Algorithm of group movement of agents with directional antennas and the related game

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Abstract. We describe one of the possible methods for organizing a control and communications system for agents such as unmanned aerial vehicles with directional antennae. A special kind of cellular automaton was proposed with an adjacency between cells changing every cycle to describe the group movement of agents forced to synchronize, as well as the corresponding game.

1. Introduction
The article will focus on one of the possible methods for organizing a control and communications system for agents such as unmanned aerial vehicles (UAVs). A system of agents that have directional antennae and, therefore, have significant restrictions on the exchange of information about their location and maintenance of the telecommunication network is described. This idea has already been briefly mentioned in the conference paper [1].

The work [2] contains a study of the use of directional antennae for organizing UAVs. Using a directional antenna with a higher gain compared to a standard omnidirectional antenna gives more than twice the communication range of the UAV. It is also obvious that the use of directional antennae increases the stealth of the communication system, which makes sense in military applications.

2. Formulation of the problem
Consider a system of \( n + 1 \) agents \( A_g = \{ a_0, a_1, \ldots, a_n \} \) (in particular, copters), one of which (denoted by \( a_0 \)) moves along a given path, and the rest should accompany him. The agents communicate over the air, the transmitting antenna being directional with the main beam width \( \theta \), \( 0 < \theta < \pi \), the receiving antenna omnidirectional. You must ensure periodic messaging between all agents.

Often the dynamics of a system of such agents is described using the equations of the “triple integrator”:

\[
\begin{align*}
\dot{p}_i(t) &= v_i(t), \\
\dot{v}_i(t) &= w_i(t), \\
\dot{w}_i(t) &= u_i(t),
\end{align*}
\]
where $p_i(t), v_i(t), w_i(t), u_i(t) \in \mathbb{R}^3$ are coordinate, speed, acceleration, and jerk of the $i$-th agent. There are also models of a “double integrator”:

$$\dot{p}_i(t) = v_i(t),$$  
$$\dot{v}_i(t) = u_i(t),$$

or “single integrator”

$$\dot{p}_i(t) = u_i(t),$$

in which the control of $u_i$ is acceleration and speed, respectively. In order to manage the escort of the 0-th agent, additional conditions are set

$$p_i(t) - p_0(t) \to P_i,$$
$$\dot{p}_i(t) - \dot{p}_0(t) \to 0,$$

where $P_i \in \mathbb{R}^3$ is the given distance vector from the escorted agent.

Model (1)–(3) can be written in the form

$$\frac{dx_i}{dt} = Ax_i + Bu_i,$$
$$y_i = Cx_i,$$

where $x_i = [p_i^T \ v_i^T \ w_i^T]^T \in \mathbb{R}^9$,

$$A = \begin{bmatrix} 0_3 & I_3 & 0_3 \\ 0_3 & 0_3 & I_3 \\ 0_3 & 0_3 & 0_3 \end{bmatrix}, \quad B = \begin{bmatrix} 0_3 \\ 0_3 \\ I_3 \end{bmatrix}, \quad C = [I_3 \ 0_3 \ 0_3].$$

Here $0_3$ is the matrix of zeros $3 \times 3$, $I_3$ is the identity matrix $3 \times 3$.

System (10)–(11) is discretized as (see, for example, [3])

$$x_{i,n+1} = A_d x_{i,n} + B_d u_{i,n},$$
$$y_{i,n} = C x_{i,n},$$

where

$$A_d = e^{Ah}, \quad B_d = \int_0^h e^{As} Bds, \quad C_d = C,$$

$h$ is a fixed time step, $e^{Ah}$ is the matrix exponent:

$$e^{Ah} = \sum_{k=0}^{\infty} \frac{1}{k!} (Ah)^k.$$

For agents that have a constant connection, control synthesis is considered in many works, for example [4]. However, if the communication of agents is possible only at certain moments, when the beams of their antennae are directed at each other, then additional synchronization is necessary.

In order to transmit information to a neighbour, an agent must turn a maximum of $2\pi/\theta$ times at an angle of $\theta$ (or rotate an antenna if technically possible). The following algorithm is proposed:
(i) Agent $a_i$ calculates a trajectory in the form of a set of control points $d_i = \{d_{i1}, \ldots, d_{il}\}$, $d_{i1}(0) = p_i(0)$.

(ii) Agent $a_i$ transfers $d_i$, $P_i$ and waits for a response from any agent $a_j$, $i < j$, if one exists, during the time $\tau$.

(iii) If there is no answer, turn around at the angle $\theta$ and return to 2, otherwise we go to 4.

(iv) We start moving along $d_i$.

Agent $ag_i$ responds to a request from another agent only once. Since $ag_i$ has information about the trajectory of the neighbouring agent and about where the neighbour is located relative to the escorted agent, he can find out the trajectory of the escorted agent and construct the corresponding trajectory.

At the point $d_{il}$, the agent $a_i$, $i \neq 0$ should

(i) “Hover”,

(ii) Wait for message from agent $a_j$, $j < i$,

(iii) Construct a new trajectory $d_i$,

(iv) Send a message with her to the agents $a_k$, $i < k$.

If $i = 0$, then 2 and 4 must be interchanged.

A system of agents will be considered to be successfully functioning if the time spent on the turns of the antennae during which conditions (8)–(9) obviously cannot be fulfilled is small compared to the time during which conditions (8)–(9) are run. The transfer of the whole trajectory, and not its closest point between the agents is needed just to minimize the need for relative orientation of the antennae. Obviously, a compromise must be sought between the length of the transmitted trajectory and the frequency of such mutual orientations of the agents. If the trajectory is too long, an obstacle may occur that will cause one of the agents to change their antennae, we must also multiply the resulting value by $\psi$ number of $n$ plus the number of turns $r$.

The neighbouring agents for a given $a_i \in Ag$ are only those agents on which the antenna of agent $a_i$ is directed and which are located no further than some distance $r$ from $a_i$. Thus, $V(\mathbf{a}_i)$ is defined as the set of all neighbouring agents for $a_i$.

“The distance” (communication) between the agents $a_i$, $a_j \in Ag$ in such a system is the number of $n_1$ rotations at the angle $\theta$ necessary to direct the antenna of agent $a_i$ to the agent $a_j$ plus the number of turns $n_2$ at the angle $\theta$ necessary to direct the antenna of agent $a_j$ to agent $a_i$. To take into account the geometric distance $L$ between the agents and the limiting range of their antennae, we must also multiply the resulting value by $\psi(L)$:

$$d(a_i, a_j) = (n_1 + n_2)\psi(L).$$

Here $\psi$ is a smooth decreasing function, $\psi(L) = 1$, $L < r$, $\psi(L) = 0$, $L + \varepsilon > r$, $\varepsilon > 0$ is a small parameter.

In a sense, one can understand such a system of agents as a cellular automaton, in which the agents themselves are the cells. The field of the cellular automaton is defined by the directed graph $\Gamma$, whose vertices are the agents. If $a_i \in V(a_j)$, $a_i$, $a_j \in Ag$, then $a_i$, $a_j$ are adjacent. “Length” of the edge will correspond to $d(a_i, a_j)$.

For the convenience of displaying such a structure, one can additionally introduce another dimension that will meet the criterion of the agents distance from the target position

$$z(a_i) = ||p_i(t) - p_0(t) + P_i|| + ||\hat{p}_i(t) - \hat{p}_0(t)||.$$
As a result, for example, of the three agents in the figure 1 we get the diagram shown in the figure 2.

For the proposed structure, the concept of adjacency will change every cycle of functioning.

3. The game-theoretic interpretation
In the language of game theory, the set of $Ag$ will correspond to the set of game participants, the pure strategies of the participants will be represented by a set

$$S = \{sr, st_1, \ldots, st_k, ss, sm, \ldots\},$$

where $sr = \text{“Rotate the antenna an angle } \theta\text{”}$, $st_i = \text{“Pass the path of length } i\text{”}$, $ss = \text{“Hover”}$, $sm = \text{“Follow trajectories”}$. Obviously, an agent can choose a strategy not arbitrarily, but
depending on a previously selected strategy. The payment function for all agents will be the
total proximity of all agents to the positions given by equations (8)–(9)

\[ F = \sum_{a_i \in Ag} z(a_i). \]

Let us define the simplified example of this game. We have \( n \) players which are agents from
\( Ag \) excluding \( a_0 \). Each player \( a_j, j = 1, n \) can

- \((s_1)\) request the player \( a_i, i = 1, n \) for synchronization, the payoff is \(-1\),
- \((s_2)\) confirm synchronization request from the player \( a_i, i = 1, n \), the payoff is \(-1\),
- \((s_3)\) follow the trajectory, the payoff is \( \frac{1}{k} - \alpha, \ 1 - \alpha > 0 \), \( \alpha > 0 \), \( k \) is the number of turns from
the last applying of \( s_1 \) which was followed by the applying \( s_2 \) by \( a_i \). If it is a first turn of
the game then the payoff is \(-\infty\),
- \((s_4)\) pass, the payoff is \(-0.5\) (the payoff size is greater than for \( s_1 \) as the player does not spend
energy to transmit a message).

This game seems to be a stochastic game which is a dynamic game with probabilistic
transitions. Denote as \( s^i_j \) the application by the player \( a_i \) the action \( s_j \). In the simplest case,
where it is always possible to confirm synchronization request and we have with two players
only, it is clear that one of the best strategy profile will be

\[
\begin{align*}
& s_1^1 s_2^2 s_3^1 s_3^2 \ldots s_3^2 \quad s_1^1 s_2^2 s_3^1 s_3^2 \ldots \\
& \text{k times until } \frac{1}{k} - \alpha > 0 \quad \text{k times until } \frac{1}{k} - \alpha > 0
\end{align*}
\]

But if we have three players, we can obtain the situation where only two players exchange
with requests and the third constantly loses. So, we should state that

- \((s_2+)\) the player \( a_j \) can confirm synchronization request from the player \( a_i \) only if the last request
confirmed was not from the \( a_j \) or if there are no other synchronization requests.

In this case, an acceptable strategy profile will be

\[
\begin{align*}
& s_1^1(2) s_2^2(1) s_3^1(3) s_3^1(2) s_3^2 s_3^2 s_3^2 \ldots s_3^2 s_3^2 s_3^2 \ldots \\
& \text{k times until } \frac{1}{k} - \alpha > 0 \quad \text{k times until } \frac{1}{k} - \alpha > 0
\end{align*}
\]

where \( s^i_j(k) \) is the application by the player \( a_i \) the action \( s_j \) to the player \( a_k \). It is not easy to
understand what would be the “best strategy” here.

4. Conclusions

A special kind of cellular automaton was proposed with an adjacency between cells changing
every cycle to describe the group movement of agents forced to synchronize, as well as the
corresponding game. It is planned to visualize such a movement.

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