ON THE MASS PROFILE OF GALAXY CLUSTER Cl 0024+1654 INFERRED FROM STRONG LENSING

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ABSTRACT

Observations of a flat-density profile in the cores of dark matter–dominated halos on the two extremes of mass for virialized objects in the universe, dwarf galaxies and galaxy clusters, present a serious challenge to the current standard theory of structure formation involving cold dark matter (CDM). By contrast, N-body simulations of halo formation in the latter indicate density profiles that are singular and steeply rising toward the center. A flat-density core on the cluster scale is indicated by gravitational lensing observations, most significantly by the strong-lensing measurements of Cl 0024+1654 by the Hubble Space Telescope. A recent reanalysis of this cluster has suggested that a uniform-density core is not demanded by the data, thereby eliminating a significant piece of the conflict between the observations and the CDM theoretical predictions. We show here, however, that the singular mass profile that that analysis reports as consistent with the lensing measurements of Cl 0024+1654 implies a velocity dispersion that is much higher than the measured value for this cluster.

Subject headings: cosmology: theory — dark matter — galaxies: clusters: individual (Cl 0024+1654) — galaxies: halos — gravitational lensing

1. INTRODUCTION

There has been a lot of recent controversy concerning the density profiles of the dark matter halos of virialized cosmological structures, from the dwarf galaxy to the galaxy cluster scale. N-body simulations of the formation of “virialized” dark matter halos associated with galaxies and clusters in a cold dark matter (CDM) model were found to be well fitted by a simple, universal form for the variation of mass density \( \rho \) with radial distance \( r \) from the center of mass, given by

\[
\rho_{\text{NFW}}(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}, \tag{1}
\]

where \( r_s \) is some characteristic radius that separates the two asymptotic power-law slopes, \( \rho \propto r^{-1} \) at \( r \ll r_s \) and \( \rho \propto r^{-3} \) at \( r \gg r_s \), and \( \rho_s \) is a characteristic density that reflects the mean density of the universe at the epoch of halo formation (Navarro, Frenk, & White 1997, hereafter NFW). More recent N-body simulations of higher resolution obtain halo profiles that agree with the NFW profile at large radii but have an even steeper inner slope, \( \rho \propto r^{-1.5} \) (Moore et al. 1999).

This prediction by the standard CDM model of singular density profiles for cosmological halos is apparently in conflict with the observed mass distributions inside dark matter–dominated halos on two extremes of the halo mass function, dwarf galaxies and galaxy clusters. As a result, these observations and their interpretation have recently come under intense scrutiny. In work going back to Flores & Primack (1994), Moore (1994), and Burkert (1995) and more recently in Kravtsov et al. (1998), Moore et al. (1999), and McGaugh & de Blok (1998), these predictions of singular halos were found to be in conflict with the halo density profiles derived from the observed rotation curves of dark matter–dominated dwarf galaxies. The latter were found, instead, to be better fitted by density profiles with flat cores. Recently, the universality of this observational requirement that dark matter–dominated galactic halos possess a uniform-density core has been challenged on the grounds that the rotation curves are not generally resolved well enough in the centers to distinguish the slowly rising rotation curve that results from a mass profile with a flat core from the more rapid rise that results from a cuspy profile (van den Bosch et al. 2000). Initially, this challenge appeared to apply primarily to low surface brightness galaxies, leaving the case for uniform-density cores in dwarf galaxies still strong. For example, the well-resolved rotation curves of the nearby dwarf galaxies DDO 154 and NGC 3109 still demanded a flat-density core (van den Bosch et al. 2000). Very recently, however, this challenge has been extended to the interpretation of dwarf galaxy rotation curves as well (van den Bosch & Swaters 2000). While the latter are still generally better fitted by halo profiles with a flat core, it can no longer be stated with much confidence that the observations demand such profiles. Until better resolution data become available, profiles with \( \rho \propto r^{-\alpha} \) near the center, with \( 0 \leq \alpha \leq 1 \), all seem to yield reasonable fits to the data. Hence, while it appears that a significant conflict remains between the CDM halos predicted by the highest resolution N-body simulations to date (which imply \( \rho \propto r^{-1.5} \) at the center) and the dwarf galaxy rotation curves, the current observations are unable to discriminate effectively between halos with inner profiles as cuspy as the NFW profile and those with a uniform-density core instead.

On the galaxy cluster scale, the case for a uniform-density core has been made most convincingly using observations of strong gravitational lensing, in which the images of background galaxies are distorted by the cluster mass to form arcs and multiple images. To date, the most spectacular example of such an observation is a Hubble Space Telescope image of multiple arcs produced by the cluster Cl 0024+1654 at \( z = 0.39 \). The relaxed structure of Cl 0024+1654, the absence of a single, central, dominant cluster galaxy, and the presence of an easily identified multiply-imaged background galaxy make this cluster...
a particularly good candidate for a determination of the halo mass profile from lensing analysis. According to Tyson, Kochanski, & Dell’Antonio (1998), the observations of this cluster require a halo mass profile with a uniform-density core, in strong conflict with the predicted cuspy profile of NFW.

These conflicts between the CDM N-body results and the observations of dwarf galaxy and cluster halo profiles have stimulated a vigorous reexamination of the theoretical underpinnings of the CDM model, including a number of suggestions for a variation of the microscopic properties of CDM that might serve to produce halos with uniform-density cores while retaining the more successful aspects of the original CDM model. These include suggestions that the dark matter is nongravitationally self-interacting (Spergel & Steinhardt 2000), warm (e.g., Sommer-Larsen & Dolgov 2000; Colin, Avila-Reese, & Valenzuela 2000; Hennestad & Scherrer 2000), fluid (Peebles 2000), decaying (Cen 2000), repulsive (Goodman 2000), fuzzy (Hu, Barkana, & Gruzinov 2000), and annihilating (Kaplinghat, Knox, & Turner 2000). In view of the importance of the dwarf galaxy and cluster halo profile observations in constraining the theoretical models, it is somewhat disappointing that the conclusions based on dwarf galaxy rotation curves are currently so ambiguous with regard to the question of the uniform-density core. This makes the conclusion of Tyson et al. (1998) regarding the core in Cl 0024+1654 all the more critical.

Recently, a new study of this cluster by Broadhurst et al. (2000) has reached a conclusion opposite to that of Tyson et al. (1998) regarding the consistency of the observed mass profile with the density profiles predicted by the N-body simulations of cluster formation in the standard CDM model. They find that the NFW mass profile is consistent with the lensing data. If this is correct, then the case against the standard CDM model is significantly weakened. The purpose of this Letter is to point out that the fit by Broadhurst et al. (2000) of the cluster lensing data with a mass distribution that follows the NFW profile implies a cluster velocity dispersion that is much larger than the value measured for this cluster by Dressler et al. (1999) of $v_c = 1150$ km s$^{-1}$.

2. OBSERVATIONAL RESULTS FOR THE CLUSTER MASS PROFILE

The projected dark matter density profile that Tyson et al. (1998) found by modeling the lensing data for Cl 0024+1654 within the arcs at radius $r_{arc} \sim 100$ h$^{-1}$ kpc is well fit by

$$\Sigma(y) = \frac{K(1 + ny^2)}{(1 + y^2)^{3/2}},$$

where $y = r/r_{core}$, $K = 7900 \pm 100$ M$_\odot$ pc$^{-2}$, $r_{core} = 35 \pm 3$ h$^{-1}$ kpc, $n = 0.57 \pm 0.02$, and $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$. Additionally, Tyson et al. (1998) rule out at a great confidence level the possibility of a good fit of the observed mass distribution by the NFW profile.

A recent paper by Broadhurst et al. (2000) obtained, instead, a good fit to the lensing data with a total mass distribution given by the NFW profile in equation (1), with $r_s \approx 400$ h$^{-1}$ kpc and $\delta \approx 8000$, where $\delta \equiv \rho_c/\rho_m(z)$; $\rho_m(z) \equiv 3H^2/(8\pi G)$, which is the critical density of the universe at the cluster redshift. This $\delta$ is directly related to the NFW concentration parameter $c \equiv r_{200}/r_s$, where $r_{200}$ is the radius within which the average density is 200 times this critical density, according to

$$\delta = \frac{200}{3} \left[ \ln(1 + c) - c/(1 + c) \right],$$

which yields $c \approx 5$ and $r_{200} \approx 2$ h$^{-1}$ Mpc. From this they concluded that there is no conflict between the observations of this cluster and the predictions of the standard CDM model.

3. CONSEQUENCES FOR THE CLUSTER VELOCITY DISPERSION

3.1. NFW Profile

These mass profiles inferred for the cluster Cl 0024+1654 based on lensing observations have implications for the velocity dispersion of its dark matter and galaxies if the cluster is assumed to be in virial equilibrium. We begin by considering the NFW profile. The same $N$-body simulations of cluster formation in the CDM universe that indicate that clusters are, indeed, in an approximate virial equilibrium that can be described by the universal mass profile suggested by NFW also yield information about the one-dimensional velocity dispersion $v_c$ and its radial dependence. Over a wide range of radii, the halos obtained in $N$-body simulations are roughly isothermal (Tormen, Bouchet, & White 1997; Eke, Navarro, & Frenk 1998). For comparison with the velocity dispersion of a cluster like Cl 0024+1654 observed within some radius, it is necessary to consider the average velocity dispersion of the NFW halo within a sphere of the same radius. The average velocity dispersion of Cl 0024+1654 was measured by Dressler et al. (1999) to be $v_c = 1150$ km s$^{-1}$ within a radius $r \sim 600$ h$^{-1}$ kpc $\approx 6r_{arc}$, based on 107 galaxy redshifts, to an uncertainty of roughly less than $\pm 100$ km s$^{-1}$. For the NFW profile proposed by Broadhurst et al. (2000) for this cluster, this radius corresponds to $r \approx r_{200}/3$. It is a relatively simple matter to estimate the predicted velocity dispersion $v_{c,NFW}$ for a given NFW density profile in terms of the circular velocity profile $v_{c,NFW}$ of the same halo, as follows.

The circular velocity profile of the NFW halo is given by

$$v_{c,NFW}(r) = \frac{GM(< r)}{r} = \frac{G\rho_0 r_s^2}{x} \left[ \ln(x + 1) - x/(x + 1) \right],$$

where $M(< r)$ is the mass enclosed by radius $r$, and $x \equiv r/r_s$. The maximum value of $v_{c,NFW}$ is $v_{max,NFW} \approx 0.465 \times (4\pi G\rho_0 r_s^2)^{1/2}$, which occurs at $x = x_{max} \approx 2.163$. The NFW velocity profile for the parameters reported by Broadhurst et al. (2000) is shown in Figure 1. We obtain $v_{max,NFW} \approx 3340$ km s$^{-1}$ for this cluster. According to the detailed analysis of numerical $N$-body results by Tormen et al. (1997), the average one-dimensional velocity dispersion within $r \approx r_{200}/3$ for simulated clusters, which are well fit by the NFW profile, is somewhat lower than $v_{max,NFW}$ but never by more than a factor of $\approx 1.5$. This factor of 1.5 agrees very well with the aperture-averaged, line-of-sight $\sigma_v$ that results from solving the Jeans equation for the variation of the radial $\sigma_v$, with $r$ inside the NFW profile, including the possible effects of anisotropic velocities (Lokas & Mamon 2000; E. L. Lokas 2000, private communication). Therefore, the NFW profile proposed for Cl 0024+1654 by Broadhurst et al. (2000) implies an average velocity dispersion for the cluster within the radius $r \sim 600$ h$^{-1}$ kpc $\approx 6r_{arc}$ of
virial temperature that result are unique functions of the mass and redshift of the formation of the object for a given background universe. According to this solution, the central density $\rho_0$ is roughly more than 500 times the density at the surface, and the core radius $r_0$ is about 1/30 of the total size. (Note that our definition of the core radius is $r_0 = \rho_0 r_0 = r_{\text{King}}/3$, where $r_{\text{King}}$ is the “King radius” defined in Binney & Tremaine 1987, p. 228.) As described elsewhere, this solution is a convenient analytical approximation for the halos that form from more realistic initial conditions in the CDM model, which reproduces many of the average structural properties of the halos found in CDM simulations, except in the very inner profile where the TIS model has a uniform-density core instead of a central cusp. As such, a fit of the TIS profile to the Tyson et al. (1998) mass model provides a plausible, physically motivated connection between this mass model with a flat-density core and the implied cluster velocity dispersion. The projected density profile of a TIS with a central density $\rho_0 \approx 0.064 h^2 M_{\odot} \text{ pc}^{-3}$ and a core radius $r_0 \approx 20 h^{-1}$ kpc provides a very close match to the Tyson et al. (1998) result for $\Sigma(r)$ discussed above. Based on this best-fit TIS, the velocity dispersion is

$$\sigma_{v, \text{TIS}} = (4 \pi G \rho_0 r_0^2)^{1/2} \approx 1200 \text{ km s}^{-1},$$

in close agreement with the measured value.

4. CONCLUSION

Two different attempts to invert the observations of strong lensing by cluster Cl 0024+1654 in order to solve for the mass profile of the cluster have reached opposite conclusions regarding the presence of a uniform-density core versus the acceptability of a central cusp like that of the NFW profile. This suggests either that there is some error in one or both of these analyses or else that there is some degeneracy in the inversion process that prevents a clean discrimination between these different solutions. While we do not claim to address the accuracy of either of the two analyses of this cluster, we point out, instead, that there may be additional constraints on the allowed mass models that can aid in distinguishing them. In particular, the measured cluster velocity dispersion, which also reflects the mass distribution of the cluster, should also be required to be consistent with the mass model derived from the inversion of the lensing data. We have shown that this is not the case for the cuspy NFW profile that Broadhurst et al. (2000) report a good fit to the lensing data. That profile, if it actually corresponds to a halo formed in the standard CDM model, predicts much too high an average velocity dispersion to be consistent with the observed value. By contrast, the mass model of Tyson et al. (1998) does not have this problem. This suggests that the lensing data for Cl 0024+1654 still favor a flattening of the density profile at small radii, in conflict with the prediction of cuspy halos by N-body simulations of the standard CDM model.

In view of the importance of this conclusion for the ongoing debate regarding the validity of the standard CDM model, it would be valuable if uncertainties in the gravitational lens models, such as that due to possible departures from spherical symmetry or substructure, were properly quantified. This uncertainty would have to be rather extreme, however, to reconcile the Broadhurst et al. (2000) profile fit with the observed velocity dispersion of Cl 0024+1654. The lensing analysis leads to NFW parameters roughly by measuring the mass interior to

![Diagram of circular velocity profiles for cluster Cl 0024+1654 implied by the mass profiles inferred from the modeling of strong gravitational lensing data by Broadhurst et al. 2000; solid curve labeled $v_{\text{TIS}}$]
the arcs, \( M(\leq r_{\text{arc}}) = 4 \pi \rho_0 r_{\text{arc}}^2 f(x_{\text{arc}}) \), where \( f(x) = \ln (1 + x) - x(1 + x)^{-1} \) and \( x \equiv r/r_\text{arc} \), and determining \( r_\text{arc} \) by matching the logarithmic slope \( \gamma \) of the density profile at \( r_{\text{arc}} \) as derived from the projected mass distribution, according to \( x_{\text{arc}} = -(1 + \gamma)/(3 + \gamma) \). Therefore, in order to adjust \( v_{\text{max,NFW}} \) downward relative to the value discussed above by a factor large enough to reconcile the NFW profile with the observed velocity dispersion while leaving the measured mass \( M(\leq r_{\text{arc}}) \) unchanged \( r_\text{arc} \) must be reduced by factor of order 4, to \( r_\text{arc} \approx 100 \, h^{-1} \) kpc. This requires an observational uncertainty so large as to change the value of \( \gamma \approx -1.3 \) reported by Broadhurst et al. (2000) into \( \gamma = -2 \), which seems very unlikely.

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