Sensitivity of nucleon–nucleus scattering to the off–shell behavior of on–shell equivalent NN potentials

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Abstract

The sensitivity of nucleon–nucleus elastic scattering to the off–shell behavior of realistic nucleon–nucleon interactions is investigated when on–shell equivalent nucleon–nucleon potentials are used. The study is based on applications of the full–folding optical model potential for an explicit treatment of the off–shell behavior of the nucleon–nucleon effective interaction. Applications were made at beam energies between 40 and 500 MeV for proton scattering from 40\textsuperscript{Ca} and 208\textsuperscript{Pb}. We use the momentum–dependent Paris potential and its local on–shell equivalent as obtained with the Gelfand–Levitan and Marchenko inversion formalism for the two nucleon Schrödinger equation. Full–folding calculations for nucleon–nucleus scattering show small fluctuations in the corresponding observables. This implies that off–shell features of the NN interaction cannot be unambiguously identified with these processes. Inversion potentials were also constructed directly from NN phase–shift data (SM94) in the 0–1.3 GeV energy range. Their use in proton–nucleus scattering above 200 MeV provide a superior description of the observables relative to those obtained from current realistic NN potentials. Limitations and scope of our findings are presented and discussed.

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I. INTRODUCTION

Theoretical studies of the two–nucleon interaction in their off–shell domain have a long standing tradition [1] and this topic is of renewed interest for designated experiments at several accelerators laboratories. Few– and many–body systems offer generally the possibilities for such studies but not seldom have such endeavors ended prematurely due to lack of statistics in data taking or an incomplete and inconclusive theory. Current interest on this issue comes from theory groups who have independently developed NN potentials which account reasonable well for the two–body phase–shifts at energies below pion production threshold [2–4]. These potentials are manifestly different in aspects such as their off–shell behavior and the device of an experimental discrimination among them would be very valuable for a comprehensive understanding of particularities for each of the currently acceptable models.

In the past much hope was given to NN bremsstrahlung since this three body reaction is within a Born approximation theoretically well defined [5,6]. As a result of this simple reaction mechanism one obtains a good perspective of the link between the \((NN\gamma)\) observables and half–off–shell NN \(t\) matrices. Using the available theoretical developments in the field of bremsstrahlung it is now known how to explain the available data with different NN potentials as far as they reproduce the NN on–shell data very well. This result has been independently validated and remains a surprise in view of the obvious differences in the half–off–shell \(t\) matrices from different model potentials. In other words, the expectation that on–shell equivalent NN potentials would provide distinctive bremsstrahlung predictions was disillusioned with more complete and reliable calculations.

It can be argued that bremsstrahlung involves only half–off–shell \(t\) matrices and thus this reaction is rather confined in phase–space. Microscopic models of nucleon–nucleus (NA) scattering in a full–folding framework [7,8] do not suffer from such limitations as the optical potential depends explicitly upon the effective interaction fully off–shell and therefore constitute a wider frame for investigating the NN interaction off–shell.

Recently, significant advances have been made in accurately handling the off–shell degrees of freedom in NA elastic scattering [9–12]. Here, studies have demonstrated that an accurate treatment of the off–shell behavior of the NN interaction is needed for a proper account of the theory. Irrespective of obvious improvements in describing the data at projectile energies below 400 MeV, the calculations still show some systematic deficiencies. In particular the misfit of spin observables at small momentum transfers, \(\lesssim 1\text{ fm}^{-1}\), is not understood. The origin of such discrepancies with the data could be attributed to intrinsic limitations of the bare NN potential with respect to the phenomenology (especially at the higher energies), to the simplifications implicit in the model for the NN effective interaction, or to the fact that the optical model potential has only been developed to its lowest order. Above 400 MeV, full folding model results quickly deteriorate as they fail to properly describe the observables [7].

Despite the significant advances in full–folding model calculations it remains difficult to identify, at the level of the scattering observables, distinctive off–shell features of NN interactions. The difficulty is mainly attributed to non negligible differences on–shell among currently available realistic NN potentials [13].

In this paper we investigate the sensitivity of NA scattering when using bare NN potentials which are equivalent on–shell. For that purpose we have chosen the Paris potential \((i.)\)
in its full momentum dependent form and (ii.) a local equivalent potential generated with Gelfand–Levitan–Marchenko inversion from the Paris potential phase–shifts and deuteron bound state. The inversion potential yields different off–shell continuations as the original Paris potential. These differences and associated correlations are investigated in the full–folding NA optical model.

We find for these two alternatives differences of about 5% overall in NA observables and this rises doubts that this sensitivity could be used to infer off–shell peculiarities of NN interactions in general. Based on this result we conclude that quantum inversion provides a practical and accurate connection between NN and NA elastic scattering. As mediating formalism may Gelfand–Levitan–Marchenko quantum inversion use only experimental NN data as input and thus link model–independent two–body with many–body data.

This article is organized as follows. In section II we outline the theoretical background for the present work. We introduce and discuss aspects and assumptions implicit in the Gelfand–Levitan and Marchenko inversion method as well as the in–medium full–folding model of the optical potential for NA elastic scattering. In section III we discuss the sensitivity of NA scattering by calculating the scattering observables from the Paris potential and from an inversion NN potential constructed from the Paris NN phase–shifts. In this way, our analysis becomes explicitly dependent only from the off–shell differences between the two potentials. In section IV we construct inversion NN potentials directly from the NN data, including approximately the NN phase–shifts above pion production threshold as a way to guide the intermediate energy properties of the potential. Calculations for proton–nucleus elastic scattering in the 40–500 MeV range and from different targets are also discussed. Finally, in section V we present a summary and draw conclusions from our work.

II. THEORETICAL BACKGROUND

A. Two Nucleon Potentials from quantum inversion

The starting point for the NN interaction is generally a relativistic scattering description and it is beyond any doubt, that a realistic potential must show strong angular momentum and spin dependencies and thus channel dependence. An explicit momentum dependence or nonlocality is predicted in all relativistic potential models. Allowance for the relativistic nature of the problem in these models is in the best cases limited to relativistic kinematics and simplified calculation of selected exchange diagrams. The full dynamics of potentials requires a derivation using quantum field theory. This not only ensures Lorentz covariance but shows the way how the actual potential must be constructed. For this purpose, different formalism have been proposed and they determine the off–energy shell or off–mass shell amplitudes. Usually the derivation of such relativistic equations also determines the propagators and the potential which contains all information about the dynamics of the interaction. An accepted four–dimensional formulation of quantum field theory is the covariant Bethe–Salpeter equation and the three–dimensional relativistic equations are obtained on the basis of quasi–potential methods. Another field theoretical methods use a generalization of the Schrödinger equation in the form of the Tomonaga–Schwinger equation and in the form of
a covariant Hamiltonian formulation. Irrespective of their foundation and motivation, the field theoretical models are using more or less phenomenology to fit data.

Well fitted and widely applied within these models are the boson exchange potentials. Representative within the boson exchange models are the Nijmegen [14] and Bonn potentials [3]. The Paris potential also belongs to this category and describes the long- and medium-range interaction with single and correlated two-pion exchange and heavier meson exchanges [2]. It fits the experimental two nucleon data reasonably well and has tradition in calculations of microscopic optical model potentials for NA scattering at low and medium energies. It is generally used in its Yukawa parameterization including an explicit momentum dependent term [2]

$$V = V_a + \frac{\hbar^2}{m} p^2 V_b + V_b \frac{\hbar^2}{m} p^2,$$  

(1)

where $m$ is the nucleon mass and $p$ the relative momentum operator. This momentum dependence emulates a hard core repulsion and produces a divergence of the phase-shift $\delta(k)$ as $k \to \infty$. When compared with data, the Paris potential fit quite well the phase shift at energies up to 280 MeV. Above this energy we notice a rapidly increasing divergence. This characteristic strong repulsion at short distances is easy to identify in cross section and spin observables of NA scattering and reactions [15]. Despite this weakness we have chosen the Paris potential since we have confidence in the numerics of our Lippmann–Schwinger and Bethe–Goldstone calculations with it.

With this investigation we aim to disclose effects which are caused by the momentum dependence and high energy phase shift discrepancies of the genuine Paris potential in off-shell $t-$ and $g-$matrices. To this purpose we use the recently developed Gelfand–Levitan–Marchenko inversion of partial wave radial Schrödinger equations to generate phase equivalent local potentials to Paris potential phase shifts. These inversion algorithms distinguish inversion for single and coupled channels cases with and without a Coulomb reference potential [16]. In other words, we obtain separately the hadronic part of the NN interaction from any set of $np$ and $pp$ phase shifts. To investigate the effect of different on-shell behaviors we also generate potentials from the latest experimental phase shift analysis SM94 by Arndt and collaborators [17].

The strong interaction inversion potentials $V(r; LSJ,T; np)$ and $V(r; LSJ,T; pp)$ are numerical solutions of Gelfand–Levitan or Marchenko integral equations,

$$K(r, r') + F(r, r') + \int K(r, s) F(s, r') ds = 0 \quad (2)$$

and

$$V(r) = \pm \frac{d}{dr} K(r, r), \quad (3)$$

for any specified radius and they are determined channel by channel. The input kernel $F(x, y)$ is computed with the spectral information, Jost functions or S-matrices including deuteron binding energy and normalization constants. We use quantum inversion as transformation of given real phase shifts, which are specified within a finite energy interval
\[ \delta(k) \sim \delta(E) = \{ \delta(k) | E = [0, E_{max}] | k = [0, k_{max}] \} . \]  

(4)

Thereafter they are smoothly extrapolated with a rational function which decays asymptotically [18]

\[ \delta(E) \sim \delta(k) = \left\{ \delta(k) | k \geq k_{max} \left| \lim_{k \to \infty} \delta(k) \sim k^{-1} \right\} . \]  

(5)

This transformation is unique for the class of potentials in which we are interested and which we assume physically significant. The resulting inversion potentials are real and local.

It is evident that this inversion procedure produces a restricted phase equivalent potential to the Paris potential. The limited data input \( \delta(E) \) for \( 0 \leq E \leq E_{max} \) is used to control the range of equivalence and the extrapolation thereafter to control the softness of the short range core interaction.

### B. In-medium full–folding optical potential

The optical potential for NA elastic scattering can be casted as the convolution of an antisymmetrized effective interaction with the target ground state single particle wave functions [19–22,9]. In momentum space this one-body operator reads

\[ U(\vec{k}', \vec{k}; E) = \int d\vec{p} d\vec{p}' \sum_{\alpha \leq \epsilon} \phi_{\alpha}^+(\vec{p}') \left\langle \vec{k}', \vec{p}' \mid F(E + \epsilon_{\alpha}) | \vec{k}, \vec{p} \right\rangle_A \phi_{\alpha}(\vec{p}) \]  

(6)

where \( E \) represents the energy of the incoming projectile and \( \{ \phi_{\alpha}, \epsilon_{\alpha} \} \) are the target ground state single–particle wave functions and corresponding energies. The momenta \( \vec{k}(\vec{k}') \) and \( \vec{p}(\vec{p}') \) correspond to the initial (final) momenta of the projectile and target struck nucleon respectively. The two–nucleon interaction in Eq. (6) accounts for multiple scattering of nucleons to all orders in the ladder approximation [19,23]. Although a general expression for this matrix can formally be defined, its practical implementation requires the device of a dynamical model for the effective two–nucleon interaction in the nucleus. The procedure we follow is that introduced in Ref. [9], where translational invariance properties of two–nucleon scattering in free space or infinite nuclear matter suggest the following \textit{ansatz} for the two-body matrix

\[ \left\langle \vec{k}', \vec{p}' \mid F(\omega) \mid \vec{k}, \vec{p} \right\rangle = \frac{1}{(2\pi)^3} \int d\vec{R} e^{i\vec{R} \cdot (\vec{Q} - \vec{Q}')} \left\langle \vec{R}' \mid f_{\vec{Q}, \vec{Q}'}(\omega ; \vec{R}) \mid \vec{R} \right\rangle . \]  

(7)

Here we have defined the initial and final two–nucleon center-of-mass (c.m.) momenta,

\[ \vec{Q} = \vec{k} + \vec{p} , \quad \vec{Q}' = \vec{k}' + \vec{p}' , \]  

(8)

and the corresponding relative momenta by

\[ \vec{r} = \frac{1}{2}(\vec{k} - \vec{p}) , \quad \vec{r}' = \frac{1}{2}(\vec{k}' - \vec{p}') . \]  

(9)

The function \( \left\langle \vec{r}' \mid f_{\vec{Q}}(\omega ; \vec{R}) \right| \vec{r} \rangle \) corresponds to the matrix elements of a reduced two-body effective interaction. In the case of no dependence of the \( f \) matrix upon the spatial coordinate
one restores total momentum conservation of the interacting pair as the radial integral in Eq. (5) leads to a c.m. momentum conserving Dirac $\delta$–function.

A calculable expression for the optical potential in medium emerges after a systematic reduction of the many body propagator when represented in terms of the target ground state spectral function [9]. To lowest order in a series expansion of the two-body propagator in a finite nucleus this interaction can be identified with the $g$ matrix solution of the Brueckner-Bethe-Goldstone equation for interacting nucleons in infinite nuclear matter evaluated at nuclear density $\rho(R)$ in the nucleus. Furthermore, it becomes convenient at this point to substitute the single particle energies $\epsilon_\alpha$ by an average value, $\bar{\epsilon}$, and to use the Slater or Campi-Bouyssy approximations [24,25] to represent the ground state mixed density, i.e.

$$\rho(p', \vec{p}) = \sum_{\alpha \leq \epsilon_p} \phi_\alpha^\dagger(p') \phi_\alpha(\vec{p}) \approx \frac{4}{(2\pi)^3} \int d\vec{R} \ e^{i(\vec{p'} - \vec{p}) \cdot \vec{R}} \rho(R) \left\{ \frac{1}{\hat{\rho}(R)} \int d\vec{P} \ \Theta[\hat{k}(R) - P] \right\} ,$$

(10)

where $\rho(R)$ is the local nuclear density at coordinate $\vec{R}$ and $\vec{P}$ represents the struck nucleon mean momentum defined by

$$\vec{P} = \frac{1}{2}(\vec{p} + \vec{p}') .$$

(11)

The local momentum function $\hat{k}(R)$ sets the range of variation of the struck nucleon mean momenta upon collisions with the projectile and is obtained from the Slater or Campi-Bouyssy [24] prescriptions. The local density function $\hat{\rho}(R)$ is defined in terms of the local momentum function by

$$\hat{\rho}(R) = \frac{2}{3\pi^2} \hat{k}^3(R) .$$

(12)

With the above considerations the optical potential can be expressed in terms of the nuclear density and a Fermi averaged effective interaction obtained from interacting nuclear matter. This interaction retains nuclear medium correlations associated with the nuclear mean fields and Pauli blocking. The in-medium full–folding optical potential then reads

$$U(\vec{k}', \vec{k}; E) = \frac{4}{(2\pi)^3} \int d\vec{R} \ e^{i\vec{q} \cdot \vec{R}} \rho(R) \frac{1}{\hat{\rho}(R)} \int d\vec{P} \ \Theta[\hat{k}(R) - P] \left\langle \frac{1}{2} (\vec{K} - \vec{P} - \vec{q}) \left| g_{\vec{k} + \vec{p}}(E + \bar{\epsilon}; \vec{R}) \right| \frac{1}{2} (\vec{K} - \vec{P} + \vec{q}) \right\rangle_A .$$

(13)

Thus, the optical potential requires the calculation of $g$ matrices off–shell as their relative momenta obey no constraints apart from those imposed by the ground state mixed density of the target. Furthermore, no assumptions are introduced on the nature of the momentum dependence of the optical potential, thus retaining all non localities arising from the genuine momentum dependence of the NN effective interaction and as prescribed by the full–folding integral. Actual calculations involve determining $g$ matrices at several densities and over a wide range of total center-of-mass momenta, features fully accounted for in the present work.

In the context of a medium independent internucleon interaction, as when the free $t$ matrix is used to represent the NN effective interaction, the integral over the spatial coordinate
in Eq. (13) can be performed separately from the motion of the target nucleons. With the use of Eq. (10) for the mixed density one recovers the expression for the full-folding optical potential in the zero density approximation [7], namely \( U(\vec{k}', \vec{k}; E) \rightarrow U_o(\vec{k}', \vec{k}; E) \), where

\[
U_o(\vec{k}', \vec{k}; E) = \int d\vec{P} \rho (\vec{P} + \frac{1}{2} \vec{q}, \vec{P} - \frac{1}{2} \vec{q}) \langle \frac{1}{2}(\vec{K} - \vec{P} - \vec{q}) | t_{K+P}(E+\tau) | \frac{1}{2}(\vec{K} - \vec{P} + \vec{q}) \rangle_A .
\]

The dependence of the optical potential on off-shell \( t \)-matrices becomes explicit in the above expression. The feasibility of the full-folding model to investigate particular signatures of the effective interaction off-shell will depend on the sensitivity of NA scattering observables to the use of \( t \) matrices with manifestly distinctive behaviors off-shell. An important constraint for such study is that the effective interactions, \( t \) matrices in this limit, agree on-shell. To the extent this constraint is met, one can attain the differences in the NA scattering observables to the differences of the interactions off-shell.

A simple kinematical effect usually overlooked, but explicitly accounted for in our calculations, is that the full-folding approach calls for matrix elements of energy \( E + \tau \) in the laboratory frame. In the limit of the free \( t \) matrix for the NN interactions, the energy of the interacting pair in its c.m. is given by \( E + \tau - \frac{1}{4m}(\vec{K} + \vec{P})^2 \). Therefore, the maximum energy of the pair in its c.m. is \( E + \tau \), the energy of the beam plus the average binding energy of the target nucleons. This is to say that in the case of optical potentials for nucleons at 500 MeV, \( t \) matrices of up to \( \sim 1 \) GeV in the laboratory frame are required. This more demanding sampling of the NN effective interaction is a result of the unconstrained kinematics allowed by the Fermi motion of the nucleons in the nucleus.

A few comments are pertinent regarding further approximations in the treatment of the \( t \) matrix which limit a clear assessment of the off-shell behavior of the NN interaction in NA scattering. A simplifying assumption, commonly used in some alternative full-folding calculations [10, 11], is that the \( t \) matrix varies very weakly with respect to the NN c.m. momentum \( \vec{K} + \vec{P} \). Thus, the magnitude of this momentum is fixed to the (asymptotic) on-shell value of the incoming projectile, \( K_o \), and the \( t \) matrix is approximated by

\[
t_{K+P}(\omega) \approx t_{K_o}(\omega) .
\]

Thus, \( t \) matrices are evaluated at a fixed energy equal in the NN c.m. to one-half the energy of the beam. Here one has neglected all effects associated with the Fermi motion in the NN c.m. momentum dependence. The resulting full-folding calculations samples the \( t \) matrix off-shell through its dependence on \( \vec{P} \) in the relative momenta exclusively (see Eq. (14)). This approximation seems adequate at beam energies near 300 MeV. Its application at lower or higher energies, however, needs further considerations as the NN \( t \) matrix exhibits a sizable NN c.m. momentum dependence [8]. In the low energy region, apart from the fact that medium effects need to be incorporated in the model, the underlying kinematics prescribed by the full-folding yields the sampling of the \( t \) matrix in regions where it varies significantly as the low energy behavior of the interactions becomes dominant. In the high energy regime, in turn, difficulties arise from the opening of inelastic channels such as those associated with pion production or \( \Delta \)-resonances. The actual merit of the theory of the optical potential needs to be assessed with a consistent incorporation of such additional degrees of freedom.
C. The NN effective interaction

In the present approach, correlations associated with interacting nucleons in the nuclear medium are obtained from the NN effective interaction defined by the Brueckner-Bethe-Goldstone equation for symmetric nuclear matter. In momentum representation, the $g$ matrix associated with interacting nucleons of total c.m. momentum $\vec{Q}$, starting energy $\omega$ and nuclear density $\rho$ satisfies

$$
\langle \vec{\kappa}' | g_{\vec{q}}(\omega; \vec{R}) | \vec{\kappa} \rangle = \int d\vec{\kappa}'' \langle \vec{\kappa}' | V | \vec{\kappa}'' \rangle \lambda_{\vec{q}}^{NM}(\vec{\kappa}''; \omega; k_F) \langle \vec{\kappa}'' | g_{\vec{q}}(\omega; \vec{R}) | \vec{\kappa} \rangle ,
$$

(16)

with $k_F$ the nuclear matter Fermi momentum determined from the nuclear density via

$$
k_F = \left( \frac{3\pi^2}{2} \rho(R) \right)^{1/3}.
$$

(17)

The two-body propagator $\lambda_{\vec{q}}^{NM}$ models both Pauli blocking and the nuclear mean field effects in the propagation of intermediate states,

$$
\lambda_{\vec{q}}^{NM}(\vec{q}; \omega; k_F) = \frac{Q(P_+; P_-; k_F)}{\omega + i\eta - \epsilon(P_+; k_F) - \epsilon(P_-; k_F)} ,
$$

(18)

with $P_\pm = \frac{1}{2} \vec{Q} \pm \vec{q}$ and $Q$ the Pauli blocking function

$$
Q(P_+; P_-; k_F) = \Theta[\epsilon(P_+; k_F) - \epsilon_F] \Theta[\epsilon(P_-; k_F) - \epsilon_F] .
$$

(19)

Here the single particle energies $\epsilon$ are defined in terms of the self-consistent nuclear matter fields, $U_{NM}$,

$$
\epsilon(k_\alpha; k_F) = \frac{k_\alpha^2}{2m} + Re[U_{NM}(k_\alpha; k_F)] ,
$$

(20)

where the mean fields $U_{NM}(k; k_F)$ are calculated self-consistently for the underlying bare NN interaction from

$$
U_{NM}(k; k_F) = \sum_{\alpha \leq k_F} \left( \frac{1}{2}(k - k_\alpha) \left| g_{\xi + \xi_\alpha}^{(1)}(\epsilon(k) + \epsilon(k_\alpha)) \right| \frac{1}{2}(k - k_\alpha) \right) ,
$$

(21)

Actual calculations of these mean fields have been made using the continuous prescription at the Fermi energy [26] and simplifying the Pauli blocking function $Q$ by its angle–averaged form.

III. PARIS VERSUS PARIS INVERSION

We have used the Paris potential to generate a set of NN phase–shifts which are taken as input to calculate the corresponding inversion potential. Thus, we make sure that Paris
and the inversion potential are equivalent on–shell within the accuracy obtained with the numeric algorithms used for implementing the quantum inversion method. The range of energies where the explicit phase–shifts are considered determine importantly the off–shell behavior of the inversion potential. This effect is observed when calculating, for example, the optical potential and observables for NA scattering from the free \( t \) matrix obtained from inversion potentials constructed from different sets of phase–shifts. Therefore, to ensure that properties of the inversion potential depend solely on the dynamical equations and not on the range of energies considered for the phase–shifts, we have verified that a set of Paris phase–shifts in the 0–1.3 GeV energy range is sufficient to construct an inversion potential that can be used up to 500 MeV in effective interaction calculations (Sec. II.C). Above 1.3 GeV, the inversion algorithm assumes a smooth decrease to zero at infinity for the phase-shifts (Eq. 5). This extrapolation departs strongly from the Paris potential behavior.

In Fig. 1 we show the Paris phase-shifts (crosses) and the phase-shifts obtained from its corresponding inversion potential (full line) for selected NN channels \((L \leq 2)\) and in the 0–1.2 GeV energy range. In general, for most of the NN channels we have considered the agreement is excellent. Some differences are observed in the (coupled) \( ^3D_1 \) state above 400 MeV. Altogether, we can conclude that the calculated inversion potential is phase–equivalent to the Paris potential. Their differences in a many–body system should come from the intrinsic properties of the two potentials and provided that the off–shell sampling is compatible with the energy range used to set the phase equivalence.

The inversion potential differs on its off–shell content from the original Paris potential as the former is static and local, whereas the latter is momentum dependent. These differences and associated correlations to all orders are now investigated in the context of the full–folding model of the optical potential for NA scattering and as described in Sec. II.B.

### A. Sensitivity to off–shell effects in NA scattering

We have calculated both \( t \) matrices and in–medium \( g \) matrices from the Paris and its inversion potentials. The inversion scheme was used for all the NN channels with total angular momentum \( J \leq 2 \). For channels with \( J > 2 \) the genuine Paris potential is used. The nucleon–nucleus optical potential was calculated using the \( g \) matrix as effective interaction since medium effects have been proved important even for nucleons with incident energies of 400 MeV [9]. The corresponding \( g \) matrices were calculated solving Eq. (16) using standard matrix inversion methods [27]. The \( \vec{R} \)–dependence in the full–folding integral was obtained by calculating \( g \) matrices at different densities as obtained from different values for \( k_F \) up to 1.4 fm\(^{-1}\). For an accurate off–shell sampling of the NN effective interaction, \( g \) matrices were calculated at several values of the total NN c.m. momentum in the 0 – 7 fm\(^{-1}\) interval, with higher density of points in the region it varies most rapidly as a function of the c.m. momentum. Contributions associated with the deuteron bound state singularity were also included [8]. The ground state nuclear densities and average binding energies (\( \bar{\epsilon} \) in Eq. (13)) are the same as in Ref. [9].

Calculations of differential cross sections \( (d\sigma/d\Omega) \) and analyzing powers \( (A_y) \) were made for proton elastic scattering from \(^{40}\text{Ca}\) and \(^{208}\text{Pb}\) in the 40–400 MeV energy range. Results for the spin rotation function have been omitted for brevity as they exhibit similar behavior...
as those observed in $A_y$. In Figs. 2 and 3 we present the calculated scattering observables, as function of the center of mass scattering angle ($\Theta_{c.m.}$) in the 40–MeV application, and as a function of the momentum transfer ($q$) at 400 MeV. The data for p$^+^{40}$Ca scattering data at 40 MeV were taken from Ref. [28,29]. In the case of p$^+^{208}$Pb, the data at 400 MeV were taken from Ref. [30]. The full and dashed curves represent results for the Paris potential and corresponding inversion respectively. Results at different energies and for different targets show comparable quantitative differences. Also, very similar differences in the observables are observed when using a free $t$ matrix as input in the full–folding calculations.

Some conclusions can be drawn. The two phase–equivalent potentials give a qualitatively similar description of the scattering observables, regardless their very different intrinsic structure. The differences between the two curves in Figs. 2 and 3 reflect the level of sensitivity to the off–shell behavior associated with each underlying NN effective interaction. Quantitatively, we observe up to a 10% difference in the magnitude of the calculated observables due to differences off–shell in the effective interaction, feature that is maintained in the energy range under study. This weak sensitivity to differences off–shell of completely different NN bare potentials suggests that the full–folding model cannot unambiguously discriminate among on–shell equivalent NN potentials. Furthermore, this equivalence indicates that a determining element in the NN potential is its on–shell content. Indeed, we have tested this by taking a restricted energy range (0 - 400 MeV) for the NN phase–shifts to construct the inversion potential. In this case, the off-shell content of the inversion potential is different as reflected by the differences observed in the NA scattering observables, more significantly above 200 MeV. We have also considered a 0–3 GeV energy range for the genuine Paris phase–shifts to construct the inversion potential. In this case we observed no further differences with respect to those observed when using the 0–1.3 GeV energy range. These results indicate that NN potentials which closely accounts for the NN phase–shift data over a wide energy range one would be able to assess the level of completeness of the optical model for NA scattering.

The relatively weak sensitivity to the off–shell effects suggest that NN potentials, constructed by means of the inversion scheme and following closely the NN data are meaningful and would provide predictions for the full–folding model very close to what would it be obtained from first principle NN potentials with comparable fit to the data.

**B. Effective interactions off–shell**

To illustrate the degree of sensitivity of the $g$ matrix upon alternative choices of bare NN potentials, we have calculated selected matrix elements relevant for the leading contribution to the optical potential. Thus, we consider diagonal $g$–matrix elements, i.e. $\langle \vec{\kappa} | g_{Q_o;\vec{k}_F}(\omega, \vec{R}) | \vec{\kappa} \rangle$, with $Q_o$ fixed to a single value by $Q_o = \sqrt{2m\omega}$. In the context of the free $t$ matrix ($k_F = 0$) this kinematics implies the on–shell relative momentum occurs for $\kappa = \frac{1}{2}Q_o$. Since $\kappa$ is in general independent of both $Q_o$ and $\omega$, the resulting function corresponds to off–shell elements of the $g$ matrix. For the Paris and its inversion equivalent potential we have solved Eq. (16) for $g$ in the cases $k_F = 0$ fm$^{-1}$ and $k_F = 1$ fm$^{-1}$ and for the state $^1S_0$. This is a good example to show the differences generally observed in most of the states for the effective interactions. In Fig. 4 we show the corresponding matrix elements,
both real and imaginary components, for \( \omega = 30, 200 \) and \( 400 \) MeV. The solid and the dashed curves represent results from the Paris and the Paris inversion potentials respectively.

Since the Paris potential and its inversion are equivalent on–shell, the solid and dashed curves for the \( t \) matrix \((k_F=0)\) must intercept each other at \( \kappa = \frac{1}{2}Q_o \) in both their real and imaginary components. This fact is indeed the case in the figures shown here as the on–shell constraint is explicitly built in the inversion method; this is not necessarily the case, however, for the \( g \) matrix as the propagator differs from that in free space by the presence of Pauli blocking and the self–consistent fields. It is interesting to note the different asymptotic behavior \((k \to \infty)\) between the Paris and the inversion potential for both \( k_F=0 \) and \( k_F=1 \) fm\(^{-1}\). Whereas all curves tend to zero as \( k \) increases, the one corresponding to the real part of the \( g \) matrix for the Paris potential does not. This result is consistent with the explicit momentum dependence introduced in the parameterization of the Paris potential, feature which becomes dominant at high momenta. The extent to which this distinctive behavior is significant in the dynamics of the collision of the projectile with the nucleus needs to be assessed in the context of the elastic scattering observables.

In Fig. 4 we also observe that the plotted matrix elements usually differ utmost in a few percents around the on–shell values. This result is consistent with the differences we have already discussed at the level of the NA observables.

**IV. NA SCATTERING FROM NN PHASE-SHIFTS**

In this section we extend the idea of NN potentials obtained directly from NN phase–shifts through the quantum inversion method and its application to NA scattering. For this purpose, we have calculated a NN inversion potential based on the SM94 phase–shift analysis of Arndt and collaborators \[17\]. The main problem we face is the choice of a meaningful energy range where the phase–shifts are to be taken from. Indeed, in order to be consistent with the inversion scheme, a set of real phase–shifts are needed. This sets a limit for the energy range at the pion production threshold \((\sim 300 \text{ MeV})\). However, our studies with the Paris potential in sect. III show that phase–shifts at much higher energies are required by the inversion method.

Our approach is as follows. In order to account approximately for the trend with energy that phase–shifts have above pion production threshold, we have neglected the imaginary component of the phase–shifts and retained only their real components. Thus we can construct a real NN inversion potential from a set of real phase–shifts. The range of energy considered for the inversion is 0–1.3 GeV. Although this approach becomes essentially qualitative, we expect that our calculations will provide a guidance on how the medium and short range parts of the NN interaction, as determined by more realistic phase–shifts, affect the description of NA scattering in the intermediate energy region.

We have calculated NN inversion potentials from the SM94 data for all NN channels with \( J \leq 2 \). In Fig. 5 we present the phase–shifts as a function of the energy for some selected channels. Dots correspond to the SM94 data (only the real part of the phase–shifts) and full curves are the results from the inversion potentials constructed from the SM94 data. We also plot the phase–shifts obtained from the Paris inversion potentials as reference (dashed curves). These results suggest two observations. One is related to the ability of the inversion
method to reproduce the input data. The level of accuracy obtained is again excellent, as in the Paris case discussed in sect. III. The other point of physical significance is the clear departure of the Paris phase–shifts from, at least, the real part of the experimental ones above pion production threshold. It is precisely the presence of these differences which should affect the overall behavior of the inversion potential and, in particular, its off–shell components.

The role of the NN inversion potentials based on the SM94 data has been tested in NA elastic scattering. We have performed in–medium (g–matrix) full–folding calculations for proton scattering on $^{40}\text{Ca}$ at several energies. In the actual calculations, the inversion potentials were used for the NN channels with $J \leq 2$. For simplicity, we have kept the components corresponding to the Paris potential for all the states with $J > 2$. In Fig. 6 we show the differential cross–section and analyzing power ($A_y$) results for $p + ^{40}\text{Ca}$ at 40 MeV. The full line corresponds to the results from the inversion potential and the dashed curve to results from the Paris inversion one. In this and following figures we include as reference the calculations done with Paris inversion potentials to have a clear comparison of the differences that may be observed. In the scheme we have followed, these differences would come directly from the different underlying set of NN phase–shifts used. The results in Fig. 6 at 40 MeV show, however, little difference. This is consistent with the fact that both potentials are comparable in their agreement with the NN data below pion production threshold (see Fig. 5). Still, some small sensitivity of the order of 5 % in the observables is observed. Since both potentials are constructed following exactly the same inversion procedure, we conclude that phase–shifts above 300 MeV still determine properties of the NN potentials which affect low–energy NA scattering. These small, mostly off–shell, differences stay in even at much smaller energies in many–body systems.

We have pursued these calculations at higher energies. In Fig. 7 we present the results for $p + ^{40}\text{Ca}$ at 200 and 300 MeV. The meaning of the curves is the same as in Fig. 6 and the data was taken from Ref. [31] at 200 MeV and from Ref. [32] at 300 MeV. At these energies we start noting a marked departure in the predictions of the two inversion potentials, with a tendency of the potential constructed from the NN phase–shift data to be relatively closer to the NA scattering data. This result reflects the disagreement existing between the data and the Paris potential phase–shifts (Fig. 5). Our findings are confirmed with calculations for $p + ^{40}\text{Ca}$ at 400 and 500 MeV. These results are shown in Fig. 8. Here, the data was taken from Ref. [33] at 400 MeV and from Ref. [30] at 500 MeV. Certainly, applications of the Paris (or its inverse) potential at 400 MeV and above constitute an extrapolation of the model. Nevertheless, these applications serve us to illustrate both the role of the NN phase–shifts above pion production threshold in determining the NN potential and the ability of the inversion method to capture that physics. The differences given by the two potentials in Fig. 8 are remarkable. In particular, it is notable the improvement obtained in describing the NA scattering observables with the inversion potential constructed from the SM94 data, mainly for $q > 1 \text{ fm}^{-1}$ in both $d\sigma/d\Omega$ and $A_y$. Furthermore, uncertainties associated with the off–shell behavior of the NN potential are smaller than the departure of the inversion potentials from the Paris to the SM94 data. This indicates that the improvement in the description of the NA data is a direct consequence of an improved account for the NN data, mainly above pion production threshold, a built–in feature in the inversion potential.
V. SUMMARY AND CONCLUSIONS

In this paper we have addressed the problem of how properties of the underlying NN bare interaction determines the dynamics of a many–nucleon system. Our approach for the NN force is based on the quantum inversion method where a local, static, channel dependent potential is constructed directly from the NN phase–shifts. By departing from NN potentials derived from a field theoretical approach and empirically modified to maximize the fit to NN data below pion production threshold, we expect to shed new light on these properties of the NN force relevant to many–body processes.

We have investigated effects associated with the genuine off–shell behavior of bare NN potentials which are equivalent from the point of view of the phase–shifts over a wide energy range. The generation of on–shell–equivalent potentials was based on the Gelfand–Levitan and Marchenko inversion method for the NN system. Applications were made with the Paris potential, where the inversion method was applied to its corresponding phase–shifts up to 1.3 GeV kinetic energy in the NN laboratory system. This fairly large energy range was required to have the off–shell behavior of the inversion potential being determined mainly by the theory and not by the choice of a particular set of phase–shifts. The corresponding NN effective interactions were found to exhibit sizable differences off–shell, particularly for relative momenta above 3 fm$^{-1}$ in the NN system. The investigation of these differences was made in the context of NA elastic scattering with the calculation of in–medium full–folding optical potentials for proton scattering from $^{40}$Ca and $^{208}$Pb and at beam energies between 40 and 400 MeV. We found a weak sensitivity of the NA scattering observables to the off–shell differences observed between the NN effective interactions provided the underlying bare NN potentials are equivalent on–shell. These differences, at most 10% at the level of the NA scattering observables, are present in the whole energy range. One striking aspect is the ability of the inversion method to generate an NN potential which is, essentially, physically equivalent to its original counterpart although, by construction, their analytic properties and asymptotic behavior are very different.

Based on the success of the inversion method, we have constructed inversion NN potentials based on the SM94 phase–shift analysis. These potentials have been applied to the calculation of full–folding optical potentials for proton elastic scattering from $^{40}$Ca in the 40–500 MeV energy range. We obtained results for the scattering observables which yield a fit to the NA scattering data at 40 MeV comparable to that obtained with the Paris potential. However, a departure from the Paris results appears above 200 MeV, with a clear improvement of the NA scattering data being achieved in the 400– and 500–MeV applications. This improvement comes as a direct consequence of the closer agreement between the inversion potential and the NN phenomenology. An important conclusion emerging from our studies is that bare NN potentials which only provide a fit to NN phase–shifts below pion production threshold and disregard their higher energy behavior are unlikely to be realistic candidates to describe nucleon–nucleon dynamic in the nuclear medium.

Despite the improvements in the description of NA scattering using inversion potentials from NN data, difficulties still remain in describing the NA scattering data at momentum transfers below 1 fm$^{-1}$. We believe the explicit treatment inelasticities of the NN interaction above pion production threshold and baryon excitation mechanisms need to be addressed as the inversion potentials from the SM94 analysis were restricted to the real part of the phase–
shifts only. On the other hand, in the lower energy applications we observe a systematic inability of the full-folding model to describe in greater detail the NA scattering data. Here, the presence of higher order processes due to the many-body nature of the problem need definitely be accounted for. Indeed, the results obtained at these energies using the Paris potential, its inversion and the SM94 inversion provide essentially the same description of the NA scattering data. Therefore, we cannot attain these limitations of the optical model to the uncertainties associated with the off-shell behavior of the NN interaction.

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FIGURES

FIG. 1. $L \leq 2$ channel phase–shifts from the Paris potential (crosses) and from corresponding inversion potential (full curves).

FIG. 2. Scattering observables calculated from the Paris (solid curves) and its corresponding inversion potential (dashed curves) for $p + ^{40}$Ca at 40 MeV.

FIG. 3. Scattering observables calculated from the Paris (solid curves) and its corresponding inversion potential (dashed curves) for $p + ^{208}$Pb at 400 MeV.

FIG. 4. Behavior of the diagonal elements of the $g$ matrix for the state $^1S_0$ as a function of the relative momentum and for $\omega=30$ MeV (top), $\omega=200$ MeV (center) and $\omega=400$ MeV (bottom). The figures on the left correspond to $k_F = 0$ ( $t$ matrix) and those on the right to $k_F = 1.0 \text{ fm}^{-1}$. The solid curves represent results from the genuine Paris potential. The dashed curves correspond to results obtained from inversion potentials based on the Paris phase–shifts.

FIG. 5. $L \leq 2$ channel real phase–shifts from SM94 data (dots), and from the inversion potentials constructed from the real part of the SM94 data (solid curves) and the Paris potential (dashed curves).

FIG. 6. Calculated and measured differential cross-section and analyzing power for $p + ^{40}$Ca elastic scattering at 40 MeV. The solid and dashed curves were obtained from full–folding using the SM94 and Paris inversion, respectively. All curves represent in-medium full–folding calculations.

FIG. 7. Calculated and measured differential cross-section and analyzing power for $p + ^{40}$Pb elastic scattering at 200 and 300 MeV. The curve patterns follow convention of Fig. 6.

FIG. 8. Calculated and measured differential cross-section and analyzing power for $p + ^{40}$Ca elastic scattering at 400 and 500 MeV. The curve patterns follow convention of Fig. 6.
