The analysis on PS wave reflection coefficient in equivalent porous fractured media

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Abstract. Combining the equivalent fractured media with Thomsen fractured media theory, the anisotropic parameters expressed in compliance are obtained. The new PS wave reflection coefficient formula presented in matrix porosity, fluid type, and fracture density were established. Numerical simulations show that the PS wave reflection coefficient in water-saturated fracture media is more sensitive to the fracture density than that in gas-saturated media; the difference between the reflection coefficients in the gas-saturated and water-saturated state greatly increases when the matrix porosity of fractured media exceeds 0.01. It shows that fracture density and matrix porosity can be used as good fluid indicators in fractured reservoir.

1. Introduction
In the rock physics analysis, the equivalent fractured media represented by HTI media has been widely studied by many scholars. The Thomsen equivalent porous model in low frequency band not only meets the seismic frequency band studied, but also conforms to the actual underground media situation because of its dual porosity characteristic, so it is worthy of further study.

The reflection coefficient is an important theory for quantitatively describing the reflection characteristics in fractured reservoirs. Rüger's research has established foundation for the study of P wave reflectivity [1]. The converted wave (PS wave) reflection coefficient in HTI media is another important research field. It started from reflection/transmission at the interface between anisotropic media discussed by Musgrave, Henneke, Keith, and Daley, but its analytical formula is complicated[2-5]. Kim et al. (1993) [6] simplified the reflection coefficient by making use of an empirical formula. Many research in this field reveals the reflection characteristics of converted wave in HTI media and lay a theoretical foundation for the quantitative interpretation of the converted wave. But, so far, the existing PS wave reflection coefficient formulas are all expressed in the form of anisotropy parameters, and not directly achieved the explicit expression with fracture parameters, lithology parameters, and filled fluid parameters, which is prone to errors in the conversion process from anisotropic parameters to the parameters of fracture, lithology and filled fluid. In addition, the anisotropic analysis based on these anisotropy parameters cannot combine matrix pores with fractured networks, so that it is only qualitative analysis and is difficult to accomplish quantitative analysis[7].

Based on the above problems, in this paper, introducing the relationship between anisotropic parameters and compliance by mean of the Thomsen model, the PS-wave reflection coefficient formula is reestablished with compliance parameters by integrating the PS wave reflection coefficient
in weakly anisotropic media derived by Cherepanov and Nefedkina (2004)[8]. Moreover, because the compliance parameters in the formula are the functions of matrix porosity, crack density, filled fluid and other parameters, they have been used as intermediate bridges to build a new reflection coefficient, which is the direct explicit relationship with the parameters such as crack density and matrix porosity for studying the effect of these parameters on the seismic shear wave reflection.

2. PS wave reflection coefficient in equivalent porous fractured media

2.1. Equivalent porosity media theory

The equivalent porous fracture model is in the middle of the linear sliding model and the Hudson fracture model, where the pores and fractures are connected. In the low frequency range of the earthquakes mostly studied, when the matrix porosity is low, this model becomes a low equivalent porous fracture model whose compliance expressions[9] are

\[ \Delta N = \frac{4\epsilon}{3g(1-g)} \left(1 - \frac{k_f}{k_0}\right) D_p \]

where,

\[ k_0 = \frac{1}{2} \frac{k_f + k_s}{(\phi_f + \phi_s)} \times (A \phi_f + A_e) \]

\[ k_s = \frac{\lambda + 2}{3} \mu = \rho c^2 (1 - \frac{2}{3} g) \]

\[ A_e = \frac{3}{4} g, \quad A_f = \frac{3}{9} g (1 - \frac{2}{3} g) \]

\[ \Delta T = \frac{16\epsilon}{3(3-2g)} \]

\[ e \text{ is crack density}; \quad \alpha, \beta \text{ are P wave and S wave velocity of media}; \quad \rho \text{ is density of media}; \quad k_f, k_s \text{ is bulk modulus of filled fluid}; \]

\( \phi_f \) is matrix porosity (the volume percentage of the pores); \( \phi_s \) is fracture porosity, that is, the volume percentage of the crack which is represented by crack density \( e \) and crack aspect ratio \( (c/a) \) as \( \phi_s = \frac{4\pi}{\pi} \xi e \); \( \Delta N \) is normal compliance, which indicates the effect of the cracks on the seismic wave in the plane perpendicular to the cracks; \( \Delta T \) is tangential compliance, which indicates the effect of the cracks on the seismic wave in the plane parallel to the crack. These compliances are both in the range of \( 0~1 \). They are not sensitive to fracture aspect ratio (characteristic of the fracture shape) \( c/a \), because \( c/a \) does not affect the elastic parameters of fracture media model[10,11].

Using the sandstone in AVO III model (Table 1) proposed by Hilterman (2001)[12] as the background parameter of the equivalent porous fractured media and setting fracture aspect ratio to 0.0005. The \( \Delta N \) and \( \Delta T \) crossplot were performed in the change of fracture density \( e = 0.01 \) and matrix porosity \( \phi_s = 0.1 \). The results are shown in Figure 1, where, the change from blue to red represents the increase of the fracture density or matrix porosity, the symbols ★ and ♦ represent the gas-saturated and water-saturated states, respectively.

| Tabel 1. AVO III model parameters |
|-----------------------------------|
| Lithology | \( \alpha(\text{km/s}) \) | \( \beta(\text{km/s}) \) | \( \rho(\text{g/cm}^3) \) |
| shale | 2.190 | 0.820 | 2.16 |
| sandstone | 1.545 | 0.900 | 1.88 |

It can be seen that the fluid type in equivalent porous fractured media is sensitive to fracture density and matrix porosity changes. When fracture density is 0.02, the \( \Delta N \) and \( \Delta T \) crossplot can distinguish fluid type in fractured media, and its ability to distinguish the fluid type is enhanced as the fracture density increases (Figure 1(a)). When matrix porosity is 0 to 0.1, the \( \Delta N \) and \( \Delta T \) crossplot can effectively distinguish the gas and water in fracture media (Figure 1(b)), and its discrimination ability increases with the increase of matrix porosity. This shows that the PS-wave reflection coefficient re-expressed after using the compliance as the intermediate parameter to introduce the parameters such as fracture density and matrix porosity, may have a certain difference in the ability to
identify the fluid type with the changes of these parameters. So, this paper re-establishes the PS wave reflection coefficient formula to analyze directly the influence characteristics of fracture density, matrix porosity and filled fluid on it, in order to explore their application in quantitative seismic interpretation.

![Fig.1 ΔN and ΔT crossplot with the changes of parameter](image)

(a) fracture density change  (b) matrix porosity change

2.2. Derivation of PS wave reflection coefficient

The PS wave reflection coefficient formula at the interface of isotropic/HTI media model were derived by Cherpanov and Nefedkina (2004)[8] based on the perturbation theory. The elastic matrix (2004) of the lower HTI media used in this formula and the generalized elastic matrix in HTI media (Musgrave, 1970)[2] are correspondingly processed and the equivalent anisotropy parameters of HTI media defined by Rüger et al. (1997)[13] are introduced into the processing result so that the anisotropy parameters , , , and in this reflection coefficient formula are expressed as compliance parameters (Bakulin et al., 2000)[10]. These anisotropic parameters are as follows:

\[ n = 4\mu (1 - g)\Delta N \quad m = \mu \Delta T \quad \Delta l = 2\mu (1 - 2g)\Delta N \]

The PS wave reflection coefficient derived by Cherpanov incorporated the expressions (1) and (2) and were sorted out to obtain the new PS wave reflection coefficient expressed by crack parameter, matrix porosity and fluid factor:

\[ R_{PS} = R_{PS}^{iso} + R_{PS}^{ani} \]

\[ R_{PS}^{iso} = \left( k + 2\frac{k_f}{k_u} \right) D_p \frac{4}{3g(1 - g)} \rho \cos^2 \phi \sin^3 \theta + \left( k + \frac{3}{3 - 2g} \right) \rho \cos^2 \phi + \sin^3 \theta \]

\[ R_{PS}^{ani} = \left( k + 2\frac{k_f}{k_u} \right) D_p \frac{4}{3g(1 - g)} \rho \cos^2 \phi \sin^3 \theta + \left( k + \frac{3}{3 - 2g} \right) \rho \cos^2 \phi + \sin^3 \theta \]

Where, \(\Delta \rho = \rho_2 - \rho_1, \rho = (\rho_2 + \rho_1)/2, \Delta \alpha = \alpha_2 - \alpha_1, \alpha = (\alpha_2 + \alpha_1)/2, \Delta \beta = \beta_2 - \beta_1, \beta = (\beta_2 + \beta_1)/2\), \(k = \beta/\alpha; \rho_1, \rho_2, \alpha_1, \alpha_2, \beta_1, \beta_2\) are the density, P wave velocity and S wave velocity of the upper and lower media, respectively; \(\theta\) is incidence angle; \(\phi\) is azimuth angle, that is, the angle between the anisotropic plane and the survey line section; \(G\), \(D_p\), \(k_f\), \(k_u\) have been described above.
This formula establishes a direct function relationship among the fracture parameter, lithological parameter, physical parameter and the PS-wave reflection coefficient, which helps to directly analyze the influence of these parameters on the reflection coefficient.

2.3. Normalized reflection coefficient

Based on the model parameters in Table 1, the lower media of the two-layer model is modified to be water-saturated and gas-saturated fractured media, whose fracture density is $\epsilon = 0.05, 0.1, 0.15$, matrix porosity is $\phi_m = 0.03$, and fracture aspect ratio is $c/a = 0.0005$. The PS wave reflection coefficients at the interface of the two-layer model are calculated at different azimuth angles, when incident angle is $0.001^\circ$ to $30^\circ$. The results are shown in Figure 2, where the blue, green and red solid lines represent respectively the crack density $\epsilon = 0.05, 0.10, 0.15$ in gas-saturated state, and the blue, green and red dashed lines have same crack density in water saturated state.

From the PS wave reflection coefficients at azimuth angle $\varphi = 0^\circ$, $30^\circ$, and $60^\circ$ in Figure 2, it can be shown that, in the same fluid-saturated state, the reflection coefficient increases rapidly with the increase of the incident angle $\theta$ when $\theta$ is within $25^\circ$, and its increasing speed slows down with the increase of $\theta$ when $\theta$ exceeds $25^\circ$, in addition, the reflection coefficient increases with the increase of crack density; in different fluid saturated states, the difference of the reflection coefficient (the solid and dashed lines with the same color) decreases with the decrease of incident angle. In order to overcome the problem that the reflection coefficient difference is too small at small incident angle, the reflection coefficient is divided by $\sin\theta$ to achieve normalization (Figure 3). It illustrates that the normalized PS-wave reflection coefficient can distinguish the gas bearing and water bearing of the media in the whole incident angle, which indicates that the ability to distinguish fluid type for the reflection coefficient is significantly improved. Therefore, the normalized PS-wave reflection coefficient has important theoretical value in study of fluid identification of fractured media at multi-directional observation and small incident angle. The effects of matrix porosity on the reflection coefficient are similar to the fracture density, and are not repeated here. The reflection coefficient mentioned in the subsequent numerical simulations means the normalized PS wave reflection coefficients.

2.4. Reflection coefficient spatial distribution
In order to investigate the spatial distribution of the reflection coefficient for the porous fracture media, keeping the background parameters of the two-layer media model unchanged, the lower fractured media parameters are changed into two cases: matrix porosity $\phi_p$ is set to 0.03 and fracture density $e$ is 0.05, 0.1, and 0.15; fracture density $e$ is 0.1 and matrix porosity $\phi_p$ is 0.001, 0.01, and 0.1. In these two cases, the PS wave reflection coefficients in water-saturated state and gas-saturated state are calculated in the range of incident angle $\theta \in (0.001^\circ \sim 30^\circ)$ and azimuth $\varphi \in (0^\circ \sim 180^\circ)$, the results are shown in Figure 4 and Figure 5. In Figure 4, (a1) and (b1) respectively correspond to the PS wave reflection coefficients which change with the fracture density change in gas-saturated and water-saturated states. In Figure 5, (a1) and (b1) correspond to the PS wave reflection coefficients which change with the matrix porosity change in gas-saturated and water-saturated states.

The obvious characteristics are as below:

1. In addition to the amplitude change significantly with offset (AVO feature), in the range of incident angles ($0^\circ \sim 25^\circ$), the PS-wave reflection coefficient has obvious azimuthal anisotropy characteristics which is that the reflection coefficient value deviates the highest from the isotropic level at azimuth angle $\varphi = 0^\circ$ or $180^\circ$, and is closest to the isotropic level at $\varphi = 90^\circ$, which illustrates that the anisotropic degree in the vertical fracture direction is higher than that in the parallel fracture direction;

2. When matrix porosity is constant, the PS-wave reflection coefficient at two-layer model interface in the same fluid-saturated state increases positively with the increase of crack density; the change rate of the reflection coefficient with the increase of fracture density in water-saturated state is higher than that in gas-saturated state.
(3) When the fracture density is constant, the reflection coefficient curved surface in the gas-saturated state decreases positively with the increase of matrix porosity while curved surface in water-saturated state does not vary with the increase of it, which shows that the matrix porosity change can only cause the reflection coefficient of gas-saturated fractured media to change. It is clear that the matrix porosity change has a significant effect on identifying fluid property, and can be used as a good fluid indicator.

3. Conclusions
The equivalent porous fractured media is an important fracture theory model. The pores and the fractures in the media are assumed to be interconnected, so the media is closer to the real fracture reservoirs. The corresponding reflection coefficient characteristic can provide a certain theoretical basis for quantitative seismic interpretation. The analysis conclusions on the PS wave reflection coefficient are as follows:

The crossplot analysis of compliance parameters $\Delta V$ and $\Delta T$ shows that fracture density or matrix porosity is sensitive to filled fluid type in fracture media. Therefore, the compliance parameters are used to establish the direct functional relationship among fluid property, fracture density, matrix porosity and the PS-wave reflection coefficient, which is conducive to analyze the direct influence of these parameters on reflection coefficient, and is helpful for the next inversion research.

The fracture density change has different extent effect on the PS-wave reflection coefficient in gas-saturated and water-saturated states, that is, as the fracture density increases, the variation ranges of the PS wave reflection coefficient in water-saturated state are greater than those in gas-saturated one. The influence degree of matrix porosity on PS wave reflection coefficient in different fluid-saturated states is obviously different.

In summary, the PS wave reflection coefficient analysis containing fracture density and matrix porosity can effectively distinguish the filled fluid property in fracture media, which can provide strong support for fluid identification of fractured reservoir, and provide the theoretical basis for lithological and fracture parameters inversion in fractured media.

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