Moduli stabilisation in early superstring cosmology

Lihui Liu

Centre de Physique Théorique, Ecole Polytechnique,
F–91128 Palaiseau Cedex, France

E-mail: lihui.liu@cpht.polytechnique.fr

Abstract. We study the cosmological evolution induced by the free energy of a string gas at finite temperature. We first consider the maximally supersymmetric heterotic string in the weak coupling regime in dimension $D \geq 4$. The free energy develops local minima associated to some gauge symmetry enhancement induced by perturbative states. This can stabilize all heterotic moduli except the dilaton. By string-string S-dualities, it follows that the dual type I moduli can be stabilized by either non perturbative D-string states or perturbative open string states. We next consider the type II string compactified on Calabi-Yau three-folds, which has $\mathcal{N} = 2$ supersymmetry in $D = 4$. The moduli space has singular loci where the Calabi-Yau space undergoes extremal transitions, accompanied with either conifold transitions or certain gauge symmetry enhancement. Also the free energy reaches local minima due to the emergence of extra non perturbative massless modes. Consequently, the type II moduli are attracted to these singular loci. Using the type II/heterotic string duality, we can stabilize the dual heterotic moduli, and in particular it is possible to stabilize the heterotic dilaton.

1. Introduction and general outline

Moduli stabilization is essential to the phenomenological application of superstring theory since the presence of moduli fields in supersymmetric compactifications of string theory leads to difficulties. First, the existence of massless scalars is in contradiction with observations of the gravitational force. Moreover, moduli are continuous parameters in the couplings and mass spectrum, and imply a loss of predictability of the theory.

In the cosmological context, once the moduli obtain masses, their late time oscillations can spoil the nucleosynthesis, which is the cosmological moduli problem. Therefore usually, one requires to generate very large masses for the moduli so that their fluctuation can eventually decay before the nucleosynthesis. Models proposed in this framework include the KKLT model and the racetrack model, where nontrivial superpotential is generated via non perturbative effects, turning the moduli into super-heavy scalar fields.

By the present work we investigate the moduli stabilization by thermal effects, where we consider a universe filled with a superstring gas at finite temperature. We study the resulting cosmology in the context of no-scale models [4], defined at classical level by backgrounds associated to vanishing minima of the scalar potential, with flat directions parameterized by the spontaneous supersymmetry breaking scale. For simplicity we consider only the thermal effect as spontaneous...
supersymmetry breaking mechanism. At the level of conformal field theory on the worldsheet, the implementation of finite temperature amounts to a Scherk-Schwarz reduction on the Euclidean time circle of radius $R_0$, with boundary conditions associated to the spacetime fermion number [5]. The string frame temperature is $\hat{T} = \beta^{-1} = 1/2\pi R_0$ and the Einstein frame temperature is $T = e^{\frac{\pi}{2\alpha'}}\phi^{(D)}\hat{T}$, where $\phi^{(D)}$ is the $D$-dimensional dilaton. The supersymmetry is thus broken spontaneously at the scale $T$. We will restrict our attention to the intermediate era between the Hagerdorn phase transition [6] and the electroweak phase transition, so we have $M_{\text{string}} \gg T \gg \Lambda_{\text{EW}}$.

To build phenomenologically viable models however, it is necessary to also include zero temperature spontaneous supersymmetry breaking. Otherwise as the temperature drops during the cosmological evolution, the supersymmetry broken by temperature will be restored, which is not desired. The case with another Scherk-Schwarz reduction performed in one of the internal dimensions is intensively studied in Refs. [1], where it is shown that the supersymmetry breaking scale $M_{\text{SUSY}}$ induced in this internal dimension evolves proportionally with $T$. It is expected that by the end of the intermediate era, when $T$ approaches $\Lambda_{\text{EW}}$, the radiative corrections induced by infrared effects start to destabilize the Higgs potential, freezing $M_{\text{SUSY}}$ at about TeV scale.

The breaking of supersymmetry gives rise to a nontrivial vacuum energy, which generically lifts flat directions. We will compute this vacuum energy at one-loop level, to be denoted by $Z$. The back-reaction of $Z$ on the spacetime background is dictated by the one-loop effective action

$$\mathcal{S} = \int d^Dx \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{2} F_{MN} \partial \Phi^M \partial \Phi^N - \mathcal{F}(T, \Phi) \right],$$

where $\Phi^M$ are the moduli, and the metric $F_{MN}$ depends on these moduli. The free energy density $\mathcal{F}$ is derived from the vacuum energy by $\mathcal{F} = -\frac{\partial}{\partial T}$, where $V$ is the Einstein frame space volume. Since $\mathcal{F}$ appears in the action as the effective potential, moduli will be attracted to its local minima.

On general grounds, when there is only temperature breaking of supersymmetry, the free energy density takes the form:

$$\mathcal{F}(T, \Phi) = -\int_0^\infty \frac{d\ell}{2\ell} \frac{1}{(2\pi T)^\frac{D-2}{2}} \sum_s e^{-\frac{\ell}{2} M_s (\Phi)^2} \sum_{k_0 \in \mathbb{Z}} e^{-\frac{(2k_0+1)^2}{2\ell^2}} = -T^D \sum_s G(M_s(\Phi)/T),$$

where $M_s$ is the tree-level mass of the $s$-th string state, which can depend on the moduli. The function $G(x)$ is defined in terms of the modified Bessel function of the second kind, $K_{\frac{D}{2}}(x)$, as

$$G(x) = 2 \sum_{k_0} \left(\frac{x}{2\pi|2k_0 + 1|}\right)^{\frac{D}{2}} K_{\frac{D}{2}}(x|2k_0 + 1|).$$

It has the following asymptotic behaviors

$$G(x) = c_D - \frac{c_D - 2}{4\pi} x^2 + \mathcal{O}(x^4) \quad \text{when} \quad x \ll 0, \quad \text{where} \quad c_D = \frac{\Gamma(D/2)}{\pi^{D/2}} \sum_{k_0} \frac{1}{|2k_0 + 1|^{D/2}};$$

$$G(x) \sim \left(\frac{x}{2\pi}\right)^{\frac{D-1}{2}} e^{-x} \quad \text{when} \quad x \gg 1.$$  

Thus $G(x)$ peaks at $x = 0$ and is exponentially suppressed at large $x$. Therefore the local minima of $\mathcal{F}$ appear at the vacuum expectation values (VEV’s) of $\Phi$ where some massive states in the string
spectrum become massless. These states can originate either from the perturbative spectrum or from non perturbative objects such as D-branes. String-string dualities can help figure out the non perturbative contribution. In particular in this work we will stabilize type I moduli using its heterotic dual and stabilize the heterotic moduli using its type II dual.

In any case, the thermal one-loop vacuum energy will induce time-dependent scalar masses, instead of constant ones. This makes the energy stored in moduli oscillations dilute before the electroweak phase transition, so that the cosmological moduli problem is avoided. To show this, we take the flat Robinson-Walker metric $ds^2 = -dt^2 + a(t)^2 dx^2$ (in the Einstein frame). Solving the resulting equations of motion about a local minimum of $\mathcal{F}$, say $\Phi_0$, we obtain the following time evolution of the scale factor, the temperature, and the thermal energy density:

$$a(t) \propto 1/T(t) \propto t^{2/D}, \quad \rho_{\text{th}} \propto T^D \propto a^{-D}. \tag{5}$$

Locally the moduli obey the equation

$$\ddot{\epsilon}^M + (D-1) H \dot{\epsilon}^M + \Lambda^M_N \epsilon^N = 0, \tag{6}$$

where we let $\bar{\Phi} = \Phi_0 + \bar{\epsilon}$, and we have the squared-mass matrix $\Lambda^M_N = (F^{MP}F_{PN}) \bar{\phi}_0$, with $F_{PN} := \partial^2 \mathcal{F} / \partial \Phi^P \Phi^N$, and $F^{MN}$ the inverse of $F_{MN}$. Plugging Eq. (2) one can show that $\Lambda^M_N \propto T^{D-2}$. Thus with Eq.(6) we have the usual scalar dynamics, but with scalar masses depending on the temperature, hence on time. This results in the late-time scalar oscillations behavior $\epsilon \sim t^{-1/2} \sin(\lambda t^{2/D} + \text{phase})$, instead of $t^{1/D-1} \sin(\lambda t + \text{phase})$ for constant mass, where $\lambda^2$ is some eigenvalue of the squared-mass matrix. Therefore the energy density stored in the scalar oscillations behaves asymptotically as

$$\rho_\Phi = \frac{1}{2} F_{MN} |\bar{\phi}_0|^2 \dot{\epsilon}^M \dot{\epsilon}^N \sim t^{4} \pi^{-3} \propto a^{2-3D/2}. \tag{7}$$

Therefore when $D \leq 5$, the cosmological evolution is radiation dominated, since compared to Eq.(5), we have $\rho_\Phi \ll \rho_{\text{th}}$. However $D = 4$ is a marginal case where we have $\rho_{\text{th}} \propto \rho_\Phi \propto a^{-4}$ so that the radiation does not dominate, yet the coherent motion of all moduli is such that the metric evolution is that of a radiation dominated universe ($\rho_{\text{total}} \propto T^2 \propto a^{-4}$). Anyhow in both cases the cosmological moduli problem is avoided. If we further include a zero temperature spontaneous supersymmetry breaking, the marginal case $D = 4$ will also lead to a radiation dominated universe [1].

### 2. Heterotic cosmology and type I dual

We start with the cosmology induced by weakly coupled $SO(32)$ heterotic string compactified on a factorized torus $\prod_{i=D}^{9} S^1(R_{hi})$, where the subscript $h$ indicates heterotic quantities. Thus we are dealing with a maximally supersymmetric model, where the metric $F_{MN}$ in Eq.(1) is exact at tree level. The moduli space is coordinatized by the $D$-dimensional dilaton $\phi_h^{(D)} := \phi_h^{(10)} - \frac{1}{2} \sum_{i,D} \ln(2\pi R_{hi})$ and all the internal radii $R_{hi}$ with $i = D, \ldots, 9$. Computing the thermal one-loop action, we find that when all radii satisfy $|R_{hi} - 1/R_{hi}| < 1/(2\pi R_{hi})$, $i = D, \ldots, 9$, the corresponding free energy density takes the form [2]:

$$\mathcal{F}_h = -T^D \left\{ n_0 c_D + \sum_{i=D}^{9} n_i G \left( 2\pi R_{hi} \left| \frac{1}{R_{hi}} - 1/R_{hi} \right| \right) + \mathcal{O}(e^{-2\pi R_{hi}}) \right\}, \tag{8}$$
where the coefficients $n_0$ and $n_1$ are positive, associated to the counting of states. The first term in the above expression is from massless states. The second term involving the $G$ function shows that $F_h$ reaches a local minimum at the self T-dual point $R_{hi} = 1$ ($i = D, \ldots, 9$), due to the states of masses $\left| \frac{1}{R_{hi}} - R_{hi} \right|$. These are just the non Cartan components responsible for the gauge symmetry enhancement $U(1) \to SU(2)$ in each internal circle. In fact in heterotic string, the correspondence between the enhancement of gauge symmetry and the local extrema of the vacuum energy is true to all loop levels [3]. Therefore the internal radii can all be stabilized at the value 1 where we have $SU(2)^{10-D}$ enhanced symmetry. Moreover for $D \geq 5$, the string coupling $\lambda^{(D)}_h = e^{\phi^{(D)}_h}$ freezes on the flat direction to some constant value determined by the initial conditions. For $D = 4$, the dilaton $\phi^{(4)}_h$ does not converge to a constant but instead decreases logarithmically with cosmological time.

We switch to the dual type I picture. If we perform naive perturbative computation $Z_i = T + K + A + M$ to obtain the free energy density, we will find no local minimum of $F_i$, since no perturbative effect can give rise to gauge symmetry enhancements in maximally supersymmetric type I string. We thus seek to include non perturbative effects which can be inferred from heterotic strings through string-string S-dualities. In dimension $D$, the duality dictionary for Einstein frame quantities is [7]

$$R_{hi} = \frac{R_{hi}}{\sqrt{\lambda_i}} \equiv R_{ti} = \frac{e^{-\frac{1}{4} \phi^{(D)}_i}}{\left( \prod_{j=D}^{9} 2\pi R_{ij} \right)^{1/4}}, \quad i = 0 \text{ or } D, \ldots, 9,$$

$$\phi^{(D)}_i = -\frac{D - 6}{4} \phi^{(D)}_i - \frac{D - 2}{8} \sum_{i=9}^{9} \ln (2\pi R_{ij})$$

(9)

where $\lambda_i$ is the type I string coupling in ten dimensions. When applying this duality map, the heterotic states that induce the local minimum in Eq.(8) are sent to non perturbative states of masses $\left| \frac{1}{R_{hi}} - R_{hi} \right|$ on the type I side. From the type I point of view, they have the natural interpretation as D (or anti-D)-strings wrapped once along the circles $S^1(R_{ti})$, with one unit of momentum. Therefore when all radii satisfy $\left| \frac{1}{R_{hi}} - R_{hi} \right| < \frac{1}{2\pi R_{ti}}$, they are attracted to $R_{ti} = \sqrt{\lambda_i}$, where we have the enhanced gauge symmetry $SU(2)^{10-D}$ due to D-string states. The type I dilaton freezes somewhere along its flat direction just as its heterotic dual except for $D = 6$ where it is stabilized while the internal space volume $\prod_{i=D}^{9} (2\pi R_{ij})$ freezes along a flat direction. This is because in $D = 6$ the duality map Eq.(9) exchanges internal volumes and string couplings. Another subtlety arising from Eq.(9) is that, since the heterotic theory is always in the weak coupling regime, the type I dual is strongly coupled for $D > 6$ and weakly coupled for $D \leq 6$. However our result is still valid at small coupling for $D > 6$ since the D-string states, responsible for the stabilization of $R_{ti}$, are BPS states whose masses are protected by supersymmetry.

The D-string state contribution can also have an E1-instanton interpretation, following the lines of Refs. [8]. For simplicity, we consider the compactification on $S^1(R_{10})$. This contrasts the zero temperature case where E1-instantons arise for $D \leq 8$. Starting from the heterotic side, we can easily express the vacuum energy as a sum over worldsheet instantons. When sending this heterotic result to the type I side using the dictionary (9), the corresponding type I vacuum energy contains
a sum of E1-instantons, which is explicitly \[2\]

\[
Z^{E1} = \frac{\hat{Y}_1^{(10)}}{(2\pi)^{10}} \sum_{\text{E1 instantons}} \frac{e^{-\hat{Y}_1}}{\hat{Y}_{12} \hat{Y}_{12}} \sum_{n=0}^{\infty} \left[ \frac{\alpha_n}{(2\pi \hat{Y}_{12})^n} \sum_{A \geq 1} b_A \left( \frac{1}{\hat{Y}_{12}} + A \frac{\hat{Y}_{12}}{\hat{Y}_{12}} \right)^{4-n} e^{2\pi i \hat{Y}_1 A} \right] + \text{c.c.} + O(e^{-4\pi R_0 \sqrt{\gamma}}),
\]

with the Kähler and complex structure moduli \(\hat{Y}_1\) and \(\hat{Y}_1\) of the torus \(S^{1}(R_{10}) \times S^{1}(R_{19})\)

\[
\begin{align*}
\hat{Y}_1 &= i \hat{Y}_{12} = i(2\hat{k}_0 + 1)R_{10} \cdot n^0 R_{19} \\
\hat{Y}_1 &= \hat{Y}_1 + i \hat{Y}_{12} = \hat{m}_9 + i \frac{(2\hat{k}_0 + 1)R_{10}}{n^0 R_{19}}, \quad n^0 > \hat{m}_9 \geq 0, \quad \hat{k}_0 \geq 0.
\end{align*}
\]

This analysis opens up the possibility to derive from a pure type I point of view the free energy responsible for the stabilization of the internal moduli.

We can further consider generic toroidal compactifications, where all the possible moduli are switched on. On the heterotic side, these moduli include the dilaton \(\phi_h^{(D)}\), the internal metric \(g^{(h)}\), the internal antisymmetric tensor \(B^{(h)}_{ij}\), and the Wilson lines \(Y^{(h)}_{(I)}\), where \(i, j = D, \ldots, 9\) and \(I = 10, \ldots, 25\). Again, all moduli except the dilaton are attracted to the values associated to some gauge symmetry enhancement, where \(F_h\) is minimized locally. In the dual type I picture, moduli stabilization is inferred from the heterotic side through the dictionary

\[
\begin{align*}
\phi_h^{(D)} &= -\frac{D-6}{4} \phi_{(i)} - \frac{D-2}{8} \ln \sqrt{g^{(h)}}, \\
g^{(h)}_{ij} &= \frac{g^{(i)}}{\lambda_i}, \quad B^{(h)}_{ij} = C_{ij}, \quad Y^{(I)}_{(h)} = Y^{(I)}_{(I)}. \tag{12}
\end{align*}
\]

where \(C_{ij}\) is the Ramond-Ramond 2-form. The subtlety is that now the dual type I moduli are stabilized by either non-perturbative D-string states in the closed string sector or perturbative states in the open string sector. For \(D \neq 6\), all type I moduli are stabilized except the dilaton, and for \(D = 6\) however, the dilaton is stabilized while the internal volume freezes on a flat direction.

As an explicit example, we examine the case of compactification on \(T^2\), where we have on the heterotic side, the moduli \(T = B_{89} + i\sqrt{g_{88}g_{99}} - \frac{27}{8} g_{89}, U = (g_{89} + i\sqrt{g_{88}g_{99}} - \frac{27}{8} g_{89})/g_{88}\) and the Wilson lines \(Y^{(I)}_{(i)}\) (\(i, j = 8, 9; I = 10, 11, \ldots, 25\)). The mass formula of string states is

\[
\begin{align*}
\mathcal{M}^{2}_{A, \bar{m}, \bar{n}, Q}(T, U, Y) &= \frac{1}{T_s U_2} \left| -m_8 U + m_9 + \bar{T} n^8 + \left( \bar{T} U - \frac{1}{2} W^I W^I \right) n^9 + W^I Q^I \right|^2 + 4A, \tag{13}
\end{align*}
\]

where \(W^I := U Y_s^I - Y_s^I \) and \(\bar{T} := T + \frac{1}{2} Y_s W^I, \bar{m}, \bar{n}\) are the internal momenta and winding numbers, and \(Q^I\) the root vector of the internal lattice \(\Gamma_{O(32)/Z_2}\). Using the mass formula we can figure out moduli attractors where there are states becoming massless. The enhanced gauge group can be determined from the Narrain lattice formed by the right-moving internal momenta of these states. For example we have the local attractor with \(SU(3) \times SO(32)\) enhanced symmetry, where the moduli are stabilized at \(Y^{(I)}_{I} = 0\), \(T = U = \frac{1}{2} + i\sqrt{3/2}\). Another less trivial example is the attractor with \(SU(2) \times SO(34)\) enhanced symmetry, where the moduli are attracted to \(T = U = i/\sqrt{2}, Y^{(10)}_{8} = 0\) and \(Y^{(10)}_{9} = -Y^{(10)}_{9} = -Y^{(12)}_{9} = \cdots = -Y^{(25)}_{9} = -1/2\). In the dual type I picture the moduli stabilization follows from the dictionary (12).
3. Type II cosmology and heterotic dual

We move on to models with less supersymmetry. We consider cosmology in type IIA strings compactified on a Calabi-Yau three-fold $M$ of Hodge numbers $(h_{11}, h_{12})$. The moduli space is a Cartesian product $\mathcal{M}_V \times \mathcal{M}_H$. The vector multiplet moduli space $\mathcal{M}_V$ of complex dimension $h_{11}$ is a special Kähler manifold, which is exact at tree level, because the dilaton is in a hypermultiplet. The hypermultiplet moduli space $\mathcal{M}_H$ of real dimension $4(h_{12} + 1)$ is a quaternionic manifold, which contains the universal hypermultiplet accommodating the dilaton. Therefore $\mathcal{M}_H$ is subject to perturbative and non perturbative corrections. When $M$ is a $K3$ fibration, a dual heterotic string theory can be available, which is compactified on $K3 \times T^2$. The string-string duality sends the type II vector multiplet moduli to the heterotic vector multiplet moduli and the same is true of the hypermultiplet moduli. Thus the stabilization of heterotic moduli can be inferred from the stabilization of the dual type II moduli.

On the type II side, the moduli space develops singular loci when the internal Calabi-Yau space undergoes extremal transitions, associated either to conifold transition or to certain non Abelian gauge symmetry enhancement. We show that these singular loci are moduli attractors.

Near the conifold locus, generally there are $R$ shrinking 2-cycles in the Calabi-Yau space $M$, among which $S$ change in Hodge numbers is $10$. Near the non Abelian locus, we have $(g-1)(N^2 - N)$ 2-cycles shrinking along a smooth curve $C$ of genus $g$. Consequently, $N^2 - N$ vector multiplets and $g(N^2 - N)$ hypermultiplets become massless, giving rise to the gauge symmetry enhancement $U(1)^{N-1} \rightarrow SU(N)$ with $g$ hypermultiplets transforming in its adjoint representation. When $g > 1$, we can construct a topologically different Calabi-Yau space $M'$ by deforming the $(g-1)(N^2 - N)$ shrinking 2-cycles into 3-cycles. The change in Hodge numbers is

$$h_{11}(M') = h_{11}(M) - S, \quad h_{12}(M') = h_{12}(M) + R - S. \quad (14)$$

Near the non Abelian locus, we have $(g-1)(N^2 - N)$ 2-cycles shrinking along a smooth curve $C$ of genus $g$. Consequently, $N^2 - N$ vector multiplets and $g(N^2 - N)$ hypermultiplets become massless, giving rise to the gauge symmetry enhancement $U(1)^{N-1} \rightarrow SU(N)$ with $g$ hypermultiplets transforming in its adjoint representation. When $g > 1$, we can construct a topologically different Calabi-Yau space $M''$ by deforming the $(g-1)(N^2 - N)$ shrinking 2-cycles into 3-cycles. The change in Hodge numbers is

$$h_{11}(M'') = h_{11}(M) - (N-1), \quad h_{12}(M'') = h_{12}(M) + (g-1)(N^2 - N) - (N-1). \quad (15)$$

In both cases, the low energy effective theory about the singular loci containing all light fields is described by a gauged $\mathcal{N}_4 = 2$ supergravity theory. The scalar fields in the light vector multiplets span a special Kähler manifold which contains $\mathcal{M}_V$, and we denote its special coordinates by $\{X^I\}$, $I = 1, \ldots, n_V$. Also the scalar fields in light hypermultiplets span a quaternionic manifold which contains $\mathcal{M}_H$, and we let its real coordinates be $q^{\Xi}$, $\Xi = 1, \ldots, 4n_H$. These scalar fields are divided into two groups: those participating in the extremal transition, and the rest which are spectators to the extremal transition. We then let $g_{ij} = g_{ij}(X^K)$ and $h_{\Lambda \Sigma} = h_{\Lambda \Sigma}(q^{\Xi})$ be the special Kähler metric and quaternionic metric. Due to the gauging, a scalar potential is generated. The supergravity action is regular in the neighborhood of singular loci since all the non perturbative light states are included.

**Attraction to conifold locus**

Near the conifold locus, the scalar fields relevant to the extremal transition are those in the vector multiplets of $U(1)^S$, $X^i$ ($i = 1, \ldots, S$), and those in the $R$ hypermultiplets charged under $U(1)^S$, $q^{Au}$ ($A = 1, \ldots, R$; $u = 1, 2, 3, 4$). The conifold locus can be parameterized by $X^i = 0 = q^{Au}$. For simplicity we switch off the spectator scalar fields. In the neighborhood of the conifold locus,
by performing power expansion in $X^i$ and $q^{Au}$ and imposing $U(1)^S$-isometry, we obtain the scalar part of the supergravity action to the lowest order:

$$S = \frac{1}{k^{(4)}_0} \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - g_{ij} \partial X^i \partial X^j - \nabla \mathcal{D}^A \nabla \mathcal{D}^A - q_{ij}^A \nabla X^i \nabla X^j q_{ij}^A \nabla \mathcal{D}^A + g_{ij}^A \mathcal{D}_i \cdot \mathcal{D}_j \right],$$

(16)

where $Q_i^A$ is the charge of the $A$-th monopole under the $i$-th $U(1)$, $g_c$ the gauge coupling constant. The Kähler potential $K_V$, the special Kähler metric and the quaternionic metric in the above action are constant, taking their values at the conifold locus. Also we have defined the $SU(2)_R$ doublet and the D-term:

$$\mathcal{D}^A = \left(-q_{ij}^{A2} + i q_{ij}^{A1}\right), \quad \bar{\mathcal{D}}_i = \sum_A Q_i^A \mathcal{D}^A \bar{\sigma} \mathcal{D}^A.$$  

(17)

The action (16) describes an $\mathcal{N} = 2$ supersymmetric Abelian gauge field theory formally coupled to gravity. We show that moduli are attracted to the conifold locus whether starting in the Coulomb branch or the Higgs branch.

- In the Coulomb branch, corresponding to the compactification on $M$, $X^i$ ($i = 1, \ldots, S$) obtain nonzero VEV’s, while $q^{Au}$ ($A = 1, \ldots, R$; $u = 1, 2, 3, 4$) have zero VEV. Sitting at this vacuum, $X^i$ are $S$ of the $h_{11}(M)$ vector multiplet moduli, and the free energy density is

$$F = -T^4 \left[ n_0 + \sum_q n_s G \left( \frac{M_s}{T} \right) \right] + \mathcal{O}(e^{-M_{\text{min}}}),$$

(18)

where $n_0$ and $n_s$ count respectively the massless states and the light monopole states. Also $M_{\text{min}}$ is the minimum mass of the states which never become massless in the neighborhood of the conifold locus. We take the temperature such that $T \ll M_{\text{min}}$. In the argument of the $G$-function, $M_s$ is the tree-level mass of the $s$-th light monopole state, which has the behavior $M_s \sim \mathcal{O}(X^i)$. Therefore at the conifold locus where $X^i = 0$, the free energy density reaches its local minimum. Thus the $S$ vector multiplet moduli $X^i$ are attracted to the conifold locus.

- In the Higgs branch, corresponding to the compactification on $M'$, $q^{Au}$ ($A = 1, \ldots, R$; $u = 1, 2, 3, 4$) have nonzero VEV’s subject to the constraints $\bar{\mathcal{D}}_i = 0$ modulo gauge orbits, so that they form $R - S$ of the $h_{12}(M') + 1$ hypermultiplet moduli. On the other hand, $X^i$ must vanish, and the vector multiplets containing $X^i$ will absorb $S$ hypermultiplets to form $S$ long massive vector multiplets. The free energy density takes the same form of Eq.(18), with $M_s \sim \mathcal{O}(q^{Au})$. Thus the $4(R - S)$ hypermultiplet moduli $q^{Au}$ are attracted to $0$, corresponding to the conifold locus in the Higgs branch.

**Attraction to the non Abelian locus**

Near the non Abelian locus the scalar fields relevant to the extremal transition are those in the $SU(N)$-vector multiplet, $X^a$ ($a = 1, \ldots, N^2 - 1$), as well as those in the $g$ hypermultiplets in the adjoint of $SU(N)$, $q^{Au}$ ($A = 1, \ldots, g$; $u = 1, 2, 3, 4$). The non Abelian loci can be parameterized as $X^a = 0 = q^{Au}$. Expanding in powers of $X^a$ and $q^{Au}$, imposing $SU(N)$ isometry, we obtain the bosonic part of the supergravity action to the lowest order:

$$S = \frac{1}{k^{(4)}_0} \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - i^2 \nabla X^a \nabla X^a - \nabla \mathcal{D}_a \nabla \mathcal{D}_a - g_{ij}^a \nabla X^i \nabla X^j q_{ij}^a \nabla \mathcal{D}_a + \frac{1}{2} \bar{\mathcal{D}}_a \cdot \mathcal{D}_a \right].$$

(19)
The $SU(2)_R$ doublet $\mathcal{D}^a_A$ and the D-term $\mathcal{D}^a$ are defined as

$$
\mathcal{D}^a_A = \begin{pmatrix} -q^{aA2} + i q^{aA1} \\ q^{aA3} + i q^{aA4} \end{pmatrix}, \quad \mathcal{D}^a = \sum_{A,b,c} i f^{abc} q^{bA1} a \mathcal{D}^c_A,
$$

(20)

where $f^{abc}$ are the structure constants of $SU(N)$. The action (19) thus describes an $\mathcal{N} = 2$ $SU(N)$ super Yang-Mills field theory formally coupled to gravity. We show that moduli can be attracted to the non Abelian locus from either the Coulomb branch or the Higgs branch.

- In the Coulomb branch, corresponding to the compactification on $M$, all Cartan components $X^\hat{a}$ and $q^{\hat{a}Au} (\hat{a} = 1, \ldots, N - 1; A = 1, \ldots, g; u = 1, 2, 3, 4)$ acquire nonzero VEV’s, while all the non Cartan components must vanish. Therefore $X^a$ form $N - 1$ of the $h_{11}(M)$ vector multiplet moduli, while $q^{\hat{a}Au}$ form $g(N - 1)$ of the $h_{12}(M) + 1$ hypermultiplet moduli. The free energy density takes the form of Eq.(18) with $M_s \sim O(X^a, q^{\hat{a}Au})$. Therefore the non Abelian locus where $X^a$ and $q^{\hat{a}Au}$ vanish is the local minimum of the free energy density. By consequence, $X^a$ and $q^{\hat{a}Au}$ are attracted to the non Abelian locus.

- In the Higgs branch, corresponding to the compactification on $M''$, $q^{\hat{a}Au}$ ($a = 1, \ldots, N^2 - 1; A = 1, \ldots, g; u = 1, 2, 3, 4$) have nonzero VEV’s subject to the constraint $\mathcal{D}^a = 0$ modulo gauge orbits, and they form $(g - 1)(N^2 - 1)$ of the $h_{12}(M'') + 1$ hypermultiplet moduli. The scalars in the $SU(N)$-vector multiplet $X^a$ must vanish. The $SU(N)$-vector multiplet will absorb one hypermultiplet in the adjoint of $SU(N)$ to become a long massive vector multiplet. The free energy density is of the form Eq.(18), with $M_s \sim O(q^{\hat{a}Au})$. Thus the $(g - 1)(N^2 - 1)$ hypermultiplet moduli $q^{\hat{a}Au}$ are attracted to 0, corresponding to the non Abelian locus in the Higgs branch.

An example

We analyze a 2-parameter example with heterotic dual, where the internal Calabi-Yau space is $M \in \mathcal{P}^{1,1,2,2,6}_{(1,1,1,1,1,3)}[12](2, 128)$. Its mirror is defined by [11]

$$
x_1^{12} + x_2^{12} + x_3^6 + x_4^6 + x_5^2 - 12 \psi x_1 x_2 x_3 x_4 x_5 - 2 \phi x_6^6 x_2^6 = 0.
$$

(21)

The coefficients $\phi$ and $\psi$ are the two Kähler moduli (from the type IIA point of view). This model has at once a conifold locus with $R = S = 1$, and an $SU(2)$-non Abelian loci with $g = 2$, which leads to a Higgs branch corresponding to the Calabi-Yau space $M'' \in \mathcal{P}^{5}_{(1,1,1,1,1,3)}[2, 6](1, 129)$. These singular loci are defined by the vanishing of [11]

$$
\Delta = \Delta_c \times \Delta_s = \left( (1 - z_1)^2 - z_2^2 z_3 \right) \times \left( 1 - 4 z_s \right),
$$

(22)

where $\Delta_c = 0$ defines the conifold locus, and $\Delta_s = 0$ the non Abelian locus, with $z_1 = \frac{1}{864} \frac{\phi}{\psi}$, $z_2 = \frac{1}{4096}$ a reparametrization of the Kähler moduli. The two singular loci intersect at two points:

$$(z_1, z_3) = \left( \frac{3}{7}, \frac{1}{4} \right) \text{ and } \left( 2, \frac{1}{8} \right).$$

The moduli are therefore attracted to these intersection points, since there is a maximal number of massless states at these points. Thus starting in the Coulomb branch, we can stabilize both of the vector multiplet moduli and 2 of the 128 + 1 hypermultiplet moduli. Also this type II model has heterotic dual on $K3 \times T^2$ in the Coulomb branch [12]. Therefore in this dual heterotic theory, no vector multiplet moduli is left free. It follows that the heterotic dilaton, living in a vector multiplet, is stabilized.
4. Conclusion

We have reviewed moduli stabilization by thermal effects in the cosmological context. The moduli attractors are the local minima of the thermal free energy, where extra massless states appear. These extra massless states can either be perturbative or non-perturbative. This approach to moduli stabilization has the advantage of inducing time-dependent scalar masses, which is crucial to avoid the cosmological moduli problem. In toroidal compactifications of heterotic string, we have shown that all moduli except the dilaton can be attracted to the values associated to certain gauge symmetry enhancement by perturbative states. Using the type I/heterotic S-duality, it follows that the NS-NS moduli, Ramond-Ramond moduli and Wilson lines are stabilized by either light D-string states or perturbative open string states. Then we considered more realistic cases with type II string compactified on Calabi-Yau three-folds. The model is not maximally supersymmetric, so that we do not know exactly the tree level action. However in the neighborhood of extremal transition loci, the knowledge of the expected gauge group and the regularity of the action allow us to figure out the local behavior of the effective $\mathcal{N}=2$ supergravity. This enables us to show the attraction of moduli to the extremal transition loci. With an explicit example with $(h_{11}, h_{12}) = (2, 128)$ we have shown that the type II moduli are attracted to the intersection points of different extremal transition loci, where all Kähler moduli are stabilized. By string-string duality, all the vector multiplet moduli in the dual heterotic strings are stabilized, including the heterotic dilaton.

In all generality, the type II moduli are attracted to values which achieve a maximum number of vanishing 2-cycles or 3-cycles, corresponding to the most singular configuration of the internal Calabi-Yau space. Therefore hopefully all type II moduli can be stabilized except those in the universal hypermultiplet, including the dilaton. It is because the universal hypermultiplet has no geometric interpretation. Therefore it will be of interest to study generalized Calabi-Yau compactifications with flux, where a solution to the stabilization of type II dilaton may be available.

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