Vehicle Packing Layout Optimization Based on Genetic Taboo Hybrid Search Algorithm

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Abstract. According to the characteristics of the actual transportation packing problem, the three-dimensional vehicle packing problem with path constraints is studied, and the optimal model of the three-dimensional vehicle packing problem with path constraints is established, and then taboo search is added to the genetic algorithm to solve the problem. The performance of the algorithm is verified by example analysis. The experimental result shows that the hybrid algorithm can obtain an approximate optimal solution of higher quality than the ordinary genetic algorithm, indicating that the algorithm has a better solution to the three-dimensional packing problem. The algorithm has a great help in improving vehicle utilization and work efficiency.

Keyword. Genetic taboo, Path optimization, Three-dimensional packing problem, Logistics.

1. Introduction

Distribution, also known as "small logistics", is the most basic activities of logistics. Goods loading and distribution path are the two basic elements of distribution, which play vital roles in the cost and service level control of distribution (even in the entire logistics). Goods loading is directly related to the utilization of space and load capacity of vehicles, and the cargo safety of transportation, thus affects the distribution cost. The optimization of the distribution path not only affects the transportation cost, but affects the timeliness (and efficiency) of the distribution, therefore, it has always been the focus of cost-saving operations for distribution companies. In the past, a lot of studies have optimized these two elements separately, however, note that they are severely related, for example, the order (or result) of cargo packing will affect the convenience and efficiency of delivery, moreover limit the order of delivery. Thus, only simply optimizing the distribution path without considering the packing results will lead to the conflicts and failure of the optimization, especially with the increase of decentralized distribution, small batch and high-frequency distribution has a greater impact on the distribution path and packing. Therefore, it is of great value to reduce the logistics cost and improve the competitiveness of enterprises to combine the two elements and make collaborative optimization from the system level.

In recent years, with the deepening understanding of distribution and its optimization value, many researchers began to consider the distribution vehicle packing problem (VRP) and vehicle routing problem (VFP) at the same time, that is to say, VFP and VRP are combined for optimization, but there are still different emphases. There are three kinds of optimization tendency that mainly focus on the packing problem, the distribution problem, or their combination. Yu et al [1] built a model aiming to
obtain a smallest number of vehicles under distribution time window constraints, and developed a genetic algorithm (GA) for solving the model. Wang et al [2] considered the route optimization problem in the process of cargo packing and designed the Memetic algorithm to optimize VFP&VRP. Kang et al [3] firstly used the ant colony algorithm to obtain the cargo matching scheme, and then used GA to verify the distribution scheme and obtained a better distribution and vehicle routing scheme. Yan et al [4] studied the vehicle routing problem with three-dimensional cargo packing constraints, and developed a two-stage method to solve the model, that is GA was used to solve the vehicle problem, and then the guided local search algorithm was used to solve the three-dimensional cargo loading problem. Taking path optimization as the first optimization object, Wang et al [5] studied the comprehensive optimization of the number of distribution vehicles and the total distance of transportation, and proposed a multi-stage hybrid algorithm architecture for comprehensive optimization of three-dimensional packing and CVRP model (3LCVRPMS). Fuellerer et al [6] studied the problem of two-dimensional vehicle-cargo matching and path optimization, in which the vehicle-cargo matching solution was firstly obtained by a heuristic algorithm with limited maximum hole degree, then the vehicle routing problem was solved by using ant colony algorithm. Doemer et al [7] used a taboo search algorithm and ant colony optimization algorithm to solve the problem of timber retailers delivering to customers in a specific area. Wei et al [8] made a combined optimization of the multi-vehicle vehicle routing problem and the three-dimensional cargo loading problem, and proposed an adaptive variable domain search (AVNs) algorithm. Based on the classic loading and unloading problem, Männel et al [9] studied the vehicle routing problem and three-dimensional packing problem considering customer requirements, load and volume constraints of multiple vehicles, and adopt a hybrid algorithm to achieve heuristic search based on binary tree. It can be seen that VRP and VFP combined optimization has become more and more popular, but the existing research on the constraints of distribution process is still insufficient, and the packing stability and sequence of goods in the packing scheme is not considered, which is a little far from the actual distribution. Secondly, the combination of multiple algorithms is also a major feature of this kind of problem. For specific cases, good results have been achieved. And for the three-dimensional packing layout optimization problem with path constraints, the GA is usually applied, however, to our best knowledge, few studies have rarely improved the shortcomings of the shortcomings of falling into the local optimal solution of GA by adding other algorithms for this problem in the past.

Therefore, in this paper, we consider more actual factors of distribution, that is, considering the distribution path, loading stability and other conditions as well as the packing layout. Under these constraints, the comprehensive optimization model is built to optimize the three-dimensional packing layout with path constraints, and a taboo genetic hybrid algorithm is proposed to solve the model. A case is introduced to illustrated the model and the algorithm. It is indicated that taboo genetic hybrid algorithm effectively improves the three-dimensional cargo efficiency and optimization of packing layout.

2. Problem and Model Formulation

2.1. Problem Formulation

Consider a distribution center with $t$ same-specification vehicles provides delivery services to $n$ customers (or nodes). Assume that each vehicle is a rectangular box, with the length $L$, width $W$, and height $H$, and the maximum load $Q$. Suppose the requirements of each customer are known, and customer $i$ ($i = 1, 2, 3, \ldots, n$) has $m_i$ items to be distributed, whose weight is $q_i$ and volume is $v_i$. Let $I_{ij}$ represent the item $j$ ($j = 1, 2, \ldots, m_i$) of customer $i$, $l_{ij}$, $h_{ij}$, and $w_{ij}$ represent the length, the width, and the height of $I_{ij}$, respectively. We need the package layout plan of the vehicles under the constraints of the traffic route and the loading of the vehicle, so that the total distance traveled by all vehicles is minimized and the utilization rate of vehicles is as high as possible. And during delivery, the cargo is not allowed to be turned upside down; circuitous transportation is not allowed in the transportation route, that is, each transportation route is only taken once; at the same time, in order to facilitate the
delivery of goods with customers, each customer can only be served by one vehicle. In addition, some characteristics of the cargo themselves also must be considered, such as pressure protection.

### 2.2. Model Formulation

To simplify the problem, we make the following assumptions.

1. A distribution center corresponds to multiple customers, and all the vehicles depart from the distribution center and finally return to the distribution center.
2. The specifications of each vehicle (length, width, height, load quality, etc.) are the same.
3. Each vehicle travels at a constant speed; and there is no traffic jam.
4. The center of gravity of each cargo is its geometric center.
5. The length-width direction of the cargo is parallel to the length-width direction of the vehicle.
6. The volume and weight of packed cargoes cannot exceed the allowable range of the vehicle.

As mentioned above, the distribution cost is related to route distance and vehicle utilization rate, so this goal can be transformed into three sub objectives, with the shortest distribution distance, the highest utilization rate of vehicle space and the highest utilization rate of vehicle load capacity. Therefore, under considering the characteristics of the goods and meeting the above constraints, we need to collect the orders of customers, through reasonable stowage, combined loading, to achieve the multi frequency, less batch distribution services. In order to better describe and optimize the loading position of goods, the coordinate system based on the origin of the lower left corner inside the vehicle carriage is established. The coordinate axes of the coordinate system correspond to the directions of the length, width and height of the carriage, as shown in figure 1. Thus use \( (x_k^i, y_k^i, z_k^i) \) to represent the coordinate of the center of gravity of cargo \( k \) of customer \( i \) in the vehicle, \( (x_{k,1}^p, y_{k,1}^p, z_{k,1}^p) \) and \( (x_{k,2}^p, y_{k,2}^p, z_{k,2}^p) \) represent the upper right corner and the lower left corner of cargo \( k \) of customer \( i \) in the carriage of vehicle \( p \).

![Figure 1. Three-dimensional packing diagram.](image)

Let \( x_{ij}^p = 1 \) if vehicle \( p \) travels from node \( i \) to \( j \); otherwise, \( x_{ij}^p = 0 \); \( y_{ij}^p = 1 \) if node \( i \) is delivered by vehicle \( p \); otherwise \( y_{ij}^p = 0 \), and \( d_{ij} \) denotes the distance between distribution node \( i \) and \( j \), \( t_{ij} \) denotes the travel time from node \( i \) to node \( j \); in order to keep vehicle stable, we set the gravity range of the carriage along the \( X, Y \) and \( Z \) axis are \([x_1, x_2], [y_1, y_2], [0, Z]\), respectively; and the allowed maximum cycle time for each distribution is \( T \), and the coefficient of support area between cargoes is \( \alpha \). We can obtain the three objective functions \( f_1, f_2 \) and \( f_3 \) as.

\[
\min f_1 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{p=1}^{V} d_{ij} x_{ij}^p
\]
\[
\text{max } f_2 = \sum_{p=1}^{n} \left( \frac{\sum_{i=1}^{n} y_i^p \cdot y_i}{LWH} \right)
\]

\[
\text{max } f_3 = \sum_{p=1}^{n} \left( \frac{\sum_{i=1}^{n} y_i^p \cdot q_i}{Q} \right)
\]

And the all restictions are expressed as follows:

\[
\sum_{p=1}^{n} y_i^p = 1, \quad i = 1, 2, \ldots, n
\]

\[
\sum_{j=1}^{n} x_{ij}^p = y_{ij}^p, \quad j = 0, 1, \ldots, n, \forall p
\]

\[
\sum_{j=1}^{n} x_{ij}^p = y_{ij}^p, \quad i = 1, 2, \ldots, n, \forall p
\]

\[
\sum_{i=1}^{n} y_i^p \cdot x_{ij}^p \leq L \cdot W \cdot H, \quad \forall p
\]

\[
\sum_{j=1}^{n} x_{ij}^p = \sum_{j=1}^{n} y_{ij}^p \leq 1, \forall p
\]

\[
\sum_{i=0}^{n} \sum_{j=0}^{n} t_{ij} \cdot x_{ij}^p + \sum_{i=1}^{n} t_{ij} \cdot y_{ij}^p \leq T, \forall p, \quad p = 1, 2, \ldots, v
\]

\[
0 \leq \overline{x}_{ik}, \quad \underline{x}_{ik} \leq L, \quad i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, m_i
\]

\[
0 \leq \overline{y}_{ik}, \quad \underline{y}_{ik} \leq W, \quad i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, m_i
\]

\[
0 \leq \overline{z}_{ik}, \quad \underline{z}_{ik} \leq H, \quad i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, m_i
\]

\[
[(\overline{x}_{ik} - \underline{x}_{ik}) - l_{ik}] \times (y_{ik}^p - \underline{y}_{ik}) - w_{ik} = 0, \forall i \in \{i \mid y_i^p = 1\}
\]

\[
\overline{z}_{ik} - \underline{z}_{ik} = h_{ik}, \forall i \in \{i \mid y_i^p = 1\}
\]

\[
X_1 = \sum_{i=1}^{n} \sum_{k=1}^{m_i} y_i^p d_{ik} x_{ik} \quad \text{and} \quad X_2 = \sum_{i=1}^{n} \sum_{k=1}^{m_i} y_i^p d_{ik} \leq X_2
\]

\[
Y_1 = \sum_{i=1}^{n} \sum_{k=1}^{m_i} y_i^p d_{ik} x_{ik} \quad \text{and} \quad Y_2 = \sum_{i=1}^{n} \sum_{k=1}^{m_i} y_i^p d_{ik} \leq Y_2
\]

\[
\sum_{i=1}^{n} \sum_{k=1}^{m_i} y_i^p d_{ik} z_{ik} \quad \text{and} \quad \sum_{i=1}^{n} \sum_{k=1}^{m_i} y_i^p d_{ik} \leq Z
\]

\[
x_1 \leq x_2 \quad y_1 \leq y_2 \quad z_1 \leq z_2
\]
Here, formula (4) (5) (6) represent the picking path constraint; formula (7) represents the vehicle volume limitation; formula (8) represents the completion of the delivery, return to the distribution center; formula (9) represents the time limit of the line; formula (10) denote the cargo must be packed inside the vehicle; formula (11) (12) denote the constraint of the direction of cargo placement; formula (13) (14) (15) represent constraint of the center of gravity of the cargo; formula (16) expresses that the cargo must not be overlapped in space, in the formula(16), $x_i = \min(x_{t_k}, x_{t_p})$, $x_2 = \max(x_{t_k}, x_{t_p})$, $y_1 = \min(y_{t_k}, y_{t_p})$, $y_2 = \max(y_{t_k}, y_{t_p})$, $z_1 = \min(z_{t_k}, z_{t_p})$, $z_2 = \max(z_{t_k}, z_{t_p})$; formula (17) represents the constraint of the support area between cargo es.

3. Genetic Taboo Hybrid Algorithm for Packing Model

Genetic algorithm has strong generality and good global optimization performance, but in practical application, there will be premature phenomenon, weak mountain climbing ability and poor convergence performance when solving the optimal solution. Taboo search has a flexible memory function in the search process, can accept poor quality solutions in the search process, has strong climbing ability, can jump out of the local optimal solution in the search process, has the ability to avoid premature, but taboo search has a strong dependence on the initial solution. To solve the Packing Model, we develop the Genetic Taboo Hybrid Algorithm by combining GA and Taboo search as follows.

3.1. Chromosome Coding

In this paper, according to the characteristics of three-dimensional packing layout, the coding adopts the sequential expression method.

For a delivery process, let 0 represent the distribution center, and 1, 2,...,n represent the customer's serial number. For the vehicle path and the loading order of the cargo, this paper also uses natural number coding to encode the path and the loading order of the items. For example, 0-2-5-6-8-0 represents a route that starts from the distribution center and then returns to the distribution center after going through the customer 2, 5, 6 and 8. For the loading order of the items, the code 3-5-7-8-1 indicates that when loading these five items, the item number 3 is loaded first, and then the items numbered 5, 7, 8 and 1 are loaded in sequence.

3.2. Fitness Function

Due to there are three optimization goals of volume utilization, weight utilization and transportation distance, we use weighting method to unify them into one objective function as follows:

$$\max f = \lambda_1 \left( \sum_{i=1}^{n} \sum_{j=0}^{n} \sum_{p=1}^{n} d_{ij} x_{ij}^p - 2 \sum_{i=0}^{n} d_{ii} \right) / R_i + \lambda_2 [\lambda_3 \left( \sum_{p=1}^{n} \frac{y_{ij}^p \cdot s_i}{LWH} \right) / R_i + \lambda_3 \left( \sum_{p=1}^{n} \frac{y_{ij}^p \cdot d_i}{D} \right) / R_i]$$

3.3. Select Operation

Using the roulette selection strategy proposed by Holland, the chromosomes are selected and put into the mating pool.

3.4. Cross Operation

To avoid getting stuck in the local optimal solution, a memory function is introduced in the crossover operator. For the individuals generated by the crossover of each pair of paired individuals, the choice is made by setting up a taboo list to retain good individuals.
3.5. Mutation Operation
We use the reverse transformation method for mutation. For example: 027|308|40 → 02780340. Repeat the above steps until the specified algebra is completed, the final distribution packing scheme can be achieved. The genetic taboo hybrid algorithm flowchart is as shown in the figure 2.

\[ \text{Figure 2. Taboo genetic algorithm process.} \]

4. Example Analysis
Set \( L =10m, \ W=2m, \ H=2m, \) \( Q \) is set to 8t, and given the speed is 45km/h. The information of customers and cargo is shown in table 1 and table 2 (refer to appendix). In table 2, the first two digits of the cargo information number indicate the customer's number, and the last two indicate the cargo type.

Table 1. Customer information.

| Serial number | X-coordinate(km) | Y-coordinate(km) |
|---------------|------------------|-----------------|
| 0(delivery center) | 25               | 25              |
| 1              | 6                | 47              |
| 2              | 14               | 43              |
| 3              | 10               | 45              |
| 4              | 25               | 48              |
| 5              | 22               | 38              |
| 6              | 33               | 41              |
| 7              | 40               | 35              |
| 8              | 30               | 31              |
| 9              | 39               | 26              |
| 10             | 23               | 28              |
| 11             | 12               | 24              |
| 12             | 9                | 13              |
| 13             | 18               | 13              |
| 14             | 20               | 12              |
| 15             | 25               | 18              |
| 16             | 34               | 11              |
| 17             | 38               | 13              |
| 18             | 33               | 7               |
| 19             | 36               | 8               |
| 20             | 48               | 10              |
Table 2. Cargo Information.

| Number | Length (mm) | Width (mm) | Height (mm) | Quantity | Total capacity (m³) | Total weight (kg) |
|--------|-------------|------------|-------------|----------|--------------------|------------------|
| 0101   | 1280        | 900        | 685         | 3        | 2.37               | 439              |
| 0102   | 820         | 430        | 280         | 7        | 0.69               | 310              |
| 0201   | 1370        | 980        | 750         | 3        | 3.02               | 533              |
| 0301   | 600         | 400        | 320         | 5        | 0.38               | 245              |
| 0302   | 900         | 400        | 280         | 4        | 0.40               | 356              |
| 0401   | 1280        | 980        | 580         | 4        | 2.91               | 400              |
| 0402   | 1280        | 1100       | 800         | 3        | 3.38               | 608              |
| 0501   | 860         | 620        | 400         | 5        | 1.07               | 288              |
| 0601   | 1250        | 910        | 930         | 3        | 3.17               | 532              |
| 0602   | 1280        | 900        | 685         | 2        | 1.58               | 290              |
| 0701   | 1320        | 900        | 780         | 3        | 2.78               | 414              |
| 0801   | 1100        | 630        | 410         | 3        | 0.85               | 300              |
| 0901   | 1000        | 730        | 680         | 3        | 1.49               | 344              |
| 1001   | 680         | 470        | 280         | 2        | 0.18               | 145              |
| 1101   | 1150        | 660        | 685         | 2        | 1.04               | 257              |
| 1201   | 820         | 450        | 485         | 3        | 0.54               | 210              |
| 1202   | 980         | 630        | 630         | 4        | 1.56               | 323              |
| 1301   | 1280        | 700        | 780         | 2        | 1.40               | 286              |
| 1302   | 900         | 500        | 430         | 6        | 1.16               | 300              |
| 1303   | 1450        | 680        | 700         | 3        | 2.07               | 468              |
| 1401   | 1420        | 570        | 380         | 2        | 0.62               | 144              |
| 1402   | 1250        | 900        | 500         | 2        | 1.13               | 322              |
| 1501   | 830         | 670        | 560         | 3        | 0.93               | 266              |
| 1502   | 640         | 720        | 550         | 3        | 0.76               | 243              |
| 1601   | 900         | 400        | 540         | 3        | 0.58               | 190              |
| 1701   | 1240        | 900        | 650         | 2        | 1.45               | 258              |
| 1702   | 1380        | 850        | 685         | 3        | 2.41               | 364              |
| 1801   | 960         | 670        | 480         | 5        | 1.54               | 255              |
| 1901   | 1480        | 900        | 680         | 4        | 3.62               | 720              |
| 2001   | 1280        | 980        | 700         | 3        | 2.63               | 380              |
| 2002   | 1280        | 760        | 800         | 4        | 3.11               | 522              |

4.1. Results

The population size of GA and Taboo GA is set to 500, the maximum number of iterations is 200, the cross-over probability is 0.4, and the mutation probability is 0.15. In the genetic taboo hybrid algorithm, the taboo table length is taken as 5, and the maximum taboo number is 5. The detailed calculation results are shown in figure 3 and figure 4.
As shown in figure 3, for all optimized goals or distribution performance indicators, the improved taboo genetic algorithm obtained better results than conventional GA. The transportation distance and transportation are decrease by 3.84% and 1.90, respectively, and the volume utilization and the weight utilization of the vehicles are increased to 32.69% and 43.03%, thus improved by 5.45% and 7.17%, respectively, compared with GA. Please see details in table 3. However, in this case, both space and load utilization of the vehicles are relatively low, and there is still a lot of room for optimization.

**Figure 3.** Calculation results of genetic algorithm and taboo genetic algorithm.

**Table 3.** Comparison of results.

| Algorithm       | Transport distance | Transportation time | Volume utilization | Weight utilization |
|-----------------|--------------------|---------------------|--------------------|--------------------|
| GA              | 312.48             | 15.81               | 27.24%             | 35.86%             |
| Taboo GA        | 300.47             | 15.51               | 32.69%             | 43.03%             |
| Improvement     | 3.84%              | 1.90%               | 5.45%              | 7.17%              |
As shown in Figure 4, the fitness of the optimal solution obtained by the improved taboo GA is 22.77, which is also higher than that of the ordinary genetic algorithm (18.79). That is to say, the improved taboo GA can better solve the optimal value of the problem when solving the three-dimensional packing layout optimization problem.

Through the optimization, the final optimum plan of distribution route and packing for the example can be both obtained. As shown in Figure 5 (a), five vehicles are needed for all customers and with specific route, for example, Customer 12, 13, 14 and Customer 15 are combined for delivery, and the delivery order is 15-13-14-12, thus, as shown in Figure 5 (b), the corresponding packing order in the vehicle is adjusted to customer 12-14-13-15 to decrease the search and carrying during delivery, namely the cargos of customer 12 are loaded in the innermost compartment.

5. Conclusion
As distribution becomes more and more fragmented, it is more and more important to carry out collaborative optimization of distribution packing and routing at the same time. In collaborative optimization, not only distribution cost and loading rate should be considered, but also packing stability should be considered. In this paper, for the three-dimensional packing problem with path constraints, we consider the distribution path, loading stability and other conditions as well as the packing layout. Both the distribution order and the topological structure of packing are obtained by optimization, which ensures the consistency of each other in the distribution. At the same time, the genetic algorithm and taboo search are combined to form a hybrid algorithm to solve the distribution packing problem. The example shows that the hybrid algorithm combines the advantages of genetic multi starting point and taboo search mountain climbing ability, and the hybrid algorithm performs better than genetic algorithm in 3D packing layout optimization, and can effectively solve the optimization problem of three-dimensional packing layout.
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