Remarks on the properties of elliptical galaxies in modified Newtonian dynamics
(Research Note)

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ABSTRACT

Context. Two incorrect arguments against MOND in elliptical galaxies could be that the equivalent circular velocity curves tend to become flat at much larger accelerations than in spiral galaxies, and that the Newtonian dark matter halos are more concentrated than in spirals.

Aims. Here, we compare published scaling relations for the dark halos of elliptical galaxies to the scaling relations expected for MONDian phantom halos.

Methods. We represent the baryonic content of galaxies by spherical profiles, and their corresponding MONDian phantom halos by logarithmic halos. We then derive the surface densities, central densities, and phase space densities and compare them with published scaling relations.

Results. We conclude that it is possible to get flat circular velocity curves at high acceleration in MOND, and that this happens for baryonic distributions described by Jaffe profiles in the region where the circular velocity curve is flat. Moreover, the scaling relations of dark halos of ellipticals are remarkably similar to the scaling relations of phantom halos of MOND.

Key words. Galaxies: elliptical and lenticular, cD – Galaxies: kinematics and dynamics – Gravitation

1. Introduction

While data on large scale structures point towards a Universe dominated by dark matter and dark energy, e.g., Komatsu et al. (2011), the nature of these is still a deep mystery (e.g., Frieman et al. 2008, Wiltshire 2008, Bertone 2010, Kroupa et al. 2010). In this context, it is good to keep in mind that this conclusion essentially relies on the assumption that gravity is correctly described by Einstein’s General Relativity in the extreme weak-field limit, a regime where the need for dark matter itself prevents the theory from being tested. Until this double dark mystery is solved, it is thus worth investigating alternative paradigms and their implications.

For instance, Modified Newtonian dynamics (Milgrom 1983) MOND naturally explains various spiral galaxy scaling relations (Tully & Fisher 1977, McGaugh et al. 2000, McGaugh 2004). The existence of a very tight baryonic Tully-Fisher relation for disk galaxies (McGaugh 2005, Trachternach et al. 2009) is for instance one of the remarkable predictions of MOND. The corresponding relation for early-type galaxies is much more difficult to investigate because they are pressure-supported systems, and the equivalent circular velocity curves determined from the velocity dispersion profiles suffer from the well-known degeneracy with anisotropy. However, some studies circumvented this problem: for instance, Kronawitter et al. (2000) used data on 21 elliptical galaxies to construct non-parametric models from which circular velocity curves, radial profiles of mass-to-light ratio, and anisotropy profiles as well as high-order moments could be computed. This led Gerhard et al. (2001) hereafter G01 to publish benchmark scaling relations for ellipticals. It was e.g. shown for the first time that circular velocity curves tend to become flat at much larger accelerations than in spiral galaxies. This would seem to contradict the MOND prescription, for which flat circular velocities typically occur well below the acceleration threshold $a_0 \approx 10^{-3}$ cm s$^{-2}$, but not at accelerations of the order of a few times $a_0$ as in ellipticals. Also Thomas et al. (2009) hereafter T09 published scaling relations for dark matter halos of 18 Coma galaxies, using similar prescriptions as G01. We remark that G01 employed spherical models while the models of T09 are axisymmetric.

Not many studies have considered the predictions of MOND in elliptical galaxies. Milgrom (1984) showed that pressure-supported isothermal systems have finite mass in MOND with the density at large radii falling approximately as $r^{-2}$. It was also shown that there exists a mass-velocity dispersion relation of the form $(M/10^{11} M_\odot) = (\sigma/100 \text{ km s}^{-1})^4$ which is similar to the observed Faber-Jackson relation (Sanders 2000, 2010), and that, in order to match the fundamental plane, MOND models must deviate from being strictly isothermal and isotropic: a radial orbit anisotropy in the outer regions is needed (Sanders 2000, Cardone et al. 2011). Tret et al. (2007) and Angus et al. (2008) also analyzed the distribution of velocity dispersion of PNe on scales of 20 kpc, and of satellites on very large scales of the order of 400 kpc around red isolated ellipticals, showing that MOND allowed to fit both scales successfully.

Hereafter, we make general remarks on the properties of spherical galaxies within MOND, and their scaling relations. We first point out a remarkable property of elliptical galaxies ex-
2. Flat circular velocity curves and the Jaffe profile

Although it has been argued that some ellipticals do not need any dark matter or enhancement of gravity (Romanowsky et al. 2003), there are many counter-examples [Magorrian & Ballantyne 2001, Richtler et al. 2004, Schubert et al. 2006, Kumar et al. 2007]. Such a recent example is the elliptical galaxy NGC 2974 where the presence of an HI disk allowed a more or less direct measurement of circular velocities (Weijmans et al. 2008). There is also evidence that elliptical galaxies exhibit flat circular velocity curves, but that, contrary to spiral galaxies, this happens in the inner regions where $g > a_0$ (e.g., G01, Weijmans et al. 2008). Such a flattening of circular velocities is a priori not expected in the strong to intermediate gravity regime in MOND, and poses the question of how to analytically interpret it.

In the intermediate gravity regime, the transition from Newtonian to MONDian dynamics is described by the $\mu$-function of MOND. Many concordant studies have recently shown that, in spiral galaxies, the “simple” transition off Jaffe & Binney (2005) is a good representation of the data (Gentile et al. 2011) for an extensive discussion). In a spherical system, with this simple transition, the enclosed (baryonic) mass $M(r)$ needed to produce the same gravitational potential in MOND as the (baryonic+dark) mass $M_N(r)$ in Newtonian gravity is:

$$M_M(r) = M_N(r) - \left( \frac{1}{M_N(r)} + \frac{G}{r^2 a_0} \right)^{-1}. \quad (1)$$

In a region where the circular velocity is constant $v_c = V$ (even if $g > a_0$), one can write $M_N(r) = V^2 r / G$, and thus after some algebra

$$M_M(r) = \frac{V^4}{a_0 G} \cdot \frac{r}{r + V^2 / a_0}. \quad (2)$$

Remarkably, this enclosed mass profile corresponds precisely to a Jaffe profile (Jaffe 1983) with scale-radius $r_j = V^2 / a_0$ (meaning that the acceleration is $a_0$ at $r_j$), and with total mass $M_{tot} = V^4 / (a_0 G)$. Indeed, as the enclosed mass $M_M(r) = M_M(r_0) + 4 \pi \int_0^{r_0} \rho(R) R^2 dR$, this enclosed mass profile corresponds locally to the density profile:

$$\rho(r) = M_{tot} \frac{r_j}{4 \pi r_j^3} \frac{r}{r + r_j^2}, \quad (3)$$

with the characteristic surface density (see also Milgrom 1984)

$$M_{tot} / r_j^2 = a_0 / G. \quad \text{This profile is of course not valid for the very inner parts of an elliptical galaxy, where } V \text{ is not constant. Let us also note that (i) it was already known that a Jaffe profile produces a flat circular velocity curve at } r \ll r_j \text{ in Newtonian gravity, which MOND generalizes to radii } r \sim r_j; \text{ (ii) } M_{tot} \text{ does not necessarily have to be the real total mass of the galaxy, as the Jaffe profile fit to the density distribution could have a cutoff in the outer parts. In that case, the constant circular velocity } V \text{ would actually fall slightly above the prediction from the baryonic Tully-Fisher relation of spiral galaxies. Interestingly, this is precisely what is observed for the G01 sample of ellipticals.}

The fact that elliptical galaxies can exhibit (equivalent) circular velocity curves that are flat in the intermediate gravity regime is thus analytically understood in MOND by the fact that the outer regions of ellipticals can be approximated by a Jaffe profile with a large scale-radius, i.e. in regions well within the intermediate gravity regime rather than in the deep-MOND regime. These flat circular velocity curves would have been impossible with exponential density profiles (as encountered in spiral galaxies), meaning that the fact that circular velocity curves become flat quicker in ellipticals does not come as a surprise in the context of MOND.

This finding looks like an interesting possibility to devise new tests of MOND based on photometry. However, in reality it might be difficult: not many spherical galaxies with a precisely measured density profile are dynamically investigated out to large radii, and have enough tracers to measure the higher order moments and constrain the anisotropy. Moreover, light might not trace the baryonic mass precisely. As an example, the circular velocity in NGC 2974, which can be traced by an HI disk, becomes constant at around 5 kpc and has the value 300 kms$^{-1}$, which would correspond to a Jaffe scale radius of 23 kpc. Unfortunately, NGC 2974 is neither spherical nor does its photometry reach large radii so that it does not serve well as a test object. In any case, Weijmans et al. (2008, their Fig. 20) showed that the reverse procedure (going from the density to the circular velocity curve) leads to a very good fit.

3. Dark matter scaling relations for phantom halos of ellipticals

We now apply the reverse procedure, and check whether the phantom halos predicted by the simple transition of MOND (Famaey & Binney 2005) comply with the observational scaling relations of dark halos of ellipticals. As stated above, Jaffe profiles are not good descriptions of the very inner parts of ellipticals. We hereafter rather choose Hernquist profiles (Famaey & de Blok 2011) for an extensive discussion). In a spherical system, with this simple transition, the enclosed (baryonic) mass $M_M(r)$ needed to produce the same gravitational potential in MOND as the (baryonic+dark) mass $M_N(r)$ in Newtonian gravity is:

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To enable the comparison with the scaling relations of G01, where the dark matter halos are adopted as logarithmic halos, we fit \( v_{\text{phantom}}(r) \) to the circular velocity \( v_{\text{log}}(r) \) of a logarithmic halo with asymptotic circular velocity \( v_0 \) and core radius \( r_0 \):

\[
v_{\text{log}}(r) = v_0 \sqrt{ \frac{r}{r_0} + \frac{r^2}{r_0^2}}. \tag{7}
\]

The fits are performed within the inner two effective radii \( r_e \). The fitted central dark matter density is then given by

\[
\rho_0 = 3v_0^2/(4\pi G). \tag{8}
\]

The characteristic central phase space density is defined (see G01) as

\[
f_{ps} = 3^{3/2} \rho_0 / v_0^3. \tag{9}
\]

The characteristic surface density within \( r_0 \) is then also defined as (see also Donato et al. 2009):

\[
S = \rho_0 \cdot r_0. \tag{10}
\]

Table 1 lists these fitted parameters for six baryonic Hernquist masses over a large mass range. The combinations of the masses and effective radii in Table 1 follow equation (5) of G01 (in accordance with the fundamental plane), where we transformed their luminosities into masses by using \( M/L_B=8 \) for all galaxy baryonic masses.

Fig. 1 shows the values of the fitted dark halo parameters derived from applying MOND to the baryonic Hernquist-profiles, together with the observational scaling relations given by G01 (dotted lines) and T09 (dashed lines). The upper panel shows the characteristic phase space density, the middle panel the central volume density, and the lower panel the characteristic surface density. Let us note that the plotted relations are indicative only, since the data (Fig. 18 of G01 and Figs. 1 and 4 of T09) show a very large scatter even when logarithmically displayed. However, within this observational uncertainty, it is remarkable that some features are perfectly reproduced, particularly the slopes of the phase space density and of the central volume density as a function of baryonic mass (given the observational scatter, the almost perfect reproduction of the central volume density of G01 might of course be partly coincidental).

As first emphasized by G01, the phase-space density values are at a given mass higher in spirals than in ellipticals, which means that under the Cold Dark Matter paradigm, dark halos of ellipticals cannot be the result of collisionless mergers of present-day spirals, but must have been assembled at a very early time, when the cosmological density was higher. In MOND this is of course not necessarily the case, as the phase-space argument does not apply to phantom halos.

One also notes a remarkable exception to the scaling relations: the fitted characteristic dark matter surface density \( S \) is fully independent from the Hernquist parameters, and it is systematically lower than in G01 and T09. We emphasize that this constancy is not related to the special relation of mass and effective radius. Varying \( R_{eff} \) by a factor of two at a given mass does not change the constant surface density significantly. This prediction of MOND thus brings the value closer to the (also constant) value of \( S \) observed in spiral galaxies, \( \log S = 2.1 \) (Donato et al. 2009). Let us note that MOND also predicts the observed constant value of \( S \) in spirals, which is somewhat lower because (i) spirals are a bit deeper into the MOND regime (Milgrom 2009) and (ii) their flattened baryonic profiles lead to a somewhat higher Newtonian gravity at a given mass, and in turn a somewhat lower MOND contribution to the phantom halo.

On the first glance one might interpret this constancy and the other scaling relations as a clear signature of MOND in ellipticals: however, CDM may also predict that the surface density within the scale radius of NFW halos weakly depends on dark matter total mass (Boyardsky et al. 2010). For spiral galaxies, this is of little interest as it is known that cuspy profiles often do not fit rotation curves (Pamela & Binney 2005; de Blok 2010; Gentile et al. 2005), the mystery then being how to erase the cusp by feedback from the baryons while keeping the product \( \rho_0 r_0 \) constant. In elliptical galaxies, the situation is less clear as NFW profiles often do fit the data equally well as cored profiles (Schubbeth et al. 2010). We thus fitted NFW profiles to the same MONDian phantom halos and found a perfect agreement. The question remains whether these NFW-halos are “cosmological” or in other words, fulfill the relation between virial mass and concentration predicted by cosmological simulations. Fig. 2 displays for our Hernquist masses the resulting concentrations of the NFW-halos (open circles) corresponding to the MONDian phantom halos, while the triangles show the concentration values expected from the equation (9) of Macciò et al. (2008), using 200 times the critical density as the mean density within the virial radius (standard cosmology: \( h=0.7, \Omega_m=0.3, \Omega_{\Lambda} = 0.7 \). One concludes that for high masses the MONDian phantom halos are not distinguishable from cosmological NFW halos, given also that the simulations predict considerable scatter. For smaller masses the difference between MONDian phantom halos and NFW cosmological halos is larger.

### Table 1

| Baryonic Mass \( [M_*] \) | \( R_{eff} \) [kpc] | \( r_0 \) [kpc] | \( v_0 \) [km s\(^{-1}\)] | \( S[M_*/pc^2] \) | \( \rho_0[M_*/pc^3] \) | \( f_{ps} \) | acc. \([\alpha_0]\) |
|------------------------|-----------------|----------------|----------------|-----------------|----------------|----------------|----------------|
| \( 10^8 \)            | 14.1            | 8.33           | 244            | 374             | 0.04           | 8.82 \times 10^4 | 1.53           |
| \( 8 \times 10^{11} \) | 11.8            | 7.60           | 228            | 439             | 0.05           | 1.19 \times 10^8 | 1.55           |
| \( 5 \times 10^{11} \) | 8.06            | 5.20           | 193            | 411             | 0.076          | 3.02 \times 10^8 | 1.91           |
| \( 2 \times 10^{11} \) | 3.84            | 2.87           | 146            | 438             | 0.143          | 1.3 \times 10^{-7} | 2.96           |
| \( 10^{11} \)         | 1.47            | 1.24           | 99             | 431             | 0.35           | 1.02 \times 10^{-6} | 8.44           |
| \( 5 \times 10^{10} \) | 1.25            | 1.04           | 90             | 431             | 0.41           | 1.6 \times 10^{-6} | 6.08           |

1. Let us note that these fits are not particularly good: the circular velocity curve \( v_c(r) = v_0/(r_0 + r) \) would have provided better fits, but the core radius of the corresponding halo would then be systematically smaller with respect to G01.
4. External field effect

Due to the non-linearity of MOND and its associated breaking of the Strong Equivalence Principle, a MONDian stellar system embedded in an external gravitational field (EF) stronger than its own internal field behaves in a quasi-Newtonian way, with an effectively higher gravitational constant (Milgrom 1983; Famaey et al. 2007). Most of the sample galaxies are located in clusters or groups where the EF might have an influence (Wu et al. 2010) for instance showed how the EF can lead to the lopsidedness of an originally axisymmetric non-isolated galaxy.

While it is beyond the scope of this research note to evaluate in detail the EF in the present sample, a very rough estimation is presented in Fig. 3 which plots for the Virgo and the Coma cluster the accelerations based on the extrapolations of the mass models cited in the figure caption (these extrapolations are only meant to give an order of magnitude estimate, but should not be taken as rigorous models). This can be compared with the internal accelerations at 2 \( R_{\text{eff}} \) for the Hernquist models in Table 1. Indicated are the projected distances of galaxies in the Virgo and Coma region. The positions of the Virgo galaxies correspond to the middle points of their NGC numbers, while the Coma galaxies are plotted as small open circles. One concludes that the EF should have no influence in the two samples at the galactocentric distances which we consider.
5. Conclusion

Here we showed that (i) in MOND, galaxies exhibit a flattening of their circular velocity curve at high gravities ($g > a_0$) if they are described by a Jaffe profile with characteristic surface density $a_0/G$ in the region where the circular velocity is constant (since this is not possible for exponential profiles, it is remarkable that such flattenings of circular velocity curves at high accelerations are only observed in elliptical galaxies); (ii) the phantom halos of ellipticals predicted by MOND (i.e., the dark halos that would produce in Newtonian gravity the same additional gravity as MOND) can be fitted by logarithmic halos which perfectly reproduce the observed scaling relations of ellipticals for phase-space densities and central volume densities $\rho_0$; (iii) these halos have a constant characteristic surface density $\rho_0 r_0$; (iv) contrary to spirals (for which there are more data in the very central parts), the phantom halos of ellipticals can as well be fitted by cuspy NFW halos, the concentration of which is in accordance with the theoretical predictions of $\Lambda$CDM for the highest masses, but in slight disagreement for baryonic masses smaller than $10^{14} M_\odot$: a modern, large, sample of elliptical galaxies, which are dynamically well investigated out to large radii and cover a large range of masses, will thus be required to get discriminating power. But in any case, and whatever the true physical reason for it, it is remarkable that a recipe (MOND) known to fit rotation curves of spiral galaxies with remarkable accuracy also apparently predicts the observed distribution of ”dark matter” in elliptical galaxies.

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Additionally, the references include a variety of astronomical and astrophysical journals and articles, covering topics such as galaxy dynamics, dark matter, and the implications of Modified Newtonian Dynamics (MOND).