Economic analysis of circulating water system based on grey system theory

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Abstract. In this paper, the multi-level grey system theory method is used to evaluate the economics of the circulating water system. Firstly, the influencing factors of each part are divided into different levels, and the weights of each factor are determined by using the analytic hierarchy process. Secondly, the influencing factors are divided into five commentary categories based on the Grey system theory. The whitening weight function of the grey class is established, and then the whitening value is calculated. Finally, the weight of the circulating water system belongs to each grey class, and the economic performance of the cooling water system is reasonably and objectively evaluated. Through the calculation and description of a specific example of the cooling water system, the economics of the whole cooling water system and various links are obtained. The economic evaluation results obtained by the analysis are in line with objective laws and provide a new way for the economic evaluation of the cooling water system.

1. Introduction

1.1. Current status of economic evaluation of circulating water system

The circulating water system is mainly composed of five parts: water pump, heat exchanger, cooling tower, pipeline and valve. The economics of the system are also the most important part of the business. Luo You-xin et al. proposed the grey decision-making guide heat exchanger from the perspective of economics [1]. But the choice of individual components does not optimize the economics of the system. On the other hand, because the evaluation information is incomplete and the amount of data is small, it is difficult to make an objective and reasonable evaluation, which also makes the results obtained by the ordinary evaluation method be different from the actual results.

1.2. Introduction to grey system theory

Grey theory is an applied mathematics discipline in which the information is partially clear, partially unclear, and with uncertainties. Traditional system theory, most of which study systems with sufficient information. For some systems with poor information. Using the black box method, we have also achieved relatively successful experience. However, some systems that have some internal information and some information are not well known are not well studied. This blank area became the birthplace of the grey system theory. Professor Deng Ju-long founded the grey system theory in 1982[2], which is mainly aimed at a new solution to such uncertainties as data and insufficient
information [3]. With in-depth research, the grey theory has gradually matured and has been widely used in various fields due to its strong applicability [4]. Based on the grey theory, this paper greys out the economic indicators of the circulating water system [5], and obtains the grey evaluation right of each indicator, thus indicating the pros and cons of the indicators.

In this paper, an economic comprehensive evaluation system is proposed for the circulating water system, and a multi-level grey evaluation method [6] is used to find out the unreasonable part of the economy. Make recommendations for unreasonable investments and minimize investment under the operating conditions of the system.

2. Analysis of economic grey evaluation of cooling water system

2.1. Establish a cooling water system evaluation system

The economic cost of circulating water systems can be divided into the following three categories:

(1) Investment cost of equipment

The investment cost of equipment mainly includes the expenses incurred in the purchase and installation of equipment.

(2) Operating expenses of equipment

The operating costs of the equipment mainly include the power consumption of the water pump and the cooling tower fan, as well as the water consumption of the system's supplementary water.

(3) Equipment maintenance cost

Due to the extremely high uncertainty of maintenance costs, investment costs and operating expenses at the same level are negligible. The following is mainly for the evaluation of investment costs and operating expenses.

The subjective evaluation system is divided into several layers (three layers are taken as examples), which are the highest level (target W), the middle layer (first-level evaluation index $U_i, i = 1, 2, 3 \ldots m$), and the lowest level (secondary level). The index layer $V_{ij}, j = 1, 2, 3 \ldots n_1$). Corresponding to the circulating water system include W layer, A layer and B layer, as shown in Figure 1 below.

$U$ represents a set of primary evaluation indicators $U_i$, denoted as $U = \{U_1, U_2 \ldots U_m\}; V_{ij}$ represents a set of secondary evaluation indicators [7].

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**Figure 1. Establishment of economic indicator system.**
2.2. Determine the weight of $\mathbf{U}_i$ and $\mathbf{V}_{ij}$

The methods for determining index weights are [8]: expert estimation method, frequency statistics method, coefficient of variation method, correlation coefficient weighting method, cosine weighting method, entropy method, subjective and objective comprehensive weighting method and analytic hierarchy process. For the economic evaluation system, the indicators are of the same dimension, using the analytic hierarchy process, and the comparison matrix is used to establish the judgment matrix, and then the index weights are obtained.

For the weight assignment of the first-level indicator $\alpha_i (i = 1, 2, 3 \ldots m)$, $\alpha_i \geq 0$ is required and $\sum_{i=0}^{m} \alpha_i = 1$ is satisfied; the weight of the second-level indicator $V_{ij}$ is assigned as $\alpha_{ij} (j = 1, 2, 3 \ldots n_i)$, $\alpha_{ij} \geq 0$ is required and $\sum_{i=0}^{n_i} \alpha_{ij} = 1$ is satisfied.

For the circulating water system, there are only three indicators of the A-level index, and the subjective weighting method is adopted; for the seven indicators under the B-layer, the analytic hierarchy process can be used. Next, the A-B judgment matrix is determined based on the data obtained above, and the magnitude of the weight is determined by judging the correlation value of the matrix. The B layer compares and determines the judgment matrix according to the consumption cost of each indicator.

**Table 1. Actual cost values.**

| 02031.des | Expected (10^4 RMB) | Actual (10^4 RMB) |
|----------|---------------------|-------------------|
| Pump cost  | 1                   | 0.8255            |
| Cooling tower cost | 15               | 14                |
| Heat exchanger cost | 3                | 2.145             |
| Pipeline cost | 220              | 216.42            |
| Valve cost   | 2                   | 1.967             |
| Pump operating cost | 4000          | 4346.4            |
| Fan operating costs | 2               | 1.27              |

In table 1, the numerical value of the relative importance of $U_i$ to $U_j$ is expressed in the following table 2.

**Table 2. Meaning of measures.**

| $b_{ij}$ value | definition                                      |
|----------------|-----------------------------------------------|
| 1              | The i factor is as important as the j factor    |
| 3              | The i factor is slightly more important than the j factor |
| 5              | The i factor is significantly more important than the j factor |
| 7              | The i factor is more important than the j factor |
| 9              | The i factor is absolutely important than the j factor |
| reciprocal     | $b_{ij} = 1/b_{ij}$                           |

The numerical value of the relative importance of $B_i$ to $B_j$ is expressed as follows in table 3, that is, the judgment matrix.

**Table 3. A-B judgement matrix.**

|     | A   | B1  | B2  | B3  | B4  | B5  | B6  | B7  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| B1  | 1.0 | 5.0 | 3.0 | 7.0 | 3.0 | 9.0 | 3.0 |    |
| B2  | 1/5 | 1.0 | 3.0 | 5.0 | 3.0 | 7.0 | 5.0 |    |
| B3  | 1/3 | 1/3 | 1.0 | 1.0 | 3.0 | 9.0 | 3.0 |    |
| B4  | 1/7 | 1/5 | 1.0 | 1.0 | 3.0 | 5.0 | 7.0 |    |
| B5  | 1/3 | 1/3 | 1/3 | 1/3 | 1.0 | 9.0 | 3.0 |    |
| B6  | 1/9 | 1/7 | 1/9 | 1/5 | 1/9 | 1.0 | 9.0 |    |
| B7  | 1/3 | 1/5 | 1/3 | 1/7 | 1/3 | 1/9 | 1.0 |    |
Add the judgment matrix by row to get:

$$\omega_i = \sum_{j=1}^{n} \frac{b_{ij}}{n}$$  \hspace{1cm} (1)

Get this vector:

$$\omega_i^T = [4.429, 3.457, 3.381, 2.927, 2.021, 1.525, 0.351]$$  \hspace{1cm} (2)

The resulting vector is normalized. The weight of each factor $U_i$ is that:

$$\alpha_i = \frac{\omega_i}{\sum_{i=1}^{n} \omega_i} \text{ (i = 1, 2, 3, ... n)}$$  \hspace{1cm} (3)

Get the weight vector of layer B:

$$A = [0.245, 0.191, 0.187, 0.162, 0.112, 0.084, 0.019]$$  \hspace{1cm} (4)

The matrix consistency is checked, $CI = \frac{\lambda_{max} - n}{n-1} = 0.404, CR = \frac{CI}{RI} = 0.299 < 1$, which satisfies matrix consistency.

2.3. Setting of rating level of evaluation index $V_{ij}$

As far as the economic evaluation of the circulating water system is concerned, the evaluation of the grey classification can be divided into five parts: low, low, medium, higher and high. And according to the indicator, each comment is given a fixed value. For the economic evaluation index of the circulating water system, when the evaluation is high or high, it is necessary to propose corresponding rectification opinions.

There are $p$ evaluators who score the scores according to the rating scale of the evaluation index $V_{ij}$, and give the score $d_{ijk} (k = 1, 2, ..., p)$. And obtain the evaluation sample matrix $D$ of the evaluated object. In this paper, five experts are combined with the expected investment to score the third-level index of circulating water. “Low” corresponds to 5 points, “Lower” corresponds to 4 points, “Medium” corresponds to 3 points, “High” corresponds to 2 points, and “High” corresponds to 1 point. When the indicator level is between the two, the corresponding score is 4.5, 3.5, 2.5, 1.5.

$$D = \begin{bmatrix}
  d_{111}, d_{112}, \ldots, d_{115} \\
  d_{121}, d_{122}, \ldots, d_{125} \\
  \vdots \\
  d_{151}, d_{152}, \ldots, d_{155} \\
  d_{211}, d_{212}, \ldots, d_{215} \\
  d_{221}, d_{222}, \ldots, d_{225}
\end{bmatrix} = \begin{bmatrix}
  5, 4.5, 5, 4.5, 5 \\
  3.3, 5, 2.5, 3, 3 \\
  4.5, 4, 3.5, 4.5, 4.5 \\
  2.5, 2, 2.5, 3, 2.5 \\
  5, 4.5, 4, 4, 4.5 \\
  1, 1, 1.5, 1, 1.5 \\
  4.5, 4, 4.5, 5, 5
\end{bmatrix}$$  \hspace{1cm} (5)

2.4. Designated evaluation grey class

Determining the evaluation of the grey class is to determine the grey number of the five parts and the whitening weight function of the grey number. Let the five parts of the grey class number be $e=1, 2, 3, 4, 5$, respectively, then the grey class and whitening weight function is as follows [9]:

The first grey class is "low" ($e = 1$), the grey number belongs to the interval $[5, \infty)$, and its whitening weight function is as follows:

$$f_1(d_i) = \begin{cases} 
0, & d \notin [0, \infty) \\
\frac{d_i}{5}, & d \in [0,5] \\
0, & d \in [5, \infty)
\end{cases}$$  \hspace{1cm} (6)
The second grey class is "low" (e = 2), the grey number belongs to the interval [0,4,8], and its whitening weight function is as follows:

\[ f_2(d_i) = \begin{cases} 0, & d \notin [0,8] \\ \frac{d_i}{4}, & d \in [0,4] \\ \frac{8-d_i}{4}, & d \in [4,8] \end{cases} \] (7)

The third grey class is "low" (e = 3), the grey number belongs to the interval [0,3,6], and its whitening weight function is as follows:

\[ f_3(d_i) = \begin{cases} 0, & d \notin [0,6] \\ \frac{d_i}{3}, & d \in [0,3] \\ \frac{6-d_i}{3}, & d \in [3,6] \end{cases} \] (8)

The fourth grey class is "low" (e = 4), the grey number belongs to the interval [0,2,4], and its whitening weight function is as follows:

\[ f_4(d_i) = \begin{cases} 0, & d \notin [0,4] \\ \frac{d_i}{2}, & d \in [0,2] \\ \frac{4-d_i}{2}, & d \in [2,4] \end{cases} \] (9)

The fifth grey class is "low" (e = 5), the grey number belongs to the interval [0,1,2], and its whitening weight function is as follows:

\[ f_5(d_i) = \begin{cases} 0, & d \notin [0,2] \\ d_i, & d \in [0,1] \\ 2 - d_i, & d \in [1,2] \end{cases} \] (10)

2.5. Solution of grey evaluation coefficient

For the evaluation index \( V_{ij} \), the grey evaluation coefficient of the e-th evaluation grey is recorded as \( X_{ij e} \)

\[ X_{ij e} = \sum_{k=1}^{p} f_e (d_{ijk}) \] (11)

Where p is the number of scorers. For evaluation index \( V_{11} \).

\[ e=1, X_{111} = \sum_{k=1}^{p} f_1 (d_{11k}) = f_1(d_{111}) + f_1(d_{112}) + f_1(d_{113}) + f_1(d_{114}) + f_1(d_{115}) \]
\[ = f_1(5) + f_1(4.5) + f_1(5) + f_1(5) + f_1(4.5) = 4.8 \]
\[ e=2, X_{112} = 4, e=3, X_{113} = 2, e=4, X_{114} = 0, e=5, X_{115} = 0 \] (12)

According to the grey theory, the grey evaluation coefficient of each grey class is normalized. All the scorers claim that the grey evaluation power of the e-th grey is \( r_{ij e} \), then:

\[ r_{ij e} = \frac{X_{ij e}}{\sum_{e=1}^{5} X_{ij e}} \] (13)

For the evaluation index \( V_{11} \), the normalization process obtains the grey evaluation right of each grey class:
Then all the grey evaluation rights of the evaluation grey class constitute a grey evaluation vector:

\[ r_{1i} = [0.4444, 0.3704, 0.1852, 0, 0] \]  

(15)

Similar to the calculation method of the first indicator \( V_{11} \), the grey evaluation right of each evaluation index \( V_{ij} \) is calculated, and the following Table 4 can be obtained:

Table 4. Indicator grey evaluation weight.

| factors                        | e=1 | e=2 | e=3 | e=4 | e=5 | Normalized processing |
|--------------------------------|-----|-----|-----|-----|-----|------------------------|
| Pump investment cost           | 4.8 | 4   | 2   | 0   | 0   | [0.4444 0.3704 0.1852 0 0] |
| Cooling tower investment cost  | 3   | 3.75| 4.667| 2.5 | 0   | [0.2156 0.2695 0.3353 0.1796 0] |
| Heat exchanger investment cost | 4.2 | 4.5 | 3   | 0.25| 0   | [0.3515 0.3766 0.2510 0.0209 0] |
| Pipeline investment cost       | 2.5 | 3.13| 4.167| 3.5 | 0   | [0.1881 0.2351 0.3135 0.2633 0] |
| Valve investment cost          | 4.4 | 4.5 | 2.667| 0   | 0   | [0.3804 0.3890 0.2306 0 0] |
| Pump operating cost            | 1   | 1.5 | 2   | 3   | 4   | [0.0870 0.1304 0.1739 0.2609 0.3478] |
| Cooling tower operating costs  | 4.6 | 4.25| 2.333| 0   | 0   | [0.4114 0.3800 0.2086 0 0] |

Then the grey evaluation vectors of all indicators constitute a grey evaluation matrix:

\[
R = \begin{bmatrix}
0.4444 & 0.3704 & 0.1852 & 0 & 0 \\
0.2156 & 0.2695 & 0.3353 & 0.1756 & 0 \\
0.3515 & 0.3766 & 0.2510 & 0.0209 & 0 \\
0.1881 & 0.2351 & 0.3135 & 0.2633 & 0 \\
0.3804 & 0.3890 & 0.2306 & 0 & 0 \\
0.0870 & 0.1304 & 0.1739 & 0.2609 & 0.3478 \\
0.4114 & 0.3800 & 0.2086 & 0 & 0 \\
\end{bmatrix}
\]  

(16)

2.6. Grey evaluation matrix

A comprehensive evaluation of \( U \), the evaluation result vector is recorded as \( B \), then

\[
B = A \cdot R = [0.304 \ 0.3125 \ 0.2515 \ 0.1028 \ 0.0292]
\]  

(17)

It can be seen from the above example that the maximum value of each ash in this condition is 0.3125, and it can be considered that 31.25% belongs to the ash "lower", so the economic evaluation for this condition can be "lower cost."
3. Conclusions

1) This paper combines the example with the theory, and describes in detail the process of applying the multi-level grey evaluation method to the economic evaluation of the cooling water system, and establishes a grey comprehensive evaluation system.

2) As shown in Table 4, the operating cost of the pump is 34.78%, and the investment cost of the pipeline and the investment cost of the cooling tower are evaluated as medium, 31.35% and 33.53%, respectively. Under the conditions, replace the relatively cheap pipeline, and optimize the cooling tower investment, you should also choose a pump that consumes less energy.

3) Through calculation, the overall economic evaluation results of circulating water project 02031 are: 30.40% is low, 31.25% is lower, 25.15% is medium, and higher and higher are 10.28% and 2.92%, respectively. The overall evaluation is lower, so the investment in the project is more reasonable. Combined with practical experience, this method has high rationality and practicability for the economic evaluation of the cooling water system.

Acknowledgement

Fund Project: Shanghai Science and Technology Commission Science and Technology Research Program (13dz1201700).

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