Entanglement Hamiltonian of the quantum Néel state

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Abstract. 2D projected entangled pair states (PEPS) provide a unique framework giving access to detailed entanglement features of correlated (spin or electronic) systems. For a bi-partitioned quantum system, it has been argued that the entanglement spectrum (ES) is in a one-to-one correspondence with the physical edge spectrum on the cut and that the structure of the corresponding entanglement Hamiltonian (EH) reflects closely bulk properties (finite correlation length, criticality, topological order, etc). However, entanglement properties of systems with spontaneously broken continuous symmetry are still not fully understood. The spin-1/2 square lattice Heisenberg antiferromagnet provides a simple example showing spontaneous breaking of SU(2) symmetry down to U(1). The ground state can be viewed as a ‘quantum Néel state’ where the classical (Néel) staggered magnetization is reduced by quantum fluctuations. Here I consider the (critical) resonating valence bond (RVB) state doped with spinons to describe such a state; this enables the use of the associated PEPS representation (with virtual bond dimension $D = 3$) to compute the EH and the ES for a partition of an (infinite) cylinder. In particular, I find that the EH is (almost exactly) a chain of a dilute mixture of heavy ($\downarrow$ spins) and light ($\uparrow$ spins) hardcore bosons, where light particles are subject to long-range hoppings. The corresponding ES shows drastic differences with the typical ES obtained previously for ground states with restored SU(2)-symmetry (on finite systems).

Keywords: Hubbard model (theory), spin liquids (theory), other numerical approaches, entanglement in extended quantum systems (theory)
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1. Introduction

It is known from early quantum Monte Carlo (QMC) simulations that the ground state (GS) of the spin-1/2 Heisenberg antiferromagnet (AFM) on the bipartite square lattice is magnetically ordered [1] and, hence, breaks the hamiltonian SU(2) symmetry. The GS can be viewed as a ‘quantum Néel state’ (QNS) where the maximum classical value $m_{stag} = 1/2$ of the staggered magnetization is reduced by (moderate) quantum fluctuations. More recent QMC simulations [2] have provided GS energy, staggered magnetization and spin–spin correlations with unprecedented accuracy. In particular, it has been established that the QNS exhibits power-law decaying spin–spin correlations characteristic of a critical state [3].

Recently, a number of powerful tools based on entanglement measures have emerged. The entanglement spectrum (ES) and its associated entanglement Hamiltonian (EH) defined via the reduced density matrix (RDM) of a bi-partioned quantum system (see definitions later) provides new insights. In particular, it has been argued that the ES is in a one-to-one correspondence with the physical edge spectrum on the cut for topological ground states [4] and low-dimensional quantum AFMs [5] and that the structure of the corresponding EH closely reflects the bulk properties (holographic principle) [6]. However, new interesting features might arise in the entanglement properties of systems with spontaneously broken continuous symmetry, such as the QNS for which SU(2) symmetry is broken down to U(1). First, the entanglement entropy (the entropy associated to the RDM) has revealed anomalous additive (logarithmic) corrections [7,8] to the area law—i.e. the linear (asymptotic) scaling of the entropy with the length of the cut. It was proposed doi:10.1088/1742-5468/2014/10/P10026
afterwards that the origin of such corrections may lie in the existence of Goldstone modes [9] associated to the spontaneously broken continuous symmetry. Note, however, that in any finite system (as in most 'exact' simulations), the SU(2) symmetry is restored by quantum fluctuations and one has a unique GS instead of a degenerate manifold. In fact, recent state-of-the-art SU(2)-symmetric Density Matrix Renormalization Group (DMRG) studies established an interesting correspondence [10], in the (singlet) GS of 2D AFMs in their magnetically-ordered phases, between the SU(2) tower of states and the lower part of the ES below an 'entanglement gap' (although DMRG does not provide information on the momenta of the ES). This suggests strongly that the above-mentioned corrections in the entropy should be associated with the tower of states structure, while the area law arises from ES levels above the entanglement gap [10].

A priori important differences may occur in the entanglement properties of a Néel-like wave-function breaking the continuous SU(2) symmetry explicitly, i.e. with a finite staggered magnetization. In particular, one expects the ES (and the EH) of a symmetry-broken QNS to differ qualitatively from those associated to the GS with restored SU(2) symmetry, computed on finite systems [10,11]. Computing the entanglement properties of a (variational) state with a finite order parameter is the main goal of this paper.

The formalism of projected entangled pair states (PEPS) [6,12] enables (i) the easy construction of symmetry broken variational states and (ii) the computation of the corresponding EH. Note that, for a given variational state and system size (one uses infinite cylinders with a finite perimeter), the calculation of the EH is fundamentally exact and provides a complete analytic expansion in terms of N-body interactions whose amplitudes are numerically computed. Here I therefore make use of a simple PEPS ansatz of the QNS in order to calculate its EH associated to a bi-partition of an infinite cylinder. The variational wave function used here is in fact the simplest PEPS (i.e. with the smallest bond dimension $D = 3$) one can construct to capture the physics of the symmetry-broken Néel state. Ansätze with a larger bond dimension will not allow the consideration of a cylinder with a large enough perimeter. Note that the PEPS formalism provides also the momentum-resolved ES. This is to be contrasted to DMRG that also gives the ES easily but without the corresponding momenta of the Schmidt states. Also an analytic form of the EH cannot be obtained in DMRG.

As shown recently using PEPS, an EH with local interactions is expected in a gapped bulk phase (with short-range entanglement), whereas a diverging interaction length of the EH is the hallmark of critical behavior in the bulk [6]. One therefore expects to see fingerprints of the critical behavior of the QNS in its EH.

2. Doped-RVB ansatz for the Néel state

I start with the square lattice RVB wavefunction defined as an equal-weight superposition of nearest-neighbor (NN) hardcore singlet coverings [13,14]. The sign structure of the wave function is fixed by imposing the condition that the singlets $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ are all oriented from one A sublattice to the other B sublattice. Such a wave function is a global spin singlet—i.e. an SU(2)-invariant state— with algebraic (i.e. critical) dimer correlations (and short-range spin correlations) [15,16]. To construct a simple ansatz for the QNS, let us
Figure 1. (a) The Néel state is represented as a spinon-doped RVB state: Singlets are oriented from the A to the B sublattice and doped spinons are polarized along $\hat{z}$ ($-\hat{z}$) on the A (B) sites. Implicitly, a sum over all singlet/spinon configurations is assumed, the average spinon density being controlled by a fugacity. (b) Under a $\pi$-rotation around $\hat{y}$ on all the B-sites, all spinons become oriented along $\hat{z}$ and singlets transform into $\frac{1}{\sqrt{2}}\{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\}$ on every NN bonds.

now assume that one breaks SU(2) symmetry down to U(1) by doping the NN RVB state with on-site spinons (i.e. spin-1/2 excitations) with opposite orientations on the two sublattices. For simplicity, I choose hereafter the staggered magnetization pointing along the $\hat{z}$-axis. Such a simple ansatz is schematically shown in figure 1(a). The average density of spinons—identical on the two sublattices—directly gives the staggered magnetization $m_{stag} (\times 2)$ and, as we will see later on, can be controlled by a fugacity $\gamma$.

Before going further, it is convenient to rewrite the Néel-RVB state in a translationally invariant form. Indeed, under a (spin) $\pi$-rotation around $\hat{y}$ on the B-sites, B-spinons transform as $|\downarrow\rangle \rightarrow |\uparrow\rangle$ and $|\uparrow\rangle \rightarrow -|\downarrow\rangle$. Under such a (unitary) transformation, the new Néel-RVB state acquires the same (average) polarization on the A and B sublattices as shown in figure 1(b). The original NN singlets are also transformed into $\frac{1}{\sqrt{2}}\{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\}$ dimers which are now symmetric with respect to the bond centers.

3. PEPS construction and energetics

Such a state can in fact be represented by a PEPS $|\Psi_{PEPS}\rangle$ with bond dimension $D = 3$, where each lattice site is replaced by a rank-5 tensor $A_{\alpha,\alpha';\beta,\beta'}^s$ labeled by one physical index, $s = 0$ or 1 and by four virtual bond indices (varying from 0 to 2) along the horizontal ($\alpha, \alpha'$) and vertical ($\beta, \beta'$) directions, as shown in figure 2(a). Physically, the absence of singlet on a bond is encoded by the virtual index being ‘2’ on that bond. I define:

$$A = R + \gamma S,$$

where $R$ is the original RVB tensor [17, 18], $S$ is a polarized spinon tensor and $\gamma \in \mathbb{R}$ is a fugacity controlling the average spinon density. To enforce the hardcore dimer constraint,
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Figure 2. (a) Local (rank-5) PEPS tensor. (b) Tensors are placed on a square lattice wrapped on a cylinder of perimeter $N_v$ and (quasi-) infinite length $N_h \gg N_v$. $B_L$ and $B_R$ boundary conditions are realized by fixing the virtual variables going out of the cylinder ends. A bipartition of the cylinder generates two L and R edges along the cut.

one takes $R^{s}_{\alpha,\alpha';\beta,\beta'} = 1$ whenever three virtual indices equal 2 and the fourth one equals $s$ and $R^{s}_{\alpha,\alpha';\beta,\beta'} = 0$ otherwise. The spinon tensor has only one non-zero element, $S^{1}_{2,2;2,2} = 1$.

The wave function amplitudes are then obtained by contracting all virtual indices (except the ones at the boundary of the system). Note that the above PEPS ansatz for the Néel state bears similarities to the one used to describe the honeycomb RVB spin liquid under an applied magnetic field [19]. However, a crucial difference is that this new ansatz is, by construction, fully U(1)-invariant in contrast to the spinon-doped RVB state of [19].

Following the usual procedure, I now place the square lattice of tensors on infinite cylinders with $N_v$ sites in the periodic (vertical) direction as shown in figure 2(b) and use standard techniques (involving exact tensor contractions and iterations of the transfer operator) to compute relevant observables. In the PEPS formulation the boundary conditions $B_L$ and $B_R$ can be simply set by fixing the virtual states on the bonds ‘sticking out’ at each cylinder end. For example, open boundary conditions are obtained by setting the boundary virtual indices to ‘2’. Generalized boundary conditions can be realized as in figure 2(b) by setting some of the virtual indices on the ends to ‘0’ or ‘1’.

I have computed the (staggered) magnetization $m_{stag}$ and the expectation values of the spin-1/2 Heisenberg exchange interactions $S_i \cdot S_j$ between NN and next-NN sites, varying $\gamma$ from zero to large values (to approach the classical Néel state). The data (normalized as the energy per site of the corresponding Heisenberg model) are displayed as a function of $m_{stag}$ in figures 3(a) and (b). The NN energy shows a broad minimum around $m_{stag} \sim 0.35$, a value a bit larger than the QMC extrapolation $\sim 0.307$ [2] for the pure NN quantum AFM. However, (i) the variational energy curve is rather flat around the minimum and (ii) the minimum energy is within $\sim 1.5\%$ of the QMC estimate, a remarkable result considering the simplicity of the 1D family of $D = 3$ PEPS. Note also that the minimum energy agrees very well with optimized $D = 3$ iPEPS [20] and finite PEPS up to $D = 6$ [21].

For completeness, I also show the next-NN energy in figure 3(b). In fact, the pure (critical) RVB state provides the lowest next-NN exchange energy, suggesting the existence of a transition, upon increasing the next-NN coupling, from the Néel state to a gapless

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4. Entanglement Hamiltonian on infinite cylinders

4.1. Bipartition and reduced density matrix

To define an EH associated to the family of Néel-RVB wavefunctions, I partition the $N_v \times N_h$ cylinder into two half-cylinders of lengths $N_h/2$, as depicted in figure 2(b). Partitioning the cylinder into two half-cylinders reveals two edges L and R along the cut. Ultimately, I aim to take the limit of infinite Néel-RVB cylinders, i.e. $N_h \to \infty$ as before.

The reduced density matrix of the left half-cylinder obtained by tracing over the degrees of freedom of the right half-cylinder, $\rho_L = \text{Tr}_R \{|\Psi_{\text{PEPS}}\rangle\langle\Psi_{\text{PEPS}}|\}$, can be simply mapped, via a spectrum conserving isometry $U$, onto an operator $\sigma_2^b$ acting only on the $D^{\otimes N_v}$ edge (virtual) degrees of freedom, i.e. $\rho_L = U^\dagger \sigma_b^2 U$ [6]. The entanglement (or boundary) Hamiltonian $H_b$ introduced above is defined as $\sigma_b^2 = \exp(-H_b)$. As $\sigma_b^2$, $H_b$ is 1D and its spectrum—the entanglement spectrum (ES)—is the same as the one of $-\ln \rho_A$. Note that the left and the right half-cylinders give identical EH. For further details on the derivation and the procedure, the reader is kindly asked to refer to [6].

For a topological state, such as the $\gamma = 0$ RVB state, the EH depends on the choice of the $B_L$ and $B_R$ cylinder boundaries that define ‘topological sectors’ [18,25]. Adding any staggered magnetization $m_{\text{stag}}$ in the PEPS immediately breaks the gauge symmetry of the tensors which is responsible for the disconnected topological sectors, as also happens in the case of field-induced magnetized RVB states [19]. Therefore, all topological sectors are mixed and $H_b$ become independent of the boundary conditions $B_L$ and $B_R$ provided $N_h \to \infty$. Note also that $H_b$ inherits the U(1) symmetry (associated to rotations around the direction of $m_{\text{stag}}$) of the Néel state.

Note that a direct transition from the Néel state to a valence bond crystal—with no intermediate gapless spin liquid phase—is also a realistic scenario [3,24].
Figure 4. Weights of the EH $H_b$ expended in terms of N-body operators. Data of several Néel-RVB wavefunctions (whose $\gamma$ values are mentioned on the plot) are shown. Calculations are done on an infinite cylinder with perimeter $N_v = 6$. As seen (e.g. in [18]), finite size effects for such integrated quantities are typically quite small.

4.2. Expansion in terms of $N$-body operators

To have a better insight of the EH, I expand it in terms of a basis of $N$-body operators, $N = 0, 1, 2, \cdots$ [6, 18]. For this purpose, I use a local basis of $D^2 = 9$ (normalized) $\hat{x}_\nu$ operators, $\nu = 0, \cdots, 8$ which act on the local (i.e. at some site $i$) configurations $\{|0\rangle, |1\rangle, |2\rangle\}$, where $|2\rangle$ is the vacuum or ‘hole’ state and $|0\rangle$ and $|1\rangle$ can be viewed as spin down and spin up particles, respectively. More precisely, $\hat{x}_0 = I$ and $\hat{x}_1 = \sqrt{3}/2 (|0\rangle\langle 0| - |1\rangle\langle 1|)$ and $\hat{x}_2 = \frac{1}{\sqrt{2}}(|0\rangle\langle 0| + |1\rangle\langle 1| - 2|2\rangle\langle 2|)$, for the diagonal matrices, complemented by $\hat{x}_3 = \hat{x}_4^\dagger = \sqrt{3}|0\rangle\langle 1|$ acting as (effective) spin-1/2 lowering/raising operators and $\hat{x}_5 = \hat{x}_7^\dagger = \sqrt{3}|2\rangle\langle 0|$ and $\hat{x}_6 = \hat{x}_8^\dagger = \sqrt{3}|2\rangle\langle 1|$ acting as particle creation/annihilation operators. On this basis $H_b$ reads [18],

$$H_b = c_0 N_v + \sum_{\nu,i} c_\nu \hat{x}_\nu^i + \sum_{\nu,\mu, r,i} d_{\nu\mu}(r) \hat{x}_\nu^i \hat{x}_\mu^{i+r} + \sum_{\lambda,\mu, r, r', i} e_{\lambda\mu\nu}(r, r') \hat{x}_\lambda^i \hat{x}_\mu^{i+r} \hat{x}_\nu^{i+r'} + \cdots,$$

where site superscript indices have been added and only the first one-body, two-body and three-body terms are shown.

The total weights corresponding to each order of the expansion of $H_b$ in terms of $N$-body operators are shown in figure 4 as a function of the order $N$ using a semi-logarithmic scale. The data clearly reveal a fast decay of the weight with the order $N$. This decay is compatible with an exponential law although more decades in the variation of the weights (i.e. larger $N_v$) would be needed to draw a definite conclusion. In any case, $H_b$ is dominated by two-body contributions in addition to the normalization constant and subleading one-body terms. The QNS is believed to be critical with power-law decay of spin–spin correlations [3]. Therefore, according to [6], one expects $H_b$ to be long-ranged.
to some degree. So, one still needs to refine the analysis and investigate further the r-dependence of the leading two-body contributions. In the next subsection, I show that $H_b$ indeed possesses long-range two-body terms.

4.3. Entanglement Hamiltonian: an effective 1D $t$–$J$ model

It is known that the EH of the $\gamma = 0$ RVB PEPS belongs to the $1/2 \oplus 0$ representation of SU(2) and its Hilbert space is the same as that of a 1D bosonic $t$–$J$ model [18], interpreting $|0\rangle$ and $|1\rangle$ states ($|2\rangle$ states) as $\downarrow$ and $\uparrow$ spins (holes). In the presence of a finite (staggered) magnetization in the bulk, the SU(2) symmetry is broken but $H_b$ keeps the unbroken $U(1)$ symmetry corresponding to spin rotations around the direction of the staggered magnetization.

The (largest) non-zero real coefficients in (2) computed on an infinitely-long cylinder of perimeter $N_v = 6$ are shown in figures 5(a)–(c) for small (a), intermediate (b) and large (c) (staggered) magnetizations. At large and intermediate values of $m_{\text{stag}}$, one finds a dominant one-body (diagonal) term which can be interpreted as a chemical potential term (up to a multiplicative factor):

$$H_2 = c_2 \sum_i \hat{x}_2^i = \frac{3}{\sqrt{2}} c_2 \sum_i (n_i - 2/3),$$

(3)

where $n_i$ counts the number of particles (i.e. ‘0’ and ‘1’ states) on site $i$. The subleading one-body operator takes the form of a Zeeman coupling:

$$H_1 = \sqrt{6} c_1 \sum_i S_i^z,$$

(4)

where $S_i^z$ is an effective spin-1/2 component (along $\hat{z}$) and $c_1 \simeq c_2/10$. 

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The leading 2-body contributions are hopping terms at all distances for the majority ‘spins’ (|1⟩ states):

\[ H_{68}(r) = d_{68}(r) \sum_i (\hat{x}_i \hat{x}_{i+r} + \hat{x}_i \hat{x}_{i+r}^r) \]

\[ = 3d_{68}(r) \sum_i (b_{i+r,1}^\dagger b_{i,1} + b_{i,1}^\dagger b_{i+r,1}) , \]

where \( b_{i,s}^\dagger, b_{i,s} \) are the canonical bosonic creation (annihilation) operators of the virtual \( s = 0, 1 \) states. Note that the minority spins (|0⟩ states) only hop at even distances (weights at odd distances are negligable) with much weaker amplitudes, \( d_{57} = d_{75} \ll d_{68} = d_{86} \). The next subleasing corrections are diagonal 2-body density-density interactions

\[ H_{22}(r) = d_{22}(r) \sum_i \hat{x}_i \hat{x}_{i+r} \]

\[ = 9d_{22}(r) \sum_i (n_i - 2/3)(n_{i+r} - 2/3) , \]

which become dominant when \( \gamma, m_{stag} \to 0 \).

Other generic operators allowed by the \( U(1) \) symmetry, such as the anisotropic XXZ chain (\( d_{11} \neq d_{34} = d_{43} \)) or mixed operators of the form \( H_{12} \propto \sum_i S_i^z(n_i + r - 2/3) \) are also present but their amplitudes turn out to be quite small. Interestingly, \( H_b \) (approximately) conserves the hole ‘2-charge’ and hence does not contain pair-field operators with sizable amplitudes, in contrast to previous studies of \( D = 3 \) PEPS [18, 19]. If one keeps to the dominant contributions (3) and (5), \( H_b \) is exactly a chain of a dilute mixture of heavy (↓ spins or |0⟩ states) and light (↑ spins or |1⟩ states) hardcore bosons, where light particles are subject to long-range hopping.

5. Entanglement spectrum

It is also of considerable interest to examine the ES in the QNS and compare it to ES obtained for GS where SU(2)-symmetry is restored on finite size systems [10]. By definition the ES is the spectrum of \( -\ln \rho_L \). Since \( \rho_L \) and \( \sigma_b^2 = \exp(-H_b) \) are related by an isometry, it is also the spectrum of the EH \( H_b \). The ES for 3 values of the fugacity \( \gamma \), as a function of the momentum along the cut, are shown in figures 6(a)–(c).

Since \( \sigma_b^2 \) conserves the total \( S_z \) of the chain (\( U(1) \) symmetry), it can be block-diagonalized using this quantum number and the eigenvalues of \( -\ln \sigma_b^2 \) are displayed in each \( S_z \) sector separately. It can be seen from figure 6(a) that the \( \gamma = 0 \) SU(2) spin multiplets are split by a small spinon density. For increasing \( \gamma \) (i.e. staggered magnetization), the splittings of the Kramer’s multiplets increase (see figures 6(b) and (c)) due to the relative increase of the amplitudes of the SU(2)-symmetry-breaking terms like (5) in the EH. In the limit of large \( \gamma \) where the classical Néel state is approached, one finds separated bands of energy levels. It may be that the ES is gapped for all \( \gamma \), but finite size effects remain too large to reach a definite conclusion. In any case, the ES of figure 6 are to be contrasted to the ES obtained in DMRG for GS with restored SU(2)-symmetry (due to the use of finite size systems). Obviously the two types of ES are very different,
with a SU(2) tower of states structure at low energy for the ES of the singlet GS [10] and a U(1) symmetric ES in the (variational) symmetry-broken Néel state.

6. Summary and discussion

In this paper, I have investigated entanglement properties of a simple 1D family of PEPS designed to describe qualitatively the GS of the square lattice AFM. These ansätze exhibit a finite staggered magnetization, i.e. they explicitly break the SU(2) symmetry down to U(1) and can be studied on infinite cylinders with a finite perimeter. The goal of this study is therefore to examine the effects of such a finite order parameter on various entanglement properties and compare them to (QMC or SU(2)-symmetric DMRG) studies where symmetry is restored in a finite system. Thanks to the PEPS structure, the EH associated to a bipartition of the cylinder can be derived exactly (for a fixed perimeter). It is found that the EH inherits the U(1) symmetry of the Néel state and possesses a very simple structure: (i) its Hilbert space is the same as that of a 1D bosonic $t$–$J$ model, interpreting the three virtual states on the edge as a $\uparrow$ spin, a $\downarrow$ spin and a hole; (ii) when expended using a local basis of operators, it shows dominant two-body interactions; and (iii) higher-order operators (three-body terms and beyond) represent less than 10% of its total weight. Examining in detail the form of the two-body interactions, I find that the dominant ones are long-range hoppings of the majority (let say $\uparrow$) spins. However, it is not possible to distinguish a power-law versus an exponential decay of these hopping terms. In any case, the associated ES is found to be qualitatively very different from the ones obtained in GS with restored SU(2)-symmetry [10] (no tower of states structure is found, as suspected). Whether the entropy exhibits additive logarithmic corrections as in [7, 8] is difficult to say. The absence of the tower of states in the ES suggests a
negative answer. However, a hypothetical power-law decay of the hopping terms in the EH (instead of exponential) might lead to some additive corrections to the entropy. It would be interesting to complement our calculation of the ES using ‘conceptually exact’ numerical methods (such as QMC or SU(2)-symmetric DMRG) on large but finite systems, adding a small external staggered field (to produce a finite order parameter), taking the limit of infinite system size first.

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References

[1] Reger J D and Young A P 1988 Monte Carlo simulations of the spin-1/2 heisenberg antiferromagnet on a square lattice Phys. Rev. B 37 5978
[2] Sandvik A W and Evertz H G 2010 Loop updates for variational and projector quantum Monte Carlo simulations in the valence-bond basis Phys. Rev. B 82 024407
[3] Sandvik A W 2010 Lecture notes for course given at the 14th training course in physics of strongly correlated systems, Salerno (Vietri sul Mare), Italy, in October 2009 AIP Conf. Proc. 1297 135
[4] Li H and Haldane F D M 2008 Entanglement spectrum as a generalization of entanglement entropy: identification of topological order in non-Abelian fractional quantum Hall effect states Phys. Rev. Lett. 101 010504
[5] Poilblanc D 2010 Entanglement spectra of quantum Heisenberg ladders Phys. Rev. Lett. 105 077202
[6] Cirac J I, Poilblanc D, Schuch N and Verstraete F 2011 Entanglement spectrum and boundary theories with projected entangled-pair states Phys. Rev. B 83 245134
[7] Kallin A B, Hastings M B, Melko R G and Singh R R P 2011 Anomalies in the entanglement properties of the square-lattice Heisenberg model Phys. Rev. B 84 165134
[8] Hyejin Ju, Kallin A B, Fendley P, Hastings M B and Melko R G 2012 Entanglement scaling in 2D gapless systems Phys. Rev. B 85 165121
[9] Metlitski M A and Grover T 2011 Entanglement entropy of systems with spontaneously broken continuous symmetry (arXiv:1112.5166)
[10] Kelley F, Depenbrock S, McCulloch I P, Schollwöck U and Alba V 2013 Entanglement spectroscopy of SU(2)-broken phases in 2D Phys. Rev. B 88 144426
[11] Luitz D J, Laflorencie N and Alet F 2014 Participation spectroscopy and entanglement Hamiltonian of quantum spin models J. Stat. Mech. P08007
[12] Schuch N, Wolf M M, Verstraete F and Cirac J I 2007 Computational complexity of projected entangled pair states Phys. Rev. Lett. 98 140506
[13] Anderson P W 1973 Resonating valence bonds: a new kind of insulator? Mater. Res. Bull. 8 153
[14] Anderson P W 1987 The resonating valence bond state in La₂CuO₄ and superconductivity Science 235 1196
[15] Albuquerque A F and Alet F 2010 Critical correlations for short-range valence-bond wavefunctions on the square lattice Phys. Rev. B 82 180408
[16] Tang Y, Sandvik A W and Henley C L 2011 Properties of resonating valence bond spin liquids and critical dimer models Phys. Rev. B 84 174427
[17] Schuch N, Poilblanc D, Cirac J I and Perez-Garcia D 2012 Resonating valence bond states in the PEPS formalism Phys. Rev. B 86 115108
[18] Poilblanc D, Schuch N, Perez-Garcia D and Cirac J I 2012 Topological and entanglement properties of resonating valence bond wave functions Phys. Rev. B 86 014404
doi:10.1088/1742-5468/2014/10/P10026
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[19] Poilblanc D, Schuch N and Cirac J I 2013 Field-induced superfluids and Bose liquids in projected entangled pair states Phys. Rev. B 88 144414
[20] Bauer B, Vidal G and Troyer M 2009 Assessing the accuracy of projected entangled-pair states on infinite lattices J. Stat. Mech. P09006
[21] Lubasch M, Cirac J I and Bañuls M-C 2014 Algorithms for finite projected entangled pair states Phys. Rev. B 90 064425
[22] Wang L, Poilblanc D, Gu Xiao-Gang Wen Z-C and Verstraete F 2013 Constructing gapless spin liquid state for the spin-1/2 J1-J2 Heisenberg model on a square lattice Phys. Rev. Lett. 111 037202
[23] Gong S-S, Zhu W, Sheng D N, Motrunich O I and Fisher M P A 2013 Plaquette ordered phase and quantum phase diagram in the spin-1/2 J1-J2 square Heisenberg model Phys. Rev. Lett. 113 027201
[24] Schulz H J, Ziman T and Poilblanc D 1996 Magnetic order and disorder in the J1J2 model J. Phys. I France 6 675–703
[25] Schuch N, Poilblanc D, Cirac J I and Perez-Garcia D 2012 Topological order in PEPS: transfer operator and boundary hamiltonians Phys. Rev. Lett. 111 090501