Non-Linear Regression Method for Instability Problems in Haptic Rendering System

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Abstract. This paper addresses instability problems in 6-DoFs haptic rendering system due do the virtual stiffness technology in haptic device by using non-linear polynomial regression analysis method for haptic feedback value. The method is based on the locally optimized generalized penetration computation algorithm which computes the minimum translational and rotational motion to separate two overlapping objects. The essence of penalty-based haptic rendering method is the stability problem which is the prospect in virtual manipulation and physical experiment simulation. The algorithm which employs the virtual coupling method in order to optimize the haptic feedback value in haptic rendering system. Combining with the non-linear regression method, we get a well-distributed output and stable force feedback. The constructed rendering algorithm can process the complexity polygon-soup models and make no assumption about the underlying geometry topology. The experiment result shows the non-linear regression analysis method can make the haptic system stable and get realistic force feedback.

1. Introduction

Haptic interface links between a human operator and the virtual environment with an artificial connection. The interaction with virtual object and get the tactile feelings is a prospect in the field of Virtual Reality (VR). [6][7][20] People are eager to achieve high immersive experience, accuracy of force feedback and stability of manipulation in digital entertainment, E-Learning, [1-2] scientific training, [3] physical simulation, computer games and etc.. The performance of haptic rendering system is an important research direction which is the reason for instability problems. Due to the complexity and shortness of haptic kinesthetic link’s virtual coupling network, people should decouple the haptic display control problem by designing virtual coupling network to enable virtual reality models form issues of mechanical stability. Since the haptic device actively generates physical energy, instabilities and limit cycle oscillation problems cannot be well resolved. [24] What’s worse, instabilities may damage hardware and even pose a physical threat to the human beings.

In highly complexity geometry objects’ haptic simulation, virtual stiffness can be a huge influence on the stability of haptic rendering system. The feedback force in Penalty-based haptic rendering method simulates as a damper-spring system. [4][5] Thus, the selected virtual stiffness is very crucial. The essence of the penalty-based haptic rendering algorithm due to how to calculate a measure of penetration depth (PD) between the movable haptic probe and fixed interacting objects, mainly including two methods: translational penetration depth and generalized penetration depth [11][12]. [8][9] According to the limit of some applications, the translational penetration depth cannot get the exactly penetration depth results, even cannot separate the two overlapping objects reasonably.
Equivalent to the translational penetration depth, generalized penetration depth could get more realistic results and more stable haptic feedback. However, existing PD calculation methods are either too slow for complicated objects [14], applicable to a certain type of objects [13], not properly defined in a mathematical sense [2], or require too much computational resources such as memory [22].

Generalized penetration depth is a computation method which can use both translation and rotation to separate two overlapping objects with a rigid motion. For computing generalized penetration depth, [13] presents an effective and fast method which is based on contact-space sampling technique. However, these computation methods for penetration depth cannot content to the requirement of complicate models and haptic rendering [14]. Lately, Min et al. [15]and etc. shows an interactive algorithm to compute penetration depth, not only can resolve some more complexity objects interaction, but also can satisfied with the ideal haptic update rate 1000Hz. Yi [9] gives a comparison of translational penetration depth and generalized penetration depth computation method in his paper. He analyzes the differences in haptic rendering by using two methods, the conclusion shows that the haptic rendering method based on generalized penetration depth has more advantages and more realistic haptic result than using translational penetration depth. Moreover, the generalized penetration depth based method can produce more stable haptic feedback. [9] This paper based on Yi’s paper, gives an optimization method on their prior work, which shows a more effective haptic rendering methods and get an approving results on both stability and accuracy.

Using statistical method to filter the possible abnormal value is the crucial issues in practical application. The filter’s selection can ensure the robustness of the model. The data fitting makes the duplication of the sample well situated and decreases the order of complexity, which has been proved feasible in Machine Learning and Mathematics [2].

In the sequel, Sec.2 points out the prior work for haptic rendering and penetration depth computation method; Sec.3 specifies the definition of generalized penetration depth and the haptic rendering method based on generalized penetration depth; Sec.4 describes the Non-linear Polynomial Regression algorithm which can decrease the instability in haptic rendering system; Sec.5 presents the different experiment scenarios results; At last, Sec.6 gives a conclusion relate to the method and deficiency of the presented algorithm, then mentions the continuous work to make the haptic rendering method more sufficiently.

2. Previous Work
In this section, give a briefly review about the former work. The presented algorithm is more relevant to penetration depth calculation method, including translational penetration depth and the generalized penetration depth.

2.1 Penetration Depth
The distance measuring method is very useful in collision detection field. Penetration depth (PD) is one of an important method in distance calculation, which defined as the minimum distance to separate two colliding objects in physical interaction. Generally, define a movable object and a fixed which are colliding with each other. Due to the different configuration of the objects in the 3D space, the penetration depth computation can be classified into two different categories: translational PD and generalized PD.

Translational PD: Translational motion for the object in 3D space is a basic movement which moving along with the x, y, z coordinate axis which get three degrees of freedom (3-DoF). [16] The translational PD can be shortly described as PDt is the minimum distance to separate two overlapping objects by translational motion in 3D space. The computation complexity of PDt needs O(n2) and for polygon soups consisting n polygons respectively. The known algorithm for PDt computation relies on basic space distance calculation--Minkowski sum [17]. Due to the high computational complexity, the existing PDt algorithms are mainly use an optimization method to get separating distance. [18] is a classical method by using a hierarchical data structure to compute the directional penetration depth. [19] C. Je presents a method called PolyDepth, since its fast computational capability and can deal
with high geometry complexities with no assumption of underlying geometry topology. [20][21] proposed the methods to compute the upper and lower bounds of PDt by using CPU and GPU respectively. By using the contact clustering technology, PDt based haptic rendering method can generate stable haptic feedback for multi-contacts.

Generalized PD: [22] More complexity, the combined translational and rotational motion in 6D space have six degrees of freedom (6-DoF). [23] The generalized penetration depth (PDg) is defined as the measure of a minimal rigid motion to separate two overlapping objects by translational and rotational motion in 6D space called generalized motion. [19] A contact-projection based technique and combined with local optimization interactive algorithm for rigid and articulated models is proposed. [9] Yi gives comparison haptic rendering methods by using both PDt and PDg in 6DoF haptic system. Comparing with this two method, PDg based method can generate more stable haptic feedback than PDt, and the torque feedback in PDg are more realistic and accurate. Since PDg estimates the amount of interpenetration more accurately than PDt, so PDg suffers less pop-through problems in penalty-based haptic system. We choose PDg in the rendering algorithm to optimized the instability problem.

2.2 Virtual Coupling

Fig.1 Haptic Rendering Processing

The technique of Virtual Coupling has great efficient 6DoFs haptic rendering system. In virtual coupling method, due to the multiple contact point of the interacting objects, the final result of the direction of haptic probe will be sent to the probe object in the end of the computation. Combined with virtual stiffness, to calculate the rendering force. Shown in Figure 1, when the feedback force beyond the limit range of the haptic device, the system will cause instability problem.

3. Generalized Penetration Depth Computation

The main difference between translational penetration depth (PDt) and generalized penetration depth (PDg) is that the PDg can using both translational and rotational motions at the same time to separate two overlapping interaction objects. Since the rigid motion in 6D space, the PDg computation is more expensive. It requires $O(n^{12})$ arrangement computation for $O(n)$ polygons. We suppose that there have two polygon objects: the movable A and fixed B which are overlapping with each other. We have the definition of generalized penetration depth (PDg) in six-degree-of-freedom (6DoF) configuration space to compute the minimum distance to separate object A from object B with some distance metric as [18]:

$$PD^g(A, B) = \min \{s, \langle q, q, q\rangle \mid \text{int error}(A(q)) \cap B = \emptyset, q \in F\}$$

(1)

In the algorithm, we should construct a contact space locally around the contact configuration, but in general, the local contact space consists of 5-dimensional non-linear manifolds which make the optimization on local contact space are very challenging to compute for interactive applications. So we are using a linearization method to approximate the local contact space, then we get the linearized local contact space (LLCS) which can simplify the computation. In conclusion, we are using the following steps to compute PDg which is called PolyDepth++ method shown in Figure 2 and Figure 3:
1. Collision free-configuration selection: We can use several methods to get an initial free configuration $q_f$ which must be closer and closer to initial world frame $o$.

2. Contact-space Projection: A translational continuous collision detection (CCD) method is used to get the projection from $q_f$ on the contact space $L$ toward the original $o$, then we get a projected configuration $q_0$ on contact space.
3. Constrained Optimization: Due to the complexity of the contact space in 6D, constructing the linearized local contact space (LLCS) around \( q_0 \) and performing constrained optimization on the LLCS to generate a new configuration \( q_1 \).

4. Re-projection: If \( q_1 \) is not on the contact space \( L \), we run re-projection step to project \( q_1 \) to \( L \), then get \( q_2 \).

5. Iteration processing: In order to get the optimized configuration iteratively runs steps 2-4 to get a contact configuration \( q_c \) which can locally minimize the object norm \( \sigma \). Then we obtain \( PD_g \equiv \sigma_A (q_c, o) \).

3.3 Haptic Rendering Method using Generalized Penetration Depth

In 6DoF haptic rendering system, we need to compute both force and torque. Since the generalized penetration depth includes both translational and rotational motion, so we naturally mapped the translational motion to force computation by using the position difference as displacement and the rotational motion to torque computation by using rotation angle and axis. Thus, we get the haptic rendering results:

\[
F = k(o' - o) \quad (2) \\
T = \xi \omega \theta \quad (3)
\]

where \( F \) is the spring force and \( T \) presents the related torque. \( k \) means the relevant control virtual stiffness and \( \xi \) is the rotational coefficient; The rotation axis \( \omega \) and angle \( \theta \) shows the rotational displacement of the coordinate in \( PD_g \). The stable range of bias data is important for the stability of the haptic force feedback. The anomaly probability would greater than the data in the stable range, so we are going to use the regression to fitting the bias data into the stable range.

4. Non-linear Polynomial Regression algorithm

Due to the mixed and disorderly of multiple local contact penetration depth output from the virtual interaction in haptic rendering system, we should get a good linearization compensation solution to stabilize the haptic feedback. Non-linear regression analysis is a powerful technique used for predicting the unknown value of a variable from the known value of one variable with high power index. The general expression form of non-linear polynomial regression model is as following,

\[
Y = \beta_0 + \beta_1 X^1 + \beta_2 X^2 + \ldots + \beta_k X^k + \varepsilon 
\]

(4)

Thereinto, \( k \) represents the number of explanatory variables, multiple regression models can accommodate many explanatory variables that may be analogous; \( \beta_0, \beta_1, \beta_2, \ldots, \beta_k \) are regression coefficient, its means the slope of the regression line and represents the influence between the independent variable \( X \) and dependent variable \( Y \); \( \varepsilon \) is error amount. The equation is also called the stochastic expression of population regression function. The non random expression is,

\[
E(Y | X) = \beta_0 + \beta_1 X^1 + \beta_2 X^2 + \ldots + \beta_k X^k 
\]

(5)

**The accomplishment process of polynomial regression in MATLAB:**

(1) \( b = \text{regress} (Y, X) \), ascertain the point estimate of regression coefficient.

\( Y \) represents the matrix of n*1; \( X \) is the matrix of (ones(n,1), x1, x2, ..., xk); Select the proper initial regression coefficient.

(2) \([b, \text{bint}, \text{rint}, \text{stats}] = \text{regress} (Y, X, \alpha)\), find the point estimation and interval estimation of regression coefficient, testing the regression model.

\( b \) means the regression coefficient; \( \text{bint} \) is the interval estimation of regression coefficient; \( \text{rint} \) and \( \text{stats} \) represent residuals and their confidence intervals; \( \text{stats} \) is the statistics for testing regression models.

(3) \( \text{rcoplot} (\text{r}, \text{rint}) \), Find the residuals and their confidence intervals.

The non-linear regression analysis usually used the numerical iteration to get the result. We suppose the feedback force direction always perpendicular to the contact surface of the colliding
object in the realistic environment. Thus, we define the normal of the contact point as the force direction \( \mathbf{v} \) on the fixed object.

\[
F_1 = -F_2 = k\mathbf{v}(o'-o)
\]  

According to the physical attribute of the haptic device, the interacting force and torque of the virtual objects may cause the instability problem. So, for increasing the stability of the haptic rendering results, we employ the definition of damping. Finally, we get the stable haptic force and torque feedback.

\[
F_1 = -F_2 = k\mathbf{v}(o'-o) - B\mathbf{v}
\]

\[
T_1 = -T_2 = \xi\omega\theta, \theta > \rho
\]

Because of we only need to think about the virtual haptic force and torque feedback on the motion object, and output by the haptic device, so we only compute the force and torque on the haptic probe.

5. Experiment Result and Discussion

Since the limitation of the experiment environment, we implement our haptic rendering optimization algorithm using C++ programming language with OpenHaptics 3.1 Academic Edition API on Core (TM) i7-8700K 3.7 GHz dual processor CPUs with 32G memory under Windows 10, X64. In order to prove the stability and accuracy of our experiment results. We are testing our haptic rendering algorithm by using some objects with different geometry complexity as shown in Table 1.

| Model Pair       | #Tri  | Fps |
|------------------|-------|-----|
| Cup & Spoon      | 1.3K/8.5K | 350 |
| Bunny & Stick    | 92/70K  | 47  |

Due to the objects with different geometry complexities have different performance. We get the performance results of each models pair shown in the following table, Table 1. The haptic rendering results for Cup&Spoo scenario as shown in Fig 4, the pink object is as the fixed object and the position of the original coordinate is not changed, the green object is controlled by the haptic probe and the cyan object presents the generalized PD results. The rendering result show that the penetration depth result will never overlapping with the interacting object and will be always contact with the surface of the fixed object. Obviously, the experiment result shows that the generalized penetration depth results have small rotation transformation on the interacting objects’ surface. Thus, the realistic tactile feeling can be get when we hold the haptic probe to interact with the virtual object.

![Fig.4 Generalized PD computation results in simpler scenario](image-url)
We assume the sample data from the above scenario has no other noise and incorrect sensor influence, so we seem the raw data as initial. We go on to build the histogram of the data distribution and data distribution spectrum for the raw data, shown in Figure 5.

From the above spectrum and histogram of data distribution, we can easily get the situation of the wide range distribution. By using the similarity analysis of data, we find that they have great similarity among each other. It is difficult to get the similarity without using statistics method. However, we are going to loop the data break from 100-20000 (beyond the 20000, we have few of data output in this scenario, and the results are serious distortion). Meantime, investigating the Euclidean distance between the single data and subsequent data. We get two data sets which shows as follows. Figure 6 shows the regression results without data compression.
The following figures show the regression results using data compression with data break in 500 and 2100 sample data, shown in Figure 7 and Figure 8.

Fig.6 The regression results without data compression

Fig.7 Fitting with data break in 500
The above figures are using non-linear regression algorithm to fitting the haptic output by using 7th order and 10th order. Due to the experiment process isn’t continuous, the curve which we get from regression method seems like unstable, but if we using linear fitting method, we would have very small slope of the curve, even the 0 slope situation. The high power order curve fitting can make sure the change has varying sensitivity, the 10th order is more sensitive than the 7th.

6. Conclusion

This paper presents an optimization methods by employing non-linear regression in order to resolve the instability problems in 6-DoF haptic rendering. Combined with stiffness analysis method based on generalized penetration depth computation for general polygonal models, the penetration depth calculation use the contact-space projection technique with iterative and local optimization method can deal with the models with tens of thousands triangles. We use the non-linear regression to get the high order curve fitting in order to optimize the feedback force value. The experiment results show that we can get the accurate and stable haptic feedback. Moreover, the motion coherence method using to select the initial collision free configuration can well avoid the pop-through problems.

According to the experiment, we get the rendering performance of in the haptic rendering between 300 and 600 fps, though it is not near the ideal haptic update rate 1000Hz, we also get the stable force feedback. We are going to explore some other good techniques to increase the performance in the future work, and think about employing the deep learning technology and neural network to investigate the advanced optimization method for penetration depth computation.

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