Twisting and buckling: A new undulation mechanism for artificial swimmers

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Abstract. Among the various locomotion strategies of the animal kingdom, the undulation locomotion is of particular interest for biomimetic applications. In this paper, we present an artificial swimmer set into motion by a new and non-trivial undulation mechanism, based on the twisting and buckling of its body. The swimmer consists of a long cylinder of ferrogel which is polarized transversely and in opposite directions at each extremity. When it is placed on a water film and submitted to a transverse oscillating magnetic field, the worm-like swimmer undulates and swims. Whereas symmetry breaking is due to the field gradient, the undulations of the worm result from a torsional buckling instability as the polarized ends tend to align with the applied magnetic field. The critical magnetic field above which buckling and subsequent swimming is observed may be predicted using elasticity equations including the effect of the magnetic torque. As the length of the worm is varied, several undulation modes are observed which are in good agreement with the bending modes of an elastic rod with free ends.

Introduction

The design of robots that mimic animal locomotion has gained in interest during the last two decades, both for their medical applications as well as for exploration tasks or search and rescue missions [1,2]. Among the various locomotion strategies of animals, the undulation locomotion is of particular interest for its simplicity and robustness, even on rough terrain. The locomotion of snakes, eels or worms have been particularly studied from fundamental as well as from biomimetic points of view [3–6] and robots inspired by their displacement methods have been proposed [2,7]. To wirelessly power and control such robots, smart materials have been developed, often actuated by magnetic fields [8,9]. Beyond the technological challenge, such swimmers also raise a more fundamental interest on the description of the non-reversible flow around the body and the prediction of the propulsive force [10–15] (see [16] for a review). The undulations of artificial swimmers are generally obtained either by oscillating a magnetic tail attached to a non-magnetic head [8,9] or by oscillating a magnetic head attached to a flexible tail [1,17].

In this paper, we present a new and non-trivial undulation mechanism that is based on the torsional buckling instability of a worm-like swimmer under an oscillating magnetic field. The experiments are conducted in an unusual intermediate regime where both inertial and viscous effects coexist.

Methods

The swimmer consists of a flexible cylinder of PVA (poly-(vinyl-alcohol), Sigma-Aldrich) based ferrogel, reticulated with glutaraldehyde (Sigma-Aldrich). Micrometric ferromagnetic particles (black iron oxide, 10\%w/w) are embedded in the gel, which has a Young modulus $E = 1.36 \text{kPa}$ (measured by indentation on a TA-XT2 texturometer). Before reticulation, the gel is poured into a cylindrical mold, with diameter $2R = 1 \text{mm}$ and length $10 < L < 50 \text{mm}$. Once the gelation is achieved, the gel is taken out of the mold and rinsed with water to remove free PVA chains. In order to provide permanent magnetic properties to the swimmer, a magnet is then placed at each of its extremities, with opposite polarization directions. Over a distance $a = 5 \text{mm}$ from each extremity, the embedded particles acquire a permanent magnetization density $M$, oriented perpendicularly to the main axis of the cylinder, with opposite direction,
Fig. 1. Overview of the experiments. (a) Sketch of the magnetic worm-like swimmer: the colored zones represent the polarized regions. (b) Experimental set-up: a flexible magnetic worm is placed at the surface of water, between two coils in Helmholtz configuration, generating a vertical magnetic field $B$. (c) Top view of the swimmer under an oscillating magnetic field ($B = 50 \text{ G}$, $f = 1 \text{ Hz}$). The images are thresholded for better visualization. The time step between successive images is 0.33 s, and the length $L$ of the swimmer is 35 mm.

see fig. 1(a). The magnetization was measured according to Foner's method [18] and equals 1.5 G.

The magnetic worm-like swimmer was then deposited at the surface of water in a Petri dish, set between two coils in Helmholtz configuration (fig. 1(b)). When we supplied with AC, the coils generated a vertical oscillating magnetic field, with adjustable magnitude $B$ and frequency $f$. The motion of the swimmer was video-recorded from above. The images were thresholded, and the shape and displacement of the worm were analysed with Matlab (Mathworks).

Results

Submitted to a constant magnetic field (i.e. for $f = 0 \text{ Hz}$), the worm drifted slowly from the center to the edge of the Petri dish, without deformation (Movie 1). The displacement velocity $V$ was very small ($\approx 0.2 \text{ mm} \cdot \text{s}^{-1}$) and proportional to the magnitude of the magnetic field $B$, fig. 2. The motion was due to the existence of a gradient of the horizontal component of the magnetic field in the radial direction, which increases by 30% from the center to the edge of the dish. A magnetic force, proportional to the field gradient, and thus to the field magnitude, acted on all the ferromagnetic particles of the worm and pushed it away from the center of the Petri dish.

Under an oscillating magnetic field, the worm undulated (fig. 1(c)) and swam toward the edge of the dish with a much higher velocity, see Movie 2. This only occurred above a critical value of the applied magnetic field, $B_c$ (fig. 2). Below this value, the swimmer drifted slowly without undulating, just as when subjected to a constant magnetic field.

In order to investigate the origin of the undulation, white dots were drawn along the worm body to serve as labels. The dots allowed to highlight the full 3D deformation of the swimmer and revealed that the worm actually twisted around its main axis when it undulated, see Movie 3. The magnetic moments of the extremities tended to align with the applied vertical magnetic field, and since they were polarized in opposite direction, this resulted in the twist of the ferrogel cylinder. Above a certain threshold, twisting of an elastic rod is known to lead to its buckling through an elastic instability, a phenomenon known as the Greenhill problem [19]: a twisted elastic rod buckles into a 3D helix. Note that as the magnetic swimmer was here constrained in the plane of the free surface of the liquid by surface tension, the deformed shape of the worm was a confined helix, a quasi-sinusoid.

Based on the observation of the twisting of the worm body, we assume that twisting leads to the buckling of the worm body and, as the magnetic field is oscillated in time, twisting occurs alternatively clockwise and counterclockwise, generating a bending wave. Because the applied magnetic field is non-homogeneous, both extremities of the swimmers are subjected to different twisting torques which breaks the head-tail symmetry and produces a propagative wave allowing the swimmer to go forward. The propagative nature of the wave is thus due to the radial gradient of the magnetic field which is here inherent to our experimental set-up. Another way of breaking the head/tail symmetry is to unbalance the magnetizations of the extremities, see Movie 4.
We now attempt to describe more precisely the undulation and swimming mechanism of the magnetic swimmer. Let us first focus on the threshold over which the undulations of the swimmer occur.

**Threshold of the instability**

The critical field $B_c$ above which the undulations of the worm occur as a function of its length $L$ ($f = 1$ Hz). Dashed line: fit of the data, of the form $B_c = 649/L$ (with $B_c$ in G and $L$ in mm). Insert: variation of velocity beyond the threshold with the applied magnetic field as a function of the length of the swimmer (dotted line: linear fit).

Fig. 3. Threshold of the instability. Critical field $B_c$ above which undulations of the swimmer occur as a function of its length $L$ ($f = 1$ Hz). Dashed line: fit of the data, of the form $B_c = 649/L$ (with $B_c$ in G and $L$ in mm). Insert: variation of velocity beyond the threshold with the applied magnetic field as a function of the length of the swimmer (dotted line: linear fit).

The swimmer is represented as an elastic rod whose shape $r(l) = (x(l), y(l), l)$ is given in the frame of material vectors $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, with $\mathbf{e}_3 = d\mathbf{r}/dl$ a tangent vector to the centerline of a rod, and $\mathbf{e}_1$ along one of the magnetic moments at the ends of the rod, see fig. 4. Apart from the magnetic torque, the balance of the twisting torque along the rod is given by the Kirchhoff model [20, 21] and reads

$$\frac{dT}{dl} + \mathbf{e}_3 \times \mathbf{F} + M \mathbf{e}_1 \times \mathbf{B} = 0,$$

where

$$\mathbf{T} = C \Omega_3 \mathbf{e}_3 + EI \frac{d\mathbf{r}}{dl} \times \frac{d^2\mathbf{r}}{dl^2}$$

and

$$\mathbf{F} = -EI \frac{d^3\mathbf{r}}{dl^3} + C \Omega_3 \frac{d\mathbf{r}}{dl} \times \frac{d^2\mathbf{r}}{dl^2} + A \frac{d\mathbf{r}}{dl}$$

are the torque and force acting on a cross-section of the rod. Here, $EI$ and $C$ are the bending and twist elastic modulus, $\Omega_3$ the twist angle along the rod, $\bar{M}$ the magnetic moment per unit length and $\mathbf{B}$ the vertical magnetic field. The term with $A$ enforces the inextensibility of the rod and is negligible in the case of small deformations. Introducing eqs. (2) and (3) into eq. (1) and projecting along $\mathbf{e}_3$ gives

$$C \frac{d\Omega_3}{dl} = -\bar{M} \mathbf{e}_2 \cdot \mathbf{B}.$$  

(4)

In the case where the rod is magnetized only at its extremities, $l = \pm L/2$, with total magnetic moment $\mathbf{m} = \pm m \mathbf{e}_1$, eq. (4) gives the twist of the rod

$$\Omega_3 = -\frac{m \mathbf{e}_2 \cdot \mathbf{B}}{C}.$$  

(5)

The force balance along the rod, $d\mathbf{F}/dl = 0$, projected along $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively, gives

$$-EI \frac{d^4x}{dl^4} - C \Omega_3 \frac{d^3y}{dl^3} = 0,$$

$$-EI \frac{d^4y}{dl^4} + C \Omega_3 \frac{d^3x}{dl^3} - ky = 0.$$  

(6)

In eq. (7), we have introduced the term $-ky$ accounting for the confinement of the rod at the surface of the liquid due to surface tension. According to the theorem of Keller [22] (which states that the total vertical force on the submerged body is given by the weight of liquid displaced by the body and the meniscus), the stiffness $k$ may be expressed in the static case as $k = 2 \rho g l_c$ with $l_c$ the capillary length and $\rho$ the density of the liquid. In the case of our experiments, as the rod is oscillated in time at 1 Hz, the thickness of the viscous layer, $\delta = \sqrt{\eta/\rho \omega}$, becomes very small compared to the capillary length. This means that the meniscus is not able to follow the shear induced by the oscillation of the rod, and driving of the fluid occurs over a distance $\delta$ rather than $l_c$. It is therefore more appropriate to use the thickness of the viscous layer in the expression of the stiffness of the interface, and we use $k = 2 \rho l g \delta$ in the following.

Integrating eq. (6) once and injecting its solution in (7) gives

$$-EI \frac{d^4y}{dl^4} - \left(\frac{C \Omega_3}{EI} \frac{d^3y}{dl^3} - ky = 0.\right.$$  

(8)

Fig. 4. Notations used in the model: $\mathbf{T}$ and $\mathbf{F}$ are the twisting torque and force acting on a cross-section of the rod, $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ are a frame of unit vectors attached to the rod with $\mathbf{e}_3$ tangent to the centerline of the rod and $\Omega_3$ is the twist angle.
In order to find the critical magnetic field necessary for the buckling of the rod, we seek for a non-trivial solution, i.e. different from \( x = y = 0 \), in the form of

\[
y = \sum_{j=1}^{4} D_j \exp \left( i \alpha_j l \right),
\]

which leads to the polynomial equation

\[
\alpha^4 - \left( \frac{C \Omega_0}{EI} \right)^2 \alpha^2 + \frac{k}{EI} = 0.
\]

The solutions are

\[
\alpha_{1,2} = \sqrt{\frac{\chi^2 \pm \sqrt{\chi^4 - kL^4/4EI}}{2}}, \quad \alpha_{3,4} = -\alpha_{1,2},
\]

with \( \chi = C \Omega_0 L/2EI \). In eq. (11) and in the following, the coordinate \( l \) has been normalized by the semi-length of the swimmer, \( L/2 \). The boundary conditions, zero bending torque \( d^2 x/dl^2 = d^2 y/dl^2 = 0 \) and zero force in the \( x \) direction \( d^2 x/dl^2 = dy/dl = 0 \) at the extremities \( l = \pm 1 \), give the following equations:

\[
\alpha_1 \tan \alpha_1 = \alpha_2 \tan \alpha_2, \quad \alpha_1 \tan \alpha_2 = \alpha_2 \tan \alpha_1,
\]

for even \( (x(l) = x(-l); y(l) = y(-l)) \) and odd \( (x(l) = -x(-l); y(l) = y(-l)) \) modes, respectively. For the even modes, we impose the values of \( \alpha_1 \) and find the associated values of \( \alpha_2 \) by solving numerically eq. (12). We then derive the corresponding values of

\[
\chi = \sqrt{\alpha_1^2 + \alpha_2^2}
\]

and

\[
\frac{kL^4}{EI} = 4 \left( \chi^4 - \left( \chi^2 - 2 \alpha_1^2 \right) \right).
\]

Figure 5 shows the dependence of the critical parameter on the capillary elasticity parameter \( kL^4/EI \). The stiffness of the interface, \( k = 2 \pi g \phi \), may be estimated to 8 N m\(^{-2} \) for \( \phi = 0.4 \) mm at 1 Hz. The capillary elasticity parameter, \( kL^4/EI \), is thus of the order of 2.10\(^4 \) (with \( L = 2 \) cm, \( R = 0.5 \) mm, \( E = 1.36 \) kPa). This leads to a critical value of \( \chi \) for the first unstable even mode of \( \chi_c = 9 \), see fig. 5. Equation (5) together with the expression of \( \chi = C \Omega_0 L/2EI \) allow us to derive the critical magnetic field over which torsional buckling occurs

\[
B_c = \frac{2 \chi_c EI}{mL}.
\]

With \( I = \pi R^4/4 \) and \( m = M \pi R^2 a \), it is more appropriately rewritten as

\[
B_c = \frac{\chi_c ER^2}{2a M L}.
\]

The critical magnetic field is found to be inversely proportional to the length of the swimmer, in agreement with the experimental results (fig. 3). The computation of \( B_c \) versus \( L \) with the above equation gives \( B_c = 2040/L \) whereas the experimental data fit with \( B_c = 649/L \) \((B_c \text{ in G, } L \text{ in mm}) \); our model overestimates by a factor of 3 the value of the instability threshold but the scaling and the order of magnitude are correct. The discrepancy might be due to a poor estimation of the Young modulus of the gel (which was not measured directly on the elastic rod), affecting the value of \( \chi_c \) derived from that of \( kL^4/EI \) (fig. 5) as well as the calculation \( B_c \) with eq. (17).

The model confirms our hypothesis that the twisting of the worm due to the interaction of its polarized extremities with the applied magnetic field is at the origin of the undulation of the swimmer. Note that the scaling of the undulation threshold, \( B_c \sim R^2/aL \), is favorable to miniaturization since it is constant when all the dimensions of the swimmer are divided by the same factor.

The full analysis of the non-linear dynamics of the swimmer, governed by eqs. (5), (6), (7) will be given elsewhere.

### Swimming dynamics

We have shown that a torsional buckling instability is at the origin of the bending wave animating the swimmer. We now focus on the swimming dynamics.

Above the critical field \( B_c \), the swimming velocity of the worm increases with the applied field, with a slope twenty times larger than below \( B_c \) (fig. 2). This increase is associated with a rise in the body curvature with \( B \), fig. 2 (insert). As the total elastic energy (~ \( EI/L \)) is inversely proportional to its length, this effect is all the more pronounced as the length of the swimmer is larger (fig. 3, insert). The increase in the slope \( \Delta V/\Delta B \) with \( L \) is due to the addition of two effects: first, the decrease of the elastic energy of the rod with \( L \); second, the fact that the gradient of the magnetic field and thus the difference of magnetic torque applied at both extremities (at the origin
of the symmetry breaking) also increases with the length of the swimmer.

Interestingly, as the length of the swimmer is increased (for a fixed value of $B$ and $f$), different undulation modes are observed and characterized by $n$, the number of antinodes, fig. 6. These successive modes of deformation are associated with an increasing curvature for increasing $L$. Note that modes 2 and 3 cannot be explained by a static twisting-buckling instability as they would require a twisting angle larger than $\pi$ which, in practice, is impossible here: if the magnetic moments of the extremities get aligned with the applied field (which happens for a torsion angle of $\pi$), the magnetic torque cancels and no further rotation can happen. Another type of explanation has therefore to be sought through the dynamic behavior of the swimmer.

The bending amplitude $A$ and swimming velocity $V$ are measured as a function of the frequency of the applied magnetic field, for fixed values of $L$ and $B$, fig. 7. Both $A$ and $V$ increase approximately linearly with $f$ up to 5 Hz, where they reach a maximum, and then decrease. The curve of the amplitude displays a pronounced peak which resembles a resonance curve. The peak frequency may be compared with the free vibration frequencies of an elastic rod. Two main modes of vibrations could be relevant here: bending modes and torsion modes (or coupling between both). For bending modes, the boundary conditions for a rod with free extremities in mode 2 (the mode observed at all frequencies in fig. 7) impose $qL = 7.85$, with $q$ the wave number. Using the dispersion relation $\omega = \beta q^2$ ($\beta = \sqrt{EI/(\rho\pi R^4)}$, with $\rho = 1050$ kg m$^{-3}$ the mass per unit volume of the rod), the eigenfrequency is found as [23]

$$f_{n=2} = \frac{9.82/\beta}{L^2}.$$  
(18)

Using the previously mentioned values of the parameters (and $L = 29$ mm), $f_{n=2}$ is here equal to 3.44 Hz, close to the experimental peak frequency at 5 Hz. The eigenfrequency of a torsion rod, for $n = 2$, is of the order of 230 Hz: the torsion modes alone are therefore not relevant to account for the dynamics of the magnetic swimmer. However, coupling between the bending modes (which are predominant) and the torsional modes is needed to explain that the peak frequency is higher than the eigenfrequency of pure bending.

Note that similar resonance-like phenomena in the swimming velocity of artificial swimmers have been re-
ported [11, 24, 25] although the underlying physics is certainly different.

We now attempt to explain qualitatively the evolution of the swimming velocity with the frequency of the applied magnetic field under simplifying hypotheses. The cutting frequency between inertial and viscous regime, \( f_c = 2\eta/\pi R^2 \) (with \( \eta = 10^{-3} \text{ Pa s}^{-1} \) the viscosity of water), is here of the order of 2.6 Hz: over the frequency range studied here, we are therefore in a mixed regime where viscous and inertial effects coexist. As the progression of the worm happens at relatively low Reynolds numbers (from 0.3 to 2 and from 1 to 20, respectively for the longitudinal and transverse motion), we use the predictions of the swimming velocity in the small Reynolds number limit although the bending motion of the swimmer is mainly dominated by inertia. Balancing the longitudinal forces acting on a sinusoidal elongated worm submitted to purely viscous friction, allows to express the progression velocity as [5, 26]

\[
V = \frac{\omega}{q} \frac{\tilde{A}^2 q^2 (c - 1)}{(2 + \tilde{A}^2 q^2 (c - 1/2))}, \quad (19)
\]

with \( \tilde{A} \), \( \omega = 2\pi f \), and \( q \), respectively the amplitude, pulsation and wave number of the propagative wave, and \( c \) the ratio of the transverse to longitudinal friction coefficients (\( c = 2 \) for a long cylinder in bulk liquid [27]). The amplitude \( \tilde{A} \) to be considered in the above equation is the amplitude of the propagative component of the bending wave animating the worm: the observed wave is indeed mainly stationary, with a small propagative part. As the propagative nature of the wave results from the gradient of magnetic field (which is 18% along the swimmer’s body), we assume that the amplitude of the propagative component is 18% of the total measured amplitude at each frequency (this is of course a very rough estimate). Moreover, in the calculation of \( V \) with eq. (19), the values of \( k \) are obtained from the wavelength measured on the superposition of the successive shapes of the swimmer (as shown in fig. 6). The main mode of deformation is mode 2 for all frequencies up to 10 Hz, but a combination of modes 2 and 3 is observed above 15 Hz, leading to two possible values of the wave number in eq. (19) at high frequency (solid line and dashed line in fig. 7(b), see legend).

The calculation of \( V \) as a function of \( f \) with eq. (19) is compared to the experimental data on fig. 7(b) and shows a reasonable qualitative agreement, although the velocity is underestimated at high frequency. Considering the mode 3 above 15 Hz (dashed line) gives a slightly better agreement than considering the mode 2 for all frequencies (solid line). An alternative model has been developed, in which viscous friction is considered for the longitudinal motion and inertial friction for the transverse motion, without great influence on the quantitative results.

Because we are in a mixed regime where viscous and inertial effects coexist, an estimate of the swimming velocity in a low Reynolds number limit allows to recover the global evolution of the velocity with the frequency although inertial bending modes also account.

**Conclusion**

We have designed a novel magnetic swimmer that moves thanks to an original undulation mechanism. A torsional buckling instability due to the interaction of the polarized extremities with the applied magnetic field gives rise to a bending wave, and the asymmetry of the twisting torques breaks the head-tail symmetry, producing net motion. The swimming dynamics is then mainly governed by the bending modes of the equivalent elastic rod.

**Supplementary material**

**Movie 1**: Drift of the worm-like swimmer under a constant magnetic field (\( f = 0 \text{ Hz}, B = 83 \text{ G}, L = 29 \text{ mm}, 10 \text{ frames per second, swimming velocity: } 0.26 \text{ mm s}^{-1} \)). The motion is here due to the radial gradient of the applied magnetic field.

**Movie 2**: Undulatory swimming of the worm-like swimmer under an oscillating magnetic field (\( f = 1 \text{ Hz}, B = 50 \text{ G}, L = 35 \text{ mm}, 10 \text{ frames per second, swimming velocity: } 2.5 \text{ mm s}^{-1} \)).

**Movie 3**: Twisting of the swimmer. First part: white dots were drawn along the worm body, revealing that the worm twists around its main axis when swimming. Second part: twisting is visualized thanks to the out-of-plane movement of thin needles pinned into the swimmer’s extremities (\( f = 1 \text{ Hz}, B = 100 \text{ G}, L = 30 \text{ mm}, 10 \text{ frames per second} \)).

**Movie 4**: Breaking the head-tail symmetry: proof of concept. The upper worm is polarized symmetrically at both extremities (but in opposite sens) and swims in the radial direction: the symmetry breaking is due to the gradient of the applied magnetic field which is here inherent to the experiental set-up. The lower worm has a magnetization two times larger at its left than at its right (magnetization length: \( a_{\text{left}} = 2a_{\text{right}} = 1 \text{ mm} \)) and does not progress: in this case, the symmetry breaking due to the polarization opposes to the symmetry breaking due to radial gradient of magnetic field, supporting the idea that both methods for breaking the head-tail symmetry of the bending waves are possible (\( f = 5 \text{ Hz}, B = 50 \text{ G}, L = 26 \text{ mm}, 10 \text{ frames per second} \)).

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1 Note that the integration is here performed at second order at all steps, leading to a slightly different expression than the one in [5].
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