Testing dissipative dark matter in causal thermodynamics

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Abstract: In this paper we study the consistency of a cosmological model, described by a novel exact analytic solution of a universe filled with a dissipative dark matter fluid, in the framework of the causal Israel-Stewart theory, testing it by using Type Ia Supernovae data. The solution is obtained when we assume for the fluid a bulk viscous coefficient with the dependence $\xi = \xi_0 \rho^{1/2}$, where $\rho$ is the energy density of the fluid. It is further considered a relaxation time $\tau$ of the form $\frac{1}{\rho p + 3\tau} = c_s^2$, where $c_s^2$ is the speed of bulk viscous perturbations, and a barotropic EoS $p = (\gamma - 1) \rho$. The constraints found for the parameters of the model allow to obtain exact solutions compatible with an accelerated expansion at late times, after the domination era of the viscous pressure and without the inclusion of a cosmological constant. Nevertheless, the fitted parameter values present drawbacks as a very large non-adiabatic contribution to the speed of sound, and some inconsistencies with the description of the dissipative dark matter as a fluid.

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I. INTRODUCTION

It is well accepted that nowadays the cosmological data consistently indicates that the expansion of the universe began to accelerate around $z = 0.64$. Thus, every model used to describe the cosmic background evolution must display this transition in its dynamics. Of course, $\Lambda$CDM presents this transition as well and it can be understood as the transition between the dark matter (DM) dominant era and the era dominated by the dark energy (DE). Nevertheless, despite the fact that the $\Lambda$CDM model has been very successful to explain the cosmological data, it presents the following weak points from the theoretical point of view: i) Why the observed value of $\Lambda$ is 120 orders of magnitude smaller than the physically anticipated value?. This is the cosmological constant problem [7,13], which can be represented mainly by the two following open questions: a) Why does the observed vacuum energy has such an un-naturally small but non vanishing value?, and b) Why do we observe vacuum density to be so close to matter density, even though their ratio can vary up to 120 orders of magnitude during the cosmic evolution? (the coincidence problem) [11-13]. This model present also serious specific observational challenges and tensions (for example, see [14] for a brief review).

As an alternative to $\Lambda$CDM, the DM unified models do not invoke a cosmological constant. In the framework of general relativity, non perfect fluids drive accelerated expansion due to the negativeness of the viscous pressure, which appears from the presence of bulk viscosity. Therefore, a cold DM viscous component is a kind of unified DM model that could, in principle, explain the above mentioned transition without the inclusion of a DE component. It is worthy mentioning that measurements of the Hubble constant show tension with the values obtained from large scale structure (LSS) and Planck CMB data, which can be alleviated when viscosity is included in the DM component [17]. The new era of gravitational waves detector has also opened the possibility to detect dissipative effects in DM and DE through the dispersion and dissipation experimented by these waves propagating in a non perfect fluid. Some constraints on those effects were found in [15]. Dissipative DM also appears as a residing component in a hidden sector, and can reproduce several observational properties of disk galaxies [19, 20].

At background level, where a homogeneous and isotropic space describes the universe as a whole, only bulk viscosity is present in the cosmic fluid and the dissipative pressure must be described by some relativistic thermodynamical approach to non perfect fluids. This implies a crucial point in a fully consistent physical description of the expansion of the universe using dissipative processes to generate the transition. Meanwhile, in the $\Lambda$CDM model the acceleration is due to a cosmological constant and the entropy remains constant, in the case of non perfect fluids it is necessary to find a solution that not only consistently describes the kinematics of the universe, but also that satisfies the thermodynamical requirements. In the case of a description of viscous fluids, the Eckart’s theory [21, 22] has been widely investigated due to its simplicity and became the starting point to shed some light in the behavior of the dissipative effects in the late time cosmology [23-25] or in inflationary scenarios [27]. Nevertheless, it is a well known result that the Eckart’s theory has non causal behavior, presenting the problem of superluminal propagation velocities and some instabilities. So, from the point of view of a consistent description of the relativistic thermodynamics of non perfect fluids, it is necessary to include a causal description such as the one given by the Israel- Stewart (IS)

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theory \[28,33\].

Our aim in this paper is to constraint the respective free parameters of novel and particular exact cosmological solutions, found in \[35\] for a universe filled only with a dissipative dark matter component. The constraint was done using the supernova Ia (SNe Ia). These solutions were found in the framework of the causal thermodynamics described by the IS theory, and it can display a transition between deceleration and acceleration expansions at background level, within a certain range of the parameters involved. Since the solution found describe a universe containing only a dissipative dark matter as the main component of the universe, it should only be considered as an adequate approximation for the late time evolution, where cold DM dominates. In this sense, this models cannot expected to be fairly representative of the early time evolution, where ultrarelativistic matter dominates.

In these solutions was assumed a barotropic EoS for the fluid that filled the universe, i.e.,
\[
p = (\gamma - 1) \rho,
\]
where \(p\) is the barotropic pressure, \(\rho\) is the energy density. Since our aim is to describe the evolution of the universe with dissipative DM with positive pressure, we shall consider that the EoS parameter lies in the range \(1 \leq \gamma < 2\), where \(\gamma = 1\) corresponds to a particular solution. Furthermore, we will use the following Ansatz for the bulk viscosity coefficient, \(\xi(\rho)\),
\[
\xi(\rho) = \xi_0 \rho^s,
\]
which has been widely considered as a suitable function between the bulk viscosity and the energy density of the main fluid, and where \(\xi_0\) must be a positive constant because of the second law of thermodynamics \[36\]. The nonlinear ordinary differential equation of the IS theory obtained with the above assumptions has been solved, for example, for different values of the parameter \(s\) in \[37\]: for \(s = 1/4\) and stiff matter in \[38\]. Inflationary solutions were found in \[39\]. Stability of inflationary solutions were investigated in \[40,41\]. For an extensive review on viscous cosmology in early and late see \[42\].

It is important to mention that in the formulation of the thermodynamical approaches of relativistic viscous fluids it is assumed that the viscous pressure must be lower than the equilibrium pressure of the fluid (the near equilibrium condition). Whenever it have solutions with acceleration at some stage, like, for example, bulk viscous inflation at early times, or transition between decelerated and accelerated expansions, at late times, the above condition is not fulfilled. Therefore, it is not exactly justified to apply the above approaches in this situations.

To overcome this issue, deviations from the near equilibrium condition have been considered within a non linear extension of IS, as it was developed in \[43\]. Using this non linear extension in accelerated eras, occurring at early times, like inflation or at late times, like phantom behavior, were investigated in \[44\] and in \[45\], respectively. The current accelerated expansion was addressed with a nonlinear model for viscosity in \[45\]. Also, a phase space analysis of a cosmological model with both viscous radiation and nonviscous dust was realized in \[47\], where the viscous pressure satisfy a nonlinear evolution equation. In \[48\] was shown that the inclusion of a cosmological constant along with a dissipative dark matter component allows to obey the near equilibrium condition within, in principle, the linear IS theory.

Our novel solution was obtained using the general expression for the relaxation time \(\tau\) \[34\], derived from the study of the causality and stability of the IS theory in \[49\]
\[
\frac{\xi}{(\rho + p)\tau} = c_s^2 b,
\]
where \(c_s\) is the speed of bulk viscous perturbations (nonadiabatic contribution to the speed of sound in a dissipative fluid without heat flux or shear viscosity). Since the dissipative speed of sound \(V\), is given by \(V^2 = c_s^2 + c_b^2\), where \(c_b^2 = (\partial p/\partial \rho)_{\rho}\), is the adiabatic contribution, then for a barotropic fluid \(c_s^2 = \gamma - 1\) and thus \(c_s^2 = c_b^2 = \epsilon (2 - \gamma)\) with \(0 < \epsilon < 1\), known as the causality condition. This exact solution generalizes the exact solution found in \[50\], where it was used the particular expression \(\tau = \xi/\rho\), taking besides the particular values \(s = 1/2\) and \(\gamma = 1\).

In a previous work, which included Eq. (3) for the relaxation time and a pressureless main fluid, the IS equation was solved using an Ansatz for the viscous pressure \[51\]. The conclusion indicates that the full causal theory seems to be disfavored. Nevertheless, in the truncated version of the theory, similar results to those of the CDM model were found for a bulk viscous speed in the interval \(10^{-11} \leq c_s^2 \leq 10^{-8}\). This last constraint on \(c_s^2\), even though was obtained with a suitable Ansatz and does not represent an exact solution of the theory, teaches us that the non-adiabatic contribution to the speed of sound must be very small to be consistent with the cosmological data.

The free parameters of the novel exact solution are \(\gamma\), \(\xi_0\) and \(\epsilon\). In the case of the assumption of a cold dark matter component from the beginning, only \(\xi_0\) and \(\epsilon\) remains free and we find the constraints to obtain a solution that presents a transition between deceleration to acceleration expansions. In the case of taking all parameters free, the constraints lead to a very unrealistic value of \(\gamma\), consistent with a hot dark matter component.

Using the constraints obtained for the parameters \(\xi_0\) and \(\epsilon\) for the both cases \(\gamma = 1\) and \(\gamma\) free, we will discuss the novel and the particular solutions from the point of view of the consistence of a fluid description during the cosmic evolution, for the dissipative dark matter component. For this aim we evaluate the constraint \(\tau H < 1\), where \(\tau\) is the relaxation time and \(H\) the Hubble parameter, in terms of the values of the parameters.

This paper is organized as follow: In section II we describe briefly the causal Israel-Stewart theory and we derive the general differential equation to be solved.
also present the constraints for the parameters of the model in order to satisfy a consistent fluid description. In section III we study the novel solution for arbitrary $\gamma$ and show that the particular solution corresponding to the particular case of dust matter found in [35]. In section IV we constraint the free parameters of the solutions with the observational supernova Ia (SNe Ia) data. In section V we recall the results from the constraint and discuss them. Finally, in section VI we present our conclusions taken into account the kinematic properties of the solutions as well as the consistence of a fluid description.

II. ISRAEL- STEWART FORMALISM

Our model of a dissipative DM component is described by the Einstein’s equations for a flat FRW metric:

$$3H^2 = \rho,$$

and

$$2\dot{H} + 3H^2 = -p - \Pi,$$

where natural units defined by $8\pi G = c = 1$ were used. In addition, in the IS framework, the transport equation for the viscous pressure $\Pi$ reads

$$\tau \dot{\Pi} + \Pi = -3\xi(\rho)\dot{H} - \frac{1}{2} \tau \Pi \left( 3H + \frac{\dot{\tau}}{\tau} - \frac{\xi(\rho)}{\xi(\rho)} - \frac{\dot{T}}{T} \right),$$

(6)

where “dot” accounts for the derivative with respect to the cosmic time, $H$ is the Hubble parameter and $T$ is the barotropic temperature, which takes the form $T = T_0 e^{(\gamma - 1)/\rho}$ (Gibbs integrability condition when $p = (\gamma - 1) \rho$), with $T_0$ being a positive parameter. Using Eqs. (1), (2) in Eq. (3) we obtain the following expression for the relaxation time

$$\tau = \frac{\xi_0}{\epsilon\gamma (2 - \gamma)} \rho^{\gamma - 1} = \frac{3^{\gamma - 1} \xi_0}{\epsilon\gamma (2 - \gamma)} H^{2(\gamma - 1)}.$$

(7)

In order to obtain a differential equation in terms of the Hubble parameter, it is necessary to evaluate the ratios $\dot{\tau}/\tau$, $\xi/\xi$, and $T/\Pi$, which appear in Eq. (6). From Eqs. (1) and (2), the expression for the viscous pressure and its time derivative can be obtained. Using the above expressions a nonlinear second order differential equation can be obtained for the Hubble parameter:

$$\ddot{H} + 3H\dot{H} + (3)^{-s}\xi_0^{-1} \epsilon\gamma (2 - \gamma) H^{2-2s} \dot{H} - \frac{(2\gamma - 1)}{\gamma} H^{-1} \dot{H}^2 + \frac{9}{4} \gamma [1 - 2\epsilon (2 - \gamma)] H^3 = 0.$$

(8)

We address the reader to see the technical details in ref. [35]. As we shall see in the next section we obtained a novel solution for the special case $s = 1/2$, which allows an important simplification of Eq. (5). In fact, in this case the simple form $H(t) = A(t_s - t)^{-1}$ is a solution of Eq. (5) with a phantom behavior, with $A > 0$, $\epsilon = 1$ and the restriction $0 < \gamma < 3/2$. Besides, the solution $H(t) = A(t - t_s)^{-1}$ can represent accelerated universes if $A > 1$, Milne universes if $A = 1$ and decelerated universes if $A < 1$, all of them having an initial singularity at $t = t_s$. [53].

As it was mentioned in Section I, an important issue that we will discuss after to constraint the parameters $\xi_0$, $\epsilon$, for the both cases $\gamma = 1$ and $\gamma$, is if the found values satisfy the condition for keeping the fluid description of the dissipative dark matter component, expressed by the constraint $\tau H < 1$. Using Eq. (4) for the case $s = 1/2$ and Eq. (5), the above inequality leads to the following constraint between the parameters $\xi_0$, $\epsilon$ and $\gamma$

$$\frac{\xi_0}{\sqrt{3\epsilon\gamma (2 - \gamma)}} < 1.$$

(9)

We will discuss later this condition using the values of $\xi_0$, $\epsilon$, with and without an election of the $\gamma$- value, obtained from the cosmological data of SNe Ia observations.

III. THE NOVEL COSMOLOGICAL SOLUTIONS

Now, we will briefly discuss the two solutions for Eq. (8) found in [35] for $s = 1/2$ and for the especial cases of $\gamma \neq 1$ and $\gamma = 1$.

1) In the case of $\gamma \neq 1$, the solution for the Eq. (8) can be written as a function of the redshift $z$ as

$$H(z) = C_3 (1 + z)^\alpha \cosh^\gamma [\beta (\ln (1 + z) + C_4)],$$

(10)

where $C_3$ and $C_4$ are constants given by

$$C_3 = \frac{H_0}{\cosh^\gamma (\beta C_4)} = H_0 \left[ 1 - \frac{(q_0 + 1 - \alpha)^2}{\gamma^2 \beta^2} \right]^{\frac{2}{\gamma}},$$

(11)

$$C_4 = \frac{1}{\beta} \arctanh \left[ \frac{(q_0 + 1) - \alpha}{\gamma \beta} \right],$$

(12)
\[ \alpha = \frac{\sqrt{3} \gamma}{2 z_0} \left( \sqrt{3} \xi_0 + \epsilon \gamma (2 - \gamma) \right), \]  
(13)

\[ \beta = \frac{\sqrt{3}}{2 z_0} \sqrt{6 \xi_0^2 \epsilon (2 - \gamma) + \epsilon^2 \gamma^2 (2 - \gamma)^2}. \]  
(14)

In the above expressions \( H_0 \) and \( q_0 \) are the Hubble and deceleration parameters at the present time, respectively, where the deceleration parameter is defined through \( q = -1 - \dot{H}/H^2 \). The initial condition \( a_0 = 1 \) is also used. This solution has a constraint that arises from Eqs. (11) and (12) that reads

\[ (\alpha - \gamma \beta) - 1 < q_0 < (\alpha + \gamma \beta) - 1. \]  
(15)

Since the value of \( q_0 \) will be taken from the observed data, we will check if the above constraints are fulfilled for the values determined for the parameters \( \xi_0, \epsilon \) and \( \gamma \) after the constraint of the SNe Ia data.

**ii)** In the case of \( \gamma = 1 \), the solution of the Eq.(8) can be written as

\[ H(z) = H_0 \left[ C_1 (1 + z)^{m_1} + C_2 (1 + z)^{m_2} \right], \]  
(16)

where \( H_0 \) is the Hubble parameter at the present time, and

\[ m_1 = \frac{\sqrt{3}}{2 \xi_0} \left( \sqrt{3} \xi_0 + \epsilon + \sqrt{6 \xi_0^2 \epsilon + \epsilon^2} \right), \]  
(17)

\[ m_2 = \frac{\sqrt{3}}{2 \xi_0} \left( \sqrt{3} \xi_0 + \epsilon - \sqrt{6 \xi_0^2 \epsilon + \epsilon^2} \right), \]  
(18)

\[ C_1 = \frac{(q_0 - 1) - m_2}{m_1 - m_2}, \]  
(19)

\[ C_2 = \frac{m_1 - (q_0 + 1)}{m_1 - m_2}. \]  
(20)

In the above equations \( q_0 \) is the deceleration parameter at the present time, and the conditions \( a_0 = 1 \) and \( C_1 + C_2 = 1 \) have been set. This solution was previously found and discussed in [50], but with a particular relation for the relaxation time of the form \( \xi_0 \rho_{\phi}^{-1} \) (which corresponds to \( \alpha = \xi_0 \) of our Ansatz), instead of the more general relation as Eq. (7), which was used in order to obtain the Eq. (11) in [33]. Even more, this solution has three different behaviors depending on the signs of the constants \( C_1 \) and \( C_2 \). So, for the fit purposes, we limit our analysis to the solution that is most similar to the ΛCDM model, and that corresponds to the Hubble parameter which fulfills the constraint

\[ m_2 - 1 < q_0 < m_1 - 1, \]  
(21)

which leads an always positive Hubble parameter.

**IV. CONSTRaining THE SOLUTIONS WITH SUPERNOVA IA DATA**

We shall constrain the free parameters of the solutions presented in the above sections with the observational supernova Ia (SNe Ia) data. To impose the constraints, we use here the joint light-curve analysis sample (JLA), which contains 740 SNe up to redshift \( z \simeq 1.3 \), coming from nine different surveys [54].

The theoretical distance modulus of SNe is defined as

\[ \mu_{th} (z, \bar{p}) = 5 \log_{10} \left( \frac{d_L (z, \bar{p})}{Mpc} \right) + 25, \]  
(22)

where the vector \( \bar{p} \) represents the free parameters of each solution, and \( d_L \) is the luminosity distance, given by

\[ d_L (z, \bar{p}) = \frac{c (1 + z)}{H_0} \int_0^z \frac{dz'}{E (z', \bar{p})}, \]  
(23)

where \( c \) is the speed of light given in units of \( km/s \), \( H_0 \) is the Hubble constant for which we consider the fixed fiducial value of 70 \( km \ s^{-1} / Mpc \), and \( E (z, \bar{p}) \) is defined as

\[ H (z, \bar{p}) = H_0 E (z, \bar{p}). \]  
(24)

On the other hand, in the JLA sample the distance estimator used assumes that supernovae with identical color, shape, and galactic environment have on average the same intrinsic luminosity for all redshifts. This hypothesis is quantified by the Tripp Formula [55] as

\[ \mu = m^*_b - (M_B - \alpha \times X_1 + \beta \times C), \]  
(25)

where \( m^*_b \) corresponds to the observed peak magnitude in rest frame \( B \) band, \( X_1 \) is the stretch parameter, \( C \) is the color parameter, and \( M_B, \alpha \) and \( \beta \) are nuisance parameters in the distance estimate. Thus, these last three parameters have to be computed and marginalized simultaneously with the free parameters present in the vector \( \bar{p} \).

To compute the best-fit parameters we use the affine invariant Markov chain Monte Carlo method (MCMC) [56], implemented in the pure-Python code emcee [57] with a likelihood given by the following Gaussian distribution,

\[ \mathcal{L} = N e^{-\chi^2/2}, \]  
(26)

where \( N \) is a normalization constant. Following [54], the distance estimate of Eq. (25) can be written in matrix notation, by forming a matrix \( \mathbf{A} \) such that

\[ \mathbf{\mu} = \mathbf{A} \mathbf{\eta} - \mathbf{M}_h, \]  
(27)

where

\[ \mathbf{\eta} = \left( (m^*_b, X_{1,1}, C_1), \ldots, (m^*_b, X_{1,n}, C_n) \right), \]  
(28)

\[ \mathbf{A} = \mathbf{A}_0 + \alpha \mathbf{A}_1 - \beta \mathbf{A}_2, \]  
(29)
are the $n$-dimensional vector and the $n \times n$ matrix, respectively, with $n = 740$ the number of SNe samples. Also, the JLA sample provides a covariance matrix $C$, which encodes the statistical and systematic uncertainties. Hence, the $\chi^2$ function of Eq. (26) has the form

$$\chi^2 = \left( \bar{\mu}(\vec{p}_J) - \mu_{th}(z,\vec{p}) \right)^{1/2} \left( \mu_{th}(z,\vec{p}) - \mu_{th}(z,\vec{p}) \right),$$

(30)

where $\vec{p}_J = (M_B, \alpha, \beta)$. This is the expression for $\chi^2$ that we will use in our MCMC analyses, a function that will be minimized in order to compute the best-fit values and confidence intervals. In this procedure, we use for the vector of parameters $\vec{p}_J$ the following priors: $-20 < M_B < -18$, $0 < \alpha < 1$ and $0 < \beta < 5$.

Because the solutions are only matter dominant, we have to impose $\Omega_m = 1$, and for the fit we use directly the expressions for the Hubble parameter given by the
FIG. 2. Joint and marginalized constraint of \( \xi_0, \epsilon \) and \( \gamma \), for the novel new analytical solution, and marginalized constraint of the light-curve parameters \( M_B, \alpha \) and \( \beta \) of the JLA sample. The admissible regions correspond to 1\( \sigma \) (68.3\%), 2\( \sigma \) (95.5\%), and 3\( \sigma \) (99.7\%) confidence level (CL), respectively. The best-fit values for each parameter are shown in Table I.

Eqs. (10) and (16). Also, dimensionless parameters for the fit are required. Considering that \( \epsilon \) and \( \gamma \) are already dimensionless, then a dimensionless \( \xi_0 \) required the following redefinition

\[
\xi_0 \rightarrow H_0^{1-2s}\xi_0, \quad (31)
\]

where, considering that the solutions are obtained for \( s = 1/2 \), then \( \xi_0 \) it is also dimensionless. Thus, in the case of the solution with \( \gamma \neq 1 \), the vector with the parameters of the model reads \( \vec{p} = (\xi_0, \epsilon, \gamma) \), for which we use the priors \( 0 < \xi_0 < 500 \), \( 0 < \epsilon < 1 \) and \( 1 < \gamma < 2 \); and in the case of the solution with \( \gamma = 1 \), the vector with the parameters of the model reads \( \vec{p} = (\xi_0, \epsilon) \), for which we use the priors \( 0 < \xi_0 < 500 \) and \( 0 < \epsilon < 1 \).

It is important to mention that in both cases we need to use the actual value of the deceleration parameter, \( q_0 = -0.6 \), as initial condition [4]. For the novel solution we need to use as a prior the restriction given by Eq. (15),...
in order to avoid a complex Hubble parameter during the fit; and in the particular solution $\gamma = 1$ we need to use as a prior the restriction given by Eq. (21), in order to obtain a positive Hubble parameter. Moreover, we have modified the $a$ parameter in the emcee code, in order to obtain a mean acceptance fraction between 0.2 and 0.5 \[57\]. The respective value of $a$ for each fit is 4 in the $\Lambda$CDM model, 2 in the novel new analytical solution and 3 in the particular solution, $\gamma = 1$.

V. RESULTS AND DISCUSSION

Both solutions will be compared with the $\Lambda$CDM model, whose respective $E (z, \vec{p})$ is given by

$$E_{\Lambda CDM} = \sqrt{\Omega_m (1 + z)^3 + \Omega_\Lambda}.$$ \hspace{1cm} (32)

whose respective vector of the parameters is given by $\vec{p} = (\Omega_m)$. 

FIG. 3. Joint and marginalized constraint of $\xi_0$ and $\epsilon$, for the particular solution, and marginalized constraint of the light-curve parameters $M_B$, $\alpha$ and $\beta$ of the JLA sample. The admissible regions correspond to 1\(\sigma\) (68.3\%), 2\(\sigma\) (95.5\%), and 3\(\sigma\) (99.7\%) confidence level (CL), respectively. The best-fit values for each parameter are shown in Table I.
TABLE I. Best-fit values for each model parameter, $\vec{p}$, as well as the respective goodness of fit criteria and light-curve parameters, $\tilde{\mu}_J$, of the JLA sample. The first row shows the best-fit values for the standard cosmological model, $\Lambda$CDM; the second and third rows correspond to the best-fit parameters for the novel new analytical solution and the particular $\gamma = 1$ case, respectively. We have focused on the Bayesian criterion information in order to determine the best model to fit the data, and on the comparison of the solutions with the reference $\Lambda$CDM model.

| Model              | $\Omega_m$ | $\xi_0$ | $\epsilon$ | $\gamma$ | $M_B$ | $\alpha$ | $\beta$ | $\chi^2_{\text{min}}$ | AIC | BIC |
|--------------------|------------|---------|------------|----------|-------|----------|---------|------------------------|-----|-----|
| $\Lambda$CDM       | 0.293$^{+0.034}_{-0.033}$ | -       | -          | 1        | -     | 19.075$^{+0.024}_{-0.022}$ | 0.136$^{+0.006}_{-0.006}$ | 3.106$^{+0.082}_{-0.080}$ | 692.1 | 700.1 | 718.5 |
| Novel solution     | 1          | 245.2$^{+17.4}_{-16.7}$ | 0.601$^{+21.9}_{-21.9}$ | -0.123   | 1.261$^{+0.189}_{-0.189}$ | -18.090$^{+0.012}_{-0.012}$ | 0.137$^{+0.006}_{-0.006}$ | 3.109$^{+0.071}_{-0.071}$ | 693.7 | 705.7 | 733.3 |
| Particular solution| 1          | 246.1$^{+18.9}_{-19.9}$ | 0.443$^{+0.666}_{-0.044}$ | 1        | -18.090$^{+0.012}_{-0.012}$ | 0.137$^{+0.006}_{-0.006}$ | 3.110$^{+0.073}_{-0.073}$ | 693.4 | 703.4 | 726.5 |

In order to compare the goodness of the fits, we will use the Akaike information criterion (AIC), which is defined as

$$AIC = 2k - 2\ln(\mathcal{L}_{\text{max}})$$

where $\mathcal{L}_{\text{max}}$ is the maximum value of the likelihood function, calculated for the best-fit parameters, and $k$ the number of free parameters of the model. In addition, we also calculate the Bayesian criterion information, defined as

$$BIC = k \ln (n) - 2 \ln (\mathcal{L}_{\text{max}})$$

where $n$ is the number of SNe samples. Both the AIC and BIC criteria try to solve the problem of maximizing the likelihood function by adding free parameters, resulting in overfitting. To resolve this problem, both criteria introduce a penalization that depends on the total number of free parameters of each model, which is higher in the BIC case that in the AIC case, because the penalization in the first one depends on the natural logarithm of the total observational data. The model favored by observations, as compared to the other, corresponds to the one with the smallest value of AIC/BIC. Hence, we focused our analysis on the BIC criterion, where in general a difference of $2 - 6$ in BIC between two models is considered as evidence against the model with the higher BIC, a difference of $6 - 10$ in BIC is already strong evidence, and a difference $> 10$ in BIC is definitely very strong evidence.

The best-fit values for each solution as well as the goodness of fit criterion are show in Table I. In Figs. 1-3 we depict the joint credible regions of the $\Lambda$CDM model and the two solutions studied in this paper, for combinations of their respective vector of parameters $\vec{p}$ and $\tilde{\mu}_J$. In principle, the value of $\xi_0$ in both solutions is not modified because only the supernova data is not enough to constrain $\xi_0$. This issue can be seen in the Figs. 2 and 3 in the distribution of $\xi_0$, where practically this parameter can be take with the same probability all the values used in the prior. This behavior that does not change if we change the prior. So, the values shown in the above mentioned figures and in the table I cannot be interpreted as the best fit value for $\xi_0$. In the same way, we have a similar problem for $\epsilon$ and $\gamma$ in the Fig. 2, and for $\epsilon$ in the Fig. 3, but only if we compared this constraint with the values of $\xi_0$. Hence, the analysis in where $\xi_0$ is involved cannot be conclusive.

As can be seen in the Table I, the $\Lambda$CDM model has the lower value of $\chi^2_{\text{min}}$, AIC and BIC, i.e. it is the model more favored by the observations; but focusing in the values of $\chi^2_{\text{min}}$, the solutions here presented are not directly discarded, because his differences in $\chi^2_{\text{min}}$ are not greater than 1.6. On the other hand, the BIC criteria is more conclusive in order to consider or discard the models. In the case of the solution with $\gamma \neq 1$, the difference in BIC with respect to the $\Lambda$CDM model corresponds to 14.8, i.e., we have a very strong evidence that the novel solution with this kind of fluid is not favored by observations. While in the case of the solution with $\gamma = 1$, the difference in BIC with respect to the $\Lambda$CDM model correspond to 8, i.e., therefore we have strong evidence that the particular solution with a pressureless fluid is not favored by observations. This is a consequence of the more greater value of $\chi^2_{\text{min}}$ and the extra free parameters of the solutions with respect to the $\Lambda$CDM model.

Despite what is indicated by the goodness of fit criteria, clearly the solutions tested here well describe the behavior given by the supernova data. Moreover, in the Fig. 4, it is possible to see that the two solutions just differ very slightly from the $\Lambda$CDM model, albeit in essence
they are actually different. Thus, the two solutions can describe the recent accelerated expansion and the transition of a decelerated to an accelerated expansion of the Universe, without cosmological constant and only with dissipative matter as the main fluid component of the Universe.

The main problem of the solutions arise in the value of $\epsilon$ and, to a lesser extent, in the value of $\xi_0$. The first one has a value of $0.443$ in the particular solution and $0.601$ in the novel solution, which are inconsistent with the value of $10^{-11} \ll \epsilon \lesssim 10^{-8}$ reported in [11], in order to be consistent with the properties of structure formation. The second one has a value of $246.1$ in the particular solution and $245.2$ in the novel solution, values that are inconsistent with the $\tau H \ll 1$ condition (see Eq. [4]), condition that will never be fulfilled considering the best fit values of the models. It is important to mention that this last sentence it is not conclusive because we are not able to assert that $\xi_0$ effectively has a best-fit value greater than $\epsilon$, because the supernova data is not enough to constraint $\xi_0$. Therefore, the only effective problem is the large value for the speed of bulk viscous perturbation.

It is worth mentioning that, we can obtain an accelerated expanding universe without cold dark matter but with warm dark matter instead, specifically with a matter with a value of $\gamma = 1.261$. The price to pay is the larger of $\epsilon$ with respect to the solution with cold dark matter. Even so, this value of $\gamma$ is very unrealistic because is too far for a cold dark matter, which is the more consistent with the structure formation. Any possibility of warm dark matter must be compatible with $\gamma \approx 1$.

VI. CONCLUSIONS

We have tested a cosmological model described by a novel analytical solution for arbitrary $\gamma$, including the particular case when $\gamma = 1$, by constraining it against Supernovae SNe Ia data. The solution gives the time evolution of the Hubble parameter in the framework of the full causal thermodynamics of Israel-Stewart. This solution was obtained considering a bulk viscous coefficient with the dependence $\xi = \xi_0 \rho^{1/2}$, and the general expression given by Eq. [3] for the relaxation time, for a fluid with a barotropic EoS $p = (\gamma - 1) \rho$. The results of the constraints still indicate that the $\Lambda$CDM model is statistically the most favored model by the observations.

The lesson that we have learned here is that unified DM models succeed to display the transition between decelerated and accelerated expansions, which is an essential feature supported by the observational data, without invoking a cosmological constant or some other form of dark energy. Nevertheless, as it was found in [31], only a very small value of $\epsilon$ is consistent with the structure formation, while the numerical value we found from the best fit to the data leads to inconsistencies with the values required at perturbative level. Our results at the background level are showing that an accelerated expansion implies a very large values of this parameter, leading to an inconsistency of the fluid description. In fact, the best fit parameters indicates that the condition $\tau H \ll 1$ cannot be fulfilled by the solution.

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