3D Computations of Flow Field in a Guide Vane Blading Designed by Means of 2D Model for a Low Head Hydraulic Turbine

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Abstract. The paper presents the main parameters of the flow field behind the guide vane cascade designed by means of 2D inverse problem and following check by means of 3D commercial program ANSYS/Fluent applied for a direct problem. This approach of using different models reflects the contemporary design procedure for non-standardized turbomachinery stage. Depending on the model, the set of conservation equation to be solved differs, although the physical background remains the same. The example of computations for guide vane cascade for a low head hydraulic turbine is presented.

1. Introduction
The chain of models applicable for a turbomachinery blading consists with 1D, 2D and 3D models as it is shown in figure 1. The complexity rises up with the number of dimensions taken into account.

![Figure 1. The chain of models for a turbomachinery designing process.](image-url)
The simplest 1D model serves generally for preliminary prediction of flow field behaviour for a given blading. It predicts the main kinematics parameters assuring the maximum efficiency. This was illustrated for low head hydraulic turbine stage in [1].

A 2D model is particularly convenient for the so-called inverse problem when the shapes of blade can be determined. The 2D model was originated by the publication of Lorenz [2] and later developed by Wu [3]. For the inverse problem 2D model was adopted by Puzyrewski, Krzemianowski [4], [5], [6], [7] and others e.g. [8].

A 3D model is recently recognized as the most powerful and enables to determine the details of the flow field including such parameters as the loss coefficients and outlet angles. These parameters are crucial for the evaluation of the blading quality expressed by the efficiency.

2. The comparison of the governing equations for the 2D and 3D models
As it was mentioned above 2D model is convenient for the inverse problem, which enables predicting the shape of blading according to the assumed boundary conditions. The boundary conditions should be determined due to the formal character of equations to be solved. Therefore, it seems reasonable to confront the forms of basic governing equations. The main assumptions for the 2D and 3D models are listed in table 1.

| Parameters          | 2D model                                                                 | 3D model                                                                 |
|---------------------|--------------------------------------------------------------------------|--------------------------------------------------------------------------|
| Geometry            | • No shapes of blades are known                                           | • Geometry of flow domain is known. Shapes of blades are given between inner and outer borders of a meridional cross-section |
|                     | • Given the expected shapes of axisymmetric stream surfaces             | • Given expected blades                                                  |
|                     | • Given inlet and outlet planes between inner and outer borders of meridional cross-section | • Given expected thickness of blades i.e. blockage factor                |
| Coordinates         | • Curvilinear axisymmetric coordinates \(x^{(1)}, x^{(2)}, x^{(3)}\)       | • Cartesian coordinates \((x, y, z)\)                                     |
|                     | • Assumed stationary flow                                               | • Assumed stationary flow                                               |
| Inlet boundary      | • Distribution of flow parameters in front of a blading is assumed      | • Distribution of velocity components ahead of a blading area is assumed |
| conditions          |                                                                          | • Pressure at the outlet of computational domain is assumed               |
| Outlet boundary     | • Outlet flow parameters results from computation                        | • Dissipation effects are computed within the frame of an applied turbulence model |
| conditions          |                                                                          |                                                                          |
| Dissipation effects | • Distribution of dissipation effects is introduced by means of an expected loss coefficient |                                                                          |

It is noteworthy that for 2D inverse problem wishful shapes of stream surfaces distributed in meridional cross-section as shown in figure 2 are assumed. In figure 2 three systems of coordinates are marked, namely: Cartesian \((x, y, z)\), Cylindrical \((r, \varphi, z)\) and Curvilinear \((x^{(1)}, x^{(2)}, x^{(3)})\). The Curvilinear system is based on the assumed stream surfaces.

The shapes of stream surfaces are chosen in the form of an assumed function as follows:

\[
r = f(x^{(1)}, x^{(3)})
\]

It is easy to note that function \(f(x^{(1)}, x^{(3)})\) does not contain the circumferential coordinate \(x^{(2)}\).

In a 2D model the intensity of dissipation process has to be introduced on realistic level in advance. For a 3D direct model the geometry of the blading channels is given as created by means of 2D model. It is free to choose the turbulent model which closes the set of governing equation for 3D computation. As a consequence the intensity of dissipation in 3D model is computed.
• **Mass conservation equation**
  ✓ 3D model:
  \[ \text{div} \vec{U} = 0 \]  \hspace{1cm} (2)
  ✓ 2D model:
  \[ \text{div} \vec{U} = 0 \]  \hspace{1cm} (3)
  turns into algebraic form \[5\]
  \[ (1 - \tau(x^{(1)}, x^{(3)})) \rho U_{x^{(3)}} \frac{f \gamma f}{\gamma f} = m(x^{(1)}) \]  \hspace{1cm} (4)
  where \( \rho \) is density, \( m(x^{(1)}) \) is mass flow rate distributed at each of a streamline.

• **Momentum conservation equation**
  ✓ 3D model [9]:
  \[ \rho \vec{U} \text{grad} \vec{U} = -\text{grad} \ p + \text{div} \ \vec{t} \]  \hspace{1cm} (5)
  where tensor \( \vec{t} \) is given as follows:
  \[ \vec{t} = \mu(\text{grad} \vec{U} + \text{grad} \vec{U}^T) - \frac{2}{3} \mu \text{div} \vec{U} \vec{I} \]  \hspace{1cm} (6)
  where \( \vec{I} \) is unit tensor, \( \mu \) is dynamic viscosity.
  ✓ 2D model:
  where body force \( \vec{F} \) replaces the differential part \( \text{div} \ \vec{t} \):
  \[ \rho \vec{U} \text{grad} \vec{U} = -\text{grad} \ p + \vec{F} \]  \hspace{1cm} (7)
  and it may be rearranged into the following forms in all three curvilinear \( (x^{(1)}, x^{(2)}, x^{(3)}) \) directions:
  \[ -\frac{\rho U_{x^{(2)}}^2}{f} + \frac{\rho U_{x^{(3)}}^2}{1 + \left( \frac{df}{dx^{(3)}} \right)^2} = -\frac{\partial p}{\partial x^{(1)}} \left( \frac{df}{dx^{(3)}} \right)^2 + \frac{\partial p}{\partial x^{(3)}} \frac{\partial f}{\partial x^{(3)}} + \rho F_{x^{(1)}} \]  \hspace{1cm} (8)
  \[ -\frac{\rho U_{x^{(3)}}}{f} \left( \frac{df}{dx^{(3)}} \right)^2 \frac{\partial}{\partial x^{(3)}} (f U_{x^{(3)}}) = \rho F_{x^{(2)}} \]  \hspace{1cm} (9)
  \[ \sqrt{1 + \left( \frac{df}{dx^{(3)}} \right)^2} \left( \frac{\partial u_{x^{(3)}}}{\partial x^{(3)}} - \frac{U_{x^{(3)}}}{1 + \left( \frac{df}{dx^{(3)}} \right)^2} \frac{\partial f}{\partial x^{(3)}} \frac{\partial f}{\partial x^{(3)}} \right) = \frac{\partial p}{\partial x^{(3)}} \sqrt{1 + \left( \frac{df}{dx^{(3)}} \right)^2} - \frac{\partial p}{\partial x^{(3)}} \sqrt{1 + \left( \frac{df}{dx^{(3)}} \right)^2} + \rho F_{x^{(3)}} \]  \hspace{1cm} (10)
  For 2D model components of body force \( \vec{F}(F_{x^{(1)}}, F_{x^{(2)}}, F_{x^{(3)}}) \) become the unknowns beside pressure \( p \) and velocity components \( \vec{U}(U_{x^{(1)}} \equiv 0, U_{x^{(2)}}, U_{x^{(3)}}) \) (\( U_{x^{(2)}} \) is tangential and \( U_{x^{(3)}} \) is meridional velocity components). As it is shown in [4] the set of the above presented equations may be reduced to hyperbolic type for pressure \( p \). Potential force is neglected because of horizontal flow.
• **Energy conservation equation**
  
  **3D model** [9]:
  
  $$\rho \text{div}(\tilde{U}h) = \bar{\tau} : \text{grad} \ U$$  
  \hspace{1cm} (11)
  
  where \( h = \frac{p}{\rho} + e \) (\( e \) is an internal energy).

  **2D model:**
  
  $$\frac{u_1^{(2)} + u_2^{(3)}}{2} + e + \frac{p}{\rho} + \Pi = \text{const}$$  
  \hspace{1cm} (12)
  
  where \( \Pi \) is gravity potential \( (\Pi = gx^{(3)}) \), \( g \) is gravitational acceleration.

• **Closing relations**

  **3D model** [9]:
  
  $$(13)$$
  
  $$(14)$$
  
  Left hand sides present the convection terms. First terms of right hand sides in the above equations represent diffusion effect. The rest terms are sources ones.

  **2D model:**
  
  The internal energy rises along the trajectory with time \( t_2 > t_1 \)
  
  $$\Delta e = e_{t_2} - e_{t_1} = \varsigma(x^{(1)}, x^{(3)}) \frac{u_1^{(2)} + u_2^{(3)}}{2} > 0$$  
  \hspace{1cm} (15)
  
  in a proportion to loss coefficient \( \varsigma(x^{(1)}, x^{(3)}) \).

  Body force component can be expressed as follows:
  
  $$F_{\xi(2)} = \delta(x^{(1)}, x^{(3)}) \left( Q_2(p, \rho, U_{x^{(2)}}, U_{x^{(3)}}, S, \ldots) + Q_2(p, \rho, U_{x^{(2)}}, U_{x^{(3)}}, S, \ldots) \frac{\partial p}{\partial x^{(2)}} + Q_2(p, \rho, U_{x^{(2)}}, U_{x^{(3)}}, S, \ldots) \frac{\partial \omega}{\partial x^{(2)}} \right)$$  
  \hspace{1cm} (16)
  
  where the shape factor \( S = \frac{\partial x^{(2)}}{\partial x^{(3)}} \) can be evaluated for skeleton surface (see details in [5]).

  The parameter \( \delta(x^{(1)}, x^{(3)}) \) is the third one beside \( \varsigma(x^{(1)}, x^{(3)}) \) and \( \tau(x^{(1)}, x^{(3)}) \) required to close conservation equations. The following shapes of functions were used in 2D computation:
  
  $$\tau(x^{(1)}, x^{(3)}) = t_1(x^{(3)})^{t_1} (1 - x^{(3)})^{t_2} (1 - t_4 x^{(1)})$$  
  \hspace{1cm} (17)
  
  $$\varsigma(x^{(1)}, x^{(3)}) = (w_0 + w_1 x^{(1)})^{n_1} (w_2 + w_3 x^{(3)})^{n_2}$$  
  \hspace{1cm} (18)
  
  $$\delta(x^{(1)}, x^{(3)}) = 0$$  
  \hspace{1cm} (19)

  where \( x^{(3)} \) is normalized axial coordinate and coefficients \( t_1, t_2, \ldots \) to be determined according to the intuition and experience of the designer. It means the necessity of some experience possessing. The value \( \delta(x^{(1)}, x^{(3)}) = 0 \) means that the body force vector is tangential to stream surface \( x^{(1)} = \text{const} \). This assumption can be omitted using method of successive approximation as presented in [5].

3. **The example of 3D computation**

A computation performed by the 2D model on the basis of assumed meridional view allows getting the skeleton surface of blading as shown in figure 3. The details of such computation have been presented in [6].
The open problem is the distribution of the thickness blade along the skeleton surface. Figure 4 presents the Göttingen 428 profile with skeleton line in the middle. Along the skeleton line the thickness can be distributed in a way defined by the distribution parameter (thickness coefficient). Along the height of blades the Göttingen 428 profile can be scaled in proportion to the length of chord.

Five cases of thickness distribution along an assumed constant skeleton were adopted (0.00, 0.25, 0.50, 0.75 and 1.00) to 3D computations as shown in figure 5. The profile with thickness coefficient 0.50 means that thickness was distributed symmetrically along skeleton (mean line). The pressure side of profile with thickness coefficient 0.00 covers the suction side with thickness coefficient 1.00.

It should be noted that 2D model is sensitive only to the thickness of blading (blockage factor) but irrelevant to the distribution parameter since no influence of circumferential direction is taken into account in this model.

Therefore the aim of 3D model computation was to establish the influence of distribution parameters upon the flow field behind the designed guide vane blade. Two factors are the matter of discussion: (1) the influence of finite number of blades and (2) the influence of thickness coefficient along the skeleton surface.

For a 3D computation the computational grid was created (example of surface mesh is shown in figure 6). The domain of calculation is much higher than domain of blading. It spreads ahead in about \( \sim 2d \) distance \((d \text{ is the diameter of blade})\).
distribution of axial velocity, mass flow rate: 235 kg/s, water density: 999.1 kg/m$^3$, dynamic viscosity: 0.00116 Pa s.

The examples of computation for are presented in the following figures. The only most characteristic figures are chosen for this presentation. The axial velocity at the front of cascade for different distribution parameter is shown in figure 7. Not too high difference one can distinguish between curves in figure 7 although the non-uniformity is noticeable (this will be commented below).

In figure 8 axial velocities at the outlet are presented. The higher blockage at the inner diameter causes the higher axial component of velocity at the root of blading in front and backside of guide vane blading. In spite of the fact that ahead of the guide vane distribution of flow components has been inserted as constant, redistribution occurred as a result of upstream influence of blading. The conclusion can be drawn for 2D model – the axial velocity ought to have characteristic inclination visible in figure 7.

The same conclusion can be drawn from figure 9 for the pressure distribution ahead of blading. For the same blockage factor pressure configuration along the height of blading depends on thickness distribution coefficient. The mean value of pressure ahead of cascade is shown in figure 10. Pressure distribution behind guide vanes cascade is shown in figure 11.

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Figure 7. Radius vs. axial velocity at the inlet of guide vanes cascade for 5 different thickness distributions used for 3D computations.

Figure 8. Radius vs. axial velocity at the outlet of guide vanes cascade for 5 different thickness distributions used for 3D computations.

Figure 9. Radius vs. pressure at the inlet of guide vanes cascade for 5 different thickness distributions used for 3D computations.
One can notice deeper expansion at the root of blading. It is also worth noting that the influence of thickness distribution coefficient is not very high upon the pressure distribution behind the cascade. The reason for this fact comes from applied boundary condition for the pressure introduced at the end of flow domain (behind the cascade).

The deeper static pressure at the root is the reason for redistribution of axial velocity between inlet to computation domain and inlet to the guide vane cascade. As it is shown in figure 7 the axial velocity at the root is higher than at the outer diameter. But at the inlet to computation domain uniform velocity was introduced. In front of the computational domain redistribution of mass flow rate occurred by inducing radial component of velocity. Figure 12 presents the radial component of velocity at the inlet to cascade generated by a deeper expansion at the root of blading.

**Figure 10.** Mean pressure vs. thickness coefficient at the inlet of guide vanes cascade for 5 different thickness distributions used for 3D computations.

**Figure 11.** Radius vs. pressure at the outlet of guide vanes cascade for 5 different thickness distributions used for 3D computations.

**Figure 12.** Radius vs. radial velocity at the inlet of guide vanes cascade for 5 different thickness distributions used for 3D computations.

### 4. Reference to 2D computational results

The main difference between 2D and 3D models consist in the fact of formal difference of the type of differential equations. The governing equations of 2D model are of hyperbolic type where the information spreads along the characteristics [10]. 3D model represented by governing equations has the ability of spreading the information in the whole domain of computation. This is the essential reason, which influences the results of computation. Due to this fact 2D model is not able to predict the change of parameters ahead of blading. The boundary conditions for both models ought to be formulated according to the character of governing equations.

The aim of guide vane blading is to generate angular momentum at the entrance of a runner at the lowest losses. This means the generation of suitable outlet angle at the exit of blading. The value of this angle results already in 2D model of computing. It is very important to know the correction of exit
angle between 2D and 3D models. On the base of presented results the correction is shown in figure 13.

![Figure 13. Outlet angle vs. thickness distribution at the inlet of guide vanes cascade for 5 different thickness distributions used for 3D computations.]

Having in mind such a value of difference shown in figure 13 the designer ought to aim when performing 2D inverse computation at lower value of outlet angle from blading. The finite number of blading in 3D computation causes the increase of outlet angle.

5. Conclusions

The design procedure of a non-standardized low head hydraulic turbine stage can be supported by the chain of typical flow models like 1D, 2D and 3D. Each of the models plays a specific role. Here the main accent was paid to the relation between 3D and 2D models:

1. 2D inverse problem serves for the design of cascade blading.
2. For the formulation of proper boundary condition for 2D inverse computation it is important to take into account the character of parameters distribution in front of cascade.
3. Non-uniformity of velocity and pressure distribution ahead of guide vanes cascade appears in a 3D direct problem computation.
4. The most important conclusion from presented comparison concerns the difference of the outlet angles between 2D and 3D models due to the finite number of blades in guide vanes cascade.

6. References

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