Anomaly Free Gauged $SU(4)_L \times U(1)_X$ Models with Little Higgs

Otto C. W. Kong

Department of Physics, National Central University, Chung-li, TAIWAN 32054
$\&$ Institute of Physics, Academia Sinica, Nankang, Taipei, Taiwan 11529

Abstract

We present an analysis showing how anomaly free fermionic spectra with consistent embeddings of the Standard Model spectrum under $SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(3)_C \times SU(N)_L \times U(1)_X$ for any $N > 2$ can be obtained, with special focus on the $N = 3$ and $4$ cases. The construction is motivated by the little Higgs mechanism. We discuss the relevancy of the fermionic spectra to the latter, concentrating on two $N = 4$ models, without fermions of exotic charges. Such models hold the promise to address and solve all the major theoretical as well as phenomenological problems of the Standard Model at the TeV scale.

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The phenomenologically overwhelmingly successful Standard Model (SM) has a few theoretically shortcomings. The latter motivates many high energy theorists looking for beyond SM structures that could help us understand basically why some of the parameters in the SM have the values they have. The Higgs sector parameters, in particular, are haunted by the hierarchy problem, or the very un-natural fine-tuning required due to the quadratically divergent quantum corrections to the Higgs boson mass. The fine-tuning problem can be alleviated, only if there is new physics at the TeV scale that guarantee the cancellation of the quadratic divergence to an acceptable level, or totally change our picture of SM physics. A guaranteed cancellation has to come from some mechanism protected by a symmetry. Candidates of the kind include supersymmetry, and the recently proposed little Higgs mechanism\[1, 2\]. The fermion sector also has a very puzzling flavor problem. Finally one may worry about the strong CP problem\[3\].

To appreciate the flavor problem, we want to emphasize first that the SM has fermions of three families each has the same set of quantum numbers intricately connected among the family by chiral gauge anomaly cancellation conditions. This is well illustrated by the following argument. Assuming the existence of a multiplet having nontrivial charges under each part of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ SM gauge group and asking simply for the minimal chiral spectrum with all the gauge anomaly canceled, one would arrive at the spectrum of a SM family as the only solution\[4\]. So, the first fundamental question of the flavor problem is why there are three families, instead of one. The next step in the direction would be to understand the values of the set of flavor parameters and the strong hierarchy among them. In this aspect, there is a notion related to the hierarchy problem (of the Higgs sector parameters). Namely, only the third family fermions may have a significant role to play in the latter problem. The first two families are simply coupled to the Higgs boson too weakly. This is suggestive that the third family might be different from the first two.

The idea of connecting the three families through gauge anomaly cancellations of an extended gauge group is related to many interesting models\[4, 5, 6, 7, 8, 9\]. For the benefit of the present perspective, we note the following two approaches. Firstly, Ref.\[4\] treats the three families on the same footing while trying to duplicate the structure of the one-family SM spectrum (as presented above) for a three-family embedding generic $SU(N) \times SU(3) \times SU(2) \times U(1)$ gauge symmetry ($N = 4$ or otherwise). The existence of family-changing gauge bosons says that such models could not be relevant till a scale of about 200 TeV.
Treating the third family (quarks) different, Refs.\[5, 6, 7\] have $SU(3)_C \times SU(3)_L \times U(1)_X$ models that leave $SU(2)_L$ singlets mostly as $SU(3)_L$ singlets. Arguably, such model spectra are not particularly appealing, though they do tie the three families together. In fact, as we will show below, the two models are only examples of a one-parameter class of infinite number of models. However, such a model has a scale of relevancy at about a TeV. None of these extended fermion models addresses the hierarchy problem, though one could always incorporate supersymmetry other wise.

On the other hand, the recent little Higgs model-building game has not been paying quite enough attention to the fermion sector and the all important issue of gauge anomaly cancellations. Looking at it as a effective field theory at the TeV scale without bothering about the strong dynamics behind it, a little Higgs model needs at least an extra top-like quark with some appropriate (global) symmetry to cancel the quadratic divergence from the top itself. Extra gauge bosons may also be needed to cancel the quadratic divergence from the electroweak gauge bosons. So, a little Higgs model does have an extended gauge symmetry with extra fermions. To figure out the flavor structure of such a model, one does need to know the full quantum numbers of the SM fields. As pointed out in our earlier paper\[10\], the construction of the full fermion spectrum is nontrivial and unavoidable. A simple family universal embedding typically leaves nonvanishing gauge anomalies. Cancelling the latter by adding extra fermionic multiplets does not guarantee that no phenomenologically unacceptable extra SM chiral fermion would be introduced. To have a complete and consistent model, we are forced to address the flavor question at the same time. Consistent model solutions then hold promise to be models that address all the major theoretical, as well as phenomenological, problems of the SM at an very accessible energy scale. Such a model spectrum typically also contains extra $SU(2)_L$ singlet neutrino states, and new interactions involving the SM neutrinos. Hence, it also provides a bonus for understanding neutrino physics at the same low energy scale. It is our focus here to look into relevant fermionic spectra of the type that do have the consistent gauge anomaly cancellations.

Following Ref.\[10\], we take the group theoretically simple little Higgs model(s) introduced in Ref.\[11\] and try to constructed compatible anomaly free fermionic spectra to complete the consistent TeV scale models, without bothering about the strong dynamics suggested to be behind the picture at the scale of tens of TeV. Ref.\[11\] starts with a model with as $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry, the consistent fermionic spectrum of which
we discussed in Ref. [10]. The $SU(3)_L$ model has a difficulty on the Higgs quartic coupling. The latter motivated the authors to extend the little Higgs construction to a model with an $SU(3)_C \times SU(4)_L \times U(1)_X$ gauge symmetry [11]. Here, we present an analysis showing how anomaly free fermionic spectra with consistent embeddings of the SM spectrum under $SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(3)_C \times SU(N)_L \times U(1)_X$ for any $N > 2$ can be obtained, with special focus on the $N = 3$ and $4$ cases. By a consistent embedding, we mean that the full fermion spectrum does yield the three families SM fermions together with extra states that are vectorlike after the gauge symmetry is broken to that of the SM. While such models can be considered to be of interest in their own right, those among them that could serve as little Higgs models are considered to be more interesting. After presenting the fermionic spectra construction, we will take a look at such little Higgs models.

As said above, a little Higgs model typically has an extra top-like quark $T$ to cancel the quadratic divergence from the top itself. The way to do it as presented in Ref. [11] is to extend the $t$-$b$ quark doublet to a fundamental representation of an $SU(N)_L$ multiplet with $T$ included. Take the $N = 4$ example. The $t$-$b$ quark doublet of $SU(2)_L \times U(1)_Y$ is to be embedded into a $SU(4)_L \times U(1)_X$ quadruplet $Q^a$ as follows

$$4_L = (t^a, b^a, T^a, T'^a)^T$$

with an appropriate $X$-charge denoted by $X_Q$. The third state is the top-light quark $T$, with the usual electric charge of $Q = \frac{2}{3}$; and $a$ represent the $SU(3)_C$ index. Here, we keep the identity of the $T'$, i.e. its electric charge, unspecified for the moment. The vectorlike QCD spectrum is to be recovered by introducing the Dirac partners in $SU(4)_L \times U(1)_X$ singlets as

$$1_L = \bar{b}_a, \quad \bar{t}_a, \quad \bar{T}_a, \quad \bar{T}'_a.$$  

Their electric charge is to be given by their $X$-charge directly. The requirement amount to nothing other than a specific choice of $X$-charge normalization.

We are about to start on the discussion on constructing the anomaly free fermionic spectra. We want to emphasize here the construction we will discuss is generic. To be explicit, we will first relax the little Higgs mechanism requirement, i.e. we do not require the $Q^a$ quadruplet to contain the $T$ quark with electric charge $\frac{2}{3}$. We will return to models of interest in view of the little Higgs mechanism discussed in Ref. [11] afterwards.
The basic approach in the kind of model-building exploits the fact that
\[ N_c = N_f ; \]
namely, the number of SM families (of fermions) \( N_f \) happens to coincide with the number of colors. While the extension of \( SU(2)_L \) doublets into complex (anti-) fundamental representations \( N_L \) or \( \bar{N}_L \) of \( SU(N)_L \) introduces nontrivial \([SU(N)_L]^3\) gauge anomaly, if the three families of quark doublets are embedded into one \( N_L \) and two \( \bar{N}_L \) representations, only one net colored-\( \bar{N}_L \) multiplet is left to contribute to the anomaly which may then be canceled by putting the three families of leptonic doublets into \( N_L \)'s. We take \( N = 4 \) here for an explicit illustration. Denote the nontrivial \( SU(4)_L \) multiplets, apart from \( Q^a \), by \( Q'^a_k \) \((k = 1 \text{ or } 2)\) and \( L_i = 4_L \) \((k = 1 \text{ to } 3)\), and their X-charges by \( X_{Q'} \) and \( X_L \) respectively. We have then
\[ X_{Q'} = \frac{1}{3} - X_Q \] (3)
and
\[ X_L = X_Q - \frac{2}{3} \] (4)
from the requirement for correct embedding of the \( SU(2)_L \) doublets, or getting the right electric charge or hypercharge differences. In fact, we can take the electric charge embedding as given by
\[ Q = \frac{1}{2} \lambda^3 + \frac{A}{3} \lambda^8 + \frac{B}{6} \lambda^{15} + X , \] (5)
with the normalization \( \text{Tr}\{\lambda^a\lambda^b\} = 2\delta^{ab} \). The correct doublet embeddings require
\[ A + B + X_Q = \frac{1}{6} \] (6)
besides the given relationship among the X-charges. The latter gives \( X_Q + 2X_{Q'} + X_L = 0 \), an equation that also guarantees the cancellation of \([SU(4)_L]^2 U(1)_X\) gauge anomaly (provided that \( N_c = N_f \)).

So far in our construction of the fermion spectrum, we are left with the freedom to specify \( X_Q \) and \( A \) (or equivalently \( B \)). Interestingly, we do not need to specify their values in order to obtain the full anomaly free spectrum if all one cares is to embed the full SM spectrum while allowing extra vectorlike pairs of \( SU(2)_L \) singlet quarks and leptons of arbitrary electric charges. All we have to do is to finish the spectrum discussed so far with \( SU(4)_L \) singlets required for the vectorlike pairings of all the fermions at the QCD and QED level. And this
works essentially in the same way for the other $N$ values. There could be some redundancy in the full spectrum so obtained. Say, one does not need to put in Dirac partners for the SM neutrinos; singlets with no $X$-charges are as good as not being there; and vectorlike $SU(4)_L$ singlet lepton pairs that come up may be removed. To convince the readers that the anomaly cancellations do work, let us outline the mathematics. The $[SU(3)_C]^3$ anomaly vanishes as QCD is kept vectorlike. The kind of embedding by putting all Dirac partners of states in nontrivial $SU(4)_L$ representations as singlets, hence with $Q = X$, also guarantee cancellation of the $[SU(3)_C]^2 U(1)_X$ gauge anomaly family by family. For the $[\text{grav}]^2 U(1)_X$ anomaly, the trace of $U(1)_X$ charges for the quarks within each family is obviously just a scale factor different from that of $[SU(3)_C]^2 U(1)_X$. We are hence left only with the leptonic contributions, which again cancel for each family. It is then obviously that the $[U(1)_X]^3$ anomaly has to vanish too — an explicit checking of the algebra could also be done.

We have illustrated above essentially the existence of infinite number of SM embeddings into $SU(3)_C \times SU(N)_L \times U(1)_X$ ($N = 4$ or otherwise) that could be phenomenologically viable. For the $N = 4$ case in particular, we still have the freedom to choose $X_Q$ and $A$ as we like. The discussion is presented in such a way that it is easy to see how similar constructions would work for any other $N$. For example, we can take $N = 3$ and $B = 0$, to remove the inadmissible $\lambda_{15}^L$ in Eq.(5). Choosing $X_Q = \frac{1}{3}$ then has $A = \frac{1}{6}$ as fixed by Eq.(6) giving the only extra quark in $Q^a$ as $T$ (electric charge $\frac{2}{3}$) and essentially the $331$ little Higgs model as constructed in Ref.[10] (see also Refs.[5, 7]). We illustrate here again the full fermion spectrum in Table I. In fact, the (first) extra quark state in $Q^a$ always has $Q = X_Q - 2A + (B) = \frac{1}{6} - 3A$ [cf. Eq.(6)]. One can freely choose the value for this electric charge. The choice here fixes the value of $A$, which in turn fixes that of $X_Q$ through Eq.(6). Say, picking $Q = \frac{5}{3}$ gives $A = \frac{1}{2}$ and $X_Q = \frac{2}{3}$ yielding the fermion spectrum of Ref.[6]. Hence, there is a one parameter class of such $331$ models each containing a different extra (exotic) singlet quark. Only the one with exactly the $T$ quark, as discussed in Ref.[10], is relevant as a little Higgs model. For $N > 4$, there have to be extra parameters similar to $A$ and $B$ in an extended Eq.(5). The structure of such anomaly free models are otherwise easy to appreciate.

Now, we focus on $SU(4)_L \times U(1)_X$ models with an interest in their compatibility with the little Higgs idea. We first note that the two extra quarks in $Q^a$ have electric charges $X_Q - 2A + B(= \frac{1}{6} - 3A)$ and $X_Q - 3B$, according to Eq.(5). We are free to choose their values,
which in turn fixed the full anomaly free spectrum of a model. The little Higgs mechanism, as discussed in Ref. [11], requires a top-like $T$ quark with $Q = \frac{2}{3}$. Doing this fixes $A = \frac{-1}{6}$. Ref. [11] suggests taking the fourth quark in $Q^a$ as having the same electric charge. Further imposing that is equivalent to taking $X_Q = \frac{5}{12}$ and $B = \frac{-1}{12}$. Following our discussion above, one can easily obtain the full fermion spectrum (see Table II for an illustration). However, as the last quark of $Q^a$ suggested in Ref. [11] does not play a role in the quadratic divergence cancellation, the choice seems to be arbitrary. Giving the wide range of possible alternatives consistent spectra$^1$, we have to consider more seriously about the question of picking the one that is most desirable from the theoretical point of view. Moreover, apart from the choice of Ref. [11], there is another choice which looks a bit special—$X_Q = \frac{1}{6}, A = \frac{-1}{6},$ and $B = \frac{1}{6}$, giving an extra bottom-like $B$ quark together with the $T$ quark. This is the second model spectrum we think may also be of special interest from the little Higgs perspective.

For the little Higgs part of such models, let us see what we have from Ref. [11], which is fully compatible with the fermion spectrum of Table II. Four scalar quadruplets of the same quantum numbers are considered. The quantum numbers are such the quadruplets (or rather their conjugates) couple to the $Q^a$ quadruplet and the $\bar{t}_a$ or $\bar{T}_a$ singlet. Under our notation here, the $4_L$ scalars have $X$-charges given by $X_Q = \frac{-2}{3}(= \frac{1}{4})$. These are nonlinear sigma model fields that may be parametrized as $[cf. \text{Eqs.}(50,51) \text{of Ref. [11]}]$}

$$
\Phi_1 = e^{i\mathcal{H}_u f_{1\bar{f}}^1} \begin{pmatrix} 0 \\ 0 \\ f_1 \\ 0 \end{pmatrix}, \quad \Phi_2 = e^{-i\mathcal{H}_u f_{1\bar{f}}^2} \begin{pmatrix} 0 \\ 0 \\ f_2 \\ 0 \end{pmatrix}
$$

$$
\Psi_1 = e^{i\mathcal{H}_d f_4^1} \begin{pmatrix} 0 \\ 0 \\ f_3 \end{pmatrix}, \quad \Psi_2 = e^{-i\mathcal{H}_d f_4^2} \begin{pmatrix} 0 \\ 0 \\ f_4 \end{pmatrix}
$$

(7)

where $\mathcal{H}_u$ and $\mathcal{H}_d$ contain electroweak symmetry breaking doublets. For instance, we have

$^1$ It may be of interest to note that one such $SU(4)_L \times U(1)_X$ model has actually been available in the literature$^2$. The model has the two extra quarks having electric charges $\frac{2}{3}$ and $\frac{5}{3}$, hence is like a combination of the two available $SU(3)_L \times U(1)_X$ models$^3, 4, 5$. 

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the first order expansions

\[ \Phi_1^\dagger = \begin{pmatrix} 0 & 0 & f_1 \end{pmatrix} + \frac{i}{\sqrt{2}} \frac{f_2}{f_{12}} \begin{pmatrix} h_u & 0 & 0 \end{pmatrix} \]

\[ \Psi_2^\dagger = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} - \frac{i}{\sqrt{2}} \frac{f_3}{f_{34}} \begin{pmatrix} h_d & 0 & 0 \end{pmatrix} , \]

with similar expressions for \( \Phi_2^\dagger \) and \( \Psi_1^\dagger \); where \( f_{12}^2 = f_1^2 + f_2^2 \) and \( f_{34}^2 = f_3^2 + f_4^2 \), and \( h_u \) and \( h_d \) are two SM Higgs doublets. The essential feature here is the vacuum misalignment between the \( \Phi_i \) pairs and the \( \Psi_i \) pairs. To keep this feature, the third and fourth states of the four scalar quadruplets must have the same vanishing hypercharge. Enforcing that is equivalent to enforcing the corresponding choice of the fermionic spectrum, \textit{i.e.} with the fourth state in \( Q^a \) being the also top-like (the \( T' \)). The little Higgs picture needs a collective symmetry breaking, with aligned VEVs for the \( \Phi_i \), as well as \( \Psi_i \), pair of Higgs multiplets. Beyond that, the existence of both aligned pair with generic, hence misaligned VEVs actually yields naturally the preserved \( U(1) \) symmetry, to be identified as the hypercharge, under which the two states in the directions of the broken symmetries have vanishing charges. The choice of fermionic spectrum with three among the four states of a quadruplet having the same electric charge is hence more natural than it may look naively. So, the question is whether this is the only option for little Higgs idea to work.

From the above discussion, we can see that to take a different choice of the fermionic spectrum, we will have to change the content of the set of the symmetry breaking Higgs quadruplets as given by Eq.(7). To preserve the VEV misalignment Ref.[11] rely on to fix the (SM)Higgs quartic coupling, the fourth states of the \( \Psi_i \) pair have to be ones with zero hypercharge. This can be fixed by a suitable choice of their \( X \)-charges. The latter now has to be different from that of the \( \Phi_i \) pair. Let us explore the possibility with the fermion spectrum of Table III. We still take the \( \Phi_i \) pairs with \( X \)-charge given by \( -\frac{1}{2} \) to take care of the top-sector Yukawa couplings. Added to that, we can take a \( \Psi_i \) pair with \( X \)-charge given by \( \frac{1}{2} \). The little Higgs mechanism together with the quartic coupling part as discussed in Ref.[11] should still work. Just like the original scenario, each of the four scalar quadruplets is related to a global \( SU(4) \) symmetry that is broken to \( SU(3) \) by its own VEV. Each pair gives rise to a SM Higgs doublet as pseudo-Nambu-Goldstone states of a collective breaking of the corresponding pair of \( SU(4) \)’s. The spectrum choice, however, dictates a different embedding of the SM gauge group and hence gives the \( h_d \) doublet a different hypercharge. In the case discussed above, and in Ref.[11], \( h_d \) actually has the same hypercharge as \( h_u \) and
does not couple to the down-sector quarks the way the latter couples to the up-sector, hence a somewhat abuse of notation. For the alternative case at hand, however, $h_d$ will be an $h_d$ literally. In fact, we have

$$
\mathcal{L}_{\text{bottom}} = y_1 \bar{b}_a \Psi_1 Q^a + y_2 \bar{B}_a \Psi_2 Q^a \\
= \left( f_3 y_1 \bar{b}_a + f_4 y_2 \bar{B}_a \right) B^a + \frac{i}{\sqrt{2}} \left[ \frac{f_4}{f_3} y_1 \bar{b}_a - f_3 y_2 y_1 \bar{B}_a \right] h_d \left( \begin{array}{c} t^a \\ b^a \end{array} \right) + \cdots \quad (9)
$$

in exact analog to the top sector Yukawa couplings (note: $\bar{b}$ and $\bar{B}$ do not denote exact Dirac partners of $b$ and $B$ here).

At the electroweak scale, the models discussed have two Higgs doublets (plus some singlets). From the phenomenological point of view, two Higgs doublet models are prefered to have natural flavor conservation. There is also some indications that a large $\tan\beta$ value is required\[12\]. The latter implies large bottom Yukawa coupling. In that case, one does have to worry about the bottom loop contribution to the Higgs mass quadratic divergence. The Higgs structure of the model spectrum given by Table III discussed above could be a little Higgs model satisfying such requirements. The $b$-$B$ quark pair has canceled quadratic divergent contributions in exactly the same way as that of the $t$-$T$ quark pair. Hence, we consider the model to be an interesting alternative little Higgs model. A possible disadvantage of the model compared to the one above is that, unlike the latter (see Ref.\[10\] for discussions of an analog case) the gauge quantum number assignments do not rule out renormalizable tree-level Yukawa couplings to SM quarks of the first two families. We hope to go into more detailed studies of the models discussed here in future publications.

To summarize, we have presented here how to construct SM extensions with viable fermionic spectra under an $SU(3)_C \times SU(N)_L \times U(1)_X$ gauge symmetry. The construction methodology is based on a family non-universal treatment, under which the gauge anomalies cancel among the three SM families in a nontrivial fashion. While such models could be of interested in their own right, our study is motivated by solving the hierarchy problem through the little Higgs mechanism. We also discussed the relevancy and basic features of some of such models as little Higgs models, with particular focus on two specific $SU(4)_L$ models. We believe that the phenomenology of the models should be studied in more details.

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Table I : Fermion spectrum for the $SU(3)_C \times SU(3)_L \times U(1)_X$ model with little Higgs. Here, we give the hypercharges of the electroweak states, with SM doublets put in [.]’s. The other states are singlets. The normalization convention is $Q = T_3 + Y$. Hence, singlets have hypercharges identical with electric charge $Q$. Other symbols are self-explanatory. Note that despite the notation, the fermions are not mass eigenstates. Mass mixings are expected.

| $U(1)_Y$-states | $U(1)_Y$-states |
|------------------|------------------|
| $(3C, 3L, \frac{1}{3})$ | $\frac{1}{6}[Q]$ $\frac{2}{3}(T)$ |
| $2(3C, 3L, 0)$ | $2 \frac{1}{6}[2Q] 2 \frac{1}{3}(D, S)$ |
| $3(lC, 3L, \frac{-1}{3})$ | $3 \frac{1}{2}[3L] 3 0(3N)$ |
| $4(3C, 1L, \frac{-2}{3})$ | $4 \frac{-1}{3}(\bar{u}, \bar{c}, \bar{t}, \bar{T})$ |
| $5(3C, 1L, \frac{1}{3})$ | $5 \frac{1}{3}(d, s, b, \bar{D}, \bar{S})$ |
| $3(lC, 1L, 1)$ | $3 1 (e^+, \mu^+, \tau^+)$ |

Table II : Fermion spectrum for a $SU(3)_C \times SU(4)_L \times U(1)_X$ model with little Higgs. Again, we give the hypercharges of the electroweak states, with SM doublets put in [.]’s. Basic notation is the same as that of Table I. Note that we separate in the last column a set of singlet quarks and leptons to which alternative choices may be a feasibility. In that case, one has to made adjustments to the some of the $U(1)_X$-charges, as discussed in the text (see also Table III).

| $U(1)_Y$-states | $U(1)_Y$-states |
|------------------|------------------|
| $(3C, 4L, \frac{5}{12})$ | $\frac{1}{6}[Q]$ $\frac{2}{3}(T)$ $\frac{2}{3}(T')$ |
| $2(3C, 4L, \frac{-1}{12})$ | $2 \frac{1}{6}[2Q] 2 \frac{1}{3}(D, S) 2 \frac{1}{3}(D', S')$ |
| $3(lC, 4L, \frac{-1}{4})$ | $3 \frac{1}{2}[3L] 3 0(3N) 3 0(3N')$ |
| $5(3C, 1L, \frac{-2}{3})$ | $4 \frac{-1}{3}(\bar{u}, \bar{c}, \bar{t}, \bar{T}) \frac{-1}{3}(\bar{D}, \bar{S})$ |
| $7(3C, 1L, \frac{1}{3})$ | $5 \frac{1}{3}(d, s, b, \bar{D}, \bar{S}) 2 \frac{1}{3}(\bar{D}', \bar{S}')$ |
| $3(lC, 1L, 1)$ | $3 1 (e^+, \mu^+, \tau^+)$ |
Table III: Fermion spectrum for another $SU(3)_C \times SU(4)_L \times U(1)_X$ model with little Higgs. Again, we give the hypercharges of the electroweak states, with SM doublets put in []’s. Basic notation is the same as that of Table II.

| $U(1)_Y$-states | $\frac{1}{6}[Q]$ | $\frac{2}{3}(T)$ | $\frac{1}{3}(B)$ |
|------------------|------------------|------------------|------------------|
| $(3_c, 4_L, \frac{1}{6})$ | $\frac{1}{6}[Q]$ | $\frac{2}{3}(T)$ | $\frac{1}{3}(B)$ |
| $2 \ (3_c, \bar{4}_L, \frac{1}{6})$ | $2 \ \frac{1}{6}[2 \ Q]$ | $2 \ \frac{1}{3}(D, S)$ | $2 \ \frac{2}{3}(U, C)$ |
| $3 \ (1_c, 4_L, \frac{1}{2})$ | $3 \ \frac{1}{2}[3 \ L]$ | $3 \ 0(3 \ N)$ | $3 \ -1(3 \ E^-)$ |
| $6 \ (\bar{3}_c, 1_L, \frac{2}{3})$ | $6 \ \frac{2}{3}(\bar{u}, \bar{c}, \bar{t}, \bar{T})$ | $2 \ \frac{2}{3}(\bar{U}, \bar{C})$ |
| $6 \ (\bar{3}_c, 1_L, \frac{1}{3})$ | $6 \ \frac{1}{3}(\bar{d}, \bar{s}, \bar{b}, \bar{D}, \bar{S})$ | $\frac{1}{3}(\bar{B})$ |
| $6 \ (1_c, 1_L, 1)$ | $6 \ 1(\bar{e}^+, \mu^+, \tau^+)$ | $3 \ 1(3 \ E^+)$ |