Impact of roughness on gas compression in inertial confinement fusion

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Abstract. The ignition is still unachieved in current schemes of inertial confinement fusion (ICF) despite significant efforts in this direction. The reason for it is unclear as the dynamics of target combine a lot of physical processes that are crucial for successful ignition. One possible limiting factor is known for a long time – hydrodynamic instabilities and mixing. Current work consider the effect of initial roughness on compression efficiency of ICF targets. The roughness is set on the ice–ablator boundary (outer ice interface). First, some analytical results on stability of accelerated perturbed interface are presented. Second, numerical simulations of ICF target show the influence of initial perturbations on hot-spot conditions and ice–ablator mixing.

1. Introduction

The energy source based on controlled thermonuclear reactions has been a dream for many years and several schemes of such reactors has been presented for today. Inertial confinement fusion as a perspective approach is extensively studied. The remarkable progress is achieved at National Ignition Facility (NIF) [1] (today the largest experimental installation in the world). Experiments at NIF are close to ignition threshold, but does not exceed it. The ignition conditions should be achieved by the special regime of DT gas compression, that is surrounded by additional fuel (DT–ice).

It is believed that the reason for the absence of ignition is due to uncontrolled growth of hydrodynamic instabilities [2], seeded by capsule imperfections, its mounting, DT–gas pumping tube, and asymmetry of irradiation due to laser–plasma instabilities. The role of hydrodynamic instabilities was studied in many works, both theoretically and experimentally. In [3, 4, 5, 6, 7] the role of material interface roughness and medium non-uniformities are studied for NIF targets. The summary of the papers is that hydrodynamic instabilities are crucial for effective compression of targets. In the current paper we consider the role of ice–ablator interface roughness on compression of perspective targets for Russian Laser Facility [8]. This facility is designed with spherical symmetry of irradiation beams, compared to cylindrical symmetry at NIF. That is why the primary scheme for irradiation is direct drive.

The paper consists of two parts. The first part contains the analysis of instability development caused by the presence of the density perturbations near the accelerated interface. The second part contains numerical simulation of direct drive target compression. The target used in simulations is proposed as primary for Russian Laser Facility [9]. The simulation is based on hydro model with a number of plasma processes that are crucial for target dynamics. Similar studies of NIF targets [10] include many effects in joint end-to-end simulation, including full set
of initial perturbations, what complicates the subsequent analysis and understanding the role of separate effects.

The current paper discusses a single source of perturbations: roughness of ice–ablator interface and its influence on the shape of compressed gas cavity. The complex structure of discontinuities arises behind the shock wave that passes through the perturbed interface (see, e.g., [11, 12, 13, 14, 15]). Our analysis shows that such structure leads to density perturbations, that holds for distances significantly larger than initial roughness length–scales behind the shock wave. We present theory that estimates the final magnitude of interface perturbation due to initial roughness.

2. Theoretical analysis
High temperatures and energy density in ICF targets allow us to use ideal plasma equation of state (EOS) for DT ice and gas (we suppose full ionization state of medium $Z = 1$).

2.1. Problem setup
Suppose the whole space is divided by initially plain surface, which separates two mediums – see Fig. 1. One medium has uniform density, another (denoted as “1”) has harmonic perturbations of density

$$\delta \rho_1 = A_{\delta \rho_1} \cos \left( k_{0z} z + k_{0y} y \right).$$

We consider the evolution of interface in the presence of the perturbations and the external uniform gravity field $g$ (directed along Z axis). Suppose that perturbations are frozen until the moment $t = 0$.

Equation of state for ideal plasma is written as:

$$P = \Gamma \rho \zeta,$$

where $\Gamma = \gamma - 1$, $\gamma$ – adiabatic index, $\zeta = C_V T$ – specific heat energy, $C_V$ – specific heat capacity.

The dynamics of each medium obeys hydrodynamics laws:

$$\frac{\partial \rho}{\partial t} + \text{div} (\rho \mathbf{v}) = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla P + \mathbf{g}, \quad \frac{dE}{dt} - \frac{P}{\rho^2} \frac{d\rho}{dt} = 0,$$
with non-penetrating conditions at the interface

$$P_1 = P_2, \quad v_{n,1} = v_{n,2}. \quad (4)$$

Indexes 1 and 2 enumerate mediums (Fig. 1).

2.2. Stationary solution
Let us start from stationary solution for non-perturbed mediums. Suppose the distribution temperature is uniform (for real systems at rest temperature is uniformed with time due to thermal conductivity). For that case equation of state (2) lead to nonuniform density that agrees with pressure. For stationary conditions the system (3) is reduced to:

$$\frac{dP}{dz} = -\rho g. \quad (5)$$

And the solution

$$\rho = \rho_{CD} \exp \{-gz/\Gamma \zeta_0\}. \quad (6)$$

Here $\rho_{CD}$ is medium density at the interface, $\zeta_0$ – constant specific heat for medium.

2.3. Linear approximation
We use linear approximation for the growth of interface perturbations. From (3) for $j$–medium ($j=1,2$) we have

$$\frac{\partial \delta \rho_j}{\partial t} + \rho_j \frac{\partial v_{z,j}}{\partial t} + \rho \left( \frac{\partial \delta v_{z,j}}{\partial z} + \frac{\partial \delta v_{y,j}}{\partial y} \right) = 0, \quad \rho_j \frac{\partial \delta v_{z,j}}{\partial t} + \frac{\partial \delta P_j}{\partial z} + \delta \rho_j g = 0,$$

$$\rho_j \frac{\partial \delta v_{y,j}}{\partial t} + \frac{\partial \delta P_j}{\partial y} = 0, \quad \frac{\partial \delta \zeta_j}{\partial t} + \delta v_{z,j} \frac{\partial \zeta_j}{\partial z} - \frac{\Gamma_j \zeta_j}{\rho} \left( \frac{\partial \delta \rho_j}{\partial t} + \delta v_{z,j} \frac{\partial \rho_j}{\partial z} \right) = 0, \quad (7)$$

$$\delta P_j - \frac{\partial \delta P_j}{\partial \rho_j} \delta \rho_j - \frac{\partial \delta P_j}{\partial E_j} \delta E_j = 0.$$

These equations are valid for general type equation of state. From first three equations in (7):

$$\frac{\partial^2 \delta \rho_j}{\partial t^2} = \left( \frac{\partial^2 \delta P_j}{\partial z^2} + \frac{\partial^2 \delta P_j}{\partial y^2} \right) - \frac{\partial \delta \rho_j}{\partial z} g = 0. \quad (8)$$

From fourth and fifth equations in (7):

$$\frac{\partial^2 \delta P_j}{\partial t^2} - c_j^2 \frac{\partial^2 \delta \rho_j}{\partial t^2} - \Gamma_j g \left( \frac{\partial \delta P_j}{\partial z} + \delta \rho_j g \right) = 0. \quad (9)$$

Where for speed of sound we use $c^2 = \sqrt{(\partial P/\partial \rho) + P/\rho^2 (\partial P/\partial E)}$ [16].

2.4. Laplace transform analysis
We use the Laplace transform [17] to solve (8) and (9). For any function $f(t)$ the transformed function $\hat{f}(s)$ of complex variable $s = s' + is''$ and inverse transformations are defined as

$$\hat{f}(s) = \int_0^\infty e^{-st} f(t) dt, \quad f(t) = \frac{1}{2\pi i} \int_{\ell-i\infty}^{\ell+i\infty} e^{st} \hat{f}(s) ds,$$
with $\ell$ – real constant. So applying EOS (2) for Eqns. (8) and (9) for $j$-medium we have

\[
\hat{s}^2 \delta \hat{P}_j - 1 - \frac{\partial^2 \delta \hat{P}_j}{\partial \hat{s}^2} + \tilde{k}_0^2 \delta \hat{P}_j - \frac{\partial \delta \hat{P}_j}{\partial \hat{s}} = 0,
\]

\[
\Delta_j \frac{\partial \delta \hat{P}_j}{\partial \hat{s}} - \hat{s} \left(\hat{s} \delta \hat{P}_j - 1\right) - \Delta_j \left(\frac{\partial \delta \hat{P}_j}{\partial \hat{s}} + \delta \hat{P}_j\right) = 0.
\]

(10)

(11)

With dimensionless variables $\hat{s} = s\sqrt{\beta}/g$, $\tilde{z} = zg/\beta$, $\tilde{k}_0 = k_0\beta/g$, $\delta \hat{P}_j = \delta \hat{P}_j(\beta^{\beta/2}, \Delta_j = \Gamma_j/(\Gamma_j + 1)$. From (10) and (11):

\[
\frac{\partial^2 \delta \hat{P}_j}{\partial \tilde{z}^2} + \frac{\partial \delta \hat{P}_j}{\partial \tilde{z}} - \frac{\Delta_j \tilde{z}^4 + \tilde{k}_0^2 (\tilde{s}^2 + \Delta_j)}{\tilde{s}^2} \delta \hat{P}_j + \frac{\Delta_j + i\tilde{k}_0 \tilde{z}}{\tilde{s}} = 0.
\]

(12)

The solution of (12) could be presented as a sum of homogeneous ($\delta \hat{P}_j^{(1)}$) and partial ($\delta \hat{P}_j^{(2)}$) solutions:

\[
\delta \hat{P}_j^{(1)} = C_{1,j}(\tilde{s}) \exp \left\{ \tilde{k}_{j,+} \tilde{z} + C_{2,j}(\tilde{s}) \exp \left\{ \tilde{k}_{j,-} \tilde{z} \right\} \right\},
\]

(13)

\[
\delta \hat{P}_j^{(2)} = B_j(s) \exp \left\{ i\tilde{k}_0 \tilde{z} + i\tilde{k}_0 \tilde{y} \right\}, \quad B_j(s) = \frac{\left(\Delta_j + i\tilde{k}_0 \tilde{z}\right)}{\Gamma_j s^4 + \left(\tilde{k}_0^2 - i\tilde{k}_0 \tilde{z}\right) s^2 + \tilde{k}_0^2 \Delta_j},
\]

(14)

with $\tilde{k}_0 = \tilde{k}_0^2 + \tilde{k}_0^2$ and $\tilde{k}_{j,\pm} = -\frac{1}{2\Psi_j} \pm \frac{1}{\tilde{s}} \sqrt{\frac{\tilde{s}^2}{4\Psi_j^2} + \frac{1}{\Psi_j} \left(\frac{\Delta_j}{\Gamma_j} \tilde{s}^4 + \tilde{k}_0^2 \tilde{s}^2 \tilde{\Psi}_j + \Delta_j\right)}$, $\tilde{\Psi}_j = \beta_j/\beta_1$.

In homogeneous solution boundary conditions (4) are taken into account.

Initial perturbations of density and temperature are present only in first medium. Using the theorem of unital value ($\lim_{t \to 0} \hat{f}(t) = \lim_{s \to \infty} s \hat{f}(s)$) we see that $C_{2,1} = C_{1,2} = 0$. So using (4) and (4), we have

\[
\delta \hat{P}_1 = C_{1,1}(\tilde{s}) \exp \left\{ \tilde{k}_{1,+} \tilde{z} + i\tilde{k}_0 \tilde{y} \right\} + B_1 \exp \left\{ i\tilde{k}_0 \tilde{z} + i\tilde{k}_0 \tilde{y} \right\},
\]

(15)

\[
C_{1,1}(\tilde{s}) = \left[ \chi(\tilde{s}) \left(\kappa_{2,-} + \frac{\Delta_2}{\Psi_2 \Gamma_2}\right) - \tilde{k}_0 \tilde{s} - \tilde{s} \Delta_1 \frac{1}{\Gamma_1} \right] \tilde{s} B_1 - 1
\]

\[
\kappa_{1,+} - \frac{\Delta_1}{\Gamma_1} - \chi(\tilde{s}) \left(\tilde{s} \kappa_{2,-} + \tilde{s} \Delta_2 \frac{1}{\Psi_2 \Gamma_2}\right), \quad \chi(\tilde{s}) = \Psi_2 \tilde{s}^2 - \Delta_2.
\]

Suppose that mediums are similar in physical properties: material constants of EOS are equal. Also initial temperature and density of materials are close to each other. So $\Delta_1 = \Delta_2 \equiv \Delta$, $\Gamma_1 = G_2 \equiv \Gamma$, $\Psi_2 = \beta_2/\beta_1 \equiv \beta_2/\beta = (1 + \eta)$, with $\eta \ll 1$. For $\delta \hat{v}_{z,1}$ we have

\[
\delta \hat{v}_{z,1} = \delta \hat{v}_{z,1}^{(1)} + \delta \hat{v}_{z,1}^{(2)},
\]

(16)

where $\delta \hat{v}_{z,j} = \delta \hat{v}_{z,j}(g/\beta)$. $\delta \hat{P}_j(0) = \delta \rho_j(0)/\rho_j$. Since we are interested in gas–ice interface perturbation, so suppose. The original function $\delta \hat{v}_{z,1}^{(1)}$ has analytical form. For $\delta \hat{v}_{z,1}^{(2)}$ we have an
integral:

$$\delta \tilde{v}_{z,1}(t) = \frac{\sqrt{\beta}}{2\pi i} \int_{\ell-i\infty}^{\ell+i\infty} \delta \tilde{v}_{z,1}(\tilde{s}) e^{\tilde{s}t} d\tilde{s}. \quad (17)$$

The integration is possible with a residue theorem. The final amplitude value is $a = \delta \tilde{v}_{z,1}t$.

Our simulations discussed below show that ice–ablator interface perturbations is carried with a wave structure that follow the shock wave that passes the interface. And gas–ice boundary is perturbed even at the acceleration stage.

Similar simplified setup of enlarged cm–sized target was studied numerically with pure hydrodynamics, without thermal transport, thermonuclear reactions etc (we do not present the simulations in the paper explicitly). Though in full target simulations we should take into account these additional physical processes, what is done in next section, the pure hydro-model is a good basis for the effect estimation. The parameters of the problem were: $g = 15000 \text{mm/\mu s}^2$, $\rho_1 = 0.0144 \text{g/cm}^3$, $\rho_2 = 0.014 \text{g/cm}^3$, $P = 300 \text{GPa}$, $\lambda_{0y} = 300 \text{\mu m}$, $\lambda_{0z} = 1 \text{cm}$.

Ice density was initially perturbed with 5% amplitude value, that results in 50 $\mu$m amplitude of interface perturbation in numerical simulations. Analytical estimation gives a similar result of 40 $\mu$m (this value should be compared with cavity size of 100 $\mu$m, what tells the significance of the effect).

### 3. Numerical simulations

Let us consider the target proposed for Russian Laser Facility [8, 9]. The target is designed for direct drive scheme and is multilayered: outer DT gas radius is at $R_{\text{gas}} = 1414 \text{\mu m}$ (density $\rho_{\text{gas}} = 10^{-3} \text{g/cm}^3$), outer DT ice radius $R_{\text{ice}} = 1563 \text{\mu m}$ ($\rho_{\text{ice}} = 0.213 \text{g/cm}^3$), CH–ablator radius $R_{\text{CH}} = 1597 \text{\mu m}$ ($\rho_{\text{CH}} = 1 \text{g/cm}^3$).

We simulate the target compression in two steps. At first step, we use the one–dimensional code with model that contains explicit laser absorption. The model includes the minimum necessary set of physical processes intended to describe processes in laser corona: two–temperature hydrodynamics, electron and ion thermal heat flow. Due to high energy density after absorption of $\sim$ MJ of laser energy the ionization state is full and ideal plasma equation of state is adequate. Laser radiation is simulated using ray–tracing model with inverse–bremsstrahlung absorption [18]. During simulation it is important to resolve region near critical electron density, that is why initial simulation domain is several times larger than target initial size. Such minimal model allows us to pass the results of simulations to two–dimensional numerical code with one–temperature plasma model.

The successful compression of the target is very sensible to the irradiation scheme. We use time dependent laser power from [9]. In one–dimensional simulations we obtain hot–spot parameters that correspond to ignition conditions: pressure of several hundred Gbar, hot–spot density $\sim 40 \text{g/cm}^3$ that is surrounded by cold shell with density $\sim 200 \text{g/cm}^3$. Using the simplified model of thermonuclear burning (without nonlocal $\alpha$–deposition) the significant part of fuel burns – 16%. The conditions in hot–spot and high burn-out of fuel shows that our minimal model adequately describes the physics of target compression and the solution is close to ignition regime.

Two–dimensional code implements one–temperature hydrodynamics with heat transport. It does not have a module for laser propagation and we simulate the effect of absorption with proper boundary conditions set at the outer radius of the target $R_{\text{CH}}$. The comparison of results by two–dimensional unperturbed simulations and one–dimensional two–temperature model show rather good agreement. The dynamics of the shell, implosion velocity are close to each other in two variants. The difference is seen only at late times, close to maximum compression time, when in 1D simulation the critical density region (and the laser energy release) goes below $R_{\text{CH}}$. 


and the approximation for problem setup via boundary conditions breaks. But physically such internal energy source if it had been taken into account in 2D simulations should strengthen the compression regime and the effects considered in the paper.

Two-dimensional simulations focus on hydrodynamic instabilities triggered by ice–ablator interface roughness. The roughness is set as a number of random short wavelength modes, that perturb an interface with amplitude $A_{\text{rough}}$. Another interfaces are also unstable: the initial inner ice interface perturbations grow only at deceleration stage of target dynamics, the outer ablator interface is unstable due to ablative Rayleigh-Taylor instability [19], what goes beyond the scope of the paper.

Two-dimensional code uses the numerical method described in [20]. This method captures multmaterial interfaces without numerical smearing on Eulerian grids. The method employs single velocity multiphase model with pressure and temperature equilibrium. Hydrosystem is solved with Godunov-type Eulerian numerical scheme with second order approximation in space and time. Fluxes at the interfaces are calculated with a special algorithm. Electron thermal flux is introduced with a splitting scheme: hydrodynamics and thermal flux are evolved on a separate substeps. The equation for heat transport of parabolic type is numerically solved by local iteration method [21], which allows to use explicit numerical scheme for any timestep.

3.1. Two-dimensional simulations
We use 2D axysymmetric problem statement and the computational domain is a sector with internal $R_{\text{in}}$ and outer $R_{\text{out}}$ radiusses and angle $\alpha$. Basic initial conditions for temperature and density distributions that correspond to 1D simulations are perturbed at the ice–ablator

Figure 2. Maximum compression moment: (a)–(c) – the whole computational domain for maximum roughness amplitudes 1 µm, 0.1 µm and 0.001 µm, (d)–(f) – zoom on gas cavity (for corresponding amplitudes: 1 µm, 0.1 µm and 0.001 µm). Colors describe the material: gray – gas, orange – ice, red – ablator, blue – mixed cells.
interface. We use six random modes to set perturbations, the maximum value of perturbation amplitude $A_{\text{rough}}$ varies between simulations and is $1 \mu m, 0.1 \mu m, 0.001 \mu m$. Such small amplitudes are not explicitly resolved by grid and are initialized by variation of volume fraction: algorithm for interface reconstruction used in simulations could resolve real interface position at a subcell level.

We use non-penetrating walls conditions for all boundaries except outer radius $R_{\text{out}}$, where pressure and temperature time dependencies are explicitly set. These dependencies are taken from 1D simulations. Other quantities have zero derivatives at outer boundary.

All 2D simulations have the following parameters: $R_{\text{inn}} = 50 \mu m$, $R_{\text{out}} = 1563 \mu m$, $\alpha = 0.578873$ rad. Grid has a polar structure and is nonuniform over radius: radial step increases over radius, so that cells stay close to square form. The number of cells in simulations is $\approx 5 \times 10^6$.

Figs.2(a)–2(f) show the distribution of materials for the moment of maximum compression. Important result is that initial outer ice interface roughness leads to the mixing of ablator material and DT ice. But the mixture is not uniform: the jets of ablator penetrate into the ice (except the case of largest initial amplitude – multiple mixed cells appear in Fig. 2(a)). The magnitude of penetration increases together with initial amplitude of roughness. The ignition scenario implies that a hot spot is mainly formed from initial DT gas, and successful ignition requires a spherical form for the hot spot and the absence of ablator mixture in it. We see that initial roughness influence the hot spot form for $A_{\text{rough}} \gtrsim 0.1 \mu m$. Moreover, the mixture of ice and ablator will interfere the subsequent burning of the shell surrounding the hot spot (that is formed from initial DT ice). Preliminary simulations with thermonuclear reactions show the $\sim 10$ times degradation of neutron yield for $A_{\text{rough}} = 1 \mu m$ (in comparison with symmetric case) and the absence of influence on yield for $A_{\text{rough}} = 0.001 \mu m$.

4. Conclusions
The large compression of ignition targets in inertial confinement fusion leads to the significant influence of hydro-instabilities for many systems. One way to control the growth of the instabilities is to clean up initial seeds. Nevertheless, instability amplification factor is so high, that even small perturbations that appear during target fabrication could have significant contribution. The paper considers direct drive target proposed in [9] and the role of initial roughness at the ice–ablator interface for its dynamics.

We present a theoretical analysis of interface instability growth seeded by density perturbations inside one medium. For multilayer systems the perturbation of one interface cause volume perturbations that follow a shock wave passing initial interface. With this mechanism perturbations are carried from one interface (ice–ablator) to another (gas–ice) even at the acceleration stage. The estimation of the effect gives 10% radial amplitude for gas cavity at the moment of maximum compression.

We numerically study the dynamics of direct drive target. The simulations are conducted in two steps. First, the dynamics of target is simulated using 1D code with explicit laser absorption model. The results show the formation of hot spot with ignition conditions and significant burn out of DT fuel. Using these simulations we formulate boundary conditions at the outer radius of the target that reproduce the initial dynamics inside. These boundary conditions are used in 2D simulations that study the role of roughness. For initial amplitude that exceed $\gtrsim 0.1 \mu m$ the interface is significantly perturbed and jets of ablator penetrate into initial ice, mixing these materials. The shape of hot spot is also altered. Together these effects lead to degradation of neutron yield for 10–times.

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