Lifelong Reinforcement Learning with Temporal Logic Formulas and Reward Machines

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Abstract
Continuously learning new tasks using high-level ideas or knowledge is a key capability of humans. In this paper, we propose Lifelong reinforcement learning with Sequential Linear temporal logic formulas and Reward Machines (LSRM), which enables an agent to leverage previously learned knowledge to fasten learning of logically specified tasks. For the sake of more flexible specification of tasks, we first introduce Sequential Linear Temporal Logic (SLTL), which is a supplement to the existing Linear Temporal Logic (LTL) formal language. We then utilize Reward Machines (RM) to exploit structural reward functions for tasks encoded with high-level events, and propose automatic extension of RM and efficient knowledge transfer over tasks for continuous learning in lifetime. Experimental results show that LSRM outperforms the methods that learn the target tasks from scratch by taking advantage of the task decomposition using SLTL and knowledge transfer over RM during the lifelong learning process.

1 Introduction

There are at least two significant abilities of human intelligence: (i) storing learned skills in memory over lifetime and leveraging them when encountering new tasks; and (ii) utilizing high-level ideas or knowledge for more efficient reasoning and learning. These abilities enable humans to adapt quickly in environments where tasks and experiences change over time. Lifelong Reinforcement Learning (LRL) (Abel et al. 2018), Brunskill and Li 2014, Garcia and Thomas 2019 formalizes the problem of building taskable agents by exploiting knowledge gained in previous tasks to improve performance in new but related tasks. Solving the LRL problem is an essential step toward general artificial intelligence as it allows agents to continuously adapt to changes in the environment with minimal human intervention, which is a key feature of human learning.

There has recently been a surge of interest in methods for achieving efficient LRL, utilizing techniques such as network consolidation (Schwarz et al. 2018) or freezing (Rusu et al. 2016), rehearsal via experience replay (Isele and Cosgun 2018), and value-function/policy initialization (Abel et al. 2018). Remarkably, the line of these works has mainly focused on the continuous learning settings, where a series of related tasks are drawn from a task distribution. Attention is restricted to subclasses of MDPs by making structural assumptions about which MDP components (rewards or transition probabilities) may change in support of the generation of tasks. While this kind of assumptions is reasonable in most real life situations, there are also scenarios when either task specification is non-Markovian and thus difficult to be expressed analytically as a reward function, or the sequential tasks cannot be generated from an underlying distribution when expressed logically using formal languages (Linz 2006, Pnueli 1977). For instance, consider a scenario when an agent has learned the task of “delivering coffee and mail to office”. When facing a new task of “delivering coffee or mail to office”, it is unclear how existing LRL methods would model these tasks as the same distribution of MDP and enable efficient transfer learning among these tasks. This limitation contradicts the human ability of compositional learning using high-level ideas or knowledge, i.e., understanding novel situations by combining and reasoning over already known primitives.

In this paper, we investigate LRL problems when the series of tasks do not necessarily share the same MDP structure, but instead are specified with high-level events using Linear Temporal Logic (LTL) (Li, Vasile, and Belta 2017, Toro Icarte et al. 2018). The basic intuition is to exploit task modularity and decomposition with higher abstraction and succinctness to facilitate transfer learning in target tasks. We first introduce the Sequential Linear Temporal Logic (SLTL), which is a supplement to LTL by adding a new operator “then”, and also prove the operator laws of SLTL to provide more flexible and rich specification of a task. In order to enable more efficient task learning, Reward Machines (RM) (Icarte et al. 2018) are utilized to express structural reward functions for tasks encoded with high-level events. Synthesizing the merits of task modularity using SLTL and policy learning over RM, we propose the Lifelong reinforcement learning with SLTL and RM (LSRM) method, which stores and leverages high-level knowledge in a memory for more efficient lifelong learning of logically specified tasks. The memory contains an RM, which can be automatically updated when facing a set of target tasks, and the high-level knowledge stored in the memory can be transferred to a new decomposed target task using a number of value composition methods. We evaluate the performance of LSRM...
in the **OFFICEworld** and **MINECRAFT** domains. Experiments show that LSRM enables the agent to learn target tasks more efficiently compared to the direct methods without transfer learning.

The remaining part of the paper is organized as follows. Section 2 provides a background introduction. Section 3 introduces the SLTL language. Section 4 presents the LSRM and Section 5 provides experimental studies. Section 6 reviews some related works, and finally, Section 7 concludes with some directions of future work.

## 2 Preliminaries

We provide a preliminary introduction to RL, LTL and RM in this section. Please refer to [Sutton and Barto (2018)](SuttonB18) and [Pnueli 1977](Pnueli77) [Toro Icarte et al. (2020)](ToroIcarte2020) for more details in these topics.

### 2.1 RL

The RL problem consists of an agent interacting with an unknown environment (Sutton and Barto 2018), which can be modeled as a *Markov Decision Process* (MDP) by a tuple $M = (S, A, r, p, \gamma)$, where $S$ is a finite set of states, $A$ is a finite set of actions, $r: S \times A \times S \to \mathbb{R}$ is a reward function, $p: S \times A \times S \to [0, 1]$ is a probabilistic transition function, and $\gamma \in [0, 1]$ is a discount factor.

A policy is a mapping $\pi: S \times A \to [0, 1]$, and $\pi(s,a)$ means the probability of choosing action $a$ in state $s$. The Q-function $Q_\pi(s,a)$ following the policy $\pi$ is the expected discounted reward of choosing action $a$ in state $s$ under policy $\pi$, i.e., $Q_\pi(s,a) = \mathbb{E}_\pi[\sum_{k=0}^{\infty} \gamma^k r_{t+k+1} | s_t = s, a_t = a]$, where $s_k, r_k, a_k$ and $\gamma$ are state, reward, action, and discount factor, respectively. The goal of RL is to learn an optimal policy $\pi^{*}$, which maximizes the expected discounted reward for each $s \in S, a \in A$, i.e., $Q^{*}(s,a) := Q_{\pi^{*}}(s,a) := \max_{\pi} Q_{\pi}(s,a)$. A well-known approach for calculating the optimal Q-function in tabular case is Q-learning (Watkins and Dayan 1992). Its one-step updating rule is given by $Q(s,a) \leftarrow r(s,a,s') + \gamma \max_{a'} Q(s',a')$, where $x \leftarrow y$ means $x \leftarrow x + \alpha(y - x)$. The action $a$ is chosen by using certain exploration strategies, such as the $\epsilon$-greedy policy, i.e., choosing a random action with probability $\epsilon$, while choosing $\arg\max_{a'} Q(s,a')$ with probability $1 - \epsilon$.

### 2.2 LTL

LTL is a propositional modal logic with temporal modalities (Pnueli 1977). It has been used to specify tasks encoded with high-level events in RL by characterizing the successful and unsuccessful executions (Toro Icarte et al. 2018) [Li, Vasile, and Bellet 2017]. Each high-level event is represented by a propositional variable, and the set of all propositional variables is denoted by $\mathcal{P}$. The set of events that occur at time $t = i$ under state $s_i$ is a label, denoted by $l_i \in \mathcal{P}$, which is given by a labelling function $L: S \to 2^\mathcal{P}$, $l_i = L(s_i)$. As an illustrative example, consider the OFFICEWORLD domain presented in Figure 1(a). The propositional variables can be $\mathcal{P} = \{c, m, o, a, b, c, d\}$, where $c$ is "getting coffee", $m$ is "getting mail", $o$ is "at office", $a$, $b$, $c$, $d$ is "at A, B, C, D", respectively. An event $p \in \mathcal{P}$ occurs if and only if the agent is located at the grid marked by $p$. Hence the labelling function is $L(s) = \{p\}$ if $s$ is marked by $p$, and $L(s) = \emptyset$ otherwise. Given the propositional variables $\mathcal{P}$, LTL formulas can be constructed from the standard Boolean operators $\land$ (and), $\lor$ (or), $\neg$ (negation), and temporal operators $\square$ (next), $U$ (until). Other operators can be derived from the operators above. For example, the operator $\diamond (\text{eventually})$ is defined as $\varphi \lor \psi \lor (\square \neg \varphi \land \psi)$ and $\diamond \varphi \equiv \square U \varphi$, respectively. The semantics of LTL is defined over an infinite sequence of labels, denoted $\lambda = l_1l_2l_3 \cdots$. We use $\lambda \models \varphi$ to denote that an LTL formula $\varphi$ is determined to be true by a sequence $\lambda$, which is formally defined as follows: (i) $\lambda \models p \iff p \in \lambda$, where $p \in \mathcal{P}$; (ii) $\lambda \models \varphi \land \psi \iff \lambda \models \varphi \land \lambda \models \psi$; (iii) $\lambda \models \varphi_1 \land \varphi_2 \iff \lambda \models \varphi_1$ and $\lambda \models \varphi_2$; (iv) $\lambda \models \square \psi \iff \lambda \models \psi$; and (v) $\lambda \models \varphi_1 U \varphi_2 \iff \exists j > 0, \lambda \models \varphi_2$ and $\forall i < j, \lambda \models \varphi_1$. The notation $\lambda^i$ denotes the postfix $l_i l_{i+1} l_{i+2} \cdots$ of the sequence. Based on the above definitions, we can define tasks using LTL formulas. For instance, the task "eventually reaching office" is defined as $\diamond o$, and the task "delivering coffee to office" is defined as $\diamond (c \land \diamond o)$. In order to specify the remaining module of a task after proceeding a sequence of labels, LTL progression (Bacchus and Kabanza 2000) has been proposed. For example, we can progress the formula $\diamond (c \land \diamond o)$ by a label $\{c\}$ and get a new formula $\diamond o$, implying that the remaining task is "going to office" after getting the coffee. Formally, an LTL progression maps an LTL formula $\varphi$ and a label $l$ to another LTL formula, denoted as $prog(\varphi, l)$, which is defined recursively as follows: (i) $prog(p,l) = \top$ if $p \in \lambda$, where $p \in \mathcal{P}$; (ii) $prog(\neg \varphi, l) = \neg prog(\varphi, l)$; (iii) $prog(\varphi_1 \land \varphi_2, l) = prog(\varphi_1, l) \land prog(\varphi_2, l)$; (iv) $prog(\square \varphi, l) = \varphi$; and (v) $prog(\varphi_1 U \varphi_2, l) = prog(\varphi_2, l) \lor prog(\varphi_1, l) \land \varphi_1 \varphi_2)$. It has been theoretically proved that a sequence of labels satisfies an LTL formula if and only if the postfix of the sequence satisfies the progressed formulas, i.e., $\lambda^i \models \varphi \iff \lambda^{i+1} \models prog(\varphi, l_i)$ (Bacchus and Kabanza 2000).

### 2.3 RM

RM can be used to reveal the structure of non-Markovian reward functions of tasks that are encoded with high-level events (i.e., propositional variables), formally defined as follows (Toro Icarte et al. 2020).

**Definition 1.** Given the propositional variables $\mathcal{P}$ and a set of all possible labels $\Sigma \subseteq 2^\mathcal{P}$, an RM is a tuple $\mathcal{R} = (U, \emptyset, V, \delta, R)$, where $U$ is a finite set of states, $\emptyset_0$ is an initial state, $F \subseteq U$ are terminal states, $\delta : U \times \Sigma \to U$ is the transition function, and $R : U \times \Sigma \to \mathbb{R}$ is the reward function.

Distinguished from the states of the environment (denoted by $s$), the states of RM (denoted by $u$) and the transitions among them are generally given by the prior knowledge of specific tasks. Figure 1(b) gives an example of RM that represents the task "eventually getting coffee and mail" in OFFICEWORLD, where the nodes are states and the edges are transitions among states. Each transition is encoded with a tuple $(l_1, l_2, \cdots, l_n | r)$, where each $l_i$ is a label and $r$ is the
output reward. The initial state is colored in yellow and the terminal state is colored in red. In order to enrich the expressiveness of RM, each state of RM can also be encoded with terminal state is colored in red. In order to induce denser rewards than binary rewards above, inductive reward shaping for RM is proposed. Reward shaping for RM.

\[ R(\psi, l) = \begin{cases} 1, & \text{if } \psi \neq \top, \delta(\psi, l) = \top; \\ 0, & \text{otherwise.} \end{cases} \]  

In order to induce denser rewards than binary rewards above, automatic reward shaping for RM is proposed. It modifies the reward function as \( R'(\psi, l) = R(\psi, l) + \gamma \Phi(\delta(\psi, l)) - \Phi(\psi) \), where \( \Phi : U \rightarrow \mathbb{R} \) is the potential function calculated by value iteration. The modified reward function of the example above is presented in Figure 1(d).

The QRM algorithm is then proposed to leverage RM to learn tasks. QRM maintains a Q-function \( Q^u \) for each state of RM \( u \in U \) (or formula), and updates all the Q-functions using internal rewards and transitions of RM with one experience \( (s, a, s') \). Formally, the Q-function of each state in RM is updated by

\[ Q^u(s, a) \leftarrow Q^u(s, a) + \alpha R(u, l) + \gamma \max_{a'} Q^{u'}(s', a'), \forall u \in U \]  

where \( l = L(s) \) is the current label, \( R(u, l) \) is the internal reward and \( u' = \delta(u, l) \) is the internal next state of RM. QRM not only converges to an optimal policy in tabular cases, but also outperforms the Hierarchical RL (HRL) methods which might converge to suboptimal policies.

3 SLTL: Sequential Linear Temporal Logic

In order to enable decomposition of sequential tasks, we add a new temporal operator \( \sim \) to the traditional LTL, resulting in the Sequential LTL (SLTL). Being compatible with LTL, SLTL provides a more succinct and flexible way to describe sequential tasks, and more importantly, enables us to exploit task modularity for more efficient transfer learning than LTL. For example, the task “eventually complete a then \( b' \) is expressed as \( \diamond(a \land \Box(b)) \) using LTL, but more directly as \( \diamond(a) \sim \diamond(b) \) using SLTL. The latter expression can be decomposed into subtasks \( \diamond(a) \) and \( \diamond(b) \), the knowledge of which can be readily transferred to the learning of target task, say \( \diamond(b) \sim \diamond(a) \). However, such straightforward manipulation cannot be readily realized using the expression of LTL, since we cannot extract \( \diamond(a) \) from the LTL formula of \( \diamond(a \land \Box(b)) \). Please refer to Appendix A. for more details in the syntax and semantics of SLTL.

We prove the laws of the operator “then” to provide different ways of decomposing a target task.

Theorem 1. For any SLTL formulas \( \varphi_1, \varphi_2, \varphi_3 \), we have the associative law:

\( (\varphi_1 \sim \varphi_2) \sim \varphi_3 = \varphi_1 \sim (\varphi_2 \sim \varphi_3) \)

and the following distribution laws:

\( (i) \ (\varphi_1 \sim \varphi_2) \land (\varphi_1 \sim \varphi_3) = (\varphi_1 \sim \varphi_2) \land (\varphi_2 \sim \varphi_3) \)

\( (ii) \ (\varphi_1 \sim \varphi_2) \lor (\varphi_1 \sim \varphi_3) = (\varphi_1 \sim \varphi_2) \lor (\varphi_2 \sim \varphi_3) \)

\( (iii) \ (\varphi_1 \sim \varphi_2) \lor (\varphi_1 \sim \varphi_3) = \varphi_1 \sim (\varphi_2 \lor \varphi_3) \)

\( (iv) \ (\varphi_1 \sim \varphi_2) \lor (\varphi_2 \sim \varphi_3) = (\varphi_1 \lor \varphi_2) \sim \varphi_3 \)

Proof. See Appendix A.

The above operator laws enable various representations of a task, leading to different ways of task decomposition and thus diverse learning efficiency. Figure 2 gives an illustration: the two formulas on the top are equivalent representations of the same target task. Suppose that formulas \( a \sim b, c, d \) (colored in white) indicate the subtasks that have been learned before (i.e., learned formulas), while the remaining formulas (colored in red) indicate the new tasks that have not been learned yet. It is clear that the left presentation has fewer new subtasks than the right one. More formally, we define the smallest representation of a task using the operator laws as follows:

Definition 2. Let \( \varphi_1, \varphi_2, \cdots, \varphi_n \) be the different representations of a target task, and \( T_1, T_2, \cdots, T_n \) be their subformulas decomposed by the \( \land, \lor, \sim \) operators. Given a set of learned formulas \( T_M \), the smallest representation \( \varphi^* \) of the target task is the one with the smallest number of subtasks that have not been learned before, i.e.,

\[ \varphi^* = \varphi_i, \text{ where } i = \arg\min_{i=1,2,\cdots,n} |T_i \setminus T_M|. \]
The consideration of the smallest representation of a target task can be attributed to the fact that when facing a new task, it is more likely to learn faster if the task can be decomposed into fewer unknown subtasks. We will provide experimental evaluations on this issue in Subsec. 5.2.

4 LSRM: Lifelong Learning with SLTL and RM

In this section, we introduce the LSRM method that combines the benefits of both SLTL and RM for efficient learning of logically specified tasks. Formally, LSRM maintains a memory \( M = (R_M, T_M, Q_M) \) for the agent over lifetime, where \( R_M = (U_M, u_0, F, \delta, R) \) is the memory RM, \( T_M \) is the set of learned SLTL formulas from previous tasks, and \( Q_M = \{Q^\varphi(s, a) \mid \varphi \in U_M\} \) is the set of Q-functions corresponding to the states in \( R_M \). In a learning phase of the lifetime, the agent is required to learn a set of target tasks, and different representations of these tasks can be generated by the operator laws. The smallest representation of the target task is then chosen according to the learned formulas \( T_M \) in the memory, and then fed into Algorithm 1 as a set of target formula(s) \( T \). Concretely, for each target formula \( \varphi \) in \( T \), LSRM first extends the memory RM and returns its new extended states denoted as \( U_{\text{new}} \) (Line 2-3). The learned Q-functions are then transferred from the memory to the new Q-function corresponding to each state in \( U_{\text{new}} \) (Line 4-5). Finally, the QRM algorithm is utilized to learn the target formulas (Line 8). When the agent starts to implement a task \( \varphi \in T \) in an episode of QRM, the initial state of the memory RM is set to be \( u_0 = \varphi \). The set of learned task \( T_M \) is then updated after the QRM learning procedure (Line 9). We show the details of extension of the memory RM and transferring knowledge in the following subsections.

Algorithm 1: The Update of the Memory in LSRM

\[ \text{Input: Set of target formula(s) } T, \text{ memory } M \]
\[ \text{Output: New memory } M \]
1: \textbf{for all} target formula \( \varphi \in T \) \textbf{do}
2: \( M.\text{ExtendRM}(\varphi) \)
3: \( U_{\text{new}} \leftarrow M.\text{ReturnNewStates()} \)
4: \textbf{for all} extended state \( \psi \in U_{\text{new}} \) \textbf{do}
5: \( Q^\psi(s, a) \leftarrow M.\text{AcquireKnowledge}(\psi) \)
6: \textbf{end for}
7: \textbf{end for}
8: \( M.\text{QRMRun}(T) \)
9: \( T_M \leftarrow (\cup_{\varphi \in T} U_{\text{new}}) \cup T_M \)

4.1 Extensions of Memory RM

In this subsection, we discuss how to extend the memory RM with a target task prescribed by an SLTL formula, so that the extended RM includes all the modules of the target task for future learning. On one hand, the target formula can be iteratively progressed in order to extract sub-formulas using the SLTL progression, inspired by \textit{LTL Progression for Off-Policy Learning} \cite{Toro Icarte et al. 2018}. On the other hand, it can also be decomposed using the “then” operator to extract sub-formulas accounting for the possibly encountered tasks in the future. Each extracted sub-formula by either progression or decomposition indicates a module of the target task. Whenever an extracted formula is new, \textit{i.e.}, not in the states \( U_M \) of the original memory RM, the formula and its corresponding transitions will be stored as an extended state of the RM.

Formally, for each input target formula \( \varphi \), the set of extracted formulas is initialized as \( T_{\text{ex}} = \{\varphi\} \) and iteratively updated by:

\[
T_{\text{ex}} \leftarrow T_{\text{ex}} \cup T_{\text{prog}} \cup T_{\text{dec}},
\]

where \( T_{\text{prog}} = \{\text{prog}(\psi, l) \mid \psi \in T_{\text{ex}}, l \in \Sigma\} \) is the set of formulas progressed by all the labels, and \( T_{\text{dec}} = \{\psi_1 \mid \psi \in T_{\text{ex}}, \psi = \psi_1 \sim \psi_2\} \) is the set of sub-formulas decomposed by the \( \sim \) operator. The iteration terminates if and only if the set \( T_{\text{ex}} \) does not change anymore. Denote the original memory RM as \( R_M = (U_M, u_0, F, \delta, R) \), then the new states are \( U_{\text{new}} = T_{\text{ex}} \setminus U_M \). Therefore, the extended RM is \( R_M = (U_M', u_0, F', \delta', R') \), where the states are \( U_M' = U_M \cup U_{\text{new}} \); the transition function is \( \delta'(\psi, l) = \delta(\psi, l) \) for original states \( \psi \in U_M \) and \( \delta'(\psi, l) = \text{prog}(\psi, l) \) for new states \( \psi \in U_{\text{new}} \); the terminal states are \( F' = U_M' \cap \{\top, \bot\} \); and the reward function \( R' \) is defined as Eq. (1). Algorithm 2 gives the procedure of extending RM and Figure 3 plots an illustrative example of such a process.

Algorithm 2: ExtendRM(\( \varphi \))

\[ \text{Input: Memory } M, \text{ the target SLTL formula } \varphi \]
\[ \text{Output: New memory } M \]
1: \textbf{if } \varphi \in U_M \textbf{ then}
2: \textbf{return}
3: \textbf{else}
4: \textbf{Initialize} queue=[\varphi], U_M \leftarrow U_M \cup \{\varphi\}
5: \textbf{while not} queue.empty() \textbf{ do}
6: \( \psi \leftarrow \text{queue.pop()} \)
7: \textbf{if } \psi = \psi_1 \sim \psi_2 \textbf{ and } \psi_1 \notin U_M \textbf{ then}
8: \( U_M \leftarrow U_M \cup \{\psi_1\}, \text{queue.append}(\psi_1) \)
9: \textbf{end if}
10: \textbf{for all } l \in \Sigma \textbf{ do}
11: \( \psi' = \text{prog}(\psi, l) \)
12: \textbf{if } \psi' \notin U_M \textbf{ then}
13: \( U_M \leftarrow U_M \cup \{\psi'\}, \text{queue.append}(\psi') \)
14: \textbf{end if}
15: \text{Store transition } \delta(\psi, l) = \psi' \textbf{ for } R_M \)
16: \textbf{end for}
17: \textbf{end while}
18: \textbf{end if}

4.2 Acquisition of Knowledge from the Memory

The key idea to acquire knowledge, \textit{i.e.}, Q-functions from the memory for a target formula \( \varphi \), is to iteratively decompose the target formula \( \varphi \) into sub-formulas until all the sub-formulas are in the learned tasks \( T_M \), such that the Q-function of the target formula \( \varphi \) can be initialized using the composed Q-values of the learned sub-formulas. If the input formula \( \psi \) has been learned, then its corresponding Q-function is returned directly. If \( \psi \) has not been learned but
Choosing the smallest representation of a task. Finally, we verify if the learning performance can be improved when the final results are averaged over 20 independent trails.

In this section, we first evaluate the performance of different composition methods (Toro Icarte et al. 2020), and the ward shaping (QRM+RS) (Toro Icarte et al. 2020), and the QRM and QRM with respect. We compare LSRM with the QRM and QRM with respect to complete the target task(s) throughout the training process. We report the performance by calculating the average steps to complete the target task(s) under every domain (Andreas, Klein, and Levine 2017). The parameters are set to be $\epsilon = 0.1$, $\gamma = 0.9$ and $\alpha = 1$ in both domains. We report the performance by calculating the average steps to complete the target task(s) throughout the training process. We compare LSRM with the QRM and QRM with respect to complete the target task(s) throughout the training process. We report the performance by calculating the average steps to complete the target task(s) throughout the training process.

Figure 3: An illustration of extensions of memory RM when faced with target tasks $\varphi_1 = (\diamond c) \sim (\diamond o)$: “deliver coffee to office” at first then $\varphi_2 = (\diamond m) \sim (\diamond o)$: “deliver mail to office” later. The white nodes (rectangles) are original states, and the red nodes and arrows denote the extended states (i.e., new formulas) and their transitions, respectively. Left: The memory RM is first initialized as a terminal state without any transitions. Middle: When learning new task $\varphi_1$, the task formula is first added to the RM as a new state, and decomposed by the $\sim$ operator, resulting in new formula $\diamond c$. Then, the formula $\varphi_1$ is progressed by each label $\{\}$, $\{c\}$, $\{m\}$, $\{o\}$, resulting in sub-formula $\varphi_2$, $\diamond o$, $\varphi_1$, $\varphi_1$, respectively. The new sub-formulas $\diamond c$, $\diamond o$ are not in the memory and thus added to the RM as new states. After that, $\diamond c$, $\diamond o$ are decomposed and progressed as above, ending up with no new formulas. Right: Similarly, after adding the task formula $\varphi_2$ as new state, only a new decomposed formula $\diamond m$ is added without no new progressed formulas.

5 Experimental Results

In this section, we first evaluate the performance of different Q-value composition methods for each operator, and then verify if the learning performance can be improved when choosing the smallest representation of a task. Finally, we implement LSRM in learning a series of SLTL tasks and evaluate its performance, compared to some direct learning methods. All the experiments are evaluated in the OFFICEWORLD domain (Toro Icarte et al. 2018) and the MINECRAFT domain (Andreas, Klein, and Levine 2017). The parameters are set to be $\epsilon = 0.1$, $\gamma = 0.9$ and $\alpha = 1$ in both domains. We report the performance by calculating the average steps to complete the target task(s) throughout the training process. We compare LSRM with the QRM and QRM with reward shaping (QRM+RS) (Toro Icarte et al. 2020), and the final results are averaged over 20 independent trails.

5.1 Evaluation of Different Composition Methods

We sample tasks from a set of source tasks i.e., learned SLTL formulas, and compose them by the operators “and”, “or” and “then” to define the target tasks. For more detailed information of the source tasks and the target tasks, see Appendix B. The RM of these target tasks is then automatically generated using the method of extending RM in Subsec. 4.1. We then initialize the Q-functions of the target tasks by composing Q-values of the source tasks using the four value composition methods in Subsec. 4.2. Finally, QRM is applied to learn each target task.

The experimental results are shown in Figure 4. In both domains, all the methods outperform QRM and QRM+RS by a large margin, except the Right Composition for target tasks composed by “then” in the OFFICEWORLD domain. The Average Composition, Max Composition, Left Composition perform best when the target tasks are composed by the “and”, “or”, “then” operators in both domains, respectively. For the “and” operator, if an action $a$ is optimal under state $s$ for both learned tasks $\varphi_1$ and $\varphi_2$, then the optimal action of the averaged Q-values under $s$ is $a$. Thus, choosing $a$ under $s$ helps completing the target task $\varphi_1 \land \varphi_2$. For the “or” operator, the best way to complete the target task $\varphi_1 \lor \varphi_2$ is to complete one of the source tasks $\varphi_1$, $\varphi_2$ with less steps. If $\max_a Q^{\varphi_1}(s, a') > \max_a Q^{\varphi_2}(s, a')$ under state $s$, then following the policy of $Q^{\varphi_1}$ helps completing the target task more quickly. The above results and analysis are consistent with the previous results in (Van Niekerk et al. 2019), although their focus is on general RL tasks that are sampled from the same MDP distribution but only differ in their reward functions, while we focus on logically specified tasks that are not necessarily Markovian. Finally, for the “then” operator, the agent has to complete $\varphi_1$ first when it implements the target task $\varphi_1 \sim \varphi_2$. Therefore, following the policy of $Q^{\varphi_1}$ helps learning the sequential target task.

5.2 Evaluation of Different Task Representations

We evaluate the performance of learning with different formulas of the same target task. The target tasks are “delivering coffee to office avoiding furniture” in the OFFICE-
Results in the OFFICE WORLD domain.

Results in the MINECRAFT domain.

Figure 4: Evaluation of different Q-value composition methods in the two domains. The learning efficiency of target tasks composed by the “and”, “or” and “then” operators are shown in the left, middle and right column, respectively.

Figure 5: Evaluation of different representations of tasks.

OFFICE WORLD domain and “making bed” in the MINECRAFT domain. The detailed representations of these target tasks are shown in Appendix.B. From the results in Figure 5 we can see that choosing the smallest representation with the fewest sub-formulas that have not been learned can improve the learning performance against other presentations. This result is reasonable as fewer unknown subtasks would bring less uncertainty during the value composition or initialization process, and thus facilitate learning of the target task.

5.3 Evaluations of LSRM in LRL Tasks

Finally, we evaluate LSRM in learning a series of tasks using the best corresponding composition method for each operator as given in Subsec. 5.1 (denoted as LSRM-best) and the worst corresponding composition method (denoted as LSRM-worst), and compare them to QRM and QRM+RS. In the OFFICE WORLD domain, we define a sequence of 6 tasks, and require the agent to learn 2 tasks per phase, where each phase contains 30,000 training steps. For example, in the first phase of 30,000 steps, the agent repeatedly learns two tasks “deliver coffee to place A avoiding furniture” and “deliver mail to place B avoiding furniture” in order. In the MINECRAFT domain, we adopt 10 tasks defined by Andreas, Klein, and Levine (2017), and also require the agent to learn 2 tasks per phase, where each phase contains 400,000 training steps. For more detailed information of the tasks and illustrations of the LSRM processes, see Appendix.B and Appendix.C, respectively. Experimental results are shown in Figure 6. As can be seen, in both domains, LSRM-best outperforms the QRM and QRM+RS baselines except for the first phase, where there is no previous knowledge to be transferred. LSRM-worst performs slightly worse than LSRM-best, but still outperforms QRM baseline since it still leverages some kind of knowledge from previous tasks, even if this knowledge transfer might not be optimal.

6 Related Work

Lifelong learning (or continual learning, multi-task learning) has received a rising interest in recent years, due to its potential to reduce agents’ training time in dynamic environments. Abel et al. (Abel et al. 2018) proposed a transfer method to realize optimal initialization of an agent’s policy or value function in LRL. Garcia et al. (Garcia and Thomas 2019) leveraged advice from previously learned tasks to enable more efficient exploration in target tasks. Ammar et al. (Ammar et al. 2015) proposed an LRL algorithm that supports efficient cross-domain transfer between tasks from different domains. An option-discovery method (Brunskill and Littman 2014) was proposed to facilitate learning by transferring the options with high sample efficiency. Other studies resorted to techniques including active learning with network consolidation (compression) (Schwarz et al. 2018), sub-network freezing (Rusu et al. 2016), or rehearsal of old data via experience replay (Isele and Cosgun 2018; Rolnick...
(a) Results in the OFFICE WORLD domain.

(b) Results in the MINECRAFT domain.

Figure 6: Evaluations of the LSRM in each phase in the OFFICE WORLD domain, and phase 1,3,5 in the MINECRAFT domain.

et al. (2018). However, these works normally assume that the series of tasks share some similar structure in MDP components such as rewards or transition probabilities, therefore it is hard for them to handle non-Markovian logically specified tasks as we did in this paper.

Formal language such as LTL allows the user to specify task constraints over sequences of events happening over time. A large number of studies have incorporated the rich expressive power of logic formulas into RL. Icarte et al. (Icarte et al. 2017) considered the use of advice expressed in LTL to guide exploration in a model-based RL algorithm. Bozkurt et al. (Bozkurt et al. 2020) applied LTL objectives to robotic learning. Li et al. proposed a formal method approach to RL that makes the reward generation process interpretable (Li et al. 2019), and proposed a variant of LTL to specify a reward function that can be optimized using RL. (Li, Vasile, and Belta 2017). De Giacomo et al. (De Giacomo et al. 2019, 2020) leveraged LTL as constraints (i.e., restraining bolts) in RL. Gao et al. (Gao et al. 2019) used LTL to specify the unknown transition probabilities for RL. However, all these studies do not target at LRL problems.

A number of studies have considered as well logic formulas for more efficient skill composition and knowledge transfer in RL. Yuan et al. (Yuan et al. 2019) proposed a modular RL approach to satisfy one-shot TL specifications in continuous state and action spaces. Li et al. (Li, Ma, and Belta 2017) also used the modularity of TL and automata together with hierarchical RL for skill composition to generalise from trained sub-tasks to complex specifications. Xu et al. (Xu and Topcu 2019) proposed the Metric Interval Temporal Logic (MITL) to specify temporal tasks and transfer knowledge between logically transferable tasks. Leon et al. (Leon, Shanahan, and Belardinelli 2020) utilized an extended observation to be fed into the network to encode new high-level knowledge. The authors in Leon et al. proposed a formal language for specifying complex control tasks. Other works reveal the relationships between tasks by constructing Markov Logic Network (MLN) (Torrey and Shavlik 2009, Mihalkova, Huynh, and Mooney 2007) and investigate the composition effectiveness given particular logical operators (Tasse, James, and Rosman 2020a,b, Van Niekerk et al. 2019). Compared to these methods that the source of sub-tasks should be specified a priori and the focus is on bottom-up composition effectiveness, our work is able to learn and decompose logically complex tasks automatically in a lifelong memory.

Finally, RM has been used to solve problems from various perspectives in RL, such as robotics training (Camacho et al. 2020), encoding a team’s task in multi-agent systems (Neary et al. 2020) and solving partially observable RL problems (Icarte et al. 2019). Unlike these existing studies, our work features a dynamic growing RM for continuous learning of future target formulas.

7 Conclusion

This paper presents an LRL method that takes advantage of the rich expressive power of temporal logic formulas for more flexible task specification and decomposition, and the knowledge transfer capability of RM for more efficient learning. Through storing and leveraging high-level knowledge in a memory, LSRM is able to achieve systematic out-of-distribution generalisation in tasks that follow the specifications of formal languages. Results in two benchmark domains show that LSRM improves the sample efficiency by a large margin compared to direct learning methods. In this paper, we evaluate the proposed method in grid-world domains with discrete actions. In the future, we will extend our approach to more complex domains, e.g., with continu-
ous states and/or actions.

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A Full Definitions and Proofs of SLTL

The syntax of SLTL formulas differs from LTL in terms of the “then” operator, i.e.,

\[ \varphi := \rho | \neg \varphi | \varphi \land \psi | \varnothing \varphi | \varphi U \psi | \varphi \sim \psi, \quad \rho \in \mathcal{P}. \] (5)

The semantic of the operator “then” is defined as

\[ \lambda \models \varphi \sim \psi \Leftrightarrow \begin{cases} 
\exists j \geq 0, \lambda^{0:j} \models \varphi, \lambda^{j+1} \models \psi, \\
\quad \text{and } \forall i < j, \lambda^{0:i} \not\models \varphi, \quad \text{if } \varphi \not\models \top; \\
\lambda \models \psi, \quad \text{if } \varphi \models \top, 
\end{cases} \] (6)

where the notation \( \lambda^{i:j} \) denotes the sub sequence \( l_{i}l_{i+1}\cdots l_{j} \).

The SLTL progression of formulas composed by the “then” operator is defined as

\[ \text{prog}(\varphi \sim \psi, l) = \begin{cases} 
\text{prog}(\varphi, l) \sim \psi, \quad \text{if } \varphi \not\models \top; \\
\text{prog}(\psi, l), \quad \text{if } \varphi \models \top, 
\end{cases} \] (7)

where \( \varphi, \psi \) are SLTL formulas and \( l \) is a label. The following theorem theoretically ensures that the progression of SLTL is well defined.

**Theorem 2.** For all SLTL formula \( \varphi \) and a label sequence denoted by \( \lambda \), we have \( \lambda^{i} \models \varphi \) if and only if \( \lambda^{i+1} \models \text{prog}(\varphi, l_{i}) \) for all \( i \geq 0 \).

**Proof.** It has been proved for LTL formulas. Therefore, we only need to prove that \( \lambda^{i} \models \varphi \sim \psi \Leftrightarrow \lambda^{i+1} \models \text{prog}(\varphi \sim \psi, l_{i}) \). If \( \varphi \models \top \), then \( \lambda^{i} \models \varphi \sim \psi \iff \exists j \geq i, \lambda^{j} \models \varphi, \forall i < j, \lambda^{i} \models \varphi \iff \lambda^{j+1} \models \text{prog}(\varphi, l_{i}) \) and \( \lambda^{i} \models \varphi, \forall i < j, \lambda^{i} \not\models \varphi \iff \lambda^{j+1} \models \text{prog}(\varphi, l_{i}) \). If \( \varphi \models \top \), then \( \lambda^{i} \models \varphi \sim \psi \iff \lambda^{i+1} \models \text{prog}(\varphi, l_{i}) \).

Finally, we give the proofs of operator laws in the following.

**Theorem 3.** For any SLTL formulas \( \varphi_{1}, \varphi_{2}, \varphi_{3} \), we have associative law: \( (\varphi_{1} \sim \varphi_{2}) \sim \varphi_{3} = \varphi_{1} \sim (\varphi_{2} \sim \varphi_{3}) \), and distribution laws (i) \( (\varphi_{1} \sim \varphi_{2}) \land \varphi_{3} = \varphi_{1} \sim (\varphi_{2} \land \varphi_{3}) \), (ii) \( (\varphi_{1} \sim \varphi_{2}) \lor \varphi_{3} = (\varphi_{1} \land \varphi_{2}) \sim \varphi_{3} \), (iii) \( (\varphi_{1} \sim \varphi_{2}) \lor (\varphi_{1} \sim \varphi_{3}) = \varphi_{1} \sim (\varphi_{2} \lor \varphi_{3}) \); and (iv) \( (\varphi_{1} \sim \varphi_{2}) \land (\varphi_{1} \sim \varphi_{3}) = (\varphi_{1} \land \varphi_{2}) \sim \varphi_{3} \). Hence, \( (\varphi_{1} \sim \varphi_{2}) \sim \varphi_{3} = \varphi_{1} \sim (\varphi_{2} \sim \varphi_{3}) \). We then prove the distribution law (i). \( \lambda \models (\varphi_{1} \sim \varphi_{2}) \land \varphi_{3} \Leftrightarrow \exists i, j \geq 0, \lambda^{0:i} \models (\varphi_{1} \sim \varphi_{2}) \land \lambda^{j+1} \models \varphi_{3} \), and \( \forall i < j, \lambda^{0:i} \not\models (\varphi_{1} \sim \varphi_{2}) \), and \( \forall k < \min(i, j), \lambda^{0:k} \not\models \varphi_{1} \land \varphi_{2} \land \varphi_{3} \Leftrightarrow \lambda \models (\varphi_{1} \land \varphi_{2}) \sim \varphi_{3} \). Hence, \( (\varphi_{1} \sim \varphi_{2}) \land (\varphi_{1} \sim \varphi_{3}) = \varphi_{1} \sim (\varphi_{2} \land \varphi_{3}) \). The proofs of (ii)-(iv) are similar.

### B Experimental Settings

The source tasks and target tasks in Section 5.1 are listed in Table 1 and 2, respectively.

In Section 5.2, the target task is “delivering coffee and mail to office avoiding furniture” in the OFFICEWORLD domain. Its smallest representation is \( (\neg w \ U c) \land (\neg w \ U o) \), and the other representation is \( (\neg w \ U c) \land (\neg w \ U o) \). In the MINECRAFT domain, the target task is “making bed”, and its smallest representation is \( (\neg c \land \neg m) \land \neg c \land c \land c \) and \( c \land c \land c \). Figure 8 gives illustrations of LSRM processes in the experiments.

### C Illustrations of the LSRM Processes in the Experiments

Figure 9 and 10 give illustrations of LSRM processes in the OFFICEWORLD and MINECRAFT domain, respectively. The nodes (rectangles) encoded with SLTL formulas are states in RM, and the black solid arrows are the transitions among them. Formulas colored in white have already been learned before, while formulas colored in red have not been learned. The red dotted arrows represent the Q-functions transferred from the learned formulas to the corresponding target formulas.
| Domain       | Source Task                                                                 | SLTL Formula |
|--------------|------------------------------------------------------------------------------|--------------|
| OfficeWorld  | deliver coffee to office avoiding furniture                                 | $\varphi_1 = (\neg \ast Uc) \sim (\neg \ast Uo)$ |
|             | deliver mail to office avoiding furniture                                    | $\varphi_2 = (\neg \ast Um) \sim (\neg \ast Uo)$ |
|             | go to B then office avoiding furniture                                       | $\varphi_3 = (\neg \ast UB) \sim (\neg \ast Uo)$ |
|             | go to B then C avoiding furniture                                           | $\varphi_4 = (\neg \ast UC) \sim (\neg \ast Uo)$ |
|             | get mail then go to D avoiding furniture                                    | $\varphi_5 = (\neg \ast Um) \sim (\neg \ast UD)$ |
|             | get mail then go to A avoiding furniture                                    | $\varphi_6 = (\neg \ast Um) \sim (\neg \ast UA)$ |

| MineCraft    | make plank                                                                  | $\psi_1 = (\Diamond a) \sim (\Diamond b)$ |
|             | make stick                                                                   | $\psi_2 = (\Diamond a) \sim (\Diamond c)$ |
|             | make cloth                                                                   | $\psi_3 = (\Diamond d) \sim (\Diamond c)$ |
|             | make rope                                                                    | $\psi_4 = (\Diamond d) \sim (\Diamond b)$ |
|             | make bridge                                                                  | $\psi_5 = (\Diamond a \land \Diamond f) \sim (\Diamond c)$ |
|             | make shears                                                                  | $\psi_6 = (\Diamond a \land \Diamond f) \sim (\Diamond c)$ |

Table 1: Source tasks in the two domains.

| Domain       | Target Task                                                                 | SLTL Formula |
|--------------|------------------------------------------------------------------------------|--------------|
| OfficeWorld  | complete source task 1 and 2                                                 | $\varphi_1 \land \varphi_2$ |
|             | complete source task 4 and 6                                                 | $\varphi_4 \land \varphi_6$ |
|             | complete source task 4 or 5                                                  | $\varphi_4 \lor \varphi_5$ |
|             | complete source task 4 or 7                                                  | $\varphi_4 \lor \varphi_7$ |
|             | complete source task 4 then 5                                                | $\varphi_4 \sim \varphi_5$ |
|             | complete source task 4 then 6                                                | $\varphi_4 \sim \varphi_6$ |
|             | complete source task 5 then 6                                                | $\varphi_5 \sim \varphi_6$ |
| MineCraft    | complete source task 1 and 3                                                 | $\psi_1 \land \psi_3$ |
|             | complete source task 4 and 5                                                 | $\psi_4 \land \psi_5$ |
|             | complete source task 4 and 6                                                 | $\psi_4 \land \psi_6$ |
|             | complete source task 1 or 3                                                  | $\psi_1 \lor \psi_3$ |
|             | complete source task 2 or 3                                                  | $\psi_2 \lor \psi_3$ |
|             | complete source task 2 then 3                                                | $\psi_2 \sim \psi_3$ |
|             | complete source task 3 then 6                                                | $\psi_5 \sim \psi_6$ |

Table 2: Target tasks in the two domains.

**Figure 7:** Representations of the task “delivering coffee and mail to office avoiding furniture” in the **OfficeWorld** domain.

**Figure 8:** Representations of the task “making bed” in the **MineCraft** domain.
| Phase | Task | The Smallest Representation |
|-------|------|-----------------------------|
| 1     | deliver coffee to A avoiding furniture | \((\neg \cdot Uc) \sim (\neg \cdot UA)\) |
|       | deliver mail to B avoiding furniture | \((\neg \cdot Um) \sim (\neg \cdot UB)\) |
| 2     | deliver coffee to office avoiding furniture | \((\neg \cdot Uc) \sim (\neg \cdot Uo)\) |
|       | deliver mail to office avoiding furniture | \((\neg \cdot Um) \sim (\neg \cdot Uo)\) |
| 3     | deliver coffee and mail to office avoiding furniture | \([((\neg \cdot Uc) \sim (\neg \cdot Uo)] \land [(\neg \cdot Um) \sim (\neg \cdot Uo)]\) |
|       | go to B then A avoiding furniture | \((\neg \cdot UB) \sim (\neg \cdot UA)\) |

Table 3: Tasks in the OFFICE WORLD domain.

| Phase | Task | The Smallest Representation |
|-------|------|-----------------------------|
| 1     | make plank | \((\Diamond a) \sim (\Diamond b)\) |
|       | make stick | \((\Diamond a) \sim (\Diamond c)\) |
| 2     | make cloth | \((\Diamond d) \sim (\Diamond e)\) |
|       | make rope  | \((\Diamond d) \sim (\Diamond b)\) |
| 3     | make bridge | \((\Diamond a \land \Diamond f) \sim (\Diamond e)\) |
|       | make bed   | \(((\Diamond a \sim (\Diamond b) \land \Diamond d)) \sim (\Diamond c)\) |
| 4     | make axe   | \(((\Diamond a \sim (\Diamond c) \land \Diamond f)) \sim (\Diamond b)\) |
|       | make shears| \((\Diamond a \land \Diamond f) \sim (\Diamond c)\) |
| 5     | get gold   | \(((\Diamond a \land \Diamond f) \sim (\Diamond e) \sim (\Diamond g)\) |
|       | get gem    | \(((\Diamond a \sim (\Diamond c) \land \Diamond f)) \sim (\Diamond b) \sim (\Diamond h)\) |

Table 4: Tasks in the MINECRAFT domain.
Figure 9: The LSRM process in the OFFICEWORLD domain. Taking Phase 2 as an example, formulas (¬*Uo), (¬*UC) ~ (¬*Uo) and (¬*Um) ~ (¬*Uo) have not been learned. Q-functions of (¬*UC) and (¬*Um) are transferred to (¬*UC) ~ (¬*Uo) and (¬*Um) ~ (¬*Uo), respectively, while there are no learned formulas transferred to (¬*Uo).
Figure 10: The LSRM process in the MINECRAFT domain. For the sake of succinctness, we use black dotted arrows to represent a hidden part of RM which includes multiple states and transitions in Phase 4-5.