Magnetic Collimation in PNe

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ABSTRACT

Recent studies have focused on the role of initially weak toroidal magnetic fields embedded in a stellar wind as the agent for collimation in planetary nebulae. In these models the wind is assumed to be permeated by a helical magnetic field in which the poloidal component falls off faster than the toroidal component. The collimation only occurs after the wind is shocked at large distances from the stellar source. In this paper we re-examine assumptions built into this “Magnetized Wind Blown Bubble” (MWBB) model. We show that a self-consistent study of the model leads to a large parameter regime where the wind is self-collimated before the shock wave is encountered. We also explore the relation between winds in the MWBB model and those which are produced via magneto-centrifugal processes. We conclude that a more detailed examination of the role of self-collimation is needed in the context of PNe studies.

Subject headings: ISM: jets and outflows — magnetic fields — magnetohydrodynamics: MHD

1. INTRODUCTION

For over 30 years now, a purely hydrodynamic theory of planetary nebulae (PNe) shaping, known as the “interacting stellar winds” (ISW) model has been studied in considerable detail. The basic idea behind the ISW model is a fast, tenuous wind which overtakes a slow, dense wind from an earlier red giant or supergiant evolutionary phase. Applications of this interacting stellar winds model to elliptical or bipolar PNe relies upon a generalization (GISW) which postulates the existence of an equatorial density enhancement in the slow wind (Kahn & West 1985; Balick 1987; Icke 1988). This density enhancement could be attributed to a binary system at the origin of the out-flowing winds (Soker 1995; Rasio & Livio 1994). This is especially true if the binary system suffers common envelope evolution. For isolated star systems other possible explanations for the equatorial density enhancement exist. One physical phenomenon possible of producing slow winds with aspherical densities is described by the “Wind Compressed Disk” model of Bjorkman & Cassinelli (1993) in which streamlines from a sufficiently rapidly rotating star converge on the equatorial plane. Another possible mechanism for producing enhanced equatorial density distributions via an initially dipolar field is described in the recent paper by Matt et al. (2000).

As successful as this GISW model has been for describing PNe, there exist some prominent features which appear in new, high resolution images that can not embraced with the purely hydrodynamic model. These features include the presence of point-symmetric knots or ansae and jets as well as complex multi-lobed outflow structures (Balick 2000; Manchado et al. 2000; Sahai 2000). While the GISW model can produce narrow jets (Icke et al. 1992; Frank & Mellema 1996; Mellema & Frank 1997) a large scale gaseous torus may be required. Thus the new features seen in observations make alternative models appear more attractive, especially for those nebulae with collimated point symmetric structures. One of the most attractive alternative models which has been proposed is based upon another generalization of the interacting stellar winds scenario as originally described by Chevalier & Luo (1994). In this process, which we will refer to as the “Magnetized Wind Blown Bubble” (MWBB) model, the fast wind carries a weak magnetic field which is strength-
enanced after passing through the wind shock. The nebula is thereby shaped by the latitudinal variation in the total pressure in the hot bubble. This model has been studied numerically by Różycka & Franco (1996) and has been subsequently generalized (García-Segura 1997; García-Segura et al. 1999) by including a latitudinal variation in the density and velocity of the slow and fast wind based upon the “Wind Compressed Disk” model of Bjorkman & Cassinelli (1993).

In this paper we critically examine assumptions built into the MWBB model. We have studied the model both analytically and numerically and we show that there is a restricted regime for which the assumptions built into the MWBB model can hold. This regime is what must be called the very weak field limit and we show that to date numerical simulations which show collimation are outside of this limit. In §2 we give a brief description of the MWBB model for the sake of completeness and clarity. In §3 we describe some of the previous numerical studies of the MWBB model and the results. In §4 we study the fast wind region appropriate for the MWBB model and show that outside of the very weak field regime the fast wind will be magnetically collimated. The conclusions of this section are applicable to the MWBB model as well as its generalization (García-Segura 1997; García-Segura et al. 1999). In §5 we discuss the relationship between MHD wind acceleration, collimation as well as its relevance to the jets observed in proto-planetary nebulae (PPNe). We summarize the results of this paper in §6.

2. THE MWBB MODEL

The purpose of this section is to provide an overview of key ingredients in the MWBB model as originally presented by Chevalier & Luo (1994). The model is based upon the interacting winds picture for planetary nebulae (for a review see Frank (1999)). It is assumed that the flow can be described by four regions separated by three surfaces: the wind shock, the contact discontinuity, and the ambient shock in order of increasing radius.

Similar to the Parker (1958) solar wind solution, the MWBB model posits that the unshocked, fast wind can be described as a spherically expanding flow with foot-points tied to a rotating stellar surface. Applying flux conservation to this quasi-hydrodynamic model results in a poloidal magnetic field \( B_p \) which scales as \( 1/r^2 \) and a toroidal magnetic field \( B_\phi \) which for \( r \gg R_\star \) scales as \( 1/r \), where \( R_\star \) is the stellar radius. For a radial wind with constant velocity \( v_w \) the density scales as \( 1/r^2 \); hence, the ratio \( \sigma = B_\phi^2/(4\pi \rho v_w^2) \) is constant in the freely expanding wind. This ratio, \( \sigma \), is the main parameter for the flow.

Next consider the region between the wind shock and the contact discontinuity, the so-called hot bubble. It is argued that the shock front in the stellar wind should occur at \( \sim 10^{17} \) cm. Using the scaling of the magnetic field with radius and reasonable values for the stellar rotation rate and fast wind velocity Chevalier & Luo (1994) argue that at radii of interest \( B_\phi/B_p \gtrsim 10^2 \). Hence they neglect the dynamical influence of the poloidal component of the magnetic field in the hot bubble. It is further assumed that the total differential \( \partial \vec{v}/\partial t \) can be neglected. Hence, the flow in the hot bubble is steady, i.e. locally independent of time, with streamlines given by straight lines in the poloidal plane. The benefit of this assumption is that the calculation of the gas pressure and toroidal field is decoupled from the velocity and the solution is readily obtained.

The region between the contact discontinuity and the ambient shock is argued to be strongly radiating. Thus it can be described in the “thin shell approximation”. The slow wind was taken to be spherically symmetric since Chevalier & Luo (1994) wanted solely to study the effect of the anisotropic (magnetic) pressure in the hot bubble in shaping the nebula.

Using a formulation by Giuliani (1982), Chevalier & Luo (1994) derived and analyzed a set of coupled time dependent equations for the system. By assuming self-similarity they were able to remove the time dependence from the equations. The resulting set of equations still requires numerical integration, though a limiting case for small parameters is analyzed analytically. The result of this limiting case suggests that deviations from spherical symmetry for the termination shock can be expected for \( \sigma \gtrsim 0.006 \). This is further supported by the results of their numerical calculations. To put numbers to this, consider the situation for the PPN phase with \( \sigma = 0.006, M = 10^{-7} M_\odot/yr \), and \( v_w = 300 \) km/s at a radius
of \( r = 10^{17} \text{ cm} \). This gives a toroidal magnetic field strength of \( B_\phi \approx 10 \mu \text{G} \). (Note that in this example we have used slower wind speeds which are appropriate to the PPN phase.)

3. NUMERICAL STUDIES

The first numerical simulations of the MWBB model were carried out by Różycka & Franco (1996). Following Chevalier & Luo (1994) they neglected the poloidal component of the magnetic field and included only a toroidal magnetic field in the fast wind. Their simulations were cylindrically symmetric, on a grid with 300 × 500 cells and a resolution of about \( 10^{-3} \text{ pc} \) in each direction. In their study they found that the resulting flow was fairly insensitive to the angular variation in the toroidal magnetic field strength. They also found that for \( \sigma < 0.05 \) the flows evolve in a homologous manner as one would expect for a purely hydrodynamic ISW model. Only for \( \sigma > 0.05 \) did the simulations develop a strong pole-ward flow in the hot bubble. The authors ascribe this phenomena to the magnetic tension associated with the shock strengthened toroidal magnetic field. The numerical and analytic studies of the MWBB model disagree on the minimum value of \( \sigma \) necessary for magnetic shaping of the hot bubble. Różycka & Franco (1996) suggest that for \( \sigma < 0.05 \), the length scale may simply be much larger than the simulation domain. Other studies of the MWBB model (García-Segura 1997; García-Segura et al. 1999) included the presence of a gaseous torus as in the GISW model. In these studies a limiting value of \( \sigma > .01 \) was found for effective magnetic collimation.

We note that in the MWBB model the freely expanding wind has an unbalanced Lorentz force associated with the helical magnetic field. This force has a component perpendicular to the wind velocity. Hence, on physical grounds, it is reasonable to expect that on some length scale and for some range of field strengths, or \( \sigma \), the fast wind will collimate on its own. The fast wind may experience significant collimation before it interacts with the wind shock. The range of initial parameters over which this occurs sets the limits on which the fast wind collimation can be reasonably neglected. It is to this question which we next direct our attention.

4. FAST WIND COLLIMATION

A basic premise of the magnetized interacting stellar wind model of Chevalier & Luo is that the fast wind velocity will remain spherically symmetric and that the \( B_\phi \) will scale as \( 1/r \) for large \( r \). This assumption can also be seen in the numerical simulations by noting that the radius of the “wind-sphere” on which the fast wind boundary conditions are set is \( \approx 10^{16} \rightarrow 10^{17} \text{ cm} \). The Lorentz force, however, is unbalanced, with components in a direction orthogonal to the velocity vector. While weak fields can not appreciably change the magnitude of the momentum, they can deflect the flow over large length scales. The question which remains to be answered is, “will the fast wind collimate on length scales of interest and how does this collimation depend upon the parameter \( \sigma \)?”

We approach this problem from a perturbation analysis in spherical polar coordinates \((r, \theta, \phi)\) with the stellar rotation axis aligned with the radial direction for \( \theta = 0 \). We will present two calculations of the deflection, both of which assume purely radial flow for the zeroth order solution. The first calculation is presented as a simple, concrete example and is based upon the Parker wind as described by Brandt (1970). This wind model treats the wind as purely hydrodynamic and neglects the dynamical influence of magnetic fields and rotational motion. The second calculation addresses concerns about the appropriateness of the analysis for small \( r \) and moderate to large field strengths and is based upon the (Weber & Davis 1967) model as described by (Barnes 1974). As we will see, both calculations are in good agreement.

4.1. THE PARKER WIND

The Parker stellar wind model is characterized by the following set of assumptions. The gas pressure, density and wind velocity are spherically symmetric \((v_\theta = v_\phi = 0)\). The magnetic field is given by

\[
B_r = B_* \left( \frac{R_*}{r} \right)^2 \\
B_\theta = 0 \\
B_\phi = -B_r \left( \frac{r \Omega \sin(\theta)}{v_r} \right) \left( 1 - \frac{R_*}{r} \right)
\]

where \( R_* \) is the stellar radius (the base of the stellar corona), \( \Omega \) the angular velocity, and \( B_* \) the
poloidal component of the magnetic field at the stellar surface. For simplicity we take $B_\star$ to be a split monopole field and assume solid body rotation.

To analyze the collimation we need to solve Newton’s equation for the motion in the $\theta$-direction. For $v_r \gg v_\theta$, Newton’s equation of motion in the $\theta$-direction is approximated by

$$\frac{\rho v_r}{r} \frac{\partial}{\partial r} (rv_\theta) = \hat{c}_\theta \cdot \frac{1}{c} (J \times B) = F^L_\theta.$$  

For $\theta \neq \pi/2$ the Lorentz force in the $\theta$-direction is given by

$$F^L_\theta = -\frac{B_\phi^2 \cot \theta}{2 \pi r}.$$ 

Since we seek a closed form solution for the wind collimation, rather than study any particular model for the wind acceleration we note that for $r \geq R_\star$

$$|B_\phi| \geq B_\star \left( \frac{r \Omega \sin(\theta)}{V_\infty} \right) \left( 1 - \frac{R_\star}{r} \right)$$

where $V_\infty$ is the asymptotic value of the radial velocity and we take $B_\star$ and $\Omega$ as positive. Using this expression for the toroidal magnetic field strength we will obtain a lower bound on the deflection. The equation of motion in the $\theta$-direction thus becomes

$$\frac{\partial}{\partial r} (rv_\theta) = -2 \sin \theta \cos \theta \left( \frac{r^2 \Omega^2 B_\star^2}{4 \pi \rho v_r V_\infty^2} \right) \left( 1 - \frac{R_\star}{r} \right)^2.$$ 

Next we note that the Michel velocity $V_m$ (Michel 1969; Belcher & MacGregor 1976) given by

$$V_m^3 = \frac{r^2 \Omega^2 B_\star^2}{4 \pi \rho v_r}$$

is a constant for steady, radial flow. Furthermore we note that following the definition of the parameter $\sigma$ we have $\sigma = (V_m / V_\infty)^3$. Collecting terms we can write

$$\frac{\partial}{\partial r} (rv_\theta) = -2 \sigma V_\infty \sin \theta \cos \theta \left( 1 - \frac{R_\star}{r} \right)^2.$$ 

The integration of this equation is straight forward. Applying the boundary condition that $v_\theta = 0$ at $r = R_\star$ we obtain

$$v_\theta = -2 \sigma V_\infty \sin \theta \cos \theta \left( 1 - x^2 + 2x \ln(x) \right)$$

where $x = R_\star / r$. Now to study the deflection or collimation of the fast wind we need to calculate the streamlines of the flow. That is, we wish to integrate the equation

$$\frac{d\theta}{dr} = \frac{v_\theta}{rv_r}.$$ 

Once again we content ourselves with a lower bound for the deflection and substitute $V_\infty$ for $v_r$ in this expression. This equation can then be integrated, resulting in a relation which describes the path taken by a particle as it moves from the point $(R_\star, \theta_\star)$ on the stellar surface to the point $(r, \theta)$ some distance away. The solution is

$$\frac{\tan \theta}{\tan \theta_\star} = x^{2\sigma + 4\sigma x} \exp\{\sigma(5 - 4x - x^2)\}$$

where again $x = R_\star / r$. For $r \gg R_\star$ this equation is well approximated as

$$\frac{\tan \theta}{\tan \theta_\star} \approx e^{5\sigma} \left( \frac{R_\star}{r} \right)^{2\sigma}.$$

![Fig. 1.— This plot shows how the collimation of the fast wind depends on the parameter $\sigma$ for two radii, ($r/R_\star$) $= 10^3$, $10^5$. This calculation shows that the fast wind will collimate for sufficiently high values of $\sigma$ and long propagation lengths. In Figure 1 we plot the angle $\theta$ (in degrees) to which the gas is deflected as a function of $\sigma$ for $\theta_\star = 45^\circ$ and ($r/R_\star$) $= 10^3$, $10^5$. For example, for $R_\star = 10^{12}$ cm Figure 1 shows that by the time gas parcels which originate at $\theta_\star = 45^\circ$ reach a radius $r = 10^{17}$ cm they will](image)
be deflected to \( \theta \approx 35^\circ, 22^\circ \) for \( \sigma = 0.02, 0.05 \) respectively.

We note that the Parker wind model is essentially a hydrodynamic wind model with the magnetic field assumed weak enough to have no dynamical influence. Thus one might worry that the preceding argument is only applicable for \( \sigma \ll 1 \). Furthermore, it is well known that for radii less than or equal to the Alfvén radius, \( R_A \), the magnetic field enforces near co-rotation of the plasma with the star. Thus there exits the concern that by assuming \( v_\phi = 0 \) we have overestimated the strength of the toroidal magnetic field at small radii and, thereby, overestimated the degree of collimation. In what follows we demonstrate that collimation also occurs when the perturbation calculation includes the plasma rotation (Weber & Davis 1967; Barnes 1974).

4.2. THE WEBER-DAVIS WIND

In this next calculation we will make use of the Weber & Davis (1967) formalism. As described by (Barnes 1974) we assume that \( v_\theta = 0 \) so that the results can be extended away from the equator. As in the previous calculation we will assume a split monopole field geometry, solid body rotation and spherically symmetric gas pressure and density for simplicity. Furthermore, we will assume that interior to the Alfvén radius that \( v_\varphi = 0 \) remains a good approximation. We will not assume that the rotational velocity, \( v_\varphi \), is zero, but instead include it self consistently with the toroidal magnetic field, \( B_\varphi \).

To analyze the collimation we solve Newton’s equation for the motion in the \( \theta \)-direction. For \( v_r \gg v_\theta \), Newton’s equation of motion in the \( \theta \)-direction is approximated by

\[
\frac{\rho v_r}{r} \frac{\partial}{\partial r} (rv_\theta) - \frac{\rho v_\varphi^2 \cot \theta}{r} = -\frac{\hat{e}_\theta}{c} \cdot (J \times B) = F_\theta^L .
\]

For \( \theta \neq \pi/2 \) the Lorentz force in the \( \theta \)-direction is given by

\[
F_\theta^L = -\frac{B_\varphi^2 \cot \theta}{2\pi r} .
\]

The equation of motion in the \( \theta \)-direction can thus be rewritten as

\[
\frac{\partial}{\partial r} (rv_\theta) = -\frac{\cot \theta}{v_r} \left( \frac{B_\varphi^2}{2\pi \rho} - v_\varphi^2 \right) .
\]

Integrating the azimuthal component of the induction equation gives

\[
B_\varphi v_r - B_r (v_\varphi - \varphi \Omega \sin \theta) = 0
\]

where again \( \Omega \) is the angular velocity at the base of the stellar corona. Integrating the azimuthal component of Newton’s equation of motion gives

\[
v_\varphi - \frac{B_r B_\varphi}{4\pi \rho v_r} = \frac{L(\theta)}{r}
\]

where \( L \) is the specific angular momentum. At the Alfvén point \((r = R_A)\) determined by the condition \( 4\pi \rho v_r^2 = B_r^2 \), these two equations are linearly degenerate giving the so-called Alfvén regularity condition, \( L = \frac{R_A^2}{\Omega} \sin \theta \). Using these relations and a bit of algebra we can solve for \( v_\varphi \) and \( B_\varphi \) in terms of parameters evaluated at the Alfvén radius. The equation of motion in the \( \theta \)-direction can then be rewritten in the form:

\[
\frac{\partial}{\partial r} (rv_\theta) = -\frac{\sin \theta \cos \theta}{v_r} \left( \frac{2\pi}{v_r} \left(1 - Y^2 \right)^2 - r^2 \Omega^2 Y^2 \right)
\]

where \( V_m \) is the Michel velocity (defined in the last section) and

\[
Y = (R_A/r)^2 - \rho/\rho_A .
\]

The parameter \( Y \) is a measure of the degree of co-rotation of the gas due to the torque applied by the magnetic field and rotational motion of the magnetic foot-points. That is, \( Y = 0 \) describes the situation of no rotational motion, \( v_\varphi = 0 \), and \( Y = 1 \) describes the situation of perfect co-rotation, \( B_\varphi = 0 \). The value of \( Y \) evaluated at the Alfvén radius, \( Y_A \), must be understood in terms of l’Hôpital’s rule and shows that \( Y_A \) is also a measure of the radial acceleration of the wind.

For \( r > R_A \), \( Y \propto (R_A/r)^2 \). Thus the equation of motion in the \( \theta \)-direction shows that at large radii the force due to the magnetic field will dominate over the centrifugal force. Unfortunately, it is difficult to proceed further without some knowledge of \( v_r \) since this controls the radial variation of the parameter \( Y \). One can show that for \( r \geq R_A \),

\[
Y \leq Y_A \left( \frac{R_A}{r} \right)^2
\]

when the radial velocity variation is slow enough that

\[
\frac{dv_r}{dr} \leq \frac{2(v_r - V_A)R_A^2}{r^2 - R_A^2} .
\]
It is under this condition that we can calculate a lower bound on the collimation of the wind with radius. The equation of motion in the \( \theta \)-direction can be rewritten as:

\[
\frac{\partial}{\partial r}(rv_\theta) = -2\sigma V_\infty \sin \theta \cos \theta \left(1 - \kappa \frac{R_A^2}{r^2}\right),
\]

where

\[
\kappa = 2Y_A + \frac{R_A^2 \Omega^2 Y_A^2}{2\sigma V_\infty^2}.
\]

Following the same procedure described for the previous calculation we obtain a relation for the streamline connecting the point \((R_A, \theta_A)\) to the point \((r, \theta)\) which describes a lower bound on the collimation. The result is

\[
\tan \frac{\theta}{\tan \theta_A} = x^{2\sigma} \exp\{\sigma(2 + \kappa - 2(1 + \kappa) x + \kappa x^2)\}
\]

where \(x = R_A/r\). For \(r \gg R_A\) this equation is well approximated as

\[
\tan \frac{\theta}{\tan \theta_A} \approx e^{(2+\kappa)\sigma} \left(\frac{R_A}{r}\right)^{2\sigma}.
\]

For comparison with the previous calculation we would like to know, order of magnitude, a reasonable number for the exponent \((2 + \kappa)\sigma\). Using the definition of \(\kappa\) we have

\[
(2 + \kappa)\sigma = 2(1 + Y_A)\sigma + \frac{R_A^2 \Omega^2 Y_A^2}{2V_\infty^2}.
\]

To estimate the first term we need an approximation for \(Y_A\). As described earlier, the parameter \(Y\) is a measure of the co-rotation of the gas. Therefore, \(Y_A\) is a measure of the force balance in the \(\theta\)-direction at the Alfvén radius. Balancing the \(\theta\)-component of the Lorentz force and centrifugal force at the Alfvén radius gives \(Y_A = (2 - \sqrt{2})\). To estimate the second term in the equation for \((2 + \kappa)\sigma\), there are two relevant limits which will be discussed more fully in a later section. In the so-called slow rotator limit the second term can be arbitrarily small. In what is called the fast rotator limit, \(V_m \approx V_\infty\) and \(Y_A \approx (2/3)V_m\) (Belercher & MacGregor 1976). Evaluating the Michel velocity at the Alfvén Radius one finds \(V_m^3 = R_A^2 \Omega^2 V_A\). Thus in the fast rotator limit we have

\[
(2 + \kappa)\sigma = 2(1 + Y_A)\sigma + \frac{3Y_A^2}{4}.
\]

Inserting \(Y_A = (2 - \sqrt{2})\), we find the exponentials in both this and the previous calculation to be of order \(\sim 1\). Thus we find good agreement for the intrinsic collimation in both the Parker and Weber-Davis model calculations.

Numerical simulations which start at large radii and assume a spherical flow are thereby constrained to small values of \(\sigma\). In order to study models with appreciable field strengths (fields beyond the "very weak limit") requires following the flow evolution at small radii. This in turn necessitates including the poloidal magnetic field since at small radii the poloidal and toroidal fields are comparable in strength. In summary, we conclude that a full treatment of the propagation and magnetic collimation effects is warranted for the study of magnetized outflows such as the MWBB model.

5. WIND ACCELERATION AND COLLIMATION

The collimation of magnetized winds has been extensively studied. In most investigations, however, both the collimation and acceleration (also known as launching) are considered together (Pudritz & Konigl 2000). In view of the rather extensive literature on the subject it is useful to attempt to put the MWBB model into the context of these investigations. This is particularly true because jets often occur in the PPNe phase (Sahai & Trauger 1998; Sahai 2000) before the star has become hot enough to radiatively drive a fast wind (Frank 2000). The launching of the wind is not addressed in the MWBB model.

The work of Weber & Davis (1967) was the first to explore the role of the magnetic field in launching a wind. This work was carried out in 1.5-D, focusing only on the launching of a wind at the equator of a rotating star. The ability of a magnetized rotor to launch and collimate a wind was explored in axisymmetric calculations in the work of Blandford & Payne (1982) and Sakurai (1985). Since that time many workers have explored magneto-centrifugal winds (see Lamers & Cassinelli (1999) or Pudritz & Konigl (2000) for a recent review of the subject). We note that a majority of the work on wind launching and collimation has focused on the role of accretion disks as the source of the wind. Recently, however, their has been some renewed interest in a star as...
the magnetized rotator source of the the magnetically launched and collimated wind (Bogovalov & Tsinganos 1999; Tsinganos & Bogovalov 2000).

A fundamental assumption of the MWBB model is that the magnetic field in the wind is sufficiently weak that intrinsic MHD wind collimation can be neglected. We now address the issue of the self-consistency of this assumption in terms of the acceleration of the wind. Belcher & MacGregor (1976) utilized the Weber-Davis model to study the transition from stellar winds in which the magnetic field is insignificant to those in which the magnetic field is dominant in driving the wind. The former and later were classified as slow and fast magnetic rotators. This classification can also be described by comparing the Michel velocity \( V_m \) to the Parker velocity \( V_p \), where \( V_p \) is defined as the velocity at infinity in the absence of rotation. Slow magnetic rotators have \( V_m \ll V_p \), while fast magnetic rotors have \( V_m \gg V_p \). The important point to note here is that fast magnetic rotators drive winds via the transfer of Poynting flux into kinetic energy flux. In the fast magnetic rotator limit \( V_\infty \approx V_m \).

Consideration of the parameters used in MWBB models shows that the majority of the MWBB simulations carried out to date (with \( \sigma < 0.1 \)) are not in the fast rotator regime. (Recall that \( \sigma = (V_m/V_\infty)^3 \).) Thus these models are consistent with the assumption that the wind is launched by some means other than conversion of Poynting flux to kinetic energy flux. Conversely the winds used in MWBB studies can not be said to have come from a form of magnetized disk/star rotator wind model since these should have \( V_m/V_\infty \approx 1 \). This is an important point as it speaks to the origin of the wind in the PPNe stage. The results of our perturbation analysis are of interest because they show that even a field that can be considered weak (i.e. \( \sigma \ll 1 \)) can produce a pre-collimation in the wind before the wind is shocked and the MWBB mechanism becomes operative.

Finally we note that other studies of magnetic winds from slow rotators have shown some degree of collimation. Tsinganos & Bogovalov (2000) found that the solar wind could achieve a significant degree of collimation at large distances when the solar rotation rate was increased by only a factor of 5. These results are consistent with our perturbation analysis.

6. CONCLUSIONS

In this paper we’ve presented an analysis of the MWBB model. We’ve shown that outside of the “very weak field” limit, the inherent magnetic collimation of the fast wind is neglected. In this sense, the numerical models which begin at large radii and include magnetic fields beyond the very weak field limit are not consistent with the assumed conditions at small radii. In addition we have explored the issue of intrinsic collimation in the MWBB winds in light of the combined launching and collimation of winds that is possible through magneto-centrifugal processes (Tsinganos & Bogovalov 2000). An analysis of conditions used in MWBB studies shows that these winds can not originate from stars lying in the fast magnetic rotator regime. This point is relevant in determining the applicability of the MWBB model PPNe. Many PPNe show collimated flows even though their central stars do not have the photon momentum flux required to drive these outflows (Alcolea et al. 2000; Sahai & Trauger 1998). The role of magnetic rotators in driving PPNe flows has recently been articulated by Blackman, Frank & Welch (2000) and Blackman et al. (2001).

We wish to emphasize, however, that the fundamental processes described by the MWBB model are sound and, in fact, represent a class of behavior that has not been given adequate attention in the literature on MHD collimation. As the previous MWBB studies have shown the post-shock hoop stresses in a magnetized wind with significant toroidal field will act as an additional collimation mechanism above and beyond the intrinsic collimation provided via the launching process.

Thus magnetohydrodynamic collimation and shaping of Planetary nebulae as well as production of such objects as the knots and jets observed within PNe is a very viable mechanism. The great promise which lies in the incorporation of magnetic fields into models of PNe formation is in the description of the shaping mechanisms. With this should also come a better understanding of the period of stellar evolution during which the mass loss is evolving from a slow to fast stellar wind.

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