Hyper-Numerical Representation of the Pollution Propagation Process in the Territorial Technosphere

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Abstract. The article is devoted to the specifics of the territorial technosphere, quantitative estimation of its conditions and the state of its subsystems (biosphere, society, technogenics) based on the parametric aggregate. The study subject is the process of the pollution propagation in the territorial technosphere. The study scope is a hyper-numerical representation of the pollution propagation. The study target is the substantiation of a hyper-numerical representation of the pollution propagation. The study results are the mathematical description and graphic illustration of the pollution propagation hyper-numerical representation. For the efficiency of the territorial technosphere condition estimation, it is proposed to “reduce” the most significant controlled parameters to a single complex indicator (integral indicator) in the form of a hyper-numeric representation on the example of the atmospheric pollution propagation process.

1. Introduction

The most important problem of ensuring the technosphere safety is the estimation of the territorial technosphere (TT) conditions, trends and forecasting the development dynamics. But given that TT is a multi-parameter object, such estimation is considered difficult from a methodological, regulatory, technical, and mathematical point of view.

In the Russian Federation, the traditional approach to estimation of TT quality is used: “footprint - condition - response”. With this approach, the footprint on the TT created by the sources of impacts (IS) is considered, its composition and intensity of formation are studied, the climatic, landscape and architectural features of the entire TT infrastructure are taken into account. After IS analysis the technogenic footprint is calculated and thus created TT conditions and response are determined. By response we mean the totality of measures and the sequence of their execution to minimize and/or exclude an unacceptable technogenic footprint. Thus the feedback between the TT quality and the control systems actions affecting this quality is realized.

The traditional approach to estimation of the TT condition is based on the use of simple correlations and the additive hazard model expressed by the modified formula of A.G. Averyanov (1957) [1], where the sum of the relative levels of controlled parameters (CP) never exceeds one, that is

\[
\frac{C_{f1}}{TLV_1} + \frac{C_{f2}}{TLV_2} + \cdots + \frac{C_{fi}}{TLV_i} \leq 1,
\]

where \( C_{fi} \) is the CP actual value; \( TLV_i \) is the top limit value of the same CP, \( i \) is the number of CPs, selected for the analysis.
The disadvantage of this approach is that the following factors are ignored:
- possible synergistic effect of several damaging factors;
- relative danger of several damaging factors, which can be taken into account through weight numbers;
- only a two-step scale (greater than and equal to 1 (dangerous) or less than 1 (safe)) is used for hazard (safety) assessment.

2. Problem statement
Based on the above, we believe that it is necessary to develop and apply a comprehensive indicator of the TT condition (integral indicator), taking into account all the above comments and reflecting the TT condition most reliably.

In global practice, the development and use of integrated environmental indicators is widespread, but takes the biological, economic, social, medical specifics and is applied by departments (national) and therefore is not common (universal).

In global practice the aggregated Living Planet Index, Ecological Footprint Indicator and Environmental Sustainability Index are used to reflect the state of the environment on the planet [2–4]. Russia has experience building integrated indicators at the regional level and using them in analytical work. There is a significant number of releases on this subject [5–10]. But mostly when conducting such studies, the quality of individual subsystems [11–22] is monitored by one or more parameters or specific indices. Therefore, an approach is needed that would allow to comprehensively take into account all the CPs selected for analysis, considering each contribution to the final result and the result evaluation criteria.

3. Theoretical part
Analysis of the atmospheric pollutants distribution by concentration is carried out in accordance with [23]. Of great importance is the elevation \( H \) of the pollutant source (PS) outlet. Figure 1 shows a polluted air stream propagation diagram in three directions (\( x, y, z \) axis) emitted out of the PS in the presence of a blowing wind flow \( v_0 \). Its action leads to the stream curvature. At a certain elevation \( H + \Delta H \), the influence of the drift flow becomes predominant, the stream turns around, its axis becomes horizontal. The plume in this case becomes a paraboloid with a vertex at point \( P \), in where fictitious PS is placed. Thus, the real picture of the pollutants propagation is replaced by a fictitious PS plume located at the elevation \( P \).

![Figure 1: Emission plume in the drift flow diagram](image)

The plume expands, reaching the ground (point \( A(x_A) \)). At some point \( M(x_M) \), the near-ground concentration of pollutants reaches a maximum of \( C_M \), tending to zero in the limit (curve 1).

The \( C_M \) value depends on the wind speed \( v_0 \). With its increase, \( \Delta H \) decreases, that is, the plume is pressed to the ground, contributing to the surface pollutant concentrations increase. On the other hand, an increase in \( v_0 \) enhances the plume dissipation process in the vertical direction, thus leading to a decrease in \( C_M \).
The maximum concentration of the -th pollutant near the ground surface is achieved along the axis of the ejection plume (in the direction of the $x$ axis) at a distance $x_{Mi}$ from the PS and should not exceed $MCL_i$ of this pollutant:

$$C_{Mlx} = \frac{AM_{Kf} m_{m} n_{f1}}{H^2 \sqrt{V T \eta}} \leq \left(MCL_i - C_{f1}\right),$$  \hspace{1cm} (2)

where $A$ is the parameter characterizing the transport properties of the atmosphere; $M$ is the pollutant weight, emitted into the atmosphere in a time unit, g/s; $k_F$ is the coefficient of the pollutants suspended particles sedimentation rate; $m$ and $n$ are the dimensionless coefficients considering the conditions for the pollutants release from the PS; $\eta$ is the dimensionless coefficient considering the land relief; $V$ is the volume of pollutant emitted by all PS, m$^3$/s; $\Delta T$ is the difference between the emitted pollutant temperature and environment temperature [23].

Near-ground pollutant concentration along the $x$ axis ($c_{xi}$) is determined by the following formula:

$$c_{xi} = s_1 C_{Mi},$$  \hspace{1cm} (3)

where $s_1$ is the dimensionless coefficient, determined by [23].

Near-ground pollutant concentration along the $y$ axis ($c_{yi}$) is determined by the following formula:

$$c_{yi} = s_2 C_{Mi},$$  \hspace{1cm} (4)

where $s_2$ the dimensionless coefficient, determined by [23].

For a stand-alone PS with elevation $H$, the pollutant concentrations $c_x$ are calculated by the following formula:

$$c_x = 0.5\left(C_{Mi}' - C_{Mi}''\right),$$  \hspace{1cm} (5)

where $C_{Mi}'$, $C_{Mi}''$ are the near-ground pollutant concentrations found by formulas (2), at various elevations.

To summarize the concentrations of the $i$-th amount of pollutants in different directions of propagation ($x, y, z$ axes), we propose using the hyper-numeric representation in the form of the octonion [24–26].

Octonions $O$ are generally expressed as:

$$O = a_0 + a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 + a_5 b_5 + a_6 b_6 + a_7 b_7,$$  \hspace{1cm} (6)

where $a_0, \ldots, a_7$ are the real octonion numbers; $b_1, \ldots, b_7$ are the imaginary octonion numbers.

In this formula, the real numbers of the octonion correspond to the pollutants concentration values, and the imaginary numbers correspond to the pollutants propagation directions in the emission plume. The resulting space points form a free group with seven components given by the values of the components $b_1, \ldots, b_7$. Therefore, at each point we can form a new private octonion, which is part of the general octonion [26], describing the pollutants concentration change in the directions $b_1, \ldots, b_7$:

$$O = \Sigma O_k = O_1 + O_2 + \cdots O_{30}$$  \hspace{1cm} (7)

where $k$ is the number of points in the coordinate grid.

Let’s formulate the first specific octonion $O_1$ at a point $ee_0$ on the axis of the real numbers:

$$O_1 = 0 + ee_0 b_1 + ee_0 b_2 + ee_0 b_3 + ee_0 b_4 + ee_0 b_5 + ee_0 b_6 + ee_0 b_7,$$  \hspace{1cm} (8)

Every member of the octonion $O_1$ is a vector, recovered from the coordinate point $ee_0$ to the specified distance. This distance is proportional to the concentration of the pollutant at the point in the complex plane.

Each point in the diagram has a complex “three-dimensional” measurement ($x, y, z$ axes in the Fig. 1). Each member of the general octonion is a rotation vector that simultaneously rotates in three axes (the axes of real and imaginary numbers, and the "proper" axis of the specific octonion).

Let’s construct the octonion $O_2$ for the point $ee_1$. The second member in this octonion is zero, as
\( n_1 = 0 \) for it. Thus, we get the following:

\[
O_2 = ee_1 + 0 + ee_1 b_2 + ee_1 b_3 + ee_1 b_4 + ee_1 b_5 + ee_1 b_6 + ee_1 b_7. \tag{9}
\]

For the point \( ee_0 \), the octonion \( O_3 \) will have two members with imaginary numbers equal to zero. Similarly, we can construct octonions for each point in the coordinate grid. Moreover, the farther the coordinate point is from the central point, the fewer terms will be there in the octonion. See figure 2 for the general view of all octonions.

![Figure 2. Octonion diagram for the positive axis of the real numbers.](image)

As a result, we got a system diagram, consisting of many subsystems of different levels. The levels of the subsystems are characterized by seven imaginary numbers \( b_1, ..., b_7 \), where \( b_1 \) is the 1 level subsystem, and the following diminish. That is, the higher is the subsystem level number, the lower is the pollutants concentration at a given point.

Thus, the hyper-numeric representation of the pollutants propagation process allows us to figuratively and capacious determine and describe their impact on the TT objects.

4. Practical significance, suggestions and results of implementation, the results of experimental studies

We propose a TT condition monitoring method using heterogeneous measurement information to normalize and evaluate the state of the TT, represented as an octonion, including the following steps, as shown in figure 3.

We define \( TLV_i \) for each CP \( C_i \) on the basis of regulatory legal documents, and, depending on the situation there can be defined several of these \( TLV_i \):

\[
C_{i}^{\text{bottom}} \leq C_i \leq C_{i}^{\text{top}}.
\]

In this case, depending on the targets of the TT condition monitoring for each CP, several top and bottom boundaries can be set, which later will allow a more “subtle” estimation of the TT condition and identification of the undesirable trends in the CP values at the early stages.

We measure and register the \( C_{fi} \) CP actual values, that is, we record them in a table or save them to the device memory.
At the next stage, we create the octonions: the maximum permissible values for the top $O_{TLV}^{\text{top}}$ and bottom $O_{TLV}^{\text{bottom}}$ boundaries of the CP limit values, as well as the octonion of the measured values $O_{mes}$ according to [26].

The tolerance field of the desired state $O_{mes}$ is limited by the values of the top $O_{TLV}^{\text{top}}$ and bottom $O_{TLV}^{\text{bottom}}$ CP boundaries. And then all the comparison (reduction) operations shown in Figure 3 are carried out on the octonions already in accordance with the algebraic rules of hypercomplex numbers given in [27].

For the purpose of the TT condition monitoring, we establish the division of the tolerance field into ranges and set a color coding for them:

- if $O_{mes} \geq O_{TLV}^{\text{top}}$ – the CPs are above the tolerance field (red);
- if $O_{TLV}^{\text{bottom}} \leq O_{mes} < O_{TLV}^{\text{top}}$ – the CPs are in the tolerance field (yellow);
- if $O_{mes} < O_{TLV}^{\text{bottom}}$ – the CPs are below the tolerance field (green).

Based on the specified color display of the TT condition monitoring results, the decision-maker needs to plan and implement measures for the TT quality management.

5. Conclusions

Thus, the pollutants propagation in the TT can be described and visualized using the hyper-numeric representation of the TT pollution propagation process. The use of octonions as a complex indicator (integral indicator) provides a higher degree of TT condition and quality description completeness, as well as ease of the final result interpretation, which is convenient for decision-making systems.

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