Switching between superluminal to subluminal velocities and Tunable slow light in a four-level atomic system

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A four-level atomic system interacting with two strong drive fields and a weak probe field has been studied. We demonstrate that the group velocity of probe field can be controlled to switch from subluminal to superluminal or even negative values around the probe resonance without any significant absorption. The basic idea is to create a gain with slow light on the probe beam resonance with one drive field while the other drive creates a dip in the gain profile of probe leading to superluminal and negative group velocities. A simple switching from slow light to negative group velocity light can be obtained by changing the relative intensities of drive fields. We also describe a control to tune the position of slow light frequency continuously simply by adjusting the relative intensities of both the driving fields.
I. INTRODUCTION

The velocity at which a peak of optical pulse travels is known as the group velocity. It is well known that the group velocity depends on susceptibility and its rate change. When spatial dispersion is ignored[10], eq(1) shows the relation between \( v_g \) and \( \chi(\omega) \) for \( \chi(\omega) \ll 1 \)

\[
v_g = \frac{c}{n_g} = Re \left( \frac{c}{1 + \frac{\chi(\omega)}{n_g} + \omega \frac{\partial \chi(\omega)}{\partial \omega}} \right)
\]  

where \( c \) is the velocity of light in vacuum, \( n_g \) is the group refractive index, \( \omega \) is the angular frequency and \( \chi(\omega) \) is the susceptibility of the medium. By modified the group refractive index we can make \( v_g < c \) which is slow light or negative i.e., \( v_g < 0 \) for \( n_g < 0 \) or even faster than light, which is superluminal velocity, for \( |n_g| < 1 \).

All the above mentioned cases can be realized by changing the dispersion of pulse \( (\frac{\partial \chi(\omega)}{\partial \omega}) \) to positive or negative values. Consider an electromagnetic radiation interacting with a two level atom non-resonantly. Then we can never have fast light because denominator is always positive and greater than or equal to one. Now consider a resonant interaction case, at the resonance electromagnetic fields are either absorbed or gained depending on the initial state of the atom. This absorption or gain of the field will be accompanied by a non zero negative or positive dispersion terms. As a result the third term in the denominator of eq(1) makes a significant contribution to group velocity leading to either slow or super luminal or negative group velocities.

In a two-level atom fast light is accompanied by strong absorption making it impossible to study. Meanwhile, we take a multi level atom which is coupled to several laser fields and making use of quantum interference we discuss the generation and control for switching from slow to negative group velocities without any absorption at resonance. One of the most famous and simplest examples for modifying group velocity by using quantum interference is Electromagnetically induced transparency(EIT)[14] where probe has very high dispersion leading to slow light.In the last decade, making use of quantum interference in a multilevel atomic system coupled with several lasers, a considerable work is done to control fast and slow light as both of them has many potential applications. For example fast light can be used in applications like radars[4][18], spectrometers[19], magnetometers[10] etc while fast light can be used in precision measurements in interferometer. Slow light is also known to enhance both rotatory[11] and longitudinal[16] photon dragging effects. However, in most of that work either fast or slow light is accompanied by absorption. There are very few papers which discuss the possibility of generation of both slow and fast light without significant absorption and being able to switch from subluminal to superluminal group velocities.

An experimental demonstration of fast light with gain is presented in [7,8]. A nice description of fast light in terms of classical interference between several different wave is given in [1]. Non-violation of causality with fast light is explained in [2,15]. Generation of light with different group velocities is discussed in[9]. In[9] control over sub luminal and superluminal group velocities with decreased absorption is achieved by controlling phase between the driving fields. Switching between subluminal and superluminal group velocities is discussed [17] in a three level lambda system by using an incoherent pump on the probe field. Decay induced interference properties are used in[5] to generate subluminal and superluminal group velocities. Simultaneous generation of slow and fast light at different frequencies is demonstrated in N-type atomic system in (Dingan han). Until now, most of the work done in controlling fast and slow light has significant absorption in different conditions while very few papers, using incoherent pump or phase differences, demonstrated both sub luminal and superluminal group velocities without any absorption.

In this paper we study a four-level system for controlling group velocity of a probe pulse from subluminal to superluminal velocities. While having the ability to tune slow light continuously and switching between slow to fast light without any absorption are the major results of this paper. It is noteworthy to mention that we can do all the controlling simply by adjusting the relative intensities of the drive fields.

II. MODEL

Figure-1 shows the schematic diagram of our system. The levels a-b have the lasing with out inversion. Levels b-c and a-d are resonantly coupled by two strong laser fields of Rabi frequencies \( \omega_{a} \) and \( \omega_{d} \) respectively. A weak probe field of Rabi frequency \( \Omega_{ab} \) couples levels a and b with detuning given by \( \delta \). Population decay channels between different levels is shown in the diagrams by \( \gamma_{ij} \), where i and j represent the atomic levels(a,b,c,d). The levels a-b are coupled by a weak probe field of frequency \( \omega_{p} \). we assume that all the initial population is in level c i.e., \( \rho_{cc}^{0} = 1 \).

Hamiltonian of the system can be written as linear combination of unperturbed \( (H_{u}) \) and perturbed Hamiltonians \( (H_{i}) \). Hamiltonian \( H = H_{u} + H_{i} \) where

\[
H_{u} = \hbar \omega_{a} |a\rangle \langle a| + \hbar \omega_{b} |b\rangle \langle b| + \hbar \omega_{e} |e\rangle \langle e| + \hbar \omega_{d} |d\rangle \langle d|
\]  

(2)
\[ H_i = \hbar |i\rangle\langle i| \left( \tilde{\Omega}_{ij} e^{i \omega_{ij} t} + \tilde{\Omega}_{ad} e^{i \omega_{ad} t} + \tilde{\Omega}_{ac} e^{i \omega_{ac} t} \right) + \langle i| d \left( \Omega_{a} e^{-i \omega_{a} t} + \Omega_{b} e^{i \omega_{b} t} + \Omega_{c} e^{i \omega_{c} t} \right) + c.c. \]

Where \( \omega_{ab}, \omega_{ac}, \omega_{cd}, \omega_{bd} \) represent the energies of bare atomic states \( a, b, c, d \) respectively. \( \Omega_{ij} = \langle i | - e r_{ij} e^{i k z} | j \rangle \) and \( \tilde{\Omega}_{ij} = \langle i | - e r_{ij} e^{-i k z} | j \rangle \) are the Rabi frequencies between the corresponding levels \( i \) and \( j \). \( \omega_{p}, \omega_{d}, \omega_{q} \) represent the angular frequencies of probe field, drive field 1 and drive field 2 respectively.

Time evolution of the coherence between different levels is given by the density matrix equation

\[ i \hbar \dot{\rho} = [H, \rho] - i \hbar \gamma \rho \]
de-phasing rates and population decay rates as follows

\begin{align}
    i\hbar\dot{\rho}_{ab} &= \rho_{cb}\left(\Omega_{bc}e^{-i\omega t} + \bar{\Omega}_{bc}e^{i\omega t}\right) + \rho_{ba}\left(\bar{\Omega}_{ab}e^{-i\omega t} + \Omega_{ab}e^{i\omega t}\right) - i\eta_{ab}\rho_{ab} - \frac{i}{\hbar}\partial_{t}\rho_{ab} \\
    i\hbar\dot{\rho}_{bc} &= \rho_{ac}\left(\Omega_{ac}e^{-i\omega t} + \bar{\Omega}_{ac}e^{i\omega t}\right) - \rho_{cb}\left(\bar{\Omega}_{bc}e^{-i\omega t} + \Omega_{bc}e^{i\omega t}\right) - i\eta_{bc}\rho_{bc} - \frac{i}{\hbar}\partial_{t}\rho_{bc} \\
    i\hbar\dot{\rho}_{bd} &= \rho_{ad}\left(\Omega_{ad}e^{-i\omega t} + \bar{\Omega}_{ad}e^{i\omega t}\right) - \rho_{db}\left(\bar{\Omega}_{bd}e^{-i\omega t} + \Omega_{bd}e^{i\omega t}\right) - i\eta_{bd}\rho_{bd} - \frac{i}{\hbar}\partial_{t}\rho_{bd}
\end{align}

The dipole de-phasing rates are given by \(\eta_{ij}\). Neglecting the collisional de-phasing rates, we can relate dipole de-phasing rates and population decay rates as follows

\begin{align}
    \eta_{ab} &= \frac{\gamma_{ab} + \gamma_{ac} + \gamma_{ad} + \gamma_{bd} + \gamma_{bc}}{2} \\
    \eta_{da} &= \frac{\gamma_{ab} + \gamma_{ac} + \gamma_{ad}}{2} \\
    \eta_{ac} &= \frac{\gamma_{ab} + \gamma_{ac} + \gamma_{ad} + \gamma_{cd}}{2} \\
    \eta_{db} &= \frac{\gamma_{be} + \gamma_{bd}}{2} \\
    \eta_{be} &= \frac{\gamma_{bc} + \gamma_{bd} + \gamma_{cd}}{2} \\
    \eta_{dc} &= \frac{\gamma_{cd}}{2}
\end{align}

applying rotating wave approximation by taking \(\rho_{ab} = \sigma_{ab}e^{-i\omega t}; \rho_{db} = \sigma_{db}e^{-i(\omega_{a} + \omega_{c})t}; \rho_{ac} = \sigma_{ac}e^{-i(\omega_{a} + \omega_{c})t}; \rho_{dc} = \sigma_{dc}e^{-i(\omega_{a} + \omega_{c})t}; \rho_{da} = \sigma_{da}e^{i\omega t}; \rho_{bc} = \sigma_{bc}e^{i\omega t}\) to density matrix equations and neglecting the terms oscillating at very high frequencies as they average to zero, the density matrix equations for coherences would transform into following equations.

\begin{align}
    i\dot{\sigma}_{ba} &= \Gamma_{ab}\sigma_{ab} + (\rho_{ab} - \rho_{aa})\Omega_{ab} + \sigma_{db}\Omega_{ad} - \sigma_{ac}\bar{\Omega}_{bc} \\
    i\dot{\sigma}_{db} &= \Gamma_{db}\sigma_{db} + \bar{\Omega}_{db}\sigma_{ab} - \sigma_{ab}\Omega_{da} - \sigma_{db}\bar{\Omega}_{bc} \\
    i\dot{\sigma}_{da} &= \Gamma_{da}\sigma_{da} + \bar{\Omega}_{da}(\rho_{aa} - \rho_{dd}) - \bar{\Omega}_{ba}\sigma_{db} \\
    i\dot{\sigma}_{ac} &= \Gamma_{ac}\sigma_{ac} + \Omega_{ac}\sigma_{bc} + \sigma_{ac}\Omega_{ad} - \sigma_{ac}\Omega_{bc} \\
    i\dot{\sigma}_{bc} &= \Gamma_{bc}\sigma_{bc} + \Omega_{bc}(\rho_{cc} - \rho_{bb}) + \bar{\Omega}_{ba}\sigma_{ac} \\
    i\dot{\sigma}_{dc} &= \Gamma_{dc}\sigma_{dc} + \bar{\Omega}_{dc}\sigma_{ac} - \Omega_{bc}\sigma_{db}
\end{align}

where \(\Gamma_{bc} = \delta_{bc} - i\eta_{bc}, \Gamma_{ab} = \delta_{ab} - i\eta_{ab}, \Gamma_{db} = \delta_{db} - i\eta_{db}, \Gamma_{da} = \delta_{da} - i\eta_{da}, \Gamma_{ac} = \delta_{ac} - i\eta_{ac}\). As the time scale of evolution of electromagnetic field is much longer than time scale in which coherences evolve, we can solve the
above equations in steady state to get the coherence between levels a and b. As the probe field is weak, we can use perturbation theory to solve only up to first order of probe field coherence while keeping the strong driving fields to all orders.

\[
\sigma_{dc}^1 = \frac{\Omega_{dc}\sigma_{ac}^1 - \Omega_{bc}\sigma_{db}^1}{-\Gamma_{dc}} \tag{27}
\]

\[
\sigma_{ac}^1 = \frac{\Omega_{ac}\sigma_{bc}^0 + \Omega_{ad}\sigma_{dc}^1 - \Omega_{bc}\sigma_{ac}^1}{-\Gamma_{ac}} \tag{28}
\]

\[
\sigma_{db}^1 = \frac{\Omega_{db}\sigma_{dc}^1 - \Omega_{ad}\sigma_{da}^0 - \Omega_{bc}\sigma_{db}^1}{-\Gamma_{db}} \tag{29}
\]

\[
\sigma_{da}^0 = \frac{\Omega_{da}(\rho_{aa}^0 - \rho_{dd}^0)}{-\Gamma_{da}} \tag{30}
\]

\[
\sigma_{bc}^c = \frac{\Omega_{bc}(\rho_{bb}^0 - \rho_{cc}^0)}{-\Gamma_{bc}} \tag{31}
\]

solving the above equations gives us the coherence between levels a and b. which is related to polarization and susceptibility in the following way

\[ P_{ij} = \omega_{ij}\rho_{ab} = \epsilon_o\chi E_e^{(kz\omega)} \Rightarrow \chi_{ab} = \frac{N\omega^2\sigma_{ab}^1}{\hbar\epsilon_o\Omega_{ab}} \tag{32} \]

where \( N \) is the density of atoms, \( \epsilon_o \) is the electric permeability of the medium, \( \omega \) is the dipole moment. Susceptibility of this system is given the equation below. Before we observing how the absorption and gain profiles look we derive the sufficient condition for getting population non-inversion and then a necessary condition for observing gain. Combining both those conditions we get the LWI which is shown at the end of the paper.

\[
\chi_{ab} = \frac{-N\omega^2}{\hbar\epsilon_o} \left[ n_{ba} - \frac{\Omega_{bc}^2}{(\Gamma_{ac}\Gamma_{bc})} + n_{ad}(\Omega_{ad}^2/(\Gamma_{db}\Gamma_{ab})) + \frac{[\Omega_{bc}^2\Omega_{ad}^2(n_{ba}/(\Gamma_{ac}\Gamma_{bc})+n_{ad}/(\Gamma_{db}\Gamma_{ab}))(1/\Gamma_{db} + 1/\Gamma_{bc})]}{-\Gamma_{db} - (\Gamma_{db} + 1/\Gamma_{bc})^2}(\Omega_{bc}^2/\Gamma_{ac} + |\Omega_{ad}|^2/\Gamma_{db})} \right] \] \[
\chi_{ab} = \frac{-N\omega^2}{\hbar\epsilon_o} \left( \frac{f}{g} \right) \tag{33} \]

\[ f = (-\Gamma_{dc}\Gamma_{db}\Gamma_{ac} + \Gamma_{ac}|\Omega_{bc}|^2 + \Gamma_{db}|\Omega_{ad}|^2)(n_{ba}\Gamma_{ac}\Gamma_{bc}\Gamma_{db}\Gamma_{da} + n_{ad}\Gamma_{db}\Gamma_{ab} - |\Omega_{bc}|^2 + |\Omega_{ad}|^2)\Gamma_{db}\Gamma_{da}\Gamma_{ac} + n_{ad}(\Gamma_{dc}\Gamma_{ac} - |\Omega_{ad}|^2 + |\Omega_{bc}|^2)\Omega_{ad}^2\Gamma_{ac}\Gamma_{bc}\Gamma_{db} \] \[
g = \{(\Gamma_{dc}\Gamma_{db}\Gamma_{ac} + \Gamma_{ac}|\Omega_{bc}|^2 + \Gamma_{db}|\Omega_{ad}|^2)(n_{ab}\Gamma_{db}\Gamma_{ac} + \Gamma_{db}|\Omega_{ad}|^2\Omega_{ac}|\Omega_{ad}|^2 - |\Omega_{bc}|^2)|\Omega_{ad}|^2(\Gamma_{ac} + \Gamma_{db})^2\} \Gamma_{db}\Gamma_{da} \tag{35} \]

III. CONTROLLING AND TUNING GROUP VELOCITY OF PROBE

As pointed in [4], We can get slow or fast light by creating a dip in absorption and gain profiles respectively. This is idea which is being in this work for creation of super-luminal light and slow without any absorption. The drive1 helps in creating a gain on the probe resulting in slow light, at the same time the drive 2 can be used to induce absorption on the probe beam by optically pumping atoms from level c to level b. This absorption of probe photon induced through drive2 creates a dip in probes gain profile which would lead to superluminal or negative group velocity light. When the drive2 is switched off then a normal gain accompanied with slow light is observed on the probe beam. As the absorption of probe photons can be controlled by the strength of drive2 we can control the dip formation in probes gain profile by controlling the intensities of driving fields and hence we can control the switching between subliminal group velocity to super luminal group velocities. Note that we have fast light with absorption but that is not useful
as the field will be simply absorbed by the medium. Hence we restrict our analysis and results only to the behavior of real part of susceptibility when there is no significant absorption.

We start with analysis of a simple condition where drive2 is zero, only drive1 and probe fields are present. In the steady state, without drive2, most of the population remains in levels a and d. Level's b and c gets populated according to the decay rate from level a. When the probe photon starts interacting with the medium in steady state there are two things that can happen one being the stimulated emission of probe photons by knocking out electrons from level a and the other one being absorption of probe photons from level b. As the population in level b tend to decay either to level d or c, the chances of a probe photon getting absorbed from level b is far less when compared with the chances of stimulated emission of probe from level a to b. As result we get gain on the probe beam which is accompanied by steep dispersion(or slow light) profile of probe. The fig below shows the absorption and dispersion profiles of probe beam when drive2 is zero. Now we focus on the case where we can create a dip in the gain profile of probe which would result in negative group velocity light. One simple way to is to increase the strength of drive1 to very high value so we can create Autler-Townes splitting in level a. As a result we get gain profile from the two split levels of level a. However this requires a very strong drive1. Fig(3) shows this effect when drive2 is zero. Real part of susceptibility shows positive slope or negative group velocity. Instead of using a very strong drive1 to create a dip in the gain profile, we can use drive2 to do the same work but with less intensity requirement. As the intensity of the drive2 can be easily controlled, we have the ability control dispersion of probe light as well as the amount of gain or absorption the probe field goes through. Fig(4) shows the scenario when both the drive fields are used. It is clear that absorption at the probe resonance is completely nullified and the probe dispersion has a positive slope(resulting in negative group velocity). As the dip formed in the gain profile depends on the strength of drive2, It is possible to obtain negative group velocity with gain. Before proceeding further it is necessary to understand the role drive2 in influencing the gain profile of probe field. Without drive(2),as shown in FIG(2), we simply have gain on probe field and once we start increasing the strength of drive2 intensity more and more atoms that are de-excited from level a
FIG. 4. $\gamma_{ab} = \gamma_{ac} = \gamma_{ad} = \gamma_{bd} = \gamma_{bc} = \gamma_{dc} = 1 \gamma, \Omega_{ad} = \gamma$ and $\Omega_{bc} = 1.35 \gamma$. Negative group velocity light with zero absorption to c will be pumped back to level b and hence the probability of absorption of probe around the resonance increases. 
This increased chance of absorption of probe field will result in decrease of gain (creating a dip in absorption profile and negative light) at the resonance and further increasing drive2 would completely cancel the gain resulting in Fig(4) and eventually leading to absorption instead of gain on probe resonance.

FIG. 5. Gradual change of probes group velocity from fast to slow with disappearance of dip in probes gain profile. $\gamma_{ab} = \gamma_{ac} = \gamma_{ad} = \gamma_{bd} = \gamma_{bc} = \gamma_{dc} = 1 \gamma, \Omega_{ad} = \gamma$ and $\Omega_{bc} = 1.35 \gamma$

Figure 6 shows the gradual disappearance of dip in probe fields gain profile when $\Omega_{bc}$ is relatively getting weaker and a gradual change of group velocity of light from negative to positive. It should be noted that probes absorption if zero or negative all the time. Another interesting possibility of this system is having ability to tune the generation of slow light continuously over a broad range frequencies. Such a possibility is demonstrated in solids by (Yoshitomo) using Brillouin scattering. In this case tuning of slow light is provided by changing wavelength one of the lasers involved while in this work slow light tuning can be done changing strength of intensities of drive fields. This can be done by increasing the strength of both the drive fields. By increasing the strength of driving fields we can continuously move the position of Autler-Townes components and as a result we can get the gain, which is accompanied by slow light, at the wings of probe field. The fig illustrates such possibility.

The fig below shows the change of absorption as a function of drive2 strength. It is clear that having a weak drive2 would result in slow light and a relatively stronger light would result in fast light. Now we go on to analyzing the requirement condition for changing from fast to slow light. 
The complete condition for getting fast light with zero absorption if given in the appendix. Here we will analyze a special case in where $\Omega_{ad} = \Omega_{ab} \gg \gamma$. As we are interested in dispersion at the probe resonance only, we can set the probe detuning to zero in eq(33). Under this condition denominator of the susceptibility is always positive and non-zero (details are shown in the appendix). We will get zero absorption when numerator is zero and we get gain when numerator is less than zero. We can write...
FIG. 6. Slow light with gain with drive1’s intensity more than drive2. \( \gamma_{ab} = \gamma_{ac} = \gamma_{ad} = \gamma_{bd} = \gamma_{bc} = \gamma_{dc} = 1 \gamma, \Omega_{ad} = 1.1 \gamma \) and \( \Omega_{bc} = \gamma \)

FIG. 7. Tunable slow light. \( \gamma_{ab} = \gamma_{ac} = \gamma_{ad} = \gamma_{bd} = \gamma_{bc} = \gamma_{dc} = 1 \gamma, \Omega_{ad} = 5 \gamma \) and \( \Omega_{bc} = 5 \gamma \)

The numerator of the transmission coefficient is given by

\[
i(|\Omega_{ab}|^2 - |\Omega_{bc}|^2)(\rho^0_{cc} - \rho^0_{ba})|\Omega_{bc}|^2 \eta_{ba} + (\rho^0_{dd} - \rho^0_{aa})|\Omega_{ad}|^2 \eta_{bc}
\]

(37)

so when \( \Omega_{ad} = \Omega_{ab} \gg \eta_{ij} \) we always have a zero or very small absorption on the probe resonance, which means we have fast light, when both the drive fields are equal. As we have already discussed changing the intensity of drive2 with respect to the drive1 would give us gain or absorption so, having a low drive2 intensity would guarantee gain on probe beam along with slow light.

IV. CONCLUSION

We discussed a four-level atomic system, which is a combination of ladder and lambda system, coupled with two driving fields and a weak probe fields for varying the group velocity of the probe pulse. It is shown that probe pulses group velocity can be changed from subluminal to super luminal and negative values continuously by changing the intensities of driving fields. We also described the role of driving fields in controlling absorption of probe pulse. Another important possibility is to able to tune the frequency of slow light continuously. This can be done simply increasing the strength of relative intensities of both the driving fields.
Appendix A: zero order solutions

The zero order steady state population equations are given by

\[ \sigma_{da} \Omega_{ad} - \sigma_{ad} \tilde{\Omega}_{da} - i(\gamma_{ab} + \gamma_{ac} + \gamma_{ad})\sigma_{aa} = 0 \quad (A1) \]

\[ \sigma_{cb} \Omega_{bc} - \sigma_{bc} \tilde{\Omega}_{cb} + i\gamma_{ab}\sigma_{aa} - i\gamma_{bd}\sigma_{bb} - i\gamma_{bc}\sigma_{bb} = 0 \quad (A2) \]

\[ \sigma_{bc} \tilde{\Omega}_{cb} - \sigma_{cb} \tilde{\Omega}_{bc} + i\gamma_{ac}\sigma_{aa} - i\gamma_{cd}\sigma_{cc} + i\gamma_{bc}\sigma_{bb} = 0 \quad (A3) \]

\[ \sigma_{ad} \tilde{\Omega}_{da} - \sigma_{da} \tilde{\Omega}_{ad} + i\gamma_{ad}\sigma_{aa} + i\gamma_{cd}\sigma_{cc} + i\gamma_{bd}\sigma_{bb} = 0 \quad (A4) \]

Using eq(29) and eq(30) we can write the following relations

\[ \sigma_{da} \Omega_{ad} - \sigma_{ad} \tilde{\Omega}_{da} = -|\Omega_{ad}|^2(1/\Gamma_{da} - 1/\Gamma^*_{da}) \quad (A5) \]

\[ \sigma_{bc} \Omega_{bc} - \sigma_{bc} \tilde{\Omega}_{cb} = -|\Omega_{bc}|^2(1/\Gamma_{bc} - 1/\Gamma^*_{bc}) \quad (A6) \]

Substituting (A5) in (A1) directly gives the relation between level a and level d zero order populations as follows

\[ \sigma_{aa} = \frac{\sigma_{dd}|\Omega_{ad}|^2(\Gamma_{bd} - \Gamma_{d})}{|\Omega_{ad}|^2(\Gamma_{da} - \Gamma_{d}) + |\Omega_{bd}|^2(i\gamma_{ab} + i\gamma_{ac} + i\gamma_{ad})} \quad (A7) \]

From (A7) we can write \( \sigma_{dd} = \Delta \sigma_{aa} \) where \( \Delta \) is the inverse of coefficient of \( \sigma_{dd} \). Using this and eq(A6) we can write (A3) and (A4) as follows

\[ \sigma_{cc} = -\frac{\gamma_{ac}k_4 + \gamma_{ab}k_2}{k_1k_4 - k_3k_2} \quad \sigma_{bb} = \frac{-\gamma_{ac}k_3 + \gamma_{ab}k_1}{k_1k_4 - k_3k_2} \sigma_{aa} \quad (A10) \]

we also know that probability of finding an atom in one of the four levels a,b,c,d is one. so using (A7) and (A10) with \( \sigma_{aa} + \sigma_{bb} + \sigma_{cc} + \sigma_{dd} = 1 \) gives the population evolution in steady state. The figure below shows the evolution of population in different levels as a function of drive 2

![FIG. 8. Continuous change of probes group velocity when there is no absorption. \( \gamma_{ab} = \gamma_{ac} = \gamma_{ad} = \gamma_{bd} = \gamma_{bc} = \gamma, \gamma_{cd} = 1, \Omega_{ad} = \gamma \) and \( \Omega_{bc} = \gamma \).](image-url)
FIG. 9. tunable slow light. $\gamma_{ab} = \gamma_{ac} = \gamma_{bd} = \gamma_{be} = \gamma_{cd} = \gamma_b \Omega_{ad} = 5\gamma$ and $\delta = 0$

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