Simulation of THz oscillations in an intrinsic Josephson junction array

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Abstract. When a voltage is applied to coupled Josephson junctions, an ac current flows because of the Josephson effect. Due to this ac current, an electromagnetic (EM) wave is emitted from junctions. For the cuprate High-Tc superconductor, frequency of the EM wave reaches to the THz regime since the energy gap is large. In order to simulate the EM wave in the superconductors, we solve the Josephson junction relations and the Maxwell equations simultaneously. Especially, we consider coupling of junctions including capacitive and inductive couplings, and spatial variations of phase differences of order parameters inside of junctions. We shows time developments of distributions of the phase differences, magnetic and electric fields in junctions in the transient state.

1. Introduction
High-Tc cuprate superconductors have several peculiar properties: Transition temperature is high and the Cooper pair has d-wave symmetry. Also, they have a layered structure where metallic and insulating layers are stacked alternately. They form intrinsic Josephson junction arrays [1-3].

When a dc voltage is applied to a Josephson junction, an ac current flows due to the Josephson effects. Because of this ac current, an electromagnetic (EM) wave is emitted from the junction. For the cuprate High-Tc superconductor, the frequency of this wave reaches up to the THz regime since the superconducting energy gap is large. Also, if oscillations in all junctions are coherent, huge EM wave is expected. Experimentally, the EM wave emission is observed in 2007 [4]. So, coherent oscillations of EM fields and phases of superconducting layers are realized.

Theoretically, Koyama and Tachiki proposed a model for the intrinsic Josephson junction array with capacitive and inductive couplings between neighboring layers [5]. Then, Koyama et al solved two-dimensional version of this model numerically and showed how the EM wave is emitted [6]. However, they only considered in-phase motion of phases and stationary EM wave. In 2016, a three-dimensional model was studied by Rudau et al [7]. They considered 700 layers and separated them into 20 segments. They assumed physical quantities are uniform inside of each segment and showed how resonant electromagnetic modes occur. Also, synchronization of flux motions in the intrinsic Josephson junction was studied theoretically using a simple model [8].

However, simulations on how such coherent oscillations of EM fields occur in a three-dimensional system with microscopic coupling between neighboring layers are missing. Therefore, we simulate the EM field in the intrinsic junction numerically using three-dimensional Koyama-Tachiki model with the finite element method. Especially, we focus on a transient state before coherent motions of EM occur.
2. Numerical Method

We consider five superconducting and insulating layers, which are stacked alternately, as a model of the intrinsic Josephson junction array. In the superconductor layer, we consider EM fields $E_x(x,y,l,t), E_y(x,y,l,t)$ and $B_z(x,y,l,t)$ where $l$ is the layer number and $t$ is a time. In the insulating layer, we consider EM fields $E_z(x,y,l,t), B_x(x,y,l,t), B_y(x,y,l,t)$, and the phase difference of the order parameter between upper and lower superconducting layers $\varphi(x,y,l,t)$. We set thickness of superconducting layers $s=3\AA$ and thickness of insulating layers $d=12\AA$. We call the bottom layer first layer.

\[
\frac{\partial}{\partial t} \varphi(x,y,l,t) = \frac{2e\delta}{\hbar} (1 - \alpha \Delta^2_z) E_z(x,y,l,t) 
\]

where $\Delta^2_z$ is second order differential in $z$ direction

\[\Delta^2_z \equiv f(l) = f(l+1) + f(l-1) - 2f(l)\]

and $\alpha$ is capacitive coupling constant, $\alpha = \frac{\epsilon u^2}{s d}$ and $\alpha \Delta^2_z E_z$ comes from coupling between the junctions.

Second, spatial variations of phase differences are given as,

\[
\frac{\partial}{\partial x} \varphi(x,y,l,t) = \frac{2ed}{hc} (1 - \eta \Delta^2_z) B_y(x,y,l,t) 
\]

\[
\frac{\partial}{\partial y} \varphi(x,y,l,t) = -\frac{2ed}{hc} (1 - \eta \Delta^2_z) B_x(x,y,l,t) 
\]

where $\eta$ is inductive coupling constant $\eta = \frac{\lambda^2}{s d}, \lambda$ is the penetration depth. Third equation comes from the Maxwell equations and is given as

\[
\frac{1}{c} \frac{\partial}{\partial t} E_z(x,y,l,t) = \frac{1}{c} \frac{\partial}{\partial y} B_y(x,y,l,t) - \frac{1}{c} \frac{\partial}{\partial x} B_x(x,y,l,t) - \frac{4\pi}{c} [J_c \sin \varphi(x,y,l,t) + \sigma E_z(x,y,l,t)] 
\]

where $J_c$ is the Josephson critical current.

Figure 1. The numerical model of the intrinsic Josephson junction. Blue layers are superconducting layers and black layers are insulating layers.

Figure 2. Finite elements for each layer

Figure 3. Area coordinates in a triangular element

In order to solve these equations, we use the finite element method (FEM). In the FEM, we divide each layer into triangular elements. In figure 2, we show how to divide each layer. Then, we define three area coordinates $A^{1} = \frac{1}{S} \mathbf{n} \cdot (\mathbf{x}_m \times \mathbf{x}_n + \mathbf{l}_{mn} \times \mathbf{x})$ in each element, which is shown in figure 3. Here $S$ is the area of the element, $(l,m,n)$ is a cyclic permutation of $(1,2,3), \mathbf{l}_{mn} = \mathbf{x}_m - \mathbf{x}_n$ and $\mathbf{n}$ is a normal vector of the surface of the element. We use this area coordinate to expand $\varphi, E_z, B_y$ and $B_x$ as follows,
\[
\varphi(x, y, l, t) = \sum_{n_l} \varphi_{l_l}^n(t) \lambda_l^x(x, y) \\
E_z(x, y, l, t) = \sum_{n_l} E_{zl}^n(t) \lambda_l^y(x, y) \\
B_\alpha(x, y, l, t) = \sum_{n_l} B_{zl\alpha}^n(t) \lambda_l^\alpha(x, y) \quad (\alpha = x, y)
\]

(2.5)

(2.6)

(2.7)

Then equations (2.1), (2.2), (2.3) and (2.4) become

\[
\begin{align*}
\sum_{i'} l_{i'i}^n \frac{\partial \varphi_{i'i}^n(t)}{\partial t} &= \frac{2ed}{\hbar} (1 - \alpha \Delta_2) \sum_{i'} l_{i'i}^n E_{zl'i}^n(t) \\
\sum_{i'} \varphi_{i'i}^n(t) J_{i'i}^{nx} &= \frac{2ed}{\hbar} c (1 - \eta \Delta_2) \left( \sum_{i'} B_{ycl'i}^n(t) l_{i'i}^n \right) \\
\sum_{i'} \varphi_{i'i}^n(t) J_{i'i}^{ny} &= -\frac{2ed}{\hbar} c (1 - \eta \Delta_2) \left( \sum_{i'} B_{ycl'i}^n(t) l_{i'i}^n \right) \\
&= -\frac{4e}{c} \int \left( \sum_{i'} \varphi_{i'i}^n(t) \lambda_{i'i}^x(x, y) \right) dS + \sigma \sum_{i'} E_{zl'i}^n(t) l_{i'i}^n
\end{align*}
\]

(2.8)

(2.9)

(2.10)

(2.11)

Here, we define \( l_{i'i}^n, j_{i'i}^{nx}, \) and \( j_{i'i}^{ny} \) as follow,

\[
l_{i'i}^n = \int \lambda_{i'i}^x(x, y) \lambda_{i'i}^y(x, y) dS
\]

(2.12)

\[
J_{i'i}^{nx} = \int (\nabla_\alpha \lambda_{i'i}^x(x, y)) \lambda_{i'i}^\alpha(x, y) dS \quad (\alpha = x, y)
\]

(2.13)

Solving equations (2.8), (2.9), (2.10), and (2.11), we simulate the EM fields in the intrinsic Josephson junction array.

3. Results

In this section, we show numerical results on oscillations of phase differences and magnetic and electric fields in junctions. Especially, we focus on the time development of these quantities in each layer. In the simulation, we set the size of each layer as 50μm×50μm, the external current \( \Im = 3.0 \times 10^{-4} \) A and the conductivity \( \sigma = 1.1 \times 10^{-4} \) S.

First, we show the distributions of phase differences at \( t = 4.0 \times 10^{-12} \) sec (figure 4) and at \( t = 12.0 \times 10^{-12} \) sec (figure 5) in the first, second, and third insulating layers. From figure 4 (a), we can see that in the first layer, there is a small uniform oscillation inside of the junction, and the direction of this oscillation is parallel to the diagonal direction. In addition to this oscillation, there is a large oscillation surrounding the junction. In the second layer (figure 4 (b)), the oscillation inside of the junction is subtle but the direction of the oscillation is parallel to the side. In addition, the oscillation surrounding the junction penetrates into inside of the junction, although amplitude of the oscillation is smaller than that of the first layer. In the third layer or the center layer (figure 4 (c)), penetration of the oscillation surrounding the junction is weaker than that in the second layer. Also, the oscillation inside
Figure 4. Distributions of phase difference at $t = 4.0 \times 10^{-12}$ sec, in the first layer (a), the second layer (b), and the third layer (c).

Figure 5. Distributions of phase difference at $t = 12.0 \times 10^{-12}$ sec, in the first layer (a), the second layer (b), and the third layer (c).

of the junction is smaller than that in the second layer. At $t = 12.0 \times 10^{-12}$ sec, phase differences in all layers develop gradually, which is shown in figure 5. In the first layer (figure 5 (a)), amplitudes of oscillations inside and surrounding the junction become large. In the second layer (figure 5 (b)), the oscillation surrounding the junction becomes large. In addition, there are some irregular structures. These structures may come from interference between outer and inside oscillations and the oscillation that is transferred from the first layer. In the third layer (figure 5 (c)), such irregular structure extends in larger region. We think this state is a transient state from the initial state to the coherent state.

Next, we show the distributions of $B_x$ and $B_y$ at $t=4.0 \times 10^{-12}$ sec (figures 6 and 8) and at $t=12.0 \times 10^{-12}$ sec (figures 7 and 9) in the first, second, and third insulating layers. At $t=4.0 \times 10^{-12}$ sec, $B_x$ (figure 6) and $B_y$ (figure 8) oscillate along x-axis and y-axis in all layer, respectively. However, in inner layers (figures 6 (b) and 6 (c) and figures 8 (b) and 8 (c)), oscillations decay rapidly. At $t=12.0 \times 10^{-12}$ sec, these oscillations of $B_y$ become large (figure 9). Oscillations of $B_x$ also becomes large (figure 7), although $B_y$ shows irregular structure inside of the junction, especially in the inner layers (figures 7(b) and 7(c)). These structures may come from interference effect of phase differences, which appears in figure 5.

Finally, we show the distributions of $E$ at $t=4.0 \times 10^{-12}$ sec (figure 10) and at $t=12.0 \times 10^{-12}$ sec (figure 11) in the first, second, and third insulating layers. At $t=4.0 \times 10^{-12}$ sec, in the first layer, large oscillation of $E$ appears around edges and small oscillation appears inside of the junction (figure 10(a)). In the second and the third layers, oscillations decay rapidly toward inside of the junction (figures 10 (b) and 10(c)). At $t=12.0 \times 10^{-12}$ sec, in the first layer, there appear an oscillation parallel to the diagonal direction (figure 11(a)) in the second and the third layers, there are irregular structures (figures 11(b) and 11(c)). We think these structures come from the interference effect in the phase differences.
Figure 6. Distributions of $B_x$ at $t = 4.0 \times 10^{-12}$ sec, in the first layer (a), the second layer (b), and the third layer (c).

Figure 7. Distributions of $B_x$ at $t = 12.0 \times 10^{-12}$ sec, in the first layer (a), the second layer (b), and the third layer (c).

Figure 8. Distributions of $B_y$ at $t = 4.0 \times 10^{-12}$ sec, in the first layer (a), the second layer (b), and the third layer (c).

Figure 9. Distributions of $B_y$ at $t = 12.0 \times 10^{-12}$ sec, in the first layer (a), the second layer (b), and the third layer (c).
Figure 10. Distributions of $E_z$ at $t = 4.0 \times 10^{-12}$ sec, in the first layer (a), the second layer (b), and the third layer (c).

Figure 11. Distributions of $E_z$ at $t = 12.0 \times 10^{-12}$ sec, in the first layer (a), the second layer (b), and the third layer (c).

4. Summary
In order to investigate time-development of coherent oscillations of EM fields in the intrinsic Josephson junctions, we have solved the generalized Josephson relations and the Maxwell equations simultaneously with the FEM. We have obtained time developments of the phase difference, magnetic and electric fields in each layer. We show these quantities in the transient state before coherent oscillations appear. In future, we will investigate coherent oscillations of EM fields, as well as the emission of EM fields.

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