POST-NEWTONIAN APPROXIMATIONS, COMPACT BINARIES, AND STRONG-FIELD TESTS OF GRAVITY

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This is an extended summary of the two parallel sessions held at MG11: PPN1 “Strong Gravity and Binaries” (chaired by L.B. and L.G.) and PPN2 “Post-Newtonian Dynamics in Binary Objects” (chaired by G.S.). The aims and contents of these sessions were close to each other and overlapping. It is natural to review both sessions in one joint contribution to the MG11 Proceedings. The summary places the delivered talks in a broader perspective of current studies in this area. One can find more details in individual contributions of the respective authors.

1. Introduction and overview

The current strong interest toward binary compact objects is largely driven by the imminent observation of gravitational waves by the presently operating or, more likely, by the upcoming advanced detectors. Many astronomical systems can emit some amounts of gravitational waves. However, it follows from the most general theoretical considerations that the amount of gravitational radiation is maximized when two massive chunks of matter are moving with respect to each other with relativistic speeds. In the Cosmos, this situation occurs naturally in tight binary systems involving the most compact objects presently known — neutron stars and black holes.

This explains the special attention to the massive compact objects orbiting each other at the late stages of their relativistic evolution — the so-called inspiralling compact binaries.\(^1\)\(^2\) It is argued\(^3\)\(^4\) that the inspiralling pairs of stellar mass black holes will probably be the first sources directly detected by the ground-based laser interferometers. The theoretical description of such binaries is usually being based on the Post-Newtonian (PN) approximations to general relativity.\(^5\)\(^7\)

In PN gravitational-wave studies it is often sufficient to treat black hole as a highly compact massive object, independently of whether the object possesses the general-relativistic event horizon or not. In the Newtonian limit, when formally \(c \to \infty\), the Schwarzschild radius \(r_g = 2Gm/c^2\) shrinks to zero, so the black hole can be viewed as a point-like particle endowed with a mass (and possibly a spin).
The PN approximation is expected to provide a good description of black holes and their relative motion in an expansion series with $c \to \infty$, that is, when the dimensionless ratios, such as $r_g/r$ and $v/c$, are sufficiently small.

The masses of black holes are assumed to be ranging from stellar masses (up to a few tens of $M_\odot$) to supermassive black holes (SMBH) in nuclei of galaxies (around $10^5 M_\odot - 10^9 M_\odot$). Although at present there is no decisive astrophysical evidence for the existence of compact objects with event horizons, the accurate PN equations of motion, as well as the emitted gravitational waveforms, are in principle capable of revealing the presence or absence of the event horizon. This is one of the features that makes these studies so exciting.

The inspiralling compact binaries in the last minutes of their orbital evolution are inherently powerful sources of gravitational waves. A crucial practical issue is the number of such systems that may populate a typical galaxy during a given interval of time, say, 1 year. Since the detectable part of gravitational radiation from such sources is a relatively short-lived event, this number is called the event rate. If the event rate were too much low, the prospect of observing these powerful sources would not be quite realistic. One would need to survey a huge volume of the Universe in order to have some confidence that at least several events occur per 1 year of observations. As a consequence, the sensitivity of the instrument would be required to be extremely high in order to guarantee the possibility of detecting the sources located in the most remote parts of this volume.

Unfortunately, the astrophysical event rates are not very certain. Sometimes their evaluation differs by orders of magnitude even in the papers of one and the same research group. This is especially true with regard to the binary systems involving black holes, for which the observational information is scarce. However, it appears that there exists growing consensus at least about the neutron star - neutron star (NS-NS) coalescences. The currently most quoted rate $3 \times 10^{-5} - 10^{-4}$ per year, for a galaxy like our own Milky Way, is at the level estimated very early on and advocated for long time by some groups.

It also follows from general arguments based on binary star evolution and their numerical simulation that the black hole - black hole (BH-BH) event rate is expected to be only an order of magnitude lower than the NS-NS rate. If this is true, the larger total mass of the BH-BH systems, in comparison with the NS-NS systems, and therefore larger gravitational wave luminosity, can more than compensate their lower event rate, when it comes to the analysis of detectability of these systems. For an instrument of a given sensitivity, such as LIGO or VIRGO, the probability of seeing a BH-BH coalescence turns out to be higher than the probability of seeing a NS-NS coalescence. Specifically, it was estimated that the BH-BH detection rate is likely to be a factor $\sim 5$ higher than the NS-NS detection rate, despite the opposite order of these systems event rates. This justifies the expectation that the first detected sources will actually be the coalescing stellar mass black holes. Theoretical and numerical studies of inspiralling and coalescing black holes are rising as a particularly interesting and important field of study.
Even the most powerful expected signals will be not more than at the level of noise in the presently available (ground-based) instruments. To extract the signal from the noise, and to determine the parameters of the radiating system, one needs to know in advance, and as accurately as possible, the theoretical signal templates for the incoming waves. It is also assumed that the response of the instrument to these waveforms is known with the equally high accuracy. The templates are being cross-correlated with the noisy output of the detector. Since the signal from inspiralling binaries is quasi-periodic, the knowledge of its phase is especially important, and the templates must remain in phase with the expected true signal as long as possible. The calculation of accurate templates from the compact binaries is now a matter of great activity.

Processes involving supermassive black holes (SMBH) provide more scientific opportunities, but add new questions and complications. Gravitational radiation from a neutron star or a stellar mass black hole inspiralling into a SMBH contains a wealth of information. A detailed waveform allows, in principle, to decide whether the central object is a black hole or some even more exotic object. Indeed, the multipole moments of a Kerr black hole satisfy unique relationships as functions of its mass and spin, whereas the multipole moments of an arbitrary body are not, in general, linked by similar relationships. The information about the multipole moments of the central object is encoded in the waveform, thus providing an example of strong gravity test with the help of gravitational waves. In practice, one could start from evaluation of the quadrupole moment of the central object (which enters as a parameter in the 2PN waveform), and check whether it satisfies the conditions for a Kerr black hole. However, the complicated trajectories of infalling masses, the uncertain radiation reaction force acting on the body, and the inevitable presence of accretion disks surrounding real (in contrast to ideal, theoretical) black holes, will make the extraction of astrophysical information not so easy.

Though most of the current activities in relativistic gravity are related to the imminent observation of gravitational waves, pulsar astronomy also has the potential of further tests of gravitational theories and more advanced insights into the Einsteinian gravity. To mention in this respect are further observations of the double pulsar to eventually measure the moment of inertia of one of the pulsars which would give information on the equation of state of neutron stars, and the construction of SKA (square kilometre array) to detect practically all pulsars in our galaxy with a chance of discovery of pulsar-black-hole binaries.

2. Theoretical modelling of compact binaries

One normally makes a convenient, but approximate, separation of the problem into several parts: internal, external, and far zone. This division is justified by the hierarchy of characteristic length scales of the binary system: size of the bodies, size of the orbit, and wavelength of the emitted gravitational radiation. The internal part of the problem is concerned with the bodies themselves, including their sizes,
shapes, internal structure and tidal gravitational effects induced by one body on another. The external problem deals with equations of motion of the centers of mass of participating bodies. As we said, in the external problem the bodies are often treated as point particles with given masses. In advanced treatments, the point particles are endowed also with spins and higher multipole moments. Finally, the far zone problem is mostly the calculation of the emitted gravitational waves. Typically this is being done in leading order in terms of the distance to the source.

Obviously, all three parts of the problem are interconnected. In particular, from the near-zone equations of motion, or from the balance relationships equating the emitted radiation and the changing orbital characteristics of the system, one finds the radiation reaction force and corrections to the Keplerian parameters of the binary, and then makes further corrections to the waveforms. It should be qualified as a remarkable success that the problem of theoretical templates has been worked out by successive approximations up to the 3.5PN approximation of general relativity, corresponding to the accuracy \((v/c)^7\), where \(v\) is the relative speed of components of the binary.

The external problem (that is, the motion of the centers of mass) has been solved at 3.5PN order independently by three groups, with completely equivalent results. One group used the Arnowitt-Deser-Misner (ADM) Hamiltonian formalism of general relativity\(^{20–23}\) and worked in a corresponding ADM-type coordinate system. Another group used a direct PN iteration of the equations of motion in harmonic coordinates.\(^{24–26}\) Both groups used a description by point particles and a self-field regularization. The end results of these two approaches have been proved to be physically equivalent.\(^{23,25}\) However, both approaches left undetermined one dimensionless parameter at the 3PN order. The appearance of this unknown parameter was related with the choice of the regularization method used to cure the self-field divergencies of point particles. The completion of the equations of motion at the 3PN order was made possible thanks to the powerful dimensional self-field regularization, which could fix up uniquely the value of the ambiguity parameter in both calculations.\(^{27,28}\) The third approach\(^{29–31}\) succeeded in obtaining the equivalent 3PN equations of motion directly, i.e. by using a “surface integral” method in which the equations of motion are written in terms of integrals on surfaces surrounding the compact bodies. This method is applicable for extended compact objects in the strong-field point particle limit. Finally, the 3.5PN terms, which constitute a 1PN relative modification of the radiation reaction force (and are relatively easier to derive), have been added in Refs.\(^{32–37}\)

The far zone problem (that is, the radiation field) has also been solved at the 3.5PN order, i.e. \((v/c)^7\) beyond the leading order given by the Einstein quadrupole formula, using a particular gravitational wave generation formalism combining multipolar expansions with the PN approximation.\(^{38}\) The crucial step is the computation of the binary’s quadrupole moment at the 3PN order which has been done by a combination of Hadamard’s self-field regularization dealing with most of the terms,\(^{39}\) and eventually completed, like in the problem of equations of motion, by
dimensional regularization able to fix the value of a few remaining ambiguity coefficients.\textsuperscript{40} The final 3.5PN templates\textsuperscript{42} take into account the values of the ambiguity parameters computed in\textsuperscript{27,28,40} (for a review, see\textsuperscript{5}).

However, there are also some difficult issues involved in this program. One should point out that the full analysis of the internal problem has not been done at 3PN order and is hard to perform. It has been carried out only for spherically-symmetric bodies and to the lowest radiative order \((v/c)^5\), \textit{i.e.} to 2.5PN order in the equations of motion. Satisfyingly, it was shown\textsuperscript{43} that the compactness parameter, characterising the size of the body \(L\) in comparison with its Schwarzschild radius \(r_g\), can be absorbed into the redefinition (or “renormalization”) of the body’s mass. Therefore, at this level of accuracy, the equations of motion are valid for compact objects of any size and structure, presumably including black holes. This dependence of the results only on integral parameters of the compact body (for example, its mass and spin) and independence on its actual shape and internal structure (for example, rearrangement of layers of matter) is sometimes called the principle of “effacement” of the internal structure.\textsuperscript{47} Obviously, the effacement principle cannot be true with arbitrarily high accuracy. It has to be violated at some sufficiently high order of PN approximations.

Despite the fact that the point-particle approach has reached a great level of rigor and sophistication, it cannot be arbitrarily accurate for real physical objects, even if it is consistent mathematically for idealized point particles. We have to worry about the magnitude of the finite-size effects, typically in the form of phenomenological parameters of an extended body and ultimately in the form of the finite gravitational radius of a black hole. Even though the PN corrections of very high level of accuracy will not be required by observers in the near future, further work is needed for precise identification of the limits of theoretical consistency of the PN program itself. It is expected, though, that the trouble will not show up too early. Indeed, it follows from the simple arguments (see \textit{e.g.}\textsuperscript{5,43}) that the tidal force in a pair of fluid bodies is of the order of \(\kappa(L/R)^5(GM^2/R^2)\), where \(L\) is linear size of the bodies, \(R\) - distance between them, and \(\kappa\) is a phenomenological parameter characterizing the “elasticity” of the bodies. In the limit where \(L\) approaches gravitational radius \(r_g\), the tidal force is a factor \(\kappa(v/c)^{10}\) smaller than the Newtonian force, \textit{i.e.} it is formally of the 5PN order and, hence, is very small. This expectation is confirmed by explicit calculations of finite-size effects for neutron star binaries\textsuperscript{41} and is consistent with the relativistic equations of motion for black holes derived by matching of the perturbed Schwarzschild solutions.\textsuperscript{47}

It should be noted, however, that the above division of the problem into the internal and external parts certainly breaks down at the merger phase of compact objects. This is especially true for merging black holes and subsequent ringdown phase of the combined black hole. This area of study requires new techniques and approaches (partially developed, see for example\textsuperscript{48,49}). In particular, the merger and ringdown of binary black holes have recently been implemented by numerical techniques.\textsuperscript{44–46} Although the amount of gravitational radiation emitted at merger and
ringdown phases is relatively modest, it may be observable by advanced detectors, opening an exciting era of new discoveries.

3. Post-Newtonian equations of motion

The introduction to the problem of testing general relativity with gravitational radiation from compact binary inspirals was given by C. Van Den Broek and B. S. Sathyaprakash (reported by C. Van Den Broeck). The speaker has emphasized that some of the PN terms in the measurable gravitational wave phase arise from the scattering of gravitational waves off the gravitational field of the source, in the vicinity of the binary. These terms are known as gravitational wave tails. Observational checks of the presence of such tail terms (using, for example, the data analysis method of 50) will be tests of the validity of PN approximations and general relativity itself. The contribution of C. Van Den Broek and B. S. Sathyaprakash has also discussed some other tests of gravitational theories, which are not necessarily based on PN approximations. The range of tested theories includes those with massive gravitons. We will briefly review these possibilities below.

S. Kopeikin spoke about the irrelevancy of the internal structure of gravitating bodies (i.e. the effacement principle) in the PN approximations of general relativity and scalar-tensor theories of gravity. He argued that in general relativity the effacing principle is violated by terms proportional to the rotational moments of inertia of the fourth order. In the scalar-tensor theories of gravity the violation begins earlier, by the terms proportional to the second order rotational moments of inertia. 51 When the effacement principle is violated, the equations of motion of extended bodies differ from those of point-like particles. Correspondingly, the emitted waveforms are also different. In the limit where the size of the body is taken to be close to the Schwarzschild radius \( r_s = 2GM/c^2 \), Kopeikin evaluates that the effacement principle is violated in the 3PN approximation in the case of scalar-tensor theories, and in the 5PN approximation (terms of the order of \((v/c)^{10}\)) in general relativity. The latter statement is in agreement with an earlier conclusion by Damour. 47

The point-particle approach inevitably encounters the necessity of regularization of the fields diverging at the world lines of the particles. There are two somewhat different methods of dealing with this problem. One is directly applicable in the employed harmonic coordinate system 24–26 while the other is based on the ADM formalism. 20–23 Both methods ultimately rely on analytical continuation of the equations of motion to the (in general, complex) \( d \neq 3 \) dimensions. 27, 28 In his talk, P. Jaranowski spoke about the dimensional regularization of the gravitational interaction in the ADM formalism. More precisely he regularized the 3PN Hamiltonian of point masses, in which the field degrees of freedom are reduced using the field equations. 27 The speaker argued that the dimensional continuation leads to a finite and unambiguous Hamiltonian in the limit \( d \rightarrow 3 \). He showed that three somewhat different methods of computation lead to the same expression for the dimensionally regularised 3PN Hamiltonian. This increases confidence in the correctness of the
3PN Hamiltonian and the associated equations of motion (an alternative method which confirmed the result $^{29-31}$).

G. Faye has derived the equations of motion of spinning compact binaries including the spin-orbit (SO) coupling terms 1PN order beyond the leading-order effect.$^{52}$ For black holes maximally spinning this corresponds to 2.5PN order. The result confirms the previous calculation of Ref.$^{53}$ The SO effects up to 2.5PN order are also computed in the conserved (Noetherian) integrals of motion, namely the energy, the total angular momentum, the linear momentum and the center-of-mass integral. The spin precession equations at 1PN order beyond the leading term are also obtained. The speaker reported then the computation (using the multipolar-PN wave generation formalism$^{38}$) of the SO contributions in the gravitational-wave energy flux and the secular evolution of the binary’s orbital phase up to 2.5PN order.$^{54}$ It was shown that the 1PN SO effects are in general numerically larger than the spin-spin (SS) effects, in terms of the number of gravitational-waves cycles, even though they appear at a formally higher PN order. These results provide more accurate gravitational-wave templates to be used in the data analysis of rapidly rotating Kerr black-hole binaries with the ground-based and space-based detectors.

L. Gergely discussed the corrections to the 2PN equations of motion of structureless point-like particles, which arise when the particles are endowed with spins and mass quadrupole moments. If the body is a neutron star with a strong magnetic field, then the representing “particle” is endowed also with a magnetic type dipole moment. The speaker has demonstrated a generalized Kepler equation$^{55}$ where the orbital elements include contributions from the spin-spin, mass quadrupole and magnetic dipole interactions. The considered effects nicely fit in the standard Kepler form with the modified parameters of the orbit.

The orbital phase of inspiralling binaries with the inclusion of the orbit’s eccentricity was discussed in the contribution of M. Vasuth. He presented the results on the change of the mean motion parameter for eccentric orbits and the evolution of the orbital phase for circular orbits. All linear effects due to spins, mass quadrupole and magnetic dipole moments are included in this derivation. The author has related the change of the mean motion and orbital frequency with the radiative energy losses. This allows one to derive the time-dependent orbital frequency and phase, and to calculate the number of relevant gravitational wave cycles. In the 2PN approximation this number includes corrections from spin-orbit (SO), spin-spin (SS), mass quadrupole - mass dipole and magnetic dipole - magnetic dipole coupling effects. A special attention was paid to the presence of a new self-interaction spin term.$^{56}$

M. Tessmer has reported on the accurate and computationally efficient derivation of waveforms produced by binaries with arbitrary eccentricity and mass ratio, and moving in slowly precessing orbits.$^{57}$ The orbital motion is restricted to be 1PN accurate, and only the quadrupole contribution to the waves’ polarization amplitudes is taken into account. The central point of the derivation is a special numerical method for solving the participating generalized Kepler equation. The
speaker has discussed the relevance of the derived waveforms for observations with the space-based interferometer LISA.\textsuperscript{61}

\textit{G. Schäfer} has reported on recent results from his research group additionally to Tessmer’s contribution, which were the phasing of gravitational waves from inspiralling eccentric binaries at the 1PN radiation-reaction order (corresponding to 3.5PN order in the equations of motion)\textsuperscript{58} and the gravitational recoil during binary black hole coalescence using the effective-one-body approach.\textsuperscript{59}

\textit{L. Blanchet} discussed the problem of the gravitational recoil of black-hole binaries using PN techniques. He reported on the recent calculation of the recoil of non-spinning black holes in the inspiralling phase at 2PN order.\textsuperscript{60} The author estimated the kick velocity accumulated during the plunge from the innermost stable circular orbit up to the horizon by integrating the momentum flux along a plunge geodesic of the Schwarzschild metric. The contribution to the total recoil due to the subsequent ringdown phase was neglected.

\section*{4. Supermassive black holes and strong field tests of gravity}

In addition to possible SMBH mergers, a promising source for LISA, called the extreme mass ratio inspiral (EMRI), is the infall of a compact object into a SMBH. The infalling body can be a neutron star or a stellar mass black hole. The detailed study of gravitational waves emitted by an EMRI event can, in principle, allow one to build a “map” of the gravitational field in a parsec radius of the galactic nucleus. It is presumed that the SMBH candidate in the nucleus is a Kerr black hole. However, the modelling of gravitational wave from EMRIs is a very challenging task as the orbits often exhibit complicated behaviour, and the EMRI parameter space is huge. In this situation, the derivation of true waveforms is practically impossible by analytical techniques and is very expensive by numerical methods.

One approach to the problem is the construction of a family of simpler, approximate waveforms, called “kludge” waveforms, which nevertheless capture the main features of the true signals.\textsuperscript{62} \textit{S. Babak} has discussed the progress in compiling a bank of detection templates which are numerical kludge (NK) waveforms. They can be generated quickly and cheaply, and they are good enough to be used as a first pass test for the parameter estimation. It is reported that satisfactory NK waveforms are available for infalling masses in the range of up to 5-6 $M_\odot$.

The derivation of sufficiently accurate waveforms always requires the proper taking into account of the radiation reaction force acting on the body and changing its trajectory. This problem is especially acute for EMRI sources where the inspiral phase lasts for long time and orbits are complicated. In SMBH studies, one normally considers the technique of small perturbations of the background space-time. Due to the emission of gravitational waves, the world line of a compact object deviates in a calculable way from the nominal geodesic line of the background (possibly, a Kerr black hole) solution.\textsuperscript{63}

In the point-particle approximation, the metric perturbations are divergent along
the particle’s world line. This requires some sort of field regularization. Although many technical problems concerning the regularization are already solved, some conceptual issues remain. Y. Mino has reported on the existing problems for the Kerr-background metric related to the choice of gauge, the restrictive assumption of linearity (weakness) of perturbations, and finally, the extraction of gravitational waveforms. One way to resolving these difficulties is a careful identification of the radiation-reaction part in the self-force expression. The author has also discussed the guides provided by the adiabatic approximation which assumes that the orbital parameters evolve slowly.

We normally take it for granted that compact binary systems are “clean”, so that the relativistic gravity is essentially the only participating interaction up to the very late stages of evolution near the final merger. However, realistic astrophysical compact objects are likely to be surrounded by accretion disks which will complicate the motion of close companions and may intervene at the level of corrections larger than the magnitude of relativistic PN effects. This seems to be especially plausible to happen in the case of EMRI sources, and this was the subject of the contribution by P. Basu. It is indeed expected that the late orbits of the infalling object will be taking place within the accretion disk of the SMBH. The infalling NS or BH will itself accrete some matter from the disk.

Typically, the specific angular momentum of the accreted matter is lower than the Keplerian angular momentum of the infalling body. Therefore, the total angular momentum of the infalling object will decrease more rapidly than expected, leading to a faster infall into the SMBH. For some accretion disks the situation can be opposite, which would lead to the slower than expected infall of the companion. Basu has outlined the gravitational field and fluid dynamics equations that should be solved simultaneously. The gravitational field of the central Kerr black hole is modelled by an effective Newtonian potential. For disks with certain parameters, the change of the companion’s angular momentum can be greater than the largest losses expected due to gravitational radiation. Qualitatively, this situation is similar to what can happen in neutron star systems when interactions other than gravity are present.

5. New research directions

The overwhelming majority of current studies in gravitational physics are based on the geometrical formulation of Einstein’s equations. For example, in PN expansions describing an isolated binary system (usually, in asymptotically Lorentzian harmonic coordinates), one normally treats the gravitational functions \( h^{\mu\nu}(t, \mathbf{x}) \) as frame-dependent pieces of curved space-time metric \( g_{\mu\nu}(t, \mathbf{x}) \), rather than components of a genuine tensorial gravitational field defined in a flat space-time with Minkowski metric.

The thrifty geometrical picture of general relativity, which combines gravity, geometry and a choice of coordinates in a single mathematical object – the curved
space-time metric tensor $g_{\mu\nu}(x^\alpha)$—is accompanied by some peculiar features. One may recall ambiguities in the description of the gravitational field energy density and gravitational-wave fluxes, constant mixing of external (coordinate) and internal (“gauge”) transformations, and, in general, certain disjointment of geometrical gravity from other field theories.

It is known for long time that the geometrical picture of general relativity is by no means compulsory or necessary. It is argued\textsuperscript{67} that the geometrical Einstein’s gravity is fully equivalent mathematically and physically (including cosmology) to a field theory in a flat space-time, \textit{i.e.} in a space-time with zero-curvature metric tensor $\gamma_{\mu\nu}(x^\alpha)$. In flat space-time, one can always choose global Lorentzian coordinates, so that the metric tensor $\gamma_{\mu\nu}(x^\alpha)$ takes on the familiar form of the Minkowski metric $\eta_{\mu\nu}$. The field-theoretical formulation of general relativity possesses all the necessary and strictly defined structures: covariant gravitational Lagrangian containing gravitational field $h^{\mu\nu}(x^\alpha)$ and its first derivatives, second-order differential non-linear field equations, gravitational energy-momentum tensor free of second and higher order derivatives of the field, universal coupling of the gravitational field to other physical fields (which allows one to combine $h^{\mu\nu}(x^\alpha)$ and $\gamma_{\mu\nu}(x^\alpha)$ into a single object $g^{\mu\nu}(x^\alpha)$ and to reinterpret the theory as a geometrical theory, where the curved space-time metric tensor $g_{\mu\nu}(x^\alpha)$ satisfies geometrical Einstein equations), conservation laws, well defined and physically distinct coordinate and gauge freedoms.

One does not expect that any new observational conclusions will arise from a reformulation of one and the same fundamental theory, but a new angle of view helps one to see problems in a different light and answer questions which otherwise could not be even properly formulated. In particular, the field-theoretical approach to gravity opens the door for natural modifications of general relativity including the concept of massive gravitons.\textsuperscript{67} It is not surprising that what we are doing in practice, “by hands”, in PN expansions is similar to traditional field-theoretical perturbative calculations, independently of whether we adhere to geometrical ideology or not.

The relevance of field-theoretical techniques for PN calculations in binary systems was emphasized by I. Rothstein and R. Porto. It was argued that the replacement of detailed internal structure of the gravitating bodies by a set of phenomenological parameters associated with a point-like particle is similar to what is routinely being done in effective field theories (EFT) where one is interested in a simple description of the influence of short-scale physics on large-scale (low energy) dynamics. The well developed methods of EFT allow one to introduce simple power-counting arguments (evaluation of the order of magnitude of various contributing terms) and employ traditional treatments of divergencies and renormalization procedures.\textsuperscript{68,69} The formalism also allows one to calculate absorptive effects for an arbitrary object in terms of the graviton absorptive cross-section.\textsuperscript{68} (In the context of gravity, the authors are using a somewhat confusing name of “Non-Relativistic General Relativity”.\textsuperscript{1}) For application to PN computations of compact binaries, these methods
should be applied twice. First, when one derives the equations of motion for point particles, and second, when one calculates the gravitational waveforms and replaces the entire binary system by a single particle with certain multipole moments. The back-reaction effects within the EFT approach should also be properly worked out.

I. Rothstein has stressed that the EFT calculations are essentially being done at the level of the Lagrangian and the action. He presented the action for a binary with slowly moving components. He argued that the divergencies arising at the \((v/c)^6\) level, that is, in the 3PN approximation, are not physical as the corresponding terms in the action can be removed by field redefinitions. This conclusion is consistent with what was stated on the grounds of calculations in the more usual PN approach.\[24,28\]

Interestingly, the same line of EFT arguments has led the speaker to the conclusion that first terms which cannot be removed by field redefinitions, and therefore provide a source of violation of the “effacement” principle, are of the order of \((v/c)^{10}\), i.e. they appear in the 5PN approximation. Again, this conclusion is consistent with other arguments, thus increasing our confidence in the internal workings of the entire PN scheme.

R. Porto has extended the EFT approach to include spin dynamics. This is achieved by adding rotational degrees of freedom to the world-line action of the point-like particles. The issue of different choices for the spin supplementary conditions was clarified. It was shown that these conditions are equivalent, in the sense of the final results, at least at the 1PN level. In the areas where the EFT approach overlaps with previous studies,\[70–73\] the final conclusions are in agreement with each other. The author has also reported new results, derived by EFT techniques, on the corrections to the spin-spin (SS) potential in the 3PN approximation.\[74\] It is argued that these corrections are easier to handle in the EFT approach, and, in general, that this approach is a powerful tool to treat in a systematic manner the higher-order PN effects.

Spin effects encapsulated in a prescribed Lagrangian were also considered in the contribution of M. Vasuth. It is assumed that the motion of the binary system is described by the Lense-Thirring Lagrangian which takes into account the spin vector of the central body and treats another body as a spinless test particle. The author considers the 1.5PN accurate motion of the test particle and concentrates on terms linear in the spin of the central body. The radial and angular dynamics of the system are treated with the help of more general results.\[75\] The outcome of calculations are both polarization components of the emitted gravitational waves, including the contributions linear in the spin. It is confirmed that the waveforms are in agreement with previous calculations.\[71\]

The geometrical general relativity requires one to be careful with such things as choice of coordinates, identification of the “gauge-independent” degrees of freedom of the gravitational field, clock synchronization, equivalence principle, and, in general, physical interpretation of the participating quantities.\[76\] L. Lusanna has discussed some of these issues in his contribution. Although these issues refer in general to the full theory of geometrical gravitation, they also manifest themselves
in approximations, such as PN gravitational-wave studies.

Returning to the review of tests of general relativity, it is important to remember that interesting tests are not necessarily associated with the strong field regime. C. Van Den Broek and B. S. Sathyaprakash have reminded us of the importance of some signatures in the regime of weak plane gravitational waves. According to general relativity, the response of an interferometer to the incoming plane wave contains not only the usual “electric” component but also the (typically, smaller) “magnetic” component. The identification of the “magnetic” component in the output data is needed for proper extraction of the radiating system’s parameters, but also as a test of the “magnetic” prediction of general relativity.

New possibilities arise in alternative theories of gravity, such as theories with massive gravitons. In particular, the detection of a “scalar” polarization state of gravitational waves, expected in such theories, would revolutionize our views on the nature of gravity and could possibly provide a gravitational explanation to some presently existing cosmological puzzles.

6. Conclusions
The current advances in observational facilities (LIGO and VIRGO on Earth, LISA in Space) have stimulated further deep insights in such problems as gravitational-wave physics in general and sources of gravitational waves in particular, relativistic celestial mechanics, tests of general relativity and alternative theories. The contributions to the PPN1 and PPN2 parallel sessions have demonstrated the depth and rigor of the continuing research in these areas. Although several of the recently derived results need to be cross-checked and placed in a unique common context, it is clear that there is no major conceptual or technical difficulties in this field. In particular the status of PN computations of equations of motion and gravitational radiation is quite satisfactory. In the coming years we will probably witness a major progress in relating and connecting the analytical PN calculations with successful numerical computations. However, it appears that there is already enough theoretical clarity and completeness at least for the level of accuracy of existing gravitational wave observations. Hopefully, further theoretical work will proceed hand in hand with successful experiments.

References
1. J. P. A. Clark, E. P. J. van den Heuvel, and W. Sutantyo, Astron. & Astrophys. 72, 120 (1979)
2. K. S. Thorne, in Three hundred years of gravitation, Eds. S. Hawking and W. Israel, Cambridge U. Press, 1987 p.330
3. V. M. Lipunov, K. A. Postnov, and M. E. Prokhorov, MNRAS 288, 245 (1997); New Astronomy 2, 43 (1997)
4. L. P. Grishchuk, V. M. Lipunov, K. A. Postnov, M. E. Prokhorov, and B. S. Sathyaprakash, Physics-Uspekhi 44, 1-51 (2001) (astro-ph/0008481)
5. L. Blanchet, in Living Reviews in Relativity 9, 4 (2006) (http://www.livingreviews.org/lrr-2006-4)
6. T. Damour, in *Three hundred years of gravitation*, Eds. S. Hawking and W. Israel, *Cambridge U. Press*, 1987 p.128
7. G. Schäfer, *Ann. Phys. (NY)* **161**, 81 (1985)
8. R. Narayan, T. Piran, and A. Shemi, *Astrophys. J.* **379**, L17 (1991)
9. E. S. Phinney, *Astrophys. J.* **380**, L17 (1991)
10. V. Kalogera, C. Kim, D. R. Lorimer, *et al.*, *Astrophys. J.* **601**, L179 (2004) [ERRATUM: *Astrophys. J.* **614**, L137 (2004)]
11. K. Belczynski, R. Tamm, V. Kalogera, F. Rasio, and T. Bulik, arXiv: astro-ph/0612032
12. V. M. Lipunov, K. A. Postnov, and M. E. Prokhorov, *Astron. & Astrophys.* **176**, L1 (1987)
13. D. Hils, P. Bender, and R. Webbink, *Astrophys. J.* **360**, 75 (1990)
14. A. V. Tutukov and L. R. Yungel’son, *Astron. Rep.* **37**, 411 (1993)
15. http://www.ligo.org, http://www.ligo.caltech.edu
16. http://www.virgo.infn.it
17. M. J. Rees and M. Volonteri, ArXiv: astro-ph/0701512
18. M. Kramer, I.H. Stairs, R.N. Manchester, M.A. McLaughlin, A.G. Lyne, R.D. Ferdman, M. Burgay, D.R. Lorimer, A. Possenti, N. D’Amico, J.M. Sarkissian, G.B. Hobbs, J.E. Reynolds, P.C.C. Freire, and F. Camilo, *Science*, **314**, 97 (2006)
19. http://www.skatelescope.org/
20. F. Jaranowski and G. Schäfer, *Phys. Rev. D*, **57**, 7274 (1998)
21. F. Jaranowski and G. Schäfer, *Phys. Rev. D*, **60**, 124003 (1999)
22. T. Damour, P. Jaranowski, and G. Schäfer, *Phys. Rev. D*, **62**, 021501R (2000)
23. T. Damour, P. Jaranowski, and G. Schäfer, *Phys. Rev. D*, **63**, 044021 (2001)
24. L. Blanchet and G. Faye, *Phys. Rev. D*, **63**, 062005 (2001)
25. V.C. de Andrade, L. Blanchet, and G. Faye, *Class. Quant. Grav.*, **18**, 753 (2001)
26. L. Blanchet and B. R. Iyer, *Class. Quant. Grav.*, **20**, 755 (2003)
27. T. Damour, P. Jaranowski, and G. Schäfer, *Phys. Lett. B*, **513**, 147 (2001)
28. L. Blanchet, T. Damour, and G. Esposito-Farèse, *Phys. Rev. D*, **69**, 124007 (2004)
29. Y. Itoh, T. Futamase, and H. Asada, *Phys. Rev. D*, **63**, 064038 (2001)
30. Y. Itoh and T. Futamase, *Phys. Rev. D*, **68**, 121501R (2003)
31. Y. Itoh, *Phys. Rev. D*, **69**, 064018 (2004)
32. B.R. Iyer and C.M. Will, *Phys. Rev. Lett.*, **70**, 113 (1993)
33. B.R. Iyer and C.M. Will, *Phys. Rev. D*, **52**, 6882 (1995)
34. P. Jaranowski and G. Schäfer, *Phys. Rev. D*, **55**, 4712 (1997)
35. M. E. Pati and C. M. Will, *Phys. Rev. D*, **65**:104008, 2002.
36. C. Königsdörffer, G. Faye, and G. Schäfer, *Phys. Rev. D*, **68**, 044004 (2003)
37. S. Nissanke and L. Blanchet, *Class. Quant. Grav.*, **22**, 1007 (2005)
38. L. Blanchet, *Class. Quant. Grav.*, **15**, 1971 (1998)
39. L. Blanchet, B. R. Iyer, and B. Joguet, *Phys. Rev. D*, **65**, 064005 (2002)
40. L. Blanchet, T. Damour, G. Esposito-Farèse, and B. R. Iyer, *Phys. Rev. Lett.*, **93**, 091101 (2004)
41. T. Mora and C. Will, *Phys.Rev. D* **69** 104021 (2004)
42. L. Blanchet, G. Faye, B. R. Iyer, and B. Joguet, *Phys. Rev. D* **65**, 061501R (2002)
43. L. P. Grishchuk and S. M. Kopeikin, *Sov. Astron. Lett.*, **9**(4), 230 (1983); in *Relativity in Celestial Mechanics and Astrometry*, Eds. J. Kovalevsky and V. A. Brumberg, *Reidel*, 1986 p.19
44. F. Pretorius, *Phys. Rev. Lett.*, **95**, 121101 (2005)
45. J. Baker, J. Centrella, D.-I. Choi, M. Koppitz, and J. van Meter, *Phys. Rev. Lett.*, **96**, 111102 (2006)
46. M. Campanelli, C. O. Lousto, P. Marronetti, and Y. Zlochower, *Phys. Rev. Lett.*, **96**
111101 (2006)

47. T. Damour, in *Gravitational Radiation*, N. Deruelle and T. Piran, editors, p. 59, Amsterdam, North-Holland Company (1983)

48. T. Damour, R. Iyer, and B. Sathyaprakash, *Phys. Rev. D* **66**, 027502 (2002)

49. A. Buonanno, Y. Chen, and T. Damour, *Phys. Rev. D* **74**, 104005 (2006)

50. L. Blanchet, and B. S. Sathyaprakash, *Phys. Rev. Lett.*, **74**, 1067 (1994)

51. S. Kopeikin and I. Vlasov, *Physics Reports*, **400**, 209 (2004); arXiv: gr-qc/0612017

52. G. Faye, L. Blanchet, and A. Buonanno, *Phys. Rev. D*, **74**, 104033 (2006)

53. H. Tagoshi, A. Ohashi, and B.J. Owen, *Phys. Rev. D*, **63**, 044006 (2001)

54. L. Blanchet, A. Buonanno, and G. Faye, *Phys. Rev. D*, **774**, 104033 (2006)

55. Z. Keresztes, B. Mikoczi, and L. Gergely, *Phys. Rev. D* **72**, 104022 (2005)

56. B. Mikoczi, M. Vasuth, and L. Gergely, *Phys. Rev. D* **71**, 124043 (2005)

57. M. Tessmer and A. Gopakumar, *MNRAS* **374**, 721 (2007)

58. C. Königsdörffer and A. Gopakumar, *Phys. Rev. D* **73**, 124012 (2006)

59. T. Damour and A. Gopakumar, *Phys. Rev. D* **73**, 124006 (2006)

60. L. Blanchet, M.S. Qusailah, and C.M. Will, *Astrophys. J.* **635**, 508 (2005)

61. http://www.lisa.jpl.nasa.gov

62. S. Babak, H. Fang, J. Gair, K. Glampedakis, and S. Hughes, ArXiv: gr-qc/0607007

63. Y. Mino, *Class. Quant. Grav.* **22**, S717 (2005)

64. Y. Mino, and H. Nakano, *Progr. Theor. Phys.* **100**, 507 (1998)

65. S. K. Chakrabarti and S. Mondal, *MNRAS* **389**, 976 (2005)

66. D. Baskaran and L. P. Grishchuk, *Class. Quant. Grav.* **21**, 4041 (2004)

67. S. V. Babak and L. P. Grishchuk, *Phys. Rev. D* **61**, 024038 (1999); *Intern. J. Mod. Physics D* **12**(10), 1905 (2003)

68. W. Goldberger and I. Rothstein, *Phys. Rev. D* **73**, 104029 (2006); *ibid*, 104030

69. R. A. Porto, *Phys. Rev. D* **73**, 104031 (2006)

70. L. Kidder, C. Will, and A. Wiseman, *Phys. Rev. D* **47**, R4183 (1993)

71. L. Kidder, *Phys. Rev. D* **52**, 821 (1995)

72. E. Poisson, *Phys. Rev. D* **57**, 5287 (1998)

73. H. Wang and C. M. Will, ArXiv: gr-qc/0701047

74. R. A. Porto and I. Rothstein, *Phys. Rev. Lett.* **97**, 021101 (2006)

75. L. A. Gergely, Z. Perjes, and M. Vasuth, *Phys. Rev. D* **57**, 876 (1998)

76. L. Lusanna, in *Current Mathematical Topics in Gravitation and Cosmology*, Karpacz Winter School, Poland, 2006 (gr-qc/0604120)