Realism and Time Symmetry in Quantum Mechanics

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Abstract

We describe a gedanken experiment with an interferometer in the case of pre- and postselection in two different time symmetric ways: We apply the ABL formalism and the de Broglie–Bohm model. Interpreting these descriptions ontologically, we get two very different concepts of reality. Finally, we discuss some problems implied by these concepts.

1 Introduction

If we are talking about the question of time symmetry/asymmetry in quantum mechanics, the following statement is quite common: The theory is time symmetric as long as it can be described by the evolution of a state vector according to the Schrödinger equation. But as soon as measurement and wave function collapse are involved, the symmetry breaks down. Time symmetry in quantum mechanics and related topics are investigated in a number of papers ([1] – [9]).

Aharonov, Bergman and Lebowitz (ABL) [1] invented a time symmetric formalism for describing quantum systems between two complete measurements. This formalism was later generalised by Aharonov and Vaidman [2]. The basic idea can be seen in a very simple experiment: (The set-up

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is taken from ref. \cite{11}. Let \( \sigma_x \) and \( \sigma_n \) be spin observables corresponding to the components of a spin-\( \frac{1}{2} \) particle along the unit vectors \( \mathbf{x} \) and \( \mathbf{n} \). The free Hamiltonian of the system shall be zero. Let us now consider a particle prepared in the state \( |\sigma_x = \frac{1}{2} \rangle \) at time \( t_1 \) and found in the state \( \langle \sigma_n = \frac{1}{2} | \) at a later time \( t_2 \). Obviously, for an intermediate measurement we get \( \text{prob}(\sigma_x = \frac{1}{2}) = \text{prob}(\sigma_n = \frac{1}{2}) = 1 \). Refs. \cite{2} and \cite{1} describe such a pre- and postselected quantum system by two state vectors (one of them forward and the other backward evolved) and provide a formula that yields probabilities for outcomes of intermediate measurements. This formalism is entirely time symmetric.

While in the standard approach the particle in the given example is completely described by the forward evolved vector \( |\sigma_x = \frac{1}{2} \rangle \) between the two measurements, this time symmetric formalism suggests that additional information about the intermediate state can be obtained from the result of the second measurement: In a generalised state between \( t_1 \) and \( t_2 \), the backward evolved vector \( \langle \sigma_n = \frac{1}{2} | \) is also taken into account. Therefore, one could wonder whether the described particle in the intermediate state somehow has fixed values for the spin in two different directions. More generally, this raises the question, of whether the time symmetric description allows any conclusions about the ontology of a pre- and postselected quantum system. In terms of causality, such an ontology could be counterintuitive (cf. \cite{5}).

While in a deterministic world (as in classical mechanics or the de Broglie–Bohm model) time symmetry and causality are in a ‘peaceful coexistence’ \cite{11}, in the probabilistic interpretation of quantum mechanics there can appear a problem with time symmetry: Timelike correlations between measurements could imply something like ‘precognitive elements’ in the quantum system.

In this paper, we shall describe an interferometer experiment with pre- and postselection (analogous to the above mentioned set-up) and apply the time symmetric formalism to it. Furthermore, we will investigate possible consequences of this formalism for the ontology or elements of reality in the described system. Finally, we shall compare these results with the de Broglie–Bohm model as a realistic interpretation of quantum mechanics.

\footnote{A. Shimony has used the same phrase with respect to nonlocality in quantum mechanics and the impossibility of superluminal signaling. (cf. ref. \cite{11})}
2 Description of the experimental set-up

We will consider an interferometer as shown in Fig. 1 with a single particle source. It shall be arranged so that different paths between two beam splitters (i.e. c and d, e and f) have exactly the same length. As the following calculations show, the set-up is composed of two balanced Mach–Zehnder type interferometers. This means that a single wave, incoming from path a (b) leads to a single wave in path e (f). And if at BS 2 there approaches a wave only from path c (d), then detector H (G) will ‘fire’ with probability 1.

The operation of every beamsplitter BS (see Fig. 2) on the state vectors $|u\rangle$ and $|v\rangle$ is given by

$$|u\rangle \xrightarrow{BS} \frac{1}{\sqrt{2}}(|x\rangle + i|y\rangle)$$

(1)
Using these conditions, the evolution of an initial state $|a\rangle$ is given by

\[ |a\rangle \xrightarrow{BS_1} \frac{1}{\sqrt{2}}(|c\rangle + i|d\rangle) \quad (\equiv |\psi_1\rangle) \]

\[ |a\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}(|c\rangle + |f\rangle) + i\frac{1}{\sqrt{2}}(|c\rangle + i|f\rangle)) = i|e\rangle \]

\[ |a\rangle \xrightarrow{BS_3} i\frac{1}{\sqrt{2}}(|g\rangle + |h\rangle) \]

\[ = \frac{1}{\sqrt{2}}(-|g\rangle + i|h\rangle). \]

3 **Element of reality in the case of pre- and post-selection**

We will now recall the time symmetric description of pre- and postselected quantum systems that was first invented by Aharonov, Bergman and Lebowitz (ABL) in 1964 [1]. Here, we shall use the generalised formalism as introduced by Aharonov and Vaidman [2].
Let us consider a quantum system prepared in a state $|\psi_1(t_1)\rangle$ at time $t_1$ and postselected in a state $\langle \psi_2(t_2) |$ at a later time $t_2$. By using the time evolution operator $U$, we get a forward evolved state vector

$$|\psi_1(t)\rangle = U(t_1, t)|\psi_1(t_1)\rangle$$

as well as a backward evolved state (denoted as a ‘bra’-vector)

$$\langle \psi_2(t) | = \langle \psi_2(t_2) | U(t, t_2)$$

for any time $t$ with $t_1 < t < t_2$. With this, the generalised state at that time is defined as a vector that includes both the backward evolved state (‘bra’) and the forward evolved state (‘ket’):

$$\Psi(t) \equiv \langle \psi_2(t) | \psi_1(t) \rangle$$

And the probability that an intermediate measurement of an operator $C$ at time $t$ yields the eigenvalue $c_n$ is given by the ABL formula

$$\text{prob}(C = c_n) = \frac{|\langle \psi_2(t) | P_{C=c_n} |\psi_1(t)\rangle|^2}{\sum_i |\langle \psi_2(t) | P_{C=c_i} |\psi_1(t)\rangle|^2},$$

where $P_{C=c_i}$ is the projection operator on the space of eigenstates with eigenvalue $c_i$.

Let us now consider the set-up of section 2 only in situations where the initial state is $|a\rangle$ and where at the end the particle is detected at $G$. Consequently, between preparation and detection we have a pre- and postselected quantum system and we can employ the above formalism.

The backward evolution at a beamsplitter (see Fig. 2) can be obtained from (1) and (2) in a straightforward calculation:

$$\langle x | \xrightarrow{BS} \frac{1}{\sqrt{2}} (|u\rangle - i|v\rangle)$$

$$\langle y | \xrightarrow{BS} \frac{1}{\sqrt{2}} (-i|u\rangle + |v\rangle)$$

And so, the postselected state $\langle g |$ evolves backwards as follows:

$$\langle g | \xrightarrow{BS_3} \frac{1}{\sqrt{2}} (|f\rangle - i|e\rangle)$$

$$\xrightarrow{BS_2} -i|d\rangle \quad (\equiv \langle \psi_2 |)$$

$$\xrightarrow{BS_1} -\frac{1}{\sqrt{2}}(|a\rangle + i|b\rangle)$$

\footnote{To prevent confusion, it should be emphasized that this term is not a scalar product!}
Therefore, as the generalised state (cf. (10)) between BS 1 and BS 2 we obtain
\[ \langle \psi_2 | \psi_1 \rangle = \frac{1}{\sqrt{2}} (|d\rangle (\langle -i|c\rangle + |d\rangle)). \] (13)

We shall now imagine the detection of the particle between BS 1 and BS 2 as an intermediate measurement with observable \(D\). Possible results are \(D = 0\) (detection in path \(c\)) and \(D = 1\) (detection in path \(d\)). So the ABL formula (cf. (11)) yields the probability of detecting the particle in path \(d\):
\[ \text{prob}(D = 1) = \frac{|\langle \psi_2 | d \rangle \langle d | \psi_1 \rangle|^2}{|\langle \psi_2 | c \rangle \langle c | \psi_1 \rangle|^2 + |\langle \psi_2 | d \rangle \langle d | \psi_1 \rangle|^2} = 1. \] (14)

An analogous calculation yields that in a similar measurement between BS 2 and BS 3 the particle would be detected in path \(e\) with probability 1.
This result of a so far purely formal description now could be interpreted ontologically. Let us therefore recall Redhead’s \[12\] “Sufficient Condition for Element of Reality”. (Originally, this condition was used in the EPR argument.) He states: “If we can predict with certainty, or at any rate with probability one the result of measuring a physical quantity at time \(t\), then at the time \(t\), there exists an element of reality corresponding to this physical quantity and having a value equal to the predicted measurement result.” As a modification of this, Vaidman \[13\] suggests to replace predict by infer. So, unlike Redhead’s condition, the statement would no longer be time biased.

Encouraged by these definitions, with our above result (14) one could conclude that it is an element of physical reality that in our gedanken experiment the particle goes through path \(d\) and \(e\) (see Fig. 3).

It should be mentioned that, apart from this ontological interpretation of the ABL formula, the time symmetric formalism can lead to other statements about an underlying reality, too. In particular, it should be worth thinking about a possible ontological meaning of the generalised state \(\overline{3}\).

But all the concepts of reality based on the above time symmetric description obviously have one complication in common: Because these ontologies depend on the performance of a particular final measurement and on its outcome, there appear to be contradictions with causality.

4 Time symmetric description of a measurement

A subtle point of the above mentioned time symmetric description is the measurement process. In particular, one could ask how time biased concepts such as detection or outcome can be compatible with time symmetry.

To see this, let us consider a measurement of the von Neumann \[14\] type: If we want to measure an observable \(A\) of a quantum system \(S\), then the interaction Hamiltonian between \(S\) and a measurement apparatus \(M\) is given by

\[
H_{\text{int}} = g(t)pA.
\]

The normalised coupling function \(g(t)\) shall be nonzero for a short time interval. The momentum \(p\) is the canonical conjugate to a pointer position \(q\). For simplicity, we assume the free Hamiltonian to equal zero.

Let the initial states of \(S\) and \(M\) be \(|\psi_i\rangle_s\) and \(|\phi_i\rangle_m\), respectively. If we denote the eigenstates of \(A\) by \(|a_k\rangle_s\) (with \(A|a_k\rangle_s = a_k|a_k\rangle_s\)), then \(|\psi_i\rangle_s\) can
be expanded as follows: $|\psi_i\rangle_s = \sum_k \alpha_k |a_k\rangle_s$. Position eigenstates of $M$ are denoted by $|q\rangle_m$.

The forward evolution of $|\psi_i\rangle_s |\phi_i\rangle_m$ during the measurement can be described in three steps:

(i) preparation of $M$ (reading of the pointer position $q_1$, collapse 1)
(ii) measurement interaction
(iii) reading the result (the pointer position $q_2$, collapse 2)

$$|\psi_i\rangle_s |\phi_i\rangle_m \xrightarrow{\text{coll.1}} |\psi_i\rangle_s |q_1\rangle_m$$

$$H\text{int} \xrightarrow{} \sum_k \alpha_k |a_k\rangle_s |q_1 + a_k\rangle_m$$

$$|a_i\rangle_s |q_1 + a_i\rangle_m \xrightarrow{\text{coll.2}}$$

The final reading yields

$$q_2 = q_1 + a_i,$$

and hence, we can deduce the result, $a_i$, of the measurement from knowledge of $q_1$ and $q_2$.

Let us now consider this process in the reversed time direction. In this case, the evolution ‘starts’ in a final state $\langle \psi_f |s \langle \phi_f |m$ with $\langle \psi_f |s = \sum k \beta_k |a_k|_s$. Here, the observer prepares $M$ by reading the position $q_2$ (collapse 2) and reads the result $q_1$ ‘afterwards’ (collapse 1):

$$\langle \psi_f |s \langle \phi_f |m \xrightarrow{\text{coll.2}} \langle \psi_f |s \langle q_2 |m$$

$$H\text{int} \xrightarrow{} \sum_k \beta_k |a_k|_s |q_2 - a_k|_m$$

$$\langle a_n|_s |q_2 - a_n|_m \xrightarrow{\text{coll.1}}$$

In this case, the result is

$$q_1 = q_2 - a_n,$$

and again, we can get the measurement result, $a_n$, if we know $q_1$ and $q_2$.

It follows from eqns. (14) and (23) that $a_i = a_n$, and hence, the result of the measurement as deduced by the forward time observer is the same as that deduced by the backward time observer. The reason for this is that it is not just the ‘final’ pointer reading that determines the measurement result, but rather the difference between the initial and final pointer reading. The
measurement process is therefore described in an entirely time symmetric way.

5 The same experiment, described in the de Broglie–Bohm model

Considering elements of reality, it shall be interesting to look at a realistic interpretation of quantum mechanics. The de Broglie–Bohm model provides such an interpretation. It describes all processes in a time symmetric and deterministic way. (In ref. 4, the derivation of Bohmian mechanics is even based on time symmetry.) Knowing the whole configuration of a system at one arbitrary instant, one can therefore exactly describe the system at any other instant by backward or forward evolution. And because in this interpretation all particles have definite positions at every time, it provides a clear ontology. As long as $\rho = |\psi|^2$ initially, this remains true at all later times. Consequently, the de Broglie–Bohm model has exactly the same predictions as standard quantum mechanics. This means in particular that the uncertainty relations hold (cf. 4). Nevertheless, in the following we will show that in our experiment of section 2 the outcome of the detection at G or H enables us to state which path the particle went according to the de Broglie–Bohm model.

So let us again look at the interferometer set-up with the initial state $|a\rangle$. Dewdney has shown how in the de Broglie–Bohm interpretation we can describe trajectories of particles (guided by wave packets) at beamsplitters and mirrors: If a wave arrives at a beamsplitter only from one path, a particle with position in the trailing (leading) half of the wave packet will be (will not be) reflected. If a wave packet is reflected, then the order of the particles will be reversed afterwards. (This can be derived using the fact that Bohm trajectories are unique and cannot intersect each other in spacetime.) According to that, at BS 1 (cf. (3)) the particle goes along path c (d), if it adopted a position within the the front (rear) half of the initial wave packet. Analogous, at BS 2 (cf. (3) – (4)) particles coming from path c (d) end up in the leading (trailing) half of the wave packet in path e. (Same length of paths c and d is needed here.) As an effect of the mirror in this path, the order of the particle positions within the wave packet gets reversed. Finally, BS 3 (cf. (4) – (5)) operates so that a particle detected at G (H) adopted a position within the rear (front) half of the arriving
wave packet. Computing all these steps, we get the result that, according to the de Broglie–Bohm interpretation, a particle preselected in state $|a\rangle$ and postselected in state $\langle g|$ between $BS_1$ and $BS_2$ always goes through path $c$. (see Fig. 4)

In order to check the above claimed time symmetry of this model, we will now look at the same experiment in a time reversed sense. That means, we have a particle ‘incoming’ in path $g$ and ‘detected’ in $a$. Assuming therefore, that the ‘initial’ state is $\langle g|$ and the ‘final’ state is $|a\rangle$, with the backward evolution (10) – (12) and considerations analogous to them in the previous paragraph, we obtain a particle trajectory going through the paths $f$ and $d$ instead of $e$ and $c$. Consequently, to get the correct ‘initial state’ in the

\footnote{Always means in the idealised case of our gedanken experiment with probability 1; we have not considered particles with positions exactly in the middle of a wave packet (a set of measure 0 in position space).}
backward picture, it is not sufficient just to know about the ‘click’ at detector $G$ in the actual experiment. Additionally, one has to take in account that at the same time an ‘empty wave’ with a certain phase difference arrives at $H$. In order to describe the time reversed experiment properly, one therefore has to evolve the state $\frac{1}{\sqrt{2}}(\langle g | - i \langle h |)$. In this way, the forward and backward picture yield the same trajectory.

This makes clear once more that the de Broglie–Bohm model is dualistic: Both particles and waves are regarded as existing in the physical world. And a description cannot be complete, if it does not entirely include both of these entities. Therefore not even ‘empty waves’ or their phases can be neglected[4]. So, in the context of our considerations about time symmetry, we can wonder, why we accept an outgoing ‘empty wave’ as natural, while we usually do not think of incoming ‘empty waves’. — Could it actually be true that in quantum mechanics there are incoming ‘empty waves’ as well as outgoing ones? Or could there perhaps be a freedom of adding ‘empty waves’ to an initial state? — This undoubtedly is impossible, because initial states with additional ‘empty waves’ would lead to (actually not observed) different outcomes. (For a related discussion see E. J. Squires [18].)

Could it therefore be that after all this time asymmetry connected with the occurrence of ‘empty waves’ only before measurements indicates a direction of time inherent in the de Broglie–Bohm model? Because the dynamical laws in this interpretation are completely time symmetric, an arrow of time (if there is one) probably cannot be based on a fundamental level. Another look at Bohm’s papers [15] shows that after measurements there are actually outgoing ‘empty waves’. But these waves are entangled with the respective states of the measurement apparatus. Because as a macroscopical system this apparatus has a large number of internal degrees of freedom, an overlap of different waves after the measurement is very unlikely. Therefore, the probability is negligibly small that an outgoing ‘empty wave’ has an influence on the actual position of the system, and for all practical purposes, we can replace the complete wave function by a new renormalised one (the

[4] The significance of ‘empty waves’ was already shown in [17] in an example where an ‘empty wave’ interacts with a particle. It is quite amusing to see that one can even get an interaction between two ‘empty waves’, if one extends the set-up of section 2 by placing the detection box A described in [17] in path $d$. If now the initial state is $|a\rangle$ and in the end the particle is detected at $G$, then (with the above considerations about Bohm trajectories) one knows that the particle went through path $c$ and that path $d$ was not blocked. From here on, the further conclusions are completely analogous to the reasoning in [17].
With this, the time asymmetry inherent in the measurement process (as described by Bohm) is ‘reduced’ to the thermodynamical arrow of time.

6 Conclusions

In this paper, we presented two different time symmetric descriptions of quantum mechanics: the ABL formalism and the de Broglie–Bohm model. As the interferometer experiment shows, these interpretations suggest entirely different ontologies. However, we cannot go as far as to conclude that these models of an underlying physical reality actually contradict each other. In fact, we are dealing with two different ontological concepts that need to be clarified much more in order to get any statements about mutual consistency or contradiction.

In section 5, we discuss particle trajectories. Our result is that in the described set-up we know (according to Bohm) the path of the particle, if we are informed about initial preparation and final detection. This path \((c)\) is actually different from the path \((d)\) where the ABL formula suggests an element of reality. However, in section 5, we also emphasized the significance of ‘empty waves’ in the de Broglie–Bohm model. With such an ‘empty wave’, this model describes something physically real going through path \(d\), too. Since particle positions are the only thing we can actually observe, one often tends to give the particles in the de Broglie–Bohm interpretation ‘more reality’ than the waves. However, one has to be very careful with this idea.

For the discussion of the quite formally defined elements of reality that were suggested in the context of the ABL formalism in section 5, it shall be useful to recall the idea of the definition: It is based on the probability for a certain outcome of an intermediate measurement. (Since we are dealing with a pre- and postselected system, this probability is neither predictive nor retrodictive, but a time symmetric inference.) If we discuss so defined elements of reality, we use this probability in particular in cases where the intermediate measurement actually is not performed. Since in general a measurement significantly changes the whole set-up, it is very question-

\[5\] to clarify this point, another look at the de Broglie–Bohm model is very instructive: If we describe the experiment of section 3 with intermediate measurement, then, of course, the ABL formula yields the correct probabilities for the results, and therefore, the particle
able, whether such a definition can have any ontological meaning. However, since Aharonov and Reznik in a recent paper [19] argue against nonlocality with the time symmetric formalism, this suggests some idea of reality behind the mathematical description. This underlying ontology could be very interesting, because the symmetric formulation includes the possibility of correlations between different times. Therefore, it shall be worth checking, if this concept of reality could even contradict causality.

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Note added in proof: Since completing this paper, we have become aware of some particularly interesting work by T. Mor, L. Goldenberg and L. Vaidman, who have also considered the double interferometer of fig. 1 in the context of pre- and postselected elements of reality.

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is always found in path d (not in path c!). The reason is that after the measurement the waves of the quantum system are entangled with those of the intermediate measurement apparatus, and therefore, the description of section is not valid for this case.
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