Thermal Freeze-out and Longitudinally Non-uniform Collective Expansion Flow in Relativistic Heavy Ion Collisions

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ABSTRACT

The non-uniform longitudinal flow model (NUFM) proposed recently is extended to include also transverse flow. The resulting longitudinally non-uniform collective expansion model (NUCEM) is applied to the calculation of rapidity distribution of kaons, lambdas and protons in relativistic heavy ion collisions at CERN-SPS energies. The model results are compared with the 200 A GeV/c S-S and 158 A GeV/c Pb-Pb collision data. The central dips observed in experiments are reproduced in a natural way.

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I. Introduction

Relativistic heavy-ion collision offers a unique opportunity to study the hot and dense matter under controlled laboratory conditions [1]. Hadronic spectra from these reactions reflect the dynamic of the hot and dense zone formed in the collision. The baryon density, established early in the reaction, is an important factor governing the evolution of the system. Comparison of model predictions with measured rapidity and transverse momentum distributions constrains the possible dynamical scenarios of the reaction, such as those for longitudinal and transverse flow. In addition, the mechanism by which the incoming nucleons lose momentum during collision (baryon stopping) is an important theoretical problem [2].

The rich physics of longitudinal and transverse flows is due to their sensitivity to the system evolution at early time. Due to the high pressure of the system created during a heavy-ion collision, particles might be boosted in the transverse and longitudinal directions. The expansion and cooling of the heated and highly compressed matter could lead to a considerable collectivity in the final state. The collective expansion implies space-momentum correlation in particle distributions at freeze-out.

In this paper we present a study of the experimental results from the central collisions of Pb-Pb at 158 A GeV/c and S-S at 200 A GeV/c measured by NA49 and NA35 at CERN SPS [4][5]. The primary goal of the study is to provide a chance to compare the stopping power and radial symmetrical flow in collision systems of different sizes. Note that at SPS energies the duration time of the evolving system will be longer in comparison with that at the AGS energies, and it is even more reasonable to assume that a thermalized system expands collectively both in the longitudinal and at the same time in the transverse direction.

A big challenge for thermal models coming from experimental data is the central dips in the rapidity distributions of heavier particles observed in the central collisions of nuclei both at AGS and at SPS energies. It is well known that models with uniform longitudinal flow [12] cannot account for such dips. In order to reproduce the experimentally observed central dips, a parameterization of baryon chemical potential $\mu_i$ as function of flow rapidity $\eta$ is introduced in Ref. [6]. However, the physical meaning of such a parametrization is unclear.

In the present paper we will construct a longitudinally non-uniform collective expansion model (NUCEM) basing on the physical argument that the fireballs produced in the nuclear collision will keep some memory on the motion of the incident nuclei. The non-uniform longitudinal flow will be parametrized using a geometrical picture proposed in Ref. [7]. The results of model calculation are compared with the experimental data on the rapidity distributions of baryons and strange particles in 200 A
GeV/c S-S and 158 A GeV/c Pb-Pb collisions.

The longitudinally non-uniform collective expansion model will be formulated in section II. The results of model calculation are presented and compared with the experimental data in section III. A short summary and conclusion will be given in section IV.

II. Longitudinally non-uniform collective expansion flow

In the non-uniform longitudinal flow model (NUFM) proposed in Ref [7] only longitudinal flow was taken into account. In fact, the one dimensional expansion presumably dominates initially because of anisotropic initial conditions. But it is inconsistent to assume that a thermalized system expands collectively only in longitudinal direction without generating transverse flow from the high pressure in the hydrodynamic system. In case of central collisions the hydrodynamic flow should be computed in at least (2+1) dimensions if azimuthal symmetry is maintained. Beside these theoretical necessities for the inclusion of transverse flow [8], there are also evidence for the existence of this flow from the experimental phenomena [9][10].

For the description of particle production in the collective expansion of fire-ball we start from the formalism of Cooper and Frye [11] which describes the single-particle spectrum as an integral over a freeze-out hypersurface, thus summing the contributions from all space-time points at which the particles decouple from the fireball:

\[ E \frac{d^3n}{d^3p} = \frac{g}{(2\pi)^3} \int_{\sigma_f} f(x,p) p' d^3\sigma_v, \]  

(1)

where \( g \) is the degeneracy factor and \( f(x,p) \) the momentum distribution at space-time point \( x \). In thermal models one takes \( f(x,p) \) as a thermal equilibrium distribution and determines \( \sigma_f \) by a freeze-out criterion for thermal decoupling [12]. At freeze-out, the Boltzmann approximation is sufficient, but we allow for a space-time dependence of the temperature \( T \), the chemical potential \( \mu \), and the flow velocity \( u^\mu \):

\[ f(x,p) = \exp \left( -\frac{p \cdot u(x) - \mu(x)}{T(x)} \right). \]  

(2)

Focusing on central collisions, we assume azimuthal symmetry of the spatial geometry and momentum distributions.

The assumption of longitudinal Bjorken flow [13] suggests to use longitudinal proper time \( \tau = \sqrt{t^2 - z^2} \) and space-time rapidity \( \eta_l = \tanh^{-1}(z/t) \) as suitable variables in the \( t-z \) plane. The transverse radial coordinate is denoted by \( r \). In an azimuthally symmetric geometry of this kind it is practical to decompose the velocity field in the
following way:

\[ u^\nu = (\cosh \eta_\ell \cosh \eta_t, \mathbf{e}_r, \sinh \eta_\ell, \cosh \eta_\ell \sinh \eta_t). \]  

(3)

Here \( \mathbf{e}_r \) is the 2-dimensional unit vector in the radial direction, \( \eta_\ell = \eta_\ell(t, r, z) \) is the longitudinal flow rapidity by which each volume element on the z-axis moves relative to the center of mass, and \( \eta_t = \eta_t(t, r, z) \) is the rapidity corresponding to the transverse flow of the volume element at position \((r, z)\) as seen from a reference point at \(z\) on the beam axis moving with the local flow velocity there.

The momentum of the particle in the center-of-fireball system can be parametrized as follows:

\[ u^\nu p^\nu = m_t \cosh(y - \eta_\ell) \cosh \eta_t - p_t \sinh \eta_\ell \cos(\phi - \varphi) \]  

(4)

Because of azimuthal symmetry in central collisions we can integrate over \( \phi \) making use of the modified Bessel function

\[ I_0(\alpha) = \frac{(2\pi)^{-1/2}}{\pi} \int_0^{2\pi} e^{\alpha \cos \phi} d\phi. \]

The geometry of the freeze-out hypersurface \( \sigma_f \) is fixed as follows: In the time direction we take a surface of constant proper time, \( \tau = \tau_f \). In \( \eta_\ell \) direction the freeze-out volume extends only to a maximum space-time rapidity \( \eta_0 \), which is required by the finite available total energy and breaks longitudinal boost-invariance proposed by Bjorken. In the transverse direction the boundary is given by \( R_f \), which describes a cylindrical fireball in the \( \eta-r \) space.

Having specified the freeze-out geometry and the distribution function at freeze-out we obtain the following thermal single-particle spectrum:

\[ \frac{d^2N}{dy dm_t^2} = \frac{g}{2\pi} m_t \tau_f \int_{\eta_0}^{\eta_0} d\eta_c dr e^{\mu/T} e^{-\tilde{\alpha} \cosh(y - \eta_\ell)} \cosh(y - \eta_\ell) I_0(\alpha), \]  

(5)

where \( \tilde{\alpha} = (m_t/T) \cosh \eta_t, \alpha = (p_t/T) \sinh \eta_t \). After integrating over \( m_t \), we get the rapidity distribution as follows:

\[ \frac{dN}{dy} = \frac{g}{4\pi} \int_{m_{t0}}^{m_{hi}} dm_t^2 \int_{-\eta_0}^{\eta_0} d\eta_t \int_0^{R_f} dr r \tau_f(r) e^{\mu/T} e^{-\tilde{\alpha} \cosh(y - \eta_\ell)} \]  

\[ \times m_t \cosh(y - \eta_\ell) I_0(\alpha). \]  

(6)

Here \( m_{t0}, m_{hi} \) denote the experimental limits in which the spectrum was measured. The freeze-out radius \( R_f \) and the longitudinal extent of the fireball is fixed via the finite interval \((-\eta_0, \eta_0)\). \( \eta_t = \tanh^{-1} \beta_t \) is the rapidity of transverse flow. Replacing the transverse flow velocity profile by its radial average, we get

\[ \frac{dN}{dy} = \frac{g\tau_f R_f^2}{8\pi} \int_{m_{t0}}^{m_{hi}} dm_t^2 m_t I_0(\alpha) \int_{-\eta_0}^{\eta_0} d\eta \cosh(y - \eta_\ell) e^{\mu/T} e^{-\tilde{\alpha} \cosh(y - \eta_\ell)}. \]  

(7)
In Eqn.(7) the collective expansions in both the longitudinal and the transverse directions have been taken into account. For the convenience of studying the non-uniform longitudinal flow, we change the order of integration in Eqn.(7)

$$\frac{dN}{dy} = \int_{-\eta_0}^{\eta_0} d\eta F(y - \eta), \quad (8)$$

$$F(y) = \frac{g_\tau R_l^2}{8\pi} \int_{m_h^2}^{m_{hi}^2} I_0(\alpha) \cosh(y)e^{\mu/T}e^{-\tilde{\alpha}\cosh(y)}m_t dm_t^2. \quad (9)$$

The function $F(y)$ plotted in Fig.1 can be interpreted as the rapidity distribution from a “single thermal fire-ball with transverse flow” and the rapidity distribution $dN/dy$ of Eqn’s. (7), (8) is the sum of the contributions of a series of such fire-balls with centers distributed uniformly in the rapidity range $[-\eta_0, \eta_0]$, cf. the schematic plot in Fig.2 and Fig.1(b).

It is well known that Eqn.(7), or equivalently Eqn’s.(8)(9), does not reproduce the central dip in the rapidity distributions of heavier particles in central relativistic heavy ion collisions observed experimentally already at the AGS energies and confirmed further in the experiments at SPS energies. In order to account for this experimental finding Ref.[6] introduced a rapidity dependence of chemical potential $\mu = \mu(\eta)$. However, the physical meaning of this dependence is not so clear.

In the following we will follow the reasoning proposed in Ref.[7], i.e. we assume that the fire-balls produced in relativistic heavy ion collisions will keep some memory

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**Fig.1**

- **(a)** Rapidity distribution from a single thermal fire-ball with transverse flow.
- **(b)** Superposition of uniformly distributed kaon-fire-balls

**Fig.2**

Schematic plot of the fire-balls distributed uniformly in longitudinal phase space
on the motion of the incident nuclei, and therefore the distribution of fire-balls, instead of being uniform in the longitudinal direction, is more concentrated in the direction of motion of the incident nuclei. This means that the distribution of fire-ball is more dense at large absolute value of rapidity, as sketched schematically in Fig.3.

Fig.3 Schematic plot of the fire-ball distribution in non-uniform longitudinal flow

![Schematic plot of the fire-ball distribution in non-uniform longitudinal flow](image)

Fig.4 The dependence of longitudinal distribution of fire-ball center $\rho(y_{le})$ on ellipticity parameter $e$

Using a simple geometrical picture to parametrize the non-uniform longitudinal flow [7], a non-flat distribution function of fire-ball center $y_{le}$

$$
\rho(y_{le}) = \sqrt{\frac{1 + \sinh^2(y_{le})}{1 + e^2 \sinh^2(y_{le})}}
$$

(10)

is introduced and the rapidity distribution (8) becomes
Fig.5 Rapidity distributions of kaon (a), net proton (b) and lambda (c) for central S-S collisions at 200 A GeV/c. Data are taken from Ref.[4]

\[ \frac{dN}{dy} = \int_{-y_{le0}}^{y_{le0}} dy_{le} \rho(y_{le}) F(y - y_{le}) \]

\[ = \frac{g \tau R_i^2}{8\pi} \int_{-y_{le0}}^{y_{le0}} \rho(y_{le}) dy_{le} \sum_{i=1}^{mbi} \int_{m_{ti}^{lo}}^{m_{ti}^{hi}} dm_i I_0(\alpha) \cosh(y - y_{le}) e^{\mu/T} e^{-\bar{\alpha} \cosh(y - y_{le})} \] (11)

In Eqn.(10) \( e \) is the “ellipticity” parameter describing the degree of non-uniformity of longitudinal flow [7]. The dependence of \( \rho(y_{le}) \) on \( e \) is shown in Fig.4. It can be seen from the figure that the larger is the parameter \( e \), the flatter is the distribution \( \rho(y_{le}) \) and the more uniform is the longitudinal-flow distribution, cf. Fig.3. When \( e \rightarrow 1 \), the longitudinal-flow distribution is completely uniform (\( \rho(y_{le}) \rightarrow 1 \)), returning back to the cylindrically symmetrical collective flow model, Eqn.(7).

III. Comparison with experiments

We present in Fig.5 (a, b and c) the rapidity distributions of kaon, net proton and lambda for central S-S collisions at 200 A GeV/c, respectively. In view of the large statistical errors of the experimental data, the fitted values of parameter \( e \) are given in a region and accordingly the rapidity distributions have a strip shape.
Fig. 6 The dependence of longitudinal distribution of fire-ball center $\rho(y_{le})$ on ellipticity parameter $e$

The rapidity limit $y_{le0}$ and the ellipticity $e$ used in the calculation are listed in Table I and illustrated in Fig. 6 (a). The parameter $T$ is chosen to be 0.12 GeV.

Table I  The values of model-parameters

| Parameter | S-S Collisions | Pb-Pb Collisions |
|-----------|----------------|------------------|
|           | $k^+$ | $p-\bar{p}$ | $\Lambda$ | $p-\bar{p}$ |
| $e$       | 0.7-0.9 | 0.3-0.45 | 0.7-0.9 | 0.56 |
| $y_{le0}$ | 1.98 | 1.98 | 1.98 | 1.96 |
| $<\beta_t>$ | 0.3c | 0.3c | 0.3c | 0.5c |

Since kaons and lambdas are produced through the interaction of colliding nuclei, they have less memory on the motion of the incident nuclei. Therefore, the values of ellipticity $e$ for kaon and lambda are bigger than that for proton, as shown in Table I.

It can be seen from the figures that our longitudinally non-uniform collective expansion model (NUCEM) reproduces the central dip of rapidity distributions for kaon, lambda and net proton very well, in agreement with the experimental findings.

The appearance or disappearance of central dip is insensitive to the rapidity limit $y_{le0}$ but depends strongly on the magnitude of the ellipticity $e$ and the mass $m$ of produced particles. When transverse flow exists there is a shallow dip also for the rapidity distribution of light particle (kaon), cf. Fig. 5, which is slightly different from the case of one-dimensional longitudinal flow.

The rapidity distribution of net protons for Pb-Pb collisions at 158 A GeV/c is shown in Fig. 7, and the longitudinal distributions of fire-ball center are shown in Fig. 6(b) for various values of ellipticity $e$. The solid line in Fig. 7 corresponds to the...
results of our model with ellipticity parameters $e = 0.56$. The fitted $\chi^2/\text{DF}$ is given in Fig.8. The value $e = 0.56$ corresponds to the minimum of $\chi^2$. The region of $e$ for $\chi^2$ to increase one unit from the minimum is between 0.40 and 0.745.

Fig.7 Rapidity distribution of net protons

Fig.8 The fitting $\chi^2$

Comparing Fig’s.5 and 7 we can see that the net-proton ($p-\bar{p}$) rapidity distribution is narrower for Pb-Pb [5], than for S-S collisions [4]. This indicates an increasing baryon stopping for Pb-Pb collisions, giving accordingly a smaller rapidity limit $y_{fo0}$ and a larger average transverse velocity and uniformity for Pb-Pb interaction. The width of the rapidity distributions is mainly controlled by the amplitude $y_{fo}$ of the longitudinal flow. For smaller colliding system (S-S collision) a single value of $y_{fo0}$ can account for the wide distribution of heavier particles (net protons and $\Lambda$) and at the same time fit the kaon-distribution well. For the larger colliding system (Pb-Pb), the $y_{fo0}$ is smaller together with a larger average transverse flow velocity. These are shown also in Table I.

IV. Summary and Conclusions

Let us begin with a discussion of the thermal freeze-out points. Freeze-out marks the transition from a strongly coupled system, which evolves from one state of local thermal equilibrium to another, to a weakly coupled one of essentially free-streaming particles. If this transition happens quickly enough, the thermal momentum distributions (superimposed by collective expansion flow) are frozen in, and the temperature and collective flow velocity at the transition point can be extracted from the measured momentum spectra. In high energy heavy-ion collisions the freeze-out process is triggered dynamically by the accelerating transverse expansion and the very rapid growth of the mean free paths as a results of the fast dilution of the matter [15]. Idealizing the kinetic freeze-out process by a single point in the phase diagram is therefore not an entirely unreasonable procedure.
In high energy heavy-ion collisions, due to the transparency of nucleus the participants will not lose the historical memory totally and the produced hadrons will carry some of their parent’s memory of motion, leading to the unequivalence in longitudinal and transverse directions, i.e. the flow of produced particles is privileged in the longitudinal direction. As the increasing of duration time and the pressure of the system at SPS energy, it is reasonable to assume that the system expands collectively both in the transverse and in the longitudinal directions. This picture has been used by lots of models [12], [14]. For asymptotically high energies, a boost-invariant longitudinal expansion model is postulated by Bjorken [13], which gives a plateau in the rapidity distribution of produced particles. For finite collision energies such as CERN SPS energy or below, a cylindrically symmetrical expansion model was firstly postulated by Schnedermann, Sollfrank and Heinz[12] by introducing a cut in rapidity. In this model a set of fire-balls with centers located uniformly in the rapidity region \([-y_{le0}, y_{le0}]\), as sketched schematically in Fig.2, represents the longitudinal flow, and at the same time there is radial flow developed simultaneously. It can account for the wider rapidity distribution when compared with the prediction of the pure thermal isotropic model but failed to reproduce the central dip in the proton and Λ rapidity distributions.

In this paper, we argue that the transparency/stopping of relativistic heavy ion collisions should be taken into account more carefully. It will not only lead to the anisotropy in longitudinal-transverse directions, but also render the fire-balls to concentrate more in the direction of motion of the incident nuclei. A non-uniform longitudinal flow model is proposed, which assumes that the centers of fire-balls are distributed non-uniformly in the longitudinal phase space. A parameter \(e\) is introduced through a geometrical parameterization which can express the non-uniformity of flow in the longitudinal direction, i.e. the centers of fire-balls of produced particles prefer to accumulate in the two extreme rapidity regions \((y_e \approx \pm y_{le0})\) in the c.m.s. frame of relativistic heavy-ion collisions, and accordingly the distribution is diluted in the central rapidity region \((y_e \approx 0)\). Apart from the non-uniform longitudinal flow, a radial flow in the transverse direction is also considered in the present paper.

It is found that the depth of the central dip depends on the magnitude of the parameter \(e\) and the mass of produced particles, i.e. the non-uniformity of longitudinal flow which is described by the parameter \(e\) determines the depth of the central dip for produced particles. Comparing with one-dimensional non-uniform longitudinal flow model [7], the rapidity distribution of lighter strange particle kaons also shows a dip due to the effect of transverse flow.

Smaller colliding systems distinguish themselves from larger ones not primarily by the achieved maximal energy density, but by the occupied collision volume in space and time. Compared to S-S collisions, Pb-Pb collisions live longer until thermal freeze-
out, expand more in the transverse direction, develop more transverse collective flow. It is found that transverse collective expansion is expected to be more prominent in the heavy Pb-Pb collision system, owing to smaller surface-to-volume ratio, the larger fireball volume and the longer duration time of expansion. On the other hand, Pb-Pb collisions develop less longitudinal flow, which suggests, together with the larger $e$, a larger stopping in the bigger colliding system.

References

[1] S.A. Bass, M. Gyulassy, H. Stöcker and W. Greiner, J. Phys. G25, (1999)R1-R57; T. Alber et al., Phys. Rev. Lett. 75, (1995) 3814.

[2] A. Capella and B. Z. Kopeliovich, Phys. Lett. B381, (1996)325; D. Kharzeev, Phys. Lett. B378, (1996) 238; K. Werner, Phys. Rep. 232,(1993)87.

[3] J. Letessier et al., Phys. Rev. D51, (1995)3408.

[4] E. Alber et al., (NA35 Collab.) Nucl. Phys. A566; Bächler et al., (NA35 Collab.) Phys. Rev. Lett. 72 (1994)1419.

[5] H. Appelshäuser et al.,(NA49 Collab) Phys. Rev. Lett. 80 (1998)4136.

[6] J. Sollfrank, Eur. Phys. J. C9, (1999) 156; C. Slotta, J Sollfrank and U. Heinz, 1995 Strangeness in Hadronic Matter, AIP Conference Proceedings 340 (Woodbury: AIP Process) p 462.

[7] Shengqin Feng, Feng Liu and Lianshou Liu, Thermal equilibrium and non-uniform longitudinal flow in relativistic heavy ion collisions, [hep-ph/0005047](http://arxiv.org/abs/hep-ph/0005047) to appear in Phys. Rev. C.

[8] K. Werner, Quark Matter 1990, Menton, France, Nucl. Phys. A525 (1991) 501c; K. Werner, P. Koch, Phys. Lett. B 242 (1990)251; A.M. Poskanzer, and S.A. Voloshin, Phys. Rev. C 58, (1998) 1671; J.-Y. Ollitrault, Nucl. Phys. A 638, (1998) 195c.

[9] H.H. Gutbrod, A.M. Poskanzer, H.G. Ritter, Rep. Prog. Phys. 52 (1989) 1267; H.A. Gustaffson et al., Phys. Rev. Lett. 53 (1984) 1590; H. Stöcker, W. Greiner, Phys. Rep. 137 (1986) 277.

[10] T. Abbott et al., Phys. Rev. Lett. 64 (1990) 847; J. Barrette et al., (E877 Collab) Phys. Rev. Lett. 73, (1994) 2543.
[11] F. Cooper, G. Frye, Phys. Rev. D 10, (1974) 186.

[12] E. Schnedermann, J. Sollfrank and U. Heinz, Phys. Rev. C 47, 1738(1993); C 48, (1993) 2462; E. Schnedermann and U. Heinz, Phys. Rev. C 50, (1994) 1675.

[13] J.D. Bjorken, Phy. Rev. D 27, (1983) 140.

[14] P. Braun-Munzinger, J. Stachel, J.P. Wessels and N. Xu, Phys. Lett. B 344, (1995) 43; P. Braun-Munzinger, J. Stachel, J.P. Wessels and N. Xu, Phys. Lett. B 365, (1996) 1; P. Braun-Munzinger, J. Stachel, Nucl. Phys. A 606, (1996) 320.

[15] U. Mayer and U. Heinz, Phys. Rev. C 56, (1997) 439; E. Schnedermann and U. Heinz, Phys. Rev. C 50, (1994) 1675; U. Heinz, Nucl. Phys. A 661, (1999) 140c.