Robust cooperative platoon control in mixed traffic flow considering the uncertainty of human-driven vehicles

Shuo Feng\textsuperscript{a}, Ziyou Song\textsuperscript{b}, Zhaojian Li\textsuperscript{c}, Yi Zhang\textsuperscript{a}, Li Li\textsuperscript{a,*}

\textsuperscript{a}Department of Automation, Tsinghua University, Beijing, 100084, China
\textsuperscript{b}Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109, USA
\textsuperscript{c}Department of Mechanical Engineering, Michigan State University, East Lansing, MI, 48824, USA

Abstract

Cooperative adaptive cruise control (CACC) is one of the promising intelligent transportation technologies. Considering that connected and automated vehicles (CAVs) and human-driven vehicles (HDVs) will coexist for a long period, the design of CACC methods in mixed traffic flow becomes critical. Theoretically, the maneuvers of CAVs can be controlled based on the prediction results of HDVs. However, considering the stochastic nature of human drivers, the prediction uncertainty cannot be avoided and could accumulate significantly. To handle the uncertainty, one method utilizes feedback control, which cannot explicitly satisfy constraints and usually requires wireless connectivity of HDVs. The other method utilizes time-triggered feedforward control (e.g., Model Predictive Control, MPC), which suffers from the huge burden of inter-vehicular communication and computation. To address these limitations, this paper proposes a robust cooperative platoon control as follows. First, the dynamics of the prediction uncertainty are formulated and analyzed. It is found that the prediction uncertainty increases significantly with the time step (e.g., quadratically for position uncertainty), which leads to the accumulation of the tracking error disturbance. Second, to mitigate and bound the accumulation, a time-triggered feedback control is designed. It is proved that the accumulated disturbance is bounded if the new prediction uncertainty of each control interval is bounded, i.e., robust. Third, an

\*This work was supported in part by the National Key Research and Development Program of China under Grant 2018YFB1600600, the National Natural Science Foundation of China (61790565), and Collaboration between China &Sweden regarding research (2018YFE0102800).

*Corresponding author

\textit{Email address:} li-li@tsinghua.edu.cn (Li Li)

Preprint submitted to Elsevier

October 17, 2019
event-triggered feedforward control is designed to handle the disturbance outside the bound. The tight constraints are satisfied by the feedforward control. Based on tube methods, the overall method can guarantee the constraints, e.g., safety, stability, and string stability. Compared with pure feedback control, the proposed method guarantees the constraints and does not require connectivity of HDVs. Compared with MPC methods, the modified feedforward control is event-triggered, and therefore the intervehicle communication and planning costs can be significantly reduced. Theoretical analysis and numerical experiments validate the robustness, efficiency, and scalability of the proposed method.

**Keywords:** Mixed traffic flow, Robust cooperative platoon control, Event-triggered method, Feedforward Control, Feedback Control

1. Introduction

Cooperative adaptive cruise control (CACC) is one of the promising intelligent transportation technologies that contribute to improving traffic flow stability, throughput, and safety (Van Arem et al. (2006); Li et al. (2014); Feng et al. (2015); Li et al. (2015); Gong et al. (2016); Zheng et al. (2018); Sun and Yin (2019); Wang et al. (2019a)). Through vehicle-to-vehicle and vehicle-to-infrastructure wireless communication, CACC can utilize more information (e.g., historical and plan information of vehicles) to better track desirable trajectories, which are coordinated with external disturbances, e.g., intersection control (Feng et al. (2018b); Yu et al. (2018); Yang et al. (2019)) or traffic maneuvers (e.g., cut-in, cut-out, and merge) (Xu et al. (2019)). To guarantee the control performance, different constraints are usually satisfied, e.g., velocity limits, acceleration limits, safety, stability, and string stability. Intuitively, stability describes vehicles converging to given trajectories, while string stability describes that the disturbances are not amplified along the string of vehicles (Feng et al. (2019b)).

Most existing CACC methods focus on the pure connected and automated vehicle (CAV) flow. However, given that CAVs and human-driven vehicles (HDVs) will coexist in the future traffic flow for a long period, it is critical to design CACC methods, which can be applied in the mixed traffic flow. Fig. 1 illustrates a sample of the mixed traffic flow, where a CAV directly follows a HDV and is connected with a predecessor CAV. Unlike CAVs whose behaviors can be well characterized and controlled, HDVs usually do not follow deterministic control laws, and therefore will compromise the performance of CACC systems. To address this issue, several CACC methods have recently been proposed for the mixed traffic flow, which usually first predict the behaviors of HDVs and then control the maneuvers of CAVs (Milanés
Various methods can be used for the prediction step, e.g., model-based methods (Chen et al. (2010); Jin and Orosz (2018); Gong and Du (2018); Zheng et al. (2018)) and model-free methods (Wang et al. (2018); Feng et al. (2018a); Wang et al. (2019b)). Considering the stochastic nature of human drivers, however, behaviors of HDVs cannot be exactly predicted, and therefore the prediction uncertainty exists.

How to model and handle the prediction uncertainty of HDVs is the major difficulty of CACC methods for mixed traffic flow. As shown in Fig. 1, because the tracking error of \( f \)-CAV is defined based on the state of \( n \)-HDV, the prediction uncertainty of \( n \)-HDV causes the disturbance between the actual tracking error and the planned tracking error of \( f \)-CAV. The planned tracking error is zero in equilibrium state or influenced by external disturbances, e.g., intersection control or traffic maneuvers. To distinguished with the external disturbance, the tracking error disturbance caused by the prediction uncertainty is denoted as internal disturbance. If the internal disturbance is not mitigated timely, it will accumulate with the time step and significantly impair the performance of platoon.

The accumulation of the internal disturbance brings great challenge to CACC methods. To address this challenge, the existing CACC methods mainly rely on two control methods, which suffer from different limitations. One method is the feedback control, which determines maneuvers of CAVs by a feedback gain of the current traffic states (Jin and Orosz (2014); Jin and Orosz (2017); Jin and Orosz (2018)). However, the feedback control cannot explicitly satisfy constraints (e.g., speed and acceleration limits) and usually requires the wireless connectivity of all HDVs. The other method is the time-triggered feedforward control (e.g., Model Predictive Control, MPC), which optimizes future maneuvers and utilizes the first input in the optimal sequence at each control interval (e.g., 0.1 second) (Li and Wang (2017); Gong and Du (2018); Chen et al. (2018)). Because the trajectory
of CAVs is replanned at each control interval, the internal disturbance is mitigated timely. However, the feedforward control usually relies heavily on the intervehicular communication and computation, which impairs the robustness of the method (Guo and Wen (2016); Wen et al. (2018)). To decrease the computational burden, a distributed algorithm was recently designed by (Gong and Du (2018)). Different with this study, the aim of our paper is to propose a new control method, which can significantly decrease the trigger number of the feedforward control and intervehicular communication.

The major idea of the new method is to integrate the feedforward control with a feedback control, which can mitigate and bound the accumulation of the internal disturbance. Because the internal disturbance is bounded by the feedback control, the feedforward control is only triggered when external disturbance emerges. Compared with the internal disturbance, the external disturbance emerges less frequently. Therefore, the trigger number of the feedforward control and intervehicular communication is much reduced, compared with MPC methods. Moreover, to overcome the limitations of the feedback control, the tube method (Mayne and Langson (2001); Langson et al. (2004); Feng et al. (2019a)) is utilized to modify the feedforward control to explicitly satisfy all constraints. Specifically, the proposed method mainly includes the following three parts, i.e., prediction uncertainty analysis, uncertainty handling by feedback control, and feedforward control with tight constraints.

First, the dynamics of the prediction uncertainty are formulated and analyzed. It is found that the prediction uncertainty increases significantly with the time step (e.g., quadratically for position uncertainty), which leads to the accumulation of tracking error disturbance. Second, to bound the accumulation, a time-triggered feedback control is designed by the discrete linear quadratic regulator method (Bender and Laub (1987)). To quantitatively measure the effectiveness of the feedback control, the minimal robust invariant positively (mRPI) set is calculated by the \( \epsilon \)-approximation method (Rakovic et al. (2005)). It is proved that if the new prediction uncertainty of each control interval is bounded, the accumulated disturbance is bounded inside the mRPI set. Third, to handle the disturbance outside the mRPI set (e.g., external disturbance), an event-triggered feedforward control is designed. The tight constraints are satisfied by the feedforward control so that the overall method can guarantee the constraints (e.g., safety, stability, and string stability). Equivalently, the feedforward control with tight constraints determines a sequence of mRPI set (i.e., the tube), which satisfies the constraints. The flowchart of the proposed control method is illustrated in Fig. 2, and an illustration of the tube is shown in Fig. 3.

From the perspective of car-following method, the \( f \)-CAV follows both its pre-
Figure 2: Flowchart of the proposed robust CACC method.
decessor CAV (p-CAV) and predecessor HDV (n-HDV). Specifically, f-CAV follows p-CAV by the feedforward control and follows n-HDV by the feedback control. Compared with the MPC methods, the integration of feedback control utilizes more information of the predecessor HDV, and therefore increases the robustness of the proposed method.

To validate the performance of the proposed method, theoretical analysis is provided and numerical experiments are designed. First, the properties of the prediction uncertainty are analyzed. Simulation results demonstrate the tendencies of the prediction uncertainty with the time step, numbers of consecutive HDVs, penetration rates of CAVs, and platoon lengths, respectively. Second, the performance of the proposed method are validated regarding safety, stability, and string stability. Compared with the MPC method, the proposed method significantly reduces the burden of computation and communication. Third, simulation results of the long mixed traffic platoon validate the scalability of the proposed method.

The rest of this paper is organized as follows. The problem formulation is proposed in Section 2. In Section 3 we analyze the properties of the prediction uncertainty of HDVs. In Section 4, a feedback control is designed to mitigate the influence of the prediction uncertainty. The overall robust CACC method is illustrated in Section 5. The feasibility and control objectives are analyzed in Section 6. Finally, we provide simulation results in Section 7 and conclusions in Section 8.
2. Problem formulation

2.1. Notations

The field of a real number is denoted by $\mathbb{R}$, whereas $\mathbb{N} = \{1, 2, \ldots \}$. For a vector $x \in \mathbb{R}^n$, its $p$-norm is given as

$$
\|x\|_p = \left( \sum_{i=1}^{n} |x_i|^p \right)^{1/p}, \quad p \in [1, \infty) \tag{1}
$$

$$
\|x\|_\infty = \max_i |x_i| \tag{2}
$$

Given a Lebesgue measurable signal $x(t) : I \to \mathbb{R}^n$, $\|x\|_{L^p}^I$ denotes its $L^p$ norm defined as

$$
\|x\|_{L^p}^I = \left( \int_I \|x(t)\|_p^p \, dt \right)^{1/p} < \infty, \quad p \in [1, \infty) \tag{3}
$$

$$
\|x\|_{\infty}^I = \sup_{t \in I} \|x\|_\infty \tag{4}
$$

where the shorthand notation $\|x\|_{\infty}^I = \|x\|_{\mathbb{L}^0}^{\infty}$ is used when $I = [0, \infty)$ (see Zhou et al. (1996) for details). A continuous function $\alpha : [0, a) \to [0, \infty), a \in \mathbb{R}^+$ is said to be of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to be of class $\mathcal{KL}$ if, for each fixed $s$, the function $\beta(\cdot, s)$ is of class $\mathcal{K}$, and for each fixed $r$, $\beta(r, \cdot)$ is decreasing and satisfies $\beta(r, s) \to 0$ as $s \to \infty$. We say $x \in \mathcal{L}_\infty$ if $\|x\|_{\mathcal{L}_\infty} < \infty$. We recall Minkowski sum for sets $A, B$ is $A \oplus B = \{x + y | x \in A, y \in B\}$ and the Pontryagin difference is $A \ominus B = \{x | x + y \in A, y \in B\}$.

2.2. Scenario description

Similar to existing studies (Gong et al. (2016); Gong and Du (2018)), a sample scenario of mixed traffic flow is studied, as shown in Fig. 1. Specifically, this scenario consists of a predecessor CAV (p-CAV), HDVs, and a following CAV (f-CAV). Intervehicular communication exists from the p-CAV to the f-CAV. On-board sensors (e.g., millimeter-wave radars) of the f-CAV can measure the distance and speed of its predecessor HDV, i.e., $n$-HDV. It is assumed that CAVs can be exactly modeled and controlled. The situation before the p-CAV is not specified. There can exit more HDVs before the p-CAV, so the scenario is a sample of a long mixed traffic flow. The p-CAV can also be the leading vehicle of the platoon and influenced by intersection control (Yu et al. (2019a); Yu et al. (2019b)) or traffic maneuvers.
2.3. Vehicle dynamics

The discrete vehicle dynamics of CAVs can be formulated by the continuous-time dynamics with Zero Order Hold method \cite{Ogata1995} and Taylor equation. Denote $s$ and $v$ as the position and speed. By determining a sampling time interval $\tau$, we obtain the discrete dynamics as

$$x_i(k + 1) = Ax_i(k) + Bu_i(k), i \in \{p, f\}$$  \hspace{1cm} (5)

where $x_p$ and $x_f$ denote the states of $p$-CAV and $f$-CAV respectively, and

$$x_i = \begin{bmatrix} s_i \\ v_i \end{bmatrix}, A = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0.5\tau^2 \\ \tau \end{bmatrix}.$$  \hspace{1cm} (6)

2.4. Tracking error

The tracking error between the $f$-CAV and its predecessor HDV (i.e., $n$-HDV) is denoted as

$$e = [e_s, e_v]^T,$$  \hspace{1cm} (7)

where $e_s$ denotes the position tracking error, and $e_v$ denotes the speed tracking error. To determine the tracking error, the range policy is important, which specifies the tracking trajectory. Several range policies have been proposed in the past decades, e.g., constant distance policy \cite{Sheikholeslam1993, Liu2001}, constant time headway policy \cite{Chien1992, Zhou2005}, and nonlinear policy \cite{Orosz2016, Santhanakrishnan2003}. In this paper, we apply the constant time headway policy as an example, so the tracking error is calculated as

$$e_s(k) = s_n(k) - s_f(k) - h \cdot v_f(k),$$
$$e_v(k) = v_n(k) - v_f(k),$$  \hspace{1cm} (8)

where $k$ denotes the discrete time step, $s_n$ and $s_f$ denote the position of the $n$-HDV and $f$-CAV respectively, $v_n$ and $v_f$ denote their speeds, and $h$ denotes the constant time headway. To make the paper concise, the tracking error is represented compactly as

$$e(k) = x_n(k) + Cx_f(k),$$  \hspace{1cm} (9)

where

$$C = \begin{bmatrix} -1 & -h \\ 0 & -1 \end{bmatrix}.$$  \hspace{1cm} (10)
2.5. Control objective

For a pure CAV flow, the control objective is to ensure all vehicles in the same group to move at a consensual speed while maintaining the desired spaces between adjacent vehicles, i.e., keep the tracking error as zero (Horowitz and Varaiya, 2000). To this end, two types of objectives have been proposed, i.e., stability and string stability. Stability describes vehicles converging to given trajectories, while string stability describes that the disturbances are not amplified along the string of vehicles, which is critical for traffic flow stability (Talebpour and Mahmassani (2016); Ke et al. (2018)). Various definitions and analysis methods of string stability have been proposed, which can be found in a detailed review (Feng et al., 2019b).

For the mixed traffic flow, however, only CAVs can be controlled, so the objectives should be modified correspondingly. Specifically, we propose the related definitions as follows:

**Definition 1.** (Stability): A mixed platoon of Eq. (5) is said to be stable if

\[ \dot{v}_n(t) = 0, \forall t \geq 0 \Rightarrow \lim_{t \to \infty} e(t) = 0, \]

where \( v_n \) denotes the speed of \( n \)-HDV.

**Definition 2.** (\( L_p \) String Stability): A mixed platoon of Eq. (5) is said to be \( L_p \) string stable if there exist class \( K \) function \( \alpha \) and constant \( c > 0 \), such that, for any initial disturbance of the \( p \)-CAV satisfying

\[ |e_{s,p}(0)| < c, \]

the solution \( e_s(t) \), exists for all \( t > 0 \) and satisfies

\[ e \in D = \left\{ e \in \mathbb{R}^2 : \|e_s(t)\|_{L_p} \leq \alpha (|e_{s,p}(0)|) \right\}, \]

where \( e_{s,p}(0) \) denotes the initial position tracking error of the \( p \)-CAV.

To better understand the definitions, we explain their properties as follows. First, the definition of stability requires that if the \( n \)-HDV keeps its speed constant, the tracking error of the \( f \)-CAV should converge to zero. This property guarantees that the controlled CAV can converge to the given trajectory if there is no new disturbance. Second, the definition of string stability requires that the tracking error of the \( f \)-CAV is bounded if the initial disturbance of the \( p \)-CAV is bounded. Generally, \( L_2 \) and \( L_\infty \) norms are utilized, which refer to the energy and maximal amplitude respectively.
Besides of the stability and string stability, the safety is another critical control objective. Moreover, the speed range and acceleration range should be considered. Therefore, the constraints of the f-CAV are summarized as

\[ e \in \mathbb{E} = \{ e \in \mathbb{R}^2 : e \in \mathbb{D}, -d_{\text{min}} \leq e_s, v_n - v_{\text{max}} \leq e_v \leq v_n - v_{\text{min}} \}; \]
\[ u_f \in \mathbb{U} = \{ u_f \in \mathbb{R} : -u_{\text{max}} \leq u_f \leq u_{\text{max}} \}; \]

(14)

where \( e \in \mathbb{D} \) is the requirement of string stability, \(-d_{\text{min}}\) represents the minimal range error for safety requirement, \(v_{\text{min}}\) and \(v_{\text{max}}\) denote the minimal and maximal speeds respectively, and \(u_{\text{max}}\) denotes the maximal acceleration.

3. Prediction uncertainty of HDVs

3.1. Prediction model of HDVs

One common step for CACC methods in mixed traffic flow is to explicitly or implicitly predict behaviors of HDVs. The transmitted information from the p-CAV is utilized to predict the future behaviors of HDVs, which provide foundation to the feedforward control (e.g., MPC) of the f-CAV.

In this paper, we apply the simple Newell car-following model (Newell (2002)) as the prediction model for proof of concept. It is assumed that the following vehicle would share the same trajectory shape of its predecessor but with spatial-temporal delay, i.e.,

\[ s^p_n(t + \tau_n) = s_{n-1}(t) - d_n, \]
\[ v^p_n(t + \tau_n) = v_{n-1}(t), \]

where \( \tau_n \) and \( d_n \) denote the temporal and spatial delay respectively, and \( s^p_n, v^p_n \) denote the predicted position and speed of the \( n \)-HDV. Therefore, when the f-CAV receives both the historical and planned trajectory information of the p-CAV, the trajectory of the \( n \)-HDV can be predicted as

\[ s^p_n(t + \sum_{i=1}^{n} \tau_i) = s_p(t) - \sum_{i=1}^{n} d_i, \]
\[ v^p_n(t + \sum_{i=1}^{n} \tau_i) = v_p(t). \]

The parameters of total temporal delay \( \sum_{i=1}^{n} \tau_i \) and spatial delay \( \sum_{i=1}^{n} d_i \) can be estimated online by various methods, e.g., the curve matching algorithm (Zhang (1994); Gong and Du (2018)) and the parallel recursive least square with inverse QR decomposition (PRLS-IQR) algorithm (Li et al. (2019)).
3.2. Prediction uncertainty

The major difference between HDV and CAV is that the HDV cannot be exactly manipulated and predicted. As the stochastic nature of human drivers, there always exists uncertainty no matter which prediction method is utilized. How to handle the uncertainty is the major difficulty for CACC methods in mixed traffic flow.

In this subsection, we analyze the temporal and spatial properties of the prediction uncertainty. Let $\tilde{x}_n$ denote the state prediction uncertainty of the $n$-HDV and $\Delta \tilde{x}_n$ denote the new uncertainty during the last control interval. First, the state uncertainty will accumulate with the time step (i.e., temporally) as

$$\tilde{x}_n(k) = A\tilde{x}_n(k - 1) + \Delta \tilde{x}_n(k - 1),$$

$$= \sum_{j=1}^{k-1} A^{k-j-1} \Delta \tilde{x}_n(j),$$

where $\Delta \tilde{x}_n(k - 1) = [\Delta \tilde{s}_n(k - 1), \Delta \tilde{v}_n(k - 1)]^T$ denotes the new prediction uncertainty at the control interval $[(k - 2)\tau, (k - 1)\tau)$. Considering

$$A^k = \begin{bmatrix} 1 & k\tau \\ 0 & 1 \end{bmatrix},$$

and Eq. (15), the absolute value of the position prediction uncertainty can be derived as

$$|\tilde{s}_n(k)| = \left| \sum_{j=1}^{k-1} \Delta \tilde{s}_n(j) + \sum_{j=1}^{k-1} (k - j - 1)\tau \Delta \tilde{v}_n(j) \right|,$$

$$\leq (k - 1) \cdot \max_j |\Delta \tilde{s}_n(j)| + \frac{(k - 1)(k - 2)\tau}{2} \max_j |\Delta \tilde{v}_n(j)|,$$

and the absolute value of the velocity prediction uncertainty can be derived as

$$|\tilde{v}_n(k)| \leq (k - 1) \cdot \max_j |\Delta \tilde{v}_n(j)|.$$

Therefore, the prediction uncertainty of position increases quadratically with the time step $k$, whereas the prediction uncertainty of velocity increases linearly.

Second, the prediction uncertainty will increase with the number of consecutive HDVs (i.e., spatially). Since the behaviors of the $n$-HDV are predicted based on the trajectories of the $p$-CAV, the uncertainty of all the $n$ consecutive HDVs contributes to the prediction uncertainty of the $n$-HDV. For example, if the uncertainty of one
HDV can be represented by a normal distribution, e.g., $\mathcal{N}(0, \sigma_i^2)$, then the uncertainty of the $n$-HDV can be represented as

$$\Delta \tilde{x}_n \sim \sum_{i=1}^{n} \mathcal{N}(0, \sigma_i^2),$$

(19)

which is the addition of the uncertainty of all consecutive HDVs.

Therefore, the prediction uncertainty of the $n$-HDV increases both temporally and spatially, which much compromises the performance of the platoon control methods.

### 3.3. Tracking error disturbance

The major influence of the prediction uncertainty lies at the tracking error disturbance. Denote the actual tracking error as $e$ and the planned tracking error as $\bar{e}$. Then the disturbance is defined as

$$\tilde{e} = e - \bar{e},$$

(20)

where $\tilde{e} = [\tilde{e}_s, \tilde{e}_v]^T$. By Eq. (9), the tracking error disturbance can be derived as

$$\tilde{e} = \tilde{x}_n + C\tilde{x}_f,$$

(21)

where $\tilde{x}_f$ denotes the prediction disturbance of the $f$-CAV, i.e., $\tilde{x}_f = x_f - \bar{x}_f$. The planned trajectory ($\bar{x}_f$) can be determined with $\bar{e} = 0$ in equilibrium or be optimized by feedforward control (e.g., MPC methods). Because the CAV is assumed exactly controlled, the prediction disturbance is zero, i.e., $\tilde{x}_f = 0$. If there is no additional control of the $f$-CAV, $\tilde{e}$ will increase with the accumulation of $\tilde{x}_n$ and eventually violate the constraints (e.g., safety, stability, and string stability) of the platoon.

Another source of the tracking error disturbance is the external disturbance, e.g., traffic maneuvers and intersection control. For example, the cut-in and cut-out maneuvers of HDVs will dramatically change the intervehicular space immediately, and therefore produce significant tracking error disturbance. For another example, when approaching intelligent intersections, CAVs could receive the guidance of trajectory, which is significantly different from the equilibrium. Compared with the disturbances caused by the prediction uncertainty, these disturbances are usually large-amplitude and occurring infrequently.

### 4. Influence mitigation of prediction uncertainty

To mitigate the influence of the prediction uncertainty of HDVs, i.e., mitigate the tracking error disturbance, the maneuver of the $f$-CAV should be fine-tuned
accordingly, as indicated in Eq. (21). Ideally, if the state disturbance of f-CAV can counteract the prediction uncertainty, i.e., $\tilde{x}_n = C\tilde{x}_f$, the tracking error disturbance is eliminated. To this end, MPC methods replan the trajectory of the f-CAV at each control interval, and therefore rely heavily on the intervehicular communication and computation. However, it is unnecessary to replan the trajectory at each control interval, considering the small amplitude of the new uncertainty ($\Delta\tilde{x}_n$). Instead, we utilize the feedback control to determine an additional control input ($\tilde{u}_f$) at each control interval, which can fine tune the planned trajectory ($\tilde{x}_f$) and mitigate the tracking error disturbance ($\tilde{e}$). Because the feedback control does not rely on the intervehicular communication and is very easy to compute, it is much more efficient than MPC methods.

4.1. Feedback control design

The aim of the feedback control is to mitigate the tracking error disturbance caused by the prediction uncertainty. According to Eq. (15, 21), the dynamics of tracking error disturbance can be derived as

$$
\tilde{e}(k+1) = \tilde{x}_n(k+1) + C\tilde{x}_f(k+1),
$$

$$= A\tilde{x}_n(k) + \Delta\tilde{x}_n(k) + C(A\tilde{x}_f(k) + B\tilde{u}_f(k)),
$$

(22)

where the last equation is derived considering $AC = CA$.

The feedback control determines the additional acceleration $\tilde{u}_f$ to mitigate the tracking error disturbance, as

$$
\tilde{u}_f(k) = K\tilde{e}(k),
$$

(23)

where $K$ denotes the feedback gain. The actual acceleration is the summation of the planned acceleration and the additional acceleration, i.e., $u_f = \bar{u}_f + \tilde{u}_f$.

To determine the feedback gain, the feedback control is formulated and solved as a discrete linear quadratic regulator problem (Bender and Laub (1987)) as Problem 1.

$$
\min_K J = \sum_0^\infty \{ ||Q\tilde{e}_n(k)||_2 + ||L\tilde{e}_v(k)||_2 + ||R\tilde{u}_f(k)||_2 \},
$$

(24)

where the weighting matrices, i.e., $Q, L,$ and $R$, are symmetric and positive definite.

Substituting the acceleration $\tilde{u}_f$ into the Eq. (22), the dynamics of the tracking error disturbance are rewritten as

$$
\tilde{e}(k+1) = A_K\tilde{e}(k) + \Delta\tilde{x}_n(k),
$$

(25)

where $A_K = A + CBK$. 
4.2. Minimal robust positively invariant set

In this subsection, we quantitatively measure the performance of the feedback control, i.e., how much the tracking error disturbance is mitigated. To this end, we first recall the following well-known definitions (Blanchini (1999)).

Definition 3. (RPI set): The set \( Z \subset \mathbb{R}^n \) is a robust positively invariant (RPI) set of the system (25) if \( A_K \tilde{e} + \Delta \tilde{x}_n \in Z \) for all \( \tilde{e} \in Z \) and all \( \Delta \tilde{x}_n \in W \), i.e., if and only if \( A_K Z \oplus W \subset Z \).

Definition 4. (Minimal RPI set): The mRPI set \( Z \) of system (25) is the RPI set in \( \mathbb{R}^2 \) that is contained in every closed RPI set of system (25).

Definition 3 indicates that if the initial tracking error disturbance is restricted by a RPI set \( Z \) and the prediction uncertainty has the bound \( W \), then the tracking error disturbance can be always restricted inside the set \( Z \) by the feedback control. For the platoon control, the initial tracking error is zero at the plan time. Therefore, if the new prediction uncertainty is bounded, the tracking error disturbance (Eq. (25)) is bounded by the feedback control.

The remaining problem is how to determine the mRPI set, which can equivalently defined as \( Z = \lim_{s \to \infty} F_s \), where

\[
F_s = \bigoplus_{i=0}^{s-1} A^K_i W, F_0 = \{0\}. \tag{26}
\]

However, it is generally impossible to obtain an explicit characterization of \( Z \) using Eq. (26) (Gayek (1991)). In this paper, the \( \epsilon \)-approximation method (Rakovic et al. (2005)) is applied to estimate a outer convex set of the mRPI set, i.e., \( Z \subseteq F \). By decreasing the difference between \( F \) and \( Z \), the outer convex set can well estimate the mRPI set. The details of the \( \epsilon \)-approximation method can be found in (Rakovic et al. (2005)).

4.3. Uncertainty bound

As shown in Eq. (26), the mRPI set is determined by the uncertainty bound (\( W \)) after the feedback control is designed. In practice, however, the bound of the prediction uncertainty could be hard to determine or even not exist, e.g., the uncertainty is changeable for different drivers and extreme behaviors could happen for the same driver.

To solve this issue, the uncertainty bound is generalized and relaxed to bound the new uncertainty most of the time (i.e., with a probability \( \theta \)) as

\[
P(\Delta \tilde{x}_n \in W) = \theta, \tag{27}
\]
where
\[ \mathbb{W} = \{ \Delta \tilde{x}_n \in \mathbb{R}^2 : |\Delta \tilde{s}_n| \leq \omega_s, |\Delta \tilde{v}_n| \leq \omega_v \} , \]
\( \omega_s \) and \( \omega_v \) denote the bound of the position and speed uncertainty respectively.

By this way, the tracking error disturbance can be bounded inside the mRPI set with the probability larger than \( \theta \) if there is no external disturbance, i.e.,
\[ P(\tilde{e} \in \mathbb{Z}) \geq \theta . \]

Note that the bounded prediction uncertainty is the sufficient but unnecessary condition for the bounded tracking error disturbance. It is the reason why the probability in Eq. (29) is larger than \( \theta \). When out-of-bound prediction uncertainty emerges, it usually needs more time to accumulate before the disturbance exceeds the mRPI set. When it happens, the feedforward control will be triggered to replan the trajectory of the \( f \)-CAV, as elaborated in next section. Actually, \( \theta \) becomes a hyper-parameter to balance the feedforward control and feedback control, which increases the flexibility of the proposed method.

5. Robust cooperative platoon control

Based on the feedback control, the feedforward control is designed to construct the robust cooperative platoon control method. As discussed above, the feedback control can mitigate the tracking error disturbance inside the mRPI set most of time. The goal of the feedforward control is to handle the large-amplitude disturbance which is defined as

**Definition 5.** (Large-amplitude Disturbance) A system is said to have a large-amplitude disturbance if the disturbance of the tracking error does not belong to the mRPI set, i.e., \( \tilde{e} \notin \mathbb{Z} \).

The large-amplitude disturbance can be caused by out-of-bound prediction uncertainty (i.e., \( \Delta \tilde{x}_n \notin \mathbb{W} \)), traffic maneuvers (e.g., cut-in or cut-out of HDVs), speed guidance from intersection control, etc. To satisfy the constraints, the feedforward control is conducted with tight constraints, which is equivalent to plan a sequence of mRPI set (i.e., the tube), as shown in Fig. 3.

5.1. Tight constraints

Instead of satisfying the original constraints, the feedforward control is required to satisfy the tight constraints. The tight constraints preserve the space for the trajectory adjustment of the feedback control. Essentially, optimizing a trajectory satisfying the tight constraints is equivalent to optimizing a tube satisfying the original
Therefore, if the actual trajectory is inside the tube, the original constraints are satisfied. As shown in Eq. (14), the original constraints are
\[ e \in E, u_f \in U. \] (30)
If denote \( Z \) as the mRPI set, the tight constraints are obtained as
\[ \mathcal{E} = E \ominus Z, \mathcal{U} = U \ominus KZ, \] (31)
where the Pontryagin difference \( \ominus \) is defined as \( A \ominus B = \{ x \mid x + y \in A, y \in B \} \).

### 5.2. Feedforward control

The goal of the feedforward control is to replan the tube to handle the large-amplitude tracking error disturbance, satisfying all constraints. To this end, the optimization problem with the tight constraints is formulated. To guarantee the stability, the terminal constraints are designed as
\[ \bar{e} \in \mathcal{E}_t = \{ \bar{e} \in \mathbb{R}^2 : \bar{e}(N_p) = 0 \}, \]
\[ \bar{u}_f \in \mathcal{U}_t = \{ \bar{u}_f \in \mathbb{R} : \bar{u}_f(N_p) = 0 \}, \] (32)
where \( N_p \) denotes the number of predictive steps. To improve the performance of the platoon, the multi-objective function is designed as
\[ J = \sum_{0}^{N_p} \left\{ \|G\bar{e}(k)\|_2 + \|F\bar{u}_f(k)\|_2 \right\}, \] (33)
where all weighting matrices, i.e., \( G, F \), are symmetric and positive.

Final, the optimization problem of feedforward control is obtained as

**Problem 2.**

\[ \min_{\bar{u}_f(1), \cdots, \bar{u}_f(N_p)} J \] (34)
subject to
\[ \bar{e}(k) \in \mathcal{E} \cap \mathcal{E}_t, \]
\[ \bar{u}_f(k) \in \mathcal{U} \cap \mathcal{U}_t, \] (35)
where \( k = 1, \cdots, N_p \).
5.3. Overall algorithm

The overall algorithm of the robust cooperative platoon control is summarized as shown in Algorithm 1, which is divided into the offline and online parts. In the offline part, the feedback gain is optimized by solving the Problem 1, the mRPI set is calculated by the $\epsilon$-approximation method after determining the prediction uncertainty bound, and then the tight constraints are computed. In the online part, the feedforward control is triggered if large-amplitude disturbances emerge or the $f$-CAV receives new planned trajectory from the $p$-CAV. The behaviors of the $n$-HDV are predicted and the Problem 2 is solved. The planned trajectory of the $f$-CAV is transmitted to the following CAV and triggers its feedforward control. Therefore the plan information is propagated along the traffic flow. Finally, the $f$-CAV is controlled by the feedback control and the feedforward control, i.e., $u_f = \bar{u}_f + \tilde{u}_f$.

Algorithm 1: The algorithm of the robust cooperative platoon control

**Offline:**
- Compute the feedback gain $K$ by solving Problem 1;
- Determine the uncertainty bound $\mathcal{W}$ after selecting the prediction method;
- Compute the mRPI set $\mathcal{Z}$ by the $\epsilon$-approximation method;
- Compute the tight constraints $\mathcal{E}, \mathcal{U}$;

**Online:**
- Initialize planned trajectory $\bar{e}(t) = 0, \bar{u}(t) = 0, t = 1, \cdots, N$;
- for $t = 1$ to $N$ do
  - Observe the actual tracking error $e(t)$;
  - Compute the tracking error disturbance $\tilde{e}(t) = e(t) - \bar{e}(t)$;
  - if $\tilde{e}(t) \notin \mathcal{Z}$ or $f$-CAV receives new planned trajectories of $p$-CAV then
    - Predict the trajectory of the $n$-HDV;
    - Compute the feedforward control $\bar{u}(k), k = t + 1, \cdots, t + N_p$ by solving the Problem 2;
    - Update the planned tracking error $\bar{e}(k), k = t + 1, \cdots, t + N_p$;
    - Transmit the historical and planned trajectory to its following CAV;
  - else
    - Compute the feedback acceleration $\tilde{u}(t) = K \cdot \tilde{e}(t)$;
    - Compute the actual acceleration $u(t) = \bar{u}(t) + \tilde{u}(t)$;
    - Implement the actual acceleration;
- end
- end
6. Theoretical analysis

In this section, we analyze the feasibility of the proposed method and the achievements of the control objectives (i.e., safety, stability, and string stability) as follows:

**Theorem 1.** The maximal uncertainty bound (i.e., \( \theta = 1 \)) increases linearly with the number of consecutive HDVs, i.e.,

\[
\omega_{s,n} = n \cdot \omega_s, \quad \omega_{v,n} = n \cdot \omega_v,
\]

where \( \omega_{s,n}, \omega_{v,n} \) denote the maximal position and speed bound of the \( n \)-HDV respectively, and \( \omega_s, \omega_v \) denote the maximal bound of one HDV.

**Theorem 2.** The Algorithm 2 is feasible if the control horizon \( N_p \) is sufficiently large and the uncertainty bound \( \bar{W} \) is sufficiently small.

**Theorem 3.** The Problem 2 is a convex optimization problem.

**Theorem 4.** The CAVs are safe, stable and string stable if the algorithm is feasible.

*Proof. See Appendix A.*

Theorem 1 indicates the tendency of the prediction uncertainty with the number of consecutive HDVs. Although only the maximal prediction uncertainty is analyzed and all HDVs are assumed identical, it indicates the linear increase speed for the prediction uncertainty. As indicated by Theorem 2, the feasibility of Algorithm 2 is guaranteed given two conditions, i.e., sufficiently large control horizon and sufficiently small uncertainty bound. The first condition can be easily satisfied by extending the control horizon. For the second condition, as indicated by Theorem 1, the uncertainty bound is determined by the uncertainty bound of each HDV and the consecutive number of HDVs. For low penetration rate of CAVs in long platoon, the number of consecutive HDVs is large. To make the algorithm feasible, a smaller probability \( \theta \) in Eq. (27) can be chosen to decrease the uncertainty bound of each HDV. Essentially, the probability \( \theta \) works as a balance between feedforward control and feedback control. Taking \( \theta = 0 \) as an example, the proposed method will degrade as the MPC method with \( Z = \emptyset \). Therefore, the introduction of \( \theta \) increases the flexibility of the proposed method.
7. Numerical experiments

7.1. Numerical experiment design

This study conducts numerical experiments to verify the control performance of the proposed robust cooperative platoon control method. First, the properties of the prediction uncertainty are analyzed. Second, the experiments verify the communicational and computational efficiency, compared with MPC methods. Third, the experiments verify the control performance regarding safety, stability, and string stability.

To achieve the above test objectives, two platoons in mixed traffic flow and two test scenarios are designed. First, besides of the platoon in Fig. 1 (denoted as P-1), a generalized platoon with another $f$-CAV is designed as shown in Fig. 4 (denoted as P-2). The P-2 can demonstrate the scalability of the proposed method for long mixed traffic flow. Second, two test scenarios are designed, i.e., the scenario 1 only has initial large-amplitude disturbance, and the scenario 2 has multiple large-amplitude disturbances with different occurring frequency. A Poisson process with different $\lambda$ is simulated to generate the large-amplitude disturbances, which can represent the influence of intersection control or traffic maneuvers.

The car-following behaviors of HDVs are simulated by the Newell car-following model with truncated normal uncertainty. Note that the verification of the prediction method is out of the scope of this paper, so the simple car-following model and simple prediction method are applied for the proof of the concept. Specifically, the normal uncertainty is designed as $\Delta \tilde{s} \sim N(0, \sigma_s^2)$ and $\Delta \tilde{v} \sim N(0, \sigma_v^2)$, and then the distribution is truncated with the interval $[-n_s, n_s]$ and $[-n_v, n_v]$ respectively. The values of the parameters can be found in Table 1.

To validate the efficiency regarding computation and communication, the trigger number of the feedforward control and intervehicular communication is compared with the MPC method, which solves the Problem 2 with $Z = \emptyset$ at each control time.
Table 1: The parameter values used in this paper.

| Parameter | Value | Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|-----------|-------|
| $n$       | 5     | $n_1$     | 3     | $n_2$     | 3     |
| $\sigma_s$ | 0.1   | $\sigma_v$ | 0.1   | $n_s$     | 1.0   |
| $n_v$     | 1.0   | $\tau$    | 0.5   | $h$       | 0.5   |
| $v_{min}$ | 0     | $v_{max}$ | 50    | $u_{max}$ | 5     |
| $Q$       | 1     | $L$       | 1     | $R$       | 1     |

interval. To study the tendency with the frequency of large-amplitude disturbances, the comparisons are conducted at the scenario 2 with different values of $\lambda$.

7.2. Uncertainty bound and mRPI set

The uncertainty bound $\mathbb{W}$ is a critical parameter to balance the control effectiveness and efficiency. Fig. 5 (a, b) show the simulation results of the prediction uncertainty of the $n$-HDV with $n = 3$ and $n = 5$ respectively. As shown in Fig. 5 (a), the bound is set $w_s = w_v = 0.2$, and the simulation results show that

$$P(\Delta \tilde{x}_3 \in \mathbb{W}) = 0.751.$$  

Similarly, as shown in Fig. 5 (b), the bound is set $w_s = w_v = 0.3$ where $\theta = 0.820$ for $n = 5$.

To study the temporal tendency of the prediction uncertainty, the prediction uncertainty is studied with the increase of time step. As shown in Fig. 5 (c), the position prediction uncertainty increases quadratically with the time step without the feedback control. To study the spatial tendency, we conduct simulations with different numbers of consecutive HDVs, i.e., from $n = 1$ to $n = 20$, and obtain the uncertainty bound $w_s = w_v$ with $\theta = 0.7$. As shown in Fig. 5 (d), the uncertainty bound increases slower than linear function of the HDV number, which ensures the scalability of the proposed method.

The penetration rate of the CAVs in the mixed traffic flow also affects the uncertainty bound. To study this influence, we simulate a platoon of total 100 vehicles with different penetration rates of CAVs from 1 to 0.1. As shown in Fig. 5 (e), as the decrease of the penetration rate, the prediction uncertainty grows significantly. It indicates that CAVs behave as stabilizers to decrease uncertainty. Fig. 5 (f) further analyzes the influence of both penetration rate and platoon length. Results show the increasing tendency of the uncertainty bound with penetration rate and platoon length.

The feedback gain is determined offline by solving the Problem 1. With the parameters in the Table 1, we obtain $K = [0.6406, 1.0192]$. After the uncertainty
Figure 5: Simulation results of the prediction uncertainty with (a) $n = 4$, (b) $n = 5$, (c) increasing time step without feedback control, (d) increasing numbers of consecutive HDVs, (d) decreasing penetration rates of CAVs, and (e) different lengths and penetration rates.
bound is determined, the mRPI sets are calculated by the \( \epsilon \)-approximation method, as shown in Fig. 6. It is obvious that the mRPI sets expand with the increase of the uncertainty bound.

7.3. Control performance evaluation for the platoon P-1

This section evaluates the performance of the proposed method as it is implemented in the platoon P-1. Fig. 7 shows the results at the scenario 1, where only initial large-amplitude disturbance exists. To validate the effectiveness of the proposed method, the feedforward control and feedback control are studied respectively. Fig. 8 shows the results at the scenario 2, where the large-amplitudes emerge following the Poisson progress with different values of \( \lambda \).

Specifically, Fig. 7(a) illustrates the planned trajectory of the \( f \)-CAV to handle the initial large-amplitude disturbance, i.e., the leading vehicle has a speed disturbance from the initial time. The planned trajectory of the \( f \)-CAV eliminates the disturbance and guarantees the safety, stability (i.e., convergence), and string stability (i.e., the disturbance is not amplified along the string of CAVs). Considering the uncertainty of HDVs, Fig. 7(b) demonstrates the actual trajectory of the \( f \)-CAV, which is manipulated simultaneously by both the feedforward control and feedback control. The control objectives of safety, stability, and string stability are achieved. To validate the effectiveness of mRPI set, all the tracking error disturbances are analyzed. As shown in Fig. 7(c), all the disturbances are bounded by the mRPI
Figure 7: Simulation results of the platoon P-1 in the scenario 1: without uncertainty (a); with uncertainty (b); error disturbances (c); and total acceleration (d).
Figure 8: Simulation results of the platoon P-1 in the scenario 2 with different values of $\lambda$: (a) $\lambda = 10$; (b) $\lambda = 7.5$; (c) $\lambda = 5$; (d) $\lambda = 2.5$.

set, i.e., the actual trajectory is tuned inside the planned tube. Fig. 7 (d) demonstrates the actual acceleration of the $f$-CAV, which is the combination of the planned acceleration $\bar{u}$ and the acceleration of the feedback control $\tilde{u}$.

Fig. 8 further demonstrates the ability of the proposed method in handling multiple large-amplitude disturbances. It shows that the proposed method can handle different occurring frequencies of large-amplitude disturbances, guaranteeing the safety, stability, and string stability. Note that though the HDVs could amplify the disturbances, the $f$-CAV can significantly reduce the disturbances, which are beneficial to the traffic stability and efficiency.

Fig. 9 compares the trigger numbers of the feedforward control and intervehicular
communication between the MPC method and the proposed method with different values of $\lambda$. Since the MPC method solves the optimization problem at each control interval, which relies on the communication information, the trigger number of the MPC method keeps equal to the total simulation steps (i.e., 150). For the proposed method, the feedforward control as well as the intervehicular communication is only triggered when the large-amplitude disturbance emerges. Therefore, with the increasing of the $\lambda$, the occurring number of the large-amplitude disturbances decreases, so the trigger number of feedforward control and communication decreases. Considering the computational burden mainly comes from the feedforward control, the proposed method is efficient regarding computation and intervehicular communication.

### 7.4. Control performance evaluation for the platoon P-2

This section further evaluates the performance of the proposed method in a long mixed traffic flow as it is implemented in the platoon P-2. Fig. 10 shows the simulation results at scenarios with different large-amplitude disturbances. To make the paper concise, the similar results of the planned trajectory, mRPI set, and acceleration profile are not analyzed again. We mainly provide the velocity results to demonstrate the effectiveness of the proposed method in the long mixed traffic flow. Specifically, Fig. 10 (a) demonstrates the performance regarding safety, stability, and string stability. Different with the HDVs which amplify the disturbances, the two CAVs behave as stabilizers which decrease the disturbance. From the perspective of $f$-CAV-2, the $f$-CAV-1 is the predecessor and plays the role of $p$-CAV. By this way, the proposed method can be applied in a long mixed traffic flow. Fig. 10 (b-d) demonstrate the performance in the scenario 2 with multiple large-amplitude disturbances. The results show that the propose method works well even for the consecutive large-amplitude disturbances, as shown in Fig. 10 (d).
Figure 10: Simulation results of the platoon P-2 in different scenarios: (a) scenario 1; (b) scenario 2 with $\lambda = 7.5$; (c) scenario 2 with $\lambda = 5$; (d) scenario 2 with $\lambda = 2.5$. 

26
8. Conclusions

In this paper, we proposed a robust cooperative platoon control method in mixed traffic flow, which leveraged the properties (i.e., amplitude bound and occurring frequency) of the prediction uncertainty of HDVs. The major idea was to integrate the event-triggered feedforward control with the time-triggered feedback control, which can mitigate and bound the accumulation of the disturbance tracking error caused by the prediction uncertainty. To measure the disturbance boundary, the mRPI set was calculated. To satisfy the constraints, tight constraints are determined for the feedforward control based on tube methods. It was proved that the proposed method can explicitly guarantee all constraints, require no wireless connectivity of HDVs, and significantly reduce the burden of intervehicular communication and computation. Theoretical analysis and numerical experiments validate the robustness, efficiency, and scalability of the proposed method.

There are several promising future studies following this research. First, the integration of advanced prediction models of HDVs can further improve the control performance. Second, extension of the proposed method into the traffic network level is promising.

Appendix A. Proof of Theorem 1-4

Proof. (Theorem 1) Consider the worst case where all HDVs have the maximal prediction uncertainty \( \Delta \bar{x} = [\omega_s, \omega_v]^T \) at the same time. Then the total prediction uncertainty \( \Delta \bar{x}_n \) is the summation of the all uncertainty as

\[
\Delta \bar{x}_n = n \cdot \Delta \bar{x}.
\]

Therefore, the maximal prediction uncertainty is determined as

\[
\omega_{n,s} = n \cdot \omega_s, \omega_v = n \cdot \omega_v,
\]

which concludes the theorem.

Proof. (Theorem 2) It is immediately obvious that two conditions are required for the feasibility of Algorithm 1: first, the mRPI set is sufficiently small such that all tight constraints exist; second, the Problem 2 is feasible. To satisfy the first condition, the mRPI set \( F_s = \bigoplus_{i=0}^{s-1} A_K^i \mathcal{W} \) is required sufficiently small. Since the \( A_K \) is unchanged, the mRPI set can be sufficiently small if the uncertainty bound \( \mathcal{W} \) is sufficiently small. To satisfy the second condition, the key is to reserve sufficient control time for the feedforward control to eliminate the initial tracking error, i.e., satisfy the terminal constraints.
Proof. (Theorem 3) Note the mRPI set computed by $\epsilon$-approximation method is convex and the set of string stability constraint, i.e., $\mathcal{D}$, is also convex. The theorem is concluded, considering that the objective function $J$ is a convex function and the sets of constraints, i.e., $\overline{E}$, $\bar{E}$, $\bar{U}$, $\bar{U}^t$, are convex sets.

Proof. (Theorem 4) First, the constraints of the CAVs, i.e., $e \in \bar{E}$ and $u_f \in \bar{U}$, are satisfied if the algorithm is feasible (Mayne and Langson (2001)). Therefore, the safety constraint is satisfied considering the prediction uncertainty, i.e., the CAV is safe. Second, the terminal constraints are satisfied by the feedforward control if there is no more uncertainty, i.e., $\dot{v}_n = 0$, which is consistent with the definition of stability. Third, the constraint of string stability is also satisfied by guaranteeing $e \in \bar{E}$.

References

Bender, D. J., Laub, A. J., 1987. The linear-quadratic optimal regulator for descriptor systems: discrete-time case. Automatica 23 (1), 71–85.

Blanchini, F., 1999. Set invariance in control. Automatica 35 (11), 1747–1767.

Chen, N., Wang, M., Alkim, T., van Arem, B., 2018. A robust longitudinal control strategy of platoons under model uncertainties and time delays. Journal of Advanced Transportation 2018.

Chen, X., Li, L., Zhang, Y., 2010. A markov model for headway/spacing distribution of road traffic. IEEE Transactions on Intelligent Transportation Systems 11 (4), 773–785.

Chien, C., Ioannou, P., 1992. Automatic vehicle-following. In: American Control Conference, 1992. IEEE, pp. 1748–1752.

Feng, S., Sun, H., Zhang, Y., Zheng, J., Liu, H. X., Li, L., 2019a. Tube-based discrete controller design for vehicle platoons subject to disturbances and saturation constraints. IEEE Transactions on Control Systems Technology.

Feng, S., Wang, X., Sun, H., Zhang, Y., Li, L., 2018a. A better understanding of long-range temporal dependence of traffic flow time series. Physica A: Statistical Mechanics and its Applications 492, 639–650.

Feng, S., Zhang, Y., Li, S. E., Cao, Z., Liu, H. X., Li, L., 2019b. String stability for vehicular platoon control: Definitions and analysis methods. Annual Reviews in Control.
Feng, Y., Head, K. L., Khoshmagham, S., Zamanipour, M., 2015. A real-time adaptive signal control in a connected vehicle environment. Transportation Research Part C: Emerging Technologies 55, 460–473.

Feng, Y., Yu, C., Liu, H. X., 2018b. Spatiotemporal intersection control in a connected and automated vehicle environment. Transportation Research Part C: Emerging Technologies 89, 364–383.

Gayek, J. E., 1991. A survey of techniques for approximating reachable and controllable sets. In: [1991] Proceedings of the 30th IEEE Conference on Decision and Control. IEEE, pp. 1724–1729.

Gong, S., Du, L., 2018. Cooperative platoon control for a mixed traffic flow including human drive vehicles and connected and autonomous vehicles. Transportation research part B: methodological 116, 25–61.

Gong, S., Shen, J., Du, L., 2016. Constrained optimization and distributed computation based car following control of a connected and autonomous vehicle platoon. Transportation Research Part B: Methodological 94, 314–334.

Guo, G., Wen, S., 2016. Communication scheduling and control of a platoon of vehicles in vanets. IEEE Transactions on Intelligent Transportation Systems 17 (6), 1551–1563.

Horowitz, R., Varaiya, P., 2000. Control design of an automated highway system. Proceedings of the IEEE 88 (7), 913–925.

Jin, I. G., Orosz, G., 2014. Dynamics of connected vehicle systems with delayed acceleration feedback. Transportation Research Part C: Emerging Technologies 46, 46–64.

Jin, I. G., Orosz, G., 2017. Optimal control of connected vehicle systems with communication delay and driver reaction time. IEEE Transactions on Intelligent Transportation Systems 18 (8), 2056–2070.

Jin, I. G., Orosz, G., 2018. Connected cruise control among human-driven vehicles: Experiment-based parameter estimation and optimal control design. Transportation research part C: emerging technologies 95, 445–459.

Ke, R., Zeng, Z., Pu, Z., Wang, Y., 2018. New framework for automatic identification and quantification of freeway bottlenecks based on wavelet analysis. Journal of Transportation Engineering, Part A: Systems 144 (9), 04018044.
Langson, W., Chryssochoos, I., Raković, S., Mayne, D. Q., 2004. Robust model predictive control using tubes. Automatica 40 (1), 125–133.

Li, F., Wang, Y., 2017. Cooperative adaptive cruise control for string stable mixed traffic: Benchmark and human-centered design. IEEE Transactions on Intelligent Transportation Systems 18 (12), 3473–3485.

Li, L., Wen, D., Yao, D., 2014. A survey of traffic control with vehicular communications. IEEE Transactions on Intelligent Transportation Systems 15 (1), 425–432.

Li, S. E., Zheng, Y., Li, K., Wang, J., 2015. An overview of vehicular platoon control under the four-component framework. In: Intelligent Vehicles Symposium (IV), 2015 IEEE. IEEE, pp. 286–291.

Li, Z., Khasawneh, F., Yin, X., Li, A., Song, Z., 2019. A new microscopic traffic model using a spring-mass-damper-clutch system. arXiv preprint arXiv:1903.04469.

Liu, X., Goldsmith, A., Mahal, S. S., Hedrick, J. K., 2001. Effects of communication delay on string stability in vehicle platoons. In: Intelligent Transportation Systems, 2001. Proceedings. 2001 IEEE. IEEE, pp. 625–630.

Mayne, D., Langson, W., 2001. Robustifying model predictive control of constrained linear systems. Electronics Letters 37 (23), 1422–1423.

Milanés, V., Shladover, S. E., Spring, J., Nowakowski, C., Kawazoe, H., Nakamura, M., 2014. Cooperative adaptive cruise control in real traffic situations. IEEE Transactions on Intelligent Transportation Systems 15 (1), 296–305.

Newell, G. F., 2002. A simplified car-following theory: a lower order model. Transportation Research Part B: Methodological 36 (3), 195–205.

Ogata, K., 1995. Discrete-time control systems. Vol. 2. Prentice Hall Englewood Cliffs, NJ.

Orosz, G., 2016. Connected cruise control: modelling, delay effects, and nonlinear behaviour. Vehicle System Dynamics 54 (8), 1147–1176.

Rakovic, S. V., Kerrigan, E. C., Kouramas, K. I., Mayne, D. Q., 2005. Invariant approximations of the minimal robust positively invariant set. IEEE Transactions on Automatic Control 50 (3), 406–410.
Santhanakrishnan, K., Rajamani, R., 2003. On spacing policies for highway vehicle automation. IEEE Transactions on Intelligent Transportation Systems 4 (4), 198–204.

Sheikholeslam, S., Desoer, C. A., 1993. Longitudinal control of a platoon of vehicles with no communication of lead vehicle information: A system level study. IEEE Transactions on vehicular technology 42 (4), 546–554.

Sun, X., Yin, Y., 2019. Behaviorally stable vehicle platooning for energy savings. Transportation Research Part C: Emerging Technologies 99, 37–52.

Talebpour, A., Mahmassani, H. S., 2016. Influence of connected and autonomous vehicles on traffic flow stability and throughput. Transportation Research Part C: Emerging Technologies 71, 143–163.

Van Arem, B., Van Driel, C. J., Visser, R., 2006. The impact of cooperative adaptive cruise control on traffic-flow characteristics. IEEE Transactions on Intelligent Transportation Systems 7 (4), 429–436.

Wang, C., Gong, S., Zhou, A., Li, T., Peeta, S., 2019a. Cooperative adaptive cruise control for connected autonomous vehicles by factoring communication-related constraints. Transportation Research Part C: Emerging Technologies.

Wang, M., Daamen, W., Hoogendoorn, S. P., van Arem, B., 2016. Cooperative car-following control: Distributed algorithm and impact on moving jam features. IEEE Transactions on Intelligent Transportation Systems 17 (5), 1459–1471.

Wang, X., Jiang, R., Li, L., Lin, Y., Zheng, X., Wang, F.-Y., 2018. Capturing car-following behaviors by deep learning. IEEE Transactions on Intelligent Transportation Systems 19 (3), 910–920.

Wang, X., Jiang, R., Li, L., Lin, Y.-L., Wang, F.-Y., 2019b. Long memory is important: A test study on deep-learning based car-following model. Physica A: Statistical Mechanics and its Applications 514, 786–795.

Wen, S., Guo, G., Chen, B., Gao, X., 2018. Cooperative adaptive cruise control of vehicles using a resource-efficient communication mechanism. IEEE Transactions on Intelligent Vehicles 4 (1), 127–140.

Xu, H., Feng, S., Zhang, Y., Li, L., 2019. A grouping based cooperative driving strategy for cavs merging problems. IEEE Transactions on Vehicular Technology.
Yang, Z., Feng, Y., Gong, X., Zhao, D., Sun, J., 2019. Eco-trajectory planning with consideration of queue along congested corridor for hybrid electric vehicles. Transportation Research Record, 0361198119845363.

Yu, C., Feng, Y., Liu, H. X., Ma, W., Yang, X., 2018. Integrated optimization of traffic signals and vehicle trajectories at isolated urban intersections. Transportation Research Part B: Methodological 112, 89–112.

Yu, C., Feng, Y., Liu, H. X., Ma, W., Yang, X., 2019a. Corridor level cooperative trajectory optimization with connected and automated vehicles. Transportation Research Part C: Emerging Technologies 105, 405–421.

Yu, C., Sun, W., Liu, H. X., Yang, X., 2019b. Managing connected and automated vehicles at isolated intersections: From reservation-to optimization-based methods. Transportation research part B: methodological 122, 416–435.

Zhang, Z., 1994. Iterative point matching for registration of free-form curves and surfaces. International journal of computer vision 13 (2), 119–152.

Zheng, F., Jabari, S. E., Liu, H. X., Lin, D., 2018. Traffic state estimation using stochastic lagrangian dynamics. Transportation Research Part B: Methodological 115, 143–165.

Zhou, J., Peng, H., 2005. Range policy of adaptive cruise control vehicles for improved flow stability and string stability. IEEE Transactions on intelligent transportation systems 6 (2), 229–237.

Zhou, K., Doyle, J. C., Glover, K., et al., 1996. Robust and optimal control. Vol. 40. Prentice hall New Jersey.

References