Question of $\eta$- and $K^-$- Nucleus Bound States

B. K. Jain$^1$, N. J. Upadhyay$^2$ and Mohammad Shoeb$^3$

$^1$ MU-DAE Centre for Excellence in Basic Sciences, Mumbai University, Kalina Campus, Mumbai-400098, INDIA
$^2$ National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824-1321, USA
$^3$ Physics Department, Aligarh Muslim University, Aligarh, INDIA
E-mail: brajeshk@gmail.com

Abstract. Interaction of the $\eta$-meson and that of the $K^-$-meson with nucleons is of special interest because both of them are strongly attractive near threshold. This raises the strong possibility that we may find in nature the bound $\eta$- and $K^-$-nuclear (quasi) bound states. This led to experimental programs to hunt for the existence of these states and theoretical studies to keep pace with them. The efforts had positive results. The $\eta$-meson studies had been there for several years by now, while those with the $K^-$-meson had been relatively recent. The talk gives a brief critical overview of the $\eta$-nuclear interaction studies, especially in context with the $\eta$-mesic state explorations.

For the $K^-$-meson we give a brief summary of the efforts in understanding the basic $K^-$-nucleon interaction and theoretical explorations for the existence of $K^-$-nucleus bound states. We critically examine the FINUDA measurements as a signal for the existence of the $K^-$-nucleus bound states, especially in context with the contribution of the single nucleon knock-out final state interaction in the ($K^-$, $p\Lambda$) reaction.

1. Introduction
It is known that most of the visible mass of cosmos is accounted for by the hadrons. Therefore, the study of hadrons and their interaction is of fundamental importance in understanding certain aspects of nature. The origin of hadron masses from the practically massless quarks is believed to lie in the non-perturbative domain of QCD. Therefore, the hadrons provide a “laboratory” where one can pursue the study of not-so-easy-to-explore non-perturbative QCD domain.

Equally important is the understanding of the interaction between hadrons, where an impressive progress has been made in describing it using chiral perturbation theories ($\chi$PT). Of special interest in these studies are the $\eta$-$N$ and $K^-$-$p$ interactions near threshold, where both of these interactions are found to be attractive and mediated by certain “doorway” states. This raises a tantalizing possibility that, if these elementary interactions generate sufficiently strong nuclear interaction when they are put in the nucleus, we might see the existence of (quasi-)bound $\eta$/K$^-$-mesic states in nature. This has generated great interest in the study of the nuclear interaction of these mesons near threshold, theoretically as well as experimentally. In the present talk we will discuss briefly the overall situation in the field of the $\eta$-meson interaction and the current state of the $K^-$-mesic studies.
2. $\eta$- Meson Interaction

The existence of $\eta$-mesic states depends sensitively on the precise quantitative knowledge of the strength of the $\eta$-N interaction, which, unfortunately, is not easily forthcoming. This is because, $\eta$-meson being unstable (decays quickly in $10^{-18}$ sec), we can not have eta beams. This makes the investigation of $\eta$-meson dynamics possible only through the $\eta$-meson producing reactions, which, unfortunately, complicates the matter, and also renders the extracted information uncertain. A large set of phenomenologically extracted $\eta$-N scattering amplitudes, therefore, exist in the literature [1, 2]. They differ in details, but all of them accept that the $\eta$-N interaction near threshold is attractive and proceeds through an intermediate $S_{11}$ $(1535)\ J^P = 1/2^-$ nucleon resonance.

Theoretically, the existence of the $\eta$-nucleus bound states in nuclei have been explored in several ways. For nuclei beyond few-nucleon systems, calculations have been done by finding complex energy solutions of the momentum-space relativistic three-dimensional integral equation for some $\eta$-nucleus optical potential [3]. This potential is essentially constructed following the “$t\rho$” approximation with the choice of the $\eta$-N amplitude guided by the phenomenologically determined on-shell amplitude extrapolated off-shell using the separable model ansatz. Existence and the position of bound states in such models depend strongly on the sub-threshold behaviour of the $\eta$-N amplitude, which it seems can not be constrained in any way. These calculations predict bound states for nuclei with mass number greater than 12. Another approach used is the QCD based quark-meson-coupling approach (QMC), where the $\eta$-meson is embedded in the nuclear medium, and couples to quarks and mixes with $\eta'$. The $\eta$ self energy obtained from such calculations is used in the local density approximation in the Klein-Gordon equation to obtain the complex energy solutions. This approach, apart from predicting the energy and width of the bound states, shows important role of $\eta - \eta'$ mixing in increasing the value of the $\eta$-N scattering length.

For few-nucleon systems existence of $\eta$ binding is explored by calculating $\eta$-nucleus scattering lengths for them. For the existence of bound states the scattering lengths need to be large and negative. For $\eta$-NN system it is calculated using relativistic Faddeev equations and some choice for potentials for binary sub-systems in it [5]. This calculation finds at best the existence of a quasivirtual state only. For an $\eta$-d system one uses a certain version of the multiple scattering formalism [6] and for $^4$He, $^3$He, $^3$He and $^4$He nuclei scattering lengths are calculated in the Finite Rank Approximation (FRA) [7]. Both these calculations need elementary $\eta$-N amplitude with a prescription for off-shell extrapolation. The outcome of these calculations obviously depends on the input quantities, which as we mentioned above are not known so well. Using the one term Yamaguchi extrapolation, these calculations suggest that if $a_{\eta N}$ is large ($\sim 0.6$ fm) it is possible to get $\eta$-nucleus bound states even for A=4 nucleus.

Experimentally, anticipated $\eta$-nucleus bound states should get reflected in a straightforward way in the measured eta producing nuclear cross sections. Such observations indeed have been made in the proton induced $\eta$ production reaction on nuclei where the measured amplitude on the deuteron target shows a sharp rise as we reach the threshold [8]. The $\eta$-$^3$He (or $\eta$-d) scattering amplitude from data has been extracted usually by writing the cross section as a product of the plane wave result and an enhancement factor, which has been parametrized in terms of the scattering length [9]. Large values of the extracted scattering lengths from such analyses are normally taken as an indicator for the existence of bound states.

This procedure of extracting the scattering lengths, however, ignores reference to any reaction mechanism for $\eta$ production and lacks totally a detailed treatment of the final state interaction. Therefore, while the extracted scattering lengths by this method may at best be indicative, their actual values may not be fully reliable for determining the bound states. A proper procedure should be to first analyze these data using a certain $\eta$ production mechanism and incorporating in detail the final state interaction as accurately as possible. Once such a procedure, using the
available $\eta N$ t-matrix, reproduces the measured cross sections, one can then infer from them the resultant $\eta$-nucleus scattering amplitudes and use them to investigate the possibility of the existence of unstable bound states.

Based on the above concept, our group carried out a systematic analysis in this direction over about the last ten years [10] to explore theoretically the possibility of the existence of the $\eta$-meson bound state.

Experimentally two measurements which give positive indication of the existence of $\eta$-mesic states have been reported in literature. One experiment is by the TAPS collaboration [11] on photo-production of $\eta$ on $^3$He, $\gamma^3$He $\rightarrow \pi^0$ p X, where one essentially sees the decay of bound $\eta$ in $^3$He through the $S_{11}$ resonance. The other is a bit more recent measurement from COSY on the $^{27}$Al (p, $^3$He)X reaction in a recoil free kinematic set-up [12], where one sees in coincidence with $^3$He, the decay of a possible bound $\eta$-$^{25}$Mg bound state, again through the $S_{11}$ resonance. This experiment was done at COSY in collaboration with the Mumbai experimental group who also designed and fabricated the Enstar detector. The COSY collaboration recently also reported the measurement of the p $^6$Li $\rightarrow ^7$Be $\eta$ reaction cross section at an excess energy around 11 MeV [13].

We proceeded to study the case of $\eta$-$^3$He system.

![Figure 1](image_url)

**Figure 1.** Comparison of the data on the $pd \rightarrow ^3He\eta$ reaction with a two step model calculation including the $\eta^3He$ final state interaction in the s-wave and s, p and d partial waves in the reaction mechanism. The scale in the upper plot has been broken at 11.2 MeV for clarity.

We analysed the $pd \rightarrow \eta^3He$ reaction for which the good quality data were available. The $\eta$ production was described by a two step process, where the beam proton interacts with the proton (or neutron) in the deuteron in the first step. This produces a deuteron and a $\pi^+$ (or $\pi^0$).
The pion in the second step interacts with another nucleon in the target deuteron and produces an $\eta$. The final state interaction (FSI) between $\eta$ and $^3\text{He}$ was incorporated by solving few body Faddeev type equations using the finite-range-approximation. These studies reproduced the available eta production data in $p\,d$ fusion near threshold for $\eta$-$^3\text{He}$ exit channel [Fig. 1] very well [14]. The $\eta$-$^3\text{He}$ t-matrix in these calculations entered through the description of the $\eta$-$^3\text{He}$ scattering wave function, $\psi^-$ in the final state,

$$
\langle \psi^- \rangle = \langle \vec{k} \rangle + \frac{d\vec{q}}{(2\pi)^3} \frac{\langle \vec{k} | T_{\eta\text{He}}(\vec{q}) | \vec{q} \rangle}{E(k) - E(q) + i\epsilon} \langle \vec{q} \rangle.
$$

Having determined the $\eta$-$^3\text{He}$ t-matrix which fit the $p\,d \rightarrow \eta$-$^3\text{He}$ data well we proceeded to explore if this t-matrix admits any bound state. For this we used the method of Wigner’s time delay [15]. Naively, this method means that in a scattering process $ab \rightarrow ab$, if there is a resonance, the scattering process will take longer time compared to the situation if there is no resonance. Equivalently, the scattering phase shift, $\delta$ will show a large variation with energy across the resonance. This method has been used in the literature for the analysis of hadron resonances successfully [16]. For our purpose a modification of Wigner’s method was done by Kelkar in [17] and we obtained an eta-mesic state in the $\eta$-$^3\text{He}$ system in agreement with experiments [17, 18].

3. $K^-$-Nuclear Interaction: Experimental

First indication of the existence of the $K^-$ bound nuclear states came in 2005 in the FINUDA measurements [19] of the stopped $K^-$ absorption in Li, C and other target nuclei. These experiments using the FINUDA spectrometer installed at the DAΦNE collider detected a $\Lambda$ hyperon and a proton pair in coincidence following $K^-$ absorption at rest in several nuclei. The emitted $\Lambda$-$p$ pair was found to emerge, predominantly back to back in all target nuclei, and had their invariant mass distributions peaking significantly below the sum of a kaon and two proton mass in free state (2.370 GeV) [Fig. 2]. If it is assumed that the $\Lambda$-$p$ pair is emitted from a “$K^-\,pp$” system in the nucleus, this mass shift implies a bound $K^-\,pp$ system in nuclei with the binding energy above 100 MeV. In a more elaborate second run of these experiments carried out recently [20] it is further reported that these mass shifts occur only for the $K^-\,pp$ module, and not for the $K^-\,np$ cluster. The absorption on an $n\,p$ pair gives $\Lambda$-$n$ and $\Sigma^-\,-p$ pairs in the final state.

Recent analysis of the old DISTO data from the Saturne accelerator on $pp \rightarrow p \,\Lambda \,K^+$ reaction too suggests the existence of a $K^-\,pp$ cluster with the binding energy around 100 MeV [21].

As the $K^-\,N$ interaction in s-state is attractive, attractive behaviour of the $K^-$-nucleus interaction was expected but the existence of such deeply bound modules came as a big surprise. The occurrence of only “$K^-\,pp$” modules is understandable because the $K^-\,N$ interaction is such that it is much stronger in $T=0$ state than in $T=1$ state. Since the $K^-\,p$ system is an admixture of $T=0$ and $T=1$ states, while the $K^-\,n$ system is a pure $T=1$ state, strong interaction between a $K^-$ and a proton is understandable. In the presence of another proton, a la Heitler-London theory of hydrogen molecules, $K^-$, by continuously shuttling between two protons, can understandably cause strong attraction between them, producing thereby bound “$K^-\,pp$” modules.

4. $K^-$-Nuclear Interaction: Theoretical

4.1. $KN$ Interaction

Theoretically, the $K^-\,p$ issue has been investigated in the framework of the SU(3) chiral perturbation theories, first developed and applied in Ref. [22]. Subsequently it was expanded in Ref. [23]. The fact that, unlike pion-nucleon and $K^+\,-N$ cases, the basic interaction here
Figure 2. Invariant mass of a $\Lambda$ and a proton in back-to-back correlation ($\cos \theta_{\text{Lab}} < -0.8$) from light targets before the acceptance correction. The inset shows the result after the acceptance correction for the events which have two protons with well-defined good tracks. Only the bins between 2.22 and 2.33 GeV/c^2 are used for the fitting. The plot is taken from Ref. [19].

This is relatively strong and is incorporated in these studies by including terms up to order $q^2$ in the chiral Lagrangian expansion. The chiral Lagrangian approach is further connected in these studies to the potential model approach. A meson-baryon potential is constructed for this purpose by constraining that in the Born approximation, this potential has the same s-wave scattering length as the effective chiral Lagrangian, up to order $q^2$. Such a potential in local representation between channel i and j with a Yukawa form comes out as,

$$V_{ij}(r) = \frac{C_{ij}\alpha_{ij}^2}{8\pi f^2} \sqrt{\frac{m_i m_j}{\omega_i \omega_j}} e^{-\alpha_{ij} r},$$

(2)

where $m_i (m_j)$ is the mass of the baryon in channel i(j), $\omega_i (\omega_j)$ is the reduced energy in channel i(j) in the centre-of-mass. $C_{ij}$ and $\alpha_{ij}$ are constants. They are obtained by iterating this potential in a set of coupled channel Lippmann-Schwinger equations and fitting the variety of low energy data of the kaon-baryon system. Such a system for $S = -1$ has six channels, $\pi^+ \Sigma^-$, $\pi^0 \Sigma^0$, $\pi^- \Sigma^+$, $\pi^0 \Lambda$, $K^- p$, $K^0 n$. This pseudo-potential for $K^- p$ and $K^- n$ systems turns out as

$$V_{K^- p}(r) = -360 e^{-ar} MeV,$$

(3)

and

$$V_{K^- n}(r) = -30 e^{-ar} MeV,$$

(4)

with $1/\alpha=0.5$ fm. This clearly shows that for the s-wave

$$V_{K^- p} >> V_{K^- n}.$$  

(5)

The amplitude resulting from these calculations for $K^- N$ system is shown in Fig. 3. Below $K^- N$ threshold it shows a characteristic resonance behaviour for the $K^- p$ system. Its imaginary part has a maximum around the resonance energy of the $\Lambda(1405)$, and the real part goes through
zero around the same energy. The $K^-n$ amplitude does not have any such behaviour. That’s the reason why the $K^-p$ potential is much stronger than the $K^-n$ potential.

Identifying that the experimentally observed $\Lambda(1405)$ is a bound state of the $K^-p$ system, the $K^-p$ potential is alternatively obtained by reproducing this bound state at the (experimental) binding energy of around 28 MeV ($1405 \text{ MeV} - K^-p$ free mass) by varying the parameters of an assumed potential.

Because of the coupling amongst different $S = -1$ channels, the $\Lambda(1405)$ also acquires a width of about 50 MeV. This width is mainly for the $\Lambda(1405) \rightarrow \Sigma\pi$ channel.

### 4.2. $K^-\text{-Nucleus Binding}$

Having learnt about the $K^-N$ potential, theoretical search for the antikaon-nucleus bound state has been made in the literature for $K^{-}$ plus 2-3 nucleons and heavier nuclei. These studies follow the variational approach [24, 25, 26, 27] and the Faddeev method [28]. All these calculations use, as input, realistic choice for the $NN$ and the $K^-N$ potentials. For the nucleon-nucleon potential, following extensive work over the years on this subject, it is always possible to make a correct choice. For the $K^-N$ potential some calculations use a phenomenological pseudo-potential, while others use a leading order chiral interaction. All these calculations, in line with the experimental observations, found $K^-pp$ bound states with about 50 MeV or more binding energies.

The variational calculations in [24, 25, 27] are done in the framework of Brueckner-Hartree-Fock or anti-symmetric molecular dynamics and use a phenomenologically constructed baryon-meson potential. We, in our calculations, use Variational Monte Carlo (VMC) method and use the leading order chiral interaction for the $K^-N$ potential. In the following we give a brief description of our calculations for a $K^-pp$ and $K^-pn$ systems.

#### 4.2.1. Variational Monte Carlo (VMC) Calculations

The VMC method, in brief, consists of constructing the general Hamiltonian for $K^-\text{-nucleus}$ system, which for the $K^-pp$ system, for example, takes the form

$$H_{N_2N_3}^{K^-} = T_{K^-}(1) + \sum_i (T_{N_i}(i) + V_{K^-N_i}(r_{1i})) + V_{N_2N_3}, \quad (6)$$
K\textsuperscript{(FSI)} in the K\textsuperscript{5. More on the Λ(1405) binding, the binding energy of this state should not be identified with the observed K\textsuperscript{-}N potential. The correlation of the mass-shift seen in the FINUDA data to the binding of the KN system anti-symmetry requires that s = σ = 0. The correlation functions f_{hh}(r) have the asymptotic behaviour, f_{hh}(r) \sim r^{-\alpha}\exp(-\kappa_{hh}r) and are obtained from the solution of the Schrodinger-type equation, a procedure developed by the Urbana group \[29\]. The energy \(-B_{KN}^{-}\) for K\textsuperscript{-}NN system is evaluated using the relation:

\[
-B_{KN}^{-} = \frac{\langle \Psi_{N3N1}^{K-N} | H_{N3N1}^{K-N} | \Psi_{N3N1}^{K-N} \rangle}{\langle \Psi_{N3N3}^{K-N} | \Psi_{N3N3}^{K-N} \rangle}.
\]

For NN pair we use very well known potential V14 of Urbana group which has repulsive soft core. It is constrained by the NN scattering data up to 350 MeV and the ground state properties of the deuteron.

Our estimates of the energy were made for 100,000 points. The statistical errors in the energies are of the order of 1%. The calculated binding energy for K\textsuperscript{-}pp and K\textsuperscript{-}np systems and r.m.s. radii for NN, \langle R_{NN}^{rms} \rangle and that of K\textsuperscript{-} w.r.t. NN centre-of-mass \langle R_{KN}^{K-N} \rangle are listed in Table 1. We find that both the systems are very strongly bound, and the separation of K\textsuperscript{-} w.r.t. NN is very small. The NN nucleon core also shrinks considerably. In the K\textsuperscript{-}np system, for example, np system shrinks by about 50% compared to deuteron (\langle R_{NN}^{rms} \rangle=3.8 fm). Our calculations include the Coulomb interaction. Ignoring it lowers the binding energy by about 2 MeV. These results are almost of the same order as found by the Japanese group earlier \[24, 25\].

Various variational calculations thus predict affirmatively about the existence of deeply bound and compact K\textsuperscript{-}NN modules. This is in accord with the experimental findings of the FINUDA group at Frascati, Rome.

However, consequent to above accord between experiments and theoretical calculations, doubts have been raised about the conclusions of the \(\chi\)PT calculations as well as the interpretation of the experimental findings. The \(\chi\)PT calculations are questioned for the legitimacy of the use of the phenomenological or the chiral leading order prescription for the \(\vec{K}N\) potential. The correlation of the mass-shift seen in the FINUDA data to the binding of the K\textsuperscript{-}pp module in nuclei is questioned for ignoring in this interpretation the final state interaction (FSI) in the \((K^{-}, p\Lambda)\) reaction.

Table 1. The calculated binding energy for K\textsuperscript{-}pp and K\textsuperscript{-}np systems for Urbana V14 potential and \langle R_{NN}^{rms} \rangle of NN pair and root mean square radius \langle R_{(NN)}^{K-N} \rangle that of K\textsuperscript{-} from c.m. of NN pair.

| K\textsuperscript{-}pp | K\textsuperscript{-}np |
|----------------------|----------------------|
| BE (in MeV) | -124.84 | -99.72 |
| \langle R_{NN}^{rms} \rangle | 1.53 | 1.85 |
| \langle R_{(NN)}^{K-N} \rangle | 0.89 | 1.24 |

where \(T_{Y}\) is the kinetic energy operator for the particle \(Y\) and \(V_{hh}\) denotes the hadron-hadron interaction for the pair \(hh\) (=K\textsuperscript{-}N and NN). The translationally invariant trial wave function for the system is taken as the product of two-body correlation functions, \(f_{hh}\) \((hh=K\textsuperscript{-}N\) and \(NN)\) and the appropriate spin function, \(\chi_{s}^{Y}\), i.e.

\[
\Psi_{N3}^{K-N} = [\Pi_{i} f_{K-N}(r_{1i})]f_{NN}(r_{23})\chi_{s}^{Y}.
\]

For the K\textsuperscript{-}pp system anti-symmetry requires that \(s = \sigma = 0\). The correlation functions \(f_{hh}(r)\) have the asymptotic behaviour, \(f_{hh}(r) \sim r^{-\alpha}\exp(-\kappa_{hh}r)\) and are obtained from the solution of the Schrodinger-type equation, a procedure developed by the Urbana group \[29\]. The energy \(-B_{KN}^{-}\) for K\textsuperscript{-}NN system is evaluated using the relation:

5. More on K\textsuperscript{-}-Nucleus Binding

What is argued is that, though the correct K\textsuperscript{-}N potential is the one which predicts or reproduces the Λ(1405) binding, the binding energy of this state should not be identified with the observed
Table 2. Binding energy \( B \) and mean distance \( \sqrt{\langle r^2 \rangle} \) for \( \bar{K}N \) two-body system for the local \( \bar{K}N \) potentials in \( I=0 \) channel derived from four variants of the chiral SU(3) coupled channel approach. The range parameter \( b \) of the potential is given in the first row.

|               | ORB [32] | HNJH [33] | BNW [34] | BMN [31] |
|---------------|----------|-----------|----------|----------|
| \( b \) [MeV] | 0.52     | 0.47      | 0.51     | 0.41     |
| \( B \) [MeV] | 12.78    | 12.25     | 11.42    | 13.51    |
| \( \sqrt{\langle r^2 \rangle} \) [fm] | 1.84 | 1.84 | 1.93 | 1.74 |
| \( b \) [MeV] | 11.1     | 10.54     | 9.60     | 11.73    |
| \( \sqrt{\langle r^2 \rangle} \) [fm] | 1.92 | 1.94 | 2.04 | 1.82 |

peak position (around 1405 MeV, i.e. B.E. \( \sim \)27 MeV) in the \( \pi\Sigma \) invariant mass distribution in the \( \bar{K}N \) scattering. This is because, in the chiral \( SU(3) \) \( \bar{K}N \) dynamics there is a strong \( \bar{K}N \leftrightarrow \pi\Sigma \) coupled-channel dynamics, which results in a two pole structure in the \( \bar{K}N \) t-matrix. One pole corresponds to the \( \bar{K}N \) quasi-bound state and another to the \( \Sigma \pi \) resonance. Because of this, the observed peak in the \( \pi\Sigma \) invariant mass distribution seen in the \( \bar{K}N \) scattering needs to be fitted by the superposition of both these poles. This, as a thorough investigation of this issue in Ref. [30] argues results in the mass of the \( \Lambda(1405) \) quasi-bound state to be 1420 MeV, and not the normally used 1405 MeV value, making thereby the \( \Lambda(1405) \) bound only by 12 MeV below \( \bar{K}N \) threshold, not 27 MeV assumed in other studies.

Recent measurements on the \( pp \rightarrow pK^+Y^0 \) reaction at COSY [31], however, seem to present experimental evidence which do not support the above two pole model for the \( \Lambda(1405) \). The shape and position of the \( \Lambda(1405) \) distribution in these measurements is reconstructed cleanly in the \( \Sigma^0 \pi^0 \) channel using invariant- and missing-mass techniques. The mass of the \( \Lambda(1405) \) is found to be \( \sim 1400 \) MeV with width \( \sim 60 \) MeV.

The 12 MeV binding for the \( \Lambda(1405) \) from the “two-pole” model, however, reduces the strength of the \( \bar{K}N \) potential considerably. The use of such a weaker potential in variational calculations then reduces the binding of the \( K^- \)-nuclear systems. The potential itself has the form

\[
V_{\bar{K}N}(\sqrt{s},r) = -V_{\bar{K}N}(\sqrt{s},0)e^{-r^2/b^2} \frac{\pi^{-3/2}b^3}{\pi^{3/2}b^3}, \tag{9}
\]

where the strength of the potential is complex and is \( \bar{K}N \) invariant mass dependent. \( \sqrt{s} \) is the total centre-of-mass energy of the system. We consider only real part of this potential. The calculated binding energy and rms distance for the two-body \( \bar{K}N \) system are given in Table 2. The use of the effective \( \bar{K}N \) potential derived from variants of the chiral SU(3) coupled channel approach [30] in our VMC calculations gives the results shown Table 3 for the \( K^-pp \) system. The binding energy in our calculations has come down to 35-40 MeV, while in calculation of Dote et al. [25] it drops to around 18-23 MeV for the same \( \bar{K}N \).

6. Final State Interaction

As regards the interpretation of the FINUDA measurements a suggestion was advanced in Ref. [35]. According to it the down-shift observed in the invariant mass of the \( p \) and \( \Lambda \) in the FINUDA experiment could be the result of the final state interaction of these particles with the recoiling nucleus. This seems quite plausible because, due to Q-value of the \( K^- + pp \rightarrow \Lambda p \) reaction
being around 317 MeV, the kinetic energies of outgoing p and Λ are 160 MeV or so. At these energies, it is well known that in the nucleon-nucleus scattering the reactive cross section mainly consists of the single nucleon knock-out channel [36]. Thus, the knock-out of one nucleon in the nucleus by the out going p or Λ can shift their energies considerably. The calculations in Ref. [35] indeed reproduce the observed mass shifts in the FINUDA experiments. However, these calculations are the computer simulations of the inter-nuclear cascade model for the nucleon-nucleus scattering. This approach describes the sequence of nucleon-nucleon collisions of the outgoing nucleon while passing through the residual nucleus in the final state in the framework of classical physics. The trajectory of each nucleon is followed. After a mean free path a N-N collision takes place and its results are computed by Monte Carlo or some similar method. Apart from the Pauli principle there are no quantum mechanical effects in this approach of describing the FSI. The nucleus too is described by the Fermi gas model, thus being totally devoid of any nuclear structure effect. In view of the crucial role played by the FSI in interpreting the FINUDA data the FSI. The nucleus too is described by the Fermi gas model, thus being totally devoid of any nuclear structure effect. In view of the crucial role played by the FSI in interpreting the FINUDA data.

Table 3. The detailed composition of the total $K^{-}pp$ binding energy (BE) using VMC calculation for $KN$ interaction in I=0 channel without coulomb potential. Auxiliary antikaon “binding energy” ($B_K$) and the centre-of-mass energy $\sqrt{s}$ of $KN$ subsystem within $K^{-}pp$ are related as $\sqrt{s} = M_N + m_K - B_K/2$, energy of two-nucleon $E_N = \langle \Psi_{N_2N_3}^{K^-} | \sum_i T_N(i) + V_{N_2N_3} | \Psi_{N_2N_3}^{K^-} \rangle$, kinetic energy $T_{nuc} = \langle \Psi_{N_2N_3}^{K^-} | \sum_i T_N(i) | \Psi_{N_2N_3}^{K^-} \rangle$ of the two-nucleon subsystem, total kinetic energy of the three-particle $T_{kin} = \langle \Psi_{N_2N_3}^{K^-} | T_{K^-}(1) + \sum_i T_N(i) | \Psi_{N_2N_3}^{K^-} \rangle$, potential energy of two-proton $V_{NN} = \langle \Psi | V_{N_2N_3} | \Psi \rangle$, and potential energy of three-particle $V_{KN} = \langle \sum_i V_{K^-N_i}(r_{1i}) + V_{N_2N_3} \rangle$. $R_{pp}$ is the rms distance between the two protons, $R_{Kp}$ between antikaon and nucleon and $R_{(pp)K}$ between cm of pp and $K$ (all energies in MeV and distances in fm).

| Effective potential | BE   | $B_K$ | $E_N$ | $T_{nuc}$ | $T_{kin}$ | $V_{NN}$ | $V_{KN}$ | $R_{pp}$ | $R_{Kp}$ | $R_{(pp)K}$ |
|--------------------|------|------|------|----------|----------|---------|---------|--------|--------|-----------|
| ORB [32]           | 39.80| 22.82| 62.61| 83.20    | 196.33   | -20.6   | -215.52 | 1.84   | 1.58   | 1.28       |
| HNJH [33]          | 36.10| 26.32| 62.64| 81.59    | 191.69   | -19.18  | -208.61 | 1.99   | 1.81   | 1.20       |
| BNW [34]           | 35.96| 18.84| 54.81| 74.31    | 173.37   | -19.46  | -189.87 | 2.01   | 1.78   | 1.17       |
| BMN [31]           | 35.34| 38.96| 74.35| 92.96    | 224.30   | -18.61  | -241.03 | 1.86   | 1.63   | 1.34       |
Therefore, in our calculations we disentangle it from the shell model dynamics of the absorbing nucleons in the nucleus. We, thus factorize the absorption probability as

\[ \omega_{\text{abs}}(p^*, \Lambda^*) = g_{\text{abs}}(q) G(Q), \]  

where \( q = (m_p p^* - m_{\Lambda} p^*)/(m_p + m_{\Lambda}) \) and \( Q = p^* + \Lambda \) are the centre-of-mass and the total momenta of the \( p \) and \( \Lambda \) respectively. \( g \) and \( G \) are respectively the absorption strength for \( K^- p p \rightarrow p \Lambda \) process in their centre of mass and the momentum probability distribution of nuclear wave functions corresponding to the total momentum, \( Q \). Because of the back-to-back emission of \( p \) and \( \Lambda \), obviously the magnitude of \( q \) is very large and that of \( Q \) is small. Due to these vastly different momentum scales for \( q \) and \( Q \), over most of the variables measured in the \( (K^-, p \Lambda) \) reaction, while \( G(Q) \) can go through a large variation, the factor \( g(q) \) should not change much.

\[ K^- + A \rightarrow p + \Lambda + X, \]

For the final state interaction (FSI) of \( p \) and \( \Lambda \) with the nucleus we consider the diagrams (a) and (b) in Fig. 4. They represent respectively the elastic scattering and the single-nucleon knock-out on the recoiling nucleus. The inclusive probability for a process like

\[ K^- + A \rightarrow p + \Lambda + X, \]

is therefore written as

\[ d\omega = d\omega_{\text{elas}} + d\omega_{\text{KO}}. \]

The sum of two terms in the above is incoherent because in principle (by making exclusive measurements) we can distinguish between elastic scattering and single scattering with one target nucleon being knocked out. We calculate the contribution of both these diagrams using fully quantum mechanical scattering theory.

6.1. Elastic Scattering

For the absorption of \( K^- \) on a pair of nucleons in the shell model orbitals \( n_1 l_1, n_2 l_2 \), \( G(Q) \) for the elastic channel comes out, (for details see [37])

\[ G(Q) = \frac{|PS| N_{l_1 l_2}}{(2l_1 + 1)(2l_2 + 1)} \times \sum_{LM} \left| \int d\vec{r}_1 \int d\vec{r}_2 \chi^{-*}(\vec{r}_1, \vec{r}_2) \phi_K(\vec{r}_1) \phi_{LM}(\vec{r}_1, \vec{r}_2) \delta(\vec{r}_1 - \vec{r}_2) \right|^2, \]

where \( \chi^{-*}(\vec{r}_1, \vec{r}_2) \) describes the elastically scattered waves in the final state. \( \phi_{LM}(\vec{r}_1, \vec{r}_2) \) is the bound state wave function of the absorbing nucleons in the target nucleus.
6.2. Single Nucleon Knock-out
The knock-out contribution for a nucleon in the shell “nl” is written as

\[ d\omega_{KO} = f(p_\gamma) \frac{\sigma_{KO}(p_\gamma, p')}{\sigma_{PB}(p')} \sigma_{PB}(p'), \]  

(14)

where \( \sigma_{PB}(p') \) is the total proton-nucleus cross section at the momentum \( |p'_\gamma| \). Momentum \( |p'_\gamma| \) is given in terms of the variables at the first vertex. Function \( f(p_\gamma) \), which describes the distribution of this momentum is also determined from the dynamics at the first vertex. The inclusive knock-out cross section is obtained using the proper scattering theory. It is given by

\[ d\sigma_{KO} = dp_\gamma \left[ \frac{2}{Q_{Rmin}} \right] \left[ \frac{d\sigma}{dQ}(\epsilon) \right]_{cm} p^N \left[ S(l) \int_{Q_{Rmin}}^{\infty} Q_R dQ_R \frac{1}{(2\pi)^{3/2}} \phi_{nl}(Q_R) \right]^2. \]  

(15)

Here \( Q_{Rmin} \) is the minimum momentum which a nucleon of binding energy \( B_N \) must have in the nucleus for the scattered proton to be observed at a scattering angle \( \theta \) with a momentum \( |p'_\gamma| \). Its value is given by

\[ Q_{Rmin}^2 = \left[ \frac{p_\gamma^2 - p'_\gamma p_\gamma \cos(\theta) + m B_N}{p'_\gamma^2 - p'_\gamma p_\gamma \cos(\theta) + m B_N} \right]^{1/2}. \]  

(16)

\( \left[ \frac{d\sigma}{dQ}(\epsilon) \right]_{cm} p^N \) is the \( pN \) elastic scattering cross section in its c.m. at an appropriate c.m. energy. \( \phi_{nl}(Q_R) \) is the single particle wave function in the nucleus.

Using this formalism in Fig. 5 we show the calculated \( p\Lambda \) invariant mass distribution along with the FINUDA measured distribution. In calculated distribution we see two peaks. The one around 2320 MeV is due to elastic scattering, while the one at 2250 MeV is due to the knock-out FSI. The latter peak agrees both in position and width with the measured distribution. The position of the \( K^-pp \) mass in the free state (2370 MeV) is indicated by the arrow in the figure.

7. Conclusions
In an overall summary, to the \( K^-pp \) cluster interpretation of the observed downshift of about 100 MeV in the FINUDA measurements of the \( \Lambda p \) invariant mass compared to its free value,
the knock-out reaction in the final state seems to be a definitive alternative for this shift. The only discomfiture in this conclusion comes from the absence of the elastic scattering peak in the observed invariant mass distribution in the FINUDA measurements.

Existence of two pole structure in the \( KN \) scattering dynamics in the \( \chi \)PT studies is also not fully accepted. Hence the value of the \( \Lambda(1405) \) binding energy is also not settled.

A way forward to make further unambiguous progress in this subject could be to have experimental and theoretical studies on the following topics:

(i) Study of the \( pp \rightarrow K^+(K^-pp) \rightarrow K^+(p\Lambda) \) reaction. As the \( K^+N \) interaction is weak, this reaction can give a direct handle on the study of the \( K^-pp \rightarrow p\Lambda \) process. This might help in settling the binding energy of the \( "K^-pp" \) module.

(ii) Study of the \( ^3He(K^-,n)p\Lambda \) reaction. This may avoid the single nucleon knock-out FSI, as it might proceed as \( K^+3He \rightarrow n + [K^-pp] \rightarrow n + (p\Lambda) \).

(iii) Availability of absolute magnitudes of the FINUDA measurements should be very useful.

References

[1] A. M. Green and S. Beech, Phys. Rev. C 71, 014001 (2005); A. Fix and H. Arenhovel, Nucl. Phys. A 697, 277 (2002); Eur. Phys. J. A. 19, 275 (2004).
[2] R. S. Bhalerao and L. C. Liu, Phys. Rev. Lett. 54, 865 (1985).
[3] Q. Haider and L. C. Liu, Phys. Lett. B 172, 257 (1986); Phys. Rev. C 66, 045208 (2002); J. Phys. G 37, 125104 (2010); L. C. Liu and Q. Haider, Phys. Rev. C 34, 1845 (1986).
[4] K. Tsushima et al., Phys. Lett. B 443, 26 (1999); S. Bass and A. W. Thomas, Phys. Lett. B 634, 368 (2006).
[5] H. Garcilazo, Phys. Rev. C 71, 048201 (2005).
[6] S. Wycech and A. M. Green, Phys. Rev. C 64, 045206 (2001); S. Wycech, Acta Phys. Polon. B 41, 2201 (2010); S. Wycech et al., Phys. Rev. C 52, 544 (1995); A. Sibirtsev et al., Eur. Phys. J. A 22, 495 (2004); J. A. Niskanen et al., Int. J. Mod. Phys. A 20, 634 (2005).
[7] S. A. Rakityansky et al., Phys. Lett. B 359, 33 (1995); S. A. Rakityansky et al., Phys. Rev. C 53, R2043 (1996); S. A. Rakityansky et al., Chin. J. Phys. 34, 998 (1996).
[8] B. Mayer et al. Phys. Rev. C 53, 2068 (1996); J. Berger et al. Phys. Lett. 61, 919 (1988).
[9] C. Wilkin, Phys. Rev. C 47, R938 (1993); G. Fadili and C. Wilkin, Nucl. Phys. A587, 769 (1995).
[10] K. P. Khemchandani, N. G. Kelkar and B. K. Jain, Nucl. Phys. A 708, 312 (2002); N. J. Upadhyay, K. P. Khemchandani, B. K. Jain and N. G. Kelkar, Phys. Rev. C 75, 054002 (2007); ibid. Mod. Phys. Lett. A 24, 2319 (2009); N. J. Upadhyay, N. G. Kelkar and B. K. Jain, Nucl. Phys. A 824, 70 (2009); K. P. Khemchandani, N. G. Kelkar and B. K. Jain, Phys. Rev. C 68, 064610 (2003); ibid. Phys. Rev. C 76, 069801 (2007).
[11] M. Pfeiffer et al., Phys. Rev. Lett. 92, 252001 (2004)
[12] A. Budzanowski et al., Phys. Rev. C 79, 012201 (2009).
[13] A. Budzanowski et al., Phys. Rev. C 82, 041001 (2010).
[14] B. K. Jain, N. J. Upadhyay, K. P. Khemchandani and N. G. Kelkar, Int. J. Mod. Phys. E 18, 1271 (2009).
[15] E. P. Wigner, Phys. Rev. 98, 145 (1955).
[16] N. G. Kelkar et al., Nucl. Phys. A730, 121 (2004).
[17] N. G. Kelkar, Phys. Rev. Lett. 99, 210403 (2007).
[18] N. G. Kelkar, K. P. Khemchandani and B. K. Jain, J. Phys. G 32, 1157 (2006); ibid, L19 (2006).
[19] M. Agnello et al., Phys. Rev. Lett. 94, 212303 (2005).
[20] H. Fujikawa et al., Nucl. Phys. A827, 303c (2009).
[21] T. Yamazaki et al., Proceedings of the International Conference on Exotic Atoms and Related Topics and International Conference on Low Energy Antiproton Physics (EXA/LEAP 2008), Vienna, Austria, 2008; arXiv:0810.5182v1.
[22] N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. A594, 325 (1995).
[23] E. Oset and A. Ramos, Nucl. Phys. A635, 99 (1998); J. A. Oller and U. G. Meiβner, Phys. Lett. B500, 263 (2001); M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002).
[24] Y. Akaiishi and T. Yamazaki, Phys. Rev. C 65, 044605 (2002).
[25] A. Dote, T. Hyodo, and W. Weise, Nucl. Phys. A804, 197 (2008); Phys. Rev. C 79 (2009).
[26] Mohammad Shoeb and B. K. Jain, DAE Symposium on Nuclear Physics, Roorkee, 53, 561 (2008).
[27] S. Wycech and A. M. Green, Phys. Rev. C 79, 014001 (2009).
[28] N. V. Shevchenko, A. Gal, and J. Mares, Phys. Rev. Lett. 98, 082301 (2007); J. Revai, Phys. Rev. C 76, 044004 (2007).
[29] I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. A359, 331 (1981).
[30] T. Hyodo and W. Weise, Phys. Rev C 77, 035204 (2008).
[31] I. Zychor et al., Phys. Lett. B660, 167 (2008).
[32] E. Oset et al., Phys. Lett. B527, 99 (2002).
[33] T. Hyodo et al., Phys. Rev. C68, 018201 (2003).
[34] B. Borasoy et al., Eur. Phys. Journ. A25, 79 (2005).
[35] V. K. Magas, E. Oset, A. Ramos, and H. Toki, Phys. Rev. C 74, 025206 (2006).
[36] N. S. Wall and P. R. Roos, Phys. Rev. C 150, 811 (1966).
[37] G. M. Pandjee, N. J. Upadhyay, and B. K. Jain, Phys. Rev. C 82, 034608 (2010).