Dendrimers are highly branched, star-shaped macromolecules with nanometer-scale dimensions. Dendrimers are defined by three components: a central core, an interior dendritic structure (the branches), and an exterior surface with functional surface groups. Topological indices are numerical numbers that help us to understand the topology of different dendrimers and can be used to predict the properties without performing experiments in the wet lab. In the present paper, we computed the Sombor index and the reduced version of the Sombor index for the molecular graphs of phosphorus-containing dendrimers, porphyrin-cored dendrimers, PDI-cored dendrimers, triazine-based dendrimers, and aliphatic polyamide dendrimers. We also plotted our results by using Maple 2015 which help us to see the dependence of the Sombor index and reduced Sombor index on the involved parameters. Our results may help to develop better understanding about phosphorus-containing dendrimers, porphyrin-cored dendrimers, PDI-cored dendrimers, triazine-based dendrimers, and aliphatic polyamide dendrimers. Our results are also useful in the pharmaceutical industry and drug delivery.

1. Introduction

Dendrimers are highly branched star-molded macromolecules with nanometer-scale measurements [1]. A dendrimer consists of three modules: a central core, an interior surface (branches), and the outer surface. A functional surface group is attached with the outer core. Various blends of these parts yield results of various shapes and sizes with protected inside centers that are an ideal contender for applications in both organic and materials sciences [2]. The characteristics of a dendrimer depend on the external group attached with the outer surface. Dendrimers have acquired a wide scope of uses in supramolecular science, especially in drug delivery, gene transfection, catalysis, energy harvesting, photo activity, molecular weight and size determination, rheology modification, and nanoscale science and technology. A dendrimer acts as a solubilizing agent in different reactions. Dendrimers have a wide range of applications in different fields of sciences [3]. The construction of dendrimers is presented in Figure 1.

Mathematical chemistry is the branch of mathematics in which mathematical tools are used to solve the problems arising in chemistry [4]. One of these tools is graphical representation of chemical compounds, and this representation is known as the molecular graph of the concerned chemical compound [5]. In the molecular graph of a chemical compound, atoms are represented as vertices, and bounds are represented as edges [6]. Topological invariants of molecular graphs are numerical numbers that enable us to collect information about concerned chemical structure and give us its hidden properties without performing experiments [7–11]. The first topological index was put forward by Wiener in 1947 [12] when he was trying to find the boiling points of alkane. This discovery led to the beginning of the theory of topological indices. The first degree-based topological index was put forward by Randić in 1975 [13]. After the success of the Randić index, Gutman introduced the Zagreb indices. There are hundreds of topological indices present in the literature [14–18]. Recently, Gutman, in 2021 [19], defined the idea of Sombor indices. Sombor
indices have two variants, Sombor index and reduced Sombor index, as follows:

\[
SO(G) = \sum_{ij \in E(G)} \sqrt{d_i^2 + d_j^2},
\]

\[
SO_{\text{red}}(G) = \sum_{ij \in E(G)} \sqrt{(d_i - 1)^2 + (d_j - 1)^2}.
\]

The aim of this paper is to study the phosphorus-containing dendrimers, porphyrin-cored dendrimers, PDI-cored dendrimers, triazine-based dendrimers, and aliphatic polyamide dendrimers. We computed the Sombor index and the reduced version of the Sombor index for the aforementioned dendrimers. We also present graphical representations of our results to see the dependence of computed indices on the involved parameters.

Throughout this paper, we consider all graphs to be simple and connected, and \(d_u\) denotes the degree of vertex \(u\) which is equal to the number of vertices at distance one to it. For the notation used in this paper but not defined, we refer to in [20, 21] and references therein.

2. Methodology

Firstly, we obtain the molecular graphs of phosphorus-containing dendrimers, porphyrin-cored dendrimers, PDI-cored dendrimers, triazine-based dendrimers, and aliphatic polyamide dendrimers. Secondly, we compute the order and size of these molecular graphs and classify their edge sets and vertex sets into different classes with respect to the degrees of vertices. Thirdly, we compute the Sombor and reduced Sombor indices for the molecular graphs of phosphorus-containing dendrimers, porphyrin-cored dendrimers, PDI-cored dendrimers, triazine-based dendrimers, and aliphatic polyamide dendrimers. Lastly, we plot our obtained results by using Maple 2015 software.

3. Main Results

In this section, we present Sombor and reduced Sombor indices for phosphorus-containing dendrimers, porphyrin-cored dendrimers, PDI-cored dendrimers, triazine-based dendrimers, and aliphatic polyamide dendrimers.

3.1. Phosphorus-Containing Dendrimers. Phosphorus-containing dendrimers have functionalities with pendant nitroxyl radicals, and these radicals show a solid attractive trade interaction. Let us consider \(D_1(m)\) to be the molecular graph of phosphorus-containing dendrimers, where \(m\) shows the generation stage of \(D_1(m)\). Figure 2 shows the molecular graph \(D_1(m)\) of phosphorus-containing dendrimers.

From Figure 2, we can observe that the order and size of \(D_1(m)\) are \(9(11 \times 2^{m+1} - 8)\) and \(6(9 \times 2^{m+2} - 13)\), respectively. If \(V'(D_1(m))\) is the vertex set, then from Figure 2, we can classify \(V'(D_1(m))\) into four subsets \(V'_1, V'_2, V'_3,\) and \(V'_{4}\) of vertices of degrees 1, 2, 3, and 4 such that \(|V'(D_1(m))| = |V'_1(D_1(m))| + |V'_2(D_1(m))| + |V'_3(D_1(m))| + |V'_{4}(D_1(m))|\). The cardinalities of \(V'_1(D_1(m)), V'_2(D_1(m)), V'_3(D_1(m)),\) and \(V'_{4}(D_1(m))\) are \(42 \times 2^{m} - 12, 96 \times 2^{m} - 39, \) and \(18 \times 2^{m} - 3\), respectively.

If \(E(D_1(m))\) represents the edge set, then Figure 2 shows that there are the following six different types of edges with respect to the degrees of end vertices present in the molecular graph of \(D_1(m)\):
Table 1: Degree-based edge partition of $D_1(m)$.

| $\varepsilon$ | $\varepsilon_{(d_u,d_v)}$ | Frequency |
|---------------|-----------------|-----------|
| $\varepsilon_1$ | $\varepsilon_{(1,3)}$ | $6 \ (2^m - 1)$ |
| $\varepsilon_2$ | $\varepsilon_{(1,4)}$ | $6 \ (5 \times 2^m - 1)$ |
| $\varepsilon_3$ | $\varepsilon_{(2,2)}$ | $18 \ (2^{m+1} - 1)$ |
| $\varepsilon_4$ | $\varepsilon_{(2,3)}$ | $6 \ (2^{m+4} - 7)$ |
| $\varepsilon_5$ | $\varepsilon_{(2,4)}$ | $(3 \times 2^{m+3})$ |
| $\varepsilon_6$ | $\varepsilon_{(3,4)}$ | $6 \ (3 \times 2^m - 1)$ |

Figure 2: $D_1(m)$ for $m = 3$.

Table 1 explains the edge partition of the edge set of $D_1(m)$ in detail. Frequency means the total number of edges in the particular class.

Theorem 1. The SO and $SO_{red}$ indices for $D_1(m)$ are as follows:

$(i)$ $SO(G) = 2^{((1/2)+m)} \sqrt{5} - 6 \sqrt{2} \sqrt{5} + 32 \times 2^{((1/2)+m)} + 48 \sqrt{2} + 6 \sqrt{13} + 2^{m+3} - 36 \sqrt{2} - 42 \sqrt{\sqrt{13} - 6 \sqrt{7}} + 90 \times 2^m - 30$

$(ii)$ $SO_{red}(G) = 24 \times 2^{((1/2)+m)} \sqrt{5} + 18 \sqrt{13} + 2^{m+3} - 36 \sqrt{2} + 42 \sqrt{\sqrt{13} - 6 \sqrt{7}} + 96 \sqrt{2} + 282 \times 2^m - 18 \sqrt{2} - 6 \sqrt{13} - 42 \sqrt{5} - 66$
\textbf{3.2. Porphyrin-Cored Dendrimers.} Figure 3 shows the molecular graph $D_2(m)$ of porphyrin-cored dendrimers, where $m$ represents different generations of $D_2(m)$.

From Figure 3, we can observe that the order and size of $D_2(m)$ are $4(2^{m+3} + 9)$ and $4(2^{m+3} + 11)$, respectively. If $\mathcal{V}(D_2(m))$ is the vertex set, then from Figure 3, we can classify this vertex set into four subsets $\mathcal{V}_1(D_2(m))$, $\mathcal{V}_2(D_2(m))$, $\mathcal{V}_3(D_2(m))$, and $\mathcal{V}_4(D_2(m))$ with respect to degrees such that $|\mathcal{V}(D_2(m))| = |\mathcal{V}_1(D_2(m))| + |\mathcal{V}_2(D_2(m))| + |\mathcal{V}_3(D_2(m))| + |\mathcal{V}_4(D_2(m))|$. The cardinalities of $\mathcal{V}_1(D_2(m))$, $\mathcal{V}_2(D_2(m))$, $\mathcal{V}_3(D_2(m))$, and $\mathcal{V}_4(D_2(m))$ are $12 \times 2^m - 8$, $12 \times 2^m + 32$, $4 \times 2^m + 16$, and $4 \times 2^m - 4$ vertices, respectively.

If $\mathcal{E}(D_2(m))$ represents the edge set, then Figure 3 shows that there are the following eight different types of edges present in the molecular graph of $D_2(m)$:

\begin{align*}
\mathcal{E}_1(D_2(m)) &= \mathcal{E}_{(1,2)}(D_2(m)) = \{e = uv \in \mathcal{E}(D_2(m)) : d_u = 1, d_v = 2\}, \\
\mathcal{E}_2(D_2(m)) &= \mathcal{E}_{(1,3)}(D_2(m)) = \{e = uv \in \mathcal{E}(D_2(m)) : d_u = 1, d_v = 3\}, \\
\mathcal{E}_3(D_2(m)) &= \mathcal{E}_{(1,4)}(D_2(m)) = \{e = uv \in \mathcal{E}(D_2(m)) : d_u = 1, d_v = 4\}, \\
\mathcal{E}_4(D_2(m)) &= \mathcal{E}_{(2,2)}(D_2(m)) = \{e = uv \in \mathcal{E}(D_2(m)) : d_u = 2, d_v = 2\}, \\
\mathcal{E}_5(D_2(m)) &= \mathcal{E}_{(2,3)}(D_2(m)) = \{e = uv \in \mathcal{E}(D_2(m)) : d_u = 2, d_v = 3\}, \\
\mathcal{E}_6(D_2(m)) &= \mathcal{E}_{(2,4)}(D_2(m)) = \{e = uv \in \mathcal{E}(D_2(m)) : d_u = 2, d_v = 4\}, \\
\mathcal{E}_7(D_2(m)) &= \mathcal{E}_{(3,3)}(D_2(m)) = \{e = uv \in \mathcal{E}(D_2(m)) : d_u = 3, d_v = 3\}, \\
\mathcal{E}_8(D_2(m)) &= \mathcal{E}_{(3,4)}(D_2(m)) = \{e = uv \in \mathcal{E}(D_2(m)) : d_u = 3, d_v = 4\}.
\end{align*}

Table 2 gives the detailed explanation about the edge partition of the edge set of $D_1(m)$.

\textbf{Theorem 2.} The SO and SO$_{\text{red}}$ indices for $D_2(m)$ are as follows:

\begin{align*}
(i) \quad & SO(D_2(m)) = 4 \times 2^{(1/2)m} \sqrt{5} + 20 \sqrt{52^m - 4 \sqrt{5} + 8^2 2^{(1/2)m} + 4 \sqrt{172^m + 4 \sqrt{132^m - 16 \sqrt{5} + 202^m + 76 \sqrt{2} - 4 \sqrt{17} + 32 \sqrt{13} - 20}} \nonumber
\quad + 30 \sqrt{172^m - 36 \sqrt{2} - 2 \times 17} - 6 \sqrt{17} + 902^m - 30, \\
(ii) \quad & SO_{\text{red}}(D_2(m)) = 8 \times 2^{(1/2)m} \sqrt{5} + 4 \sqrt{52^m - 8 \sqrt{2} \sqrt{5} + 4 \times 2^{(1/2)m} + 4 \sqrt{132^m + 32 \sqrt{5} + 242^m + 44 \sqrt{2} - 4 \sqrt{13} - 20}} \nonumber
\quad + 30 \sqrt{172^m - 36 \sqrt{2} - 2 \times 17} - 6 \sqrt{17} + 902^m - 30.
\end{align*}
we have the following computations of SO and SO\textsubscript{red} indices:

\[ \text{SO(D}_2) = \sum_{i,j} \sqrt{d_i^2 + d_j^2} = \sqrt{1^2 + 2^2 (4 \times 2^m)} + \sqrt{1^2 + 3^2 (4 \times 2^m - 4)} + \sqrt{1^2 + 4^2 (4 \times 2^m - 4)} + \sqrt{2^2 + 2^2 (4 \times 2^m + 20)} + \sqrt{2^2 + 3^2 (4 \times 2^m + 32)} + \sqrt{2^2 + 4^2 (8 \times 2^m - 8)} + \sqrt{3^2 + 3^2 (12)} + \sqrt{3^2 + 4^2 (4 \times 2^m - 4)} = 4 \times 2^{(1/2) + m} \sqrt{5 + 20 \sqrt2 \sqrt5 - 4 \sqrt2 \sqrt{15} + 82 \sqrt{12} + 4 \sqrt{172} - 4 \sqrt{173} - 16 \sqrt{5} + 202^2 + 76 \sqrt2 - 4 \sqrt{173} + 32 \sqrt{113} - 20}, \]

\[ \text{SO\textsubscript{red}(D}_2) = \sum_{i,j} \sqrt{(d_i - 1)^2 + (d_j - 1)^2} = \sqrt{(1 - 1)^2 + (2 - 1)^2 (4 \times 2^m)} + \sqrt{(1 - 1)^2 + (3 - 1)^2 (4 \times 2^m - 4)} + \sqrt{(1 - 1)^2 + (4 - 1)^2 (4 \times 2^m - 4)} + \sqrt{(2 - 1)^2 + (2 - 1)^2 (4 \times 2^m + 20)} + \sqrt{(2 - 1)^2 + (3 - 1)^2 (4 \times 2^m + 32)} + \sqrt{(2 - 1)^2 + (4 - 1)^2 (8 \times 2^m - 8)} + \sqrt{(3 - 1)^2 + (3 - 1)^2 (12)} + \sqrt{(3 - 1)^2 + (4 - 1)^2 (4 \times 2^m - 4)} = 8 \times 2^{(1/2 + m)} \sqrt{5 + 4 \sqrt{52} - 8 \sqrt{2} \sqrt{5} + 4 \times 2^{(1/2) + m}} + 4 \sqrt{172} \sqrt{5} + 202^2 - 4 \sqrt{173} + 32 \sqrt{113} - 20 + 242^2 + 44 \sqrt{2} - 4 \sqrt{173} - 20. \]
3.3. PDI-Cored Dendrimers. The water-dissoluble PDI-cored dendrimers have various accommodations, containing low cytotoxicity, solid red fluorescence, high quantum yield, amazing photostability, and flexible surface alteration. These dendrimers have numerous applications in different fields such as fluorescence live-cell imaging and labeling. Let $D_3(m)$ be the molecular graph of PDI-cored dendrimers; then, Figure 4 shows the 2D graph of $D_3(m)$.

From Figure 4, we can observe that the order and size of $D_3(m)$ are $20 \times 2^m + 20$ and $20 \times 2^{2m} + 20$, respectively. If $\mathcal{E}(D_3(m))$ is the vertex set, then by observing Figure 4, we can classify this vertex set into three subsets $\mathcal{E}_1(D_3(m))$, $\mathcal{E}_2(D_3(m))$, and $\mathcal{E}_3(D_3(m))$ such that $|\mathcal{E}(D_3(m))| = |\mathcal{E}_1(D_3(m))| + |\mathcal{E}_2(D_3(m))| + |\mathcal{E}_3(D_3(m))|$. The cardinalities of $\mathcal{E}_1(D_3(m))$, $\mathcal{E}_2(D_3(m))$, and $\mathcal{E}_3(D_3(m))$ are $2 \times 2^{m+1} + 4$, $6 \times 2^{m+1}$, and $2 \times 2^{m+1} + 16$, respectively.

If $\mathcal{E}(D_3(m))$ represents the edge set, then Figure 4 shows that there are the following five different types of edges present in the molecular graph of $D_3(m)$:

\[
\begin{align*}
\mathcal{E}_1(D_3(m)) &= \mathcal{E}(D_3(m)) = \{ e = uv \in \mathcal{E}(D_3(m)) : d_u = 1, d_v = 2 \}, \\
\mathcal{E}_2(D_3(m)) &= \mathcal{E}(D_3(m)) = \{ e = uv \in \mathcal{E}(D_3(m)) : d_u = 1, d_v = 3 \}, \\
\mathcal{E}_3(D_3(m)) &= \mathcal{E}(D_3(m)) = \{ e = uv \in \mathcal{E}(D_3(m)) : d_u = 2, d_v = 2 \}, \\
\mathcal{E}_4(D_3(m)) &= \mathcal{E}(D_3(m)) = \{ e = uv \in \mathcal{E}(D_3(m)) : d_u = 2, d_v = 3 \}, \\
\mathcal{E}_5(D_3(m)) &= \mathcal{E}(D_3(m)) = \{ e = uv \in \mathcal{E}(D_3(m)) : d_u = 3, d_v = 3 \}.
\end{align*}
\]

Table 3 gives the detailed explanation about the edge partition of the edge set of $D_3(m)$.

**Theorem 3.** The SO and SO\textsubscript{red} indices for $D_3(m)$ are as follows:

(i) $SO(D_3) = 2^{(3/2)m+3} + 5 \times 2^{(m+1)} + 12 \times 2^{(1/2)m} + 10 \sqrt{132m} + 4 \sqrt{10} + 68 \sqrt{2}$

\[
SO_{\text{red}}(D_3) = \sum_{ij} \sqrt{(d_i - 1)^2 + (d_j - 1)^2} = \sqrt{(1-1)^2 + (2-1)^2(2^{m+1})} + \sqrt{(1-1)^2 + (3-1)^2(2^{m+1})} + \sqrt{(2-1)^2 + (2-1)^2(2^{m+1})} + \sqrt{(2-1)^2 + (3-1)^2(22)}
\]

\[
= 6 \times 2^{(1/2)m} + 10 \sqrt{52m} + 6 \sqrt{2} + 45 \sqrt{2} + 8
\]

3.4. Triazine-Based Dendrimers. The divergent method is used for the synthesis of triazine-based dendrimers. Triazine-based dendrimers are less toxic and can be further studied as drug carriers. Let $D_4(m)$ represent the molecular graph of triazine-based dendrimer drug carriers in the future. Figure 5 shows the molecular graph of $D_4(m)$.

From Figure 5, we can observe that the order and size of $D_4(m)$ are $(2(5 \times 2^{2m+2} + 1)/3)$ and $7 \times 2^{2m+1} + 1$, respectively. If $\mathcal{E}(D_4(m))$ is the vertex set, then by observing Figure 5, we can classify this vertex set into four subsets $\mathcal{E}_1(D_4(m))$, $\mathcal{E}_2(D_4(m))$, and $\mathcal{E}_3(D_4(m))$ such that $|\mathcal{E}(D_4(m))| = |\mathcal{E}_1(D_4(m))| + |\mathcal{E}_2(D_4(m))| + |\mathcal{E}_3(D_4(m))|$. The cardinalities of $\mathcal{E}_1(D_4(m))$, $\mathcal{E}_2(D_4(m))$, and $\mathcal{E}_3(D_4(m))$ are $2^{2m+1}$, $2^{2m+1} + (7 \times 4^{m+1}/6) + (4^{m+1}/3)$, and $4 + (5 \times 4^{m+1}/6) - (10/3)$, respectively.

If $\mathcal{E}(D_4(m))$ represents the edge set, then Figure 5 shows that there are the following four different types of edges present in the molecular graph of $D_4(m)$:
Theorem 4. The SO and SO\textsubscript{red} indices for $D_4(m)$ are as follows:

(i) $SO(D_4) = \sqrt[2]{2^{m+1}} + \frac{32}{3}\sqrt[2]{24^m} - \frac{14}{3}\sqrt[2]{2} + \frac{22}{3}\sqrt[2]{132^m} + \frac{8}{3}\sqrt[2]{13}$

(ii) $SO_{\text{red}}(D_4) = 4 \times 2^m + 6\sqrt[2]{24^m} - \frac{8}{3}\sqrt[2]{2} + \frac{22}{3}\sqrt[2]{13}$

Proof. From the edge partition of the edge set of $D_4(m)$ given in Table 4, we have the following computations for SO and SO\textsubscript{red} indices:
Table 4: Degree-based edge partition of $D_5(m)$.

| $\mathcal{E}_i$ | $\mathcal{E}'_{(d,v)}$ | Frequency |
|-----------------|----------------------|-----------|
| $\mathcal{E}_1$ | $\mathcal{E}'_{(1,2)}$ | $2^{2m+1}$ |
| $\mathcal{E}_2$ | $\mathcal{E}'_{(2,2)}$ | $(2(5 \times 2^{2m} - 2)/3)$ |
| $\mathcal{E}_3$ | $\mathcal{E}'_{(2,3)}$ | $(2(11 \times 2^{2m} + 4)/3)$ |
| $\mathcal{E}_4$ | $\mathcal{E}'_{(3,3)}$ | $(2 \times 2^{2m+1} - 1)/3$ |

SO$(D_4) \sum_{i<j} \sqrt{d_i^2 + d_j^2 + \sqrt{2^2 + 2^2(3 \times 2^{2m+1} + 1)}} = \sqrt{1^2 + 2^2(2^{2m+1})} + \sqrt{2^2 + 2^2\left(\frac{2(5 \times 2^{2m} - 2)}{3}\right)}$

$+ \sqrt{2^2 + 3^2\left(\frac{2(11 \times 2^{2m} + 4)}{3}\right)} + \sqrt{3^2 + 3^2\left(\frac{2 \times 2^{2m+1} - 1}{3}\right)}$

$\sqrt{52^{(2m+1)}} + \frac{32}{3} \sqrt{24^m} - \frac{14}{3} \sqrt{2} + \frac{22}{3} \sqrt{132^m} + \frac{8}{3} \sqrt{13}$,

SO$_{red}(D_4) = \sum_{i<j} (d_i - 1)^2 + (d_j - 1)^2 = \sqrt{(1 - 1)^2 + (2 - 1)^2(2^{2m+1})}$

$\sqrt{(2 - 1)^2 + (2 - 1)^2\left(\frac{2(5 \times 2^{2m} - 2)}{3}\right)}$

$+ \sqrt{(2 - 1)^2 + (3 - 1)^2\left(\frac{2(11 \times 2^{2m} + 4)}{3}\right)}$

$+ \sqrt{(3 - 1)^2 + (3 - 1)^2\left(\frac{2 \times 2^{2m+1} - 1}{3}\right)}$

$4 \times 2^m + 6 \sqrt{24^m} = \frac{8}{3} \sqrt{2} + \frac{22}{3} \sqrt{52^m} + \frac{8}{3} \sqrt{5}$.

3.5. Aliphatic Polyamide Dendrimers. Recently, Jishkariani, for the first time, studied aliphatic polyamide dendrimers containing ethylenediamine and piperazine. These dendrimers are enzymatically and hydrolytically stable. Let $D_5(m)$ represent the molecular graph of the aliphatic polyamide-based dendrimer. Figure 6 shows the molecular graph of $D_5(m)$.

From Figure 6, we can observe that the order and size of $D_5(m)$ are $2(2^{2m+1} - 5)$. If $\mathcal{V}'(D_5(m))$ is the vertex set, then by observing Figure 6, we can classify this vertex set into three subsets $\mathcal{V}'(D_5(m))$, $\mathcal{V}'(D_5(m))$, and $\mathcal{V}'(D_5(m))$ such that $|\mathcal{V}'(D_5(m))| = |\mathcal{V}'(D_5(m))| + |\mathcal{V}'(D_5(m))| + |\mathcal{V}'(D_5(m))|$. The cardinalities of $\mathcal{V}'(D_5(m))$, $\mathcal{V}'(D_5(m))$, and $\mathcal{V}'(D_5(m))$ are $4(3 \times 2^{2m+1} - 1)$, $4(3 \times 2^{2m+1} - 1)$, and $2^{2m+2}(2^m - 1)$, respectively.

If $\mathcal{E}'(D_5(m))$ represents the edge set, then Figure 6 shows that there are the following four different types of edges present in the molecular graph of $D_5(m)$:

| $\mathcal{E}_1'(D_5(m))$ | $\mathcal{E}'_{(1,2)}(D_5(m))$ | $\mathcal{E}_2'(D_5(m))$ | $\mathcal{E}'_{(2,3)}(D_5(m))$ | $\mathcal{E}_3'(D_5(m))$ | $\mathcal{E}'_{(1,3)}(D_5(m))$ | $\mathcal{E}_4'(D_5(m))$ | $\mathcal{E}'_{(1,4)}(D_5(m))$ |
|------------------------|-------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| $d_u = 1, d_v = 2$    | $d_u = 2, d_v = 3$    | $d_u = 1, d_v = 3$    | $d_u = 1, d_v = 4$    |

Table 5 gives the detailed explanation about the edge set of $D_5(m)$.

Theorem 5. The SO and SO$_{red}$ indices for $D_5(m)$ are as follows:
Table 5: Degree-based edge partition of $D_5(m)$.

| $E$   | $E_{(d_i,d_j)}$ | Frequency |
|-------|----------------|-----------|
| $E_1$ | $E_{(1,2)}$   | $2^{m+1}$ |
| $E_2$ | $E_{(2,3)}$   | $2^{m+1}$ |
| $E_3$ | $E_{(1,3)}$   | $2(2^m - 1)$ |
| $E_4$ | $E_{(1,4)}$   | $2(2^m - 1)$ |
| $E_5$ | $E_{(2,2)}$   | $2(2^m - 1)$ |
| $E_6$ | $E_{(3,4)}$   | $2(2^m - 1)$ |
| $E_7$ | $E_{(3,3)}$   | $2$       |
| $E_8$ | $E_{(2,4)}$   | $2^{m+2} - 4$ |

(i) SO  $(D_5) = 2^{((3/2)+m)} \sqrt{5} + 10\sqrt{52^m} + 2^{(m+1)} \sqrt{17} - 2\sqrt{10} + \sqrt{132^{(m+1)}} + 4 \times 2^{((1/2)+m)} - 2\sqrt{17} - 8\sqrt{5} + 2\sqrt{2} + 10 \times 2^m - 10$

(ii) SO$_{red}$ $(D_5) = 6\sqrt{132^m} + \sqrt{52^m+1} + 2^{((3/2)+m)} + 18 \times 2^m + 2\sqrt{2} - 6\sqrt{13} - 14$

Proof: From the edge partition of the edge set of $D_5(m)$ given in Table 5, we have the following computations of SO and SO$_{red}$ indices:

\[
SO(D_5) = \sum_{ij} \sqrt{(d_i^2 + d_j^2 + 2^2(2^m - 1) + \sqrt{17} - 2\sqrt{10} + \sqrt{132^{(m+1)}} + 4 \times 2^{((1/2)+m)} - 2\sqrt{17} - 8\sqrt{5} + 2\sqrt{2} + 10 \times 2^m - 10,}
\]

\[
SO_{red}(D_5) = \sum_{ij} \sqrt{(d_i - 1)^2 + (d_j - 1)^2 + \sqrt{17} - 2\sqrt{10} + \sqrt{132^{(m+1)}} + 4 \times 2^{((1/2)+m)} - 2\sqrt{17} - 8\sqrt{5} + 2\sqrt{2} + 10 \times 2^m - 10,}
\]

(11)
4. Conclusion

Topological indices found numerous applications in many regions of material science, arithmetic, informatics, biology, and so on. However, their most important use is in the nonexact quantitative structure-property relationships (QSPR) and quantitative structure-activity relationships (QSAR). Topological indices have an interconnection with the structure of the chemical structure. In this paper, we computed the newly introduced Sombor indices for phosphorus-containing dendrimers, porphyrin-cored dendrimers, PDI-cored dendrimers, triazine-based dendrimers, and aliphatic polyamide dendrimers. Figures 7 and 8 give the graphical comparison of computed results for the aforementioned dendrimers.

Data Availability

All the data required for this research are included within this paper.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors’ Contributions

Shahid Amin proved the main results, Abaid Ur Rehman Virk designed the problem, M.A. Rehman verified the results, and Nehad Ali Shah wrote the whole paper.

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