Abstract—Multi-view learning improves the learning performance by utilizing multi-view data: data collected from multiple sources, or feature sets extracted from the same data source. This approach is suitable for primate brain state decoding using cortical neural signals. This is because the complementary components of simultaneously recorded neural signals, local field potentials (LFPs) and action potentials (spikes), can be treated as two views. In this paper, we extended broad learning system (BLS), a recently proposed wide neural network architecture, from single-view learning to multi-view learning, and validated its performance in monkey oculomotor decision decoding from medial frontal LFPs and spikes. We demonstrated that medial frontal LFPs and spikes in non-human primate do contain complementary information about the oculomotor decision, and that the proposed multi-view BLS is a more effective approach to classify the oculomotor decision, than several classical and state-of-the-art single-view and multi-view learning approaches.

Index Terms—Broad learning system, local field potentials, action potentials, multi-view learning, primate oculomotor decision.

I. INTRODUCTION

MULTI-view learning attempts to improve the learning performance by utilizing multi-view data, which can be collected from multiple data sources, or different feature sets extracted from the same data source. For example, in an invasive brain-machine interface (BMI) using electrodes [1], effective BMI cursor control can be achieved using action potentials (spikes), which are high-pass filtered neural signals, or local field potentials (LFPs), which are low-pass filtered neural signals measured from the same electrodes. The spikes and LFPs can represent two views of the same task.

There have been a few studies on applying multi-view learning to human brain state decoding. Kandemir et al. [2] combined multi-task learning and multi-view learning in decoding a user’s affective state, by treating different types of physiological sensors (e.g., electroencephalography, electrocardiography, etc.) as different views. Pasupa and Szedmak [3] used tensor-based multi-view learning to predict where people are looking in images (saliency prediction), by treating the image and the user’s eye movement as two views. Spyrou et al. [4] used multi-view learning to integrate spatial, temporal, and frequency signatures of electroencephalography signals for interictal epileptic discharges classification. However, to our knowledge, no one has applied multi-view learning to non-human primate brain state decoding using invasive signals like LFPs and spikes (we will discuss this in detail in Section IV-E).

A broad learning system (BLS) [5] is a flexible neural network, which can incrementally adjust the number of nodes for the best performance. It has achieved comparable performance, with much less computational cost, to deep learning approaches in two applications [5]. The main difference between a BLS and a deep learning model is that BLS improves the learning performance by increasing the width, instead of the depth, of the neural network. This paper proposes a multi-view BLS (MvBLS), which extends BLS from traditional single-view learning to multi-view learning, and applies it to monkey oculomotor decision decoding from both LFP and spike features. By using features from different views in generating the enhancement nodes, the proposed MvBLS can significantly outperform some classical and state-of-the-art single-view and multi-view learning approaches.

The main contributions of this paper are:

1) We proposed three different MvBLS architectures, which have comparable performances but different computational cost.

2) We applied MvBLS to monkey oculomotor decision decoding using neural signals recorded in the medial frontal cortex, and demonstrated that it outperformed some classical and state-of-the-art single-view and multi-view learning approaches.

3) We verified through extensive experiments that combining LFP and spike features can improve the decoding performance in monkey oculomotor decision classification. This shows that, at least in this context, LFPs and spikes in the medial frontal cortex contain complementary information about oculomotor decisions.

The remainder of this paper is organized as follows: Section II introduces the BLS and our proposed MvBLS. Section III describes the neurophysiological dataset used in this work, and the experimental results. Section IV presents some additional discussions. Finally, Section V draws conclusions.
II. BLS AND MVBLS

This section introduces the single-view BLS, and our proposed MvBLS, for multi-class classification.

A. Broad Learning System (BLS)

Single-layer feed-forward neural networks are universal approximators [6], and have been used in numerous applications. Random vector functional neural networks (RVFLNNs) accelerate single-layer feed-forward neural networks by randomly generating the weight matrix [7]. BLS is a further improvement of the RVFLNN.

In an RVFLNN, the input and output layers are directly connected. In a BLS, the input layer first passes through a feature extractor for dimensionality reduction and noise suppression. Due to the use of sparse auto-encoders, the extracted features are more diverse. This helps improve the generalization performance.

Let \( \mathbf{X} \in \mathbb{R}^{N \times M} \) be the data matrix, where \( N \) is the number of observations, and \( M \) the feature dimensionality. Let \( \mathbf{Y} \in \mathbb{R}^{N \times C} \) be the one-hot coding matrix of the labels of \( \mathbf{X} \), where \( C \) is the number of classes. The architecture of a BLS is shown in Fig. 1. It first constructs feature nodes \( \mathbf{Z} \) from \( \mathbf{X} \), and then enhancement nodes \( \mathbf{H} \) from \( \mathbf{Z} \). Finally, BLS estimates \( \mathbf{Y} \) from both \( \mathbf{Z} \) and \( \mathbf{H} \).

The steps to build a BLS are:

1) Construct the linear feature nodes \( \mathbf{Z} \). Let \( n \) be the number of groups of feature nodes, and \( m \) be the number of features nodes in each group. We first concatenate \( \mathbf{X} \) with an all-one bias vector \( \mathbf{1} \in \mathbb{R}^{N \times 1} \) to form the augmented data matrix \( \mathbf{X}' = [\mathbf{X} \ 1] \), then construct each of the \( n \) groups of feature nodes, \( \{\mathbf{Z}_i\}_{i=1}^n \), individually. For the \( i \)-th group of feature nodes \( \mathbf{Z}_i \), we first randomly generate uniformly distributed feature weights \( \mathbf{W}_e \in \mathbb{R}^{(M+1) \times m} \) and compute the random feature nodes \( \mathbf{Z}_r = \mathbf{X}' \mathbf{W}_{e} \), then use least absolute shrinkage and selection operator (LASSO) to obtain sparse weights \( \mathbf{W}_{e_s} \in \mathbb{R}^{(M+1) \times m} \).

\[
\mathbf{W}_{e_s} = \arg\min_{\mathbf{W}} \left( \frac{1}{2} ||\mathbf{Z}_r - \mathbf{X}'\mathbf{W}_{e}||^2_F + \lambda_1 ||\mathbf{W}||_{1,1}^T \right),
\]

where \( ||\mathbf{W}||_{1,1} = \sum_{i=1}^m \sum_{j=1}^{M+1} |w_{ij}| \), and \( \lambda_1 \) is the L1 regularization coefficient. Alternating direction method of multipliers (ADMM) is applied to solve (1). Then, we construct \( \mathbf{Z}_i = \mathbf{X}' \mathbf{W}_{e_s} \), and \( \mathbf{Z} = [\mathbf{Z}_1, ..., \mathbf{Z}_n] \).

2) Construct the \( k \) nonlinear enhancement nodes \( \mathbf{H} \). Let \( \xi \) be the hyperbolic tangent sigmoid function, i.e.,

\[
\xi(x) = \frac{2}{1 + e^{-2x}} - 1
\]

then

\[
\mathbf{H}' = [\mathbf{Z}, \mathbf{1}] \mathbf{W}_h,
\]

\[
\mathbf{H} = \xi \left( \frac{s \mathbf{H}'}{\max(|\mathbf{H}'|)} \right),
\]

where \( \mathbf{W}_h \in \mathbb{R}^{(nm+1) \times k} \) is a matrix of the orthonormal bases of a randomly generated uniformly distributed weight matrix in \( \mathbb{R}^{(nm+1) \times k} \), \( s \) is a scalar normalization factor, and \( \max(|\mathbf{H}'|) \) is the maximum absolute value of all elements in \( \mathbf{H}' \). The goal of \( \frac{s \mathbf{H}'}{\max(|\mathbf{H}'|)} \) is to constrain the input to \( \xi \) to \([s, -s] \), i.e., it performs normalization.

3) Calculate \( \mathbf{W}_o \in \mathbb{R}^{(nm+k) \times C} \), the weights from \([\mathbf{Z}, \mathbf{H}] \) to \( \mathbf{Y} \). Ridge regression is used to compute \( \mathbf{W}_o \), i.e.,

\[
\mathbf{W}_o = \arg\min_{\mathbf{W}} \left( ||[\mathbf{Z}, \mathbf{H}]\mathbf{W} - \mathbf{Y}||^2_F + \lambda_2 ||\mathbf{W}||^2_F \right)
\]

\[
= (\lambda_2 \mathbf{I} + [\mathbf{Z}, \mathbf{H}]^T[\mathbf{Z}, \mathbf{H}])^{-1} [\mathbf{Z}, \mathbf{H}]^T \mathbf{Y},
\]

where \( \lambda_2 \) is the L2 regularization coefficient.

The pseudocode of BLS is given in Algorithm 1. Through \( n \) random feature weight matrices \( \mathbf{W}_r \) and L1 regularization, BLS extracts multiple sets of diverse linear de-noised features \( \mathbf{Z} \) (which help increase its generalization ability). Then, orthogonal mapping and sigmoid functions are used to construct the enhancement nodes \( \mathbf{H} \) to introduce more nonlinearity (which help increase its model fitting power). Finally, \( \mathbf{Z} \) and \( \mathbf{H} \) are concatenated as the features for predicting \( \mathbf{Y} \).

B. Multi-View Broad Learning System (MvBLS)

BLS has achieved comparable performance, with much less computational cost, with deep learning approaches on two single-view image datasets [5]. However, it is not optimized for multi-view data. This subsection extends single-view BLS to multi-view.

The architecture of the proposed MvBLS is shown in Fig. 2. Without loss of generality, we only consider two views. The extension to more than two views is straightforward.

The general idea is to construct the linear de-noised feature nodes of each view separately, concatenate the feature nodes from all views to construct the nonlinear enhancement nodes, and...
Algorithm 1 The BLS training algorithm [5].

Input: \( X \in \mathbb{R}^{N \times M} \), the training data matrix; 
\( Y \in \mathbb{R}^{N \times C} \), the corresponding one-hot coding label matrix of \( X \); 
\( n \), the number of feature node groups; 
\( m \), the number of feature nodes in each group; 
\( k \), the number of enhancement nodes; 
\( s \), the normalization factor; 
\( \lambda_1 \), the L1 regularization coefficient for determining \( W_{ei} \); 
\( \lambda_2 \), the L2 regularization coefficient for determining \( W_o \).

Output: BLS weight matrices \( W_{ei} \in \mathbb{R}^{(M+1) \times m} \) (\( i = 1, \ldots, n \)), \( W_h \in \mathbb{R}^{(nm+1) \times k} \), and \( W_o \in \mathbb{R}^{(nm+k) \times C} \).

Construct \( X' = [X, 1] \) for \( i = 1 \) to \( n \) do
- Initialize \( W_r \) randomly;
- Calculate \( Z_{ei} = X'W_{ei} \);
- Calculate \( W_{ei} \) using (1);
- Calculate feature nodes \( Z_i = X'W_{ei} \);
end for

Construct \( Z = [Z_1, \ldots, Z_n] \);
Construct an orthonormal basis matrix \( W_h \in \mathbb{R}^{(nm+1) \times k} \) from a randomly generated matrix in \( \mathbb{R}^{(nm+1) \times k} \);
Calculate the enhancement nodes \( H \) using (3) and (4);
Calculate \( W_o \) using (5).

finally fuse the feature nodes and enhancement nodes together for prediction. By separating the two views in the first layer of the MvBLS and optimizing \( Z^A \) and \( Z^B \) separately, we may obtain better features than optimizing \( Z = [Z^A, Z^B] \) directly (as in the case that we concatenate \( X^A \) and \( X^B \) and feed them altogether into a single BLS), because \( Z \) may be too long to be optimized effectively.

![Fig. 2. Architecture of the proposed MvBLS. Diverse linear de-noised features \( Z^A \) and \( Z^B \) are extracted from Views A and B, respectively; \( Z^A \) and \( Z^B \) are then mapped into nonlinear features \( H \). \( Z^A \), \( Z^B \) and \( H \) are next concatenated to predict \( Y \).](image)

Let the two views be \( A \) and \( B \), the corresponding data matrices be \( X^A \in \mathbb{R}^{N \times MA} \) and \( X^B \in \mathbb{R}^{N \times MB} \) (\( MA \) and \( MB \) are the feature dimensionality of Views \( A \) and \( B \), respectively), and the shared label matrix be \( Y \in \mathbb{R}^{N \times C} \). The procedure for constructing the MvBLS is:

1) Construct the feature nodes \( Z^A = [Z^A_1, \ldots, Z^A_n] \) for View \( A \), and \( Z^B = [Z^B_1, \ldots, Z^B_n] \) for View \( B \), using Step (1) of Algorithm 1.
2) Construct the enhancement nodes \( H \), using the concatenated feature nodes \( [Z^A, Z^B] \) from both views and Step (2) of Algorithm 1.
3) Calculate \( W_o \in \mathbb{R}^{(2nm+k) \times C} \), the weights from \( [Z^A, Z^B, H] \) to \( Y \). Again, ridge regression is used to compute \( W_o \). Let \( Z' = [Z^A, Z^B, H] \). Then,

\[
W_o = \arg \min_W \|Z'W - Y\|_F^2 + \lambda_2||W||_F^2 = (\lambda_2I + Z'^TZ')^{-1}Z'^TY, \tag{6}
\]

where \( \lambda_2 \) is the L2 regularization coefficient.

The pseudocode for MvBLS is shown in Algorithm 2.

Algorithm 2 The proposed MvBLS for two views.

Input: \( X^A \in \mathbb{R}^{N \times MA} \), the training data matrix for View \( A \); 
\( X^B \in \mathbb{R}^{N \times MB} \), the training data matrix for View \( B \); 
\( Y \in \mathbb{R}^{N \times C} \), the corresponding one-hot coding label matrix;
\( n \), the number of feature node groups;
\( m \), the number of feature nodes in each group;
\( k \), the number of enhancement nodes;
\( s \), the normalization factor;
\( \lambda_1 \), the L1 regularization coefficient for determining \( W_{ei} \);
\( \lambda_2 \), the L2 regularization coefficient for determining \( W_o \).

Output: MvBLS weight matrices \( W_{ei} \in \mathbb{R}^{(MA+1) \times m} \) (\( i = 1, \ldots, n \)), \( W_h \in \mathbb{R}^{(MB+1) \times k} \), and \( W_o \in \mathbb{R}^{(2nm+k) \times C} \).

Calculate \( W_{ei} \) and \( W_h \) using \( X^A, X^B, Y \), and Step (1) of BLS to construct the feature nodes \( Z^A \) and \( Z^B \);
Calculate \( W_h \) using \( Z^A, Z^B \), and Step (2) of BLS;
Calculate \( W_o \) using (6).

III. EXPERIMENT AND RESULTS

This section applies BLS and MvBLS to monkey oculomotor decision classification, and compares their performance with those using several classical and state-of-the-art single-view and multi-view learning approaches.

A. The Neurophysiology Experiment

The invasive neurophysiological experimental setup used here and animal behavior were reported in [6]. All animal care and experimental procedures were in compliance with the US Public Health Service policy on the humane care and use of laboratory animals, and were approved by Johns Hopkins University Animal Care and Use Committee.

Two male rhesus monkeys (Monkey A: 7.5 kg; Monkey I: 7.2 kg) were trained to perform an oculomotor gambling task, as shown in Fig. 3. In each trial, the monkeys chose between two gamble options by making an eye movement (saccade) towards one of the visual cues. Two visual cues were randomly presented in two of four fixed locations (top right, bottom right, top left, and bottom left). Each cue was comprised of two
colors (from a four-color library of cyan, red, blue, and green) and each color was associated with an amount of reward (1, 3, 5 to 9 units of water respectively, where 1 unit equaled 30 µL). The background color of a visual cue was cyan (small reward), and the foreground color was either red, or blue, or green (larger reward). The proportion of the two colors represented the probability of winning the corresponding reward. For the red/cyan target in Fig. 3 there was a 60% probability of having one unit of water (cyan color), and a 40% probability of having three units of water (red color). The expected reward value would then be $0.6 \times 1 + 0.4 \times 3 = 1.8$ units of water. There were a total of seven gamble options, representing three different expected reward values, as shown in Fig. 4.

In both types of trials, neural signals from the monkeys’ supplementary eye field in the medial frontal cortex were recorded with one or more tungsten electrodes, and the monkeys’ corresponding choices were recorded using an eye tracking system (Eye Link, SR Research Ltd, Ottawa, Canada).

The goal of our study was to investigate the task of decoding eye movements (choice intention) from neural signals in the primate medial frontal cortex, which are causally involved in risky decisions [10].

### B. Datasets

Forty-five datasets [9] were recorded from 45 different days of experiments from the two monkeys (33 from Monkey A, and 12 from Monkey I). Their statistics are shown in Table I, where $d_1$-$d_4$ denote the four different saccade directions (classes), $nE$ the number of electrodes in recording the LFPs, and $nC$ the number of cells in recording the spikes.

For each recording, electrodes were lowered into the monkeys’ supplementary eye field using electric microdrives. While the monkeys were preforming the task, activity was recorded extracellularly using 1 to 4 tungsten microelectrodes with an impedance of 2-4 MΩ (Frederick Haer, Bowdoinham, ME, USA) spaced 1-3 mm apart. Neural activity was measured against a local reference, a stainless steel guide tube, which carried the electrode array and was positioned above the dura.

At the preamplifier stage, signals were processed with 0.5 Hz 1-pole high-pass and 8k Hz 4-pole low-pass anti-aliasing Bessel filters, and then divided into two streams for the recording of LFPs and spiking activity. The stream used for LFP recording was amplified (500-2000 gain), processed by a 4-pole 200 Hz low-pass Bessel filter, and sampled at 1000 Hz. The stream used for spike detection was processed by a 4-pole Bessel high-pass filter (300 Hz), a 2-pole Bessel low-pass filter (6000 Hz), and was sampled at 40k Hz. Up to four template spikes were identified using principal component analysis. The spiking activity was subsequently analyzed offline to ensure only single units were included in consequent analyses. Finally, spikes were counted within each millisecond bin. Thus, its sampling rate was reduced to 1000 Hz.

### C. LFP and Spike Feature Extraction

In this study, single-unit spikes were smoothed through 100-point moving average. The LFPs and processed spikes were then epoched to [0, 400) ms after target onset for each electrode/cell. The eye movement reaction times (the time between target onset and eye movement onset) for the monkeys ranged from 100 ms to 300 ms. Therefore, the monkeys finished their eye movement before the end of each trial. The spike view had $408 \cdot nC$ features, where $nC$ is the number of cells for spikes in Table I. For each LFP trial from each electrode, power spectrum density of the 400 samples were computed by the Welch’s method with a Hamming window of 88 ms and 50% overlap. Then, log-power in eight frequency bands (theta, 4-8 Hz; alpha, 8-12 Hz; beta 1, 12-24 Hz; beta 2, 24-34 Hz; gamma 1, 34-55 Hz; gamma 2, 65-95 Hz; gamma 3, 130-170 Hz; gamma 4, 170-200 Hz), as used in [11], were calculated and concatenated with the 400 time domain samples as the features. Therefore, the LFP view had $408 \cdot nE$ features, where $nE$ is the number of electrodes for LFPs in Table I. Finally, both spike and LFP features were $z$-normalized.
TABLE I
STATISTICS OF THE 45 DATASETS.

| Dataset | Number of trials | nE | nC |
|---------|------------------|----|----|
| 1       | 218 226 199 182 191 | 43.35 | 4  |
| 2       | 255 259 195 182 225 | 37.64 | 3  |
| 3       | 275 310 256 195 259 | 48.17 | 3  |
| 4       | 198 187 197 141 181 | 26.96 | 3  |
| 5       | 203 210 146 153 178 | 33.16 | 3  |
| 6       | 218 231 213 190 213 | 17.11 | 3  |
| 7       | 159 184 161 142 162 | 17.25 | 3  |
| 8       | 170 193 188 164 179 | 13.94 | 3  |
| 9       | 194 183 197 177 188 | 9.36  | 3  |
| 10      | 222 249 220 195 222 | 22.07 | 3  |
| 11      | 224 235 242 212 228 | 13.12 | 3  |
| 12      | 121 114 140 129 126 | 11.17 | 2  |
| 13      | 193 177 178 189 184 | 7.97  | 3  |
| 14      | 251 220 201 192 216 | 26.09 | 2  |
| 15      | 227 211 184 176 200 | 23.67 | 2  |
| 16      | 207 188 183 156 184 | 21.05 | 2  |
| 17      | 225 203 131 173 183 | 40.69 | 2  |
| 18      | 228 208 194 188 205 | 17.77 | 2  |
| 19      | 185 166 149 148 162 | 17.42 | 3  |
| 20      | 169 147 145 164 156 | 12.04 | 1  |
| 21      | 170 151 117 134 144 | 22.38 | 1  |
| 22      | 163 144 102 126 134 | 26.00 | 1  |
| 23      | 193 192 171 182 185 | 10.28 | 1  |
| 24      | 196 183 164 172 179 | 13.89 | 1  |
| 25      | 148 138 104 138 132 | 19.25 | 2  |
| 26      | 209 166 129 182 172 | 33.43 | 2  |
| 27      | 218 168 133 210 230 | 39.48 | 2  |
| 28      | 183 161 118 164 157 | 27.45 | 1  |
| 29      | 198 173 111 170 163 | 36.87 | 3  |
| 30      | 188 181 144 143 164 | 23.85 | 3  |
| 31      | 216 189 174 212 198 | 19.81 | 3  |
| 32      | 213 206 137 213 192 | 36.98 | 3  |
| 33      | 198 174 116 179 167 | 35.38 | 3  |
| 34      | 212 188 115 178 173 | 41.37 | 3  |
| 35      | 304 274 168 274 255 | 59.70 | 3  |
| 36      | 274 222 134 245 219 | 60.37 | 3  |
| 37      | 263 202 114 204 196 | 61.41 | 3  |
| 38      | 244 202 160 202 202 | 34.29 | 3  |
| 39      | 249 240 134 223 212 | 52.78 | 3  |
| 40      | 273 253 182 220 232 | 39.86 | 3  |
| 41      | 133 136 134 123 132 | 5.80  | 2  |
| 42      | 138 154 141 117 138 | 15.33 | 3  |
| 43      | 283 248 232 246 252 | 21.70 | 3  |
| 44      | 194 197 162 168 172 | 29.41 | 3  |
| 45      | 146 187 145 124 151 | 26.36 | 3  |
| Avg     | 208 196 160 171 185 | 27.85 | —  |

D. Algorithms

We compared the performance of different decoding algorithms, including both single-view learning and multi-view learning approaches:

1) **Support vector machine (SVM)**, which uses error-correcting output codes (ECOC) [12] for multi-class classification. SVM is a classical statistical machine learning approach, and has achieved outstanding performance in numerous applications. The box constraint $C$ was chosen from $\{10^{-6}, 10^{-4}, \ldots, 10^0\}$ by nested cross-validation. Each binary SVM classifier was solved by sequential minimal optimization (SMO) [13], and the optimization stopped if the gradient difference between upper and lower violators obtained by SMO was smaller than 0.001.

2) **Ridge classification (Ridge)**, which performs ridge regression to approximate the $\{0, 1\}$ output of each class, and then classifies the input to the class with the largest output. The L2 regularization coefficient $\lambda_2$ was chosen from $\{10^{-6}, 10^{-4}, \ldots, 10^0\}$ by nested cross-validation.

3) **BLS**, which has been introduced in Section II-A. We used normalization factor $s = 0.8$ and L1 regularization coefficient $\lambda_1 = 0.001$, and selected the number of feature node groups $n$ from $\{10, 20\}$, the number of feature nodes in each group $m$ from $\{10, 20\}$, the number of enhancement nodes $k$ from $\{100, 500\}$, and the L2 regularization coefficient $\lambda_2$ from $\{10^{-6}, 10^{-4}, \ldots, 10^0\}$, using nested cross-validation. The alternating direction method of multipliers [8] used to solve (1) for feature nodes construction was iterative, and it terminated after 50 iterations.

4) **Multi-view discriminant analysis with view-consistency (MvDA)** [14], which extends classical single-view linear discriminant analysis to multi-view learning, and adds a regularization term to enhance the view-consistency.

5) **Multi-view modular discriminant analysis (MvMDA)** [15], which exploits the distance between class centers across different views.

6) **MvBLS**, which has been introduced in Section II-B. Its parameter tuning was the same as that for BLS.

Note that the first three algorithms can be used for both single-view learning and multi-view learning. When they were used in multi-view learning, we simply combined the features from different views as a single view input to the classifier. The last three approaches were used in multi-view learning only. The subspace dimensionality of MvDA and MvMDA was set to three (the number of classes minus one). After subspace alignment, the subspace features of all views were concatenated and fed into an ECOC-SVM classifier. Linear kernel was employed in all SVMs.

We randomly partitioned each dataset into three subsets: 60% for training, 20% for validation, and the remaining 20% for test. We repeated this process 30 times on each of the 45 datasets, and recorded the test classification accuracies as our performance measure.

E. Classification Using only the LFPs

In the first experiment, we used only the LFPs in classification. The classification accuracies in the 45 sessions, each averaged over 30 cross-validation runs, are shown in the bar graph in the top panel of Fig. 5 and also in the box plot in the top-left panel of Fig. 6. The last group of the bar plot also shows the average accuracies across the 45 sessions, whose numerical values are given in Table III. Each standard deviation showed in Table III was computed from 30 average accuracies of the 45 sessions. On average BLS slightly outperformed SVM and Ridge, which was also true in 29 and 32 out of the 45 individual sessions, respectively.

To find out whether there were statistically significant differences between different algorithms, non-parametric multiple pairwise comparison tests using Dunn’s procedure [16], with a $p$-value correction using the False Discovery Rate method [17], were performed on the cross-validation accuracies. The null hypothesis in each pairwise comparison was the probability of
observing a randomly selected value from the first group that is larger than a randomly selected value from the second group equals $0.5$, and it was rejected if $p \geq \alpha/2$, where $\alpha = 0.05$. The $p$-values, when only the LFP features were used, are shown in the first part of Table III. There was no statistically significant difference between any pair of algorithms.

In summary, we have shown that when only the LFP features were used, SVM, Ridge and BLS achieved comparable classification performances (BLS may be slightly better, but there was no statistically significant difference).

\textit{F. Classification Using only the Spikes}

In the second experiment, we used only the spikes in classification. The classification accuracies in the 45 sessions, each averaged over 30 cross-validation runs, are shown in the bar graph in the middle panel of Fig. 5 and also in the box plot in the top-right panel of Fig. 6. The last group of the bar graph shows the average accuracies across the 45 sessions, whose numerical values are also given in Table II. For all
algorithms, using spikes only achieved better classification accuracy than using LFPs only. On average Ridge slightly outperformed BLS, which was also true in 32 individual sessions. Interestingly, the opposite held when the LFP features were used. This may indicate that LFPs and spikes encode non-identical information about the oculomotor decision.

Non-parametric multiple comparison tests were also performed, and the \( p \)-values are shown in the second part of Table III. There was no statistically significant difference between any pair of algorithms.

In summary, we have shown that when only the spike features were used, SVM, Ridge and BLS again achieved comparable classification performance (Ridge may be slightly better, but there was no statistically significant difference).

G. Classification Using both LFPs and Spikes

In the third experiment, we used both LFPs and spikes in classification. The classification accuracies in the 45 sessions, each averaged over 30 cross-validation runs, are shown in the bar graph in the bottom panel of Fig. 5 and also the box plot in the bottom panel of Fig. 6. The last group of the bar graph shows the average accuracies across the 45 sessions, whose numerical values are also given in Table II. Observe that:

1) On average the two subspace multi-view learning algorithms, i.e., MvDA and MvMDA, performed much worse than the three single-view algorithms, i.e., SVM, Ridge, and BLS.
2) On average our proposed MvBLS outperformed the two subspace multi-view learning algorithms. This suggests that MvBLS can extract more discriminative features and better fuse them than the other two approaches.
3) On average our proposed MvBLS also outperformed the three single-view learning algorithms. This suggests that fusing the two views in a more sophisticated way may be more advantageous than simply concatenating and feeding them into a single-view classifier.

Non-parametric multiple comparison tests were also performed, and the \( p \)-values are shown in Table IV, where the statistically significant ones are marked in bold. There was statistically significant difference between MvBLS and each of the other five algorithms.

We also compared MvBLS with random guess. The results are shown in Fig. 7. Note that the random guess approach obtained slightly different accuracies in different sessions (not always exactly 25% in 4-class classification), because different classes had different numbers of trials. On average the classification accuracy of MvBLS was about twice of that of random guess (47.94% vs 25.04%), suggesting that a sophisticated machine learning approach like MvBLS can indeed mine useful information from LFPs and spikes.

To validate if LFPs and spikes do contain complementary information, we counted the number of sessions that LFPs+spikes achieved better performance than LFPs or spikes only, and show the results in Table V. Regardless of which classifier was used, LFPs+spikes always outperformed LFPs or spikes alone, in most sessions. Moreover, when MvBLS was used, LFPs+spikes outperformed the best LFPs performance (among SVM, Ridge and BLS) in 40 sessions (88.89%), and the best spikes performance in 32 sessions (71.11%).

In summary, we have shown that LFPs and spikes contain complementary information about the brain’s oculomotor decision, and our proposed MvBLS can better fuse these features than several classical and state-of-the-art single-view and multi-view learning approaches.

IV. DISCUSSIONS

This section presents some additional discussions on the proposed MvBLS.

A. Classification Accuracy versus the Number of Electrodes/Cells

Figs. 5 and 7 show that sometimes the classification accuracy was very low, e.g., close to 25% (random guess). The
main reason is that the number of electrodes/cells was small in these cases. For example, the top panel of Fig. 5 also shows the number of LFP electrodes in different sessions. It has a strong correlation with the classification accuracy, regardless of which classification algorithm was used. Particularly, the sessions with the lowest classification accuracy (Sessions 20–24) had the smallest number of electrodes. The middle panel of Fig. 5 shows the number of cells in different sessions. Similar patterns can be observed.

Next, we performed a deeper investigation on how the number of electrodes in LFPs and the number of cells in spikes affected the performance of BLS.

The LFPs were studied first. We identified all datasets with three or more electrodes, and considered each one separately. Let’s use a dataset with three electrodes to illustrate our experimental procedure. We randomly partitioned the dataset into 60% training, 20% validation, and 20% test. Then, we increased the number of chosen electrodes $k$ from one to two and then to three; for each $k$, we used LFPs from the corresponding electrodes to trained a BLS and compute its test accuracy. All possible combinations of $k$ electrodes were considered, and the average test accuracy was computed. We then repeated the data partition 30 times and computed the grand average test accuracies, as shown in Fig. 8(a). Clearly, the classification accuracy increased with the number of electrodes. We can imagine that much higher classification accuracy could be obtained if many more electrodes were used.

The spikes were studied next, and the results are shown in Fig. 8(b). The experimental procedure was very similar to that for the LFPs, except one subtle difference: the number of cells associated with different electrodes were generally different, so the total number of cells from $k$ electrodes could have different values. As a result, unlike in Fig. 8(a), where each dataset has only one curve, in Fig. 8(b) each dataset may have multiple branches leading to the same end-point. To make the curves more distinguishable, we only show the results for 10 datasets in Fig. 8(b). Generally, the classification accuracy increased with the number of cells, which is intuitive.

We studied LFPs and spikes separately. Each time there was only one view, and hence MvBLS degraded to BLS.

**B. MvBLS Parameter Sensitivity**

MvBLS has three structural parameters and three normalization/regularization parameters ($n$, $m$, $k$, $s$, $\lambda_1$ and $\lambda_2$ in Algorithm 2). It is important to know the sensitivity of MvBLS to them, which will provide valuable guidelines in selecting these parameters in future applications.

By default $n = 15$, $m = 15$, $k = 300$, $s = 0.8$, $\lambda_1 = 0.001$ and $\lambda_2 = 1$. When studying the sensitivity of MvBLS to $n$, we fixed $m$, $k$, $s$, $\lambda_1$ and $\lambda_2$ at their default values, and varied $n$ from 10 to 100. For each $n$ on each dataset, we trained 30 MvBLSs on 30 different partitions of the dataset (80% for training and 20% for test), and recorded the average test
accuracy across the 30 runs. Finally we took the average of the 45 datasets, and show the results in the top-left panel of Fig. 9. Similarly, we varied the 45 datasets, and show the results in the top-left panel of accuracy across the 30 runs. Finally we took the average of 0.5 numbers and shown in Table VI. The platform was a platform was a

**Fig. 9. MvBLS classification accuracy versus its parameters.**

**C. Computational Cost**

It is also interesting to compare the computational cost of different algorithms, as in practice a faster algorithm is preferred over a slower one, given similar classification accuracies.

We recorded the mean running time (including training, validation and test time) of 45 sessions for different classifiers, when different features were used. Since this process was repeated 30 times, we obtained 30 mean running time for each algorithm-feature combination. The mean and standard deviation for each combination were computed from these 30 numbers and shown in Table VI. The platform was a Linux workstation with Intel Xeon CPU (E5-2699@2.20GHz) and 500-GB RAM. SVM was the most efficient single-view learning algorithm, and MvBLS the most efficient multi-view learning algorithm. Particularly, MvBLS was several times faster than the other two state-of-the-art multi-view learning approaches, and it also achieved the best performance. In summary, our proposed MvBLS is both effective and efficient.

It is also important to point out that the model training is time-consuming, because a large number of parameters need to be optimized; however, once the optimal model parameters are found, all models can be run very fast in testing, which involves mostly matrix operations.

**D. Additional MvBLS Approaches**

In addition to the MvBLS model in Fig. 2, other MvBLS architectures can also be configured, by constructing the inputs to Y differently. Two additional configurations are shown in Fig. 10 and denoted as MvBLS2 and MvBLS3, respectively. Compared with MvBLS in Fig. 2, MvBLS2 in Fig. 10(a) first constructs enhancement nodes $H^A$ from $Z^A$ and $H^B$ from $Z^B$, and then feeds all of them into Y; so, it has more nodes and weights than MvBLS. Compared with MvBLS2, MvBLS3 in Fig. 10(b) further constructs enhancement nodes $H$ from $Z^A$ and $Z^B$, and then feeds $H$, $H^A$, $H^B$, $Z^A$ and $Z^B$ into Y. So, MvBLS3 has even more nodes and weights than MvBLS2.

**Fig. 10. Two additional configurations of MvBLS. (a) MvBLS2; (b) MvBLS3.**

The performances of MvBLS, MvBLS2 and MvBLS3 in the 45 sessions are shown in Fig. 11. Their classification accuracies were almost identical, which is interesting, considering that MvBLS2 and MvBLS3 have more parameters and connections. Indeed, non-parametric multiple comparisons showed that there was no statistically significant difference between any two of them. Since MvBLS has much simpler...
configuration and is easier to train, it is preferred in our application.

E. Related Work

Both LFPs and spikes contain information about the monkeys’ oculomotor decision, and there has been independent research on both. Spikes are high-pass filtered neural signals, which can be decoded into high-performance movement control signals [18], [19]. However, since spikes often deteriorate as electrodes degrade over time, more stable LFPs, which are low-pass filtered neural signals, are used in long term BMIs [20]–[25].

Because LFPs and spikes can be recorded from the same electrodes [26], and they convey complementary information [27]–[30], a natural approach is to combine them for more accurate decoding [1], [11], [31]–[34].

Bokil et al. [31] trained two macaque monkeys to perform a memory-saccade task and collected LFPs and spikes from the lateral intraparietal area. Two-dimensional Fourier transforms were performed to extract the features. Saccade prediction was achieved by maximizing the log-likelihood function of the observed neural activity. This approach was novel in that it did not use trial start time or other trial-related timing information. However, the performance degraded when switching from the preferred-or-anti-preferred binary classification to four-direction and eight-direction classifications.

Bansal et al. [34] trained two male macaque monkeys to perform reach-and-grasp tasks in three dimensions, and collected 192-channel LFPs and spikes from primary and ventral premotor areas. Linear Gaussian state-space representation and Kalman filter were then used to decode the reach-and-grasp kinematics. The decoding was first conducted for each channel, then about 30 channels were iteratively chosen based on the decoding performance (the Pearson correlation coefficient between the measured and the reconstructed kinematics). This approach required a large number of channels to be chosen from, which may not available in many human and non-human primate studies, including ours.

Hsieh et al. [11] trained one adult rhesus macaque to perform a center-out-and-back task and collected 137-channel LFPs and spikes from dorsal premotor cortex and ventral premotor cortex of both hemispheres. They then developed a multi-scale encoding model, a multi-scale adaptive learning algorithm, and a multi-scale filter for decoding the millisecond time-scale of spikes and slower LFPs. This approach solved a trajectory regression problem, whereas we focused on oculomotor decision classification.

Most studies, except [34] and [11] introduced above, however, have not shown significant improvements in decoding performance, compared with using LFPs or spikes alone. Our research has shown that sophisticated machine learning approaches like MvBLS can better use LFPs and spikes, and hence achieve significant decoding performance improvements.

V. Conclusion

Multi-view learning is very suitable for primate brain state decoding using medial frontal neural signals. This is because these simultaneously recorded neural signals comprise both low-frequency LFPs and high-frequency spikes, which can be treated as two views of the brain state. In this paper, we have extended single-view BLS to MvBLS, and validated its performance in monkey oculomotor decision decoding from medial frontal LFPs and spikes. We demonstrated that primate medial frontal LFPs and spikes do contain complementary information about the oculomotor decision, and that the proposed MvBLS is a more effective approach to use these two types of information in decoding the decision, than several classical and state-of-the-art single-view and multi-view learning approaches. Moreover, we showed that MvBLS is fast, and robust to its parameters. Therefore, we expect that MvBLS will find broader applications in other primate brain state decoding tasks, and beyond.

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|          | SVM   | Ridge | BLS  | MvDA | MvMDA | MvBLS |
|----------|-------|-------|------|------|-------|-------|
| LFPs     | 7.72±0.30 | 4.40±0.53 | 37.52±2.24 | −     | −     | −     |
| Spikes   | 19.05±2.92 | 585.54±52.87 | 85.77±5.42 | −     | −     | −     |
| LFPs + Spikes | 24.84±3.19 | 1033.14±89.47 | 102.40±6.83 | 748.06±66.55 | 741.87±68.08 | 129.66±7.93 |

Table VI

Mean and standard deviation of running time (seconds) when different classifiers and features were used.
Fig. 11. Classification accuracies of the three different MvBLSs.

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