RETAIL OUTSOURCING STRATEGY IN COURNOT & BERTRAND RETAIL COMPETITIONS WITH ECONOMIES OF SCALE

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ABSTRACT. This paper investigates a manufacturer’s retail outsourcing strategies under different competition modes with economies of scale. We focus on the effects of market competition modes, economies of scale and competitor’s behavior on manufacturer’s retail outsourcing decisions, and then we develop four game models under three competition modes. Firstly, we find the channel structure where both manufacturers choose retail outsourcing cannot be an equilibrium structure under the Cournot competition. The Cournot competition mode is less profitable to the firm than the Bertrand competition when the products are complements. Secondly, under the hybrid Cournot-Bertrand competition mode, there is only one equilibrium supply chain structure where neither manufacturer chooses retail outsourcing cannot be an equilibrium structure under the Cournot competition. The Cournot competition mode is less profitable to the firm than the Bertrand competition when the products are complements. Secondly, under the hybrid Cournot-Bertrand competition mode, there is only one equilibrium supply chain structure where neither manufacturer chooses retail outsourcing, when the substitutability and complementarity levels are not sufficiently high. In addition, setting price (quantity) contracts as the strategic variables is the dominant strategy for the direct-sale manufacturer who provides complementary (substitutable) products. Thirdly, both competitive firms will benefit from the situation where they choose the same competition mode. When the products are substitutes (complements), both of them choose the Cournot (Bertrand) competition mode. Finally, we show that the economies of scale have little impact on the equilibrium of the outsourcing structure but a great impact on the competition mode equilibrium.

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1. Introduction. Outsourcing is a common practice for both manufacturers and service providers, largely due to increased supply chain competitiveness in the global environment (see, e.g., [5, 45]). Manufacturers undertake a variety of outsourcing initiatives and strategies to improve their own financial and operational performance, including supply chain partnerships and alliances, integration of material flows, production outsourcing (e.g., product research and design, product component manufacturing, etc.), and retail outsourcing such as product distribution. Herein, our paper focuses on retail outsourcing. Retail outsourcing is a form of business operation where a firm produces the products that are sold and delivered to consumers through an intermediary retailer (middleman). Consequently, the firm is regarded as a manufacturer who acts as the Stackelberg-game leader in the supply chain. Furthermore, if the firm adopts the outsourcing strategy, the decisions between the two parties are decentralized. Otherwise, they are considered as centralized supply chain decisions when the products are produced and directly sold by the firm herself. Manufacturers have been very successful in harvesting the benefits of this outsourcing strategy by reducing service time, increasing product flexibility and agility, and improving responsive speed to customer and channel competitiveness. A representative example is Dell, which is known for its consistent direct-sales mode. This mode allows a competitive advantage in retail pricing, but the communication between manufacturers and consumers is weakened. By outsourcing its retail operations to Wal-Mart, Dell not only improves the quality of service to customers in the sales process, but also reduces the risks of interferences caused by non-marketing factors (see, [32]).

Although the importance of retail outsourcing is widely recognized, the existing results indicate that it does not necessarily lead to a satisfactory performance but occasionally results in a corporate breakup of outsourcing relationships (see, [40]). Additionally, retail outsourcing can also lead to the increase of inventory and financial risks including firm’s financial distress and bankruptcy, and its complexity may go far beyond expectations. For example, some enterprises have reclaimed the downstream retailers’ dealership to sell products independently after encountering difficulties in controlling the behavior of third-party retailers. Therefore, a firm should first evaluate whether retail outsourcing should be employed by weighing its pros and cons. Cachon and Harker [7] echo in literature that making the best use of retail outsourcing can make the firm more competitive.

Our paper contributes to exploring the impact of a firm’s competition mode (Cournot or Bertrand) choice on the retail outsourcing strategy. We design three competition modes: Cournot competition, Bertrand competition, and hybrid Cournot-Bertrand competition. In the small-car industry, for example, Honda and Subaru dealers engage in quantity competition and adjust prices to clear the market accordingly, while Scion and Saturn dealers compete in pricing and then fulfill the orders (see, [38]). Several recent articles focus on comparing Cournot and Bertrand competition modes in terms of the firm’s profit. For example, in real business world, the profits are higher in quantity competition than those in price competition in almost all cases (see, [15]). In a hybrid duopoly the outcomes show that quantity competition is higher than price competition, resulting in a lower profit for the private firm, which is contrary to the case of a private duopoly (see, e.g., [17, 22]). In addition, the selection between Cournot and Bertrand competitions can be driven by other factors. For example, Bian et al. [6] show that retailers can engage in Bertrand, Cournot, or hybrid Bertrand-Cournot competition modes in equilibrium,
depending on the upstream manufacturers’ collusion incentives and the value of future cash flow. Our study differs from the above literature in that we focus on exploring the retail outsourcing strategy under the hybrid Bertrand-Cournot competition, which is a case as in Honda vs. Scion. We show that when the levels of substitutability and complementarity of the products are not sufficiently high, there exists only one equilibrium strategy where neither manufacturer chooses retail outsourcing. Besides, our research investigates the impacts of economies of scale on the competition mode selections and the equilibrium retail outsourcing strategy.

The fundamental research questions in this study include: (i) how to evaluate the value of the firm’s outsourcing strategy, (ii) how to analyze the impact of competition modes (Cournot and Bertrand) on the equilibrium outsourcing decisions, and (iii) how to guide firms to choose the optimal outsourcing strategies under different competition modes with economies of scale. To obtain the value of the firm’s outsourcing strategy, we develop four game models based on the following supply chain structures (let $C$ and $D$ represent the cases without and with retail outsourcing option, respectively): (i) the $CC$ structure: neither of the two manufacturers outsources retailing, (ii) the $DD$ structure: both manufacturers outsource their retailing, and (iii) and (iv) the $CD$ and $DC$ structure: one firm outsources its retailing while the other does not. Intuitively, the decentralized supply chain is systematically inefficient due to double marginalization, while the centralized decisions should lead to the optimal channel profit for the entire supply chain. Therefore, a series of supply chain management articles focus on the design of incentive contracts to achieve the supply chain coordination where the manufacturer’s profit is maximized. However, when competitors join the game in the same market, how does that influence the decisions of the manufacturers? In particular, we are interested in the following questions: (i) when the competing manufacturer adopts a certain outsourcing strategy under a certain competition mode, what outsourcing strategy and competition mode should the original manufacturer choose to use? (ii) How does the product-to-product relationship affect the manufacturer’s decision making about the outsourcing strategy and the competition mode? (iii) When the economies of scale are considered in the duopoly market, what strategic decisions should the manufacturer make to gain higher profits?

The remainder of this paper is organized as follows: Section 2 gives the literature review. Section 3 describes the basic problem description and model assumption. Section 4 discusses the equilibrium supply chain structures with economies of scale and compares the profits under the pure competition modes. Section 5 studies the equilibrium supply chain structures under the hybrid Cournot-Bertrand competition mode, and the equilibrium competition mode under different outsourcing strategies. Section 6 features a numerical study that outlines the impact of the retail outsourcing strategy on the manufacturer’s profit under different competition modes with economies of scale. In Section 7, we summarize the conclusions and points out the future research directions.

2. Literature review. Our work is closely related to two areas, outsourcing strategy and market competition. In the following two subsections, we review the representative and relevant articles in these two topics, compare them with our research, and highlight the differences and our contributions.

2.1. Outsourcing strategy. Outsourcing strategies have been studied extensively in literature. The extant work primarily focuses on supplier selection, supplier
relationships, supplier management, outsourcing risks and benefits, and procurement strategies to reduce risks and to improve benefits ([18]). When facing supply uncertainty, buyers can use an alternative outsourcing strategy to obtain products from an external source which are then provided to their customers ([8]). For other related work, one can refer to various issues pertaining to outsourcing, such as the advantages and disadvantages of outsourcing ([42]), outsourcing under exchange rate risk and competition ([29]), outsourcing services for manufacturers that compete for product quality ([41]), insufficient outsourcing capabilities ([36]) and outsourcing quality risks ([23, 25]). Overall, outsourcing has been widely adopted by firms as a means to reduce costs, improve responsiveness, reduce cycle times, increase agility and flexibility, and enhance overall competitiveness (e.g., [24, 27]). Cost and strategic factors are the main drivers of outsourcing ([26]).

Arya et al. [3] study the outsourcing decision in the presence of retail competition, and show that the manufacturer may opt to outsource even if the cost of outsourcing is higher than that of vertical integration. In addition, it indicates the retail rival is not willing to pay as much under Bertrand as under Cournot competition to secure such access because of the less intense retail competition that prevails under Bertrand competition, when production is outsourced to the vertically integrated producer. Our paper differs from their work in that we consider a hybrid Cournot-Bertrand competition mode where the retail business is outsourced to the downstream retailer. From the manufacturer’s perspective, under the hybrid Cournot-Bertrand competition she will not outsource her retailing if the retail competition is very intense.

From the existing literature, we can see that most work focuses on the outsourcing of supply or production, and that little considers the outsourcing of logistics or retailing. Meanwhile, it is also very difficult to evaluate the outsourcing performance due to its highly tacit nature. In this paper, we investigate the retail outsourcing where the manufacturing firm manufactures the products by herself and then sells the products through a retailer. Unlike the existing results, we develop four supply chain game models according to the different supply chain structures, and obtain the equilibrium structure under different competition environments. We investigate how different outsourcing strategies and competition modes influence the profits of the competing firms. Moreover, we consider the impact of economies of scale on the equilibrium results of the supply chain structure and competition mode.

2.2. Market competition. Market competition generally begins with production quantity competition. Fang and Shou [13] study the Cournot competition between two supply chains and examine how the levels of supply uncertainty and competition intensity affect the equilibrium decisions of ordering quantity, contract offering, and centralization choice. Chen et al. [9] investigate the impacts of Cournot competition on order decision and find it results in the less retail price and the lower performance for the whole system. Considering duopolistic retailers with a Cournot competitive behaviour, Sun et al. [37] compare these optimal solutions with other competitive behaviour patterns and examine the effects of certain parameters in a special case and a general case, respectively. To improve firms’ business performance in market competition, Goli et al. [21] focus on the issue of demand prediction. With the rise of the profit-oriented private enterprises, the competitive market is enriched by a variety of competition modes. Farahat and Perakis [15] compare equilibrium profits of the Bertrand (price) and the Cournot (quantity) competitions in oligopolies with
differentiated substitutable products. They find that the total profit of the industry under the Cournot competition is at least as high as that under the Bertrand competition if the competition intensity is less than 0.909. Matsumura and Ogawa [30] consider the cases in which goods are complements and find that the price contract is a dominant strategy for both firms in mixed duopoly, whether the goods are substitutes or complements. Besides, in order to remain competitive in industries, the effective production scheduling is essential, which is a tool for making the best use of available resources ([20]).

The choice of price versus quantity ([14]) is investigated under patent licensing in Din and Sun [12], and for substitute products both Cournot and mixed price-quantity competitions may lead to the equilibrium outcomes. However, the authors do not consider the impact of the product-product relationship, especially the complementarity, and the channel structure on the equilibrium outcomes of the competition mode. In our paper, however, all of these factors are considered. For example, we find that when the products are complements, the Bertrand competition dominates the Cournot competition in the case where both firms choose the direct sales mode. Besides, our paper also discusses the impact of economies of scale on the optimal decisions.

The economies of scale on production are ubiquitous in the oligopolistic market, and the linear cost is usually used in the enterprise outsourcing and channel structure literature to enrich the analysis. Cachon and Harker [7] investigate the balanced channel structure and a major driver of competitive supply chains. The fierce competition cases studied in their papers lighten the important role of economy of scale in market economy. However, the case where the firms follow a quantity contract (the Cournot competition) has not been considered in their paper. In practice, there exist some Cournot-type firms, such as Honda and Subaru in the small car market. Moreover, most of the existing literature assumes that there is one manufacturer selling products to exclusive buyers in the supply chain and focuses on how competition affects the balance of supply chain decisions. The earlier research by McGuire and Staelin [31] show that if two supply chains are highly competitive, operating in a decentralized mode is the main strategy. Different from the current literature, we consider the supply chain competition with the retail outsourcing strategy and study how the market intensity, competition modes and economies of scale influence the optimal retail outsourcing strategy of the firms. The main objective of our paper is to assess the firms’ performance under the economic operations modes and the motivation of the retail outsourcing.

Evidences show that it is not clear whether a firm’s outsourcing decisions under different competition modes are always consistent with her competitive strategy ([16]). In practice, the trend of individual manufacturing has put a lot of pressure on the original manufacturer(s) and results in intensive market competition. When the goal of retail outsourcing strategy is to achieve a competitive advantage, it deserves some more attentions and more questions should be explored. For example, is it necessary to employ a retail outsourcing strategy? If yes, how should the firm make the retail outsourcing decision to align it to its overall competition modes?

Compared with the extant literature, our model is innovative in the following aspects. (a) We consider four supply chain structures (i.e., $CC$, $CD$, $DC$ and $DD$) under three competition modes. Additionally, we analyze and compare the equilibrium competition modes when the firms employ the retail outsourcing strategy. (b) When the firms use different contracts (price or quantity), we study how they make
optimal decisions on their outsourcing strategies. (c) Considering the economies of scale that is the underlying characteristics of duopoly, we explore whether the equilibrium results on the competition modes and supply chain structures will be affected. (d) By comparing the firm’s profit with her competitor’s profit, we provide the firm a guidance towards her decision in response to the competitor’s economic strategy, such as which contract to choose (price or quantity) and which outsourcing strategy to adopt (direct-sale or retail outsourcing). Table 1 contrasts our research with the extant works. It helps understand how we bridge the gaps and contribute to the literature landscape. Note. All tables are at the end of the paper.

3. Problem description and model assumption. Some of the key assumptions to our game models as well as how to achieve firms’ equilibria strategies in different contexts are discussed in this section. First, assume that there are two competitive symmetric supply chains, each of which consists of one manufacturer and one retailer. The manufacturer in each supply chain may choose to adopt either a quantity ($Q$) or price ($P$) contract (or competition mode), individually. Each manufacturer can also determine whether or not to outsource their retailing function. Therefore, there are four abovementioned supply chain structures ($CC$, $CD$, $DC$ and $DD$) under different economic competition modes ($QQ$, $QP$, $PQ$ and $PP$), where $C$ denotes the case that the manufacturer does not outsource retailing (i.e., the centralized decisions); and $D$ denotes the case that manufacturer outsources retailing (i.e., the decentralized decisions). Considering that duopoly provides a positive condition for the formation of economies of scale, we develop our basic model on the linear cost. In addition, the following assumption should be firstly made for the basic profit model.

Assumption 1. Both firms compete on the production quantity in the market with a linear inverse demand function under the Cournot competition mode.

\[ p_i = a - q_i - bq_j, \quad i, j = 1, 2, \quad i \neq j \quad \text{and} \quad -1 < b < 1. \]  

(1)

This assumption is common in literature ([13, 43]). Where $a$ is the size of the market demand, $p_i$ and $q_i$ represents the retail price and quantity of supply chain $i$, respectively. $b$ is the substitution/competition ($0 \leq b < 1$) /complementation ($-1 < b < 0$) coefficient, and reflects the degree of product differentiation. Intuitively the higher the value of $b$, the more intense the market competition will be. When $b \rightarrow 1$ (or $b \rightarrow -1$), it means that both products tend to be perfect substitutable (or perfect complementary).

According to Eq.(1), we can derive the demand function under the Bertrand competition mode as follows.

\[ q_i = \frac{a(1-b) - p_i + bp_j}{1 - b^2}, \quad i, j = 1, 2 \quad \text{and} \quad i \neq j. \]  

(2)

Once a manufacturer chooses to outsource her retailing function to a downstream retailer, the centralized supply chain will become a decentralized one. In this decentralized supply chain, the manufacturer, acting as the Stackelberg leader, provides the downstream retailer with a wholesale price per unit of product, and then the retailer, as the Stackelberg follower, determines the retail price through the Nash game of retailing market competition.

Assumption 2. The unit cost for each manufacturer is $c_i(q_i) = c_0 - \theta q_i$, $(i, j = 1, 2)$, where $c_0 > 0$ and the total cost for $q$ units is $qc(q)$. When $\theta = 0$, the case of linear cost has been discussed in [31]. The case of $\theta < 0$ corresponds to a convex
cost function (diseconomies of scale), while the case of $0 < \theta < \frac{c_i}{q_i}$ results in an increasing concave cost function (economies of scale).

For simplicity, we introduce following function notations, where $i = 1, 2$.

Function notations: $X = CC, CD, DC, DD$.

- $\pi^{QQ}_{X_{Mi}}$: Profit of manufacturer $i$ under $X$ structure in pure Cournot competition;
- $\pi^{PP}_{X_{Mi}}$: Profit of manufacturer $i$ under $X$ structure in pure Bertrand competition;
- $\pi^{QP}_{X_{Mi}}$: Profit of manufacturer $i$ under $X$ structure in hybrid Cournot-Bertrand competition.

For the above four supply chain structures, we provide three economic competitive settings, respectively. (i) Pure Cournot competition mode ($QQ$): both firms perform the production quantity contract in this game, where they play as quantity takers. (ii) Pure Bertrand competition mode ($PP$): both firms perform the price contract and they play as price setters in the game. (iii) Hybrid Cournot-Bertrand competition ($QP$): one of firms performs the production quantity contract, while the other plays as a price setter. If a firm acts as a quantity taker or performs the production quantity contract, it means that this firm captures market share by dominating the production quantity. Meanwhile, if a firm acts as a price setter or performs the price contract, this firm competes on the price with her competitor to gain the market share.

4. Pure competition mode.

In this section, we assume that both firms play a symmetric competition game, Cournot or Bretrand, where they behave as either quantity takers or price setters. Under each competition mode, there will be four supply chain structures, namely $CC, CD$ ($DC$) and $DD$. We investigate the equilibrium supply chain structures under $QQ$ and $PP$ competition modes and provide some managerial implications for the firms.

4.1. Pure Cournot competition mode. When both firms play as quantity takers, they will compete on the quantity to yield the profit maximization. The linear inverse demand function and unit cost for the two firms are $p_i = a - q_i - bq_j$ and $c_i(q_i)$, respectively.

**Scenario 1: Structure CC**

We first consider a special supply chain structure: structure $CC$ in which both manufacturers use self-producing and self-selling (SPSS) mode and both supply chains are centralized. They decide on the production quantity independently and simultaneously. According to Eq.(1), each manufacturer can make her best decisions from the following optimization problem.

$$\max_{q_i} \pi^{QQ}_{CC_{Mi}}(q_i | q_j) = [a - q_i - bq_j - c_i(q_i)]q_i, i, j = 1, 2 \text{ and } i \neq j.$$  

**Scenario 2: Structure DD**

Second, structure $DD$ is considered when both manufacturers make products by themselves but outsource the retailing to their respective downstream retailers. Under the $QQ$ mode, manufacturer $i$ decides on the unit wholesale price $w_i$, and then both retailers compete on the order quantity. Based on the Stackerberg game between manufacturer $i$ and retailer $i$, manufacturer $i$’s decision can be obtained from the following two-stage programming.

$$\max_{w_i} \pi^{QQ}_{DD_{Mi}}(w_i) = [w_i - c_i(q_i)]q_i$$

$$\max_{q_i} \pi^{QQ}_{DD_{Ri}}(q_i | q_j, w_i) = [a - q_i - bq_j - w_i]q_i, i, j = 1, 2 \text{ and } i \neq j.$$  

To obtain the equilibrium solutions of Eq.(4), we use the backwards technology as follows.

(i) Both retailers provide the equilibrium order quantity from the first-order conditions, respectively;
(ii) Based on the best response of both retailers, both manufacturers engage in solving the wholesale prices;
(iii) All the equilibrium results are obtained by substitution method.

**Scenario 3: Structure CD or DC**

Next, we consider the case of CD (DC). In CD, we assume that manufacturer 2 (M2) employs the retail outsourcing while manufacturer 1 (M1) does not. In this game model, M2 (or M1) first makes a decision on the unit wholesale price, based on which R2 (or R1) and M1 (or M2) then compete on the order quantity. Therefore, the optimal decision of M2 can be made by the following two-stage programming.

\[
\max_{w_2} \pi_{QQ}^{CD}_{M2}(w_2) = [w_2 - c_2(q_2)]q_2
\]

\[
\rightarrow \left\{ \begin{array}{l}
\max_{q_2} \pi_{QQ}^{CD}_{R2}(q_2 | q_1, w_2) = (a - q_2 - bq_1 - w_2)q_2 \\
\max_{q_1} \pi_{QQ}^{CD}_{M1}(q_1 | q_2, w_2) = [a - q_1 - bq_2 - c_1(q_1)]q_1.
\end{array} \right.
\]

To obtain the equilibrium solutions of Eq.(5), we use the backwards technology as follows.

(i) R2 and M1 solve the order quantity from the first-order conditions;
(ii) Based on the results of Part (i), M2 solves the wholesale price;
(iii) All the equilibrium results are provided by substitution method.

**Corollary 1.** Under pure Cournot competition mode, the equilibrium solutions for the four different structures are presented in Table 2.

Note. All simplified symbols are presented by Part A and proofs of all corollaries and propositions are given by Part B in Appendix.

From Corollary 1, we can derive the underlying relationships of profits between the two competitive supply chains, which are presented in the following proposition.

**Proposition 1.** For the equilibrium supply chain structures,

(1) When economies of scale are not considered (i.e., \( \theta = 0 \)), we have

\[
\pi^{*QQ}_{CC_{M2}(\theta=0)} \geq \pi^{*QQ}_{CD_{M2}(\theta=0)} \forall b \in (-1,1);
\]

\[
\pi^{*QQ}_{DD_{M1}(\theta=0)} \leq \pi^{*QQ}_{CD_{M1}(\theta=0)}
\]

(2) When economies of scale are considered (i.e., \( 0 < \theta < \frac{c_0}{2q} \)), we have

\[
\pi^{*QQ}_{CC_{M2}} \geq \pi^{*QQ}_{CD_{M2}} \text{ if and only if } f_1(\theta, b) \geq 0
\]

\[
\pi^{*QQ}_{DD_{M1}} \leq \pi^{*QQ}_{CD_{M1}} \forall b \in (-1,1) \text{ and } \forall \theta \in (0, c_0/2q)
\]

Under the pure quantity competition (Honda vs. Subaru), given that a fixed cost is for each unit product (i.e., part (1)), we can see one manufacturer should always choose direct-sales mode to maximize its profit, no matter what outsourcing strategy her competitor chooses (direct-sales or retail outsourcing). The result is consistent with that in [13], in which supply chain centralization is always the dominant strategy under supply uncertainty and chain competition. In practice, on the one hand, the direct-sales mode always benefits customers through higher product availability, larger variety and lower price. On the other hand, direct-sales strategy is absolutely favored in the quantity competition market and even
in the intense market. Specially, from the perspective of equilibrium supply chain structure, we find that the CC structure is an equilibrium structure while the DD structure is not an equilibrium structure under the pure Cournot competition. It implies when a manufacturer chooses to directly sell her products, the optimal operation for the other manufacturer is to sell directly as well.

In addition, we can see from Corollary 1 the order quantity (price) of one manufacturer who uses SPSS is more (lower) than that of the other (i.e. $q_{CC}(\theta=0) > q_{CD}(\theta=0)$, $p_{CC}(\theta=0) > p_{CD}(\theta=0)$, $q_{CD}(\theta=0) < q_{DD}(\theta=0)$, $p_{CD}(\theta=0) > p_{DD}(\theta=0)$), regardless of the competitor’s sales mode (direct selling or distribution). Clearly, manufacturers have strong incentives to use SPSS under the pure Cournot competition mode. In part (2), we consider the factor of economies of scale and assume that $f_1(\theta, b) = (\pi^{*QQ}_{CC}M^2 - \pi^{*QQ}_{DD}M^2)/(a - c_0)^2$ where $f_1(\theta, b)$ is a function of two variables, $b$ and $\theta$. When $f_1(\theta, b) = 0$, we can roughly draw the diagram of $\theta$ and $b$ as shown in Fig 1. Alternatively, we can consider the diagram (curve) as a projection of 3-dimensional image $f_1(\theta, b)$ onto the plane $f_1(\theta, b) = 0$. EOS in the figure represents the equilibrium supply chain structure.

![Figure 1](image_url)

**FIGURE 1.** Possible regions for equilibrium structures in the pure Cournot competition

Due to the constraints of strict concavity of the total cost function, CC is the only equilibrium supply chain structure (EOS) as indicated in the shaded area, while DD fails to be an EOS in the feasible region. From Fig 1 we can draw the following insights:

(i) Given that the demand is stable the manufacture is not motivated to outsource her retailing to share the marketplace, even though the substitutability between products is very high. As a result, DD fails to be an EOS. In other words, the production quantity competition is capable of softening the competition pressure from the lower average unit production cost. In addition, the figure is near symmetric with respect to the $\theta$ axis. The impact of relatively high complementarity and substitutability on the EOS is similar on both sides.

(ii) From Corollary 1, we find that the order quantity when $\theta > 0$ is always higher than that when $\theta = 0$, in the production quantity competition mode. The main reason is that the economies of scale stimulate the demand. On the one hand, it is the inevitable result of economies of scale that the retail price of the product declines due to the lower unit production cost ($\theta > 0$). On the other hand, the quantity competition drives the two manufacturers to expand their production capacities to capture the marketplace in the first place.
4.2. Pure Bertrand competition mode. When both firms play as price setters, they compete on the retail prices. Similarly, we will use the criteria of maximizing manufacturers’ profits to discuss the question about four supply chain structures under the pure Bertrand competition mode.

Scenario 1: Structure CC
For structure CC in which both manufacturers decide the retail prices independently and simultaneously, according to Eq.(2), the decision of each manufacturer can be obtained from the following optimization problem.

\[
\max_{p_i} \pi_{CC, Mi}(p_i | p_j) = \left[ p_i - c_i(q_i) \right] \left[ a(1 - b) - p_i + bp_j \right] \frac{1}{1 - b^2}.
\]  

Scenario 2: Structure DD
For structure DD where both manufacturers outsource retailing under competing on the retail prices, the optimal decision of manufacturer \( i \) can be obtained from the following two-stage programming.

\[
\max_{w_i} \pi_{DD, Mi}(w_i) = \left[ w_i - c_i(q_i) \right] \left[ a(1 - b) - p_i + bp_j \right] \frac{1}{1 - b^2} \rightarrow \max_{p_i} \pi_{DD, Ri}(p_i | w_i, p_j) = \left( p_i - w_i \right) \left[ a(1 - b) - p_i + bp_j \right] \frac{1}{1 - b^2}.
\]  

We employ backwards technology to obtain the equilibrium results of Structure DD. That is,
(i) We first derive the retail price decisions of both retailers from the first-order conditions;
(ii) By substituting the solutions of Part (i) into \( M_i \)'s profit, we obtain the wholesale price decisions of \( M_i \);
(iii) Based on Parts (i) and (ii), all the equilibrium results can be obtained by substitution method.

Scenario 3: Structure CD or DC
In structure CD (or DC), \( M_2 \) (or \( M_1 \)) sets a unit wholesale price to \( R_2 \) (or \( R_1 \)), based on which the downstream \( R_2 \) (or \( R_1 \)) and \( M_1 \) (or \( M_2 \)) compete on the retail price. The optimal unit wholesale price of \( M_2 \) in the CD mode can be obtained from the following two-stage programming, and that of \( M_1 \) in the DC mode can be derived similarly.

\[
\max_{w_2} \pi_{CD, M_2}(w_2) = [w_2 - c_2(q_2)]q_2 \rightarrow \max_{p_2} \pi_{CD, R_2}(p_2 | w_2, p_1) = \left( p_2 - w_2 \right) \frac{a(1 - b) - p_2 + bp_1}{1 - b^2} \]  

\[
\max_{p_1} \pi_{CD, M_1}(p_1 | w_2, p_2) = \left( p_1 - c_1(q_1) \right) \frac{a(1 - b) - p_1 + bp_2}{1 - b^2}.
\]  

Similarly, we solve this programming backwards as follows.
(i) We first derive the retail price decisions of \( R_2 \) and \( M_1 \) from the first-order conditions;
(ii) By substituting the solutions of Part (i) into \( M_2 \)'s profit, we compute the wholesale price decision of \( M_2 \);
(iii) All the equilibrium results are obtained by substitution method.

Corollary 2. Under pure Bertrand competition mode, the equilibrium solutions for the four different structures are presented in Table 3.
According to Corollaries 1 and 2, we can derive some interesting comparisons in Proposition 2. It implies the key factors to determine the profitability of manufacturers include the degree of product substitution or complementation \( b \) and the degree of cost concavity in the competitive supply chains.

**Proposition 2.** Under different pure competition modes (QQ or PP), the equilibrium results can be compared as follows.

When economies of scale are not considered (i.e., \( \theta = 0 \)), we have

1. **for the CC structure,** \( \pi_{CC_{cc}}^{QQ}(\theta=0) \geq \pi_{CC_{cc}}^{PP}(\theta=0) \) if \( \theta \in [0,1] \),

2. **for the DD structure,** \( \pi_{DD_{dc}}^{QQ}(\theta=0) \geq \pi_{DD_{dc}}^{PP}(\theta=0) \) if \( \theta \in [0,1] \),

When economies of scale are considered (i.e., \( 0 < \theta < \frac{c}{\eta} \)), we have

1. **for the CC structure,** \( \pi_{CC_{cc}}^{QQ}(\theta=0) > \pi_{CC_{cc}}^{PP}(\theta=0) \) if \( \theta \in (0,1) \) and \( \forall \theta \in (0, c_0/2q) \),

2. **for the DD structure,** \( \pi_{DD_{dc}}^{QQ}(\theta=0) < \pi_{DD_{dc}}^{PP}(\theta=0) \) for \( \forall \theta \in (-1,1) \) and \( \forall \theta \in (-1,1) \).

In part (1), the firm sets a higher equilibrium retail price under the pure Cournot than under the pure Bertrand competition mode \( (p_{CC_{cc}}^{QQ}(\theta=0) > p_{CC_{cc}}^{PP}(\theta=0)) \), while the firm orders a lower equilibrium quantity under the pure Cournot than under the pure Bertrand mode \( (q_{CC_{cc}}^{QQ}(\theta=0) < q_{CC_{cc}}^{PP}(\theta=0)) \), regardless of whether the products are substitutes or complements. Therefore, Cournot-type firms are more likely to trade sales for higher prices, while Bertrand-type firms are more likely to trade prices for larger sales. However, when the products are substitutable \( (b \in [0,1]) \), we can find that the quantity strategy (the pure Cournot mode) is more profitable than the price strategy (the pure Bertrand mode). If the products are complements \( (b \in (-1,0)) \), we will have the opposite conclusion. Clearly, this result is consistent with that of [35]. It implies that the relationship between two products determines which pure competition mode can bring higher profit. Similar to part (1), under DD structure we can see the pure Cournot competition will make the firm less (more) profitable if the products are complements (substitutes). Similar results have been verified by [11]. In other words, the firms’ profitability not only depends on the competition modes (Cournot or Bertrand), but also the relationship between the two products (substitute or complement).

Considering economies of scale, we assume \( f_2(\theta, b) = (\pi_{CC_{cc}}^{QQ} - \pi_{CC_{cc}}^{PP})/(a - c_0)^2 \), and the projection of \( f_2(\theta, b) \) onto the plane, \( f_2(\theta, b) = 0 \), within the feasible region is represented in Fig 2(a). Simultaneously, the projection of \( f_3(\theta, b) = (\pi_{DD_{dc}}^{QQ} - \pi_{DD_{dc}}^{PP})/(a - c_0)^2 \) is visually displayed in Fig 2(b).

In the case where economies of scale are considered, for the CC structure we find a similar conclusion as in part (1), where the degree of cost concavity is in the feasible region. The addition of economies of scale is slightly influencing the comparison result: which mode helps make a higher profit, Cournot or Bertrand? Nevertheless, for the DD structure, the result is significantly different from part (2). When the products are substitutes and the market intensity is relatively low, it is more profitable for the Bertrand rather than the Cournot competition. The cost per unit for larger production quantities declines through economies of scale and
consumers generally pay a lower unit price for the product. The manufacturer whose downstream retailer plays as a quantity taker is less likely to benefit from economies of scale. On the contrary, if the market is very intense, the Cournot competition mode dominates the Bertrand competition mode. In the Cournot competition, the downstream retailer acting as a price receiver competes on the order quantity with the other retailer, and the intense market has no immediate effect on the price taking. Meanwhile, since the intense market can cut the retail prices within the retailers who are competing on price, the wholesale price and profit of the manufacturer will be affected as well. Furthermore, in the very intense market, it is more profitable under the Cournot than the Bertrand competition mode.

Furthermore, we can explore the equilibrium supply chain structures under the pure Bertrand competition mode based on Corollary 2. When a fixed cost is considered for every unit product (i.e., $\theta = 0$), we can obtain a similar but more generalized conclusion in [28], as follows.

**Proposition 3.** For the equilibrium supply chain structures with the pure Bertrand competition mode, we have

- For the pure parallel price competition model, we have $\pi^{PP}_{CC_M2}(\theta = 0) \geq \pi^{PP}_{CD_M2}(\theta = 0)$ for $b \in (-1, 1)$;
- For the direct direct-price competition model, we have $\pi^{PP}_{DD_M2}(\theta = 0) \leq \pi^{PP}_{CD_M1}(\theta = 0)$ if $b \in [-b_5, b_5]$ and $\pi^{PP}_{DD_M1}(\theta = 0) > \pi^{PP}_{CD_M2}(\theta = 0)$ otherwise,

where the threshold is uniquely determined by the equation $f_4(b_5) = 0$, and the function $f_4(\cdot)$ is given in Part A of Appendix.

Under the pure Bertrand competition mode (Scion vs. Saturn), if the competing firm ($M_2$) chooses the direct-sales mode, the manufacturer ($M_1$) prefers to follow
(use the same mode), no matter the products are substitutes or complements. If $M_2$ chooses to outsource retailing and when the market competition is highly intense or the products are close to perfect complements, $M_1$ will choose to employ retail outsourcing as well. On the one hand, the intense retail competition and the highly-complementary products stimulate market demand, thus leading to the supply increase. On the other hand, each manufacturer benefits from an outsourcing strategy that mitigates retail competitiveness when faced with fierce price competition. In contrast, when the retail competition is not intense, the manufacturers will have no incentive to outsource their retailing because the negative effect outweighs the positive effect of double marginalization on retail competition. Finally, we can see that $CC$ is the equilibrium structure for all $b$ values, while $DD$ cannot be an equilibrium supply chain structure when $b$ satisfies the certain condition.

When the factor of economies of scale is considered (i.e., $0 < \theta < \frac{c_0}{2q}$), we explore the conditions under which $CC$ or $DD$ is an equilibrium structure for different values of $b$ and $\theta$. The results are shown graphically in Fig 3. It is easy to see that the $CC$ structure is the only EOS in the region below the boundary and above the boundary $\pi_{CC,P}^{PP} = \pi_{CC,D}^{PP}$. Meanwhile, $DD$ is the EOS in the region below the curve of $\pi_{DD,M}^{PP} = \pi_{CD,M}^{PP}$.

![Figure 3. Possible regions for equilibrium structure in pure Bertrand competition](image)

According to Fig 3, we can draw some interesting insights below.

(i) Compared with the case where $\theta = 0$, the $CC$ structure is not an EOS in the whole regions. This is because we enforce the condition $\theta \in (0, \frac{c_0}{2q}]$ to ensure the cost concavity of profit functions. Therefore, the feasible region of the $CC$ equilibrium structure is influencing by economies of scale.

(ii) From Fig 3, when the market is fierce or the complementation between the two products is high, the $DD$ structure becomes an EOS, which is similar to the case where a fixed cost $c_0$ is for each unit product. The difference is, the higher the concavity of the total production cost is, the more unlikely $DD$ is an EOS. On the one hand, although the average unit production cost becomes lower, the relatively stable market environment (both products are lower substitutes) makes manufacturers reluctant to outsource their retailing. On the other hand, the price competition intensifies double marginalization when the cost concavity is high.
(iii) Overall, the addition of the cost concavity does not make a big difference. In addition, comparing to part (2) of Proposition 1, we find it is only in the pure Bertrand competition that the DD equilibrium supply chain structure exists. The price competition can stimulate the diversity of market channels more than the quantity competition. Meanwhile, the quantity competition can help the firms bear higher competitive stress from market. Therefore, the Bertrand competition may be a relatively stable mode.

5. Hybrid Cournot-Bertrand competition mode. In this section, we will investigate the equilibrium supply chain structures when one firm plays as a price setter and the other as a quantity taker (QP or PQ) under the equilibrium competition modes in the CC, CD (DC) and DD structures. In order to show the results in a concise and logical way, we first explore the case where a fixed cost is used for every unit product (i.e., \( \theta = 0 \)). Then, under the hybrid Cournot-Bertrand competition (QP) mode (we only need to show the case of QP due to the symmetry), the equilibrium solutions for the CC, CD, DC and DD structures are presented under the following scenarios.

5.1. Models without economies of scale. Under the hybrid Cournot-Bertrand competition (QP) mode, the four structures are presented as follows.

**Scenario 1: Structure CC**

In the CC structure, \( M_1 \) and \( M_2 \) make decisions independently and simultaneously by solving the following optimization problem defined by Eq.(9), where \( M_1 \) as a quantity taker chooses \( q^{*\text{QP}}_{CC1} \) in order to maximize her profit and \( M_2 \) as a price setter sets \( p^{*\text{QP}}_{CC2} \) in order to maximize her profit.

\[
\begin{align*}
q^{*\text{QP}}_{CC1} &= \arg \max_{q_1} \pi^{\text{QP}}_{CC1} (q_1 | p_2) = [(1 - b)a - (1 - b^2)q_1 + bp_2 - c_0]q_1 \\
p^{*\text{QP}}_{CC2} &= \arg \max_{p_2} \pi^{\text{QP}}_{CC2} (p_2 | q_1) = (p_2 - c_0)(a - bq_1 - p_2).
\end{align*}
\]  
(9)

**Scenario 2: Structure DD**

In the DD structure, both manufacturers determine the unit wholesale prices first. Then retailer \( R_1 \) determines the order quantity and retailer \( R_2 \) makes a decision on the retail price simultaneously. The optimal unit wholesale price of \( M_i (i = 1, 2) \) can be obtained from the following two-stage nonlinear programming.

\[
\begin{align*}
q^{*\text{QP}}_{DD1} &= \arg_{q_1} \max \pi^{\text{QP}}_{DD1} (q_1 | p_2) = [(1 - b)a - (1 - b^2)q_1 + bp_2 - c_0]q_1 \\
p^{*\text{QP}}_{DD2} &= \arg_{p_2} \max \pi^{\text{QP}}_{DD2} (p_2 | q_1) = (p_2 - c_0)(a - bq_1 - p_2).
\end{align*}
\]  
(10)

We solve this programming backwards as follows.

(i) Retailers solve the equilibrium order quantity and retail price from the first-order conditions, respectively;

(ii) Based on the best response of both retailers from Part (i), both manufacturers engage in solving the equilibrium wholesale price decisions;

(iii) All the equilibrium results are obtained by substituting.

**Scenario 3: Structure CD**

In the CD structure, \( M_2 \) determines the unit wholesale price first, and then \( M_1 \) as a quantity taker determines the order quantity in order to compete with \( R_2 \) (remember that \( R_2 \) sets the retail price to maximize his profit in the Nash game).
The optimization problem can be presented in the following two-stage nonlinear programming.

\[
\max_{w_2} \pi_{CDM2}^{QP}(w_2) = (w_2 - c_0)q_2 \\
\rightarrow \begin{cases} 
q_{CD}^{*QP} = \arg_{q_1} \max \pi_{CDM1}^{QP}(q_1 | p_2) = [(1 - b)a - (1 - b^2)q_1 + b p_2 - c_0]q_1 \\
p_{CD}^{*P} = \arg_{p_2} \max \pi_{CDM2}^{QP}(p_2 | q_1) = (p_2 - w_2)(a - b q_1 - p_2).
\end{cases}
\]

(11)

We solve above programming backwards as follows.
(i) \(M_1\) and \(R_2\) solve the equilibrium order quantity and retail price from the first-order conditions, respectively;
(ii) Based on the results of Part (i), we can obtain the equilibrium wholesale price of \(M_2\);
(iii) All the equilibrium results are obtained by substituting.

**Scenario 4: Structure DC**

In the **DC** structure, after \(M_1\) determines the unit wholesale price, \(M_2\) as a quantity taker competes with \(R_1\) who sets the retail price to maximize his profit. The optimization problem can be presented in the following two-stage nonlinear programming.

\[
\max_{w_1} \pi_{DCM1}^{QP}(w_1) = (w_1 - c_0)q_1 \\
\rightarrow \begin{cases} 
q_{DC}^{*QP} = \arg_{q_1} \max \pi_{DCM1}^{QP}(q_1 | p_2) = (p_2 - c_0)(a - b q_1 - p_2) \\
p_{DC}^{*P} = \arg_{p_2} \max \pi_{DCM2}^{QP}(p_2 | q_1) = [(1 - b)a - (1 - b^2)q_1 + b p_2 - w_1]q_1.
\end{cases}
\]

(12)

Similar to Structure **CD**, the equilibrium results can be obtained by employing the backwards technology.

**Corollary 3.** Under the hybrid Cournot-Bertrand competition mode, the equilibrium solutions for the four supply chain structures are presented in Table 4.

From Corollary 3, we can compare manufacturers’ profits under the hybrid competition mode and Proposition 4 presents the results for the **CC** and **DD** structures.

**Proposition 4.** Under the **QP** mode, we have

1. **under the CC structure,**
   \[
   \begin{cases} 
   q_{CC}^{*QP} \leq q_{CC}^{*QP} \quad \text{and} \\
p_{CC}^{*QP} \geq p_{CC}^{*QP} \quad \text{if} \ b \in (-1, 0) \\
p_{CC}^{*QP} \geq p_{CC}^{*QP} \quad \text{if} \ b \in [0, 1) \\
q_{DD}^{*QP} \geq q_{DD}^{*QP} \quad \forall b \in (-1, 1)
   \end{cases}
   \]

2. **under the DD structure,**
   \[
   \begin{cases} 
   w_{DD}^{*QP} < w_{DD}^{*QP} \quad \text{if} \ b \in (-1, 0) \\
w_{DD}^{*QP} \geq w_{DD}^{*QP} \quad \text{if} \ b \in [0, 1)
   \end{cases}
   \]

and \(\pi_{DDM1}^{QP} \geq \pi_{DDM2}^{QP}\).

When both manufacturers adopt the same outsourcing strategy (either **CC** or **DD**) and perform different contracts (**QP** in this case), we can see from part (1) the order quantity (price) from the firm following the Bertrand competition is higher (lower) than that from the firm following the Cournot competition. In other words, the Bertrand competition mode improves the efficiency of the firm through more supply and lower price than the Cournot competition mode does. However, the lower retail price and higher order quantity sometimes result in a lower profit, when the products are substitutes. The characteristic of substitutable products weakens
the advantage of price (Bertrand) competition. This conclusion is similar to that in part (1) of Proposition 2. The competitor’s competition mode matters to the firm as the profit will be changed accordingly.

According to part (2), the relationship between the wholesale prices set by both manufacturers depends on whether the products are substitutable or complementary. Specifically, if the products are complements, the wholesale price from a quantity taker is lower than that from a price setter. This result is obviously different from part (1). The retail outsourcing strategy strengthens the impact of product-product relationship (substitutes or complements) on prices. In addition, the middleman also stimulates the more order quantity under the Cournot competition. From the manufacturer’s perspective, the Cournot competition mode is the most profitable under the DD structure, no matter the products are substitutes or complements. On the one hand, the manufacturer’s profit is propelled by the higher market demand, because the quantity competition has grabbed more market share. On the other hand, the decentralized supply chain experiences system inefficiency due to double marginalization, which has a negative impact on the firm who chooses the price competition mode.

From Section 4, we know that in the pure competition modes (QQ or PP) there exist the equilibria supply chain structures (CC and DD) under certain conditions. For the hybrid QP mode, a natural question is if an equilibrium supply chain structure exists. The next proposition provides an answer to this question.

**Proposition 5.** In the hybrid Cournot-Bertrand competition mode, we have

\begin{equation}
\begin{cases}
q_{CC}^{QP} > q_{DC}^{QP}, & q_{CC}^{QP} < p_{DC}^{QP} \\
q_{CC}^{QP} > q_{CD}^{QP}, & q_{CC}^{QP} < p_{CD}^{QP}
\end{cases}
\end{equation}

\begin{equation}
\forall b \in (-1,1).
\end{equation}

\begin{equation}
\begin{cases}
\pi_{CC}^{QP} \geq \pi_{DC}^{QP} & \text{if } b \in [-2/\sqrt{5}, 2/\sqrt{5}]
\pi_{CC}^{QP} < \pi_{DC}^{QP} & \text{otherwise}
\end{cases}
\end{equation}

\begin{equation}
\forall b \in (-1,1)
\end{equation}

and

\begin{equation}
\begin{cases}
\pi_{DD}^{QP} \leq \pi_{CD}^{QP} & \text{if } b \in [-b_6, b_6]
\pi_{DD}^{QP} > \pi_{CD}^{QP} & \text{otherwise}
\end{cases}
\end{equation}

\begin{equation}
\forall b \in (-1,1)
\end{equation}

where the threshold \( b_6 \) is uniquely determined by the equation \( f_5(b_6) = 0 \), and the function \( f_5(\cdot) \) is given in Part A of Appendix.

According to part (2), when one firm chooses to compete on the quantity and the other chooses the price competition mode like Honda vs. Scion, it is easy to find that supply chain structure CC is an equilibrium under the condition with \( b \in [-\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}] \) where product substitutability is for \( b \in [0, \frac{2}{\sqrt{5}}] \) or product complementarity is for \( b \in [-\frac{2}{\sqrt{5}}, 0] \). The finding is slightly different from the results in Propositions 1 (1) and 3 (1) in which the equilibrium supply chain structure CC is established for any \( b \in (0,1) \). The main reason for this result is that the two players make different choices of competition modes. Specifically, if the substitutability (complementarity) level of products is sufficiently high, the manufacturer competing on the quantity prefers outsourcing her retail business to the downstream retailers (i.e., \( \pi_{CC}^{QP} < \pi_{DC}^{QP} \), for \( b \in (-1, -\frac{2}{\sqrt{5}}] \) or \( b \in (\frac{2}{\sqrt{5}}, 1) \)), while the other firm competing on the price tends to sell directly (i.e., \( \pi_{CC}^{QP} > \pi_{DC}^{QP} \) for \( b \in (-1,1) \)).
From part (2), we also find that DD structure cannot be an equilibrium, because the profit of $M_2$ who competes on the price with retail outsourcing is always lower than that without outsourcing (i.e., $\pi_{QP}^{\star} < \pi_{QC}^{\star}$). Comparing with Proposition 3 (2), the firm believes that the competition mode chosen by her rival can have a noticeable impact on her outsourcing strategy. Apparently, $M_1$ is better off by choosing the quantity competition and the retail outsourcing strategy when the substitutability is sufficiently high (i.e., $b \in [0.861, 1]$) or the complementarity is relatively high (i.e., $b \in (-1, -0.861]$). Meanwhile, $M_2$ should select the price competition mode and the outsourcing strategy. Thus, both the product-to-product relationship and the competition mode have a significant impact on the equilibrium supply chain structure decision.

Moreover, for decision variables, we discuss the relationship between the prices (order quantities) of both players. The manufacturer who adopts a retail outsourcing strategy has a higher price (lower order quantity) than the one who uses direct sales no matter which competition mode she chooses. Thus, the retail outsourcing strategy has a greater impact on the price and order quantity than the competition mode. It implies that the selection of competition mode has a complicated influence on profits.

We have so far obtained the equilibrium solutions under the different competition modes (QQ, PP and QP) and outsourcing strategies (CC, DD and CD). The equilibrium competition modes can be derived accordingly. It implies that the firms should adopt different contracts (price or quantity contract) with respect to a certain outsourcing structure to earn higher profits. Table 5 summarizes the conditions that the equilibrium competition modes need to satisfy.

Based on previous results, we compare these equilibrium profits under different competition modes and obtain the following corollary about the equilibrium competition modes.

**Corollary 4.** For the equilibrium competition modes, when $\theta = 0$, we have

1. **Under the CC structure,**
   
   \[\begin{align*}
   &\pi_{CC}^{\star} \leq \pi_{CQ}^{\star} \quad \text{if } b \in (-1, 0] \\
   &\pi_{CC}^{\star} > \pi_{CQ}^{\star} \quad \text{if } b \in (0, 1)
   \end{align*}\]

2. **Under the DD structure,**
   
   \[\begin{align*}
   &\pi_{DC}^{\star} \leq \pi_{DD}^{\star} \quad \text{if } b \in (-1, 0] \\
   &\pi_{DC}^{\star} > \pi_{DD}^{\star} \quad \text{if } b \in (0, 1)
   \end{align*}\]

3. **Under the CD structure,**
   
   \[\begin{align*}
   &\pi_{CD}^{\star} \leq \pi_{QP}^{\star} \quad \text{if } b \in (-1, 0] \\
   &\pi_{CD}^{\star} > \pi_{QP}^{\star} \quad \text{if } b \in (0, 1)
   \end{align*}\]

4. **The results under the DC structure and the CD structure are symmetric.**

From part (1) we find that under the CC structure there exist two equilibria competition modes (PP and QP). Specifically, when the products are substitutes, QQ
equilibrium competition mode is tenable. Both competitive manufacturers will benefit from the situation where they choose the same competition mode QQ. On the contrary, when the products are complements, PP equilibrium competition mode is established. For interdependent manufacturers, once one of them adopts a price contract, the other should follow rather than use a quantity contract. Similarly, part (2) indicates there are PP and QQ equilibria competition modes. If the products are substitutes (i.e., \( b \in (0,1) \)), the equilibrium competition mode only includes QQ. Reconsidering the conclusion of Proposition 2 (1) and (2), we believe that the Cournot absolutely dominates the Bertrand competition mode, when the products sold in the market are substitutes. The main reason is that in the Cournot competition mode consumers are regarded as price takers. In the market of substitutes, the Bertrand competition mode is less profitable than the Cournot competition mode. Therefore, in this case firms will be advised to choose a quantity contract over a price contract. Part (3) explores the equilibrium competition modes under the CD structure. Surprisingly, we can see the result is similar to part (1) and absolutely depends on the product-product relationship. Therefore, unilateral conversion on the outsourcing strategy (i.e., from CC to DC or CD structure) does not change the equilibrium results.

5.2. Models with economies of scale. In this section, we assume that the economies of scale are considered in the duopoly market during the long-term production. This economic phenomenon has spread mainly in the communications industry. Its most immediate influence includes the formation of fierce competition among a few firms and the difficult admittance for some incipient firms.

In order to show the impact of the concavity of the total production cost on both equilibrium supply chain structure and competition modes (Cournot and Bertrand), we derive the equilibrium solutions of the four structures under economies of scale as follows.

**Corollary 5.** Under the hybrid Cournot-Bertrand competition mode with economies of scale, the equilibrium profits for the CC, CD, DC and DD structures are presented as Table 6.

First, we can use Table 6 to explore when either the CC or DD can be an equilibrium supply chain structure for different \( b \) (level of product substitution or complementation) and \( \theta \) (degree of cost concavity) values in the competitive supply chains. The result is plotted graphically in Fig 4. Here CC is an equilibrium structure within the region embodied by the solid curves, which are the conditions \( \bar{\pi}_{CC}^{QP} \geq \bar{\pi}_{DC}^{QP} \) and \( \bar{\pi}_{CC}^{QP} \geq \bar{\pi}_{CD}^{QP} \). The constraint \( \theta < \frac{c_0}{2q} \) defines the boundary. DD cannot form an equilibrium structure in the feasible region, where \( b \) satisfies \( b \in (-1,1) \) and \( \theta \) satisfies \( 0 < \theta < \frac{c_0}{2q} \).

From Fig 4, comparing this case with the case without economies of scale, we derive the following insights.

(i) When the level of product substitution or complementation is sufficiently high, and the degree of cost concavity is relatively low, the CC structure cannot be an equilibrium (i.e., \( \bar{\pi}_{CC}^{QP} < \bar{\pi}_{DC}^{QP} \)). That is, in the Cournot-Bertrand competition mode, if a manufacturer sells directly, it is not optimal for the other manufacturer to follow. For an industry which captures the market share by dominating the production quantity, it is advisable to sell strongly competitive products through a downstream retailer when it is faced with a competitor competing on the price.
(ii) Similar to the case without economies of scale, when the level of product substitution or complementation is sufficiently high, we always have $\tilde{\pi}^{*_{QP}}_{DP_{M1}} > \tilde{\pi}^{*_{QP}}_{CD_{M1}}$ but $\tilde{\pi}^{*_{QP}}_{DD_{M2}} < \tilde{\pi}^{*_{QP}}_{DC_{M2}}$. Under this condition, the Cournot-type firm is willing to adopt a retail outsourcing strategy, while the Bertrand-type firm prefers selling directly. However, when $b$ is approaching to zero, it means the Bertrand-type firm prefers outsourcing her retail business to a downstream retailer, while the Cournot-type firm tends to follow a different outsourcing strategy. Therefore, the DD structure cannot be an equilibrium supply chain structure.

(iii) Finally, there is only one equilibrium structure, the CC structure, within the region where the conditions, $\tilde{\pi}^{*_{QP}}_{CC_{M1}} > \tilde{\pi}^{*_{QP}}_{DC_{M1}}$ and $\tilde{\pi}^{*_{QP}}_{CC_{M2}} > \tilde{\pi}^{*_{QP}}_{CD_{M2}}$, are satisfied. Based on the conclusion of Proposition 5, we can see that the addition of economies of scale does not change much of the equilibrium results of the supply chain structure. Therefore, for an industry, the selection of the competition mode has a greater impact on the outsourcing strategy than the factor of economies of scale.

Next, we investigate the equilibrium competition mode (ECM) under the CC and DD structures. Clearly, the concavity of the total production cost has an impact on the ECM results. The results are presented in Fig 5.

(i) From part (1) of Corollary 4, we know QQ and PP are the ECMs under the CC structure for $\theta = 0$, when the products are substitutes and complements, respectively. Similarly, due to the concavity of total production cost (i.e., $\theta > 0$), if PP is the ECM, the condition $\tilde{\pi}^{*_{QP}}_{CC_{M1}} > \tilde{\pi}^{*_{QP}}_{DP_{M1}}$ should be satisfied, while if QQ is the ECM, the condition $\tilde{\pi}^{*_{QP}}_{CC_{M2}} > \tilde{\pi}^{*_{QP}}_{DP_{M2}}$ should be satisfied. In particular, the addition of $\theta$ does not change the equilibrium condition of the competition mode on the substitutability and complementarity of products. It implies the quantity competition is preferred by the firm if one of them adopts a quantity contract in selling the substitutable products directly. Direct sales strategy is a relatively simple economic model for a firm, and the impacts of some economic factors on her profit are also relatively straightforward. Thus, based on Fig 5 (a) we can see the impact of the cost concavity on the equilibrium condition of the competition mode is negligible.
(ii) Under the DD structure, we find when one of firm sells the strongly complementary products in a price contract, it is the optimal strategy for the other firm to follow the same strategy. In other words, there exists a PP equilibrium competition mode in the region where \( \tilde{\pi}^{PP}_{DDM_1} > \tilde{\pi}^{QP}_{DDM_1} \). Comparing with the results in part (2) of Corollary 4, we find that the factor of economies of scale has an impact on the region for PP equilibrium. The economies of scale enhance the capability of the middleman to weaken the price competition, when the level of product complementation is relatively low. Therefore, QQ equilibrium exists in the region where the product-to-product relationship is complementary. Moreover, when the market intensity is relatively low, the firm prefers a price competition mode rather than a quantity mode when her competitor uses a quantity contract.

(iii) The adoption of a retail outsourcing strategy by an industry means allowing the middlemen to join in the supply chain, which complicates its structure. When an increase in the production quantity leads to a decrease in the average cost (i.e., economies of scale), the industry will choose an appropriate contract based on the following cases: (a) if the product-to-product relationship is strongly complementary, we advise the industry to follow a price contract; (b) if the product-to-product relationship is strongly substitutable, a quantity contract should be suggested; (c) if the products are relatively low complements, the industry can capture market share by dominating the production quantity; (d) if the products are relatively low substitutes, theoretically, there is no equilibrium competition mode. The industry may benefit from adopting a competition mode differing from the competitor’s.
6. **Numerical analysis.** In this section, we study the impact of the retail outsourcing strategy on the manufacturer’s profit under different competition modes with economies of scale. Specifically, we investigate the following two scenarios.

(i) When the competitor \( (M_2) \) chooses the Cournot competition mode with or without the retail outsourcing strategy, what strategy should the manufacturer \( (M_1) \) use?

(ii) When \( M_2 \) chooses the Bertrand competition mode with or without the retail outsourcing strategy, what strategy should \( M_1 \) use?

Some management implications are also provided accordingly.

6.1. **When the competitor chooses the Cournot competition mode.** In this case, the manufacturer faces two competitive settings where the competitor adopts the Cournot competition mode with and without the retail outsourcing decision. Herein, to show the impact of \( M_1 \)'s retail outsourcing strategy, let \( \tau \) represent the improvement rate of \( M_1 \)'s strategy change from the “direct-sales” mode to another mode. We define,

(i) If the competitor \( (M_2) \) does not employ the retail outsourcing strategy,

\[
\tau^Y_Z = \frac{\pi^{Y_Q\text{DM}}_{CM} - \pi^{Q\text{C}}_{CM}}{\pi^{Q\text{C}}_{CM}},
\]

(ii) If the competitor \( (M_2) \) employs the retail outsourcing strategy,

\[
\tau^Y_Z = \frac{\pi^{Y_Q\text{DM}}_{CM} - \pi^{Q\text{CDM}}_{CM}}{\pi^{Q\text{CDM}}_{CM}}, \text{ where } Y = Q, P \text{ and } Z = C, D.
\]

For example, \( \tau^Q_D \) means the improvement rate of \( M_1 \)'s strategy change from “direct-sales with Cournot competition” mode to “retail outsourcing with Cournot competition” mode. Specifically, \( \tau^Q_C = 0 \) implies the strategy stays the same. Besides, we assume that \( a = 100 \) and \( c_0 = 10 \). The following figures illustrate the impacts of \( M_1 \)'s strategy change.

![Figure 6](image)

(a) For the complementary products  
(b) For the substitutable products

**Figure 6.** The effect of the retail outsourcing strategy on \( M_1 \)'s profit when \( M_2 \) uses direct sales

Fig.6 illustrates the impact of \( M_1 \)'s choice of the retail outsourcing strategy under different competition modes, when the competitor adopts the Cournot competition mode without retail outsourcing (or by selling directly). On the one hand, if \( M_1 \) follows the same competition mode (i.e., Cournot), the direct-sales strategy is more conducive to increase her profit than the retail outsourcing strategy, since \( \tau^Q_D < 0 \).
On the other hand, if $M_1$ uses the different competition mode (i.e., Bertrand), it is not the optimal for her to choose the outsourcing retail due to $\tau^P_D < 0$, whether the products are complements or substitutes. In addition, when the product is close to perfect complementation, $M_1$ prefers to using a price contract with a direct-sales mode (i.e., $\tau^P_C > 0$). In this setting, the profit of $M_1$ decreases with the level of product substitution, while increases with the level of product complementation. According to Fig.6, if the products are complements, we suggest the manufacturer choose direct sales strategy with a price contract. If the products are substitutes, we suggest the manufacturer keep the initial strategy: direct sales with a quantity contract.

![Figure 7. The effect of the retail outsourcing strategy on $M_1$’s profit when $M_2$ uses retail outsourcing](image)

From Fig.7(a), we can see that $M_1$ prefers to selling directly under a different competition mode from $M_2$ (i.e., Bertrand), while $M_2$ sells through her downstream retailer with a quantity contract. Thus, for the complementary products, direct sales strategy and price competition mode are more profitable. Nevertheless, according to Fig.7(b), we find that when the substitutability of product is approaching to zero, or the intensity of market is infinitely low, $M_1$ tends to outsource her retail business to the downstream retailer with a price contract. Moreover, when the market competition (the substitutability of product) is relatively high, $M_1$ will choose to sell directly and follow a quantity contract as $M_2$, because $\tau^Q_D < 0$. In the same time, the strategy of retail outsourcing with the price competition becomes less favorable for $M_1$ (i.e., $\tau^P_D < \tau^P_C < \tau^Q_D < \tau^Q_C$). Interestingly, enterprises should be encouraged to produce the complements rather than the substitutes, since the profits in Fig.7(a) are generally higher than those in Fig.7(b).

### 6.2. When the competitor chooses the Bertrand competition mode.

In this case, $M_2$ adopts the Bertrand competition mode with or without the retail outsourcing strategy to compete with $M_1$. We can obtain the following definitions.

(i) If the competitor ($M_2$) does not employ the retail outsourcing strategy,

$$\tau^Y_Z = \frac{\pi^{YP}_{ZCM_1} - \pi^{PP}_{ZCM_1}}{\pi^{PP}_{ZCM_1}}$$

(ii) If the competitor ($M_2$) employs the retail outsourcing strategy,

$$\tau^Y_Z = \frac{\pi^{YP}_{ZDM_1} - \pi^{PP}_{ZDM_1}}{\pi^{PP}_{ZDM_1}}$$

where $Y = Q, P$ and $Z = C, D$. 
Next, we assume that $a = 100$ and $c_0 = 10$. The following figures illustrate the impacts of $M_1$’s strategy change when $M_2$ adopts the Bertrand competition mode without or with the retail outsourcing strategy.

**Figure 8.** The effect of the retail outsourcing strategy on $M_1$’s profit when $M_2$ uses direct sales

Fig.8(a) demonstrates the optimal choice of $M_1$ is to sell directly with a price contract, when $M_2$ adopts the Bertrand competition mode with direct sales. Besides, when the level of product complementation is sufficiently high, we find that $M_1$ who adopts a quantity contract tends to outsource her retail business (i.e., $\tau_{Q_D}^Q > \tau_{Q_C}^Q$). For the substitutable products (i.e., Fig.8(b)), if the level of product substitution is relatively low, it is the optimal for $M_1$ to sell directly with a quantity contract due to $\tau_{Q_C}^Q > 0$. On the contrary, if the level of product substitution is relatively high, $M_1$ prefers to use the retail outsourcing strategy with a Cournot competition mode, which is different from $M_2$. Thus, the factors influencing the strategy of $M_1$ include not only the level of the product-product relationship but also the choice of her own competition mode. For example, since $\tau_{Q_D}^P < 0$, $M_1$ should not be encouraged to choose the retail outsourcing when competing on prices with $M_2$, who sells directly.

**Figure 9.** The effect of the retail outsourcing strategy on $M_1$’s profit when $M_2$ uses retail outsourcing
From Fig.9(a), since $\tau_P^P, \tau_D^Q, \tau_C^Q < 0$, we believe that it is optimal for $M_1$ to sell directly with a price contract. When the level of product complementation is approaching to -1, $M_1$ is more likely to choose the retail outsourcing with a price contract adopted by her retailer. Comparing with Figs.6(a) or 7(a), we find that the profit of $M_1$ under the Cournot competition mode decreases with the level of product complementation. Therefore, for strongly complementary products, it is not desirable to capture the market share by dominating the production quantity, when the competitor adopts a price contract. When the level of product substitution is sufficiently high, from Fig.9(b) $M_1$ has a chance to choose selling products through the downstream retailer. On the contrary, if the level of product substitution is relatively low, $M_1$ would rather sell directly with a price or quantity contract.

7. Conclusions. In this paper, we investigate how different supply chain structures and the different competition modes of the firm influence the decisions and profit of the firm. Based on the manufacturer’s retail outsourcing strategy, we consider the equilibrium supply chain structures under three competition modes. Similarly, based on the firm’s price or quantity contract, the equilibrium competition modes are explored likewise. A basic conclusion is drawn that if both firms use the price contracts, there exists an equilibrium structure $DD$ in the intense market. If the firms both adopt the quantity contracts, then the channel structure where both manufacturers outsource their retail to downstream retailers cannot be an equilibrium supply chain structure. In other words, the double marginalization takes more effect under the price contract, which effectively alleviates the pressure caused by fierce price competition. Compared with the price contract, the quantity contract is more stable in a competition environment.

The strategic choice between price and quantity contracts is aimed for the firm to gain a competitive advantage through either mitigating the competition or behaving more aggressively in the market. We find that under the $CC$ structure both firms as quantity takers can be more profitable than them as price setters with the substitutable products. When one firm performs the price contract and the other performs the quantity contract, we find that the quantity taker gains higher profit than the price setter with the substitutable products. The clear-cut conclusion is overturning the conventional wisdom. The main reason is that we do not mimic the competitor (rival)’s choice, that is, firms tend to choose different contracts to compete with each other. Similarly, under the $DD$ structure we have distinct results between the case when both firms choose the same competition mode and the case when they choose different modes. It is necessary for us to observe the rival’s strategy before acting. For the equilibrium competition mode, we find that the $CC$ structure contributes to the stable equilibrium competition mode. Furthermore, under $DD$ structure, $QQ$ sometimes cannot be dominant.

Considering the economies of scale from the duopoly market, we develop four models in the context and compare them with the basic models. Some different results are obtained. The lower average unit production cost can result in a fierce market and the firms would like to choose the outsourcing strategy to relax the competition. Particularly under the pure Bertrand competition, the $CC$ structure fails to be an equilibrium in the whole region due to the constraint from economies of scale, while the $DD$ equilibrium structure sometimes is established. What is more, under the $CC$ structure when the products are substitutes, $PP$ cannot be an equilibrium competition mode. The firm who aims at enhancing her competitiveness
in the market should not only focus on the type of contracts, but also on the appropriate outsourcing strategy. Meanwhile, the average unit production cost and the levels of the product differentiation also influence the selection of contracts and outsourcing strategies.

We now discuss the limitations and some possible future directions of our research. First, our paper mainly discusses the supply chain decision-making with symmetric information. One can introduce asymmetric information to incorporate the private information of the damaged requirements and to achieve channel coordination. The results of our study are likely to change when the supply chain retailers hold the main information of the market and the retailers do not share with the manufacturers about the true demand. Second, our study assumes that the demand functions are deterministic under the linear price-dependent or quantity-dependent scenarios. In addition, we can consider the impact of service competition on the market demand ([1]) and the relevant researches are going on. The demand uncertainty also affects industries’ outsourcing decisions, and therefore one possible extension is to consider a similar fuzzy demand as shown in Goli and Malmir [19]. Finally, according to Pahleven et al. [34], sustainability is a source of profit creation (or economic opportunity). It would be interesting to examine the outsourcing and competition mode strategies of the manufacturers in a sustainable supply chain network.

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Appendix.

Part A

The following notations are used to simplify the results.

\[ A = 2(1 - \theta) \quad S = (1 - b^2)(4 - b^2) + 2\theta(b^2 - 2) \quad U = 3b^2 - 4 + 4\theta \]

\[ T = b^2 - 2 + 2\theta \quad M = 1 - b^2 - 2\theta \quad N = 1 - b^2 - \theta \]

The solutions \( b_i \) presented in positions are determined by equations as follows.

\[ f_4(b_5) = 4(2 - b_6^2)^2(-3b_5^4 + 12b_5^2 - 8) - b_5^3, \]

\[ f_5(b_6) = 8(4 - 3b_5^2)(2 - b_6^2)^2 - (1 - b_6^2)(16 - 9b_6^2)^2, \]

\[ f_6(b_7) = 2(2 - b_7)(4 - 3b_6^2)(16 - 9b_7^2)^2 - (2 + b_7)(2 - b_7^2)(4 - b_7^2)^2(5b_7^2 + 2b_7 - 8)^2. \]

Part B

B.1 Pure Cournot competition with economies of scale

(i) The CC structure:

The inverse demand function of manufacture \( i \) is \( p_i = a - q_i - bq_j, (i, j = 1, 2; i \neq j) \).

The profit function of manufacture \( i \) can be presented as \( \pi^Q_{CC, i} = [p_i - c_i(q_i)]q_i, i = 1, 2 \) and \( c_i(q_i) = c_0 - \theta q_i \).

The optimization problem of supply chain \( i \) is \( \max_{q_i, \pi^Q_{CC, i}} q_i \pi^Q_{CC, i} = \max_{q_i, \pi^Q_{CC, i}} (\pi_{CC, i} q_i) \). Since \( \partial^2 \pi^Q_{CC, i} / \partial q_i^2 = 2\theta - 2 < 0 \), \( \pi^Q_{CC, i} \) is concave over \( q_i^Q \).

From the first-order differentiation conditions, we have \( \partial^2 \pi^Q_{CC, i} / \partial q_i = (2\theta - 2)q_i - bq_i + a - c_0 = 0 \).
Due to the symmetry, we obtain
\[ q_{CD}^{QQ} = \frac{a - c_0}{A + b}. \] (b1)

Substituting Eq. (b1) into \( p_i = a - q_i - bq_j \), we have
\[ x_{CD}^{QQ} = \frac{(1 - 2\theta)a + (1 + b)c_0}{A + b}. \] (b2)

From Eqs. (b1-b2), the equilibrium solution of profit is \( \pi_{CDM_1}^{QQ} = (1 - \theta)(q_{CD}^{QQ})^2 \).

(ii) The CD structure:
The profit function of \( M_1 \) is \( \pi_{CDM_1}^{QQ} = [p_i - c_1(q_1)]q_1 \) and \( c_1(q_1) = c_0 - \theta q_1 \). The optimization problem is \( \max_{q_1} \pi_{CDM_1}^{QQ} \). Similar to the CC structure, we can easily prove that the function \( \pi_{CDM_1}^{QQ} \) is also concave over \( q_1 \). We have
\[ \partial \pi_{CDM_1}^{QQ}/\partial q_1 = a - 2q_1 + 2\theta q_1 - bq_2 - c_0 = 0. \] (b3)

The profit function of \( R_2 \) is \( \pi_{CDM_2}^{QQ} = (p_2 - w_2)q_2 \). Given the unit wholesale price \( w_2 \) offered by her upstream \( M_2 \), \( R_2 \)'s optimal problem is written as \( \max_{q_2} \pi_{CDM_2}^{QQ} \). Obviously, \( \partial \pi_{CDM_2}^{QQ}/\partial q_2 = a - 2q_2 - bq_1 - w_2 \) and \( \partial^2 \pi_{CDM_2}^{QQ}/\partial q_2^2 = -2 < 0 \), therefore, we have \( \pi_{CDM_2}^{QQ} \) is concave over \( q_2 \). We have
\[ \partial \pi_{CDM_2}^{QQ}/\partial q_2 = a - 2q_2 - bq_1 - w_2 = 0. \] (b4)

From Eqs. (b3-b4), we can derive
\[ q_{CD}^{QQ} = \frac{(b - 2)a - 2c_0 + bw_2}{4(1 - \theta) - b^2} \text{ and } q_{CD}^{QQ} = \frac{2(1 - \theta)(a - w_2) + b(c_0 - a)}{4(1 - \theta) - b^2}. \] (b5)

The profit function of \( M_2 \) is \( \pi_{CDM_2}^{QQ} = [w_2 - c_2(q_2)]q_2 \). The optimization problem of \( M_2 \) is \( \max_{q_2} \pi_{CDM_2}^{QQ} \).

From \( \partial^2 \pi_{CDM_2}^{QQ}/\partial q_2^2 = -2(1 - \theta)[4(1 - \theta)^2 - 2b^2 + 4(1 - \theta)] < 0 \) the function \( \pi_{CDM_2}^{QQ} \) is concave over \( q_2 \). Then, from the first-order condition, we have
\[ \partial \pi_{CDM_2}^{QQ}/\partial q_2 = (1 + \theta)q_2 \text{ and } (w_2 - c_0 + \theta q_2) \partial q_2 / \partial w_2 = 0. \] (b6)

From Eqs. (b5-b6), we can derive the equilibrium solutions as follows.
\[ w_{CD}^{QQ} = \frac{(A^2 - b^2)(A - b)a + A(2A - b^2)^2 + (A^2 - b^2)b}{A(A^2 + 2A - 2b^2)}, \]
\[ q_{CD}^{QQ} = \frac{(A^2 + 2A - b^2 - Ab)(a - c_0)}{A(A^2 + 2A - 2b^2)}, q_{CD}^{QQ} = \frac{(A - b)(a - c_0)}{A(A^2 + 2A - 2b^2)}, \]
\[ p_{CD}^{QQ} = a - q_{CD}^{QQ} = bq_{CD}^{QQ}, p_{CD}^{QQ} = a - q_{CD}^{QQ} = bq_{CD}^{QQ}, \]
\[ \pi_{CDM_1}^{QQ} = (1 - \theta)(q_{CD}^{QQ})^2 \text{ and } \pi_{CDM_2}^{QQ} = \frac{(2 - \theta)(A - b^2)(q_{CD}^{QQ})^2}{A}. \]

(iii) The DD structure:
The profit function of \( M_i \) and \( R_i \) in supply chain \( i \) are \( \pi_{DDM_i}^{QQ}(w_i) = [w_i - c_i(q_i)]q_i \) and \( \pi_{DDM_i}^{QQ}(q_i | q_j, w_i) = (a - q_i - bq_j - w_i)q_i \) \( (i = 1, 2) \), respectively. Similarly, it is easy to show that \( \pi_{DDM_i}^{QQ} \) is concave over \( w_{DD}^{QQ} \) and that \( \pi_{DDM_i}^{QQ} \) is concave over \( q_{DD}^{QQ} \).
Given that $w_{DD_i}^{QQ}$ is offered by $M_i$, $R_i$’s optimal problem is $\max_{q_i} \pi_{DD_i}^{QQ}$. From the first-order conditions, we have

$$q_{DD_i}^{QQ} = a - 2q_{DD_i}^{QQ} - b q_{DD_i}^{QQ} - w_{DD_i}^{QQ}, \quad (i, j = 1, 2; i \neq j). \quad (b7)$$

Then we have $\partial q_{DD_i}^{QQ}/\partial w_{DD_i}^{QQ} = (1 + \theta_i 2\theta_i - 4)q_{DD_i}^{QQ} + (w_{DD_i}^{QQ} - c_0 + \theta q_{DD_i}^{QQ}) 2_i = 0$ from the $M_i$’s perspective. Therefore, we obtain

$$w_{DD_i}^{*QQ} = \frac{(b^2 + 4\theta - 4)a - 2(b + 2)c_0}{(b - 4)(b + 2) + 4\theta}. \quad (b8)$$

Substituting Eq.(b8) into Eq.(b7), we can derive the equilibrium solutions due to the symmetry. $q_{DD_i}^{*QQ} = 2(\theta - a) / (b - 4(b + 2) + 4\theta)$ and $p_{DD_i}^{*QQ} = (b^2 + 4\theta - 4)a - 2(1 + b)c_0$ and $\pi_{DD_i}^{*QQ} = (4 - 2\theta - b^2) / (q_{DD_i}^{*QQ})^2$.

\[\Box\]

B.2 Pure Bertrand competition with economies of scale

(i) The CC structure:

The demand function of manufacturer $i$ is $q_i = \frac{a(1-b) - p_i + b p_i}{1-b_\theta^2}, \quad (i, j = 1, 2; i \neq j).$

The profit function of manufacturer $i$ can be presented by $\pi_{CC_{M_i}}^{PP} = [p_i - c_i(q_i)]q_i, i = 1, 2$ and $c_i(q_i) = c_0 - \theta q_i$.

The optimization problem of supply chain $i$ is $\max p_{CC_{M_i}}^{PP} (p_i | p_j)$. Because $\partial \pi_{CC_{M_i}}^{PP} / \partial p_i = (1 - \frac{2\theta}{1-b_\theta^2})q_i - \frac{p_i - c_0}{1-b_\theta^2}$ and $\partial^2 \pi_{CC_{M_i}}^{PP} / \partial p_i^2 = -\frac{2}{1-b_\theta^2} < 0$, $\pi_{CC_{M_i}}^{PP}$ is concave over $p_{CC_i}^{PP}$.

From the first-order conditions, we have $\partial \pi_{CC_{M_i}}^{PP} / \partial p_i = (1 - \frac{2\theta}{1-b_\theta^2})q_i - (p_i - c_0) \frac{1}{1-b_\theta^2} = 0$.

Due to the symmetry, we obtain

$$q_{CC_i}^{*PP} = \frac{a - c_0}{(2 - b)(1 + b) - 2\theta}. \quad (b9)$$

Substituting Eq.(b9) into $p_i = a - q_i - b q_i$, we have

$$p_{CC_i}^{*PP} = \frac{Ma + (1 + b)c_0}{(2 - b)(1 + b) - 2\theta}. \quad (b10)$$

From Eqs.(b9-b10), the equilibrium profit will be denoted as $\pi_{CC_{M_i}}^{PP} = N(q_{CC_i}^{*PP})^2$.

(ii) The CD structure:

The profit function of $M_1$ is $\pi_{CD_{M1}}^{PP} = [p_1 - c_1(q_1)]q_1$ and $c_1(q_1) = c_0 - \theta q_1$. The optimization problem is $\max p_{CD_{M1}}^{PP}$. Similar to CC structure, we can easily prove that the function $\pi_{CD_{M1}}^{PP}$ is concave over $p_{CD_{D1}}^{PP}$. We have

$$\partial \pi_{CD_{M1}}^{PP} / \partial p_{CD_{D1}}^{PP} = (1 - \frac{2\theta}{1-b_\theta^2})q_1 - (p_1 - c_0) \frac{1}{1-b_\theta^2} = 0. \quad (b11)$$

The profit function of $R_2$ is $\pi_{CD_{R2}}^{PP} = (p_{CD_{D2}}^{PP} - w_{CD_{D2}}^{PP})q_{CD_{D2}}^{PP}$. Given the unit wholesale price $w_{CD_{D2}}^{PP}$ offered by his upstream $M_2$, $R_2$’s optimal problem is written as $\max \pi_{CD_{R2}}^{PP}$. Obviously, $\partial \pi_{CD_{R2}}^{PP} / \partial p_2 = \frac{a(1-b) - 2p_2 + b p_1 + w_2}{1-b_\theta^2}$ and $\partial^2 \pi_{CD_{R2}}^{PP} / \partial p_2^2 = \frac{2}{1-b_\theta^2} < 0$, therefore, we have $\pi_{CD_{R2}}^{PP}$ is concave over $p_{CD_{D2}}^{PP}$.

We have

$$\partial \pi_{CD_{R2}}^{PP} / \partial p_{CD_{D2}}^{PP} = \frac{a(1-b) - 2p_2 + b p_1 + w_2}{1-b_\theta^2} = 0 \quad (b12)$$
From Eqs. (b11-b12), we can derive

\[
p_{CD1}^* = \frac{(2 + b)(1 - b)(1 - b^2 - 2\theta)a + 2(1 - b^2)c_0 + b(1 - b^2 - 2\theta)w_2}{(4 - b^2)(1 - b^2) + 2\theta(b^2 - 2)} \text{ and }
\]

\[
p_{CD2}^* = \frac{[(2 + b)(1 - b) - 2\theta](1 - b^2)a + b(1 - b^2)c_0 + 2(1 - b^2 - \theta)w_2}{(4 - b^2)(1 - b^2) + 2\theta(b^2 - 2)}. \quad (b13)
\]

The profit function of \(M_2\) is \(\pi_{CDM_2}^* = (w_2 - c_2)q_2\). The optimization problem of \(M_2\) is max \(\pi_{CDM_2}^*\).

From \(\partial^2 \pi_{CDM_2}^*/\partial w_2^2 = \frac{(S + 2\theta T + 1)T}{S} < 0\), the function \(\pi_{CDM_2}^*\) is concave over \(w_2\). Then, from the first-order condition, we have

\[
\partial \pi_{CDM_2}^*/\partial w_2 = (1 + \theta \frac{\partial q_2}{\partial w_2})q_2 + (w_2 - c_0) \frac{\partial q_2}{\partial w_2} = 0. \quad (b14)
\]

From Eqs. (b13-b14), we can derive the equilibrium solutions,

\[
w_{CD2}^* = \frac{(S + 2\theta T)(T + b)a + (ST - Sb - 2\theta Tb)c_0}{S}, \quad q_{CD1}^* = \frac{-(T + b)a + b(c_0 + T w_{CD2}^*)}{S}, \quad q_{CD2}^* = \frac{b(T - b)c_0 + T w_{CD2}^*}{S},
\]

\[
p_{CD1}^* = \frac{(b^2 - 1)(b^2 - 2) \theta - (1 - b^2)(c_0 + T w_{CD2}^*)}{2}, \quad P_{CD2}^* = \frac{(b^2 - 1)(b^2 - 2)(\theta - c_0) + 2Nw_{CD2}^*}{2b},
\]

\[
\pi_{CDM_1}^* = N(q_{CD1}^*)^2 \quad \text{and} \quad \pi_{CDM_2}^* = \frac{(4 - b^2)(1 - b^2)}{2 - b^2} (q_{CD2}^*)^2. \quad \square
\]

(iii) The DD structure:

The profit function of \(M_i\) and \(R_i\) in supply chain \(i\) are \(\pi_{DDM_i}^* (w_i) = [w_i - c_i(q_i)]q_i\) and \(\pi_{DDM_i}^* (p_i, p_j, w_i) = (a - q_i - b q_j - w_i)q_i (i = 1, 2, \ldots)\), respectively. Similarly, it is easy to show that \(\pi_{CDM_1}^*\) is concave over \(w_{DD_i}\) and that \(\pi_{DDM_i}^*\) is concave over \(w_{DD_i}\).

Given that \(w_{DD_i}^*\) is offered by \(M_i, R_i\)'s optimal problem is max \(\pi_{DDM_i}^*\). From the first-order conditions, we have

\[
p_{DD_i}^* = \frac{(1 - b)(2 + b)a + bw_{DD_i}^* + w_{DD_i}^*}{4 - b^2} (i, j = 1, 2; i \neq j). \quad (b15)
\]

Then we have \(\partial \pi_{DDM_i}^*/\partial w_{DD_i}^* = 1 + \frac{2\theta(b^2 - 2)}{(4 - b^2)(1 - b^2)} q_{DD_i}^* + (w_{DD_i}^* - c_0) \frac{b^2 - 2}{(4 - b^2)(1 - b^2)} = 0\) for \(M_i\). Therefore, we obtain

\[
w_{DD_i}^* = \frac{S a - (1 + b)(2 - b)(b^2 - 2)c_0}{S - (1 + b)(2 - b)(b^2 - 2)}. \quad (b16)
\]

Substituting Eq. (b16) into Eq. (b15), we can derive the equilibrium solutions due to the symmetry. \(q_{DD_i}^* = \frac{a - w_{DD_i}^*}{(1 + b)(2 - b)}\), \(p_{DD_i}^* = \frac{(1 - b)a + w_{DD_i}^*}{2 - b}\) and

\[
\pi_{DDM_i}^* = \frac{(1 - b^2)(4 - b^2) + (b^2 - 2)\theta}{2 - b^2} (q_{DD_i}^*)^2. \quad \square
\]

B.3 Hybrid competition modes with economies of scale

(i) The CC structure:

The optimization problem of chain 1 is max \(\pi_{CCM_1}^*\). Because \(\partial \pi_{CCM_1}^*/\partial \tilde{q}_1 = 2(b^2 - 1 + \theta) \tilde{q}_1 + \hat{b} \tilde{p}_2 + a(1 - b) - c_0\) and \(\partial^2 \pi_{CCM_1}^*/\partial \tilde{q}_1^2 = 2(b^2 - 1 + \theta) < 0\), \(\pi_{CCM_1}^*\) is concave over \(\tilde{q}_{CC1}^*\).
From the first-order conditions, we have
\[ \frac{\partial \tilde{\pi}_{CCM_1}^{QP}}{\partial \tilde{q}_1} = 2(b^2 - 1 + \theta)\tilde{q}_1 + b\tilde{p}_2 + a(1 - b) - c_0 = 0. \] (b17)

The optimization problem of chain 2 is \( \max_{\tilde{p}_2} \tilde{\pi}_{CCM_2}^{QP} \). Because \( \frac{\partial \tilde{\pi}_{CCM_2}^{QP}}{\partial \tilde{p}_2} = b(2\theta - 1)\tilde{q}_1 + (2\theta - 1)\tilde{p}_2 + a(1 - 2\theta) + c_0 = 0 \) and \( \frac{\partial^2 \tilde{\pi}_{CCM_2}^{QP}}{\partial \tilde{p}_2^2} = 2\theta - 2 < 0 \), \( \tilde{\pi}_{CCM_2}^{QP} \) is concave over \( \tilde{p}_2^{QP} \).

From the first-order conditions, we have
\[ \frac{\partial \tilde{\pi}_{CCM_2}^{QP}}{\partial \tilde{p}_2} = b(2\theta - 1)\tilde{q}_1 + (2\theta - 1)\tilde{p}_2 + a(1 - 2\theta) + c_0 = 0. \] (b18)

From Eqs.(b17-b18), we have \( \tilde{q}_{CC_1}^{QP} = \frac{(T+b-\theta)(\alpha-c_0)}{AT+\beta} \) and \( \tilde{q}_{CC_2}^{QP} = \frac{(T+b)(\alpha-c_0)}{AT+\beta} \).

Then, the equilibrium solutions of prices and profits are given as follows,
\[ \hat{p}_{CC_1} = a - \hat{\pi}_{CC_1}^{QP} - b\hat{q}_{CC_1}^{QP}, \quad \hat{p}_{CC_2} = a - \hat{\pi}_{CC_2}^{QP} - b\hat{q}_{CC_2}^{QP}, \]
\[ \hat{\pi}_{CCM_1} = (1 - b^2 - \theta)(\hat{q}_{CC_1}^{QP})^2 \text{ and } \hat{\pi}_{CCM_2} = (1 - \theta)(\hat{q}_{CC_2}^{QP})^2. \]

(ii) The DD structure:
The profits of supply chain 1 include \( \tilde{\pi}_{DDM_1}^{QP} = [\tilde{w}_1 - c_1(\tilde{q}_1)]\tilde{q}_1 \) and \( \tilde{\pi}_{DD_{R1}}^{QP} = (\tilde{p}_1 - \tilde{w}_1)\tilde{q}_1 \). The optimization problems of are \( \max_{\tilde{w}_1} \hat{\pi}_{DDM_1}^{QP} \) and \( \max_{\tilde{q}_1} \hat{\pi}_{DD_{R1}}^{QP} \). It is easy to prove that \( \hat{\pi}_{DDM_1}^{QP} \) and \( \hat{\pi}_{DD_{R1}}^{QP} \) are concave over \( \tilde{w}_{DD_1} \) and \( \tilde{q}_{DD_1} \), respectively. Similarly, the profits of supply chains 2 include \( \tilde{\pi}_{DDM_2}^{QP} = [\tilde{w}_2 - c_1(\tilde{q}_2)]\tilde{q}_2 \) and \( \tilde{\pi}_{DD_{R2}}^{QP} = (\tilde{p}_2 - \tilde{w}_2)\tilde{q}_2 \). And the optimization problems of are \( \max_{\tilde{w}_2} \hat{\pi}_{DDM_2}^{QP} \) and \( \max_{\tilde{q}_2} \hat{\pi}_{DD_{R2}}^{QP} \), which are concave over \( \tilde{w}_{DD_2} \) and \( \tilde{p}_{DD_2} \), respectively.

From the first-order conditions, we have
\[ \frac{\partial \hat{\pi}_{DD_{R1}}^{QP}}{\partial \tilde{q}_1} = a(1 - b) + 2(b^2 - 1)\tilde{q}_1 + b\tilde{p}_2 - \tilde{w}_1 = 0 \]
\[ \text{and} \quad \frac{\partial \hat{\pi}_{DD_{R2}}^{QP}}{\partial \tilde{p}_2} = a - b\tilde{q}_1 - 2\tilde{p}_2 + \tilde{w}_2 = 0. \] (b19)

Then we can obtain
\[ \tilde{q}_1 = \frac{(2 - b)a - 2\tilde{w}_1 + b\tilde{w}_2}{4 - 3b^2} \text{ and } \tilde{p}_2 = \frac{(2 - b - b^2)a + 2(1 - b^2)\tilde{w}_2 + b\tilde{w}_1}{4 - 3b^2}. \] (b20)

Based on Eqs.(b19-b20), we derive that
\[ \frac{\partial \hat{\pi}_{DDM_1}^{QP}}{\partial \tilde{w}_1} = \frac{(4 - 3b^2 - 4\theta)}{4 - 3b^2} \tilde{q}_1 - \frac{2}{4 - 3b^2}(\tilde{w}_1 - c_0) = 0 \]
\[ \text{and} \quad \frac{\partial \hat{\pi}_{DDM_2}^{QP}}{\partial \tilde{w}_2} = \frac{[1 + \frac{2b(2 - b^2)}{4 - 3b^2}]\tilde{q}_2 + \frac{b^2 - 2}{4 - 3b^2}(\tilde{w}_2 - c_0)}{4 - 3b^2} = 0. \] (b21)

From Eqs.(b20-b21), the equilibrium solutions are given as follows,
\[ \tilde{w}_{DD_1}^{*,QP} = \frac{U(b - b^3 + 5b^2 + 2b - 8 + 2(2 - b^2)\alpha - (2 - b^2)(5(3b^2 - 4) + 4(2 - b^2) + b)}{3b^2 - 4(9b^2 - 16) + 2b(4(2 - b^2))(5b^2 - 8) + 8b(2 - b^2)}, \]
\[ \tilde{w}_{DD_2}^{*,QP} = \frac{U(b - 2)\alpha - 2(3b^2 - 4)c_0 + 4(3b^2 - 4 + 2b^2)\tilde{w}_{DD_1}^{*,QP}}{Ub}, \]
\[ \tilde{q}_{DD_1} = \frac{2(\tilde{w}_{DD_1}^{*,QP} - c_0)}{4 - 3b^2 - 4\theta}, \quad \tilde{q}_{DD_2} = \frac{2(b(\tilde{w}_{DD_2}^{*,QP} - c_0))}{4 - 3b^2 + 2b(2 - b^2)}. \]
\[ p_{DD_1}^* = a - \tilde{q}_{DD_1}^* - bq_{DD_2}^*, \quad \tilde{p}_{DD_1}^* = a - \tilde{q}_{DD_1}^* - bq_{DD_1}^* \]

\[ \tilde{\pi}_{DD_{M1}}^* = \left( \frac{4-\theta}{2} \right)^2 \left( \tilde{q}_{DD_1}^* \right)^2 \text{ and } \tilde{\pi}_{DD_{M2}}^* = \left( \frac{4-\theta}{2} \right)^2 \left( \tilde{q}_{DD_2}^* \right)^2. \]

**B.4 Hybrid competition modes without economies of scale**

(i) The CD structure:

The profit function of \( M_1 \) is \( \pi_{CD_{M1}}^{QP} = (p_1 - c_0)q_1 \). The optimization problem is max \( \pi_{CD_{M1}}^{QP} \). Similar to the CC structure, we can easily prove that the function \( \pi_{CD_{M1}}^{QP} \) is also concave over \( q_{CD_1}^* \). We have

\[ \partial \pi_{CD_{M1}}^{QP} / \partial q_1 = (1 - b)a + 2(b^2 - 1)q_1 + bp_2 - c_0 = 0. \] (b22)

The profit function of \( R_2 \) is \( \pi_{CD_{R2}}^{QP} = (p_2 - w_2)q_2 \). Given the unit wholesale price \( w_2 \) offered by its upstream \( M_2 \), \( R_2 \)’s optimal problem is written as max \( \pi_{CD_{R2}}^{QP} \). Obviously, \( \partial \pi_{CD_{R2}}^{QP} / \partial p_2 = a - bq_1 - 2p_2 + w_2 = 0 \) and \( \partial^2 \pi_{CD_{R2}}^{QP} / \partial p_2^2 = -2 < 0 \), and therefore, \( \pi_{CD_{R2}}^{QP} \) is concave over \( p_2 \). We have

\[ \partial \pi_{CD_{R2}}^{QP} / \partial p_2 = a - bq_1 - 2p_2 + w_2 = 0. \] (b23)

From Eqs.(b22-b23), we can derive

\[ q_{CD_1}^* = \frac{(2 - b)a - 2c_0 + bw_2}{4 - 3b^2} \text{ and } p_{CD_2}^* = \frac{(2 - b - b^2)a + bc_0 + 2(1 - b^2)w_2}{4 - 3b^2}. \] (b24)

The profit function of \( M_2 \) is \( \pi_{CD_{M2}}^{QP} = (w_2 - c_0)q_2 \). The optimization problem of \( M_2 \) is max \( \pi_{CD_{M2}}^{QP} \). It is easy to prove that \( \pi_{CD_{M2}}^{QP} \) is concave over \( w_2 \). Then, from the first-order condition, we have

\[ \partial \pi_{CD_{M2}}^{QP} / \partial w_2 = q_2 + (w_2 - c_0)(-b \partial q_1 / \partial w_2 - \partial p_2 / \partial w_2) = 0. \] (b25)

From Eqs.(b24-b25), we can derive the equilibrium solutions,

\[ u_{CD_2}^* = \frac{(1-b)(2+b)-a-(1+b)(b-2)c_0}{2(2-b^2)}, \quad q_{CD_1}^* = \frac{(2-b)a - 2c_0 + bw_{CD_2}^*}{4 - 3b^2}, \]

\[ q_{CD_2}^* = \frac{(2-b^2-a+bc_0+b^2-2)c_{CD_2}^*}{4 - 3b^2}, \]

\[ \pi_{CD_{M1}}^{QP} = (1 - b^2)(q_{CD_1}^*)^2 \text{ and } \pi_{CD_{M2}}^{QP} = \frac{4-\theta}{2} \left( q_{CD_2}^* \right)^2. \]

(ii) The DC structure:

The profit function of \( M_2 \) is \( \pi_{DC_{M2}}^{QP} = (p_2 - c_0)q_2 \). The optimization problem is max \( \pi_{DC_{M2}}^{QP} \). Similar to the CD structure, we can easily prove that the function \( \pi_{DC_{M2}}^{QP} \) is also concave over \( p_{DC_2}^* \). We have

\[ \partial \pi_{DC_{M2}}^{QP} / \partial p_2 = a - bq_1 - 2p_2 + c_0 = 0. \] (b26)

The profit function of \( R_1 \) is \( \pi_{DC_{R1}}^{QP} = (p_1 - w_1)q_1 \). Given that the unit wholesale price \( w_1 \) is offered by its upstream \( M_1 \), \( R_1 \)’s optimal problem is written as
max \pi_{DC}^{QP}_{1}. From the first-order condition we have

\[ \frac{\partial \pi_{DC1}^{QP}}{\partial q_1} = (1 - b) a + 2(b^2 - 1)q_1 + bp_2 - w_1 = 0. \]  

(b27)

From Eqs.(b26-b27), we can derive

\[ q_{DC1}^{*} = \frac{(2 - b)a + bc_0 - 2w_1}{4 - 3b^2} \quad \text{and} \quad q_{DC2}^{*} = \frac{(1 - b)(2 + b)a + 2(1 - b^2)c_0 + bw_1}{4 - 3b^2}. \]  

(b28)

The profit function of \( M_1 \) is \( \pi_{DC1}^{QP} = (w_1 - c_0)q_1 \). The optimization problem of \( M_1 \) is max \( \pi_{DC1}^{QP} \). It is easy to prove that \( \pi_{DC1}^{QP} \) is concave over \( w_1 \). Then, from the first-order condition, we have

\[ \frac{\partial \pi_{DC}^{QP}}{\partial w_1} = q_1 - 2(w_1 - c_0) = 0. \]  

(b29)

From Eqs.(b28-b29), we can derive the equilibrium solutions,

\[ u_{DC1}^{*} = \frac{(2-b)a+2(b+c_0)}{4}, \quad \pi_{DC1}^{*} = \frac{(2-b)a+2bq_1^{*}}{4-3b^2}, \]

\[ q_{DC2}^{*} = \frac{(2-b^2-b)(b^2-2)c_0+bc_0^{*}}{4-3b^2}, \]

\[ \pi_{DC1}^{*} = \frac{4-3b^2}{2}(q_{DC1}^{*})^2 \quad \text{and} \quad \pi_{DC2}^{*} = (q_{DC2}^{*})^2. \]

Proof of Proposition 1(1).

Based on Appendix B.1, we let \( \theta = 0 \) and obtain

\[ \pi_{CC}^{QQ}(\theta=0) - \pi_{CD}^{QQ}(\theta=0) = (a-c_0)^2 \left[ \frac{a-b+4}{2}\right] - \frac{b^2}{3} \left( a-c_0 \right)^2. \]

Since \( b \in (-1, 1) \), we have \( \pi_{CC}^{QQ}(\theta=0) - \pi_{CD}^{QQ}(\theta=0) \geq 0. \)

Similarly,\( \pi_{DC}^{QQ}(\theta=0) - \pi_{CD}^{QQ}(\theta=0) = \frac{2(2-b)(a-b)^2}{4(b+2)^2} - \frac{8b^2-b}{16(4-b^2)^2} > 0. \)

Proof of Proposition 1(2).

Based on Appendix B.1, we have

\[ \pi_{CC}^{QQ}(\theta=0) = (a-c_0)^2 f_1(\theta, b) = (a-c_0)^2 \left[ \frac{(1-\theta)(A+b)^2}{(A+2)^2} - \frac{(A-b)^2(2-\theta)(A-b^2)}{(A+2A-2b)^2A^2} \right]. \]

Given that \( f_1(\theta, b) \) is a function of two variables, we can roughly draw a projection of 3-dimensional image \( f_1(\theta, b) \) onto the plane \( f_1(\theta, b) = 0 \). Specifically, (i) when \( b = -1 \), we can derive \( \theta = 0.6031 \) from \( f_1(\theta, -1) = 0 \); (ii) when \( b = 0 \), we can derive \( \theta \rightarrow 1 \) from \( f_1(\theta, 0) = 0 \); (iii) when \( b = 1 \), we can derive \( \theta = 0.6031 \) from \( f_1(\theta, 1) = 0 \).

By a similar approach, we can derive \( \pi_{CD}^{QQ} \leq \pi_{CD}^{QQ}. \)

Proof of Proposition 2(1)-(2). Based on Appendix B.2, let \( \theta = 0 \) and some results are easily available by subtraction, so we skip the details.

Proof of Proposition 2(3)-(4). Based on Appendix B.2, we use the similar analysis to derive these results.

Proof of Proposition 3. The proof is similar to that of Proposition 1(1).

Proof of Proposition 4-5. Based on Appendix B.3 and B.4, we use a similar approach to Proposition 2 to obtain these results.
Table 1. Compare and contrast our model with the extant literature

| Article                  | Model                                      | Market competition mode | Economies of scale | Product complementation | Outsourcing decision | Structure equilibrium | Competition mode equilibrium |
|--------------------------|--------------------------------------------|-------------------------|--------------------|-------------------------|----------------------|-----------------------|-----------------------------|
| Chen & Lee [10]          | Price vs. quantity under R&D competition   | ✓                       | ✓                  | ✓                       |                      |                       |                             |
| Narin et al [33]         | Cournot competition                        |                         | ✓                  | ✓                       |                      | ✓                     |                             |
| Arya et al [2]           | Price vs. quantity competition             | ✓                       | ✓                  | ✓                       |                      |                       |                             |
| Farahat & Perakis [15]   | Equilibrium                                | ✓                       | ✓                  | ✓                       |                      |                       |                             |
| Matsunuma & Ogawa [30]   | Price or a quantity contract in a mixed duopoly competition | ✓                       | ✓                  | ✓                       | ✓                    | ✓                     |                             |
| Fang & Shou [13]         | Pricing of complementary products in dual channel chain | ✓                       | ✓                  | ✓                       |                      |                       |                             |
| Zhao et al [44]          | Competitive supply chains with generalised supply costs | ✓                       | ✓                  | ✓                       |                      | ✓                     |                             |
| Atkins & Liang [4]       | Comparing price and quantity competition  | ✓                       | ✓                  | ✓                       |                      |                       |                             |
| Haraguchi & Matsunuma [22]| Choice of prices versus quantities with patent licensing pricing problem of complementary products | ✓                       | ✓                  | ✓                       |                      |                       |                             |
| Din & Sun [12]           | Equilibrium analysis under the supply chain competition | ✓                       | ✓                  | ✓                       | ✓                    | ✓                     | ✓                           |
| Wang et al [39]          |                                           | ✓                       | ✓                  | ✓                       | ✓                    | ✓                     | ✓                           |
| Our paper                |                                           | ✓                       | ✓                  | ✓                       | ✓                    | ✓                     | ✓                           |
Table 2. Equilibrium solutions for the four different structures

| X   | \( q_{X_1}^{eq} \) | \( \pi_{X_1}^{eq} \) |
|-----|-----------------|-----------------|
| CC  | \( \frac{a-c_0}{2(b-1)} \) | \( (1-\theta)q_{C1}^{eq} \) |
| CD  | \( \frac{a-2b}{A(A^2+2A-2b^2)} \) | \( (1-\theta)q_{C2}^{eq} \) |
| DD  | \( \frac{a-c_0}{2(b-1)} \) | \( (1-\theta)q_{D1}^{eq} \) |

Table 3. Equilibrium solutions for the four different structures

| X   | chain | \( u_{X_1}^{IP} \) | \( q_{X_1}^{IP} \) | \( \pi_{X_1}^{IP} \) |
|-----|-------|-----------------|-----------------|-----------------|
| CC  | i     | NA              | \( \frac{(2-b)(1+b)-2\theta}{(1-b)(2+b)a+(b^2-2)c_0+bw_{CD}^{*P,P}} \) | \( 1-b^2-\theta)(q_{C1}^{IP})^2 \) |
| CD  | 1(2)  | NA              | \( \frac{S}{(T+S)T} \) | \( -(T+b)a+bC_{D2}^{*P,P} \) | \( (1-b^2-\theta)(q_{C2}^{IP})^2 \) |
| DD  | 2(1)  | \( \frac{S-a-(1+b)(2-b)(b^2-2)c_0}{S-(1+b)(2-b)(b^2-2)c_0} \) | \( \frac{a-w_{CD}^{*IP}}{(1+b)(2-b)} \) | \( (1-b^2)(1-b^2) - \theta)(q_{D1}^{IP})^2 \) |
Table 4. Equilibrium solutions for the four supply chain structures

| Chain | \( w_{X_1}^{QP} \) | \( P_{X_1}^{QP} \) | \( q_{M_1}^{QP} \) | \( \pi_{M_1}^{QP} \) | \( \pi_{X_2}^{QP} \) |
|-------|----------------|----------------|----------------|----------------|----------------|
| **CC** | 1 NA | \( \frac{(b^2 - 1)(a - b)}{16(1 - b^2)(1 - b^2)(2 + b^2)} \) | \( \frac{(b^2 - 1)(a - b)}{16(1 - b^2)(1 - b^2)(2 + b^2)} \) | \( \frac{(1 - b^2)(q_{CC_1}^{QP})^2}{2} \) | NA |
| 2 NA | \( \frac{(b^2 - 1)(a - b)}{16(1 - b^2)(1 - b^2)(2 + b^2)} \) | \( \frac{(b^2 - 1)(a - b)}{16(1 - b^2)(1 - b^2)(2 + b^2)} \) | \( \frac{(1 - b^2)(q_{CC_2}^{QP})^2}{2} \) | NA |
| **CD** | 1 NA | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) |
| 2 \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) |
| **DC** | 1 NA | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) |
| 2 \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) |
| **DD** | 1 NA | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) |
| 2 \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) | \( \frac{(1 - b^2)^2 - 2b^2}{16 - 9b^2} \) |

Table 5. Payoff table for the strategic competition choice of the X supply chain structure

| Supply chain 1 | Cournot | Bertrand |
|----------------|---------|---------|
| Supply chain 2 | Cournot | Bertrand |
| (\( \pi_{X_1}^{QP} \), \( \pi_{X_2}^{QP} \), \( \pi_{X_1}^{QP} \)) | (\( \pi_{X_1}^{QP} \), \( \pi_{X_2}^{QP} \), \( \pi_{X_1}^{QP} \)) | (\( \pi_{X_1}^{QP} \), \( \pi_{X_2}^{QP} \), \( \pi_{X_1}^{QP} \)) |
| (\( \pi_{X_1}^{QP} \), \( \pi_{X_2}^{QP} \), \( \pi_{X_1}^{QP} \)) | (\( \pi_{X_1}^{QP} \), \( \pi_{X_2}^{QP} \), \( \pi_{X_1}^{QP} \)) | (\( \pi_{X_1}^{QP} \), \( \pi_{X_2}^{QP} \), \( \pi_{X_1}^{QP} \)) |
Table 6. Equilibrium solutions for the four structures with economies of scale

| Chain i | $\hat{w}^{QP}_{X_1}$ | $\hat{q}^{QP}_{X_2}$ | $\hat{q}^{QP}_{X_M}$ |
|---------|----------------------|----------------------|----------------------|
| **CC**  |                      |                      |                      |
| 1       | NA                   | $\frac{(T+1-b)^2(1-q)}{A^T_b+\theta}$ | $(1-b^2-\theta)(\hat{q}^{QP}_{CC})^2$ |
| 2       | NA                   | $\frac{(T+1-b)^2(1-q)}{A^T_b+\theta}$ | $(1-\theta)(\hat{q}^{QP}_{CC})^2$ |
| **CD**  |                      |                      |                      |
| 1       | NA                   | $\frac{(U-2\theta T)(T+1)+[(U(T+b)+2\theta T)]q}{2T(U-\theta T)}$ | $(1-b^2-\theta)(\hat{q}^{QP}_{CD})^2$ |
| 2       | NA                   | $\frac{(A-b)u+bq-A\hat{q}^{QP}_{CD}}{2A[U+6\theta(1-b^2-\theta)]}$ | $(\frac{U}{\theta}-\theta)(\hat{q}^{QP}_{CD})^2$ |
| **DC**  |                      |                      |                      |
| 1       | $\frac{2b\theta^2-U}{2+b(1-b^2-2)+b\theta q^{QP}_{DC}}$ | $\frac{2b\theta^2-U}{2+b(1-b^2-2)+b\theta q^{QP}_{DC}}$ | $(\frac{U}{\theta}-\theta)(\hat{q}^{QP}_{DC})^2$ |
| 2       | NA                   | $\frac{2b\theta^2-U}{2b\theta^2-U}$ | $(\frac{U}{\theta}-\theta)(\hat{q}^{QP}_{DC})^2$ |
| **DD**  |                      |                      |                      |
| 1       | $\frac{(U-U^3+5b^2-25-b^2+8\theta(2-b^2)_{[2-(2-b^2)]}5(U+6\theta)]}{3(U^3-4)(5b^2-16)+2\theta(4-b^2)(5b^2-8)+8\theta(2-b^2)}$ | $\frac{2b\theta^2-U}{2+b(1-b^2-2)+b\theta q^{QP}_{DD}}$ | $(\frac{U}{\theta}-\theta)(\hat{q}^{QP}_{DD})^2$ |
| 2       | $\frac{2b\theta^2-U}{2b\theta^2-U}$ | $\frac{2b\theta^2-U}{2b\theta^2-U}$ | $(\frac{U}{\theta}-\theta)(\hat{q}^{QP}_{DD})^2$ |
REFERENCES

[1] S. M. Ali, M. H. Rahman, T. J. Tumpa, A. A. M. Rifat and S. K. Paul, Examining price and service competition among retailers in a supply chain under potential demand disruption, *Journal of Retailing and Consumer Services*, 40 (2018), 40–47.

[2] A. Arya, B. Mittendorf and D. E. M. Sappington, Outsourcing, vertical integration, and price vs. quantity competition, *International Journal of Industrial Organization*, 26 (2008), 1–16.

[3] A. Arya, B. Mittendorf and D. E. M. Sappington, The make-or-buy decision in the presence of a rival: Strategic outsourcing to a common supplier, *Management Science*, 54 (2008), 1747–1758.

[4] D. Atkins and L. Liang, A note on competitive supply chains with generalized supply costs, *European Journal of Operational Research*, 207 (2010), 1316–1320.

[5] P. Bajec and M. Zanne, The current status of the Slovenian logistics outsourcing market, its ability and potential measures to improve the pursuit of global trends, *International Journal of Logistics Systems & Management*, 18 (2014), 436–448.

[6] J. Biau, K. K. Lai, Z. Hua, X. Zhao and G. Zhou, Bertrand vs. Cournot competition in distribution channels with upstream collusion, *International Journal of Production Economics*, 204 (2018), 278–289.

[7] G. P. Cachon and P. T. Harker, Competition and outsourcing with scale economies, *Management Science*, 48 (2002), 1314–1333.

[8] K. Chen and T. Xiao, Outsourcing strategy and production disruption of supply chain with demand and capacity allocation uncertainties, *International Journal of Production Economics*, 170 (2015), 243–257.

[9] K. Chen, R. Xu and H. Fang, Information disclosure model under supply chain competition with asymmetric demand disruption, *Asia-Pacific Journal of Operational Research*, 33 (2016), 1650043, 35 pp.

[10] J. Chen and S.-H. Lee. Cournot-bertrand comparison under R&D competition: Output versus R&D subsidies, *Cogent Business & Management*, (2021), https://mpra.ub.uni-muenchen.de/107949/.

[11] L. K. Cheng, Comparing Bertrand and Cournot equilibria: A geometric approach, *The Rand Journal of Economics*, 16 (1985), 146–152.

[12] H.-R. Din and C.-H. Sun, Welfare improving licensing with endogenous choice of prices versus quantities, *The North American Journal of Economics & Finance*, 51 (2020), 100859.

[13] Y. Fang and B. Shou, Managing supply uncertainty under supply chain Cournot competition, *European Journal of Operational Research*, 243 (2015), 156–176.

[14] L. Fanti and M. Scrimitore, How to compete Cournot versus Bertrand in a vertical structure with an integrated input supplier, *Southern Economic Journal*, 85 (2019), 796–820.

[15] A. Farahat and G. Perakis, A comparison of Bertrand and Cournot profits in oligopolies with differentiated products, *Operations Research*, 59 (2011), 507–513.

[16] E. Garaventa and T. Tellefsen, Outsourcing: The hidden costs, *Review of Business*, 22 (2001), 28–31.

[17] A. Ghosh and M. Mitra, Comparing Bertrand and Cournot in mixed markets, *Economics Letters*, 109 (2010), 72–74.

[18] B. C. Giri and B. R. Sarker, Improving performance by coordinating a supply chain with third party logistics outsourcing under production disruption, *Computers & Industrial Engineering*, 103 (2017), 168–177.

[19] A. Goli and B. Malmir, A covering tour approach for disaster relief locating and routing with fuzzy demand, *International Journal of Intelligent Transportation Systems Research*, 18 (2020), 140–152.

[20] A. Goli, E. B. Tirkolaee and N. S. Aydin, Fuzzy integrated cell formation and production scheduling considering automated guided vehicles and human factors, *IEEE Transactions on Fuzzy Systems*, (2021).

[21] A. Goli, H. K. Zare, R. T. Moghaddam and A. Sadeghieh, A comprehensive model of demand prediction based on hybrid artificial intelligence and metaheuristic algorithms: A case study in dairy industry, *Journal of Industrial and Systems Engineering*, 11 (2018), 190–203.
[22] J. Haraguchi and T. Matsumura, Cournot-Bertrand comparison in a mixed oligopoly, *Journal of Economics*, 117 (2016), 117–136.
[23] M. Huang, J. Tu, X. Chao and D. Jin, Quality risk in logistics outsourcing: A fourth party logistics perspective, *European Journal of Operational Research*, 276 (2019), 855–879.
[24] B. Jiang, J. A. Belohlav and S. T. Young, Outsourcing impact on manufacturing firms’ value: Evidence from Japan, *Journal of Operations Management*, 25 (2007), 885–900.
[25] M. Kaya and Ö. Özer, Quality risk in outsourcing: Noncontractible product quality and private quality cost information, *Naval Research Logistics*, 56 (2009), 669–685.
[26] T. Kremic, O. I. Tukel and W. O. Rom, Outsourcing decision support: A survey of benefits, risks, and decision factors, *Supply Chain Management*, 11 (2006), 467–482.
[27] J. R. Kroes and S. Ghosh, Outsourcing congruence with competitive priorities: Impact on supply chain and firm performance, *Journal of Operations Management*, 28 (2010), 124–143.
[28] Y. J. Lin, Oligopoly and vertical integration: Note, *The American Economic Review*, 78 (1988), 251–254.
[29] Z. Liu and A. Nagurney, Supply chain outsourcing under exchange rate risk and competition, *Omega: International Journal of Management Science*, 39 (2011), 539–549.
[30] T. Matsumura and A. Ogawa, Price versus quantity in a mixed duopoly, *Economics Letters*, 116 (2012), 174–177.
[31] T. W. McGuire and R. Staelin, An industry equilibrium analysis of downstream vertical integration, *Marketing Science*, 2 (1983), 161–191.
[32] S. M. Pahlevan, S. Hosseini and A. Goli, Sustainable supply chain network design using products’ life cycle in the aluminum industry, *Environmental Science and Pollution Research*, (2021).
[33] N. Singh and X. Vives, Price and quantity competition in a differentiated duopoly, *The Rand Journal of Economics*, 15 (1984), 546–554.
[34] S. Sinha and S. P. Sarmah, Supply chain coordination model with insufficient production capacity and option for outsourcing, *Mathematical and Computer Modelling*, 46 (2007), 1442–1452.
[35] H. Sun, Y. Wan, Y. Li, L. L. Zhang and Z. Zhou, Competition in a dual-channel supply chain considering duopolistic retailers with different behaviours, *Journal of Industrial & Management Optimization*, 17 (2021), 601–631.
[36] V. J. Tremblay, C. H. Tremblay and K. Isariyawongse, Cournot and Bertrand competition when advertising rotates demand: The case of Honda and Scion, *International Journal of the Economics of Business*, 20 (2013), 125–141.
[37] L. Wang, H. Song, D. Zhang and H. Yang, Pricing decisions for complementary products in a fuzzy dual-channel supply chain, *Journal of Industrial & Management Optimization*, 15 (2019), 343–364.
[38] C. Y. Wong and N. Karia, Explaining the competitive advantage of logistics service providers: A resource-based view approach, *International Journal of Production Economics*, 128 (2010), 51–67.
[39] T. Xiao, Y. Xia and G. P. Zhang, Strategic outsourcing decisions for manufacturers competing on product quality, *IIE Transactions*, 46 (2014), 313–329.
[40] T. Xiao, Y. Xia and G. P. Zhang, Strategic outsourcing decisions for manufacturers that produce partially substitutable products in a quantity-setting duopoly situation, *Decision Sciences*, 38 (2007), 81–106.
[41] Q. Yang, X. Zhao, H. Y. J. Yeung and Y. Liu, Improving logistics outsourcing performance through transactional and relational mechanisms under transaction uncertainties: Evidence from China, *International Journal of Production Economics*, 175 (2016), 12–23.
[42] J. Zhao, X. Hou, Y. Guo and J. Wei, Pricing policies for complementary products in a dual-channel supply chain, *Applied Mathematical Modelling*, 49 (2017), 437–451.
[45] W. Zhu, S. C. H. Ng, Z. Wang and X. Zhao, The role of outsourcing management process in improving the effectiveness of logistics outsourcing, *International Journal of Production Economics*, **188** (2017), 29–40.

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