Magnetic monopole and the nature of the static magnetic field

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We investigate the factuality of the hypothetical magnetic monopole and the nature of the static magnetic field. It is shown from many aspects that the concept of the massive magnetic monopoles clearly is physically untrue. We argue that the static magnetic field of a bar magnet, in fact, is the static electric field of the periodically quasi-one-dimensional electric-dipole superlattice, which can be well established in some transition metals with the localized d-electron. This research may shed light on the perfect unification of magnetic and electrical phenomena.

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I. INTRODUCTION

The general concept of symmetry plays an important role in physics and other fields of science. It is well known that the behavior of electric and magnetic fields can be completely described by Maxwell’s equations. For time-varying fields, the differential form of these four important equations in cgs (short for centimeter, gram, second) is given by

\begin{align}
\nabla \cdot \mathbf{E} &= 4\pi \rho_e, \\
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \\
\nabla \times \mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{J}_e,
\end{align}

where \( \mathbf{E} \) is the electric field, \( \mathbf{B} \) is the magnetic field, \( \rho_e \) is the electric charge density, \( \mathbf{J}_e \) is the electric current density and \( c \) is the speed of light in a vacuum.

Without electromagnetic sources (\( \rho_e = 0; \mathbf{J}_e = 0 \)), we can see clearly that the set of Eq. (1) will remain invariant under the following duality transformations

\[ \mathbf{E} \rightarrow \mathbf{B}; \quad \mathbf{B} \rightarrow -\mathbf{E}. \]

This implies that electric and magnetic fields are symmetrical and equivalent in this special case. Obviously, the electric-magnetic duality symmetry is no longer true when \( \rho_e \neq 0 \) (or \( \mathbf{J}_e \neq 0 \)). However, Dirac believed that the electromagnetic laws should have the “dual nature” under any circumstances, in other words, the electric and magnetic fields may have a general intrinsic symmetry and the Maxwell’s equations of Eq. (1) are incomplete. In 1931 [1], Dirac claimed that the mathematical introduction of magnetic monopole (a basic unit of magnetic charge) into the Maxwell’s equations would lead to a symmetrical form of the Maxwell-Dirac equations

\begin{align}
\nabla \cdot \mathbf{E} &= 4\pi \rho_m, \\
\nabla \cdot \mathbf{B} &= 4\pi \mathbf{J}_m, \\
\nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} - \frac{4\pi}{c} \mathbf{J}_m, \\
\nabla \times \mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{J}_e.
\end{align}

where \( \rho_m \) is the magnetic charge density and \( \mathbf{J}_m \) is the magnetic current density. The above four equations would also be invariant under the following transforma-
Dirac’s monopole theory [1] results into the following relation between an electric charge \( e \) and magnetic charge \( g \)

\[
E \rightarrow B; \quad B \rightarrow -E,
\]

\[
\rho_e \rightarrow -\rho_m; \quad \rho_m \rightarrow -\rho_e,
\]

\[
J_e \rightarrow J_m; \quad J_m \rightarrow -J_e.
\]

(4)

Dirac’s monopole theory results into the following relation between an electric charge \( e \) and magnetic charge \( g \)

\[
eg g = \frac{\hbar c}{4\pi n} = \frac{\hbar c}{2n}, \quad (n = 1, 2, 3, \cdots)
\]

(5)

where \( \hbar \) is the Plank’s constant, \( \hbar = h/2\pi \) and \( c \) is the speed of light.

It should be pointed out that the magnetic monopole is merely a hypothetical particle whose existence is postulated based on the duality symmetry. The equivalence of the electric charge (electron) and the magnetic charge (monopole) is explicitly shown in Fig. 1. Interestingly, Dirac linked the magnetic monopoles with the quantization of electric charge by Eq. (5). Such appealing proposal exhilarated a number of theoretical and experimental investigations since then. The numerous attempts of experimental search for these magnetic monopoles at accelerators and in cosmic rays have been done. And various techniques of detection in the experiments to search for magnetic monopole have been developed, for instance, the magnetometer SQUID. Unfortunately, up to now, no positive evidence for its existence has been found. In theoretical physics, ’t Hooft [2] pointed out that a unified gauge theory in which electromagnetism is embedded in a semisimple gauge group would predict the existence of the magnetic monopole as a soliton with spontaneous symmetry breaking. Wu and Yang [3] first described magnetic monopoles in terms of a principal of fiber bundle. Seiberg and Witten [4] developed the famous magnetic monopole equations. The standard SU(5) model predicts that the magnetic monopoles are extremely heavy with a mass at least \( 10^{16} \text{ GeV}/c^2 \) (the mass about \( 10^{15} \) protons), moreover, whose mass is estimated to be even higher (up to the Planck mass of \( 10^{19} \) GeV/\( c^2 \)) by the Kaluza-Klein model.

What we are most concerned about is why no magnetic monopoles have been detected after it had been hypothesized for 77 years. The experimental status of monopoles had led Dirac to doubt his theory: “I am inclined now to believe that monopoles do not exist” [3]. In fact, several errors of the Dirac monopole theory have been pointed out a long time ago [6, 7, 8]. In this paper, we provide a solid argument that the hypothetical magnetic monopoles aren’t naturally real or the concept of magnetic monopole is only a well-known particle (electron) of different representation.

II. MAGNET: ELECTRIC CHARGES OR MAGNETIC CHARGES?

It is well accepted that if a bar magnet is cut in half repeatedly, then each half of the magnet becomes a separate magnet with its own north and south poles, as shown in Fig. 2 (a). Since the discovery of the peculiar feature of the magnetic materials, many people are curious about what it would look like if there was the smallest magnet, moreover, can the smallest magnet be further isolated? According to Dirac’s opinion, similar to electric charges, there would have net magnetic charges (a magnet with only one pole) in the universe, as shown in Fig. 2 (b). Although the assumption of the existence of the magnetic monopoles sounds interesting, there are two fatal problems with this idea.

First, if the hypothetical magnetic particle is true in the natural world, apparently, there should be plenty of the Dirac’s magnetic monopoles inside the permanent magnet materials. Hence, there is much more possibility to detect the magnetic monopoles in the magnet materials than in the accelerators and cosmic rays. In our opinion, no evidence for the monopole’s existence in the sources (magnet materials) for the magnetic monopoles may indicate that monopoles do not exist at all.

Second, assuming there occurs a magnetic to non-magnetic transition in a material, how and where are the magnetic monopoles going? If there are some magnetic monopoles escaping from the material during the
transition, as a result, the mass of the material should be greatly reduced due to the theoretical predication of the massive magnetic monopoles. Of course, if one considers that all the magnetic monopoles still remain in the material after the transition, then he has to explain what are the differences between the monopole’s states before and after the transition and why these differences can not be experimentally detected.

From the viewpoint of the objectiveness and rationality of physics, when a permanent magnet material is cut in the way of Fig. 2, there is no doubt that ultimately we will obtain one positively charged proton and one negatively charged electron, rather than the hypothetic magnetic monopoles, as shown in Fig. 2 (c). Now the question turn out to be “Can the real particles of proton and electron be used to interpret the extremely common natural phenomenon described in Fig. 2 (a)?” In the following sections, we will try to answer this important question in a very intuitive way.

III. MAGNETIC FIELD OR ELECTRIC FIELD?

According to the traditional physics, the magnetic field lines of a bar magnet form closed lines. The field direction is taken to be outward from the North pole (N) and in to the South pole (S) of the magnet, as shown in Fig. 3. The magnetic field lines, which can be traced out with the use of the compasses (see also Fig. 3), are clearly more concentrated around the two poles of the magnet. Basically, the space with a denser magnetic field lines indicates a stronger magnetic field in that region.

If there really exist the monopoles with Dirac magnetic charges $+g$ and $-g$, then the magnetic field lines associated with a magnetic dipole can be readily obtained, as shown in Fig. 4(a). As a comparison between the magnetic field of the artificial magnetic dipole and the electric field of the real electric dipole, in Fig. 4(b) we plot the well-known electric field lines for the electric dipole. Similar to the case of the magnetic dipole of Fig. 4(a), the electric field lines produced by positive charge $+e$ will end in the negative charge $-e$. It is not difficult to find that the two figures are identical. In fact, there is no effective experimental means which can be used to distinguish between the magnetic field of the so-called magnetic dipole and the electric field of the electric dipole. In our opinion, the physical definition of the static magnetic field is essentially an electric-dipole field. Namely, the widely accepted physical concept of the static magnetic field most likely do not exist in practice, it is therefore unnecessary to discuss the possible existence of the magnetic monopoles (the sources of the static magnetic field) in the nature.

IV. THE NATURE OF THE STATIC MAGNETIC FIELD

In order to make our argument of the nature of the static magnetic field sounded, we try to design a bar “magnet” and some compass needles with the positive and negative charges. As shown in Fig. 5, the “magnetic” bar and the compasses have a superlattice structure comprising some pairs of layers of positive and negative electric charges. With an appropriate “magnetic” bar (structure, size and shape), the exactly same mag-
The electric field lines of a charge-superlattice bar

V. CONCLUSION

In this paper, on one hand, we have studied the possibility of the existence of the Dirac’s magnetic monopoles, on the other hand, we have attempted to uncover the physical nature of the static magnetic field generated by a bar magnet. It was shown clearly that the concept of the massive magnetic monopoles is physically untrue. The hypothetical particle is likely to be the well-known electron. This result indicates that any attempts to search for the magnetic monopole in the universe will be proved to be in vain. We have found that the traditional static magnetic field of a bar magnet, in fact, is the static electric field of the periodically quasi-one-dimensional electric-dipole superlattice. It seems that we had misdefined the static electric field of the electric-dipole lattice as the magnetic field of the magnet (or the magnetic monopoles). Interestingly, this new concept of periodic structure of the positive and negative charges may prove to be true in some transition metals with the intrinsic magnetic property. Further related research is being conducted.

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