PROBABILISTIC PERIODIC REVIEW \( \langle Q_m, N \rangle \) INVENTORY MODEL USING LAGRANGE TECHNIQUE AND FUZZY ADAPTIVE PARTICLE SWARM OPTIMIZATION

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Received 2013-09-30; Revised 2014-06-11; Accepted 2014-08-09

ABSTRACT

The integration between inventory model and Artificial Intelligent (AI) represents the rich area of research since last decade. In this study we investigate probabilistic periodic review \( \langle Q_m, N \rangle \) inventory model with mixture shortage (backorder and lost sales) using Lagrange multiplier technique and Fuzzy Adaptive Particle Swarm Optimization (FAPSO) under restrictions. The objective of these algorithms is to find the optimal review period and optimal maximum inventory level which will minimize the expected annual total cost under constraints. Furthermore, a numerical example is applied and the experimental results for both approaches are reported to illustrate the effectiveness of overcoming the premature convergence and of improving the capabilities of searching to find the optimal results in almost all distributions.

Keywords: Inventory System, Periodic Review Model, Particle Swarm Optimization, Fuzzy Adaptive Particle Swarm Optimization

1. INTRODUCTION

In some cases while a few customers are ready to wait till the next arrival of stock (backorder case), the remaining may be impatient and would persist on satisfying their demand immediately from some other sources (lost sales case). Inventory models which involve both backorders and lost sales are known as models with a mixture shortage. First solution to such a model was derived by (Montgomery et al., 1973). A similar model for variable lead time with fixed reorder point was analyzed by (Quyang and Wu, 1996). Hariga and Ben Daya (1999) discussed both periodic and continuous review models with a mixture of backorders and lost sales in case of full and partial demand information. Abuo-El-Ata et al. (2002) studied probabilistic multi-item Inventory model with varying order cost under two restrictions. Fergany (2005) described periodic review model with zero lead time under constraints and varying order cost. Fergany and Elwakeel (2006) introduced constrained probabilistic lost sales inventory system with normal distribution and varying order cost.

After appear the shortcomings of the traditional methods to deal with complexities of nonlinear programming with low and high dimensions, Particle Swarm Optimization (PSO) appears which has exhibited good performance for solving problems in wide range of applications such as in engineering design and computer science whereas the PSO is easy to implement in computer simulations. The PSO has fewer operators to adjust in the implementation and it can be flexibly combined with other optimization techniques to build a hybrid algorithm, the mechanism of PSO facilitates better convergence performance than some other optimization procedures.

A new optimizer using particle swarm theory derived by (Eberhart and Kennedy, 1995). Tasgetiren and Liang (2003) presents a binary particle swarm optimization algorithm for lot sizing problem. Andries (2005)
discussed the fundamentals of computational swarm intelligence. Kang et al. (2006) studied a novel Fuzzy Adaptive Strategy Optimization (FAPSO) for the particle swarm algorithm. Xiaobin et al. (2007) described fuzzy economic order quantity inventory models without backordering. Parsopulos et al. (2008) investigate particle swarm optimization for tackling continuous review inventory models. Kannan et al. (2009) introduced analysis of closed loop supply chain using genetic algorithm and PSO. Taleizadeh et al. (2010) studied a particle swarm optimization approach for constraint joint single buyer—single vendor inventory problem with changeable lead time and (r, Q) policy in supply chain. Hiremath et al. (2010) discussed optimization for efficient supply chain using swarm intelligence: An Outsourcing Perspective. Piperagkas et al. (2011) applying PSO and DE on multi-item inventory problem with supplier selection.

In this study, we investigate a probabilistic Single-Item Single-Source (SISS) inventory model with varying mixture shortage cost under two restrictions, which one is on the expected backorder cost and the other is on the expected lost sales cost. The optimal maximum inventory level $Q_m$, the optimal time between reviews $N^*$ and the minimum Expected Total Cost (min E (TC)) are obtained, some special cases are deduced, some distributions are implemented and results of numerical computations for optimum parameters of this model using Lagrange multiplier technique and fuzzy adaptive particle swarm optimization and their comparisons are presented.

The rest of this study is organized as follows: Section 2 presents constrained probabilistic single item periodic review model $<Q_m, N>$ model with Varying Mixture Shortage. Section 3 presents Standard Particle Swarm Optimization (SPSO). Section 4 the solution procedure of FAPSO is proposed. Section 5 presents experiments and results of numerical example to test the validity and performance of the approach. Section 6 presents a comparative study between two approaches. Section 7 concludes the study and future work.

2. CONSTRAINED PROBABILISTIC SINGLE ITEM PERIODIC REVIEW $<Q_m, N>$ MODEL WITH VARYING MIXTURE SHORTAGE

The following assumptions are made for developing the model:

- A sufficient quantity is ordered to bring the inventory level up to level $Q_m$, where $Q_m$ is the maximum inventory level
- The varying backorder cost for the item per period is $C_bN^2$, where $C_b$ is the backorder cost per period, the varying lost sales cost for the item per period is $C_L N^2$, where $C_L$ is the lost sales cost per period and the cost is independent of the length of time for which the backorder and lost sales exists and $\beta$ are constant real numbers selected to provide the best fit of estimated expected cost function
- The behavior of the periodic review system with partial backorders and lost sales case shown in Fig. 1.

The expected annual total cost is the sum of the expected review cost, expected ordering cost, expected holding cost, expected backorder cost and the expected lost sales cost respectively Equation 1:

$$E(\text{TC}) = E(\text{RC}) + E(\text{OC}) + E(\text{HC}) + E(\text{BC}) + E(\text{LC})$$

Where:

$$E(\text{RC}) = \frac{C_r}{N}, \text{the cost of making a review}$$

$$E(OC) = \frac{C_o}{N}$$

$C_r$ the cost of placing an order (ordering cost) per period

The expected number of backorders incurred per year $E(\text{BC})$ is the expected number of backorders incurred per period multiplied by the number of orders per year. Consider the first case where $L$ is constant (where $L$ the lead time between the placement of an order and its receipt). The expected number of backorders incurred per period is $\int_{0}^{L} (x - Q_m) f(x; L + N) dx$, where $x$ is the demand between the time an order is placed and the time this order arrives (the demand during lead time), $f(x; L+N)$ is the probability density function of the lead time demand $x$ during the time interval of length $L+N$.

Suppose now that the lead time $L$ is a random variable with density $g(L)$. Let $L_{\text{min}}$, $L_{\text{max}}$ be the lower and upper limits respectively to the possible range of the lead time values. If $L_1$, $L_2$ are the lead times for the orders placed at times $t$ and $t+N$ respectively, then the expected number of backorders incurred per period is given by:

$$\mathbb{E}(Q_m) = \int_{L_{\text{min}}}^{L_{\text{max}}} \int_{Q_m}^{Q_2} \int_{Q_m}^{Q_2} \int_{0}^{L_2} (x - Q_m) f(x; L_2 + N) g(L_2) dx dL_2 dL_1 = \int_{Q_m}^{Q_2} \int_{0}^{L_2} (x - Q_m) h(x; N) dx$$
Where:

\[
\int_{\min}^{\max} g(L)\,dL = 1, h(x; N)
\]

\[
= \int_{\min}^{\max} f(x; L_2 + N)g(L_2)\,dL_2
\]

It is necessary to known that \( L_{\max} < L_{\min} + N \), hence the expected backorder cost incurred per year Equation 2 and 3:

\[
E(BC) = C_h N^{\beta-1} \mathbb{S}(Q_n)
\]

\[
= C_h N^{\beta-1} \mathbb{V} \int_{0}^{\infty} (x - Q_n) h(x; N)\,dx
\]

\[
E(LC) = C_L (1 - \gamma) N^{\beta-1} \mathbb{S}(Q_n)
\]

\[
= C_L (1 - \gamma) N^{\beta-1} \int_{0}^{\infty} (x - Q_n) h(x; N)\,dx
\]

where, \( \gamma \) is a fraction of unsatisfied demand that will be backorder while the remaining fraction \((1 - \gamma)\) is completely lost.

\[
E(HC) = \frac{C_h I}{N}, \quad C_h \text{ is the holding (carrying) cost of the item per period and } T \text{ the average inventory level per period. Let } \mathcal{E}(x; Q_n) \text{ be the on hand inventory when the procurement arrives. If the lead time demand is } x \text{ and the maximum inventory level immediately after the arrival of the procurement is } Q_n, \text{ then:}
\]

\[
\mathcal{E}(x; Q_n) = \begin{cases} 
Q_n - x, & x \geq 0 \\
0, & Q_n - x < 0 
\end{cases}
\]

And the expected on hand inventory when the procurement arrives (safety stock) is:

\[
\mathcal{E}(x; Q_n) = \int_{0}^{Q_n} \mathcal{E}(x; Q_n) h(x; N)\,dx = \int_{0}^{Q_n} (Q_n - x) h(x; N)\,dx
\]

\[
= Q_n - \mu + \int_{Q_n}^{\infty} (x - Q_n) h(x; N)\,dx
\]

\( \mu \) is the expected lead time demand.

Hence if the expected on hand inventory immediately after the arrival of a procurement is \( S \), it is therefore \( S - DN \) gust prior to the arrival of a procurement in the next period. Thus the expected on hand inventory varies between \( S \) and \( S - DN \). where \( DN \) is the expected demand during the time \( N \) between reviews and \( D \) is the average demand rate. Then the average inventory level per period is given by:

\[
\mathcal{T} = (\text{the expected inventory level (at the beginning of an inventory period) at the end of the period)}/2:
\]

\[
\mathcal{T} = N \left( Q_n - \mu - \frac{DN}{2} + (1 - \gamma) \int_{Q_n}^{\infty} (x - Q_n) h(x; N)\,dx \right)
\]

Which yields Equation 4:

\[
E(HC) = \frac{C_h}{N} \int_{0}^{Q_n} \mathcal{E}(x; Q_n) h(x; N)\,dx
\]

\[
= C_h \left( Q_n - \mu - \frac{DN}{2} + (1 - \gamma) \mathbb{S}(Q_n) \right)
\]

Fig. 1. The behavior of the periodic review system with partial backorders and lost sales case
Then the expected annual total cost is given by:

$$E(TC(Q_m,N)) = \frac{C_s}{N} + \frac{C_w}{N} + C_i \left( Q_m - \frac{DN}{2} \right)$$

$$+ \left[ C_y N^{\beta - 1} + \left( C_i N^{\beta - 1} + C_b \right) (1 - \gamma) \right]$$

$$\int_{0}^{\infty} (x - Q_m) h(x;N) dx$$

(5)

$$= \frac{C_s}{N} + \frac{C_w}{N} + C_i \left( Q_m - \frac{DN}{2} \right)$$

$$+ \left[ C_y N^{\beta - 1} + \left( C_i N^{\beta - 1} + C_b \right) (1 - \gamma) \right] \mathcal{S}(Q_m)$$

The objective is to determine the optimal values $Q_m^*$, $N'$ that minimize the expected annual total cost $E(\text{TC}(Q_m, N))$ under the following constraints:

$$E(BC) \leq K_b \quad E(\text{LC}) \leq K_L$$

(6)

To solve this primal function which is a convex programming problem, Equation 5 and 6 can be written in the following form:

$$\text{Min} \left[ E(\text{TC}(Q_m, N)) \right]$$

$$= \frac{C_s}{N} + \frac{C_w}{N} + C_i \left( Q_m - \frac{DN}{2} \right)$$

$$+ \left[ C_y N^{\beta - 1} + \left( C_i N^{\beta - 1} + C_b \right) (1 - \gamma) \right]$$

$$\int_{0}^{\infty} (x - Q_m) h(x;N) dx$$

Subject to Equation 7:

$$C_y N^{\beta - 1} \mathcal{S}(Q_m) \leq K_b \quad C_i (1 - \gamma) N^{\beta - 1} \mathcal{S}(Q_m) \leq K_L$$

(7)

where, $\min \left( E(\text{TC}(Q_m, N)) \right)$ is the minimum expected annual total cost function.

To find the optimal values $Q_m$ and $N'$ which minimize Equation 5 under the two constraints (6), we can use the Lagrange multipliers technique with the Kuhn-Tacker conditions as follows:

$$G(Q_m,N) = \frac{C_s}{N} + \frac{C_w}{N} + C_i \left( Q_m - \frac{DN}{2} \right)$$

$$+ \left[ C_y N^{\beta - 1} + \left( C_i N^{\beta - 1} + C_b \right) (1 - \gamma) \right]$$

$$\int_{0}^{\infty} (x - Q_m) h(x;N) dx$$

$$+ \lambda_0 \left( C_y N^{\beta - 1} \mathcal{S}(Q_m) - K_b \right)$$

$$+ \lambda_\beta \left( C_i (1 - \gamma) N^{\beta - 1} \mathcal{S}(Q_m) - K_L \right)$$

(8)

where, $\lambda_0$ and $\lambda_\beta$ are the Lagrange multipliers.

The optimal values $Q_m$ and $N'$ can be calculated by setting the corresponding first partial derivatives of Equation 8 equal to zero, as follows:

$$\frac{\partial G(Q_m,N)}{\partial Q_m} \bigg|_{Q_m=Q_m^*,N'=N'} = 0$$

Hence the optimal maximum inventory level is the solution of the following Equation 9:

$$\int_{0}^{\infty} h(x;N) dx = \frac{C_s}{C_i} \left( \left(1 + \lambda_0 \right) C_y N^{\beta - 1} + (1 - \gamma) \left( \left(1 + \lambda_\beta \right) C_i N^{\beta - 1} + C_b \right) \right)$$

(9)

Clearly, there is no closed form solution of Equation 9 so we minimize the expected annual total cost numerically. The following algorithm is used:

Step 1: Assume that any initial value for $N$, put $\lambda_0, \lambda_\beta$ equal to zero then from Equation 9 we have the initial maximum inventory level $Q_m$ as follows:

$$\int_{0}^{\infty} h(x;N) dx = \frac{C_s}{C_i} \left( \left(1 + \lambda_0 \right) C_y N^{\beta - 1} + (1 - \gamma) \left( \left(1 + \lambda_\beta \right) C_i N^{\beta - 1} + C_b \right) \right)$$

Step 2: Assume different values for $N$ and varying $\beta$ then substituting about them in Equation 9 hence we get different values of the maximum inventory level i.e., for $N_i$ we get $Q_m$, $i=1,...,m$.

Step 3: Substituting $N_i, Q_m$ in Equation 5 then we have different values of total cost until we get the minimum total cost then the correspondence value of $N$ is the optimal value i.e., $N'$.

Step 4: The procedure is to vary $\lambda_0$ and $\lambda_\beta$ in steps 2 and 3 using $N'$ until the smallest value of $\lambda_0>0$ and $\lambda_\beta>0$ that achieves the constraints for different values of $\beta$. Hence we get $Q_m^*$ that gives the minimum annual expected total cost numerically.

2.1. Special Cases

Case (1): Let $\beta = 0$, $\lambda_0 = 0$ and $\gamma = 1$. Thus Equation 9 becomes:

$$\int_{0}^{\infty} h(x;N) dx = \frac{C_s N}{C_i}$$
This is unconstrained periodic review inventory model for backorder case with constant units of cost, which are the same results as in (Hadley and Whitin, 1963).

Case (2): Let \( \beta = 0 \), \( \lambda_r = 0 \) and \( \gamma = 0 \). Thus, Equation 9 become:

\[
\int_{\alpha_m}^{\infty} h(x; N) dx = \frac{C \cdot N}{C_L + C_H}
\]

This is unconstrained periodic review inventory model for lost sales case with constant units of cost, which are the same results as in (Hadley and Whitin, 1963).

2.2. The Model with Normally Lead Time Demand

In this section, we assume that the lead-time demand follows normal distribution with mean \( \mu \) and standard deviation \( \sigma \), i.e.:

\[
E(x) = \mu, \quad V(x) = \sigma^2 f(Q_m) = \phi \left( \frac{Q_m - \mu}{\sigma} \right)
\]

where, \( \phi(z) = N(z; 0, 1) \) is the probability density function of the standard normal distribution, \( \Phi(z) \) is the cumulative distribution function of the standard normal distribution. Where \( z = \frac{Q_m - \mu}{\sigma} \). Substituting in Equation 9, the optimal maximum inventory level is the solution of the following Equation 10:

\[
1 - \Phi \left( \frac{Q_m - \mu}{\sigma} \right) = \frac{C_L}{(1 + \lambda_r) C_L \cdot N^{\beta - 1} + (1 - \gamma) \left[(1 + \lambda_r) C_L \cdot N^{\beta - 1} + C_h\right]}
\]

In addition, the expected number of backorder incurred per period will be:

\[
\overline{S} (Q_m) = \int_{0}^{\infty} (x - Q_m) h(x; N') dx = \frac{1}{\sigma \sqrt{2 \pi}} \int_{0}^{\infty} h(x; N') dx \cdot \left[ 1 - \Phi \left( \frac{Q_m - \mu}{\sigma} \right) \right] + \Phi \left( \frac{Q_m - \mu}{\sigma} \right)
\]

Substituting from Equation 11 to 5 then the expected annual total cost will be:

\[
\begin{align*}
\min (E(TC(Q_m, N))) & = \frac{C_L}{N} + \frac{C_H}{N} + C_L \left( Q_m - DI - \frac{DN}{2} \right) \\
& + \left[ C_L \cdot N^{\beta - 1} + (1 - \gamma) \left[(1 + \lambda_r) C_L \cdot N^{\beta - 1} + C_h\right] \right] \\
& \left[ \Phi \left( \frac{Q_m - \mu}{\sigma} \right) + \left( \mu - Q_m \right) \left[ 1 - \Phi \left( \frac{Q_m - \mu}{\sigma} \right) \right] \right]
\end{align*}
\]

Subject to:

\[
C_h \cdot N^{\beta - 1} \overline{S} (Q_m) \leq K_L \quad (1 - \gamma) \cdot N^{\beta - 1} \overline{S} (Q_m) \leq K_L \quad (12)
\]

2.3. The Model with Uniform Lead-Time Demand

If the lead-time demand follows the Uniform distribution over range from zero to \( b \) then, the probability of the shortage and the expected shortage quantity will be Equation 13:

\[
P(Q_m) = 1 - F(x) = 1 - \frac{Q_m}{b} = \frac{C_m b}{\left[1 + \lambda_r \right] C_L \cdot N^{\beta - 1} + (1 - \gamma) \left[(1 + \lambda_r) C_L \cdot N^{\beta - 1} + C_h\right]}
\]

Also, since:

\[
\overline{S} (Q_m) = \int_{0}^{Q_m} (x - Q_m) f(x) dx = \frac{Q_m^2}{2b} + \frac{b}{2} - Q_m
\]

Substituting from Equation (13) in (14) by \( Q_m \) we get:

\[
\overline{S} (Q_m) = \left[ \frac{C_m}{\left[1 + \lambda_r \right] C_L \cdot N^{\beta - 1} + (1 - \gamma) \left[(1 + \lambda_r) C_L \cdot N^{\beta - 1} + C_h\right] \right] \overline{S} (Q_m)
\]

Hence:

\[
\begin{align*}
E(TC(Q_m, N)) & = \frac{C_L}{N} + \frac{C_H}{N} + C_L \left( Q_m - \mu - \frac{DN}{2} \right) \\
& + \left[ C_L \cdot N^{\beta - 1} + (1 - \gamma) \left[(1 + \lambda_r) C_L \cdot N^{\beta - 1} + C_h\right] \right] \\
& \left[ \Phi \left( \frac{Q_m - \mu}{\sigma} \right) + \left( \mu - Q_m \right) \left[ 1 - \Phi \left( \frac{Q_m - \mu}{\sigma} \right) \right] \right]
\end{align*}
\]

Subject to:

\[
C_h \cdot N^{\beta - 1} \overline{S} (Q_m) \leq K_L \quad (1 - \gamma) \cdot N^{\beta - 1} \overline{S} (Q_m) \leq K_L
\]
2.4. The Model with Exponential Lead-Time Demand

If the lead time demand follows the Exponential distribution then:

\[ f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \quad x \geq 0, \quad E(x) = \theta \]

The probability of the shortage and the expected shortage will be Equation 16:

\[ P(Q_n) = \int_{0_n}^{\infty} f(x) dx = \frac{C_h}{\left[(1 + \lambda_s)C_s N^{\beta - 1} + (1 - \gamma)(1 + \lambda_s)C_s N^{\beta - 1} + C_n\right]} \]

i.e.,

\[ P(Q_n) = \int_{0_n}^{\infty} \frac{1}{\theta} e^{-\frac{x}{\theta}} dx = \frac{C_h}{\left[(1 + \lambda_s)C_s N^{\beta - 1} + (1 - \gamma)(1 + \lambda_s)C_s N^{\beta - 1} + C_n\right]} \]

\[ = \frac{1}{\theta} \int_{0_n}^{\infty} e^{-\frac{x}{\theta}} dx = \frac{0_n}{\theta} \]

\[ = \frac{C_h}{\left[(1 + \lambda_s)C_s N^{\beta - 1} + (1 - \gamma)(1 + \lambda_s)C_s N^{\beta - 1} + C_n\right]} \]

By taking In of two sides Equation 17 and 18:

\[ \therefore Q_n = -\theta \ln \frac{C_h}{\left[(1 + \lambda_s)C_s N^{\beta - 1} + (1 - \gamma)(1 + \lambda_s)C_s N^{\beta - 1} + C_n\right]} \]  

(17)

\[ S(Q_n) = \int_{0_n}^{\infty} (x - Q_n) f(x) dx = \theta e^{-\frac{0_n}{\theta}} \]  

(18)

Substituting from (17) by \( Q_n \) in (18) Equation 19:

\[ S(Q_n) = \theta e^{-\frac{1}{\theta}0_n} = \theta \left(\frac{C_h}{\left[(1 + \lambda_s)C_s N^{\beta - 1} + (1 - \gamma)(1 + \lambda_s)C_s N^{\beta - 1} + C_n\right]}\right) \]

Hence:

\[ E\left[TC(Q_n, N)\right] = \frac{C_s}{N} + \frac{C_N}{N} + C_h \left(Q_s - \mu - \frac{DN}{2}\right) \]

+ \left[C_s N^{\beta - 1} + (1 - \gamma)(1 + \lambda_s)C_s N^{\beta - 1} + C_n\right] \]

\[ \times \theta \left(\frac{C_h}{\left[(1 + \lambda_s)C_s N^{\beta - 1} + (1 - \gamma)(1 + \lambda_s)C_s N^{\beta - 1} + C_n\right]}\right) \]

Subject to:

\[ C_s N^{\beta - 1} S(Q_n) \leq K_s \]

2.5. The Model with Laplace Lead-Time Demand

If the lead time demand follows the Laplace distribution then:

\[ f(x) = \frac{1}{2\theta} e^{-\frac{|x|}{\theta}}, \quad -\infty < x < \infty \]

And \( E(x) = \mu \)

The probability of the shortage and the expected shortage quantity will be Equation 20:

\[ P(Q_n) = \int_{0_n}^{\infty} f(x) dx = \frac{C_h}{\left[(1 + \lambda_s)C_s N^{\beta - 1} + (1 - \gamma)(1 + \lambda_s)C_s N^{\beta - 1} + C_n\right]} \]

(20)

But:

\[ f(x) = \frac{1}{2\theta} e^{-\frac{|x|}{\theta}} \]

Then:

\[ Q_n = \mu - \theta* \ln \left(\frac{2C_h}{\left[(1 + \lambda_s)C_s N^{\beta - 1} + (1 - \gamma)(1 + \lambda_s)C_s N^{\beta - 1} + C_n\right]}\right) \]

Also, since Equation 22:

\[ S(Q_n) = \int_{0_n}^{\infty} (x - Q_n) f(x) dx = \frac{1}{2} \theta e^{-\frac{|0_n - \mu|}{\theta}} \]

(22)
Substituting from (21) by $Q_m$ in (22) then. Since:

$$Q_m = \mu - \theta^* \ln \left( \frac{2C_0}{(1+\lambda_c)C_r yN^{\beta-1} + (1-\gamma)(1+\lambda_c)C_r N^{\beta-1} + C_s} \right)$$

Then Equation 23:

$$\bar{S}(Q_m) = \frac{1}{2} \theta e^{\left(\frac{Q_m - \mu}{\theta}\right)}$$

Hence:

$$E(\{T^C(Q_{m})\}) = \frac{C_s}{N} + \frac{C_r}{N} + C_0 + \left[\frac{C_s yN^{\beta-1} + (C_r N^{\beta-1} + C_s)(1-\gamma)}{C_r yN^{\beta-1} + (1-\gamma)(1+\lambda_c)C_r N^{\beta-1} + C_s} \right]$$

Subject to:

$$C_s yN^{\beta-1} \bar{S}(Q_m) \leq K_b$$
$$C_r (1-\gamma)N^{\beta-1} \bar{S}(Q_m) \leq K_r$$

3. STANDARD PARTICLE SWARM OPTIMIZATION (SPSO)

The Particle Swarm Optimization (PSO) algorithm is a population-based search algorithm based on the simulation of the social behavior of birds within the flock and fish school proposed by Kennedy and Eberhart. The population is called a swarm, while the search points are called the particles. The particles are initialized randomly in the search space and have an adaptable velocity; each particle has a memory remembering the best position of the search space it has ever visited. Let we have D-dimensional search space, the swarm is a set of $i^{th}$ particle represented as $X_i = (x_{i1}, x_{i2}, \ldots, x_{id})$ and its velocity for the $i^{th}$ particle is represented as $V_i = (v_{i1}, v_{i2}, \ldots, v_{id})$ The Particle swarm optimization concept consists of at each iteration, changing the velocity and location of each particle toward its $P_{best}$ (best value of each particle so far) and $g_{best}$ (best previous position and towards the best particle in the whole swarm) locations according to the following Equation 24 and 25:

$$v_i(t) = wv_i(t-1) + c_1r_1(x_{pi} - x_i(t)) + c_2r_2(x_i(t) - x(t))$$

$$x_i(t) = x_i(t-1) + dv_i(t)$$

In the iteration $t$, the velocity $v_i(t)$ has update to pull the particle $i^{th}$ towards its own best position $x_{pi}$ and the best position for all the particles $x_i$ that has the best fitness value until the preceding generation, $r_1$, $r_2$ are random variables uniformly distributed between 0 and 1 this two random values are generated independently, $c_1$, $c_2$ are referred to as the cognitive and social parameter and $w$ is the inertia weight. Equation 25 updates each particle’s position in the solution hyperspace. Then evaluate the fitness for each particle to find best previous position and global best to update the velocity and the position while the stopping criterion is achieved.

4. FUZZY ADAPTIVE PARTICLE SWARM OPTIMIZATION (FAPSO)

In this section, we introduce a velocity update approach (Liu et al., 2007) for the particles in PSO and analyze its effect on the particle’s behavior during the D-dimensional search space. One of the main effects is the premature convergence that occurs when the $v_i$ arrives to zero or near to zero, but this does not mean that the particle arrives to the global or local best particle but mean the best position particle. In the FAPSO (Elhefnawy et al., 2007), the minimum velocity $v_{c1}$ can be tuned adaptively by using the fuzzy control parameter $\alpha$ in the solution procedure to overcome the previous case. If a particle’s velocity decreases to a threshold $v_{c1}$, a new velocity is assigned
using Equation 26 to drive those lazy particles and let them explore better solutions. Thus, we present the FAPSO using the following velocity update:

\[
\begin{align*}
    v_i(t) = \begin{cases} 
        v_i & \text{if } |v_i| \geq v_{it} \\
        u(-1,1)v_{\text{max}}/\rho & \text{if } |v_i| < v_{it}
    \end{cases}
\end{align*}
\]

(26)

where, \( u(-1,1) \) is the random number, uniformly distributed with the interval \([-1, 1]\) and \( \rho \) is the scaling factor to control the domain of the particle’s oscillation according to \( v_{\text{max}} \), where The value of \( v_{\text{max}} \) is \( \rho s \), with \( 0.1 \leq \rho \leq 1.0 \) and is usually chosen to be \( s \), i.e., \( \rho = 1 \). Figure 2 illustrates the trajectory of a single particle in FAPSO using the fuzzy control parameter \( a \). Also, shows the effects of the different fuzzy control parameter \( a \) on the behavior of the solution procedure, respectively.

The Procedure of the FAPSO can be Explained as follows:

Step1: Generate a set of initial solutions of the Probabilistic \(<Q_m, N>\) Inventory Model with Varying Mixture Shortage

Step2: Constructing the membership function for particle's velocity

Step3: Determine the control parameter \( a \) to obtain \( v_{it} \) that may cause the premature convergence

Step4: Evaluate the fitness function of each particle

Step5: If the particle does not remain in feasible solution region (divergence particle), discard it and mutated again with \( x_i = x_{pi} \) go to step 8

Step6: The particle’s velocity can be updated based on the following equation:

\[
\begin{align*}
    v_i(t) = \begin{cases} 
        v_i(t) & \text{if } |v_i(t)| \geq v_{i1} \\
        u(-1,1)v_{\text{max}}/\rho & \text{if } |v_i| < v_{it}
    \end{cases}
\end{align*}
\]

Where:

\[
v_i(t) = uw_i(t-1) + c_1r_1(x_{pi} - x_i(t)) + c_2r_2(x_g - x_i(t))
\]

Step7: The position of each particles can be updated according to the following equations \( x_i(t) = cx_i(t-1) + dv_i(t) \)

Step8: Save the new fitness values in the repository

Step9: If the no. of generation reached go to step 10. Otherwise, go to step 4.

Step10: Stop.

Figure 3 represents the flow chart for the suggested multi-objective FAPSO algorithm.

5. NUMERICAL EXAMPLE

Consider the following data for solving the probabilistic periodic review \(<Q_m, N>\) inventory model with mixture shortage given in Table 1.
Fig. 3. Flow chart of the suggested FAPSO algorithm
A warehouse follows a policy of reviewing all items periodically every 1.94 month \((N^*)\). The lead-time \(L\) is nearly constant and its value is 6 months. A fraction of unsatisfied demand that will be backorder \(r\) is 0.56.

Each simulation run was carried by using the following parameters when solved by FAPSO approach:

| Parameter                  | Value     |
|----------------------------|-----------|
| Number of generations      | 500       |
| Population size            | 80        |
| Self-recognition coefficient \(c_1\) | 1.49      |
| Social coefficient \(c_2\)  | 1.49      |
| Inertia weight \(\omega\)  | 0.9       |

Each optimization experiment was run 10 times with different random seeds. The optimal review period \((N^*)\), optimal maximum inventory level \((Q^m)\) and constant real numbers \((\beta)\) which will minimize the expected annual total cost \(E(TC)\) recorded in the Table 3-9.

5.1. Test of the Problem Using Lagrange Multiplier Technique and FAPSO

To make sure (examine) the efficiency and fitness of the algorithm of Fuzzy Adaptive Particle Swarm Optimization (FAPSO) we aim at fixing \(N, \beta\) between the interval \([0,1]\) and test the results between FAPSO and Lagrange multiplier technique. The FAPSO give us results close to the results of Lagrange multiplier and give us an impression for the validity of the algorithm but in this case we restrict the FAPSO algorithm. For this reason, we examine the FAPSO algorithm when \(N, \beta\) are variables and compare the results of FAPSO with the similar of Lagrange multiplier technique using Mathematica program. Hence we found that FAPSO gives results better than Lagrange multiplier technique, which discussed in section 5.2.

Table 2-5 represent the results of two techniques at \(N = 1.94\) month and a constant real number selected to provide the best fit of Estimated expected cost function \((\beta)\) between 0.1 and 0.9.

5.2. Comparative study

5.2.1. The Solution for the (Normal Distribution)

Let the demand in the time \(L+N\) can be represented quite well by a normal distribution with mean \(D (L+N) = 600 \ (L+N)\) and variance \(\sigma^2 (L+N) = 900 \ (L+N)\).

It is desired to determine to optimal values \(Q^m, N^*\) and the minimum total cost:

- The expected demand in time \(L+N\) is \(E(x) = 397.002\)
- The variance of the demand in this time \(\sigma^2 (L+N) = 595.5\)
- Then the standard deviation is \(\sigma\sqrt{L+N} = 24.4029\)
- Hence the results using Mathematica program can be summarized as follows

The optimum values for different values of \(\beta\) and the total cost based on the Lagrange multiplier technique under two constraints when the lead time demand follows normal distribution with mean \(\mu\) and standard deviation \(\sigma\) is given by Table 6.

However, the results using Fuzzy Adaptive Particle Swarm Optimization can be summarized as follows.

Consider that \(N, \beta\) and \(Q_m\) are variables then the optimum values of the time between reviews, the maximum inventory level and the total cost using FAPSO approach under two constraints when the lead-time demand follows normal distribution with mean \(\mu\)
and standard deviation $\sigma$ is given by Table 7. Moreover, clarify of the results is shown in Fig. 4.

### 5.3. The Solution for the (Uniform Distribution)

Let $f(x) = \frac{1}{50}$, hence $E(x) = 25$

Also:

$$S(Q_m) = \frac{Q_m^2}{2b} + \frac{b}{2} - Q_m = \frac{Q_m^2}{100} + \frac{50}{2} - Q_m$$

The optimum values of the maximum inventory level and the total cost based on the Lagrange multiplier technique when the lead-time demand follows uniform distribution is given by Table 8.

Consider that $N, \beta$ and $Q_m$ are variables then the optimum values of the time between reviews, the maximum inventory level and the total cost using FAPSO approach when the lead-time demand follows uniform distribution is given by Table 9. Moreover, clarify of the results displayed in Fig. 5.

### 5.4. The Solution for the (Exponential Distribution)

Let:

$$f(x) = \frac{1}{25} e^{-\frac{x}{25}}, x \geq 0, \theta = 25$$

Hence:

$$E(x) = 25 \overline{S}(Q_m) = \theta e^{-\frac{Q_m}{\theta}} = 25 e^{-\frac{Q_m}{25}}$$

### Table 1. Presents the value of the parameters

| Parameters | Value |
|------------|-------|
| $C_o + C_r$ | 25 $ |
| $C_h$ | 3 $ |
| $C_b$ | 25 $ |
| $C_L$ | 25 $ |

### Table 2. The results using mathematica program and FAPSO for Normal distribution

| $\beta$ | $Q_{\alpha}$ | $E(\text{TC})$ | $Q_m^{-1}$ | $E(\text{TC})$ |
|----------|--------------|----------------|-------------|----------------|
| 0.1      | 445.715      | 473.31 $        | 407.9091    | 202.3958       |
| 0.2      | 444.109      | 468.567        | 408.2650    | 208.3604       |
| 0.3      | 442.455      | 463.655        | 408.2995    | 223.4091       |
| 0.4      | 440.670      | 458.393        | 408.3293    | 240.7190       |
| 0.5      | 438.864      | 453.068        | 408.3504    | 253.1674       |
| 0.6      | 437.001      | 447.598        | 408.4786    | 264.2640       |
| 0.7      | 435.105      | 442.051        | 408.5786    | 273.3417       |
| 0.8      | 433.194      | 436.481        | 408.0180    | 302.4184       |
| 0.9      | 431.172      | 430.612        | 407.9785    | 295.6521       |

The optimum values of the maximum inventory level and the total cost based on the Lagrange multiplier technique when the lead-time demand follows exponential distribution is given by Table 10.

Consider that $N, \beta$ and $Q_m$ are variables then the optimum values of the time between reviews, the maximum inventory level and the total cost using FAPSO approach when the lead time demand follows Exponential distribution is given by Table 11 and explain of the results is exhibit in Fig. 6.

### 5.5. The Solution for the (Laplace Distribution)

$$E(x) = 25 \overline{S}(Q_m) = \frac{1}{2} \theta e^{-\frac{(Q_m-\mu)}{\theta}} = \frac{1}{2} \times 10.206 \times e^{-\frac{(Q_m-25)}{10.206}}$$

The optimum values of the maximum inventory level and the total cost based on the Lagrange multiplier technique when the lead-time demand follows Laplace distribution is given by Table 12.

Consider that $N, \beta$ and $Q_m$ are variables then the optimum values of the time between reviews, the maximum inventory level and the total cost using FAPSO approach when the lead-time demand follows Laplace distribution is given by Table 13. Moreover, illustrate of the results is shown in Fig. 7.

### 5.6. Remark

From the previous results it becomes clear that when we Use FAPSO approaches it leads toward the global optimum instead of trapping into local peaks in all the pervious distributions.
Table 3. The results using Mathematica program and FAPSO for Uniform distribution

| β  | $Q_w^{*}$ | E(TC)          | $Q_w^*$ | E(TC) |
|----|-----------|----------------|---------|-------|
| 0.1| 48.85270  | 203.657$       | 49.29800| 205.7859 |
| 0.2| 48.74330  | 203.333        | 49.46144| 205.8740 |
| 0.3| 48.62340  | 202.977        | 48.62533| 204.5115 |
| 0.4| 48.49210  | 202.588        | 48.87756| 204.6693 |
| 0.5| 48.34823  | 202.163        | 48.71286| 205.0256 |
| 0.6| 48.19070  | 201.697        | 49.46144| 205.8740 |
| 0.7| 48.01810  | 201.188        | 48.62533| 204.5115 |
| 0.8| 47.82910  | 200.631        | 48.71286| 205.0256 |
| 0.9| 47.62200  | 200.022        | 48.62533| 204.5115 |

Table 4. The results using Mathematica program and FAPSO for Exponential distribution

| β  | $Q_w^{*}$ | E(TC)          | $Q_w^*$ | E(TC) |
|----|-----------|----------------|---------|-------|
| 0.1| 95.2259   | 413.223$       | 94.93518| 411.9810 |
| 0.2| 90.6704   | 399.703        | 89.87002| 400.7742 |
| 0.3| 86.1149   | 386.212        | 88.49079| 387.0650 |
| 0.4| 81.5594   | 372.756        | 82.73436| 373.4911 |
| 0.5| 77.0039   | 359.342        | 83.55936| 362.7138 |
| 0.6| 72.4485   | 345.979        | 76.05411| 345.3453 |
| 0.7| 67.8929   | 332.676        | 73.58664| 335.8071 |
| 0.8| 63.3375   | 319.446        | 65.25027| 321.6515 |
| 0.9| 58.7820   | 306.303        | 57.99037| 304.7468 |

Table 5. The results using Mathematica program and FAPSO for Laplace distribution

| β  | $Q_w^{*}$ | E(TC)          | $Q_w^*$ | E(TC) |
|----|-----------|----------------|---------|-------|
| 0.1| 57.009    | 255.276$       | 64.64356| 251.1807 |
| 0.2| 55.1493   | 249.755        | 62.81299| 245.6890 |
| 0.3| 53.2895   | 244.246        | 58.65988| 233.2296 |
| 0.4| 51.4297   | 238.751        | 58.92988| 234.0396 |
| 0.5| 49.5700   | 233.273        | 55.32898| 223.2369 |
| 0.6| 47.7103   | 227.815        | 51.73142| 212.4443 |
| 0.7| 45.8504   | 222.381        | 50.58961| 209.0188 |
| 0.8| 43.9908   | 216.977        | 50.32193| 208.1888 |
| 0.9| 42.1310   | 211.607        | 50.13726| 207.6618 |

Table 6. The optimal results of model 2-1

| β  | $\lambda_w^{*}$ | $\lambda_w^{-}$ | $Q_w^*$ | E(TC) |
|----|------------------|------------------|---------|-------|
| 0.1| 0.002            | 0.00260          | 445.715 | 473.31$ |
| 0.2| 0.035            | 0.02547          | 444.109 | 468.567 |
| 0.3| 0.075            | 0.03815          | 442.455 | 463.655 |
| 0.4| 0.107            | 0.03818          | 440.670 | 458.393 |
| 0.5| 0.145            | 0.03820          | 438.864 | 453.068 |
| 0.6| 0.182            | 0.03820          | 437.001 | 447.598 |
| 0.7| 0.224            | 0.03820          | 435.105 | 442.051 |
| 0.8| 0.276            | 0.03815          | 433.194 | 436.481 |
| 0.9| 0.3147           | 0.03820          | 431.172 | 430.612 |
Table 7. The results using FAPSO

| $\beta$  | $N^*$ | $Q_m^*$ | E (TC)$^*$ |
|-----------|-------|---------|------------|
| 1.35E-02  | 0.27994 | 483.6817 | 91.04408   |
| 0.254816  | 0.26443 | 475.0849 | 110.66580  |
| 0.121196  | 0.276644 | 481.7898 | 117.30190  |
| 3.50E-03  | 0.279011 | 482.6383 | 118.09550  |
| 2.69E-03  | 0.259179 | 469.9347 | 134.35520  |
| 5.88E-03  | 0.282823 | 484.7993 | 134.78630  |
| 0.289517  | 0.180891 | 422.2667 | 139.28720  |
| 0.338156  | 0.296493 | 494.7425 | 166.28250  |
| 4.63E-02  | 0.281428 | 481.3388 | 315.68550  |

Table 8. The optimal results of the model 2-2

| $\beta$ | $\lambda_0$ | $\lambda_L$ | $Q_m^*$ | E(TC) |
|---------|-------------|-------------|---------|-------|
| 0.1     | 0.00450     | 0.00382     | 48.85270| 203.657 $ |
| 0.2     | 0.17380     | 0.00382     | 48.74330| 203.333 |
| 0.3     | 0.35870     | 0.00382     | 48.62340| 202.977 |
| 0.4     | 0.56080     | 0.00382     | 48.49210| 202.588 |
| 0.5     | 0.78155     | 0.00376     | 48.34283| 202.163 |
| 0.6     | 1.02270     | 0.00382     | 48.19070| 201.697 |
| 0.7     | 1.28620     | 0.00382     | 47.98180| 201.188 |
| 0.8     | 1.57390     | 0.00382     | 47.82910| 200.631 |
| 0.9     | 1.88780     | 0.00382     | 47.62200| 200.022 |

Table 9. The results using FAPSO

| $N$     | $\beta$ | $Q_m^*$ | E(TC)$^*$ |
|---------|---------|---------|------------|
| 0.855797| 0.630314| 58.38030| 20.50866   |
| 0.693346| 0.197418| 54.16220| 25.58115   |
| 0.578352| 0.481714| 42.89127| 27.59362   |
| 0.560751| 0.649507| 47.76340| 30.36465   |
| 0.664848| 0.147211| 53.94762| 30.51989   |
| 0.59093 | 0.303433| 51.05595| 32.25129   |
| 0.588301| 0.120997| 52.01549| 36.96916   |
| 0.541767| 0.097877| 50.53540| 41.61468   |
| 0.549307| 0.124951| 51.50322| 43.60950   |
| 0.758902| 0.133017| 32.24886| 45.07503   |

Table 10. The optimal results of the model 2-3

| $\beta$ | $\lambda_0^*$ | $\lambda_L^*$ | $Q_m^*$ | E(TC) |
|---------|---------------|---------------|---------|-------|
| 0.1     | 0.041000      | 0.038190      | 95.2259 | 413.223 $ |
| 0.2     | 0.037350      | 0.038180      | 90.6704 | 399.703 |
| 0.3     | 0.032960      | 0.038180      | 86.1149 | 386.212 |
| 0.4     | 0.027700      | 0.038180      | 81.5594 | 372.756 |
| 0.5     | 0.021380      | 0.038180      | 77.0039 | 359.342 |
| 0.6     | 0.013810      | 0.038180      | 72.4485 | 345.979 |
| 0.7     | 0.004710      | 0.038180      | 67.8929 | 332.676 |
| 0.8     | 0.003805      | 0.025460      | 63.3375 | 319.446 |
| 0.9     | 0.000720      | 0.012728      | 58.7820 | 306.303 |
Table 11. The results using FAPSO

| β        | N*   | Qᵐ*  | E(TC)A |
|----------|------|------|--------|
| 0.4058053| 0.97973 | 57.43209 | 42.78574 |
| 0.935998 | 0.98157 | 59.75097 | 42.85972 |
| 0.624991 | 0.95466 | 49.19647 | 49.08882 |
| 9.81E-02 | 0.95122 | 54.91412 | 49.72275 |
| 0.642026 | 0.99140 | 69.49759 | 50.95656 |
| 0.471227 | 0.93516 | 54.96149 | 51.88449 |
| 0.229297 | 0.95286 | 65.71236 | 54.66653 |
| 0.31015  | 0.92877 | 49.52028 | 56.46565 |
| 0.203667 | 0.59067 | 76.79328 | 154.6951 |

Table 12. The optimal results of the model 2-4

| β | λ₀* | λ₂* | Qᵐ*  | E(TC) |
|---|-----|-----|------|-------|
| 0.1 | 0.106650 | 0.038200 | 57.0090 | 255.2768 |
| 0.2 | 0.103000 | 0.003818 | 55.1493 | 249.755 |
| 0.3 | 0.098600 | 0.003818 | 53.2895 | 244.246 |
| 0.4 | 0.093330 | 0.003818 | 51.4297 | 238.751 |
| 0.5 | 0.087020 | 0.003818 | 49.5700 | 233.273 |
| 0.6 | 0.079460 | 0.003818 | 47.7103 | 227.815 |
| 0.7 | 0.070340 | 0.003818 | 45.8504 | 222.381 |
| 0.8 | 0.059436 | 0.003818 | 43.9908 | 216.977 |
| 0.9 | 0.04635 | 0.003818 | 42.1310 | 211.607 |

Table 13. The results using FAPSO

| β        | N*   | Qᵐ*  | E(TC)A |
|----------|------|------|--------|
| 0.235573 | 0.471201 | 34.38432 | 10.53058 |
| 0.190189 | 0.901405 | 64.60248 | 11.33112 |
| 0.538946 | 0.848705 | 61.44213 | 11.47725 |
| 0.629335 | 0.593022 | 44.74105 | 12.42682 |
| 0.680409 | 0.400022 | 30.52506 | 19.13798 |
| 0.350657 | 0.616576 | 48.75258 | 19.31785 |
| 0.315997 | 0.462833 | 37.51333 | 22.13034 |
| 0.61585  | 0.594566 | 49.71379 | 27.00388 |
| 2.31E-02 | 0.841329 | 93.55012 | 109.16580 |
| 0.895867 | 0.668046 | 95.88036 | 149.85670 |

Fig. 4. Display the results of normal distribution

Fig. 5. Display the results of uniform distribution
6. CONCLUSION

In this study, we developed a probabilistic Single-Item Single-Source (SISS) inventory model with varying mixture of backorders and lost sales under two restrictions, which the first on the expected backorder cost and the other on the expected lost sales cost. We reached the optimal review period and optimal maximum inventory level that minimized the expected annual total cost under constraints using Lagrange multiplier technique and the Fuzzy Adaptive Particle Swarm Optimization (FAPSO). We overcome the problems that meet us when we use Lagrange multiplier technique by using FAPSO whereas most algorithms tend to be stuck to a sub-optimal solution, an algorithm efficient in solving one optimization problem may not be efficient in solving another one and these techniques such as Lagrange multiplier technique are useful over a relatively narrow range. FAPSO proved good results and performance when applied to solve complexities problems. Using FAPSO algorithm in the inventory model promise to achieve the global solution with decreasing the number of iteration that required for arrive to it. The algorithm has been tested using a numerical example; the results show the algorithms described in this study perform well.

In the future, we want to increase the number of constraints to contain all the costs and implement Fuzzy Adaptive Particle Swarm Optimization (FAPSO) on the constrained $<Q_m,N>$ model with mixture varying shortage when all components of costs are fuzzy number.

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