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A Consistent Relaxation of the Consumption Invariant Rate in the Discounted-Utility Model

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ABSTRACT

The main theoretical contribution of this paper is a mechanical result that relates growth rates across \( n \) commodities. We exemplify through a 2-commodity economy of health and money where the results of current health economic theory are confirmed using this technology. The applications are, however, broad; both with regards to spacial discount rates by making relevant assumptions about interpersonal/international comparability and to sustainable growth by envisioning scenarios for the future.

Keywords: Matrix Algebra; Discount Rate; Health-Economics

1. Introduction

Time preferences are common, important and often life-saving. Sustainability of factors of production [1], ethical fairness towards future generations [2,3] and cost-effectiveness analyses of preventive programs [4] are a few of the well-known economic debates revolving around different approaches towards valuing of the future. The standard model of intertemporal choice is the discounted utility model (DU-model), which was first introduced by Samuelson [5]. Since then, the axiomatic derivation of the model has been numerous [6-10]. An important assumption of the DU-model is that the discount rate is positive and invariant across time and across all forms of consumptions [11]. However, it is often empirically remarked that individuals do not coordinate their intertemporal preferences with pricing choices [12-15]. Frederick and Loewenstein [13], among others, found that intertemporal preferences generally did not have the expected mapping properties on intertemporal willingness-to-pay. In the health sector, for example, current arguments in Health Technology Assessments (HTAs) concern the theoretical foundation of differential discounting for costs and health outcomes [16,17]. To that end, we relax the assumption of an invariant rate.

We assume a set of \( n \)-commodity-specific discount functions and provide a matrix-vector representation of marginal substitutions based on a model consistent expectation. Since we are concerned with marginals, one need not necessarily specify the “absolute” functions, per se, but rather the relative change of a function compared to another. Our model, being valid in cases of negative discount rates as well, derives from a more general concept. Our conceptualisation gained popularity, mostly among physicists, from Einstein’s general theory of relativity in which he considered a set of coordinate system where the metric tensor defined the type of space, flat or curved etcetera. In our case, we note that some function of well-being across time for an \( n \)-commodity economy can be geometrically represented by a function of the \( n \)-dimensional commodity space where each coordinate changes with time according to some specific functions. With regards to the coordinate transformations, we shall assume that production processes of \( n \) different commodities are equivalent to some value-gaining processes such as Rae’s instruments [18], where the value gaining processes are, possibly, dissimilar. Thus a single point can be infinitely characterised by alternative time-dependent coordinate systems. Section 2 provides a theorem of the representation which we prove by induction.

Using arguments such as consistency in intertemporal choices and intercommodity wise, we then have a cyclical representation of marginal valuations. Cyclical mechanisms describing the economy have, for the past few decades, gained the attention of several economists [19,20]. The simplicity of the mathematics of input-output systems has led to extending such systems to...
open ones as well as to intertemporal ones by adding an additional growth term. An important facet in input-output systems generally (if not always) includes some arguments of equivalence relation. In our case, we firstly equate commodity i’s current input quantities to its future quantity specified by some growth function. Although the quantities of commodities vary stochastically, as a first approach, we propose a model-consistent expectation that assumes that commodities evolve along deterministic (expected) functions of time. Section 3 provides a visual derivation based on some uniform measure of the commodities or purely on physical quantities of the commodities. Our representation however allows for differential discounting which is a major ethical debate in health economics. As such one might want to investigate the cost-effectiveness with regards to net-monetary gains as well as increased life expectancy or increased standards of life. Section 4 introduces illustrations with such marginal valuations and Section 5 concludes.

2. A Covariance Representation of Coordinate Transformations

Suppose that we have an n-dimensional function, say \( o(q) \) described by two sets of coordinate systems, say \( (q^i_1, q^i_2, \ldots, q^i_n) \) and \( (q^j_1, q^j_2, \ldots, q^j_n) \) where the coordinate transformation is given by \( q^j_i = f(q^i_i, T) \) with \( i = 1, \ldots, n \), say. Since a differential, \( dq \), is uniquely characterised in the \( q^j \)-frame of reference as well as uniquely characterised in the \( q^i \)-frame of reference, one could consider a differential, \( dq^i \leftrightarrow dq^j_1, dq^j_2, \ldots, dq^j_n \) which can also be characterised by \( dq^i_1, dq^i_2, \ldots, dq^i_n \). Assuming that we know the set of partial derivatives in the \( q^i \)-frame of reference, \( \frac{\partial q^i_j}{\partial q^i_i}, i, j = 1, \ldots, n \), as well as in the \( q^j \)-frame of reference, \( \frac{\partial q^j_i}{\partial q^j_j}, i, j = 1, \ldots, n \), we can form a matrix with those known components such that the eigenvector of that matrix, corresponding to an eigenvalue 1, represents the bijective coordinate transformation components, \( \frac{\partial q^j_i}{\partial q^i_i} = i = 1, \ldots, n \). We then have a matrix-vector representation of partial derivatives since we are simply concerned with the derivative of an axis with respect to another (not necessarily orthogonal here).

Theorem 2.1. Suppose that we have an n-dimensional space characterized by 2 sets of n-dimensional coordinates, \( q^i \) and \( q^j \), then, defining \( v^i_j \) by \( \frac{\partial q^j_i}{\partial q^i_i} \), the matrix \( X = (x^i_j) \), given by,

1) \( x^i_j = v^i_j \) if \( i < j \)
2) \( x^i_j = v^j_i \) if \( i > j \)
3) \( x^i_j = 1 - \sum^i_{j=1} v^i_j - \sum^n_{j=1} v^j_j \) if \( i = j \)

has an eigenvalue of 1 that corresponds to the eigenvector \( e = \left( \frac{\partial q^j_1}{\partial q^i_1}, \frac{\partial q^j_2}{\partial q^i_1}, \ldots, \frac{\partial q^j_n}{\partial q^i_1} \right) \).

That is

\[
\begin{align*}
1 - \sum^n_{j=1} v^i_j & \quad \cdots \quad v^n_i \\
1 - \sum^n_{j=1} v^j_j & \quad \cdots \quad v^n_j \\
\vdots & \quad \vdots \quad \vdots \\
1 - \sum^n_{j=1} v^i_j & \quad \cdots \quad 1 - \sum^n_{j=1} v^j_j
\end{align*}
\]

Proof. Suppose there exists a matrix, \( X \) that is non-zero and non-diagonal, given by

1) \( x^i_j = v^i_j \) if \( i < j \)
2) \( x^i_j = v^j_i \) if \( i > j \)
3) \( x^i_j = 1 - \sum^n_{j=1} v^i_j - \sum^n_{j=1} v^j_j \) if \( i = j \)

such \( X e = e \) where \( e = (f(q^j_1), f(q^j_2), \ldots, f(q^j_n)) \)

with \( f(q^j_i) = \frac{\partial q^j_i}{\partial q^i_i} \). \( f(q^j_i) \) and \( v^i_j = \frac{\partial q^j_i}{\partial q^i_i}, i, j = 1, \ldots, n, t = 0, T \).

Then, \( f(q^j_i) \) satisfies

\[
\sum^n_{j=1}{x^i_j} f(q^j_i) = f(q^i_i), \text{ i.e.}
\]

\[
\sum^n_{j=1}{x^i_j} f(q^j_i) = 1 - \sum^n_{j=2} f(q^j_i) + v^i_1 f(q^j_1) + \cdots + v^n_i f(q^j_n)
\]

\[
= \frac{\partial q^j_i}{\partial q^i_i} + \sum^n_{j=2} \frac{\partial q^j_i}{\partial q^j_i} f(q^j_i) + \cdots + \frac{\partial q^j_i}{\partial q^i_i} f(q^j_i)
\]

\[
= \frac{\partial q^j_i}{\partial q^i_i} f(q^j_i)
\]

Assume that \( \sum^n_{j=1} x^i_{i-1} f(q^j_i) = f(q^j_{i-1}) \) is true.

Then \( \sum^n_{j=1} x^i_j f(q^j_i) \), given by

---

1For example, Sraffa’s system equates the value (price times quantity) of a product of each industry to the value of all goods and services absorbed by this same industry.

2Throughout this paper, subscripts shall denote commodities and superscripts shall denote time.
\[ \sum_{j=1}^{\infty} x_j f(q_j^T) = \sum_{j=1}^{\infty} x_j f(q_j^T) + \sum_{j=1}^{\infty} x_j f(q_j^T) = \sum_{j=1}^{\infty} \frac{\sum_{i=j}^{\infty} \partial q_j^T}{\partial q_j^T} + \left(1 - \sum_{j=1}^{\infty} \frac{\partial q_j^T}{\partial q_j^T} \right) f(q_j^T) + \sum_{j=1}^{\infty} \frac{\partial q_j^T}{\partial q_j^T} \]

is also true. By induction, X, given by the theorem is deduced.

We now note some properties of the matrix-vector system\(^1\) which makes the usage fairly attractive for economists.

1) All the elements of our matrix are partial differentials valued with time being constant (i.e. each component of the matrix are specific to one and only one time point, either \( t = 0 \) or \( t = T \)).

2) The elements of the vector are partial differentials relating to a single axis (i.e. each of the elements of the vector is specific to one and only one commodity \( i, i = \) either 1 or 2 or 3 or \( n \)). Furthermore, the growth vector, say \( f(q_j^T) = \frac{\partial q_j^T}{\partial q_j^T} \) is not specified and the chain rule allows \( f(q_j^T) \) to be, say, an exponential growth while \( f(q_j^T) \) to be, say, a linear growth, and the general solution of the system of equations is given by:

\[ \frac{v_j^T}{v_j^0} = f(q_j^T) / f(q_j^T) \]  

3. A Derivation with Commodities

Suppose that we have an \( n \)-commodity economy where the measures of each commodity evolve through time \( t \), each according to specific growth functions, \( f(q_j^T) \), \( i = 1, \ldots, n \). Without loss of generality, we suppose that

\[ \text{the commodity bundle is } \kappa' \in \Omega^n, \text{ the } n \text{-dimensional Euclidean orthant and the physical quantity of commodity } i \text{ at time } t \text{ is denoted } q_i^T. \]

Given that we have a closed economy, the ratio of any arbitrary commodity \( i \) to another arbitrary commodity \( j, i, j = 1, \ldots, n \) is fully specified at all times. Alternatively, given a system of ratios of all commodities to other commodities at different times, a unique vector of growths exists for each of the \( n \) commodities through time. Our quest in this section is to specify a matrix whose entries are the ratios of commodity \( i \) to commodity \( j, i, j = 1, \ldots, n \) at specific times, say \( t = 0 \) and \( t = T \) that would correspond to the growth functions \( f(q_j^T) \), \( i = 1, \ldots, n \).

Suppose that we now, time \( t = 0 \), have \( q_i^0 \) of commodity \( i, i = 1, \ldots, n \) which grows up to time \( T \) to \( q_i^T \). We define the growth function, \( f(q_j^T), i = 1, \ldots, n \) to be the ratio of the future quantity of commodity \( i \) to its current quantity.

\[ f(q_j^T) = \frac{q_j^T}{q_j^0} \]  

Let \( \Delta q_i^0 \) be the quantity of commodity \( i \) at time \( t = 0 \) that will make up for the part commodity \( j \) in the future. At that future time, \( T \), the quantity of commodity \( j, q_j^T, j = 1, \ldots, i, \ldots, n \), is composed of the total parts from all \( n \) commodities, which at time \( t = 0 \) were allocated to its future production, \( \Delta q_i^0, i = 1, \ldots, n \), each forwarded to time \( T \) at their respective growth rates, \( f(q_j^T), i = 1, \ldots, n \). The quantity of commodity \( j \) at time \( T \) is then given by:

\[ q_j^T = \sum_{j=1}^{n} f(q_j^T) \Delta q_j^0 \]  

Now, from Equations (2) and (3), we have

\[ f(q_j^T) = \sum_{j=1}^{n} f(q_j^T) \Delta q_j^0 = \frac{\sum_{j=1}^{n} f(q_j^T) \Delta q_j^0}{\sum_{j=1}^{n} \Delta q_j^0} \]  

and from Equations (2) and (4), we have

\[ f(q_j^T) = \frac{\sum_{j=1}^{n} f(q_j^T) \Delta q_j^0}{\Delta q_j^0} = \frac{\sum_{j=1}^{n} f(q_j^T) \Delta q_j^0}{\sum_{j=1}^{n} \Delta q_j^0} \]  

Equating Equation (5) with (6),
\[
\sum_{j=1}^{n} f(q^t_j) \Delta q^0_j = \sum_{j=1}^{n} f(q^t_j) \Delta q^0_j \\
\rightarrow \sum_{j=1}^{n} f(q^t_j) \Delta q^0_j = \sum_{j=1}^{n} f(q^t_j) \Delta q^0_j = 0 \\
\rightarrow \sum_{j=1}^{n} f(q^t_j) \Delta q^0_j - f(q^t_j) \Delta q^0_j = 0
\]

Let us consider the solution
\[ f(q^t_j) \Delta q^0_j = f(q^t_j) \Delta q^0_j \forall i, j \]. This solution is the equivalence relation that we shall investigate. In the next section, the continuous version of the equivalence relation, \( \Delta q^0_j = \Delta q^0_j \forall i, j \), is shown to be a direct consequence of a major assumption in time preferences, namely the welfare-preserving rate. However, restricting ourselves to physical quantities for the time being, assuming that \( f(q^t_j) \Delta q^0_j = f(q^t_j) \Delta q^0_j \forall j \), we have
\[ \frac{f(q^t_j) \Delta q^0_j}{\Delta q^0_j} = \frac{f(q^t_j) \Delta q^0_j}{\Delta q^0_j} = \cdots = \frac{f(q^t_j) \Delta q^0_j}{\Delta q^0_j} \]

implying that we can write \( f(q^t_j) \), rather than a ratio of sums, as a sum of ratios, \( \Delta q^0_j \), i.e. Equation (6) becomes \( f(q^t_j) = \sum_{j=1}^{n} (\Delta q^0_j / \Delta q^0_j) f(q^t_j) \). Thus, with
\[ \frac{f(q^t_j) \Delta q^0_j}{\Delta q^0_j} = f(q^t_j) \forall j \], we have a system of linear equations which we can write in terms of a matrix of ratios, say \( X = (x_j) \) where \( x_j = \frac{\Delta q^0_j}{\Delta q^0_j} i, j = 1, \ldots, n \) and a vector of growths, say \( e = (e_1, e_2, \ldots, e_n) = (f(q^t_1), f(q^t_2), \ldots, f(q^t_n)) \), such that \( X e = e \). We shall use the transpose notation to denote column vectors. Further, note that all elements of the Matrix \( X \) are ratios valued in the present time only, that is time \( t = 0 \).

Furthermore, given that we have a closed economy, with
\[ \frac{f(q^t_j) \Delta q^0_j}{\Delta q^0_j} = \frac{f(q^t_j) \Delta q^0_j}{\Delta q^0_j} = f(q^t_j) , \]
our columns are also specified. That is, given an entry in the upper triangular matrix, say \( x_{ji}, j > i \), an entry in the lower triangular matrix, \( x_{ij} \), is also specified such that \( x_{ji}f(q^t_j) = x_{ji}f(q^t_j) \forall i, j \). Since it is more of our interest to consider how a single ratio changes, we wish to maintain the numerator and denominator so that they represent the ratio of the same commodities; but at different times. Thus \( x_{ji} = \frac{\Delta q^r_j}{\Delta q^r_j} j > i \), and remembering that \( x_{ji}f(q^t_j) = x_{ji}f(q^t_j) \forall i, j \), we let
\[ x_{ji} = \frac{\Delta q^r_j}{\Delta q^r_j} \]
so that it is only the time at which we consider the ratios that is changed. Next, with
\[ f(q^t_j) = 1. \]
\[ f(q^t_j) \] implying that a unit of commodity \( j \) grows to \( f(q^t_j) \), we add a further constraint on the diagonal entries. Given that we have \( e = e \), then we firstly require that \( X = (x_j) i, j = 1, \ldots, n \) has an eigenvalue of one corresponding to the eigenvector, \( e = (f(q^t_1), f(q^t_2), \ldots, f(q^t_n)) \). We thus require that each of the columns of our matrix sum to 1, which is a consequence of equation 3. We therefore condition on the diagonal entries so that \( x_{ji} \) equal to \( \sum_{i,j=1}^{n} x_{ij} \). Thus we have specified all entries of our matrix of ratios such that \( X e = e \) that is
Without loss of generality, we could consider partial dif-

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1This follows from Euclid’s proposition 12, The Elements, Book V. That is: “If any number of magnitudes be proportional, as one of the antecedents is to one of the consequents, so will all the antecedents be to all the consequents”.

2Analogous to Staffa’s note relating to a system in a self-replacing state (page 5 of the production of commodity by means of commodity), our formulation presupposes a system undergoing indefinite growth. As a result such a state is feasible merely by changing the ratios in which the individual equations enter it.

3To maintain numerator and denominator of \( x_i \) and \( x_j \), note that \( \frac{\Delta q^r_j}{\Delta q^r_j} \) can be achieved by forwarding the numerator of the time zero ratio or by discounting the denominator of the time t ratio, i.e. \( \frac{\Delta q^r_j}{\Delta q^r_j} f(q^t_j) \).

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ferentials rather than ratio of quantities. Furthermore, such continuity assumptions often allow the incorporation of some economic definitions fairly well. So, we shall let \( f(q_i^t) = \partial q_i^t / \partial q_i^t \), \( i = 1, \ldots, n \), \( t = 0, T \) and consequently redefine the terms of our matrix,
\[
\Delta q_{ij}^t = \partial q_i^t / \partial q_j^t, i, j = 1, \ldots, n, t = 0, T
\]
in the traditional partial differential sense. Since the entries of \( X \), only involve the pairwise exchanges, \( \Delta q_{ij}^t, i, j = 1, \ldots, n, t = 0, T \), i.e. the change in quantity of commodity \( i \) at time \( t \) that comes from commodity \( j \) at time 0 divided by the change in quantity of commodity \( j \) at time \( t \) that comes from commodity \( i \) at time 0, then given that \( \partial q_i^t / \partial q_j^t = f(q_j^t), \Delta q_{ij}^t \) can effectively be stated in a much simpler fashion; i.e. the change in commodity \( j \) with respect to commodity \( i \) at time \( t \), \( \partial q_i^t / \partial q_j^t \).

Next, suppose we wish to find the matrix inputs at a given time \( t = T \) that corresponds to the vector \( e \). Letting \( v_j^t = \partial q_i^t / \partial q_j^t \), the partial derivative of commodity \( j \) with respect to commodity \( i \) at the specific time \( T \), we have the representation theorem.

4. An Illustration in Welfare

The non-specificity of a functional “absolute” form opens doors for the use of the matrix system in various other sectors and for different other purposes. The mathematics of marginal rate of transformation and of marginal rate of substitution, being simply that of marginals or partial differentials, in this subsection, we wish to consider the term “marginal value” as input in our system. In order to justify the matrix approach for marginal substitutions, it is necessary to make various simplifying assumptions: we recall firstly that we have an economy that is closed and is composed of \( n \) commodities only; and we further require that our fictitious society satisfies the necessary assumptions required for the existence of an intercommodity and intertemporal indifference curve such as the Von Newman and Morgenstern [21] rationality axioms and the axioms presented by Ok and Masatlioglu [22], for intercommodity and intertemporal conditions respectively.

Definition 4.1. Let the social welfare function, at time \( t \), be \( W' = W(q_i^t, q_j^t, \ldots q_k^t) \) where the commodity bundle is \( K' \in \Omega^t \), the \( n \)-dimensional Euclidean orthant defined before.

Definition 4.2. Let the (negative) marginal substitution for commodity \( j \) between its future quantity and its current quantity be denoted as \( f(Q^t) = W_i^t / W_{ij}^t = -\partial q_i^t / \partial q_j^t \), \( \frac{\partial q_i^t}{\partial q_j^t} \rightarrow 0 \).

where \( W_i^t = \partial W / \partial q_i^t \).

Definition 4.3. We further denote the (negative) social marginal rate of substitution between commodity \( j \) and commodity \( i \) at time \( t \), as, \( v_j^t = \partial W / \partial q_i^t / \partial q_j^t \).

Remarks. Definition 4.3, having a bijective nature, should remind us of the equivalence relation from the previous section; \( f(q_j^t) = \Delta q_{ij}^t \) \( \forall i, j \). However, in order to use the representation for preferences, we shall require an independence condition introduced by Leontief [23]. To do so, we impose the equivalence lemma of Stigum [24]:

Definition 4.4. Let \( \hat{K}^t, K^t \) be a partition of the set of variables, \( K \), and let \( \hat{K}^t \) be any set of non-negative quantities of the variables in \( K^t \). The group of variables, \( K^t \) is separable in \( W \) from a variable \( K_i \), if and only if the correspondence, \( \beta \), defined by \( \beta(\hat{K}^t, K^t) = \{ K^t | W(\hat{K}^t, K^t) > W(\hat{K}^t, K_i) \} \) is independent of \( q_i \), the quantity of the variable \( K_i \).

Remarks. This definition is equivalent to the condition that the marginal rates of substitution at time \( t \),
\[
\frac{\partial W}{\partial q_i^t} / \frac{\partial W}{\partial q_j^t} = 0,
\]
is independent of \( q_i^t \forall i, j \neq k \) [25]. If \( W(K) \) is twice differentiable, then the condition is equivalent to \( \frac{\partial v_j^t}{\partial q_i^t} = 0 \).

Since we again have two sets of partial differentials where each set, in turn, relate to each other through partial differentials, we can write, with such definitions,
\[
\sum_{j=1}^{i} x_j f(q_i^t) = \sum_{j=1}^{i} x_j \beta(q_i^t) + x_i f(q_i^t) + \sum_{j=i+1}^{n} x_j f(q_i^t)
\]
where \( x_j \) is defined as in theorem 2.1. and we have a consistent representation of marginal substitutions. Although the above definitions assume the welfare preserving rate, with other welfare models, adjustments for \( dw \) can be made ex-ante since our representation builds on the basis differential, \( dq \).

\footnote{For example, \( \frac{\partial q_i^t}{\partial q_j^t} = \frac{\partial q_i^t}{\partial q_i^t} \).}

\footnote{The use of partial differentials in the system provides a broader set of possibilities of definitions. However, restricting ourselves to the purpose of this paper, we treat the usage as being a continuity of proportional change.}

\footnote{It is necessary for the chain-rule to hold.}
A Representation for Health Economists

Although there have been several debates in literature regarding differential time preferences, we focus on discounting of health outcomes in this subsection and validate our representation with current health-economic literature. While education, for example, is commonly known, with several empirical evidences, to boost both health and income, other economic activities are known, on the one hand, to be favorable to economic growth, while, on the other hand, to impact negatively on the population’s health. As Myrdal stated, production is a circular and cumulative sequence of causations [26]. Historians such as [27], for example, also noted that “all forms of economic growth exert intrinsically negative population health effects among the communities that are most directly involved in the transformations which they entail”.

Consequently, in order to represent current health economic discounting with our system, we shall restrict ourselves to a 2-commodity economy; namely income and the QALY, a unique measure of health outcome which is the combination of quality of life and life years. To restricting ourselves, again, to welfare preserving rates for money and health, in order to illustrate our matrix approach, we let commodity 1 and 2 be health, $h$, and money, $m$, respectively and use a one year time period, $(t = 0, t = T = 1)$, for this illustration. Let our social welfare function be summarised over only money streams and health streams, say $W(m', h')$. Letting the marginal value of future money in terms of current money be denoted by $W_m = \frac{\partial W}{\partial m}$ and let the marginal value of future health in terms of current health be denoted by $W_h = \frac{\partial W}{\partial h}$.

The reason why the matrix method is similar to the NPV method is that they are both solutions to the same problem. The aim was to find a relationship between $v^0$ and $v^1$ with the given constraints that $\partial h^0 = \partial h^1 (1+r_h)\delta h^0$, $\partial m^1 = \partial m^0 (1+r_m)$, and $v^1 = \frac{W_m}{W_h}$ . Thus in order to be indifferent to the gain of $(1 + r_h)$ of future health at time $t = 1$, consistency requires that we could now, at time zero, hold either 1 unit of health only or $\left(\frac{v^0}{v^1}\right)$ units of income only or, in our case, we hold both income and health in the proportions: $(1 - v^1)$ units of health and $v^0$ units of wealth. We see that, in the example given by Gravelle and Smith, in order to have $\Delta h^1$ of health at time $t = 1$, they either hold $v^0 \Delta h^1 \frac{1}{(1 + r_m)}$ of health now or $v^1 \Delta h^1 \frac{1}{(1 + r_h)}$ of income now. Thus, our $2 \times 2$ matrix and the NPV approaches provide the same results.

5. Discussion

Our approach resembles, to some degree, that of the original cyclical mechanisms that were proposed by Que snay Tableau economique [28]. We, however, rather than equating the “physical quantity on the side of the means of production to that on the side of the product, both of which consist of the same product” [20], allow for a non-fixed timing of the production process similar to Rae’s instruments, which we equate through Euclid’s proposition 12. It might not be unimportant to note that,^{11}Gravelle and Smith considered a period of time $t = 0$ and a period of time $t = 1$ and the definition they used was two-period while we use specific times $t = 0$ and $t = 1$ and hence assume a single period. The derivation however follows.

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while this paper addresses consistency in preferences, using growths in physical quantities of a single product, investigations on a sustainable production-consumption cycle seem fairly attractive. Alternatively, plugging in rates of time preferences as growth parameters of different commodities might aid in investigating consistency among social discount rates.

As Riccardo’s methodology in devising rates of profits of a farmer by singling out corn as a ‘basic’ commodity, we choose to, rather, single out health measures as basic commodity. Analogous to Riccardo’s conclusion with that regards, we propose that “it is the growth in health commodity. Analogous to Riccardo’s conclusion with we choose to, rather, single out health measures as basic of

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