Boson’s field renormalization prescription

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We discuss the problem of the present boson’s field renormalization prescription induced by the imaginary parts of the unstable boson’s propagation amplitudes and how to resolve it.

PACS numbers: 11.10.Gh, 11.55.-m

I. INTRODUCTION

The field renormalization prescription has been present for a long time, but at present it encounters some new problems for unstable fermions [1, 2, 3]. The conventional field renormalization prescription isn’t suitable for unstable fermions, but the revised field renormalization prescription in Ref.[4] leads to the physical amplitude gauge-parameter dependent [2]. Furthermore this prescription leads to the decay width of a physical process gauge-parameter dependent (see the appendix) which further proves the fermion field renormalization prescription of Ref.[4] isn’t suitable for the standard model. D. Espriu et al. suggest to introduce two set independent Field Renormalization Constants (FRC) for the external-line fermion fields which can guarantee the physical amplitude gauge independent [2].

But for unstable boson the corresponding discussion is still not carried out. In this manuscript we will discuss this problem. In the follows we firstly discuss what problem exists in the present boson’s field renormalization prescription. Then we discuss how to construct a reasonable boson’s field renormalization prescription. In section 4 we illustrate the consistence of the present boson’s field renormalization prescription with the gauge theory in standard model by the calculations of the physical amplitude of $Z \to d_i \bar{d}_i$, i.e. the gauge boson $Z$ decaying into a pair of down-type $i$ quarks. Lastly we give the conclusion.

II. PROBLEM OF THE PRESENT BOSON’S FIELD RENORMALIZATION PRESCRIPTION

Since the scalar boson’s field renormalization prescription can be easily obtained from the vector boson’s one, here we mainly discuss the vector boson’s field renormalization prescription. The vector boson’s FRC can be introduced as

$$
\Phi^\mu_{0i} = \sum j Z^\frac{1}{2} ij \Phi^\mu j, \quad \Phi^{\mu\dagger}_{0i} = \sum j \Phi^{\mu\dagger} j \bar{Z}^\frac{1}{2} ji,
$$

(1)

where two set vector boson’s FRC have been introduced. Obviously they should satisfy the hermitian conjugation relationship

$$
\bar{Z}^\frac{1}{2} ij = Z^{\frac{1}{2} \dagger} ji.
$$

(2)

The renormalized vector boson’s two-point function can be written as

$$
\hat{\Gamma}_{ij}^{\mu \nu}(p) = -ig^{\mu \nu} \sum_{k,l} Z^\frac{1}{2} ik \left[(p^2 - m_{0k}^2)\delta_{kl} + \Sigma_T^{\mu \nu}(p^2)\right] Z^{\frac{1}{2} l j} - i p^{\mu} p^{\nu} \Sigma_L^{\mu \nu}(p^2),
$$

(3)

where the shaded circle is the sum of the 1PI diagrams, $m_{0k}$ is the bare mass of vector boson $k$, and $\Sigma_T^{\mu \nu}$ is the transverse vector boson’s two-point function removed the external-line FRC. The conventional vector boson’s field renormalization prescription is

$$
\hat{\Gamma}^{\mu \nu}_{ij}(p)\varepsilon^{\nu}(p)|_{p^2=m_i^2} = 0, \quad \hat{\Gamma}^{\mu \nu}_{ij}(p)\varepsilon^{\nu}(p)|_{p^2=m_j^2} = 0,
$$

$$
\lim_{p^2 \to m_i^2} \frac{1}{p^2 - m_i^2} \hat{\Gamma}^{\mu \nu}_{ii}(p)\varepsilon^{\nu}(p) = -\varepsilon^{\mu}(p).
$$

(4)

But it’s well known that Eqs.(4) cannot be satisfied for unstable vector bosons since there is imaginary part in the transverse two-point function. In order to be suitable for unstable vector bosons and satisfy the constraint of Eq.(2)
A. Denner revised Eqs.(4) as follows

\[ Re \hat{\Gamma}^{\mu\nu}_{ij}(p) \varepsilon_\nu(p)|_{p^2=m_i^2} = 0, \quad Re \hat{\Gamma}^{\mu\nu}_{ij}(p) \varepsilon_\nu(p)|_{p^2=m_j^2} = 0, \]

\[ \lim_{p^2 \to m_i^2} \frac{1}{p^2 - m_i^2} Re \hat{\Gamma}^{\mu\nu}_{ii}(p) \varepsilon_\nu(p) = -\varepsilon^\mu(p), \]  

(5)

where \( Re \) takes the real part of the two-point function.

Of course Eqs.(5) are suitable for unstable vector bosons, and also satisfy the constraint of Eq.(2). But we find it leads to physical amplitudes gauge-parameter dependent. We calculate a physical process of \( Z \to d_i \bar{d}_i \), i.e. the gauge boson \( Z \) decaying into a pair of down-type \( i \) quarks, to illustrate this problem. At one-loop level we have

\[
\mathcal{M}(Z \to d_i \bar{d}_i) = \left[ -\frac{e}{6} \delta Z_{\gamma\gamma} + \frac{e(2\gamma^2_W + 1)}{12 c_W s_W} (\delta Z_{\gamma\gamma,ii} + \delta Z_{\gamma\gamma,ii}^{dl}) \right] A_L - \left[ -\frac{e}{6} \delta Z_{\gamma\gamma} + \frac{e s_W}{6 c_W} (\delta Z_{\gamma\gamma,ii} + \delta Z_{\gamma\gamma,ii}^{dr}) \right] A_R \\
+ \mathcal{M}^{amp}(Z \to d_i \bar{d}_i),
\]

(6)

where the vector boson’s FRC have been expanded as \( Z_{\gamma\gamma,ii}^{\pm} = \delta_{ij} + \frac{1}{2} \delta Z_{ij,ii}, \delta Z_{\gamma\gamma,ii}^{dr} \) et al. are \( d_i \) quark’s FRC, \( e \) is the electron’s charge, \( s_W \) and \( c_W \) are the sine and cosine of the weak mixing angle, and

\[
A_L = \bar{u}(p_{d_i}) \psi_1 \gamma_L \nu(p_{\bar{d}_i}), \quad A_R = \bar{u}(p_{d_i}) \psi_2 \gamma_R \nu(p_{\bar{d}_i}),
\]

(7)

with \( \gamma_L \) and \( \gamma_R \) the left- and right- handed helicity operators, and \( \mathcal{M}^{amp} \) is the amplitude of the amputated diagrams shown in Fig.1. Our numerical calculation has shown that the real part of the physical amplitude is gauge-parameter independent. So in the follows we only need to discuss the gauge dependence of the imaginary part of the physical amplitude. Firstly the gauge-dependent imaginary parts of \( d_i \) quark’s FRC are

\[
Im(\delta Z_{ii}^{dr} + \delta Z_{ii}^{dl})|_k = 0, \quad Im(\delta Z_{ii}^{dl} + \delta Z_{ii}^{dr})|_\xi = \frac{e^2}{16 \pi s_W^2 x_{d,i}} \sum_j |V_{ji}|^2 (x_{d,i} - x_{u,j} - \xi_W) B \theta[m_{d,i} - m_{u,j} - \sqrt{\xi_W M_W}],
\]

(8)
where the subscript $\xi$ takes the gauge-dependent part, $\theta$ is the Heaviside function, $m_{d,i}$ and $m_{u,j}$ are the masses of $d_i$ quark and up-type $j$ quark, $M_W$ and $\xi_W$ are the mass and gauge parameter of gauge boson $W$, $V_{ji}$ is the CKM matrix element \[2\], $x_{d,i} = m_{d,i}^2/M_W^2$ and $x_{u,j} = m_{u,j}^2/M_W^2$, and

$$B = \sqrt{\xi_W^2 - 2(x_{d,i} + x_{u,j})\xi_W + (x_{d,i} - x_{u,j})^2}. \quad (9)$$

On the other hand using the cutting rules \[3\] we obtain

$$ImM^{amp}(Z \to d_i\bar{d}_i)|_\xi = \left[\frac{e^3}{384\pi c_W^3 s_W^3} (1 - 4c_W^2\xi_W)^{3/2} \theta[M_Z - 2\sqrt{\xi_W}M_W] - \frac{e^3(2c_W^2 + 1)}{192\pi c_W s_W^3 x_{d,i}} \sum_j [V_{ji}]^2 (x_{d,i} - x_{u,j} - \xi_W) B \theta[m_{d,i} - m_{u,j} - \sqrt{\xi_W}M_W] - \frac{e^3}{192\pi c_W^3 s_W^3} ((\xi_W - 1)^2 c_W^4 - 2(\xi_W - 5)c_W^2 + 1) C \theta[M_Z - M_W - \sqrt{\xi_W}M_W] \right] A_L. \quad (10)$$

with $M_Z = M_W/c_W$ the mass of gauge boson $Z$ and

$$C = \sqrt{(\xi_W - 1)^2 c_W^4 - 2(\xi_W - 5)c_W^2 + 1}. \quad (11)$$

We note that the result of Eq.(10) coincides with the results of the conventional loop momentum integral algorithm \[4\] and the causal perturbative theory \[5\]. According to Eqs.(5) the boson’s FRC $\delta Z_{\gamma Z}$ and $\delta Z_{ZZ}$ don’t contain imaginary part. So from Eqs.(6,8,10) we obtain

$$ImM(Z \to d_i\bar{d}_i)|_\xi = \left[\frac{e^3}{384\pi c_W^3 s_W^3} (1 - 4c_W^2\xi_W)^{3/2} \theta[M_Z - 2\sqrt{\xi_W}M_W] - \frac{e^3}{192\pi c_W^3 s_W^3} ((\xi_W - 1)^2 c_W^4 - 2(\xi_W - 5)c_W^2 + 1) C \theta[M_Z - M_W - \sqrt{\xi_W}M_W] \right] A_L. \quad (12)$$

This means the physical amplitude of $Z \to d_i\bar{d}_i$ is gauge dependent under the boson’s field renormalization prescription of Ref.\[2\].

**III. CONSTRUCT A REASONABLE BOSON’S FIELD RENORMALIZATION PRESCRIPTION**

From the above discussion we find in order to keep physical amplitudes gauge invariant the constrain of Eq.(2) must be discarded. In fact the hermitian conjugation relationship of Eq.(2) is broken by the imaginary parts of the unstable boson’s propagation amplitudes \[2,3\] (we can also see this point in the following discussions). So in the follows we will discard the constrain of Eq.(2) and treat $\bar{Z}^\pm$ and $Z^\pm$ as independent quantities.

We firstly discuss how to construct the off-diagonal vector boson’s FRC. Consider the integrality of physical amplitudes the off-diagonal field renormalization conditions of Eqs.(4) should keep unchanged, i.e.

$$\hat{\Gamma}_{\mu \nu}^{ij}(p) \epsilon_{\nu}(p)|_{p^2 = m_i^2} = 0, \quad \hat{\Gamma}_{\mu \nu}^{ij}(p) \epsilon_{\nu}(p)|_{p^2 = m_j^2} = 0, \quad \text{for } i \neq j. \quad (13)$$

From Eq.(3) the solutions of Eqs.(13) can be written as

$$\tilde{Z}^\pm_{ij} = \frac{1}{(m_{0i}^2 - m_{ij}^2)\tilde{Z}^\pm_{ij}} \left(\sum_{k \neq j} \tilde{Z}^\pm_{ik}(m_i^2 - m_{0k}^2)\tilde{Z}^\pm_{kj} + \sum_{k,l} \tilde{Z}^\pm_{ik} \Sigma_{kl}(m_i^2)\tilde{Z}^\pm_{lj}\right), \quad \text{for } i \neq j, \quad (14)$$

One can easily see that the loop levels of the vector boson’s FRC of the r.h.s. of Eqs.(14) are less than the ones of the l.h.s. of Eqs.(14). So we can use Eqs.(14) to determine the vector boson’s off-diagonal FRC order by order by recursion algorithm. At one-loop level Eqs.(14) become

$$\tilde{Z}^\pm_{ij(1)} = \frac{\Sigma^T_{ij(1)}(m_i^2)}{m_{ij}^2}, \quad \tilde{Z}^\pm_{ij(1)} = \frac{\Sigma^T_{ij(1)}(m_j^2)}{m_{ij}^2}, \quad \text{for } i \neq j, \quad (15)$$
where the subscript (1) represents the one-loop-level parts of the quantities. From Eqs.(15) one can easily see that the hermitian conjugation relationship of Eq.(2) is broken by the imaginary part of the unstable vector boson’s two-point function $\Sigma^{T}_{ij(1)}$.

For the diagonal vector boson’s FRC we cannot use the renormalization conditions of Eqs.(4) since the diagonal unstable vector boson’s self energy cannot be renormalized to zero at physical mass point. We should determine it by the LSZ reduction formula [4]. However, the LSZ reduction formula has only been proven for stable particles [3]. So we need to postulate a generalization of the LSZ reduction formula to unstable particles. Under the postulation the LSZ reduction formula also holds true for unstable particles [4]. When expanding the vector boson’s propagation amplitude at physical mass pole we have from Eq.(3)

$$
\sum_{k,l} Z^{T}_{ik}(p^2 - m_{ok}^2)\delta_{kl} + \Sigma^{T}_{kl}(p^2)\frac{\partial}{\partial p^2}, \quad \frac{-i \epsilon}{(p^2 - m_{ik}^2\Sigma^{T}_{kl}(m_{ik}^2)\Sigma^{T}_{il}(m_{ik}^2) + i\epsilon},
$$

where $\Sigma^{T}_{kl} = \frac{\partial \Sigma^{T}_{kl}}{\partial p^2}$, and $\epsilon = \sum_{k,l} Z^{T}_{ik}(m_{ik}^2 - m_{ok}^2)\delta_{kl} + \Sigma_{kl}(m_{ik}^2)\Sigma^{T}_{il}(m_{ik}^2)/i$ is a small quantity. From Eq.(16) the unit residue condition requires

$$
Z^{T}_{ii} Z^{T}_{ij} = 1 - \sum_{k\neq i} Z^{T}_{ik} Z^{T}_{kj} - \sum_{k,l} Z^{T}_{ik} \Sigma^{T}_{kl}(m_{ik}^2) Z^{T}_{li}.
$$

To one-loop level Eq.(17) becomes

$$
Z^{T}_{ii} Z^{T}_{ij} = 1 - \Sigma^{T}_{ii(1)}(m_{ij}^2).
$$

Obviously there is some freedom left in the definition of the diagonal vector boson’s FRC.

In order to completely determine the diagonal vector boson’s FRC we need to carefully investigate the feature of the renormalized vector boson’s two-point function $\tilde{\Gamma}^{\mu\nu}_{ij}$. In fact $\tilde{\Gamma}^{\mu\nu}_{ij}$ is symmetric about its indexes, i.e.

$$
\tilde{\Gamma}^{\mu\nu}_{ij}(p) = \tilde{\Gamma}^{\mu\nu}_{ji}(p), \quad \text{for } i \neq j.
$$

For the problem concerned we only need to consider the transverse part of $\tilde{\Gamma}^{\mu\nu}_{ij}$. Putting Eqs.(15) into Eq.(3) we find that Eq.(19) is automatically satisfied at one-loop level since $\Sigma^{T}_{ij(1)}(p^2) = \Sigma^{T}_{ji(1)}(p^2)$. At two-loop level the transverse part of $\tilde{\Gamma}^{\mu\nu}_{ij}(p)$ becomes from Eq.(3)

$$
\tilde{\Gamma}^{T}_{ij(2)}(p) = (p^2 - m_{ij}^2)\tilde{\Gamma}^{T}_{ij(2)} + Z^{T}_{ij(2)}(p^2 - m_{ij}^2) - \delta m_{ij}^2 Z^{T}_{ij(1)} - Z^{T}_{ij(1)} \delta m_{ij}^2 + \sum_{k \neq i,j} Z^{T}_{ik(1)}(p^2 - m_{k}^2) Z^{T}_{kj(1)} + Z^{T}_{ik(1)}(p^2 - m_{ij}^2) Z^{T}_{ij(1)} + Z^{T}_{j}(p^2 - m_{ij}^2) Z^{T}_{j}(1) + \Sigma^{T}_{ij(2)}(p^2)
$$

$$
+ \sum_{k \neq i,j} Z^{T}_{ik(1)}(p^2 - m_{ij}^2) Z^{T}_{kj(1)} + \sum_{k \neq i,j} \Sigma^{T}_{ik(1)}(p^2) - \sum_{k \neq i,j} \Sigma^{T}_{ik(1)}(p^2) Z^{T}_{kj(1)}, \quad \text{for } i \neq j,
$$

where the superscript $T$ in the l.h.s. of the equation takes the transverse part of the two-point function. In order to make Eq.(20) satisfy Eq.(19) for arbitrary momentum $p$ the terms in Eq.(20) containing the two-point function $\Sigma^{T}_{ij}(p^2)$ must be symmetric about the indexes $i$ and $j$. This leads to (from Eqs.(15) and the fact that $\Sigma^{T}_{ij}(p^2)$ is symmetric about the indexes $i$ and $j$)

$$
Z^{T}_{ii(1)} = Z^{T}_{ij(1)}, \quad Z^{T}_{ij(1)} = Z^{T}_{ij(1)}.
$$

From Eqs.(14,15,21) we easily obtain

$$
Z^{T}_{ij(2)} = Z^{T}_{ij(2)}, \quad \text{for } i \neq j.
$$

Obviously Eq.(20) satisfies Eq.(19) under Eqs.(15,21,22). From Eqs.(15,21) we also find that

$$
Z^{T}_{ij(0)} = Z^{T}_{ij(0)}, \quad Z^{T}_{ij(1)} = Z^{T}_{ij(1)}.
$$

Eqs.(23) manifests that the two vector boson’s FRC matrices $\tilde{Z}^{T}$ and $Z^{T}$ satisfy transposition relationship between each other to one-loop level. In fact we can prove this relationship to all loop levels using the condition of Eq.(19)
by recursion algorithm. Basing on the results of Eqs.(23) we only need to prove the conclusion: if the transposition relationship between $\hat{Z}_i^+(m)$ and $\hat{Z}_j^+(m)$ is true to $n$-loop level, it will be also true to $n+1$-loop level. Under the condition

$$Z_{ij(m)} = Z_{ji(m)}^+, \quad for \ m = 0, 1, \cdots, n, \quad (24)$$

we can easily have from Eqs.(14)

$$Z_{ij(n+1)}^+ = Z_{ji(n+1)}^+, \quad for \ i \neq j, \quad (25)$$

i.e. the off-diagonal $n+1$-loop-level part of $\hat{\Gamma}$ i.e. the off-diagonal

$$n+2$$-loop-level part of $\hat{\Gamma}$

we have completely determined the vector boson’s FRC.

Thus we have

$$\sum_{k,l} \sum_u Z_{ik(u)} \Sigma_{kl(v)}^T(p^2) Z_{lj(n+2-u-v)}^+ \quad for \ v = 1, \cdots, n+2. \quad (27)$$

From Eq.(24) we find that Eq.(27) is automatically satisfied for $v \geq 2$. For $v = 1$ the l.h.s. of Eq.(27) becomes

$$\sum_{k,l} \sum_u Z_{ik(u)} \Sigma_{kl(v)}^T(p^2) Z_{lj(n+1-u)}^+ = (Z_{ii(n+1)}^+ + Z_{jj(n+1)}^+ + \sum_{k \neq i} Z_{ik(n+1)}^+ \Sigma_{kj(1)}^T(p^2) + \sum_{k \neq j} Z_{ik(n+1)}^+ \Sigma_{kj(1)}^T(p^2)$$

$$+ \sum_{k \neq j} \Sigma_{ik(1)}^T(p^2) Z_{kj(n+1)}^+ + \sum_{k,l} \sum_{u=1}^n Z_{ik(u)} \Sigma_{kl(v)}^T(p^2) Z_{lj(n+1-u)}^+ \quad (28)$$

From Eqs.(24,25) we find that the last three terms of the r.h.s. of Eq.(28) are symmetric about the indexes $i$ and $j$. Therefore in order to make Eq.(28) satisfy Eq.(27) we only need and must need the conditions

$$Z_{ii(n+1)}^+ = Z_{ii(n+1)}, \quad Z_{jj(n+1)}^+ = Z_{jj(n+1)}^+. \quad (29)$$

Since $i$ and $j$ are arbitrary, we have proven that the diagonal $n+1$-loop-level vector boson’s FRC also satisfy the transposition relationship. Thus we have proven the recursion condition mentioned above. Combined Eqs.(23) this means to all loop levels

$$Z_{ij}^+ = Z_{ji}^+. \quad (30)$$

Especially we have

$$Z_{ii}^+ = Z_{ii}. \quad (31)$$

Obviously Eq.(30) is consistent with Eqs.(14) and satisfies Eq.(19) (see Eq.(3)).

From Eqs.(17,31) we have

$$Z_{ij} = Z_{ii} = 1 - \sum_{k \neq i} Z_{ik}^+ Z_{kj}^+ - \sum_{k,l} \Sigma_{kl}^T(m_k^2) Z_{li}^+. \quad (32)$$

Thus we have completely determined the vector boson’s FRC.

IV. GAUGE INVARIANCE OF PHYSICAL AMPLITUDES UNDER THE PRESENT BOSON’S FIELD RENORMALIZATION PRESCRIPTION

In this section we give an example of the calculation of physical amplitude to see whether the present boson’s field renormalization prescription keeps physical amplitude gauge invariant. We calculate the physical amplitude of
$Z \rightarrow d_i d_i$. Part of the result has been calculated in section 2. Here we only need to calculate $\delta Z_{\gamma Z}$ and $\delta Z_{ZZ}$. From Eqs.(15) and Eqs.(18,31) we have at one-loop level

$$\delta Z_{\gamma Z} = -\frac{2\Sigma^\gamma_T(m_Z^2)}{m_Z^2}, \quad \delta Z_{ZZ} = -\frac{\partial}{\partial p^2} \Sigma^T_{ZZ}(m_Z^2).$$

(33)

In Fig.2 we show the one-loop gauge-parameter dependent $Z \rightarrow \gamma$ diagrams which are used to calculate the gauge-parameter-dependent imaginary part of $\delta Z_{\gamma Z}$. Using the cutting rules we obtain from Eqs.(33)

$$\text{Im } \delta Z_{\gamma Z}|\xi = \frac{e^2}{96\pi c_W s_W} (1 - 4c_W^2 \xi_W)^{3/2} [M_Z - 2\sqrt{\xi_W M_W}]$$

$$- \frac{e^2 s_W}{48\pi c_W} (\xi_W^2 - 1)^2 c_W^4 - 2(\xi_W - 5)c_W^2 + 1) C \theta[M_Z - M_W - \sqrt{\xi_W M_W}],$$

(34)

where $C$ has been shown in Eq.(11). On the other hand, in Fig.3 we show the one-loop gauge-parameter dependent $Z \rightarrow Z$ diagrams which are used to calculate the gauge-parameter-dependent imaginary part of $\delta Z_{ZZ}$. Using the cutting rules we obtain from Eqs.(33)

$$\text{Im } \delta Z_{ZZ}|\xi = -\frac{e^2}{96\pi c_W s_W} (1 - 4c_W^2 \xi_W)^{3/2} [M_Z - 2\sqrt{\xi_W M_W}]$$

$$+ \frac{e^2}{48\pi c_W} (\xi_W^2 - 1)^2 c_W^4 - 2(\xi_W - 5)c_W^2 + 1) C \theta[M_Z - M_W - \sqrt{\xi_W M_W}],$$

(35)
Putting Eqs.(34,35) and Eqs.(8,10) into Eq.(6) we finally obtain
\[ \text{Im} \mathcal{M}(Z \to d_i \bar{d}_i) \bigg|_{\xi} = 0. \] (36)
This means the present boson’s field renormalization prescription keeps the physical amplitude of \( Z \to d_i \bar{d}_i \) gauge-parameter independent.

V. CONCLUSION

In summary, we firstly discuss the present boson’s field renormalization prescriptions and find the prescription of Ref.[4] leads to the physical amplitude gauge-parameter dependent. Then we postulate a generalization of the LSZ reduction formula to unstable particles and use the symmetry of the boson’s two-point function about its particle’s indexes to construct a reasonable boson’s field renormalization prescription. The calculation of the physical amplitude of \( Z \to d_i \bar{d}_i \) shows that the present boson’s field renormalization prescription is consistent with the gauge theory in standard model.

Acknowledgments

The author thanks Prof. Xiao-Yuan Li for his useful guidance and Prof. Cai-dian Lu for the fruitful discussions with him.

Appendix

In the appendix we calculate the gauge dependence of the decay width of \( t \to c Z \), i.e. top quark decaying into charm quark and gauge boson Z, under the fermion field renormalization prescription of Ref.[4]. At one-loop level we have
\[ \mathcal{M}(t \to c Z) = \frac{e(4s_W^2 - 3)}{12s_Wc_W} (\delta Z^L_{ci} + \delta Z_{ci}^L) \bar{c} \gamma_L \gamma_5 t + \frac{e s_W}{3c_W} (\delta Z^R_{ci} + \delta Z_{ci}^R) \bar{c} \gamma_R \gamma_5 t + \mathcal{M}^{amp}(t \to c Z), \] (37)
where \( \mathcal{M}^{amp} \) is the amplitude of the one-loop amputated diagrams shown in Fig.4, and the quark’s FRC \( \delta Z^L_{ci} \) et al. are listed in Ref.[2] which satisfy the relationship \( \delta \bar{Z}_{ij}^L = \delta Z_{ij}^L \) and \( \delta \bar{Z}_{ij}^R = \delta Z_{ij}^R \) under the fermion field renormalization prescription of Ref.[2]. Our numerical result has shown the quasi-real part, which takes the real part of the loop momentum integrals appearing in the amplitude but not of the coupling constants appearing there \[4\], of Eq.(37) is gauge-parameter independent. So we only need to calculate the gauge dependence of the quasi-imaginary part, which takes the imaginary part of the loop momentum integrals appearing in the amplitude but not of the coupling constants appearing there \[6\], of Eq.(37). According to Eqs.(3.20) of Ref.[4] the quasi-imaginary parts of the quark’s FRC are equal to zero, so we only need to calculate the quasi-imaginary part of \( \mathcal{M}^{amp}(t \to c Z) \). Using the cutting rules \[6\] we obtain
\[ \text{Im} \mathcal{M}(t \to c Z) \bigg|_{\xi} = \bar{c} \gamma_L \gamma_5 t \sum_i \frac{V_{ti} V_{ci}^* e^3(4s_W^2 - 3)}{384\pi c_W s_W^3} \left[ \right. \]
\[
\frac{x_c - \xi_W - x_{d,i}}{x_c} \sqrt{x_c^2 - 2(\xi_W + x_{d,i})x_c + (\xi_W - x_{d,i})^2} \theta[m_c - m_{d,i} - M_W \sqrt{\xi_W}]
+ \frac{x_t - \xi_W - x_{d,i}}{x_t} \sqrt{x_t^2 - 2(\xi_W + x_{d,i})x_t + (\xi_W - x_{d,i})^2} \theta[m_t - m_{d,i} - M_W \sqrt{\xi_W}],
\] (38)

where \( \tilde{I}m \) takes the quasi-imaginary part of the amplitude, \( V_{2i} \) and \( V_{3i} \) are the CKM matrix elements, \( m_c \) and \( m_t \) are the masses of charm quark and top quark, and \( x_c = m_c^2/M_W^2 \), \( x_t = m_t^2/M_W^2 \). We note that the result of Eq.(38) coincides with the results of the conventional loop momentum integral algorithm and the causal perturbative theory. Since there is no tree level contribution, the result of Eq.(38) directly have contribution to the cross section of the physical process. In other words the decay width of \( t \to cZ \) is gauge-parameter dependent under the fermion field renormalization prescription of Ref.[4].

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