Virtualization of 5G Cellular Networks as a Hierarchical Combinatorial Auction

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Abstract—Virtualization has been seen as one of the main evolution trends in the forthcoming fifth generation (5G) cellular networks which enables the decoupling of infrastructure from the services it provides. In this case, the roles of infrastructure providers (InPs) and mobile virtual network operators (MVNOs) can be logically separated and the resources (e.g., subchannels, power, and antennas) of a base station owned by an InP can be transparently shared by multiple MVNOs, while each MVNO virtually owns the entire BS. Naturally, the issue of resource allocation arises. In particular, the InP is required to abstract the physical resources into isolated slices for each MVNO who then allocates the resources within the slice to its subscribed users. In this paper, we aim to address this two-level hierarchical resource allocation problem while satisfying the requirements of efficient resource allocation, strict inter-slice isolation, and the ability of intra-slice customization. To this end, we design a hierarchical combinatorial auction mechanism, based on which a truthful and sub-efficient resource allocation framework is provided. Specifically, winner determination problems (WDPs) are formulated for the InP and MVNOs, and computationally tractable algorithms are proposed to solve these WDPs. Also, pricing schemes are designed to ensure incentive compatibility. The designed mechanism can achieve social efficiency in each level even if each party involved acts selfishly. Numerical results show the effectiveness of the proposed scheme.

Index Terms—5G cellular, massive MIMO, wireless network virtualization, resource allocation, mechanism design, hierarchical combinatorial auction, incentive compatibility, winner determination problem (WDP).

I. INTRODUCTION

The next generation cellular wireless networks (i.e., 5G networks) are expected to be deployed around 2020 which are envisioned to provide higher data rate, lower end-to-end latency, improved spectrum/energy efficiency, and reduced cost per bit. In general, addressing these requirements will require significantly larger amount of spectrum, more aggressive frequency reuse, extreme densification of small cells, and the wide use of several enabling technologies (e.g., full-duplex, massive MIMO, C-RAN, and wireless virtualization) [1]. In this paper, we will focus on the issue of wireless virtualization which has been receiving increasing attentions from both academia and industry [2]–[4].

The main idea of wireless virtualization is to enable resource sharing and to decouple the infrastructure from the services it provides. Accordingly, the role of infrastructure provider (InP) needs to be logically separated from the role of service provider. And the InP can provide the infrastructure as a service (IaaS) to mobile virtual network operators (MVNOs) who may not have their own infrastructure and/or wireless network resources. Specifically, the physical resources (e.g., infrastructure, spectrum, power, backhaul/fronthaul, and antennas) of a base station (BS) owned by an InP are abstracted into isolated virtual resources (i.e., slices) which are then transparently shared among different MVNOs. Each MVNO virtually owns the entire BS with the resources provided in the allocated slice.

Several benefits can be achieved through such decoupling and sharing (i.e., virtualization). First, the resource utilization can be improved through moderating the dynamic requirements of users from different MVNOs (i.e., statistical multiplexing gains). Second, the capital expenses (CapEx) and operation expenses (OpEx) can be reduced through sharing. Third, lower entry barrier for small service providers could enrich the services provided to users.

A significant challenge for wireless virtualization is resource allocation which addresses the problem of how to slice the physical resources for virtual networks of MVNOs to accommodate the dynamic demands of their subscribed users, while satisfying the requirements of efficient resource allocation, inter-slice isolation, and the ability of intra-slice customization. The problem is more challenging if the agents involved are self-interested. That is, how to design a mechanism that can achieve a desirable social efficiency even when each agent acts selfishly. Numerical results show the effectiveness of the proposed scheme.

In general, there are two types of implementation schemes for resource allocation in wireless virtualization. In the first type, the InP plays the central role who directly allocates the physical resources to users of different MVNOs according to certain requirements (e.g., predetermined resource sharing ratios). In the second type, the MVNOs are also involved which makes the resource allocation problem a hierarchical (i.e., two-level) problem. In this case, the InP is only responsible for allocating the resources to each MVNO, while each MVNO manages the resource allocation for its users.

Most of the existing work on resource allocation for wireless virtualization can be categorized into the first type. Specifically, in [5]–[8], optimization-based dynamic resource allocation schemes were proposed. In [9], a stochastic game based scheme was proposed. The schemes in these work can achieve high resource utilization. However, since the MVNOs are not involved in the resource allocation, the capability of intra-slice customization for each MVNO cannot be easily achieved. Besides, the computation complexity for InP is high consider-
ing that the optimal resource allocation has to be obtained directly for all users. A few work considered the problem of resource allocation to MVNOs. For example, in [10], an opportunistic sharing based resource allocation scheme was proposed. In [11], a bankruptcy game was proposed for dynamic wireless resource allocation among multiple operators. However, in these work the users are not involved. Besides, most of the existing work on wireless virtualization do not consider techniques such as massive MIMO [17], which will be a key enabler for 5G networks. Also, the social efficiency was not considered when agents play selfishly.

To jointly address the two-level resource allocation problem, we design a hierarchical auction mechanism consisting of two hierarchical auction models (i.e., a single-seller multiple-buyer model as shown in Fig. 1 and an extended multiple-seller multiple-buyer model as shown in Fig. 2), allocation procedures, and the corresponding pricing schemes. Note that auction approaches have been widely used in the literature for resource allocation problem in wireless systems [12]. This is due to the efficiency in both process and outcome. With the advent of the concepts of wireless network virtualization and spectrum secondary market in cognitive radio networks, the middlemen (e.g., MVNOs) play more important roles in bridging the supply from resource owners with the demands from end users. However, almost all existing work on application of auction mechanisms for resource allocation only consider single-level auctions without involving the middlemen. To the best of our knowledge, the work in [13] is the first work on this topic in which some qualitative analyses were provided but without application. Besides, there is a social planner controlling the entire resource allocation of all players. In this case, it cannot satisfy the requirement of intra-slice customization for wireless virtualization. The work in [14] proposed a three-stage auction mechanism for spectrum trading. However, the middleman can submit bids for multiple items but can acquire at most one. The work in [13] investigated the hierarchical resource allocation through an auction in the upper level and a price-demand method in the lower level. Besides, all these work only consider the single seller case. The main differences of the proposed scheme with existing work are summarized in Table I in Appendix A. Note that compared with hierarchical game based approach (e.g., Stackelberg game in [16]) in which the social welfare under equilibrium strategies may not be the optimal ones in sub-games, the proposed hierarchical auction mechanism achieves social efficiency at each level.3

Based on the proposed hierarchical auction mechanism, a truthful and sub-efficient semi-distributed resource allocation framework for wireless virtualization is provided. Specifically, the proposed hierarchical auction models consist of two levels of combinatorial auctions. In the lower-level auction, the users act as the bidder and each MVNO acts as a seller, while in the upper-level auction, the MVNOs act as the bidders and the InP acts as the seller. The role of an MVNO can be regarded as that of a middleman. Note that the two-level auctions are dependent considering the fact that the MVNOs do not have intrinsic demands and valuations, which however depend on the demands and resale gains from users.

To determine the optimal amount of resources allocated to each bidder, winner determination problems (WDPs) are formulated, and the corresponding solution algorithms are proposed. Also, pricing schemes are designed to ensure incentive compatibility, which is critical for achieving social efficiency. The proposed scheme can satisfy the three requirements mentioned above. First, efficient resource allocation can be achieved in each level by allocating resources to bidders with higher valuations. Second, we can achieve flexible but strict inter-slice isolation in the sense that once the resources are allocated to an MVNO, these resources can only be allocated to the users of that MVNO (i.e., strict), while the amount of resources allocated to each MVNO can be dynamically changed (i.e., flexible). Third, intra-slice customization can be achieved due to the involvement of MVNOs which can individually decide how the resources within the slice can be allocated. Also, compared with totally centralized allocation schemes (e.g., schemes of the first type), the computational complexity can be reduced considering two facts. First, the computation is distributed among InP and MVNOs, each of which only needs to calculate the allocation for its own bidders. Second, the dimension of the winner determination problem faced by each MVNO is relatively small considering that the available resources are only a subset of the entire BS resources.

The novelty and main contributions of this paper can be summarized as follows:

- A hierarchical combinatorial auction mechanism is designed to jointly address the hierarchical resource allocation for wireless virtualization in massive MIMO networks.
- The proposed scheme can satisfy the three requirements of wireless virtualization. Also, it jointly considers the feasibility, admission control, and allocation problems in a unified resource allocation framework.
- Several desirable properties of the hierarchical auction mechanism (i.e., incentive compatibility, individual rationality, and allocation efficiency in each level) can be achieved with appropriate design of allocation and pricing schemes.
- The computations are migrated to different parties which lowers the complexity for each party. Also, computationally tractable algorithms are proposed for solving the WDPs.
- While most of the existing auction-based resource allocation schemes only consider one dimensional resource (e.g., in most cases the spectrum), we consider the allocation with more degrees of freedom (i.e., frequency, power, and spatial). Also, most existing work only consider single-minded bidder, while we consider both single-minded and general valuation bidders.

The rest of the paper is organized as follows. Section II describes the system model, assumptions, and presents

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3Note that there is a gap between the allocation efficiency achieved by a general sharing scheme (i.e., the InP directly allocates resources to all users) and that achieved by the hierarchical auction mechanism. Therefore, we use the term ‘sub-efficient’ for the entire hierarchical auction. This can also be seen as the cost due to the introduction of middlemen.

4The definitions of single-minded and general valuations will be given in Section III.
the proposed hierarchical combinatorial auction models. In Section III, the resource allocation for wireless virtualization is investigated. WDP formulations are given and the corresponding solution algorithms and pricing schemes are presented. Also, theoretical analysis of the auction properties is provided. In Section IV, the auction model is extended to consider multi-seller multi-buyer case. Numerical results and analysis are presented in Section V. Section VI concludes the paper.

II. SYSTEM MODEL, ASSUMPTIONS, AND PROPOSED HIERARCHICAL COMBINATORIAL AUCTION MODELS

A. Channel Model and Assumptions

For the system model, we consider the downlink transmission of an OFDMA-based cellular system with an InP providing infrastructure services (including base stations and wireless resources) to a set of $\mathcal{M} = \{1, 2, \ldots, M\}$ MVNOs. Each MVNO $m$ then provides services to $K_m$ subscribed users in the considered cell. The InP owns a set of $C = \{1, 2, \ldots, C\}$ subchannels each with bandwidth $W$. Universal frequency reuse is considered for all cells. The base station (BS) in each cell is equipped with $A$ antennas and each user equipment has a single antenna (i.e., multi-user MIMO). We assume $A$ to be large (e.g., several hundreds) to achieve massive MIMO effect which scales up traditional MIMO by orders of magnitude.

The multi-cell system is operated in time-division duplexing (TDD) mode and we assume that all BSs and UEs are perfectly synchronized. For achieving all the benefits of massive MIMO, the base station requires channel state information for precoding. To this end, channel reciprocity is exploited. That is the downlink channel is obtained by the Hermitian transpose of the uplink channel, which can be estimated by the BS from the uplink pilots transmitted by each user. Note that by operating in TDD mode, the massive MIMO system is scalable in the sense that the time required for pilots is proportional to the number of users served per cell and is independent of the number of antennas [17]. Also, the channel responses are assumed to be invariant during the symbol time.

Massive MIMO uses spatial-division multiplexing. In this case, the BS can serve multiple users in the same time-frequency resource block. We assume that for each subchannel, the maximum number of users can be served simultaneously is $J$, which is limited by the coherence time and accordingly the number of orthogonal pilots.

The downlink received signal for a user $k$ of MVNO $m$ on subchannel $n$ in the considered cell is given by

$$s_k^n = \sqrt{p_k^n} d_k g_k h_k^T(n) f_k(n) s_k^n + \sum_{j \neq k} \sqrt{p_j^n} d_k g_k h_k^T(n) f_k(n) s_j^n + z_k^n,$$

where the first term in the right hand side of (1) denotes the desired signal for user $k$, while the second and the third terms represent the subchannel reuse interference within the cell and the interference from other cells, respectively. Specifically, $p_k^n$ is the transmit power for the link from the BS to user $k$ in subchannel $n$, $d_k$ and $g_k$ represent the path-loss and the shadowing gain, respectively. $h_k(n) \in \mathbb{C}^{A \times 1}$ denotes the small-scale fading between the BS and user $k$ on subchannel $n$ which is the Hermitian transpose of the uplink channel, where $A_m$ is the number of antennas allocated to MVNO $m$. $f_k(n) \in \mathbb{C}^{A \times 1}$ represents the precoding vector used by the BS. $s_k^n$ is the transmit signal, and $z_k^n$ denotes the additive noise with distribution $\mathcal{CN}(0, N_0)$, where $N_0$ is the noise power spectral density. Accordingly, the received signal-to-interference-plus-noise ratio (SINR) for user $k$ in subchannel $n$ can be expressed as

$$\Gamma_k^n = \frac{p_k^n d_k g_k h_k^T(n) f_k(n) h_k(n) h_k^T(n) f_k(n)}{W N_0 + I_{\text{re}use} + I_{\text{other cell}}},$$

where $I_{\text{re}use}$ and $I_{\text{other cell}}$ represent the interference terms. The ergodic achievable downlink rate for user $k$ in subchannel $n$ can be obtained as

$$r_k^n = W \mathbb{E} \left[ \log(1 + \Gamma_k^n) \right].$$

The above ergodic achievable rate is difficult to calculate for finite system dimensions. Instead, to achieve a tight approximation for finite systems, an asymptotic analysis is performed in [18] assuming that the number of antennas $A_m$ and the number of users $K_m$ approach infinity while keeping a finite ratio. Based on the results in [18], ignoring estimation noise and considering inter-user interference to be negligible compared with noise and pilot contamination in large scale multiuser (MU)-MIMO systems (the user channels decorrelate with large number of BS antennas, and strong desired signal can be received with little inter-user interference [19]), a deterministic approximation of SINR for user $k$ of MVNO $m$ in subchannel $n$ can be obtained as

$$\hat{\Gamma}_k(n) = \frac{1}{\rho_k(n) A_m + \alpha (L - 1)},$$

where $\rho_k(n)$ represents the transmit SNR, $L = 1 + \alpha (L - 1)$ and $L$ represents the number of cells, $\alpha$ represents the intercell interference factor, $\alpha (L - 1)$ is the pilot contamination caused by the reuse of pilot sequences in other cells, which primarily limits the performance of massive MIMO systems [20]. Accordingly, the approximate achievable downlink rate for user $k$ can be expressed as

$$r_k = \sum_{n \in C_m} y_k(n) W \log(1 + \hat{\Gamma}_k(n)),$$

where $C_m$ is the set of subchannels allocated to MVNO $m$, $y_k(n)$ is the assignment indicator with $y_k(n) = 1$ indicating subchannel $n$ is assigned to user $k$, and $y_k(n) = 0$ otherwise.

Note that equation (4) is obtained without assuming $A_m \gg K_m$ (i.e., assume $A_m \to \infty$ while keeping $K_m$ fixed when analyzing the SINR). Instead, it considers more practical settings where the number of BS antennas is not extremely large compared with the number of users. This is suitable for our system model since the BS antennas need to be partitioned for different MVNOs.

B. Wireless Virtualization Model

For wireless network virtualization, isolation among different virtualized wireless networks for different MVNOs is a basic requirement which can be done at different levels (e.g., flow level [21], [22] and physical resource level [5]-[8].}
In general, isolation at a higher level is simpler for implementation while at the cost of possible inefficient allocation and non-strict isolation. In contrast, isolation at a lower level could achieve better resource utilization at the cost of higher computational complexity. In this work, we consider the isolation to be performed at physical resource (i.e., subchannel, power, and antenna) level.

Also, isolation at the physical resource level can be implemented in different manners. The first is a static fixed sharing scheme with which the MVNOs are preassigned a fixed subset of physical resources in different domains, and the access is restricted within this fixed subset. The second is a dynamic general sharing scheme with which there is no restriction on the resource access, while the isolation is achieved by guaranteeing certain pre-determined requirements (e.g., minimum share of the resources). In this work, we adopt a hybrid isolation scheme in between. Specifically, the InP reserves certain amounts of resources for each MVNO according to pre-determined service agreements, while the leftover resources can be dynamically shared by all MVNOs (e.g., through auctions). Note that this model can also be applied to the case that some MVNOs own certain resources.

C. The Proposed Hierarchical Auction Models

In general, an auction process involves the following entities: a) bidders who want to buy certain commodities, b) sellers who want to sell certain commodities, and c) an auctioneer who hosts and directs the auction process. An auction mechanism mainly involves the following procedures:

- **Bidding procedure**: each bidder $i$ places a bid $b_i$ according to its own valuation $v_i$ of the item/items to be auctioned. The valuation is a private information which represents the maximum a bidder is willing to pay for the item/items. Different bidders could have different valuations for the same item/items.

- **Allocation procedure**: after collecting the bids from all participating bidders, the auctioneer needs to determine how to allocate the item/items among the bidders for achieving certain objectives. A bidder is a winning bidder if the resource requirement in her bid is satisfied.

- **Pricing procedure**: after determining the allocation, the auctioneer also needs to determine the price $q_i$ charged to each winning bidder $i$.

In this work, two hierarchical (i.e., two-level) combinatorial auction models are proposed for wireless network virtualization as shown in Fig. 1 and Fig. 2.

1) Single-seller multiple-buyer hierarchical auction model: We first consider a single-seller multiple-buyer case (i.e., the model in Fig. 1) with which the bidders (e.g., users) can only acquire resources from a single seller. The entire hierarchical auction consists of sub-auctions in two levels. Specifically, in the upper level, the InP, who owns the physical resources, holds a sub-auction and acts as the seller as well as the auctioneer. The MVNOs act as the bidders (i.e., buyers). In the lower level, each MVNO then holds a sub-auction acting as the seller and the subscribed users act as the bidders (accordingly, there are $M$ sub-auctions in the lower level).

As stated in Section II-B, we consider a hybrid isolation scheme. In this case, each MVNO $m$ reserves $C^\text{res}_m$ number of subchannels, $P^\text{res}_m$ amount of power, and $A^\text{res}_m$ number of antennas, where $\sum_{m=1}^M C^\text{res}_m \leq C$, $\sum_{m=1}^M P^\text{res}_m \leq P$, and $\sum_{m=1}^M A^\text{res}_m \leq A$. The leftover $C^\text{up} = C - \sum_{m=1}^M C^\text{res}_m$ number of subchannels, $P^\text{up} = P - \sum_{m=1}^M P^\text{res}_m$ amount of power, and $A^\text{up} = A - \sum_{m=1}^M A^\text{res}_m$ number of antennas are the commodities to be auctioned among the MVNOs in the upper level auction. Denote by $C_m \geq 0$, $P_m \geq 0$, and $A_m \geq 0$ the resources obtained by MVNO $m$ in the upper-level auction, then for each user of MVNO $m$ in the lower-level auction, the available resources are $\hat{C}_m = C^\text{res}_m + C_m$, $\hat{P}_m = P^\text{res}_m + P_m$, and $\hat{A}_m = A^\text{res}_m + A_m$.

Note that the hierarchical auction model is not a simple combination of two levels of separated auctions due to the involvement of middlemen. Specifically, unlike the users (i.e., bidders) in the lower-level auction, the MVNOs as middlemen do not have intrinsic demand\(^5\). Furthermore, the MVNOs do not have intrinsic valuations of the resources, and the valuation depends on the resale revenue which is also shown in [13]. In this case, the two level auctions are interrelated and should be studied jointly (e.g., in a way similar to that for analyzing a hierarchical game).

![Fig. 1. A single-seller multiple-buyer hierarchical auction model.](image)

In the proposed hierarchical auction model, we consider the bidders to be self-interested each of which carefully chooses the bidding strategy to maximize her own utility under the given designed auction mechanism (including allocation and pricing procedures). Specifically, we consider a quasilinear utility of the following form:

$$u_i = v_i - q_i,$$

which represents the difference between valuation and the price charged. The equilibrium bidding behavior can be analyzed by using noncooperative game theory (e.g., a Bayesian game considering that the valuations are private information). Note that there are several desirable properties when designing an auction mechanism: 1) individual rationality; 2) incentive compatibility; 3) allocation efficiency. The definitions for these are given below.

**Definition 1**: An auction mechanism is **individual rational** if for any bidder $i$, $u_i \geq 0$.

The individual rationality indicates that a bidder will never charged more than her valuation of the received resource.

**Definition 2**: An auction mechanism is **incentive compatible** (truthful) if and only if for every bidder $i$ with true valuation 

\(^5\)Note that the demand of an MVNO is not simply the sum of the demands from all users, since it may not be optimal for her.
we have \( u_i(v_i, \tilde{b}_{-i}) \geq u_i(\hat{b}_i, \tilde{b}_{-i}) \), where \( \hat{b}_i \neq v_i \) is any non-truthful bidding strategy.

That is, truth telling is a dominant strategy for each bidder no matter how other bidders place their bids.

Definition 3: An allocation is efficient if the sum of valuations of all accepted bids is maximized.

In the proposed scheme, these properties are achieved by the use of combinatorial auction [26] with appropriate design of allocation and pricing schemes. Specifically, for the sub-auction mechanism in each level, we adopt combinatorial auction which allows the bidders to express preferences over bundles or combinations of resources (e.g., bundles of sub-channels, power, and antennas for a slice). The consideration of bidding for bundles of resources stems from the fact that the value of a bundle of items may not be equal to the sum of individual value of each item in the bundle. That is, there are possible substitution and complementarity properties among the items. In this case, the combinatorial auction can lead to more efficient allocations (in terms of both auction process and outcome) compared with the case where a traditional single-item auction is repeated for each item in the bundle.

In a combinatorial auction, for each bidder \( i \), the valuation \( v_i \) is a mapping from a bundle of items \( S_i \) to a real value. A bidder is a winning bidder and receives value \( v_i(S_i) \) if all the items in the bundle \( S_i \) are allocated, otherwise the bidder receives nothing and the received value is zero (i.e., \( v_i(\emptyset) = 0 \)). The valuation function should also satisfy the monotone property. That is, for any \( S \subseteq T \) we have that \( v_i(S) \leq v_i(T) \).

Note that different from single-item auction, there could exist multiple winning bidders in a combinatorial auction.

The social welfare obtained by a combinatorial auction is denoted by

\[
V = \sum v_i(S_i),
\]

and a socially efficient allocation is an allocation with maximum social welfare among all allocations such that

\[
V^* = \max \sum v_i(S_i).
\]

In our scheme, the use of combinatorial auction can achieve social efficiency in each level if all bidders bid truthfully, while pricing schemes are designed for ensuring the incentive compatibility. The details of incentive compatible pricing schemes will be introduced in Section III.

![Fig. 2. A multiple-seller multiple-buyer hierarchical auction model.](image)

2) **Multiple-seller multiple-buyer hierarchical auction model:** We then extend the above single-seller multiple-buyer model to a multiple-seller multiple-buyer model as shown in Fig. 2. The main difference is that there exists multiple sellers at each level and each bidder could acquire resources from one of the multiple sellers. Compared with the single-seller model, the multiple-seller model provides more flexibility for bidders which accounts for service selection (or user association) and accordingly induces higher efficiency in resource allocation. For example, in the lower level, the user association is not fixed for all \( K = \sum K_m \) users. Instead, a user will be associated with the MVNO that can satisfy her resource requirement. Accordingly, the competition among MVNOs is explicitly captured in the sense that the MVNO with larger amount of resources could attract more users.

It is worth noting that different from the single-seller model, the MVNOs and InPs are not acting as the auctioneers in the multiple-seller model. Instead, service brokers are introduced as external auctioneers at each level on behalf of the sellers (i.e., the MVNOs and the InPs). The bidders in each level submits bids to the corresponding service broker who then determines the resource allocation and pricing.

### III. Wireless Network Virtualization as a Single-Seller Multiple-Buyer Hierarchical Combinatorial Auction

In this section, we will show how the proposed single-seller multiple-buyer hierarchical combinatorial auction model can be applied for virtualization of wireless resources in OFDMA-based 5G networks with massive MIMO. Specifically, we will answer the following questions: 1) how do the bidders (i.e., users in the lower-level auction and MVNOs in the upper-level auction) place their bids? 2) how to determine the set of winning bids (i.e., the winner determination problem in the context of combinatorial auction)? 3) how to solve the WDP in a computationally tractable manner? 4) how to price the winning bidders such that incentive compatibility can be achieved? Besides, theoretical analysis of the properties of the proposed hierarchical auction mechanism is also provided.

#### A. How to Place a Bid?

1) **Bids for users:** We consider two cases.

**Case I:** We consider that the users explicitly express their intrinsic physical resource demands (which is the case considered in most existing work) when applying auction schemes for wireless resource allocation. In this case, each user explicitly requests a bundle of resources \( S_k \) from her associated MVNO \( m \) with

\[
S_k = \{y_k, p_k, A_k\},
\]

where \( y_k = [y_k(n)]^{C_m} \), \( p_k = [p_k(n)]^{C_m} \), and \( A_k \) are the subchannel request vector, power request vector, and antenna request, respectively. Note that in this paper, energy efficiency is not considered. Therefore, all the available antennas of an MVNO will be activated for achieving the best performance. Accordingly, we have \( A_k = A_m \), \( \forall k \in K_m \).

We assume that the users are single-minded [24] who are only interested in a specific set of commodities, and the

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6We assume that a user will not be served simultaneously by multiple MVNOs.
valuation will be a specified scalar value if the whole set is allocated, and will be zero otherwise. The definition is given as follows:

**Definition 4:** A bidder \(k\) is single-minded if there is a set of commodities \(S_k\) and a scalar value \(a\) such that the valuation

\[
v_k(S) = \begin{cases} 
    a, & \forall S \supseteq S_k, \\
    0, & \text{otherwise}.
\end{cases}
\]  

(10)

For a single-minded user \(k\) with explicit resource demand, the bid can be simply expressed as a pair \(\{S_k, b_k(S_k)\}\). The valuation of the requested resources depends on the application type and could be different for different users. In this paper, we consider a linear function \(v_k(r_k(S_k)) = \delta_k r_k(S_k)\), where \(\delta_k\) is a constant and \(r_k(S_k)\) is the achievable rate if the requested resource bundle \(S_k\) is allocated. The consideration of valuation functions for different types of applications is straightforward and will not change the formulation and analysis. Also, the valuation function considering desirability of the resources can be designed.

**Case II:** We consider that each user \(k\) implicitly expresses her resource demand by simply indicating an intrinsic target rate of \(\hat{R}_k\). How the resources in different dimensions are allocated to satisfy the rate requirements is left with the MVNOs who have much stronger computation power.

In this case, the bid pair for user \(k\) with implicit resource demand is expressed as \(\{\hat{R}_k, b_k(\hat{R}_k)\}\) and the same valuation function applies for the implicit case.

2) Bids for MVNOs: For MVNOs, the bids are different from that of the users since the MVNOs are middlemen who do not have intrinsic demands and valuations. Accordingly, they cannot be considered to be single-minded. Instead, in the upper-level auction, each MVNO will need to submit bids for all possible set of resource bundles (i.e., general valuations\(^7\)). Also, we consider that the MVNOs explicitly express their physical resource demands to the InP, and the bid for each MVNO can be expressed as a combination of pairs \(\{S_m, b_m(S_m)\}\), \(\forall S_m \in \Omega_m\), where \(S_m = \{C_m, P_m, A_m\}\) is a tuple representing the requested number of subchannels \(C_m\), the amount of transmit power \(P_m\), and the number of antennas \(A_m\), and \(\Omega_m = \{C_m, P_m, A_m\}\) represents the set of all possible resource bundles.

Note that although an MVNO can submit combinations of bids, we adopt XOR-bid with which at most one bid can be accepted for each MVNO. For an MVNO \(m\) with auctioned resource tuple \(S_m\), let us denote by \(q_k(S_m) \geq 0\) the price charged to user \(k\) in the lower-level auction. Then the valuation of resource \(S_m\) can be expressed as

\[
v_m(S_m) = \sum_{k \in K_m} q_k(S_m) - q_m^{\text{res}},
\]  

(11)

where \(q_m^{\text{res}}\) is the cost for reserved resources.

A summary of the key elements of the hierarchical auction model is provided in Table I.

### B. How to Determine the Winning Bids?

After collecting the bids from the bidders, the auctioneer needs to determine which set of bids to be accepted. For the proposed hierarchical auction model, we need to formulate the winner determination problems (WDPs) for both InP as the auctioneer in the upper-level auction and MVNOs as auctioneers in the lower-level auctions.

1) WDP formulation for the InP: Each MVNO \(m\) submits a bid combination \(\{S_m, b_m(S_m)\}\) for all possible \(S_m \subseteq \Omega_m\) and the objective of the InP is to maximize the sum value of accepted bids. Accordingly, the WDP for the InP in the upper-level auction is formulated as

\[
\max \sum_{m \in M} \sum_{S_m \subseteq \Omega_m} b_m(S_m) x_m(S_m) \quad \text{s.t.}
\]

\[
\begin{align*}
\sum_{S_m \subseteq \Omega_m} x_m(S_m) C_m(S_m) & \leq C_{\text{up}}, \\
\sum_{S_m \subseteq \Omega_m} x_m(S_m) A_m(S_m) & \leq A_{\text{up}}, \\
\sum_{S_m \subseteq \Omega_m} x_m(S_m) P_m(S_m) & \leq P_{\text{up}}, \\
\sum_{S_m \subseteq \Omega_m} x_m(S_m) & \leq 1, \forall m \in M, \\
x_m(S_m) & \in \{0, 1\}, \forall S_m, m,
\end{align*}
\]  

(12)-(16)

where \(x_m(S_m)\) is a binary variable with \(x_m(S_m) = 1\) indicating that the requested resource bundle \(S_m\) is accepted and \(x_m(S_m) = 0\) otherwise. \(C_m(S_m), P_m(S_m),\) and \(A_m(S_m)\) represent the number of subchannels, the amount of power, and the number of antennas requested in \(S_m\). The constraints (12)-(14) ensure that the accepted resource demands do not exceed the available capacity, while the constraint (15) ensures that no bidder receives more than one set of resources.

**Remark** Since the power is divisible, having general valuations for each specific amount of power is impossible. In this case, in the upper level auction, we consider the power to be discretized based on certain unit, and each MVNO specifies the number of power units required in each resource bundle. While such discretization is not necessary in the lower-level auction since the user is single-minded.

2) WDP formulation for the MVNOs: The WDP for each MVNO \(m\) in the lower-level auction considering single-minded users with explicit resource requirements (i.e., Case

### Table I

| Demand | Valuation | Bid | Utility | Price |
|-------|-----------|-----|---------|-------|
| UEex | \(S_k\) | \(v_k(S_k)\) | \(b_k(S_k)\) | \(v_k - q_k\) | \(q_k\) |
| UEen | \(R_k\) | \(v_k(R_k)\) | \(b_k(R_k)\) | \(v_k - q_k\) | \(q_k\) |
| MVNO | N.A. | \(\sum_k q_k - q_m^{\text{res}}\) | \(\{b_m(S_m)\}\) | \(\sum_k q_k - q_m - q_m^{\text{res}}\) | q_m |
| InP | N.A. | N.A. | N.A. | \(\sum_m q_m\) | N.A. |
I) is formulated as

$$\max_{x_k} \sum_{k \in K_m} b_k(S_k) x_k$$

s.t. \( \sum_{k \in K_m} p_k(S_k) x_k \leq \hat{P}_m, \quad (17) \)

$$\sum_{k \in K_m} x_k \leq J, \quad \forall n, \quad (18)$$

$$x_k \in \{0, 1\}, \quad \forall k, \quad (19)$$

where \( x_k \) is the decision variable indicating whether the resource demand from user \( k \) can be satisfied, \( p_k(S_k) = \sum_{n \in \mathcal{C}_m} p_k(n) \) is the total power requested by user \( k \), and \( S_k \ni n \) if \( y_k(n) = 1 \). The objective is also to maximize the sum value of accepted bids, and thus achieving efficient resource allocation as required by wireless virtualization. The first constraint ensures that the sum power of all accepted bids does not exceed the available total power. The second constraint indicates that the number of users sharing a subchannel cannot exceed \( J \).

Note that the WDPs for MVNOs with explicit resource requirement from users are simpler comparing with that for the InP since the users are single-minded and therefore do not require general valuations.

Similarly, the WDP for each MVNO \( m \) considering single-minded users with implicit resource requirements (i.e., Case II) is formulated as

$$\max_{x_k, z_k(\hat{S}_k)} \sum_{k \in K_m} b_k(\hat{R}_k) x_k$$

s.t. \( r_k(\hat{S}_k(\hat{y}_k, \hat{p}_k, A_m)) = \hat{R}_k, \forall \hat{S}_k \in \Omega_k, \quad (20) \)

$$\sum_{k \in K_m} \sum_{\hat{S}_k \in \Omega_k} z_k(\hat{S}_k) \leq J, \forall n, \quad (21)$$

$$\sum_{k \in K_m} \sum_{\hat{S}_k \in \Omega_k} p_k(\hat{S}_k) z_k(\hat{S}_k) \leq \hat{P}_m, \quad (22)$$

$$z_k(\hat{S}_k) \leq 1, \forall \hat{S}_k \in \Omega_k, \quad (23)$$

$$x_k, z_k(\hat{S}_k) \in \{0, 1\}, \quad (24)$$

where \( x_k = 1 \) indicates the rate demand of user \( k \) can be satisfied, and \( x_k = 0 \) otherwise, \( \hat{S}_k = \{\hat{y}_k, \hat{p}_k, A_m\} \) is the tuple representing a resource allocation profile considering the freedoms in power, frequency, and spatial domains. Specifically, \( \hat{y}_k = [y_k(n)]_{C_m} \) is the subchannel allocation vector with \( y_k(n) = 1 \) indicating subchannel \( k \) is assigned to user \( k \), \( \hat{p}_k = [p(n)]_{C_m} \) is the power allocation vector. Equation (20) is the rate constraint which requires the resource allocation profile \( \hat{S}_k \) should satisfy the user rate requirement. The achievable rate can be calculated from (5). The set of all resource allocation profiles \( \hat{S}_k \) for user \( k \) constitutes the set \( \Omega_k \). \( z_k(\hat{S}_k) \) is an indicator with value 1 indicating the allocation profile \( \hat{S}_k \) is accepted, and 0 otherwise. Apparently \( x_k = 1 \) if there exists \( z_k(\hat{S}_k) = 1 \) for all \( \hat{S}_k \).

Note that the fairness among users is not considered.

Note that if minimum rate requirement is considered, the ‘=’ in (20) will be replaced by ‘>’. The use of minimum rate requirement will not change the analysis since it only increases the number of allocation strategies for satisfying the rate requirement. The achievable rate can be calculated from (5). The set of all resource allocation profiles \( \hat{S}_k \) for user \( k \) constitutes the set \( \Omega_k \). \( z_k(\hat{S}_k) \) is an indicator with value 1 indicating the allocation profile \( \hat{S}_k \) is accepted, and 0 otherwise. Apparently \( x_k = 1 \) if there exists \( z_k(\hat{S}_k) = 1 \) for all \( \hat{S}_k \).

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indicates that the number of users sharing a subchannel cannot exceed \( J \). Equation (23) indicates that for a user \( k \), at most one allocation profile \( S_k \in \Omega_k \) can be accepted.

Remark It can be observed that the above optimization problem is fundamental for resource allocation which jointly considers feasibility, admission control, and allocation problems in a unified framework. Specifically, given certain degrees of freedom in resource allocation, the solution of the above problem gives the answers on whether the system can satisfy the rate requirements of all users (i.e., feasibility problem). And if not, which users should be accepted (i.e., admission control problem), and how the resources can be allocated so that the sum utility of all users is maximized (i.e., allocation problem).

C. How to Solve the WDPs in the Hierarchical Auction?

Similar to solving a hierarchical game (e.g., Stackelberg game), the method of backward induction can be used for solving the proposed hierarchical auction problem. In this case, we start with solving the WDP for an MVNO \( m \) in the lower-level auction.

1) Solving the WDP in the lower-level auction: It has been shown in the literature that the winner determination problem of combinatorial auctions is an integer programming problem which is NP-hard even for the single-minded cases [32]. For simplifying the original problem, we consider two assumptions as follows:

Assumption 1: We assume the subchannels to be homogeneous for each end user (i.e., the channel gains of different subchannels are the same for a user, while they can be different for different users). Accordingly, equal power is allocated to each assigned subchannel.

Assumption 2: The achievable rate of a user is independent of which other users are sharing the same channel.

These assumptions are practical for massive MIMO systems since the small-scale fading are averaged out and only the large-scale fading (e.g., path-loss and shadowing) affects. Accordingly, the SINR is constant with respect to frequency since the slow fading coefficients are independent of frequency [20]. Also, the users’ channels decorrelate with the increasing number of BS antennas [19].

Based on these two assumptions, the WDP for MVNO with explicit user resource request can be reformulated as

$$\max_{x_k} \sum_{k \in K_m} b_k(S_k) x_k$$

s.t. \( \sum_{k \in K_m} p_k(S_k) x_k \leq \hat{P}_m, \quad (25) \)

$$\sum_{k \in K_m} c_k(S_k) x_k \leq \hat{C}_m J, \quad (26)$$

$$x_k \in \{0, 1\}, \quad \forall k, \quad (27)$$

where \( c_k \in [0, \hat{C}_m] \) and \( p_k \in [0, \hat{P}_m] \) represent the requested number of subchannels and power, respectively. Note that in this reformulation, we can use \( c_k \) and \( p_k \) instead of \( y_k \) and \( p_k \) since we consider the channels to be homogeneous. Therefore, the expression for the resource bundle requested by user \( k \) is simplified as \( S_k = \{c_k, p_k, A_m\} \).
It can be seen that the simplified problem is also an integer program which is still NP hard. Accordingly, there are fundamental tradeoffs between the social efficiency (optimality) and computational complexity. In general, there are two possible ways for solving the reformulated WDP in a computationally tractable manner. The first is to find the exact optimal solution for a small problem (e.g., through dynamic programming or branch-and-bound method) at the cost of possible high computational complexity. The second is to design low-complexity algorithms to find approximate optimal solutions for large scale problems.

We first propose a dynamic programming-based algorithm for obtaining the exact solution. The main idea is to divide the original problem WDP\(_m(K_m, P_m, C_m)\) into similar subproblems which can be solved recursively. Specifically, we partition the resource allocation into \(K_m\) stages, and denote by WDP\(_m(k, e(k))\) the subproblem which considers the resource allocation to \(k\) users with available resources \(e(k) = [e_c(k), e_p(k)]^T\), where \(e_c(k)\) and \(e_p(k)\) denote, respectively, the available subchannels and power at stage \(k\), which can also be regarded as the state variables. In each stage \(k\), the MVNO \(m\) allocates \(u_c(k)\) subchannels and \(u_p(k)\) units of power to user \(k\). Denote by \(u(k) = [u_c(k), u_p(k)]^T\). Accordingly, the state transition equation can be expressed as
\[
e(k+1) = e(k) - u(k).
\]

Also, denote by \(x^*(k, e(k)) = [x_1^*, \ldots, x_k^*]\) the optimal solution to the subproblem WDP\(_m(k, e(k))\) with the corresponding optimal value function \(f(k, e(k))\). Accordingly, we can have
\[
f(k, e(k)) = \max\{f(k - 1, e(k)), f(k - 1, e(k) - u(k)) + b_k(S_k)\}, \quad k = 2, \ldots, K_m.
\]
The initial condition is given by
\[
f(1, e(1)) = \begin{cases} b_1(S_1), & e_c(1) \geq c_1(S_1), e_p(1) \geq p_1(S_1), \\ -\infty, & \text{otherwise}. \end{cases}
\]

Note that for single-minded users, we only need to consider the state transition for fixed \(u_c(k)\) and \(u_p(k)\) at each state. Specifically, \(u_c(k) = c_k(S_k)\) and \(u_p(k) = p_k(S_k)\) if \(x_k = 1\), and \(u_c(k) = 0\) and \(u_p(k) = 0\) if \(x_k = 0\). The details are given in Algorithm 1.

Algorithm 1 A dynamic programming-based algorithm to solve the WDP for MVNO with explicit user resource request

1. Initialization: set \(c = 0\) and \(p = 0\).
2. For each state and each possible state calculate the optimal value function \(f(k, e(k))\).
3. Output: Find \(f(K_m, C_m, P_m)\) and obtain the corresponding optimal allocation in each state by using
\[
x_k = \arg\max_{x_k} \{f(k - 1, e(k)), f(k - 1, e(k) - u(k)) + b_k(S_k)\}.
\]

Compared with exhaustive enumeration with time-complexity of \(O(2^{K_m})\), the time-complexity of the dynamic programming-based algorithm is of \(O(K_m \Theta_m^2)\), where \(\Theta_m = \max\{C_m, P_m\}\). The significant reduction in time-complexity stems from the fact that the optimal value for each stage and each state is stored and calculated only once, while it needs to be calculated repeatedly in exhaustive enumeration.

We also implement a polynomial-time greedy algorithm to obtain an approximate optimal solution which is based on the algorithm proposed in [23, 28] for WDP with single-minded bidders. The main idea of the greedy algorithm is to allocate the resource to bidders with larger normalized value. Specifically, after collecting all the bids from the users, the MVNO as the auctioneer sorts the bids in a decreasing order of \(\frac{b_k}{\sqrt{\sum S_k}}\) which is viewed as the normalized value of a bid. Note that since the requested resource bundle \(S_k\) consists of multiple dimensional resources, we consider \(|S_k| = \omega_c c_k + \omega_p p_k\), which is a weighted sum of the number of different type of resources requested. The details of the greedy algorithm are presented in Algorithm 2.

Algorithm 2 A greedy algorithm to solve the WDP for an MVNO with explicit user resource request

1. Initialization: set \(c = 0\) and \(p = 0\).
2. For each submitted bid pair \(\{S_k, b_k(S_k)\}\), calculate \(b_k(S_k)/\sqrt{|S_k|}\). Re-index all bid pairs such that \(b_1(S_1)/\sqrt{|S_1|} \geq b_2(S_2)/\sqrt{|S_2|} \geq \cdots \geq b_{K_m}(S_{K_m})/\sqrt{|S_{K_m}|}\).
3. For \(k = 1 : K_m\), if \(c + c_k \leq C_m J\) and \(p + p_k \leq P_m\), then allocate \(c_k\) number of subchannels and \(p_k\) units of power to corresponding user \(k\).

Note that with this algorithm we may only obtain an approximate optimal solution while the dynamic programming algorithm can obtain the exact solution. This difference will have impacts on the choice of pricing scheme to guarantee the incentive compatibility which will be shown next.

For the WDP of MVNO with implicit user resource request, although the users are single-minded, there exist combinations of resource allocation strategies for achieving the target rate due to the freedoms in multiple domains. Accordingly, it is equivalent to a WDP formulation with general valuations. Given assumptions 1 and 2, the WDP for the MVNO with implicit user resource request can be reformulated as a WDP with general valuations as follows:

\[
\begin{aligned}
\max \sum_{k \in K_m} \sum_{S_k \in \Omega_k} b_k(\hat{R}_k) x_k(\hat{S}_k) \\
\text{s.t.} \quad r_k(\hat{S}_k(\hat{c}_k, \hat{p}_k, A_m)) = \hat{R}_k, & \forall \hat{S}_k \in \Omega_k, \\
\sum_{k \in K_m} \sum_{S_k \in \Omega_k} p_k(\hat{S}_k) x_k(\hat{S}_k) \leq P_m, & (29) \\
\sum_{k \in K_m} \sum_{S_k \in \Omega_k} e_k(\hat{S}_k) x_k(\hat{S}_k) \leq \hat{C}_m J, & (30) \\
\sum_{\hat{S}_k \in \Omega_k} x_k(\hat{S}_k) \leq 1, & (31) \\
x_k(\hat{S}_k) \in \{0, 1\}. & (32)
\end{aligned}
\]

To solve the above problem, we first need to find the set of resource allocation strategies (i.e., \(\Omega_k\)) which satisfy the rate requirement. Specifically, all the antennas available will be activated. Also, we assume the channels to be homogeneous
for a user. In this case, for each number of requested subchannels, the power required for satisfying the rate requirement can be calculated according to \[\text{(5)}.\] After obtaining the set \(\Omega_k\), we extend the previous dynamic programming algorithm and greedy algorithm for single-minded bidders to accommodate the general valuation case.

Specifically, for the dynamic programming algorithm, the main difference is that when calculating the optimal value function, it is required to consider all possible state transitions due to the general valuations. Specifically,

\[
f(k, \mathbf{e}) = \max_{\mathbf{u}(k)} \{b_k(\mathbf{u}(k)) + f(k-1, \mathbf{e}(k) - \mathbf{u}(k))\},
\]

where \(u_c(k) \in [0, \bar{C}_m]\) and \(u_p(k) = p_k(u_c(k))\).

To extend the greedy algorithm for the WDP with general XOR bid, we consider the bid combinations \(\{\tilde{S}_k, b_k(\tilde{S}_k)\} \forall \tilde{S}_k \in \Omega_k\) submitted by user \(k\) as combinations of bid pair \(\{\tilde{S}_k, b_k(\tilde{S}_k)\}\) submitted by virtual single-minded bidders the number of which is equal to the number of all possible bid combinations. For example, there exists \(|\Omega_k|\) number of virtual bidders with user \(k\). Note that due to the XOR bid, at most one virtual single-minded bidder of each user can be accepted. To address this problem, we introduce the concept of virtual commodity corresponding to a user. The bid combinations of user \(k\) are extended as \(\{\tilde{S}_k \bigcup k, b_k(\tilde{S}_k)\}\). Since the virtual commodity \(k\) can only be allocated to one winning bidder, the original XOR bid can be realized. The extended greedy algorithm is given in Algorithm 3.

**Algorithm 3**

A greedy algorithm to solve the WDP for an MVNO with implicit user resource request

1. Initialization: set \(c = 0\), \(p = 0\) and \(x_k = 0\) for each user \(k\).
2. For each submitted bid pair \((\tilde{S}_k, b_k(\tilde{S}_k))\), calculate \(b_k(\tilde{S}_k)/\sqrt{|\tilde{S}_k|}\). Re-index all bid pairs such that

\[
\frac{b_1(\tilde{S}_1)}{\sqrt{|\tilde{S}_1|}} \geq \frac{b_2(\tilde{S}_2)}{\sqrt{|\tilde{S}_2|}} \geq \ldots \geq \frac{b_T(\tilde{S}_T)}{\sqrt{|\tilde{S}_T|}},
\]

where \(T = \sum_k |\Omega_k|\).
3. For \(k = 1 : T\), if \(c + c_k(\tilde{S}_k) \leq \bar{C}_mJ\), \(p + p_k(\tilde{S}_k) \leq \bar{P}_m\), and \(x_k = 0\), then allocate \(c_k(\tilde{S}_k)\) number of subchannels and \(p_k(\tilde{S}_k)\) amount of power to corresponding user \(k\) and set \(x_k = 1\).

2) **Solving the WDP in the upper-level auction:** Compared with users in the lower-level auction, the MVNOs are not single-minded which results in combinations of XOR bids. Note that the valuation of each resource bundle in the upper-level auction depends on the resale gain in the lower-level auction, which can be obtained by solving the lower-level auction supposing this resource bundle is allocated. After obtaining the valuations of all resource bundles, similar algorithms for solving the WDP with implicit resource request can be applied. For example, with the dynamic programming-based algorithm, the resource allocation is partitioned into \(M\) stages, and the optimal value function of each stage is

\[
f(m, \mathbf{e}(m)) = \max_{\mathbf{u}(m)} \{b_m(\mathbf{u}(m)) + f(m-1, \mathbf{e}(m) - \mathbf{u}(m))\},
\]

with \(\mathbf{e}(m) = [e_c(m), e_p(m), e_v(m)]^T\) and \(\mathbf{u}(m) = [u_c(m), u_p(m), u_v(m)]^T\), where \(u_c(m) \in [0, \bar{C}_m]\), \(u_p(m) \in [0, \bar{P}_m]\), and \(u_v(m) \in [0, \bar{A}_m]\).

Note that in a practical situation the number of MVNOs may not be large (e.g., \(m = 3, 4\)). Also, it is reasonable that the InP sells the resources to MVNOs only in a grouped manner (e.g., a group of 5 or 10 subchannels). Furthermore, the InP could impose restrictions on the maximum allowable groups of subchannels each MVNO can bid for. Accordingly, the bid combinations can be significantly reduced and the complexity of finding the exact optimal solutions can be reduced. Note that, apparently, restricting the bid combinations would also incur a tradeoff between the computational complexity and social efficiency.

### D. How to Price the Winning Bidders?

1) Pricing scheme with exact solution for the WDP: The design of pricing scheme of is of critical importance for achieving incentive compatibility. With all bidders bidding truthfully, the above WDPs which aim to maximize the sum of accepted bids can also achieve social optimality at each level since \(b_k = v_k\).

For single-commodity auctions, the second-price auction (i.e., Vickrey auction \[\text{(29)}\]) has been shown to be an incentive compatible scheme with which the winning bidder (with highest bid) pays the second highest bid. The VCG scheme \[\text{(29)}-\text{(31)}\], as a generalization of the second-price auction to multiple commodities, preserves the incentive compatibility. The intuitive idea of VCG pricing is that a bidder should pay the potential loss they impose to other bidders. Specifically, with VCG pricing, a bidder \(k\) will be charged

\[
q_k^{\text{VCG}} = \sum_{j \neq k} v_j(S_j^*) - \sum_{j \neq k} v_j(S_k^*),
\]

where \(S_j^*\) and \(S_k^*\) represent, respectively, the resources obtained by bidder \(j\) when bidder \(k\) is not participating and is participating. For a winning bidder, the VCG price is the decrease of welfare of all other bidders caused by her presence. Note that the VCG prices are nonnegative.

Although the VCG pricing can achieve incentive compatibility, it is not designed for maximizing the seller’s revenue. In some cases, the resulting revenue (e.g., \(\sum q_k^{\text{VCG}}\)) can be even far from the optimal one. For example, if there are sufficient resources such that the requirements of all bidders can be satisfied, the VCG price for each bidder is zero. This could cause problem for MVNOs whose valuations depend on the revenue gained from resale. Specifically, with VCG pricing, the valuation \(v_m(S_m)\) could decrease with an increasing number of resources in \(S_m\) which may motivate the MVNO to lease less resources.

To address this problem while guaranteeing the incentive compatibility, we jointly use the VCG pricing with a base access price. Specifically, each type of resource has a base access price, and a user who is admitted (i.e., a successful
bidder) will be charged the larger of the base price and the VCG price. That is
\[
q_k = \max\{q_k^{\text{base}}, q_k^{\text{vcg}}\}. \tag{36}
\]

Note that the base price is known to all users. If a bidder is aware that the valuation of the requested resources is even less than the base price, she will not place the bid. In this pricing scheme, the base price can guarantee certain revenue for the MVNO, while the VCG price could represent the impact of satisfying the resource requirement of a user to the other users. For example, if there are sufficient resources, the VCG price for a bidder can be low. However, when there is an intense resource competition, the more resource a user requests, the higher will be the probability that other users will not be admitted, and the higher will be the VCG price charged to this user.

Although the objective of each auctioneer is to maximize the social welfare, with such a pricing scheme, we can also achieve approximate optimal seller’s revenue. Also, the incentive compatibility can be preserved.

2) Pricing scheme with approximate solution for the WDP:
The VCG price can preserve incentive compatibility only if the WDP is solved exactly (i.e., optimal solution is obtained), while it is incompatible with approximate algorithms in general [24]. That is, if we obtain sub-optimal \( S^* \) and \( S^* \) in using approximate algorithms, the corresponding VCG price \( q_k \) is not incentive compatible.

Accordingly, the corresponding pricing scheme needs to be designed for the greedy algorithm. Specifically, we propose to jointly use the base access price with a VCG-like pricing. We first give the definition of blocking as follows:

**Definition 5:** Assume bidder \( k \) with bid \( b_k \) is a winning bidder while a bidder \( j \) with bid \( b_j \) is not accepted. Then the bidder \( k \) uniquely blocks bidder \( j \) if the bidder \( j \) is a winning bidder without bidder \( k \)’s participation in the auction.

Denote by \( B_k \) the set of bidders blocked by bidder \( k \). The main idea is to charge bidder \( k \) according to the highest value bid it blocks. Note that this value is also normalized as that in the greedy algorithm. The VCG-like price for a winning bidder \( k \) is
\[
q_k^{\text{greedy}} = \max_{j \in B_k} \frac{b_j}{\sqrt{|S_j| \sqrt{|S_k|}}}. \tag{37}
\]

Accordingly, the winning bidder will be charged the larger of the base access price and the VCG-like price as follows:
\[
q_k = \max\{q_k^{\text{base}}, q_k^{\text{greedy}}\}. \tag{38}
\]

These two pricing schemes can be applied in both upper level and lower level auctions in accordance with the solving algorithms used.

**E. Analysis of Properties of the Proposed Hierarchical Auction Mechanism**

In this part, we analyze the properties of the proposed hierarchical auction mechanism. We first show that the proposed mechanism can achieve individual rationality.

**Theorem 3.1:** The proposed hierarchical auction mechanism is individual rational for all truthful bidders in both upper and lower level auctions.

**Proof** The VCG scheme together with exact WDP solving algorithms has been shown to be individual rational [31]. For the greedy algorithm with corresponding pricing scheme, we consider two cases. First, if there is no bidder blocked by a winning bidder \( k \) (i.e., \( B_k = \emptyset \)), then \( q_k = \max\{q_k^{\text{base}}, 0\} = q_k^{\text{base}} \). Then, \( b_k \geq q_k^{\text{base}} \) and accordingly \( u_k \geq 0 \). Second, if the set \( B_k \neq \emptyset \), according to the pricing scheme, the price charged for bidder \( k \) is
\[
q_k = \max\{q_k^{\text{base}}, \max_{j \in B_k} \frac{b_j}{\sqrt{|S_j| \sqrt{|S_k|}}} \}. \tag{39}
\]

While according to the allocation scheme, we have
\[
b_k \geq \max_{j \in B_k} \frac{b_j}{\sqrt{|S_j| \sqrt{|S_k|}}} \tag{40}
\]

Accordingly, we can have \( b_k \geq q_k \) and \( u_k \geq 0 \).

We also show the property of allocation efficiency in the following theorem.

**Theorem 3.2:** With the proposed dynamic programming algorithms for exact solution of the WDPs, the proposed hierarchical auction mechanism achieves allocation efficiency with truth-telling bidders at each level.

This result can be obtained immediately from the WDP formulation which aims to maximize the sum of accepted bids. Note that the property of allocation efficiency is not preserved for the entire hierarchical auction which will be shown in the numerical results. Also, similar observation was made in [13].

In the following, we will analyze the incentive compatibility of the proposed mechanism. To this end, we first introduce the concepts of monotone and critical value as follows:

**Definition 6:** The allocation scheme of an auction is monotone if a bidder \( k \) with bid \( b_k(S_k), S_k \) is a winning bidder, then all bidders \( j \) with \( b_j(S_j), S_j \geq b_k(S_k), S_k \) are also winning bidders.

The notation \( \succeq \) denotes the preference order bid pairs. Specifically, \( \{b_j(S_j), S_j\} \succeq \{b_k(S_k), S_k\} \) if \( b_j(S_j) \geq b_k(S_k) \) for \( |S_j| = |S_k| \) or \( |S_j| \leq |S_k| \) for \( b_j(S_j) = b_k(S_k) \).

The monotonicity indicates that the chance for obtaining a required bundle of resources can only be increased by either increasing the bid or decreasing the amount of resources required.

**Definition 7:** For a given monotone allocation scheme, there exists a critical value \( \hat{q}_k \) of each bid pair \( b_k(S_k), S_k \) such that \( \forall b_k \geq \hat{q}_k \) will be a winning bid, while \( \forall b_k < \hat{q}_k \) is a losing bid.

The critical value can be seen as the minimum a bidder has to bid for obtaining the requested bundle of resources. With the concepts of monotonicity and critical value, we have the following lemma [33]:

**Lemma 3.3:** An auction mechanism is incentive compatible if the allocation scheme is monotone and each winning bidder pays the critical value.

The VCG scheme has been shown to be incentive compatible [31] for allocation algorithms which solve the WDP exactly. Accordingly, we can have the following theorem.

**Theorem 3.4:** For the sub-auction at each level of the proposed hierarchical auction, the mechanism consisting of proposed dynamic programming algorithm for WDP and VCG pricing with base access price achieves incentive compatibility (for both single-minded and general valuation cases).
We will show that for single-minded users with explicit resource request, the greedy algorithm with corresponding designed pricing scheme can also achieve incentive compatibility in the lower-level auction.

Theorem 3.5: The proposed auction mechanism with greedy algorithm is incentive compatible with any combination of incentive valuations. Although there exist schemes which preserve incentive compatibility in the pricing scheme, the corresponding auction mechanism is incentive compatible.

Proof The proof of monotonicity can be immediately obtained from the allocation algorithm. Specifically, a bidder can increase her order in the ranking by either increasing the bid value or reducing the amount of required resources. For example, two users requesting the same number of subchannels and power, the user with higher achievable rate will be preferred.

Then we will find the critical value which is the minimum a bidder has to bid to win the requested bundle of resource. Denote by $j$ the blocked bidder with highest normalized valuation who would win if bidder $k$ is not participating in the auction. Accordingly, the minimum bid the bidder $k$ needs to place is $\frac{y_j}{\sqrt{|S_j|}}$, which is just the payment of bidder $k$ in the pricing scheme.

With monotonicity and critical payment property, it is straightforward that the incentive compatibility can be achieved according to Lemma 3.3.

However, note that the result on incentive compatibility for the greedy algorithm and pricing is only valid for single-minded cases and it will not hold for bidders with general valuations. Although there exist schemes which preserve incentive compatibility for general valuations, the worst-case performance is much inferior than that for the greedy algorithm.

Regarding the incentive compatibility of the entire hierarchical auction mechanism, we have the following theorem.

Theorem 3.6: The proposed hierarchical auction mechanism is incentive compatible with any combination of incentive compatible sub-auctions at each level.

The proof is similar to that in [13] and is omitted here.

IV. EXTENSION TO A MULTIPLE-SELLER MULTIPLE-BUYER HIERARCHICAL AUCTION MODEL

We now extend the single-seller multiple-buyer hierarchical auction to a multiple-seller multiple-buyer model. In this case, the users are not restricted to only one provider but can freely choose among several MVNOs. Similarly, the MVNOs can choose from different InPs.

A. WDP Formulations

The bids are the same as that in the single-seller model. The WDP formulation for the service broker as the auctioneer in the upper-level auction is expressed as

$$\max_{x_m(S_m)} \sum_{i \in \mathcal{I}} \sum_{m \in \mathcal{M}} \sum_{S_m} b_m(S_m)x_{im}(S_m)$$

s.t. $\sum_{S_m} x_{im}(S_m) C_m(S_m) \leq C_i - \sum_{m \in \mathcal{M}} C_{res}^{im}, \forall i$,

$\sum_{S_m} x_{im}(S_m) A_m(S_m) \leq A_i - \sum_{m \in \mathcal{M}} A_{res}^{im}, \forall i$,

$\sum_{S_m} x_{im}(S_m) P_m(S_m) \leq P_i - \sum_{m \in \mathcal{M}} P_{res}^{im}, \forall i$,

$x_{im}(S_m) \leq 1, \forall m \in \mathcal{M},$

$x_{im}(S_m) \in \{0, 1\}, \forall S_m, m, i,$

where $\mathcal{I}$ is the set of InPs and $x_{im}(S_m) = 1$ indicates the resource request for $S_m$ from MVNO $m$ to InP $i$ is accepted. Note that the MVNO can only lease resources from one of the InPs.

Similarly, the WDP for the service broker in the lower-level auction with explicit user resource request is expressed as:

$$\max_{x_m(S_m)} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} b_k(S_m)x_{mk}$$

s.t. $\sum_{k \in \mathcal{K}} p_k(S_m)x_{mk} \leq \bar{P}_m, \forall m$,

$\sum_{k \in \mathcal{K}} c_k(S_m)x_{mk} \leq \bar{C}_m J_m, \forall m$,

$\sum_{m \in \mathcal{M}} x_{mk} \leq 1, \forall k$,

$x_{mk} \in \{0, 1\}, \forall m, k,$

where $\mathcal{K} = \bigcup K_m$.

The WDP for the service broker in the lower-level auction with implicit user resource request is expressed as:

$$\max_{x_m(S_m)} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} \sum_{S_m \in \Omega_{mk}} b_k(\tilde{S}_m)x_{mk}(\tilde{S}_m)$$

s.t. $r_k(\tilde{S}_m) = R_k, \forall \tilde{S}_m \in \Omega_{mk},$

$\sum_{k \in \mathcal{K}} p_k(\tilde{S}_m)x_{mk}(\tilde{S}_m) \leq \bar{P}_m, \forall m,$

$c_k(\tilde{S}_m) \leq \bar{C}_m, \forall m,$

$\sum_{k \in \mathcal{K}} c_k(\tilde{S}_m)x_{mk}(\tilde{S}_m) \leq \bar{C}_m J_m, \forall m,$

$\sum_{m \in \mathcal{M}} x_{mk}(\tilde{S}_m) \leq 1, \forall k,$

$x_{mk}(\tilde{S}) \in \{0, 1\}$.  

B. Allocation and Pricing Schemes

The above WDPs are equivalent to multiple multidimensional knapsack problems. For such problems, the dynamic programming-based methods require huge memory [14]. In this case, branch-and-bound method can be applied to find the exact solutions. The key challenge for applying branch-and-bound approaches is to find a tight upper bound of the problem with which then standard branch-and-bound algorithms can be
used (e.g., [35]). Therefore, in this part, we will focus on the derivation of the upper bound of the WDPs.

We use the WDP in the lower-level auction with explicit user resource request as an example. To obtain the upper bound, surrogate relaxation is used.\footnote{Note that continuous relaxation is not used here since the upper bound obtained by continuous relaxation is dominated by the bound obtained by surrogate relaxation.} Given a set of positive vector of multipliers \( \pi \), the standard surrogate relaxation of the original WDP problem can be expressed as

\[
\begin{align*}
\max_{x_{mk}} & \quad S = \sum_{m \in M} \sum_{k \in K} b_k(S_k) x_{mk} \\
\text{s.t.} & \quad \sum_{m \in M} \pi_m \sum_{k \in K} p_k(S_k) x_{mk} \leq \sum_{m \in M} P_m, \\
& \quad \sum_{m \in M} \pi_m \sum_{k \in K} c_k(S_k) x_{mk} \leq \sum_{m \in M} \hat{C}_m J_m, \\
& \quad \sum_{m \in M} x_{mk} \leq 1, \quad \forall k, \\
& \quad x_{mk} \in \{0,1\}, \quad \forall m,k.
\end{align*}
\]

Denote by \( \hat{\pi} \) the optimal value of the above relaxed problem. Accordingly, \( \hat{\pi} \) is an upper bound of the original problem for arbitrary nonnegative multiplier vector of \( \pi \). To achieve a tight upper bound, the optimal multiplier should be chosen such that \( S \) is minimized. That is

\[
\pi^* = \arg \min_{\pi} S(\pi).
\] (41)

Compared with Lagrangian relaxation the optimal multipliers of which can only be obtained numerically (e.g., through subgradient methods), the optimal value of the multipliers for surrogate relaxation for the formulated problem can be easily obtained as shown in the following lemma [35].

**Lemma 4.1:** For any instance of multiple knapsack problem, the optimal vector of multipliers for \( S(\pi) \) is \( \pi^*_m = \zeta \) for all \( m \), where \( \zeta \) is any positive constant.

With the optimal vector of multipliers, the relaxed problem becomes

\[
\begin{align*}
\max_{x_k} & \quad \sum_{k \in \mathcal{K}} b_k \hat{x}_k \\
\text{s.t.} & \quad \sum_{k \in \mathcal{K}} p_k(S_k) \hat{x}_k \leq \sum_{m \in M} P_m, \\
& \quad \sum_{k \in \mathcal{K}} c_k(S_k) \hat{x}_k \leq \sum_{m \in M} \hat{C}_m J_m, \\
& \quad \hat{x}_k \in \{0,1\}, \quad \forall k.
\end{align*}
\]

Such relaxation can be viewed as considering only one knapsack with larger capacity. A tight upper bound can be obtained by solving this relaxed problem, for which the previously proposed dynamic programming algorithm can be used considering the similarity in problem structure. Also, based on the algorithm provided in [35], we can have a low-complexity polynomial-time (of \( O(n^2) \)) heuristic algorithm for obtaining an approximation solution. The details can be found in Appendix B.

For the pricing, since branch-and-bound approach can obtain the exact solution of the WDP problems, the previous joint use of VCG pricing and base access price can be applied here which preserves the incentive compatibility, individual rationality, and allocation efficiency in each level. Similarly, corresponding pricing schemes for approximate solution can be applied.

V. PERFORMANCE EVALUATION

For numerical analysis, we consider a hexagonal system with \( L = 7 \) cells. We consider that an InP owns \( C = 100 \) subchannels and the BS of the InP is equipped with \( A = 200 \) antennas. Also, the total transmit power of the BS is equally divided into 500 power units. There are two MVNOs each of which reserves 30 subchannels, 150 power units, and 50 antennas (i.e., \( C^\text{res} = 30, P^\text{res} = 150, \) and \( A^\text{res} = 50 \)), and the leftover resources are available for auction in the upper level. Each MVNO has 50 subscribed users. And in the simulation, each user requests \( p \) units of power and \( c \) subchannels, where \( p \) and \( c \) are integer random variables uniformly distributed in the interval \([0, 10]\), and \([0, 2]\), respectively. All the results are obtained by averaging over 1000 simulation runs.

For solving the single-seller multiple-buyer hierarchical combinatorial auction problem, we adopt both dynamic programming algorithm and greedy algorithm. For convenience, we use the terms ‘DPA’ and ‘GA’ to represent the use of dynamic programming-based algorithm and the use of greedy algorithm in both levels, respectively.

For comparison purpose, we consider a fixed sharing scheme, where each MVNO reserves half of the resources. This fixed sharing can also be viewed as the case where there is no wireless virtualization for resource sharing (e.g., each MVNO has its own infrastructure and fixed resources). Accordingly, the hierarchical auction degenerates to single lower-level combinatorial auctions held by each MVNO. Also, we consider a general sharing scheme as the benchmark, where the MVNOs are not involved and the InP directly holds a single-level combinatorial auction for resource allocation. We will use the terms ‘FS’ and ‘GS’ to represent the results obtained by fixed sharing and general sharing, respectively. We also solve the multiple-seller multiple-buyer problem and compare the results with those obtained for the single-seller model. For fair comparison, we consider the same setting as that in the single-seller model while allowing the users to access the services from one of the MVNOs. We will use the term ‘MS1’ and ‘MS2’ to represent the exact and approximate solutions obtained for the multiple-seller problem, respectively.

For numerical analysis, we mainly consider three performance metrics for resource allocation: average social welfare (i.e., the sum value of all accepted bids), average resource utilization (i.e., the proportion of resources utilized), and average user satisfaction (i.e., the ratio of users whose resource requests are satisfied).

We first consider the average social welfare achieved by different algorithms as shown in Fig. 3. It is straightforward that general sharing (GS) provides the largest social welfare which is used as the benchmark. For dynamic programming algorithm, we consider two group sizes (i.e., the resources is auctioned in a grouped manner). There are tradeoffs in selecting the group size in terms of performance and complexity. We can see that the DP-Algorithm with group size 1
(i.e., DPA1) outperforms that with group size 5 (i.e., DPA2). We can also observe that the social welfare obtained by DPA1 is less than that obtained by general sharing. This indicates that although the social efficiency can be achieved for each level, there could exist a gap between the social welfare obtained by the entire hierarchical auction with the global optimal one obtained by general sharing. This gap represents the tradeoff between global social efficiency and the flexibility of intra-slice customization. Also, we can see that the greedy algorithm provides good solutions compared with the dynamic programming-based algorithm. We can also observe that the welfare obtained for multiple-seller setting is larger than that for single-seller problem. This is due to the flexibility introduced by allowing dynamic user association. We can observe that all the proposed dynamic resource sharing schemes (i.e., DPA, GA, and MS) outperform the fixed sharing scheme. This indicates the resource utilization gain by having wireless virtualization.

![Normalized average social efficiency](image)

Fig. 3. Normalized average social efficiency achieved by different algorithms. "FS" = Fixed sharing, "DPA1" = DP-Algorithm with group size 1, "DPA2" = DP-Algorithm with group size 5, "GA" = Greedy algorithm, "GS" = General sharing, "MS1" = Multiple seller exact solution, "MS2" = Multiple seller approximate solution.

We then investigate the average resource utilization achieved by different algorithms as shown in Fig. 4. Here we use the subchannel utilization as an example. The comparison results are similar to those for the social welfare.

![Average subchannel utilization](image)

Fig. 4. Average subchannel utilization achieved by different algorithms.

We also compare the average user satisfaction obtained by different algorithms and compare the results for explicit resource request case and implicit resource request as shown in Fig. 5. We can see that the implicit resource request model can achieve better user satisfaction. This is due to the benefits from general valuation compared with single-minded model. Specifically, for implicit resource request, these exist multiple resource allocation strategies to achieve the target rate, and if anyone of these strategies is accepted, the user request is satisfied.

![Average user satisfaction](image)

Fig. 5. Average user satisfaction achieved by different algorithms.

The impact of different number of MVNOs on the average resource utilization is investigated in Fig. 6. Note that the total number of resources of the InP and the total number of users are fixed for different number of MVNOs. We can observe that the utilization decreases with the increased number of MVNOs. This is due to the fact that increasing the number of MVNOs will decrease the number of resources allocated as well as the number of users associated to each MVNO, and the statistical multiplexing gain will decrease.

![Average resource utilization](image)

Fig. 6. Average resource utilization achieved by different algorithms with different number of MVNOs.

VI. CONCLUSION

We have proposed a hierarchical combinatorial auction mechanism to jointly address the two-level resource allocation problem for virtualization of massive MIMO-based 5G cellular networks. Specifically, we have considered both single-seller and multiple-seller hierarchical models, and the corresponding winner determination problems in two levels have been formulated and studied. Algorithms have been proposed to solve the WDPs in a computationally tractable manner and different pricing schemes have been designed. The properties of the proposed hierarchical auction mechanism have also been analyzed. The proposed scheme satisfies the requirements of efficient resource allocation, strict inter-slice isolation, and the ability of intra-slice isolation. Numerical results have been presented which show the effectiveness of the proposed scheme. For the future work, a valuation function taking into account the fairness and desirability can be designed.
TABLE II
A SUMMARY OF MAIN DIFFERENCES OF PROPOSED SCHEME WITH EXISTING WORK

| Schemes | Differences of the model | Pricing scheme | Multi-seller case | Dimensions of auctioned items |
|---------|--------------------------|----------------|------------------|-----------------------------|
| [13]    | There is a social planner controlling the resource allocation in all tiers | First price or VCG-Pricing | No | Single dimension |
| [14]    | A middleman can submit bids for multiple items but win at most one | Designed pricing with reserve price in the upper-tier, the same price charged to SUs in the lower-tier | No | Single dimension |
| [15]    | Multi-layered spectrum trading through an auction in the upper level and a price demand method in the lower level | VCG pricing and two variants of uniform pricing | No | Single dimension |
| Proposed | Multi-item combinatorial auction at both tiers | VCG pricing or VCG-like pricing with reserved price | Yes | Multiple dimensions |

APPENDIX A: SUMMARY OF MAIN DIFFERENCES OF PROPOSED SCHEME WITH EXISTING WORK

The main differences between existing work and the proposed scheme is shown in Table 1.

APPENDIX B: HEURISTIC ALGORITHM FOR SOLVING THE MULTI-SELLER MULTI-BUYER PROBLEM

We provide a low-complexity (of $O(n^2)$) heuristic algorithm by extending the algorithm in [35] for single dimensional items to multiple dimensions. Specifically, in the initialization phase, we re-index all users and all MVNOs. Then an initial feasible solution can be obtained by applying the greedy algorithm. After obtaining the initial solution, the algorithm improves the solution through local exchanges. The details of the algorithm are described as follows.

Algorithm 4 Initialization

1. Set $|S_k| = \omega_c c_k + \omega_p p_k$, $\forall k$;
2. Re-index all users such that
   \begin{equation}
   \frac{b_1(S_1)}{|S_1|} \geq \frac{b_2(S_2)}{|S_2|} \geq \cdots \geq \frac{b_K(S_K)}{|S_K|};
   \end{equation}
3. Set $\hat{C}_m = \omega_c \hat{C}_m + \omega_p \hat{P}_m$, $\forall m$;
4. Re-index all MVNOs such that
   \begin{equation}
   \hat{C}_1 \leq \hat{C}_2 \leq \cdots \leq \hat{C}_M.
   \end{equation}

Algorithm 5 Greedy Algorithm

Denote by $x_k$ the assignment index. Specifically, $x_k = 0$ if user $k$ is currently unassigned, and $x_k$ equals the index of the MVNO it is assigned to, otherwise.

Initialization: set $x_k = 0$, $\forall k$; $\hat{C}_m = \hat{C}_m$, $\forall m$ and $\hat{P}_m = \hat{P}_m$, $\forall m$;
for $k := 1$ to $K$ do
  if $x_k = 0$, $p_k \leq \hat{P}_m$, and $c_k \leq \hat{C}_m$ then
    $x_k := m$;
    $\hat{C}_m = \hat{C}_m - c_k$; \quad $\hat{P}_m = \hat{P}_m - p_k$;
  end if
end for

Algorithm 6 Initial Solution

$z := 0$;
for $k := 1$ to $K$ do
  $x_k := 0$;
end for
for $m := 1$ to $M$ do
  Call Greedy Algorithm
end for

Algorithm 7 Rearrangement

$z := 0$; $m := 1$;
for $k := K$ to 1 do
  if $x_k > 0$ then
    let $l$ be the first index in $\{m, \ldots, M\} \cup \{1, \ldots, m - 1\}$ such that $c_k \leq \hat{C}_l$ and $p_k \leq \hat{P}_l$;
    if no such $l$ then
      $x_k := 0$
    else
      $x_k := l$; \quad $\hat{C}_l = \hat{C}_l - c_k$; \quad $\hat{P}_l = \hat{P}_l - c_k$; \quad $z := z + b_k$;
    end if
  end if
  if $l < m$ then
    $m := l + 1$;
  else
    $m := 1$;
  end if
end for
for $m := 1$ to $M$ do
  Call Greedy Algorithm
end for

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Algorithm 8 First improvement

for $k := 1$ to $K$
do
  if $x_k > 0$
do
    for $j := k + 1$ to $K$
do
      if $0 < x_j 
eq x_k$
do
        $h := \arg \max \{c_j, c_k\}$; $l := \arg \min \{c_j, c_k\}$;
        $d_c := c_h - c_l$; $d_p := p_h - p_l$;
        if $d_c \leq C_{x_k}$ and $C_{x_k} \geq \min\{w_u : x_u = 0\}$
do
          $t := \arg \max \{b_h : x_u = 0 \text{ and } w_u \leq C_{x_h} + d_c\}$;
          $C_{x_h} := C_{x_h} + d_c - c_t$; $C_{x_k} := C_{x_k} - d_c$
          $P_{x_h} := P_{x_h} + d_p$; $P_{x_k} := P_{x_k} - d_p$
          $x_h := x_h$; $x_k := x_k$; $x_l := x_l$; $z := z + b_t$
        end if
      end if
    end for
  end if
end for

Algorithm 9 Second improvement

for $k := 1$ to $K$
do
  if $x_k > 0$
do
    $C := C_{x_k} + c_k$; $\tilde{P} := P_{x_k} + p_k$; $Y := \emptyset$
  end if
  for $j := 1$ to $K$
do
    if $x_k = 0, c_k \leq C$, and $p_k \leq \tilde{P}$
do
      $Y := Y \cup j$; $C := C - c_k$; $P := \tilde{P} - p_k$
    end if
  end for
  if $\sum_{y \in Y} b_y > b_k$
do
    for each $k \in Y$
do
      $x_l := x_k$; $C_{x_k} := C$; $P_{x_k} := \tilde{P}$; $x_k := 0$
      $Z := Z + \sum_{j \in Y} b_j - b_k$
    end for
  end if
end for

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