Comment on ‘Using cold atoms to measure neutrino mass’

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Abstract. The proposal by Jerkins et al (2010 New J. Phys. 12 043022) to determine the neutrino mass from a complete kinematic reconstruction of the $\beta$-decay of trapped, cold tritium atoms is reanalyzed here. It is found that this innovative concept will hardly lead to competitive results because of the following shortcomings that have been identified: (i) a viable concept for a $\beta$-spectrometer with the required performance is missing; (ii) a factor of $10^6$ in the event rate is missing; and (iii) controlling spurious electromagnetic potentials to the level of $10^{-13}$ Tm and $10^{-9}$ V is hardly conceivable.

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1. Introduction

Current attempts to improve the experimental upper limit on the absolute neutrino mass of 2 eV are still following the traditional approach of measuring the $\beta$-spectrum of suitable nuclides close to the endpoint, where the sensitivity of fitting the neutrino mass to the data maximizes. The KATRIN experiment under construction is designed to reach a mass limit of 0.2 eV from tritium decay [1], whereas the MARE collaboration is exploring the possibility of reaching the same limit at the extremely low decay energy of $^{187}$Re [2]. Meeting this goal would be equally...
relevant for both particle and astrophysics, but both experiments require large investment and long-term R&D input. Therefore it makes much sense searching for different approaches. In particular, one might watch out for radically new instruments, which would be capable of measuring the complete kinematics of $\beta$-decay with sufficient sensitivity and precision to reconstruct the missing neutrino mass from the data of each individual event. Along these lines, Jerkins et al [3] recently published a proposal regarding such instruments on the grounds of the astonishing possibilities now offered by atomic physics and quantum optics. It is the purpose of this comment to analyze this proposal further and to point to some critical circumstances. The latter seems to render rather impossible the ambitious goal of a competitive neutrino mass measurement—in spite of the innovative ideas.

2. Neutrino mass and $\beta$-decay kinematics

Determining the neutrino mass ($m_\nu$) from its measured total energy ($E_\nu$) and momentum ($p_\nu$) requires either low neutrino energy (‘slow’ neutrinos) or enormous precision of the input data, because the quadratic form of the relativistic relation (using the units $c = \hbar = 1$)

$$m^2_\nu = E^2_\nu - p^2_\nu$$

boosts the resulting $m^2_\nu$-uncertainty by factors ($E_\nu$) and ($p_\nu$), respectively:

$$\sigma_{m^2,\nu} = 2 \sqrt{E^2_\nu \sigma^2_E + p^2_\nu \sigma^2_p} \rightarrow 2E_\nu \sqrt{\sigma^2_E + \sigma^2_p}.$$  

This is particularly annoying with respect to irreducible systematic uncertainties $\sigma_{\text{syst}}$ of the input data. Jerkins et al consider in [3] the kinematic reconstruction of the three-body decay

$$T \rightarrow \text{He}^+ + \beta^- + \nu, \quad (Q = 18.6 \text{ keV})$$

of tritium atoms that are almost at rest ($T = 1 \mu\text{K}$) within an almost point-like magnetic trap ($\phi = 100 \mu\text{m}$). It is worth mentioning that a still simpler kinematics is met in the (quite rare) two-body decay of tritium $T \rightarrow \text{He}^- + \nu$, where the $\beta^-$ sticks to the daughter, and the neutrino carries off most of the decay energy, namely $E_\nu = Q - E_{\text{recoil}} \approx Q - Q^2/2M_\text{He}$. It recoils to the daughter a momentum of $p \approx 18.6 \text{ keV}$, which might be measured by time-of-flight in the following situation: the decay populates a suitable excited daughter state, which emits a prompt photon as the start signal and decays into a metastable state whose energy serves for emitting an electron from the detector surface as the stop signal. Inserting $E_\nu$ as derived from the known $Q$-value $Q = \Delta M(T,^{3}\text{He}) = 18 589.8(1.2) \text{ eV}$ [4] and the measured $p_\nu$ into (1) yields straight $m^2_\nu$. But the unreasonably large neutrino energy will boost $\sigma_{m^2,\nu}$, through (2) far into a non-competitive region, even if one considers the smallest conceivable uncertainties $\sigma_Q, \sigma_{p,\nu}$. This is the summary of the various discussions on this topic over the years [5–7].

Coming back to the three-body decay (3), Jerkins et al consider measuring both momenta $\vec{p}_{\text{He}^+}$ and $\vec{p}_\beta$ in order to reconstruct $p_\nu$, but only up to $E_{\nu,\text{max}} = 500 \text{ eV}$ in order to avoid the calamity discussed above. However, it still does not make sense calculating $E_\nu$ from the present $Q$-value in view of its large uncertainty of 1.2 eV; rather the authors seem to use the $\beta$-endpoint ($E_{\text{e0}}$) as fitted to their measured $\beta$-spectrum, yielding much better $\sigma_{E,\nu}$. 

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3. Proposed experiment and analysis

The proposed ultra-cold T-source shall be an array of a large number of microscopic traps containing $10^{13}$ atoms altogether. It shall be viewed by three detectors (see figure 1 in [3]): (i) a multichannel plate measuring the direction and amount of $\vec{p}_{\text{He}^+}$; (ii) a zero-energy-loss track chamber measuring the direction of $\vec{p}_p$; and (iii) an electrostatic spectrometer measuring the kinetic energy $E_{k,\beta}$. The tracking chamber is based on an innovative and challenging idea: the medium shall consist of sheets of highly excited Rydberg atoms trapped in optical lattices. Electrons passing will induce transitions to a neighboring state with negligible energy transfer. Each transition shall be traced precisely by further optical and ion-optical manipulations. The realization of this chamber would certainly open a new era in the instrumentation of low-energy physics. The spectrometer following the track chamber shall have an energy resolution of 5 meV at $E_{k,\beta}$ slowed down to 900 eV. The authors propose a spectrometer of hemispherical type, without discussing the details. Instead, they refer to a current device. However, this turns out to be inapplicable, since it owes its high resolution to unacceptably narrow entrance and exit slits. Rather, one would use the entrance coordinates from the track chamber and exit coordinates from another track chamber downstream and calculate the $\beta$-energy with the help of the transfer function of the bending field in-between, as is usual in particle physics experiments. Still, it is questionable whether this alternative would allow for the accuracy required (see below). Anyway, (ii) and (iii) together shall provide $\sigma_{p,\beta}$ ranging from 2.8 to 40 meV. And somewhat smaller numbers seem to be expected for $\sigma_{p,\text{He}}$.

Considering a one-year measuring time, the source will produce about $10^{12}$ decays. Assuming detector efficiencies of 100% for simplicity and an angular acceptance $\Delta \Omega / 4\pi = 7 \times 10^{-5}$ (limited by the solid angle of the micro-channel plate (MCP)), $7 \times 10^7$ decays may be observed. Out of these about $N = 2000$ will fall into the accepted energy window $E_v \leq 500$ eV and form the total data set.

Jerkins et al present their expected data in figure 2 of [3] by means of a simulated two-dimensional event plot upon the $(E_v, m^2_v)$-plane. Its structure seems quite plausible and allows for a simple estimation of the statistical mass sensitivity $\sigma_{m^2,\text{stat}}$. Apart from some fluctuations, it looks like a ridge whose height rises proportionally to $E_v$ as well as the width of its Gaussian slopes along the $m^2_v$-direction. The latter is expected from (2) assuming constant input uncertainties. The linear rise in both dimensions makes the number of events $\Delta N$ within a certain energy bin $\Delta E_v$ rise like $E_v^2 \Delta E_v$, as required for a $\beta$-spectrum below its endpoint. Hence we may write

$$\Delta N = \frac{aE_v \Delta E_v}{b \sqrt{2\pi}} \int \exp \left( -\frac{1}{2} \left( \frac{m^2_v}{bE_v} \right)^2 \right) dm^2_v = aE_v^2 \Delta E_v \Rightarrow a = 3N/E^3_v\max, \quad (4)$$

with number-(width-)parameters $a$ ($b$), respectively. Furthermore, we recognize that an energy bin $\Delta E_v$ contributes a partial $m^2_v$-uncertainty independent of energy

$$\sigma^2_{m^2,\Delta E_v} = bE_v / \sqrt{\Delta N} = b/ \sqrt{aE_v}, \quad (5)$$

yielding for the whole ensemble $N$ the simple result

$$\sigma_{m^2,\text{stat}} = b/ \sqrt{aE_v\max} = bE_v\max/ \sqrt{3N}. \quad (6)$$

From figure 2 of [3], one reads off $bE_v\max \approx 1500\text{ eV}^2$. A data set of 2000 events would then yield the statistical uncertainty of the final result

$$\sigma_{m^2,\text{stat}} = 20\text{ eV}^2. \quad (7)$$

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This exceeds the aspired limit of 0.02 eV\(^2\) (equivalent to a neutrino mass sensitivity of 0.2 eV at 2\(\sigma\)) by a factor of 1000. Hence a factor of 10\(^6\) in the event rate is missing. Where is it hiding? It is noteworthy that Jerkins et al obtain, in their paper [3], the same result from the same decay rate as quoted in their preprint [7], although the latter considers an MCP area that is 44 times larger. Assuming in addition that they forgot to apply the reduction factor by the energy window, the missing event rate might indeed have been found.

With respect to systematic uncertainties of the input parameters, Jerkins et al content themselves with the remark that these should be mitigated by adequate control measurements. One can still discuss the limits necessary to restrict the systematic \(m_2\)-uncertainty to the aspired level of 0.02 eV\(^2\): the accepted events will occur at average neutrino energy \(\langle E_\nu \rangle \approx 370\) eV. Then (2) requires for \(\sigma_{p_\nu}\) and \(\sigma_{E_\nu}\) an upper limit as sharp as \(2.7 \times 10^{-5}\) eV. Any change of the momentum of the charged recoil ion and \(\beta\)-particle by uncontrolled electromagnetic fields along their path

\[
\Delta \vec{p}_B = e \int (\vec{B} \times d\vec{s}) \quad \text{and} \quad \Delta \vec{p}_E = e \int \vec{E} dt = e \int \vec{E} v^{-1} ds \approx e (v^{-1}) \int \vec{E} ds,
\]

would have to respect this limit, resulting in

\[
\left| \int (\vec{B} \times d\vec{s}) \right| \leq 10^{-13} \text{Tm}, \quad \left| \int \vec{E} ds \right| \leq 1 \text{nV for } ^3\text{He}^+ \quad \text{and} \quad \leq 1 \mu\text{V for } \beta^-,
\]

respectively. (8)

Respecting such extreme limits is hardly conceivable, as stated e.g. in [6]. With regard to the proposed setup, (9) requires the control of the magnetic field to the level of 100 pico-Gauss along a flight path of 7 m altogether—a tremendous challenge, considering the suppression of any spurious magnetizations or electrical currents, even inside a multilayer \(\mu\)-metal shield. The chances to control voltage on the level of nV and \(\mu\)V, respectively, are still much worse: in the case of the ion arm, fluctuations of the contact potential of surfaces inside the apparatus will exceed the nV level by many orders of magnitude. The \(\beta^+\), on the other hand, shall pass a 17.6 keV retarding potential ahead of momentum analysis. Controlling its relative stability and spatial distribution to \(10^{-10}\), as required by (9), surpasses the metrological state of the art by about four orders of magnitude.

4. Conclusion

In spite of the interesting and innovative ideas presented, the proposed experiment for measuring the neutrino mass by kinematic reconstruction of tritium \(\beta\)-decay will not achieve its ambitious goal due to the following shortcomings.

1. The proposed \(\beta\)-spectrometer does not match the design specifications of the experiment.
2. A factor of 10\(^6\) in the event rate is missing.
3. Controlling spurious electromagnetic potentials to the required level of \(10^{-13}\) Tm and \(10^{-9}\) V seems inconceivable.

At present, only problem (1) seems surmountable. Still, atomic tritium traps and Rydberg track chambers—once they are realized—might open promising routes to further fundamental research on nuclear \(\beta\)-decay, for instance angular correlations that are currently a hot topic.
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