A new direction for dark matter research: intermediate-mass compact halo objects

George F. Chapline\textsuperscript{a} and Paul H. Frampton\textsuperscript{b,1}

\textsuperscript{a}Lawrence Livermore National Laboratory, P.O. Box 808, Livermore, CA, U.S.A.
\textsuperscript{b}15 Summerheights, 29 Water Eaton Road, Oxford OX2 7PG, U.K.

E-mail: george.chapline@gmail.com, paul.h.frampton@gmail.com

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Abstract. The failure to find evidence for elementary particles that could serve as the constituents of dark matter brings to mind suggestions that dark matter might consist of massive compact objects (MACHOs). In particular, it has recently been argued that MACHOs with masses $>15\,M_\odot$ may have been prolifically produced at the onset of the big bang. Although a variety of astrophysical signatures for primordial MACHOs with masses in this range have been discussed in the literature, we favor a strategy that uses the potential for magnification of stars outside our galaxy due to gravitational microlensing of these stars by MACHOs in the halo of our galaxy. We point out that the effect of the motion of the Earth on the shape of the micro-lensing brightening curves provides a promising approach to testing over the course of next several years the hypothesis that dark matter consists of massive compact objects.

Keywords: dark matter experiments, dark matter theory

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1 Introduction

Astronomical observations have led to a consensus that the energy make-up of the visible universe is approximately 70% dark energy (DE), 25% dark matter (DM) and 5% normal matter [1]. While DE apparently represents a vacuum energy, the physical nature of DM remains mysterious [2]. Up to the present time the most popular theories of DM have been that it consists of either weakly interacting elementary particles with a mass between 1 GeV and 1 TeV (WIMPs) or very weakly interacting invisible axions [3].

The hypothesis that DM consists of WIMPs is plausible in the sense that the existence of WIMP-like particles is expected in any fundamental theory of elementary particles that possesses a supersymmetry between bosons and fermions. Supersymmetry gives rise to an R-parity which is needed to prevent unobserved proton decay and gave rise to the expectation that a light R=-1 particle, the neutralino, is the WIMP responsible for DM. Unfortunately, all attempts to identify DM WIMPs have so far failed. Most notably at the CERN LHC, where supersymmetry was widely expected to make an appearance if dark matter consists of WIMPs, there is no sign of any WIMP; the best limits on WIMP detection are from liquid xenon detectors including LUX [4] and PANDA-X [5]. Apparently supersymmetry is broken at an energy scale not accessible to the LHC [6]. Another type of elementary particle that has been much discussed as a candidate for DM is the invisible axion, with a mass $10^{-6} - 10^{-3}$ eV.

Because it appeared that it might be possible to detect a dark matter density of invisible axions using their signature conversion to 2 photons in a strong magnetic field, experiments were initiated to search for DM axions. However, to date no evidence for invisible axions has appeared, although it should be emphasized that only a modest fraction of the allowed mass range has been scanned. Taken together with basic theoretical questions about the axion idea, a reasonable conclusion at this time is that alternatives to elementary particle models for DM should be seriously considered [7]. In this paper we consider the alternative idea that DM consists of massive primordial compact objects in the form of either black holes [8], gravastars [9], or dark energy stars [10].

The first dedicated searches for dark matter massive compact objects within our galaxy (MACHOs), carried out in the 1990’s [11, 12], used the gravitational microlensing technique, so-called because of the very small angular sizes of the lenses as seen from the earth, first suggested by Paczynski [13] and Griest [14]. Although a number of microlensing events were recorded, the observed number of such events could not account for the total density dark matter known to exist in the Milky Way including its halo. The original MACHO search looked for events with durations ranging between about two hours and two hundred days, corresponding to MACHO masses between approximately $10^{-6} M_\odot$ and $15 M_\odot$. However, to
our knowledge, there is no fundamental reason why the technique of looking for gravitational
microlensing used in the original [11, 12] and subsequent MACHO searches [15, 16] cannot
be extended to higher mass MACHOs.

MACHOs were originally conceived of as the end products of stellar evolution i.e. white
dwarfs, neutron stars, black holes, — or possibly brown dwarfs — and as a result of this
prejudice it was not expected that MACHO masses would exceed $\sim 15M_\odot$, corresponding
to the mass of a black hole formed by the collapse of the core of a very massive star whose
nuclear fuel has been exhausted. On the other hand it was suggested many years ago by
one of us [17] that DM might consist of primordial black holes (PBHs). Although it was
initially thought that these PBHs would typically have masses much smaller than the mass
of the Sun [8], theoretical arguments have been advanced [18–20] that PBHs with masses
$> 15M_\odot$ might have been prolifically produced at the onset of the big bang. Indeed there are
cosmological models where primordial production of a DM density of either PBHs [21, 22] or
dark energy stars [23] with masses $> 15M_\odot$ is favored. We will continue to use the acronym
MACHO for these objects because for masses $< 10^5 M_\odot$ such objects can contribute to the
dark matter halo of our galaxy. In the following we will use the acronym IM MACHO to
denote MACHOs with masses in the range

$$15M_\odot < M_{\text{MACHO}} < 10^5 M_\odot \quad (1.1)$$

i.e. intermediate between the masses of black holes formed as a result of stellar evolution
and the supermassive compact objects found at the centers of galaxies. Although the 1990s
search concluded that number of galactic MACHOs with masses $< 15M_\odot$ was insufficient to
explain the density of DM in our galaxy, we believe that the time is ripe to reconsider the
idea that dark matter consists of MACHOs, particularly in the mass range (1). It might be
noted that the recent LIGO detection of gravitational radiation from coalescing $30M_\odot$
black holes is entirely consistent with the hypothesis that DM is made up of objects in this mass
range [24]. In section 2 we will consider the question whether there are other astrophysical
observations that bear on whether DM consists of IM MACHOs. Our general conclusion
is that the question as to whether DM consists of IM MACHOs is best settled by using
gravitational micro-lensing to limit the density of IM MACHOs in our galaxy.

Our main objective in this paper is to point out how the microlensing search techniques
used in previous MACHO searches might be extended to definitively test over the next several
years the hypothesis that the DM mass density is dominated by MACHOs with masses in the
range in eq. (1.1). Our strategy for directly detecting these MACHOs, discussed in more
detail in section 3 is to search for a transient brightening of stars beyond our galaxy that
is consistent with gravitational micro-lensing, and then follow these candidate microlensing
events over a period of a year or more to observe the parallax effect of the earth’s motion
on the microlensing. In section 3 we comment on how well this type of observing program
maps onto the capabilities of astronomical resources that are available either currently or in
the immediate future.

2 Can DM consist of IM MACHOs?

A well known theoretical argument that DM cannot consist of IM MACHOs with masses
in the range eq. (1.1) hinges on the consequences of accretion of interstellar gas onto a
DM density of primordial MACHOs just after the time when electrons and nuclei in the
interstellar gas recombine to form atoms ($z_{\text{rec}} \approx 10^5$). The X-rays emitted by the accretion of
this interstellar gas onto a DM population would heat the interstellar gas, which in turn would cause via the inverse Compton effect a distortion in the CMB both with regard to its spectrum and isotropy. A widely cited attempt to calculate this effect [25] employed the Bondi-Hoyle model for spherical accretion onto black holes, and carries through the computation all the way up to a point of comparison with limits on CMB spectral distortions derived from the COBE satellite observations. In particular it was claimed in [25] that the observed limits on the CMB distortion constrain the density of massive PBHs to only a tiny fraction \(< 10^{-4}\) of the DM density. However the Bondi-Hoyle model for accretion ignores angular momentum conservation. Although this may be a good approximation before recombination when the angular momentum of accreting matter is rapidly dissipated by Compton viscosity [26], it becomes a questionable approximation after \(z_{\text{rec}} \approx 10^3\) when the DM mass-energy density is already much larger than that of the CMB. Indeed it is well known [27] that in the observable universe the Bondi-Hoyle model fails badly as a model for accretion onto the super-massive compact objects at the centers of galaxies — which in fact could themselves be primordial in origin. It was also noted in [25] that the limits on the contribution of massive PBHs to DM is sensitive to the fraction of time that the accretion onto the black hole is interrupted due to the Eddington limit for accretion. However if the IM MACHO is a dark energy star rather than a black hole, the intermittency of the accretion onto the primordial MACHOs would be significantly increased due to a dramatic lowering of the accretion rate where the Eddington limit is reached.

It has also been suggested that the existence of binary stars with wide separations might exclude [28] the existence of a DM density of IM MACHOs; however, the argument is indirect, and at the present time it is uncertain whether the observed abundance of wide binaries is inconsistent with a DM density of IM MACHOs [29]. One argument that can be interpreted to be supportive of the hypothesis that DM does consist of IM MACHOs is that numerical simulations [30] of the spontaneous formation of inhomogeneous DM cosmic structures after \(Z \sim 3000\), which typically use as a model for DM point-like masses with masses in range \(10^3 - 10^4 M_\odot\), reproduce quite nicely the observed large scale inhomogeneous structure of the universe [31]. Of course, the masses used in these simulations were not chosen for any fundamental reason but for computational convenience. On the other hand, the fact that these numerical simulations provide a good account for observed large scale cosmic structures provides a kind of existence proof that, at least in first approximation, IM MACHOs can explain DM.

It is also worth noting that when the DM gravitational dynamics simulations are amended by adding gas dynamics the observed morphologies of galaxies can be explained if the \(10^3 - 10^4 M_\odot\) point masses are supplemented with \(10^5 M_\odot\) point masses [32]. This is consistent with the growing suspicion that the supermassive compact objects at the centers of galaxies may be primordial in origin, although this interpretation of the origin of the supermassive compact objects is still subject to debate. It might also be mentioned that there are theoretical arguments based on observed entropy of the universe [33, 34] that DM should consist of massive compact objects.

3 Searching for IM MACHOs

We are interested in the gravitational lensing of stars just outside our galaxy (e.g. in the Magellanic Clouds) by IM MACHOs within our Galaxy. The possible use of gravitational microlensing to search for MACHOs with masses much higher than \(10 M_\odot\), was first discussed in detail by Gould [35]. From the perspective of an Earth observer the microlensing event sought
Figure 1. Microlensing light curves as a function of time for the MACHO to cross the Einstein ring and the MACHO impact parameter seen by an Earth observer on the Earth. The circle is the Einstein radius. Based on a figure by Penny Sackett published in Sass-Fee Advanced Course 33.

in these searches is simply described as a characteristic transient brightening of the source star, where the time dependence arises from the motion of the lens relative to the observer’s line of sight to the source. Some brightness amplification curves for some selected values of the projected distance of the passing MACHO from the star being imaged are shown in figure 1.

The $x$-axis in figure 1 is the Einstein ring crossing time $t_E \equiv R_E/v$, where $R_E = (4GMD/c^2)^{1/2}$ is the radius of the Einstein ring and $v$ is the MACHO velocity perpendicular to the line of sight to the star, which for a MACHO in the galactic halo can be approximated as the virial velocity for the halo; i.e. $\sim 210\text{km/s}$. $D$ is related to the distance from the microlensing object to the star $d_s$ and distance from the Earth to the microlensing object $d_L$ (L=lens) by $D = d_L d_s/(d_L + d_s)$. For sources in the Magellanic Clouds $D$ would typically be $\approx 10$ kpc. This tells us that the duration of the microlensing brightening will be on the order of the Einstein ring crossing time $t_E \approx 0.2\sqrt{(M/M_\odot)}$ years, so for $100M_\odot < M < 10^4M_\odot$ one would have $2yr < t_E < 20yr$. Since the maximum practical duration of a survey is perhaps 10 years, a challenge faced by our proposal is that many of the events we are interested in may have traversed only a fraction of their full light curve during the survey time.

A pertinent question is how many micro-lensing events could one hope to detect during a survey of say 5 years duration? The number of microlensing events in progress at any time,
defined as those with a source inside the Einstein ring of the lens, is given by \( N_{\text{event}} = N\tau \) where \( N \) is the number of source stars being monitored and \( \tau \) is the micro-lensing optical depth. \( \tau \) is independent of \( M \), depending only on the structure of the Galaxy and the assumption that the dark matter which determines its kinematics consists of MACHOs. A value of \( \tau \sim 5 \times 10^{-7} \) is a reasonable estimate for the optical depth to the Magellanic Clouds [11]. Determining \( N \) is more difficult, since it depends not only on the actual number of stars in the survey stellar field to some limiting magnitude, but also on the capabilities of the algorithms to identify individual stars. As reference points, the MACHO survey [11] monitored \( 10^7 \) stars in the central region of the LMC while the OGLE survey [16] monitored \( 8 \times 10^7 \) stars in both Magellanic Clouds. Both surveys monitored stars brighter than visual magnitude 20.5, a limit which unfortunately missed the bulk of the Magellanic Cloud’s main sequence stars. With the 4 meter Blanco telescope one could probably reach \( V = 23.5 \text{ mag} \), while with the 8 meter LSST one could do even better, going to \( V = 24.5 \text{ mag} \). If we assume that we can resolve all the stars in the Magellanic Clouds down to \( V = 24.5 \text{ mag} \), then conservatively \( N > 10^8 \) could yield \( N_{\text{event}} > 50 \). This task will be challenging, given that for IM MACHOs near to the top of the mass range (1) the stellar brightness will vary over the survey duration by amounts which are not much larger than the photometric errors (For the LSST: 0.001mag per exposure up to \( V=20\text{mag} \); 0.1mag per exposure up to \( V=24.5\text{mag} \)), and that there will be other sources of stellar variability, both due to astrophysical effects and systematic errors in photometry.

For the purposes of eliminating spurious variations in stellar brightening, the previously used technique of looking for achromatic variation will still be useful. One advantage of monitoring a candidate microlensing event over a period longer than 1 year though is that this provides an opportunity to use the motion of the Earth around the Sun. As first suggested by Gould [35] it is possible to the parallax effect of the Earth’s motion on the microlens brightening curve to not only distinguish microlensing from intrinsic brightening, but also say quite a bit about the phase space density of MACHOs. Figure 2 shows the effect of the Earth’s motion on the micro-lensing magnification curve as a function of MACHO mass. This parallax effect, first observed by the MACHO collaboration in [36], would allow one to distinguish microlensing brightening from intrinsic stellar variability or background sources (e.g. variable stars, supernova, etc.) that may exhibit brightening but no parallax effect. In figures 2 and 3 we illustrate how the parallax effect depends on MACHO galactic velocity and distance to the MACHO. Such information would be invaluable for confirming that the incipient brightening has the properties expected if IM MACHOs indeed form the DM halo of our galaxy. Further details about the parallax effect are provided in [37].

It also was suggested by Gould [38] that the masses of the microlensing objects can in principle be estimated by directly observing the Einstein image of the background star. As candidates are identified, their compact nature may be unambiguously determined using adaptive optics imaging from the ground. For masses between \( 10^3 M_\odot \) and \( 10^6 M_\odot \), the angular size of the Einstein ring is \( 0.025 < \theta_E < 0.8 \text{ arcsec} \). This is comparable to the demonstrated 0.032 arcsec angular resolution of Southern Hemisphere Magellan adaptive optics system, so direct imaging of the Einstein images for the most massive IM MACHOs may already be a possibility. However, direct imaging of the Einstein images over the full mass range in eq. (1.1) will probably have to wait until the first light for the Thirty Meter or Giant Magellan Telescopes.

Of course, in order to identify an incipient microlensing event the brightness of each individual star with must be recorded with sufficient accuracy with each exposure to deter-
Figure 2. An illustration of the differential effect of Earth’s orbital motion around the sun on observed micro-lensing curves as a function of the lens distance.

mine if the star has brightened in a manner consistent with micro-lensing. This accuracy will be limited by the duration of each exposure and how often the exposure is made; i.e. the “cadence”. Among existing astronomical facilities, the Blanco Dark Energy Camera in Chile may be the best candidate for achieving the \( \sim 1\% \) level of photometry accuracy per exposure that would needed to identify IM MACHO candidates in the next few years. The LSST, which is expected to be able to measure stellar brightness with an accuracy of \( \sim 1\% \) for 100 30-second exposures at all magnitudes up to \( V=24.5 \), would almost certainly be capable of identifying IM MACHO candidates after it becomes available as a user facility. Using the LSST to monitor the Magellanic Clouds, we estimate that a 30 second exposure every 2 weeks ought to suffice to produce \( > 100 \) micro-lensing candidates over a period of \( \sim 5 \) years after first light if DM does indeed consist of IM MACHOs.

4 Summary

As noted in introduction the recent LHC results have cast doubt on the hypothesis that DM consists of WIMPs, and have refocused attention on the idea that primordial MACHOs may be responsible for DM. While it as been argued that a DM density of MACHOs could be discerned in the statistics of gravitationally weakly bound structures or limited by the
Figure 3. An illustration of the differential effect of Earth’s orbital motion around the sun on observed micro-lensing curves as a function of MACHO velocity and distance. This effect may allow one to determine the MACHO mass spectrum.

observed homogeneity of the cosmic microwave background, methods based on the gravitational lensing of light offer a more direct approach to determining whether for DM consists of massive MACHOs. Although it may be possible to detect IM MACHOs using existing instruments such as Blanco DECam and Magellani AO system. the LSST when it comes online circa 2022 will provide a definitive capability for rapidly detecting and determining the mass spectrum of DM MACHOs, if they exist.

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