Supersymmetric Decays of the $Z'$ Boson

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Abstract

The decay of the $Z'$ boson into supersymmetric particles is studied. We investigate how these supersymmetric modes affect the current limits from the Tevatron and project the expected sensitivities at the LHC. Employing three representative supersymmetric $Z'$ models, namely, $E_6$, $U(1)_{B-L}$, and the sequential model, we show that the current limits of the $Z'$ mass from the Tevatron could be reduced substantially due to the weakening of the branching ratio into leptonic pairs. The mass reach for the $E_6$ $Z'$ bosons is about $1.3 - 1.5$ TeV at the LHC-7 ($1$ fb$^{-1}$), about $2.5 - 2.6$ TeV at the LHC-10 ($10$ fb$^{-1}$), and about $4.2 - 4.3$ TeV at the LHC-14 ($100$ fb$^{-1}$). A similar mass reach for the $U(1)_{B-L}$ $Z'$ is also obtained. We also examine the potential of identifying various supersymmetric decay modes of the $Z'$ boson because it may play a crucial role in the detailed dynamics of supersymmetry breaking.
I. INTRODUCTION

Existence of extra neutral gauge bosons has been predicted in many extensions of the standard model (SM) \cite{1}. String-inspired models and grand-unification (GUT) models usually contain a number of extra $U(1)$ symmetries, beyond the hypercharge $U(1)_Y$ of the SM. The exceptional group $E_6$ is one of the famous examples of this type \cite{2}. These extra $U(1)$’s are broken at some intermediate energy scales between the GUT and the electroweak scales. Phenomenologically, the most interesting option is the breaking of these $U(1)$’s at around TeV scales, giving rise to extra neutral gauge bosons observable at the Tevatron and the Large Hadron Collider (LHC). Recent developments in model buildings also result in new models that contain extra gauge bosons. For example, little Higgs models \cite{3} with additional gauge groups predict a number of new gauge bosons; the SM gauge bosons propagating in the extra dimensions after compactification can give rise to Kaluza-Klein towers of gauge bosons \cite{4}; Stueckelberg $Z'$ model connecting a hidden sector to the visible sector in the context of dark matter \cite{5}, just to name a few. We will denote these neutral extra gauge bosons generically by $Z'$ for the general discussion in this introduction section.

Collider experiments such as CDF \cite{6} and DØ \cite{7} at the Tevatron have been searching for the neutral extra gauge bosons $Z'$, mainly through its leptonic decay modes. The leptonic mode is a very clean channel to probe for $Z'$ since it may give rise to a discernible peak above the Drell-Yan background right at the $Z'$ mass, provided that the size of the coupling strength to SM quarks and leptons is not too small. Currently, the best limit comes from the negative search at the Tevatron. The lower mass bound on $Z'$ is about $800 - 900$ GeV for a number of $Z'$ bosons of the $E_6$ type and a stronger bound of almost $1050$ GeV for the sequential $Z'$, which has exactly the same coupling strength and chiral couplings as the SM $Z$ boson that can be served as a benchmark. The LHC with just an integrated luminosity of about $40$ $\text{pb}^{-1}$ has already set limits on $Z'$’s \cite{8} almost as good as those from the Tevatron. With more luminosity accumulated in the current LHC run the limit on $Z'$ will improve substantially in the near future.

Most of previous studies on $Z'$ bosons focused on the decays into SM fermions, and the corresponding limits were obtained based on the decay into leptons. This scenario is not necessarily a must, but just for simplicity and fewer choices of parameters. Indeed, when the mass of $Z'$ is more than a TeV or even larger it has chances of decaying into
other exotic particles that must be included in the model for various theoretical reasons. For instance, in GUT models or in little-Higgs models there are other fermions needed to cancel the anomalies, the $Z'$ could decay into these exotic fermions if their masses are not too heavy. Another example is the minimal supersymmetric standard model (MSSM) with electroweak-scale SUSY partners and a $Z'$ which could also decay into sfermions and Higgsinos. In a recently proposed $Z'$-mediated SUSY-breaking model [9], supersymmetry breaking in the hidden sector is communicated by a $Z'$ boson to the visible sector. The low-energy spectrum includes a $Z'$ boson of a few TeV and light gauginos, such that the $Z'$ can also decay into SUSY particles other than the SM fermions.

In this work, we consider a scenario of $Z'$ boson, which arised from $U(1)$ symmetry breaking at around TeV scale, in the context of weak-scale supersymmetry, in which all SUSY partners are relatively light (a few hundred GeV) except for the squarks (may of order $O(1)$ TeV). Such a $Z'$ may come from breaking of one of the $U(1)$'s in $E_6$, $U(1)_{B-L}$ [10], or $U(1)_{B}$ [11] etc. Once we specify the $U(1)$ charges for the matter superfields and Higgs superfields, the couplings of $Z'$ to MSSM particles are determined. We study the decays of $Z'$ and its production at the Tevatron and the LHC. The decays of $Z'$ will be modified when the SUSY particle masses are only of order a few hundred GeV, which will then affect the leptonic branching ratio of the $Z'$. We investigate how much the limits from the Tevatron will be affected, because of the reduction in the leptonic branching ratio. We also study how much the sensitivity at the LHC will be reduced when the SUSY decay modes are open for the $Z'$ boson. Finally, we study the prospect of using the SUSY decay modes of the $Z'$ to search for the $Z'$ boson itself and investigate its properties.

We note that some related works had appeared in literature, for example in Refs. [12–14]. However substantial improvements over these previous works have been made in this work. These include

1. Ref. [12] focused on how the presence of supersymmetric and other exotic particles of $E_6$ models in the $Z'$ decay can affect the $Z'$ boson discovery at the Tevatron and the LHC. In our work, we used the most current updated limit on $\sigma(Z') \times B(Z' \rightarrow \mu^+\mu^-)$ to put limits on the mass of various $Z'$ bosons. We have illustrated the case of decaying into SM particles only and the case of including both SUSY and SM particles. We have shown that the $Z'$ mass limits have to be relaxed by $20 – 30$ GeV if including
SUSY particles in the decay. Ref. [12] was written in 2004 and certainly our paper used the newest 2011 data. For the LHC sensitivity we worked out the more realistic energy-luminosity combinations (7 TeV, 10 TeV, 14 TeV). Refs. [12, 13] did not know about the options of 7 and 10 TeV at their time.

2. Ref. [13] focused on how the discovery potential of sleptons can be improved via the decay of $Z'$ into a slepton pair. They also studied various lightest supersymmetric particle (LSP) scenarios and used distributions to determine the masses of sleptons, gauginos, and the $Z'$ boson. In our work, in addition to the slepton-pair, we also studied the $Z'$ decays into a chargino pair and into a neutralino pair. We have shown clearly that the presence of $Z'$ is visible in the transverse-mass spectrum. One can therefore measure the transverse-mass spectrum and determine if there is a $Z'$ boson. This spectrum can also be utilized to estimate the mass differences, and couplings of $Z'$ to sleptons, neutralinos, and charginos. This can help us to understand the underlying supersymmetry breaking mechanism.

3. In this work, we study $E_6$ models, $U(1)_{B-L}$, and the sequential $Z'$ model, while Ref. [12] studied only the $E_6$ models and Ref. [13] studied only the $U(1)_{B-L}$ model.

The organization of the paper is as follows. In the next section, we write down the interactions and briefly describe a few $Z'$ models and their extensions to include supersymmetry. In Sec. III, we calculate the branching ratios of the $Z'$ boson in various models. In Sec. IV, we show the shift of the limits for the masses of the $Z'$ in various models due to opening of supersymmetric particles. We estimate the 5$\sigma$ discovery reach at the LHC, including the SUSY decay modes in Sec. V. We further discuss in Sec. VI the SUSY decay modes of the $Z'$ boson. We conclude in Sec. VII. Feynman rules that are related to the $Z'$ are collected in the appendix.
II. $Z'$ INTERACTIONS AND REPRESENTATIVE MODELS

A. $Z'$ Interactions

Following the notation of Refs. [1, 15], the Lagrangian describing the neutral current gauge interactions of the standard electroweak $SU(2)_L \times U(1)_Y$ and extra $U(1)\text{'s}$ is given by

$$- L_{\text{NC}} = e J_{\text{em}}^\mu A_\mu + \sum_{\alpha=1}^n g_\alpha J_{\alpha}^\mu Z_\alpha^0,$$

where $Z_1^0$ is the SM $Z$ boson and $Z_\alpha^0$ with $\alpha \geq 2$ are the extra $Z$ bosons in the weak-eigenstate basis. For the present work we only consider one extra $Z_2^0$ mixing with the SM $Z_1^0$ boson. Thus the second term of the Lagrangian in Eq. (1) can be rewritten as

$$- L_{Z_1^0 Z_2^0} = g_1 Z_{1\mu}^0 \left[ \sum_f \bar{\psi}_f \gamma^\mu (g_L^f P_L + g_R^f P_R) \psi_f \right] + g_2 Z_{2\mu}^0 \left[ \sum_f \bar{\psi}_f \gamma^\mu (Q_L^f P_L + Q_R^f P_R) \psi_f \right],$$

where for both quarks and leptons

$$g_{L,R}^f = T_{3L}^f - x_w Q_f,$$

and $Q_{L,R}^f$ are the chiral charges of fermion $f$ to $Z_2^0$ and $P_{L,R} = (1 \mp \gamma_5)/2$. Here $T_{3L}^f$ and $Q_f$ are, respectively, the third component of the weak isospin and the electric charge of the fermion $f$. The chiral charges of various $Z'$ models are listed in Tables I and II. The overall coupling constant $g_1$ in Eq. (1) is the SM coupling $g/\cos \theta_w$, while in grand unified theories (GUT) $g_2$ is related to $g_1$ by

$$\frac{g_2}{g_1} = \left( \frac{5}{3} x_w \lambda \right)^{1/2} \simeq 0.62 \lambda^{1/2},$$

where $x_w = \sin^2 \theta_w$ and $\theta_w$ is the weak mixing angle. The factor $\lambda$ depends on the symmetry breaking pattern and the fermion sector of the theory, which is usually of order unity.

The mixing of the weak eigenstates $Z_1^0$ and $Z_2^0$ to form mass eigenstates $Z$ and $Z'$ are parametrized by a mixing angle $\theta$:

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} Z_1^0 \\ Z_2^0 \end{pmatrix}.$$

The mass of $Z$ is $M_Z = 91.19$ GeV.
After substituting the interactions of the mass eigenstates $Z$ and $Z'$ with fermions are

$$-\mathcal{L}_{ZZ'} = \sum_f g_f \left[ Z_\mu \bar{\psi}_f \gamma^\mu (l_s^f P_L + r_s^f P_R) \psi_f + Z'_\mu \bar{\psi}_f \gamma^\mu (l_n^f P_L + r_n^f P_R) \psi_f \right], \quad (6)$$

where

$$l_s^f = g_f^L + \frac{g_2}{g_1} \theta Q_{fL}, \quad r_s^f = g_f^R + \frac{g_2}{g_1} \theta Q_{fR}, \quad (7)$$

$$l_n^f = \frac{g_2}{g_1} Q_{fL} - \theta g_f^L, \quad r_n^f = \frac{g_2}{g_1} Q_{fR} - \theta g_f^R. \quad (8)$$

Here the subscript “s” denotes the observed SM $Z$ boson and “n” denoted the new heavy gauge boson $Z'$. We have used the valid approximation $\cos \theta \approx 1$ and $\sin \theta \approx \theta$. In the following, we ignore the mixing ($\theta = 0$) such that the precision measurements for the SM $Z$ boson are not affected, unless stated otherwise.

### B. Including Supersymmetry

The superpotential $W$ involving the matter and Higgs superfields in a $U(1)'$ extended MSSM can be written as

$$W = \epsilon_{ab} \left[ y_u^{ij} Q_{ij} H_u^c U^c_i - y_d^{ij} Q_{ij} H_d^b D^c_i - y_l^{ij} L_{ij} H_u^b E^c_i + h_s S H_u^a H_d^b \right], \quad (9)$$

where $\epsilon_{12} = -\epsilon_{21} = 1$, $i, j$ are family indices, and $y^u$ and $y^d$ represent the Yukawa matrices for the up-type and down-type quarks respectively. Here $Q, L, U^c, D^c, E^c, H_u,$ and $H_d$ denote the MSSM superfields for the quark doublet, lepton doublet, up-type quark singlet, down-type quark singlet, lepton singlet, up-type Higgs doublet, and down-type Higgs doublet respectively, and the $S$ is the singlet superfield. Note that we have assumed other exotic fermions are very heavy. The $U(1)'$ charges of the fields $H_u, H_d$, and $S$ are related by $Q'_{H_u} + Q'_{H_d} + Q'_{S} = 0$ such that $SH_u H_d$ is the only term allowed by the $U(1)'$ symmetry beyond the MSSM. Once the singlet scalar field $S$ develops a VEV, it generates an effective $\mu$ parameter: $\mu_{\text{eff}} = h_s \langle S \rangle$. The case is very similar to NMSSM, except we do not have the cubic term $S^3$. The singlet field will give rise to a singlet scalar boson and a singlino, which will mix with other particles in the Higgs sector and neutralino sector, respectively. In this work, we are contented with the assumption that the singlet scalar field and the singlino are heavy enough that it is out of reach at the LHC and the mixing effects are negligible. Detailed phenomenological studies involving the singlet field will be presented in a future
work. Furthermore, we also take the superpartner, dubbed as $Z'$-ino, of the $Z'$ boson to be heavy. Phenomenology involving the singlet scalar boson, singlino, and the $Z'$-ino of various singlet-extended MSSM can be found in Refs. [16].

Below the TeV scale the particle content is the same as the MSSM plus a $Z'$ boson. Thus, the superpotential and the soft breaking terms are the same as in MSSM. Extra couplings of the $Z'$ boson with the MSSM particles are coming from the gauge interactions of the extra $U(1)$ and the corresponding supersymmetric vertices of Yukawa interactions.

The gauge interactions involving the fermionic and scalar components, denoted generically by $\psi$ and $\phi$ respectively, of each superfield are

$$\mathcal{L} = \bar{\psi} i\gamma^\mu D_\mu \psi + (D^\mu \phi)^\dagger (D_\mu \phi),$$

where the covariant derivative is given by

$$D_\mu = \partial_\mu + ieQA_\mu + \frac{ig}{\sqrt{2}}(\tau^+ W^+_\mu + \tau^- W^-_\mu) + ig_1(T_{3L} - Q_xw)Z_\mu + ig_2Z'_\mu Q'.$$

Here $e$ is the electromagnetic coupling constant, $Q$ is the electric charge, $\tau^\pm$ are the rising and lowering operators on $SU(2)_L$ doublets and $Q'$ is the chiral charges of the $U(1)$ associated with the $Z'$ boson. The interactions of $Z'$ with all MSSM fields go through Eqs. (10) and (11).

Details and conventions are given in the appendix and the Feynman rules that involve the $Z'$ boson are listed there as well.

C. Representative models

1. The Sequential model

The sequential $Z'_{SM}$ model is a reference model of extra $Z$ bosons. It has exactly the same chiral charges as the SM $Z$ boson but at a larger mass. The gauge coupling constant is also taken to be the same as the SM one, i.e., $g_2 = g_1 = g/\cos\theta_w$. Note, however, that when SUSY modes are open, the $Z'_{SM}$ can also decay into sfermions, neutralinos, charginos, and Higgs bosons. In general, experimental constraints on the sequential model are the strongest, because the other $E_6$ models, for example, have smaller gauge coupling constant, as in Eq. (4).
2. The $E_6$ models

Two most studied $U(1)$ subgroups in the symmetry breaking chain of $E_6$ occur in

$$E_6 \rightarrow SO(10) \times U(1)_\psi, \quad SO(10) \rightarrow SU(5) \times U(1)_\chi.$$  

In $E_6$ each family of the left-handed fermions is promoted to a fundamental $27$-plet, which decomposes under $E_6 \rightarrow SO(10) \rightarrow SU(5)$ as

$$27 \rightarrow 16 + 10 + 1 \rightarrow (10 + 5^* + 1) + (5 + 5^*) + 1.$$  

Each $27$ contains the SM fermions, two additional singlets $\nu^c$ (conjugate of the right-handed neutrino) and $S$, a $D$ and $D^c$ pair ($D$ is the exotic color-triplet quark with charge $-1/3$ and $D^c$ is the conjugate), and a pair of color-singlet SU(2)-doublet exotics $H_u$ and $H_d$ with hypercharge $Y_{H_u,H_d} = \pm 1/2$. In the supersymmetric version of $E_6$, the scalar components of one $H_{u,d}$ pair can be used as the two Higgs doublets $H_{u,d}$ of the MSSM. The chiral charges $U(1)_\psi$ and $U(1)_\chi$ for each member of the $27$ are listed in the third and fourth columns in Table [I]. In general, the two $U(1)_\psi$ and $U(1)_\chi$ can mix to form

$$Q'({E_6}) = \cos \theta_{E_6} Q'_\chi + \sin \theta_{E_6} Q'_\psi,$$  

where $0 \leq \theta_{E_6} < \pi$ is the mixing angle. A commonly studied model is the $Z'_\eta$ model with

$$Q'_{\eta} = \sqrt{\frac{3}{8}} Q'_\chi - \sqrt{\frac{5}{8}} Q'_\psi,$$  

which has $\theta_{E_6} = \pi - \tan^{-1} \sqrt{5/3} \sim 0.71\pi$. There are also the inert model with $Q'_{I} = -Q'({E_6}) = \tan^{-1} \sqrt{3/5} \sim 0.21\pi$), the neutral $N$ model with $\theta_{E_6} = \tan^{-1} \sqrt{15} \sim 0.42\pi$, and the secluded sector model with $\theta_{E_6} = \tan^{-1} \sqrt{15}/9 \sim 0.13\pi$. The chiral charges for each member of the $27$ are also listed in the last four columns in Table [I] for these four variations of $Z'$ models within $E_6$. Here we take the assumption that all the exotic particles, other than the particle contents of the MSSM, are very heavy and well beyond the reaches of all current and planned colliders.

3. $B - L$ models

The extra $U(1)$ symmetry here is the $U(1)_{B-L}$ with $Q'_f = \frac{1}{2}(B - L)^f$, where $B$ and $L$ are the baryon and lepton numbers, respectively. The theory is vector-like, thus the
TABLE I. The chiral charges of the left-handed fermions for various $Z'$ bosons arised in $E_6$. Note that $Q'_{f_R} = -Q'(f^c)$ since all the right-handed SM fermions are necessarily converted into left-handed charge-conjugated fermions in order to put them into the irreducible representation of 27 of $E_6$.

| $SO(16)$ | $SU(5)$ | $2\sqrt{10}Q'_{\chi}$ | $2\sqrt{6}Q'_{\psi}$ | $2\sqrt{15}Q'_{\eta}$ | $2Q'_{I}$ | $2\sqrt{10}Q'_{N}$ | $2\sqrt{15}Q'_{\text{sec}}$ |
|-----------|---------|------------------|------------------|------------------|---------|------------------|------------------|
| 16        | 10      | $-1$             | $1$              | $-2$             | $0$     | $1$              | $-1/2$           |
|           | 5*      | $3$              | $1$              | $1$              | $-1$    | $2$              | $4$              |
|           | $\nu^c$ | $-5$             | $1$              | $-5$             | $1$     | $0$              | $-5$             |
| 10        | 5       | $2$              | $-2$             | $4$              | $0$     | $-2$             | $1$              |
|           | 5*      | $-2$             | $-2$             | $1$              | $1$     | $-3$             | $-7/2$           |
| 1         | 1S      | $0$              | $4$              | $-5$             | $-1$    | $5$              | $5/2$            |

Table II. The chiral charges of the left-handed fermions for the $Z'_{B-L}$ boson. Note that $Q'_{f_R} = -Q'(f^c)$ since charge-conjugated fields are used here.

| $Q = (u, d)$ | $1/6$ |
| $u^c$ | $-1/6$ |
| $d^c$ | $-1/6$ |
| $L = (\nu, e^-)$ | $-1/2$ |
| $e^c$ | $1/2$ |

III. DECAYS OF $Z'$

In general, the $Z'$ boson can decay into all SM fermion, squark, slepton, sneutrino, neutralino, chargino, and Higgs-boson pairs. The decays into neutralino and chargino pairs go via the couplings to Higgsinos. The Higgs-boson pairs include $Ah^0$, $AH^0$, and $H^+H^-$. All decay modes are subjected to kinematic threshold. We have chosen typical input parameters
for the supersymmetric particles:

Set(A) : \[ \tan \beta = 5 \], \[ m_{\tilde{q}} = O(\text{a few TeV}) \], \[ M_{\tilde{L}} = 200 \text{ GeV} \], \[ M_A = 500 \text{ GeV} \], \[ M_1 = 100 \text{ GeV} \], \[ M_2 = 200 \text{ GeV} \], \[ \mu = 150 \text{ GeV} \], \[ A = 100 \text{ GeV} \]. \hspace{1cm} (14)

With this set of choices the \( Z' \) boson will not decay into squark pairs, but can decay into all other sfermion pairs, neutralino and chargino pairs, and Higgs-boson pairs.

We show in Figs. 1 to 8 the decay branching ratios of various \( Z' \) bosons. In these figures, we only show \( e^+e^- \) – one of the charged lepton modes, which is the most direct discovery mode of the \( Z' \) boson, \( \tilde{e}_L \tilde{e}_L^* + \tilde{e}_R \tilde{e}_R^* \) – one of the slepton modes, \( \tilde{\nu}_L \tilde{\nu}_L^* \) – one of the sneutrino modes, but the sum of neutralino and chargino pairs. The other charged-lepton modes and slepton modes are approximately the same as the corresponding one shown in the figures. The top-quark pair and the other Higgs-boson pairs are also shown. Note that the \( Z'_{B-L} \) does not couple to the Higgs fields.

We also select another set of SUSY parameters that we call set (B), typically it is a large \( \tan \beta \) case.

Set(B) : \[ \tan \beta = 40 \], \[ m_{\tilde{q}} = O(\text{a few TeV}) \], \[ M_{\tilde{L}} = 300 \text{ GeV} \], \[ M_A = 500 \text{ GeV} \], \[ M_1 = 200 \text{ GeV} \], \[ M_2 = 400 \text{ GeV} \], \[ \mu = -300 \text{ GeV} \], \[ A = 300 \text{ GeV} \]. \hspace{1cm} (15)

We shall show the results for the limits from the Tevatron and the sensitivities at the LHC with the two choices of set (A) and set (B).

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1 There are additional \( D \)-term contributions to the sfermion and Higgs boson masses from the breaking of the \( U(1)' \) symmetry, which further depend on the details of the VEV of additional Higgs fields that break the \( U(1)' \) symmetry. Instead, we used the physical masses as inputs to our study.

2 We choose the squark mass to be heavy enough such that the decay into squark pair is not open. We have set the soft parameters \( M_{\tilde{L}}^2 = M_{\tilde{E}}^2 = (200 \text{ GeV})^2 \) such that the physical slepton masses are about 205 and 204 GeV for the left-handed and right-handed sleptons, but about 190 GeV for the sneutrino. We have ignored the additional \( D \)-term contribution from the \( U(1)' \).
FIG. 1. Decay branching ratios for some decay modes of the sequential $Z'$ model.

FIG. 2. Same as Fig. 1 but for $Z'_{\chi}$ model.

FIG. 3. Same as Fig. 1 but for $Z'_{\psi}$ model.
FIG. 4. Same as Fig. 1 but for $Z'_{\eta}$ model.

FIG. 5. Same as Fig. 1 but for $Z'_{I}$ model.

FIG. 6. Same as Fig. 1 but for $Z'_{N}$ model.
IV. CURRENT LIMITS FROM THE TEVATRON

The current limits quoted from the Tevatron [6, 7] and the LHC [8] are based on the theoretical calculation of $\sigma(Z') \times B(Z' \rightarrow \ell^+ \ell^-)$, where $\ell = e$ or $\mu$. There the decay of $Z'$ includes only the SM fermions. The most updated limits from the Tevatron range from about 0.8 to about 1 TeV, depending on the models. Here we are going to demonstrate that because of the additional decay channels of the $Z'$, including the sfermions, neutralinos, and charginos, the mass limits from the Tevatron are reduced by a nontrivial amount. From the plots shown in the last section we saw that the branching ratio into charged leptons decreases substantially because of the opening of the SUSY modes. Therefore, the product $\sigma(Z') \times B(Z' \rightarrow \ell^+ \ell^-)$ decreases. We show in Fig. 3 the limits for (a) the sequential $Z'$ boson and (b) the $Z'_{N}$ boson for decays into SM fermions only and for decays into SM fermions and SUSY particles. The choice of SUSY parameters is given in Eq. (14) for set (A). With these
FIG. 9. Exclusion limits on (a) the sequential SM $Z'$ boson and (b) the $Z'_{N}$ boson. The lines are the theoretical production cross section of $\sigma(Z') \times B(Z' \rightarrow \mu^+\mu^-)$ for (i) decays into SM fermions only and (ii) decays into SM fermions and SUSY particles. The solid (red) line shows the most current upper limit from the CDF.

The $Z'$ cannot decay into squarks but can decay into slepton pairs, sneutrino pairs, neutralino and chargino pairs, plus the original SM fermions. Note that the $Z'$ can also decay into Higgs-boson pairs such as $Ah^0$, $AH^0$, and $H^+H^-$. The limit for the sequential $Z'$ boson shifts from 1012 GeV down to 989 GeV (a difference of 23 GeV). The other $Z'$ models behave similarly. We show the $Z'$ mass limits for various models when the SUSY decay modes are open in Fig. [10]. We summarize the changes in limits in Table III for the parameter sets (A) and (B). Compared with the case of SM fermions only, the reduction in the $Z'$ mass limits ranges between 11 (5) and 32 (18) GeV for various models with the SUSY parameters of set (A) (set (B)).

TABLE III. Table showing the down shift of the limits on the $Z'$ boson mass due to the inclusion of the SUSY decay modes of $Z'$. The choices of SUSY parameters are given in set (A) and in set (B).

|          | $Z'_{N}$ | $Z'_{X}$ | $Z'_{\psi}$ | $Z'_{\eta}$ | $Z'_{sec}$ | $Z'_{I}$ | $Z'_{SM}$ | $Z'_{B-L}$ |
|----------|----------|----------|-------------|-------------|------------|----------|-----------|------------|
| SM fermions only | 896      | 882      | 910         | 898         | 811        | 770      | 1012      | 987        |
| SUSY included [Set (A)] | 864      | 866      | 879         | 882         | 793        | 759      | 989       | 966        |
| SUSY included [Set (B)] | 880      | 874      | 892         | 890         | 802        | 765      | 997       | 973        |
FIG. 10. Exclusion limits on $Z'$ bosons of various models described in Sec. II when the SUSY decay modes are open. The SUSY parameters chosen are listed in Eq. (14). The lines are the theoretical production cross section of $\sigma(Z') \times B(Z' \rightarrow \mu^+\mu^-)$. The solid (red) line shows the most current upper limit from the CDF.

V. PROJECTED SENSITIVITIES AT THE LHC

Here we investigate the sensitivities at the LHC with $\sqrt{s} = 7, 10, \text{and } 14 \text{ TeV}$ with projected luminosities of 1, 10, and 100 fb$^{-1}$, respectively, using the conventional search channel, $pp \rightarrow Z' \rightarrow \ell^+\ell^-$. Since the $Z'$ boson is a narrow resonance state, we adopt the following procedures to obtain the 5$\sigma$ discovery potential of each $Z'$ model.

1. Take a $Z'$ boson of mass, say, 800 GeV. We pick a conservative bin-size resolution of order 100 GeV in the $M_{\ell\ell}$ distribution around TeV. Therefore, we are looking at the window $(800 - 50, 800 + 50)$ GeV.

2. We calculate the expected Drell-Yan background in this mass window with the center-of-mass energy and the corresponding luminosity, say $N$ events.

3. If $N > 10$ we can use Gaussian distribution, and the standard deviation is given by $\sqrt{N}$. The 5$\sigma$ signal corresponds to $5 \times \sqrt{N}$ events. If $N < 10$ then we should use Poisson statistics to determine the 5$\sigma$ signal events.
4. Check the $\sigma(Z') \times B(Z' \to \ell^+\ell^-)$ if it is larger or smaller than the 5$\sigma$ events. If $\sigma(Z') \times B(Z' \to \ell^+\ell^-)$ is larger, it means that the discovery sensitivity can go further; otherwise, the discovery sensitivity cannot go further and the current $M_{Z'}$ is the 5$\sigma$ discovery limit, and the procedures stop here.

5. Repeat with an increment of, say 10 GeV, for $M_{Z'}$ and go back to step (1).

The 5$\sigma$ discovery sensitivity for LHC-7, LHC-10, and LHC-14 with luminosities 1, 10, and 100 fb$^{-1}$, respectively, are shown in Table IV for the decay into SM particles only, and for decay into SM and SUSY particles with SUSY parameter sets (A) and (B). The difference in sensitivity reach between set (A) and (B) is very small here, because the mass of the $Z'$ boson near the sensitivity reach is so large that the masses of the sleptons and gauginos involved are relatively negligible. The mass reach for the $E_6$ $Z'$ bosons is about $1.3 - 1.5$ TeV at the LHC-7 with 1 fb$^{-1}$, and could be up to $2.5 - 2.6$ TeV at the LHC-10 with 10 fb$^{-1}$, and 4.2 - 4.3 TeV at the LHC-14 with 100 fb$^{-1}$ when supersymmetry is included in the decay of the $Z'$. On the other hand, the mass reach for $Z'_{SM}$ and $Z'_{B-L}$ is about one to a few hundred GeVs better than the $Z'$ in $E_6$ models. We have checked that the number of signal events at the mass-reach limit, say for the $Z'_{N}$ model, is 5 at the LHC-7 against a Drell-Yan background of 0.09 events, and is 4 at the LHC-10 and LHC-14 against a Drell-Yan background of 0.04 and 0.03 events, respectively. The probability for background fluctuation is $\lesssim 10^{-7}$, such that it is a 5$\sigma$ discovery. Note that we only count one channel of the charged lepton (electron). If both electron and muon are counted, there will be 8 - 10 charged-lepton pair events for discovery, which should be clean enough against negligible Drell-Yan background.

VI. SUPERSYMMETRIC DECAY MODES

Identification of supersymmetric decay modes of the $Z'$ has its own interests, namely, to understand the role of $Z'$ in the SUSY breaking. Moreover, if the $Z'$ boson decays frequently into SUSY particles, we can make use of the SUSY channels to probe for the $Z'$. So far, in the models that we illustrate the branching ratio into charged leptons is not negligible, such that the best discovery mode is still the charged-lepton mode, which cleanly shows the peak in the invariant-mass distribution. Nevertheless, there exist models, e.g, Refs. [5], in which
TABLE IV. The 5σ discovery sensitivity reach in $M_{Z'}$ (TeV) for various $Z'$ models at the LHC-7, -10, and -14 with luminosities 1, 10, and 100 fb$^{-1}$, respectively. The cases with decay into SM particles only, and decay into SM and SUSY particles with parameter sets (A) and (B) are shown.

|               | $Z'_N$ | $Z'_\chi$ | $Z'_\psi$ | $Z'_\eta$ | $Z'_{sec}$ | $Z'_I$ | $Z'_{SM}$ | $Z'_{B-L}$ |
|---------------|--------|-----------|-----------|-----------|------------|--------|----------|----------|
| **LHC-7 at 1 fb$^{-1}$** |        |           |           |           |            |        |          |          |
| SM           | 1.50   | 1.53      | 1.46      | 1.35      | 1.53       | 1.52   | 1.83     | 1.74     |
| SUSY [Set (A)] | 1.36   | 1.49      | 1.32      | 1.31      | 1.48       | 1.46   | 1.68     | 1.60     |
| SUSY [Set (B)] | 1.37   | 1.49      | 1.33      | 1.31      | 1.48       | 1.47   | 1.69     | 1.61     |
| **LHC-10 at 10 fb$^{-1}$** |        |           |           |           |            |        |          |          |
| SM           | 2.65   | 2.69      | 2.61      | 2.53      | 2.67       | 2.65   | 3.16     | 3.03     |
| SUSY [Set (A)] | 2.50   | 2.62      | 2.45      | 2.45      | 2.59       | 2.56   | 3.03     | 2.82     |
| SUSY [Set (B)] | 2.51   | 2.63      | 2.46      | 2.46      | 2.59       | 2.56   | 3.04     | 2.83     |
| **LHC-14 at 100 fb$^{-1}$** |        |           |           |           |            |        |          |          |
| SM           | 4.47   | 4.51      | 4.44      | 4.34      | 4.44       | 4.41   | 5.21     | 5.04     |
| SUSY [Set (A)] | 4.25   | 4.41      | 4.21      | 4.23      | 4.33       | 4.27   | 5.03     | 4.89     |
| SUSY [Set (B)] | 4.26   | 4.41      | 4.22      | 4.23      | 4.33       | 4.27   | 5.03     | 4.89     |

the charged-lepton decay mode is highly suppressed. One could also imagine that a $Z'$ does not couple to fermions or sfermions but only to the Higgs sector, such that it couples solely to Higgs bosons and Higgsinos. In such an extreme the $Z'$ would substantially decay into Higgsinos (or the physical neutralinos and charginos after mixings). In other words, the supersymmetric decay modes of the $Z'$ boson could be sizable and useful for understanding the SUSY breaking.

Typically, the SUSY decay modes include (i) $Z' \rightarrow \ell\ell^* \rightarrow \ell^+\ell^-\tilde{\chi}_1^0\tilde{\chi}_1^0$, (ii) $Z' \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0 \rightarrow \ell^+\ell^-\tilde{\chi}_1^0\tilde{\chi}_1^0$, (iii) $Z' \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- \rightarrow \ell^+\ell^-\nu\bar{\nu}\tilde{\chi}_1^0\tilde{\chi}_1^0$, etc. Such leptonic modes give rise to a signature consisting of a charged-lepton pair and large missing energies. It is clean and we can construct the cluster transverse mass $M_T$ of the lepton pair and the missing energy. The transverse mass would indicate a broad peak structure, which is sensitive to the intermediate $Z'$ boson mass. Here we only give a taste of what one can do to see the presence of the $Z'$ via the supersymmetric decays. Detailed studies including various SUSY spectra and decay
modes, and branching ratios will be given in a future publication.

A. Slepton-pair production

Let us first investigate slepton-pair production in MSSM and in MSSM plus a $Z'$. Electroweak production of $\tilde{e}_L\tilde{e}^*_L$ or $\tilde{e}_R\tilde{e}^*_R$ goes through the $\gamma$, $Z$, $Z'$ exchanges. The differential cross section for the subprocess $q\bar{q} \rightarrow \tilde{e}_\alpha\tilde{e}^*_\alpha$ ($\alpha = L, R$) is given by

$$\frac{d\hat{\sigma}}{d\cos \theta} = \frac{\beta}{96\pi \hat{s}} \left( |M_{L\alpha}|^2 + |M_{R\alpha}|^2 \right) (\hat{u}t - m_{\tilde{e}_\alpha}^4),$$

(16)

where

$$M_{L\alpha} = \frac{e^2 Q_q Q_{\tilde{e}_\alpha}}{\hat{s}} + \frac{g_1^2}{\hat{s} - m_{Z}^2 + im_{Z}\Gamma_Z} g_L^2 g_{\tilde{e}^*_\alpha} + \frac{g_2^2}{\hat{s} - m_{Z'}^2 + im_{Z'}\Gamma_{Z'}} Q_{qL} Q'_{L\alpha} Q'_{L\alpha},$$

(17)

$$M_{R\alpha} = \frac{e^2 Q_q Q_{\tilde{e}_\alpha}}{\hat{s}} + \frac{g_1^2}{\hat{s} - m_{Z}^2 + im_{Z}\Gamma_Z} g_R^2 g_{\tilde{e}^*_\alpha} + \frac{g_2^2}{\hat{s} - m_{Z'}^2 + im_{Z'}\Gamma_{Z'}} Q_{qR} Q'_{R\alpha} Q'_{R\alpha},$$

(18)

and $\beta = \sqrt{1 - 4m_{\tilde{e}_\alpha}^2/\hat{s}}$. Here the electric charge $Q_{\tilde{e}_\alpha} = Q_e$, the $Z$ charge $g_{\tilde{e}^*_\alpha} = g_{\alpha}^e$, and the $Z'$ charge $Q'_{\alpha} = Q'_{\alpha}$. The subprocess cross section is then folded with parton distribution functions to obtain the total cross section. The so-produced $\tilde{e}_\alpha\tilde{e}^*_\alpha$ will decay into the electron and positron and the lightest neutralinos under the normal hierarchy of SUSY masses.

Thus, the final state consists of a charged-lepton pair and a missing energy. We can construct the cluster transverse mass given by

$$M_T = \left[ \left( \sqrt{p_{Te+e^-}^2 + M_{e^+e^-}^2} + \vec{p}_T \right)^2 - E_T^{Te+e^-} \right]^{1/2},$$

(19)

where the second equality is because $\vec{p}_{Te+e^-} = -\vec{p}_T$. We show the distribution for this cluster transverse mass in Fig. 11. We have imposed a set of leptonic cuts before we construct the cluster transverse mass:

$$p_{Te} > 30 \text{ GeV}, \quad |\eta_e| < 3, \quad p_T > 50 \text{ GeV}.$$

(20)

The $Z'$ models shown in Fig. 11 are $Z'_N$, $Z'_\eta$, $Z'_\chi$, and $Z'_B-L$. The other $Z'$ models show similar features. The underneath curve is the MSSM contribution only with $\gamma$ and $Z$ exchanges.

3 In this work, we will ignore all the Higgs exchange diagrams since they are suppressed by light quark masses.

4 Here we assume the lightest neutralino is the lightest supersymmetric particle (LSP) and the usual order of SUSY masses: $m_{\tilde{e}_L} \approx m_{\tilde{e}_R} > m_{\chi^0_1}$. 
while the upper curve includes also the contribution from $Z'$. The $Z'$ peak becomes broad because of the missing energies from the two neutralinos involved. Nevertheless, the sharp edge of the peak is sensitive to the mass difference between the $Z'$ and the slepton masses.

**FIG. 11.** The differential cross section versus the cluster transverse mass $M_T$ defined in Eq. (19) for $pp \rightarrow \gamma, Z, Z' \rightarrow \tilde{e}_L \tilde{e}^*_L + \tilde{e}_R \tilde{e}^*_R \rightarrow e^+ e^- \tilde{\chi}^0_1 \tilde{\chi}^0_1$. We have applied the leptonic cuts given in Eq. (20). Both $\tilde{e}_L \tilde{e}^*_L$ and $\tilde{e}_R \tilde{e}^*_R$ are included. The underneath curve is without the $Z'$ boson while the upper curve includes the $Z'$ boson. The $Z'$ models shown here are $Z'_N$, $Z'_\eta$, $Z'_\chi$, and $Z'_{B-L}$. The masses are $M_{Z'} = 1.5$ TeV, $m_{\tilde{e}_{L,R}} = 200$ GeV, and $m_{\tilde{\chi}^0_1} = 80$ GeV.

**B. Neutralino-pair production**

Next, we study the production of a neutralino pair $\tilde{\chi}^0_1 \tilde{\chi}^0_2$. Assuming $\tilde{\chi}^0_1$ is the LSP, then $\tilde{\chi}^0_2$ can decay into $\tilde{\chi}^0_1 \ell^+ \ell^-$ via a virtual $Z$ boson or a virtual slepton. The final state consists of a charged lepton pair and a missing energy. We can again construct the cluster transverse mass as in Eq. (19).

Electroweak production of $\tilde{\chi}^0_1 \tilde{\chi}^0_2$ goes through the $Z$ and $Z'$ exchanges. We assume that the squarks are much heavier such that the $t$-channel squark exchanges are suppressed. The
differential cross section for the subprocess $q(p_1)\bar{q}(p_2) \rightarrow \tilde{\chi}_0^1(k_1)\tilde{\chi}_0^2(k_2)$ is given by

$$\frac{d\hat{\sigma}}{d\cos \hat{\theta}} = \frac{\beta'}{96\pi\hat{s}} \left\{ (m_{\tilde{\chi}_0^1}^2 - \hat{u}) (m_{\tilde{\chi}_0^2}^2 - \hat{u}) \left( |M_{LL}(\hat{s})|^2 + |M_{RR}(\hat{s})|^2 \right) ight.$$

$$+ (m_{\tilde{\chi}_0^1}^2 - \hat{t}) (m_{\tilde{\chi}_0^2}^2 - \hat{t}) \left( |M_{LR}(\hat{s})|^2 + |M_{RL}(\hat{s})|^2 \right)$$

$$+ \left( \Re \left[ M_{LL}(\hat{s}) M_{RL}^*(\hat{s}) \right] \right) + \left( \Re \left[ M_{RR}(\hat{s}) M_{LR}^*(\hat{s}) \right] \right) \right\},$$

(21)

where

$$M_{\alpha\beta}(\hat{s}) = \frac{g_2^2}{\hat{s} - m_Z^2 + im_Z\Gamma_Z} O_{12}^{\alpha\beta} + \frac{g_2^2}{\hat{s} - m_{Z'}^2 + im_{Z'}\Gamma_{Z'}} O_{12}^{\alpha\beta} C_{12}^{\alpha\beta} Q_{\beta}^\theta,$$

(22)

$$\alpha, \beta = L, R \text{ and }$$

$$\beta' = \left\{ \left[ 1 - (m_{\tilde{\chi}_0^1}^2 + m_{\tilde{\chi}_0^2}^2)/\hat{s} \right]^2 - \left( 2 m_{\tilde{\chi}_0^1} m_{\tilde{\chi}_0^2}/\hat{s} \right)^2 \right\}^{1/2}.$$

(23)

Here the chiral couplings $O_{12}^{\alpha\beta}$ of the $Z$ boson and the chiral couplings $C_{12}^{\alpha\beta}$ of the $Z'$ boson to the neutralinos are, respectively, given by

$$O_{12}^{\alpha L} = -\frac{1}{2} N_{13} N^*_{23} + \frac{1}{2} N_{14} N^*_{24},$$

$$O_{12}^{\alpha R} = -O_{12}^{\alpha L*},$$

(24)

and

$$C_{12}^{\alpha L} = -Q'_{H_d} N_{13} N^*_{23} - Q'_{H_u} N_{14} N^*_{24},$$

$$C_{12}^{\alpha R} = -C_{12}^{\alpha L*},$$

(25)

where $N$ is the mixing matrix of the neutralinos defined in the appendix. Numerically, with the choice of SUSY parameters of set (A), we obtain the masses $m_{\tilde{\chi}_0^1} = 74.8$ GeV and $m_{\tilde{\chi}_0^2} = 130.6$ GeV, and the mixing parameters $N_{13} = 0.52$, $N_{23} = -0.41$, $N_{14} = -0.33$, and $N_{24} = 0.38$.

We apply the same set of leptonic cuts as in Eq. (20) and construct the cluster transverse mass. We show the cluster transverse-mass spectrum in Fig. 12. The $Z'$ models shown in Fig. 12 are $Z'_N$, $Z'_\chi$, $Z'_I$, and $Z'_\text{sec}$. The underneath curve is the MSSM contribution only with the $Z$ exchange while the upper curve includes also the contribution from $Z'$. The $Z'$ peak becomes broad because of the missing energies from the two neutralinos involved. Nevertheless, the edge of the peak is sensitive to the mass difference between the $Z'$ and the neutralinos.
FIG. 12. The differential cross section versus the cluster transverse mass $M_T$ defined in Eq. (19) for $pp \rightarrow Z, Z' \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ followed by the leptonic decay of $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 e^+e^-$. We have applied the leptonic cuts given in Eq. (20) and assumed the branching ratio $B(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 e^+e^-) = 0.1$. The underneath curve is without the $Z'$ boson while the upper curve includes the $Z'$ boson. The $Z'$ models shown here are $Z'_N, Z'_\chi, Z'_I$, and $Z'_{sec}$. The masses are $M_{Z'} = 1.5$ TeV, $m_{\tilde{\chi}_1^0} = 74.8$ GeV, and $m_{\tilde{\chi}_2^0} = 130.6$ GeV.

C. Chargino-pair production

Lastly, we have electroweak production of $\tilde{\chi}_1^+\tilde{\chi}_1^-$ that goes through the $\gamma, Z,$ and $Z'$ exchanges. The differential cross section for the subprocess $q(p_1)\bar{q}(p_2) \rightarrow \tilde{\chi}_1^+(k_1)\tilde{\chi}_1^-(k_2)$ is given by

$$\frac{d\hat{\sigma}}{d\cos \theta} = \frac{\beta}{96\pi s} \left\{ (m_{\tilde{\chi}_1^+}^2 - \hat{s})^2 \left( |M_{LL}(\hat{s})|^2 + |M_{RR}(\hat{s})|^2 \right) \\
+ (m_{\tilde{\chi}_1^+}^2 - \hat{t})^2 \left( |M_{LR}(\hat{s})|^2 + |M_{RL}(\hat{s})|^2 \right) \\
+ 2m_{\tilde{\chi}_1^+}^2 \hat{s} \left( \text{Re} \left[ M_{LL}(\hat{s}) M_{RL}^*(\hat{s}) \right] \\
+ \text{Re} \left[ M_{RR}(\hat{s}) M_{LR}^*(\hat{s}) \right] \right) \right\},$$

(26)
where
\[
M_{\alpha\beta}(\hat{s}) = \frac{e^2 Q_{\tilde{\chi}^+} Q_q}{\hat{s}} + \frac{g_1^2 O_{11}^\alpha g_3^q}{\hat{s} - m_Z^2 + i m_Z \Gamma_Z} + \frac{g_2^2 C_{11}^\alpha g_2^q}{\hat{s} - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'}} ,
\] (27)
\[\alpha, \beta = L, R, \text{ and } \beta = \sqrt{1 - 4 m_{\tilde{\chi}_1^+}^2 / \hat{s}}.\]
Here the chiral couplings \(O_{11}^\alpha\) of the \(Z\) boson and the couplings \(C_{11}^\alpha\) of the \(Z'\) boson to charginos are, respectively,
\[
O_{11}^L = |V_{11}|^2 + \frac{1}{2} |V_{12}|^2 - x_w ,
\]
\[
O_{11}^R = |U_{11}|^2 + \frac{1}{2} |U_{12}|^2 - x_w ,
\] (28)
and
\[
C_{11}^L = Q'_{H_u} |V_{12}|^2 ,
\]
\[
C_{11}^R = -Q'_{H_d} |U_{12}|^2 ,
\] (29)
where \(U\) and \(V\) are the mixing matrices of the charginos. Numerically, with the choice in set (A) for SUSY parameters we have \(V_{12} = 0.75, V_{11} = -0.67, U_{12} = 0.89, U_{11} = -0.46\) and \(m_{\tilde{\chi}_1^+} = 109.5\) GeV.

We only calculate the production of \(\tilde{\chi}_1^+ \tilde{\chi}_1^-\), because the second chargino is about twice as heavy as the first one. Each of the charginos decays via a virtual \(W, \tilde{\nu}\), or \(\tilde{\ell}\) into a charged lepton, a neutrino, and the lightest neutralino (if going through the virtual \(W\), light quarks are also possible). Therefore, there will two charged leptons plus missing energies in the final state. Just as the same as the case of slepton-pair or neutralino-pair production, we reconstruct the cluster transverse mass as in Eq. (19). We show the distribution of cluster transverse mass in Fig. 13 for \(Z'_N, Z'_\eta,\) and \(Z'_{sec}\). It is easy to see the bump due to the presence of the \(Z'\) boson, though the bump is not as discernible as the previous two cases of slepton-pair and neutralino-pair production.

VII. CONCLUSIONS

In this work, we have studied the possible supersymmetric decay modes of an additional neutral gauge boson \(Z'\), which is currently limited to be at least 1 TeV. Grand unified theories have predicted one or more such \(Z'\) bosons along the path through which the GUT symmetry is broken down to the electroweak symmetry. When supersymmetry is included in the theory, such a \(Z'\) boson can decay not only into the SM particles but
FIG. 13. The differential cross section versus the cluster transverse mass $M_T$ defined in Eq. (19) for $pp \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ followed by the leptonic decay of $\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 e^+ \nu_e$. We have applied the leptonic cuts given in Eq. (20) and assumed the branching ratio $B(\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 e^+ \nu_e) \times B(\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 e^- \bar{\nu}_e) = 0.1$. The underneath curve is without the $Z'$ boson while the upper curve includes the $Z'$ boson. The $Z'$ models shown here are $Z'_N$, $Z'_\eta$, and $Z'_\text{sec}$. The masses are $M_{Z'} = 1.5$ TeV and $m_{\tilde{\chi}_1^+} = 109.5$ GeV.

also the supersymmetric partners. We have used $E_6$, $U(1)_{B-L}$, and the sequential models to illustrate how the decays of the $Z'$ are affected. In particular, the golden search mode–charged leptons– for the $Z'$ will have a smaller branching ratio as the supersymmetric modes open. We have shown that the current limits obtained at the Tevatron and the LHC will be reduced by a noticeable amount, of order 20 GeV. We have also estimated the $5\sigma$ discovery sensitivities of the $Z'$ at the LHC, including the effect of supersymmetric decay modes. Finally, we demonstrated that even though the $Z'$ decays into supersymmetric particles, giving rise to missing energies, one can still reconstruct the cluster transverse mass (using the observable charged leptons) to identify the existence of the $Z'$. We believe further studies along this direction is worthwhile since it can help us to fully understand the role of the $Z'$ in the supersymmetry-breaking and the symmetry breaking pattern.
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Appendix A: Feynman Rules

As explained in Sec. IIB, the extra term $h_s S H_u H_d$ allowed by the $U(1)'$ symmetry will generate the effective $\mu$ term when the singlet scalar field $S$ develops a VEV. With the assumption that the singlet scalar and singlino fields, $Z'$-ino, and other exotic fermions are very heavy, the low-energy particle content includes the $Z'$ boson and those of MSSM. The effective superpotential $W_{\text{eff}}$ involving the matter and Higgs superfields is the same as the MSSM’s, and given by

$$W_{\text{eff}} = \epsilon_{ab} \left[ y_u^{ij} Q_j^a H_u^b U_i^c - y_d^{ij} Q_j^a H_d^b D_i^c - y_l^{ij} L_j^a H_u^b E_i^c + \mu H_u^a H_d^b \right],$$  \hspace{1cm} (A1)

where $\epsilon_{12} = - \epsilon_{21} = 1$, $i, j$ are family indices, and $y^u$ and $y^d$ represent the Yukawa matrices for the up-type and down-type quarks respectively. Here $Q, L, U^c, D^c, E^c, H_u, \text{ and } H_d$ denote the superfields for the quark doublet, lepton doublet, up-type quark singlet, down-type quark singlet, lepton singlet, up-type Higgs doublet, and down-type Higgs doublet respectively. The scalar interactions are obtained by calculating the $F$- and $D$-terms of the superpotential, and by including the following soft-SUSY-breaking terms

$$\mathcal{L}_{\text{soft}} = -M_1 \tilde{B} \tilde{B} - M_2 \tilde{W} \tilde{W} - M_3 \tilde{g} \tilde{g} - M_Q \tilde{Q} \tilde{Q} - M_U \tilde{U} \tilde{U}^c - M_D \tilde{D} \tilde{D}^c - M_L \tilde{L} \tilde{L} - M_E \tilde{E} \tilde{E}^c$$ \hspace{1cm} (A2)

$$\mathcal{L}_{\text{tril}} = \epsilon_{ab} \left[ y_u^{ij} A_{ij}^a Q_j^a H_u^b U_i^c - y_d^{ij} A_{ij}^a Q_j^a H_d^b D_i^c - y_l^{ij} A_{ij}^a L_j^a H_u^b E_i^c + \mu BH_u^a H_d^b \right],$$ \hspace{1cm} (A3)

where the $\mathcal{L}_{\text{soft}}$ represents the soft mass terms for the gauginos and sfermions, and $\mathcal{L}_{\text{tril}}$ represents the trilinear $A$ terms.

The gauge interactions for the fermionic and scalar components, denoted generically by $\psi$ and $\phi$ respectively, of each superfield mentioned above are given by

$$\mathcal{L} = \bar{\psi} i \gamma^\mu D_\mu \psi + (D_\mu \phi)^\dagger (D_\mu \phi),$$ \hspace{1cm} (A4)

There should also be a soft gaugino mass term for the $Z'$-ino. However, we will assume it is heavy and let it decouple from the low energy spectrum in the present work.
where the covariant derivative is defined as usual

\[ D_\mu = \partial_\mu + ieQ A_\mu + ig \frac{g}{\sqrt{2}} (\tau^+ W^\mu_\mu^+ + \tau^- W^\mu_-) + ig_1 (T_{3L} - x_w Q) Z_\mu + ig_2 Z'_\mu Q'. \quad (A5) \]

Here \( g_1 = g / \cos \theta_w \) with \( g \) the \( SU(2)_L \) gauge coupling and \( \theta_w \) the weak mixing angle, \( g_2 \) is the gauge coupling for the extra \( U(1) \), \( \tau^\pm \) and \( T_{3L} \) are the ladder operators and the third component of the \( SU(2)_L \) generators, \( Q \) and \( Q' \) are the charges of the two \( U(1) \)'s in unit of the electromagnetic charge \( e \) and \( g_2 \) respectively, and finally, \( x_w = \sin^2 \theta_w \). Since we only consider supersymmetric \( U(1) \) symmetry for the \( Z' \) boson and due to the Majorana nature of the \( Z' \)-ino, there is no coupling between the \( Z' \)-ino and the \( Z' \) boson. Simply from Eqs. \((A4)\) and \((A5)\) we obtain the interactions of \( Z' \) with fermions, sfermions, neutral and charged Higgsinos (which become the physical neutralinos and charginos after mixing effects are taken into account), and the Higgs bosons.

The rotation of neutral bino, wino, and Higgsinos into the physical neutralinos is given by

\[
\begin{pmatrix}
\tilde{\chi}_1^0 \\
\tilde{\chi}_2^0 \\
\tilde{\chi}_3^0 \\
\tilde{\chi}_4^0
\end{pmatrix}
= N
\begin{pmatrix}
\tilde{B} \\
\tilde{W}^3 \\
\tilde{H}_d^0 \\
\tilde{H}_u^0
\end{pmatrix}, \quad (A6)
\]

where \( N \) is an orthogonal matrix. The rotation of the charged wino and Higgsino into the physical charginos is via bi-unitary transformation

\[
\begin{pmatrix}
\tilde{\chi}_1^+ \\
\tilde{\chi}_2^-
\end{pmatrix}
= V
\begin{pmatrix}
\tilde{W}^+ \\
\tilde{h}_u^+
\end{pmatrix}, \quad \begin{pmatrix}
\tilde{\chi}_1^- \\
\tilde{\chi}_2^-
\end{pmatrix}
= U
\begin{pmatrix}
\tilde{W}^- \\
\tilde{h}_d^-
\end{pmatrix}, \quad (A7)
\]

where \( U \) and \( V \) are unitary matrices. The mixing angles involved in physical Higgs bosons \((h^0, H^0, H^\pm, A^0)\) can be read off from the following decompositions of the Higgs boson fields \( H_u \) and \( H_d \):

\[
H_u = \begin{pmatrix}
h^0_u \\
h^0_u
\end{pmatrix}
= \begin{pmatrix}
\frac{v \sin \beta}{\sqrt{2}} + \frac{1}{\sqrt{2}} (h^0 \cos \alpha + H^0 \sin \alpha) + i (A^0 \cos \beta + G^0 \sin \beta) \\
H^+ \cos \beta + G^+ \sin \beta
\end{pmatrix}, \quad (A8)
\]

\[
H_d = \begin{pmatrix}
h^0_d \\
h^0_d
\end{pmatrix}
= \begin{pmatrix}
\frac{v \cos \beta}{\sqrt{2}} + \frac{1}{\sqrt{2}} (-h^0 \sin \alpha + H^0 \cos \alpha) + i (A^0 \sin \beta - G^0 \cos \beta) \\
H^- \sin \beta - G^- \cos \beta
\end{pmatrix}, \quad (A9)
\]

where the angle \( \alpha \) is the mixing of the neutral CP-even Higgs bosons \( h^0 \) and \( H^0, G^{0,\pm} \) are the Goldstone bosons, and \( v = 246 \text{ GeV} \).
Lastly, there are Yukawa-type interactions between the gauginos and the scalar $\phi$ and fermionic $\psi$ components of the matter superfield of the following form

$$\mathcal{L} = -\sqrt{2}g^a \phi^* T^a \tilde{\chi}^a \psi + \text{h.c.}$$  

(A10)

where $a$ is the group index of the $U(1)_Y$, $SU(2)_L$, $SU(3)_C$, or the extra $U(1)_{Z'}$. The interactions involving the $Z'$-ino are

$$\mathcal{L}_{Z'} = -\sqrt{2}g_2 \left[ \tilde{Q}^\dagger \tilde{Z}' Q^c Q + \tilde{U}^\dagger \tilde{Z}' Q'^c U^c + \tilde{D}^\dagger \tilde{Z}' Q'^c D^c + \text{h.c.} \right. $$

$$+ \tilde{L}^\dagger \tilde{Z}' Q'^c L + \tilde{E}^\dagger \tilde{Z}' Q'^c E^c + \text{h.c.} $$

$$+ H_u^\dagger \tilde{Z}' Q'^c H_u + H_d^\dagger \tilde{Z}' Q'^c H_d + \text{h.c.} \right].$$

(A11)

Note that the $Z'$-ino will mix with the $\tilde{H}_u$ and $\tilde{H}_d$ Higgsinos when the $H_u^0$ and $H_d^0$ take on vacuum expectation values. Thus, we will have a $5 \times 5$ neutralino mass matrix. However, we decouple the $Z'$-ino in this work by setting the $Z'$-ino mass heavy. We will come back to this in later work.

In the following we list the Feynman diagrams and the corresponding Feynman rules involving the $Z'$ boson and the MSSM particles. For each model, one should use the corresponding coupling strength and chiral charges. The coupling strength $g_2$ and the $Z'$ charges are given in Tables II and III for the $E_6$ and $U(1)_{B-L}$ models respectively. For the sequential $Z'$ model, replace the coupling strength $g_2$ by $g_1$ while the corresponding chiral charges are given by the SM values: $Q'_i(Z'_{SM}) = T_{3i} - x_w Q_i$. The chiral couplings of the charginos and neutralinos with the $Z'$ boson are, respectively, given by

$$C'_{ij}^L = Q'_{H_u} V_{i2} V'^*_{j2}, \quad C'_{ij}^R = -Q'_{H_d} U_{i2}^* U_{j2}^*,$$

$$C''_{ij}^L = -Q'_{H_d} N_{i3} N'^*_{j3} - Q'_{H_u} N_{i4} N'^*_{j4}, \quad C''_{ij}^R = -C''_{ij}^L.$$  

(A12)

These coupling coefficients are the same for the three $Z'$ models that we have studied in this work, as long as we use the corresponding $Q'$ charges for the two Higgs doublet fields.
\[ -i g_2 \gamma^\mu (Q'_{f_L} P_L + Q'_{f_R} P_R) \]

\[ -i g_2 Q'_{f_L} (p + p')^\mu \]

\[ -i g_2 Q'_{f_R} (p + p')^\mu \]

\[ -i g_2 \gamma^\mu (C'_{ij}^L P_L + C'_{ij}^R P_R) \]
\[ i g_2 \gamma^\mu (C''_{ij} P_L + C''_{ij} P_R) \]

\[ -g_2 (Q'_{H_u} \cos \alpha \cos \beta - Q'_{H_d} \sin \alpha \sin \beta) (p + p')^\mu \]

\[ -g_2 (Q'_{H_u} \sin \alpha \cos \beta + Q'_{H_d} \cos \alpha \sin \beta) (p + p')^\mu \]

\[ i g_2 (Q'_{H_d} \sin^2 \beta - Q'_{H_u} \cos^2 \beta) (p + p')^\mu \]
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