Numerical simulation and analysis of complex fracture propagation during SC-CO₂ fracturing using a thermal-hydro-mechanical coupling model

Zhifeng Luo¹, Long Cheng¹, Liqiang Zhao¹

¹State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu, 610500, China.

Abstract. Supercritical CO₂ has been widely used in shale fracturing due to the low interfacial tension and high diffusion characteristics. However, under the high pressure and temperature condition of shale reservoirs, the properties evolution of SC-CO₂ cannot be accurately described with the cubic equations of state. And on the other hand, owing to the effects of sedimentation and geological tectonism the lamellation is commonly developed in the shale, which has a great impact on the fracture propagation. In order to explore the mechanism of complex fracture evolution during SC-CO₂ fracturing of shale reservoirs, a thermal-hydro-mechanical coupling model is established with the extended finite element method, and the Span–Wagner equation of state is employed to obtain the SC-CO₂ properties under high pressure. With the proposed model, a series of numerical examples are presented and discussed. It is found that fracture is inclined to propagate along the lamellation, and a complex fracture networks is easier to obtain in shale reservoirs developing complex lamellation. And in comparison with the water and oil, the fracture morphology generated by SC-CO₂ is more complex than water and oil because of the high density and low viscosity.

Keywords: SC-CO₂ fracturing; Thermal-hydro-mechanical coupling; Fracture propagation; Lamellation; Extended finite element method

1 Introduction

For the sake of meeting the growing energy demand of oil and gas, the shale gas and oil with highly rich resources have been the major exploitation and development reservoirs recently. And hydraulic fracturing is the most effective technology for stimulating shale oil and gas production due to the extremely tight reservoirs and heterogeneity. With the development of fracturing technology, CO₂ fracturing has been the popular technique for shale oil and gas because of the effective diffusivity and
flowability and lower reservoir damage, comparing with other fracturing fluid, such as water and oil. On the other hand, the CO₂ transport and displacement during CO₂ fracturing can contribute to the CO₂ geological sequestration and oil and gas recovery. And many researches about CO₂ fracturing have been reported.[1–5].

Throughout these studies, the CO₂ properties evolution caused by pressure and temperature variation during fracturing is only described by cubic equations of state in most researches, which is suitable for the low pressure or high temperature conditions and cannot obtain the CO₂ properties under high pressure and temperature. Besides, fracturing in underground reservoirs is actually a thermal-hydro-mechanical (THM) coupling process,[6–9] accompanying rock deformation, fluid flow and temperature variation. The pure fracture propagation and pressure evolution in fracture under isothermal condition cannot accurately characterize the fracturing mechanism. Moreover, natural crack and lamellation are usually developed in shale rocks due to the effects of geological deposition and geological structure, and these discontinuous structures have a great impact on the rock mechanics properties of shale rocks, rock fracturing mechanism as well as fracture propagation path. Different with the natural crack with strong discontinuity, as a kind of weak plane the lamellation developed in shale rocks will highly affect the fracture propagation direction and path because of its own lower strength. And as for the studies around the fracture propagation in reservoirs developing lamellation, most researches are mainly concentrate on the effects of formation layer on the fracture height propagation.[10–12] The fracture propagation mechanism in shale rocks developing lamellation are still unclear, and the effects of lamellation properties on fracture propagation patterns, complex fracture networks development need to be investigated.

Therefore, in order to analyze interaction mechanism of fracture propagation and lamellation during SC-CO₂ fracturing, this paper establishes a thermal-hydro-mechanical coupling model considering the thermal strain effect and poroelastic theory, including rocks deformation equation, compressible fluid flow equation and heat transport equation. The CO₂ properties evolution is described by S-W equation of state and transport equation. And the THM coupling model is solved by the extended finite element method (XFEM), and the mesh refinement and mesh reconstruction are all avoided with XFEM, which highly improves the numerical calculation efficiency. The interaction integral method is used to obtain the stress intensity factors and the maximum energy release rate criterion is employed for the fracture propagation. With the proposed model, some numerical examples that emphasize the effects of lamellation and fracturing fluid types are discussed.

2 Governing equations

2.1 Governing Equation of stress THM coupling
The stress equilibrium equation in two dimensions is given as following for fractured porous media saturated fluid when considering the poroelastic theory and thermal stress effect.

\[ \nabla \cdot (\sigma' + \alpha I_m p_m - \beta \Delta T) + f = 0 \]  \hspace{1cm} (1)

where, \( \sigma' \) is the effective stress. \( \alpha \) is the Biot coefficient. \( p_m \) is the pore pressure. \( I_m \) is the unit
vector in two dimensions. \( \beta_f = \alpha_f E/(1-2\nu) \), \( \alpha_f \) is the thermal expansion coefficient. \( E \) is the Young's modulus. \( \nu \) is the Poisson's ratio. \( \Delta T \) is the temperature difference. \( f \) is the body force vector.

And as for CO\(_2\) flow, the continuity equations in matrix and fracture are given as:

\[
\phi_a \rho_a C_e \frac{\partial p_a}{\partial t} - \nabla \left( \rho_a \frac{k_m}{\mu_g} \nabla p_a \right) + \alpha \nabla \cdot \left( \rho_a \frac{k_m}{\mu_g} \nabla u \right) = 0 \tag{2}
\]

\[
b \phi_f \rho_f C_f \frac{\partial p_f}{\partial t} - \frac{\partial}{\partial x} \left( b \rho_f \frac{k_f}{\mu_g} \frac{\partial p_f}{\partial x} \right) = 0 \tag{3}
\]

where, \( p_a \) is the fluid pressure in matrix. \( k_m \) is the matrix permeability. \( \phi_f \) is the porosity of fracture. \( p_f \) is the fluid pressure in fracture. \( k_f \) is the fracture permeability, \( k_f = b^2/12 \). \( b \) is the fracture width. \( \rho_f \) is the CO\(_2\) density. \( \mu_g \) is the CO\(_2\) viscosity. \( C_e \) is the compression coefficient of CO\(_2\). \( C_e = \frac{1}{\rho_e} \left( \frac{\partial \rho_e}{\partial p} \right) \).

The governing equations of heat transport in matrix and fracture are written as following.

\[
\left[ \rho_c \left( 1 - \phi_a \right) + \rho_c \phi_a \right] \frac{\partial T_m}{\partial t} = \nabla \cdot \left[ \left( \lambda_c \left( 1 - \phi_a \right) + \lambda_c \phi_a \right) \nabla T_m \right] + Q \tag{4}
\]

\[
w_f \rho_f c_f \frac{\partial T_f}{\partial t} = -w_f \rho_f c_f \frac{\partial T_f}{\partial x} + \nabla \cdot \left( w_f \lambda_f \frac{\partial T_f}{\partial x} \right) + Q \tag{5}
\]

where, \( \rho_c \) is the rock density. \( c_c \) is the specific heat capacity of rock. \( T_m \) is the temperature in matrix. \( \lambda_c \) is the heat conductivity of rock. \( \rho_f \) is the density of fracturing fluid. \( c_f \) is the specific heat capacity of fracturing fluid. \( \lambda_f \) is the heat conductivity of fracturing fluid. \( Q \) is the heat exchange of matrix and fracture. \( T_f \) is the temperature in fracture.

2.2 Equation of state for CO\(_2\)

In this study, in order to accurately describe the state of CO\(_2\) under high pressure and temperature, the Span-Wagner (S-W) equation proposed by Span et al\([13]\), is chosen for CO\(_2\) density. The S-W equation is based on Helmholtz free energy, and the CO\(_2\) density is related to the residual part of S-W equation of state.

\[
\phi^* = \sum_{i=1}^{2} \eta_i \delta \tau_i \epsilon_i + \sum_{i=4}^{34} \eta_i \delta \tau_i \epsilon_i e^{-\delta \gamma_i} + \sum_{i=35}^{39} \eta_i \delta \tau_i \epsilon_i e^{-\eta_i \delta \gamma_i} T_i \gamma_i + \sum_{i=40}^{42} n_i \Delta \delta \epsilon_i e^{-\eta_i \delta \gamma_i} T_i \gamma_i
\]

where, \( \delta = \rho / \rho_c \), \( \delta \) is the reduced density. \( \rho_c \) is the critical density of CO\(_2\). \( \tau = T_c / T \), \( \tau \) is the inverse reduced temperature. \( T_c \) is the critical temperature. \( n_i, d_i, l_i, c_i, a_i, e_i, \beta_i, \gamma_i, b_i, C_i, D_i, A_i, B_i, a_i \) are all constant. \( \Delta = \left( 1 - \tau \right)^2 + \left( \delta - 1 \right)^2 + \left( \delta - 1 \right)^2 \).

Then the CO\(_2\) density can be obtained by the following implicit equation,

\[
p(\delta, \tau) = \rho R T \left( 1 + \delta \phi^*_f \right) \tag{8}
\]

where, \( R \) is the universal gas constant. \( \phi^*_f \) is the first derivative of \( \phi^* \) with respect to \( \delta \). And the
compression coefficient of CO₂ in Eq (2) can be written as \( C_s = \frac{1}{\rho_s} \left( \frac{1}{RT} \left( 1 + 2\phi'_{\delta} + \phi''_{\delta} \right) \right) \). \( \phi_{\delta} \) is the second derivative of \( \phi \) with respect to \( \delta \).

The CO₂ viscosity evolution during SC-CO₂ fracturing can be described by transport equation proposed by Fenghour and Vessovic et al.\(^{[14]}\),

\[
\mu_s (\rho, T) = \mu_e (\rho, T) + \Delta \mu (\rho, T) + \Delta \mu_e (\rho, T)
\]

where, \( \mu_0 (T) \) is the viscosity under the zero-density limit. \( \Delta \mu (\rho, T) \) is the excess viscosity. \( \Delta \mu_e (\rho, T) \) is the critical enhancement accounting for the increase in viscosity at the immediate vicinity of the critical point. The details of the three parts can be found in Fenghour and Vessovic’s study.

### 2.3 Numerical method

In the light of the Galerkin principle, the weak forms of THM coupling are given as follows.

\[
\begin{align*}
\int_{\Omega} \varepsilon (v) \cdot \sigma (u) & d\Omega - \int_{\Omega} \varepsilon (v) : \alpha I_m \lambda_n d\Omega - \int_{\Omega} \varepsilon (v) \cdot \beta_j I_m \lambda T d\Omega - \int_{\Gamma} \delta u \cdot f d\Gamma \\
- \int_{\Gamma} \delta u \cdot i d\Gamma - & \int_{\Gamma} \| \delta u \| p_j d\Gamma = 0
\end{align*}
\]

(10)

\[
\begin{align*}
\int_{\Omega} \delta p \phi_n \rho_s \phi_\delta \frac{\partial p}{\partial t} + & \int_{\Gamma_r} \delta p \alpha N \cdot \frac{\partial u}{\partial t} d\Gamma + \int_{\Gamma_s} \delta p \rho_s g_s d\Gamma & = 0
\end{align*}
\]

(11)

With the extended finite element method, the displacement approximation, pressure approximation and temperature approximation are expressed as.

\[
\begin{align*}
\mathbf{u} (x) & = \sum_{i = 1}^{N_e} N_i (x) \mathbf{u}_i + \sum_{i = 1}^{N_e} \sum_{k = 1}^{N_{\nu}} \mathbf{N}_i (x) \left[ \mathbf{H} (x) - \mathbf{H} (x_i) \right] \mathbf{a}_k + \sum_{i = 1}^{N_e} \sum_{k = 1}^{N_{\nu}} N_i (x) \left[ J (x) - J (x_i) \right] \mathbf{d}_k \\
\mathbf{p} (x) & = \sum_{i = 1}^{N_e} \mathbf{N}_i (x) \mathbf{p}_i + \sum_{i = 1}^{N_e} \sum_{k = 1}^{N_{\nu}} \mathbf{N}_i (x) \left[ \mathbf{H} (x) - \mathbf{H} (x_i) \right] \mathbf{p}_{\nu} + \sum_{i = 1}^{N_e} \mathbf{N}_i (x) \sum_{a = 1}^{3} \mathbf{B}^a (x_i) \mathbf{B}^a (x) \mathbf{p}_{\nu} \\
\mathbf{T} (x) & = \sum_{i = 1}^{N_e} \mathbf{N}_i (x) \mathbf{T}_i + \sum_{i = 1}^{N_e} \sum_{k = 1}^{N_{\nu}} \mathbf{N}_i (x) \left[ \mathbf{J} (x) - \mathbf{J} (x_i) \right] \mathbf{T}_{\nu}
\end{align*}
\]

(12)

(13)

(14)

where, \( N_i \) is the finite element shape function. \( N^0 \) is the standard nodes set. \( N^{disc} \) is the node set enriched by Heaviside function. \( N^{tip} \) is the node set enriched by tip enrichment function. \( N^{junc} \) is the node set enriched by the junction enrichment function. \( H(x) \) is the Heaviside function. \( B_0(x) \) is the tip enrichment function. \( R^c(x) \) is the Ramp function. \( \Psi(x) \) is the level set enrichment function for lamellation element. \( J(x) \) is the junction enrichment function for junction element.
Submit XFEM approximations into the weak forms, and the finite difference method is employed for time discretization. Then the numerical model of THM coupling is given as.

\[ K u = F \] (15)

\[ \Delta H + D \rho^{n+1} = \Delta t F Q + D p^n - L (u^{n+1} - u^n) \] (16)

\[ \Delta G + N T^{n+1} = \Delta t F T + NT^n \] (17)

where, \( K \) is the total stiffness matrix, \( F \) is the total force vector, \( H \) is the total seepage matrix, \( D \) is the total compression matrix, \( L \) is the total strain matrix, \( G \) is the total diffusion matrix, and \( N \) is the total transport matrix. In this study, the maximum energy release rate criterion is used for the fracture propagation criterion, and the more details about the energy release rate and stress intensity factors can be found in Cheng et.al study [15].

3 Numerical results
In this section, some numerical results of SC-CO\(_2\) fracturing in shale rocks are studied. And the effects of lamellation and different fracturing fluid are emphatically discussed. The physics model of fracturing is shown in Figure 1, and the top and right boundaries are subjected to the horizontal stress. The left and right boundaries are fixed to rollers, and fracturing fluid is injected at the center of the left boundary. Other model parameters are presented in Table 1.

![Figure 1](image_url). The physics model of SC-CO\(_2\) fracturing in shale rocks developing lamellation.

Table 1. Model parameters.

| Parameter                        | Value         | Parameter                        | Value         |
|----------------------------------|---------------|----------------------------------|---------------|
| \( \sigma_H \)                   | 50MPa         | \( \sigma_b \)                   | 40MPa         |
| Young’s modulus of rock          | 40GPa         | Young’s modulus of lamellation   | 20GPa         |
| Poisson’s ratio of rock          | 0.25          | Poisson’s ratio of lamellation   | 0.32          |
| Permeability of matrix           | 0.001mD       | Permeability of lamellation      | 0.01mD        |
| Porosity of matrix               | 0.1           | Porosity of lamellation          | 0.2           |
| Initial pore pressure            | 30MPa         | Injection rate                   | 4m\(^3\)/min |
| Water viscosity                  | 1mPa·s        | Oil viscosity                    | 360mPa·s      |
| critical energy release rate of  | 0.1MPa·m      | critical energy release rate of   | 0.05MPa·m     |
| rock                             |               | lamellation                      |               |
|               |               |                                  |               |
3.1 Effects of lamellation

The fracture propagation in shale rocks developing lamellation during SC-CO₂ fracturing is firstly analyzed. Figure 2 and Figure 3 present the fracture propagation paths and pressure distributions at horizontal, vertical and orthogonal lamellation, respectively. It can be seen that fracture is inclined to propagate along the lamellation at horizontal and vertical lamellation due to the lower Young’s modulus and lower critical energy release rate than matrix rock, and the lamellation shall capture the fracture as result of lower energy consumption. And the horizontal lamellation will contribute to the fracture propagation along the horizontal direction while vertical lamellation will contribute to the propagation along the vertical direction. Orthogonal lamellation will capture the fracture and fracture could initiate at the orthogonal points, which will lead to the generation of crossed fracture. And complex fracture networks will be obtained at the orthogonal lamellation condition. According to the simulations, the interaction patterns of fracture and lamellation would mainly include propagation along lamellation, penetrating lamellation and offset lamellation.

![Figure 2. The fracture propagation paths at different lamellation orientation.](Image)

![Figure 3. The pressure distributions at different lamellation orientation.](Image)

3.2 Effects of fracturing fluid

As a common and cheap fracturing fluid, water has been widely used for hydraulic fracturing in shale rocks to stimulating oil and gas production. However, the water shortage, environmental pollution and reservoir damage caused by water when hydraulic fracturing are more and more concerned. Recently years, CO₂ has been gradually used for fracturing fluid due to the effective flowability and diffusivity, and the injection of CO₂ will displace the shale gas and contribute to the CO₂ storage. In this section, we investigated fracture propagation mechanism at different injection fluid, and the fracture propagation paths and pressure distributions at different fracturing fluid are displayed in Figure 4 and Figure 5. Among the three fracturing fluids, the most complex fracture is obtained when injecting CO₂, water comes second, and the simplest fracture morphology is gotten by oil. The propagation paths of CO₂ and water are similar due to the approximate density, but the propagation path of CO₂ is more complex and fracture scale is larger than the water’s as a result of the lower viscosity of CO₂. So it can
be inferred that a complex fracture network is easy to obtain with CO₂. And in Figure 5, it is found that as the CO₂ viscosity is lower than water and oil, the flowability and diffusivity of CO₂ are better than water and oil, and the sweep region when injecting CO₂ is largest among three fracturing fluids, which will promote the rock fracture and generation of complex fracture.

![Figure 4. The fracture propagation paths at different fracturing fluid.](image)

![Figure 5. The pressure distributions at different fracturing fluid.](image)

4 Conclusions
In this study, a thermal-hydro-mechanical coupling model is developed to simulate fracture propagation in shale rocks developing lamellation based on the poroelastic theory and thermal strain effect. The CO₂ density and viscosity evolution is described by S-W equation and transport equation. And the extended finite element method is employed to solve the THM coupling model. With the proposed model, numerical results are simulated and analyzed.

The numerical examples show that lamellation has a great impact on fracture propagation path. The lamellation will dominate fracture propagation, and horizontal lamellation can contribute to the fracture propagating along horizontal direction, while vertical lamellation can contribute to the fracture propagating along vertical direction. And the orthogonal lamellation will promote the generation of complex fracture network. Besides, the propagating along lamellation, penetrating lamellation and offset lamellation are the mainly interaction patterns of fracture and lamellation. Moreover, comparing with water and oil, CO₂ with high flowability and diffusivity can generate larger sweep region and contribute to the complex fracture networks propagation because of the lower viscosity.

Acknowledgements
This paper was supported by the National Natural Science Foundation of China (No. 51974264).

References
[1] Middleton R, Viswanathan H, Currier R, Gupta R. CO₂ as a fracturing fluid: Potential for
commercial-scale shale gas production and CO2 sequestration. *Energy Procedia* 2014, 63: 780–4.

[2] Mostafa Mollaali, Vahid Ziaei-Rad, Yongxing Shen. Numerical modeling of CO2 fracturing by the phase field approach. *Journal of Natural Gas Science and Engineering*. 2019, 70: 102905.

[3] Bailong Liu, Anna Suzuki, Takatoshi Ito. Numerical analysis of different fracturing mechanisms between supercritical CO2 and water-based fracturing fluids. *International Journal of Rock Mechanics and Mining Sciences*. 2020, 132: 104385.

[4] Heqian Zhao, Kan Wu, Zhongwei Huang, Zhengming Xu, Huaizhong Shi, Haizhu Wang. Numerical model of CO2 fracturing in naturally fractured reservoirs. *Engineering Fracture Mechanics*. 2021, 244: 107548.

[5] C.P. Zhang, P. Cheng, Z.Y. Ma, P.G. Ranjith, J.P. Zhou. Comparison of fracturing unconventional gas reservoirs using CO2 and water: An experimental study. *Journal of Petroleum Science and Engineering*. 2021, 203: 108598.

[6] Khoei A. R, Mouleemi, S, Haghighat E. Thermo-hydro-mechanical modeling of impermeable discontinuity in saturated porous media with X-FEM technique. *Engineering Fracture Mechanics*. 96(2012), 701-723.

[7] Ingrid Tomac, Marte Gutierrez. Coupled hydro-thermo-mechanical modeling of hydraulic fracturing in quasi-brittle rocks using BPM-DEM. *Journal of Rock Mechanics and Geotechnical Engineering*, 9(2017), 92-104.

[8] Wenzheng Liu, Qingdong Zeng, Jun Yao. Numerical simulation of elasto-plastic hydraulic fracture propagation in deep reservoir coupled with temperature field. *Journal of Petroleum Science and Engineering*, 171(2018), 115-126.

[9] Ying Xin, Zhiyue Sun, Li Zhuang, Jun Yao, Kai Zhang, Dongyan Fan, Kelvin Bongole, Tong Wang, Chuanyin Jiang. Numerical simulation of fluid flow and heat transfer in EGS with thermal-hydraulic-mechanical coupling method based on a rough fracture model. *Energy Procedia*, 158(2019), 6038-6045.

[10] Jun Xie, Jizhou Tang, Rui Yong, Yu Fan, Lihua Zuo, Xing Chen, Yuwei Li. A 3-D hydraulic fracture propagation model applied for shale gas reservoirs with multiple bedding planes. *Engineering Fracture Mechanics*. 2020, 228: 106872.

[11] Chao Sun, Heng Zheng, Wei David Liu, Wenting Lu. Numerical simulation analysis of vertical propagation of hydraulic fracture in bedding plane. *Engineering Fracture Mechanics*. 2020, 232: 107056.

[12] Xu Chang, Hongbo Zhao, Long Cheng. Fracture propagation and coalescence at bedding plane in layered rocks. *Journal of Structural Geology*. 2020, 141: 104213.

[13] R Span, W Wagner. A New Equation of State for Carbon Dioxide Covering the Fluid Region from the Triple-Point Temperature to 1100 K at Pressures up to 800 MPa. *Journal of Physical and Chemical Reference Data*. 1996, 25(6): 1509-1596.

[14] Vesovic V, Wakeham W A, Olchowy G A et al. The transport properties of carbon dioxide. *Journal of Physical and Chemical Reference Data*, 1990, 19(3): 763-808.
[15] Cheng Long, Luo Zhifeng, Yu Yang, Zhao Liqiang, Zhou Changlin. Study on the interaction mechanism between hydraulic fracture and natural karst cave with the extended finite element method. *Engineering Fracture Mechanics*, 2019, 222:106680.