Bottom-Tau Unification in SUSY SU(5) GUT and Constraints from $b \rightarrow s\gamma$ and Muon $g-2$

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Abstract

An analysis is made on bottom-tau Yukawa unification in supersymmetric (SUSY) SU(5) grand unified theory (GUT) in the framework of minimal supergravity, in which the parameter space is restricted by some experimental constraints including Br($b \rightarrow s\gamma$) and muon $g-2$. The bottom-tau unification can be accommodated to the measured branching ratio Br($b \rightarrow s\gamma$) if superparticle masses are relatively heavy and higgsino mass parameter $\mu$ is negative. On the other hand, if we take the latest muon $g-2$ data to require positive SUSY contributions, then wrong-sign threshold corrections at SUSY scale upset the Yukawa unification with more than 20 percent discrepancy. It has to be compensated by superheavy threshold corrections around the GUT scale, which constrains models of flavor in SUSY GUT. A pattern of the superparticle masses preferred by the three requirements is also commented.

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I. INTRODUCTION

Unification of interactions of elementary particles \[1\] is a very appealing idea which has attracted much attention. Supersymmetric (SUSY) SU(5) Grand Unified Theory (GUT) \[2\] is a prototype of such, in which the standard model gauge group is unified into a SU(5) group, and quarks and leptons belong to 10 and 5 representations.

Gauge coupling unification is the most renowned and tested prediction of the SUSY GUT \[3\]. In fact renormalization group analyses show that the unification indeed occurs up to less than a few \% errors \[4\]. This means that one needs this amount of corrections coming from physics around the GUT scale and/or above, and it can easily be accounted for by superheavy thresholds around the GUT scale, which may include possible SU(5)-violating non-renormalizable contributions to the gauge coupling constants \[5\]. In this sense, the gauge coupling unification is very successful in the SUSY GUT scenario.

The grand unification generically has yet another prediction, *i.e.* the bottom-tau Yukawa unification \[6–10\]. There is some expectation that the bottom quark and the tau lepton which belong to the same SU(5) multiplet have a unified Yukawa coupling at the GUT scale. Apparent difference between the two Yukawa couplings are then due to renormalization group effects from that scale down to low energy. In addition, it is known that finite radiative corrections involving superparticles in the minimal SUSY standard model (MSSM) give important contribution to the bottom Yukawa coupling \[9\]. The sign and magnitude of the threshold corrections at the SUSY scale strongly depend on superparticle masses. On the other hand, the superparticle masses now suffer from several experimental constraints. They include mass bounds on the superparticles and Higgs boson from direct searches, an inclusive decay rate of \(b \rightarrow s\gamma\), and recent measurement of muon \(g - 2\) or muon anomalous magnetic moment \(a_\mu \equiv (g - 2)_\mu/2\).

In this paper, we shall study whether the Yukawa unification occurs under these experimental constraints. We perform a numerical analysis based on minimal supergravity (mSUGRA) and its slight modifications, but most of our results apply to other cases. We
will clarify how the Yukawa coupling unification depends on the superparticle mass spectrum and how it relates to the experimental constraints. We will show that for positive \( \mu \)-parameter where SUSY contribution to muon \( g - 2 \) is positive, threshold corrections at the SUSY scale make the Yukawa coupling of bottom quark more than 20 percentage smaller than that of tau. On the other hand, for negative \( \mu \)-parameter, the SUSY threshold corrections help achieve the bottom-tau unification, while the \( \text{Br}(b \to s\gamma) \) bound is satisfied for relatively heavy superparticles. This case may, however, be confronted with the latest muon \( g - 2 \) data, if its apparent deviation from the standard model prediction requires positive SUSY contribution.

Our analysis will provide a useful guide for building models of flavor. Deviation from the Yukawa unification, if any, has to be compensated by corrections at high energy scale around the GUT scale, if one insists the view of grand unification. The high energy corrections will be model dependent and thus the issue of the Yukawa unification will help discriminate models of flavor in SUSY GUT.

Recently, similar analyses were presented in Refs [11]. Their concern is the top-bottom-tau Yukawa unification in SO(10) GUT and thus they focused on \( \tan \beta \gtrsim 50 \). In this paper, we investigate wider regions of the parameter space since we are interested in general SUSY GUT models.

The organization of the paper is the following. In section 2, we will briefly review some issues on the bottom-tau unification. In section 3, we will explain the experimental constraints used in our analysis. Our numerical analysis is given in section 4. We will summarize our results in section 5.

**II. BOTTOM-TAU YUKAWA UNIFICATION**

In this section, we review some of the important issues on the bottom-tau Yukawa unification. In the SU(5) GUT, the SU(2) doublet bottom and the singlet tau belong to \( \Phi(10) \) and the singlet bottom and the doublet tau (combined with tau neutrino) belong to \( \Psi(\bar{5}) \).
If the Yukawa coupling comes from the Higgs $\bar{H}(5)$ as

$$\mathcal{L} = y\Phi(10)\bar{\Psi}(\bar{5})\bar{H}(5)$$  

then the bottom and tau have a unified Yukawa coupling $y$. It is the one given at the GUT scale, and renormalization group (RG) effects mainly due to QCD make the bottom Yukawa coupling much larger than the other one. In RG, large Yukawa couplings of top and bottom quarks play an important role to somewhat lower the bottom Yukawa coupling. It was recognized some time ago, the QCD RG effect is too strong, predicting a too large bottom quark mass, unless either of the Yukawa couplings are very large, i.e. $\tan \beta$ is close to unity, or very large $\gtrsim 50$. Here $\tan \beta \equiv \langle H_2 \rangle/\langle H_1 \rangle$ is the ratio of the vacuum expectation values (VEVs) of the two Higgs bosons $H_1$ and $H_2$. It was also realized that there exist large threshold corrections at the SUSY scale, namely finite radiative corrections involving superparticles in the MSSM give significant contributions to the bottom mass. This is because in the SUSY limit only one of the Higgses $H_1$ has Yukawa coupling to the bottom quarks while SUSY breaking allows a new coupling of the other Higgs $H_2$ to the bottom quarks. The threshold corrections can be enhanced especially for large $\tan \beta$. They include two main contributions: one from a gluino loop and the other from a chargino loop. They are approximately written as

$$\frac{\delta m_{\tilde{b}}}{m_b} \approx \frac{2\alpha_3}{3\pi} M_3 \mu I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, M_3^2) \tan \beta, \quad (2)$$

$$\frac{\delta m_{\tilde{\chi}^\pm}}{m_b} \approx -\frac{y_t^2}{16\pi^2} \mu A_t I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2) \tan \beta, \quad (3)$$

where $\alpha_3$ is the strong coupling constant, $y_t$ is the top Yukawa coupling, $M_3$ is the gluino mass, $\mu$ is the supersymmetric Higgsino mass, $A_t$ is the trilinear SUSY-breaking coupling of the stops to $H_2$, $m_{\tilde{b}_{1,2}}, m_{\tilde{t}_{1,2}}$ are sbottom and stop mass eigenvalues, respectively. The function $I$ is given

$$I(x, y, z) = \int_0^\infty \frac{udu}{(u + x)(u + y)(u + z)}, \quad (4)$$

which is characterized by $I(x, x, 0) = 1/x$, $I(x, x, x) = 1/2x$. 

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Of the two threshold corrections the gluino loop dominates generically unless the trilinear coupling $A_t$ is very large. Note that minimal supergravity (mSUGRA) boundary conditions give roughly

$$A_t \approx 0.2 A_0 + 2 M_0$$

(5)

where $A_0$ is the universal trilinear coupling given at the GUT scale and $M_0$ is the universal gaugino mass at the GUT scale. The small coefficient in the first term is due to that the top Yukawa coupling is close to its infra-red fixed point. Eq. (5) implies that the trilinear stop coupling $A_t$ is mostly controlled by the universal gaugino mass and thus related to the gluino mass as far as $A_0$ and $M_0$ are in the same order of magnitude.

In this generic case, the sign of the SUSY threshold corrections is solely determined by the sign of $\mu M_3$. Recalling that the RG effects tend to give too large bottom quark mass, we find the naive Yukawa unification at the GUT scale requires this sign to be negative. This argument given here is rather general, though we will focus on the mSUGRA and its variants in our numerical analysis.

Another important point to the SUSY threshold corrections is that they are non-decoupling effects. In fact taking all superparticle masses infinity gives a finite contribution. We will see the importance of this property later on.

We use the $\overline{MS}$ mass for the bottom quark in the following range

$$m_{\overline{MS}}(m_b) = 4.2 \pm 0.2 \text{ GeV}.$$ 

(6)

Note that recent analysis based on perturbative QCD yields an estimate of the bottom mass with a much smaller error. At this moment, however, other errors coming mainly from the uncertainty of the top quark mass have comparable effects as Eq. (6), so that reducing the error of the bottom mass alone does not reduce the errors in the estimate of the bottom-tau unification. It is important to note, however, that the pinpoint evaluation of the bottom quark mass will become important when future experiments reduce the error of the top mass.
III. EXPERIMENTAL CONSTRAINTS

Here we summarize experimental constraints used in our analysis.

Null results of superparticle searches and Higgs boson searches give significant constraints on the allowed parameter space. As for the mass bounds of superparticles, we impose $m_{\tilde{\chi}^-_1} > 100\text{GeV}$ and $m_{\tilde{\tau}^-} > 80\text{GeV}$. The Higgs mass bound $m_h > 113.5$ GeV from the LEP II [16] is also important, which constrains $\tan \beta$ and the scale of superparticle masses.

Another important constraint comes from $b \to s\gamma$. The inclusive branching rate $\text{Br}(b \to s\gamma)$ is estimated from experimental measurements as $(3.21 \pm 0.43 \pm 0.27^{+0.18}_{-0.10}) \times 10^{-4}$, where the errors are of statistical, systematic and theoretical, respectively [15]. It is known that besides the standard model contribution which is consistent with the experimental value, SUSY gives two additional contributions, one from a charged Higgs loop, the other from superparticle loops. The former always gives an additive contribution to the standard model. The latter is dominated by a chargino/stop loop, which can have either sign, depending on the sign of $A_t\mu$. It turns out that the new contributions tend to cancel for $A_t\mu \propto M_3\mu > 0$. This case is preferred if we recall that the standard model contribution already explains the experimental data. On the other hand, when $M_3\mu < 0$ both the two contributions have the same sign as the standard model contribution. To survive the experimental constraint, the superparticle mass scale should be large enough, so that all the additional contributions decouple. Note that this decoupling behavior is contrasted to the case of the bottom quark mass in which the SUSY contribution remains finite in the decoupling limit.

In the following, we conservatively take the allowed range for $\text{Br}(b \to s\gamma)$ to be

$$2 \times 10^{-4} \leq \text{Br}(b \to s\gamma) \leq 4.5 \times 10^{-4},$$

(7)

to constrain the parameter space.\footnotemark

\footnotetext{We should keep in mind that the $b \to s\gamma$ process changes flavor, and thus non-trivial generation mixing in squark masses coming from unknown flavor physics may change our estimate of the
The other constraint we consider in this analysis is $a_\mu$. The latest result of the E821 experiment at Brookhaven National Laboratory reported a possible deviation from the standard model by 2.6-$\sigma$ [17]:

$$a_\mu(E821) - a_\mu(SM) = 43(16) \times 10^{-10}.$$ (8)

This may be a signal of new physics beyond the standard model. If it is all accounted for by supersymmetry, the SUSY contribution $a_\mu(SUSY)$ should be positive, and of order $10^{-9}$ [18]. In particular the positiveness requires $M_3\mu > 0$. In our analysis, we will include it as an possibly important constraint, though it may be premature to conclude that the latest data is a clear evidence of new physics, when one takes into account statistical significance and uncertainties in the evaluation of the standard model contributions.

IV. ANALYSIS

In this section, we would like to present our numerical results in the mSUGRA case.

A. Procedure

What we will do is to compute the Yukawa couplings of the bottom quark and the tau lepton at the GUT scale, by using their values at low energy scale to extrapolate them at the GUT scale following renormalization group flow.

We take $\alpha_3^{\overline{MS}}(M_Z) = 0.118$, and include SUSY threshold effects to obtain $\alpha_3^{\overline{DR}}(M_Z)$ [19].

We fix $\overline{DR}$ Yukawa couplings at $M_Z$ scale as follows. Having fixed $m_b^{\overline{MS}}(m_b)$, we evolve the running mass from $m_b$ to $M_Z$ using the three-loop $\overline{MS}$ RGE’s for the Standard Model. Next we convert $m_b$ to the $\overline{DR}$ scheme using a one-loop correction factor [20]. Then we include SUSY threshold effects explained before to extract the bottom Yukawa coupling in supersymmetric limit. As for the tau Yukawa coupling, we use $m_{\tau}^{\overline{MS}}(M_Z) = 1.746$ GeV branching ratio.
which is obtained from the pole mass with two-loop QED corrections [21]. For the top Yukawa, we take the pole mass $m_{t}^{pole} = 175\text{GeV}$. We will discuss how change of the top Yukawa affects our results later on.

To obtain the gauge and Yukawa couplings at the unification scale, we use two-loop level RGE’s for the minimal supersymmetric standard model (MSSM) [22]. We define $M_{\text{GUT}}$ to be a point where $\alpha_1$ and $\alpha_2$ meet. Here it is convenient to define $\Delta_{b-\tau} \equiv (Y_b(M_{\text{GUT}}) - Y_{\tau}(M_{\text{GUT}}))/Y_b(M_{\text{GUT}})$ to characterize how well the Yukawa unification achieves.

To evaluate the SUSY threshold corrections, the Higgs mass, $\text{Br}(b \to s\gamma)$ and SUSY contribution to $a_\mu$, we need to know the SUSY breaking masses at the weak scale. In our analysis, these are computed by using one-loop level RGE’s for the MSSM. (For the gauge and Yukawa couplings, we use two-loop level RGE’s.) We evaluate the parameter $\mu$, the masses of the charged Higgs and the pseudoscalar Higgs at the energy scale of a geometrical mean of stop masses $\sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}}$ with one-loop effective potential. We calculate the lightest Higgs boson mass using an approximate formula including two-loop corrections, $\text{FeynHiggsFast}$ [25]. Since the bottom Yukawa coupling receive the large SUSY threshold, we have to take this effect into account to calculate $\text{Br}(b \to s\gamma)$ in the large $\tan\beta$ case. We follow the NLO formalism that can be applicable to large $\tan\beta$ case [23]. And for the SUSY contribution $\Delta a_\mu$ to the muon anomalous moment, we use the formula in the one loop approximation [24].

\[ ^2 \text{At the } M_{\text{GUT}}, \alpha_3 \text{ is smaller than } \alpha_1 \text{ and } \alpha_2 \text{ by up to } 4 \%. \text{ We do not care this small discrepancy here.} \]

\[ ^3 \text{This approximate result can be different from the exact one by up to } 2 \text{ GeV} \text{ [23]. The lower limit on the SUSY scale from the Higgs boson mass constraint changes by a few tens GeV because of this uncertainty.} \]
B. Supergravity Boundary Conditions

In our analysis, we give the boundary conditions of the soft SUSY breaking masses at the GUT scale, and discard possibly large contributions above it [26]. At the GUT scale, the scalar masses of 10, 5 representations of SU(5) $m^2_{\phi}$, $m^2_{\psi}$, respectively, and the Higgs boson masses are assumed to be in the form

$$m^2_{\phi}(M_{GUT}) = m^2_{\phi0}, \quad m^2_{\psi}(M_{GUT}) = m^2_{\psi0}, \quad m^2_{H_1}(M_{GUT}) = m^2_{H_{10}}, \quad m^2_{H_2}(M_{GUT}) = m^2_{H_{20}}.$$  

(9)

As for trilinear couplings, we take the following form,

$$A_u(M_{GUT}) = A_d(M_{GUT}) = A_e(M_{GUT}) = A_0.$$  

(10)

Three gaugino masses in the MSSM are unified to $M_0$ at the GUT scale. We take a convention that $M_0 > 0$. By solving the RGE’s and imposing the radiative electroweak symmetry breaking, one obtains all SUSY parameters at the weak scale. For later convenience, we give approximate formulas of the Higgsino mass $\mu$, scalar masses in the third generation and the charged Higgs mass since SUSY contribution to the muon $\Delta a_{\mu}$, $b \rightarrow s\gamma$ and the lightest Higgs mass depend on these masses. For $\tan \beta \lesssim 10$ where effects of the bottom and tau Yukawa couplings to RG running of scalar masses are neglected, one finds,

$$\mu^2 \simeq -0.64m^2_{H_{20}} + 0.72m^2_{\phi0} + 2.7M_0^2 + 0.42M_0A_0,$$  

(11)

$$m^2_{H^-} \simeq m^2_{H_{10}} - 0.64m^2_{H_{20}} + 0.72m^2_{\phi0} + 3.2M_0^2 + 0.42M_0A_0,$$  

(12)

$$m^2_{t_L} \simeq 0.76m^2_{\phi0} - 0.12m^2_{H_{20}} + 5.5M_0^2 - 0.14M_0A_0,$$  

(13)

$$m^2_{t_R} \simeq 0.52m^2_{\phi0} - 0.24m^2_{H_{20}} + 3.9M_0^2 - 0.28M_0A_0.$$  

(14)

Sbottom and stau masses are the same as the first two generations. In the mSUGRA case where all scalar masses are taken to be universal, $m^2_{\phi0} = m^2_{\psi0} = m^2_{H_{10}} = m^2_{H_{20}} = m^2_0$, we find

$$\mu^2 \simeq 0.08m_0^2 + 2.7M_0^2 + 0.42M_0A_0,$$  

(15)

$$m^2_{H^-} \simeq 0.36m_0^2 + 3.2M_0^2 + 0.42M_0A_0.$$  

(16)
These equations show that coefficients in front of $m^2_0$ get small, especially for $\mu^2$, because of top Yukawa effects. Hence in the mSUGRA case these values are mostly determined by the universal gaugino mass, and larger than the Bino mass $M_1 \simeq 0.4M_0$ or Wino mass $M_2 \simeq 0.8M_0$.

**C. Results**

Here we present our numerical results of the bottom-tau unification, $\Delta_{b-\tau} = (Y_b(M_{GUT}) - Y_\tau(M_{GUT}))/Y_b(M_{GUT})$ under the constraints from the Higgs mass, $b \to s\gamma$, and $a_\mu$ in the mSUGRA case.

Firstly we would like to discuss how the bottom-tau unification as well as the experimental constraints are sensitive to the sign of the $\mu$ parameter. For illustration, we take $\tan \beta = 30$, $A_0 = 0$.

In Fig. 1, we draw contours on the bottom-tau unification for negative $\mu$. We find that the unification achieves within 5 percent in the large region of the parameter space. Notice that the present uncertainties of the bottom mass and the top quark mass will generate the error of about 5 percent, and also we expect that a few percentage superheavy thresholds are easily obtained. Thus in this case, we conclude that the bottom-tau unification is successful.

A large part of the parameter space is, however, eliminated by the experimental constraints. The Higgs mass bound is satisfied when the gaugino mass $M_0$ is larger than about 300 GeV, while the requirement from $\text{Br}(b \to s\gamma)$ is a bit severer. In fact it requires relatively heavy superparticles, for example, $M_0 \gtrsim 700$ GeV for small scalar mass, or $m_0 \gtrsim 1,200$ GeV for small gaugino mass. This is because, in the negative $\mu$ case, the SUSY contribution as well as the charged Higgs contribution to the $b \to s\gamma$ process are additive to the standard model, and thus these contributions have to decouple in order to be consistent with the
experimental data. Note that even with such heavy superparticle masses the bottom-tau unification can be explained as one sees in Fig. 1.

On the other hand, the SUSY contribution to the muon anomalous magnetic moment is always negative in this case, which is disfavored by the data from the E821 experiment. Thus if $a_\mu$ requires a positive SUSY contribution, then the $\mu < 0$ case is ruled out. More conservatively if one allows, for instance, 3-$\sigma$ deviation from the central value of their result, the $\mu < 0$ case is allowed. In Fig. 1, we plot a contour of the SUSY contribution $\Delta a_\mu = -5 \times 10^{-10}$, corresponding to the 3-$\sigma$ deviation. This gives a much severer constraint than $b \rightarrow s\gamma$, and the bottom-tau unification (within 5 %) is achieved only for very heavy superparticle mass spectrum in which sleptons, for instance, weigh much more than 1 TeV.

Next we will consider the positive $\mu$ case. Compared to the negative $\mu$ case, the Higgs mass bound gives more or less a similar constraint, while the constraint from $b \rightarrow s\gamma$ becomes much weaker. This is because the new contributions tend to cancel each other, and thus a wider region gets allowed than the previous case. The SUSY contribution to $a_\mu$ is positive, which can easily be accommodated to the E821 data. Now the bottom-tau unification becomes in bad shape, namely the exploration of the Yukawa couplings from the low energy data to the GUT scale using the RGE’s with the SUSY threshold included yields discrepancy of typically more than 20 percent. In fact for $\mu > 0$ the SUSY threshold to the bottom quark mass is always positive, which makes the bottom Yukawa in the SUSY limit small. This in turn gives a too small bottom Yukawa at the GUT scale after the renormalization group flow.

Let us next discuss how the bottom-tau unification depends on $\tan \beta$. To see this, we randomly generated parameters in the range $100$GeV $\leq m_0 \leq 1500$GeV, $100$GeV $\leq M_0 \leq 1500$GeV, and required that $m_h \geq 113.5$GeV, $m_{\tilde{\chi}^\pm} \geq 100$GeV, $m_{\tilde{\tau}} \geq 80$GeV, $2 \times 10^{-4} \leq \text{Br}(b \rightarrow s\gamma) \leq 4.5 \times 10^{-4}$. In Fig. 3, we plot parameter points which survive the constraints given above for the positive $\mu$ case. Points marked with square ($\Box$) are those in which $\Delta a_\mu$ is consistent with the result of the E821 experiment at the 2$\sigma$ level. The upper end of the band corresponds to heavier SUSY scale. We find the large discrepancy from the naive
expectation of the bottom-tau unification ($\Delta_{b-\tau} = 0$), especially for large $\tan \beta$. Notice that the points which satisfy the 2-$\sigma$ constraint on $a_{\mu}$ tend to give larger deviation in the bottom-tau unification. This can be understood in the following way. Recall that the dominant SUSY contribution to $a_{\mu}$ comes from sneutrino-chargino loop. To make the SUSY contribution large, we therefore require that the sneutrino is relatively light, which means that in the context of the mSUGRA the sbottoms and the gluino cannot be very heavy. This results in a large SUSY threshold correction, which makes the bottom-tau unification worse.

The discrepancy should be explained by high energy thresholds, such as threshold corrections by superheavy GUT-particles and/or effects from non-renormalizable operators. It is highly model dependent whether we can get more than 20 percentage corrections and thus this will provide a critical test of models of flavor in the SUSY GUT.

A similar plot is given for $\mu < 0$ in Fig. 4. For $\tan \beta \lesssim 35$, the bottom-tau unification works well within 10%. In this case as we anticipate, there is no point which give the 2-$\sigma$ $\Delta a_{\mu}$. There are, however, points where $\Delta a_{\mu}$ is consistent with the E821 result within 3-$\sigma$ level, which are marked with circle ($\circ$) in Fig. 4.

So far we have discussed the mSUGRA case. One characteristic feature of the mSUGRA model is that $\mu$ is almost determined by $M_0$ and is insensitive to $m_0$, as far as $m_0$ and $M_0$ is in the same order of magnitude. But once we relax the mSUGRA boundary conditions and suppose, for instance, that the Higgs boson masses and the sfermion masses are different at the GUT scale, then the situation can change. For example, we consider the case with the following boundary conditions

$$m_{\psi}^2(M_G) = m_{\phi}^2(M_G) = m_0^2 I, \quad m_{H_u}^2(M_G) = m_{H_d}^2(M_G) = (1.5m_0)^2. \quad (19)$$

Then $\mu$ parameter, the charged Higgs mass and stop masses are approximately estimated as

$$\mu^2 \simeq -0.72m_0^2 + 2.7M_0^2 + 0.42M_0A_0, \quad (20)$$

$$m_{H^+}^2 \simeq 1.5m_0^2 + 3.2M_0^2 + 0.42M_0A_0, \quad (21)$$
Comparing to the mSUGRA case, we find that $\mu^2$ can be smaller, thanks to the negative coefficient in front of $m_0^2$. Since the SUSY threshold correction to the bottom mass is proportional to $\mu$, it will be suppressed. The light Higgsino mass, on the other hand, does not suppress $\Delta a_\mu$ since a dominant SUSY contribution to it comes from a chargino/sneutrino loop in which both the Wino and Higgsino propagate. In addition, the Higgs mass and $b \rightarrow s\gamma$ constraints do not change so much since the charged Higgs boson mass and the stop mass mostly depend on the universal gaugino mass as the mSUGRA case. A numerical result for $\mu > 0$ is shown in Fig. 5. We find that the difficulty of the bottom-tau unification for the positive $\mu$ case is somewhat ameliorated, i.e. the discrepancy can be as small as 15%.

Finally we wish to comment on uncertainties coming from the bottom mass and the top mass. As was discussed in a previous section, we conservatively took the error of the bottom mass estimate to be 5%. This error almost linearly reflects the uncertainty in the bottom-tau unification. On the other hand, the uncertainty from the top mass is more subtle. We shifted the pole mass of the top quark of 175 GeV by $\pm$5 GeV, and found that the resulting change in the bottom-tau unification is up to 5%. These uncertainties should be kept in mind when discussing this issue.

**V. CONCLUSIONS AND DISCUSSION**

In this paper, we have analyzed the bottom-tau unification in the SUSY SU(5) scenario. The sign of the SUSY threshold corrections plays an important role. It is determined by the sign of the $\mu$ parameter, in the mSUGRA scenario where the gaugino masses have the same

\[ m^2_{\tilde{t}_L} \simeq 0.49m_0^2 + 5.5M_0^2 - 0.14M_0A_0, \quad (22) \]

\[ m^2_{\tilde{t}_R} \simeq -0.02m_0^2 + 3.9M_0^2 - 0.28M_0A_0 \quad (23) \]

\footnote{Notice that the shift of the top quark mass also affects the lightest Higgs boson mass by about 10 GeV. Hence for $m^\text{pole}_t = 180$ GeV, a lower $\tan \beta$ will be allowed.}
sign and the SUSY-breaking trilinear coupling of stop-stop-Higgs $A_t$ is strongly correlated
with the sign of the gluino. In fact the bottom-tau unification works well for $\mu < 0$. On the
other hand, the consistency of the measured branching ratio of $b \rightarrow s \gamma$ with the standard
model prediction implies that the additional contributions in the MSSM should be small.
This favors the case where the charged Higgs contribution is partially cancelled with the
stop/chargino loop contribution, which occurs in the positive $\mu$ case. We showed that the
bottom-tau unification is in accord with the $b \rightarrow s \gamma$ constraint for $\mu < 0$ if the superparticle
masses are relatively heavy. In this case the SUSY contributions to the $b \rightarrow s \gamma$ process
decouples. Since the SUSY threshold to the bottom mass does not have this decoupling
property, the bottom-tau unification does not occur in $\mu > 0$.

Inclusion of the measurement of $a_\mu$ at the E821 experiment makes the bottom-tau uni-
ification difficult. The positive SUSY contribution to $a_\mu$ suggested by the experiment favors
$\mu > 0$, opposite to the sign preferred by the Yukawa unification. In fact, we showed that
the discrepancy of the unification is larger than 20 % for $\mu > 0$ in the mSUGRA. The shift
of the soft mass for Higgses does not significantly improve the situation. Thus we conclude
that the bottom-tau unification in the mSUGRA-type SUSY breaking will conflict with the
$a_\mu$ measurement, if the deviation is confirmed in future.

Before closing, we would like to discuss what kind of SUSY breaking pattern is needed
to explain all three, namely the bottom-tau unification, $b \rightarrow s \gamma$, and the discrepancy of $a_\mu$.
We infer that the following superparticle mass spectrum will be satisfactory:

- The hypothesis of the universal gaugino mass is abandoned, and the gluino mass and
the Wino mass should have an opposite sign.

- To obtain a sizable effect to $a_\mu$, the sneutrino and the charginos are relatively light.

- To suppress the additional contributions to $b \rightarrow s \gamma$, we need the heavy charged Higgs
and the heavy stop. This may be realized if the soft mass to the Higgs is large and
different from the other scalar masses, and the gluino is heavy so that the stop acquires
mass via the renormalization group effect.
To illustrate the first point, a plot is given in Fig. 6 for the case that the sign of the gluino mass is opposite to the Bino and the Wino mass, \( M_1(M_{GUT}) = M_2(M_{GUT}) = -M_3(M_{GUT})(> 0) \), while all scalar masses are universal at the GUT scale. In this case the bottom-tau unification can be achieved for \( M_3 \mu < 0 \) while the E821 result can be explained by SUSY effects since \( M_2 \mu > 0 \). The negative sign of the gluino mass results in negative value of \( A_t \), therefore the \( b \to s \gamma \) constraint becomes severer. However, as shown in Fig. 6, we find a region where all constraints are satisfied, in which the muon \( g - 2 \) is consistent with the E821 result at 2-\( \sigma \) level, and the bottom-tau unification is achieved within 10 %. If we impose 1-\( \sigma \) constraint on \( a_\mu \), however, the allowed region will disappear. It is interesting to note that future improvement of the \( a_\mu \) measurement may be able to eliminate the allowed region \([27]\).

We expect that this type of the soft masses may be provided in some classes of mediation mechanisms of SUSY breaking, including the case where SUSY is broken in an SU(5) violating 3-brane and thus non-universal gaugino masses may arise \([28]\). Further discussion along this line should be encouraged in particular when the deviation in \( a_\mu \) is confirmed in future.

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FIG. 1. The solid lines are contours of $\Delta_{b-\tau}$ on $m_0$ s $M_{1/2}$ plane for $\tan \beta = 30$ and $\mu < 0$ with mSUGRA boundary condition. The lightly shaded region is the parameter space where $\text{Br}(b \to s\gamma) > 4.5 \times 10^{-4}$ and thus excluded by the experimental results. $m_h = 113.5$ GeV line is shown as the dotted line. The region above the dashed line is consistent with the E821 result at 3-$\sigma$ level, i.e., $\Delta a_\mu > -5 \times 10^{-10}$. Electroweak symmetry breaking does not occur correctly in the darkly-shaded region.

FIG. 2. Contours of $\Delta_{b-\tau}$ for $\tan \beta = 30$ and $\mu > 0$. In the lightly-shaded region $\text{Br}(b \to s\gamma) < 2 \times 10^{-4}$, thus this region is excluded. The dashed lines are for $\Delta a_\mu = 11 \times 10^{-10}$, $\Delta a_\mu = 43 \times 10^{-10}$ and $\Delta a_\mu = 75 \times 10^{-10}$ from the bottom-left.
FIG. 3. Plots of $\Delta b_{-\tau}$ versus $\tan \beta$ for the positive $\mu$ case with mSUGRA boundary condition. All points are consistent with the lower limit of the lightest Higgs mass, chargino mass, stau mass, and the $b \to s\gamma$ result. Points marked with square are consistent with the E821 result at 2-$\sigma$ level.

FIG. 4. Same plot as the Fig. 3, for the negative $\mu$ case. In this figure, points marked with circle are consistent with the E821 result at 3-$\sigma$ level.
FIG. 5. Same as Fig. 3 except that $m^2_{H_1} = m^2_{H_2} = (1.5 m_0)^2$, $m^2_{\phi} = m^2_{\psi} = m^2_{0}$ at the GUT scale. The plot is for $\mu > 0$.

FIG. 6. Same as Fig. 3 except that $M_1(M_{GUT}) = M_2(M_{GUT}) = -M_3(M_{GUT})$ at the GUT scale. The soft scalar masses are assumed to be universal at the GUT scale and $\mu > 0$. 