Research on Three Phase Permanent Magnet Synchronous Motor Control Algorithm Based on Sliding Mode Variable Structure

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Abstract. For the speed and position control of permanent magnet synchronous motor, an improved exponential reaching law is proposed. Firstly, the mathematical model of permanent magnet synchronous motor in rotating coordinate system is established, and the speed loop sliding mode controller and position loop sliding mode controller are designed. Lyapunov function is established to determine the stability of the designed controller. The control model of position loop, speed loop and current loop is used to simulate the permanent magnet synchronous motor. The effectiveness of the algorithm is verified by comparing with the traditional sliding mode controller.

1. Introduction

Permanent magnet synchronous motor (PMSM) is an important driving device with high efficiency and stable performance. It has been successfully applied in industry, robotics, automobile and other fields [1]. Because permanent magnet synchronous motor is a non-linear and strongly coupled multi-variable system, when the control system is affected by external disturbances or the internal parameters of the motor change, the traditional PI control method can not meet the actual requirements [2].

Sliding mode control (SMC) is widely used in the field of motor control because of its fast response and insensitive to parameter changes [3]. In reference [4], the exponential reaching law is improved by using saturation function, and a sliding mode controller is designed. The algorithm is verified by simulation. In reference [5], an adaptive variable-speed exponential reaching law is proposed, and an adaptive terminal sliding mode control is designed to control the speed of the motor. In reference [6], an integral sliding mode variable structure control strategy is proposed, and a load torque observer is designed to solve the load disturbance problem in the control process. In steady state, the velocity fluctuation is small and the chattering phenomenon is suppressed. In reference [7], Based on the global sliding mode theory, a global sliding mode disturbance sensor is proposed. The robustness of DC motor is improved by using the optimal control theory of linear quadratic regulator.

In the above literature, sliding mode control is used to control the speed of motor and improve the stability of motor operation. It does not involve the control of motor position. In order to improve the position response and accuracy of motor control, an improved speed loop sliding mode control (ISSMC) and position loop sliding mode control (IPSMC) are proposed. The effectiveness of the proposed algorithm is verified by simulation.
2. Mathematical model of three-phase permanent magnet synchronous motor

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It is assumed that the three-phase PMSM is an ideal motor and satisfies the following conditions: neglecting the saturation of the motor core; neglecting the eddy current and hysteresis loss in the motor; and the current in the motor is a symmetrical three-phase sinusoidal current [8]. Then the three-phase voltage equation of PMSM in natural coordinate ABC is as follows.

\[ u_s = R i_s + \frac{d}{dt} \psi_s \]  

Flux linkage equation.

\[ \psi_s = L_s i_s + \varphi_m \cdot F_s(\theta) \]  

Motor torque equation.

\[ T_e = \frac{1}{2} p_n \frac{\partial}{\partial \theta_m} (i_s^2 \cdot \psi_s) \]  

Motion equation of motor.

\[ J \frac{d\omega_m}{dt} = T_e + T_L + B \omega_m \]

In the formula, \( \psi_s \) is the total flux of three-phase winding. \( u_s, R \) and \( i_s \) are the phase voltage, resistance and current of three-phase winding. \( L_s \) is the inductance of three-phase winding. \( F_s(\theta) \) is the flux of three-phase winding. \( \varphi_m \) is the permanent magnet flux. \( \theta_m \) is the mechanical angular displacement. \( p_n \) is the polar logarithm of three-phase PMSM. \( \omega_m \) is the mechanical angular velocity. \( J \) is the inertia. \( B \) is the damping coefficient. \( T_L \) is the load torque.

The formula of coordinate transformation from natural coordinate system ABC to stationary coordinate system \( \alpha \beta \) is as follows.

\[
\begin{bmatrix}
  f_a \\
  f_b \\
  f_c
\end{bmatrix}
= \frac{2}{3}
\begin{bmatrix}
  1 & 1 & 1 \\
  2 & \sqrt{3} & \sqrt{3} \\
  \sqrt{2} & \sqrt{2} & \sqrt{2}
\end{bmatrix}
\begin{bmatrix}
  f_d \\
  f_q \\
  f_s
\end{bmatrix}
\]

The formula of coordinate transformation from stationary coordinate system \( \alpha \beta \) to synchronous rotating coordinate system \( d-q \) is as follows.
According to the voltage equation, torque equation, motion equation and coordinate transformation formula of the motor, the PMSM model in the coordinate system $d-q$ is obtained as follows [9-11].

\[
\begin{align*}
\frac{d u_d}{dt} &= R_i d_i + L_i \frac{di_d}{dt} - p_n \omega_n L_{dq} \Phi_q \\
\frac{d u_q}{dt} &= R_i q_i + L_i \frac{dq_i}{dt} + p_n \omega_n L_{dq} i_q + p_n \omega_n \Phi_m \\
J \frac{d \omega_n}{dt} &= \frac{3}{2} p_n \Phi_m i_q - T_i
\end{align*}
\]

In the formula, $u_d$ and $u_q$ are respectively the $d-q$ component of the stator voltage $i_d$ and $i_q$ are the $d-q$ axis component of the stator current. $L_d$ and $L_q$ are the $d-q$ axis inductance component. $p_n$ is the number of pole pairs.

3. Design of Speed Loop Sliding Mode Controller

The surface mounted PMSM can adopt $i_d = 0$ the rotor field oriented control method, and the mathematical model of speed loop can be obtained as follows.

\[
\frac{d \omega_n}{dt} = \frac{1}{J} \left(-T_i + \frac{3}{2} p_n \Phi_m i_q \right)
\]

State Variables of PMSM Velocity Loop.

\[
\begin{align*}
x_3 &= \omega_{ref} - \omega_n \\
\dot{x}_3 &= \dot{\omega}_{ref} - \dot{\omega}_n
\end{align*}
\]

According to formulas (8) and (9), it can be obtained.

\[
\begin{align*}
\dot{x}_2 &= \dot{\omega}_{ref} + \frac{1}{J} \left( T_i - \frac{3}{2} p_n \Phi_m i_q \right) \\
\ddot{x}_2 &= \ddot{\omega}_{ref} - \frac{3}{2J} p_n \Phi_m i_q
\end{align*}
\]

To reduce the influence of disturbance on motor speed, the following sliding surface is defined.

\[
s = cx_3 + \dot{x}_3
\]

In formula, $c > 0$, derivation of formula (11).

\[
\dot{s} = cx_3 + \dot{x}_3
\]

The traditional exponential reaching law is as follows.

\[
\dot{s} = -c \text{sgn}(s) - qs
\]
An improved exponential reaching law is proposed.

\[
\dot{s} = -\varepsilon \frac{1}{1 + D_1 |x_2|^{\frac{1}{h_1}}} \text{sgn}(s) - (q + D_2 |x_2|^{\frac{1}{h_2}})s
\]

(14)

The \( u = i_s \) of the controlled object is defined, and the expressions of the controllers obtained from the formulas (10), (11), (12) and (14) are as follows.

\[
u = \frac{2J}{3p_s \phi_m} \left[ c\dot{x}_2 + \varepsilon \frac{1}{1 + D_1 |x_2|^{\frac{1}{h_1}}} \text{sgn}(s) + (q + D_2 |x_2|^{\frac{1}{h_2}})s + \dot{\phi}_m \right]
\]

(15)

The reference current of the design q-axis satisfies the following control law.

\[
i_q^* = \frac{2J}{3p_s \phi_m} \int_0^\tau \left[ c\dot{x}_2 + \varepsilon \frac{1}{1 + D_1 |x_2|^{\frac{1}{h_1}}} \text{sgn}(s) + (q + D_2 |x_2|^{\frac{1}{h_2}})s + \dot{\phi}_m \right] d\tau
\]

(16)

The Lyapunov function of position loop sliding mode control is taken as follows.

\[
V = \frac{1}{2} s^2
\]

(17)

Derivation of equation (17).

\[
\dot{V} = s \cdot \dot{s} = -\varepsilon \frac{1}{1 + D_1 |x_2|^{\frac{1}{h_1}}} \text{sgn}(s)s
\]

\[-(q + D_2 |x_2|^{\frac{1}{h_2}})s^2\]

(18)

In the formula, \( s \text{sgn}(s) \geq 0, \varepsilon > 0, q > 0, D_1, D_2, h_1, h_2, g_1, g_2 \) and \( g_2 \) are all greater than zero.

\[
\dot{V} < 0
\]

(19)

Therefore, the system is stable. The improved sliding mode control is suitable for PMSM speed loop control.

4. Design of Sliding Mode Controller for Position Loop

For the actual operation of permanent magnet synchronous motor system, the following position loop state variables are defined.

\[
x_1 = \theta_m - \theta_n
\]

\[
x_1 = \dot{\theta}_m - \dot{\theta}_n
\]

(20)

In order to reduce the position error and improve the accuracy of position control, the following sliding surface is defined.
Derivation of Sliding Mode Surface Function.

\[ s = Cx_i \]  
\[ \dot{s} = C\dot{x}_i = C(\theta_{ref} - \theta) \] \hspace{1cm} (22)

An improved exponential reaching law is proposed.

\[ \dot{s} = -\varepsilon_1 \frac{1}{1 + D_3 |x_i|^{h_3/e_3}} \text{sgn}(s) - p |x_i|^{h_3/e_3} \text{sgn}(s) \] \hspace{1cm} (23)

The \( \dot{\theta}_o = -f(\theta, t) + bu \) of the controlled object is defined. The control law is obtained as follows.

\[ u = \frac{1}{bC} (\varepsilon_i \frac{1}{1 + D_3 |x_i|^{h_3/e_3}} \text{sgn}(s) + p |x_i|^{h_3/e_3} \text{sgn}(s)) + \frac{1}{b} (\dot{\theta}_{ref} + f(\theta, t)) \] \hspace{1cm} (24)

The Lyapunov function of position loop sliding mode control is as follows.

\[ V = \frac{1}{2} s^2 \] \hspace{1cm} (25)

Derivation of formula (25).

\[ \dot{V} = s \cdot \dot{s} = -\varepsilon_1 \frac{1}{1 + D_3 |x_i|^{h_3/e_3}} \text{sgn}(s)s \]
\[ - p |x_i|^{h_3/e_3} \text{sgn}(s)s \] \hspace{1cm} (26)

In the formula, \( s \text{sgn}(s) \geq 0, \varepsilon_i > 0, p > 0, D_3, h_3, h_4, g_3 \) and \( g_4 \) are all greater than zero.

\[ \dot{V} < 0 \] \hspace{1cm} (27)

Therefore, the system is stable. The improved sliding mode control is suitable for PMSM position loop control.

5. PMSM algorithm simulation

Fig. 1 is a block diagram of PMSM vector control, in which APR is a position loop controller, ASR is a speed loop controller, ADR is a current loop d-axis current PI controller, and AQR is a current loop q-axis current PI controller.
Figure 1. Block diagram of three closed-loop vector control for three-phase permanent magnet synchronous motor

See the following table for the parameters of the simulation prototype.

| Table 1. PMSM parameters |
|---------------------------|
| **Number of pole-pairs** | $p_s = 4$ |
| **Stator d-axis inductance** | $L_d = 5.25mH$ |
| **Stator q-axis inductance** | $L_q = 12mH$ |
| **Stator resistance per phase** | $R_s = 0.1827Wb$ |
| **Flux linkage** | $J = 0.003kg\cdot m^2$ |
| **Rotor inertia** | $R = 0.958Ohm$ |
| **Rotor damping** | $B = 0.008N\cdot m\cdot s$ |

5.1. Speed loop simulation

Double closed loop control mode of speed loop and current loop is adopted. Current loop adopts PI control, and speed loop adopts ISSMC and SMC respectively. SMC adopts traditional exponential approach rate. The ISSMC parameters are $c = 50$, $e = 200$, $q = 200$, $D_1 = 10$, $D_2 = 1$, $h_1 = 2$, $h_2 = 6$, $g_1 = 1$ and $g_2 = 5$. The step response speed is given at 400r/min and 300r/min. As can be seen from the fig.2 and fig.3, under the low speed condition, the speed loop algorithm in this paper can make the speed fast tract the given value. The speed loop algorithm in this paper has faster response than the traditional SMC.
5.2. Position loop simulation

A three-loop control mode of position loop, speed loop and current loop is adopted, the current loop adopts PI control, the speed loop adopts improved SMC control, and the position loop adopts improved SMC and SMC respectively. Improved SMC Position Loop Control Parameters: 

\[ b = 1, \quad C = 6.8, \quad \epsilon_i = 50, \quad D_i = 10, \quad p = 20, \quad h_i = 2, \quad h_i = 6, \quad g_i = 1, \quad g_i = 5. \]

Set the given position so that. Fig. 4 shows the change of position error with time. It can be seen from the figure that the given position can be quickly reached by using the position loop control in this paper. Compared with the traditional SMC algorithm, the precision of the position loop control in this paper is higher.
6. Conclusion
In this paper, an improved exponential reaching law is proposed. The reaching law involves the system state variables. The reaching speed can be adaptively improved according to the size of the state variables at different times. In the low speed state, the speed loop improvement algorithm improves the speed response of PMSM, and the position loop improvement algorithm improves the position response and position accuracy. Using the improved reaching law, the speed loop sliding mode controller and position loop sliding mode controller are designed. The simulation results verify the effectiveness of the method.

References

[1] Li Y, Chen Y Q, Podlubny I. Stability of fractional-order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag–Leffler stability [J]. Computers & Mathematics with Applications, 2010, 59(5): 1810-1821.

[2] Zhang Xiaoguang, Zhao Ke, Sun Li, et al. Dynamic Quality Control of Sliding Mode Variable Structure Speed Regulation System for Permanent Magnet Synchronous Motors [J]. Journal of China Electrical Engineering, 2011, 31(15): 47-52.

[3] Tang Wenxiu, Xi Wenlong, Li Zhipeng, et al. DC Motor Position Control Based on Sliding Mode Variable Structure and High Gain State Observer [J]. Journal of China University of Science and Technology, 2018.

[4] Li Yongteng, Lawrence Wang, Fan Liangzhong. Control Method of Permanent Magnet Synchronous Motor with Improved Sliding Mode Variable Structure [J]. Power Grid and Clean Energy, 2016(8): 57-61.

[5] Xu bo, Zhu Xuan Qiu. adaptive nonsingular terminal sliding mode control and its application in BPMSM [J]. control and decision, 2014(5): 833-837.

[6] Tu Qunzhang, Huang Hao, Jiang Chengming, Pan Ming, Li Pei, Xue Jinhong. Servo Motor Integral SMC Speed Control Strategy [J]. Journal of National Defense University of Science and Technology, 2019,41(02): 150-157.

[7] Zhang H, Ge L, Shi M, et al. Research of Compound Control for DC Motor System Based on Global Sliding Mode Disturbance Observer [J]. Mathematical Problems in Engineering, 2014, 2014: 1-7.

[8] Yuan Lei, Hu Bingxin, Wei Keyin, et al. Control Principle of Modern Permanent Magnet Synchronous Motor and MATLAB Simulation [M]. Beijing University of Aeronautics and Astronautics Press, 2016.
[9] Cheng Fan, Yu Haitao, Li Zhongkun, Mingfei Xu. Research on Control Strategy of Improved Terminal Sliding Mode Permanent Magnet Linear Synchronous Motor [J]. Motor and Control Applications, 2016, 43(08): 31-35.

[10] Luo Q W , Huang S D , Cao G Z . Sensorless vector control of permanent magnet synchronous motors based on the improved sliding mode observer[C]// International Conference on Power Electronics Systems & Applications. IEEE, 2014.

[11] Walambe R A , Joshi V A , Apte A A , et al. Study of sensorless control algorithms for a permanent magnet synchronous motor vector control drive[C]// International Conference on Industrial Instrumentation & Control. IEEE, 2015.