The baryon magnetic moments of the octet and the decuplet using different limits of the SU(3) flavor group

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Abstract

Working within the non relativistic quark model a two parameter fit to the magnetic moments of the baryon octet is presented. The model is based on taking different limits of the SU(3) flavor group to describe different magnetic moments. Using the values extracted from the fit the magnetic moments of the baryon decuplet have been predicted and an excellent agreement with the experimental measurements has been found.

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1 Introduction

There has recently been a renewed interest in the magnetic moments and spin structure of baryons within a variety of models. For example, the chiral quark model [1,2], quenched lattice gauge theory [3] and the $1/N_c$ expansion [4] to name a few. These models are more ambitious than the non-relativistic quark model (NQM). Nonetheless it has been argued that due to some subtle cancellations the NQM is a good approximation to the magnetic moments [5], so a simple model could be able to extract the physics of the problem more easily than a complicated one.
It is well known since a long time that the magnetic moments of the octet baryons can be described approximately via a SU(3) flavor group [6]. Using this approach within the NQM, it is assumed that the breaking of the flavor symmetry acts equally in all states of the octet. Thus, for example, the agreement of the predicted ratio of the magnetic moment of the proton to that of the neutron with the experimental result is taken as a coincidence.

On the other hand it is evident that even knowing that the breaking of SU(3) flavor symmetry is about 30% in the octet, it is naive to expect such a discrepancy between the NQM and each of the magnetic moments. Take for example the case of the 20 in SU(4). There the flavor symmetry breaking is quite bigger than in the case of SU(3). Nonetheless there is no reason why this should greatly affect the predictions of the magnetic moment of the proton or the neutron within a model in SU(4) with respect to SU(3).

Using these thoughts as guidelines, the effect of considering different limits of the SU(3) flavor group was studied and applied to calculate the magnetic moments of the octet. It was found that using a two parameter fit to the experimental data a better agreement in terms of $\chi^2$ was obtained, than other two parameter fits in the literature and a comparable agreement to fits using four parameters (see for example [7,8]). Using the value obtained for the parameters the magnetic moments for the baryons in the decuplet have been predicted. The comparison to the existing experimental values is quite satisfactory.

The current status on the experimental side is as follows. Seven of the magnetic moments are measured with around 1% accuracy or better [9]. The transition magnetic moment for $\Sigma^0 \rightarrow \Lambda$ is known to a 5% precision [10]. From the decuplet, the $\Omega^-$ was measured some time ago [11] and recently a new measurement has been presented [12]. Finally the magnetic moment of the $\Delta^{++}$ has also been measured [13].

### 2 The magnetic moments within different limits of the SU(3) flavor group

To date the way to explain the magnetic moments have been to look for models to break the SU(3) symmetry. Here it was decided to, on the one hand, to keep the flavor symmetry exact (in the following this case will be labeled SU(3)$^e$),
on the other hand to let only the mass of the strange quark to go to $\infty$ and keep $m_u = m_d$ (in the following this case will be labeled SU$(2)^\infty$). The driving idea behind this approach is the Ansatz that baryons, and with them their magnetic moments, prefer to stay near symmetric states. In this case it means that for example the proton and the neutron not knowing anything about the strange quark, may prefer to remain in a SU$(2)^\infty$ state more than staying in the exact SU(3) flavor state.

The magnetic moments for the octet of baryons within NQM are given in table 1. The case of exact SU(3) flavor symmetry requires the masses of the quarks to be equal: $m_3 \equiv m_u = m_d = m_s$. This implies that the magnetic moments $\mu_i$ obey the following equalities: $\mu_u = -2\mu_d = -2\mu_s$. Under these conditions the magnetic moments for the octet are given as shown in the third column of table 1 where the definition $\mu_3 \equiv \mu_s$ has been used.

The masses in the SU$(2)^\infty$ scenario fulfill $m_2 \equiv m_u = m_d \ll m_s$. The magnetic moments of the baryon octet were calculated in this case and then the limit $m_s \to \infty$ is taken. The expressions obtained from this procedure are shown in the last column of table 1 where the definition $\mu_2 \equiv \mu_d$ has been used.

Now, one could write the formulas for both cases just one step before taking the limit. For example the case of SU$(2)^\infty$ yields the equations presented in the second column of table 2. Note that in terms of $\mu_3/\mu_2 = m_2/m_3$ the magnetic moments could be group as those with small corrections $p$, $n$, $\Sigma^+$, $\Sigma^-$, $\Sigma^0 \to \Lambda$ and those with big corrections $\Lambda$, $\Xi^0 \Xi^-$. Repeating the exercise for these three last magnetic moments in the case of SU(3)$^e$ the equations of the last column of table 2 are found. Here the correction factors are again small.

From here one concludes that the magnetic moments of the baryons in the first group like the SU$(2)^\infty$ limit better, whereas the rest prefers to be close to the SU(3)$^e$ limit. To test this model a fit to the experimental values—shown in table 3—was performed. In this fit the SU$(2)^\infty$ formulas were used for the magnetic moments of the baryons $p$, $n$, $\Sigma^+$, $\Sigma^-$ and the transition $\Sigma^0 \to \Lambda$ and the SU(3)$^e$ equations for $\Lambda$, $\Xi^0$ y $\Xi^-$. 

There is an important technical point, while performing the fit. The magnetic moments of both the proton and the neutron have a very small experimental error. This precision of more than one part per million is huge when compared to the accuracy of the isospin symmetry of the $(p,n)$ doublet. This turns
meaningless a $\chi^2$ approach to the fit. To avoid this problem, it was proposed in [14] to add in quadrature a common absolute error to all the moments. Following this lead (see also [8]) an absolute error of $\sigma = 0.03\mu_N$ has been added in quadrature to the real experimental error, and then the fit has been performed.

This two parameter fit can be viewed as two independent one parameter fits. For the case of the SU(2)$^\infty$ limit a $\chi^2$ per degree of freedom of 0.42 was found. The SU(3)$^e$ case yield $\chi^2/\text{ndf}=1.9$. These produce a total $\chi^2/\text{ndf}=1.4$ when considering all eight magnetic moments together. The fitted values of the parameters are $\mu_2 = -0.930 \pm 0.007$ and $\mu_3 = -0.628 \pm 0.013$. The values obtained for the magnetic moments using these parameters are shown in table 3. The errors shown are the maximum spread in the values of the magnetic moments obtained by varying the parameters within their errors.

3 Discussion

1. To be able to perform the fit an extra error of $\sigma = 0.03\mu_N$ has been added in quadrature to the experimental error. This value makes sense as much as in the size of accuracy of considering the proton and the neutron as a isospin doublet, as in comparison to the errors of the other experimental errors. Nonetheless to study the sensitivity of the results to this error, its value was changed to 0.02 and 0.04 $\mu_N$. As expected, the main effect was in the $\chi^2/\text{ndf}$ which changed from 1.4 to 2.6 and 0.9 respectively. The value of the parameters remained the same and their errors varied between $\pm 0.93$ to $\pm 0.17$ for $\mu_2$ and $\pm 0.005$ to $\pm 0.009$ for $\mu_3$. This shows that the fit is quite stable under variation of this assumption. It must be noted that other analysis have used this extra error to equalize the weights, within the fit, of the different magnetic moments and to force a $\chi^2/\text{ndf}$ of the order of one [7,15].

2. From this analysis it is clear that the effects of the flavor symmetries and their breaking is different for different members of the octet. This can be explained in a natural way from the wave function of the baryons. Those grouped near the SU(3)$^e$ limit have either two strange quarks (the $\Xi$s) or the influence of light quarks tends to cancel each other ($\Lambda$). On the same way the rest of the baryons do not have a dominance of the strange quark in their wave
function and thus cluster around the SU(2)$^\infty$ limit.

3. A similar analysis can be carried in the decuplet, either studying their wave functions or writing their magnetic moments as a function of the SU(2)$^\infty$, respectively SU(3)$^e$, limit plus a correction term. In this case as all the wave functions are symmetric, there is no cancellation in the light quark sector, as there were in the case of the Λ, and the grouping is quite natural. It is seen that the magnetic moments of the Δs and the Σ'ss group near SU(2)$^\infty$ and the Ξ's and the Ω prefer the SU(3)$^e$ flavor limit. The predicted magnetic moments, using the parameters found from the analysis of the octet are shown in table 4. An excellent agreement with the experimental measurements is found.

4 Conclusions

A two parameter fit to the magnetic moments of the baryon octet has been shown. In terms of $\chi^2$ the model presented here has an accuracy of the same order than other 4 parameter fits in the literature. The value of the parameters has been used to predict the magnetic moments of the decuplet and an excellent agreement with the measured values has been found.

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Table 1
Expressions for the magnetic moments of the octet baryons for the NQM, and the SU(3) and SU(2) flavor limits.

| Magnetic moment | NQM | SU(3) | SU(2)∞ |
|-----------------|-----|-------|-------|
| p               | $\frac{1}{3}(4\mu_u - \mu_d)$ | $-3\mu_3$ | $-3\mu_2$ |
| n               | $\frac{1}{3}(4\mu_d - \mu_u)$ | $2\mu_3$ | $2\mu_2$ |
| $\Lambda$       | $\mu_s$ | $\mu_3$ | 0 |
| $\Sigma^+$      | $\frac{1}{3}(4\mu_u - \mu_s)$ | $-3\mu_3$ | $-8/3\mu_2$ |
| $\Sigma^-$      | $\frac{1}{3}(4\mu_d - \mu_s)$ | $\mu_3$ | $4/3\mu_2$ |
| $\Xi^0$         | $\frac{1}{3}(4\mu_s - \mu_u)$ | $2\mu_3$ | $2/3\mu_2$ |
| $\Xi^-$         | $\frac{1}{3}(4\mu_s - \mu_d)$ | $\mu_3$ | $-1/3\mu_2$ |
| $\Sigma^0 \rightarrow \Lambda$ | $\frac{1}{\sqrt{3}}(\mu_d - \mu_u)$ | $\sqrt{3}\mu_3$ | $\sqrt{3}\mu_2$ |

Table 2
Expressions for the magnetic moments of the octet baryons for the SU(3) and SU(2)∞ limits and a first correction to them.

| Magnetic moment | SU(2)∞ | SU(3) |
|-----------------|--------|-------|
| p               | $-3\mu_2(1 + 0)$ | |
| n               | $2\mu_2(1 + 0)$ | |
| $\Lambda$       | 0 | $\mu_3(1 + 0)$ |
| $\Sigma^+$      | $-8/3\mu_2(1 + \frac{\mu_3}{3\mu_2})$ | |
| $\Sigma^-$      | $4/3\mu_2(1 - \frac{\mu_3}{3\mu_2})$ | |
| $\Xi^0$         | $2/3\mu_2(1 + \frac{2\mu_3}{\mu_2})$ | $2\mu_3[1 - \frac{1}{3}(1 - \frac{\mu_3}{\mu_2})]$ |
| $\Xi^-$         | $-1/3\mu_2(1 - \frac{4\mu_3}{\mu_2})$ | $\mu_3[1 + \frac{1}{3}(1 - \frac{\mu_3}{\mu_2})]$ |
| $\Sigma^0 \rightarrow \Lambda$ | $\sqrt{3}\mu_2(1 + 0)$ | |

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Table 3
Measured values for the magnetic moments of the baryon octet, along with the prediction of the model.

| Baryon   | $\mu_{\text{exp}}$   | $\mu_{\text{model}}$ |
|----------|-----------------------|-----------------------|
| $p$      | 2.79±6.3x10$^{-8}$    | 2.79±0.02             |
| $n$      | -1.91±4.5x10$^{-7}$   | -1.86±0.01            |
| $\Lambda$| -0.613±0.004          | -0.63±0.01            |
| $\Sigma^+$| 2.46±0.01             | 2.48±0.02             |
| $\Sigma^-$| -1.16±0.025           | -1.24±0.01            |
| $\Xi^0$  | -1.25±0.014           | -1.26±0.02            |
| $\Xi^-$  | -0.651±0.0025         | -0.63±0.01            |
| $\Sigma^0 \rightarrow \Lambda$ | -1.61±0.08          | -1.61±0.01            |

Table 4
Prediction of the magnetic moments of the baryon decuplet and comparison with the measured values.

| Baryon   | Magnetic moment | $\mu_{\text{mod}}$ | $\mu_{\text{exp}}$ |
|----------|-----------------|---------------------|---------------------|
| $\Delta^{++}$ | $-6\mu_2$       | 5.58±0.04           | 4.52±0.95           |
| $\Delta^+$  | $-3\mu_2$       | 2.79±0.02           | –                   |
| $\Delta^0$  | 0               | 0                   | –                   |
| $\Delta^-$  | $3\mu_2$        | -2.79±0.02          | –                   |
| $\Sigma^{*+}$ | $-4\mu_2$       | 3.72±0.03           | –                   |
| $\Sigma^{*0}$ | $-\mu_2$       | 0.93±0.01           | –                   |
| $\Sigma^{*-}$ | $2\mu_2$        | -1.86±0.01          | –                   |
| $\Xi^{*0}$  | 0               | 0                   | –                   |
| $\Xi^{*-}$  | $3\mu_3$        | -1.88±0.04          | –                   |
| $\Omega^-$  | $3\mu_3$        | -1.88±0.04          | -2.02±0.06          |