Which Fundamental Constants for CMB and BAO?

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March 23, 2015

Abstract

We study the Cosmic Microwave Background using the three-scale framework of Hu et al. to derive the dependence of the CMB temperature anisotropy spectrum on the fundamental constants. We show that, as expected, the observed spectrum depends only on dimensionless combinations of the constants, and we emphasize the points that make this generally true for cosmological observations. Our analysis suggests that the CMB spectrum shape is mostly determined by $\alpha^2 m_e/m_p$ and the proton-CDM-particle mass ratio, $m_p/m_\chi$, with a sub-dominant dependence on $(Gm_\chi m_e/hc)\alpha^\beta$ with $\beta \sim -7$. The distance to the last-scattering surface depends on $Gmpm_\chi/hc$, so published CMB observational limits on time variations of the constants, besides making assumptions about the form of the dark-energy, implicitly assume the time-independence of this quantity. On the other hand, low-redshift $H_0$, BAO and large-scale structure data can be combined with the shape of the CMB spectrum to give information that is largely independent of the dark-energy model. In particular we show that the pre-recombination values of $Gm^2_\chi/hc$ and $\alpha^2 m_e/m_\chi$ could not have differed from their present values by more than of order 25%.

The Cosmic Microwave Background (CMB) anisotropy spectra are primarily used to determine cosmological parameters [11 2] but the spectra can also give information on the values of the fundamental constants in the early universe. Building on studies of the BOOMeranG and MAXIMA data [3 4], limits on the difference of the pre-recombination value of the fine structure constant, $\alpha$, and its present value were derived using the WMAP data [5]. These limits were extended to combined limits on $(\alpha, m_e)$ using WMAP data [6 19] and Planck data [7]. The limits are based on the effects of $(\alpha, m_e)$ on the recombination process [8 3 9]. While the procedure used to obtain these limits is not obviously incorrect, the publication of a limit on the variation of $m_e$ is perplexing since it is generally admitted that only dimensionless fundamental constants are physically
meaningful \[10\]. This is manifestly true for laboratory measurements which consist of comparing quantities of a given dimension with standards of the same dimension \[11\]. It is less obviously true for cosmological measurements where two times are typically involved. For example CMB measurements concern the time of recombination, \(t_{\text{rec}}\), and the measurement time, \(t_0\), and one can form dimensionless quantities like \(m_c(t_{\text{rec}})/m_c(t_0)\). In fact, the results of \[6, 7\] are expressed as limits on the deviation from unity of this dimensionless quantity. Similarly, limits from other studies on time variations of Newton’s constant \(G\) \[12\] are typically expressed as measurements of \(G(t)/G(t_0)\). In this paper we will show how a proper analysis gives only measurements of equal-time dimensionless quantities like \(m_c(t)/m_p(t)\).

Part of the problem with using CMB data is that the phenomenology is rather complicated so it is difficult to include the effects of all relevant fundamental constants in compact formulas. This is one reason that results are expressed in terms of dimensionful constants like \(m_c\). In any case, the analyses leading to limits in \((m_c, \alpha)\) space all assume the time-independence of the proton mass and \(G\), meaning that limits on \(m_c\) should be interpreted as limits on \(Gm_c^2/\hbar c = (Gm_p^2/\hbar c)(m_c/m_p)^2\) with \(Gm_p^2/\hbar c\) held constant\[7\]. The analysis presented here suggests that the natural dimensionless variables for studying the shape of the spectrum are \(\alpha^2 m_c/m_p, m_p/m_\chi\) and \(Gm_\chi m_c/\hbar c\), where \(m_\chi\) is the mass of the CDM particles. Clearly, the analyses leading to limits on \((m_c, \alpha)\) also suppose the time dependence of \(Gm_\chi^2/\hbar c\). The introduction of \(m_\chi\) into the problem reminds us that not even the present values of all relevant fundamental constants are known. However, this will not prevent us from studying their time variation.

In the following analysis, Section 1 defines the cosmological and physical parameters and section 2 applies the model of Hu et al. \[16, 17\] to the CMB spectrum. Section 3 and 4 describe the analysis of CMB and BAO data, highlighting how limits on variations of dimensionless constants can be derived in the presence of degeneracies with cosmological parameters. Section 5 concludes with some thoughts on why cosmological observations always conspire to give information only on dimensionless constants.

## 1 The fundamental constants and cosmological parameters

We first define the physical and cosmological model that we will use. For the CMB, the five most important coupling constants and masses are:

\[
\alpha \quad G \quad m_\chi \quad m_p \quad m_e \quad (1)
\]

Since we will allow for time variations, the current values will be given with a zero subscript, e.g. \(m_{p0}\). Of the five, only \(\alpha\) is dimensionless and our goal will be to show that observable quantities depend only on dimensionless combinations of the last four like \(m_c/m_p\) and \(Gm_\chi^2/\hbar c\).

As emphasized, for example, in \[12\], simply knowing the dependence of observable quantities on fundamental constants in the absence of time-variations does not mean
that one can reliably relate variations in those constants to observable effects. This is because the physical introduction of time-variations of constants generally requires the introduction of extra degrees of freedom, like time-varying scalar fields. This will add additional terms to the Friedman equation, modifying the expansion rate. In the absence of a specific model, one has to avoid these complications by making simplifying assumptions. As was done in the WMAP and Planck studies [6, 7], we assume that time variations of fundamental constants are such that they are effectively time-independent at high redshift, where they determine the recombination process. They then quickly “relax” to their post-recombination values where they determine the distance to the last-scattering surface and provide standards for local measurements of the CMB temperature, \( T_0 \), and the expansion rate, \( H_0 \). We ignore the modifications of the expansion dynamics that necessarily occur during the relaxation.

There are several possible choices for cosmological parameters. For simplicity, we will assume a flat universe containing vacuum energy, cold-dark-matter, baryonic matter, photons and neutrinos. If one is willing to entertain the possibility of time-dependent fundamental constants, it is best to define the cosmology as much as possible in terms of time-independent cosmological parameters that make no reference to a particular epoch. Therefore, instead of the standard \( \Omega_B \) and \( \Omega_{CDM} \), we use \( \eta_b \), the baryon-photon number density ratio and \( \eta_\chi \), the same quantity for dark-matter particles. We suppose throughout this paper that \( \eta_b \) and \( \eta_\chi \) are time-independent. To these two density parameters, we add the current expansion rate and CMB temperature so the cosmological parameters are

\[
\eta_\chi \quad \eta_b \quad H_0 \quad T_0
\]

The two \( \eta \)'s are to be determined by the analysis of the CMB anisotropies, \( T_0 \) by the CMB wavelength spectrum, and \( H_0 \) by distance-ladder techniques.\(^1\)

Local measurements give \( H_0 \) and \( T_0 \) in SI units. For example, most distance-ladder measurements of \( H_0 \) are based on radar distance to the Sun. This distance is than scaled up via parallax measurements and calibrated candle measurements. The CMB temperature \( T_0 \) involves COBE wavelength measurements of CMB photons with a Michaelson interferometer. Since values of fundamental constants are also known in SI units, we can express \( H_0 \) and \( T_0 \) in terms of measured fundamental constants. In units where \( \hbar = c = k_B = 1 \), \( H_0 \) and \( T_0 \) have dimension of mass or energy and it will turn out to be convenient to use the proton mass as the standard. We will thus express results in terms of the measured dimensionless quantities:

\[
\frac{H_0}{m_p0} = (1.66 \pm 0.05) \times 10^{-42} \quad \frac{T_0}{m_p0} = (2.503 \pm 0.001) \times 10^{-13}
\]

for \( H_0 = 73.0 \pm 2.4 \) km s\(^{-1}\) Mpc\(^{-1}\) \([13]\) and \( T_0 = (2.726 \pm 0.0013) \) K \([14]\). Because \( m_\chi \) is unknown, standard cosmological analyses replace \( m_\chi \eta_\chi \) with \( \Omega_{CDM} H_0^2 \):

\[
\Omega_{CDM} H_0^2 = 2.04 G m_\chi \eta_\chi T_0^3.
\]

\(^1\)The CMB spectrum also can be used to measure \( H_0 \) but in the presence of time-varying constants, this is perhaps asking too much. We therefore prefer to use distance-ladder measurements.
Note that for a flat universe, the vacuum energy density is determined by $H_0$, $T_0$ and $m_\chi \eta_\chi$: $\Omega_\Lambda H_0^2 = H_0^2 - \Omega_M H_0^2$.

Throughout, we will suppose a flat universe with CDM dominating over baryonic matter, sometimes neglecting $\eta_b m_p$ with respect to $\eta_\chi m_\chi$ when appropriate. Similarly, we will ignore the presence of neutrons and nuclei, equating the baryon, proton and electron number densities. More generally, the results presented here will emphasize “first-order” effects of the fundamental and cosmological parameters. This is not a shameful restriction because at some order all physical and cosmological parameters must affect all observations.

2 The CMB anisotropy spectrum

To understand the CMB spectrum, we use the qualitative model of Hu et al [16, 17] based on three length scales that are imprinted on the spectrum. The scales are the Hubble length at matter-radiation equality, the acoustic scale giving the distance a sound wave can travel before recombination, and the damping scale due to photon random walks near photon-matter decoupling. Each of the three scales ($r_{eq}$, $r_A$, $r_{dec}$) gives a “feature” on the anisotropy power spectrum, $C_\ell$, at $\ell \sim \pi D(z_{dec})/r$, where $D(z_{dec})$ is the co-moving distance to the last-scattering surface. For models consistent with the observed CMB spectrum, the values are ($\ell_{eq}$, $\ell_A$, $\ell_{dec}$) $\sim$ (150, 300, 1300) [17]. The matter-radiation equality scale, $r_{eq}$, determines the minimum $\ell$ that benefited from radiation driving (early-time Sachs-Wolfe effect), resulting an enhancement of the temperature anisotropies over the primordial value $\Delta T/T \sim 10^{-5}$. The acoustic scale, $r_A$ determines the positions of the peaks in the spectrum. Finally, the decoupling scale determines the minimum $\ell$ that were damped by photon diffusion, resulting in a suppression of power for $\ell > \pi D/r_{dec}$. (Formally, this scale is the geometric mean of the photon mean free path and Hubble scale at decoupling but at this time the two are forced to be of the same order of magnitude.)

The three scales ($r_{eq}$, $r_A$, $r_{dec}$) are closely related to the Hubble lengths at matter-radiation equality, $1/H_{eq}$, at baryon-photon equality, $1/H_{p\gamma}$, and photon-matter decoupling, $1/H_{dec}$. They have the relatively simple dependencies of fundamental and cosmological parameters shown in Table 1. The first two columns give the temperatures and squared expansion rates at the redshift where the scales are defined. The third column gives the inverse scales redshifted to present epoch where, along with the distance $D(z_{dec})$, they determine the observed spectrum. We note the important fact that the redshift results in only dimensionless combinations of fundamental constants being present in the third column.

The first scale is found by equating the matter density to the photon plus neutrino density, giving

$$T_{eq} = \frac{m_\chi \eta_\chi + m_p \eta_b}{2.7(1 + 0.68 N_\nu/3)} \Rightarrow r_{eq}^{-1} \sim \sqrt{G(m_\chi \eta_\chi + m_p \eta_b)T_0}$$  \hspace{1cm} (5)

where $N_\nu \sim 3$ is the number of neutrino species, A similar relation defines baryon-photon
\[
T \sim m_\chi \eta_x \quad G(m_\chi \eta_x)^4 \quad r_{eq}^{-1} \sim \sqrt{Gm_\chi^2 \eta_x} T_0
\]
\[
T_{\gamma} \sim m_p \eta_b \quad Gm_\chi \eta_x (m_p \eta_b)^3 \quad r_{\gamma}^{-1} \sim \sqrt{Gm_\chi m_p \sqrt{\eta_x \eta_b}} T_0
\]
\[
T_{dec} = \alpha^2 m_e f_{dec} \quad Gm_\chi \eta_x (\alpha^2 m_e f_{dec})^3 \quad r_{\text{dec}}^{-1} \sim \sqrt{Gm_\chi m_e \alpha^2 f_{dec} \sqrt{\eta_x}} T_0
\]
\[
D^{-1} \sim \sqrt{G_0 m_{\chi 0} m_{\rho 0} \sqrt{\eta_x} T_0 / m_{\rho 0}} T_0
\]

Table 1: The Hubble scales relevant for the CMB temperature anisotropy spectrum and their simplified dependencies on cosmological and fundamental parameters. The first column gives the temperature scale, and the second the squared expansion rate. The third redshifts the associated inverse of the distance scale, $1/H(T)$, to present epoch. The redshifting leaves only dimensionless combinations of fundamental constants. The subscript zero refers to present values and its absence refers to pre-recombination values. In the second and third columns, numerical factors are omitted and CDM domination is assumed ($m_\chi \eta_x \gg m_p \eta_b$). The factor $f_{\text{dec}} \sim 0.02$ is a logarithmic function of cosmological and fundamental parameters, eqn (9). The fourth line gives the distance to the last-scattering surface.
equality giving
\[
r_A^{-1} \sim r_p^{-1} \sim \sqrt{G(m_\chi \eta_\chi + m_p \eta_b)m_p \eta_b T_0}
\]

(6)

A more accurate estimate of the horizon includes the fact that acoustic waves continue to propagate, at reduced speed, until decoupling, resulting in a logarithmic increase in the sound horizon with time. The resulting sound horizon is [15]:

\[
r_A \sim r_p \gamma \ln \left[ \frac{\sqrt{1 + R_{\text{dec}}} + \sqrt{R_{\text{dec}} + R_{\text{eq}}}}{1 + \sqrt{R_{\text{eq}}}} \right]
\]

(7)

where the baryon-to-photon energy density ratio is given by

\[
R(T) = \frac{3\rho_B}{4\rho_\gamma} = 0.278 \frac{m_p}{T} \eta_b
\]

(8)

The temperature at decoupling, \(T_{\text{dec}}\) has an even more complicated dependence on the cosmological and fundamental parameters. In table 1, this dependence is hidden in the factor \(f_{\text{dec}} \equiv T_{\text{dec}}/\alpha^2 m_e\), which scales the decoupling temperature from its natural value of \(\sim \alpha^2 m_e/2 = 13.6\) eV to the calculated value of \(\sim 0.2\) eV. It can be estimated in a model where perfect equilibrium between photons and electrons is maintained so the free-electron density is determined by the Saha equation. The decoupling temperature is defined by equating the photon-electron (Thompson) scattering rate, \(n_e \sigma_T c\), and the expansion rate. Using \(\sigma_T = \frac{(8\pi/3)}{2} \frac{m_e}{\alpha^2} \) we get

\[
f_{\text{dec}}^{-1} - 3 \ln f = 2 \ln \left[ \frac{8\pi}{3(2\pi)^{3/2}} \frac{\eta_b}{\eta_\chi} \frac{\alpha^7}{G m_\chi m_e} \right].
\]

(9)

More complete calculations (see e.g. [3]) integrate the Boltzmann equation to find the decoupling temperature. Analyses using Planck and WMAP data use the RECFAST code [9] which can be modified to include all expected dependencies on the recombination process on fundamental constants. Presumably, such calculations would give a slowly varying dependence of \(f_{\text{dec}}\) on fundamental constants as in equation (9). The combination would necessarily be dimensionless and [9] suggests that it would be \(Gm_\chi m_e\) times a power of \(\alpha\).

The use of the angular positions of the features induced by these three scales requires the introduction of the fourth length scale, the distance to the last-scattering surface. For flat \(\Lambda\)CDM models, this is give by

\[
D(z_{\text{dec}}) = \frac{1}{\sqrt{\Omega_M H_0^2}} \int_{0}^{z_{\text{dec}}} \frac{dz}{\left[ (1 - \Omega_M)/\Omega_M + (1 + z)^3 \right]^{1/2}}
\]

(10)

Most of the integral is in the matter dominated redshift range and the integral is not far from it’s value, 1.94, for \(\Omega_M = 1\). We therefore write

\[
D(z_{\text{dec}}) = \frac{1.94}{\sqrt{\Omega_M H_0^2}} [1 - f_0(\Omega_M)]
\]

(11)
where the small correction ranges from $f_0(1) = 0$ to $f_0(0.2) = 0.13$. For simplicity, we will ignore the dependence on $f_0$ and, using (11), we get

$$D(z_{\text{dec}})^{-1} = 0.82T_0 \left( G_0 (m_\chi 0 \eta_\chi + m_p 0 \eta_b) m_p 0 \frac{T_0}{m_p 0} \right)^{1/2}$$

(12)

The distance depends on the dimensionless combinations $G_0 m_\chi 0 m_p 0$ and $G_0 m_p^2 0$ and on the measured ratio of the temperature and the proton mass.

The observed spectrum is determined by the ratios of the scales in the third column of Table 1. The overall shape, independent of the angular scale, is determined by the ratios of the first three scales. Two ratios depend on $\eta_b$:

$$\frac{r_p}{r_{eq}} \sim \left( \frac{m_\chi}{m_p} \right)^{1/2} \left( \frac{\eta_\chi}{\eta_b} \right)^{1/2} \quad \text{and} \quad \frac{r_p}{r_{\text{dec}}} \sim \left( \frac{\alpha^2 m_e f_{\text{dec}}}{m_p} \right)^{1/2} \eta_b^{-1/2}$$

(13)

The ratio of the equality and decoupling scales is independent of $\eta_b$:

$$\frac{r_{\text{dec}}}{r_{eq}} \sim \left( \frac{m_\chi \eta_\chi}{\alpha^2 m_e f_{\text{dec}}} \right)^{1/2}$$

(14)

The spectrum shape reflected in these ratios is determined by the pre-recombination values of dimensionless combinations of constants and the time-independent values of $\eta_\chi$ and $\eta_b$.

The angular positions of the features are the ratios between the length scales and $D(z_{\text{dec}})$. Usually, one refers to the peaks in $\ell$-space which are near harmonics of $D(z_{\text{dec}})/r_A$. Using (12) and (7) and neglecting $m_p \eta_b$ compared to $m_\chi \eta_\chi$, we get

$$\frac{D(z_{\text{dec}})}{r_A} \sim \left( \frac{G m_\chi m_p}{G_0 m_\chi 0 m_p 0} \right)^{1/2} \left( \frac{\eta_b}{T_0/m_p 0} \right)^{1/2}$$

(15)

where for clarity we have suppressed the logarithmic factor in $r_A$ (7). The angular scale thus depends on the ratio of $G m_\chi m_p$ in the early universe to the same quantity today.

3 Analysis of CMB spectra

We now reverse the discussion in the previous section and discuss the information that can be obtained from the study of the observed CMB spectrum. The shape can be fit to determine the ratios (13) and (14), so if we impose the locally measured values of $m_p$, $m_e$, $\alpha$ and $T_0$, the fit determines $\Omega_B H_0^2 \propto m_p \eta_b$ and $\Omega_\chi H_0^2 \propto m_\chi \eta_\chi$. This is consistent with the well-known fact that the CMB shape determines precisely these two cosmological parameters.

On the other hand, if independent information on $\eta_\chi$ and $\eta_b$ is imposed in (13) and (14), we can determine the pre-recombination values of certain fundamental constants, which can then be directly compared with their present values. The two ratios (13) depend on $\eta_b$, whose value determined with low-redshift observations is uncertain to a factor $\sim 2$ [15].
The decoupling-to-equality ratio (14) is more useful since the most uncertain factor in its present value is \( m_\chi \eta_\chi \propto \Omega_M H_0^2 \). At low redshift, this can be measured to a precision of \( \sim 20\% \) using the shape of the matter power spectrum (giving \( \Omega_M H_0 \)) and distance-ladder measurement of \( H_0 \). The concordance of the low-redshift values and CMB values at the 20% level means that \( m_\chi \eta_\chi/\alpha^2 m_e \) could not have changed by more than \( \sim 40\% \) since recombination. (We neglect the logarithmic dependence of \( f_{\text{dec}} \) on other dimensionless constants.) Since \( \eta_\chi \) is assume to be time-independent, we conclude conservatively that

\[
\frac{d}{dt} \ln[\alpha^2 m_e/m_\chi] < \sim 10^{-10} \text{yr}^{-1}
\]

(16)

averaged over the intervening time, \( \sim 10^{10} \text{yr} \). To our knowledge, this is the first limit on the stability of a fundamental constant whose value is unknown. Since it depends only on the shape of the CMB spectrum, this limit does not depend on estimates of \( D(z_{\text{dec}}) \), and is therefore largely independent of the dark-energy model.

Published studies [6, 7] perform global fits to the spectrum and therefore include not only the shape but also the angular positions of the features, making them sensitive to \( D(z_{\text{dec}}) \). The most prominent features are the positions of the peaks which appear near harmonics of \( D(z_{\text{dec}})/r_A \). Using (15) and \( T_0/m_{p0} = 2.52 \times 10^{-13} \), we get

\[
\frac{D(z_{\text{dec}})}{r_A} = 96 \left( \frac{G m_\chi m_p}{G_0 m_{\chi0} m_{p0}} \right)^{1/2} \left( \frac{\eta_b}{5.95 \times 10^{-10}} \right)^{1/2}
\]

(17)

where the numerical factors correspond to the Planck ΛCDM model [2].

Equation (17) can be analyzed in two ways. If one assumes that the quantity \( G m_\chi m_p \) is time-independent, the measured \( D/r_A \) determines \( \eta_b \). The ratio \( r_{\text{dec}}/r_{\gamma} \) in equation (13) then determines \( \alpha^2 m_e/m_p \) at the last-scattering surface, which can then be compared with the \( (\alpha^2 m_e/m_p)_0 \). This is a simplified, but conceptually clearer, version of what is done in traditional CMB analyses of time variations [6, 7]. The WMAP analysis [6] confirms that in the \((\alpha, m_e)\) space, the best determined combination is indeed \( \sim \alpha^2 m_e \). (Their analysis assumes a fixed \( m_p \).) The Planck data is sufficiently precise to give tight constraints on the orthogonal combination of \((\alpha, m_e)\).

On the other hand, if \( \eta_b \) is known from other sources, equation (17) can be used to set a limit on the time variation of \( G m_\chi m_p \). Nucleosynthesis estimates can be used but that would involve more hypotheses on the variations of the constants in the early universe. On the other hand, low-redshift estimates of \( \eta_b \) are within a factor of two of the cosmological value [18]. The agreement of the CMB peak position with the pre-factor in (17) then indicates that \( \sqrt{G m_\chi m_p} \) could not have changed by more than a factor two since recombination, i.e. by less than \( \sim 10^{-10} \text{yr}^{-1} \). Of course, the limit applies only within the framework that we have supposed, i.e. a flat ΛCDM universe with time-independent fundamental constants in the pre-recombination and late-time epochs. A tighter and more model-independent limit on the time variation of \( G m_\chi^2 \) will be derived below by combining CMB and BAO data.
4 BAO

Searches for time variations of fundamental constants have also been performed using the BAO peak in the matter correlation function at a redshift \( z \approx 0.57 \) \[19\]. The BAO peak can be seen in both transverse and radial directions. It is simplest to analyze in the radial direction where the peak is at \( \Delta z_{BAO} = r_A/H(z) \). Assuming matter dominance at the observed redshift, \( z \), one finds

\[
\frac{\Delta z_{BAO}}{(1 + z)^{3/2}} \sim \left( \frac{G_0 m_X m_0}{G m_X m_p} \right)^{1/2} \left( \frac{T_0}{m_p} \right)^{1/2} \eta_b
\]  

(18)

where we assume no variations of fundamental parameters between the present epoch and the measured redshift. The dependence on fundamental and cosmological parameters is similar to that in the CMB peak position \[15\].

A more interesting use of BAO uses the lowest possible redshift \( z \approx 0.1 \) \[20\] where distances between galaxies are given in units of \( c/H_0 \). (Higher precision at \( z = 0.1 \) can be found by using the deceleration parameter derived from supernova data). The position of the BAO peak in the correlation function then gives a direct measurement of \( r_A \) in units of \( c/H_0 \); i.e. of

\[
\tilde{r}_A \equiv \frac{r_A}{c/H_0}
\]

(19)

Since \( H_0 \) is known from distance-ladder techniques to \( \sim 3\% \) precision, \( r_A = (c/H_0)\tilde{r}_A \) is known in this way to \( \sim 10\% \) precision.

This result can then be imposed on the analysis of the CMB spectrum. The ratio of the acoustic and decoupling CMB scales becomes

\[
\frac{r_A}{r_{dec}} = \sqrt{\frac{G m_X \eta_X \alpha^2 m_e f_{dec}}{\tilde{r}_A^{-1} H_0/T_0}}
\]

(20)

The l.h.s. is determined by the shape of the CMB spectrum while the denominator on the r.h.s. is determined by low-redshift measurements. The numerator, \( G m_X \eta_X \alpha^2 m_e f_{dec} \) is thus determined at the 10% level of precision. This pre-recombination measurement can be then compared with local measurements:

\[
(G m_X \eta_X \alpha^2 m_e f_{dec})_0 = (\alpha^2 m_e f_{dec})_0 \Omega_M H_0^2 / (2.04 T_0^3)
\]

(21)

where \( \Omega_M H_0^2 \) is understood to refer to a low-redshift measurements. Such measurements, e.g. with the galaxy power spectrum \[21\], agree with CMB-based measurements at the 20% level. Since \( \eta_X \) is assumed to be time-invariant, and ignoring the logarithmic dependence of \( f_{dec} \) on the fundamental constants, we can conclude that the quantity \( \alpha^2 G m_X m_e / \eta c \) has not changed by more than of order 40% since recombination. Conservatively, this corresponds to

\[
\frac{d}{dt} \ln[\alpha^2 G m_X m_e / \eta c] < 10^{-10} \, \text{yr}^{-1}
\]

(22)
Since the limit depends only on local measurements and the shape of the CMB spectrum, it is independent of the dark-energy model. Performing the same manipulations with \( r_A/r_{eq} \) we determine the pre-recombination value of \( G(m_\chi \eta_\chi + m_p \eta_b)^2 \) and comparing with low-redshift measurements of \( m_\chi \eta_\chi \) gives

\[
\frac{d}{dt} \ln[Gm_\chi^2/\hbar c] < 10^{-10} \text{yr}^{-1}
\]  

where to simplify we have ignored the baryonic component. We note that this limit can be derived independently by combining the two previous limits (16) and (22). We can conclude that the gravitational couplings on the dominant dark-matter component have been relatively constant since recombination.

5 Conclusion

The most striking result of this study is seen in the third column of Table 1: all three length scales of the CMB spectrum, after redshifting to the present epoch, depend on dimensionless combinations of constants in the pre-recombination universe. Before the redshifting, the dimensionality was contained in the fundamental constants. The redshifting transferred the inverse-length dimension to \( T_0 \).

The distance to the last scattering surface in the fourth line of Table 1 also only depends on a dimensionless combination, this time at the present epoch. This came about by the “trick” of writing \( Gm_\chi_0 T_0 \) as \( Gm_\chi_0 m_{p0} \times T_0/m_{p0} \). This just corresponds to our freedom to express measured quantities like \( T_0 \) as multiples of fundamental quantities. In fact, this “freedom” is an obligation since it takes into account the dependence of our SI standards on fundamental constants. Expressing results in such manifestly dimensionless forms avoids all discussion about what units are being used.

The transfer of the inverse-length dimension to \( T_0 \) works for any standard ruler, so our conclusion that only dimensionless combinations are relevant for length scales is quite general. A similar reasoning works for standard candles [22]. For example, if one can express the total energy output of a supernova, \( Q_{SN} \), in terms of fundamental constants, then one can also work with the dimensionless energy output, \( Q_{SN}/\alpha^2 m_e \). This quantity gives the number of photons that would be produced if all energy were converted to \( \text{Ly}_\alpha \) photons. It can be related to the true number of photons by scaling by \( R \), the directly measured ratio of the mean observed supernova photon energy to the observed energy of \( \text{Ly}_\alpha \) photons from the same redshift. The total number of photons detected by an ideal detector at a distance, \( D(z) \), from the supernova is then

\[
N = (Q_{SN}/\alpha^2 m_e) \frac{A_{det}}{4\pi D^2} R
\]  

where \( A_{det} \) is the detector area. Only the dimensionless combination \( Q_{SN}/\alpha^2 m_e \) determines the number of detectable photons.

The CMB observables are the ratios of the three distances scales to each other (13) and to \( D(z_{dec}) \) (15). These three ratios provide a tidy way of summarizing the first-order
cosmological and physical information contained in the CMB spectrum. The combinations of parameters seen in these ratios reflect the degeneracies between fundamental and cosmological parameters emphasized by the groups performing the studies [6, 7]. Combination of CMB data with low-redshift measurements of cosmological parameters can lead to more reliable constraints like those on $\alpha^2 m_e/m_\chi$ (16) and on $Gm^2/\bar{h}c$ (23). It will be a challenge to incorporate these qualitative results into a rigorous analysis of the CMB spectrum.

I thank Nicolas Busca, Sylvia Galli, Jean-Christophe Hamilton, Claudia Scóccola, Douglas Scott, and especially Jean-Philippe Uzan for helpful comments and suggestions.

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