Strong decays analysis of excited nonstrange charmed mesons: Implications for spectroscopy

Keval Gandhi* and Ajay Kumar Ra†
Department of Applied Physics,
Sardar Vallabhbhai National Institute of Technology,
Surat 395007, Gujarat, India.

(Dated: November 27, 2019)

The strong decays of $D_{s1}(2420)^0$, $D_0^*(2460)^+$, $D_2^*(2460)^+$, $D_0^*(2550)^0$, $D_2^*(2600)^0$, $D(2740)^0$, $D_1^*(2750)^+$, $D_2^*(2750)^-$, $D_1(3000)^0$, $D_2^*(3000)^0$ and $D_2^*(3000)^0$ resonance states are analyzed in the heavy quark mass limit of Heavy Quark Effective Theory (HQET). The individual decay rates and the branching ratios among the strong decays determine their spin and parity. From such states the Regge trajectories are constructed in $(J, M^2)$ and $(n_c, M^2)$ planes and further predict the masses of higher excited states. Their masses and decay widths are measured precisely (with statistical and systematic uncertainties) and coming from the experimentalists to search these states into their respective decay modes. Moreover, the ratio of the branching fractions measurement of strong decay modes and the branching ratios of these higher excited states are also examined, which can help the experimentalists to search these states into their respective decay modes.

I. INTRODUCTION

Remarkable progress has been made in the field of charmed meson spectroscopy recently by experimental observations as well as theoretical computations. Different experimental facilities have provided new informations in this sector like masses, decay widths, branching ratios, isospin mass splittings, spin, parity, polarization amplitude etc.. At latest, the LHCb Collaboration has studied the amplitude contribution in $B^+ \to D^+ \pi^-$ decay using the Dalitz plot analysis technique [1]. They found that the main contributions are coming from the $D_2^*(2460)^0$, $D_1^*(2680)^0$, $D_2^*(2760)^0$ and $D_2^*(3000)^0$ resonances which are decaying into $S$-wave $D^* \pi^-$. Their masses and decay widths are measured precisely (with statistical and systematic uncertainties) and make a spin parity assignment of $D_2^*(3000)^0$ as $2^-$ first time. The LHCb group in their earlier analysis of decay $B^0 \to D^0 \pi^+ \pi^-$ has measured $D_0^*(2400)^0$ and $D_2^*(2460)^-$ mesons and identified the $D_2^*(2760)^-$ with a spin parity $3^-$ in the squared invariant mass region of $D^*\pi^-\pi^-$. In 2013, the LHCb detector found $D^+ \pi^-, D^0 \pi^+$ and $D^{*+} \pi^-$ final state mass spectra at the centre-of-mass energy $7\text{ TeV}$ of $pp$ collision [3]. They have observed the rich spectrum of nonstrange charmed mesons, $D_2^*(2580)^0$ and $D_2^*(2740)^0$ with unnatural parity ($0^-, 1^+, 2^-, ...$) in the $D^{*+} \pi^-$ decay mode. The mass spectra analysis of $D^+ \pi^-, D^0 \pi^+ \pi^-$ and $D^{*+} \pi^-$ reconstruct the masses and the decay widths of $D_2^*(2460)$. The $D_2^*(2650)^0$ and $D_2^*(2760)^0$ are found with the natural parity ($0^+, 1^-, 2^+ ,...$) in the $D^{*+} \pi^-$ mass spectra. Along with these they have also got the resonant structures in a region around $3\text{ GeV}$. The $D_2^*(3000)^0$ was observed in $D^{*+} \pi^-$ decay mode with unnatural parity and the $D_2^*(3000)^0$ in $D^+ \pi^-$ with natural parity [3].

Earlier, the BABAR experiment had collected the data sample of excited $D$ mesons resonances corresponding to an integrated luminosity 454 $\text{fb}^{-1}$ of $e^+e^-$ collision at the center-of-mass energy $10.58\text{ GeV}$ [4]. The masses and decay widths of the observed $D$ mesons ($D(2550)^0$, $D^*(2600)^0/+$, $D(2750)^0$ and $D^*(2760)^0/+)$, are reconstructed from $D^+ \pi^-, D^0 \pi^+$ and $D^{*+} \pi^-$ decay resonances. Moreover, the helicity distribution analysis identified $D(2550)^0$ and $D^*(2600)^0$ as a $2S$ doublet of spin-parity $0^-$ and $1^-$ respectively; and the states $D(2750)^0$ and $D^*(2760)^0/+)$ belong to $L = 2$ ($L$ is the orbital angular momentum). The masses, decay widths, spin-parity observed by the experimental groups LHCb [1,4] and the BABAR [4] are presented in Table I with their respective observed decay modes.

Experimentally, the Dalitz plot model in the $B$ decay production determines the spin-parity and the prompt production analysis differentiate the hadrons with natural and unnatural parity. Moreover, the ratio of the branching fractions measurement of strong decay modes can help to classify the decaying mesons. It is very crucial to assign the spin-parity of hadrons which facilitate the determination of experimental properties. According to the latest Review of Particle Physics (RPP) by Particle Data Group (PDG), the $JP$ ($J$ is the total spin and $P$ is parity) values of $D_1(2420)^\pm$, $D(2550)^0$, $D_2^*(2600)^0$, $D^*(2640)^0$, $D(2740)^0$ and $D(3000)^0$ mesons are not yet confirmed from the known experimental measurements [1]. Many theoretical groups have computed the excited state masses of charmed mesons with the help of various potential models. Recently, Jiao-Kai Chen obtained the radial and orbital Regge trajectories by applying the Bohr-Sommerfeld quantization approach [6]. Other variants include semi-relativistic approach [2], Godfrey-Isgur (GI) relativized quark model [8], relativistic quark model [9], Lakhina and Swanson proposed nonrelativistic constituent quark model [10], the Quantum Chromodynamics (QCD) motivated relativistic quark model based on the quasipotential approach [11], relativistic quark model [12].
| Meson | LHCb(2016) | LHCb(2015) | LHCb(2013) | BABAR(2010) | Decay mode |
|-------|------------|------------|------------|-------------|------------|
| $D_1(2420)^0$ | 2419.6 ± 0.1 ± 0.7 | 35.2 ± 0.4 ± 0.9 | 2420.1 ± 0.1 ± 0.8 | $D^{+}\pi^{-}$ |
| $D_1^+(2420)^0$ | 2463.7 ± 0.4 ± 0.4 ± 0.6 | 47.0 ± 0.8 ± 0.9 ± 0.3 | 2460.4 ± 0.1 ± 0.1 | $D^{+}\pi^{-}$ |
| $D_2^+(2460)^0$ | 2463.1 ± 0.2 ± 0.6 | 48.6 ± 1.3 ± 1.9 | | $D^0\pi^+$ |
| $D_2^-(2460)^-$ | 2468.6 ± 0.6 ± 0.3 | 47.3 ± 1.5 ± 0.7 | | $D^0\pi^-$ |
| $D(2550)^0$ | 2579.5 ± 3.4 ± 5.5 | 177.5 ± 17.8 ± 46.0 | 2539.4 ± 4.5 ± 6.8 | $D^{+}\pi^{-}$ |
| $D_1^+(2600)^0$ | 2681.1 ± 5.6 ± 4.9 ± 13.1 | 186.7 ± 8.5 ± 8.6 ± 8.2 | 2608.7 ± 2.4 ± 2.5 | $D^{+}\pi^{-}$ |
| $D_2^-(2750)^0$ | 2775.5 ± 4.5 ± 4.5 ± 4.7 | 95.3 ± 9.6 ± 7.9 ± 33.1 | 2760.1 ± 1.1 ± 3.7 | $D^{+}\pi^{-}$ |
| $D_3^+(2750)^0$ | 2771.7 ± 1.7 ± 3.8 | 66.7 ± 6.6 ± 10.5 | 2769.7 ± 3.8 ± 1.5 | $D^0\pi^+$ |
| $D_3^-(2750)^-$ | 2798 ± 7 ± 1 ± 7 | 105 ± 18 ± 6 ± 23 | | $D^0\pi^-$ |
| $D_J(3000)^0$ | 2971.8 ± 8.7 | 188.1 ± 44.8 | | $D^{++}\pi^{-}$ |
| $D_J^+(3000)^0$ | 3008.1 ± 4.0 | 110.5 ± 11.5 | | $D^+\pi^-$ |
| $D_J^+(3000)^0$ | 3214 ± 29 ± 33 ± 36 | 186 ± 38 ± 34 ± 63 | | $D^+\pi^-$ |
TABLE II. Spectra of nonstrange charmed mesons obtained from different models (in MeV).

| $N^{2S+1}L_J$ | $J^P$ | Ref. [6] | Ref. [7] | Ref. [8] | Ref. [9] | Ref. [10] | Ref. [11] | Ref. [12] | Ref. [13] | Ref. [14] |
|----------------|--------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $1^1S_0$       | 0$^+$  | 1869    | 1884    | 1877    | 1874    | 1867    | 1871    | 1868    | 1874    | 1865    |
| $1^3S_1$       | 1$^−$  | 2002    | 2010    | 2041    | 2038    | 2010    | 2010    | 2005    | 2006    | 2027    |
| $2^1S_0$       | 0$^+$  | 2562    | 2582    | 2581    | 2583    | 2555    | 2581    | 2589    | 2540    |         |
| $2^3S_1$       | 1$^−$  | 2616    | 2655    | 2643    | 2645    | 2636    | 2632    | 2692    | 2601    |         |
| $3^1S_0$       | 0$^+$  | 2970    | 3186    | 3110    | 3068    | 3062    | 3141    | 3062    | 3141    | 2904    |
| $3^3S_1$       | 1$^−$  | 3004    | 3239    | 3068    | 3111    | 3096    | 3226    | 3096    | 3226    | 2947    |
| $1^1P_0$       | 0$^+$  | 2319    | 2357    | 2399    | 2398    | 2252    | 2406    | 2377    | 2341    | 2325    |
| $1^3P_1$       | 1$^+$  | 2411    | 2425    | 2456    | 2457    | 2402    | 2426    | 2417    | 2417    | 2489    |
| $1^3P_2$       | 2$^+$  | 2456    | 2461    | 2502    | 2501    | 2466    | 2460    | 2460    | 2477    | 2743    |
| $2^1P_0$       | 0$^+$  | 2976    | 2931    | 2932    | 2752    | 2919    | 2949    | 2919    | 2949    | 2758    |
| $2^3P_1$       | 1$^+$  | 3016    | 2924    | 2933    | 2866    | 2932    | 2995    | 2932    | 2995    | 2792    |
| $2^3P_2$       | 2$^+$  | 3034    | 2961    | 2952    | 2926    | 3021    | 3045    | 3021    | 3045    | 2802    |
| $2^3P_3$       | 2$^+$  | 2893    | 3039    | 2957    | 2971    | 3012    | 3035    | 3012    | 3035    | 2860    |
| $3^1P_0$       | 0$^+$  | 3536    | 3343    |         |         | 3346    | 3346    |         |         | 3050    |
| $3^3P_1$       | 1$^+$  | 3567    | 3328    |         |         | 3365    | 3365    |         |         | 3082    |
| $3^3P_2$       | 1$^+$  | 3582    | 3360    |         |         | 3461    | 3461    |         |         | 3085    |
| $3^3P_3$       | 2$^+$  | 3214    | 3584    | 3353    |         | 3407    | 3407    |         |         | 3142    |
| $1^1D_1$       | 1$^−$  | 2775    | 2755    | 2817    | 2816    | 2740    | 2788    | 2795    | 2750    |         |
| $1^3D_2$       | 2$^−$  | 2789    | 2754    | 2816    | 2827    | 2693    | 2806    | 2775    | 2639    |         |
| $1^3D_3$       | 3$^−$  | 2737    | 2783    | 2845    | 2834    | 2789    | 2850    | 2833    | 2727    |         |
| $1^3D_4$       | 3$^−$  | 2796    | 2788    | 2833    | 2833    | 2719    | 2863    | 2799    | 2633    |         |
| $2^1D_1$       | 1$^−$  | 3315    | 3231    | 3231    | 3.168   | 3228    | 3228    | 3052    |         |         |
| $2^3D_2$       | 2$^−$  | 3318    | 3212    | 3225    | 3.145   | 3259    | 3259    | 2997    |         |         |
| $2^3D_3$       | 2$^−$  | 3341    | 3248    | 3235    | 3.215   | 3307    | 3307    | 3029    |         |         |
| $2^3D_4$       | 3$^−$  | 3355    | 3226    | 3226    | 3.170   | 3335    | 3335    | 2999    |         |         |
| $1^3F_2$       | 2$^+$  | 3105    | 3132    | 3132    |         | 3090    | 3090    | 3091    |         |         |
| $1^3F_3$       | 3$^+$  | 3087    | 3108    | 3123    |         | 3129    | 3129    | 3074    |         |         |
| $1^3F_4$       | 3$^+$  | 2998    | 3143    | 3129    |         | 3145    | 3145    | 3123    |         |         |
| $1^3F_5$       | 4$^+$  | 3073    | 3132    | 3113    |         | 3187    | 3187    | 3101    |         |         |
including the leading order corrections in $1/m$, the Blankenbecler-Sugar equation in the framework of heavy-light interaction models, the lattice QCD etc.,

We summarize the predicted mass spectra in Table I (the symbol $N^{2S+1}L_J$ is used to represent the meson quantum state: where $N$, $L$ and $S$ denote the radial, orbital and the intrinsic spin quantum number respectively). Here, we take their comparison with experimental data and make following conclusions,

i. Two $1S$ states ($D$ and $D^*$) and the four $1P$ states ($D_0^*(2300)$, $D_1(2420)$, $D_1(2430)$ and $D_2^*(2460)$) are well established with their respective $J^P$ values.

ii. $D(2550)^0_0$ was observed by experimental groups LHCb and BABAR. They both suggested that such a state has unnatural parity (but the PDG-2018 need more confirmation). The theoretical studies identified its quantum state $2^1S_0$.

iii. $D^*_0(2600)^0$ and $D^*_2(2640)^0$ are probably the same state. From LHCb and BABAR its $J^P$ value is consistent with natural parity and it can be a candidate of $2^1S_1$.

iv. $D(2740)^0$ was observed in a single experiment LHCb with unnatural parity and it can be a candidate of $1^3D_2$ or $2^1D_2$ state.

v. $D^*_1(2750)$ belongs to $1^3D_3$ quantum state. Experimentally, the LHCb determined its $J^P$ value $3^-$. Yet the state $D^*_1(2750)$ is not observed experimentally.

vi. So far the nature of $D_J(3000)^0$, $D^*_J(3000)^0$ and $D^*_J(3000)^0$ mesons are unsolved theoretically. According to LHCb the $D_J(3000)^0$ has unnatural parity. So it can be a candidate of $3^1S_0$ and $2^1P_0$ states. $D^*_J(3000)^0$ has natural parity and may belongs to $3^3S_1$, $2^3F_2$, $1^3F_3$ and $1^3F_4$ quantum states. The LHCb measured the spin parity of $D^*_J(3000)^0$ as $2^+$ and can belongs to quantum states $3^3F_2$ and $1^3F_2$.

In Ref. S, S. Godfrey and K. Moats are used $^3P_0$ quark-pair-creation (QPC) model, and identified $D_J(2550)^0$, $D^*_J(2600)^0$, $D^*_1(2760)^0$, $D^*_2(2760)^0$, $D^*_J(2750)^0$, $D^*_J(3000)^0$, and $D^*_J(3000)^0$ states as $2^1S_0$, $2^3S_1$, $1^3D_1$, $1^3D_3$, $1^3D_2$, $3^1S_0$, and $1^3F_4$ respectively; through their strong decays analysis. Y. Sun et al. calculated the strong decays of $3S$, $2P$, $2D$, and $1F$ states of $D$ mesons in the $^3P_0$ QPC model. They assigned $D_J(3000)^0$ and $D^*_J(3000)^0$ as $2P(J^P = 1^+)$ and $2^1P_0$ respectively. Also, the Ref. used the same model and examined: $D(2550)$ as $2^1S_0$, $D(2750)$ (or $D(2760)$) as $1^3D_3$ state, and the state $D(2600)$ identify as the low-mass mixing of $1^3D_1 - 2^3S_1$ states. Refs. 15, 16 are determined the strong decay rates of excited heavy-light mesons in the chiral constituent quark model. They predict $D(2760)$ as $1^3D_3$ and $D^*_J(3000)^0$ as $1^3F_4$, and $D(2600)$, $D(2750)$ and $D_J(3000)^0$ are found to be a mixed state of $1^3D_1 - 2^3S_1$, $1^3D_2 - 1^3D_1$ and $2^1P_1 - 2^3P_1$ states respectively.

In this work, we analyze the strong decays of excited nonstrange charmed mesons observed by LHCb and BABBARN Collaborations using the Heavy Quark Effective Theory (HQET) in the leading order approximations. On the basis of the strong decay widths and the branching fractions predictions of $D_J(2420)$, $D(2550)$, $D(2740)$, $D_J(3000)$, and $D^*_J(3000)^0$ and their spin partners $D^*(2640)$, $D^*_J(2600)^0$, $D^*_J(2750)^0$, and $D^*_J(3000)$ we have assigned their spin and parity. Also, the strong coupling constants are determined by comparing the computed strong decay widths with experimental measurements. Similar kind of studies have been done by LHCb and LHCb to identify the higher charmed mesonic states. The spectroscopy of a system containing one light (up ($u$) or down ($d$) or strange ($s$) and one heavy (charm ($c$) or bottom ($b$)) quark provides an excellent base to study the Quantum Chromodynamics (QCD) in the low energy regime. Additionally, our tentative spin-parity assignment of nonstrange charmed mesons allow to construct the Regge trajectory in ($J, M^2$) and ($n_c, M^2$) planes, where $J$ is the total spin, $n_c$ is the radial principal quantum number and $M^2$ is the square of the meson mass. They estimate the masses of $1^3D_2$, $1^3D_3$, $3^1S_0$, $3^3S_1$, $1^3F_3$, $1^3F_4$, $2^3D_3$, $3^1P_2$ and $2^1F_2$ states. Their strong decay rates and branching fraction studies can guide to the experimentalists for searching them in a respective decay channels.

This paper is arranged as follows: after the introduction, section II is a brief description of HQET used to study the strong decays. Section III presents results and discussion, where we attempt to identify the spin and parity of experimentally known excited nonstrange charmed mesons. In section IV, we plot the Regge trajectories in ($J, M^2$) and ($n_c, M^2$) planes using the masses from PDG-2018. Further, we analyzed the strong decay rates and the branching fractions of $1^3D_2$, $1^3D_3$, $3^1S_0$, $3^3S_1$, $1^3F_3$, $1^3F_4$, $2^3D_3$, $3^1P_2$ and $2^1F_2$ states lying on the Regge lines. Finally, the conclusions are presented in section V.

II. THEORETICAL FRAMEWORK

In the framework of heavy quark effective theory (HQET) the properties of heavy-light mesons can be determined systematically by considering infinite mass of one heavy quark, i.e. $m_Q \rightarrow \infty$. The heavy quark spin and the flavor symmetry arising from the QCD are demolished in this heavy quark (HQ) mass limit and classify the heavy-light mesons according to the total angular momentum of the light antiquark $\bar{s}_Q = \bar{s}_q + \bar{t}$, where $\bar{s}_q$ and $\bar{t}$ are the spin and the orbital angular momentum of the light antiquark respectively.

Here we discuss the $D$ mesons doublets corresponding to $s$, $p$, $d$ and $f$ waves for $l = 0$, 1, 2 and 3 respec-
TABLE III. The strong decay widths of nonstrange charmed mesons with possible quantum state assignments (in MeV).

| Meson       | $N^{2S+1L_J}$ | Decay mode | LHCb(2016) | LHCb(2015) | LHCb(2013) | BABAR(2010) |
|-------------|---------------|------------|------------|------------|------------|-------------|
| $D_s(2420)^0$ | $1^P_1$       | $D^+\pi^-$  | 56.2711$r_1^2$ | 56.6228$r_1^2$ |            |             |
|             |               | $D^{0}\pi^0$ | 29.3228$r_1^2$ | 29.5040$r_1^2$ |            |             |
|             |               | $D_s^+K^-$   | --          | --          |            |             |
|             |               | $D^{*0}\eta$ | --          | --          |            |             |
|             |               | Total        | 85.5939$r_1^2$ | 86.1268$r_1^2$ | 0.641      | 0.604       |
| $D_s^+(2460)^0$ | $1^P_2$       | $D^+\pi^+$  | 127.9786$r_1^2$ | 126.52$h_1^2$ |            |             |
|             |               | $D^{0}\pi^0$ | 66.8656$r_1^2$ | 66.1147$r_1^2$ |            |             |
|             |               | $D_s^+K^-$   | 55.3938$r_1^2$ | 55.4757$r_1^2$ |            |             |
|             |               | $D^{*0}\eta$ | 28.4944$r_1^2$ | 29.2442$r_1^2$ |            |             |
|             |               | Total        | 273.0859$r_1^2$ | 277.355$h_1^2$ | 0.409      | 0.427       |

| $D_s^-(2460)^-$ | $1^P_2$ | $D^0\pi^-$  | 131.875$h_1^2$ |            |            |             |
|                |         | $D^-\pi^0$  | 63.6968$h_1^2$ |            |            |             |
|                |         | $D_s^-K^0$  | 56.0702$h_1^2$ | 28.4944$h_1^2$ |            |             |
|                |         | $D^{\ast0}\eta$ | --          | --          |            |             |
|                |         | $D_s^{\ast-}\pi^0$ | 27.968$r_1^2$ | 29.2442$r_1^2$ |            |             |
|                |         | $D_s^{\ast-}K^0$ | --          | --          |            |             |
|                |         | $D^{\ast-}\eta$ | 0.0975       | 0.368       |            |             |
|                |         | Total        | 295.664$r_1^2$ | 30.2238$r_1^2$ |            |             |

| $D(2550)^0$   | $2^P_0$  | $D_s^{\ast0}\pi^0$ | 864.734$g_{ll}^{12}$ | 709.405$g_{ll}^{12}$ |            |             |
|               |         | $D_s^{\ast0}\eta$  | 44.692$g_{ll}^{12}$  | 363.314$g_{ll}^{12}$ |            |             |
|               |         | Total               | 1310.30$g_{ll}^{12}$ | 1072.72$g_{ll}^{12}$ | 0.368      | 0.348       |

continued...
TABLE III. The strong decay widths of nonstrange charmed mesons with possible quantum state assignments (in MeV).

| Meson       | $N^{2S+1L_J}$ | Decay mode | LHCb(2016) [1] | LHCb(2015) [2] | LHCb(2013) [3] | BABAR(2010) [4] |
|-------------|---------------|------------|----------------|----------------|----------------|----------------|
| $D^+_s(2600)^0$ | $2^3S_1$      | $D^+\pi^-$ | 680.382$g_{12}^1$ | 541.421$g_{12}^2$ |
|             |               | $D^0\pi^0$ | 345.515$g_{12}^2$ | 274.992$g_{12}^2$ |
|             |               | $D_s^+ K^-$ | 199.173$g_{12}^1$ | 104.757$g_{12}^1$ |
|             |               | $D_s^0 K^0$ | 47.9068$g_{12}^1$ | 29.1069$g_{12}^1$ |
|             |               | $D_s^{++}\pi^-$ | 886.679$g_{12}^2$ | 656.589$g_{12}^2$ |
|             |               | $D_s^{*0}\pi^0$ | 450.689$g_{12}^2$ | 334.839$g_{12}^2$ |
|             |               | $D_s^{*+}K^-$ | 78.3291$g_{12}^2$ | 8.24532$g_{12}^2$ |
|             |               | $D_s^{*0}\eta^*$ | 31.0273$g_{12}^1$ | 1949.95$g_{12}^2$ |
|             |               | Total         | 2719.709$g_{12}^1$ | 0.218          |
| $D^+_s(2600)^0$ | $2^3S_1$      | $D^{++}\pi^-$ | 781.919$g_{12}^2$ |
|             |               | $D^{*+}\pi^0$ | 397.965$g_{12}^2$ |
|             |               | $D^{*+}K^-$ | 33.6215$g_{12}^1$ |
|             |               | $D^{*0}\eta$ | 19.8058$g_{12}^2$ |
|             |               | $D^+\pi^0$ | 617.246$g_{12}^1$ |
|             |               | $D^0\pi^0$ | 313.774$g_{12}^1$ |
|             |               | $D_s^+ K^-$ | 155.109$g_{12}^1$ |
|             |               | $D_s^0 K^0$ | 39.2693$g_{12}^1$ |
|             |               | Total         | 2358.71$g_{12}^1$ |
| $D(2740)^0$   | $1^3D_2$      | $D^{++}\pi^-$ | 126.986$g_{12}^1$ |
|             |               | $D^{*+}\pi^0$ | 65.8248$g_{12}^1$ |
|             |               | $D^{*+}K^-$ | 1.92685$g_{12}^1$ |
|             |               | $D^{*0}\eta$ | 1.30793$g_{12}^1$ |
|             |               | Total         | 196.046$g_{12}^1$ |
| $D(2750)^0$   | $1^3D_3$      | $D^+\pi^-$ | 190.520$g_{12}^1$ |
|             |               | $D^0\pi^0$ | 98.5331$g_{12}^2$ |
|             |               | $D_s^+ K^-$ | 20.954$g_{12}^1$ |
|             |               | $D_s^0 K^0$ | 7.03403$g_{12}^1$ |
|             |               | $D^{++}\pi^-$ | 99.8604$g_{12}^1$ |
|             |               | $D^{*0}\pi^0$ | 45.5805$g_{12}^1$ |
|             |               | $D_s^{++}K^-$ | 2.88624$g_{12}^1$ |
|             |               | $D^{*0}\eta$ | 1.53565$g_{12}^1$ |
|             |               | Total         | 472.95$g_{12}^1$ |
| $D_s(2750)^0$ | $1^3D_3$      | $k_Y$ | 0.449 |
|             |               |              | 0.420 |
|             |               | $1^3D_3$      | 0.376 |
| $D_s(2750)^0$ | $1^3D_3$      | $k_Y$ | 0.419 |
|             |               |              | 0.423 |

continued...

dagger ($P^1, P^{1*}, ...$) and for $n = 3$ they are ($P^1, P^{1*}, ...$). Hence, each doublet contains two states (or two spin partners) with total spin $J = s_l \pm \frac{1}{2}$ and parity $P = (-1)^J$. and can be described by the superfields $H_\alpha$, $S_\alpha$, $T_\alpha$, $X_\alpha$, $Y_\alpha$, $Z_\alpha$ and $R_\alpha$, written as $28, 29$, 

$$H_\alpha = \frac{1 + \gamma_5}{2} [P_{\alpha\mu} \gamma^\mu - P_\alpha \gamma_5],$$  

(1)
### TABLE III. The strong decay widths of nonstrange charmed mesons with possible quantum state assignments (in MeV).

| Meson          | $N^{2S+1L_J}$ | Decay mode | LHCb(2016) [1] | LHCb(2015) [2] | LHCb(2013) [3] | BABAR(2010) [4] |
|----------------|---------------|------------|---------------|---------------|---------------|----------------|
| $D_s^*(2750)^+$| $1^3D_3$      | $D^0\pi^+$ | 191.164k$^2_Y$ | 188.68k$^2_Y$ |               |                |
|               |               | $D^+\pi^0$ | 93.4520k$^2_Y$ | 92.2321k$^2_Y$ |               |                |
|               |               | $D^+K^0$   | 19.4484k$^2_Y$ | 18.8051k$^2_Y$ |               |                |
|               |               | $D^{*+}\pi^+$ | 6.4305k$^2_Y$ | 6.291k$^2_Y$  |               |                |
|               |               | $D^{*+}\pi^0$ | 99.3632k$^2_Y$ | 97.7744k$^2_Y$ |               |                |
|               |               | $D^{*+}K^0$ | 48.8147k$^2_Y$ | 48.0319k$^2_Y$ |               |                |
|               |               | $D^{++}\eta$ | 2.46429k$^2_Y$ | 2.35189k$^2_Y$ |               |                |
|               |               | $D^{*+}\eta$ | 1.35745k$^2_Y$ | 1.31015k$^2_Y$ |               |                |
|               |               | Total       | 462.494k$^2_Y$ | 455.476k$^2_Y$ |               |                |
|               |               | $k_\gamma$  | 0.380         | 0.366         |               |                |
| $D_s^*(2750)^-$| $1^3D_3$      | $D^0\pi^-$  | 226.341k$^2_Y$ |               |               |                |
|               |               | $D^+\pi^0$  | 110.734k$^2_Y$ |               |               |                |
|               |               | $D^+K^0$    | 26.6515k$^2_Y$ |               |               |                |
|               |               | $D^{*+}\pi^+$ | 8.49284k$^2_Y$ |               |               |                |
|               |               | $D^{*+}\pi^0$ | 122.268k$^2_Y$ |               |               |                |
|               |               | $D^{*+}K^0$ | 60.1023k$^2_Y$ |               |               |                |
|               |               | $D^{*+}\eta$ | 4.34870k$^2_Y$ |               |               |                |
|               |               | Total        | 561.045k$^2_Y$ |               |               |                |
|               |               | $k_\gamma$  | 0.433         |               |               |                |
| $D_J(3000)^0$ | $3^1S_0$      | $D^{*+}\pi^-$ | 3216.82g$^2_H$ |               |               |                |
|               |               | $D^{*0}\pi^0$ | 1623.35g$^2_H$ |               |               |                |
|               |               | $D^{*+}K^-$  | 1434.74g$^2_H$ |               |               |                |
|               |               | $D^{*0}\eta$ | 305.64g$^2_H$  |               |               |                |
|               |               | Total        | 6580.55g$^2_H$ |               |               |                |
|               |               | $g_{J^P}$    | 0.169         |               |               |                |
| $D_J(3000)^0$ | $2^3P_1$      | $D^{*+}\pi^-$ | 3315.44h$^{12}_S$ |               |               |                |
|               |               | $D^{*0}\pi^0$ | 1669.56h$^{12}_S$ |               |               |                |
|               |               | $D^{*+}K^-$  | 2409.03h$^{12}_S$ |               |               |                |
|               |               | $D^{*0}\eta$ | 515.393h$^{12}_S$ |               |               |                |
|               |               | Total        | 7909.42h$^{12}_S$ |               |               |                |
|               |               | $h_{J^P}$    | 0.154         |               |               |                |
| $D_J(3000)^0$ | $3^3S_1$      | $D^+\pi^-$  | 1493.41g$^{12}_H$ |               |               |                |
|               |               | $D^0\pi^0$  | 753.344g$^{12}_H$ |               |               |                |
|               |               | $D^+K^-$    | 867.203d$^{12}_H$ |               |               |                |
|               |               | $D^0\eta$   | 170.321d$^{12}_H$ |               |               |                |
|               |               | $D^{*+}\pi^-$ | 2338.80g$^{12}_H$ |               |               |                |
|               |               | $D^{*0}\pi^0$ | 1179.62g$^{12}_H$ |               |               |                |
|               |               | $D^{*+}K^-$  | 1116.37g$^{12}_H$ |               |               |                |
|               |               | $D^{*0}\eta$ | 233.034d$^{12}_H$ |               |               |                |
|               |               | Total        | 8152.1g$^{12}_H$ |               |               |                |
|               |               | $g_{J^P}$    | 0.116         |               |               |                |
| $D_J(3000)^0$ | $2^3P_2$      | $D^+\pi^-$  | 2003.50h$^{12}_S$ |               |               |                |
|               |               | $D^0\pi^0$  | 1018.38h$^{12}_S$ |               |               |                |
|               |               | $D^+K^-$    | 782.29h$^{12}_S$ |               |               |                |
|               |               | $D^0\eta$   | 177.739h$^{12}_S$ |               |               |                |
|               |               | $D^{*+}\pi^-$ | 1904.84h$^{12}_S$ |               |               |                |
|               |               | $D^{*0}\pi^0$ | 967.421h$^{12}_S$ |               |               |                |
|               |               | $D^{*+}K^-$  | 537.317h$^{12}_S$ |               |               |                |
|               |               | $D^{*0}\eta$ | 134.843h$^{12}_S$ |               |               |                |
|               |               | Total        | 7526.33k$^{12}_S$ |               |               |                |
|               |               | $h_{J^P}$    | 0.121         |               |               |                |

continued...
TABLE III. The strong decay widths of nonstrange charmed mesons with possible quantum state assignments (in MeV).

| Meson          | $N^{2S+1L_J}$ | Decay mode | LHCb(2016) | LHCb(2015) | LHCb(2013) | BABAR(2010) |
|----------------|---------------|------------|------------|------------|------------|-------------|
| $D^+_s(3000)^0$ | $1^3F_2$      | $D^+\pi^-$ | 1031.35k^2_2 | 1031.35k^2_2 | 1031.35k^2_2 | 1031.35k^2_2 |
| $D^0\pi^0$     |               |            | 527.38k^2_2 | 527.38k^2_2 | 527.38k^2_2 | 527.38k^2_2 |
| $D^+K^-$       |               |            | 380.85k^2_2 | 380.85k^2_2 | 380.85k^2_2 | 380.85k^2_2 |
| $D^0\eta$      |               |            | 101.56k^2_2 | 101.56k^2_2 | 101.56k^2_2 | 101.56k^2_2 |
| $D^{*+}\pi^0$  |               |            | 354.65k^2_2 | 354.65k^2_2 | 354.65k^2_2 | 354.65k^2_2 |
| $D^{*+}K^-$    |               |            | 181.01k^2_2 | 181.01k^2_2 | 181.01k^2_2 | 181.01k^2_2 |
| $D^{*0}\eta$   |               |            | 28.13k^2_2  | 28.13k^2_2  | 28.13k^2_2  | 28.13k^2_2  |
| Total          |               |            | 1043.40k^2_2 | 1043.40k^2_2 | 1043.40k^2_2 | 1043.40k^2_2 |
| $D^+_s(3000)^0$ | $1^3F_4$      | $D^+\pi^-$ | 2414.64k^2_2 | 2414.64k^2_2 | 2414.64k^2_2 | 2414.64k^2_2 |
| $D^0\pi^0$     |               |            | 1246.21k^2_2 | 1246.21k^2_2 | 1246.21k^2_2 | 1246.21k^2_2 |
| $D^+K^-$       |               |            | 426.29k^2_2 | 426.29k^2_2 | 426.29k^2_2 | 426.29k^2_2 |
| $D^0\eta$      |               |            | 129.61k^2_2 | 129.61k^2_2 | 129.61k^2_2 | 129.61k^2_2 |
| $D^{*+}\pi^0$  |               |            | 645.51k^2_2 | 645.51k^2_2 | 645.51k^2_2 | 645.51k^2_2 |
| $D^{*+}K^-$    |               |            | 123.48k^2_2 | 123.48k^2_2 | 123.48k^2_2 | 123.48k^2_2 |
| $D^{*0}\eta$   |               |            | 44.79k^2_2  | 44.79k^2_2  | 44.79k^2_2  | 44.79k^2_2  |
| Total          |               |            | 6284.05k^2_2 | 6284.05k^2_2 | 6284.05k^2_2 | 6284.05k^2_2 |
| $D^+_s(3000)^0$ | $3^3P_2$      | $D^+\pi^-$ | 3844.03h^2_2 | 3844.03h^2_2 | 3844.03h^2_2 | 3844.03h^2_2 |
| $D^0\pi^0$     |               |            | 1946.41h^2_2 | 1946.41h^2_2 | 1946.41h^2_2 | 1946.41h^2_2 |
| $D^+K^-$       |               |            | 1977.59h^2_2 | 1977.59h^2_2 | 1977.59h^2_2 | 1977.59h^2_2 |
| $D^0\eta$      |               |            | 412.51h^2_2 | 412.51h^2_2 | 412.51h^2_2 | 412.51h^2_2 |
| $D^{*+}\pi^0$  |               |            | 4062.86h^2_2 | 4062.86h^2_2 | 4062.86h^2_2 | 4062.86h^2_2 |
| $D^{*+}K^-$    |               |            | 2055.30h^2_2 | 2055.30h^2_2 | 2055.30h^2_2 | 2055.30h^2_2 |
| $D^{*0}\eta$   |               |            | 386.09h^2_2 | 386.09h^2_2 | 386.09h^2_2 | 386.09h^2_2 |
| Total          |               |            | 16435.2h^2_2 | 16435.2h^2_2 | 16435.2h^2_2 | 16435.2h^2_2 |
| $D^+_s(3000)^0$ | $1^3F_2$      | $D^+\pi^-$ | 2622.01k^2_2 | 2622.01k^2_2 | 2622.01k^2_2 | 2622.01k^2_2 |
| $D^0\pi^0$     |               |            | 1334.12k^2_2 | 1334.12k^2_2 | 1334.12k^2_2 | 1334.12k^2_2 |
| $D^+K^-$       |               |            | 1280.27k^2_2 | 1280.27k^2_2 | 1280.27k^2_2 | 1280.27k^2_2 |
| $D^0\eta$      |               |            | 306.43k^2_2 | 306.43k^2_2 | 306.43k^2_2 | 306.43k^2_2 |
| $D^{*+}\pi^0$  |               |            | 1043.40k^2_2 | 1043.40k^2_2 | 1043.40k^2_2 | 1043.40k^2_2 |
| $D^{*+}K^-$    |               |            | 529.91k^2_2 | 529.91k^2_2 | 529.91k^2_2 | 529.91k^2_2 |
| $D^{*0}\eta$   |               |            | 108.87k^2_2 | 108.87k^2_2 | 108.87k^2_2 | 108.87k^2_2 |
| Total          |               |            | 7655.84k^2_2 | 7655.84k^2_2 | 7655.84k^2_2 | 7655.84k^2_2 |
FIG. 1. Strong decay widths of $D_1(2420)^0$ (in MeV) changing with the square of the coupling $h_f^2$ in HQET. The masses of $D_1(2420)^0$ observed (in the decay mode $D^* \pi^-$) by LHCb(2013) [3] (upper) and BABAR(2010) [4] (lower) are used.

$$S_a = \frac{1 + \gamma}{2} \left[ P_{1a}^\mu \gamma^\mu \gamma_5 - P_{0a}^\mu \right],$$  \hspace{1cm} (2)

$$T^\mu_a = \frac{1 + \gamma}{2} \left\{ P_{2a}^{\mu \nu} \gamma_\nu - P_{1a}^{\mu \nu} \sqrt{\frac{3}{2}} \gamma_5 \left[ g_{\mu \nu} - \gamma^\nu (\gamma^\mu - v^\mu) \right] \right\},$$  \hspace{1cm} (3)

$$X^\mu_a = \frac{1 + \gamma}{2} \left\{ P_{3a}^{\mu \nu \sigma} \gamma_\sigma - P_{4a}^{\mu \nu \sigma} \sqrt{\frac{5}{3}} \gamma_5 \right\},$$  \hspace{1cm} (4)

$$Y^\mu_{\alpha \beta} = \frac{1 + \gamma}{2} \left\{ P_{5a}^{\mu \nu \sigma} \gamma_\alpha - P_{6a}^{\mu \nu \sigma} \sqrt{\frac{5}{3}} \right\} \left\{ g_{\alpha \beta}^g - g_{\alpha \beta}^g \gamma_5 (\gamma^\mu - v^\mu) \right\} - \frac{g_{\alpha \beta}^g \gamma_5 (\gamma^\mu - v^\mu)}{5},$$  \hspace{1cm} (5)

FIG. 2. Strong decay widths of $D_2(2460)^0$ (in MeV) changing with the square of the coupling $h_f^2$ in HQET. The masses of $D_2(2460)^0$ observed (in the decay mode $D^+ \pi^-$) by LHCb(2016) [1] (upper), LHCb(2013) [3] (middle) and BABAR(2010) [4] (lower) are used.
FIG. 3. Strong decay widths of \(D_2^*(2460)^+\) (left) and \(D_2^*(2460)^-\) (right) (in MeV) changing with the square of the coupling \(h_T^2\) in HQET. The mass of \(D_2^*(2460)^+\) observed (in the decay mode \(D_0^+\pi^+\)) by LHCb(2013) \[3\] (upper) and the mass of \(D_2^*(2460)^-\) observed (in the decay mode \(D_0^-\pi^-\)) by LHCb(2015) \[2\] (lower) are used.

\[Z^\mu_\alpha = \frac{1}{2} \left[ P^\mu_{3a} \gamma_\alpha \gamma_5 - P_{2a}^\alpha \gamma_5 \right] \left( \frac{g_\alpha^\mu g_\beta^\nu - g_\beta^\mu g_\alpha^\nu}{5} - g_\alpha^\mu g_\beta^\nu \gamma_\alpha \gamma_\beta \frac{(\gamma^\mu + \nu\nu)}{5} \right) \] (6)

\[R^\mu_\nu = \frac{1}{2} \left\{ P_{4a}^{\mu\nu\rho\sigma} \gamma_\alpha \gamma_\beta \gamma_\gamma - P_{2a}^{\alpha\beta} \sqrt{\frac{7}{4}} \left[ g_\alpha^\mu g_\beta^\nu - g_\beta^\mu g_\alpha^\nu \right] \right\} \] (7)

where \(a = u, d\) or \(s\) is the \(SU(3)\) light quark flavor representation and \(\nu\) gives the meson four velocity and is conserved in strong interactions. The heavy meson field operators \(P\) and \(P^*\) (see Eqs. (1) to (7)) contain a

FIG. 4. Strong decay widths of \(D(2550)^0\) (in MeV) changing with the square of the coupling \(g_H^2\) in HQET. The masses of \(D(2550)^0\) observed (in the decay mode \(D^*+\pi^-\)) by LHCb(2013) \[3\] (upper) and BABAR(2010) \[4\] (lower) are used.

\[Z^\mu_\alpha = \frac{1}{2} \left[ P^\mu_{3a} \gamma_\alpha \gamma_5 - P_{2a}^\alpha \gamma_5 \right] \left( \frac{g_\alpha^\mu g_\beta^\nu - g_\beta^\mu g_\alpha^\nu}{5} - g_\alpha^\mu g_\beta^\nu \gamma_\alpha \gamma_\beta \frac{(\gamma^\mu + \nu\nu)}{5} \right) \] (6)

\[R^\mu_\nu = \frac{1}{2} \left\{ P_{4a}^{\mu\nu\rho\sigma} \gamma_\alpha \gamma_\beta \gamma_\gamma - P_{2a}^{\alpha\beta} \sqrt{\frac{7}{4}} \left[ g_\alpha^\mu g_\beta^\nu - g_\beta^\mu g_\alpha^\nu \right] \right\} \] (7)

where \(a = u, d\) or \(s\) is the \(SU(3)\) light quark flavor representation and \(\nu\) gives the meson four velocity and is conserved in strong interactions. The heavy meson field operators \(P\) and \(P^*\) (see Eqs. (1) to (7)) contain a
FIG. 5. Strong decay widths of $D^*_J(2600)^0$ (in MeV) changing with the square of the coupling $g_{H}^{2}$ in HQET. The masses of $D^*_J(2600)^0$ observed (in the decay mode $D^{*+}\pi^-$) by LHCb(2016) 1 (upper left) and BABAR(2010) 4 (upper right), and (in the decay mode $D^{*+}\pi^-$) by LHCb(2013) 3 (lower) are used.

The strong decays take place with the emission of light pseudoscalar octet mesons. We write the matrix $M$ of light pseudoscalar mesons described by the fields $\xi = e^{i \phi} \frac{f_{\pi}}{\sqrt{m_{Q}}} \sqrt{m_{Q}}$ having a mass dimension $\frac{3}{2}$, which annihilate the mesons with four-velocity $\nu$. Eq. (1) is for $s$ wave mesons; Eq. (2) and (3) for $p$ wave mesons; Eq. (4) and (5) for $d$ wave mesons, and Eq. (6) and (7) for $f$ wave mesons. The strong decays take place with the emission of light pseudoscalar octet mesons. We write the matrix $M$ of light pseudoscalar mesons described by the fields $\xi = e^{i \phi} \frac{f_{\pi}}{\sqrt{m_{Q}}} \sqrt{m_{Q}}$ having a mass dimension $\frac{3}{2}$, which annihilate the mesons with four-velocity $\nu$. Eq. (1) is for $s$ wave mesons; Eq. (2) and (3) for $p$ wave mesons; Eq. (4) and (5) for $d$ wave mesons, and Eq. (6) and (7) for $f$ wave mesons. The strong decays take place with the emission of light pseudoscalar octet mesons. We write the matrix $M$ of light pseudoscalar mesons described by the fields $\xi = e^{i \phi} \frac{f_{\pi}}{\sqrt{m_{Q}}} \sqrt{m_{Q}}$ as,

$$M = \left( \begin{array}{ccc} K^+ & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ \pi^- & K^0 & K^0 \\ K^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & -\frac{1}{2}\eta \end{array} \right) \right) \ (8)$$

where $f_{\pi} = 130.2$ MeV. Refs. 31, 36 also study the strong decays of heavy mesons along with the light vector.
mesons ($\rho$, $\omega$, $K$, and $\phi$). The effective heavy meson chiral Lagrangians $\mathcal{L}_H$, $\mathcal{L}_S$, $\mathcal{L}_T$, $\mathcal{L}_X$, $\mathcal{L}_Y$, $\mathcal{L}_Z$ and $\mathcal{L}_R$ describe the two body strong interactions by an exchange of light pseudoscalar mesons, are taken from (32),

\[
\mathcal{L}_H = g_H Tr[\bar{H}_a H_b \gamma_\mu \gamma_5 A_\mu^a], 
\]

\[
\mathcal{L}_S = g_S Tr[\bar{H}_a S_b \gamma_\mu \gamma_5 A_\mu^a] + H.C.,
\]

\[
\mathcal{L}_T = \frac{g_T}{\Lambda} Tr[\bar{H}_a T_0^a (iD_\mu A^\mu + iD^\mu A_\mu)]_{ba} \gamma_5] + H.C.,
\]

\[
\mathcal{L}_X = \frac{g_X}{\Lambda} Tr[\bar{H}_a X_0^a (iD_\mu A^\mu + iD^\mu A_\mu)]_{ba} \gamma_5] + H.C.,
\]
FIG. 7. Strong decay widths of $D^*_3(2750)^0$ (in MeV) changing with the square of the coupling $k_Y^2$ in HQET. The masses of $D^*_3(2750)^0$ observed (in the decay mode $D^+\pi^-$) by LHCb(2016) [1] (upper left), LHCb(2013) [3] (upper right) and BABAR(2010) [4] (lower) are used.

where vector and axial-vector operators are, 
\[ V_{\mu\alpha b} = \frac{1}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)_{\alpha b}, \quad (16) \]
\[ A_{\mu\alpha b} = i \frac{1}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)_{\alpha b}; \quad (17) \]
and the operator, \( D_{\mu\alpha b} = -\delta_{\alpha b} \partial_\mu + V_{\mu\alpha b} \). Also here, \((D_\mu, D_\nu) = D_\mu D_\nu + D_\nu D_\mu\) and \((D_\mu, D_\nu, D_\rho) = D_\mu D_\nu D_\rho + D_\mu D_\rho D_\nu + D_\nu D_\rho D_\mu + D_\nu D_\mu D_\rho + D_\rho D_\mu D_\nu + D_\rho D_\nu D_\mu\). \( \Lambda_\chi \) is the chiral symmetry breaking scale and is fixed to 1 GeV.

The mass parameters $\delta m_S = m_S - m_H$, $\delta m_T = m_T - m_H$, $\delta m_X = m_X - m_H$, $\delta m_Y = m_Y - m_H$, $\delta m_Z = m_Z - m_H$, and $\delta m_R = m_R - m_H$ represent the mass splittings between the higher and the lower mass doublets described by the field $H_a$ (see Eq. (11)). The strong running coupling constants $g_H$, $h_S$, $h_T$, $k_X$, $k_Y = k_Y^1 + k_Y^2$, $k_Z = k_Z^1 + k_Z^2$, and $k_R = k_R^1 + k_R^2$ can be fitted to the experimental data. For $n = 2$ the coupling constants are denoted by $g_H^1$, $h_S^1$, $h_T^1$, $k_X^1$, $k_Y^1$, $k_Z^1$, and $k_R^1$ and for $n = 3$ they are $g_H^2$, $h_S^2$, $h_T^2$, $k_X^2$, $k_Y^2$, $k_Z^2$, and $k_R^2$. $g_H$ (in Eq. (9))
FIG. 8. Strong decay widths of $D_3^* (2750)$ (in MeV) changing with the square of the coupling $k_2^2$ in HQET. The masses of $D_3^* (2750)$ observed (in the decay mode $D_3^* + \pi^-$) by LHCb (2013) [3] (upper) and BABAR (2010) [4] (lower) are used.

controls the $s$ wave decays, $h_S$ and $h_T$ (in Eqs. [10] and [11]) are governs the $p$ wave decays, $k_X$ and $k_Y$ (in Eqs. [12] and [13]) describe the $d$ wave decays, and $k_Z$ and $k_R$ (in Eqs. [14] and [15]) are responsible for the $f$ wave decays. Such chiral Lagrangians can determine the expressions of strong decays of heavy-light mesons into the lower mass charged and neutral $D^{(*)}$ and $D_S^{(*)}$ mesons along with the light pseudoscalar mesons ($\pi$, $\eta$ and $K$).

I. Decaying $s$ wave doublet ($P, P^*$) or ($P^\dagger, P^{\dagger*}$) or ($P^\dagger\dagger, P^{\dagger\dagger*}$):

\[
\Gamma(P^\dagger \to P^* P) = C_P \frac{g^2_H}{2 \pi f^2_\pi} P_1^* |\vec{P}_P|^3
\]

\[
\Gamma(P^\dagger* \to P^* P) = C_P \frac{g^2_H}{6 \pi f^2_\pi} P_1^* |\vec{P}_P|^3
\]

\[
\Gamma(P^\dagger \to P^* P) = C_P \frac{g^2_H}{2 \pi f^2_\pi} P_1^* |\vec{P}_P|^3
\]

\[
\Gamma(P^\dagger* \to P^* P) = C_P \frac{g^2_H}{6 \pi f^2_\pi} P_1^* |\vec{P}_P|^3
\]

\[
\Gamma(P^\dagger \to P^* P) = C_P \frac{g^2_H}{3 \pi f^2_\pi} P_1^* |\vec{P}_P|^3
\]

\[
\Gamma(P^\dagger* \to P^* P) = C_P \frac{g^2_H}{3 \pi f^2_\pi} P_1^* |\vec{P}_P|^3
\]

TABLE VIII. The masses of nonstrange charmed meson states (in GeV) lying on the $1^3 S_0$, $1^3 S_1$ and $1^3 P_2$ Regge lines in $(n_r, M^2)$ plane.

| State | $1^3 S_0$ | $1^3 S_1$ | $1^3 P_2$ |
|-------|-----------|-----------|-----------|
| Present | 1.865 [5] 2.564 [5] 3.110 | 2.007 [5] 2.623 [5] 3.120 | 2.460 [5] 3.008 [5] 3.470 |
| Ref. [8] | 1.877 2.581 3.068 | 2.041 2.643 3.110 | 2.502 2.957 3.353 |
| Ref. [9] | 1.874 2.583 3.062 | 2.038 2.645 3.111 | 2.501 2.957 3.340 |
| Ref. [11] | 1.871 2.581 3.062 | 2.010 2.632 3.096 | 2.460 3.012 3.407 |
| Ref. [12] | 1.868 2.589 2.775 | 2.005 2.692 3.226 | 2.460 3.035 3.340 |
FIG. 9. Strong decay widths of $D_0^*(2750)^+$ (upper) and $D_0^*(2750)^-$ (lower) (in MeV) changing with the square of the coupling $k_Y^2$ in HQET. The masses of $D_0^*(2750)$ observed (in the decay mode $D^0\pi^+$) by LHCb(2013) [3] (upper left) and BABAR(2010) [4] (upper right), and (in the decay mode $D^0\pi^-$) by LHCb(2015) [2] (lower) are used.

\[
\Gamma(P_1^{\dagger}\rightarrow P^*P) = C_P \frac{2h_{S}}{3\pi f_{\pi}^{2}} \frac{P^*}{P_1} |\vec{P}_P|^5
\]  

(24)

\[
\Gamma(P_1 \rightarrow P^*P) = C_P \frac{2h_{S}^2}{3\pi f_{\pi}^{2}} \frac{P^*}{P_1} |\vec{P}_P|^5
\]  

(25)

\[
\Gamma(P_2^* \rightarrow PP) = C_P \frac{4h_{I}^2}{15\pi f_{\pi}^{2}} \frac{P^*}{P_2} |\vec{P}_P|^5
\]  

(26)

\[
\Gamma(P_2^* \rightarrow P^*P) = C_P \frac{2h_{I}^2}{5\pi f_{\pi}^{2}} \frac{P^*}{P_2} |\vec{P}_P|^5
\]  

(27)

\[
\Gamma(P_2^{\dagger} \rightarrow P^*P) = C_P \frac{4h_{I}^2}{15\pi f_{\pi}^{2}} \frac{P^*}{P_2} |\vec{P}_P|^5
\]  

(28)

\[
\Gamma(P_2^{\dagger} \rightarrow PP) = C_P \frac{2h_{I}^2}{5\pi f_{\pi}^{2}} \frac{P^*}{P_2} |\vec{P}_P|^5
\]  

(29)
TABLE IX. Strong decay widths (in MeV), ratio and branching fraction of nonstrange charmed mesons lying on the Regge lines with possible quantum number assignments.

| $\mathcal{N}^{2S+1}L_J$ | Decay mode | Decay width | Ratio | Branching fraction |
|-----------------|------------|-------------|-------|--------------------|
| $1^1 D_2$       | $D^{*+}\pi^-$ | 1772.11k$^2_\chi$ | 1     | 52.3               |
|                 | $D^{*0}\pi^o$ | 901.22k$^2_\chi$ | 0.51  | 26.6               |
|                 | $D^{*+}K^-$  | 553.089k$^2_\chi$ | 0.31  | 16.32              |
|                 | $D^{*0}\eta$ | 160.782k$^2_\chi$ | 0.09  | 4.74               |
|                 | Total        | 3388.20k$^2_\chi$ |       |                    |
| $1^3 D_3$       | $D^{*+}\pi^-$ | 290.925k$^2_\gamma$ | 1     | 37.89              |
|                 | $D^{*0}\pi^o$ | 149.953k$^2_\gamma$ | 0.52  | 19.53              |
|                 | $D^{*+}K^-$  | 45.1974k$^2_\gamma$ | 0.16  | 5.89               |
|                 | $D^{*0}\eta$ | 13.6889k$^2_\gamma$ | 0.05  | 4.7                |
|                 | $D^{*+}\pi^0$ | 167.347k$^2_\gamma$ | 0.58  | 21.79              |
|                 | $D^{*0}\pi^0$ | 86.1774k$^2_\gamma$ | 0.3   | 29.62              |
|                 | $D^{*+}K^-$  | 10.3364k$^2_\gamma$ | 0.04  | 3.55               |
|                 | $D^{*0}\eta$ | 4.24088k$^2_\gamma$ | 0.01  | 1.46               |
|                 | Total        | 767.860k$^2_\gamma$ |       |                    |
| $3^1 S_0$       | $D^{*+}\pi^-$ | 4389.38g$^2_H$ | 1     | 46.14              |
|                 | $D^{*0}\pi^o$ | 2210.97g$^2_H$ | 0.5   | 23.24              |
|                 | $D^{*+}K^-$  | 2427.41g$^2_H$ | 0.55  | 25.52              |
|                 | $D^{*0}\eta$ | 484.987g$^2_H$ | 0.11  | 11.05              |
|                 | Total        | 9512.75g$^2_H$ |       |                    |
| $3^1 S_1$       | $D^{+}\pi^-$ | 1837.20g$^2_H$ | 1     | 17.23              |
|                 | $D^{0}\pi^o$ | 925.499g$^2_H$ | 0.5   | 8.68               |
|                 | $D^{+}K^-$  | 1180.56g$^2_H$ | 0.64  | 11.07              |
|                 | $D^{0}\eta$ | 224.751g$^2_H$ | 0.12  | 2.11               |
|                 | $D^{*+}\pi^-$ | 2987.19g$^2_H$ | 1.62  | 28.01              |
|                 | $D^{*0}\pi^0$ | 1504.51g$^2_H$ | 0.82  | 14.11              |
|                 | $D^{*+}K^-$  | 1671.63g$^2_H$ | 0.91  | 15.67              |
|                 | $D^{*0}\eta$ | 332.810g$^2_H$ | 0.18  | 3.12               |
|                 | Total        | 10664.15g$^2_H$ |       |                    |
| $1^1 F_3$       | $D^{*+}\pi^-$ | 3211.24k$^2_Z$ | 1     | 48.85              |
|                 | $D^{*0}\pi^o$ | 1629.54k$^2_Z$ | 0.51  | 24.79              |
|                 | $D^{*+}K^-$  | 1384.61k$^2_Z$ | 0.43  | 21.06              |
|                 | $D^{*0}\eta$ | 348.403k$^2_Z$ | 0.11  | 5.3                |
|                 | Total        | 6573.79k$^2_Z$ |       |                    |
| $1^3 F_4$       | $D^{+}\pi^-$ | 6784.48k$^2_Z$ | 1     | 34.1               |
|                 | $D^{0}\pi^o$ | 3480.78k$^2_Z$ | 0.51  | 17.49              |
|                 | $D^{+}K^-$  | 1841.32k$^2_Z$ | 0.27  | 9.25               |
|                 | $D^{0}\eta$ | 490.200k$^2_Z$ | 0.07  | 2.46               |
|                 | $D^{*+}\pi^-$ | 4145.64k$^2_Z$ | 0.61  | 20.84              |
|                 | $D^{*0}\pi^0$ | 2121.85k$^2_Z$ | 0.31  | 10.66              |
|                 | $D^{*+}K^-$  | 794.006k$^2_Z$ | 0.12  | 3.99               |
|                 | $D^{*0}\eta$ | 236.237k$^2_Z$ | 0.03  | 1.19               |
|                 | Total        | 19806.51k$^2_Z$ |       |                    |

continued...

\[ \Gamma(P_2^{1*} \rightarrow P \pi) = C_P \frac{4h_1^{12}}{15\pi f_{\pi}^{2} P_2^{3}} |\vec{P}_F|^{5} \]  \hspace{1cm} (30)

\[ \Gamma(P_2^{1*} \rightarrow P^* \pi) = C_P \frac{2h_1^{12}}{5\pi f_{\pi}^{2} P_2^{3}} |\vec{P}_{P^*}|^{5} \]  \hspace{1cm} (31)

III. Decaying d wave doublets ($P_1^{*}, P_2$) and ($P_2', P_3'$):

\[ \Gamma(P_2 \rightarrow P^* \pi) = C_P \frac{2k_{x}^{3}}{3\pi f_{\pi}^{2} P_2^{2}} |m_{\pi}^{2} + |\vec{P}_{P^*}|^{2}||\vec{P}_{P}||^{3} \]  \hspace{1cm} (32)
TABLE IX. Strong decay widths (in MeV), ratio and branching fraction of nonstrange charmed mesons lying on the Regge lines with possible quantum number assignments.

| $N^{2S+1}L_J$ | Decay mode | Decay width | Ratio | Branching fraction |
|--------------|------------|-------------|-------|-------------------|
| $2^3D_3$     | $D^+\pi^-$ | $3130.22k_{12}^2$ | 1     | 26.65             |
|              | $D^0\pi^0$ | $1590.47k_{12}^2$ | 0.51  | 13.54             |
|              | $D^+_sK^-$ | $1436.88k_{12}^2$ | 0.46  | 12.23             |
|              | $D^0\eta$  | $315.14k_{12}^2$  | 0.1   | 2.68              |
|              | $D^{*+}\pi^-$ | $2668.36k_{12}^2$ | 0.85  | 22.72             |
|              | $D^{*0}\pi^0$ | $1353.63k_{12}^2$ | 0.43  | 11.52             |
|              | $D^{*+}K^-$ | $1012.98k_{12}^2$ | 0.32  | 8.62              |
|              | $D^{*0}\eta$ | $238.64k_{12}^2$  | 0.08  | 2.03              |
|              | Total       | $11746.32k_{12}^2$ |       |                   |
| $3^3P_2$     | $D^+\pi^-$ | $7478.96k_{12}^2$ | 1     | 20.93             |
|              | $D^0\pi^0$ | $3774.68k_{12}^2$ | 0.5   | 10.56             |
|              | $D^+_sK^-$ | $4684.67k_{12}^2$ | 0.63  | 13.11             |
|              | $D^0\eta$  | $916.58k_{12}^2$  | 0.12  | 2.69              |
|              | $D^{*+}\pi^-$ | $8640.15k_{12}^2$ | 1.16  | 24.18             |
|              | $D^{*0}\pi^0$ | $4357.18k_{12}^2$ | 0.58  | 12.19             |
|              | $D^{*+}K^-$ | $4886.89k_{12}^2$ | 0.65  | 13.68             |
|              | $D^{*0}\eta$ | $993.32k_{12}^2$  | 0.13  | 2.78              |
|              | Total       | $35732.4k_{12}^2$ |       |                   |
| $2^3F_4$     | $D^+\pi^-$ | $64151k_{12}^2$   | 1     | 26.75             |
|              | $D^0\pi^0$ | $32588k_{12}^2$   | 0.51  | 13.59             |
|              | $D^+_sK^-$ | $31378.1k_{12}^2$ | 0.49  | 13.08             |
|              | $D^0\eta$  | $6879.94k_{12}^2$ | 0.11  | 2.87              |
|              | $D^{*+}\pi^-$ | $51500.9k_{12}^2$ | 0.8   | 21.48             |
|              | $D^{*0}\pi^0$ | $26110.5k_{12}^2$ | 0.41  | 10.89             |
|              | $D^{*+}K^-$ | $22101.5k_{12}^2$ | 0.34  | 9.21              |
|              | $D^{*0}\eta$ | $5103.52k_{12}^2$ | 0.08  | 2.13              |
|              | Total       | $239813k_{12}^2$  |       |                   |

\[\Gamma(P_2^s \rightarrow P^*P) = C_\Gamma \frac{4k^2_{2Y}}{15\pi f_{\pi}^2 P_2^s} |\vec{P}_P|^7 \] \hspace{1cm} (33)

\[\Gamma(P_3^s \rightarrow PP) = C_\Gamma \frac{4k^2_{2Y}}{35\pi f_{\pi}^2 P_3^s} |\vec{P}_P|^7 \] \hspace{1cm} (34)

\[\Gamma(P_3^s \rightarrow P^*P) = C_\Gamma \frac{16k^2_{2Y}}{105\pi f_{\pi}^2 P_3^s} |\vec{P}_P|^7 \] \hspace{1cm} (35)

\[\Gamma(P_3^{1s} \rightarrow PP) = C_\Gamma \frac{4k^2_{1Y}}{35\pi f_{\pi}^2 P_3^{1s}} |\vec{P}_P|^7 \] \hspace{1cm} (36)

\[\Gamma(P_3^{1s} \rightarrow P^*P) = C_\Gamma \frac{16k^2_{1Y}}{105\pi f_{\pi}^2 P_3^{1s}} |\vec{P}_P|^7 \] \hspace{1cm} (37)

IV. Decaying f wave doublets ($P_2^s, P_3$) and ($P_3', P_3^*$):

\[\Gamma(P_2'^s \rightarrow PP) = C_\Gamma \frac{4k^2_{2Y}}{25\pi f_{\pi}^2 P_2^s} |m_P^2 + |\vec{P}_P|^2||\vec{P}_P|^5 \] \hspace{1cm} (38)

\[\Gamma(P_3'^s \rightarrow PP) = C_\Gamma \frac{4k^2_{2Y}}{75\pi f_{\pi}^2 P_3'^s} |\vec{P}_P|^9 \] \hspace{1cm} (39)

\[\Gamma(P_3'^s \rightarrow P^*P) = C_\Gamma \frac{8k^2_{2Y}}{75\pi f_{\pi}^2 P_3'^s} |m_P^2 + |\vec{P}_P|^2||\vec{P}_P|^5 \] \hspace{1cm} (40)

\[\Gamma(P_3^{1s} \rightarrow PP) = C_\Gamma \frac{4k^2_{1Y}}{35\pi f_{\pi}^2 P_3^{1s}} |\vec{P}_P|^9 \] \hspace{1cm} (41)

\[\Gamma(P_3^{1s} \rightarrow P^*P) = C_\Gamma \frac{16k^2_{1Y}}{75\pi f_{\pi}^2 P_3^{1s}} |\vec{P}_P|^9 \] \hspace{1cm} (42)

\[\Gamma(P_4'^s \rightarrow PP) = C_\Gamma \frac{16k^2_{2Y}}{35\pi f_{\pi}^2 P_4'^s} |\vec{P}_P|^9 \] \hspace{1cm} (43)

\[\Gamma(P_4'^s \rightarrow P^*P) = C_\Gamma \frac{4k^2_{1Y}}{75\pi f_{\pi}^2 P_4'^s} |\vec{P}_P|^9 \] \hspace{1cm} (44)
For the decay mode $P_a \rightarrow P_b + P$ we have $|\mathcal{F}| = \sqrt{m_{P_a}^2 + m_{P_b}^2 + m_{P}^2 - 2m_{P_a} m_{P_b} - 2m_{P_a} m_{P} - 2m_{P_b} m_{P}} / \sqrt{2m_{P_a}}$; where $m_{P_a}$, $m_{P_b}$ and $m_{P}$ are their respective masses. The coefficients $P$ of the light pseudoscalar mesons are: $C_{\pi^\pm}, C_{K^\pm} = 1$, $C_{\rho} = \frac{1}{3}$ and $C_{\sigma} = \frac{1}{2}$. The masses of the light pseudoscalar mesons and the ground state charmed mesons are taken from PDG-2018 [5]. $M_{\pi^\pm} = 139.57061$ MeV, $M_{\rho} = 134.9770$ MeV, $M_{K^\pm} = 493.677$ MeV, $M_{K^0} = 497.611$ MeV, $M_{\eta} = 547.862$ MeV, $M_{D^\pm} = 1869.65$, $M_{D^0} = 1864.84$ MeV, $M_{D_s^\pm} = 2010.26$ MeV, $M_{D_s^0} = 2006.85$ MeV, $M_{D_{s1}^\pm} = 1969.0$ MeV, $M_{D_{s1}^0} = 2112.2$ MeV. In the heavy quark mass limit, the spin and flavor violations of order $\frac{1}{m_h^2}$ are not taken into the consideration in this present study to avoid introducing new unknown coupling constants. The strong decay widths can provide some useful informations and are used for the classification of various mesonic states according to their total spin and parity. Also the ratio and the branching fractions among the decay widths, independent of the coupling constants, can help to identify the heavy mesons.

### III. RESULTS AND DISCUSSION

Using the Eqs. (18) to (44), the strong decay rates of nonstrange singly charmed mesons $(D_s^*(2460), D(2550)^0, D_s^*(2600)^0, D(2740)^0, D_s^*(2750)^0, D_s(3000)^0, D_s^*(3000)^0$ and $D_s(3000)^0$ observed by the experimental Collaborations LHCb [1-3] and BABAR [4] are computed. That are presented in Table III in terms of the ratio of the strong decays rates that are changing with respect to the square of the couplings with their respective experimentally observed decay widths listed in Table I. We determine the strong coupling constants which are presented in Table III.

The branching ratios avoid the unknown hadronic couplings and are compared with experimental observations where available. The branching ratio,

$$BR_{D_s^*(2460)^0} = \frac{\Gamma(D_s^*(2460)^0 \rightarrow D_s^+ \pi^-)}{\Gamma(D_s^*(2460)^0 \rightarrow D^+ \pi^-)} \approx 2.3,$$

calculated from Ref. [1, 3] and [4]. It is in good agreement with the measurements of CLEO Collaboration 2.3 ± 0.8 [41], underestimated to ZEUS 2.8 ± 0.8 [41] and overestimated to BABAR 1.47 ± 0.03 [4] and ZEUS 1.4 ± 0.3 [42]. The ratio,

$$BR_{D_s^*(2460)^+} = \frac{\Gamma(D_s^*(2460)^+ \rightarrow D_s^0 \pi^+)}{\Gamma(D_s^*(2460)^+ \rightarrow D^0 \pi^+)} \approx 0.70 \text{ from Refs. [1, 2, 4]},$$

and is close to 0.62 ± 0.02 of $BABAR$ measurement [42]. The branching ratio,

$$BR_{D_s^*(2460)^+} = \frac{\Gamma(D_s^*(2460)^+ \rightarrow D_s^+ \pi^-)}{\Gamma(D_s^*(2460)^+ \rightarrow D^+ \pi^-)} \approx 2.3,$$

from [2], which is nearer to 1.9 ± 1.1 of CLEO measurement [41] and underestimated to ZEUS 1.1 ± 0.4 [42]. And, also the ratio

$$BR_{D_s^*(2460)^+} = \frac{\Gamma(D_s^*(2460)^+ \rightarrow D_s^0 \pi^+)}{\Gamma(D_s^*(2460)^+ \rightarrow D^0 \pi^+)}$$
FIG. 11. Strong decay widths of $D_J^*(3000)^0$ as $3^3S_1$ (upper left), $2^3P_2$ (upper right), $1^3F_2$ (lower left) and $1^3F_4$ (lower right) changing with the square of the couplings $g_{H}^{12}$, $h_{T}^{12}$, $k_{Z}^{2}$ and $k_{R}^{2}$ respectively. The mass of $D_J^*(3000)^0$ observed (in the decay mode $D^+\pi^-$) by LHCb(2013) [3] is used. Therefore, the charmed mesons $D_J^*(2460)$ and $D_J^*(2750)$ belonging to $1^3P_2$ and $1^3D_3$ are dominant in $D\pi$ decay mode and, $D_J^*(2600)$ with $2^3S_1$ dominant in $D^+\pi^-$ decay. That are in accessible with the experimental observations. Moreover, the $D_1(2420)$, $D(2550)$ and $D(2740)$ are found to be spin partners of $D_J^*(2460)$, $D_J^*(2600)$ and $D_J^*(2750)$ respectively. So we write,

\[
(D_1(2420), D_J^*(2460)) = (1^+, 2^+)^{3/2} = (1^1P_1, 1^3P_2), \tag{45}
\]

\[
BR_{D_J^*(2600)^0} = \frac{\Gamma(D_J^*(2600)^0 \to D^+\pi^-)}{\Gamma(D_J^*(2600)^0 \to D^+\pi^-)} \approx 0.8
\]

\[
BR_{D_J^*(2750)^0} = \frac{\Gamma(D_J^*(2750)^0 \to D^+\pi^-)}{\Gamma(D_J^*(2750)^0 \to D^+\pi^-)} \approx 1.9
\]

calculated from Ref. [1], [2] and [4], which are overestimated to the BABAR measurements $BR_{D_J^*(2600)^0} = 0.32 \pm 0.02 \pm 0.09$ and $BR_{D_J^*(2750)^0} = 0.42 \pm 0.05 \pm 0.11$ [4].

\[\approx 0.7 \text{ from } [2] \text{ close to Ref. } [4].\]
FIG. 12. Strong decay widths of $D^{*+}(3000)^0$ as $3^3P_2$ (upper) and $1^3F_2$ (lower) changing with the square of the couplings $h_T^{2}$ and $k_Z^{2}$ respectively. The mass of $D^{*+}(3000)^0$ observed (in the decay mode $D^+\pi^-$) by LHCb(2016) [1] is used.

\[
(D(2550), D_J^+(2600)) = (0^-, 1^-)_{\frac{1}{2}} = (2^1S_0, 2^3S_1),
\]

\[
(D(2740), D_J^+(2750)) = (2^-, 3^-)_{\frac{1}{2}} = (1^3D_2, 1^3D_3).
\]

The mass difference $M_{D_J^+(3000)^0} - M_{D_J^+(3000)^0} \approx 36$ MeV. They might be from the same wave family. Experimentally, $D_J^+(3000)^0$ is measured with unnatural parity and $D_J^+(3000)^0$ with natural parity. So they can have an isodoublet state either $(0^-, 1^-)_{\frac{1}{2}}$ or $(1^+, 2^+)_{\frac{3}{2}}$. The

FIG. 13. Regge trajectory of nonstrange charmed mesons in $(J, M^2)$ plane with natural parity (upper) unnatural parity (lower).

FIG. 14. Regge trajectory of nonstrange charmed mesons in $(n_r, M^2)$ plane.
\((D_J(3000), D_J'(3000))\) is not of an isodoublet \((1^+, 0^+)\) because the \(J^P = 0^+\) of \(D_J'(3000)\) is not possible to be heavier than \(J^P = 1^+\) of \(D_J(3000)\). The \(D_J'(3000)^0\) as \(3^3S_1\) has

\[
BR_{D_J'(3000)^0} = \frac{\Gamma(D_J'(3000)^0 \to D^+\pi^-)}{\Gamma(D_J'(3000)^0 \to D^{++}\pi^0)} \approx 0.64,
\]

that means, the decay mode \(D^+\pi^-\) is dominant over \(D^{+}\pi^0\). The \(D_J'(3000)^0\) as \(2^3P_2\) has \(BR_{D_J'(3000)^0} \geq 1\), which is in agreement with the experimental measurement. Hence,

\[
(D_J(3000), D_J'(3000)) = (1^+, 2^+) = (2^3P_1, 2^3P_2).
\]

The mass difference between \(D_J'(3000)^0\) and \(D_J'(3000)^0\) is approximately 206 MeV. Such a large mass difference indicate \(D_J(3000)^0\) state is not of \(2P\) state. Experimentally, its observed spin-parity is \(2^+\). So it can be a candidate of \(3^3P_2\) or \(1^3F_2\). For \(D_J'(3000)^0\) as \(3^3P_2\),

\[
BR_{D_J'(3000)^0} = \frac{\Gamma(D_J'(3000)^0 \to D^+\pi^-)}{\Gamma(D_J'(3000)^0 \to D^{++}\pi^0)} \approx 0.95,
\]

i.e. the decay \(D^+\pi^-\) is more dominant than \(D^{+}\pi^0\). For \(1^3F_2\) state, the \(BR_{D_J'(3000)^0}\) is 2.51, which is most favorable to decay in \(D^+\pi^-\) and, it is in accordance with the experimental measurement. So,

\[
D_J'(3000)^0 = (2^+) = (1^3F_2).
\]

IV. REGGE TRAJECTORY

Spin and parity assignments of excited D mesons from the strong decays analysis are presented in Table IV with their respective PDG-2018 [5] world average masses. Using these we construct the Regge trajectory in which the total spin (or principal quantum number \(n\)) and the mass of hadrons are related. This can help in predicting the possible quantum states of hadrons. An investigation of meson spectrum in the non-perturbative regime of quark-gluon interactions has a great importance for understanding the dynamics of strong interactions (for details see Refs. [15, 46]). We are using the following definitions:

I. the Regge trajectory in \((J, M^2)\) plane,

\[
J = \alpha M^2 + \alpha_0;
\]

II. and the Regge trajectory in \((n_J, M^2)\) plane,

\[
n_J = \beta M^2 + \beta_0;
\]

where \(\alpha, \beta, \alpha_0, \beta_0\) are slopes and \(n_J(= n - 1) = 0, 1, 2, \ldots\) is the radial principal quantum number. The Regge trajectory in \((J, M^2)\) plane are available with the evenness and oddness of the total spin \(J\) are respectively distinguished according to their parity \(P = (-1)^J\) called natural parity and \(P = (-1)^{J-1}\) called unnatural parity. Figures 13 and 14 shows the plots of Regge trajectories in \((J, M^2)\) and \((n_J, M^2)\) planes which are usually called Chew-Frautschi plots. The D meson states are fitted on the Regge line with sufficiently good accuracy. The parameters like Regge slopes and the intercepts are extracted from the Regge trajectories (see in Table V and VI), that estimate the masses of the states lying on these Regge trajectories. The Regge slope is assumed to be same for all D meson multiplets lying on the single Regge line.

The masses of \(1^1D_2, 1^3D_3, 3^1S_0, 3^3S_1, 1^1F_3, 1^3F_4, 2^1D_3, 3^3P_2\) and \(2^3F_4\) states are estimated (see in Table VII and VIII). The 2.843 GeV of \(1^3D_2\) is overestimated to \(D_J(2570)^0\) by a mass difference of 79 MeV. Also, the helicity distribution disfavors the identification of \(D_J(2750)^0\) as a \(1^3D_3\) [45]. But we tentatively identify \(D_J'(2750)^0\) as \(3^3P_2\) with \(n = 1\). For \(1^1D_3, 1^3F_4, 2^1D_3, 2^3F_4\) and \(3^3S_1\), our results are in agreement with D. Ebert et al. [11] and are overestimates to the predictions of Refs. [8, 9, 12]. Such heavier masses agree with the argument that slopes of Regge trajectories decrease with quark mass increase [45–51]. The partial strong decay rates of these predicted states are calculated and presented in Table IX. These are also shown in Figures 15 to 23 where the strong decay rates change with respect to the square of the couplings. The decay mode \(D^{+}\pi^-\) is dominant in the states \(1^1D_2, 3^1S_0, 3^3S_1, 1^1F_3\) and \(3^3P_2\) with branching fractions 52.30%, 46.14%, 28.01%, 48.85% and 24.18% respectively. And, for the \(1^3D_3, 1^3F_4, 2^1D_3\) and \(2^3F_4\) states the \(D^+\pi^-\) decay is dominant with branching fractions 37.87%, 34.09%, 26.64% and 26.74% respectively.

V. CONCLUSIONS

In this paper, we have examined the nonstrange charmed mesons \(D_1(2420)^0, D_2(2460)^0, D(2550)^0, D_J(2600)^0, D(2740)^0, D_J(2750), D_J(3000)^0, D_J'(3000)^0\) and \(D_J'(3000)^0\) observed by the LHCB [4] and \(BABAR\) [4] Collaborations according to their spin, parity and masses. Their strong decays into ground state charmed mesons \((\pi, \eta, K)\) are analyzed in the HQET. The branching ratios among the strong decays tentatively identify the quantum numbers of nonstrange charmed mesons.

The strong decay widths are retained with the square of the coupling constants \(g_{TT}, g_{TH}, g_{HR}, g_{HS}, g_{TR}, g_{RS}, g_{TR}, g_{RS}\) and \(k_{2Z}\), which are determined comparing those with the widths observed by experimental groups given in Table III. We identify the states \(D(2550)^0, D_J(2600)^0, D(2740)^0\) and \(D_J'(2750)\) with spin-parity \(0^-, 1^-, 2^-\)
and $3^-$ respectively. They are in agreement with the strong decays analysis done by Refs. [22–26]. An unclear resonance structures near 3 GeV region motivated our present study. We tentatively assign the quantum states of $D_J(3000)^0$, $D_J^*(3000)^0$ and $D_2^*(3000)^0$ as $2^3P_1$, $2^3P_2$ and $1^3F_2$ respectively. The states $D_J(3000)^0$ and $D_2^*(3000)^0$ are in accordance with the predictions of P. Gupta and A. Upadhyay [26]. J.-K. Chen [6] assigned the states $D_J(3000)^0$, $D_J^*(3000)^0$ and $D_2^*(3000)^0$ as $3^1S_0$, $3^3S_1$ and $3^3P_2$ respectively. S. Godfrey and K. Moats identified $D_J(3000)^0$ as $3^1S_0$ state and $D_J^*(3000)^0$ as $1^3F_4$. To identify its nature, we expect some more experimental
FIG. 19. Strong decay widths of $1^1 F_3$ (in MeV) nonstrange charmed meson state (lying on the Regge line $1^1 S_0$ in $(J, M^2)$ plane) changing with the square of the coupling $k_2^2$ in HQET.

FIG. 20. Strong decay widths of $1^3 F_4$ (in MeV) nonstrange charmed meson state (lying on the Regge line $1^3 S_1$ in $(J, M^2)$ plane) changing with the square of the coupling $k_2^2$ in HQET.

FIG. 21. Strong decay widths of $2^3 D_3$ (in MeV) nonstrange charmed meson state (lying on the Regge line $2^3 S_1$ in $(J, M^2)$ plane) changing with the square of the coupling $k_2^2$ in HQET.

Using these spin and parity assignments of experimentally observed $D$ mesons, we construct the Regge trajectories in $(J, M^2)$ and $(n_r, M^2)$ planes. By fixing the slopes and intercepts of the Regge lines we estimate the masses of higher excited states $1^1 D_2$, $1^3 D_3$, $3^1 S_0$, $3^3 S_1$, $1^1 F_3$, $1^3 F_4$, $2^3 D_3$, $3^3 P_2$ and $2^3 F_4$ of $D$ mesons. Their strong decays analysis conclude that the $D^{*+} \pi^-$ is dominant decay mode for $1^1 D_2$, $1^3 S_0$, $3^3 S_1$, $1^1 F_3$, $3^3 P_2$ states, and the decay mode $D^+ \pi^-$ is dominant for $1^3 D_3$, $1^3 F_4$, $2^3 D_3$, $2^3 F_4$ states. This study can help the experimentalists for searching these higher excited states in such decay modes. We would like to extend this scheme for the study of strong decays of excited strange charmed mesons in future.

[1] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 94, 072001 (2016).
[2] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 92, 032002 (2015).
[3] R. Aaij et al. (LHCb Collaboration), JHEP 09, 145 (2013).
[4] P. del Amo Sanchez et al. (BABAR Collaboration) Phys. Rev. D 82, 111101(R) (2010).
[5] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update.
[6] J.-K. Chena, Eur. Phys. J. C 648, 78 (2018).
[7] V. Kher, N. Devlani, and A.K. Rai, Chin. Phys. C 41,
FIG. 22. Strong decay widths of $3^3P_2$ (in MeV) nonstrange charmed meson state (lying on the Regge line $1^1P_1$ in $(n_r,M^2)$ plane) changing with the square of the coupling $\hat{h}^2$ in HQET.

FIG. 23. Strong decay widths of $2^3F_4$ (in MeV) nonstrange charmed meson state (lying on the Regge line $2^3S_1$ in $(n_r,M^2)$ plane) changing with the square of the coupling $\hat{k}^2$ in HQET.

[8] S. Godfrey and K. Moats, Phys. Rev. D 93, 034035 (2016).
[9] Y. Sun, X. Liu, and T. Matsuki, Phys. Rev. D 88, 094020 (2013).
[10] D.M. Li, P.F. Ji, and B. Ma, Eur. Phys. J. C 71, 1582 (2011).
[11] D. Ebert, R.N. Faustov, and V.O. Galkin, Eur. Phys. J. C 66, 197 (2010).
[12] M. Di Pierro and E. Eichten, Phys. Rev. D 64, 114004 (2001).
[13] T.A. Lahde, C.J. Nyfalt, and D.O. Riska, Nucl. Phys. A 674, 141 (2000).
[14] K. Cichy, M. Kalinowski, and M. Wagner, Phys. Rev. D, 94 094503 (2016).
[15] X.-H. Zhong, Phys. Rev D 82, 114014 (2010).
[16] L.-Y. Xiao and X.-H. Zhong, Phys. Rev D 90, 074029 (2014).
[17] P. Colangelo, F. De Fazio, and R. Ferrandes, Phys. Lett. B 634, 235 (2006).
[18] P. Colangelo, F. De Fazio, and S. Nicotri, Phys. Lett. B 642, 48 (2006).
[19] P. Colangelo, F. De Fazio, S. Nicotri, and M. Rizzi, Phys. Rev. D 77, 014012 (2008).
[20] P. Colangelo and F. De Fazio, Phys. Rev. D 81, 094001 (2010).
[21] P. Colangelo, F. De Fazio, F. Giannuzzi, and S. Nicotri, Phys. Rev. D 86, 054024 (2012).
[22] Z.G. Wang, Phys. Rev. D 83, 014009 (2011).
[23] Z.G. Wang, Commun. Theor. Phys. 57, 93 (2012).
[24] Z.G. Wang, Phys. Rev. D 88, 114003 (2013).
[25] M. Batra and A. Upadhyay, Eur. Phys. J. C 75, 319 (2015).
[26] P. Gupta and A. Upadhyay, Phys. Rev. D 97, 014015 (2018).
[27] M. Neubert, Phys. Rep. 245, 259 (1994).
[28] A. F. Falk, Nucl. Phys. B 378, 79 (1992).
[29] A. F. Falk and M. E. Luke, Phys. Lett. B 292, 119 (1992).
[30] R. Casalbuoni et al., Phys. Rep. 281, 145 (1997).
[31] S. Campanella, P. Colangelo, and F. De Fazio Phys. Rev. D 98, 114028 (2018).
[32] R. Casalbuoni et al., Phys. Lett. B 302, 95 (1993); F. De Fazio, Phys. Rev. D 79, 054015 (2009); Z. G. Wang, Eur. Phys. J. A 47, 94 (2011); Z. G. He, X. R. Lu, J. Soto and Y. Zheng, Phys. Rev. D 83, 054028 (2011); Z.G. Wang, Int. J. Theor. Phys. 51, 1518 (2012); Mod. Phys. Lett. A 27, 1250197 (2012).
[33] B. Singh et al. (PANDA Collaboration), Phys. Rev. D 95, 032003 (2017); Eur. Phys. J. A 52, 325 (2016); Nucl. Phys. A 954, 323 (2016); Eur. Phys. J. A 51, 107 (2015); J. Phys. G 46, 045001 (2019).
[34] G. Barucca et al. et al. (PANDA Collaboration), Eur. Phys. J. A 55, 42 (2019).
[35] P. Colangelo, F. De Fazio, G. Nardulli, N. Di Bartolomeo, and R. Gatto, Phys. Rev. D 52, 6422 (1995).
[36] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, F. Feruglio, R. Gatto, and G. Nardulli, Phys. Rep. 281, 145 (1997).
[37] Z. G. Wang and S. L. Wan, Phys. Rev. D 74, 014017 (2006).
[38] Z. G. Wang, Nucl. Phys. A 796, 61 (2007).
[39] P. Z. Huang, L. Zhang, and S. L. Zhu, Phys. Rev. D 81, 094025 (2010).
[40] P. Avery et al. (CLEO Collaboration), Phys. Rev. D 41, 774 (1990).
[41] S. Chekanov et al. (ZEUS Collaboration), Eur. Phys. J C 60, 25 (2009).
[42] W. Abramowicz et al. et al. (ZEUS Collaboration), Nucl.
Phys. B 866, 229 (2009).

[43] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 79, 112004 (2009).

[44] T. Bergfeld et al. (CLEO Collaboration), Phys. Lett. B 340, 194 (1994).

[45] P.D. Collins, An introduction to Regge theory and high energy physics (Cambridge University Press, 1977).

[46] S. Godfrey and J. Napolitano, Rev. Mod. Phys. D 66, 1411 (1999).

[47] P. del Amo Sanchez et al., Phys. Rev. D 82, 111101 (2010).

[48] X.-H. Guo, K.-W. Wei, and X.-H. Wu, Phys. Rev. D 78, 056005 (2008).

[49] A. Zhang, Phys. Rev. D 72, 017902 (2005).

[50] D.-M. Li, B. Ma, Y.-X. Li, Q.-K. Yao, and H. Yu, Eur. Phys. J. C 37, 323 (2004).

[51] M. M. Brisudova, L. Burakovsky, and T. Goldman, Phys. Rev. D 61, 054013 (2000).