Noncommutative Quantum Mechanics based on Representations of Exotic Galilei Group

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Abstract. Using elements of symmetry, we constructed the Noncommutative Schrödinger Equation from a representation of Exotic Galilei Group. As a consequence, we derive the Ehrenfest theorem using noncommutative coordinates. We also have showed others features of quantum mechanics in such a manifold. As an important result, we find out that a linear potential in the noncommutative Schrödinger equation is completely analogous to the ordinary case. We also worked with harmonic and anharmonic oscillators, giving corrections in the energy for each one.

1. Introduction

The concept of continuous group was introduced by the Norwegian mathematician Sofus Lie [1] while working with differential equations in second half of nineteenth century. Then it has gained great visibility in 1928 with the work of H. Weyl [2], who linked group theory to atomic spectroscopy. Thus it was established, for the first time, a direct relation between quantum theory and group theory. The importance of such a subject increased enormously when Wigner applied the theory of representations of Lie groups to physical systems. Particularly fundamental particles can be described by means of irreducible representations of kinematical groups, such as De Sitter, Galilei and Poincaré groups. Hence fundamental interactions are implemented by gauge symmetries. Later, in order to find an appropriate Hamiltonian structure for atomic systems, P. Dirac also used the theory of representation of Lie groups to investigate such systems [3]. Therefore group theory, which is an important branch of mathematics, became an indispensable tool to understand and formalize symmetries in physics.

The notion of non-commutativity began in physics with the uncertain principle which is a well known relation between coordinates and momenta operators. In a letter addressed to Peierls in 1930, Heisenberg considered that this very feature could be valid also between coordinates operators, in such a way that it could precludes the well known divergences of the self-energy terms calculated for particles. Since then the idea of noncommutative spatial coordinates propagated throughout the scientific community until it reached H. Snyder, a former Ph.D.
student of Oppenheimer [4, 5, 6]. Thus, at least formally, Snyder was the first one who stated that spatial coordinates would not commutate with each other at small distances, in his work published in 1947 [7, 8]. He proposed a whole new paradigm in which the spacetime should be understood as a collection of tiny cells of minimum size, where there is no such a concept as the idea of a point. Thus, once the minimum size is reached, in the realm of some high energy phenomenon, the position should be given by the noncommutative coordinate operators. As a direct consequence it would be impossible to precisely measure a particle position.

After Snyder’s pioneering, those ideas remained forgotten, mainly because the success achieved by the so called renormalization procedure, which has led to an entire new branch of research in physics, on dealing with ultra-violet divergences of scattering integrals in the context of quantum field theory. Then in the last years the interest of the scientific community on noncommutative geometry has increased due to works on non-abelian theories [9], gravitation [10, 11, 12], standard model [13, 14, 15] and on quantum Hall effect [16]. More recently the discovery that the dynamics of an open string can be associated with noncommutative spaces has contributed to the last revival of noncommutative theories [17].

From the mathematical point of view, the simplest algebra obeyed by the coordinates operators $\hat{x}^\mu$ is

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta_{\mu\nu},$$

where $\theta^{\mu\nu}$ is a skew-symmetric constant tensor. It worths to point out that the mean values of the position operators do correspond to the actual position observed, thus it is said that such operators are hermitian ones. It is well known in quantum mechanics that a noncommutative relation between two operators lead to a specific uncertain relation, hence the above expression yields

$$\Delta\hat{x}^\mu\Delta\hat{x}^\nu \geq \frac{1}{2}\vert\theta^{\mu\nu}\vert.$$

Following the ideas introduced by Snyder, it possible to associate the minimum size with a distance of the $\sqrt{\vert\theta^{\mu\nu}\vert}$ order of magnitude. Thus the noncommutative effects turn out to be relevant at such scales. Usually the non-commutativity is introduced by means of the Moyal product defined as [7]

$$f(x) \star g(x) = \exp\left(\frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu}\right) f(x)g(y)|_{y \to x}$$

$$= f(x)g(x) + \frac{i}{2} \theta^{\mu\nu} \partial_{\mu} f \partial_{\nu} g + \frac{1}{2!} \left(\frac{i}{2}\right) \theta^{\mu\nu_1\nu_2} (\partial_{\mu_1} \partial_{\nu_2} f)(\partial_{\nu_1} \partial_{\nu_2} g) + \cdots$$

with a constant $\theta^{\mu\nu}$. Then the usual product is replaced by the Moyal product in the classical lagrangian density.

In this work we study the representation of Galilei group in noncommutative space. Particularly, we analyse the exotic structure of Galilei algebra, that appears in special case of 2+1 dimensions [18, 19]. This exotic structure shows us some physical properties which are not shared by another dimensions, for instance, it allows a natural association between noncommutative coordinates and the second central extension of the algebra. In this sense, the exotic extension of Galilei algebra is responsible for the emergence of noncommutative coordinates [20, 21].

The paper is organized as follows. In Section 2 we construct the noncommutative Galilei algebra in 2+1 dimensions and write the Schödinger equation in noncommutative plane. As a result, we derive the Ehrenfest Theorem in noncommutative coordinates. In section 3 we study the linear potential in noncommutative plane. Then in section 3 we work with the harmonic oscillator in the noncommutative plane. In section 4 we present the anharmonic oscillator using noncommutative coordinates in two dimensions. We obtain a correction in the energy using perturbation theory. Finally in the last section we present our concluding remarks.
2. Noncommutative Galilei-Lie Algebra

In this section, we study the noncommutative Galilei algebra in $2+1$ dimension. For this proposal, we construct the operators

\[
\hat{x}_i = x_i - \frac{i}{2} \theta_{ij} \frac{\partial}{\partial x_j},
\]

\[
\hat{p}_i = i\hbar \frac{\partial}{\partial x_i},
\]

\[
\hat{K}_i = m \hat{x}_i - \hbar \hat{p}_i,
\]

\[
\hat{L} = \hat{x}_i \hat{p}_j - \hat{x}_j \hat{p}_i,
\]

and

\[
\hat{H} = \frac{1}{2m} \sum_{i=1}^{2} \hat{p}_i^2.
\]

From this set of unitary operators we obtain, after some long but simple calculations, the following set of commutation relations,

\[
[\hat{p}_i, \hat{p}_j] = 0,
\]

\[
[\hat{K}_i, \hat{p}_j] = i\hbar m \delta_{ij},
\]

\[
[\hat{L}, \hat{p}_i] = i\hbar \epsilon_{ij} \hat{p}_j,
\]

\[
[\hat{K}_i, \hat{K}_j] = im^2 \theta_{ij},
\]

\[
[\hat{p}_i, \hat{H}] = 0,
\]

\[
[\hat{L}, \hat{H}] = 0.
\]

This is the extended exotic Galilei algebra, where $\hat{L}_i$ stand for rotations, $\hat{p}_i$ for translations and $\hat{K}_i$ for boosts. The central extension is given by $[\hat{K}_i, \hat{p}_j] = i\hbar m \delta_{ij}$. However, in the plane, the Galilei group admits a second extension, highlighted by the non-commutativity of boost generators. This extension was studied first by Levy-Leblond [18], and this algebra is called the Exotic Galilei Group. In general, the second extension of Galilei algebra is associated to noncommutative coordinates. Here, we show that noncommutativity between coordinates leads to exotic extension. To obtain the physical content, we first notice that $\hat{p}$ transform under the boost as the physical momentum, i.e.,

\[
\exp \left( -i \hat{v} \cdot \frac{\hat{K}}{\hbar} \right) \hat{p}_j \exp \left( i \hat{v} \cdot \frac{\hat{K}}{\hbar} \right) = \hat{p}_j + m v_j \mathbf{1}.
\]

Furthermore, the operators $\hat{x}$ and $\hat{p}$ do not commute with each other, that is,

\[
[\hat{x}_i, \hat{p}_j] = i\hbar \delta_{ij}.
\]

Therefore, $\hat{x}$ and $\hat{p}$ can be taken to be the physical observables of position and momentum. To be consistent, generators $\hat{L}$ are interpreted as the angular momentum operator, and $\hat{H}$ is taken as the Hamiltonian operator. The Casimir invariants of the Lie algebra are given by

\[
I_1 = \hat{H} - \frac{\hat{p}^2}{2m} \quad \text{and} \quad I_2 = \hat{L} - \frac{1}{m} \hat{K} \times \hat{p},
\]
where $I_1$ describes the Hamiltonian of a free particle and $I_2$ is associated with the spin degrees of freedom. First, we study the scalar representation; i.e. spin zero. Particularly, $\hat{H}$ can be identify as generator of temporal translation. In this way, the time evolution of the system state $\psi(x, t) = \langle x | \psi(t) \rangle$ is given by

$$\psi(x, t) = e^{\frac{-i}{\hbar} \hat{H} t} \psi(x, 0).$$

(6)

Differentiating Eq.(6) with respect to $t$, we obtain

$$i\hbar \partial_t \psi(x, t) = \frac{\nabla^2 \psi(x, t)}{2m} + V(x_i - \frac{i}{2} \theta_{ij} \partial_{x_j}) \psi(x, t),$$

(7)

which is the noncommutative Schrödinger equation. Note that Eq.(7) was obtained here from representation theory of symmetry group.

As a important result, we derive the Ehrenfest theorem for the noncommutative coordinates. For this proposal, we take the expansion

$$V\left(x_k - \frac{1}{2\hbar} \theta_{kl} \hat{p}_l\right) = V(x_k) - \frac{1}{2\hbar} \theta_{kl} \hat{p}_l \frac{\partial V}{\partial x_k} + o(\theta^2),$$

and to use in the equation $\frac{d}{dt} \langle A \rangle = \frac{i}{\hbar} \langle [H, A] \rangle + \langle \frac{\partial A}{\partial t} \rangle$, we obtain

$$\frac{d}{dt} \langle x_k \rangle = \left\langle \frac{\partial V}{\partial x_k} \right\rangle - \frac{\theta_{kl}}{2\hbar} \left\langle \frac{\partial^2 V}{\partial x_l \partial x_j} \right\rangle,$$

(8)

$$\frac{d}{dt} \langle p_k \rangle = \left\langle -\frac{\partial V}{\partial x_k} \right\rangle + \frac{\theta_{jl}}{2\hbar} \left\langle \frac{\partial^2 V}{\partial x_l \partial x_j} \right\rangle.$$

(9)

Observe that if we take $\theta = 0$, we obtain the same result for commutating coordinates. In sequence, we prove an important theorem valid for linear potential in noncommutative coordinates.

3. Linear Potential

In this section we analyse the linear potential in noncommutative coordinates. The problem of linear potential is applied for example in quantum chromodynamics for studies of gluon condensates (H. G. Dosch). Our focus is in the two dimensional case $V(x, y) = \alpha x + \beta y$, where $\alpha$ and $\beta$ are complex constants. In this sense, the noncommutative Schrödinger equation with linear interaction can be written by ($\hbar = m = 1$)

$$\frac{\partial^2 \psi(x, y)}{\partial x^2} + \frac{\partial^2 \psi(x, y)}{\partial y^2} + \alpha x \psi(x, y) + \beta y \psi(x, y) - \frac{i\alpha}{2} \frac{\theta \partial \psi(x, y)}{\partial y} + \frac{i\beta}{2} \frac{\partial^2 \psi(x, y)}{\partial x} = E \psi(x, y).$$

(10)

Observe that we utilize the anti-symmetric tensor $\theta$ as

$$\theta_{ij} = \begin{bmatrix} 0 & \theta \\ -\theta & 0 \end{bmatrix}.$$

To solve Eq. (10) firstly we note that its imaginary part yields the equation

$$\beta \frac{\partial \psi(x, y)}{\partial x} - \alpha \frac{\partial \psi(x, y)}{\partial y} = 0,$$
Performing a change of variables we would like to analyze the isotropic case which is settled by the condition \( \omega = \omega_x = \omega_y = \omega \). Thus we obtain

\[
(\alpha^2 + \beta^2) \frac{\partial^2 \psi(z)}{\partial z^2} + z \psi(z) - E \psi(z) = 0. \tag{11}
\]

Observe that Eq.(11) is of the general form

\[
\frac{d^2 u}{dx^2} - xu = 0,
\]

that is the Airy’s differential equation. So, the solutions of Eq.(11) are given by

\[
\psi(x, y) = \frac{2^{2/3}}{2\pi} Ai(ax + by + E), \tag{12}
\]

where \( Ai(w) \) is the Airy function. Observe that the solution does not depend on the noncommutative parameter \( \theta \), which shows that the Linear Potential does not have noncommutative corrections.

### 4. Two Dimensional Noncommutative Isotropic Harmonic Oscillator

In this section we will deal with the two dimensional isotropic harmonic oscillator in noncommutative coordinates. The Hamiltonian for just for the 2D harmonic oscillator is given by

\[
\hat{H} = \frac{1}{2m} \hat{p}_x^2 + \frac{1}{2m} \hat{p}_y^2 + \frac{1}{2} m (\omega_x^2 x^2 + \omega_y^2 y^2), \tag{13}
\]

If we take \( m = \hbar = 1 \) then the Hamiltonian reads

\[
\hat{H} = \frac{1}{2m} \hat{p}_x^2 + \frac{1}{2m} \hat{p}_y^2 + \frac{1}{2} \omega_x^2 (x - \frac{i}{2} \theta_{12} \partial_y - \frac{i}{2} \theta_{21} \partial_x)^2 + \frac{1}{2} \omega_y^2 (y - \frac{i}{2} \partial_{21} \partial_x)^2. \tag{14}
\]

Once we rearrange the terms it yields

\[
\hat{H} = \frac{1}{2} \left[ \frac{\theta_{12}^2 \omega_y^2}{4} \right] \hat{\partial}_x^2 + \left( 1 + \frac{\theta_{12}^2 \omega_x^2}{4} \right) \hat{\partial}_y^2 + \frac{1}{2} (\omega_x^2 x^2 + \omega_y^2 y^2) + \frac{i}{2} \theta_{12} (\omega_y^2 \partial_y - \omega_x^2 \partial_x). \tag{15}
\]

Thus we would like to analyze the isotropic case which is settled by the condition \( \omega_x = \omega_y = \omega \). Performing a change of variables \( z = \frac{1}{2} (\omega_x^2 x^2 + \omega_y^2 y^2) \) which implies, as in the previous section, that we are seeking for real solutions, we obtain

\[
\hat{H} \psi = -\frac{1}{2} \left( 1 + \frac{\theta_{12}^2 \omega^2}{4} \right) \left( 2 \omega^2 \frac{d^2 \psi}{dz^2} + 2z \omega^2 \frac{d^2 \psi}{dz^2} \right) + z \psi, \tag{16}
\]

we recall that \( \theta_{12} = -\theta_{21} = \theta \). Therefore the Schrödinger equation reads

\[
-\beta \left( \frac{d^2 \psi}{dz^2} + \frac{d \psi}{dz} \right) + z \psi = E \psi, \tag{17}
\]
where $\beta = \omega^2 \left( 1 + \frac{2\epsilon^2}{4} \right)$. If we seek by solutions of the form $\psi = e^{-\xi/\omega}\phi$, where $\xi = \beta z$, then we find

$$\xi \frac{d^2 \phi}{d\xi^2} + \left( 1 - \frac{2\xi}{\omega} \right) \frac{d\phi}{d\xi} + \frac{1}{\omega} \left( -1 + \frac{\epsilon}{\omega} \right) \phi = 0,$$

where $\epsilon = E\omega/\sqrt{\beta}$. Performing another change of variables $\zeta = 2\frac{\xi}{\omega}$ in order to obtain a non-dimensional equation, we get

$$\zeta \frac{d^2 \phi}{d\zeta^2} + \left( 1 - \zeta \right) \frac{d\phi}{d\zeta} + \frac{1}{2} \left( -1 + \frac{\epsilon}{\omega} \right) \phi = 0,$$

the above equation is known as Kummer equation and its solution is the confluent hypergeometric function. If we look for polynomial solutions, then we get

$$\psi(z) = c \exp(-z/\beta^{1/2}) F(-n; 1; 2z/\beta^{1/2}),$$

where $c$ is a normalization constant, $n$ is an integer and

$$\epsilon = \omega(2n + 1).$$

Using the properties of the confluent hypergeometric functions, it is possible to write the solution as

$$\psi(x, y) = \frac{\omega}{\beta^{1/4} 2^{(n_1 + n_2)/2} \sqrt{n_1! n_2!}} \exp \left[ -\frac{\omega^{2}(x^2 + y^2)}{2\beta^{1/2}} \right] H_{n_1} \left( \frac{\omega x}{\beta^{1/4}} \right) H_{n_2} \left( \frac{\omega y}{\beta^{1/4}} \right),$$

where $n = \frac{(n_1 + n_2)}{2}$ and $H_n(x)$ are the Hermite polynomials. Thus the energy is given by

$$E_{n_1, n_2} = \left( 1 + \frac{\theta^2 \omega^2}{4} \right)^{1/2} \omega (n_1 + n_2 + 1),$$

we can see that the correction due to the non-commutativity in the coordinates is given by the first factor of the above expression.

5. Two Dimensional Noncommutative Anharmonic Oscillator

In this section we will analyze the anharmonic oscillator in two dimensions under, taking into account the feature of non-commutativity between coordinates. This system consists of small deviations from the normal oscillator. The Hamiltonian that describes it just reads

$$\hat{H} = \frac{1}{2m} \hat{p}_x + \frac{1}{2m} \hat{p}_y + \frac{1}{2} m (\alpha x^2 y^2 + \alpha y^2 x^2) + \alpha (\hat{x}^3 + \hat{y}^3) + \gamma (\hat{x}^4 + \hat{y}^4),$$

the quantities $\alpha$ and $\gamma$ are coupling constants, they are supposed to be very small, in such a way that it is possible to apply perturbation theory to calculate the corrections in the energy. Thus given a Hamiltonian of the form $\hat{H} = \hat{H}_0 + \Delta \hat{H}$ where $\hat{H}_0 \Psi_0 = E_0 \Psi_0$, it is possible to evaluate the correction by the expression $\Delta E = \langle \Psi_0 | \Delta \hat{H} | \Psi_0 \rangle$. If we take $m = \hbar = 1$, then we get

$$\hat{H}_0 = \frac{1}{2} \hat{p}_x + \frac{1}{2} \hat{p}_y + \frac{1}{2} \omega_x^2 (x - i \frac{\theta_{12}}{2} \partial_y)^2 + \frac{1}{2} \omega_y^2 (y - i \frac{\theta_{21}}{2} \partial_x)^2$$

and

$$\Delta \hat{H} = \alpha \left[ (x - i \frac{\theta_{12}}{2} \partial_y)^3 + (y - i \frac{\theta_{21}}{2} \partial_x)^3 \right] + \gamma \left[ (x - i \frac{\theta_{12}}{2} \partial_y)^4 + (y - i \frac{\theta_{21}}{2} \partial_x)^4 \right].$$
From last section we identify \( E_0 \) as \( E_{n_1n_2} = \sqrt{b}(n_1 + n_2 + 1) \) and \( \Psi_0 \) as the function into expression (21).

In order to calculate the mean values of the quantities appearing in the Hamiltonian, it is necessary to present some integrals. Thus using the following relations

\[
\int_{-\infty}^{\infty} e^{-x^2}H_n^2(x)dx = 2^n n! \sqrt{\pi},
\]

\[
\int_{-\infty}^{\infty} x e^{-x^2}H_n(x)H_m(x)dx = \sqrt{\pi}2^n 2n! \left[ \frac{1}{2} \delta_{n-1,m} + (n+1) \delta_{n+1,m} \right],
\]

\[
\int_{-\infty}^{\infty} x^2 e^{-x^2}H_n(x)H_m(x)dx = \sqrt{\pi}2^n n! \left[ n + \frac{1}{2} \delta_{n,m} + (n+2)(n+1) \delta_{n+2,m} + \frac{1}{4} \delta_{n-2,m} \right],
\]

we get

\[
\int_{-\infty}^{\infty} x^3 e^{-x^2}H_n^3(x)dx = 0,
\]

\[
\int_{-\infty}^{\infty} x^3 e^{-x^2}H_n(x)H_{n-1}(x)dx = 3 \sqrt{\pi} 2^{n-2} n^2 (n-1)!,
\]

\[
\int_{-\infty}^{\infty} x^4 e^{-x^2}H_n^2(x)dx = 3 \sqrt{\pi} 2^{n-2}(2n+2n+1) n!,
\]

\[
\int_{-\infty}^{\infty} e^{-x^2/2}H_n(x) \left( \frac{\partial^2}{\partial x^2} \right) e^{-x^2/2}H_n(x)dx = \sqrt{\pi}2^n \left( n - \frac{1}{2} \right) n!,
\]

\[
\int_{-\infty}^{\infty} e^{-x^2/2}H_n(x) \left( \frac{\partial^4}{\partial x^4} \right) e^{-x^2/2}H_n(x)dx = \frac{3}{2} \sqrt{\pi}2^n \left( 3n^2 - 7n + \frac{1}{2} \right) n!.
\]

Hence we have

\[
\langle x^4 \rangle = \frac{3\beta}{2\omega^{7/2}} \left( n_1^2 + n_1 + \frac{1}{2} \right),
\]

\[
\langle y^4 \rangle = \frac{3\beta}{2\omega^{7/2}} \left( n_2^2 + n_2 + \frac{1}{2} \right),
\]

\[
\langle x^2 \frac{\partial^2}{\partial y^2} \rangle = \left( n_1 + \frac{1}{2} \right) \left( n_2 - \frac{1}{2} \right),
\]

\[
\langle y^2 \frac{\partial^2}{\partial x^2} \rangle = \left( n_1 - \frac{1}{2} \right) \left( n_2 + \frac{1}{2} \right),
\]

\[
\langle \frac{\partial^4}{\partial x^4} \rangle = \frac{3\omega^4}{2\beta} \left( 3n_1^2 - 7n_2 + \frac{1}{2} \right),
\]

\[
\langle \frac{\partial^4}{\partial y^4} \rangle = \frac{3\omega^4}{2\beta} \left( 3n_2^2 - 7n_1 + \frac{1}{2} \right).
\]

Therefore the above expressions lead to the following results

\[
\langle x^4 \rangle = 0,
\]

\[
\langle y^4 \rangle = 0,
\]

\[
\langle x^4 \rangle = \frac{3\beta}{2\omega^{7/2}} \left( n_1^2 + n_1 + \frac{1}{2} \right) - \frac{3\theta^2}{2} \left( n_1 + \frac{1}{2} \right) \left( n_2 - \frac{1}{2} \right) + \frac{3\theta^4 a^4}{32\beta} \left( 3n_2^2 - 7n_2 + \frac{1}{2} \right),
\]

\[
\langle y^4 \rangle = \frac{3\beta}{2\omega^{7/2}} \left( n_2^2 + n_2 + \frac{1}{2} \right) - \frac{3\theta^2}{2} \left( n_1 - \frac{1}{2} \right) \left( n_2 + \frac{1}{2} \right) + \frac{3\theta^4 a^4}{32\beta} \left( 3n_1^2 - 7n_1 + \frac{1}{2} \right).
\]
Then the first order correction of the energy is given by

\[
\langle \Delta E \rangle = \gamma \left\{ \frac{3\beta}{2\omega^{7/2}} \left( n_1^2 + n_2^2 + n_1 + n_2 + 1 \right) - \frac{3\theta^2}{2} \left[ \left( n_1 + \frac{1}{2} \right) \left( n_2 - \frac{1}{2} \right) + \left( n_1 - \frac{1}{2} \right) \left( n_2 + \frac{1}{2} \right) + \frac{3\theta^4\omega^4}{32\beta} \left[ 3 \left( n_2^2 + n_1^2 \right) - 7 \left( n_2 + n_1 \right) + 1 \right] \right] \right\}.
\] 

(23)

We stress out that the corrected energy has, itself, correction in terms of the noncommutative parameter \( \theta \). This means that we recover what is known for the anharmonic oscillator plus a new behavior due to the non-commutativity between coordinates.

6. Concluding Remarks

In this paper we have dealt with the noncommutative Galilei group which is also known as Exotic Galilei group. Such a group only has consistency when we work with two spatial dimensions and one temporal dimension (2+1). In this framework we obtain the noncommutative Schrödinger equation from a group structure rather than the well known procedure consisting on to change the normal product by the Moyal product into the Lagrangian. Then as applications we analyze the noncommutative Schrödinger equation or the case of a linear potential, harmonic oscillator and anharmonic oscillator. We find no corrections for the first case. On the other hand we give corrections for the energy in terms of the noncommutative parameter \( \theta \) for both oscillators (harmonic and anharmonic). We point out that all corrections obtained in this paper goes to the known quantities in the limit \( \theta \to 0 \).
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