A mathematical programming model for the recycling of sustainable food waste

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Abstract. Food waste is a primary component of municipal solid waste (MSW), which can be recycled or disposed of depending on how it has been treated. The cost and environmental impact of treating food waste are common concerns of local authorities. In this study, a mathematical model is presented which is designed to assist local authorities in determining the treatment plan. In addition to data of waste generation and treatment capacity, the level of local demand for recycled products is also included in the developed model. With the use of the proposed model, the recycling and treatment scheme can contribute to local sustainability by reducing the dependence on virgin materials for local industries. Four food waste treatments including wet feeding, dried feeding, composting and incineration have been examined for their cost-effectiveness and environmental impact. A case study of Taichung City, Taiwan is presented to demonstrate the applicability of the developed model. The results revealed the trade-off costs and environmental impacts associated with each recycling scenario. Also, the results indicate that establishing new facilities may reduce the impact on the environment, as well as the total cost.

1. Introduction
Food waste is usually a primary component of municipal solid waste (MSW). Due to its composition of typically biodegradable organic matter of high moisture, so neither landfilling nor incinerating is appropriate. Recycling or disposal treatments of food waste, such as wet feeding, dry feeding, composting and incinerating, are common options of local authorities in charge of municipal solid waste management (MSWM). Environmental impacts and cost are two essential concerns for local authorities in determining which treatment is to be employed and probably with different weightings among local authorities. Both Chen and Lin [1], and Lee et al, [2] have indicated that municipal solid waste practices should be modified depending on local features. If resultant products from food waste are back into material recirculation in local industries, the dependency on relevant virgin materials, costs and environmental impact due to transportation can be reduced. Local authorities usually lack the expertise and resources to maintain large or innovative facilities. As a result, small facilities providing typical treatment options are preferred. Bastin and Longden [3] examined centralized waste-for-energy facilities with large treatment capacities and distributed MSW facilities with limited capacities. They also concluded that distributed facilities are superior in terms of the levels of energy consumption and greenhouse gas emissions. Food waste recycling can be assessed in a similar way with the local utilization of recycled products. In addition to determining the locations and capacities...
of distributed facilities, use of recycled products should be taken into consideration. An analytical procedure based on the foregoing concept to lower the dependency on virgin materials and relevant environmental impacts is developed in this study, and its purpose is aimed to assist local authorities to determine a food waste recycling plan focused on local sustainability and material recirculation.

Life cycle analysis (LCA) procedures have been widely employed to evaluate the environmental impacts of different treatment methods. For instance, Kim and Kim [4] assessed the global warming potential of four food waste treatments, including dry feeding, wet feeding, composting and landfilling, and concluded that wet feeding is the best option. In addition to environmental impact, the cost is also an important factor for local authorities to determine a recycling scheme. Cost includes but is not limited to the capital, maintenance and profit from recycled products. The tradeoffs between cost and environmental impact can be optimized by employing mathematical programming models (MPMs), which are widely applied in MSW management problems, e.g. [5–8]. Typically, the objective function and constraints incorporated differ in each MPM and are dependent on the purpose of the MPM. To our knowledge, the study by Solano et al., [6] might be the first one that has established the comprehensive MPM which incorporates LCA impact and cost to evaluate the optimal integrated MSWM scheme. They assigned each component of the MSW to with the sole treatment/recycling method and found the optimal scheme of overall MSWM problem. Collection and treatment stages of MSW are primary concerns of the model. To improve the local sustainability of material recirculation, the use stage of a recycled product should also be taken into account. In attempting to address this need, the proposed model, which aims to determine a food waste recycling scheme, has been developed and presented in this study.

2. Materials and methods

2.1. LCA and food waste treatments

![Figure 1. LCA system boundary of four treatments for food waste.](image)

Figure 1 presents the system boundary of LCA in this study. The four treatments, including wet feeding, dry feeding, composting and incinerating, were assessed. The function unit in this study was one kilogram of food waste. In addition, the zero burden assumption was applied, i.e. the life cycle of
food waste begins as it has been discharged. To satisfy the requirements of food recycling techniques, the food wastes are classified into plate waste and prep waste in this study. The plate food waste indicates the cooked food discarded, such as unwanted leftovers or out-of-date canned food. Prep waste indicates the remains from food preparation, which can be further distinguished to as edible prep food waste or inedible food waste. Edible prep food waste mainly comprises originally edible food with more nutrients than inedible ones, such as overripe fruits or decade vegetables, which are suitable for feeding treatments. The impact of transportation is evaluated independently via each ton-km unit for model use. The inventory of environmental pollution, energy and material consumed during each treatment and commercial products were investigated in actual local plants, local technical reports. The avoided impacts of commercial products were also included in this study. To evaluate the environmental impact, software Simapro 7.1 [9] is implemented with the forgoing datum. The damaged oriented Eco-indicator 99 [10] with a hierarchist view was selected, which contains 11 indices and can be classified into three categories: human health, ecosystem quality and resources. Detailed accounts of data inventory and the LCA applied in this study can be found in a previous study [11].

2.2. Model development

The proposed model is used to evaluate both the environmental impact from the LCA and the cost of a food waste treatment scheme.

Objective functions:

\[
\begin{align*}
\min & \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{k=1}^{K} PT \cdot D_{ij}(fww_{i,j,t} + fwd_{i,j,t} + fwe_{i,j,t}) + \\
& \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{t=1}^{T} PT \cdot D_{jk}fwrp_{j,k,t} + \sum_{j=1}^{J} \sum_{t=1}^{T} PP_{ij}t_{rj} - \sum_{k=1}^{K} \sum_{t=1}^{T} PA_{it}f_{ptk} \\
\end{align*}
\]

subject to

\[
\begin{align*}
\sum_{j=1}^{J} \sum_{t=1}^{T} fwrp_{j,k,t} &= W_{i}rdr_{i} \quad \forall i \\
\sum_{j=1}^{J} \sum_{t=1}^{T} fww_{i,j,t} &= W_{i}rw_{i} \quad \forall i \\
\sum_{j=1}^{J} \sum_{t=1}^{T} fwe_{i,j,t} &= W_{i}rede_{i} \quad \forall i \\
\sum_{i=1}^{I} (fww_{i,j,t} + fwd_{i,j,t} + fwe_{i,j,t}) &= tr_{j,t} \quad \forall (j,t) \\
\alpha_{t}tr_{j,t} &= \sum_{k=1}^{K} fwrp_{j,k,t} \quad \forall (j,t) \\
f_{ptk} &= \sum_{j=1}^{J} \sum_{t=1}^{T} fwrp_{j,k,t} \quad \forall (j,t) \\
S_{j,t} &\leq R_{t} \cdot y_{j,t} \quad \forall (j,t) \\
tr_{j,t} &\leq s_{j,t} \quad \forall (j,t) \\
\beta_{t} \cdot f_{ptk} &\leq Dem_{k,t} \quad \forall (k,t)
\end{align*}
\]

where \(i, j\) and \(k\) are the location indices of a food waste collection depot, treatment facility, and recycled product to be consumed, respectively; \(t\) is the index of a recycled product (technique); \(PT\)
represents the unit environmental impact of transportation (pt/ton-km); \( D_{ij} \) and \( D_{ik} \) represent the distance from collection depot \( i \) to facility \( j \) and facility \( j \) to location \( k \), respectively; \( f_{wri} \), \( f_{wde} \), and \( f_{wdr} \) represent plate, edible prep and inedible prep food waste transported from collection depot \( i \) to facility \( j \) by \( t \) treatment, respectively; \( f_{wRP} \) represents the amount of recycled product \( t \) which is generated by facility \( j \) and is to be consumed at location \( k \); \( PP_t \) represents the impacts (pt) caused by \( t \) treatment to process one ton of food waste; \( tr_{ij} \) represents the amount of food waste processed by facility \( j \) using \( t \) treatment; \( PA_t \) represents the avoided impacts of saving one ton of regular product \( t \); \( \alpha_t \) represents the equivalent utility of regular product by utilizing one ton of recycled product \( t \); \( f_{p} \) indicates the quantity of recycled product \( t \) delivered to location \( k \); \( CT \) represents the unit cost of transportation (NTD/ton-km); \( F_{cstf} \) represents the capital recovery factor; \( CP \) and \( CS \) represent the fixed cost and variable cost, respectively, to install \( t \) treatment in a facility; \( \gamma \) is a binary variable to determine if the \( t \) treatment at \( j \) facility available; \( s_{ij} \) represents the maximal capacity of the \( t \) treatment at \( j \) facility (tons/month); \( CP \) represents the cost (NTD/ton) by applying \( t \) treatment to process one ton of food waste; \( CA_t \) represents the price of another respective regular product (NTD/ton) for which recycled product \( t \) is a substitute; \( Wi \) represents the amount of food waste at collection depot \( i \); \( Rwi \), \( Rdei \) and \( Rdri \) represent the ratio of plate, edible prep and inedible prep food waste at collection depot \( i \); respectively; \( \alpha_t \) represents the output ratio that the amount of recycled products generated by using one ton of food waste by treatment \( t \); \( R \) represents the upper bound of maximal capacity for treatment \( t \); \( Dem_{kj} \) represents the demand of regular product \( t \) at location \( k \); \( TLw \) and \( TUw \) represent the upper bound and lower bound of plate food waste ratios allowed for treatment \( t \), respectively; in a like manner, \( TLd_r \), \( TUd_r \), \( TLd_t \), and \( TUd_t \) represent those for inedible prep food waste and edible prep food waste ratios allowed for treatment \( t \), respectively. Where Equation 1 is for environmental impacts and Equation 2 for the cost. Equation 3 ensures that inedible prep food waste transported from a collection depot to all treatment facilities is equivalent to the total amounts of food wastes at the collection depot multiplied by the inedible prep food waste ratio. Equation 4 and Equation 5 represent amounts of edible prep food waste and plate food waste, respectively. Equation 6 ensures that the amounts of the three types of food wastes from all depots are equivalent to the amount processed by the specific treatment method and facility. Equation 7 and 8 are mass balance equations to ensure the amount of food waste and resultant products, resultant products and consumption districts, respectively. Equation 9 to Equation 11 defines the capacity of a treatment plant and local demand for a resultant product.

3. Materials and methods

3.1. Case background

In order to demonstrate the applicability of the proposed model, a case study was conducted. Taichung City is the third largest metropolis in Taiwan. Its area is approximately 2,214 square kilometers, with about 2.65 million inhabitants in 29 districts. The recycling of food waste comprised more than 188 tons/day according to the local statistics, which was equal to 8.9 % of the MSW of Taichung City [12]. The typical recycling treatments for the collected food waste include wet feeding for swine, dry feeding for poultry and composting. In addition, food waste not recycled is incinerated along with other MSW.

3.2. LCA analysis

Figure 2 presents the impact in three impact categories via four technologies for treating one kilogram of food waste, where positive values indicate environmental loss and negative mean benefit. The total point of treatment is its lump sum of three categories. Incineration has impacts on all of the three categories. Impacts of swine feeding and dry feeding are similar, beneficial in “human health” and “resources” categories by substituting commercial feeds and negative in “ecosystem quality” because of decreasing the use of grains in feeds and losing its relevant carbon sequestration. Dry feeding in our case is to substitute commercial feeds for poultry which typically have higher ratios of grains,
consequently leading to higher negative impacts than wet feeding for swine. In terms of LCA, composting is the best technology among the four treatments as it offers the most negative total point and has environmental benefit in three impact categories.

3.3. Model analysis
The developed model was implemented to designate the food waste management plan. Figure 3 presents the trade-off between environmental impacts and costs of different scenarios, negative values of vertical axis mean profit and positive ones represent the cost. The frontier represents the Pareto set of model results after a typical multi-objective solving procedure. Five scenarios, CP and P# are marked for readability, which represents the current plan (CP), the minimal environmental impact plan (P1), an optimized current plan (P2), a self-liquidating plan (P3) and the maximal profit plan (P4), respectively. Table 1 presents the details of each scenario, including the percentages of food waste treated by each technology, environmental impact and cost. Obviously, the alternatives in the lower right offer higher environmental impacts and cost than the upper left ones, and vice versa. Since incineration has no advantage when compared to the other three recycling technologies, it was selected by none of the alternatives on the frontier edge. Typically, composting has a lower environmental impact and is preferred in the upper left scenarios, while dry feeding with significant profit potential is preferred for those in the lower right. In addition, wet feeding is widely applied among all five scenarios because of its superiority both in terms of cost and environmental impact.

Figure 2. LCA results of four treatment methods.

Figure 3. A trade-off between environmental impact and cost of scenarios.
Scenario CP is not in the frontier, indicating CP was not efficient. Scenario P2 has both less cost and environmental impact than Scenario CP, which can be used by the local authorities to modify the current plan. The optimized routing scheme by the developed model is helpful to save the cost. For illustration, Figure 4 and Figure 5 present the inter-district transportation of compost production and consumption of Scenarios CP and P2, respectively. Districts with darker colour represent higher demands for resultant product. Food waste treatment and resultant product consumption not in identical districts are represented by solid and dashed directed line, respectively. Typically, the demands of compost mainly exist on the outskirts of Taichung City, where most of the region’s farms are located. The demands of compost in these areas are greater than the amounts of supply and thus the consumption of composts outside districts where it has been produced is limited. It can be seen that all districts (except District 22) either consume all the composts or do not produce compost. The inefficient location of facilities and collection routes can be improved by establishing new facilities, which are marked by unfilled circles. Table 1 also presents the difference of the cost between scenario CP and P2, with slight change of wet feeding and compost ratio, and appropriate locations for compost plants, the cost can be reduced to 80% of the current plan and the environmental impacts can be improved, too.

Table 1. Model results of marked scenarios.

| Treatment ratio (%) | CP   | P1   | P2   | P3   | P4   |
|---------------------|------|------|------|------|------|
| Wet feeding         | 40%  | 40%  | 45%  | 59%  | 44%  |
| Composting          | 60%  | 60%  | 55%  | 38%  | 3%   |
| Dry feeding         | 0%   | 0%   | 0%   | 3%   | 53%  |

Environmental impacts (pt/month)

|                        | Food waste | Transportation | 2823 | 539 | 914 | 1089 | 1367 |
|------------------------|------------|----------------|------|-----|-----|------|------|
| Processing             |            |                | 9196 | 9196| 10261| 13384| 14691|
| Recycling Product      |            |                |      |     |      |      |      |
| Transportation         |            |                | 813  | 154 | 242 | 479  | 634  |
| Utilization            |            |                | -9478| -9478| -9242| -6365| 32843|
| Total                  |            |                | 3354 | 411 | 2175 | 8587 | 49535|
| Ratio                  |            |                | 100% | 12% | 65% | 256% | 1477%|

Cost (1000 NTD/month)

|                        | Food waste | Transportation | 617  | 117 | 200 | 238  | 299  |
|------------------------|------------|----------------|------|-----|-----|------|------|
| Processing             |            |                | 7020 | 8530| 7650| 7311 | 11169|
| Recycling Product      |            |                |      |     |     |      |      |
| Transportation         |            |                | 178  | 34  | 53  | 105  | 139  |
| Utilization            |            |                | -4950| -4950| -5604| -8273| -22437|
| Total                  |            |                | 2865 | 3731| 2299| -619 | -10830|
| Ratio                  |            |                | 100% | 130%| 80% | -22% | -378%|
4. Conclusions
This study has established a procedure to determine a food waste treatment plan based on local sustainability and preference. The developed mathematical model which incorporates a multi-objective decision process can efficiently analyze the environmental impact and cost-effective alternatives, which are flexible enough to allow the local authorities to determine an optimized recycling plan based on local features. The proposed model takes into account local demands for recycled products while significantly reducing the need for virgin materials and relevant environmental impact and cost resulting from transportation. In short, the proposed model can be used to improve the level of local sustainability and material recirculation. Finally, a typical LCA procedure was analyzed in this study to demonstrate the process for evaluating the environmental impact of various treatments, which can then be determined and implemented in order to meet the needs of local authorities.

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