Chapter

Dark Matter as Cold Atomic Hydrogen in Its Lower Ground State

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Abstract

This chapter presents a novel theory of dark matter made plausible by several astronomical observations reported in 2018 and 2019. The author introduced this theory to colleagues invited to the Dark Matter Workshop at the World Science Festival in May of 2019, and its first publication was in a peer-reviewed physics journal in July of 2019.

Keywords: dark matter, atomic hydrogen, interstellar medium, cosmic dawn, Wouthuysen-Field effect, cosmology theory, Milky Way galaxy

1. Introduction

The theory [1], simply stated, is that what we currently refer to as “cold dark matter” is, in actuality, slow-moving interstellar and intergalactic neutral atomic hydrogen in its lower 1 s ground state. Its exceedingly low density within the vacuum of space can be quantified by measuring the intensity of its signature spectral hyperfine 21-cm absorption line in lines of site to stellar objects at known distances. At an average HI density of approximately one atom per cubic centimeter ($1.67 \times 10^{-21} \text{ kg m}^{-3}$) within the vast, cold, and remote interstellar vacuum of the Milky Way (MW), it is very nearly collisionless and thus mostly unperturbed. And, given its current nearly perpetual lower ground state condition, it cannot emit light. Whenever and wherever hydrogen is mostly above this ground state, and significantly more concentrated, it is readily visible and we call it something else (a cold, warm, or hot gas cloud, for instance).

Following a brief review of the historical evidence for the existence of dark matter, its key observations reported in 2018 and 2019 will be summarized and its current constraints elaborated. The author’s calculations, in the context of these observations, will then be presented in the Results section, and a Discussion section with a table based upon these findings will follow.

2. Historical background

It is generally agreed that astronomer Fritz Zwicky, in 1933, was the first scientist to apply the virial theorem to infer the existence of dark matter. He referred to it as “dunkle materie” (i.e., “dark matter”) [2, 3]. Unfortunately, Zwicky’s dark matter proposal was largely ignored at that time.
Beginning in 1970, this problem of “missing matter” was further elucidated and essentially proven by the detailed studies of galactic rotation by Vera Rubin and William Ford [4, 5], although it took considerable time for them to receive due recognition for this achievement.

With gradual acceptance of the observational implications, what has followed in the ensuing decades has been a stepwise progression of tightening constraints on the nature and quantity of dark matter. As a consequence, much like a horse race with changing leads, various creative and exotic theories of the nature of dark matter (WIMPs, MACHOs, axions, sterile neutrinos, supersymmetry partners, SIMPs, GIMPs, etc.) have fallen in and out of favor [6]. Given the difficulty of its detection, there have even been attempts to discard the idea of dark matter altogether in favor of modifying Newtonian celestial mechanics (Modified Newtonian Dynamics (MOND)) [7, 8].

A review of these various theories, and a discussion of their current plausibility, is beyond the scope of this chapter. Whole books have been written about them. Suffice it to say, in view of the many continuing exotic dark matter detector failures, there is room for a new theory such as the one presented herein. The following section will summarize key constraints on dark matter as of 2020.

3. Current observational constraints

Upon establishing the likelihood of an abundance of cosmic matter which, in its current state, does not emit light, astronomers and astrophysicists have attempted to quantify it with respect to the visible matter (i.e., stars, gas clouds, and cosmic dust). The 2018 Planck Collaboration report [9] indicates a cosmic dark matter-to-visible matter ratio of approximately 5.4:1. This is in close agreement with a ratio of approximately 5:1 established by a 2019 Gaia-Hubble survey report [10] on the Milky Way galaxy. The Gaia report indicates a total virial MW mass of approximately 1.5 trillion solar masses which include a visible matter mass of approximately 250 billion solar masses. Based upon these and other studies, dark matter is currently believed to comprise about 85% of all cosmic matter. Thus, although it appears, by gravitational lensing, to be predominantly within and haloed around the visible galaxies, dark matter is most likely ubiquitous and therefore a key structural (i.e., “scaffold”) component of the universe. In this context, it is worth noting that the Planck Collaboration study of the cosmic microwave background (CMB) anisotropy documents the presence and gravitational influence of dark matter within the hot and dense early universe during the recombination/decoupling epoch. So what we now tend to think of as “cold dark matter” (CDM) was once hot, and very possibly light-emitting, in its past excited state.

Although relatively few in number, MW halo stars at various known distances beyond the galactic disk can provide for line-of-site spectral analysis and a rough MW halo vacuum density determination of interstellar neutral atomic hydrogen in its lower ground state. Specifically, the intensity of the hyperfine 21-cm absorption line gives us some idea of the number of these particular atoms per unit volume of the column of intervening interstellar space. Best estimates of this sort, made over a number of decades, have indicated an average density within the MW interstellar vacuum of roughly one of these atoms per cubic centimeter [11–13].

Making use of some initial Gaia survey data released in 2018, Posti and Helmi reported results [14] which allow one to deduce a ratio of dark matter-to-visible matter within a 20 kpc (i.e., 65,000 light-years) radius halo sphere of the MW (see schematic Figure 1). This halo sphere is represented in black in the figure and is
roughly to scale with respect to the 50,000 light-year radius MW disk (in white). The disk averages approximately 1000 light-years in thickness. The relatively few halo stars well beyond the disk are also schematically represented in the figure. As mentioned, these are useful for density measurements of cold hydrogen in the lower ground state within the halo vacuum.

The total virial mass of their sphere was reported by Posti and Helmi to be $1.91 \times 10^{11} \, M_{\odot}$ (solar masses), of which the mass of dark matter was reported to be $1.37 \times 10^{11} \, M_{\odot}$. This would imply that the MW 20 kpc sphere ratio of dark matter-to-visible mass is about 2.54:1. Therefore, if we normalize the MW visible mass to the 250 billion $M_{\odot}$ value given in the 2019 Gaia survey report, this Posti and Helmi ratio would imply a corresponding dark matter mass of approximately 635 billion $M_{\odot}$ within the same 20 kpc radius halo sphere. These numbers will be compared in the subsequent Results section.

Aside from the inability of dark matter to emit light, observations have confirmed that it is nearly collisionless. It appears to be composed of particles with a low scattering cross section. This can be deduced from Tucker's early observations of the bullet cluster [15] and subsequent observations of other colliding galaxies.

Dark matter, at present, is also believed to be cold (i.e., slow-moving). A predicted Maxwell-Boltzmann particle velocity distribution ranging from roughly 0 to 600 km/sec, and peaking at roughly 220–230 km/sec, is the theoretical basis for optimizing a variety of cold dark matter particle detectors [16]. Unfortunately, none of these experiments to date has produced a positive result of an exotic (i.e., non-baryonic) dark matter particle. Intriguingly, however, the 2018 EDGES study [17] of the hyperfine 21-cm absorption line of neutral atomic hydrogen corresponding to cosmological redshifts of $15 < z < 20$ (cosmic dawn) has reported a strong signal consistent with a hydrogen gas temperature in the low single digits of the Kelvin temperature scale. This is considerably lower than the cosmic dawn CMB radiation temperature and
produces strong constraints on the nature of dark matter. This CMB decoupling phase during cosmic dawn indicates that whatever we are currently referring to as dark matter has been particularly cold since at least the time of early cosmic dawn, has a particle mass of no more than about 2–3 GeV, and has a scattering cross-sectional $\sigma_1$ value of at least $1.5 \times 10^{-21}$ cm$^2$. If the EDGES observations of cosmic dawn are, in fact, the result of dark matter cooling of warmer (i.e., CMB-equilibrated) hydrogen atoms, the proposed WIMPs and all but one baryon (namely, *colder* atomic hydrogen in its lower ground state) are effectively ruled out as dark matter candidates.

Figure 3 on page 9 of Barkana’s review [18] related to the EDGES study findings summarizes the new cosmic dawn dark matter constraints with a log graph of the implied baryon-dark matter (b-DM) cross-sectional $\sigma_1$ and the minimum possible 21-cm brightness temperature ($T_{21}$) on the two vertical axes and the corresponding implied dark matter particle mass $M_X$ on the horizontal axis. All constraint values indicated in the graph correspond to the strong signal measured at $z = 17$, which corresponds to a redshifted 21-cm hyperfine hydrogen absorption line detectable at a frequency of 78.9 MHz. To fully comprehend the significance of these dark matter constraints, the reader should obtain this reference and pay particular attention to the dark matter particle mass corresponding to a cross-sectional $\sigma_1$ value of $10^{-20}$ cm$^2$ and a 21-cm brightness temperature log$_{10}$ value (in mK) of 2.32. Please note that these values correspond to a cold dark matter particle fitting with neutral atomic hydrogen, which has a similar low velocity scattering cross section and a mass energy of 0.938 GeV. Furthermore, it should be remembered that the 21-cm absorption line is the signature of atomic hydrogen in its lower ground state. These new cosmic dawn constraints on dark matter will be a major focus in the following Discussion section, particularly with respect to the Wouthuysen-Field effect.

Without specifically naming any particular non-excluded baryons, physicist Stacy McGaugh published a brief note [19] at the time of the EDGES publication (March 2018) which strongly supports the idea that the cosmic dawn observations are to be, in his words, “expected for a purely baryonic universe.” He begins the note with the observation that the strength of the redshifted hyperfine 21-cm absorption line at $z = 17$ is anomalously strong for $\Lambda$CDM, which proposes non-baryonic dark matter. He also points out that current knowledge in atomic physics would indicate that a maximum possible $T_{21}$ signal should occur when the neutral hydrogen fraction $X_{HI} = 1$ and spin temperature $T_S = T_K$. McGaugh’s cogent arguments and interpretation of the EDGES cosmic dawn data are strongly supportive of the theory presented herein.

An additional constraint on dark matter has to do with the “cusp-core problem,” specifically why some galaxies have a distinctly cuspy distribution of dark matter and others do not. A 2019 report on dark matter distribution within dwarf galaxies, by Read et al. [20], offers a clue. It shows that galaxies which stopped forming stars over 6 billion years ago tend to be cuspiest than those with more extended star formation. This is equivalent to saying that the extended star formation dwarf galaxies have shallower dark matter cores. Thus, their findings agree well with models where dark matter is presumably heated up by bursty star formation. This means that any plausible theory of dark matter must explain why extended and bursty star formation is correlated with a so-called “cored” dark matter distribution.

One obvious possible interpretation of the Read observations is simply that bursts of highly energetic particles and photons, produced by a concentration of new stars in and around active galactic centers, would tend to heat up and eject cold dark matter from their vicinity. If this is the correct interpretation, then a self-interacting dark matter (SIDM) model becomes unnecessary to explain the “cusp-core problem.” In fact, all sorts of bizarre non-baryonic properties of dark matter then become unnecessary.
4. Results (calculation)

Given the new dark matter theory as briefly summarized in the Introduction section, a simple calculation can be made on the Posti and Helmi 20 kpc MW halo sphere, as a test of this theory. If we start with the current best estimate of an average of only one atom of atomic hydrogen in the lower ground state per cubic centimeter of the Posti and Helmi 20 kpc halo sphere, that assumes a vacuum hydrogen density of \( 1.67 \times 10^{-21} \) kg m\(^{-3}\). If we then multiply that number by the volume of the 20 kpc sphere (\( 9.85 \times 10^{62} \) m\(^3\)), the total mass of atomic hydrogen in the bottom ground state is \( 1.645 \times 10^{42} \) kg. That is the equivalent of \( 827 \) billion M\(_{\odot}\). This is 3.3 times the 2019 Gaia survey MW galaxy visible mass! Even allowing for only 0.75 atom of atomic hydrogen in the bottom ground state per cubic centimeter of the 20 kpc halo sphere, the Posti and Helmi dark matter-to-visible matter ratio of 2.54 can be met.

5. Discussion: interstitial hydrogen, cosmic dawn, and the Wouthuysen-Field effect

Observations of the CMB anisotropy map suggest the following cosmic evolution scenario since the CMB emission epoch:

Denser regions of the primordial hydrogen distribution, already subject to the positive feedback of gravity, further aggregated into the hot stars, warm gas clouds, galaxies, quasars, and filaments. In contrast, due to adiabatic cosmic expansion, the primordial hydrogen within the low gravity interstices of the CMP map progressively became exceedingly sparse and cold (i.e., CMB-equilibrated). These interstices we know today as the vast interstellar and intergalactic space, including the voids.

The expanding and cooling universe, after CMB emission, was completely dark before the first dense clusters of primordial hydrogen underwent nuclear fusion. This period, known as the cosmic “dark age,” merged into the “cosmic dawn” reionization epoch at around 100 million years after the big bang. The “cosmic dawn” epoch is named as such because this is when the first stars are thought to have formed.

As documented by the EDGES study, a strange phenomenon occurred during the period of cosmic dawn. For about 150 million years, corresponding roughly to the cosmological redshift range of \( 15 < z < 20 \), the temperature \( T_G \) of the vast interstitial primordial hydrogen gas was decoupled from the CMB radiation temperature \( T_R \). At the peak of this phenomenon, at roughly \( z = 17 \), this primordial hydrogen appears to have been in the low single digits of the Kelvin temperature scale. Thereafter, the hydrogen gas gradually warmed back up to the CMB temperature at roughly \( z = 15 \).

Figure 2 illustrates this phenomenon. On this graph, \( z = 20 \) corresponds to about 100 million years after the big bang, \( z = 17 \) corresponds to about 180 million years after the big bang, and \( z = 15 \) corresponds to about 250 million years after the big bang.

This phenomenon of “cosmic dawn CMB decoupling” is most commonly attributed to a b-DM scattering interaction, whereby dark matter is presumed to have chilled faster than primordial hydrogen during the cosmic dark age, to the point where it could then interact with and chill the CMB-equilibrated interstitial hydrogen and decouple it from the CMB radiation temperature.

The problem with this particular explanation of the EDGES study observations is to explain why the beginning of the CMB decoupling phenomenon coincided with the first stars at the crack of cosmic dawn. How is it that dark matter had cooled sufficiently to enable b-DM scattering and CMB decoupling of primordial hydrogen
just when the first stars were forming? Could there be a simpler explanation for cosmic dawn CMB decoupling without requiring a non-baryonic intermediary?

Fully in keeping with McGaugh’s bold assertion of a purely baryonic mechanism, this cosmic dawn coincidence may have been entirely due to the Wouthuysen-Field (WF) effect on CMB-equilibrated primordial atomic hydrogen. If unfamiliar with this radiation effect on atomic hydrogen, the reader is encouraged to read an excellent and brief summary of the WF effect on the Wikipedia page entitled “Wouthuysen-Field Coupling” [21]. A more extensive and highly technical summary is also found on the AstroBaki website [22]. Briefly, the Lyman-alpha ultraviolet (UV) radiation of the first stars was of sufficient energy to have caused a redistribution of the balance of the two hydrogen electron hyperfine 21-cm ground states such that the primordial hydrogen gas could effectively bypass its “forbidden transition” (from parallel to antiparallel electron spin) and easily reach the lower ground state. The net effect of this process would have been to decouple primordial hydrogen from the CMB radiation temperature, producing the strong 21-cm absorption line signal observed. Thus, it appears that an exotic, non-baryonic, form of dark matter was completely unnecessary for cosmic dawn CMB decoupling. The mysterious dark matter at cosmic dawn could simply have been the first of the interstitial hydrogen to be chilled and decoupled by the Lyman-alpha radiation. The process then, over millions of years, would have extended to the rest of the CMB-equilibrated hydrogen, peaking at a cosmic redshift of $z = 17$.

The key dark matter features, including observational constraints achieved over the last few years, and the correlating features of interstitial atomic hydrogen in the lower HI ground state, can now be brought together into a table (Table 1) for comparison.

These correlations are striking and strongly suggest that interstitial cold atomic hydrogen in its lower ground state is what we have been calling dark matter over the last few decades.
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It has long been assumed that the average atomic density of the “nearly empty” vacuum of interstellar space beyond the visible stars, gas clouds, and cosmic dust can be ignored in galactic mass calculations. While this might be true for the confines of the galactic disk and bulge, where visible matter is particularly concentrated, it is definitely not true for the galactic halo in close proximity to the disk. The sheer vastness of space belies the assumption mentioned above. It appears that this mistaken assumption has been a key foundational error behind the long-standing mystery of dark matter. The simple calculation in the Results section supports this conclusion. Even a single stray baryonic atom per cubic centimeter of interstellar space within the 20 kpc MW halo of Posti and Helmi can dwarf the combined mass of all visible stars, clouds of gas, and cosmic dust!

The fact that the particular atom in question appears now not to be in the least bit exotic but, instead, the most common structural element in the universe is indeed ironic. In a sense, because of the many distractions and obscurations provided by the highly visible warm and hot hydrogen atoms, cold interstitial hydrogen, because of its remote location, extremely low density, low velocity, and prolonged lower ground state, has been essentially hiding in plain sight. Observations of the 21-cm hyperfine absorption line (its signature) have been noted for decades but only recently connected to phenomena attributed to dark matter. The simple calculation in the Results section supports this conclusion. Even a single stray baryonic atom per cubic centimeter of interstellar space within the 20 kpc MW halo of Posti and Helmi can dwarf the combined mass of all visible stars, clouds of gas, and cosmic dust!

Any useful physical theory should be falsifiable and predictive. The falsifiability of this particular theory is obvious. This theory would be falsified if a particle $M_X$ of 0.938 GeV becomes excluded from dark matter constraints, or current best estimates of the average MW halo vacuum density of cold atomic hydrogen are subsequently proven to be severely overestimated. However, a minor correction to approximately 0.5–0.75 atom per cubic centimeter is entirely consistent with this theory. As for observations to further strengthen this theory, the following predictions are made:

1. There will be tightening dark matter constraints around a particle $M_X$ value of 0.938 GeV (i.e., the mass energy of neutral atomic hydrogen).

2. Computer simulations of galaxy formation and evolution which incorporate this theory will show excellent correlations with observations, including the coring effect of heating and ejecting cold interstellar hydrogen from active galactic centers with bursty star formation.

Table 1.
Dark matter features vs. interstitial HI cold hydrogen.

| Dark matter features | Interstitial HI cold hydrogen | References |
|---------------------|-------------------------------|------------|
| Cold (0–600 km/sec) | Cold (0–600 km/sec)           | [16]       |
| Dark (no emissions) | Lower ground state (cannot emit) | [2-5]     |
| Cross-section $\sigma_1 > 1.5 \times 10^{-21}$ cm | $\sigma_1 > 1.5 \times 10^{-21}$ cm (at low velocity) | [15, 17, 18] |
| Baryon (strongest 21-cm signal) | Baryon for $X_{HI} = 1$ and $T_S = T_X$ | [19] |
| Mass-Energy less than 3 GeV | Mass-Energy = 0.938 GeV | [17, 18] |
| Mass 20 kpc Halo = 635 Billion $M_\odot$ | Mass 20 kpc Halo = 827 Billion $M_\odot$ | [14, 10] |
| Central DM heating (“coring”) | Ejected and loses ground state | [20] |
| CMB decoupling at cosmic dawn | Wouthuysen-Field Effect | [21, 22] |
| Structural scaffold | Most abundant atom | [9] |
| Existence at CMB emission | Most abundant atom | [9] |
3. No exotic non-baryonic particles fitting the observed qualitative and quantitative constraints will ever be discovered.

6. Summary and conclusion

To summarize, this chapter has introduced the reader to a plausible new theory of dark matter which appears to match current observational constraints. The theory, simply stated, is that what we currently refer to as “cold dark matter” is, in actuality, slow-moving interstellar and intergalactic neutral atomic hydrogen in its lower 1 s ground state. So long as it stays in this lower ground state, it cannot emit light. Furthermore, it is currently so sparse as to be nearly collisionless. Whenever and wherever hydrogen is mostly above this ground state, and significantly more concentrated, it is readily visible and we call it something else (a cold, warm, or hot gas cloud, for instance).

Dark matter observations corresponding to the cosmic dawn epoch, which were reported in 2018 and 2019, have provided the necessary constraints on dark matter to favor this theory above all others at the present time. In particular, the Bowman (i.e., EDGES) and Barkana references point to a cold dark matter particle with features quite consistent with cold atomic hydrogen. Furthermore, a convincing case has been made by McGaugh that the strong hydrogen absorption signal at cosmic dawn is the signature of a baryonic universe. The obvious mechanism for such signal strength, and its coincidence with cosmic dawn, is the Wouthuysen-Field effect. From the foregoing discussion, it becomes apparent that exotic (i.e., non-baryonic) matter is not necessary to explain dark matter observations to date.

We conclude by asking the following question:

If interstitial cold atomic hydrogen in its lower ground state is qualitatively and quantitatively sufficient to explain dark matter observations to date, do we really need to spend more of our time and money continuing to look for anything else?
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