Transient Response of Grounding Electrode using the Wire Antenna Theory Approach

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Abstract. The paper deals with an assessment of transient current induced along the vertical grounding electrode applying the thin wire antenna model in the frequency domain. The formulation is based on the homogeneous Pocklington integro-differential equation. The influence of an imperfectly conducting half-space is taken into account via the corresponding reflection coefficient arising from the Modified Image Theory (MIT). The Pocklington equation is solved using numerical and analytical approach respectively. The numerical solution is carried out via the Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM). The transient response is obtained by means of the Inverse Fourier Transform (IFFT). Some illustrative computational examples for the transient current induced at the center of a vertical grounding electrode are presented in the paper.

1. Introduction
Transient analysis of vertical grounding electrode, as an important component in many realistic grounding systems, such as wind turbines (WTs) is always of interest in electromagnetic compatibility (EMC) [1], [2]. Transient analysis of complex grounding systems for wind turbines has been already reported elsewhere, e.g. in [1], where a combination of the ring, vertical rods and horizontal electrodes is considered. Nevertheless, a study of vertical electrode behavior, as an individual element, still remains a rather important issue.

Transient behavior of wire structures in EMC can be carried out using the approximate Transmission Line (TL) approach, or using the more rigorous wire antenna theory (AT) model [3], [4]. A trade-off between AT and TL approach can be found elsewhere, e.g. in [4]. Frequency domain numerical/analytical antenna theory study of vertical electrodes as individual elements, using Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM) and analytical solution, respectively, has been reported in [2].

This paper deals with a transient response of a straight vertical electrode, thus extending the frequency domain analysis presented in [2]. First, the current distribution along the electrode is obtained by using both the GB-IBEM and analytical solution procedure presented in [2].

The transient response of the vertical electrode is obtained by means of Inverse Fourier Transform (IFFT). Some illustrative computational results are given in the paper.

2. Formulation
Fig. 1 shows a vertical grounding electrode of length $L$ and radius $a$, buried in a lossy medium at depth $d$ and energized by an equivalent current generator with one terminal connected to the electrode and the other one grounded at infinity.
The governing integral expressions for the current and scattered voltage induced along the vertical electrode can be readily derived from Maxwell’s equations by enforcing the continuity conditions for the tangential components of the electric field along the electrode and utilizing the concept of generalized Telegrapher’s equations.

The current distribution along the grounding electrode is governed by a homogeneous integro-differential equation of the Pocklington type [2]:

$$\int_{d}^{-d} I(z') g(z,z') dz' - \frac{1}{j4\pi\omega\epsilon_{eff}} \frac{\partial}{\partial z} \int_{d}^{-d} \frac{\partial I(z')}{\partial z'} g(z,z') dz' + Z_s(z) I(z) = 0. \quad (1)$$

The complex permittivity of the lossy ground $\epsilon_{eff}$ is

$$\epsilon_{eff} = \epsilon_0 \epsilon_r + \frac{\sigma}{j\omega}, \quad (2)$$

where $\epsilon_r$ and $\sigma$ are the soil permittivity and conductivity, respectively.

The total Green function is of the form [2]:

$$g(z,z') = g_0(z,z') - \Gamma_{ref} g_i(z,z'), \quad (3)$$

where $g_0(z,z')$ and $g_i(z,z')$ are given by:

$$g_0(z,z') = \frac{e^{-\gamma R_1}}{R_1}, \quad g_i(z,z') = \frac{e^{-\gamma R_2}}{R_2} \quad (4)$$

and the propagation constant of the lossy ground is:

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2 \mu_0 \epsilon_0}, \quad (5)$$

where $R_1$ and $R_2$ are the distances from the source and the image to the observation point, respectively.

Within the numerical solution the Fresnel reflection coefficient (RC) is used [4]:

Fig. 1. Vertical grounding electrode excited by the equivalent current generator.
\[
\Gamma_{\text{MIT}}^{\text{ref}} = \frac{1}{n} \cos \theta - \frac{1}{n} \sin^2 \theta ; \quad \theta = \arctg \left( \frac{x-x'}{2d} \right) ; \quad n = \frac{\varepsilon_{\text{eff}}}{\varepsilon_0}.
\]

Furthermore, a simplified reflection coefficient arising from modified image theory is used within the analytical solution and is given by [4]:

\[
\Gamma_{\text{MIT}}^{\text{ref}} = -\frac{\varepsilon_{\text{eff}} - \varepsilon_0}{\varepsilon_{\text{eff}} + \varepsilon_0}.
\]

The current source is inserted into the Pocklington integro-differential equation formulation in terms of the following conditions at the electrode ends [2]:

\[
I(-d) = I_g, \quad I(-d - L) = 0,
\]

where \(I_g\) stands for the unit current source excitation.

It is worth noting that the knowledge of the electrode current provides the assessment of the scattered voltage.

The scattered voltage along the electrode can be obtained by integrating the horizontal component of the electric field (normal to the electrode and tangential to the ground-air interface) from the remote soil to the electrode surface [5].

Performing certain mathematical manipulations and assuming the scalar potential in the remote soil to be zero [5], yields:

\[
V^{\text{sct}}(z) = -\int_{-L}^{d} E^{\text{sct}}_s(x,z)dx = -\frac{1}{j4\pi\omega\varepsilon_{\text{eff}}} \int_{-L}^{d} \frac{\partial I(z')}{\partial z'} g(z, z')dz'
\]

Note that the tedious integration from infinity to the electrode surface is avoided in the final expression at the right hand side. The procedure is presented in detail in [5].

3. Solution procedures

The space-time dependent current and voltage along the electrode are obtained by solving (1) and (9) using numerical/analytical solution procedures. The full mathematical description of the solution procedures can be found elsewhere, e.g. in [2] and [4].

The transient response of the vertical electrode is calculated by using the IFFT procedure [6].

Thus the discrete frequency response is obtained by sampling analytical solution in the frequency domain:

\[
I(x, \omega) = \sum_{j=1}^{N} I(x, \omega_j) \delta(\omega - \omega_j)
\]

Transient current induced at the center of the electrode is calculated using the following IFFT formula:

\[
I(x, t_j) = \frac{1}{N} \sum_{i=1}^{N} I(x, \omega_i) \omega_i^{(j-1)(i-1)}
\]

Having performed extensive numerical testing, \(2^{16}\) samples with maximum frequency of 500 MHz were found to be optimal to ensure accurate results within a reasonable time frame.
4. Results
Some illustrative computational examples for the transient current induced at the center of 1 m and 10 m long electrode obtained by means of analytical and numerical approach, respectively are presented in Figs. 2 and 3. Radius of the electrode is $a=5 \text{ mm}$, ground relative permittivity is, $\varepsilon_r=10$, burial depth is $d=0.5 \text{ m}$. The electrode is excited by the double-exponential $0.1/1 \mu\text{s}$ pulse.

![Fig. 2. Transient current at the center of the grounding electrode, $L=1 \text{ m}$, $\sigma=10 \text{ mS/m}$.](image2)

![Fig. 3. Transient current at the center of the grounding electrode, $L=10 \text{ m}$, $\sigma=1 \text{ mS/m}$.](image3)

Note that in both cases the results obtained via different approaches agree quite satisfactorily.

5. Conclusion
Transient behavior of the vertical grounding electrode has been analyzed using the thin wire antenna model. The formulation is based on the homogeneous Pocklington equation in the frequency domain. The influence of a dissipative half-space is taken into account via the corresponding reflection coefficient. The Pocklington equation is solved using numerical and analytical approach respectively. The numerical solution is carried out via the Galerkin-Bubnov variant of the Indirect Boundary Element Method (GB-IBEM). The transient response is calculated by means of the Inverse Fourier Transform (IFFT). Some illustrative computational examples for the transient current induced at the center of 1 m and 10 m long vertical grounding electrode are presented in this work. The results obtained via different approaches are found to be in a satisfactory agreement.

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