Theoretical analysis and numerical simulation of square honeycombs

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Abstract. Aluminum honeycomb is widely used in energy absorption field for its special structure and excellent performance. The three-dimensional finite element models of aluminum honeycombs are developed in order to explore the mechanical behaviors under out-of-plane and in-plane compression. The simulation method is validated based on the experimental results of hexagonal honeycombs. Based on the simplified super folding element theory, we study the platform stress of square honeycomb under out-of-plane and in-plane compression. The theoretical formulas of platform stress under two conditions are provided in this study. Finally, compared with the simulation results of square honeycomb compression, the maximum error between the simulation and the theory for the platform stress under out-of-plane and in-plane compressions is less than 10%, and 15%, respectively. Both the results predicted by the theoretical formulas show high precision.

1. Introduction

As for its excellent physical properties such as high ratio of stiffness and strength, aluminum honeycombs have been widely used in aircrafts, ships and vehicles and other fields. In the process of compression, the collapse platform of aluminum honeycomb will absorb most of the energy, which is often the focus of researchers, as it determines the energy dissipated. In order to evaluate the energy absorption performance of aluminum honeycombs, the average crushing strength is an important evaluation index. In order to develop the theoretical model of predicting the average strength, a lot of research has been carried out.

There are many studies on the mechanical and physical properties of honeycombs under out-of-plane compression. For instance, McFarland [1] developed a semi-empirical model for studying the average crushing strength of hexagonal honeycomb under axial compression load. Gibson and Ashby [2] proposed another theoretical model to study the average crushing strength, which was verified by experimental data. Wierzbicki [3] studied the crushing behavior of hexagonal honeycomb under quasi-static axial load and established the general folding element composed of toroidal, trapezoidal, conical and cylindrical surfaces with moving hinge lines. This study obtained the formula for the out-of-plane average stress of single and double wall thickness hexagonal honeycombs, which makes a great contribution to the study of the mechanical properties of honeycomb structures. The super folding theory was simplified by Chen and Wierzbicki [4]. Bai et al. [5] and Zhang et al. [6] used this simplified model
to successfully study the average crushing strength of hexagonal and square multi-cell thin-walled structures. The analytical solution shows an excellent agreement with the numerical result. On the basis of the validation on the simulation modeling method, Yang et al [7] proposed a new novel bio-inspired aluminum honeycomb to improve the loading-carrying capacity of the traditional honeycomb. Xu et al [8] used Hypermesh and Pamcrash softwares to establish the finite element model of aluminum honeycombs and conduct simulation under out-of-plane compression. The results of the simulation were verified against the experimental results.

Some researchers have studied the mechanical properties under in-plane compression. Liu et al [9] studied the in-plane dynamic crushing behavior of triangular honeycombs and square honeycombs and disclosed the internal relationship between the dynamic responses and the cell configuration. Hu et al [10] studied the mechanical behavior of hexagonal aluminum honeycomb with different cell wall angles by using simulation methods. The results show that the geometric factors of cells play a leading role in the load-carrying capacity of aluminum honeycombs. Sun et al [11] analyzed the influence of velocity on the impact performance of aluminum honeycomb by simulation.

Compared with other honeycomb configurations, hexagon honeycomb is widely used because of its mature manufacturing technology. Therefore, most of the above theoretical studies are focused on hexagon honeycomb, but few on other honeycomb configurations, such as square honeycomb. In this study, the theoretical analysis and numerical simulation of energy absorption characteristics of square aluminum honeycomb are investigated, which can provide some references for the design of energy-absorbing device with aluminum honeycomb.

2. Numerical simulation of aluminum honeycombs under compression

2.1. Finite element model of Square honeycombs

Due to the large plastic deformation of honeycomb during axial dynamic compression, in this study, we establish a hexagonal aluminum honeycomb model based on HyperMesh and LS-DYNA platform for simulation analysis. The model is meshed by 4-node shell elements with 5 integration points through the cell wall thickness. In order to determine the appropriate element size, a convergence study was performed among different element sizes. The element size is 0.3 mm finally. The finite element model has 695580 nodes and 694888 elements. As shown in figure 1, the hexagonal honeycomb was located between two flat rigid plates. In compression, the bottom rigid plate was constrained by fixing all the degrees of freedom. The upper rigid plate has only one degree of freedom in axial direction. To simulate the deformation of the honeycomb under axial compression, it moves downwards at a prescribed velocity. In order to prevent the penetration of the model surface, automatic self-contacts were defined among the cell walls of the honeycomb. In addition, the dynamic friction coefficient for the contact of the honeycomb walls was set to be 0.1; and the dynamic friction coefficient between the honeycomb and the rigid plate was assigned 0.3.

![Figure 1. The FE model of the hexagonal honeycomb.](image_url)
The rigid plate is assigned with MAT_20, and the honeycomb material is simulated with MAT_24. The mechanical parameters are obtained from the test of Al3003 honeycomb matrix material. As shown in figure 2 and table 1, the mechanical parameters of honeycomb were obtained from the test of Al3003 honeycomb matrix material.

![Figure 2. The experimental results of matrix material.](image)

**Table 1.** Material parameters of the aluminum honeycomb.

|                | Young’s modulus E (GPa) | Density $\rho$ (kg/m$^3$) | Poisson’s ratio $\nu$ | Yield stress $\sigma_y$ (MPa) | ultimate stress $\sigma_u$ |
|----------------|-------------------------|---------------------------|-----------------------|-------------------------------|-----------------------------|
| Al3003         | 70                      | 2700                      | 0.35                  | 200                           | 228                         |

2.2. *Compression test on honeycombs*

The honeycomb specimen is obtained by the wire-electrode cutting method, and its relative density is 0.27. Specimens employed in tests were designed as cube with height of 77 mm and cross-section of $77 \times 77$mm$^2$. The specimen contains at least 21 complete cells to eliminate the size effect for the purpose that the specimen can represent the real mechanical properties of the whole honeycomb. The geometric configuration of a cell and sample honeycomb structure are shown in figure 3. The short side length $c$ is 6mm, the long side length $l$ is 10.4mm, the cell expanding angle $\alpha$ is 120° and the wall thickness $t$ is 0.045mm.

![Figure 3. The geometric configuration of a cell and compression experiment.](image)

In this study, we have carried out honeycomb compression tests under different strain rates. The quasi-static compression stress-strain curve and the picture of honeycomb after compressed are shown in figure 4. It is seen that the collapse process of honeycombs obviously shows three stages: linear elasticity, platform section and densification.
2.3. Validation of the finite element models

In order to validate the finite element model of honeycomb, the numerical results are compared with experimental results. Considering the calculation cost, the compression condition with a strain rate of 10/s is simulated in this section. The modeling technique is the same as the description in Section 2.1. In addition, the size and material parameters of this model are consistent with those mentioned above.

Figure 5(a) exhibits the compressive stress-strain curves between the simulation and the experimental test. At this strain rate, the speed is relatively fast. In order to ensure the safety of the testing machine, the displacement of the test is smaller than that of the simulation. It is obvious that the experimental results show good consistency with the numerical results. The value of platform stress is fairly close.

In order to further verify the FE model, this study also compares the simulation and experimental results of the honeycomb in the literature [12]. In the literature, the cell of honeycomb is a regular hexagon with side length of 3mm and wall thickness of 0.06mm. The material parameters and external dimensions of honeycomb specimens are also consistent with those described in the literature [12]. Figure 5(b) exhibits the results of stress-strain curves between the simulation and experimental test at the compression rate of 28m/s. It is also obvious that the experimental results show good consistency with the numerical results. In addition, the deformation process of the honeycomb is consistent with the experiment. Therefore, the finite element model is reliable enough.

![Comparison of the platform stress between the experiment and simulation](image1)

![Comparison of the platform stress between the experiment data in literature [12] and simulation](image2)

Figure 5. Comparison of experimental and simulation results.
3. Theoretical analysis of the mechanical behavior of square honeycomb

3.1. Theoretical model of the platform stress under out-of-plane compression

As mentioned in section 1.2, the honeycomb compression process is obviously divided into three sections, in which the platform section absorbs energy through plastic collapse deformation, which is an important stage of honeycomb deformation. The platform stress $\sigma_m$ is an important index to characterize the energy absorption capacity of honeycomb structure.

According to the simplified super folding element theory [4], the deformation energy of folding element is composed of two parts: bending hinge and membrane deformation energy absorption. The simplified basic folding element is shown in figure 6.

![Figure 6. The simplified basic folding element.](image)

The basic folding element of the square honeycomb under out-of-plane compression can be simplified as a cross cell composed of two cell walls with thickness of $t$, length of $l$ and height of $2H$, as shown in figure 7.

![Figure 7. Cross cell of square honeycomb.](image)

For a complete collapse process of a basic folding element, according to the energy equilibrium of the system, the external work done by compression will be dissipated by plastic deformation in bending and membrane as equation (1).

$$\kappa \cdot P_m \cdot 2H = E_b + E_m$$

(1)

where $H$ is half of the folding wavelength, and $P_m$ is the average crushing force. $E_b$ and $E_m$ are the bending energy and the membrane energy, respectively. In fact, an actual folding element can never be completely flattened. $\kappa$ is effective compression stroke coefficient, whose value is taken as 75% in the following derivation.

The bending energy $E_b$ can be determined by calculating the energy dissipation at stationary hinge lines. 3 horizontal stationary hinge lines are involved for each flange. As shown in figure 6(a), the three
horizontal stationary hinge lines are located on the half-fold line and both sides of the basic folding element respectively, in which the corresponding rotation angles are $\pi$, $\pi/2$ and $\pi/2$. The bending energy absorption can be expressed as:

$$E_b = 4 \sum_{i=1}^{3} M_0 \theta_i c$$  \hspace{1cm} (2)

where $M_0 = \sigma_0 t^2/4$ is the full plastic bending moment; $\theta_i$ is the rotation angle at each hinge line and $c$ denotes the length of each hinge line. $\sigma_0$ is the effective flow stress of the matrix material, which can be calculated by the following equation:

$$\sigma_0 = \frac{\sigma_y \sigma_u}{1 + n}$$  \hspace{1cm} (3)

where $\sigma_y$ and $\sigma_u$ denote the yield stress and the ultimate stress of the matrix material, respectively; and $n$ is the strain hardening exponent. This value is taken as 0.07 in the following derivation.

The membrane energy $E_m$ dissipated during one wavelength crushing can be evaluated by integrating the extensional and compressional area (shade area in figure 6(b)).

$$E_m = 2 \int_A (\sigma_n t) \, ds$$  \hspace{1cm} (4)

where $A$ is the shade area. Considering the uncertainty of the folding process, here we add a coefficient between plastic region area $S$ and $H^2$ as $\gamma[5]$. By substituting it into the calculation, the membrane energy $E_m$ of a cross cell is: $E_m = 2\sigma_0 tyH^2$. The average crushing force $P_m$ can be calculated by the following equation:

$$P_m = 8M_0 \pi l + 4\sigma_0 tyH^2$$  \hspace{1cm} (5)

The wavelength $H$ can be determined according to the stationary condition of the mean crushing force as equation (6).

$$\delta P_m / \delta H = 0$$  \hspace{1cm} (6)

$$H = \sqrt{\frac{2M_0 \pi l}{\sigma_0 ty}}$$ can be obtained. The mean crushing force for a cross cell can be calculated by equation (7) by substituting $H$ into equation (5).

$$P_m = 3.8\sigma_0 \sqrt{l^3}$$  \hspace{1cm} (7)

As shown in equation (8), the platform stress $\sigma_m$ can be obtained by dividing $P_m$ by the area A.

$$\sigma_m = P_m / l^2 = 3.8\sigma_0 \left(\frac{l}{T}\right)^{\frac{3}{2}}$$  \hspace{1cm} (8)

This theoretical model is a simplified model based on the super folding element theory and elastic buckling failure theory. The influence of the local buckling of the honeycomb was not considered, which should be further investigated in future.

3.2. Theoretical model of the platform stress under in -plane compression

The platform stress is determined by cell failure mechanism. Under in-plane compression, for a small $t/l$ value, the cell wall buckles elastically. The following is the derivation of the platform stress $\sigma_e$ according to the elastic buckling failure theory. Under in-plane compression, the square honeycomb can be simplified as an I-shape unit, as shown in figure 8.
When the external stress is $\sigma_1$, it can be obtained from the stress balance that the force in vertical cell wall is

$$T = \sigma_1 lh$$  \hspace{1cm} (9)

When elastic buckling occurs, the Euler buckling load of a compression bar is:

$$P_{cr} = \frac{n^2 \pi^2 EI}{l^2}$$  \hspace{1cm} (10)

where $I$ is moment of inertia, for the vertical cell wall, $I = ht^3/12$. $n$ is the end constraint factor. For square honeycombs, this value is taken as one in the following derivation.

By substituting equation (9) into equation (10), the elastic buckling load of square honeycombs along the cell wall direction was calculated by equation (11).

$$\sigma^*_e = \frac{n^2}{12} \left( \frac{t}{l} \right)^3$$  \hspace{1cm} (11)

4. Comparison between simulation and theoretical models

In order to verify the theoretical model in Section 3, the simulation method described in Section 1 was adopted to simulate the compression of square honeycomb in out-of-plane and in-plane directions. In order to ensure the reliability of the verification, the aluminum honeycombs with different configurations and parameters under compression were simulated by the finite element model and computed by the theoretical model. The comparison results in the platform stress between the simulation and theoretical results under out-of-plane and in-plane compression are listed in Table 2 and Table 3, respectively. It is seen that the relative error is fairly small.

**Table 2.** Comparison of the platform stress between simulation and theoretical model under out-of-plane compression.

| Wall thickness $t$ (mm) | Side length $l$ (mm) | Wall thickness-length ratio $t/l$ | Simulation results (MPa) | Theoretical results (MPa) | Relative error % |
|------------------------|----------------------|----------------------------------|--------------------------|---------------------------|-----------------|
| 0.06                   | 6                    | 0.01                             | 0.86                     | 0.79                      | 8.1             |
| 0.1                    | 6                    | 0.167                            | 1.8                      | 1.7                       | 5.5             |
| 0.08                   | 8                    | 0.01                             | 0.82                     | 0.79                      | 3.6             |
| 0.1                    | 8                    | 0.0125                           | 1.15                     | 1.1                       | 3.3             |
| 0.075                  | 9.67                 | 0.00775                          | 0.56                     | 0.54                      | 3.6             |
| 0.12                   | 9.67                 | 0.0124                           | 1.11                     | 1.08                      | 2.7             |
Table 3. Comparison of the platform stress between simulation and theoretical model under in-plane compression.

| Wall thickness \(t\) (mm) | Side length \(l\) (mm) | Wall thickness-length ratio \(t/l\) | Simulation results (MPa) | Theoretical results (MPa) | Relative error % |
|--------------------------|------------------------|-----------------------------------|--------------------------|---------------------------|------------------|
| 0.06                     | 6                      | 0.01                              | 0.036                    | 0.037                     | -2.7             |
| 0.08                     | 8                      | 0.01                              | 0.039                    | 0.037                     | 5.1              |
| 0.1                      | 8                      | 0.0125                            | 0.071                    | 0.072                     | -1.4             |
| 0.075                    | 9.67                   | 0.00775                           | 0.02                     | 0.017                     | 15               |
| 0.12                     | 9.67                   | 0.0124                            | 0.063                    | 0.07                      | -11.1            |

5. Conclusion
Based on the simplified super folding element theory, we study the platform stress of square honeycomb under out-of-plane and in-plane compression. The theoretical formulas of platform stress under two conditions are given in this study. The finite element models of the aluminum square honeycombs are developed to explore its mechanical behavior. The simulation results are validated against compression experiments. The aluminum honeycombs with different configurations and parameters under compression were simulated by the finite element model and computed by the theoretical model. The maximum error of the platform stress under out-of-plane compression is less than 10%, and that under in-plane compression is less than 15%. It shows that both theoretical models exhibit high precision. This study can provide some references and guidance for the design of energy-absorbing device with aluminum honeycomb.

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References
[1] McFarland R K 1963 Hexagonal cell structures under post-buckling axial load AIAA Journal 1(6) 1380-5
[2] Gibson L J and Ashby M F 1997 Cellular solid: Structure and properties 2nd ed. London: Cambridge University Press
[3] Wierzbicki T 1983 Crushing analysis of metal honeycombs Int. J. Impact Eng. 1(2) 157-74
[4] Chen W and Wierzbicki T 2001 Relative merits of single-cell, multi-cell and foam-filled thin-walled structures in energy absorption Thin Wall. Struct. 39(4) 287-306
[5] Bai Z, Guo H, Jiang B, Zhu F and Cao L 2014 A study on the mean crushing strength of hexagonal multi-cell thin-walled structures Thin Wall. Struct. 80 38-45
[6] Zhang X, Cheng G and Zhang H 2006 Theoretical prediction and numerical simulation of multi-cell square thin-walled structures Thin Wall. Struct. 44(11) 1185-91.
[7] Yang X, Sun Y, Yang J and Pan Q 2018 Out-of-plane crashworthiness analysis of bio-inspired aluminum honeycomb patterned with horseshoe mesostructure Thin Wall. Struct. 125 1-11.
[8] Xu T S, Sun Y B, MA S Q, Li M W and Bi C 2019 Simulation and verification of aluminum honeycomb out-of-Plane crushing behavior based on pamcrash Mech. Res. Appl. 032(003) 66-69.
[9] Liu Y and Zhang X C 2009 The influence of cell micro-topology on the in-plane dynamic crushing of honeycombs Int. J. Impact Eng. 36(1) 98-109.
[10] Hu L and You F 2012 Dynamic mechanical properties of aluminum honeycomb and its effect factors Explosion and Shock Waves 32(01) 26-31.
[11] Sun D Q and Zhang W H 2008 In-plane impact properties of aluminum double-walled honeycomb cores J. Vib. Shock (07) 69-74.
[12] Tan S, Hou B, Li Y and Zhao H 2015 Effect of base materials on the dynamic enhancement of aluminium honeycombs Explo. Shock Waves 35(01) 16-21.