Using the Optical Glauber Monte Carlo Model to Calculate $R_{AA}$ Related Results Produced by the ATLAS Collaboration

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ABSTRACT

The energy loss during jet quenching due to the existence of Quark Gluon Plasma (QGP) is calculated by Optical Glaube Monte Carlo model with data collected by ATLAS Collaboration using the LHC detector. An energy loss formula for this situation was modeled and took the form $\Delta p_T = \sqrt{L} \cdot e^{\ln p_T \cdot \frac{c + \ln p_T}{30 \pi^2}}$. The nuclear modification factor, $R_{AA}$, for jets in a $^{208}$Pb + $^{208}$Pb nucleus collision with rapidity interval of $|y| = 2.8$ and the initial transverse momentum of $50 \text{ GeV} \leq p_T \leq 1000 \text{ GeV}$, are compared with various data plots produced by ATLAS Collaboration. $R_{AA}$ results are plotted in different centrality bins, which are defined by the distribution of number of participating nucleons $N_{\text{part}}$. The $R_{AA}$ value was found to slowly increase at lower transverse momenta and flatten out at higher transverse momenta. The model’s theoretical calculation results turned out to be similar to the plots produced by the ATLAS Collaboration using data from the LHC with small differences for higher systematic uncertainty events.

1. INTRODUCTION

High energy ultra relativistic heavy-ion collisions between two atomic nuclei in particle accelerators can result in a condensed substance that exhibits fluid-like properties known as Quark Gluon Plasma (QGP) [1], which is mainly constituted of colour charge carriers (e.g. quarks and gluons). Quantum Chromodynamics (QCD) describes the strong interactions between elementary particles such as quark and gluons, which carry colour charges. Unlike in Quantum Electrodynamics (QED), where field lines originating from charged particles spread infinitely throughout space, field lines between interacting quarks only exist in a thin tube between them; this has been determined experimentally and through simulations. Therefore, independent free
quarks can only exist under special circumstances, as the energy required to separate confined quarks found in hadrons would scale indefinitely with the distance of separation, and the energy inputted would make it favourable for the creation of a quark anti-quark pair due to vacuum fluctuations.

To better understand the properties and the compositions of QGP, particle colliders are used both to generate the plasma and to read their signatures through processes similar to scattering experiments, which can be done by observing and measuring properties of hard and soft probes exiting out of the QGP [2]. Jets, like vector-bosons and quarkonia, are a form of hard probes, for their signatures going through QGP are different compared to their signatures when moving through vacuum. Jets produced at nucleon-nucleon collision sites are subjected to energy modifications due to the effects imposed by the QGP. It has been observed that the energies of jets produced in Pb + Pb collisions are lower than those of jets produced in pp collisions; this phenomenon has been termed as jet quenching [3, 4], as the jets’ passage through QGP lowers their energies. The energy by which a jet loses is dependent on its initial transverse momentum \( p_T \) and the distance through the QGP that the jet travels, which is modeled by the distance through the participating nucleons that the jet penetrates (\( L \)). This energy suppression could be observed by comparing the jet \( p_T \) distribution in Pb + Pb collisions to the jet yield of pp collisions.

The characterization of the distribution of the number of participating nucleons, \( N_{\text{part}} \), achieved by grouping these events into different centrality bins that correspond to different ranges of impact parameters (for example, smaller impact parameters can mean higher numbers of collisions and larger impact parameters can mean smaller numbers of collisions), can categorize data for further calculation regarding \( N_{\text{part}} \) as shown in Table 1. Figure 1 displays the distribution of \( N_{\text{part}} \) over 100,000 samples, separated into different centrality bins.

The measurement of the jet yield and nuclear modification factor \( R_{AA} \) represents the relationship between the jet energy loss and the initial transverse momenta with respect to different centralities in a collision.

![Figure 1](https://doi.org/10.4236/ns.2020.123010)  
**Figure 1.** The distribution of the number of participating nucleons \((N_{\text{part}})\) separated into different centrality bins. Starting at the y-axis where the centrality is about 100%, every interval between each vertical colored line categorizes a centrality range of 10%. (From left to right the first centrality range is 90% to 100%; the second range is 80% to 90%, etc. The lowest range is 0% - 10%.) \( N_{\text{part}} \) has a mode at 2, and the frequency at which each \( N_{\text{part}} \) occurs gradually decreases as \( N_{\text{part}} \) increases, until falling to 0 after \( N_{\text{part}} \) reaches 416, the maximum number of participants for a \( ^{208}\text{Pb} + ^{208}\text{Pb} \) collision. Note the use of a logarithmic scale for the y-axis.
The mean number of participants, \( N_{\text{part}} \), and their uncertainties for different centrality intervals.

| Centrality Range | \( N_{\text{part}} \) |
|------------------|------------------|
| 70% - 80%        | 25               |
| 60% - 70%        | 43               |
| 50% - 60%        | 67.5             |
| 40% - 50%        | 99               |
| 30% - 40%        | 139              |
| 20% - 30%        | 190              |
| 10% - 20%        | 256              |
| 0% - 10%         | 354.5            |

In order to incorporate the Optical Glauber Monte Carlo Model, the following interpretation of \( R_{AA} \), given in Equation (2), is used to simplify and optimize the calculations.

\[
R_{AA} = \frac{\int \frac{d^2 N_{\text{jet}}}{dp_T dy} \left|_{\text{cent}} \right|}{\left\langle T_{AA} \right\rangle \frac{d2\sigma_{\text{jet}}}{dp_T dy} \left|_{pp} \right|}
\]

where \( \psi(p_T) \) is the frequency distribution of transverse momentum, \( p_T \).

Using data provided by the ATLAS Collaboration from the LHC detector, this paper aims to analyze the relationship between \( p_T \) and \( R_{AA} \) under different centrality ranges, attempt to find a formula for the jet energy loss, and reproduce some of the plots presented in the paper by the ATLAS Collaboration [5] through the use of computer simulations built under the Optical Monte Carlo Model.

2. METHOD

The program setup is based on the Optical Glauber Monte Carlo approach [6-8], which is a simplistic, geometry based method of approximating \( N_{\text{part}} \) (the number of participating nucleons) and \( N_{\text{coll}} \) (the number of binary nucleon-nucleon collisions), given an impact parameter \( b \) and a certain nucleus radius \( r \) of a nucleus-nucleus collision.

Within this model, it is assumed that the velocities of the two colliding lead nuclei are exactly opposite of each other. For the purposes of this project, it was assumed that the protons and neutrons are geometrically identical and that the charge that the protons carry has no effects on the collisions. The initial locations of nucleons are determined by assuming that the density distribution of the nucleons with respect to the distance to the center of the nucleus is rectangular, but disregarding the effects of nuclear charge density and minimum inter-nucleon separation distance. As the experiment using the ATLAS detector at the LHC indicated, the \(^{208}\text{Pb} \) nucleus, containing 208 nucleons each with a radius of 0.546 fm, has a radius of 6.62 fm.
The event of a Pb-Pb collision is simulated using the computer program ROOT, which utilizes random number generators to generate the positions of the nucleons in two spheres to simulate two Lead nuclei. The position of each nucleon is determined randomly by selecting arbitrary points within a sphere of radius 6.62 fm; this step is repeated for each of the 208 nucleons in each nucleus. Coordinates of these three-dimensional spherical nuclei, separated by the impact parameter (which is determined randomly given the distribution $\sigma_{\text{PP}} = 2\pi b$), are then projected onto a two-dimensional plane. If the distance between the centers of two arbitrary nucleons is less than the diameter of a nucleon (which is 1.092 fm), they are registered as collided and are participants. If a given nucleon does not collide with any other nucleon in the other nucleus, the nucleon would be regarded to as a spectator. **Figure 2** shows an example of a simulated collision with this model.

This model treats the nuclei collision as numbers of independent binary nucleon-nucleon collisions. The model also assumes that all the nucleons carry ultra-relativistic momentum and that they travel in a straight trajectory according to their initial positions and can collide with multiple nucleons without any path change or deflection. These properties indicate that the inelastic nucleon-nucleon cross section is not disturbed by other collisions made by the same nucleon. Due to the fact that the elastic cross sections do not contribute to a significant amount of energy loss, they are not considered here in this model.

The Optical Glauber Monte Carlo model allows for the derivation of centrality ranges from the $N_{\text{part}}$ distribution, which characterize whether the collision is central or peripheral. Dijets, two jets that start at the same position but facing the opposite directions, are produced, initiating at the centers of the nucleon-nucleon collisions going at a random angle $\theta$. It is important to note that, in this model, these jets are infinitely thin and will not be deflected or curved during their contact within the Quark Gluon Plasma. The amount of QGP that each jet goes through is calculated by taking the sum of the total distance of participating nucleons the jet penetrates through ($L_1$ and $L_2$ for the distances that the dijet goes through on each side). **Figure 3** shows the distribution of $L_1$ and $L_2$ over 4 different centrality bins: 0% - 10%, 20% - 30%, 40% - 50%, 60% - 70%.

**Figure 2.** A collision between two randomly generated lead nuclei with an impact parameter of 6.62 fm, projected onto the transverse plane. The origin is the midpoint of the centers of the nuclei. Nucleons from the nucleus on the left are colored in blue, while nucleons from the nucleus on the right are colored in red. Spectator nucleons have dotted edges, while participating nucleons have solid edges. The outline of the nuclei is shown in large dotted circles.
Figure 3. The result of the Jet distance calculation using the Optical Glauber Monte Carlo model bounded within different centrality ranges over 1,000,000 samples each. The right color bar demonstrates the intensity of the distribution at a particular point. (a) Centrality range: 0% - 10%; (b) Centrality range: 20% - 30%; (c) Centrality range: 40% - 50%; (d) Centrality range: 60% - 70%.

However, the model has certain downsides regarding the physics behind it. It assumes that all the nuclei and nucleons are completely spherical, thus disregarding most of the ultra-relativistic and quantum affects. According to experimental data, it is theorized that, during process of jets traveling through the QGP, different kinds of interactions will contribute to an energy loss. At present this process is not completely understood, but many researchers have done numerous works on this energy loss formula [9, 10]. We propose an energy loss formula written in the form as given by Equation (3) in order to compensate for all the above causes. The use of the equation will be demonstrated later in the paper.

$$\Delta p_T = \sqrt{L} \cdot \ln p_T \cdot \left( \frac{e + \ln p_T}{30\pi^2} \right)$$

3. RESULTS AND DISCUSSIONS

Equation (3) presents the formula for the energy lost for a jet with transverse momentum $p_T$ traveling through QGP a distance of $L$. Using this energy loss formula, we produced different graphs of the amount of energy lost based on the initial transverse momentum at different centrality ranges. Figure 4 presents several graphs of energy loss at centrality ranges of 0% - 10%, 20% - 30%, 40% - 50%, and 60% -
70%.

Similar to the results obtained by the ATLAS Collaboration, Figure 5 shows the nuclear modification factor $R_{AA}$ as a function of Jet $p_T$ for eight centrality ranges. The $R_{AA}$ value is calculated by jets with $|y|=2.8$ and $p_T$ between 50 GeV and 1000 GeV. Following the ATLAS results, we have modified our bins using variable binning to compress higher $p_T$ jet intervals before evaluating $R_{AA}$. $R_{AA}$ values increase slowly as $p_T$ values increase for all events at different centrality ranges until flattening out at high $p_T$ values. $R_{AA}$ predictions yield by the Optical Glauber Monte Carlo model is fairly close to the actual experimental data collected by the LHC and presented by ATLAS. Most of the data points presented by the model can potentially lie directly on top of the ATLAS data if luminosity uncertainty is considered. This is a proof that the energy loss formula is sufficient to compensate most the ultra-relativistic quantum effects that are not considered in the Optical Glauber Monte Carlo model.

Figure 6 shows a relationship between $R_{AA}$ and $N_{part}$ for $p_T$ range ($100 < p_T < 126$ GeV) of jets with $|y| \leq 2.8$. Compared with data from ATLAS, our paper produced similar results: $R_{AA}$ decreases on a smooth curve as number of participants increases. The model corresponding to the characteristics of the ATLAS results because both of them showed tendency for peripheral collisions to produce higher $R_{AA}$ values.

Figure 4. The energy that a jet would lose based on the transverse momentum of the jet and bounded into different centrality, 0% - 10% centrality for the graph above. The right colour bar demonstrates the intensity of the distribution at a particular point. (a) Centrality range: 0% - 10%; (b) Centrality range: 20% - 30%; (c) Centrality range: 40% - 50%; (d) Centrality range: 60% - 70%.
The results seem like that both the initial $p_T$ and the amount of QGP will influence the energy loss during the collision. When the initial $p_T$ become higher, the influences of QGP on jets will tend to be constant. Therefore, we predict that the QGP will exert a stronger effect on jets with lower transverse momentum, which means the jets with lower $p_T$ will lose relative more energy than the jets with higher $p_T$ due to the QGP. It is also explicit that the jet energy loss is dependent on the amount of QGP (and subsequently, the number of participants and the impact parameter), and the amount of QGP will become more crucial factor as $p_T$ increases.

Figure 5. Plots reproduced in this project. The colored boxes are results obtained ATLAS Collaboration. The grey box labeled “Glauber Model Data” corresponds to the results according to the name centralities. (a) Centrality intervals 0% - 10%, 20% - 30%, 40% - 50%, and 60% - 70%; (b) Centrality intervals 10% - 20%, 30% - 40%, 50% - 60%, and 70% - 80%.
Figure 6. $R_{AA}$ with respect $N_{\text{part}}$ to with $100 < p_T < 126$ GeV. The white dots represent data from the ATLAS Collaboration, while the stars represent the data outputted from the Optical Glauber Monte Carlo Model.

4. CONCLUSION

Nuclear Modification Factor ($R_{AA}$) predictions, calculated using the Optical Glauber Monte Carlo model and random Dijet generation functions with transverse momentum ($p_T$) data for Pb-Pb nucleon-nucleon collisions at center-of-mass energy of 5.02 TeV provided by the ATLAS detector at the LHC, are consistent with the data yield from ATLAS results. From the plots, one can see that the effects of Quark Gluon Plasma (QGP) on jets vary with $p_T$ and the different centrality ranges, which correspond to the $N_{\text{part}}$. These results represent the success of the Optical Glauber Monte Carlo Model at replicating the actual experimental data. So the Optical Glauber Monte Carlo Model turns out to be a plausible theoretical model for the evaluation of jet quenching.

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CONFLICTS OF INTEREST

The authors declare no conflicts of interest regarding the publication of this paper.

REFERENCES

1. Busza, W., Rajagopal, K. and Wilke, V.D.S. (2018) Heavy Ion Collisions: The Big Picture, and the Big Questions. *Annual Review of Nuclear and Particle Science*, 68, 339-376. [https://arxiv.org/abs/1802.04801](https://arxiv.org/abs/1802.04801)

2. Qin, G.Y. and Wang, X.N. (2015) Jet Quenching in High-Energy Heavy-Ion Collisions. *International Journal of Modern Physics E*, 24, Article ID: 1530014. [https://doi.org/10.1142/S0218301315300143](https://doi.org/10.1142/S0218301315300143)

3. Aaboud, M., Aad, G., Abbott, B., Abdinov, O., Abeloos, B., Abhayasinghe, D.K., Abreu, H., et al. (2018) Observation of Centrality-Dependent Acoplanarity for Muon Pairs Produced via Two-Photon Scattering in Pb + Pb Collisions at $\sqrt{s_{NN}} = 5.02$ TeV with the ATLAS Detector. *Physical Review Letters*, 121, Article ID: 212301.
4. Brewer, J., Milhano, J.G. and Thaler, J. (2018) Sorting Out Quenched Jets. *Physical Review Letters*, **122**, Article ID: 222301. https://doi.org/10.1103/PhysRevLett.122.222301

5. Collaboration, A. (2014) Measurements of the Nuclear Modification Factor for Jets in Pb + Pb Collisions at $\sqrt{s_{NN}} = 5.02$ TeV with the ATLAS Detector. *Physics Letters B*, **790**, 108.

6. Loizides, C., Nagle, J. and Steinberg, P. (2014) Improved Version of the Phobos Glauber Monte Carlo. *SoftwareX*, **1-2**, 13-18. https://doi.org/10.1016/j.softx.2015.05.001

7. Alver, B., Baker, M., Loizides, C. and Steinberg, P. (2008) The PHOBOS Glauber Monte Carlo. https://arxiv.org/abs/0805.4411

8. Miller, M.L., Reygers, K., Sanders, S.J. and Steinberg, P. (2007) Glauber Modeling in High-Energy Nuclear Collisions. *Annual Review of Nuclear and Particle Science*, **57**, 205-243. https://arxiv.org/abs/nucl-ex/0701025v1 https://doi.org/10.1146/annurev.nucl.57.090506.123020

9. D’Enterria, D. (2010) 6.4 Jet Quenching. *Landolt-Bornstein*, **23**, 471-520. https://doi.org/10.1007/978-3-642-01539-7_16

10. Djordjevic, M. (2009) Theoretical Formalism of Radiative Jet Energy Loss in a Finite Size Dynamical QCD Medium. *Physical Review C*, **80**, Article ID: 064909.