OPTIMAL INVENTORY POLICY FOR FAST-MOVING CONSUMER GOODS UNDER E-COMMERCE ENVIRONMENT

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Abstract. Coming up with effective inventory-ordering strategies for fast-moving consumer goods (FMCGs) through online channels has a major characteristic that the goods are promoted frequently. In this paper, a multi-period inventory model is employed wherein each period represents the promotion period, and the inventory level can be adjusted by replenishing or salvaging the inventory at the beginning of each promotion period. A two-threshold ordering policy is proven to be optimal for each promotion period. The benefits of salvaging can be significantly high for decision makers. This study contributes to the literature of inventory management that products are frequently promoted under an e-commerce environment.

1. Introduction. Fast-moving consuming goods (FMCGs) has occupied most of the shelves in the online supermarket. For example, Yihaodian is the earliest and one of the largest online supermarkets in China. Nowadays, it has been fully controlled by Walmart which is the leading traditional retailer in the world. Walmart bought 51% of Yihaodian’s stakes in 2008 and bought out the remaining stakes in 2015. Since then, Yihaodian has become the e-commerce arm of Walmart in China. Yihaodian sells over 180,000 kinds of FMCGs and offers a one-stop-shopping service to its customers. Meanwhile, an operation manager must have an efficient inventory control policy for FMCGs, which is crucial in effectively handling Yihaodian. FMCGs are products that are sold and consumed quickly at a relatively low price. Generally, the shelf life of FMCGs is comparatively shorter because most of these products are not durable and are often consumed in a few days, weeks, or months. Typical FMCGs include dairy goods, toiletries, cigarettes, alcohol, and soft drinks.

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Traditionally, FMCGs are provided in local groceries, supermarkets, or hypermarkets. With the development of e-commerce, FMCGs have come to take a large market share for the products sold in Yihaodian. In fact, FMCGs contribute more than 80% of its sales. Table 1 shows the top 3 SKU numbers of FMCGs categories from Yihaodian. We can see the SKU of FMCGs has abundant variety. However, research about the inventory control problem for FMCGs, which are sold through online channels is rare.

Table 1. SKU of FMCGs from Yihaodian

| Categories                     | 27th Week | 28th Week | 29th Week | 30th Week |
|-------------------------------|-----------|-----------|-----------|-----------|
| Food and beverage             | 12,949    | 12,926    | 12,799    | 12,811    |
| Maternal and infant products  | 5,936     | 5,301     | 5,296     | 5,291     |
| Kitchen and cleaning products | 5,217     | 5,231     | 5,155     | 5,188     |

1 Above data are collected during the period from 2011-6-27 to 2011-7-24.

FMCGs in online channels have four main characteristics. (1) **High inventory turnover rate**: FMCG are goods that are replaced or consumed over a few days, weeks, or months, hence the short shelf life of FMCGs forces retailers to curtail the replenishment cycle. (2) **High promotion frequency**: The promotion frequency is high under an e-commerce environment because of three reasons. First, the promotion setup fee is quite low for online channels. Yihaodian provides free technical support for the online sales promotion of its registered retailers. Second, many promotion methods are existing. On November 2015, Yihaodian employed three promotion strategies: (a) downloading the mobile apps of www.yhd.com, (b) following the Twitter account of Yihaodian, which frequently promotes its products and provides information about these goods, and (c) downloading the shopping client of www.yhd.com to mobile devices. On November 2015, Yihaodian promoted its products three times. From November 1st to 3rd, thousands of products with 50% discount were advertised. In the second period from November 11th to 13th, Yihaodian promoted group buying for hundreds of products with half the original price. In the third period from November 21st to 23th, many popular items were sold for half the original price. During the main stream of sales promotion, small-scale promotional activities on soft drinks, consumer electronics, general daily goods, etc., were carried out. Other e-commerce companies, such as Taobao, Alibaba, and Amazon, employ the same strategies. (3) **Low consumer-switching cost**: Customers who shop online will easily switch to buy goods from another company if their demand is not immediately satisfied by a company. Thus, the unsatisfied demand is a loss for that e-commerce company. (4) **Online retailers are aggressive**: Retailers can negotiate with the suppliers because they can adjust the inventory level after every promotion period. At the end of each promoting period, product managers will negotiate with their suppliers to salvage excess inventory. One frequently used method is returning the remaining goods at a lower price to suppliers.

This approach of selling FMCGs online motivates us to investigate the optimal inventory policy for FMCGs. In this paper, we consider a multi-period problem that a retailer decides order quantity and salvage quantity in each promotion period. Our formulation for the inventory management problem has some similarities with the formulation of capacity adjustment management, e.g., [24], and the stochastic cash balance problem, e.g., [8]. In this paper, we consider the optimal inventory ordering policy for an e-commerce company who can salvage and replenish inventory quickly...
at the beginning of each promotion period. First, the optimal ordering policy is a two-threshold structure \( Q_l(t), Q_h(t) \) that, if the inventory level is less than \( Q_l(t) \), the retailer increases the inventory level to \( Q_l(t) \). If the inventory level is larger than \( Q_h(t) \), the retailer returns the products to the suppliers to reduce the inventory level to \( Q_h(t) \). If the inventory level belongs to the interval \( [Q_l(t), Q_h(t)] \), the retailer keeps the inventory level unchanged. Finally, the sensitivity analysis of the value of the salvage option are presented by numerical experiments.

2. Literature review. Two previous works are closely related to our research, namely, [24] and [8]. Ye and [24] consider a multi-period capacity adjustment problem that the capacity can be dynamically increased or decreased with setup costs. Our model differs from [24]'s model because they consider the perishable inventory with only one period life-time. In our model, the inventory of FMCGs is long-lived and could be carried over. Moreover, we do not consider the capacity constraint. [8] study an inventory problem where cash or inventory changes can be either positive or negative in each period, and the firm can purchase or salvage inventory with fixed and variable costs. Nevertheless, [8] focus on cash balance rather than inventory control and demand satisfaction, which are the focus of our paper.

This problem has been extensively studied in the literature of cash management from the perspective of cash balance rather than demand satisfaction. The latter investigates a model where a company faces a non-stationary stochastic demand and adjusts its capacity to demand over time by purchasing or salvaging capacity. However, the company cannot carry inventory from one period to the next and it only considers the adjustment of capacity level rather than the inventory level.

The first stream of literature related to our model focuses on inventory management, which deals with seasonal products, which may have overstock inventory. [18] considered the return policy for perishable products and showed that the entire supply chain can be coordinated by properly designing the contract. The salvage value that the retailer obtains can be considered the return price ascertained in the contract. The inter-temporal price policy for retail stores selling seasonal products was studies in [3], while [2] considered the pricing policy when the customers are forward-looking. [14] consider the multi-echelon inventory management problem with seasonal products.

The second stream of research is about inventory control with product or component return. [17] proved that (s, S) policy is still optimal when a single-product and finite-horizon inventory system system faces uncertain returns in addition to demand. [13] extended this result to an infinite-horizon case. [10] provided the conditions where a base-stock policy is optimal when the inventory system faces a fixed fraction of return demand. [11] presented the conditions where a single-product and multiple-component assembly system with some used components that are recovered is equivalent to a series system with returns. [12] extended the previous results to a case with multiple products. Some previous works considered a closed-loop supply chain with remanufactured products. [12] focused on a single-product and periodic-review inventory system with various types of returnable products. The researcher presented an optimal manufacturing-remanufacturing disposal policy. [26] extended the results by including pricing decisions, and [15] conducted a comprehensive review on the closed-loop supply chain. However, most works that considered product or component return assumed that certain or uncertain return flows are exogenous. By contrast, the product return in the current paper is an endogenous decision of a retailer. Furthermore, the retailer could employ salvaging
inventory to adjust the inventory. [9] consider the jointly inventory and pricing problems over multi-period with costly adjustment. [6] uses the level crossing techniques to resolve the classic cash balance problem. However, their model settings are different from ours.

Another closely related stream is the research on the FMCG supply chain. [19] empirically studied the effect of retailers’ and its competitor’s decisions on the long-term effectiveness of promoting FMCG categories. The researcher determined that smaller brands are disadvantaged compared with leading brands, and their promotion scheme benefits their competitors more. [4], the researchers quantitatively assessed the effect of RFID technology and electronic product code system in the FMCG supply chain, while [20] considered an FMCG supply chain, in which goods can be stored in a location near the factory or transported to an intermodal terminal where the products can be stocked before being sent to the final destination. The researchers evaluated the performance of a floating stock delivery strategy. However, most of these papers studied FMCGs from a marketing perspective or a purely logistic angle.

The literature in operation management under an e-commerce environment is still in its emerging phase. We recommend [1] and [23] for extensive review. [16] summarized the types of e-commerce their effects on operations management from the conceptual level. Meanwhile, [5] discussed the influence of operational execution on repeated purchasing for heterogeneous customer segments. The relationship between the multi-class inventory control problem and the pricing problem for seasonal products is described in [7] and the references therein.

The rest of this paper is organized into sections. The problem of formulation is presented in Section 3. The structural properties of the expected profit function and the optimal inventory control policy are also analyzed. Section 4 describes the various numerical experiments, and Section 5 concludes this paper.

3. Model. In this section, we consider an inventory model that a retailer promotes products multiple times over a single selling season. The promotion times are indexed by 0, 1, ..., T. The demand between t and t+1 is a random variable $D_t$ where $t = 0, 1, ..., T$. The demands are assumed to be independent of each other. At the beginning of each period, the retailer has to decide the number of products to order or salvage. The purchasing cost per unit is $c_t$, and the salvage cost per unit is $s_t$ at the beginning of period $t$. Ordering and selling can happen instantaneously, i.e., the lead time is assumed to be zero. Positive lead time will not change our results. The exogenous retail price at period $t$ is denoted by $p_t$. We assume the salvage cost decreases with time that $p_t \geq c_t \geq s_t \geq s_{t+1}$. First, salvage cost must be lower than ordering cost to avoid the arbitrage behaviors, i.e., $c_t \geq s_t$. Second, $s_t \geq s_{t+1}$ holds because of discount factor of time. The salvage value decreases through time.

The sequence of events is as follows. At the beginning of the each period, the retailer receives the quantity of products ordered. The demand is realized over the promotion period. At the end of the each period, the leftover inventory is carried to the next period, and the unsatisfied demand is lost. If the retailer wants to replenish some inventories, an ordering cost proportional to order quantity with $c_t$ per unit will incur. Simultaneously, the retailer could salvage some inventory to the supplier and obtain a salvage value proportional to the salvaged quantity with $s_t$ per unit. All the remaining inventories are then assumed to be sold at a salvage value with $s_{t+1}$ per unit at the end of the next period.
Let $x_t$ be the on-hand inventory at the beginning of period $t$. The retailer decides the order-up-to level $y_t$. The inventory dynamics can be written as $x_{t+1} = (y_t - D_t)^+$. At the end of period $t$, a holding cost $h_t^1$ charges for each remaining item and a penalty cost $h_t^2$ for each unsatisfied demand, where $h_t^1 \geq 0$ and $h_t^2 \geq 0$. Thus, the holding and penalty cost is $h_t(x) = h_t^1 x^+ + h_t^2 x^-$, where $a^+$ denotes $\max(a, 0)$ and $a^-$ denotes $\min(a, 0)$. Note that $h_t(x)$ is convex in $x$. We assume $p_{t-1} + h_{t-1}^1 - c_t \geq 0$. This assumption implies that the retailer will satisfy all the demands as much as possible. Suppose the seller has one unit product on hand and one unit demand arrives in period $t - 1$. If the seller does not satisfy the demand at period $t - 1$, the margin $p_{t-1} + h_{t-1}^1$ will be lost and an item with value $c_t$ will be saved. The assumption $p_{t-1} + h_{t-1}^1 - c_t \geq 0$ excludes the trivial case. At the end of last period $T$, the salvage value is $s_{T+1} x$ when there are $x$ items remain. We use $\Pi_t(x)$ to denote the optimal expected overall profit from period $t$ to period $T$.

$$\Pi_t(x) = \max_{y_t} \mathbb{E}\{p_t(y_t \wedge D_t) + s_t(x_t - y_t)^+ - c_t(y_t - x_t)^+ - (y_t - D_t)^+\} + \gamma \Pi_{t+1}((y_t - D_t)^+) \tag{1}$$

where $a \wedge b$ represents $\min(a, b)$ and $\gamma$ is the discount factor, $0 < \gamma \leq 1$. The above equation can be rewritten as

$$\Pi_t(x) = s_t x_t + \max_{y_t} \{p_t(y_t - s_t) y_t - (c_t - s_t)(y_t - x_t)^+ + \mathbb{E}\{-p_t(y_t - D_t)^+ - h_t(y_t - D_t) + \gamma \Pi_{t+1}((y_t - D_t)^+)\}\}$$

Denote $f_t(x) = \Pi_t(x) - s_t x_t$.

$$f_t(x) = \max_{y_t} \{p_t(y_t - s_t) y_t - (c_t - s_t)(y_t - x_t)^+ + J_t(y_t)\} \tag{2}$$

and

$$J_t(y_t) = \mathbb{E}\{-p_t(y_t - s_t + 1)((y_t - D_t)^+ - h_t(y_t - D_t) + \gamma f((y_t - D_t)^+))\} \tag{3}$$

Moreover, since $\Pi_{T+1}(x) = s_{T+1} x$, then $f_{T+1}(x) = 0$. To characterize the optimal ordering policy, we need the following proposition.

**Proposition 1.** For any period $t$, $0 \leq t \leq T$,

(i): $J_t(x)$ and $f_t(x)$ are both concave in $x$.

(ii): $f_{t+1}(x) - (p_t - s_{t+1} + h_t^1) x$ is decreasing in $x$.

**Proof.** We first prove the statement (i). We prove it by backward induction. Clearly, $f_{T+1}(x)$ is concave in $x$, and $f_{T+1}(x) - (p_T - s_{T+1} + h_T^1) x$ is decreasing in $x$. Suppose that $f_{t+1}(x)$ is concave in $x$. Then, $f_{t+1}(x) - (p_t - s_{t+1} + h_t^1) x$ is concave in $x$. Denote $g(x) = f_{t+1}(x) - (p_t - s_{t+1} + h_t^1) x$. Then, $g(x)$ is a decreasing and concave function. Note that for any realized value of $D_t$ and real values $\alpha \in [0, 1]$, $y_1$ and $y_2$, we have

$$g((\alpha y_1 + (1 - \alpha)y_2 - D_t)^+) = g((\alpha (y_1 - D_t) + (1 - \alpha)(y_2 - D_t))^+) \geq g(\alpha(y_1 - D_t)^+ + (1 - \alpha)(y_2 - D_t)^+) \geq \alpha g((y_1 - D_t)^+) + (1 - \alpha) g((y_2 - D_t)^+)$$

which implies that $g((x - D_t)^+)$ is concave in $x$. Here the first inequality is attributed to the decreases of $g(x)$ and $(a + b)^+ \leq a^+ + b^+$ for any real values $a$ and $b$. The second inequality holds because $g(x)$ is concave. Based on Equation (3) and the fact that $h^2_t(x - D_t)^-$ is convex in $x$, then $J_t(y) = \mathbb{E}\{-h^2_t(y - D_t)^- + g((y - D_t)^+)\}$ is concave in $y$. 

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Next, $f_t(x)$ is proved to be concave in $x$ if $J_t(y)$ is a concave function of $y$. Suppose Equation (2) attains the maximum at $y = y_t$ for $x = x_i$, $i = 1, 2$, respectively. For any real value, $\alpha \in [0, 1],$

$$\alpha f_t(x_1) + (1 - \alpha) f_t(x_2)$$

$$= \alpha [(p_t - s_t)y_1 - (c_t - s_t)(y_1 - x_1) + J_t(y_1)]$$

$$+ (1 - \alpha) [(p_t - s_t)y_2 - (c_t - s_t)(y_2 - x_2) + J_t(y_2)]$$

$$= (p_t - s_t)[\alpha y_1 + (1 - \alpha)y_2] - (c_t - s_t)[\alpha(x_1 - y_1) + (1 - \alpha)(y_2 - x_2)]$$

$$+ \alpha J_t(y_1) + (1 - \alpha) J_t(y_2)$$

$$\leq (p_t - s_t)[\alpha y_1 + (1 - \alpha)y_2] - (c_t - s_t)\alpha y_1 + (1 - \alpha)y_2$$

$$- \alpha(x_1 - (1 - \alpha)x_2) + J_t(\alpha y_1 + (1 - \alpha)y_2)$$

$$\leq f_t(\alpha x_1 + (1 - \alpha)x_2).$$

where the first inequality is attributed to the fact that $J_t(y)$ is concave function and $(a + b)^+ \leq a^+ + b^+$. The second inequality holds by the definition of $f_t(x)$. Thus, $f_t(x)$ is concave in $x$.

Finally, we prove the statement (ii) that $f_{t+1}(x) - (p_{t+1} - s_{t+1} + h_{t+1}^1)x$ is decreasing in $x$. Note that $(p_t - s_t)(y - c_t) - (p_{t-1} - s_t + h_{t-1}^1)x = (p_t - c_t)y - (p_{t-1} - s_t + h_{t-1}^1)x$ is decreasing in $x$ if $x \geq y$. Hence, for any $y$, $(p_t - s_t)(y - c_t) - (p_{t-1} - s_t + h_{t-1}^1)x + J_t(y)$ is decreasing in $x$. Therefore, $f_{t+1}(x) - (p_{t+1} - s_{t+1} + h_{t+1}^1)x = \max_{y \geq 0}\{(p_t - s_t)y - (c_t - s_t)(y - x) + J_t(y)\}$ is decreasing in $x$. Thus, the result holds for period $t$, $0 \leq t < T$. The proof is complete. 

Based on the concavity of $J_t(y)$, both the left-hand derivative $J'_t(y^+)$ and the right-hand derivative $J'_t(y^-)$ can be determined. Define $Q_t(y) = \sup\{y \geq 0 : J'_t(y^-) \geq c_t - p_t\}$, $Q_h(t) = \inf\{y \geq 0 : J'_t(y^+) \leq s_t - p_t\}$. Let $H_t(x, y) = (p_t - s_t)yx - (c_t - s_t)(y - x) + J_t(y)$, then $f_t(x) = \max_{y \geq 0} H_t(x, y)$. In particular, if the initial inventory of the first period is zero, the optimal ordering quantity of the first period is defined as $Q_0 = Q_{10}(t)$. With above definitions, we can present the optimal ordering policy as follows.

**Theorem 3.1.** Given the inventory level $x$ at the beginning of period $t$, $Q_t(x) < Q_h(t)$ such that the optimal ordering policy has the following structure.

(i): If $x < Q_t(x)$, the retailer increases the inventory level to $Q_t(x)$.

(ii): If $x > Q_h(t)$, the retailer decreases the inventory level to $Q_h(t)$.

(iii): If $x \in (Q_t(x), Q_h(t))$, the retailer orders nothing.

**Proof.** By concavity of $J_t(y)$, both $J'_t(y^+)$ and $J'_t(y^-)$ could be determined. For any $y_1 < y_2$, $J'_t(y_1^-) \geq J'_t(y_1^+) \geq J'_t(y_2^-) \geq J'_t(y_2^+)$. Define $Q_t(y) = \sup\{y \geq 0 : J'_t(y^-) \geq c_t - p_t\}$, and $Q_h(t) = \inf\{y \geq 0 : J'_t(y^+) \leq s_t - p_t\}$. By $c_t \geq s_t$ and above argument, we know $Q_t(t) \leq Q_h(t)$. Moreover, $(p_t - c_t)y + J_t(y)$ is increasing in $y$ if $y \leq Q_t(t)$ and decreasing in $y$ if $y > Q_t(t)$. $(p_t - s_t)y + J_t(y)$ is decreasing in $y$ if $y \leq Q_h(t)$ and increasing in $y > Q_h(t)$.

**Case 1.** Under condition $x < Q_t(t)$, then $H_t(x, y) = (p_t - c_t)y + (c_t - s_t)x + J_t(y)$ is increasing in $y$ if $y > Q_t(t)$. While, if $y \leq Q_t(t)$,

$$H_t(x, y) = \begin{cases} (p_t - c_t)y + (c_t - s_t)x + J_t(y), & \text{if } x < y \leq Q_t(t) \\ (p_t - s_t)y + J_t(y), & \text{if } y \leq x \end{cases}.$$

is increasing in $y$. Hence, $y = Q_t(t)$ is the optimal solution.
Case 2. Under the condition $x > Q_h(t)$, if $y \leq Q_h(t)$, then $H_t(x, y) = (p_t - s_t)y + J_t(y)$ is increasing in $y$. If $y \geq Q_h(t)$, we have

$$H_t(x, y) = \begin{cases} (p_t - s_t)y + J_t(y), & \text{if } Q_h(t)y < x; \\ (p_t - c_t)y + (c_t - s_t)x + J_t(y), & \text{if } y \geq x. \end{cases}$$

is decreasing in $y$. Hence, $y = Q_h(t)$ is the optimal solution.

Case 3. Under the condition $Q_l(t) \leq x \leq Q_h(t)$, if $y \leq x$, $H_t(x, y) = (p_t - s_t)y + J_t(y)$ is increasing in $y$ because $y \leq Q_h(t)$. If $y > x$, then $H_t(x, y) = (p_t - c_t)y + (c_t - s_t)x + J_t(y)$ is increasing in $y$ because $y > Q_l(t)$. Thus, the optimal decision is to order nothing. The proof is complete.

Based on Theorem 3.1, the optimal ordering policy can be characterized by two thresholds $Q_l(t)$ and $Q_h(t)$. However, the thresholds are state dependent. We propose the following Proposition to describe the properties of the thresholds.

**Proposition 2.** For any period $t$, $0 \leq t \leq T$,

(i): Both $Q_l(t)$ and $Q_h(t)$ increase with $p_t$.

(ii): $Q_l(t)$ decreases with $c_t$ and $Q_h(t)$ is independent of $c_t$.

(iii): $Q_l(t)$ increases with $s_t$ and $Q_h(t)$ decreases with $s_t$.

(iv): Both $Q_l(t)$ and $Q_h(t)$ decreases with $h_t^1$ and increases with $h_t^2$.

(v): $\Pi_t(x)$ increases with $p_t$, $s_t$, and decreases with $c_t$, $h_t^1$, and $h_t^2$.

**Proof.**

• Proof of the statement (i): Based on the formulation of $J_t(y)$ and $f_{t+1}(x)$ is independent of $p_t$, $\frac{\partial}{\partial p_t} J_t(y) = \frac{\partial}{\partial p_t} (p_t - c_t + f_{t+1}(y)) = 1 - F_D(y) \geq 0$ where $F_D$ is the cumulative distribution function of demand $D$. Thus, $(p_t - c_t) + J_t'(y^-)$ is increasing in $p_t$. Similarly, $(p_t - c_t) + J_t'(y^+)$ is increasing in $p_t$ as well. Then, based on the definition of $Q_l(t)$ and $Q_h(t)$ that $Q_l(t) = \sup \{ y \geq 0 : J_t'(y^-) \geq c_t - p_t \}$, $Q_h(t) = \inf \{ y0 : J_t'(y^+) \leq s_t - p_t \}$. Notice that $J_t'(y^-)$ and $J_t'(y^+)$ are decreasing in $y$. Therefore, $Q_l(t)$ and $Q_h(t)$ increase in $p_t$.

• Proof of the statement (ii): Note that $J_t(y)$ is independent of $c_t$. Thus, by definitions of $Q_l(t)$ and $Q_h(t)$ and the fact that $J_t'(y^-)$ is decreasing in $y$, $Q_l(t)$ is known to decrease in $c_t$ and $Q_h(t)$ is independent of $c_t$.

• Proof of the statement (iii): Note that $J_t(y)$ is independent of $s_t$. According to the definition of $x_1$ and the fact that $J_t'(y^-)$ decreases in $y$, $Q_l(t)$ is known to be independent of $s_t$. Note that $J_t'(y^-) + p_t - s_t$ decreases in $s_t$. Thus, based on the definitions of $Q_h(t)$ and the fact that $J_t'(y^+)$ decreases in $y$, then $Q_h(t)$ decreases in $s_t$.

• Proof of the statement (iv): Note that $h_t(x) = h_t^1 x^+ + h_t^2 x^-$, both $J_t'(y^-)$ and $J_t'(y^+)$ decrease in $h_t^1$ and $h_t^2$. Thus, based on the definitions of $Q_h(t)$ and the fact that $J_t'(y^-)$ and $J_t'(y^+)$ decrease in $y$, then Both $Q_l(t)$ and $Q_h(t)$ decreases with $h_t^1$ and increases with $h_t^2$.

• Proof of the statement (v): Based on Equation (1) by substituting $t$ with $t + 1$, $\Pi_{t+1}(x)$ is determined to be independent of parameters $p_t$, $c_t$, $s_t$, $h_t^1$, $h_t^2$. According to Equation (1) and $h_t(x) = h_t^1 x^+ + h_t^2 x^-$, $\Pi_t(x)$ increases in $p_t$, $s_t$, and decreases with $c_t$, $h_t^1$, and $h_t^2$. The proof is complete.

To evaluate the value of salvage option, we consider a basic setting that the retailer only decides the ordering quantity at the end of each promotion period but does not have salvage option. The basic setting is the standard multi-period periodic review inventory system. The optimal ordering policy is known as a base-stock type. In such a case, we denote the value function at period $t$ with inventory
x as \( \Pi'_t(x) \). The dynamic programming equation of the basic setting is as follows.

\[
\Pi'_t(x) = \max_{y_t} E \left\{ p_t(y_t \wedge D_t) - c_t(y_t - x_t)^+ - h_t(y_t - D_t) + \gamma \Pi'_{t+1}((y_t - D_t)^+) \right\}
\]

and \( \Pi'_{T+1}(x) = s_{T+1}x \).

**Proposition 3.** For any period \( t, 0 \leq t \leq T \), \( \Pi_t(x) \geq \Pi'_t(x) \) holds. It is always beneficial to provide the option of salvage.

**Proof.** The result is proven by the backward induction. Obviously, the result in the last period holds, \( \Pi_{T+1}(x) = \Pi'_{T+1}(x) = s_{T+1}x \). Suppose that \( \Pi_{t+1}(x) \geq \Pi'_{t+1}(x) \). Then,

\[
\Pi'_t(x) = \max_{y_t \geq x_t} E \left\{ p_t(y_t \wedge D_t) - c_t(y_t - x_t)^+ - h_t(y_t - D_t) + \gamma \Pi'_{t+1}((y_t - D_t)^+) \right\}
\]

and

\[
\Pi'_t(x) = \max_{y_t \geq x_t} E \left\{ p_t(y_t \wedge D_t) + s_t(y_t - x_t)^- - c_t(y_t - x_t)
- h_t(y_t - D_t) + \gamma \Pi'_{t+1}((y_t - D_t)^+) \right\}
\]

By induction, the proof is complete.

4. **Numerical studies.** We study our model numerically and have the following results. First, we use an example to show the path of two thresholds change over time horizon. Next, we numerically test the impact of the initial inventory, the salvage value, the price elasticity, the holding cost, the penalty cost, and the discount factor. Our results are robust under varying parameters. Third, we use numerical examples to study the effect of salvaging frequency. The results show that the decreasing salvaging frequency indicates that the replenishment decisions between the interval of adjusting inventory increase. The mismatch of uncertain demand and on-hand inventory has been smoothed by the inventory replenishment decisions in the interval.

4.1. **Two-threshold policy structure.** Although model parameters \( p_t, s_t, c_t, h_t^1 \) and \( h_t^2 \) can change with time, the parameters used in this section do not vary with time. For the basic model, the parameters are set as follows: period number \( T = 10 \), selling price \( p_t = p = 5 \), salvage value \( s_t = s = 2.5 \), purchase cost \( c_t = c = 3 \), holding cost \( h_t^1 = 1 \) and the penalty cost \( h_t^2 = 4 \) where \( t = 0, 1, ..., T \). Moreover, the salvage value at the end of the sales horizon is assumed to be also \( s \), i.e., \( s_{T+1} = s \) and the discount factor is \( \gamma = 0.8 \). When the sensitivity of one parameter is analyzed, other parameters are assumed to be unchanged. The demands in different periods are assumed to be independent and identically distributed with the following discrete distribution:

\[
D = \min(\max(\lfloor 30z \rfloor + 10, 0), 100),
\]

where \( z \sim N(0, 1) \), and \( \lfloor y \rfloor \) denotes the largest integer smaller than or equal to \( y \). In the basic setting, Figure 1 shows the variation of the two thresholds in time.
4.2. Sensitivity analysis. In this subsection, we determine how the benefit of salvaging $\Delta$ varies with initial inventory, salvage cost, price elasticity, holding cost, penalty cost and discount factor. The results are shown in Figure 2.

Figure 2. The relationship of the benefit of salvaging $\Delta$ and model parameters
Figure 2(a) shows that the variation in benefits of salvaging $\Delta$ depending on the initial inventory level $x_0$. The figure indicates that when the initial inventory level is low, the option of salvaging does not seem to be advantageous. However, with increasing initial inventory level, the option of salvaging begins to demonstrate its merits. Still, salvaging could be beneficial because it provides the retailer an opportunity to reduce inventory to avoid unnecessary holding cost. If the initial inventory is too high, the retailer must take the excessive inventory over the selling season and pay the additional holding cost. Thus, the benefit of salvage increases with the initial inventory. From Figure 2(b) to Figure 2(f), three cases are considered where the initial inventory $x_0$ is equal to 60, 80, or 100. The benefit of salvaging, which increases with the initial inventory level, could be observed in Figure 2(b), Figure 2(c), and Figure 2(d). The results are consistent with the observation in Figure 2(a).

First, salvage value $s$ varies from zero to three, which is increased by 0.1. Note that $s = 0$ corresponds to the extreme case where salvaging is not profitable and $s = c = 3$ corresponds to the extreme case, in which the retailer can adjust the inventory level to maximize the overall expected profit by maximizing the expected profit of the current period. Figure 2(b) shows how the benefit of salvaging $\Delta$ varies with salvaging value $s$ under different levels of initial inventory. The figure demonstrates that the benefit of salvaging increases with the increase in salvaging value $s$. Aside from the flexibility of adjusting inventory level brought by the salvage option, increasing the salvage value also enhances the advantage of the salvage option.

Next, the effect of pricing strategy is determined. In the previous discussion, the retailer is assumed to use a fixed-price policy, which sets each product to five. Now, the retailer is assumed to employ a markdown pricing strategy. Particularly, $p_t = p_0 - et$, where $p_0 = 5 + 5.5e$ and allow price elasticity $e$ to vary from zero to one, which is increased by 0.1. The prices are chosen, such that the overall average price is five. Figure 2(c) shows that the benefit of salvaging $\Delta$ increases with price elasticity $e$. Higher price elasticity leads to a lower holding value for the inventory. Thus, the value of salvaging increases with price elasticity.

For holding cost, holding cost $h^1_t = h^1$ varies from zero to two, increased by 0.1. Figure 2(d) indicates that the benefit of salvaging $\Delta$ increases with holding cost $h^1$. The higher the inventory holding cost, the higher the value of the chance of salvaging. For penalty cost, $h^2_t = h^2$ is set to vary from four to five, which is increased by 0.1. Figure 2(e) shows that the benefit of salvaging is insensitive to penalty cost $h^2$. Penalty cost is associated with the tradeoff of demand loss and on-hand inventory, which is unrelated with the option of reducing inventory. Thus, salvaging is beneficial at this instance.

Finally, discount factor $\gamma$ is set to vary from zero to one, which is increased by 0.1. Note that $\gamma = 0$ corresponds to the extreme case, in which a retailer makes a myopic optimal decision and $\gamma = 1$ corresponds to the benchmark case. Figure 2(f) indicates that the benefit of salvaging, $\Delta$, is insensitive to the discount factor when the initial inventory is low and sensitive to the discount factor when the initial inventory is high. The advantage of salvaging is associated with the value of inventory. If the discount factor is high, the retailer prioritizes the control of on-hand inventory. Hence, the value of salvaging option improves. However, when the initial inventory level is low, the value of salvaging option converges to zero, and the effect of the discount factor is less evident.
4.3. Effect of salvaging frequency. The salvaging activities may not be carried out in each period. Suppose that the salvage activities can be performed at period \(kt, t = 1, 2, \cdots, \left\lfloor \frac{T}{kT} \right\rfloor\). We let \(k = 1, 2, 5, \text{ and } 10\). Here, \(k = 1\) corresponds to the basic model, in which the retailer can employ salvaging at each promotion period. Here, \(k = 2\) denotes that a retailer can use the salvaging option once every two periods, and \(k = 5\) means that the retailer can salvage once every five periods.

Table 2 shows that the benefit of salvaging increases with salvaging frequency and initial inventory. If the interval increases each time the salvaging option is used, the improvement decreases dramatically. Table 2 shows that the timely inventory adjustment operation could significantly increase the profit of the retailer. Comparatively, the decreasing salvaging frequency indicates that the replenishment decisions between the interval of adjusting inventory increase. The mismatch of uncertain demand and on-hand inventory has been smoothed by the inventory replenishment decisions in the interval.

| \(k\) | 1     | 2     | 5     | 10    |
|------|-------|-------|-------|-------|
| \(x_0 = 60\) | 4.5483 | 2.0535 | 0.1946 | 0.0085 |
| \(x_0 = 80\) | 32.6130 | 16.7936 | 2.4027 | 0.0760 |
| \(x_0 = 100\) | 84.8075 | 49.5771 | 9.9460 | 0.3748 |

4.4. Effect of correlation demands. In the basic model, the main results are build on the assumption that the demand distribution of each period are identical and independent of each other. In this subsection, we consider a more complicated cases that the demands are correlated. For simplification, we still assume the parameters are stationary that they will not change with time. The parameters are set as follows: period number \(T = 5\), selling price \(p = 5\), salvage value \(s = 2.5\), purchase cost \(c = 3\), holding cost \(h_1^t = 1\) and the penalty cost \(h_2^t = 4\) where \(t = 0, 1, \ldots, T\). The discount factor is set as \(\gamma = 0.8\). In addition, we set the initial inventory as \(x_0 = 100\). Since we only consider two periods, we use bivariate normal distribution to generate the demand sequence for simulation. To avoid the negative demand cases, we normalize the demands as follows:

\[
D = \min(\max(30z_k + 50, 0), 200),
\]

where \((z_k, z_{k+1})\) follows a standard bivariate normal distribution where \(k = 1, 2, \ldots, T\), and \((z_k, z_{k+1}) \sim N(Mu, Sigma)\) where \(Mu = [0, 0]\) and \(Sigma = [1, \tau; \tau, 1]\) where \(\tau\) indicates the value of covariance. In the basic setting, Figure 1 shows the variation of the two thresholds over time. We tried extensive numerical experiments and find the structures of optimal policy under every sample paths are still two thresholds type. We show how the average gap between the optimal thresholds varies with the covariance in the following table.

Table 3 indicates that the average gap between two thresholds are very robust among the various covariance values. In particular, we find the average gap decreases with the number of period. In other words, the optimal thresholds get closer if the decision epoch increases. It also implies the optimal policy performs more precise for a longer time horizon.
Table 3. Average gap between two thresholds varies with covariance

| τ       | T = 4 | T = 10 | T = 20 | T = 100 |
|---------|-------|--------|--------|---------|
| 0.99    | 7.50  | 7.60   | 7.05   | 7.34    |
| 0.50    | 7.50  | 7.20   | 7.35   | 7.17    |
| 0.00    | 7.25  | 7.10   | 6.80   | 7.00    |
| -0.50   | 7.00  | 7.60   | 7.10   | 6.99    |
| -0.99   | 7.50  | 7.70   | 6.80   | 6.97    |

5. Conclusions. In this paper, we model the procurement of FMCGs sold by an e-commerce company as a two-period inventory problem, in which the products have salvage value at the end of both periods. The salvage value at the end of the first period is larger than that at the end of the second period. We prove that the optimal ordering policy has a two thresholds structure. Finally, numerical examples are employed to examine the relationship between the benefit of salvaging and model parameters. Future studies could determine whether the retailer and the supplier can design a contract to coordinate the whole supply chain and how the parameters should be designed, particularly the sharing of salvage value between the supplier and the retailer.

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