Detection Feasibility of Cluster-Induced CMB Polarization

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ABSTRACT

Galaxy clusters can potentially induce sub-$\mu$K polarization signals in the CMB with characteristic scales of a few arcminutes in nearby clusters. We explore four such polarization signals induced in a rich nearby cluster and calculate the likelihood for their detection by the currently operational SPTpol, advanced ACTpol, and the upcoming Simons Array. In our feasibility analysis we include instrumental noise, primordial CMB anisotropy, statistical thermal SZ cluster signal, and point source confusion, assuming a few percent of the nominal telescope observation time of each of the three projects. Our analysis indicates that the thermal SZ intensity can be easily mapped in rich nearby clusters, and that the kinematic SZ intensity can be measured with high statistical significance toward a fast moving nearby cluster. The detection of polarized SZ signals will be quite challenging, but could still be feasible towards several very rich nearby clusters with exceptionally high SZ intensity. The polarized SZ signal from a sample of $\sim 20$ clusters can be statistically detected at $S/N \sim 3$, if observed for several months.

Key words: galaxies: clusters: general – cosmic background radiation – polarization

1 INTRODUCTION

A wealth of information on the temperature anisotropy of the cosmic microwave background (CMB) and its E-mode polarization has been accumulated over the past two decades by multiple ground-based and balloon-borne telescopes, the WMAP, and Planck satellites. In particular, Planck cosmic-variance-limited measurements mapped the temperature anisotropy down to $\sim 4'$ scales, and the South Pole Telescope (SPT), and the Atacama Cosmology Telescope (ACT) have already mapped the CMB temperature across a sky patch down to the sub-arcminute resolution.

Numerous other ongoing ground-based CMB experiments, e.g. QUIET, POLARBEAR, and its successor – the Simons Array (SA) – will measure the CMB polarization to unprecedented precision on scales $\gtrsim 3'$. This effort continued with the upgrades of SPT and ACT – SPTpol and Advanced ACTpol (AdvACTpol) – that recently detected the CMB polarization on arcminute scales (e.g. Keisler et al. 2015, Louis et al. 2017b), making the detection of polarization signals produced in gravitationally-bound systems potentially feasible.

Temperature anisotropy in directions to galaxy clusters is induced by scattering of the CMB off moving electrons; random and ordered motions give rise to the thermal and (the typically weaker) kinematic components of the SZ effect, respectively. As is well established, the SZ effect is a powerful diagnostic tool that can yield important information on cosmological and cluster parameters (e.g. Planck Collaboration et al. 2016b, de Haan et al. 2016, Sifón et al. 2016). Additional information can be extracted from the polarization state of the scattered radiation, which probes other combinations of cosmological and cluster parameters.

A prerequisite for cluster-induced polarization is the presence of a non-vanishing quadrupole in the incoming radiation when viewed in the scattering electron rest frame; upon Compton scattering off intracluster (IC) gas, the CMB becomes polarized. Since polarized Compton scattering blocks all moments other than the quadrupole at the rest frame of the scattering electron, CMB polarization towards individual galaxy clusters is a pristine measurement of the local quadrupole moment in the CMB. This local quadrupole could be either of primordial origin or, alternatively, yield information on IC gas properties, i.e. either the cluster geometry, its peculiar velocity in the Hubble frame, gas temperature, or combination thereof. Two such quadrupole sources, linear in the optical depth, are explored here: The primordial CMB quadrupole, and quadrupole anisotropy associated with the transversal second order Doppler component of the bulk motion of the cluster. Two other relevant po-
Polarization components arise from double scattering in the same cluster (Sunyaev & Zeldovich 1980, Sazonov & Sunyaev 1999). First scattering induces temperature anisotropy either via the thermal or kinematic SZ effect. If this temperature anisotropy contains a local quadrupole moment, polarization is induced upon second scattering. For a typical Thomson optical depth of IC gas, \( \tau \sim 0.01 \), these \( \tau^2 \)-dependent components are clearly very weak, with the thermal effect sourcing the largest of the two. It has also been suggested that cluster polarization signals could possibly provide a way to increase the sampling of the primordial quadrupole by the additional linear polarization it induces, thereby lowering the statistical uncertainty of measurement of this quadrupole moment (Kamionkowski & Loeb 1997, Bunn 2006), a possibility that was further elaborated upon recently by Hall & Challinor (2014) and Louis et al. (2017a).

A quantitative study of cluster-induced polarization was begun by Sunyaev & Zeldovich (1980), and continued with the works of Sazonov & Sunyaev (1999) and Audit & Simmons (1999). Polarization levels and spatial distribution were determined in a detailed analysis of a cluster simulated with the hydrodynamical Enzo code by Shimon et al. (2006). Power spectra of the statistical (all-sky) cluster-produced polarization, which is clearly much smaller than the primordial CMB polarization on the relevant angular scales, was also studied (e.g. Cooray et al. 2004, Shimon et al. 2006).

In order to assess the likelihood of detection of the typically weak polarization signals from possible cluster targets of current and anticipated very high resolution ground-based polarization-sensitive experiments, a realistic feasibility analysis is needed. This provides the main motivation of the work presented in this paper.

As stated above, the main objective of the work presented here is to carry out detailed estimates of cluster polarization signals and to assess their detectability towards nearby rich clusters by three ground-based telescope projects - SPTpol, AdvACTpol, and the SA. These will clearly be nearby rich clusters by three ground-based telescope projects

\[
\Delta T \over T = y f(x),
\]

\[
f(x) = x \coth(x/2) - 4, \quad y = \int \sigma_T n_e \frac{k_B T_e}{m_e c^2} dl,
\]

where \( x = h \nu / (k_B T) \) is the dimensionless frequency, \( y \) is the comptonization parameter, \( \sigma_T \) is the Thomson cross section, \( n_e \) and \( T_e \) are the electron number density and temperature, and the integration is along the line of sight (los). The constants \( h, k_B, m_e, \) and \( c \), are Planck’s and Boltzmann’s constants, the electron mass, and the speed of light, respectively.

Hereafter we denote the dimensionless gas temperature as \( \Theta \equiv k_B T_e / (m_e c^2) \).

The typically smaller kinematic SZ effect is proportional to the los velocity component of the cluster, \( v_r \), and since it is a first order Doppler shift of the CMB, the temperature change it induces is frequency-independent,

\[
\Delta T \over T = - \int \sigma_T n_e \beta_r dl,
\]

with \( \beta_r = \frac{v_r}{c} \). Compton scattering can polarize incident radiation if it has a quadrupole moment in the rest frame of the scattering electron; the quadrupole is the only multipole transmitted by scattering (assuming photons are soft in the electron rest frame, which is virtually always the case for the thermal electron populations considered here). The incident CMB radiation has a global quadrupole moment in addition to a small non-vanishing quadrupole moment \( (O(10^{-5})) \) which is induced by (first) scattering in the cluster, i.e. the SZ effect. The degree of linear polarization and its orientation are determined by the two Stokes parameters

\[
Q = \frac{3 x f}{16 \pi} \int n_e dl \int \sin^2 \theta \cos(2\phi) T(\theta, \phi) d\Omega,
\]

\[
U = \frac{3 x f}{16 \pi} \int n_e dl \int \sin^2 \theta \sin(2\phi) T(\theta, \phi) d\Omega,
\]

where \( \theta \) and \( \phi \) define the relative directions of the incoming and outgoing photons, \( d\Omega \) is an element of integration over the solid angle, and \( T(\theta, \phi) \) is the temperature of the incident radiation. (Temperature-equivalent units are used throughout.) Since the los is taken to be along the z-axis for convenience, the angles \( \theta \) and \( \phi \) are actually defined with respect to the outgoing photon in this system. The average electric field defines the polarization plane with a direction given by

\[
\alpha = \frac{1}{2} \tan^{-1} \left( \frac{U}{Q} \right),
\]

and the total polarization is defined as

\[
P = \sqrt{Q^2 + U^2}.
\]

In the following we describe the polarization generated by Compton scattering when a quadrupole moment is induced by either electrons moving at the cluster radial velocity, or electrons randomly moving in the hot gas with a velocity dispersion which is characterized by the gas temperature \( T_e \).

### 2.1 Polarization of Isotropic Incident Radiation

Scattering of the CMB in a cluster at rest in the CMB frame results in local anisotropy due to the different pathlengths of photons arriving from various directions to a given point (e.g. Chluba et al. 2014). This anisotropy provides the requisite quadrupole moment; second scatterings then polarize the radiation (Sunyaev & Zeldovich 1980, Sazonov & Sunyaev 1999). By symmetry, the measured net polarization (essentially) vanishes if the cluster is not resolved. Nonetheless, and since we consider here the performance of the arcmin-resolution SPTpol and AdvACTpol, it is useful to explore the signal associated with double scattering since it is a priori expected to dominate over the other polarization signals...
in rich clusters (Shimon et al. 2006) for which $\tau^2$ is not negligibly small.

The CMB appears anisotropic in the frame of a non-radially moving cluster; consequently, scattering by IC electrons polarizes the radiation. Two polarization components are induced; the first is linear in the cluster velocity component transverse to the line of sight, $v_t \equiv \beta \vec{v}$, but quadratic in $\tau$; the second is linear in $\tau$ but quadratic in $\beta_e$. The quadrupole moment for the latter is induced by the (transversal) second order Doppler effect. The spatial patterns of the various polarization components can be readily determined when the gas distribution is spherically symmetric, in the ‘hard sphere’ approximation for gas density and temperature, e.g. Sunyaev & Zeldovich (1980), Sazonov & Sunyaev (1999).

More realistic gas distributions, substructure and high internal velocities result in complicated polarization patterns (Lavaux et al. 2004, Shimon et al. 2006).

2.1.1 The Kinematic Polarization Components

The degree of polarization induced by double scattering in a non-radially moving cluster was determined by Sunyaev & Zeldovich (1980) in the simple case of uniform gas density,

$$P = \frac{1}{40} \tau^2 \beta_t.$$  

(7)

This polarization component is frequency-independent in equivalent temperature units since the requisite quadrupole is generated by the kinematic SZ effect, a first order Doppler shift. A more complete calculation of this and the other polarization components was carried out by Sazonov & Sunyaev (1999). Viewed along a direction $\vec{n} = (\theta, \phi)$, the temperature anisotropy at a point $(X, Y, Z)$, $\Delta T(X, Y, Z, \theta, \phi)$, leads to polarization upon second scattering. The Stokes parameters are calculated from Eq. (4),

$$Q \pm iU(X, Y) = \frac{3\sqrt{3}}{16\pi} \int dZ n_e(X, Y, Z)$$

$$\times \int d\Omega \sin^2(\theta) e^{i\pm2\theta} \Delta T(X, Y, Z, \theta, \phi)$$

$$= \frac{3\sqrt{3}}{4\pi} \int dZ n_e(X, Y, Z)$$

$$\times \int d\Omega Y^{2 \pm 2}_{\ell}(\theta, \phi) \Delta T(X, Y, Z, \theta, \phi),$$

(8)

where $Y^{2 \pm 2}_{\ell}$ are the $\ell = 2$ and $m = \pm 2$ spherical harmonics and we recall that $\theta$ and $\phi$ are defined by a system centred at the scattering electron. The temperature change resulting from first scatterings is

$$\frac{\Delta T(X, Y, Z, \theta, \phi)}{T} = \sigma_T \int d\tilde{l}(X', Y', Z', \theta, \phi)$$

$$\times n_e(X', Y', Z', \theta, \phi) \hat{n} \cdot \beta(X', Y', Z'),$$

(9)

and the optical depth through the point $(X, Y, Z)$ in the direction $(\theta, \phi)$ is

$$\tau(X, Y, Z, \theta, \phi) = \sigma_T \int n_e(X', Y', Z') d\tilde{l}(X', Y', Z', \theta, \phi).$$

(10)

Thus, $Q(X, Y)$ and $U(X, Y)$ fully describe the linear polarization field.

The second kinematic polarization component is $\propto \tau \beta^2_t$; it is generated by virtue of the fact that the radiation appears anisotropic in the electron frame if the electron motion has a non-vanishing transverse component. The polarization of singly-scattered radiation is then calculated (Sazonov & Sunyaev 1999) by using Eq. (4). The Stokes parameters for this polarization component are

$$Q = \frac{1}{20} \tau (\beta^2_x + \beta^2_y) g(x)$$

$$U = \frac{1}{10} \tau (\beta_x \beta_y) g(x)$$

(11)

where

$$g(x) \equiv x \coth \frac{x}{2}.$$  

(12)

By applying a rotation in the sky (X-Y) plane, i.e. working in a polar frame in which $U_r$ identically vanishes, it can be shown that $Q = \frac{2}{5} \tau (\beta^2_x + \beta^2_y) g(x)$ and $U = 0$.

The polarization direction in our arbitrary coordinate system is

$$\alpha = \frac{1}{2} \tan^{-1} \left( \frac{2\beta_x \beta_y}{\beta^2_x - \beta^2_y} \right).$$

(13)

2.1.2 The Thermal Polarization Component

As with the $\tau^2 \beta_t$ component discussed above, double scattering off electrons moving with random thermal velocities can induce polarization that is proportional to $\tau^2 \Theta$. The anisotropy introduced by single scattering is the thermal component of the SZ effect with temperature change $\Delta T$. Its dependence on frequency is quantified by the spectral function $f(x)$ as defined in Eq. (2). In particular, this spectral function changes sign at the crossover frequency $x_0 = 3.83$, consequently $Q$ and $U$ flip signs, and the polarization pattern locally rotates on the sky by 90°. Globally, this results in the polarization pattern changing from radial at $x < x_0$, to tangential at $x > x_0$ (Sazonov & Sunyaev 1999). Actual numerical calculation of the effect for a given cluster configuration requires a 4D integration on the simulation box which we do not specify here and is described in Shimon et al. (2006).

2.2 Polarization of Anisotropic Incident Radiation

The large scale anisotropy of the CMB includes a global quadrupole moment at the level of $\approx 15 \mu K$, as measured by COBE, WMAP and Planck all-sky surveys (Planck Collaboration et al. 2016a, and references therein). Knowing that the probability for generating polarization by scattering in clusters is a fraction $\approx 1/\ell$ of the incident quadrupole, and that $\tau \sim 0.01$, we expect the resulting polarization signal to be $\approx 15 \text{nK}$, tiny by all measures, including the currently projected high sensitivity levels. A recent study of the detectability prospects of this effect has concluded that this component could be marginally detected at $\sim 2\sigma$ confidence level from a joint analysis of polarization maps of 550 clusters (Hall & Challinor 2014). It seems reasonable to expect that other cluster-induced CMB polarization signals and source confusion will likely degrade this projected detection level.
3 DETECTABILITY OF THE POLARIZATION SIGNAL

The main objective of this work is to determine values of the Signal-to-Noise (S/N) for detection of polarized CMB radiation induced by a rich cluster. Assuming temperature and two polarization (Q & U Stokes parameters, or the equivalent E & B modes) maps of the cluster are available, one can use their auto- and cross-correlations to estimate the S/N for cluster detection. The cluster is placed at various redshifts in the range \( z \in [0.02, 0.2] \), and its gas density and temperature are described by a polytropic equation of state with a \( \beta \)-profile.

Polarization maps and polarization levels for the cluster are produced for all components discussed above \( \propto \tau Q_{\text{prim}}, \propto \tau^2 \beta_r, \propto \tau^2 \Theta_r \) and \( \propto \tau^2 \beta_t \) in our analysis. These have different characteristic spatial and spectral signatures and therefore the feasibility of their measurement will obviously depend on the telescope sensitivity and angular resolution. Since ground-based experiments normally observe through only a few narrow atmospheric spectral bands, the main difference between the projected S/N achievable with the different telescopes will largely depend on their nominal instrumental noise levels. Our analysis does not account for beam systematics as it aims at assessing the expected nominal S/N independently of modelling uncertainties. Typically, beam systematics are dominant on sub-beam scales and they usually leak total-to polarized-intensity, and therefore our estimates would in this sense provide over-idealized bounds on the expected measured polarization signal. We do account for point source confusion, as specified below.

The polarization maps are decomposed into their E- and B-modes. This is required because the primordial CMB contribution is typically given in terms of the pure-parity E- and B-modes. For the same definite-parity property of the E- and B-modes it is advantageous to work in this basis as part of the primordial CMB temperature anisotropy and polarization, detector noise, integrated SZ (temperature only), and point sources.

Assuming certain linear Wiener-type filtering is applied to the observed T, E, and B maps that de-weights modes according to their respective S/N (per mode), the expected S/N from using all possible auto- and cross-correlations and all observed modes at all spectral bands is, e.g., Tegmark & Efstathiou (1996), Herranz et al. (2002), Lanz et al. (2010)

\[
\left( \frac{S}{N} \right)^2 = \sum_{\ell} \int \frac{d^2 \ell}{(2\pi)^2} S_\ell(\nu) N_\ell(\nu)^{-1} S_\ell(\nu)^1, \tag{14}
\]

where \( S_\ell = (T_\ell, E_\ell, B_\ell) \), and \( N_\ell(\nu)^{-1} \) is the inverse of the covariance matrix

\[
N_\ell(\nu) = \begin{pmatrix}
C_{TT} & C_{TE} & C_{TB} \\
C_{ET} & C_{EE} & C_{EB} \\
C_{BT} & C_{BE} & C_{BB}
\end{pmatrix}.	ag{15}
\]

Overall there are 6 possible pairings: TT, EE, BB, TE, TB, and EB. Here \( N_{\ell} \) should be understood as a 3x3 symmetric matrix per each multipole, whose XY component is the power spectrum \( C_{XY} \) at that \( \ell \). Since \( C_{TBB,\text{prim}} \) and \( C_{EEB,\text{prim}} \) vanish in the standard cosmological model. The detector noise \( C_{NN,\text{det}} \) ideally vanishes whenever \( X \neq Y \), the all-sky integrated polarization from the entire cluster population is negligible (Baumann & Cooray 2003, Shimon et al. 2006), and so is the expected EB-TB correlation. Since the temperature-polarization correlation of point sources is not known, we assume that all cross-correlation power spectra of the noise contributions except for \( C_{TE} \) are negligible.

Then, Eq. 14 significantly simplifies to

\[
\left( \frac{S}{N} \right)^2 = \sum_{\ell} \int \frac{d^2 \ell}{(2\pi)^2} \left[ C_{TT} C_{EE} - 2\text{Re}(T_\ell E_\ell) C_{TE} + |E_\ell|^2 C_{TT} \right. \left. + |B_\ell|^2 \right], \tag{16}
\]

where here we suppressed the frequency-dependence (explicitly shown in Eq. 14) for notational clarity, and the various \( C_\ell \) are the power spectra characterizing the various noises contributions to both temperature and polarization. In practice, these noise power spectra are obtained from the map itself – here we only adopt the expected noise power spectra; e.g., for the primordial CMB we use the output of the Boltzmann code CAMB (Lewis et al. 2000). From the telescope specifications (Austermann et al. 2012, Calabrese et al.)
where \( r \) is the (gas) core radius; typical values of \( r_c \) are around 200 kpc and 2/3, respectively. We construct a 3D semi-analytic simulation of the gas distribution through \( n_c \) through eq (1), we adopt the commonly used value of \( n_c = 2/3 \). A \( \beta \)-profile \footnote{where \( \beta \approx 2/3 \) and \( r_c \approx 2/3 \), respectively.} is taken for the gas spatial distribution

\[
\rho_g = \rho_{g,0} \left[ 1 + \left( \frac{r_c}{r_c} \right)^2 \right]^{-3/2} \beta
\]

where \( r_c \) is the (gas) core radius; typical values of \( r_c \) and \( \beta \) (deduced from measurement of the cluster X-ray surface brightness) are \( r_c \approx 200 \) kpc and 2/3, respectively. We construct a 3D semi-analytic simulation of the gas distribution with around 200 cubical cells with a minimum (resolution) side of 20 kpc; since the cluster virial radius is \( R_V = 2.2 \) Mpc, it is fully contained in the simulation volume.

The gas temperature and its profile are determined from the expression for the (ideal gas) pressure \( P_g = \rho g k_B T_g / \mu \),

\[
T_g = T_{g,0} \left( \frac{P_g}{\rho_{g,0}} \right)^{\frac{1}{2}},
\]

where \( \mu \) is the mean molecular weight (≈ 0.6 in a fully ionized cosmic gas), and \( T_{g,0} \) is the central temperature, whose fiducial value is set to \( T_{g,0} = 10^8 \) K. The central density is obtained from the total mass (within the virial radius) and the gas mass fraction \( f_g \),

\[
\rho_{g,0} = f_g M_{\text{tot}} \left[ 4\pi \int_0^{R_V} r^2 \left( 1 + (r/r_c)^2 \right)^{-1/2} dr \right]^{-1}.
\]

The electron number density is \( n_e \approx 6 \rho_g / (7 m_p) \). In our numerical estimates we take \( f_g = 0.12 \) \footnote{e.g. Dvorkin & Rephaeli 2015}, and consider the cluster to be moving at a velocity of 1000 km s\(^{-1}\) both along and across the los.

4 Results

We calculated the expected S/N for both total and polarized SZ intensities in a rich cluster with a total mass of \( M_{\text{tot}} = 2 \times 10^{15} M_{\odot} \), at an observed redshift in the range 0.02 \( \leq z \leq 0.2 \).

IC gas is described by a polytropic equation of state

\[
\rho_g = \rho_{g,0} (\gamma - 1) (P_g / \rho_{g,0})^{\gamma - 1}
\]

where \( P_g \) is the gas pressure, \( P_{g,0} \) is the central pressure, \( \rho_g \) is the gas density, \( \rho_{g,0} \) is the central gas density, and \( \Gamma \) is the adiabatic index (which is related to the polytropic index \( n \) through \( \Gamma = 1 + 1/n \)). We adopt the commonly used value of \( \Gamma = 1.2 \) (\( n = 5 \)).

The gas temperature and its profile are determined from the expression for the (ideal gas) pressure \( P_g = \rho g k_B T_g / \mu \),

\[
T_g = T_{g,0} \left( \frac{P_g}{\rho_{g,0}} \right)^{\frac{1}{2}},
\]

where \( \mu \) is the mean molecular weight (≈ 0.6 in a fully ionized cosmic gas), and \( T_{g,0} \) is the central temperature, whose fiducial value is set to \( T_{g,0} = 10^8 \) K. The central density is obtained from the total mass (within the virial radius) and the gas mass fraction \( f_g \),

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The projected temperature, gas density, and optical depth maps in our fiducial rich cluster are shown in Figure 1. At 150 GHz the thermal component is larger than the kinematic component by a factor of \( \sim 4 \), and is somewhat more centrally concentrated due to its dependence on both temperature and density; whereas the kinematic component is \( \propto \sigma \) (Figure 1(c)).

The predicted S/N of both total and polarized intensities include the confusion due to emission by (unclustered) radio and infrared galaxies. We adopt the currently deduced upper bounds on the power spectra \( \langle C_l \rangle \) of point sources, 13 nK\(^2\) \footnote{Keisler et al. 2011, Crites et al. 2015}, and 0.28 nK\(^2\) \footnote{Crites et al. 2015}, respectively. Estimates of the S/N levels for SZ intensities towards the model cluster if observed for the specified integration times of each of the three telescope projects are plotted as function of redshift in Figure 2.

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constant detection level for the various redshifts when the detector noise is dominant, unlike the case of detection of the full effect. This also explains the sharp decrease in S/N levels at low redshifts, where contribution from lower multipoles is more dominant. As the redshift of the cluster increases, the signal becomes prominent on the noisy range of the multipoles, until the redshift increases above $\sim 0.04$ and S/N levels rise, levelling-off at $z \gtrsim 0.2$. We see again that the SA is advantageous at the CVL, while there is no real benefit to increased observation time with the other two experiments. It then has very similar detection levels compared with AdvACTpol due to both having a third channel.

Estimates of S/N levels of SZ polarization towards the model cluster, when observed for the specified integration times of each of the three telescope projects, are plotted as function of redshift in Figure 4. As expected, the measurement of cluster polarized signals is challenging, with only marginal detection significance of clusters at $z \lesssim 0.04$ by all three experiments and all observation times. Point source emission and detector noise contribute most to the detection uncertainty. For longer observation time point source contamination becomes more prominent at high multipoles (mostly affecting clusters at high redshift), which results in a relatively slow S/N falloff with redshift due to the constant (in $\ell$-space) point source contamination. The overall shape of the polarization S/N curves is different than those for the respective total intensity curves due to the relative contributions of point sources and instrument noise compared to the integrated thermal SZ contribution to the noise budget of total intensity, and their different dependence on angular scale. The impact of these sources of noise is much stronger on the polarization S/N curves, especially for more distant clusters, so much so that S/N values monotonously decrease (rather than increase) with redshift. In other words, the re-
The component is the one proportional to calculations. It has been appreciably lower than the values assumed in our analysis. The polarized point source and instrument noise contributions would have been similar in shape had the currently deduced upper limit. Observation times and corresponding line styles are as in Figure 2. Point source noise contamination at the deduced upper limit (specified in the text).

At 150 GHz, the most prominent polarization component is the one proportional to \( \tau \beta^2 \), followed by the thermal double scattering component proportional to \( \tau^2 \theta_0 \). At frequencies close to \( \nu_c \sim 218 \) GHz the latter component is insignificant. The polarization orientations for all components are similar to those determined from the simplified hard-sphere cluster analysis carried out by Sazonov & Sunyaev (1999), as shown in their Figure 2.

We also calculated the expected S/N in measurement of the kinematic polarization components, which depend on the tangential velocity of the cluster; results are shown in Figure 5. As seen in the figure, the S/N curves are almost identical to those for detection of the total polarization signal, but with appreciably lower likelihood. Even though the kinematic component contributes the largest polarization signal, the fact that the second largest component is not localized in the centre of the map indicates that it contributed differently to the overall S/N, sufficiently so that without its contribution the total signal is too faint to be detected. Due to the reduced thermal S/N signal in the \( \sim 218 \) GHz channel, both AdvACTpol and the SA benefit from the capability to observe at this frequency, leading to higher S/N levels than that of SPTpol in the CVL.

It is also of interest to assess the likelihood of detecting SZ polarization by cross-correlation of the polarization and total intensity signals. Due to the much higher amplification of the latter, the cross-correlated S/N is not sufficiently sensitive to the polarization signals, and therefore no extra information can be deduced from it on cluster-induced polarization of the CMB.

Since the point source contamination has a major impact on the feasibility of detecting SZ polarization towards clusters, and given the fact that the current upper limit may well be too conservative, it is only reasonable to gauge the improved detection likelihood when its level is assumed to be appreciably lower. To do so we re-calculated S/N values with point source power and no detector noise (CVL case in Figure 6) a slight increase in S/N around \( z \sim 0.05 \) is apparent; this resembles the behaviour of the S/N curves for intensity detection.

5 DISCUSSION

There is considerable interest in polarization measurements of individual super-galactic systems in order to determine the spectral and spatial properties of what are the dominant sources that constitute the inherent confusing foreground in
CMB measurements. Reasonably detailed knowledge of microwave foregrounds is needed for a correct identification of any residual CMB signals. As the largest bound systems, galaxy clusters are the most important contributors to the foreground on scales larger than a few arcminutes. This further enhances the intrinsic interest in spectral and spatial SZ mapping of individual clusters which is diagnostically important for determining basic cluster properties – IC gas pressure profile, the gas and total mass of the cluster, and its velocity. The transverse velocity component of the cluster can only be determined from measurement of the kinematic polarization signals described in Section II.

We calculated the total and polarized intensities resulting from scattering in a fiducial rich cluster whose total mass is \(2 \times 10^{15} M_\odot\) with polytropic IC gas whose mass fraction is \(f_g = 0.12\); the cluster is assumed to have a radial and tangential velocity components of 1000 km s\(^{-1}\). For the three leading CMB polarization projects we calculated the predicted S/N of the thermal and kinematic total and polarized SZ intensities as function of the cluster redshift. Given the high measurement sensitivity of all three telescope systems, it is only to be expected that mapping of the thermal SZ component across nearby clusters would be readily accomplished. Dedicated long measurements with ground-based telescope arrays will make it possible to determine the gas pressure across most of the cluster extent, not just the central region. This will yield the gas density profile and, consequently, also the total mass profile, thereby resulting in a more precise determination of the cluster mass within the virial radius.

More demanding but still quite likely is the projected capability to measure the kinematic SZ component towards fast moving clusters. As is apparent from Figure 3, observing a cluster moving radially with a velocity of \(\sim 1000\) km s\(^{-1}\) by AdvACTpol, SA, and SPTpol for more than \(\sim 1\%\) of their respective observing time will result in a very significant detection of the kinematic SZ component. Significant detection of this component is likely also for slower moving clusters (when observed for the same times). For example, at \(\sim 500\) km s\(^{-1}\), detection at \(S/N > 3\) is likely by the AdvACTPol and SA for clusters at redshifts in the full range considered here (\(z < 0.2\)); for SPTpol the corresponding redshift is somewhat narrower since \(S/N \sim 3\) for \(0.03 \leq z \leq 0.07\). These estimates indicate that the radial velocity component of nearby clusters will be measured at a high level of precision by AdvACTpol and the SA, especially so for clusters at \(z > 0.04\).

In this study the fiducial rich cluster was taken to be spherically symmetric, relaxed, and in hydrostatic equilibrium. Clearly, the predicted level of induced polarization depends very much on the actual gas morphology and its velocity field, in addition to the temperature and density profiles. The former two characteristics can be very different in a merging cluster from those in a relaxed cluster, so much so that the feasibility of detection of the kinematic and total SZ polarization signals could be significantly higher than estimated here. Whereas a simple scaling of the kinematic polarization signals towards a merging cluster based on the possibly higher value of the transverse velocity component, as compared to the value assumed here (\(\sim 1000\) km s\(^{-1}\)), would seem to be adequate for estimating the enhanced kinematic polarization signals, a more realistic analysis would be needed in order to account for the non-spherical distribution of the gas and the spatial profile of the velocity field. This can be reliably accomplished based on detailed mapping of these quantities by a high resolution hydrodynamical simulation of a merging cluster.

The statistical cluster polarization signal is very small, as is clearly apparent from the predicted polarization power spectra calculated by (e.g.) Shimon et al. (2006). Ground-based observations aimed at detection of the predicted inflationary B-mode polarization typically target only few percent of the sky. Even though these (radio-quiet) patches are optimally chosen, polarization signals induced by individual clusters can still have an overall effect of more than the conservative level of a few percent residual contamination after subtraction of the statistical cluster signal estimated in the latter paper. In principle, the cluster polarization power spectra could also affect the precision of global parameter estimation, but these are too weak to impact the overall error in deduced parameter values (Shimon et al. 2009). A possible exception could be their effect on the residual lensing-induced B-mode signal, which can amount to a few percent, depending on the accuracy with which the lensing signal can be removed.

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