Chiral approach to massive higher spins

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We propose a new, chiral description for massive higher-spin particles in four spacetime dimensions, which facilitates the introduction of consistent interactions. As proof of concept, we formulate three theories, in which higher-spin matter is coupled to electrodynamics, non-Abelian gauge theory or gravity. The theories are chiral and have simple Lagrangians, resulting in Feynman rules analogous to those of massive scalars. Starting from these Feynman rules, we derive tree-level scattering amplitudes with two higher-spin matter particles and any number of positive-helicity photons, gluons or gravitons. The amplitudes reproduce the arbitrary-multiplicity results that were obtained via on-shell recursion in a parity-conserving setting, and which chiral and non-chiral theories thus have in common. The presented theories are currently the only examples of consistent interacting field theories with massive higher-spin fields.

I. INTRODUCTION

The study of higher-spin fields is a formidable subject. In the massless case, such fields possess rich gauge symmetry, starting from the familiar case of electromagnetism, see e.g. [1, 2] for recent reviews. The standard approach is to introduce fields of spin s as symmetric traceless tensors $\Phi^{\mu_1...\mu_s}$ or spinors $\Psi^{\mu_1...\mu_s-1/2}$. Massive higher-spin fields [3, 4] are more subtle and require a host of auxiliary fields that prevent propagation of unphysical degrees of freedom in interacting theories [7, 8].

Massive higher-spin particles do exist in nature as composite states of elementary particles (see e.g. [9]). One should therefore be able to describe their physics by suitable effective field theories involving an infinite hierarchy of higher-dimensional operators. Although only a finite subset of such operators contribute at any given order in the energy-scale cutoff, it is a laborious task to even list them in a manner consistent with no ghost propagation.

For these reasons, the space of massive higher-spin theories seems vast and riddled with obstacles. It seems plausible, however, that there are hidden gems among such theories, as indicated by recent on-shell studies. For these reasons, the space of massive higher-spin theories seems vast and riddled with obstacles. It seems plausible, however, that there are hidden gems among such theories, as indicated by recent on-shell studies. For instance, at the level of 3-point scattering amplitudes, various possible on-shell spinor structures for an electromagnetically interacting higher-spin particle of mass $m$ have been classified by Arkani-Hamed, Huang and Huang (AHH), and one out of them was singled out [10] due to its relatively tame behavior in the massless limit, namely

$$A(1^{s}, 2^{(b)}, 3^+) = \frac{\langle 1^{a_1} 2^{(b_1)} ... 1^{a_{2s}} 2^{(a_{2s})} \rangle}{m^{2s}} A(1, 2, 3^+)$$

(1)

Here $A(1, 2, 3^+)$ is the positive-helicity photon emission amplitude in scalar quantum electrodynamics (QED), whereas all higher-spin information is encoded into two sets of SU(2) little-group indices, $\{a_1, ..., a_{2s}\}$ and $\{b_1, ..., b_{2s}\}$, via $2s$ copies of the same chiral product $\langle 1^{a} 2^{b} \rangle := \epsilon_{\alpha\beta} |1^{a}|^\alpha (2^{b})^\beta$ of massive-spinors.

Interestingly, the AHH amplitude may be extended [11] to include $(n-2)$ positive-helicity photons, gluons or gravitons instead of one, while still having the same massive-spin structure as Eq. (1), see e.g. Eq. (32) below for gauge theory. These amplitudes can be derived from mere knowledge of their factorization limits by naive use of on-shell (BCFW) recursion [13, 14] — in exactly the same way that is known to work for their spin-1/2 counterparts in quantum chromodynamics (QCD) [15]. Unlike mixed-helicity configurations, where the same approach produces answers afflicted by unphysical poles [10, 16], the like-helicity amplitudes involving a pair of higher-spin particles seem absolutely healthy. Another argument in favor of the like-helicity results is that they can be derived [12] using two distinct BCFW constructions: either complex-shifting two massless momenta [14] or one massless and one massive [17, 18].

It is well-known that on-shell recursion fails when the desired amplitudes have bad boundary behavior, i.e. they do not vanish as the complex BCFW-shift parameter $z$ is taken to infinity. This is something that should absolutely be expected from generic effective theories, where the action is built out of higher-dimensional vertex operators, typically with a growing number of derivatives. However, the existence of healthy $n$-point amplitudes that are constructible from their factorization limits (albeit in a restricted helicity sector) suggests that they should belong to a well-defined massive higher-spin theory — one that is intimately related to the minimally coupled scalar theory and thereby satisfies the boundary-behavior condition allowing for on-shell recursion.

This is precisely the kind of theories that we present in this paper. We start with Sec. [1] where we review the common difficulties in higher-spin field theory and show how they are avoided by a new chiral Lagrangian description of a free massive higher-spin field in four dimensions.
Minimal gauge interactions are then added to this theory in Sec. II. After that, we present the corresponding chiral theory of gravitationally interacting higher-spin field Sec. IV. Finally, we conclude by discussing some implications of our theories in Sec. V.

II. FREE HIGHER-SPIN THEORY

In this section we discuss the basics of describing a free higher-spin particle. For simplicity, let us temporarily concentrate on integer-spin particles. In the standard approach [4], integer-spin fields constitute symmetric traceless rank-s tensors $\Phi_{\mu_1...\mu_s}$. For instance, in electromagnetism, i.e. the theory of a free massless spin-1 boson, the off-shell field is $A_{\mu}$, and its connection to the on-shell particle description relies on the notion of polarization vector $\varepsilon_{a\mu}$, which converts the Lorentz index into the helicity label. Helicity governs the particle’s irreducible representation of the little group U(1) for a given massless momentum $p$, and raised with the two-dimensional Levi-Civita tensor.

For $p^2 = m^2 \neq 0$, one can employ symmetric rank-2s tensors as irreducible representations of the massive little group SU(2) with exactly (2s + 1) degrees of freedom. For $s = 1$, we therefore use polarization vector $\varepsilon^a = \varepsilon^{a1}$, where $a_1$ are fundamental SU(2) indices, whereas free-spin-higher-spin fields are naturally expanded in terms of the polarization tensors

$$\varepsilon_{\mu_1...\mu_s} := \varepsilon^{a_1...a_s} \varepsilon^{(a_1...a_s)}.$$

Symmetry in the Lorentz indices is then obvious, and tracelessness $\eta^{\mu\nu} \varepsilon_{\mu_1...\mu_s} = 0$ follows from the spin-1 orthonormality

$$\varepsilon_{\mu\nu} \varepsilon_{\rho\sigma} = -\varepsilon_{\nu\rho} \varepsilon_{\sigma\mu}, \quad \varepsilon_{\mu\nu} \varepsilon_{\rho\sigma} := \varepsilon_{\mu\nu\rho\sigma} = (\varepsilon_{\mu\nu})^\ast,$$

where have used the fact that SU(2) indices are lowered and raised with the two-dimensional Levi-Civita tensor. All external wavefunctions of quantum fields can be built from basic fields $\Phi_{\alpha1...\alpha_{2s}}$ in the chiral (2s,0) representation of the Lorentz group instead of $(s,s)$. As we will shortly see, this essentially trivializes the transition between the off-shell symmetry group SL(2, C) and the on-shell little-group SU(2).

To be more specific, we employ the massless spinor-helicity formalism [10 27], which provides perfect building blocks for this construction. Namely, we use chiral and antichiral on-shell spinors $|p\rangle$ and $|\bar{p}\rangle$, such that

$$m = 0 \quad \Rightarrow \quad |p\rangle_\alpha |p\rangle_\beta := p_\mu \sigma^\mu_{\alpha\beta},$$

$$m \neq 0 \quad \Rightarrow \quad |p^\dagger\rangle_\alpha |p^\dagger\rangle_\beta := p_\mu \sigma^\mu_{\alpha\beta}. $$

All external wavefunctions of quantum fields can be built out of these spinors. For instance, the massive polarization vector can be constructed as

$$\varepsilon_{p,\mu} := i\langle p|^{(0)} |\mu\rangle / \sqrt{2m}.$$

Therefore, the free field equations that one needs to impose on $(s,s)$-representation tensors are

$$\left(\partial^2 + m^2\right)\Phi_{\mu_1...\mu_s} = 0, \quad \partial^\mu\Phi_{\mu\mu_2...\mu_s} = 0. \quad (5)$$

We have referred to the symmetric traceless tensor representation by the numbers of its chiral and antichiral SL(2, C) indices upon the application of the spinor map

$$\Phi_{\alpha1...\alpha_s\beta1...\beta_s} := \Phi_{\mu_1...\mu_s} \epsilon^{\mu_1}_{(0)} \cdots \epsilon^{\mu_s}_{(0)}\sigma^{\alpha_s}_{\alpha_s\beta_s}, \quad (6)$$

where $\sigma^\mu = (1, \sigma^1, \sigma^2, \sigma^3)$ are the Pauli matrices. In fact, it is easy to see in this spinor language that, although $(s,s)$ is irreducible under SL(2, C) $\cong SO(1,3)$, it is highly reducible under SU(2) $\subset SL(2, C)$ and decomposes into symmetric SU(2) tensors of rank $0, 2, \ldots, 2s$.

The Klein-Fock-Gordon equation in Eq. (5) can be obtained from a simple Lagrangian. However, there is no action principle for $s \geq 1$ that also generates the second equation in Eq. (5), which is required to ensure irreducibility under Wigner’s little group, unless a host of auxiliary fields is introduced [3]. For instance, the Singh-Hagen approach [4 5] relies on introducing $(s-1)$ symmetric traceless tensor fields of rank 0, 1, ..., $s-2$. Alternatively, Zinoviev’s approach [6] involves s such fields of rank 0, 1, ..., $s-1$, that are also subject to the doubleTrace condition $\Phi^{a1}_{b1\mu_{s-2}} = 0$. All these fields vanish on shell but serve to ensure the free-field expansion of $\Phi_{\mu_1...\mu_s}$ in terms of the physical polarization tensors [2].

Here, our radically simple proposal, as inspired by the higher-spin amplitudes [1] and by chiral higher-spin gravity [27], is to take basic fields $\Phi_{\alpha1...\alpha_{2s}}$ in the chiral (2s,0) representation of the Lorentz group instead of $(s,s)$. As we will shortly see, this essentially trivializes the transition between the off-shell symmetry group SL(2, C) and the on-shell little-group SU(2).

1 The extension of the discussion of the first few paragraphs of Sec. II to half-integer spins is straightforward and can be found e.g. in [10].

2 More concretely, for a generic Lorentz transformation $L \in SO(1,3)$ we have

$$\epsilon_{p,\mu} := \epsilon_{p,\mu}, \quad (4)$$

where $U(L) \in SU(2)$ in principle depends on the choice of the spin quantization axis for any given $p$.

3 The Weyl spinor indices are raised and lowered with two-dimensional Levi-Civita tensors $\epsilon^{\alpha\beta} = \epsilon^{\alpha\beta} = \epsilon^{ab} = (0, 1) = \epsilon_{\alpha\beta} = \epsilon_{\alpha\beta} = \epsilon_{ab},$ just as SU(2) indices. Spinor products like $[12] := \epsilon^{\alpha\beta}\epsilon^{[1\alpha]\beta} = (1\check{\nu} \check{b})$, which appeared in Eq. 1, are hence antisymmetric. The spinors obey an array of properties [10 15]. e.g. the Weyl/Dirac equation $p_\mu \sigma^\mu_{\alpha\beta} |p\rangle_\beta = m |p\rangle_\alpha$.
and all of the desired properties, such as Eq. 3 and transversality, follow automatically.

For a higher-spin field in representation \((2s, 0)\), the free field equations reduce to the Klein-Fock-Gordon equation

\[
(\partial^2 + m^2)\Phi_{a_1 \ldots a_{2s}} = 0,
\]

required to define the mass shell. Indeed, the number of degrees of freedom no longer needs to be artificially reduced, so the only thing that we need from the corresponding external wavefunctions is the orthonormality and completeness relations for spin 1/2.

The Lagrangian implying the free field equation (11) is

\[
\mathcal{L}_0 = \frac{1}{2} (\partial_{\mu} \Phi_{a_1 \ldots a_{2s}})(\partial^{\mu} \Phi_{a_1 \ldots a_{2s}}) - \frac{m^2}{2} \Phi_{a_1 \ldots a_{2s}} \Phi_{a_1 \ldots a_{2s}},
\]

where we treat \(\Phi\) as a real/hermitian field. The reality has a literal meaning in Euclidean or split signature spaces. This is precisely what the massive spinors (7b) are good for, so the external higher-spin wavefunctions are

\[
\langle -p, a_1, \ldots, a_{2s} = 1 \frac{m^{a_{1} \ldots a_{2s}}}{m^2} \Phi_{a_1 \ldots a_{2s}} \rangle
\]

Here the mass prefactor is needed to absorb the mass dimension, which is 1/2 for momentum spinors. The free-field expansion is therefore

\[
\Phi_{a_1 \ldots a_{2s}}(x) = \int \frac{d^{2s}p}{(2\pi)^{2s}} \left[ \frac{|p^{a_1}|_a \cdots |p^{a_{2s}}|_{a_{2s}} a_{a_1 \ldots a_{2s}}(\vec{p}) e^{-ip \cdot x}}{m^2} + (-1)^{2s} \frac{|p^{a_1}|_a \cdots |p^{a_{2s}}|_{a_{2s}} a_1 \cdots a_{2s}(\vec{p}) e^{ip \cdot x}}{m^2} \right] |p^a = \sqrt{p^2 + m^2}|
\]

\[13\]

The expansion for the spin-1/2 field \(\Phi(x)\) coincides with the chiral part of the Majorana field \(\Psi_M(x)\) as written in [10]. The standard properties

\[
(p^a p^b) = -me^{ab}, \quad |p^a|_a |p^b|_b = -m\delta^a_b
\]

of the massive spinors are then equivalent to the orthonormality and completeness relations for spin 1/2.

\[16\]

In particular, the relationship between Lorentz and little-group transformations for \(\epsilon_{\rho \mu \nu}^a\) is now a simple consequence of the analogous relations for spinors:

\[
S_\alpha^\beta (p^a) = U^\alpha(s)(p^a)\beta, \quad (9a)
\]

\[
[p^a]_\beta (S^a) = U^\alpha(s)(p^a)\beta\alpha. \quad (9b)
\]

Here the little-group transformation \(U\) only depends on \(S\) instead of \(L\), but they are of course subject to the two-to-one correspondence \(\text{SL}(2, \mathbb{C}) \cong \text{SO}(1, 3)\):

\[
L^{\mu \nu} = \frac{1}{2} \text{tr}(\sigma^\mu S_\nu S_5), \quad (10)
\]

\[12\]

Note that, when all indices are raised, the propagator numerator in Eq. (19) is consistent with the completeness relation above.

\[19\]

\[20\]

### III. GAUGE THEORY

In this section we include minimal gauge interactions in the chiral higher-spin theory [18]. Since complex conjugation switches between the chiral and antichiral representations of the Lorentz group, we choose \(O(N)\) as the generic gauge group, which encompasses other interesting cases, such as \(SU(N) \subset O(2N)\).

We take the Lagrangian to be simply

\[
\mathcal{L}_g = \frac{1}{2} (D_{\mu} \Phi^{(\alpha)})(D^{\mu} \Phi^{(\alpha)}) - \frac{m^2}{2} \Phi^{(\alpha)} \Phi^{(\alpha)} - \frac{1}{2} \text{tr}[A_{\mu} A^\mu - (t^A, t^B)] f^{ABC} t^C. \quad (21)
\]

\[22\]
involves antisymmetric O(N) generators in the antihermitian convention \( t^A_{ij} = -t^A_{ji} \). The higher-spin field interacts with the gauge field via the 3-point vertex

\[
A^A_{3\mu} = g t^A_{ij} \delta(\beta_1 \ldots \beta_2 \alpha_3)(p_1 - p_2)^\mu
\]

and the 4-point vertex

\[
A^A_{4\mu} A^{\nu}_{4} = -ig^2 \left[t^A_{ik} T^B_{kj} + t^B_{ik} T^A_{kj}\right] \delta(\beta_1 \ldots \beta_2 \alpha_3 \alpha_4)(p_1 - p_2)^\mu
\]

The crucial feature of these vertices is that they equal those of a scalar minimally coupled to a gauge field times the \((2s,0)\)-identity operator. This guarantees that all of the tree-level scattering amplitudes in this theory will essentially factorize onto those of massive scalar QCD.

For a single pair of higher-spin particles, we have

\[
A(1,2,3) = \frac{(1,2)^{2s}}{m^{2s}} A(1,2,3) = \frac{(1,2)^{2s}}{m^{2s}} A(1,2,3)
\]

where \( A(1,2,3) \) is the \((n-2)\)-photon emission amplitude in scalar QED, and \( \odot \) denotes the symmetrized tensor product for massive spinors:

\[
(1,2)^{2s} \odot (3,4)^{2s} = (1,2)(3,4)(5,6) \cdots (1,2)^{2s}(3,4)^{2s}
\]

Since there are no vertices with more than two massive fields, we can easily write similar relations for amplitudes with multiple pairs of higher-spin particles. For instance,

\[
A(1,2,3,4,5,6,\ldots,n) = \frac{(1,2,3)^{2s}}{m^{2s}} A(1,2,3,4,5,6,\ldots,n)
\]

Comparing with the original parity-conserving AHH amplitudes \cite{10} for gauge theory \cite{7}

\[
A_{AHH}(1^{(a)},2^{(b)},3^{(c)}) = 2g t_{ij} \frac{(1,2)^{2s}}{m^{2s}} (p_1 \cdot \epsilon_3^+), \quad (29a)
\]

\[
A_{AHH}(1^{(a)},2^{(b)},3^{(c)}) = 2g t_{ij} \frac{(1,2)^{2s}}{m^{2s}} (p_1 \cdot \epsilon_3^+), \quad (29b)
\]

we observe agreement for positive helicity and mismatch for negative helicity. This reflects the intrinsic chirality of the higher-spin theory \cite{21}, see Sec. \[V\] for the discussion of how parity could be restored. Here we concentrate on the (incoming) positive-helicity gluons. Such states correspond to the self-dual sector of Yang-Mills theory in the sense that the part of the linearized field strength \( F_{\alpha \beta \beta} := F_{\mu \nu} \sigma_{\alpha \beta} \sigma_{\alpha \beta} = F_{\alpha \beta} \epsilon_{\alpha \beta} + F_{\alpha \beta} \epsilon_{\alpha \beta} \), which gets Wick-contracted with them, is

\[
\tilde{F}_{\alpha \beta} (p) = i 2 \sqrt{2} \pi [\sigma_{\alpha} \sigma_{\beta} \delta^{+} (p^2) a_{+} (p) - \delta^{-} (p^2) a_{-} (p)]
\]

satisfies \( \frac{1}{2} \delta_{\mu \nu \rho \sigma} F^{+ \rho \sigma} = i F^{+ \rho \sigma} \). Much is known about the self-dual sector \cite{32,33}. Strictly speaking, self-dual Yang-Mills theory (SDYM) includes both gluonic helicities, and in the on-shell approach one may deal with it simply by switching off one of the 3-gluon amplitudes:

\[
A(1^+,2^+,3^-) = - \frac{1}{2} g f^{ABC} (12^+) \left[ \frac{23}{23} \right] \quad \text{SDYM} \quad \Rightarrow 0.
\]

In full Yang-Mills theory, \( L_{YM} = -\frac{1}{4} F^{A}_{\mu \nu} F^{A \mu \nu} \), both amplitudes \[13,31\] appear in factorization limits. Even if we couple it to our chiral higher-spin matter \cite{21}, the existence of BCFW shifts with good boundary behavior is guaranteed \cite{17} by the scalar version of this theory e.g. if we choose to shift only gluonic momenta \cite{22}. The ensuing on-shell recursion relations \cite{13,14} allow to build up the entire tree-level scattering matrix, which will by construction be related to that of scalar QCD via such identities as Eqs. \[23\] and \[26\].

In this theory, there is a subset of amplitudes such that only amplitudes \[29a\] and \[31a\] appear in their factorization limits. Such amplitudes coincide with those in

\[6\]

\[7\]

\[8\] As usual, massless polarization vectors can be constructed from massless momentum spinors \[1a\] as

\[
\varepsilon_{\mu \nu} = \frac{q \sigma_{\mu} \sigma_{\nu}}{\sqrt{2} (q p)}, \quad \varepsilon_{\mu \nu} = \frac{p \sigma_{\mu} \sigma_{\nu}}{\sqrt{2} (q p)}.
\]

In contrast to Eq. \[6\], the presence of extraneous reference spinors \( q \) and \( p \) encodes the linearized gauge freedom \( \varepsilon_{\mu \nu} \rightarrow \varepsilon_{\mu \nu} + f_{\mu \nu} \).

\[8\] In \[12,16\] spin-s amplitudes, such as \[28\] and \[32\], carried an additional prefactor \(-1)^{\epsilon(s)}\) due to the massive polarization vector \[3\] being conventionally spacelike, which is irrelevant for our chiral theories here.

\[8\] The simplest argument in favor of good boundary behavior in massive scalar QCD starts with pure Yang-Mills theory in higher dimensions, for which such shifts do exist, as discussed e.g. in \[40\]. After reducing it to four dimensions and leaving only the massless spectrum, the extra-dimensional gluonic states behave as adjoint-representation scalars, whereas the low-energy manipulation of introducing masses in their propagators cannot change their boundary behavior, which is a high-energy feature.
SDYM coupled to our massive higher-spin matter \[21\]. In this chiral theory, all purely gluonic amplitudes automatically vanish at tree level except for Eq. \[31\]. Amplitudes with matter do not vanish but are significantly simpler than those in the full theory due to the integrability of the self-dual gauge interaction. The non-trivial overlap of the full and the self-dual theories consists of the amplitudes involving a single pair of massive higher-spin particles, to which we have referred in the introduction. Their color-ordered versions are explicitly \[12\]

\[
A(1^{(a)}, 2^+, 3^+, \ldots, (n-1)^+, n^{(b)}) = \frac{i^2(1^{(a)}n^{(b)})^{2s}}{m^{2s-2}} \frac{2^{n-3}}{\prod_{j=2}^{n-2} (j(j+1)(s_{1\ldots j} - m^2))} \left[ n-1 \right].
\]

(32)

Here we have written \( P_{1\ldots j} = p_1 + \ldots + p_j \) for momentum sums and \( s_{1\ldots j} \) for their Lorentz squares. The factors involving slashed matrices in the numerator are \( \{ (P^{\mu}_{1\ldots j}, \sigma, j) \} \left( P^{\mu}_{1\ldots j}, \sigma, j \right) + (s_{1\ldots j} - m^2) \delta_{\sigma} \), and their order of multiplication is such that \( j \) increases from left to right. The amplitudes (32) are consistent with those for massive scalars first derived in \[12\], as well as with those for massive quarks \[15\] and \[43\] and gauged spin-1 matter obtained from Yang-Mills theory via the Higgs mechanism \[14\]. We have also performed additional numerical checks through 8 points that the Feynman rules \(23a,b\) and \(23b\), in combination with the standard 3- and 4-gluon vertices, give the same answers as the formula (32).

In the important special case of SO(2), for which \( f^{ABC} = 0 \) and \( f_{ij}^A = e^{ij} \), we combine the fields into

\[
\Phi^{(\alpha)} := \frac{1}{\sqrt{2}} (\Phi^{(\alpha)}_{j=1} + i \Phi^{(\alpha)}_{j=2}), \quad \bar{\Phi}^{(\alpha)} := \frac{1}{\sqrt{2}} (\Phi^{(\alpha)}_{j=1} - i \Phi^{(\alpha)}_{j=2}),
\]

(33)

where the charge conjugation avoids the chirality switch. The resulting chiral higher-spin QED Lagrangian is

\[
\mathcal{L}_Q = (D^{\mu} \Phi^{(\alpha)})(D^{\mu} \Phi^{(\alpha)}) - m^2 \Phi^{(\alpha)} \Phi^{(\alpha)},
\]

(34)

where the covariant derivative \( D_\mu := \partial_\mu - iQ A_\mu \) involves the coupling constant renamed to \( Q \). A straightforward application of the resulting Feynman rules

\[
A^\mu_1 = iQ \delta^{(\beta_1 \ldots \beta_{2s})}_{(p_2 - p_1)} \mu,
\]

\[
\bar{\Phi}^{(\alpha_1 \ldots \alpha_{2s})} \times \frac{1}{\sqrt{2}} (\Phi^{(\alpha_1 \ldots \alpha_{2s})} + i \Phi^{(\alpha_1 \ldots \alpha_{2s})}) = 2Q^2 \delta^{(\beta_1 \ldots \beta_{2s})}_{(p_2 - p_1)} \eta^{\mu \nu},
\]

(35)

leads to the explicit amplitudes (see e.g. \[45\])

\[
A(1^{(a)}, 2^+, 3^+, \ldots, (n-1)^+, n^{(b)}) = -i(2Q)^{n-2}
\]

\[
\times \left( \frac{1}{n^{(b)}} \frac{m^{2s}}{2} \sum_{\sigma = S_{n-2}(2,3,\ldots,n-1)} \frac{\prod_{j=2}^{n-2} (P_{1j}(\sigma) \cdot e^+_{\sigma(j)})}{\prod_{j=2}^{n-2} (s_{1j}(\sigma) - m^2)} \right).
\]

(36)

We have checked numerically through 8 points that they are consistent with the color-dressed analogue of Eq. \(32\) under the usual QCD-to-QED projection

\[
A(1^{(a)}, 2, 3, \ldots, n-1, n^{(b)}) = (\sqrt{2}Q)^{-n-2} \sum_{\sigma = S_{n-2}} A(1^{(a)}, \sigma, n^{(b)}).
\]

(37)

Note that all gluonic factorization channels non-trivially cancel here, leaving only the massive ones as in Eq. \(36\).

**IV. GRAVITY**

In this section we minimally couple our chiral higher-spin theory to gravity. The Lagrangian is

\[
\mathcal{L}_G = \sqrt{-g} \left\{ \frac{1}{2} (\nabla_\mu \Phi^{(\alpha)})(\nabla^\mu \Phi^{(\alpha)}) - m^2 \Phi^{(\alpha)} \Phi^{(\alpha)} \right\},
\]

(38)

where we have included the metric dependency of the integration measure. The covariant derivatives may in general act on the Lorentz indices via the spin connection:

\[
\nabla_\mu \Phi^{(\alpha_1 \ldots \alpha_{2s})} = \partial_\mu \Phi^{(\alpha_1 \ldots \alpha_{2s})} + 2s \omega_{\mu, \alpha_1 \beta_1} \Phi^{(\alpha_2 \ldots \alpha_{2s})}_\beta \beta_2 \ldots \beta_{2s}.
\]

(39)

This prevents the vertices from factorizing onto those of the massive-scalar theory. So unlike in the gauge-theoretic case, we cannot make helicity-independent statements of the type \(24\).

However, much like in gauge theory, there is a clear connection between positive-helicity gravitons and the self-duality condition of the Riemann tensor. Moreover, it is equally true for the spin connection: the chiral part of the spin connection is known to vanish in self-dual gravity (SDGR) \[37\] [40]:

\[
\omega_{\mu, \alpha \beta} := \frac{1}{2} \omega_{\mu \rho \alpha} \epsilon^\rho \alpha \beta \Rightarrow 0,
\]

(40)

where the frame indices are displayed with hats. So restricting to the self-dual gravitational sector, we can easily write the 3-point interaction vertex

\[
\frac{b^\mu_1 \beta_1}{2} \times \frac{b^\mu_2 \beta_2}{2} = \frac{1}{m^2} \left( \frac{\rho(p_2, p_1)}{2} + \frac{m^2}{2} \eta^{\mu \nu} \right),
\]

(41)

e.g. in terms of the perturbation of the “gothic inverse metric” \(\sqrt{-g} \eta^{\mu \nu} = \eta^{\mu \nu} - \kappa b^{\mu \nu} \) \[49\] [50] and the coupling constant \(\kappa = \sqrt{32\pi G_{(\text{Newton})}}\). In this formulation
of self-dual gravitational perturbation theory, all vertices involving two massive fields and multiple gravitons come exclusively from the mass term due to

\[ \sqrt{-g} = 1 - \frac{k}{2} e^{\lambda} + \frac{k^2}{8} (\partial^2 - 2b^{\mu \nu} \partial_{\mu \nu}) + O(k^3), \tag{42} \]

where \( \lambda := b^{\mu \nu} \partial_{\mu \nu} \). In any case, in view of the vanishing anti-self-dual spin connection \( 10 \), all of the Feynman rules clearly factorize onto those for a massive scalar.

Therefore, for positive-helicity gravitational amplitudes involving two higher-spin matter particles, we do have the factorization property

\[ \mathcal{M}(1_a, 2_i, 3^+, \ldots, n^+) = \frac{(1_a e^{ib} \sigma)^2}{m^{2s}} \mathcal{M}(1, 2, 3^+, \ldots, n^+). \tag{43} \]

Equivalently, these amplitudes are constructible \( 11 \) from their factorization limits via on-shell recursion starting from the two 3-point amplitudes

\[ \mathcal{M}(a_1, b_2, 3^+) = -i \kappa (\frac{1}{2} \frac{(12)_6}{(23)_2 (31)_2} ) (p_1 \cdot e_3)^2 \tag{44} \]

and

\[ \mathcal{M}(1^+, 2^+, 3^-) = -i \kappa \frac{(12)_6}{(23)_2 (31)_2} \Rightarrow 0. \tag{45a} \]

but with no reference to the second 3-graviton amplitude

\[ \mathcal{M}(1^-, 2^-, 3^+) = -i \kappa \frac{(12)_6}{(23)_2 (31)_2} \Rightarrow 0. \tag{45b} \]

V. SUMMARY AND DISCUSSION

We have presented a chiral approach to massive higher-spin fields and formulated three theories, in which such fields interact strongly, electromagnetically or gravitationally. We have focused on the self-dual sectors of these interactions, where scattering amplitudes coincide with those derived previously assuming parity conservation. On the one hand, all three theories may be consistently truncated down to their respective self-dual sectors. On the other hand, they may be extended by introducing additional vertex operators to their Lagrangians.

The problem of healthy interactions for higher-spin fields is important for various reasons. Higher-spin states might be indispensable for building consistent quantum gravity models, as indicated by string theory, the AdS/CFT correspondence and higher-spin gravities. Moreover, massive higher-spin particles can model many real physical systems within the effective field theory approach. A suitable implementation of the classical limit \( 11 \) \( 30 \) \( 31 \) \( 60 \) \( 65 \) even allows to model gravitational dynamics of spinning black holes or other compact objects.

For marginal spins, the problem of consistent interactions has a solution for massive spin-1 fields, which always result from a spontaneously broken Yang-Mills theory, and consistent massive spin-2 theories are known as massive gravity \( 66 \) \( 69 \). For higher spins, this problem can be roughly subdivided into two: self-interactions of massive higher-spin fields and their gauge or gravitational interactions. The latter is more important for such applications as black-hole scattering. No solution to either problem has existed beyond spin-2, and the present paper aims to provide a new way forward.

Let us compare our chiral-field approach to other descriptions of massive higher-spin fields. In the Singh-Hagen formulation \( 4 \) \( 5 \), the auxiliary fields are fine-tuned to give enough differential consequences of the field equations to guarantee the correct number of physical degrees of freedom (p.d.o.f.), which is almost equivalent to a tedious analysis of Hamiltonian constraints. Zinoviev’s approach \( 6 \) has a more transparent pattern of auxiliary fields intertwined by gauge symmetries à la Stueckelberg, and the remaining challenge is of a purely technical nature \( 70 \) \( 75 \). If one is concerned only with field equations, more economic approaches to control the number of p.d.o.f. can be applied, see e.g. \( 76 \) \( 79 \). Furthermore, the light-front approach starts out with the correct p.d.o.f., see e.g. \( 80 \) \( 81 \) and especially \( 82 \) for the four-dimensional case, but the study beyond the cubic order can still be quite laborious. For twistorial constructions involving mass see e.g. \( 83 \) and the references therein.

Our present proposal for massive higher-spin particles is inspired by the recent attempts to bootstrap their on-shell amplitudes with as little off-shell input as possible \( 10 \) \( 16 \) \( 61 \) \( 85 \), which works surprisingly well for like-helicity configurations of the force-carrier bosons \( 11 \) \( 12 \). The chiral description of massive higher-spin interactions is very close in spirit to the ideas originating from twistor theory \( 84 \) \( 88 \) and from the covariantization of chiral higher-spin gravity \( 23 \) \( 26 \) \( 27 \) \( 91 \), where a chiral field \( \Phi_{a_1 \ldots a_2} \) is used to describe massless spin-s states. It is also close to the light-cone gauge in not having redundant
degrees of freedom, the advantage being in maintaining manifest Lorentz symmetry. For spin-1/2 fields, the chiral description can be understood as integrating out half of the fermion components out of QCD [72], and likewise for spin-1 [93] after first integrating in an auxiliary field. However, no such interpretation is available for higher spin fields, while a parent action [94] may still exist.

Having presented the three chiral theories, we can already comment on their extensions. Similarly to the self-dual theories, our theories, while being consistent on their own, generate a subset of the amplitudes of their would-be non-chiral completions. In the case of SDYM (with matter) and SDGR the completions to full Yang-Mills theory (with matter) and gravity are known [35, 93, 95], and it would be interesting to find such completions for our proposals. Aiming at restoring parity can also be motivated by the applications to spinning black holes. Indeed, the gravitational AHH amplitude [41] was shown [30, 31] to contain the same spin-induced multipole moments as a spinning black hole [96–98], but only in combination with its antichiral version

\[ \mathcal{M}_{AHH}(1^a, 2^{16}, 3^-) = -i\kappa \frac{1 \bar{\epsilon} \gamma_5 \gamma_k \Phi^\alpha \gamma_5 \gamma_k \Phi^\beta \gamma_5 \gamma_k \Phi^\gamma}{m^2} (p_1 \cdot \epsilon^-)^2. \] (47)

On the classical-gravity side, the tower of multipole moments can be extracted directly from the linearized Kerr solution [63]. We can easily write the general form of the interaction terms that could modify the 3-point amplitude [29b] without spoiling [29a]:

\[ (D_{\alpha_1} \gamma_1 \cdots D_{\alpha_k} \gamma_k \Phi^\alpha \cdots \delta^{\omega} \gamma_2)\epsilon_{\alpha_{k+1} \beta_{k+1} \cdots \epsilon_{\alpha_{2z-1} \beta_{2z-1} \cdots \epsilon_{\alpha_{2z}} \beta_{2z}} F_{\alpha_2 \beta_2} \times (D_{\beta_1} \gamma_1 \cdots D_{\beta_k} \gamma_k \Phi^\beta \cdots \delta^{\omega} \gamma_2). \] (48)

A single such term \(\Phi^\alpha F_{\alpha} \Phi^\beta\) is in fact known to be sufficient for restoring parity to the entire spin-1/2 theory, which then constitutes a chiral formulation of QCD with massive quarks due to Chalmers and Siegel [92]. As for higher spins, we will explore their gauge and gravitational interactions in more detail elsewhere.

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