Real–time counting of single electron tunneling through a T–shaped double quantum dot system

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Real–time detection of single electron tunneling through a T–shaped double quantum dot is simulated, based on a Monte Carlo scheme. The double dot is embedded in a dissipative environment, and the presence of electrons on the double dot is detected with a nearby quantum point contact. We demonstrate directly the bunching behavior in electron transport, which leads eventually to a super–Poissonian noise. Particularly, in the context of full counting statistics, we investigate the essential difference between the dephasing mechanisms induced by the quantum point contact detection and the coupling to the external phonon bath. A number of intriguing noise features associated with various transport mechanisms are revealed.

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I. INTRODUCTION

To control and manipulate electronic dynamics in nanoscale devices it requires knowledge of the involving transport processes at single–electron level. The spectrum of current fluctuations, which characterizes the degree of correlation between charge transport events, serves as an essential tool superior to the average current in distinguishing various transport mechanisms. Full counting statistics (FCS) of current has also been measured, owing in particular to the development of highly sensitive on–chip detection of single–electron tunneling technique. All statistical cumulants of the number of transferred particles can now be extracted experimentally.

Current fluctuations would obey a Poissonian process if the tunneling events were statistically independent. However, non-Poissonian fluctuation is in general a reality. In the case of transport through a localized state, the Pauli exclusion principle suppresses the noise, leading to a sub-Poissonian statistics. Systems of multiple non-local states such as coupled quantum dots are more interesting. The intrinsic quantum coherence and many–particle interactions there result in different sources of correlations. The fascinating super-Poissonian noise thus occurs in various contexts, and thereby has been attracting a wide interest recently.

A representing system, which will be studied in this work, is a T–shaped double quantum dot (TDQD) system as schematically shown in Fig. 1. The system is of particular interest, as it can be mapped to a structure of quantum well in presence of an impurity inside, which has been investigated experimentally. The source and drain electrodes of the TDQD are in such a configuration that maximizes locality versus nonlocality contrast. In addition, the TDQD is also influenced by an inevitable dissipative environment. The nearby quantum point contact (QPC) that serves as a charge detector is asymmetrically coupled to the dots. The current through the QPC depends on the charge state, for excess electrons on two dots individually. The TDQD is also coupled to an inevitable dissipative phonon environment (not shown explicitly).

![FIG. 1: Schematics of electron transport through a TDQD system, monitored continuously by the QPC current that depends on the charge state for excess electrons on two dots individually. The TDQD is also coupled to an inevitable dissipative phonon environment (not shown explicitly).](image-url)
The electronic parts are written in terms of electron annihilation (creation) operator in the QD1 (σ only one spin–degenerate level (σ = 2). Each quantum dot is assumed to have \( \hat{n}_l \sigma = \frac{1}{2}(\hat{n}_{l\uparrow} - \hat{n}_{l\downarrow}) \) and \( \hat{n}_{l\uparrow} \). The interdot coupling strength is Ω. The electron Hamiltonian for the coupled dots system reads

\[
H_{\text{sys}} = \sum_{l} (\hat{\mathcal{Q}}_{l\sigma}^c + \hat{\mathcal{Q}}_{l\sigma}) + \sum_{l} U_l \hat{n}_{l\uparrow} \hat{n}_{l\downarrow} + U' \hat{n}_{l\uparrow} \hat{n}_{l\downarrow},
\]

where \( \hat{Q}_{l\sigma} \equiv \hat{d}_{l\uparrow}^\dagger \hat{d}_{l\sigma} - \hat{d}_{l\sigma}^\dagger \hat{d}_{l\downarrow} \), \( \hat{Q}_{l\sigma} \equiv \hat{d}_{l\uparrow}^\dagger \hat{d}_{l\sigma} + \hat{d}_{l\sigma}^\dagger \hat{d}_{l\downarrow} \), \( \hat{n}_{l\uparrow} = \sum_{\alpha} \hat{n}_{l\alpha \uparrow} \), and \( \hat{n}_{l\downarrow} = \hat{d}_{l\uparrow}^\dagger \hat{d}_{l\uparrow} \). The electron annihilation (creation) operator in the QD1 (\( l = 1 \)) or QD2 (\( l = 2 \)) quantum dot. Each quantum dot is assumed to have only one spin–degenerate level (\( \sigma = \uparrow \) or \( \downarrow \)) in the bias window. The level detuning between the two dots is \( \epsilon = \epsilon_1 - \epsilon_2 \). The interdot coupling strength is \( \Omega \). The intradot and interdot Coulomb interactions, \( U_l \) and \( U' \), are both assumed to be much larger than the Fermi levels. We shall be interested in the double-dot Coulomb blockade regime, i.e. at most one electron resides in the TDQD. It can be realized by proper tuning the gate and bias voltages. The environment is of \( H_{\text{env}} = h_{\text{ph}} + \sum_{\alpha} h_{\alpha} + h_{\text{QPC}} \). It is composed of the phonon bath, the electron reservoirs of the source and drain (\( \alpha = \text{L} \) and \( \text{R} \)) electrodes, as well as the QPC detector. Each of them is modeled as a collection of noninteracting particles. The phonon bath assumes \( h_{\text{ph}} = \sum_{j} \hbar \omega_j (\hat{b}_{j\uparrow}^\dagger \hat{b}_{j\uparrow} + \hat{b}_{j\downarrow}^\dagger \hat{b}_{j\downarrow}) \). The electron reservoirs are modeled with \( h_{\alpha} = \sum_{k,\sigma} \epsilon_{\alpha k} \hat{c}_{\alpha k \sigma}^\dagger \hat{c}_{\alpha k \sigma} \) and \( h_{\text{QPC}} = \sum_{p,\sigma} \epsilon_{p\sigma} \hat{c}_{p\sigma}^\dagger \hat{c}_{p\sigma} + \sum_{q,\sigma} \epsilon_{q\sigma} \hat{c}_{q\sigma}^\dagger \hat{c}_{q\sigma} \). These electronic parts are written in terms of electron creation and annihilation operators in the \( \alpha \)-electrode and the QPC reservoirs states.

The system–environment coupling can be written as

\[
H_{\text{sys–env}} = \sum_{\sigma} \hat{Q}_{\sigma}^c F_{\text{ph}} + \sum_{\alpha,\sigma} (d_{\alpha \sigma}^\dagger f_{\alpha \sigma} + f_{\alpha \sigma}^\dagger d_{\alpha \sigma}) + \sum_{s = 0, 1, 2} \hat{n}_s \epsilon_{\sigma} F_{\text{QPC}}^s.
\]

The first term describes the coupling with phonon bath, in which \( F_{\text{ph}} \equiv \sum_{j} \lambda_j \hat{b}_j \). This term is responsible for the dot level energy fluctuations. The effect of phonon bath on the double–dot system is characterized by the phonon interaction spectral density, \( J_{\alpha \sigma}(\omega) = \sum_{\lambda} |\lambda_j|^2 \delta(\omega - \omega_j) \). The second term describes the transfer coupling between QD1 and the leads, in which \( f_{\alpha \sigma} \equiv \sum_{k} t_{\alpha k \sigma} c_{\alpha k \sigma} \). The third term describes the interaction between the QPC detector and the measured system, in which \( F_{\text{QPC}}^s \equiv \sum_{p,\sigma} \epsilon_{p\sigma} c_{p\sigma}^\dagger c_{p\sigma} + \text{H.c.} \). The amplitude of electron tunneling through the QPC depends on the TDQD charge state \( \hat{n}_s = |s\rangle \langle s| \), with \( s = 0 \) for no excess electrons, and \( s = 1 \) or \( 2 \) for one excess electron in QD1 or QD2, respectively. In other words, the current through the QPC sensitively depends on the charge states of the TDQD, and thus can be used to measure single electron tunneling events. The effects of these electron reservoirs components on the double–dot system are characterized individually by their interaction spectral densities: \( J_{\alpha \sigma}(\omega) = \sum_{k,\sigma} t_{\alpha k \sigma}^2 \delta(\omega - \epsilon_{\alpha k}) \), and \( J_{\text{QPC}}^s(\omega) = \sum_{p,\sigma} |t_{p\sigma\sigma}|^2 \delta(\omega - \epsilon_{p\sigma} - \epsilon_{\sigma}) \). In what follows we adopt flat bands in the electrodes, which yields energy independent couplings \( \Gamma_{\alpha} \). Analogously, the coupling with the QPC is described by the rate \( T_{\alpha} = 2\pi \sum_{k} |t_{\alpha k \sigma}|^2 V_{\text{QPC}} \), where \( V_{\text{QPC}} \) is the QPC bias voltage, \( g_p \) and \( g_{\sigma} \) are the density of states in the QPC reservoirs. We assume here–after the density of states to be constant, and \( t_{pq\sigma} \) reservoir states independent. Thus \( T_{\alpha} \) just depends on the system charge occupation state \( |s\rangle \langle s| \), with \( s = 0 \) being for zero excess electron, and \( s = 1 \) or \( 2 \) for one excess electron in QD1 or QD2, respectively. For the QPC current, \( I_s = e T_{\alpha} \), we have \( I_0 > I_2 > I_1 \), as implied in the scheme of Fig. I.

### B. Quantum master equation theory

To describe the quantum measurement, we exploit the reduced density operator \( \rho^{(n)}(t) \) of the TDQD system, for the specified number \( n \) electrons having passed through the QPC detector and being recorded up to the given time. The related conditional quantum master equation can be derived, which is greatly simplified under the Born–Markov approximation. Here, instead of using the “\( n \)”–resolved evolution directly, it is convenient to introduce its \( \chi \)–space counterpart via \( \rho(\chi, t) = \sum_n e^{i n \chi} \rho^{(n)}(t) \), where \( \chi \) is the so–called count-
ing field. Let the quantum master equation be formally

$$\frac{\partial}{\partial t} \rho(\chi, t) = -i(L + R_\chi)\rho(\chi, t),$$  \tag{3}

where $L \equiv [H_{\text{sys}}, \cdot]$ is the system Liouvillian, and $R_\chi$ is the dissipation superoperator to be specified soon. Hereafter, we set unit of $\hbar = e = 1$ for the Planck constant and electron charge, unless where is needed for clarity.

To that end, let us recast the reduced density matrix in the vector notation,

$$\rho = (\rho_{00}, \rho_{11}, \rho_{22}, \rho_{12}, \rho_{21}, \rho_{\bar{1}\bar{2}}, \rho_{\bar{2}1}, \rho_{\bar{2}\bar{1}})^T,$$  \tag{4}

with $\rho_{ss'} \equiv \langle s \uparrow | \rho | s \uparrow \rangle$ and $\rho_{s\bar{s}} \equiv \langle s \downarrow | \rho | s \downarrow \rangle$. There are 9 nonzero elements of the reduced density matrix in the Coulomb blockade regime. The other 16 elements, describing coherence between different spin states, are all zeroes as the system–environment coupling consider here does not cause spin flip. The dissipation superoperator $R_\chi$ in Eq. (3) is simply a $9 \times 9$ matrix.

In the large voltage limit and Coulomb blockade regime, the involving Fermi functions can be approximated by the step function of either one or zero. Under the Born-Markov approximation for weak tunnel coupling and second–order perturbation theory in the electron–phonon coupling, the dissipation superoperator $R_\chi$ matrix is obtained explicitly

$$R_\chi = \begin{pmatrix}
2\Gamma_L + \tau_0 \chi(q) & -\Gamma_R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\Gamma_L & \Gamma_R + \tau_1 \chi(q) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \Gamma_+ & \Gamma_- & 0 & 0 & 0 & 0 & 0 \\
0 & \Gamma_- & \Gamma_+ & \Gamma_- & 0 & 0 & 0 & 0 & 0 \\
-\Gamma_L & 0 & 0 & 0 & \Gamma_R + \tau_1 \chi(q) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \Gamma_+ & \Gamma_- & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \Gamma_+ & \Gamma_- & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_+ & \Gamma_- \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_d
\end{pmatrix}.  \tag{5}
$$

Here we have introduced

$$\Gamma_\pm = \frac{\pi}{2} \Omega J_{\text{ph}}(\Delta) \left[1 \pm \frac{\epsilon}{\Delta} \coth \left(\frac{\Delta}{2k_BT}\right)\right],$$  \tag{6}

$$\Gamma_d = \frac{1}{2} (\Gamma_R + \gamma_{\text{ph}} + \gamma_\chi),$$  \tag{7}

and

$$\gamma_{\text{ph}} = 2\pi \frac{\Omega^2}{\Delta^2} J_{\text{ph}}(\Delta) \coth \left(\frac{\Delta}{2k_BT}\right),$$  \tag{8}

$$\gamma_\chi = (\sqrt{T_1} - \sqrt{T_2})^2 + 2\sqrt{T_1T_2}q(\chi),$$  \tag{9}

with $q(\chi) \equiv 1 - e^{i\chi}$, $\Delta \equiv \sqrt{\epsilon^2 + 4\Omega^2}$ the Rabi frequency of the double–dot system, $k_B$ the Boltzmann constant, and $T$ the temperature. The physical processes described by the transfer elements in Eq. (5) are clear.

Consider $R_{11,00} = -\Gamma_L$, $R_{00,11} = -\Gamma_R$, and their opposite spin counterparts, as inferred from Eq. (5). These identities agree with the sequential tunneling picture, with $\Gamma_L$ being the rate of electron tunneling from the left electrode to system, and $\Gamma_R$ being that from system to the right electrode; both via QD1.

The parameter $\Gamma_\pm$ [Eq. (6)] denotes the nonsecular elements for the population–to–coherence transfers: $\Gamma_+ \equiv R_{12,11}^\chi = R_{21,11}^\chi$, $\Gamma_– \equiv R_{12,22}^\chi = R_{21,22}^\chi$, and their opposite spin counterparts, as denoted in Eq. (5). They are purely due to the phonon bath environment.

The parameter $\Gamma_d$ [Eq. (7)] is the total decoherence rate between two levels: $\Gamma_d \equiv R_{12,12}^\chi = R_{21,21}^\chi$ and the opposite spin counterparts, as denoted in Eq. (5). The total decoherence rate is composed of not just $\Gamma_R$ due to the electron depopulation to collector, but also $\gamma_{\text{ph}}$ [Eq. (8)] and $\gamma_\chi$ [Eq. (9)], due to the phonon bath coupling and the QPC detection, respectively. Note that $\gamma_d \equiv \gamma_{\lambda=0} = (\sqrt{T_1} - \sqrt{T_2})^2$ denotes the dephasing rate induced by the existence of QPC ensemble.

In the numerical demonstrations below, we set the phonon bath spectral density $J_{\text{ph}}(\omega) = 2\eta\omega e^{-\omega/\omega_c}$. Here, the dimensionless parameter $\eta$ reflects the strength of dissipation and $\omega_c$ is the Ohmic high energy cutoff.

III. MONTE CARLO SIMULATION OF SINGLE ELECTRON TUNNELING

Experimentally, the most intuitive method for measuring the FCS of electron transport is to count electrons passing one by one through the conductor. The real–
time detection of single electron transport enables direct evaluation of the probability distribution function of the number of electrons transferred through device within a given time period. In addition to the current and the
malized state, conditioned by the definite number of

temperatures

\[ U = 2 \rho \equiv \text{for single electron tunneling through device.} \]

order cumulants. The solution to the reduced state

\[ \rho \chi, t = \rho(t_0) \otimes U(t_1 - t_0) \]

space is obtained via the inverse Fourier transform

\[ \rho = \frac{2}{\pi} \int d\chi \chi \delta(t_0 - t_0) \equiv \mathcal{U}(n, \delta t) \rho(t_0). \]  

The involving propagator \( \mathcal{U}(n, \delta t) \) is completely determined by the dynamic structure of the master equation \( \mathcal{L} \), regardless of the initial state. Therefore, we can numerically evaluate it by a “one–time task”, such as fast Fourier transform, which results in an efficient real–time simulation.

Specifically, consider the evolution of state \( \rho(t_j) \) to \( \rho^{(n_j)}(t_j + \tau) \) at \( t_j + \tau \) \( \rho^{(n_j)}(t_j + \tau) = \mathcal{U}(n_j, \tau) \rho(t_j) \), and denote \( \text{Pr}(n_j) = \text{Tr}[\rho^{(n_j)}(t_j + \tau)] \) that is the probability of having \( n_j \) electrons passed through QPC during the time interval \( [t_j, t_j + \tau] \). If the measurement is made but the result is ignored, then the (mixture) state reads

\[ \rho(t_j + \tau) = \sum_{n_j} \rho^{(n_j)}(t_j + \tau) = \sum_{n_j} \text{Pr}(n_j) \rho^c(n_j, t_j + \tau). \]  

Here, \( \rho^c(n_j, t_j + \tau) = \rho^{(n_j)}(t_j + \tau)/\text{Pr}(n_j) \) is the normalized state, conditioned by the definite number of \( n_j \) electrons having passed through QPC during \( [t_j, t_j + \tau] \).

The second equality of Eq. (11) implies that if we stochastically generate \( n_j \) according to \( \text{Pr}(n_j) \) for each time interval \( [t_j, t_j + \tau] \), and collapse the state definitely onto \( \rho^c(n_j, t_j + \tau) \), we have in fact simulated a particular realization for the selective state evolution conditioned on the specific measurement results.

For the output current in a particular real–time measurement, we have

\[ I_{\text{QPC}}(t) = I_0 \delta_{n_0} + I_1(\rho^{c}_{11} + \rho^{c}_{12}) + I_2(\rho^{c}_{22} + \rho^{c}_{23}) + \xi(t). \]  

The first three terms determine the conditional evolution of the charge state. The last term \( \xi(t) \) originates from the intrinsic noise of detector. Here, we consider in the diffusive regime, where \( \xi(t) \) is a Gaussian variable with zero mean value and the spectral density \( S_\xi = 2e \langle I_{\text{QPC}} \rangle \), with \( \langle I_{\text{QPC}} \rangle \) the average stationary QPC current. Accordingly, we can stochastically generate \( n_j \), the number of electrons having passed through QPC during \( [t_j, t_j + \tau] \), via \( n_j = \int_{t_j}^{t_j + \tau} dt' I_{\text{QPC}}(t') = I_0 \delta_{n_0} + I_1(\rho^{c}_{11} + \rho^{c}_{12}) + I_2(\rho^{c}_{22} + \rho^{c}_{23}) + \xi(t) \), where \( dW(t_j) \) is the Wiener increment during \( [t_j, t_j + \tau] \).

A typical example of simulated real–time detector current \( I_{\text{QPC}} \) is displayed in Fig. 2 for (a) \( T = 100 \text{ mK} \) and (b) \( T = 800 \text{ mK} \), respectively. The temperatures are \( T = 100 \text{ mK} \) and \( T = 800 \text{ mK} \), respectively. Other parameters are: \( \Gamma_R = 4\Gamma_L = \Omega = 20 \text{ kHz}, \epsilon = -4\Omega, \eta = 2 \times 10^{-6}, \) and \( \omega_c = 0.4 \text{ meV} \). The time step used is \( \tau = 0.002 \text{ ms} \), such that the minimum characteristic time scale of the system can be clearly resolved.

FIG. 2: Time traces of the QPC current fluctuations correspond to different charge states in the TDQD (shown on the right). The arrows indicate transitions where an electron is entering the QD1 from the left lead. (a) and (b) correspond to different temperatures \( T = 100 \text{ mK} \) and \( T = 800 \text{ mK} \), respectively. Other parameters are: \( \Gamma_R = 4\Gamma_L = \Omega = 20 \text{ kHz}, \epsilon = -4\Omega, \eta = 2 \times 10^{-6}, \) and \( \omega_c = 0.4 \text{ meV} \). The time step used is \( \tau = 0.002 \text{ ms} \), such that the minimum characteristic time scale of the system can be clearly resolved.
on the right. We choose rates of tunneling to the left and right leads as $\Gamma_L = 5 \text{ kHz}$ and $\Gamma_R = 20 \text{ kHz}$, such that all the tunneling rates are within the bandwidth ($\sim 30 \text{ kHz}$) of the QPC detector. The QPC conductance (without excess electrons in TDQD) is $G_0 = 0.02 e^2/h$, and QPC bias voltage $V_{\text{QPC}} = 0.15 \text{ mV}$, which corresponds to a QPC current $I_0 \approx 0.12 \text{ nA}$. The conductance is assumed to decrease by 4% or 2%, when an electron occupies on the QD1 or QD2, respectively. The corresponding QPC current distribution. The central quantity is then dephasing due to QPC charge detection of electrons passing through the device during the counting measurement. The associated cumulant generating function (CGF) $g(\varphi)$ reads

$$e^{g(\varphi)} = \sum_N P(N, t_c) e^{-iN\varphi},$$

where $\varphi$ is the counting field on a specified TDQD lead. All cumulants of the current can be obtained from the CGF by performing derivatives with respect to the counting field

$$\langle I^k \rangle = -\langle -i\partial_\varphi \rangle^k g(\varphi)|_{\varphi=0}. $$

The first three cumulants are related to the average current, the (zero-frequency) current noise, and the skewness, respectively.

To evaluate the CGF, let us consider $g(\varphi, t) \equiv \sum_N R^{(N)}(t)e^{iN\varphi}$. Its equation of motion reads

$$\dot{g}(\varphi, t) = L_\varphi g(\varphi, t).$$

The involving generator $L_\varphi$ is completely determined by the associated conditional master equation, which is similar to Eq. (3) but with the number of electrons $N$, $t_c$ is completely determined by $L_\varphi$, $\varphi$, $t_c$. The formal solution to Eq. (15) is $g(\varphi, t) = e^{L_\varphi t}g(\varphi, 0)$. Straightforwardly, the CGF is determined as $g(\varphi) = -\ln\langle \text{Tr} R^{(N)}(t_c) \rangle$. Actually, we are most interested in the zero-frequency limit, i.e. the counting time $t_c$ is much longer than the time of tunneling through the system. The CGF then simplifies to $g(\varphi) = \lambda_{\min}(\varphi)t_c$, where $\lambda_{\min}(\varphi)$ is the minimal eigenvalue of $L_\varphi$ that satisfies $\lambda_{\min}(\varphi \to 0) \to 0$. With the knowledge of CGF, the distribution function can be readily obtained via

$$P(N) = \int_0^{2\pi} d\varphi \frac{d\varphi}{2\pi} e^{-g(\varphi)-iN\varphi}. $$

V. RESULTS AND DISCUSSIONS

In what follows, we focus our analysis on electron tunneling from the left lead to QD1. The relevant $N$ is then the number of electrons entering the dot from left lead, and $\varphi$ is the corresponding counting field. Note the same calculations apply to the counting for the right lead. The numerical results for the probability distribution are plotted by the solid lines in Fig. 3. It shows a striking agreement with the histograms obtained by Monte Carlo simulation; thus, it also verifies the validity of our Monte Carlo method for real-time measurement. The two distributions in Fig. 3(a) and (b) are rather different. The latter shows a broader and more asymmetric distribution.
than the former. We characterize the differences quantitatively based on the FCS analysis, as follows.

Consider first the situation without electron-phonon interaction \((\eta = 0)\). The first current cumulant gives the average current \(I = 2\Gamma_L \Gamma_R / \Gamma_{\text{eff}}\), with \(\Gamma_{\text{eff}} = 4\Gamma_L + \Gamma_R\) the total effective tunneling width. It is independent of level detuning \(\epsilon\), interdot coupling \(\Omega\), and QPC charge detection induced dephasing \(\gamma_d\). In contrast to the current, valuable information can be extracted in the second cumulant (zero-frequency shot noise). By expressing it in terms of the Fano factor \(F \equiv \langle I^2 \rangle / \langle I \rangle\), we readily obtain

\[
F = 1 - \frac{8\Gamma_L \Gamma_R}{\Gamma_{\text{eff}}^2} + 2\Gamma_L^2 \Gamma_R (\gamma_d + \Gamma_R)^2 + 4\epsilon^2 (\gamma_d + \Gamma_R) \Gamma_{\text{eff}}^2 / \Gamma_{\text{eff}}^2. \tag{17}
\]

It thus allows us to get more knowledge than the current on the processes involved in the electronic transport. In Fig. 4(a), the Fano factor \(F\) is plotted against the level detuning \(\epsilon\) for different QPC-induced dephasing rates \(\gamma_d\).

Noticeably, a significantly enhanced Fano factor is expected when the dot levels are far from resonance. In this case, electron transitions between the two dots are suppressed. For instance, if an electron is tunneled into QD2, it will dwell on it for a long time. In the strong Coulomb blockade regime, the electron in QD2 will block the current until it is removed and tunneled out to the right lead. Consequently, a mechanism of dynamical channel blockade is developed, i.e. the noise is reduced with rising dephasing rate, particularly in the regime far from resonance. On the other hand, the measurement gives rise to the so-called quantum “Zeno” effect, which dominates in the regime of large dephasing rate, and results in a strong dynamic charge blockade behavior. The noise is finally enhanced with increasing dephasing rate, as we have checked (not shown in Fig. 4).

The Fano factor studied so far proves to be much sensitive than the average current, it only reveals, however, limited information about the QPC charge detection induced dephasing mechanism. We thus expect more information to be extracted in the next order cumulant, i.e. the skewness. The numerical results for the normalized skewness \(S \equiv \langle I^3 \rangle / \langle I \rangle\) is displayed in Fig. 4(b) as a function of level detuning \(\epsilon\). Without QPC charge detection induced dephasing, the skewness reaches the maximum when the dot levels are in resonance \((\epsilon = 0)\). As the dephasing rate grows, it turns into a local minimum. At the edges of the resonance a double maximum structure symmetric around \(\epsilon = 0\) is observed. The local maxima shifting away from resonance with increasing dephasing rates, as clearly demonstrated in Fig. 4(b).

Now let us turn to the influence of phonon heat bath that induces dephasing between two dots. We will reveal the essential difference between the dephasing induced via QPC charge detection and that by phonon coupling. The former can be modified via the coupling between the TDQD and the QPC, while the latter is generated with emission and absorption of phonons and increases with rising temperature. Here, we limit our discussions to the temperatures well below the Coulomb charging energies and the bias voltage. Therefore the temperature acts solely due to the coupling to the phonon heat bath.

The calculated Fano factor and normalized skewness versus level detuning for different QPC–induced dephasing rates. The electron charge is set to be \(\epsilon = 1\). The temperature is \(T = 100\text{mK}\), and the other parameters are the same as in Fig. 4 but in the absence of phonon bath; i.e., \(\eta = 0\).
creases with rising temperature, as displayed in Fig. 5(a).

If the TDQD is coupled only to the QPC, the cumulants are symmetric around $\epsilon = 0$, as shown in Fig. 4. However, with non–zero coupling to the heat bath the noise exhibits a clear asymmetry (see Fig. 5). This is essentially due to another consequences of phonon coupling: The phonon mediated transition can partially resolve the dynamical charge blockade. For instance, at zero temperature and $\epsilon > 0$, spontaneous phonon emission can partially lift the localization caused by dephasing, and the spectrum turns out to be asymmetric. Similar argument applies to the regime of finite temperature, where both phonon absorption and emission take place. The fact that emission is more likely than absorption explains the observed asymmetry.

The skewness of the current distribution, which was not explored in Ref. 24, is found to be more sensitive to thermal phonon bath–induced dephasing. Without coupling to the phonon bath, the skewness shows a symmetric double maximum structure. The spectral becomes asymmetric in the presence of electron–phonon interaction. With increasing temperature, the phonon absorption takes place more frequently at $\epsilon < 0$, and thus greatly enhance the local maximum. While in the opposite regime of $\epsilon > 0$ the local maximum is reduced due to suppressed phonon emission.

VI. SUMMARY

We have investigated electron transport through a T–shaped double quantum dot system by utilizing a quantum master equation approach. The major advantage of the present approach is its simplicity of treating properly the dephasing mechanism of the QPC charge detection and that due to an external heat bath. In addition, this approach has the merit of dealing with other sources of dephasing, such as that entailed by anti-resonances. Particularly, based on the Monte Carlo scheme, real-time detection of single electron tunneling is simulated by exploiting the sensitivity of a current, passing through a nearby quantum point contact, to the fluctuating charge on the quantum dots. Owing to the interplay between the Coulomb interactions and the dephasing mechanisms, a strong bunching behavior in the charge transfer was detected, which leads eventually to a super–Poissonian noise.

Furthermore, full counting statistics of the transport current is analyzed based on the probability distribution, which is determined by ensemble average over a large number of single trajectories. It is demonstrated that the dephasing mechanism of the QPC charge detection and that owing to the external heat bath give rise to distinct and intriguing features. It thereby enables us to achieve a clear identification of different dephasing sources. Investigations of various processes involved in the electronic transport through similar devices are highly desirable in experiments.

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