Model-aided Deep Neural Network for Source Number Detection

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Abstract—Source number detection is a critical problem in array signal processing. Conventional model-driven methods, e.g., Akaike's information criterion (AIC) and minimum description length (MDL), suffer from severe performance degradation when the number of snapshots is small or the signal-to-noise ratio (SNR) is low. Performance analyses and improvements of AIC and MDL are provided in [7], [8].

In practical systems, coherent sources would exist, e.g., multipath propagation and smart jammer scenarios. Unfortunately, AIC and MDL related methods cannot be directly applied to the coherent source scenarios due to the rank-deficient covariance matrix. One solution is to employ forward-backward spatial smoothing (FBSS) to “decorrelate” the source signals. By partitioning the whole array into sub-arrays and averaging of the sub-array received covariance matrices, the equivalent source covariance matrix becomes nonsingular. The FBSS scheme is widely-used since it can be regarded as a preprocessing before applying AIC and MDL as well as DOA estimation algorithms.

Recently, deep learning (DL) has drawn growing attentions for its great success in various areas, such as computer vision, automatic speech recognition and wireless communications [10]–[14]. Apart from the excellent learning capability, remarkable generalization property and low computational complexity of DL are both attractive advantages that promote its applications. In this paper, we exploit the deep neural network (DNN) with the aid of signal modeling information to estimate the number of sources. We first develop the eigenvalue based regression network (ERNet) and classification network (ECNet) for estimating the number of sources. Then, we extend the ERNet and ECNet for estimating the number of coherent sources, where the forward-backward spatial smoothing (FBSS) scheme is adopted to improve the performance of ERNet and ECNet. Numerical results demonstrate the outstanding performance of ERNet and ECNet over the conventional AIC and MDL methods as well as their excellent generalization capability, which also shows their great potentials for practical applications.

Index Terms—Source number detection, deep neural network, model-driven, non-coherent sources, coherent sources

I. INTRODUCTION

Estimation of the source number from noisy received data is a fundamental problem in array signal processing and related applications, such as radar [11], acoustic signal processing [2], wireless communications [3], Internet of Things (IoT) [4], etc. In particular, most direction-of-arrival (DOA) estimation algorithms (e.g., MUSIC and ESPRIT) [5] rely on the prior knowledge of accurate source number and would fail when such knowledge is unavailable.

In the last three decades, various methods have been proposed to estimate the number of sources, among which the Akaike's information criterion (AIC) and the minimum description length (MDL) are the two most popular ones. In these two methods, the estimated number of sources is the value that minimizes the AIC or the MDL criterion [6]. However, AIC and MDL exhibit performance degradations when the number of snapshots is small or the signal-to-noise ratio (SNR) is low. Performance analyses and improvements of AIC and MDL are provided in [7], [8].

In practical systems, coherent sources would exist, e.g., multipath propagation and smart jammer scenarios. Unfortunately, AIC and MDL related methods cannot be directly applied to the coherent source scenarios due to the rank-deficient covariance matrix. One solution is to employ forward-backward spatial smoothing (FBSS) to “decorrelate” the source signals. By partitioning the whole array into sub-arrays and averaging of the sub-array received covariance matrices, the equivalent source covariance matrix becomes nonsingular. The FBSS scheme is widely-used since it can be regarded as a preprocessing before applying AIC and MDL as well as DOA estimation algorithms.

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II. SIGNAL MODEL

Consider \( K \) signals emitted from the far field impinging on an array of \( M \) antennas, the \( n \)-th snapshot of the array can be written as

\[
\mathbf{r}(n) = \mathbf{A}(\mathbf{\theta}) \mathbf{s}(n) + \mathbf{w}(n), \quad (1)
\]

where \((\cdot)^T\) denotes the transport operator, \( \mathbf{s}(n) = [s_1(n), s_2(n), \ldots, s_K(n)]^T \) is the signal vector, \( \mathbf{w}(n) = [w_1(n), w_2(n), \ldots, w_M(n)]^T \) is the noise vector, \( \mathbf{\theta} = [\theta_1, \theta_2, \ldots, \theta_K]^T \) is the DOA vector of all \( K \) users, and \( \mathbf{A}(\mathbf{\theta}) = [\alpha(\theta_1), \alpha(\theta_2), \ldots, \alpha(\theta_K)] \) is the \( M \times K \) matrix of the steering vectors. Suppose the number of snapshots is \( N \). The covariance matrix of \( \mathbf{r}(n) \) is estimated from the limited received data as

\[
\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{r}(n)\mathbf{r}^H(n). \quad (2)
\]
B. Architectures of ERNet and ECNet

The proposed ERNet and ECNet adopt the fully-connected neural network (FNN) architecture with $L$ layers, including one input layer, $L - 2$ hidden layers, and one output layer as shown in Fig. 1. The input of both ERNet and ECNet is $x_E = [\lambda_1, \cdots, \lambda_M]^T$. (4)

The output of FNN is a cascade of nonlinear transformation of $x_E$, i.e.,

$$\hat{y} = \text{NET}(x_E, \Omega) = f^{(L-1)} \left( \cdots f^{(1)} (x_E) \right),$$

(5)

where $L$ is the number of layers and $\Omega \triangleq \{W^{(l)}, b^{(l)}\}_{l=1}^{L-1}$ is the network parameters to be trained. Moreover, $f^{(l)}$ is the nonlinear transformation function of the $l$-th layer and can be written as

$$f^{(l)}(x) = g^{(l)} \left( W^{(l)}x + b^{(l)} \right), 1 \leq l \leq L - 1,$$

(6)

where $W^{(l)}$ is the weight matrix associated with the $(l-1)$-th and $l$-th layers, while $b^{(l)}$ and $g^{(l)}$ are the bias vector and the activation function of the $l$-th layer, respectively. The activation function for the hidden layers is selected as the rectified linear unit (ReLU) function $g^{(l)}(z) = \max(0, z)$, where $[z]_p$ denotes the $p$-th entry of the vector $z$, $\text{len}(z)$ represents the length of the vector $z$, and $p = 1, 2, \cdots, \text{len}(z)$.

For ERNet, the supervise label is the number of sources, $K$; The estimated number of sources, $\hat{K}_{ER}$, is the output of ERNet rounded to the nearest integer; The activation function for the output layer is the linear function, i.e., $g^{(L)}(z) = z$; The loss function is the $L_2$ norm function, i.e.,

$$\text{Loss}_{L_2}(\Omega) = \frac{1}{V} \sum_{v=0}^{V-1} \left\| \hat{y}(v) - y(v) \right\|_2^2,$$

(7)

where $\| \cdot \|_2$ denotes the $L_2$ norm, $V$ is the batch size, and $v$ denotes the index of the $v$-th training sample.

For ECNet, the supervise label is the one-hot encoding vector related to $K$ and $M$; The estimated number of sources is the position index of the maximum element of the output vector $\hat{y}_{EC}$, i.e., $\hat{K}_{EC} = \arg \max_p [\hat{y}^{\text{EC}}]_p$; The activation function for the output layer is the Softmax function, i.e.,

$$[g^{(L)}(z)]_p = \frac{e^{[z]_p}}{\sum_{p=1}^{\text{len}(z)} e^{[z]_p}}, \quad p = 1, 2, \cdots, \text{len}(z);$$

(8)

The loss function is the categorical cross entropy function, i.e.,

$$\text{Loss}_{\text{EC}}(\Omega) = \frac{1}{V} \sum_{v=0}^{V-1} \sum_{p=1}^{\text{len}(y)} [y(v)]_p \log \left[ \hat{y}(v) \right]_p.$$

(9)

The DL based algorithm has two stages, i.e., the off-line training and the on-line testing stages. In the off-line training stage, the inputs and the supervise labels are collected as training samples. The optimal $\Omega$ can be obtained by minimizing the difference between the outputs and the labels through off-line

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1 Batch size is the number of samples in one training batch.

2 One-hot encoding vector related to $N$ and $M$ is an $M$-dimension vector with its $(N+1)$-th element being 1 and other elements being 0.
training. The adaptive moment estimation (ADAM) algorithm [15] is adopted to minimize the loss function until the loss converges, and then the optimal $\Omega$ can be obtained. In the testing stage, $\Omega$ is fixed, and the network directly outputs the estimates of the labels based on the input data.

C. Detection in the Case of Coherent Sources

When the sources are coherent, which is the common in the real environment, the signal covariance matrix is rank-deficient, and therefore the performance of eigenvalue based methods degrades [16]. To solve the problem, the FBSS scheme is adopted [9], where the total array of $M$ antennas is divided forward/backward into overlapping sub-arrays with size $M_0$. The number of forward/backward sub-arrays is $T = M - M_0 + 1$. Note that the $M_0 > K$ and $T \geq K$ should be satisfied as proved in [16]. Then, the forward/backward averaged covariance matrix is given by

$$\hat{R}_{\text{ave}} = \frac{1}{2T} \sum_{t=1}^{T} \left( \hat{R}_f^{(t)} + \hat{R}_b^{(t)} \right),$$

(10)

where $\hat{R}_f^{(t)}$ and $\hat{R}_b^{(t)}$ are the covariance matrices of the received signals that are estimated from the $t$-th ($1 \leq t \leq T$) forward/backward sub-arrays, respectively. Details about FBSS scheme can be found in [16].

With the FBSS scheme, we refine the input of ERNet and ECNet as

$$\mathbf{x}_{\text{FE}} = [\lambda_{\text{ave},1}, \cdots, \lambda_{\text{ave},M_0}]^T,$$

(11)

where $\lambda_{\text{ave},1}, \cdots, \lambda_{\text{ave},M_0}$ are the eigenvalues of the $\hat{R}_{\text{ave}}$, and the training approach is the same.

D. Complexity Analysis

Denote $n_l$ as the number of neurons in the $l$-th layer, and the number of layers, $L$, is set to be 4 in the proposed networks. The computational costs of ERNet, ECNet, AIC and MDL methods are compared, as listed in Tab. I. Since the eigenvalue decomposition is required in all the above-mentioned method, the computational cost of the eigenvalue decomposition is omitted in the Tab. I. Note that the Softmax in ECNet is unnecessary in the testing stage since the estimate of $K$ can be directly obtained by finding the index of the maximum element in the output vector. Therefore, the computational cost of Softmax is not included in ECNet. As shown in the Tab. I, the complexity of both AIC and MDL estimators is $O(M^2)$ while the the complexity of ERNet and ECNet is $O(M)$, which implies both ERNet and ECNet have lower complexities than AIC/MDL at the cost of off-line training.

IV. SIMULATION RESULTS

In this section, a uniform linear array (ULA) with 10 antennas is used to received the signals. Unless otherwise specified, the simulation parameters are set as follows: The number of snapshots $N$ is 20; The number of sources $K$ is generated randomly over $[0, 5]$; The direction of each source is generated randomly over $[0, 2\pi]$; The number of neurons in the hidden layer are all $(8,8)$ for ERNet and ECNet. Keras 2.2.0 is employed as the DL framework. The initial learning rate of the ADAM algorithm is 0.001. The batch size is 128. The parameters of all the networks are initialized as truncated normal variable with normalized variance. The number of training samples is 8,000, and the number of epochs is 400.

It needs to be mentioned that the ERNet and ECNet are both trained at varying SNRs rather than trained at each SNR separately. This is because that training and testing the networks at each SNR separately is unpractical. In real systems, SNR tends to be varying or even unknown, and therefore it is almost impossible to obtain perfect training and testing sets, i.e., the statistics mismatches between the testing and training sets is unavoidable. By exploiting the generalization capability of DNN, we train the networks with samples generated at different SNRs and then test the networks with samples generated at a fixed SNR. In this way, we can directly collect samples generated with different SNRs as training samples without the knowledge of SNRs and the networks only need one-time training, which is more feasible than training at each SNR in practical applications. To generate a single training sample, the value of SNR is set as a number randomly selected from $[0, 40]$ dB.

A. Source Number of Non-coherent Sources

In the case of non-coherent sources, AIC and MDL estimators are used as benchmarks.

Fig. 2 depicts the accuracy performance of the ERNet, ECNet, AIC and MDL versus the number of snapshots, where SNR is set to be 5 dB in the testing stage. As shown in Fig. 2 AIC and MDL exhibit similar performance while the ERNet and ECNet both significantly outperform the AIC and MDL estimators. Besides, when the number of snapshots is less than 20, the ECNet achieves slightly better performance than the ERNet and AIC.

3The truncated normal distribution is a normal distribution bounded by two standard deviations from the mean.

4The weights of neurons in the $l$-th layer are initialized as truncated normal variables with variance $1/n_l$. 
TABLE I
Comparison of computational complexity

| Methods         | ERNet | ECNet | AIC  | MDL  |
|-----------------|-------|-------|------|------|
| Eigenvalue decomposition | Yes   | Yes   | Yes  | Yes  |
| The number of multiplications/divisions | $M_{n_1} + n_2$ | $M(n_1 + n_2)$ | $M^2 + 7M$ | $M^2 + 7M$ |
| The number of additions/subtractions | $n_1 + n_2 + 1$ | $n_1 + n_2 + M$ | $0.5(M^2 + M)$ | $0.5(M^2 + M)$ |
| The number of logarithmic operators | 0     | 0     | $2M$ | $M$  |
| The number of comparisons | 0     | $M$   |      |      |

Fig. 3. The accuracy performance versus SNR, non-coherent sources, $K=100$.

than ERNet. When the number of snapshots goes beyond 100, both ERNet and ECNet almost yield perfect number detection, even when SNR is as low as 5 dB. Besides, the covariance matrix based network directly estimates the source number based on $\hat{R}$ and performs much worse than MDL and AIC, which validates the analyses in Section III-A.

Fig. 4 compares the accuracy performance of the ERNet, ECNet, AIC and MDL versus SNR when the number of snapshots is set as 20. As shown in Fig. 3, the performance of ERNet and ECNet is consistently good in the low SNR region while ECNet achieves better performance than ERNet when SNR is higher than 20 dB. Both ERNet and ECNet significantly outperform the AIC and MDL estimators, especially when SNR is lower than 20 dB, which demonstrates the remarkable superiority and the excellent generalization capability of ERNet/ECNet.

B. Source Number of Coherent Sources

In the case of coherent sources, the FBSS based AIC, and the FBSS based MDL methods are used as benchmarks. To generate a single training sample, the number of coherent sources is generated randomly over $[0, K-1]$, and each coherent source is set to be identical with one of independent sources (randomly selected). The size of sub-arrays is 5. It should be noted that when the number of coherent sources is $K-1$, the sources are non-coherent.

As shown in Fig. 4, the FBSS based AIC achieves slightly better performance than the FBSS based MDL when SNR is lower than 5 dB while both ERNet and ECNet significantly outperform the FBSS based AIC and MDL, especially when SNR is lower than 25 dB. When SNR is 0 dB, the accuracy of the FBSS based AIC and MDL is about 0.7 while the accuracy of both ERNet and ECNet is higher than 0.95, which shows the effectiveness of ERNet/ECNet in the case of coherent sources. Furthermore, ECNet achieves better performance than ERNet, especially when SNR is lower than 5dB or higher than 20 dB.

Moreover, it can be observed from Figs. 3 and 4 that when SNR is higher than 20 dB, the proposed networks achieve similar performance in both the non-coherent and coherent cases. While in the low SNR region, the proposed networks achieve better performance in the non-coherent case than in the coherent case.

V. CONCLUSION

In this paper, we proposed the model-aided data-driven networks, i.e., ERNet and ECNet, to estimate both the number of non-coherent sources and coherent sources. The comparisons of computational complexities among the proposed and the conventional model-driven methods were presented in closed-form. Simulation results have demonstrated that the proposed ERNet and ECNet significantly outperform the conventional model driven methods and have remarkable generalization capability with respect to SNRs. Both the above properties make ERNet and ECNet great candidates in real-world applications.

REFERENCES

[1] M. Khan, K. Iftekharuddin, E. McCracken, K. Islam, S. Bhurtel, L. Wang, and R. Kozma, “Autonomous wireless radar sensor mote for target material classification,” Digital Signal Process., vol. 23, no. 3, pp. 722–735, 2013.
[2] Y. Wu, Z. Hu, H. Luo, and Y. Hu, “Source number detectability by an acoustic vector sensor linear array and performance analysis,” *IEEE J. Oceanic Engineering*, vol. 39, no. 4, pp. 769–778, Oct. 2014.

[3] B. Wang, F. Gao, S. Jin, H. Lin, and G. Y. Li, “Spatial- and frequency-wideband effects in millimeter-wave massive MIMO systems,” *IEEE Trans. Signal Process.*, vol. 66, no. 13, pp. 3393–3406, Jul. 2018.

[4] Y. Mohamedani, B. Fergani, J. Laheurte, and B. Poussot, “DOA estimation techniques applied to RFID tags using receiving uniform linear array,” in *IEEE Int. Symp. Antennas and Propagation USNC/URSI National Radio Science Meeting*, Vancouver, Canada, Jul. 2015, pp. 1760–1761.

[5] B. Vikas and D. Vakula, “Performance comparison of MUSIC and ESPRIT algorithms in presence of coherent signals for DoA estimation,” in *Proc. Int. Conf. Electron., Commun. Aerospace Technol. (ICECA)*, Apr. 2017, vol. 2, pp. 403–405.

[6] D. Williams and V. Madisetti, “Detection: Determining the number of sources,” *Digital Signal Processing Handbook*, 1999.

[7] Q. Ding and S. Kay, “Inconsistency of the MDL: On the performance of model order selection criteria with increasing signal-to-noise ratio,” *IEEE Trans. Signal Process.*, vol. 59, no. 5, pp. 1959–1969, May 2011.

[8] L. Huang and S. Wu, “Low-complexity MDL method for accurate source enumeration,” *IEEE Signal Process. Lett.*, vol. 14, no. 9, pp. 581–584, Sep. 2007.

[9] S. Shirvani Moghaddam and S. Jalaei, “Determining the number of coherent sources using FBSS-based methods,” *Frontiers in Science*, vol. 2, pp. 203–208, Dec. 2012.

[10] Z. Qin, H. Ye, G. Y. Li, and B. F. Juang, “Deep learning in physical layer communications,” *IEEE Wireless Commun.*, vol. 26, no. 2, pp. 93–99, Apr. 2019.

[11] H. He, S. Jin, C. Wen, F. Gao, G. Ye Li, and Z. Xu, “Model-driven deep learning for physical layer communications,” *IEEE Wireless Commun. (early access)*, pp. 1–7, 2019.

[12] Y. Yang, F. Gao, X. Ma, and S. Zhang, “Deep learning-based channel estimation for doubly selective fading channels,” *IEEE Access*, vol. 7, pp. 36579–36589, Mar. 2019.

[13] Z. Jia, W. Cheng, and H. Zhang, “A partial learning based detection scheme for massive MIMO,” *IEEE Wireless Commun. Lett.*, pp. 1–1, Apr. 2019.

[14] Y. Yang, F. Gao, G. Y. Li, and M. Jian, “Deep learning based downlink channel prediction for FDD massive MIMO system,” *IEEE Commun. Lett.*, pp. 1–1, 2019.

[15] D. Kingma and J. Ba, “Adam: A method for stochastic optimization,” *arXiv preprint arXiv:1412.6980*, 2014.

[16] S. Li and B. Lin, “On spatial smoothing for direction-of-arrival estimation of coherent signals in impulsive noise,” in *Proc. IEEE Adv. Inf. Technol., Electron. Automation Control Conf. (IAEAC)*, Chongqing, China, 2015, pp. 339–343.