ON THE ANDREWS-CURTIS CONJECTURE AND
ALGORITHMS FROM TOPOLOGY

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Abstract. We relate the Andrews-Curtis conjecture to the triviality problem
for balanced presentations using algorithms from 3-manifold topology. Implement-
ing this algorithm could lead to counterexamples to the Andrews-Curtis
conjecture.

The Andrews-Curtis conjecture \[1\] says that a balanced presentation of the triv-
ial group can be transformed to a standard presentation \(\langle \alpha_1, \ldots, \alpha_n; \alpha_1, \ldots, \alpha_n \rangle\) by
using ‘Andrews-Curtis moves’, i.e. multiplying one relation by another, inverting a
relation and conjugating a relation by a generator. Here, we use algorithmic meth-
ods from 3-manifold topology to relate this conjecture to a fundamental algorithmic
question in group theory, namely the triviality problem for balanced presentations.

Thus, our main result is

**Theorem 0.1 (3.9).** At least one of the following holds

- There is an algorithm to recognise balanced presentations of the trivial group,
or
- The (balanced) Andrews-Curtis conjecture is false.

Moreover, our methods could, after extensive computations, show the failure
of the (balanced) Andrews-Curtis conjecture for balanced presentations, and also
‘Property-R’ for links.

Our approach is to construct an algorithm, using algorithms from 3-manifold
topology, in particular the Rubinstein-Thompson algorithm \([3, 4, 5]\) for
recognising the 3-sphere, to recognise certain presentations of the trivial group.
To do this, we associate with a group presentation a ‘handle-structure’, a natural
generalisation of a handle-decomposition of a 3-manifold. Just as the Rubinstein-
Thompson-Matveev recognises a certain class of handle-decompositions of homo-
topy 3-spheres, namely those corresponding to a 3-sphere, our algorithm recognises
handle-structures corresponding to certain presentations of the trivial group. It is
not clear exactly what this class of presentations is, but one can see that it includes
those related to the standard presentations by the Andrews-Curtis moves.

We can immediately conclude that either the Andrews-Curtis conjecture is false,
or we have an algorithm to solve the triviality problem for balanced presentations.
Indeed, if the Andrews-Curtis conjecture were true, then our algorithm would recog-
nise the trivial group given a balanced presentation.

Further, we can apply the algorithm to candidate counterexamples. If the answer
is in the negative, then we know that we have a presentation that violates the
Andrews-Curtis conjecture. In particular, if such a presentation comes from a Kirby diagram for $S^3$ then property-R for links is false.

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1. Handle-structures

A connected 3-manifold with non-empty boundary has a handle-decomposition with a 0-handle, some 1-handles and some 2-handles. We call 0, 1 and 2-handles balls, beams and plates respectively. A beam is homeomorphic to $D^2 \times [0,1]$ and is attached to the ball along $D^2 \times \{0\}$. We shall call $D^2 \times \{0,1\}$ the ‘sticky end’ of a beam. Likewise a plate is homeomorphic to $D^2 \times [0,1]$ and is attached to the union of balls and beams along $S^1 \times [0,1]$, which we shall call the ‘sticky end’ of the plate. The components of intersection of beams and plates with the ball shall be called islands and bridges respectively.

It is clear that a handle-decomposition as above is completely determined by the beams and plates together with their ‘sticky ends’, for there is an essentially unique way to stick the ‘sticky ends’ to the 0-handle. Further, suppose we are given beams and plates, with the plates glued to beams along strips in their sticky end as they would be in a 3-manifold. Then these are the beams and plates of a handle-decomposition of a 3-manifold iff the sticky end of these is planar. This is an unnecessary assumption for many applications of 3-manifold techniques, in particular many results in normal surface theory. This motivates the definition

**Definition 1.1.** A handle-structure is a union of beams and plates, which intersect in unions of discs that when viewed as subsets in either the beams or the plates are of the form $\alpha \times [0,1]$ where $\alpha$ is an arc in $D^2$. The horizontal boundaries $D^2 \times \{0,1\}$ of the beams together with the portion of the horizontal boundary of the plates that is not glued to a beam form a surface consisting of islands and bridges that we call the **sticky end** (see figure 1). The space consisting of the beams and plates will be called the **total space**.

Our aim in introducing handle-structures is to associate these with finite presentations of groups. We shall use the notation $X/Y$, where $X$ and $Y$ are topological spaces with $Y \subset X$, to denote the quotient space obtained from $X$ by identifying $Y$ to a point. Briefly, we can associate with any presentation a singular 3-manifold, with the only singularities being transversal intersections and self-intersections of the bridges. We delete from this the closure of the ball to get a non-singular open 3-manifold. The completion of this gives a handle-structure, with the sticky-part being the points added to complete the space.

**Proposition 1.1.** Given a finite presentation of a group $G$, we can associate to it a handle-structure. This construction is canonical up to finitely many choices. Further, if $M$ is the total space and $S$ the sticky end, then $\pi_1(M/S) = G$.

**Proof.** Take a beam corresponding to each generator and a plate corresponding to each relation. Successively glue the plates to the beams according to the word that the relation represents. Namely, start with a base point that will not be glued to any 1-handle, and for each successive letter in the word of the relation attach a
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strip in the horizontal boundary $S^1 \times [0, 1]$ of the plate to the corresponding beam, with the direction determined by the sign of the exponent. Thus, strips attached to 1-handles alternate with bridges as in figure 1, with the attaching maps being governed by the word that the 2-handle represents. The only essential choices involved are in choosing where to attach the strip to the beam relative to previous intersections with plates.

Clearly $M/S$ deformation retracts onto a 2-complex that is well known to have fundamental group $G$. More precisely, each one handle deformation retracts to its core, and in the quotient the two islands of a 1-handle are identified to a single point. Thus, the 1-handles deformation retract onto a wedge of circles. The 2-handles in turn deformation retract on to their cores, which are 2-discs. By construction, the attaching maps of the disc corresponding to a given 2-handle to the wedge of circles formed by the 1-handles gives the corresponding relation in the fundamental group.

**Example 1.1.** For the presentation $\langle \alpha_1, ..., \alpha_n; \rangle$ of a free group on $n$ generators, we have $n$ 1-handles with $2n$ islands, 2 on each 1-handle, and no 2-handles. Thus $M/S$ is a thickened wedge of $n$ circles (see figure 2).

**Example 1.2.** For the presentation $\langle \alpha; \alpha \rangle$ of the trivial group, we have one 1-handle and a 2-handle, with the intersection of these connected. Thus $M$ is the union of two 3-balls glues together along a 2-disc, and is therefore a 3-ball. $S$ consists of two islands joined together by a bridge, and hence is $B^3$ (see figure 2).

Figure 1. 1-handles and 2-handles in a handle structure
M/S for a free group

(M,S) for the trivial group

Example 1.3. The standard balanced presentation of the trivial group with \( n \) generators and relations gives a handle-structure with \( n \) components, each of them as in the previous example.

We shall recognise handle-structures that correspond to a class of presentations of the trivial group.

2. The Algorithm

We assume for the next two sections that we are given a balanced presentation of a group. To this we associate a finite collection of handle structures as above. We apply the following algorithm to each of these.

Algorithm. Given a handle-structure, let \( M \) be the total space and \( S \) the sticky end.

1. Attach thickened discs to \( M \) along each component of \( \partial S \) to get a closed manifold. Define the sticky part of this to be the union of the components of the boundary that intersect \( S \).
2. If each component of the resulting manifold is \( S^2 \times [0, 1] \), with one of its boundary components being the sticky part, then answer ‘Yes’ and terminate.
3. Cut \((M, S)\) along a maximal family of pairwise disjoint and non-parallel annuli such that each annulus has exactly one boundary component in \( S \). If there are no such annuli, answer ‘No’ and terminate.
4. Repeat from step 3 using the new \((M, S)\).
We shall call annuli which are as in step 3 of the algorithm vertical annuli.

**Remark 2.1.** Step 2 can be achieved by using the Rubinstein-Thompson-Matveev algorithm to recognise $S^3$. Step 3 uses basic normal surface theory. The algorithm terminates because the boundary gets simpler at each iteration.

**Lemma 2.2.** If the algorithm terminates with a ‘Yes’, then the $\pi_1(M/S)$ is the trivial group.

**Proof.** This follows by induction on the number of steps needed. If $M$ is as in step 2 then this is true. It is easy to see that this remains true if $S^2 \times [0,1]$ is obtained after cutting along vertical annuli and filling in discs as in steps 3 & 1. We need to see that if $\pi_1(M/S)$ is trivial after cutting along an annulus or capping off, then it is trivial before making these changes. In the latter case, this follows as there is no relation added - the boundary of the disc is already trivial as it is in $S$. In the case where we cut along a vertical annulus, let $M'$ and $S'$ be obtained after cutting. Now an arc that in $M$ that intersects the annulus can be homotoped in the quotient off this - it breaks into an arc ending in $S$ just before the annulus and an arc starting from $S$ just after the annulus. This is in $M'$, hence trivial in $M'/S'$, and so all the more in $M/S$.

We thus have an algorithm to recognise a certain class of presentations of the trivial group. Namely, given a presentation, we apply the above algorithm to all the handle-structures associated to it. If the algorithm says ‘Yes’ for at least one of these, we conclude that the presentation corresponds to the trivial group. We shall call presentations where this happens, and also the handle-structures for which the algorithm says ‘Yes’ sphere-like.

**Remark 2.3.** In case the presentation and the handle-structure come from a 3-manifold, after capping off $S$ is a union of 2-spheres, and handle-structure is sphere-like iff the manifold is a sphere.

### 3. Relation to the Andrews-Curtis moves

We have constructed above an algorithm to recognise certain presentations of the trivial group, which we call sphere-like. For this to be useful, we need to see that this class of presentations is reasonably large. In fact, we shall see

**Theorem 3.1.** A balanced presentation of the trivial group that comes from the standard one by Andrews-Curtis moves is sphere-like.

**Proof.** We shall show this in two stages. First, we multiply relations merely by concatenation without any cancellation. We shall make a choice of handle-structures that correspond to the new (in general not reduced) presentation such that the new presentation is sphere-like. We then consider the effect of cancellation and show that even after cancellation, the handle-structure is sphere-like.

The first step is accomplished by induction on the number of moves. In the case of the standard presentations, $(M, S) = \bigcup(D^2 \times [0,1], D^2 \times 0)$, which is sphere-like.

We shall have to consider two basic transformation of the pairs $(M, S)$ – gluing and puncturing.

**Definition 3.1.** Let $x$ and $y$ be two points on $\partial S$. Then by gluing $x$ and $y$ we mean the identification of disjoint regular neighbourhoods of these points, with the new $S$ being the image of $S$ under the quotient map.
Note that the new total space and the new sticky part are obtained from the old ones by taking a $\partial$-connected sum.

**Definition 3.2.** Let $\gamma$ be an arc properly embedded in $M$ with exactly one endpoint in $S$. By *puncturing* along $\gamma$ we mean deleting a regular neighbourhood of $\gamma$.

The arc above need not be unknotted. We shall call such an arc a *vertical arc*.

**Lemma 3.2.** On making an appropriate choice of handle-structure corresponding to a presentation obtained by multiplying words without cancellation, the effect of the Andrews-Curtis moves is a sequence of gluings and puncturings.

**Proof.** Firstly, the inversion of a relation has no effect on the associated handle-structures (with obvious choices).

Next consider conjugation of a relation by a generator. This, again with an obvious choice of handle structure, amounts to a handle slide together with puncturing a hole along an arc with one boundary point on the sticky part. It may be helpful to consider the change in handle structure on making the Andrews-Curtis move $\langle \alpha, \beta; \alpha, \beta \rangle \rightarrow \langle \alpha, \beta; \alpha, \alpha^{-1} \beta \alpha \rangle$. The resulting $(M, S)$ is $(\text{Annulus} \times [0, 1], \text{Annulus})$ (see figure 4). The transformation in general is very similar to this particular case. If we attach a thick disc along the new bridge that forms a loop, we see that the resulting $(M, S)$ is obtained from the original one by gluing. Puncturing along the arc dual to the thick disc now gives us the handle-structure obtained by conjugation.

Next, we consider the effect of multiplying a relation by another. First consider the particular example $\langle \alpha, \beta; \alpha, \beta \rangle \rightarrow \langle \alpha, \beta; \alpha, \alpha \beta \rangle$. The new handle-structure is $(M, S) = (D^3, D^2)$ (see figure 3) and is clearly obtained by a gluing. Suppose that we are replacing the relation $r_1$ by $r_1 r_2$. If $r_2$ has only one bridge, then we have a similar situation. Namely, in the new handle-structure, $r_1$ has a bridge replaced by two bridges and has an additional component of intersection with the only beam that $r_2$ intersects. Choose the new handle structure so that this component of intersection is adjacent to the attachment of $r_2$ to the beam. Then the handle-structure is clearly obtained topologically by gluing.

Suppose now $r_2$ has more than one bridge, i.e., bridges in addition to the one along which gluing has taken place. We attach the relations other than $r_1$ as before. The new $r_1$ is the old one with additional bridges and gluings to 1-handles. Namely first pick bridges of $r_2$ and $r_1$ corresponding to the base points, and replace the bridge of $r_1$ by two bridges to the islands bounding the bridge of $r_2$ and adjacent to this bridge. Then attach $r_1$ to the 1-handles adjacent to the where $r_2$ has been attached. The new bridges of $r_1$ will thus be adjacent to those of $r_2$. Attaching thick discs to these gives, as in the example above, a handle-structure obtained by gluing. As in the case of conjugation, the final structure is obtained by puncturing along dual arcs to these discs.

We shall need some further transformations to take into account cancellations.

**Definition 3.3.** A *bigon* in $(M, S)$ is a properly embedded disc in $M$ whose boundary has exactly one component of intersection with each of $S$ and $\partial M \setminus S$.

**Remark 3.3.** The image of the discs identified in gluing forms a bigon in the resulting manifold. Thus, we shall sometimes refer to gluing as gluing along a bigon.
Figure 3. The presentation $\langle \alpha, \beta; \alpha, \alpha \beta \rangle$

Figure 4. The presentation $\langle \alpha, \beta; \alpha, \alpha^{-1} \beta \alpha \rangle$

**Definition 3.4.** A **quadrilateral** in $(M, S)$ is a properly embedded disc in $M$ whose boundary has exactly two components of intersection with each of $S$ and $\partial M \setminus S$.

**Lemma 3.4.** The handle-structure obtained after cancelling a pair of adjacent letters that are inverses of each other is obtained by cutting along a quadrilateral and then along a bigon.

**Proof.** To see the above, note that when we have a cancelling pair of letters, the 2-handle goes over a 1-handle and then returns over the same 1-handle immediately. Thus, successive bridges of the 2-handle are one ending in the 1-handle, one beginning and ending in the 1-handle (which shall often be referred to as a loop in
what follows) and then one beginning in the 1-handle. We cut along a quadrilateral (see figure 5) consisting of separating arcs along the first and last of these bridges and arcs in the vertical boundary of the 2-handle. Then we cut along a bigon whose boundary has an arc in the bridge that begins and ends in the 1-handle. The result is homeomorphic to a handle-structure corresponding to the cancelled presentation. Note that in the figure, after splitting along the quadrilateral, we declare the left side, i.e., the side away from the bigon as part of the sticky end. Thus what we have left is indeed a bigon.

It may be helpful here to see these in figure 4. Here, The quadrilateral splits the two bridges joining the two 1-handles. It is thus the dotted arc joining these bridges in the figure thickened. The side towards the lower handle in the figure becomes part of $S$. The bigon is the dotted line joining the third bridge to the quadrilateral thickened.

We have thus reduced the problem to showing that puncturing, gluing and cutting along bigons and quadrilaterals coming from cancellations takes sphere-like pairs $(M, S)$ to sphere-like pairs.

**Lemma 3.5.** Gluing and puncturing take sphere-like pairs $(M, S)$ to sphere-like pairs.

**Proof.** The effect of puncturing along a vertical arc is undone in the first step of the algorithm, where the components of $S$ are capped off by attaching discs. Thus, there is nothing to prove in this case.

When we glue along a bigon, there are two cases to consider, namely where we glue two distinct boundary components and where we glue a boundary component to itself. In the latter case, the manifold after capping off does not change, we merely glue two discs instead of one. In the former case, after gluing and capping off, we end up with an essential vertical annulus. This is because gluing together and capping off has the same effect as attaching a cylinder between the two boundary components. After cutting along this and capping off, we end up with the manifold
obtained by capping off the original annulus. Finally, we note that as though the maximal family of vertical annuli need not be unique, the complementary region does not depend on the family chosen. Hence we may assume that the family chosen in the algorithm contains the abovementioned annulus.

Lemma 3.6. Cutting along a bigon takes a sphere-like pair \((M, S)\) to a sphere-like pair.

Proof. Note that there are at most two punctures intersecting the bigon, corresponding to the two ‘vertices’ of the bigon. If the vertices are on different components of \(\partial S\), then we see that cutting along the bigon and capping off has the same effect as capping off the original manifold.

Thus, we need to consider only the case where both vertices are on the same component. We first consider the case where we have a punctured product \((\tilde{S} \times I) \setminus \text{punctures}\), with the only punctures being ones that intersect the bigon. We shall then reduce to this case. Note that we can assume that punctures disjoint from the bigon were made after cutting along the bigon and thus the latter part of the reduction is immediate.

Thus, the vertices are either on a puncture or a component \(\gamma\) of \(\partial \tilde{S}\). In the former case, we call a neighbourhood of the puncture the boundary annulus and in the latter case we use this term for \(\gamma \times I\). We shall see that we have two possibilities, a bigon parallel to an (in general) knotted puncture and a possibly essential one in a product (i.e. with vertex on \(\partial \tilde{S}\) or an unknotted puncture).

We have a natural vertical annulus in the manifold capped off at the component of \(\partial S\) containing the vertex (see figure 6). Namely, we add to the bigon the neighbourhood of an arc in \(\partial S\), i.e., a strip in the boundary annulus. If this is an essential vertical annulus, then as the arc along which the manifold was punctured is a vertical arc on this annulus, puncturing and then cutting along the bigon still gives a product. Otherwise, the boundary curves of the annulus must bound discs, giving a 2-sphere. This must bound a 3-ball (which follows from irreducibility of the punctured product). Thus, cutting along the bigon gives one component \(D^2 \times I\) and the other homeomorphic to the original manifold. In either case, the manifold is still sphere-like.

Now to reduce to the case considered, we need to consider the intersection of the annulus with the bigons along which gluing has taken place. This intersection, after making things in general position, is a collection of inessential arcs (i.e. those with both endpoints on the same boundary component of the annulus), vertical arcs and horizontal circles. We take an outermost such component. If it is an inessential arc, then we can isotope the annulus to remove this intersection. To do this, note that the arc separates a disc in each of the annulus and the bigon, and these together form a compressing disc. As the boundary is incompressible, we have a sphere, which must bound a ball by irreducibility. If we have a horizontal circle, then this bounds a disc in the annulus, and we see that we have a bigon parallel to a puncture.

Finally, in case of a vertical arc, our bigon decomposes into a piece between two parallel punctures (perhaps the same one and a bigon with simpler intersection. Again, the first bigon is as in one of the above cases, and in either case, it is easy to see that we get a sphere-like pair.
Lemma 3.7. Cutting along a quadrilateral corresponding to a cancellation takes a sphere-like pair $(M, S)$ to a sphere-like pair.

First, observe that when cutting along a quadrilateral, there is a choice involved. Namely, the disc along which $M$ has been cut now corresponds to two discs in $\partial M$, and one of these is to be included in $S$. If the wrong choice is made, the result is not even homologically $S \times I$. It is easy to see that in our case we do have a product homologically, and so the choice has been made correctly. Namely, if we consider the handle-structure obtained with the other choice, then we have two bridges, one corresponding to the earlier loop and one a new one, so that the bridges end on both sides in the two islands corresponding to the one-handle where the cancellation takes place. For each bridge, we can take an arc along it and close it up in the one handle to get a closed curve. These curves bound an annulus in $M$, and are thus homologically equivalent. But an arc across one of the bridges intersects one of the curves once, but not the other, hence the curves are not homologous in $S$. Thus we do not have a (homological) product with this wrong choice.

With the correct choice, this claim is well known in the case of products (and more generally for taut sutured manifolds [2]). In our situation, we claim that cutting along a quadrilateral has the effect of cutting along two bigons and then...
gluing along a bigon. More specifically, we can find $\partial$-compressing disc for the quadrilateral, i.e., a disc whose boundary consists of an arc in the quadrilateral and an arc in $S$ (see figure 7) at some stage in the algorithm. Boundary-compressing along such a disc gives two bigons, and it is easy to see that cutting along the quadrilateral is equivalent to cutting along each of these discs and then re-gluing along the $\partial$-compressing disc.

We show that such a disc exists by induction. In the case of a product, this follows by cutting up $M$ along vertical annuli and discs. An outermost disc of the vertical discs and annuli cut by the quadrilaterals gives the compression, except in the trivial case where the quadrilateral can be made disjoint from these. Similarly, if we have a quadrilateral after gluing along bigons, we look at outermost arcs of the bigons after cutting by the quadrilateral to reduce to the situation before gluing along the bigon.

As with bigons, we may assume that punctures not involving the vertices of the quadrilateral were made after cutting along the quadrilateral. Further, we may again assume that we are in a product. For, we again consider the intersections with the bigon. Inessential arcs can be handled as before. An arc that separates one vertex from the other three is like a vertical arc in the case of the bigons. An arc that separates the vertices into two pairs separates a disc in the bigon which gives a compression, and so we are done in this case.

To find the compression, consider an arc $p$ along the quadrilateral joining a pair of edges in $S$ (see figure 7). This is homotopic to an essentially unique arc in $S$ before puncturing (as we are in a product).

We claim that the arc is unknotted, i.e., it is isotopic to this arc and unlinked from the punctures on which the vertices lie. This follows since the quadrilateral corresponds to a cancellation, and hence we can find an embedded, once-punctured disc that the arc $p$ together with an arc in $S$ bounds. Namely, we can join the end points of this arc by an arc in $S$ running through the bridges that have been cut and the common island in which these end. The original arc together with this bound a tube, one half of which is in the chopped off portion of the plate and the other in the beam corresponding to the cancellation. The other boundary of this tube is a circle in $S$, whose boundary is an arc along the bridge that forms a loop and the other in the other island that our beam bounds.

If the total space is $F \times I$ with $F$ a punctured sphere, as the arc is unknotted and unlinked from the punctures containing its vertices, the arc bounds a compressing disc. In the case of a general product, there exists a family of annuli, which we can assume are disjoint from the arc and the circle in $S$ that are part of the boundary of the quadrilateral, so that the complement of these is $F \times I$, with $F$ a punctured sphere. By considering outermost components of intersection as before, we reduce to the previous case.

Thus, we have reduced to the case where we cut along bigons, which is the previous lemma.

We conclude that some handle-structure corresponding to the presentation obtained by applying the Andrews-Curtis moves is sphere-like, i.e., the presentation is sphere-like.

To summarise,
Theorem 3.8. There is an algorithm that recognises a class of presentations of the trivial group that includes all those obtained from the standard one by Andrews-Curtis moves.

An immediate corollary is

Theorem 3.9. At least one of the following holds

• There is an algorithm to recognise balanced presentations of the trivial group, or
• The (balanced) Andrews-Curtis conjecture is false.

One may use the algorithm given here to attempt to find particular presentations that violate the (balanced) Andrews-Curtis conjecture. To do so, we take a candidate presentation of the trivial group and apply the algorithm to it. If it turns out not to be sphere-like we have a counterexample.

In particular, this method could show that the Andrews-Curtis conjecture (in the balanced case) is false. Furthermore, if this method works, for instance, for presentations corresponding to the Akbulut-Kirby 4-sphere, we would be able to conclude that property-R for links is false.

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