EXCLUSIVE PROTON–ANTIPROTON ANNIHILATION INTO TWO PHOTONS AT LARGE $S$ \\

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Exclusive proton–antiproton annihilation into two photons can be viewed as the Compton process in the crossed channel. At large $s \approx 10 \text{ GeV}^2$ and $|t|, |u| \sim s$ this process can be described by a generalized partonic picture, analogous to the “soft mechanism” in wide-angle real Compton scattering. The two photons are emitted in the annihilation of a single fast quark and antiquark (“handbag graph”). The transition of the $p\bar{p}$ system to a $\bar{q}q$ pair through soft interactions is described by double distributions, which can be related to the timelike proton elastic form factors as well as, by crossing symmetry, to the usual quark–antiquark distributions in the nucleon. We estimate that this reaction should be observable with reasonable statistics at the proposed 1.5...15 GeV high-luminosity antiproton storage ring (HESR) at GSI. (Talk presented at the Workshop on Exclusive Processes at High Momentum Transfer, Jefferson Lab, Newport News, VA, May 15–18, 2002) 

Compton scattering, i.e., the scattering of real or virtual photons off hadrons, has proven to be a useful tool for investigating the structure of the nucleon. Of particular interest are certain extreme kinematical situations where the reaction mechanism simplifies. In the limit of large virtuality of the incoming photon and fixed $t$ (“deeply virtual Compton scattering”) QCD implies that the amplitude factorizes into a hard photon–quark amplitude and a generalized parton distribution, representing the information about the structure of the nucleon. Similarly, it has been argued that real Compton scattering at large $s \approx 10 \text{ GeV}^2$ and $|t|, |u| \sim s$ (wide–angle scattering) is dominated by contributions in which the photon scatters off a single quark/antiquark in the nucleon (“handbag graph”). The soft interactions responsible for the emission and absorption of the active quark/antiquark by the nucleon are parametrized by double distributions, which can be related to both the usual quark/antiquark distributions measured in inclusive deep–inelastic scattering and the elastic form factors of the proton. This so-called “soft mechanism” describes well the existing
data for the total cross section,\textsuperscript{4} and also recent results for the spin asymmetry from the JLAB Hall A experiment.\textsuperscript{5} The hard scattering mechanism, in which the struck quarks rescatters via gluon exchange of virtuality $\sim t$, is relevant only at asymptotically large $t$ and falls short of the measured cross section at JLAB energies.\textsuperscript{1}

Exclusive proton–antiproton annihilation into two photons, $p\bar{p} \rightarrow \gamma \gamma$, can be regarded as the Compton process in the crossed channel. This reaction could be studied with the proposed high–luminosity 1.5...15 GeV antiproton storage ring (HESR) at GSI.\textsuperscript{6} In this talk I would like to argue that at large $s$ and $|t|, |u| \sim s$ (wide–angle scattering) this process can be described by a generalized partonic picture analogous to the “soft mechanism” in wide–angle real Compton scattering. The results reported here have been obtained in collaboration with A. Freund (Regensburg U.), A. V. Radyushkin (Jefferson Lab and Old Dominion U.), and A. Schäfer (Regensburg U.). A detailed account will be published shortly.\textsuperscript{7} Similar ideas have been presented by P. Kroll at this meeting.\textsuperscript{8,9}

To motivate the partonic picture of proton–antiproton annihilation let us recall $e^+e^-$ annihilation into two photons in QED. This process proceeds via the $t$ (or $u$) channel exchange of a virtual electron/positron with spacelike momentum. In $p\bar{p} \rightarrow \gamma \gamma$ annihilation in QCD, the exchanged system consists of at least three quarks. At large momentum transfer such an exchange should be strongly suppressed by the proton and antiproton wave functions. In this situation the most efficient way of accommodating a large momentum transfer is the handbag diagram shown in Fig. 1. In the first part the proton–antiproton system makes a transition to a quark–antiquark pair by exchanging a virtual $qq$ (“diquark”) type system, denoted by the blob in the diagram, whose spacelike virtuality is limited by the bound–state wave functions. In the second part, the quark–antiquark pair annihilates into two photons by exchanging a highly virtual quark/antiquark, exactly as in $e^+e^-$ annihilation in QED. This picture is consistent: Because of the limit on the virtuality of the exchanged diquark–type system the active quark (antiquark) carries a significant fraction of the proton (antiproton).
momentum, which in turn makes for a large virtuality in the quark propagator connecting the photon vertices. In short, our picture states that the dominant contribution to exclusive $p\bar{p} \rightarrow \gamma\gamma$ at large $s,|t|$ and $u$ comes from the annihilation of “fast” quarks and antiquarks in the proton and antiproton, respectively.

The matrix element describing the transition of the $p\bar{p}$ system to a $q\bar{q}$ pair through “soft” interactions can be parametrized by double distributions, which are the timelike analogue of the functions parametrizing the matrix element in wide–angle Compton scattering. These distributions measure the momenta of the active quark and antiquark in terms of the proton and antiproton momenta, or, equivalently, their average and difference, $p$ and $r$, see Fig. 2. Transverse momenta are neglected here, and the variables $\tilde{x}$ and $\alpha$ obey the “partonic” restrictions $|\tilde{x}| + |\alpha| < 1$. There are two types of matrix elements, corresponding to Dirac structures $(\gamma_{\mu})_{ij}$ and $(\gamma_{\mu}\gamma_5)_{ij}$ in the quark spinor indices. Following Refs.\textsuperscript{10}, we parametrize them by two double distributions $F_a(\tilde{x}, \alpha; s)$ and $G_a(\tilde{x}, \alpha; s)$, depending on the partonic variables $\alpha$ and $\tilde{x}$ as well as on $s$ (the subscript $a = u, d$ represents the quark flavor); details will be given elsewhere.\textsuperscript{7} When integrating over the spectral variables and summing over the contributions of the different quark flavors, the new double distribution $F_a(\tilde{x}, \alpha; s)$ reduces to the timelike proton Dirac form factor,

$$\sum_a e_a^2 \int \int_{|\tilde{x}| + |\alpha| < 1} d\tilde{x} d\alpha F_a(\tilde{x}, \alpha; s) = F_1(s); \quad (1)$$

a similar relation holds for $G_a(\tilde{x}, \alpha; s)$ and the axial nucleon form factor. (For simplicity we neglect here components of the matrix element corresponding to the Pauli and pseudoscalar form factors; these components should be included in a more complete treatment.) Another useful relation is found in the limit $s \rightarrow 0$, where one can use crossing symmetry to relate the annihilation–type double distribution of Fig. 2 to the usual scattering–type double distributions of wide–angle Compton scattering. In particular,
this implies
\[
\int_{-1+|\tilde{x}|}^{1-|\tilde{x}|} d\alpha \ F_a(\tilde{x}, \alpha; s = 0) = f_a(\tilde{x}),
\] (2)
where \( f_a(\tilde{x}) = \theta(\tilde{x}) q_a(\tilde{x}) - \theta(\tilde{x}) \bar{q}_a(-\tilde{x}) \) is the usual unpolarized quark/antiquark distribution of flavor \( a \) in the proton, as measured in deep-inelastic scattering. In a similar way the function \( G_a(\tilde{x}, \alpha; s) \) reduces to the polarized distribution. These “reduction relations” provide constraints for models of the double distributions.

To construct an explicit model for the double distributions we factorize them into an \( s \)-independent double distribution and a “cutoff function” containing the \( s \)-dependence,
\[
F_a(\tilde{x}, \alpha; s) = f_a(\tilde{x}, \alpha) S(\tilde{x}, \alpha; s).
\] (3)

The \( s \)-independent double distribution we model as \( f_a(\tilde{x}, \alpha) = f_a(\tilde{x}) h(\tilde{x}, \alpha) \), where \( f_a(\tilde{x}) \) is the usual quark/antiquark distribution, and \( h(\tilde{x}, \alpha) \) a normalized profile function; a particularly simple choice is \( h(\tilde{x}, \alpha) = \delta(\alpha). \) The distributions refer to a scale of the order \( |t| \sim 1 \text{ GeV}^2 \).

The cutoff function \( S(\tilde{x}, \alpha; s) \), which is defined to be unity at \( s = 0 \), implements the restriction on the virtuality of the exchanged “diquark”–type system in the handbag diagram of Fig. 1:
\[
\frac{[(1 - \tilde{x})^2 - \alpha^2] s}{4\tilde{x}(1 - \tilde{x})} < \lambda^2,
\] (4)
where \( \lambda^2 \) is a parameter. For the scattering–type double distributions appearing in wide–angle Compton scattering this cutoff could be derived from the overlap of light–cone wave functions; \(^1,^2\) such an interpretation is no longer possible in the annihilation channel. In practice, we choose a Gaussian cutoff and fix the parameter \( \lambda^2 \) by fitting the proton form factor, cf. Eq.(1); for details we refer to the forthcoming publication.\(^7\)

The helicity–averaged differential cross section obtained from our simple model is
\[
\frac{d\sigma}{d \cos \theta} = \frac{2\pi\alpha_{em}^2}{s} R^2_V(s) \cos^2 \theta + R^2_A(s) \sin^2 \theta,
\] (5)
where \( \theta \) is the scattering angle in the center–of–mass system. The information about the structure of the proton is contained in generalized form factors, defined as
\[
R_V(s) = \sum_a e_a^2 \int_{|\tilde{x}|+|\tilde{\alpha}| < 1} d\tilde{x} \ d\alpha \frac{F_a(\tilde{x}, \alpha; s)}{\tilde{x}} \;
\] (6)
Figure 3. The form factors $R_V^2(s)$ (solid line) and $R_A^2(s)$ (dashed line) calculated from the double distribution model, cf. Eq.(6).

$R_A(s)$ is given by the corresponding integral over the double distribution $G_f(\tilde{x}, \alpha; s)$. The integral here differs from the one in the proton elastic form factor, Eq.(1), by an additional factor $1/\tilde{x}$ in the integrand; this is just the “remnant” of the quark propagator in the hard $q\bar{q} \to \gamma\gamma$ scattering amplitude in the handbag graph of Fig. 1. For $R_V(s) \equiv R_A(s) \equiv 1$ the expression (5) would reproduce the Klein–Nishina formula for the $e^+e^- \to \gamma\gamma$ cross section in QED. Note that our partonic picture is applicable only for $|t|, |u| \sim s$, which implies that $\theta$ should be sufficiently far from 0 or $\pi$ (wide–angle scattering). The numerical results for the form factors are shown in Fig. 3.

It is interesting to make a quick estimate of the counting rate for $p\bar{p} \to \gamma\gamma$ annihilation expected for the proposed 1.5...15 GeV antiproton storage ring (HESR) at GSI. With a fixed solid target the luminosity could be as high as $L = 2 \times 10^{32} \text{cm}^{-2} \text{s}^{-1} = 2 \times 10^6 \text{fm}^{-2} \text{s}^{-1}$. Since our partonic picture applies only for $|t|, |u| \sim s$ we have to exclude the region of $\theta$ close to 0 or $\pi$. For $s = 10 \text{GeV}^2$, the range $45^{\circ} < \theta < 135^{\circ}$ would correspond to $|t|, |u| > 1.5 \text{GeV}^2$, which is hopefully sufficient for our approximations to work. The integrated cross section over this region is $1.3 \times 10^{-10} \text{fm}^2$, which corresponds to a counting rate of $2.6 \times 10^{-4} \text{sec}^{-1}$, that is $\sim 700$ events per month. Note that already slightly lower values of $s$ would increase the counting rate dramatically, cf. Fig. 3. The process should thus be measurable with reasonable statistics at the proposed facility.

To summarize, we have discussed exclusive annihilation $p\bar{p} \to \gamma\gamma$ in a
generalized partonic picture. Our approach represents an attempt to extend the “soft mechanism”, which successfully describes wide–angle Compton scattering at JLAB energies, to the annihilation channel. The timelike double distributions describing the transition of the $p\bar{p}$ system to a $q\bar{q}$ pair are strongly constrained by data for the timelike proton form factor and the quark/antiquark distributions in the proton, which have been obtained from independent measurements. Our estimate of the cross section, based on a simple model for the double distributions, suggests that this reaction could be observed with reasonable statistics at the proposed GSI HESR facility. This would offer the exciting possibility of studying the hadronic Compton process in the annihilation channel. The results could also be compared to data on hadron production in photon–photon collisions observed in $e^+e^-$ experiments.

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