Confinement transition in a Kitaev-like honeycomb model with bond anisotropy

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The honeycomb $K - \Gamma$ model is known to have both deconfined spin liquid and confined phases. We study here a confinement transition between the Kitaev spin liquid and a dimerized phase in the limit of strong bond anisotropy. By partially projecting out Majorana states we are able to map the model onto a model of weakly coupled Ising chains in a transverse field. Within this mapping the ordered Ising phase corresponds to the condensation of $Z_2$ fluxes, or confinement. Our results may improve our understanding of the extensively studied spin liquid candidate material $\alpha$-RuCl$_3$, where $K - \Gamma$ interactions are dominant.

One of the defining characteristics of quantum spin liquid phases is the high level of quantum entanglement in their ground state $|1\rangle$. These states are commonly associated with deconfined gauge degrees of freedom on a lattice $|2|\text{-}3\rangle$. More commonly, however, spin systems tend to order magnetically, in which cases their ground states are continuously connected to product states with zero entanglement entropy. By tuning a given system away from its spin liquid ground state towards a magnetically ordered state one must cross a confinement transition followed by breaking of time reversal symmetry.

A good understanding of such transitions is of great importance in studying possible spin liquid materials. One such candidate spin liquid material is $\alpha$-RuCl$_3$, a layered honeycomb Mott insulator which orders magnetically at temperatures well below its Curie-Weiss scale, most likely due to frustrated magnetic interactions $|6|\text{-}7\rangle$. Furthermore, a broad continuum in inelastic neutron scattering spectra may indicate the existence of fractionalized excitations $|8\rangle\text{-}11\rangle$. Much of the recent experimental effort has focused on the intriguing paramagnetic phase obtained by applying an in-plane magnetic field $|12|\text{-}21\rangle$. Due to $\alpha$-RuCl$_3$’s lattice structure Kitaev interactions are dominant, and therefore it has been proposed that it is parametrically close to a Kitaev spin liquid (KSL) $|22|\text{-}23\rangle$. Specifically, one scenario suggests that by applying a magnetic field the system undergoes a deconfinement transition into a spin liquid phase. Kitaev interactions are not, however, the only dominant spin-spin interactions, and off-diagonal symmetric, or $\Gamma$ interactions, are not, however, the only dominant spin-spin interactions.

The Kitaev $-\Gamma$, or $K - \Gamma$, model is known to exhibit deconfined spin liquid and confined phases. The nature of the two phases is completely different. The spin liquid phase is characterized by deconfined $Z_2$ gauge degrees of freedom and fractionalized Majorana excitations $|23\rangle$, whereas the magnetically ordered phase is confined in the gauge theory sense and its excitations are standard spin waves. Between the two phases one thus expects a confinement transition, associated with the condensation of $Z_2$ fluxes. Although the $K - \Gamma$ model has been studied using a variety of approaches $|31|\text{-}38\rangle$, there is no known framework which gives an accurate account of such confinement transitions. In the following we show that it is possible to work out the details of a confinement transition in the case of strong bond anisotropy. Specifically, we propose to study the anisotropic $K - \Gamma$ model

$$H = \sum_{\alpha=x,y,z} K_\alpha \sum_{\langle i,j \rangle \in z} S^\alpha_i S^\alpha_j + \Gamma_z \sum_{\langle i,j \rangle \in z} (S^x_i S^y_j + S^y_i S^x_j),$$

where $S^\alpha_i$ are spin-$1/2$ operators located at sites $i$ of a honeycomb lattice, and $\langle i, j \rangle$ denote bonds of types $x, y$ or $z$ extending from a site $i$ on sublattice 1 to site $j$ on sublattice 2, see Fig. 1. We focus on the case of strong anisotropy in the Kitaev interaction, $|K_z| \gg |K_x|, |K_y|$. In the extreme limit, $K_z = K_y = 0$, each of the $z$-bonds is isolated, with a product state for the ground state. We take $\Gamma_z > 0$, and for the most part consider the case $K_z < 0$, where the ground state is dimerized,

$$|0\rangle = \prod_{\langle ij \rangle \in z} \frac{1}{\sqrt{2}} \left( |\uparrow\uparrow\rangle_{ij} - i |\downarrow\downarrow\rangle_{ij} \right),$$

and is obviously confined in the gauge theory sense. How-
ever, unlike the magnetically ordered case, it does not break time reversal symmetry. This state is stable to small perturbations, such as small $K_x$ and $K_y$, since it is non-degenerate and gapped to all excitations. In the opposite limit, $\Gamma_z = 0$, we arrive at Kitaev’s model, which, being in a deconfined phase, has a highly entangled ground state. We note that the ground state of the anisotropic Kitaev model has the same symmetries as the dimer state in Eq. (2), and in fact, it is specifically this high entanglement which differentiates between them.

Our plan is to study the confinement transition between the Kitaev and dimerized states in the limit of small $K_{x,y}$ and $\Gamma_z$ via an effective low-energy Hamiltonian. To this end we follow Kitaev [22], and rewrite $H$ using four Majorana fermion operators on each site

$$S_i^\alpha \rightarrow \frac{1}{2} i b_i^\alpha c_i.$$  \hspace{1cm} (3)

Thus,

$$H \rightarrow \tilde{H} = \frac{1}{4} \sum_{\alpha=x,y,z} K_{\alpha} \sum_{\langle i,j \rangle \in \alpha} i b_i^\alpha b_j^\alpha ic_i c_j - \frac{\Gamma_z}{4} \sum_{\langle i,j \rangle \in z} (ib_i^z b_j^z + ib_i^z b_j^z) ic_i c_j.$$  \hspace{1cm} (4)

The Hamiltonian $\tilde{H}$ operates in a larger Hilbert space than that of the original Hamiltonian $H$, a fact reflected by its gauge symmetry. Explicitly, it is invariant under local $Z_2$ gauge transformations, $D_i \tilde{H} D_i = \tilde{H}$, where $D_i \equiv b_i^x b_i^y b_i^z c_i$. However, only those states of the extended Hilbert space $|\psi\rangle$ for which $D_i |\psi\rangle = |\psi\rangle$ correspond to physical spin states.

The eigenstates of the Kitaev model, defined here by setting $\Gamma_z = 0$, are tensor products $|u\rangle \otimes |\phi_u\rangle$ of states $|u\rangle$ in the $b^\alpha$-fermions’ sector, and states $|\phi_u\rangle$ in the $c_i$-fermions’ sector. The former are simultaneously eigenstates of the bond operators $ib_i^\alpha b_j^\alpha$, where $\langle i, j \rangle \in \alpha$, with eigenvalues $u_{ij} = \pm 1$. For a given set $u$ of bond values the $|\phi_u\rangle$ are eigenstates of the free Majorana Hamiltonian,

$$H_u = -\frac{1}{4} \sum_{\alpha} K_{\alpha} \sum_{\langle i,j \rangle \in \alpha} u_{ij} ic_i c_j,$$  \hspace{1cm} (5)

obtained by replacing the $ib_i^\alpha b_j^\alpha$ in $\tilde{H}$ by $u_{ij}$. In the limit $K_x = K_y = 0$ they are eigenstates of the operators $ic_i c_j$ on z-bonds with eigenvalues $\pm 1$, and the degenerate ground-state manifold of $\tilde{H}$ consists of products $|\psi_u\rangle = |u\rangle \otimes |\phi_u\rangle$ obeying $u_{ij} ic_i c_j = K_z/|K_z|$ for $\langle i, j \rangle \in z$. Since excited $c$-fermion states are gapped their effect on the low-energy physics is captured by standard degenerate perturbation theory in $K_{x,y}$ and $\Gamma_z$. The first non-trivial contribution of $K_{x,y}$ to the effective Hamiltonian reads $H_{\text{eff}}(\Gamma_z = 0) = -\Delta_v \sum_p w_p$, where $w_p = \prod_{\langle i,j \rangle \in p} u_{ij} = \pm 1$ measures the $Z_2$ gauge-invariant flux through a hexagonal plaquette $p$ and $\Delta_v = K_z^2 K_y^2 / 64 |K_z|^{3}$ is the energy of a vison, i.e., a $w_p = -1$ flux excitation $22$.

In order to evaluate the leading contribution of the $\Gamma_z$ term to $H_{\text{eff}}$ we need its matrix elements between states in the degenerate manifold. For what follows we find it convenient to fix the gauge and consider only $|\psi_u\rangle$ for which $u_{ij} = 1$ on all z-bonds. Other states are related to this form by the action of gauge transformations $D_i$, which change $u_{ij} \rightarrow -u_{ij}$ and $c_i \rightarrow -c_i$. The invariance $D_i \tilde{H} D_i = \tilde{H}$ implies the invariance of the matrix elements under such transformations. Thus, within the chosen gauge sector, a matrix element of the $\Gamma_z$ term on a given bond $\langle ij \rangle \in z$ takes the form

$$\Gamma_{ij}(u', u) \equiv \frac{\Gamma_z}{4} \langle \psi_u | (ib_i^z b_j^z + ib_j^z b_i^z) ic_i c_j |\psi_u\rangle = \frac{\Gamma_z K_z}{4|K_z|} \langle u'|ib_i^z b_j^z + ib_j^z b_i^z |u\rangle,$$  \hspace{1cm} (6)

provided that $u$ and $u'$ share the same set of $u_{ij}$ for all $\langle ij \rangle \in z$, and vanishes otherwise. Here we have used the condition that determines the $|\psi_u\rangle$ manifold.

Eq. (6) indicates that while the presence of $\Gamma_z$ interactions does not change the fact that the $u_{ij}$‘s are good quantum numbers for z-bonds, it renders the $u_{ij}$‘s on $x$ and $y$-bonds truly dynamical effective spin-1/2 degrees of freedom. Consequently, it is useful to formulate $H_{\text{eff}}$ in terms of operators acting on the latter. For this purpose we note that the operator $ib_i^z b_j^z$ operates only on $u_{i+\hat{x}}$ and $u_{j-\hat{y}}$, see Fig. 1 and that it has the same matrix elements as $\sigma_{j}^{z} \sigma_{j}^{z} - \sigma_{j}^{x} \sigma_{j}^{ar{x}}$, where $\sigma_{j}^{z}$ is the Pauli matrix $\sigma^z$ acting on $u_{kl}$, and where matrices associated with different bonds commute. Similarly, $ib_i^z b_j^z$ can be replaced by $\sigma_{j}^{z} \sigma_{j}^{z} + \sigma_{j}^{ar{x}} \sigma_{j}^{ar{x}}$. In addition, Pauli matrices acting on $u_{kj}$ states can also be used to measure the flux on a given plaquette, namely, $w_p = \prod_{\langle i,j \rangle \in p} \sigma_3$. Combining these results we can cast $H_{\text{eff}}$ in the form

$$H_{\text{eff}} = -\Delta_v \sum_p \sum_{\langle i,j \rangle \in p} \sigma_3 \langle \psi_u | \frac{\Gamma_z K_z}{4|K_z|} \sum_{\langle i,j \rangle \in z} (\sigma_{i+\hat{x}}^{z} \sigma_{j-\hat{y}}^{ar{x}} + \sigma_{i+\bar{x}}^{z} \sigma_{j+\hat{y}}^{ar{x}}) |\psi_u\rangle.$$  \hspace{1cm} (7)

Further progress is facilitated by employing the constraint $D_i D_j = 1$ for $\langle i,j \rangle \in z$, which partially projects the model onto the physical space. In particular, it eliminates states with unphysical energies on the isolated bond. In terms of the Majorana operators it becomes

$$ib_i^z b_j^z + ib_j^z b_i^z = -ib_i^z b_j^z ic_i c_j,$$

$$\sigma_{i+\hat{x}}^{z} \sigma_{j-\hat{y}}^{ar{x}} \sigma_{i+\bar{x}}^{z} \sigma_{j+\hat{y}}^{ar{x}} = -(K_z/|K_z|) I,$$  \hspace{1cm} (8)

where in the last step we have evaluated the operators in the $|\psi_u\rangle$ subspace.

At this point it is possible to collapse the z-bonds onto points on a rhombic lattice, and consider the $u_{ij}$ degrees of freedom on the remaining x and y-bonds, see Fig. 2.
Denoting the points on the new lattice by $r$ we recast $H_{\text{eff}}$ into

$$H_{\text{eff}} = -\Delta_v \sum_r \sigma_{r,r+\hat{x}}^3 \sigma_{r+\hat{x},r+\hat{z}}^3 \sigma_{r+\hat{z},r+\hat{y}}^3 \sigma_{r+\hat{y},r+\hat{\hat{z}}}^3 \sigma_{r+\hat{\hat{z}},r}^3$$

where each site $r$ has four connected bonds, denoted by $r, r\pm \hat{x}$ and $r, r\pm \hat{y}$. The constraint, Eq. (8), now becomes

$$\sigma_{r,r+\hat{x}}^2 \sigma_{r+\hat{x},r+\hat{z}}^2 \sigma_{r+\hat{z},r+\hat{y}}^2 \sigma_{r+\hat{y},r}^2 = -(K_z/|K_z|)I.$$  

(10)

Within this constraint, half of the bonds can be treated as dependent degrees of freedom [39]. Consider, for example, site $r$ in Fig. 2. We can use Eq. (10) to express $\sigma_{r,r+\hat{y}}^2$ in terms of $\sigma_{r,r+\hat{x}}^2, \sigma_{r,r+\hat{z}}^2$ and $\sigma_{r,r+\hat{y}}^2$. In turn, the constraint on site $r' = r - \hat{x}$ implies that $\sigma_{r,r+\hat{z}}$ is given by $\sigma_{r',r+\hat{y}}^2, \sigma_{r',r+\hat{2z}}$ and $\sigma_{r',r+\hat{z}}^2$, and so forth. Consequently, for all points $r$ on sublattice $A$, see Fig. 2 $\sigma_{r,r+\hat{x}}^2$ and $\sigma_{r,r+\hat{z}}^2$ are expressible as products of $\sigma_{r,r+\hat{\hat{z}}}$ and $\sigma_{r,r-\hat{\hat{y}}}$ on other sites of the $A$ sublattice. Hence, we take the former to be dependent on the latter. The roles are reversed for points $r$ on sublattice $B$. Once the dependent degrees of freedom have been eliminated in favor of the independent bonds, the effective Hamiltonian commutes with $\sigma^3$ on the dependent bonds and we can simply set them to $\sigma^3 = 1$. The result for $K_z < 0$ is

$$H_{\text{eff}} = -\sum_{r \in A} \left[ \Delta_v \sigma_{r,r+\hat{z}}^3 \sigma_{r+\hat{z},r+\hat{y}}^3 \sigma_{r+\hat{y},r}^3 - \frac{\Gamma_z}{2} \sigma_{r,r+\hat{z}}^2 \sigma_{r,r+\hat{y}}^2 \right]$$

$$-\sum_{r \in B} \left[ \Delta_v \sigma_{r,r+\hat{y}}^3 \sigma_{r+\hat{y},r+\hat{z}}^3 \sigma_{r+\hat{z},r}^3 - \frac{\Gamma_z}{2} \sigma_{r,r+\hat{y}}^2 \sigma_{r,r+\hat{z}}^2 \right].$$  

(11)

When $K_z > 0$, the $\Gamma_z$ term is expected to additionally scale as $(K_{x,y}/K_z)^{4}$ since lower order terms cancel each other when applying Eq. (10) to Eq. (9).

The final stage of the calculation consists of a duality transformation which maps $H_{\text{eff}}$ onto a model of disconnected quantum Ising chains, with spin-1/2 degrees of freedom on the points of the dual lattice, see Fig. 2. To do so we define a new set of dual spin-flip operators in terms of the independent bond operators

$$\mu_r^1 = \begin{cases} \sigma_{r,r+\hat{z}}^3 \sigma_{r+\hat{z},r+\hat{y}}^3 \sigma_{r+\hat{y},r}^3 & r \in A \\ \sigma_{r,r+\hat{y}}^3 \sigma_{r+\hat{y},r+\hat{z}}^3 \sigma_{r+\hat{z},r}^3 & r \in B \end{cases}.$$  

(12)

These two sets of operators correspond to the $e$ and $m$ excitations in Kitaev’s work [22]. At the same time, $\mu_r^3$ is defined as a product of $\sigma^2$ operators on a semi-infinite string, running parallel to the original $z$-bonds, and which terminates at the plaquette above $r$

$$\mu_r^3 = \begin{cases} \prod_{l \geq 0} \sigma_{r-l\hat{y}-\hat{z},r-l\hat{y}}^2 \sigma_{r-l\hat{y}-\hat{z},r-l\hat{x}}^2 & r \in A \\ \prod_{l \geq 0} \sigma_{r-l\hat{x}-\hat{y},r-l\hat{x}}^2 \sigma_{r-l\hat{x}-\hat{y},r-l\hat{y}}^2 & r \in B \end{cases}.$$  

(13)

see Fig. 2. Here we have defined the displacement parallel to the strings as $\hat{a} = \hat{x} + \hat{y}$. It is possible to verify that the dual operators obey the same (anti-)commutation relations as spin-1/2 operators, $[\mu_r^1, \mu_r^1] = 2I\delta_{a03}$, and $[\mu_r^3, \mu_r^3] = 0$ for $r \neq r'$. Noting that

$$\mu^3 = \begin{cases} \sigma_{r,r+\hat{z}}^2 \sigma_{r+\hat{z},r+\hat{y}}^2 & r \in A \\ \sigma_{r,r+\hat{y}}^2 \sigma_{r+\hat{y},r+\hat{z}}^2 & r \in B \end{cases},$$  

(14)

we finally obtain the dual Hamiltonian

$$H_d = \frac{\Gamma_z}{2} \sum_r \mu_r^3 \mu_r^3 - \Delta_v \sum_r \mu_r^1,$$  

(15)

describing a set of decoupled quantum Ising chains. When $\Gamma_z/2 > \Delta_v$, this dual model is known to transition into an ordered phase [40], which corresponds to condensation of visons in the original model, and therefore, to confinement. For $K_z < 0$ this happens at

$$\frac{\Gamma_z}{|K_z|} \approx 2\Delta_v \approx \frac{1}{32} \left( \frac{K_{x,y}}{K_z} \right)^4.$$  

(16)

This is the main result of our calculation. To support it we performed density-matrix renormalization-group (DMRG) [41] calculations of the $K - \Gamma_z$ model on a cylindrical $6 \times 16$ site system and calculated the entanglement entropy $S$ for a bi-partition cutting only $x$ and $y$-bonds. In the dimerized phase, for weak $K_{x,y}$, the $z$-bond dimers are only weakly entangled and $S$ is expected to be small. In the deconfined phase, however, there is a contribution to the entanglement entropy coming from the gauge...
sector. Specifically, for the Kitaev honeycomb spin liquid the gauge sector (b-fermions) entanglement entropy for a bi-partition cutting 2L bonds is given by

\[ S_G \approx (L - 1) \ln 2 \]

for different values of \( K_{x,y} \). At low \( \Gamma_z \) \( S \) is slightly above 2 \( \ln 2 \) as expected for a Kitaev spin liquid on a cylinder with a 6 bond circumference. The excess entropy comes from the gapped c-fermions. A confinement transition to the dimerized state is seen as a drop in \( S \) from the gapped phase. An approximate follow Eq. (16), in support of our calculation. In the following we discuss some generalizations, which go beyond our model, Eq. (1), with points obtained from (a).

**Tuning towards the isotropic limit.** As \( K_{x,y} \) increase, terms of orders higher than \( (K_{x,y}/K_z)^4 \) must be included in Eq. (1). They translate into \( \mu_i \mu_j \) terms, which decrease with \( |r_i - r_j| \) and introduce weak interactions between the otherwise decoupled Ising chains in Eq. (1). Thus, the critical behavior is expected to belong to the three-dimensional Ising universality class. Since a pair of neighboring visons have a lower energy than the two isolated visons, it is likely that as the model is tuned towards the isotropic point the critical \( \Gamma_z \) will be lower than twice the bare vison gap \( 2\Delta_0 \). Note, however, that \( \Delta_0 \) itself grows rapidly with \( K_{x,y} \), making the deconfined region larger in the isotropic limit. Eventually, the c-fermions become gapless and our treatment breaks down.

**Other spin-spin interactions.** The Heisenberg interaction \( J(S_i^x S_j^x + S_i^y S_j^y) \) on \( z \)-bonds can be treated on equal footing to the \( \Gamma_z \) term. Its order \( J \) contribution to \( H_{\text{eff}} \) vanishes within the physical space when \( K_z < 0 \), but survives for \( K_z > 0 \) and couples the Ising chains. On the other hand, \( \Gamma \) and Heisenberg terms on \( x \) and \( y \)-bonds do not preserve the choice of gauge, which we have used, and therefore lie outside the scope of the present framework.

**Effect of a magnetic field.** Consider, for example, the perturbation \( -h \sum_i (S_i^z + S_i^0) \) due to a magnetic field \( \hat{h} = (h, h, 0) \). For \( K_z < 0 \), and to leading order in \( h \) within the physical space it shifts the strength of the \( \Gamma_z \) term to \( \Gamma_z - 4h^2/(K_z) \). Thus, when starting in the \( \Gamma_z > 2\Delta_0 \) confined dimerized phase, increasing the magnetic field suppresses the \( \Gamma_z \) term and may induce a deconfinement transition. We note that this effect, of order \( h^2 \), is more relevant than the time reversal symmetry breaking \( h^3 \) terms considered in studies focusing on a possible chiral spin liquid phase in \( \alpha \)-RuCl\(_3\) [44, 50].

**Conclusion.** Starting from a model of anisotropic local spin interactions on a honeycomb lattice, we determined the phase boundary between a deconfined KSL phase and a confined dimerized phase. It is difficult to directly apply our results to \( \alpha \)-RuCl\(_3\) since the bond anisotropy is not expected to be as large as considered here. Furthermore, non-negligible Heisenberg interactions probably exist in the material, stabilizing a magnetically ordered confined phase rather than the dimerized confined phase of the model above. A dimerized state has been observed in \( \alpha \)-RuCl\(_3\) under pressure [51, 53], but it is most likely not the same as in Eq. (2). Nevertheless, we do expect the framework outlined here to be a good starting point for more detailed discussions of confinement transitions in \( \alpha \)-RuCl\(_3\) and similar materials.

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[1] L. Savary and L. Balents, “Quantum spin liquids: a review,” Rep. Prog. Phys. 80, 016502 (2017)
[2] N. Read and S. Sachdev, “Large-N expansion for frustrated quantum antiferromagnets,” Phys. Rev. Lett. 66, 1773 (1991)
[3] T. Senthil and M. P. A. Fisher, “Z2 gauge theory of electron fractionalization in strongly correlated systems,” Phys. Rev. B 62, 7850 (2000)
[4] R. Moessner, S. L. Sondhi, and E. Fradkin, “Short-ranged resonating valence bond physics, quantum dimer models, and Ising gauge theories,” Phys. Rev. B 65, 024504 (2001)
[5] X.-G. Wen, “Quantum orders and symmetric spin liquids,” Phys. Rev. B 65, 165113 (2002)
[6] J. A. Sears, M. Songvilay, K. W. Plumb, J. P. Clancy, Y. Qiu, Y. Zhao, D. Parshall, and Y.-J. Kim, “Magnetic order in \( \alpha \)-RuCl\(_3\); A honeycomb-lattice quantum magnet with strong spin-orbit coupling,” Phys. Rev. B 91, 144420 (2015)
[7] H. B. Cao, A. Banerjee, J.-Q. Yan, C. A. Bridges, M. D. Lumsden, D. G. Mandrus, D. A. Tennant, B. C. Chakoumakos, and S. E. Nagler, “Low-temperature crystal and magnetic structure of \( \alpha \)-RuCl\(_3\),” Phys. Rev. B 93, 134423 (2016)
[8] A. Banerjee, C. A. Bridges, J.-Q. Yan, A. A. Aczel, L. Li, M. B. Stone, G. E. Granroth, M. D. Lumsden, Y. Yiu,
J. Knolle, S. Bhattacharjee, D. L. Kovrizhin, R. Moessner, D. A. Tennant, D. G. Mandrus, and S. E. Nagler, “Proximate Kitaev quantum spin liquid behaviour in a honeycomb magnet,” Nat. Mater. 15, 733 (2016)

[9] A. Banerjee, J. Yan, J. Knolle, C. A. Bridges, M. B. Stone, M. D. Lumsden, D. G. Mandrus, D. A. Tennant, R. Moessner, and S. E. Nagler, “Neutron scattering in the proximate quantum spin liquid α-RuCl₃,” Science 356, 1055 (2017)

[10] A. Little, L. Wu, P. Lampen-Kelley, A. Banerjee, S. Patankar, D. Rees, C. A. Bridges, J.-Q. Yan, D. Mandrus, S. E. Nagler, and J. Orenstein, “Antiferromagnetic resonance and terahertz continuum in α-RuCl₃,” Phys. Rev. Lett. 119, 227201 (2017)

[11] S.-H. Do, S.-Y. Park, J. Yoshitake, J. Nasu, Y. Motome, Y. S. Kwon, D. T. Adroja, D. J. Voneshen, K. Kim, T.-H. Jung, J.-H. Park, K.-Y. Choi, and S. Ji, “Incarnation of Majorana fermions in Kitaev quantum spin lattice,” arXiv:1703.01081

[12] J. A. Sears, Y. Zhao, Z. Xu, J. W. Lynn, and Y.-J. Kim, “Phase diagram of α-RuCl₃ in an in-plane magnetic field,” Phys. Rev. B 95, 184411 (2017)

[13] A. U. B. Wolter, L. T. Corredor, L. Janssen, K. Nenkov, S. Schönecker, S.-H. Do, K.-Y. Choi, R. Albrecht, J. Hunger, T. Doert, M. Vojta, and B. Bünchner, “Field-induced quantum criticality in the Kitaev system α-RuCl₃,” Phys. Rev. B 96, 041405 (2017)

[14] S.-H. Baek, S.-H. Do, K.-Y. Choi, Y. S. Kwon, A. U. B. Wolter, S. Nishimoto, J. van den Brink, and B. Bünchner, “Evidence for a field-induced quantum spin liquid in α-RuCl₃,” Phys. Rev. Lett. 119, 037201 (2017)

[15] J. Zheng, K. Ran, T. Li, J. Wang, P. Wang, B. Liu, Z.-X. Liu, B. Normand, J. Wen, and W. Yu, “Gapless spin excitations in the field-induced quantum spin liquid phase of α-RuCl₃,” Phys. Rev. Lett. 119, 227208 (2017)

[16] R. Hentrich, A. U. B. Wolter, X. Zotos, W. Brenig, D. Nowak, A. Isaeva, T. Doert, A. Banerjee, P. Lampen-Kelley, D. G. Mandrus, S. E. Nagler, J. Sears, Y.-J. Kim, B. Bünchner, and C. Hess, “Unusual phonon heat transport in α-RuCl₃: Strong spin-phonon scattering and field-induced spin gap,” Phys. Rev. Lett. 120, 117204 (2018)

[17] Y. J. Yu, Y. Xu, K. J. Ran, J. M. Ni, Y. Y. Huang, J. H. Wang, J. S. Wen, and S.Y. Li, “Ultralow-temperature thermal conductivity of the Kitaev honeycomb magnet α-RuCl₃ across the field-induced phase transition,” Phys. Rev. Lett. 120, 067202 (2018)

[18] Y. Kasahara, K. Sugii, T. Ohnishi, M. Shimozawa, M. Yamashita, N. Kurita, H. Tanaka, J. Nasu, Y. Motome, T. Shibatachi, and Y. Matsuda, “Unusual thermal Hall effect in a Kitaev spin liquid candidate α-RuCl₃,” Phys. Rev. Lett. 120, 217205 (2018)

[19] L. Y. Shi, Y. Q. Liu, T. Lin, M. Y. Zhang, S. J. Zhang, L. Wang, Y. G. Shi, T. Dong, and N. L. Wang, “Field-induced magnon excitation and in-gap absorption in the Kitaev candidate α-RuCl₃,” Phys. Rev. B 98, 094414 (2018)

[20] R. Hentrich, M. Roslova, A. Isaeva, T. Doert, W. Brenig, B. Bünchner, and C. Hess, “Large thermal Hall effect in α-RuCl₃: Evidence for heat transport by Kitaev-Heisenberg paramagnons,” arXiv:1803.08162

[21] Y. Nagai, T. Jinno, Y. Yoshitake, J. Nasu, Y. Motome, M. Hoh, and Y. Shimizu, “Two-step gap opening across the quantum critical point in a Kitaev honeycomb magnet,” arXiv:1810.05379

[22] A. Kitaev, “Anyons in an exactly solved model and beyond,” Ann. Phys. (N.Y.) 321, 2 (2006)

[23] G. Jackeli and G. Khaliullin, “Mott insulators in the strong spin-orbit coupling limit: From Heisenberg to a quantum compass and Kitaev models,” Phys. Rev. Lett. 102, 017205 (2009)

[24] H.-S. Kim, V. Shankar, A. Catuneanu, and H.-Y. Kee, “Kitaev magnetism in honeycomb α-RuCl₃ with intermediate spin-orbit coupling,” Phys. Rev. B 91, 241110 (2015)

[25] H.-S. Kim and H.-Y. Kee, “Crystal structure and magnetism in α-RuCl₃: An ab initio study,” Phys. Rev. B 93, 155143 (2016)

[26] S. M. Winter, T. Li, H. O. Jeschke, and R. Valentí, “Challenges in design of Kitaev materials: Magnetic interactions from competing energy scales,” Phys. Rev. B 93, 214431 (2016)

[27] R. Yadav, N. A. Bogdanov, V. V. Katukuri, S. Nishimoto, J. Brink, and L. Hozoi, “Kitaev exchange and field-induced quantum spin-liquid states in honeycomb α-RuCl₃,” Sci. Rep. 6, 37925 (2016)

[28] S. M. Winter, A. A. Tsirlin, M. Daghoffer, J. van den Brink, Y. Singh, P. Gegenwart, and R. Valentí, “Models and materials for generalized Kitaev magnetism,” J. Phys.: Condens. Matter 29, 493002 (2017)

[29] P. Lampen-Kelley, S. Rachel, J. Reuther, J.-Q. Yan, A. Banerjee, C. A. Bridges, H. B. Cao, S. E. Nagler, and D. Mandrus, “Anisotropic susceptibilities in the honeycomb Kitaev system α-RuCl₃,” Phys. Rev. B 98, 100403 (2018)

[30] J. Chaloupka and G. Khaliullin, “Hidden symmetries of the extended Kitaev-Heisenberg model: Implications for the honeycomb-lattice iridates A₂IrO₄,” Phys. Rev. B 92, 024413 (2015)

[31] J. G. Rau, E. K.-H. Lee, and H.-Y. Kee, “Generic spin model for the honeycomb iridates beyong the Kitaev limit,” Phys. Rev. Lett. 112, 077204 (2014)

[32] I. Rousochatzakis and N. B. Perkins, “Classical spin liquid instability driven by off-diagonal exchange in strong spin-orbit magnets,” Phys. Rev. Lett. 118, 147204 (2017)

[33] S. M. Winter, K. Riedl, P. A. Maksimov, A. L. Chernyshev, A. Honecker, and R. Valentí, “Breakdown of magnons in a strongly spin-orbital coupled magnet,” Nat. Commun. 8, 1152 (2017)

[34] A. Catuneanu, Y. Yamaji, G. Wachtel, Y. B. Kim, and H.-Y. Kee, “Path to stable quantum spin liquids in spin-orbit coupled correlated materials,” npj Quantum Mater. 3, 23 (2018)

[35] M. Gohlke, G. Wachtel, Y. Yamaji, F. Pollmann, and Y. B. Kim, “Quantum spin liquid signatures in Kitaev-like frustrated magnets,” Phys. Rev. B 97, 075126 (2018)

[36] S. M. Winter, K. Riedl, D. Kaib, R. Coldea, and R. Valentí, “Probing α-RuCl₃ beyond magnetic order: Effects of temperature and magnetic field,” Phys. Rev. Lett. 120, 077203 (2018)

[37] J. Knolle, S. Bhattacharjee, and R. Moessner, “Dynamics of a quantum spin liquid beyond integrability: The Kitaev-Heisenberg-Γ model in an augmented parton mean-field theory,” Phys. Rev. B 97, 134432 (2018)

[38] A. M. Samarakoon, G. Wachtel, Y. Yamaji, D. A. Tennant, C. D. Batista, and Y. B. Kim, “Classical and
quantum spin dynamics of the honeycomb Γ model,” Phys. Rev. B 98, 045121 (2018)
[39] J. B. Kogut, “An introduction to lattice gauge theory and spin systems,” Rev. Mod. Phys. 51, 659 (1979)
[40] P. Pfeuty, “The one-dimensional Ising model with a transverse field,” Ann. Phys. (N.Y.) 57, 79 (1970)
[41] S. R. White, “Density matrix formulation for quantum renormalization groups,” Phys. Rev. Lett. 69, 2863 (1992)
[42] H. Yao and X.-L. Qi, “Entanglement entropy and entanglement spectrum of the Kitaev model,” Phys. Rev. Lett. 105, 080501 (2010)
[43] B. Dóra and R. Moessner, “Gauge field entanglement in Kitaev’s honeycomb model,” Phys. Rev. B 97, 035109 (2018)
[44] M. Gohlke, R. Moessner, and F. Pollmann, “Dynamical and topological properties of the Kitaev model in a [111] magnetic field,” Phys. Rev. B 98, 014418 (2018)
[45] D. C. Ronquillo and N. Trivedi, “Signatures of fractionalization in the field orientation dependent spin dynamics of the Kitaev honeycomb model,” [arXiv:1805.03722]
[46] Z. Zhu, I. Kimchi, D. N. Sheng, and L. Fu, “Robust non-Abelian spin liquid and a possible intermediate phase in the antiferromagnetic Kitaev model with magnetic field,” Phys. Rev. B 97, 241110 (2018)
[47] P. A. McClarty, X.-Y. Dong, M. Gohlke, J. G. Rau, F. Pollmann, R. Moessner, and K. Penc, “Topological magnons in Kitaev magnets at high fields,” Phys. Rev. B 98, 060404 (2018)
[48] J. Nasu, Y. Kato, Y. Kamiya, and Y. Motome, “Successive Majorana topological transitions driven by a magnetic field in the Kitaev model,” Phys. Rev. B 98, 060416 (2018)
[49] H.-C. Jiang, C.-Y. Wang, B. Huang, and Y.-M. Lu, “Field induced quantum spin liquid with spinon Fermi surfaces in the Kitaev model,” [arXiv:1809.08247]
[50] L. Zou and Y.-C. He, “Field-induced neutral Fermi surface and QCD-Chern-Simons quantum criticalities in Kitaev materials,” [arXiv:1809.09091]
[51] G. Bastien, G. Garbarino, R. Yadav, F. J. Martinez-Casado, R. Beltrán Rodríguez, Q. Stahl, M. Kusch, S. P. Limandri, R. Ray, P. Lampen-Kelley, D. G. Mandrus, S. E. Nagler, M. Roslova, A. Isaeva, T. Doert, L. Hozoi, A. U. B. Wolter, B. Büchner, J. Geck, and J. van den Brink, “Pressure-induced dimerization and valence bond crystal formation in the Kitaev-Heisenberg magnet α-RuCl₃,” Phys. Rev. B 98, 060416 (2018)
[52] G. Li, X. Chen, Y. Gan, F. Li, M. Yan, S. Pei, Y. Zhang, L. Wang, H. Su, J. Dai, Y. Chen, Y. Shi, X. Wang, L. Zhang, S. Wang, D. Yu, F. Ye, J.-W. Mei, and M. Huang, “Raman evidence for dimerization and mott collapse in α-RuCl₃ under pressures,” [arXiv:1807.08211]
[53] R. Yadav, S. Rachel, L. Hozoi, J. van den Brink, and G. Jackeli, “Strain- and pressure-tuned magnetic interactions in honeycomb Kitaev materials,” Phys. Rev. B 98, 121107 (2018)