Prediction of the Probability of Earthquake in Seismic Risk Analysis Using Bayesian Method

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Abstract. Because of the randomness and uncertainties of earthquake and its structures, it is a natural choice to analyze and evaluate the risk of earthquake in the way of seismic risk analysis. Effective prediction of earthquakes can minimize property losses and protect citizens’ safety. In this article, Bayes rule was used to make a simple prediction of the risk of a strong earthquake in a certain area. Then we conducted probabilistic seismic risk analysis (PSHA) to tackle the problem of a specific earthquake about where the earthquake will occur, how big the earthquake will be and when will the earthquake occur. Furthermore, we dived into spatial uncertainty, temporal uncertainty, and size uncertainty, respectively. Since earthquake intensity is significant for seismic vulnerability analysis, PGA (peak ground acceleration) was used as the index of earthquake intensity in this article. To be more specific, Wenchuan city in Sichuan province in China was chosen as the subject. Finally, seismic risk analysis was performed and the failure probability of the system was gained.

1. Introduction
Strong ground vibration triggered by earthquakes and associated ground cracks and deformation can cause damages to buildings, equipment and facilities. For example, during and after earthquakes, traffic and communication are usually interrupted and other lifeline engineering facilities are often destroyed. Also, earthquakes may lead to fire, explosion, plague, toxic substances leaking, radioactive pollution and other severe disasters, resulting in animal and human casualties and property losses. Therefore, the prediction of earthquakes is certainly important for the whole world.

Previously, scholars used a long-period, time-independent model to evaluate the risk and harmfulness of earthquakes. The characteristic of this model is that the evaluation result will not change with the time, thus, a fixed constant. Cornell first came up with the concept of probabilistic earthquake harmfulness (PSHA) in 1968. [1]. PSHA model draws on two main theories, the first improvement is to take the uncertainty into improved risk model. After a long academic discussion, scholars finally reached an agreement, in the scope of using the method of logical tree description model and determine the accuracy of analysis results, and solved the choice of data sets and the weight distribution of logical tree branches. The second improvement is the addition of time variables into the simulation of seismic recurrence hazards on crustal faults. Assume that the fracture stress is completely released, it is unlikely that another major earthquake will occur in the same fault zone for a long time after the earthquake under the condition of less disturbance of other tectonic stresses. Based on previous experience and physical calculations, a time-varying effect based seismic recurrence model is proposed and applied in PSHA [2].

The following sections begin by discussing the expression for the probability of a strong earthquake in a period of time at a certain region given a Beta prior distribution and the number of strong and weak
earthquakes. The method of predicting the probability of the occurrence of earthquakes accurately is to apply probabilistic seismic hazard analysis (PSHA) considering three major uncertainties, namely spatial, size, and temporal. The seismic vulnerability analysis is also included to determine the potential damage to structures. Finally, a real-life earthquake happened in Sichuan Province in China is examined to validate the method and draw the conclusion about the failure probability of the system.

2. Bayes Rule and Its Application in Predicting Earthquake Hazard

In recent years, with the development of the relevant industries, Bayes learning and its statistical inference have attracted extensive attention. Bayes theorem takes the expected value and the sample mean of the prior distribution as weighted average according to their respective accuracy and reasonably synthesizes the prior information and sample information. The characteristic of Bayes learning is using probability to represent uncertainty, and the result of Bayes learning can be expressed as the probability distribution of random variable. Bayes method can avoid the subjective bias which may be caused by only using prior information and the absence of sample information. Therefore, it is applicable to the problem of rare or expensive samples [3].

2.1. Bayes Theorem

The density function of variable X when x is taken can be expressed as p(x), if p(x) is dependent on the unknown parameter \( \theta \), the density function can be expressed as p(X|\( \theta \)), which represents the conditional density function when random variable \( \theta \) is given a value x.

Suppose the prior distribution is \( \pi(\theta) \), the joint conditional probability density function is generated according to the population distribution when a sample X=(X1, X2, ..., Xn) is independent:

\[
p(x|\theta) = \prod_{i=1}^{n} p(x_i|\theta), (i = 1, 2, \ldots, n)
\]

where \( \theta \) is an unknown parameter, p(x) represents the density function of variable X and p(X|\( \theta \)) refers to the density function when p(x) is dependent on the unknown parameter \( \theta \). This function combines the population information with the sample information and is called the likelihood function.

2.2. Conjugate Prior Distribution

Prior probability is the probability obtained according to previous experience and analysis and posterior probability is the probability of outcomes of an experiment after it has been performed and a certain event has occurred. If the prior distribution \( \pi(\theta) \) and the posterior distribution \( \pi(\theta|x) \) have the same form, then \( \theta \) is a conjugate distribution.

The posterior probability can be calculated by prior probability and the likelihood function according to Bayes theorem. In particular, if the posterior distribution and the prior distribution are the same distribution, the calculation process can be simplified.

Next, we take the binomial distribution as an example to analyze the prior distribution and the posterior distribution [4].

If \( X_1 \sim b(n_1, \theta) \), the prior distribution of \( \theta \) is the Beta distribution:

\[
\pi(\theta) = Be(a\beta) = \frac{\Gamma(a+\beta)}{\Gamma(a)\Gamma(\beta)} \theta^{a-1}(1-\theta)^{\beta-1}
\]

Bayes formula can be used to calculate the posterior which is still a Beta distribution:

\[
\pi(\theta|X_1) = \frac{\Gamma(a_1+\beta_1)}{\Gamma(a_1)\Gamma(\beta_1)} \theta^{a_1-1}(1-\theta)^{\beta_1-1}\pi(\theta|X_1) = \frac{\Gamma(a_1+\beta_1)}{\Gamma(a_1)\Gamma(\beta_1)} \theta^{a_1-1}(1-\theta)^{\beta_1-1}
\]

\[
a_1 = a_0 + X_1, \beta_1 = \beta_0 + n_1 - X_1 a_1 = a_0 + X_1, \beta_1 = \beta_0 + n_1 - X_1
\]

The mean of the posterior distribution \( \pi(\theta|x_1) \) is:
\[ \theta = E(\theta | x_1) = \frac{\alpha_1}{\alpha_1 + \beta_1} = \frac{n_1}{n_1 + \frac{X_1}{n_1}} \cdot \frac{\alpha_0 + \beta_0}{\alpha_0 + \beta_0 + n_1} \cdot \frac{\alpha_0}{\alpha_0 + \beta_0} \]

\[ = \gamma \frac{X_1}{n_1} + (1 - \gamma) \frac{\alpha_0}{\alpha_0 + \beta_0} \]

(5)

where \( \gamma = \frac{n_1}{\alpha_0 + \beta_0 + n_1} \), \( \frac{X_1}{n_1} \) is the sample mean, and \( \frac{\alpha_0}{\alpha_0 + \beta_0} \) is the prior mean.

Thus, the resulting estimate of \( \theta \) is a weighted average of the prior mean and the sample mean. In addition, when \( n_1 \) and \( X \) are both large and the sample mean is close to some constant:

\[ E(\theta | x_1) \approx \frac{X_1}{n_1} \]

(6)

This means that when the sample size increases, the posterior mean mainly depends on the sample mean. As \( n_1 \) increases proportionally to \( X_1 \), the prior information will have less influence on the posterior. Therefore, the posterior obtained by using Bayes formula does carry out a reasonable synthesis of prior information and sample information, and the results obtained out formed the one using prior information or sample information alone.

Known from the previous discussion, the obtained posterior information can be taken as the prior of a new round of calculation, and then combined with the new sample information to obtain the next posterior information. After repeating this process, the obtained posterior information will get closer to the actual one [5].

3. Probabilistic Seismic Hazard Analysis

Probabilistic seismic risk analysis (PSHA) was developed in the 1970s as a method to deal with most of the uncertainties involved with seismic hazard. [6]. The major purpose of PSHA is to evaluate the hazard of seismic ground motion at a given site by considering all possible earthquakes in the area, estimating the associated shaking at the site, and calculating the probabilities of these occurrences. And the method of PSHA intends to capture the problem of a specific earthquake about where the earthquake will occur, how big the earthquake will be, and when the earthquake will occur. These problems directly lead to the unknowable aspects of the space, the size and the time of earthquakes, which can be defined as three major uncertainties. Furthermore, it is vital in any estimation of probabilities of earthquake to make a statement about the degree of confidence in the final results as well as engineering programs. Therefore, in this section, we are going to dive into spatial uncertainty, temporal uncertainty, and size uncertainty respectively [7].

3.1. Spatial Uncertainty

Spatial uncertainty deals with the question about where the earthquake will occur. With the assumption that an earthquake has the same or uniform likelihood of occurring in possible location along seismic source, we use a uniform distribution to calculate and analyse spatial uncertainty. Generally, in computing the likelihood, the seismic source is divided up into small segments, so that each segment could be computed and summed by using integral. All too often, a seismic source can range from a point source, which can be seen as a volcano, to a line source, or even to a source of two-dimensional flat surface.

3.1.1. Point Source

A point source can be simplified as a point, which is regarded as the particle version of a structure, with another point nearby as site of the centre of an earthquake. And the minimum distance from the site to the source is a line that connects them in between. Figure 1(a) is a simplified graphic of an example of volcano, in which the probability is associated with the only one distance \( r_s \). In this case, the PDF can be computed and drafted with one bin.
3.1.2. Line Source
Figure 1(b) is an example of line source. A line source is the simplified graphic of a long fault with a site of earthquake. In this case, there is a minimum distance from the earthquake site to the source as well as a maximum distance. Thus, the probability density function is going to range between the minimum and the maximum where anything to the left of the minimum or to the right of the maximum equals to zero.

3.1.3. Area Source
As it is shown in Figure 1(c), an area source is consisted of an earthquake site and a two-dimensional surface. Given that the location of the fault in this grid area could be anywhere, we typically assume that the distance could be between any one of these squares and the earthquake site. So, the probability distribution of this type of source is computed as the summation of individual probability of each distance, where all the probabilities are between the minimum and the maximum distance.

3.2. Size Uncertainty
Size uncertainty usually deals with how big the earthquake will be, which is related to the magnitude. To analyse this type of uncertainty, we use recurrence laws to evaluate how often a certain magnitude of earthquake will repeat itself in order for engineers to do risk analysis during decision-making process. In particular, there are three general types of recurrence laws that are commonly used in PSHA, including Slip-Dependent Laws, Gutenberg-Richter Laws, and Characteristic Earthquake Laws. Next, each recurrence law will be further described and illustrated with graphics.

3.2.1. Slip-Dependent Recurrence Laws
Slip-Dependent Recurrence laws are assigned to faults that are known to have an approximate average annual slip rate, where it is used to predict how much slip is going to occur given a certain amount of time. Generally, there are certain faults that have shown a tremendous amount of regularity in the slip that they produced, and given the amount of slip and a certain magnitude, we can simply calculate the return period from Figure 2. [8].
Figure 2. Effect of fault slip rate and earthquake magnitude on return period. (Slemmons, 1982.)

3.2.2. Gutenberg-Richter Recurrence Laws

Gutenberg-Richter recurrence laws have been around for a while, but they are still used widely by engineers. They essentially state that every single seismic source tends to follow the same pattern, suggesting that the number of earthquakes occurring annually from a given source is a log-linear function of the magnitude. [9]

Figure 3. An application of Gutenberg-Richter law to worldwide seismicity data. (Esteva, 1970.)

In Figure 3(b), the vertical axis is marked by the annual rate of exceedance, $\lambda_m$, which represents the number of earthquake larger than a specific magnitude that occurs each year on average. And in Figure 3(a), the data of log $\lambda_m$ where the line crosses would be a magnitude zero event, which equals to $10^a$, and b is the slope of the linear function. Given the meaning of a and b parameters, we can predict the mean annual rate of exceedance for any given magnitude of interest:

$$\log \lambda_m = a - bm$$

$$\lambda_m = 10^{a-bm}$$

(7)

Where m is the moment magnitude of interest. Also:

$$\lambda_m = \exp (\alpha - \beta m)$$

(8)

where $\alpha=2.303a$ and $\beta=2.303b$.

3.2.3. Characteristic Earthquake Recurrence Laws

In the 1980s, paleoseismologists began to notice that some faults seem to have a “characteristic earthquake”, meaning that they tend to rupture with the same magnitude event repeatedly, instead of a linear distribution of big earthquakes.
Figure 4. Characteristic EQ model for some faults (Youngs And Coppersmith, 1985) [8].

Figure 4 illustrates that earthquakes with small magnitude were still linearly distributed, whereas earthquakes will larger magnitude were not. According to the graph, we can observe that the seismicity data tend to follow Gutenberg-Richter linear pattern, but saw a slight drop-off at low-magnitude events. In other words, due to the reason that faults are locked up on the asperity, a large repeated number of small magnitude event is often not observable. In contrast, instead of having data at high-magnitude events, it has a gap between about m=5 and m=7 where there are no data recorded of earthquakes. In addition, there is a geologic data area that correspond to a maximum magnitude. In conclusion, characteristic earthquake laws present that large and medium intensity earthquakes tend to happen repeatedly while for earthquakes between are missing, which clarified the paleoseismologists’ finding [10].

3.3. Temporal Uncertainty
Temporal uncertainty is the uncertainty associated with when an earthquake of a given size will occur [11]. Because earthquakes tend to occur (hundreds or thousands of years in a row) infrequently relative to the lifetime of structural designs (typically for a 50-year lifespan), we can treat them as random and independent processes. In this case, the Poisson Probability Model can be applied to analyse temporal uncertainty:

\[ P[Y_T>y^*] = 1 - e^{-\lambda y^* T} \]  (9)

where \( P[Y_T>y^*] \) = the probability of exceeding \( y^* \) in a specified time frame, \( T \) refers to the time frame of interest in years, \( \lambda y^* \) is the mean annual rate of exceeding \( y^* \)(the mean annual rate of exceeding any peak acceleration value at their site).

However, the Poisson model is valid unless when it comes to these situations: 1)The structure has an unusually long design life, 2) Previous seismicity shows strong time-dependence between events, and 3) One or more of the significant sources is well overdue. Nonetheless, the error is negligible in the majority of cases because the lifespan of structure is far shorter than the return period of earthquakes.

3.4. Correction of Uncertainty in Seismic Hazard Analysis
Because of the incompleteness of seismic and geological data and the lack of knowledge of the regularity of earthquakes, there are uncertainties in every step of seismic risk analysis. The uncertainty of seismic risk analysis mainly comes from three aspects.

First is the uncertainty of determination of potential source area. To a large extent, this uncertainty is caused by the incomplete understanding of the underground structure and seismic activity.

Second is the uncertainty of seismic activity parameter valuation. The study shows that among the parameters such as the annual average occurrence rate and the upper limit value of earthquake magnitude, the change of the upper limit value of earthquake magnitude has the most significant influence on the
seismic risk analysis results, especially when the earthquake magnitude is large.

For the above two kinds of uncertainties, a variety of scheme combinations can be used to calculate the seismic risk analysis results of different schemes respectively, and the mean value and variance of the seismic risk analysis results can be obtained by comprehensively considering the variation range of the results of all the calculation schemes.

And the last is the uncertainty of the empirical relationship between ground motion attenuation and fault length and magnitude. The calculation and analysis show that the ground motion peak with a given probability can be changed by 20%~60% after considering the uncertainty of attenuation relationship.

3.5. Bayes Rule in Predicting Future Seismic Risk

The steps of Bayesian prediction can be stated as follows: 1) Define random variables and find the likelihood function of sample observations, 2) Determine the prior distribution, 3) Bayes theorem is used to calculate the posterior distribution density function, and 4) Take inferences based on the calculated posterior.

Taking above factors into consideration, we can use Bayes’ rule to make a simple prediction of the risk of a strong earthquake in a certain area.

Suppose n1 and n2 are the number of strong earthquakes and weak earthquakes occurred in a certain region during the time period Ci (i = 1, n), we can predict the probability of a strong earthquakes occurring in this region during the time period Cn+1. The seismic outcome variable during the time period Ci is Xi (i = 1, n+1), set of observed values D=(X1 =x1, Xn = xn), where

\[ X_i = \begin{cases} 1, \text{strong} \\ 0, \text{weak} \end{cases}, X_i \sim b(n, \theta), n_1 = \sum_{i=1}^{n} X_i, n_2 = n - n_1 \quad (10) \]

Bayes formula is used to obtain the probability distribution function with a given D:

\[ p(\theta \mid D) = \frac{p(\theta)^{1-\theta} p(D)}{p(D)} \quad (11) \]

\[ p(D) = \int p(D \mid \theta) p(\theta) d\theta \quad (12) \]

where \( p(\theta) \) is the prior probability density function of \( \theta \), so the problem is converted to calculate \( p(X_{n+1} \mid D) \) and \( p(D \mid \theta) \) is the sample likelihood function. If the parameter \( \theta \) is known, the observed values in D are independent, and the probability of a strong earthquake occurring at any time is \( \theta \), and the probability of a strong earthquake occurring at any time is \((1-\theta)\), then there is:

\[ p(\theta \mid D) = \frac{p(\theta)^{1-\theta} (1-\theta)^{n_2}}{p(D)} \quad (13) \]

Taking the average of all possible values of \( \theta \) as the probability of a strong earthquake occurring during the period Cn+1:

\[ p(X_{n+1} = S \mid D) = \int p(X_{n+1} = S \mid D) p(\theta \mid D) d\theta \]

\[ = \int \theta p(\theta \mid D) d\theta = E_{p(\theta \mid D)}(\theta) \quad (14) \]

\[ E_{p(\theta \mid D)}(\theta) \text{ represents the mathematical expectation of the distribution } \]

\[ p(\theta) = Be(\theta \mid \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} \quad (15) \]

Taking the prior distribution:

Among them, \( \alpha > 0, \beta > 0 \), they are the parameters of Beta distribution, called super parameters. Since the Beta distribution is a conjugate distribution, the obtained posterior distribution is also a Beta distribution:

\[ \text{posterior distribution:} \]
For posterior distribution, the mathematical expectation of $\theta$ is

$$E(\theta | D) = \theta \cdot Be(\theta | \alpha_0, \beta_0) d\theta = \frac{\alpha_0}{\alpha_0 + \beta_0}$$

Thus, given a Beta prior, we can obtain a simple expression for the probability of a strong earthquake during the time period $C_{n+1}$:

$$p(X_{n+1} = S | D) = \frac{\alpha_0 + n_1}{\alpha_0 + \beta_0 + n_1}$$

$\theta$ is unknown parameter.

$n_1$ is the number of strong earthquakes and weak earthquakes.

$n_2$ is the number of strong earthquakes and weak earthquakes.

$\alpha_0$ is the parameters of Beta distribution, super parameters.

$\beta_0$ is the parameters of Beta distribution, super parameters.

$\alpha_1$ is the parameters of Beta distribution, super parameters.

$\beta_1$ is the parameters of Beta distribution, super parameters.

4. Seismic Vulnerability

The seismic vulnerability is the method that deals with the potential damage, which consists of earthquake intensity and seismic fragility. In addition to these, the seismic fragility statistic model can be helpful to increase the accuracy by computing fragility in a statistical aspect. [1].

4.1. Earthquake Intensity

Earthquake intensity is significant for seismic vulnerability analysis. We pick PGA (peak ground acceleration) as the index of earthquake intensity in this report. The model of John Zhao has the most accurate forecast of the peak acceleration and short period spectrum acceleration in the earthquake in Wenchuan, therefore we choose his model as the PGA model in this case:

$$\ln (Y_{i,j}) = aM + bx_{i,j} - \ln (r_{i,j}) + e(h - h_c)\delta_k + F_R + S_I + S_S + S_{SL} \ln (x_{i,j}) + C_k + \xi_{i,j} + \eta_i$$

$$r_{i,j} = x_{i,j} + c \exp (dM)$$

The notations are illustrated as follow:

- $Y_{i,j}$ is the geometric mean of PGA (peak ground acceleration) in two horizontal direction. (unit : gal)
- $M$ is the moment magnitude which is 7.9 in the earthquake at WENCHUAN.
- $x$ is the minimum fault distance. I set the minimum distance from the fault to road or bridge as the minimum fault distance in this report.
- $h$ is the depth of source
- $h_c = 15$km is the constant term of the depth of source
- $\delta_k$ is the modify term of the depth of source which is 0 in this case
- $F_R$ is the reverse fault event coefficient which is 0.251 in this case
- $S_I$ is interface event coefficient which is 0 in this case
- $S_S$ is the subduction slab event coefficient 0 in this case
- $S_{SL}$ is modify term considering the propagation path of earthquake which is 0 in this case
- $C_k$ is the site type coefficient. In this case we set all the site as hard soil site so $C_k = 0.293$
- $\xi_{i,j}$ is the error within earthquakes which is 0 in this case
- $\eta_i$ is the error among earthquakes which is 0 in this case
Other coefficients are given by regression results as follow:
\[ a = 1.101 \quad b = -0.00564 \quad c = 0.0055 \quad d = 1.080 \quad e = 0.01412 \]

4.2. Seismic Fragility
Seismic fragility is defined as the probability that the damage of system, structure, or component exceeds a specific damage state \( y \) for a given level of seismic hazard.

\[
\text{Fragility} = P(\text{Damage} > y | \text{Seismic Hazard})
\]  

Variable \( y \) represents actual responses of the structure under seismic excitations, which depend not only on the GMP but also on the characteristics of the structure, e.g., materials and nonlinear response. Variability in \( Y \) is often assumed to be lognormally distributed, the PDF of \( Y \):

\[
f_Y(y) = \frac{1}{\zeta_Y \sqrt{2\pi}} \exp \left( -0.5 \left( \frac{\ln y - \lambda_Y}{\zeta_Y} \right)^2 \right) \quad 0 < y < \infty
\]

where \( \zeta_Y \) and \( \lambda_Y \) are two parameters of lognormal distribution. They can be computed from \( \sigma_Y \) and \( \mu_Y \).

\[
\lambda_Y = E(\ln y) = \ln \mu_Y - \frac{1}{2} \zeta_Y^2
\]

\[
\zeta_Y^2 = \text{Var}(\ln y) = \ln \left( 1 + \left( \frac{\mu_Y}{\sigma_Y} \right)^2 \right) = \ln \left( 1 + \delta_Y^2 \right)
\]

where \( E(\ln y) \) is the mean of variable \( \ln Y \) and \( \text{Var}(\ln y) \) is the variance of variable \( \ln Y \). We set \( \delta_Y = \sigma_Y / \mu_Y \) as the coefficient of variation. And we assume \( \zeta_Y \approx \delta_Y \) if \( \delta_Y < 0.3 \).

With the assumption that \( Z = \ln Y \sim N(\lambda_Y, \zeta_Y^2) \), we set \( \phi(s) \) as the cumulative distribution function (CDF) of random variable \( Y \) which is lognormally distributed.

\[
F_Y(y) = \phi(s)
\]

\[
S = \frac{Z - \mu_Z}{\sigma_Z} = \frac{\ln y - \lambda_Y}{\zeta_Y}
\]

We assume \( c \) as the median of random variable \( Y \),

\[
\lambda_Y = \ln c
\]

\[
c = \exp(\lambda_Y)
\]

We usually use (median) \( c \) and (log-standard deviation) \( \zeta \) as two parameters of lognormal distribution function.

4.3. Seismic Fragility Statistic Model
Usually we build fragility model and fragility curves for each damage state of the road or bridge independently. The states are 1) slightly damage, 2) medium damage, 3) serious damage and 4) completely destroyed. Then we use maximum likelihood estimator to estimate two parameters of the function. But in this report, we only set two damage states, which are undamaged and damaged for the time and knowledge reason [12][13].

\[
L(c, \zeta) = \prod_{i=1}^{N} F(a_i)^{x_i} [1 - F(a_i)]^{1-x_i}
\]

\( F(*) \) is the fragility curve of a certain damage state.
\( a_i \) is the PGA (peak ground acceleration) of bridge \( i \).
\( x_i \) is the \( x_i \) of Bernoulli trial. \( x_i = 1 \) if it’s damaged and \( x_i = 0 \) if it’s undamaged.
\( N \) is the bridge amount.
\( F(a) \) is defined as follow

\[
F(a) = \phi\left( \frac{\ln a}{\zeta} \right)
\]

where \( a \) is the PGA, \( \phi(*) \) is standard normal distribution.
Then we got estimators of $c$ and $\zeta$.

5. Case Study

I chose Wenchuan, a city in Sichuan province in China, as the example to explain the seismic risk analysis. Due to the limitation of time and data, I simplified the example in diagram and parameters and some computational procedure. More specific simplify procedure will be explained below.

Figure 5. An actual map of example area

Figure 6. Simplified diagram of example area

5.1. PSHA

Figure 2 simplifies the irregular road in Figure 1 into a regular geometric shape for easier analysis. Suppose that $R_i$ is road $i$, $P_i$ is the failure probability of $R_i$, $F_i$ is the fragility curve of road $i$, $H_i$ is the seismic hazard of road $i$, $P(sys)$ is the failure probability of the system, i.e., the probability that transportation between $A$ and $B$ will be interrupted. The road has a failure probability according to fragility model if the fault rupture crosses it. Other part of the highway is not vulnerable to earthquakes. So $P_2=P_5=0$. Earthquake is caused by a 1 km long rupture occurs on the fault. Assume the rupture is equally likely to occur anywhere along the fault, but not extending beyond its ends, $P_i = H_i \ast F_i$,

\[
\frac{d \ln L}{dc} = \frac{d \ln L}{d\zeta} = 0
\]  

(31)

Then we got estimators of $c$ and $\zeta$.

Because it is not easily accessible to the data I mentioned above. So I supposed that $H_i$ is the probability of fault rupture crossing the road. Then I got:

\[
P(sys) = P_1[P_3(1 - P_4) + P_4(1 - P_3) + P_3P_4]
\]  

(32)

$R_i$ is road $i$, $P_i$ is the failure probability of $R_i$, $F_i$ is the fragility curve of road $i$. Because it is not easily accessible to the data I mentioned above. So I supposed that $H_i$ is the probability of fault rupture crossing the road. Then I got:
\[
H1 = \frac{1+1}{1+1+0.5+1} = 0.57 = H3 = H4
\] (33)

5.2. Seismic Vulnerability Analysis
The article uses the model data in the paper (Seismic Vulnerability Analysis for Highway Bridges in Wenchuan Region) [14].

| Table 1. Data fragility model for each road (10 pt regular) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| road            | 1               | 2               | 3               | 4               | 5               |
| \(c\):Median(g) | 0.8901          | 1.0309          | 0               | 1.0235          | 0               |
| \(\zeta\):Log-standard deviation | 0.3933          | 0.3498          | 0               | 0.3566          | 0               |

I only set two damage states in this report as I mentioned above, so \(F_i\) is the probability that the road will be damaged at a certain PGA. Then we set PGA as 1 here to simplify the computation.

\[
F_i = F(a) = \phi\left[ \frac{\ln\left( \frac{a}{\zeta} \right)}{\zeta} \right] \quad (34)
\]

And \(F_1 = \phi\left[ \frac{\ln\left( \frac{1}{0.8901} \right)}{0.3933} \right] = 0.65\), similarly \(F_3 = 0.63\), \(F_4 = 0.62\)

Then we can compute:

\[
P_1 = H1F1 = 0.37, P3 = H3F3 = 0.36, P4 = H3F3 = 0.35
\]
\[
P(sys) = P1[P3(1 - P4) + P4(1 - P3) + P3P4] = 0.21608
\]

Finally, we can calculate the failure probability of the system, the failure probability of the system.

6. Conclusion
Given a Beta prior and using Bayes’ rule, we can make a simple expression of the risk of a strong earthquake in a certain area. Spatial uncertainty deals with the question about where will the earthquake occur. Generally, in computing the likelihood, the seismic source is divided up into small segments, so that each segment could be computed and summed by using integral. All too often, a seismic source can range from a point source, which can be seen as a volcano, to a line source, or even to a source of two-dimensional flat surface. Size uncertainty usually deals with how big the earthquake will be, which is related to the magnitude. To analyse this type of uncertainty, we use recurrence laws to evaluate how often a certain magnitude of earthquake will repeat itself in order for engineers to do risk analysis during decision-making process. Temporal uncertainty is the uncertainty associated with when an earthquake of a given size will occur. As for system analysis, we know that the road has a failure probability according to fragility model if the fault rupture crosses it. Wenchuan, a city in Sichuan province in China, is used as an example to predict the probability that the road will be damaged at a certain PGA.

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