Freeze-in generation of lepton asymmetries after baryogenesis in the $\nu$MSM

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Abstract: The $\nu$MSM—an extension of the Standard Model by three relatively light singlet Majorana fermions $N_{1,2,3}$—allows for generation of lepton asymmetry which is several orders of magnitude larger than the observed baryon asymmetry of the Universe. The lepton asymmetry is produced in interactions of $N_{2,3}$ (with masses in the GeV region) at temperatures below the sphaleron freeze out $T \lesssim 130$ GeV and can enhance the cosmological production of dark matter (DM) sterile neutrinos $N_1$ (with mass of the keV scale) happening at $T \sim 200$ MeV due to active-sterile neutrino mixing. In this work we address the question of the magnitude of late-time asymmetry (LTA) generated by the heavy neutral leptons $N_{2,3}$ during their freeze-in at $T \sim 20$ GeV and study how much of it can survive down to the lower temperatures relevant for the sterile neutrino DM creation. We find that this LTA could result in production of a sizeable fraction of dark matter. We also examine a role played by magnetic fields and the Abelian chiral anomaly in generation of LTA, not accounted for in the previous studies. We argue that the production of LTA can be increased significantly and make an estimate of the influence of this effect.
1 Introduction

Despite its remarkable success the Standard Model (SM) of particle physics fails to explain neutrino oscillations, origin of matter-antimatter asymmetry of the Universe, and the nature of Dark Matter (DM). It has been suggested in refs. [1, 2] that all these shortcomings of the SM can be simultaneously addressed in its minimal extension with three singlet Majorana fermions, the $\nu$MSM. The lightest of the three, $N_1$, is the DM particle with mass of the keV scale [3–8]. The other two, $N_2$ and $N_3$ are responsible for both the active neutrino masses via the see-saw mechanism [9–14] and the generation of baryon asymmetry of the Universe (BAU) through coherent oscillations of heavy neutrinos [2, 15]. The latter ones are called heavy neutral leptons, HNLs in short. Notably, even after the freeze-out of sphalerons the $N_2$ and $N_3$ can keep producing lepton asymmetry [16, 17], which we refer to as late-time lepton asymmetry (LTA).

This asymmetry is crucial for the resonantly enhanced mechanism of sterile neutrino DM production [4]. If the concentration of DM is zero\footnote{This is not necessarily the case. As has been found recently in ref. [8], DM sterile neutrinos can be created after inflation by universal four-fermion interaction in Einstein-Cartan gravity. Similar conclusions can be reached if the $\nu$MSM is supplemented by higher dimensional operators [18].} at $T \simeq 100$ GeV, the resonant
production seems to be the only option in the $\nu$MSM framework, since the astrophysical X-ray bounds on active-sterile neutrino mixing and mass bound from structure formation rule out (see, e.g. [19]) the non-resonant production mechanism of [3]. The LTA has to be quite large, of order $10^{-5}$ in units $n/s$, where $n$ is a number density and $s$ is the entropy density. This asymmetry is generated after the freeze-out of the sphalerons so that its value doesn’t contradict the observed BAU. To be relevant, it has to survive until temperatures around $\sim 200$ MeV, when the resonant production of DM takes place [4–7].

Baryogenesis in the $\nu$MSM has attracted considerable attention from both theoretical (an incomplete list of related refs. [16, 17, 20–52]) and experimental (see, e.g. [53–68]) sides. One of the reasons for such an interest is the testability of the model in the current and planned experimental facilities, such as LHC [64, 66, 69, 70], NA62 [62, 67, 68], SHiP [65, 71], MATHUSLA [72], CODEX-b [73], FASER [74, 75], and ANUBIS [76].

On the contrary, the study of the LTA generation has been performed only in a few works, ref. [17, 26, 48, 50]. In fact, if one accepts that both BAU and resonantly produced DM are the consequence of the $\nu$MSM see-saw Lagrangian [9–14], the requirement of the generation of LTA big enough to explain the sterile neutrino DM abundance is the most restrictive one [17, 26, 50]. Given the potential of the forthcoming experiments—especially SHiP—to study a large portion of the parameter space below B-meson mass, it is important to clarify if the HNLs responsible for both BAU and LTA production are within the experimental reach.

Let us briefly summarize a possible scenario of the evolution of the Universe within the $\nu$MSM. Right after inflation the baryon and lepton numbers of the Universe as well as the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Interaction rates of two HNLs as functions of temperature (HNL mass is 1 GeV, see figure 2 for more details). In this and the other similar plots time goes from right to left. There are three characteristic temperatures at which the HNL rates cross the Hubble rate and large asymmetry can in principle be generated: $T_{in}$, $T_{out}$, and $T_{dec}$. In this work we consider asymmetry generation around $T_{in}$.}
\end{figure}
number of HNLs may well be zero, and we will assume in the present paper that this is indeed the case [77] (see, however, ref. [8] and footnote 1). The baryon asymmetry of the Universe is produced in a set of processes including coherent oscillations of HNLs, exchange of the lepton number between the HNLs and active leptons, and anomalous sphaleron transitions [2, 15]. The behaviour of HNLs after baryogenesis can be qualitatively understood from figure 1 showing their equilibration rates. It is seen that HNLs enter in thermal equilibrium at $T_a$, which exceeds a few tens of GeV, and subsequently go out of equilibrium at $T_{out} \sim 1$ GeV.

Owing to Sakharov non-equilibrium conditions, the lepton asymmetries can be generated at three instances: at freeze-in (temperature $T_{in}$), at freeze-out $T_{out}$, and during the HNL decays, at $T_{dec}$. The latter two possibilities have been considered in [16, 17]. At the time these papers were written, the freeze-in LTA produced at $T_{in}$ was believed to be inessential. The arguments [16, 17] were based on the fact that at $T_{out} < T < T_{in}$ the HNLs are in thermal equilibrium and thus all asymmetry which could have been produced at $T_{in}$ will be erased.

In fact, the situation happened to be more complicated [41, 42]. In spite of the fact that the HNL equilibration is much faster than the rate of the Universe expansion, some combination of lepton numbers and HNL asymmetries remains approximately conserved, and thus is protected from wash-out. As a result, one can expect that part of LTA produced above $T_{in}$ survives till the low temperatures. [41, 42]. So the question arises whether this asymmetry is enough for the DM production. The present study aims at providing a quantitative answer to this question.

The paper is organised as follows. We start section 2 from a brief description of the main ingredients entering the calculation of asymmetries in the $\nu$MSM. In section 3 we quantify the effect of entropy injection caused by the HNL decays. In section 4 we analyse the regime in which one can expect large LTA in the freeze-in scenario. In section 5 the previous considerations are put on the quantitative level by performing a scan of the parameter space. We also discuss possible uncertainties of the kinetic description of the system. In section 6 we address the question whether the presence of Abelian chiral anomaly and possible asymmetry transfer into helical magnetic fields may play a role in LTA generation. We argue that such effects may indeed be important and therefore should be systematically accounted for. In particular, the maximal value of the electron asymmetry reached during the evolution of the system could influence the final asymmetry. Therefore in section 7 we perform a scan of the parameter space looking for the maximal electron asymmetry. In section 8 we discuss possible uncertainties of the state-of-the-art approach based on the kinetic equations which we adopt in this work. Finally, section 9 contains our conclusions and outlook.

## 2 Generation of asymmetry

In this section we summarize the main ingredients required for the calculation of the BAU and LTA in the $\nu$MSM. All notations coincide with those of [46] to which we refer for further details.
The Lagrangian of the system is the well known see-saw one [9–14]. We present it below in order to fix the notations. In the basis where charged lepton Yukawa couplings and the Majorana mass term for the right-handed neutrinos are diagonal the Lagrangian can be written in the following form.

\[
\mathcal{L} = \mathcal{L}_{SM} + i\bar{\nu}_R \gamma^\mu \partial_\mu \nu_R - F_{\alpha I} \bar{L}_\alpha \Phi \nu_R - \frac{M_{IJ}}{2} \bar{\nu}^c_R \nu_R + h.c.,
\]

where \( \mathcal{L}_{SM} \) is the SM Lagrangian, \( \nu_R \) are right-handed neutrinos, \( I, J = 1, 2, 3 \), \( F_{\alpha I} \) is the matrix of Yukawa couplings, \( L_\alpha \) are the left-handed lepton doublets, \( \alpha = e, \mu, \tau \) and \( \Phi = i\sigma_2 \Phi^* \), \( \Phi \) is the Higgs doublet. Upon diagonalising the mass matrix following from (2.1) one finds three light mass eigenstates \( \nu_i \) and three heavy mass eigenstates \( N_I \). At the leading order of the see-saw mechanism \( N_I = \nu_R \), whereas \( \nu_{L_\alpha} = U_{\alpha i} \nu_i + \Theta_{\alpha I} N^c_I \), \( U_{\alpha i} \) is the PMNS matrix. In the last equation we have introduced the mixing angle \( \Theta_{\alpha I} = v_0 F_{\alpha I}/M_I \), where \( v_0 = 174 \text{ GeV} \) is the Higgs vacuum expectation value. see, e.g. ref. [78].

The lightest right-handed neutrino is the dark matter candidate. The values of its Yukawa coupling constants are significantly constrained by astrophysical observables. As a consequence, the contributions to the active neutrino masses are negligibly small [1]. Therefore only two heavier HNLs provide neutrino masses and participate in the generation of both BAU and LTA. In what follows we will limit our consideration to these two heavier HNLs, \( N_2 \) and \( N_3 \). The only input from the lightest sterile neutrino is the value of the LTA required to boost the DM production. This value is not unique and depends upon the active-sterile mixings and the mass of \( N_1 \). The thorough study of DM production can be found in ref. [79].

Not all choices of Yukawas are compatible with the measured masses and mixings of active neutrinos. A convenient way of accounting for the oscillation data is given by the Casas-Ibarra parametrisation [80]. In this parametrisation all Yukawas consistent with the observed oscillation data are determined by the following 6 parameters: the common mass \( M \) of two HNLs; mass splitting \( \Delta M \); two CP violating phases of the PMNS matrix \( \delta \) and \( \eta^2 \); real and imaginary parts of a complex angle \( \omega \). The real part of \( \omega \) enters all expressions as \( \exp(-i \text{Re} \omega) \). The imaginary part \( \text{Im} \omega \) controls the size of the Yukawas, namely \( |F_{\alpha I}| \propto \exp(\text{Im} \omega) \) for large positive \( \text{Im} \omega \) and \( |F_{\alpha I}| \propto \exp(-\text{Im} \omega) \) for large negative \( \text{Im} \omega \).

Description of the evolution of the system comprising two HNL species and the SM degrees of freedom is rather complicated. The reason is that the asymmetry production is a genuinely non-equilibrium phenomenon and it involves many processes, such as scatterings and decays of HNLs, their coherent oscillations, transfer of the asymmetry to leptons and their back reaction, and redistribution of the asymmetry among the SM degrees of freedom. These processes can be systematically accounted for in the integro-differential kinetic equations [2, 16] which have to be solved numerically. In this work we use the equations

\footnote{In the case of two HNLs, the PMNS matrix contains only one independent Majorana phase. Using PDG conventions one can identify the phase \( \eta \) with \((\alpha_{21} - \alpha_{31})/2\) in the case of normal hierarchy and \( \alpha_{21}/2 \) in the case of inverted hierarchy.}
of ref. [46]. These equations provided a unified description of low-scale leptogenesis models, including both resonant leptogenesis and baryogenesis via oscillations [51]. They can be written in terms of the matrix of densities $\rho_N$ of two HNLs ($\rho_{\bar{N}}$ for HNLs of opposite helicity) and the densities of the combinations $\Delta_\alpha = L_\alpha - B/3$ for $\alpha = e, \mu, \tau$ which are not affected by the sphaleron processes.

$$\frac{idn_{\Delta_\alpha}}{dt} = -2i\frac{\mu_\alpha}{T} \int \frac{d^3k}{(2\pi)^3} \Gamma_{\nu\alpha} f_\nu (1 - f_\nu) + i \int \frac{d^3k}{(2\pi)^3} \left( Tr[\tilde{\Gamma}_{\nu\alpha} \rho_{\bar{N}}] - Tr[\tilde{\Gamma}_{\nu\alpha}^* \rho_{N}] \right), \quad (2.2a)$$

$$\frac{id\rho_N}{dt} = [H_N, \rho_N] - \frac{i}{2} \{ (\Gamma_N, \rho_N - \rho_{\bar{N}}^{eq}) - \frac{i}{2} \sum_\alpha \tilde{\Gamma}_N^\alpha \left[ 2\frac{\mu_\alpha}{T} f_\nu (1 - f_\nu) \right] \}, \quad (2.2b)$$

$$\frac{id\rho_{\bar{N}}}{dt} = [H_{\bar{N}}, \rho_{\bar{N}}] - \frac{i}{2} \{ (\Gamma_{\bar{N}}, \rho_{\bar{N}} - \rho_N^{eq}) + \frac{i}{2} \sum_\alpha (\tilde{\Gamma}_N^\alpha)^* \left[ 2\frac{\mu_\alpha}{T} f_\nu (1 - f_\nu) \right] \}, \quad (2.2c)$$

where $f_\nu = \left( e^{k/T} + 1 \right)^{-1}$ is the Fermi-Dirac distribution function of a massless neutrino in equilibrium, $\rho_{\bar{N}}^{eq} = diag(1, 1) \left( e^{E_{\bar{N}}/T} + 1 \right)^{-1}$ is the matrix of densities of HNLs in equilibrium and $E_N = \sqrt{k^2 + M^2}$. Chemical potentials to $\Delta_\alpha$ are related to the number densities as $\mu_\alpha = \omega_{\alpha\beta}(T)n_{\Delta_\beta}$, where $\omega_{\alpha\beta}(T)$ is the (inverse) susceptibility matrix, see, e.g. [32, 43]. Notice that equations (2.2) agree with linearised equations from refs. [42, 48, 49]. All the processes listed above are encoded in the rates entering the kinetic equations. Computation of these rates poses a theoretical challenge on its own [23, 42]. Here we use the results of the state-of-the-art computations of ref. [47].

Solving integro-differential kinetic equations is very time consuming, so to allow for a scan of the parameter space we simplify them. Namely, we assume that the matrix of densities of the HNLs is proportional to the equilibrium one. This ansatz allows reducing the infinite system of integro-differential equations to a set of 11 ordinary differential equations with averaged rates. However, one needs to keep in mind that this ansatz is rather ad hoc and can be justified only by solving the full system. This complicated and computer resource-demanding task has been performed in refs. [24, 45]. Results of these works suggest that there is a reasonable agreement between the exact and averaged calculations. The asymmetries obtained from the averaged equations are typically within a factor of two agreement with the accurate ones.

At the end of this section we introduce for convenience the so-called yields $Y_X$

$$Y_X \equiv \frac{n_X}{s}, \quad n_X = \int \frac{dk^3}{(2\pi)^3} \rho_X, \quad (2.3)$$

where we use the entropy density $s$ as computed in refs. [81, 82]. These quantities are not affected by the expansion of the Universe without extra entropy injection.

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These equations have been derived in the relativistic approximation. As a result, the Boltzmann suppression is not present in the terms proportional to $f_\nu(1 - f_\nu)$. We have confirmed numerically that this suppression does not change the results for the whole mass range considered here. The more general form of kinetic equations can be found in ref. [51].
3 HNL decays and injection of entropy

The HNLs that have participated in the generation of BAU and LTA eventually decay into the SM particles. Slow decays of the HNLs eject additional entropy, thus diluting the otherwise conserved quantities, such as $Y_B$ or $n_{DM}/s$ [83, 84]. This effect can be accounted for directly in eqs. (2.2) [48]. However, since the freeze-in asymmetry generation takes place well before the decays of HNLs, one can facilitate the numerics by computing the effect of the entropy injection separately. The HNL decays take place at low temperatures and the corresponding rates can be approximated [17] by zero-temperature vacuum decay widths of the HNLs [85, 86].\footnote{Our results slightly differ from those in ref. [48]. The reason is that the we use the zero-temperature width of HNLs which accounts for all decay channels [85, 86], whereas an effective number of flavours is used in [48].}

The diagonal elements of the matrix $\Gamma_N$ are given by [17]

$$
(\Gamma_N^{\text{dec}})_{II} = (F^\dagger F)_{II} \frac{M_I}{E_N} \sum_{X_\alpha} \frac{\Gamma(N_I \rightarrow X_\alpha)}{|F_{aI}|^2}, \quad (3.1)
$$

where the sum goes over all allowed final states $X_\alpha$ containing a lepton of flavour $\alpha$ and $\Gamma$ is the partial decay width in the rest frame of an HNL. We add expression (3.1) to $\Gamma_N$ which is mostly determined by “indirect” contributions (i.e. those proportional to Higgs vev rather than to temperature, see e.g. ref. [32]). Owing to the factor $M_I/E_N$ the decay rates (3.1) decrease at high temperatures. As an example, the total rates for $M = 1$ GeV HNLs are shown in the upper panel of figure 2.

One can expect from figure 2 that the HNLs with $|\text{Im}\, \omega| \ll 1$ will deviate from equilibrium at $T = O(1)$ GeV. They will start to decay at temperatures around $O(10)$ MeV when $\Gamma_I$, which at low temperatures are determined by the vacuum decays and do not explicitly depend on temperature, surpass the Hubble rate again. These decays cause extra entropy injection [83, 84] as shown in the lower panel of figure 2. In order to quantify this statement we follow the procedure of ref. [48] using the rates as in eq. (3.1). We solve the corresponding kinetic equations from temperature $T_{in} = 5$ GeV till $T_{fin} = 0.1$ MeV numerically for different masses of the HNLs and compute the dilution factor $sa^3$, where $a$ is the scale factor. As introduced in the section 2 $Y_X$ is a conserved quantity if there are no processes changing the number density of particle species $X$ and the entropy in the co-moving volume. In fact the baryon to entropy ratio $Y_B$ freezes out at $T < T_{sph} \simeq 130$ GeV, and the generation of lepton asymmetries $Y_{\Delta_\alpha}$ also stops at temperatures below $\sim 10$ GeV. It is the entropy injection diminishes both $Y_B$ and $Y_{\Delta_\alpha}$ subsequently. Since this dilution happens after HNLs cease generating $Y_B$ and $Y_{\Delta_\alpha}$, it is sufficient to compute $sa^3$ independently and then use the fact that $Y_X(T_{fin}) = Y_X(T_{in})(sa^3)_{in}/(sa^3)_{fin}$.

The dilution factor for various values of $\text{Im}\, \omega$ is shown in figure 3, in which we normalized $(sa^3)_{in} = 1$. It is large if the HNL decays are slow, that corresponds to small mixings and small masses. For a given mass the mixing takes its smallest value for $\text{Im}\, \omega = 0$. Note that the upper black curve in figure 3 should be treated with caution. Indeed, a very light HNL may be long-lived, thus causing problems with Big-Bang nucleosynthesis [87–91].
Figure 2. Upper panel: the ratio \( \Gamma_I/H \) as a function of temperature. Green and orange curves correspond to \((\Gamma_N)_{22}\) and \((\Gamma_N)_{33}\). Horizontal line is placed at \( \Gamma_I/H = 1 \). Central panel: Actual number density of HNLs normalized to the entropy density (solid lines) and the equilibrium one (dashed line). HNLs deviate from equilibrium when the rates cross the Hubble rate. Lower panel: dilution factor \( s a^3 \) as a function of temperature. Out-of-equilibrium decays of HNLs increase the value of \( s a^3 \). The parameters are fixed to the values \( M = 1 \text{ GeV}, \Delta M = 10^{-11} \text{ GeV}, \text{Im} \omega = 2 \times 10^{-3}, \text{Re} \omega = 0.5\pi, \delta = 1.5\pi, \) and \( \eta = 0.5\pi \).

4 The most efficient asymmetry generation

The dynamics described by system (2.2) depends on the parameters in a complicated way. For example, as we have already mentioned, \( \exp (\text{Im} \omega) \) controls the size of the Yukawas and hence the size of the damping rates \( \Gamma_N, \Gamma_{\nu_3} \), and the efficiency of the asymmetry transfer from HNL to neutrino sector and back. Therefore large \( |\text{Im} \omega| \) causes efficient asymmetry generation and, at the same time, its efficient wash-out. In this section we qualitatively describe a regime in which large lepton asymmetry can be produced after the electroweak phase transition. The findings of this section are quantitatively confirmed in the next one.

The major obstacle to large asymmetries at low temperatures is the wash-out. However,
the wash-out is not always efficient. Since there are many different rates, let us illustrate the statement considering the following parameter set.

\[
\begin{align*}
M &= 2.0 \text{ GeV}, \quad \Delta M = 0.983 \times 10^{-11} \text{ GeV}, \quad \text{Im} \omega = -2.754 \times 10^{-3}, \\
\text{Re} \omega &= 0.551 \pi, \quad \delta = 0.993 \pi, \quad \eta = 1.479 \pi.
\end{align*}
\]

(4.1)

The HNL damping rates $\Gamma_N$ for the parameter set (4.1) are shown in the upper panel of figure 4, in which the HNL rates cross the Hubble rate around $50 - 100 \text{ GeV}$. The other rates exhibit very similar patterns. Soon after $\Gamma_N/H$ becomes larger than $1$, HNL densities start to follow equilibrium line $Y_{eq} \simeq 0.0025$ (the second panel in figure 4). Let us, however, consider the total asymmetry in the HNL sector

\[
\Delta_N = \left[ \int \frac{d^3k}{(2\pi)^3} \text{Tr} (\rho_N - \rho_{\bar{N}}) \right].
\]

(4.2)

Evolution of $\Delta N/s$ is shown in the third panel of figure 4 (red dashed curve). One can see that the asymmetry is much smaller than $Y_{eq} \simeq 0.0025$ but it is non-vanishing. In fact, the value $(\Delta N/s)/Y_{eq} \sim 10^{-3}$ is surprisingly large given that the HNL rates exceed the Hubble rate by several orders of magnitude.

In order to understand why the asymmetry $\Delta_N$ is partially preserved during the period of equilibrium, it is instructive to rewrite the kinetic equations in the matrix form

\[
\frac{dn(T)}{dT} = A(T)n(T) + n^S(T),
\]

(4.3)

where $n(T)$ is a column with 11 real entries (3 lepton asymmetries $Y_{\Delta a}$, diagonal elements of hermitian matrices $Y_{\pm}$, and real and imaginary parts of the off-diagonal elements of $Y_{\pm}$), $n^S(T)$ is a column of source terms associated with time derivative of $\rho_{N}^{eq}$. Above we
Figure 4. An example of generation of large LTA. The first panel displays two eigenvalues of the HNL equilibration rate $\Gamma_N$ divided by the Hubble rate $H$ as functions of temperature $T$. The second panel shows the HNL yields as functions of $T$. The green dotted line indicates the equilibrium yield. The third panel shows the evolution of the lepton asymmetries (solid lines) and HNL asymmetry (dashed line) (4.2). The fourth panel shows the number of HNL oscillations (see eq. (4.5)). Having $T_{osc}/T \simeq 1$ in the region where the eigenvalues of the rate $\Gamma_N$ cross the Hubble rate maximizes generated asymmetry. The model parameters are specified in eq. (4.1).

introduced $Y_+ = (Y_N + Y_{\bar{N}})/2 - Y_{eq}$ and $Y_- = Y_N - Y_{\bar{N}}$, see [46] for details. For the HNL masses we are considering here, the source term is irrelevant\footnote{Precisely speaking, the deviation from equilibrium caused by the expansion of the Universe could drive the asymmetry generation, see, e.g. [51]. However, the asymmetries generated in this way are at most of order $\sim 10^{-7}$, i.e., several orders of magnitude smaller than the required ones.} and the whole information is contained in the matrix $A(T)$. The imaginary parts of the eigenvalues of $A(T)$ describe oscillations, whilst the real parts correspond to production and wash-out of asymmetries.
The real parts of the smallest and the largest eigenvalues of $A$ as functions of temperature are shown in figure 5. As one can see, the smallest eigenvalue exceeds the Hubble rate only in a relatively short temperature interval. This means that the linear combination of the variables corresponding to this eigenvalue is almost conserved. The weight coefficients of this linear combinations vary with temperature. We have performed a numerical scan with $M \leq 5$ GeV and found two combinations

$$L_\pm \simeq \Delta_N \mp \sum_\alpha \Delta_\alpha,$$

which are nearly conserved either at $T \gtrsim 150$ GeV ($L_-$) or $T \lesssim 20$ GeV ($L_+$). In the intermediate region the linear combination corresponding to the smallest eigenvalue is more complicated and contains all 11 variables $n(T)$ entering equation (4.3). Combinations (4.4) correspond to the approximately conserved quantum numbers identified in ref. [41] (notation in (4.4) is chosen to match that of ref. [41]). Indeed, as it has been shown in refs. [41] and [42], the time derivative of $L_-$ is proportional to fermion number violating (helicity-conserving) rates which are suppressed in the symmetric phase. In turn, the time derivative of $L_+$ is proportional to the fermion number conserving rate [41] which becomes small at low temperatures. One particular consequence of the approximate conservation of $L_+$ is that $\Delta_N \simeq -\Delta_\alpha$ at low temperatures. Numerical solution of the kinetic equations confirms this observation, as can be seen in the third panel of figure 4.

We see that the presence of an almost conserved combination corresponding to the smallest eigenvalue of $A$ protects the asymmetry from being completely washed out. As we have already mentioned, the value of $|\text{Im} \omega|$ controls the size of Yukawa couplings. The values of Yukawas determine in turn the eigenvalues of the matrix $A$. Therefore, if we are interested in the presence of a small eigenvalue, $|\text{Im} \omega|$ should be close to zero.

Now once we have clarified how the asymmetry can survive, we need to understand how it can be generated. Thermally produced asymmetry is enhanced by the oscillations of the HNLs [1, 15]. This mechanism is the most efficient around the first few oscillations.

**Figure 5.** The real parts of the smallest (blue) and the largest (orange) eigenvalues of the matrix $A$ normalized to the Hubble rate as functions of temperature.
The number of oscillations is given by $T_{\text{osc}}^3 / T^3$, where

$$T_{\text{osc}} \simeq (M \delta M M_0 / 3)^{1/3}.$$  \hspace{1cm} (4.5)

The physical mass difference of HNLs $\delta M$ is given by

$$\delta M = \frac{E_N}{2M} \Delta \lambda,$$  \hspace{1cm} (4.6)

with $\Delta \lambda = \lambda_2 - \lambda_1$, where $\lambda_{1,2}$ are the eigenvalues of the effective Hamiltonian $H_N$ entering eqs. (2.2). In order to clarify the parametric dependence of the physical mass difference, we present the corresponding expression at zero temperature (in the case of normal hierarchy of neutrino masses, NH for short).

$$(\delta M)^2 \simeq \Delta M^2 + \Delta M (m_3 - m_2) \cos(2 \text{Re} \omega) + \frac{1}{4} (m_3 - m_2)^2 + \mathcal{O} \left( \left( \frac{\Delta M}{M} \right)^2 \right),$$  \hspace{1cm} (4.7)

where $m_{2,3}$ are the masses of active neutrinos and $m_3 - m_2 \simeq \sqrt{m_{\text{atm}}^2} \simeq 5 \times 10^{-11}$ GeV$^6$. The similar expression in the case of inverted hierarchy (IH) can be obtained by replacing $m_2 \rightarrow m_1, m_3 \rightarrow m_2$.

The terms containing active neutrino masses at zero temperature in (4.7) originate from the Yukawa interactions in the Lagrangian. If there are no cancellations between Majorana $\Delta M$ and Yukawa contributions to the physical mass difference, the physical mass splitting is larger than the atmospheric mass difference. In this case the oscillation temperature is bounded from below by

$$T_{\text{osc}} \gtrsim 180 \text{ GeV} \left( \frac{M}{1 \text{ GeV}} \right)^{1/3}.$$  \hspace{1cm} (4.8)

This temperature exceeds $T_{\text{sph}} \simeq 130$ GeV. Therefore to generate large lepton asymmetry which would not contradict the measured value of BAU, one has to lower $T_{\text{osc}}$. This can be achieved if $\text{Re} \omega \simeq \pi / 2$. In this case the second term in (4.7) is negative and can offset two other terms.

The last requirement is that the first oscillation has to take place when the rates are large but HNLs are not in equilibrium yet, like in the lower panel of figure 4. This can be achieved for each value of $M$ by a specific choice of $\Delta M$, $\text{Re} \omega$, $\text{Im} \omega$ and two phases. In order to identify such choices we have performed a scan of the parameter space which is described in the next section.

5 Late-time lepton asymmetries

In this section we present the main results obtained through the numerical solution of equations (2.2) integrated over momentum. First, we briefly discuss the resonant mechanism of sterile neutrino dark matter production and required late-time lepton asymmetries. Next, we describe the numerical procedure and present the main results and comment on them.

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$^6$In the framework of the $\nu$MSM the mass of the lightest active neutrino is negligibly small [1].
5.1 Required LTA

As we have already mentioned, thermal DM production \[3\] via mixing with active neu-
trinos cannot be responsible for 100\% of DM abundance \[6\] given the astrophysical X-ray
bounds on the active-sterile mixing. Large lepton asymmetry, if present at temperatures
\(T_{\text{prod}} \sim 200\ \text{MeV}\), can boost the DM production \[4\]. Computation of the asymmetry that is
needed for producing the correct abundance of DM have been performed in refs. \[4–7, 79, 92\].
The recent analysis of ref. \[93\] confirmed the findings of ref. \[79\]. Moreover, the results de-
pend upon the flavour structures of the neutrino Yukawa couplings and different types of
pre-existing lepton asymmetries. Here we follow ref. \[79\], which uses \(M_{\text{DM}} = 7.1\ \text{keV}\) as a
benchmark. According to table 1 of ref. \[79\], the minimal initial asymmetry at \(T = 4\ \text{GeV}\)
yielding the correct dark matter abundance is \(Y_{\nu_e} = Y_{\nu_\mu} = Y_{\nu_\tau} \approx 11 \times 10^{-6}\). Note that all
lepton asymmetries are equal at low temperatures owing to presence of the two types of
the rates \[41\]. This translates into

\[ Y_L \equiv \sum_\alpha Y_{\Delta_\alpha} \simeq 66 \times 10^{-6}, \tag{5.1} \]

where we have defined the total lepton asymmetry \(Y_L\).\(^7\) Theoretical errors, mainly coming
from hadronic uncertainties are expected at the level 10 – 20\% \[79\].

Note that the value (5.1) is not taking into account the effect of the entropy dilution
discussed in section 3. If the two HNLs are sufficiently light and feeblely coupled, their slow
out-of-equilibrium decays which take place at \(T \ll T_{\text{prod}}\) inject additional entropy and, as a
consequence, the otherwise conserved quantities, like \(Y_B\) or \(Y_{\text{DM}}\) become smaller. We will
address this effect below.

5.2 Numerical procedure and the main results

Now our aim is to clarify whether the asymmetry at level (5.1) can be indeed generated via
the mechanism described in section 2. To answer this question we have performed a scan of
the parameter space. Equations (2.2) were solved numerically using the Fortran code based
on LSODE \[94\], see ref. \[46\] for details. The parameters considered in the scan are listed in
table 1. First we have scanned over the broad range of the parameters and identified the
most interesting region. In this region (dubbed “large LTA” in table 1) we performed more
detailed scans. It is interesting to note that even for \(\delta = 0, \eta = 0\) large asymmetry can be
generated. Let us stress that sufficiently large LTA can only be generated in the case of
normal hierarchy of neutrino masses.

Our numerical analysis confirmed qualitative considerations of section 4 for \(M < 5\ \text{GeV}\).
Namely, \(|\text{Im} \omega| \ll 1\) to ensure the smallness of the rates; \(\text{Re} \omega \simeq \pi/2\) to allow for cancel-
lations between Majorana and Higgs contributions to the physical mass difference, see
figure 6. LTA as a function of \(\text{Im} \omega\) and Majorana mass difference \(\Delta M\) is shown in figure 7.
As one can see from the figure, large asymmetry \(Y_L = \sum_\alpha Y_{\Delta_\alpha}\) can be generated if \(\text{Im} \omega\)
is sufficiently small. In this region the mixing angles between active neutrinos and HNLs

\(^7\)The total asymmetry as it defined here also contains the BAU contribution. However, since \(Y_B \ll Y_L\)
in all interesting cases, this contribution can be safely neglected.
Table 1. Parameters of the theory: common mass; Majorana mass difference; imaginary and real parts of $\omega$; Dirac and Majorana phases. In the second line we indicate the full ranges of these parameters which were considered in this work, whereas the third line corresponds to more restricted region where large LTA can be generated.

| $M$, GeV | $\log_{10}(\Delta M/\text{GeV})$ | $\text{Im} \omega$ | $\text{Re} \omega$ | $\delta$ | $\eta$ | range: |
|---------|-----------------|-----------------|-----------------|---------|-------|-------|
| $[0.1 - 30]$ | $[-17, -9]$ | $[-2, 2]$ | $[0, 2\pi]$ | $[0, 2\pi]$ | $[0, 2\pi]$ | broad range |
| $[0.1 - 30]$ | $[-12, -10]$ | $[-0.2, 0.2]$ | $[0.4\pi, 0.6\pi]$ | $[0, 2\pi]$ | $[0, 2\pi]$ | large LTA |

For example, the mixing summed over the lepton flavours and HNL generations reads

$$|U|^2 \equiv \sum_{\alpha, l} |\Theta_{\alpha l}|^2 = \frac{m_2 + m_3}{M} [\exp(2\text{Im} \omega) + \exp(-2\text{Im} \omega)].$$

In figure 8 we show the maximal value of LTA as a function of HNL mass. LTA can be indeed large for quite light HNLs, but such HNLs cannot generate enough asymmetry by the time of the sphaleron freeze-out and thus BAU is smaller than the observed value. Generated asymmetry can still be large even if we require that the observed amount of BAU is generated, as is shown by the green shape in the figure.

The generated LTA boosts the DM production which takes place at $T \sim 100$ MeV. Figure 8 shows that the correct amount of DM can be produced for $M \simeq 2$ GeV HNLs if all uncertainties are accounted for in an optimistic way. However, the decays of the very same HNLs which happen at $T \sim 1$ MeV reduce both DM and BAU. To quantify the magnitude of dilution we show the LTA divided by the dilution factor shown in figure 3 as a function of mass. The result is shown in the lower panel of figure 8. Note that dilution takes place after the conversion of lepton asymmetry to DM, so the lower panel of figure 8 shows an effective LTA. Such effective description is valid since the DM abundance depends approximately linearly on the LTA in the region of interest [79]. The DM abundance computed with this effective LTA can be directly compared with the present day value. If all factor of 2 uncertainties are pushed in the direction which maximizes generation of BAU and DM production, the $M \simeq 2$ GeV HNLs can provide \( \simeq 50\% \) of the observed DM abundance.

As it is clear from figure 9, asymmetries are much smaller in the IH case. This can be understood by considering the detailed structure of the rates $\Gamma_N$ entering the kinetic equations (2.2). We address this in appendix A.

Below we qualitatively explain the behaviour observed in figures 8 and 9 in the two distinct mass regions.

In the region $M < 5$ GeV the qualitative picture of section 4 is confirmed. The rates increase with mass so above $M \simeq 2$ GeV the wash-out becomes efficient.

Another potentially interesting region revealed by the numerical scan is the one of relatively heavy HNLs with $M \gtrsim 10$ GeV. Figures 8 and 9 exhibit that very large lepton asymmetry can be generated in this case. However, the production of such asymmetry due to the freeze-in of HNLs takes place around temperatures of sphaleron freeze-out $T_{sph} \simeq$
Figure 6. Dependence of the total lepton asymmetry on the parameters $\Delta M$ (upper panel), $\text{Im } \omega$ (middle panel), and $\text{Re } \omega$ (lower panel). Every point corresponds to a certain parameter set in the scan. Dependence on the Majorana and Dirac phases is rather flat and therefore we do not show it here.

130 GeV. It means the BAU also turns out to be much larger than the observed value. We
Figure 7. LTA as a function of Im $\omega$ and Majorana mass difference $\Delta M$. Large value of LTA can be obtained for Im $\omega$ close to zero for sufficiently light HNLs ($M = 2$ GeV, upper panel). For heavier HNLs ($M = 30$ GeV, lower panel) another mechanism is operative and hence Im $\omega$ deviates from zero and mass splitting is smaller, see the main text for more detailed explanation.

The main findings of our numerical study are the following.

- Large ($\sim 10^{-5}$) LTA can be generated during freeze-in in the case of normal hierarchy (NH) of neutrino masses.\textsuperscript{8}

- All parameter sets leading to large LTA are concentrated close to the see-saw line Im $\omega \simeq 0$. In terms of the mixing between active neutrinos and HNLs summed over

\textsuperscript{8}The situation is different if the asymmetry is generated during freeze-out or decays of HNLs (see figure 1). This mechanism allows for production of 100% DM in both NH and IH cases if Im $\omega \sim O(1)$ and the physical mass splitting is very tiny [17, 50].
Figure 8. Total lepton asymmetry as a function of $M$. Upper panel: at temperature $T = 10$ GeV. Lower panel: after dilution. The grey shaded region indicates the total lepton asymmetry which is needed for $N_1$ to compose 100% of DM. Thickness of the curves indicates the possible uncertainties. Normal hierarchy.

- If the theoretical uncertainty is accounted for in a generous way (that is, if the actual values of BAU and LTA are twice larger than those obtained with the momentum averaged equations), the total lepton asymmetry as large as $\simeq 6 \times 10^{-5}$ and the observed value of BAU can be generated simultaneously.
- Slow decays of the HNLs after generation of DM inject additional entropy thus diluting both DM abundance and BAU.
- We were able to find points where, after accounting for the entropy dilution, the $\nu$MSM can explain up to $\sim 50\%$ of 7 keV DM. Note that the value of required LTA depends on the mass and mixing angle of the DM.

\[ |U|^2 \simeq 6 \times 10^{-11} \frac{\text{GeV}}{M}. \] (5.3)
The results above are obtained using the kinetic equations (2.2) averaged over momentum with the rates as derived in refs. [42, 47]. We consider possible sources of uncertainty in section 8. Apart from these uncertainties, there might be new physical effects which are not accounted for in eqs. (2.2). In particular, we consider the processes related to the Abelian part of the anomaly in the next section.

To sum up, if (i) the dark matter abundance is vanishing at $T \sim 100$ GeV, (ii) theoretical uncertainties do not exceed the values used in this section, and (iii) effects not accounted for in our equations (2.2) are not important, we can conclude that the lepton asymmetry generated in freeze-in is not enough to explain DM and BAU simultaneously.\footnote{Note that findings of ref. [17, 50] confirm that both DM and BAU could be explained if asymmetry is generated during freeze-out and decays of HNLs.} In the next section we examine validity of the assumption (iii).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9.png}
\caption{The same plots as figure 8 but for the inverted hierarchy.}
\end{figure}
Figure 10. Total lepton asymmetry (red) and individual asymmetries (blue, orange, and green) as functions of temperature. The plot is logarithmic in both positive and negative directions of $Y$. The black dashed vertical line indicates the sphaleron freeze-out temperature $T_{sph} \simeq 130 \text{ GeV}$ where the baryon asymmetry freezes out. The total asymmetry and hence the baryon asymmetry are already large at $T_{sph}$.

6 Chiral asymmetry and magnetic fields

Up to date, all quantitative studies of the low-scale leptogenesis were based on kinetic equations (2.2). These equations seem to account for all relevant processes except those related to the Abelian part of the anomaly (second term in the equation below) in the leptonic $j_L^\mu$ and baryonic $j_B^\mu$ currents,

$$\partial_\mu j_L^\mu = \partial_\mu j_B^\mu = \frac{n_f}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \left( \frac{1}{2} g^2 F_{\mu\nu}^a F_{\rho\sigma}^a + \frac{1}{2} g'^2 F_{\mu\nu} F_{\rho\sigma} \right).$$

(6.1)

Here $n_f = 3$ is the number of fermionic generations, $F_{\mu\nu}^a$ and $F_{\mu\nu}$ are the $SU(2)$ and $U(1)$ gauge field strengths respectively. The Abelian field contribution to the divergence of the currents does not lead to the irreversible change of the baryon and lepton numbers, since for $U(1)$ gauge fields the Chern-Simons (CS) charge for vacuum configurations is zero. Still, if the $U(1)$ field is massless (as is the case for the hyper-charge field in the symmetric phase or electromagnetic field in the Higgs phase), the non-vacuum configurations with non-zero (hyper) magnetic field may carry CS number and therefore the transfer of baryon or lepton numbers into CS “condensate” of (hyper) magnetic fields may take place. And, indeed, if there were primordial helical hyper-magnetic fields, they could be converted into baryon asymmetry [96, 97]. Or, the large fermionic asymmetries can induce instabilities in the gauge sector and lead to generation of magnetic fields [98, 99].

We will assume that there are no primordial (hyper) magnetic fields. If true, the effects associated with the Abelian part of the anomaly equation can be neglected for analysis of the baryogenesis. Indeed, in the symmetric phase of the SM the rate of the $SU(2)$ anomalous processes is much higher than that related to the $U(1)$ group. Also, the generation of magnetic fields is a non-linear effect [96, 99] which requires large fermionic asymmetries.
However, they are very small at the sphaleron freeze-out (we know this empirically from the measured value of the baryon asymmetry) and the system stays in linear regime.

The generation of (large) lepton asymmetry can only occur in the Higgs phase of the SM, below the sphaleron decoupling. Here the massless gauge degree of freedom which can give rise to long-ranged field corresponds to the electromagnetic field $A_\mu$ which is a mixture of the $SU(2)$ and $U(1)$ hypercharge fields. We will denote the corresponding magnetic and electric field strengths by $\vec{B}$ and $\vec{E}$. The right-hand side of eq. (6.1) does not contain a product $\vec{B}\vec{E} \propto F_{EM}\tilde{F}_{EM}$ meaning that the baryon and lepton numbers cannot be converted into the CS condensate of the electromagnetic field. The combination $F_{EM}\tilde{F}_{EM}$ still appears as an anomaly in the chiral current of leptons,

$$\partial_\mu j^\mu_{5\alpha} = m_\alpha \bar{\Psi}_\alpha \Psi_\alpha + \frac{1}{32\pi^2} e^{\mu\nu\rho\sigma} \frac{1}{2} e^2 F_{EM}\tilde{F}_{EM},$$

(6.2)

where $m_\alpha$ is the mass of a lepton of generation $\alpha$ and $e$ is the electric charge. The reactions which change the lepton chiralities can occur due to perturbative spin flip (the first term in (6.2)) or due to the processes in which the CS number of electromagnetic field is changed (the second term in (6.2)). The rate of the first type of reactions at temperatures higher than the mass of the corresponding lepton is at least of the order of $\Gamma_{\text{flip}} \sim \alpha^2 \frac{m_\alpha^2}{4\pi} T$. These are just electromagnetic reactions: the Compton process $l\gamma \rightarrow l\gamma$ and annihilation $l^+l^- \rightarrow \gamma\gamma$.

The rate of the second type of reactions is known to be non-zero in the presence of external magnetic fields. An evaluation based on magnetohydrodynamics (MHD) reads [96, 98] $\Gamma_{\text{anom}} = \frac{12\alpha^2 B^2}{\pi\sigma} \frac{T}{T}$, where $\sigma \propto T/\alpha$ is the conductivity of the plasma. This estimate accounts only for the fluctuations of the electromagnetic field on the scales larger than the mean free path of the particles in the plasma. The recent lattice simulations [101, 102] incorporating the short scale fluctuations indicated that the actual rate has the same parametric dependence on the magnetic field and the fine-structure constant $\alpha$, but is larger by a factor of 10. It is not excluded that the non-perturbative processes can occur even in the absence of the magnetic fields (see the discussion in [101]) with the rate of the order of $\alpha^6 T$. These processes are of the similar nature as the weak SU(2) sphaleron processes.

In the first type of processes the fermionic chirality is lost forever, while in the second it is transferred into Chern-Simons number of the $U(1)$ field (it can be called CS condensate). If the chirality flip rate is inferior to the one due to anomaly, the entire chiral charge is converted into the CS condensate. It is the electron flavour which is the most important as the perturbative chirality flip rate for it is much smaller than for the other flavours. The evolution of the chiral charge and of the magnetic field is quite peculiar, it has been studied in [98, 99]. Basically, the system enters in the steady state non-linear evolution in which the chiral charge and the CS condensate of electromagnetic fields change much slower than one would expect from the linear analysis and from the rates of different reactions: the

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10It has been shown in the recent work [100] that this rate is in fact much larger.

11Yet another contribution is the Higgs decays, $h \rightarrow l^+l^-$, which freezes out at temperature $\simeq 8$ GeV for electron flavour [99] must be considered as well.

12The importance of accounting for fluctuations of Abelian U(1) field in anomalous processes was also stressed in [103].
anomaly pumps the chiral charge into the CS condensate, the CS condensate decreases due to plasma conductivity and releases back the chiral charge into the plasma. These two processes nearly compensate each other, leading to approximate conservation of the chiral charge.

The phenomena discussed above may result in modification of the evolution of leptonic asymmetries which we described up to now by equations (2.2) below the sphaleron freeze-out. The detailed study of this problem would require three-dimensional magneto-hydro-dynamical simulations including helical and non-helical magnetic fields, unified with analysis of the thermal fluctuations of electromagnetic fields at smaller scales. We leave this for a future work. Instead, we will discuss at the quantitative level what kind of physical effects one can expect, and how they can modify the computation of LTA.

The important point is that HNLs interact directly only with the left-handed leptons. Therefore, their CP-violating interactions produce not only the leptonic asymmetry, but also the asymmetry in chiral charges, and, in particular, in the chiral lepton densities. At temperatures where the production of the lepton asymmetries is the most efficient, $T \sim 20$ GeV, the created chiral charge may be large enough to lead to the generation of the helical magnetic field, and the anomalous rate $\Gamma_{\text{anom}}$ exceeds the perturbative chirality flip rate $\Gamma_{\text{flip}}$ for electron, but $\Gamma_{\text{anom}} \lesssim \Gamma_{\text{flip}}$ for muon and tau leptons. This means that the chiral asymmetry in muons and tau flavours is destroyed, but that sitting in the electron flavour is transferred to the CS condensate. If it is sufficiently large, the system enters into the non-linear regime as above with effectively conserved chiral asymmetry in the electronic flavour. The processes with HNLs will redistribute this asymmetry among the other flavours forming a long-living configuration carrying a net lepton number density and magnetic helicity. In the most optimistic scenario the maximal asymmetry in the electronic flavour, attained during the time evolution which does not account for magnetic fields, will survive until the late times and amplify the sterile neutrino DM production.

Having these physics considerations in mind it is possible to write “phenomenological” equations accounting for CS condensate. To this end let us introduce as usual the chemical potentials $\mu_i$ for slowly varying leptonic numbers $L_i$, for electric charge $Q$ and baryon number $B$: $\mu_q$ and $\mu_B$, and, to account for CS condensate, the chemical potential for the number density $R_0$ of the right-handed electrons, $\mu_R$ (when the processes with chirality flip are in thermal equilibrium and anomaly is absent $\mu_R = 0$). The standard procedure allows to express $\mu_i$ and $\mu_R$ via $L_i$ and $R_0$ (we put $Q = 0$, and also $B = 0$ since the baryon asymmetry is too small to lead to any effects). The result is

$$\mu_1 = \frac{6}{271 T^2} (143L_1 + 10L_2 + 10L_3 + 128R_0), \quad (6.3)$$

$$\mu_2 = \frac{2}{271 T^2} (30L_1 + 311L_2 + 40L_3 - 30R_0), \quad (6.4)$$

$$\mu_3 = \frac{2}{271 T^2} (30L_1 + 40L_2 + 311L_3 - 30R_0), \quad (6.5)$$

$$\mu_R = \frac{12}{271 T^2} (64L_1 - 5L_2 - 5L_3 + 207R_0). \quad (6.6)$$

The expressions for $\mu_i$ should be used in equations (2.2) written in previous chapters of this...
work. To account phenomenologically for magnetic fields, we should add an equation for \( R_0 \). In the absence of the CS condensate and of anomalous reactions, the equation for \( R_0 \) has the form

\[
\frac{\partial R_0}{\partial t} = -\Gamma_R \mu_R ,
\]

(6.7)

where \( \Gamma_R \) is the perturbative chirality flip rate. This equation will drive \( \mu_R \) to zero, and the results for the previous sections are reproduced. Now, if the CS condensate and anomalous reactions are present, \( \mu_R \) will be driven to some fraction \( \kappa \) of the asymmetry in \( L_1 \), resulting in the change of eq. (6.7) to

\[
\frac{\partial R_0}{\partial t} = -\Gamma_C (\mu_R - \kappa \mu_1) ,
\]

(6.8)

where \( \Gamma_C \) is the effective rate that accounts for non-linear dynamics of the condensate and the chiral charge. When the combination \( (\mu_R - \kappa \mu_1) \) approaches zero, the system should enter in the steady state solution with non-zero value of \( R_0 \). This is achieved if \( \Gamma_C \) also goes to zero when \( (\mu_R - \kappa \mu_1) \to 0 \). Assuming \( \Gamma_C = \gamma_C (\mu_R - \kappa \mu_1)^2 \) we arrive at an effective equation

\[
\frac{\partial R_0}{\partial t} = -\gamma_C (\mu_R - \kappa \mu_1)^3 ,
\]

(6.9)

where \( \gamma_C \) does not depend on the chemical potentials, whereas the sign “minus” and power “3” was chosen to ensure the qualitatively correct behaviour of the solution.

This equation is to be added to eqs. (2.2). This new system contains two new parameters, \( \gamma_C \) and \( \kappa \), the estimate of which goes beyond the scope of this paper. So, we simply considered the evolution of the system in different cases. A particular example with \( \kappa = 0.1 \) is shown in Fig. 11. In this example the rate \( \gamma_C \) is much larger than Hubble at high temperatures and then quickly drops at the temperature where electron asymmetry is maximal. The number density of right-handed electrons freezes at its maximum value. The resulting lepton asymmetry still “feels” the effects of HNLs which freeze a bit later. This leads to the partial redistribution of electron asymmetry in agreement with (6.6). Still, as one can see from figure 11, the resulting electron asymmetry is much larger than asymmetries in the other lepton flavours, whereas if the dynamics of chiral condensate is not accounted for, all three asymmetries are the same (cf. the dashed line in figure 11).

As we have already discussed, the most optimistic case corresponds to the pattern in which the asymmetry in electron flavour reaches its maximum at some temperature and then freezes around this value. By varying the coefficients \( \gamma_C \) and \( \kappa \) we were able to see that this type of behaviour can always be achieved. So, for the scan of the parameters in this most optimistic case we were solving the original equations (2.2) and reading off the maximal asymmetry in the electron flavour attained during the time evolution. This procedure is discussed in greater detail in the next section.

\[\text{In this section we consider temperatures below } T_{\text{sph}} \text{ and do not distinguish lepton asymmetries } L_\alpha \text{ and } L_\alpha - B/3 \text{ owing to the fact } B/s \sim 10^{-10} \text{ whereas the lepton asymmetries interesting from the DM production perspective are 5 orders of magnitude larger.}\]
Figure 11. Evolution of the lepton asymmetries in the presence of chiral condensate (solid curves). For comparison we show the evolution described by the equations (2.2) without (6.9) (dashed curves). In the presence of chiral condensate the electron asymmetry freezes at much larger value. The lower panel shows the zoomed in region $T < 50$ GeV.

7 Maximal electron asymmetry

In the previous section we have seen that the processes related to the Abelian part of the anomaly can significantly alternate dynamics of asymmetry production if large electron asymmetry is generated at some temperature.

It is premature to perform a parameter scan using eqs. (2.2) plus (6.9) since we don’t know the exact values (and temperature dependence) of the coefficients $\gamma_C$ and $\kappa$. Still it is important to clarify the potential significance of the effects related to the anomaly. To this end we have asked a different question: what is the maximal value of the electron asymmetry which can be reached in the system described by eqs. (2.2). This maximal asymmetry can be partially conserved as in the example of figure 11. Let us stress that the generation of maximal asymmetry does not guarantee that the final asymmetry will be the largest. In fact, there are two processes which are important: (i) generation of the chiral asymmetry and (ii) its wash-out due to interactions with the HNLs. Therefore, the parameter leading to the largest value of the maximal electron asymmetry does not necessarily leads to the largest value of the final asymmetry, since redistribution between the other flavours can be
more efficient as well. Nevertheless, a study of the maximal possible electron asymmetry is a first step in this direction. The results of the scan of the parameter space are presented in figure 12. In this figure we show the maximal value of the asymmetry in the electron flavour as a function of the mass of the HNLs. As one can see, the maximal asymmetry can be of the correct magnitude for HNLs with \( M > 2 \) GeV. Even though this fact doesn’t guarantee that the \( \nu \)MSM can account for 100\% of DM without fine-tunings of ref. [17], it indicates the importance of the non-perturbative effects which have not been accounted for so far.

\[
\begin{align*}
\text{Figure 12. Maximal electron asymmetry as a function of M. Upper panel: normal hierarchy.}\n\text{Lower panel: inverted hierarchy. The grey shaded region indicates the electron asymmetry which is needed for } N_1 \text{ to compose 100\% of DM. The span over } \Delta_e \text{ is due to different assumptions about DM mixings. Thickness of the orange and green curves indicates the possible uncertainty of the averaging procedure.}\n\end{align*}
\]

8 Possible uncertainties of kinetic equations

We have already mentioned in section 2 that our analysis contains the inherent uncertainty related to integration of the kinetic equations over momentum. We have also stressed that
the non-perturbative effects related to Abelian chiral anomaly are not accounted for in the kinetic equations. In this section we discuss the other possible sources of uncertainties.

First, let us comment on the role of the rates entering kinetic equations (2.2). In the broken phase these rates can be split into “direct” (proportional to $T$) and “indirect” (proportional to the Higgs vev $v_0$) parts [32]. The “indirect” rates depend on the thermal neutrino interaction rates. Recently, one of these rates has been computed at next-to-leading order [104]. It has been found that the NLO rate is 15...40% smaller than the leading order one. Even though it has been shown in ref. [104] that the resulting lepton asymmetry is affected only at 1% level, it is still reasonable to question the sensitivity of LTA to variation of the other rates. To partially clarify this issue we have computed LTA multiplying the rates $\Gamma_{\nu\alpha}, \tilde{\Gamma}_{\nu\alpha}, \Gamma_N, \tilde{\Gamma}_N$ entering kinetic equations (2.2), by a constant factor. Result of this procedure is shown in figure 13. The resulting LTA is surprisingly stable against the factor of 2 variations of the rates. Note however, that the value of the BAU is sensitive to this variations. Namely, if the rates are larger or smaller one will need to change the other parameters of the theory in order to obtain the correct amount of BAU, this in turn will change the resulting lepton asymmetry. Further, figure 13 shows that larger rate variations completely change the dynamics. Also, as one can see from the right panel, the maximal value of the electron asymmetry is more sensitive to the rates. Therefore further refinement of the rates is an important task.

A more serious concern is related to the effective Hamiltonian. Indeed, the derivation of equations (2.2) in refs. [41, 46] was based on the separation of time scales. Namely, all quantities proportional to Majorana mass difference of HNLs $\Delta M$ or to Yukawas $F_{\alpha I}v_0$ were treated as small compared to combinations of energies of neutrinos and HNLs, such as $E_N + E_\nu$ or $E_N - E_\nu$: \[ \Delta M, F_{\alpha I}v_0 \ll E_N + E_\nu, E_N - E_\nu. \] (8.1)

---

**Figure 13.** Evolution of the total lepton asymmetry (left panel) and the electron asymmetry (right panel) computed using the rates multiplied by a constant factor $\Gamma \rightarrow \kappa \cdot \Gamma$, where $\Gamma$ denotes all rates $\Gamma_{\nu\alpha}, \tilde{\Gamma}_{\nu\alpha}, \Gamma_N, \tilde{\Gamma}_N$. The solid green line corresponds to the original rates. The vertical grey line in the left plot indicates the sphaleron freeze-out temperature $T_{sph} \simeq 131.7$ GeV. The parameters are fixed to the values $M = 1$ GeV, $\Delta M = 1.26 \times 10^{-11}$ GeV, $\text{Im} \omega = -0.14$, $\text{Re} \omega = 0.52 \pi$, $\delta = 1.30 \pi$, and $\eta = 1.02 \pi$. 
This allowed us to integrate out the fast processes and derive equations (2.2) in terms of slowly varying quantities. In the mass region of interest, \( M \lesssim 35 \) GeV, assumption (8.1) is justified at temperatures above the sphaleron freeze-out, \( T_{\text{sph}} \simeq 131.7 \) GeV. In this regime thermal corrections to active neutrino energy are proportional to \( T \) and large \([105]\). However, (8.1) might not be valid at temperatures around a few tens of GeV. The reason is that the thermal correction to active neutrino at some point changes its sign so the energy levels of HNLs and active neutrinos do cross. This effect has not been accounted for in our rates in the Higgs phase.

While the LTA calculations are rather robust to changes of the rates, the role of the effective Hamiltonian \( H_N \) is far more drastic. The effective Hamiltonian can be decomposed into fermion number conserving and violating parts: \( H_N = H_+ + H_- \) (following notations of refs. \([41, 46]\)). For an illustration we switched off the \( H_+ \) part (see the precise expressions in \([41, 46]\)) completely and solved the kinetic equations, see figure 14.\(^\text{14}\) It shows that

![Figure 14](image-url)

Figure 14. Evolution of the total lepton asymmetry (left panel) and the electron asymmetry (right panel) computed with (blue line) and without (orange line) \( H_+ \) part of the effective Hamiltonian. The parameters are fixed to the values \( M = 1 \) GeV, \( \Delta M = 1 \times 10^{-12} \) GeV, \( \text{Im} \omega = \log(3) \), \( \text{Re} \omega = 13/16\pi \), \( \delta = 29/16\pi \), and \( \eta = 22/16\pi \).

production of the asymmetry is greatly enhanced if \( H_+ = 0 \). The reason is that \( H_- \sim (E_N - E_\nu) \) crosses zero at some temperature around \( 20 - 30 \) GeV, precisely where the rates peak. Around the zero of \( H_- \) the physical mass difference is determined only by the Majorana part, and can be small enough to ensure the resonant amplification of the asymmetry production. Similar observations about the importance of the thermal mass corrections have been made in ref. \([47]\).

Figure 14 illustrates the importance of the effective Hamiltonian. Both LTA and maximal electron asymmetry can vary by many orders of magnitude depending on the behaviour of \( H_I \) around the point where \( E_N = E_\nu \). Therefore a dedicated derivation of the kinetic equations accounting for the level crossing is very desirable. We leave this for future work.

\(^\text{14}\)Let us note in passing that computing BAU for the parameter set used in figure 14 requires one to use the approach of ref. \([43]\), since the total lepton asymmetry changes the sign right before the sphaleron freeze-out. This approach allows for accurate tracking of BAU without enlarging the set of equations and making it stiffer. To this end one needs to solve a separate kinetic equation for sphalerons using the lepton asymmetries as an input.
9 Discussion

In this work we have studied the freeze-in generation of the late time asymmetry in the $\nu$MSM. More specifically, we have systematically investigated production of asymmetry which takes place before decays of HNLs. Using the momentum averaged kinetic equations based on (2.2) with the state-of-the-art rates taken from [47] we have found that large lepton asymmetry can be generated at temperatures above a few GeV. The mechanism differs from the one considered in refs. [17, 26, 50]. Considered mechanism is operative due to presence of both fermion number violating and conserving rates [41, 42]. Even though the generated asymmetry could be quite large, the subsequent decays of the HNLs reduce the created DM abundance. Using the benchmarks from ref. [79] we found that the asymmetry generated during freeze-in can eventually be responsible for $\sim 50\%$ of DM abundance. It is important to note that large LTA can be generated only for small values of $\text{Im}\omega$. This means that the values of the total mixing $|U|^2$ is very close to the see-saw ones. Interestingly, Higgs inflation in Einstein-Cartan formulation provides an independent mechanism of DM production [8]. In this mechanism the only constraint on the $\nu$MSM parameters comes from BAU, as no LTA is required.

We identify further directions.

- A regime of the asymmetry generation during freeze-out and decays have been studied in refs. [17, 26]. There were many significant improvements in understanding of the system since these works have been performed. The recent study [50] has demonstrated that production of the correct amount of DM in the $\nu$MSM is possible also accounting all the improvements and effects, such as the entropy dilution. Still, an updated study of the parameter space is missing.

- The results here were obtained using the kinetic equations averaged other momentum. It is generally believed that results obtained by averaged equation do not deviate significantly from those obtained by solving the full integro-differential system [24, 47]. However, repeating the current analysis with the original equations, if technically possible, would be very desirable.

- The other possible uncertainties related to the interaction rates and especially the effective Hamiltonian could be large. These points need to be further clarified.

- Last, but not the least, as we have discussed in section 6 non-perturbative effects associated with Abelian anomaly may play an important role in the dynamics of the system. Therefore two important tasks arise: (i) derivation of a kinetic equation (6.9) from the first principles and (ii) determination of the parameters $\gamma_C$ and $\kappa$ entering this equations.

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A Late time asymmetries and Inverted hierarchy

In this section we provide more details on the inverted hierarchy case. As has been discussed in section 5, the asymmetry in the IH case is much smaller than in the NH case. In order to understand this fact, we need to examine the rates entering kinetic equations (2.2). Namely, let us consider the HNL washout rate $\Gamma_N$. It can be written as

$$\Gamma_N = \gamma_+ \sum_{\alpha} \begin{pmatrix} h_{\alpha 3} h_{\alpha 3}^* & -h_{\alpha 3} h_{\alpha 2}^* \\ -h_{\alpha 2} h_{\alpha 3}^* & h_{\alpha 2} h_{\alpha 2}^* \end{pmatrix} + \gamma_- \sum_{\alpha} \begin{pmatrix} h_{\alpha 2} h_{\alpha 2}^* & -h_{\alpha 3} h_{\alpha 2}^* \\ -h_{\alpha 2} h_{\alpha 3}^* & h_{\alpha 3} h_{\alpha 3}^* \end{pmatrix} ,$$

(A.1)

where $\gamma_\pm$ are functions of the temperature and momentum, whereas $h_{\alpha I}$ are Yukawas in the flavour basis (see, e.g. ref. [46]). The combinations of Yukawas entering (A.1) can be expressed using the parameter $\omega$ as

$$\sum_{\alpha} h_{\alpha 2} h_{\alpha 2}^* = \frac{(m_h + m_l) M}{2v^2} \exp(2\text{Im}\omega),$$

$$\sum_{\alpha} h_{\alpha 3} h_{\alpha 3}^* = \frac{(m_h + m_l) M}{2v^2} \exp(-2\text{Im}\omega),$$

$$\sum_{\alpha} \text{Re}(h_{\alpha 2} h_{\alpha 3}^*) = \frac{(m_h - m_l) M}{2v^2} \cos(2\text{Re}\omega),$$

$$\sum_{\alpha} \text{Im}(h_{\alpha 2} h_{\alpha 3}^*) = \frac{(m_h - m_l) M}{2v^2} \sin(2\text{Re}\omega),$$

(A.2)

where $v = 173.1$ GeV is the Higgs vev, $m_h$ is the mass of the heaviest active neutrino, and $m_l$ is the mass of the lighter one.

$$m_h + m_l^{\text{NH}} = m_{\text{atm}} + m_{\text{sol}} \simeq 5.86 \times 10^{-11} \text{ GeV},$$

$$m_h + m_l^{\text{IH}} = m_{\text{atm}} + \sqrt{m_{\text{atm}}^2 - m_{\text{sol}}^2} \simeq 9.86 \times 10^{-11} \text{ GeV},$$

$$m_h - m_l^{\text{NH}} = m_{\text{atm}} - m_{\text{sol}} \simeq 4.13 \times 10^{-11} \text{ GeV},$$

$$m_h - m_l^{\text{IH}} = m_{\text{atm}} - \sqrt{m_{\text{atm}}^2 - m_{\text{sol}}^2} \simeq 7.51 \times 10^{-13} \text{ GeV}.$$  

(A.3)

As we have discussed in section 4, the asymmetry peaks at $\text{Im}\omega \simeq 0$. In this case the diagonal entries in (A.1) are equal. As a consequence, the difference between the eigenvalues of $\Gamma_N$ is proportional to the magnitude of the off-diagonal elements. We can see from the expressions above that the off-diagonal entries of $\Gamma_N$ are $\sim 55$ times smaller in the IH case. In physical terms it means that the equilibration rates of two HNLs are practically the same in the IH case (cf figures 2 and 4 in ref. [41]). This implies that less asymmetry can build up between the moments when two HNLs enter equilibrium, since it happens almost simultaneously. In the NH case the equilibration rates are different, and therefore more asymmetry can survive in the HNL sector.
Yet another consequence of eq. (A.2) is related to the $CP$ violation in the oscillations of HNLs. The effective Hamiltonian $H_N$ describing these oscillations depends on the same combinations of Yukawas as (A.1). The $CP$ violating effects are associated with the imaginary part of $H_N$ which—as one can see from eqs. (A.2) and (A.3)—is also smaller in the IH case.

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