Simplified Simulation Method of Round Steel Bar Cooling
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Applying the finite difference method for numerical solution of one dimensional heat transfer equation provides a simple way for cooling process simulation of hypoeutectoid low alloy steel rod (round long length bar). The obtained results are comparable with ones calculated using finite element modeling commercial software. The temperature difference of the proposed model and literature experimental data for 50...270 mm diameter bars cooled in still air and conventional oil does not exceed 25°C for the whole examined time-temperature regions before the phase transformation.

KEY WORDS: modeling and simulation; steels; finite difference method (FDM); quenching.

1. Introduction

Application of heat treatment during manufacturing process is one of the most effective ways to improve main service properties of hypoeutectoid low alloy steel products: fire resistance,1) susceptibility to hydrogen induced cracking and sulfide stress cracking,2) abrasive wear resistance,3) strength and toughness.4) Depending on the cooling intensity of the metastable austenite even before its phase transformation different microstructures with different properties are obtained,5) especially throughout the cross-section of the products with large diameters.6) To predict final microstructure and properties it is worth to obtain the exact thermal history of exact section of the product by means of direct temperature measurements,7) numerical calculations8) or combined experiments.9)

In case of cooling hypoeutectoid low alloy steel rod (pipe and any rotationally symmetrical part) it becomes possible in a very simple and fast way to calculate the temperature distribution versus time at different distances from the round surface, subjected to cooling media. In the present paper we demonstrate the numerical approach to the temperature distribution calculation and compare obtained results with experimental data from literature and finite element modeling results obtained by Deform 3D software with the same input data.

2. Methodology

Calculation is based on the numerical solution of one dimensional heat transfer equation using finite difference method developed by E. Schmidt.10) Assumption of one dimensional heat transfer between cooling media and rod surface simplifies the calculation routine greatly and though makes it possible to apply the model for cooling long length round bars, pipes or any other cylinders with length to diameter ratio more than three.11) In this case the differential equation of heat transfer transforms to the following:

\[ \frac{\partial t}{\partial \tau} = a \left[ \frac{\partial^2 t}{\partial r^2} + 1/r \frac{\partial t}{\partial r} \right], \quad \text{(1)} \]

where \( t \) is temperature, °C; \( \tau \) is time, s; \( r \) is cylinder radius, m; \( a \) is thermal diffusivity coefficient, m²/s.

The application of finite difference method assumes that partial derivative is replaced by finite differences ratio,12) thus Eq. (1) can be rewritten as follows:

\[ \frac{\Delta t_{\text{const}}}{\Delta \tau} = a \left[ \frac{\Delta t_{\text{const}}}{\Delta r^2} + 1/r \frac{\Delta t_{\text{const}}}{\Delta r} \right], \quad \text{(2)} \]

where \( \Delta t_{\text{const}} \) is a temperature difference during \( \Delta \tau \) time at exact point from the surface of round bar cross-section, °C; \( \Delta (t_{\text{const}}) / \Delta r^2 \) is a difference of the temperature differences at time \( \tau \) at points located on \( \Delta r \) distance from exact point towards surface and center of a cylinder, °C/m²; \( \Delta t_{\text{const}} / \Delta r \) is a difference of the temperature at time \( \tau \) at exact point and point located on \( \Delta r \) distance towards cylinder center, °C/m.

To solve the proposed Eq. (2) numerically a round bar radius is divided into several equal parts \( m \) using Schmidt’s mesh as it is shown in Fig. 1. Meanwhile \( \Delta r \) determines \( \Delta \tau \) by the following statement:

\[ \Delta \tau = \Delta r^2 / (f \mu a), \quad \text{(3)} \]

where \( f \) is a template dimension (in our calculations \( f = 6 \)); \( \mu \) is an empirical fitting coefficient for \( a \), for the first time suggested in the present paper to obtain better match between calculated and experimental data.

During cooling the temperature field of round bar cross-section at any point of the radius is described in the following way:

\[ t_{(n+1)\Delta r, i\Delta \tau} = t_{(n+1)\Delta r, (i-1)\Delta \tau} + [f_{\Delta r, i\Delta \tau} - 2f_{\Delta r, (i-1)\Delta \tau} + f_{\Delta r, (i-2)\Delta \tau} + 1/(2m)(t_{(n+1)\Delta r, (i-1)\Delta \tau} - t_{(n+1)\Delta r, (i-2)\Delta \tau})]/f, \quad \text{(4)} \]

where \( n \) is time step number; \( i \) is the number of calculated layer.

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Fig. 1. Rod radius under consideration and its meshing.
The next required step consists of boundary condition statement. It is usual to apply the third kind boundary conditions,13 when it is assumed that dependence of heat-transfer coefficient versus temperature is known from literature14 or was determined using various experimental procedures.15 In differential form using μ we introduce the main boundary conditions as follows:

\[ \alpha(t_{0},\Delta t_{0}) = \mu ac_{p}(\bar{\tau} / \bar{r}), \] (5)

where \( \alpha \) is a heat-transfer coefficient of the cooling media, W/m²K; \( t_{0} \) is a product surface temperature, °C; \( \bar{\tau} \) is a temperature of the cooling media, °C; \( c_{p} \) is a heat capacity of the product, J/g K; \( \bar{r} \) is a density of the product, g/m³.

Using finite difference method and boundary conditions (5) the product surface temperature can be described by the following equation:

\[ t_{0} = \frac{\alpha(\Delta t_{0}) + \mu ac_{p}t_{0}}{\mu ac_{p} + \alpha}. \] (6)

Thus Eqs. (4) and (6) are the basic for calculation temperature distribution during cooling of low alloy steel rod (round long length bar).

The input data of the proposed model include of the product diameter at ambient temperature; thermal diffusivity coefficient, heat capacity and density of the specified steel at austenization temperature. These thermo physical properties are assumed to be constant during cooling process and so there is no need to conduct special calculations or expensive measurements of these characteristics at different temperatures. Furthermore calculation of steel cooling from austenite temperature region requires austenite properties only, because the properties of resulted structures are determined by the origin and kinetics of the previous phase transformation. The heat transfer coefficient of cooling media is the only temperature dependent input characteristic used in the calculations.

To determine \( \mu \) which has no physical origin at all we applied some literature data from well-known handbook16, p. 273, 274 Time-temperature curves obtained during oil and air cooling at different distances from the surface of round steel bars with diameters 100, 150, 180 and 270 mm were used. After introducing different integer values of \( \mu \) the results of modeling were compared with experimental data and in some cases modeling results do not match experimental data in the time-temperature region under consideration. Similar results were obtained for 50 mm diameter steel bar cooled in still air (Fig. 3). Comparison with experimental data18 demonstrates that application of the proposed model provides a remarkable prediction of cooling curve before the beginning of austenite transformation and the difference does not exceed 25°C. For the Deform 3D data the difference is not lesser and in some cases modeling results do not match experimental data in the time-temperature region under consideration.

3. Results and Discussion

Modeling results for 130 and 210 mm diameter bars cooled in oil were compared with experimental data16 and finite element modeling results obtained by Deform 3D software with the same input data (product diameter at ambient temperature; thermal diffusivity coefficient \( \alpha = 4.6 \times 10^{-6} \) m²/s, heat capacity \( c_{p} = 0.72 \) J/g*K and density \( \rho = 7.8 \times 10^{3} \) g/m³ of the specified steel at austenization temperature \( t_{0} = 865 \)°C; temperature dependence of the heat transfer coefficient of the cooling media7). The Deform 3D calculation parameters were the following:
- Heat Treatment module (only heat transfer modeling was applied);
- 2D axisymmetric model;
- material – steel C40, 4140;
- geometry of finite elements – 4-node quadrilateral (Fig. 1);
- number of finite elements – 5000;
- boundary conditions – temperature dependence of the heat transfer coefficient of the cooling media, as used for proposed simplified model;
- temperature of cooling media – 40°C for oil, 25°C for air;
- time step of calculation was set to be 1 s.

Cooling curves for various distances from the surface of bars are presented in Fig. 2.

To determine the temperature difference between modeling results and experimental data at any time the section-

![Fig. 2](https://via.placeholder.com/150)

Experimental (dots) and calculated (curves) temperature histories at different distances from the surface of: (a) 130 and (b) 210 mm diameter bars cooled in oil. (Online version in color.)
rial temperature history.

After the beginning of austenite transformation coupled thermo-mechanical and phase transformation analysis of the finite element modeling commercial software usually provided good match between experimental and calculated data. But coupled calculation techniques require a number of the temperature dependent material properties. In some cases an urgent and simplified estimation of the intensity of the cooling media or determination of the time needed to reach specified temperature in the steel bar of the given size are demanded. It is critical for control rolling, thermo-mechanical treatment and operations with restricted time tables.

The proposed simulation method can be used for solving an inverse problem of heat transfer when the cooling curves were obtained during experiments and the temperature dependence of heat transfer coefficient is to be determined (e.g. for cooling capacity control). In this case the following calculation algorithm is suggested:

1. The first iteration of calculation using available heat transfer coefficient dependence on temperature, e.g. from handbooks [14, 16] or other sources.
2. Comparison of calculated and experimental cooling curves using mathematical and statistical tools (Fisher criterion, standard deviation etc.).
3. Further iterations using adjusted heat transfer coefficient dependence on temperature until the best fit between the calculated and experimental cooling curves is obtained.

The calculated temperature dependence of heat transfer coefficient can be applied for subsequent modeling of cooling processes.

4. Conclusions

(1) Introducing an empirical fitting coefficient for thermal diffusivity coefficient in finite difference method calculations of temperature distribution during cooling in round bars provides temperature difference with experimental steel bar cooling curves lesser than 25°C before austenite phase transformation. Proposed model is valid for conventional oil and still air cooling of hypoeutectoid low alloy steel bars of 50, 100, 130, 150, 180, 210, 270 mm diameter.

(2) Input data for the calculations include product diameter at ambient temperature; thermal diffusivity coefficient, heat capacity and density of the specified steel at the austenitization temperature. The heat transfer coefficient of the cooling media is the only temperature dependent input characteristic used in the calculations.

(3) The algorithm of the proposed finite difference method model is considered to be useful for solving an inverse problem of heat transfer when the cooling curves were obtained during experiments and the temperature dependence of heat transfer coefficient is to be determined. Calculations seem to be valid not only for round steel bars, but also for cylinders of other metals and alloys.

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