We experimentally study the motion of atoms interacting with a periodically pulsed near resonant standing wave. For discrete pulse frequencies we observe a comb-like momentum distribution. The peaks have widths of $\approx 0.3\,\hbar k$ and a spacing which is an integer multiple of the recoil momentum $\hbar k$. The atomic population is trapped in ground states which periodically evolve to dark states each time the standing wave is switched on.

Ultra cold atoms form an ideal system to study the evolution of matter waves. The cold atoms have a large spatial coherence and can be manipulated by atom-light interaction. Using laser light, motional quantum states of atoms can be precisely prepared and observed. Very recently, Bloch oscillations and the Wannier-Stark ladder have been demonstrated in fascinating experiments, where laser cooled atoms interacted with an accelerating standing wave potential. A similar system has been used to study the relation between quantum evolution and the underlying classical dynamics. These experiments were performed in a regime where dissipation due to spontaneous emission was negligible.

In this work we study the evolution of atomic matter waves interacting with a periodically pulsed standing wave in the presence of dissipation. We prepare atoms via spontaneous emission in non-stationary quantum states, which are superpositions of different momentum eigenstates. We monitor their evolution and observe revivals of states which are decoupled from the light field by varying the time between successive pulses. The periodic interaction with the standing wave leads to an accumulation of the atoms in a comb-like momentum distribution with peaks narrower than the recoil momentum of a single photon. Our scheme can thus be regarded as a new approach to sub-recoil cooling. In velocity selective coherent population trapping (VSCPT) the velocity selection is achieved during the interaction with either a continuous light field or with a velocity-selective Raman pulse. For both techniques a long interaction time with the light field results in narrow momentum distributions. In our scheme the kinetic evolution of the atoms between two successive pulses leads to the selection of sharply defined velocity classes. This is similar to Ramsey spectroscopy, where the energy difference between atomic states is determined using two or more separated interaction regions instead of a single large one.

To understand the interaction of the atoms with the pulsed standing wave let us recall the concept of dark states. Consider an atomic transition which has an integer total angular momentum $F$ both in the ground and excited state manifolds. If this transition is driven with polarized light, there is one ground state of the $2F+1$ ground states which is not coupled to the excited state manifold. This state is called a dark state and it can be populated by optical pumping.

If the atom interacts with a continuous one dimensional standing wave field with a polarization gradient a slightly more complex situation arises. We then have to include the motion of the atom along the standing wave axis. For the special case of an atom with a $F = 1 \rightarrow 1$ transition there is a dark state which is also an eigenstate of the kinetic energy. This stationary dark state facilitates VSCPT cooling. For atoms with larger angular momenta stationary dark states are not found in the considered light field, i.e. there are no dark states which are also eigenstates of the kinetic energy. Consequently the atom cannot remain decoupled from the light field.

In the case of the periodically pulsed standing wave the atom kinetically evolves in the time between successive pulses. The kinetic evolution can lead to a revival of the dark state, where the revival time depends on the atomic velocities along the standing wave axis. This revival of the dark state can be periodic. An atom can therefore propagate through the pulsed standing wave without being excited if it is in a state that evolves to a dark state each time the standing wave is switched on. These states are superpositions of states with different momenta and will be referred to as propagating dark states. They exist only for discrete pulse frequencies (for $F > 1$) and have discrete momenta. The propagating dark states are populated by optical pumping and we expect the atomic population to accumulate in these states after a sufficiently long sequence of pulses. The atomic momentum distribution will then show the momenta of propagating dark states, which results in a comb-like distribution.

To be more specific consider a $F = 2 \rightarrow 2$ transition interacting with a periodically pulsed standing wave. The
total Hamiltonian of this system is given by
\[ H = P^2/(2M) + H_A + \eta(t)V, \]
where \( P \) is the atomic momentum operator, \( M \) the atomic mass and \( H_A \) the Hamiltonian of the internal atomic states. The function \( \eta(t) \) is plotted in the lower part of Fig. 1 and describes the time dependence of the atom-light interaction \( V \). It is periodically switched on for time intervals of the length \( \tau \) separated by longer intervals of the length \( T \). The standing wave consists of two counter propagating waves with circular polarizations inducing \( \sigma^+ \) and \( \sigma^- \) transitions. The atom-light interaction can be written as a sum over different momentum families [12]:
\[ V = \frac{1}{2} \hbar \Omega e^{-\omega_L t} \sum_p V_p/\sqrt{6} + \text{h.c.,} \]
where \( \Omega \) is the resonant Rabi coupling and \( \omega_L \) the laser frequency. The term \( V_p \) describes the coherent coupling within a momentum family. It consists of a W-type and a M-type coupling scheme, as illustrated in the upper part of Fig. 1. We will neglect the W-type system (thin lines) since it does not contribute to the dark state and is depopulated by optical pumping. The M-type coupling scheme (bold lines) is given by
\[ V_p = \sqrt{2} |e_{p,-1}\rangle \langle g_{p,-2}| - \sqrt{3} |e_{p,-1}\rangle \langle g_{p,0}| + \sqrt{3} |e_{p,+1}\rangle \langle g_{p,0}| - \sqrt{2} |e_{p,+1}\rangle \langle g_{p,+2}|, \]
where \( |g_{p,m}\rangle \) (\( m = \pm 2, 0 \)) describes a ground state with the magnetic quantum number \( m \) and the momentum \( p + m \hbar k \) (where \( k \) is the wave vector). Correspondingly, \( |e_{p,m}\rangle \) (\( m = \pm 1 \)) describes an excited state with the magnetic quantum number \( m \) and the momentum \( p + m \hbar k \). The magnetic quantum numbers label the eigenstates of the projection of the total angular momentum \( \hat{F} \) on the axis of the standing wave. Different momentum families are coupled only via spontaneous emission.

Let us now consider the ground state \( \psi_p \)
\[ |\psi_p\rangle = e^{-i \phi_{p,-2}} \sqrt{\frac{2}{3}} |g_{p,-2}\rangle + e^{-i \phi_{p,0}} \sqrt{\frac{1}{2}} |g_{p,0}\rangle + e^{-i \phi_{p,+2}} \sqrt{\frac{1}{3}} |g_{p,+2}\rangle. \]
It is a superposition of three states with different momenta and different magnetic quantum numbers, which belong to the same momentum family \( p \). If the phases \( \phi_{p,0} \) and \( \phi_{p,+2} \) have the same value (modulo \( 2\pi \)), the state \( \psi_p \) is a dark state and does not couple to the light field, i.e. \( V|\psi_p\rangle = 0 \). It is not an eigenstate of the kinetic energy and therefore not a stationary dark state. The free kinetic evolution of \( \psi_p \) can be described by the time dependent phases \( \phi_{p,0}(t) \) and \( \phi_{p,+2}(t) \) which evolve according to the kinetic energy of the corresponding momentum states. These phases are given by \( \phi_{p,0}(t) = (p/(\hbar k))^2 \omega_r t \) and \( \phi_{p,\pm 2}(t) = \phi_{p,0}(t) + 4 \omega_r t \pm 4 \omega_r p/(\hbar k)t \), where \( \omega_r = \hbar k^2/(2M) \) is the recoil frequency. Here we assume that the state \( \psi_p(t) \) is in a dark state at \( t=0 \) with \( \phi_{p,0}(t=0) = \phi_{p,\pm 2}(t=0) = 0 \). After a revival time \( T \) the state \( \psi_p \) is again a dark state if the phases satisfy \( \phi_{p,0}(T) - \phi_{p,-2}(T) = 2m \pi \) and \( \phi_{p,+2}(T) - \phi_{p,0}(T) = 2\pi(m+n) \), where \( m \) and \( n \) are integer numbers. Both conditions are satisfied for times \( T_n = n \tau /8 \) (where \( \tau = 2\pi/\omega_r \) is the recoil time), if \( \psi_p \) belongs to a momentum family with \( p = m^{(n)} = \frac{m}{2} \hbar k + \hbar k \). The kinetic evolution of a state \( \psi_p \) (with \( p = m^{(n)} \)) is periodic and it is a dark state at times \( t = 0, T_n, 2T_n, 3T_n, \ldots \). If the standing wave is switched on only at these times (i.e. when \( \psi_p \) is a dark state), \( \psi_p \) is a propagating dark state and its interaction with the standing wave field vanishes. For \( n = 1 \) the propagating dark states have momenta which are odd integer multiples of \( \hbar k \). For \( n = 2 \) the propagating dark states have momenta which are even and odd integer multiples of \( \hbar k \) [13]. For \( n > 2 \) propagating dark states can also have momenta which are integer fractions of \( \hbar k \) [14].

The lifetime of a propagating dark state in the periodically pulsed standing wave is limited by the intervals \( \tau \) during which the standing wave is switched on. During this time the kinetic Hamiltonian leads to an effective coupling of the dark state to the light field [12]. This results in a lifetime \( 1/\Gamma'' \) of the dark state which is proportional to \( \frac{|g|^2}{\Omega^2} (1 + \frac{\Delta}{\Gamma})^{-1} \) (where \( \Gamma^{-1} \) is the excited state life time and \( \Delta \) the detuning between laser and atomic transition frequency). To minimize the excitation of a propagating dark state it is necessary to choose \( \tau \ll 1/\Gamma'' \). A high Rabi coupling effectively hinders the evolution of the propagating dark state [13]. We therefore expect the longest lifetime for propagating dark states when the time between successive standing wave pulses is \( T = T_n \) (rather then \( \tau + T = T_n \)). This corresponds to the assumption that we can neglect the evolution of the propagating dark state while the standing wave is switched on.

Atoms which are not in propagating dark states are excited by the pulsed standing wave. Each spontaneous emission of a photon changes the momentum of the atom, and the atom can decay to a propagating dark state. There it experiences a strongly reduced excitation rate. We therefore expect that atoms accumulate in propagating dark states during the interaction with the pulsed standing wave. The momentum distribution will then show peaks at the momenta of propagating dark states. The width of the peaks can become narrower than the single photon recoil. This velocity selectivity is due to the kinetic evolution of atoms between two pulses. Consider an atom in a state \( \psi' \) which is in a dark state at \( t=0 \) and has a center momentum \( p' = p^{(m)} + \delta p \) (\( \delta p \ll \frac{\hbar}{2} \)). During the free kinetic evolution the state \( \psi' \) will increasingly deviate from the propagating dark state \( \psi_p \).
(with $p = p_{an}$). When the standing wave is switched on again after a time $T = T_n$, the state $\psi_{p'}$ has not completely evolved to a dark state. This increases the probability that $\psi_{p'}$ is excited by the standing wave and it reduces the atomic population in the momentum family $p'$. 

To investigate the dynamics of propagating dark states, we have performed an experiment with $^{87}$Rb atoms (Fig. 3). A cloud of $\approx 10^8$ magneto-optically trapped atoms is accelerated downwards using optical molasses cooling to a moving reference frame. After 17 cm of flight the cloud arrives with a speed of 3.2 m/s and a density of $n \approx 10^8$ cm$^{-3}$ in the interaction region, which is shielded with mu-metal against magnetic fields to below 0.5 mG. On their ballistic trajectory downwards the atoms interact with a horizontally aligned $\sigma^+ - \sigma^-$ standing wave having a vertical Gaussian waist of 1.37 mm. The light field is tuned $\Delta = 14 \Gamma = 2 \pi \cdot 40$ MHz to the blue of the $F = 2 \rightarrow 2$ transition of the D$_1$ line, where $\Gamma^{-1} = 28$ ns is the life time of the excited states. Each running wave has a peak intensity of 9.4 mW/cm$^2$, which corresponds to a resonant Rabi coupling of $\Omega = 1.3 \Gamma$ and to an excitation rate of $\Gamma' = 8 \cdot 10^{-3} \Gamma = (4 \mu$s$)^{-1}$ (on the $F = 2 \rightarrow 2$ transition).

An accusto-optical modulator is used to switch the standing wave on for intervals of $\tau = \tau_n/100 = 3 \mu$s alternating with dark intervals of $T = \tau_n/8 = 35 \mu$s (corresponding to $n = 1$). During the interaction time of 1.1 ms the atoms are subjected to 28 standing wave pulse waves. To recycle atoms that have decayed to the $F = 1$ ground state manifold a continuous standing wave of 1.6 mm waist overlaps. It is tuned to the $F = 1 \rightarrow 2$ transition of the D$_2$ line. Its single pass Rabi coupling is $\Omega_{12} = 0.2 \Gamma$, which corresponds to an excitation rate of $0.021 \Gamma = (1 \mu$s$)^{-1}$. The interaction time with this additional light field is 1.5 ms. Both light fields are derived from grating stabilized laser diodes. The laser beams are spatially filtered to achieve Gaussian modes. The standing waves are formed by retroreflection off a mirror, which is 13 cm away from the interaction region. A quarter wave plate in front of the mirror is used to provide the $\sigma_+ - \sigma_-$ polarization of the standing wave.

To determine the momentum distribution we place a pinhole of 75 $\mu$m diameter 5 mm below the standing wave axis. The atoms that pass through the pinhole ($\approx 3 \cdot 10^3$ atoms) expand horizontally in two dimensions according to their transversal momentum. A transversal momentum of $1 \hbar k$ translates to a 170 $\mu$m transversal displacement in a plane 9.6 cm below the pinhole. The spatial distribution of the atoms in this plane is imaged by recording the fluorescence in a sheet of light with a CCD camera. The sheet of light is formed by a standing wave, which is resonant with the closed $F = 2 \rightarrow 3$ transition of the D$_2$ line. This allows detection of atoms in the $F = 2$ ground state. Optionally, an additional laser beam 2 mm above the sheet of light optically pumps the atoms from the $F = 1$ to the $F = 2$ ground state manifold and can be used to additionally detect atoms in the $F = 1$ ground state. We have used this beam to verify that no atoms leave the interaction region in the $F = 1$ ground state manifold. The overall momentum resolution of the detection system has been improved to $\sigma = 0.3 \hbar k$ (where $\sigma$ is the $e^{-1/2}$ half width), as compared to previous experiments done with the same apparatus [16]. The measurements presented here are integrated over 200 atom clouds extracted from the magneto-optical trap at a rate of 1 s$^{-1}$. The stray light background has been measured in an identical repetition of the experiment but without atoms and has been subtracted from the data. To yield one dimensional momentum distributions we have integrated the two dimensional images along the direction perpendicular to the cooling axis.

In Fig. 3(a) the momentum distribution measured for a pulse spacing $T = 35 \mu$s is shown. The comb-like structure with $2\hbar k$ spacing between the sub-recoil cold peaks stems from atoms trapped in propagating dark states. The peaks occur at odd multiples of $\hbar k$, as we expect for $n = 1$ ($T = 1\tau_n/8 = 35 \mu$s). Each propagating dark state contributes to three neighboring peaks. The width of these peaks is determined by a best Gaussian fit as $\sigma = 0.3 \hbar k$. The envelope of the momentum distribution has a width of $\sigma \approx 4.4 \hbar k$. It results from broadening of the initial distribution, which has for geometrical reasons a width of $\sigma = 1.7 \hbar k$ (measured with all fields permanently switched off in the interaction region). We experimentally varied the length $\tau$ of the light pulses. For pulses up to a factor of two longer the observed momentum distribution did not change significantly, whereas shorter pulses lead to a reduced contrast in the momentum spectrum. This is in agreement with the calculated $\Gamma' = 0.8$ excitation cycles per light interval. Fig. 3(b) and Fig. 3(d) show the momentum distributions for pulse spacings $T = 30 \mu$s and $T = 40 \mu$s. The sharp peaks smear out since in both cases the pulse spacing deviates from the resonance condition $T = 35 \mu$s. Propagating dark states which are completely decoupled from the standing wave do not exist for these pulse spacings. A further increase of the pulse spacing to $T = 60 \mu$s completely washes out the comb-like structure. For $T = 2\tau_n/8 = 70 \mu$s [see Fig. 3(c)] a momentum comb appears again with a spacing of $\hbar k$, as expected for the $n = 2$ resonance.

Our scheme can be extended to two and three dimensions and to transition schemes of the type $F \rightarrow F$ and $F \rightarrow F - 1$ (for $F > 1$). For $A$-type coupling schemes the non-coupling state is a superposition of two momentum states, so that for any time between successive pulses a propagating dark state exists. As a cooling technique our scheme might find particular interest for atoms which have transition frequencies that can only be excited using pulsed laser-sources, e.g. the Lyman-$\alpha$ transition of
hydrogen. As our scheme provides continuous cooling it can be applied to atomic beams so that another tempting application appears in atom lithography. There the width of the fabricated nanostructures is mainly limited by the transversal momentum distribution of the atomic beam.

In conclusion, we have experimentally shown that propagating dark states can be populated if the standing wave is pulsed with a frequency of $\frac{1}{8} \tau_r$ or $\frac{1}{8} \tau_r$. For larger time intervals ($n \tau_r/8$, $n \geq 3$) between the pulses we expect the atoms to accumulate in momentum states which are spaced by fractional multiples of $\hbar k$.

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