Abstract. A precise measurement of the vector and axial-vector form factors difference $F_V - F_A$ in the $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ decay is presented. About 95$^3$K events of $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ are selected in the OKA experiment. The result is $F_V - F_A = 0.134 \pm 0.021$(stat) $\pm 0.027$(syst). Both errors are smaller than in the previous $F_V - F_A$ measurements.

1 Introduction

Radiative kaon decays are sensitive to hadronic weak currents in low-energy region and provide a good testing for the chiral perturbation theory ($\chi PT$). The amplitude of the $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ decay includes two terms: internal bremsstrahlung (IB) and structure dependent term (SD) [1]. IB contains radiative corrections for $K^+ \rightarrow \mu^+ \nu_\mu$ decay. SD is sensitive to the electroweak structure of the kaon.

The differential decay rate can be written in terms of standard kinematic variables $x = 2E_\gamma^*/M_K$ and $y = 2E_\mu^*/M_K$ [2], which are proportional to the photon $E_\gamma^*$ and muon $E_\mu^*$ energy in the kaon rest frame ($M_K$ is the kaon mass). It includes IB, SD$^\pm$ terms and their interference INT$^\pm$. The SD$^\pm$ and INT$^\pm$ contributions are determined by two form factors $F_V$ and $F_A$.

The general formula for the decay rate is as follows:

$$\frac{d\Gamma}{dx dy} = A_{IB}f_{IB}(x,y) + A_{SD}[(F_V + F_A)^2f_{SD^+}(x,y) + (F_V - F_A)^2f_{SD^-}(x,y)]$$
\[-A_{INT}[(F_V + F_A)f_{INT}^+(x, y) + (F_V - F_A)f_{INT}^-(x, y)],\]

where
\[f_{IB}(x, y) = \left[\frac{1 - y + r}{x^2(x + y - 1 - r)}\right][x^2 + 2(1 - x)(1 - r) - \frac{2xr(1 - r)}{x + y - 1 - r}],\]
\[f_{SD}^+(x, y) = [(x + y - 1 - r)[(1 - x)(1 - y) + r]],\]
\[f_{SD}^-(x, y) = [(x - y + r)[(x + y - 1)(1 - x) - r]],\]
\[f_{INT}^+(x, y) = \frac{1 - y + r}{x(x + y - 1 - r)}[(1 - x)(1 - x - y) + r],\]
\[f_{INT}^-(x, y) = \frac{1 - y + r}{x(x + y - 1 - r)}[(x^2 - (1 - x)(1 - x - y) - r)],\]

and
\[r = \frac{1}{M_K^2}, A_{IB} = \Gamma_K \frac{\alpha}{2\pi} \frac{1}{(1 - r)^2}, A_{SD} = \Gamma_K \frac{\alpha}{8\pi} \frac{1}{r(1 - r)^2} \frac{M_K}{F_K},\]
\[A_{INT} = \Gamma_K \frac{\alpha}{2\pi} \frac{1}{(1 - r)^2} \frac{M_K}{F_K}.\]

Here \(\alpha\) is the fine structure constant, \(F_K\) is \(K^+\) decay constant (\(F_K = 155.6 \pm 0.4\,\text{MeV}\ [3]\)) and \(\Gamma_K\) is the \(K\mu^2\) decay width.

Fig. 1 shows the kinematic distribution for IB, INT\(^-\), INT\(^+\), SD\(^-\) and SD\(^+\). The main goal of the analysis is to measure \(F_V - F_A\) by extracting the INT\(^-\) term. Other terms are either suppressed by backgrounds or give negligible contribution to the total decay rate with respect to IB. In the lowest order of \(\chi PT\ O(p^4)\), \(F_V\) and \(F_A\) are constant and \(F_V - F_A = 0.052\) [2]. The first measurement of \(F_V - F_A\) was made by the ISTRA\(^+\) experiment: \(F_V - F_A = 0.21 \pm 0.04\,(\text{stat}) \pm 0.04\,(\text{syst})\) [4].

### 2 OKA detector and separated kaon beam

The OKA setup, Fig. 2, is a double magnetic spectrometer.

The OKA detector includes:

- Beam spectrometer consisting of the magnet M2, 7 beam proportional chambers BPC, 4 beam scintillation counters S and 2 threshold Cherenkov counters \(C_1,2\) for the kaon identification;
- 11 m long He filled decay volume DV with the guard system (GS) containing 670 Lead-Scintillator calorimetric modules \(20 \times (5\,\text{mm Sc} + 1.5\,\text{mm Pb})\) with WLS readout;
- Main magnetic spectrometer on the basis of \(200 \times 140\,\text{cm}^2\) wide aperture magnet SP-40A with a field integral 1 Tm, complemented by 13 planes of proportional chambers (PC), straw (ST) and drift tubes (DT);
- 2 gamma detectors: electromagnetic calorimeter GAMS-2000 and large angle detector EGS (EGS is used to supplement GS as a gamma veto at large angles);
- Hadron calorimeter GDA-100 and 4 muon scintillation counters \(\mu C\) (marked as MC in Fig. 2) used for muon identification;
- Pad (Matrix) Hodoscope MH for the trigger and track reconstruction.
Figure 1: Kinematic distribution of different terms: (a) IB, (b) INT−, (c) SD− and SD+, (d) INT+.

More details can be found in [5].

The data acquisition system of the OKA setup [6] operates at $\sim 25$ kHz event rate with the mean event size of $\sim 4$ kByte.

The OKA beam is a separated secondary beam of the U-70 Proton Synchrotron of NRC "Kurchatov Institute"-IHEP, Protvino [7]. RF-separation with the Panofsky scheme [8] is implemented. The beam contains up to 12.5% of kaons with an intensity of about $5 \times 10^5$ kaons per 3 sec U-70 spill. The beam momentum was 17.7 GeV/c during the data taking period used for the analysis (November 2012). The present study uses about 1/2 of the statistics collected in 2012, where 504M events were stored on tape.

3 Trigger streams and primary selection

The following trigger was used for the analysis: $T_{GAMS} = beam \cdot \bar{C}_1 \cdot \bar{C}_2 \cdot \bar{S}_{bk} \cdot E_{GAMS}$, where $beam = S_1 \cdot S_2 \cdot S_3 \cdot S_4$ is a coincidence of four beam scintillation counters, $\bar{C}_{1,2}$ - threshold Cherenkov counters ($\bar{C}_1$ selects pions, $\bar{C}_2$ - pions and kaons), $S_{bk}$ ("beam killer") - two scintillation counters on the beam axis after the magnet aimed to suppress undecayed beam particles. The analog amplitude sum in the GAMS-2000 is required be higher than $E_{GAMS}$ ($E_{GAMS}$ is chosen to be above the average MIP energy deposit).
times prescaled minimum bias trigger $T_{kaon} = beam \cdot \overline{C}_1 \cdot \overline{C}_2 \cdot \overline{S}_{bk}$ was used for the trigger efficiency measurement $\epsilon_{\text{trig}} = (T_{GAMS} \cap T_{kaon}) / T_{kaon}$ (Fig. 3). This trigger efficiency was applied during the Monte Carlo (MC) simulation.

To select the decay channel the following requirements are applied:

- 1 primary track;
- 1 secondary track identified as muon in GAMS-2000, GDA-100 and $\mu_C$;
- 1 electromagnetic shower in GAMS-2000 with energy $E_{\text{tot}} > 1 \, \text{GeV}$ not associated with charged track;
- GS energy deposition $E_{GS} < 10 \, \text{MeV}$;
- EGS energy deposition $E_{EGS} < 100 \, \text{MeV}$;
- Decay vertex inside the decay volume DV.

4 Event selection

The main background to the $K^+ \to \mu^+ \nu_{\mu} \gamma$ decay comes from 2 decay modes: $K^+ \to \mu^+ \nu_{\mu} \pi^0$ ($K\mu3$) and $K^+ \to \pi^+ \pi^0$ ($K2\pi$) with one $\gamma$ lost from $\pi^0 \to \gamma \gamma$ decay and $\pi$ misidentified as $\mu$. Additional contribution at $y > 1$ is given by the decay mode $K^+ \to \mu^+ \nu_{\mu}$ with an accidental $\gamma$. At low $y$ values there is a small contribution from the $K^+ \to \pi^+ \pi^- \pi^+$ ($K3\pi$) decay.

The MC simulation of the OKA setup is done within the GEANT3 framework [9]. Signal and background events are weighted according to corresponding matrix elements.

The $K^+ \to \mu^+ \nu_{\mu} \gamma$ event selection strategy is based on the ISTRA+ approach [4]. Signal extraction procedure starts with dividing all kinematic ($x$, $y$) region into strips in $x$ with $\Delta x = 0.05$ width. The following steps are implemented for each $x$-strip:

- Fill the $y$ plot.
- Select the signal region by a cut $y_{\text{min}} < y < y_{\text{max}}$ and fill $\cos \theta_{\mu\gamma}^*$ plot, where $\theta_{\mu\gamma}^*$ is an angle between $\mu$ and $\gamma$ in the kaon rest frame. $y_{\text{min}}$ and $y_{\text{max}}$ are selected from the maximization of signal significance defined as $S/\sqrt{S+B}$ where $S$ is the signal and $B$ is the background.
\begin{itemize}
\item Put a cut on $\cos \theta_{\mu \gamma}^*$ to reject background and fill $m_k$ plot. $m_k^2 = (P_\mu + P_\nu + P_\gamma)^2$, where $P_\mu, P_\nu, P_\gamma$ are 4-momenta of decay particles in the laboratory frame, $\vec{p}_\nu = \vec{p}_K - \vec{p}_\mu - \vec{p}_\gamma$, $E_\nu = |\vec{p}_\nu|$. $m_k$ peaks at the kaon mass for the signal.
\item The last step is a simultaneous fit of all 3 histograms $(y, \cos \theta_{\mu \gamma}^*, m_k)$ with the MINUIT tool [10] where the signal and backgrounds normalization factors are the fit parameters.
\end{itemize}

For the correct estimation of the statistical error $\sigma_{\text{exp}}$, only the $m_k$ histogram is used. The MINOS program [10] is run once with the initial parameter values equal to those obtained in the simultaneous fit. Statistical errors were extracted from the MINOS output.

Fig. 4 shows the selected kinematic region for the extraction of the INT$^-$ term. For the further analysis 10 $x$-strips were selected in the $0.1 < x < 0.6$ region. The $y$-width varies from 0.12 to 0.30 inside $x$-strips.

![Figure 3: Trigger efficiency $\epsilon_{\text{trig}}$ as the function of the GAMS total energy deposition. Black points - data, colored curves - fit by the third degree polynomial in four intervals.](image1)

![Figure 4: INT$^-$ Dalitz-plot density and selected kinematic region (area contoured by the black line).](image2)

The result of the simultaneous fit for the strip 2 ($0.15 < x < 0.2$) is shown in Fig. 5. Both signal and background shapes are taken from the MC simulation. The total normalization of the MC to data is made to the $K\mu3$ decay at $y < 0.6$, where the contribution of other backgrounds is very small. The relative normalization of other backgrounds is done according to their branching ratios. For the $K^+ \to \mu^+ \nu_{\mu \gamma}$ decay, only IB term is included in the simultaneous fit. The simultaneous fit gives a reasonable agreement between data and MC with $\chi^2$ from 1.3 to 1.7 for different $x$-strips.

5 $F_V - F_A$ calculation

For each $x$-strip the number of signal events $N_{\text{Data}}$ is extracted from the simultaneous fit and the IB event number $N_{IB}$ is obtained from MC. Their ratio is plotted as a function
of $x$ (Fig. 6). For the signal containing IB only this ratio would be equal to 1. It is the case for small $x$, when the IB is dominating and INT$^-$ is negligible. For large $x$ the INT$^-$ term gives significant negative contribution resulting in smaller values of $N_{Data}/N_{IB}$.

The $N_{Data}/N_{IB}$ distribution is fitted with a function $p_{signal}(x) = p_0(1+p_1(\varphi_{INT^-}(x)/\varphi_{IB}(x)))$, where $p_0$ is normalization factor, $p_1 = F_V - F_A$ is the difference of vector and axial-vector form factors, $\varphi_{INT^-}(x)$ is the $x$-distribution for the reconstructed MC-signal events taken with the weights $(M_K/F_K)f_{INT^-}(x_{true}, y_{true})$, $\varphi_{IB}(x)$ is a similar distribution for the same MC sample, but with the weights $f_{IB}(x_{true}, y_{true})$. Here $x_{true}, y_{true}$ are "true" MC values of $x$ and $y$.

The result of the fit is $F_V - F_A = 0.134 \pm 0.021$. The normalization factor is $p_0 = 1.000 \pm 0.007$. The total number of selected $K^+ \rightarrow \mu^+\nu\gamma$ decay events is $95428 \pm 309$.

In the next order $\chi PT O(p^6)$ $F_V$ linearly depends on the momentum transfer $q^2$ [11]
with the following parametrization [12]: $F_V = F_V(0)(1 + \lambda(1 - x))$, $F_A = \text{const}$. The theoretical prediction is tested in three ways:

- The final fit is performed with $F_V$ and $F_A$ fixed from $\chi PT O(p^6)$ prediction: $F_V(0) = 0.082$, $F_A = 0.034$, $\lambda = 0.4$. This fit has bad compliance with $\chi^2/NDF = 28.0/9$.
- $F_V(0)$ and $F_A = 0.034$ are taken from $\chi PT O(p^6)$, $\lambda$ is a fit parameter. It gives $\lambda = 2.28 \pm 0.53$ with $\chi^2/NDF = 15.8/8$ (Fig. 7).
- $F_V(0)$ is fixed from $\chi PT O(p^6)$. $F_A$ and $\lambda$ are used as fit parameters. $(F_V, \lambda)$ correlation is shown in Fig. 8. The theoretical prediction (red star) is slightly out of $3\sigma$-ellipse.

![Figure 7: $\chi PT O(p^6)$ fit, $F_V(0)$ and $F_A$ are taken from theory. The fit gives $\lambda = 2.279 \pm 0.528$.](image)

![Figure 8: $(F_V, \lambda)$ correlation plot. $F_V(0)$ is taken from theory. Red star is theory prediction.](image)

6 Systematic errors

The obtained value of $F_V - F_A$ depends on the width of $x$-strips, $y$ and $\theta_{\mu\gamma}$ cuts and the fit procedure. The following sources of the systematic errors were investigated:

- Non ideal description of signal and background by the MC.
  For the estimation of this systematics, the statistical error in each bin of Fig. 6 was scaled by the factor $\sqrt{\chi^2/NDF}$, where $\chi^2/NDF$ is obtained from the simultaneous fit in each $x$-strip. A new fit of $N_{Data}/N_{IB}$ with the same function $p_{\text{signal}}(x)$ gives the best description with $\chi^2/NDF = 7.8/8$ compared to $\chi^2/NDF = 12.3/8$ of the main fit. The new value of $F_V - F_A = 0.138$ is consistent with the main one but the fit error $\sigma_{\text{fit}} = 0.026$ is larger. Assuming $\sigma_{\text{fit}}^2 = \sigma_{\text{shape}}^2 + \sigma_{\text{stat}}^2$ the systematic error is $\sigma_{\text{shape}} = 0.015$.

- The fit range in $x$ (number of $x$-strips in the fit).
  The ratio $N_{Data}/N_{IB}$ was refitted by removing one or two bins on the left (right) edge. For the estimate of systematics the average difference between the new $F_V - F_A$ values and the nominal one is taken. The error is negligible: $\sigma_x < 0.006$.  

• Width of $x$-strips.
The $F_V - F_A$ calculation is repeated for 2 different values of $x$-binning: $\Delta x = 0.035$, $\Delta x = 0.07$. The deviation of the new $F_V - F_A$ value with respect to the main one gives $\sigma_{\Delta x} = 0.011$.

• $y$ limits in $x$-strips.
The events inside FWHM of the $y$-distribution for the signal MC are selected. Such limits are tighter than those used in the main analysis. The difference between the new value and main one gives systematic error $\sigma_y = 0.008$.

• Possible contribution of INT$^+$. The INT$^+$ term is added to the final fit (see Section 5). The value $|F_V + F_A| = 0.165 \pm 0.013$ measured by the E787 experiment is used [13]. Two fits were repeated for the minimal ($-0.178$) and maximal ($+0.178$) possible values of $F_V + F_A$. The fitting function was:

$$p_{\text{signal}}(x) = p_0(1 + (F_V + F_A)(\varphi_{\text{INT}^+}(x)/\varphi_{\text{IB}}(x)) + (F_V - F_A)(\varphi_{\text{INT}^-}(x)/\varphi_{\text{IB}}(x)))$$

where $\varphi_{\text{INT}^+}(x)$ is the $x$-distribution similar one as $\varphi_{\text{INT}^-}(x)$. The maximal difference between obtained values of $F_V - F_A$ and the main one measured in Section 5 is $\sigma_{\text{INT}^+} = 0.018$.

Summing up quadratically all the systematic errors the total error is found to be 0.027.

7 Conclusion

The largest statistics of about 95K events of $K^+ \rightarrow \mu^+\nu\gamma$ is collected by the OKA experiment. The INT$^-$ term is observed and $F_V - F_A$ is measured:

$$F_V - F_A = 0.134 \pm 0.021(\text{stat}) \pm 0.027(\text{syst})$$

The result is $2.4 \sigma$ above $\chi PT O(p^4)$ prediction.

A recent calculation in the framework of the gauged nonlocal effective chiral action ($E\chi A$) gives $F_V - F_A = 0.081$ [14]. The OKA result is $1.6 \sigma$ above the $E\chi A$ prediction.

The obtained value of $F_V - F_A$ is in a reasonable agreement with a similar analysis of the ISTRA+ experiment: $F_V - F_A = 0.21 \pm 0.04(\text{stat}) \pm 0.04(\text{syst})$ [4].

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9 References

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