A REGION-BASED MULTI-ISSUE NEGOTIATION PROTOCOL FOR NONMONOTONIC UTILITY SPACES

MIGUEL A. LOPEZ-CARMONA, IVAN MARSA-MAESTRE, ENRIQUE DE LA HOZ, AND JUAN R. VELASCO

Computer Engineering Department, Universidad de Alcala, Madrid, Spain

Nonmonotonic utility spaces are found in multi-issue negotiations where the preferences on the issues yield multiple local optima. These negotiations are specially challenging because of the inherent complexity of the search space and the difficulty of learning the opponent's preferences. Most current solutions successfully address moderately complex preference scenarios, while solutions intended to operate in highly complex spaces are constrained by very specific preference structures. To overcome these problems, we propose the Region-Based Multi-issue Negotiation Protocol (RBNP) for bilateral automated negotiation. RBNP is built upon a nonmediated recursive bargaining mechanism which efficiently modulates a region-based joint exploration of the solution space. We empirically show that RBNP produces outcomes close to the Pareto frontier in reasonable negotiation times, and show that it provides a significantly better performance when compared to a generic Similarity-Based Multi-issue Negotiation Protocol (SBNP), which has been successfully used in many negotiation models. We have paid attention to the strategic issues, proposing and evaluating several concession mechanisms, and analyzing the equilibrium conditions. Results suggest that RBNP may be used as a basis to develop negotiation mechanisms in nonmonotonic utility spaces.

Key words: multi-agent systems, automated negotiation, multi-issue, nonmonotonic utility spaces.

1. INTRODUCTION

Automated negotiation provides an important mechanism to reach agreements among distributed decision makers (Rosenschein and Zlotkin 1994; Beer et al. 1999; Lai et al. 2004). It may be seen as a paradigm to solve coordination and cooperation problems in complex systems (Kraus, Sycara, and Evenchick 1998; Jennings 2001), providing a mechanism for autonomous agents to reach agreements on, e.g., task allocation, resource sharing, or surplus division (Kersten and Noronha 1998; Fatima, Wooldridge, and Jennings 2004; Buttner 2006). Of particular interest is multi-issue negotiation, which is concerned with reaching an agreement on a deal involving multiple issues, where it is possible to make issue trade-offs to search for win–win solutions. This allows to avoid suboptimal deals; namely, agreements that one party can modify to obtain better payoff without negatively impacting the others (Raiffa 1982).

Multi-issue negotiation scenarios may involve complex nonmonotonic preference spaces (Ito, Klein, and Hattori 2008), where reaching Pareto-efficient agreements may be computationally hard (Marsa-Maestre et al. 2009b). For example, in a sensor-network scenario, we can have a robot which performs information aggregation tasks for different service providers. These aggregation tasks depend on the data gathered by different sensors which are distributed throughout the environment. For instance, in a forest environment we may have temperature, humidity, smoke detection, wind speed, and rain gauge sensors, to name just a few. We may also find some more specific kind of sensors, like those used in honey farms to monitor the state of the bees by analyzing the buzz and flight patterns. Different providers rely on different kinds of sensors to provide their services. In the forest example, nature protection squads may rely on temperature, humidity, and smoke detection sensors, meteorology stations may use temperature, wind and rain gauge sensors, and bee farmers may mainly use humidity and specific bee sensors. Assuming that the sensors are too widespread...
to allow routing among all them, the aggregation robot will only be able to gather data from a subset of the sensors (the nearest ones). Therefore, each provider has a different preference on the optimal location of the aggregation robot, since its distance from the different sensors affects the quality of the services they provide. This scenario defines a nonlinear preference space on three dimensions. For a given service provider, proximity of the aggregation robot to each specific sensor will be more or less useful, and thus each possible robot location will yield a given utility to the provider depending on the combination of its distances from the different sensors. More dimensions may be added to the problem if we consider, for instance, temporal variations of utility (due to variations on service demand) or sensor uptime or remaining battery.

Another scenario where nonmonotonic utility spaces appear is the problem of negotiating complex contracts, which was first introduced by Klein et al. (2003). Contracts are defined as a set of issues or clauses, each of which may have a value. The aforementioned authors limited clauses to binary values, meaning that the clause was or not present in a given contract. Even with such restriction, the domain of the solution space may become very large. For instance, a negotiation scenario with 50 possible clauses would yield a search space of about $10^{15}$ possible contracts. Unless all clauses are totally independent (which is not likely), the preferences of an agent about the different potential contracts will be nonmonotonically distributed through the utility space. Furthermore, most contracts may have nonbinary clauses. In a rental agreement, for instance, clauses may state the rent, the security deposit, or the length of the lease. A labor agreement may include different insurance options. Such issues may have a larger domain, which can greatly increase the solution and preference space complexity.

The complexity of nonmonotonic scenarios arises at least from three main factors: the difficulty of reaching Pareto-optimal outcomes between self-interested agents in an incomplete information environment (i.e., parties know nothing about each other), the computational overhead imposed by local exploration of the preference space, and the complexity of deducing and learning each other’s preferences. The main objective of this work is therefore to develop effective mechanisms for negotiating under complex preference spaces.

In multi-issue negotiation, there may exist different offers which provide an agent the same utility level. Which offer to propose is usually nontrivial, and should be based on selecting the offer that maximizes the opponent’s utility, so that the opponent is more likely to accept the offer. In order to select this offer, similarity criteria have been widely used to approximate the preferences of the opponents in many negotiation models (Sycara 1998; Choi, Liu, and Chan 2001; Faratin, Sierra, and Jennings 2002; Lau, Tang, and Wong 2004). When using this approach, it is expected that the more similar is an offer to the previous opponent’s offers, the higher probability exists for the offer to be accepted. Thus, this idea has been successfully tested mainly in monotonic preference scenarios (Faratin et al. 2002; Coehoorn and Jennings 2004; Lopez-Carmona et al. 2010). Our first objective is to test the performance of similarity-based negotiation under nonmonotonic settings. The approach we take to test this issue is based on the generic Similarity-Based Negotiation Protocol (SBNP) for bilateral negotiations proposed by Lai and Sycara (2009), which is built on an iterative constrained optimization mechanism. Though Lai’s work is focused on the analysis of SBNP using monotonic utility functions, the protocol and decision mechanisms they propose can be applied to nonmonotonic scenarios. To adapt the decision mechanisms to different types of preference structures, the only requirement is to use an adequate optimizer. For example, with monotonic preferences we may use gradient-based optimization techniques, while in nonmonotonic or discontinuous preference scenarios derivative-free optimization will be a better choice. It is important to note that we are talking about optimization at an individual agent level, i.e., each agent performs optimization local processes in her utility space to
find potential contracts. A detailed description and discussion of SBNP is provided in Section 4.

It is expected that nonmonotonicity of preferences makes similarity a weak criterion because of the lack of information about the opponent’s preference structure. Thus, our second objective is to propose a negotiation protocol which can efficiently operate in complex utility spaces where the similarity-based approach fails. Our proposal is the region-based automated multi-issue negotiation protocol (RBNP), which is built upon a recursive non-mediated bargaining mechanism. RBNP is inspired by pattern search optimization methods (Lewis, Torczon, and Trosset 2000). Pattern search methods are characterized by a series of exploratory moves that consider the behavior of the objective function at a pattern of points, all of them lying on a rational lattice or mesh around a current reference point. The exploratory moves consist of a systematic strategy for visiting points in the lattice in the immediate vicinity of the current iterate. After polling, the algorithm changes the value of the mesh size. The default is doubling the current mesh size after a successful poll, and halving after an unsuccessful poll. A poll is considered successful when there is at least one mesh point which improves the reference point value. In this case, the next iteration reference point is the mesh point which has provided a better improvement in the objective function value. Optimization finishes when the mesh size is less than a given mesh tolerance. The main advantage of direct search methods like pattern search is that they do not need to compute derivatives, so they usually perform well with nonmonotonic and nondifferentiable functions.

In this work we decided to extend some of the pattern search principles to the automated negotiation domain. Taking this into account, RBNP captures the idea of mesh expanding and contracting in order to perform a region-based distributed search on the solution space. Note, however, that the problem to solve in a nonmediated negotiation scenario is significantly different from classical single-objective or multi-objective optimization. In nonmediated negotiation, the objective functions (i.e., the agents’ utility functions) are not revealed to the opponents, and, thus, there is not a centralized optimization process. To incorporate the iterative exploration fundamentals of pattern search into RBNP, we propose switching from a contract to a region exchange-based interaction protocol. The joint exploration of the solution space is recursive. This means that when agents agree on an offer (a region proposed by an agent), a new bargaining is performed using lower-sized regions within the previously agreed region. This process may be seen as an iterative zoom in or mesh contraction on the solution space. The end is reached as soon as agents decide that the agreed region is small enough to be considered a contract. In a similar way to mesh expansion, agents may zoom out on the solution space to avoid zones of no agreement and therefore reinitiate the bargaining process on different higher sized regions.

The rationale behind the approach also bears some similarities with the iterative narrowing protocol described in Hattori, Klein, and Ito (2007), though their approach relies on a very particular constraint-based preference space, and the flexibility of the search algorithm is limited, since the number of rounds is fixed to three and the volumes of the regions in each round are given by the structure of the agent preference space. RBNP, in contrast, does not impose requirements on the agents preference space, which makes it suitable for generic scenarios. In addition, the parametric design of RBNP allows to control the search depth and the size of the regions, among other negotiation parameters, making it possible to adapt the negotiation mechanism to the singularities of the different specific scenarios we may deal with.

SBNP and RBNP have been tested under monotonic and nonmonotonic scenarios. Results show that RBNP outperforms SBNP, and that RBNP results in approximate Pareto-optimal outcomes obtained in reasonable times. We have paid attention to the utility
concession strategies played by the negotiating agents in a negotiation encounter (Faratin, Sierra, and Jennings 1998; Fatima, Wooldridge, and Jennings 2005). Thus, we propose three different concession strategies for RBNP and evaluate the strategic issues and equilibrium conditions of the protocol when agents using the strategies interact.

The rest of the article is organized as follows. In Section 2 we review the related research. Section 3 describes how we have simulated the monotonic and nonmonotonic negotiation scenarios. Section 4 evaluates the behavior of SBNP in monotonic and nonmonotonic negotiation scenarios. Section 5 presents RBNP, and in Section 6 it is empirically evaluated. Finally, Section 7 reviews the strategic issues, and the last section summarizes our conclusions and sheds light on some future research.

2. RELATED RESEARCH

Klein et al. (2003) present, as far as we are aware, the first negotiation mechanisms intended for complex preference spaces. They propose a simulated annealing-based approach, a refined version based on a parity-maintaining annealing mediator, and an unmediated version of the negotiation protocol for bilateral negotiation with binary issues and binary dependencies. Soft constraints have been reported as a very useful framework to represent preferences (Zhang and Pu 2004). Hence, there are several works which propose a fuzzy constraint-based framework for bilateral multi-issue negotiations (Luo et al. 2003; Lai and Lin 2004). Although these approaches may operate with nonmonotonic preferences, they assume that the whole preference space can be ordered as a set of hard constraints prior to negotiation, which limits their application to more general preference models (Lopez-Carmona and Velasco 2006; Lopez-Carmona, Velasco, and Marsa-Maestre 2007). Ito et al. (2008) proposed a multi-party bidding-based protocol to deal with complex utility spaces generated using weighted constraints. In contrast to the fuzzy constraint-based works, weighted constraints need not to be ordered. The preference space admits high-order dependencies, and issues are not restricted to a binary domain. In Marsa-Maestre et al. (2009a,b) we showed that the performance of the multi-party bidding-based protocol decreases drastically in highly nonmonotonic utility scenarios. Hence, we defined a set of mechanisms for the bidding and deal identification processes which improve the previous approaches in terms of optimality and scalability. The mechanisms take into account both the utility of a bid for an agent and its viability (a measure of the likelihood of the bid to yield a deal). The balance between bid utility and deal probability yields a significant improvement in terms of optimality rate and failure rate in highly nonmonotonic scenarios. In addition, we proposed a probabilistic search mechanism for the mediator which lowers the scalability problem while achieving acceptable optimality rates, and an iterative expressive negotiation protocol to give feedback to the agents in case no deals have been found with the initial bids. The main disadvantage of these approaches is that they are restricted to constraint-based utility spaces and cannot be used with other types of utility functions or in nonmediated scenarios.

Another interesting approach to solve the computational cost and complexity of nonmonotonic spaces is to transform the negotiation space. Hindriks, Jonker, and Tykhonov (2006) propose a weighted approximation technique to simplify the utility space. They show that for smooth utility functions the application of this technique results in an outcome that closely matches the outcome based on the original utility structure. The limitation of this approach is that it is only applicable to smooth utility functions, and that it is necessary to estimate a region within the utility space where the actual outcome is expected to be (i.e., it is assumed that this region is known a priori by the agents). Fatima, Wooldridge, and Jennings (2009) analyze bilateral multi-issue negotiation involving nonlinear utility functions. They
consider the case where issues are divisible and there are time constraints in the form of deadlines and discounts, and show that it is possible to reach Pareto-optimal agreements by negotiating all the issues together, and that finding an equilibrium is not computationally easy if the agents’ utility functions are nonlinear. In order to overcome this complexity they investigate two solutions: approximating nonlinear utility spaces with linear functions, and using a simultaneous procedure where the issues are discussed in parallel but independently of each other. They show that the equilibrium solution can be computed in polynomial time. However, their work is focused on symmetric negotiations where the agent’s preferences are identically distributed, and the utility functions are separable in nonlinear polynomials of a single variable. In Robu, Somefun, and La Poutré (2005) utility graphs are used to model issue interdependencies for binary-valued issues. Utility graphs are inspired by graph theory and probabilistic influence networks to derive efficient heuristics for nonmediated bilateral negotiations about multiple issues. The underlying idea is to decompose highly nonlinear utility functions in sub-utilities of clusters of inter-related issues. They show how utility graphs can be used to model an opponent’s preferences. In this approach agents need prior information about the maximal structure of the utility space under consideration. Authors argue that this prior information could be obtained through a history of past negotiations or the input of domain experts.

There are also several proposals which employ genetic algorithms to learn opponent’s preferences according to the history of the counteroffers based on stochastic approximations. In Choi et al. (2001) a system based on genetic algorithms for electronic business is proposed. The utility functions are restricted to take the form of a product combination (i.e., utility of an outcome is the product of the utility values of the different issues). The objective function used is based on the comparison of the changes in consecutive offers. Small changes in an issue suggest that this issue is more important. For each new population, the protocol enforces that generated candidates cannot be better than the previous offer. Unlike other negotiation models based on genetic algorithms, this proposal adapts to the environment by dynamically modifying its mutation rate. Another negotiation mechanism for nonmediated negotiation based on genetic algorithms can be found in Lau et al. (2004). The proposed fitness function relies on three aspects: an agent’s own preference, the distance of a candidate offer from the previous opponent’s offer, and time pressure. In this work, agents’ preferences are quantified by a linear aggregation of the issue valuations. However, nonmonotonic and discontinuous preference spaces are not explored. In Chou, Fu, and Liu (2007) a genetic algorithm is proposed which is based on a joint elitism operation and a joint fitness operation. In the joint elitism operation an agent stores the latest offers received from the opponent. The joint fitness operation combines the agent’s own utility function and the euclidean distance to the opponent’s offer. In this work, two different negotiation scenarios are considered. In the first one, utility is defined as the weighted sum of the different issue values (i.e., issues are independent). The second scenario defines a utility function where there is a master issue and a set of slave issues. Utility is calculated as the weighted sum of the different issue values, where the weights of the slave issued are set according to the value of the master issue.

Finally, in Yager (2007) a mediated negotiation framework for multi-agent negotiation is presented. This framework involves a mediation step in which the individual preference functions are aggregated to obtain a group preference function. The main interest is focused on the implementation of the mediation rule where they allow a linguistic description of the rule using fuzzy logic. A notable feature of their approach is the inclusion of a rewarding mechanism for agents open to alternatives which are not their preferred choices. The negotiation space and utility values are assumed to be arbitrary (i.e., preferences can be nonmonotonic). However, the set of possible solutions is defined a priori and is fixed. Moreover, the preference function needs to be provided to the mediation step in the negotiation
process, and Pareto optimality is not considered. Instead, a stopping rule is considered, which determines when the rounds of mediation stop.

In summary, in the existing research nearly all the models which assume nonmonotonic utility spaces rely on binary valued issues, low-order dependencies, or a fixed set of defined a priori solutions. Simplification of the negotiation space has also been reported as a valid approach for simple utility functions. Finally, the existing works that assume highly non-monotonic utility spaces are focused on constraint-based preference structures and mediated solutions. Therefore, new approaches are needed if automated negotiation is to be applied to nonmediated settings involving nonmonotonic preference spaces.

3. MONOTONIC AND NONMONOTONIC NEGOTIATION SCENARIOS

In this section we describe how the monotonic and nonmonotonic preference scenarios have been simulated. In both scenarios we consider two agents, each one with its own private utility function, which automatically negotiate a deal on two or more issues.

3.1. Monotonic Negotiation Scenarios

The monotonic negotiation scenarios are simulated with Constant Elasticity of Substitution (CES) utility functions (Mas-Colell, Whinston, and Green 1995). Their functional form is widely used in economics as a production function, and in consumer theory as a utility function.

Definition 1. The CES utility functions for two agents $A_b$ and $A_s$, and a contract space of $n$ issues, are respectively defined by

$$U_b(s) = \left( \sum_{i=1}^{n} \alpha_{b,i} \cdot x_i \right)^{1/\beta_b},$$

$$U_s(s) = \left( \sum_{i=1}^{n} \alpha_{s,i} \cdot (100 - x_i)^{\beta_s} \right)^{1/\beta_s},$$

where $s = (x_1, x_2, \ldots, x_n)$ is a contract and $x_i$ the $i$th issue, $\alpha_{x,i}$ is the share parameter, and $\beta_x$ is the elasticity of substitution parameter. The CES utility functions we use satisfy that: $\sum_{i=1}^{n} \alpha_{x,i} = 1$, $0 \leq \alpha_{x,i} \leq 1$, and $0 < \beta_x \leq 1$. To ensure conflicting interests, we assume increasing preferences on the issues for agent $A_b$, and decreasing preferences for agent $A_s$.

Note that the monotonic negotiation scenario does not leave room for compatible issues, i.e., where both agents have increasing or decreasing preferences in the same issue, and thus, all the attributes are in opposition. Although it would be possible to add compatible issues, preliminary experiments confirm that when there is no interdependency among the agent’s preferences for the attributes, compatible issues do not influence the negotiation results.

In order to provide diversity to the instantiation of a monotonic scenario, the followed strategy is to generate four different utility function pairs by randomly choosing the $\alpha$ and $\beta$
parameters:

\[
\text{CES scenario} = \begin{bmatrix}
U_1^b(s) & U_1^s(s) \\
U_2^b(s) & U_2^s(s) \\
U_3^b(s) & U_3^s(s) \\
U_4^b(s) & U_4^s(s)
\end{bmatrix}.
\]

The evaluation of a CES scenario implies the execution of a predefined number of negotiations over each pair of utility functions \((U_k^b(s)U_k^s(s))\).

3.2. Nonmonotonic Negotiation Scenarios

The nonmonotonic negotiation scenarios are created using an aggregation of Bell functions. We have chosen this type of utility functions because they capture the intuition that agents’ utilities for a contract usually decline gradually, rather than stepwise, with distance from their ideal contract. Bell functions are ideally suited to model, for instance, spatial and temporal preferences. In addition, they provide us with the capability of configuring different negotiation scenarios in terms of different nonmonotonicity degrees. Specifically, we can easily modulate the negotiation space correlation length. Correlation length is defined as the minimum distance between samples in the utility space which makes the correlation between those samples drop below 0.5. Therefore, a low correlation length implies a highly rugged preference space. This measure has been also used to assess fitness landscape complexity in evolutionary computation (Weinberger 1990).

**Definition 2.** A Bell function is defined by a center \(c\), height \(h\), and a radius \(r\). Let \(\|s - c\|\) be the euclidean distance from the center \(c\) to a contract \(s\), then the Bell function is defined as

\[
f_{\text{bell}}(s, c, h, r) = \begin{cases} 
    h - 2h \|s - c\|^2 \bigg/ r^2 & \text{if } \|s - c\| < r/2, \\
    2h r^2 \|s - c\| - r)^2 \bigg/ r^2 & \text{if } r > \|s - c\| \geq r/2, \\
    0 & \text{if } \|s - c\| \geq r
\end{cases}
\]

and the Bell utility function as

\[
U_{b,s}(s) = \sum_{i}^{nb} f_{\text{bell}}(s, c_i, h_i, r_i),
\]

where \(c_i \in [0, 100]^n\), \(h_i \in [h_{\text{min}}, h_{\text{max}}]\) and \(r_i \in [r_{\text{min}}, r_{\text{max}}]\), are randomly generated within the predefined intervals to construct an instance of utility function, and \(nb\) is the number of generated bells. Thus, to generate a Bell utility function we randomly generate \(nb\) centers and assign to each center a height and a radius obtained from the uniformly distributed intervals \([h_{\text{min}}, h_{\text{max}}]\) and \([r_{\text{min}}, r_{\text{max}}]\), respectively. The correlation length of the negotiation space can be modulated by varying the different intervals and the number of bells.

Without loss of generality, and for simplicity, bell functions define hyperspheres to bound the contracts within the bell. It would be possible to use different geometry shapes, ellipses for example, which modulate the influence of each issue on a bell, or to restrict the dependence of a bell on a subset of issues. We can simulate the same effects by varying the bell generation parameters.
To generate nonmonotonic scenarios with different levels of complexity, we fix $nb$ and $[h_{\min}, h_{\max}]$, and vary the interval $[r_{\min}, r_{\max}]$. To make the relative complexity of different scenarios invariant with the number of issues, the range $[r_{\min}, r_{\max}]$ must change with the number of issues. Let us consider first a one-dimensional space, and a range $[p_{\min} \cdot |D|, p_{\max} \cdot |D|]$ for the radius generation, where $|D|$ represents the domain length and $p_{\min}$, $p_{\max}$ are two factors such that $0 < p_{\min} \leq 1$, $0 < p_{\max} \leq 1$, and $p_{\min} < p_{\max}$. The question now is how to obtain the corresponding intervals to maintain the level of complexity for high dimensional spaces. The answer is that we need to keep constant the ratio between the volume of the whole negotiation domain and the hyperspheres defined by the radius generation interval. The volume of a $n$-dimensional space, under the assumption of an equal length for each dimension, is simply $|D|^n$. The volume of a hypersphere is calculated as

$$V_n(r) = \frac{r^n \cdot \pi^{n/2}}{\Gamma(n/2 + 1)},$$

where $r$ is the radius of the hypersphere, and $\Gamma(.)$ is the Gamma function. Thus, we need to satisfy

$$p_{\min,\max} = \frac{|D|^n}{V_n(p_{\min,\max} \cdot |D|)},$$

from which we define the $r(n, p_{\min,\max})$ function used to calculate the radius intervals

$$r(n, p_{\min,\max}) = \left(\frac{p_{\min,\max} \cdot |D|^n \cdot \Gamma(n/2 + 1)}{\pi^{n/2}}\right)^{1/n}.$$

In order to cope with the inherent diversity of this type of environments, to provide different opposition levels, and to instantiate a particular nonmonotonic scenario we generate 10 pairs of Bell utility functions. We considered two nonmonotonic scenarios: the Bell Smooth(BELLs) scenario that describes smooth nonmonotonic preferences, and the BELL Complex(BELLc) scenario that defines a much more complex structure of preferences with many local optima:

BELLs scenario =

$$\begin{bmatrix}
U_{b}^{1s}(s) & U_{s}^{1s}(s) \\
\ldots & \ldots \\
U_{b}^{10s}(s) & U_{s}^{10s}(s)
\end{bmatrix}$$

BELLC scenario =

$$\begin{bmatrix}
U_{b}^{1c}(s) & U_{s}^{1c}(s) \\
\ldots & \ldots \\
U_{b}^{10c}(s) & U_{s}^{10c}(s)
\end{bmatrix}.$$
Figure 1 represents four different utility functions for the particular case of a bidimensional negotiation space. The linear additive and CES utility functions represent an example of monotonic preferences (note that the linear additive function corresponds to a CES function where $\beta_i = 1$). The BELL Smooth utility function shows a nonmonotonic preference space with a relatively simple shape, while the BELL Complex utility function exhibits a much more complex shape with many peaks and valleys. BELL Smooth has been generated with 50 peaks, a radius interval between 20 and 35, and a height between 0.1 and 1. BELL Complex has also been generated with 50 peaks and a height between 0.1 and 1, but the radius of the different peaks lies within the interval 5 and 10.

The CES and BELL utility functions cover a wide spectrum of possible negotiation scenarios. It is worth noting, however, that research on automated multi-issue negotiation has devoted a great effort to develop negotiation protocols operating under constraint-based utility spaces (Luo et al. 2003; Lopez-Carmona and Velasco 2006; Lopez-Carmona et al. 2007; Ito et al. 2008; Marsa-Maestre et al. 2009b). From these works, the most closely related to RBNP is Ito et al. (2008) that proposed a mediated auction-based negotiation protocol (ABNP) for multi-party negotiations. ABNP works only in constraint-based utility spaces, taking advantage of the constrained structure of the preference space, and thus, it is not able to operate under the more generic CES or BELL scenarios. Although we cannot compare RBNP and ABNP using continuous utility functions (e.g., CES) or utility functions with removable discontinuity (e.g., BELL), we still can compare both approaches in a constraint-based negotiation space. Therefore, Appendix includes an evaluation of RBNP under constraint-based utility spaces, and a comparative analysis between RBNP and ABNP.
4. SBNP IN MONOTONIC AND NONMONOTONIC NEGOTIATION SCENARIOS

In order to show the efficiency of the similarity-based negotiation under monotonic and nonmonotonic scenarios, we have implemented and tested an instance of SBNP, based on a generic similarity-based alternating offers negotiation protocol described in Lai and Sycara (2009). In their work the authors describe the structure of the negotiation mechanisms, although they do not specify which optimization techniques are used to implement them. In the following, our proposal of SBNP instance is described in detail.

4.1. SBNP Protocol

SBNP divides the decision mechanisms of an agent into three components: conceding, proposing, and responding. For the conceding mechanism, we use a time-dependent strategy (Faratin et al. 1998). In this strategy for each period $T$ (iteration of the alternating offers protocol), each agent sets its reservation utility (i.e., the minimum utility which should give an opponent’s offer in order to be accepted) using the expression:

$$U_{rs}^i(t) = 1 - (1 - U_{th}^i) \left( \frac{t}{T} \right)^{\frac{1}{\beta}}$$

where $U_{rs}^i(t)$ is the reservation utility of agent $i$ in period $t$; $T$ is the negotiation deadline in periods for the agent; $U_{th}^i$ is a minimum utility threshold; and $\beta > 0$ represents the strategy parameter of agent $i$. If $\beta < 1$, agent $i$ concedes slowly at the beginning but quickly when the deadline approaches; if $\beta > 1$, the agent concedes quickly at the beginning but slowly when the deadline approaches; and if $\beta = 1$, the agent concedes evenly during the whole negotiation.

In the proposing mechanism the agent obtains the most similar contract $s_j^i$ to the opponent’s last offer $s_{j-1}^j$, from the set of available contracts $C_i$, which yields the reservation utility given by the concession strategy. This is basically a constrained optimization problem, which can be formally defined as follows:

$$s_j^i = \min_{s_i \in C_i} \| s_i - s_{j-1}^j \| \quad s.t. \ U_i(s_i) = U_{rs}^i(t).$$

That is, to prepare an offer an agent minimizes the euclidean distance to the opponent’s last offer, constrained to the contracts lying in the isocurve defined by her current reservation utility. We have used two different optimization techniques depending on the negotiation scenario. For scenarios where the utility functions are continuous and have continuous first derivatives we use a gradient-based method (Nocedal and Wright 2006). Specifically, a sequential quadratic programming (SQP) method is used which does not need the input of a user-supplied Hessian. It computes a quasi-Newton approximation to the Hessian of the Lagrangian. This optimization technique is applied in monotonic scenarios (i.e., where agents use CES utility functions). In nonmonotonic scenarios (i.e., where agents have BELL utility functions) we use a pattern search algorithm, which is a derivative-free optimization method. The Bell utility functions considered do not have continuous first derivatives.

Finally, in the responding mechanism an agent accepts an opponent’s offer if its utility is higher than her current reservation utility.

$$\text{response}_j^i(s_{j-1}^i) = \begin{cases} \text{accept}, & \text{if } U_{rs}^i(t) \leq U_i(s_{j-1}^i) \\ \text{reject}, & \text{otherwise.} \end{cases}$$

Figure 2 shows the dynamics of SBNP in a two-dimensional contract space for two different scenarios. In Figure 2(a) the utility functions of the two negotiating agents, buyer
and seller, are monotonic, while in Figure 2(b) the utility functions are clearly nonmonotonic. In both cases, figures show a set of indifference curves (i.e., isocurves) corresponding to different utility levels, and the isocurves which correspond to the agents’ reservation values (with thick lines). In the monotonic scenario it can be seen how the Pareto frontier is defined by the set of points which correspond to joint tangent lines of a pair of indifference curves. The negotiation starts with agents exchanging proposals at a high aspirational level (i.e., the agents’ reservation utilities are high). As agents exchange offers and concede, because of the similarity-based optimization process which generates the offers, the negotiation easily gets near to the Pareto frontier and negotiation ends with a deal. In the nonmonotonic scenario the distribution of isocurves makes the convergence of the negotiation much more difficult. The example shows how the initial offers lie within different basins of attraction with respect to the region where a final agreement is reached. Fortunately, for the agents in the example, the exchange of offers has conducted the negotiation to basins of attraction where there exists a zone of agreement. However, SBNP does not guarantee that the mutual attraction imposed by the similarity-based mechanism lead the agents to zones of agreement. Intuitively, the initial offers chosen will have a strong influence in the success of the negotiations with SBNP.

It is worth noting that, to our knowledge, there are no studies of the performance of similarity-based negotiation protocols under nonmonotonic preference scenarios. Next, we
fill this gap with an evaluation of SBNP in monotonic and nonmonotonic scenarios using the CES utility function type, and the BELLs and BELLc utility function types, respectively.

### 4.2. SBNP in Monotonic Scenarios

We test negotiation scenarios with 2, 5, 10, and 20 issues. In each negotiation, the agents’ parameters are randomly varied according to the following uniform distributions:

- \( U^b_{th} = \text{unifrnd}[0.1, 0.3] \)
- \( U^s_{th} = \text{unifrnd}[0.1, 0.3] \)
- \( T^b_{th} = \text{unifrnd}[10, 20] \)
- \( T^s_{th} = \text{unifrnd}[10, 20] \)
- \( \beta^b = \text{unifrnd}[0.5, 1.5] \)
- \( \beta^s = \text{unifrnd}[0.5, 1.5] \).

We have measured the mean distance of the outcome utilities from the Pareto frontier, the negotiation time, the failure rate, and the number of negotiation rounds. To compute the Pareto frontier we used a genetic multi-objective optimization algorithm. We ran 25 negotiations for each CES utility function pair, which implies the execution of 100 negotiations for each scenario.

Table 2 summarizes the results of the experiments, which are statistically significant within the \( p < 0.05 \) range. Results show that the outcome utilities are close to the Pareto frontier and that the failure rate is 0%. The negotiation time linearly increases with the number of issues, which makes of SBNP a scalable protocol in terms of the number of negotiated issues. Results do not show dependence of the number of negotiation rounds on the number of issues.

|                      | Number of issues | 2    | 5    | 10   | 20   |
|----------------------|-----------------|------|------|------|------|
| **CES**              |                 |      |      |      |      |
| Negotiation Time(s)  |                 | 2.95 | 9.45 | 18.60| 23.32|
| Distance             |                 | 0.00 | 0.02 | 0.04 | 0.07 |
| Rounds               |                 | 7.83 | 8.17 | 8.64 | 7.64 |
| Failures             |                 | 0%   | 0%   | 0%   | 0%   |
| **BELLs**            |                 |      |      |      |      |
| Negotiation Time(s)  |                 | 21.41| 34.72| 54.82| -    |
| Distance             |                 | 0.06 | 0.18 | 0.46 | -    |
| Rounds               |                 | 4.69 | 7.26 | 4.34 | -    |
| Failures             |                 | 0%   | 2%   | 60%  | 100% |
| **BELLe**            |                 |      |      |      |      |
| Negotiation Time(s)  |                 | 43.43| 64.77| -    | -    |
| Distance             |                 | 0.21 | 0.21 | -    | -    |
| Rounds               |                 | 8.61 | 6.04 | -    | -    |
| Failures             |                 | 5%   | 28%  | 96%  | 100% |
4.3. SBNP in Nonmonotonic Scenarios

To evaluate SBNP in the BELLs and BELLc scenarios we ran 10 negotiations for each Bell utility function pair, which implies the execution of 100 negotiations for each scenario. In each negotiation the SBNP agents’ negotiation parameters are randomly varied as in the previous evaluation of SBNP for the monotonic scenarios. The results are reported in Table 2. It can be seen how the performance of SBNP is very poor for more than two issues. Even for two issues, in the BELLc scenario the mean distance from the Pareto frontier is 0.21, and the negotiation time goes to 43 seconds. For more than five issues the failure rate drastically increases, and for 20 issues the failure rate is 100%.

5. RBNP

In this section we present RBNP. Section 5.1 provides an overview of the negotiation problem by describing the structure of preferences of the negotiation space. In Sections 5.2, 5.3 and 5.3.3 we elaborate the negotiation protocol, the decision mechanisms, and propose three different concession strategies for the agents. Finally, in Section 5.4 a summary of the negotiation parameters is presented.

5.1. Overview of the Problem and Structure of Preferences

We define the issues under negotiation as a finite set of variables \( X = \{x_i | i = 1, \ldots, n\} \), where each issue \( x_i \) can be normalized to a continuous or discrete range \( d_i = [0, 100] \). Without loss of generality we will assume a continuous range. Accordingly, the negotiation domain can be denoted by \( D = [0, 100]^n \), where a contract is a vector \( s = \{x_i^s | i = 1, \ldots, n\} \) defined by the issues’ values. The aim of the agents is to reach an agreement on a contract.

Each agent \( A_i \in \{b, s\} \) owns a utility function \( U_i : D \rightarrow \mathbb{R} \) that gives the payoff the agent assigns to a contract. The utility function can be described as any mapping function between the negotiation space contracts and the set of real numbers. In contrast to prior works, which usually assume that agents have relatively simple preferences on the issues (e.g., can be characterized by strictly concave utility functions), we make a more general assumption that the preference of each agent can be nonmonotonic and nondifferentiable. We only require the preferences to be rational:

**Definition 3.** The ordinal preference \( \prec_i \) of agent \( A_i \) in the negotiation domain is rational if it satisfies the following conditions:

1. **Strict preference is asymmetric:** There is no pair of \( x \) and \( x' \) in \( X \) such that \( x \prec_i x' \) and \( x' \prec_i x \);
2. **Transitivity:** For all \( x, x', \) and \( x'' \) in \( X \), if \( x \lesssim_i x' \) and \( x' \lesssim_i x'' \), then \( x \lesssim_i x'' \);
3. **Completeness:** For all \( x \) and \( x' \) in \( X \), either \( x \lesssim_i x' \) or \( x' \lesssim_i x \), where \( x \lesssim_i x' \) (or \( x < x' \)) indicates that the offer \( x' \) is at least as good as (or better than) \( x \) for agent \( i \).

The first two conditions ensure the consistency of agents’ preferences in the negotiation domain, and the third condition ensures that any pair of points in the negotiation domain can be compared. When dealing with strictly concave utility functions, for any solution \( x \), the set of solutions that an agent prefers to \( x \) is strictly convex. This implies that each

---

1 With \( \{b, s\} \) we mean buyer or seller, which is a usually adopted notation in the bilateral negotiation research literature.
Pareto-efficient solution of a multi-issue negotiation is a joint tangent hyperplane of a pair of indifference curves or surfaces of the two agents, where an indifference curve (surface) or isocurve (isosurface) of an agent consists of the points that are indifferent to the agent (i.e., give the same payoff to the agent). This condition makes it tractable to find (near) Pareto-optimal solutions (Kersten and Noronha 1998). However, for nonmonotonic utility functions this condition does not hold, and then the approximation of Pareto-optimal solutions under incomplete information settings turns harder.

5.1.1. Region and Overall Satisfaction Degree of a Region (OSD). The proposed negotiation protocol involves the exchange of offers defined as regions within the negotiation space. As described in detail in Section 5.3, the generation of these regions implies an optimization process, which is carried out at an individual agent level and it is constrained by a higher sized region or parent region. Therefore, the shape of the parent region will have a strong influence on the optimization performance. For simplicity and efficiency, in RBNP a region takes the form of a hypercube (see an example of region for a two-dimensional negotiation space in Figure 3). The main advantage of using hypercubes instead of any other volume is that with hypercubes we only need to perform a bounded local optimization, while with any other hyperpolyedron or an hypersphere, a nonlinear constrained optimization is needed, and bounded optimization is much faster than nonlinear constrained optimization. We formally define a region as follows:

**Definition 4.** A region $R_i$ of the n-dimensional negotiation space of agent $A_i$ is formed by the set of contracts lying within the hypercube defined as a 2-tuple $R_i = < c, r >$, where $c \in D$ and $r \in \mathbb{R}$ define the center and the edge length of the hypercube. We name the region $R_i = < c, r >$ as a region of size $r$.

In the negotiation protocol we propose, agents’ offers are regions within the contract space. Hence, an evaluation mechanism is needed to assess regions. We define the overall satisfaction degree (OSD) function to provide this evaluation mechanism.
Definition 5. The OSD of a region $R_i$ for an agent $A_i$ is defined as a ratio of the number of contracts with a utility value above a reservation utility. Let $S_{R_i} = \{ s_k \in D \mid k = 1, \ldots, nsc \}$ be a set of $nsc \in \mathbb{N}^+$ randomly sampled contracts in $R_i$, $U_{rs}^i$ the reservation utility, and $S_{rs}^i$ the subset of acceptable contracts in $S_{R_i}$ which satisfy $U_i(s_k) \geq U_{rs}^i$. An agent $A_i$ computes the OSD of a region $R_i$ as:

$$\text{OSD}(R_i) = \frac{|S_{rs}^i|}{|S_{R_i}|}.$$ 

Under complete uncertainty about the preferences of the opponent, and from the individual perspective of an agent, the OSD of a region $R_i$ for an agent $A_i$ may be understood as an estimate of the probability of finding agreements within this region. The rationale behind this is straightforward. The probability of finding an agreement in a region proposed by an agent where she has many acceptable contracts will be higher than if there are fewer acceptable contracts.

5.2. Negotiation Protocol

The RBNP negotiation protocol is based on a recursive bargaining mechanism which is performed concurrently and synchronously by two agents. RBNP may be seen from the perspective of communicative acts as a protocol with two communication channels. Considering two agents $A_b$ and $A_s$, one of the channels is used by agent $A_b$ to submit offers (i.e., regions) and by agent $A_s$ to respond to $A_b$. Agent $A_s$ may, for example, accept or reject the offer, or respond with a more elaborated communicative act. In a similar way, the other channel is used by agent $A_s$ to submit proposals and by agent $A_b$ to respond. We can see that with this configuration both channels may work concurrently. One of the agents may be exploring a region of the negotiation space, while the other agent may explore a completely different area. However, we enforce a synchronization rule whereby two offers (one in each channel) must be followed by the corresponding two responses before a new set of offers is to be sent. By means of this simultaneous-offer game we balance the agents’ negotiation power. Note that there would exist the possibility to operate both channels without any synchronization rule. In this case, we could have channels operating at a different message rate. Also note that we could omit one of the channels in the implementation of the protocol and then to configure an asymmetric protocol, where one of the agents always makes the proposals and the other agent only responds.

Previous description of the protocol is only a piece of the puzzle. At a higher level, as described in Section 5.2.2, a set of Negotiation Process Rules enforce the current size of the proposals (regions) and the area where such proposals must be contained when agents exchange offers and responses. The size of the offers and the exploration area are always shared by both channels. The size of the proposals (i.e., the size of the regions proposed by the agents), and the area where the proposals must be contained change according to a set of transition rules. These transition rules are fired by agreements reached in one or both channels, or when agents consider that it is unfeasible to reach an agreement on one or both of the channels. In this section the high-level operation of the protocol is presented. Section 5.3 presents the low-level description of the protocol, which is focused on the bargaining mechanisms.

We have formalized the protocol as a negotiation dialogue composed of bargaining threads (BTHs).

Definition 6. A negotiation dialogue $\text{NegD} = \{ b_{t_0}^i \rightarrow b_{t_1}^i \rightarrow \cdots \}$ is a sequence of BTHs, where each thread starts in a period $t_n$. Each BTH...
is a sequential exchange of offers (regions) and responses to the offers. The pair \((R_b, R_s)_{t+1}^{t+a}\) represents the exchange of offers of size \(r_{im}\) in period \(t_n + a\), and \((res_b, res_s)_{t+1}^{t+a+1}\) the responses to these offers. The dialogue admits three types of responses: Accept, Reject, and Request, and each BTH is restricted to the exchange of offers of size \(r_{im}\) (we denote a thread exchanging offers of size \(r_{im}\) as a BTH of size \(r_{im}\)).

Note that any pair \(R_x, res_x\) will be driven through one of the communication channels, where one of the agents submits \(R_x\) and the other one responds with \(res_x\). Before a negotiation dialogue begins, it is assumed that agents agree on a finite set of region sizes

\[
\text{RegS} = \{r_i | i = 1, \ldots, m; \forall l < k, r_l > r_k\},
\]

where \(r_m\) represents the lowest size and \(r_1\) the highest size. We name \(m\) the search depth of the negotiation dialogue. To simplify the characterization of RegS we have defined the function

\[
F_{\text{RegS}}(x) = \left(\frac{1}{e^{\tau_x} - 1}\right) \cdot (e^{\tau_x} - 1) \cdot (r_1 - r_m) + r_m,
\]

which generates the distribution of region sizes. Parameter \(x\) is an element from a set of \(m\) equally spaced points from 1 to 0, and the curvature parameter \(\tau_x\) modulates the distribution of region sizes between \(r_1\) and \(r_m\). The distribution is more linear as \(\tau_x\) approximates 0. For \(\tau_x > 0\) we have a higher density of lower sized regions, while for \(\tau_x < 0\) the density is higher for higher sized regions. Accordingly, only the \(m, r_1, r_m,\) and \(\tau_x\) values need to be agreed. We define a one-shot prenegotiation mechanism to find an agreement on these parameters. Agents exchange their preferred values, and then the following rules are applied: (a) \(m = \text{mean}(m^b, m^s)\); (b) \(r_1 = \text{max}(r_1^b, r_1^s)\); (c) \(r_m = \text{min}(r_m^b, r_m^s)\); and (d) \(\tau_x = \text{mean}(\tau_x^b, \tau_x^s)\).

### 5.2.1. Negotiation Process.

Figure 4 shows an example of the dynamics of RBNP in monotonic and nonmonotonic negotiation scenarios. It can be seen how offers are regions within the contract space instead of contracts, and how the size of the regions decreases until an agreement is found. Negotiation starts with a BTH of size \(r_1\) (i.e., the highest size). The main goal of the agents is to reach a final agreement on a region of size \(r_{im}\) (note that \(r_{im}\) is the tolerance defined by the agents for a region to be considered as a contract). Every time a region (offer) is accepted by the opponent in any of the communication channels, the current BTH ends, and negotiation moves toward a new thread of lower size. The search in the new thread is restricted to the domain of the reached agreement (i.e., the domain of the accepted region) in the previous thread. If agents abort the dialogue in the current thread because of the impossibility to reach an agreement, they return to negotiate higher sized regions.

Transitions among BTHs are controlled by a state diagram, which structures the exploration of the negotiation space. We name this state diagram negotiation search tree. Its topology is agreed by the agents prior to negotiation, and depends on the set of region sizes RegS and a set \(\text{NumB} = \{nbt_{r_1}, nbt_{r_2}, \ldots, nbt_{r_{m-1}}\}\), which defines the number \(nbt_{r_{im}}\) of BTHs of size \(r_{im}\) which may be derived from any parent BTH of size \(r_{im-1}\). Note that \(nbt_{r_m}\) is not included in \(\text{NumB}\) because its value is always 1, as we will see in the example below. The procedure to define a shared search tree topology consists in the minimization of the tree topologies revealed by the agents before negotiating:

\[
\text{NumB} = \{\min(nbt_{r_1}^b, nbt_{r_1}^s), \min(nbt_{r_2}^b, nbt_{r_2}^s), \ldots, \min(nbt_{r_{m-1}}^b, nbt_{r_{m-1}}^s)\}.
\]
Figure 4. Example of RBNP negotiations in monotonic (a) and nonmonotonic (b) scenarios. The solid squares represent the sellers’ offers and the dashed squares the buyers’ offers.

Figure 5(a) shows an example of search tree for a set of regions sizes \( \text{RegS} = \{r_1, r_2, r_3\} \) and \( \text{NumB} = \{2, 2\} \). The node \( D \) represents the global negotiation domain, and the \( r_{im} \) nodes represent BTHs of size \( r_{im} \) (i.e., bargaining threads where agents try to find an agreement on a region of size \( r_{im} \)). Braces show the domain relationship between BTHs. In the example, there will be as much as two BTHs of size \( r_2 \) derived from a BTH of size \( r_1 \). The solid lines show descending transitions that become active when agents agree on an offer, and the dashed lines ascending transitions that trigger when the current thread is unfeasible. The general rule that governs the movement through the negotiation search tree is that each node can be visited only once. Also, we can see that there is no reason to have more than one child per node of size \( r_2 \), so that \( nb_{trm} \) is not included in \( \text{NumB} \). To prove this, let us assume that agents are in a BTH of size \( r_2 \), and that we have added a child to each node of size \( r_2 \). A transition is fired as soon as agents find an agreement. This transition takes the agents to the first child node of size \( r_3 \). Once there, we may encounter two possibilities: either agents reach an agreement on an outcome of size \( r_3 \) and negotiation ends, or agents abandon the current \( r_3 \) node and jump to a new BTH of size \( r_2 \), the latter meaning that the added child node is not reachable.
Finally, to clarify the operation of the negotiation process, Figure 5(b) provides an example of negotiation. Agents begin to negotiate regions of size $r_1$ (transition 1). In transition 2, an agreement on a region of size $r_1$ has been found, and then, agents engage in a BTH of size $r_2$, where they find a new agreement (transition 3). Now, agents negotiate in a BTH of size $r_3$, however, they are not able to find an agreement, and move upward to a new BTH of size $r_2$. Note that this BTH is derived from the previous agreement on a region of size $r_1$. Again, they are not able to find an agreement and move to a new BTH of size $r_1$ (transition 5). Negotiation ends with transitions 6, 7, and 8, which represent consecutive agreements on regions of size $r_1$, $r_2$, and $r_3$, respectively.

5.2.2. Negotiation Process Rules. In the following, a detailed description of the protocol rules which refine how the negotiation progresses through the different BTHs is provided, i.e., how the expansion/contraction of regions operate. We assume that agents share a negotiation search tree.

**BTHr1: Starting Rule:** A negotiation begins with $b^{0}_{r_1}$, where agents negotiate on regions which are restricted by the whole domain $D$.

**BTHr2: General Transition Rule:** For any transition between two threads $b^{l}_{r_l} \rightarrow b^{l+1}_{r_k}$, $r_l$ and $r_k$ must satisfy $|l - k| = 1$. This means that any transition is performed between adjacent region sizes.

**BTHr3: Acceptance Transition Rule:** A descending transition $b^{l}_{r_{im}} \rightarrow b^{l+1}_{r_{im+1}}$, is fired when in any given period in $b^{l}_{r_{im}}$, $R^{l+2}_{b_{im}}$, $R^{l+2}_{s_{im}}$, or both $R^{l+2}_{b_{im}}$ and $R^{l+2}_{s_{im}}$ are accepted as a solution by $A_s$, $A_b$ or both $A_s$ and $A_b$. Once an agent’s offer in $b^{l}_{r_{im}}$ has been accepted by the opponent, the offer is considered the child of the most recent accepted region of size $r_{im-1}$ in $b^{l}_{r_{im-1}}$. In case of simultaneous acceptance of regions (i.e., both agents accept the opponents’ proposals), only a region is chosen as a valid agreement. In order to balance the agents’ negotiation power, the protocol enforces that each agent alternatively selects the region which will be considered the valid agreement (i.e., her own offer or the opponent’s offer).
BTHr4: Unfeasibility Transition Rule: An ascending transition \( b_{rim}^i \rightarrow b_{rim-1}^{i+1} \) is fired when in any given period in \( b_{rim}^i, A_b, A_s \), or both \( A_b \) and \( A_s \) consider that it is unfeasible to find an agreement on regions of size \( r_{im} \) in the current thread. The unfeasibility condition of a thread is considered below in the description of the decision mechanisms in Section 5.3.

BTHr5: Negotiation Domain Rule: The exchange of offers in a BTH \( b_{rim}^i \) is confined to the negotiation space constrained by the accepted offer in the most recent thread of size \( r_{im-1} \) (i.e., \( b_{rim-1}^{a,1} \)). This rule enforces the recursive search in the negotiation space. For \( b_{rim}^i \), the search is confined to the global domain \( D \).

BTHr6: Ending Rule: Under the assumption of bounded duration of the BTHs, the transition rules BTHr2, BTHr3, BTHr4, and the search tree topology defined by RegS and NumB guarantee the completion of the negotiations. A negotiation ends once the agents have reached an agreement in a BTH \( b_{rim}^i \). In this case, the negotiation ends before completing the exploration of the search tree. If agents complete the exploration without an agreement in some \( b_{rim}^i \), we say that the negotiation has failed.

The transition rules described above determine the joint exploration strategy in the negotiation space. We have seen that the negotiation protocol is founded on a sequence of bargaining threads, and that this sequence is a recursive process of exploration. This is the high-level view of the protocol. The acceptability of a region and the unfeasibility of agreements in a bargaining thread govern the transitions between different threads. This is the low-level perspective of the protocol. The analysis of these elements within the scope of a bargaining thread is covered in the next section. Specifically, we will define how an agent generates an offer, when an offer is accepted or rejected, and how a received offer is evaluated in order to suggest a new derived offer. These mechanisms have been defined in order to facilitate the search of socially desirable regions. This search emerges through the negotiation dialogue. In any negotiation, a socially desirable and successful agreement usually emerge from concession and when agents are able to compensate the values of the different contract attributes to satisfy the opponents and at the same time their own preferences. RBNP applies this principle with the use of MovementRequest messages (see Section 5.3). An agent \( A_b \) submits a region, and the other agent \( A_s \) responds suggesting the movement of the region in a specific direction (we assume that \( A_s \) has evaluated this direction and has found good contracts for her). Agent \( A_b \) evaluates the request and if she also finds good contracts, then she attends the request and submits a new proposal (region) in the direction proposed by \( A_s \). This way, both agents will be moving to a jointly preferred area in the negotiation space.

5.3. Decision Mechanisms

We divide the decision mechanisms of an agent into three components: responding, proposing, and concession. The responding mechanism determines whether an agent should accept, reject, or suggest the movement of an offer proposed by the opponent. The proposing mechanism determines which offers should be proposed to the opponent. The concession mechanism controls the dynamics of the reservation utility \( U_{irs}^i \) of an agent \( i \) during the negotiation dialogue. Next, we describe these three components in detail.

5.3.1. Responding Mechanism. This mechanism is based on the OSD evaluation of the opponent’s offer. Based on the OSD evaluation, the response of an agent \( i \) depends on two sets of thresholds: the acceptance thresholds \( (ATH^i) \) and the quality thresholds \( (QTH^i) \). For simplicity, the quality thresholds are derived from the acceptance thresholds by means of a quality threshold factor \( q_{thf} \in \mathbb{R} \). Thus, each agent \( i \) privately defines acceptance and
quality thresholds for each region size in RegS:

\[
ATH^i = \{ath_{r_1}^i, ath_{r_2}^i, \ldots, ath_{r_m}^i\},
\]

\[
QTH^i = qthf \cdot ATH^i.
\]

As with RegS, to simplify the characterization of \(ATH^i\), we define the function

\[
F_{ATH}^i(x) = ath_{r_1}^i - (ath_{r_1}^i - ath_{r_m}^i) \cdot \left(1 - \frac{x - r_m}{r_1 - r_m}\right) \cdot e^\frac{\tau_{ia} \cdot \left( r_m - x \right)}{r_1 - r_m}.
\]

It creates the acceptance threshold distribution as a function of \(ath_{r_1}^i, ath_{r_m}^i\), and a curvature factor \(\tau_{ia}\), where \(x\) is a region size from RegS. The parameter \(\tau_{ia}\) modulates the distribution curvature, so that the distribution is linear as \(\tau_{ia}\) approaches to 0. For \(\tau_{ia} > 0\), \(F_{ATH}^i(x)\) is convex, and the curvature is higher in the lower sized regions. For \(\tau_{ia} < 0\), \(F_{ATH}^i(x)\) is concave, and the curvature is higher in the higher sized regions. Figure 6 gives several examples of threshold and region size distributions.

Now, let us assume that the current exchange of offers within the BTH \(b_{rim}^{rb}\) is \((R_b, R_s)^{r_{rim}^{sa}}\).

The response strategy for agent \(A_b\) will be (for \(A_s\) the strategy is similar):

**Response Strategy:**

\[
\begin{align*}
\text{Accept} & \quad \text{If } OSD((R_s)^{r_{rim}^{sa}}) \geq ath_{rim}^b, \\
\text{MovementRequest} & \quad \text{If } OSD((R_s)^{r_{rim}^{sa}}) < ath_{rim}^b \text{ AND } OSD((R_s)^{r_{rim}^{sa}} - (R_s)^{r_{rim}^{sa}}) \geq qth_{rim}^b, \\
\text{Reject} & \quad \text{Otherwise.}
\end{align*}
\]

An agent accepts an opponent’s offer if its OSD is higher than \(ath_{rim}^b\), so that the current BTH ends and the negotiation progresses following the negotiation protocol (see rule BTHr3). If the opponent’s offer is not accepted, the agent evaluates the quality of the offer, and if the measured quality is above the quality threshold an offer movement request is
generated. To evaluate the quality of the received offer, an agent computes the OSD of the offer surroundings, which is the space between the offer and a concentric region with the parent domain size. Finally, if the measured quality is below $q_{th_{rin}}$, the offer is rejected. Note that the $q_{thf}$ factor controls the tendency to suggest the movement of the opponents’ offers.

The offer movement request is defined as a vector $\vec{v}_q(R_s)_{rin} \in D$. This vector defines the preferred direction an agent wants the opponent to move his next offer. In order to obtain $\vec{v}_q$, we use the center of mass of the filtered samples $S_{rs}^{R_s}$ (those above the reservation utility $U_{rin}^s$) taken in the quality evaluation of the opponent’s offer:

$$\vec{v}_q(R_s)_{rin} = \text{norm} \left( \frac{\sum_{s_k \in S_{rs}^{R_s}} (U_b(s_k) \cdot s_k)}{\sum_{s_k \in S_{rs}^{R_s}} U_b(s_k)} - c_s^{rin} \right).$$

The first term is the center of mass, and the second term $c_s^{rin}$ is the center of the opponent’s offer $(R_s)_{rin}$. The norm function converts the difference into a unit vector.

Figure 7 shows an example of operation of the responding mechanism. For ease of presentation, only the proposals of agent $A_b$ are shown. We assume that the reservation utility $U_{i}^{rs}$ is equal to the threshold utility $U_{i}^{th}$, and that it does not change during negotiation.
In Figure 7(a), all the contracts within $R^{t+1}$ give agent $A_s$ a payoff below her reservation utility. In addition, $R^{t+1}$ is far from the reservation utility isocurve. Hence, $A_s$ rejects $R^{t+1}$. In period $t_n + 3$ agent $A_b$ offers $R^{t+1}$. This offer provides $A_s$ an OSD which falls below $\alpha th$. However, when $A_s$ evaluates the quality of the offer, there are several sample contracts which provide her a utility above her reservation utility. If we assume that the computed OSD is higher than $qth$, then the offer is considered of good quality and a vector $v_q$ is obtained and sent to agent $A_b$ as a movement request. Figure 7(b) shows that agent $A_b$ has considered the movement request by proposing $R^{t+5}$. This offer is finally accepted by agent $A_s$.

5.3.2. Proposing Mechanism. To make an offer, an agent generates one or more regions until a region with an OSD above the current $\alpha th$ is found. The generation of regions is governed by a set of local rules which control the search of these regions. More specifically, regions during a BTH are generated on the basis of the history of passed offers. Thus, the proposing mechanism is based on three submechanisms by which an agent generates regions derived from a previous offer if exists (reference offer), and a set of rules which govern the movement through the history of passed offers. Note that we distinguish between region and offer. A region is a candidate to be an offer, and an offer is a region that has been or is going to be proposed to the opponent.

Let $b^{t_n}$ be the current thread, and let us consider the proposing mechanism from the perspective of agent $A_b$. The three submechanisms are formally defined as follows:

Root Region Generation: This submechanism is applied when there are not passed offers that may be taken as a reference to generate a new region. In this case, $A_b$ applies simulated annealing to her utility function to find a local optimum $s^{t+1}$, and generates the region $$(R^{t+1})_{t_n} = < s^{t+1}, r_{im} > .$$

This region is named root region. Depending on the annealing temperature used by the optimizer, an agent controls the local optima search performance. For a high temperature the probability of finding regions with a high OSD increases. However, an agent searching for root regions with a very high OSD can get stuck in a reduced set of local optima, and successively generate the same proposals to the opponent. If the set of proposals are comprised within a zone of no agreement, the negotiation will fail. In order to mitigate this problem an agent must take care of the variability of her offers, applying a coherent concession strategy with respect to the root region generation. This issue is covered when describing the concession mechanisms in Section 5.3.3.

Directed Child Region Generation: In this submechanism agents are assumed to have a previous offer as a reference to generate a new region. The term ‘directed’ means that agents derive the new region from the center of the reference offer in the direction proposed by the opponents in a MovementRequest message. Therefore, $A_b$ generates a child region $$(R^{t+2})_{t_n} = < s^{t+2}, r_{im} > ,$$

where $s^{t+2} = s^{t+1} + \max(r_{im}, 0.1 \cdot r_{im-1}) \cdot \vec{v_q} ((R^{t+1})_{t_n})$. The child region is generated in the direction $\vec{v_q}$ proposed by the opponent, at a distance $\max(r_{im}, 0.1 \cdot r_{im-1})$ from the center $s^{t+1}$. We say that $(R^{t+2})_{t_n}$ is a directed child region of $(R^{t+1})_{t_n}$. The $\max$ operator guarantees that the movement is at least equal to the region edge length. For low-sized regions it guarantees that the movement is at least the 10% of the parent domain length. Note that the child region concept within the proposing mechanism is different from the parental relation of regions in the BTHs context. In the BTHs context, a parent is a specific
domain which restricts the bargaining within a BTH. In this mechanism, a child region is a region which has been generated from other region of equal size.

**Random Child Region Generation:** In this submechanism agents are assumed to have a previous offer as a reference to generate new regions. However, in contrast to the ‘directed’ generation, the new region is generated in a random direction from the center of the reference offer. Therefore, $A_b$ generates a child region

$$(R_b)_{rim}^{a+a+2} = s_{a+a+2}^{rim}$$

where $s_{a+a+2} = s_{a+a} + \max(rim \cdot 0.1 \cdot rim^{-1}) \cdot vq_{random}$. In contrast to a directed child region, the random child region is generated on a random direction from the parent region center. We say that $(R_b)_{rim}^{a+a+2}$ is a random child region of $(R_b)_{rim}^{a+a}$.

To prepare an offer, an agent generates a region $(R_i)_{rim}$ by means of any of these generation submechanisms, and then evaluates whether its OSD is above the current acceptance threshold $ath_{rim}$, which is obtained from the set $ATH_i$. It means that we use the same thresholds to generate offers and to accept opponent’s offers. However, it must be noted that an agent could use different thresholds with a strategic purpose.

The rules which govern the generation and history of offers within a BTH, and consequently, which of the three submechanisms described above is used during negotiation are as follows:

**OGr1: First Region Rule:** The first region in a BTH is always a root region.

**OGr2: Unacceptable Region Rule:** Any unacceptable region (note that what we mean is that the agent generates a region with an OSD below the current $ath_{rim}$ threshold) is discarded and then a new search is performed in order to find a new region. If the unacceptable region is a root then the agent searches for a new root region; otherwise, the agent generates a new random child region.

**OGr3: Accepted Offer Rule:** The acceptance by the opponent of an offer implies that the current BTH ends, and that the negotiation progresses following the negotiation protocol to a new BTH or to a deal.

**OGr4: Rejected Offer Rule:** The rejection by the opponent of an offer implies that the agent moves to the rejected offer’s parent, and then searches for a new random child region. If the rejected offer is a root the agent searches for a new root region to prepare a new offer.

**OGr5: Movement Request Rule:** An agent tries to generate a directed child region upon the reception of a movement request.

**OGr6: Root Offer Limits Rule:** A configurable parameter $nR \in N$ bounds the number of root offers and the number of trials when searching for root regions in a BTH. A configurable parameter $nD \in N$ bounds the number of descendants of a root offer. A BTH is considered *unfeasible* when the $nR$ limit is reached. If an agent exceeds $nD$, a new root offer search is performed.

**OGr7: Child Region Limits Rule:** For a reference offer, the parameter $nC \in N$ bounds: the number of unacceptable child regions (i.e., generated regions whose OSD is below $ath_{rim}$), the number of child offers, and the number of rejected children. When this limit is reached, the offer is discarded and the agent moves upward in the history of offers to set a new reference offer and continue with the search process. If the considered offer is a root, the agent performs a root offer search.

It can be easily shown that a negotiation dialogue ends in a finite number of periods (i.e., exchange of offers) if both agents assign finite values to the parameters $nR$ and $nD$. 
According to the negotiation protocol the number of BTHs is bounded by NumB. Thus, the negotiation dialogue ends if the number of periods within a BTH is bounded. The \( nR \) parameter limits the number of root offers and regions, and it is established that the thread is considered unfeasible (i.e., the thread ends) when this limit is reached. The parameter \( nD \) limits the number of descendants of a root offer, and if this limit is reached the agent withdraws from the current root offer’s chain of descendants in order to search for another root offer. This in turns guarantees that for the worst case the \( nR \) limit is reached, and then a BTH ends. Hence, a negotiation dialogue ends in a finite number of periods.

5.3.3. Concession Strategies. A concession mechanism updates the aspirational utility level of an agent during a negotiation process. In SBNP, the aspirational level is identified by the agent’s reservation utility \( U_{rs}^i \), which directly controls the generation and acceptance of offers. The variation of the reservation utility during a negotiation depends on the strategy followed by the agent, and it is usually lower bounded by a utility threshold value \( U_{th}^i \). In RBNP, however, the generation and acceptance of offers depend on both the OSD operator, which is a function of the reservation utility, and on the acceptance and quality thresholds (i.e., \( ATH^i \) and \( QTH^i \)). In particular, \( ATH^i \) modulates the expected deal probability when accepting or generating offers, \( QTH^i \) regulates the generation of movement requests, while \( U_{rs}^i \) defines the aspirational utility level.

RBNP is based on a region search tree in which the negotiation process evolves through the consecution of partial agreements on regions. Hence, the concession strategies influence the search of partial agreements on regions. Assuming that the acceptance and quality thresholds remain unchanged, the concession strategy will depend on the variation of the reservation utility, which influences the dynamics of acceptance and quality evaluation of regions through the OSD. We can think of the acceptance and quality thresholds as the tools used to set the assumed risk when an agent accepts a region or evaluates its quality for a given reservation utility.

We have developed three different concession strategies for RBNP, which we believe are able to cover a wide range of possibilities: static, monotonic, and adaptive. Next we describe these strategies in detail.

**Static.** This is a purely cooperative strategy. The reservation utility is fixed to the threshold utility and does not change during the negotiation:

\[
U_{rs}^i = U_{th}^i.
\]

**Monotonic.** This is a competitive strategy. For each BTH an agent begins to negotiate at her highest reservation utility (i.e., 1), and then applies a monotonic concession protocol to update \( U_{rs}^i \). Agents concede when they find difficulties in the generation of offers, or when the opponent rejects a previous offer. A concession step factor controls the concession speed. Formally, the protocol is subject to the following four updating rules:

1. \( U_{rs}^i(t_n) = 1 \), which means that an agent begins to negotiate at each BTH at her highest aspirational level.
2. \( U_{rs}^i(t) = U_{rs}^i(t - 1) - \delta \) when the generation of a root region fails (see the Root Region Generation mechanism and the OGr2 rule). The \( \delta \) parameter is a real-valued updating step factor which determines the concession rate.
3. \( U_{rs}^i(t) = U_{rs}^i(t - 1) - \delta \) when the opponent rejects a previous offer (see the OGr4 rule).
4. \( U_{th}^i(t) \geq U_{rs}^i(t - 1) - \delta \), which means that the objective utility is lower bounded by a reservation value. Once \( U_{rs}^i = U_{th}^i \) the second and third rules deactivate.
Adaptive. This is a semi-cooperative strategy. Agents dynamically adjust their reservation utilities depending on the evolution of the negotiation dialogue. The rationale behind this concession strategy is to act in a semi-cooperative way, conceding when the negotiation dialogue turns out to be harder, and increasing the aspirational level when the exploration of the search space is satisfactory. The protocol is subject to the following rules:

1. $U_{rs}^i(t_0) = 1$, which means that an agent begins to negotiate in the first BTH of a negotiation at her higher aspirational level. This rule is different from the first rule in the monotonic concession protocol, where the reservation utility is updated to the highest aspiration level at each new BTH.
2. $U_{rs}^i(t) = U_{rs}^i(t - 1) - \delta$ when the generation of a root region fails (see the Root Region Generation mechanism and the OGr2 rule).
3. $U_{rs}^i(t) = U_{rs}^i(t - 1) - \delta$ when the opponent rejects the previous offer (see the OGr4 rule).
4. $U_{rs}^i(t) = U_{rs}^i(t - 1) + \delta$ when the opponent accepts the previous offer (see the OGr3 rule).
5. $U_{rs}^i(t) = U_{rs}^i(t - 1) + \delta$ when the opponent sends a movement request (see the OGr5 rule).
6. $U_{rs}^i(t) \geq U_{th}^i$, which means that the reservation utility is lower bounded by a threshold value. When $U_{rs}^i = U_{th}^i$ the second and third rules provisionally deactivate until $U_{rs}^i > U_{th}^i$.

Figure 8 shows how agents update their reservation utilities during a negotiation dialogue for each of the proposed concession strategies.

5.4. Summary of Negotiation Parameters

We group all the negotiation parameters into a common negotiation profile shared by both agents and an individual negotiation profile which defines the behavior of a single agent. The common negotiation profile is agreed and known by both agents prior to the negotiation dialogue. For the individual negotiation profile we assume an incomplete information setting, which means that an agent does not know the opponent’s profile.

Common negotiation profile

- Issues under negotiation $X$ and the corresponding negotiation space $D$.
- Set RegS defined by $m$, $r_1$, $r_m$, and $\tau_r$.
- Set NumB.

Individual negotiation profile

- Utility function $U_i$.
- Utility threshold $U_{th}^i$ (i.e., a minimum utility value).
- Reservation utility updating factor $\delta_i$.
- Number $nsc_i$ of sample contracts used in the computation of the OSD.
- Limits for the proposing mechanism ($nR_i$, $nD_i$, $nC_i$).
- Acceptance thresholds $\text{ATH}^i$ defined by $\text{ATH}^i_{r_1}$, $\text{ATH}^i_{r_m}$, and $\tau_a^i$.
- Quality factor $\text{qthf}^i_i$. 
6. EXPERIMENTAL EVALUATION OF RBNP

In this section, we test the performance of RBNP and show that RBNP outperforms SBNP under monotonic and nonmonotonic negotiation scenarios. We have evaluated the CES, BELLs, and BELLc negotiation scenarios, for the Static, Adaptive, and Monotonic strategies. For the sake of brevity we only show the detailed experiments for the CES and BELLc scenarios when both agents play the Static strategy. The detailed results for the rest of the strategies and scenarios can be found at http://it.aut.uah.es/miguellop/links.

We first describe the experimental setup. We then show the results for the CES and BELLc scenarios when using the Static strategy, and the summary of results for all the scenarios and strategies. Finally, an in-depth evaluation of the influence of the search depth and the number of issues is covered in Section 6.3.

6.1. Experimental Setup

For tractability, in order to obtain the optimal negotiation profiles for RBNP, we first consider symmetric negotiation encounters (i.e., both agents use the same strategies and the same negotiation parameters), which model purely cooperative (Static vs. Static), semi-cooperative (Adaptive vs. Adaptive), and purely competitive (Monotonic vs. Monotonic) strategic scenarios (asymmetry is considered in Section 7). Each combination of negotiation scenario and strategy is evaluated using different negotiation profiles. To generate the
different negotiation profiles, we base our analysis on a full-factorial designed experiment. Thus, six factors have been chosen: search depth $m$, acceptance threshold for the highest sized regions $ath_{r_1}$, curvature parameter of acceptance thresholds $\tau_a$, quality threshold $qthf$, region size curvature parameter $\tau_r$, and number of BTH NumB. These factors may vary as follows:

1. $m = \{1, 2, 5, 10\}$ defines four search depths. The instance $m = 1$ is used as a control experiment, so that agents negotiate on contracts or points in the solution space from the beginning of the bargaining, and there is no recursivity in the joint exploration of the solution space.

2. $ath_{r_1} = \{0.5\%, 2\%, 10\%\}$ ranges from low to high expected deal probability in the acceptance and generation of the highest sized offers.

3. $\tau_a = \{1, 10, 20\}$ ranges from quasi-linear distribution of acceptance and quality thresholds to highly curved distribution.

4. $qthf = \{\text{off, 1, 0.5}\}$ defines three possibilities for the operation of the offer movement request mechanism: deactivated, low sensibility, and high sensibility.

5. $\tau_r = \{0.01, 2, -2\}$ defines three configurations for the distribution of region sizes: linear, more dense distribution of lower sized regions, and more dense distribution of higher sized regions.

6. $\text{NumB}_{\text{house}} = \{3, 3, 3, 1, \ldots\}$, $\text{NumB}_{\text{rocket}} = \{3, 1, 1, \ldots, 3, 3\}$, $\text{NumB}_{\text{uniform}} = \{3, 1, 1, \ldots\}$, $\text{NumB}_{\text{binary}} = \{3, 2, 2, \ldots, 2\}$ define four different search tree topologies. The house search tree exhibits a high number of nodes in the upper levels. Note that we limit the number of children nodes to three in order to not excessively increase the negotiation times. The rocket configuration has a higher density of nodes in the lower levels, while the uniform tree defines the same number of nodes at each level. Finally, the binary configuration proposes a binary search tree.

The rest of the negotiation profile parameters are fixed as follows:

- $n = 10$ issues.
- $nR = 50$; $nD = 10$; $nC = 2$.
- $\text{nsc} = 32 \cdot n$ makes the number of sampled contracts in the computation of the OSD proportional to the number of issues.
- $r_1 = 50$; $r_m = 1e - 10$ defines the highest sized regions as hypercubes with an edge of length 50 (i.e., 50% of the negotiation domain), and the lowest sized regions as hypercubes with an edge length of $1e - 10$. It means that a region with an edge length of $1e - 10$ may be considered as a contract or point in the solution space.
- $U_{th} = [0.1, 0.3]$ defines a range for the agents’ utility thresholds. For each experiment, an agent randomly picks up a utility threshold within this range. The highest utility level for an agent is 1.
- $ath_{r_m} = 1$ specifies that the OSD of the lowest sized region $r_m$ has to be 100%. Recall that a region of size $r_m$ is assumed to be a contract or point in the solution space, and thus, all the points within the region should have the same properties.

This configuration provides $4 \times 3 \times 3 \times 3 \times 3 \times 4 = 1296$ experimental runs, where each experimental run tests a negotiation profile. In the monotonic scenario, we run eight negotiations for each CES utility function pair (i.e., 32 negotiations). In the nonmonotonic scenarios, we run three negotiations for each utility function pair (i.e., 30 negotiations).
We measure the distance of the outcome utilities from the Pareto frontier, the failure rate, the negotiation time, and the number of negotiation rounds. The Pareto frontier is computed with a genetic multi-objective optimizer. To analyze the different measures obtained in the experiments, we use a six factors N-way ANOVA with fixed effects, or a Kruskal–Wallis test for nonparametric one-way analysis of variance (Montgomery and Runger 1996). The ANOVA and Kruskal–Wallis analysis determine whether there are any differences among the measured variable means of the different negotiation profiles (i.e., the different factor levels). To compare the different means we use a Tukey–Kramer test (Montgomery and Runger 1996), and represent the results using multiple comparison test plots. A multiple comparison test plot represents in the vertical axis the different factors under evaluation, and in the horizontal axis the means of the evaluated measurements. For a given combination of parameters (factor level), the circle and the horizontal line represent the mean value and the confidence interval, respectively. In Figure 10 are shown several multiple comparison test plots for the Pareto distance. Figure 10(d) represents the different combinations of factor levels \( m \) and \( ath \), and their corresponding Pareto distance mean values.

For all the experiments, we first try to apply ANOVA for three-way interaction effects. We then apply an iterative procedure to omit the interaction terms whose \( p \)-value is larger than 0.05 (i.e., the term is not significant) and pool their effects into the error term. To validate the ANOVA model, we use a normal probability plot to test residual normality. To test homogeneity of variance, we plot the residuals versus the corresponding predicted values, and the residuals versus the factor levels. If these plots do not satisfy the variance hypothesis, then we check that the ratios of residual variances among the different groups do not exceed the ratio 3:1. If the hypotheses do not hold, we try with \( \log, \sqrt{\cdot} \), and \( \text{range} \) transformations. The \( \text{range} \) transformation computes the ranks of the values in the dependent variable. If any of the values are tied, their average rank is computed. If the model is not valid after applying the transformations, we limit the analysis to a Kruskal–Wallis one-way test analysis of variance. In these cases, a good strategy to add information to the analysis of variance is to represent the corresponding histograms and to obtain conclusions from them. For the analysis of negotiation time and number of negotiation rounds we will apply this method.

To simplify the presentation of results, the model validation plots have been omitted, and only the most significative Tukey–Kramer test plots are presented (i.e., the plots which show the most significative results in terms of the performance of the measured effect, failure rate, Pareto-distance, negotiation time, or number of rounds). Shown ANOVA tables are those obtained as a result of the described iterative procedure that pools the nonsignificant terms into the error term.

Experiments were coded in Matlab and run on a 2.4-Ghz Intel Core 2 Duo processor with 2 GB memory using Mac OS X 10.5.8.

6.2. Results

6.2.1. CES-Static Results

Failure rate. The failure rate is 0.

Pareto distance. The ANOVA table in Figure 9 shows that the distance to the Pareto frontier depends on the search depth \( m \), the curvature parameters \( \tau_a \) and \( \tau_r \) (\( \tau_{aua} \) and \( \tau_{aur} \) in the ANOVA table), and the acceptance threshold \( ath_{\tau_1} \) (\( ath \) in the ANOVA table). However, the \( qthf \) parameter has no influence on the outcomes. The rationale behind this is that offers
are rapidly accepted by the opponent and the movement request mechanism does not activate. Finally, the Tukey–Kramer test plots in Figure 10 show that the most significative parameter is the search depth (see Figure 10 (a)), and that the two-factor interactions do not provide a significant improvement in the results.

**Negotiation time.** Figure 11(a) shows the negotiation time histogram for the whole data set, and Figure 11(b) for those negotiations where \( m = 10 \). Of note in Figure 11(a) are the four peaks, which are expected to correspond to the negotiation time distributions for the different search depths. A nonparametric one-way ANOVA confirms the strong influence of search depth on negotiation time. In Figure 12 is the multiple comparison of negotiation time mean ranks. The following experiments are performed for \( m = 10 \), i.e., for the search depth where negotiators reach the best outcomes.

The ANOVA table for the range transformation of negotiation time is shown in Figure 13, and in Figure 14 is the most significative multiple comparison test. Negotiation time improves for distributions of acceptance thresholds (\( \tau_a = \{10, 20\} \)) and a more dense distribution of lower sized regions (\( \tau_r = 2 \)).

**Negotiation rounds.** The analysis is performed on the whole data set. In Table 3 is the cross-tabulation of depth search versus number of rounds, which shows that the number of rounds is proportional to the search depth. For example, when the search depth is 10, in 10327 negotiation tests, it takes 10 negotiation rounds to reach an agreement.

### 6.2.2. BELLc-Static Results

#### 6.2.2.1. Failure rate.

Figure 15 shows the ANOVA table for the number of failures per negotiation profile test. We can see that the number of failures depends on the search depth \( m \) and the acceptance threshold \( ath \), and that both parameters interact. To evaluate the influence of the parameter values on the mean failure rate, we perform a Tukey–Kramer multiple comparison test. Figure 16 shows the most significative multiple comparison test plots. We detect a strong influence of \( ath \) on failure rate. The best results are obtained for \( m = \{2, 5, 10\} \) and \( ath = \{0.02 - 1, 0.005 - 1\} \). For this factor (parameter value) combination the failure rate approaches to 0%. If we make \( ath_{\text{target}} = 0.1 \) the failure rate goes to 50% (note that the number of failures is around 15, and the number of experiments is 30). For the 648 negotiation profiles in which \( m = \{2, 5, 10\} \) and \( ath = \{0.02 - 1, 0.005 - 1\} \), the tabulation of the number of failures versus negotiation profile gives the following results: 16 negotiation profiles where there is only one failure, three negotiation profiles with two failures, and one negotiation profile with three failures.
FIGURE 10. Tukey–Kramer multiple comparison tests for Pareto distance in CES-Static scenarios.
FIGURE 11. Negotiation time histograms for the whole data set (a), and for negotiations where $m = 10$ (b) in CES-Static scenarios.

FIGURE 12. Nonparametric multiple comparison test of negotiation time mean ranks versus search depth $m$, in CES-Static scenarios.

FIGURE 13. ANOVA table for negotiation time in CES-Static scenarios and $m = 10$.

6.2.2.2. Pareto distance. We consider the whole data set (i.e., we evaluate the Pareto distance for all the negotiation profiles, including those where the number of failures is high). Figure 17 shows the ANOVA table, and in Figure 18 are the most significative multiple comparison test plots. The ANOVA table shows that Pareto distance depends on the search depth, the distribution of acceptance thresholds, and the distribution of region sizes. Search
FIGURE 14. Tukey–Kramer multiple comparison tests for negotiation time in CES-Static scenarios and $m = 10$.

TABLE 3. Cross-tabulation of Depth Search versus Number of Rounds in the CES-Static Scenarios.

| Rounds | 1 | 2 | 3 | 5 | 6 | 7 | 10 | 11 |
|--------|---|---|---|---|---|---|----|----|
| 1      | 10,356 | 12 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2      | 0 | 10,351 | 17 | 0 | 0 | 0 | 0 | 0 |
| 5      | 0 | 0 | 0 | 10,328 | 38 | 2 | 0 | 0 |
| 10     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10,327 | 41 |

FIGURE 15. ANOVA table for the number of failures in BELLc-Static scenarios.

FIGURE 16. Multiple comparison test for the number of failures in BELLc-Static scenarios. The $ath$ factor includes the two thresholds from ATH: $ath_{r_1} = (0.1, 0.02, 0.005)$ and $ath_{r_m} = 1$. 
FIGURE 17. ANOVA table for Pareto distance in BELLc-Static scenarios and the whole data set.

FIGURE 18. Most significative Tukey–Kramer multiple comparison test for Pareto distance in BELLc-Static scenarios and the whole data set.

depth interacts with the rest of the parameters, and there exists a small interaction between the acceptance threshold and the decay factor of acceptance thresholds. Looking at the multiple comparison test plots, we can conclude that the best results are obtained for $m = 10$ and $\tau_a = taua = 1$. Of note in Figure 18(a) is the good performance in terms of Pareto distance when $ath = 0.1 - 1$. However, as we have seen in the analysis of the number of failures, a
high acceptance threshold for higher sized regions (i.e., $ath_r$) negatively impacts the failure rate.

6.2.2.3. Negotiation time. The nonparametric one-way ANOVA analysis confirms the strong influence of $m$ on negotiation time. To evaluate negotiation time, we first consider the whole data set (i.e., we evaluate all the negotiation profiles), then we restrict the evaluation to the negotiation profiles where we obtained the best results in terms of failure rate and Pareto distance. Figure 19(a) shows the negotiation time histogram for the whole data set, and Figure 19(b) for those negotiations with the best configuration ($m = 10$, $ath = \{0.02 - 1, 0.005 - 1\}$, $tau_a = 1$). For the best configuration, we can see that the mean negotiation time is around 5 seconds, and that most negotiations have a duration of no more than 10 seconds. Finally, and restricted to the best configuration for failure rate, Pareto distance, and negotiation time, the three-way ANOVA for the range transformation of negotiation time does not show dependency on the $qthf$, $taur$, and $nbth$ factors.

6.2.2.4. Negotiation rounds. Figure 20 shows the negotiation rounds histograms for the following configurations: (a) Whole data set; and (b) the best configuration considering the failure rate and Pareto distance ($m = \{2, 5, 10\}$, $ath = \{0.02 - 1, 0.005 - 1\}$, $\tau_a = 1$).

The nonparametric multiple comparison test for the data set (b) is shown in Figure 21. It confirms that the main effect in the number of negotiation rounds is the search depth.

6.2.3. Summary of Results. This section summarizes the experimental results for the RBNP and SBNP protocols. It describes, for the different negotiation strategies and scenarios, the corresponding optimal parameter configurations and their results. The different values are statistically significant within the $p < 0.05$ range.

6.2.3.1. Failure rate. Table 4 summarizes the failure rate results for the RBNP and SBNP protocols. RBNP shows a failure rate of 0% in the CES and BELLs scenarios. In BELLc, a high $ath_r$ negatively impacts on the failure rate. For low $ath_r$ acceptance thresholds and
$m = 2, 5, 10$ the failure rate drops to 0%. SBNP performance is good in the CES scenarios, but decreases drastically in the nonmonotonic environments.

6.2.3.2. Pareto distance. Table 5 shows the Pareto-distance results for the RBNP and SBNP protocols. In RBNP, Pareto distance improves with the increase of search depth. For the adaptive and monotonic concession strategies, the activation of $q_{thf}$ significantly improves the outcomes. When using the static strategy, agents’ offers are quickly accepted by the opponents. It means that the movement request mechanism rarely activates, and hence $q_{thf}$ has a small influence on the negotiation process.

If we take into consideration the failure rate and Pareto-distance measures, a good configuration would be: high search depth $m$, activation of offer movement request ($q_{thf} = 0.5$), low acceptance thresholds for higher sized BTHs ($ath_{r_1} = 0.5\%$), more dense distribution of lower sized regions ($\tau_{r} = 2$), and a quasi-linear distribution of acceptance thresholds ($\tau_a = 1$).
TABLE 4. Failure Rate Results for the RBNP and SBNP Protocols.

|        | Failure rate               |
|--------|---------------------------|
| CES    |                           |
| RBNP-Static | 0%                        |
| RBNP-Adaptive | 0%                       |
| RBNP-Monotonic | 0%                       |
| SBNP   | 0%                        |
| BELLs  |                           |
| RBNP-Static | 0%                        |
| RBNP-Adaptive | 0%                       |
| RBNP-Monotonic | 0%                       |
| SBNP   | 60%                       |
| BELLc  |                           |
| RBNP-Static | 50% for ath = 0.1–1       |
| RBNP-Adaptive | 0% for m = 2,5,10 ath = 0.02–1,0.005–1 |
| RBNP-Monotonic | 70% for ath = 0.1–1       |
| SBNP   | 96%                       |

TABLE 5. Pareto-distance Results for the RBNP and SBNP Protocols.

|        | Pareto distance |
|--------|----------------|
| CES    |                |
| RBNP-Static | m = 10         |
| RBNP-Adaptive | m = 10 qthf = 0.5,1 |
| RBNP-Monotonic | m = 10 qthf = 0.5,1 |
| SBNP   | 0.0424         |
| BELLs  |                |
| RBNP-Static | m = 10 τ_a = 1 |
| RBNP-Adaptive | m = 10 qthf = 0.5 |
| RBNP-Monotonic | m = 10 qthf = 0.5 |
| SBNP   | 0.4598         |
| BELLc  |                |
| RBNP-Static | m = 10 ath = 0.02–1,0.005–1 τ_a = 1 |
| RBNP-Adaptive | m = 10 ath = 0.02–1,0.005–1 (qthf = 0.5 or τ_a = 1) |
| RBNP-Monotonic | m = 10 ath = 0.005–1 qthf = 0.5,1 |
| SBNP   | 0.2176         |

The SBNP and RBNP performances are quite similar in the CES scenarios. In BELLs, SBNP performs much worse than RBNP, with a 60% failure rate and very poor Pareto-distance results. In BELLc scenarios SBNP does not work.

6.2.3.3. Negotiation time and negotiation rounds. Table 6 represents the negotiation time and negotiation round results for the RBNP and SBNP protocols. The most influential factor in the negotiation time is the search depth. Low $ath_r$ values improve the performance,
Table 6. Negotiation Time and Negotiation Round Results for the RBNP and SBNP Protocols.

| Protocol            | Time Rounds |
|---------------------|-------------|
| CES                 |             |
| RBNP-Static         | $m = 10$ $\tau_a = 10,20$ $\tau_r = 2$ | 0.8583 10 |
| RBNP-Adaptive       | $m = 10$ $qthf = 0.5,1$ $ath = 0.1-1$ $\tau_r = 2$ | 1.2320 15 |
| RBNP-Monotonic      | $m = 10$ $qthf = 0.5,1$ ($ath = 0.005–1$ or $\tau_a = 10,20$) | 5.4536 30 |
| SBNP                |             | 18.604 9 |
| BELLs               |             |
| RBNP-Static         | $m = 10$ $\tau_a = 1$ | 4.2635 15 |
| RBNP-Adaptive       | $m = 10$ $qthf = 0.5$ ($ath = 0.02–1, 0.005–1$ or $\tau_a = 10,20$) | 5.3643 20 |
| RBNP-Monotonic      | $m = 10$ $qthf = 0.5$ $ath = 0.005–1$ $\tau_a = 10,20$ | 45.2966 60 |
| SBNP                |             | 54.829 5 |
| BELLc               |             |
| RBNP-Static         | $m = 10$ $ath = 0.02–1,0.005-1$ $\tau_a = 1$ | 6.7232 60 |
| RBNP-Adaptive       | $m = 10$ $ath = 0.005–1$ ($qthf = 0.5$ or $\tau_a = 1$) | 11.4517 60 |
| RBNP-Monotonic      | $m = 10$ $ath = 0.005–1$ $qthf = 0.5,1$ $\tau_a = 10,20$ | 52.3414 70 |
| SBNP                |             |

Table 7. Summary of Preferred Configurations for Different Scenarios and Strategies.

| m = high, qthf = low, $\tau_r = high$, nbth = any | ath$_{r_1}$ | $\tau_a$ |
|--------------------------------------------------|-------------|-----------|
| CES-Static                                       | High        | High      |
| CES-Adaptive                                     | High        | High      |
| CES-Monotonic                                    | Low         | High      |
| BELLs-Static                                     | Low         | Low       |
| BELLs-Adaptive                                   | Low         | High      |
| BELLs-Monotonic                                  | Low         | High      |
| BELLc-Static                                     | Low         | Low       |
| BELLc-Adaptive                                   | Low         | High      |
| BELLc-Monotonic                                  | Low         | High      |

with the exception of the CES-Adaptive scenario. High $\tau_a$ values improve the results, though in the nonmonotonic scenarios and with the static strategy, we saw that Pareto distance improves with $\tau_a = 1$. The $\tau_r$ factor improves the performance when it defines a more dense distribution of lower sized regions, i.e., $\tau_r = 2$.

The number of negotiation rounds is mainly influenced by the search depth and the concession strategy. As expected, the number of rounds increases with the search depth and with the level of competitiveness of the agents’ strategies.

Finally, Table 7 summarizes the best configurations in the different scenarios and for the different strategies. To consider a configuration as optimal, we have taken the failure rate as the most relevant measure, then the Pareto distance, and finally the negotiation time. To simplify the characterization of the different factors, we use the terms ‘high’ and ‘low’ to define value trends. In those cases where a factor has no influence on the performance of the protocol, we have chosen a concrete value to make the table as general as possible. In monotonic negotiation scenarios, agents perform well with a convex function of acceptance
thresholds (i.e., $\tau_a = 20$) and high initial acceptance thresholds. Only when using the monotonic strategy agents should make $ath_{r_1}$ smaller. In nonmonotonic scenarios, $ath_{r_1}$ must take a low value in order to avoid negotiation failures in the higher sized BTHs. With the adaptive and monotonic strategies agents perform better with a convex function of acceptance thresholds ($\tau_a = 20$). However, with the static strategy it is better to use quasi-linear functions.

6.3. Number of Issues and Search Depth in RBNP

We have already obtained the best configuration of negotiation parameters for the different strategies and scenario types. The experiments have shown that the search depth and the concession strategies play a key role in the performance of RBNP. It is of interest now to test the RBNP performance when using search depths higher than 10. Thus, taking as a basis the best configurations shown in Table 7, we extend the evaluation of RBNP for search depths $m = \{15, 20, 25, 30, 35, 40\}$. In addition, we perform the experiments under negotiation scenarios of 2, 5, 10, and 20 issues to evaluate scalability. Figures 22, 23, and 24 report the performance of the RBNP and SBNP protocols for the CES, BELLs, and BELLc scenarios, respectively.
6.3.1. CES. For two issues, SBNP performs slightly better than RBNP. It can be seen in the upper-left graphic in Figure 22 that the dashed line representing the distance to the Pareto frontier (i.e., Pareto distance) for the SBNP protocol is below the other lines, which means that the agreements with SBNP are closer to the Pareto frontier. However, as the number of issues increases (see the upper row in Figure 22), and for a search depth higher than 10, SBNP performs worse when compared to RBNP-Adaptive and RBNP-Monotonic. Overall, RBNP performs better than SBNP in monotonic scenarios. The only exception is in the number of rounds, where SBNP performance is clearly better.

When comparing the different strategies in RBNP (see the three marked lines in the upper row within Figure 22), the Pareto-distance measures are better with RBNP-Monotonic, and this is more evident as the number of issues increases. The worst results in terms of Pareto distance are obtained, as expected, with the Static strategy, where agents do not compete for high utility aspirational levels. The RBNP-Adaptive protocol obtains slightly worse results than those obtained with RBNP-Monotonic. However, the negotiation time and number of rounds performance with the Adaptive strategy are much better (see the second and third rows in Figure 22). Another expected result is that as the number of issues increases, agents need to negotiate with a higher search depth to obtain good results. This is evident if we look at the Pareto-distance graphics in Figure 22, where it can be seen that for 2, 5, and 10 issues
the minimum Pareto distance is obtained for a search depth of 15 and higher, while for 20
issues the minimum value is obtained for a search depth of 35 and higher. However, for a
given scenario, there exists an optimal search depth value such that there is no benefit in
using a higher one. Considering that there is a trade-off between search depth and negotiation
time, it is convenient to use that search depth value. A search depth value between 10 and 20
seems to be a good trade-off between Pareto distance, time, and number of rounds. Another
interesting property of RBNP when working in the CES scenarios is that the negotiation
time does not significantly depend on the number of negotiated issues. Finally, we can see
how none of the negotiation protocols fail in the CES environment (see the bottom row in
Figure 22).

6.3.2. BELLs. Figure 23 shows the performance results for the BELLs scenario. Over-
all, we can see that the performance of SBNP decreases. Thus, for 10 and 20 issues the
failure rate is, respectively, 60% and 100%, and for 2, 5, and 10 issues the Pareto distance
and negotiation time are sensibly worse than for RBNP. As in the CES scenarios, the only
exception is in the number of rounds, where SBNP is significantly better.
When comparing the different strategies in RBNP, we can see that the Pareto-distance measures are quite similar with the Adaptive and Monotonic strategies, and as in the CES scenarios, that the RBNP-Adaptive and RBNP-Monotonic performances are sensibly better than with RBNP-Static. Also, the negotiation time and number of negotiation rounds with the Adaptive strategy are much better than with the Monotonic strategy. Moreover, for 20 issues and the Monotonic strategy, RBNP exhibits a high failure rate, while with RBNP-Adaptive the failure rate approaches to zero. In the BELLs scenarios, a search depth value between 10 and 20 seems to be a good trade-off between Pareto distance, time, and number of rounds.

6.3.3. BELLc. Figure 24 shows the performance results for the BELLc scenario. Here, the SBNP performance decreases drastically in the BELLc scenarios. Thus, the failure rates for 2, 5, 10, and 20 issues are, respectively, of 5%, 28%, 96%, and 100%, and for 2 and 5 issues the Pareto distance and negotiation time are sensibly worse than with RBNP. As in the CES scenarios, the only exception is in the number of rounds, where SBNP is significantly better.

When comparing the different strategies in RBNP, we can see that the best Pareto-distance performances for 2, 5, and 10 issues are obtained when using the Monotonic strategy. With the Adaptive strategy the results are slightly worse than those obtained with RBNP-Monotonic, but RBNP-Adaptive shows two advantages: the failure rate for 20 issues is below 20%, and the negotiation time and number of rounds are significantly lower. As in the BELLs scenarios, a search depth value between 10 and 20 seems to be a good trade-off between Pareto distance, time, and number of rounds.

7. STRATEGY ANALYSIS

So far, we have evaluated scenarios where agents use the same negotiation strategies. However, we need to analyze the potential consequences of the strategic behavior of the negotiating agents, analyzing the dynamics of the negotiation process when agents under incomplete information settings and with different strategies interact. The questions we want to answer are which strategy should an agent play, whether there exists a dominant strategy, and whether individual rationality may lead to situations of low social welfare. The notion of price of anarchy (PoA) may be used to measure the loss of social efficiency. The price of anarchy was first introduced in Papadimitriou (2001) in the context of selfish routing, as a measure of loss of social efficiency due to selfish behavior. In the context of a problem of social welfare maximization, PoA can be defined as follows:

Definition 7. The price of anarchy (PoA) in a given game is defined as the ratio between the social welfare of the best possible outcome of the game and the social welfare of the worst Nash equilibrium in the game:

$$\text{PoA} = \frac{\max_{s \in S} \text{sw}(s)}{\min_{s \in S_{\text{Nash}}} \text{sw}(s)},$$

where $S$ is the set of all possible outcomes of the game, $S_{\text{Nash}} \subseteq S$ is the set of all possible outcomes induced by a Nash equilibrium in the game, and $\text{sw}(s)$ is the social welfare of a given outcome $s$. 
Defined in this way, PoA gives an indication of the potential loss in a given game when individually rational agents are confronted. A PoA of 1 indicates that there is no social welfare loss, while on the other side, a PoA of $\infty$ indicates that the minimum social welfare of the Nash equilibria is zero.

In this section, we measure the RBNP performance when agents use different concession strategies in a given negotiation encounter. We evaluate the CES, BELLs, and BELLc scenarios for 10 and 20 issues. It is assumed that each agent uses the best configuration of negotiation parameters and a search depth of 20. Note that under an incomplete information setting, an agent does not know the opponent’s utility function and negotiation strategy. Thus, in the selection of a negotiation profile an agent considers her own strategy, not the opponent’s strategy (which is not known).

7.1. CES

Figure 25 depicts the boxplots of utilities for the different combinations of strategies in the CES scenarios. Each graphic shows a whisker plot with one box for each negotiating agent. The boxes have lines at the lower quartile, median, and upper quartile values (i.e., 50% of the agreements provide the agent a utility value within the upper and lower quartile values). The whiskers are lines extending from each end of the boxes to show the extent of the rest of the data. Thus, for example, StaticAdaptive in the upper left corner in Figure 25 means that agent $A_b$ plays RBNP static and agent $A_s$ RBNP Adaptive. In this case, 50% of the agreements provide $A_s$ a utility between 0.65 and 0.8, while for $A_b$ 50% of the agreements fall within the range 0.4–0.54. This is evidence of better performance of RBNP Adaptive when playing against RBNP Static.

Results show that an agent playing Monotonic against Static or Adaptive always receives a higher payoff (i.e., higher utility) than playing any other strategy. In addition, we can see that if both agents play the Monotonic strategy, there is no evidence of a decrease in the social welfare when compared to the Static–Static and Adaptive–Adaptive pairs. It means that there exists a unique Nash equilibrium in which each agent plays her strictly dominant strategy, the Monotonic strategy. We can conclude that in the CES scenario, the Monotonic–Monotonic pair is at the same time the unique Nash equilibrium and the maximum social welfare pair, which means that PoA is 1.

We have considered the strategic analysis from the perspective of utility. If we take into consideration negotiation time or the number of rounds, it may be necessary for an agent to play Static or Adaptive to speed up an agreement, at the cost of a decrease in the utility of the outcomes.

7.2. BELLs

The boxplots of utilities under the BELLs scenarios are shown in Figure 26. With 10 issues, the results are similar to those obtained under the CES environment. The strictly dominant strategy is to play Monotonic, with a PoA of 1.

With 20 issues, when both agents play Monotonic the failure rate is very high (note that the Monotonic–Monotonic boxplot for 20 issues in Figure 26(b) shows high utility levels because it does not gather the negotiation failures). If we consider that a negotiation failure provides the agents utility 0, the payoff matrix for a game with the Adaptive and Monotonic strategies is as shown in Table 8, where each number represents the median of utility. Note that the Static strategy is not being considered because there is no individual or social benefit in using it. This payoff matrix is known as the game of chicken, and is characterized by the existence of two Nash equilibria, corresponding to the Monotonic–Adaptive pairs. There is
no dominant strategy, which means that the optimal strategy for an agent depends on the opponent’s strategy. If the opponent plays Adaptive, the agent should play Monotonic, and if the opponent plays Monotonic, the agent should play Adaptive. However, if agents do not know the strategies played by the opponents, risk attitude plays a main role in this scenario. A risk-prone agent may play Monotonic, assuming the risk of a negotiation failure, but with the
likelihood of obtaining the highest payoff if the other agent plays Adaptive. On the other hand, a risk-averse agent should play Adaptive in order to obtain a payoff of at least 0.2. Finally, the PoA is the ratio between the pairs: Adaptive–Monotonic and Adaptive–Adaptive. For the sum of utilities $\text{PoA} = \frac{0.5 + 0.5}{1} = 1.25$, and for the product of utilities $\text{PoA} = 2.08$. 

Figure 26. Boxplots of utilities for the different pairs of strategies under the BELLs scenarios.
TABLE 8. Payoff Matrix for the Adaptive and Monotonic Strategies in the BELLs and 20 Issues Scenario.

| Median of utility | $A_a$ Adaptive | $A_b$ Monotonic |
|------------------|---------------|-----------------|
| $A_a$ Adaptive    | 0.5           | 0.6             |
| $A_b$ Adaptive    | 0.5           | 0.2             |
| $A_b$ Monotonic   | 0.2           | 0.6             |
|                   | 0.2           | 0.0             |

7.3. BELLc

Figure 27 shows the boxplots of utilities under the BELLc scenario. For 10 issues, as in the CES and BELLs scenarios, the Monotonic strategy is dominant, and the unique Nash equilibrium is the Monotonic–Monotonic pair. In the BELLc scenario with 20 issues, the failure rate is 100% when at least one of the agents plays Monotonic. The intuition behind this is that in highly complex negotiation spaces only a cooperative behavior lead agents to an agreement. Regarding the Static and Adaptive strategies, there is not strong evidence of dominance. However, as can be seen in Figure 24, the negotiation time and number of rounds are significantly better with the Adaptive–Adaptive pair.

8. CONCLUSION

In this article, we analyze the problem of automated negotiation in complex nonmonotonic preference spaces. We propose the Region-Based Multi-issue Negotiation Protocol (RBNP) for bilateral automated negotiation. RBNP is built upon a nonmediated recursive bargaining mechanism which efficiently modulates a region-based joint exploration of the solution space. The proposed region expansion and contraction method within the bargaining mechanism avoid zones of no-agreement in an efficient manner. We pay attention to the strategic issues, where concession mechanisms play a fundamental role, incorporating three different concession strategies, purely cooperative, purely competitive, and semi-cooperative in order to cover a wide range of agent behaviors.

We first show that the generic Similarity-Based Multi-issue Negotiation Protocol (SBNP), which has been successfully used in many negotiation models, has a poor performance in nonmonotonic spaces. The main problem with SBNP is that agents’ utility concessions may lead to situations where agents’ offers are similar but it is not possible to make further concessions (i.e., agents get stuck in a zone of no agreement). RBNP solves these situations by means of region expansion and contraction. In addition, offer movement request is identified as a useful technique which complements expansion and contraction by letting agents to express their preferences for the opponent’s offers. This technique significantly improves the results under complex preference settings.

We perform an exhaustive empirical analysis of RBNP under monotonic and nonmonotonic scenarios, and obtain the optimal negotiation parameter values. It is shown that RBNP produces outcomes close to the Pareto frontier in acceptable negotiation times, and that it clearly outperforms SBNP both in monotonic and in nonmonotonic negotiation scenarios. RBNP only fails when both agents play a monotonic concession strategy under a nonmonotonic negotiation scenario with 20 issues.
Search depth (i.e., the number of region sizes used in a negotiation encounter) plays a main role in the negotiation protocol performance. In general, Pareto optimality increases with search depth at the cost of an increase in negotiation time. Thus, there is a trade-off between Pareto optimality and negotiation time which may be controlled with the search depth.

The strategy analysis we perform evaluates the consequences of the strategic behavior of the negotiating agents. The experimental analysis shows that the dominant strategy in monotonic scenarios is to play a monotonic concession strategy. With the Monotonic–Monotonic pair we obtain also the maximum social welfare solution, which means that the PoA is 1. In nonmonotonic negotiation scenarios, and for 10 issues, the dominant strategy is again the monotonic strategy, and the corresponding PoA is 1. However, for medium-level complexity scenarios and many issues (BELLs and 20 issues), there is no dominant strategy, and we have two Nash equilibrium solutions for the Adaptive–Monotonic strategy pairs. Finally, in the highly complex BELLc and 20 issues scenario the failure rate goes to 100%
when at least one of the agents plays the monotonic strategy. It means that only the adaptive or static strategies lead agents to agreements.

RBNP has shown to be an effective protocol which performs well under monotonic and nonmonotonic scenarios. The compromise between Pareto optimality and negotiation time can be easily controlled by means of the search depth. Only under extremely complex preference spaces RBNP fails. Regarding the strategic issues, in most practical cases the monotonic strategy will be the right choice, and only when negotiation time is critical or in complex preference settings it will be necessary to change to another strategy. We have considered in this work a static environment. In dynamic environments, however, autoadaptive mechanisms will be needed, so that agents can improve their configuration during the course of the negotiation. We believe that these results open the door to a new set of negotiation algorithms, and that the concepts of region and recursivity could be applied in more complex negotiation settings such as multi-party and mediated negotiations.

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APPENDIX

This appendix provides an experimental evaluation, which compares RBNP and the Auction-Based Negotiation Protocol (ABNP) proposed by Ito et al. (2008). ABNP is a one-shot mediated negotiation protocol that works only on constraint-based utility spaces. Thus, the evaluation is provided using constraint-based nonlinear utility spaces as those defined in Ito et al. (2008).

Constraint-based Nonlinear Utility Spaces can be described by using different categories of functions (Zhang and Pu 2004). In Ito et al. (2008) agents’ utility functions are described by defining weighted constraints. Each constraint represents a region with one or more dimensions, which has an associated utility value. The number of dimensions of the space is given by the number of issues \( n \) under negotiation, and the number of dimensions of each constraint must be lesser than or equal to \( n \). The utility yielded by a given potential solution (contract) in the utility space for an agent is the sum of the utility values of all the constraints that are satisfied by that contract. More formally, we can define the issues under negotiation as a finite set of variables \( x = \{ x_i | i = 1, \ldots, n \} \), and a contract (or a possible solution to the negotiation problem) as a vector \( s = \{ x_i^s | i = 1, \ldots, n \} \) defined by the issues’ values. Issues may take values from the domain of integers or real numbers. Agent utility space is defined as a set of constraints \( C = \{ c_k | k = 1, \ldots, l \} \). Each constraint is given by a set of intervals which define the region where a contract must be contained to satisfy the constraint. In this way a constraint \( c \) is defined as \( c = \{ I^c_i | i = 1, \ldots, n \} \), where \( I^c_i = [x^\text{min}_i, x^\text{max}_i] \) defines the minimum and maximum values for each issue to satisfy the constraint. Each constraint \( c_k \) has an associated utility value \( u(c_k) \). A contract \( s \) satisfies a constraint \( c \) if and only if \( x^s_i \in I^c_i \forall i \). For notation simplicity, this is denoted as \( s \in x(c_k) \), meaning that \( s \) is in the set of contracts that satisfy \( c_k \). An agent’s utility for a contract \( s \) is defined as \( u(s) = \sum_{c_k \in C | s \in x(c_k)} u(c_k) \), that is, the sum of the utility values of all constraints satisfied by \( s \). Figure A1 shows two examples of constraint-based utility functions for a two-dimensional negotiation space and two different complexity levels. The low-complexity utility function in (a) has been generated using 10 constraints (five unary and five binary constraints) with a random width in the range \( I^c_i = [30, 70] \). The high-complexity utility function in (b) has been generated using 50 constraints (25 unary and 25 binary constraints) within the range \( I^c_i = [5, 10] \).

The ABNP protocol consists on the following four steps:
(1) Sampling: Each agent takes a fixed number of random samples from the contract space, using a uniform distribution.

(2) Adjusting: Each agent tries to find a local optimum in its neighborhood using simulated annealing. This results in a set of high-utility contracts.

(3) Bidding: Each agent generates a bid for each high-utility, adjusted contract. The bids are generated as the intersection of all constraints which are satisfied by the contract. Each agent sends its bids to the mediator, along with the utility associated with each bid. The number of bids issued plays an important role in the quality of the agreements and the negotiation speed.

(4) Deal identification: The mediator employs breadthfirst search with branch cutting to find overlaps between the bids of the different agents. The regions of the contract space corresponding to the intersections of at least one bid of each agent are tagged as potential solutions. The final solution is the one that maximizes joint utility, defined as the sum of the utilities for the different agents.

In the experimental evaluation, we have negotiation scenarios with 2, 5, 10, and 20 issues and two different complexity levels: low-complexity and high-complexity. In the low-complexity scenario we randomly generate \( n \cdot 5 \) pairs of utility functions with \( n \cdot 5 \) constraints (five unary constraints, five binary constraints, five ternary constraints, etc.) within the range \( I_c^i = [30, 70] \). In the high-complexity scenario we use \( n \cdot 25 \) constraints within the range \( I_c^i = [5, 10] \). We run 10 experiments for each pair of utility functions, which means 100 negotiations per number of issues and complexity level.

To test RBNP we use the Adaptive concession strategy, the best configuration of negotiation parameters for nonlinear scenarios (i.e., BELL scenarios), and a search depth \( m = 10 \). ABNP is evaluated using two different configurations differing in the number of bids issued by the agents. We name the first configuration ABNP fast, and is based on using the minimum number of bids so that the failure rate is zero. It is expected that this configuration provides fast negotiations. The second configuration is named ABNP slow, which consists of adjusting
FIGURE A2. Pareto distance and negotiation time boxplots for RBNP with Simple and Complex Constraint-based Utility Functions.

FIGURE A3. Pareto distance and negotiation time boxplots for ABNP fast and ABNP slow with Simple and Complex Constraint-based Utility Functions.
the number of bids so that the negotiation time is in the same order of magnitude than for the RBNP experiment results. It is expected that this approach provides slower negotiation processes but better results in terms of Pareto efficiency.

Figures A2 and A3 show the Pareto distance and negotiation time results for the RBNP and ABNP fast/slow protocols. From the results we can see that ABNP fast performs badly with both simple and complex utility functions. Even for two issues and the simple scenario the Pareto-distance median is close to 0.2, while for five and more issues it goes beyond 0.4. When using RBNP and ABNP slow the results clearly improve. In terms of Pareto distance, for 2, 5, and 10 issues and both with simple and complex utility functions RBNP outperforms ABNP slow. However, for 20 issues ABNP slow slightly outperforms RBNP. Overall, we can conclude that RBNP may be a valid nonmediated alternative to ABNP for automated bilateral negotiations under constraint-based utility spaces.