Tosto, M. G., Petrill, S. A., Malykh, S., Malki, K., Haworth, C. M. A., Mazzocco, M. M. M., Thompson, L., Opfer, J., Bogdanova, O. Y., & Kovas, Y. (2017). Number sense and mathematics: which, when and how? Developmental Psychology, 53(10), 1924-1939. https://doi.org/10.1037/dev0000331
Number Sense and Mathematics: Which, When and How?

Maria G. Tosto  
Tomsk State University

Stephen A. Petrill  
The Ohio State University

Sergey Malykh  
Psychological Institute of RAE, Moscow, Russia

Karim Malki  
King’s College London at the Institute of Psychiatry, Psychology and Neuroscience (IOPPN)

Claire M. A. Haworth  
University of Bristol

Michele M. M. Mazzocco  
University of Minnesota

Lee Thompson and John Opfer  
The Ohio State University

Olga Y. Bogdanova  
Tomsk State University

Yulia Kovas  
Tomsk State University, and Goldsmiths, University of London

Individual differences in number sense correlate with mathematical ability and performance, although the presence and strength of this relationship differs across studies. Inconsistencies in the literature may stem from heterogeneity of number sense and mathematical ability constructs. Sample characteristics may also play a role as changes in the relationship between number sense and mathematics may differ across development and cultural contexts. In this study, 4,984 16-year-old students were assessed on estimation ability, one aspect of number sense. Estimation was measured using 2 different tasks: number line and dot-comparison. Using cognitive and achievement data previously collected from these students at ages 7, 9, 10, 12, and 14, the study explored for which of the measures and when in development these links are observed, and how strong these links are and how much these links are moderated by other cognitive abilities. The 2 number sense measures correlated modestly with each other ($r = .22$), but moderately with mathematics at age 16. Both measures were also associated with earlier mathematics; but this association was uneven across development and was moderated by other cognitive abilities.

Keywords: mathematics, number sense, symbolic estimation, nonsymbolic estimation

Supplemental materials: http://dx.doi.org/10.1037/dev0000331.supp

“Number sense” is a term used to describe a wide range of mathematically relevant concepts, with up to 30 different constructs falling under this broad definition (e.g., Berch, 2005). Estimation, one aspect of number sense, is associated with quantifying and representing number magnitudes and numerosities (discrete items in a set). Estimation is itself heterogeneous, involving

This article was published Online First July 31, 2017.

Maria G. Tosto, Department of Psychology, Tomsk State University; Stephen A. Petrill, Department of Psychology, The Ohio State University; Sergey Malykh, Psychological Institute of RAE, Moscow, Russia; Karim Malki, King’s College London at the Institute of Psychiatry, Psychology and Neuroscience (IOPPN); Claire M. A. Haworth, School of Experimental Psychology and The Medical Research Council Integrative Epidemiology Unit, School of Social and Community Medicine, University of Bristol; Michele M. M. Mazzocco, Institute of Child Development, University of Minnesota; Lee Thompson and John, Opfer, Department of Psychology, The Ohio State University; Olga Y. Bogdanova, Department of Psychology, Tomsk State University; Yulia Kovas, Department of Psychology, Tomsk State University, and Department of Psychology, Goldsmiths, University of London.

We gratefully acknowledge the ongoing contribution of the participants in the Twins Early Development Study (TEDS) and their families. TEDS is supported by a program grant [G0901245; and previously G0500079] from the United Kingdom Medical Research Council; our work on environments and academic achievement is also supported by grants from the U.S. National Institutes of Health [HD044454, HD046167 and HD059215]. YK’s, SM’s, and MGT’s research is supported by Tomsk State University, Competitiveness Improvement Programme grant 8.1.11.2017.

This article has been published under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited. Copyright for this article is retained by the author(s). Author(s) grant(s) the American Psychological Association the exclusive right to publish the article and identify itself as the original publisher.

Correspondence concerning this article should be addressed to Yulia Kovas, Department of Psychology, Goldsmiths, University of London, London SE14 6 North West, United Kingdom. E-mail: y.kovas@gold.ac.uk
different abilities, such as nonsymbolic estimation and symbolic estimation (see Cohen Kadosh, Lammertyn, & Izard, 2008). These skills have been associated with mathematics, although questions remain about the extent to which this association varies depending on specific estimation tasks and periods of development.

**Nonsymbolic Estimation and Its Relationship With Mathematics**

Nonsymbolic estimation involves nonverbal processing of quantities and numerosities without using numerals. For example, this ability enables us to select a queue with fewer people without counting. Research suggests that this type of numerosity processing depends on the absolute number of items in a set: Evaluation of individual sets including fewer items is more accurate compared with those containing more items (set-size effect; e.g., Gordon, 2004; Whalen, Gallistel, & Gelman, 1999). Furthermore, discrimination between two sets is more difficult when the discrepancy between the number of items in the sets is smaller (distance effect; e.g., Feigenson, Carey, & Hauser, 2002; Holloway & Ansari, 2009; Moyer & Landauer, 1967). These two effects are encompassed by Weber’s law, with the Weber Fraction indexing the minimum ratio between two sets reliably discernible by individuals (Weber, 1834).

Numerosity processing can be carried out without formal knowledge of numbers or formal instruction (e.g., Pica, Lemer, Izard, & Dehaene, 2004) and, in humans, this skill improves with development. For example, 6-month-old babies can successfully discriminate only between large ratios, such as 8 versus 16 (ratios 1:2), with corresponding Weber Fraction of 1 ((12 − 1)/1); e.g., Libertus & Brannon, 2010). Adults can discriminate larger numerosities and smaller ratios (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012).

People differ greatly in the speed and accuracy of estimation (e.g., Halberda, Mazzocco, & Feigenson, 2008). Individual differences in nonsymbolic estimation, assessed using different nonsymbolic tasks, have been found in preschoolers, school-age children, and adults (e.g., Barth et al., 2006; Gilmore, McCarthy, & Spelke, 2010; Halberda et al., 2012; Nys & Content, 2012). A few studies that looked at potential sex differences in nonsymbolic estimation found no average differences between males and females (e.g., 3–5-year-olds, Bonny & Lourenco, 2013; 5–6-year-olds, Gilmore et al., 2010; 4-year-olds, Libertus, Feigenson, & Halberda, 2011; 14–15-year-olds, Mazzocco, Feigenson, & Halberda, 2011a). However, one study reported a small male advantage in 4-year-olds (Soltész, Szícs, & Szücs, 2010).

Several longitudinal studies showed an association between individual differences in nonsymbolic estimation and mathematical performance in preschool children (Gilmore et al., 2010; Mazzocco, Feigenson, & Halberda, 2011b) and older children (Halberda & Feigenson, 2008), with evidence suggesting a causal association (Wang, Odic, Halberda, & Feigenson, 2016). However, other studies have failed to find a significant correlation (e.g., Holloway & Ansari, 2009; Roussele & Noël, 2007; Sasanguie, Defever, Maertens, & Reynvoet, 2014).

Despite inconsistencies across individual studies, meta-analyses have shown that nonsymbolic estimation is prospectively and retrospectively, weakly, associated with mathematics across development ($r = .24$ prospectively and $r = .17$ retrospectively, Chen & Li, 2014; $r = .22$, Fazio, Bailey, Thompson, & Siegler, 2014; $r = .24$, Schneider et al., 2017). Discrepancies across individual studies may have stemmed from: differences in age of participants (Fazio et al., 2014; Schneider et al., 2017); measures of estimation used (for a discussion see Clayton, Gilmore, & Inglis, 2015); specific mathematics skills with which estimation is being correlated (Mazzocco et al., 2011a); mathematics achievement level of the participants (Bonny & Lourenco, 2013; Mazzocco et al., 2011a); and overall lack of statistical power to detect weak associations.

**Symbolic Estimation and Its Relationship With Mathematics**

Symbolic estimation relies on symbols, such as Arabic numerals (Booth & Siegler, 2006; Cohen Kadosh et al., 2008). For example, by relying on symbolic estimation people can tell that the solution to a numerical problem is incorrect without calculating an exact answer. The size and ratio effects observed for nonsymbolic estimation are also observed for symbolic estimation. Overall, people are faster in comparing two small numbers (1 and 2) than two large numbers (8 and 9) even when the distance between them is kept constant, suggesting that it is easier to process small numbers (Moyer & Landauer, 1967). Moreover, adults and children are faster and more accurate in judging the difference between two numerical magnitudes when the numerical distance between the numerals is larger (1 vs. 9) than when it is smaller (6 vs. 8; e.g., Dehaene, Dupoux, & Mehler, 1990). The presence of size and ratio effects in symbolic estimation has been taken as indirect evidence that symbolic representation of numbers builds on the approximate representation of nonsymbolic numerosity (Feigenson, Dehaene, & Spelke, 2004). The closeness between symbolic and nonsymbolic estimation seems also supported by reliance on partially overlapping neuronal activity in the intraparietal sulcus (IPS) and prefrontal cortex (for a discussion see Nieder & Dehaene, 2009). IPS areas are activated when attending to numerosity stimuli (e.g., Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004) or manipulating Arabic number symbols (e.g., Pinel, Dehaene, Riviere, & LeBihan, 2001). Different neurons in parietal regions, respond to a specific numerosity (tuning function); such tuning functions are organized sequentially, preserving the order of cardinality (numerosity of a set size) and following the Weber law (Nieder & Merten, 2007). However, some neural pathways show differential activation during encoding of numerical magnitudes gathered from symbolic and nonsymbolic stimuli (Holloway, Price, & Ansari, 2010). Furthermore, there is evidence of lateralization in IPS response to symbolic and nonsymbolic processing (Holloway, Battista, Vogel, & Ansari, 2013).

It is thought that, as numerals are acquired, they map onto existing nonsymbolic representations and become mentally represented along a mental “number line” (e.g., Restle, 1970; Siegler & Opfer, 2003). This line is organized in ascending order, following a left-to-right direction in English-writing participants and right-to-left in Arabic-writing participants (Dehaene, Bossini, & Giraux, 1993; cf. Ito & Hatta, 2004). It is hypothesized that numbers on the mental number line are initially logarithmically compressed (e.g., Dehaene & Mehler, 1992). With age, a gradual shift seems to occur from the less accurate logarithmic mental number representation to a more precise linear representation. The linear representation becomes dominant from the age of 6 to 8 years, as evidenced...
by improved performance on the number line task (Siegler & Booth, 2004). However, performance on this task may be based on strategies such as reliance on midpoint (knowing that 50 is half of 100; Ashcraft & Moore, 2012) and reliance on proportion-judgment, as the position of a number on a number line is estimated relatively to the size of the whole line (Barth & Paladino, 2011). Therefore, developmental changes may be due to the increasing use of a reference point rather than a log-to-linear shift. Another explanation for the increased accuracy on number line tasks takes into account familiarity with number symbols (e.g., Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008; Moeller, Pixner, Kaufmann, & Nuerk, 2009). These explanations are not mutually exclusive (Dackermann, Huber, Bahnmueller, Nuerk, & Moeller, 2015).

Several studies in different cultures have found a correlation between performance on number line tasks and mathematics skills (e.g., Booth & Siegler, 2006; Fazio et al., 2014; Fuchs et al., 2010a; Geary, 2011; Siegler & Booth, 2004; Siegler & Mu, 2008). The mechanisms of the association are unclear. Research suggests that experience with numbers, such as playing numerical board games, can improve children’s estimation abilities on the number line (Siegler & Booth, 2004). In turn, improvement of magnitude processing on the number line was found to be causally related to better arithmetic (addition problems) skills (Booth & Siegler, 2008). However bidirectional effects are also likely. For example, it was found that access to numerical instruction can improve nonsymbolic estimation skills in Western adults (Nys et al., 2013). In children, the association between nonsymbolic estimation and mathematics was found to be mediated by symbolic estimation skills, such as knowledge of number words and Arabic numerals and of their meaning (cf. Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009; van Marle, Chu, Li, & Geary, 2014). It is possible that number line activities contribute to the knowledge of symbolic quantities, which is one of the most powerful predictors of later achievement (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009).

Similar to nonsymbolic estimation, there is evidence pointing to a small male advantage in number line estimation (Hannula, 2003; LeFevre et al., 2010), although these results are not consistent (Gunderson, Ramirez, Beilock, & Levine, 2012; Thompson & Opfer, 2008).

**Nonsymbolic and Symbolic Estimation and Other Cognitive Abilities**

A wealth of previous research has found associations between mathematics and other non-numerical abilities, such as working memory (e.g., Bull, Johnston, & Roy, 1999; Geary, 2011; McLean & Hitch, 1999; Siegel & Ryan, 1989; Swanson & Sachse-Lee, 2001); speed of processing (Bull & Johnston, 1997; Bull et al., 1999; Case, Kurland, & Goldberg, 1982); and reading and general cognitive factors (e.g., Dirks, Spyer, van Lieshout, & de Sonnevile, 2008; Fuchs et al., 2010b; Kovas, Harlaar, Petrill, & Plomin, 2005; Kovas, Haworth, Petrill, & Plomin, 2007). Less is known about the role of these abilities in the link between mathematics and nonsymbolic and symbolic estimation.

One study found that the correlation between a nonsymbolic (dot) discrimination task at age 14 and mathematical ability at age 8 remained significant after controlling for 16 cognitive measures assessed at age 8, including visuospatial reasoning, working memory, reading, word knowledge, and object perception (Halberda & Feigenson, 2008). Similarly, nonsymbolic estimation skills were significantly correlated with mathematics in over 10,000 11-to 85-year-old participants, after controlling for age, sex, as well as measures of science, writing and computer ability (Halberda et al., 2012). In preschoolers nonsymbolic estimation skills were associated with mathematical abilities, but not with vocabulary or letter identification in early primary school (Mazzocco et al., 2011b). However, another study found that a nonsymbolic (dot) discrimination task correlated only with short-term memory but not with counting and number knowledge in 4–7-year-olds (Soltész et al., 2010). Number line estimation has been linked to individual differences in IQ and in aspects of working memory in 7–8-year-old children (Geary, Hoard, Nugent, & Byrd-Craven, 2008) and with visuospatial skills (Bachtot, Gevers, Fias, & Roeyers, 2005).

**The Present Study**

The body of knowledge on the links between estimation, other cognitive abilities, and mathematics is growing. However, most of the studies into symbolic and nonsymbolic estimation have been conducted in early to middle childhood. Furthermore, most studies have used only a few measures, and therefore meta-analyses draw conclusions based on widely differing measures and ages (see Schneider et al., 2017). Previous research provided inconsistent findings regarding the presence of sex differences in estimation abilities. It is therefore unclear whether sex differences in estimation, if found, may contribute to the observed sex differences in mathematical ability (e.g., Spelke, 2005).

The present study is a large-scale multivariate investigation into the relationship between two aspects of number sense and formal mathematics across development. The study has three major aims: (1) to examine the relationship between nonsymbolic and symbolic estimation abilities, as assessed by a dot estimation and a number line tasks at age 16; (2) to assess whether estimation abilities measured at age 16 are related with mathematical abilities measured at ages 7, 9, 10, 12, 14, and 16; (3) to assess whether the links between mathematical ability and estimation are present after accounting for a number of verbal and non-verbal abilities measured in the same children at 7, 9, 10, 12, 14, and 16 years of age. The large sample used in the study affords a statistically powerful evaluation of potential sex differences in estimation and in the extent to which sex differences in estimation are associated with sex differences in mathematical ability.

**Method**

**Participants**

Participants were drawn from the longitudinal, U.K. representative Twins Early Development Study (TEDS) sample (Haworth, Davis, & Plomin, 2013). Families of twins born between 1994 and 1996 in England and Wales were identified through birth records. Out of the 16,810 families recruited into the study, over 12,000 remain active. The project received approval from the King’s College London Institute of Psychiatry ethics committee. For each assessment, informed consent was obtained from parents before data collection and the twins gave their assent.
The current report is based on cognitive abilities and school achievement data collected when the twins were 7 (M\text{age} = 7.12, SD = .25), 9 (M\text{age} = 9.03, SD = .28), 10 (M\text{age} = 10.09, SD = .28), 12 (M\text{age} = 11.65, SD = .68), 14 (M\text{age} = 14.08, SD = .57), and 16 (M\text{age} = 16.58, SD = .30) years old. Data were excluded from twins for whom English is not their first language and those with severe medical conditions, psychiatric disorders, and perinatal complications. These criteria generated a sample of 17,882 individuals (9,175 females, from 8,941 families) who contributed at least one data point.

Not all twins were tested at each assessment wave (see supplementary online materials [SOM] for further details). This led to only partially overlapping samples across ages; therefore, homogeneity and representativeness of the samples over time were assessed in order to ensure meaningful comparisons across ages. First, quantile regressions assessed (1) whether the strength of association was similar across the 25\textsuperscript{th}, 50\textsuperscript{th}, and 75\textsuperscript{th} quantiles of each measure (details in SOM and Figures S5 and S6); and (2) the stability of the associations across development. These analyses showed very similar patterns across the quantiles (homogeneity), justifying the use of mean analyses. Furthermore, the associations were stable across ages, showing very similar results in the partially overlapping samples.

Second, we compared socioeconomic status (SES), assessed when the twins were about one and a half years old, across the partially overlapping groups at ages of 7, 9, 10, 12, 14, and 16 years. These analyses (detailed in Table S1, SOM) showed significant but very small mean SES differences between ages 7 and 12, 7 and 14, and 7 and 16, with effect size ranging between .07 and .12 when computed in r, and between .10 and .24 in Cohen’s d. In the comparison of all groups, the effect size, computed in r, ranged between .03 and .12, and in Cohen’s d ranged between .07 and .24, suggesting little effects of the missing data. Furthermore, the TEDS sample has shown to be representative of the same age U.K. population over the years (Haworth et al., 2013). Overall, these analyses suggest that it is unlikely that the results are affected by the different composition of the samples. Given the diverse causes and refer to them as “dot estimation.” The Number Line task, adapted from Opfer and Siegler (2007), assesses understanding of numerical magnitudes and ability to estimate the size of numbers. A line, with the left edge marked with “0” and the right edge marked with “1000,” is presented with a numeral above it. Participants indicate the position of numerals (22 in this test) by dragging and releasing a cursor along the line, using a computer mouse. The numbers on the number line are programmed as deviations in pixels from “0”; participants’ scores represent the mean of deviations in pixels from the correct position of each number on the line.

### Measures

#### Measures age 16

Data measuring symbolic and nonsymbolic estimation, mathematics, and a range of cognitive abilities were collected using 11 computerized tests administered online, briefly described below and summarized in Table 1. More details about these tests and recruitment of the sample at age 16 can be found in SOM. The age of 16 corresponds to the end of the compulsory education in the U.K., and students take a public examination (GCSE: General Certificate of Secondary Education). We used the mathematics GCSE scores as a further measure of mathematical ability at this age.

#### Estimation Ability

was measured with two tasks. The Dot Task, adapted from Halberda and Feigenson (2008), is used to assess nonsymbolic approximate estimation of large numerosities. The task consists of 150 trials depicting arrays with interspersed yellow and blue dots. These stimuli remain on the screen for 400 ms, during which time the participant selects whether the display contains more yellow or blue dots, by pressing “Y” for more yellow and “B” for more blue dots. A Weber Fraction score was derived as a measure of the numerical ratio at which a participant’s numerical discrimination is reliably accurate, which in turn indicates the precision of numerical estimation (details in SOM). Weber Fraction scores correlated over 98% with accuracy (proportion of correct answers) on this task; analyses conducted using both accuracy and Weber Fraction scores yielded very similar results. Here we report only results on the Weber Fraction scores and refer to them as “dot estimation.”

#### Number Line

The Number Line task, adapted from Opfer and Siegler (2007), assesses understanding of numerical magnitudes and ability to estimate the size of numbers. A line, with the left edge marked with “0” and the right edge marked with “1000,” is presented with a numeral above it. Participants indicate the position of numerals (22 in this test) by dragging and releasing a cursor along the line, using a computer mouse. The numbers on the number line are programmed as deviations in pixels from “0”; participants’ scores represent the mean of deviations in pixels from the correct position of each number on the line.

#### Table 1

Summary Measures from Age 7 to 16 Years

| Domains assessed               | Age 7 | Age 9 | Age 10 | Age 12 | Age 14 | Age 16 |
|--------------------------------|-------|-------|--------|--------|--------|--------|
| Number Line estimation         | Web   | Web   | Web    | Web    | Web    | Web    |
| Dot estimation                 | Web   | Web   | Web    | Web    | Web    | Web    |
| Mathematics                    | Teach. Quest. | Teach. Quest. | Teach. Quest.; Web | Teach. Quest.; Web | Teach. Quest. | Web |
| Verbal ability                 | Telephone | Child Quest. | Web | Web | Web | Web |
| Non-verbal ability             | Telephone | Child Quest. | Web | Web | Web | Web |
| Reading fluency                | Telephone | Web | Web | Web | Web | Web |
| Reading comprehension          | Web | Web | Web | Web | Web | Web |
| Language                       | Web | Web | Web | Web | Web | Web |
| Spatial ability                | Web | Web | Web | Web | Web | Web |
| Memory                         | Web | Web | Web | Web | Web | Web |
| Speed of processing            | Web | Web | Web | Web | Web | Web |

Note. Data gathered using the following: Telephone testing (Telephone); Child Questionnaire (Child Quest.); Teacher Questionnaire (Teach. Quest.); Web testing (Web); Exams results (Exams).
The scores were normalized with a log-10 transformation prior to the analyses. Scores on this task are referred to as “number line estimation.”

**Mathematical Performance** was measured with two Web tests and one postal questionnaire. The Problem Verification Task, adapted from Murphy and Mazzocco (2008), assesses calculation fluency—the efficiency with which the veracity of an arithmetic solution is evaluated and basic facts of arithmetic are retrieved. The test consists of 48 arithmetic problems such as 28 ÷ 16 = 2. Participants are asked to quickly indicate, by key-press, whether the answer is correct. Number of correct answers was used for the analyses. Understanding Numbers measures mathematical skills according to the achievement level required by the U.K. National Curriculum at age 16 (e.g., Tosto, Ashbury, Mazzocco, Petrill, & Kovas, 2016). Items are 18 problems selected from the National Foundation for Educational Research (NFER) booklets (levels 1 to 8; nferNelson, 1994, 1999, 2001). For some questions such as “Work out the value of x: 6x + 9 = 8x,” response is given by clicking on the correct solution from five choices. For some problems the answer needs to be typed in. Number of correct answers was used in the analyses. The two mathematics Web tests correlated .70 and were combined together in a single score, Mathematics web, by averaging their standardized means. Mathematics GCSE scores were collected by questionnaires sent to the families, soon after the release of school examination results. Mathematics GCSE is graded from G (lowest) to A* (A-star, the highest). These grades were coded on an 8-point scale, from 4 to 11, respectively.

**General Cognitive Ability** was assessed with four tests. Corsi Tapping Block, adapted from Farrell Pagulayan, Busc, Medina, Bartok, and Krikorian (2006), measures visuospatial working memory. Stimuli consist of 9 small cubes arranged inside a black square. The cubes glow one at a time in a sequential pattern. Participants are asked to reproduce the pattern by clicking on the cubes with a mouse. Number of correct responses was used in the analyses. Reaction Time, adapted from Deary, Der, and Ford (2001), assesses speed of processing as measured by response reaction time (RT). Participants are asked to complete 40 trials in a fixed order by pressing 1, 2, 3, and 4 on the keyboard as soon as one of these numbers is presented on the screen. Prior to analyses, to account for speed-accuracy trade-off, efficiency scores were derived by dividing the median RT of correct responses by the proportion of correct answers. Efficiency scores were then normalized with a log-10 transformation. Raven’s Progressive Matrices, adapted from Raven, Court, and Raven (1996), assesses non-verbal (fluid) intelligence. Participants are administered a maximum of 30 trials where they complete a matrix by clicking on the missing pattern among the choice of 8. Number of correct responses was used in the analyses. Mill Hill Vocabulary, adapted from Raven, Raven, and Court (1998), assesses verbal ability. Participants complete 33 trials, selecting which of 6 words is similar in meaning to the target word presented on the screen. Number of correct answers was used in the analyses.

**Language Ability** was measured with the semantics Figurative Language subtest adapted from the Test of Language Competence (Wiig, Secord, & Sabers, 1989). The test assesses the interpretation of metaphors or figures of speech and the understanding of such nonliteral language. The stimuli consist of 15 figurative expressions referring to a situation presented in oral and written format (e.g., A boy talking about his girlfriend says, “She is easily crushed”). Participants select a matching expression from a choice of 4 (such as the following: Her bones break quite easily; She must be handled with care; She can handle anything; She has a crush on me) by clicking on it with a mouse. Number of correct responses was used in the analyses.

**Reading Ability** was measured with two tests (r = .4), combined into a reading composite by averaging their standardized means. The Reading Fluency test, adapted from Woodcock-Johnson III (Woodcock, McGrew, & Mather, 2001), consists of 98 questions requiring yes/no answers. Participants have 2 minutes and 30 seconds to answer as many questions as possible by clicking with a mouse on the “Yes” or “No” buttons appearing on the screen together with the question. Number of correct responses was used in the analyses. Reading comprehension test, developed by Hayiou-Thomas & Dale (available from the authors) is based on two passages of written text. Participants read the passages and answer 13 multiple choice questions for each passage. Number of correct responses was used in the analyses.

**Validation.** Prior to the main data collection, the tasks were piloted and tested for reliability and suitability for Web administration using samples of 16-years-old singleton and twin students. All tests proved to be suitable for Web administration (see SOM for details) and showed good internal consistency and test–retest reliability (see Table 2).

**Measures age 7 to 14.** Measures used at the ages 7, 9, 10, 12, and 14 are briefly listed below and summarized in Table 1. More details are presented in SOM. Detailed descriptions of the tests at these ages and their validation can be found elsewhere (e.g., Haworth et al., 2007; Kovas et al., 2007).

**7 years.** Data for cognitive abilities (Verbal Ability, Non-Verbal Ability, and Reading) were collected using telephone testing. Mathematics school achievement was collected using teacher questionnaires.

**9 years.** Data for cognitive abilities (Verbal Ability and Non-Verbal Ability) were collected using child-completed postal booklets. Mathematics school achievement was collected using teacher questionnaires.

**10 years.** Data for cognitive abilities (Verbal Ability, Non-Verbal Ability, Mathematics Web and Reading) were collected using an online test battery. Mathematics school achievement was collected using teacher questionnaires.

**12 years.** Data for cognitive abilities (Verbal Ability, Non-Verbal Ability, Mathematics Web, Spatial Ability, Language and Reading) were collected using a Web-based test battery. Mathematics school achievement was collected using teacher questionnaires.

**14 years.** Data for cognitive abilities (Verbal Ability and Non-Verbal Ability) were collected using a Web-based test battery. Mathematics school achievement was collected using teacher questionnaires.

**Results**

All measures were corrected for age and standardized to a mean of 0.0 and a standard deviation of 1.00; scores ± 3 standard deviations (SDs) were excluded. Descriptive statistics for the whole sample and for males and females separately are presented.
in Table 3 for measures at age 16, and in SOM Table S3, for measures at ages 7–14.

All tables present the results for one half of the sample. Results from the replication sample are available from the authors. As expected, the two samples were nearly identical in terms of means and distributions for all variables. The symbol ◦ indicates results that were statistically significantly different between the two samples, suggesting weak/unreliable effects.

Number line estimation and dot estimation correlated with each other modestly, \( r = .22 \), 95% CI [18; .26] (Table S4, SOM); we further explored their association by entering them into an exploratory factor analysis together with all the cognitive abilities measured at age 16 (Table S2, SOM). The method of the eigenvalues greater than one, suggested the extraction of two factors; however, because the initial extraction identified a third factor with an eigenvalue of .91 and the scree plot allowed the extraction of a third factor, we conducted analyses extracting two and three factors. In a two-factors model number line and dot estimation clustered together, with a three-factors model they loaded in two distinct factors (details of the analysis in SOM). The modest correlation and the results of the factor analysis suggest heterogeneity within the estimation domain, at least when assessed with a dot estimation and a number line task at age 16.

### Table 3

**Means, Standard Deviations, and Effects of Sex on Variables at Age 16**

| Measure (scores on the test) | All (M, SD) | All (M (N), SD) | Females (M (N), SD) | Males (M (N), SD) | ANOVA-effects of sex |
|------------------------------|-------------|----------------|-------------------|----------------|---------------------|
|                              | M | SD | M (N) | SD | M (N) | SD | M (N) | SD | p | \( \eta^2 \) | \( R^2 \) |
| Number line estimation       | 36.66 | 15.62 | -0.01 (n = 2792) | .98 | .03 (n = 1614) | .98 | -.05 (n = 1178) | .98 | .03 | .00 | .00 |
| Dot estimation (Weber Fraction) | 28 | .13 | -.12 (n = 2437) | .76 | -.11 (n = 1421) | .75 | -.14 (n = 1016) | .76 | .42 | .00 | .00 |
| Mathematics GCSE scores      | 8.87 | 1.26 | .03 (n = 5707) | .97 | -.01 (n = 3032) | .98 | -.06 (n = 2675) | .96 | .01 | .00 | .00 |
| Mathematics web scores       | 15.35 | 4.29 | -.02 (n = 2697) | .95 | -.02 (n = 1564) | .95 | -.03 (n = 1133) | .95 | .71 | .00 | .00 |
| Verbal ability scores         | 15.18 | 8.77 | -.03 (n = 2943) | .97 | -.03 (n = 1207) | .96 | -.02 (n = 983) | .94 | .22 | .00 | .00 |
| Reading comprehension scores  | 10.28 | 2.57 | .02 (n = 2563) | .95 | .03 (n = 1490) | .94 | .02 (n = 1073) | .95 | .74 | .00 | .00 |
| Speed of Processing scores    | 37.56 | 1.82 | -.06 (n = 2412) | .84 | -.07 (n = 1404) | .82 | -.04 (n = 1008) | .87 | .31 | .00 | .01 |
| Memory (Corsi) scores         | 5.50 | 2.03 | .03 (n = 2445) | .97 | -.03 (n = 1427) | .92 | .13 (n = 1018) | 1.0 | .00 | .01 | .01 |

Note. \( n \) = sample size based on one randomly selected twin in the pair; \( M \) = mean; \( SD \) = Standard deviation; \( p \) = p-value of the effects of sex on variables; \( \eta^2 \) = partial eta-squared; \( R^2 \) = variance explained by sex. Standardized variables have been cleared of outliers scores (±3 standard deviations). Mean and standard deviation on raw data for the Number Line test represent the average error in estimation. The mathematics web test and reading scores are composites obtained by averaging the standardized means of two tests scores, therefore no raw data is provided for these composites. Descriptive statistics on speed of processing are presented for efficiency scores derived from the reaction time test; the column with raw data reports mean and standard deviation for accuracy on the test. Boys and girls showed different variance in non-verbal ability and memory (significant Levene’s test), however, these differences contributed to 1% of variance (\( R^2 \)) in memory and less than 1% in non-verbal ability.
Robust correlations among cognitive abilities and achievement were observed over time (see Table S4, SOM). Smaller scores for number line estimation, dot estimation, and speed of processing index better performance, therefore correlations of these three measures are positive among each other and negative with all other measures. On average, scores from both estimation measures correlated substantially with mathematics at all ages (average measures are positive among each other and negative with all other number line estimation, dot estimation, and speed of processing were observed over time (see Table S4, SOM). Smaller scores for mathematics age 16 predicted by number line estimation and dot estimation as statistical predictors and mathematics scores as criterion variables. The results, presented in Table 4, show that symbolic and nonsymbolic estimation were significantly associated with mathematics, concurrently and retrospectively. Overall, number line estimation was more strongly associated with mathematics than dot estimation (average $r = -.29$ and $-.16$ for number line and dot estimation, respectively). The association between number line and the mathematics variables (as indexed by beta-coefficients) was overall very similar across ages. The only significant differences were found between the strength of the association at age 16 and the

### Table 4

**Regressions Method Forced Entry. Mathematics at Each Age Predicted by Number Line Estimation and Dot Estimation Measured at Age 16**

| Model | DV = Mathematics school achievement teacher rated & GCSE scores (age 16) | | DV = Mathematics web test |
|-------|---------------------------------------------------------------|------------------|---------------------------------------------------------------|
| Mathematics age 7 predicted by the scores | | | |
| Number line estimation | $-0.22$ | $-9.36^{**}$ | | |
| Dot estimation | $-0.17$ | $-7.09^{**}$ | | |
| | $R^2 = 0.10$; $F(2, 1651) = 86.64; p < .001$ | | |
| Mathematics age 9 predicted by | | | |
| Number line estimation | $-0.26$ | $-9.91^{**}$ | | |
| Dot estimation | $-0.15$ | $-5.69^{**}$ | | |
| | $R^2 = 0.11$; $F(2, 1381) = 80.50; p < .001$ | | |
| Mathematics age 10 predicted by | | | |
| Number line estimation | $-0.29$ | $-11.73^{**}$ | $-0.25$ | $-10.58^{**}$ |
| Dot estimation | $-0.11$ | $-4.48^{**}$ | $-0.15$ | $-6.08^{**}$ |
| | $R^2 = 0.11$; $F(2, 1477) = 92.38; p < .001$ | $R^2 = 0.10$; $F(2, 1653) = 91.11; p < .001$ | |
| Mathematics age 12 predicted by | | | |
| Number line estimation | $-0.27$ | $-8.30^{**}$ | $-0.31$ | $-13.74^{**}$ |
| Dot estimation | $-0.14$ | $-4.14^{**}$ | $-0.20$ | $-8.82^{**}$ |
| | $R^2 = 0.11$; $F(2, 894) = 54.38; p < .001$ | $R^2 = 0.16$; $F(2, 1768) = 166.72; p < .001$ | |
| Mathematics age 14 predicted by | | | |
| Number line estimation | $-0.34$ | $-5.78^{**}$ | | |
| Dot estimation | $-0.11$ | $-1.86$ | | |
| | $R^2 = 0.14$; $F(2, 275) = 24.36; p < .001$ | | |
| Mathematics age 16 predicted by | | | |
| Number line estimation | $-0.31$ | $-15.11^{**}$ | $-0.35$ | $-18.28^{**}$ |
| Dot estimation | $-0.16$ | $-7.90^{**}$ | $-0.23$ | $-11.81^{**}$ |
| | $R^2 = 0.15$; $F(2, 2104) = 179.60; p < .001$ | $R^2 = 0.21$; $F(2, 2259) = 298.50; p < .001$ | |

**Note.** DV = Dependent variable; $\beta$ = standardized beta; $t$ = t-value of $\beta$, significant t-values are reported in bold characters. Regressions based on one randomly selected twin in each pair. Analyses were repeated using the second half of the sample, with very similar results (available from the authors). $^{**} p < .001$. 

### Estimation and Mathematics Over Time

Although we had longitudinal measures of mathematical ability from age 7 to 16, estimation was only measured at age 16. To address the second aim of the study we examined the retrodictive predictions from estimation to mathematics at each age in separate regressions, entering number line and dot estimation as statistical predictors and mathematics scores as criterion variables. The results, presented in Table 4, show that symbolic and nonsymbolic estimation were significantly associated with mathematics, concurrently and retrospectively. Overall, number line estimation was more strongly associated with mathematics than dot estimation (average $r = -.29$ and $-.16$ for number line and dot estimation, respectively). The association between number line and the mathematics variables (as indexed by beta-coefficients) was overall very similar across ages. The only significant differences were found between the strength of the association at age 16 and the
strength of association shown at age 7, 10 and 12 (Figure S1 in SOM). A similar pattern was observed for the associations between dot estimation and mathematics over time. However, for dot estimation the differences were significant only between Teacher assessed mathematics at 14 and Mathematics web at 16 (Figure S2 in SOM). These results suggest that the association between mathematics and estimation abilities changes with the changes in mathematics phenotype; they also suggest that the association between them strengthens over time, potentially due to reciprocal influences.

Next, we explored whether early mathematical ability and achievement explained additional variance in individual differences in estimation abilities at 16, beyond concurrent mathematics. As evidence suggests that there is a strong relationship between early mathematics achievement and early number knowledge (see Geary, Hoard, Nugent, & Bailey, 2013), we used the earliest measure of mathematics in our sample to test whether it was related to estimation skills at age 16. These analyses were conducted on over 1,400 participants with complete data. Number line estimation and dot estimation were entered as criterion variables in separate stepwise regressions, mathematics at age 16 (GCSE and Web) was entered in the first step, and mathematics at age 7 was added in the second. For number line estimation, both measures of mathematics at age 16 and mathematics at 7 were significant predictors with betas on the second step as follows: $\beta = -10, t = -2.68, p < .01$ for GCSE age 16, $\beta = -30, t = -8.71, p < .001$ for Web assessed mathematics at age 16 and $\beta = -09, t = -3.31, p < .01$ for teacher assessed mathematics at age 7. Both mathematics measures at 16 were significant predictors of dot estimation in the first step. In the second step mathematics Web assessed at age 16 (but not GCSEs) was a significant predictor of dot estimation ($\beta = -20, t = -5.24, p < .001$, second step) together with mathematics at age 7 ($\beta = -10, t = -3.46, p < .01$, second step). Overall, these results suggest lasting links between late estimation and early mathematics. As estimation was available only at age 16, it is unclear whether this would also be true for early dot and number line estimation. An inspection of the 95% CI of the beta coefficients derived from these analyses (Figures S3 and S4 in SOM) suggests that teacher assessed mathematics at age 7 and exam assessed mathematics at age 16 have similar association with estimation (both measures) at age 16; furthermore, these associations were significantly different from the association between Web assessed mathematics at age 16 and estimation at age 16.

It is possible that classroom based mathematics builds on skills that are responsible for the association between mathematics with estimation ability; some of these early abilities may be more relevant for dot estimation skills (hence the association of dot estimation with teacher mathematics at 7 but not with GCSE). It is unclear whether the contemporaneous association between dot estimation and mathematics (both measured at age 16) was restricted to the Web assessed mathematics because of shared methods (e.g., both collected with online tests). Because Web assessed mathematics was not available at age 7, we cannot differentiate whether (1) web assessment taps into some abilities that emerge at a later age (age 16) and are important for estimation or (2) web assessments draw on some abilities unimportant for estimation. However, if shared methods were a source of association we should also observe the link of number line only with Web assessed mathematics and not with GCSE.

**Associations Among Estimation, Mathematics, and Related Cognitive Abilities Over Time**

The third aim of the study was to examine whether the links between mathematical ability and estimation are present after accounting for a number of verbal and nonverbal abilities measured in the same children at ages 7, 9, 10, 12, 14, and 16 years of age.

First, mathematics at each age (Web, teacher assessed, GCSE scores at 16) were separately entered into multiple regressions as criterion variables. Number line and dot estimation were entered as predictors together with other cognitive abilities. The twins’ sex was also entered in each regression as predictor of mathematics and estimation (see Table 5).

In the presence of other cognitive abilities, number line estimation was a significant predictor of mathematics (teacher and Web assessed) at each age (average 11% of the variance explained). Conversely, dot estimation was a significant predictor of Web assessed mathematics at age 16 only, and of teacher assessed mathematics at ages 7, 9, and 10 (average 5% of the variance explained). Other cognitive abilities explained between 7% (non-verbal abilities scores at age 7) and 32% (non-verbal ability scores at 16) of the variance in mathematics.

The next set of analyses examined whether estimation at 16 was best predicted by mathematics as opposed to other cognitive skills. Number line and dot estimation were entered as dependent variables, with mathematics and general cognitive abilities at ages 7, 9, 10, 12, 14, and 16 entered as independent predictors (see Table 6). Separate regressions were run for teacher rated (and GCSE) and Web assessed mathematics. The significance level for these regressions was adjusted for multiple testing ($.05 ÷ 9 = .006, p < .01$).

In the presence of other cognitive abilities, all measures of mathematics explained between 8% (age 7) and 17% (age 16) of the variance in number line estimation. However, other abilities were also significant predictors of number line estimation: non-verbal abilities at age 9, 10, and 16 (average 7% variance explained); reading at age 7 and 12 (average 6% of the variance; only when mathematics Web was included at age 12); spatial ability at age 12; and memory scores at age 16 (respectively explaining 8% and 5% of variance).

The pattern was overall similar for dot estimation, although its association with mathematics was uneven. Mathematics measured at age 7, 9, 10, and 12 was a significant predictor of dot estimation at age 16 (average variance explained 5%). At age 16 only Web assessed mathematics added independent variance (10%). After correction for multiple testing, other cognitive abilities explained independent variance in dot estimation: non-verbal abilities at age 9, 10, 14, and 16 (average 5%); reading at age 7 (2%); spatial ability at age 12 (4%, only when teacher assessed mathematics was included), and speed of processing scores at age 16 (4%: average contribution when GCSE and Web scores were included).

**Sex Differences in Estimation and Mathematics**

As shown in Table 3, boys and girls performed very similarly on both measures of estimation assessed at age 16. Mean differences
Table 5

Summary of Multiple Regressions, Method Forced Entry. Mathematics Predicted at Each Age by Number Line and Dot Estimation Measured at Age 16 and Cognitive Abilities Measured at the Same Age of Mathematics

| Model | DV = Mathematics school achievement: teacher rated & GCSE scores (age 16) | DV = Mathematics web tests |
|-------|---------------------------------------------------------------------------|---------------------------|
|        | \( \beta \)                       | \( t \)       | \( \eta^2 \) | \( \beta \)                       | \( t \)       | \( \eta^2 \) |
| Mathematics age 7 predicted by | | | |
| Number line estimation at 16 | \(-.12\) | \(-5.51^{***}\) | \(.08\) | | | |
| Dot estimation at 16 | \(-.09\) | \(-4.41^{***}\) | \(.05\) | | | |
| Verbal ability at 7 | \(.15\) | \(6.37^{***}\) | \(.15\) | | | |
| Non-verbal ability at 7 | \(.08\) | \(3.70^{***}\) | \(.07\) | | | |
| Reading at 7 | \(.41\) | \(17.75^{***}\) | \(.29\) | | | |
| Sex | \(.07\) | \(3.25^{***}\) | \(.00\) | | | |
| | | \(R^2 = .34; F(6, 1526) = 130.84; p < .001\) | | | \(R^2 = .41; F(6, 1520) = 177.66; p < .001\) |

| Mathematics age 9 predicted by | | | |
| Number line estimation at 16 | \(-.17\) | \(-6.40^{***}\) | \(.09\) | | | |
| Dot estimation at 16 | \(-.11\) | \(-4.41^{***}\) | \(.04\) | | | |
| Verbal ability at 9 | \(.19\) | \(6.35\) | \(.10\) | | | |
| Non-verbal ability at 9 | \(.23\) | \(8.31^{***}\) | \(.15\) | | | |
| Sex | \(.07\) | \(2.92^{**}\) | \(.00\) | | | |
| | | \(R^2 = .20; F(5, 1257) = 43.00; p < .001\) | | | \(R^2 = .23; F(6, 1166) = 58.62; p < .001\) |

| Mathematics age 10 predicted by | | | |
| Number line estimation at 16 | \(-.20\) | \(-7.33^{***}\) | \(.09\) | | | |
| Dot estimation at 16 | \(-.07\) | \(-2.75^{**}\) | \(.03\) | | | |
| Verbal ability at 10 | \(.15\) | \(4.75\) | \(.13\) | | | |
| Non-verbal ability at 10 | \(.11\) | \(3.60^{**}\) | \(.11\) | | | |
| Reading at 10 | \(.19\) | \(6.02^{***}\) | \(.13\) | | | |
| Sex | \(.05\) | \(1.76\) | \(.00\) | | | |
| | | \(R^2 = .23; F(6, 1166) = 58.62; p < .001\) | | | \(R^2 = .41; F(6, 1520) = 177.66; p < .001\) |

| Mathematics age 12 predicted by | | | |
| Number line estimation at 16 | \(-.09\) | \(-2.46^{**}\) | \(.11\) | | | |
| Dot estimation at 16 | \(-.06\) | \(-1.67\) | \(.04\) | | | |
| Verbal ability at 12 | \(.09\) | \(2.00\) | \(.17\) | | | |
| Non-verbal ability at 12 | \(.05\) | \(1.25\) | \(.12\) | | | |
| Reading at 12 | \(.25\) | \(6.20^{***}\) | \(.22\) | | | |
| Language at 12 | \(.17\) | \(3.74^{***}\) | \(.19\) | | | |
| Spatial ability at 12 | \(.09\) | \(2.46^{**}\) | \(.08\) | | | |
| Sex | \(-.01\) | \(-.21\) | \(.00\) | | | |
| | | \(R^2 = .30; F(8, 647) = 35.97; p < .001\) | | | \(R^2 = .52; F(8, 1401) = 190.26; p < .001\) |

| Mathematics age 14 predicted by | | | |
| Number line estimation at 16 | \(-.23\) | \(-4.17^{***}\) | \(.15\) | | | |
| Dot estimation at 16 | \(.00\) | \(.01\) | \(.05\) | | | |
| Verbal ability at 14 | \(.29\) | \(5.16^{***}\) | \(.23\) | | | |
| Non-verbal ability at 14 | \(.32\) | \(5.58^{***}\) | \(.23\) | | | |
| Sex | \(.11\) | \(2.21^{*}\) | \(.00\) | | | |
| | | \(R^2 = .39; F(5, 237) = 32.07; p < .001\) | | | \(R^2 = .52; F(8, 1401) = 190.26; p < .001\) |

| Mathematics age 16 predicted by | | | |
| Number line estimation at 16 | \(-.14\) | \(-7.27^{***}\) | \(.12\) | | | |
| Dot estimation at 16 | \(-.02\) | \(-.97\) | \(.06\) | | | |
| Speed of processing at 16 | \(-.09\) | \(-4.52^{***}\) | \(.10\) | | | |
| Memory (Corsi) at 16 | \(.11\) | \(5.83^{***}\) | \(.11\) | | | |
| Verbal ability at 16 | \(.13\) | \(6.09^{***}\) | \(.19\) | | | |
| Non-verbal ability at 16 | \(.21\) | \(10.39^{***}\) | \(.24\) | | | |
| Reading at 16 | \(.19\) | \(8.62^{***}\) | \(.25\) | | | |
| Language at 16 | \(.16\) | \(7.47^{***}\) | \(.24\) | | | |
| Sex | \(.02\) | \(1.27\) | \(.00\) | | | |
| | | \(R^2 = .43; F(9, 1830) = 155.52; p < .001\) | | | \(R^2 = .51; F(9, 2038) = 232.66; p < .001\) |

Note. DV = Dependent variable; \( \beta \) = standardized beta; \( t \) = t-value of \( \beta \), significant t-values are reported in bold characters; \( \eta^2 \) = partial eta-squared. The symbol * indicates results significant in one sample of twins and non-significant in the co-twins. Regressions based on one randomly selected twin in each pair. 

\( ^* p < .05. \quad ^{**} p < .01. \quad ^{***} p < .001. \)

were significant for number line estimation only. However, the effects of sex on both estimation measures were negligible (\( \eta^2 = .00 \) for both). No meaningful variance or mean sex differences were observed for other measures. Sex was included as a predictor of mathematics at each age in all regressions presented in Table 5 and explained between 0% and 3% of the variance. When sex was included as a predictor of number line estimation and dot estimation, it was not a significant predictor (see Table 6).
### Table 6

**Summary of Multiple Regressions, Method Forced Entry, Number Line, and Dot Estimation Measured at Age 16 Predicted by Mathematics and Cognitive Abilities Measured at Age 16 and at Previous Ages**

| Model | DV = Number line estimation at age 16 | DV = Dot estimation at age 16 |
|-------|--------------------------------------|-------------------------------|
|        | $\beta$ | $t$ | $\eta^2$ | $\beta$ | $t$ | $\eta^2$ |
| **Age 7 predictors of estimation** Mathematics teacher at 7 | -.21 | $-7.43^{***}$ | .08 | -.18 | $-5.95^{***}$ | .05 |
| Verbal ability at 7 | -.03 | $-1.06$ | .02 | .01 | .25 | .01 |
| Non-verbal ability at 7 | -.04 | $-1.80$ | .02 | -.04 | $-1.62$ | .01 |
| Reading at 7 | -.08 | $-3.05^{**}$ | .05 | -.08 | $-2.73^{**}$ | .02 |
| $R^2 = .08$; $F(5, 1746) = 32.17$; $p < .001$ | $R^2 = .06$; $F(5, 1546) = 19.40$; $p < .001$ |
| **Age 9 predictors of estimation** Mathematics teacher at 9 | -.22 | $-8.07^{***}$ | .09 | -.17 | $-5.74^{***}$ | .04 |
| Verbal ability at 9 | -.01 | $-0.50$ | .02 | .05 | $1.83$ | .00 |
| Non-verbal Ability at 9 | -.13 | $-4.45^{**}$ | .05 | -.15 | $-5.06^{**}$ | .03 |
| $R^2 = .09$; $F(4, 1423) = 37.46$; $p < .001$ | $R^2 = .06$; $F(4, 1270) = 21.37$; $p < .001$ |
| **Age 10 predictors of estimation** Mathematics teacher at 10 | -.22 | $-7.79^{***}$ | .09 | -.13 | $-4.00^{***}$ | .03 |
| Verbal ability at 10 | -.03 | $-0.96$ | .05 | .03 | $.99$ | .02 |
| Non-verbal ability at 10 | -.11 | $-3.52^{***}$ | .06 | -.14 | $-4.23^{***}$ | .04 |
| Reading at 10 | -.06 | $-1.76$ | .05 | -.05 | $1.34$ | .03 |
| $R^2 = .11$; $F(5, 1311) = 33.30$; $p < .001$ | $R^2 = .05$; $F(5, 1178) = 13.09$; $p < .001$ |
| **Age 10 predictors of estimation** Mathematics web at 10 | -.21 | $-7.06^{***}$ | .09 | -.10 | $-3.01^{***}$ | .04 |
| Verbal ability at 10 | -.03 | $-1.03$ | .05 | .01 | $.98$ | .02 |
| Non-verbal ability at 10 | -.08 | $-2.82^{**}$ | .06 | -.13 | $-3.98^{**}$ | .04 |
| Reading at 10 | -.06 | $-2.04^{*}$ | .05 | -.05 | $1.46$ | .03 |
| $R^2 = .10$; $F(5, 1717) = 40.17$ $p < .001$ | $R^2 = .05$; $F(5, 1537) = 17.17$; $p < .001$ |
| **Age 12 predictors of estimation** Mathematics teacher at 12 | -.14 | $-3.45^{**}$ | .11 | -.09 | $-2.03^{*}$ | .04 |
| Verbal ability at 12 | -.05 | $-1.14$ | .06 | -.02 | $.42$ | .03 |
| Non-verbal ability at 12 | -.13 | $-3.13^{**}$ | .07 | -.05 | $1.09$ | .04 |
| Reading at 12 | -.09 | $-2.15^{*}$ | .06 | -.02 | $.52$ | .04 |
| Language at 12 | -.04 | $-0.83$ | .05 | -.05 | $.90$ | .03 |
| Spatial ability at 12 | -.10 | $-2.64^{**}$ | .08 | -.12 | $-2.85^{**}$ | .04 |
| $R^2 = .16$; $F(7, 723) = 20.89$; $p < .001$ | $R^2 = .05$; $F(7, 655) = 6.80$; $p < .001$ |
| **Age 12 predictors of estimation** Mathematics web at 12 | -.20 | $-6.13^{***}$ | .13 | -.15 | $-4.05^{***}$ | .07 |
| Verbal ability 12 | -.05 | $-1.50$ | .06 | -.04 | $.10$ | .03 |
| Non-verbal ability at 12 | -.06 | $-1.94$ | .07 | -.08 | $-2.50^{*}$ | .04 |
| Reading at 12 | -.06 | $-3.22^{**}$ | .06 | -.06 | $-1.86$ | .04 |
| Language at 12 | -.05 | $-1.48$ | .05 | .03 | $.77$ | .03 |
| Spatial ability at 12 | -.11 | $-3.84^{**}$ | .08 | -.05 | $1.54$ | .04 |
| $R^2 = .14$; $F(7, 1587) = 36.47$; $p < .001$ | $R^2 = .07$; $F(7, 1414) = 16.51$ $p < .001$ |
| **Age 14 predictors of estimation** Mathematics teacher at 14 | -.31 | $-4.47^{***}$ | .15 | -.08 | $-1.12$ | .05 |
| Verbal ability at 14 | -.11 | $-1.66$ | .06 | -.09 | $-1.24$ | .04 |
| Non-verbal 14 | -.08 | $-1.24$ | .08 | -.24 | $-2.99^{***}$ | .06 |
| $R^2 = .17$; $F(4, 267) = 15.22$; $p < .001$ | $R^2 = .09$; $F(4, 240) = 6.89$; $p < .001$ |
| **Age 16 predictors of estimation** Mathematics GCSE at 16 | -.21 | $-7.85^{***}$ | .12 | -.05 | $-1.87$ | .06 |
| Verbal ability at 16 | .04 | $1.62$ | .03 | -.02 | $.80$ | .03 |
| Non-verbal ability at 16 | -.14 | $-5.52^{***}$ | .09 | -.13 | $-4.82^{***}$ | .07 |
| Reading at 16 | -.04 | $-1.52$ | .06 | -.00 | $.05$ | .03 |
| Language at 16 | -.04 | $-1.68$ | .06 | -.06 | $-2.22^{*}$ | .05 |
| Speed of processing at 16 | -.05 | $2.43$ | .04 | .17 | $7.23^{***}$ | .07 |
| Memory (Corsi) at 16 | -.08 | $-3.57^{***}$ | .05 | -.05 | $-2.00^{*}$ | .03 |
| $R^2 = .15$; $F(8, 1973) = 44.83$; $p < .001$ | $R^2 = .08$; $F(8, 1841) = 20.95$; $p < .001$ |
| **Age 16 predictors of estimation** Mathematics web at 16 | -.32 | $-12.17^{***}$ | .17 | -.15 | $-5.34^{***}$ | .10 |
| Verbal ability at 16 | .04 | $1.41$ | .03 | -.02 | $.65$ | .03 |
| Non-verbal ability at 16 | -.08 | $-3.41^{***}$ | .09 | -.09 | $-3.39^{***}$ | .07 |
| Reading at 16 | -.05 | $-1.83^{*}$ | .06 | -.02 | $.87$ | .03 |
| Language at 16 | -.00 | $.16$ | .06 | -.05 | $.18$ | .05 |
| Speed of processing at 16 | .04 | $-1.94$ | .04 | .17 | $7.44^{**}$ | .01 |
| Memory (Corsi) at 16 | -.07 | $-3.39^{***}$ | .05 | -.05 | $-2.29^{*}$ | .03 |
| $R^2 = .18$; $F(8, 2210) = 61.08$; $p < .001$ | $R^2 = .10$; $F(8, 2057) = 29.33$; $p < .001$ |

**Note.** DV = Dependent variable; $\beta$ = standardized beta; $t$ = t-value of $\beta$; significant t-values are reported in bold characters; $\eta^2$ = partial eta-squared. The symbol * indicates results significant in one sample of twins and non-significant in the co-twins. Regressions based on one randomly selected twin in each pair. Sex was included as a predictor in each model. However, as it did not significantly predict number line or dot estimation, the measure is not shown in this table. The number of degrees of freedom reflects the presence of sex as variable in the regression.

* $p < .05$  ** $p < .01$  *** $p < .001$  .001 $p$-values < .05 are not considered significant after correction for multiple testing.
ANOVAs were conducted to assess the effects of sex on the two mathematics measures at age 16 after controlling for number line and dot estimation scores, separately for each measure. In these analyses, the partial eta-squared were almost identical (η²p = .00 and .03 for GCSEs and Web scores respectively) to the partial eta-squared for the mathematics measures shown in Table 3. This suggests that the small sex differences observed in mathematics at age 16 may not be related to estimation.

**Discussion**

This longitudinal study used a large U.K. representative sample of students to investigate number sense abilities and their association with mathematics. Specifically, the study examined the extent to which symbolic and nonsymbolic estimation abilities are associated to each other at the age of 16. It also investigated the relationship between these two aspects of estimation with concurrent and earlier mathematics achievement. The specificity and continuity of this relationship was assessed controlling for a number of cognitive abilities measured across the school years. A particular strength of the design was the employment of a discovery-replication approach, by generating two matching samples of the same cohort at different ages. The association between number line estimation measured at 16 was detected in all samples. Conversely the association between mathematics and dot estimation was less consistent over time, supporting previous research (Chen & Li, 2014; Schneider et al., 2017).

Because dot estimation was only measured at age 16, it remains unclear whether the links between dot estimation and mathematics are stronger earlier in development. The observed developmental pattern is likely to reflect the heterogeneity of the mathematical domain. It is possible that when mathematics becomes more complex and abstract it may rely more strongly on spatial and other non-verbal cognitive abilities than on nonsymbolic estimation.

Other studies have suggested that different aspects of mathematics may be more closely related to dot estimation than others (e.g., Mazzocco et al., 2011a). Accordingly, our Web assessed mathematics at 16, but not school GCSE scores, correlated with dot estimation. Web assessed mathematics, which includes the component of fluency, correlated also with speed of processing at the same age, to which dot estimation was also correlated. This pattern of association between dot estimation, mathematical fluency, and speed of processing is of particular interest. Another study found that growth in nonsymbolic estimation abilities predicted mathematical fluency but not mapping or mathematical reasoning in first grade children (Toll, Van Vierson, Kroesbergen, & Van Luit, 2015). Previous research also suggests that automaticity in retrieval of the basic arithmetic facts (speed of processing) is important in mathematical learning (Bull & Johnston, 1997; Hitch & McAuley, 1991). It is possible that nonsymbolic estimation skills may be involved only at early stages of mathematics learning, in preschool or early school years, (e.g., Bonny & Lourenco, 2013; Mazzocco et al., 2011a), contributing to acquiring automaticity in basic arithmetic.

However, it remains unclear whether successful automaticity reflects foundational abilities that promote mathematics learning, or whether achieving automaticity supports later learning and therefore mediates the relationship between early estimation abilities and later mathematics. For example, once automaticity has been achieved, nonsymbolic estimation may no longer be necessary, and plays a less significant role in subsequent achievement gains. This idea is supported by one study that found no association between nonsymbolic estimation performance and mathematical ability in adults. Interestingly, however, individuals with higher mathematical skills had a more automatic access to nonsymbolic numerosity, reflected in slower performance on a numer-
necessary. The results were consistent with previous reports that reported associations of number line estimation with IQ (e.g., Bachot et al., 2005; Geary et al., 2008). Dot estimation at age 16 was also predicted by non-verbal ability at ages 9 and 10, reading at age 7, and spatial ability at age 12. These results counter the view of nonsymbolic estimation as a numerical specific process. The results are consistent with recent reports of reduced but greater than zero correlations between nonsymbolic estimation and mathematics after controlling for inhibitory control in young children (that was not controlled for in our design; Keller & Libertus, 2015) and after controlling for other cognitive abilities (Chen & Li, 2014). Together with the results of the factor analysis, this evidence points to estimation as related to the multifaceted domains of intelligence (e.g., the three-stratum model, Carroll, 1993). Further research is needed to explore the nature of the specific associations observed in this study. For example, dot estimation at age 16 was uniquely (beyond mathematics and other abilities) associated with early reading (age 7) that was assessed by word recognition/decoding tests. Evaluation of small numerosity arrays containing 2 or 3 items (subitizing) is a perceptual process that may not require counting; Gelman and Gallistel (1978) note that in children, estimation of numerosities up to 6 can rely on recognition of patterns of the possible configuration of 2, 3, 4, 5, and 6 elements. As word recognition relies on pattern recognition, it is possible that the observed association between dot estimation and early reading is partially related to pattern recognition processes. Although the processes underlying nonsymbolic estimation need to be fully understood, studies have shown that estimation does not involve numerical processing exclusively but relies on other visual cues present in the stimuli (e.g., Clayton et al., 2015; Gebuis & Reynvoet, 2012). Pattern processing may be at the core of the correspondence between numerosity of a set and its symbolic representation of number, a process that has been proposed as vital for mathematical learning (Butterworth, 2005; Gelman & Gallistel, 1978). It is also possible that pattern recognition important in nonsymbolic estimation contributes to early number symbols learning, which in turn influences future mathematics achievement (van Marle et al., 2014). Finally, the study tested whether the small but significant male advantage in mathematics found at age 16 was related to estimation. No meaningful sex differences were found in either of the two estimation tasks at age 16, suggesting that some early differences may disappear by this age. Therefore, the observed sex differences in mathematics at age 16 are not explained by estimation differences, but cognitive (e.g., spatial, Wei, Chen, & Zhou, 2016) or other noncognitive (e.g., academic anxiety; Wang et al., 2014). Further research is needed to understand whether the inconsistencies in the literature regarding sex differences in estimation are related to differences in samples, specific measures used, or developmental patterns.

The present investigation addressed “which, when, and how” questions about the relationship between number sense and mathematics. Although our study looked at the development of mathematics between the age of 7 and 16 years, estimation was measured only at age 16. This type of data has allowed only correlational analyses, limiting the understanding of directionality of effects. Also, the variability of the sample size and composition, together with the diverse measures of cognitive abilities and mathematics used, may have contributed to some of the uneven association of the dot task and mathematics. The results suggest that different measures of number sense at age 16 are partially independent constructs and are differentially related to mathematics. This supports the theory that symbolic and nonsymbolic estimation follow partially different developmental paths (Lyons, Ansari, & Beilock, 2012). More longitudinal research is needed to explore the directionality of the associations between different aspects of number sense and mathematical ability. As with other constructs related to mathematics, it is likely that the influences between estimation and mathematics are reciprocal (Carey, Hill, Devine, & Szücs, 2016).

Most covariance between mathematics and estimation was shared with other cognitive skills. Such results are consistent with previous research whereby a symbolic and a nonsymbolic task, although distinct from each other, contributed uniquely to mathematics achievement and the strength of their association with mathematics depended on the type of mathematical task (Mazzocco et al., 2011a). Importantly, much of the variance in participants’ estimation performance at age 16 remained unexplained, suggesting that estimation is a complex construct. At age 16, when estimation skills are relatively mature (Halberda et al., 2012), only 18% and 10% of the variance in number line and dot estimation respectively was explained by all other variables examined at this age. More research is needed to identify sources of the wide variability in estimation (Halberda et al., 2012).

Taken together, the results from this study indicate that relationship between number sense and mathematics depends on which specific aspects are considered and at which age.

References

Ashcraft, M. H., & Moore, A. M. (2012). Cognitive processes of numerical estimation in children. Journal of Experimental Child Psychology, 111, 246–267. http://dx.doi.org/10.1016/j.jecp.2011.08.005

Bachot, J., Geyvers, W., Fias, W., & Roeyers, H. (2005). Number sense in children with visuospatial disabilities: Orientation of the mental number line. Psychology Science, 47, 172–183.
Barth, H., La Mont, K., Lipton, J., Dehaene, S., Kanwisher, N., & Spelke, E. (2006). Non-symbolic arithmetic in adults and young children. *Cognition, 98*, 199–222. doi.org/10.1016/j.cognition.2004.09.011

Barth, H. C., & Paladino, A. M. (2011). The development of numerical estimation: Evidence against a representational shift. *Developmental Science, 14*, 125–135. doi.org/10.1111/j.1467-7687.2010.00962.x

Berch, D. B. (2005). Making sense of number sense: Implications for developmental Neuropsychology, 15, 387–398. http://dx.doi.org/10.1016/j.pneurobio.2007.11.001

Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology, 42*, 189–201. doi.org/10.1037/0016-6085.42.2.189

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development, 79*, 1016–1031. http://dx.doi.org/10.1111/j.1467-8624.2008.01173.x

Bull, R., & Johnston, R. S. (1997). Children’s arithmetical difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of Experimental Child Psychology, 65*, 1–24. http://dx.doi.org/10.1006/jecp.1996.2358

Bull, R., Johnston, R. S., & Roy, J. A. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children’s arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology, 15*, 421–442. http://dx.doi.org/10.1080/8756564909540759

Butterworth, B. (2005). Developmental dyscalculia. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 455–467). Hove, England: Psychology Press.

Carey, E., Hill, F., Devine, A., & Szucs, D. (2016). The chicken or the egg? The direction of the relationship between mathematics anxiety and mathematics performance. *Frontiers in Psychology, 6*, 1987. http://dx.doi.org/10.3389/fpsyg.2015.01987

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor-analytic studies*. New York, NY: Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511571312

Case, R., Kurland, D. M., & Goldberg, J. (1982). Operational efficiency and its relation to early math achievement: Evidence from the preschool years. *Journal of Experimental Child Psychology, 34*, 375–388. http://dx.doi.org/10.1016/0022-1647(82)90030-L

Deary, I. J., Der, G., & Ford, G. (2001). Reaction times and intelligence differences: A population-based cohort study. *Intelligence, 29*, 389–399. http://dx.doi.org/10.1016/S0160-2896(01)00662-9

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: Human Perception and Performance, 19*, 626–641. http://dx.doi.org/10.1037/0016-9563.19.3.626

Dehaene, S., & Mehler, J. (1992). Cross-linguistic regularities in the frequency of number words. *Cognition, 43*, 1–29. http://dx.doi.org/10.1016/0010-0277(92)90030-L

Dirks, E., Spyer, G., van Liershout, E. C., & de Sonneville, L. (2008). Prevalence of combined reading and arithmetic disabilities. *Journal of Learning Disabilities, 41*, 460–473. http://dx.doi.org/10.1177/0022219408321128

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., . . . Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*, 1428–1446. http://dx.doi.org/10.1037/0012-1646.43.6.1428

Ebersbach, M., Luwel, K., Frick, A., Onghena, P., & Verschaffel, L. (2008). The relationship between the shape of the mental number line and familiarity with numbers in 5- to 9-year-old children: Evidence for a segmented linear model. *Journal of Experimental Child Psychology, 99*, 1–17. http://dx.doi.org/10.1016/j.jecp.2007.08.006

Farrell, Pagulayan, K., Busch, R. M., Medina, K. L., Bartok, J. A., & Krikorian, R. (2006). Developmental normative data for the Corsi Block-Tapping task. *Journal of Clinical and Experimental Neuropsychology, 28*, 1043–1052. http://dx.doi.org/10.1080/13803950500350977

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology, 123*, 53–72. http://dx.doi.org/10.1016/j.jecp.2014.01.013

Feigenson, L., Carey, S., & Hauser, M. (2002). The representations underlying infants’ choice of more: Object files versus analog magnitudes. *Psychological Science, 13*, 150–156. http://dx.doi.org/10.1111/1467-9280.00427

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307–314. http://dx.doi.org/10.1016/j.tics.2004.05.002

Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., & Bryant, J. D. (2010a). The contributions of numerosity and domain-general abilities to school readiness. *Child Development, 81*, 1520–1533. http://dx.doi.org/10.1111/j.1467-8624.2010.01489.x

Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., Seethaler, P. M., . . . Schatschneider, C. (2010b). Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? *Developmental Psychology, 46*, 1731–1746. http://dx.doi.org/10.1037/a0020662

Fuchs, M. W., & McNeil, N. M. (2013). ACSs and maths ability in preschoolers from low-income homes: Contributions of inhibitory control. *Developmental Science, 16*, 136–148. http://dx.doi.org/10.1111/desc.12013

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology, 47*, 1539–1552. http://dx.doi.org/10.1037/a0025510

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2013). Adolescents’ functional numeracy is predicted by their school entry number system knowledge. *PLoS ONE, 8*, e54651. http://dx.doi.org/10.1371/journal.pone.0054651
Holloway, I. D., Battista, C., Vogel, S. E., & Ansari, D. (2013). Semantic
Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto
Haworth, C. M. A., Davis, O. S., & Plomin, R. (2013). Twins Early
Hannula, M. S. (2003). Locating fraction on a number line. In N. A.
Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2011b). Preschoolers’
Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N.,
Gelman, R., & Gallistel, C. R. (1978). The child’s understanding of
Gordon, P. (2004). Numerical cognition without words: Evidence from
GOAL. (2002).
gordian, C. M. A., Harlaar, N., Kovas, Y., Haworth, C. M., Petrill, S. A., & Plomin, R. (2007). Mathematics performance.
Koval, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. Developmental Psychology, 48, 1229–1241.
Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the “number sense”. The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. Developmental Psychology, 44, 1457–1465.
Halberda, J., Ly, R., Wilmer, J. B., Naiman, D. Q., & Germine, L. (2012). Number sense across the lifespan as revealed by a massive Internet-based sample. Proceedings of the National Academy of Sciences of the United States of America, 109, 11116–1120. http://dx.doi.org/10.1073/pnas.1200196109
Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with mathematics achievement. PLoS ONE, 8(6), e67374. http://dx.doi.org/10.1371/journal.pone.0067374
Gilmore, C., McCarthy, S. E., & Spelke, E. S. (2010). Non-symbolic arithmetic abilities and mathematics achievement in the first year of formal schooling. Cognition, 115, 394–406. http://dx.doi.org/10.1016/j.cognition.2010.02.002
GOAL. (2002). GOAL Formative Assessment: Key Stage 3. London, England: Hodder & Stoughton.
Gordon, P. (2004). Numerical cognition without words: Evidence from Amazonia. Science, 306, 496–499. http://dx.doi.org/10.1126/science.1094492
Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. Developmental Psychology, 48, 1229–1241.
Hamill, D. D., Brown, V. L., Larsen, S. C., & Wiederholt, J. L. (1994). Test of adolescent and adult language. Austin, TX: Pro-Ed.
Hannula, M. S. (2003). Locating fraction on a number line. In N. A. Pateman, B. J. Dougherty, & J. Zilliox (Eds.), In Proceedings of the 27th conference of the international group for the psychology of mathematics education (Vol. 3, pp. 17–24). Berlin, Germany: International Group for the Psychology of Mathematics Education.
Haworth, C. M. A., Davis, O. S., & Plomin, R. (2013). Twins Early Development Study (TEDS): A genetically sensitive investigation of cognitive and behavioral development from childhood to young adulthood. Twin Research and Human Genetics, 16, 117–125. http://dx.doi.org/10.1017/thg.2012.91
Haworth, C. M., Harlaar, N., Kovas, Y., Davis, O. S., Oliver, B. R., Hayiou-Thomas, M. E., . . . Plomin, R. (2007). Internet cognitive testing of large samples needed in genetic research. Twin Research and Human Genetics, 10, 554–563. http://dx.doi.org/10.1375/twin.10.4.554
Hitch, J. G., & McAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. British Journal of Psychology, 82, 375–386. http://doi.org/10.1111/j.2044-8295.1991.tb02406.x
Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto
Holloway, I. D., Battista, C., Vogel, S. E., & Ansari, D. (2013). Semantic
linguistic fMRI adaptation study. Journal of Cognitive Neuroscience, 25, 388–400. http://dx.doi.org/10.1162/jocn_a_00323
Holloway, I. D., Price, G. R., & Ansari, D. (2010). Common and segregated neural pathways for the processing of symbolic and nonsymbolic numerical magnitude: An fMRI study. Neuroimage, 49, 1006–1017. http://dx.doi.org/10.1016/j.neuroimage.2009.07.071
Ito, Y., & Hatta, T. (2004). Spatial structure of quantitative representation of numbers: Evidence from the SNARC effect. Memory & Cognition, 32, 662–673. http://dx.doi.org/10.3758/BF03195857
Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. Developmental Psychology, 45, 850–867. http://dx.doi.org/10.1037/a0014939
Kaplan, E., Fein, D., Kramer, J., Delis, D., & Morris, R. (1999). The WISC-III as a process instrument. San Antonio, TX: The Psychological Corporation.
Keller, L., & Liburtus, M. (2015). Inhibitory control may not explain the link between approximation and math abilities in kindergartners from middle class families. Frontiers in Psychology, 6, 685. http://dx.doi.org/10.3389/fpsyg.2015.00685
Koenker, R. (2016). Quantreg: Quantile regression (R package version 5.29) [Computer software]. Available from http://www.r-project.org
Kolkman, M. E., Kroesbergen, E. H., & Leseman, P. P. (2013). Early numerical development and the role of non-symbolic and symbolic skills. Learning and Instruction, 25, 95–103. http://dx.doi.org/10.1016/j.learninstruc.2012.12.001
Kovas, Y., Harlaar, N., Petrill, S. A., & Plomin, R. (2005). “Generalizer genes” and mathematics in 7-year-old twins. Intelligence, 33, 473–489. http://dx.doi.org/10.1016/j.intell.2005.05.002
Kovas, Y., Haworth, C. M., Petrill, S. A., & Plomin, R. (2007). Mathematical ability of 10-year-old boys and girls: Genetic and environmental etiology of typical and low performance. Journal of Learning Disabilities, 40, 554–567. http://dx.doi.org/10.1177/0022219407040060601
LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamwar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. Child Development, 81, 1753–1767. http://dx.doi.org/10.1111/j.1467-8624.2010.01508.x
Liberts, M. E., & Brannon, E. M. (2010). Stable individual differences in number discrimination in infancy. Developmental Science, 13, 900–906. http://dx.doi.org/10.1111/j.1467-7687.2009.00948.x
Liberts, M. E., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. Developmental Science, 14, 1292–1300. http://dx.doi.org/10.1111/j.1467-6787.2011.01080.x
Lyons, I. M., Ansari, D., & Beilock, S. L. (2012). Symbolic estrangement: Evidence against a strong association between numerical symbols and the quantities they represent. Journal of Experimental Psychology: General, 141, 635–641. http://dx.doi.org/10.1037/a0027248
Markwardt, F. C. (1997). Peabody Individual Achievement Test—Revised/Normative Update manual. Circle Pines, MN: American Guidance Service.
Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011a). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). Child Development, 82, 1224–1237. http://dx.doi.org/10.1111/j.1467-8624.2011.01608.x
Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011b). Preschoolers’ precision of the approximate number system predicts later school mathematics performance. PLoS ONE, 6, e23749. http://dx.doi.org/10.1371/journal.pone.0023749
McCarthy, D. (1972). Manual for the McCarthy Scales of Children’s Abilities, New York, NY: Psychological Corporation.
McLean, J. F., & Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. Journal of Experimental Psychology: Learning, Memory, and Cognition, 25, 1099–1109. http://dx.doi.org/10.2327/117223
inental Child Psychology, 74, 240–260. http://dx.doi.org/10.1006/jecp.1999.2516
Moeller, K., Pixon, S., Kaufmann, L., & Nuerk, H.-C. (2009). Children’s early mental number line: Logarithmic or decomposed linear? Journal of Experimental Child Psychology, 103, 503–515. http://dx.doi.org/10.1016/j.jecp.2009.02.006
Moyer, R. S., & Landauer, T. K. (1967). Time required for Judgements of Numerical Inequality. Nature, 215, 1519–1520. http://dx.doi.org/10.1038/2151519a0
Murphy, M. M., & Mazzocco, M. M. (2008). Mathematics learning disabilities in girls with fragile X or Turner syndrome during late elementary school. Journal of Learning Disabilities, 41, 29–46. http://dx.doi.org/10.1177/0022219407311038
NferNelson. (1994). Maths 5–14 series. London, England: Author.
NferNelson. (1999). Maths 5–14 series. London, England: Author.
NferNelson. (2001). Maths 5–14 series. London, England: Author.
Nieder, A., & Dehaene, S. (2009). Representation of number in the brain. Annual Review of Neuroscience, 32, 185–208. http://dx.doi.org/10.1146/annurev.neuro.051508.135550
Nieder, A., & Merten, K. (2007). A labeled-code line for small and large numerosities in the monkey prefrontal cortex. The Journal of Neuroscience, 27, 5986–5993. http://dx.doi.org/10.1523/JNEUROSCI.1056-07.2007
Nys, J., & Content, A. (2012). Judgement of discrete and continuous quantity in adults: Number counts! The Quarterly Journal of Experimental Psychology, 65, 675–690. http://dx.doi.org/10.1080/17470218.2011.619661
Nys, J., Ventura, P., Fernandez, T., Querido, L., Leybaert, J., & Content, A. (2013). Does math education modify the approximate number system? A comparison of schooled and un schooled adults. Trends in Neuroscience and Education, 2, 13–22. http://dx.doi.org/10.1016/j.tine.2013.01.001
Oliver, B. R., & Plomin, R. (2007). Twins’ Early Development Study (TEDS): A multivariate, longitudinal genetic investigation of language, cognition and behavior problems from childhood through adolescence. Twin Research and Human Genetics, 10, 96–105. http://dx.doi.org/10.1375/twin.10.1.96
Opfer, J. E., & Siegler, R. S. (2007). Representational change and children’s numerical estimation. Cognitive Psychology, 55, 169–195. http://dx.doi.org/10.1016/j.cogpsych.2006.09.002
Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. Neuron, 44, 547–555. http://dx.doi.org/10.1016/j.neuron.2004.10.014
Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approxi mate arithmetic in an Amazonian indigene group. Science, 306, 499–503. http://dx.doi.org/10.1126/science.1102085
Pinel, P., Dehaene, S., Rivière, D., & LeBihan, D. (2001). Modulation of parietal activation by semantic distance in a number comparison task. NeuroImage, 14, 1013–1026. http://dx.doi.org/10.1006/nimg.2001.0913
Räsänen, P., Savainen, J., Wilson, A. J., Aunio, P., & Dehaene, S. (2009). Computer-assisted intervention for children with low numeracy skills. Cognitive Development, 24, 450–472. http://dx.doi.org/10.1016/j.cogdev.2009.09.003
Raven, J. C., Court, J. H., & Raven, J. (1996). Manual for Raven’s Progressive Matrices and Vocabulary Scales. New York, NY: Oxford University Press.
Raven, J., Raven, J. C., & Court, J. H. (1998). Mill Hill Vocabulary Scale. Oxford, England: Oxford Psychologists Press.
R Core Team. (2014). R: A language and environment for statistical computing [Computer software]. Vienna, Austria: R Foundation for Statistical Computing. Retrieved from http://www.R-project.org/
Restle, F. (1970). Speed of adding and comparing numbers. Journal of Experimental Psychology, 83, 274. http://dx.doi.org/10.1037/h0028573
Rousseau, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. Cognition, 102, 361–395. http://dx.doi.org/10.1016/j.cognition.2006.01.005
Sasanguie, D., Defever, E., Maertens, B., & Reynvoet, B. (2014). The approximate number system is not predictive for symbolic number processing in kindergarteners. The Quarterly Journal of Experimental Psychology, 67, 271–280. http://dx.doi.org/10.1080/17470218.2013.803581
Schneider, M., Beeres, K., Coban, L., Merz, S., Schmidt, S. S., Stricker, J., & De Smetd, B. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. Developmental Science, 20, e12372.
Siegel, L. S., & Ryan, E. B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. Child Development, 60, 973–980. http://dx.doi.org/10.1111/j.1467-8624.2004.00684.x
Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. Child Development, 75, 428–444. http://dx.doi.org/10.1111/j.1467-8624.2004.00684.x
Siegler, R. S., & Mu, Y. (2008). Chinese children excel on novel mathematics problems even before elementary school. Psychological Science, 19, 759–763. http://dx.doi.org/10.1111/j.1467-9280.2008.02153.x
Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. Psychological Science, 14, 237–250. http://dx.doi.org/10.1111/1467-9280.2003.02438
Smith, P., Fernandes, C., & Strand, S. (2001). Cognitive Abilities Test 3 (CAT3). Windsor, England: NferNelson.
Smith, P., & Lord, T. (2002). Spatial reasoning 6–14 series, a teacher’s guide. London, England: NferNelson.
Soltesz, F., Szucs, D., & Szucs, L. (2010). Relationships between magnitude representation, counting and memory in 4- to 7-year-old children: A developmental study. Behavioral and Brain Functions, 6, 13. http://dx.doi.org/10.1186/1744-9081-6-13
Speelke, E. S. (2005). Sex differences in intrinsic aptitude for mathematics and science?: A critical review. American Psychologist, 60, 950–958. http://dx.doi.org/10.1037/0003-066x.60.9.950
Swanson, H. L., & Sachse-Lee, C. (2001). Mathematical problem solving and working memory in children with learning disabilities: Both executive and phonological processes are important. Journal of Experimental Child Psychology, 79, 294–321. http://dx.doi.org/10.1006/jecp.2000.2587
Thompson, C. A., & Opfer, J. E. (2008). Costs and benefits of representational change: Effects of context on age and sex differences in symbolic magnitude estimation. Journal of Experimental Child Psychology, 101, 20–51. http://dx.doi.org/10.1016/j.jecp.2008.02.003
Toll, S. W., Van Viersen, S., Kroesbergen, E. H., & Van Luit, J. E. (2015). The development of (non-) symbolic comparison skills throughout kindergarten and their relations with basic mathematical skills. Learning and Individual Differences, 38, 10–17. http://dx.doi.org/10.1016/j.lindif.2014.12.006
Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (1999). Test of Word Reading Efficiency (TOWRE). Austin, TX: Pro-Ed.
Tosto, M. G., Asbury, K., Mazzocco, M. M. M., Petrill, S. A., & Kovas, Y. (2016). From classroom environment to mathematics achievement: The mediating role of self-perceived ability and subject interest. Learning and Individual Differences, 50, 260–269. http://dx.doi.org/10.1016/j.lindif.2016.07.009
Tosto, M. G., Tikhomirova, T., Galajinsky, E., Akimova, K., & Kovas, Y. (2013). Development and validation of a mathematics-number sense Web-based test battery. Procedia: Social and Behavioral Sciences, 86, 423–428. http://dx.doi.org/10.1016/j.sbspro.2013.08.591
van Marle, K., Chu, F. W., Li, Y., & Geary, D. C. (2014). Acuity of the approximate number system and preschoolers’ quantitative develop-
Wechsler, D. (1992). Wechsler Intelligence Scale for Children (3rd ed.). New York, NY: The Psychological Corporation.

Wei, W., Chen, C., & Zhou, X. (2016). Spatial ability explains the male advantage in approximate arithmetic. Frontiers in Psychology, 7, 306. http://dx.doi.org/10.3389/fpsyg.2016.00306

Whalen, J., Gallistel, C., & Gelman, R. (1999). Nonverbal counting in humans: The psychophysics of number representation. Psychological Science, 10, 130–137. http://dx.doi.org/10.1111/1467-9280.00120

Wiig, E. H., Secord, W., & Sabers, D. (1989). Test of Language Competence: Expanded Edition. San Antonio, TX: Psychological Corporation.

Woodcock, R., McGrew, K., & Mather, N. (2001). Woodcock–Johnson III (WJ-III). Itasca, IL: Riverside.

Received February 10, 2016
Revision received January 16, 2017
Accepted March 7, 2017

Call for Nominations

The Publications and Communications (P&C) Board of the American Psychological Association has opened nominations for the editorships of the Journal of Experimental Psychology: Animal Learning and Cognition, Neuropsychology, and Psychological Methods for the years 2020 to 2025. Ralph R. Miller, PhD, Gregory G. Brown, PhD, and Lisa L. Harlow, PhD, respectively, are the incumbent editors.

Candidates should be members of APA and should be available to start receiving manuscripts in early 2019 to prepare for issues published in 2020. Please note that the P&C Board encourages participation by members of underrepresented groups in the publication process and would particularly welcome such nominees. Self-nominations are also encouraged.

Search chairs have been appointed as follows:

- Journal of Experimental Psychology: Animal Learning and Cognition, Chair: Stevan E. Hobfoll, PhD
- Neuropsychology, Chair: Stephen M. Rao, PhD
- Psychological Methods, Chair: Mark B. Sobell, PhD

Candidates should be nominated by accessing APA’s EditorQuest site on the Web. Using your browser, go to https://editorquest.apa.org. On the Home menu on the left, find “Guests/Supporters.” Next, click on the link “Submit a Nomination,” enter your nominee’s information, and click “Submit.”

Prepared statements of one page or less in support of a nominee can also be submitted by e-mail to Sarah Wiederkehr, P&C Board Editor Search Liaison, at swiederkehr@apa.org.

Deadline for accepting nominations is Monday, January 8, 2018, after which phase one vetting will begin.