Compact stars and the symmetry energy

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Abstract.

The effect of the symmetry energy on some properties of compact stars which contain strange degrees of freedom is discussed. Both the onset of hyperons or kaon condensation will be considered. The hyperon-meson couplings are chosen according to experimental values of the hyperon nuclear matter potentials and possible uncertainties are considered. It is shown that a softer symmetry energy affects the onset of strangeness, namely neutral (negatively charged) strange particles set on at larger (smaller) densities, and gives rise to a smaller strangeness fraction as a function of density. A softer symmetry energy will possibly give rise to maximum mass configurations with larger masses. Hyperon-meson couplings have a strong effect on the mass of the star. It is shown that, for stars with masses above 1 M⊙, the radius of the star varies linearly with the symmetry energy slope L.

1. Introduction

The measurement with high precision of the mass of the millisecond pulsar PSR J1614-2230, 1.97±0.04 M⊙ [1], puts strong constraints on the appearance of exotic degrees of freedom at high densities inside a compact star, namely strange degrees of freedom. In particular, calculations using microscopic approaches predict too small maximum masses if hyperons are included in the equation of state (EoS) [2], and not even the inclusion of hyperonic three-body forces seem to solve the problem [3].

The fraction of strangeness in a compact star is strongly dependent on the EoS that describes the hadronic matter: if the nucleonic EoS is very stiff the onset of strangeness occurs at smaller densities and the fraction of strangeness is larger, leading as a result to a softer EoS. Therefore, the fraction of the strangeness in a star is intrinsically related to some properties of the nucleonic EoS, namely its incompressibility at large densities, its density dependence of the symmetry energy and its effective mass.

In the present work we want to understand how sensitive are some properties of compact stars to the symmetry energy and its slope at saturation. We will discuss the strangeness content, the mass and radius of the maximum mass configuration and the onset of the direct...
Urca process. The structure of the pasta phase and the crust-core transition are also influenced by the symmetry energy but this topic will not be discussed in the present contribution [4], [5].

Stars with a mass in the range $1.0 \, M_\odot < M < 1.4 \, M_\odot$ have a central density that goes from 1.5 $\rho_0$ to 2-3 $\rho_0$, and, therefore, it is of particular interest the study of these stars because we will be testing the equation of state at suprasaturation densities which may be attained in the near future at FAIR.

The work is performed in the relativistic mean field approximation and we consider different parametrizations of the non-linear Walecka model (NLWM) [6], which allow us to discuss the role of the density dependence of the symmetry energy on the star properties. We also present results obtained with the quark-meson-coupling (QMC) model [7], an effective nuclear model that takes into account the internal structure of the nucleon explicitly. Within the QMC model, matter at low densities and temperatures is a system of nucleons interacting through meson fields, with quarks and gluons confined within MIT bags. Finally, we discuss the possible onset of a kaon condensate.

2. Formalism

We describe hadronic matter within the framework of NLWM [6]. The nuclear interaction is described by the interchange of the $\sigma$, $\omega$ and $\rho$ mesons. The Lagrangian density includes several non-linear terms in order to described adequately the saturation properties of nuclear matter.

In order to study the effect of the density dependence of the symmetry energy, a $\rho - \omega$ meson coupling term is introduced as in [8, 9]. The Lagrangian density reads

$$
\mathcal{L} = \sum_{j=1}^{8} \bar{\psi}_j \left[ \gamma_{\mu} (i \partial^\mu - g_{\omega j} \omega^\mu - g_{\rho j} \rho^\mu) - m_j^* \right] \psi_j + \sum_{l=1}^{2} \bar{\psi}_l (i \gamma_{\mu} \partial^\mu - M_l) \psi_l
$$

$$
+ \frac{1}{2} \partial_{\mu} \sigma \partial^\mu \sigma - \frac{1}{2} m^2_{\sigma} \sigma^2 - \frac{1}{3!} k \sigma^3 - \frac{1}{4!} \Lambda \sigma^4 - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m^2_{\omega} \omega^\mu \omega^\mu + \frac{1}{4} \xi g^4_{\omega}(\omega^\mu \omega^\mu)^2
$$

$$
- \frac{1}{4} \bar{R}_{\mu\nu} \cdot \bar{R}^{\mu\nu} + \frac{1}{2} m^2_{\rho} \rho^\mu \cdot \rho^\mu + \Lambda_{\nu} (g^2_{\rho} \bar{\rho}_{\mu} \cdot \rho^\mu) (g^2_{\omega} \omega^\mu \omega^\mu),
$$

(1)

where $m_j^* = m - g_{\sigma j} \sigma$ is the effective mass of baryon $j$, $\Omega_{\mu\nu} = \partial_{\mu} \omega_{\nu} - \partial_{\nu} \omega_{\mu}$, $\bar{R}_{\mu\nu} = \partial_{\mu} \bar{\rho}_{\nu} - \partial_{\nu} \bar{\rho}_{\mu} - g_{\rho} (\bar{\rho}_{\mu} \times \bar{\rho}_{\nu})$, $g_{ij}$ are the coupling constants of mesons $i = \sigma, \omega, \rho$ with baryon $j$, $m_i$ is the mass of meson $i$ and $l$ stands for the leptons $e^-$ and $\mu^-$. The couplings $k$, $\Lambda$ and $\xi$ define the contribution of the non-linear $\sigma$ and $\omega$ terms and $\bar{\tau}$ is the isospin operator.

The sum over $j$ in (1) extends over the octet of lightest baryons \{n, p, $\Lambda$, $\Sigma^-$, $\Sigma^0$, $\Sigma^+$, $\Xi^-$, $\Xi^0$\}.

In dense matter, due to the Pauli principle the nucleon Fermi energy increases and, if the Fermi energy of nucleons becomes larger than the hyperon masses, energy and pressure are lowered by conversion of some nucleons to hyperons. This softens the equation of state and has some direct consequences on the properties of compact stars as exemplified in Fig. 1: maximum star masses become smaller and neutrino fractions in neutrino trapped matter are larger.

Within the NLWM, we consider the following parametrizations, see Fig. 2: NL3 [10] and TM1 [11] both with a quite large symmetry energy and incompressibility at saturation and fitted in order to reproduce the ground state properties of both stable and unstable nuclei, TM1 has a quartic term on the $\omega$-meson in order to reproduce Dirac-Brueckner results at high densities; FSU [8] and IUFSU [9], which were accurately calibrated to simultaneously describe the GMR in $^{90}$Zr and $^{208}$Pb, and the IVGDR in $^{208}$Pb and still reproduce ground-state observables of stable and unstable nuclei. FSU is very soft at high densities. GM1 and GM3 [12] are generally used to describe stellar matter, and are softer than NL3, and NL$\rho$ [13], which has at high densities, a behavior between GM1 and GM3.
keeping the symmetry energy fixed at the density $0.1 \text{ fm}^{-3}$ a modified version of the IUFSU, TM1 and QMC parametrizations: we change $g$ terms.

In general, the energy per particle of asymmetric nuclear matter characterized by the asymmetry parameter $\alpha = (\rho_n - \rho_p) / \rho$, where $\rho_p$, $\rho_n$ and $\rho$ are respectively the proton, the neutron and the total densities, may be expanded in powers of $\alpha$

$$
E_{A}(\rho, \alpha) = \epsilon_{SNM}(\rho) + \epsilon_{sym}(\rho) \alpha^2 + \mathcal{O}(4),
$$

where $\epsilon_{SNM}$ is the energy per particle of symmetric nuclear matter and $\epsilon_{sym}$ is the symmetry energy

$$
\epsilon_{sym}(\rho) = \frac{1}{2} \left. \frac{\partial^2 E_{A}}{\partial \alpha^2} \right|_{\alpha=0} = E_{sym} + L x + \frac{K_{sym}}{2} x^2 + \frac{Q_{sym}}{6} x^3 + \mathcal{O}(4) \quad x = \frac{\rho - \rho_0}{3\rho_0}.
$$

In the last expression, an expansion of $\epsilon_{sym}$ around the saturation density $\rho_0$ was done. $E_{sym}$, the symmetry energy at saturation, is constrained by experiments to be within $31 \leq E_{sym} \leq 35$ MeV [14]. The quantities $L$, $K_{sym}$, and $Q_{sym}$ are related to its slope, curvature, and third derivative at saturation [4].

In the QMC model, the nucleon in nuclear medium is assumed to be a static spherical MIT bag in which quarks interact with the scalar ($\sigma$) and vector ($\omega$, $\rho$) fields, and those are treated as classical fields in the mean field approximation [7, 15]. The quark field, $\psi_{qi}$, inside the bag then satisfies the equation of motion:

$$
\left[ i \slashed{\partial} - (m_q^0 - g_\sigma^q \sigma) - g_\omega^q \omega \gamma^0 + \frac{1}{2} g_\rho^q \tau_z \rho_0 \gamma^0 \right] \psi_{qi}(x) = 0, \quad q = u, d, s
$$

where $m_q^0$ is the current quark mass, and $g_\sigma^q$, $g_\omega^q$ and $g_\rho^q$ denote the quark-meson coupling constants. The effective mass of a nucleon bag at rest is taken to be equal to the energy of the bag, $M_i^* = E_i^{bag}$. The equilibrium condition for the bag is obtained by minimizing the effective mass, $M_i^*$ of baryon $i$ with respect to the bag radius, see [7, 15]. The energy density and pressure of hadronic matter within QMC is equal to the one obtained for NLWM with the effective mass replaced by the energy of the bag, $E_i^{bag}$, and excluding the non-linear $\sigma$ and $\omega$ terms.

In order to study the effect of the isovector channel in the star properties we also consider a modified version of the IUFSU, TM1 and QMC parametrizations: we change $g_\rho$ and $\Lambda_i$, keeping the symmetry energy fixed at the density $0.1 \text{ fm}^{-3}$ while the isoscalar channel remains unchanged. We generate a set of models that differ in their value of the symmetry energy and corresponding slope at saturation but have the same isoscalar properties.
As extremes we take the parametrization IUFSU, with \( L = 47 \) MeV, and \( L = 99 \) MeV both within the experimental values obtained from isospin diffusion in heavy ion reactions [17]. Values of \( L \) below 47 MeV would give unacceptable EOS because they would predict that neutron matter is bound. In a second study we consider the TM1 parametrization with \( L = 110 \) MeV and a modified version of TM1 with \( L = 55 \) MeV corresponding to \( \Lambda_\nu = 0.03 \). For QMC we take values of \( \Lambda_\nu \) between 0 and 0.1 corresponding to \( L = 95.6 - 39 \) MeV.

We consider different sets of hyperon-meson couplings. Within the coupling set A [18] the \( \omega \) and \( \rho \) meson-hyperon coupling constants are obtained using the SU(6) symmetry:

\[
\frac{1}{2}g_{\omega \Lambda} = \frac{1}{2}g_{\omega \Sigma} = g_{\omega \Xi} = \frac{1}{3}g_{\omega N}, \quad \frac{1}{2}g_{\rho \Sigma} = g_{\rho \Xi} = g_{\rho N}, \quad g_{\rho \Lambda} = 0,
\]

where \( N \) means ‘nucleon’ \( \langle g_N \equiv g_i \rangle \). The coupling constants \( \{g_{ij}\}_{j=\Lambda,\Sigma,\Xi} \) of the hyperons with the scalar meson \( \sigma \) are constrained by the hypernuclear potentials in nuclear matter to be consistent with hypernuclear data [19]. We take \( V_j = x_{\omega j}V_\omega - x_{\sigma j}V_\sigma \) where \( x_{ij} \equiv g_{ij}/g_i \), \( V_\omega \equiv g_{\omega \omega \Lambda} \) and \( V_\sigma \equiv g_{\rho \sigma 0} \) are the nuclear potentials for symmetric nuclear matter at saturation with (see Ref. [19]), \( V_\Lambda = -28 \) MeV, \( V_\Sigma = 30 \) MeV, \( V_\Xi = -18 \) MeV. However, while the binding of the \( \Lambda \) in symmetric nuclear matter is well settled experimentally, the binding values of the \( \Sigma^- \) and \( \Xi^- \) still have a lot of uncertainties [20]. We, therefore, test also the effect of the cascade-meson couplings by taking \( V_\Xi = -10 \), 0, \(+18 \) MeV.

In QMC the couplings of the hyperons to the \( \sigma \)-meson do not need to be fixed because the effective masses of the hyperons are determined self-consistently at the bag level. We obtain \( x_{\omega B} \) from the hyperon potentials in nuclear matter, \( V_j = -(M_j^* - M_j) + x_{\omega j}g_{\omega \omega 0} \), and find \( x_{\omega \Lambda} = 0.743 \), \( x_{\omega \Sigma} = 1.04 \) and \( x_{\omega \Xi} = 0.346 \). \( x_{\rho B} = 1 \) for all the baryons [21].

In order to show how results are sensitive to the hyperon couplings we consider one of the coupling sets proposed in [12], set B, with \( x_\omega = 0.8 \) and equal for all the hyperons. The fraction \( x_\omega \) is determined using \( V_j = V_\Lambda = -28 \) MeV for all the hyperons. For the hyperon-\( \rho \)-meson coupling we consider \( x_\rho = x_\sigma \). This choice considers that the interaction of all hyperons in symmetric nuclear matter is attractive, and is restricted by acceptable maximum mass star configurations.

Strangeeness may also occur in the interior of compact stars through kaon condensation. Kaplan and Nelson [22] suggested that the interaction of the \( K^- \) with the nuclear medium reduces its mass within chiral perturbation theory favoring a boson condensation in a zero momentum state which would replace electrons as the neutralizing agent in charge neutral matter. The existence of a kaon condensate has strong implications on the star properties, namely stronger neutrino fluxes or a late low mass black-hole formation are expected. Kaons
may be included in a RMF theory through a kaon effective lagrangian density [23]

\[ \mathcal{L}_K = \mathcal{D}_\mu K^* \mathcal{D}^\mu K - m_K^* K^* K, \]

with \( \mathcal{D}_\mu = \partial_\mu + i g_\omega \omega_\mu + \frac{ig_\rho}{2} \rho_\mu \mathbf{\tau} \cdot \mathbf{b}_\mu \), and \( m_K^* = m_K - g_{\sigma K} \sigma \). The dispersion relation \( \omega_K = m_K^* - g_{\omega K} \omega_0 - \frac{i}{2} g_{\rho K} \rho_0 \) is obtained replacing \( K = f_K e^{iE_t} \) in the kaon equation of motion. The parameters of the kaon lagrangian density are the vacuum mass 497 MeV, the vector mesons coupling fixed using a simple quark model and isospin counting rule \( g_\omega = \frac{3}{2} g_\omega \), \( g_{\rho K} = g_\rho \), and the scalar coupling calculated from the optical potential of the kaon in symmetric nuclear matter \( V_K = -g_{\sigma K} \sigma - g_{\omega K} \omega_0 \). According to kaonic atom data \( V_K(\rho_0) = -(50 - 200)\) MeV [20]. In the following we consider \( V_K(\rho_0) = -125 \) MeV, a value suggested by chiral models [24]. Charge neutrality in matter with a kaon condensate requires \( \rho_\rho = \rho_e + \rho_\mu + \rho_K \), while \( \beta \)-equilibrium imposes \( \mu_K = \mu_e \) and defines the kaon condensate amplitude. The chemical potential \( \mu_e \) increases with density while the effective kaon mass \( m_K^* \) decreases. Eventually \( \mu_K = \mu_e \), kaons become energetically favorable and condense. If a further increase of the density occurs, \( m_K^* \) decreases, thus \( \mu_K \) and \( \rho_e \) decrease: since all kaons can condense in the lowest energy state (s wave), they become energetically more favorable than electrons as the neutralizing agent of positive charge. At finite temperature, thermal kaons should also be included.

3. Results

The stellar matter EoS is obtained after solving the meson field equations together with the chemical equilibrium and charge neutrality conditions. Chemical equilibrium is expressed in terms of the relations \( \mu_B = \mu_\mu - q_B (\mu_e - \mu_{\mu e}) \) and \( \mu_n - \mu_{\mu e} = \mu_\mu - \mu_{\mu e} \), where \( \mu_\mu \) is the chemical potential of baryon (B), neutron (n), electron (e), muon (\( \mu \)) and neutrino (\( \nu \)), \( q_B \) is the electric charge of baryon B, and the equilibrium is attained through the weak interaction. In neutrino free matter \( \mu_{\mu e} = \mu_{\nu e} = 0 \), and if trapped neutrinos are included we fix the fraction of electron leptons \( Y_{Le} = (\rho_e + \rho_{\nu e})/\rho \) and take the fraction of muon leptons equal to zero. The maximum value of \( Y_{Le} \) is 0.3–0.4. Charge neutrality is expressed by \( \sum_{\text{baryons}} q_B \rho_B + \sum_{\text{leptons}} q_l \rho_l = 0 \).

The effect of the symmetry energy on the strangeness fraction is seen in Figs. 3: the strangeness content is sensitive to the model and the meson-hyperon couplings. In general, the softer the nucleonic EoS the larger the strangeness onset density if the \( \Lambda \) is the first hyperon to set on, but the contrary will occur for the \( \Sigma^- \), and the smaller the strangeness content. A large meson-hyperon vector coupling, as occurs in set B, hinders the formation of hyperons. However, we

![Figure 3.](image-url)
should point out that when comparing different models we are comparing not only the effect of the density dependence of the symmetry energy but also the density dependence of the isoscalar channel, namely the incompressibility and effective mass. This explains why FSU, a softer EOS, has a smaller strangeness content than IUFSU, a model with a softer symmetry energy than FSU above saturation density. In the middle panel of Fig. 3 the strangeness content for the modified IUFSU models is shown for \( L = 47 \) MeV and 99 MeV: a smaller symmetry energy slope hinders the formation of hyperons because it gives rise to a softer EoS. The conditions for the onset of hyperons depend on the charge of the hyperon: \( \Lambda \) is the first hyperon to appear with set A and occurs at larger densities for a smaller slope \( L \), on the contrary, with set B, \( \Sigma^- \) will occur first and at smaller densities for smaller values of \( L \).

The influence of the symmetry energy on the onset density of the hyperons is seen in the right panel of Fig. 3: a smaller \( L \) implies a decrease (increase) of the onset density of the \( \Sigma^- \) (\( \Lambda \)). A convenient choice of the magnitude of the hyperon potentials may induce an interchange between the onset of \( \Lambda \) and \( \Sigma^- \) hyperons when the slope \( L \) increases. Large values of \( L \) (small values of \( \Lambda \)) favor the neutral hyperon.

Cooling of the star by neutrino emission can occur relatively fast if the direct Urca process, \( n \rightarrow p + e^- + \bar{\nu}_e \), is allowed [25]. This occurs when the proton fraction exceeds a critical value \( x_{DU} \) [25]. Cooling rates of neutron stars seem to indicate that this fast cooling process does not occur and, therefore, a constraint is set imposing that the direct Urca process is only allowed in stars with a mass larger than \( 1.5 \, M_{\odot} \), or a less restrictive limit, \( 1.35 \, M_{\odot} \) [26]. Since the onset of the direct Urca process is closely related with the density dependence of the symmetry energy, this constraint gives information on the isovector channel of the EOS.

In figures 4 the onset density of the nucleon direct Urca (DU) process is plotted as a function of the slope \( L \) for the IUFSU and modified versions in the left panel, and for NL3, GM1, GM3, NL\( \rho \), FSU and IUFSU models in the right panel. We conclude that the symmetry energy and the hyperon content affect the DU onset density according to a) for non-strange matter the larger the slope \( L \) the smaller the neutron-proton asymmetry above the saturation density and, therefore, the smaller the DU onset density. In fact, a larger slope \( L \) corresponds to a harder symmetry energy and, therefore, larger fractions of protons are favored; b) generally, for a low value of \( L \) the presence of hyperons decreases the onset density. This is always true if the first hyperon to occur is negatively charged because the proton fraction increases. However, if \( \Lambda \) is the first hyperon to set on, both the proton and the neutron fractions decrease and it is the net effect that defines the behavior.

Integrating the Tolman-Volkoff-Oppenheimer [27] equations we have obtained the mass-radius curves for the families of stars without hyperons and with hyperons with the meson-hyperon sets A and B. Both sets give similar results [28].

**Figure 4.** Onset of the direct Urca process in stellar matter for the modified IUFSU models (left panel) and NLWM (right panel), for non-hyperon matter (red triangles), hyperon coupling set A (green squares) and hyperon coupling set B (blue stars).
Figure 5. Star radius dependence on the slope of the symmetry energy for stars with maximum mass (squares), 1.4 $M_\odot$ (circles), 1 $M_\odot$ (triangles): non-hyperons (left panel), with hyperons and coupling A (middle panel). The empty symbols are for the set of different models considered. The full symbols are for the modified IUFSU models. Right panel: Mass/radius curves using set B for the hyperon couplings.

All the models except FSU are able to describe the mass of pulsar J1614-2230 [1] if hyperons are not included [28]. The star properties are determined by the underlying relativistic mean field (RMF) models: the harder models like NL3 and GM1 predict larger masses and radius, the softest EOS, FSU, has the smallest mass. It is also seen that the smaller the value of $L$ the smaller the radius [28]. This last property is clearly seen in the left panel of Fig. 5 were the radius of maximum mass stars (squares), 1.4 $M_\odot$ stars (circles) and 1.0 $M_\odot$ stars (triangles) are plotted as a function of the symmetry energy slope for nucleonic stars. The full symbols are for the modified IUFSU models and the empty ones for the NL3, GM1, GM3, NL $\rho$, FSU models. The modified IUFSU models show that the radius decreases when $L$ decreases. This reduction is larger for 1.0 $M_\odot$ stars The RMF models chosen also show the same trend, but, since the isoscalar properties differ among them the linear behavior is not present.

Including hyperons in the EOS makes the EOS softer at large densities and the mass of the maximum mass stars is smaller [29]. As an example, in the right panel of Fig. 5 the mass-radius curves obtained with hyperon-meson coupling set B is plotted. It is clearly seen that it is important to have correct couplings since the star masses are sensitive to the hyperon couplings. With set A only the NL3 model is able to describe the mass of the pulsar PSR J1316-2230 [1], while within set B NL3, GM1, GM3 and NL$\rho$ describe this star. The trend discussed above between the star radius and $L$ is still present in these stars. We also conclude that the mass of the maximum mass configuration is quite insensitive to the symmetry energy slope [28], although one might expect that the strangeness content could affect the maximum mass configuration. As we will see, this conclusion depends on the hadronic model considered.

We next discuss the results obtained within the framework of the QMC model. In Fig. 6 we plot the QMC EoS with nucleons only, with nucleons and hyperons [15] and different values of the coupling parameter $\Lambda_v$. We also include the empirical EOS obtained by Steiner et al. [30]. The agreement of the calculated EOS with the empirical one when hyperons are included is defined by the hyperon-meson interaction and the $\Lambda_v$ coupling, or, equivalently, by the symmetry energy. The QMC $pn$ EOS satisfies the constraints. However, the inclusion of hyperons with the hyperon couplings obtained for the hyperon nuclear potentials taking $V_\Lambda = -28$ MeV, $V_\Sigma = 30$ MeV and $V_\Xi = -18$ MeV makes the EOS too soft. Increasing $\Lambda_v$ makes the EOS harder bringing the EOS closer to the constraints defined by the empirical EOS. A larger $\Lambda_v$ corresponds to a smaller symmetry energy at supersaturation densities, giving rise to a softer $pn$ EOS at high densities and, therefore, hindering the onset of hyperons, and the drastic softening due to the
presence of these extra degrees of freedom. The effect of a less attractive $\xi\Lambda$ potential is also clear: the EoS becomes harder because the onset of hyperons occurs at larger densities and a mechanism that hinders the formation of hyperons makes the EOS harder.

The mass/radius curve for stars with a mass larger than $1\, M_\odot$ are shown in the middle panel of Fig. 6. First let us discuss the effect of the symmetry energy and the hyperon couplings on the mass/radius curve. A larger $\Lambda_\epsilon$ gives rise to a softer EOS and, therefore, a smaller radius. It is seen that when going from $\Lambda_\epsilon = 0$ to 0.1 the radius of stars with a mass $M = 1 - 1.5\, M_\odot$ decreases by $\sim 0.3$ Km. A similar effect was obtained with the NLWM \cite{28, 21}. However, within the NLWM models the maximum mass did not depend so much on $L$, while in the framework of the QMC model there is a clear effect of almost $0.1\, M_\odot$ if $\Lambda_\epsilon$ increases from 0 to 0.1. This is mainly due to the smaller strangeness fraction inside the star as seen in the right panel of Fig. 6. The reduction of the attractiveness of $\xi\Lambda$ has a similar effect on the maximum mass of the star, i.e., the mass increases $\sim 0.2\, M_\odot$ if $\xi\Lambda$ increases from -18 to 0 MeV. There is still a large uncertainty on the coupling of hyperons to nuclear matter and, therefore, there is still room for a very massive star such as the pulsar J1614-2230, even including hyperons in the EOS.

Figure 6. Equation of state for QMC and different values of $\xi\Lambda$ and of the $\Lambda_\epsilon$ coupling (left panel), corresponding mass/radius curve for the families of compact stars (middle panel), and strangeness content (right panel).

Figure 7. EoS and strangeness fraction for TM1 without and with hyperons (see text for discussion) the dots identify the maximum mass configuration.
In order to better understand the effect of the symmetry energy together with the uncertainty on the hyperon interaction on the strangeness content, the mass and radius of the stars, we consider the parametrization TM1, a parametrization that satisfies the heavy-ion flow constraints for symmetric matter at $2-3\rho_0$ [16] (see Fig. 2). TM1 has a value of $L = 110$ MeV at saturation density, a value which is presently considered too high, and, therefore, we also take a second parametrization changing the isovector channel of TM1 by making the symmetry energy slope softer at suprasaturation densities and choosing $L = 55$ MeV ($\Lambda_v = 0.03$). Using these two nucleonic EoS we test different hyperon interactions in nuclear matter. For the hyperon potentials in symmetric nuclear matter at saturation we take $V_\Lambda = -28$ MeV, $V_\Sigma = 30$ MeV and $V_\Xi = \pm 18$ MeV. The two values of $V_\Xi$ take into account some uncertainty on the experimental data [20]. Finally, we also consider the inclusion of the strange vector meson $\phi$. The strange scalar meson is not included because according to recent experimental $\Lambda - \Lambda$-hypermultiplet data, the $\Lambda - \Lambda$ interaction is only weakly attractive [31]. The results of the calculations are shown in Fig. 7 and 8.

We conclude that a) a smaller $L$ makes the nucleonic EoS softer giving rise to less massive stars with smaller radius as discussed in [32]; b) the inclusion of hyperons softens the EoS but if the $\phi$ meson is included the central densities attained in a hyperonic star may be as large as the one in a nucleonic star, and, although the strangeness fraction as a function of density is smaller, the total strangeness content in the star will be larger than when $\phi$ is not included; c) an attractive or repulsive optical potential of the hyperons, in this case the $\Xi$, clearly affects the fraction of strangeness as a function of density. A large fraction of strangeness at smaller densities makes the EoS too soft and smaller maximum masses and central densities are obtained; d) a softer symmetry energy hinders the appearance of hyperons, the onset is shifted to larger densities and at a given density the fraction of strangeness, with the appearance of the $\Lambda$, is smaller. As a result, although the initial nucleonic EoS is softer, if hyperons are included the EoS becomes harder and larger densities are attained at the center of the star. The total strangeness content of stars with the same mass is smaller; e) a consequence of the last comment is that the maximum mass configuration of a star with a softer symmetry energy maybe slightly larger $1.76 M_\odot$ instead of $1.75 M_\odot$ for $V_\Xi = 18$ MeV or $1.69 M_\odot$ instead of $1.68 M_\odot$ for $V_\Xi = -18$ MeV. This effect was also observed in the calculation with the QMC model. This is not anymore true when the strange meson is included because it brings extra repulsion to the EoS, that is stronger the
larger the hyperon density. Therefore an EoS with more strange content may become harder. In this schematic study the larger mass obtained for hyperonic stars was \(1.89 M_\odot\), below \(1.97 \pm 0.04 M_\odot\) of the pulsar PSR J16142230. However, if a slightly harder at supra-saturation densities EoS had been used, and this is possible still within the heavy-ion flow data constraints [16], we would have obtained a larger mass.

The occurrence of kaon condensation is another possible mechanism that gives rise to the appearance of strange degrees of freedom. In order to discuss the effect of the symmetry energy slope we consider again the TM1 parametrization and the TM1 modified parametrization with \(\Lambda_v = 0.03\) for the nucleonic EoS and fix the kaon-meson coupling. The couplings are obtained from a kaon optical potential in symmetric nuclear matter at saturation \(V_K = -125\) MeV, a value suggested by chiral models [24]. A more attractive potential lowers the onset density and softens the EoS as shown in [23], but we are not concerned with this point here. In the left panel of Fig. 9 we plot the fraction of particles in \(\beta\)-equilibrium matter with the possibility of kaon condensation for two EoS: the dash lines correspond to TM1 with a symmetry energy slope \(L = 110\) MeV and the full lines to a modified TM1 model with \(L = 55\) MeV. A larger \(L\) corresponds to a larger symmetry energy at suprasaturation densities and therefore, less neutrons and more protons and leptons. The kaon condensate replaces the leptons and a larger fraction of kaons occurs for a larger \(L\). However, the onset of kaons occurs at lower densities if \(L = 55\) MeV. In this case the coupling \(g_\rho\) is larger and \(\omega_K\) decreases faster with density. Although a larger fraction of kaons is expected for the EoS with \(L = 110\) MeV, this EoS is harder within the range of densities of interest, e.g. below the central density of the maximum mass configuration, therefore its central density is smaller and the star contains less exotic matter. These properties are reflected in the total strangeness content of the star (middle panel of Fig. 9) and the mass-radius star curves, right panel of Fig. 9. It is interesting to notice that with the present choice of parameters at the level of the nucleon EoS and the kaon interaction the mass of the millisecond pulsar PSR J16142230 would be obtained. The softer EoS with the smaller slope \(L\) gives rise to stars with smaller radius and with a larger total strangeness content and may favor the appearance of a larger content of exotic matter because larger central densities are possible and the onset density is smaller. A second conclusion is that the pulsar PSR J16142230 does not rule out kaon-condensation inside a compact star. The uncertainties existing at the level of the nucleon EoS at suprasaturation densities are enough to allow the appearance of exotic degrees of freedom.

Figure 9. Left panel: fraction of particles in stellar matter with a kaon condensate. The full (dashed) lines correspond to \(L = 110\) MeV (\(L = 55\) MeV). The dots identify the maximum mass configurations; middle panel: the strangeness number over the baryonic number inside the two families of stars; right panel: the mass radius curves for stars with a mass above \(M_\odot\).
4. Conclusions

The density dependence of the symmetry energy and the hyperon couplings have both a strong effect on the mass and radius of the star. We have tested different hyperon-meson parametrizations, using information from hypernuclei to fix the couplings. We took advantage of the fact that QMC predicts the hyperon effective masses without being necessary to fix the hyperon-σ couplings, in one of the parametrizations.

A softer symmetry energy gives rise to smaller stars. Also the hyperon fraction is affected: softer symmetry energy corresponds to a slower increase of the hyperon fraction with density [28, 21]. However, the onset of strangeness depends on the charge of the hyperons. Negatively charged hyperons or antikaons set on at smaller densities while neutral hyperons at larger densities for smaller values of $L$. The EoS of matter with less hyperons may become harder above hyperons onset, and larger central densities may be attained. The total strangeness fraction is generally smaller, but since larger central densities may occur, it may also happen that the total strangeness content is relatively larger. Generally a smaller strangeness content gives rise to larger maximum mass configurations, although this result depends on the model used to describe nucleonic matter and the hyperon interaction. Even including hyperons in the QMC EOS we could explain the mass of the pulsar J1614-2230 if the cascade nuclear potential is set to be very little attractive. More data on hypernuclei is needed to constrain the hyperon-meson couplings.

The kaon condensation was studied. In this case the EoS with a softer symmetry energy has shown a total strangeness fraction much larger because much larger central densities were obtained. Also in this case using TM1 and a optical potential of $K^-$ in nuclear matter at saturation equal to -125 MeV, of the order of the ones predicted by chiral models, a mass of the order of 2 $M_\odot$ was obtained. A similar conclusion was taken in [33] with a different model. The present uncertainties on the kaon interaction in the medium and the EoS at high densities do not allow the exclusion of a kaon condensation in compact stars.

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