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Explicit probability of fixation formula for mutual competitors in a stochastic population model under competitive trade-offs

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Abstract

Competition is ubiquitous in nature, and mathematicians have a long history of studying general models for ecological competition, most notably the deterministic Lotka-Volterra competition model. However, deterministic modeling struggles to capture the effects of small fitness differences. In this talk, we consider the two-species stochastic Lotka-Volterra competition model, which allows for a more nuanced interpretation of the competitive advantage conferred by fitness differences. In particular, we study the probability that one species outcompetes the other, called the probability of fixation, by analyzing the associated backward Kolmogorov equation (BKE). By identifying and exploiting a natural slow timescale, we derive an approximation to the BKE that allows us to find a closed form expression for the probability of fixation, through which we can easily examine the effects of parameter changes. Finally, we use our result to study fitness tradeoffs within a competitive environment and show that certain tradeoff strategies are beneficial while the population exists at high frequencies, but harmful at low frequencies, and vice versa. As a specific biological example, we show that our results agree with the gut-invasion strategy of Salmonella Typhimurium.