Shocks and Tides Quantified in the “Sausage” Cluster, CIZA J2242.8+5301 Using N-body/Hydrodynamical Simulations

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Abstract

The colliding cluster, CIZA J2242.8+5301, displays a spectacular, almost 2 Mpc long shock front with a radio based Mach number $M \approx 5$, that is puzzlingly large compared to the X-ray estimate of $M \approx 2.5$. The extent to which the X-ray temperature jump is diluted by cooler unshocked gas projected through the cluster currently lacks quantification. Here we apply our self-consistent N-body/hydrodynamical code (based on FLASH) to model this binary cluster encounter. We can account for the location of the shock front and also the elongated X-ray emission by tidal stretching of the gas and dark matter between the two cluster centers. The required total mass is $8.9 \times 10^{14} M_{\odot}$ with a 1.3:1 mass ratio favoring the southern cluster component. The relative velocity we derive is $\approx 2500 \text{ km s}^{-1}$ initially between the two main cluster components, with an impact parameter of 120 kpc. This solution implies that the shock temperature jump derived from the low angular resolution X-ray satellite Suzaku is underestimated by a factor of two, due to cool gas in projection, bringing the observed X-ray and radio estimates into agreement. Finally, we use our model to generate Compton-γ maps to estimate the thermal Sunyaev–Zel’dovich (SZ) effect. At 30 GHz, this amounts to $\Delta S_\gamma = -0.072 \text{ mJy/arcmin}^2$ and $\Delta S_\alpha = -0.075 \text{ mJy/arcmin}^2$ at the locations of the northern and southern shock fronts respectively. Our model estimate agrees with previous empirical estimates that have inferred the measured radio spectra of the radio relics can be significantly affected by the SZ effect, with implications for charged particle acceleration models.

Key words: galaxies: clusters: general – galaxies: clusters: individual (CIZA J2242.8+5301) – methods: numerical

1. Introduction

Massive pairs of colliding clusters display extreme physical effects, including large X-ray shock fronts that are often traced by radio “relics” of large-scale diffuse synchrotron emission (Enßlin 1999; for a recent review, see Feretti et al. 2012). Cluster collisions are often recognized by a clear bimodal distribution of member galaxies and dark matter mapped by weak and strong lensing effects. The shocked gas in the iconic “bullet cluster” (1E0657-56) clearly implies that two massive clusters have just passed centrally through each other with a high relative velocity of $\gtrsim 3000 \text{ km s}^{-1}$ implied by the clearcut Mach cone found in its X-ray image (Markevitch et al. 2002). Velocities derived from X-ray observations using the shock jump conditions and N-body/hydrodynamical simulations support this interpretation (Markevitch et al. 2004; Springel & Farrar 2007; Mastropietro & Burkert 2008; Molnar et al. 2013a), with a wide range of model velocities derived, $2700–4500 \text{ km s}^{-1}$ (for a recent review, see Molnar 2015). The bullet cluster provides model independent support for the simple, collisionless assumption regarding dark matter, because the bimodal weak lensing pattern is coincident with the galaxy distribution (Clowe et al. 2006), and hence, the dark matter must be essentially collisionless, like the member galaxies that have passed by each other during the first core passage.

The most massive cluster collisions have been uncovered in Sunyaev–Zel’dovich (SZ) sky surveys (ACT: Atacama Cosmology Telescope, Sifón et al. 2013; SPT: South Pole Telescope, Reichardt et al. 2012 Planck Satellite, Planck Collaboration et al. 2011), demonstrating that SZ selection favors clusters caught in collision when large columns of high pressure compressed gas are generated. A prime example is the strong SZ source, “El Gordo,” which is a binary colliding cluster discovered by ACT (ACT-CL J0102-4915, $z = 0.87$; Menanteau et al. 2012). Using N-body/hydrodynamical simulations, we have shown that its cometary X-ray appearance can be readily reproduced by a collision that is not head on, generating two long, parallel tails of compressed gas, one of which has been pushed under gas pressure away from the most massive cluster component (Molnar & Broadhurst 2015). The other tail is tidally compressed gas lying between the two cluster centers. This compressed gas in “El Gordo” lies interior to several relatively small radio “relics” that appear to mark shock boundaries at the interfaces of gas belonging to each cluster. They lie at large radii, where the pressure jump can be a factor of several but the relatively low gas density means the shock transitions are much less visible in X-rays compared to the dense central, compressed gas.

Much larger radio relics have been uncovered in other colliding clusters (e.g., 1RXS J0603.3+4214, the “Toothbrush” cluster, Ogrean et al. 2013b; CIZA J2242.8+5301, the “Sausage” cluster, van Weeren et al. 2010; Ogrean et al. 2013a, 2014; Akamatsu et al. 2015). In CIZA J2242.8+5301, a Mpc-scale “sausage” shaped radio structure was found, which clearly corresponds to the gas collision shock front in the north, with its outer edge coincident with an X-ray temperature jump detected by Akamatsu et al. (2015). This shock front is the most clearly defined example known, spanning an angle of 80°. Its highly polarized radio emission implies the presence of a cluster scale magnetic field strength of $\approx 5 \mu \text{G}$ (van Weeren et al. 2010). The sausage also shows another large, but more diffuse radio relic in the south that lies close, but somewhat interior to the location of the southern X-ray shock claimed by Akamatsu et al. (2015). This X-ray shock front is expected to be generated in the
“reverse” direction, opposite the faster moving infalling cluster that has moved through to the north after the first core passage (Ricker & Sarazin 2001; Molnar et al. 2012; Molnar & Broadhurst 2015). No self-consistent hydrodynamical model has been constructed for CIZA J2242.8+5301 to date, and here we will also discuss this reverse shock front based on our simulations. (Note that after we submitted our paper, Donnert et al. 2017 presented a model of CIZA J2242.8+5301 based on N-body/hydrodynamical simulations.)

Several collision shocks have Mach number estimates from both radio and X-ray observations, e.g., A2255; 1RXS J0603.3+4214, the “Toothbrush” cluster; CIZA J2242.8+5301, the “Sausage” cluster (van Weeren et al. 2010; Ogrene et al. 2013a, 2013b, 2014; Akamatsu et al. 2015, 2017), with a pattern emerging that the radio estimates are generally significantly larger than the X-ray estimates. In principle, the X-ray estimates simply relate the physical temperature change across the shock to the standard Rankine–Hugoniou jump conditions. However, X-ray observations detect projected temperatures, which are derived from the observed spectrum as an integral through the entire cluster, so that temperature estimates at any projected radius can comprise gas with a spread of temperatures. Also, in practice, the X-ray emission is weak in the outer regions where the shocks are often seen (e.g., Akamatsu et al. 2015) requiring large area binning, and hence a large projected column of unshocked, cooler gas in each X-ray bin may flatten the temperature jump, biasing the Mach number low. This problem is exacerbated by the presence of very hot and luminous shocked gas, so that a model of the gas distribution in the line of sight (LOS) is beneficial to help estimate the effects of projection and derive the physical quantities of interest, including the density and temperature distribution of the gas. Such a model can be derived using full N-body/hydrodynamical simulations constrained by observations.

Here we generate a comprehensive set of self-consistent N-body/hydrodynamical simulations of binary merging clusters of galaxies constrained by observations of member galaxies, gas, and dark matter to model CIZA J2242.8+5301, and thus obtain the three-dimensional (3D) structure of the shocks. We base the gas dynamics on the FLASH Adaptive Mesh refinement (AMR) codes because of the presence of shocks that are not well captured by coarse grids or the “smooth particle hydrodynamics” approximation.

The structure of this paper is as follows. In Section 2, we summarize results from previous analyses of CIZA J2242.8+5301 based on multifrequency observations and numerical simulations. We describe our simulation setup for modeling CIZA J2242.8+5301 as a binary merger in Section 3. Section 4 presents our results, and provides a physical interpretation of the morphology of multifrequency observations of CIZA J2242.8+5301. In this section, we also discuss quantitatively the merging shocks in CIZA J2242.8+5301, as well as the biases in Mach numbers, derived from X-ray observations, and in flux measurements of radio relics due to SZ contamination from shocks. Section 5 contains our conclusions.

2. CIZA J2242.8+5301: A Merging Cluster with the Prototype of Relics

The most prominent radio relic is the northern relic in CIZA J2242.8+5301 (the “Sausage” Cluster; van Weeren et al. 2010). This is a bright radio relic, about 1.7 Mpc long, has a smooth bow or sausage shape (hence the name), and it is only about 55 kpc wide, apparently delineating a merger shock. Assuming a diffusive shock acceleration (DSA) model (the most popular model to explain synchrotron radio emission from relics), van Weeren et al. (2010) derived a Mach number of 4.6. On the opposite side of this merging cluster, on the southern part, there are some scattered relics with irregular shapes.

CIZA J2242.8+5301, at a redshift of z = 0.19 (Jee et al. 2015), has been studied extensively. Detailed spectroscopic observations of CIZA J2242.8+5301 were carried out by Dawson et al. (2015). They found that the two subclusters have very similar redshifts, suggesting that the collision is close to the plane of the sky. XMM-Newton and Chandra observations of CIZA J2242.8+5301 were carried out to study the X-ray morphology and search for shocks and discontinuities in the X-ray emission by Ogrene et al. (2013a, 2014).

Numerical simulations aiming to model the radio emission, and constrain the initial parameters of the merging clusters, were carried out by van Weeren et al. (2011). They used N-body/hydrodynamical simulations (FLASH), but assumed fixed potentials for both subclusters. van Weeren et al. 2011 concluded that CIZA J2242.8+5301 is a binary merging cluster after the first core passage; the collision is close to the plane of the sky (∼10° from the plane of the sky) with a mass ratio of ∼2:1 (the northern subcluster being more massive), and an impact parameter of ∼400 kpc. Weak lensing observations were used to derive the total mass of the system and the masses of the components of CIZA J2242.8+5301. Jee et al. (2015) found a total mass of \( M_{200,\text{total}} \sim 2.1 \times 10^{15} M_\odot \), a ratio ∼1:1, and masses: \( M_{200,\text{l}} = 1.10^{+0.7}_{-0.5} \times 10^{14} M_\odot \) and \( M_{200,\text{r}} = 9.8^{+3.8}_{-2.5} \times 10^{14} M_\odot \) (200c refers to a density 200 times the critical mass density). Okabe et al. (2015) derived a total virial mass of \( M_{\text{vir,\text{total}}} \sim 1.9 \times 10^{15} M_\odot \) (\( M_{200,\text{total}} \sim 1.6 \times 10^{15} M_\odot \)), a mass ratio ∼2:1, and masses: \( M_{\text{vir,l}} = 1.24^{+0.8}_{-0.7} \times 10^{14} M_\odot \) and \( M_{\text{vir,r}} = 9.6^{+4.2}_{-1.4} \times 10^{14} M_\odot \), the southern subcluster being more massive. Northern and southern shocks were found around the relics with Mach numbers of 2.7±0.4 and 1.7±0.3, respectively, using Suzaku observations by Akamatsu et al. (2015).

Recently, steepening of the spectra of the northern relic in CIZA J2242.8+5301, and the relic in 1RXS J0603.3+4214, the “Toothbrush” cluster, have been found (Stroe et al. 2016), which excludes simple DSA models. Several solutions have been proposed to solve this problem using the northern relic in CIZA J2242.8+5301 as a prototype. Kang & Ryu (2015, 2016) proposed that fossil electrons (accelerated by the DSA mechanism) are reaccelerated as the shock runs them over; Fujita et al. (2016) suggested that cosmic rays are reaccelerated by turbulence generated behind shocks; Donnert et al. (2016) offered the explanation that magnetic turbulence generated by the amplification of the magnetic field is responsible for the particle reacceleration. An alternative solution was suggested by Basu et al. (2016): the curved radio spectrum is a consequence of the SZ effect, which lowers the measured radio flux at the observed radio wavelengths (<217 GHz).

3. Modeling CIZA J2242.8+5301 Using Hydrodynamical Simulations

The main goal of our project was to obtain a reasonable physical model for CIZA J2242.8+5301 and study the merging shocks, not to carry out a systematic search for the initial
parameters and determine their errors (since that would require many more simulations).

3.1. Details of the Simulations, Initial Setup

We model CIZA J2242.8+5301 using 3D self-consistent \( N \)-body/hydrodynamic numerical simulations of binary galaxy cluster mergers including dark matter and intracluster gas. The simulations have been carried out using the publicly available parallel Eulerian AMR code FLASH, developed at the Center for Astrophysical Thermonuclear Flashes at the University of Chicago (Fryxell et al. 2000 and Ricker 2008).

We adopted a large box size, 13.3 Mpc on a side, for our simulations to capture the outgoing merger shocks. We used our well-established method to initialize and run the merging cluster simulations (Molnar et al. 2012, 2013a, 2013b; Molnar & Broadhurst 2015). We reach the highest resolution, 12.7 kpc, at the centers of the clusters and at the merger shocks, as well as in the turbulent regions behind the shocks. Our FLASH simulations were adiabatic including shock heating, which is the most important irreversible process for merging clusters.

We assumed spherical clusters with cutoffs at the virial radius \( r \leq R_{\text{vir}} \) for the the dark matter and the intracluster gas. Our initial distribution of the dark matter was the NFW model (Navarro et al. 1997),

\[
p_{\text{DM}}(r) = \frac{\rho_0}{x(1 + x)^2},
\]

where \( x = r/r_c \) and \( \rho_0, r_c = R_{\text{vir}}/c_{\text{vir}} \) are scaling parameters for the radius and the density, and \( c_{\text{vir}} \) is the concentration parameter, and for the gas density distribution we adopted a non-isothermal \( \beta \) model,

\[
p_{\text{gas}}(r) = \frac{\rho_0}{(1 + y^2)^{3/2}},
\]

where \( y = r/r_{\text{core}} \), is the scaling parameters for the radius, \( \rho_0 \), is the density at the center, and \( r_{\text{core}} \) is the scaling parameter for the radius for the gas distribution.

We assumed hydrostatic equilibrium and derived the temperature from the equation of hydrostatic equilibrium adopting the ideal gas equation of state with \( \gamma = 5/3 \). We assumed \( \beta = 1 \), suggested by cosmological numerical simulations for the large-scale distribution of the intracluster gas, removing the filaments (Molnar et al. 2010). We used a gas fraction of 0.14, and represented the stellar matter in galaxies with collisionless particles, since galaxies can be considered collisionless for our purposes. The velocities of the dark matter particles were drawn from a Maxwellian distribution with a velocity dispersion as a function of distance from the cluster center obtained from the Jeans equation assuming isotropic velocity dispersion (Lokas & Mamon 2001). The distribution of the direction of the velocity vectors was assumed to be isotropic. More details of the setup for our simulations can be found in Molnar et al. (2012).

3.2. FLASH Runs

We performed a series of FLASH simulations changing the masses, concentration parameters, impact parameter, and infall velocity of the system to find the best model for CIZA J2242.8+5301. Although the main goal of our study was not to carry out a systematic parameter search, in order to describe qualitatively the effect of changing initial parameters, we also show representative simulations that do not resemble CIZA J2242.8+5301. We list the initial parameters for those simulations we present in this paper in Table 1. The IDs of our runs are listed in the first column indicating the impact parameter \( P \) in kpc and infall velocity \( V \) in 100 km s\(^{-1}\) of each run as \( \text{PijkVlm} \), the virial masses (in units of \( 10^{14} M_\odot \)) and concentration parameters of the two subclusters are shown in columns 2–5. In columns 6 and 7, we list the impact parameters in kpc and infall velocities in km s\(^{-1}\).

4. Results and Discussion

4.1. Deprojecting CIZA J2242.8+5301

We used the X-ray morphology, the positions of the merging shocks (van Weeren et al. 2010; Ogrean et al. 2014; Akamatsu et al. 2015), and the positions of the dark matter centers of the two subclusters derived from weak lensing measurements of CIZA J2242.8+5301 (Okabe et al. 2015), to constrain the masses, the concentration parameters, the impact parameter, and the infall velocity of the system.

After each run, we rotated the system with an angle, \( \theta \), out of the plane of the sky (assumed to coincide with the main plane of the collision containing the two cluster centers and the relative velocity vector) to match the projected distance to that observed (Okabe et al. 2015). The second rotation around the axis ("roll angle") connecting the two dark matter centers with an angle, \( \varphi \), was constrained by the observed X-ray morphology (Ogrean et al. 2014). We choose those outputs (epochs), which could be rotated in a way that the two merging shocks could be projected to match the observed positions (Ogrean et al. 2014; Akamatsu et al. 2015). The shocks were located based on projected pressure gradients. A detailed description of our method to generate the X-ray and mass surface density maps can be found in Molnar & Broadhurst (2015).

We display images of CIZA J2242.8+5301 based on multi-wavelength observations and our best model in Figure 1. The first panel (from the left) shows an image from Chandra observations (Figure 1 from Ogrean et al. 2014) overlaid on the shock positions based on Suzaku observations proposed by Akamatsu et al. (2015; yellow lines). Comparing the black and gray data points of the temperature around the southern relic on

| ID  | \( M_{\text{vir}} \) | \( c_{\text{vir}} \) | \( M_{\text{vir}} \) | \( c_{\text{vir}} \) | \( P \) | \( V \) |
|-----|----------------|----------------|----------------|----------------|-----|-----|
| P120V25 | 5.0           | 8              | 3.9           | 8              | 120 | 2500 |
| P050V25 | 5.0           | 8              | 3.9           | 8              | 50  | 2500 |
| P180V25 | 5.0           | 8              | 3.9           | 8              | 180 | 2500 |
| P100V25 | 5.0           | 8              | 3.0           | 8              | 100 | 2500 |
| P150V20 | 5.0           | 5              | 4.5           | 6              | 150 | 2000 |
| P150V30 | 4.0           | 8              | 3.5           | 8              | 150 | 3000 |
| P200V18 | 16.0          | 6              | 6.0           | 8              | 200 | 1800 |

Notes.

* ID of the runs indicates the impact parameters in kpc and infalling velocities in 100 km s\(^{-1}\) and the mass of the main cluster in \( 10^{14} M_\odot \).
* Virial mass in \( 10^{14} M_\odot \) and concentration parameter for the main cluster (1).
* Virial mass in \( 10^{14} M_\odot \) and concentration parameter for the infalling cluster (2).
* Impact parameter, \( P \), in units of kpc.
* Infall velocity of cluster 2, \( V \), in km s\(^{-1}\).
the left panel of Figure 7 of Akamatsu et al. (2015), we suggest that the southern shock is farther out than their estimate. We locate it at the outer edge of the relics, shown as the light green line in our figure (Figure 1). The second panel displays a Subaru image (gi color) with the galaxy number density contours (white contours), the XMM-Newton X-ray luminosity (red contours), and the WSRT radio emission (green contours) overlaid (Figure 1 from Dawson et al. 2015). We show our best model for CIZA J2242.8+5301, which has a total virial mass of $M_{\text{vir}} = 8.9 \times 10^{14} M_\odot$, a mass ratio of 1.3:1, an impact parameter of $P = 120 \text{ kpc}$, and an infall velocity, $V_{\text{in}} = 2500 \text{ km s}^{-1}$ (with rotations: $\theta = 15^\circ$ and $\varphi = 20^\circ$) in the third panel of Figure 1 (run P120V25). This model has a similar elongated—tidally stretched—X-ray morphology to that of CIZA J2242.8+5301, the positions of the dark matter peaks and the shocks also match the observations. The epoch of this model, the best match to the data, is 0.4 Gyr after the first core passage. All cases at later epochs, such as infalling after turnover and second core passage can be excluded, because by those times the shocks have already run out of the clusters. In our best model, the shocks run out about 0.5 Gyr after first core passage. Earlier epochs can be excluded based on their compact X-ray morphology and the short distance between the shocks. We illustrate the sensitivity of the location of the peaks of the projected dark matter distribution, the X-ray emission, and the location of the two shocks in two panels in the last row in Figure 2. The left panel shows our best model 0.1 Gyr before the best epoch (0.4 Gyr). At this time, the X-ray emission is too compact, and the shocks are too close to each other. The right panel displays our best model 0.1 Gyr after the best epoch. At this time, the X-ray emission is too thin and filamentary, and the shocks are too far from each other and running out of the clusters.

In Figure 2, we show models that do not fully resemble CIZA J2242.8+5301, but deviate significantly in one or more ways from the body of constraining data. The panels in the first row show models with the same initial parameters as our best model, but with an impact parameter that is either too small (50 kpc) or too large (180 kpc), resulting in a bulky X-ray core or a second peak in the north, respectively. In the second and third rows we show models that have too large or too small mass ratios, or a very high infall velocity with a small mass ratio that produces the southern shock tilted more to the west than the observed one (first panel in the third row). Our model with a more massive main cluster (the total mass being about the same as the one derived by Okabe et al. (2015), but with a somewhat higher mass ratio, 2.66:1 versus 2:1) shows a thick X-ray bridge between the two dark matter centers not a thin, elongated feature as in CIZA J2242.8+5301 (second panel in the third row). For the input parameters of all models, see Table 1. In general, very massive merging clusters ($M_{\text{vir}} \sim 10^{15} M_\odot$) keep their gas and no thin tidal bridge would form between the two dark matter centers, and thus they do not look like CIZA J2242.8+5301.

4.2. Properties of Merging Shocks in CIZA J2242.8+5301

Our simulations clearly demonstrate that the merging cluster, CIZA J2242.8+5301, is being seen just after the first core passage (in agreement with van Weeren et al. 2011) and before any subsequent core passage. The less massive infalling cluster, moving north, has passed the core of the main cluster and is driving forward a bow shock currently located at the north of the cluster (Figure 1). We predict a back shock propagating south. The two shocks are not expected to be symmetrical due to the different sizes of the merging clusters. Our simulations suggest that the northern subcluster is less massive, in agreement with Okabe et al. (2015), and differing from van Weeren et al. (2011). Who suggest that the northern subcluster is more massive based on the argument that it has larger relics.

The pre-shock gas that lies ahead of the bow shock (the northern shock; to the right in Figure 4) will be driven northward by the gas pressure of the infalling cluster, but...
jump conditions is given by
\[
\frac{T_2}{T_1} = \frac{5M^2 + 14M^2 - 3}{16M^2},
\]
where \( T_1 \) and \( T_2 \) are the pre- and post-shock temperatures, and \( M \) is the Mach number (e.g., Akamatsu et al. 2015; for a review, see Markevitch & Vikhlinin 2007). Using the temperature jump from our simulations at the shocks in Equation (3), we obtain Mach numbers for the northern and southern shocks (the bow shock and back shock): \( M_{n,\text{simu}} = 6.5 \) and \( M_{s,\text{simu}} = 7.8 \).

Based on our set of simulations, we find that, in general, the back shocks are stronger than the bow shocks in binary merging clusters.

Our simulations predict spatially smooth shock fronts for the bow shock and the back shock (the northern and southern shocks as shown in Figure 1). For CIZA J2242.8+5301, the morphology of the radio emission associated with the bow shock is smooth, and similar in curvature to our simulations. In the south, the angular location of our predicted shock front always lies at a somewhat larger radius than the radio relics. Interestingly, the observed X-ray shock front is also claimed to lie at somewhat larger radius than the southern radio relics, though the precision of this position could be improved with deeper X-ray data. We also note that the observed radio structures that lie close the predicted back shock do not form a continuous sharp arc, but appear bifurcated with a more irregular morphology. A possible reason for this imperfect correspondence in the radio might be that the backward shock is impeded by subsequently infalling gas associated with the same filament that was connected to the infalling northern cluster. The next wave of shocks then breaks up, producing patchy relics that lag behind the predicted location of our ideal simulations. However, this needs further investigations via numerical simulations, and it is beyond the scope of this study.

### 4.3. Bias in Mach Numbers Derived from X-Ray Observations

Some recent observations of merging clusters of galaxies show that the shock Mach numbers based on radio observations are about twice as large as those derived from X-ray observations (e.g., A2255: \( M_{\text{radio}} = 2.7 \), \( M_{\text{X-ray}} = 1.4 \), Akamatsu et al. 2017; RX J0603.3+4214 (the “Toothbrush” cluster): \( M_{\text{radio}} = 2.8 \), \( M_{\text{X-ray}} = 1.2 \), van Weeren et al. 2016; the northern shock in CIZA J2242.8+5301 (the “Sausage” cluster): \( M_{\text{radio}} = 4.6 \), \( M_{\text{X-ray}} = 2.7 \), van Weeren et al. 2010; Stroe et al. 2013; Akamatsu et al. 2015).

However, the physical gas temperature jump, which should be used in Equation (3), is not observed. Only the projected temperature, which is the LOS integrated temperature weighted by the emission measure and convolved with the response function of the X-ray detector, can be derived from X-ray observations directly (i.e., without assuming a model for the LOS distribution of the gas). The usual method is to assume a simple geometry of the shock and use that model to deproject the observed image and derive the temperature jump (e.g., Menanteau et al. 2012). Some corrections are used occasionally to deal with the fall of the temperature with distance from the cluster center of the undisturbed (pre-shocked) cluster gas (e.g., Akamatsu et al. 2015). In principle, the best way to deproject a merging cluster, and derive the Mach numbers for the shocks is to model the system using a full N-body/hydrodynamical code. We demonstrate the

![Figure 2](image-url). Models that do not match in detail CIZA J2242.8+5301. The color coding is the same as in the third panel in Figure 1. The two panels in the first row show simulations with the same initial parameters as our best model, but different impact parameters: \( P = 50 \), and 180 kpc (runs P050V25 and P180V25), the panels in the second and third row display images of runs with varying mass ratios, impact parameters and infall velocities (runs P100V25, P150V20, P150V30, and P200V18; see Table 1 for input parameters for our models). The two panels in the last row show the best model for CIZA J2242.8+5301 (run P120V25) 0.1 Gyr before and after the best epoch (0.4 Gyr after the first core passage) with green lines marking the shock locations.

Belongs to the main cluster. The pre-shocked gas behind the opposite shock (the southern shock) belongs to the infalling cluster and this back shock is moving faster relative to the pre-shocked gas than the bow shock, and has a higher temperature. The position of this southern shock is depicted by the solid line in Figure 4.

The Mach number derived from the temperature jump at the shock in X-ray observations based on the Rankine–Hugoniot
power of a full numerical simulation by comparing the Mach number we derived using the best model to that inferred from Suzaku observations of CIZA J2242.8+5301 (Akamatsu et al. 2015). The derivation of Mach numbers from Suzaku observations is difficult, because, in addition to the above mentioned problems with deprojection, it also has a low angular resolution of 1′.6. Also, the shocked gas can reach a temperature of 20–50 keV, but the effective area of Suzaku’s X-ray Imaging Spectrometer (XIS) cuts off at about ~8 keV. Note that this is also a problem for Chandra and XMM-Newton.

In Figure 3, we display temperature maps based on our best model for CIZA J2242.8+5301 (run P120V25). The left panel shows the physical temperature (2D cut through the main plane of the collision), the middle panel shows a temperature map taking into account projection effects (spectroscopic temperature of Mazzotta et al. 2004), and the right panel displays a mock Suzaku observation (spectroscopic temperature convolved with the point-spread function, PSF of Suzaku). From this figure, we can see that projection effects and a convolution with a low-resolution PSF soften the shocks and they seem to be closer to the center of the merging clusters. Our estimate for the position of the southern shock taking into account the effect of the low resolution of Suzaku is marked with a green line in Figure 1.

Figure 4 shows the temperature profile across the bow and the back shocks associated with the infalling and main cluster after the first core passage. Solid, dashed, and dashed-dotted lines represent the physical gas temperature, the predicted spectroscopic temperature, the Mach number of Mazzotta et al. 2004, and a mock Suzaku observation based on the spectroscopic temperature (the spectroscopic temperature convolved with the PSF of Suzaku). From this figure, we can see that, due to projection effects and the low angular resolution, the measured temperature jump at the shocks by Suzaku would be biased low by a factor of two (compare the solid and dashed-dotted temperature profiles).

Akamatsu et al. (2015), for example, measured a Mach number of $M_{n,A} = 2.7^{+1.1}_{-0.7}$ at the northern shock (bow shock) using Suzaku observations. Our N-body/hydrodynamical simulations suggest a correction of about $\Delta M \sim +2$ for Suzaku observations. With this correction, the Mach number for the northern shock in CIZA J2242.8+5301 would be $M_{n,A,\text{corrected}} \sim 4.7$. This corrected Mach number for the northern shock coincides with $M_{n,\text{radio}} = 4.6^{+1.3}_{-0.9}$, the value derived from radio observations by van Weeren et al. (2010).

However, averaging the projected temperature over a larger solid angle further lowers the measured shock temperature, and thus the Mach numbers for the bow (northern) and back (southern) shocks. Thus the true Mach number for the northern shock, $M_n$, will be even higher. Note that our best model suggests $M_{n,\text{simu}} = 6.5$ and $M_{n,\text{simu}} = 7.8$ (see Section 4.2).

We illustrate the bias in the Mach number due to measurement errors in the post-shock temperature in Figure 5. In this figure, using Equation (3), we show the Mach number as a function of post-shock temperature, $T_2$, holding the pre-shock temperature fixed at a value expected in the outer parts of a cluster, $T_1 = 2.5$ keV. This figure suggests that, in general, if $T_2$ is underestimated by a factor of two, the Mach number is going to be biased low by $\Delta M \sim 2$.

4.4. Bias in Flux Measurements of Radio Relics Due to SZ Contamination from Shocks

Radio relics, found in the outskirts of clusters of galaxies, are elongated synchrotron radio sources with a length of the order of 1 Mpc. The radio emission of relics is highly polarized and has a steep spectrum. The relativistic electrons emitting the radiation are assumed to be accelerated to high energies by shocks in merging clusters (for a recent review, see Feretti et al., 2012).

The main physical mechanism responsible for the particle acceleration at cluster merger shocks has not been identified yet. It is a subject of active research. The radio flux and the spectral index as a function of distance from the shock front

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**Figure 3.** Temperature maps in units of keV from our best model for CIZA J2242.8+5301 (run P120V25). The high temperature regions (red/yellow) on the north and south mark the merging shocks moving outward. Note that the collision axis is aligned vertically to match the shock geometry. Left panel: physical temperature, 2D cut through the main plane of the collision. Middle panel: spectroscopic temperature (Mazzotta et al. 2004). Right panel: mock Suzaku observation.

**Figure 4.** Temperature profiles across the bow shock (northern shock) and the back shock (southern shock) associated with the infalling and main cluster after the first core passage based on our best model (from Figure 3). Solid, dashed, and dashed-dotted lines represent the physical gas temperature, the spectroscopic temperature, and a mock Suzaku observation.
provides important constraints on particle acceleration models, because they are related to “spectral aging” of the relativistic electrons due to synchrotron and inverse Compton energy losses. Since in CIZA J2242.8+5301 the projected distance from the shock gives a good approximation to the physical distance (the collision is close to the plane of the sky), this cluster has been used to test different particle acceleration models, e.g., the DSA mechanism (Stroe et al. 2014a; Stroe et al. 2016) also analyzed 1RXS J0603.3+4214, the “Toothbrush” cluster; fossil electrons (accelerated by the DSA mechanism) reaccelerated as the shock runs over them (Kang & Ryu 2015); turbulent cosmic-ray reacceleration (Fujita et al. 2016); and magnetic turbulence generated by the amplification of the magnetic field (Donnert et al. 2016).

DSA acceleration predicts a synchrotron power-law spectrum. However, recent radio observations have found curved spectra of some relics, which is inconsistent with the prediction of a simple DSA model (e.g., 1RX J0603.3+4214, CIZA J2242.8+5301: Stroe et al. 2013, 2014b, 2016; A2256: Trasatti et al. 2015). Apart from other explanations (listed in the previous paragraph) it has been suggested that around the shock the thermal SZ effect may contaminate the radio flux measurements by lowering the measured flux from radio relics, and as a result, the radio spectrum becomes curved (e.g., Basu et al. 2016).

In Figure 6, we show the Compton-$y$ map of our best model for the merging cluster CIZA J2242.8+5301 (run P120V25). The infalling cluster passed the main cluster, it is moving north. The maximum of the Compton-$y$ parameter is around the core of the main cluster. Note that the collision axis is also aligned vertically, as in Figure 3.

Figure 7. Compton-$y$ profile from Figure 6 across the two shocks as a function of distance from the cluster center (in Mpc). The infalling cluster, moving to the right, passed the core of the main cluster, its center is located at about +0.6 Mpc. The center of the main cluster is at −0.5 Mpc. The sharp drops in the Compton-$y$ parameter associated with the bow shock (northern shock) at +1.25 Mpc (ahead of the infalling cluster moving to the right) and the back shock (southern shock) at −1.3 (behind the main cluster moving in the left).

In the Compton-$y$ parameter, following Basu et al. (2016), we estimate the decrease in the radio flux density due to the SZ effect at the shocks using

$$\frac{\Delta S}{\text{mJy/arcmin}^2} = \frac{1}{340} \frac{\Delta T_{\text{RJ}}}{\text{mK}} \left( \frac{\nu}{\text{GHz}} \right)^2,$$

where $\nu$ is the frequency in GHz, $\Delta T_{\text{RJ}} = -2yT_{\text{CMB}}$ is the thermal SZ decrement in Rayleigh–Jeans radiation temperature in mK, and the drop of the radio flux is in mJy/arcmin$^2$ (e.g., Birkinshaw 1999). We find that at the northern and southern shocks at 30 GHz, the radio flux drops $\Delta S = -0.072$ mJy/arcmin$^2$ and $\Delta S = -0.075$ mJy/arcmin$^2$.

5. Conclusions

We have performed a wide set of self-consistent $N$-body/hydrodynamical simulations (using FLASH) to seek a
representative solution for CIZA J2242.8+5301, and to help understand better the level of deprojection on the Mach numbers of shocks affecting X-ray observations of this and other similar clusters. We have modeled CIZA J2242.8+5301 as a binary merger, constraining the initial parameters using lensing, X-ray, and radio observations. The X-ray morphology and the locations of the two lensing centroids we find help constrain the impact parameter and the infall velocity. The positions of the outgoing shocks were constrained by X-ray and radio observations.

Our numerical simulations represent the first attempt to model CIZA J2242.8+5301 using self-consistent N-body/hydrodynamical simulations. We can appreciate from these simulations how tidal effects influence the gas distribution lying between the two clusters allowing us to identify suitable combinations of these initial parameters using the detailed X-ray morphology. Note that other models have not been able to benefit from this. For example, van Weeren et al. (2011) assumed, for simplicity, fixed non-interacting shapes for the gravitational potentials of the two merging clusters moving on pre-calculated paths. Most recently, after we submitted our paper, Donert et al. posted an article to the archive (Donnert et al. 2017) using N-body/hydrodynamical simulations to model CIZA J2242.8+5301.

We have demonstrated that low angular resolution X-ray telescopes (e.g., Suzaku) underestimate the shock temperature as much as a factor of two, and thus the resulting Mach number may be biased low by \( \Delta M \sim 2 \). Adding this correction to the Suzaku result for the Mach number of the northern shock, \( M_{n,A} \sim 2.7 \) (Akamatsu et al. 2015), we obtain \( M_{n,A,\text{corrected}} \sim 4.7 \), which coincides with the result from radio observations, \( M_{n,\text{radio}} = 4.6 \) (van Weeren et al. 2010).

We have suggested that the relics around the northern and southern shocks in CIZA J2242.8+5301 look different. The southern relics, in contrast to our simulations, are patchy and irregular in shape, perhaps because a filament of gas that follows the infalling northern cluster has impeded and disturbed the back shock on its way out toward the south.

The main mechanism for particle acceleration around shocks in merging galaxy clusters is an active subject of research. The shape of the radio spectrum from radio relics located around shocks provides constraints on particle acceleration models associated with shocks. However, the thermal SZ effect associated with the dense shocked gas may act to significantly lower the measured radio flux at shocks, as a function of frequency. We have simulated Compton-\( y \) maps based on our best model for CIZA J2242.8+5301, and shown that the drops in the Compton-\( y \) parameter due to the merging shocks may reach more than one order of magnitude. Our model indicates that at the northern and southern shocks at 30 GHz, the radio flux drops \( -0.072 \, \text{mJy arcmin}^{-2} \) and \( -0.075 \, \text{mJy arcmin}^{-2} \). This sharp discontinuity, however, is smoothed out by the PSF of the observing radio telescopes. In agreement with Basu et al. (2016), we find that merging shocks may considerably reduce the measured flux of radio relics depending mainly on the masses and relative velocities of the merging clusters. However, large-scale diffuse SZ signals are resolved out by interferometers, therefore only the sharp edges are important. These sudden drops can lead to ripples in the spatial domain rather than a simple flux density offset.

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