Driving nanomechanical resonators by phonon flux in superfluid $^4$He

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We report on nanomechanical resonators with very high-quality factors operated as mechanical probes in liquid helium $^4$He, with special attention to the superfluid regime down to millikelvin temperatures. Such resonators have been used to map out the full range of damping mechanisms in the liquid on the nanometer scale from 10 mK up to $\sim$ 3 K. The high sensitivity of these doubly-clamped beams to thermal excitations in the superfluid $^4$He makes it possible to drive them using the momentum transfer from phonons generated by a nearby heater. This so-called “phonon wind” is an inverse thermomechanical effect that until now has never been demonstrated, and provides the possibility to perform a new type of optomechanical experiments in quantum fluids.

I. INTRODUCTION

Nanoelectromechanical systems (NEMS) are finding an increasing range of application owing to their small size, high operation frequencies, high intrinsic quality factors [1] and exceptionally small masses [2] which endow them with very high force sensitivities [3]. These properties make such resonators the perfect candidates for probing quantum liquids on length scales comparable to the coherence lengths in superfluid $^3$He and the de Broglie wavelengths of thermal excitations in superfluid $^4$He at sub-mK temperatures. $^4$He is the most investigated quantum fluid, with a well-understood spectrum of thermal excitations [4] and topological defects [5]. This makes it an ideal starting point for investigation of the behavior of high-frequency nanomechanical resonators in superfluids. Moreover, recent theoretical and experimental research in optomechanics combined with superfluid $^4$He [6–8] makes them favorable candidates for such investigations.

In the following, we characterize the doubly clamped beams in vacuum and then discuss their use in superfluid $^4$He in the temperature range from 10 mK up to $\sim$ 3 K. The very high force sensitivity of the nanobeams over the covered temperature range has allowed us to study in detail all the dissipation mechanisms over five orders of magnitude of damping. Finally, the high responsiveness of these devices has enabled us to drive a resonator by illuminating it with a modulated beam of phonons generated by a heater in the superfluid $^4$He. The phonon pressure sensed by the nanobeam is analogous to the photon pressure studied in the seminal experiment by P. N. Lebedev [9], where the pressure of photons has been measured for the first time.

II. THE NANOMECHANICAL RESONATORS AND LOW TEMPERATURES

CHARACTERIZATION IN VACUUM

The nanoelectromechanical resonators we use here are silicon nitride doubly-clamped beams coated with a layer of aluminum to provide a conducting path. The current experimental setup allows to characterize two nanomechanical resonators in one cooldown either independently or simultaneously. The first beam has a thickness of $t = 130 \text{nm}$, width $w = 300 \text{nm}$ and length $l = 150 \text{m}$ with a fundamental mode frequency in vacuum of 1.6 MHz. The second is similar but has a length of $l = 30 \text{m}$ and a fundamental frequency of 11.6 MHz.

The devices were fabricated on commercially-available undoped silicon wafers covered with a 100 nm thick silicon nitride ($\text{Si}_3\text{N}_4$) layer and a 30 nm thick deposited aluminum layer. The aluminum was patterned by electron-beam lithography to create the nanobeams and the on-chip wiring, to be used as the mask for dry-etching the $\text{Si}_3\text{N}_4$. The doubly-clamped beams were finally suspended by an undercut in the silicon substrate by selective etching in XeF$_2$.

A scanning electron image of the longer $150 \text{m}$ doubly-clamped beam is shown in Fig. 1, along with a schematic of the measurement setup for magnetomotive measurements. In our experiments the beams are exposed to a constant perpendicular magnetic field, $B$, and the Lorentz force from an AC current passed along the beam induces transverse oscillatory motion. The motion is detected by the emf generated across the device which can be measured by a vector network or spectrum analyzer.

All the measurements described here were taken in a brass cell thermalized at the mixing chamber of a dilution refrigerator with a base temperature of $T_0 = 10 \text{mK}$. The cell temperature is inferred from a calibrated RuO$_2$ thermometer anchored to the mixing chamber. Filling capillaries allow us to perform measurements either in...
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of magnetization effects in the beam and is predicted to
dashed lines in the figure. Such a dependence is a result
Q on magnetic field as
magnetomotive loading starts to dominate, and depends
less of the beam length, in the higher magnetic fields,
larger and remain field-independent up to 1 T. Regard-
Fig. 2(a). Internal losses for the 30 \( \mu \text{m} \)-long composite aluminum on silicon nitride
film becomes normal and the damping increases up to
\( \sim 10 \text{mK} \) in a magnetic field of 10 mT, where the aluminum film
is in the superconducting state.

vacuum or in a liquid helium environment by condensing
superfluid helium on the surface of sintered-silver heat
exchangers.

We initially characterized the nanobeams in vacuum
using a magnetomotive scheme, to determine the intrin-
sic losses of the resonators, both in the normal and super-
conducting states of the aluminum. The inset in Fig. 1
shows the mechanical resonance in vacuum of the 150 \( \mu \text{m} \)-long beam in the superconducting state at the base tempera-
ture in a magnetic field of 10 mT. The high quality fac-
tor shows that under these conditions the internal losses
in the nanobeam are very low, \( Q_{\text{int}}^{-1} \approx 2 \times 10^{-7} \). We
should note that cooling the device in vacuum relies on
the thermal link to the aluminum clamping leads, which
provide a contact to the external thermal bath. With
the aluminum in the superconducting state that should
be relatively poor. However, the results are consistent
with those taken in higher fields where the aluminum is
in the normal state and we are confident that the beam
is indeed at \( T_0 = 10 \text{mK} \).

At higher magnetic fields of \( \approx 50 \text{mT} \), the aluminum
film becomes normal and the damping increases up to
\( \sim 10^{-6} \). Such damping is independent of the magnetic
field and remains dominant up to 150 mT, as shown in
Fig. 2(a). Internal losses for the 30 \( \mu \text{m} \)-long beam are
larger and remain field-independent up to 1 T. Regard-
less of the beam length, in the higher magnetic fields,
magnetomotive loading starts to dominate, and depends
on magnetic field as \( Q_{\text{int}}^{-1} \propto B^2 \) [10], as shown by the
dashed lines in the figure. Such a dependence is a result
of magnetization effects in the beam and is predicted to
be temperature independent at low temperatures [11].

III. MEASUREMENTS IN LIQUID \(^4\text{He} \), THE
DAMPING MECHANISMS

In the next stage we operated the nanobeams in liquid
\(^4\text{He} \) over the temperature range from 10 mK to \( \sim 3 \text{K} \).
This is not trivial, since over this interval the damping
arising from the ambient liquid varies over four orders of
magnitude. We therefore had to adjust the magnetomotive
force according to the level of damping, meaning that
measurements had to be made over a range of magnetic
fields up to 5 T.

Both the 150 \( \mu \text{m} \) and 30 \( \mu \text{m} \)-long beams demonstrated
an order of magnitude larger damping in superfluid \(^4\text{He} \)
at 10 mK than in vacuum, even in low magnetic fields
where magnetomotive losses are negligible. This is rather
counter-intuitive: the Stokes’ drag from interaction with
the superfluid normal component should be negligible at
these temperatures, since the density of thermally excited
phonons and rotons is insignificant [13].

The measurements map out rather consistently the
damping arising from the liquid over the whole tempera-
ture range. In order to clarify the damping mechanisms
in the liquid helium, the temperature dependence of the
quality factor of the 150 \( \mu \text{m} \)-long beam was measured,
as shown in Fig. 2(b). As can be seen in the figure, in
addition to the beam’s internal, \( Q_{\text{int}}^{-1} \), and magnetomo-
tive, \( Q_{\text{mm}}^{-1} \), losses, there are three distinct temperature
regimes distinguished by the dominant damping mechani-
isms originating in the liquid. These are: (with increasing
temperature) acoustic emission \( Q_{\text{ac}}^{-1} \), ballistic scat-
tering of thermal excitations (phonons, \( Q_{\text{ph}}^{-1} \), and rotons,
\( Q_{\text{rot}}^{-1} \)) and finally, above 1 K, hydrodynamic losses \( Q_{\text{hy}}^{-1} \).
These mechanisms together constitute the total damping:

\[
Q_{\text{tot}}^{-1} = Q_{\text{int}}^{-1} + Q_{\text{mm}}^{-1} + Q_{\text{ac}}^{-1} + Q_{\text{ph}}^{-1} + Q_{\text{rot}}^{-1} + Q_{\text{hy}}^{-1}.
\]

(1)

It is remarkable that all these distinct behaviors can
be extracted with a single device. The hydrodynamic
Stokes’ drag by the normal fluid component is substan-
tial at temperatures above 1 K and can be described in
the framework of the phenomenological two-fluid model
of superfluid \(^4\text{He} \) [14].

To fit the damping at lower temperatures, we note that
since from 0.1 K to 1 K, the mean free paths of thermal
excitations in the superfluid (rotons and phonons) are
larger than the effective nanobeam diameter, while the
de Broglie wavelengths are much smaller [13], we can take
the dissipation as arising from the scattering of individual
phonons and rotons at the beam surface. The phonon
drag force exerted per unit length of the beam is obtained
by summing the momentum of phonons colliding with the
beam per unit time [13], giving the damping contribution:

\[
Q_{\text{ph}}^{-1} = A \frac{k_B^4}{45h^4df_0(\rho + \rho_s)c_{\text{ph}}^4} T^4,
\]

(2)

with \( d \approx 150 \text{nm} \) the effective beam diameter, \( f_0 \) the
resonance frequency of the beam, \( c_{\text{ph}} = 229 \text{ms}^{-1} \) the
phonon velocity [15], \( \rho \) and \( \rho_s \) are the densities of the
beam material and superfluid, respectively, and $A = 2.67$ is a constant taking into account the “cylindrical” beam geometry [13].

Following similar arguments, the damping arising from roton scattering, $Q_{\text{rot}}^{-1}$ becomes:

$$
Q_{\text{rot}}^{-1} = \frac{4p_0^4}{3\pi^2 k_B^2 f_0 (\rho + \rho_s) c_{\text{rot}}} \sqrt{\frac{m^*}{8\pi^3 k_B T}} \exp \left( -\frac{\Delta}{k_B T} \right),
$$

with $p_0$, $m^*$, and $\Delta$ the Landau roton parameters [13] and $c_{\text{rot}} = 160 \text{ m s}^{-1}$ the roton velocity [16].

The combined damping from the phonon and roton scattering, $Q_{\text{ph}}^{-1} + Q_{\text{rot}}^{-1}$, is shown by the blue dashed line in Fig. 2(b), and describes well the envelope of the experimental data in the temperature range 0.4 K – 0.8 K with no fitting parameters.

At lower temperatures, acoustic losses begin to dominate and the damping saturates. These losses depend on the nanobeam frequency, reaching $Q_{\text{ac}}^{-1} \approx 5 \times 10^{-6}$ for the 150 $\mu$m-long, and $Q_{\text{ac}}^{-1} \approx 2 \times 10^{-4}$ for the 30 $\mu$m-long nanobeams and can be described in the framework of dipole emission [12] as:

$$
Q_{\text{ac}}^{-1} = \frac{\pi^3 \rho_s}{2} \left( \frac{df_0}{c_{\text{ph}}} \right)^2. 
$$

The solid line in Fig. 2(b) shows the resulting sum of the damping from thermal excitations in the helium and the acoustic emission from the nanobeam which fits the 40 mT data well down to the lowest temperatures, where at this field the magnetomotive losses are negligible as shown by red circles in Fig. 2(a). In the inset of Fig. 2(a) the dashed line shows the expected losses from acoustic emission obtained from Eq. (4) again without fitting parameters, as compared with the measured values for the two beams.

This is quite a different behavior to that expected at zero frequency where the condensate behaves essentially as a mechanical vacuum. These acoustic losses could be reduced by utilizing lower frequency detectors, or beams with a smaller diameter, for example, carbon nanotubes. Alternatively, the mechanical resonators can be enclosed in a cavity, with a size smaller than half of the acoustic wavelength of the emitted sound along the axis of the nanobeam.
FIG. 3. Diagram of the experimental setup for driving the nanomechanical resonator by a phonon flux in liquid $^4$He. The 30 $\mu$m-long beam, operated off-resonance, is used as a heater. It is located $\sim 5$ mm away from the 150 $\mu$m-long beam, which is used as a detector. Both beams are placed in superfluid $^4$He at 10 mK in a magnetic field 1.3 T (The working point shown by the arrow in Fig. 2(a)). The heater is driven by an AC current at half the detector resonance frequency. The generated emf on the detector beam is monitored by a spectrum analyzer.

dipole emission. This will restrict the number of acoustic modes available for emission. For the frequency of $\sim 1.6$ MHz, the characteristic size of the acoustic cavity should be $\sim 50 \mu$m.

IV. DRIVING A NANOBEAM WITH A MODULATED PHONON FLUX IN THE SUPERFLUID

Now we turn to the phonon-generated excitation of the nanobeams. Figure 3 shows the experimental setup used. The 150 $\mu$m and 30 $\mu$m-long beams, separated by a distance $\sim 5$ mm, are immersed in superfluid helium at 10 mK in a magnetic field 1.3 T perpendicular to the conducting plane of the nanobeams.

In this arrangement the 30 $\mu$m-long beam acts an ohmic heater, since the aluminum is in the normal state in this magnetic field, which injects phonons into the superfluid. The 150 $\mu$m beam serves as the detector. To set the detector beam into resonance the phonon flux from the heater must be modulated at the fundamental frequency of the detector beam, 1.622 MHz. The AC current to the heater must thus be operated at half this frequency to emit bursts of phonons at the detector fundamental. The generated phonons propagate ballistically through the helium and scatter on the surface of the detector beam thus transferring their momentum and setting the beam into oscillation. This movement is perpendicular to the ambient magnetic field, hence generating an emf across the beam which we detect with a spectrum analyzer to yield the velocity amplitude. In other words, the ohmic heater sets up a pulsed “phonon wind” which excites the detector motion.

The power spectral density (PSD) of the generated emf signal is shown in Fig. 4(a) as a function of frequency for three different values of the heater power. The PSD has a clear peak at 1.6221 MHz indicating the resonant driving of the nanobeam detector. We note that the quality factor $Q = 9300$ of the detector as measured this way is relatively owing to the fairly large magnetomotive loading at this ambient field. The corresponding operating point is indicated in Fig. 2(a), by the arrow labeled “Phonon wind”.

The total detected power $P_F$ is calculated as an integral over the resonance curve and is found to be linearly proportional to the applied power, as seen in Fig. 4(b). Above an applied heater power of $P_h \approx 100$ pW the response falls below the linear relation as the liquid begins to heat substantially.

The value of the induced emf, $E = \sqrt{P_F Z_0}$, is obtained from the total detected power and the characteristic impedance, $Z_0 \approx 50 \Omega$, of the detection scheme. The emf is proportional to the velocity, $v$, of the beam motion in the magnetic field since $E = Blv$ and must be proportional to the magnitude of the resonant driving force, $F_{ph}$, of the driving phonon wind. Thus, the actuating force of the phonon wind can be written:

$$F_{ph} = 2\pi m f_0 \frac{P_F Z_0}{QB},$$

where $m \approx 1.79 \times 10^{-14}$ kg is the effective mass of the detector. This gives $F_{ph} \approx 0.5$ N at 0.5 aW and $F_{ph} \approx 1.3$ N at 3.5 aW of detected power. The sensitivity of the response, $\sim 22$ aN Hz$^{-1/2}$, is also obtained from the PSD measurements.

The actuating force can also be obtained using molecular kinetic theory with a few basic (and fairly loose) assumptions. Suppose that the non-equilibrium phonons incident on the detector surface have a density $n$, and behave as point-like non-interacting particles with the same average momentum $p_{ph} = k_B T_{ph}/c_{ph}$, corresponding to the effective temperature of the phonon beam. A characteristic phonon energy of $k_B T_{ph}$ in the heater vicinity,
FIG. 4. (a) The power spectral density (PSD) of the generated emf signal as a function of frequency for three different values of the heater power. The solid line is a Lorentzian fit of the experimental data. (b) The integrated emf power, $P_E$, from the detector as a function of the heater power, $P_H$. The dashed line is a guide for the eye showing the linear dependence. The arrows indicate the values of power at which the response curves plotted in panel (a) were measured.

The density of non-equilibrium phonons probed by the detector is obtained from the combination of Eq. (5) and Eq. (6). Substitution of the experimental data into this solution shows that the density remains almost constant at $n \approx 3 \times 10^{19} \text{m}^{-3}$ for any power applied to the heater from 20 pW up to 100 pW. This fact is qualitatively confirmed by the linear dependence $P_E(P_H)$ in Fig. 4(b), since $n \propto \sqrt{P_E/P_H}$.

It is interesting to compare this density with the equilibrium phonon concentration at the base temperature of the experiment [17]:

$$n_{ph} = 8\pi \zeta(3) \left(\frac{k_B T_0}{\hbar c_{ph}}\right)^3 \approx 2 \times 10^{19} \text{m}^{-3},$$

which is only 1.5 times lower than that probed by the detector. Nonetheless, the effectiveness of the phonon detection arises from driving the beam from one side only, compared with the uniform pressure of ambient phonons.

Our experiment shows that the heater can emit phonon bursts with a high repetition frequency of MHz, which is interesting on its own right. That said, we must assume the thermal time constant for the heater arrangement is shorter than can be expected for millikelvin temperatures. The action of forcing a current through the heater beam initially heats the electron gas locally. For this excess power to be injected into the helium it must be transformed into a form which can cross the aluminum-
helium boundary. That implies that an excess of high energy phonons must be accumulated in the metal which then cross the boundary into phonon states in the liquid. However, at 10 mK the phonon density of states in the helium is very small, since the Debye temperature is about 25 K [18]. In the aluminum the situation is even worse, since the Debye temperature is about 400 K. The very poor densities of phonon states on both sides of the barrier would initially suggest that the process is rather slow. This finding requires further experimental investigation.

V. CONCLUSION

We have presented the novel method of driving nanomechanical resonators by a modulated phonon flux. This so-called “phonon wind” is an inverse thermomechanical effect that until now has never been demonstrated, and provides the possibility to perform a new type of optomechanical experiments in quantum fluids. Furthermore, we have successfully used nanomechanical resonators as probes of dumping mechanisms in superfluid ⁴He down to millikelvin temperatures. We have demonstrated that acoustic dissipation is the main damping mechanism for high-frequency nanobeams at low magnetic fields in quantum fluids. These acoustic losses could be much reduced by utilizing lower frequency detectors, or by enclosing the nanobeams in a small cavity, to restrict the number of acoustic modes available for sound emission. The sensitivity of the detector to thermal excitations in quantum fluids could be significantly improved by the addition of a cryogenic amplifier to the measurement scheme. Such modifications would allow similar experiments with the detectors in the superconducting state to give extremely high Q-factors, an essential ingredient for advances in optomechanical systems exploiting quantum media. By incorporating superconducting nanomechanical resonators in in quantum circuits, e.g. single-Cooper-pair transistors, SQUIDs, or QUBITs, etc., a new class of quantum instruments for probing the quantum fluids, ⁴He and ³He, can be built.

ACKNOWLEDGMENTS

We would like to acknowledge the excellent technical support of A. Stokes, M. G. Ward, M. Poole and R. Schanen and very useful scientific discussions with S. Autti, E. Laird, A. Jennings, M. T. Noble and T. Wilcox. This research was supported by the UK EPSRC Grants No. EP/L000016/1 and No. EP/I028285/1, and the European Microkelvin Platform.

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