Examining Beam Oscillations in the Space of Technogenic and Seismic Impact Parameters (Part II)

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Abstract. The proposed paper (Part II) is a continuation of another paper under the same name (Part I), published in this volume. Part II explores two types of forced oscillations: harmonic (polyharmonic) and random. The sources of oscillations are kinematic and dynamic effects which are concentrated on the boundaries and in the span. The mathematical problem of oscillations is presented as a boundary value problem from the basic equation in hyperbolic partial derivatives of the fourth order in spatial coordinates, of second order in time. The technical theory of bending oscillations of rods based on Bernoulli's hypothesis is used. Practical conclusions have been made.

1. Introduction

Nowadays beam oscillations have an extensive bibliography as they are widespread elements of buildings and structures and are used in the equipment of almost all industries. In recent years, the widespread use of robots, automated plants and workshops has resulted in beams experiencing unconventional, non-classical technogenic external influence of dynamic and kinematic character. To date, transverse and longitudinal beam oscillations and rods in general have an extensive bibliography \cite{1-4}. But the scientific and technical revolution of the last decades and the requirements of the digital economy of the last years require more and more accurate structural calculations as well as beams. Although there is a vast amount of literature on harmonic and random beam oscillations, some questions remain unclear. These include oscillations under the influence of multidimensional vector kinematic and dynamic disturbances such as seismic and technogenic loads due to their random nature strongly bound to the uncertainties at the place of occurrence of earthquakes and multiparameter intensity of their course. Detailed solutions to the problems of random oscillations show that the dynamic behaviour of structures essentially depends on the parameters of the effects forming a large area in a multidimensional space (mathematical!) of parameters. The multidimensional space of the impact parameters in this paper implies the sum of subspaces of the Euclidian space $\mathbb{R}^n$, which contain the corresponding points of the sources of mechanical oscillations: frequency, dominant frequency, amplitude, phase, force direction, point of application, spectral structure, bandwidth, correlation coefficient, mathematical expectation, dispersion, etc.
2. Mathematical model of oscillations

The substantive formulation of the problem is based on the calculation scheme of a compressed beam depicted in Fig. 1 and bearing two discrete masses. Two dynamic concentrated forces \( f_2(t), f_3(t) \) as well as two kinematic movements of the support \( f_1(t), f_4(t) \) impact the beam. A compression force \( P \) acts on the beam in the axial direction, and in the transverse direction – static forces from the natural and bearing mass \( m \) and \( q \) which are evenly distributed. The beam supports are flexible and resilient with stiffness coefficients \( c_1, c_2 \). We will consider the lower beam to which the dynamic concentrated forces \( f_2(t), f_3(t) \) are transferred from the auxiliary upper beam. As a result, the beam is subject to kinematic seismic and/or technogenic impacts in the form of a vector process \( F [ f_1(t), f_2(t), f_3(t), f_4(t)] \).

The vector components can be in-phase, antiphase or independent in a deterministic case, correlated or uncorrelated to the correlation coefficients in the range \( k \in [-1, 1] \) under random influences.

Proceeding to the mathematical model of oscillations, we will use the technical theory of bending oscillations of rods. Then, the basic equation in traditional designations has the form of a homogeneous hyperbolic partial differential equation [2]

\[
bu^{\infty} + Pu^\infty + r \ddot{u} + \mu m \dddot{u} = 0, \quad b = EJ, \quad r = m + q, \quad x \in (0, l), \quad t > -\infty.
\] (1)

Dashes in the upper index indicate the differentiation of the deflection function by the \( x \) coordinate, dots over the characters – differentiation by time \( t \).

The presence of discrete masses \( M_1 \) and \( M_2 \) leads to discontinuity of the second and third derivatives of the function \( u(x, t) \) at the points of their location, which forces to include block connection conditions of the sites to the left and right of them in the mathematical model of oscillations. Let us consider the situation near the mass \( M_1 \) (Fig. 2).

![Figure 1. Beam on resilient supports.](image1)

![Figure 2. Block connection.](image2)

The basic equation (1) is joined by boundary conditions corresponding to Fig. 1, and block connection conditions on the left and right of the discrete masses.

Boundary conditions

\[
u''(0, t) = 0, \quad bu''(0, t) - c_1 u(0, t) = 0, \quad u''(l, t) = 0, \quad bu''(l, t) + c_2 u(l, t) = 0, \quad t > -\infty.\] (2)

\( c_1, c_2 \) – stiffness coefficients of resilient supports.

Some block connection conditions are kinematic and consist in the fact that the function \( u(x, t) \) and its first- and second-order derivatives must be smooth at the location points of discrete masses \( x_i \). Their implementation is obvious from the point of view of continuity and structural integrity. Other conditions are dynamic and arise due to the presence of force factors at the junction of adjacent areas (Fig. 2): bending moments on the left and right \( M_l, M_r \), transverse forces \( Q_l, Q_r \), longitudinal forces \( N \), D’Alambert’s inertia force \( D \).

The cross section at the given point is rotated by a small value and it follows that \( M_l \approx M_r \). The mass moment of inertia resulting from the rotation is negligibly small. As a result, the second
derivatives on the left will be equal and their continuity will be ensured. Transverse forces \( Q_i \) and D’Alambert’s inertia force \( D \) act in a vertical direction and create a progressive mass movement. The forces must be balanced

\[
D - Q_i + Q_i = 0.
\]  

Here, D’Alambert’s inertia force is \( D = M_i \ddot{u}(x_i, t) \). It follows from the above that equation (3) must be included in the connection conditions. Being more concrete, we will get

\[
M_i \ddot{u}(x_i, t) = b[u^n(x_i - 0, t) - u^n(x_i + 0, t)], \quad k = 1, 2.
\]  

The stiffness of the function \( u(x_i, t) \) on the third-order derivative is not provided, and this must be taken into account in the forthcoming calculations. Equation (1), boundary conditions (2) and connection conditions (4) form a mathematical model that allows to determine the function \( u(x_i, t) \).

3. Forced polyharmonic oscillations

The problem of harmonic oscillations is considered for two reasons:
1. They can actually occur when there are technogenic disturbances on beam supports.
2. The results of this problem-solving make it much easier to set and solve problems on random oscillations.

In the calculation scheme in Fig. 1, we now assume that the impact are as follows

\[
f_i(t) = a_i \exp\left[j(\omega_i t + \varphi_i)\right], \quad i = 1, 2, 3, 4.
\]  

Here, \( a_i \) – is disturbance amplitudes, \( \omega_i \) – disturbance frequency, \( \varphi_i \) – initial phase of disturbances, \( j \) – an imaginary unit.

Substitution of (5) in the boundary conditions (2) gives

\[
\begin{align*}
u^n(0, t) &= 0, \quad bu^n(0, t) - c_i u(0, t) = a_i \exp\left[j(\omega_i t + \varphi_i)\right], \\
u^n(l, t) &= 0, \quad bu^n(l, t) + c_i u(l, t) = a_i \exp\left[j(\omega_i t + \varphi_i)\right],
\end{align*}
\]  

\( t > \infty \).  

Conditions (4) take the form of

\[
b[u^n(x_i - 0, t) - u^n(x_i + 0, t)] - M_i \ddot{u}(x_i, t) = f_i(t), \quad k = 1, 2.
\]  

The system under consideration is linear, so the result of autonomous actions of the sources \( f_1, f_2, f_3, f_4 \) on the beam can be summed up by the principle of superposition. Consider the established forced oscillations which are unabated harmonic and their initial phase does not play a role. The corresponding function therefore appears in the method of separation of variables

\[
u(x_i, t) = X(x_i) \exp(\lambda t), \quad \lambda = j\omega_i, \quad i = 1, 2, 3, 4.
\]  

Here, \( X(x) \) is the amplitude function. Similarly to the case of free oscillations using the methods of separation of variables and finite differences, we will obtain a system of linear algebraic equations, however, a heterogeneous one

\[
B(\lambda) Y = d,
\]  

where \( B(\lambda) \) is a square matrix of order \( n \), \( d \) – a column vector, \( Y \) – a column-vector of discrete arguments \( Y = \{y_1, y_2, ..., y_n\} \), \( y_i = X(x_i) \). Matrix \( B \) is described in detail in Part I and is not shown here. Non-zero elements of the vector \( d \) which meet the boundary conditions (6) have the following values

\[
d_1 = 2h^2 c_1 e^{\omega b} / b, \quad d_{n-1} = 2h^2 c_2 e^{\omega b} / b.
\]  

Similar components of \( d \), for connection points take the form of

\[
d_k = 2h^2 e^{\omega k b} / b, \quad k \text{ is the number of connection points}.
\]
Calculations will be made for two cases of a specific beam.

Example 1. Input data: \( l = 6 \text{ m} \), \( E = 210 \text{ GPa} \), \( J = 2550 \text{ cm}^4 \), \( m = 24 \text{ kg/m} \), \( M_1 = 8000 \text{ kg} \), \( M_2 = 5000 \text{ kg} \), \( q = 3000 \text{ kN/m} \), \( P = 10 \text{ kN} \), \( c_1 = 200 \text{ kN/m} \), \( c_2 = 100 \text{ kN/m} \), \( \mu = 0.1 \text{s}^{-1} \), \( n = 600 \),
\[
a = \{10 \text{ cm} \ 150 \text{ kN} \ 120 \text{ kN} \ 15 \text{ cm} \}, \omega_k = \{21.72 \ 48.04 \ 137.66 \} \text{ s}^{-1}, \omega = \{15 \ 20 \ 28 \ 48 \ 137 \} \text{ s}^{-1}.
\]
Here, \( \omega_k \) – eigenfrequency.

1. \( \varphi = \{0 \ 0 \ 0 \ 0\} \text{ rad} \). Let us build graphs of the functions \( Y(x) \) for in-phase oscillations with frequency \( \omega \). The results of the calculation are shown in Fig. 3. It is visible that as disturbance frequency grows, the character of oscillations changes significantly, the influence of eigenfrequencies and natural forms of free oscillations discussed in part I is detected.

![Figure 3](image_url)

**Figure 3.** The effect of disturbance frequencies.

![Figure 4](image_url)

**Figure 4.** The effect of disturbance phase shift.

2. \( \varphi = \{0 \ \pi/3 \ 2\pi/3 \ \pi\} \text{ rad} \). Let us study the impact of the initial phase of \( \varphi \) disturbance on oscillations. The corresponding vector will now be non-zero. The results of the calculations shown in Fig. 4, although similar to the ones in Fig 3, are very different in their magnitude. Hence the need to take account of the phase shift in the calculations of such beams.

4. Forced random calculations

Seismic disturbances are always, and technogenic ones are often random processes over time. In most cases, however, the stationary part of such processes lasts longer than others in time and poses the greatest danger to structures. Therefore, the relevant oscillations will be studied as stochastic in the theory of stationary random processes. This approach has been implemented in the tasks of the monograph [5]. For this purpose, the mathematical model for harmonic oscillations given above will be modified for random oscillations. It will only refer to additional conditions, and namely, the input vector function \( F(f_1, f_2, f_3, f_4) \). Its components will now be interpreted as components of a centered stationary random vector represented by their spectral densities in the form of symmetrical matrix

\[
S_f(\omega) = \begin{pmatrix}
s_{11} & 0 & 0 & s_{14} \\
0 & s_{22} & s_{23} & 0 \\
0 & s_{32} & s_{33} & 0 \\
s_{41} & 0 & 0 & s_{44}
\end{pmatrix}, \quad s_{ij}(\omega) = s_{ji}(\omega).
\]  \hspace{1cm} (10)
Zero elements of the matrix correspond to uncorrelated mixed pairs of kinematic and dynamic effects.

The task is to find dispersions (or standard deviations) in the output process using a given spectral matrix of an input random process. The elements of the matrix (10) have a form for the most frequently occurring earthquake and technogenic processes

\[ s_{i j}(\omega) = \frac{49 \theta^2 \sigma_i \sigma_j k_{ij}}{\pi((\omega^2 - \theta^2)^2 + 49^2 \omega^2)} , \quad \omega \in [0, \infty), \quad \theta^2 = \theta^2 + \phi^2 , \quad i, j = 1, 2, 3, 4. \]  

(11)

Here, \( \theta \) and \( \phi \) – are parameters of bandwidth and characteristic frequency, \( \sigma_i \) – standard deviations in processes \( f_i(t) \), \( k_{ij} \) – elements of a normalized correlation matrix

\[ K_f = \begin{pmatrix}
k_{11} & 0 & 0 & k_{i4} \\
0 & k_{22} & k_{23} & 0 \\
0 & k_{32} & k_{33} & 0 \\
k_{41} & 0 & 0 & k_{44}
\end{pmatrix}. \]

As is known, a continuous random centered stationary process can be represented as a Fourier row each summand of which is a harmonic oscillation. This makes it possible to present a continuous spectral density of type (11) (Fig. 5) as a discrete linear sector (Fig. 6) consisting of individual elementary dispersions \( D_i \). Each discrete frequency \( \omega_i \) will then be matched with the elementary dispersion \( S(\omega) d\omega \), shaded in Fig. 5. If it is taken instead of the amplitude of the input harmonic process \( f_i(t) \), the output of the task will result in an elementary dispersion of deviations \( dD_x(\omega_x, x) \). Subsequent summing of \( \omega_i \) gives the dispersion of deviations at the discrete point of the beam \( D_x(x) \).

![Figure 5. Spectral density.](image)

![Figure 6. Discrete spectrum.](image)

**Example 2.** Input data as in example 1. Parameters of random disturbances added: \( l = 6 \text{ m}, \ E = 210 \text{ GPa}, \ J = 2550 \text{ cm}^4, \ m = 24 \text{ kg/m}, \ M_1 = 8000 \text{ kg}, \ M_2 = 5000 \text{ kg}, \ q = 3000 \text{ kg/m}, \ P = 10 \text{ kN}, \ c_1 = 200 \text{ kN/m}, \ c_2 = 100 \text{ kN/m}, \ \mu = 0.1 \text{ s}^{-1}, \ n = 601, \ \sigma_i = \{10 \text{ cm} 150 \text{ kN} 120 \text{ kN} 15 \text{ cm}\}, \ \omega_k = \{21, 72, 48, 04, 137, 66\} \text{ s}^{-1}, \ \phi = \{15, 20, 28, 48, 137\} \text{ s}^{-1}, \ \theta = 0.1 \text{ s}^{-1}.

1. \( K_f = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}, \quad 2. \ K_f = \begin{pmatrix}
1 & 0 & 0 & -1 \\
0 & 1 & 0.707 & 0 \\
0 & 0.707 & 1 & 0 \\
-1 & 0 & 0 & 1
\end{pmatrix}.

The whole variety of correlation and spectral matrices cannot be considered. So, only the typical cases will be taken. For this purpose, let us simplify the spectral matrix (11) to the form of

\[ s_{i j}(\omega) = \frac{49 \theta^2 \sigma_i \sigma_j k_{ij}}{\pi((\omega^2 - \theta^2)^2 + 49^2 \omega^2)} , \quad \omega^2 = \theta^2 + \phi^2 , \quad i, j = 1, 2, 3. \]
First, we take the stochastic analogue of the example 1 (Fig. 3) of the in-phase oscillations. The in-phase character of random oscillations from four external impacts will be that the correlation matrix $K_f$ has the form of 1 with the equality of frequencies in the spectral planes.

All four components are uncorrelated to each other. The results obtained are shown in Fig. 7. They have many similarities with the graphs in Fig. 3 taking into account the non-negativity of standard deviations. The parameters for random disturbances have been chosen so that the random oscillations are as close to harmonic as possible. The aim was that it would be possible to check the two results simultaneously by comparing them. A simple visual examination of Fig. 3 and Fig. 7 reveals many similarities in oscillations. For example, the values of amplitudes and standard deviations are approximately the same. The waviness of curves at random deviations is greater than the amplitude. The logic behind this fact is that the spectrum of random disturbances contains an infinite number of frequencies, including those coinciding with eigenfrequencies, and therefore resonant oscillations appear. Apparently, the standard deviations in curve 3 in Fig. 7 significantly exceeded the amplitude of curve 3 in Fig. 3 for this reason. In general, as the dominant frequency of exposure $\phi$ increases, the forms of oscillations change. First, they occur in their own first form, then in the second, third, etc.

Such overlapping dynamics of oscillations observed with free, forced harmonic and random oscillations are a certain kind of verification of mathematical models, algorithms and programmes for all these types of oscillation.

Let us consider a more general case of random oscillations when all four disturbances are correlated with each other, to which the correlation matrix 2 corresponds. The results of the calculation are shown in Fig. 8. The first conclusion from the analysis of the graphs is that there was some reduction in the standard deviation in comparison with the previous case. Curves 3 and 4 have changed significantly. The obvious reason is the correlation of the components of the vector random disturbance process which leads to mutual interference, 'competition' between them.

Based on the results of the two examples, it is clear that the vertical component must be taken into account in beam calculations.

5. Conclusions

1. A significant dependence of the oscillation amplitudes on the disturbance frequencies was detected, both in terms of the numerical values of the deviations and the curvature of the centre lines, on which the strength and reliability of the beams during their operation depends.
2. The correlated components of the vector random process significantly influence beam deviations during oscillations. It should therefore be determined from experimental data and taken into account in beam calculations.

3. Current standards for designing construction beams should take into account the properties of technogenic and seismic impacts in more detail.

4. It is necessary to study the effect of phase shifts on the oscillation amplitudes as they must be taken into account in the calculations when designing the beams.

5. Studying beam oscillations in the parameter space of oscillation system requires a significant number of examples solved. This will provide valuable practical results.

6. The vertical component of seismic and technogenic disturbances must be taken into account in the regulations on calculations and design of beams.

6. References

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