A new estimator for gravitational lensing using galaxy and intensity mapping surveys

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We propose a new tomographic estimator for the gravitational lensing potential, based on a combination of intensity mapping (IM) and galaxy number counts. The estimator can be written schematically as IM×galaxy – galaxy×IM; this combination allows to greatly reduce the contamination by density-density correlations, thus isolating the lensing signal. As a pure cross-correlation estimator, it is additionally less susceptible to systematic effects. We show that the new estimator strongly suppresses cosmic variance and consequently improves the signal-to-noise ratio (SNR) for the detection of lensing, especially on linear scales and intermediate redshifts. For cosmic variance dominated surveys the SNR of our estimator is a factor 30 larger than the SNR obtained from the correlation of galaxy number counts only. Shot noise and interferometer noise reduce the SNR. For the specific example of DES cross-correlated with HIRAX, the SNR is around 4, whereas for Euclid cross-correlated with HIRAX it reaches 52. This corresponds to an improvement of a factor 4-5 compared to the SNR from DES alone. For Euclid cross-correlated with HIRAX the improvement with respect to Euclid alone strongly depends on the redshift. We find that the improvement is particularly important for redshifts below 1.6, where it reaches 5. This makes our estimator especially valuable to test dark energy and modified gravity, that are expected to leave an impact at low and intermediate redshifts.

Introduction. Gravitational lensing is a powerful probe of the matter distribution in our Universe. It describes the deflection of light rays by metric perturbations along the photon trajectory from their distant sources. Weak gravitational lensing refers to the regime where the deflections are small enough to not induce caustics. The most common approach to observe weak lensing is through the distortion of the observed shape of galaxies, which generates correlations between their ellipticity. This effect, referred to as cosmic shear, has been detected for the first time in the early 2000s and has been subsequently measured in various surveys providing tests of the consistency of the ΛCDM model. But weak lensing also modifies the observed number of distant galaxies, via the effect of magnification bias: weak lensing modifies on one hand the observed size of the solid angle in which we count how many galaxies we detect, consequently diluting the number of galaxies per unit of solid angle. On the other hand, weak lensing modifies the observed luminosity of galaxies, enhancing consequently the number of galaxies that are above the magnitude threshold of a given survey. These two effects combine to distort the number counts of galaxies.

One challenge in measuring cosmic shear comes from the fact that it requires precise images of galaxies. Magnification bias has the advantage of not relying on precise imaging, since the effect is measured from the galaxy number counts. However, it is affected by intrinsic fluctuations in the number counts of galaxies, which generally strongly dominate over magnification bias. This can be overcome by correlating galaxies at widely separated redshifts. In this case, density fluctuations become uncorrelated, and the only correlated signal comes from lensing. Magnification bias has been robustly measured using this technique: for example has measured the cross-correlation of quasars at redshift 1 < z < 2.2, with foreground galaxies at mean redshift 0.3 in SDSS; whereas has used the cross-correlation of background galaxies at redshift 0.7 < z < 1 with foreground galaxies at 0.2 < z < 0.4 in the Dark Energy Survey (DES).

A new approach to map the large-scale structure of the Universe up to high redshifts is intensity mapping with radio telescopes or interferometers. These surveys will observe the intensity fluctuations of some emission line, typically the 21 cm line emitted by neutral hydrogen that is expected to trace the fluctuations in the galaxy distribution. Various existing or planned post-reionisation radio surveys, like BINGO, HIRAX, CHIME, MeerK- LASS and the SKA will detect these fluctuations and measure the power spectrum of the 21 cm radiation.

In this letter we propose a novel method to measure gravitational lensing with magnification bias, by correlating the fluctuations in 21 cm intensity mapping (or intensity mapping of other lines) with the galaxy number counts, in such a way as to isolate gravitational lensing. The main idea is that gravitational lensing affects the galaxy number counts, but has no impact on intensity mapping – at least at linear order in perturbation

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As a consequence, the following schematic estimator isolates the gravitational lensing contribution

\[ \text{IM}(z_f) \times \text{galaxy}(z_b) - \text{galaxy}(z_f) \times \text{IM}(z_b), \]  

where \( z_b \) refers to the background redshift, and \( z_f < z_b \) is the foreground redshift. In the first term of Eq. (1), galaxies in the background are lensed by the presence of foreground matter perturbations, that are responsible for the 21 cm signal. In the second term, however, the 21 cm intensity in the background is not lensed. By subtracting the two terms, we cancel the density-density correlations that affect both terms in the same way (up to bias differences), while keeping the gravitational lensing contribution. This method, therefore, provides a way to isolate gravitational lensing, without restricting ourselves to wide redshifts separations. In the next section, we elaborate on this idea, and we show how our estimator increases the signal-to-noise of gravitational lensing by a factor of \( \sim 30 \) for cosmic variance dominated surveys, and a factor of 4-5 for specific examples like DES×HIRAX and Euclid×HIRAX.

Estimator. The galaxy number counts in direction \( \mathbf{n} \) and redshift \( z \) are given by

\[ \Delta_g(\mathbf{n}, z) = b_g(z) \left[ \delta(\mathbf{n}, z) + (2 - 5s(z)) \phi(\mathbf{n}, z) \right], \]  

where \( b_g \) is the galaxy bias, \( s \) is the slope of the luminosity function and \( \phi \) denotes the density matter fluctuations. The second contribution is the so-called magnification bias contribution, proportional to the lensing potential

\[ \phi(\mathbf{n}, z) = -\int_0^r dr' \frac{r - r'}{r'^2} \Delta_\Omega(\Phi + \Psi), \]  

with \( r \) the conformal distance to the source, \( \Phi \) and \( \Psi \) the two metric potentials, and \( \Delta_\Omega \) the angular part of the Laplacian.

Intensity mapping is generically expressed in terms of the brightness temperature, whose fluctuations are a biased tracer of matter density

\[ \Delta_{\text{HI}}(\mathbf{n}, z) = b_{\text{HI}}(z) \delta(\mathbf{n}, z), \]  

where \( b_{\text{HI}} \) is the bias of neutral hydrogen. We neglect in Eq. (4) and (1) the contribution from redshift space distortions since we will average our estimator over thick redshift bins. We also neglect the contribution from relativistic effects [13] [15] [18] which are subdominant in the regime we are interested in.

We can expand the number counts and the brightness temperature fluctuations in spherical harmonics

\[ \Delta_X(\mathbf{n}, z) = \sum_{\ell m} a^{X}_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \]  

with \( X = g, \text{HI} \). We now define our estimator which cross-correlates galaxies and 21 cm intensity mapping:

\[ \hat{E}_\ell^x \equiv \hat{C}^{g\text{HI}}(z_f, z_b) - \hat{C}^{g\text{HI}}(z_f, z_b) \]

\[ = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} \left[ a^{\text{HI}}_{\ell m}(z_f) a^{g}_{\ell m}(z_b) - a^{g}_{\ell m}(z_f) a^{\text{HI}}_{\ell m}(z_b) \right]. \]

Here \( \hat{C}_\ell \) is a general estimator for the angular power spectrum, and the second equality holds for the standard full-sky estimator. The expectation value of \( \hat{E}_\ell^x \) is

\[ E_\ell^x \equiv \langle \hat{E}_\ell^x \rangle = \frac{1}{2} b_{\text{HI}}(z_f)(2 - 5s(z_b)) \mathcal{C}^{\phi\phi}(z_f, z_b) \]

\[ - \frac{1}{2} b_{\text{HI}}(z_b)(2 - 5s(z_f)) \mathcal{C}^{\phi\phi}(z_f, z_b) \]

\[ + \left[ b_{\text{HI}}(z_f)b_g(z_b) - b_g(z_f)b_{\text{HI}}(z_b) \right] \mathcal{C}^{\phi\phi}(z_f, z_b). \]

The first line is the contribution we want to measure: it represents the lensing potential of background galaxies generated by a foreground density at \( z_f \). The second line, which contains the correlation between the lensing potential in the foreground and the density in the background, is negligible. The last line is a residual contamination from density fluctuations. If the two biases would have the same redshift dependence, this term would exactly vanish, allowing us to perfectly isolate gravitational lensing. In practice however the two biases evolve differently, and a small density contribution remains.

We compare our estimator with the standard estimator used to measure magnification bias \( \hat{E}_\ell = \mathcal{C}^{\phi\phi}(z_f, z_b) \), whose expectation value is

\[ E_\ell^x \equiv \langle \hat{E}_\ell^x \rangle = \frac{1}{2} b_g(z_f)(2 - 5s(z_b)) \mathcal{C}^{\phi\phi}(z_f, z_b) \]

\[ + \frac{1}{2} b_g(z_b)(2 - 5s(z_f)) \mathcal{C}^{\phi\phi}(z_f, z_b) \]

\[ + \frac{1}{4} (2 - 5s(z_f))(2 - 5s(z_b)) \mathcal{C}^{\phi\phi}(z_f, z_b) \]

\[ + b_g(z_f)b_g(z_b) \mathcal{C}^{\phi\phi}(z_f, z_b). \]

The first and third line correspond to the lensing signal that we want to measure. The third line is due to the fact that both the background and foreground galaxies are lensed by the same structures in front of the foreground galaxies. This contribution is absent in \( \hat{E}_\ell^x \) because 21 cm is not lensed. As before, the second line is negligible. Finally the last line represents the contamination from density fluctuations. The standard way of minimising this contamination consists of choosing \( z_b \) and \( z_f \) sufficiently far away to make it negligible.

Contamination and signal-to-noise ratio. Let us now study two questions: does our estimator reduce the con-
tamination from density fluctuations? And does our estimator improve the SNR of lensing? The standard estimator is indeed constructed to minimize the density contamination in the signal, but this contamination reappears in the variance, where it dominates. We will see that our estimator has the advantage of strongly reducing the density contribution also in the variance, consequently increasing the SNR.

We split the signal into a lensing contribution, that we want to measure, and the contamination from density: \( E_\ell^x = E_\ell^x \text{len} + E_\ell^x \text{cov} \) and \( E_\ell^s = E_\ell^s \text{len} + E_\ell^s \text{cov} \). The lensing contribution corresponds to the terms in Eqs. (7) and (8) that involve the lensing potential \( \phi \), while the contamination are the terms proportional to \( C_\ell^{\delta \delta} \). To give a quantitative example of how the contamination is reduced for \( E_\ell^x \), we specifically use the bias and slope \( s \) of the optical survey DES\(^2\) and of HIRAX\(^3\). For the cosmological parameters, we use throughout the paper the values from Planck\(^22\). For redshift pairs separated by \( \Delta z = 0.25 \), we find typically that the contamination is about 1% for \( E_\ell^x \), whereas it is 30–40% for \( E_\ell^s \). A figure can be found in the supplemental material. The estimator \( \hat{E}_\ell^x \) allows us therefore to extract lensing from closer pairs than \( \hat{E}_\ell^s \). This is due to the fact that in \( \hat{E}_\ell^x \), the contamination is doubly suppressed: first by the fact that the density correlation quickly decreases with redshift separation, and second by the bias difference. The second suppression is especially effective at small redshift separation, when the bias has not evolved much between \( z_f \) and \( z_b \).

We give the full expression of the covariance due to cosmic variance in the supplemental material. Here we discuss the diagonal part of the covariance, given by \( z'_f = z_f \) and \( z'_b = z_b \), corresponding to the variance for the redshift pair \( (z_f, z_b) \). It is dominated by the density contribution taken at the same redshift. Neglecting the lensing contribution, which is sub-dominant in those terms, we obtain for the standard estimator

\[
\text{var}[\hat{E}_\ell^x (z_f, z_b)] \simeq \frac{b_f^2(z_f) b_g^2(z_b)}{(2\ell + 1)f_{\text{sky}}} C_\ell^{\bar{\delta} \delta}(z_f) C_\ell^{\bar{\delta} \delta}(z_b),
\]

and for our new estimator

\[
\text{var}[\hat{E}_\ell^x (z_f, z_b)] \simeq \frac{1}{(2\ell + 1)f_{\text{sky}}} \times [b_{\text{HI}}(z_f)b_g(z_b) - b_g(z_f)b_{\text{HI}}(z_b)]^2 C_\ell^{\bar{\delta} \delta}(z_f) C_\ell^{\bar{\delta} \delta}(z_b).
\]

This confirms that \( \hat{E}_\ell^x \) has the advantage of suppressing the density contribution not only in the mean of the estimator, but also in its variance, thanks to the bias difference which appears in Eq. (10).

\(^2\) https://www.darkenergysurvey.org/ For the bias we used \(^1\) for a we followed \(^20\), who used the fitting formula derived in \(^21\) for the Euclid photometric survey \(^22\).

\(^3\) https://gitlab.com/radio-fisher/bao21cm/tree/master/radiofisher

FIG. 1. Top panel: |SNR| per \( \ell \) mode for DES×HIRAX with cosmic variance only, for the redshift pair \( z_f = 0.8 \) and \( z_b = 1.3 \). Bottom panel: Same as the top panel but including shot-noise and thermal noise using the three cases discussed in the text.

In Fig. 1 (top panel) we plot the SNR per multipole \( \ell \) of \( \hat{E}_\ell^x \) and \( \hat{E}_\ell^s \) for \( z_f = 0.8 \) and \( z_b = 1.3 \), for DES×HIRAX. We assume a sky coverage of 5000 deg\(^2\) for both estimators, since HIRAX will overlap with DES. For this case, the contamination is less than 0.01% in both estimators, so that the signal is simply given by the lensing term. The cumulative SNR, for this redshift pair, from \( \ell_{\text{min}} = \pi / \theta_{\text{sky}} = 5 \) to \( \ell_{\text{max}} = 1000 \) is 2.4 for \( \hat{E}_\ell^s \) and 54 for \( \hat{E}_\ell^x \). If we reduce \( \ell_{\text{max}} \) to 200, to exclude non-linear scales, the cumulative SNR is 0.6 for \( \hat{E}_\ell^s \) and 12 for \( \hat{E}_\ell^x \). Our estimator therefore improves the detection of gravitation lensing by a factor of \( \sim 20 \) with respect to the conventional method.

The SNR calculated above corresponds to a survey which is cosmic variance limited over the whole range of multipoles. In reality, two additional sources of errors contribute to the variance. First, galaxies are discrete objects, which generate a shot noise contribution to the variance. Shot noise affects both the galaxy number counts and 21 cm intensity mapping. However, for the latter it has been shown that shot noise is always negligible with respect to the interferometer noise \(^24\). As a consequence we simply replace in the expression for the covariance \( C_\ell^{\delta \delta}(z, z') \rightarrow C_\ell^{\delta \delta}(z, z') + \delta z, z'/\bar{n}(z) \), where \( \bar{n} \)
denotes the mean number of galaxies per redshift bin and per steradian.

For interferometer noise, we concentrate on the Hydrogen Intensity mapping and Real time Analysis experiment (HIRAX) which will measure the neutral hydrogen distribution in the redshift range of $z \sim 0.8$ to 2.5 covering 15000 square degrees of the southern sky [9]. In the literature we can find several expressions for the noise: two widely used prescriptions are [24] and [26, 27]. We discuss them in some detail in the supplemental material. We find that for the specific case of HIRAX the two expressions differ by four orders of magnitude, which has a significant impact on the forecasts. The main difference is that [24] assumes that each field of view is observed sequentially, whereas [26, 27] assume that the whole sky is observed at once. In our forecasts, we show results for both scenarios, the first one labelled as “pessimistic” and the second one as “optimistic”. Finally, preliminary simulations of the HIRAX interferometer noise based on [28, 29] find a noise curve which is about a factor 10 better than the pessimistic scenario. We also include results for this case that we label as “realistic”.

In Fig. 1 (bottom panel), we plot the SNR for $\hat{E}_x^\ell$ and $\hat{E}_t^\ell$, including shot noise and interferometer noise, for the three cases discussed above. We use the galaxy number density from DES [19]. We see that shot noise and interferometer noise significantly reduce the SNR of $\hat{E}_x^\ell$ at large $\ell$. Since cosmic variance is larger for $\hat{E}_t^\ell$, the impact of shot noise is less relevant for this estimator. The two vertical lines in Fig. 1 correspond respectively to $\ell_{\text{min}} = \pi/\theta_{\text{sky}} = 5$ and $\ell_{\text{min}} = \pi/\theta_{\text{FOV}} = 99$. The latter applies in the case where the calibration of each field of view (FOV) is not known, such that only modes smaller than the FOV of the interferometer can be observed. The cumulative SNR up to $\ell_{\text{max}} = 1000$ is 3.9 for $\hat{E}_x^\ell$ and 2.0 for $\hat{E}_t^\ell$. For $\ell_{\text{max}} = 200$ we find 2.4 for $\hat{E}_x^\ell$ and 0.6 for $\hat{E}_t^\ell$. This improvement by a factor of 4 allows a marginal detection of lensing with $\hat{E}_x^\ell$ from a single redshift pair, for which $\hat{E}_x^\ell$ cannot detect anything. This is particularly useful to follow the redshift evolution of dark energy or modified gravity.

**Forecasts on the lensing amplitude $A_L$.** As an application of our novel estimator, we now forecast the precision with which we will be able to measure the amplitude of the lensing potential $\phi$. For this we replace $\phi \rightarrow A_L \cdot \phi$ in Eq. (5) and we forecast the error on $A_L$, with fiducial value $A_L = 1$. We fix all cosmological and astrophysical parameters to their fiducial value, and we compute the Fisher element for $A_L$

$$\mathcal{F}_{A_L}^x = \sum_{z_f,z_b} \sum_{\ell_{\text{min}}} \frac{\partial E_x^\ell}{\partial A_L}(z_f,z_b) \cdot \frac{\partial E_x^\ell}{\partial A_L}(z_f',z_b') \cdot \text{Cov}^{-1}[E_x^\ell(z_f,z_b),E_x^\ell(z_f',z_b')], \quad (11)$$

and similarly for $\hat{E}_t^\ell$. We use Gaussian redshift bins of width $\sigma_z = 0.05$, spaced by $\Delta z = 0.1$, and we sum over all possible pairs of redshifts accessible in the survey, with the condition that the contamination for each pair is below 1%. This means that in both cases, we exclude pairs with redshift difference smaller than 0.3. For $\hat{E}_x^\ell$ we could consider pairs down to a difference of 0.25 but that would require finer redshift bins than the current analysis. This negligible amount of contamination ensures that we measure $A_L$ in a model-independent way, i.e. without having to model the evolution of density fluctuations.

We compute the Fisher matrix for both estimators for two combinations of surveys: DES×HIRAX, and Euclid (photometric)×HIRAX. For the second case, we use $f_{\text{sky}} = 3/2$, $f_{\text{sky}}$, Euclid where $f_{\text{sky}}$, Euclid ≃ 0.36) for $E_x^\ell$ since HIRAX will not completely overlap with Euclid. We focus on the common redshift range, which is $z \in [0.8, 1.3]$ for DES×HIRAX and $z \in [0.8, 2.5]$ for Euclid×HIRAX. For the standard estimator we also use this range as a point of comparison. Our goal is indeed to understand how $E_x^\ell$ improves over $\hat{E}_x^\ell$ over the common range. An ideal analysis would then combine $\hat{E}_x^\ell$.
over the common range, with $\hat{E}^\text{int}$ over the rest of the optical range. We restrict the $\ell$-range to $\ell_{\text{max}} = 200$, where linear perturbation theory is valid.

In Fig. 2 we show the precision on the measurement of $A_L$, $\sigma_{A_L} = 1/\sqrt{F_{A_L}}$, as a function of the maximum redshift included in Eq. (11), for DES×HIRAX (top panel) and Euclid×HIRAX (bottom panel). For the case $\ell_{\text{min}} = 5$, we see that our estimator allows to measure the lensing amplitude with a precision of 0.3 (using the realistic noise curve). This corresponds to an improvement of a factor 4.7 compared to the standard estimator in that same redshift range. If we use $\ell_{\text{min}} = \pi/\theta_{\text{FOV}}$, the improvement is slightly smaller, but still interesting: 3.7 in the realistic case. This clearly shows that, over the common redshift range, our estimator is an optimal tool to measure gravitational lensing. To reach a similar improvement using a single survey we would need to increase the sky coverage by a factor 14 in that redshift range.

Using Euclid×HIRAX, we reach $\sigma_{A_L} = 0.05$ for $z_{\text{max}} = 1.6$ and $\sigma_{A_L} = 0.02$ if we include pairs up to $z_{\text{max}} = 2.5$ (realistic case, and $\ell_{\text{min}} = 5$). Comparing with the standard estimator, in the same redshift range, we find an improvement of a factor 3.6 at $z_{\text{max}} = 1.6$, whereas at $z_{\text{max}} = 2.5$ the standard estimator is slightly better. This is due to the fact that at high redshift, the signal becomes larger for $\hat{E}^\text{int}$ than for $\hat{E}^\text{vis}$, due to the lensing-lensing contribution, $C_{\phi\phi}$, which strongly increases with redshift and which is present in $\hat{E}^\text{int}$ but not in $\hat{E}^\text{vis}$. Our estimator is therefore mainly valuable at intermediate redshift, where it allows a clean and better measurement of $C_{\ell}^{\phi\phi}$ on its own. Moreover, combining the two estimators would provide separate measurements of $C_{\ell}^{\phi\phi}$ and $C_{\ell}^{\phi\phi}$, which is particularly useful to measure the evolution of the lensing potential, and study the impact of dark energy and modified gravity as a function of redshift.

Conclusion. We have constructed a new estimator to measure weak gravitational lensing, using the correlation of galaxy clustering and 21 cm intensity mapping. Our estimator improves over standard magnification bias estimators from galaxy clustering in several ways. First, it allows us to significantly reduce the contamination from density correlations, when the foreground and background redshifts are close. Second, the cosmic variance of our new estimator is significantly reduced with respect to the standard estimator. For cosmic variance limited surveys the new estimator improves the SNR by a factor of 30, compared to the standard estimator. Shot noise and interferometer noise reduce this improvement, but it still reaches a factor 4-5 at intermediate redshift. Finally, as our estimator involves only cross-correlations, we expect it to exhibit an improved resistance to systematic effects from galaxy surveys, which should be uncorrelated with intensity mapping fluctuations. Note that it is possible to extend the estimator to remove the density contribution completely, a further development that we will discuss in a future publication. This letter already shows that the fundamental idea of combining intensity mapping and galaxy surveys to isolate lensing holds considerable promise for upcoming surveys.

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Appendix A: Covariances

Here we give the full covariances from cosmic variance for the two estimators. For the standard estimator, $E^\text{vis}_\ell$, it reads
\[ \text{cov} \left[ \hat{E}^\text{int}_\ell(z_f, z_b) \hat{E}^\text{vis}_\ell(z'_f, z'_b) \right] = \frac{1}{(2\ell + 1)f_{\text{sky}}} \times \left[ C^{gg}_\ell(z_f, z'_f)C^{gg}_\ell(z_b, z'_b) + C^{\phi\phi}_\ell(z_f, z'_f)C^{\phi\phi}_\ell(z_b, z'_b) \right]. \] (A1)

For our cross-estimator, $E^\text{int}_\ell$, we obtain
\[ \text{cov} \left[ \hat{E}^\text{int}_\ell(z_f, z_b) \hat{E}^\text{int}_\ell(z'_f, z'_b) \right] = \frac{1}{(2\ell + 1)f_{\text{sky}}} \times \left[ C^{HHH}_\ell(z_f, z'_f)C^{gg}_\ell(z_b, z'_b) \right. \]
\[ \left. - C^{HHH}_\ell(z_f, z'_f)C^{\phi\phi}_\ell(z_b, z'_b) + C^{\phi\phi}_\ell(z_f, z'_f)C^H_{\ell \nu}(z_b, z'_b) \right. \]
\[ \left. + C^{\phi\phi}_\ell(z_f, z'_f)C^{\phi\phi}_\ell(z_b, z'_b) \right]. \] (A2)

Appendix B: Contamination

In Fig. 3 we show a plot of the ratio of contamination for the case DES×HIRAX and for the specific redshift pair $z_f = 1$ and $z_b = 1.25$.

Appendix C: Interferometer noise

Reference [25] gives the noise spectrum as
\[ C^{\text{interf}}_\ell(z) = \frac{T^2_{\text{sys}} S_{\text{area}} \lambda(z)^4}{n_{\text{pol}} t_{\text{tot}} \Delta \nu N_{\text{beam}} A_{\text{eff}}^2 \theta_b^2 n(u = \ell/2\pi)}, \] (C1)
where $T_{\text{sys}} = T_{\text{antenna}} + T_{\text{sky}}$ is the addition of antenna and sky temperature, $S_{\text{area}} = 4\pi f_{\text{sky}}$ is the observed area.
of the sky, $\lambda(z)$ is the observed wavelength of 21cm line

total observation time, $\Delta \nu$ is the frequency bin corresponding to the redshift bin width, $N_{\text{beam}}$ is the beam number, $A_{\text{eff}} = 0.7\pi D_{\text{dish}}^2/4$ is the effective area of each dish and the factor 0.7 is the efficiency of the dish, $\theta_0 = D_{\text{dish}}/\lambda$ is the beam of the telescope, and $n(u)$ is the number density of baselines in the $uv$ plane. Expression (C1) assumes that each field of view (FOV) of the interferometer is observed sequentially. If on the other hand one assumes that the whole sky area is observed at once (instantaneous FOV), we obtain the expression presented in [26, 27]

$$C_{\ell}^{\text{interf}} = \frac{(2\pi)^3 T^2_{\text{sys}}}{\Delta \nu t_{\text{tot}} f_{\text{cover}} \ell_{\text{max}}^2 (\nu)^2},$$

where $\ell_{\text{max}} = 2\pi D_{\text{tel}}/\lambda(z)$ and $D_{\text{tel}}$ is the diameter of the telescope array, $f_{\text{cover}} = N_{\text{dish}} A_{\text{eff}}/(\pi D_{\text{tel}}^2/4)$ is the effective collecting area of the telescope. For HIRAX the difference between the two noise curves is of the order of $10^4$.

[1] D. J. Bacon, A. R. Refregier, and R. S. Ellis, Mon. Not. Roy. Astron. Soc. 318, 625 (2000), arXiv:astro-ph/0003008 [astro-ph]
[2] D. M. Wittman, J. A. Tyson, D. Kirkman, A. Challinor, and A. Challinor, Phys. Rev. D87, 043528 (2013), arXiv:1212.0728 [astro-ph.CO]
[3] M. A. Troxel et al. (DES), Phys. Rev. D98, 043528 (2018), arXiv:1708.01538 [astro-ph.CO]
[4] H. Hildebrandt, F. Kohlinger, J. L. van den Busch, B. Joachimi, C. Heymans, A. Kannawadi, A. H. Wright, M. Asgari, C. Blake, and H. Hoekstra, arXiv e-prints , arXiv:1812.06076 (2018), arXiv:1812.06076 [astro-ph.CO]
[5] R. Scranton et al. (SDSS), Astrophys. J. 633, 589 (2005), arXiv:astro-ph/0504510 [astro-ph]
[6] M. Garcia-Fernandez et al. (DES), Mon. Not. Roy. Astron. Soc. 476, 1071 (2018), arXiv:1801.10326 [astro-ph.CO]
[7] J. R. Pritchard and A. Loeb, Rept. Prog. Phys. 75, 086901 (2012), arXiv:1109.6012 [astro-ph.CO]
[8] R. A. Battye, I. W. A. Browne, C. Dickinson, G. Heron, B. Maffei, and A. Pourtsidou, Mon. Not. Roy. Astron. Soc. 434, 1239 (2013), arXiv:1209.0543 [astro-ph.CO]
[9] L. B. Newburgh et al., Proceedings, Ground-based and Airborne Telescopes VI: Edinburgh, United Kingdom, June 26-July 1, 2016, Proc. SPIE Int. Soc. Opt. Eng. 9906, 99065X (2016), arXiv:1607.02059 [astro-ph.IM]
[10] L. B. Newburgh et al., Proc. SPIE Int. Soc. Opt. Eng. 9145, 4V (2014), arXiv:1406.2267 [astro-ph.IM]
[11] M. G. Santos et al. (MeerKLAAS), in Proceedings, MeerKAT Science: On the Pathway to the SKA (MeerKAT2016): Stellenbosch, South Africa, May 25-27, 2016 (2017), arXiv:1709.06099 [astro-ph.CO]
[12] P. Bull, Astrophys. J. 817, 26 (2016), arXiv:1509.07562 [astro-ph.CO]
[13] A. Hall, C. Bonvin, and A. Challinor, Phys. Rev. D87, 064026 (2013), arXiv:1212.0728 [astro-ph.CO]
[14] M. Jalilvand, E. Majerotto, R. Durrer, and M. Kunz, JCAP 1901, 020 (2019), arXiv:1807.01351 [astro-ph.CO]
[15] J. Yoo, A. L. Fitzpatrick, and M. Zaldarriaga, Phys. Rev. D80, 083514 (2009), arXiv:0907.0707 [astro-ph.CO]
[16] C. Bonvin and R. Durrer, Phys. Rev. D84, 063505 (2011), arXiv:1105.5280 [astro-ph.CO]
[17] A. Challinor and A. Lewis, Phys. Rev. D84, 043516 (2011), arXiv:1105.5292 [astro-ph.CO]
[18] D. Jeong, F. Schmidt, and C. M. Hirata, Phys. Rev. D85, 023504 (2012), arXiv:1107.5427 [astro-ph.CO]
[19] A. Font-Ribera, P. McDonald, N. Mostek, B. A. Reid, H.-J. Seo, and A. Slosar, JCAP 1405, 023 (2014), arXiv:1308.4164 [astro-ph.CO]
[20] S. Yang and A. R. Pullen, Mon. Not. Roy. Astron. Soc. 481, 1441 (2018), arXiv:1807.05639 [astro-ph.CO]
[21] F. Montanari and R. Durrer, JCAP 1510, 070 (2015), arXiv:1506.01369 [astro-ph.CO]
[22] R. Laurejs, J. Amiaux, S. Ardini, J. L. Augerues, J. Brinchmann, R. Cole, M. Cropper, C. Dabin, L. Duvert, and A. Ealet, arXiv e-prints , arXiv:1110.3193 (2011), arXiv:1110.3193 [astro-ph.CO]
[23] Planck Collaboration, N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A. J. Banday, R. B. Barreiro, and N. Bartolo, arXiv e-prints , arXiv:1105.3026 (2011), arXiv:1105.3026 [astro-ph.CO]
[24] E. Castorina and F. Villasecas-Navarro, Mon. Not. Roy. Astron. Soc. 471, 1788 (2017), arXiv:1609.05157 [astro-ph.CO]
[25] P. Bull, P. G. Ferreira, P. Patel, and M. G. Santos, Astrophys. J. 803, 21 (2015), arXiv:1405.1452 [astro-ph.CO]
[26] M. Zaldarriaga, S. R. Furlanetto, and L. Hernquist, Astrophys. J. 608, 622 (2004), arXiv:astro-ph/0311514 [astro-ph]
[27] A. Pourtsidou and R. B. Metcalf, Mon. Not. Roy. Astron. Soc. 439, L36 (2014), arXiv:1311.4484 [astro-ph.CO].

[28] J. R. Shaw, K. Sigurdson, U.-L. Pen, A. Stebbins, and M. Sitwell, Astrophys. J. 781, 57 (2014), arXiv:1302.0327 [astro-ph.CO].

[29] J. R. Shaw, K. Sigurdson, M. Sitwell, A. Stebbins, and U.-L. Pen, Phys. Rev. D91, 083514 (2015), arXiv:1401.2095 [astro-ph.CO].