Abstract. Unruh’s detector calculation is used to study the effect of the defect angle $\beta$ in a space-time with a cosmic string for both the excitation and deexcitation cases. It is found that a rotating detector results in a non-zero effect for both finite (small) and infinite (large) time.
I. INTRODUCTION

Different aspects of cosmic strings are studied in many papers and reviews. One can give References 1, 2, 3 as a good point to start learning about this ever developing field. Among new physical processes where the effects of cosmic strings are studied one can cite references 4 and 5. Here stimulated and spontaneous emission near cosmic strings are studied. The presence of the cosmic string gives rise to modifications in the rates of these processes.

Here we do the similar calculation as in these references for different physical processes, using the model of a particle detection due to Unruh 6 and De Witt 7.

Section II is devoted to the review of the method and the results already known. We first go over the Davies-Sahni 8 results for the detector at rest and oscillating in the r and z directions. Note that if the detector switches on for a finite time \( T \), the response function depends on \( T \), the excitation energy \( E \), and the distance from the string \( R \). Here we will stick to the the standard method of 10, performing the calculation for infinite time.

In Section III we study the case when the detector revolves around the string at distance \( R \) with constant angular velocity \( \omega \). This case was also studied by Davies and Sahni 8, with no definite result. We do the computation both for finite (small) and infinite (large) time and we find a change in the detector response function for both cases. For the deexcitation amplitude, we find extra poles if the string parameter \( \beta \) is less than a definite value. We conclude with a few remarks.

II. REVIEW OF PREVIOUS RESULTS

As described in references 6, 7, and 10, here we assume that an idealized point particle, acting as a detector with internal energy levels labelled by the energy \( E \) is coupled via a monopole interaction with a scalar field \( \phi \). The particle detector moves along a world line described by the function \( x^\mu(\tau) \), where \( \tau \) is the detector’s proper time. The detector-field interaction is described by an interaction Lagrangian \( g m(\tau) \phi(x(\tau)) \), where \( g \) is a small coupling constant and \( m(\tau) \) is the detector’s monopole moment operator.

The calculation is performed in first order perturbation theory. We square the first order amplitude and sum over all the energies and scalar field excited states to get the
transition probability
\[ g^2 \sum_E |\langle E|m(0)|E_0 \rangle|^2 F(E - E_0) \] (1)

where the detector response function is given by \( F(E - E_0) \)
\[ F(E - E_0) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \exp \left[ -i(E - E_0)(\tau - \tau') \right] G^+(x(\tau), x(\tau')). \] (2)

Here \( G^+ \) is the Wightman function of a scalar particle for the metric in question; \( \tau \) is the proper time and \( E - E_0 > 0 \), is the excitation of the detector. If we consider the transition probability per unit time, we have to consider
\[ g^2 \sum_E |\langle E|m(0)|E_0 \rangle|^2 \int_{-\infty}^{\infty} d(\Delta \tau) e^{-i(E - E_0)\Delta \tau} G^+(\Delta \tau) \] (3)

where \( \Delta \tau = \tau - \tau' \).

We ascribe a certain trajectory to the detector and look for possible non-zero response. The effect depends not only on the metric, but also on the worldline followed by the detector. If we get zero response for a certain trajectory, this does not at all mean that we will get zero response for other trajectories for the same metric. In fact quite the contrary is known to be true. It is well known that when the detector accelerates in Minkowski space, we get a nonzero response, the Unruh effect, whereas when the detector is stationary, or moving with constant velocity, we get null result in the same space. On the other hand, in de Sitter space and for other non-flat metrics or in the presence of thermal radiation \(^{8,10}\), even a stationary trajectory gives a non-zero result.

Note that the results above refer to a detector calculation for infinite time. As a limiting case of the detector response for finite time, in Section 3.2 we study the behaviour of the integrand near \( \Delta \tau = 0 \). We show that the “pole” as \( (\Delta \tau - i\epsilon)^{-2} \to 0 \) depends on \( \beta \omega R \) only, hence a “background contribution” can be subtracted to regularize the integral. Then the first nonzero term in the Laurent expansion of the regularized integrand will be nonzero for \( \beta \neq \frac{1}{k} \), \( k \) integer, and zero otherwise.

Here we will study the response function in the cosmic string background. We anticipate variation from the Minkowski result if the presence of the cosmic string actually
changes the physics of the problem. We introduce the Wightman function to this formalism, already calculated by many authors \cite{11}. We use the form given by the expression

\[ G^+ = \frac{1}{(2\pi)^2 \beta r_1 r_2} \left[ \frac{1 - \Delta^2}{1 + \Delta^2} \right] \]

where

\[ r_1 = \left[-(t - t' - i\epsilon)^2 + (z - z')^2 + (r - r')^2 \right]^{\frac{1}{2}}, \]

\[ r_2 = \left[-(t - t' - i\epsilon)^2 + (z - z')^2 + (r + r')^2 \right]^{\frac{1}{2}}, \]

\[ \Delta = \left( \frac{r_2 - r_1}{r_2 + r_1} \right), \]

for the metric

\[ ds^2 = dt^2 - dr^2 - dz^2 - \beta^2 r^2 d\phi^2. \]

Here $\beta$ is a constant satisfying $0 < \beta \leq 1$.

In calculating $F(E)$, we first study a detector at rest. It is shown that at the coincidence limit for $z, r$ and $\phi$, the response function $F(E)$ is per unit time proportional to

\[ \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} d(\Delta t) \frac{e^{-i(E-E_0)(t-t')}}{\beta(-i(t-t' - i\epsilon))\sqrt{4r^2 - (t-t' - i\epsilon)^2}} \]

\[ \times \frac{\{ \sqrt{4r^2 - (t-t' - i\epsilon)^2} + i(t-t' - i\epsilon) \}^{\frac{1}{2}} + \{ \sqrt{4r^2 - (t-t' - i\epsilon)^2} - i(t-t' - i\epsilon) \}^{\frac{1}{2}}}{\{ \sqrt{4r^2 - (t-t' - i\epsilon)^2} + i(t-t' - i\epsilon) \}^{\frac{1}{2}} - \{ \sqrt{4r^2 - (t-t' - i\epsilon)^2} - i(t-t' - i\epsilon) \}^{\frac{1}{2}}} \]

As noted in reference 6 this expression has poles at $(t-t') = i\epsilon$. There are no cuts in the lower half plane. If we close the contour in the lower half plane, as we should since $E - E_0 > 0$ we get zero, the Minkowski result. Reference 9 shows that performing a finite integral in proper time gives non zero results.

To study the accelerating case we first note that in this space the equations of motion are

\[ \ddot{x}^\phi + \frac{1}{r} \dot{x}^\phi \dot{x}^r = F^\phi, \]

\[ \ddot{x}^r - \beta^2 r \dot{x}^\phi \dot{x}^\phi = F^r, \]

\[ \ddot{x}^t = F^t, \]

\[ \ddot{x}^z = F^z. \]
If the force is harmonic, i.e.,
\[ F_t = \frac{t}{\alpha^2}, \quad F_r = \frac{r}{\alpha^2}, \quad F_\phi = 0, \quad F^z = 0, \]
then we obtain
\[ t = \alpha \sinh \frac{\tau}{\alpha}, \quad r = \alpha \cosh \frac{\tau}{\alpha}. \]  

Then the response function reads
\[ \frac{F(E)}{T} = \frac{1}{\beta (2\pi)^2} \int_{-\infty}^{\infty} \frac{d(\Delta \tau) e^{-i(E-E_0)\Delta \tau}}{4\alpha^2 \sinh(\frac{\Delta \tau}{2\alpha}) - i\epsilon \cosh(\frac{\Delta \tau}{2\alpha})} \frac{A^\dagger + B^\dagger}{A^\dagger - B^\dagger}. \]  

where
\[ A = \cosh \frac{\tau + \tau'}{2\alpha} + i \sinh \left( \frac{\tau - \tau'}{2\alpha} - i\epsilon \right) \]
\[ B = \cosh \frac{\tau + \tau'}{2\alpha} - i \sinh \left( \frac{\tau - \tau'}{2\alpha} - i\epsilon \right). \]

In these expressions $-i\epsilon$ are put inside the hyperbolic cosine and sine functions. This is only correct when $\tau$ approaches $\tau'$, the only point where $\epsilon$ has any meaning.

Since $\epsilon > 0$, we do not get any cuts in the lower half plane.

We can perform the contour integration. The poles are at points where $\sinh(\frac{\Delta \tau}{2\alpha} - i\epsilon)$ vanish. Expanding the expression about the poles result in,
\[ \frac{F(E)}{T} = \frac{1}{(2\pi)^2 \beta} \int_{-\infty}^{\infty} \frac{d(\Delta \tau) e^{-i(E-E_0)\Delta \tau}}{\cosh(\frac{\tau}{2\alpha})} [2 + \sinh^2 \frac{\Delta \tau}{2\alpha} - i\epsilon \cosh \frac{\Delta \tau}{2\alpha} - i\epsilon \frac{1}{\beta} (\frac{1}{\beta} - 1) + \cdots]. \]  

All reference to $\beta$ nicely cancel. We end up with
\[ \frac{F(E)}{T} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{d(\Delta \tau) e^{-i(E-E_0)(\Delta \tau)}}{\alpha^2} \frac{E - E_0}{2\pi} \frac{E - E_0}{e^{2\pi (E-E_0)\alpha} - 1}. \]

which is the Minkowski result for an accelerating detector, or for a particle in a heat bath with temperature $T = \frac{1}{2\pi \alpha k_B}$, here $k_B$, is the Boltzman constant/8.

We can also accelerate our detector parallel to the string. The above result does not change if we take $z = \alpha \cosh \frac{\tau}{\alpha}, t = \alpha \sinh \frac{\tau}{\alpha}$ which corresponds to taking $F_r = 0, F_\phi = 0, F_t = \frac{t}{\alpha^2}, F_z = \frac{z}{\alpha^2}$. When we set $\phi = \phi', r = r'$, we get the same result, namely
\[ \frac{F(E - E_0)}{T} = \frac{1}{2\pi e^{2\pi (E-E_0)\alpha} - 1}. \]  


Note that moving paralel or perpendicular to the string does not matter.

These results are true for the infinite contour when we do not impose a cut-off on the interaction. If the interaction is switched off after a certain time, one finds that a finite effect due to the presence of the string is detected as shown in reference 9.

We only review the results prior to the publication of reference 9 in this section and refer to the original article for the situation for finite time in the stationary string case. We will, however, treat a new case with finite contour in the next section and show that there exists, indeed, a finite effect for this example.

For all these cases the integrand near $\Delta \tau = 0$ is studied and there is a nonzero qualitative effect. The case $\beta = 1/k$ is indistinguishable from $\beta = 1$ provided that certain other parameters are kept constant.

III. ROTATING DETECTOR

We consider now a detector rotating around the cosmic string in a plane perpendicular to the $z$ axis. In this case

$$F^\phi = F^t = F^z = 0, \quad F^r = -A_\beta^2 \beta^2 \omega^2 R,$$

with

$$A_\beta = \frac{A}{\sqrt{1 - \omega^2 \beta^2 R^2}}.$$  

Here $A$ is a constant and $R$ is the distance from the cosmic string. It can be seen that then the trajectory is given by $x^r = \text{const.}, \ x^t = \text{const.}, \ x^z = \text{const.}$ and $x^\phi = \omega t$, with $t = A_\beta \tau$.

We take $z = z', \ r = r' = R, \ \phi - \phi' = \omega(t - t')$. Then

$$r_1 = i(t - t' - i\epsilon),$$
$$r_2 = \sqrt{4R^2 - (t - t' - i\epsilon)^2},$$
$$\cos(\phi - \phi') = \cos \omega(t - t').$$

Writing $\Delta = \frac{\Delta_+}{\Delta_-}$ with $\Delta_\pm = r_2 \pm r_1$, \quad $F(E - E_0)/T$ reduces to
\[
\frac{F(E - E_0)}{T} = \frac{1}{(2\pi)^2 \beta} \times \int_{-\infty}^{\infty} \frac{d(\Delta(\tau))e^{-i(E - E_0)\Delta\tau}}{i(\Delta t - i\epsilon) \sqrt{4R^2 - (\Delta t - i\epsilon)^2}} \frac{(\Delta_+^\beta + \Delta_-^\beta)(\Delta_+^\beta - \Delta_-^\beta)}{\Delta_+^\beta + \Delta_-^\beta - 2\Delta_+^\beta \Delta_-^\beta \cos \omega(t - t')} \tag{22}
\]

Note that for finite \(\epsilon\), the integrand is technically non-divergent, however for small \(\epsilon\) the above integral cannot be computed numerically. In Section 3.1 we shall introduce a coordinate transformation to convert the integral over the real line to a contour integral in the complex plane and use residue calculus to obtain the result. In Section 3.2, we shall study the behaviour of the integrand near \(\Delta \tau = 0\) and we will use a Laurent expansion of the integrand around the “pole” to study the divergence.

### 3.1 Infinite (large) time behaviour.

To simplify the integral (22), we make the change of variable

\[
t - t' - i\epsilon \to 2R \sin(z)
\]

where \(z\) is the complex variable, \(z = x + iy\). Then \(r_2 = \pm 2R \cos z\), however it can be seen that the integrand is independent of the sign of \(r_2\). Taking the positive sign, the new parametrization gives

\[
\Delta_+ = 2Re^{iz}, \quad \Delta_- = 2Re^{-iz}.
\]

Using also \(\Delta \tau = \Delta t/A_\beta\), our integral simplifies to

\[
I = \frac{1}{R(2\pi)^2 \beta A_\beta} \int dz e^{-i\frac{E - E_0}{A_\beta}(2R \sin(z) + i\epsilon)} \frac{\sin \frac{\Delta^\beta}{\beta} \cos \frac{\Delta^\beta}{\beta}}{\sin z \left( \cos \frac{2\Delta}{\beta} - \cos (2\omega R(\sin z + i\epsilon)) \right)} \tag{23}
\]

**The case \(E - E_0 > 0\).**

We first consider the case \(E - E_0 > 0\), hence we close the contour in the lower half plane. The imaginary part of the expression

\[
\Delta t - i\epsilon = 2R(\sin(x) \cosh(y) + i \cos(x) \sinh(y)). \tag{24}
\]

defines the contour of integration and its real parts determine the lines of constant \(\Delta \tau\). It can be seen that the integrand vanishes as \(y \to -\infty\), hence the contour integral can be
evaluated using residues. For small \( \epsilon \), the product \( \cos(x) \sinh(y) \) is small hence \( x \to \pi/2 \) as \( y \to -\infty \). Thus the contour of integration looks like the union of straight lines \( \{ x = \pm \pi/2 \} \), \( y < 0 \) joined by a curve just below the \( x \)-axis.

The poles inside the contour are the zeros of \( \cos^{2z_\beta} - \cos(2 \omega R (\sin z + i \epsilon)) \). Using the identity \( \cos p - \cos q = -2 \sin(\frac{p-q}{2}) \sin(\frac{p+q}{2}) \) with \( p = \frac{2z_\beta}{\beta} \), \( q = 2 \omega R (\sin z + i \epsilon) \), it can be seen that the zeros of poles correspond to

\[
\frac{x}{\beta} \pm \omega R \sin x \cosh y = -k\pi, \quad \frac{y}{\beta} \pm \omega R \cos x \sinh y = \pm \epsilon/2 = 0.
\]

However, as for \( y \) and \( \sinh y \) have the same sign, for small \( \epsilon \) the second equation can be satisfied only with the negative sign. Furthermore it can also be seen that \( k \) has to be positive.

In the formulation \( \epsilon \) was introduced to avoid the poles at on the real axis. Hence after restricting the poles to the ones that occur for \( \epsilon > 0 \) we can take the limit \( \epsilon \to 0 \) and the only poles are now given by

\[
x_k - \beta \omega R \sin x_k \cosh y_k = -\beta k\pi, \quad k > 0, \quad y_k - \beta \omega R \cos x_k \sinh y_k = 0. \tag{25}
\]

It can also be seen that as \( x \to -x \) the integrand for \( z = x + iy \) goes to negative of its complex conjugate, hence the contour integral is real. The integral around the contour can be evaluated using residues as

\[
I = \frac{-1}{4 \pi i} \frac{2\pi i}{R(2\pi)^2 \beta A_\beta} \sum_{k=-\infty}^{k=+\infty} \frac{e^{-i(E-E_0)/A_\beta}(2R \sin z_k)}{\sin z_k \sin(\frac{2z_k}{\beta} + \omega R \sin z_k) \cos(\frac{2z_k}{\beta} - \omega R \sin z_k) (\frac{1}{\beta} - \omega R \cos z_k)}
\]

\[ \tag{26} \]

At the poles \( \sin(\frac{2z_k}{\beta} - \omega R \sin z_k) = 0 \) hence \( \cos(\frac{2z_k}{\beta} - \omega R \sin z_k) = (-1)^k \) and it can be seen that \( \sin(\frac{2z_k}{\beta} + \omega R \sin z_k) = \sin \frac{2z_k}{\beta} \cos(k\pi) \). Thus the integral is simplified to

\[
I = \frac{i}{8 \pi R A_\beta} \sum_{k=-\infty}^{k=+\infty} \frac{e^{-i(E-E_0)/A_\beta}(2R \sin z_k)}{\sin z_k \sin z_k} \frac{1}{1 - \beta \omega R \cos z_k} \tag{27}
\]

From this expression it is clear that the explicit dependence on \( \beta \) is through the location of the poles. The contribution from the pole corresponding to \( k = 0 \) is the dominant one and it depends on \( \beta \omega R \) only.
We now show that the summation above is convergent. Using symmetry properties of the integrand we can see that the residues for $k < 0$ are the negative complex conjugates of the residues for $k > 0$. Hence the convergence of the series is determined by the convergence of
\[
\frac{-i}{8\pi RA\beta} \sum_{k=0}^{\infty} \frac{1}{\sin z_k} \frac{1}{1 - \beta\omega R \cos z_k}
\]
for large $y$. We can take $x = \pi/2$, $\sinh y = -e^{-y}/2$, $\cosh y = e^{-y}/2$, and $e^y = \frac{\beta\omega R}{\pi - 2\beta \pi}$. Thus by comparison with the $\sum \frac{1}{k^2}$, it can be seen that the series is convergent.

We give below numerical values of the residues for typical values of the parameters. As a physically realistic case we take
\[
\beta = 0.9, \quad R = 1, \quad \beta\omega R = 0.6, \quad E - E_0 = 1.
\]
Then the contributions from the first few poles are given below.
\[
-0.007334, \quad -0.00010600, \quad 5.1 \times 10^{-7}, \quad 2.59 \times 10^{-6}, \quad -6.16 \times 10^{-8},
\]
\[
-3.37 \times 10^{-7}, \quad -3.92 \times 10^{-8}
\]

The contributions from the residues for large $\Delta \tau$ become quickly comparable with computational precision and it is not meaningful to attempt a computation for large $\Delta \tau$ using residues only. Hence as an approximation for the integral for infinite time we use a contour consisting of the union of the original contour with $\Delta \tau < 1000$ and the horizontal line joining the two end points. The integral for finite but very large time is obtained as the sum of residues inside the contour minus the value of the integral along the horizontal line.

We have obtained the plots of these integrals for various combinations of $\beta$, $\omega$ and $R$ values. By numerical integration one can verify that the integral converges to a finite value as the range of integration is increased. We calculated the value of the integral for $\omega R = 0.6$ and for $\omega R = 0.8$ for $\beta$ ranging between unity and 0.61. These values are plotted in Figures 1.a and b. Here the value of the integral for $\beta = 1$ is subtracted from the value found for a particular $\beta$. We find that these two figures can be fitted to the function
\[
A_{\omega R}[\exp(14\pi(1 - (\omega R)^2)^{1/3}\omega R(\beta - 1)\sqrt{1 - (\omega \beta R)^2}) - 1]
\]
where the \( A_\omega R \) varies with \( \omega R \) as given in Fig.2. We also calculated the behaviour of the integral for constant \( \beta \omega R \) as \( \beta \) ranges from unity to .61. This behaviour is seen in Fig.3 and can be fitted to the function

\[
A_{\beta \omega R}(\exp a_{\beta \omega R}(\beta - 1) - 1).
\]

Here we find that \( a_{\beta \omega R} \) is much smaller than the constant in the previous case; its sign is different and is only five percent of that number in magnitude.

In both cases we conclude that there is a distinct difference when the cosmic string is present compared to the case when it is absent. We see that the general behaviour does not change considerably as time ranges from small to large values.

**The case** \( E - E_0 < 0 \).

In the previous calculation we assumed \( E > E_0 \). If \( E < E_0 \), still using the change of variables \( t - t' - i\epsilon \to 2R\sin z \), the integral over the real line is mapped to the contour determined by the imaginary part of (24), but in this case we cannot close the contour as \( y \to -\infty \), as the integrand does not vanish there. We can however use the following symmetry argument to obtain a closed contour and use Cauchy’s theorem to evaluate the integral. The contour of integration in the lower half plane can be deformed to a nearly rectangular path consisting of the union of the lines \( \{x = \pm\pi/2, y < \epsilon\} \) and \( \{y = -\epsilon, -\pi/2 < x < \pi/2\} \). Note that the integrand is invariant under \( y \to -y \) when \( x = \pm\pi/2 \). We consider the contour consisting of the union of the lines \( \{x = \pm\pi/2, y > -\epsilon\} \), and \( \{y = -\epsilon, -\pi/2 < x < \pi/2\} \). The integral over the line segments \( \{x = \pm\pi/2, -\epsilon < y < \epsilon\} \) arise as additional terms in the integral over this new contour but these terms go to zero as \( \epsilon \to 0 \) provided that there are no poles on these lines. The new contour can be closed as \( y \to \infty \) and residue calculus can be used to compute the integral.

We looked for the poles of the integrand in this region. Since equations (25) are invariant for \( -y \) going to \( y \), we get the same number of poles as the previous case. There are extra ones, though. There is a pole at \( z = 0 \) for all values of \( \beta \) including unity. The existence of extra poles when \( \beta \) is less than one depends strongly on what this value is. For values of \( \beta \) close to unity, we could not find any new poles. We found the first pole for
\( \beta < \frac{\pi}{2(\pi-1)} \) if we set \( \omega R \) equal to unity. For values of angular velocity less than unity, we find poles for smaller values of \( \beta \). If \( \beta = 1/2 \) a new pair of poles exist for any finite value of \( \omega R \).

We checked the presence of other poles in the rectangular region by studying the equations carefully and by performing contour integrations around finite regions numerically which gave zero within sensible limits.

When \( \beta > \frac{\pi}{2(\pi-1)} \) we have only one extra pole at \( z \) equals zero. We can evaluate the residue corresponding to this pole. In the presence of the string we get

\[
\frac{F_1}{T} = \frac{(E_0 - E)\Theta(E_0 - E)}{32\pi R(1 - \omega^2 \beta^2 R^2) \Lambda^2_\beta}.
\] (29)

When there is no string we get the similar expression where \( \beta = 1 \). Then we see that the expressions given for these two cases are identical if we take only the extra poles into account.

If we have \( \beta \) taking values which seems to be excluded by experiments, however, we get the signature of the string in the residue of two new extra poles.

If \( 1/2 \leq \beta < \frac{\pi}{2(\pi-1)} \), we may have a new pair of poles for appropriate values of \( \omega R \). If \( \beta = 1/2 \), then any positive value of \( \omega R \) allows one pair of new poles. For \( \beta = 1/2 \) and \( 2\omega R \sin \frac{3\pi}{8} = \pi/2 \), then we can evaluate the residue. The extra contribution is given by

\[
\frac{1}{32\pi R \Lambda_\beta^2} \frac{\Theta(E_0 - E)}{\sin \frac{3\pi}{8}} \sin \left( \frac{(E - E_0)R \sin 3\pi/4}{\Lambda_\beta} \right).
\] (30)

In this expression both the value of \( \beta \) and \( \omega R \) are fixed by the equations given above.

If \( \beta < 1/2 \), a second pair of poles may come up depending on the value \( \omega R \) takes. If \( \beta = 1/3 \), we have the second pair of poles for any value of \( \omega R \). Only the value of the residue depends on \( \omega R \). Similar behaviour goes on. For \( 1/4 < \beta < 1/3 \) another pair of poles is possible. For \( \beta = 1/4 \), we have the new pair for any \( \omega R \), etc,. At the end we got a formula like the one given in reference 8, also given in ref’s 4 and 9, for similar processes. This extra contribution reads

\[
F_{extra}/T = \Theta(E_0 - E) \sum_{i=1}^{p-1} C_i \cdot \frac{2RA_\beta}{\Lambda_\beta} \sin \left( (E - E_0)R \frac{\sin 2\theta_i}{\Lambda_\beta} \right).
\] (31)
where \( C_i, \theta_i \) are constants depending on the location of the pole. Here \( \frac{1}{\beta} = p \) where \( p \) is an integer. If \( 1/\beta \) is not an integer then the sum goes up to the integer less than \( 1/\beta \) or \( 1/\beta - 1 \) depending on the value of \( \omega R \).

3.2 Dedector response for finite (small) time.

In this section we study the behaviour of the integrand

\[
I_\beta(\Delta \tau) = \frac{1}{(2\pi)^2} \frac{1}{\beta r_1 r_2} e^{-i \Delta E \Delta \tau} \frac{1 - \Delta^2/\beta}{1 + \Delta^2/\beta - 2\Delta^{1/\beta} \cos(\Delta \phi)}
\]

near \( \Delta \tau = 0 \).

Let \( p = \Delta t - i\epsilon \). We shall express the integrand in terms of \( p \) and obtain first three terms of its Laurent expansion around \( p = 0 \). At the coincidence limits \( \Delta z = 0, \Delta r = 0 \), we have

\[
\begin{align*}
    r_1 &= ip, \\
    r_2 &= \sqrt{4R^2 - p^2}, \\
    \Delta \phi &= \omega(p + i\epsilon), \\
    \Delta \tau &= \frac{1}{A_\beta}(p + i\epsilon).
\end{align*}
\]

Inserting these in the integrand we obtain an expression \( I_\beta(p) \). We obtain the Laurent expansion of \( I_\beta(p) \) using REDUCE as follows.

\[
I_\beta(p) = e^{\Delta E \epsilon / A_\beta} \frac{1}{4\pi^2(\beta^2 \omega^2 R^2 - 1)} \frac{1}{p^2} - i \frac{e^{\Delta E \epsilon / A_\beta}}{4\pi^2 A_\beta (\beta^2 \omega^2 R^2 - 1)} \frac{1}{p} + \frac{e^{\Delta E \epsilon / A_\beta}}{48\pi^2 \beta^2 R^2} \left[ (\beta \omega R)^4 + 2\beta^2 (\beta \omega R)^2 - 2(\beta \omega R)^2 - \beta^2 + 1 \right] + \frac{e^{\Delta E \epsilon / A_\beta}}{48\pi^2 A_\beta} [\Delta E^2 (1 - (\beta \omega R)^2)] + \ldots
\]

(34)

Note that in the limit \( \epsilon \to 0 \), \( p \) is proportional to \( \Delta \tau \) and the \( \frac{1}{p} \) term has no contribution when integrated over a symmetric interval. Thus the divergence is due to the \( \frac{1}{p^2} \) term.

Now let \( \beta \omega R \) be fixed and consider the difference \( I_\beta(p) - I_{1/k}(p) \). In this case, as the coefficients of \( \frac{1}{p^2} \) and \( \frac{1}{p} \) do not depend on \( \beta \) the divergences are eliminated. Then we have

\[
I_\beta(p) - I_{1/k}(p) = e^{\Delta E \epsilon / A_\beta} \frac{1 - \beta^2 k^2}{48\pi^2 \beta^2 R^2} + O(p).
\]

(35)

Thus the difference \( I_\beta(p) - I_{1/k}(p) \) with \( \beta \omega R \) fixed is regular at \( \Delta \tau = 0 \) and there is a qualitative difference between the cases \( \beta = 1/k \) and \( \beta \neq 1/k \).
Repeating a similar Laurent expansion for a stationary detector, it can be seen that the difference \( I_\beta(p) - I_{1/k}(p) \) is nonzero for \( \beta \neq 1/k \). This result agrees with Reference 9, where there is nontrivial detector response for a stationary detector when the interaction time is finite.

The method of obtaining a Laurent expansion of the integrand around a pole can be applied without specifying the trajectory explicitly. We briefly outline the method but omit explicit calculations. We assume that \( r_1, r_2, \Delta \tau \) and \( \Delta \phi \) are certain analytic functions of \( p \) such that

\[
\lim_{p \to 0} r_1 = 0, \quad \lim_{p \to 0} r_2 \neq 0, \quad \lim_{p \to 0} \phi = O(\epsilon) \quad \lim_{p \to 0} \Delta \tau = O(\epsilon).
\]

We can then obtain the Laurent expansion of \( I_\beta(p) \) with straightforward but messy computations. Under these general assumptions, the Laurent expansion starts with \( p^{-2} \) term and with additional symmetry assumptions it is possible to ensure that \( p^{-1} \) term has no contribution if integrated over a symmetrical interval. The integrand is regularized by subtracting the “background” contribution and the constant term in the regularized integrand in proportional to \( 1 - \beta^2 k^2 \) as before.

The “background” for each trajectory has to taken a spacetime with \( \beta = 1 \) with trajectory parameters such that the expressions involved in the divergent terms are kept fixed. For example, in the case of a detector in a spacetime with \( \beta = 0.9, R = 1, \omega = 0.6 \), so that \( \beta \omega R = 0.54 \), the “background” has to be a detector moving in the Minkowski space with say \( R = 1, \omega = 0.54 \).

CONCLUSION

Here we first reviewed the already known material concerning whether the presence of a cosmic string can be felt by an Unruh detector, in a new formalism. We then extended our calculations to the undecided case when a detector revolves around a string with a constant angular velocity. This case was first studied in reference 8 and a definite answer was not obtained. We found that both the integrand and the resulting integral are different from the expressions we get in the absence of the cosmic string. We find a qualitative difference when we study the integrands, though. For \( \beta \neq 1/k \), the subtracted
expression does not vanish as the argument approaches zero, whereas the contrary is true when \( \beta = 1/k \). This effect can be seen from the integrand expression for the detector by performing the integration over a small interval, and dividing the integral by the interval. In the limit the interval goes to zero, we will get zero if \( \beta = 1/k \) and a non zero result if \( \beta \neq 1/k \).

If we study the case when \( E_0 \) is greater then \( E \), which describes the deexcitation of the detector, we find extra poles if \( \beta \) is less than a certain value. For the critical value of \( \beta < \frac{\pi}{2(\pi-1)} \), there is a new contribution which occurs only for very fast particles with velocities close to that of light . When \( \beta = 1/2 \), a particle with any finite velocity will sense this effect. For even smaller values of \( \beta \) we have additional contributions which first occur for fast or slow particles depending upon the value of \( \beta \).

A quantization condition is seen to set in for \( \beta = 1/p \), where \( p \) is an integer. An extra contribution is possible only if we pass a new threshold. To be certain of the new contribution, for any non zero value of \( \omega R \), we must go to the next value, \( \beta = \frac{1}{p+1} \). This behaviour reminds us of the Bohr-Sommerfeld condition of fitting an integer number of waves on the cone, this number depending on \( p \). If we are between \( p \) and \( p + 1 \) we may be able to fit another one depending on the value of \( \omega R \) which decides where on the cone this wave will be located.

We think that this quantization phenomena depending on the value of \( \beta \) and the “new phenomena ” that occurs at a critical value of \( \beta \) where we first have a new pair of poles should be investigated in other physical processes as well.

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Fig 1a. Numerical integration results for the detector response function when $\beta$ varies from 0.61 to 1 and $wR = 0.6$.

Fig 1b. Numerical integration results for the detector response function when $\beta$ varies from 0.61 to 1 and $wR = 0.8$.

Fig 2. The variation of the coefficient as a function of $wR$.

Fig 3. Numerical integration results for the detector response function when $\beta$ varies from 0.61 to 1 and $w\beta R = 0.6$. 
This figure "fig1-1.png" is available in "png" format from:

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