Comparative Study of Optimization of Plate Fin Heat Exchanger and Pressure Vessel Design using mTLBO Algorithm

A S Pendse1* and A G Kamble2

1Student, School of Mechanical and Civil Engineering, MIT Academy of Engineering, Alandi, Pune, Maharashtra, India
2Associate Professor, School of Mechanical and Civil Engineering, MIT Academy of Engineering, Alandi, Pune, Maharashtra, India
*Corresponding author. E-mail: aspendse@mitaoe.ac.in
*Corresponding author. Contact: 9370186299

ABSTRACT

Teaching Learning Based Optimization (TLBO) algorithm has been proved beneficial in many engineering applications. This algorithm is free from any algorithm specific parameters and can be adapted to all types of design problems. However, there are some drawbacks like convergence to local optimal solution, large computational time and slow convergence rate for complex functions. Some modifications were introduced to overcome these drawbacks in modified Teaching Learning Based Optimization (mTLBO) algorithm. In this paper mTLBO has been applied to optimize plate fin heat exchanger and pressure vessel design. The performance of mTLBO algorithm was compared with original algorithm and other population based techniques such as Particle Swarm Optimization, Generic Algorithm and Artificial Bee Colony. It was found that mTLBO gives the least value of entropy generation units that is 7.22% less than the value obtained using TLBO. Also cost of pressure vessel design using mTLBO is 3.2% lower than that of TLBO design.

KEYWORDS

Plate fin heat exchanger, Pressure vessel design, Teaching Learning Based Optimization, mTLBO.

1. INTRODUCTION

Optimization is the selection of parameters within range to get the best solution possible while satisfying all constraints. It is used in various fields like engineering, medicine, economics, etc. There
are many techniques available for optimization. Linear programming, non-linear programming, quadratic programming, dynamic programming, geometric programming, generalized reduced radiant method, etc. are some of the Traditional optimization techniques. These algorithms are commonly used for solving simple optimization problems. But, whenever we fail to solve any problem with traditional techniques, we find new ways to solve such problems. Many researchers have developed algorithms based on natural phenomenon. Genetic Algorithm (GA) is inspired by Charles Darwin’s theory of natural evolution. This algorithm uses the survival of the fittest criterion [1]. Evolution Strategies (ES) also use principles of natural evolution [2]. Simulated Annealing (SA) is based on simulation of the annealing of solids and the problem of solving large combinatorial optimization problems [3]. Ant Colony Optimization (ACO) is based on pheromone-based communication of biological ants [4]. There are some other non-traditional algorithms like Differential Evolution (DE), Particle Swarm Optimization (PSO), Evolutionary Programming (EP), etc. [5].

Many evolutionary algorithms have been proved good for solving optimization problems. PSO was used to optimize various types of heat exchangers [6]. Artificial Bee Colony (ABC) algorithm was used to optimize mechanical draft counter flow wet-cooling tower [7]. The performance of these algorithms greatly depends upon their own algorithm-specific parameters. For example, PSO requires tuning of inertia weight and cognitive and social parameters, GA require tuning of mutation probability, selection operator and cross-over probability, etc. Also there are other issues like computation time and cost, convergence rate, population or swarm size, etc. are of much concern. There were several modifications and variants made to overcome these problems.

TLBO algorithm was proposed to overcome parameter dependency of other algorithms [8]. Main advantage of this algorithm is that, it is independent of any algorithm specific parameters. It was successfully used for optimization of various engineering applications like casting process, flat plate solar air heater sterling engine, etc. Recently TLBO algorithm was used for thermo-economic analysis and optimization of a solar micro CCHP [9]. Also TLBO along with DE was used for optimization of critical parameters of PEM fuel cell [10]. However, there are some drawbacks of this algorithm like convergence to local optimal solution, slow convergence rate and large computational time for complex functions. Therefore, some modifications in the original algorithm were suggested in mTLBO algorithm [11]. In this paper mTLBO is applied to Plate Fin Heat Exchanger design and Pressure Vessel Design. These two problems are earlier solved using various techniques. The results obtained using mTLBO are compared with basic TLBO and some other population based algorithms. Also, working of TLBO [12] and mTLBO is explained briefly. MATLAB software is used for the programming. This paper is organized as follows: Section 2 provides the basic idea of TLBO
algorithm while section 3 explains the modification made in TLBO algorithm. Design problems and results are discussed in sections 4 and 5 respectively. Finally, paper is concluded in section 6.

2. TEACHING LEARNING BASED OPTIMIZATION

TLBO method is based on a relationship between teacher and students. Sample population is considered for optimization which can be considered as a group of students. Various design variables are considered for evaluation, similar to different subjects taught in a class. The best solution so far is analogous to Teacher in TLBO. Working of TLBO can be divided into 5 parts:

[i] Function Definition
[ii] Initialization
[iii] Teacher Phase
[iv] Learner Phase
[v] Algorithm Termination

2.1 Function Definition

Mathematical modelling of given problem is required for its optimization. Population size (N) is determined according to complexity of problem, number of design variables (D), constraints (G), etc. Population size and number of design variables are analogous to number of students and number of subjects respectively.

2.2 Initialization

Population of i rows and j columns is generated randomly using equation 1:

\[ X_{(i,j)} = X_{j}^{min} + \left( X_{j}^{max} - X_{j}^{min} \right) \ast rand \]  \hspace{1cm} (1)

Where, \( rand \) represents a uniformly distributed random variable within the range(0,1), \( X_{j}^{min} \) and \( X_{j}^{max} \) represent minimum and maximum value for \( j^{th} \) parameter respectively. Initial solution is obtained by using these values in objective function and constraints.


2.3 Teacher Phase

Mean of each subject is calculated. The learner which gives the least objective function value (for minimization problem) is considered as the teacher for respective iteration. In this phase mean of learners is shifted towards their teacher. Difference mean is calculated using following equation 2:-

\[
\text{Diff}_{-}\text{mean}_X(i,j) = r \left( \text{Teacher}_X(j) - \text{mean}_X(j) \right)
\]  

(2)

This difference mean is added to respective \(X(i,j)\) and new function values are obtained. Initial solution and newly obtained solution is compared and the best function values are selected. This modified solution is the Teacher Phase solution.

2.4 Learner Phase

In this phase the learners interact with one another. The process of mutual random interactions tends to improve his or her knowledge. For a given learner \(X^g(i)\), another learner \(X^g(r)\) is randomly selected\((i \neq r)\). The \(i^{th}\) parameter of the matrix \(X_{new}\) in the learner phase is given by equation 3

\[
X_{new}^g(i) = X^g(i) + \text{rand} \ast \left( X^g(i) - X^g(r) \right) \text{ if } X^g(i) < X^g(r)
\]  

\[
\text{or}
\]

\[
X_{new}^g(i) = X^g(i) + \text{rand} \ast \left( X^g(r) - X^g(i) \right) \text{ if } X^g(r) < X^g(i)
\]  

(3)

New function values are obtained and newly obtained solution is compared with teacher phase solution and the best function values are selected. This modified solution is the Learner Phase solution.

2.5 Algorithm Termination

After some iteration, final solution is obtained and algorithm is terminated.

3. MODIFIED TLBO

In this algorithm teacher phase remains same. Only learner phase is modified. Concept of tutorials given by teacher to students is used to improve the final solution. TLBO may not always give global optimum solution. It sometimes prematurely converges to local optimum solution. To obtain global
solution author added an extra term in teacher phase equation 3. This extra term is analogous to the tutorials in a class. In normal teaching learning scenario, students learn from their teacher and also through interaction amongst them. But, if tutorials are used, students may learn better. Similar concept is used in mTLBO. Learner phase of TLBO is modified by incorporating an extra term representing tutorials. Hence, \( X_{new} \) can be obtained using equation 4,

\[
X_{new}^{(i)} = X^{(i)} + \text{rand} \times \left( X^{(r)} - X^{(i)} \right) + 0.5 \times (1 + \text{rand}) \times \left( X_{Teacher}^{g} - X^{(i)} \right) \text{ if } X^{(i)} < X^{(r)}
\]

or

\[
X_{new}^{(i)} = X^{(i)} + \text{rand} \times \left( X^{(r)} - X^{(i)} \right) + 0.5 \times (1 + \text{rand}) \times \left( X_{Teacher}^{g} - X^{(i)} \right) \text{ if } X^{(i)} < X^{(r)}
\]

Where, \( X_{Teacher}^{g} \) is the minimum objective function value after teacher phase. This added term helps in obtaining global solution.

4. DESIGN PROBLEMS

4.1 Plate Fin Heat Exchanger

Standard application is considered for optimization [13]. Cross flow plate fin heat exchanger with heat duty of 160 kW (as shown in Figure 1) has air as a fluid on both the sides. Design of heat exchanger is to be optimized for minimum entropy generation. Fluid a and Fluid b enter the heat exchanger with the flow rate of 0.8962 kg/s and 0.8296 kg/s at a temperature of 513 K and 277 K respectively. The schematic of plate fin heat exchanger is as shown in Fig. 1 and Fig. 2.

![Fig. 1 Schematic of Plate Fin Heat Exchanger](image1)

![Fig. 2 Design Parameters for Offset Fins](image2)
Number of entropy generation units ($N_s$) or indicates the amount of power lost due to system irreversibilities. Finite temperature difference heat transfer in the fluid streams and the pressure drops along them cause irreversibilities in a heat exchanger. Optimization of heat exchanger using this method means minimizing the amount of lost power while satisfying all constraints. For more heat transfer effectiveness with better performance number of entropy generation units should be minimized [14]. So, the objective is to find out the heat exchanger dimensions giving the required heat duty for minimum entropy generation. Methodology of Bejan is used for obtaining number of entropy generation units [15]. So the objective function may be given as equation 5,

$$\min \left( N_s \right) = \frac{c_a}{c_{max}} \left\{ \ln \left[ 1 - e^{\frac{c_{min}}{c_a}} \left( \frac{1 - \frac{T_{h,1}}{T_{a,1}}}{} \right) \right] - \frac{p_a}{c_p} \ln \left( 1 - \frac{\Delta P_a}{\rho_{a,n}} \right) \right\} + \frac{c_b}{c_{max}} \left\{ \ln \left[ 1 + e^{\frac{c_{min}}{c_b}} \left( \frac{T_{a,1}}{T_{b,1}} - 1 \right) \right] \right\} -$$

$$\frac{p_b}{c_p} \ln \left( 1 - \frac{\Delta P_b}{\rho_{b,n}} \right)$$

Subjected to following constraints:

$$g_1(X) \rightarrow 0.1 \leq L_a \leq 1$$

$$g_2(X) \rightarrow 0.1 \leq L_b \leq 1$$

$$g_3(X) \rightarrow 0.002 \leq H \leq 0.01$$

$$g_4(X) \rightarrow 100 \leq n \leq 1000$$

$$g_5(X) \rightarrow 0.0001 \leq t \leq 0.0002$$

$$g_6(X) \rightarrow 0.001 \leq l \leq 0.01$$

$$g_7(X) \rightarrow 1 \leq N_a \leq 10$$

$$g_8(X) \rightarrow \varepsilon(X) - Q = 0$$

where, $\varepsilon(X)$ is the heat duty computed while $Q$ is the required heat duty. Various equations and details required for this problem are given below,

Rate of entropy generation for 2 fluid streams can be expressed as $\dot{S} = m_a \left[ C_{p_a} \ln \frac{T_{a,2}}{T_{a,1}} - R_a \ln \frac{\rho_{a,2}}{\rho_{a,1}} \right] + m_b \left[ C_{p_b} \ln \frac{T_{b,2}}{T_{b,1}} - \ln \frac{\rho_{a,2}}{\rho_{b,1}} \right]$
Number of entropy generation units:

\[ N_s = \frac{C_a}{C_{\max}} \left( \ln \left( 1 - \frac{T_b}{T_{a,1}} \right) - \frac{R_a}{C_p} \ln \left( 1 - \frac{\Delta P_a}{P_{a,1}} \right) \right) \]

\[ + \frac{C_b}{C_{\max}} \left( \ln \left( 1 + \frac{T_{a,1}}{T_b} - 1 \right) - \frac{R_b}{C_p} \ln \left( 1 - \frac{\Delta P_b}{P_{b,1}} \right) \right) \]

Effectiveness can be given as

\[ \varepsilon = \frac{C_a(T_{a,1} - T_{a,2})}{C_{\min}(T_{a,1} - T_{b,1})} = \frac{C_b(T_{b,2} - T_{b,1})}{C_{\min}(T_{a,1} - T_{b,1})} \]

Now,

\[ T_{a,2} = T_{a,1} - \frac{C_{\min}}{C_a}(T_{a,1} - T_{b,1}) \]

\[ T_{b,2} = T_{b,1} + \frac{C_{\min}}{C_b}(T_{a,1} - T_{b,1}) \]

\[ P_{a,2} = P_{a,1} - (P_{a,1} - P_{a,2}) = P_{a,1} - \Delta P_a \]

\[ P_{b,2} = P_{b,1} - (P_{b,1} - P_{b,2}) = P_{b,1} - \Delta P_b \]

\[ \varepsilon = 1 - \exp \left( \left( \frac{1}{C_r} \right) NTU^{0.22} \exp(-C_r * NTU^{0.78}) - 1 \right) \]

where,

\[ C_r = \frac{C_{\min}}{C_{\max}} \]

\[ \frac{1}{NTU} = \frac{C_{\min}}{UA} = \frac{1}{(hA)_a} + \frac{1}{(hA)_b} \]

\[ \frac{1}{NTU} = \frac{C_{\min}}{j_a C_p a Pr_a^{-2/3} m_a} + \frac{1}{j_b C_p b Pr_b^{-2/3} m_b} \frac{A_{ff b}}{A_b} \]

Free flow areas may be calculated as,

\[ A_{ff a} = (H_a - T_a)(1 - n_a t_a)L_a N_a \]

\[ A_{ff b} = (H_b - T_b)(1 - n_b t_b)L_a N_b \]

Similarly heat transfer areas for 2 sides can be obtained as,

\[ A_a = L_a L_b N_a[1 + 2n_a(H_a - t_a)] \]

\[ A_b = L_a L_b N_b[1 + 2n_b(H_b - t_b)] \]
Total heat transfer area,

\[ A_{HT} = A_a + A_b \]

Rate of heat transfer can be calculated as,

\[ Q = \varepsilon C_{min} (T_{a,1} - T_{b,1}) \]

Frictional Pressure drop on both sides,

\[
\Delta P_a = \frac{2f_a L \rho \overline{d}^2}{\rho_a D_a A_{ff} a} \\
\Delta P_b = \frac{2f_b L \rho \overline{d}^2}{\rho_b D_h A_{ff} b}
\]

When \( Re \leq 1500 \)

\[
j = 0.53 (Re)^{-0.5} (l/D_b)^{-0.15} [s/(H - t)]^{-0.14} \\
f = 8.12 (Re)^{-0.74} (l/D_b)^{-0.41} [s/(H - t)]^{-0.02}
\]

When \( Re \geq 1500 \)

\[
j = 0.21 (Re)^{-0.4} (l/D_b)^{-0.24} [t/D_h]^{0.02} \\
f = 1.12 (Re)^{-0.36} (l/D_b)^{-0.65} [t/D_h]^{0.17}
\]

\[ R_e = \frac{GD_h}{\mu} = \frac{mD_h}{A_{ff} \mu} \]

Hydraulic diameter,

\[ D_h = \frac{2(s - t)(H - t)}{s + (H - t) + (H - t)t/l} \]

\[ s = \frac{1}{n - t} \]

### 4.2 Pressure Vessel Design

Standard pressure vessel design problem is considered for optimization [16]. Objective is to minimize the total cost \( f(X) \) of a pressure vessel. As shown in Fig. 3, there are four design variables: thickness of the shell (\( X_1 \)), thickness of the head (\( X_2 \)), inner radius (\( X_3 \)) and length of the cylindrical section of the vessel (\( X_4 \)). Due to restriction of available thicknesses of rolled steel plates \( X_1 \) and \( X_2 \) are integer...
multiples of 0.0625 inches while $X_3$ and $X_4$ are continuous variables. This problem can be formulated as equation 6:

\[
\min(f) = 0.6224 X_1 X_3 X_4 + 1.7781 X_2 X_3^2 + 3.1661 X_1^2 X_4 + 19.84 X_1^2 X_3
\]  

(6)

This problem has a nonlinear objective function with one non-linear inequality constraint and three linear constraints. Details of this problem are given below,

Minimize cost,

\[
\min(f) = 0.6224 X_1 X_3 X_4 + 1.7781 X_2 X_3^2 + 3.1661 X_1^2 X_4 + 19.84 X_1^2 X_3
\]

Subject to the following constraints:

\[
g_1 = -X_1 + 0.0193 X_3 \leq 0
\]

\[
g_2 = -X_2 + 0.00954 X_3 \leq 0
\]

\[
g_3 = -\pi X_1^2 X_3^2 - \left(4\pi X_3^3\right)/3 + 1296000 \leq 0
\]

\[
g_4 = X_4 - 240 \leq 0
\]
Where, \((1 \times 0.0625) \leq X_1 \leq (99 \times 0.0625)\), \((1 \times 0.0625) \leq X_2 \leq (99 \times 0.0625)\), \(10 \leq X_3 \leq 200\), \(10 \leq X_4 \leq 240\)

5. RESULTS AND DISCUSSIONS

5.1 Plate Fin Heat Exchanger

Solution obtained using mTLBO algorithm for the optimization is given in Table 1. Also the results obtained in other techniques like GA, PSO, ABC and TLBO are given for comparison [17].

| Parameters | GA       | PSO      | ABC      | TLBO     | mTLBO    |
|------------|----------|----------|----------|----------|----------|
| \(L_a\) (m) | 0.994    | 0.985    | 0.957    | 0.934    | 0.931    |
| \(L_b\) (m) | 0.887    | 0.996    | 0.984    | 0.967    | 0.971    |
| \(H\) (mm)  | 9.53     | 9.8      | 9.6      | 9.9      | 9.88     |
| \(n\) (fins/m) | 534.9    | 442.9    | 474.4    | 466.87   | 451.39   |
| \(t\) (mm)  | 0.146    | 0.1      | 0.12     | 0.1      | 0.1      |
| \(l\) (mm)  | 6.6      | 9.8      | 9.7      | 10       | 10       |
| \(Na\)      | 8        | 10       | 10       | 10       | 10       |
| \(Q\) (kW)  | 159.99   | 159.99   | 159.99   | 159.99   | 159.99   |
| \(\Delta Pa\) (N/m²) | 5287.7   | 3331.3   | 2179.4   | 1861.1   | 1759.5   |
| \(\Delta Pb\) (N/m²) | 2216.9   | 1834.5   | 1234.4   | 1120.5   | 1089.4   |
| \(N_s\)     | 0.063332 | 0.053028 | 0.0503   | 0.04945  | 0.045878 |

From Table 1 it is clear that the best solution is obtained in case of mTLBO algorithm. Least value of entropy generation units along with least pressure drop values is obtained using mTLBO algorithm.
5.2 Pressure Vessel Design

Pressure vessel design problem was solved by various optimization algorithms already. Liu H, et al. solved this problem using particle swarm optimization with differential evolution [18]. He Q., et al. used co-evolutionary particle swarm optimization algorithm [19]. Mezura-Montes E, et al. used Evolutionary algorithm [20]. Huang F. Z., et al. solved this problem using co-evolutionary differential evolution algorithm [21]. Parsopoulos K, et al. used Unified Particle Swarm Optimization [22]. Akay B, et al. used Artificial Bee Colony algorithm to solve this problem [23]. Rao solved this problem using Teaching Learning Based Optimization algorithm and achieved the best mean results [24]. In this paper we have used modified Teaching Learning Based Optimization algorithm proposed by Suresh C. Satapathy and Anima Naik [11]. Comparison of solution obtained by all methods is given below in Table 2.

| Algorithm | Best   | Mean   | Evaluations |
|-----------|--------|--------|-------------|
| PSO-DE    | 6059.701 | 6379.938 | 42100       |
| CPSO      | 6061.077 | 6147.133 | 200000      |
| (μ + λ)-ES| 6059.701 | 6379.938 | 30000       |
| CoDE      | 6059.734 | 6085.230 | 240000      |
| UPSO      | 6544.270 | 9032.550 | 100000      |
| ABC       | 6059.714 | 6245.308 | 30 000      |
| TLBO      | 6059.714 | 6059.714 | 10000       |
| mTLBO     | 5850.383 | 5967.472 | 10000       |

Using mTLBO algorithm global minimum of 5850.383 (Best) is obtained when \(X_1 = 0.75\), \(X_2 = 0.375\), \(X_3 = 38.86\) and \(X_4 = 221.37\).
Also the convergence rate of TLBO and mTLBO is compared in following graph Fig. 4.

![Convergence Graph](image)

**Fig. 4** Convergence Graph

From above graph, it is clear that, mTLBO has faster convergence rate than TLBO. It gives optimum solution after 12-13 iterations where as TLBO requires 18-20 iterations.

### 6. CONCLUSIONS

The following conclusions are drawn from the present work:

I. From results it can be concluded that TLBO gives better results than other algorithms except mTLBO.

II. It is clear that mTLBO gives the least value of entropy generation units 0.045878 which is 7.22% less than that of TLBO.

III. Therefore better heat transfer can be achieved using mTLBO design while maintaining low pressure drop on both sides.

IV. Cost of pressure vessel design using mTLBO is 3.2% lower than that of TLBO design.

V. Best solutions are obtained in both problems while satisfy all constraints.

VI. Also it can be noted that, mTLBO has faster convergence rate than TLBO algorithm.

VII. mTLBO can be applied to any engineering problem with some modifications.
REFERENCES

[1] Holland J H 1975 *Adaptation in Natural and Artificial Systems* (Ann Arbor: University of Michigan Press) p 183

[2] Bäck T, Hoffmeister F and Schwefel H 1991 *Proc. of 4th Int. Conf. on Genetic Algorithms* (San Diego: Morgan Kaufmann) pp 2–9

[3] Van Laarhoven P J M and Aarts E H L 1987 *Simulated Annealing: Theory and Applications* (Dordrecht: Springer) pp 7-15

[4] Dorigo M, Birattari M and Stutzle T 2006 *IEEE Computational Intelligence Magazine* 1(4) pp 28-39

[5] Pratihar D K 2012 *Computational Optimization and Applications: Traditional vs. non-traditional optimization tools* (New Delhi: Narosa Publishing House Pvt. Ltd.) pp 25-33.

[6] Rao R V and Patel V 2010 *Applied Thermal Engineering* 30 pp 1417-25

[7] Rao R V and Patel V 2011 *Energy Conversion and Management* 52(7) pp 2611-22

[8] Rao R V, Savsani V J and Vakharia D P 2011 *Computer-Aided Design* 43 pp 303-15

[9] Azizimehr B, Assareh E and Moltames R 2020 *Energy Sources: Recovery, Utilization, and Environmental Effects* A pp 1747-61

[10] Guo C, Lu J, Tian Z, Guo W and Darvishan A 2019 *Energy Conversion and Management* pp 149-58

[11] Satapathy S C and Naik A 2013 *Recent Patents on Computer Science* 6 pp 60-72

[12] Rao R V 2016 *Decision Science Letters* 5 pp 1–30

[13] Mishra M, Das P K and Sarangi S 2009 *Int. J. Heat Exchanger* 5(2) pp 379-402

[14] Bejan A, Tsatsaronis G, Moran M 1996 *Thermal Design and Optimisation* (New York: John Wiley & Sons, Inc.)

[15] Bejan A 1977 *ASME J. Heat Transfer* 99 pp. 374–80

[16] Gexiang Z, Jixiang C, Marian G and Qi M 2013 *Applied Soft Computing* 13 pp 1528–42

[17] Rao R V and Patel V 2011 *Int. J. Advances in Thermal Sciences and Engineering* 2(2) pp 91-96

[18] Liu H, Zixing C and Yong Wang 2010 *Applied Soft Computing* 10 pp 629–40

[19] He Q and Wang L 2007 *Engineering Applications of Artificial Intelligence* 20 pp 89–99

[20] Mezura-Montes E and Coello C A C 2005 *LNCS: Advances in Artificial Intelligence* 3789 pp 652-62

[21] Huang F Z, Ling W and Qie H 2007 *Applied Mathematics and Computation* 186(1) pp 340–56
[22] Parsopoulos K and Vrahatis M 2005 LNCS: Advances in natural computation 3612 pp 582–91

[23] Akay B and Karaboga D 2010 J. Intelligent Manufacturing 23 pp. 1001–14

[24] Rao R V 2015 Teaching Learning Based Optimization and its Engineering Applications (London: Springer)