DETECTION OF NETWORK STRUCTURE IN THE LAS CAMPANAS REDSHIFT SURVEY

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ABSTRACT
We employ a percolation technique developed for pointwise distributions to analyze two-dimensional projections of the three northern and three southern slices in the Las Campanas Redshift Survey. One of the goals of this paper is to compare the visual impressions of the structure within distributions with objective statistical analysis. We track the growth of the largest cluster as an indicator of the network structure. We restrict our analysis to volume-limited subsamples in the regions from 200 to 400 $h^{-1}$ Mpc, where the number density of galaxies is the highest. As a major result, we report a measurement of an unambiguous signal, with a high signal-to-noise ratio (at least at the level of a few $\sigma$), indicating significant connectivity of the galaxy distribution which in two dimensions is indicative of a filamentary distribution. This is in general agreement with the visual impression and typical for the standard theory of the large-scale structure formation based on gravitational instability of initially Gaussian density fluctuations.

Subject headings: galaxies: distances and redshifts — large-scale structure of universe — methods: statistical

1. INTRODUCTION
For decades cosmologists have been developing methods for characterizing and quantifying the geometry and topology of structure in the local galaxy distribution as supplied to them by astronomers. Numerous statistics have been employed and refined in this endeavor, with the most successful being percolation analysis (Shandarin & Zeldovich 1983; Einasto et al. 1984) and the genus statistic (Gott, Melott, & Dickinson 1986; Pearson et al. 1997). 1 A more general approach that in principle accommodates both percolation and genus statistics is based on measuring the Minkowski functionals (Mecke, Buchert, & Wagner 1994; Schmalzing & Buchert 1997). The filling factor and the genus (which apply to the entire distribution) are two of the four Minkowski functionals ($M_0$, $M_3$ correspondingly), and the volume of the largest cluster statistic is another ($M_0$ for the largest cluster only).

By differing techniques, these statistics have produced compatible results describing the structure of the local universe in the IRAS 1.2 Jy survey (Yess, Shandarin, & Fisher 1997; Protogerous & Weinberg 1997; Kerscher et al. 1997). Percolation analysis gives similar results for Gaussian distributions but significantly differs from the genus statistic for the non-Gaussian distributions (Sahni, Sathyaprakash, & Shandarin 1997). In studies of geometry and topology, the major limiting factors were the shot noise in the analysis of pointwise distributions or resolution in the analysis of density fields derived from galaxy positions (Yess et al. 1997), and the small size of the survey. The relatively high galaxy number density in the Las Campanas Redshift Survey (LCRS) reduces the discreteness effects. Also, the size of the survey promises that a fair sample of the universe is being probed. Visually, at least, there are no structures comparable to the size of the slices. For the first time, a redshift survey has reached the scale where the universe looks roughly homogeneous apart from the obvious inhomogeneity of a magnitude-limited survey. The extent of the upcoming Sloan Digital Sky Survey (see, e.g., Gunn & Weinberg 1995) promises equally unequivocal results over even larger regions.

The particulars of the LCRS and our utilization of the survey are detailed in § 2 of this paper. In addition, the standard Poisson distributions and their application are explained in § 2. The parameters used to characterize the galaxy distributions are described in § 3, along with the percolation method for pointwise distributions. In § 4 the percolation results are presented and explained. Conclusions are also drawn in § 4, with suggestions for further investigations.

2. THE LCRS AND POISSON STANDARDS
There are approximately 25,000 galaxies with redshift positions in the LCRS. They are distributed over six slices, three northern and three southern. The geometry of slices, schematically depicted in Figure 1, are strips of the sky 1.5 thick and 80° wide which are separated by 3°. The northern slices are centered at declinations of $-3°$ (North1), $-6°$ (North2), and $-12°$ (North3) and the southern slices at $-39°$ (South1), $-42°$ (South2), and $-45°$ (South3). All slices are probed to a depth of 60,000 km s$^{-1}$ ($600 h^{-1}$ Mpc for $H_0 = h$ km s$^{-1}$ Mpc$^{-1}$) for galaxies of $m = 17.75$, the limiting magnitude. For the details of the LCRS and clustering properties see Shectman et al. (1996) and references therein. The survey volume of the LCRS allows for a fair assessment of the connectivity of the galaxy distribution given that cosmic structures are on the scale of 100 $h^{-1}$

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1 As reported in a recent paper by Bhavsar & Splinter (1996), the percolation properties can be reconstructed from the minimal spanning tree.

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Fig. 1.—Schematic of the LCRS slices. The orientation and curvature of the north and south slices are illustrated for clarity. The sectors that the galaxy positions are projected onto are shown above.

Fig. 2.—Map of the LCRS slice, North1. There are 4495 galaxies plotted (initial FF of 0.016). Top right: The two-dimensional Poisson distribution. Bottom: The distributions corrected for projection (right) and selection and projection effects (left). The number of points is the same in every panel. The axis units are Mpc.
Mpc and the number of galaxies contained in the survey gives a signal that, given the sensitivity of percolation analysis, overcomes random noise for a large portion of the survey volume.

In order to characterize the topologies of the LCRS slices, standards typifying random distributions need to be constructed. Since the galaxy positions of the slices are projected to a central conical surface of each slice, the standards have to account for the local galaxy number density of the LCRS as well as the projection effects.

Figure 1 schematically shows the central conical surfaces of the six slices of the LCRS. The northern slices are nearly

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**Fig. 3.**—LCS results for the LCRS slices. The top panels are the results for the full slices ($R \leq 600 \, h^{-1} \, \text{Mpc}$), and the middle and bottom panels are the results for the magnitude ($60 \, h^{-1} \, \text{Mpc} \leq R \leq 400 \, h^{-1} \, \text{Mpc}$) and volume-limited samples, respectively ($200 \, h^{-1} \, \text{Mpc} \leq R \leq 400 \, h^{-1} \, \text{Mpc}$). The solid lines are the survey results (one lightest, three darkest, for both northern and southern slices). The dashed lines are the LCS for corresponding Poisson distributions corrected for selection and projection effects, while the dotted lines are uncorrected Poisson distribution results for reference. The error bars are $1 \sigma$ deviations over four realizations in all cases but one. The final comparison should be done with the dashed lines and corresponding error bars in the bottom panels, which have been obtained from 50 realizations.

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**TABLE 1**

| SLICE      | Declination (deg) | $N$   | $\text{FF}_0$ | $\text{FF}_0$ | $\text{FF}_0^*$ |
|------------|-------------------|-------|---------------|---------------|---------------|
| North1.....| -3                | 4495  | 0.016         | 0.029         | 0.019         |
| North2.....| -6                | 2589  | 0.009         | 0.018         | 0.009         |
| North3.....| -12               | 5217  | 0.018         | 0.030         | 0.018         |
| South1.....| -39               | 4339  | 0.015         | 0.025         | 0.014         |
| South2.....| -42               | 4728  | 0.017         | 0.027         | 0.015         |
| South3.....| -45               | 4140  | 0.015         | 0.024         | 0.012         |

* Magnitude- and volume-limited regions are bordered by $60 \, h^{-1} \, \text{Mpc} < R < 400 \, h^{-1} \, \text{Mpc}$ and $200 \, h^{-1} \, \text{Mpc} < R < 400 \, h^{-1} \, \text{Mpc}$, respectively.
flat and the southern ones are significantly curved. We analyze the distributions obtained by projecting the galaxies on these surfaces. Since the conical surface does not have inner curvature, it can be unrolled onto a plane surface without distortion (shown at the top of Fig. 1).

Figure 2 shows the two-dimensional flat map of a northern slice, North1, of the LCRS in the upper left panel. Its corresponding Poisson distributions, with corrections sequentially applied, are also shown to illustrate the effect of each correction: the projection increases the two-dimensional density toward the outer border (bottom right panel) and the selection effect toward the central part of the slice (bottom left panel). Poisson distributions created to correspond to an appropriate selection function and corrected for projection effects serve as standards for pointwise distributions. It is worth noting that all panels in Figure 2 have the same number of points. The selection function chosen to approximate the distributions of the six slices was taken from Lin et al. (1996) and based on the subsample of galaxies from both north and south slices termed NS112. Note that it is easy for the eye to discern the survey distribution from the random distributions.

3. PERCOLATION

The first step in the percolation of pointwise distributions is to superpose a grid on the sector geometry and then locate the galaxy positions on the lattice. In projecting the galaxy positions to a flattened, central plane, we have accentually ignored the effects of curvature associated with the geometry of the initial survey, which may possibly have a greater consequence for the southern slices, since they are more strongly curved. Our percolation analysis is performed on a two-dimensional lattice of cells 1 Mpc$^2$ in area and the same size for all slices and standards. The positions of galaxies are equated with filled lattice cells. Filled cells that share a common side are considered neighbors. Through the stipulation that "any neighbor of my neighbor is my neighbor," clusters composed of adjacent cells are defined and grow. Our percolation method for pointwise distributions allows for two means by which clusters can grow. Circles of specified radius are constructed around the initially filled cells (galaxy positions) in the distribution. The radii of these circles are incrementally increased to encompass adjacent sites. Cells enveloped by expanding circles are labeled filled and are considered neighbors of the initial cell at the center of the circle. If two or more circles come to overlap while expanding, the members of the overlapping clusters merge into a single combined cluster. As the radii increase, clusters will grow in size and generally diminish in number due to mergers. This process will continue until the largest cluster is the only cluster in the distribution. In an infinite space, the largest cluster emerges as the infinite cluster. For details of the pointwise percolation method see Klypin & Shandarin (1993).

In this study, we track two percolation parameters as functions of the increasing circle radius. The filling factor (FF) is defined as the fraction of filled cells in the total area. The second parameter, the largest cluster statistic (LCS), is the relative size of the largest cluster to the total area of filled cells. Because the size of the largest cluster is reported in units of the FF, its initial value should be small when the largest cluster is one of many clusters, and its maximum value is 1.0 when the largest cluster spans the space and incorporates all the filled cells of the distribution. The relative area of the largest cluster is reported as a function of the FF in comparisons between galaxy and Poisson distributions (see Fig. 3).

A rapid rise in the LCS is indicative of the percolation condition\(^1\) (Klypin & Shandarin 1993). However, it is not important for this study to determine the exact FF\(_p\) associated with percolation. The exact FF\(_p\) is a noisier statistic than the LCS and less discriminating. In our method, the LCS, over its full range, is used to characterize the nature of the distribution (Yess & Shandarin 1996). In general, the faster the LCS grows, the more the curve shifts to the left and the more connected the distribution. A distribution for which the LCS grows more rapidly than a comparable Poisson model is described as an example of a network topology, and a distribution for which the LCS grows more slowly is considered to be clumpy or have a "meatball" topology.

For a direct comparison of percolation results, pointwise distribution standards need to have initial FFs equivalent to those of the distributions they are characterizing. If the initial FF of a standard distribution is too high (FF\(_0\) \(\approx\) 0.1), the resolution of the percolation parameters will not be sufficient to detect the onset of percolation. The Poisson standards in this study are random distributions adjusted, as a function of radius, for selection and projection effects with corresponding initial filling factors well below the resolution limit. The initial filling factors of the survey slices fall in the 1%-3% range, which is much smaller than the percolation transitions (see Fig. 3 and Table 1). In addition, it is easy to see that because of our percolation method the resolution of the LCS can depend strongly on the value of the initial FF. For instance, for distributions that initially have well-isolated galaxies the majority of the clusters will contain only one filled cell. In this case, the first iteration of the expanding circles will add four nearest neighbors to virtually every cluster, causing the FF to increase by a factor of nearly 5. The initial FFs of the LCRS slices are well below the level where this effect would negate the results.

4. RESULTS AND CONCLUSIONS

Shown in Figure 3 are the results of the LCRS percolation analysis. The left column shows the results for the three northern slices of the survey while the right column shows the results of the southern slices. We present the results separately, as the north and south slices have significantly different geometries (see Fig. 1) and also different patterns of regions observed with 50 fiber and 112 fiber spectrographs. The southern slices have a quasi-periodic pattern of differently observed fields.

The solid lines are the results for the survey slices, with the lightest being North1 (South1) and the heaviest North3 (South3). In all graphs, the dotted line to the far right is the result from a statistically homogeneous two-dimensional network.

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\(^2\) The term "cluster" as used in percolation analysis does not imply cluster of galaxies in the astronomical sense.

\(^3\) The maximum rate of the growth of the largest cluster volume can be used as an indicator of the percolation threshold; see, e.g., de Lapparent, Geller, & Huchra (1991).
Poisson distribution with the appropriate initial FF and geometry (see Fig. 2, top right panel). This result is shown for reference. The dashed line is the result from an appropriate three-dimensional Poisson distribution projected on the two-dimensional conical surface and corrected for selection effects (see Fig. 2, bottom left panel). The observational bias due to the differences in the observation strategies has not been modeled.

In all but one case the error bars on the reference curves represent 1σ deviations over four realizations. The final comparison is made with the mean curve (dashed lines with 1σ error bars in the bottom panels) obtained from 50 realizations of three-dimensional Poisson distributions projected on the two-dimensional conical surface and corrected for the selection effect.

In the top panels of Figure 3 the corrected Poisson results are indistinguishable from the survey results. An explanation for this is that the distortion caused by the selection function produces statistically inhomogeneous distribution. Mixing the regions with very different number densities of galaxies at the first glance looks like the shift to the left, but it completely smears out the characteristic percolation transition. As a result the LCS curve looks like a featureless almost straight line. Thus, we conclude the percolation analysis requires more statistically homogeneous sampling than geometrically blind statistics.

By separating the survey into two regions (60  ≤  R  ≤  400 h⁻¹ Mpc and 400  ≤  R  ≤  600 h⁻¹ Mpc), the effect of the selection function is considerably reduced and the LCS of the Poisson distribution shifts to the right and acquires a characteristic form (see Fig. 3, middle panels). The reason is that in a magnitude-limited survey the selection function is dependent on the distance from the observer and in the case of the LCRS peaks at a value of R  ≈  200 h⁻¹ Mpc. The middle panels show the results of a magnitude-limited sample in the region 60  ≤  R  ≤  400 h⁻¹ Mpc. There is now a clear distinction between survey and corrected Poisson distribution results especially for the north slices. Results for the region 400  ≤  R  ≤  600 h⁻¹ Mpc (not shown) are qualitatively similar to those for the inner region shown. The slices percolate at lower filling factors than the random standard in all cases. More homogeneous subsamples of the survey reveal a clear and unambiguous signal for a connected topology in the region analyzed.

The bottom panels show the results for volume-limited subsamples (see Fig. 4) derived from the survey in the region 200  ≤  R  ≤  400 h⁻¹ Mpc.¹ We have restricted our analysis to the most dense central regions of the slices where the combined selection and projection effects are the least. Once again there is a clear signal at a few σ level for a connected topology for both the north and south subsamples. The results above illustrate the necessity for volume-limited

¹ Volume-limited samples with the same geometry as the magnitude-limited samples were also analyzed. The results for those samples where similar to the results shown and the conclusions are the same.

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**Fig. 4.—Maps of the volume-limited distributions for all LCRS slices (200 h⁻¹ Mpc ≤ R ≤ 400 h⁻¹ Mpc). The axis units are Mpc. The arcs show the boundaries of the zones used in the analysis.**
samples. Our method contrasts with the method of de Lapparent et al. (1991) to remove the distortion effects of the selection function in their work with the CfA slices. They increase the size of outer-lying grid cells in order to maintain a similar mean number of galaxies per grid cell throughout the survey. In essence, they may have traded one distortion for another; certainly the effects of their method are not understood and a comparison with their results is difficult. They also employed a nonstandard definition of a neighbor.

Sources of distortion in this study are statistically inhomogeneous distributions due to the selection function, two-dimensional analysis of three-dimensional surveys, the curvature of the slices, and possible inherent galaxy incompleteness in the survey design. We can substantially reduce the selection effects by analyzing volume-limited surveys and the projection effects by generating random two-dimensional reference catalogs that are similar to the surveys. Even then, the volume-limited northern slices show somewhat stronger connectivity than the southern slices when the LCS is examined over its entire range, owing to curvature effects or to regional variation in the topology at these scales. A three-dimensional study may determine the cause of this difference.

The results of this study imply a connected topology for all slices consistent with a filamentary geometry. However, in a survey having the thin slice geometry, distinguishing between filaments in three dimensions and pancakes is not easy by means only of percolation. A recent two-statistics comparison, one has to generate the model "galaxy" catalogs that are similar to the results. Fingers-of-God distortions are well known to astronomers, but recent work by Pratón & Melott (1997) has examined distortions that result in linear structures perpendicular to the line of sight. A comparison between percolation of simulated galaxy catalogs in real space and their redshift space counterparts may help address this problem. In addition, the resolution of the LCRS must be optimized in terms of grid cell size and grid orientation effects must be made. et al. Landy et al. (1996) have reported an enhancement of the power spectrum on length scales of roughly $100 \ h^{-1} \ Mpc$. This signal is associated with identifiable structures in the survey and is highly directional. Any effects on percolation results due to the directionality of the $k$-space fluctuations needs to be assessed by rotating the grid. Also, the consequences of possible redshift distortions must be evaluated and incorporated into the results. Fingers-of-God distortions are well known to astronomers, but recent work by Pratón & Melott (1997) has examined distortions that result in linear structures perpendicular to the line of sight. A comparison between percolation of simulated galaxy catalogs in real space and their redshift space counterparts may help address this problem. In addition, the resolution of the LCRS must be optimized in terms of grid cell size and uncertainties in the redshift positions. We intend to report on these studies in a forthcoming paper.

The major result of this paper is the measurement of the significant signal indicating the network structures in the galaxy distribution. We do not compare it with the predictions of cosmological scenarios. In order to make such a comparison, one has to generate the model "galaxy" catalogs similar to the LCRS and then apply these statistics. We would be happy to provide the necessary software.

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5 Using the core sampling statistics, Doroshkevich et al. (1996) estimated that about 60% of galaxies are in the sheetlike structures.