Modelling and Closed-Loop System Identification of a Quadrotor-Based Aerial Manipulator

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Abstract. This paper presents the modelling and system identification of a quadrotor-based aerial manipulator. The aerial manipulator model is first derived analytically using the Newton-Euler formulation for the quadrotor and Recursive Newton-Euler formulation for the manipulator. The aerial manipulator is then simulated with the quadrotor under Proportional Derivative (PD) control, with the manipulator in motion. The simulation data is then used for system identification of the aerial manipulator. Auto Regressive with eXogenous inputs (ARX) models are obtained from the system identification for linear accelerations $\ddot{x}$ and $\ddot{y}$ and yaw angular acceleration $\ddot{\phi}$. For linear acceleration $\ddot{z}$, and pitch and roll angular accelerations $\dot{\theta}$ and $\dot{\phi}$, Auto Regressive Moving Average with eXogenous inputs (ARMAX) models are identified.

1. Introduction
Aerial manipulators have the ability not only to move in three-dimensional space, but to physically interact with their environment. Potential applications for aerial manipulators include inspection by contact of structures such as bridges, repair and maintenance of high-voltage overhead power lines, precision agriculture, and geological sampling of volcanic rock. While research has been conducted in the modelling and control of aerial manipulators; system identification of aerial manipulators has not received attention. System identification has, however, been performed for a quadrotor alone without a manipulator. For instance, Zhang et al., [1] surveyed various modelling and identification methods employed for quadrotors.

A number of authors first derived the quadrotor model using either the Newton-Euler or Euler-Lagrange formulations and then identified parameters such as thrust and drag parameters. Derafa et al., [2] identified the inertia matrix, dynamic friction coefficients and translational drag coefficients for a quadrotor. They created a rotational pendulum rig by suspending the quadrotor on wires to measure the inertia, and translational drag parameters. Dong et al., [3] used the Newton-Euler formulation to develop the dynamic model of the quadrotor. They calculated the thrust and aerodynamic drag of the quadrotor and used a three-wire pendulum set-up to measure the inertia of the quadrotor. Elsamanty et al., [4] also used the Newton-Euler formulation for the quadrotor dynamic model. They used a Computer-Aided Design (CAD) model to calculate the mass moment of inertia and geometrical parameters of a quadrotor. They then identified the propeller angular velocity using a microphone as a tachometer. Thrust and drag moment were identified using a lever arm test rig. Araar et al., [5] identified the thrust factor and drag factor of a quadrotor using a didital scale and tachometer. They obtained the inertia matrix from the CAD model. Hatamleh et al., [6] performed a simulation study to
investigate the dynamic model parameters of a quadrotor in the presence of noisy feedback signals. They compared three estimation methods and found that the hybrid of artificial neural network and iterative bi-section shooting had a higher accuracy in estimating the parameters.

Black-box system identification has been performed by various authors, using both simulated quadrotors and physical quadrotors. Lee et al., [7] used the Prediction Error Method (PEM), to estimate a linearised discrete-time state-space model for a quadrotor. They used a six Degree of Freedom (DoF) test-rig to perform tests in addition to actual flight tests. Stanculeanu and Borangiu, [8] used a discrete-time linear state-space model for the quadrotor. They performed black-box system identification for the quadrotor in near hover flight using the PEM. Schreurs et al., [9] developed a simulation model of a quadrotor and used the PEM for system identification from which they obtained a 12th-order Auto Regressive Moving Average with eXogenous inputs (ARMAX) model. Bergamasco et al., [10] performed flight experiments for a hovering quadrotor and logged the flight data. They used the flight data to derive a continuous-time model using subspace model identification methods. Pedro and Crouse, [11] performed system identification for a simulated quadrotor using radial basis function neural networks. Black-box system identification for a quadrotor, using dynamic neural networks was also performed by Pedro and Dangor [12].

While previous studies have focused on the system identification of quadrotors, none have attempted the system identification of quadrotor-based aerial manipulators. This paper therefore presents the system identification of a quadrotor-based aerial manipulator with a three-DoF manipulator. The remainder of this paper is structured as follows: section 2 presents the dynamic model of the quadrotor-based aerial manipulator. Section 3 describes the system identification performed and results obtained. Conclusions are then presented in section 4.

2. Aerial Manipulator Dynamic Modelling

The quadrotor model is developed using the Newton-Euler formulation. For the manipulator, the Recursive Newton-Euler formulation is used. By modelling the manipulator with the quadrotor as its floating base, the two models are combined to form the model of the aerial manipulator.

2.1. Quadrotor Model

Figure 1 shows the quadrotor system reference frames: the earth-inertial frame (E-frame) represented by \((E_X, E_Y, E_Z)\) and the quadrotor body frame (B-frame) represented by \((B_x, B_y, B_z)\). The inertial and body frames are related by three successive rotations about the \(z\), \(y\) and \(x\) axes. These are given by the Euler \((z\ y\ x)\) rotations, where roll \(\phi\) is the rotation about the \(x\)-axis, pitch \(\theta\) is the rotation about the \(y\)-axis, and yaw \(\psi\) is the rotation about the \(z\)-axis. The rotation from the body frame to the inertial frame is given by the rotation matrix \(^{E}_B R_{b}\). \(^{E}_BT_{b}\) transforms the angular velocities in the body frame to the time variation of the Euler angles. In matrix form, the Newton-Euler formulation for the quadrotor is:

\[
\begin{bmatrix}
{\mathbf{F}}_b \\
{\mathbf{T}}_b
\end{bmatrix} =
\begin{bmatrix}
{m}_Q {I}_3 & \mathbf{0}_3 \\
\mathbf{0}_3 & {I}_Q
\end{bmatrix}
\begin{bmatrix}
{\mathbf{V}}_b \\
{\mathbf{\omega}}_b
\end{bmatrix} +
\begin{bmatrix}
{\mathbf{\omega}}_b \times {m}_Q {\mathbf{V}}_b \\
{\mathbf{\omega}}_b \times {I}_Q {\mathbf{\omega}}_b
\end{bmatrix}
\]

(1)

where: \(^{E}_B F\) is the total force acting on the center of mass; \(^{E}_B T\) is the total torque; \(m_Q\) is the mass of the quadrotor; \(I_Q\) is the moment of inertia of the quadrotor; \(I_3\) is a \(3 \times 3\) identity matrix; \(\mathbf{0}_3\) is a \(3 \times 3\) zero matrix; \(^{E}_B V\) is the linear velocity of the quadrotor in the body frame; and \(^{E}_B \omega\) is the angular velocity in the body frame.

\[
^{E}_B \omega = \begin{bmatrix} p & q & r \end{bmatrix}^T
\]

(2)
The controllable forces and moments of the quadrotor are thrust force $U_1$, yaw moment $U_4$, pitch moment $U_3$, roll moment $U_2$. The control inputs can be written as:

$$
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4
\end{bmatrix} =
\begin{bmatrix}
\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 \\
\omega_4^2 - \omega_2^2 \\
\omega_3^2 - \omega_1^2 \\
\omega_2^2 - \omega_3^2 + \omega_2^2 - \omega_1^2
\end{bmatrix}
$$

where $\omega_1$, $\omega_2$, $\omega_3$, $\omega_4$ are the rotor’s angular velocities respectively.

Neglecting drag force, there are two forces acting on the quadrotor, the gravitational force and thrust force. The total torque is a result of the gyroscopic moments due to inertia of the rotors and thrust moments. In the body frame the gravitational force experienced by the quadrotor is:

$$
^B F_G = -m_q g \begin{bmatrix} -\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta \end{bmatrix}^T
$$

The thrust force experienced by the quadrotor in the body frame is:

$$
^B F_T = k [0 \hspace{2cm} 0 \hspace{2cm} U_1]^T
$$

where $k$ is the thrust factor. The thrust moment in the body frame is:

$$
^B \tau_T = [lk U_2 \hspace{1cm} lk U_3 \hspace{1cm} c_q U_4]^T
$$

where $c_q$ is a constant of proportionality; and $l$ is the length of the quadrotor arms. The gyroscopic moment due to the rotating rotors is:

$$
^B \tau_G = l_r [q \bar{\omega}_r \hspace{1cm} -p \bar{\omega}_r \hspace{1cm} 0]^T
$$

where

$$
\bar{\omega}_r = -\omega_1 + \omega_2 - \omega_3 + \omega_4
$$

2.2. Manipulator Model
The Denavit-Hartenberg convention as found in [13] and [14] is used to describe the manipulator kinematics. A reference frame is attached to each link of the manipulator. The position of the origin of coordinate frame $O_i$, (with axis $(x_i, y_i, z_i)$), relative to coordinate frame $O_{i-1}$, (with axis $(x_{i-1}, y_{i-1}, z_{i-1})$), can be denoted by vector $^{i-1}p_i$. The orientation of frame $O_i$ relative to frame $O_{i-1}$ can be denoted by the rotation matrix $^{i-1}R_i$. The recursive Newton-Euler formulation as found in [13] is used in this paper to describe the manipulator dynamics.

2.2.1. Forwards recursion. In the Newton-Euler forward recursion the following are found [13]:

\[ ^i \omega_i = ^{i-1}R_i^T (^{i-1}i \omega_{i-1} + \ddot{\theta}_i z_0) \]  

(9)

\[ ^i \omega_i \] is the angular velocity of link $i$ expressed in frame $i$:

\[ ^i \dot{\omega}_i = ^{i-1}R_i^T (^{i-1}i \dot{\omega}_{i-1} + \dot{\theta}_i \dot{\theta}_i ^{i-1}i \omega_{i-1} \times z_0) \]  

(10)

\[ ^i \dot{p}_i \] is the linear acceleration of link $i$ expressed in frame $i$:

\[ ^i \dot{p}_i = ^{i-1}R_i^T ^{i-1}i \dot{p}_{i-1} + ^i \dot{\omega}_i \times ^i r_{i-1,i} + ^i i \omega_i \times (^{i}i \omega_i \times ^i r_{i-1,i}) \]  

(11)

\[ ^i \dot{p}_{ci} \] is the linear acceleration of center of mass of link $i$ relative to coordinate frame $O_i$:

\[ ^i \dot{p}_{ci} = ^i \dot{p}_i + ^i \dot{\omega}_i \times ^i r_{i,ci} + ^i i \omega_i \times (^{i}i \omega_i \times ^i r_{i,ci}) \]  

(12)

2.2.2. Backwards recursion. In the backward recursion, the following are found [13]:

\[ ^i f_i \] is the force exerted by link $i$ relative to link $i$ expressed in frame $i$:

\[ ^i f_i = ^i R_{i+1} i+1 ^i f_{i+1} + m_i ^i \dot{p}_{ci} \]  

(13)

\[ ^i \mu_i \] is the moment exerted by link $i$ relative to frame $O_{i-1}$ expressed in frame $i$:

\[ ^i \mu_i = -^i f_i \times (^{i}r_{i-1,i} + ^i r_{i,ci}) + ^i R_{i+1} i+1 ^i \mu_{i+1} + i R_{i+1} i+1 ^i f_{i+1} \times ^i r_{i,ci} + ^i I_i ^i \dot{\omega}_i + ^i i \omega_i \times \]  

\[ (^{i}i \omega_i) \]  

(14)

\[ \tau_i \] is the torque of link $i$:

\[ ^i \mu_i ^{i-1}R_i^T z_0 \]  

(15)

2.3. Aerial Manipulator Model

Figure 1 shows the aerial manipulator consisting of a quadrotor, and a three-DoF manipulator (two-DoF arm and one-DoF wrist). Combining the quadrotor and the manipulator means that the manipulator no longer has a fixed base but has a floating base, that is the quadrotor. For a manipulator with a floating base, the initial linear velocity $^0 \dot{p}_0$, linear acceleration angular velocity $^0 \dot{\omega}_0$, and angular acceleration $^0 \ddot{\omega}_0$ are those of the quadrotor:

\[ ^0 \dot{p}_0 = B R_0 E R_B E \dot{\Gamma} \]  

(16)

\[ ^0 \ddot{p}_0 = B R_0 E R_B (E \ddot{\Gamma} - [0 \ 0 \ -g]^T) \]  

(17)

\[ ^0 \dot{\omega}_0 = B R_0 B \omega \]  

(18)

\[ ^0 \ddot{\omega}_0 = B R_0 B \ddot{\omega} \]  

(19)
where $^b\mathbf{\Gamma}$ is the position of the quadrotor in the inertial frame; and $^b\mathbf{R}_0$ is the rotation matrix from the manipulator base frame to the quadrotor body frame. The center of mass $C_M$ of the aerial manipulator system in the body frame is:

$$
C_M = \frac{C_{M0} m_Q + \sum_{i=1}^{n} C_{Mi} m_i}{m_Q + \sum_{i=1}^{n} m_i}
$$  \hspace{1cm} (20)

where $C_{M0}$ is the center of mass of the quadrotor expressed in the quadrotor body frame; and $C_{Mi}$ is the center of mass of link $i$ expressed in the quadrotor body frame. Expressing the moment of inertia of each link about the aerial manipulator center of mass in the quadrotor body frame:

$$
I_i = I_{cm_i} + m_i S(C_i) S(C_i)^T
$$  \hspace{1cm} (21)

where $S$ is the skew symmetric matrix of $C_i$

$$
C_i = C_{Mi} - C_M
$$  \hspace{1cm} (22)

The moment of inertia $I_S$ of the aerial manipulator is thus:

$$
I_S = \sum_{i=1}^{n} I_i + I_Q + m_Q S(C_Q) S(C_Q)^T
$$  \hspace{1cm} (23)

For the combined aerial manipulator system, the Newton-Euler formulation for the dynamics can be written as:

$$
\begin{bmatrix}
^{b}\mathbf{F} \\
^{b}\mathbf{\tau}
\end{bmatrix} + \begin{bmatrix}
^{b}\mathbf{f}_1 \\
^{b}\mathbf{\mu}_1
\end{bmatrix} = \begin{bmatrix}
\mathbf{m}_Q I_3 \\
\mathbf{0}_3
\end{bmatrix} \begin{bmatrix}
^{b}\dot{\mathbf{V}} \\
^{b}\dot{\mathbf{\omega}}
\end{bmatrix} + \begin{bmatrix}
^{b}\mathbf{\omega} \times m^{b}\mathbf{V} \\
^{b}\mathbf{\omega} \times I_S^{b}\mathbf{\omega}
\end{bmatrix}
$$  \hspace{1cm} (24)

$$
^{b}\mathbf{f}_1 = ^b\mathbf{R}_1 ^1\mathbf{f}_1
$$  \hspace{1cm} (25)

$$
^{b}\mathbf{\mu}_1 = ^b\mathbf{R}_1 ^1\mathbf{\mu}_1
$$  \hspace{1cm} (26)

3. System Identification

Modelling and system identification of the aerial manipulator was performed in Matlab, with the full nonlinear model of the quadrotor, combined with the manipulator dynamics, being used for the simulations. The aerial manipulator was simulated with the quadrotor altitude and attitude controlled using Proportional-Derivative (PD) control. The quadrotor was controlled to try maintain a constant $Z$ height of 1.5 m, while trying to keep the roll $\phi$, pitch $\theta$ and yaw $\psi$ angles at 0 rad. The manipulator joint angles were set to perform trapezoidal velocity profiles resulting in the joint angle trajectories shown in figure 2a. For validation the manipulator angle trajectories shown in figure 2b were used.

The quadrotor linear accelerations $\ddot{X}$, $\ddot{Y}$, $\ddot{Z}$ and angular accelerations $\ddot{\phi}$, $\ddot{\theta}$, $\ddot{\psi}$ in the inertial frame were then identified given the control inputs $U_1$ to $U_4$ and the manipulator joint angular positions, velocities and accelerations.

First, Auto Regressive with eXogenous inputs (ARX) models of orders one to thirty were attempted for each of the linear and angular accelerations. If ARX models were not successful, then ARMAX models of orders one to thirty and nonlinear ARX models were also attempted. For the nonlinear ARX models, both wavelet and sigmoid networks nonlinearity estimators were attempted.
3.1. Results

For the linear acceleration $\ddot{X}$ a third-order ARX model was identified as shown in figure 3. Both validation and estimation data had over 99% fit.

Similarly, for the linear acceleration $\ddot{Y}$ a third-order ARX model was identified as shown in figure 4. As with $\ddot{X}$ both validation and estimation data for $\ddot{Y}$ had over 99% fit.

The linear acceleration $\ddot{Z}$ had poor results for ARX models. Nonlinear ARX models were also investigated, however they did not significantly improve on the results of the linear ARX models. A fifth-order ARMAX was identified as shown in figure 5. In this case, estimation data had a 69.5% fit and validation had a 62% fit. It should be noted that the $\ddot{Z}$ linear acceleration had, overall, a significantly smaller magnitude than that of the linear accelerations $\ddot{X}$ and $\ddot{Y}$.

For roll angular acceleration $\ddot{\phi}$, a third-order ARMAX model was identified with estimation and validation fits of over 79% as shown in figure 6. A third-order ARX model was also identified with estimation and validation fits of over 78.8%.
Figure 5. System Identification Results - Z acceleration.

Figure 6. System Identification Results - Roll acceleration.

For pitch angular acceleration $\ddot{\theta}$, a sixth-order ARMAX model was identified with an estimation fit of 87.5% and validation fit of 84.9% as shown in figure 7.

Figure 7. System Identification Results - Pitch acceleration.

For yaw angular acceleration $\dddot{\psi}$ a third-order ARX model resulted in a fit of 88.3% estimation fit and an 86.1% validation fit as shown in figure 8.

Figure 8. System Identification Results - Yaw acceleration.
4. Conclusions and Future Work
This paper presents the system identification of a simulated quadrotor-based aerial manipulator. The aerial manipulator dynamics model for a quadrotor with a two-link manipulator with a one-DoF wrist was successfully identified for the case of the quadrotor under altitude and attitude control with the manipulator in motion. ARX models were identified for linear accelerations $\ddot{x}$ and $\ddot{y}$ and yaw angular acceleration $\dot{\psi}$. For linear acceleration $\ddot{z}$ and angular accelerations $\dot{\phi}$ and $\dot{\theta}$, ARMAX models were identified.

Future work includes performing system identification from actual flight data of a quadrotor-based aerial manipulator. While satisfactory results were obtained for linear acceleration $\ddot{z}$, non-linear ARMAX models could also be investigated for system identification of the aerial manipulator to improve the identification results of $\ddot{z}$.

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