Limits on the Two Higgs Doublet Model from meson decay, mixing and CP violation

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Abstract
We calculate the rate of $\pi^+$, $K^+$, $D^+$ and $B^+ \to \mu^+\nu_\mu$ decays, the branching ratio corresponding to $H^+ \to \tau^+\nu_\tau$, and the box diagrams of $B^o \leftrightarrow \bar{B}^o$, $K^o \leftrightarrow \bar{K}^o$ and $D^o \leftrightarrow \bar{D}^o$ mixing in the Two Higgs Doublet Model (Model II). Using the experimental data on meson decay rates, mixing, and CP violation in the $K^o$ and $B^o$ systems we set competitive upper and lower limits to the parameter $\tan \beta$ as a function of the mass of the charged Higgs $m_H$.

1 Introduction
The Standard Model of quarks and leptons is here to stay. This theory is based on principles: special relativity, locality, quantum mechanics, local symmetries and renormalizability[1]. Therefore the predictions of the Standard Model “are precise and unambiguous, and generally cannot be modified ‘a little bit’ except in very limited specific ways. This feature makes the experimental success especially meaningful, since it becomes hard to imagine that the theory could be approximately right without in some sense being exactly right.”[1] Among the extensions of the Standard Model that respect its principles and symmetries, that are compatible with present data within a region of parameter space, and are of interest at the large particle colliders, is the addition of a second doublet of Higgs fields. Higgs doublets can be added to the Standard Model without upsetting the $Z/W$ mass ratio; higher dimensional representations upset this ratio. A second Higgs doublet could make the three running coupling constants of the Standard Model meet at the Grand Unified Theory (GUT) scale. A second Higgs doublet is necessary in Supersymmetric extensions of the Standard Model[2]. In this article
we explore the limits that present data place on the parameters of the Two Higgs Doublet Model (Model II).[3] In particular we consider meson decay, mixing and CP violation.

All of our analysis is based on the “tree-level Higgs potential”[3]. The physical spectrum of the Two Higgs Doublet Model (Model II) contains five Higgs bosons: one pseudoscalar $A^o$ (CP-odd scalar), two neutral scalars $H^o$ and $h^o$ (CP-even scalars), and two charged scalars $H^+$ and $H^-$. The masses of the Higgs bosons, the mixing angle $\alpha$ between the two neutral scalar Higgs fields, and the ratio of the vacuum expectation values of the two neutral components of the Higgs doublets, $\tan \beta > 0$, are free parameters of the theory.

$$\Phi_1 = \left( \begin{array}{c} \Phi_1^o \\ -\Phi_1^- \end{array} \right), \quad \Phi_2 = \left( \begin{array}{c} \Phi_2^+ \\ \Phi_2^o \end{array} \right), \quad \tan \beta \equiv \frac{\langle \Phi_2^o \rangle}{\langle \Phi_2^o \rangle}.$$  

Using the experimental data on meson decay rates, mixing and CP violation we set limits to the parameter $\tan \beta$ as a function of the mass of the charged Higgs $m_H$. This article is an update of [4]. The reason for this update is that the recent measurements of $\sin(2\beta_{CKM})$ by the B-factories Belle[5] and BaBar[6] permit us to set more stringent limits on $\tan \beta$. $\beta_{CKM}$ is an angle of the “unitarity triangle”.[7]

2 Theory

Consider the $(B^o, \bar{B}^o)$ system. $B^o \leftrightarrow \bar{B}^o$ mixing occurs because of the box diagrams illustrated in Figure I. The difference in mass of the two eigenstates that diagonalize the Hamiltonian can be written in the form

$$\Delta m_B = \frac{\beta_B G_F m_W^2 f_B^2 m_B}{6\pi^2} \left| \sum_{i,j} \xi_i \xi_j \left[ S_{WW}^i - 2\cot^2 \beta \cdot S_{HW}^i + \frac{1}{4} \cot^4 \beta \cdot S_{HH}^i \right] \right|.$$  

(1)

The functions

$$S_{WW}^i (x_W^i, x_W^j), \quad S_{HW}^i (x_W^i, x_W^j, x_H^i, x_H^j, x_W^j) \quad \text{and} \quad S_{HH}^i (x_H^i, x_H^j, x_W^j)$$

are obtained from the box diagrams and are written in Appendix A. The Feynman rules for $H^\pm$ are listed in Appendix B. We have derived[8] $S_{WW}$ in agreement with the literature[9]. The derivation of $S_{HW}$ and $S_{HH}$ is given in [10]. The variables of these functions are

$$x_W^i \equiv \frac{m_W^2}{m_W^2}, \quad x_H^i \equiv \frac{m_H^2}{m_H^2}, \quad \text{and} \quad x_W^j \equiv \frac{m_W^2}{m_H^2}.$$
where \( i = u, c, t \), \( \xi_i \equiv V_{ib}V_{is}^* \). The notation for the remaining symbols in (1) is standard [7]. To obtain the Standard Model [9], omit \( S^{WW} \) and \( S^{HH} \). \( \beta_B \) is a factor of order 1. Estimates of \( \beta_B \) using “vacuum intermediate state insertion” [9], “PCAC and vacuum saturation” [9], “bag model” [9], “QCD corrections” [11, 12], and the “free particles in a box” [8] models span the range \( \approx 0.4 \) to \( \approx 1 \). \( f_B \) is the decay constant that appears in the decay rate for \( B^+ \to \mu^+ \nu_\mu \) [8] which at tree level in the Two Higgs Doublet Model (Model II) is:

\[
\Gamma_{B^+} = \frac{|V_{ub}|^2}{8\pi} G_F^2 f_B^2 m_{B^+} \left( 1 - \frac{m_\mu^2}{m_{B^+}^2} \right)^2 \left[ f_B - g_B \frac{m_{B^+}^2}{m_H^2} \tan^2 \beta \right]^2
\]  

(2)

In the derivation of (2) we have substituted \( \bar{v} (\bar{b}) \gamma^\mu (1 - \gamma^5) u (u) \to p^\mu f_B \),

\[
\bar{v} (\bar{b}) (1 - \gamma^5) u (u) \to -\frac{m_{B^+}^2}{m_b} g_B
\]

which defines the decay constants \( f_B \) and \( g_B \). \( \bar{v} (\bar{b}) \) and \( u (u) \) are spinors, see Appendix B. We expect \( f_B \approx g_B \): for a scalar meson with the quark and antiquark at rest \( f_B = \frac{m_\mu}{m_b} g_B \). The decays \( B^+ \to \mu^+ \nu_\mu \) and \( D^+ \to \mu^+ \nu_\mu \) are not yet accessible to experiment so that \( f_B \) and \( f_D \) are unknown. \( f_B \) is estimated using sum rules [13], or the \( B^* - B \) mass difference [14], or a phenomenological model [15], or the MIT bag model [16]. These estimates span the range \( \approx 0.06 GeV \) to \( \approx 0.2 GeV \) with the convention used in reference [8] and in Equation (2).

In the “free particles in a box” [8] model \( \beta_B = 1 \) (after correcting [8] by a color factor 4/3) and the volume of the box, \( i.e. \) the meson, is \( V = 8 |f_B|^2 f_B^2 \).

For the \( (B^o_s, B^o_s) \) system: \( \xi_s \equiv V_{ib}V_{is}^* \) where \( i = u, c, t \); in (1) replace subscript \( B \) by \( B_s \). For the \( (K^o, K^o) \) system: \( \xi_s \equiv V_{is}V_{id}^* \) where \( i = u, c, t \); in (1) replace subscript \( B \) by \( K \). The CP violation parameter \( \varepsilon \) in the \( (K^o, K^o) \) system in the Two Higgs Doublet Model is given by:

\[
\varepsilon = e^{i \phi} \cdot \frac{\text{Im} \left( \sum_{i,j} \xi_i \xi_j \left[ S^{WW} - 2 \cot^2 \beta \cdot S^{HW} + \frac{1}{4} \cot^4 \beta \cdot S^{HH} \right] \right)}{2\sqrt{2} \cdot \sum_{i,j} \xi_i \xi_j \left[ S^{WW} - 2 \cot^2 \beta \cdot S^{HW} + \frac{1}{4} \cot^4 \beta \cdot S^{HH} \right]}
\]  

(3)

For the \( (D^o, D^o) \) system: \( \xi_s \equiv V_{is}V_{id}^* \) where \( i = d, s, b \); in (1) replace subscript \( B \) by \( D \) and replace \( \cot \beta \) by \( \tan \beta \) (leave \( \tan \beta \) as is in (2)).

The branching ratio for \( H^+ \to \tau^+ \nu_\tau \) for \( m_H < m_i \) is given by

\[
B (H^+ \to \tau^+ \nu_\tau) \approx \frac{m_\tau^2 \tan^2 \beta}{|V_{cs}|^2 a + |V_{cb}|^2 b + m_i^2 \tan^2 \beta}
\]  

(4)
Figure 1: Feynman diagrams corresponding to $B^o \leftrightarrow \bar{B}^o$ mixing in the Two Higgs Doublet Model. $q = d$ or $s$ and $i, j = u, c, t$. The diagrams on the right side interfer with a “−” sign.
with \( a \equiv 3 [m_s^2 \tan^2 \beta + m_c^2 \cot^2 \beta] \) and \( b \equiv 3 [m_b^2 \tan^2 \beta + m_c^2 \cot^2 \beta] \). From the measured limit \([17]\) on \( m_H \) as a function of the branching ratio and \((4)\) we obtain a lower bound of \( m_H \) for each \( \tan \beta \).

Let us finally mention that the time-dependent CP-violating asymmetry \( A \equiv (\Gamma - \bar{\Gamma})/(\Gamma + \bar{\Gamma}) \), where \( \Gamma \) (\( \bar{\Gamma} \)) is the rate of the decay \( B^o \rightarrow J/\psi + K_s \) (\( B^o \rightarrow J/\psi + K_s \)), measured by CDF, Belle and BaBar is given by \( \sin(2\beta_{CKM}) \cdot \sin(\Delta M t) \) in both the Standard Model and in the Two Higgs Doublet Model (Model II). This is because the dominating terms of \( \xi_i \xi_j S_{HW} \) and \( \xi_i \xi_j S_{HH} \) have \( i = j = t \).

3 Limits

All experimental data are taken from \([7]\). In order to obtain limits we assume conservatively \( 0.4 < \beta_x < 1.8 \), and \( f_x = g_x \) with \( x = B, B_s, D, K, \pi \). These assumptions are not critical since the upper (lower) limits on \( \tan \beta \) depend on terms \( \propto \tan^4 \beta (\propto \cot^4 \beta) \) in \((1)\) or \((2)\). We take the magnitude of the elements of the CKM matrix from \([7]\) and leave the phase \( \Delta V_{ub} \) as a free parameter. The following calculations are made for each \((m_H, \tan \beta)\). The measured value of the parameter \( \varepsilon \) determines the phase \( \Delta V_{ub} \) of the CKM matrix, and hence \( \beta_{CKM} \). This phase is required to be within the experimental bounds: \( 0.325 < \tan(\beta_{CKM}) < 0.862 \) at 95\% confidence.\([7]\)

The measured decay rates \( \Gamma_K \) and \( \Gamma_\pi \) determine \( f_K \) and \( f_\pi \) using \((2)\). The experimental upper bounds on \( \Gamma_B \) and \( \Gamma_D \) determine upper bounds on \( f_B \) and \( f_D \) using \((2)\). The measured \( \Delta m_B \) and \( \Delta m_K \) determine \( \beta_B f_B^2 \) and \( \beta_K f_K^2 \) using \((2)\). The experimental upper bound on \( \Delta m_D \) determines an upper bound on \( \beta_D f_B^2 \). The experimental lower bound on \( \Delta m_{B_s} \) determines a lower bound on \( \beta_{B_s} f_{B_s}^2 \). From the preceding information we obtain \( \beta_K \) and a lower bound on \( \beta_B \). Then the requirements \( 0.4 < \beta_K < 1.8 \), \( \beta_B < 1.8 \) and \( 0.325 < \tan(\beta_{CKM}) < 0.862 \) place limits on \( \tan \beta \) for each \( m_H \) as listed in Table \([1]\). The confidence level of these limits is 95\%. It turns out that the lower limit on \( \tan \beta \) is determined by the experimental lower limit of \( \tan(\beta_{CKM}) \), and the upper limit on \( \tan \beta \) is determined by \( \beta_B < 1.8 \).

4 Conclusions

Using measured meson decay rates, mixing and CP violation we have obtained lower and upper bounds of \( \tan \beta \) for each \( m_H \). These limits are compared with the results of direct searches in Figures \([4]\). Note that the measurements of \( \sin(2\beta_{CKM}) \) by the Belle and BaBar collaborations have
Table 1: Limits on $\tan \beta$ for several $m_H$ from measurements of meson decay, mixing and CP violation. These limits correspond to 95% confidence.

| $m_H$ (GeV) | $1.74 < \tan \beta < 67$ | $1.36 < \tan \beta < 134$ | $1.13 < \tan \beta < 202$ | $0.58 < \tan \beta < 672$ |
|-------------|---------------------------|-----------------------------|-----------------------------|-----------------------------|
| $m_H = 100$  | $1.74 < \tan \beta < 67$ | $1.36 < \tan \beta < 134$ | $1.13 < \tan \beta < 202$ | $0.58 < \tan \beta < 672$ |
| $m_H = 200$  | $1.36 < \tan \beta < 134$ | $1.13 < \tan \beta < 202$ | $0.58 < \tan \beta < 672$ |
| $m_H = 300$  | $1.13 < \tan \beta < 202$ | $0.58 < \tan \beta < 672$ |
| $m_H = 1000$ | $0.58 < \tan \beta < 672$ |

Figure 2: Lower and upper limits on $\tan \beta$ as a function of the mass of the charged Higgs $m_H$ from meson decay, mixing and CP violation (continuous curve) compared to limits obtained by CDF\cite{18}, D0\cite{19} and LEP2\cite{20}, all at 95% confidence.
raised the lower bound on \( \tan \beta \) by a factor \( \approx 5 \) with respect to our previous calculation.[4]

A Functions \( S_{WW}, S_{HW}, \) and \( S_{HH} \).

If \( i \neq j \):

\[
S_{WW}(x_i^W, x_j^W) = \frac{x_i^W + x_j^W - \frac{11}{4} x_i^W x_j^W}{(1 - x_i^W)(1 - x_j^W)} + \frac{1}{x_i^W - x_j^W} \left[ G(x_i^W, x_j^W) - G(x_j^W, x_i^W) \right]
\]

(5)

where

\[
G(x_i^W, x_j^W) = \frac{(x_i^W)^2 \ln(x_i^W)}{(1 - x_i^W)^2} \left[ 1 - 2x_i^W + \frac{1}{4} x_i^W x_j^W \right].
\]

(6)

If \( i = j \):

\[
S_{WW}(x_i^W, x_i^W) = \frac{x_i^W}{(1 - x_i^W)^2} \left[ 3 - \frac{19}{4} x_i^W + \frac{1}{4} (x_i^W)^2 \right]
\]

\[
+ \frac{2x_i^W \ln(x_i^W)}{(1 - x_i^W)^2} \left[ 1 - \frac{3}{4} \frac{(x_i^W)^2}{1 - 4(1 - x_i^W)} \right].
\]

(7)

If \( i \neq j \):

\[
S_{HH}(x_i^H, x_j^H, x_W^H) = \frac{x_i^H x_j^H}{x_W^H} \left[ \frac{J(x_i^H) - J(x_j^H)}{x_i^W - x_j^W} \right]
\]

(8)

with

\[
J(x_i^H) = \frac{1}{1 - x_i^H} + \frac{(x_i^H)^2 \ln(x_i^H)}{(1 - x_i^H)^2}.
\]

(9)

If \( i = j \):

\[
S_{HH}(x_i^H, x_i^H, x_H^W) = \frac{(x_i^H)^2}{x_H^W} \left[ 1 - (x_i^H)^2 + 2x_i^H \ln(x_i^H) \right].
\]

(10)
For $i \neq j$:

\[
S^{\text{HW}}(x_W^i, x_W^j, x_H^i, x_H^j, x_W^j) = \frac{x_H^i x_H^j}{(x_W^i - 1)(x_H^i - 1)(x_H^j - 1)} \left[ 1 - \frac{1}{8x_W^i} \right] + \frac{x_H^ix_H^j}{(x_W^i - 1)(x_H^i - x_W^j)(x_H^j - x_W^j)} \left[ \frac{3}{4} \ln(x_H^i) - \frac{7}{8} \right] + \frac{(x_H^i)^2 x_H^j}{(x_H^i - x_W^j)(x_H^j - x_W^j)(x_H^j - 1)} \left[ \ln(x_H^j) \left( 1 - \frac{1}{4} x_W^j \right) + \left( \frac{1}{8} x_W^j - 1 \right) \right].
\]

For $i = j$:

\[
S^{\text{HW}}(x_W^i, x_W^i, x_H^i, x_H^i, x_W^i) = (x_W^i)^2 \left[ \frac{\ln(x_H^i)}{(x_W^i - 1)(x_H^i - 1)^2} \left( 1 - \frac{1}{4x_W^i} \right) \right] - \frac{3}{4} \frac{x_W^i \ln(x_W^i)}{(x_W^i - 1)(x_H^i - x_W^i)^2} + \frac{1}{(x_H^i - 1)(x_H^i - x_W^i)} \left( 1 - \frac{1}{4} x_W^i \right). \tag{12}
\]

B  Feynman rules of the charged Higgs in the Two Higgs Doublet Model

The effective Lagrangian corresponding to the $H^+ f \bar{f}'$ vertex is:

\[
L = \frac{g}{2\sqrt{2}m_W} \left[ H^+ V_{f f'} \bar{u}_f (A + B \gamma^5) v_{f'} + h.c. \right]. \tag{13}
\]

where $A \equiv (m_f \tan \beta + m_f \cot \beta)$ and $B \equiv (m_f \tan \beta - m_f \cot \beta)$, $f = \text{fermion (quark or lepton)}$ and $\bar{f}' = \text{antifermion (antiquark or antilepton)}$. $V_{f f'}$ is an element of the CKM matrix.

The charged-Higgs propagator is: $i/ (K^2 - m_H^2 + i\varepsilon)$.

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