Light quark decays of doubly heavy baryons in light front approach

Hang Liu\textsuperscript{1,}\textsuperscript{a}, Zhi-Peng Xing\textsuperscript{2,}\textsuperscript{b}, Chang Yang\textsuperscript{1,2,}\textsuperscript{c}

\textsuperscript{1} INPAC, Key Laboratory for Particle Astrophysics and Cosmology (MOE), Shanghai Key Laboratory for Particle Physics and Cosmology, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China
\textsuperscript{2} Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China

Received: 22 October 2022 / Accepted: 23 January 2023 / Published online: 6 February 2023
© The Author(s) 2023

Abstract We explore the semileptonic and nonleptonic decays of doubly heavy baryons ($\Omega_{cc}^{+}$, $\Omega_{bb}^{0}$, $\Omega_{bc}^{(*)-}$, $\Omega_{bc}^{(*)0}$) induced by the $s \to u$ transition. Hadronic form factors are parametrized by transition matrix elements and are calculated in the light front quark model. With the form factors, we make use of helicity amplitudes and analyze semileptonic and nonleptonic decay modes of doubly heavy baryons. Benchmark results for partial decay widths, branching fractions, forward–backward asymmetries and other phenomenological observables are derived. We find that typical branching fractions for semileptonic decays into $\ell \bar{\nu}$ are at the order $10^{-7} - 10^{-8}$ and the ones for nonleptonic decays are at the order $10^{-5}$, which are likely detectable such as in LHCb experiment. With the potential data accumulated in future, our results may help to shape our understanding of the decay mechanism in the presence of two heavy quarks.

1 Introduction

Weak decays of quarks can play an important role in testing the standard model of particle physics, and shaping our understanding of CP violation in our universe. Since most quarks in nature are bounded, the study of weak decays of quarks inside a hadron also provides a platform for the exploration of strong interactions. In the past decades there have been dramatic progresses made on both experimental and theoretical sides, and the unprecedented precise experimental measurements and theoretical calculations of heavy $B$ and $D$ meson decays lead to a stringent test of the standard model \cite{1–3}.

In addition to the bottom/charm mesons and baryons that consist of one heavy quark, baryons composed of two heavy quarks are of special interests and provide a unique arena to explore the QCD dynamics in the presence of two heavy quarks. In 2017, the LHCb collaboration has firstly observed the doubly heavy baryon $\Xi_{cc}^{++}$ via the final state $\Lambda_{cc}^{+} K^{-} \pi^{+} \pi^{+}$ \cite{4}. Subsequently, properties including the mass and lifetime has been precisely determined, and a series of decay channels were discovered in sequence \cite{5–10}. Very recently, a new decay mode $\Xi_{cc}^{++} \to \Xi_{cc}^{*+} \pi^{+} \pi^{+}$ was reported by the LHCb collaboration \cite{11}. Meanwhile many properties of doubly heavy spectroscopy have been investigated in theory, such as the hadron spectrum \cite{12–16}, the “decay constant” or the so-called pole residue \cite{17}, and so on \cite{18–20}. Focusing on the weak decays of doubly heavy baryons, there have been intense explorations ranging from the flavor SU(3) symmetry analyses \cite{21} to the model-dependent determinations of form factors and decay widths \cite{22–33}.

Aside from decays where the heavy quark undergoes a weak transition, there is also a class of decays in which the heavy quarks act as a spectator while the light quark explode in a weak transition. In particular a previous analysis which mainly focused on the light-quark decays of heavy hadrons with one heavy quark can be found in Ref. \cite{34}. But such decay processes of double heavy baryons are not previously explored, and the aim of this work is to fill this blank. Due to the very small phase space in this class of decays, only limited channels are kinematically allowed. In this work, we will explore the $s \to u$ transition of doubly heavy baryons decays, and consider explicitly

- the spin-1/2 to spin-1/2 decays,

$$\Omega_{cc}^{+} \to \Xi_{cc}^{++} , \quad \Omega_{bb}^{0} \to \Xi_{bb}^{0} , \quad \Omega_{bc}^{(*)-} \to \Xi_{bc}^{(*)-} , \quad \Omega_{bc}^{(*)0} \to \Xi_{bc}^{(*)0}.$$
the spin-3/2 to spin-1/2 decays,

$$\Omega_{cc}^{*+} \rightarrow \Xi_{cc}^{++}, \quad \Omega_{bb}^{*-} \rightarrow \Xi_{bb}^{0}, \quad \Omega_{bc}^{*0} \rightarrow \Xi_{bc}^+.$$  

While the small phase space will substantially suppress these decay branching fractions, and make them very difficult to be observed in experiments, the small phase space allows for solid theoretical predictions. On the one hand, measurements of semileptonic decay widths can help to provide rather reliable constraints on the form factors since the recoil is small. With the knowledge of form factors, nonleptonic decays could give a direct exploration of theoretical tools such as factorization. On the other hand, due to the small phase space, the helicity suppression amplitudes in certain semileptonic decays are uplifted by the muon mass. These effects are absent in semi-leptonic decays with electrons in the final state. Thus these classes of decays induced by $s \rightarrow u$ provide a new platform for studying the helicity suppression amplitudes in the experiment and offer possibilities of significant new physics (NP) contributions.

In the bottom-charm baryons, the spin of the $bc$ system can be both 0 or 1. For convenience we use the $\Xi_{bc}^+/\Xi_{bc}^0$ to denote the doubly heavy baryons consisting of spin-1 bc system and the $\Xi_{bc}^+/\Xi_{bc}^0$ to denote the doubly heavy baryons consisting of spin-0 bc system. At this stage, it is not clear which of the two hadrons is the ground state, and thus we will consider the decays of both hadrons. In addition to the spin-1/2 initial state, we will also consider the spin-3/2 doubly heavy baryons for a comparison, though whose main decay mode might be induced by the electromagnetic transition.

The rest of this paper is organized as follows. In Sect. 2, we give the theoretical framework in detail. After the parametrization of the form factors with both spin-1/2 to spin-1/2 and spin-3/2 to spin-1/2 processes, we will present the explicit calculation in the light-front approach. Numerical results for form factors and phenomenological analysis including decay widths, branching ratios and forward–backward asymmetry are given in Sect. 3. A brief summary is given in the last section.

2 Theoretical framework

Semileptonic and nonleptonic decays of the $s$ quark are induced by the quark-level transition $s \rightarrow u \ell \bar{\nu}$ and $s \rightarrow u \bar{u}d$. The corresponding Feynman diagrams for the processes to be investigated are shown in Fig. 1. For the nonleptonic decay processes, we only consider the factorizable contributions, which are less reliable but can give benchmark estimate of decay branching fractions.

The effective Hamiltonians for $s \rightarrow u \ell \bar{\nu}$ and $s \rightarrow u \bar{u}d$ are given as

$$\mathcal{H}(s \rightarrow u \ell \bar{\nu}) = \frac{G_F}{\sqrt{2}} V_{us} \left[ \bar{u} \gamma_\mu (1 - \gamma_5) s \right] \left[ \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu \ell \right],$$

$$\mathcal{H}(s \rightarrow u \bar{u}d) = \frac{G_F}{\sqrt{2}} V_{us} V_{ud}^*.$$
the form factors are defined as hadron matrix elements which can be parameterized by the coefficients at $m_s$.

Using the Hamiltonian, one can derive the semileptonic and color-allowed nonleptonic decay amplitudes as

\[
i \mathcal{M}(B_{Q_1 Q_2} \to B'_Q Q_2 \ell \bar{\nu}_\ell) = \frac{G_F}{\sqrt{2}} V_{ud} \bar{u} \gamma^\mu (1 - \gamma_5) u (P, S) \\
\times (B'_Q Q_{2u}(P', S'_2) \bar{u} \gamma_\mu (1 - \gamma_5) s) |B_{Q_1 Q_2s}(P, S_2)\rangle,
\]

\[
i \mathcal{M}(B_{Q_1 Q_2} \to B'_Q Q_2 \pi^-) = \frac{G_F}{\sqrt{2}} V_{ud} V_{ud}^* \times a_1 (\pi^-(P_3)) |\bar{u} \gamma_\mu (1 - \gamma_5) u(0)\rangle \\
\times (B'_Q Q_{2u}(P', S'_2) \bar{u} \gamma_\mu (1 - \gamma_5) s) |B_{Q_1 Q_2s}(P, S_2)\rangle,
\]

with $Q_{1,2} = b, c, a_1 = C_1/N_c + C_2$. The $C_i$ are Wilson coefficients at $m_s$ scale whose values can be taken from Ref. [35]: $C_1 = -0.742, C_2 = 1.422$. In the above the $P$ and $P'$ are the momentum of doubly heavy baryons $B'_Q Q_2$ and $B_{Q_1 Q_2}$ respectively.

Hadronic parts of these processes are represented by the hadron matrix elements which can be parameterized by the form factors. For the spin-1/2 to spin-1/2 processes, the form factors are defined as

\[
\left[ B_{Q_1 Q_2 u} (P', S') | u \gamma_\mu \right] (P, S = \frac{1}{2}, S_z = \frac{1}{2}) \\
\times (1 - \gamma_5) s |B_{Q_1 Q_2 s}(P, S = \frac{1}{2}, S_z = \frac{1}{2})\rangle
\]

\[
= \bar{u}(P', S'_2) \left[ \frac{M y_{\mu}}{M} f_1 \frac{1}{2} - \frac{1}{2} (q^2) + \frac{P_{\mu}}{M} f_2 \frac{1}{2} - \frac{1}{2} (q^2) \right] u(P, S_z) \\
+ \frac{P_{\mu}}{M} f_3 \frac{1}{2} - \frac{1}{2} (q^2) u(P, S_z),
\]

where $M = M - M'$. For the spin-3/2 to spin-1/2 process, the form factors are defined as

\[
\left[ B_{Q_1 Q_2 u} (P', S') | u \gamma_\mu \right] (P, S = \frac{3}{2}, S_z = \frac{1}{2}) \\
\times (1 - \gamma_5) s |B_{Q_1 Q_2 s}(P, S = \frac{3}{2}, S_z = \frac{1}{2})\rangle
\]

\[
= \bar{u}(P', S'_2) \left[ \frac{M y_{\mu}}{M} \frac{1}{2} - \frac{1}{2} (q^2) + \frac{P_{\mu}}{M} \frac{1}{2} - \frac{1}{2} (q^2) \right] u(P, S_z) \\
+ \frac{P_{\mu}}{M} f_3 \frac{1}{2} - \frac{1}{2} (q^2) + g_{\mu\alpha} f_4 \frac{1}{2} - \frac{1}{2} (q^2) \right] u(P, S_z),
\]

2.1 Light-front quark model

With the aid of the quark–diquark approximation, baryons are similar to the mesonic systems. In the weak transition, the two spectator quarks act as anti-quark. In doubly heavy decays, this approximation has been widely used [36–39]. Recently, a three-body vertex function in LFQM has been investigated [40]. Taking the $\Lambda_b \to \Lambda_c$ and $\Sigma_b \to \Sigma_c$ transitions as the example, the authors have demonstrated that the three-body vertex function can give consistent results with the diquark picture, and thus have validated the diquark approximation from a certain viewpoint.

The four vector in the light front frame is represented as: $v^\mu = (v^+, v^- , v_\perp)$ with $v^\pm = v^0 \pm v^3$. In LFQM, a baryon state can be presented by the internal quark and diquark, and one can expand the hadron state in terms of the momenta space function and flavor-spin function. For the spin-1/2 baryon state, it is

\[
[B(P, S, S_z)] = \int \{d^3 p_1|d^3 p_2\} 2(2\pi)^3 \delta^3(\vec{P} - \vec{p}_1 - \vec{p}_2) \\
\times \sum_{\lambda_1, \lambda_2} \psi_{Ss_c}(\vec{p}_1, \vec{p}_2, \lambda_1, \lambda_2) |q(p_1, \lambda_1)(di)(p_2, \lambda_2)\rangle,
\]

where $q$ denotes the light $s/u$ quark and “(di)” corresponds to the diquark shown in Fig. 2. Their helicities are denoted by $\lambda_1$ and $\lambda_2$. The momenta of the baryon, quark and diquark are $P$, $p_1$ and $p_2$, respectively. The momenta $\vec{P}$, $\vec{p}_1$ and $\vec{p}_2$ are the three-dimensional momenta, with the notation $\vec{P} = (p^+, p_\perp)$. Obviously, the on-shell momentum has only three degrees of freedom with four components, thus the minus component of the momentum is fixed as $p^- = (m^2 + p^2_\perp)/p^+$. 

\[
\Omega_{s_c}^{++} (P) \quad \text{diquark}(p_2)
\]

\[
\begin{array}{c}
s(p_1) \\
\longrightarrow \\
\longrightarrow \\
\longrightarrow
\end{array}
\]

\[
\begin{array}{c}
o(\vec{p}_1) \\
\longrightarrow \\
\longrightarrow \\
\longrightarrow
\end{array}
\]

\[
\begin{array}{c}
\Xi_{s_c}^{++}(P') \\
\longrightarrow \\
\longrightarrow
\end{array}
\]

\[
\text{Fig. 2} \quad \text{The diquark approximation for the baryonic transition}
\]
The wave function $\Psi$ in Eq. (5) can be generally decomposed as the combination of spin and momentum space:

$$\Psi^{sS}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \frac{1}{\sqrt{2(1 + \lambda_1 + \lambda_2)}}\right|\right|_{\lambda_1 = \lambda_2 = 1} \Gamma_S(\tilde{P}, S_c)\phi(x, k_\perp).

(6)

As we mentioned above, the diquark can be a spin-0 scalar and spin-1 axial-vector. For the scalar diquark, the interaction vertex $\Gamma$ is:

$$\Gamma = \frac{\gamma_5}{\sqrt{3}} \left( \phi^*(p_2, \lambda_2) - \frac{M_0 + m_1 + m_2}{p \cdot x_2 + m_2 M_0} \phi^*(p_2, \lambda_2) \cdot \tilde{P} \right),

(7)

Here $m_1$ and $m_2$ are the masses of quark and the spectator diquark. The $\tilde{P}$ is the sum of the on-mass-shell momentum of the light quark $q$ and diquark. It satisfies the condition: $\tilde{P} = p_1 + p_2$ and $\tilde{P}^2 = M_0^2$. The $M_0$ is the invariant mass of $P$ and is different from the baryon mass $M$. That is because the quark, the diquark and the baryon they composed cannot be on their mass shells simultaneously. The momentum $\tilde{P}$ and mass $M$ of baryon will satisfy the physical mass-shell condition: $M^2 = \tilde{P}^2$. Obviously, the momentum $\tilde{P}$ is not equal to the $P$.

For the spin-3/2 baryon states the momenta space wave function are similar with the spin-1/2 wave function [37]:

$$\Psi^{sS}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \frac{1}{\sqrt{2(1 + \lambda_1 + \lambda_2)}}\right|\right|_{\lambda_1 = \lambda_2 = 1} \Gamma_S(\tilde{P}, S_c)\phi(x, k_\perp).

(8)

In this situation, the spin of diquark can only be 1. The coupling vertex $\Gamma_A^\alpha$ is then:

$$\Gamma_A^\alpha = -\left( \epsilon^\alpha(p_2, \lambda_2) - \frac{p_2^\alpha}{p \cdot x_2 + m_2 M_0} \epsilon^*(p_2, \lambda_2) \cdot \tilde{P} \right).

(9)

The $\phi$ in Eq. (7) is a Gaussian-type function which is constructed as

$$\phi = 4 \left( \frac{\pi}{\beta^2} \right)^{3/4} \sqrt{\frac{e_1 e_2}{x_1 x_2 M_0}} \exp\left( -\frac{k_\perp^2}{2\beta^2} \right),

(10)

where $e_1$ and $e_2$ represent the energy of quark $q$ and diquark in the rest frame of $\tilde{P}$. $x_1$ and $x_2$ are the light-front momentum fractions which satisfy $0 < x_2 < 1$ and $x_1 + x_2 = 1$. The internal motion of the constituent quarks are described by the internal momentum $\tilde{k}$.

$$k_i = (k_1^-, k_1^+, k_i \perp) = (e_i - k_{i+}, e_i + k_{i-}, k_i \perp)

= \left( \frac{m_1^2 + k_{i+}^2}{x_1 M_0}, x_i M_0, k_i \perp \right),

p_{1+} = x_1 \tilde{P}^-, \ p_{1-} = x_1 \tilde{P}^+, \ p_{1 \perp} = x_1 \tilde{P} \perp + k_{i \perp},

p_{2+} = x_2 \tilde{P}^+ + k_{i \perp}, \ k_i \perp = -k_{2 \perp} = k_{2 \perp}.

(11)

The $\tilde{k}$ in the Eq. (10) is the internal three-momentum vector of diquark presented as $\tilde{k} = (k_2 \perp, k_2 \perp) = (k_\perp, k_\perp)$. The parameter $\beta$ in Eq. (10) describes the momentum distributions among the constituent quarks. Using the definition of internal three-momentum vector, we can deduce the invariant mass square $M_0^2$ as a function of the $(x_i, k_i \perp)$:

$$M_0^2 = \frac{k_{i \perp}^2 + m_1^2}{x_1} + \frac{k_{i \perp}^2 + m_2^2}{x_2}.

(12)

The expressions of the energy $e_i$ and $k_\perp$ can also be presented in terms of the internal variables $(x_i, k_i \perp)$ as:

$$e_i = \frac{x_i M_0}{2} + \frac{m_1^2 + k_{i \perp}^2}{2 x_1 M_0} = \sqrt{m^2 + k_{i \perp}^2 + k_{i \perp}^2},

k_{i \perp} = \frac{x_i M_0}{2} - \frac{m_1^2 + k_{i \perp}^2}{2 x_1 M_0}.

(13)

In the following, we use the notation $x = x_2$ and $x_1 = 1 - x$.

### 2.2 Form factors

The hadron matrix element for the spin-1/2 to spin-1/2 processes becomes:

$$\left\langle B_{Q_1} Q_{2u} \left( p', \frac{1}{2}, S'_c \right) | \bar{u} \gamma^\mu (1 - \gamma_5)s \right| B_{Q_1} Q_{2s} \left( p, \frac{1}{2}, S_c \right) \right\rangle

= \int (d^3p_2) \frac{\phi^\alpha(x', k'_\perp)}{2\sqrt{p_{1+}^2 + m_1 M_0}(p_{1+}^2 + m_1 M_0)} \times \sum_{k_\perp} \tilde{u}(\tilde{P}' , S'_c) \left[ \Gamma_A^\alpha(p_1^2 + m_1^2)\gamma^\mu (1 - \gamma_5)s(p_1^2 + m_1) \Gamma_A \right]

\times u_0(\tilde{P}, S_c).

(14)

For the spin-3/2 to spin-1/2 processes, the hadron matrix element is:

$$\left\langle B_{Q_1} Q_{2u} \left( p', \frac{1}{2}, S'_c \right) | \bar{u} \gamma^\mu (1 - \gamma_5)s \right| B_{Q_1} Q_{2s} \left( p, \frac{3}{2}, S_c \right) \right\rangle

= \int (d^3p_2) \frac{\phi^\alpha(x', k'_\perp)}{2\sqrt{p_{1+}^2 + m_1 M_0}(p_{1+}^2 + m_1 M_0)} \times \sum_{k_\perp} \tilde{u}(\tilde{P}' , S'_c) \left[ \Gamma_A^\alpha(p_1^2 + m_1^2)\gamma^\mu (1 - \gamma_5)s(p_1^2 + m_1) \Gamma_A^\alpha \right]

\times u_0(\tilde{P}, S_c).

(15)

and

$$m_1 = m, \ m_1' = m, \ m_2 = m(\alpha), \ \tilde{P}' = p_1' + p_2', \ \tilde{P}' = p_1' + p_2', \ \tilde{M}_0^2 = \tilde{P}'^2.

(16)
The momenta of $s$ quark in the initial baryon and $u$ quark in the final baryon are $p_1$ and $p'_1$, respectively. The $p_2$ is the momentum of the spectator diquark. $P$ and $P'$ are the four-momentum of the initial baryons $B_{1,2}$ and the final baryon states $B'_{1,2}$, respectively. $M$ and $M'$ are the physical masses of them.

It is noted that one can also define the invariant masses of the initial and final baryon states $M_0$ and $M'_0$ with $\tilde{P}(0^2) = M_0^{(02)}$. The $\Gamma$ in Eq. (14) is defined as

$$\Gamma_\alpha = \frac{1}{\sqrt{3}} \left( -\gamma_2(p_2, \lambda_2) + \frac{M'_0}{p'_2 + p_2 + m_2 M'_0} \epsilon(p_2, \lambda_2) \right) \gamma_5.$$

(17)

Compared with the LFQM results for the hadron matrix element, the form factors defined in Eqs. (3) and (4) can be extracted by solving the following equations as

$$\text{Tr} \left\{ (B' + M') \left[ \frac{M_{\gamma \mu}}{M} f_1^{1+1/2}(q^2) + \frac{P'_\mu}{M} f_2^{1+1/2}(q^2) \right] + \frac{P'_\mu}{M} f_1^{1+1/2}(q^2) \left( P + M \right) \left[ \Gamma_\mu \right] \right\} = H_1^3,$$

$$\text{Tr} \left\{ (B' + M') \left[ \frac{M_{\gamma \mu}}{M} g_1^{1+1/2}(q^2) + \frac{P'_\mu}{M} g_2^{1+1/2}(q^2) \right] + \frac{P'_\mu}{M} g_1^{1+1/2}(q^2) \left( P + M \right) \left[ \Gamma_\mu \right] \right\} = H_1^3,$$

$$\text{Tr} \left\{ (B' + M') \left[ \frac{M_{\gamma \mu}}{M} f_1^{1+1/2}(q^2) + \frac{P'_\mu}{M} f_2^{1+1/2}(q^2) \right] + \frac{P'_\mu}{M} f_1^{1+1/2}(q^2) \left( P + M \right) \left[ \Gamma_\mu \right] \right\} = H_1^3.$$
where \( s_{\pm} = (M \pm M^*)^2 - q^2 \). The expressions of form factors \( g_i \) and \( i \) can also be obtained by applying the transformation as

\[
\begin{align*}
M' &\rightarrow -M', \ M \rightarrow M, \ H_j \rightarrow K_j, \ i = 1 \\
M' &\rightarrow -M', \ M \rightarrow M, \ H_j \rightarrow -K_j, \ i = 2, 3, 4.
\end{align*}
\]  

(23)

Since the diquark can be scalar or axis-vector, the physical baryon state is the combination of the baryon state with scalar diquark and axis-vector in LFQM. Thus the physical form factor can also be written as

\[
[\text{formfactor}]_{\text{physical}} = c_S \times [\text{formfactor}]_S + c_A \times [\text{formfactor}]_A.
\]  

(24)

The coefficients \( c_S \) and \( c_A \) are flavor-spin factors that can be calculated by the flavor-spin wave function of baryons in the diquark basis. The flavor-spin wave functions of the spin-1/2 baryons are

\[
\begin{align*}
|B_{Q_1Q_2}, S_z = \frac{1}{2} \rangle &= \frac{1}{\sqrt{2}} (Q_1 Q_2 + Q_2 Q_1) q \\
&\times \left( \frac{1}{\sqrt{6}} (\uparrow \downarrow \uparrow + \downarrow \uparrow \downarrow - 2 \uparrow \uparrow \downarrow) \right) = [Q_1 Q_2]_A q,
\end{align*}
\]  

(25)

and the flavor-spin wave function of the spin-3/2 baryons are

\[
\begin{align*}
|B_{Q_1Q_2}, S_z = \frac{3}{2} \rangle &= \frac{1}{\sqrt{2}} (Q_1 Q_2 + Q_2 Q_1) q \\
&\times \left( \frac{1}{\sqrt{3}} (\uparrow \uparrow \downarrow + \uparrow \downarrow \uparrow + \downarrow \uparrow \uparrow) \right) = [Q_1 Q_2]_A q.
\end{align*}
\]  

(26)

For the doubly heavy baryons \( B_{bc} \), there is another set of the flavor-spin wave functions in which the bc system is scalar. Their flavor-spin wave functions are

\[
\begin{align*}
|B_{bc}, S_z = \frac{1}{2} \rangle &= \frac{1}{\sqrt{2}} (bc - bc) q \\
&\times \left( \frac{1}{\sqrt{2}} (\uparrow \downarrow \uparrow - \downarrow \uparrow \downarrow) \right) = [bc]_S q,
\end{align*}
\]  

(27)

where the flavor wave functions of the diquark basis are

\[
\begin{align*}
[Q_1 Q_2]_A &= \frac{1}{\sqrt{2}} (Q_1 Q_2 + Q_2 Q_1), \\
[Q_1 Q_2]_S &= \frac{1}{\sqrt{2}} (Q_1 Q_2 - Q_2 Q_1).
\end{align*}
\]  

(28)

Thus the overlapping factors are extracted and shown in Table 1.

3 Numerical results

After deriving the analytic expression in the LFQM, we now give the numerical results. In the calculation, the masses of quarks are taken from Refs. [41–43]

\[
m_u = m_d = 0.25 \text{ GeV}, \quad m_s = 0.37 \text{ GeV}, \quad m_c = 1.4 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}.
\]  

(29)

The diquark masses can be approximatively given as

\[
m_{[bb]} = 2m_b, \quad m_{[cc]} = 2m_c, \quad m_{[bc]} = m_b + m_c.
\]  

(30)

The baryon masses, lifetime, and input parameter \( \beta_{Q_1Q_2\ell q} \) are shown in Table 2.

3.1 Form factors

From the parametrization of form factors in Eqs. (3) and (4), one can notice that the form factors are the functions of \( q^2 \). To access the \( q^2 \) dependence of the form factors, we take the following parametrization scheme called the single pole model as

\[
F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m_{\text{pole}}^2}},
\]  

(31)

where \( F(0) \) is the numerical results of form factor at \( q^2 = 0 \). In this formula, the \( m_{\text{pole}} \) is set as \( m_{\text{pole}} = 0.494 \text{ GeV} \) for the form factor \( g_1 \) and \( m_{\text{pole}} = 0.845 \text{ GeV} \) for \( f_1 \). It can be understood by inserting a set of hadron complete states in the transition matrix element as

\[
\langle B' | \bar{u} \gamma^\mu (1 - \gamma_5) s | B \rangle
\]  

\[
= \sum_\lambda \int \frac{d^4k}{(2\pi)^4} \frac{1}{q^2 - m_\lambda^2} \langle B' \lambda | B \rangle \langle 0 | \bar{u} \gamma^\mu (1 - \gamma_5) s | \lambda \rangle
\]  

\[
= \sum_\lambda \int \frac{d^4k}{(2\pi)^4} \left[ f_{K^* K^*} \langle B' K^* | B \rangle - \frac{f_{K^* K^*}}{m_{K^*}^2} \langle B' K^* | B \rangle \right]
\]  

\[
\cdots = \frac{f_{K^* q^\mu}}{q^2 - m_{K^*}^2} \mathcal{M}(B \rightarrow B' K^*) - \frac{f_{K q^\mu}}{q^2 - m_{K}^2} \mathcal{M}(B \rightarrow B' K)
\]  

(32)

\[
\langle 0 | \bar{u} \gamma^\mu \gamma_5 s | K^* (q) \rangle = f_{K^* q^\mu},
\]  

\[
\langle 0 | \bar{u} \gamma^\mu s | K (q) \rangle = f_K q^\mu.
\]  

(33)

It can be easily seen that the \( q^2 \) dependence behavior of the transition matrix element will have a pole at \( q^2 = m_{K^*}^2 \) for the axis-vector current and \( q^2 = m_{K}^2 \) for the vector current. Therefore it is reasonable to set \( m_{\text{pole}} = m_{K} \) for the form factors \( g_1 \) and \( m_{\text{pole}} = m_{K^*} \) for \( f_1 \). Numerical results for the form factors are collected in Tables 3 and 4.

The form factors can also be extrapolated by fitting the functions since the form factors can be calculated in the un-physical region \( q^2 \leq 0 \) in LFQM [37]. However, in principle the region of

### Table 1

| Channel | \( c_S \) | \( c_A \) |
|---------|---------|---------|
| \( B_{Q_1Q_2} \rightarrow B_{Q_1Q_2} \) | 0 | 1 |
| \( \Omega_{bc}^0 \rightarrow \Omega_{bc}^+ \) | 1 | 0 |
| \( \Omega_{bc}^+ \rightarrow B_{Q_1Q_2}^0 \) | 0 | -1 |
−q^2 in our fit should be \(-q^2 << (m_B - m_G)^2\) for the accuracy of our calculation. Due to the small phase space, the uncertainty of our calculation will influence our extrapolated form factors very much.

### Table 3 Numerical results for the form factors of spin-1/2 to spin-1/2 transitions. The \(F^*(0)\) indicates the form factors simplified in the heavy quark limit in Eq. (35)

| Channel          | Form factor | \(F(0)\) | \(F^*(0)\) |
|------------------|-------------|----------|------------|
| \(\Omega_{bc}^+ \to \Xi_{bc}^+\) | \(f_1^{1/2}\) | \(-1.53 \pm 0.27 \pm 0.28 \pm 0.71\) | \(-1.510\) |
|                  | \(f_2^{1/2}\) | \(1.68 \pm 0.10 \pm 0.11 \pm 0.31\) | \(2.57\) |
|                  | \(f_3^{1/2}\) | \(-0.091 \pm 0.18 \pm 0.18 \pm 0.42\) | \(-1.04\) |
|                  | \(g_1^{1/2}\) | \(-1.30 \pm 0.02 \pm 0.01 \pm 0.04(10^{-2})\) | \(0\) |
|                  | \(g_2^{1/2}\) | \(-3.95 \pm 1.07 \pm 0.11 \pm 0.02\) | \(-4.53\) |
|                  | \(g_3^{1/2}\) | \(3.57 \pm 1.08 \pm 0.10 \pm 0.002\) | \(3.85\) |
| \(\Omega_{bb} \to \Xi_{bb}^0\) | \(f_1^{1/2}\) | \(0.47 \pm 0.36 \pm 0.34 \pm 0.89\) | \(0.48\) |
|                  | \(f_2^{1/2}\) | \(0.89 \pm 0.14 \pm 0.14 \pm 0.43\) | \(1.81\) |
|                  | \(f_3^{1/2}\) | \(-1.36 \pm 0.22 \pm 0.20 \pm 0.47\) | \(-2.31\) |
|                  | \(g_1^{1/2}\) | \(-4.80 \pm 0.10 \pm 0.01 \pm 0.05(10^{-3})\) | \(0\) |
|                  | \(g_2^{1/2}\) | \(-16.34 \pm 3.7 \pm 0.07 \pm 0.08\) | \(-17.04\) |
|                  | \(g_3^{1/2}\) | \(15.92 \pm 3.7 \pm 0.06 \pm 0.07\) | \(16.27\) |
| \(\Omega_{bc}^0 \to \Xi_{bc}^+\) | \(f_1^{1/2}\) | \(0.30 \pm 0.37 \pm 0.36 \pm 0.87\) | \(0.31\) |
|                  | \(f_2^{1/2}\) | \(0.99 \pm 0.15 \pm 0.15 \pm 0.41\) | \(1.90\) |
|                  | \(f_3^{1/2}\) | \(-1.28 \pm 0.23 \pm 0.21 \pm 0.47\) | \(-2.22\) |
|                  | \(g_1^{1/2}\) | \(-6.90 \pm 0.20 \pm 0.03 \pm 0.10(10^{-3})\) | \(0\) |
|                  | \(g_2^{1/2}\) | \(-11.78 \pm 2.5 \pm 0.13 \pm 0.08\) | \(-12.49\) |
|                  | \(g_3^{1/2}\) | \(11.37 \pm 2.5 \pm 0.12 \pm 0.08\) | \(11.72\) |
| \(\Omega_{bc}^0 \to \Xi_{bc}^+\) | \(f_1^{1/2}\) | \(0.46 \pm 0.37 \pm 0.35 \pm 0.88\) | \(0.47\) |
|                  | \(f_2^{1/2}\) | \(0.91 \pm 0.15 \pm 0.15 \pm 0.41\) | \(1.82\) |
|                  | \(f_3^{1/2}\) | \(-1.36 \pm 0.23 \pm 0.21 \pm 0.48\) | \(-2.30\) |
|                  | \(g_1^{1/2}\) | \(2.08 \pm 0.06 \pm 0.01 \pm 0.03\) | \(0\) |
|                  | \(g_2^{1/2}\) | \(35.34 \pm 7.5 \pm 0.38 \pm 0.25\) | \(37.48\) |
|                  | \(g_3^{1/2}\) | \(-34.11 \pm 7.6 \pm 0.36 \pm 0.23\) | \(-35.17\) |

### 3.2 Heavy quark limit

The transition form factors for the doubly heavy baryon decays can be greatly simplified in heavy quark limit. In this limit, the initial baryon mass \(M\) and the final baryon mass \(M'\) become
Table 4 Numerical results for the form factors in spin-$3/2$ to spin-$1/2$ doubly heavy baryon decays induced by $s \to u$. The $F^*(0)$ indicates the form factors simplified in the heavy quark limit in Eq. (37)

| Channel | Form factor | $F(0)$  | $F^*(0)$ |
|---------|-------------|---------|----------|
| $\Omega_{cc}^{++} \to \Xi_{cc}^{++}$ | $f_1^{1\to 1/2}$ | $3.40 \pm 0.03 \pm 0.09 \pm 0.08(10^{-3})$ | 0 |
|         | $f_2^{1\to 1/2}$ | $0.063 \pm 0.005 \pm 0.007 \pm 0.06$ | 0.06 |
|         | $f_3^{1\to 1/2}$ | $-0.063 \pm 0.005 \pm 0.007 \pm 0.06$ | -0.07 |
|         | $f_4^{1\to 1/2}$ | $-2.19 \pm 0.04 \pm 0.13 \pm 0.24$ | -2.19 |
|         | $g_1^{1\to 1/2}$ | $-0.33 \pm 0.42 \pm 0.41 \pm 3.26$ | -0.31 |
|         | $g_2^{1\to 1/2}$ | $0.30 \pm 0.16 \pm 0.28 \pm 2.36$ | 0.30 |
|         | $g_3^{1\to 1/2}$ | $-0.018 \pm 0.27 \pm 0.13 \pm 0.94$ | -0.049 |
|         | $g_4^{1\to 1/2}$ | $-1.13 \pm 0.07 \pm 0.12 \pm 0.95$ | -1.12 |
| $\Omega_{bc}^{++} (bbs) \to \Xi_{bc}^{0}$ | $f_1^{1\to 1/2}$ | $1.20 \pm 0.01 \pm 0.05 \pm 0.02(10^{-3})$ | 0 |
|         | $f_2^{1\to 1/2}$ | $0.035 \pm 0.002 \pm 0.003 \pm 0.01$ | 0.035 |
|         | $f_3^{1\to 1/2}$ | $-0.035 \pm 0.003 \pm 0.003 \pm 0.01$ | -0.037 |
|         | $f_4^{1\to 1/2}$ | $-5.93 \pm 0.05 \pm 0.28 \pm 0.04$ | -5.93 |
|         | $g_1^{1\to 1/2}$ | $-1.77 \pm 0.34 \pm 0.24 \pm 0.66$ | -1.77 |
|         | $g_2^{1\to 1/2}$ | $1.23 \pm 0.17 \pm 0.18 \pm 0.48$ | 1.23 |
|         | $g_3^{1\to 1/2}$ | $0.53 \pm 0.18 \pm 0.07 \pm 0.18$ | 0.52 |
|         | $g_4^{1\to 1/2}$ | $-1.54 \pm 0.06 \pm 0.06 \pm 0.18$ | -1.54 |
| $\Omega_{bc}^{0} \to \Xi_{bc}^{+}$ | $f_1^{1\to 1/2}$ | $1.75 \pm 0.01 \pm 0.06 \pm 0.02(10^{-3})$ | 0 |
|         | $f_2^{1\to 1/2}$ | $0.039 \pm 0.002 \pm 0.003 \pm 0.009$ | 0.039 |
|         | $f_3^{1\to 1/2}$ | $-0.039 \pm 0.003 \pm 0.003 \pm 0.001$ | -0.042 |
|         | $f_4^{1\to 1/2}$ | $-3.93 \pm 0.03 \pm 0.14 \pm 0.04$ | -3.93 |
|         | $g_1^{1\to 1/2}$ | $-1.71 \pm 0.37 \pm 0.27 \pm 0.66$ | -1.69 |
|         | $g_2^{1\to 1/2}$ | $1.15 \pm 0.18 \pm 0.19 \pm 0.48$ | 1.15 |
|         | $g_3^{1\to 1/2}$ | $0.54 \pm 0.19 \pm 0.08 \pm 0.19$ | 0.52 |
|         | $g_4^{1\to 1/2}$ | $-1.51 \pm 0.07 \pm 0.07 \pm 0.18$ | -1.50 |

$M' \approx m_Q, \quad M \approx m_Q + \bar{M}$.

where the $m_Q$ is the mass of heavy diquark. Then the formula of the form factors in Eqs. (21) and (22) can be expanded in $1/m_Q$. In the first step, the dimensionless functions $\hat{H}_i$ and $\hat{K}_i$ are introduced as

$$
\hat{H}_1^{1/2} = \frac{H_1^{1/2}}{(M + M')M}, \quad \hat{H}_2^{1/2} = \frac{H_2^{1/2}}{(M + M')^2M},
$$

$$
\hat{H}_3^{1/2} = \frac{H_3^{1/2}}{(M + M')^2M}, \quad \hat{K}_1^{1/2} = \frac{K_1^{1/2}}{M (M + M')},
$$

$$
\hat{K}_2^{1/2} = \frac{K_2^{1/2}}{M^2}, \quad \hat{K}_3^{1/2} = \frac{K_3^{1/2}}{M^3}.
$$

Using these dimensionless function, one can simplify the form factors at $q^2 = 0$ in the heavy quark limit as

$$
f_1^{1\to 1/2} = \frac{1}{2}(-\hat{H}_1^{1/2} + \hat{H}_2^{1/2} + \hat{H}_3^{1/2})
$$

$$
+ \frac{M}{4m_Q^2} (\hat{H}_1^{1/2} - 2\hat{H}_3^{1/2}) + O(\hat{M}^2/m_Q^2),
$$

$$
f_2^{1\to 1/2} = \frac{1}{4}(\hat{H}_1^{1/2} + \hat{H}_2^{1/2} - 3\hat{H}_3^{1/2})
$$

$$
+ \frac{M}{8m_Q^2} (6\hat{H}_3^{1/2} - \hat{H}_1^{1/2}) + O(\hat{M}^2/m_Q^2),
$$

$$
f_3^{1\to 1/2} = \frac{1}{4}(\hat{H}_1^{1/2} - 3\hat{H}_2^{1/2} + \hat{H}_3^{1/2})
$$

$$
+ \frac{M}{8m_Q^2} (\hat{H}_1^{1/2} - 6\hat{H}_2^{1/2}) + O(\hat{M}^2/m_Q^2),
$$

$$
8_1^{1\to 1/2} = 0 + O(\hat{M}^2/m_Q^2),
$$

$$
8_2^{1\to 1/2} = \frac{1}{4}(\hat{K}_1^{1/2} + 3\hat{K}_2^{1/2} - 3\hat{K}_3^{1/2}).
$$
Table 5 Numerical results of decay widths and branching fractions in doubly heavy baryon semileptonic decays using the form factor \( F(q^2) \) in Eq. (31)

| Channel | \( \Gamma(\times 10^{-19} \text{ GeV}) \) | \( \Gamma_L/\Gamma_T \) | Br(%) | \( R_{\Xi_Q} \) |
|---------|-----------------------------------|-----------------|------|----------|
| \( \Omega_c^+ \rightarrow \Xi_{cc}^{++} e^- \bar{\nu}_e \) | 4.62 ± 0.92 ± 0.92 ± 2.13 ± 0.04 | 0.56 | 1.26 ± 0.25 ± 0.25 ± 0.58 ± 0.01(10^{-5}) | 0.31 |
| \( \Omega_c^+ \rightarrow \Xi_{cc}^{++} \mu^- \bar{\nu}_\mu \) | 1.45 ± 0.11 ± 0.10 ± 0.15 ± 0.01 | 2.41 | 3.95 ± 0.29 ± 0.28 ± 0.42 ± 0.04(10^{-6}) | |
| \( \Omega_b^0 \rightarrow \Xi_{bb}^{0} e^- \bar{\nu}_e \) | 1.72 ± 1.39 ± 1.38 ± 2.65 ± 0.03 | 0.70 | 2.09 ± 1.68 ± 1.67 ± 3.20 ± 0.03(10^{-5}) | 0.65 |
| \( \Omega_b^0 \rightarrow \Xi_{bb}^{0} \mu^- \bar{\nu}_\mu \) | 1.13 ± 0.52 ± 0.51 ± 0.68 ± 0.02 | 4.69 | 1.36 ± 0.65 ± 0.63 ± 0.83 ± 0.02(10^{-5}) | 0.57 |
| \( \Omega_b^0 \rightarrow \Xi_{bb}^{0} \mu^- \bar{\nu}_\mu \) | 3.20 ± 0.67 ± 0.70 ± 0.73 ± 0.03 | 2.64 | 1.07 ± 0.23 ± 0.24 ± 0.24 ± 0.01(10^{-5}) | |
| \( \Omega_b^0 \rightarrow \Xi_{bb}^{0} \mu^- \bar{\nu}_\mu \) | 1.82 ± 0.29 ± 0.28 ± 0.27 ± 0.02 | 8.87 | 6.08 ± 0.97 ± 0.94 ± 0.90 ± 0.06(10^{-6}) | |
| \( \Omega_b^0 \rightarrow \Xi_{bb}^{0} \mu^- \bar{\nu}_\mu \) | 11.93 ± 0.36 ± 0.53 ± 1.59 ± 0.17 | 0.79 | 3.98 ± 0.12 ± 0.18 ± 0.53 ± 0.06(10^{-5}) | 0.32 |
| \( \Omega_b^0 \rightarrow \Xi_{bb}^{0} \mu^- \bar{\nu}_\mu \) | 3.84 ± 0.26 ± 0.33 ± 0.38 ± 0.06 | 1.66 | 1.28 ± 0.08 ± 0.11 ± 0.13 ± 0.02(10^{-5}) | |

For the spin-3/2 to spin-1/2 processes form factors, the dimensionless functions \( \hat{H}_i \) and \( \hat{K}_i \) are defined as

\[
\hat{H}_i^{1/2} = \frac{H_i^{1/2}}{(M + M')M^2}, \quad \hat{H}_i^{3/2} = \frac{H_i^{3/2}}{M^4},
\]

\[
\hat{K}_i^{1/2} = \frac{K_i^{1/2}}{(M + M')M^2}, \quad \hat{K}_i^{3/2} = \frac{K_i^{3/2}}{(M + M')^2M^2}.
\]

Then the form factors at \( q^2 = 0 \) can be expressed in the heavy quark limit as

\[
f_i^{3/2} = 0 + \mathcal{O}(M^2/m_Q^2).
\]
3.3 Semileptonic decays

In this subsection, the semileptonic decays are studied with the help of helicity amplitudes. The full decay amplitude can be decomposed in terms of hadron helicity amplitude and lepton helicity amplitude as

\[ i \mathcal{M}(B_{Q_1 Q_2} \to B'_{Q_1 Q_2} e^\nu) = \frac{G_F}{\sqrt{2}} V_{us} H_{S,\lambda'}^{S,\lambda} \lambda_w \lambda_w, \]

\[ H_{S,\lambda'}^{S,\lambda} = H_{S,\lambda'}^{S,\lambda} - H_{S,\lambda'}^{S,\lambda}. \]  

The helicity amplitudes \( H_V, H_A \) and \( L \) are defined as

\[ H_V^{S,\lambda} = (B'_{Q_1 Q_2} \to B_{Q_1 Q_2} e^\nu)(B(S, \lambda))e_{\nu}^{\mu}(\lambda_w), \]

\[ H_A^{S,\lambda} = (B'_{Q_1 Q_2} \to B_{Q_1 Q_2} e^\nu)(B(S, \lambda))e_{\nu}^{\mu}(\lambda_w), \]

\[ L_{\lambda_w}^{\lambda'} = \tilde{l}_{\gamma_e}(1 - \gamma_S) \gamma_l(\lambda_e) e_{\nu}^{\mu}(\lambda_w). \]  

where \( e_{\nu}^{\mu} \) is the polarization vector of the virtual propagator and \( \lambda_w \) means the polarization of the virtual propagator. The \( \lambda, \lambda' \), and \( \lambda_e \) are the helicities of the initial baryon, the final baryon and the lepton. \( S \) denotes the spin of the initial baryon states.

Then the differential decay width of semileptonic decays is

\[ \frac{d^2\Gamma}{dq^2 d\cos \theta} = \frac{G_F^2 |V_{CKM}|^2 \sqrt{s - m_t}}{512\pi^3 M^3} (1 - m_\ell^2) \times \left[ L_1 + L_2 \cos \theta + L_3 \cos 2\theta \right]. \]  

(41)

Here the coefficients \( L_j \) can be expressed by the helicity amplitudes \( H_{S,\lambda'}^{S,\lambda}. \) For the spin-1/2 to spin-1/2 processes, the coefficients are

\[ L_1 = \frac{q^2}{2}(1 - m_\ell^2) \left[ (3 + m_\ell^2) \left( |H_{2,1}^{1/2,1/2}|^2 + |H_{2,1}^{1/2,-1/2}|^2 \right) + 2(1 + m_\ell^2) \left( |H_{1,1}^{1/2,1/2}|^2 + |H_{1,1}^{1/2,-1/2}|^2 \right) + 4m_\ell^2 \left( |H_{2,1}^{1/2,1/2}|^2 + |H_{2,1}^{1/2,-1/2}|^2 \right) \right]. \]

\[ L_2 = \frac{q^2}{2}(1 - m_\ell^2) \left[ \left( |H_{1,1}^{1/2,1/2}|^2 + |H_{1,1}^{1/2,-1/2}|^2 \right) + 2m_\ell^2 \left[ \mathcal{R}_e \left( H_{2,1}^{1/2,1/2} H_{2,1}^{1/2,-1/2} \right) + \mathcal{R}_e \left( H_{2,1}^{1/2,-1/2} H_{2,1}^{1/2,1/2} \right) \right] \right]. \]

\[ L_3 = \frac{q^2}{2}(1 - m_\ell^2) \left[ \left( |H_{1,1}^{1/2,1/2}|^2 + |H_{1,1}^{1/2,-1/2}|^2 \right) - 2 \left( |H_{2,1}^{1/2,1/2}|^2 + |H_{2,1}^{1/2,-1/2}|^2 \right) \right]. \]  

(42)

For the spin-3/2 to spin-1/2 processes, the coefficients can be written as

\[ L_1 = \frac{q^2}{4}(1 - m_\ell^2) \left[ (3 + m_\ell^2) \left( |H_{3,1}^{3/2,3/2}|^2 + |H_{3,1}^{3/2,-3/2}|^2 \right) + |H_{2,1}^{3/2,3/2}|^2 + |H_{2,1}^{3/2,-3/2}|^2 + 2(1 + m_\ell^2) \left( |H_{1,1}^{3/2,3/2}|^2 + |H_{1,1}^{3/2,-3/2}|^2 \right) + 4m_\ell^2 \left( |H_{2,1}^{3/2,3/2}|^2 + |H_{2,1}^{3/2,-3/2}|^2 \right) \right]. \]

\[ L_2 = q^2(1 - m_\ell^2) \left[ \left( |H_{1,1}^{3/2,3/2}|^2 + |H_{1,1}^{3/2,-3/2}|^2 \right) + 2m_\ell^2 \left[ \mathcal{R}_e \left( H_{2,1}^{3/2,3/2} H_{2,1}^{3/2,-3/2} \right) + \mathcal{R}_e \left( H_{2,1}^{3/2,-3/2} H_{2,1}^{3/2,3/2} \right) \right] \right]. \]

\[ L_3 = \frac{q^2}{4}(1 - m_\ell^2) \left[ \left( |H_{1,1}^{3/2,3/2}|^2 + |H_{1,1}^{3/2,-3/2}|^2 \right) - 2 \left( |H_{2,1}^{3/2,3/2}|^2 + |H_{2,1}^{3/2,-3/2}|^2 \right) \right]. \]  

(43)
Fig. 3 The differential decay width $\frac{d\Gamma}{dq^2}$ of spin-$1/2$ to spin-$1/2$ doubly heavy baryon decay processes. The $B_L$ and $B_T$ represent the contribution of longitudinal polarisation and transverse polarisation in branching fractions in Eq. (44).
For the spin-3/2 to spin-1/2 processes, the differential decay widths are

\[
d\Gamma_L/dq^2 = \frac{G_F^2|V_{CKM}|^2}{2} \frac{\sqrt{s - x_t} (1 - \hat{m}_e^2)}{384\pi M^3} q^2 (1 - \hat{m}_e^2) \\
\times \left[ (\hat{m}_e^2 + 2) \left( |H_{1/2,0}^{3/2}|^2 + |H_{1/2,1}^{3/2}|^2 \right) \\
+ 2\hat{m}_e \left( |H_{1/2,0}^{1/2}|^2 + |H_{1/2,1}^{1/2}|^2 \right) \right],
\]

\[
d\Gamma_T/dq^2 = \frac{G_F^2|V_{CKM}|^2}{2} \frac{\sqrt{s - x_t} (1 - \hat{m}_e^2)}{384\pi M^3} q^2 (1 - \hat{m}_e^2) \\
\times \left[ (\hat{m}_e^2 + 2) \left( |H_{1/2,0}^{3/2}|^2 + |H_{1/2,1}^{3/2}|^2 \right) \right],
\]

\[
d\Gamma_R/dq^2 = \frac{G_F^2|V_{CKM}|^2}{2} \frac{\sqrt{s - x_t} (1 - \hat{m}_e^2)}{384\pi M^3} q^2 (1 - \hat{m}_e^2) \\
\times \left[ (\hat{m}_e^2 + 2) \left( |H_{1/2,0}^{3/2}|^2 + |H_{1/2,1}^{3/2}|^2 \right) \right].
\]

For the spin-1/2 to spin-1/2 doubly heavy baryon decay processes. The R represent the results using F(q^2) in Eq. (31)

After integrating out the q^2, one can give the partial decay widths and the branching fractions which are presented in Tables 5 and 6. Most of the partial decays are at the order O(10^{-19}) GeV, which corresponds to a small branching fraction.

For these s \rightarrow u decay processes, one can find that the phase space is small. Thus in our analysis, we also give the results for decay widths by setting the form factors F(q^2) = F(0). The decay widths deduced from F(q^2) are in Table 5, and the decay widths from F(0) are in Table 6. Using the Eqs. (44) and (45), we study the q^2 distributions of (B, B_L, B_T) which correspond to the decay widths (\Gamma, \Gamma_L, \Gamma_T). The distributions are shown in Figs. 3 and 5, and the dash line reflects the upper and lower limits of the error.

One can see that the q^2 distributions of decay widths are much different for different lepton types (\ell = \mu, e). For the \ell = e case, branching ratios B_L are obviously enhanced in the low-q^2 region. As shown in Eqs. (A1) and (A2), because the helicity amplitudes \mathcal{H}_{s=0}^{S_{\ell,\bar{u}}} and \mathcal{H}_{s=1}^{S_{\ell,\bar{u}}} contain the 1/\sqrt{q^2} term, the branching ratio B_L which includes the H_0 and H_1 is enhanced in the q^2 \rightarrow m_e^2 region. However, this effect does not exist in the \ell = \mu case. The difference comes from the phase space integral which includes (1 - \hat{m}_e^2) term. This term will suppress the differential decay width when q^2 \rightarrow m_e^2. For the \ell = \mu, the phase space factor has a more severe impact since the mass of \mu is much larger than m_e.
Fig. 5 The differential decay width $\frac{d\Gamma}{dq^2}$ of spin-3/2 to spin-1/2 doubly heavy baryon decay processes. The $B_L$ and $B_T$ represent the contribution of longitudinal polarisation and transverse polarisation in branching fractions in Eq. (45).

Especially in these $s \rightarrow u$ semileptonic decays the phase space is very small and $q^2$ is comparable to the $m_\mu$ for these processes.

Differences in decay widths with two different final lepton types ($\mu$, $e$) can be used as a platform to explore the lepton flavor universality. Therefore one can define the value $R_{Xi1QQ}$ as

$$R_{Xi1QQ} = \frac{\Gamma(\Omega_{Xi1QQ} \rightarrow \Xi_{Xi1QQ} \mu \bar{\nu}_\mu)}{\Gamma(\Omega_{Xi1QQ} \rightarrow \Xi_{Xi1QQ} e \bar{\nu}_e)}.$$  

(46)

The values of these ratios are shown in Tables 5 and 6. Since these semileptonic decay processes in our work are sensitive to the lepton mass, the value of the ratio $R_{Xi1QQ}$ will be far from the 1. Namely, in these processes the muon mass can uplift the helicity suppression of certain semi-leptonic decay amplitudes which are unobservable in decays with electron in the final state. In the presence of new physics that is sensitive to the lepton flavor, these ratios can serve as a platform for searching new physics effects in future. For more information on the lepton universality, one can also define the differential ratio $\frac{dR_{Xi1QQ}}{dq^2}$ as

$$\frac{dR_{Xi1QQ}}{dq^2} = \frac{d\Gamma(\Omega_{Xi1QQ} \rightarrow \Xi_{Xi1QQ} \mu \bar{\nu}_\mu)/dq^2}{d\Gamma(\Omega_{Xi1QQ} \rightarrow \Xi_{Xi1QQ} e \bar{\nu}_e)/dq^2}.$$  

(47)

The behaviors of the differential ratio $\frac{dR_{Xi1QQ}}{dq^2}$ are shown in Figs. 4 and 6.

For further studying these processes, one can also explore the $\theta$ distribution through Eq. (41). The normalized forward-backward asymmetry can be defined as

$$\frac{dA_{FB}}{dq^2} = \frac{\left[ f_1^0 - f_{-1}^0 \right] d\cos \theta \frac{d^2\Gamma}{dq^2 d\cos \theta_\lambda}}{\left[ f_1^0 - f_{-1}^0 \right] d\cos \theta \frac{d\Gamma}{d\cos \theta_\lambda}} \propto L_2.$$  

(48)
The normalized forward–backward asymmetries of these processes are shown in Figs. 7 and 8. One can find that the behavior of $A_{FB}$ is proportional to the coefficient $L_2$ in Eq. (41). As the helicity amplitude shown in Eq. (A1), the first part of $L_2$ in Eqs. (42) and (43) are the interference terms between the vector helicity amplitude $HV$ and the axis-vector helicity amplitude $HA$. If we set $m_\ell \to 0$, the coefficient $L_2$ should be proportional to $f_{\pi_1}$ and the $A_{FB}$ should be positive. However in Figs. 3 and 4 the $A_{FB}$ is not always positive in the three channels $\Omega_{cc}^+ \to \Xi_{cc}^{++} \mu \bar{\nu}$, $\Omega_{bc}^0 \to \Xi_{bc}^{0+} \mu \bar{\nu}$ and $\Omega_{cc}^+ \to \Xi_{cc}^{++} \mu \bar{\nu}$. We can see that when $m_\ell = m_\mu$, the assumption $m_\ell \to 0$ is inappropriate and the second part which is proportional to $m_\ell^2$ in $L_2$ should be taken into count. The second part in $L_2$ is the interference term of different polarisation helicity amplitude which can induce a negative contribution which depends on the form factor in hadron transition matrix element. As we mentioned above when the $m_\ell = m_\mu$, the muon mass can uplift the helicity suppression of certain semi-leptonic decay amplitudes. Therefore the forward–backward asymmetry $A_{FB}$ may show some different behavior with muon in the final state and it would be a new method to study the hadron transition matrix element and form factors in the future.

3.4 Nonleptonic decays

Nonleptonic decays with pion emission can be estimated with the helicity amplitude method. In Eq. (2), the local matrix element $(\pi^- (P_\pi) | \hat{d} \gamma^\mu (1 - \gamma_5) u(0))$ can be expressed by the decay constant $f_{\pi}$ as

$$
(\pi^- (P_\pi) | \hat{d} \gamma^\mu (1 - \gamma_5) u(0)) = i f_{\pi} \, P^\mu_u,
$$

where $f_\pi = 0.130$ GeV. Then the amplitude of nonleptonic decays becomes

$$
i M(B_1 Q_1 Q_2 \to B'_1 Q_1 Q_2 \pi^-) = \frac{i G_F}{\sqrt{2}} f_{\pi} \, V_{u_3} V^*_{ud} \theta_1
$$

$$
\times \langle B'_1 Q_1 Q_2 (P', S'_z) | \bar{u} P_\pi (1 - \gamma_5) u | B_1 Q_1 Q_2 (P, S_z) \rangle,
$$

$Q_{1,2} = b, c.$ (50)

The helicity amplitudes in the nonleptonic decay processes are defined as

$$
HV^S_{\lambda,\lambda'} = \langle B'_1 Q_1 Q_2 (P', \lambda') | \bar{u} P_\pi s | B_1 Q_1 Q_2 (P, \lambda) \rangle,
$$

$$
HA^S_{\lambda,\lambda'} = \langle B'_1 Q_1 Q_2 (P', \lambda') | \bar{u} P_\pi s \gamma_5 s | B_1 Q_1 Q_2 (P, \lambda) \rangle.
$$

With the help of helicity amplitude, the total decay width of the spin-1/2 to spin-1/2 and the spin-3/2 to spin-1/2 processes can be expressed as

$$
\Gamma_{\frac{1}{2} - \frac{1}{2}} = \mathcal{P} \frac{\sqrt{4 + \frac{5}{3}}}{32 \pi M^3} \left( |H_{\frac{3}{2}, \frac{1}{2}}|^2 + |H_{\frac{3}{2}, -\frac{1}{2}}|^2 \right),
$$

$$
\Gamma_{\frac{3}{2} - \frac{1}{2}} = \mathcal{P} \frac{\sqrt{4 + \frac{5}{3}}}{64 \pi M^3} \left( |H_{\frac{1}{2}, 0}|^2 + |H_{\frac{1}{2}, \frac{1}{2}}|^2 \right),
$$

$$
\mathcal{P} = \frac{G_F^2}{2} |V_{u_3} V^*_{ud}|^2 a_1^2 f_{\pi}^2.
$$

(52)
Fig. 7 The $dA_{FB}/dq^2$ of spin-1/2 to spin-1/2 doubly heavy baryon decay processes. $dA_{FB}/dq^2$ is depend on $F(q^2)$.
Fig. 8 The $dA_{FB}/dq^2$ of spin-3/2 to spin-1/2 doubly heavy baryon decay processes. $dA_{FB}/dq^2$ is depend on $F(q^2)$

Table 7 Numerical results of decay width and branching fraction in doubly heavy baryon nonleptonic decays using $F(q^2)$ when $N_c = 3, 2, \infty$

| Channel | $N_c = 3$ | $N_c = 2$ | $N_c = \infty$ |
|---------|-----------|-----------|----------------|
|         | $\Gamma(\times 10^{-17}$ GeV) | $\mathcal{B}(\%)$ | $\Gamma(\times 10^{-17}$ GeV) | $\mathcal{B}(\%)$ | $\Gamma(\times 10^{-17}$ GeV) | $\mathcal{B}(\%)$ |
| $\Omega_{c\bar{c}}^+ \to \Xi_{c\bar{c}}^{++} \pi^-$ | 4.21 $\times 10^{-2}$ | 1.15 $\times 10^{-3}$ | 3.37 $\times 10^{-2}$ | 0.92 $\times 10^{-3}$ | 6.17 $\times 10^{-2}$ | 1.68 $\times 10^{-3}$ |
| $\Omega_{b\bar{b}} \to \Xi_{b\bar{b}}^{++} \pi^-$ | 3.72 $\times 10^{-2}$ | 4.51 $\times 10^{-3}$ | 2.98 $\times 10^{-2}$ | 3.61 $\times 10^{-3}$ | 5.45 $\times 10^{-2}$ | 6.61 $\times 10^{-3}$ |
| $\Omega_{b\bar{b}}^{0} \to \Xi_{b\bar{b}}^{+} \pi^-$ | 6.68 $\times 10^{-2}$ | 2.23 $\times 10^{-3}$ | 5.35 $\times 10^{-2}$ | 1.78 $\times 10^{-3}$ | 9.79 $\times 10^{-2}$ | 3.26 $\times 10^{-3}$ |
| $\Omega_{b\bar{b}}^{0} \to \Xi_{b\bar{b}}^{-} \pi^-$ | 7.59 $\times 10^{-2}$ | 2.53 $\times 10^{-3}$ | 6.07 $\times 10^{-2}$ | 2.02 $\times 10^{-3}$ | 11.12 $\times 10^{-2}$ | 3.71 $\times 10^{-3}$ |
| $\Omega_{b\bar{b}}^{0} \to \Xi_{b\bar{b}}^{-} \pi^-$ | 2.85 $\times 10^{-2}$ | 2.28 $\times 10^{-3}$ | 4.18 $\times 10^{-2}$ | – | – | – |
| $\Omega_{b\bar{b}}^{0} \to \Xi_{b\bar{b}}^{-} \pi^-$ | 8.78 $\times 10^{-2}$ | 7.03 $\times 10^{-3}$ | 12.87 $\times 10^{-2}$ | – | – | – |
| $\Omega_{b\bar{b}}^{0} \to \Xi_{b\bar{b}}^{-} \pi^-$ | 1.30 $\times 10^{-2}$ | 1.04 $\times 10^{-3}$ | 1.91 $\times 10^{-2}$ | – | – | – |
Though in our estimation the nonfactorizable contributions which shown in the second line of Fig. 1 is not take into count, we can roughly estimate its effect by varying the $N_c$. This strategy is used in Ref. [47], it has been suggested that the nonfactorizable contributions in $B$ decays can be estimated through a variation of $N_c$ in Wilson coefficient $a_1 = C_1/N_c + C_2$. The effect are estimated in Table 7, and $O(40\%)$ corrections are found.

### 4 Summary

The weak decays of doubly heavy baryons induced by $s \rightarrow u$ are studied in this work. Not only the spin-1/2 to spin-1/2 processes but also spin-3/2 to spin-1/2 processes are considered. Using the light-front-quark model, we use the diquark picture in which the two spectator heavy quarks can be seen as a scalar or an axis vector diquark and the baryon state can be treated as the meson state. Using the form factors defined in the hadron matrix element, we estimate the decay width branching fractions of semileptonic and two-body nonleptonic decays and give their phenomenological applications.

To obtain the phenomenological results, the helicity amplitude are used. With the help of the lifetime of doubly heavy baryons shown in Table 2, we have predicted the branching fraction in Tables 5, 6, 8 and 9. The $q^2$ distribution and angular distribution are also studied which is shown in Figs. 3, 5, 7, and 8. Since the integral region in phase space is very small which is comparable to the $m_{\pi}^2$, there are many strange phenomenological results in these channel such as branching fraction, forward–backward asymmetry and $R_{\Xi_{Q''}/Q}$. The decay processes induced by $s \rightarrow u$ with $\mu$ leptons in the final state offer possibilities of significant NP contributions not present in processes with electron. Therefore these phenomenological results are excited and they may become a new method to search new physics in the future.

### Acknowledgements

We thank Prof. Wei Wang for useful discussions. This work was supported in part by NSFC under Grant Nos.12147147, 11735010, 11911530088, U2032102, 12125503.

### Data Availability Statement

This manuscript has no associated data or the data will not be deposited. [Authors’ comment: All data of this manuscript are displayed in the tables and figures, and this manuscript has no other associated data.]

### Open Access

This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

Funded by SCOAP³. SCOAP³ supports the goals of the International Year of Basic Sciences for Sustainable Development.

### Appendix A: Helicity amplitude

The helicity amplitude can be expressed by the form factors defined in hadron matrix element. Let $(M^2 - M_{q}^2) \pm q^2 = M_{h}^2$ and for the spin-1/2 to spin-1/2 semileptonic and nonleptonic decay processes, the helicity amplitudes are

\[
HV_{\frac{1}{2} \rightarrow \frac{1}{2} +1} = \frac{-i M \sqrt{2s}}{M} f_{\frac{1}{2} \rightarrow \frac{1}{2} +1}.
\]
\[
HV_{1/2}^{1/2-1/2} = -\frac{i\sqrt{\gamma_s}}{2Mqf_2}\left[ 2M(M + M')f_1^{1/2-1/2} + s_f f_2^{1/2} + f_3^{1/2-1/2} \right], \\
HV_{1/2}^{1/2-1/4} = -\frac{i\sqrt{\gamma_s}}{2Mqf_2}\left[ 2M\hat{M}f_1^{1/2-1/4} + \hat{M}_q f_2^{1/2} + \hat{M}_q f_3^{1/2-1/4} \right], \\
HV_{1/2}^{1/2+1/2} = -\frac{i\sqrt{\gamma_s}}{2Mqf_2}\left[ 2M\hat{M}f_1^{1/2+1/2} + \hat{M}_q f_2^{1/2} + \hat{M}_q f_3^{1/2+1/2} \right], \\
HV_{1/2}^{1/2-1/2} = -\frac{i\sqrt{\gamma_s}}{2Mqf_2}\left[ 2M(M + M')f_1^{1/2-1/2} + \hat{M}_q f_2^{1/2} + \hat{M}_q f_3^{1/2-1/2} \right], \\
HV_{1/2}^{1/2-1/4} = -\frac{i\sqrt{\gamma_s}}{2Mqf_2}\left[ 2M(M + M')f_1^{1/2-1/4} + \hat{M}_q f_2^{1/2} + \hat{M}_q f_3^{1/2-1/4} \right], \\
HV_{1/2}^{1/2+1/2} = -\frac{i\sqrt{\gamma_s}}{2Mqf_2}\left[ 2M(M + M')f_1^{1/2+1/2} + \hat{M}_q f_2^{1/2} + \hat{M}_q f_3^{1/2+1/2} \right].
\]

References

1. M. Ablikim et al., BESIII. Phys. Rev. Lett. 112 (2014), 132001. https://doi.org/10.1103/PhysRevLett.112.132001. arXiv:1308.2760 [hep-ex]
2. R. Aaij et al., LHCb. Phys. Rev. Lett. 113, 151601 (2014). https://doi.org/10.1103/PhysRevLett.113.151601. arXiv:1406.6482 [hep-ex]
3. R. Aaij et al. [LHCb], Phys. Rev. Lett. 115 (2015), 111803 (2015) [erratum: Phys. Rev. Lett. 115, no.15, 159901 (2015)] https://doi.org/10.1103/PhysRevLett.115.111803. arXiv:1506.08614 [hep-ex]
4. R. Aaij et al., LHCb. Phys. Rev. Lett. 119 (2017), 112001. https://doi.org/10.1103/PhysRevLett.119.112001. arXiv:1707.01621 [hep-ex]
5. R. Aaij et al., LHCb. Phys. Rev. Lett. 121 (2018), 052002. https://doi.org/10.1103/PhysRevLett.121.052002. arXiv:1806.02744 [hep-ex]
6. R. Aaij et al., LHCb. Sci. China Phys. Mech. Astron. 63 (2020), 221062 (2020). https://doi.org/10.1007/s11433-019-1471-8. arXiv:1909.12273 [hep-ex]
7. R. Aaij et al., LHCb. Chin. Phys. C 44 (2020), 022001. https://doi.org/10.1088/1674-1137/44/022001. arXiv:1910.11316 [hep-ex]
8. R. Aaij et al., LHCb. JHEP 10, 124 (2019). https://doi.org/10.1007/JHEP10(2019)124. arXiv:1905.02421 [hep-ex]
9. R. Aaij et al., LHCb. Chin. Phys. C 45 (2021), 093002 (2021). https://doi.org/10.1088/1674-1137/ac0c70. arXiv:2104.04759 [hep-ex]
10. R. Aaij et al., LHCb. JHEP 12, 107 (2021). https://doi.org/10.1007/JHEP12(2021)107. arXiv:2109.07292 [hep-ex]
