Successive identification of surface heat flux and thermophysical properties of plasma facing components inside the JET tokamak: numerical feasibility

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Abstract. We present here the numerical feasibility of using two thermal diagnostics (IR surface and embedded thermocouple temperature measurements), that outfit the same carbon tile inside the JET Fusion reactor and whose combination enables to identify, on one hand, a surface heat flux history and, on the other hand, the spatial and time variation of the thermal resistance of an unknown deposited surface carbon layer (SCL). The Conjugate Gradient Method (CGM) and the adjoint state were applied to perform these two identifications.

1. Introduction
Inside magnetic confinement fusion machines (tokamaks), some internal components are submitted to very high heat fluxes. For example, in the Joint European Torus (JET), several MW/m² are coupled to plasma facing components (PFC) for about 20 seconds. A large part of this power is directed towards inertially cooled carbon tiles. In the JET experiment and in most of the Tokamaks using carbon PFC, the eroded carbon is circulating in the plasma and is redeposited elsewhere. This leads at some locations, to the formation of surface carbon layers (SCL) usually poorly attached to the PFC. In JET experiments, for better understanding and controlling the heat transfer between the plasma to the surrounding wall, it is very important to estimate, in transient state, the imposed heat flux on PFC (with and without surface carbon layer).

Reference [1] presents a linear approach of the 2D heat flux estimation on carbon tiles of outer and inner side of the JET divertor. This identification was based on the simultaneous use of surface temperature coming from an infrared camera and temperature measurements provided by one thermocouple (TC) embedded inside each tile (1cm depth). If the tile is free of surface carbon layer, the surface heat flux can be identified either by using infrared data (as input for a thermal model based) or by using the embedded thermocouple information.

For tiles with surface carbon layers, the unknown thermophysical properties of such layers prevent us from identifying the surface heat flux. The in-situ measurement of these thermophysical properties with photothermo thermal method is impossible, and it is also impossible to extract a carbon tile for a laboratory analysis. Then, in presence of surface carbon layer, there are differences between the heat flux identified with IR and TC data, because of the unmodeled carbon deposit. Moreover, thermal properties of SCL are suspected to vary with time during
a plasma experiment. We propose here to model this surface carbon layer and to identify the spatial and time variation of the thermal resistance of this layer, by using the Conjugate Gradient Method (CGM) and minimizing a quadratic criterion on the IR data. The thermal resistance of the SCL including the thermal contact resistance $R_c$ is defined as following:

$$R_{th}(x, t) = \frac{e}{\lambda_{eq}(x, t)}$$

with

$$\lambda_{eq} = \frac{1}{\frac{1}{\lambda_{scl}} + \frac{R_c}{e}}$$

where $e$ is the thickness of the SCL (about $100\,\mu m$) and $\lambda_{eq}$ the equivalent thermal conductivity of the SCL. Eq. (1) is valid in steady state, while our study is fundamentally unsteady. But the diffusion time of the SCL being half the time step of our experiment implies that errors of the computed surface temperatures are less than the measurement errors, making relevant the use of that model for SCL. Before each SCL thermal resistance identification, the surface heat flux needs to be estimated in order to be used as a prescribed boundary condition. This is done using the embedded thermocouple signal and an a priori heat flux surface shape $f(x)$ (see Fig. (1)).

After a presentation of the virtual experiment design, the successive surface heat flux and thermal resistance identifications using the CGM will be explained. Finally the results of the method will be discussed.

2. Virtual experiment design

![Fig. 1. Geometry and dimension of the virtual experiment design.](image)

Fig. (1) shows the geometry and the dimensions, for the transient heat transfer problem considered in this virtual experiment. The geometry has two domains: the substrate composed of Carbon Fiber Composite (CFC) (in blue) and the SCL (in red). Fig. (2) shows the evolution of the thermal properties of the CFC versus temperature, we note the strong variation of the thermal properties, especially the thermal diffusivity which is divided by 4 between low and high temperature, this strong variation leads to a strong non linearity of the transient heat transfer problem. The equivalent thermal conductivity of the SCL can vary temporally. The density and heat capacity of the SCL are assumed equivalent to the CFC. On the interface $\Gamma_4$ we assume perfect contact between the substrate and the SCL, as the thermal contact resistance is included in the estimated thermal resistance. The two domains are initially at the same temperature $\theta_0 = 393K$, then the boundary surface $\Gamma_2$ is subjected to a heat flux $\phi(x, t)$, whose the $x$ variations are assumed to be known as given by other diagnostics. The substrate and the SCL exchange a radiative heat flux with the ambient that is assumed to be a blackbody at $\theta_{amb} = 393K$, the
emissivity of the CFC and the SCL is known ($\varepsilon = 0.83$). The convection exchanges are neglected because the tokamak is kept under vacuum ($P \simeq 10^{-5} Pa$). The simulated time is 20s with a time step of $dt = 5.10^{-2} s$. The mathematical formulation of this transient heat conduction problem is given as follows (for clarity we note $\theta = \theta(x,y,t)$):

**CFC region**

$$
\rho C p_{cfc}(\theta) \frac{\partial \theta}{\partial t} - \nabla \cdot \left( \lambda_{cfc}(\theta) \nabla (\theta) \right) = 0 \quad \text{in } \Omega_{cfc}
$$

(2a)

$$-\lambda_{cfc}(\theta) \frac{\partial \theta}{\partial n_1} = \varepsilon \sigma (\theta^4 - \theta_{amb}^4) \quad \text{on } \Gamma_1
$$

(2b)

**SCL region**

$$
\rho C p_{scl}(\theta) \frac{\partial \theta}{\partial t} - \nabla \cdot \left( \lambda_{scl}(\theta) \nabla (\theta) \right) = 0 \quad \text{in } \Omega_{scl}
$$

(3a)

$$\theta_{scl} = \theta_{cfc} \quad \text{on } \Gamma_4
$$

(3b)

$$\theta = \theta_0 \quad \text{at } t=0
$$

(3c)

This mathematical formulation is solved by the finite element method with the software CAST3M [3]. The geometry is composed of 352 elements in the SCL part and 1050 elements in the substrate part, the elements are quadratic elements at 8 nodes. It gives the unsteady temperature field in the substrate and the SCL, at the SCL surface and at the TC location.

3. Inverse problem

The two inverse problems concerning the identification of the surface heat flux and the thermophysical properties of the SCL will be explained simultaneously. We denote $p$ the function to be determined, in our case this function will be successively the heat flux $\phi(x,t)$ on $\Gamma_2$ and

$$p(x,t) = \phi(x,t) \quad \text{with} \quad \phi(x,t) = f(x) Q(t) \quad \text{or} \quad p(x,t) = \lambda_{eq}(x,t)
$$

(4)

We note $Y(x,y,t)$ the temperature data taken by the TC for the identification of $\phi(x,t)$, or by an infrared camera for the identification of $\lambda_{eq}(x,t)$. The solution of Eqs. (2)-(3) obtained for the given parameters $p$ is noted $\theta(x,y,t;p)$. The inverse heat conduction problem consists in determining $p(x,t)$ such that $\| \theta(x,y,t;p) - Y(x,y,t) \|$ is minimal. We define the cost function which is the following quadratic criterion measuring the Euclidean distance between data and model.

$$J(p) = \frac{1}{2} \int_0^{t_f} \sum_{i=1}^{N_s} \left( \theta(x_i,y_i,t;p) - Y_i(x_i,y_i,t) \right)^2 \ dt \quad \text{with} \quad Y_i = Y_{exact}(x_i,y_i,t;p_{exact}) + \omega_i \sigma
$$

(5)

with $N_s$ the number of sensors (1 (TC) for the identification of $\phi(x,t)$ and 22 (pixels) for the identification of $\lambda_{eq}(x,t)$). $Y_{exact}$ is the simulated noisy measurement data solution of Eqs. (2)-(3) with the exact evolutions of $\phi(x,t)$ and $\lambda_{eq}(x,t)$, $\sigma$ is the standard deviation of measurements, and $\omega$ is a random variable with a Gaussian distribution, zero mean and variance equal to 1. The Conjugate Gradient Method was used to minimize the cost function Eq. (5) by estimating the best estimation of the parameters $p$.

3.1. Conjugate Gradient Method

The Conjugate Gradient Method is an iterative process to estimate the parameters $p$ by minimizing the cost function $J(p)$. The CGM calculates the new iterate $p^{n+1}$ from the previous iteration $p^n$ (with $n$ the iteration number), by:

$$p^{n+1} = p^n - \gamma^n d^n
$$

(6)
where $\gamma^n$ is the step size, determined by the resolution of the sensitivity problem (§3.2), and $d^n$ is the direction of descent given by:

$$d^n = \nabla J(p^n) + \beta^n d^{n-1}$$

(7)

where $\nabla J(p^n)$ is the gradient of the cost function, determined by the resolution of the adjoint problem (§3.3). The conjugation coefficient $\beta^n$ is computed with the Polak-Ribière-Polyak’s version of the CGM [4]:

$$\beta^n_{P RP} = \frac{\int \int \left[ \nabla J(p^n) \left( \nabla J(p^n) - \nabla J(p^{n-1}) \right) \right] d\Omega dt}{\| \nabla J(p^{n-1}) \|^2} \quad \text{and} \quad \beta^0_{P RP} = 0$$

(8)

Others versions of the CGM have been tested like Fletcher-Reeve’s, Liu-Storey’s or Powell-Beale’s. Polak-Ribière-Polyak’s version appeared as the most efficient version in terms of convergence rate and stability of convergence.

3.2. Sensitivity problem and step size search

The sensitivity function $\delta\theta(x, y, t)$ describes the temperature rise resulting from of parameters variations of $\eta \delta p$, it is defined by [3]:

$$\delta\theta(x, y, t) = \lim_{\eta \to 0} \frac{\theta(x, y, t; p + \eta \delta p) - \theta(x, y, t; p)}{\eta}$$

(9)

The sensitivity problem is the system describing the evolution of the sensitivity function (for clarity we note $\delta\theta = \delta\theta(x, y, t)$) [4]:

**Step 1: identification of $p(x, t) = \phi(x, t)$**

**CFC region**

$$\frac{\partial p C_{p_c f c}(\theta) \delta \theta}{\partial t} - \nabla \cdot \left( \lambda_{c fc}(\theta) \nabla (\delta \theta) \right) = 0 \quad \text{in} \quad \Omega_{c fc}$$

(10a)

$$\frac{\partial \lambda_{c fc}(\theta) \delta \theta}{\partial n_1} = \left[ 4 \varepsilon \sigma \theta^3 \right] \delta \theta \quad \text{on} \quad \Gamma_1$$

(10b)

**SCL region**

$$\frac{\partial p C_{p_{scl}}(\theta) \delta \theta}{\partial t} - \nabla \cdot \left( \lambda_{eq}(x, t) \nabla (\delta \theta) \right) = 0 \quad \text{in} \quad \Omega_{scl}$$

(11a)

$$\frac{\partial \lambda_{eq}(x, t) \delta \theta}{\partial n_2} = 4 \varepsilon \sigma \theta^3 \delta \theta - \delta p \quad \text{on} \quad \Gamma_2$$

(11b)

$$\frac{\partial \lambda_{eq}(x, t) \delta \theta}{\partial n_3} = 4 \varepsilon \sigma \theta^3 \delta \theta \quad \text{on} \quad \Gamma_3$$

(11c)

$$\frac{\partial \lambda_{eq}(x, t) \delta \theta}{\partial n_4} = \frac{\partial \lambda(\theta) \delta \theta}{\partial n_4} \quad \text{on} \quad \Gamma_4$$

(11d)

$$\delta \theta_{scl} = \delta \theta_{c fc} \quad \text{on} \quad \Gamma_4$$

(11e)

$$\delta \theta = 0 \quad \text{at} \quad t = 0$$

(11f)

**Step 2: identification of $p(x, t) = \lambda_{eq}(x, t)$**

**CFC region**

$$\frac{\partial p C_{p_{scl}}(\theta) \delta \theta}{\partial t} - \nabla \cdot \left( \lambda_{eq}(x, t) \nabla (\delta \theta) \right) = 0 \quad \text{in} \quad \Omega_{scl}$$

(13a)

$$\frac{\partial \lambda_{eq}(x, t) \delta \theta}{\partial n_2} = 4 \varepsilon \sigma \theta^3 \delta \theta - \delta p \quad \text{on} \quad \Gamma_2$$

(13b)

$$\frac{\partial \lambda_{eq}(x, t) \delta \theta}{\partial n_3} = 4 \varepsilon \sigma \theta^3 \delta \theta \quad \text{on} \quad \Gamma_3$$

(13c)

$$\frac{\partial \lambda_{eq}(x, t) \delta \theta}{\partial n_4} = \frac{\partial \lambda(\theta) \delta \theta}{\partial n_4} \quad \text{on} \quad \Gamma_4$$

(13d)

$$\delta \theta_{scl} = \delta \theta_{c fc} \quad \text{on} \quad \Gamma_4$$

(13e)

$$\delta \theta = 0 \quad \text{at} \quad t = 0$$

(13f)
The coefficient $\gamma^n$, determines the step size between iteration $n$ and $n+1$, computed by \[3\]:

$$
\gamma^n = \left[ \int_0^{t_f} (\theta(x, y, t) - Y(x, y, t)) \delta \theta(x, y, t) dt \right] \left[ \int_0^{t_f} (\delta \theta(x, y, t))^2 dt \right]^{-1}
$$

### 3.3. Adjoint method

The adjoint method is used to calculate the gradient $\nabla J(p)$ of the cost function $J(p)$. For that, we define the Lagrangian $L(\theta, \psi, p)$, associated to the minimization problem defined by Eq\[4\], and constrained by the system of equations Eqs\[19\]-\[22\]. The expression of the Lagrangian is:

$$
L(\theta, \psi, p) = J(p) + \int_0^{t_f} \int_\Omega \left( \rho C_p \frac{\partial \theta}{\partial t} - \nabla (\lambda \nabla (\theta)) \right) \psi d\Omega dt
$$

Nullify the derivative of this Lagrangian \[5\] gives the following equations (for clarity we note $\psi = \psi(x, y, t)$):

**Step 1**: identification of $p(x, t) = \phi(x, t)$

**CFC region**

$$
\rho C_{p,cfc}(\theta) \frac{\partial \psi}{\partial t} + \nabla \left( \lambda_{cfc}(\theta) \nabla (\psi) \right) = E \quad \text{in} \quad \Omega_{cfc} \quad \text{(16a)}
$$

$$
- \lambda_{cfc}(\theta) \frac{\partial \psi}{\partial n_1} = [4\varepsilon \sigma \theta^3] \psi \quad \text{on} \quad \Gamma_1 \quad \text{(16b)}
$$

**SCL region**

$$
\rho C_{p,scl}(\theta) \frac{\partial \psi}{\partial t} + \nabla \left( \lambda_{scl}(\theta) \nabla (\psi) \right) = 0 \quad \text{in} \quad \Omega_{scl} \quad \text{(17a)}
$$

$$
- \lambda_{scl}(\theta) \frac{\partial \psi}{\partial n_2} = [4\varepsilon \sigma \theta^3] \psi \quad \text{on} \quad \Gamma_2 \quad \text{(17b)}
$$

$$
- \lambda_{scl}(\theta) \frac{\partial \psi}{\partial n_3} = [4\varepsilon \sigma \theta^3] \psi \quad \text{on} \quad \Gamma_3 \quad \text{(17c)}
$$

$$
- \lambda_{scl}(\theta) \frac{\partial \psi}{\partial n_4} = -\lambda_{cfc}(\theta) \frac{\partial \psi}{\partial n_4} \quad \text{on} \quad \Gamma_4 \quad \text{(17d)}
$$

$$
\psi_{scl} = \psi_{cfc} \quad \text{on} \quad \Gamma_4 \quad \text{(17e)}
$$

$$
\psi = 0 \quad \text{at} \quad t=t_f \quad \text{(17f)}
$$

where $t_f$ is the final time and the errors terms $E$ is defined by:

$$
E(t) = (\theta(x, y, t; p) - Y(x, y, t)) \delta (x-x_s, y-y_s)
$$

where $\delta(x-x_s, y-y_s)$ is the Dirac delta function equal to 1 at the sensors locations and 0 elsewhere. The expression of the gradient for the cost function $J(p)$ is \[5\]:

**Case :** identification of $p(x, t) = \phi(x, t)$

$$
\nabla J(p) = \psi \quad \text{(21)}
$$

**Case :** identification of $p(x, t) = \lambda_{eq}(x, t)$

$$
\nabla J(p) = -\left( \frac{\partial \theta}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \theta}{\partial y} \frac{\partial \psi}{\partial y} \right) \quad \text{(22)}
$$
3.4. Stopping criterion
In agreement with the "discrepancy principle" [9], the iterations go on as long as the cost function is not exceeding the threshold value \( J_{\text{threshold}} \), using the standard deviation of measurement noise \( \sigma \) assumed to be known:

\[
J_{\text{threshold}} = \frac{1}{2} \int_0^{t_f} \sum_{s} (\sigma)^2 \, dt
\]  

(23)

4. Results and discussions
The objective of this study is to show the accuracy of the present approach in predicting \( \phi(x,t) \) and \( \lambda_{eq}(x,t) \) with no prior information on the time dependence of the unknown functions. Prior information are given on dependence of \( \phi \) with respect to \( x \) (Fig. 1) and of \( \lambda_{eq} \) with respect to \( x \) and \( y \) (uniform function per pixel). The direct, adjoint and sensitivity problems were numerically solved using a finite element method with the software CAST3M. Eqs (2)–(3) are used to simulate the TC and IR measurements with realistic evolutions [2] of heat flux \( \phi(x,t) \) and thermal resistance \( R_{th}(x,t) \) (Fig. 3 and Fig. 5) and realistic noise (\( \sigma = 0.5K \) for the TC and \( \sigma = 3K \) for the IR). The initial guess of the parameters in the case of the identification of \( \phi(x,t) \) is set equal to zero, and to \( 10^{-2}m^2.K/W \) in the case of the identification of \( R_{th}(x,t) \) (which is an error of 2 order of magnitude from the exact values going from 2 to \( 5.10^{-2}m^2.K/W \)). For the estimation of \( \lambda_{eq}(x,t) \), results are presented through the resulting thermal resistance \( R_{th}(x,t) = \frac{e}{\lambda_{eq}(x,t)} \).

**Figure 3.** Evolution of the maximum heat flux \( Q(t) \) \( (f(x)=1) \). (—) Exact evolution of \( Q(t) \). (—) Optimal estimation of \( Q(t) \) (with exact evolution of \( \lambda_{eq}(x,t) \)). (—) Estimated evolution of \( Q(t) \) at the first iteration. (—) Estimated evolution of \( Q(t) \) at the second iteration.

**Figure 4.** Evolution of the temperature \( \theta(t) \) at the TC location. (—) Noisy measurement of \( \theta(t) \) with \( \sigma = 0.5K \). (—) Computed evolution of \( \theta(t) \) with the estimated evolutions of \( \phi(x,t) \) and \( R_{th}(x,t) \) at the second iteration.

The estimation of \( \phi(x,t) \) and \( R_{th}(x,t) \) are performed successively. Step 1 is the heat flux \( \phi(x,t) \) estimation by CGM (17 iterations) without modeling the SCL, the solution is the heat flux prescribed in the step 2 consisting to estimate the thermal resistance \( R_{th}(x,t) \) of the SCL, by the CGM (22 iterations). These two steps are considered as the first iteration. The next iterations use the previous estimated values of \( R_{th}(x,t) \) and \( \phi(x,t) \) to identify respectively new values of \( \phi(x,t) \) and \( R_{th}(x,t) \). There, only two iterations are necessary to converge to the best estimations.

Fig. 3 shows that, at the second iteration, the estimated heat flux coincides with the optimal estimation of the heat flux. We note the small difference between the first and second iteration in the estimation of the heat flux, which shows the small sensitivity of the temperature at the
TC location to the thermal resistance of the SCL, this sensitivity is proportional to the heat flux. Magnitude and time behavior of the heat flux are well recovered despite of the use of an embedded TC. Fig. (4) presents the noisy measurement of the TC and the temperature computed at the second iteration, the two evolutions coincide and the residuals between this two evolutions have the same statistical properties as the noise used to synthesise the measurements. Fig. (5) and (6) show that the spatial and time variations of the thermal resistance are well estimated in the time range between 1.5s and 14.5s. The estimation of the thermal resistance can not take place in the whole time range (0 − 20s), because it is necessary to have a significant heat flux to estimate the thermal resistance of the SCL. In practice it is assumed that the thermal resistance outside the range 1.5 − 14.5s are constants and equals to the nearest estimated values at \( t = 1.5s \) or \( t = 14.5s \). Fig. (7) and (8) show the estimated thermal resistance respectively for the pixel n°8 and 22. There are a lot of differences between the first and second iteration, particularly for the pixel n°8 which is the pixel in front of the maximum heat flux. This shows the high sensitivity to errors made in the estimation of the heat flux to estimate the thermal resistance. Fig. (8) shows
a bias for the lowest values of the thermal resistance, this bias is induced by the fact that there is just 10% of the heat flux which impacts this pixel and the thermal resistance varies here during time with a factor of about 2.5 much higher than the pixel n°8.

Fig. 9 and 10 show the surface temperature of the SCL, respectively the noisy measurement and the computed temperature with the heat flux and the thermal resistance estimated at the second iteration. We note that we have a good correspondence between the two maps despite the fact that we cannot estimate the thermal resistance in the all time range.

![Figure 9](image-url)  ![Figure 10](image-url)

**Figure 9.** Mapping of the simulated noisy measurement of the SCL surface temperature with $\sigma = 3K$. Used as input.

**Figure 10.** Mapping of the computed SCL surface temperature with the estimated evolutions of $\phi(x,t)$ and $R_{th}(x,t)$ at the second iteration.

5. Conclusion

In this study, 2D nonlinear unsteady calculations are used with the Conjugate Gradient Method and the adjoint state, for the estimation of a field of heat flux and thermal resistance. The method can estimate a field of heat flux and thermal resistance with realistic evolutions expected in tokamak environment. The estimation converges despite significant errors on the initial values of the parameters, and the method is robust in regard of spatial and time evolutions of the parameters. But the calculation time of the estimated values is high, in this case (401 parameters for the heat flux and 261×22 parameters for $R_{th}$) the method takes about 20h to estimate the two fields of parameters.

The method is now numerically validated and ready to be applied on experimental data from the JET tokamak.

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