MATHEMATICAL MODEL OF THE PUMPING UNIT OF MACHINE WATER LIFTING SYSTEMS

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Abstract. Pumping stations have been investigated as objects of control and energy saving. Methods for determining energy-efficient processes of functioning of water-lifting pumping stations with the formation of their energy-saving modes are given. Mathematical methods have been developed for describing and modeling a pumping unit for machine water lifting systems.

1. Introduction

Nowadays, one of the main sources of meeting the growing demand of the economy of the Republic of Uzbekistan for electricity should be energy and resource conservation. Therefore, the tasks of developing energy-saving technologies in the sectors of the economy of the republic, in particular, at such energy-intensive objects as irrigation pumping stations of machine water lifting systems acquire a decisive importance. It is known that the main technical and economic indicators of the operation of a pumping unit as indeed of the entire pumping station are the efficiency of the pumping unit (station) and the specific power consumption spent on the supply (pumping) of a unit of water volume into the hydraulic pressure network [1]. It was determined [2] that when forming a mathematical model of a pumping unit for machine water lifting systems to ensure energy-saving modes of operation, as a criterion for assessing the efficiency of its work, it is necessary to take into account the cavitation-abrasive wear of its working bodies can be determined by the expression [4]:

\[ \eta_i = K_{wear} \cdot \eta \]  

where

- \( K_{wear} \) – the wear factor of the pump’s working bodies, calculated as \( K_{wear} = e^{-0.00000833 \cdot t} \);
- \( \eta \) – the value of the efficiency of the pump determined for each specific operating condition of the pump unit in accordance with [5].

The current value of the efficiency of the pump by taking into account the cavitation-abrasive wear of its working bodies can be determined by the formula:

\[ \eta_{D,i} = \frac{1}{1 + (1/\eta_{D,r} - 1) \cdot (K_{Z,i} + A_T/K_{Z,i})/(1 + A_T)} \]  

where

- \( \eta_{D,r} \) – therated value efficiency of the drive motor;
- \( A_T \) – loss factor of the electric motor.

For an asynchronous motor:

- \( \text{atn} \leq 1000 \text{ RPM} A_{r}=0.5 \);  
- \( \text{atn} \geq 1000 \text{ RPM} A_{r}=0.7 \);  

and for a synchronous motor:

- \( \text{atn} \leq 1000 \text{ RPM} A_{r}=1 \);  
- \( \text{atn} \geq 1,000 \text{ RPM} A_{r}=2 \).

The value of the efficiency of the pump drive motor by taking into account its load can be determined by the formula:

\[ \eta_{D,i} = \eta_i \cdot \eta_{D,i} \]  

where

- \( \eta_{D,i} \) – efficiency of \( i \)-th pump unit as part of a pumping unit;
- \( \eta_i \) – pump efficiency of \( i \)-th pump unit;
- \( \eta_{D,i} \) – efficiency of the drive motor \( i \)-th pump unit;
- \( \eta_{T,i} \) – transmission efficiency.

2. Experimental research

As is known [3], the specific power consumption for the supply of 1 million m³ of pumped water to a height of 1 meter of water column for pumping units is determined by the following formula:

\[ \Delta E = 2,724/\eta_{PU,i}, \text{ kW} \cdot \text{h/mln. m}^3 \cdot \text{m}, \]  

where

\[ \eta_{PU,i} = \eta_i \cdot \eta_{D,i} \cdot \eta_{T,i}. \]  

There \( \eta_{PU,i} \) – efficiency of \( i \)-th pump unit; \( \eta_i \) – pump efficiency of \( i \)-th pump unit; \( \eta_{D,i} \) – efficiency of the drive motor \( i \)-th pump unit; \( \eta_{T,i} \) – transmission efficiency.

The current value of the efficiency of the pump by taking into account the cavitation-abrasive wear of its working bodies can be determined by the expression [4]:

\[ \eta = K_{wear} \cdot \eta, \]  

\[ \eta_{D,i} = \frac{1}{1 + (1/\eta_{D,r} - 1) \cdot (K_{Z,i} + A_T/K_{Z,i})/(1 + A_T)} \]  

\[ \eta_{PU,i} = \eta_i \cdot \eta_{D,i} \cdot \eta_{T,i}. \]  

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the rated power of the drive electric motor of the pump unit; 
P_{M,i} – the mechanical power of the pump is determined by the expression (7) [5]:

\[ P_{M,i} = A_{P_i} \times Q_i - B_{P_i} \times Q_i^2 + C_{P_i} \]

where 
\( A_{P_i}, B_{P_i}, C_{P_i} \) - constant approximation coefficients of the ith pump; 
\( Q_i \) - capacity (performance) of the ith pump at the steady-state operating point. 

The combination of the above formulas makes it possible to calculate using the expression (1) the specific consumption of electrical energy of each of the pumping units, combined for joint operation into a common hydraulic pressure network as a part of a pumping unit. The specific flow rate of the pumping unit of machine water lifting systems as a whole can be determined as:

\[ \Delta E = 2.724/ \eta_{PU}, \text{kw} \times \text{h/mln. m}^3 \times \text{m}, \quad (8) \]

where 
\( \eta_{PU} \) - the efficiency of the pumping unit. 
An expression \( \eta_{PU} \) can be obtained for any number of operating pumping units, structurally combined for joint parallel operation into a common hydraulic pressure network. 

\[ \eta_{PU} = \frac{\sum_{i=1}^{N} Q_i}{\sum_{i=1}^{N} Q_i \times \eta_{HAi}}, \quad (9) \]

where 
\( N \) – the number of operating pumping units as part of a pumping unit. 

Thus, the expression (8), obtained on the basis of (9), allowing to calculate the efficiency of the pumping unit depending on the number of jointly functioning pumping units, as well as taking into account the equations that determine the corresponding performance characteristics of the pumps [5] and (1), (4) for the efficiency of the pump and the drive motor of each of the pumping units, together with the restrictions imposed on the parameters of the pumping unit functioning in the form of:

\[ Q \geq Q_{\text{schedule}}; \]

\[ h_{\text{min}} \leq h_{i} \leq h_{\text{max}}; \quad (10) \]

where 
\( h_{i} = H_{CT} + R_{HE} \times Q_{i}^2; \)
\( Q_{\text{se edule}} \) - therequired amount of water flow in accordance with the schedule; 
\( H_{CT}, H_{\text{max}}, H_{\text{min}} \) - current, minimum and maximum values of the created pressure permissible under the conditions of the pumping unit functioning; 
\( n_{\text{min}} \) - minimum speed of the pump unit in accordance with the set operating mode of the pump unit.
The correction factor is in accordance with the formula:  
\[ K_{COR.I} = 0.8144 \times \beta_{i} \times 0.0884 \]  
where  
\[ \beta_{i} \]  
the speed of water movement in the pipeline of the  
\[ i \] -th pump unit, which is defined as:  
\[ \beta_{i} = \frac{Q_{i}}{[0.785 \times (D_{i}/1,000)^{2}]} \],  
\( i \)

where  
\[ D_{i} \]  
inner diameter of the suction line of the  
\[ i \] -th pump unit.

The specific resistance of a steel pipeline is calculated by the expression:  
\[ A_{REL.I} = 0.001478 / (D_{i} / 1,000)^{2.226} \] (16)

Head loss in communication pipeline of the  
\[ i \] -th pump unit is defined as:  
\[ \Delta h_{i} = R_{k,i} \times Q_{i}^{2} \] ;  
\( i \)

where  
\[ R_{k,i} \]  
pressure head communication line resistance of the  
\[ i \] -th pump unit;  
\[ K_{M} \]  
coefficient that takes into account pressure losses in local resistances (gate valve, check valve, places of smooth turning, gradual expansion or narrowing, etc.) of pressure pipelines;  
\[ L_{k,i} \]  
length of pressure communication pipeline of the  
\[ i \] -th pump unit.

For this type of discharge line  
\[ A_{REL.I} \]  
and  \[ K_{COR.I} \]  
are calculated by formulas (16) and (14), respectively, using the size (in mm) of the inner diameter of the communication pipeline  
\[ D_{k,i} \].

Due to the complexity of the hydraulic phenomena occurring in the pressure pipelines of the pumping station of machine water lifting systems, each local resistance is characterized by its own loss factor, which is usually determined empirically or, in some cases, can be calculated from theoretical data. Wherein  \[ K_{M} \]  
for each specific pumping unit of machine water lifting systems taking into account the design of the entire complex of the pipeline pressure network and bearing in mind the presence of certain types of local resistances, which, as a rule, are determined from special reference literature [6,7,8]. Head loss in the supply pressure pipeline of the  
\[ i \] -th pump unit is calculated in the same way as above using the formula:  
\[ h_{P,i} = R_{P,i} \times Q_{i}^{2} \] ;  
\( i \)

where  
\[ R_{P,i} \]  
the pressure supply line resistance of the  
\[ i \] -th pump unit;  
\[ L_{P,i} \]  
length of the supply pressure line of the  
\[ i \] -th pump unit.

For the considered type of design of the pressure pipeline  
\[ K_{COR.I} \]  
and  \[ A_{REL.I} \]  
are calculated in the same way by formulas (14) and (16) taking into account the size of the inner diameter of the supply pressure pipeline  
\[ D_{i} \].

Based on the ratios obtained above, the averaged total head losses in the pipelines of the pumping unit of machine water lifting systems to their point of connection to the common pressure network is determined by the expression:

\[ h^\Sigma_{S} = \frac{\Sigma_{n}(h_{P,i} + h_{k,i} + h_{c,i})}{N} = \frac{Q_{i}^{2}}{N} + \sum_{i=1}^{N}(R_{P,i} + R_{k,i} + R_{c,i}) \] (21)

Provided that all pumping units are equipped with the same type and identical hydraulic power equipment with the appropriate characteristics, the total head loss  
\[ h^\Sigma_{S} \]  
can be expressed through the flow rate  
\[ Q_{c,pumping \ unit} \]  
in the following form:

\[ h^\Sigma_{S} = \frac{1}{N} \times Q_{c,pumping \ unit}^{2} \] (22)

The head loss in the general hydraulic pressure network of the pumping unit of the machine water lifting systems is expressed by the equation:  
\[ h^\Sigma_{S} = R_{i} \times Q_{i}^{2} \],  
\( i \)

where  
\[ R_{cmn} \]  
the resistance of the common pressure network of the pumping unit of the machine water lifting systems;  
\[ L_{cmn} \]  
the length of the general pressure network of the pumping unit of the machine water lifting systems.

When a pumping unit of machine water lifting systems is operating, the operating point of its operation is determined by the total flow rate-pressure characteristic of the pumping unit and the characteristic of the pipeline hydraulic pressure network in accordance with the equality  
\[ H_{curr} = H_{RP} \]. This means that at a given operating point, the pressure developed by the pumping unit of machine water lifting systems is equal to the pressure required by the pipeline network in order to ensure the required supply. Therefore, the equation of the regime point, taken into account in the constraints (10), will take the form:

\[ H_{e} = H_{S} + \left[ R_{cmn} + \frac{1}{N} \times \sum_{i=1}^{N}(R_{P,i} + R_{k,i} + R_{c,i}) \right] \times Q_{i}^{2} \] (25)

4. Conclusion

Thus, all the necessary analytical expressions required for the formation of a mathematical model of a pumping unit for machine water lifting systems have been determined in the form of the dependence of the specific consumption of electrical energy on the design, operational and technological parameters of its functioning, which allow to optimize and study the operating modes of the pumping unit to ensure energy-resource saving of the system machine water lifting.

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