Random motion of quantum reactive harmonic oscillator.

Thermodynamics of vacuum of asymptotic subspace

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Abstract

The system of oscillator interacting with vacuum is considered as a problem of random motion of quantum reactive harmonic oscillator (QRHO). It is formulated in terms of a wave functional regarded as complex probability process $\Psi_{stc}(x, t | W(t))$ in the extended space $\Xi = \mathbb{R}^1 \otimes R\{W(t)\}$. This wave functional obeys some stochastic differential equation (SDE). Based on the nonlinear Langevin type SDE of second order, introduced in the functional space $R\{W(t)\}$, the variables in original equation are separated. The general measure in the space $R\{W(t)\}$ of the Fokker-Plank type is obtained and expression for total wave function (wave mixture) $\Psi^{br}(n; x, t)$ of random QRHO is constructed as functional expansion over the stochastic basis set $\Psi_{stc}^+(n; x, t | W(t))$. The pertinent transition matrix $S^{br}$ is constructed. For Wiener type processes $W(t)$ the exact representation for “vacuum-vacuum” transition probability $\Delta_{0->0}$ is obtained. The thermodynamics of vacuum is described in detail for the asymptotic space $\Xi_s = \mathbb{R}^1 \otimes R\{W_s\}$. The exact values for Energy, shift and expansion of ground state of oscillator and its Entropy are calculated.

1. Introduction

All the processes, described by standard quantum mechanical approach, are stochastic processes from the point of view of classical dynamics. The natural equivalence between Schrödinger and Fokker-Plank equations was used for formulation of quantum mechanics as stochastic theory [1], and the procedure of quantization was introduced [2], that takes into account the influence of stochastic processes on dynamics. For solution of quantum problems different numerical algorithms were proposed for stochastic dynamics (see [3]). Note, that in all above approaches the formulation of the main quantum object, that is the wave function, was deterministic. We must underline, that deterministic features of the physical theory are the outcome of the symmetry of its main equations with the change of the sign of time evolution.

At same time there is a lot of evidences for quantum deterministic description violation both in physics (see [4]) and in chemistry [5-6].

In several papers of the authors [7-9], nonstationary multichannel scattering in collinear three-body system was formulated as a problem of wave packet evolution in a system of
body fixed reper, that makes in general case complex, some times chaotic, motion on the induced Riemann manyfold. It was shown, that for three-body system there exist an "internal time" describing the evolution of the system, and in which the equation of motion is not symmetric with the change of sign of time. It means, that at certain conditions the wave function can be the object of probability description.

In present paper we propose a simple, but nontrivial problem of stochastic quantum mechanics - the problem of QRHO under Brownian motion. It was shown by the authors [9], that such a model can correspond for example to the description of bimolecular chemical reaction, that goes via the resonance complex.

2. Description of the problem

In the case of the random QRHO the equation for the wave function can be written in following form

\[ i\delta t \Psi = \hat{H} (x, t|W(t)) \Psi, \quad -\infty < x, t < +\infty, \]  

\[ \hat{H} (x, t|W(t)) = \frac{1}{2} [-\partial^2_x + \Omega^2 (t|W(t)) x^2], \quad \partial^2_x \equiv \partial^2 /\partial x^2, \quad \hbar = 1. \]  

with the frequency \( \Omega (t|W(t)) \), and the wave state \( \Psi_{stc} (x, t|W(t)) \) being the functional of, in general, complex Markovian process \( W(t) \). We shall denote by \( \delta t \) the total derivative in view of process \( \Psi_{stc} (x, t|W(t)) \) (see (3.2) ). We shall suppose also, that the frequency and wave functional are subjected to following boundary conditions

\[ \lim_{t \to \pm \infty} \Omega (t|W(t)) = \Omega_{in(out)} > 0, \]  

\[ \lim_{|x| \to +\infty} \Psi_{stc} (x, t|W(t)) = \lim_{|x| \to +\infty} \partial_x \Psi_{stc} (x, t|W(t)) = 0. \]

In particular case of frequency, being the regular function of "t" , that is \( \Omega (t|W(t)) = \Omega_0 (t) \), the equation (2.1) with initial condition (2.2) has exact solution (see [9],[10]).

So in our new approach the equation (2.1) is SDE for complex stochastic process \( \Psi_{stc}^+ (n; x, t|W(t)) \), determined in the extended space \( \Xi = \mathbb{R}^{1} \otimes \mathbb{R}_{\{W(t)\}} \).

It should be noted that asymptotic behavior of functional \( \Psi_{stc}^+ (n; x, t|W(t)) \) for the time \( t \to -\infty \) according to (2.2) have a following form

\[ \Psi^+ (n; x, t|W(t)) \to \Psi_{in} (n; x, t) = \]  

\[ = \left[ \frac{(\Omega_{in}/n)^{1/2}}{2n!} \right]^{1/2} \exp \left\{ -i(n+\frac{1}{2})\Omega_{in} \tau - \frac{1}{2} \Omega_{in} x^2 \right\} H_n \left( \sqrt{\Omega_{in}} x \right) \]

Our main problems are:

a) to find the conditions on Markovian process \( W(t) \), for which the variables in equation (2.1) are separated and so the detailed solution \( \Psi_{stc} (x, t|W(t)) \) is found;

b) the evaluation of evolution of average wave function

\[ \Psi_{br}^+ (n; x, t) = \left\langle \Psi_{stc}^+ (n; x, t|W(t)) \right\rangle_{\{W(t)\}} \]

that describes the state of random QRHO (where \( \langle \ldots \rangle_{\{W(t)\}} \) denote the functional integration over the total Fokker-Plank measure including the integration over the distribution of stationary process \( W_s = \lim_{t \to +\infty} W(t) \));
c) computation of corresponding transition $S^{br}$-matrix, and representation for "vacuum-vacuum" transition probability $\Delta^{br}_{0\to0}$ for a case of Wiener type process;
d) investigation of vacuum thermodynamics for the asymptotic space $\Xi_s = R^1 \otimes R_{\{W_s\}}$ (calculation of shift and width of ground state energy, Entropy and Free energy of the oscillator, interacting with vacuum).

3. Solution of SDE for complex process-wave functional $\Psi_{stc} (x, t|W(t))$

Let us start from the equation of classical oscillator under Brownian motion

$$\ddot{\xi} + \Omega^2 (t|W(t)) \xi = 0, \quad \dot{\xi} = \delta \xi (t|W(t)), \quad (3.1)$$

with boundary condition (2.2). Note, that dot over functional $\xi (t|W(t))$ is total derivative of Ito type

$$\dot{\xi} = \delta \xi (t|W(t)) = \partial_t \xi + \frac{1}{2} \delta^2 \xi + (\delta W \xi) d_t W(t), \quad (3.2)$$

where $\delta W$ -stands for functional derivative

$$\delta_W \xi = \left\{ \delta \xi (t|W(t)) / \delta W (t') \right\}_{t=t'}. \quad (3.3)$$

Like the following from (2.2) condition, the solution (3.1) have a following asymptotic behavior

$$\xi (t|W(t)) \rightarrow \exp(i\Omega_{in} t) \quad (3.4)$$

**Theorem:** If the etalon SDE (3.1) take place, then the SDE (2.1) for complex process have an exact solution

$$\Psi_{stc}^\pm (n; x, t|W(t)) = \left[ \frac{(\Omega_{in}/\pi)^{1/4}}{2 \pi^{1/4} n!} \right]^{1/2} \times$$

$$\times \exp \left\{ -i(n+\frac{1}{2})\Omega_{in} \int_{-\infty}^{t} dt' + i \frac{\dot{\xi}^2}{2 \Omega_{in}} \right\} H_n \left( \sqrt{\Omega_{in} \xi} \right), \quad (3.5)$$

that due to condition (3.4) goes to asymptotic state (2.4) in the limit $t \rightarrow -\infty$.

**The Prove.**

The solution of equation (3.1) can be represented in the following form

$$\xi (t|W(t)) = \sigma (t|W(t)) \exp [ir (t|W(t))], \quad \sigma (t|W(t)) = |\xi (t|W(t))|. \quad (3.6)$$

It is obvious, that differentials of Ito type exist also for $\sigma (t|W(t))$ and $r (t|W(t))$ functionals.

For further analytic investigation of the problem it is useful to introduce the scales of length $\sigma (t|W(t))$ and time $\tau = r (t|W(t))/\Omega_{in}$. How one can see, these scales have a stochastic character unlike the case of regular problem of parametric quantum oscillator. Going to investigation of SDE (2.1) let us make transformation $x \rightarrow y = x/\sigma (t|W(t))$, then equation (2.1) will as follows:

$$\hat{L} (y, t|W(t)) \tilde{\Psi}_{stc} = 0, \quad \tilde{\Psi}_{stc} (y, t|W(t)) = \Psi_{stc} (x, t|W(t)), \quad (3.7)$$
\[ \hat{L} (y, t|W(t)) = i\delta_t - i\frac{\dot{\sigma}}{\sigma} y\delta_y + \frac{1}{2\sigma^2}\delta_y - \frac{\sigma^2}{2}\Omega^2 (t|W(t)) y^2. \]  

(3.8)

Representing the solution of equation (3.7) in a following form

\[ \tilde{\Psi}_{stc} (y, t|W(t)) = \left\{ \exp \left[ \frac{2\Lambda (t|W(t)) y^2}{\sigma (t|W(t))} \right] \right\}^{1/2} \Phi \left( y, \int_{-\infty}^{t} \frac{dt'}{\sigma^2(t'|W(t'))} \right) \]  

(3.9)

and after transformations \( t \to \tau = r (t|W(t)) / \Omega_{in} \) we get from (3.7)-(3.8):

\[ i \left[ \Lambda - \frac{1}{2}(\frac{\dot{\sigma}^2}{\sigma}) \right] (\Phi + 2y\delta_y\Phi) + i \left( \frac{\dot{r}^2}{\Omega_{in}} \right) \delta \Phi = -\frac{1}{2} \left\{ \delta_y^2 - \sigma^2 \left[ 2\Lambda - 4\dot{\sigma} \Lambda + 4\sigma^{-2}\Lambda^2 + \sigma^2\Omega^2 (t|W(t)) y^2 \right] \right\} \Phi \]  

(3.10)

Thus after transformation \((x, t) \to (y, \tau)\) and substitution (3.9) from (2.1) we arrive to equation (3.10), where the functionals \( \sigma (t|W(t)), r (t|W(t)) \) and \( \Lambda (t|W(t)) \) still remain to be determined. For their determination let us subject them to realization of following conditions:

\[ \dot{r} (t|W(t)) = \Omega_{in}/\sigma^2 (t|W(t)), \]  

(3.11)

\[ \Lambda (t|W(t)) = \dot{\sigma} (t|W(t)) \sigma (t|W(t))/2, \]  

(3.12)

\[ 2\Lambda - 4\dot{\sigma} \Lambda + 4\sigma^{-2}\Lambda^2 + \sigma^2\Omega^2 (t|W(t)) = \Omega_{in}^2/\sigma^2. \]  

(3.13)

If we assume that first variation of \( r (t|W(t)) \) functional due to \( W(t) \) process is equal to zero, i.e. \( \delta_W r (t|W(t)) = 0 \), then from equation (3.11) it follows, that stochastic time \( \tau \) is coupled with natural parameter (usual time) \( t \) via the following integral transformation:

\[ \tau = \int_{-\infty}^{t} \frac{dt'}{\sigma^2(t'|W(t'))}. \]  

(3.14)

As to equations (3.12) and (3.13) it is easy to show, that their combination bring the equation (3.1) for complex process \( \xi (t|W(t)) \). By taking into account expressions (3.11)-(3.13) from (3.10) one can obtain the following equation:

\[ \hat{L}_0 (y, \tau) \Phi (y, \tau) = 0, \]  

(3.15)

\[ \hat{L}_0 (y, \tau) = i\delta_t + \frac{1}{2}\delta_y^2 - \frac{1}{2}\Omega_{in}^2 y^2. \]  

(3.16)

It is clear that equations (3.15) and (3.16) describe autonomic quantum system, but on the stochastic space-time continuum. Solution of (3.15)-(3.16) have the following form:

\[ \Phi (y, \tau) = \left[ \frac{(\Omega_{in}/\pi)^{1/2}}{2^nn!} \right]^{1/2} \exp \left\{ -i(n+\frac{1}{2})\Omega_{in}\tau - \frac{1}{2}\Omega_{in}y^2 \right\} H_n \left( \sqrt{\Omega_{in}}y \right) \]  

(3.17)
Combining (3.9) and (3.17) for the complex process Ψ\textsuperscript{stc}(n; x, t|W(t)) one can obtain final expression (3.5), that had to be proved. It is clear from (3.5), that had stochastic process Ψ\textsuperscript{stc}(n; x, t|W(t)) at the limit t → −∞ goes to asymptotic state (2.4).

In conclusion let us pay attention to the following important feature of complex stochastic process Ψ\textsuperscript{stc}(n; x, t|W(t)):

\[
\langle \Psi\textsuperscript{+}_{stc}(m; x, t|W(t))\Psi\textsuperscript{+}_{stc}(n; x, t|W(t)) \rangle_x = \delta_{mn}, \quad \langle ... \rangle_x = \int_{-\infty}^{\infty} ... \, dx,
\]

that shows the fact, that wave functionals make up the full orthonormal basis.

4. The derivation of Langevin equation for the real stochastic process θ(t)

Now, after determination of the basis in the space of complex functionals Ψ\textsuperscript{+}_{stc}(n; x, t|W(t)), we can pass to construction of expression for averaged wave state Ψ\textsuperscript{br}(n; x, t) of quantum random oscillator. For this purpose at first it is necessary to determine the measure of functional space \( R\{W(t)\} \) on which stochastic process Ψ\textsuperscript{+}_{stc}(n; x, t|W(t)) will be averaged.

Returning to equation (3.1) let us note, that in general case its analysis is very difficult and for its further analytical investigation it is necessary to finalize some features of ξ(t|W(t)).

**Theorem**: If the functional ξ(t|W(t)) is subjected to the conditions

\[
\delta_W \xi(t|W(t)) = 0, \quad \delta_W \{\partial_t \xi(t|W(t))\} \neq 0,
\]

then the stochastic equation (3.1) turns in to nonlinear equation of Langevin type

\[
\dot{\theta} + \theta^2 + \Omega^2_0(t) + F(t|W(t)) = 0,
\]

where \( F(t|W(t)) \) is the generator of stochastic force.

**The Prove:**

The solution of model equation (3.1) can be represented in the following form:

\[
\xi(t|W(t)) = \xi_0(t) \exp \left( \int_{-\infty}^{t} \Phi(t'|W(t')) \, dt' \right),
\]

where \( \xi_0(t) \) is the solution of equation (3.1) with regular frequency \( \Omega_0(t) \).

After substitution (4.1) into (3.1) with regular frequency \( \Omega_0(t) \), one gets for Φ(t|W(t)) the stochastic nonlinear equation of Langevin type

\[
\dot{\Phi} + 2\dot{\xi}_0\xi^{-1}_0\Phi + \Phi^2 + F(t|W(t)) = 0,
\]

\[
\Omega^2(t|W(t)) = \Omega^2_0(t) + F(t|W(t)).
\]

After transformation

\[
\Phi(t|W(t)) = \theta(t|W(t)) - \dot{\xi}_0(t)/\xi_0(t)
\]

we pass from (4.4) to the equation (4.2). Let us note, that transformation (4.5) is equivalent to transition to regular moving coordinate system in complex functional space.
As to \( \theta (t|W(t)) \) functional, it belongs to real functional space \( R_{\theta(t)} \). Thus, the theorem is proved.

### 5. Investigation of Fokker-Plank equation. Determination of the measure of functional space \( R_{\{\theta(t)\}} \)

Let us pass to derivation of the evolutional equation for condition probability \( P(\theta, t|\theta', t') \). We shall study the functional of the form

\[
P(\theta, t|\theta', t') = \left< \delta[\theta(t) - \theta(t')] \right>_{W(t)},
\]

where \( \theta(t) \) is the solution of nonlinear Langevin equation (4.2). After differentiating (5.1) over the time and using (4.2) one can obtain

\[
\partial_t P(\theta, t|\theta', t') = -\partial_\theta \left< \delta[\theta(t) - \theta(t')] \right>_{W(t)} = 
\]

\[
= \partial_\theta \left\{ [\theta^2 + \Omega_0^2(t)] P(\theta, t|\theta', t') + \left< F(t|W(t)) \delta[\theta(t) - \theta(t')] \right>_{W(t)} \right\}.
\]

The second member in the rhs of equation (5.2) still remains undetermined. For its calculation it is necessary to definite the type of stochastic force generator \( F(t|W(t)) \). As in most interesting cases the \( F(t|W(t)) = F(t) \) functional is the gaussian function, that in considered problem changes more quickly than \( \xi(t|W(t)) \), the choice of the model of ”white noise” for stochastic is quite suitable

\[
\left< F(t) F(t') \right> = 2\varepsilon \delta(t - t'), \quad (F(t)) = 0, \quad \varepsilon > 0.
\]

Now using the Vick theorem (see [11])

\[
\left< F(t) N(F(t)) \right>_{\{F(t)\}} = 2 \left< \frac{\delta N}{\delta F} \right>_{\{F(t)\}},
\]

where \( N(F(t)) \) is arbitrary functional of \( F(t) \), one can write the following expression

\[
\left< F \delta [\theta(t) - \theta(t')] \right> = -2 \left< \delta \theta(t) / \delta F(t) \right> \partial_\theta \delta [\theta(t) - \theta(t')]_{W(t)} = 
\]

\[
= -2\partial_\theta \left< \delta \theta(t) / \delta F(t) \right> \delta [\theta(t) - \theta(t')]_{W(t)}.
\]

Variational derivative of \( \theta(t) \) due to stochastic force \( F(t) \) equals to \( \varepsilon \cdot \text{sgn}(t - t') + O(t - t') \). After regularization by standard procedure (in sense of Fourie decomposition) one can find it value for \( t = t' \): \( \varepsilon \cdot \text{sgn}(0) = \frac{1}{2}\varepsilon \). Taking into account the above said notations now we can obtain now the final expression for Fokker-Plank equation for conditional probability:

\[
\partial_t P(\theta, t|\theta', t') = \partial_\theta \left\{ [\theta^2 + \Omega_0^2(t)] + \varepsilon \partial_\theta \right\} P(\theta, t|\theta', t').
\]

Note, that (5.6) determines the diffusional process, for which \( \theta(t) \) is continuous.
Let the probability be subjected to boundary condition \( P(\theta, t|\theta', t) = \delta(\theta - \theta') \), then, for small time intervals the solution of equation (5.6) is straightforward [12]:

\[
P(\theta, t|\theta', t') = (2\pi \varepsilon \Delta t)^{-1/2} \exp \left\{ -\frac{\theta - \theta' - (\theta^2 + \Omega_0^2(t)) \Delta t^2}{2\varepsilon \Delta t} \right\}, \quad t = t' + \Delta t.
\] (5.7)

It is clear, that the evolution of the system in the functional space \( R_{\{\theta(t)\}} \) is governed by the regular shift with the speed \( (\theta^2 + \Omega_0^2(t)) \) modulated by quantum Gaussian fluctuations with constant correlations \( \varepsilon \).

After this we can establish some properties of the trajectory \( \theta(t) \) in the space \( R_{\{\theta(t)\}} \).

It is given by the formula (see [12])

\[
\theta(t + \Delta t) = \theta(t) + (\theta^2(t) + \Omega_0^2(t)) \Delta t + F(t) \Delta t^{1/2},
\] (5.8)

and it is not difficult to show, that the trajectory are continuous everywhere, that is \( \theta(t + \Delta t) \xrightarrow{\Delta t \to 0} \theta(t) \), but has no derivative anywhere due to the member \( \sim \Delta t^{1/2} \) in (5.8).

Suppose, that \( \Delta t = t/N \), with \( N \to \infty \), then eq. (5.7) can be regarded as transition probability for \( \theta(t') = \theta_k \to \theta_{k+1} = \theta(t) \) at a time \( \Delta t \) in the model of Brownian motion. The eq. (5.7) thus gives the total Fokker-Plank measure of the space \( R_{\{\theta(t)\}} \).

Now we can construct the full wave function of random QRHO. Using expression (2.5) and turning to moving coordinate system by means of regular shift (4.5) for the wave functional the final expression is obtained:

\[
\Psi_{stc}^+(n; x, t) = \left\langle \Psi_{stc}^+(n; x, t|\theta(t)) \right\rangle_{\{\theta(t)\}} = \int D\mu\{\theta(t)\} \Psi_{stc}^+(n; x, t|\theta(t)).
\] (5.9)

In (5.9) by \( D\mu\{\theta(t)\} \) the measure of functional space \( R_{\{\theta(t)\}} \) is denoted:

\[
D\mu\{\theta(t)\} = \alpha^{-1} d\mu\{\theta_0\} \times d\mu\{\theta_t\} \lim_{N \to \infty} \{ (2\pi \varepsilon t/N)^{-N/2} \times \]

\[
\prod_{k=0}^{N} \exp \left[ -\frac{1}{2\varepsilon} (\theta_{k+1} - \theta_k - (\theta_k^2 + \Omega_0^2(t)) t/N)^2 \right] d\theta_{k+1},
\]

where \( \alpha, d\mu\{\theta_0\} \) and \( d\mu\{\theta_t\} \) are determines, accordingly, by the following expressions:

\[
\alpha = \int D\mu\{\theta(t)\},
\] (5.11)

\[
d\mu\{\theta_0\} = \delta \left( \theta_0 - \dot{\xi}_0(t)/\xi_0(t) \right) d\theta_0,
\] (5.12)

\[
d\mu\{\theta_t\} = P(\theta, t|0, 0) d\theta_t.
\] (5.13)

In the formulae (5.9)-(5.13) \( \alpha \) is normalization constant for the functional integral (5.9) with full Fokker-Plank measure, nonequal to one, integration over \( d\mu\{\theta_0\} \) measure provides transition to moving coordinate system and integration over \( d\mu\{\theta_t\} \) measure provides, accordingly, the process of averaging by coordinate distribution \( \theta \) in a moment of time \( t \). By integration over \( d\mu\{\theta_0\} \) measure in expression (5.9) it is possible to get the factorization of regular and chaotic motion. Then the wave function of Brownian particle will be rewritten as follow:
\[ \Psi^+_{br}(n; x, t) = \Psi^+(n; x, t) \int D\mu\{\theta(t)\} \Psi^+_{stc}(n; x, t|\theta(t)), \quad (5.14) \]

where \( D\mu\{\theta(t)\} = D\mu\{\theta(t)\}/d\mu\{\theta_0\} \).

**6. Solution of the equation for distribution function of stationary Markovian process**

Let's consider the probability \( P(\theta, t|0, 0) = Q(\theta, t) \) that characterizes the distribution of the coordinate \( \theta \) in the \( R_{\{\theta(t)\}} \)-space as a function of time "\( t \)". In this case (5.6) should be interpreted as the conservation law for probability density

\[ \partial_t Q(\theta, t) + \partial_\theta J(\theta, t) = 0, \quad J(\theta, t) = -(\theta^2 + \Omega^2(\theta^2)) Q(\theta, t) - \varepsilon \partial_\theta Q(\theta, t), \quad (6.1) \]

with the initial and boundary conditions

\[ \lim_{t \to -\infty} Q(\theta, t) = \delta(\theta), \quad \lim_{|\theta| \to +\infty} Q(\theta, t) = 0. \quad (6.2) \]

For the boundary fluxes one has \( J_0 = J_0(-\infty, t) = J_0(+\infty, t) \) and it does not vanish since \((\theta^2 + \Omega^2(\theta^2))\) on that boundaries turns to be infinity. At the limit \( t \to +\infty \) the flux density turns to its limit value

\[ J_{0f} = \lim_{t \to +\infty} \{ J(\theta, t) \text{ sign}(\dot{\theta}(t)) \}, \quad J_{0f} = J_0(\Omega^2_{out}). \quad (6.3) \]

From equation (4.2) it follows that \( \lim_{t \to +\infty} \dot{\theta}(t) < 0 \) and as a consequence \( J_{0f} > 0 \). As a result the equation for probability distribution \( Q_s(\theta) \) for stationary process can be derived from equations (6.1) and (6.3)

\[ J_{0f} = (\theta^2 + \Omega^2_{out}) Q_s + \varepsilon d_\theta Q_s, \quad d_\theta = d/d\theta. \quad (6.4) \]

It may be easily solved, giving

\[ Q_s(\varepsilon, \Omega_{out}; \theta) = \varepsilon^{-1/3} \widetilde{Q}_s(\lambda, \gamma; \overline{\theta}) = \frac{J_{0f}}{\varepsilon^{2/3}} \exp \left( \frac{\theta^3}{3} - \lambda \gamma \overline{\theta} \right) \int_{-\infty}^{\overline{\theta}} dz \exp \left( \frac{z^3}{3} + \lambda \gamma z \right) \quad (6.5) \]

where \( \lambda = (\Omega_{in}/\varepsilon^{1/3})^2, \gamma = (\Omega_{out}/\Omega_{in})^2, \overline{\theta} = \theta/\varepsilon^{1/3}. \)

The constant \( J_{0f} \) may be calculated from the normalization condition and has the form [13]:

\[ J_{0f}^{-1} = \pi \varepsilon^{-1/3} \mathcal{J}_{0f}^{-1} = \pi^{1/2} \varepsilon^{-1/3} \int_0^{\infty} dz \, z^{-1/2} \exp \left( -\frac{z^3}{12} - \lambda \gamma z \right). \quad (6.6) \]

For the \( \mathcal{J}_{0f} \) one can obtain another representation via the special functions. It may be done by passing to Fourier components in the equation (6.4) [14]:

\[ J_{0f}^{-1} = \pi \varepsilon^{-1/3} \mathcal{J}_{0f}^{-1} = \pi \varepsilon^{-1/3} \left[ A t^2 (-\lambda \gamma) + B t^2 (-\lambda \gamma) \right], \quad (6.7) \]
where $Ai(x)$ and $Bi(x)$ are linear independent solutions of Airy equation [15]:

$$y'' - xy = 0.$$  \hfill (6.8)

Numerical calculations of function of distribution $\bar{Q}_s(\lambda; \bar{\theta}) \equiv \bar{Q}_s(\lambda, \gamma = 1; \bar{\theta})$ in dependence of $\bar{\theta}$ from (6.5) and (6.6) for some values of parameter $\lambda$ when $\gamma = 1$ are shown on fig. 1.

Fig. 1. Distribution of stationary process $\bar{Q}_s(\lambda; \bar{\theta})$ over $\bar{\theta}$ in dependence of parameter $1/\lambda \sim \varepsilon$.

It is visible, that when $\varepsilon \to 0$, i.e. when passing to regular case in initial problem (2.1), the function of distribution of stationary process turn to delta-function of Dirac.

$$\lim_{\varepsilon \to 0, \gamma \to \infty} \bar{Q}_s(\lambda, \gamma; \bar{\theta}) = \delta(\bar{\theta}).$$ \hfill (6.9)

7. Calculation of transition amplitude for the random QRHO

The transition matrix for the random QRHO will be evaluated as a limit $t \to +\infty$ of the projection of the total averaged wave function (5.9) on the asymptotic wave function $\Psi_{\text{out}}(m; x, t)$

$$S_{mn}^{br} = \lim_{t \to +\infty} \langle \Psi_{\text{out}}(m; x, t) \Psi_{br}^+(n; x, t) \rangle_x.$$ \hfill (7.1)

Taking into account, that measure in the functional integral is real and positively defined (5.7) we can change the order of integration in the expression (7.1) and represent the transition matrix in the following form:

$$S_{mn}^{br} = \lim_{t \to +\infty} \langle \bar{S}_{mn}^{stc}(t|\xi(t)) \rangle_{\{\xi(t)\}} = \lim_{t \to +\infty} \langle S_{mn}^{stc}(t|\theta(t)) \rangle_{\{\theta(t)\}},$$ \hfill (7.2)

were $\bar{S}_{mn}^{stc}(t|\xi(t))$ is a stochastic transition matrix,
\[ \tilde{S}_{mn}^{stc} (t|\xi(t)) = \left\langle \Psi_{out}^+ (m; x, t) \Psi_{stc}^+ (n; x, t|\xi(t)) \right\rangle_x , \quad \left[ \xi(t) \right] \equiv \xi(t|W(t)). \quad (7.3) \]

It should be noted, that when Hamiltonian (2.1) is a real function, stochastic matrix \( \tilde{S}_{mn}^{stc} (t|\xi(t)) \) as well as its averaged value \( S_{mn}^{br} \) are unitary ones. If Hamiltonian is a complex function the unitarity of this matrix breaks down.

Now we will pass to the calculation of expression for transition probability. Taking into account completeness and orthogonality of functional basis \( \Psi_{stc}^+ (n; x, t|\xi(t)) \) (see (3.6)) calculation of stochastic matrix elements \( \tilde{S}_{mn}^{stc} (t|\xi(t)) \) is convenient to carry out by generating functionals method. Let us construct generating functional in following form:

\[ \Psi_{stc}^+ (z, x, t|\xi(t)) = \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} \Psi_{stc}^+ (n; x, t|\xi(t)) , \quad (7.4) \]

where \( z \) is some subsidiary complex function. After substitution of expression for \( \Psi_{stc}^+ (n; x, t|\xi(t)) \) from (3.5) to (7.4) and carrying out summation \([10, 15]\) we find the following equation:

\[ \Psi_{stc}^+ (z, x, t|\xi(t)) = \left( \frac{\Omega_m}{\pi} \right)^{\frac{1}{2}} \frac{1}{\sqrt{\xi}} \exp \left\{ -\frac{1}{2} (ax^2 - 2bx + c) \right\} , \quad (7.5) \]

where \( a, b \) and \( c \) have following type:

\[ a (t|\xi(t)) = -i \frac{\dot{\xi}(t|W(t))}{\xi(t|W(t))} , \quad b = \sqrt{2\Omega_{in}} \frac{z}{\xi(t|W(t))} , \quad c = z^2 \exp (-2ir(t|W(t))). \quad (7.6) \]

As it seen from (7.6) the generating functional dependence over the \( x \) coordinate is stochastic gaussian packet. In a limit \( t \to -\infty \) (7.6) turns over to the ordinary gaussian packet

\[ \Psi_{stc}^+ (z, x, t|\xi(t)) \quad t \to -\infty \Psi_{in} (z, x, t) = \]

\[ = \left( \frac{\Omega_{in}}{\pi} \right)^{\frac{1}{2}} \exp \left\{ -\frac{1}{2} \left( \Omega_{in}x^2 - 2\sqrt{\Omega_{in}z}xe^{-i\Omega_{in}t} + z^2 e^{-i2\Omega_{in}t} + i\Omega_{in}t \right) \right\} . \quad (7.7) \]

The generating function of \((out)\) state can be obtained by making in (7.7) formal substitutions \( \Omega_{in} \to \Omega_{out} \) and \( z \to z_1 \).

Now we will consider the following integral:

\[ I (z_1, z_2; t|\xi(t)) = \left\langle \Psi_{out}^+ (z_1, x, t) \Psi_{stc}^+ (z_2, x, t|\xi(t)) \right\rangle_x . \quad (7.8) \]

Substituting expressions (7.5) and (7.9) to (7.8) and carrying out integration by \( x \) coordinate for the generating functional and generating function of \((out)\) asymptotic space one can obtain the following equation:

\[ I (z_1, z_2; t|\xi(t)) = (\Omega_{in}\Omega_{out})^{\frac{1}{2}} \left( \frac{2}{A\xi} \right)^{\frac{1}{2}} \exp \left\{ -\frac{1}{2} \left( C - \frac{B^2}{A} \right) \right\} , \quad (7.9) \]

where the following notations are made:
\[ A(t|\xi(t)) = -i\xi^{-1} + \Omega_{\text{out}}, \]

\[ B(t|\xi(t)) = \sqrt{2}\Omega_{\text{in}}\xi^{-1}z_2 + \sqrt{2}\Omega_{\text{out}} \exp(i\Omega_{\text{out}}t)z_1, \quad (7.10) \]

\[ C(t|\xi(t)) = \exp(-i2r)z_2^2 + \exp(i2\Omega_{\text{out}}t)z_1^2 - i\Omega_{\text{out}}t, \]

where \( z = |z| \exp(i\arg z) \), \( \bar{z} = |z| \exp(-i\arg z) \).

As it is easy to see, that the \( I(z_1,z_2;t|\xi(t)) \) integral is generating functional for matrix element \( \tilde{S}_{mn}^{\text{stc}}(t|\xi(t)) \)

\[ I(z_1,z_2;t|\xi(t)) = \sum_{m,n=0}^{\infty} \frac{z_1^m z_2^n}{\sqrt{m!n!}} \tilde{S}_{mn}^{\text{stc}}(t|\theta(t)). \quad (7.11) \]

Decomposing \( I(z_1,z_2;t|\xi(t)) \) into Taylor power series over \( z_1 \) and \( z_2 \) from (7.11) we find the following final expression

\[ \tilde{S}_{mn}^{\text{stc}}(t|\xi(t)) = \frac{1}{\sqrt{m!n!}} \left\{ \partial_{\bar{z}_1}^m \partial_{z_2}^n I(z_1,z_2;t|\xi(t)) \right\}_{z_1=z_2=0}. \quad (7.12) \]

Below the expressions for some first stochastic matrix elements are shown without some phases irrelevant for scattering process,

\[ \tilde{S}_{00}^{\text{stc}}(t|\xi(t)) = \sqrt{\sigma} (\Omega_{\text{in}}/\Omega_{\text{out}})^{\frac{1}{2}} (-i\xi/\Omega_{\text{out}} + \xi)^{-\frac{1}{2}}, \quad \tilde{S}_{10}^{\text{stc}}(t|\xi(t)) = \left( \tilde{S}_{00}^{\text{stc}}(t|\xi(t)) \right)^3, \]

\[ \tilde{S}_{02}^{\text{stc}}(t|\xi(t)) = \tilde{S}_{00}^{\text{stc}} \left[ -1 + \left( \frac{\Omega_{\text{in}}}{\Omega_{\text{out}}} \right)^{\frac{1}{2}} \frac{1}{|\xi|^2} \left( \tilde{S}_{00}^{\text{stc}} \right)^2 \right] \exp \left( -i2\Omega_{\text{in}} \int_{-\infty}^{t} dt' \right), \quad (7.13) \]

\[ \tilde{S}_{20}^{\text{stc}}(t|\xi(t)) = \tilde{S}_{00}^{\text{stc}} \left[ -1 + \left( \frac{\Omega_{\text{in}}}{\Omega_{\text{out}}} \right)^{\frac{1}{2}} \left( \tilde{S}_{00}^{\text{stc}} \right)^2 \right]. \]

The matrix elements (7.13) after regular shift (4.5) in functional space \( R(\theta(t)) \) have the following form:

\[ S_{00}^{\text{stc}}(t|\theta(t)) = \sqrt{2} (\Omega_{\text{in}}/\Omega_{\text{out}})^{\frac{1}{2}} (-i\theta/\Omega_{\text{out}} + 1)^{-\frac{1}{2}} \exp \left( -\frac{1}{2} \int_{-\infty}^{t} \theta(t') dt' \right), \]

\[ S_{11}^{\text{stc}}(t|\theta(t)) = \left( S_{00}^{\text{stc}}(t|\theta(t)) \right)^3, \quad S_{20}^{\text{stc}}(t|\theta(t)) = S_{00}^{\text{stc}} \left[ -1 + \left( \frac{\Omega_{\text{in}}}{\Omega_{\text{out}}} \right)^{\frac{1}{2}} \left( S_{00}^{\text{stc}} \right)^2 \right], \quad (7.14) \]

\[ S_{02}^{\text{stc}}(t|\theta(t)) = S_{00}^{\text{stc}} \left[ -1 + \left( \frac{\Omega_{\text{in}}}{\Omega_{\text{out}}} \right)^{\frac{1}{2}} \left( S_{00}^{\text{stc}} \right)^2 \exp \left( -2 \int_{-\infty}^{t} \theta(t') dt' \right) \right] \times \]

\[ \times \exp \left( -i2\Omega_{\text{in}} \int_{-\infty}^{t} e^{-2\int_{\theta(t')}^{t'} e^{-\theta(t'')}} dt' \right). \]
As it visible from expressions (7.13) and (7.14) for stochastic matrix elements the only transitions possible are transitions with same evens non-depending from value of fluctuation constant \( \varepsilon \). As to symmetry of matrix elements by oscillation quantum numbers of initial and final channels \( n \) and \( m \), it breaks down as a result of irreversible character of quantum mechanics constructed here.

To demonstrate the proposed approach we shall represent the results of evaluation of the "vacuum-vacuum" transition probability under the condition of Wiener’s process. Using equations (5.9)-(5.11) and (6.5)-(6.6) from (7.1) one obtains:

\[
S_{00}^{br} (\lambda, \rho) = (1 - \rho)^{\frac{d}{2}} \{ I_1 (\lambda, \gamma) - i I_2 (\lambda, \gamma) \},
\]

(7.15)

\[
I_1 (\lambda, \gamma) = \int_{-\infty}^{+\infty} d\theta \frac{1}{d} \sqrt{\frac{d+1}{2}} \bar{Q}_s (\lambda, \gamma; \bar{\theta}),
\]

(7.16)

\[
I_2 (\lambda, \gamma) = \int_{-\infty}^{+\infty} d\theta \frac{1}{d} \sqrt{\frac{d-1}{2}} \bar{Q}_s (\lambda, \gamma; \bar{\theta}), \quad d (\lambda, \gamma; \bar{\theta}) = \left( 1 + \frac{\bar{\theta}^2}{\lambda \gamma} \right)^{\frac{1}{2}}.
\]

(7.17)

Here \( \rho \) is a reflection coefficient of the correspondent one-dimensional quantum problem (see [16]), \( \gamma (\rho) \) is denoted by the barrier shape i.e. by the frequency \( \Omega_0(t) \). In the case of the step-shape (fig. 2) barrier one has:

\[
\gamma (\rho) = \left( \frac{\Omega_{out}}{\Omega_{in}} \right)^2 = \left( \frac{1 + \rho^{1/2}}{1 - \rho^{1/2}} \right)^2.
\]

(7.18)

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\]

(7.18)

Fig. 2. Barrier model of dependence of frequency \( \Omega \) over time \( t \). It is clear, that changes of \( \delta \) in range of \( 0 \leq \delta < \infty \) cause changes of reflection coefficient \( \rho \) in range of \( 0 \leq \rho \leq 1 \).

As a result, using equations (6.5)-(6.7) and (7.14)-(7.17), for the probability of "vacuum-vacuum" transition one obtains:

\[
\Delta_{0\rightarrow 0}^{br} (\lambda, \rho) = \left| S_{00}^{br} (\lambda, \rho) \right|^2,
\]

(7.19)
\[ |S_{00}^{br}(\lambda, \rho)|^2 = \sqrt{1 - \rho \{ I_1^2(\lambda, \gamma) + I_2^2(\lambda, \gamma) \}}. \] (7.20)

The result of calculation of the transition probability (7.18)-(7.19) are represented on (fig. 3-4) as a function of \( \rho \) and \( \lambda \).

Fig. 3. "Vacuum-vacuum" transition probability in dependence of \( \lambda \) and \( \rho \).

Fig. 4. Dependence of "vacuum-vacuum" transition probability over \( \lambda \) in the case when \( \rho = 0 \).

As it visible from fig. 3, the probability of "vacuum-vacuum" transition starting from some value of \( \lambda \) (or \( \varepsilon \)) have nonmonotonic behavior in depending over reflection coefficient \( \rho \). This fact distinguish the stochastic problem from regular one.
8. The vacuum thermodynamics for the asymptotic space \( \Xi_s \subset \Xi \)

It is well known, that the principal object of interest for quantum statistical mechanics is the density matrix \( \rho(x, x') \), that after Dirac and von Neyman (see, [17]) is determined by expression

\[
\rho(x, x') = \sum_k P_k \varphi_k(x) \overline{\varphi_k(x')}, \tag{8.1}
\]

with distribution function for canonical distribution \( P_k = \exp(-\beta E_k) \), and wave function being the solution of Schrödinger equation \( \hat{H} \varphi_k = E_k \varphi_k \), \( \beta = (kT)^{-1} \) with \( T \) being the temperature of the system and \( k \) being the Boltzmann constant.

Since for our problem the wave function of the system is the complex stochastic process it is natural to turn to thermodynamic description. It is possible to develop such picture even for the group of states, corresponding to quantum number "\( n \)".

Here we discuss the thermodynamics of vacuum in the asymptotic space \( \Xi_s = R^1 \otimes R_{\{W_s\}} = \lim_{t \to +\infty} R^1 \otimes R_{\{W(t)\}} \).

Definition 1. The stochastic density matrix for vacuum in the space \( \Xi_s \)

\[
\rho_{ste} (x, t; \theta(t) \mid x', t'; \theta(t')) = \left\{ \Psi_{ste}^+(0; x, t \mid \theta(t)) \overline{\Psi_{ste}^+(0; x', t' \mid \theta(t'))} \right\}. \tag{8.2}
\]

Definition 2. The expectation value of stochastic operator \( \hat{A}(x, t \mid \theta(t)) \) in the vacuum have a next form

\[
\langle \hat{A} \rangle_{vac} = \lim_{t \to +\infty} \left\{ Tr_x \left( \langle \hat{A}\rho_{ste} \rangle_{\{\theta(t)\}} \right) / Tr_x \left( \langle \rho_{ste} \rangle_{\{\theta(t)\}} \right) \right\}, \tag{8.3}
\]

with being the trace over the coordinate \( x \).

Definition 3. The nonequilibrium partition function of the vacuum and quantum oscillator system is

\[
\vartheta_{osc vac} (\varepsilon, \Omega_{as}; t) = Tr_x \left\{ \langle \rho_{ste} \rangle_{\{\theta(t)\}} \right\}. \tag{8.4}
\]

Knowing the partition Function it is easy to determine all the thermodynamic properties of the system:

a) average internal energy

\[
U_{vac} (\varepsilon, \Omega_{as}) = \lim_{t \to +\infty} U_{vac} (\varepsilon, \Omega_{as}; t), \quad U_{vac} (\varepsilon, \Omega_{as}) = -\partial_\varepsilon \left\{ \ln \vartheta_{osc vac} (\varepsilon, \Omega_{as}, t) \right\}, \tag{8.5}
\]

b) free Helmholtz energy

\[
F_{vac} (\varepsilon, \Omega_{as}) = -\varepsilon^{-1} \lim_{t \to +\infty} \left\{ \ln \vartheta_{vac} (\varepsilon, \Omega_{as}; t) \right\}, \tag{8.6}
\]

c) the Entropy

\[
S_{vac} (\varepsilon, \Omega_{as}) = \varepsilon k \left\{ U_{vac} (\varepsilon, \Omega_{as}) - F_{vac} (\varepsilon, \Omega_{as}) \right\}. \tag{8.7}
\]

The practical computations start from the stochastic density matrix
\[
\rho_{\text{stc}} (x, t; \theta(t) | x', t'; \theta(t')) = (\Omega_{\text{as}}/\pi)^{1/2} \exp\left\{ -\Omega_{\text{as}} (x^2 + x'^2)/2 - \frac{1}{2} \int_{-\infty}^{t} \theta(\tau) d\tau - \frac{1}{2} \int_{-\infty}^{t'} \theta(\tau) d\tau - i [\theta(t)x^2 - \theta(t')x'^2] \right\},
\]

(8.8)

with \(\Omega_{\text{as}}\) being the frequency in the asymptotical space \(\Xi_s\). The stochastic Hamiltonian (2.1) is discussed as an example of stochastic operator \(\hat{A}(x, t | \theta(t))\). After the nondifficult calculation from (8.3) with taking into account (8.8) one can obtain the energy of vacuum+oscillator system

\[
E (\lambda; \Omega_{\text{as}}) = \frac{1}{2} \Omega_{\text{as}} \int_{0}^{\infty} dzz^{-3/2} \exp (-z^3/12 - \lambda z) + \frac{1}{2} \Omega_{\text{as}} \left\{ 1 - \frac{1}{\chi} \int_{0}^{\infty} dzz^{3/2} \exp (-z^3/12 - \lambda z) + \langle F \rangle / 4\Omega_{\text{as}}^2 \right\} + i \frac{1}{2\sqrt{\lambda}} \Omega_{\text{as}} \int_{0}^{\infty} dzz^{1/2} \exp (-z^3/12 - \lambda z).
\]

(8.9)

As it clear from (8.9), the first term in the energy expression diverges, corresponding to the infinite energy of the vacuum. The second term corresponds to the oscillator energy, that is shifted by the interaction with vacuum

\[
E_{\text{osc}}^{\text{vac}} (\lambda; \Omega_{\text{as}}) = \frac{1}{2} \Omega_{\text{as}} \left\{ 1 - \frac{1}{\lambda} \partial_\alpha [(1 + \alpha) \partial_\alpha \ln A (-\lambda + \alpha)] \right\} |_{\alpha=0},
\]

(8.10)

\[
A (-\lambda + \alpha) = Ai^2 (-\lambda + \alpha) + Bi^2 (-\lambda + \alpha),
\]

and \(\langle F \rangle\) in (8.10) in our situation is zero. Note, that second term in (8.10) is analog the Lamb shift of the energy level it is well-known from the standard quantum electrodynamics [18]. The third term in (8.9) corresponds to the width of the energy ground state and is inverse proportional to its decay time

\[
\Delta t = 2 \sqrt{\lambda \Omega_{\text{as}}} \left\{ \partial_\alpha \ln A (-\lambda + \alpha) \right\} |_{\alpha=0}.
\]

(8.11)

Now let us calculate the basic thermodynamical function of the system - the Entropy \(S_{\text{vac}} (\lambda; \Omega_{\text{as}})\).

We start from the expression for partition function (8.4)

\[
\vartheta_{\text{vac}} (\varepsilon; t) = Tr_x \left\{ \langle \rho_{\text{stc}} \rangle_{\theta(t)} \right\} = B_0(t)B_1(\varepsilon, \Omega; t),
\]

(8.12)

with \(B_0(t)\) and \(B_1(\varepsilon, \Omega; t)\) being determined by the conditions

\[
B_0(t) = \left\langle \exp \left(- \int_{-\infty}^{t} \theta (t') dt' \right) \right\rangle_{\{\theta(t)\}} = \int_{-\infty}^{\infty} d\theta u(\theta, t),
\]

(8.13)

\[
B_1(\varepsilon, \Omega_{\text{as}}; \theta, t) = \int_{-\infty}^{\infty} d\theta Q(\varepsilon, \Omega_{\text{as}}; \theta, t),
\]

(8.14)
and \( u(\theta, t) \) in (8.14) by Feynman-Kac theorem being the solution of parabolic equation [19]

\[
\partial_t u(\theta, t) = \frac{1}{2} \partial^2_\theta u(\theta, t) - \theta u(\theta, t),
\]

(8.15)

with initially and boundary conditions of the type (6.2).

It is easy to show, that those solutions do not depend upon the volume of \( \varepsilon \) and the limit of \( B_0(t) \) at \( t \to +\infty \) is \( 2^{-1/3} \). But the function \( B_1(\varepsilon, t) \) is clearly dependent on \( \varepsilon \).

The general solution of (6.1) with additional conditions (6.2) can be represented in the form

\[
Q(\varepsilon, \Omega_{as}; \theta, t - t') = \sum_{k=0}^{\infty} e^{-\lambda_k (t-t')} Q_s^k(\varepsilon, \Omega_{as}; \theta), \quad t \succ t', \quad t' \to -\infty,
\]

(8.16)

with \( \lambda_0 = 0 \), and \( Q_s^0(\varepsilon, \Omega_{as}; \theta) = Q_s(\lambda, \gamma; \theta) \). So after differentiating of (8.12) and taking into account (8.14)-(8.16) in the limit of \( t \to +\infty \) one gets for the average internal energy the expression

\[
U_{vac}(\varepsilon, \Omega_{as}) = -\int_{-\infty}^{\infty} d\theta \varepsilon Q_s(\varepsilon, \Omega_{as}; \theta).
\]

(8.17)

After straightforward computation we have

\[
U_{vac}(\varepsilon, \Omega_{as}) = U_{vac}(\lambda) = \frac{1}{3\varepsilon} \{ 1 + 2\lambda \partial_\alpha \ln(A(-\lambda + \alpha)) \} |_{\alpha=0}.
\]

(8.18)

Taking into account (8.12)-(8.15) it is possible to have the expression also for Helmholtz Free Energy

\[
F_{vac}(\varepsilon, \Omega_{as}) = \frac{1}{3\varepsilon} \ln 2,
\]

(8.19)

and for the Entropy of the vacuum can be represented in the form

\[
S_{vac}(\varepsilon, \Omega_{as}) = S_{vac}(\lambda) = \frac{2k\lambda}{3} \{ \partial_\alpha \ln(A(-\lambda + \alpha)) \} |_{\alpha=0} + \frac{k}{3} (1 - \ln 2).
\]

(8.20)

That give the expressions for all thermodynamical potentials of quantum oscillator in the ground state, interacting with vacuum. The fig. 5. shows dependence of energy of oscillator ”ground state”, its shift and entropy of vacuum over parameter \( \lambda \) in units of Boltzmann constant \( k \). It is visible, that when system turn to balance state (\( \varepsilon \to 0 \), i.e. \( \lambda \to \infty \)), the entropy aspire to maximum value.
9. Conclusion

Chaos in quantum systems was observed first by one of the founders of quantum mechanics Wigner, when he studied the nucleus energy spectrum [4]. However, Wigner and many other researchers associated this phenomena with nonclear and exotic nature of nucleus interactions, and they have exit from this difficult situation by introducing some urge parameters to the nucleus statistical theory. But, as it was shown by further investigations, the chaos arise in spectrum of quantum systems with some particular interaction potentials, for example, hydrogen atom in strong magnetic field. Some modern researches of modeling of bimolecular chemical reaction [5-6] showed, that chaos affects the wave function of quantum system. In other words, we have obvious example of violation of deterministic principle related to the basic object of quantum mechanics - wave function. To overcome this difficulty the authors in the framework of internal time idea (see [9], [20]) have developed new representation for multichannel scattering. It was proved, that system, including three or more particles, in general case have chaotic internal time. The last shows, that constructed quantum theory in general case being irreversible in relation to that time. It must be noticed, that chaos may be caused not only by the difficult dynamics of the quantum system, but also by the strong interaction of system with thermostat (with vacuum in our case). This situation was investigated in framework of one-dimensional random QHRO model. The main idea consist in representation of the wave function as a complex probabilistic process $\Psi_{stc}(x, t|W(t))$ on the extended space $\Xi = R^1 \otimes R_{\{W(t)\}}$. Using the model one-dimensional nonlinear Langevin equation the separation of variables in initial SDE for wave function was made and stochastic basis set $\Psi_{stc}^+(n; x, t|W(t))$ of quantum system was obtained. One of the very important features of such representation is that at least for the closed system ”vacuum+oscillator” the nonlinear Langevin SDE generate real full Fokker-Plank measure in the functional space $R_{\{W(t)\}}$. This circumstance provide exact mathematical basis of the constructed mixed functional-wave representation of random QHRO wave function $\Psi_{br}^+(n; x, t)$. The developed theory unificate two inconsistent concepts: the quantum analog of Arnold’s transformation, that don’t admit arising of chaos inside the trajectories beam,that described by the similar topology, and functional integral method, that allows to run over the current tubes of arbitrary topology in functional space $R_{\{W(t)\}}$ and to generate chaos. In another words,
the proposed theory allows to establish the connection between the chaotic classical and chaotic quantum regions. In this work for the case of Wiener measure exact expression for the amplitude of "vacuum-vacuum" transition probability $\Delta_{0-0}^{w}(\lambda, \rho)$ was constructed and it was shown, that behavior of this probability by changing the reflection coefficient $\rho$ of one-dimensional quantum problem is nonmonotonic. It is in detail investigated the properties of "vacuum+oscillator" system in asymptotic space $\Xi_s = R^1 \otimes R\{W_s\}$ and it was shown, that the ground state of oscillator is described by innumerable basis set in Hilbert space, unlike the case of standard quantum mechanics. The thermodynamics of vacuum in asymptotic space $\Xi_s$ is studied in detail and the energy of oscillators ground state with analog of Lamb shift and level width calculated, the expression for the Entropy of system in dependence of couple constant $\lambda$ is constructed. Let us remind, that the Lamb shift of energy levels in hydrogen atom from the point of view of quantum electrodynamics is obtained in framework of perturbation theory, but in proposed theory the analog of Lamb shift is obtained without including of perturbation theory. The last feature of this approach indicate its nonperturbative nature, as was expected early [11]. And in the end let pay attention to the principle difference of this theory from any other quantum approach, that is the possibility of decay of ground state.

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