Cosmic ray positron to electron ratio in the Galaxy: results of the fractional diffusion approach

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Abstract. The unexpected rise of the positron to electron ratio with energy in the 20 ÷ 100 GeV range has been recently observed by PAMELA. Later, it was also confirmed by Fermi-LAT. In the last experiment it was additionally detected that the positron fraction continues to rise between 100 and 200 GeV. We report the results of new calculations of the positron to electron ratio in the wide energy range. Fractional diffusion model was implemented to describe the particles propagation from sources.

It is shown that a self-consistent description of the experimental data can be obtained if we assume that both positrons and electrons are injected into the interstellar medium by the sources with the same spectral exponent $p \approx 2.85$. We have also found that the positron to electron ratio increases to a constant value of $\sim 0.22$ for energies $E > 100$ GeV. This flattening of the positron fraction obtained in our model for $E > 100$ GeV can be examined in the near future by the AMS-02 experiment.

Introduction

The experimental results of direct observations of the cosmic ray electrons and positrons obtained in the last decade using new generation of instruments have opened a new phase in study of their origin, acceleration and propagation.

The ATIC balloon-borne experiment [1] found a prominent spectral feature at $\sim 300 ÷ 800$ GeV in the total electron spectrum. This feature was also observed by PPB-BETS [2] and Fermi-LAT [3]. Furthermore, the H.E.S.S. [4] atmospheric Cherenkov telescope reported a significant steepening of the electron spectrum above $\sim 600$ GeV.

In addition to electron measurements, another independent indication of the presence of possible contradictions from the standard scenario came from the measurements of the positron to electron ratio, $e^+/(e^++e^-)$, between 1.5 and 100 GeV by the PAMELA satellite experiment [5]. PAMELA found that the positron fraction changes slope at around 10 GeV and begins to increase steadily up to 100 GeV. A similar trend was also indicated by Fermi-LAT [6]. In the last experiment it was additionally detected that the positron fraction continues to rise between 100 and 200 GeV.

This behavior in the electrons and positrons spectra is very different from that predicted for standard scenario of origin, acceleration and propagation for the cosmic rays leptonic component.

On the basis of identified deviations we offer the possible directions of an expansion for the standard scenario: (i) replacement of the stationary sources model with the non-stationary...
model, (ii) inclusion of the primary positrons sources, (iii) taking into account of the nonlocal character of the particles propagation throughout turbulent (fractal-like) medium.

The main goals of this paper are calculation of the cosmic ray electrons and positrons energy spectra in the fractal-like galactic medium under different assumptions about sources as well as establishment of conditions for which the theoretical results give the self-consistent description of the contemporary experimental data.

1. Fractional diffusion model

The propagation’s equation of cosmic rays electrons and positrons with energy \( E \) from galactic sources with a distribution density \( N(r, t, E) \) in a fractal medium can be written as [7]

\[
\frac{\partial N}{\partial t} = -D(E, \alpha)(-\Delta)^{\alpha/2}N(r, t, E) + \frac{\partial(b(E)N(r, t, E))}{\partial E} + S(r, t, E). \tag{1}
\]

Here \( D(E, \alpha) \) is the anomalous diffusivity [8] and \( b(E) \) is the mean rate of continuous energy losses. The fractional Laplacian (called ‘Riss operator’) \((-\Delta)^{\alpha/2}\) [9, 10] reflects a nonlocality of the particles diffusion process in the interstellar medium.

The rate of change of the electrons energy, as well as of positrons, \( b(E) \), during their propagation in the medium is attributed to ionization, inverse Compton losses, bremsstrahlung, and synchrotron radiation. Here we write \( b(E) \) as \( b(E) = b_0 + b_1 E + b_2 E^2 \approx b_2 (E + E_1)(E + E_2) \), where \( b_0 = 3.06 \cdot 10^{-16} \text{ n (GeV s}^{-1} \text{)}, \ b_1 = 10^{-15} \text{ n (s}^{-1} \text{)}, \) and \( b_2 = 1.38 \cdot 10^{-16} \text{ (GeV s}^{-1} \) \) (for the magnetic field intensity \( B = 5 \mu \text{G} \) and background photon density \( \omega = 1 \text{ (eV cm}^{-3} \text{)} \), whereas \( E_1 = b_0/b_1 \) and \( E_2 = b_1/b_2 \).

The equation for Green’s function \( G(r, t; E; E_0) \) describing electrons and positrons diffusion under condition that the particles started from origin \( r_0 = 0 \) at the time \( t_0 = 0 \) with the energy \( E_0 \) has the form [7]

\[
\frac{\partial G}{\partial t} = -D(E, \alpha)(-\Delta)^{\alpha/2}G + \frac{\partial(b(E)G)}{\partial E} + \delta(r)\delta(t)\delta(E - E_0). \tag{2}
\]

The Green’s function of the problem was derived using standard Syrovatskiii substitutions [11] and Fourier transform:

\[
G(r, t; E; E_0) = \frac{g_3^{(\alpha)}(|r|^{-1/\alpha})}{\lambda^{3/\alpha}(1 - b_2 t(E + E_2))^2} \delta \left( E_0 - \left\{ \frac{E + E_1}{1 - b_1 t(E + E_2)/(E_2 - E_1)} - E_1 \right\} \right) \times H(1 - b_2 t(E + E_2))H(t).
\]

Here, \( g_3^{(\alpha)}(r) \) is the probability density of three-dimentional spherialaly-symmetrical stable distribution [10], and \( E_0(t) = \frac{E + E_1}{1 - b_1 t(E + E_2)/(E_2 - E_1)} - E_1, \ \lambda(E, E_0) = \int_E^{E_0(t)} \frac{D(E', \alpha)}{b(E', \alpha)} dE' \).

A solution of equation (1) \( N(r, t, E) \) for a point impulse and point steady sources, which simulate generation of electrons in astrophysics sources, are given in [12].

2. Energy spectrum of electrons and positrons

The particles intensity from all galactic sources was presented as

\[
J(r, t, E) = J_L(r, t, E) + J_G(r, t, E) = \frac{v}{4\pi} N(r, t, E), \tag{3}
\]

where

\[
N(r, t, E) = \sum_{r_j < 1 \text{ kpc}} \int_{t_j < 10^{9} \text{yr}} N(r_j, t_j, E) + \int_{r = 1 \text{ kpc}}^\infty \int_{t = 10^{6} \text{yr}}^\infty dt N(r, t, E).
\]
Figure 1. Comparison of the positron to electron ratio calculated within the frameworks of fractional diffusion model with the experimental data. Dash-dotted line correspond to the case (4). Full line is the result of the fractal diffusion model for the case (5). Results of calculations for the standard scenario with the GALPROP code [15] are also presented in this figure (dotted line). References to the experimental data are given in [14]. The new Fermi-LAT data [6] is also presented.

In (3) $J_L$ is the local component, i.e. the contribution nearby ($r < 1$ kpc) young ($t < 10^6$ yr) sources and $J_G$ is the global spectrum component determined by the multiple old ($t \geq 10^6$ yr) distant ($r \geq 1$ kpc) sources.

Distribution of the sources in the area $r \geq 1$ kpc ($G$-component) was described according to standard scenario (system of steady-state sources). To take into account an effect of solar modulation we used the model proposed in [13], with the potential $\Phi = 600$ MV.

To calculate the electrons and positrons spectra from nearby young sources ($L$-component), simulation of the Poisson ensemble of sources was carried out. The Poisson distribution parameter (average number of the sources in the local space) was chosen $\sim 10$. Coordinates and times of birth of the sources were generated randomly and uniformly in the space region $100 \leq r < 10^3$ pc and in the time interval $10^4 \leq t < 10^6$ yr. Duration of the particle generation by the local sources was assumed to be $T \approx 10^4$ yr.

Detailed discussion of the key model’s parameters (the exponent $\alpha$, the anomalous diffusivity and the exponent of injection spectrum $p$ for both positrons and electrons) was presented in [12]. In this paper following parameters was accepted: $\alpha = 1.4$, $D(E, \alpha) = D_0(\alpha) E^\delta$ with $D_0(\alpha) \approx 2 \cdot 10^{-4}$ pc$^{1.4}$/yr, $\delta \approx 0.27$. The exponent of injection spectrum changes from $p = 2.6$ to $p = 2.85$ in energy region $E \in [0.1; 10^3]$ GeV [12, 14].

Results and conclusions

We present results of calculations of the spectra of electrons and positrons under the assumption that observed spectra of the particles are formed by multiple old distant sources (spectra of primary particles from this group of sources are denoted as $(e^+_{pr})_G$, $(e^-_{pr})_G$) and nearby young ones (their spectra are $(e^+_{pr})_L$ and $(e^-_{pr})_L$). To accommodate the contribution of secondary positrons and electrons ($(e^+_{sec}$ and $e^-_{sec}$ respectively) produced in the collision of cosmic rays nuclei with the interstellar medium, the GALPROP code [15] was used with the parameters stated in our
model.

In figure 1 we present results of calculation of the positron to electron ratio for the following model assumptions.

- For $E < 1$ GeV main contribution to observed spectrum of the particles is due to secondary particles as well as due to electrons and positrons injected by old distant sources

$$R \approx \frac{e_{sec}^+ + (e_{pr}^+)G}{e_{sec}^+ + (e_{pr}^+)G + e_{sec}^- + (e_{pr}^-)G}.$$  \hspace{1cm} (4)

- In the range $E \in [1 : 100]$ GeV the contribution of the nearby young sources becomes significant

$$R = \frac{e_{sec}^+ + (e_{pr}^+)G + (e_{pr}^+)L}{e_{sec}^+ + (e_{pr}^+)G + (e_{pr}^+)L + e_{sec}^- + (e_{pr}^-)G + (e_{pr})L}.$$  \hspace{1cm} (5)

- For $E \gg 100$ GeV, as follows from the equation (5), observed spectrum of the positrons is formed by the local sources only and the positron to electron ratio increases to a constant value

$$R \approx \frac{(e_{pr}^+)L}{(e_{pr}^+)L + (e_{pr})L} \approx \text{const.}$$  \hspace{1cm} (6)

It can be seen that a self-consistent description of the experimental data can be obtained if we assume that both positrons and electrons are injected into the interstellar medium by the sources with the same spectral exponent $p$. We have also found that the positron to electron ratio increases to a constant value of $\sim 0.22$ for energies $E > 100$ GeV. This flattening of positron fraction obtained in our model can be examined in the near future by the PAMELA and the AMS-02 experiments.

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