Comparison of Models for GPS Kinematic Data Processing

LIU Lilong  WEN Hongyan  LIU Bin

Abstract  The characteristics of three GPS kinematic data processing models, Least Squares (LS), Kalman filtering and \( H_{\infty} \) filtering are discussed and their advantages and disadvantages are compared. With observational data and pertinent data processing software, the applicable condition, context and effect of the three models are experimented. Results show that when the mobile platform is in uniform motion, the accuracy of the three models are almost equal; when the mobile platform is in stochastic acceleration, the accuracy of \( H_{\infty} \) filtering model is superior to that of LS, while that of Kalman filtering is the worst.

Keywords  LS; Kalman filtering; \( H_{\infty} \) filtering

CLC number  P228.4

Introduction

In general kinematic positioning algorithms, LS method is first and widely used as a recursive estimator of the unknown position because its solutions have fine statistical quality and the distributing quality of its data need not be known. Because a model of the disturbances of observational data cannot be constructed by the method, the transcendent information of observational data cannot be fully utilized. Therefore, more processing time and observational data are needed for the solution of unknown position. By augmenting data capacity, the data processing time will increase exponentially, making it a great disadvantage (Kim, 1999; Liu, 2005). Thus, the Kalman filtering method is explored to process GPS kinematic positioning. As a linear recursive estimator capable of estimating data disturbance, the Kalman filtering method has shown great advantages in GPS kinematic positioning compared to the LS. Due to the limits of computer word length and the accumulation and transfer of rounding error and truncation error in computation, the error variance matrix loses its symmetry positive definite and leads to the instability of the numerical value. To improve the stability and the computing efficiency, square root filtering, UD (upper-triangular-diagonal) decomposing filtering and SVD (singular value decomposition) filtering, etc were presented (Dong, 1997). Normal Kalman filtering is based on the condition that the function model is precise and the statistics of the stochastic disturbances is known. In practice, the function model is always uncertain, and the statistics of disturbances is not fully known. These uncertain factors greatly decreases the estimating precision and even cause the filtering to be divergent, making it a severe inferiority for normal Kalman filtering method. In recent years, the idea of robust control is introduced into filtering, and the theory of robust filtering is formed, among
which $H_\infty$ robust filtering is the representative (Sueo, 1999; Zhao, 2002; Shinya, 2002). In the following, the merits and demerits of the three models are first compared, and with practical observational data and pertinent software, the applying condition, context and effects of the three models are experimented.

1 Data processing models for GPS kinematical positioning

1.1 LS method

LS is a basic data processing tool in modern survey techniques, and it is also widely used in GPS parameter estimates. The method is characterized by its simplicity in algorithm because statistical information of the estimator and the observational value need not be known in computation.

1.1.1 Normal LS

Suppose that $X$ is a parameter vector and its dimension is $n$. Generally, $X$ cannot be measured directly, but the observational vector $L$ can be measured which is constructed by $X$. So the first step of LS is to constitute the observational equation connecting $L$ with $X$:

$$L = f(X) + v$$

(1)

Where $f(X)$ is modeled observation and $v$ the observational error. Linearization observational Eq.(2) will be obtained when Eq.(1) is developed with Thaler theorem.

$$L = f(X_0) + A_dX + V$$

or

$$Z = L - f(X_0) = A_dX + V$$

(2)

Where $A$ is a coefficient matrix constituted by the observational equation and is usually called design matrix. $a_{ij} = dl_i/dx_j$, the element of $A$, is the partial derivative of observational value $i$ to estimated parameter $j$. $f(X_0)$ is theoretical observational vector computed with the transcendent parameter. $v$ is the filtered residual error, and $dX$ the small revised vector to transcendent parameter.

The ordinary Least Squares criterion is to minimize the sum of squares of differences between observational value and estimator, that is:

$$J(\hat{X}) = (Z - Ad\hat{X})^T (Z - Ad\hat{X}) = V^TV = \text{min}$$

(3)

When $A$ is a non-singularity matrix, equation (3) is minimal, and the estimator can be obtained:

$$\hat{X} = X_0 + d\hat{X} = X_0 + (A^T A)^{-1} A^T Z$$

(4)

If the residual error is a stochastic vector, the average value is “0” and the variance $R$, then the ordinary Least Squares method has the following characteristics.

1) The ordinary Least Squares is an unbiased estimate.

2) The mean error matrix of the ordinary Least Squares is $(A^T A)^{-1} A^T RA(A^T A)^{-1}$.

1.1.2 Weighting LS

The precision of the estimator usually is not high if the normal LS is directly employed, because the quality of observational vectors is not considered. If the quality of observational vector is known, weighting means can be employed to improve the precision. Bigger weight value is given to high quality observational value and smaller to low quality value. According to this principle, the criterion of weighting LS can be obtained:

$$J(\hat{X}) = (Z - Ad\hat{X})^T W (Z - Ad\hat{X}) = V^TWV = \text{min}$$

(5)

According to the criterion, the estimator of the parameter vector $X$ can be obtained:

$$\hat{X} = X_0 + d\hat{X} = X_0 + (A^T WA)^{-1} A^T WZ$$

(6)

Where $Z = L - f(X_0)$. If the residual error $V$ is a stochastic vector, the average value is “0” and the variance $R$, and $W = R^{-1}$, the weighting LS of $X$ is:

$$\hat{X} = X_0 + d\hat{X} = X_0 + (A^T R^{-1} A)^{-1} A^T R^{-1} Z$$

(7)

Mean error weight matrix is:

$$P = (A^T R^{-1} A)^{-1}$$

(8)

The above weighting LS is also called Markov estimate. It can be proved that the mean error of Markov estimate is smaller than any other weighting LS. Therefore, the Markov estimate is the best one in weighting LS.

1.2 Kalman filtering method

In GPS kinematic positioning, the stochastic line-
arity discrete system model can be established as follows:

\[ X_k = \Phi_k X_{k-1} + \Gamma_{k,k-1} W_{k-1} \]  \hspace{1cm} (9a)

\[ Z_k = H_k X_k + V_k \]  \hspace{1cm} (9b)

Where \( X_k \) is the \( n \) dimension state vector of the system, \( Z_k \) the \( m \) dimension observational vector, \( W_k \) the \( p \) dimension random disturbance vector, \( V_k \) the \( m \) dimension observational noise vector, \( \Phi_k \) the \( n \times n \) dimension transition matrix, \( \Gamma_k \) the \( n \times p \) dimension input disturbed matrix, and \( H_k \) the \( m \times n \) dimension observational matrix.

Based on its characteristics, the Kalman filtering recursion equation can be obtained:

\[ \hat{X}_{k|k-1} = \Phi_{k|k-1} \hat{X}_{k-1|k-1} \]  \hspace{1cm} (10)

\[ \hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k [Z_k - H_k \hat{X}_{k|k-1}] \]  \hspace{1cm} (11)

\[ K_k = P_{k|k-1} H_k^T [H_k P_{k|k-1} H_k^T + R_k]^{-1} \]

or

\[ K_k = P_{k|k-1} H_k^T R_k^{-1} \]  \hspace{1cm} (12)

\[ P_{k|k-1} = \Phi_{k|k-1} P_{k-1|k-1} \Phi_{k|k-1}^T + \Gamma_{k,k-1} Q_{k-1} \Gamma_{k,k-1}^T \]  \hspace{1cm} (13)

\[ P_{k|k} = (I - K_k H_k) P_{k|k-1} (I - K_k H_k)^T + K_k R_k K_k^T \]  \hspace{1cm} (14)

or

\[ P_{k|k} = (I - K_k H_k) P_{k|k-1} \]  \hspace{1cm} (15a)

\[ P_{k|k} = (P_{k|k-1} + H_k^T H_k)^{-1} \]  \hspace{1cm} (15b)

1.3 \( H_\infty \) filtering method

In view of the uncertainty of the model and external disturbance in filtering, \( H_\infty \) norm is introduced into the filter to make \( H_\infty \) norm minimum from the disturbed input to the output of filtering error. \( H_\infty \) filtering makes no assumptions about the frequency spectrum of disturbances (this is opposite to the normal Kalman filtering), and the error of the estimator is minimum even in the worst disturbance situation with the filter.

Stochastic linearity discrete time system is used in the \( H_\infty \) filter and its equations are the same as Eqs.(9a) and (9b).

Suppose that the original state of the system is \( X_0 \) and \( \hat{X}_0 \) is the estimator to \( X_0 \), then the initial error estimation matrix is defined as:

\[ P_0 = E\{[X_0 - \hat{X}_0][X_0 - \hat{X}_0]^T\} \]  \hspace{1cm} (16)

No supposition is made here about the natural properties of process noise \( W_k \) and observational noise \( W_k \) of the system. \( X_0, W_k \) and \( V_k \) are input as unknown disturbance.

Now look for the optimal \( H_\infty \) estimator \( \hat{z}_k = F_j(y_0, y_1, \cdots, y_k) \), which makes \( \sum_{j=0}^k (F_j) \) minimum, that is

\[ \gamma^2 = \inf_{F_j} \sum_{j=0}^k (F_j) \]

\[ = \inf_{F_j} \sup_{x, u, w, y, \gamma} \left\{ ||x - X_0|| + ||w|| + ||v|| \right\} \]  \hspace{1cm} (17)

Where \( P_0 \) is a positive definite matrix.

2 Comparison of the three models for GPS kinematic positioning

After the ambiguity has been fixed, the carrier phase observational value can be used to obtain high precision positioning. In this section, as a case study, LS, Kalman filtering and \( H_\infty \) filtering models are compared in the accuracy of GPS positioning and in resisting carrier maneuver error.

2.1 Static experiment (simulating kinematic positioning)

Two GPS double-frequency receivers made by JAVAD Corporation were used in this experiment, and the data renewal rate was 1Hz (that is, sampling rate is 1s). Eight public satellites (PRN02 03 08 11 13 15 27 31) were observed in China from 9:00~11:00 a.m. on Dec.10, 2003. After the ambiguity was fixed, 1000 epoch numbers were taken and processed respectively with the above three models. Fig.1 to Fig.3 represent respectively \( X, Y, Z \) orientation errors of the
LIU Lilong, et al./Comparison of Models for GPS Kinematical

baseline. (The whole data during the observational period was taken. With TGO software of Trimble Corporation, the following results are obtained, the baseline length $9560.375 \text{ m}$, $\Delta x = -8669.165 \text{ m}$, $\Delta y = -657.074 \text{ m}$, $\Delta z = -3976.759 \text{ m}$. With these values as true values, we compare the results of the above three methods). In the Figures, $K$ represents Kalman filtering method; $L$, weighting least squares method; and $H$, $H_\infty$ filtering method.

It can be seen from these figures that the accuracy of Kalman filtering method is the worst and its observational noise is not a white one, which may be due to the influence of double-difference multi-path error, residual error of troposphere and ionosphere. The accuracy of weighting LS and $H_\infty$ filtering are almost the same.

2.2 Kinematic Experiment

Two GPS double-frequency receivers made by JAVAD Corporation were used in this experiment, one was set up on the reference station, the other was set up in a car, its velocity being about 15 m/s, and the data renewal rate was 1Hz (namely, sampling rate is 1s). Data were collected in some place in China from 9:00~11:00 a.m. on Dec.10, 2003. Thirty minutes were spent on static initialization at the begin-

referring to the experiment in order to use the result calculated by the TGO software of Trimble Company as a reference in the data processing. Variables of the three coordinate orientation calculated by TGO software are presented in Fig.4 (1000 epoch data are chosen randomly to analyze.)
mobile acceleration is very poor, which may be due to the larger model error in calculation. This means that Kalman filtering is not sensitive to acceleration and is unable to meet the requirement of high precision kinematical positioning. As a contrast, the results of Least Squares method and $H_\infty$ filtering are more robust.

3 Conclusions

1) In white noise condition, the differences in precision between the three data processing models are very small. However, in GPS kinematic positioning, the precision of Kalman filtering is very poor due to the influences of multi-path effect, double-difference ionosphere and the residual error of the troposphere. The precision of LS method and that of $H_\infty$ filtering are almost equal.

2) In stochastic acceleration condition, the differences in precision between the three data processing models are quite big. The accuracy of $H_\infty$ filtering is superior to that of LS, while that of Kalman filtering is the worst.

References

[1] Liu Lilong (2005) The research on the precision of KINRTK theory and its applications[D]. Wuhan: Wuhan University

[2] Donghyun K, Richard B (1999) An optimized least-squares technique for improving ambiguity resolution and computational efficiency[C]. The 12th International Technical Meeting of the Satellite Division of the Institute of Navigation, Nashville

[3] Dong Xurong, Tao Daxin (1997) An efficient Kalman filtering algorithm and its application in kinematic GPS data processing[J]. Acta Geodaetica et Cartographica Sinica, 26:221-227

[4] Zhao Wei, Yuan Xin, Lin Xueyuan (2002) Research on Complete GPS/INS Integration Using $H^\infty$ Filter[J]. Acta Aeronautica et Astronautica Sinica, 23:265-267

[5] Sugimoto S, Kubo Y, Kindo T, etc (1999) Static carrier phase differential positioning by applying the $H_\infty$ filter[C]. The 12th International Technical Meeting of the Satellite Division of the Institute of Navigation, Nashville

[6] Maruo S, Kubo Y, Uratan C, etc (2002) Kinematic carrier phase differential GPS positioning by applying the $H_\infty$ filter[C]. The 15th International Technical Meeting of the Satellite Division of the Institute of Navigation, Portland, Oregon