Velocity effects on an accelerated Unruh–DeWitt detector

Shohreh Abdolrahimi

Institut für Physik, Universität Oldenburg, Postfach 2503 D-26111 Oldenburg, Germany

E-mail: shohreh.abdolrahimi@uni-oldenburg.de

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Abstract
We analyze the response of an Unruh-DeWitt detector moving along an unbounded spatial trajectory in a two-dimensional spatial plane with constant independent magnitudes of both the four-acceleration and of a timelike proper time derivative of the four-accelration. In a Fermi-Walker frame moving with the detector, the direction of the acceleration rotates at a constant rate around a great circle. This is the motion of a charge in a uniform electric field when in the frame of the charge there is both an electric and a magnetic field. We compare the response of this detector to a detector moving with constant velocity in a thermal bath of the corresponding temperature for non-relativistic velocities and two regimes: ultraviolet and infrared. In the infrared regime, the detector in the Minkowski space-time moving along the spatially two-dimensional trajectory should move with a higher speed to keep up with the same excitation rate of the inertial detector in a thermal bath. In the ultraviolet regime, the dominant modification in the response of this detector compared with the blackbody spectrum of Unruh radiation is the same as the dominant modification perceived by a detector moving with constant velocity in a thermal bath.

Keywords: quantum field theory, moving detector, blackbody radiation

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1. Introduction

A uniformly accelerated observer in Minkowski spacetime, i.e., a linearly accelerated observer with constant proper acceleration, associates a thermal bath of Rindler particles to the no-particle state of inertial observers. This is the Unruh effect [1–3]. It implies the conceptually important result that the particle content of a field theory is observer dependent. The Unruh effect is important in its own right, perhaps having experimental applications.
relevant for particle accelerators [4, 6, 23, 24], electrons in Penning traps [8, 26], atoms in microwave cavities [9, 10], or hadronic collisions [11–13] and as a tool to investigate other phenomena such as the thermal emission of particles from black holes [14, 15] and cosmological horizons [16]. For a review of the Unruh effect and applications see [17]. Recently, Martín-Martínez, Fuentes, and Mann have shown [18] that a detector acquires a Berry phase as a result of its motion in spacetime, and this fact can be used for the direct detection of the Unruh effect in regimes physically accessible with current technology.

Unruh has introduced a detector model consisting of a small box containing a non-relativistic particle satisfying the Schrödinger equation [1]. The system is said to have detected a quanta if the particle in the box jumps from the ground state to some excited state. DeWitt [19] has introduced a detector which consists of a two-level point monopole. In this paper we use an idealized point detector with internal energy levels labelled by energy $E_0$ and $E > E_0$, coupled via a monopole interaction with a scalar field $\phi$, known as an Unruh-DeWitt detector in the literature.

An eternal uniformly accelerated Unruh-DeWitt detector, moving along a linear spatial trajectory in the Minkowski vacuum with constant magnitude of its four-acceleration $a$, perceives a radiation rate which is equivalent to that perceived by a detector at rest in a thermal bath of Minkowski particles at the temperature $T = \hbar a (2 \pi c k_b)$, where $h$ is the reduced Planck constant, $c$ is the speed of light, and $k_b$ is the Boltzmann constant. This detector will experience a time-independent situation and hence settle in a stationary state. However, for other time-like curves the geodesic interval between two points along the curve depends only on the proper time interval. Letaw [20] has classified these so-called stationary curves into six categories. The vacuum excitation spectra of detectors on a representative sample of such stationary world lines have been calculated, and some of which were presented only numerically.

The corresponding vacuum states have also been classified. Letaw and Pfautsch [21] showed that the corresponding vacuum states are found to be restricted to two possibilities: those in coordinate systems without event horizons are the Minkowski vacuum; those in coordinate systems with event horizons are the Fulling vacuum.

The analog of the Unruh effect for spatially circular trajectories has been discussed in particular with relation to polarization effects of electrons in storage rings and for electrons circulating in a cavity [22–29]. Gutti, Kulkarni, and Srimukumar have shown that the response of the rotating detector can be computed exactly (albeit numerically) even when coupled to a field governed by a nonlinear dispersion relation [30]. Korsbakkena and Leinaasa [31] related the excitation spectrum of a detector moving along planar stationary trajectories to the properties of the Minkowski vacuum in the accelerated frame and defined an effective temperature in terms of the transition rate of a detector into up or down states. Barbado and Visser analyzed the response function of an Unruh-DeWitt detector moving with time-dependent acceleration along a one-dimensional trajectory in Minkowski spacetime [34].

In this paper, we consider a special case of a stationary trajectory. We consider a detector moving along an unbounded spatial trajectory in a two-dimensional spatial plane with constant independent magnitudes of both the four-acceleration and of a timelike proper time derivative of four-accelration, such that in a Fermi-Walker frame moving with the detector, the direction of the acceleration rotates at a constant rate around a great circle. This is the motion of a charge in a uniform electric field when in both an electric and a magnetic field are within the frame of the charge.

We choose a special coordinate for describing the motion, in which one of the components of the 4-velocity, $w = dy/d\tau$ (\(\tau\) is the proper-time of the detector) is constant. We
calculate explicitly the response of an Unruh-DeWitt detector moving along such a trajectory in the non-relativistic limit. In the non-relativistic limit, the zero order term is of course the thermal spectrum of Unruh radiation, but we are interested to find the next dominant term in the response of the detector, proportional to the square of the four-velocity component $w^2$.

We also consider a detector moving with constant non-relativistic speed $\tilde{w}$ in a thermal bath of temperature corresponding to the Unruh temperature $T = \hbar a/(2\pi c k_B)$. The first dominant term in the response of this detector is the Planckian spectrum of a thermal bath. We find the next dominant term in the response of this detector to be proportional to $\tilde{w}^2$ and compare this dominant term to the one we find from the accelerating detector moving in Minkowski space-time in two regimes, which we call ultraviolet and infrared.

Now that we have reviewed the definitions of the physical quantities involved in the description of an Unruh-DeWitt detector we continue in Sec. II with a description of a trajectory of the detector and calculate the response of an Unruh-DeWitt detector following the described trajectory. We compare the response of this Unruh-DeWitt detector with that of a uniformly accelerated detector ($w = 0$) moving along a spatially straight line and also with that of a detector moving with constant velocity in a thermal bath. We use the system of units where $\hbar = c = k_B = 1$.

2. The detector

Suppose we have a pointlike two-level system (detector) moving along a worldline described by the functions $x^\mu (\tau) = (t(\tau), x(\tau))$, where $\tau$ is the detector’s proper time and $\mu$ labels the coordinates in space-time. Assume that this two-level system has two internal energy levels labelled identified as the energy $E_0$ and $E > E_0$ and is coupled to a quantum scalar field $\varphi$ via a monopole interaction, $V = m q (\tau) \varphi [x(\tau)]$, where $q (\tau)$ is the monopole moment operator [1, 19, 32, 33] and $m$ is the interaction constant. Then the system, i.e., the two-level detector, and the quantum field are described by the following Hamiltonian:

$$\hat{H} = \hat{H}_0^{(o)} + \hat{H}_0^{(f)} + \hat{V}, \tag{1}$$

where $\hat{H}_0^{(o)}$ is the Hamiltonian of the free two-level system, $\hat{H}_0^{(f)}$ is the Hamiltonian of the free quantum scalar field, and $\hat{V}$ defines the interaction. Assume that $|A\rangle$’s are the eigenvectors of the orthonormal basis of the Hilbert space of the states of the system without interaction,

$$\hat{H}_0^{(o)} |A\rangle = E_0^{(o)} |A\rangle, \quad \hat{H}_0^{(f)} |A\rangle = \hat{H}_0^{(o)} + \hat{V}_0^{(f)}. \tag{2}$$

For a general trajectory, the system of the two-level detector and the field will not always remain in its ground state $E_0^{(o)}$ but will instead undergo a transition to an excited state $E_0^{(f)} > E_0^{(o)}$. If we assume that the interaction constant $m$ is small, in the first-order approximation of the perturbation theory the probability amplitude of the transition from the initial state $|A\rangle$ to the final state $|B\rangle$ at the proper time $\tau$ is given by

$$\mathcal{A}_{BA} = -i \, m \int_{-\infty}^{\tau} d\tau^* V_{BA}(\tau^*), \tag{3}$$

1 When the acceleration is set to zero this detector corresponds to one moving with constant velocity in the Minkowski vacuum; such a detector perceives no temperature.
Let the states $|n\rangle$ and $|N\rangle$ be the eigenstates of the non-interacting free Hamiltonian of the two-level detector and the non-interacting free Hamiltonian of the free field,

$$\hat{H}_0^{(n)} |n\rangle = E_n |n\rangle, \quad \hat{H}_0^{(N)} |N\rangle = \omega_N |N\rangle,$$

respectively. Then the states $|A\rangle$ of the free system of the two-level detector and field can be written as

$$|A\rangle = |n\rangle |N\rangle.$$

The two-level detector is either in the ground state $|0\rangle$ with energy $E_0$ or in the excited state $|1\rangle$ with energy $E$. The probability amplitude of the transition (3) can be derived by using (4), which in the basis (6) can be written as

$$V_{BA} = V_{mn, NO} = q_{mn} e^{i(E_m - E_0)\tau} \langle M | \hat{\phi}[x(\tau)] | N \rangle,$$

where $q_{mn} = \langle m | q(0) | n\rangle$. Suppose that the field $\phi$ is initially in vacuum state $|0_M\rangle$, where the subscript $M$ stands for Minkowski vacuum, and the two-level system is in ground state $E_0$. Let us consider mental copies of the above two-level Unruh-DeWitt detector, where these copies differ only in the value of their second energy level $E$. Assume that all of these detectors are prepared in the same initial state and following identical trajectories (see [34] for a discussion about physical construction of such a system of detectors). The transition probability to all possible $|M\rangle$ and $|1\rangle$’s (of different value of energy $E$) for this ensemble of detectors is

$$p_{\text{total}} = m^2 \sum_k q_{mn} \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dt'' e^{i(E_m - E_0)\Delta t}$$

$$\times G^+(x(t'), x(t'')),$$

where $G^+$ is the positive frequency Wightman function,

$$G^+(x(t'), x(t'')) = \langle 0 | \hat{\phi}[x(t')] \hat{\phi}[x(t'')] | 0 \rangle,$$

which for massless scalar field reads

$$G^+(x(t'), x(t'')) = \frac{1}{4\pi^2 \left[(t'' - t' - it)^2 - |x' - x'|^2\right]},$$

where $\epsilon \ll 0$. Note that $(t', x')$ and $(t'', x'')$ are functions of the proper time. If $G^+(x(t'), x(t''))$ can be written as $G^+(\Delta t, r)$, where $r = |x' - x'|$ and $\Delta t = t'' - t'$. Here, we have used the conventional representation of the Wightman function [32].

Calculations of the Unruh effect based on idealized particle detectors are usually made in a way that involves integrations along the entire detector trajectory up to the infinitely remote future. This is the case we consider here. Improper treatment of the $ie$ regulator (the standard regularization $t \rightarrow t - ie$ of the correlation function) can lead to unphysical results in some cases. For example mishandling of the regularization procedure may lead to an apparently time-dependent transition rate on a stationary trajectory ([35] and the appendix of [36]). However, analysis of [37] obtains a regulator-free integral formula for the total excitation probability by using smooth switching functions to turn the interaction on and off and shows that an instantaneous transition rate can be recovered in a suitable limit; see also [38, 39] 2.
We do not discuss these interesting issues in this paper, but we stress that our method of calculation reaches the same results that would be obtained from the one in these papers, and does not introduce the spurious time-dependence discovered in [35].

In our experimental case in which the integrand in (8) depends only on $\Delta \tau$ and we can write (8) in the following form:

$$
\int \int \sum_{k} q_{k} \int_{-\infty}^{\infty} d \Delta \tau e^{i(E-E_{0})\Delta \tau} G^{\pm}(\Delta \tau).
$$

The transition probability per unit proper time is

$$
\frac{d\rho_{\text{Minkowski}}}{d\tau} = m^{2} \sum_{k} |q_{k}|^{2} \mathcal{F}(E),
$$

where

$$
\mathcal{F}(E) = \int_{-\infty}^{\infty} d \Delta \tau e^{i(E-E_{0})\Delta \tau} G^{\pm}(\Delta \tau),
$$

is the response function per unit proper time and is independent of the detailed structure of the detector. If the quantum scalar field is initially in the thermal state rather than the Minkowski vacuum state, then the response function $\mathcal{F}$ has to be replaced by

$$
\mathcal{F}_{\text{th}}(E) = \int_{-\infty}^{\infty} d \Delta \tau e^{i(E-E_{0})\Delta \tau} G_{\text{th}}^{\pm}(\Delta \tau),
$$

where $G_{\text{th}}^{\pm}$ is the Wightman thermal Green function, which for the case of massless scalar field is [33]

$$
G_{\text{th}}^{\pm}(\Delta \tau, r) = G^{\pm}(\Delta \tau, r) + \frac{1}{4\pi^{2} \left( \Delta \tau^{2} - r^{2} \right)},
$$

$$
\frac{1}{8\pi \beta r} \left[ \coth \left( \pi (r + \Delta \tau)/\beta \right) + \coth \left( \pi (r - \Delta \tau)/\beta \right) \right],
$$

where $\beta = 1/T$ is the inverse temperature and $\Delta \tau = t' - t$. In what follows we shall consider a two-level Unruh-DeWitt detector as described in this section. The response function of this detector per unit proper time can be calculated by using (13) if the detector is moving in Minkowski vacuum or by using (14) if the detector is coupled to the thermal quantum scalar field.

3. Motion of the detector

Consider an Unruh-DeWitt detector as explained in the previous section moving in Minkowski space-time along an unbounded spatial trajectory in a two-dimensional spatial plane with a constant square of magnitude of four-acceleration $\alpha, \alpha' = a^{2}$, where $a'' = d^{2}x^{\mu}/d\tau^{2}$, and constant magnitude of a timelike proper-time derivative of four-acceleration $(da/d\tau)(da'/d\tau)$ and having as a component of the four-velocity, $dy/d\tau = w$, a constant, namely a detector moving along the following worldline

2 Note that one can consider a smooth switching function $\chi(\tau)$ of compact support vanishing before the interaction and positive during the interaction. In that case, $V = m \phi(\tau)f(\tau)\chi(\tau)$. This smooth switching function has been shown to cure the non-Lorentz invariance of the conventional $\epsilon$ regularization of the correlation function [39].
\[ x^a(\tau) = \left( \frac{a}{\alpha^2} \sinh (\alpha \tau), \frac{a}{\alpha^2} \cosh (\alpha \tau), \omega \tau, 0 \right), \quad (16) \]

\[ \alpha = \frac{a}{\sqrt{1 + \omega^2}} > 0, \quad (17) \]

where \( x^a = (t, x, y, z) \) are the Minkowski coordinates. This is the motion of a charge \( (e/m = 1) \) in a uniform electric field when both an electric and a magnetic field are within the frame of the charge. Motion of this charge in a frame where the electric field is in \( x \) direction, \( E = (E, 0, 0) \), and the magnetic field is zero is described by

\[ x^a(\tau) = \left( b \sinh (E \tau) + b v_{yo} \cosh (E \tau) - b v_{yo} + t_0 \right), \]

\[ b \cosh (E \tau) + b v_{yo} \sinh (E \tau) - b + x_0, \]

\[ b E v_{yo} \tau + y_0, \]

\[ b E v_{yo} \tau + z_0, \quad (18) \]

where \( b := \gamma_0/E, \)

\[ \gamma_0 = \left( 1 - v^2_{yo} - v^2_{xo} - v^2_{z0} \right)^{-1/2}. \quad (19) \]

Here, \( v_0 = (v_{xo}, v_{yo}, v_{z0}) \) is the initial velocity, and \( x_0 = (x_0, y_0, z_0) \) is the initial position of the particle. Trajectory (16) is a special case of (18), with \( v_0 = (0, v_{yo}, 0), x_0 = (0, 0, 0), \) and \( t_0 = 0. \) The trajectory (16) is related to (18) by

\[ a = bE^2, \quad \omega = bE v_{yo}. \quad (20) \]

The magnitude of the Fermi-Walker derivative of the acceleration for the motion (16) is

\[ |D_a| = \left( D^\mu_{\nu} \left[ a \right] D^{\mu\nu} \left[ a \right] \right)^{1/4} = \frac{a^2 w}{\sqrt{1 + \omega^2}}. \quad (21) \]

The parameter \( \eta = |D_a|/a^2 \) is less than one. In a Fermi-Walker frame moving with the detector, the direction of the acceleration rotates at a constant rate \( \eta \) around a great circle. In the general case for the trajectory (18), \( \eta \) is given by

\[ \eta = \left( \frac{P}{1 + P} \right)^{1/2}, \quad P := \gamma_0^2 \left( v^2_{xo} + v^2_{z0} \right). \quad (22) \]

For a circular trajectory rather than (16), which gives \( \eta > 1, \) one needs to replace the uniform electric field with a uniform magnetic field and take the charge moving so that both a magnetic and an electric field are in the frame of the charge.

The trajectory (16) is one of the trajectories considered in [20]. They calculate the spectrum numerically, showing the analytical behavior for large values of \( w \). In this paper we consider analytic analysis of the response function of the detector for \( w \ll 1 \). Note that they use the following definition of the Wightman function

\[ G^+(\tau, \tau') = \left[ x_0(\tau) - x_0(\tau') \right] \left[ x^0(\tau) - x^0(\tau') \right]. \quad (23) \]

For \( w = 0, \) the trajectory is

\[ x^a = x^a(\tau) = \left( a^{-1} \sinh (\alpha \tau), a^{-1} \cosh (\alpha \tau), 0, 0 \right), \quad (24) \]

which is a trajectory of a detector moving along a spatially straight line along the \( x \) direction with constant magnitude of four-acceleration \( a^a a^a = a^2. \)
Note that if instead of the trajectory (16), with a component of the four-velocity \( w = dy/dt = const. \), we have considered the component of the three-velocity \( d\gamma/d\tau = v \) to be constant, the response of the detector would have been completely equivalent to that of a detector which is moving along a spatially straight line with constant magnitude of four-acceleration, (24), as such observers can be related to the ones moving along the trajectory (24) by Lorentz transformations.

3.1. Motion of the detector in the Minkowski vacuum

For the trajectory (16), the positive frequency Wightman Green function (10) for a massless scalar field reads

\[
G^+(\Delta\tau) = -\frac{\alpha^2}{16\pi^2 a^2} \left[ \sinh^2 \left( \frac{a\Delta\tau}{2} - i\frac{\epsilon\alpha^2}{a} \right) - \frac{w^2\alpha^4}{4a^2} \Delta\tau^2 \right]^{-1},
\]  

(25)

Here, we have absorbed a positive function of \( \tau \) and \( \tau' \), \([\sinh(\alpha\tau) - \sinh(\alpha\tau')] / [\sinh(\Delta\tau/2\alpha) \cosh(\Delta\tau/2\alpha)] \), into \( \epsilon \). With this redefinition of \( \epsilon \), i.e., ‘regularization of \( \epsilon \) term’, the Wightman Green function (25) depends only on \( \Delta\tau \).

Note that for \( w = 0 \) we have \( \alpha = \alpha \) and (25) immediately converts to the positive Wightman Green function of a detector moving along a spatially straight line in the direction (24), with constant magnitude of the four-acceleration (spatially one-dimensional)

\[
G^{+(1d)}(\Delta\tau) = -\frac{\alpha^2}{16\pi^2} \left[ \sinh^2 \left( \frac{a\Delta\tau}{2} - i\alpha \right) \right]^{-1}.
\]  

(26)

Here and in what follows the \( (1d) \) index is used to distinguish the quantities such as the Wightman Green function or the response function calculated for the spatially one-dimensional trajectory (24) as opposed to the index \( (2d) \) for the quantities associated with the spatially two-dimensional trajectory (16). The transition probability per unit proper time (12) for the detector following trajectory (24) is

\[
p_{\Delta\tau} = m^2 \sum \epsilon_q \lfloor 2F^{-(1d)}(E) \rfloor,
\]  

(27)

\[
F^{-(1d)}(E) = \frac{\Delta E}{2\pi e^{\Delta E/\Delta \tau} - 1},
\]  

(28)

where \( \Delta E = E - E_0 \). This is the usual blackbody excitation rate, indicating that the excitation rate of an accelerated detector coupled to the field \( \phi \) in the state \( |0_{\mu} \rangle \) is the same as that of a detector, unaccelerated, at rest in a bath of thermal radiation at temperature \( T = 1/\beta = a/(2\pi) \).

In the ‘infrared limit’ \( \Delta E \ll 1 \), the blackbody excitation rate (28) has the following dominant behavior:

\[
F^{-(1d)}(E) \sim \frac{1}{2\pi \beta}.
\]  

(29)

In the ‘ultraviolet limit’ \( \Delta E \gg 1 \), the blackbody excitation rate (28) has the following dominant behavior:

\[
F^{-(1d)}(E) \sim \frac{\Delta E}{2\pi} e^{-\Delta E}.\]

(30)

We calculate the response function per unit proper time (12) of the detector following the trajectory (16) for non-relativistic velocities \( \gamma \ll 1 \)
\[ v_\gamma = \frac{dy}{dt} = \frac{w}{\sqrt{1 + w^2 \cosh (\alpha \tau)}}. \]  

or \( w \ll 1 \). Note that here we are not considering the ultra-relativistic limit because for ultra-relativistic velocities \( v_\gamma \to 1 \) or \( w \to \infty \) the response function of the detector (16) is suppressed; that is, the Unruh effect is suppressed as

\[ \mathcal{F}^{(\omega)}(E) \sim \frac{\Delta E}{2\pi} \left[ e^{\beta \Delta E/\hbar} - 1 \right] \frac{1}{w^2}. \]  

The Wightman Green function (25) in the leading order for \( w \ll 1 \) reads

\[ G^{(2d)}(\Delta \tau) = (1 - 2w^2) G^{(1d)}(\Delta \tau) - \frac{a^2}{16\pi^2} \left( \sinh (a\Delta \tau - 2i\alpha) \left( \frac{a\Delta \tau}{4} - i\alpha \right) + \frac{a^2\Delta \tau^2}{4} \right) \times \left( \sinh \left( \frac{a\Delta \tau}{2} - i\alpha \right) \right)^{-1} w^2 + O(w^4). \]  

To calculate the response function per unit proper time (13), we use the following identity:

\[ \sinh x = x \prod_{k=1}^{\infty} \left( 1 + \frac{x^2}{k^2 \pi^2} \right). \]  

and calculating the integral (13), we arrive at

\[ \mathcal{F}^{(2d)}(E) = \mathcal{F}^{(1d)}(E) + \mathcal{F}^{(\omega)}(E)w^2 + O(w^4), \]  

where

\[ \mathcal{F}^{(\omega)}(E) = -\frac{e^{\beta \Delta E/\hbar} \Delta E^2}{12\pi^2 \left[ e^{\beta \Delta E/\hbar} - 1 \right]^2} \times \left[ \frac{8\pi^2}{\Delta E^2} + 9\beta^2 - \beta \Delta E \left( \frac{4\pi^2}{\Delta E^2} + \beta^2 \right) e^{\beta \Delta E/\hbar} - 1 \right]. \]  

For the ‘infrared’ tail of the spectrum \( \Delta E \ll 1 \), the response function is

\[ \mathcal{F}^{(2d)}(E) = \frac{1}{2\pi \beta} \left[ 1 - \left( \frac{7}{6} - \frac{\pi^2}{9} \right) w^2 \right] + O(\Delta E). \]  

For the ‘ultraviolet’ tail of the spectrum \( \Delta E \gg 1 \), the excitation rate (36) has the following dominant behavior:

\[ \mathcal{F}^{(\omega)}(E) \sim \frac{\Delta E^3}{12\pi} e^{-\beta \Delta E} \beta^2. \]  

### 3.2. Motion of the detector in the thermal bath

We now consider a detector moving along a spatially straight line with a constant component of its four-velocity \( \vec{w} \):

\[ x^\gamma(\tau) = \left( \sqrt{1 + \vec{w}^2} \tau, 0, \vec{w} \tau, 0 \right). \]
in a thermal bath of temperature \( T \) (see [40, 41]), corresponding to the temperature that an accelerated Unruh-DeWitt detector moving along a spatially straight line with a constant magnitude of four-acceleration, trajectory (24), in the Minkowski vacuum perceives, i.e., \( T = a/(2\pi) \). Here and in what follows, by ‘thermal bath’ we mean thermal quantum scalar field. We are interested to see if any relation exists between \( \mathcal{F}^{(\text{id})}(E) \), (35), and the response function of the detector moving along a spatial line with constant non-relativistic speed \( v = \hat{v}/\sqrt{1 + \hat{w}^2} \ll 1 \) or \( w \ll 1 \) in a thermal bath of corresponding temperature \( T \). We consider a detector with the same parameters as that of the previous subsection. From (15) the Wightman thermal Green function for a detector following trajectory (24) is

\[
G^+_\beta(\Delta \tau) = \frac{1}{4\pi^2(\Delta \tau - i\epsilon)^2} + \sqrt{1 - v^2} \left[ \coth \left( \frac{\gamma_{\tau} \Delta \tau}{\beta} \right) + \frac{1}{4\pi^2 \Delta \tau^2} \right],
\]

where \( \gamma_{\tau} = \sqrt{(1 + v)/(1 - v)} \), and \( \beta = 1/T \). The response function per unit time of this detector is

\[
\mathcal{F}^{(\text{th})}(E) = \frac{\sqrt{1 - v^2}}{4\pi\beta v} \ln \left[ \frac{1 - e^{-\beta \Delta E}}{1 - e^{-\beta \Delta E_v}} \right],
\]

where \( \Delta E = E - E_0 \) is the difference between the ground state and the excited state of the detector.

For non-relativistic velocities \( w \ll 1 \) the leading behavior of the response function (41) is

\[
\mathcal{F}^{(\text{th})}(E) = \mathcal{F}^{(\text{id})}(E) + \mathcal{F}^{(v)}(E)\hat{w}^2 + O(\hat{w}^4),
\]

where

\[
\mathcal{F}^{(v)}(E) = -\frac{e^{\beta \Delta E} \Delta E^2}{12\pi\beta \left( e^{\beta \Delta E} - 1 \right)} \left[ 3\beta^2 - \beta^3 \Delta E \left( \frac{e^{\beta \Delta E} + 1}{e^{\beta \Delta E} - 1} \right) \right],
\]

and \( \mathcal{F}^{(\text{id})}(E) \) is the same as (28).

For the infrared tail of the spectrum \( \Delta E \ll 1 \), the \( \mathcal{F}^{(\text{th})} \), (42), is

\[
\mathcal{F}^{(\text{th})}(E) = \frac{1}{2\pi\beta} \left[ \frac{1}{6} - \frac{1}{\Delta E} \right] + O(\Delta E).
\]

\(^{3}\) It is easy to understand (45). Consider two observers immersed in the blackbody radiation. Observer \( O \) is at rest relative to the radiation; thus this observer sees strictly isotropic blackbody radiation, and the other observer, \( O' \), is moving with speed \( v \) along the \( x \)-axis of the first observer. The moving observer carries with him a detector with collecting area \( A \), with its normal at angle \( \theta \) to the axis. It has been shown [42-44] that the Lorentz transformation changes the radiation temperature \( T \) to an effective directional radiation temperature \( T' \)

\[
T'(T, v, \theta) = \frac{T\sqrt{1 - v^2}}{1 - v \cos \theta},
\]

but the observer \( O' \) looking in the fixed direction \( \theta \) still would map out a blackbody spectrum. Even though the validity of this conclusion has been questioned in [41], the author has considered this conclusion to be valid for the infrared sector of radiation \( \Delta E \ll T \). An Unruh-DeWitt detector perceives only the radiation over whole angles.
The expression (45) can be mapped to the expression (37) if we relate the speed of the detector moving in a thermal bath to the one following the trajectory (16) in Minkowski space by the following transformations:

\[ \tilde{w} = w \sqrt{21 - 2 \pi^2 / \sqrt{3}} = 0.65 w. \] (46)

The detector in the Minkowski space-time moving along trajectory (16) should move with a higher speed \( \tilde{w} = 1.54 \tilde{w} \) to keep up with the same excitation rate of the inertial detector in a thermal bath in the infrared limit. For the ultraviolet tail of the spectrum \( E \gg 1 \), the excitation rate (42) has the following dominant behavior:

\[ \mathcal{F}^{(v)}(E) \sim \frac{\Delta E^3}{12 \pi} e^{-\Delta E \beta} \beta^2. \] (47)

This is the same as (38). Therefore, in an ultraviolet regime, the dominant modification in the response of the detector following trajectory (16) compared with the blackbody spectrum of Unruh radiation is the same as the dominant modification perceived by a detector moving with constant four-velocity component \( \tilde{w} \) in a thermal bath along the trajectory (39).

4. Conclusion

We have considered the response of an Unruh-DeWitt detector moving along an unbounded spatial trajectory in a two-dimensional spatial plane with constant independent magnitudes of both the four-acceleration \( a \) and of a timelike proper time derivative of four-acceleration and having a component of four-velocity \( w = dy/d\tau \) constant. This is the motion of a charge in a uniform electric field when both an electric and a magnetic field are in the frame of the charge. We have compared the response function of this detector (35) and (36) with that of a detector moving with constant velocity in a thermal bath of the corresponding temperature \( \pi = Ta/(2\pi) \) in the non-relativistic limit, in the ultraviolet limit, and in the infrared limit.

The first dominant term in the response function is the Planck distribution, equivalent to a detector moving along a spatially straight line with constant magnitude of four-acceleration \( a \). The second dominant term \( \mathcal{F}^{(v)} \) in the response function of this detector can be mapped to the second dominant term \( \mathcal{F}^{(v)} \) of a detector moving non-relativistically in a thermal bath via (46) in the infrared limit.

To map the response functions of these two detectors in these different situations, the detector in Minkowski space-time moving along trajectory (16) should move with a higher speed \( w = 1.54 \tilde{w} \) to keep up with the same excitation rate of the inertial detector in a thermal bath in the infrared limit.

We also have shown that in an ultraviolet regime, the dominant modification in the response of the detector following trajectory (16) compared with the blackbody spectrum of Unruh radiation is the same as that of the dominant modification perceived by a detector moving with constant four-velocity component \( \tilde{w} \) in a thermal bath along the trajectory (39).

Integrating (44) over the solid angle, the average temperature perceived by an Unruh-DeWitt detector is Lorentz transformed according to \( T' = T \left( 1 - \tilde{v}^2 / 6 \right) \). In other words, \( \beta' = \beta \left( 1 + \tilde{v}^2 / 6 \right) \) or \( \beta' = \beta \left[ 1 + \tilde{w} \left( 6 - 1 - \tilde{w}^2 \right) \right] \).

Plug this transformation of \( \beta' \) into the blackbody excitation rate and also consider the non-relativistic limit \( \tilde{w} \ll 1 \); then, the infrared limit \( \Delta E \ll T \) expression (45), will then be reproduced.
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