Effect of sampling interval on the output statistics of large 3D discontinuity surfaces generated by a multiscale random field model

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Abstract. The scale effect is known to hinder reliable shear strength estimation of large-scale discontinuities. Recently, a stochastic approach was proposed to predict shear strength of large discontinuities directly at problem scale, thereby bypassing the scale effect. One aspect of the stochastic approach seeks to use the available roughness information from the 1D profile of a discontinuity to create a series of statistically representative 3D synthetic rock surfaces, via a rigorous random field model. The application procedure for producing such synthetic surfaces was validated at small scale; however, preliminary large-scale applications were not quite satisfactory. It was found that the absence of consideration for the multiscale nature of discontinuity roughness contributed to the issues encountered. This paper presents the details of the revised multiscale-based 2D LAS approach for producing representative large-scale synthetic surfaces with an emphasis on the effect of sampling interval, called segment length, on the statistics of the synthetic surfaces.

1. Introduction

The estimation of shear strength is a significant factor in design and stability analysis encompassing engineering scale rock masses, however, its prediction for in-situ discontinuities at this scale is still challenging. This is largely to the hindrance encountered by the limited surface roughness information available from in situ discontinuities and acknowledged scaling effects.

Recently, Casagrande et al. [1] proposed a new approach for estimating the shear strength of large in situ discontinuities based on stochastic analysis. Essential to this approach is the ability to generate a series of synthetic surfaces, via a random field model using roughness statistics of the daylighting discontinuity profile as input parameters. In its original version, the method for generating the synthetic surfaces did not require to distinguish different scales of roughness. However, Buzzi and Casagrande [2] identified an issue with the generation of large synthetic surfaces in that the distribution of gradients of synthetic surfaces did not reflect the distribution of the visible reference profile, a problem that was not met when generating small scale surfaces. Due to the mismatch of gradient distributions, the semi-analytical model for shear strength estimation which is reliant on the gradients for the estimation of shear resistance, resulted in very large shear strength prediction errors. It is essential, for the reliability and robustness of the stochastic method for shear strength estimation, to be able to generate large scale synthetic surfaces with roughness statistics matching the input statistics as best as possible.
Rock joint roughness is commonly recognised (from a mechanical response perspective) to comprise two scales: a large scale waviness, which influences joint dilation during shearing and small scale unevenness that contributes to shear strength and the gouge material generation during shearing [3-5]. These scales of roughness have also been referenced as first and second order asperities, primary and secondary roughness or just undulation and roughness [6-9]. Guidance to the accurate quantification and geometric representation of roughness scales is limited. ISRM methods [3] provides some basic lengths for use with the visual descriptions, Patton [6] separated the first and second order asperities based on wavelength (\(\lambda\)) magnitudes, whereas, McMahon [10] distinguishes large scale joint surface roughness to three levels based on a wavelength (\(\lambda\)) and wavelength to joint length ratio (\(\lambda / L\)) criteria.

This paper presents a practical and efficient multiscale random field approach for the generation of large and rough synthetic surfaces consisting of three scales of roughness (after the observations of McMahon [5]). The method has the ability to accurately estimate and distinguish the three scale of joint roughness in order to provide inputs parameters for a random field model. The paper outlines the new approach for simulating large scale synthetic joint surfaces and compares the statistics of the generated composite surfaces (defined by different segments lengths) to the seed trace, to gain an understanding of prediction performance. At this time, the choice of segment length for defining the roughness profiles is a user input. The objective of this paper is to explore how the choice of segment length influences the final roughness statistics of generated synthetic composite surfaces and how well they reflect their seed trace statistics.

2. Producing 3D synthetic rock joint surfaces with consideration for multiscale roughness.

2.1. Overview

After surveying a discontinuity trace, that trace is decomposed into three daughter profiles equivalent to three scales of roughness (called large, intermediate and small, see figure 1a). The asperity height of each daughter profile is then statically analysed and its statistics form the input of the 2D Local average subdivision (LAS) random field [11-14], used to create synthetic surfaces at the corresponding scale level. The daughter synthetic surfaces hence produced at the different scale levels are then superimposed to create composite multiscale synthetic surfaces, as is illustrated in figure 1b.

![Figure 1](image-url)

**Figure 1.** Conceptual demonstration (a) roughness decomposition into the relative scale roughness profiles and (b) composite surface construction by superimposition of simulated daughter surfaces.
2.2. **Defining Daughter Profiles**

The procedure for defining daughter profiles at large and intermediate scales is to fit a piecewise linear trend to a given profile, which comprises of a continuous series of the segment lengths (that are as factorial of the profile length), segmentally fitted to the profile via a least squares algorithm.

The first piecewise linear trend fitted to the seed trace, referred to as the large scale daughter profile $z_L(x)$, is defined by a series large scale segment lengths ($SL_L$) and estimates the global form of the large scale roughness, as illustrated in figure 2a. The residual profile or deviation between the seed trace ($z(x)$) and the large scale profile ($z_L(x)$), represents the combined roughness profile of the intermediate and small scales (figure 2b). The Residual profile is attained by subtracting the large scale daughter profile from the seed trace via:

\[
\text{Residual}(x) = z(x) - z_L(x)
\]

The intermediate scale daughter profile, $z_I(x)$ is obtained by a similar method; a continuous linear piecewise function of a chosen intermediate scale segment length ($SL_I$), is fitted to the Residual height profile (figure 2c). Noting that the magnitude of $SL_I$ will always be smaller than $SL_L$ but greater than the spatial resolution of the seed trace (in this case 1 mm). The intermediate scale daughter profile is subtracted from the Residual profile to obtain the small scale roughness profile, $z_S(x)$ via equation (2) (see figure 2d).

\[
z_S(x) = \text{Residual}(x) - z_I(x)
\]

**Figure 2.** Illustration of the decoupling process of a seed trace into daughter profiles. a) Seed trace (continuous black line) with piecewise linear fitting using segment length of 200 mm, b) Decoupled large scale daughter profile (continuous red line) and residual profile (continuous grey line), c) Residual profile (continuous black line) with intermediate scale with fitted linear piecewise trend defined by 20 mm segment lengths, d) Decoupled intermediate scale daughter profile (continuous red line) and small-scale daughter profile (continuous grey line).
2.3. Simulation and Superposition

Details of the LAS random field application for generating 3D surface and require parameter inputs are presented in detail in Casagrande et al. [1]. In this study, the correlation length of the asperity height ($\theta_z$), a parameter that defines the distance over which asperity heights are spatially correlated is estimated by fitting the Gaussian correlation formulation (equation (3)) to the experimental autocorrelation function, $ACF(d)$ (given by equation (4)), via a regression over $\theta_z$.

$$\rho(d) = e^{-\frac{d^2}{\pi \theta_z^2}}$$  \hspace{1cm} (3)

$$ACF(d) = \frac{1}{n-k} \sum_{i=0}^{n-k} (z(iSL_X) - \mu_z)(z(i+k)SL_X) - \mu_z$$ \hspace{1cm} (4)

where, $n$ is the total number of data points, $z$ is a height data along the profile (at any roughness scale), $SL_X$ is the selected segment length at a given scale with $X=L, I$ or $S$; $\mu_z$ and $\sigma_z^2$ are the mean and variance of profile heights respectively and $k$ is the period index of lag distance (i.e. lag, $d = k \cdot SL_X$).

In order to create a composite surface from the simulated daughter surfaces via superposition, the field resolutions must be of the same size. By using a bilinear interpolation algorithm, the resolution of large and intermediate surfaces are increased to match that of the small scale. A composite surface can then be created by superimposing the three synthetic daughter surfaces:

$$z_C(x,y) = z_L(x,y) + z_I(x,y) + z_S(x,y)$$ \hspace{1cm} (5)

$z_C(x,y), z_L(x,y), z_I(x,y)$ and $z_S(x,y)$ are the heights of node $(x,y)$ on the composite surface, the synthetic large, intermediate and small scale surface, respectively.

3. Study Surfaces & Seed Traces

This study uses three rock surfaces from a range of geological settings in New South Wales, Australia:

- Martin Lime Quarry (MLQ) a 2.8 m x 2.4 m, freshly exposed coarse limestone fracture
- Wallabadah Rock Surface (WRS) a 2.0 m x 3.0 m, old, exposed rhyolitic joint surface
- Terrigal Skillion Failure (TSF) a 2.8 m x 3.6 m, freshly exposed sandstone joint surface

The surfaces were digitally reconstructed as 3D point cloud via photogrammetry; using a series of accurately surveyed surface ground control points (GCP), between 60 and 100 digital photographs (Canon EOS 7D camera with a 24 mm lens) taken from multiple perspectives and the commercial structure for motion software package ‘Agisoft Photoscan Professional’. The surfaces were reconstructed with x, y and z modelling errors typically less than 500 μm. The point cloud was structured into a grid with a 1 mm spatial increment. The reconstructed surfaces are presented in figure 3. For this study, two seed traces were selected (one in each direction) from each surface (refer figure 3), ensuring a variety of surface features, amplitudes and trace lengths were included.
4. Selection of Large and Intermediate scale segment lengths

A systematic approach has been used to test the sensitivity of output statistics to the selection of SL_L and SL_I combinations. The following combinations of segment lengths were used for the decoupling process:

- \( SL_L = 50\text{mm}, 100\text{mm}, 200\text{mm} \) and \( 400\text{mm} \) (500mm only used for WRS Trace Y)
- \( SL_I = 20\text{mm}, 40\text{mm} \)
- \( SL_S = 10\text{mm} \)

After each decomposition, 50 synthetic composite surfaces were created and the mean statistics were computed for comparison with the input statistics.

5. Results

When using random field models, it is important to appreciate that the output of a single simulation is likely not to reproduce input statistics. In order to determine the quality of input statistics reproduction, the average of a large series of simulations needs to be used. Therefore, for this study, 50 composite surface were generated and statistically analysed for each input combination. The standard deviation of heights and of gradients were calculated for each of the 50 composite simulations and for the whole series of composite surfaces (referred to as “mean standard deviation”). Figure 4 first presents the mean standard deviation of gradients \(<\sigma_i>\) against the mean standard deviation of heights \(<\sigma_z>\) for each of the SL_L-SL_I combination series. Also reported on the figure are the standard deviation of heights and of gradients of each trace \((\sigma_i \text{ and } \sigma_z, \text{ full red diamonds})\). Figure 5 presents the calculated relative errors on standard deviations between the seed trace statistics and the mean statistic of 50 composite surfaces for each combination series.

Figure 4 clearly shows that the output statistics are not very far off the input statistics but also that they are sensitive to the segment length used at both scales. In addition, figure 5 shows that the possible changes in relative error experienced from different combinations of segment length can be quite significant. It is also interesting to note that the choice of spatial increment generally seems to affect the gradients more than the heights.

For MLQ Trace X, increasing \( SL_L \) contributes to an underestimation of the standard deviation of heights, regardless of the choice of \( SL_I \). The relative error ranges from -3% to -15% for an \( SL_I \) of 50mm and of 400 mm respectively. Increasing \( SL_L \) from 20 mm to 40 mm generates larger standard deviation of gradients. For MLQ Trace Y, changes to \( SL_L \) do not lead to significant changes in relative error in the standard deviation of height, as the majority of values are within ±5%. Additionally, beside a general overestimation of 5 to 10%, changes to \( SL_I \) do not lead to significant changes in relative error in the standard deviation of gradients.

![Figure 3. 3D contoured digital representation of the study surfaces, overlaid with selected seed traces (black lines). a) Martin Lime Quarry, b) Wallabadah Rock Surface, c) Terrigal Skillion Failure. Dimensions on contour maps are all in mm. Colour scale indicates surface height, z in mm.](image-url)
Figure 4. Standard deviation of gradients vs standard deviation of heights for seed trace and the average of 50 synthetic composite surfaces. Different symbols correspond to a different large and intermediate scale segment length combinations Column a): Martin lime Quarry (MLQ); b): Wallabadah Rock (WRS); c): Terrigal Skillion Failure (TSF). Top row: Trace X, bottom row: Trace Y.

Figure 5. Relative error on standard deviation of gradients vs relative error on standard deviation of heights. Each point corresponds to a different large and intermediate scale segment length combination series used for the analysis (presented as $SL_L$–$SL_I$ in the legend). Column a): Martin lime Quarry (MLQ); b): Wallabadah Rock (WRS); c): Terrigal Skillion Failure (TSF). Top row: Trace X, bottom row: Trace Y.

For WRS Trace X, the standard deviation of height is quite sensitive to changes in $SL_L$: the larger $SL_L$, the larger the error (negative error being an underestimation). The relative error increase by about 11% when $SL_L$ increases from 50 mm to 400 mm. Changing $SL_I$ does not lead to significant changes in the relative error of the standard deviation of gradients, sitting at about 5%. For WRS Trace Y, augmenting $SL_L$ leads to a greater underestimation of the standard deviation of height. Generally the underestimation is within $\pm 10\%$ for lengths up to 200 mm, with 500 mm contributing to a relative error of -15%. For all but the combination series defined by a 500 mm $SL_L$, the gradients output statistics are not sensitive to changes in value $SL_I$, all plotting within a narrow width range around $\pm 6\%$. The change
in SL₄ value from 20 mm to 40 mm, generally increased the standard deviation of gradients by approximately 1° or a relative error change of 10%.

For TSF Trace X, regardless of the choice of SL₄ and SL₆, there is are no significant changes in the relative error in standard deviation of heights (relative error is ±5%). However, it can be seen that this simulated standard deviation of gradients are sensitive to the choice of SL₄ rather than SL₆. The results show that adopting a shorter SL₄ (eg 50 mm or 100 mm) contributes to an overestimation of the gradient statistic (relative error of between +10-30%). Conversely, adopting a SL₄ of 400 mm causes an underestimation of approximately -15%. Finally, the output statistics of surfaces generated from TSF Trace Y are quite sensitive to changes in both SL₄ and SL₆. Changes in segment length at the large scale affects the relative error on the standard deviation of heights, although it remains within ±10%. In contrast, changing the SL₄ to affect the relative error on the standard deviation of gradients, which remains within ±15%.

6. Discussion
The results show that the proposed approach has the ability to generate large scale composite surfaces with asperity height and gradient statistics that reflect the seed trace statistics (or within an acceptable error range). The results reveal that the average final composite surface statistics are sensitive to the chosen large and intermediate scale segment length, however there is no consistent trend, across all surfaces considered, as to the effect of each segment length on output statistics. Some surfaces may be more sensitive to changes in one scale than another, other may be rather sensitive to both. Surface TSF seems to be the most sensitive to changes of segment length. Surprisingly, the two selected seed traces for that surface are the smoothest (sandstone texture). At this time, it is not fully understood the cause, drivers or full extent of the sensitivity, or how to readily compensate for it.

The results have showed that establishing and refining a convenient ‘one size fits all’ SL₄ and SL₆ combination approach is unlikely to consistently render realisations that are statically representative of their seed trace. This is because in some cases the segment lengths will be undersized, thus overestimating the amount of roughness assigned to a particular daughter roughness profile, in turn reducing the overall roughness of the residual profile. Conversely, oversizing segment lengths will have the opposite effect. Consecutive under-sizing (or oversizing) is likely to lead to noticeably smoother (or rougher surfaces) than that of the seed trace characterised by a large negative (or positive) standard deviation of gradient differential between the simulation series average and their respective seed trace. The current trial and error process used in this paper to identify an appropriate segment length at each scale is not an efficient and a more efficient approach needs to be developed, which constitutes further research.

7. Conclusions
Casagrande et al.’s stochastic approach for estimating shear strength of large scale in situ rock joints relies on creating of a series of 3D simulated synthetic surfaces using 2D LAS algorithm that reflect the roughness statistics of 2D seed trace. The original method for generating large scale rough surfaces was not able to produce surfaces with suitable statistics, thus requiring an adjustment of the inputs values to achieve the desired statistics. This paper presents a refined approach for creating large scale 3D synthetic rock joint surfaces from a seed trace and the application of 2D LAS random field algorithm. The approach has the ability to produce surfaces statistics similar to that of the seed trace, without the need for any adjustments. The key to the approach is the consideration that roughness is multiscale in nature. The approach incorporates a method for decoupling the seed roughness profile into three daughter scales (large intermediate and small) using a fitted equally segmented linear piecewise trend. A final composite surface is then created by superimposing a synthetic surface realisation for each of the daughter scales using the 2D LAS algorithm.

The approach was applied to six seed traces (ranging in length from 2 m to 3.6 m) from three digitally reconstructed rock surfaces of different geological settings from across New South Wales, Australia. The sensitivity analysis consisted of applying the approach using different combinations of large and
intermediate scale segment lengths for defining the forms of the large and intermediate scale daughter profiles to see the influence they have on the final surface roughness statics. The study revealed that the final roughness statics (standard deviation of asperity heights and gradients) of the generated composite surfaces are sensitive to the magnitude of the segment length chosen at both scales. Additionally, the magnitude of sensitivity varies between seed traces, highlighting that the ‘correct’ choice of the segment length is critical to success and meaningful applications. A practical method of identifying suitable segment lengths at large and intermediate scales needs to be established.

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