The study of the accuracy of the robot movement along a given path considering the workspace boundaries, velocity and inertial properties of the drive

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Abstract. The paper considers the accuracy characteristics of a robot machine with a parallel structure as a control object. A specifically developed model of a planar 3-RPR mechanism with an electric drive based on a DC motor made it possible to experimentally study the dependence of the following robot output link accuracy according to the speed, weight and size characteristics of the drive and mechanism.

1. Introduction
The accuracy and speed of functioning of a parallel-structure robot are its most important consumer properties, largely determined by the control system used. The control of parallel mechanisms in order to ensure their movement along a given trajectory is explained, for example, in [1–9]. In [10], a two-stage robot control method was proposed, which includes, at the first stage, constructing a safe trajectory of its output link located at a sufficient distance from the boundaries of the working space (taking into account singularity zones and restrictions introduced by the dimensions of workpiece), and its transition into the coordinates of the mechanism’ input link by solving the inverse problem of its kinematic analysis. At the second control stage [11], the mechanism moves along a given trajectory. Moreover, since the trajectory satisfies the above safety requirements, its implementation is ensured by a synchronous change in the input coordinates of the mechanism in accordance with the task. The present study is aimed at determining the necessary distance of the trajectory from the theoretical boundaries of the working space of a robot of parallel structure, depending on the speed and mass-size characteristics of the drive and mechanism.

2. Model of a parallel robot control system
To conduct the planned experiments in MATLAB, a model of a control system for a parallel robot was built (Figure 1).
The model of the mechanism consists of three identical rods LEG-1, LEG-2 and LEG-3, which are controlled independently and provide a change in the length of each of them according to a pre-calculated algorithm that ensures the movement of the moving platform center along a given path. The model in this form is universal and can describe the control of various robots of parallel structure with three rods of variable length. A model of a planar 3-RPR mechanism (Figure 2) was built using the Sim-scape Multibody library.

![Figure 1. Model of a parallel robot control system](image1)

![Figure 2. The planar 3RPR mechanism](image2)

Each of the rods (Figure 3) consists of two rods connected by a Prismatic Joint. At the opposite end, one of them is attached using the Revolute Joint hinge to one of the points of the fixed base (A₁, A₂ or A₃ in Figure 2). Similarly, the other rod is pivotally attached to the movable platform at one of the points (B₁, B₂ or B₃). As a drive motor required changing the working length of the rod, a DC motor is used. When modeling it, the Simscape Electrical library was used. To control the engine, the blocks are used Con-trolled PWM Voltage and H-Bridge. At the same time, the control action formed by the regulator is decomposed into two components: its absolute value is limited and used to form the PWN voltage reference, and the sign determines the reverse signal.

![Figure 3. Model of a contour for regulating the length of the rod (subsystem LEG-3)](image3)

![Figure 4. The determination of the coordinates of point C by the known li, α, β and the coordinates of point Ai](image4)
Rotational motion is converted into translational using the screw-nut transmission. In reality, it is preferable to use ballscrews, but this is not the case in MATLAB, and the Leadscrew block (from Simscape Driveline) is quite suitable for describing ballscrews. By analogy with the Rev-Rot Interface [12], a Prism-Trans Interface block was developed to coordinate the applied force and the speed of movement of the prismatic hinge. The feedback signal of the drive hinge, which is compared with the reference, is used in feedback. To control the accuracy of regulation, the results of each of the local circuits are displayed together with the corresponding tasks. To build the system trajectory in the output coordinates (x, y) in the LEG-3 subsystem, the state signals α and β of the swivel joints are additionally removed. They are used to quickly calculate the output coordinates of the mechanism - the position of the center of the moving platform. For the calculation, formulas are used, which are easy to obtain from Figure 4. In the reference system associated with the moving platform, its center has constant coordinates:

\[ x_c'' = r \cos \left( \pi / 3 \right), \quad y_c'' = r \sin \left( \pi / 3 \right), \]

where \( r \) is the distance between points B and C (radius of the circle described near the triangle B1B2B3 in Figure 2), \( \pi / 3 \) is the angle determined by the geometry of the system.

In the reference system associated with the progressively moving end of the rod, the coordinates of point C are \( x_c' = r \cos \left( \beta + \pi / 3 \right), \quad y_c' = r \sin \left( \beta + \pi / 3 \right) \), where \( \beta \) is the angle between the axes \( x'' \) and \( x' \).

Finally, in the global frame of reference associated with a fixed base, the points B and C have the coordinates \( x_{Bi} = x_{A1} + l_i \cos \alpha, \quad y_{Bi} = y_{A1} + l_i \sin \alpha \), respectively; where \((x_{A1}, y_{A1})\) - known coordinates of the point \( A_i \), \( l_i \) - variable distance between the points \( A_i \) and \( B_i \), \( \alpha \) – the angle between the axes \( x' \) and \( x \),

\[
\begin{align*}
    x_c &= x_{Bi} + r \cos \left( \alpha + \beta + \pi / 3 \right) = x_{A1} + l_i \cos \alpha + r \cos \left( \alpha + \beta + \pi / 3 \right), \\
    y_c &= y_{Bi} + r \sin \left( \alpha + \beta + \pi / 3 \right) = y_{A1} + l_i \sin \alpha + r \sin \left( \alpha + \beta + \pi / 3 \right).
\end{align*}
\]

The result of the system - the trajectory of the point C - can be controlled using the XY Graph block.

3. Trajectory selection and modeling

We set some trajectory safe from the point of view of a possible exit from the working space of the mechanism and implement the movement along it with the help of the constructed model. For simplicity of accuracy control, a path is chosen, which is a circle in the \((x, y)\) plane, the center of which coincides with the center of the fixed base. The radius of the circle \((0.6 \text{ m})\) is determined in accordance with the restrictions [13]. In this case, the rotation angle of the plate-form (\( \phi \)) is fixed at a certain constant value. The selected trajectory is represented as a discrete sequence of nodal points \((x, y, \phi)\), for each of which, by solving the inverse kinematic analysis problem, the corresponding point in the input coordinates \((l_1, l_2, l_3)\) is determined. Switching tasks (steps of the trajectory) should be synchronous and can be carried out, for example, in time. Modeling allows you to track the change in the length of the drive links of the mechanism in time (Figure 5) and its trajectory in the output coordinates (Figure 6). From the figures it can be seen that the actual lengths of the rods correspond to the specified ones, and the center of the moving platform moves along an ideal curve. Since the given trajectory is a circle, a final estimate of the accuracy of the sequence can be given by calculating the distance from the center of the fixed base \((0, 0)\) to the current point \((x, y)\) and comparing it with the given radius of the trajectory. The correspondence graph (Figure 7) shows that the maximum modulus of the dynamic error is less than 0.3 mm, i.e. less than 0.05% of the radius of the circle.
4. **The study of the dependence of the accuracy of following a given trajectory on the inertial properties of the robot**

During the experiment, the weight and size characteristics of the mechanism were varied (the parameters of the rods and the movable platform changed separately). As a result, curves were obtained that reflect the dependence of the error on the listed parameters. The mass of the platform was increased by 10, 100 and 1000 times, Figure 7 is supplemented by the corresponding error graphs. Figure 8 shows that the magnitude of the dynamic error increases with increasing mass. The dependence of the maximum modulus error on the platform mass is shown in Figure 9. On a logarithmic scale, the graph is linear, which corresponds to a power function. It can be assumed that the maximum error is directly proportional to the square root of the platform mass. The weight of the rods also increased by 10, 100 and 1000 times, Figure 7 was supplemented by the corresponding error graphs. From Figure 10 (yellow - initial mass, blue - x10, red - x100, green - x1000) it can be seen that the dynamic error increases with increasing mass. The dependence of the maximum absolute value of the error on the mass of the rods is shown in Figure 11. It should be noted that the dependence is
different: with an increase in mass of 10 and 100 times, the error remains almost unchanged, with a further increase it increases sharply. The result obtained is quite explainable: with a comparable mass, the moment of inertia of the rods is much less than the moment of inertia of the platform.

Figure 9. The dependence of the maximum modulus error on the mass of the platform.

Figure 10. Dynamic error when changing the mass of the rods (yellow - initial mass, blue - x10, red - x100, green - x1000)

Figure 11. The dependence of the maximum modulus of error on the mass of the rods

Figure 12. Delay growth as a result of adjusting the parameters of the regulators

In the course of the next experiment, the inertial characteristics of the electric drive (DC motor and transmission) were varied. Since the frequency of the task change was selected in accordance with the inertial properties of the control object, an increase in the inertia of the electric drive leads to an increase in the dynamic error. Correction (reduction) of regulator coefficients can contribute to maintaining system accuracy. This leads to a noticeable delay (Figure 12) but reduces the error (Figure 13). However, a further increase in inertia leads to the fact that the drive does not have time to track the task (Figure 14). Correction of the regulator coefficients will not solve the problem: an increase in the step duration is necessary. The study showed that reducing the inertia of the electric drive does not lead to a significant increase in accuracy, but it allows you to speed up the system by proportionally reducing the duration of each step.
5. Conclusion
The paper describes a model of a planar 3-RPR mechanism developed in the MATLAB environment with an electric drive based on a direct current motor, an experiment was conducted to study the dependence of the following the robot output link accuracy along a given path from the speed, weight and size characteristics of the drive and mechanism. Assessment of the magnitude of the dynamic error and its dependence on the speed and weight and size characteristics of the drive and mechanism will allow you to determine the necessary distance of the trajectory of the robot parallel structure from the theoretical boundaries of its working space.

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