Study of effect of magnetohydrodynamics and couple stress on steady and dynamic characteristics of porous exponential slider bearings

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Abstract: Exponential slider bearings with porous facing is analysed in this article. The modified Reynolds equation is derived for the Exponential porous slider bearing with MHD and couple stress fluid. Computed values of Steady film pressure, Steady load capacity, Dynamic stiffness and Damping coefficient are presented in graphical form. The Steady film pressure, Steady load capacity, Dynamic stiffness and Damping coefficient decreases with increasing values of permeability parameter and increases with increasing values of couplestress parameter and Hartmann number.

1. Introduction
In engineering practice, slider bearings are designed for supporting transverse load. Cameron [1] suggested an exponential form of slider bearing as the nearest shape to bend owing to elastic, thermal or uneven wear effects. Based upon the thin lubrication theory, for an exponential film shape, the characteristics of wide slider bearing have been studied by Lin and Hung [2]. They stated for greater values of the inlet-outlet film ratio, the steady and dynamics of characteristics of wide slider bearing is significantly high when compared with inclined plane bearing. Also, Lin et al [3] proposed the dynamic characteristics of the same bearing lubricated with non-Newtonian couplestress fluid on the account of Stokes microcontinuum model [4], which permits for the presence of body couples, couple stresses and non-symmetric tensors. They noticed that with increasing couplestress parameters, the dynamic characters are more pronounced.

As is well known, Magneto-hydrodynamic (MHD) deals the dynamics of the flow of electrically conducting fluid (CF) in the presence of a magnetic field. On the basis of MHD, Lin and Lu [5] investigated the dynamic thin film characteristics of a wide exponential slider bearing with non-Newtonian fluid. They suggested that the applied magnetic field provides a significant increase in the load-carrying capacity, the stiffness coefficient and the damping coefficient when compared with non-conducting lubricant (NCL).

Advances in technology, many researchers have investigated the effect of MHD on characteristics of conducting couplestress lubricant film bearing such as for different types Finite plates by Fathima et al [6] and Naduvinamani et al [7,8]. The results obtained in these studies showed that the load carrying capacity increases as increasing the strength of applied magnetic field. Also found that there is a significant increase in load with increase in the value of couplestress parameter. Taking these effects in
to account, the steady and dynamic characteristics for Plane Slider bearing studied by Hanumagowda [9], Rayleigh Step slider bearing by Naduvinamani et al [10]. They observed that the applied magnetic field and the presence of couplestresses in fluid provide higher values for static and dynamic characteristics of the bearing. More recently, Naduvinamani et al [11] discussed the dynamic characteristics of Slider bearing considering MHD couple stresses for the exponential film shape. Bhat [12] analysed the hydro-dynamic lubrication of porous slider bearing considering an exponential film profile. Use of porous matrix decreased the load capacity and friction force on the slider bearing. In order to improve these defects, the steady and dynamic characteristics of exponential porous slider bearing lubricated with electrically conducting couple stress fluid in the presence of transverse magnetic field is presented theoretically.

2. Mathematical formulation

The configuration of the MHD porous slider bearing lubricating with couplestress fluid exponentially shaped under the investigation is shown in figure 1. The bearing has length L, breadth N, where L<<N .It moves with a uniform velocity U in the x-direction and a transverse applied magnetic field \( B_0 \) which is a function of x. The stator lies along the x-axis and has a porous matrix of uniform thickness \( \delta \) and a squeezing velocity \( dh/dt \), where \( t \) is the time in the y-direction. For an exponentially shaped slider bearing, the film thickness \( h(x,t) \) is expressed by

\[
h(x,t) = h_0(t) \exp \left\{-\frac{x}{L} \ln \left( \frac{h_1(t)}{h_m(t)} \right) \right\}, \quad \text{where } 0 \leq x \leq L \text{ and } h_1(t) = d + h_m(t)
\]

In the above relation, \( h_1(t) \), \( h_m(t) \) and \( d \) are the steady inlet, outlet film thickness, and shoulder height respectively.

It is also assumed that (i) the fluid film is thin, (ii) an induced magnetic field is small compared with the applied magnetic field, (iii) inertia effects are ignored compared to viscous force, and (iv) the Lorentz force are the only body forces acting on the fluid.

The appropriate equations govern MHD flow for a couplestress fluid can be written as:

\[
\begin{align*}
\mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} - \sigma B_0^2 u &= \frac{\partial p}{\partial x} + \sigma E_z B_0 \\
\frac{\partial p}{\partial y} &= 0 \\
\frac{\partial u}{\partial x} + \frac{\partial \nu}{\partial y} &= 0 \\
\int_{z=0}^{h} (E_z + B_0 \nu) \, dy &= 0
\end{align*}
\]

Where \( \mu, \eta, \sigma, B_0 \) and \( E_z \) present viscosity of lubricant, a constant associated with the couplestress, electrical conductivity, applied magnetic field and an induced electric field respectively.

The boundary conditions for the fluid components of velocity are given as:

\[
\text{at } y = h: \quad u = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0, \quad \nu = \frac{\partial h}{\partial t};
\]
at $y = 0 : u = U, \quad \frac{\partial^2 u}{\partial y^2} = 0, \quad v = v^*$. 

In the porous matrix, the components of velocity $u^*$ and $v^*$ follows Darcy's law are given as:

$$u^* = -\frac{k}{\mu(1 - \beta)} \left( \frac{\partial p^*}{\partial x} + \sigma E_B B_0 \right), \quad \frac{\partial u^*}{\partial y} = 0,$$  \label{eq:7}

$$v^* = -\frac{k}{\mu(1 - \beta)} \frac{\partial p^*}{\partial y}, \quad \frac{\partial v^*}{\partial x} = 0. \quad \label{eq:8}$$

In the above, the symbols $p^*$ and $k$ represent the pressure distribution and the permeability parameter in the porous region respectively, $\delta$ is the porous layer thickness and $\beta$ is the ratio of microstructure size to pore size $\left(\frac{\eta}{\mu}/k\right)$.

**Figure1.** Geometry of exponential porous slider bearing lubricated with an electrically conducting fluid.

The boundary conditions for the pressure $p^*$ in the porous matrix one can consider as

$$\frac{\partial p^*}{\partial y} \bigg|_{y=0} = 0, \quad \text{(Solid blocking)} \quad \label{eq:9}$$

$$p^*(x,0) = p(x,0). \quad \text{(Continuity of pressure at the interface)} \quad \label{eq:10}$$

The continuity equation in the porous region is given by

$$\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} = 0 \quad \label{eq:11}$$
Using the velocity components $u^*$ and $v^*$ in Eq. (11) and integrating once with respect to $y$ over the porous layer thickness with conditions (9) and (10), gives

$$v^*\big|_{y=0} = \frac{k\delta}{\mu c^2} \frac{\partial}{\partial x} \left( \frac{\partial p^*}{\partial x} + \sigma E_z B_0 \right)$$

(12)

where $c = \left( 1 - \beta + \frac{k\sigma B_0^2}{\mu m} \right)^{1/2}$

Solving (1) subject to the boundary condition (5) and (6), the expression for the velocity component one can express as

$$u = -\frac{U}{2} g_1 - \frac{h_{w0}}{2l\mu M_0^2} \frac{\partial p}{\partial x} g_2$$

(13)

where

$$g_1 = g_{11} - g_{12}, \quad g_2 = g_{13} - g_{14} \quad \text{for} \quad 4M_0^2 I^2 / h_{w0}^2 < 1$$

(14a)

$$g_1 = g_{21} - g_{22}, \quad g_2 = g_{23} - g_{24} \quad \text{for} \quad 4M_0^2 I^2 / h_{w0}^2 = 1$$

(14b)

$$g_1 = g_{31} - g_{32}, \quad g_2 = g_{33} - g_{34} \quad \text{for} \quad 4M_0^2 I^2 / h_{w0}^2 > 1$$

(14c)

In the above, $M_0$ represents the Hartmann number defined as $M_0 = B_0 h_{w0} \left( \sigma / \mu \right)^{1/2}$ and $l$ represents couplestres parameter defined as $l = \sqrt{n/\mu}$.

The MHD Reynold’s equation for porous slider bearing with exponential profile lubricated with non-Newtonian fluid one can obtained as

$$\frac{\partial}{\partial x} \left( f(h,l,M_0) \frac{\partial \bar{h}}{\partial x} \right) = 6U \frac{\partial \bar{h}}{\partial t} + 12 \frac{\partial \bar{h}}{\partial t}$$

(15)

where

$$f(h,l,M_0) = \begin{cases} \frac{6h_{w0}^2}{\mu M_0^2} \left( \frac{A^2 - B^2}{B} \right) \left( \frac{1 + k\delta M^2}{h_{w0}^2} \right) - \frac{2l}{h} & \text{for} \quad 4M_0^2 I^2 / h_{w0}^2 < 1 \\
\frac{6h_{w0}^2}{\mu M_0^2} \left( \frac{2 \left( \cosh \left( h/\sqrt{2l} \right) + 1 \right)}{3 \sqrt{2} \sinh \left( h/\sqrt{2l} \right) - h/l} \right) \left( \frac{1 + k\delta M^2}{h_{w0}^2} \right) - \frac{2l}{h} & \text{for} \quad 4M_0^2 I^2 / h_{w0}^2 = 1 \\
\frac{6h_{w0}^2}{\mu M_0^2} \left( \frac{M_0 \left( \cos B_h + \cos A_h \right) h_{w0}^2}{h_{w0}^2 (A \sin B_h + B_h \sin A_h) \left( \frac{1 + k\delta M^2}{h_{w0}^2} \right) - \frac{2l}{h}} \right) & \text{for} \quad 4M_0^2 I^2 / h_{w0}^2 > 1 \end{cases}$$

(16)

Introducing dimensionless physical quantities and parameters expressed as follows

$$x^* = \frac{x}{L}, \quad t^* = \frac{t}{h_{w0}}, \quad P^* = \frac{p^* h_{w0}^2}{\mu U L}, \quad I^* = \frac{2l}{h_{w0}}, \quad h^*(x^*, t^*) = h_{w0}(x^*) \exp \left\{ -x^* \ln \left( d^* + 1 \right) \right\}$$

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and \( c = \left( 1 - \beta + \frac{k\sigma B_0^2}{\mu m} \right)^{1/2} \)

Substituting above values in (15) one can express

\[
\frac{\partial}{\partial x^*} \left\{ f(h^*, l^*, \psi, M_0) \frac{\partial P}{\partial x^*} \right\} = 6h^*_m(l^*) \frac{d}{dx^*} \left[ \exp \left\{ -x^* \ln (d^* + 1) \right\} \right] + 12 \exp \left\{ -x^* \ln (d^* + 1) \right\} V^*
\]

where

\[
f^*(h^*, l^*, \psi, M_0) = \begin{cases} 
\frac{12h^*}{l^* M_0^2} \left[ \frac{(A^2 - B^2)}{2} \right] & \text{for } M_*^2 l^2 < 1 \\
\frac{12h^*}{l^* M_0^2} \left[ \frac{1 + \cosh \left( \sqrt{2h^*} / l^* \right)}{3 \sqrt{2} \sinh \left( \sqrt{2h^*} / l^* \right) - (h^* / l^*)} \right] & \text{for } M_*^2 l^2 = 1 \\
\frac{12h^*}{l^* M_0^2} \left[ \frac{M_0 \cosh \left( h^* / l^* \right)}{A^2 \sinh \left( h^* / l^* \right) + B^2 \sinh \left( A^2 / h^* \right)} \right] & \text{for } M_*^2 l^2 > 1 
\end{cases}
\]

The MHD film pressure in dimensionless form is obtained by integrating the MHD dynamic Reynold’s equation twice with respect to \( x^* \) under the following conditions:

\( P^* = 0 \) at \( x^* = -1 \) and \( x^* = 0 \)

Namely,

\[
p^* = \left[ 6h^*_m(l^*) - \frac{12V^*}{\ln (d^* + 1)} \right] I_1(0, x^*) + \left[ \frac{6h^*_m(l^*) - \frac{12V^*}{\ln (d^* + 1)}}{I_2(0, -1)} \right] I_2(0, x^*) \tag{17}
\]

Where

\[
I_1(0, x^*) = \int_{x^*}^{\infty} \frac{1}{f^*(h^*, l^*, \psi, M_0)} dx^* \quad ; \quad I_1(0, -1) = \int_{x^*}^{\infty} \frac{1}{f^*(h^*, l^*, \psi, M_0)} dx^* \tag{18a}
\]

\[
I_2(0, x^*) = \int_{x^*}^{\infty} \frac{1}{f^*(h^*, l^*, \psi, M_0)} dx^* \quad ; \quad I_2(0, -1) = \int_{x^*}^{\infty} \frac{1}{f^*(h^*, l^*, \psi, M_0)} dx^* \tag{18b}
\]

Let \( W \) represents the friction force on the slider bearing and its dimensionless expression can be written as:

\[
W^* = \frac{Wh^*_m}{\mu UL^2 B_0} = - \int_{x^*}^{\infty} p^* dx^*
\]

Substitution for \( p^* \) and integration gives
The MHD steady load carrying capacity on porous slider bearing in dimensionless form is reduced as from the equation (17) can reduce to form:

\[ W^* = \left\{ 6h_m^*(\tau) - \frac{12V^*}{\ln(d^* + 1)} \right\} I_1(0, -1) - \left\{ \frac{6h_m^*(\tau) - \frac{12V^*}{\ln(d^* + 1)}}{I_2(0, -1)} \right\} I_4(0, -1) \]  

\[ I_3(0, -1) = -\int_{0}^{\infty} \int_{0}^{x} \frac{\exp\left\{ -x^* \ln\left( d^* + 1 \right) \right\}}{f^*(h^*, l^*, \psi, M_h)} dx^* dx^* \]  

The MHD dynamic stiffness coefficients in dimensionless form can be expressed as:

\[ K^*_d = \left( \frac{\partial W^*}{\partial h_m^*} \right) = 6h_m^*(\tau) I_5(0, -1) + 6I_1(0, -1) - 6 \left\{ \frac{I_2(0, -1) I_4(0, -1) - \left( \frac{h_m^*(\tau)}{l_1(0, -1)} \right) I_4(0, -1)}{I_5(0, -1)} \right\} I_4(0, -1) \]

\[ I_5(0, -1) = -\int_{0}^{\infty} \int_{0}^{x} \frac{\exp\left\{ -x^* \ln\left( d^* + 1 \right) \right\}}{f^*(h^*, l^*, \psi, M_h)} dx^* dx^* \]

In the above discussion, we have
\[
\frac{\partial f^*(h^*,l^*,M_0)}{\partial h^*_{m}} = 12\exp\{-x^* \ln (d^* + 1)\} \left\{ \xi_1 \left( 2h^* + \frac{\psi M_0^2}{c^2} \right) - \xi_2 \left( h^* + \frac{\psi M_0^2}{c^2} \right) - \Gamma \right\} (24)
\]

\[
\xi_1 = \xi_{11}, \quad \xi_2 = \xi_{12} \text{ for } M_0^2 l^2 < 1
\]

\[
\xi_1 = \xi_{21}, \quad \xi_2 = \xi_{22} \text{ for } M_0^2 l^2 = 1
\]

\[
\xi_1 = \xi_{31}, \quad \xi_2 = \xi_{32} \text{ for } M_0^2 l^2 > 1
\]

The associated relations for the expressions (25a),(25b) and (25c) are as follows:

\[
\xi_{11} = \frac{(A^2/B^2) \tanh (B^* h^*/l^*) - (B^2/A^2) \tanh (A^* h^*/l^*)}{A^2 - B^2} (26a)
\]

\[
\xi_{12} = \frac{(A^2/l^*) \text{Sech}^2 (B^* h^*/l^*) - (B^2/l^*) \text{Sech}^2 (A^* h^*/l^*)}{A^2 - B^2} (26b)
\]

\[
\xi_{21} = \frac{(3/\sqrt{2}) \tan h (h^*/\sqrt{2}l^*) - (h^*/\sqrt{2}l^*) \text{Sech}^2 (h^*/\sqrt{2}l^*)}{(1/l^*) + (h^*/\sqrt{2}l^*) \tan h (h^*/\sqrt{2}l^*)} (26c)
\]

\[
\xi_{22} = \text{Sech}^2 (h^*/\sqrt{2}l^*) \left\{ (1/l^*) + (h^*/\sqrt{2}l^*) \tan h (h^*/\sqrt{2}l^*) \right\} (26d)
\]

\[
\xi_{31} = \frac{(B^* - A^* \text{Cot} \Theta^*) \text{SinB}^* h^* + (A^* + B^* \text{Cot} \Theta^*) \text{SinhA}^* h^*}{M_0 (\text{CosB}^* h^* + \text{CoshA}^* h^*)} (26e)
\]

\[
\xi_{32} = \frac{2}{l^*} \left[ \frac{1 + \text{Cot} \Theta^* \text{SinB}^* h^* \text{SinhA}^* h^* + \text{CosB}^* h^* \text{CoshA}^* h^*}{(\text{CosB}^* h^* + \text{CoshA}^* h^*)^2} \right] (26f)
\]

The dimensionless form of MHD dynamic damping coefficients can expressed as:

\[
D^*_d = -\left( \frac{\partial W^*}{\partial V^*} \right) = -\frac{12}{\ln (d^* + 1)} \left[ I_3 (0,-1) - \left\{ \frac{I_1 (0,-1)}{I_2 (0,-1)} \right\} I_4 (0,-1) \right] (27)
\]

3. Results and discussion

On the basis of MHD thin-film lubrication theory and the Stokes micro-continuum theory, the steady and dynamic characteristics of exponentially shaped porous slider bearing lubricated with electrically conducting couplestress fluid has been considered theoretically. In the present analysis, the MHD bearing dynamics are given for the value of the physical quantities permeability parameter \( (\psi = 0 \sim 0.01) \), couplestress parameter \( (l^* = 0, 0.2, 0.4) \), profile parameter \( (d^* = 1.2) \), steady minimum film height \( (h^*_{m} = 0.8) \), and Hartmann number \( (M_0 = 0, 2, 3) \).

3.1 MHD steady film pressure:

Figure 2 illustrate that the variant of the dimensionless steady MHD film pressure \( P^*_s \) with the dimensionless coordinate \( x^* \) for several values of \( M_0 \) and \( l^* \) under the fixed values porosity.
It is observed that the effects of MHD and couplestresses enhances the steady film pressure in presence of porous medium when compared with comparing with non-magnetic and NCL case.

3.2 MHD steady load capacity:
Figure 3 presents the dimensionless steady MHD load capacity $W_s^*$ verses profile parameter $d^*$ for several values of $M_0$ and $l'$ under the fixed values porosity $m = 0.06$, $\delta^* = 0.01$ and $\beta = 0.2$. Comparing with non-magnetic and NCL case, The MHD load capacity of the bearing is observed to increase with the effect of applied magnetic field in the presence of conducting lubricant.
3.3 MHD dynamic stiffness coefficient:
The variation of the dimensionless stiffness coefficient with profile parameter $d^*$ for various values $M_0$ and $l'$ under the fixed values porosity $m = 0.06, \delta^* = 0.01$ and $\beta = 0.2$ is displayed in Figure 4. Comparing with non-magnetic and NCL case, it is observed that the values of $K_d^*$ increase significantly as increasing values of $d^*$ until it reaches maximum and thereafter decreases gradually.

![Figure 4](image.png)

Figure 4 Variation of dynamic stiffness coefficient $K_d^*$ with $d^*$ for different values of $M_0$ and $l'$ with $h_m^* = 0.8, m = 0.06, \nu = 0.001, \beta = 0.2, \delta = 0.01$

3.4 MHD dynamic damping coefficient:
The dimensionless damping coefficient $D_d^*$ verses profile parameter $d^*$ for several values of $M_0$ and

![Figure 5](image.png)

Figure 5 Variation of dynamic damping coefficient $D_d^*$ with $d^*$ for different values of $M_0$ and $l'$ with $h_m^* = 0.8, m = 0.06, \nu = 0.001, \beta = 0.2, \delta = 0.01$
Comparing with non-magnetic and NCL case, it is interesting to note that the values of $D_d^*$ increases with decreasing values of $d^*$. It is found that in the presence of porous medium, the values of MHD damping coefficient decreases significantly with the effect of couplestress effects as compared to NCL case.

**Table 1:** Tabulated values for $P_s^*$, $W_s^*$, $K_d^*$, $D_d^*$ with $d^* = 0.8$, $h_m^* = 0.8$, $m = 0.06$, $\delta^* = 0.01$, $\beta = 0.2$, $x^* = -0.6$.

|        | $M_0 = 0$          | $M_0 = 2$          |
|--------|-------------------|-------------------|
|        | $\psi = 0$        | $\psi = 0.001$    | $\psi = 0.01$ | $\psi = 0$        | $\psi = 0.001$ | $\psi = 0.01$ |
| $P_s^*$| $l^* = 0$         | 0.308523          | 0.305098       | 0.277723          | 0.337157       | 0.336562       | 0.336499       |
|        | $l^* = 0.2$       | 0.334337          | 0.332066       | 0.298119          | 0.365598       | 0.364906       | 0.364832       |
| $W_s^*$| $l^* = 0$         | 0.242811          | 0.239481       | 0.213666          | 0.263286       | 0.262737       | 0.262678       |
|        | $l^* = 0.2$       | 0.266074          | 0.261999       | 0.230965          | 0.288507       | 0.287852       | 0.287782       |
| $K_d^*$| $l^* = 0$         | 0.607029          | 0.586419       | 0.439816          | 0.609337       | 0.606401       | 0.606089       |
|        | $l^* = 0.2$       | 0.721026          | 0.693951       | 0.50665           | 0.725662       | 0.721869       | 0.721467       |
| $D_d^*$| $l^* = 0$         | 1.03273           | 1.01857        | 0.908774          | 1.11982        | 1.11748        | 1.11723        |
|        | $l^* = 0.2$       | 1.13168           | 1.11434        | 0.98235           | 1.22709        | 1.2243         | 1.22401        |

4. Conclusions

The modified MHD Reynolds type is derived to analyze the steady and dynamic characteristics of exponential porous slider bearing in presence transverse magnetic field by considering couplestress fluid as lubricant. According to the earlier discussion, the following results are summarized.

- The MHD steady pressure of squeeze film porous slider bearing as increases the couplestress ($l^*$) effects or strength of transverse magnetic field ($M_0$).
- The MHD steady load capacity in the case of porous slider bearing also increases as the parameters $l^*$ and $M_0$ increases and there exists a maximum load for $l^*$ and $M_0$.
- The force of friction on porous the slider bearing is very small on the effect of applied magnetic field.
- The MHD steady pressure($P_s^*$), load carrying capacity($W_s^*$), dynamic stiffness ($K_d^*$), and damping coefficient ($D_d^*$) significantly increasing with the effect of non-Newtonian couplestress fluid for exponential porous slider bearing and same is presented in Table 1 when compared with Newtonian case.

It is therefore suggested that the MHD exponential bearing dynamics can be predicted when bearing is lubricated with non-Newtonian couplestress fluid in presence of porous medium.
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Nomenclature:

$B_0$ applied magnetic field

$d$ difference between the inlet and outlet film thickness (i.e. shoulder height)

$K_d^*$ dynamic damping coefficient (non-dimensional)

$E_z$ induced electric field in the $z$-direction

$W$ film force

$W^*$ $\frac{F_{h_{ms}}^2}{\mu U L^2 B_0}$ film force (non-dimensional)

$h(x,t)$ film thickness

$h^*(x^*,t^*)$ $h(x,t)/h_{ms}$ film thickness (non-dimensional)

$h_{ms}(t)$ minimum squeezing film thickness

$h_{ms}^*(t^*)$ minimum squeezing film thickness (non-dimensional)

$h_{ms}$ steady state reference minimum film thickness at outlet

$h_i$ inlet film thickness

$L, N$ length and breadth of the bearing

$l$ couple stress parameter $S$

$l^*$ $2l/h_{ms}$ (non-dimensional)
\[ M = \frac{B_f h_{ms} (\sigma / \mu)^{1/2}}{P} \]  Hartmann number

\[ P \]  film pressure

\[ p_s \]  steady film pressure

\[ p^* = \frac{\rho h_{ms}^2}{\mu UL} \]  film pressure (non-dimensional)

\[ p^*_s = \frac{p_s h_{ms}^2}{\mu UL} \]  steady film pressure(non-dimensional)

\[ p_{sm} \]  steady maximum film pressure (non-dimensional)

\[ S_d \]  dynamic damping coefficient (non-dimensional)

\[ t \]  time,

\[ t^* = \frac{t}{h_{n0}} \]  time (non-dimensional)

\[ U \]  sliding velocity of lower part

\[ V \]  squeezing velocity

\[ V^* \]  squeezing velocity(non-dimensional)

\[ u, v \]  velocity components in x and y directions

\[ W_s \]  steady load

\[ W^*_s \]  steady load(non-dimensional)

\[ x, y \]  Cartesian coordinates

\[ x^* = \frac{x}{L} \]  coordinate(non-dimensional)

\[ \delta \]  profile parameter \( \delta = d / h_{ms} \)

\[ \eta \]  constant responsible associate with couple stress

\[ \sigma \]  conductivity of the lubricant

\[ \mu \]  lubricant viscosity