Label Aware Graph Convolutional Network

Not All Edges Deserve Your Attention

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Abstract—Graph classification is practically important in many domains. To solve this problem, one usually calculates a low-dimensional representation for each node in the graph with supervised or unsupervised approaches. Most existing approaches consider all the edges between nodes while overlooking whether the edge will bring positive or negative influence to the node representation learning. In many real-world applications, however, some connections among the nodes can be noisy for graph convolution, and not all the edges deserve your attention. In this work, we distinguish the positive and negative impacts of the neighbors to the node in graph node classification, and propose to enhance the graph convolutional network by considering the labels between the neighbor edges. We present a novel GCN framework, called Label-aware Graph Convolutional Network (LAGCN), which incorporates the supervised and unsupervised representations of nodes in graphs have been proved extremely useful as feature inputs for a wide variety of prediction and graph analysis tasks[1], [2], [3], [4], [5]. These representations can then be fed into downstream machine learning models and facilitate various tasks. Recently, deep learning models, especially Convolutional Neural Networks (CNNs), have revolutionized various machine learning tasks with grid-like input data, such as image classification[5] and machine translation[17]. By utilizing local connection and weight sharing, CNNs are able to pursue translational invariance of the data. In many other contexts, however, the input data are lying on irregular or non-Euclidean domains, such as graphs which encode the pairwise relationships, like social networks[8], protein interfaces[9], and 3D meshes[10]. Thus, it is important to investigate how to aggregate the information of the nodes by applying convolution operations on irregular graphs.

Graph convolutional networks (GCNs) [11] generalize convolutional neural networks (CNNs) [12] to graph structured data. The “graph convolution” operation applies the same convolutional transformation to the neighbors of each node, followed by mean pooling and non-linearity[13]. By stacking multiple graph convolution layers, the latent embedding of the node can contain more information from distant neighbors. However, GCN suffers from the heavy computational complexity of matrix multiplication, which constrains its application on large-scale graphs. Variants like GraphSAGE[8] and FastGCN[14] introduce node-wise and layer-wise sampling method separately to enable the convolutional operation to be applied and optimized with sampled mini-batch nodes. However, aggregators of these methods give the same weight to all the neighbors, and they suffer from the variance caused by the unstable sampling.

Fig. 1. Illustration of the impact of the negative neighbors. The curve line in red denotes the decision boundary of the downstream node classification function.

To compute the weight of the neighbors dynamically, Graph Attention Network (GAT)[15] employs the multi-head attention mechanisms introduced in[7] as a graph aggregator to solve the node classification problem. AGCN [16] combines node-wise, layer-wise sampling, attention mechanisms and variance reduction techniques to achieve better performance. Attention based models obtain the weights of each neighbor by computing the similarity between the representations of the node and corresponding neighbor, but overlook whether the node and the corresponding neighbor have the same label. However, the label information of the neighbors is important for the node prediction. As shown in fig.1 (a), the empty blue circle connects to two blue filled circles and the other five filled circles of the other colors. The embedding of the nodes and the decision border is shown in fig.1 (b). If we aggregate the neighbors of the center node without filtering the neighbors with different labels, the convolutional operation which averages the representation of the neighbors will give the wrong prediction.

There are three challenges in using the label information for graph convolution networks. First, related works except attention based models only optimize the parameter matrix of each layer. They do not deal with the appearance of the different labeled neighbors. Second, for attention based models, they only change the weights of existing neighbors and do not enrich the neighbors’ information of the nodes which have no neighbors or fewer neighbors. The weights,
which are computed by the attention mechanism by weighting the similarity between nodes, are also ambiguous and do not reflect whether two nodes have a same label. Third, predicting whether two nodes have the same label is difficult. It is challenging to enhance the prediction of the center node when the prediction accuracy of the neighbor label is low.

To address the above challenges, we propose a novel Label-Aware GCN framework called LAGCN. We first decompose the convolution operation of GCN-based methods into a label-aware view of convolution by splitting the neighbors of a center node into positive neighbors and negative neighbors. We analyze the label-aware view of convolution and demonstrate that we can enhance the prediction by increasing the number of positive neighbors and reducing the number of negative neighbors. Then we simplify the prediction of neighbors’ label into an edge label prediction, and give a simple and efficient implementation of the edge label predictor. Based on the label-aware view of convolution, we give the lower bound of the edge label predictor. Finally, we propose the importance sampling and aggregation method and make the original GraphSAGE model achieve the state-of-the-art performance.

We evaluate the performance of our framework on three node-classification benchmarks. We verify our motivation by analyzing deep into the prediction of Cora dataset. Moreover, we take numerical experiments with artificial controlled edge predictors to justify the lower bound of LAGCN. Our main contributions can be summarized as follows:

- To the best of our knowledge, this is the first deep graph convolutional network that emphasizes the importance of the nodes labels in the convolution operation. This framework can be easily adapted to various GCN-Based methods.
- We propose a label-aware framework to select the positive neighbors and filter the negative ones to enhance the original GraphSAGE by a significant margin.
- We give the lower bound of the edge label predictor by a theoretical analysis under our assumption. Numerical experiments verify the lower bound.

II. RELATED WORK

When dealing with graph structured data, it is important to design efficient graph models to learn the embedding for each node. These methods can be categorized into supervised learning methods and unsupervised learning methods depending on whether utilize the information of the training labels. In supervised learning methods, convolution-based methods have achieved better performance by aggregating the information from localized neighbors. The spectral and localized approach was first proposed in [17], which defined the graph convolution operation in the Fourier domain. Later, [18] and [19] introduced the localized filters and Chebyshev expansion to avoid the eigen-decomposition. The GCN proposed in [11] simplifies the previous convolution operations as a matrix multiplication of the normalized adjacent matrix and the hidden features, which restricted that the convolution to be computed layer by layer and activated with non-linear functions at each layer. To accelerate the training and inference process of the GCN, SGC [20] reduce the excess complexity of GCNs by repeatedly removing the nonlinearities between GCN layers and collapsing the resulting function into a single linear transformation. The authors verified that SGC exhibits comparable performance as GCN.

To further enhance the performance of GCN, two types of methods are proposed: 1) sampling and 2) attention. There are two kinds of sampling-based methods including GraphSAGE [8] and FastGCN [14], they introduced node-wise sampling and layer-wise sampling, separately. GraphSAGE computed node representation with sampling the 1-step and 2-step neighbors of the center node to construct a sub-graph for the center node, and fused the information of the sub-graph with convolutional aggregators. The FastGCN model interpreted graph convolutions as a layer-wise integral transformation and sampled the nodes in each layers independently.

Another class of methods can enhance the GCN network using the attention mechanism, GAT [15] first applies the idea of self-attention to graph representation learning. GAT gives different weights to the neighbors of the center node by weighting the similarity between each neighbor and the center node. However, the computation of attention consumes a large amount of time. Gaan [21] accelerate the training and prediction of GAT by using the node-wise sampling method mentioned in GraphSAGE. ASGCN [16], [22] ensemble the node-wise sampling, layer-wise sampling and the attention mechanism to achieve a better performance. However, these methods are not aware of the labels of the edges, which may aggregate the information from the neighbors whose labels are different. For nodes which have fewer neighbors than others, these methods also suffer from lacking of localize information.

Besides the supervised methods, unsupervised learning methods can learn the graph representation without using the label information. The unsupervised learning methods can be classified into three main classes: 1) factorization-based approaches, 2) random walk approaches and 3) convolutional approaches. Early methods for graph representations learning are mainly inspired by dimensionality reduction, and are largely based on matrix-factorization approaches [23]. Methods in this class aim to make the representation of the connected nodes more close than nodes have no connection between them [24], [5], [25]. Many recent methods are based on word embedding methods which are widely used in natural language processing (NLP). Random walk methods [2] generate random walks from the graph, and then treat the nodes as words and the whole walk as a sentence, and then use word2vec [26] to learn the representation of nodes. The convolution-based approaches are to train convolution-based encoders to be contrastive between representations that capture statistical dependencies of interest and those that do not. These methods usually aggregate the localized information like normal GCN-based methods, and then optimize the encoder with unsupervised objective functions [27], [28], [29].

III. BACKGROUND

In this section, we will first demonstrate the notations shared by graph convolutional networks, and then briefly review
several types of graph convolutional networks. Finally, we
describe the limitations of the existing GCN structures and
introduce our framework.

A. Notations

In GCN learning tasks, we are given a set of node features,
\( X = \{x_1, x_2, \cdots, x_N\} \), where \( N \) is the number of nodes in
the graph, \( x_i \in \mathbb{R}^F \) represents the features of node \( i \), and \( F \)
is the feature size. For nodes in training set, we also have the
labels \( Y = \{Y_1, Y_2, \cdots, Y_t\} \) of the nodes, where \( Y_i \in \mathbb{R}^C \)
denotes the label of node \( i \), and \( C \) is the number of classes. We are
also provided with relational information between these
nodes in the form of an adjacency matrix, \( A \in \mathbb{R}^{N \times N} \). A
may consist of arbitrary real numbers (or even arbitrary edge
features). \( A_{i,j} = 0 \) denotes there is no edge between \( i \) and \( j \).
In practice, it is common to add a loop edge for each node
by setting \( A_{i,i} = 1 \) for \( 1 \leq i \leq N \). Then, the objective
of the graph convolutional network is to learn an embedding
function which projects \( X \) and \( A \) to hidden representation \( \mathcal{H} \),
such as \( \mathcal{E}(X, A) = \mathcal{H} = \{h_1, h_2, \cdots, h_N\} \), which \( h_i \in \mathbb{R}^{F'} \)
denotes the high-level representations and \( F' \) is the size of
representation space. Then the prediction \( \hat{Y}_i \) of the nodes \( i \)
is achieved by applying a predicted function \( f \) on the final
representation like \( f(h_i) \).

B. Spectral Graph Convolution

The basic GCN uses the 1-step connectivity structure of
the graph as the convolutional filter to extract higher level
representation from neighborhoods. The architecture can be
elegantly summarized as follows,
\[
H^{(l+1)} = \sigma(\hat{A}H^{(l)}W^{(l)}),
\]
where \( \hat{A} \) is the normalized adjacency matrix, \( H^{(l)} \) denotes
embedding of the nodes in the \( l \)-th layer where \( H^{(0)} = X \), \( W^{(l)} \)
denotes the parameter matrix of layer \( l \), and \( \sigma \) denotes the
non-linearity activation function.

The simplify GCN(SGC) hypothesizes that the non-
linearity between two successive GCN layers is not critical,
but that the majority of the benefit arises from the local
averaging. SGC therefore removes the nonlinear transition
functions between each layer. Thus, the resulting model is
linear, but still has the same increased receptive field of a \( K \-
layer GCN,
\[
H^{(K)} = \hat{A}^KX, \quad \hat{Y} = \text{softmax}(\hat{A}^KX\Theta),
\]
where \( K \) denotes the number of the simplify graph convolution
layers and \( \Theta \) is the reparameterized weights of \( \Theta = \prod_{l=0}^{K} W^{(l)} \).

C. Spatial Graph Convolution

To accelerate the computation and enable the mini-batch
training, the layer-wise convolution operation can be inter-
preted as node-wise aggregation process \cite{8}, \cite{14}, \cite{16}.
\[
h^{(l+1)}_i = \sigma \left( FC_{\theta(j)} \left( \sum_{j \in \mathcal{N}_i} w_{i,j}^{(l)} h^{(l)}_j \right) \right),
\]
where \( FC_{\theta(j)} \) denotes a fully-connect layer with the parameter
\( \theta(j) \), which works just the same as \( W^{(l)} \), and \( \mathcal{N}_i \) denotes
the direct neighbors of node \( i \). \( w_{i,j}^{(l)} \) denotes the importance of the
representation from node \( j \) which are set as \( 1/(\text{Degree of } j) \)
in GCN. In Fig 2(a), we present a single convolution layer as
a GCN aggregator. This aggregator treats the node’s neighbors
the same importance.

In Fig 2(b), we present the attention aggregator, in which
the weight of neighbors \( w_{i,j}^{(l)} \) is computed by multi-head self-
attention, which can be written as follows,
\[
w_{i,j}^{(l)} = \frac{\exp \left( LReLu \left( a^T [W_{a}h_i^{(l)} || W_{a}h_j^{(l)}] \right) \right)}{\sum_{k \in \mathcal{N}_i} \exp \left( LReLu \left( a^T [W_{a}h_i^{(l)} || W_{a}h_k^{(l)}] \right) \right)},
\]
where \( a \) denotes the weight vector of the attention mechanism,
\( W_a \) is the shared linear transformation of the attention
mechanism, and \( || \) is the concatenation operation.

Although the attention aggregator can compute the weight
of the neighbors dynamically, the attention mechanism cannot
utilize the label information between the center node and
the neighbors. Besides, the weights are highly depended on
the similarity of the representation of nodes such as: \( h_i \) and
\( h_j \) where \( j \in \mathcal{N}_i \). The accuracy of the representation will
influence the similarity computation \( w_{i,j}^{(l)} \) and the prediction \( \hat{Y}_i \)
simultaneously. As shown in Fig 1, the representation of node
\( i \) is similar to the representations of neighbors with different

Fig. 2. Comparison of different graph aggregators. The aggregators are drawn for two aggregation step. The nodes in red are center nodes and the nodes in other colors are neighboring nodes. The nodes in light gray meaning the nodes are not considered by the aggregator. The arrow-lines show the feature flow of the aggregators. The inner oval in dash line shows the boundary of the 1-step neighbors and the outer oval in dash lines shows the boundary of the 2-step neighbors.
class labels, and the attention computed by \( h_i \) and \( h_j \) will further exacerbate the wrong aggregation.

In order to control the computation complexity and enable the mini-batch training, GraphSAGE adopts node-wise sampling method to control the number of the neighbors of each aggregator. As shown in Fig 2(c), GraphSAGE first samples the 1-step neighbors of the center node and then samples the 2-step neighbors for the sampled 1-step neighbors.

From Fig 2, we find the sampling methods suffer from the presence of the neighbors with different labels. Besides, the above methods may also suffer from the absence of the neighbors with the same label. As shown in Fig 2(c), the center node have more different labeled neighbors (which are denoted by triangle) than same labeled neighbors (which are denoted by circle). After the sampling process of GraphSAGE, the aggregator aggregates more information from the different labeled neighbors than the same labeled neighbors, causing bias in the representation learning of node \( i \).

### D. LAGCN Framework

In order to solve this problem, we propose the Label-Aware GCN framework. We first get the node embedding matrix with an unsupervised learning method, and then simplify the neighbors’ label prediction into an edge label prediction problem to predict whether the labels of the center node and its neighbors are the same. Finally, as described in Fig 2(d), we propose a sampling and aggregation method which can filter the different labeled edges and absorb the same labeled 2-step neighbors to enhance the GCN network.

### IV. LABEL-AWARE GCN

In this section, we first represent the label-aware graph convolution operation and demonstrate how to enhance the graph convolution by adding the positive neighbors (neighbors with the same class label) and filtering the negative neighbors (neighbors with different class label). Then we propose an effective edge label predictor and give the lower boundary of the prediction accuracy to make a positive influence on the graph convolutional network. Finally, we introduce the sampling and aggregation method to enhance the Graph Convolutional Network with the limited accuracy edge prediction result.

#### A. Label-Aware Graph Convolution

As discussed before, existing convolutional aggregators aggregate the neighbors’ representation without considering whether their labels are the same to the center node. In order to analyze the impact of the different labeled neighbors, we can rewrite the aggregator formula in (3) as follows,

\[
\begin{align*}
\mathbf{h}^{(l+1)}_i &= \sigma \left( \mathbf{FC}_{\theta_i} \left( \sum_{j \in N^+_i} \mathbf{w}^{(l)}_{ij} \mathbf{h}^{(l)}_j + \sum_{j \in N^-_i} \mathbf{w}^{(l)}_{ij} \mathbf{h}^{(l)}_j \right) \right)
\end{align*}
\]

where \( N^+_i \) denotes the neighbors which have the same label as the center node \( i \), called positive neighbors. And \( N^-_i \) denotes the neighbors which have different labels with node \( i \), called negative neighbors.

To illustrate that the result of the node classification can be impacted by the number of the positive neighbors and the negative neighbors, we make the following assumptions. First, for the center node \( i \) and its direct neighbor \( j \in N'_i \), we consider they are either positive neighbors and negative neighbors. We assume there is a linear function \( f \) that projects the representation of the neighbor nodes or the center node to a real number, and the real number denotes which class the node belongs to. Let \( \tau \) denote the threshold of the classification, we assume if \( f(h_j) > \tau \), we will classify node \( j \) into the positive class. Thus, if \( f(h_j) \leq \tau \), we classify node \( j \) into the negative class. Considering the probability of misclassification, we assume that \( f(h_j) \sim Norm(\mu^+, \sigma^2) \) for \( j \in N^+_i \) and \( f(h_j) \sim Norm(\mu^-, \sigma^2) \) for \( j \in N^-_i \). Empirically, we regard \( \mu^+ \) as a real number larger than \( \tau \), and \( \mu^- \) as a real number smaller than \( \tau \). Actually, the assumption of the Normal Distribution is unnecessary, we use the assumption of Normal Distribution for simplicity.

We simplify the GCN aggregator by discarding the activation layer. Then the aggregation of node \( i \) which is denoted by \( h_i \) turns to be the mean average of \( h_j \) for \( j \in N'_i \).

\[
\mathbf{h}_i = \frac{\sum_{j \in N^+_i} \mathbf{h}_j + \sum_{j \in N^-_i} \mathbf{h}_j}{n_i^+ + n_i^-} \tag{6}
\]

where \( n_i^+ \) denotes the number of the positive neighbors and \( n_i^- \) denotes the number of the negative neighbors. By making the above assumption, the GCN aggregator is greatly simplified. Then we can get the following theorem.

**Theorem 1:** Let \( P(f(h_i) > \tau) \) be the probability of correctly classifying the center node \( i \). \( P(f(h_i) > \tau) \) can be increased by increasing \( n_i^+ \) and reducing \( n_i^- \).

**Proof:** As the representation \( h_i \) can be derived with Formula (6), we can get the prediction by applying the linear transformation \( f \) on \( h_i \), \( f(h_i) \) can be written as follows.

\[
f(h_i) = \frac{\sum_{j \in N^+_i} f(h_j) + \sum_{j \in N^-_i} f(h_j)}{n_i^+ + n_i^-} \tag{7}
\]

As we supposed above, the linear transform of \( f(x_i) \) obeys the normal distribution. And the expectation of \( f(h_i) \) can be computed as follows.

\[
f(h_i) \sim Norm(\mu_i, \sigma^2) \]

\[
\mu_i = \frac{n_i^+ \mu^+ + n_i^- \mu^-}{n_i^+ + n_i^-} \tag{8}
\]

\[
\sigma = \frac{\sqrt{n_i^+ + n_i^-}}{n_i^+ + n_i^-} \sqrt{\mu^+ - \mu^-} \]

Then the probability of the accuracy prediction can be rewritten as follow.

\[
P(f(h_i) > \tau) = \Phi\left( \frac{\mu_i - \tau}{\sigma} \right) \tag{9}
\]

As described in the assumption, \( \mu^+ \) is a real number larger than \( \tau \) and \( \mu^- \) is a real number smaller than \( \tau \). To increase the prediction accuracy, we can increase the positive parameter.
\( n_i^+ \) and decrease the negative parameter \( n_i^- \), without making any difference on the value of \( \mu^+ \) and \( \mu^- \).

Thus the above theorem guarantees that we can enhance the graph convolutional network by increasing the positive neighbors and reducing the negative neighbors.

### B. Edge Label Predictor

To increase the positive neighbors and reduce the negative neighbors for increasing the prediction accuracy, we transform the node label prediction problem into the edge label prediction problem. Considering the target to filter the negative neighbors and add more positive neighbors, the edge label predictor should be efficient enough to be optimized. Considering the large number of the 1-step and 2-step connections of a node, we introduce a simple and efficient edge label predictor to predict whether the two nodes forming an edge have the same label. Considering the versatility and the simplicity of the edge predictor, we first get the unsupervised node embedding vector \( e_i \) for each node \( i \), and then use it as feature for the edge label classification task.

Given the edge \((i, j)\), the edge predictor is designed to predict whether \( Y_i \) equals to \( Y_j \). Thus, when we input the representation vectors of node \( i \) and node \( j \), the edge label predictor \( E(i, j) \) will return a real number reflect the probability of \( i \) and \( j \) having the same label. Specifically, we design a multi-layer perceptron, which takes \( e_i \) and \( e_j \) as input. The input features are designed as the concatenation of the absolute difference \(|e_i - e_j|\), the summation \( e_i + e_j \) and the Hadamard product \( e_i \odot e_j \). Thus, the prediction of the edge satisfies the law of commutation, which means the output of the edge \((i, j)\) is the same as \((j, i)\). Let \( Y_{i,j} \) denotes the label of the edge. \( Y_{i,j} = 1 \) if \( Y_i = Y_j \) and \( Y_{i,j} = 0 \) if \( Y_i \neq Y_j \). The predicted edge label \( \hat{Y}_{i,j} \) is as follows.

\[
\hat{Y}_{i,j} = E(i, j) = MLP( |e_i - e_j| \parallel (e_i + e_j) \parallel (e_i \odot e_j))
\]

(10)

As described in the label-aware graph convolution, we can enhance the graph convolutional network by adding more positive neighbors and filtering negative ones. Thus the performance of LAGCN can be largely affected by edge label prediction accuracy, and the edge label predictor with low accuracy may hurt the performance. Next, we give the lower boundary analysis on the edge label predictor.

To decrease the number of the negative neighbors of the center node, we can filter the nodes which are predicted as negative neighbors. But as the prediction may not be accurate, we may delete the positive neighbors by mistake. Thus, we have the following theorem to show the minimum requirements of the edge label predictor for the negative neighbors filtering process.

**Theorem 2:** Let \( p_{sen} = P(\hat{Y}_{i,j} = 1|Y_{i,j} = 1) \) and \( p_{spe} = P(\hat{Y}_{i,j} = 0|Y_{i,j} = 0) \) be the sensitivity and the specificity of the edge label predictor, respectively. We filter the negative neighbors \( j \) of center node \( i \) with \( \hat{Y}_{i,j} = 0 \). Then the mean value of the probability of correctly classifying the node \( i \) \( P(f(h_i) > \tau) \) after the filtering will be enlarged as long as \( p_{sen} + p_{spe} > 1 \).

**Proof:** After filtering the direct neighbors of node \( i \) with the edge label predictor, the value of the positive neighbors \( n_i^+ \) and the negative neighbors \( n_i^- \) can be rewritten as follows,

\[
n_i^+ \leftarrow n_i^+ - (1 - p_{sen})n_i^+ \\
n_i^- \leftarrow n_i^- - p_{spe}n_i^-
\]

(11)

Following equation \( (8) \), the mean value of \( P(f(h_i) > \tau) \), which is denoted as \( \mu_i \), can be written as follows,

\[
\mu_i = \frac{p_{sen}n_i^+}{p_{sen}n_i^+ + (1 - p_{spe})n_i^-} \mu^-
\]

(12)

Then the difference between the two mean values, \( \mu_i' - \mu_i \) can be written as follows,

\[
\mu_i' - \mu_i = \frac{n_i^+ n_i^- (p_{sen} - (1 - p_{spe})) (\mu^+ - \mu^-)}{(p_{sen}n_i^+ + (1 - p_{spe})n_i^-) (n_i^+ + n_i^-)}
\]

(13)

As assumed above, \( \mu^+ \) is a real number larger than \( \mu^- \). Then the mean of the prediction for node \( i \), \( P(f(x_i) > \tau) \), will be greater than the original \( \mu \) as long as \( p_{sen} + p_{spe} > 1 \).

Thus, with a edge label predictor whose sensitivity and specificity both satisfy Theorem \( \square \), we can enhance the graph convolution network by filtering negative neighbors.

However, the filtering operation may further aggravate the lack of direct neighbors problem for the center nodes which have a small number of neighbors. With the help of the edge label predictor, we can select and add more 2-step neighbors whose labels are predicted to be the same as the center node. But due to the inaccuracy of the edge label predictor, we may mistakenly add some negative neighbors. Following theorem shows the minimum requirements of the edge label predictor for the positive neighbors adding process.

**Theorem 3:** Let \( p_{pre} = P(\hat{Y}_{i,j} = 1|Y_{i,j} = 1) \) be the precision of the edge predictor. We add the positive neighbors \( j \) from the 2-step neighbors with \( \hat{Y}_{i,j} = 1 \). Supposing the number of the added neighbors is \( n_i^+ \), the mean value of the probability of correctly classifying node \( i \) \( P(f(h_i) > \tau) \) after adding the positive neighbors will be enlarged as long as \( p_{pre} > n_i^+ (n_i^+ + n_i^-) \).

**Proof:** After adding the 2-step neighbors, the value of positive neighbors \( n_i^+ \) and negative neighbors \( n_i^- \) can be rewritten as follows,

\[
n_i^+ \leftarrow n_i^+ + p_{pre}n_i^+ \\
n_i^- \leftarrow n_i^- + (1 - p_{pre})n_i^-
\]

(14)

Following equation \( (8) \), the mean value of \( P(f(h_i) > \tau) \), which is denoted as \( \mu_i \), can be written as follows,

\[
\mu_i = \frac{(n_i^+ + p_{pre}n_i^+)^- + (n_i^- + (1 - p_{pre})n_i^+)\mu^-}{n_i^+ + n_i^- + n_i^+}
\]

(15)

Then the difference between the mean values \( \mu_i' - \mu_i \) can be written as follows:

\[
\mu_i' - \mu_i = \frac{n_i^+ (p_{pre}n_i^- - (1 - p_{pre})n_i^+) (\mu^+ - \mu^-)}{(n_i^+ + n_i^- + n_i^+)(n_i^+ + n_i^-)}
\]

(16)

As assumed above, \( \mu^+ \) is a real number larger than \( \mu^- \). Then the mean of the prediction for node \( i \), \( P(f(x_i) > \tau) \), will be
larger than the original $\mu$ as long as $p_{pre} > n_1^i/(n_1^i + n_{-1})$.

Thus, with a edge label predictor whose precision satisfies the Theorem\[3\] we can enhance the graph convolution network by adding positive neighbors for the center node whose degree is fewer than others.

C. Importance Sampling and Aggregation

As mentioned above, the edge label predictors have a limit performance, then we develop the original sampling and aggregation method to an importance sampling and aggregation method. Considering two situations, if node $i$ have sufficient neighbors, we can eliminate the impact of the negative nodes by giving the negative nodes a low importance(probability) to be sampled. If node $i$ has few neighbors, in spite of the low probability of the negative nodes, the may be sampled due to the lack of neighbors. To minimize the impact of the negative neighbors, we will give the negative neighbors a low importance(weight) during the aggregation process. The key point of the importance sampling and aggregation is the construction of the adjacent probability matrix $A^{(e)}$. In our work, we begin from the original adjacent matrix $A^{(e)} = A$. The predicted label $E(i, j)$ of edge $(i, j)$ lower than a threshold will be considered as negative. In this paper we empirically set the threshold as 0.5. Then we set $A^{(e)}_{i, j}$ as the prediction value of the edge label predictor $E(i, j)$. For the edge between the center node $i$ and its 2-step neighbor $j$ $(i, j)$, if the prediction $E(i, j)$ is higher than the threshold 0.5, we consider it as positive. Then we set $A^{(e)}_{i, j}$ as the prediction value of the edge label predictor $E(i, j)$.

After the construction of the adjacent probability matrix $A^{(e)}$. The sampling probability of the neighbor nodes can be written as follows,

$$P(j \in N_i^+ \setminus A^{(e)}_{i, j} > 0 \setminus A^{(e)}_{i, j} = 0 (17)$$

where $N_i^+$ denotes the sampled neighbor set.

Thus, we use the alias sampling method\[30\] to sample the neighbors for each center node. Besides sampling, we also change the weight $w_{i, j}$ of the aggregator depending on whether the labels of the neighbor and the center node are the same. In our framework, the positive neighbors have a higher impact on the center node, and the negative neighbors have a lower impact on the center node. Thus, we use the connection weight $A^{(e)}_{i, j}$ as the weight $w_{i, j}$. The label-aware aggregator can be written as follows,

$$h_{i}^{(l+1)} = \sigma \left( FC_{\theta(l)} \left( \sum_{j \in N_i^+ \setminus A^{(e)}_{i, j} > 0 \setminus A^{(e)}_{i, j} = 0} \frac{A^{(e)}_{i, j}}{\sum_{k \in N_i^+} A^{(e)}_{i, j}} h_{j}^{(l)} \right) \right). (18)$$

V. EXPERIMENT

A. Datasets

In this section, We evaluate the performance of our method on the following benchmarks: Cora, Citeseer and Pubmed\[31\].

These graphs vary in sizes from small to large. Particularly, the number of nodes in Cora and Citeseer are of scale $O(10^5)$, while Pubmed contains more than $10^4$ vertices. More details of the benchmark datasets are shown in Table I.

B. Experiment Settings

In order to demonstrate the effectiveness of our proposed framework, we adopt the LAGCN framework on the GraphSAGE model, and name it LA-GSAGE. By using several different unsupervised learning methods to compute the node representation for the edge label predictor, there are four types of LA-GSAGE. The hidden dimensions of the four unsupervised learning methods are all set as 128.

- **GF-GSAGE**: GF-GSAGE uses Graph Factorization\[24\] as the unsupervised learning method to get the node representation for the edge label predictor. Graph factorization is only provided with the adjacent matrix.
- **DW-GSAGE**: DW-GSAGE uses DeepWalk\[2\] as the unsupervised learning method to get the node representation for the edge label predictor. DeepWalk first generates random walks on the graph and then uses word2vec\[26\] to get the embedding of the nodes. DeepWalk is only provided with the adjacent matrix.
- **N2V-GSAGE**: N2V-GSAGE uses Node2Vec\[1\] as the unsupervised learning method to get the node representation for the edge label predictor. Similar to DeepWalk, Node2Vec introduces another two parameters to control the backward or the forward walking process. Node2Vec is only provided with the adjacent matrix.
- **DGI-GSAGE**: DGI-GSAGE uses DGI\[32\] as the unsupervised learning method to get the node representation for the edge label predictor. DGI is an unsupervised graph convolutional network, which achieves a competitive performance in node classification benchmarks. DGI is provided with the adjacent matrix and the feature matrix.

In order to demonstrate the effectiveness of our proposed approach, we compare it against several baseline methods, including basic regression methods, unsupervised regression methods and supervised convolutional methods. Especially, we demonstrate the performance margin between the LA-SAGE and GraphSAGE. For GraphSage, we use the implement in PyTorch\[23\]. For GCN\[11\], GAT\[15\] and ASGCN\[16\], we use the implement in Tensorflow\[34\]. All the methods are evaluated by the best performance on the three datasets. We show

C. Experiment Results

Table I indicates the experiment results of baseline methods over the evaluation accuracy on the three datasets. We show
Fig. 3. Histogram of unbalance of the neighbor node labels for the Cora dataset. Histogram (a) and (c) represent the distribution of the positive neighbor number minus the negative neighbor number for the center nodes. Histogram (b) and (d) represent the distribution of the positive neighbor number divided of the GraphSAGE.

Furthermore, our method performs much better than GAT model, demonstrating that the label-aware structure significantly outperforms the similarity based attention mechanism. Thus, the result states the label-aware structure acts more effective in graph classification task.

D. Edge Label Prediction Analysis

In this subsection, we analyze the edge label prediction results of the GraphSAGE model on the Cora dataset. We find that if a center node has more positive neighbors than negative ones, the node is more likely to be correctly classified and vice versa. In Fig 3 (a), we plot the histogram of the difference between the amount of positive neighbors and the negative ones for each node. In Fig 3 (b), we plot the histogram of the ratio of the amount of positive neighbors and negative ones for each node. These two histograms indicate that for the misclassified center nodes, the difference between its positive and negative neighbors is mostly less than 3, while the number difference for the correctly classified center nodes are mostly larger than or equal to 3.

Then we analyze whether our edge prediction can increase the positive neighbors and decrease the negative ones. In Fig 3 (c), we compare the histogram of the difference between positive and negative neighbors for the wrong prediction nodes before and after the label-aware enhancement. In Fig 3 (d), we compare the histogram of the ratio of the amount of positive neighbors and negative ones for the wrong prediction nodes before and after the label-aware enhancement. The histograms in blue bars show the original distribution of the wrong classification nodes. The histograms in orange bars and red line show the distribution of the misclassified nodes after the edge adding processing, respectively. We can find that the filtering process and the adding process can both effectively decrease the unbalance.
As shown in Theorem 2, the mean probability of correctly classifying the node $i$ $P(f(h_i) > \tau)$ after the filtering will be enlarged if $p_{\text{sen}} + p_{\text{spe}} > 1$. Thus, we implement several experiments on Cora dataset to test the theorem. We artificially design several edge label predictors, whose $p_{\text{sen}}$ varies from 0.0 to 0.9 and $p_{\text{spe}}$ varies from $\max(0.0, p_{\text{sen}} - 0.3)$ to $\min(0.9, p_{\text{sen}} + 0.3)$. Then we build the LA-GSAGE model, which only uses the edge label predictors to filter the negative neighbors. Fig 2 (a) gives the performance of LA-SAGE with edge filter. We find that the accuracy presents a linear increasing trend with the increase of the summation $p_{\text{sen}} + p_{\text{spe}}$. When $p_{\text{sen}} + p_{\text{spe}} = 1$, LA-GSAGE achieves a similar performance as the original GraphSAGE. When $p_{\text{sen}} + p_{\text{spe}} > 1$, the LA-GSAGE outperforms GraphSAGE.

In this paper, we introduce a novel approach that allows the graph convolution networks to consider the label of neighbors. LAGCN model can enhance the graph convolution networks like GraphSAGE by a significant margins. Our theoretical analysis provides insight into the impact of the positive neighbors and the negative neighbors. To further investigate the performance of the LAGCN, we give the lower bound of the edge label predictor. A number of potential extensions are possible, such as extending LAGCN to other graph convolutional networks or other edge label predictors.

VI. CONCLUSION

In this paper, we introduce a novel approach that allows the graph convolution networks to consider the label of neighbors. LAGCN model can enhance the graph convolution networks like GraphSAGE by a significant margins. Our theoretical analysis provides insight into the impact of the positive neighbors and the negative neighbors. To further investigate the performance of the LAGCN, we give the lower bound of the edge label predictor. A number of potential extensions are possible, such as extending LAGCN to other graph convolutional networks or other edge label predictors.

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