Dissociation of Heavy Quarkonia in the Quark-Gluon Plasma

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Abstract

Using a temperature-dependent potential obtained from lattice gauge calculations of Karsch et al., we study the stability of heavy quarkonia in the quark-gluon plasma. We find that only the $\Upsilon(1S)$ and the $\eta_b(1S)$ are bound in the quark-gluon plasma, and have a small binding energy. The quark-gluon plasma may be revealed by an $\Upsilon(1S)$ dilepton peak with an invariant mass close to twice the current $b$ quark mass, which is lower than the $\Upsilon(1S)$ mass in free space. The quarkonia $\Upsilon(1S)$ and $\eta_b(1S)$ can dissociate by collision with quarks and gluons in the quark-gluon plasma. The $\Upsilon(1S)$ and the $\eta_b(1S)$ can also dissociate spontaneously at temperatures above the dissociation temperature $1.11T_c$, where $T_c$ is the quark-gluon plasma phase transition temperature. At temperatures slightly above the dissociation temperature these states appear as resonances, which provides another signature for the quark-gluon plasma.
I. INTRODUCTION

The suppression of heavy quarkonium production in a quark-gluon plasma has been a subject of intense interest since the pioneering work of Matsui and Satz [1]. Initial insight into the dissociation temperatures of heavy quarkonium was further provided by Karsch, Mehr, and Satz [2]. Recently, using the temperature-dependent $Q\bar{Q}$ interaction inferred from lattice gauge calculations of Karsch et al. [3], Digal, Petreczky, and Satz [4,5] reported theoretical results for the dissociation temperatures of heavy quarkonia in hadronic matter ($T < T_c$) and in the quark-gluon plasma ($T \geq T_c$), where $T_c$ is the quark-gluon plasma phase transition temperature. Subsequent analysis of the dissociation temperatures of heavy quarkonia at $T < T_c$ using different interactions and selection rules was given in detail in [6] and summarized in [7]. The dissociation of heavy quarkonia by thermalization and by collisions with particles in the medium has also been investigated [6–19].

For $T > T_c$, the equilibrium state of matter is the quark-gluon plasma. If the interaction between a $Q\bar{Q}$ and the QGP is weak, the $Q\bar{Q}$ system can be color-singlet or color-octet. Using perturbation theory as a guide, Digal et al. [5] extracted the color-singlet potential $V_1(r,T)$ and color-octet potential $V_8(r,T)$ of a heavy quark pair from the Polyakov loop $\langle L(0)L^\dagger(r) \rangle$. They used the relation [20]

$$\frac{\langle L(0)L^\dagger(r) \rangle}{\langle L^2 \rangle} = \frac{1}{9} \exp \left[ \frac{-V_1(r,T)}{T} \right] + \frac{8}{9} \exp \left[ \frac{-V_8(r,T)}{T} \right]$$

(1.1)

where $\langle L^2 \rangle$ denotes $\langle L(0)L^\dagger(r) \rangle$ at $r \to \infty$. This relation is valid when the $Q\bar{Q}$ interacts weakly with a perturbative color medium so that color-singlet and color-octet $Q\bar{Q}$ states are approximate eigenstates of the system. They also made the assumptions

$$V_1(r,T) = -\frac{4}{3} \frac{\alpha(T)}{r} e^{-\mu(T)r},$$

(1.2)

and

$$V_8(r,T) = \frac{1}{6} \frac{c(T)}{r} \frac{\alpha(T)}{r} e^{-\mu(T)r},$$

(1.3)

as suggested by perturbation theory (for which $C(T) = 1$). They assumed that $\mu(T)$ was $1.15T$ from the systematics at high temperatures and chose functions $\alpha(T)$ and $c(T)$ to fit the Polyakov loop results of Karsch et al. [3]. They examined color-singlet heavy quarkonia in the quark-gluon plasma, even for temperatures slightly greater than $T_c$ [5].

We shall restrict our interest to the region of temperatures slightly greater than $T_c$, which is the most important region for the investigation of the stability of heavy quarkonia in the quark-gluon plasma. In this temperature region, the screening effects are non-perturbative. Evidence for non-perturbative screening comes from numerical lattice gauge calculations of Karsch et al. [21] who show that the Debye screening mass is significantly larger than the leading-order perturbative value of $m_D = gT \sqrt{1 + n_f/6}$ and that the screening mass remains about three times larger than the perturbative Debye mass $m_D$ even up to temperatures as high as $100T_c$. Similarly, the $rT$ dependence as shown in Eqs. (1.1)-(1.3) with the perturbative value of $C(T) = 1$ is approximately valid only for $T > 3T_c$ [21].
For the region of temperatures close to $T_c$ in which we shall be interested, $C(T)$ obtained from the procedures of Ref. [4] is about 0.1 which deviates significantly from the perturbative value of $C(T) = 1$, and the system cannot be described as a weakly coupled and perturbatively screened system [21]. Non-perturbative screening effects are important and the extrapolation procedure in the non-perturbative region employed in Ref. [4] is ambiguous. Specifically, the Polyakov loop provides a single number for a given $r$ and $T$, but the above separation procedure yields quantitative determination for the physical quantities of $V_1$, $V_8$, and the (dynamical) relative weight of the color-singlet and color-octet components. Many assumptions are needed to extract these three physical quantities out of a single quantity of the Polyakov loop. Without theoretical non-perturbative results to guide us, these additional assumptions are not unique. Different assumptions will lead to different extracted quantities in the non-perturbative region.

Related to the question of the separation is the question whether purely color-singlet and color-octet $Q\bar{Q}$ states are approximate eigenstates in the non-perturbative region of temperatures in QGP, when $T$ is just slightly greater than $T_c$. Purely color-singlet and color-octet $Q\bar{Q}$ states are approximate eigenstates of the system in the high-temperature, perturbative region for which Eq. (1.1) is valid. This however cannot be the case in the non-perturbative region of $T \gtrsim T_c$. A constituent of a $Q\bar{Q}$ in a deconfined medium can interact with a constituent of the medium ($\bar{q}$, $q$, and $g$) by exchanging a single gluon. Such a gluon exchange alters the color of the $Q\bar{Q}$ from a color-singlet state to a color-octet state and vice versa. The color of the medium will undergo corresponding changes so that the color of the whole system is neutral. Such a color-changing interaction mixes the color states of the heavy quarkonium. We shall show below in Section II that the degree of mixing increases with the magnitude of the ratio of the color-changing interaction matrix element $\lambda$ to the energy difference $\Delta$ between the unperturbed color-octet and color-singlet states. In the non-perturbative region of $T$ in the QGP, $|\lambda/\Delta|$ is not small and the eigenstates of the system become states of mixed colors. A purely color-singlet or purely color-octet state cannot be a stationary state of the non-perturbative system.

Even though a $Q\bar{Q}$ state in the non-perturbative region of temperatures cannot be separated into eigenstates of pure color, it is still a meaningful question to find out how the $Q$ interacts with the $\bar{Q}$ in such a system, whatever the color admixture of the resultant $Q\bar{Q}$ may be. In lattice gauge calculations the expectation value of the Polyakov loop operator is evaluated allowing all possible configurations of the gauge fields and fermion fields at all lattice points and links, and this corresponds to all color states of the $Q\bar{Q}$ and the medium. The gauge field variables along the path of the Polyakov loop provide information on the evolution of the heavy-quark system. The greatest contributions to the Polyakov loop at a given $Q$-$\bar{Q}$ separation will come from those gauge fields and quark fields which adjust themselves in magnitude and in color admixture to give states of lowest energies of a $Q$-$\bar{Q}$ in the medium. As the eigenstates of the system are states with mixed colors (see Section II), the Polyakov loop provides information on the free energy of mixed-color states. Carrying this calculations for different separations between the $Q$ and the $\bar{Q}$, one obtains the free energy as a function of the separation, which provides the spatial variation of the effective interaction between the $Q$ and the $\bar{Q}$. It can be used to find out if the interaction is strong enough to bind the $Q$ with the $\bar{Q}$ in the non-perturbative deconfined medium. Therefore, in this non-perturbative medium with strong interactions, the effective interaction potential
$V_{QQ}(r,T)$ between the $Q$ and the $\bar{Q}$, is related to the expectation value of the Polyakov loop from lattice gauge theory by

$$\frac{\langle L(0)L^\dagger(r) \rangle}{\langle L^2 \rangle} = \exp \left[ \frac{-V_{QQ}(r,T)}{T} \right].$$

(1.4)

The quantity $V_{QQ}(r,T)$ was extracted previously from the lattice gauge calculations by Karsch et al. [3]. The binding energies of quarkonia as a function of the temperature will provide useful plasma diagnostic information on the dissociation temperatures and quarkonia masses in the nonperturbative quark-gluon plasma phase.

As a heavy quarkonium becomes unbound, it appears as a resonance in the continuum. Such a resonance occurs only in the quark-gluon plasma at a certain range of temperatures and may be used as a signature of the quark-gluon plasma [23]. Following the method used previously by Calogero [22] and by Wong and Chatterjee [23], we shall study the resonance structure of heavy quarkonia in the quark-gluon plasma.

Quarkonia can also dissociate by collision with particles in the medium [6–19]. In particular, we will study $\Upsilon(1S) + g \to b + \bar{b}$. Such a dissociation process is similar to the photodissociation of a deuteron after absorbing a photon. This dissociation process can be studied by using a multipole expansion of the gluon field, analogous to the electromagnetic case discussed in detail by Blatt and Weisskopf [24]. As the gluon dissociation cross section has also been obtained previously by Peskin and Bhanot [25,26] and subsequently used by Kharzeev and Satz [12], we show in the Appendix the equality of the cross sections obtained by the two formalisms for the simple case considered by Peskin and Bhanot.

In Section II, we discuss the color structure of a $Q\bar{Q}$ and its dependence on the interaction between the constituents of the $Q\bar{Q}$ and the constituents of the medium. In Section III, we describe the Schrödinger equation used to calculate the energies and wave functions of quarkonium states. We introduce an analytical form of the effective screening potential $V_{QQ}$ to represent the results of lattice gauge calculations. The quarkonium energies and the dissociation temperatures are calculated in Section IV. In Section V, we consider continuum states and follow the resonances as functions of the quark-gluon plasma temperature. In Section VI we evaluate the cross section of the dissociation process $\Upsilon(1S) + g \to b + \bar{b}$. A discussion of our results is given in Section VII.

II. COLOR STRUCTURE OF A $Q\bar{Q}$ IN THE QGP

The color structure of a $Q\bar{Q}$ placed in a deconfined medium depends on the interaction between the constituents of the $Q\bar{Q}$ and the constituents of the deconfined medium. We can consider first two extreme cases of very weak and very strong interactions as illustrated in Fig. 1.

If the interaction between the $Q$ and the $\bar{Q}$ of the quarkonium with $g,\bar{g}$, and $g$ is weak as in a perturbative medium, [Fig. 1(a)], the $Q$ and the $\bar{Q}$ can combined together to form pure color-singlet and color-octet eigenstates. The concept of a purely color-singlet or color-octet state is useful as an approximate description, when the interaction between the $Q$ and the
$Q$ with the quark matter particles can be ignored.

(a) Perturbative Medium

(b) Highly Non-perturbative Medium

Fig. 1. $Q$ and $\bar{Q}$ in a medium with perturbatively weak interactions (a), and in a highly non-perturbative medium with strong interactions (b). Open circles represent quarks, antiquarks, and gluons in the plasma.

In the other extreme of very strong interactions as illustrated in Fig. 1(b), the $Q$ and the $\bar{Q}$ are intimately linked to the medium particles by gluons. The isolation of the $Q\bar{Q}$ system is impossible and one cannot speak unambiguously of a purely color-singlet or color-octet state.

We discuss how we can pass from one limit to the other limit with a schematic model. We consider a $Q\bar{Q}$ system in a medium and use $(Q\bar{Q})^{(c)}$ basis states where $c = 0$ is for color-singlet and $c = 8$ for color octet. We consider for simplicity the medium represented by $M^{(c)}$ where $c$ is again the color index. As the color of the whole system consisting of both $(Q\bar{Q})^{c}$ and $M^{c}$ must be color neutral, a state of the system is described by

$$|\psi\rangle = a_0 (Q\bar{Q})^{(0)} M^{(0)} + a_8 [(Q\bar{Q})^{(8)} M^{(8)}]^{(0)}$$

One can represent this admixture by a column vector with elements $a_0$ and $a_8$. In the absence of the interaction between the $Q$ and the $\bar{Q}$ with the medium particles, the color-singlet state and the color-octet state can be separated and we denote the energy difference between the unperturbed color-octet state and the color-singlet state by $\Delta = E^{(8)} - E^{(0)}$. 

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By the interaction $v_{Aa}(A + a) \cdot g \cdot A' + a'$ in which the constituent $A$ in $(Q \bar{Q})^{(c)}$ exchange a gluon $g$ with the constituent $a$ in $M^{(c)}$ to become $A'$ and $a'$, the color of both $(Q \bar{Q})^{(c)}$ and $M^{(c)}$ will change. We define the color-changing transition matrix element $\lambda$ as

$$\lambda = \langle (Q \bar{Q})^{(8)} M^{(8)} | v_{Aa} | (Q \bar{Q})^{(0)} M^{(0)} \rangle$$  \hspace{1cm} (2.2)$$

The Hamiltonian for such a system is

$$H = \begin{pmatrix} -\Delta/2 & \lambda \\ \lambda & -\Delta/2 \end{pmatrix}.$$  \hspace{1cm} (2.3)$$

The lowest energy eigenstate of the system is then

$$|\psi\rangle_0 = \left(\frac{1 + (1 + 4\lambda^2/\Delta^2)^{1/2}}{-2\lambda/\Delta}\right)\frac{1}{\sqrt{2[1 + 4\lambda^2/\Delta^2 + (1 + 4\lambda^2/\Delta^2)^{1/2}]^{1/2}}}.$$  \hspace{1cm} (2.4)$$

The excited eigenstate is

$$|\psi\rangle_1 = \left(\frac{2\lambda/\Delta}{1 + (1 + 4\lambda^2/\Delta^2)^{1/2}}\right)\frac{1}{\sqrt{2[1 + 4\lambda^2/\Delta^2 + (1 + 4\lambda^2/\Delta^2)^{1/2}]^{1/2}}}.$$  \hspace{1cm} (2.5)$$

Thus, the color-structure of the system depends on the ratio $|\lambda/\Delta|$. In the confined phase, $\Delta$ is very large, and so $|\lambda/\Delta|$ is very small; the lowest eigenstate is purely color singlet. In the perturbative deconfined phase at very high temperatures, $\lambda$ is small and $|\lambda/\Delta|$ is also small; the eigenstates are nearly pure in color.

On the other hand, if $|\lambda/\Delta|$ is large, the eigenstates of the system is an admixture of color-singlet and octet state. The nonperturbative region of temperature is a case of strong interactions between the $Q$ (or $\bar{Q}$) and the medium particles. We can estimate the ratio $|\lambda/\Delta|$ which determines the color admixture. The energy difference $\Delta$ between color-octet and color-singlet states is approximately the dissociation energy of the color-singlet state. Earlier estimates of the dissociation energy of $J/\psi$ in the hot hadron phase show that it decreases with increasing temperatures, resulting in zero dissociation energy [3] or nearly zero dissociation energy (see Fig. 3 of [3]) for $J/\psi$ at $T \lesssim T_c$. For $T \gtrsim T_c$, screening of deconfined quark matter will tend to weaken further the $c\bar{c}$ attraction. (Calculations in Section IV is consistent with a vanishing value of the dissociation energy of $J/\psi$.) Thus, $\Delta$ is zero or nearly so for $c\bar{c}$. For $b\bar{b}$, the dissociation energy is about 0.15 GeV for $\Upsilon(1S)$ [3] at $T \lesssim T_c$. For $T \gtrsim T_c$, screening in the quark-gluon plasma makes the dissociation energy even smaller. (In Section VI, the dissociation energy is found to be of order 0.01 GeV.)

On the other hand, for a QGP at $T = 200$ MeV, the average separation between the quanta in the QGP is approximately $d = 0.6$ fm. The transition matrix element is of order $\lambda \sim \alpha_s \exp\{-m_{th}d\}/d$ where the thermal mass $m_{th} \sim gT$ and $g^2 = \alpha_s/4\pi$. We find that for $\alpha_s = 0.2$, $\lambda$ is of order 0.06 GeV. The ratio $|\lambda/\Delta|$ is also quite large for $b\bar{b}$, if we take $\Delta \sim 0.01$ GeV. Therefore, the $Q\bar{Q}$ system is strongly mixed in color in the nonperturbative deconfined region of temperatures slightly greater than $T_c$.

Even though a $Q\bar{Q}$ state in the non-perturbative region of temperatures cannot be separated into eigenstates of pure color, it remains a meaningful question to find out how the $Q$ interacts with the $\bar{Q}$ in such a system. As explained in the Introduction, the expectation value of the Polyakov loop operator gives the free energy of the system of the $Q\bar{Q}$ coupled to the medium in a mixed-color state. It provides the effective interaction potential $V_{Q\bar{Q}}(r, T)$ between the $Q$ and the $\bar{Q}$ in in the region of $T \gtrsim T_c$. We shall use this interaction potential to examine the stability of the $Q\bar{Q}$ system in the next few sections.
III. SCHRÖDINGER EQUATION FOR HEAVY QUARKONIUM STATES

The energy $\epsilon(T)$ of a heavy quarkonium state $(Q\bar{Q})_{JLS}$, measured relative to the mass of the $Q$ and the $\bar{Q}$, can be obtained by solving for the associated eigenvalue of the Schrödinger equation

$$\left\{ -\frac{\hbar^2}{2\mu_{Q\bar{Q}}} \nabla^2 + V_{Q\bar{Q}}(r, T) \right\} \psi_{JLS}(r, T) = \epsilon(T) \psi_{JLS}(r, T), \quad (3.1)$$

where $\mu_{Q\bar{Q}} = m_Q/2$ is the reduced mass. The quarkonium is bound if $\epsilon(T)$ is negative, and is unbound and dissociates spontaneously into a $Q$ and a $\bar{Q}$ if $\epsilon(T)$ is positive.

The color charges of the $Q$ and the $\bar{Q}$ are screened in the quark-gluon plasma. We can conveniently represent $V_{Q\bar{Q}}(r, T)$ by a screened Yukawa potential with an effective strength $\alpha_{\text{eff}}(T)$ and a screening mass parameter $\mu(T)$. We shall be interested in temperatures slightly greater than $T_c$. The $Q-\bar{Q}$ interaction obtained from the lattice calculations of Karsch et al. \[3\] for $1.2T_c > T > T_c$ in the quark-gluon plasma phase can be approximated by

$$V_{Q\bar{Q}}(r, T) = -\frac{4}{3}\alpha_{\text{eff}}(T) \frac{e^{-\mu(T)r}}{r} \quad (3.2)$$

where

$$\alpha_{\text{eff}}(T) = 0.20, \quad \mu(T) = \mu_0 + \mu_T (T/T_c - 1), \quad (3.3)$$

$$\mu_0 = 0.323 \text{ GeV}, \quad \text{and} \quad \mu_T = 3.04 \text{ GeV}.$$

Fits to $V_{Q\bar{Q}}(r, T)$ using the above form are shown in Fig. 2. We include the color-singlet factor $(-4/3)$ in the above definition (3.2) for $\alpha_{\text{eff}}(T)$ in order to have a comparison of the strength of the $Q-\bar{Q}$ interaction in the quark-gluon plasma phase with the color-singlet $Q-\bar{Q}$ interaction seen at $T < T_c$ \[6\]. The strength parameter $\alpha_{\text{eff}}(T)$ has very weak temperature dependence, and can be adequately represented by the constant $\alpha_{\text{eff}}(T) = 0.20$. This value of $\alpha_{\text{eff}}(T) = 0.20$ in the quark-gluon plasma phase is smaller than the value of $\alpha_s \sim 0.32$ for charmonium and $\alpha_s \sim 0.24$ for bottomonium in the screened color-Coulomb potential for temperatures below $T_c$ \[6\]. The screening mass parameter $\mu_0$ for $T \geq T_c$ is greater than the screening mass parameter of $\mu_0 = 0.28$ GeV used for $T < T_c$ in \[6\]. One concludes that because of the screening due to deconfined quarks and gluons, the effective potential between the $Q$ and the $\bar{Q}$ in the quark-gluon plasma remains attractive, but its strength is weaker and its range is shorter than the color-Coulomb interaction seen in hadronic matter.
The range of the screened potential decreases with increasing temperatures.

**Fig. 2.** The effective potential $V_{Q\bar{Q}}(r,T)$ between the $Q$ and the $\bar{Q}$ in a quark-gluon plasma at various temperatures. The symbols represent the results of the lattice gauge calculations of [3] reported in [5]. The curves in the main figure are from the parameterization of Eqs. (3.2) and (3.3). The inserted figure shows the effective potential at $T = 0$ (for charmonium) and $T = 1.03T_c$.

**IV. BOUND STATES AND DISSOCIATION TEMPERATURE**

The eigenvalues of the Hamiltonian can be obtained by integrating the Schrödinger equation numerically and requiring an exponentially decaying wave function at large distances. For our numerical calculations we use a current $c$ quark mass $m_c = 1.25$ GeV, and a current $b$ quark mass $m_b = 4.2$ GeV [27].

We find that the potential of Eqs. (3.2) and (3.3), as determined by the lattice gauge calculations of Karsch et al., is too shallow to hold a bound charmonium state. Because of the large $b$ quark mass, we can neglect the spin-orbit and hyperfine interactions. In this approximation, the $L = 0$ $\Upsilon(1S)$ and $\eta_b(1S)$ states are degenerate, as are the $L = 1 \chi_{b}(1P)$ and $h_b(1P)$ states. For $b\bar{b}$ the $L = 1 \chi_{b}(1P)$ and $h_b(1P)$ states are unbound, and the only bound states are the $L = 0 \Upsilon(1S)$ and $\eta_b(1S)$ states. The energies of the $\Upsilon(1S)$ and $\eta_b(1S)$ states as functions of the quark-gluon plasma temperature are shown in Fig. 3. They become
less bound with increasing temperature. The dissociation temperature of $\Upsilon(1S)$ and $\eta_b(1S)$ is $T_d = 1.11 T_c$, as determined by the condition $\epsilon = 0$. The root-mean-squared radius of $\Upsilon(1S)$ at $T = T_c$ is 0.80 fm, indicating a large radius because of the weak binding. The rms radius increases with temperature. At $T = 1.10 T_c$ which is close to the dissociation temperature, the rms radius has increased to 3.3 fm.

![Energy vs Temperature Graph](image)

**Fig. 3.** The energies of $\Upsilon(1S)$ and $\eta_b(1S)$ states as a function of temperature.

Previously we found that the $\chi_b$ and $\eta_b$ states were stable against dissociation for $T < T_c$. In the quark-gluon plasma phase, however, screening by the deconfined $q$, $\bar{q}$, and $g$ alters the interaction between $Q$ and $\bar{Q}$. The interaction does not need to be a continuous function of the temperature at $T_c$, because a new type of screening arises from deconfined $q$, $\bar{q}$, and $g$ in the quark-gluon plasma phase for $T \geq T_c$. Consequently, the quarkonium energies do not need to be continuous across the QGP phase transition boundary.

For completeness we show the heavy quarkonium dissociation temperatures in Table I where we also include those shown in Table III of [6]. The dissociation temperature of $T_d/T_c = 1.00+$ for $\chi_b$ and $h_b$ in Table I denotes $T_d/T_c = 1.00$ in the quark-gluon plasma phase. The dissociation temperatures obtained here are substantially lower than the values of 1.13$T_c$ for the color-singlet $\chi_b$ and 2.31$T_c$ for the color-singlet $\Upsilon(1S)$ obtained by Digal et al. [5] using the color-singlet potential $V_1(r,T)$. These differences arise because the present investigation assumes a $Q\bar{Q}$ pair that is immersed in and interacting strongly with the color medium of the quark-gluon plasma, and the work of Digal et al. [5] considers the dissociation of a color-singlet $Q\bar{Q}$ pair in the quark-gluon plasma. Hence there are significant differences
in the dissociation temperatures.

Table I. The dissociation temperatures $T_d$ in units of $T_c$ for various heavy quarkonia

| Heavy Quarkonium | $\psi'$ | $\chi_c^2$ | $\chi_c^1$ | $J/\psi$ | $\Upsilon''$ | $\chi_b^2$ | $\chi_b^1$ | $\Upsilon'$ | $\chi_b, h_b$ | $\Upsilon, \eta_b$ |
|------------------|---------|------------|------------|-----------|-------------|------------|------------|-------------|--------------|----------------|
| $T_d/T_c$        | 0.50    | 0.91       | 0.90       | 0.99      | 0.57        | 0.82       | 0.82       | 0.96        | 1.00+        | 1.11          |
| $T_d/T_c$        | 0.1-0.2 | 0.74       | 0.74       | 1.10      | 0.75        | 0.83       | 0.83       | 1.10        | 1.13         | 2.31          |
| (Digal et al. [5]) |         |            |            |           |             |            |            |             |              |               |

V. QUARKONIUM STATES IN THE CONTINUUM

To study the decay of a heavy quarkonium resonance, it is of interest to examine the related $Q$-$\bar{Q}$ scattering process. The scattering phase shifts with the screened potential $V_{QQ}(r, T)$ will provide information regarding the locations and the widths of heavy quarkonium resonances in the continuum.

To obtain the continuum wave function in the scattering problem, we use the phase-angle method discussed in detail by Calogero [22] and used previously by Wong and Chatterjee [23]. We write the wave function as

$$\psi_{JLS}(r) = R_{JLS}(r)Y_{JLS}^{\hat{K}} = \alpha_{L}(r)\hat{D}_{L}(kr)\sin(\hat{\delta}_{L}(kr) + \delta_{L}(r)),$$

where $\alpha_{L}(r)$ and $\delta_{L}(r)$ depend on the interaction $U(r) = 2\mu_{QQ}V_{QQ}(r)$. The equation for $\delta_{L}(r)$ is given by

$$\frac{d}{dr}\delta_{L}(r) = -\frac{U(r)}{k}\hat{D}_{L}^{2}(kr)\left\{\sin[\hat{\delta}_{L}(kr) + \delta_{L}(r)]\right\}^{2}.$$

After the function $\delta_{L}(r)$ is evaluated, the amplitude $\alpha_{L}(r)$ can be obtained from $\delta_{L}(r)$ by

$$\alpha_{L}(r) = \exp\left\{\frac{1}{2k}\int_{0}^{r}ds U(s)\hat{D}_{L}^{2}(ks)\sin 2[\hat{\delta}_{L}(ks) + \delta_{L}(s)]\right\}.$$
Numerical integration of Eq. (5.5) gives the asymptotic phase shift $\delta_L = \delta_L(\infty)$ and the continuum wave function. We show the phase shift $\delta_0$ in Fig. 4(a) and $\sin^2 \delta_0$ in Fig. 4(b) for the $L = 0$ b$\bar{b}$ state as a function of energy $\epsilon$ at various temperatures. The phase shift $\delta_0$ equals $\pi$ at $\epsilon = 0$ for $T < 1.11T_c$, in accordance with the Levinson’s theorem [which states that $\delta = n\pi$ at $\epsilon = 0$, where $n$ is the number of bound states, except when an $S$-wave resonance occurs at zero energy, in which case $\delta = (n + 1/2)\pi$]. At temperatures below the dissociation temperature $T_d = 1.11T_c$ the phase shift decreases as a function of energy, and is equal to $\pi/2$ at some energy $\epsilon$. Such an occurrence of $\delta_0 = \pi/2$ with $d\delta(\epsilon)/d\epsilon < 0$ does not represent a resonance; it is an “echo” of the bound state which lies just below the continuum [28]. At the location $\sin^2 \delta_0 = 1$ the scattering cross section is enhanced when a bound state lies just below the continuum, as was first noted by Wigner and discussed by Landau and Lifshitz [29].

At the $\Upsilon(1S)$ dissociation temperature $T_d = 1.11T_c$, $\delta_0 = \pi/2$ at $\epsilon = 0$, and the $L = 0$ $\Upsilon(1S)$ and $\eta_b(1S)$ resonances occur at $\epsilon = 0$. The phase shift decreases fairly rapidly as a
function of energy. The energy difference between the location of the maximum of $\sin^2 \delta_0$ and the half maximum is about 0.2 GeV, indicating a resonance with a half width of about this magnitude.

For temperatures higher than $T_d$ the phase shift is zero at $\epsilon = 0$. It increases rapidly with $\epsilon$ and reaches a maximum before it decreases slowly. One can associate the location $\epsilon$ of the maximum phase shift in $\epsilon$ with the position of the $\Upsilon(1S)$ and $\eta_b(1S)$ “resonances”. Strictly speaking the enhancement at the phase shift maximum does not represent a resonance, as the magnitude of the maximum phase shift is less than $\pi/2$. However when the transition temperature is slightly greater than $T_d$, the magnitude of the maximum phase shift is close to $\pi/2$, and it decreases continuously as the temperature increases. The location $\epsilon$ of the maximum phase shift is a smooth continuation of the bound state energy. We can therefore speak of these enhancements as “resonances”, as they are the continuation of the bound states into the positive energy domain. We can choose to identify the position of the maximum of the phase shift for $T > T_d$ with the location of the resonances as a function of temperature. The maximum value of $\sin^2 \delta_0$ decreases as the temperature increases. The width of $\sin^2 \delta_0$ increases, showing a broader enhancement of the cross section as the temperature increases. For example, at $T = T_d + 0.02T_c$ the maximum occurs at $\epsilon = 0.008$ GeV, and the separation between the maximum and the half maximum on the higher energy side is about 0.30 GeV. At $T = T_d + 0.04T_c$ the maximum occurs at $\epsilon = 0.018$ GeV, and the separation between the maximum and the half maximum on the greater energy side is about 0.43 GeV. The function $\sin^2 \delta_0$ is not symmetrical with respect to the maximum. It initially rises rapidly, but then decreases slowly as a function of $\epsilon$. The width increases rapidly with temperature.

We show in Fig. 5 the phase shift $\delta_1$ and $\sin^2 \delta_1$ for $L = 1$ $Q$-$\bar{Q}$ scattering. For $T$ slightly greater than the phase transition temperature, the phase shift has a maximum at $\epsilon \sim 0.1-0.2$ GeV. The maximum magnitude of the phase shift is 0.54 at $\epsilon = 0.12$ GeV for $T/T_c = 1.01$, and is 0.39 at $\epsilon = 0.265$ GeV for $T/T_c = 1.05$. The maximum phase shift does not reach $\pi/2$. The quantity $\sin^2 \delta_1$ has a maximum value of 0.22 at $T/T_c = 1.01$, and a value of 0.14 at $T/T_c = 1.05$. These enhancements are quite broad. For example, for $T/T_c = 1.01$, the quantity $\sin^2 \delta_1$ decreases slowly after it reaches the maximum value, and the separation
between the maximum and the half maximum on the higher energy side is about 1.7 GeV.

![Graph showing phase shift and sin^2(δ1) for various T/Tc](image)

**Fig. 5.** (a) The phase shift δ1 and (b) sin^2δ1 for the scattering of Q and Q̄ in the L = 1 state as a function of the energy ε for various T/Tc.

### VI. CROSS SECTION FOR Υ(1S) + g → b + b̄

The quarkonia Υ(1S) and ηb(1S) are stable in the quark-gluon plasma, but have small binding energies (from 0 to about 0.0175 GeV). They can dissociate into b and b̄ by collision with quarks and gluons.

We shall focus our attention on the dissociation reaction Υ(1S) + g → b + b̄ through gluon absorption; the dissociation of η(1S) can be treated in a similar way. For the dissociation processes Υ(1S) + g → b + b̄, the initial quarkonium state is a 1S bound state, and the final state is a b̄b state in the continuum. This process is similar to the photodissociation of a deuteron after photon absorption. The dissociation process can be studied using a multipole expansion of the electromagnetic or chromodynamic fields, and the lowest multipole orders are M1 and E1 absorption processes. For low energy photons or gluons, these processes can
be treated in the long wavelength limit, discussed in detail by Blatt and Weisskopf \[24\].

Following Eq. (XII.4.33) of Blatt and Weisskopf \[24\], the $E1$ dissociation cross section for a transition from the initial bound $1S$ state with energy $\epsilon_i = -B = -\gamma^2/m_Q$ to a final $1P$ continuum state with energy $\epsilon = k^2/m_Q$, after absorbing a gluon of energy $\epsilon - \epsilon_i$, is given by

$$\sigma_{\text{dis}}^{E1} = 4 \times \frac{\pi}{3} \alpha_{gQ}(k^2 + \gamma^2)k^{-1}I^2,$$

where the factor of 4 on the right hand side has been included for color electric dipole contributions from both the $Q$ and the $\bar{Q}$, (as distinct from the deuteron case, in which only the proton contributes to the electric dipole matrix element).

The quantity $I$ of Eq. (6.1) is the radial overlap integral for the $E1$ electric dipole matrix element,

$$I = \int_{0}^{\infty} u_{1P}(r)u_{1S}(r)dr,$$

where the wave function $u_{1P}(r)$ is given by Eqs. (5.2)-(5.6). The $1S$ bound state wave function in Eq. (6.2) is normalized according to

$$\int_{0}^{\infty} |u_{1S}(r)|^2dr = 1.$$  (6.3)

The quantity $\alpha_{gQ}$ is an effective strong interaction coupling constant for a gluon interacting with a heavy quark or antiquark. Specifically, similar to Eq. (3) of the Appendix, we have

$$4\pi \alpha_{gQ} = |\langle f | g \frac{\lambda^c}{2} | i \rangle|^2,$$

where $|i\rangle$ is the initial state and $|f\rangle$ the final state after the absorbing a gluon. For our problem with a mixed-color heavy quarkonium system in the QGP, we assume that $\lambda \gg \Delta$ in Eq. (2.4) and so the initial and final states are characterized by $a_0 = 1/\sqrt{2} = -a_8$. Then we obtain

$$4\pi \alpha_{gQ} = \frac{g^2}{6}.$$  (6.5)

For these mixed-color states, if we represent the interaction between $Q$ and $\bar{Q}$ by a gluon exchange, the associated coupling constant will be $g^2[(-4/3) + (1/6)]/2$. We identify this coupling constant with the coupling constant extract from lattice calculations, $|(-4/3)\alpha_{\text{eff}}|$ of Eq. (3.2). Hence, the coupling constant for the absorption of a gluon is then

$$\alpha_{gQ} = \frac{1}{6} \times \frac{12}{7} \times | -\frac{4}{3} \alpha_{\text{eff}} |.$$  (6.6)

Previously, the cross section for the dissociation of a color-singlet heavy-quarkonium in collision with a gluon was obtained in the short-distance approach by Peskin and Bhanot \[25,26\] and subsequently used by Kharzeev and Satz \[12\], for the simple case when the heavy-quarkonium can be treated as a hydrogenic system. The calculation of Peskin and Bhanot uses the operator product expansion, dispersion relations, and the sum of a large number
of diagrams in the large $N_c$ limit. The leading order cross section involves the square of the color electric field $E^2$, which corresponds to the dissociation by an electric dipole radiation. As the physical process included in the evaluation of the dissociation cross section in Peskin et al. [25,26] and in Eq. (6.1) of Blatt and Weisskopf [24] are the same, they should give the same dissociation cross section. We shall show the equality of both results explicitly for the simple case considered by Peskin and Bhanot in the Appendix.

The analytical result of Peskin and Bhanot is useful only for the idealized case of a hydrogenic $1S$ initial color-singlet state and a non-interacting $1P$ final state in the large $N_c$ limit. The wave functions of the heavy quarkonium systems of interest are not hydrogenic. The energy and the wave functions of the initial and final states also depend sensitively on temperature. The large $N_c$ limit is only a crude approximation for $N_c = 3$. We need to use the more general results of Eq. (6.1) for our problem in which both the initial and the final state wave functions have been obtained numerically, without using the large $N_c$ limit.

Fig. 6. The $E1$ cross section for the dissociation process $\Upsilon(1S) + g \rightarrow b + \bar{b}$ after absorbing a gluon.

With the $L = 1$ phase shift $\delta_1(r)$ and amplitude $\alpha_1(r)$ obtained in Section IV and the $L = 0$ bound state wave function obtained in Section III, the overlap integral (5.2) can be evaluated to give the dissociation cross section of $\Upsilon(1S)$ by gluon absorption. We show in
Fig. 6 the dissociation cross section of $\Upsilon(1S)$ by gluon absorption as a function of the final continuum state energy $\epsilon$. The cross section is zero at $\epsilon = 0$, rises to a maximum and then decreases with increasing $\epsilon$. The dependence of the $E1 \Upsilon(1S)$ dissociation cross section on energy is similar to that of the $E1$ deuteron photodissociation cross section as shown in Fig. XII.4.1 of [24].

Because the mass of the $b$ quark is large, the $M1$ cross section is small relative to the $E1$ cross section, and can be neglected.

It should be noted that the dissociation cross section has been calculated for the exclusive channel $g + \Upsilon(1S) \rightarrow b + \bar{b}$. As the energy increases, the cross section for this specific channel decreases (Fig. 6) and other reaction channels such as $g + \Upsilon(1S) \rightarrow b + \bar{b} + n g + n'(q + \bar{q})$ will make important contributions to the dissociation of the $\Upsilon(1S)$. It will be of great interest to evaluate these cross sections in future work.

VII. DISCUSSION

We have studied the dissociation of a heavy quarkonium state in a quark-gluon plasma using the finite-temperature potential $V_{Q\bar{Q}}(r)$ inferred from lattice gauge calculations of Karsch et al. [3].

Focusing our attention on states that can be directly or indirectly detected by dilepton decay, we can classify quarkonia into three groups: (1) quarkonia which dissociate spontaneously in hadronic matter: $\psi'$, $\chi_c$, $J/\psi$, $\Upsilon''$, $\chi_b$, $\Upsilon'$; (2) quarkonia which are stable in hadronic matter but dissociate spontaneously in the quark-gluon plasma: $\chi_b$; and (3) quarkonia which are stable in hadronic matter and the quark-gluon plasma below a dissociation temperature: $\Upsilon(1S)$ with the dissociation temperature of $1.11T_c$.

From this classification, one notes that the $\chi_{b1}$ and $\chi_{b2}$ mesons are good indicators of a quark-gluon plasma, as they are stable in hadronic matter but dissociate spontaneously in a quark-gluon plasma. The $\Upsilon(1S)$ (and $\eta_b(1S)$) have the distinction that they are the only stable quarkonia in the quark-gluon plasma, although the binding energies are rather small.

For a quark-gluon plasma at a temperature below the $\Upsilon(1S)$ dissociation temperature $T_d$, a direct observation of the position of the $\Upsilon(1S)$ using decays to dileptons is of great interest. The mass of the $\Upsilon(1S)$ state in the quark-gluon plasma is $2m_b + \epsilon(1S)$, where $\epsilon(1S)$ lies between $-0.0175$ GeV and $0$ GeV (Fig. 3), and the current $b$ quark mass $m_b$ is $4.2 \pm 0.2$ GeV [27]. The invariant mass of the dilepton peak from the decay of the $\Upsilon(1S)$ in the quark-gluon plasma will be substantially lower than the dilepton invariant mass of 9.46 GeV seen in the decay of the $\Upsilon(1S)$ in free space. The fraction of dileptons from the decay of the $\Upsilon(1S)$ inside the plasma depends on the lifetime of the quark-gluon plasma and the dilepton decay partial width of $\Upsilon$ in the plasma. This fraction may be rather small, making the observation of the dilepton energy peak shift a difficult task.

We have examined the dissociation cross section for the exclusive channel $g + \Upsilon(1S) \rightarrow b + \bar{b}$ through a color $E1$ transition. We have also studied the phase-shifts for $Q\bar{Q}$ collisions in the continuum. We find that there are $\Upsilon(1S)$ and $\eta_b(1S)$ resonances at $\epsilon = 0$ for $L = 0$ at the dissociation temperature $T_d = 1.11T_c$, with a half width of $\Gamma/2 \sim 0.2$ GeV. The occurrence of these $L = 0$ resonances just above $\epsilon = 0$ is a characteristic of the quark-gluon plasma, as they are not expected from other sources. These resonances will decay in the quark-gluon plasma and appear as separated $b$ and $\bar{b}$ quarks. After the quark-gluon plasma cools to become hadronic matter, the $b$ and $\bar{b}$ quarks will hadronize into $B$ and
B mesons. The invariant mass of this pair will show a resonance at low relative kinetic energies. However, direct detection of B and B̅ mesons is a difficult task whose prospects may improve with future advances in detector technology.

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Appendix: Approximate gluon dissociation cross section for a color-singlet heavy quarkonium

We would like to show how Eqs. (6.1)-(6.2) lead to the result of Peskin and Bhanot [25,26] for the special case they considered. They assumed that the initial ground state of the color-singlet heavy quarkonium can be approximately described by a hydrogenic 1S wave function

$$u_{1S}(r) = 2\gamma^{3/2}re^{-\gamma r}, \quad (1)$$

where $$\gamma^2 = 2\mu_{Q\bar{Q}}B, B = \mu_{Q\bar{Q}}\alpha_{\text{singlet}}^2/2,$$ and $$\gamma = |\mu_{Q\bar{Q}}\alpha_{\text{singlet}}|.$$ Here, $$\alpha_{\text{singlet}}$$ is the color-singlet coupling constant, relating to the strong interaction coupling constant $$\alpha_s$$ by

$$\alpha_{\text{singlet}} = -\frac{N_c^2 - 1}{2N_c}\alpha_s, \quad (2)$$

where $$N_c = 3$$ is the number of colors. Peskin and Bhanot used the large $$N_c$$ approximation and the above color-singlet effective coupling constant for $$N_c = 3$$ is approximated in this large $$N_c$$ limit by

$$\lim_{\text{large } N_c} \alpha_{\text{singlet}} = -\frac{N_c}{2}\alpha_s = -\frac{3}{2}\alpha_s. \quad (3)$$

Peskin and Bhanot considered the final color-octet state to be approximated by the 1P wave function as in the free-particle case. The corresponding wave function is

$$u_{1P}(r) = \frac{\sin kr}{kr} - \cos kr. \quad (4)$$

The overlap integral $$I$$ of Eq. (6.2) then gives

$$I = \int_0^\infty u_{1P}(r)u_{1S}(r)rdr = \frac{16\gamma^{5/2}k^2}{(k^2 + \gamma^2)^{3/2}}. \quad (5)$$

Using $$k^2 = 2\mu_{Q\bar{Q}}\epsilon$$ and the relation between $$\gamma$$ and the binding energy $$B,$$ Eq. (6.1) from Blatt and Weisskopf gives the dissociation cross section

$$\sigma_{\text{dis}}^{E_1}((Q\bar{Q})_{1S} + g \rightarrow Q + \bar{Q}) = \frac{4\pi}{3}(32)^2\alpha_s\frac{\alpha_{\text{singlet}}^2m_Q^2}{\alpha_{\text{singlet}}^2m_Q^2}(\epsilon/B)^{3/2}(\epsilon/B + 1)^{\gamma/2}. \quad (6)$$
The quantity $\alpha_{gQ}$ in the above equation is the effective coupling constant for the interaction of a gluon with the $Q$ (or $\bar{Q}$) of the quarkonium. In the problem studied by Peskin et al. \cite{25,26}, the initial $1S$ state is in the color-singlet state

$$|1\rangle = \frac{1}{\sqrt{3}} \sum_{ij} |3i, \bar{3}j\rangle,$$  \hspace{1cm} (7)

and the final $1P$ state is in a color-octet state with color component $c$

$$|8\rangle = \frac{1}{\sqrt{2}} \sum_{ij} \lambda^c_{ij} |3i, \bar{3}j\rangle.$$ \hspace{1cm} (8)

The fermion-(gauge boson) vertex in a Feynman diagram is associated with $-ie\gamma^\mu$ in QED and with $g\gamma^\mu\lambda^c/2$ in QCD where $c$ is the color index of the gluon. Thus, in going from the QED case of Blatt and Weisskopf to the QCD case of Peskin and Bhanot, $e$ in QED is replaced by $g\lambda^c/2$ in QCD. The value of $e^2 = 4 \pi \alpha_{QED}$ in QED of Blatt and Weisskopf is replaced in QCD by

$$|\langle 8|g\frac{\lambda^c}{2}|1\rangle|^2 = 4 \pi \alpha_{gQ}$$ \hspace{1cm} (9)

where $\lambda^c$ acts only on the $Q$ (or $\bar{Q}$). Eqs. (7)-(9) gives

$$\alpha_{gQ} = \frac{g^2}{4 \pi \times 6} = \frac{\alpha_s}{6}. \hspace{1cm} (10)$$

The above equations lead to

$$\sigma((Q\bar{Q})_{1S} + g \rightarrow Q + \bar{Q}) = \frac{2}{3} \pi \left(\frac{32}{3}\right)^2 \left(\frac{16 \pi}{3g^2}\right) \frac{1}{m_Q^2} \frac{(e/B)^{3/2}}{(e/B + 1)^5}, \hspace{1cm} (11)$$

which is the same as Eq. (4.4) of Bhanot and Peskin (Ref. \cite{26}), demonstrating the equivalence of the descriptions of Blatt and Weisskopf and Peskin and Bhanot.

It should be pointed out that the large $N_c$ limit is only a crude approximation for $N_c = 3$. If one does not invoke the large $N_c$ limit, we then have for $N_c = 3$, $\alpha_{\text{singlet}} = -(4/3)\alpha_s$. Hence, we obtain a more accurate dissociation cross section from Eqs. (1)-(10)

$$\sigma((Q\bar{Q})_{1S} + g \rightarrow Q + \bar{Q}) = \left(\frac{9}{8}\right)^2 \frac{2}{3} \pi \left(\frac{32}{3}\right)^2 \left(\frac{16 \pi}{3g^2}\right) \frac{1}{m_Q^2} \frac{(e/B)^{3/2}}{(e/B + 1)^5}, \hspace{1cm} (12)$$

which is $(9/8)^2$ times the approximate large-$N_c$ limit result of Peskin and Bhanot \cite{25,26}.

One notes that from this cross section expression (12), the peak of the dissociation cross section is located at $\epsilon = 3B/7$ and the maximum dissociation cross section from Eq. (12) is

$$\sigma_{\text{max}}((Q\bar{Q})_{1S} + g \rightarrow Q + \bar{Q}) = \frac{18.90}{\alpha_s m_Q^2}. \hspace{1cm} (13)$$

If one uses the hydrogenic description for $J/\psi$ with $\alpha_s = 0.3$, then $B = 0.06$ GeV and the maximum of the gluon dissociation cross section is located at $\epsilon = 0.025$ GeV with a maximum of 10.90 mb. For a color-singlet $\Upsilon$ with $\alpha_s = 0.2$, then $B = 0.089$ GeV and the maximum cross section is located at 0.038 GeV with a maximum of 1.5 mb.
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