Electronic band structure in $n$-type GaAs/AlGaAs wide quantum wells in tilted magnetic field

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Oscillations of the real component of AC conductivity $\sigma_1$ in a magnetic field were measured in the $n$-AlGaAs/GaAs structure with a wide (75 nm) quantum well by contactless acoustic methods at $T=20-500$ mK. Deep oscillations corresponding to the integer and half-integer filling factors were observed. The appearance of oscillations with half-integer filling factors $\nu > 5/2$ is associated with the two-subband electron spectrum, namely the symmetric (S) and antisymmetric (AS) subbands formed due to electrostatic repulsion of electrons. A change of the integer and half-integer filling factors oscillations amplitude in tilted magnetic field observed in the experiments occurs due to the crossings of Landau levels of different subbands (S and AS) at the Fermi level. The theory developed in this work shows that these crossings are caused by the difference in the cyclotron energies in the S and AS subbands induced by the in-plane magnetic field.

I. INTRODUCTION

Transport investigations of two-dimensional (2D) high-mobility structures demonstrate a variety of effects taking place in different ranges of magnetic fields and temperatures. In moderate fields, the Shubnikov-de Haas oscillations are observed, while in stronger fields, the integer and fractional quantum Hall effects take place [1]. In some systems the Wigner crystal is formed [1]. The most interesting results are mainly obtained on $n$-type GaAs/AlGaAs structures due to high mobility of carriers of up to $10^7$ cm$^2$/Vs in these heterostructures.

Since the general description of the phenomena observed in 2D systems in high magnetic fields is well developed, the interest is now shifting to more exotic systems and situations. At large half-integer filling factors $\nu > 5/2$ and low temperatures $T < 150$ mK, the formation of the stripe phases due to strong electron-electron interaction has been predicted [2–4]. This phase is characterized by a strong anisotropy of longitudinal conductivity $\sigma_{xx}$, and the stripe formation explains the oscillations in ultra-high-mobility heterojunctions [5]. Recently it has been demonstrated experimentally that the stripes can be re-oriented by the in-plane component of a magnetic field, i.e., in a tilted magnetic field [6].

The conductivity in tilted magnetic fields is rather non-trivial in 2D structures with a wide quantum well (WQW). In such systems, electrons are localized near the interfaces due to electrostatic repulsion. As a result, a formed two-layer system is similar to a pair of quantum wells. Due to an inter-layer tunneling, the electronic energy structure consists of two subbands, the symmetric (S) and antisymmetric (AS), separated by an energy gap $\Delta_{SAS}$ [7]. In low magnetic fields, conductivity oscillations caused by elastic scattering between the S and AS subbands were observed which are different from the Shubnikov-de Haas oscillations [8]. The presence of two conductivity channels in WQWs results in conductivity magnetooscillations corresponding to filling factors $\nu$ with even denominators. At small filling factors $\nu = 1/2, 3/2$, the fractional quantum Hall effect has been observed in some WQWs. A phase diagram has been constructed demonstrating conditions of stable observation of the fractional quantum Hall effect at $\nu = 1/2$ at various electron densities, quantum well widths and values of $\Delta_{SAS}$, see Ref. [9] and references therein. In other WQWs, a gas of composite fermions has been observed and investigated in detail, see, e.g., Ref. [10].

The conductivity magnetooscillations can be strongly affected by the in-plane field component. In systems with one 2D subband, the Landau level crossing in tilted magnetic fields occurs because the orbital splitting is determined by the perpendicular field component, while the spin splitting is given by the total magnetic field. It results in crossing of spin sublevels with opposite spin orientation and related to different Landau levels. This crossing is allowing determination of the $g$-factor from the magnetoconductivity measurements, a method proposed long time ago [11] and widely used up to nowadays. The situation is much richer in systems with two electronic subbands, in particular in WQWs, due to a presence of the $\Delta_{SAS}$ gap. It has been pointed out in Refs. [12, 13] that the crossing of Landau levels from different subbands (S and AS) can occur in this case in a tilted field, since the value of $\Delta_{SAS}$ can depend on the in-plane field component. Calculations of the energy spectrum in parallel field $B||$ show that the gap $\Delta_{SAS}$ increases at small $B||$ while in stronger fields $B|| > 2$ T, the inter-layer coupling is suppressed, and $\Delta_{SAS} \rightarrow 0$ [14]. This result demonstrates a great potential of experiments on high-mobility WQWs in tilted magnetic fields.

In the present work, we study the conductivity of a WQW structure in tilted magnetic fields. In a perpendicular magnetic field $0.5 < B_\perp < 1.5$ T, we have observed deep regular conductivity oscillations corresponding to half-integer filling factors. However, in contrast to
II. EXPERIMENT

We studied a multilayered $n$-GaAlAs/GaAs/GaAlAs structure with a 75 nm wide GaAs quantum well. The WQW was $\delta$-doped on both sides and located at the depth $\approx 197$ nm below the surface of the sample. While cooling the sample down to 15 K and illuminating it with infrared light of an emitting diode, we achieved the electron mobility $\mu = 2.2 \times 10^7$ cm$^2$/Vs and density $n = 1.4 \times 10^{11}$ cm$^{-2}$ (at $T = 0.3$ K). In the absence of a magnetic field, the S-AS energy splitting in this structure was $\Delta_{\text{SAS}} = 0.42$ meV [8].

In our contactless acoustic experiments, a surface acoustic wave (SAW) propagates along a surface of a piezoelectric crystal LiNbO$_3$ while the sample is slightly pressed onto this surface on which the interdigital transducers are deposited to generate and detect the wave. The ac electric field accompanying the SAW penetrates into the 2D channel located in the semiconductor structure. The field produces electrical currents which, in turn, cause Joule losses. As a result of the interaction of the SAW electric field with charge carriers in the quantum well, the SAW attenuation $\Gamma$ and its velocity shift $\Delta \nu$ are governed by the complex high-frequency conductance, $\sigma^{\text{AC}} = \sigma_1(\omega) - i\sigma_2(\omega)$. The SAW technique has shown to be a very efficient tool as it allows studying both real $\sigma_1$ and imaginary $\sigma_2$ components of the high-frequency conductance without any needs for electrical contacts [15].

The measurements in magnetic field perpendicular to the sample plane were carried out in a dilution refrigerator. Dependences of the absorption coefficient $\Gamma$ and the velocity change $\Delta \nu$ on the magnetic field at $B \leq 18$ T were measured in the temperature interval 20-500 mK. The SAW frequency was changed from 28 to 307 MHz. The ac conductivity $\sigma^{\text{AC}}$ was deduced with help of the expressions of Ref. [15]. The measurements in tilted magnetic fields were done at SAW frequency of 30 MHz, in the same magnetic field range of 18 T, and at the temperature 0.3 K. The samples were mounted on a one-axis rotator, which enabled us to change the angle between the normal to the structure and the magnetic field.

In the present work, we analyzed dependences of the real component of the conductivity $\sigma_1 \gg \sigma_2$ (in this case $\sigma_1 \approx \sigma^{\text{DC}}$) on the magnetic field. We studied the interval $B_{\perp} = 0.5 - 1.5$ T, where the oscillations corresponding to the half-integer filling factors $\nu = 3/2, 5/2, 7/2, 9/2, ... 21/2$ (at $T = 310$ mK) were observed. Fig. 1 presents the oscillations of $\sigma_1$ in the range $B_{\perp} = 0.5 - 2.5$ T. The oscillation at $\nu = 3/2$ (observed at $B=3.9$ T and shown here) can be attributed to the quantum Hall effect. However the oscillations at half-integer $\nu \geq 5/2$ have the same form as at integer $\nu$. Note that the filling factors were determined by the formula $\nu = n\hbar/eB_{\perp}$, where $n$ is the total electron density in the WQW. The oscillations presented in Fig. 1 exist up to $T = 400$ mK.

![Fig. 1. Dependence of the real part of the ac conductivity $\sigma_1$ on magnetic field $B_{\perp}$ at $T = 20$ mK. SAW frequency $f = 30$ MHz. The filling factors are denoted near oscillations.](image)

In order to understand the nature of the oscillations we consider a fan of Landau levels for the two-subband system under study, Fig. 2. We see that the magnetic fields where the Fermi level crosses the Landau levels of the lower (S) subband correspond to integer filling factors while those for the AS subband correspond to half-integer filling factors. Here we do not take into account the Fermi energy oscillations in magnetic field. The minima of the conductivity at half-integer $\nu$ are observed at temperatures up to 400 mK which allowed us to study their temperature dependence in the range 20 – 400 mK. This dependence shown in Fig. 3 demonstrates that the conductivity in minima has the activation character. The same takes place at integer filling factors. We conclude that the oscillations corresponding to half-integer filling factors in the WQW under study are caused not by the stripe phase formation but by the existence of two size-quantized subbands.

The unusual oscillation pattern in the WQW is even more intriguing in tilted magnetic fields. The results of corresponding measurements are presented in Fig. 4. This figure shows that at some tilt angles of the magnetic field the minima of the conductivity oscillations are changed to maxima. The experimental results presented in Fig. 4 were obtained at the in-plane component of the magnetic field $B_{\parallel}$ oriented along the SAW propagation direction. We also performed experiments in another configuration and observed that the minima of conduc-
FIG. 2. Landau levels in the WQW with the tunneling splitting $\Delta_{SAS} = 0.42$ meV. The fans are crossed by the energy $E^\text{tot}$ corresponding to the total electron density in the WQW $n = 1.4 \times 10^{11} \text{ cm}^{-2}$. The arrows indicate the filling factors $\nu$. The numbers $n$ and $n'$ are the Landau level numbers in the subbands S and AS, respectively. The Landau levels of the lower (S) and AS subbands shown by red and blue colors, respectively. Lines related to $\nu=8$ and $\nu=17/2$ are intentionally omitted due to the figure space limitations.

FIG. 3. Temperature dependence of the conductivity in oscillations minima at half-integer $\nu$ shown near the curves. The SAW frequency $f = 85$ MHz.

III. THEORY

The Hamiltonian of a doped WQW in a parallel magnetic field is given by

$$\mathcal{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + \frac{1}{2m} \left( p - \frac{e}{c} A \right)^2 + e\phi(z). \quad (1)$$

Here $z$ is the axis normal to the WQW, $p$ is the in-plane electron momentum, $A = z(B_y, -B_x, 0)$ is the vector potential of the magnetic field, and $\phi(z)$ is the electrostatic potential.

We performed self-consistent calculations of the electrostatic potential and electron wavefunctions. First, the wave functions are calculated in the tight-binding approach [16]. Then the electron wave functions are used to calculate the electron density distribution in the quantum well as explained in Refs. [8, 17]. The value of the total electron density extracted from our experiment is $n = 1.4 \times 10^{11} \text{ cm}^{-2}$. The electrostatic potential $\phi(z)$ corresponding to the charged QW is found...
from the numerical solution of the Poisson equation at zero in-plane momentum $\mathbf{p} = 0$. We account for the in-plane magnetic field $B_\parallel$ through the effective potential $\Delta_P = e(B_\parallel d/c)^2/2m$, see Eq. (1). We add the term $e\phi(z) + e\phi_B$ to the structure potential and compute the next approximation for the electron wave functions of the levels in the WQW. The procedure is repeated until the self-consistency of the electron wave functions and electrostatic potential is reached. In accordance with previous studies [7, 8], we obtained that the carriers are located near the WQW edges in the narrow layers separated by a distance $d = 53$ nm. At $B_\parallel = 0$, $\Delta_{\text{SAS}} = 0.42$ meV, and the in-plane magnetic field changes this value due to the diamagnetic term $e\phi_B$. We discuss this dependence in the next section. Here we note that $\Delta_{\text{SAS}}$ has weak dependence on $B_\parallel$ increasing by just about 40% at $B_\parallel = 2$ T.

Not taking into account $\Delta_{\text{SAS}}$, we consider the states in the left (L) and right (R) layers separately assuming the layers’ width to be infinitely small. Then, in the presence of the in-plane magnetic field $B_\parallel$, the energy dispersions of the L and R states are anisotropic:

$$
E_{\text{LR}}(p) = \frac{p^2}{2m} + \frac{1}{2m} \left( eB_\parallel d \frac{2c}{2c} \right)^2 \pm \frac{ed}{2mc}(B_\parallel \times p)_z, \quad (2)
$$

Here the second term is the diamagnetic shift equal for both states, and the separation of the subbands’ minima in the momentum space, $\Delta p = eB_\parallel d/c$, is given by the last term. In a tilted magnetic field, this separation results in the difference of centers of cyclotron circles $\Delta r = d B_\parallel / B_\perp$, Fig. 5.

Now we take into account that, due to tunneling, the carrier trajectories are not just two shifted circles but are more complicated ones. This fact results in a difference of the cyclotron energies. Indeed, the cyclotron period $T$ in the perpendicular field $B_\perp$ is related to the area in the momentum space bounded by the trajectory, $A$, by

$$
T = \frac{c}{|eB_\perp|} \frac{\partial A}{\partial E}, \quad A = \int d\varphi \frac{p^2(\varphi)}{2}, \quad (3)
$$

where $E$ is the carrier energy and $\varphi$ is an angle between $p$ and $B_\parallel$. The electron momentum absolute values at a fixed energy $E$ for $0 < \varphi < \pi/2$ are found from Eq. (2)

$$
p_{\text{LR}}(\varphi, E) = \left| \frac{eB_\parallel d}{2c} \sin \varphi \pm \sqrt{2mE - \left( \frac{eB_\parallel d}{2c} \right)^2 \cos^2 \varphi} \right|, \quad (4)
$$

and the momenta at other values of $\varphi$ are obtained from this expression by appropriate rotations. With account for tunneling, the S and AS states are formed, and the electron trajectories are different from simple circles [14, 18, 19]. The cyclotron orbits are shown schematically in Fig. 5. With account for tunneling, the electrons spend by a half of their cyclotron periods in the left and right layers. As a result, at $|\varphi| < \pi/2$ $p = p_L(\varphi)$ in the S subband and $p = p_R(\varphi)$ in the AS subband. Substitution to Eq. (3) yields

$$
\omega_{\text{S,AS}} = 2 \int_0^{\pi/2} d\varphi p^2_{\text{LR}}(\varphi), \quad (5)
$$

and for the cyclotron periods in the S and AS subbands:

$$
T_{\text{S,AS}} = \frac{2\pi}{\omega_{\text{c,0}}} (1 \pm b), \quad b = \frac{2}{\pi} \arcsin \left( \frac{eB_\parallel d}{2c\sqrt{2mE_F}} \right). \quad (6)
$$

Here $\omega_{\text{c,0}} = |eB_\perp|/mc$ is the cyclotron frequency in the absence of parallel field, and we took the electron energy $E$ equal to the Fermi energy $E_F$ assuming $E_F \gg \Delta_{\text{SAS}}$. As a result, we have different cyclotron frequencies in two subbands:

$$
\omega_{\text{c,AS}} = \frac{\omega_{\text{c,0}}}{1 \pm b}. \quad (7)
$$

The cyclotron periods are different due to tunneling and separation of the cyclotron trajectory centers.

We see that the cyclotron frequencies in the S and AS subbands depend on the tilt angle $\theta$ via $B_\parallel = B_\perp \tan \theta$.

**IV. DISCUSSION**

The maxima in the conductivity in the activation regime are caused by crossings of some Landau levels at the Fermi level. These levels correspond to S and AS subbands and to opposite spin orientations. Energies of the $n$th level of the S subband with spin down (↓) and $n$’th level of the AS subband with spin up (↑), are given
(a) and (b) demonstrate a very good agreement of the level crossings at experimentally found and theoretically obtained angles corresponding to the Landau level positions on the tilt angle \( \theta \). We estimated the value of \( g = 9.0 \pm 2.0 \) for the factor \( g \) and \( \mu_B \) is the Bohr magneton and \( B \) is the total magnetic field. Equation (8) demonstrates three reasons for reconstruction of the energy levels by parallel magnetic field. First, the tunneling gap \( \Delta_{\text{SAS}} \) changes in the parallel field [14]. We have calculated the dependence of \( \Delta_{\text{SAS}} \) on \( B_\parallel \) as described in the previous section of this paper. The results of calculations are shown in the inset to Fig. 6. We see that the dependence of \( \Delta_{\text{SAS}} \) on the parallel field \( B_\parallel \) is weak, so that \( \Delta_{\text{SAS}} \) in the WQW under study is small in the whole range \( 0 < B_\parallel < 2 \, \text{T} \). Our analysis shows that for the dependence of \( \Delta_{\text{SAS}} \) on the parallel field \( B_\parallel \) shifts the crossing points of the Landau level for \( \nu = 10 \) just by 0.2 meV which is 0.45% only.

Then, the Zeeman terms in Eqs. (8) result in the dependence of the Landau levels on the tilt angle at a fixed perpendicular field via \( B = B_\perp \cos \theta \). However, for explanation of the level crossings at experimentally found angles one should take the values of \( g \sim 30 \ldots 40 \) which are unrealistic.

Finally, we examined the dependencies of the cyclotron energies in the S and AS subbands on \( B_\parallel \). According to Eq. (7), the cyclotron energy decreases in the S subband and increases in the AS subband with \( \nu \). It results in a dependence of the Landau level positions on \( \theta \) via the factors \( b(\theta) \), see Eqs. (8). We plot the Landau level positions at the filling factor \( \nu = 10 \) (see Fig. 6) taking the inter-layer separation in the WQW under study as \( d = 53 \, \text{nm} \) [7, 8] and the Fermi energy in each subband as \( E_F = 2.5 \, \text{meV} \). Fig. 6 demonstrates that at \( B_\parallel = 0.565 \, \text{T} \), the 4th Landau level of the AS subband \( (n' = 4) \) crosses at the Fermi level with the 5th Landau level of the S subband \( (n = 5) \) at \( \theta \approx 7^\circ \). At \( \theta \approx 44^\circ \), the Landau levels with \( n' = 3 \) and \( n = 6 \) cross at the Fermi level.

We plot the Landau levels as in Fig. 6 for various filling factors. From the analysis of odd \( \nu = 3, 5, 7, 9 \) and half-integer \( \nu = 11/2, 13/2 \) we obtain that the Landau-level crossings occur at \( \theta_c = 30^\circ \) and \( 54^\circ \), while for even \( \nu = 6, 8, 10 \) we get \( \theta_c = 7^\circ, 44^\circ, \) and \( 59^\circ \). These values of \( \theta_c \) are shown in Fig. 7 by vertical lines. We do not analyze the crossings at higher angles where \( B_\parallel > 1.5 \, \text{T} \) because the developed theory does not describe this regime.

Fig. 7 shows the angular dependencies of the conductivity on the tilt angle \( \theta \) for various filling factors \( \nu \). One can see that the conductivity maxima positions coincide with the theoretically obtained angles corresponding to Landau level crossings at the Fermi level. We have adjusted the \( g \)-factor values for better agreement of theory and experiment. The \( g \)-factor values determined by that procedure were \( g = 9.0 \pm 2.0 \). This large spreading of the \( g \)-factor values is explained by an error in the determination of the initial value of the tilt angle \( \theta = 0^\circ \). The obtained \( g \)-factor agrees with the data available in the literature determined from transport experiments in various \( n \)-type GaAs/AlGaAs structures where the values are \( g = 8 \ldots 11 \), see Ref. [20] and references therein.

Figs. 7 (a) and (b) demonstrate a very good agreement between theory and experiment obtained with one adjustable parameter, namely the \( g \)-factor.

V. CONCLUSION

In the high-mobility \( n \)-GaAs/AlGaAs structure with a 75-nm wide quantum well, we have observed deep regular conductivity oscillations at integer and half-integer filling factors in magnetic fields \( B_\perp = 0.5 \ldots 1.5 \, \text{T} \) in the temperature range \( T = 20 \ldots 400 \, \text{mK} \). Analysis of the oscillation minima positions in a magnetic field shows that oscillations with half-integer filling factors are caused by the two-subband electron spectrum (S and AS subbands) formed due to electrostatic repulsion of electrons. In tilted fields, a strong increase of the conductivity leading to a disappearance of the minima took place at some tilt angles different for different minima. This effect is shown to be caused not by a variation of \( \Delta_{\text{SAS}} \) with \( B_\parallel \) which is very weak in the studied structure but by crossings of the Landau levels from the S and AS subbands. The developed theory demonstrates that the crossings in the range \( B_\parallel = 0 \ldots 2 \, \text{T} \) occur due to different dependencies of cyclotron energies in the subbands on the parallel field. Namely, the cyclotron energy in the S (AS) subband decreases (increases) with \( B_\parallel \). We estimated the value of
the electron $g$-factor by fitting theoretical dependences to experimental data.

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