Shifted $\mu$-hybrid inflation, gravitino dark matter, and observable gravity waves

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Abstract

We investigate supersymmetric hybrid inflation in a realistic model based on the gauge symmetry $SU(4)_c \times SU(2)_L \times SU(2)_R$. The minimal supersymmetric standard model (MSSM) $\mu$ term arises, following Dvali, Lazarides, and Shafi, from the coupling of the MSSM electroweak doublets to a gauge singlet superfield which plays an essential role in inflation. The primordial monopoles are inflated away by arranging that the $SU(4)_c \times SU(2)_L \times SU(2)_R$ symmetry is broken along the inflationary trajectory. The interplay between the (above) $\mu$ coupling, the gravitino mass, and the reheating following inflation is discussed in detail. We explore regions of the parameter space that yield gravitino dark matter and observable gravity waves with the tensor-to-scalar ratio $r \sim 10^{-4} - 10^{-3}$.

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I. INTRODUCTION

In its simplest form supersymmetric (SUSY) hybrid inflation [1, 2] is associated with a
gauge symmetry breaking $G \to H$, and it employs a minimal renormalizable superpotential
$W$ and a canonical Kähler potential $K$. Radiative corrections and soft SUSY breaking terms
together play an essential role [3–6] in the inflationary potential that yields a scalar spectral
index in full agreement with the Planck data [7]. In this minimal model the symmetry
breaking $G \to H$ occurs at the end of inflation, and the symmetry breaking scale $M$ is
predicted to be of the order of $(2 - 3) \times 10^{15}$ GeV [1, 3–6]. One simple extension of this
minimal model retains a minimal $W$ but invokes a nonminimal $K$ [8], such that the correct
scalar spectral index is obtained without invoking the soft SUSY breaking terms. Nonmin-
imal Kähler potentials are also used to realize symmetry breaking scales comparable to the
grand unified symmetry (GUT) scale $M_{\text{GUT}}$ ($\sim 2 \times 10^{16}$ GeV) [9], and to predict possibly
observable gravity waves [10, 11].

If the symmetry breaking $G \to H$ produces topological defects such as magnetic
monopoles, a more careful approach is required in order to circumvent the primordial
monopole problem. The first such example is provided by the so-called ‘shifted-hybrid infla-
tion’ [12, 13], in which the monopole producing Higgs field actively participates in inflation
such that, during inflation, $G$ is broken to $H$ and the monopoles are inflated away.

In this paper we explore inflation and reheating in the framework of the gauge symmetry
$SU(4)_c \times SU(2)_L \times SU(2)_R$ ($G_{4-2-2}$) [14]. A SUSY model based on this symmetry including
hybrid inflation was first explored in Ref. [15]. However, the primordial monopole problem
was not resolved, but it was subsequently addressed and successfully rectified in Ref. [12]
based on shifted hybrid inflation. In the model proposed here, we employ the mechanism
invented in Refs. [15, 16] for generating the MSSM $\mu$ term, and we exploit shifted hybrid
inflation to overcome the monopole problem. We implement this scenario using both minimal
and nonminimal Kähler potentials, and address in both cases important issues related to
the gravitino problem [17]. For a discussion of leptogenesis via right-handed neutrinos in
models where the dominant inflaton decay channel yields Higgsinos, see Ref. [18].

The plan of the paper is as follows: In Sec. II, we present the SUSY $G_{4-2-2}$ model in-
cluding the superfields, their charge assignments, and the superpotential which respects a
$U(1)_R$ symmetry. In Sec. III, the inflationary setup is described. This includes the scalar
potential for global SUSY as well as the one including supergravity (SUGRA). The shifted \( \mu \)-hybrid inflation (\( \mu \)HI) scenario with minimal Kähler potential and its compatibility with the gravitino constraint [19] is studied in Sec. IV. The analysis is extended by employing a nonminimal Kähler potential in Sec. V, discussing again the gravitino problem and the bounds it imposes on reheat temperature, and focusing on solutions with observable gravity waves. Our conclusions are summarized in Sec. VI.

II. THE SUPERSYMMETRIC \( SU(4)_c \times SU(2)_L \times SU(2)_R \) MODEL

The matter superfields \( F_i \) and \( F^c_i \) belong in the following representations of \( G_{4-2-2} \),

\[
F_i = (4, 2, 1) \equiv \begin{pmatrix} u_{ir} & u_{ig} & u_{ib} & \nu_{id} \\ d_{ir} & d_{ig} & d_{ib} & e_{il} \end{pmatrix}; \quad F^c_i = (\bar{4}, 1, 2) \equiv \begin{pmatrix} u^c_{ir} & u^c_{ig} & u^c_{ib} & \nu^c_{id} \\ d^c_{ir} & d^c_{ig} & d^c_{ib} & e^c_{il} \end{pmatrix},
\]

where the index \( i(=1, 2, 3) \) denotes the three families of quarks and leptons, and the subscripts \( r, g, b, l \) are the four colors in the model, namely red, green, blue, and lilac. The GUT Higgs superfields \( H^c \) and \( \overline{H}^c \) are represented as follows:

\[
H^c = (\bar{4}, 1, 2) \equiv \begin{pmatrix} u^c_{Hr} & u^c_{Hg} & u^c_{Hb} & \nu^c_{Hl} \\ d^c_{Hr} & d^c_{Hg} & d^c_{Hb} & e^c_{Hl} \end{pmatrix}; \quad \overline{H}^c = (4, 1, 2) \equiv \begin{pmatrix} \overline{u}^c_{Hr} & \overline{u}^c_{Hg} & \overline{u}^c_{Hb} & \overline{\nu}^c_{Hl} \\ \overline{d}^c_{Hr} & \overline{d}^c_{Hg} & \overline{d}^c_{Hb} & \overline{e}^c_{Hl} \end{pmatrix},
\]

and acquire nonzero vacuum expectation values (vevs) along the right-handed sneutrino directions, that is \( |\langle \nu^c_{Hl} \rangle| = |\langle \overline{\nu}^c_{Hl} \rangle| = v \neq 0 \), to break the \( G_{4-2-2} \) gauge symmetry to the standard model (SM) gauge symmetry \( G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y \), around the GUT scale \( (\sim 2 \times 10^{16} \text{ GeV}) \), while preserving low scale SUSY [20]. The electroweak breaking is triggered by the electroweak Higgs doublets, \( h_u \) and \( h_d \), which reside in the bidoublet Higgs superfield \( h \) represented as follows:

\[
h = (1, 2, 2) \equiv (h_u, h_d) = \begin{pmatrix} h^+_u & h^0_d \\ h^0_u & h^-_d \end{pmatrix},
\]

Note that such doublets can remain light because of appropriate discrete symmetries [21].

A gauge singlet chiral superfield \( S = (1, 1, 1) \) is introduced, which triggers the breaking of \( G_{4-2-2} \) and whose scalar component plays the role of the inflaton. A sextet Higgs
TABLE I: Superfields together with their decomposition under the SM and their $R$ charge.

| Superfields | $4_e \times 2_L \times 2_R$ | $3_e \times 2_L \times 1_Y$ | $q(R)$ |
|-------------|-----------------------------|-----------------------------|--------|
| $F_i$       | $(4, 2, 1)$                  | $Q_{ia}(3, 2, 1/6)$          | 1      |
|             |                             | $L_i(1, 2, -1/2)$            |        |
| $F_i^c$     | $(4, 1, 2)$                  | $u_{ia}^c(3, 1, -2/3)$       | 1      |
|             |                             | $d_{ia}^c(3, 1, 1/3)$        |        |
|             |                             | $\nu_i^c(1, 1, 0)$           |        |
|             |                             | $e_i^c(1, 1, 1)$             |        |
| $H^c$       | $(4, 1, 2)$                  | $u_{Ha}^c(3, 1, -2/3)$       | 0      |
|             |                             | $d_{Ha}^c(3, 1, 1/3)$        |        |
|             |                             | $\nu_H^c(1, 1, 0)$           |        |
|             |                             | $e_H^c(1, 1, -1)$            |        |
| $\overline{H}^c$ | $(4, 1, 2)$              | $\overline{u}_{Ha}^c(3, 1, 2/3)$ | 0    |
|             |                             | $\overline{d}_{Ha}^c(3, 1, -1/3)$ |        |
|             |                             | $\overline{\nu}_H^c(1, 1, 0)$ |        |
|             |                             | $\overline{e}_H^c(1, 1, -1)$ |        |
| $S$         | $(1, 1, 1)$                  | $S(1, 1, 0)$                 | 2      |
| $G$         | $(6, 1, 1)$                  | $g_a(3, 1, -1/3)$            | 2      |
|             |                             | $g_a^c(3, 1, 1/3)$           |        |
| $h$         | $(1, 2, 2)$                  | $h_u(1, 2, 1/2)$             | 0      |
|             |                             | $h_d(1, 2, -1/2)$            |        |

superfield $G = (6, 1, 1)$, which under the SM splits into the color-triplet Higgs superfields $g = (3, 1, -1/3)$ and $g^c = (\overline{3}, 1, 1/3)$, is introduced to provide superheavy masses to the color-triplet pair $d_H^c$ and $\overline{d}_H^c$ [15]. The superfields with their representations, transformations under $G_{4-2-2}$, decompositions under $G_{SM}$, and charge assignments are shown in Table I.

The main part of the superpotential of our model that is compatible with $G_{4-2-2}$ and the $R$-symmetry $U(1)_R$ is given by

$$W = \kappa S(\overline{H}^c H^c - M^2) + \lambda S h^2 - S \left( \beta_1 \frac{(\overline{H}^c H^c)^2}{\Lambda^2} + \beta_2 \frac{(\overline{H}^c)^4}{\Lambda^2} + \beta_3 \frac{(H^c)^4}{\Lambda^2} \right)$$

$$+ \lambda_{ij} F_i^c F_j^c h + \gamma_{ij} \frac{\overline{H}^c H^c}{\Lambda} F_i^c F_j^c + a G H^c H^c + b G \overline{H}^c \overline{H}^c,$$

where $\kappa, \lambda, \beta_{1,2,3}, \lambda_{ij}, \gamma_{ij}, a,$ and $b$ are real and positive dimensionless couplings and $M$ is a mass parameter of the order of $M_{GUT}$. We assume the superheavy scale $\Lambda$ to be in the range $10^{16}$ GeV $\lesssim \Lambda \lesssim m_P$, where $m_P$ denotes the reduced Planck scale ($2.4 \times 10^{18}$ GeV).
The first three terms in the superpotential are of the standard $\mu$HI case as discussed in Refs. [19, 22]. The first two and the fourth term characterize the ‘shifted case’ by providing additional inflationary tracks to avoid the monopole problem. The third term $\lambda S h_u h_d$ yields the effective $\mu$ term. Indeed assuming gravity-mediated SUSY breaking [23, 24], the scalar component of $S$ acquires a nonzero vev proportional to the gravitino mass $m_{3/2}$ and generates a $\mu$ term with $\mu = -\lambda m_{3/2}/\kappa$, thereby resolving the MSSM $\mu$ problem [16]. The $\lambda_{ij}$-terms contain the Yukawa couplings, and hence provides masses for fermions. The $\gamma_{ij}$-terms yield large right-handed neutrino masses, needed for the see-saw mechanism. The other possible couplings similar to $\gamma_{ij}$-terms which are allowed by the symmetries are $FFH^cH^c$, $FF\bar{H}^c\bar{H}^c$, and $F^cF^cH^cH^c$. The last two terms in the superpotential involving the sextuplet superfield $G$ are included to provide superheavy masses to $d^c_H$ and $\bar{d}^c_H$.

It is important to mention here that the matter-parity symmetry $Z_{2}^{\text{mp}}$, which is usually invoked to forbid rapid proton decay operators at renormalizable level, is contained in $U(1)_R$ as a subgroup. The superpotential $W$ is invariant under $Z_{2}^{\text{mp}}$ and this symmetry remains unbroken. There is no domain wall problem and the lightest SUSY particle (LSP) is stable and consequently a plausible candidate for dark matter (DM).

III. $\mu$-HYBRID INFLATION IN $SU(4)_c \times SU(2)_L \times SU(2)_R$

The relevant part of the superpotential for shifted $\mu$HI contains the terms

$$\delta W = \kappa S(\bar{H}^c H^c - M^2) + \lambda S h^2 - \xi \frac{\kappa S(\bar{H}^c H^c)^2}{M^2},$$

(5)

where $\xi = \beta_1 M^2/\kappa \Lambda^2$ is a dimensionless parameter. We ignore the $\beta_{2,3}$-terms in our future discussions as they become irrelevant in the $D$-flat direction, that is the direction where the $D$-term contributions vanish (i.e. with $|\nu_{H_d}^c| = |\nu_{H_u}^c|$ and all other components zero). For simplicity, the superfields and their scalar components will be denoted by the same notation.

The global SUSY minimum obtained from Eq. (5) is given as

$$\langle S \rangle = 0, \quad \langle h \rangle = 0, \quad v^2 = \langle H^c H^c \rangle = \frac{M^2}{2\xi}(1 \pm \sqrt{1 - 4\xi}),$$

(6)

which requires that $\xi \leq 1/4$ for real values of $v$. 

5
The global SUSY scalar potential obtained from the superpotential in Eq. (5) is

\[ V = \left| \kappa \left( \frac{|H^c|^2}{M^2} - M^2 - \xi \right) \left( \frac{|H^c|^2}{M^2} \right) \right|^2 + \lambda |h|^2 + \kappa^2 |S|^2 \left( |H^c|^2 + |H^c|^2 \right) 1 - 2 \xi \left( \frac{|H^c|^2}{M^2} \right)^2 + D\text{-terms}. \tag{7} \]

Taking the $D$-flat direction the scalar potential takes the form:

\[ V = \left| \kappa \left( |H^c|^2 - M^2 - \xi \right) \right|^2 + \lambda |h|^2 + \kappa^2 |S|^2 |H^c|^2 1 - 2 \xi \left( \frac{|H^c|^2}{M^2} \right)^2. \tag{8} \]

Rotating the complex field $S$ to the real axis by suitable transformations, we can identify the normalized real scalar field $\sigma = \sqrt{2}S$ with the inflaton. Introducing the dimensionless fields

\[ w = \frac{|S|}{M}, \quad u = \frac{|H^c|}{M}, \quad z = \frac{|h|}{M}, \tag{9} \]

the normalized potential $\tilde{V}$ takes the form

\[ \tilde{V} = \frac{V}{\kappa^2 M^4} = (u^2 - 1 - \xi u^4 + \lambda z^2)^2 + 2w^2 u^2 (1 - 2\xi u^2)^2. \tag{10} \]

The extrema of the above potential with respect to $u$ are given as:

\[ u_1 = 0, \tag{11} \]
\[ u_2 = \pm \frac{1}{\sqrt{2\xi}}, \tag{12} \]
\[ u_3^\pm = \frac{1}{\sqrt{2\xi}} \sqrt{1 - 6w^2 - \xi \pm \sqrt{-4 \xi + 36 \xi^2 w^4 - 8 \xi^2 w^2 + 4 \lambda \xi^2 z^2 + 1}}. \tag{13} \]

From now on we assume the system to be stabilized along a particular direction with $z = 0$. These extrema can be visualized with the help of the potential $\tilde{V}(u, w, z = 0)$, plotted in Fig. 1, for various values of $\xi$.

In Fig. 1, the standard $\mu$HI case with $\xi = 0$ is reproduced in plot (a). In this case, $u = 0$, $w > 1$ is the only inflationary valley available. It evolves at $w = 0$ into a single pair of global SUSY minima with vev $v = \pm M$. For $\xi \neq 0$, in addition to the standard track at $u = u_1$, two shifted local minima appear at $u = u_2$ for $w > \sqrt{1/8\xi - 1/2}$. In plot (b) for $\xi < 1/8$, the shifted tracks lie higher than the standard track. Following Ref. [12], in order to have suitable initial conditions for realizing inflation along the shifted tracks, we assume $\xi \geq 1/8$. The normalized scalar potential $\tilde{V}$ is shown in plots (c)-(e) for some realistic
FIG. 1: The normalized scalar potential $\tilde{V}(w, u, z = 0) = V/\kappa^2 M^4$, where $w = |S|/M$, $u = |H^c|/M$. The standard $\mu$HI case is reproduced in plot (a). Here $u = 0$, $w > 1$ is the only inflationary valley available in this case and evolves at $w = 0$ into a single pair of global SUSY minima with vev $v = \pm M$. For $\xi \neq 0$, in addition to the standard track at $u = u_1$, there are two shifted trajectories at $u = u_2 = \pm 1/\sqrt{2\xi}$, for $w > \sqrt{1/8\xi - 1/2}$. Plot (b) shows the undesirable situation where the shifted tracks lie higher than the standard track for $\xi < 1/8$. Plots (c)-(e) are for $\xi = 1/8$, $\xi = 1/6$, and $\xi = 1/4$, respectively. The case $\xi > 1/4$ is shown in plot (f), but it is disfavored since SUSY is broken in the vacuum. So any feasible choice for $\xi$ lies in the region $[1/8, 1/4]$. 
values of $\xi$, namely for $\xi = 1/8$, $\xi = 1/6$, and $\xi = 1/4$. In the last plot (f) with $\xi > 1/4$, we
obtain $V_{\text{min}} \neq 0$, and therefore SUSY will be broken at high scale after inflation. So for our
analysis, it is appropriate to keep $\xi$ within the interval $[1/8, 1/4]$.

As the inflaton slowly rolls down the inflationary valley and enters the waterfall regime at
$k = 1/8\xi - 1/2$, inflation ends due to fast rolling and the system starts oscillating about
a vacuum at $k = 0$. Note that in the $H^c$ direction there are actually two pairs of vacua at
[see Eq. (13)]

$$(u_3^\pm)^2 \xrightarrow{w=0} v_\pm^2 = \frac{1}{2\xi} [1 \pm \sqrt{1 - 4\xi}]. \tag{14}$$

However, the path leading to $v_-$ appears before the one leading to $v_+$, as explained in
Ref. [12]. The necessary slope for realizing inflation in the valley with $w > 1/8\xi - 1/2$, $u = u_2$, $z = 0$ is generated by the inclusion of the one-loop radiative corrections, the
SUGRA corrections, and the soft SUSY breaking terms. The one-loop radiative corrections
$V_{\text{loop}}$, arising as a result of SUSY breaking on the inflationary path, are calculated using the
Coleman-Weinberg formula [25]:

$$V_{\text{loop}} = \frac{1}{64\pi^2} \sum_i (-1)^{F_i} M_i^4 \ln \left( \frac{M_i^2(S)}{Q^2} - \frac{3}{2} \right) = \kappa^2 m^4 \left[ \frac{k^2}{4\pi^2} F(x) + \frac{\lambda^2}{4\pi^2} F(y) \right], \tag{15}$$

where $F_i$ and $M_i^2$ are the fermion number and squared mass of the $i$th state. The function
$F(x)$ is given by

$$F(x) = \frac{1}{4} [(x^4 + 1) \ln \left( \frac{x^4 - 1}{x^4} \right) + 2x^2 \ln \left( \frac{x^2 + 1}{x^2 - 1} \right) + 2 \ln \left( \frac{2\kappa^2 m^2 x^2}{Q^2} \right) - 3], \tag{16}$$

$y = \sqrt{\gamma/2} x$ with $\gamma = \lambda/\kappa$, and $x$ is defined in terms of the canonically normalized real
inflaton field $\sigma$ as $x = \sigma/m$ with $m^2 = M^2(1/4\xi - 1)$. The function $F(y)$ exhibits the
contribution of the $\mu$ term in the superpotential $W$, and for $\gamma \gtrsim 1$, is expected to play an
important role in the predictions of inflationary observables. The renormalization scale $Q$
is set equal to $\sigma_0$, the field value at the pivot scale $k_0 = 0.05$ Mpc$^{-1}$ [7].

The soft SUSY breaking terms are added in the inflationary potential as:

$$V_{\text{soft}} = m_{3/2} \left[ z_i \frac{\partial W}{\partial z_i} + (A - 3)W + \text{h.c.} \right], \tag{17}$$

where $A$ is the complex coefficient of the trilinear soft-SUSY-breaking terms.
The $F$-term SUGRA scalar potential is evaluated using

$$V_{\text{SUGRA}} = e^{K/m_P^2}(K^{-1}_{ij}D_{z_i}W D_{z_j} W^* - 3m_P^{-2}|W|^2),$$  \hspace{1cm} (18)

where $z_i \in \{S, H^c, \overline{H^c}, h, \ldots\}$ and

$$K_{ij} = \frac{\partial^2 K}{\partial z_i \partial z_j^*}, \quad D_{z_i} W \equiv \frac{\partial W}{\partial z_i} + m_P^{-2} \frac{\partial K}{\partial z_i} W, \quad D_{z_i}^* W^* = (D_{z_i} W)^*, \hspace{1cm} (19)$$

The Kähler potential $K$ is expanded in inverse powers of $m_P$:

$$K = K_c + \kappa_S \frac{|S|^4}{4m_P^2} + \kappa_H \frac{|H^c|^4}{4m_P^2} + \kappa_{\overline{H^c}} \frac{| \overline{H^c} |^4}{4m_P^2} + \kappa_h \frac{|h|^4}{4m_P^2} + \kappa_{SH^c} \frac{|S|^2|H^c|^2}{m_P^2} + \kappa_{SH} \frac{|S|^2|h|^2}{m_P^2} + \kappa_{SS} \frac{|S|^6}{6m_P^4} + \ldots, \hspace{1cm} (20)$$

where the minimal canonical Kähler potential $K_c$ is given by

$$K_c = |S|^2 + |H^c|^2 + |\overline{H^c}|^2 + |h|^2. \hspace{1cm} (21)$$

The inflationary potential along the D-flat direction with $|H^c| = |\overline{H^c}|$, stabilized along the $h = 0$ direction, and incorporating the SUGRA corrections [24], the radiative corrections [1], and the soft-SUSY-breaking terms [3, 4], is given by

$$V(x) \simeq V_{\text{SUGRA}} + V_{\text{loop}} + V_{\text{soft}} \simeq \kappa^2 m^4 \left(A + \frac{1}{2}B \left(\frac{m}{m_P}\right)^2 x^2 + \frac{1}{4}C \left(\frac{m}{m_P}\right)^4 x^4 \right) + \frac{\kappa^2}{4\pi^2} F(x) + \frac{\lambda^2}{4\pi^2} F(y) + a \frac{m_{3/2}^2}{\sqrt{2} \kappa m} x + \frac{m_{3/2}^2}{2k^2 m^2} x^2 + \frac{m_{3/2}^2 M^2}{k^2 m^4} \right). \hspace{1cm} (22)$$

Here $A$, $B$, and $C$ are the coefficients of the constant, quadratic, and quartic SUGRA terms, respectively, and are defined in terms of $H_P = (M/m_P)/\sqrt{2}\kappa$ as

$$A = 1 + 2c_0 H_P^2 + 2c_1 H_P^4, \quad B = -\kappa_S + 2c_2 H_P^2, \quad C = \frac{\gamma_S}{2}, \hspace{1cm} (23)$$

where $\gamma_S = 1 + 2\kappa_S^2 - 3\kappa_{SS} - 7\kappa_S/2$ [26]. The independently varying parameters $c_0$, $c_1$, and $c_2$ for the nonminimal case are similar to the ones given in Ref. [26]. Our choice for these
parameters will be shown in the relevant sections. The parameter $a$ depends on $\arg S$ as follows:

$$a = 2 \left| 2 - A + \frac{A}{2\xi} \right| \cos[\arg S + \arg(2 - A + \frac{A}{2\xi})]. \quad (24)$$

Assuming negligible variation in $\arg S$, with $a = -1$, the scalar spectral index $n_s$ is expected to lie within the experimental range $[4, 26]$. This could also be achieved by taking an intermediate-scale, negative soft mass-squared term for the inflaton [27]. But with the nonminimal terms in the Kähler potential, one can also obtain the central value of $n_s$ with TeV-scale soft masses even for $a = 1$ [8, 9]. The variation in $\arg S$ with general initial condition has been studied in Refs. [3, 6, 9].

The slow-roll parameters are defined by

$$\epsilon = \frac{m_p^2}{2m^2} \left( \frac{V'}{V} \right)^2, \quad \eta = \frac{m_p^2}{m^2} \left( \frac{V''}{V} \right), \quad \zeta^2 = \frac{m_p^4}{m^4} \left( \frac{V'V'''}{V^2} \right), \quad (25)$$

where the primes denote derivatives with respect to $x$. The scalar spectral index $n_s$, the tensor-to-scalar ratio $r$, the running of the scalar spectral index $dn_s/d\ln k$, and the scalar power spectrum amplitude $A_s$, to leading order in the slow-roll approximation, are as follows:

$$n_s \simeq 1 - 6\epsilon + 2\eta, \quad r \simeq 16\epsilon, \quad \frac{dn_s}{d\ln k} \simeq 16\epsilon \eta - 24\epsilon^2 - 2\zeta^2, \quad A_s(k_0) = \frac{1}{12\pi^2} \left( \frac{m}{m_P} \right)^2 \left| \frac{V^3}{V'} \right| \bigg|_{x_0}, \quad (26)$$

where $A_s(k_0) = 2.196 \times 10^{-9}$ and $x_0$ denotes the value of $x$ at the pivot scale $k_0 = 0.05 \text{ Mpc}^{-1}$ [7]. For the numerical estimation of the inflationary predictions, these relations are used up to second order in the slow-roll parameters.

Assuming a standard thermal history, the number of $e$-folds $N_0$ between the horizon exit of the pivot scale and the end of inflation is

$$N_0 = \left( \frac{m}{m_P} \right)^2 \int_{1}^{x_0} \left( \frac{V}{V'} \right) dx = 53 + \frac{1}{3} \ln \left( \frac{T_r}{10^9 \text{ GeV}} \right) + \frac{2}{3} \ln \left( \frac{\sqrt{\kappa m}}{10^{15} \text{ GeV}} \right). \quad (27)$$

The reheat temperature $T_r$ is approximated by

$$T_r \approx \sqrt{\frac{90}{\pi^2 g_*}} \sqrt{\Gamma_S m_P}, \quad (28)$$

where $g_* = 228.75$ for MSSM and $\Gamma_S$ is the inflaton decay width. From the $\mu$-term coupling
\( \lambda S h^2 \) in Eq. (5), we see that the inflaton can decay into a pair of Higgsinos \( \tilde{h}_u, \tilde{h}_d \) with a decay width

\[
\Gamma_S(S \rightarrow \tilde{h}_u \tilde{h}_d) = \frac{\lambda^2}{8\pi} m_{\text{infl}},
\]

where

\[
m_{\text{infl}} = \sqrt{2} \kappa v \left( 1 - \frac{2\xi v^2}{M^2} \right) = 2\kappa m \sqrt{1 - \sqrt{1 - 4\xi}}
\]

is the inflaton mass [12]. The reheat temperature, the inflaton decay width, and the inflaton mass defined above in Eqs. (28)-(30) are used together with Eq. (27) in order to derive the numerical predictions for the present inflationary scenario.

### IV. \( \mu \)-HYBRID INFLATION WITH MINIMAL KÄHLER POTENTIAL

The inflationary potential corresponding to the minimal Käehler potential \( K_c \) in Eq. (21) is easily transcribed from Eq. (22) as follows:

\[
V(x) \simeq \kappa^2 m^4 \left( 1 + 2 \left( \frac{M}{\sqrt{2} \xi m_p} \right)^2 + 2 \left( \frac{M}{\sqrt{2} \xi m_p} \right)^4 + \left( \frac{m}{m_p} \right)^2 x^2 + \frac{1}{8} \left( \frac{m}{m_p} \right)^4 x^4 \right)
\]

\[
+ \frac{\kappa^2}{4\pi^2} F(x) + \frac{\lambda^2}{4\pi^2} F(y) + a \frac{m_{3/2}}{\sqrt{2}\kappa m} x + \frac{m_{3/2}^2}{2k^2 m^2} x^2 + \frac{m_{3/2}^2 M^2}{2k^2 m^4 \xi}
\]

since, in this case, \( C = 1/2, c_0 = c_1 = c_2 = 1 \) and, thus, the coefficients \( A = 1 + 2(H_P^2 + H_P^4), B = 2H_P^2 \). Following the same line of argument as in Refs. [19, 22], the shifted \( \mu \)HI with minimal \( K \) is analyzed for the following three cases:

1. a stable gravitino LSP;

2. an unstable long-lived gravitino with mass \( m_{3/2} < 25 \text{ TeV} \);

3. an unstable short-lived gravitino with mass \( m_{3/2} > 25 \text{ TeV} \).

First we consider the possibility that the LSP is the gravitino. In Fig. 2, the upper thick solid-magenta, dashed-blue, dot-dashed-green curves show the variation of the gravitino mass constrained by inflation, for \( \xi = 0.125, 0.148, 0.245 \) respectively. The lower bound on the reheat temperature \( T_r \gtrsim 2 \times 10^{10} \text{ GeV} \) is obtained for a gravitino mass \( m_{3/2} \gtrsim 1 \text{ TeV} \). If
FIG. 2: Plot of the gravitino mass $m_{3/2}$ versus the reheat temperature $T_r$. The thick solid-magenta, dashed-blue, dot-dashed-green curves correspond to $\xi = 0.125$, 0.148, 0.245 respectively for the minimal Kähler potential with the conditions $n_s \approx 0.964$, $\gamma = 2$, and $a = -1$. The thin curves represent the corresponding varying gluino mass $m_{\tilde{g}}$ (see Eq. (32)). The LHC lower bound on the gluino mass ($m_{\tilde{g}} \gtrsim 2\text{ TeV}$), shown by a gray vertical line at $T_r \sim 1.4 \times 10^{11}$ GeV, excludes the shaded region. For the unstable gravitino scenario, $m_{3/2} = 25\text{ TeV}$ corresponds to $T_r \sim 10^{11}$ GeV as shown by the vertical dashed-gray line.

If the gravitino is the LSP and hence constitutes DM, the DM relic abundance $\Omega_{DM} h^2 = 0.12$ [7] can be used to obtain the variation of the gluino mass $m_{\tilde{g}}$ with $T_r$:

$$\frac{m_{\tilde{g}}}{2\text{ TeV}} = \left[0.46 \left(\frac{10^{10}\text{ GeV}}{T_r}\right) \left(\frac{m_{3/2}}{1\text{ TeV}}\right)^{\frac{1}{2}}\right].$$

This variation is depicted by the lower-thin-faded curves in Fig. 2 (again for $\xi = 0.125$, 0.148, 0.245), which are cutoff at 2 TeV, thus complying with the LHC bound on the gluino mass. The shaded region is excluded by the LHC lower bound on gluino mass ($m_{\tilde{g}} \gtrsim 2\text{ TeV}$). Since the thick curves representing the gravitino mass are above the gluino mass depicted by thin-faded curves in the region above the LHC cutoff, the gravitino cannot be the LSP.

In the second case, the long-lived unstable gravitino will decay after big bang nucleosynthesis (BBN), and so one has to take into account the BBN bounds on the reheat temperature
which are the following [28–30]:

\[ T_r \lesssim 3 \times (10^5 - 10^6) \text{ GeV}, \quad m_{3/2} \sim 1 \text{ TeV}, \]
\[ T_r \lesssim 2 \times 10^9 \text{ GeV}, \quad m_{3/2} \sim 10 \text{ TeV}. \]  

(33)

The bounds on the reheat temperature from the inflationary constraints for gravitino masses 1 and 10 TeV are \( T_r \gtrsim 2.2 \times 10^{10} \text{ GeV} \) and \( 7.5 \times 10^{10} \text{ GeV} \) respectively (see Fig. 2). These are clearly inconsistent with the above mentioned BBN bounds, and so the unstable long-lived gravitino scenario is not viable.

Lastly, for the unstable short-lived gravitino case, we compute the LSP lightest neutralino (\( \tilde{\chi}_1^0 \)) density produced by the gravitino decay and constrain it to be smaller than the observed DM relic density. For reheat temperature \( T_r \gtrsim 10^{11} \text{ GeV} \) with \( m_{3/2} > 25 \text{ TeV} \) (see Fig. 2), the resulting bound on the neutralino mass \( m_{\tilde{\chi}_1^0} \) comes out to be inconsistent with the lower limit set on this mass \( m_{\tilde{\chi}_1^0} \gtrsim 18 \text{ GeV} \) in Ref. [31]. To circumvent this, the LSP neutralino is assumed to be in thermal equilibrium during gravitino decay, whereby the neutralino abundance is independent of the gravitino yield. For an unstable gravitino, the lifetime is

\[ \tau_{3/2} \simeq 1.6 \times 10^4 \left( \frac{1 \text{ TeV}}{m_{3/2}} \right)^3 \text{ sec}. \]  

(34)

Now for a typical value of the neutralino freeze-out temperature, \( T_F \simeq 0.05 m_{\tilde{\chi}_1^0} \), the gravitino lifetime is estimated to be

\[ \tau_{3/2} \lesssim 10^{-11} \left( \frac{1 \text{ TeV}}{m_{\tilde{\chi}_1^0}} \right)^2 \text{ sec}. \]  

(35)

Comparing Eq. (34) and Eq. (35), we obtain a bound on \( m_{3/2} \),

\[ m_{3/2} \gtrsim 10^8 \left( \frac{m_{\tilde{\chi}_1^0}}{2 \text{ TeV}} \right)^{2/3} \text{ GeV}. \]

(36)

Thus, minimal shifted \( \mu \text{HI} \) conforms with the conclusion of the standard case [19, 22] by requiring split-SUSY with an intermediate-scale gravitino mass and reheat temperature \( T_r \gtrsim 10^{13} \text{ GeV} \) (see Fig. 2). To check whether the shifted \( \mu \text{HI} \) scenario is also compatible with low reheat temperature (i.e. \( T_r \lesssim 10^{12} - 10^8 \text{ GeV} \) [33]) and TeV-scale soft SUSY breaking, we employ nonminimal Kähler potential in the next section.
V. $\mu$-HYBRID INFLATION WITH NONMINIMAL KÄHLER POTENTIAL

The nonminimal Kähler potential used in the following analysis is

$$K = K_c + \kappa_S \frac{|S|^4}{4m_p^2} + \kappa_{SS} \frac{|S|^6}{6m_p^4},$$

which includes only the nonminimal couplings of interest $\kappa_S$ and $\kappa_{SS}$. (For a somewhat different approach to $\mu$-hybrid inflation with nonminimal $K$, see Ref. [34]). Thus, for the nonminimal scenario we take $c_0 = c_1 = 1$ and $c_2 = 1 - \kappa_S$ in Eq. (23) [26]. Using these values the potential of the system can easily be read off from Eq. (22).

It is worth noting that with the nonminimal Kähler potential we can realize the central value of $n_s$ with TeV-scale soft masses even for $a = 1$ [8, 9]. Our study is conducted in two parts, described separately in the following subsections, first with $\kappa_{SS} = 0$ and then by allowing $\kappa_{SS}$ to be nonzero. The appearance of a negative mass term with a single nonminimal coupling $\kappa_S$ in the potential in Eq. (22) is expected to lead to red-tilted inflation with low reheat temperature, as for standard $\mu$HI (see Ref. [22]). Furthermore for nonzero $\kappa_{SS}$, the possible larger $r$ solutions leading to observable gravity waves are also anticipated. These expectations along with the impact of an additional parameter $\xi$ on inflationary predictions are discussed below.

A. Low reheat temperature and the gravitino problem

Incorporating the inflationary constraints and the nonminimal $K$ in Eq. (37) with $\kappa_{SS} = 0$, we summarize some of the results depicting the main features of nonminimal shifted $\mu$HI in Figs. 3–5. From these figures it is clear that with low reheat temperature we can obtain a higher gauge symmetry breaking scale $M$ ranging from $5 \times 10^{15}$ GeV to the string scale $5 \times 10^{17}$ GeV. The reheat temperature is lowered by nearly half an order of magnitude in the shifted $\mu$HI as compared to the standard $\mu$HI (see Ref. [22]), as can be seen from Fig. 3. Also, it is not surprising that around $\kappa \sim 10^{-3}$ the system is oblivious to the gravitino mass, since the contribution of the linear term becomes less important compared with the SUGRA or radiative corrections [8]. The interesting new feature is due to the presence of another parameter $\xi$, whose effect is to increase the range of symmetry breaking scale $M$. For a particular value of $\kappa$, say $\kappa \sim 10^{-6}$, and $m_{3/2} = 1$ TeV, a wider range
FIG. 3: The symmetry breaking scale $M$ versus the reheat temperature $T_r$ and $T_r$ versus $\kappa$, for gravitino mass equal to 1 TeV (thick-green curves), 10 TeV (dot-dashed-red curve), and 100 TeV (thin-blue curves). We fix the scalar spectral index $n_s = 0.9655$, $\kappa_S = 0.02$, $\kappa_{SS} = 0$, and $\gamma = 2$. The parameter $\xi = 0.125$, 0.167, and 0.245 corresponding to the solid, dashed, and dotted curves respectively.

Of $M \simeq 5 \times (10^{15} - 10^{16})$ GeV exists, corresponding to $\xi$ in the range $0.125 \leq \xi \leq 0.245$ (see Fig. 4). So there is an order of magnitude increase in the spread of $M$, compared with standard $\mu$HI, where the maximum value is $M \sim 8 \times 10^{15}$ GeV corresponding to the
FIG. 4: The symmetry breaking scale $M$ versus $\kappa$ and the tensor-to-scalar ratio $r$, for gravitino mass equal to 1 TeV (thick-green curves) and 100 TeV (thin-blue curves). We fix the scalar spectral index $n_s = 0.9655$, $\kappa_S = 0.02$, $\kappa_{SS} = 0$, and $\gamma = 2$. We consider three values of $\xi$, namely $\xi = 0.125$, 0.167, and 0.245 corresponding to the solid, dashed, and dotted curves respectively.

The lowest reheat temperature $T_r \sim 6 \times 10^6$ GeV, with gravitino of mass 1 TeV [22]. This maximum value has now increased to $M \simeq (9 \times 10^{15} - 7 \times 10^{16})$ GeV with $\xi$ in the range $0.125 \leq \xi \leq 0.245$. Also, the lower plot of Fig. 4 shows the variation of $M$ with respect
FIG. 5: The symmetry breaking scale $M$ versus the running of spectral index $-\frac{dn_s}{d \ln k}$ and $\kappa_S$, for gravitino mass of 1 TeV (thick-green curves) and 100 TeV (thin-blue curves). We fix the scalar spectral index $n_s = 0.9655$, $\kappa_S = 0.02$, $\kappa_{SS} = 0$, and $\gamma = 2$. The parameter $\xi = 0.125$, 0.167, and 0.245 corresponding to the solid, dashed, and dotted curves respectively.

to the tensor-to-scalar ratio $r$ with $r \lesssim 10^{-9}$, which is experimentally inaccessible in the foreseeable future [35–38].

As Fig. 5 shows, the running of the scalar spectral index $dn_s/d \ln k$ also turns out to
be small in the present scenario, namely $10^{-10} \lesssim -\frac{d n_s}{d \ln k} \lesssim 10^{-4}$, which is a common feature of small field models. The nonminimal Kähler coupling $\kappa_S$ remains constant in the low reheat temperature range as can be seen from the lower plot of Fig. 5, since the radiative and the quartic-SUGRA corrections can be neglected in this regime. The scalar spectral index $n_s$ in the low reheat temperature region is $n_s \simeq 1 - 2\kappa_S$ [15], and so for the central value of the scalar spectral index $n_s = 0.9655$, one obtains $\kappa_S = 0.0173$, as exemplified by Fig. 5. To explore larger values of $r$, we will make use of the freedom provided by the second nonrenormalizable coupling $\kappa_{SS}$ in the next section. Note that the number of e-folds $N_0$ in Eq. (27) generally ranges between about 47 and 56.

Proceeding next to the role of the gravitino in cosmology, one can read off the lower bounds on the reheat temperature $T_r$ from Fig. 3. Since, at low reheat temperatures, inflation occurs near the waterfall region (with $x_0$ close to 1), we devised a criterion by allowing only 0.01% fine-tuning on the difference $x_0 - 1$. This yields

$$T_r \gtrsim 2 \times 10^6, \; 7 \times 10^5, \; \text{or} \; 2 \times 10^5 \text{ GeV} \quad \text{for} \quad m_{3/2} = 1, 10, \text{or} \; 100 \text{ TeV}. \quad (38)$$

For the first scenario with the gravitino being the LSP in shifted $\mu$HI with nonminimal Kähler potential, the upper bounds on the reheat temperature obtained in Ref. [22] (see Fig. 3 and Eq. (30) in this reference) are $T_r \lesssim 2 \times (10^{10}, \; 10^9, \; 10^8) \text{ GeV}$ for $m_{3/2} = 1, 10, \; 100 \text{ TeV}$ respectively. These upper bounds on $T_r$ are consistent with the lower bounds in Eq. (38), and so the scenario with the gravitino as LSP can be consistently realized in the nonminimal Kähler case.

For the second possibility, namely an unstable long-lived gravitino (with $m_{3/2} \lesssim 25 \text{ TeV}$), comparison of Eqs. (33) and Eq. (38) reveals that an 1 TeV gravitino is marginally ruled out but a 10 TeV gravitino lies comfortably within the BBN bounds.

For the third scenario of a short-lived gravitino (for instance with mass $m_{3/2} = 100 \text{ TeV}$), the gravitino decays before BBN, and so the BBN bounds on the reheat temperature no longer apply. The gravitino decays into the LSP neutralino $\tilde{\chi}_1^0$. We find that the resulting neutralino abundance is given by

$$\Omega_{\tilde{\chi}_1^0} h^2 \simeq 2.8 \times 10^{11} \times Y_{3/2} \left( \frac{m_{\tilde{\chi}_1^0}}{1 \text{ TeV}} \right), \quad (39)$$
where the gravitino yield
\[ Y_{3/2} \simeq 2.3 \times 10^{-12} \left( \frac{T_r}{10^{10} \text{ GeV}} \right) \]

is acceptable over the range \( T_r \sim 10^5 \text{ GeV} - 10^{12} \text{ GeV} \) [32]. The LSP (lightest neutralino) density produced by the gravitino decay should not exceed the observed DM relic density \( \Omega_{DM} h^2 \simeq 0.12 \) [7]. The resulting bound on the lightest neutralino mass
\[ m_{\tilde{\chi}_1^0} \lesssim (18 - 10^6) \text{ GeV} \text{ for } 10^{11} \text{ GeV} \gtrsim T_r \gtrsim 2 \times 10^5 \text{ GeV} \]

turns out to be less restrictive than the corresponding bound from the abundance of the lightest neutralino from the gravitino decay in the case of standard \( \mu \text{HI} \). Indeed, the non-LSP gravitino with \( m_{3/2} \sim 100 \text{ TeV} \) is acceptable in a larger domain, namely \( 10^5 \text{ GeV} \lesssim T_r \lesssim 10^{11} \text{ GeV} \). There is nearly an order of magnitude decrease in the acceptable lower reheat temperature as compared with the standard \( \mu \text{HI} \). Note that the lower limit on the neutralino mass, \( m_{\tilde{\chi}_1^0} \gtrsim 18 \text{ GeV} \), is obtained in Ref. [31] by employing a minimal set of theoretical assumptions. In conclusion the shifted \( \mu \text{HI} \) is successful with \( m_{3/2} \sim 1 - 100 \text{ TeV} \) and low reheat temperatures.

**B. Large r solutions or observable gravity waves**

The canonical measure of primordial gravity waves is the tensor-to-scalar ratio \( r \) and the next-generation experiments are gearing up to measure it. One of the highlights of PRISM [35] is to detect inflationary gravity waves with \( r \) as low as \( 5 \times 10^{-4} \) and a major goal of LiteBIRD [36] is to attain a measurement of \( r \) within an uncertainty of \( \delta r = 0.001 \). Future missions include PIXIE [37], which aims to measure \( r < 10^{-3} \) at 5 standard deviations, and CORE [38], which forecasts to lower the detection limit for the tensor-to-scalar ratio down to the \( 10^{-3} \) level. As seen in previous sections, with \( \kappa_{SS} = 0 \), the tensor-to-scalar ratio remains in the undetectable range \( r \lesssim 10^{-6} \). It is therefore instructive to explore our model further to look for large-\( r \) solutions, which, as it turns out, yield \( r \)'s in the \( 10^{-4} - 10^{-3} \) range. To achieve this, we employ nonzero \( \kappa_{SS} \) in addition to a nonzero \( \kappa_S \), and the results are presented in Figs. 6–9, for a range of values of the field \( S \) at horizon crossing of the pivot scale \( S_0 = (0.1 - 1) m_P \). In addition, the variation of the parameter \( \xi \) is also depicted in these figures by plotting results with \( \xi = 0.125 \) and \( \xi = 0.2 \).
FIG. 6: The symmetry breaking scale $M$ versus the tensor-to-scalar ratio $r$ for $\xi = 0.125$ and $\xi = 0.2$ in the upper and lower plot respectively. The gravitino mass $m_{3/2} \sim 1 - 100$ TeV, $n_s = 0.9655$, $\gamma = 2$, and $S_0 = (0.1 - 1) m_P$. The solid-gray lines are the constant reheat temperature curves ranging from $10^5 - 10^{12}$ GeV. The dashed-gray line represents the fine-tuning bound, and the double-dot-dashed line represents either the upper bound on $\kappa_{SS}$ or the points where $M = m_P$.

The curves corresponding to field values $S_0$ close to $m_P$ are terminated since, at some point, either the nonminimal coupling $|\kappa_{SS}|$ takes unnatural values $\approx 10$ (see Fig. 9) or $M$ reaches $m_P$. Indeed, for $\xi = 0.125$, the coupling $|\kappa_{SS}|$ can exceed the bound of 10 on curves
FIG. 7: The symmetry breaking scale $M$ versus the running of the scalar spectral index $d n_s / d \ln k$ for $\xi = 0.125$ and $\xi = 0.2$ in the upper and lower plot respectively. The gravitino mass $m_{3/2} \sim 1 - 100$ TeV, $n_s = 0.9655$, $\gamma = 2$, and $S_0 = (0.1 - 1)$ $m_P$. The solid-gray lines are the constant reheat temperature curves ranging from $10^5 - 10^{12}$ GeV. The dashed-gray line shows the fine-tuning bound, and the double-dot-dashed line shows either the upper bound on $\kappa_{SS}$ or the points where $M = m_P$.

with $S_0 \geq 0.8$ $m_P$ and, for $\xi = 0.2$, the symmetry breaking scale $M$ can exceed $m_P$ on curves with $S_0 \geq 0.5$ $m_P$. We see that the symmetry breaking scale $M$ is not independent of $\xi$. In fact, as $\xi$ increases from $\xi = 0.125$ to $\xi = 0.2$, the symmetry breaking scale also
FIG. 8: The symmetry breaking scale $M$ versus $\kappa$ for $\xi = 0.125$ and $\xi = 0.2$ in the upper and lower plot respectively. The gravitino mass $m_{3/2} \sim 1-100$ TeV, $n_s = 0.9655$, $\gamma = 2$ and $S_0 = (0.1-1) m_P$. The solid-gray lines are the constant reheat temperature curves ranging from $10^5$ – $10^{12}$ GeV. The dashed-gray line represents the fine-tuning bound, and the double-dot-dashed line represents either the upper bound on $\kappa_{SS}$ or the points where $M = m_P$. The curves are terminated at their left end due to the fine-tuning bound that we used in the numerical work. The solid-gray lines in Figs. 6–8 are the constant reheat temperature lines, starting from the upper cutoff.

increases (this is observed in the $\kappa_{SS} = 0$ case as well).
FIG. 9: The variation of the couplings $\kappa_S$ and $\kappa_{SS}$ for $\xi = 0.125$ and $\xi = 0.2$ in the upper and lower plot respectively. The gravitino mass range $m_{3/2} \sim 1 - 100$ TeV, $n_s = 0.9655$, $\gamma = 2$, and $S_0 = (0.1 - 1) m_P$.

at $T_r = 10^{12}$ GeV and going down to values as low as $10^4 - 10^5$ GeV.

The upper bound on $r$ as can be read off from Fig. 6 is $r \lesssim 0.001$ for the choice $S_0 = m_P$ and $r \lesssim 10^{-5}$ for $S_0 \sim 0.1 m_P$. The Fig. 6 also shows that $r \lesssim 10^{-6} - 10^{-3}$ from the requirement that $T_r \lesssim 10^{11}$ GeV for circumventing the gravitino problem. The running of the scalar spectral index $dn_s/d\ln k$ remains small namely $10^{-7} \lesssim -dn_s/d\ln k \lesssim 4 \times 10^{-3}$, as shown in Fig. 7. The variation of the symmetry breaking scale $M$ with $\kappa$ is shown in Fig. 8, where we find values of $\kappa$ up to $5 \times 10^{-4}$ for large values of $M$ ($\sim 10^{17} - 10^{18}$ GeV). The respective variation in the couplings $\kappa_S$ and $\kappa_{SS}$ is shown in Fig. 9. They remain acceptably small and well within the bound $|\kappa_S|, |\kappa_{SS}| \lesssim 1$, for natural values of $S_0 = 0.5 m_P$ or less.
Although the plots in Figs. 6–9 are for gravitino mass $m_{3/2} = 1$ TeV, the curves, for these larger $r$ solutions, are independent of the gravitino mass and are valid for a gravitino mass range $m_{3/2} = 1 - 100$ TeV.

VI. CONCLUSION

We have implemented a version of SUSY hybrid inflation in $SU(4)_c \times SU(2)_L \times SU(2)_R$, a well motivated extension of the SM. This maximal subgroup of Spin(10) contains electric charge quantization and arises in a variety of string theory constructions. The MSSM $\mu$ term arises, following Dvali, Lazarides, and Shafi, from the coupling of the electroweak doublets to a gauge singlet superfield playing an essential role in inflation, which takes place along a shifted flat direction. The scheme with minimal Kähler potential leads to an intermediate scale gravitino mass $m_{3/2} \gtrsim 10^8$ GeV with the gravitino decaying before the freeze out of the LSP neutralinos and with reheat temperature $T_r \gtrsim 10^{13}$ GeV [19]. This points towards split SUSY. In the nonminimal Kähler case, we have realized successful inflation with reheat temperatures as low as $10^5$ GeV. This is favorable for the resolution of the gravitino problem and compatible with a stable LSP and low-scale ($\sim$TeV) SUSY. Compared with standard $\mu$ hybrid inflation [22], the reheat temperature is lowered by half an order of magnitude and, due to the additional parameter $\xi$, an order of magnitude increase in the spread of $M$ is seen. We have discussed how primordial monopoles are inflated away and provided a framework that predicts the presence of primordial gravity waves with the tensor-to-scalar ratio $r$ in the observable range ($\sim 10^{-4} - 10^{-3}$). This is realized with the $G_{4,2,2}$ symmetry breaking scale approaching values that are comparable to the string scale ($\sim 5 \times 10^{17}$ GeV) and a gravitino mass lying in the $1 - 100$ TeV range. It is worth noting that the inflaton field values do not exceed the Planck scale, which may be an additional desirable feature in view of the swampland conjectures [39, 40]. For a recent discussion and additional references see Ref. [41].

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