Energy-aware Graph Job Allocation in Software Defined Air-Ground Integrated Vehicular Networks

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Abstract—The software defined air-ground integrated vehicular (SD-AGV) networks have emerged as a promising paradigm, which realizes the flexible on-ground resource sharing to support innovative applications for UAVs with heavy computational overhead. In this paper, we investigate a vehicular cloud-assisted graph job allocation problem in SD-AGV networks, where the computation intensive jobs carried by UAVs, and the vehicular cloud are modeled as graphs. To map each component of the graph jobs to a feasible vehicle, while achieving the trade-off among minimizing UAVs’ job completion time, energy consumption, and the data exchange cost among vehicles, we formulate the problem as a mixed integer non-linear programming problem, which is Np-hard. Moreover, the constraint associated with preserving job structures poses addressing the subgraph isomorphism problem, that further complicates the algorithm design. Motivated by which, we propose an efficient decoupled approach by separating the template (feasible mappings between components and vehicles) searching from the transmission power allocation. For the former, we present an efficient algorithm of searching for all the subgraph isomorphisms with low computation complexity. For the later, we introduce a power allocation algorithm by applying convex optimization techniques. Extensive simulations demonstrate that the proposed approach outperforms the benchmark methods considering various problem sizes.

Index Terms—graph job allocation, power allocation, vehicular cloud, software defined air-ground integrated vehicular networks.

1 INTRODUCTION

BENEFITING from the considerable support in military, public and civil services [1], unmanned aerial vehicles (UAVs, also known as drones) as one of the fastest growing techniques have achieved over 126% increasing of annual market scale in 2016, and estimated to generate 3 billion dollars of global revenues [2]. Innovative applications related to UAVs such as transportation management, disaster relief (e.g., rescue missions and target detections) and smart surveillance facilitate both safety and convenience to people’s work and life [3]. Moreover, UAVs have realized significant values in incidents such like the radiation leakages of the Fukushima nuclear power plant in 2011 [3], [4], and the search work during the basketball legend Kobe Bryant’s helicopter crash, in 2020 [5].

However, computational overhead required by the aforementioned applications and use cases pose major challenges to the UAVs with limited processing capabilities and battery lives (e.g., ≤ 90 minutes without on-board processing in current marketplace [6]). Promoted by extensive research and development efforts on connected smart cars, vehicular cloud computing (VCC) technology [7], [8] facilitates a revolution of the resource sharing between the on-ground vehicles with surplus resources, and the UAVs with heavy workloads. Specifically, vehicles (service providers, SPs) form a cloud (vehicular cloud, VC) via vehicle-to-vehicle (V2V) communications to support efficient collaborative computing, while UAVs (service requestors) are encouraged to offload the computation-intensive applications to SPs through air-to-ground (A2G) links.

Nevertheless, the traditional network architecture can hardly meet different QoS requirements imposed by diverse services, and various communication techniques in networks containing air segment (UAVs) and ground segment (vehicles) [9], [10]. To enable a dynamic and adaptable air-ground integrated network with cost-effectivity, software defined networking (SDN) has been applied as an emerging architecture. Specifically, SDN separates the control plane and the data plane, introduces a logically centralized control with a global view of the network, while facilitating network programmability/reconfiguration through open interfaces [11].

Motivated by which, we consider the software defined air-ground integrated vehicular (SD-AGV) network architecture, to implement agile and flexible support for heterogenous applications. Under such framework, the graph-based-representation [7], [8] is utilized to characterize the non-negligible internal structures associated with the applications of UAVs. Each application¹ is modeled as a graph, where the vertices (components) represent either data sources or data processing units, while weighted edges describe the dependency among the vertices. Moreover, virtual machine (VM)-based representation is utilized to quantify available resources on SPs. In this paper, we study an interesting energy-aware graph job allocation problem in a SD-AGV network architecture. Concretely, the com-

¹ The “application” is interchangeable with “job” or “graph job” for the rest of this paper.
ponents of graph jobs carried by UAVs can be mapped to feasible on-ground SPs, while achieving the trade-off upon minimizing the job completion time and the energy consumption of UAVs, as well as the data exchange cost among SPs which incurred by the required data interactions among job components. Major challenges are summarized below:
1) obtaining feasible mappings between the components of graph jobs and the SPs requires solving the subgraph isomorphism problem, which is NP-complete [12];
2) considering UAVs’ energy consumption requires addressing the transmission power optimization problem, which is generally formulated with non-convex feature;
3) the energy-aware graph job allocation problem stands for a coupling of obtaining the mappings between the components and the SPs, and solving the transmission power optimization problem, which are challenging to be solved parallely.

Addressing the abovementioned challenges represents our main motivation. Specifically, we investigate a novel decoupled approach to solve the energy-aware graph job allocation problem, by separating components mapping from power allocation. For the former, we propose an efficient template search algorithm, where each template stands for a feasible mapping between the components of graph jobs and the SPs. For the later, we introduce a power allocation algorithm via applying convex optimization techniques.

1.1 Related work
SD-AGV networks: Several works have been dedicated to the architectures applying SDN to vehicular networks or AGV networks, and the related applications [9]–[11], [13]. A collaborative edge computing solution was introduced by Wang et al. [9] under the architecture of SD-vehicular networks. A new smart identifier networking paradigm and the customized solution enabling crowd collaborations under software defined vehicular networks architecture were proposed by Quan et al. [10]. A software defined space-air-ground integrated vehicular network framework, and the relevant challenges and solutions were presented in [11] by Zhang et al. Luo et al. [13] studied a novel cooperative data sharing problem in edge computing assisted SD-vehicular networks.

Computation-intensive job allocation: There are existing studies devoted to the job allocation problem, which can be roughly divided into two categories based on the modeling of jobs: a) jobs that are represented by bit streams without concerning the inherent dependencies, such as [1], [14]–[16]; b) jobs under the graph-based-representation upon considering the inner dependencies among components such like [7], [8], [17]–[24], which stands for the main focus of this paper. The bit stream-represented job allocation problem has been widely discussed in UAV networks. Messous et al. in [1] focused on the computation offloading problem in a mobile edge computing (MEC)-assisted UAV network, by establishing a non-cooperative theoretical game with multi-players and three pure strategies. In [14], a sequential game containing three player types (UAV, base station and MEC server) was adopted by Messous et al., where the existence of a Nash equilibrium was proved. Bai et al. [15] conceived an energy-efficient computation offloading technique for UAV-MEC systems, with an emphasis on physical-layer security. By regarding UAVs as computing servers, Hu et al. [16] proposed an architecture of UAV clustering to enable the efficient offloading of multimodal multi-tasks of on-ground users. Existing literature of graph job allocation can be further classified into three types according to the dynamism of the network topology: static [19], semi-static [17], [20]–[24], and dynamic [7], [8], [18]. However, the graph-represented job models have rarely been concerned in UAV related networks. Considering static topologies of both computing servers and users in cloud computing context, Ghaderi et al. [19] proposed a randomized graph job scheduling algorithm by considering job arrivals/departures, which facilitated a smooth trade-off between the average execution cost and queue length. For the semi-static network environments where neither the servers or the users is considered as mobile, an energy-efficient graph job allocation framework in geo-distributed cloud networks was proposed by Hosseinalipour et al. [17], where solutions were obtained for data center networks of various scales. Considering jobs as directed graphs, Huang et al. [20] proposed a Lyapunov optimization-based dynamic offloading approach. In [21], aiming to minimize the job completion time while considering energy consumption, the problem of scheduling embarrassingly parallel jobs composed of a set of independent tasks was studied by Shi et al. Goudarzi et al. [22] proposed a fast hybrid multisite computation offloading algorithm by modeling each application as a weighted related graph. Geng et al. addressed the energy-efficient computation offloading problem on multicore-based mobile devices in [23] by formulating a mixed-integer nonlinear programming problem and applying a heuristic algorithm, where applications were modeled as directed graphs. In [24], Sun et al. studied a VC-based computation offloading mechanism where computing missions were modeled as tasks with interdependency and executed in different vehicles to minimize overall response time, and thus alleviates the heavy workloads of edge clouds.

The graph job allocation problem in dynamic networks has rarely been investigated, as the limitations of opportunistic server-user communications and the interdependency among components pose substantial challenges to solution designs. We are among the few that work on the problem considering such dynamic network environments. A randomized graph job allocation mechanism based on hierarchical tree decomposition was proposed in our previous work [7], through which, feasible mappings between components and SPs were obtained. In [8], we presented a novel multi-task offloading problem under graph-representation by considering the potential inter-component competition due to task concurrency. In [33], we studied a truthful auction-based graph job allocation problem in vehicular cloud-assisted networks while considering the resource reutilization. However, concerning energy consumption especially for battery-conscious devices are not yet considered in the abovementioned previous researches. To conform green computing [25], an energy-aware graph job allocation problem in VC-assisted IoV was studied in our latest study [18]. Specifically, a hierarchical tree based random matching ap-
approach was applied to determine the assignment of a single graph job over vehicular cloud; while a structure-preserved simulated annealing algorithm was proposed to solve the power allocation problem.

1.2 Novelty and contributions

This paper studies the energy-aware allocation problem of mapping the components of multiple graph jobs carried by UAVs to on-ground SPs. Factors such as multiple concurrent jobs and complicated A2G channels present additional challenges related to the problem size and algorithm efficiency. Moreover, the potential competitions among UAVs caused by communication overlaps are necessarily considered. To the best of our knowledge, this paper is among the first to study the energy-aware graph job allocation problem under the SD-AGV network architecture. The major contributions in this paper are summarized below:

- We present a novel framework of energy-aware graph job allocation in the SD-AGV network, where the SDN controller achieves the efficient orchestration of undeveloped on-ground resources. The reliable integration of air segment and ground segment enables resource sharing between the UAVs with computation intensive graph jobs and the vehicles with idle resources, under the logically centralized control of the SDN controller.
- We study a novel job allocation problem under SD-AGV network architecture, where each job is modeled as a graph with components and weighted edges. Through solving the problem, the components of graph jobs of UAVs can be efficiently mapped to feasible on-ground vehicles, while achieving the trade-off upon minimizing the job completion time and energy consumption of the UAVs, as well as the data exchange cost among the SPs.
- We formulate the afore-mentioned problem as a mixed integer non-linear programming (MINLP) problem, which is NP-hard. Moreover, one of the constraints associated with preserving the graph job structures requires addressing the subgraph isomorphism problem, which further complicates the algorithm design. Thus, we propose an ingenious decoupled approach by separating the template search stage from the power allocation stage. The problem in the former stage is formulated as the search for all the subgraph isomorphisms between the graph jobs and the VC, for which we present an efficient template search algorithm. For the latter stage, we introduce a practical power optimization algorithm by applying convex optimization techniques.
- Based on thorough numerical analysis and comparative evaluations, we demonstrate that the performance of the proposed decoupled approach can outperform the baseline methods considering various problem sizes, while providing a low computation complexity in most cases.

The rest of this paper is organized as follows. The system model is introduced in Section 2. We formulate the energy-aware graph job allocation problem as a MINLP problem in Section 3. In Section 4, we propose an efficient decoupled approach. The performance evaluation through comprehensive simulations is introduced in Section 5 before drawing the conclusion in Section 6.

2 System Model

2.1 The framework of graph job allocation in SD-AGV networks

The SD-AGV networks is seen as an emerging network architecture of the late years. Specifically, SDN decouples the control plane from the data plane [11], introduces logically centralized control with a global view of networks, while facilitating network programmability/reconfiguration through open interfaces. Combine with the VCC technology, a manageable and cost-effective marketplace is established to orchestrate on-ground resources

![Diagram](image-url)
for the UAVs with computing requirements, achieving an efficient collaborative computing system. The framework of VC-assisted graph job allocation in SD-AGV networks and the relevant coordinate system are shown in Fig. 1(a) and Fig. 1(b), respectively.

Data plane and available resources: In this framework, each UAV or vehicle serves as an SDN switch that abided by unified scheduling and follows the Openflow protocol commonly used in SDN [13]. The UAVs are service requesters with heavy workloads, while vehicles serve as service providers with available resources. Both parties are following the schedule of the SDN controller. Decoupling data transmission and processing from control stands for one of the key features in this framework, which enables efficient orchestration of undeveloped resources.

Control plane and the SDN controller: This framework effectively facilitates the independency between the physical communication channels of control plane and that of the data plane, where the SDN controller can capture the status information [9] reported periodically by UAVs and vehicles (e.g., channel state information, locations, computing service requirements, current available resources, etc.). Specifically, a feasible energy-aware graph job allocation decision generated by the SDN controller will be distributed to the related mobile devices. Then, the data of graph jobs will be transmitted from UAVs to on-ground vehicles according to the allocation decision via data plane.

2.2 The model of the vehicular cloud

Suppose a VC covers a region containing SP set $S = \{s_k | k \in \{1, 2, \cdots, |S|\}\}$, where each $s_k \in S$ owns different number of fully connected idle VMs for leasing, and every VM provides the computational capability related to the execution time $t_{exec}$ for processing one component of a graph job. Notably, an available VM can only process one component at a time. Correspondingly, the VC is represented as a graph $G^* = \{S, E^*, W^*\}$, where $S$ is the set of SPs and each $s_k \in S$ can provide a set of available VMs denoted by $V_k$. $E^* = \{e_{k,k'} | s_k, s_{k'} \in S, s_k \neq s_{k'}\}$ represents the edge set where $e_{k,k'}$ indicates the edge between $s_k$ and $s_{k'}$, namely, a one-hop V2V communication link between these SPs. Moreover, $W^* = \{w_{k,k'} | s_k, s_{k'} \in S, s_k \neq s_{k'}\}$ denotes the associated weight of the edge set describing the corresponding parameters of the exponential distribution of V2V connections, which will be introduced in the following Section 2.5.

2.3 The model of the graph job

Consider an UAV set $U = \{u_m | m \in \{1, 2, \cdots, |U|\}\}$, where each $u_m \in U$ carries a computation-intensive graph job requires to be offloaded to on-ground SPs for execution. The job of $u_m$ is modeled as a graph $G^{um} = \{V^{um}, E^{um}, W^{um}\}$, where $V^{um} = \{v_{n,m} | n \in \{1, 2, \cdots, |V^{um}|\}\}$ denotes the components set in the graph job, and $v_{n,m}$ indicates the $n^{th}$ component of the graph job of $u_m$, with data size $D_{n,m} = D (\text{bit})^3$. $E^{um} = \{e_{n,n'} | n, n' \in \{1, 2, \cdots, |V^{um}|\}, n \neq n'\}$ and $W^{um} = \{w_{n,n'} | n, n' \in \{1, 2, \cdots, |V^{um}|\}, n \neq n'\}$ represent the edge set and the related weight set respectively, where

| Table 1 | Major notations |
|-----------------|----------------|
| $G^*, S, E^*, W^*$ | the VC graph, the set of SPs, edges and weights |
| $s_k, e_{k,k'}, w_{k,k'}$ | the $k^{th}$ SP, the edge and weight between SPs $s_k$ and $s_{k'}$ |
| $U, u_m, R_m$ | the set of UAVs, the $m^{th}$ UAV, the SPs set covered by $u_m$ |
| $G^{um}, V^{um}, E^{um}, W^{um}$ | the graph job of $u_m$, the set of components, edges, and weights of $G^{um}$ |
| $v_{n,m}, e_{n,n'}, w_{n,n'}$ | the $n^{th}$ components in $V^{um}$, the edge and weight between components $v_{n,m}$ and $v_{n',m}$ |
| $x_{k,m}$ | the indicator of mapping $v_{n,m}$ to $s_k$, the allocated power of $u_m$ to $s_k$ |
| $x_{k,m}$ | the matrix of $x_{k,m}$ and $q_{k,m}$ |
| $d_{k,k'}, d_{k,k'}^{AG}$ | the distance between $s_k$ and $s_{k'}$ and the distance between $u_m$ and $s_k$ |
| $c_{n,n'}$ | the channel gain and the data transmission rate between $u_m$ and $s_k$ |
| $D_{n,m}, t_{exec}$ | the data size of component $v_{n,m}$, the execution time of each VM |
| $\alpha_1, \alpha_2$ | the probabilistic parameters |
| $X, X_s$ | the template set, the $z^{th}$ template in set $X$ (used in stage 1) |
| $S_{x_s, E^s, W^s}$ | the set of SPs, edges and weights associated with template $X_s$ (used in stage 1) |
| $e_{x_s}^{k,k'}, w_{x_s}^{k,k'}$ | the corresponding SP edge and weight in sets $S_{x_s}, E^s$ and $W^s$ (used in stage 1) |
| $N$ | the component exploration sequence |
| $D^*(v_{n,m}), D^{map}(v_{n,m})$ | the degree, and the component mapping degree of $v_{n,m}$ |
| $D^*(s_k)$ | the current available degree of $s_k$ |
| $Pred(v_{n,m})$ | the set of predecessors of $v_{n,m}$ |
| $V^u, E^u, W^u$ | the set of components, edges and weights regardless of any particular UAV or template (used in stage 2) |
| $v_{n,m}, e_{n,n'}, w_{n,n'}$ | the corresponding component, edge and weight in sets $V^u, E^u$ and $W^u$ (used in stage 2) |
| $S_{u_m}, E^u, W^u$ | the set of SPs, edges and weights regardless of any particular template (used in stage 2) |
| $s_k, e_{k,k'}, w_{k,k'}$ | the corresponding SP edge and weight in sets $S, E^* and W^*$ (used in stage 2) |
\(e^{u_m}\) and \(w^{u_m}\), denote the required data flow, and connect duration between components \(v_{n,m}\) and \(v_{n',m}\). A graph job \(G^{u_m}\) describes the internal dependencies of how the computation split among the components in \(V^{u_m}\). For notational simplicity, let \(V^U \triangleq \bigcup_{m \in U} V^{u_m}\) be the union set of the components of graph jobs.

### 2.4 The template

Observe that for any graph job, there exist several ways (an exponential large number) in which the job can be distributed over SPs. Note that multiple graph jobs may exist in the network, a template \(X\) corresponds to a feasible mapping from the components set \(V^U\) to a subset of \(S\) in the related VC. An example of the template is given in Fig. 1(a). Notably, a mapping fails to preserve the structures or meet the weight requirements among edges of the graph jobs cannot be a template.

### 2.5 The model of communication

For analytical simplicity, let binary indicator \(x_{k,m} = 1\) denote the assignment where component \(v_{n,m}\) is mapped to SP \(s_k\); otherwise, \(x_{k,m} = 0\). Note that different UAVs may have various communication ranges, let \(R_m\) be the set of SPs that are covered by the communication radius of UAV \(v_m\), and the components of \(G^{u_m}\) can only be mapped to the SPs in \(R_m\). Also, we assume that each UAV stays hovering or moves slightly in the sky, which enables the SPs in each set \(R_m\) remaining unchanged during graph job allocation.

#### V2V channel model:

The contact duration between SPs \(s_k\) and \(s_{k'}\) obeys an exponential distribution [7], [8], [26] with parameter \(\alpha_{d,k,k'}\). Thus, the probability of the contact duration \(\Delta t_{k,k'}\) between \(s_k\) and \(s_{k'}\) exceeding a certain period \(\Delta t\) is given by \(\Pr(\Delta t_{k,k'} \geq \Delta t|w^s_{k,k'}) = e^{-\frac{\Delta t}{\alpha_{d,k,k'}}}\), where the larger \(\Pr(\Delta t_{k,k'} \geq \Delta t|w^s_{k,k'})\) can bring more assurance for the required data exchange duration between different SPs.

The path loss of a V2V communication link is considered by following the dual-slope model [27], which is defined as a piece-wise function of the distance \(d_{k,k'}\) between \(s_k\) and \(s_{k'}\).

\[
pl(d_{k,k'}) = \begin{cases} 
pl_0 + 10\eta_1 \log_{10} \left( \frac{d_{k,k'}}{d_0} \right) + X_\sigma, & \text{if } d_0 \leq d_{k,k'} \leq d_B \\
pl_0 + 10\eta_1 \log_{10} \left( \frac{d_{k,k'}}{d_B} \right) + 10\eta_2 \log_{10} \left( \frac{d_{k,k'}}{d_B} \right) + X_\sigma, & \text{if } d_{k,k'} > d_B 
\end{cases}
\]

where \(d_0\) is the reference distance, \(pl_0\) is the path loss at \(d_0\), \(X_\sigma\) denotes a zero-mean normally distributed random variable with a standard deviation of \(\sigma\). Notation \(\eta_1\) and \(\eta_2\) denote the path loss exponent before and after distance \(d_B\), respectively, and \(d_B\) indicates the breakpoint distance which is calculated as (2).

\[
d_B = \frac{4h_t h_r}{\lambda} - \frac{w}{4},
\]

where \(h_t\) and \(h_r\) are the transmitter’s and the receiver’s height, and \(w\) denotes the wavelength. Here, let \(h_t = h_r\) owing to the possible intermediate data exchange that makes \(s_k\) and \(s_{k'}\) into both transmitter and receiver. Rely on the uncertain channel conditions of V2V communication links, the case where two connected components are mapped to different SPs can bring intermediate data exchange cost, which captures the cost incurred from traffic exchange among different SPs in a VC. Correspondingly, \(c_{k,k'}^{\text{cost}}\) is defined in (3).

\[
c_{k,k'}^{\text{cost}} = \begin{cases} 
f(pl(d_{k,k'})), & s_k \neq s_{k'} \\
0, & \text{otherwise}
\end{cases}
\]

where \(f()\) is a monotone increasing function. Apparently, a larger value of \(pl(d_{k,k'})\) will bring a higher cost for intermediate data exchange among different vehicles. Let \(c_{k,k',m}^{n,n'}\) describe the data exchange cost incurred when two connected components \(v_{n,m}\) and \(v_{n',m}\) of \(G^{u_m}\) are assigned to different SPs, which is given in (4).

\[
c_{k,k',m}^{n,n'} = \begin{cases} 
c_{k,k'}^{\text{cost}}, & \forall e_{n,m} \in E^{u_m}, \text{if } x_{k,m} \times x_{k,m}' = 1 \\
0, & \text{otherwise}
\end{cases}
\]

#### A2G channel model:

As shown in Fig. 1(a), we consider several multi-antenna UAVs capable for offloading the components of a graph job to SPs. Moreover, each SP relies on full-duplex techniques and the self-interference is ignored. Denote the channel gain between \(u_m\) and \(s_k \in R_m\) as \(g_{k,m}\), which is assumed to be dominated by the line of sight (LoS) path [29], shown in (5).

\[
g_{k,m} = g_1 \times \left( \frac{d_{A2G}^{u_m}}{d_{k,m}} \right)^{-\eta_3},
\]

where \(g_1\) corresponds to the channel gain at the reference distance of 1 meter, \(d_{A2G}^{u_m}\) indicates the A2G distance between \(u_m\) and \(s_k\) and \(\eta_3\) denotes the path loss exponent of the LoS path. The data transmission rate \(r_{k,m}\) between UAV \(u_m\) and SP \(s_k\) is a function of the transmission power \(q_{k,m}\) that \(u_m\) allocates to \(s_k\), calculated by (6), where \(B\) denotes the channel bandwidth and \(N_0\) represents the background noise power.

\[
r_{k,m} = B \log_2 \left( 1 + \frac{q_{k,m} \times g_{k,m}}{N_0} \right)
\]

### 2.6 The model of computation and energy consumption

The completion time \(t_m\) of graph job \(G^{u_m}\) is composed by the time of data transmission, execution and the resulting feedback. Notably, the delay on resulting feedback from the SP to the UAV can be ignored owing to a much smaller output data size [7], [8]. Apparently, \(t_m\) relies on the slowest processed component of graph job \(G^{u_m}\).

\[
t_m = \max \left[ \sum_{n=1}^{\lvert V^{u_m} \rvert} x_{k,n,m} \times D_{n,m}, \frac{\sum_{n=1}^{\lvert V^{u_m} \rvert} x_{k,n,m} \times D_{n,m}}{r_{k,m}} \right] + t_{\text{exec}},
\]

where \(\sum_{n=1}^{\lvert V^{u_m} \rvert} x_{k,n,m} \times D_{n,m}\) and \(\frac{\sum_{n=1}^{\lvert V^{u_m} \rvert} x_{k,n,m} \times D_{n,m}}{r_{k,m}}\) represent the total amount of data, and the relevant transmission time of components from \(u_m\) to \(s_k\), respectively. Moreover,

4. Each SP can receive the graph job data from the UAV while exchanging the intermediate data with other SPs [28].
the UAV would incur extra overhead in terms of energy when transmitting data to SPs via wireless access. Thus, the energy consumption $e_m$ of $u_m$ is calculated as:

$$
e_m = \sum_{k=1}^{\lvert S \rvert} \sum_{n=1}^{\lvert V^w_m \rvert} x_{k,n}^m \left( \frac{q_{k,m} \times D_{n,m}}{r_{k,m}} \right) + \ell,$$

where $\ell$ indicates the tail energy [30] given that the UAV will hold the channel for a while even after data transmission. We summarize the major notations and the related definitions in Table 1.

## 3 Problem Formulation

Consider a network containing a set of vehicles $S$ as service providers and a set of UAVs $U$ as service requestors, the relevant constraints are listed below.

- **Available resource limitation** imposes restrictions on idle VMs for each SP:

$$\sum_{m=1}^{\lvert U \rvert} \sum_{n=1}^{\lvert V^w_m \rvert} x_{k,n}^m \leq \lvert V_k \rvert, \forall s_k \in R_m. \quad (C1)$$

- **Soft opportunistic V2V connection** poses a soft constraint that if two connected components $v_{n,m}$ and $v_{n',m}$ of $u_m$ with weight $w_{n,m}$ are mapped to different SPs $s_k$ and $s_{k'}$, the probability of the contact duration between $s_k$ and $s_{k'}$ being larger than $\frac{D_{n,m}}{r_{k,m}} - \frac{D_{n',m}}{r_{k',m}} + w_{n,m}$ should be greater than a threshold $\alpha_1 (0 < \alpha_1 < 1)$. Notably, $\frac{D_{n,m}}{r_{k,m}} - \frac{D_{n',m}}{r_{k',m}}$ denotes the absolute value of the transmission time difference between component $v_{n,m}$ and $v_{n',m}$ and the order of components transmission is ignored.

$$e^{-\left(\frac{D_{n,m}}{r_{k,m}} - \frac{D_{n',m}}{r_{k',m}} + w_{n,m}\right) \times w_{s_k,s_{k'}}} \geq \alpha_1,$$

if $s_k \neq s_{k'}$ and $x_{k,n}^m \times x_{k',n'}^m = 1$, $\forall v_{n,m} \in F_{U^w,m}$ and $u_m \in U$ and $s_k, s_{k'} \in R_m$. (C2)

- **Transmission power limitation** prevents the case where the total allocated power may exceed $u_m$’s upper limit $Q_m$:

$$\sum_{k=1}^{\lvert S \rvert} \sum_{n=1}^{\lvert V^w_m \rvert} x_{k,n}^m \times q_{k,m} \leq Q_m, \forall u_m \in U \quad (C3)$$

- **UAV’s coverage limitation** only allows each UAV to offload components to SPs within its communication radius.

$$q_{k,m} \leq 0, \forall u_m \in U \text{ and } s_k \notin R_m \quad (C4)$$

For notational simplicity, let $x = [x_{n,m}^k]_{1 \leq k \leq \lvert S \rvert} \leq \lvert U \rvert, 1 \leq m \leq \lvert V^w_m \rvert$ denote the matrix of binary variable $x_{n,m}^k$ with size $\lvert V^w \rvert \times \lvert S \rvert$, where $\lvert V^w \rvert$ and $\lvert S \rvert$ indicate the total number of components and SPs, respectively. Let $q = [q_{k,m}]_{1 \leq k \leq \lvert S \rvert} \leq \lvert U \rvert$ be the power allocation matrix of size $\lvert U \rvert \times \lvert S \rvert$, where $\lvert U \rvert$ represents the number of UAVs in the system. Correspondingly, aiming to minimize the value of the objective function $F(x,q)$ given in (9), we formulate the proposed optimization problem of energy-aware graph job allocation as $P$ in (10).

$$\mathcal{F}(x,q) = \omega_1 \sum_{m=1}^{\lvert U \rvert} l_m + \omega_2 \sum_{m=1}^{\lvert U \rvert} c_m + \frac{\omega_3}{2} \sum_{k=1}^{\lvert S \rvert} \sum_{k'=1}^{\lvert S \rvert} \sum_{m=1}^{\lvert U \rvert} \sum_{m'=1}^{\lvert V^w_m \rvert} \sum_{n=1}^{\lvert V^w_m \rvert} x_{k,n}^m x_{k',n'}^m c_{n,n',m,m'},$$

$$\text{s.t.} \quad (C1), (C2), (C3), (C4).$$

$\mathcal{F}(x,q)$ stands for the weighted sum of job completion time, energy consumption and data exchange cost, where $\omega_1$, $\omega_2$ and $\omega_3$ represent the non-negative coefficients. $\sum_{m=1}^{\lvert U \rvert} l_m$ and $\sum_{m=1}^{\lvert U \rvert} c_m$ denote the total job completion time and energy consumption of UAVs, respectively. $\sum_{k=1}^{\lvert S \rvert} \lvert S \rvert - \sum_{k'=1}^{\lvert S \rvert} \sum_{m=1}^{\lvert U \rvert} \sum_{m'=1}^{\lvert V^w_m \rvert} \sum_{n=1}^{\lvert V^w_m \rvert} c_{n,n',m,m'}$ indicates the total data exchange cost among SPs in the VC, where the normalization factor 1/2 is considered since the cost will be calculated twice due to the undirected graph model.

$$\mathcal{P} : \arg \min_{x,q} \mathcal{F}(x,q) \quad (10)$$

Notably, $\mathcal{P}$ stands for a non-trivial MINLP problem which is NP-hard with coupled binary variables $x_{n,m}^k \in x$ and consecutive variables $q_{k,m} \in q$, that both needed to be optimized. Moreover, constraint (C2) requires solving the subgraph isomorphism problem, which further poses challenges to the algorithm design [7], [12], [17], [18]. In principle, solutions can be obtained through exhaustive search, which, however, is practically infeasible due to high complexity. For example, determining templates of mapping components of UAVs to SPs through exhaustive search results in high computational complexity of $O(2^{(\lvert V^w \rvert) \times (\sum_{k=1}^{\lvert S \rvert} \lvert V_k \rvert)})$; and for each feasible mapping, the optimization problem of power allocation needs to be solved. Consequently, the system can rarely identify the IoV extemporaneously, as the running time required to solve large and real-life network cases increases sharply with increasing vehicular and UAV’s density. As a result, we propose an efficient decoupled approach for solving $\mathcal{P}$ in the next section, which can offer a low computation complexity.

## 4 Solving the Energy-Aware Graph Job Allocation Problem: A Decoupled Approach

The significance of preserving the structures of both the VC and the graph jobs complicates the simultaneous allocation of job components and transmission power among SPs. In this section, we propose an efficient approach by decoupling the template search problem from the power allocation problem, which mainly contains two stages. For the former stage, an efficient template search algorithm is proposed aiming to search for all the feasible mappings between the graph jobs and the SPs. Then, given the templates obtained
from stage 1, a power allocation algorithm is presented via applying convex optimization techniques.

### 4.1 Stage 1: The proposed template search algorithm

The template stands for one of the key concerns in this paper, which is formulated as the search for all the subgraph isomorphisms [12] between the graph jobs and the VC. For analytical simplicity, let \( X = \{ X_z | z \in \{1, 2, \cdots |X|\} \} \) be the set of templates, where \( X_z \) is the \( z \)-th template in set \( X \). Note that not every SP can be selected; we define \( S_z \subseteq S \) as a subset of \( S \) associated with template \( X_z \). Specifically, let \( \tilde{s}_k^z \in \tilde{S}_z \) be the \( k \)-th SP in set \( S_z \), and \( \tilde{v}_k \) be the related VM set of \( \tilde{s}_k^z \). The edge and weight set corresponding to \( S_z \) are denoted as \( E^{s,z} = \{ \tilde{e}^z_{k,k'} | \tilde{s}_k^z, \tilde{s}_k'^z \in \tilde{S}_z \} \) and \( \tilde{W}^{s,z} = \{ \tilde{w}^{s,z}_{k,k'} | \tilde{s}_k^z, \tilde{s}_k'^z \in \tilde{S}_z \} \), where \( \tilde{E}^{s,z} \subseteq E^s \) and \( \tilde{W}^{s,z} \subseteq W^s \). Thus, each template \( X_z \in X \) can be represented as \( X_z = (x^{u,m}_k(z))_{u,m} \in U, v_{n,m} \in V^{u,m}, \tilde{s}_k^z \in \tilde{S}_z \), where \( x^{u,m}_k(z) \) indicates the assignment from component \( v_{n,m} \) to SP \( \tilde{s}_k^z \in \tilde{S}_z \), in template \( X_z \). The problem of searching for the templates is formulated as \( P_1 \) in (11).

\[
P_1 : X
\]

s.t.

\[
\sum_{m=1}^{U} \sum_{n=1}^{V^{u,m}} x^{u,m}_k(z) \leq \tilde{V}_{k}^{z}, \forall \tilde{s}_k^z \in \tilde{S}_z \text{ and } \forall X_z \in X,
\]

(C5)

\[
x^{u,m}_k(z) \neq 0, \forall \tilde{s}_k^z \notin \mathcal{R}_m, X_z \in X
\]

(C6)

\[
e^{-w^{u,m}_k(z) + \rho^{u,m}_k} \geq \alpha_2,
\]

if \( \exists \forall u, n, m \in E^{u,m} \) and \( x^{u,m}_k(z) \times x^{u,m}_k(z) = 1, \forall X_z \in X. \)

(C7)

Similar with (C1), constraint (C5) imposes restrictions on available VMs of each \( \tilde{s}_k^z \in \tilde{S}_z \). Constraint (C6) refers to the coverage limitation of each UAV, that is similar with (C4). Constraint (C7) poses a hard restriction that every template preserves the graph job structures; and a soft restriction which ensures that the probability of the contact duration between different SPs \( \tilde{s}_k^z \) and \( \tilde{s}_k'^z \) (processing connected components \( v_{n,m} \) and \( v_{n',m} \), respectively) being larger than \( w_{n,m} \) should be greater than a threshold \( \alpha_2 \) \((0 < \alpha_2 < 1)\). Notably, the air-to-ground data transmission time is not concerned in \( P_1 \) owing to the unknown power allocation solution (which is introduced in the following Stage 2).

The coverage overlaps may bring competitions among UAVs of available VMs. Thus, an efficient template search algorithm is proposed to solve 3P1, while achieving conflict avoidance of reoccupations of the same VM among different UAVs. Our proposed algorithm is divided into two steps: preprocess for obtaining the component exploration sequence, and search for the templates. The first step finds a preprocessing of the graph jobs, aiming to obtain an exploration sequence to determine the next candidate component during mapping. The second step aims to obtain the set of templates \( X \) under an efficient manner.

**Step 1. Preprocess for obtaining the component exploration sequence:** to prioritize the components that are more rare and constrained in graph jobs [12], step 1 defines the order relationship by generating the component exploration sequence \( N \). Specifically, an exploration sequence \( N \) denotes a permutation of the components in set \( V^U \), and is applied in step 2 to determine the next candidate component. In this paper, the rarerness of a component \( v_{n,m} \in V^{u,m} \) relies on the degree of component \( D^c(v_{n,m}) \), shown below.

**Definition 1 (the degree of component \( D^c \)):** the degree \( D^c(v_{n,m}) \) of component \( v_{n,m} \) is calculated as the number of edges related to \( v_{n,m} \) in graph job \( G^{u,m} \). In this paper, \( D^c(v_{n,m}) \) stands for the rarerness of component \( v_{n,m} \), where the larger value of \( D^c \) represents a rarer component.

To preserve the structure of each graph job, the order of components in \( N \) is determined by computing the connections of a component with the components those are already in \( N \). Correspondingly, we define the component mapping degree \( D^{map}(v_{n,m}) \) of a component \( v_{n,m} \) as below:

**Definition 2 (the component mapping degree \( D^{map} \)):** the component mapping degree \( D^{map}(v_{n,m}) \) of \( v_{n,m} \) equals to the number of edges between component \( v_{n,m} \) and all the components that are already inside \( N \).

Therefore, the procedure of generating \( N \) firstly depends on the component mapping degree \( D^{map} \) of each component; if two or more components have the same \( D^{map} \), they are sorted according to the value of \( D^c \); if both \( D^{map} \) and \( D^c \) are equal, the choice is done randomly. Specifically, the component with the largest \( D^c \) is selected as the first component in \( N \); if more than one component have the same \( D^c \), randomly choose one of them to be the first in \( N \). The second column of Table 2 depicts an example of the component exploration sequence related to the graph jobs.

| Step | \( N \) | Predecessor | Component | Candidate | Mapping | Update |
|------|--------|------------|-----------|----------|---------|-------|
| 1    | [F]    | –          | F         | \{ s_1, s_5, s_6 \} | \( D^c(s_3) = D^c(s_5) = 6, D^c(s_6) = 7 \) |
| 2    | [F]    | Pred(F) = \{ \} | E         | \{ s_1, s_5, s_6, s_7 \} | \( D^c(s_3) = D^c(s_5) = 6, D^c(s_7) = 0 \) |
| 3    | F, E   | Pred(E) = \{ F \} | H         | \{ s_6 \} | \( D^c(s_3) = 0, D^c(s_3) = D^c(s_5) = 5 \) |
| 4    | F, E, H | Pred(H) = \{ E, F, G \} | I         | \{ s_3, s_5 \} | \( D^c(s_3) = D^c(s_5) = 4 \) |
| 5    | F, E, H, G | Pred(G) = \{ F \} | I         | \{ s_3, s_5 \} | \( I \rightarrow s_3 \) | \( D^c(s_3) = D^c(s_5) = 3 \) |
| 6    | F, E, H, G, I | Pred(I) = \{ F, G \} | B         | \{ s_2, s_3, s_4, s_8 \} | \( A \rightarrow s_5 \) | \( D^c(s_3) = 6, D^c(s_8) = 4, D^c(s_3) = D^c(s_8) = 3 \) | \( D^c(s_3) = 4 \) |
| 7    | F, E, H, G, I, A | Pred(A) = \{ \} | D         | \{ s_2, s_3, s_4, s_8 \} | \( B \rightarrow s_2 \) | \( D^c(s_3) = 2, D^c(s_4) = D^c(s_4) = 2, D^c(s_3) = 3 \) |
| 8    | F, E, H, G, I, A, B | Pred(B) = \{ A \} | C         | \{ s_2, s_3, s_4 \} | \( C \rightarrow s_2 \) | \( C \rightarrow s_4 \) | \( D^c(s_2) = D^c(s_4) = 2, D^c(s_4) = 0 \) |
| 9    | F, E, H, G, I, A, B, C | Pred(C) = \{ A \} | D         | \{ s_4 \} | \( D \rightarrow s_5 \) | \( D^c(s_3) = 1, D^c(s_5) = 0 \) |
| 10   | F, E, H, G, I, A, B, C, D | Pred(D) = \{ A \} | The related template \( X : \{ A, B, C, D, E, F, G, H, I \} \rightarrow \{ s_5, s_2, s_4, s_5, s_7, s_5, s_6, s_8, s_3 \} \) | |

**TABLE 2:** Examples of the component exploration sequence, the related predecessors, and how a template is generated.
Definition 5 (the candidate of component): a SP \( s_n \) is a candidate of component \( v_{n,m} \) as defined in Table 2 (the third column).

Algorithm 1: Preprocess for obtaining the component exploration sequence (Step 1)

Input : \( G_{u,m} = \{V_{u,m}, E_{u,m}, W_{u,m}\} \), \( \forall u,m \in U \)
Output: \( N \)
1 Initialization: \( N \leftarrow \emptyset \), calculate \( D^c(v_{n,m}) \) for all \( v_{n,m} \in V_{u,m}, u,m \in U \)
2 \( N \leftarrow \) the component with largest value of \( D^c \), if two or more components have the same \( D^c \), randomly choose one,
3 for \( i = 1 \) to \( |V_U| - 1 \) do
4 for all \( v_{n,m} \notin N \) do
5 calculate \( D_{map}(v_{n,m}) \),
6 \( N \leftarrow N \cup \) the component with the largest \( D_{map}(v_{n,m}) \); if two or more components have the same value of \( D_{map} \), choose the one with the largest \( D^c \); if two or more components have the same values of both \( D_{map} \) and \( D^c \), randomly choose one,
7 \( i = i + 1 \),
8 end algorithm

Definition 3 (the predecessor of component): the predecessor \( Pred(v_{n,m}) \) of component \( v_{n,m} \) is defined as a set of components that have one-hop connection with \( v_{n,m} \), and located before \( v_{n,m} \) in \( N \). Notably, some of the components may have no predecessor, such as the first component in sequence \( N \).

To preserve the graph job structures, while ensuring the efficiency of the proposed template search algorithm, a component \( v_{n,m} \) can only be mapped to a SP that can meet the related degree requirements, and the structure constraints (C6) and (C7) with the components in set \( Pred(v_{n,m}) \). Correspondingly, we define the current available degree \( D^s(s_k) \) of a SP \( s_k \) in \( R_m \), and the candidate of a component \( v_{n,m} \) as follows:

Definition 4 (the current available degree \( D^s \) of the SP): the current available degree \( D^s(s_k) \) of SP \( s_k \) is calculated as the sum of the current available VMs of \( s_k \), and that of the SPs which have one-hop connection with \( s_k \). Notably, if there is no local VM available on \( s_k \), \( D^s(s_k) = 0 \).

Definition 5 (the candidate of component): a SP \( s_k \) can be a candidate of component \( v_{n,m} \) if and only if the following two conditions are both satisfied:
1. \( D^s(s_k) \geq D^c(v_{n,m}) - D_{map}(v_{n,m}) \),
2. map component \( v_{n,m} \) to SP \( s_k \) can meet all the edge and weight constraints with the components in set \( Pred(v_{n,m}) \).

Take the graph jobs shown in Fig. 1(a) as example. Under the exploration sequence \( N = [F, E, H, G, I, A, B, C, D] \) shown in Table 2, the current available degree \( D^s(s_k) \) of \( s_k \) before graph job allocation is \( D^s(s_k) = |V_6| + |V_3| + |V_5| + |V_7| = 2 + 2 + 3 + 1 = 8 \). After allocate component \( E \) to \( s_6 \), the current available degree of which is updated as \( D^s(s_6) = (2 - 1) + 2 + 3 + (1 - 1) = 6 \) due to that the VM of \( s_7 \) has been occupied by \( E \). Accordingly, \( D^s(s_7) = 0 \).

Given an exploration sequence \( N \), the major steps of the proposed template search algorithm are given in Algorithm 2. The main idea is to sequentially assign each component in set \( N \) to an unmapped candidate at a time, until all the templates are searched out. Notably, the computation complexity also relies on both the graph job and the VC structures. For example, consider the VC structure as a complete graph (there exist an edge between any two SPs), the computation complexity of the proposed algorithm may rise to the same level with exhaustive search. Thus, the proposed template search algorithm can provide a best computation complexity performance of \( O(|V_U|) \), but a worst case equals to the exhaustive search algorithm. However, the proposed algorithm works on both the preprocess of graph jobs, and the selection of candidate SPs for each component, which greatly reduces the searching space during mapping.

Owing to the flexible topologies of the graph jobs as well as the VC structures in real-life applications and networks, we can make a weak assumption that in most cases, the proposed algorithm will offer a low computation complexity.

Algorithm 2: Search for the templates (Step 2)

Input : \( N, G_{u,m} = \{V_{u,m}, E_{u,m}, W_{u,m}\} \), \( \forall u,m \in U \), \( R_m \), \( \forall u,m \in U, G^s \)
Output: \( X \)
1 Initialization: get \( Pred(v_{n,m}) \) for all \( v_{n,m} \in V_U \),
2 in each iteration \( z \),
3 for \( i = 1 \) to \( |N| \) do
4 get the candidate set of \( N[i] \), \%\( N[i] \) denotes the \( i \)th component in \( N \),
5 assign \( N[i] \) to the first unmapped candidate SP,
6 put the related component-SP pair into \( X_z \),
7 \( i = i + 1 \),
8 \( X \leftarrow X_z \cup X \),
9 until finish searching for all unmapped candidate SP of each component in each iteration, by following \( N \),
10 end algorithm when all templates are searched out

To better analyze our proposed template search algorithm, a walk through example is provided in Table 2 (from the forth column to the seventh column), showing how a template is generated given the graph job and VC structures shown in Fig. 1(a), under the given exploration sequence \( N = [F, E, H, G, I, A, B, C, D] \).

4.2 Stage 2: The proposed power allocation algorithm

In this section, we study an effective transmission power allocation algorithm under the given templates obtained from stage 1. Owing to that the proposed algorithm works indistinguishably for various templates, and the transmission power allocation is independent among different UAVs,
where the indexes \( z \) and \( m \) referring to a unique template \( \mathcal{X}_z \) and UAV \( u_m \) are ignored. Hereafter, symbols \( x^u_k \), \( D_n \), \( r_k \), and \( Q \) as the substitutions of \( x^m_k(z) \), \( D_{n,m} r_{k,m} \), \( q_k \), and \( Q_n \), and \( V^u \) as \( \{ V_{n,n} \} \) the data transmission rate \( r_k \) is a function of \( q_k \). Thus, the proof can be obtained by verifying the concavity of \( -\frac{q_k}{r_k q_k} \) [31], which makes \( \mathcal{P}_3 \) non-convex.

In consequence, the change-of-variable technique is applied to transform \( \mathcal{P}_3 \) into a convex optimization problem \( \mathcal{P}_4 \), by introducing a set of substitute variables \( \rho \) as follows. 

\[
\mathcal{P}_4 : \arg \min_{\rho} \sum_{k=1}^{\bar{s}} \left( |W_{1,k}(\rho_k)^p + W_{2,k} \rho_k \left( \frac{2\pi f_k}{r_k} - 1 \right) \right)
\]

s.t.
\[
\sum_{k=1}^{\bar{s}} 1 - 2 \pi r_k = 0,
\]
\[
\rho_k - \rho_k^* + C_{n,m}^p \leq 0, \forall \bar{s}_k \neq \bar{s}_k',
\]
\[
\rho_k - \rho_k^* - C_{n,m}^p \leq 0, \forall \bar{s}_k \neq \bar{s}_k',
\]
\[
\forall \bar{e}_{n,m}' \in E^u \text{ and } \forall \bar{s}_k, \bar{s}_k' \in \bar{S},
\]
\[
|S| = \sum_{k=1}^{\bar{s}} q_k \leq Q,
\]
\[
(C8), (C9).
\]

Although we may concentrate on obtaining the optimal solution of \( \mathcal{P}_3 \), there still remains difficulties featured by non-convexity shown in the following lemma.

**Lemma 1:** \( \mathcal{P}_3 \) represents a non-convex optimization problem.

**Proof.** According to equation (6), the data transmission rate \( r_k \) is a function of \( q_k \). Thus, the proof can be obtained by verifying the concavity of \( -\frac{q_k}{r_k q_k} \) [31], which makes \( \mathcal{P}_3 \) non-convex.

Due to that there may exist multiple pairs of connected components being mapped to different SPs, constraint (C12) is thus represented as a inequality constraints set \( C \), shown as (C13):

\[
C = \left\{ f_i(\rho_k, \rho_k^*, C_{n,m}^p) \leq 0 \mid i \in \{1, \ldots, |C|\}, \forall \bar{s}_k, \bar{s}_k' \right\}
\]

and \( x_k^n \times x_{k'}^{n'} = 1 \) and \( \forall e_{n,m}' \in E^u \text{ and } \forall \bar{s}_k, \bar{s}_k' \in \bar{S},
\]

where \( e_{n,m}' \) stands for the number of UAV 1 shown in Fig. 1 (a) connected components been mapped to different SPs \( \bar{s}_k \) and \( \bar{s}_k' \). For any given template, the value of each \( C_{n,m}^p \) is also fixed (namely, each \( C_{n,m}^p \) is a constant). Thus, (C11) can be rewritten as (C12) owning to \( C_{k,k'}^p \leq 0 \).

\[
C_{n,m}^p = \frac{D_{k,k'}^p}{s_k x_{k'}^{n'} + x_{k,k'}^n},
\]

where \( D_{k,k'}^p \) denotes the parameter when two connected components \( u_v, u_{v'} \) are mapped to different SPs \( \bar{s}_k \) and \( \bar{s}_k' \). For any given template, the value of each \( C_{n,m}^p \) is also fixed (namely, each \( C_{n,m}^p \) is a constant). Thus, (C11) can be rewritten as (C12) owning to \( C_{k,k'}^p \leq 0 \).

\[
(f_1(\rho_k, \rho_k^*, C_{n,m}^p)) \leq 0, \forall \bar{s}_k \neq \bar{s}_k',
\]

\[
(C13).
\]

5. For example, the constraint set \( C \) of UAV 1 shown in Fig. 1 (a) contains two in-equations: 

\[
f_1(q_{k_1} - q_{k_2}, q_{k_3} - q_{k_4}) = (q_{k_1} - q_{k_2})^2 - (q_{k_3} - q_{k_4})^2 \leq 0,
\]

\[
f_2(q_{k_4} - q_{k_5}, q_{k_6} - q_{k_7}) = (q_{k_4} - q_{k_5})^2 - (q_{k_6} - q_{k_7})^2 \leq 0.
\]

Here, \( A, B, C \) and \( D \) are used to indicate components of the graph job for notational simplicity.
spondingly, the problem $\mathcal{P}_4$ represents a convex optimization problem, as proved in Lemma 2.

**Lemma 2.** $\mathcal{P}_4$ represents a convex optimization problem.

**Proof.** Let function $y(\rho) = y_1(\rho) + y_2(\rho)$, where $y_1(\rho) = \mathcal{W}_{1,k}(\rho)\rho$ and $y_2(\rho) = \mathcal{W}_{2,k}\rho(2\pi\rho - 1)$. The second-order derivative of $y(\rho)$ can be given by:

$$
\frac{d^2 y(\rho)}{d\rho^2} = y''(\rho) + y''(\rho),
$$

(16)

where $y''(\rho) = p(p-1)\mathcal{W}_1\rho^{p-2} \geq 0$, and $y''(\rho)$ is calculated as:

$$
y''(\rho) = \frac{d^2 y_2(\rho)}{d\rho^2} = \frac{d^2 (W_{2,k}\rho)}{d\rho^2} = 2\pi\rho^{-1} \ln 2 - \frac{2}{\rho^2}.
$$

(17)

Thus, we have $y''(\rho)$ shown in (18).

$$
y''(\rho) = \frac{d^2 y_2(\rho)}{d\rho^2} = 2\pi\rho^{-1} \ln 2 - \frac{2}{\rho^2}.
$$

(18)

Apparently, $\frac{d^2 y(\rho)}{d\rho^2} > 0, \forall \rho > 0$, and $y(\rho)$ represents a convex function of variable $\rho$. Since $\mathcal{P}_4$ aims at minimizing a summation of convex functions $y(\rho_k)$, where the constraint (C10) is convex with $(2\pi\rho')'' > 0(\forall \rho > 0)$. Moreover, each inequality constraint in (C13) is convex, $\mathcal{P}_4$ is proved to be a convex optimization problem.

In consequence, the power allocation vector can be obtained numerically by using convex optimization solvers such as MATLAB function fmincon and CVX [32].

### 5 Numerical Results and Performance Evaluation

This section presents numerical results that illustrate the validity of the proposed decoupled approach (abbreviate to “Proposed” for simplicity). In the following, the performance of the proposed template search and power allocation algorithms comparing with several baseline methods, are analyzed in detail. Moreover, various problem sizes are investigated considering different graph job types and VC structures, as well as various numbers of UAVs and SPs.

#### 5.1 Simulation setup

We consider a simulation space of $1000m \times 1000m \times 100m$ (length×width×height), wherein the height of the UAV is randomly chosen from 80m to 100m. The graph job types considered in this simulation are depicted in Fig. 1(b). The monotone increasing function $f(pl(d_{k,v})) = 0.15 \times pl(d_{k,v}) + 0.001$ is applied to determine the data exchange cost among different SPs. The main simulation parameters are randomly obtained from the following intervals: $D \in [500Kb, 600Kb]$, $Q \in [1.5Watts, 2Watts]$, $N_0 \in [4mWatts, 5mWatts]$, $B \in [10MHz, 12MHz]$, $\omega_1 = \omega_3 = 1/3$, $w_{n,n'} \in [0.1, 0.3]$, $\nu_{sec} \in [0.1, 0.2]$, $w_{k,v} \in [0.05, 0.06]$ for small problem size cases, and $w_{k,v} \in [0.01, 0.02]$ for large problem size cases.

#### 5.2 Performance comparisons of template searching

This section presents performance comparisons of the running time, and the number of templates (use “templates count” instead for notational simplicity) between the proposed template search algorithm and baseline methods listed below:

1) **Exhaustive search algorithm (ESA)** [8]: check through all the mappings between the graph jobs and the VC structure, where the feasible ones are regarded as templates.

2) **Random search algorithm (RSA)**: randomly select a component and randomly match it to a SP at a time, until find a template. Notably, we consider different number of iterations: 10000 (RSA1), 20000 (RSA2), and 30000 (RSA3), to better demonstrate the performance evaluation.

#### Table 3

| Graph job type | Proposed | 7/8/9 | 7/12/11 | 10/14/14 | 12/16/15 | 14/18/1le |
|---------------|----------|-------|---------|----------|----------|----------|
| SPs/VMS/edges |          |       |         |          |          |          |
| ESA           | 16       | 1112  | 216     | 36       | 24       |          |
| RSA1          | 1        | 9     | 5       | 3        | 1        |          |
| RSA2          | 2        | 13    | 9       | 5        | 1        |          |
| RSA3          | 2        | 32    | 17      | 6        | 3        |          |
| RSA4          |          |       |         |          |          |          |

The running time and the template count performance comparisons in small problem size containing one UAV and a couple of SPs, are shown in Fig. 2 and Table 3, respectively. The 10-based logarithm representation is applied in Fig. 2, since the gap between the running time of various algorithms becomes large as the graph job and VC structures become more complicated (e.g., upon increasing the number of components, SPs/VMs and edges). As can be seen in Fig. 2, the running time of ESA rises sharply owing to that ESA relies heavily on the number of components, available VMs and the complexity of graph job as well as VC structures, which makes the ESA inappropriate for fast-changing and large-scale networks. Comparatively, the running time of the proposed algorithm remains approximately
Running time(ESA) = 6853.1120 seconds
Running time(Proposed) = 44.3959 seconds
Templates count = 28800

The ith edge
Possible candidate
The ith edge

48 seconds (RSA1), 1 second (RSA2) and 1.5 seconds of the RSA mainly depends on the number of iterations, at a certain order of magnitude of $10^{-2} \sim 1$ second when considering small problem sizes. Since the running time of the RSA mainly depends on the number of iterations, the performance of which remains slightly fluctuant near 0.5 seconds (RSA1), 1 second (RSA2) and 1.5 seconds (RSA3). Table 3 presents the comparisons of templates count between different algorithms in small problem size cases. Apparently, our proposed algorithm can search for all the subgraph isomorphisms between the graph jobs and the VC, while offering a much lower computation complexity than ESA according to Fig. 2. The templates count of RSA under different numbers of iteration are far less than that of the proposed algorithm due to the randomness. In fact, failures and repetitions of mappings are common during the RSA procedure, owing to the structure preservation constraint.

Fig. 3 presents the performance comparisons of running time and templates count in large problem size cases: a). considering graph job type 1 and type 2; b). considering graph job type 2 and type 3; c). considering graph job type 1, type 2 and type 3.

![Diagram](image)

Fig. 4. Running time performance comparisons between the VC structures with the same number of SPs and VMs (including two UAVs, each UAV carries a graph job type 2): a). a VC contains 12 SPs/14 VMs/13 edges, where the 4th edge locates between $s_4$ and $s_8$; b). a VC contains 12 SPs/14 VMs/13 edges, where the 4th edge locates between $s_1$ and $s_3$; c). a VC contains 12 SPs/14 VMs/22 edges.

To investigate the factors that influence the running time of our proposed algorithm, we focus on two UAVs with the same signal coverage as service requestors, each carries a graph job (type 2). Based on which, Fig. 4 presents the performance comparisons of running time and templates count between three VCs with the same number of SPs and VMs, but different topologies (e.g., different number of edges). Specifically, each red dotted rectangle indicates a possible candidate of the component with $D^c = 5$ in graph job type 2. Fig. 4(a) and Fig. 4(b) focus on the VCs with the same number of SPs/VMs/edges (12/14/13), and the different topologies reflected by edge 4 and edge 4' (the yellow edges). Apparently, the same amount of resources (SPs/VMs) and V2V connections (edges) can bring various number of possible candidates to components during mapping, which leads to significant differences of both the running time, and the templates count. Theoretically, the more candidates of a component will bring a larger
searching space during mapping, and thus leads to higher running time and more templates even under the same amount of resources (e.g., the same number of SPs and VMs). Comparatively, Fig. 4(c) shows a further complicated VC structure containing 12 SPs, 14 VMs and 22 edges, which enables more candidates of the component with $D_c = 5$ in graph job type 2. Correspondingly, obtaining all the templates of mapping the considered graph jobs to the VC in Fig. 4(c) results in a larger running time. Compared with the proposed algorithm, the running time of ESA in Fig. 4(a), Fig. 4(b) and Fig. 4(c) stay around 7000 seconds owing to that the computation complexity of which relies mainly on the total number of components, SPs and VMs.

5.3 Performance comparisons of the value of $F(x, q)$

Given the templates obtained from the proposed template search algorithm, Fig. 5 presents the performance comparison of the value of the objective function $F(x, q)$ given in (9), between the proposed power allocation algorithm, and several baseline methods listed below.

1) **Uniform allocation (UA)**: for each given template, the transmission power is uniformly allocated to each SP. The algorithm fails when cannot meet constraint (C13).

2) **Random allocation (RA)**: for each given template, the transmission power is randomly allocated to each SP, until find a feasible allocation solution that meets constraint (C13).

3) **Channel condition preferred allocation (CCPA)**: for each given template, a A2G channel with better condition (e.g., larger SNR) is allocated with more transmission power, while meeting constraint (C13).

4) **Structure-preserved simulated annealing (SPSA) [18]**: for each given template, the transmission power is allocated to each SP via simulated annealing algorithm, while meeting constraint (C13).

In various small problem size cases where each considers one UAV and a couple of SPs, Fig. 5(a), Fig. 5(b) and Fig. 5(c) reveal that the performance of the value of $F(x, q)$ greatly outperform the baseline methods UA, RA, CCPA, and SPSA, when applying the proposed power allocation algorithm with different values of $p$. Particularly, the cases where $p = 3$ achieve better performance of the value of $F(x, q)$ than those consider $p = 1$, which commendably prove the theoretical idea of (13) (given in Section 4.2). Namely, a larger $p$ enables the value of a vector’s $p$-norm to approach that of the $\infty$-norm. The values of $F(x, q)$ of RA fluctuate irregularly owing to the randomness factor; while that of CCPA often stay at high values due to the deficiency of balancing various A2G channel conditions, which thus leads to larger data transmission rates and unsatisfactory job completion time. Furthermore, our proposed algorithm
obtains far better performance than SPSA since the process of generating a new state in SPSA only considers discrete values of power, which pose difficulties searching for the whole solution space. The performance comparisons considering large problem size cases with more UAVs and SPs, as well as complicated VC structures are depicted in Fig. 5(d), Fig. 5(e) and Fig. 5(f). Similarly, our proposed algorithm can approach better performance than the baseline methods under both situations when \( p = 1 \) and \( p = 3 \).

5.4 Performance evaluation considering different values of \( p \)

As mentioned in Section 4.2, the larger value of \( p \) will bring a better performance on power allocation. Concretely, since a vector’s \( \infty \)-norm describes the largest value (peak value) in this vector, while a vector’s \( p \)-norm can approach the relevant \( \infty \)-norm upon increasing the value of \( p \). Correspondingly, this section depicts the impact on \( F(x, q) \) when considering various \( p \). Note that the proposed power allocation algorithm works indistinguishably and independently among different templates and UAVs, we focus on single UAV scenarios as examples. Fig. 6 demonstrates that for different graph job types and VC structures, upon increasing the value of \( p \) can always bring a better solution for problem \( \mathcal{P}_2 \) given in (12), and thus achieve a satisfying performance of the objective function \( F(x, q) \).

Fig. 7 shows two examples of the changing process on the value of \( F(x, q) \), upon having various \( p \) and different templates. Fig. 7(a) is associated with graph job type 1 and VC3 of Fig. 6(a), where 48 templates are obtained from applying the proposed template search algorithm. Apparently, under each given template, a larger \( p \) achieves a lower \( F(x, q) \), and the best graph job and power allocation solution can be obtained by comparing through all the templates. Similar conclusion can be found from Fig. 7(b) under 36 templates, which is related to graph job type 3 and VC1 shown in Fig. 6(c).

6 Conclusion

This paper studies the energy-aware graph job allocation in SD-AGV networks. To achieve the trade-off upon minimizing the job completion time and energy consumption, as well as the data exchange cost, the problem is formulated as a MINLP problem which is NP-hard. An efficient decoupled approach is proposed by separating the template search stage from the power allocation stage. In the former stage, an effective algorithm is presented to search for all the subgraph isomorphisms between the graph jobs and the VC structure. For the later stage, we introduce an applicable power allocation optimization algorithm by applying convex optimization techniques. The effectiveness of the proposed approach is revealed through comprehensive simulations. Several future research directions could involve improving the computation efficiency of the template search algorithm, and considering the optimization of the UAV’s path trajectory to achieve better job allocation performance.

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