On the non-Abelian extension of the aether term

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Abstract

In this paper we prove that an aether term can be generated from radiative corrections of the (non-minimal) coupling between the gauge and the matter fields, in a Lorentz-breaking extended Yang-Mills theory. Furthermore, we show that the path integral quantization in the Landau gauge is still consistent according to the Gribov-Zwanziger framework.

1 Introduction

The possibility of a Lorentz symmetry breaking is being recently discussed within many contexts. It was first proposed in the 90s in the context of QED by Carroll, Field and Jackiw (CFJ) \cite{1}. The authors for the first time proposed a consistent Lorentz-breaking extension of a known field theory model involving a constant axial vector $b_\mu$, which introduces a privileged space-time direction. Soon after, various Lorentz-breaking extensions of the standard model have been proposed \cite{2}, and many nontrivial issues related with these theories have been discussed. Among them, we can emphasize an unusual wave propagation including birefringence and rotation of polarization plane of electromagnetic field in a vacuum (cf. \cite{3}) which has been shown to take place in various Lorentz-breaking extensions of QED (cf. \cite{4, 5}), ambiguity of quantum corrections (cf. \cite{6}) and perturbative generating new Lorentz-breaking terms (cf. \cite{1}). Many possible impacts of Lorentz symmetry breaking have been measured experimentally in different cases \cite{7}. The renormalizability of Lorentz violation in QED was discussed in \cite{8}.

As mentioned before the Lorentz symmetry breaking was treated in the context of the QED and naturally one can ask about the possibility of a non-Abelian extension of the Lorentz-breaking terms. The non-Abelian CFJ term has been generated perturbatively in \cite{9}, and some consequences of including this term were discussed in \cite{10, 11, 12}. The first example of such a theory studied in a systematic manner is the four-dimensional Lorentz-breaking Yang-Mills (YM) theory, originally formulated in \cite{13} whose Lagrangian is the sum of the standard Yang-Mills Lagrangian and the non-Abelian generalization of the Carroll-Field-Jackiw (CFJ) term, which essentially represents itself as a four-dimensional extension of the well-known non-Abelian Chern-Simons term. In that paper the authors explored the one-loop renormalization of YM-CFJ system. The renormalizability of non-Abelian systems involving Lorentz

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symmetry breaking was explored [10, 11, 12]. The study of the path integral quantization of the YM+CFJ system was recently developed by the authors in [14].

Naturally, one can ask about the possibility of a non-Abelian generalization of others Lorentz-breaking terms. The natural candidate for such a generalization is the aether term, which unlike the CFJ term it does not break the CPT symmetry. Classical aspects of this term, together with its possible implications within the extra dimensions concept, were intensively discussed in [15]. The Abelian perturbative generation of the aether term has been carried out for the first time in [16], and in [17] the aether term was shown to be strongly ambiguous. Up to now the non-Abelian generalization of the aether term has never been studied. It is of course important to investigate the existence of an effective model that would takes into account such a term.

After proving the existence of an aether-like radiative correction in Yang-Mills theories with Lorentz-breaking terms, it is important to study the effects of such a term in a path integral quantization procedure, within the Landau gauge. The Landau gauge is a very interesting special case since it makes possible to properly understand the Gribov problem, [18]. Recently, the Gribov issue has been consistently treated within the wider class of Linear Covariant Gauges, [19]. The Gribov ambiguities will be treated within the modern approach developed by D. Zwanziger, called the Gribov-Zwanziger (GZ) prescription, and a sufficiently detailed introduction to this framework is provided in section 3, for those not familiar with Zwanziger’s approach.

The structure of the paper looks like follows. In the section 2, for the first time, the perturbative generation of the non-Abelian aether term will be obtained. In the section 3, we review the Gribov-Zwanziger approach to the Gribov problem within the Landau gauge. In the section 4, we carry out the path integral quantization of the Yang-Mills-aether system in the Landau gauge and deal with the Gribov copies. Finally, in the section 5 we present a summary where the results and perspectives will be discussed.

2 The generation of the non-Abelian aether term

Now, let us start with the discussion of the non-Abelian generalization of the aether term. Originally the Abelian aether term within the Lorentz-breaking extended QED has been proposed in [15] in the five-dimensional space-time, having the form

\[ \mathcal{L}_{aether} = u^\mu u_\nu F_{\mu\lambda} F^{\nu\lambda}, \]  

(1)

where \( u_\mu \) is a some constant vector (in [15] it was assumed to be directed along the extra dimension; nevertheless, the aether term can be naturally generalized to any space-time dimension), and \( F_{\mu\nu} \) is the usual stress tensor of the electromagnetic field. Actually, this term, being CPT-even, represents itself as a particular form of the general CPT-even term

\[ \mathcal{L}_{even} = \kappa^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}, \]  

(2)

proposed in [2] as an ingredient of the Lorentz-breaking extended standard model, in the case of a special expression of the constant tensor \( \kappa^{\mu\nu\lambda\rho} \) (cf. [20], for some classical issues related to this CPT-even term, including its impacts for the plane wave solutions of equations of motion).

In [16] the generation of the Abelian aether term was proposed. This term was shown to arise as a one-loop quantum correction in a theory that involves a magnetic coupling of the fermion to an electromagnetic field. The generalization of the scheme used in [16] for the non-Abelian case is straightforward. In order
to proceed with the non-Abelian generalization, we start with the following classical action,

\[ S = \int d^4x \sum_{i,j=1}^{N} \bar{\psi}_i(i\delta_{ij}\partial - g'\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}^a_{ij}a_\lambda\gamma_\rho(T^a_{ij}) - m\delta_{ij})\psi_j. \]  

(3)

Here \( F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + igf^{abc}A_\mu^bA_\nu^c \) is the usual non-Abelian stress tensor; \( g \) is the gluon self-coupling constant; \( g' \) is the coupling constant responsible for the non-minimal (magnetic) coupling between the gauge and matter fields. The axial vector \( a_\lambda \) is responsible for the implementation of the Lorentz symmetry breaking\(^1\). In order to generate the non-Abelian aether term we need to make some remarks. First, the non-minimal (magnetic) coupling is sufficient to generate the aether effective action (cf. \[16\] for the Abelian case). Second, the generation of the triple and quartic terms in the non-Abelian case is a non-trivial problem which we left for a future work. Here we restrict ourselves to demonstrate the possibility of obtaining the effective non-Abelian aether action. In our case it is much simpler, since we need only two coupling vertices in order to deal with it (to the best of our knowledge, the perturbative generation of the effective non-Abelian aether action has never been performed).

So, to generate the non-Abelian aether term, we should expand the functional trace

\[ \Gamma^{(1)} = i\text{Tr} \ln(i\delta_{ij}\partial - g'\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}^a_{ij}a_\lambda\gamma_\rho(T^a_{ij}) - m\delta_{ij}) \]  

(4)

up to the second order in \( F_{\mu\nu}^a \). The result, being the analogue of the expression from \[16\], is

\[ S_2(p) = \frac{g'^2}{2}\text{tr}(T^aT^b)\epsilon^{\mu\nu\lambda\rho}\epsilon^{\mu'\nu'\lambda'\rho'}a_\mu F_{\nu\lambda}^a(p)a_{\mu'}F_{\nu'\lambda'}^b(-p) \int \frac{d^4k}{(2\pi)^4}\text{Tr}[\gamma_\rho S(k)\gamma_\rho' S(k + p)]. \]  

(5)

The explicit form of this expression, at \( p = 0 \), for \( \text{tr}(T^aT^b) = \delta^{ab} \), is

\[ S_2(p) = -\frac{g'^2}{2}\epsilon^{\mu\nu\lambda\rho}\epsilon^{\mu'\nu'\lambda'\rho'}a_\mu F_{\nu\lambda}^a(p)a_{\mu'}F_{\nu'\lambda'}^b(-p) \int \frac{d^4k}{(2\pi)^4}\frac{1}{[k^2 - m^2]^2}\text{Tr}[m^2\eta_{\rho\rho'} + k^\alpha k^\beta(\eta_{\alpha\rho}\eta_{\beta\rho'} - \eta_{\alpha\beta}\eta_{\rho\rho'})]. \]  

(6)

Proceeding with the calculation of the trace over the Dirac matrices, we arrive at

\[ S_2(p) = -2g'^2\epsilon^{\mu\nu\lambda\rho}\epsilon^{\mu'\nu'\lambda'\rho'}a_\mu F_{\nu\lambda}^a(p)a_{\mu'}F_{\nu'\lambda'}^b(-p) \int \frac{d^4k}{(2\pi)^4}\frac{1}{[k^2 - m^2]^2}\left[m^2\eta_{\rho\rho'} + k^\alpha k^\beta(\eta_{\alpha\rho}\eta_{\beta\rho'} - \eta_{\alpha\beta}\eta_{\rho\rho'})\right]. \]  

(7)

This expression is known to be finite and ambiguous since the integral

\[ I_{\rho\rho'} = \int \frac{d^4k}{(2\pi)^4}\frac{1}{[k^2 - m^2]^2}\left[m^2\eta_{\rho\rho'} + k^\alpha k^\beta(\eta_{\alpha\rho}\eta_{\beta\rho'} - \eta_{\alpha\beta}\eta_{\rho\rho'})\right] \]  

(8)

depends on the manner of calculation (see the discussion in \[17\]). Finally, one has

\[ S_2 = Cm^2g'^2\epsilon^{\mu\nu\lambda\rho}a_\lambda F_{\mu\nu}^a, \]  

(9)

where \( C \) is a constant equal to \( \frac{1}{4\pi^2} \) or zero. Other values of the constant \( C \) can be obtained within different calculation schemes as well, see the discussion in \[21\].

\(^1\)In the paper \[16\], this vector has been denoted as \( b_\lambda \); here we denote it as \( a_\lambda \) in order to match notations used in our previous paper \[14\].
## 3 The Gribov-Zwanziger quantization procedure in the Landau gauge

In [18] V. N. Gribov showed that the Coulomb gauge is plagued by a gauge fixing ambiguity, as well as the Landau gauge. Just after Gribov’s paper, I. M. Singer claimed that such gauge fixing ambiguity should be present for any gauge condition if some compactness requirement on the space-time is imposed, [22]: it is not possible to continuously select one gauge field configuration at each gauge orbit.

Originally, Gribov proposed a mechanism to get rid of such ambiguities (known as the Gribov copies), [18]. His proposal is to restrict the functional integration of the gauge field to the region where the Faddeev-Popov (FP) operator is free of zero-modes, the so-called first Gribov region. This restriction amounts to consider only gauge field configurations corresponding to a positive definite FP operator’s eigenvalue. Since the FP operator is closely related to the ghost-anti-ghost two point function, Gribov proposed to investigate the influence of the gauge field on such function by computing it up to one-loop.

In this section we provide a consistent introduction to the framework by Zwanziger, intended to provide an all order approach to get rid of the Gribov copies. Just as in the original proposal by Gribov, in Zwanziger’s approach there is also a self-consistency condition, known as the gap equation, that must be satisfied. It means that, according to Gribov-Zwanziger (GZ) the gap equation must be satisfied, in order to consistently perform the path integral quantization of a non-Abelian gauge field theory, in the Landau gauge (but not only).

### 3.1 The Gribov-Zwanziger framework

In 1982, D. Zwanziger generalized Gribov’s proposal to all orders [23]. Zwanziger realized that Gribov’s restriction of the gauge field configuration space to the region where the FP operator is positive definite boils down to consider only gauge field configurations corresponding to the lowest (non-trivial) eigenvalue of the FP operator. Namely, it means

\[
Z = \int \mathcal{D}\phi \, \delta(\lambda_{\text{min}}[A]) \, e^{-\left(S_{YM} + S_{gf}\right)},
\]

where \(\lambda_{\text{min}}[A]\) accounts for the trace over the matrix of all the lowest lying eigenvalue of the FP operator and \(\phi\) accounts for all quantum fields involved.

Zwanziger’s idea is to impose the “positive definite” condition on the summation of all lowest lying eigenvalues of the Faddeev-Popov operator, by considering a perturbative technique. Namely, the Faddeev-Popov operator is written as

\[
\mathcal{M}^{ab} = \mathcal{M}_0^{ab} + \mathcal{M}_1^{ab} = -\delta^{ab} \partial^2 + g f^{abc} A^c_\mu \partial_\mu .
\]

Then, after identifying all the lowest lying eigenvalues\(^2\) associated to the “non-perturbed” operator \(\mathcal{M}_0^{ab}\), the eigenvalue equation for the full (or “perturbed”) operator is written in a matrix notation such as

\[
\mathcal{M} S = S \Lambda_{\text{min}}
\]

with \(\mathcal{M}\) standing for the full FP operator; \(S\) is the matrix composed by the eigenstates of \(\mathcal{M}\), lying on the columns, related to the lowest lying eigenvalues; \(\Lambda_{\text{min}}\) stands for the diagonal matrix of the lowest lying eigenvalues of \(\mathcal{M}\).

\(^2\)Remember that the trivial lowest eigenvalue is not being accounted.
The eigenstate matrix $S$ is treated perturbatively, as well as is $\Lambda_{\text{min}}$, with respect to the coupling constant. Namely,

$$S = \sum_{n=0}^{\infty} S_n \quad \text{and} \quad \Lambda_{\text{min}} = \sum_{n=0}^{\infty} \Lambda_n ,$$

with $S_0$ standing for the eigenstate related to the lowest lying eigenvalue, $\Lambda_0$, of the “unperturbed FP operator” $\mathcal{M}_0$. Therefore, the zero order term reads

$$\mathcal{M}_0 S_0 = S_0 \Lambda_0 .$$

Making use of orthogonality condition applied to each subspace, $\mathcal{H}_n$, generated by the eigenstates of $S_n$, with respect to the zero order subspace $\mathcal{H}_0$, Zwanziger could solve the eigenvalue equation at each order.

Taking the infinite volume limit some simplifications take place so that a general expression for the lowest lying eigenvalues can be derived. Zwanziger then substitutes the stronger condition that “the sum of every lowest lying eigenvalue shall be positive” to the weaker condition that “the trace of the sum must be positive”. Therefore, with the general expression for the eigenvalues, one can derive the trace, obtaining

$$\text{Tr} \Lambda = 2 \left( \frac{2\pi}{L} \right)^2 \left( d(N^2 - 1) - \frac{1}{V} \int d^4x d^4y \left[ M^{-1} \right]^{cl} A^d_\mu(y) \delta(x-y) \right) > 0 .$$

Therefore, the condition should be implemented in the partition function as

$$Z_{GZ} = \int D\phi \theta(dV(N^2 - 1) - H(A)) e^{-\left(S_{YM} + S_{fr}\right)} ,$$

with

$$H(A) = \frac{1}{V} \int d^4x d^4y g^2 f^{abc} f^{adl} A^b_\mu(x) \left[ M^{-1} \right]^{cl} A^d_\mu(y) \delta(x-y)$$

being called the horizon function.

The partition function (15) represents an uniform ensemble, where only gauge field configurations satisfying $H(A) \leq dV(N^2 - 1)$ are accounted. That is, it assigns non-zero probability for physical configurations whose energy lies within an specific range; for physical configurations whose energy lies out of this range, it assigns zero probability.

Making use of a geometric result, that the volume defined by an hypersurface, such as $H(A) = dV(N^2 - 1)$, becomes concentrated in the hypersurface as long as the volume increases. Such a fact can be realized through simple examples, as the 3-sphere infinitesimal volume: $dxdydz = 4\pi r^2 dr$. So, the bigger the radius of the sphere is, the greater is the contribution of the surface $4\pi r^2$ to its volume. Therefore, it is not difficult to see that, in the thermodynamic limit the uniform ensemble becomes a microcanonical ensemble. The partition function of a microcanonical ensemble reads

$$Z_{GZ} = \int D\phi \delta(dV(N^2 - 1) - H(A)) e^{-\left(S_{YM} + S_{fr}\right)} .$$

That is, only gauge field configurations satisfying the condition $H(A) = dV(N^2 - 1)$ is assigned with non-zero probability. Such condition is named horizon condition.

The integral representation of a $\delta$-function leads us to

$$Z_{GZ} = \int_{-\infty + i\epsilon}^{\infty + i\epsilon} \frac{d\beta}{2\pi i} e^{-f(\beta)} ,$$

5
with \( f(\beta) = -\ln W(\beta) \) and

\[
W(\beta) = \int D\phi \ e^{-\ln \beta} e^{-[S_{YM} + S_{gf} + \beta(H(A) - dV(N^2 - 1))]}. \tag{19}
\]

Let us make use of the saddle point approximation in order to compute the integral (18). The necessary condition for use of the saddle point approximation method is that

\[
\left. \frac{df(\beta)}{d\beta} \right|_{\beta = \beta_*} = 0, \tag{20}
\]

so that, once this condition is satisfied, the approximation becomes exact in the infinite volume limit. Namely,

\[
Z_{GZ} = e^{-f(\beta_*)}. \tag{21}
\]

The saddle point necessary condition (20) is called the gap equation.

Finally, the partition function becomes

\[
Z_{GZ} = e^{-f(\beta_*)}, \tag{22}
\]

describing a canonical ensemble, or Boltzmann ensemble, in the thermodynamic limit.

From now on in this paper, the Gribov parameter \( \beta \) will be replaced by \( \gamma^4 \), for simplicity and to keep track of the mass dimension of the Gribov parameter.

### 3.2 The gap equation

As a consequence of the Gribov restriction, a non-local mass term of the gauge field is introduced into the action, accounting for non-perturbative effects. Therefore, in order to proceed any perturbative computation a localized version of the GZ action would be needed, e.g. the gauge field propagator and the gap equation.

In order to obtain a localized version of the action a couple of auxiliary fields, called Gribov’s ghosts, will be consistently introduced, in the sense that they are BRST-doublets. Namely,

\[
s\varphi^{ab}_\mu = \omega^{ab}_\mu, \quad s\omega^{ab}_\mu = 0 \tag{23}
\]

\[
s\bar{\omega}^{ab}_\mu = \bar{\varphi}^{ab}_\mu, \quad s\bar{\varphi}^{ab}_\mu = 0. \tag{24}
\]

The localized Gribov-Zwanziger action reads,

\[
S_{GZ} = S_{YM} + S_{gf} + S_0 + S_\gamma, \tag{25}
\]

with

\[
S_0 = \int d^4x \left( \varphi^{ac}_\mu (\partial_\nu D^{ab}_\nu)\varphi^{bc}_\mu - \omega^{ac}_\mu (\partial_\nu D^{ab}_\nu)\omega^{bc}_\mu - gf^{amb}(\partial_\nu \omega^{ac}_\mu)(D^mp_c)p_c^{bc}_\mu \right), \tag{26}
\]

and

\[
S_\gamma = \gamma^2 \int d^4x \left( gf^{abc}A^a_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) \right) - 4\gamma^4V(N^2 - 1). \tag{27}
\]

Within perturbation theory, the gap equation (i.e. equation (20)) and the gluon two point function can be explicitly computed at tree-level. To that end, it boils down to consider only quadratic terms in the
quantum fields, as well as constant terms, of the local action (25). Performing a Fourier transformation one ends up with

$$Z_{GZ}^{\text{quad}} = \int [dA] \left[ \det -\partial^2 \right] \exp \left\{ -\frac{1}{2} \int \frac{d^dq}{(2\pi)^d} A_\mu^a(q) K_{\mu\nu}^{ab} A_\nu^b(-q) - 4V\gamma^4(N^2 - 1) \right\}. \quad (28)$$

with

$$K_{\mu\nu}^{ab} = \delta^{ab} \left[ \left( q^2 + \frac{2Ng^2\gamma^4}{q^2} \right) \delta_{\mu\nu} + \left( \frac{1}{\alpha} - 1 \right) q_\mu q_\nu \right] \quad (29)$$

Note that in equation (28) the FP ghosts are integrated out, as well as the Gribov ghosts; the parameter $\alpha$ stands for the FP gauge fixing parameter, so that if $\alpha \to 0$ the Landau gauge is recovered. The Gribov parameter $\gamma^4$ represents the specific value that solves the gap equation (20), so that in the thermodynamic limit the saddle point approximation becomes exact, leading us to

$$Z_{GZ}^{\text{quad}} = e^{-f(\gamma^4)}. \quad (30)$$

After some algebraic manipulations, one is able to derive the following expression for $f(\gamma)$,

$$f(\gamma) = 4\gamma^4V(N^2 - 1) - \ln \gamma^4 - \frac{3V(N^2 - 1)}{4} \int \frac{d^4p}{(2\pi)^4} \ln \left( p^2 + \frac{2\gamma^4Ng^2}{p^2} \right). \quad (31)$$

From the saddle point method condition

$$\left. \frac{df(\gamma)}{d\gamma^2} \right|_{\gamma^2 = \gamma^2} = 0,$$

and in the thermodynamic limit, one ends up to the explicit expression for the gap equation,

$$1 = \frac{3Ng^2}{8} \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^4 + 2\gamma^4Ng^2}. \quad (32)$$

Therefore, it must be clear that the gap equation (32) has to be solved, so that the Yang-Mills theory makes sense. Such condition comes from the saddle point necessary condition, (20), used in the thermodynamic limit to implement the Gribov restriction.

Since our effective Lorentz-broken YM theory, accounting for the CPT-even aether term, still displays gauge freedom, it is interesting and important study the effects of such an aether term in the gap equation in the Landau gauge.

4 Yang-Mills non-Abelian aether Lorentz violating term within the Gribov restriction

Since, as we have just seen, a CPT-even coupling term (i.e. the aether-like term) arises from radiative corrections to the 1PI-two-point function of the gauge field, by considering Lorentz violation and CPT-odd coupling terms between the gauge and the fermionic fields in Yang-Mills theories, it seems reasonable to investigate the influence of such an aether-like term in the gauge field propagator (at tree level). For
this, we will consider an effective model, where the aether-like term is present in a Lorentz-broken Yang-Mills theory within the Landau gauge. The non-Abelian Lorentz-breaking action can be described with the YM term and the aether term. Namely,

$$S^{\text{Mink}} = \int d^4x \left( -\frac{1}{4} (F^a_{\mu\nu})^2 - \frac{1}{2} a^\mu F^a_{\mu\nu} a_\delta F^{\delta a} \right),$$

(33)

where $a_\mu$ is the Lorentz-breaking constant vector, which is dimensionless in four dimensions. The Euclidean action reads,

$$S = \int d^4x \left( \frac{1}{4} (F^a_{\mu\nu})^2 + \frac{1}{2} a^\mu F^a_{\mu\nu} a_\delta F^{\delta a} \right).$$

(34)

As we only have gauge invariant terms, the path integral is still plagued by the gauge redundancy and as pointed out before this means that by the Gribov copies also. Along the paper we work in the Landau gauge. By following the procedure described in the section 3.1 we have,

$$S = \int \frac{d^4k}{(2\pi)^4} \left( -\frac{1}{2} \tilde{A}_\mu^a(k) Q^{ab}_\mu \tilde{A}_\nu^b(-k) \right),$$

(35)

where

$$Q^{ab}_\mu = \delta^{ab} \left( k^2 + \alpha(a \cdot k)^2 + \frac{\gamma^4}{k^2} \right) \delta_{\mu\nu} + \left( \frac{1}{\alpha} - 1 \right) k_\mu k_\nu - (a \cdot k) (k_\mu a_\nu + a_\mu k_\nu) + k^2 a_\mu a_\nu,$$

(36)

where $\gamma^4 = \frac{2N_c g^2}{N_f (N_f - 1)}$ is known as the Gribov parameter and $\alpha$ is used to implement to Landau gauge. By finding the inverse of (36) we obtain the propagator,

$$\langle A^a_\nu(k) A^b_\delta(k) \rangle = \delta^{ab} F(k) \left[ \delta_{\nu\delta} - \frac{k_\nu k_\delta}{k^2} \right] - \frac{(a \cdot k)^2}{(L + k^2 a^2 - (a \cdot k)^2)} \frac{k_\nu k_\delta}{k^2}$$

$$- \frac{k^2}{L + k^2 a^2 - (a \cdot k)^2} a_\nu a_\delta + \frac{(a \cdot k)}{L + k^2 a^2 - (a \cdot k)^2} a_\nu k_\delta$$

$$+ \frac{(a \cdot k)}{(L + k^2 a^2 - (a \cdot k)^2)} a_\delta k_\nu,$$

(37)

where $L(k) = k^2 + (a \cdot k)^2 + \frac{\gamma^4}{k^2}$ and

$$F(k) = \frac{k^2}{\zeta(a, \phi) k^4 + \gamma^4}.$$

(38)

with $\zeta(a, \phi) = 1 + a^2 \cos^2 \phi$. As the pole structure relies on the definition of $F(k)$ we can see that the aether-like term does not affect it:

$$F(k) = \frac{k^2}{\zeta(a, \phi) k^4 + \gamma^4} = \frac{1}{2\gamma i} \left( \frac{1}{\sqrt{\zeta(a, \phi) k^2 + i\gamma^2}} - \frac{1}{\sqrt{\zeta(a, \phi) k^2 - i\gamma^2}} \right).$$

(39)

It means that we still have the propagation of unphysical modes preserving the Gribov propagator.

The reason why this happened is that because the aether term in (33) does not introduce a new massive parameter to our theory (the vector $a_\mu$ is dimensionless). We could left in (33) explicitly the parameters $m^2$ and $g^2$, presented in (9), as the aether coupling constants (instead of reabsorbing them in the vector Lorentz symmetry breaking as we did). However from a dimensional analysis these parameters are dimensionless and would appear in the definition of $\zeta(a, \phi)$ not changing then the structure in (39). This fact can also be seen in the computation of the gap equation presented in the next subsection.

\[3\]In this section we absorb the constants in (9) in the Lorentz breaking vector: $a_\mu \rightarrow (\sqrt{C m^2})a_\mu$
4.1 The gap equation

In this section we compute the gap equation in the presence of an aether term based on the steps presented in the section 3.1. By starting with (34), the gluon propagator, as in (31), reads

\[
\langle A^a_\mu(k)A^b_\nu(p) \rangle = \delta(p + k)N \int \frac{d\beta}{2i\pi\beta} e^{\beta/2} (\det Q^{ab}_{\mu\nu})^{-1/2}(Q^{ab}_{\mu\nu})^{-1},
\]

(40)

by computing the determinant of (36) we have,

\[
(\det Q^{ab}_{\mu\nu})^{-1/2} = \epsilon^\frac{i}{2} \text{det} \ln Q_{\mu\nu} = \epsilon^\frac{i}{2} \text{Tr} \ln Q_{\mu\nu}.
\]

(41)

The trace of the functional logarithm can be computed by using the expansion \( \ln(1 + x) = x - \frac{x^2}{2} + \ldots \),

\[
\text{Tr} \ln Q_{\mu\nu} = (N^2 - 1) \text{Tr} \ln \left( L \delta_{\mu\nu} \right) \left( \delta_{\kappa\nu} + \frac{1}{L} k_\kappa k_\nu - \frac{1}{L} \left( (a \cdot k) k_\kappa a_\nu + a_\kappa (a \cdot k) k_\nu - k^2 a_\kappa a_\nu \right) \right).
\]

(42)

Taking the trace and summing again \( x - \frac{x^2}{2} + \ldots = \ln(1 + x) \) back we have

\[
\text{Tr} \ln Q_{\mu\nu} = (N^2 - 1) \left[ d \sum_k \ln L \right.
\]

\[
\left. + \sum_k \ln \left( 1 + \left( \frac{1}{\alpha} - 1 \right) \frac{k^2}{L} - \frac{1}{L} \left( 2(a \cdot k)^2 - k^2 a^2 \right) \right) \right]
\]

\[
= (N^2 - 1) \left[ d \sum_k \ln L - \sum_k \ln \left( \frac{k^4 + (a \cdot k)^2 k^2 + \gamma^4}{k^2} \right) \right.
\]

\[
\left. + \sum_k \ln \left( \frac{\lambda}{k^2} \right) \right],
\]

(43)

where we have used \( L(k) = k^2 + (a \cdot k)^2 + \frac{\gamma^4}{k^2} \) and defined \( \lambda = k^4 a^2 - k^2 (a \cdot k)^2 + \gamma^4 \). The last term can be worked out as

\[
\sum_k \ln \left( \frac{\lambda}{k^2} \right) = \sum_k \left( \lambda + \frac{k^4}{\alpha} \right) + \sum_k \ln k^2
\]

\[
= V \int \frac{d^d k}{(2\pi)^d} \ln \left( \frac{k^2}{\sqrt{\alpha}} + i\sqrt{\lambda} \right) + V \int \frac{d^d k}{(2\pi)^d} \ln \left( \frac{k^2}{\sqrt{\alpha}} - i\sqrt{\lambda} \right)
\]

\[
\sim \alpha^{d/4},
\]

(44)

where \( \int d^d k \ln k^2 \) is zero in dimensional regularization. In the Landau gauge limit, \( \alpha \to 0 \), the trace becomes

\[
\text{Tr} \ln Q_{\mu\nu} = (N^2 - 1) \left[ d \sum_k \ln L - \sum_k \ln \left( \frac{k^4 + \alpha (a \cdot k)^2 k^2 + \gamma^4}{k^2} \right) \right]
\]

\[
= (N^2 - 1) V (d - 1) \int \frac{d^d k}{(2\pi)^d} \ln \left( \frac{k^2 + \alpha (a \cdot k)^2 + \gamma^4}{k^2} \right).
\]

(45)
Therefore, (41) reads
\[
\text{(det } Q_{\mu \nu}^{ab})^{-1/2} = \exp \left[ -\frac{(d-1)}{2} (N^2 - 1) V \int \frac{d^d k}{(2\pi)^d} \ln \left( k^2 + (a \cdot k)^2 + \frac{\gamma^4}{k^2} \right) \right],
\]
so that the new version of (31) reads
\[
f(\beta) = \beta - \ln \beta - \frac{d-1}{2} (N^2 - 1) V \int \frac{d^d k}{(2\pi)^d} \ln \left( k^2 + (a \cdot k)^2 + \frac{\beta Ng^2}{N^2 - 1} \frac{2}{dV k^2} \right).
\]
In the thermodynamic limit, the saddle point approximation condition, w.r.t. \( \beta \), requires \( f'(\beta_0) = 0 \), where \( \beta_0 \) is the value of \( \beta \) that minimizes the vacuum energy. Using the definition in (39) we obtain,
\[
0 = 1 - \frac{1}{\beta_0} - \frac{d-1}{d} Ng^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{\zeta(a, \phi)k^4} \left( \frac{1}{\zeta(a, \phi)k^4 + \frac{\beta_0 Ng^2}{N^2 - 1} \frac{2}{dV k^2}} \right).
\]
The \( 1/\beta_0 \) term can be neglected\(^5\) and we obtain,
\[
1 = \frac{d-1}{d} Ng^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{\zeta(a, \phi)k^4 + \frac{\beta_0 Ng^2}{N^2 - 1} \frac{2}{dV k^2}}.
\]
By defining \( q^4 = \zeta(a, \phi)k^4 \) we obtain,
\[
1 = \frac{d-1}{d} Ng^2 \zeta(a, \phi)^{d/4} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^4 + \frac{\beta_0 Ng^2}{N^2 - 1} \frac{2}{dV}}.
\]
For the same reasons pointed out after (39) we can see that the aether term does not affect the gap equation.

5 Conclusion

In this paper we have treated the generation of the non-Abelian aether term and the properly path integral quantization of the Lorentz-broken YM-aether system in the Landau gauge.

We have studied the radiative generation of an aether-like term in a Lorentz-broken YM theory with a non-minimal coupling between the gauge and matter fields. In this paper the radiative correction of this non-minimal coupling was performed up to one-loop in perturbation theory, and with this we have shown that it is possible to obtain an effective field theory displaying a CPT-even Lorentz-breaking term also in a non-Abelian gauge theory, known as the aether-like term.

Furthermore, we proved that such an effective CPT-even Lorentz-broken YM theory can be consistently quantized in the Landau gauge, according to the Gribov-Zwanziger quantization prescription. It means that, following the GZ approach to get rid of the Gribov copies in the Landau gauge, we proved that the gap equation exists and still can be solved within our effective theory displaying CPT-even Lorentz-broken symmetry, since it is formally equivalent to the gap equation of the Lorentz symmetric YM theory.

Finally, it would be interesting to study the renormalizability of the effective Lorentz-broken YM-aether theory, within the Algebraic Renormalization prescription, and to restudy this procedure by including a Higgs field to see its impact on the gap equation and on the poles of the gluon propagator.

\(^4\)Here the Gribov parameter has been redefined to \( \gamma^4 = \frac{\beta Ng^2}{N^2 - 1} \frac{2}{dV} \).

\(^5\)The spacetime volume is infinity: \( V \sim \infty \). If we set \( \beta_0 \sim V \) we keep the term finite and non-null.
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