TWO-DIMENSIONAL HYDRODYNAMIC EROSION MODEL APPLIED TO SPUR DYKES

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Abstract

With the advances in the field of computing, robust CFD models have evolved in the last two decades. Initially, one and two-dimensional models were used but these days, three-dimensional models are used frequently that produce more accurate results. However, the solution of 3D models is expensive not only in terms of computational costs but is time-consuming. In this work, a two-dimensional CFD model that is based on shallow water equations coupled with an erosion model is presented. The equations are solved using finite volume formulation and high-resolution shock capturing methods. This study is an attempt to cover accuracy issues with 2D models by incorporating high-resolution shock capturing methods as compared to 3D models, the solution of which is based on conventional schemes.

The model is initially used to simulate dam-break problems over fixed and mobile beds to assess the model stability and hydraulic performance in terms of simulating the flow and bed morphology. The assessment has shown the model to be stable throughout the simulation and the produced results have shown the hydro-dynamic capability of the model. The model is then applied to simulate flow over an erodible sediment bed in a channel with spur dykes on its flood plain. The simulated results are compared with experimental results and numerical results of a 3D model. The comparison has shown a close agreement both with experimental and numerical 3D model results that show that the model could be applied to study bed morphology confidently.

Keywords: CFD, High Resolution, Shock Capturing, Mobile Beds.
I. Introduction

Spur dikes are river training structures with one end situated on the bank while the other end is projected into the current, mainly provided to protect stream banks from erosion by reducing the current along the banks. The hydro-engineering structures are sometimes built at right angles or inclined angles to the flow depending on the site conditions and desired function. It prevents the erosion of the banks by slowing down the velocity of currents while also encouraging the growth of aquatic habitats in the still pools formed between consecutive structures.

Though these structures have been used for ages for protecting the river banks they also undergo failures in a variety of ways depending on the failure mechanism generated within the structure. In general, the most common reasons for spur dikes failures are:

1. Run-over or wave overflow where the height of the spur dike is too low as compared to the water level and allows the water to flow over the crest resulting in erosion of the crest surface and infiltration into the material below which may result in the land side of the structure;
2. Instability of outer slope protection where the material used to protect the embankment (masonry, pitching, or rockfill) is eroded and eventually removed and washed away by the waves causing the formation of holes or exposure of the material underneath and ultimately leading to failure of structure;
3. Piping where the local groundwater may flow into the structure and erode it by the movement of sediments. The sediment transportation may also occur due to high water levels which force the water to seep through the embankment and cause erosion inside the embankment either below or behind the dike on the land side;
4. Horizontal sliding where the structure is completely pushed aside by high water pressure.

Initially, most of the research regarding scouring and spur dikes has been carried out using experimental modeling [VI], [XVIII], [XIV], [XVI]. These researchers aimed to identify the reasons for erosion and to enhance the protection of these structures by carrying out tests in different conditions and modes of failure. With the advances in sophisticated and more robust computing technologies, numerical modeling has become a reliable tool for analyzing physical problems in the field of computational fluid dynamics. Experimental work has been replaced with numerical modeling up to a significant extent in the last two decades. Such models include simulation of flow over mobile beds by coupling shallow water models with erosion models. Initially, 1D models were used and with the availability of more robust computing technologies, these days 2D and 3D models are extensively used. In general, extensive literature on studies of 1D, 2D and 3D numerical models for simulation of flow over mobile beds are available [XXIV], [XII], [XXIX], [I], [XIII], [III], [XIX], [IV], [IX], [XV], [XII], [I], [X], [XXI], [V], [VIII]. Though the 3D models are more accurate and produce results reflecting the true physical phenomenon, however, they have proved to be time-consuming and computationally expensive.

This study presents a 2D model coupled with an erosion model. The solution of the model is based on finite volume formulation and using shock-capturing
techniques. The model is applied to simulate flow over a mobile sediment bed in a channel with spur dykes. The results show that the 2D model when solved on high-resolution shock capturing methods produces results comparable with 3D models.

II. Mathematical Formulation

Governing Equation

The shallow water equations that consist of the mass and the momentum conservation equations in two dimensions are derived from Navier Stokes Equations. The source terms due to the bed mobility, both deposition and erosion, are incorporated in the equations as proposed by [XXIX].

The mass conservation for the water-sediment mixture for two-dimensional sets of the equation can be written as:

$$\frac{\partial}{\partial t} h + \frac{\partial}{\partial x} (hu) + \frac{\partial}{\partial y} (hv) = \frac{E - D}{1 - p}$$  (1)

The momentum conservation, in the longitudinal direction, of the shallow water equation for water-sediment mixture can be expressed as:

$$\frac{\partial}{\partial t} (hu) + \frac{\partial}{\partial x} \left( hu^2 + \frac{1}{2} gh^2 \right) + \frac{\partial}{\partial y} (huv) = -gh \left( \frac{\partial z_b}{\partial x} + S_{fx} \right) - \frac{(\rho_s - \rho_f)gh^2 c}{2\rho} \frac{\partial c}{\partial x} + \frac{(\rho_s - \rho_f)(E - D)u}{\rho(1 - p)}$$  (1)

Similarly, the other component of the momentum conservation equation, transverse, is expressed as:

$$\frac{\partial}{\partial t} (hv) + \frac{\partial}{\partial x} (huv) + \frac{\partial}{\partial y} \left( hv^2 + \frac{1}{2} gh^2 \right) = -gh \left( \frac{\partial z_b}{\partial y} + S_{fy} \right) - \frac{(\rho_s - \rho_f)gh^2 c}{2\rho} \frac{\partial c}{\partial y} + \frac{(\rho_s - \rho_f)(E - D)v}{\rho(1 - p)}$$  (2)

The mass conservation of the sediment material can be expressed by the equation

$$\frac{\partial}{\partial t} (hc) + \frac{\partial}{\partial x} (huc) + \frac{\partial}{\partial y} (hvc) = E - D$$  (3)

where

- \text{h = water depth, } u = \text{depth averaged velocity along x-axis, } v = \text{depth averaged velocity along y-axis, } z_b = \text{elevation of the channel bed, } \rho_s = \text{density of the bed material, } \rho_f = \text{density of the fluid, } c = \text{sediment concentration, } E = \text{erosion, } D = \text{deposition, } S_{fx} = \text{friction slope in x direction, } S_{fy} = \text{friction slope in y direction.}
Model Closure

Empirical equations are required to use the above equations in the numerical model. Empirical relations that incorporate Manning’s \( n \) are used for the friction slope. That is:

\[
S_{fx} = \frac{n^2 u^2}{h^{4/3}} \tag{5}
\]

\[
S_{fy} = \frac{n^2 v^2}{h^{4/3}} \tag{6}
\]

Empirical functions introduced by [XXVII] for sediment deposition and bed erosion are used in the model. That is:

\[
D = \omega_0 (1 - C_a)^m C_a \tag{7}
\]

And

\[
E = \frac{160}{R^{0.8}} \frac{(1-p)(\theta - \theta_c) d U_\infty}{\theta_c h} \tag{8}
\]

where \( \omega_0 \) = settling velocity of the sediment particle; \( C_a \) = sediment concentration in the vicinity of the channel bed; \( m \) = an exponent; \( R \equiv \sqrt{sg \delta d / \nu} \); \( s = \rho_s / \rho_w - 1 \); \( \nu \) = kinematic viscosity of fluid; \( U_\infty \) = free surface velocity; \( \theta \equiv U_\infty^2 / s g D = \) shields parameter; \( U_\ast \) = friction velocity; and \( \theta_c \) = critical shields parameter responsible for initiating sediment movement.

Equations (7) and (8) have been modified by approximating \( C_a = \alpha c \), where \( \alpha = \min[2, (1 - p)/c] \) and \( U_\infty = 7u/6 \) [XXIX]. Also, the following values were incorporated:

\[
D = \alpha \omega_0 (1 - \alpha C_a)^m \tag{9}
\]

And

\[
E = \begin{cases} (\theta - \theta_c) u h^{-1} d^{-0.2} & \text{if } \theta \geq \theta_c \\ 0 & \text{else} \end{cases} \tag{10}
\]

III. Numerical Scheme

The shallow water equations discussed in the aforementioned section are numerically solved using finite volume formulation that solves the partial differential equations on the unstructured triangular mesh using the MUSCL-Hancock Scheme [II]. This scheme ensures high order accuracy both in space and time. This scheme updates the element variable in three different steps, that are:

- **Step 1 - Data Reconstruction**: The data is reconstructed through interpolation at the sides of each control volume/element. For the interpolation of data Maximum Limited Gradient (MLG) limiter is used that was developed by Batten [XVIII]. MLG limiter is ordered four limiters and the control volume uses the information of the three surrounding elements as shown in Error! Reference source not found..

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Figure 1. Data reconstruction - the shaded elements are the three surrounding elements of the control volume.

- **Step 2 - Evolution of State variables:** In this step, a transitional solution is found by advancing the interpolated data in step 1 by half a time step. This step is vital to achieving a higher-order solution in time.

- **Step 3 - Solution of the piecewise constant data - Riemann problem:** The intermediate solution in step 2 computes the state variables in each control volume. At this stage the states variable at the sides of each control volume using the limiter as discussed in step 1. The resulted in two values of each state variable on the left and right of the sides of each control volume define the Riemann problem. The solution to this Riemann problem is achieved by an HLLC solver that is the improved form of Harten-Lax-Van Leer (HLL) solver [VII].

**IV. Application**

In this section of the work, first, the model is tested by simulated a dam break scenario both for fixed and erodible sediment beds to assess the model performance. Then the model is applied to simulated the flow of the sediment-water mixture over a mobile sediment bed in a channel with spur dykes installed on the flood plain of the channel.

**Dam Break Test**

This is test is performed to assess the model performance and stability before the model is applied to spur dykes, the main objective of this study. The test consists of a dam break simulation over fixed and erodible mobile beds with dry beds on the d/s of the dam. The setup for the test consists of a rectangular area of 1000 meters long by 500 meters wide with the location of the dam in the middle of the area along the longitudinal direction. The initial condition used for the simulation is given in Table 1.
Table 1: Initial Conditions for the dam break test

|     | u/s reservoir depth | d/s reservoir depth | Manning’s n | Inflow to the reservoir | Sediment Diameter |
|-----|---------------------|---------------------|-------------|-------------------------|-------------------|
|     | 4 m                 | 0 m                 | 0.03        | 0 m³/s                  | 4 mm              |

First, the test is performed on the fixed bed and over the erodible bed with all the initial conditions being the same.

Figure 2 and Figure 3 show results of the aforementioned tests on fixed and erodible beds respectively. By comparing the results, it is evident that the wavefront traveling along the mean direction of the flow for the fixed bed is considerably faster than that for the erodible bed while for the transverse direction the wave is faster for the erodible bed as compared with the bed fixed. The obvious reason behind this is definitely that the erosion pit on the downstream of the dam transfers a part of the longitudinal momentum is transferred the transverse direction. This observation was also made by Scott [III].

Figure 2. Wave front of Dam Break on Fixed Flat Bed after 20 Seconds of the Dam Break

Figure 3. Wave front of Dam Break on Erodible Flat Bed after 20 Seconds of the Dam Break

Figure 4 shows the erosion pattern in the vicinity of the dam break while Figure 5 shows the velocity vectors for the flood wave as a result of the dam break.

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Both the patterns of erosion and velocity vectors in the locality are as expected with no abnormalities, spikes and negative values.

Figure 4. Bed erosion near the dam for the dam break over an erodible bed

Figure 5. Velocity vectors of the wavefront of the dam break over an erodible bed

The results of the aforementioned test show the reliability of the proposed model and could be confidently applied to spur dykes that are discussed in the proceeding section.

Application to Spur Dykes

In this section results produced by the proposed model are compared with experimental data by Muto [XXVI] and numerical results of a three-dimensional model by Nakagawa [XI] for flow over erodible sediment bed in a channel with spur dykes on the flood plain. The schematic diagram of the channel along with spur dykes are shown in Figure 6.

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Figure 6. Setup of the channel with spur dykes. The longitudinal slope of the channel is 1/700 [XXVI]

The hydraulic initial and boundary conditions used for the experimental setup of Muto [XXVI] and the 3D model by Nakagawa [XI] are adopted for the simulation which is given in Table 2.

**Table 2: Hydraulic conditions for the experiment and simulation**

| Discharge  | Water Depth | Mean Vel | Froude Number | Friction Vel |
|------------|-------------|----------|---------------|--------------|
| 8.23 l/s   | 4.30 cm     | 27.34 cm/s | 0.42          | 2.45 cm/s    |

The experimental results produced significant scour holes near the first two spur dykes as shown in Figure 7 while for the rest of the spur dykes the scour holes are much smaller and less significant. The 3D model proposed by Nakagawa [XI] had shown a similar pattern as shown in Figure 8.

Figure 7. Experimental results by [XXV] of scouring pattern around the first two spur dykes.

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The proposed 2D model has also resulted in almost the same pattern of the bed morphology, as shown in Figure 9. Comparison of the results of the proposed model with experimental and numerical 3D model leads to the following observations:

- The location of the deepest hole in the channel bed by the proposed model is the same as that by the experiment and the 3D model.
- The experiment and 3D model have resulted in the deepest scour holes that are 4.37cm and 3.99cm respectively while the value resulting from the proposed model is 3.29cm. The comparison is shown in Figure 11.
- In addition to the aforementioned observation, the velocity vector of the flow patterns resulted from the proposed model is shown in Figure 10. The velocity vectors show that part of the flow near the heads of the spur dykes is diverted towards the main channel and the rest circulates in the bay between the two spurs with negligible velocity.

Figure 9. Numerical results of the proposed 2D model, showing the scoring pattern around the first two spur dykes.
Figure 10. Velocity vectors of the flow near the first two spur dykes in the channel were produced as a result of the proposed model.

Figure 11. Comparison of maximum scour depths resulted by experiment, 3D and 2D models.

IV. Conclusions

In this study, a two-dimensional model that is based on shallow water equations coupled with an erosion model was presented. The integrated model was solved on the unstructured triangular mesh using finite volume formulation and high-resolution shock-capturing techniques. The model was initially tested by simulating a dam-break flow over fixed and mobile beds to assess the performance and stability of the model. The results significantly reflected the true physical expected scenario for both cases. The comparison of the patterns of the wave propagation in the longitudinal and transverse directions for fixed and mobile beds revealed interesting and realistic physical phenomena.

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The model was successfully applied to simulate the flow over a mobile sediment bed in a channel with spur dykes. From the simulated results of the flow in the channel, the following points are concluded:

- The smaller value of the scour hole could be due to the absence of the third component of velocity. Also, the turbulence term is not incorporated in the two-dimensional model.
- The slope of the scour hole from the numerical results is also in very close agreement with the experimental results in contrast to the 3D model where the slope is smaller as compared to the experimental results. This could be attributed to the high-resolution shock-capturing technique adopted for the solution of the 2D model that can track discontinuities with higher accuracy in comparison with the traditional methods.

In a nutshell, it is concluded that the proposed 2D model gives significantly close results as compared with the 3D turbulent model. The accuracy is achieved through high-resolution shock-capturing techniques. The proposed model could be reliably adapted for flow over mobile beds where 3D models are computationally expensive and time-consuming.

Conflict of Interest

There was no relevant conflict of interest regarding this paper.

References

I. A. Canestrelli, M. Dumbser, A. Siviglia, and E. F. Toro, “Well-balanced high-order centered schemes on unstructured meshes for shallow water equations with fixed and mobile bed,” Adv. Water Resour., vol. 33, no. 3, pp. 291–303, 2010.

II. B. van Leer, “Towards the ultimate conservative difference scheme. V. A second-order sequel to Godunov’s method,” J. Comput. Phys., vol. 32, no. 1, pp. 101–136, 1979.

III. C. F. Scott and F. A. Khan, “Two-Dimensional Dam Break Hydraulics Over an Erodible Bed,” Annual Conference on Hydraulic Engineering, Dresden, 2010.

IV. C. Juez, J. Murillo, and P. García-Navarro, “A 2D weakly-coupled and efficient numerical model for transient shallow flow and movable bed,” Advances in Water Resources, vol. 71, pp. 93–109, 2014.
V. D. Santillán, L. Cueto-Felgueroso, A. Sordo-Ward, and L. Garrote, “Influence of Erodible Beds on Shallow Water Hydrodynamics during Flood Events,” Water, vol. 12, no. 12, p. 3340, 2020.

VI. E. Elawady, M. Michiue, and O. Hinokidani, “Experimental Study of Flow Behavior Around Submerged Spur-Dike On Rigid Bed,” Proc. Hydraul. Eng., vol. 44, pp. 539–544, 2000.

VII. E. F. Toro, “Shock-capturing methods for free-surface shallow flows,” 2001.

VIII. F. Bahmanpouri, M. Daliri, A. Khoshkonesh, M. Montazeri Namin, and M. Buccino, “Bed compaction effect on dam break flow over erodible bed; experimental and numerical modeling,” J. Hydrol., 2020.

IX. G. Kesserwani, A. Shamkhalchian, and M. J. Zadeh, “Fully Coupled Discontinuous Galerkin Modeling of Dam-Break Flows over Movable Bed with Sediment Transport,” J. Hydraul. Eng., vol. 140, no. 4, 2014.

X. H. Hu, J. Zhang, and T. Li, “Dam-Break Flows: Comparison between Flow-3D, MIKE 3 FM, and Analytical Solutions with Experimental Data,” Appl. Sci., vol. 8, no. 12, Dec. 2018.

XI. H. Nakagawa, H. Zhang, and Y. Muto, “Modeling of sediment transport in alluvial rivers with spur dykes,” in Ninth International Symposium on River Sedimentation, Yichang, China, pp. 18–21, 2004.

XII. J. H. Almedeij and P. Diplas, “Bedload Transport in Gravel-Bed Streams with Unimodal Sediment,” J. Hydraul. Eng., vol. 129, no. 11, pp. 896–904, 2003.

XIII. J. Xia, B. Lin, R. A. Falconer, and G. Wang, “Modelling dam-break flows over mobile beds using a 2D coupled approach,” Adv. Water Resour., vol. 33, no. 2, pp. 171–183, 2010.

XIV. M. Ghodsian and M. Vaghefi, “Experimental study on scour and flow field in a scour hole around a T-shape spur dike in a 90° bend,” Int. J. Sediment Res., vol. 24, no. 2, pp. 145–158, 2009.

XV. M. J. Creed, I.-G. Apostolidou, P. H. Taylor, and A. G. L. Borthwick, “A finite volume shock-capturing solver of the fully coupled shallow water-sediment equations,” Int. J. Numer. Methods Fluids, vol. 84, no. 9, pp. 509–542, 2017.

XVI. M. Vaghefi, S. Solati, and C. Abdi Chooplou, “The effect of upstream T-shaped spur dike on reducing the amount of scouring around downstream bridge pier located at a 180° sharp bend,” Int. J. River Basin Manag., 2020.

XVII. P. Batten, C. Lambert, and D. M. Causon, “Positively conservative high-resolution convection schemes for unstructured elements,” Int. J. Numer. Methods Eng., 1996.
XVIII. R. A. Kuhnle, C. V. Alonso, and F. D. Shields, “Local Scour Associated with Angled Spur Dikes,” J. Hydraul. Eng., vol. 128, no. 12, pp. 1087–1093, 2002.

XIX. S. Zhang and J. G. Duan, “1D finite volume model of unsteady flow over mobile bed,” J. Hydrol., vol. 405, no. 1–2, pp. 57–68, 2011.

XX. Sepehr Mortazavi Farsani, Najaf Hedayat, Nelia Sadeghi Khoveigani, : Numerical Simulation of the effect of simple and T-shaped dikes on turbulent flow field and sediment scour/deposition around diversion intakes, J. Mech. Cont.& Math. Sci., Vol.-14, No.-4, July-August (2019) pp 197-215

XXI. T. Uchida and S. Fukuoka, “Quasi-3D two-phase model for dam-break flow over movable bed based on a non-hydrostatic depth-integrated model with a dynamic rough wall law,” Adv. Water Resour., vol. 129, pp. 311–327, 2019.

XXII. Uzair Ali, Syed Shujaat Ali, : SIMULATION OF RIVER HYDRAULIC MODEL FOR FLOOD FORECASTING THROUGH DIMENSIONAL APPROACH, J. Mech. Cont.& Math. Sci., Vol.-15, No.-1, January (2020) pp 275-282

XXIII. W. Wu and S. S. Wang, “One-Dimensional Modeling of Dam-Break Flow over Movable Beds,” J. Hydraul. Eng., vol. 133, no. 1, pp. 48–58, 2007.

XXIV. X. Liu, A. Mohammadian, and J. Á. Infante Sedano, “A numerical model for three-dimensional shallow water flows with sharp gradients over mobile topography,” Comput. Fluids, vol. 154, pp. 1–11, 2017.

XXV. X. Zhang, P. Wang, and C. Yang, “Experimental study on flow turbulence distribution around a spur dike with different structure,” in Procedia Engineering, vol. 28, pp. 772–775, 2012.

XXVI. Y. Jia and S. S. Y. Wang, “Numerical Model for Channel Flow and Morphological Change Studies,” J. Hydraul. Eng., vol. 125, no. 9, pp. 924–933, Sep. 1999.

XXVII. Y. Muto, K. Kitamura, A. Khaleduzzaman, and H. Nakagawa, “Flow and bed topography around impermeable spur dykes,” Adv. River Eng. JSCE, vol. 9, 2003.

XXVIII. Z. Cao, “Equilibrium near-bed concentration of suspended sediment,” J. Hydraul. Eng., vol. 125, pp. 1270-1278, 1999.

XXIX. Z. Cao, G. Pender, S. Wallis, and P. Carling, “Computational Dam-Break Hydraulics over Erodible Sediment Bed,” J. Hydraul. Eng., vol. 130, no. 7, pp. 689–703, 2004.