On Vortices and Phase Coherence in High $T_c$ Superconductors

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We show that an array of Josephson coupled Cooper paired planes can never have long range phase coherence at any finite temperature due to an infrared divergence of phase fluctuations. The phase correlations decay in a slow enough manner providing enough local phase coherence as to make possible the nucleation of vortices. The planes then acquire Kosterlitz-Thouless topological order with its intrinsic rigidity and concomitant superfluidity. We thus conclude that the high temperature superconducting cuprates are topologically ordered superconductors rather than phase ordered superconductors. For low enough superfluid densities, as in the underdoped cuprates, the transition temperature, $T_c$, will be proportional to the superfluid density corresponding to vortex-antivortex unbinding, and not to disappearance of the Cooper pairing amplitude. Above $T_c$, but below the BCS pairing temperature $T_p$, we will have a dephased Cooper pair fluid that is a vortex-antivortex liquid. The AC and DC conductivities measured in this region are those corresponding to flux flow. Furthermore there will be vortices above $T_c$ which will lead to Nernst vortexlike response and there will be a measurable depairing field $H_{c2}$ above $T_c$ as evidenced by recent experiments.

In the present letter we propose that the high $T_c$ superconducting cuprates are topologically ordered superconductors exactly like the two dimensional (2D) Kosterlitz-Thouless superfluids and not long range phase ordered superconductors like regular 3D bulk superconductors. We show that thermal phase fluctuations destroy phase coherence for even an infinite array of Josephson coupled superconducting planes. One could naturally conclude that such systems cannot be superfluid. This conclusion would be erroneous as the planes can acquire the rigidity necessary for supporting persistent supercurrents through the topological order originally discovered by Kosterlitz and Thouless.

High-temperature superconductors exhibit the pseudogap phenomenon. In underdoped cuprates, the transition temperature, $T_c$, out of the superconducting state is proportional to the superfluid density, $n_s$. This is consistent with the transition being an order parameter phase instability and not a pairing amplitude instability as in conventional BCS superconductors.

The superconducting cuprates are quite anisotropic materials. In the continuum approximation, a 3D superconductor cannot lose long range phase coherence due to thermal phase fluctuations (see Appendix) regardless of anisotropy and superfluid density. In 3D, superconductors can lose phase coherence if the Cooper pairs have a Bose condensation temperature, $T_c$, which is lower than the pairing temperature, $T_p$. This would predict the relation $T_c \sim n_s^{2/3}$ contrary to experiment.

The superconducting cuprates consist of Copper-Oxygen planes well separated in space where all the action happens. They are layered superconductors described as an array of discrete Josephson coupled planes: a discrete Lawrence-Doniach model. We obtain that such an array of coupled planes does not have long range phase coherence (no ODLRO) at any finite temperature (see Appendix) due to an infrared divergence of thermal phase fluctuations. This could lead one to conclude that superconductivity is impossible for such a system as the Meissner effect and infinite conductivity are consequences of the phase rigidity of the ordered ground state.

On the other hand, in 2D there also cannot exist off-diagonal long range order (ODLRO) at any finite temperature due to the infrared divergence of thermal phase fluctuations (see Appendix). The long distance phase correlations of a discrete layered superconductor decay in the same manner as for 2D superconductors.

In 2D quantum states corresponding to different flows are topologically distinct. When an energy barrier prevents tunneling or thermal hopping between topologically distinct states, the system is topologically ordered. This topological order gives rigidity to the superfluid ground state and stability to persistent currents. Such a 2D superfluid is said to be in the Kosterlitz-Thouless (KT) phase.

The KT phase raises interesting questions about the existence of vortices. We now review the old answers to these questions. In superconductors, it is impossible to nucleate vortices in the absence of a pairing amplitude. The more important concern is if one can nucleate vortices in the absence of phase coherence. The screening supercurrent intrinsic to a vortex excitation precludes the possibility of vortex nucleation in the absence of phase coherence at least up to some finite distance.

In 2D, despite the absence of ODLRO, the phase correlations decay slowly enough to provide sufficient short range phase coherence as to make possible the existence...
of vortices. In superconductors, a similar phenomenon occurs in a vortex liquid phase in which vortex flow destroys ODLRO, yet there is enough short range order to nucleate vortices. The vortex degrees of freedom and a particular form of their interaction are necessary for the stability of the topologically ordered KT phase.

The KT phase is a vortex insulating fluid of vortex-antivortex bound “dipoles”. The inverse distance attraction, i.e. logarithmic interaction, among vortices and antivortices is responsible for dipole binding and thus for an energy barrier to excite free vortices in the KT phase. The scarcity of free vortices at low enough temperature provides for the stability against decay of supercurrents and ultimately for the topological ordered ground state and its concomitant rigidity. The KT phase becomes thermodynamically unstable when entropy gain makes vortex-antivortex unbinding favorable at a temperature, $T_p$, which is proportional to $n_s$, the superfluid density.

At first, the KT phase was not believed to exist for 2D superconductors as vortices interact through an inverse square force law when sufficiently far apart. Soon after, it was remembered that vortices within a penetration depth of each other interact logarithmically with distance. Therefore the KT phase can occur for 2D superconducting films with large penetration depths. For superconductors with small enough superfluid density, $T_c$, is smaller than the BCS Cooper pairing temperature, $T_p$, above $T_c$ and below $T_p$, there is a dephased Cooper pair fluid which is not superconducting: it is not a perfect diamagnet nor does it exhibit resistanceless conduction. The system thus exhibits the pseudogap phenomenon.

The dephased superfluid, existing above $T_c$ and below $T_p$, is actually a neutral vortex-antivortex “metal” or “plasma”. The resistivity for this dephased superfluid is provided by flux flow in the vortex-antivortex fluid present at those temperatures. Moreover, a magnetic field acts as a chemical potential creating an imbalance between + and − vortices so that the vortex-antivortex plasma is no longer neutral. We have a vortex liquid in the presence of a magnetic field. In the presence of a thermal gradient these vortices will flow with their Josephson electric field producing a Nernst effect signal. It also follows that there is a depairing field $H_{c2}$ above $T_c$ which collapses to zero at $T_p$.

In underdoped BSCCO, measurements of the DC conductivity above $T_c$ agree with the flux flow conductivity $(k_B T / n_s D \Phi_0^2$ with $\Phi_0 = hc/2e$ the flux quantum and $D$ the vortex diffusivity) expected of a vortex liquid. The behavior just described is exactly that of the phase above $T_c$ and below the pairing temperature, $T_p$, in a KT superconductor. Furthermore, the AC conductivity has the universal scaling form expected above a KT superconducting phase.

Recently there have been reports of vortexlike Nernst effect signals characteristic of a vortex liquid phase in underdoped high temperature superconducting cuprates. In particular, N. P. Ong and collaborators also measure $H_{c2}$ for underdoped cuprates above $T_c$. From $H_{c2}$ the coherence length was determined according to $\xi_0 = \sqrt{\Phi_0/2\pi H_{c2}}$. From photoemission experiments the coherence length was determined according to $\xi_0 \sim \hbar c/\Delta_0$, with $\Delta_0$ the maximum gap. The two lengths agree closely with each other and track each other with doping. Mobile vortices and a depairing field above $T_c$ is the behavior of a dephased KT superfluid below the pairing temperature $T_p$.

In summary, we have shown that layered superconductors like the high temperature superconducting planes cannot have long range phase coherence at any finite temperature just like 2D superfluids. For low enough superfluid densities, as in the underdoped regime, the transition out of the superconducting state is through destruction of topological order by vortex-antivortex unbinding at a temperature $T_c \sim n_s$, smaller than the Cooper pairing temperature $T_p$, thus exhibiting the pseudogap phenomenon just like 2D KT superconductors. Above $T_c$ and below $T_p$, there is dephased Cooper pair fluid whose dynamical response is mainly that of a vortex liquid in the same manner as KT superconductors.

We thus conclude that layered superconductors like the superconducting cuprates are topologically ordered Kosterlitz-Thouless superconductors. Just like in 2D, the presence of vortex-antivortex dipoles at low enough temperatures stabilizes Kosterlitz-Thouless topological order which provides the necessary rigidity for a superconducting ground state. In the absence of this topological order, superconductivity would be impossible for 2D and for layered superconductors due to the absence of ODLRO.

A layered KT superconductor above its KT transition temperature will consist of planes of a dephased Cooper pair fluid with finite resistance for transport in the plane. For interplane charge transport, electrons have to be ripped out of the Cooper pair fluid and hopped from plane to plane. Thus c-axis transport will exhibit a “semiconducting” gap. This behavior is the one observed in the superconducting cuprates in AC conductivity measurements.

The physics proposed in this work should be universal for all dephased KT superfluids. In particular, artificially engineered dephased superconducting systems as studied in the experiments of Dynes and collaborators should exhibit similar properties. In such experiments we thus predict the existence of vortex excitations above $T_c$ on the “underdoped” regime in the presence of a magnetic field. If the granularity does not cause strong enough pinning, these could be observed via Nernst signals as we then expect the vortex phase to be a vortex liquid characteristic of a dephased KT superfluid as in the Ong experiments. If the vortices are pinned and hence not mobile, they should be observable with an STM.
also predict that a depairing field $H_{\text{c}}$ can be measured above $T_c$.

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**APPENDIX: OFF-DIAGONAL LONG RANGE ORDER**

In this Appendix we study the effect of thermal phase fluctuations on long range phase coherence or ODLRO in anisotropic 3D superconductors, 2D superconductors, and layered superconductors. We will work at the level of Ginzburg-Landau (GL) concentrating in a Ginzburg-Landau phase model only. Thus the GL order parameter is taken to be $\psi(\vec{x}) = \psi_0 e^{i\varphi(\vec{x})}$ where $\psi(\vec{x}) = \psi_0 = \sqrt{n_x}$ is the order parameter amplitude that minimizes the GL free energy.

The GL phase only Hamiltonian is given by

$$\mathcal{H}[\varphi(\vec{x})] = \frac{\hbar^2 \psi_0^2}{2m} \int [ (\nabla_{\perp} \varphi(\vec{x}))^2 + \frac{m}{M} (\partial_x \varphi(\vec{x}))^2 ] \, d\vec{x} \tag{1}$$

where $\nabla_{\perp}$ is the gradient operator in the x-y plane. Since the cuprates are anisotropic, in order to model them by a continuous GL model, we introduce anisotropy by considering a different mass in the z direction than in the x-y direction\,[11]. The separation between the planes $c$ is larger than the Copper-Oxygen plane lattice constant $a$.

We test for the existence of ODLRO by studying the fixed point properties of the order parameter correlation function $G(\vec{x}_1 - \vec{x}_2) = \langle \psi(\vec{x}_1) \psi^*(\vec{x}_2) \rangle$. There will be ODLRO if the correlation function is finite as $|\vec{x}_1 - \vec{x}_2| \to \infty$ due to broken gauge invariance in the superfluid state\,[22].

Generalizing to the 3D anisotropic superconductor, and stealing from the results of Rice and others\,[13], we have

$$G(\vec{x}_1 - \vec{x}_2) = \frac{1}{Z} \int \prod d\varphi(\vec{x}) \psi_0^2 \times \exp \left( i \langle \varphi(\vec{x}_1) - \varphi(\vec{x}_2) \rangle - \beta \mathcal{H}[\varphi(\vec{x})] \right) \tag{2}$$

where $Z$ is the GL partition function. We define the Fourier transform $\varphi(\vec{x}) = \sum_\k \Phi_\k e^{i\k \cdot \vec{x}}$, with $\Phi_\k \equiv \varphi_\k + i\chi_\k$, where $\varphi_\k$ and $\chi_\k$ are both real. The reality of $\varphi(\vec{x})$ implies $\Phi_\k = \Phi_\k^*$, or $\varphi_\k = \varphi_{-\k}$ and $\chi_\k = -\chi_{-\k}$.

The correlation function (2), after plenty of massaging, becomes\,[13]

$$G = \psi_0^2 \exp \left( -\frac{1}{2V} \frac{1}{\psi_0^2} \beta \left[ \sum_\k \frac{1 - \cos(\vec{k} \cdot \vec{X})}{(k_x^2 + k_y^2 + (m/M)k_z^2)} \right] \right) \tag{3}$$

where $V$ is the volume of the system, and $\vec{X} \equiv \vec{x}_1 - \vec{x}_2$. Since fixed point properties are cut-off independent, the momentum sums are cut-off at the ultraviolet anisotropically: $Q_x = \sqrt{M/m}Q_{x,y}$.

For convenience we make the integrand isotropic by defining $\vec{q} \equiv (k_x, k_y, k_z\sqrt{M/m})$, and $\vec{R} \equiv (x, y, z\sqrt{M/m})$ so that $\vec{q} \cdot \vec{R} = k \cdot \vec{X}$. This allows us to write the sum in the exponent as $I = \sum_\k \frac{1 - \cos(q \cdot \vec{R})}{q^2} = \sum_\k \frac{1 - \cos(q \cdot \vec{R})}{q^2} \sin \theta \, d\theta \, dq$, or $I = \frac{1}{4\pi} \int_0^\infty \frac{1 - \cos(q \cdot \vec{R})}{q^2} \sin \theta \, d\theta \, dq$. The Q in front of the integral can be subsumed in an unobservable superfluid density renormalization. The large $Q$ fixed point limit is now a finite cut-off independent number for arbitrary $R$. Hence the order parameter correlator is finite for infinite separation. Therefore there is ODLRO in 3D irrespective of anisotropy and superfluid density as long as the superconductor is paired. The transition out of such a superconducting state is necessarily a depairing BCS transition, unless the superfluid density is small enough for the Cooper pairs to Bose uncondensate at a temperature below $T_p$.

In the case of a two dimensional superconductor\,[13], the correlation function is

$$G = \psi_0^2 \exp \left( -\frac{1}{2A\psi_0^2} \beta \left[ \sum_\k \frac{1 - \cos(\vec{k} \cdot \vec{X})}{(k_x^2 + k_y^2)} \right] \right) \tag{4}$$

where now $A$ is the area of the system. In the thermodynamic limit and fixed point limit, $Q >> 1$, the sum in the exponent becomes $I = \frac{A}{2\pi} \int_0^\infty \frac{1 - \cos(kX t)}{k} \, dk \, dt$, or with $\frac{A}{2\pi} \int_0^1 \frac{1 - \cos(qX t)}{t} \, dq \, dt$, with $t = q/Q$. The integral becomes cutoff independent in the limit $Q X \to \infty$ since $J_0 (Q X t) \to 0$. In this limit the integral has an infrared logarithmic divergence. That is, phase correlations are destroyed by infrared thermal phase fluctuations. Therefore there is no ODLRO in the usual sense in 2D.

In order to examine ODLRO in the cuprates, which are layered superconductors, we evaluate the correlator for discrete Josephson coupled planes. We consider planes, numbered by $n$, which are separated by a distance $c$ in the $z$ direction, so that the vector $\vec{X} = (x, y, z) \equiv (x, y, nc) \equiv (\vec{x}_\perp, nc)$. The GL Hamiltonian is given by\,[10, 11].
\[ \mathcal{H}[\varphi(\vec{x}_\perp,nc)] = \frac{\hbar^2}{2m} \int \left( \sum_{n=-N_z}^{N_z} \left[ (\partial_{\vec{x}_\perp} \varphi(\vec{x}_\perp,nc))^2 \right. \right. \\
+ \frac{c}{\sqrt{2\pi}} \left( \varphi(\vec{x}_\perp, (n+1)c) - \varphi(\vec{x}_\perp,nc) \right)^2 \right) d\vec{x}_\perp \] (5)

The Fourier transform of the phase is again \( \phi(\vec{x}_\parallel, nc) = \sum_k \Phi_k e^{i\vec{k} \cdot \vec{x}} \) but this time \( k_x = \frac{2\pi m}{\hbar^2} m \in [-N_z, N_z]; k_y = \frac{2\pi p}{ \hbar^2 y}; \) \( l \in [-N_y, N_y]; k_z = \frac{2\pi p}{ \hbar^2 z}, p \in [-N_z, N_z]; l_x = N_x a, l_y = N_y a, l_z = N_z c \) are half the length of the sample in the \( x, y \) and \( z \) directions, and \( k \vec{X} = k_x \vec{x} + k_y \vec{y} + k_z nc = \vec{k}_c \vec{x} + k_z nc \).

The correlator for the layered superconductor is

\[ G(\vec{x}_1 - \vec{x}_2) = \psi_0^2 \times \exp \left( \frac{-1}{2\beta(\hbar^2/2m)\psi_0^2 V} \sum_{k_x} \frac{1 - \cos(k_x \vec{X})}{k_x^2 + k_y^2 + \frac{\pi}{2} [1 - \cos(k_z c)]} \right) \] (6)

We look for the in-plane correlation function (\( \vec{X} = (\vec{x}_\perp - \vec{x}_\perp, 0) \)) which are the more relevant ones as they are stronger than the out-of-plane ones. The sum over \( k_z \) becomes \( \sum_{k_z} (k_x^2 + k_y^2 + \frac{\pi}{2} [1 - \cos(k_z c)]) \) or

\[ \sum_{r=1}^{1} \frac{1}{g^2 + (1 - \cos(\pi r))}, \] where \( r = p/N_z, \) and \( y^2 = \frac{\pi^2}{2} (k_x^2 + k_y^2), \) and the summation is done is steps of \( 1/N_z. \nIn the limit \( N_z \rightarrow \infty \) the sum is transformed into an integral to yield

\[ \frac{\pi}{2} N_z \int_{-1}^{1} \frac{1}{y^2 + (1 - \cos(\pi r))} dr \] \[ = l_z c \frac{2}{y\sqrt{2 + y^2}}. \] (7)

The correlation function now becomes

\[ G(\vec{x}_1 - \vec{x}_2) = \psi_0^2 \times \exp \left( \frac{-\sqrt{2l_z}}{2\beta(\hbar^2/2m)\psi_0^2 V} \sum_{k_x} \frac{1 - \cos(k_x \vec{X})}{k_x^2 + k_y^2 + \frac{\pi}{2} (k_x^2 + k_y^2)} \right) \]

Let \( q = \sqrt{k_x^2 + k_y^2}. \) In the thermodynamic limit the sum is evaluated by transforming it into an integral with cut-off \( Q. \) This yields

\[ G(\vec{x}_1 - \vec{x}_2) = \psi_0^2 \times \exp \left( \frac{-2\sqrt{2l_z}l_y l_z}{(2\pi)^2 \beta(\hbar^2/2m)\psi_0^2 V} \int_0^Q \int_0^{2\pi} \frac{1 - \cos(qX \cos \theta)}{q\sqrt{2 + \frac{\pi^2}{2} q^2}} q dq d\theta \right) \] (8)

In order to study in a controlled manner the fixed point behavior of the integral in the exponent, we make the change of variables \( t = q/Q \) obtaining

\[ Q \int_0^1 \frac{1 - J_0(qX)}{\sqrt{1 + (\frac{\pi}{2} q)^2}} dq \] (9)

This is exactly the 2D integral for an isolated plane which has a logarithmic infrared divergence and becomes cut-off independent for \( QX \gg 1. \) Thus the planes decouple, impeding the existence of long range phase coherence for layered superconductors.

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