ORTHOGONAL POLYNOMIALS AND LIE SUPERALGEBRAS

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Abstract. For \( \mathfrak{o}(2n+1) \), in addition to the conventional set of orthogonal polynomials, another set is produced with the help of the Lie superalgebra \( \mathfrak{osp}(1|2n) \). Difficulties related with expression of Dyson’s constant for the Lie superalgebras are discussed.

§0. Introduction

0.1. History. In 1962 while studying statistical mechanics Dyson [D] considered the constant term in the expression

\[
\prod_{i \neq j}(1 - \frac{x_i}{x_j}) \text{ for } k \in \mathbb{N},
\]

depending on indeterminates \( x_1, \ldots, x_n \). Dyson conjectured the explicit form of this constant term. His conjecture was soon related with the root system of \( \mathfrak{sl}(n) \), generalized to other root systems of simple Lie algebras and proved. The expressions obtained for the Dyson constant are called Macdonald’s identities, see [M].

Let us briefly recall the main results. Let \( g \) be a simple (finite dimensional) Lie algebra, \( \mathcal{R} \) its root system, \( \mathcal{P} \) the group of weights; \( A = \mathbb{C}[\mathcal{P}] \) the group of formal exponents of the form \( e^\lambda \), where \( \lambda \in \mathcal{P} \); let \( W \) be the Weyl group of \( g \) and

\[
\Delta = \frac{1}{|W|} \prod_{\alpha \in \mathcal{R}} (1 - e^\alpha).
\]

On \( A \), define the scalar product by setting

\[
(f, g) = [f \bar{g} \Delta]_0,
\]

(0.0)

where \( \bar{e}^\lambda = e^{-\lambda} \) and \([f]_0\) is the constant term of the power series \( f \).

It turns out that

the characters \( \chi_\lambda \) of finite dimensional irreducible representations of \( g \) are uniquely determined by their properties

(i) to form an orthogonal (with respect to the form (0.0)) basis in \( A^W \), the algebra of \( W \)-invariant elements of \( A \);

(ii) \( \chi_\lambda = e^\lambda + \text{ terms with exponents } \mu \text{ such that } \mu < \lambda \).

Here for Lie superalgebras I consider the following problem: what are the analogs of the scalar product (0.0) (hence, of \( \Delta \) and \( W \)) for which (i) and (ii) hold? If (i) and (ii) do not hold as stated, how to modify the definitions and the statement to make them reasonably interesting?

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I am thankful to D. Leites, who raised the problem, for support and help.
0.2. Main result. So far, there is not much that can count as a result, actually. I consider this note as a remark on the results from [M] and a report on the work in progress.

It turns out that for Lie superalgebras there is no function $\Delta$ (understood as a formal distribution) such that the characters of irreducible representations would satisfy (0.1), i.e., were orthonormal. With one exception: the series $\mathfrak{osp}(1|2n)$. Thanks to this exception, the main results of this note are:

1) For $\mathfrak{osp}(1|2n)$ I reproduce an observation of Rittenberg and Scheunert [RS] on a correspondence between irreducible $\mathfrak{osp}(1|2n)$-modules and $\mathfrak{o}(2n + 1)$-modules. (I also give a short and lucid demonstration of this correspondence. [J]) From this correspondence I deduce in the $\mathfrak{o}(2n + 1)$ case the existence of another set of orthogonal polynomials in addition to the set described in [M].

2) For any simple Lie superalgebra $\mathfrak{g}$ I can produce a function $\Delta$ for which the characters of the typical representations are orthonormal with respect to (0.0). I hope to return to this topic elsewhere.

Remark. Observe that for the simple Lie algebras, $\Delta$ appears in the Weyl integration formula: if $f$ is a class function on a compact Lie group $G$ such that $\text{Lie}(G) \otimes \mathbb{C} \cong \mathfrak{g}$ and $T \subset G$ is a maximal torus, then

$$\int_G f \, dg = \int_T f \Delta dt. \tag{0.2}$$

For the general Lie superalgebras the analog of identity (0.2) is unknown to me. Here are several little problems: not every simple Lie superalgebra (supergroup) over $\mathbb{C}$ has a compact form; the volume of those that have may vanish identically, cf. [B].

§1. The orthogonality of the characters of $\mathfrak{osp}(1|2n)$-modules

We recall some basic facts from the representation theory of $\mathfrak{osp}(1|2n)$ (see, e.g., [K]) and (for convenience) $\mathfrak{o}(2n + 1)$.

1.1. $\mathfrak{osp}(1|2n)$, its roots and characters. Set

$$R_0 = \{ \pm \varepsilon_i \pm \varepsilon_j \text{ for } i \neq j; \pm 2\varepsilon_i \}, \quad R_1 = \{ \pm \varepsilon_i \};$$

$$S_0 = \{ \pm \varepsilon_i \pm \varepsilon_j \text{ for } i \neq j \} \subset R_0;$$

$$2\rho_0 = \sum_{i<j}(\varepsilon_i - \varepsilon_j + \varepsilon_i + \varepsilon_j) + \sum_i 2\varepsilon_i = 2 \sum_{i<j} \varepsilon_i + 2 \sum_i \varepsilon_i = 2 \sum (n - i + 1)\varepsilon_i.$$ 

For the Lie superalgebras

$$\rho = \rho_0 - \rho_1, \text{ where } \rho_1 = \frac{1}{2} \sum_i \varepsilon_i.$$

All $\mathfrak{osp}(1|2n)$-modules are typical. The invariant bilinear form is $\text{str}(\text{ad}(x)^2)$. Explicitly, the restriction of this form onto Cartan subalgebra reads as follows:

$$\text{str}(\text{ad}(x)^2) = \sum (\pm \varepsilon_i \pm \varepsilon_j)^2 + \sum (\pm 2\varepsilon_i)^2 - \sum (\pm \varepsilon_i)^2 =$$

$$2 \sum (\varepsilon_i \pm \varepsilon_j)^2 + 4 \sum (\pm \varepsilon_i)^2 - \sum (\pm \varepsilon_i)^2 =$$

$$2 \sum ((\varepsilon_i + \varepsilon_j)^2 + (\varepsilon_i - \varepsilon_j)^2) + 6 \sum (\varepsilon_i)^2 =$$

$$(4n + 2) \sum (\varepsilon_i)^2.$$

Leites informed me, that this demonstration basically coincides with the one Deligne communicated to Leites in 1991 (unpublished).
The supercharacter of the finite dimensional irreducible $\mathfrak{osp}(1|2n)$-module $V^\lambda$ with highest weight $\lambda$ is

$$\text{sch}V^\lambda = \frac{\sum_{w \in W} \text{sign}'(w)e^{w(\lambda+\rho)}}{L},$$

where

$$L = \prod_{\alpha \in R_0^+} (e^{\alpha/2} - e^{-\alpha/2}) \prod_{\beta \in R_1^+} (e^{\beta/2} - e^{-\beta/2})$$

and $\text{sign}'(w)$ is equal to $-1$ to the power equal to the number of reflections in the even roots $\alpha$ except those $\alpha$ for which $\frac{\alpha}{2} \in R_1$.

The Weyl group $W$ of $\mathfrak{osp}(1|2n)$ is equal to $\mathfrak{S}_n \circ (\mathbb{Z}/2)^n$ and

$$\text{sign}'(\sigma \cdot \tau_1 \ldots \tau_n) = \text{sign}(\sigma)$$

for any $\sigma \in \mathfrak{S}_n$ and $\tau_i$ from the $i$-th copy of $\mathbb{Z}/2$.

Observe that

$$L = \sum_{w \in W} \text{sign}'(w)e^{w\rho}.$$ 

Indeed, apply the character formula to the trivial module.

In other words, everything is the same as for $\mathfrak{o}(2n + 1)$ but instead of the character sign on $W$ we now take $\text{sign}'$.

The unique, up to $W$-action system of simple roots in $\mathfrak{g}$ is of the form

$$\Pi = \{\varepsilon_1 - \varepsilon_2, \ldots, \varepsilon_{n-1} - \varepsilon_n, \varepsilon_n\}.$$ 

Observe that

$$\lambda + \rho = \sum (\lambda_i + n - i + \frac{1}{2})\varepsilon_i.$$ 

Since

$$L = \prod_{\alpha \in R_0^+} (e^{\alpha/2} - e^{-\alpha/2}) \prod_{\beta \in R_1^+} (e^{\beta/2} - e^{-\beta/2})$$

and

$$L = \prod_{\alpha \in R_0^+} (e^{\alpha/2} - e^{-\alpha/2}) \prod_{\beta \in R_1^+} (e^{\beta/2} + e^{-\beta/2}).$$

We will use the latter expression of $L$ as well.

1.2. $\mathfrak{o}(2n + 1)$, its roots and characters. Clearly, $R(\mathfrak{o}(2n + 1)) = S(\mathfrak{osp}(1|2n))_0 \cup R(\mathfrak{osp}(1|2n))_1$ and $\rho$ is the half-sum of the positive roots; the restriction of the Killing form is proportional to $\sum \varepsilon_i^2$, the Weyl group is $W = \mathfrak{S}_n \circ (\mathbb{Z}/2)^n$ and for the nontrivial homomorphism $\text{sign} : \mathbb{Z}/2 \rightarrow \{\pm 1\}$ we have

$$\text{sign}(\sigma \cdot \tau_1 \ldots \tau_n) = \text{sign}(\sigma)\text{sign}(\tau_1) \ldots \text{sign}(\tau_n)$$

for any $\sigma \in \mathfrak{S}_n$ and $\tau_i$ from the $i$-th copy of $\mathbb{Z}/2$.

The system of simple roots is the same as for $\mathfrak{osp}(1|2n)$; the character of the finite dimensional irreducible $\mathfrak{o}(2n + 1)$-module $V^\lambda$ with highest weight $\lambda$ is

$$\text{ch}V^\lambda = \frac{\sum_{w \in W} \text{sign}(w)e^{w(\lambda+\rho)}}{L},$$
where

\[ L = \prod_{\alpha \in S(\mathfrak{osp}(1|2n))_0} (e^{\alpha/2} - e^{-\alpha/2}) \prod_{\alpha \in \mathcal{R}(\mathfrak{osp}(1|2n))_1} (e^{\alpha/2} - e^{-\alpha/2}). \]  

(1.2)

Set

\[ (a, q)_\infty = (1 - a)(1 - aq)(1 - aq^2) \ldots \]

Set, further (for an indeterminate \( \epsilon \) such that \( \epsilon^2 = 1 \); it corresponds to the 1-dimensional odd superspace; we hope that the reader will not confuse \( \epsilon \) with the root \( \varepsilon \)):

\[ \Delta(q, t, \epsilon) = \frac{\prod_{\alpha \in S(\mathfrak{osp}(1|2n))_0} (e^{\alpha}, q)_\infty}{(te^\alpha, q)_\infty} \cdot \frac{\prod_{\alpha \in \mathcal{R}(\mathfrak{osp}(1|2n))_1} (e^{\epsilon \alpha}, q)_\infty}{(te^{\epsilon \alpha}, q)_\infty} \quad \text{for} \quad \alpha \in \mathcal{R}(\mathfrak{o}(2n + 1)). \]

Let

\[ t = q^k \quad \text{for} \quad k \geq 0. \]

Then

\[ \Delta(q, t, \epsilon) = \prod_{\alpha \in S(\mathfrak{osp}(1|2n))_0} \prod_{r=0}^{k-1} (1 - q^r e^\alpha) \cdot \prod_{\alpha \in \mathcal{R}(\mathfrak{osp}(1|2n))_1} \prod_{r=0}^{k-1} (1 - \epsilon q^r e^\alpha) = \prod_{\alpha \in \mathcal{R}} \prod_{r=0}^{k-1} (1 - e^{p(\alpha)} q^r e^\alpha), \]

where \( p(\alpha) = 0 \) for \( \alpha \in S(\mathfrak{osp}(1|2n))_0 \) and \( p(\alpha) = 1 \) for \( \alpha \in \mathcal{R}(\mathfrak{osp}(1|2n))_1 \).

Let \( P \) be the group of weights of \( \mathfrak{osp}(1|2) \) and let \( A = \mathbb{C}[P] \) be the group of formal exponents of the form \( e^\lambda \), where \( \lambda \in P \). Recall that \( \lambda \in P \) if and only if \( \lambda = \sum n_i \varepsilon_i \), where \( n_i \in \mathbb{Z} \) for all \( i \).

The Weyl group \( \mathcal{W} = \mathfrak{S}_n \circ (\mathbb{Z}/2)^n \) of \( \mathfrak{osp}(1|2n) \) acts on \( P \), hence, on \( A \), as follows: \( \mathfrak{S}_n \) permutes the \( \varepsilon_i \) and \( (\mathbb{Z}/2)^n \) changes their signs.

Observe that for \( \mathfrak{o}(2n + 1) \) the group of weights is larger than same for \( \mathfrak{osp}(1|2n) \): the former includes the half-integer \( n_i \).

4.2. Theorem. For \( \mathfrak{o}(2n + 1) \) there exists a unique (up to a constant factor) basis of \( A^\mathcal{W} \) consisting of \( \lambda \in P^+ \) such that

a) \( F_\lambda = m_\lambda + \sum_{\mu < \lambda} u_{\lambda, \mu} m_\mu \), where \( m_\lambda = \sum_{\nu \in \{ \text{the orbit of} \lambda \}} e^\nu \) and the coefficients \( u_{\lambda, \mu} \) are rational functions in \( t \) and \( \epsilon \);

b) for \( f, g \in A \) define the pairing \( (f, g) \) by means of formula (0.0), where \( \Delta \) is determined by (1.3). Then

\[ (F_\lambda, F_\mu) = 0 \quad \text{if} \quad \lambda \neq \mu. \]

Proof. Uniqueness. Since \( F_\lambda = m_\lambda + \sum_{\mu < \lambda} u_{\lambda, \mu} m_\mu \), it follows that the transition matrix from \( F_\lambda \) to \( F_\lambda \) is an upper triangular one, i.e., \( m_\lambda = F_\lambda + \sum_{\mu < \lambda} a_{\lambda, \mu} F_\mu \). If \( F_\lambda' \) is another set of elements from \( A \) with the properties needed, then the transition matrix from \( m_\lambda \) to \( F_\lambda' \) is also an upper triangular one, so \( F_\lambda' = \sum_{\mu \leq \lambda} b_{\lambda, \mu} F_\mu \). If both the bases are orthogonal, i.e.,

\[ (F_\lambda, F_\mu) = (F_\lambda', F_\mu') = 0 \quad \text{for} \quad \lambda \neq \mu, \]

this means that \( F_\lambda = F_\lambda' \cdot C_\lambda \).

Existence. It suffices to prove the existence of an operator \( D : A^\mathcal{W} \rightarrow A^\mathcal{W} \) such that

i) \( (Df, g) = (f, Dg) \);

ii) \( Dm_\lambda = \sum_{\mu \leq \lambda} c_{\lambda, \mu} m_\mu \);

iii) if \( \lambda, \mu \in P^+ \) are distinct, then \( c_{\lambda, \lambda} \neq c_{\mu, \mu} \).
Set
\[ \Delta_+ = \prod_{\alpha \in R^+} \prod_{r=0}^{k-1} (1 - e^{p(\alpha)} q^r e^\alpha). \]

In the Cartan subalgebra \( \mathfrak{h} \subset \mathfrak{g} \) select \( h_1 = \varepsilon_1^* \) which plays the role of a miniscule weight for the dual root system. More exactly, for \( R^+ = \{ \varepsilon_i - \varepsilon_j; \varepsilon_i + \varepsilon_j; \varepsilon_i \} \) we have \( \alpha(h_1) = 0 \) or \( 1 \) for any \( \alpha \in R^+ \).

Define the action of the operator \( T = T_{h_1} \) by setting
\[ Te^\lambda = q^{\lambda(h_1)} e^\lambda \]
and define \( D \) by setting
\[ Df = \sum_{w \in W} w(\Delta_+^{-1} T(\Delta_+ f)). \]

The operator \( D \) is self-adjoint. Indeed,
\[ \Delta = w\Delta = w\Delta_+ w\Delta_+ \text{ for any } w \in W. \]

Further on,
\[
\begin{align*}
(Df, g) &= \sum \left( w(\Delta_+^{-1} T(\Delta_+ f)), g \right) = \\
&= \frac{1}{|W|} \sum_{w \in W} \left[ w \left( T(\Delta_+ f) \right) \cdot \tilde{g} \cdot w\Delta_+ w\Delta_+ \right]_0 = \\
&= \frac{1}{|W|} \sum_{w \in W} \left[ w(T(\Delta_+ f)) \cdot \tilde{g} w\Delta_+ \right]_0 = \frac{1}{|W|} \sum_{w \in W} \left[ w(T(\Delta_+ f)) \cdot \tilde{g} \Delta_+ \right]_0 = \\
&= (\text{since } g^w = g) = \frac{1}{|W|} \sum_{w \in W} \left[ w(T(\Delta_+ f)) \cdot \tilde{g} \Delta_+ \right]_0 = \\
&= (\text{since the constant term is always } W\text{-invariant}) = \left[ T(\Delta_+ f) \cdot \tilde{g} \Delta_+ \right]_0.
\end{align*}
\]

Similarly,
\[ (Dg, f) = [T(\Delta_+ g) \cdot \tilde{f} \Delta_+]_0. \]

But, as is easy to verify,
\[ [T(f) \cdot \tilde{g}]_0 = [T(g) \cdot \tilde{f}]_0. \]

Therefore,
\[ (Df, g) = (f, Dg) \text{ for any } f, g \in A^W. \]

Let us show that \( D \) sends \( A \) into \( A \). Set \( \Phi = \Delta_+^{-1} T(\Delta_+) \). Then \( D = \sum_{w \in W} w(\Phi T(f)) \) since \( T \)
is an automorphism.

Let us compute \( \Phi \). Since \( \alpha(h_1) = 0 \) or \( 1 \) for any \( \alpha \in R^+ \), it follows that
\[
T(e^\alpha) = \begin{cases} 
q e^\alpha & \text{for } \alpha(h_1) = 1 \\
e^\alpha & \text{for } \alpha(h_1) = 0.
\end{cases}
\]
The case ε = 1 is considered in [M1]. Therefore, in what follows we assume that ε = −1.

We have:

\[
\Phi = \frac{T(\Delta_+)}{\Delta_+} = T(\prod_{\alpha \in R^+} \prod_{r=0}^{k-1} (1 - \epsilon^{p(\alpha)} q^r e^\alpha))
\]

\[
\prod_{\alpha \in R^+} \prod_{r=0}^{k-1} \frac{1 - \epsilon^{p(\alpha)} q^{\alpha(h_1)} q^r e^\alpha}{1 - \epsilon^{p(\alpha)} q^r e^\alpha} = \prod_{\alpha \in R^+, \alpha(h_1)=1} \prod_{r=0}^{k-1} \frac{1 - \epsilon^{p(\alpha)} q^{r+1} e^\alpha}{1 - \epsilon^{p(\alpha)} q^r e^\alpha} = (t = q^r)
\]

\[
\prod_{\alpha \in R^+, \alpha(h_1)=1} k^{-1} \prod_{r=0}^{k-1} \frac{1 - \epsilon^{p(\alpha)} t^{-\alpha(h_1)} e^{-\alpha}}{1 - \epsilon^{p(\alpha)} e^{-\alpha}} = \prod_{\alpha \in R^+, \alpha(h_1)=1} k^{-1} \prod_{r=0}^{k-1} \frac{1 - \epsilon^{p(\alpha)} t^{-\alpha(h_1)} e^{-\alpha}}{1 - \epsilon^{p(\alpha)} e^{-\alpha}} =
\]

\[
t^{\alpha(2p)} \prod_{\alpha \in R^+, \alpha(h_1)=1} k^{-1} \prod_{r=0}^{k-1} \frac{1 - \epsilon^{p(\alpha)} t^{-\alpha(h_1)} e^{-\alpha}}{1 - \epsilon^{p(\alpha)} e^{-\alpha}} =
\]

where

\[
d = \sum_{\psi \in \Psi} \text{sign}'(\psi) e^{w\rho}.
\]

Observe that \( w\delta = \text{sign}'(w) \). Observe also that

\[
d = \prod_{\alpha \in R^+} (e^{\alpha/2} - \epsilon^{p(\alpha)} e^{-\alpha/2}).
\]

For any \( X \subset R^+ \) set

\[
\sigma(X) = \sum_{\alpha \in X} \alpha.
\]

If we simplify (1.4) by eliminating parentheses, then Φ takes the form

\[
\Phi = t^{2q(h_1)} e^\rho \delta^{-1} \sum_{X \subset R^+} \varphi_X(t) e^{-\sigma(X)},
\]

where

\[
\varphi_X(t) = (-1)^{|X|} \epsilon^{p(X)} t^{-\sigma(X)(h_1)}
\]

and \( p(X) = \#(\text{odd roots that occur in } X) \). Let us calculate \( De^\mu \) for \( \mu \in P \). Observe that

\[
w(Te^\mu) = q^{\mu(h_1)} e^{w\mu},
\]

hence,

\[
De^\mu = \delta^{-1} q^{\mu(h_1)} t^{2q(h_1)} X_{\subset R^+} \varphi_X(t) \sum_{\psi \in \Psi} \text{sign}'(\psi) e^{w(\mu+\rho-\sigma(X))} =
\]

\[
q^{\mu(h_1)} t^{2q(h_1)} \sum_{X \subset R^+} \varphi_X(t) \left( \delta^{-1} \sum_{w \in W} \text{sign}'(w) e^{w(\mu+\rho-\sigma(X))} \right) =
\]

\[
q^{\mu(h_1)} t^{2q(h_1)} \sum_{X \subset R^+} \varphi_X(t) \chi_{\mu-\sigma(X)}.
\]

Here

\[
\chi_{\mu-\sigma(X)} \neq 0 \iff \text{there exists a } \nu \in P^+ \text{ such that } \nu + \rho = w(\mu - \sigma(X) + \rho).
\]
The last result does not depend on the choice of the one-dimensional character (sign or sign') on \( W \) because the orbit of the weight \( \lambda \) contains a dominant weight if and only if the \( \lambda_i^2 \) are pairwise distinct. Therefore,

\[
Dm_\lambda = \sum_{\mu \in W_\lambda} D e^\mu = t^{2\rho(h_1)} \sum_{X \subseteq R^+} \varphi_X(t) \sum_{\mu \in W_\lambda} q^{\mu(h_1)} \chi_{\mu - \sigma(X)}.
\]  

(1.5)

But

\[
\rho - \sigma(X) = \frac{1}{2} \sum_{\alpha \in R^+} \varepsilon_\alpha \alpha
\]

(1.6)

where \( \varepsilon_\alpha = \pm 1 \); hence, \( w(\rho - \sigma(X)) \) is of the same form (1.6) and, therefore,

\[
w(\rho - \sigma(X)) = \rho - \sigma(Y)
\]

for some \( Y \subset R^+ \).

Thus,

\[
\nu = w\mu - \sigma(Y) \leq w\mu \leq \lambda.
\]

Consequently, \( Dm_\lambda = \sum_{\nu \leq \lambda} C_{\lambda\nu}(h_1) \cdot m_\nu \) and a) is verified.

Let us prove b). In (1.4), \( \nu = \lambda \) if and only if \( Y = \emptyset \) and

\[
w\mu = \lambda \iff w(\rho - \sigma(X)) = \rho, \ \mu = w^{-1}\lambda, \ X = R^+ \cap (-wR^+).
\]

Therefore,

\[
C_{\lambda\lambda}(h_1) = t^{2\rho(h_1)} \sum_{X \subseteq R^+} q^{w^{-1}\lambda(h_1)} \chi_{R^+ \cap (-wR^+)}(t)
\]

because

\[
\chi_{R^+ \cap (-wR^+)} = (-1)^{R^+ \cap (-wR^+)}(-1)^{p(R^+ \cap (-wR^+))} t^{w^{-1}\rho - \rho}(h_1) = \sign'(w) t^{w^{-1}\rho - \rho}(h_1).
\]

Hence,

\[
C_{\lambda\lambda}(h_1) = t^{\rho(h_1)} \sum_{w \in W} q^{\lambda(wh_1)} t^{\rho(wh_1)},
\]

as in [M]. Therefore (see (6.14) in [M]), if \( \lambda \neq \mu \) and \( \lambda, \mu \in P^+ \), then \( C_{\lambda\lambda} \neq C_{\mu\mu} \).  

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