Fractal Dimension Fitting of Virtual Material and Experimental Verification

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Abstract: Virtual material is a dynamic model of mechanical contact surface, which is suitable for finite element simulation. This method can characterize the stiffness and damping of the contact surface well by using fractal geometry theory to describe the contour of the contact surface. Based on standard W-M function, we verify the accuracy of the fractal dimension calculated by wavelet transform, and then fit the fractal dimension of machined surfaces with different roughness. Using the fractal dimension, we calculate the virtual material parameters to establish the finite element model of the unit specimen. Finally, we carry out the experimental modal analysis to examine reproduce the virtual material method.

1. Introduction
The mechanical structure is generally assembled by multiple parts, so the contact surfaces widely exist in it. When modeling and simulating the dynamics of the mechanical structure, the error of the contact surface model is the main source of the error of the mechanical structure model \cite{1}, therefore establishing a joint model suitable for finite element simulation calculation has become the research hotspot. The finite element model of the joint based on the equivalent virtual material is convenient for computer simulation and has high precision, so it has been widely used in finite element modal analysis in recent years.

The virtual material model, just as the name implies, means adding a thin layer entity in the simulation, which does not exist in the physical contact surface. By setting the material parameters of the thin layer entity, the stiffness and damping of the contact surface can be effectively characterized. In addition to the material parameters of the paired parts and the bolt preload, some contact surface parameters are also required. In order to describe precisely the profile of the contact surface, surface roughness alone is obviously not enough. Many scholars have tried to apply fractal geometry theory to describe the complexity of the profile of the contact surface through fractal dimension \cite{2}.

However, the fractal dimension is not a measured physical quantity, but fitted by the measurable surface profile parameters \cite{3}. In order to better characterize the stiffness and damping of the contact surface and improve the accuracy of the virtual material model, it is necessary to further study the fitting of fractal dimension, and to carry out experimental verification.

2. Fitting of Fractal Dimension
The Weierstrass function is continuous but non-derivative, and has obvious details in arbitrary intervals. The contour of a rough surface is such a self-affine curve. This function is also used to describe the earliest fractal phenomenon. It can be expressed as
\[ Z(x) = G^{(D-1)} \sum_{n=1}^{\infty} y^{(D-2)} n \cos(2\pi y^2 x) \]  

where \( D \) is the fractal dimension; \( G \) is the characteristic scale coefficient.

### 2.1. Fitting methods of fractal dimension

We use the power spectral density method and the wavelet transform method to fit the fractal dimension of the W-M function respectively, and verify the fitting error of the two methods in contrast to the theoretical fractal dimension of the W-M function.

For the profile curve, its spatial frequency components and corresponding amplitude can be calculated from its power spectral density. Therefore, we can obtain the power spectral density function of W-M function, given by

\[ P(\omega) = \frac{g^2(D-1)}{2\ln y} \omega^{2D-5} \]  

In dual logarithmic coordinate system, the power spectral density obeys a linear increase behavior as a function of the frequency \((\log \omega - \log P)\), which slope \( k_1 \) is related to the fractal dimension \( D \), shown as

\[ D = 2.5 + 0.5k_1 \]  

Wavelet transform is an important tool for signal processing. By separating signals of different frequencies through bandpass filters, the fractal curve can be decomposed into a set of wavelet functions. In this paper, we choose Daubechies wavelet as basis function. The discrete wavelet transform decomposes the signal into low-pass and high-pass sub-bands, and the results of the low-pass sub-bands continue to be subjected to discrete wavelet transform [4].

Take the average \( \bar{d}_m \) of the detail coefficients \( d^m \) at all levels, and this average increase linearly with the series in semi-logarithmic coordinate system \((m - \log_2 d_m)\). Its slope \( k_2 \) is also related to the fractal dimension and satisfy

\[ D = 2.5 + k_2 \]  

We calculate the fractal dimension of the standard W-M function through the above two methods, the results are shown in the table.

| Theoretical Value of Fractal Dimension | Calculated Value of Power Spectral | Relative Error | Calculated Value of Wavelet Transform | Relative Error |
|----------------------------------------|------------------------------------|----------------|--------------------------------------|---------------|
| 1.2                                    | 1.590                              | 32.5%          | 1.21                                 | 0.8%          |
| 1.4                                    | 1.596                              | 14%            | 1.37                                 | -2.1%         |
| 1.5                                    | 1.598                              | 6.7%           | 1.45                                 | -3.3%         |
| 1.6                                    | 1.602                              | 0.1%           | 1.53                                 | -4.4%         |
| 1.8                                    | 1.610                              | -10.6%         | 1.68                                 | -6.7%         |

It can be seen that, for the power spectral method, the fitting results increase with the increase of the theoretical fractal dimension of the W-M function, but the fitting error of power spectral is larger when the fractal dimension is not close to 1.6. Correspondingly, the accuracy of the wavelet transform method is much higher than that of the power spectral density method.

### 2.2. Fitting of fractal dimension of machined surface

In order to calculate the fractal dimension of the surface profile with different roughness, we machine three specimens, whose surface roughness are Ra1.6, Ra3.2 and Ra6.3 respectively. We use the Olympus LEXT OLS5100 laser scanning confocal microscope to measure the surface profile. The sampling length is 3 mm, the number of sampling points is 4800, and the sampling interval is 0.625 μm. The measurement process is shown in the figure 1.
Figure 1 Surface Profile Measurement

The scan results is shown in figure 2.

Figure 2 Surface Profile Measurement Results

On the one hand, we calculate the power spectral density function of the measured data, and perform the first-order polynomial fitting of the least square method on the power spectral density function in dual logarithmic coordinates. On the other hand, we perform multi-level discrete wavelet transform on the measured data, and fit the first-order polynomial of least squares on the one-norm of the detail coefficients.

Figure 3 Fractal Dimension Fitting

We calculate the fractal dimension of each roughness surface, and the fitting results are shown in Table 2.

| Roughness | Calculated Value of Power Spectral | Calculated Value of Wavelet Transform |
|-----------|-----------------------------------|--------------------------------------|
| Ra1.6     | 1.28                              | 1.26                                 |
| Ra3.2     | 1.44                              | 1.54                                 |
| Ra6.3     | 1.52                              | 1.87                                 |

As an important parameter for the surface profile, and fractal dimension and its fitting result can be used in the subsequent dynamics simulation. And this result also verifies the conclusion of the existing research [4], that is, as the surface roughness increases, the surface profile becomes more complex, and the calculated fractal dimension also increases significantly.
3. Modal Analysis of Unit Specimen

The two parts of the unit specimen are made by different metal materials respectively, 45 steel and HT250 cast iron, which can explore the accuracy of the virtual material model of the joint of different materials.

3.1. Finite Element Simulation of Element Specimen

Import the 3D model into ANASYS Workbench. The 3D model consists of two specimens and a virtual material layer in the middle. The bolts and the holes on the specimen can be simplified and deleted. Each contact surface is bonded, and modal analysis is performed to calculate the natural frequencies and modal shapes.

![Finite Element Simulation Geometry](image)

The material parameters of 45 and HT250 cast iron are shown in the table.

| Parameter     | HT250 | 45 Steel |
|---------------|-------|----------|
| Density/\(kg \cdot m^{-3}\) | 7340  | 7833     |
| Elastic Modulus/\(GPa\)     | 174   | 206      |
| Poisson's Ratio            | 0.27  | 0.3      |

The surface roughness of the specimen is Ra1.6, and the pre-tightening force of the two bolts is 30 Nm, that means, the normal load of the joint surface is 25000 N. According to the surface material parameters of the specimen and the contact surface parameters, we can calculate the virtual material parameters from the normal and tangential contact stiffness model of the joint [5].

![Finite Element Simulation Geometry](image)

| Parameter     | Value     |
|---------------|-----------|
| Normal Elastic Modulus/\(GPa\) | 36.56     |
| Tangential Elastic Modulus/\(GPa\) | 12.41     |
| Shear Modulus/\(GPa\) | 3.85      |
| Density/\(kg \cdot m^{-3}\) | 1170.84   |

The finite element calculation obtains the natural frequencies of the first six modes, which are 491.60 Hz, 716.38 Hz, 997.20 Hz, 2140.8 Hz, 2282.4 Hz, 2766.7 Hz. The results provide a reference for the details of experiment modal analysis.

3.2. Experiment Modal Analysis of Element Specimen

In the experiment, we use the equipment of China Orient Institute of Noise and Vibration: the INV9312 test force hammer for excitation, the INV9822 acceleration sensor for collecting response signals, and DASP-V11 software for performing data processing for modal analysis.

Considering the support of the specimen, the simulation results show that the first-order natural frequency of the specimen is about 500 Hz. In this paper, we use sponge for support, and its support
stiffness is much smaller than that of the specimen.

Considering the material of the force hammer head, the simulation results show that the sixth-order natural frequency of the specimen is close to 3000 Hz, and the material of the specimen itself is also steel. So it is necessary to use a force hammer with a steel hammer head for excitation [6].

The sampling frequency of the excitation signal is 51.2 kHz. Using the variable time base function, we set the sampling frequency of the hammer 4 times higher than that of the acceleration sensor. The sampling frequency of the acceleration sensor is 12.8 kHz.

In this experiment, we select 68 measuring points to build the basic outline of the model, and adopt the multi-input and multi-output method: the positions of the three unidirectional acceleration sensors are fixed, and the force hammer is used to hit all the measuring points respectively. The positions of unidirectional acceleration sensors are shown in figure 3.

The natural frequencies of the test results are shown in Table 03.

| Mode | Natural Frequency/Hz | Damping Ratio | Mode Shape |
|------|----------------------|---------------|------------|
| 1    | 504.38               | 1.157%        | Rotate around the z-axis |
| 2    | 659.90               | 0.477%        | Rotate around the y-axis |
| 3    | 927.65               | 0.4605%       | Rotate around the x-axis |
| 4    | 2233.43              | 0.435%        | Translate along the y-axis |
| 5    | 2360.89              | 0.650%        | Translate along the x-axis |
| 6    | 2767.24              | 0.616%        | Translate along the z-axis |

The modal assurance criterion (MAC) evaluates the correlation between two modal vectors. The MAC value usually represent the quality of the experimental results. The lower MAC value means the irrelevant between two vectors. It can be seen from Figure 6 that the MAC values on the diagonal are all 1, and the values on the off diagonal are all less than 0.2. This result indicates the modes are independent of each other and the experimental results are reliable.

The comparison between the simulation and the experiment are shown in the table.

| Mode | Experiment/Hz | Simulation/Hz | Relative Error |
|------|---------------|---------------|----------------|
| 1    | 504.38        | 491.60        | -2.53%         |
| 2    | 659.90        | 716.38        | 8.56%          |
| 3    | 927.65        | 997.20        | 7.50%          |
| 4    | 2233.43       | 2140.8        | -4.15%         |
| 5    | 2360.89       | 2282.4        | -3.32%         |
| 6    | 2767.24       | 2766.7        | -0.02%         |
The relative errors of the first six-order simulation and the experiment are all within 10%, and the calculation results are relatively accurate. This result verifies the accuracy of the joint surface contact stiffness model and the transversely isotropic virtual material model, and also discusses applicability of this method to the contact surface of different materials.

4. Conclusion
In order to verify the model accuracy of the contact surface virtual material method, we start from the fitting of the surface fractal dimension and uses modal analysis to verify it. The fractal dimension of the standard W-M function is calculated by using the power spectral method and the wavelet transform method respectively. The results show that the wavelet transform method has less error. The machined surface profiles with different roughness are measured and their fractal dimensions are fitted based on the two methods above. Finally, we carry out the modal analysis on the unit specimen containing only one contact surface, including the finite element simulation and experimental modal analysis. The comparison results implies that the natural frequency errors of the finite element model are all less than 10%. In conclusion, the virtual material method combined with wavelet transform realizes high accuracy and provides a powerful tool for dynamic simulation of large complex assembly.

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