FRB 121102 Burst Pairs

J. I. Katz, 1*

1 Department of Physics and McDonnell Center for the Space Sciences, Washington University, St. Louis, Mo. 63130 USA

ABSTRACT

The repeating FRB 121102 emitted a pair of apparently discrete bursts separated by 37 ms and another pair, 131 days later, separated by 34 ms, during observations that detected bursts at a mean rate of $\sim 2 \times 10^{-4}/s$. Here I assume that these events are separate bursts rather than multiple peaks from longer single bursts and consider their implications. They then are inconsistent with Poissonian statistics. The measured burst intervals constrain any possible periodic modulation underlying the highly episodic emission. If more such short intervals are measured a period may be determined or periodicity may be excluded. Narrow wandering beams are predicted to produce an excess of pairs (or higher multiples) of bursts spaced at intervals much less than the mean interval.

Key words: radio continuum: transients

1 INTRODUCTION

The repeating FRB 121102 is unique. Its repetitions permitted accurate localization and identification with a star-forming dwarf galaxy at a redshift $z = 0.193$ (Chatterjee et al. 2017; Tendulkar et al. 2017; Bassa et al. 2017). Its redshift resolved all doubt that (if it is representative of FRB, aside from its repetition rate, as it appears to be) FRB are at “cosmological” distances, implying high emitted instantaneous power, despite a small duty factor. Repetition demonstrated that FRB, unlike GRB, are not the product of catastrophic events. Observations of repetitions over several years exacerbated the stringent requirements on the energy available, and test theoretical models, such as extreme pulsar pulses and SGR outbursts (Katz 2016). Comparatively frequent bursts offer the opportunity to obtain information, such as the distribution of intervals between bursts, unavailable from FRB not observed to repeat.

Scholz et al. (2017) at the Green Bank Telescope and Hardy et al. (2017) at Effelsberg have observed pairs of radio bursts from the repeating FRB 121102 separated by 37.3±0.3 ms and 34.1±0.3 ms, respectively (error estimates from Hardy et al. (2017), assumed the same for Scholz et al. (2017), and propagated as root-sum-of-squares). Using the mean rate of burst detections of $\sim 2 \times 10^{-4}/s$ in both studies, if Poissonian statistics applied only a fraction $\lesssim 10^{-5}$ of bursts would be found in such close pairs. The detection of two such pairs in observations comprising (together) 25 bursts is thus extraordinarily unlikely unless bursts are correlated on very short time scales. Many possible models are consistent with correlation. No single model is specifically indicated, but any successful model must admit such short-time correlations.

In this paper I consider implications of the observations of these short intervals between bursts. Scholz et al. (2016) show one burst (10) with either two components separated by about 10 ms or a FWHM of about 10 ms; the structure is frequency-dependent and the noise level is significant. The greatest FWHM shown by Scholz et al. (2017) is about 3 ms while the greatest FWHM indicated by the Gaussian fits of Hardy et al. (2017) is about 5 ms. The signals shown in these latter two papers, with high S/N, have no indication of any power in the $\sim 30$ ms between the two closely spaced peaks. This makes it unlikely that these peaks $\sim 35$ ms apart are substructure of a single broad burst. I therefore assume that the reported bursts are in fact separate bursts.

I first consider the hypothesis that SGR are analogous to giant pulsar pulses (Katz 2016) in which the radiated power is drawn from the rotational energy of a neutron star. Bursts of FRB 121102 have been reported to have fluxes as high as 0.8 Jy (Hardy et al. 2017), though this may be enhanced by scintillation or lensing Cordes et al. (2017). A conservative lower bound might be 0.1 Jy. This implies, assuming isotropic emission, a power of $1.5 \times 10^{34}$ ergs/s. If spindown power is converted to coherent GHz radiation with efficiency $\epsilon$ the maximum spin period would be $120\epsilon^{1/4}B_{15}^{1/2}$ ms. Camilo et al. (2006) have found separations of sub-pulses in the radio emission from an AXP of $\sim 0.1$ of the spin period. Unless $B_{15} \gtrsim 10\epsilon^{-1/2}$, requiring an unprecedentedly high $\epsilon$ (the frequency of giant pulses of the Crab pulsar sharply decreases for $\epsilon \gtrsim 10^{-5}$ (Karuppusamy, Stappers & van Straten 2010) and $B$ orders of magnitude greater than in any known magnetar, it is not possible to explain the $\approx 35$ ms burst intervals as multi-
ple phase windows within a single longer rotation period of a rotation-powered FRB.

SGR-like models, dissipating magnetostatic energy, are not bound by these constraints, and Hardy et al. (2017) suggested that the short intervals may represent multiple rotational phases of a single (slower) rotation. Such models are disfavored by the absence of a FRB in a fortuitous radio observation during the extraordinary 2004 outburst of SGR 1806-20; its Galactic distance of ~15 kpc would imply a brightness about 110 dB greater than that of FRB at cosmological distances, more than compensating for a 70 dB sidetone suppression at 35° from the beam, yet none was detected (Tendulkar, Kaspi & Patel 2016). This argument depends on the unverified assumption that FRB emit roughly isotropically; if strongly beamed, the argument is vitiated.

If the burst source has an underlying periodicity with extensive nulling, like RRAT Rotating Radio Transients (McLaughlin et al. 2006), a template (if scaled up by many orders of magnitude in energy) for a pulsar-based model of FRB, then these observations constrain possible periods. If there is a single rotational phase of emission the period must be an integer fraction of all the observed pair intervals. The observation of more than one such interval requires also that the period be an integer fraction of the differences between each of the pair intervals, a strong constraint. The observation of multiple pair intervals could either unambiguously determine a millisecond period or exclude the possibility of such underlying periodicity.

2 THE PERIOD

Spitler et al. (2016); Hardy et al. (2017) pointed out that the discovery of repeating pulses offers an opportunity to determine if they are periodic, and their period if so, even if (as in RRAT) pulses are observed very infrequently. This method becomes less effective if the periods are very much shorter than the intervals between observed pulses, and may fail entirely if the period varies so that the observed phase cannot be maintained between successive observations. This is likely if the FRB is produced by a very fast and high-field neutron star that rapidly spins down, if there are orbital Doppler shifts or glitches, or if the emission region is in a surrounding nebula and moves. Phase may be lost if the intervals between observations \( t_{\text{int}} \gtrsim \sqrt{T/\nu} \sim \sqrt{\Delta T/\nu} \) where \( T \) is the spindown time. For hypothetical \( T \sim 100 \) y and \( \nu \sim 500/\text{s} \) phase may be maintained over intervals of 1–2 hours, comparable to the intervals observed during periods of apparent activity (Cordes et al. 2017), provided irregular timing noise is small. Phase may be maintained over longer intervals if the spindown rate follows a simple function like a power law that can be fitted.

Suppose a strictly periodic underlying phenomenon in which an overwhelming majority of possible pulses (more than 99.9999% in the recent observations) are nulled below the detection limit. This (with less extreme nulling) is the generally accepted model of RRAT. It would also describe giant pulsar pulses were there sufficiently high thresholds of detectability, and many pulsars show more or less frequent nulling. Pairs of pulses must be separated by an integer multiple of the period, so that the period

\[
P = \frac{\Delta T_i}{n_i},
\]

for some positive integer \( n_i \), where \( \Delta T_i \) is the interval between two bursts in the \( i \)-th pair. In addition, for all pairs \((i, j)\)

\[
P = \frac{\Delta T_i - \Delta T_j}{k_{i,j}}
\]

for some positive integer \( k_{i,j} \). These resemble Diophantine equations, but are complicated by the fact that the \( \Delta T_i \) have errors of measurement.

Measurement uncertainty limits the efficacy of period determination. A single pair of bursts with an interval \( t_{\text{int}} \sim 5000 \) s, timed to an accuracy \( \delta t \sim 0.3 \) ms, is consistent with \( N_\nu \sim t_{\text{int}} \delta t \nu^2 \sim 4 \times 10^5 \) distinct possible spin periods of frequency \( \nu = O(500/\text{s}). \) As Spitler et al. (2016); Hardy et al. (2017) pointed out, the extant data are not sufficient to determine a fast spin period, if there be one, or to exclude its existence. Each additional independently measured interval of length \( \sim 5000 \) s prunes the set of possible periods by a factor \( \sim \nu \delta t \sim 0.1 \), so that many intervals with \( t_{\text{int}} \sim 5000 \) s must be measured to determine a unique last period.

For the close pairs observed by Scholz et al. (2017) and Hardy et al. (2017), Eq. 2 is the strongest constraint, and implies that \( P \) must be an integer fraction of \( 3.2 \pm 0.4 \) ms (propagating errors as root-sum-of-squares). The integer is unlikely to be more than about five because a neutron star has a (somewhat uncertain) minimum rotational period of about 0.6 s. Periods permitted by the extant data are \( 3.2 \pm 0.4 \) ms, \( 1.6 \pm 0.2 \) ms, \( 1.07 \pm 0.13 \) ms, \( 0.81 \pm 0.10 \) ms, \( 0.65 \pm 0.08 \) ms, \( 0.54 \pm 0.07 \) ms etc. If periodic at all, the period must be \( \lesssim 3 \) ms. Such short periods are also required to meet the energetic requirements of pulsar models (Katz 2016) provided they are not narrowly beamed and their instantaneous radiated power does not exceed the spindown power (see, however, Katz (2017a,b) for speculative alternatives).

These uncertainties correspond to phase lags of many cycles over a ~5000 s interval, so that the longer intervals, however accurately measured, cannot be used to select a valid, or exclude an invalid, short period directly from those permitted by the 34.1 and 37.3 ms intervals. It may be possible to “ladder up” through a series of measured intervals \( \Delta T_n \), \( n = 1, 2, \ldots \) satisfying

\[
\frac{\Delta T_{n+1}}{\Delta T_n} \lesssim \frac{P}{2\delta t}.
\]

where the \( T_n \) can be differences between measured intervals. For periods \( \sim 1 \) ms this ratio is not large. Suitable intervals have not yet been measured.

3 CLOSE PAIRS

If burst arrival times are described by Poissonian statistics with a mean rate \( \tau^{-1} \), then the a priori likelihood that an interval between bursts is less than \( T \) is \( 1 - e^{-T/\tau} \approx T/\tau \) (if \( T/\tau \ll 1 \)). For the close pairs observed by Scholz et al. (2017); Hardy et al. (2017), \( \tau \sim 5000 \) s and \( T/\tau \sim 7 \times 10^{-6} \).
The \textit{a posteriori} choice of the observed intervals as the criterion $T$ introduces a bias that invalidates the quantitative applicability of the \textit{a priori} likelihood, but it is still apparent that the process is far from Poissonian.

The distribution of intervals may be consistent with a model in which a narrow beam executes a random walk in direction, whether the beam is emitted by a neutron star or a black hole accretion disc (Katz 2017a,c). In such models the statistics of recurrence are non-Poissonian. If the beam once points to the observer, producing an observable burst, the interval before the next burst is likely to be much less than its mean for Poissonian statistics, the reciprocal of the mean burst rate. This may be estimated from the probability that a two dimensional random walk in angle (the space of angular deviations is nearly Cartesian for small deviations) will return to a particular direction in the interval $t$ to $t + dt$ after its previous visit to that direction. That probability density, for small deviations, is $\propto t^{-1}$ (because the dispersion $\sigma \propto t^{-1/2}$ in each of two orthogonal directions). Hence the likelihood of a return within a time $T$

\begin{equation}
    P(T) = \frac{\int_{T_0}^{T} dt \frac{dt}{\tau_{\text{max}}}}{\int_{T_0}^{T_{\text{max}}} dt} = \frac{\ln(T/T_0)}{\ln(T_{\text{max}}/T_0)},
\end{equation}

where $T_0$ is a lower cutoff corresponding to the time required for the beam to wander its own width and $T_{\text{max}}$ an upper cutoff on the recurrence time (in this model, corresponding to the time for the beam to return to a direction to the observer following diffusion to the outer bounds of its angular range). Neither of these parameters is well known (the observed pulse widths set an upper bound to $T_0$ but are broadened by scintillation, imperfect de-dispersion and instrumental response), but the dependence of $P(T)$ on them is only logarithmic. For $T_0 = 1 \text{ ms}$ (a plausible upper limit) and $T_{\text{max}} = \tau \approx 5000 \text{ s}$, $P(50 \text{ ms}) \approx 0.25$. This result should not be taken quantitatively, but indicates that in a wandering beam model short recurrence times occur orders of magnitude ($10^3$–$10^5$ for our parameters) more frequently than would be indicated by Poissonian statistics.

The data are shown in Fig. 1. Each subfigure shows the intervals reported in the indicated paper, binned in widths of $\sqrt{10}$ on a logarithmic scale. The solid lines show the predictions of Poissonian statistics with the mean burst rate observed during the period over which the recurrences were observed. This rate varies over times of hours, days and longer (Table 1 of Spitler et al. 2016) and Table 2 of Scholz et al. (2016) indicate periods of greater and lesser apparent (Cordes et al. 2017) activity like those of SGR (Laros et al. 1987)) so the predictions are not quantitative, but confirm the conclusion that the existence of repetition intervals $\sim 35 \text{ ms}$ is strong evidence against the Poissonian model even during periods of greater activity (when nearly all the observed bursts occur and intervals are measured). The dotted lines show the predictions of the random walk model (Eq. 5), with rollofs allowing for the finite length of continuous observations (intervals longer than the time from a burst to the end of the observation cannot be observed).

This model is statistically consistent with the observations of millisecond intervals, although there may be a significant deficiency of intervals between 0.1 s and hundreds of seconds that is not explained by the model.

4 DISCUSSION

The remarkable discoveries by Scholz et al. (2017); Hardy et al. (2017) of closely spaced pairs of bursts from the repeater FRB 121102 offer important clues to FRB mechanisms. If there is an underlying periodic clock, such as neutron star rotation, with period short enough that the pairs cannot be structure of single bursts then these pairs strongly constrain possible periods. Because every pair of closely spaced bursts provides an additional independent $\Delta T_i$, the number of constraints provided by Eq. 2 grows quadratically with the number of pairs observed. The discovery of one or two more pairs with separations comparable to those recently observed could either conclusively demonstrate the existence of a periodicity and determine its period (and hence demonstrate the validity of pulsar-like models) or demonstrate the absence of periodicity (and
J. I. Katz

hence disprove such models in which the emission region is stable in phase).

The observation of close pairs with highly non-Poissonian statistics requires explanation. Even in a periodic model, the observation of close pairs requires that, apart from the periodicity, the source’s activity be correlated in time. Many astronomical objects do have time-clustered non-Poissonian statistics, with SGR as perhaps the most dramatic examples (Laros et al. 1987). While these clearly distinguish periods of activity from less active periods, they do not resemble the close pairs of FRB 121102. Unlike SGR and other episodically active objects (and phenomena outside astronomy such as earthquake swarms), a general period of enhanced activity is not sufficient to explain the close pairs of FRB 121102. Perhaps they can be explained, with statistical plausibility, if a collimated beam executes a random walk in direction. It returns to the observer more often shortly after having pointed in that direction than long after, when it has wandered far away.

REFERENCES

Bassa, C. G., Tendulkar, S. P., Adams, E. A. K. et al. 2017 ApJ 843, L8.
Camilo, F., Ransom, S. M., Halpern, J. P. et al. 2006 Nature 442, 892.
Chatterjee, S., Law, C. J., Wharton, R. S. et al. 2017 Nature 541, 58.
Cordes, J. M., Wasserman, I., Hessels, J. W. T. et al. 2017 ApJ 842, 35.
Hardy, L. K., Dhillon, V. S., Spitler, L. G. et al. 2017 arXiv:1708.06156.
Karuppusamy, R., Stappers, B. W. & van Straten, W. 2010 A&A 515, 36.
Katz, J. I. 2016 Mod. Phys. Lett. A 31, 1630013.
Katz, J. I. 2017a MNRAS 467, L96.
Katz, J. I. 2017b MNRAS 469, L39.
Katz, J. I. 2017c MNRAS 471, L92 doi:10.1093/mnrasl/slx113 arXiv:1704.08301.
Laros, J. G., Fenimore, E. E., Klebesadel, R. W. et al. 1987 ApJ 320, L111.
McLaughlin, M. A., Lyne, A. G., Lorimer, D. R. et al. 2006 Nature 439, 817.
Scholz, P., Spitler, L. G., Hessels, J. W. T. et al. 2016 ApJ 833, 177.
Scholz, P., Bogdanov, S., Hessels, J. W. T. et al. 2017 ApJ 846, 80 arXiv:1705.07824.
Spitler, L. G., Scholz, P., Hessels, J. W. T. et al. 2016 Nature 531, 202.
Tendulkar, S. P., Kaspi, V. M. & Patel, C. 2016 ApJ 827, 59.
Tendulkar, S. P., Bassa, C. G., Cordes, J. M. et al. 2017 ApJ 834, L7.

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.