Chaotic Mapping Based Advanced Aquila Optimizer With Single Stage Evolutionary Algorithm

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ABSTRACT The intelligent optimization techniques have been introduced by carefully observing the behavior of various hunters like a whale, grey wolf, Aquila, and lizards for estimating global optimum solutions in fair time by forming appropriate mathematical models. However, hunting-based algorithms suffer from slow and pre-requisite convergence and get caught up in local minimum. Aquila optimizer (AO) is one of the recently developed hunting-based methods that encounter a similar type of shortcoming in a few situations. This research introduces the concept of chaotic mapping to the standard AO in order to increase the convergence speed. Also to maintain the balance of exploration performed by AO with its exploitation capability, a single stage evolutionary algorithm is also integrated with it. The performance of standard AO and modified AO are tested for well-defined unimodal and multimodal Benchmark functions. The proposed framework produces one population by standard AO and a new population by single stage genetic algorithm based evolutionary concept in which binary tournament selection, roulette wheel selection, shuffle crossing over and displacement mutation occur to generate a new population. The chaotic mapping criteria are then applied to obtain various variants of the standard AO technique. The general results obtained from the proposed novel chaotic mapping-based advanced AO with single stage evolutionary algorithm shows that it outperforms the standard AO. This advanced technique is thus applied to real-world design engineering problems to study its significance from an industrial point of view.

INDEX TERMS Hunting-based algorithm, evolutionary algorithm, Aquila optimizer (AO), chaotic maps.

I. INTRODUCTION
Several metaheuristic methods are utilized for addressing numerous real-world applications. For example, routing of vehicles [1], robot path tracker, robot gripping [2], electric motor geometry optimization [3], and design of distribution system of water [4].

The development in metaheuristic techniques has gained pace due to its flexibility, simplicity, ability to achieve accuracy while solving linear, non-linear or complex real world optimization problems. These metaheuristic techniques are developed on the basis of different inspirations. The search strategies in different optimization can be Stochastic based, Enumerative based or Calculus based. Based on the search methods, the classification of state-of-the-art optimization methodologies [5], [6], [7] is presented in Figure 1.

A. MOTIVATION AND PRIOR WORK
Out of so many inspirations, stochastic search based have raised the interest of researcher for obtaining advancement
in the performance of existing frameworks. By doing so, the updated versions of existing metaheuristic frameworks become more efficient for solving many complex real world problems [8], [9], [10], [11], [12], [13], [14], [15]. The evolutionary algorithms are based on the laws of natural evolutions. The typical evolutionary algorithms are Genetic Algorithms (GA) [16], Genetic Programming (GP) [17], Evolutionary Strategy (ES) [18], Differential Evolution (DE) [19], and Evolutionary Programming (EP) [20]. For obtaining single string of solution, single stage evolutionary algorithm of GA is the most appropriate among all [21].

Apart from this, stochastic search based algorithms include physics based [22], [23], social based [24], biology based [25], swarm behavior [26], chemical based [27], music based [28], plant based [7], mathematics based [29], sports based [30], water based [31] algorithms.

The Aquila Optimizer (AO) is one of the new technique which is recently designed based on the hunting behavior of Aquila. The genus of ‘eagle’, i.e., Aquila is suspected to be carrying Zeus’s thunderbolt in Roman mythology. Aquila is a proficient hunter. AO algorithm is designed by sub-dividing the exploration as well as exploitation phases on the basis of expanded and narrowed vision. This unique feature guarantees that AO traverse the search region with diverse vision. However, it exhibits few limitations of showing disproportion in the speed of convergence during stochastic search. The other drawbacks are falling for locally optimized region due to discrimination issues and imbalance between exploration-exploitation phases of AO.

The evolutionary algorithm, GA is designed using a strong base of biology inspired mechanism. Originally, GA is capable of generating reprints of procedures of natural evolutions, the concept of which was introduced by Charles Darwin. Although the concept of biological evolution is obscure, the experimental support exists for few facts related to it; i.e.,

(1) The concept of evolution is applicable more to genotype (chromosomes) than to phenotype (organisms) themselves. Those genotypes are considered to be the tools responsible for coding the life and the corresponding biological form is deciphered using the information possessed by the chromosomes.

(2) The mechanism of natural selection is responsible for the relationship between genotypes and phenotypes. It gives rise to better expectation of life and enables those individuals to reproduce that are the most adapted.

(3) The evolution occurs during the phase of reproduction.

Contrarily, the technicalities of biological theory based concepts of natural selection, reproduction, and mutation which are an integral part of the basic genetic algorithm (GA) show sluggish elongated search time if applied solo to optimization problems. However, its high accuracy, robustness,
and efficiency of solving complex problems make it a proficient contender to implementation in the integrated structure [32].

The three main reasons, which arise limitations in the stochastic computations in origins of AO and GA are given as:

- The populations are dynamic in nature. Thus, a variation is observed in the individuals’ number, which is in contrary to the fixed population used in both AO and GA.
- The crossover and mutation processes, which occur during the phase of reproduction in GA, are totally chaotic and are independent of any parameter.
- The initial population does not possess completely random characteristics. If there are chaos in reproduction, the chaos will be present in the population as well.

In [33] and [34], the authors introduced the algorithms possessing variable populations which is defined by chaos. The performance of metaheuristic algorithms gets improved using this dynamic configuration.

In Chaotic map theory, the study of dynamical systems is carried out which corresponds to the systems that get evolved over time. In chaotic mapping, the present state depends upon the characteristics of the previous state. It is in contrast to the random systems, in which the prediction of the upcoming state is not possible due to lack of correlation between the previous and upcoming state.

The use of chaotic maps in the generation of initial population of AO helps to avoid convergence to a locally optimized region. The integration of single stage evolutionary algorithm, GA with AO helps to increase the convergence speed and avoid imbalance between exploration-exploitation phase of AO. As per the literature, there have been few researchers who attempted to improve the performance of AO using different techniques. Table 1 shows the brief overview of modified versions of AO indicating innovations and research gaps.

The GA, on the other hand, was introduced in 1992 [25]. Although, there have been various variants of the basic genetic algorithm since then shown in [35], from a classical point of view, the basic steps are used to update the current solution as per the single-stage evolutionary genetic algorithm due to the following advantages:

- A binary tournament selection strategy, being easier to implement, ensures perseverance of population diversity.
- A simulated binary crossover strategy, ensures better exploration.
- The polynomial mutation strategy, ensures manifoldness.

Hence in order to improve the performance of standard AO, the evolutionary genetic algorithm is integrated with the standard AO to propose seven novel variants of AOs by introducing chaos to it. The proposed novel technique is named as chaotic mapping-based modified Aquila Optimizer with single-stage evolutionary (CMAOE) algorithm.

By single-stage evolutionary algorithm, it is meant that only one stage of the overall genetic algorithm containing selection, crossover, and mutation is being implemented in the proposed CMAOE method in this article.

| Algorithm       | Authors       | Year | Innovations                                                                 | Gaps                                      |
|-----------------|---------------|------|-------------------------------------------------------------------------------|-------------------------------------------|
| IHAOHHO*        | Wang et. al.  | 2021 | Based on hybridization of AO and HHO*                                       | Originality of AO is compromised.          |
| IAO*            | Ma et. al.    | 2021 | Concept of quasi-opposition learning and wavelet mutation strategies is used | Additional uncertainty in AO is observed. |
| AOAAO*          | Zhang et. al. | 2022 | Based on hybridization of AO and AO*                                        | High computation time                      |
| IAO             | Ewees et. al. | 2022 | Search strategy of AO is boosted using operators of WOA*                     | Originality of WOA and AO are compromised.|
| AGWO*           | Ma et. al.    | 2022 | Based on hybridization of WOA and AO                                        | Originality of both techniques are not retained.|
| enAO*           | Ekinci et. al.| 2022 | Enhanced version of AO is developed using modified opposition-based learning and Nelder-Mead simplex search method | Additional complexity in computation is increased. |

1Improved Hybrid Aquila Optimizer and Harris Hawks Optimization (IHAOHHO)
2Harris Hawk Optimization (HHO)
3Improved Aquila Optimizer (IAO)
4Arithmetic Optimization Algorithm with Aquila Optimizer (AOAAO)
5Arithmetic Optimization Algorithm (AOA)
6Whale Optimization Algorithm (WOA)
7Aquila Grey Wolf Optimization (AGWO)
8Grey Wolf Optimization (GWO)
9enhanced Aquila Optimizer (enAO)

B. AIM

The primary goal of this article is to study the utilization of Chaotic maps for generating the initial population of CMAOE. The performance of the proposed CMAOE algorithm is compared with standard AO for constrained Benchmark functions. The best out of all proposed variants of AO is thus tested on a real world design engineering problem. In addition, the entropy in the initial population is measured and analyzed.

C. CONTRIBUTIONS

This article contributes the following novel features of the proposed CMAOE technique:

- The dynamical system of the stochastic initial population has consistent irregularity in several solutions. This happens during the expanded and narrowed diversification stage of standard AO and reproduction stage of GA which is entirely chaotic in nature. Hence, the
chaotic mapping systems are introduced to strengthen the prominence of the initial population.

- The originality of standard AO is retained by not altering the parameters of AO.
- The performance evaluation of the CMAOE method is carried out by employing it for 13 benchmark functions consisting of unimodal and multimodal functions.
- The effectiveness of variants of AO is compared with the standard AO technique by comparing their respective convergence profiles.
- The potency of CMAOE is inspected by implementing it for a real-world designed optimization problem which shows the significance of research carried out in this work from an industrial point of view.

D. PAPER ORGANIZATIONS

The paper organization is as follows: In addition to the introduction prescribed in section I, Section II presents a detailed explanation of the methods and materials exploited in achieving the goals of this work. In section III, modified AO with chaotic mapping and evolutionary genetic algorithm is explained. Section IV presents empirical studies of proposed CMAOE along with the performance analysis results and discussions. Section V shows the application of the proposed algorithm to solve the real-world design engineering of electric motors. The results, obtained by the proposed CMAOE are compared with the analytical results obtained by the means of finite element analysis. The concluding remarks and future scope of this research are listed in section V.

II. METHODS AND MATERIALS

This section presents the details of the materials and methods utilized in order to achieve the aims of this study.

A. STANDARD AO

The standard AO begins with the generation of a stochastic population consisting of competent solutions \((x)\) belonging to the feasible spectrum of decision variables of the problem (Equation (1)).

\[
x = \begin{bmatrix} x_{1,1} & \cdots & x_{1,D} \\ \vdots & \ddots & \vdots \\ x_{n,1} & \cdots & x_{n,D} \end{bmatrix}
\]  

(1)

where \(x_{n,D}\) represents value of \(D^{th}\) variable in the \(n^{th}\) solution, where \(D\) denotes the number of variables. The present solution \(x\) is computed using Equation (2), where \(i = 1, 2, \ldots n\) and \(j = 1, 2, \ldots D\), \(L_j\) is the lower bound of solution \(x\) value, \(U_j\) is the upper bound of solution \(x\) value and \(r\) is a random constant.

The standard AO takes into consideration the hunting characteristics of Aquila in mathematical form, expressed in four separate stages to follow.

\[
x_{ij} = L_j + r \times (U_j - L_j)
\]  

(2)

- **Stage-I: Steep soar with upright dip**

At first, the apt area for prey hunting is selected by exploring the search zonal area from steep soar with upright sag. The mathematical form of this action is expressed as Equation (3):

\[
x_1 (t + 1) = x_b (t) \times \left(1 - \frac{t}{T}\right) + x_m (t) \times r
\]  

(3)

where at \(t^{th}\) iteration, \(x_b (t)\) and \(x_m (t)\) are the best and the mean solution respectively, \(x_1 (t + 1)\) is the revised solution due to Stage-I and factor \(\left(1 - \frac{t}{T}\right)\) is controlling the D-ph in accordance with the iteration where \(T\) represents the upper limit of iterations. Equation (4) determines the term \(x_m (t)\).

\[
x_m (t) = \frac{1}{n} \sum^n_{i=1} x_j(t); \quad j = 1, 2, \ldots D
\]  

(4)

- **Stage-II: Contour navigation with concise drift attack**

Now a circular contour is formed above the target prey thereby narrowing the zonal area for hunting. The action is named ‘contour navigation with concise drift attack’, represented as Equation (5):

\[
x_2 (t + 1) = x_b (t) \times L (D) + x_r (t) + (y - x) \times r
\]  

(5)

where at \(t^{th}\) iteration, \(x_r (t) \in [1, n]\) is a randomly selected solution, \(L (D)\) is a flight levy function defined by Equation (6) and \(x_2 (t + 1)\) is the revised solution due to Stage-II. Factor \((y - x)\) is controlling the spiral contour shape in the course of exploration, where \(x\) and \(y\) are represented by Equations (8) and (9).

\[
L (D) = \frac{s \times u \times \sigma}{|v|^\beta}
\]  

(6)

\[
\sigma = \frac{\Gamma(1 + \beta) \times \sin \pi \beta}{\Gamma\left(\frac{1+\beta}{2}\right) \times \beta \times 2^{-\frac{\beta-1}{2}}}
\]  

(7)

\[
x = r \times \sin \theta
\]  

(8)

\[
y = r \times \cos \theta
\]  

(9)

\[
r = r_1 + p \times d_1
\]  

(10)

\[
\theta = -\omega \times d_1 + \theta_1
\]  

(11)

where \(u, v \in [0, 1]\) are random numbers obtained from standard normal distribution, \(s, \beta, r_1, p, \omega\) and \(\theta_1\) are constant values and \(d_1 \in [1, D]\) represents an integer vector.

- **Stage-III: Lowering navigation with gradual plunge attack**

After stating the prey area, the lower navigation is executed with a gradual drop. The area of prey is widely exploited by this action called ‘lowering navigation with gradual plunge attack’, given by Equation (12).

\[
x_3 (t + 1) = (x_b (t) - x_m (t)) \times a + r + ((U - L) \times r + L) \times \delta
\]  

(12)

where \(a\) and \(\delta\) are controlling parameters of wide exploitation and \(x_3 (t + 1)\) is the revised solution due to Stage III.
Stage-IV: Stroll and catch prey

When prey is nearby located, it is strolled and grabbed corresponding to its random movement. This action is represented by Equation (13).

\[ x_4(t + 1) = Q \times x_4(t) - (g_1 \times x(t) \times x) - (g_2 \times L(D)) + r \times g_1 \]  

where at \( t^{th} \) iteration, quality function \( Q \) is computed using Equation (14) and \( x_4(t + 1) \) is the revised solution due to Stage-IV. Parameters \( g_1 \) and \( g_2 \) are controlling the track path of the prey using Equations (15) and (16) respectively.

\[ Q(t) = \frac{2^{(1-r)^2}}{1+T} \]  
\[ g_1 = 2 \times r - 1 \]  
\[ g_2 = 2 \times (1 - \frac{r}{T}) \]  

B. PROPOSED CMAOE ALGORITHM

The CMAOE algorithm consists of replacement of random procedures of generating initial population by Chaotic maps. The chaotic map functions, for example, the Henon map [36], build a numerical series to produce the population by following Equation (17):

\[ x_n = 1 - A \times x_{n-1}^{2} + B \times x_{n-1} \]  

where \( x_n \) represents each individual member of population \( x \) (described in Equation (1)) whose position depends on the position of the previous individual, \( x_{n-1} \).

In CMAOE, the standard AO is upgraded by integrating the single stage GA to further escalate the search precision and accuracy.

Each revised solution vector at \( t^{th} \) iteration obtained at stage IV of standard AO is treated as the population of candidate solutions for further steps of GA workflow, having the following steps:

**Step (i) of GA in CMAOE:**

In this step, binary tournament selection is implemented and thus, the parent solution is obtained. This is a recurring process for the selection of the agents as long as the CMAOE algorithm is administered.

**Step (ii) of GA in CMAOE:**

Two-parent agents \( P_1 = \{P_{1,1}, P_{1,2}, ..., P_{1,D}\} \) and \( P_2 = \{P_{2,1}, P_{2,2}, ..., P_{2,D}\} \) are picked randomly by Simulated Binary Crossover (SBX). The generation of each element of offspring \( O_1 \) and \( O_2 \) is done using Equation (18).

\[ O_1 = 0.5 \times [(1 + \mu) \times P_1 + (1 - \mu) \times P_2] \]  
\[ O_2 = 0.5 \times [(1 - \mu) \times P_1 + (1 + \mu) \times P_2] \]  

\[ \mu = \begin{cases} 
(2r)^{\frac{1}{\eta_{m}+1}} & \text{if } r \leq 0.5 \\
\frac{1}{2(1-r)} & \text{otherwise} 
\end{cases} \]  

where \( \mu \) is the crossover SBX controlling factor, computed by Equation (19), where \( r \) is a random number, \( \eta_c = 20 \) is the crossover probability distribution mean.

**Step (iii) of GA in CMAOE:**

This step involves the mutation of offspring. Each offspring agent is mutated to produce mutated offspring \( O_m \) from offspring \( O \) using Equation (20).

\[ O_m = O + (U - L) \times \Delta \]  

where \( \Delta \) is the mutation control factor, calculated using Equation (21).

Where \( r \) is a random number, \( \eta_m = 20 \) is the mutation probability distribution mean. Thereafter, the combined population is obtained and sorted non-dominantly. Thus, the best solution vector of dimension \([1, n]\) is updated.

\[ \Delta = \begin{cases} 
(2r)^{\frac{1}{\eta_m+1}} - 1; & \text{if } r < 0.5 \\
1 - [2(1 - r)]^{\frac{1}{\eta_m+1}}; & \text{if } r \geq 0.5 
\end{cases} \]  

The other parameters related to CMAOE, such as size of population, number of generation, Levy flight control parameters, crossover probability, mutation probability etc. are kept constant. Table 2 shows the parameter settings of the CMAOE algorithm. The process flow diagram of chaotic mapping-based advanced Aquila Optimizer with single-stage evolutionary algorithm (CMAOE) is presented in Figure 2.

**TABLE 2. Parameter setting for experimental analysis.**

| Parameter   | Value | Parameter | Value |
|-------------|-------|-----------|-------|
| Size of population | 20 | \( p \) | 0.0265 |
| Number of generations | 1000 | \( \omega \) | 0.005 |
| Crossover probability | 0.8 | \( \theta_1 \) | 3\pi/2 |
| Mutation probability | 0.2 | \( s \) | 0.1 |
| \( \beta \) | 1.5 | \( r_1 \) | 10 |

C. BENCHMARK FUNCTIONS

The Benchmark Functions are set of functions, defined by Siva Sathya and Radhika [37] to assess the performance of algorithms. These functions are classified by the; (i) ability to be non-separable or separable functions, (ii) dimensionality of search region, (iii) multimodality. The separable functions having \( m \) variables can be re-written in the form of sum of \( m \) functions having only one variable. In case of random distribution of local optima in the search region, complexity of searching for the global optimum point becomes more complex. This complexity is tested using multimodal functions. The dimensionality of search space also affects the performance of algorithms. It signifies the accuracy in computation times of particular algorithm. This work uses the benchmark functions as presented in Figure 3. The unimodal functions used are Sphere, Schwefel 2.22, Rotated hyper ellipsoid, Schwefel 2.21, Rosenbrock, Step and Quartic. The multimodal functions used are Schwefel, Rastrigin, Ackley, Griewank, Penalized1 and Levy N.13.

D. INITIAL POPULATION

The CMAOE method implements the chaotic processes for the generation of initial populations. The generation of the...
initial population is carried out by eleven chaotic functions, having parameters given in Table 3. By selecting different chaotic maps, the different variants of the proposed algorithm are developed. The corresponding names of different algorithms are also mentioned in Table 3 along with the chaotic map chosen for generation of initial population. For each execution of the algorithm, the initial population is computed using chaos and its corresponding entropy is also computed. For enhancing the reusability of this work, the proposed CMAOE is presented in the form of pseudo code in Figure 4.

E. COMPLEXITY ANALYSIS OF PROPOSED METHOD

The complexity of an algorithm relies on three main processes: initialization, fitness function computation and solution updating process. The process of CMAOE involves the computation of initial population using a particular chaotic map, calculation of fitness function and updating of solution using hunting behavior of Aquila and the usual steps of evolution, like selection, crossover, mutation. The complexity analysis of initialization process which involves chaotic maps is carried out using Entropy computation in the upcoming section. Apart from initialization process, the complexity of CMAOE is $O(n \times (T \times D + 1)) + O(T \times (n \times D + n \times D + n))$. Here, $n$ is the population size, $T$ is number of generations and $D$ is the dimension size. Therefore, the overall order of computational complexity is $O(T \times n \times D)$. Since, the overall order of complexity is not disturbed. Hence, the proposed method is flexible in producing better optimum solution without the increase in complexity.

III. EXPERIMENTAL RESULTS ON BENCHMARK FUNCTIONS

The statistical analyses are carried out for two experiments consisting of various benchmark functions. For fair comparison, the start point is same for all the algorithms. The various fitness results obtained by simulating variants of CMAOE are presented in the form of a table. The convergence speeds of various variants of CMAOE are compared in the form of figures as a function of simulation size. The coding of all the programs were carried out in Matlab R2016a and the corresponding executions were performed on a LENOVO Slim 3 with 10th Gen Intel® Core i3 CPU @ 1.20 GHz.
Experiment 1: Seven unimodal functions are selected. These possess a single optimal value. The parameters for simulations are taken as: dimension size $= 10$, population size $= 20$, simulation size $= 1000$. Eleven variants of AO are simulated for solving Sphere function, Schwefel 2.21 function, Schwefel 2.22 function, Rotated hyper ellipsoid function, Rosenbrock function, Step function, and Quartic function.
Discussions: Table 4 presents the optimal results obtained by simulating standard AO and its eleven AO variant algorithms designed with different chaotic maps for different unimodal benchmark functions.

By observing the performance of CMAOE algorithms with different chaotic maps, following inferences are realized:

- The logistic chaotic map-based CMAOE6 outperforms the standard AO and other variants for most of the functions such as the Sphere function, Rotated hyper ellipsoid function, Rosenbrock function, and step function.

- Other chaotic maps like sine, Chebyshev, and iterative maps generate initial populations which enhance the diversification capability of standard AO for solving Quartic, Sphere, and Step functions and thus secure higher ranks from 2 to 4.

- The maps such as henon, circle, gauss, and tent in CMAOE algorithms produce nearly similar behavior in producing the best optimal results for most of the unimodal functions.

Hence, by taking reference to standard AO performance, three categories of chaotic maps embedded in CMAOE algorithms, showing best, better, and good performance are generated. Table 5 presents the categorical distribution of chaotic maps for their selection in CMAOE algorithms for solving unimodal functions.

Experiment 2: In the second experiment, six multimodal functions are selected. These possess various local optima and a single global optimal value. The parameters for simulations are taken as: dimension size = 10, population size = 20, simulation size = 1000. Eleven variants of AO are simulated for solving the Rastrigin function, Schwefel function, Ackley function, Griewank function, Penalized1 function, and Levy N.13 function.

Discussions: The optimal results obtained by simulating the proposed eleven CMAOE algorithms and standard AO for solving different multimodal benchmark functions are presented in Table 6.

Following presumptions are realized by observing the performance of CMAOE algorithms with different chaotic maps for solving multimodal functions:

- While solving the Rastrigin function, Ackley function, and Griewank function, all the variants of CMAOE display the identical global optimum.

- For multimodal functions like Penalized1 function, Schwefel function, and Levy N.13, CMAOE embedded with iterative, Chebyshev, logistic, and singer chaotic maps are found to be outperforming other algorithms.

- The CMAOE with gauss, circle, sine, and sinusoidal mapping exhibits decent performance in locating the global optimum and hence secure better than the AO category.
TABLE 4. Performance analysis results present the best optimal value for unimodal benchmark functions; Ranks (R) from 1 to 12 are awarded based on their respective performance.

| Algorithm | Sphere | Schwefel 2.22 | Rotated hyper ellipsoid | Schwefel 2.21 | Rosenbrock | Quartic |
|-----------|--------|---------------|-------------------------|--------------|------------|---------|
| CMAOE1    | 9.508e-274 | 4.438e-142 | 3 | 7.903e-277 | 2 | 5.241e-136 | 4 | 7.083e-5 | 5 | 3.580e-5 | 9 |
| CMAOE2    | 3.735e-287 | 1.618e-140 | 5 | 9.864e-263 | 5 | 1.054e-115 | 8 | 6.578e-5 | 4 | 2.449e-5 | 3 |
| CMAOE3    | 2.510e-257 | 2.656e-105 | 7 | 2.625e-266 | 4 | 1.398e-140 | 3 | 5.517e-4 | 12 | 3.614e-5 | 7 |
| CMAOE4    | 2.089e-270 | 1.146e-147 | 1 | 1.932e-237 | 6 | 3.139e-124 | 7 | 4.864e-4 | 11 | 8.855e-6 | 1 |
| CMAOE5    | 3.582e-281 | 1.839e-134 | 7 | 7.833e-226 | 7 | 5.979e-136 | 5 | 6.525e-5 | 3 | 3.006e-5 | 5 |
| CMAOE6    | 9.266e-296 | 1.158e-143 | 2 | 2.998e-282 | 1 | 6.596e-135 | 6 | 4.617e-5 | 1 | 2.502e-5 | 4 |
| CMAOE7    | 2.632e-293 | 2.032e-139 | 6 | 9.617e-268 | 3 | 2.615e-100 | 10 | 5.532e-5 | 3 | 6.090e-5 | 10 |
| CMAOE8    | 1.947e-200 | 1.913e-97  | 11 | 4.543e-200 | 9 | 1.426e-144 | 1 | 1.653e-4 | 7 | 2.140e-5 | 2 |
| CMAOE9    | 1.892e-194 | 1.409e-92  | 12 | 4.721e-189 | 11 | 2.215e-98  | 11 | 3.756e-4 | 10 | 2.203e-4 | 12 |
| CMAOE10   | 7.743e-190 | 8.348e-122 | 8 | 2.438e-187 | 12 | 5.396e-98  | 12 | 2.233e-4 | 8 | 1.939e-4 | 11 |
| CMAOE11   | 4.688e-220 | 1.075e-140 | 4 | 1.205e-193 | 10 | 6.963e-141 | 2 | 1.537e-4 | 6 | 4.602e-5 | 8 |
| AO        | 1.704e-199 | 1.155e-115 | 9 | 1.394e-222 | 8 | 5.236e-113 | 9 | 3.329e-4 | 9 | 3.518e-5 | 6 |

TABLE 5. Categorical distribution of chaotic maps based on performance analysis results obtained by implementing the corresponding CMAOE for solving unimodal functions.

| Category | Chaotic map                                      | Inference         |
|----------|------------------------------------------------|-------------------|
| 1        | Logistic, Sine, Chebyshev, iterative             | Best performance  |
| 2        | Henon, Gauss, Circle, Tent                       | Better than standard AO |
| 3        | Piecewise, Singer, Sinusoidal                    | Good and/or similar to AO |

- The chaotic maps using sinusoidal, henon, tent, and piecewise distribution present almost similar results as compared to the results produced by standard AO.

As a result, the categorical distribution of chaotic maps for best, better and good performance with respect to standard AO performance is concluded. Table 7 presents the corresponding categorical distribution of chaotic maps.

Experiment 3: In this experiment, the study of the importance of using single stage evolutionary search in the proposed technique is performed. The Wilcoxon test [38] has been performed to compute the performance factor, p-value, to test the null hypothesis that the solution vectors obtained from CMAOE and from standard AO samples from continuous distributions have equal medians.

Discussions: The Wilcoxon test result for the sphere benchmark function is obtained as p-value = 1.59e-5. As per the 5% significance level, the rejection of the null hypothesis is obtained [39]. This indicates the two sample solutions had different distributions [40]. Other benchmark functions were also studied using this test and the null hypothesis is found out to be rejected indicating that the solutions obtained are diverse and have dissimilar distributions. The Sphere benchmark function is plotted in Figure 5 along with the evolutionary search plot presenting the offspring solutions and parent solutions with respect to simulation size.

Experiment 4: In this experiment, the convergence speeds of designed algorithms are compared and presented in the form of a log |Best| vs simulation size plot. Four unimodal and two multimodal functions are utilized to obtain various plots.

Discussions: By observing convergence rate of optimal solutions obtained by simulating CMAOE algorithms, shown in Figure 6, it is concluded that the sluggishness in diversification and intensification stages of AO has been improved to large extent using the advanced CMAOE techniques.

Experiment 5: This experiment shows the comparative analysis of the proposed CMAOE with some of the other existing optimization algorithm. For fair comparison, the population size of 20, dimension of 10 and generations of 1000 are kept constant. For unimodal Benchmark functions, CMAOE6 secured Rank 1 for most of the functions.

Hence, it is used for comparison with other prescribed algorithm. Similarly, for multimodal Benchmark functions, CMAOE2 secured Rank 1 for most of the Benchmark functions, so it is compared with the prescribed algorithms.

Discussions: The best solution obtained by the proposed algorithm is compared with that produced by the renowned optimization algorithms, like Grasshopper Optimization Algorithm (GOA), Equilibrium Optimizer (EO), Particle Swarm Optimization (PSO), Dragonfly Algorithm (DA), Ant Lion Optimizer (ALO), Grey Wolf Optimizer (GWO), Marine Predators Algorithm (MPA), Salp Swarm Algorithm (SSA), Sine Cosine Algorithm (SCA), Whale
TABLE 6. Performance analysis results present the best optimal value for multimodal Benchmark functions; Ranks (R) from 1 to 12 are awarded based on their respective performance.

| Algorithm | Schwefel | Rastrigin | Ackley | Griewank | Penalized1 | Levy N.13 |
|-----------|----------|-----------|--------|----------|------------|-----------|
|           | Best     | R         | Best   | R        | Best       | R         |
| CMAOE1    | -4104.6  | 11        | 0.000  | 1        | 8.882e-16  | 3         |
| CMAOE2    | -4193.3  | 2         | 0.000  | 1        | 8.881e-16  | 1         |
| CMAOE3    | -4222.7  | 10        | 0.000  | 1        | 8.880e-16  | 2         |
| CMAOE4    | -4103.1  | 12        | 0.000  | 1        | 8.881e-16  | 1         |
| CMAOE5    | -4188.4  | 1         | 0.000  | 1        | 8.884e-16  | 4         |
| CMAOE6    | -4147.8  | 7         | 0.000  | 1        | 8.885e-16  | 5         |
| CMAOE7    | -4230.3  | 9         | 0.000  | 1        | 8.881e-16  | 1         |
| CMAOE8    | -4160.5  | 6         | 0.000  | 1        | 8.883e-16  | 4         |
| CMAOE9    | -4197.4  | 3         | 0.000  | 1        | 8.883e-16  | 4         |
| CMAOE10   | -4172.4  | 4         | 0.000  | 1        | 8.886e-16  | 6         |
| CMAOE11   | -4143.5  | 8         | 0.000  | 1        | 8.881e-16  | 1         |
| AO        | -4169.9  | 5         | 0.000  | 1        | 8.886e-16  | 6         |

TABLE 7. Categorical distribution of chaotic maps based on performance analysis results obtained by implementing the corresponding CMAOE for solving multimodal functions.

| Category | Chaotic map | Inference |
|----------|-------------|-----------|
| 1        | Logistic, Iterative, Chebyshev, Singer | Best performance |
| 2        | Circle, Gauss, Sine | Better than standard AO |
| 3        | Piecewise, Henon, Sinusoidal | Good and/or similar to AO |

Optimization Algorithm (WOA) and Slime Mold Algorithm (SMA). It is observed from Table 8 variant of CMAOE is producing better performance for unimodal functions as well as for multimodal functions as compared to other algorithms.

Experiment 6: In this experiment, the computational time of the proposed algorithm is computed. The overall computational time of CMAOE is compared with that of other hybrid versions of standard AO. (say, running time of one instruction is $t_i$)

Discussions: The computational time of algorithms is computed for 1000 iterations, population size of 20 and dimensional size of 10. For fair comparison, the time complexity of other four hybrid versions of AO are computed and compared in Table 9. It is found that CMAOE has less burden on CPU computational time complexity for reaching the global optimum.

IV. APPLICATION OF CMAOE TO AN INDUSTRIAL REAL-WORLD PROBLEM

In this section, an optimization problem of obtaining minimum supplementary cogging torque in a new surface inset type of permanent magnet synchronous motor (si-PMSM) is formulated.

In this motor, the drastic variation in cogging torque is obtained by varying inter-polar length in the rotor. The reasons for applying the proposed CMAOE algorithms to solve this problem are listed below [41], [42], [43], [44], [45], [46]:

- For the higher electromagnetic performance of si-PMSM, the necessity to suppress the cogging supplement in its torque potential is worth addressing since it procreates noisy behavior of the motor.
- The inter-polar length $l_i$ or the gap between rotor PMs in si-PMSMs is of original rotor material and induces the enhanced distortion in torque behavior if not optimized at the design stage. Hence, the dependency of cogging torque over $l_i$ is derived by the assistance of surrogacy.

Recently, the design and optimization methods done on the basis of approximation have gained thorough recognition. This approach, generally, approximates the comprehensive operations to a suitable and simpler form of models analytically. Such models are called ‘Meta-models’ or ‘surrogate models’ and the concept of constructing meta-models is
called ‘meta-modelling’ or ‘surrogate modelling’. Using the obtained meta-models, optimization procedure is applied in order to search the optimum solution. Barthelemy and Haftka [47] and Haftka et al. [48] has discussed about the relationship between experiments and optimization.

An extensive consolidated review is elucidated by Simpson et al. regarding meta-models by exploring different sampling methods, meta-modelling procedures, approximating models, practical systems for experiments and their applications [49].
TABLE 8. Comparative analysis results for the best optimal solution of classical benchmark functions obtained by different algorithms.

| Algorithms | Unimodal Functions |
|------------|--------------------|
|             | Sphere        | Schwefel 2.22 | Rotated hyper ellipsoid | Schwefel 2.21 | Rosenbrock | Quartic |
| CMAOE6      | 9.266E-296    | 1.158E-143   | 2.998E-282              | 6.569E-135    | 4.617E-5   | 2.502E-5 |
| GOA         | 4.531E-06     | 4.283E-02    | 7.667E-01               | 8.747E-02     | 3.592E-06  | 5.452E-02 |
| EO          | 4.334E-58     | 4.456E-34    | 3.282E-30               | 7.250E-22     | 5.300E+00  | 7.183E-04 |
| PSO         | 2.887E+21     | 1.850E-10    | 3.020E-07               | 2.093E-05     | 1.038E-01  | 3.963E-03 |
| DA          | 9.406E-01     | 1.117E+00    | 4.937E+01               | 1.589E+00     | 1.348E+02  | 7.388E-03 |
| ALO         | 1.393E-08     | 2.056E-05    | 7.705E-01               | 1.046E-03     | 5.995E+00  | 1.644E-02 |
| GWO         | 3.395E-51     | 5.233E-29    | 5.841E-22               | 2.361E-16     | 6.237E+00  | 3.952E-04 |
| MPA         | 4.828E-31     | 1.559E-17    | 4.328E-15               | 2.306E-13     | 8.756E+00  | 3.949E-04 |
| SSA         | 4.525E-10     | 7.858E-06    | 5.501E-06               | 1.617E-05     | 7.893E+00  | 1.398E-02 |
| SCA         | 2.296E+15     | 2.267E-11    | 8.789E-06               | 4.378E-06     | 7.278E+00  | 1.477E-03 |
| WOA         | 1.296E-74     | 2.326E-54    | 1.804E+01               | 2.103E-03     | 6.824E+00  | 1.993E-03 |
| SMA         | 0.0000E+00    | 4.080E-185   | 0.0000E+00              | 4.153E-184    | 1.398E-01  | 6.398E+05 |

| Algorithms | Multimodal Functions |
|------------|----------------------|
|             | Schwefel  | Rastrigin  | Ackley  | Griewank | Penalized1 | Levy N.13 |
| CMAOE2      | -4193.3   | 0.000      | 8.881E-16 | 0.000    | 6.852E-8   | 2.863E-7  |
| GOA         | -3.321E+03 | 2.885E+01  | 1.037E-03 | 1.958E-01 | 8.349E-04  | 8.249E-04 |
| EO          | -3.594E+03 | 0.000E+00  | 4.440E+15 | 0.000E+00 | 5.979E-17  | 4.027E-16 |
| PSO         | -2.593E+03 | 1.989E+00  | 3.206E+10 | 1.008E-01 | 2.041E-21  | 8.612E-22 |
| DA          | -2.998E+03 | 1.558E-01  | 1.427E+00 | 6.515E+02 | 7.582E-07  | 2.887E-01 |
| ALO         | -3.222E+03 | 1.591E+01  | 8.175E-05 | 1.206E-01 | 2.627E-01  | 2.253E-07 |
| GWO         | -3.141E+03 | 0.000E+00  | 7.993E+15 | 0.000E+00 | 2.741E-06  | 6.550E-06 |
| MPA         | -3.951E+03 | 0.000E+00  | 4.440E+15 | 0.000E+00 | 1.168E-11  | 3.587E-11 |
| SSA         | -3.259E+03 | 5.969E+00  | 9.227E-06 | 1.326E-01 | 5.146E-11  | 5.780E-10 |
| SCA         | -2.100E+03 | 2.714E-12  | 5.642E+08 | 6.632E+12 | 6.601E-02  | 3.353E-01 |
| WOA         | -3.821E+03 | 0.000E+00  | 8.881E-16 | 0.000E+00 | 1.482E-03  | 2.499E-03 |
| SMA         | -4.189E+03 | 0.000E+00  | 8.881E-16 | 0.000E+00 | 8.967E-05  | 6.699E-03 |

The state-of-the-art categorization of different approximation methods based on selection, sampling, and curve fitting is presented in Table 10. A model of si-PMSM having 9 slots and 6 poles is developed having a concentrated winding configuration. The dimensions of the initial design are chosen to comply with sufficient electric vehicle powertrain conditions. Table 11 displays the model specifications. The problem, in this work, is formulated using approaches based on finite element (FE) and regression analysis of supervised learning via surrogate modelling (SMO) concepts. The FE technique helps to collect the cogging torque response of the motor at different inter-polar lengths. The least square method is the simplest and efficient approximation methods to derive the desired relationship of input design parameters with the output response, i.e., cogging torque of the motor. Figure 7 depicts the decision variable and its variation performed to obtain ten experimental results for cogging torque. The SMO method assists to derive the relationship between the design parameter ($l_i$) and the response of interest (i.e., supplementary cogging torque). The various FE responses for different values of $l_i$ are treated as the design of experiments (DOEs) of SMO.

In design industries, by the administration of FE analysis to complex system architecture, a steep level of accuracy is ensured while studying the influence of critical factors such as structural parameters and their respective materialistic properties on the system performance. FE modeling safeguards the user-friendly environment to encounter susceptibility and versatility of the design. For different inter-polar lengths, the corresponding FE analysis results for ten experiments are registered in Table 12.
Using the regression analysis through the least square approach, the coefficients of the meta-model of cogging torque is estimated using the MATLAB 2016a tool \[50], \[51]. Thus, the polynomial response surface (RS) model is derived. The appropriateness of the RS model is inspected via the root mean square error (RMSE) test. The lower the value of RMSE value \(\approx 0.9577\) which is less than 1 since the coefficient of determination \(R^2 = 0.9991\) and adjusted coefficient of determination \(R^2_{adj} = 0.9990\) are nearly equal with the error of \(\sim 0.0001\). It ensures that no unwanted terms are included in the designed RS model. The real world objective problem is thus formulated as a cogging segment of torque \(f_1\) denoted by Equation (22). Here, \(T_{out} \geq 11.18 \, \text{Nm}\) is the output torque produced at the shaft by the motor which is used as one of the inequality constraints due to the fact that the optimized model should be able to produce at least 11.18 Nm of torque, produced by the initial model.

\[
\begin{align*}
\text{min} \quad f_1(l) & = -5.664 + 8.4925l_i - 0.4661l_i^2 \\
\text{subjected to:} \quad 1.5 \, \text{mm} & \leq l_i \leq 15.94 \, \text{mm} \\
T_{out} & \geq 11.18 \, \text{Nm} \tag{22}
\end{align*}
\]

V. VALIDATION TEST OF CMAOE ALGORITHMS FOR DESIGNED PROBLEM

In this section, first the effect of entropy is evaluated for various variants of CMAOE by implementing it on Equation (22). Then, a validation test is employed to compare the change in performance of the initial model and the optimized model.

Entropy measures randomness in a population statistically. It is used to characterize the texture of the input information. By normalizing each solution in \(x\) (see equation (1)), the entropy of the initial population is computed in the CMAOE algorithm (see Figure 4). Following inferences were observed from this study (see Figure 8).

- The bar (l) in Figure 8 corresponds to the entropy computed by the random function used in standard AO to generate the initial population. The Henon (a), Piecewise (g), and Tent (k) maps show similar behavior as that of the bar (l) and have the entropy concentration of a lower range, i.e., \(< 3\).
- The populations generated using maps Circle (c), Gauss (d), Sine (h), and Sinusoidal (j) exhibit entropy concentrations of medium-range, i.e., \(3 - 5\).
- The map Logistic (f) performs the best performance with respect to the highest entropy concentration of 6.8955. Maps Chebyshev (b), Iterative (e), and singer (i) also present the entropy concentration range of high values, i.e., \(> 5\).

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- The map Logistic (f) performs the best performance with respect to the highest entropy concentration of 6.8955. Maps Chebyshev (b), Iterative (e), and singer (i) also present the entropy concentration range of high values, i.e., \(> 5\).

### TABLE 9. Computational time comparison.

| Algorithms    | Total computational time = (initialization time + time to update solution + fitness calculation time) |
|---------------|-------------------------------------------------------------------------------------------------|
| CMAOE         | \(10t_i + 1000 \times 51t_i + 1000 \times 20t_i\)                                             |
| HHAOHIO       | \(10t_i + 1000 \times 61t_i + 1000 \times 20t_i\)                                             |
| AOAAO         | \(10t_i + 1000 \times 62t_i + 1000 \times 20t_i\)                                             |
| AGWO          | \(10t_i + 20t_i + 1000 \times 41t_i + 1000 \times 20t_i\)                                     |

### TABLE 10. Categorization of related work to meta-models.

| Meta-model selection | Sampling | Curve fitting                      |
|----------------------|----------|------------------------------------|
| Polynomial           | Classic techniques | Least square regression (weighted) |
| • Linear             | Factorial/fractional |                                  |
| • Quadratic          | Box–Behken     |                                    |
| • Higher order       | Plackett–Burman |                                    |
| • Least              | Central composite|                                    |
| • interpolating      | Alphabetic optimum |                              |
| spline               |                      |                                    |
| Splines              | Space-filling      | Best linear predictor             |
| • Linear             | Simple grids      |                                    |
| • Cubic              | Orthogonal arrays |                                    |
| • Rational B splines | Hammersley sequence |                               |
| ( NurBS )            |                      |                                    |
| • Multi-variate      | Minimax and      |                                    |
| adaptive regression  | maximin          |                                    |
| spline               | Uniform designs   |                                    |
| ( MARS )             |                      |                                    |
| Gaussian process     | Hybrid techniques | Best linear unbiased predictor (BLUP) |
| Kriging model        | Human or random selection | Multipoint approximation (MPA) |
| Radial basis function (RBF) | Based on importance | Log likelihood |
| Hybrid model         | Directional simulation | Back propagation (in ANN) |
| Support vector machine (SVM) | Adaptive methods | Entropy (for inductive learning on decision tree) |
| Knowledge based or decision tree | Based on discrimination |                                      |

### TABLE 11. Specifications of the si-PMSM model (HP = Horse power, RPM = Revolutions per minute, EMF = Electromotive force).

| Specification | Value | Unit |
|---------------|-------|------|
| Rated Output Power | 3 | HP   |
| Rated Speed    | 3000 | Rpm  |
| Operation Frequency | 150 | Hz   |
| Input Voltage  | 200  | V    |
| Stator Inner Diameter | 67.6 | Mm   |
| Stator Outer Diameter | 110 | Mm   |
| Rotor Outer Diameter | 66.6 | Mm   |
| Stack Length   | 50   | Mm   |
| Air gap Length | 2.5  | Mm   |
| Coil Turns     | 30   |      |
| Inter-polar length | 15.34 | Mm |
| Electrical Steel | M400-50A | -    |
| Permanent Magnet | NdFe35 | -    |
The optimized model shows a noticeable improvement in its electromagnetic performance with respect to the initial model, shown in Figure 9. The validation test analysis results are reported in Table 13. It is observed that the cogging torque of the optimized model obtained by AO is reduced by 51.5% while that of the model obtained by CMAOE is reduced by 53%. It is also found that the optimized model is able to produce enhanced output torque at the shaft ($T_{out}$) by 41% using AO while that 43.4% using CMAOE.

VI. DISCUSSIONS ON PERFORMANCE OF CMAOE

On the basis of source of inspiration, all the hybrid versions of Aquila Optimizers have been collected for the first time to encourage the future research. The chaotic maps are integrated to many metaheuristic algorithms like Bird Swarm algorithms with chaotic mapping [52], Chaotic Bee Colony Algorithms for global numerical optimization [53], Chaotic Optics inspired optimization algorithms as global solution search approach [54], Chaotic League Championship algorithm [55], and Chaotic Harmony Search Algorithms [56], etc., in order to generate random numbers involved in the computations of converging solutions. However, this paper integrates the chaotic mappings to generate initial population to hybrid version of AO for the first time.

Thus, the proposed CMAOE presents the following features highlighting certain pros and cons:
- It has higher diversity in the population search region using the hybridization of AO with GA.
Through comparison of entropy of initial population, generated via integration of chaotic maps, with that generated via stochastic method, it is found that the performance of the algorithm has been improved.

The order of computational complexity has not been affected as compared to the standard AO.

As compared to the other hybrid versions of AO, the CMAOE shows inferior burden on CPU computational time. However, as compared to the standard AO, the computational time of proposed CMAOE has been increased to solve certain complex multimodal functions like Rastrigin and Penalized1 etc.

The hybridization of algorithms includes the computation of extra variables which require extra CPU memory.

The proposed CMAOE has following practical advantages:

- The high accuracy of CMAOE helps the design engineers to apply the proposed hybrid version of AO to design optimization of electric motors.
- The improvement in convergence speed of CMAOE ensures that the proposed algorithm can be applied to high constrained problems like power system optimal power flow.

The limitations of the research work are listed below:

- To use a particular chaotic map for generation of initial population in CMAOE, the complexity of the problem has to be considered. It effects the overall computational time of the algorithm [57].
- To formulate the real world problem using methods other than least square method, the feasibility ranges of the design variables have to be thoroughly analyzed.

VII. CONCLUSION

This paper successfully illustrates the following accomplishments:

- A novel advanced version of the standard AO algorithm, obtained by incorporating chaotic mapping and single-stage evolutionary algorithm concept, is developed and successfully implemented to optimize problems efficiently.
- The proposed CMAOE technique is executed for retrieving the optimal value of various benchmark functions.
- On the basis of its performance for unimodal and multimodal functions, the chaotic maps are categorized in ‘Best’, ‘Better’, and ‘Good’ levels.
- For unimodal functions, CMAOE with Logistic, sine, Chebyshev and Iterative chaotic maps are found to be giving the results, hence are categorized as “Best” performance category.
- For multimodal functions, CMAOE with Logistic, Iterative, Chebyshev and Singer chaotic maps are giving the results falling in the “Best” category.
- The convergence profiles of CMAOE algorithms are found to be more efficient than the standard AO framework.
- The solution samples obtained from standard AO and CMAOE show dissimilarity in distribution which is verified using Wilcoxon null hypothesis test.
- A real-world industrial effective motor optimization problem is formulated using FE and SMO-based realizations. The analyzed results are thus obtained by both AO and CMAOE algorithms.
- It is found that the CMAOE technique produces 53% reduction in cogging torque and 43.4% increment in torque output of the motor. Whereas, standard AO produces 51.5% reduction in cogging torque and 41% increment in torque output of the motor.

Thus, this paper illustrates that the proposed CMAOE algorithms can efficiently solve variety of problems. For future works, the authors plan to examine the evolution in the average entropy. Also, the effects of multifractals can be monitored during the formation of initial populations.

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