mPa·s within a variety of channel sizes.

5. Conclusion
In this numerical study of particle inertial focusing, the COMSOL Multiphysics laminar flow module was used in calculating the lateral force and equilibrium position of a particle. The agreement between
Figure 3. Equilibrium position per channel half-width versus the parameters of (a) fluid density squared, (b) one over fluid viscosity squared, (c) mean fluid velocity squared, (d) one over channel half-width and (e) particle radius cubed.

Simulated results and previous experimental studies indicate high accuracy of the simulation model. The results of the study can be further applied in designing microfluidic systems that utilize the inertial focusing effect in separating particles of different properties.

Acknowledgments
This work has been supported by the Thailand Center of Excellence in Physics and the Development and Promotion of Science and Technology Talents Project (DPST), Thailand.

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