Spin splittings among charmed hadrons
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The mass differences between spin-1/2 and spin-3/2 baryons are compared to the mass differences between spin-0 and spin-1 mesons. Results of simulations for charmed hadrons in the quenched approximation from a tadpole-improved anisotropic action are discussed in the context of other lattice calculations, quark model predictions, heavy quark symmetry predictions and experimental data.

1. MOTIVATION

The mass differences between the lightest vector and pseudoscalar mesons containing one or two heavy quarks are persistently smaller in quenched lattice simulations than the experimental values\cite{1,3}. Studies of the unquenched theory have found that the discrepancy persists\cite{4}.

Fewer lattice simulations have been performed for the mass differences between spin-3/2 and spin-1/2 baryons containing one or more heavy quarks\cite{1,2,5,6}. It is useful to compare these baryon mass differences to the meson mass differences mentioned above, since both are colour hyperfine (spin-spin) effects. Quantitative relationships between the meson and baryon splittings have been claimed using experimental data, quark models or heavy quark effective theory. If these relations are respected by lattice QCD simulations then the baryon spin splittings will be smaller than experiment (just like mesons), but if lattice results for the baryons agree with experiment then the relations must clearly be violated. In either case, the information will be helpful when seeking to understand the existing discrepancy between lattice QCD simulations and experiment.

2. PHENOMENOLOGY

A simple quark model for the hadron masses uses the operators

\[
M_{\text{meson}} = m_1 + m_2 + \frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2},
\]

\[
M_{\text{baryon}} = m_1 + m_2 + m_3 + \sum_{i>j} \frac{\alpha_{ij} \vec{s}_i \cdot \vec{s}_j}{m_i m_j}.\tag{2}
\]

It follows that the spin splittings for heavy-light mesons and for singly and doubly heavy baryons vanish like \(1/m_Q\),

\[
M_{3/2} - M_{1/2} = \frac{3c'}{4c}(M_u - M_p) \sim \frac{1}{m_Q},
\]

where \(c'\) and \(c\) contain wave function information. The determination of \(c'/c\), using a relation between the wave function at the origin and the derivative of the potential (plus some other postulates), has been discussed by Lipkin\cite{7} and by Lipkin and O’Donnell\cite{8}.

For doubly heavy baryons the pair of heavy quarks forms a colour anti-triplet, so \(c \approx c'\) and

\[
[M_{3/2} - M_{1/2}]_{QQq} = \frac{3}{4}(M_u - M_p).
\]

A more elegant derivation of this same result comes from incorporating the anti-triplet diquark directly into heavy quark effective theory\cite{9}. The resulting Lagrangian has a superflavour symmetry among heavy quarks and heavy diquarks:

\[
\mathcal{L} = \chi^\dagger \left[iD^0 + \frac{\lambda D^2}{2m_Q} + \frac{g_s \vec{\Sigma} \cdot \vec{B}}{2m_Q} + O \left(\frac{1}{m_Q^2}\right)\right] \chi \tag{5}
\]

where

\[
\chi^T = (Q_\uparrow, Q_\downarrow, \{QQ\}_1, \{QQ\}_0, \{QQ\}_{-1}),
\]

\[
\lambda = \text{diag} \left(1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right).
\]
Hadron creation operators.

\[ D = Q^\dagger \gamma_5 u \]
\[ D^* = Q^\dagger \gamma_\mu u \]

\[ A_Q = \frac{1}{\sqrt{6}} e^{abc} (2 [u^T_a C \gamma_5 d_b] Q_c + [u^T_a C \gamma_5 Q_b] d_c - [d^T_a C \gamma_5 Q_b] u_c) \]
\[ \Sigma_Q = e^{abc} [u^T_a C \gamma_5 Q_b] d_c \]
\[ \Xi_Q = e^{abc} [u^T_a C \gamma_5 Q_b] s_c \]
\[ \Xi'_{Q} = \frac{1}{\sqrt{6}} e^{abc} ([u^T_a C \gamma_5 Q_b] s_c + [s^T_a C \gamma_5 Q_b] u_c) \]
\[ \Omega_Q = e^{abc} [s^T_a C \gamma_5 Q_b] s_c \]
\[ \Xi_{QQ} = e^{abc} [u^T_a C \gamma_5 Q_b] Q_c \]
\[ \Omega_{QQ} = e^{abc} [s^T_a C \gamma_5 Q_b] Q_c \]

spin-3/2 baryons

\[ \Sigma^*_{Q} = e^{abc} [u^T_a C \gamma_\mu d_b] Q_c \]
\[ \Xi'_{Q} = e^{abc} [u^T_a C \gamma_\mu s_b] Q_c \]
\[ \Omega^*_Q = e^{abc} [s^T_a C \gamma_\mu s_b] Q_c \]
\[ \Xi_{QQ} = e^{abc} [Q^T_a C \gamma_\mu Q_b] u_c \]
\[ \Omega_{QQ} = e^{abc} [Q^T_a C \gamma_\mu Q_b] s_c \]

and \( \Sigma \) is the block-diagonal spin vector for quarks and diquarks. A direct calculation from this Lagrangian leads to Eq. (4).

3. LATTICE CHOICES AND METHOD

As reported in Ref. [1], our simulations used an anisotropic tadpole-improved relativistic action for gauge fields and for quarks. The gauge action includes a \( 1 \times 2 \) rectangular plaquette which remove \( O(a^2) \) classical errors, and the \( D234 \) quark action has no \( O(a) \) nor \( O(a^2) \) classical errors.

The hadron creation operators are listed in Table I. Sink-smeared and local correlators were built from these operators and fit simultaneously to single and double exponential functions. The “spin-3/2” operators in Table I actually contain an admixture of spin-1/2 as well. The spin-3/2 component is isolated by a judicious choice of Lorentz components.

In our simulations, the bare anisotropy was set to \( \alpha_s/\alpha_t = 2 \), tadpole improvement was implemented via the mean link in Landau gauge, and quarks have Dirichlet time boundaries. Configuration details are given in Table 2.

4. COMPUTED HADRON MASSES

As shown in Fig. 4, the experimental ratio of \( Qq \) meson spin splittings to \( Qqq \) baryon spin splittings is remarkably constant, where \( q \) is a light quark and \( Q \) is light or strange or charmed. Our lattice results, without any extrapolation in \( a \) or \( \kappa \), are also independent of \( m_Q \), but significantly smaller than experiment.

Fig. 3 shows the ratio of spin splittings for \( \bar{Q}q \) mesons and \( QQq \) baryons. Once again, the lattice results are independent of \( m_Q \) and smaller than the experimental point. Interestingly, they are near the prediction of Eq. (4).

Some representative lattice computations of the heavy-light meson spin splittings are shown in Fig. 4 for \( Q \) from light quarks through to bottom. The experimental results are linear in the average mesons mass to a few percent, but the lattice results show a significant slope. The lattice bottom splitting is about half of the experimental value. Heavy quark symmetry requires that \( (M_u - M_d)M \to \infty \), but Fig. 4 does not allow us to determine what the heavy quark limiting value is.

In contrast to the meson situation, the lattice results for the \( Qqq \) baryon spin splittings, plotted in Fig. 5, are not significantly smaller than experiment; in fact, some lattice results are larger than experiment.

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Figure 1. Ratio of spin splittings: mesons/singly heavy baryons.

Figure 2. Ratio of spin splittings: mesons/doubly heavy baryons.

Figure 3. Spin splittings among heavy-light mesons. “Lattice literature” is Refs. [2,3].

Figure 4. Spin splittings among singly heavy baryons. “Lattice literature” is Refs. [2,5,6].

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