About the Electron Charge Accelerated in the Small-size Betatron MIB-4

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Abstract. It is assumed that the electron charge accelerated in small-size betatrons is of the same order as that in the classical betatron. However, the parameters of the interpolar space of small-size betatrons significantly differ from the parameters of the classical betatron. We can expect that the value of the accelerated electron charge will be different. The paper presents the results of the measurements of the electron charge accelerated in a small-size betatron MIB-4. It is shown that the electron charge accelerated is this betatron is larger than that in the classical betatron.

1. Introduction

Depending on the requirements to radiation parameters and applications, betatrons can be divided into three types – stationary, high-current, and small-size betatrons. There are also special types of betatrons with a constant magnetic field, iron-free betatrons, cylindrical betatrons and modified betatrons, however, they are not widespread. The so-called "classical" betatrons with energies up to 18–35 MeV designed for industrial radiographic non-destructive testing are referred to stationary betatrons. High-current betatrons produce energies up to 15–50 MeV with the radiation intensity 2–3 orders of magnitude higher than that of the "classical" betatrons of the same energy. A significant increase in the radiation intensity is due to special technical solutions used in high-current betatrons, which allocates them to a separate type of accelerators. These systems are used for imaging fast and explosive processes in enclosed volumes and for activation analysis of materials. Small-size betatrons produce energies up to 10 MeV [1]. Small-size betatrons are characterized by relatively low weight and high mobility. At the moment, this type of betatrons is most common and most demanded on the Russian and world markets. Small-size betatrons are widely used in industrial radiographic non-destructive testing, industrial tomography [2], medicine [3] and inspection systems [4] which have become particularly widespread in recent years.

Along with other parameters, the practical value of a betatron is assessed by the bremsstrahlung intensity. The betatron bremsstrahlung intensity is proportional to the quantity of the accelerated electron charge. Each type of betatrons provides the maximum accelerated charge, and its value is determined by the dimensions of the interpolar space and injection voltage. In the classical betatron, the accelerated electron charge is \(5 \cdot 10^9\) electrons or 0.8 nC. In the high-current betatron, the accelerated electron charge is \(10^3\) times greater and amounts to \(5.56 \cdot 10^{12}\) electrons or 890 nC. The data on the electron charge accelerated in small-size betatrons is not provided in the literature. The electron charge accelerated in small-size betatrons is assumed to be of the same order as that in the classical betatron. However, small-size betatrons are essentially different from classical betatrons in terms of the interpolar space parameters. Hence, the value of the accelerated electron charge is expected to be different.

This paper presents the measurement results obtained for the electron charge accelerated in the small-size betatron MIB-4.
2. Theoretical part

At the beginning of acceleration, the maximum equilibrium charge which can be maintained by the magnetic field of the betatron after the end of the injection process is determined by the relation

$$Q = \frac{E_i}{2eR_0} \left( \frac{E_i}{E_0} \right)^2 - 1 \right) S_0$$

where \(e\) is electron charge; \(R_0\) is equilibrium orbit radius; \(E_0\) is the rest energy of the electron; \(E_i\) is the total energy of injected electrons; \(E_i = E_0 + eU_i\), \(U_i\) is injection voltage; \(S_0\) is the cross-sectional area of the equilibrium beam, i.e. the electron beam for which the Coulomb repulsion forces are balanced by the focusing forces of the magnetic field.

Voltage injection for MIB-4 and most small-size betatrons is 40 kV. The radius of the equilibrium orbit \(R_0\) and the cross-sectional area of the equilibrium beam \(S_0\) can be calculated by constructing a potential function of the magnetic field. The potential function is the most explicit and complete characteristic of the focusing properties of the betatron magnetic field. The expression of the potential function \(V\) through the vector potential \(A\) is written in the form

$$V = \frac{e}{2mc^2} \cdot A^2 = \frac{e}{2mc^2} \left( \frac{r \cdot \bar{H}(r)}{2} \right)$$

where \(\bar{H}\) is the mean value of the magnetic field in a circle of radius \(r\).

The radius of the equilibrium orbit can be determined by the minimum of the potential function. Since the potential function has a minimum in the equilibrium radius \(R_0\), the graphs of the potential function are referred to as "potential wells". An essential characteristic of this potential well is the area of its cross section by the radial plane, i.e. the area covered by the equipotential line. The equipotential line equation takes the form

$$z = \frac{1}{k} \text{arccosh} \left( \frac{A}{A_0 \left[ aJ_1(kr) + bN_1(kr) \right]} \right)$$

where \(J_1(kr)\) is a Bessel function of the first kind; \(N_1(kr)\) is the Neumann function of the first kind; \(A_0\) is the vector potential at the equilibrium radius.

The equipotential at the injection level sets the boundaries of the equilibrium charge zone, i.e. the area of the equilibrium beam cross-section.

After measuring the parameters of the magnetic field, the maximum equilibrium charge was calculated which is able to maintain the magnetic field of the betatron MIB-4.

The actually accelerated electron charge was measured by the method which implies measuring the electron current directly from a tungsten target with the phase shift from the equilibrium orbit corresponding to energy of 1 MeV. At electron energies significantly higher than those specified, the current target will be different from the true one for several reasons: firstly, the electron path in the material will be larger than the thickness of the target, therefore, a portion of the accelerated electrons will pass through the target and will not be recorded with the measuring instrument; secondly, under electron bombardment of the target, the target material releases secondary electrons. If some of these electrons escape from the target, the measuring instrument will record the decreased current from the target. In this case, we used a tungsten target with a thickness of 0.6 mm. For the energy of 1 MeV, the average linear electron path in the target material is 0.4 mm, which fully satisfies the measurement requirements.

In order to determine the true number of electrons collided with the target, the electrons reflected from the target material are to be considered. The reflection of electrons with energies varying from several tens of keV to several MeV for target materials in a wide range of \(Z\) has been studied in detail and the reflection coefficients has been provided in a number of studies [5–7].

The reflection coefficient of electrons is defined as following:

$$\beta = \frac{(n_{\text{reg}} - n_{\text{init}})}{n_{\text{init}}} = \frac{n_{\text{reg}}}{n_{\text{init}}} - 1$$

where \(n_{\text{init}}\) is the number of electrons recorded by the detector with no reflecting sample (in our case, it is the target) and \(n_{\text{reg}} = n_{\text{init}} + n_{\text{refl}}\) is the number of particles recorded by the same detector at the same point with regard to the reflected electrons, that is in case of the presence of the reflecting sample (target); \(n_{\text{refl}}\) is the number of reflected electrons.
The reflection coefficient is $\beta=0.8$ for tungsten and the energy of accelerated electrons of 1 MeV. The correction for reflection in the form of $(1-\beta)^{-1}$ should be introduced in the expression to determine the charge dumped onto the target. Then, assuming that the current pulse from the target is sinusoidal, the charge of the dumped beam can be calculated with the formula

$$Q_m = \frac{I_m t}{1.57(1 - \beta)} = \frac{U_{mt}}{1.57 R (1 - \beta)}$$

where $I_m$ is peak value of the current pulse from the target; $t$ is duration of the current pulse; $\beta = 0.8$ is reflection coefficient of the electrons from the tungsten target; $U_m$ is peak value of the voltage drop across resistance $R$ through which with the measured current of the electrons "dropped" onto the target flows.

3. Experimental part

The measuring disc shown in figure 1 was used to measure the magnetic field in the pole gap of the betatron electromagnet.

![Figure 1. Measuring disc.](image)

The conductors with the bundled ends in the form of twisted pairs are fixed in the grooves of the measuring disk. At one point of time, the emf values can be obtained from one measuring coil only. To measure the voltage in the coils, a digital oscilloscope WP 725Zi was used to record the data from two coils, namely the coil being measured at the given moment and the supporting coil common to other coils. The objective of the approach is to eliminate errors due to the field amplitude fluctuations from one pulse to another. The recorded arrays in increment of 2 ns were transferred to PC for processing.

The graph for the potential function of the magnetic field of the betatron MIB-4 is presented in figure 2, and the graphs of the magnetic field equipotentials are shown in figure 3. The graph shows that the function minimum is observed at radius $R=4.5$ cm, and thus the radius of equilibrium orbit of the betatron MIB-4 is $R_0=4.5$ cm.

![Figure 2. Graph of potential function of the magnetic field of betatron MIB-4.](image)

The measurements of the magnetic field characteristics of betatron MIB-4 showed that the area of equilibrium beam is $S_0=3 \cdot 10^{-4}$ m$^2$, and the maximum charge the field is capable of retaining at the time of injection is $Q=2.68 \cdot 10^{-9}$ C that corresponds to the number of particles $N=1.68 \cdot 10^{10}$.  

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The measurements of the accelerated electron charge were carried out directly from the tungsten target of the small-size betatron MIB-4. The small-size betatron MIB-4 is a commercially available accelerator. The kinetic energy of accelerated electrons in the small-size betatron MIB-4 is 3.5 MeV, the total energy is 4 MeV, respectively. The betatron is equipped with soldered acceleration chamber RBK3-3. The chamber is designed to generate bremsstrahlung and it is equipped with the tungsten target of 0.6 mm thick attached to the anode injector. After the electrons reach the given energy, pulse is generated in the bias coil to extend the equilibrium orbit and to "dump" the accelerated electrons onto the tungsten target to generate X-ray bremsstrahlung. The measurements were carried out at the phase shift corresponding to the energy of accelerated electrons equal to 1 MeV in order for all accelerated electrons to be "decelerated" by the target material and recorded by measuring apparatus.

Oscillogram of the current pulse from the target of small-size betatron MIB-4 under electron "bombardment" with the energy of 1 MeV is shown in figure 4.

The voltage pulse from the target of the small-size betatron MIB-4 generated by the electron current across the resistance of 51 Ω, with the bias phase of accelerated electrons corresponding to 1 MeV, has the amplitude equal to 0.0875V with the duration of 1.2 microseconds. In this case, the electron charge is:

\[ Q_m = \frac{U_m t}{1.57 R (1 - \beta)} = 1.6 \cdot 10^{-9} \text{C} \]

That corresponds to the number of accelerated electrons \( N = 1.0 \times 10^{10} \).

4. Conclusion
The principle of geometric similarity was applied to develop a classical betatron, that is when a basic well-established accelerator model is taken as a basis, and accelerators with different energy are developed by changing geometric dimensions. The principle of geometric similarity proved to be inapplicable to develop small-size betatrons. If the classical betatron B-18 with energy of 18 MeV is taken as a basis to develop a geometrically reduced replica betatron with the energy of 4 MeV, the cross-section of acceleration chamber will be unacceptably small causing significant decrease in the accelerated electron charge. The reduced
dimensions of the acceleration chamber will affect its electric strength and will not allow application of sufficiently high voltage injection which will also cause decrease in the accelerated electron charge. Small-size betatrons were developed after the development of high-current betatrons. To develop small-size and high-current betatrons, the optimum dimensions of the interpolar space for the classical betatrons had to be withdrawn volume for the focusing power action. Small-size betatrons are geometrically reduced replica accelerators of high-current betatrons in terms of the parameters of the interpolar space.

The effective electron capture into acceleration allows bringing the accelerated electron charge to the possible maximum for the given volume of the focusing power action and voltage injection. Special schemes called contractors have been developed for this purpose. At the initial stage of acceleration the radius of electron rotation differs from that of the equilibrium orbit, and some time is required for the positions of radii to coincide. The contractor coils are installed above and below the equilibrium orbit. The magnetic flux generated by the current pulses flowing through the contractor coils makes it possible to affect the rate of change in the electron rotation radius at the moment of injection, thereby affecting the efficiency of electron capture into acceleration. The contractors have shown high efficiency. They are used in all newly manufactured small-size betatrons.

Thus, the increased dimensions of the interpolar space to volume of the focusing power action and the contractor application, thereby increasing the capture efficiency, made it possible to bring the accelerated electron charge in small-size betatrons up to $10^{10}$ particles. The number of accelerated electrons in small-size betatrons is significantly greater than that achieved in the majority of best examples of classical betatrons ($5 \cdot 10^9$ electrons), but less than in high-current betatrons ($10^{12}$ electrons). However, the accelerated charge in the small-size betatron MIB-4 is significantly greater than the charge which the betatron magnetic field is able to retain, namely $Q=2.68 \cdot 10^{-9}$C that corresponds to the number of particles $N=1.68 \cdot 10^{10}$.

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