Epistemic Logic Programs: A Different World View

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Epistemic Logic Programs (ELPs), an extension of Answer Set Programming (ASP) with epistemic operators, have received renewed attention from the research community in recent years. Classically, evaluating an ELP yields a set of world views, with each being a set of answer sets. In this paper, we propose an alternative definition of world views that represents them as three-valued assignments, where each atom can be either always true, always false, or neither. Based on several examples, we show that this definition is natural and intuitive. We also investigate relevant computational properties of these new semantics, and explore how other notions, like strong equivalence, are affected.

1 Introduction

Answer Set Programming (ASP) is a generic, fully declarative logic programming language that allows users to encode problems such that the resulting output of the program (called answer sets) directly corresponds to solutions of the original problem [20]. Negation in ASP is generally interpreted according to the stable model semantics [11, 12], that is, as negation-as-failure, also called default negation. The negated version \( \neg a \) of an atom \( a \) is true if there is no justification for \( a \) in the same answer set. Hence, default negation is a kind of “local” operator in the sense that it is defined relative to the answer set that is currently considered.

Epistemic Logic Programs (ELPs) are an extension of ASP with epistemic operators. In this paper we will mainly consider the epistemic negation operator \( \text{not} \), defined in [21]. It allows for a form of meta-reasoning, that is, reasoning over multiple answer sets. Intuitively, an epistemically negated atom \( \text{not} a \) expresses that \( a \) is false in at least one answer set, that is, that it is not true everywhere. Thus, epistemic negation is defined relative to a collection of answer sets, which [21] refer to as a world view. Deciding whether such a world view exists is known to be \( \Sigma^P_3 \)-complete [21], one level higher on the polynomial hierarchy than deciding answer set existence [5].

Originally introduced as two modal operators \( K \) (“known” or “provably true”) and \( M \) (“possible” or “not provably false”) by Gelfond [8, 9], epistemic negation in ASP has received renewed interest (c.f. e.g. [10, 23, 13, 7, 21, 22, 14, 6]), with refinements of the semantics and proposals of new language features. Further, the development of efficient solving systems is underway with several working systems now available [15, 22, 1].

While ELPs offer the same advantages as ASP, namely a fully declarative, intuitive language for modelling and problem encoding, in this paper we would like to argue that the notion of the world view, as proposed in works on ELPs up to date, is not an intuitive construct that end users of the ELP language can directly use. Consider the following example:

**Example 1.** We consider a classical example for the need of epistemic operators, investigated in [8, 10, 27], namely a slightly simplified version of the scholarship eligibility problem. For a given student, we would like to decide whether they are eligible for a scholarship or ineligible. The rules say that someone with a high GPA is eligible while with a low GPA they are ineligible.
Further, we know that the student under consideration has either a high GPA or a fair one, but not a low one.

- highGPA ∨ fairGPA

Finally, we use epistemic negation to say that if we can neither prove that the student is always eligible or always ineligible, we want to interview them.

- interview ← not eligible, not ineligible

According to [21, Example 3] there is exactly one world view, namely the set of answer sets \{M_1, M_2\} where the answer set \(M_1 = \{\text{fairGPA, interview}\}\) and answer set \(M_2 = \{\text{highGPA, eligible, interview}\}\).

In the above example, it is not easy to extract the relevant information for the end user from the world view. In fact, they would have to check each answer set to answer their original question about whether students are eligible, ineligible, or should be interviewed. In the example above, interview is true in all answer sets of the world view, and hence, the student should be interviewed. In addition, the individual answer sets contain a lot of information not directly relevant to the question of the user. A result that directly represents the desired results, that is, a different version of the world view that simply states that interview is always true, while ineligible is always false, would, in the authors’ opinion, be a more intuitive representation of the result of the ELP of Example 1 as it would directly answer the question whether the student under consideration is eligible, ineligible, or should be interviewed. The aim of this paper is to propose such a notion, and to investigate its consequences for evaluating ELPs.

**Contributions.** The results and contributions presented in this paper can be summarized as follows.

- We provide a different definition of the notion of “world view” for ELPs, such that relevant information about the solution to an ELP is represented directly, and hence easily accessible to an end user. This is done by defining the world view not as a collection of answer sets, but as a three-valued model that encodes this information.

- We investigate the relationship of our new world view definition with the semantics of ELPs provided by Shen and Eiter [21] and establish a close connection between the notion of the epistemic guess proposed by these authors, and our new definition of a world view.

- We investigate notions of equivalence under our new semantics. Here, we show that equivalence under our definition of a world view is more general than the one under the world view definition of [21], but that for strong equivalence, the two semantics coincide.

- We study the computational complexity of the world view existence problem, and of equivalence testing.

- Finally, as a case study, we investigate the problem of QSAT-solving, and show that our notion of the world view can more intuitively capture the relevant part of the solution to a given QSAT problem.

**Structure.** The remainder of the paper is structured as follows. In Section 2, we provide an overview of ASP and ELPs. In Section 3, our novel definition of the world view is proposed and investigated. Notions of equivalence are studied in Section 4, and a case study is performed in Section 5. We conclude with a summary in Section 6.
2 Preliminaries

This section provides relevant details on Answer Set Programming and Epistemic Logic Programs.

2.1 Answer Set Programming (ASP)

A ground logic program with double negation (also called answer set program, ASP program, or, simply, logic program) is a pair $\Pi = (\mathcal{A}, R)$, where $\mathcal{A}$ is a set of propositional (i.e. ground) atoms and $R$ is a set of rules of the form

$$a_1 \lor \cdots \lor a_l \leftarrow a_{l+1}, \ldots, a_m, \neg \ell_1, \ldots, \neg \ell_n;$$

where the comma symbol stands for conjunction, $0 \leq l \leq m$, $a_i \in \mathcal{A}$ for all $1 \leq i \leq m$, and each $\ell_i$ is a literal, that is, either an atom $a$ or its (default) negation $\neg a$ for any atom $a \in \mathcal{A}$. Note that, therefore, doubly negated atoms may occur. Each rule $r \in R$ of form (1) consists of a head $H(r) = \{a_1, \ldots, a_l\}$ and a body $B(r) = \{a_{l+1}, \ldots, a_m, \neg \ell_1, \ldots, \neg \ell_n\}$. We denote the positive body by $B^+(r) = \{a_{l+1}, \ldots, a_m\}$.

An interpretation $I$ (over $\mathcal{A}$) is a set of atoms, that is, $I \subseteq \mathcal{A}$. A literal $\ell$ is true in an interpretation $I \subseteq \mathcal{A}$, denoted $I \models \ell$, if $a \in I$ and $\ell = a$, or if $a \notin I$ and $\ell = \neg a$; otherwise $\ell$ is false in $I$, denoted $I \not\models \ell$.

Finally, for some literal $\ell$, we define that $I \models \neg \ell$ if $I \not\models \ell$. This notation naturally extends to sets of literals. An interpretation $M$ is called a model of $r$, denoted $M \models r$, if, whenever $M \models B(r)$, it holds that $M \models H(r)$.

We denote the set of models of $r$ by mods($r$); the models of a logic program $\Pi = (\mathcal{A}, R)$ are given by mods($\Pi$) = $\bigcap_{r \in R}$ mods($r$). We also write $I \models r$ (resp. $I \models \Pi$) if $I \in$ mods($r$) (resp. $I \in$ mods($\Pi$)).

The GL-reduct $\Pi^l$ of a logic program $\Pi = (\mathcal{A}, R)$ w.r.t. an interpretation $I$ is defined as $\Pi^l = (\mathcal{A}, \mathcal{R}^l)$, where $\mathcal{R}^l = \{H(r) \leftarrow B^+(r) | r \in R, \forall \ell \in B(r) : I \not\models \neg \ell\}$.

**Definition 2.** [17] $M \subseteq \mathcal{A}$ is an answer set of a logic program $\Pi$ if (1) $M \in$ mods($\Pi$) and (2) there is no subset $M' \subset M$ such that $M' \in$ mods($\Pi^M$).

The set of answer sets of a logic program $\Pi$ is denoted by AS($\Pi$). The consistency problem of ASP, that is, to decide whether for a given logic program $\Pi$ it holds that AS($\Pi$) $\neq \emptyset$, is $\Sigma^p_1$-complete [6], and remains so also in the case where doubly negated atoms are allowed in rule bodies [19].

**Strong Equivalence for Logic Programs.** Two logic programs $\Pi_1 = (\mathcal{A}, R_1)$ and $\Pi_2 = (\mathcal{A}, R_2)$ are equivalent iff they have the same set of answer sets, that is, AS($\Pi_1$) = AS($\Pi_2$). The two logic programs are strongly equivalent iff for any third logic program $\Pi_3 = (\mathcal{A}, R)$ it holds that the combined logic program $\Pi_1 \cup \Pi_3 = (\mathcal{A}, R_1 \cup R)$ is equivalent to the combined logic program $\Pi_2 \cup \Pi_3 = (\mathcal{A}, R_2 \cup R)$. Note that strong equivalence implies equivalence, since the empty program $\Pi = (\emptyset, 0)$ would already contradict strong equivalence for two non-equivalent programs $\Pi_1$ and $\Pi_2$.

An SE-model [24] of a logic program $\Pi = (\mathcal{A}, R)$ is a two-tuple of interpretations $(X, Y)$, where $X \subseteq Y \subseteq \mathcal{A}$, $Y \models \Pi$, and $X \models \Pi^Y$. The set of SE-models of a logic program $\Pi$ is denoted $\mathcal{SE}(\Pi)$. Note that for every model $Y$ of $\Pi$, $(Y, Y)$ is an SE-model of $\Pi$, since $Y \models \Pi$ implies that $Y \models \Pi^Y$.

Two logic programs (over the same atoms) are strongly equivalent iff they have the same SE-models [16] [24]. Checking whether two logic programs are strongly equivalent is known to be coNP-complete [24] [19].

2.2 Epistemic Logic Programs (ELPs)

An epistemic literal is a formula $\text{not } \ell$, where $\ell$ is a literal and $\text{not}$ is the epistemic negation operator. A ground epistemic logic program (ELP) is a tuple $\Pi = (\mathcal{A}, R)$, where $\mathcal{A}$ is a set of propositional atoms
and \( \mathcal{R} \) is a set of rules of the following form:
\[
a_1 \lor \cdots \lor a_k \leftarrow \ell_1, \ldots, \ell_m, \xi_1, \ldots, \xi_j, \neg \xi_{j+1}, \ldots, \neg \xi_n,
\]
where \( k \geq 0, m \geq 0, n \geq j \geq 0 \), each \( a_i \in \mathcal{A} \) is an atom, each \( \ell_i \) is a literal, and each \( \xi_j \) is an epistemic literal, where the latter two each use an atom from \( \mathcal{A} \). Such rules are also called ELP rules.

Similar to logic programs, let \( H(r) = \{a_1, \ldots, a_k\} \) denote the head elements of an ELP rule, and let \( B(r) = \{\ell_1, \ldots, \ell_m, \xi_1, \ldots, \xi_j, \neg \xi_{j+1}, \ldots, \neg \xi_n\} \), that is, the set of elements appearing in the rule body. The union (or combination) of two ELPs \( \Pi_1 = (\mathcal{A}_1, \mathcal{R}_1) \) and \( \Pi_2 = (\mathcal{A}_2, \mathcal{R}_2) \) is the ELP \( \Pi_1 \cup \Pi_2 = (\mathcal{A}_1 \cup \mathcal{A}_2, \mathcal{R}_1 \cup \mathcal{R}_2) \). The set of epistemic literals occurring in an ELP \( \Pi \) is denoted \( \text{elit}(\Pi) \).

Shen and Eiter \[21\] define the semantics of ELPs as follows: let \( \Pi = (\mathcal{A}, \mathcal{R}) \) be an ELP. A subset \( \Phi \subseteq \text{elit}(\Pi) \) is called an epistemic guess (or, simply, a guess). Given such a guess, the epistemic reduct of \( \Pi \) w.r.t. \( \Phi \), denoted \( \Pi^\Phi \), consists of the rules \( \{r^- \mid r \in \mathcal{R}\} \), where \( r^- \) is defined as the rule \( r \in \mathcal{R} \) where all occurrences of epistemic literals not \( \ell \in \Phi \) are replaced by \( \top \), and all remaining epistemic negation symbols \( \neg \) are replaced by default negation \( \neg \). Note that, after this transformation, \( \Pi^\Phi \) is a logic program without epistemic negation \[1\].

We are now ready to give the following, central definition:

**Definition 3.** Let \( \Pi = (\mathcal{A}, \mathcal{R}) \) be an ELP. A set \( \mathcal{M} \) of interpretations over \( \mathcal{A} \) is a Shen–Eiter candidate world view (SE-CWV) of \( \Pi \) if there is an epistemic guess \( \Phi \subseteq \text{elit}(\Pi) \) such that \( \mathcal{M} = AS(\Pi^\Phi) \) and the following conditions hold:

1. \( \mathcal{M} \neq \emptyset \);
2. for each epistemic literal \( \neg \ell \in \Phi \), there exists an answer set \( M \in \mathcal{M} \) such that \( M \not\models \ell \); and
3. for each epistemic literal \( \neg \ell \in \text{elit}(\Pi) \setminus \Phi \), for all answer sets \( M \in \mathcal{M} \) it holds that \( M \models \ell \).

The set of all SE-CWVs of an ELP \( \Pi \) is denoted by se-cwv(\( \Pi \)). Following the principle of knowledge minimization, \[21\] define a Shen–Eiter world view as a SE-CWV that has a maximal guess.

**Definition 4.** Let \( \Pi = (\mathcal{A}, \mathcal{R}) \) be an ELP. An SE-CWV \( \mathcal{C} \) is called a Shen–Eiter world view (SE-WV) if its associated guess \( \Phi \) is subset-maximal w.r.t. \( \text{elit}(\Pi) \).

The main reasoning task for ELPs is checking whether they are consistent, that is, whether it holds that \( \text{se-cwv}(\Pi) \neq \emptyset \). This problem is also referred to as **world view existence problem**\[2\]. This problem is known to be \( \Sigma_3^P \)-complete \[21\].

### 3 A Different World View

In this section, we will introduce our novel world view definition and explore the relationship with the existing world view definition of Shen and Eiter \[21\], as defined in Section 2. We also look at the computational complexity of deciding whether a world view exists.

#### 3.1 The World View as a Model

The idea underlying our definition is to treat world views not as collections of answer sets, which are hard to interpret from an intuitive perspective, but, instead, as three-valued models of ELPs. We first define the notion of a world interpretation, which will be in a similar relationship to our new world view, as an interpretation is to a model in classical logic.

\[1\] Note that \( \Pi^\Phi \) may contain triple-negated atoms \( \neg
\neg
\neg a \). But such formulas can always be replaced by \( \neg a \) \[17\].

\[2\] SE-CWV-existence and SE-WV-existence are equivalent.
Definition 5. Let $\Pi = (\mathcal{A}, \mathcal{R})$ be an ELP, and let $\mathcal{L}$ be the set of all literals built from atoms in $\mathcal{A}$. Then, we call a consistent subset $I \subseteq \mathcal{L}$ a candidate world interpretation (CWI) for $\Pi$.

This definition gives rise to a total, three-valued truth assignment to the atoms $\mathcal{A}$ of an ELP $\Pi = (\mathcal{A}, \mathcal{R})$; hence, we will sometimes treat a CWI $I$ as a triple of disjoint sets $(I^p, I^n, I^u)$, where $I^p = \{a \mid a \in I\}$, $I^n = \{a \mid \neg a \in I\}$ and $I^u = (\mathcal{A} \setminus I^p) \setminus I^n$.

Note the immediate intuitive meaning of this three-valued assignment: an atom $a$ in $I^p$ (i.e. $a \in I$) means that it is always true, in $I^n$ (i.e. $\neg a \in I$) it means it is always false, and in $I^u$ it is unknown in the sense that it is neither always false nor always true. Note further that CWIs naturally represent consistent epistemic guesses in the semantics of Shen and Eiter: assume an ELP $\Pi = (\mathcal{A}, \mathcal{R})$ contains exactly two epistemic literals, not$a$ and not$\neg a$. A-priori, from the definitions in [21], one may think that there are four possible epistemic guesses: $\Phi_1 = \{nota, not\neg a\}$, $\Phi_2 = \{nota\}$, $\Phi_3 = \{not\neg a\}$, and $\Phi_4 = \emptyset$. But it turns out that $\Phi_4$ will never give rise to a SE-CWV: we cannot both believe that nota is false and that also not$\neg a$ is false, because that would mean that in the epistemic reduct $\Pi^{\Phi_4}$, $a$ has to be true in every answer set of the epistemic reduct, but also that $\neg a$ has to be true in every such answer set. This is only possible if $\Pi^{\Phi_4}$ has no answer set, but then $\Phi_4$ can, by definition, not lead to a SE-CWV. Hence $\Phi_4$ is an inconsistent guess, which is naturally excluded by our definition of CWIs.

The other three epistemic guesses, $\Phi_1$, $\Phi_2$, and $\Phi_3$, however, correspond to three distinct truth assignments to $a$. $\Phi_2$ represents that we believe nota but not not$\neg a$. The former says that there must be an answer set of $\Pi^{\Phi_2}$ where $a$ is false, and the latter says that in all answer sets of $\Pi^{\Phi_2}$, $\neg a$ must be true. Hence, $\Phi_2$ represents the epistemic guess that says that $a$ must be false everywhere. Similarly, $\Phi_3$ says that $a$ must be true everywhere. Finally, $\Phi_1$ says that there must be an answer set of $\Pi^{\Phi_1}$ where $a$ is true, and one where $a$ is false, and hence, that $a$ shall neither be true everywhere, nor false everywhere. Since the intuitive meaning of these three guesses can only be discerned by referring back to the set of epistemic literals that appear in $\Pi$. In fact, for ELPs where nota does not appear, the meaning of guess $\Phi_2$ would be different: instead of saying that $a$ must be false everywhere, it would then say that $a$ must be false somewhere (but not necessarily everywhere). It is thus our contention that epistemic guesses, as defined in [21], represent their intuitive meaning in a rather convoluted and hard-to-understand way. Now note that the three guesses $\Phi_1$, $\Phi_2$, and $\Phi_3$ naturally correspond to the intuitive meaning of our CWIs: assuming that the universe of atoms $\mathcal{A}$ contains only the atom $a$, $\Phi_1$ corresponds to the CWI $\emptyset$, $\Phi_2$ to the CWI $\{\neg a\}$ and $\Phi_3$ to the CWI $\{a\}$. Thus, in the authors’ opinion, CWIs capture this intuitive meaning better than the notion of the epistemic guess.

Before we can give our novel definition of a world view, we define the notion of the epistemic reduct w.r.t. a CWI, in the same style as in [21], as follows:

Definition 6. Let $\Pi = (\mathcal{A}, \mathcal{R})$ be an ELP, and let $I$ be a CWI for $\Pi$. Then, let $\Pi^I$ denote the epistemic reduct of $\Pi$ w.r.t. $I$, defined as follows: $\Pi^I = (\mathcal{A}, \mathcal{R}^I)$, where $\mathcal{R}^I = \{r^I \mid r \in \mathcal{R}\}$ and $r^I$ denotes the rule $r$ where each epistemic literal nota is replaced by $\neg \ell$ if $\ell \in I$, and by $\top$ otherwise.

We now define when a set of interpretations is compatible with a CWI.

Definition 7. Let $\mathcal{I}$ be a set of interpretations over a set of atoms $\mathcal{A}$. Then, a CWI $I$ is compatible with $\mathcal{I}$ if the following conditions hold:

1. $\mathcal{I} \neq \emptyset$;
2. for each atom $a \in I^p$, it holds that for each interpretation $J \in \mathcal{I}$, $a \in J$;
3. for each atom $a \in I^n$, it holds that for each interpretation $J \in \mathcal{I}$, $a \notin J$;
4. for atom $a \in I^u$, it holds that there are two interpretations $J, J' \in \mathcal{I}$, such that $a \in J$, but $a \notin J'$. 
With these definitions in place, we can now give the main definition of this work, our novel definition of a world view:

**Definition 8.** Let \( \Pi = (\mathcal{A}, \mathcal{R}) \) be an ELP, and let \( I \) be a CWI for \( \Pi \). Then, \( I \) is a candidate world view (CWV) of \( \Pi \) iff the set of answer sets \( \text{AS}(\Pi^I) \) is compatible with \( I \). A CWV \( I \) is called a world view (WV) if it is subset-minimal, that is, there is no other CWV \( J \subset I \) of \( \Pi \). The set of CWVs (resp. WVs) of an ELP \( \Pi \) is denoted \( \text{cwv}(\Pi) \) (resp. \( \text{wv}(\Pi) \)).

To illustrate the usefulness of our newly proposed notion of WVs, let us return to our introductory example from Section 1.

**Example 9.** Consider the ELP \( \Pi = (\mathcal{A}, \mathcal{R}) \), where the set of atoms \( \mathcal{A} \) is the set of all the atoms appearing in Example \( \square \) that is, \( \mathcal{A} = \{\text{eligible}, \text{ineligible}, \text{highGPA}, \text{lowGPA}, \text{fairGPA}, \text{interview}\} \), and where \( \mathcal{R} \) contains the rules from Example \( \Box \). As stated in Section \( \square \) this ELP has precisely one SE-CWV (and hence SE-WV). This SE-CWV corresponds to the epistemic guess \( \Phi \), where \( \Phi = \{\text{not eligible}, \text{not ineligible}\} \). As stated in Example \( \square \) both answer sets in this SE-CWV contain the fact interview, and hence the candidate should be interviewed.

Using our new notion of CWVs, we again see that there is exactly one CWV (and hence WV) \( W \), where we have that \( W = \{\text{interview}, \neg \text{ineligible}\} \); or, equivalently, where \( W^P = \{\text{interview}\} \), \( W^N = \{\text{ineligible}\} \), and \( W^U = (\mathcal{A} \setminus W^P) \setminus W^N \). From our CWV \( W \), we can immediately recognize the solution to the scholarship eligibility problem; namely, in this case, to interview the student (since the fact interview is true in \( W \), and hence, intuitively, “true in every possible world”). For SE-CWVs, the individual answer sets first need to be examined and compared to each other to draw that conclusion.

Emphasized by the above example, we would argue that our novel definition of a world view is much more intuitive than the standard definition as a set of answer sets. This is mainly because it has the distinct advantage of being immediately interpretable: it is simply a truth assignment to all the atoms in the ELP, with the additional information that some atoms are both true and false somewhere (i.e. unknown). This information is suitable to be used directly by end-users of ELP solvers.

With our main definitions in place, next we will investigate the relationship between SE-CWVs and our novel CWVs as defined above.

### 3.2 Relationship to SE-CWVs

In this subsection, we will show that there is a close relationship between SE-CWVs and CWVs. As is obvious from the definitions, the conditions that define these notions are similar. The main difference is that our CWVs correspond to the epistemic guesses in \[21\] instead of to the actual SE-CWVs, those being sets of answer sets. However, for an ELP \( \Pi = (\mathcal{A}, \mathcal{R}) \), note that our CWVs are actually total three-valued assignments to the atoms in \( \mathcal{A} \); that is, each atom is either always true, always false, or neither (i.e. unknown). Epistemic guesses, on the other hand, operate only on those epistemic literals that actually appear in \( \Pi \). Nevertheless, we can show that epistemic guesses and our CWVs are closely related. Firstly, note that we can always modify an ELP such that all possible epistemic literals actually appear in \( \Pi \).

**Lemma 10.** Let \( \Pi = (\mathcal{A}, \mathcal{R}) \) be an ELP and let \( \Pi' = (\mathcal{A}, \mathcal{R}') \) be an ELP over the same atom domain, but with \( \mathcal{R}' \supset \mathcal{R} \) and \( \mathcal{R}' \setminus \mathcal{R} = \{r_1\} \), where \( r_1 \) is the rule \( \bot \leftarrow a, \neg a, \text{not } \ell \), where \text{not } \ell \) does not appear in \( \mathcal{R} \) and \( \ell \) is a literal over the atom \( a \in \mathcal{A} \). Then, \( \text{se-cwv}(\Pi) = \text{se-cwv}(\Pi') \).

**Proof.** Note that the literal \( \ell \) must be over some atom from \( \mathcal{A} \). Note further that the rule \( r_1 \) is clearly tautological, that is, it will always be satisfied. Now, consider any epistemic guess \( \Phi \subseteq \text{elit}(\Pi) \), and let
\( \Phi' = \Phi \cup \{ \text{not} \ell \} \). Observe that \( \Pi^{\Phi'} = \Pi^{\Phi} \), since, in the epistemic reduct, rule \( r_1 \) reduces to \( \bot \leftarrow a, \neg a \) for both guesses \( \Phi \) and \( \Phi' \). Hence, let \( \mathcal{M} = \text{AS}(\Pi^{\Phi}) \). Since \( r_1 \) is tautological, it has no influence on the answer sets, and we conclude that \( \mathcal{M} = \text{AS}(\Pi^{\Phi}) \).

Now, assume that \( \mathcal{M} \) is an SE-CWV for \( \Pi \) w.r.t. the epistemic guess \( \Phi \). Then, it is not difficult to see that it also satisfies the conditions of Definition 3 for \( \Pi' \) w.r.t. either the epistemic guess \( \Phi \) or \( \Phi' \) (since either \( \ell \) is true in all answer sets in \( \mathcal{M} \), or there is one answer set where \( \ell \) is false). Hence, \( \mathcal{M} \) is also an SE-CWV for \( \Pi' \), either w.r.t. guess \( \Phi \) or \( \Phi' \).

Conversely, assume that \( \mathcal{M} \) is not an SE-CWV for \( \Pi \) w.r.t. guess \( \Phi \). Then, either \( \mathcal{M} = \emptyset \), or there is an epistemic literal \( \text{not} \ell' \in \Phi \) that violates Condition 2 or 3 of Definition 3. But then, this violation also holds for \( \mathcal{M} \) w.r.t. the ELP \( \Pi' \) and both of the guesses \( \Phi \) and \( \Phi' \). Hence, \( \mathcal{M} \) is also not an SE-CWV for \( \Pi' \).

We are now ready to state our main correspondence theorem between our notion of world views and the notion by Shen and Eiter \[21\]. In fact, we show that there is a one-to-one correspondence between SE-CWVs and our CWVs.

**Theorem 11.** Let \( \Pi = (\mathcal{A}, \mathcal{R}) \) be an ELP. Then, it holds that for each CWV of \( \Pi \) there is exactly one SE-CWV of \( \Pi \), and for each SE-CWV of \( \Pi \) there is exactly one CWV of \( \Pi \).

**Proof (Sketch).** Firstly, note that by repeated applications of Lemma 10 we can always assume that the rules of \( \Pi \) contain all the epistemic literals that can be build from \( \mathcal{A} \).

To prove the first direction, assume that \( W \) is a CWV of \( \Pi \). Let \( \Phi \) be the following epistemic guess for \( \Pi \): for each \( a \in W^P \), let \( \Phi \cap \{ \text{not}a, \text{not} \neg a \} = \{ \text{not}a \} \); for each \( a \in W^N \), let \( \Phi \cap \{ \text{not}a, \text{not} \neg a \} = \{ \text{not}a \} \); and for each \( a \in W^U \), let \( \Phi \cap \{ \text{not}a, \text{not} \neg a \} = \{ \text{not}a \} \). It can be verified that the epistemic reduct \( \Pi^W \) is equal to the Shen-Eiter epistemic reduct \( \Pi^{\Phi} \). Hence, \( \mathcal{M} = \text{AS}(\Pi^W) = \text{AS}(\Pi^{\Phi}) \).

Since \( W \) is, by assumption, a CWV of \( \Pi \), \( \mathcal{M} \) fulfills the conditions of Definition 3 w.r.t. \( W \). But then it is not difficult to see that, by construction of \( \Phi \), \( \mathcal{M} \) also satisfies the conditions of Definition 3 and hence is an SE-CWV of \( \Pi \) w.r.t. guess \( \Phi \).

For the other direction, assume that \( \mathcal{M} \) is an SE-CWV for \( \Pi \) w.r.t. some epistemic guess \( \Phi \). Now, construct CWI \( W \) as follows (recall that \( \Phi \) is a subset of all possible epistemic literals over \( \mathcal{A} \)): for each \( a \in \mathcal{A} \), if \( \Phi \cap \{ \text{not}a, \text{not} \neg a \} = \{ \text{not}a \} \) then let \( a \in W^P \); if \( \Phi \cap \{ \text{not}a, \text{not} \neg a \} = \{ \text{not}a \} \) then let \( a \in W^N \); and if \( \Phi \cap \{ \text{not}a, \text{not} \neg a \} = \{ \text{not}a, \text{not} \neg a \} \) then let \( a \in W^U \). By a similar argument to the one above, we can show that \( W \) is indeed a CWV of \( \Pi \), since, by construction of \( W \), the epistemic reducts \( \Pi^{\Phi} \) and \( \Pi^W \) coincide.

The proof for the theorem above gives a reduction between SE-CWV-existence and CWV-existence and vice-versa. From this reduction, and the fact that SE-CWV-existence is \( \Sigma^p_3 \)-complete in general \[21\], the statement below follows immediately:

**Theorem 12.** Checking whether an ELP has at least one CWV (or, equivalently, at least one WV) is \( \Sigma^p_3 \)-complete.

The statement above also holds for WVs, because if a CWV exists then, clearly, also a subset-minimal CWV (that is, a WV) exists. Furthermore, any WV is also a CWV. This concludes our investigation of the novel world view definition proposed in this section.
4 Equivalence of ELPs

In this section, we will look at different equivalence notions for ELPs. But before we start, let us first explore several properties of ELPs that we will make use of in this section.

We start by re-stating a folklore result from the world of ASP, namely that the universe of atoms of an answer set program can be extended without changing its answer sets.

**Proposition 13.** Let $(\mathcal{A}, \mathcal{R})$ be a logic program and let $\Pi = (\mathcal{A}', \mathcal{R})$ be a logic program with the same set of rules, but with $\mathcal{A}' \supset \mathcal{A}$. Then, $AS(\Pi) = AS(\Pi')$.

It is easy to see that the above proposition holds by noting that any atom $a \in \mathcal{A}' \setminus \mathcal{A}$ can clearly not appear in the rules $\mathcal{R}$, and hence any such $a$ must be false in any answer set of $\Pi'$ (i.e. for all $M \in AS(\Pi')$ it holds that $a \notin M$). From this result, it is easy to obtain a similar result for ELPs:

**Proposition 14.** Let $\Pi = (\mathcal{A}, \mathcal{R})$ be an ELP and let $\Pi' = (\mathcal{A}', \mathcal{R})$ be an ELP with the same set of rules, but with $\mathcal{A}' \supset \mathcal{A}$. Then, $cwv(\Pi') = \{ W \cup \{ \neg a \} \mid W \in cwv(\Pi) \}$.

**Proof.** Note that, clearly, any atom $a \in \mathcal{A}' \setminus \mathcal{A}$ cannot appear anywhere in $\mathcal{R}$. Hence, for every CWV $I$ over $\mathcal{A}$, $a$ also does not appear in the rules of the epistemic reduct $\Pi^I$ and thus also not in any answer set of $\Pi^I$. But then, $AS(\Pi^I) = AS(\Pi^I)$ (via Proposition 13). Then, $a$ is false in every one of these answer sets. It is therefore trivial that if $W$ is a CWV of $\Pi$, then $W \cup \{ \neg a \}$ is a CWV of $\Pi'$.

From the above, we can see that the atom domain of an ELP can be arbitrarily extended, and atoms that are in the atom domain of an ELP but do not appear in its rules will always be false in every CWV of the ELP. With these results we are now ready to investigate notions of equivalence for ELPs under the semantics proposed in Section 3.

In a recent paper that investigates strong equivalence for ELPs [6], the authors define equivalence of ELPs under the SE-CWV semantics as follows: two ELPs are **SE-CWV-equivalent** iff their SE-CWVs are the same and **SE-WV-equivalent** iff their SE-WVs are the same. We can define a similar notion of equivalence using our definition of CWVs as follows:

**Definition 15.** Two ELPs $\Pi_1$ and $\Pi_2$ are CWV-equivalent (resp. WV-equivalent), denoted $\Pi_1 \equiv_{CWV} \Pi_2$ (resp. $\Pi_1 \equiv_{WV} \Pi_2$) if and only if $cwv(\Pi_1) = cwv(\Pi_2)$ (resp. $wv(\Pi_1) = wv(\Pi_2)$).

Recall that our new notion of CWVs abstracts from the answer sets, since the individual answer sets themselves no longer appear directly within them. Hence, our notion of equivalence turns out to be strictly more general than the one in [6]: we can find an ELP that is equivalent under our semantics, but not equivalent under their notion of equivalence, which is based on SE-CWVs, as the following theorem states.

**Theorem 16.** CWV-equivalence (resp. WV-equivalence) strictly generalizes SE-CWV-equivalence (resp. SE-WV-equivalence).

**Proof.** Clearly, SE-(C)WV-equivalence implies (C)WV-equivalence, since when the SE-(C)WVs are the same, then so are the (C)WVs. To see the other direction, we can construct two ELPs $\Pi_1$ and $\Pi_2$, such that they both have exactly one CWV (and hence WV, SE-CWV, and SE-WV), and that the SE-CWV of $\Pi_1$ is $\{ \{a\}, \{b\}, \{c\} \}$. Hence, the corresponding CWV $W$ assigns all three atoms $a$, $b$, and $c$ to unknown (i.e. $W = \{a, b, c\}$). Now, we can construct an ELP $\Pi_2$ in such a way that it has the CWV $W$ (that is, all three atoms are still unknown), but the corresponding SE-CWV of $\Pi_2$ is $\{ \{a\}, \{b, c\} \}$. This shows that $\Pi_1$ and $\Pi_2$ are (C)WV-equivalent, but not SE-(C)WV-equivalent, as desired.
With the definition for equivalence in place, we can now also investigate the notion of strong equivalence. Strong equivalence is a well-studied problem in plain ASP, with useful applications for program simplification \[16, 3, \text{18, 4}\]. For ELPs, several works deal with strong equivalence by abstracting into epistemic extensions of Heyting’s logic of here-and-there (HT) \[25, 7\]. A recent paper deals with a direct characterization of strong equivalence for the SE-CWV semantics \[6\]. Similarly to the definition of ordinary equivalence, we will see how we can generalize their characterization to apply to our semantics.

We begin by defining strong equivalence in our context:

**Definition 17.** Two ELPs \(\Pi_1\) and \(\Pi_2\) are strongly CWV-equivalent (resp. strongly WV-equivalent) if and only if for any third ELP \(\Pi\), it holds that \(\Pi_1 \cup \Pi \equiv_{\text{CWV}} \Pi_2 \cup \Pi\) (resp. \(\Pi_1 \cup \Pi \equiv_{\text{WV}} \Pi_2 \cup \Pi\)).

Along the lines of \[6\], we characterize the notion of strong equivalence via a so-called SE-function, defined as follows:

**Definition 18.** The SE-function \(\mathcal{E}_{\Pi}(\cdot)\) of an ELP \(\Pi = (\mathcal{A}, \mathcal{R})\) maps CWIs \(I\) over \(\mathcal{A}\) to sets of SE-models as follows.

\[
\mathcal{E}_{\Pi}(I) = \begin{cases} 
\mathcal{E}(\Pi^I) & \text{if } I \text{ is compatible with some subset } \mathcal{J} \subseteq \text{mods}(\Pi^I) \\
\emptyset & \text{otherwise.}
\end{cases}
\]

It turns out that this modified version of the SE-function precisely characterizes strong equivalence in our setting, and that the two notions of strong equivalence given in Definition 17 coincide, as the following theorem states:

**Theorem 19.** Let \(\Pi_1\) and \(\Pi_2\) be two ELPs. Then, the following statements are equivalent:

1. \(\Pi_1\) and \(\Pi_2\) are strongly CWV-equivalent;
2. \(\Pi_1\) and \(\Pi_2\) are strongly WV-equivalent; and
3. \(\mathcal{E}_{\Pi_1} = \mathcal{E}_{\Pi_2}\).

**Proof (Sketch).** Via Proposition 14, we can always assume that \(\Pi_1\) and \(\Pi_2\) have the same atom domain. Now, \((1) \Rightarrow (2)\) follows from Definition 17 and the fact that every WV is a CWV. \((3) \Rightarrow (1)\) follows by the same argument as \((5) \Rightarrow (1)\) in the proof of Theorem 15 in \[6\]. Finally, \((2) \Rightarrow (3)\) can be established by showing the contrapositive. We can construct \(\Pi\) in such a way that it realizes a particular CWI in the SE-function as a CWV for both \(\Pi_1 \cup \Pi\) and \(\Pi_2 \cup \Pi\) (which, by construction of the SE-function, is always possible). We chose a CWI \(I\) where the SE-function contains a difference. We then make use of the fact that, for this CWI \(I\), there is a difference in the SE-models of the epistemic reducts \((\Pi_1 \cup \Pi)^I\) and \((\Pi_2 \cup \Pi)^I\). However, simply realizing this difference is not enough to show non-strong-equivalence (as opposed to the SE-CWV semantics, where this would already suffice). Instead, we need to make sure that \(I\) actually becomes a CWV only in \(\Pi_1 \cup \Pi\) but not for \(\Pi_2 \cup \Pi\) (w.l.o.g., by symmetry). This can be done by introducing a new atom, say \(a\), not appearing in \(\Pi_1\) or \(\Pi_2\), and constructing \(\Pi\) in such a way that this atom is true precisely in the answer set created from the SE-model that marks the difference in the SE-functions of \(\Pi_1\) and \(\Pi_2\). Thus, since this atom \(a\) is false in every answer set of \((\Pi_1 \cup \Pi)^I\), but true in exactly one answer set of \((\Pi_2 \cup \Pi)^I\), clearly, \(I\) cannot be a CWV of both, showing \((2) \Rightarrow (3)\) via the contrapositive.

This shows that the two notions of strong WV-equivalence and strong CWV-equivalence coincide. Interestingly, since our construction of the SE-function mirrors the one proposed in \[6\], we observe that their different notions of strong equivalence and the ones proposed in this paper coincide. Hence, Corollary 16 from \[6\] also holds for our setting:
Corollary 20. ELPs are strongly equivalent if and only if they have the same SE-function.

This is somewhat surprising, since, as we have seen, our notion of (ordinary) equivalence is a more general one. From this observation, the following complexity result immediately follows from Theorem 20 in [6]:

Theorem 21. Checking whether two ELPs are strongly equivalent is \( \text{CO}^{\text{NP}} \)-complete.

5 QSAT Solving: A Case Study

In this section, we will look at a brief case study to see how our new semantics can be used to obtain intuitive information about problem solutions from an ELP encoding for that problem in a much more straightforward way than with SE-CWVs.

To this end, we will be looking at the problem of solving quantified satisfiability problems. In particular, we will look at the problem of solving SAT formulas of the form

\[ \exists X \forall Y \exists Z \Psi, \]

that is, existential QSAT formulas with three quantifier blocks. We will further assume that the formula \( \Psi \) is of the form \( \bigwedge_{j=1}^{n} (\ell_j^1 \lor \ell_j^2 \lor \ell_j^3) \), that is, a formula in conjunctive normal form with three literals each. Each literal \( \ell_j^i \) may be of the form \( w \) or \( \neg w \), where \( w \in X \cup Y \cup Z \). We will follow the reduction from the proof of Theorem 5 in [21], and assume, w.l.o.g., that when all the variables in \( Y \) are set to \( \top \), then the formula becomes a tautology (independent of the variables \( X \) and \( Z \)). We will write our encoding in non-ground form, since this should make it easier to read, and the intuitive meaning of the non-ground rules is easier to grasp. Note that, to obtain the ground version of the encoding, which then directly corresponds to the construction in [21], we simply substitute all the variables in the encoding with all possible combinations of constants. This process is also called \textit{grounding}; cf. e.g. [21, Section 2.2.2].

The Set of Facts. Assume that the QSAT formula is represented by the following facts:

- \( \text{var}_1(x) \), for each \( x \in X \);
- \( \text{var}_2(y) \), for each \( y \in Y \);
- \( \text{var}_3(z) \), for each \( z \in Z \); and
- \( \text{clause}(w_1, \eta_1, w_2, \eta_2, w_3, \eta_3) \), for each clause \( \ell_j^1 \lor \ell_j^2 \lor \ell_j^3 \), \( 0 < j \leq n \), in \( \Psi \), where for \( 0 < i \leq 3 \), \( \eta_j = 0 \) if \( \ell_j^i = w_i \), or \( \eta_j = 1 \) if \( \ell_j^i = \neg w_i \).

The Rules. Now, let us construct the rules to actually solve this problem. Note that the rules are non-ground, that is, they may contain variables. Rules with variables can be seen as an abbreviation for multiple copies of these rules, where each variable is replaced by some constant from the atom domain. In this way, a non-ground ELP can be turned into a (ground) ELP as defined in Section 2.

- First, using epistemic negation, we guess an assignment for the variables in \( X \) of our formula:

\[
\begin{align*}
\text{assign}(X, 0) & \leftarrow \text{not assign}(X, 1), \text{var}_1(X), \\
\text{assign}(X, 1) & \leftarrow \text{not assign}(X, 0), \text{var}_1(X).
\end{align*}
\]
• Then, we guess an assignment for the variables in $Y$:

$$assign(Y, 0) \lor assign(Y, 1) \leftarrow var_2(Y).$$

• For the variables in $X$, we use the standard ASP modelling technique of saturation [5], in order to quantify over the variables in $X$, as follows:

$$assign(Z, 0) \lor assign(Z, 1) \leftarrow var_3(Z),$$

$$assign(Z, 0) \leftarrow sat,$$

$$assign(Z, 1) \leftarrow sat,$$

$$esat \leftarrow not esat, not \neg sat.$$ 

This completes the construction. Note that this non-ground encoding is a straight-forward lifting of the ground version of this encoding presented in [21]. Correctness hence follows from their paper. However, we would like to point out the main difference between the SE-CWV semantics and the semantics proposed in this paper. In the case of SE-CWV semantics, an ELP solver would output a large set of answer sets, grouped together by their participation in an SE-CWV. This makes it tedious to interpret the result, since this possibly very large set of answer sets has to be carefully parsed and post-processed in order to ascertain which atoms are always true, which are always false, and to extract a truth assignment for the variables in $X$ of the original QSAT formula. While this process can be done without much difficulty by a computer, the information is not directly accessible by a human user. In contrast, our semantics would simply yield a number of CWVs, where, by construction, each CWV would contain, for each variable $x \in X$, exactly one of the two atoms $assign(x, 0)$ or $assign(x, 1)$, allowing a hypothetical end-user to directly extract a “solution” to the QSAT formula in the form of the truth assignment to the variables in $X$. In addition, every CWV under our semantics represents exactly one valid truth assignment to the variables of $X$. In the authors’ opinion, this seems like a much more intuitive representation of the solutions of the QBF than the SE-CWVs.

6 Conclusions

In this paper, we have presented a novel take on the semantics of epistemic logic programs. When dealing with such ELPs under the semantics proposed by Shen and Eiter [21], we argued that the information that is actually of interest to an end user is “hidden” in the so-called epistemic guess, and is only implicitly present when SE-CWVs (which are sets of answer sets) are computed. We therefore propose a novel notion of “world view” that directly encodes this information as a three-valued assignment to the atoms of an ELP, presenting immediately accessible information to the end user about which atoms are always true, always false, or neither. We investigated complexity questions and notions of equivalence between ELPs, as well as the relationship between our new semantics and the one in [21].

Future work will include implementing our semantics into an ELP solving system, and investigating, whether practical solving shortcuts can be found, since, as opposed to the SE-CWV semantics, not all the answer sets of the epistemic reducts need to be computed. Further, note that our new definition of the world view is not restricted to the semantics proposed in [21], but can also be directly applied to other semantics, e.g. the ones proposed by Gelfond in [8][10], or by Kahl et al. in [13][15]. We would like to do a similar investigation w.r.t. these semantics, as the one done in this paper w.r.t. the semantics from [21].
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