Spin Josephson effect in ferromagnet/ferromagnet tunnel junctions

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Abstract. – We consider the tunnel current between two ferromagnetic metals from a perspective similar to the one used in superconductor/superconductor tunnel junctions. We use fundamental arguments to derive a Josephson-like tunnel spin current $I_{\text{spin}} \propto \sin(\theta_1 - \theta_2)$. Here the phases are associated with the planar contribution to the magnetization, $\langle c_\uparrow c_\downarrow \rangle \sim e^{i\theta}$. The crucial step in our analysis is the fact that the $z$-component of the spin is canonically conjugate to the phase of the planar contribution: $[\theta, S_z] = i$. This is the counterpart to the commutation relation $[\phi, N] = i$ in superconductors, where $\phi$ is the phase associated with the superconducting order parameter and $N$ is the Cooper pair number operator. We briefly discuss the experimental consequences of our theoretical analysis.

In recent years the possibility of using the spin degrees of freedom in addition to the charge degrees of freedom in electronic devices has open new perspectives in condensed matter physics. Indeed, the field now known as spintronics or magnetoelectronics is rapidly developing and there are even commercial devices available [1]. If the current density due to an electron of spin $\sigma$ is $J_\sigma$, then the usual electric current is just the sum of the electronic currents for each spin degree of freedom, $j_{\text{charge}} = \sum_\sigma j_\sigma$. The spin current density, on the other hand, can be defined by $j_{\text{spin}} = \sum_\sigma \sigma j_\sigma$. Although such a definition of spin current density is used very often, it should be kept in mind that the true spin current density is actually a tensor and not simply a vector like the charge current density. Indeed, if $\mathbf{M}$ is the magnetization, the continuity equation for the spin current is given by $\partial M_i / \partial t + \partial_\mu J_{i\mu}^{\text{spin}} = 0$, where $J_{i\mu}^{\text{spin}}$ is the spin current density tensor.

The most familiar situation in the theory of electronic transport corresponds to the case where a charge current flows in the system and the spin current is zero. However, the opposite situation is also possible in some ferromagnetic systems, i.e., a spin current flowing in the absence of any charge current [2–4]. A relevant problem to study in this context is the behavior of the spin current across a tunnel junction between two ferromagnetic metals [5], especially whether there is phase coherence of the spin current across the junction. Phase coherent charge transport is a well known feature in tunnel junctions between superconductors and is responsible for the Josephson effect [6]. In this paper we will study a similar effect for a spin...
current in ferromagnet/ferromagnet (FM/FM) tunnel junctions. Recently it was pointed out that a charge Josephson-like effect would be possible in a spin valve [7], but our analysis below does not confirm such a prediction. The spin Josephson effect in itself is not new and was observed some time ago in $^3$He-B weak links [8,9]. The observation of this effect establishes the existence of spin supercurrents in $^3$He-B. It is plausible that a similar effect may occur in other systems, like in tunnel junctions involving ferromagnetic metals. Another possibility is to use thin film helimagnets, where spin transport without dissipation was discussed recently [10].

The main result of this letter will be a microscopic derivation of the spin Josephson effect in FM/FM tunnel junctions. The key observation in our analysis is that in a ferromagnetic metal spin flip averages like $\langle c^\dagger_\uparrow c_\downarrow \rangle$ are nonzero and play a role analogous to the superconducting order parameter $\langle c^\dagger_\uparrow c^\dagger_\downarrow \rangle$. This same analogy was used recently in discussions of a Josephson-like effect [11–14] observed in double layer quantum Hall pseudo-ferromagnet [15]. There the spin label is associated with the layer such that an analogy with exciton superfluids [16] is also possible [17].

In order to make our analysis of the spin Josephson effect self-contained, we first recall some basic facts about superconductor/superconductor (SC/SC) tunnel junctions. There the phase plays a crucial role leading to the celebrated Josephson effect [18]. Indeed, the Josephson effect is a consequence of the fact that the phase $\varphi$ of the order parameter is canonically conjugate to the Cooper pair number operator $N$, $[\varphi, N] = i$, which implies the uncertainty relation $\Delta \varphi \Delta N \sim 1$ [18]. The superconducting current is given by $I_{SC} = \frac{2e}{C}(N_1 - N_2)$, where $e$ is the electric charge and $N_1$ and $N_2$ are the Cooper pair number operators for the superconductors in each side of the tunnel junction. Using then the Hamiltonian for the tunnel junction

$$H = -E_J \cos(\varphi_1 - \varphi_2) + \frac{2e^2}{C}(N_1 - N_2)^2,$$

(1)

where $C$ is the capacitance of the tunnel junction, and treating $N_i$ and $\varphi_i$ as classical canonically conjugate variables, we easily obtain $\mathcal{F}^{SC}_J = (E_J/e) \sin \Delta \varphi$, $\Delta \varphi = 2eV$, where $\Delta \varphi \equiv \varphi_1 - \varphi_2$ and $V = 2e(N_1 - N_2)/C$ [19].

In the case of FM/FM tunnel junctions it is the $z$-component of the spin that is canonically conjugated to the phase: $[\theta, S^z] = i$. This follows from writing the raising and lowering spin operators as [20]

$$S_+ = \exp(i\theta) \sqrt{\left(S + \frac{1}{2} \right)^2 - \left(S^z + \frac{1}{2} \right)^2},$$

$$S_- = \sqrt{\left(S + \frac{1}{2} \right)^2 - \left(S^z + \frac{1}{2} \right)^2} \exp(-i\theta).$$

(2)

Thus, we obtain the uncertainty relation $\Delta \theta \Delta S^z \sim 1$. Furthermore, we can derive the commutation relation $[S^z, e^{\pm i \theta}] = \pm e^{\pm i \theta}$ which in turn implies the commutation relations defining the algebra of spin operators. Using the representation (2) we can establish a classical Hamiltonian for the FM/FM tunnel junction out of which a spin current $I_{J\text{spin}} = \mu_B(N_1 - N_2)$ is derived:

$$H = -E_J S^z \cos(\theta_L - \theta_R) + \frac{\mu_B^2}{2C_s}(S^z_L - S^z_R)^2,$$

(3)
Table I – Analogies between SC/SC and FM/FM tunnel junctions

| SC/SC junction          | FM/FM junction          |
|-------------------------|-------------------------|
| $[\phi, N] = i$          | $[\theta, S_z] = i$     |
| Voltage $V$             | Spin voltage $V_s$      |
| $\Delta \dot{\phi} = 2eV$ | $\Delta \dot{\theta} = 2\mu_B V_s$ |
| $I^{SC}_J = (E_J/e) \sin \Delta \phi$ | $I^{\text{spin}}_J = (2E_J S^2 / \mu_B) \sin \Delta \theta$ |

where for convenience we have introduced the “spin capacitance” $C_s$, which will be specified later in the discussion of the microscopic origin of the effect. The coefficient $E_J S^2$ of the first term of Eq. (5) corresponds in fact to an exchange energy between the magnetizations in both sides of the tunnel junction. The value of $E_J S^2$ will be determined later in terms of parameters of the microscopic model.

Using the classical Hamiltonian equations of motion we easily derive:

$$\Delta \dot{\theta} = 2\mu_B V_s, \quad I^{\text{spin}}_J = (2E_J S^2 / \mu_B) \sin \Delta \theta,$$

where we have defined the spin voltage as $V_s = (\mu_B / C_s)(S_z^L - S_z^R)$. Thus, a spin current in a FM/FM tunnel junction would behave in the same way as the superconducting current in the SC/SC tunnel junction, see Table I.

Now that we have motivated the problem on the basis of a straightforward physical analogy, we are ready to derive microscopically the spin Josephson effect in FM/FM tunnel junctions. Before starting, it is worth to compare once more the role of the phases in superconductors and ferromagnets. In superconductors a phase transformation of the electron destruction operators involves the particle number operator:

$$c_\sigma(\phi) = \exp \left[ i \frac{\phi}{2} (n - 1) \right] c_\sigma(0) \exp \left[ -i \frac{\phi}{2} (n - 1) \right] = e^{i\phi/2} c_\sigma(0),$$

where $n = \sum_\sigma c_\sigma^\dagger c_\sigma$ and from the first to the second line we have just solved the equation of motion $\partial c_\sigma / \partial \phi = i((n - 1)/2, c_\sigma)$. Similarly, for a ferromagnetic metal the relevant transformation is

$$c_\sigma(\theta) = \exp \left[ i \frac{\theta}{2} (n_\uparrow - n_\downarrow) \right] c_\sigma(0) \exp \left[ -i \frac{\theta}{2} (n_\uparrow - n_\downarrow) \right] = e^{i\theta/2} c_\sigma(0).$$

Eq. (5) is thus associated to gauge transformations $c_\sigma = e^{i\phi/2} \tilde{c}_\sigma$ out of which we obtain that $\langle c_\uparrow c_\downarrow \rangle = e^{-i\phi} \langle \tilde{c}_\uparrow^\dagger \tilde{c}_\downarrow^\dagger \rangle$. Eq. (6), on the other hand, yields $\langle c_\uparrow c_\downarrow \rangle = e^{-i\theta} \langle \tilde{c}_\uparrow^\dagger \tilde{c}_\downarrow^\dagger \rangle$.

We will consider the ferromagnetic metals in each side of the junction in the simple framework of the Stoner model for itinerant ferromagnetism [21]. The interaction is given simply by

$$H_I = U \int d^3 r n_\uparrow(r)n_\downarrow(r),$$

which can be rewritten as
\[ H'_L = \int d^3r \left\{ \frac{3}{2U} [m^2(r) + |\Delta(r)|^2] - c^\dagger(r)c(r)\Delta(r) \right. \\
- \left. \Delta^\dagger(r)c^\dagger(r)c(r) - m(r)[n_\uparrow(r) - n_\downarrow(r)] \right\}, \tag{8} \]

upon a Hubbard-Stratonovich transformation. It is easy to see from the equations of motion for the auxiliary fields that mean-field theory corresponds to

\[ \Delta^\dagger = (2U/3)\langle S_+ \rangle, \quad \Delta = (2U/3)\langle S_- \rangle, \quad m = (2U/3)\langle S_z \rangle, \tag{9} \]

where the spin operators are defined in terms of fermion operators as \( S_+ = c^\dagger \sigma c \) and \( S_z = (n_\uparrow - n_\downarrow)/2 \).

The electron field operators in the left and right subsystems will be denoted as \( c_\sigma \) and \( f_\sigma \), respectively. The bosonic auxiliary fields in the left and right subsystems will be distinguished respectively by labels \( L \) and \( R \) and assumed to be uniform in a mean-field approximation. Furthermore, for simplicity we will also assume that the left and right ferromagnetic metals are the same. Thus, we will take \( m_L^2 = m_R^2 = m^2 \) and \( |\Delta_L| = |\Delta_R| = |\Delta| \), but in general \( m_L \neq m_R \), since we can have \( m_L = m_0 \) and \( m_R = -m_0 \). Also, the phases of \( \Delta_L \) and \( \Delta_R \) are generally not the same. The tunnel Hamiltonian is given as usual by

\[ H_T = \sum_{k,p} \sum_\sigma \sigma T_{kp} c_{k\sigma} f_{p\sigma} + h.c. \],

where \( T_{kp} \) is the tunnel matrix element. The charge current is given by \( I_{\text{charge}} = -e\langle \dot{N}_L(t) \rangle \), where \( N_L = \sum_{k,\sigma} c^\dagger_{k\sigma} c_{k\sigma} \) and \( \dot{N}_L = i \sum_{k,p} \sum_\sigma (T_{kp} c^\dagger_{k\sigma} f_{p\sigma} - T_{kp} f^\dagger_{p\sigma} c_{k\sigma}) \), while the spin current is given by \( I_{\text{spin}} = -\mu_B \langle \dot{S}_L^z \rangle \) where \( \dot{S}_L^z = i \sum_{k,p} \sum_\sigma \sigma (T_{kp} c^\dagger_{k\sigma} f_{p\sigma} - T_{kp} f^\dagger_{p\sigma} c_{k\sigma}) \).

We will evaluate the spin current using linear response theory. In this approximation we have \( I_{\text{spin}} = 2\mu_B \text{Im}[\Xi(eV + i\delta)] \), where \( \delta \to 0^+ \). In terms of the Matsubara formalism we have

\[ \Xi(i\omega) = \frac{1}{\beta} \sum_{\omega_1} \sum_{k,p} |T_{kp}|^2 \left[ \sum_\sigma \sigma G^\sigma_L(k, i\omega_1) G^\sigma_R(p, i\omega_1 - i\omega) \right. \\
+ \left. F^\dagger_L(k, i\omega_1) F^\dagger_R(p, i\omega_1 - i\omega) - F^\dagger_R(k, i\omega_1) F^\dagger_L(p, i\omega_1 - i\omega) \right]. \tag{10} \]

The Green functions appearing in Eq. (10) are given in the mean-field approximation by

\[ G^\sigma_L(k, i\omega) = \frac{i\omega - \epsilon_k + \sigma m_L}{(i\omega - \epsilon_k)^2 - m^2 - |\Delta|^2}, \]
\[ F^\dagger_L(k, i\omega) = \frac{\Delta_L}{(i\omega - \epsilon_k)^2 - m^2 - |\Delta|^2}, \]
\[ F^\dagger_R(k, i\omega) = \frac{\Delta^\dagger_L}{(i\omega - \epsilon_k)^2 - m^2 - |\Delta|^2}, \tag{11} \]

with similar expressions for the right side Green functions. The dispersion energies are \( \epsilon_k = \epsilon_k - \mu_L \) and \( \epsilon_p = \epsilon_p - \mu_R \), where \( \mu_L - \mu_R = eV \). The spin current can be separated into two parts, one due to direct spin current tunneling and another one due to spin flip tunneling. It is the latter that is responsible for the spin Josephson effect and arises from the product.
between the $F_{L,R}^{αβ}$ Green functions. The direct spin current vanishes at zero voltage while the spin flip current does not. Explicit evaluation gives

$$I_ν^{spin} = \frac{πμ_B|Δ|^2|T|^2}{2(m^2 + |Δ|^2)} S(eV, \sqrt{m^2 + |Δ|^2}) \sin Δθ$$

(12)

where $Δθ$ is the phase difference between the left and right subsystem as before. The function $S(a,b)$ is given by

$$S(a,b) = \int du ∫ dv ρ(u) ρ(v) \left[ f(v+b) + f(v-b) - f(u+b) - f(u-b) \right]$$

$$\frac{f(v+b) - f(u+b)}{a+2b+v-u} - \frac{f(v-b) - f(u+b)}{a-2b+v-u},$$

(13)

where $ρ(ε)$ is the density of states and $f(ε)$ is the Fermi-Dirac distribution function. The function $S(a,b)$ satisfy the properties $S(a,0) = 0$ and $S(0,b) ≠ 0$. Eq. (12) establishes microscopically the spin Josephson effect in FM/FM tunnel junctions. The Josephson-like spin tunnel current is nonzero only if $|Δ| ≠ 0$. Furthermore, it is maximum for $m = 0$, which corresponds to the case of a tunnel junction between planar ferromagnets.

From Eq. (12) we can easily determine the exchange magnetic energy across the junction, since $I_ν^{spin} = ∂E/∂Δθ$, where $E$ is the energy of the tunnel junction. In this way we obtain that the coefficient $E_j S^2$ appearing in Eq. (3) is given by the coefficient of the sine of Eq. (12) with $V = 0$.

The spin Josephson effect as given by Eq. (12) implies a spin current even in the absence of voltage. Now we will show that when $m_L - m_R$ is nonzero the above effect is actually an AC-like effect. In order to show this we have to use a gauge transformation based on Eq. (6) in the Lagrangians for the left and right subsystems, i.e., we set $c_{kσ} = e^{iσθ_L/2}c_{kσ}$ and $f_{kσ} = e^{iσθ_R/2}f_{kσ}$. For instance, we obtain for the left subsystem

$$L_{L}^{MF} = ∑_{kσ} c_{kσ}^† \left( i\partial_t - \frac{σ}{2} \partial_t θ_L + σm_L + \frac{eV}{2} \right) c_{kσ} - H_{L}^{MF},$$

(14)

with a similar expression for the right subsystem. In analogy with the Josephson effect in superconductors, we can compensate for the phase transformation by writing $∂t θ_L = 2m_L$ and $∂t θ_R = 2m_R$, out of which we obtain

$$Δθ = 2(m_L - m_R) = 2μ_B V_s,$$

(15)

where we have defined a “spin voltage” $V_s ≡ (m_L - m_R)/μ_B$. Using the expression for $m$ in Eq. (9) for the left and right systems and comparing Eq. (15) with Eq. (4), we obtain that the “spin capacitance” is given by

$$C_s = \frac{3μ_B^2}{2 U}.$$  

(16)

It is worth to discuss once more the similarities between FM/FM and SC/SC tunnel junctions. In the latter the time evolution of the phase difference is proportional to the average particle number difference [22], while in the former it is proportional to the difference of the $z$-projection of the magnetization across the junction. Note that in superconductors a Hubbard-Stratonovich transformation similar to the one in Eq. (3) holds with the spin operators replaced by the pseudo-spin operators $S_+ = c_↑c_↑$, $S_- = S_↑$, $S_z = (n_↑ + n_↓ - 1)/2$. 

[References: Karl-Heinz Bennemann and Flavio S. Nogueira: Spin Josephson effect]
If $m_L^2 = m_R^2 = m^2$, the AC-like spin Josephson effect occurs only if the left and right subsystems have the spin projection on the $z$-axis anti-parallel to one another. By solving Eq. (15) with $m_L = -m_R = m$ and using Eq. (12) we obtain the important result

$$I_{\text{spin}}^J = \frac{\pi \mu_B |\Delta|^2 |I|^2}{2(m^2 + |\Delta|^2)} S(eV, \sqrt{m^2 + |\Delta|^2}) \sin(\Delta \theta_0 + 4mt).$$

(17)

The spin Josephson effect can be probed by trying to detect the induced electric fields associated with it [23]. It can be seen from Maxwell equations that a spin current should induce an electric field in the same way that a charge current induces a magnetic field. Indeed, in absence of voltage the induced electric field should satisfy

$$\partial_x E_y - \partial_y E_x = -4\pi \mu_B \partial \langle S_z \rangle / \partial t = 4\pi I_{\text{spin}}^J,$$

(18)

where we have assumed that $\partial H / \partial t = 0$.

In conclusion, we have shown that a spin Josephson effect should occur in FM/FM tunnel junctions. In the more general case where the FM order parameter is three-dimensional, the effect is necessarily an AC-like one, with the oscillations in time being associated to the $z$-component of the magnetization. Finally, we would like to stress that our analysis is completely different from Josephson effects discussed previously in FM/FM junctions where a charge current is discussed [7]. Here we have considered the phase coherence of a spin tunnel current. Our point of view is similar to the one of Ref. [10] where averages like $\langle c_\uparrow c_\downarrow \rangle$ also play a crucial role. In Ref. [10] the phase coherence arises from the fact that a thin film helimagnet is being considered while in our case it is due to the existence of a boundary between two ferromagnetic metals.

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