Flow and Heat Transfer Characteristics of Turbulent Gas Flow in Microtube with Constant Heat Flux

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Abstract. Local friction factors for turbulent gas flows in circular microtubes with constant wall heat flux were obtained numerically. The numerical methodology is based on arbitrary-Lagrangian-Eulerian method to solve two-dimensional compressible momentum and energy equations. The Lam-Bremhorst’s Low-Reynolds number turbulence model was employed to calculate eddy viscosity coefficient and turbulence energy. The simulations were performed for a wide flow range of Reynolds numbers and Mach numbers with different constant wall heat fluxes. The stagnation pressure was chosen in such a way that the outlet Mach number ranged from 0.07 to 1.0. Both Darcy friction factor and Fanning friction factor were locally obtained. The result shows that the obtained both friction factors were evaluated as a function of Reynolds number on the Moody chart. The values of Darcy friction factor differ from Blasius correlation due to the compressibility effects but the values of Fanning friction factor almost coincide with Blasius correlation. The wall heat flux varied from 100 to 10000 W/m\(^2\). The wall and bulk temperatures with positive heat flux are compared with those of incompressible flow. The result shows that the Nusselt number of turbulent gas flow is different from that of incompressible flow.

1. Introduction

The number of applications of micro-electro mechanical systems (MEMS) has increased the need for understanding heat transfer in micro-geometries. In the case of micro-tube gas flows, it is found that the flow accelerates due to the gas expansion and thermal energy converts to kinetic energy. This results in a static temperature decrease of the gas [1].

Since the experimental work by Wu and Little [2], who measured the friction coefficient and Nusselt number for nitrogen, argon and helium flows in silicon or glass micro-channels, many experimental and numerical investigations have been undertaken.

Chen and Kuo [3] solved numerically the compressible turbulent boundary layer equations with using Baldwin-Lomax model. Their calculated friction factor is higher than that of Blasius correlation and calculated temperature dropped well below the wall temperature at the outlet due to energy conversion into the kinetic energy.

Hara et al. [4] experimentally investigated heat transfer for high pressure ratio of air flow in square mini-channels whose size was 0.3 to 2.0 mm and length was 10 mm to 100 mm. They reported that
the heat transfer coefficient was about 7.3 times greater than Dittus and Boelter correlation for fully developed turbulent pipe flow. Turner et al. [5] performed experimental investigations to measure the inlet and outlet temperatures and the micro-channel wall temperature in thermal boundary conditions of constant temperature gradient along the micro-channel length. The experimental measurements of the inlet and outlet gas temperature and the micro-channel wall temperature were used to validate a two dimensional numerical model for the laminar gaseous flow in micro-channel. Asako [6], Asako and Toriyama [7] and Hong and Asako [8] performed numerical investigations to obtain the heat transfer characteristics of laminar gas flows in a micro-channel and in a micro-tube with constant wall temperature, whose wall temperature is lower or higher than the inlet temperature. In the case of slow flow (Ma_{in}<0.3), identical heat transfer coefficients are obtained for both heated and cooled cases. However, in the case of fast flow, different heat transfer coefficients are obtained for each cooled and heated case. The way to predict of the heat transfer rate of gaseous flow in a micro-channel and in a micro-tube has been proposed. Recently, Hong et al. [9] measured total temperatures at the outlet of a micro-tube of D=163-242 µm with constant wall temperature whose temperature is higher than the inlet temperature in the laminar flow regime in order to validate numerical results by Hong and Asako [8]. The bulk temperatures obtained experimentally are in excellent agreement with those of numerical results. They found that measured total temperatures at the outlet are higher than the bulk temperatures of incompressible due to the additional heat transfer from the wall to the gas. Chen et al. [10] investigated experimentally forced convective heat transfer characteristics of air and CO₂ in micro stainless steel tube with inside diameter of 920 µm. The tube wall was heated by electric current. They used a non-contacted liquid crystal thermography (LCT) temperature measurement method and thermocouples to measure the surface temperature of micro-tube. They concluded that the conventional heat transfer correlation of laminar and turbulent flow can be well applied for prediction of heat transfer performance in a micro-tube. However, Yang et al. [11] investigated experimentally and numerically the characteristics of nitrogen gas convective heat transfer in commercial stainless steel micro tubes with inner diameter of 175µm and 750 µm. Hong et al. [12] and Isobe et al. [13] investigated experimentally heat transfer characteristics of turbulent gas flow in micro tube with constant wall temperature. The experiments were performed for nitrogen gas flow through a micro-tube with 243 µm and 342 µm in diameter and 50 mm in length. The wall temperature was maintained at 310-350 K. They reported that heat transfer characteristics of turbulent gas flow in micro-tube with constant wall temperature are different from those of the incompressible flow. Additional heat transfer for turbulent flow is more dominant than that for laminar flow. As can be seen from the literature survey, there seems to be few numerical investigations on the heat transfer of turbulent gas flow through a micro-tube. Chen and Kuo [3] numerically investigated the heat transfer of turbulent gas flow through a micro-tube. However, in their paper, the effect of compressibility on turbulent heat transfer is not clear. Therefore, in turbulent flow regime the effect of compressibility on heat transfer has not been clarified totally yet. This has motivated the present numerical study to obtain heat transfer characteristics of turbulent gas flow in microtubes with constant heat flux.

2. Formulation

2.1. Description of the problem and conservation equations

The problem modelled in this study is depicted schematically in Figure. 1. The characteristics of turbulent gas flow in a microtube will be determined. A reservoir at the stagnation temperature, T_{stg}, and the stagnation pressure, p_{stg}, is attached to the upstream section of the microtube. The gas flows into the microtube and flows out to atmosphere. The wall temperature of the tube is assumed to be constant. The
flow assumed to be steady, axisymmetric and turbulent. Two-dimensional coordinates are used for computations. The fluid is assumed to be an ideal gas. The governing equations to be considered are the time-averaged compressible continuity, momentum, and energy equations [14]. An eddy viscosity model is used to account for the effect of turbulence. The model chosen is the Lam-Bremhorst low-Reynolds number form of the k-\( \varepsilon \) (turbulence kinetic energy-turbulence dissipation rate) model [15]. Constant thermophysical properties are assumed. The Reynolds stress of compressible flow is modeled by

\[
-\rho u_i u_j = \mu_1 \left\{ \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial \bar{u}_k}{\partial x_k} \right\} - \frac{2}{3} \delta_{ij} \bar{\rho} \bar{k}
\]

(1)

The governing equations can be expressed as

\[
\frac{\partial \rho u_i}{\partial x_j} + \frac{1}{r} \frac{\partial (\rho \bar{u}_i \bar{v})}{\partial r} = 0
\]

(2)

\[
\frac{\partial \rho \bar{v}_i}{\partial x} - \frac{\partial \rho \bar{u}_i}{\partial r} = \frac{\partial}{\partial x} \left( \bar{p} + \frac{2}{3} \bar{p} \bar{k} \right) + \frac{\partial \tau_{xx}}{\partial x} + \frac{1}{r} \frac{\partial \tau_{r\tau}}{\partial r} (\tau_{r\tau})
\]

(3)

\[
\frac{\partial \rho \bar{v}_i}{\partial x} + \frac{1}{r} \frac{\partial (\rho \bar{v}_i \bar{v})}{\partial r} = -\frac{\partial}{\partial r} \left( \bar{p} + \frac{2}{3} \bar{p} \bar{k} \right) + \frac{\partial \tau_{xx}}{\partial x} + \frac{1}{r} \frac{\partial \tau_{r\tau}}{\partial r} (\tau_{r\tau}) - \frac{\tau_{00}}{r}
\]

(4)

where the components of viscous stress tensor are

\[
\tau_{xx} = (\mu + \mu_i) \left\{ \frac{2}{3} \frac{\partial \bar{u}}{\partial x} - \frac{1}{r} \left( \frac{\partial \bar{u}}{\partial x} + \frac{1}{r} \frac{\partial \bar{v}}{\partial r} \right) \right\}, \quad \tau_{rr} = (\mu + \mu_i) \left\{ \frac{2}{3} \frac{\partial \bar{v}}{\partial r} - \frac{1}{r} \left( \frac{\partial \bar{v}}{\partial r} + \frac{1}{r} \frac{\partial \bar{v}}{\partial r} \right) \right\}
\]

(6)

\[
\tau_{00} = (\mu + \mu_i) \left\{ \frac{2}{3} \frac{\partial \bar{v}}{\partial r} - \frac{1}{r} \left( \frac{\partial \bar{v}}{\partial r} + \frac{1}{r} \frac{\partial \bar{v}}{\partial r} \right) \right\}
\]

\[
\phi = 2 \left\{ \frac{\partial \bar{u}}{\partial x} \right\}^2 + \left( \frac{\bar{v}}{r} \right)^2 + \left( \frac{\partial \bar{v}}{\partial r} \right)^2 - \frac{2}{3} \left( \frac{\partial \bar{u}}{\partial x} + \frac{1}{r} \frac{\partial \bar{v}}{\partial r} \right)^2 + \left( \frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{v}}{\partial r} \right)^2
\]

(7)

\[
\bar{\gamma}_i = \frac{R}{\gamma - 1} \bar{T} = \frac{1}{\gamma - 1} \bar{p}
\]

(8)

2.2. Turbulence model
In *k*-ε turbulence model, turbulence kinetic energy *k* and viscosity coefficient *µ* are determined solving the equations of transportation below

\[
\frac{\partial \rho \overline{u} k}{\partial x} + \frac{1}{r} \frac{\partial r \overline{u} k}{\partial r} = \frac{\partial}{\partial x} \left( \mu + \mu_t \right) \frac{\partial k}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( \mu + \mu_t \right) \frac{\partial k}{\partial r} + \left( \frac{\partial}{\partial x} \rho + \frac{\partial}{\partial r} \rho \right)
\]

\[\text{(9)}\]

\[
\frac{\partial \rho \epsilon}{\partial x} + \frac{1}{r} \frac{\partial r \rho \epsilon}{\partial r} = \frac{\partial}{\partial x} \left( \mu + \mu_t \right) \frac{\partial \epsilon}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( \mu + \mu_t \right) \frac{\partial \epsilon}{\partial r} + c_1 f_1 \frac{\rho \epsilon^2}{k} - c_2 f_2 \frac{\rho \epsilon^{\frac{3}{2}}}{k}
\]

\[\text{(10)}\]

\[
P_t = \mu \rho - \frac{2}{3} \overline{\rho} k \left( \frac{\partial \overline{u} k}{\partial x} + \frac{1}{r} \frac{\partial r \overline{u} k}{\partial r} \right)
\]

\[\text{(11)}\]

\[
\mu_t = C_{\mu} f_\mu \rho k^2
\]

\[\text{(12)}\]

The various ways have been presented to model the coefficients appearing these equations [16]. In this paper, the Lam- Bremhorst low Reynolds number model was adopted and the following values are used for coefficients

\[
C_t = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.3, \quad C_1 = 1.44, \quad C_2 = 1.92
\]

\[\text{(13)}\]

\[
f_1 = 1 + \left( \frac{0.05}{f_\mu} \right)^3, \quad f_2 = 1 - e^{-R_T^{1/2}}, \quad f_\mu = \left( 1 - e^{-0.163 R_T} \right) \left( 1 + \frac{20.5}{R_T} \right)
\]

\[\text{(14)}\]

\[
R_T = \frac{\overline{\rho} k^2}{\epsilon^{\frac{3}{2}}}, \quad R_y = \frac{\overline{\rho} k y_w}{\mu}
\]

\[\text{(15)}\]

where *y_w* is defined as the minimum distance from the tube wall. Turbulent thermal conductivity in Eq. (5) is

\[
\lambda_t = \frac{C_p \rho H_t}{\frac{Pr}{Pr_t}}
\]

\[\text{(16)}\]

where *Pr_t* is the turbulent Prandtl number and its value is 0.9.

### 2.3. Boundary conditions

In the range of the micro tube sizes considered in this paper, Knudsen number is less than 0.001 for the atmospheric pressure and the room temperature so that the effect of rarefaction can be neglected. Therefore no-slip condition at the wall is used for velocity. Also, it is assumed that the tube wall is constant heat flux and at the tube inlet the distributions of velocity, pressure, temperature and density are uniform. From the assumptions above, the boundary conditions are described as follows

- on the wall: \( u = v = 0, \quad \frac{\partial T}{\partial r} = \dot{q} \)
- on the symmetric axis: \( \frac{\partial u}{\partial r} = v = 0, \quad \frac{\partial T}{\partial r} = 0 \)
- at the inlet: \( u = u_{in}, \quad v = 0, \quad p = p_{in}, \quad \rho = \rho_{in}, \quad T = T_{in} \)
- at the outlet: \( p = p_{out} \)

The values of velocity, pressure, temperature and density at the inlet are evaluated by the method presented by Kariki [17] as described below. For the cells adjacent to the symmetric axis, pressure of the *n*-th cell from the tube inlet is expressed as *p_n*. At the time *t*, pressure of the cell adjacent to the inlet, (*p_{n+1}\_guess*), is extrapolated from *p_2* and *p_3*. Then using this (*p_{n+1}\_guess* and (*p_1\_old* at the time *t*, the inlet pressure *p_{n+1} at the time *t + Δt* is calculated as

\[
p_{in} = \omega (p_{n+1\_old}) + (1 - \omega)(p_{n+1\_guess})
\]

\[\text{(17)}\]
where $\omega$ is relaxation factor and the value of 0.9 is chosen in this paper. Assuming isentropic change from stagnation point to the tube inlet, $\rho_{in}$, $T_{in}$ and $u_{in}$ at the time $t + \Delta t$ are calculated using $p_{in}$ of Eq. (17) as

$$\frac{p_{in}}{p_{stg}} = \left(\frac{\rho_{in}}{\rho_{stg}}\right)^{\gamma} = \left(\frac{T_{in}}{T_{stg}}\right)^{\gamma/(\gamma - 1)}$$  \hspace{1cm} (18)

$$u_{in} = \sqrt{\frac{2\gamma RT_{stg}}{\gamma - 1} \left[1 - \left(\frac{p_{in}}{p_{stg}}\right)^{(\gamma - 1)/\gamma}\right]}$$  \hspace{1cm} (19)

$k$ and $\varepsilon$ at the inlet is determined as follows

$$k_{in} = \frac{3}{2} (t_u u_{in})^2$$  \hspace{1cm} (20)

$$\varepsilon_{in} = 0.1 k_{in}^2$$  \hspace{1cm} (21)

where $t_u$ is ranges from 1% to 6%.

2.4. Dimensionless variables

Attention will now be focused on the calculation of the Reynolds and Mach numbers that will be defined as

$$Re = \frac{2\dot{m}}{\mu} = \frac{\rho_{ave} u_{ave} D}{\mu}, \hspace{0.5cm} Ma = \frac{u_{ave}}{\sqrt{\gamma(\gamma - 1)\rho_{ave}}}$$  \hspace{1cm} (22)

where $\dot{m}$ is the mass flow rate per unit depth and $D$ is the tube diameter. $u_{ave}$, $\rho_{ave}$ and $i_{ave}$ are the average velocity, density and specific internal energy at a cross-section.

$$u_{ave} = \frac{1}{A} \int u dA, \hspace{0.5cm} \rho_{ave} = \frac{1}{A} \int \rho u dA / \int u dA, \hspace{0.5cm} p_{ave} = \frac{1}{A} \int p dA, \hspace{0.5cm} i_{ave} = \frac{1}{\gamma - 1} \frac{p_{ave}}{\rho_{ave}}$$  \hspace{1cm} (23)

The product of friction factor and Reynolds number is called Poiseuille number. The friction factors based on the Darcy’s and Fanning definitions will be introduced. The Darcy friction factor is defined as

$$f_d = \frac{-2D}{\rho_{ave} u_{ave}^2} \left(\frac{dp_{ave}}{dx}\right)$$  \hspace{1cm} (24)

The modified Fanning friction factors (four times of Fanning friction factor) is based on the wall shear stress and is defined as

$$f_f = \frac{4\tau_w}{(1/2)\rho_{ave} u_{ave}^2} = 2D \left(\frac{dp_{ave}}{dx}\right) \left[\frac{1}{\rho_{ave}} - \frac{1}{\rho_{ave} u_{ave}^2}\right]$$  \hspace{1cm} (25)

2.5. Numerical solution

The numerical methodology is based on the Arbitrary-Lagrangian-Eulerian (ALE) method proposed by Amsden et al. [18]. The details of the ALE method are documented in the literature and will not be given here. The computational domain is divided into quadrilateral cells. The velocity components are defined at the vertices of the cell and other variables such as pressures, specific internal energy and density are assigned at the cell centers. The number of cells in the $x$-direction was 200. The cell size gradually increased in the $x$-direction to the mid of the tube and it gradually decreases to the outlet of the tube. The number of cells in $r$-direction was fixed at 40 for all the components. The ALE method
Table 1  Tube diameter, length, p_{stag} Re and Ma

| # | D (µm) | L (m) | p_{stag} (kPa) | p_{out} (kPa) | Re | Ma_{in} | Ma_{out} | flow     |
|---|--------|-------|----------------|--------------|----|---------|---------|----------|
| 1 | 200    | 0.04  | 1450           | 1400         | 4497 | 0.070   | 0.075   | slow     |
| 2 | 300    | 100   | 2947           | 0.228        | 0.660 | fast    |
| 3 | 350    | 3575  | 0.238          | 0.778        |      |         |         |          |
| 4 | 400    | 4208  | 0.246          | 0.889        |      |         |         |          |
| 5 | 400    | 0.08  | 200            | 100          | 3770 | 0.218   | 0.446   | fast     |
| 6 | 250    | 5180  | 0.242          | 0.591        |      |         |         |          |
| 7 | 300    | 6541  | 0.255          | 0.725        |      |         |         |          |
| 8 | 350    | 7862  | 0.264          | 0.846        |      |         |         |          |
| 9 | 400    | 9143  | 0.269          | 0.951        |      |         |         |          |

is a time marching method. The value of 10^{-4} was used for convergence criterion of Newton-Raphson iteration in the interior loop of the ALE method.

3. Results and discussion

The computations were performed for the tube with the constant heat flux of \( \dot{q} = 10^2, 10^3 \) and \( 10^4 \) W/m². The working fluid was nitrogen and it was assumed to be ideal gas. The thermo-physical properties of \( R = 296.7 \text{ J/(kg·K)} \), \( \gamma = 1.399, \mu = 1.787\times10^{-5} \text{ Pa·s} \) and \( \lambda = 0.0260 \text{ W·(m·K)}^{-1} \) at 300K were used for the computations. The diameter of the tube was 200 and 400 µm and the aspect ratio of the tube height and length is 200. The stagnation temperature was kept at \( T_{stag} = 300 \text{ K} \). The stagnation pressure, \( p_{stag} \) was chosen in such away that the Mach number at the exit ranges from 0.1 to 1.0. The outlet pressure maintained at atmospheric condition, \( p_{out} = 100 \text{ kPa} \). Additionally, in order to obtain turbulent slow flow, the higher outlet pressure maintained with the 1400 kPa for D=200 µm was computed. The tube diameter, the tube length, the stagnation pressure and the corresponding Reynolds & Mach numbers at the inlet and outlet for \( \dot{q} = 10^4 \text{ W/m}^2 \) are listed in table 1. The Reynolds number obtained ranges from 2947 to 9143, and the Mach number at the outlet ranges from 0.070 to 0.951 widely.

3.1. Velocity vectors and contour plot of temperature

Velocity vectors for the case #1 and #4 in table 1 are presented in Figure 2. Contour plots of temperature are also presented in Figure 3. These are typical velocity vectors and temperature contour plots for the combination of slow and fast flows. Figure 2 shows the velocity vectors of gas flow with uniform velocity at the inlet. The flows are turbulent since the shape of the velocity profile except near the inlet is almost flat. Also, obtained Reynolds number is well lager than the critical Reynolds number. The gaseous flow is accelerated in a microtube. Therefore, Mach number at the outlet is greater than that at the inlet, \( Ma_{in} < Ma_{out} \). The flow whose Ma at the outlet is less than 0.3 is called as “slow flow” and the flow whose Ma at the outlet is greater than 0.3 is called as “fast flow”. As can be seen in Figure 3 (a), in the case of the slow flow (\( Ma_{out} < 0.3 \)), the temperature rises gradually along the tubes through the influence of the constant heat flux. This is the similar temperature contour to that of the incompressible flow since the temperature contour begins to develop along the tube length with parabolic curves depending on the constant heat flux. On the other hand, in the case of the fast
Figure 2. Velocity vector for slow (a) and fast flow (b)

Figure 3. Contour plots of temperature for slow (a) and fast flow (b)

Figure 4. Ma as a function of x
flow (Ma_out > 0.3), the temperature fall can be seen near the outlet due to the energy conversion into the kinetic energy caused by the flow acceleration in the core region of the tube (Figure 3 (b)). The wall temperature decreases approaching to the outlet due to the temperature fall in the core region. This is the typical temperature contour of the compressible flow.

3.2. Mach number and friction factor

The product of the friction factor and Reynolds number, $f_1 \cdot Re$ and $f_d \cdot Re$ of two cases for Figures 2 and 3 are plotted as a function of x in Figures 4 (a) and (b), respectively. Corresponding Mach numbers are also plotted in Figure 5. As can be seen in Figures 4 (a) and (b), the tube can be divided into two
regions. The region where the friction factor decrease sharply for both cases can be considered the entrance region. In the case slow flow, the region after the entrance region can be considered as fully developed region. However, in the case of fast flow, the region after the entrance region can be considered as the quasi-fully developed region. Then the Mach number increases along the tube due to acceleration of the flow. The value of $f \cdot Re$ of fast flow almost coincide with that of slow flow since the viscous wall friction loss for turbulent flow depends on Reynolds number. The value of $f_d \cdot Re$ of fast flow is higher than that of slow flow since $f_d \cdot Re$ includes the acceleration loss. And the value of $f_l \cdot Re$ of fast flow agrees with that of slow flow since $f_l \cdot Re$ do not include flow acceleration loss. The $f_l$ and $f_d$ obtained for all case of the present numerical calculations plotted as a function on Moody chart in Figure 6. Then, the $f_l$ and $f_d$ at the locations where Mach number indicated just 0.3, 0.4, 0.5, 0.6 and 0.7 are extracted from numerical results. The solid line and dashed line represent conventional correlation of friction factor for incompressible flow ($f = \frac{64}{Re}$ for laminar flow and $f = 0.316 Re^{0.25}$ (Blasius correlation) for turbulent flow). $f_l$ is almost coincide with conventional value given by Blasius correlation. As can be seen in Figure 2, gradient of the normalized velocity at the wall is almost constant even if Mach number changes, so $f_l$ depends only on Reynolds number. However, the plotted values of $f_d$ at constant Mach number are aligned on a straight lines. These lines are parallel to the line of Blasius correlation ($f = 0.316 Re^{0.25}$) and shift upwards as Mach number increases. The ratio of $f_d$ to $f_l$ or Blasius correlation is dependent on only Mach number and independent on Reynolds number.

3.3. Wall temperature and Nusselt number of incompressible flow

In the case of the incompressible flow, the local Nusselt number, based on the temperature difference between the wall and the bulk temperatures, of a simultaneously developing flow in a duct with the constant heat flux is defined as

$$Nu_x = \frac{\dot{q}D}{(T_w - T_b)k}$$  \hspace{1cm} (26)

If the flow is incompressible, the bulk temperature is expressed by the function of $x$ as

$$T_b = T_m + \frac{2\dot{q}x}{\rho C_p \int u dA}$$  \hspace{1cm} (27)
The wall temperature of an incompressible flow in a duct is expressed as a function of a local Nusselt number and the location as

\[
T_w = \left( 4X^* + \frac{1}{\text{Nu}_x} \right) \left( \frac{\dot{q}D}{k} \right) + T_{in}
\]

where \(X^*\) is the inverse of Graetz number defined by

\[
X^* = \frac{x}{D \text{ Re Pr}}
\]

Equation (28) can be rewritten as

\[
\frac{T_w - T_{in}}{\dot{q}D/k} = 4X^* + \frac{1}{\text{Nu}_x}
\]

The dimensionless wall temperature of an incompressible flow in a duct is a function of \(X^*\) and \(\text{Nu}_x\). The turbulent heat transfer characteristics for incompressible duct flows has been investigated by many researchers and Nusselt numbers were reported in literatures (e.g., Kays and Crawford [19]). In the present study, the obtained \(\text{Nu}\) for the turbulent fully developed flow by Kays and Crawford [19] defined by

\[
\text{Nu} = 0.022 \text{Re}^{0.8} \text{Pr}^{0.5}
\]

was used for the calculation of the wall temperature for the incompressible flow.

3.4. Estimation of wall temperature

The dimensionless wall temperature, \((T_w - T_{in})/(\dot{q}D/k)\), for two cases of the tubes of D=200 µm for \(\dot{q}=10^4\) W/m² is plotted as a function of \(X^*\) in Figure 7. The dimensionless wall temperature for the incompressible flow obtained by Equation (30) is also plotted in the figure. As can be seen in Figure 2 in the case of slow flow (#1), the dimensionless wall temperature increases along the dimensionless length and it coincides with that of incompressible flow. On the other hand, in the case of fast flow (#4), the dimensionless wall temperature increases gradually along the dimensionless length and level off and decreases quickly approaching to the outlet due to the conversion of the thermal energy into the kinetic energy. The qualitative same tendency can be seen for the tubes with D=400µm. Therefore, in the case of fast flow (\(\text{Ma}_{out}>0.3\)), the wall temperature of the gas flow in the microtube
The dimensionless wall temperature differences, \((T_w - T_{w,icomp}) / (\dot{q} D / \kappa)\), between the gas flow and incompressible flow for two cases of the tubes of \(D=200\, \mu m\) for \(\dot{q}=10^4\, W/m^2\) are plotted as a function of \(X^*\) in Figure 8 (a). Also, the dimensionless bulk temperature differences, \((T_b - T_{b,icomp}) / (\dot{q} D / \kappa)\), between the gas flow and incompressible flow are plotted as a function of \(X^*\) in Figure 8 (b). In the case of slow flow, the dimensionless wall and bulk temperatures coincide with those of incompressible flow. Therefore, there is no difference between them as shown in Figure 8 (a) and (b). On the other
hand, in the case of fast flow, the dimensionless wall and bulk temperature difference decreases approaching to the outlet.

Note that the bulk temperature difference between the gas flow and incompressible flow represents the dynamic temperature of the fluid, \( T_k \), defined as

\[
T_k = \frac{\int \rho u u^2 dA}{2 \int \rho C_p u dA}
\]

(32)

The gas temperature decreases near the outlet due to the energy conversion into the kinetic energy as shown in Figure 3. If the thermal conductivity of the gas is extremely high, the wall temperature falls corresponding to the fall of the bulk temperature. However, in actual situation, the thermal conductivity of the gas is low, therefore the fall of the wall temperature is smaller than that of the bulk temperature as shown in Figure 8 (a) and (b). Then, the ratio of wall temperature difference between the gaseous flow and incompressible flow and the dynamic temperature, \( \zeta \) is defined as

\[
\zeta = \frac{T_{w,\text{comp}} - T_w}{T_k}
\]

(33)

The ratio, \( \zeta \) calculated from Equation (33) for fast flow cases of Table 1 is plotted in Figure 9 as a function of the Mach number at the outlet. The ratio calculated for \( q^\prime = 10^2 \) W/m\(^2\) and \( q^\prime = 10^3 \) W/m\(^2\) are plotted in the figure. For the case of slow flow, the flow can be assumed to be an incompressible flow, and the temperature differences between the wall temperature and those of incompressible flow and the dynamic temperature are quite small, therefore, it is not plotted in Figure 9. The dashed line represent the ratio for laminar flow obtained by the previous study (Hong et al. [20]). The ratio, \( \zeta \) is independent of hydraulic diameter and decreases with increasing the Ma at the outlet. Eq. (33) can be rewritten as

\[
T_w = T_{w,\text{comp}} - \zeta T_k
\]

(34)

Then, the wall temperature of tubes with constant heat flux can be predicted from the wall temperature of the incompressible flow, the ratio, \( \zeta \) and the dynamic temperature. However, the ratios for \( q^\prime = 10^2 \) W/m\(^2\) and \( q^\prime = 10^3 \) W/m\(^2\) scattered since their heat flux are relatively not large for the turbulent fast flow (Ma\(_{\text{out}}\) > 0.3) of D \( \geq 200 \) \( \mu \)m.
4. Concluding remarks

Two-dimensional compressible momentum and energy equations are solved to obtain the heat transfer characteristics of turbulent gas flow in microtubes with constant heat flux. The Lam-bremhorst Low Reynolds number turbulence model was adopted to simulate turbulent flow cases. The following conclusions are obtained.

1. For turbulent flow, Fanning friction factor, \( f_f \) is almost equal to the value given by Blasius correlation. However, Darcy friction factor, \( f_d \) with acceleration loss is higher than \( f_f \) or Blasius correlation.

2. In the case of slow flow, the identical temperature profiles normalized by heat flux are obtained with those of incompressible flow. However, in the case of fast flow in a microtube, different temperature profiles are obtained.

3. The wall temperature of a microtube can be predicted from the wall temperature of the incompressible flow, the ratio, \( \zeta \) and the dynamic temperature.

\[
T_w = T_{w,comp} - \zeta T_k
\]

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