Teleportation of an arbitrary mixture of diagonal states of multiqubits via classical correlation and classical communication

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We propose a protocol in which the faithful and deterministic teleportation of an arbitrary mixture of diagonal states is completed via classical correlation and classical communication. Our scheme can be generalized straightforwardly to the case of $N$-qubits by using $N$ copies of classical correlated pairs and classical communication. Moreover, a varying scheme by using the generalized classical correlated state within a multiqubit space is also presented. In addition, the arbitrary mixed state whose set of eigenvectors is known are a direct application of our protocol.

PACS numbers: 03.67.Hk, 03.67.-a, 03.65.Ud

I. INTRODUCTION

Since Bennett, Brassard, Crépeau, Jozsa, Peres, and Wootters (BBCJPW) proposed quantum teleportation in 1993 [1], many theoretical protocols were suggested and some experimental implementations were proposed. Recently, the teleportation of multiqubits has been well studied [2]. Quantum teleportation has become one of the most important and the most influential achievements in quantum theory, in particular, in the end of twenty century.

Quantum teleportation transports an unknown quantum state from Alice (sender) to Bob (receiver) via quantum correlation (Einstein-Podolsky-Rosen’s pair) and classical communication. When transporting a known quantum state, one can used the remote state preparation (RSP) which was proposed by Lo [3] and Pati [4]. As to the protocol of transporting a partially known quantum state, sometimes it is called as teleportation and sometimes it is called the remote state preparation either. Whatever if the quantum state one wants to transport is known, partially known or unknown, it is very interesting and important to know what resources will be costed at least and which resources can be replaced by the other ones or be traded off among the used resources, as well as how new resources can be exploited. This is just our main motivation. In addition, we would like to show the action of classical correlation in quantum information processing and further help for understanding the nature of quantum and classical correlations produced in quantum theory.

In this paper, we propose a protocol in which the faithful and deterministic teleportation of an arbitrary mixture of diagonal states is completed via classical correlation and classical communication. Here, a classical correlated pair of two qubits can be written as

$$C_{AB}^n = \frac{1}{2} (|00\rangle_{AB} \langle 00| + |11\rangle_{AB} \langle 11|)$$

Its name is from that it does not violate local hidden variable (LHV) theory [5]. In fact, it is a separable mixed state of two qubits and then there is no any quantum entanglement. In transporting and distributing quantum state, it has played a substituting and active role. For example, Cubitt et al. [6] used it to distribute entanglement. Toner and Bacon [7] constructed a protocol showing that the teleportation of a single qubit admits a local hidden variable theory. Ghosh et al. [8] provided an alternative simple proof of the necessity of entanglement in quantum teleportation and tried to show that it is sufficient to have a classical correlated channel in order to teleport any commuting qubits.

Our scheme can be generalized straightforwardly to the case of multiqubits. Teleporting an arbitrary mixture of diagonal states of $N$-qubits needs to using $N$ copies of classical correlated pairs and classical communication. Actually, we also can use the generalized classical correlated state within a multiqubit space to carry out our teleportation. In addition, we discuss the application of our protocol to the arbitrary mixed state whose set of eigenvectors is known.

II. ONE QUBIT

First, let us describe how to teleport a mixture of diagonal states of one qubit via a classical correlation and classical communication [8, 9]. As a class of mixed state, such a state can be written as

$$\rho^d_X (1) = a_0 |0\rangle_X \langle 0| + a_1 |1\rangle_X \langle 1|$$

where for a density matrix, $a_0$ and $a_1$ ought to be real and positive, as well as their summation is 1.
Initially, Alice and Bob shared a classical correlated pair. Thus, the joint system of them is just

\[ \rho_{\text{ini}}(1) = \rho_A^X(1) \otimes C_B^p \] (3)

where the first two qubits (denoted by the subscripts \(XA\)) belong to Alice and the third one (denoted by the subscript \(B\)) belongs to Bob. Our protocol can be divided into two steps.

Step one: Alice performs the operation \(\text{O}_{\text{Alice}}(1) = (\sigma_0^X \otimes H^A \otimes \sigma_0^B). (\text{CNOT}_0(X, A) \otimes \sigma_0^B)\) (4)

where \(\sigma_0 = 2 \times 2\) unit matrix, \(H\) is a Hadamard transformation and \(\text{CNOT}_0(X, A)\) is a controlled NOT of two neighbor qubits in which \(A\) is a control qubit lying at the latter. The definitions of these operations are respectively

\[ H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \text{CNOT}_0(X, A) = \sigma_0^X \otimes |0\rangle_A \langle 0| + \sigma_1^X \otimes |1\rangle_A \langle 1| \] (5)

where \(\sigma_1\) is Pauli matrix. The transformation skill (4) is actually from M. A. Nielsen and I. L. Chuang’s idea [10] in order to change Bell’s basis measurement into the computation basis measurement. Here, our aim is to make that Alice can execute the measurement in the computation basis.

It is easy to obtain the transformed result as the following:

\[
\begin{align*}
\frac{1}{4} (|00\rangle_X A (|00\rangle + |01\rangle_X A (|01\rangle)) \otimes (a_0 |0\rangle_B + a_1 |1\rangle_B (|1\rangle)) \\
+ \frac{1}{4} (|10\rangle_X A (|10\rangle + |11\rangle_X A (|11\rangle)) \otimes (a_1 |0\rangle_B + a_0 |1\rangle_B (|1\rangle)) \\
+ \frac{1}{4} (|00\rangle_X A (|01\rangle + |00\rangle_X A (|00\rangle)) \otimes (a_0 |0\rangle_B - a_1 |1\rangle_B (|1\rangle)) \\
+ \frac{1}{4} (|10\rangle_X A (|11\rangle + |11\rangle_X A (|10\rangle)) \otimes (a_1 |0\rangle_B - a_0 |1\rangle_B (|1\rangle))
\end{align*}
\] (6)

Step Two: Alice executes the measurement in the computation basis: \{\(|00\rangle_X A |00\rangle, \ |01\rangle_X A |01\rangle, \ |10\rangle_X A |10\rangle, \ |11\rangle_X A |11\rangle\} and Bob has the probability 0.25 of reducing to one of them. For the first two basis, she can send a cbit 0 to Bob and for the later two basis, she can send a cbit 1 to Bob and then Bob has to apply a \(\sigma_1\) (NOT) transformation to obtain \(a_0 |0\rangle_B + a_1 |1\rangle_B (|1\rangle)\) as required.

III. TOW QUBITS

Now, let us consider the case of an arbitrary mixture of diagonal states of two qubits. The state to be teleport reads

\[ \rho_{X_1 X_2}^d(2) = \frac{1}{4} \sum_{x_1, x_2 = 0}^1 a_{x_1 x_2} |x_1 x_2\rangle_X A \langle x_1 x_2| \] (7)

Alice and Bob have to share two classical correlated pairs, that is, the joint system of them can be written as

\[ \rho_{\text{ini}}(2) = \rho_{X_1 X_2}^d(2) \otimes C_{A_1 B_2}^p \otimes C_{A_2 B_2}^p \] (8)

where four qubits \(X_1 X_2\) and \(A_1 A_2\) belong to Alice, and two qubits \(B_1 B_2\) belong to Bob.

Similar to the case of one qubit, we use two steps to teleport \(\rho_{X_1 X_2}^d(2)\) from Alice to Bob.

Step one: Alice performs the operation

\[
\text{O}_{\text{Alice}}(2) = \left( \sigma_0^X \otimes \sigma_0^X \otimes H_A \otimes \sigma_0^B \otimes H_B \otimes \sigma_0^B \right)
\times \left( \text{CNOT}_1(X_1, A_1) \otimes \sigma_0^B \otimes \sigma_0^A \otimes \sigma_0^B \right)
\times \left( \sigma_0^X \otimes \text{CNOT}_2(X_2, A_2) \otimes \sigma_0^B \right)
\] (9)

where the control NOT of parting \(n\) qubits is defined by

\[ \text{CNOT}_n(X_i, A_i) = \sigma_0^X \otimes \sigma_0^\otimes n \otimes |0\rangle_{A_i} \langle 0| + \sigma_1^X \otimes \sigma_0^\otimes n \otimes |1\rangle_{A_i} \langle 1|, \quad (i = 1, 2; \ n \geq 1) \] (10)
It is not difficult to calculate out the transformed state $\rho_{AO}$

$$\rho_{AO} = \frac{1}{16} \sum_{i} \sum_{\alpha, \alpha'} \sum_{\beta, \beta'} a_{\beta, \beta'} |x_{i}x_{j}⟩⟨x_{i}x_{j}|$$

\(\otimes |\alpha⟩_{1}(|\alpha'⟩_{1} \otimes |\sigma_{x}⟩_{1} |β⟩_{1} |σ_{x}⟩_{1} \otimes |1 - \alpha'⟩σ_{0} + \alpha'σ_{3})

\(\otimes |\alpha⟩_{2}(|\alpha'⟩_{2} \otimes |1 - \beta⟩σ_{0} + \betaσ_{3}) |σ_{x}⟩_{2} |σ_{x}⟩_{2} \otimes (1 - \alpha'σ_{0} + \alpha'σ_{3})$$

(11)

It has 256 terms, however, only 64 diagonal terms are important because the non-diagonal terms will not appear after measurement. Obviously, the diagonal terms with the forms as the following:

$$\frac{1}{16} |x_{i}x_{j}⟩⟨x_{i}x_{j}| \otimes \sum_{\beta, \beta'} a_{\beta, \beta'} \left[ |α⟩_{1} |α⟩_{1} \otimes |σ_{x}⟩_{1} |β⟩_{1} |σ_{x}⟩_{1} \otimes |α⟩_{2} |α⟩_{2} \otimes |σ_{x}⟩_{2} |β⟩_{2} |σ_{x}⟩_{2} \right]$$

(12)

where $x_{1}, x_{2}, α, α' = 0, 1$.

Step Two: Alice executes the measurement in the 16 computation basis: \{ $|α⟩_{1} |α⟩_{1} \otimes |σ_{x}⟩_{1} |β⟩_{1} |σ_{x}⟩_{1} \otimes |α⟩_{2} |α⟩_{2} \otimes |σ_{x}⟩_{2} |β⟩_{2} |σ_{x}⟩_{2} \right]$. The diagonal terms of $\rho_{AO}$ include in to four groups respectively to correspond to apply a corresponding transformation listed in the following table to obtain the teleported state $\rho_{AO}^{d}$.

| Alice's measurement | Bob's operation |
|---------------------|-----------------|
| $|00⟩_{X_{1}X_{2}}$ | $|0⟩_{AO} \otimes σ_{0}$ |
| $|01⟩_{X_{1}X_{2}}$ | $|0⟩_{AO} \otimes σ_{1}$ |
| $|10⟩_{X_{1}X_{2}}$ | $|0⟩_{AO} \otimes σ_{0}$ |
| $|11⟩_{X_{1}X_{2}}$ | $|0⟩_{AO} \otimes σ_{1}$ |

IV. N-QUBITS

The generalization to the case of $N$-qubits is straightforward but significant. In order to simplify our notions, we do not write obviously the symbol of direct product and unit matrix. Suppose the state to be teleported is

$$\rho_{X_{1}X_{2}...X_{N}}^{d}(N) = \frac{1}{2N} \sum_{x_{1}, x_{2}, ..., x_{N}} a_{x_{1}, x_{2}, ..., x_{N}} |x_{1}, x_{2}, ..., x_{N}⟩⟨x_{1}, x_{2}, ..., x_{N}|$$

(13)

Initially, the joint system of Alice and Bob reads

$$\rho_{mi}(N) = \rho_{X_{1}X_{2}...X_{N}}^{d}(N) \prod_{i=1}^{N} C^{p}(A_{i}, B_{i})$$

(14)

where $2N$ qubits $X_{1}, X_{2}, ..., X_{N}; A_{1}, A_{2}, ..., A_{N}$ belong to Alice, and $N$ qubits $B_{1}, B_{2}, ..., B_{N}$ belong to Bob. Note that $N$ copies of classical correlated pairs $\prod_{i=1}^{N} C^{p}(A_{i}, B_{i})$ are shared by Alice and Bob.

Step one: Alice performs the operation

$$O_{Alice}(N) = \prod_{i=1}^{N} |H⟩_{A_{i}} |H⟩_{A_{i}}$$

(15)

That is, taking each $A_{i}$ as the control qubit, $X_{i}$ as the corresponding target qubit, Alice first performs $N$ CNOT operations; then for every $A_{i}$ qubit, Alice always applies a Hadamard transformation $|H⟩_{A_{i}}$. The diagonal terms of transformed state becomes

$$\frac{1}{4N} |x_{1}x_{2}...x_{N}⟩⟨x_{1}x_{2}...x_{N}| \sum_{β_{1}, β_{2}, ..., β_{N}} a_{β_{1}, β_{2}, ..., β_{N}} \prod_{i=1}^{N} (|α⟩_{i} ⟨α|_{i}) (σ_{x_{i}} |β⟩_{i} ⟨β|_{i} σ_{x_{i}})$$

(16)

Step two: Alice executes the measurements in the $4^{N}$ computation basis $|x_{1}x_{2}...x_{N}⟩⟨x_{1}x_{2}...x_{N}| \otimes |α_{1}α_{2}...α_{N}⟩⟨α_{1}α_{2}...α_{N}| (x_{1}, x_{2}, ..., x_{N} = 0, 1; α_{1}, α_{2}, ..., α_{N} = 0, 1)$ and send a relevant cbit to Bob. Bob performs the corresponding operation $\prod_{i=1}^{N} σ_{x_{i}}$ and obtains the teleported state $\rho_{X_{1}X_{2}...X_{N}}^{d}(N)$.

Based on our previous scheme, it is easy to draw out its quantum circuit:
FIG. 1: Quantum circuit of teleportation of an arbitrary mixture of diagonal states of $N$-qubits

V. A VARYING SCHEME

It is worthy of pointing out that there is a varying scheme of our above protocol by using the generalized classical correlated state which is constructed by moving all of odd positions of $N$-copies of classical correlated pairs to the front. In fact, such a transformation can be completed by a series of swapping operations. Introducing a swapping transformation of two neighbor qubits ($2 \times 2$ matrix) defined by

$$S(X, Y) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Its action is

$$S(X, Y)|\alpha_X\beta_Y\rangle = |\beta_Y\alpha_X\rangle, \quad S(X, Y)(Q_X \otimes Q_Y)S(X, Y) = Q_Y \otimes Q_X$$

Thus,

$$S \left( \prod_{i=1}^{N} |\alpha_i\beta_i\rangle \right) = \left( \prod_{i=1}^{N} |\alpha_i\rangle \right) \left( \prod_{j=1}^{N} |\beta_j\rangle \right), \quad S \left( \prod_{k=1}^{N} Q_{A_k}Q_{B_k} \right) S = \left( \prod_{i=1}^{N} Q_{A_i} \right) \left( \prod_{j=1}^{N} Q_{B_j} \right)$$

where

$$S = \prod_{j=1}^{N-1} \left( \prod_{i=1}^{N-j} S(B_i, A_{N-j+1}) \right)$$

Note that we have used the fact $S = S^{-1}$.

Therefore, a generalized classical correlated state of $N$-qubit space can be written as

$$C^{s}_{A_1A_2 \cdots A_N B_1 B_2 \cdots B_N} (N) = S \left( \prod_{i=1}^{N} C^{p}_{A_i, B_i} \right) S$$

In the experimental implementation, it is different from the $N$-copies of classical correlation pairs, and so it is only not a problem of notion. Just as one considered a generalized four particle entanglement different from two pairs of
We show that the teleportation of an arbitrary mixture of diagonal states of states is completed via thus, the joint system of Alice and Bob is the teleported state complete this teleportation.

Step two: Alice executes the measurements in $4^N$ computation basis $\prod_{i=1}^{N} |x_i \alpha_i \rangle \langle x_i \alpha_i | (x_1, x_2, \cdots, x_N = 0, 1; \alpha_1, \alpha_2, \cdots, \alpha_N = 0, 1)$ and send a relevant cbit to Bob. Bob performs a corresponding operation $\prod_{j=1}^{N} \sigma_j x_j$ and obtain the teleported state $\rho_{X_1, X_2 \cdots X_N}^d(N)$.

VI. DISCUSSIONS AND CONCLUSIONS

Actually, our protocol has more applications. For example, it is applicable to the teleportation of a general mixed state whose set of eigenvectors is known (the eigenvalues remain unknown). Alice can teleport it by first performing a diagonalized unitary transformation which can be constructed by the set of eigenvectors of this mixed state, then do our above protocol, and finally, Bob also has to performs this diagonalized unitary transformation in order to complete this teleportation.

We have proposed a protocol in which the faithful and deterministic teleportation of an arbitrary mixture of diagonal states is completed via classical correlation and classical communication, however, without quantum entanglement. We show that the teleportation of an arbitrary mixture of diagonal states of $N$-qubits needs $N$-copies of classical correlated pairs. It must be emphasized that the teleported state in our protocol is not fully unknown but partially known. This is a reason why our protocol does not need any quantum entanglement. It reminds that there is an extreme case of trade-off between quantum correlation and classical correlation under some preconditions. In our point
of view, quantum entanglement is still necessary for a quantum teleportation of a fully unknown state. A complete quantum teleportation needs both quantum entanglement and quantum measurement. Our protocol is, at most, a kind of partially quantum teleportation since only using quantum measurement. Just because our protocol depends on quantum measurement, it is not a classical teleportation one. Of course, the results presented here offer an intriguing glimpse into the nature of correlations produced in quantum theory, and show that classical correlation also is an important and useful resource in quantum information processing.

Acknowledgement

We particularly thank Wan Qing Niu for his earlier work about an arbitrary mixture of diagonal states of one qubit, and Liang Qiu for his surveying many related references. We are grateful all of collaborators of our quantum theory group in the institute for theoretical physics of our university. This work was founded by the National Fundamental Research Program of China with No. 2001CB309310, partially supported by the National Natural Science Foundation of China under Grant No. 60573008.

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