Sense and Nonsense on Parton Distribution functions of the Photon

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Abstract

The organization of finite order QCD approximations to \( F_2 \) based on the separation of pure QED contribution from those of genuine QCD nature is discussed.

1 Introduction

Using the recently completed calculation of order \( \alpha_s^3 \) parton-parton and order \( \alpha_s^2 \) photon-parton splitting functions [1,2] the standard NLO and NNLO approximations for the photon structure function \( F_2 \) exhibit very small difference when the latter is evaluated in the \( \overline{\text{MS}} \) factorization scheme (FS). This stands in contrast to the large difference in this FS, noted in [3], between the LO and NLO approximations. In order to cure this, and related problems, the so called DIS\( \gamma \) FS has been proposed in [4].

In this talk I will argue that this large difference between the standard LO and NLO approximations to \( F_2 \) in the \( \overline{\text{MS}} \) FS disappears provided these approximations are defined with respect to genuine QCD contribution. I will furthermore argue that the way DIS\( \gamma \) FS is introduced violates the factorization scheme invariance of \( F_2 \) and is thus theoretically flawed. For lack of space only brief outline of the arguments can be given in this written version, for details see [2,6].

The source of the problem in the standard treatment of \( F_2 \) can be traced back to the interpretation of the behaviour of PDF of the photon in perturbative QCD. In [5] I have discussed this point at length and argued that PDF of the photon behave like \( \alpha \), rather than \( \alpha/\alpha_s \) as usually claimed. To see the point, let us recall basic facts concerning the theoretical analysis of \( F_2 \).

For simplicity we shall restrict ourselves to the nonsinglet channel, where

\[
\frac{F_{2,\text{NS}}(Q^2)}{x} = q_{\text{NS}}(M) \otimes C_q(Q/M) + \delta_{\text{NS}} C_\gamma
\]

with \( q_{\text{NS}} \) satisfying the evolution equation

\[
\frac{dq_{\text{NS}}(x, M)}{d \ln M^2} = \delta_{\text{NS}} k_q + P_{\text{NS}} \otimes q_{\text{NS}}
\]

and \( \delta_{\text{NS}} = 6n_f \left( \langle e^4 \rangle - \langle e^2 \rangle^2 \right) \). The splitting functions \( P_{\text{NS}} \) and \( k_q \) are given as power expansions in \( \alpha_s(M) \):

\[
k_q = \frac{\alpha}{2\pi} \left[ k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi} k_q^{(1)}(x) + \cdots \right],
\]

\[
P_{\text{NS}} = \frac{\alpha_s(M)}{2\pi} P_{\text{NS}}^{(0)}(x) + \cdots.
\]

The leading order splitting functions \( k_q^{(0)}(x) = x^2 + (1-x)^2 \) and \( P_{\text{NS}}^{(0)}(x) \) are unique, whereas all higher order ones depend on the choice of the factorization scheme (FS). The coefficient functions \( C_q, C_\gamma \) admit perturbative expansions of the form

\[
C_q(x, Q/M) = \delta(1-x) + \frac{\alpha_s(\mu)}{2\pi} C_q^{(1)} + \cdots,
\]

\[
C_\gamma(x, Q/M) = \frac{\alpha}{2\pi} \left[ C_\gamma^{(0)} + \frac{\alpha_s(\mu)}{2\pi} C_\gamma^{(1)} + \cdots \right],
\]

where coefficient function \( C_\gamma^{(0)} \)

\[
C_\gamma^{(0)} = k_q^{(0)}(x) \ln \frac{Q^2(1-x)}{M^2 x} + 8x(1-x) - 1
\]

similarly as \( k_q^{(0)} \) is of the pure QED origin. To simplify the formulae I will in the following drop the subscript “NS” and set \( \delta_{\text{NS}} = 1 \) everywhere.

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2 PDF of the photon

The general solution of the evolution equation (1) can be written as the sum of a particular solution of the full inhomogeneous equation and the general solution of the corresponding homogeneous one, called hadronic. The subset of the solutions of (1) resulting from the resummation of the contributions of diagrams in Fig. 1 and vanishing at some definite \( M_0 \), are called pointlike solutions. In the LLA, i.e. taking only the first terms \( k^{(0)}(0) \) and \( P^{(0)}(0) \) on the r.h.s. of (1), it has in momentum space the closed form

\[
q_{PL}(n,M_0,M) = \frac{4\pi a(n)}{\alpha_s(M)} \left[ 1 - \left( \frac{\alpha_s(M)}{\alpha_s(M_0)} \right)^{1-2P^{(0)}(n)/\beta_0} \right].
\]

(2)

As the principal difference between PDF of the photon and hadrons result from the contribution to the former of these pointlike solutions, only they will be discussed further and the specification “pointlike” will be dropped. It must, however, be kept in mind that the separation of PDF of the photon into hadronic and pointlike parts is not unique. In our simplified case it depends on the choice of \( M_0 \). The smaller \( M_0 \), the more important is the pointlike part relative to the hadronic one and vice versa.

The fact that \( \alpha_s(M) \) appears in the denominator of (2) is usually, and unfortunately, interpreted literally, i.e. as implying that the pure QED contribution \( C^{(0)}_{\gamma} \) is of “higher order” than quark distribution function \( q \). This, in turn, implies that the former is not included in the standard “LO” approximation to \( F_2^\gamma(x,Q^2) \)

\[
F_{2,\text{LO}}^\gamma(x,Q^2) = x q_{\text{NS}}(x,M)
\]

which also “behaves” as \( \alpha/\alpha_s \). This is obvious nonsense as if we switch off QCD we must get pure QED contribution. Indeed, keeping \( M \) and \( M_0 \) fixed and switching QCD off by sending \( \Lambda \to 0 \) the expression (2) approaches

\[
q(M, M_0) \to q_{\text{QED}} = \frac{\alpha}{2\pi} k^{(0)}(0) \ln \frac{M^2}{M_0^2},
\]

(4)

corresponding to the first, pure QED, diagram in Fig. 1. To see what is wrong with the claim that \( q \propto \alpha/\alpha_s \) consider the pure QED evolution equation

\[
\frac{dq_{\text{NS}}(x,M)}{d \ln M^2} = \frac{\alpha}{2\pi} k_\gamma^{(0)}(x)
\]

(5)

There is no trace of QCD, but we can formally rewrite it in terms of the derivative with respect to QCD couplant \( \alpha_s \) (taking for simplicity \( \beta \)-function to the lowest order)

\[
\frac{dq(n,Q)}{d \alpha_s} = -\frac{4\pi}{\beta_0} \frac{\alpha}{2\pi} k^{(0)}(n) \frac{\alpha_s(Q)}{\alpha_s^2}.
\]

(6)

This equation can be trivially solved to get

\[
q(n,Q) = \frac{\alpha}{2\pi} \frac{4\pi}{\beta_0} k^{(0)}(n) + A,
\]

(7)

where \( A \) stands for arbitrary integration constant specifying the boundary condition on the solution of (6), which can be chosen as \( A = 0 \). The above expression (7) might, but should not, mislead us to the usual claim that \( q \propto \alpha/\alpha_s \), because we know that we are still within pure QED! Indeed, if we evaluate the difference at two scales and insert the explicit expression for \( \alpha_s(Q) \) we get back the starting, purely QED expression

\[
q(n,Q_1) - q(n,Q_2) = \frac{\alpha}{2\pi} k^{(0)}(n) \ln \frac{Q_1^2}{Q_2^2}.
\]

(8)

There is another argument demonstrating that PDF of the photon behave as \( O(\alpha) \).
Consider the Mellin moments\(^a\) of \(F^\gamma(x) \equiv F_{NS}(x,Q)/x\)

\[
F^\gamma(Q) = q(M)C_q(Q/M) + C_\gamma(Q/M) \tag{9}
\]

where \(q(M)\) vanishes at \(M_0\): \(q(M_0) = 0\). As \(F^\gamma(Q)\) is independent of the factorization scale \(M\), we can take any \(M\) to evaluate \([9]\). For instance \(M_0\). However, for \(M = M_0\) the first term in \([9]\) vanishes and we get

\[
\frac{2\pi}{\alpha} F^\gamma(Q) = C^{(0)}_\gamma \left( \frac{Q}{M_0} \right) + \frac{\alpha_s(\mu)}{2\pi} C^{(1)}_\gamma \left( \frac{Q}{M_0} \right) \\
+ \left( \frac{\alpha_s(\mu)}{2\pi} \right)^2 C^{(2)}_\gamma \left( \frac{Q}{M_0}, \frac{Q}{\mu} \right) + \cdots \tag{10}
\]

i.e. manifestly the expansion in powers of \(\alpha_s(\mu)\), which starts with \(O(\alpha_s)\) pure QED contribution \((\alpha/2\pi)C^{(0)}_\gamma\) and includes standard QCD corrections of orders \(\alpha^k_s, k \geq 1\), which vanish when QCD is switched off, and no trace of the supposed “\(\alpha/\alpha_s\)” behaviour.

3 From semantics to substance

The claim that \(q \propto \alpha/\alpha_s\) stems in part from inappropriate terminology. Although a matter of convention, it is wise to define the terms “leading”, “next–to–leading” etc. in a way which guarantees that they have the same meaning in different processes. For the case of the ratio

\[
R_{e^+e^-}(Q) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{3 \sum_{i=1}^{n_f} e_i^2}{(1 + r(Q))} \tag{11}
\]

it is general practice to apply the terms “leading” and “next–to–leading” to genuine QCD effects as described by \(r(Q)\), i.e. subtracting from \([11]\) the pure QED contribution. Unfortunately, this practice is ignored in most analyses of \(F^\gamma\) \([3, 4]\).

In \([3]\) I have proposed the definition of QCD approximations to \(F^\gamma\) which follows closely QCD analysis of quantities like \([11]\). It starts with writing the quark distribution function \(q(M)\) as the sum of the purely QED contribution \([1]\) and the QCD corrections, which start at order \(\alpha_s\) and are generated by the terms in splitting and coefficient functions proportional to positive powers of \(\alpha_s\). This implies significant difference in the definition of fixed order approximations compared to those in the standard approach. Whereas the standard LO approximation to \(F^\gamma_2\) includes only the lowest order splitting functions \(k^{(0)}_q\) and \(F^{(0)}_{NS}\), we include four more terms. \(C^{(0)}_\gamma\), which is of pure QED origin and is closely related to \(k^{(0)}_q\), as well as three terms appearing at order \(\alpha_s\): \(k^{(1)}_q\), \(C^{(1)}_\gamma\) and \(C^{(1)}_q\). At the NLO the difference is even more pronounced (see table 1 of \([6]\)). The numerical importance of the individual contributions of these additional terms as well as the overall difference between the standard and our definition of the LO approximation to \(F^\gamma_2\) has been discussed by J. Hejbal \([7]\). The latter is phenomenologically relevant and comparable to errors of existing data.

4 What is wrong with the DIS\(_\gamma\) factorization scheme?

In \([6]\) I discussed how the freedom in the definition of quark distribution functions of the photon is related to the non-universality of the coefficient functions \(C^{(j)}_q, j \geq 1\). The QED contribution to \(F^\gamma\) is of order \(\alpha^0_s = 1\), but since in the standard approach the quark distribution function is assigned to the order \(1/\alpha_s\), the QED photonic coefficient function \(C^{(0)}_\gamma\) appears as “NLO” contribution and is therefore treated in a similar way as the lowest order QCD coefficient function \(C^{(1)}_q\). In \([3]\) the authors introduced the so called DIS\(_\gamma\) factorization scheme by absorbing in the redefined quark distribution function of

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the photon the pure QED term $C^{(0)}_\gamma$ according to (see eq. (5) of [4])

$$\gamma(M, M_0) \equiv q(M, M_0) + \frac{\alpha}{2\pi} C^{(0)}_\gamma(1). \quad (12)$$

and imposing “the same boundary conditions for the pointlike LO and HO distributions”

$$q^{\gamma}_\text{PL}(x, Q^2_0) = \gamma^{\gamma}_\text{PL}(x, Q^2_0) = G^{\gamma}_\text{PL}(x, Q^2_0) = 0 \quad (13)$$
as in the original FS [3]. The redefinition (12) is legitimate procedure but is in conflict with the assumption (13). To see why remember that (12) involves QED quantity $C^{(0)}_\gamma$, and as the notion of quark distribution function inside the photon is well-defined also in pure QED, the above procedure must make sense even in absence of QCD effects. Moreover, as the QCD contribution depends on the numerical value of $\alpha_s$ it cannot cure any problem of the pure QED part.

In pure QED the contribution to $F^{\gamma}$ coming from the box diagram regularized by explicit quark mass $m_q$ reads

$$F^{\gamma}_{\text{QED}}(Q) = \frac{\alpha}{2\pi} C^{(0)}_\gamma(Q/m_q) = \frac{\alpha}{2\pi} \left[ k^{(0)} \ln \frac{Q^2}{m_q^2} + C^{(0)}_\gamma(1) \right]. \quad (14)$$

Introducing an arbitrary scale $M$, we can split it into quark distribution function

$$q^{\text{QED}}(M) \equiv \frac{\alpha}{2\pi} k^{(0)} \ln \frac{M^2}{m_q^2} \quad (15)$$

and $C^{(0)}_\gamma(Q/M)$

$$F^{\gamma}_{\text{QED}}(Q) = q^{\text{QED}}(M) + \frac{\alpha}{2\pi} C^{(0)}_\gamma(Q/M). \quad (16)$$

We can redefine $q^{\text{QED}}(M)$ by adding to it an arbitrary function $f(n)$ (the DIS, FS of [3] corresponds to $f = C^{(0)}_\gamma(1)$) according to

$$q^{\gamma, f}(M) \equiv q^{\text{QED}}(M) + \frac{\alpha}{2\pi} f. \quad (17)$$

In order to keep the sum

$$F^{\gamma}_{\text{QED}}(Q) = q^{\gamma, f}(M) + \frac{\alpha}{2\pi} C^{(0)}_\gamma(Q/M)$$

independent of $f$ we must change $C^{(0)}_\gamma$ accordingly

$$C^{(0)}_\gamma(Q/M) \equiv C^{(0)}_\gamma(Q/M) - f. \quad (18)$$

We can write down the evolution equation

$$\frac{d q^{\gamma, f}(M)}{d \ln M^2} = \frac{\alpha}{2\pi} k^{(0)}, \quad (19)$$

which is the same in all $f$-schemes but we are not allowed to use the same boundary condition for all $q^{\gamma, f}(M)$. If we do that and impose the boundary condition $q^{\gamma, f}(M = m_q) = 0$ for all $f$ we get

$$F^{\gamma}(Q) = \frac{\alpha}{2\pi} \left( C^{(0)}_\gamma(Q/m_q) - f \right) \quad (20)$$

which depends on the choice of $f$ and only for $f = 0$ coincides with (14).

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