Electromagnetic Mass Difference of Heavy Mesons

P. Colangelo\textsuperscript{a}, M. Ladisa\textsuperscript{a,b}, G. Nardulli\textsuperscript{a,b}, T. N. Pham\textsuperscript{c}

\textsuperscript{a} Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy
\textsuperscript{b} Dipartimento di Fisica, Università di Bari, Italy
\textsuperscript{c} Centre de Physique Théorique, Centre National de la Recherche Scientifique, UPR A0014, Ecole Polytechnique, 91128 Palaiseau Cedex, France

Abstract

Using the Cottingham formula, we give an estimate of the electromagnetic mass splitting of pseudoscalar heavy mesons in the beauty and charm sector. We include in the dispersion relation the Born term, the $1^-$ resonance and the positive parity $1^+$ resonance. We also evaluate the contribution to the mass difference from the isospin breaking quark mass differences. Our results: $m_{B^+} - m_{B^0} = -0.83 \pm 0.34 \text{ MeV}$ and $m_{D^+} - m_{D^0} = +4.33 \pm 0.37 \text{ MeV}$, are in agreement with the experimental measurements: $m_{B^+} - m_{B^0} = -0.35 \pm 0.29 \text{ MeV}$ and $m_{D^+} - m_{D^0} = +4.78 \pm 0.10 \text{ MeV}$. We also compute the mass differences in the infinite heavy quark mass limit, which show small deviations from the finite mass results for the $B$ case and 30% effects in the charm case.
1 Introduction

The mass difference between $B^+$ and $B^0$ mesons is an interesting physical quantity, whose precise knowledge might be of primary importance at the future B-factories. As a matter of fact, the ratio $\frac{BR(\Upsilon(4S) \to B^0\bar{B}^0)}{BR(\Upsilon(4S) \to B^+B^-)}$ determines the relative abundance of neutral and charged $B$ mesons produced at such accelerators and is strongly dependent on the $B^+ - B^0$ mass difference, since the $B$ pair production threshold is very close to the $\Upsilon(4S)$ mass.

The experimental determination of $\delta m(B^+ - B^0)$ changed significantly during the last ten years, from the value $\delta m = -2.0 \pm 1.1 \pm 0.3$ MeV [1] to the values $\delta m = +0.9 \pm 1.2 \pm 0.5$ MeV (ARGUS) [2] and $\delta m = +0.4 \pm 0.6 \pm 0.5$ MeV (CLEO) [3]. A recent measurement by the CLEO Collaboration [4] gives a negative mass difference $\delta m = -0.41 \pm 0.25 \pm 0.19$ MeV, and the combined CLEO-ARGUS result is $\delta m = -0.35 \pm 0.29$ MeV [5]. Such a small value has to be compared to the analogous figure for the $D^+ - D^0$ mass difference: $\delta m(D^+ - D^0) = +4.78 \pm 0.10$ MeV.

In the theoretical understanding of these values an important role is played by the isospin symmetry breaking effects related to the current quark masses. According to the modern picture, which incorporates the old tadpole mechanism of Coleman and Glashow [6], the strong isospin breaking is due to the intrinsic $u - d$ mass difference. By making the $d$ quark heavier than the $u$ quark, one can explain, at least qualitatively, all the known meson and baryon electromagnetic mass differences [7]. In particular, the large value for the $D^+ - D^0$ mass difference can be explained by the combined effects of the $u - d$ mass difference and the repulsive Coulomb energy between the $c$ and $\bar{d}$ quark.

According to heavy quark symmetry, the effect due to the $u - d$ mass difference $(q_m)$ is independent of the heavy quark mass; therefore, in the case of $B^+ - B^0$ the quark mass term gives a large negative contribution and would cancel out the repulsive Coulomb electrostatic energy resulting in a small $B^+ - B^0$ mass difference. Such a simple picture, however, has to be implemented quantitatively, and this is the aim of the present letter. We begin by giving in Section 2 an estimate of the contribution to $\delta m$ arising from the $u - d$ mass difference, using $SU(3)$ flavour symmetry and data on $B_s - B$ mass differences. Since the tiny $B^+ - B^0$ mass difference arises from the sum of two terms, comparable in size (a few MeV), but opposite in sign, it is desirable to have an estimate of the electromagnetic contribution as accurate as possible. This task is afforded by using the covariant Cottingham formula [8], a method employed for the calculation of
the electromagnetic mass differences in the light hadron sector. By the Cottingham approach one relates the electromagnetic (e.m.) mass difference to the forward Compton scattering amplitudes $T_1$ and $T_2$, which satisfy dispersion relations (DR) and can be put in a form which contains integration over space-like photon momenta $q^2 = -Q^2 < 0$.

The application of the Cottingham formula to the evaluation of electromagnetic mass differences has a long story. Previous (prior to QCD) attempts to use the Cottingham formula for evaluating electromagnetic hadron mass differences encountered two problems: the first one is the convergence of the $Q^2$ integral and the second one is the convergence of the DR satisfied by $T_i$. The current approach to these problems involves a cut-off of the $Q^2$ integral at a maximum value $Q_{max}^2 = \mu^2$, where $\mu$ represents a scale coinciding with the onset of the QCD scaling behaviour: this point is discussed in Section 3. As for the convergence of the DR, the different contributions to $\text{Im} T_i$ can be related to different Feynman graphs of an effective theory including hadrons and photons. For light mesons this can be done by using chiral perturbation theory, and recently some determinations of $\delta m(\pi^+ - \pi^0)$ and $\delta m(K^+ - K^0)$ by chiral perturbation theory have appeared in the literature. In this approach, the subtraction constant can be computed directly from the Feynman amplitudes. A similar effective theory was developed also for heavy mesons (for a review see), and we will use it in our description of the electromagnetic coupling of the heavy mesons involved in the calculations (the low-lying $B$ and $B^*$ mesons and the first excited positive parity resonances). Therefore, also in our approach the subtraction constant in the DR is directly evaluated from the Feynman amplitude. These points are discussed in Section 4.

We conclude the paper by computing in Section 5 the meson mass differences in the $m_Q \to \infty$ limit. This calculation allows a remarkable simplification of the formalism, with a clear view of the mechanism producing $\delta m$. We find that the infinite limit can be well applied to the $B$ case, whereas in the charm case the deviation due to the finite heavy quark mass is of the order of 30%.

Previous attempts to estimate the electromagnetic contribution to $\delta m(B^+ - B^0)$ used the quark model and QCD sum rules; in the result $\delta m(B^+ - B^0) = -1.5 \text{ MeV}$, including quark mass effects, was obtained. This issue has been investigated using the Cottingham formula in and . In the elastic contribution is considered with a different treatment of the light quark currents, disregarding inelastic contributions. In the calculations are carried out in the $N_c \to \infty$ limit. The numerical results are in agreement with the results of this paper.
2 Quark mass contributions

The contribution $\delta m(B^+ - B^0)_{qm}$ and $\delta m(D^+ - D^0)_{qm}$ from the strong isospin breaking $u - d$ quark mass difference: $m_u - m_d$, can be computed observing that the (approximate) $SU(3)$ flavour symmetry allows us to write:

$$\delta m^2(B^+ - B^0)_{qm} = (m_u - m_d) < B^+ | \bar{u}u | B^+ > \simeq (m_u - m_d) < B_s | \bar{s}s | B_s > .$$ (1)

Considering the quark mass contribution to the $B_s - B^0$ and $B_s - B^+$ mass differences, we have similarly

$$\delta m^2(B_s - B^+)_{qm} + \delta m^2(B_s - B^0)_{qm} = 2 \left[ m_s - \frac{m_u + m_d}{2} \right] < B_s | \bar{s}s | B_s > .$$ (2)

For $\delta m(B_s - B)$ we can assume that the quark mass contribution basically coincides with $\delta m$, since it is of the order of the strange quark mass ($\simeq 100 MeV$), i.e. much larger than the expected electromagnetic mass difference (of the order $\alpha \Lambda_{QCD} \simeq$ a few MeV). Writing

$$\delta m^2(B_s - B)_{qm} \simeq \delta m^2(B_s - B)$$ (3)

we obtain

$$\delta m^2(B^+ - B^0)_{qm} = \left[ \delta m^2(B_s - B^+) + \delta m^2(B_s - B^0) \right] \frac{m_u - m_d}{2m_s - (m_u + m_d)} .$$ (4)

A similar formula holds for $\delta m^2(D^+ - D^0)_{qm}$. Using experimental data for $\delta m^2(B_s - B^+)$, $\delta m^2(B_s - B^0)$, $\delta m^2(D_s - D^+)$, $\delta m^2(D_s - D^0)$ [5] and the result given in [16] for the (scale independent) ratios of current quark masses from chiral perturbation theory:

$$\frac{m_s - (m_u + m_d)/2}{m_d - m_u} = 40.8 \pm 3.2 ,$$ (5)

we obtain:

$$\delta m(B^+ - B^0)_{qm} = -2.23 \pm 0.27 MeV$$ (6)

$$\delta m(D^+ - D^0)_{qm} = +2.54 \pm 0.21 MeV .$$ (7)

In the limit $m_Q \to \infty$ we expect $\delta m(B^+ - B^0)_{qm} = -\delta m(D^+ - D^0)_{qm}$ regardless of $m_Q$, a prediction which is supported by the results [5,7].
3 The Cottingham formula

Let us consider the mass splitting of heavy mesons due to the electromagnetic interaction. To be definite, we consider the $B$ meson; its electromagnetic mass shift can be derived by computing:

$$\delta m^2 = \frac{ie^2}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{g_{\mu\nu} T^{\mu\nu}(q,p)}{q^2 + i\epsilon}$$

where

$$T^{\mu\nu}(q,p) = i \int d^4x e^{-iqx} < B(p) | T(J^{\mu}(x)J^{\nu}(0)) | B(p) > ;$$

$J^{\mu}$ is the electromagnetic current. The Compton amplitude can be decomposed in terms of gauge invariant tensors:

$$T^{\mu\nu}(q,p) = D^{\mu\nu}_1 T_1(q^2, \nu) + D^{\mu\nu}_2 T_2(q^2, \nu)$$

($\nu = p \cdot q$), where

$$D^{\mu\nu}_1 = -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}$$

$$D^{\mu\nu}_2 = \frac{1}{m_B^2} \left( p^\mu - \frac{\nu}{q^2} q^\mu \right) \left( p^{\nu} - \frac{\nu}{q^2} q^{\nu} \right) ,$$

$m_B$ being the meson mass.

The first step in the calculation of the integral [8] consists of a rotation in the complex plane and a change of variables. Let us consider the meson rest frame, $\nu = m_B q_0$. Since the singularities in $T^{\mu\nu}$ are located just below the positive real axis and just above the negative real axis in the complex $q_0$ plane, the integration over $q_0$ may be rotated to the imaginary axis $q_0 = i k_0$ without encountering any singularity. After this transformation, the integral involves only spacelike momenta for the photon, i.e. $q^2 = -Q^2 = -(q_0^2 + q^2)$. After a change of variables from $(|q|, q_0)$ to $(Q^2, k_0)$, one obtains the Cottingham formula [8]:

$$\delta m^2 = \frac{e^2}{16\pi^3} \int_0^{\mu^2} \frac{dQ^2}{Q^2} \int_{-\sqrt{Q^2}}^{+\sqrt{Q^2}} dk_0 \sqrt{Q^2 - k_0^2} \times$$

$$\times \left[ -3 T_1(-Q^2, ik_0) + \left( 1 - \frac{k_0^2}{Q^2} \right) T_2(-Q^2, ik_0) \right] .$$

In eq.(13) we have introduced a cut-off in the $Q^2$ integration at $Q^2_{max} = \mu^2$. Its origin is as follows (see [13] for a detailed discussion). To take into account possible ultraviolet (UV) divergences, the Cottingham formula has to be renormalized. The renormalization
is accomplished by a regularization of the $Q^2$ integral and the inclusion of counterterms in the lagrangian describing electromagnetic and strong interactions of quarks, gluons and photons. Both the strong coupling constant $\alpha_s$ and the quark masses $m_q$ have to be specified at a renormalization mass scale $\mu$; since the counterterms cancel the infinite contribution induced by virtual particles with momenta larger than $\mu$, the net effect is analogous to a cut-off of the $Q^2$ integral at $Q^2_{\text{max}} = \mu^2$. There is a residue smooth dependence on $\mu$, but it should be canceled by the $\mu$–dependence of the renormalized quark masses and strong coupling constant. Typical values of $\mu$ are in the range of 1-2 GeV, corresponding to the onset of the scaling behaviour of QCD. The presence of heavy quarks does not change this procedure since the relevant mass scale, in the infinite heavy quark mass limit, is the residual energy release and the onset of scaling is again at a few GeV in this variable.

The Compton amplitudes $T_i (i = 1, 2)$ satisfy dispersion relations (DR) in the $\nu = p \cdot q$ variable with $T_1$ requiring one subtraction \[8\], as follows:

\begin{align*}
T_1(q^2, \nu) &= T_1(q^2, 0) + \frac{\nu^2}{\pi} \int_0^\infty \frac{dv'^2}{\nu'^2 - \nu^2} \text{Im} T_1(q^2, \nu') \\
T_2(q^2, \nu) &= \frac{1}{\pi} \int_0^\infty \frac{dv'^2}{\nu'^2 - \nu^2} \text{Im} T_2(q^2, \nu').
\end{align*}

(14)

(15)

By employing these DR, the integral over $k_0$ in eq.(13) can be performed explicitly, with the result:

\begin{align*}
\delta m^2 &= \frac{\alpha}{4\pi} \int_0^\mu dQ^2 \left[ -\frac{3}{2} T_1(-Q^2, 0) + 3 \int_0^\infty \frac{dv'^2}{\nu'^2} W_1(-Q^2, \nu') \Lambda_1 \left( \frac{\nu'^2}{m_B^2 Q^2} \right) \\
&\quad + \int_0^\infty \frac{dv'^2}{m_B^2 Q^2} W_2(-Q^2, \nu') \Lambda_2 \left( \frac{\nu'^2}{m_B^2 Q^2} \right) \right],
\end{align*}

(16)

where

\begin{align*}
\frac{1}{\pi} \text{Im} T_i(q^2, \nu) &= W_i(q^2, \nu)
\end{align*}

(17)

and

\begin{align*}
\Lambda_1(y) &= \frac{1}{2} + y - y \sqrt{1 + \frac{1}{y}} \\
\Lambda_2(y) &= -\frac{3}{2} - y + (1 + y) \sqrt{1 + \frac{1}{y}}.
\end{align*}

(18)

(19)
4 B and D meson electromagnetic mass differences

In order to evaluate the DR (14),(15) we consider the contribution of the Born term (the B meson), the $J^P = 1^-$ resonance $B^*$, and the positive parity resonance $J^P = 1^+ B_1$. We notice that the Born term pole and the $B^*$ belong to the supermultiplet $s_ℓ^P = \left(\frac{1}{2}\right)^−$ of the Heavy Quark Effective Theory (HQET) ($s_ℓ^P$ is the total angular momentum of the light degrees of freedom), whereas $B_1$ is the $J^P = 1^+$ partner of the $s_ℓ^P = \left(\frac{1}{2}\right)^+$ supermultiplet (the other partner, with $J^P = 0^+$, has no electromagnetic coupling to the B meson). Let us observe explicitly that we do not introduce the $s_ℓ^P = \left(\frac{3}{2}\right)^+$ supermultiplet of heavy mesons containing the $J^P = 1^+$ and $J^P = 2^+$ states, for which we do not have sufficient phenomenological information at the moment.

To compute the electromagnetic contribution to the B meson mass difference, we consider the following matrix elements ($q = p' - p$):

\[
<B(p')|J_µ^{em}|B(p)> = f(q^2)(p+p')^µ
\]
\[
<B^*(p',ε)|J_µ^{em}|B(p)> = ih(q^2)ε^{µνσρ}ε_νpq_ρ
\]
\[
<B_1(p',ε)|J_µ^{em}|B(p)> = -\frac{1}{2m_B}[K_1[g^{µσ}(p^2 - m_B^2) - q^σ(p+p')^µ] + K_2[q^2g^{µσ} - q^σq^µ]]ε_σ
\]

where $ε$ is the $B^*$ or $B_1$ polarization vector and $f$, $h$, $K_1$, $K_2$ are electromagnetic form factors. In general they contain two terms, describing the couplings of the electromagnetic current to the heavy $Q = b, c$ and light $q = u, d, s$ quarks, respectively:

\[
f(q^2) = \frac{e_Qξ(ω)}{1 - q^2/m_ν^2}
\]
\[
h(q^2) = \frac{e_Qξ(ω)}{Λ_Q} + \frac{e_q}{Λ_q(1 - q^2/m_ν^2)}
\]
\[
K_1(q^2) = 2e_Qτ_{1/2}(ω) + \frac{e_qσ}{1 - q^2/m_ν^2}
\]
\[
K_2(q^2) = 2e_Qτ_{1/2}(ω)
\]

where $ω = ν · ν'$, and $ν$ and $ν'$ are the heavy particle four velocities. We note explicitly that, e.g. for $B^+ = u$${b}$, one has $e_q = \frac{2}{3}$, $e_Q = e_b = +\frac{1}{3}$. $ξ(ω)$ is the Isgur-Wise form factor [17] and $τ_{1/2}(ω)$ is the analogous form factor describing the transitions between the $(0^−, 1^-)$ and the $(0^+, 1^+)$ doublets of heavy mesons [18]. From HQET [17], at the leading order in $1/m_Q$:

\[
<B(ν')|\bar{b}γ^µb|B(ν)> = m_B(ν^µ + ν'^µ)ξ(ω)
\]
\[
< B^*(v', e) | \bar{b} \gamma^\mu b | B(v) > = \text{im}_{BB} \xi(\omega) e^{\mu \nu \rho \sigma} \epsilon^*_\lambda v'_\rho v_\sigma \tag{28}
\]
\[
< B_1(v', e) | \bar{b} \gamma^\mu b | B(v) > = 2 \sqrt{m_B m_{B_1}} \tau_{1/2}(\omega) [(1 - \omega) g^{\mu \nu} + v'\nu v] \epsilon^*_\sigma \tag{29}
\]

We can write \(\xi(\omega)\) as
\[
\xi(\omega) = \left[ \frac{2}{1 + \omega} \right]^{2 \rho^2} \tag{30}
\]
using the normalization condition \(\xi(1) = 1\). The experimental determination of the slope \(\rho^2\) contains several uncertainties (see for example the discussion in [19]). A value \(\rho^2 = 1 \pm 0.3\) encompasses most of the theoretical predictions, while being in agreement with the data [20]. Therefore we shall take in the following \(\rho^2 = 1\), which means that we can take for the Isgur-Wise function the following expression (with \(\omega = 1 - \frac{q^2}{2m_B^2}\) in this case):
\[
\xi(\omega) = \left[ \frac{2}{1 + \omega} \right]^{2} = \frac{1}{[1 - q^2/4m_B^2]^2}. \tag{31}
\]

For \(\tau_{1/2}(\omega)\) we take the QCD sum rule results given in [21]; we shall discuss the uncertainties related to this choice below. For the light quarks part of the electromagnetic current, we assume Vector Meson (\(\rho, \omega\)) Dominance of the form factor; under this hypothesis the constants \(\Lambda\) and \(\sigma\) can be estimated as follows: \(\Lambda_Q = m_B, \Lambda_q \simeq 0.5\) GeV [22], \(\sigma = 2\sqrt{2} \frac{g_V f_V}{m_V^2} |\mu| \sqrt{m_B m_{B_1}} \simeq 2.7\) (here \(g_V \simeq 5.8, f_V \simeq 0.17\) GeV\(^2\), \(m_V\) is the \(\rho\) meson mass, and \(|\mu| \simeq 0.1\) GeV\(^{-1}\) parametrizes the \(BB_1V\) vertex [15].

Using the matrix elements and the coupling constants just introduced, we can calculate the electromagnetic contribution to the mass splitting of heavy mesons. The contributions of the different terms to the DR are as follows; the subtraction term \(T_1(q^2, 0)\) is given by:
\[
T_1(q^2, 0) = -2 \left[ f_1^2(q^2) - f_0^2(q^2) \right] + 2m_B^2 \left[ h_1^2(q^2) - h_0^2(q^2) \right] \\
- \frac{q^4}{4m_B^2 \nu_R} \left[ (K_{1,+} + K_{2,+})^2 - (K_{1,0} + K_{2,0})^2 \right], \tag{32}
\]
where
\[
\nu_R = \frac{q^2 + m_B^2 - m_{B_1}^2}{2}. \tag{33}
\]
As for the two structure functions \(W_{1,2}(q^2, \nu)\) that appear in the dispersion relations for \(T_i\), they are given by:
\[
W_1(q^2, \nu) = -\frac{q^4}{2} \left[ h_1^2(q^2) - h_0^2(q^2) \right] \left[ \frac{q^2}{4} - m_B^2 \right] \delta \left( \nu^2 - \frac{q^4}{4} \right)
\]
\[ W_2(q^2, \nu) = -2m_B^2 q^2 \left[ f_+^2(q^2) - f_0^2(q^2) \right] \delta \left( \nu^2 - \frac{q^4}{4} \right) + \frac{q^4 m_B^2}{2} \left[ h_+^2(q^2) - h_0^2(q^2) \right] \delta \left( \nu^2 - \frac{q^4}{4} \right) + \left\{ \left[ q^2(K_{1,+} - K_{2,+})^2 - 4m_{B_1}^2 K_{1,+}^2 \right] - \left[ q^2(K_{1,0} - K_{2,0})^2 - 4m_{B_1}^2 K_{1,0}^2 \right] \right\} \delta \left( \nu^2 - \frac{q^4}{4} \right) \] 

(34)

(35)

where \( h_+ \), \( f_+ \), \( K_{j,+} \) refer to \( B^+ \) (resp. \( D^+ \)) and and \( h_0 \), \( f_0 \), \( K_{j,0} \) to \( B^0 \) (resp. \( D^0 \)).

The \( 1^- \) resonance is quite narrow (less than 1 keV); on the contrary, the \( 1^+ \) axial vector resonance is broad enough to require the convolution of the mass difference term, depending upon \( m_{B_1} \), with a lorentzian distribution centered on \( m_{B_{1,\text{aver}}} = 5.732 \) GeV. Experimental data suggest a width \( \Gamma = 145 \) MeV. The \( D \) case is computed in full analogy with the \( B \) one. The numerical results of this analysis are reported in Table I, for \( \mu = 1 \) GeV and (in parentheses) \( \mu = 2 \) GeV. It may be useful to stress that the results are remarkably insensitive to variations of the cut-off \( \mu \); for example varying \( \mu \) in the range \( \mu = 2 - 5 \) GeV introduces an uncertainty of less than 8%. Another possible source of error is in the slope of the Isgur-Wise function \( \rho^2 \). We find an uncertainty of \( \pm 1\% \) for \( \delta m(D^+ - D^0) \) and negligible for \( \delta m(B^+ - B^0) \) when \( \rho^2 \) varies between 0.80 and 1.20. Also the uncertainties related to the choice of \( \tau_{1/2} \) are negligible, given the smallness of the \( 1^+ \) contribution, and we do not expect significant contributions from the \( s_\ell = \left( \frac{3}{2} \right)^+ \) poles.

To compare our result to the experimental data we have to add the quark mass contribution computed in Section 2; the different terms and the total theoretical prediction are reported in Table II which shows a good agreement with experiment within the errors.

5 Electromagnetic mass difference in the \( m_Q \to \infty \) limit

We wish now to evaluate the electromagnetic mass difference \( B^+ - B^0 \) in the infinite heavy quark mass limit, which allows a remarkable simplification of the formulae and a deeper understanding on the underlying physics.
In the $m_b \to \infty$ limit, since
\begin{equation}
\delta m^2(B^+ - B^0) = 2m_B \delta m(B^+ - B^0) ,
\end{equation}
we get, in the $m_b \to \infty$ limit, from previous formula, the result:
\begin{equation}
\delta m(B^+ - B^0)_{\text{em}} \to \delta m_{\text{Born}} + \delta m_V + \delta m_{\text{subtr}} \quad (m_b \to \infty) ,
\end{equation}
where $\delta m_{\text{Born}}$ is the contribution from the Born term and is given by
\begin{equation}
\delta m_{\text{Born}} = \frac{\alpha m_V}{6\pi} \left[ 5 \arctan \frac{\mu}{m_V} + \frac{\mu/m_V}{1 + \mu^2/m_V^2} \right].
\end{equation}

As explained above, $m_V \simeq 770$ MeV is the $\rho$ mass and $\mu$ is the cut-off; for $\mu = 1$ GeV and $\mu = 2$ GeV, we get $\delta m_{\text{Born}} = 1.5$ MeV and 1.9 MeV respectively. The remaining contributions in (37) arise from the subtraction term $T_1(-Q^2, 0)$ : $\delta m_{\text{subtr}}$, and from the vector meson $1^-$ dispersive contribution to $W_1$ and $W_2$. $\delta m_V$ (the contribution from the $1^+$ pole vanishes). The two terms are:
\begin{equation}
\delta m_{\text{subtr}} = \frac{\alpha \mu^2}{8\pi \Lambda_q^2} \frac{m_B}{1 + \mu^2/m_V^2} ,
\end{equation}
\begin{equation}
\delta m_V = \frac{\alpha \mu^2}{8\pi \Lambda_q^2} \frac{m_B}{1 + \mu^2/m_V^2} - \frac{\alpha m_V^3}{12\pi \Lambda_q^2} \left[ \arctan \frac{\mu}{m_V} - \frac{\mu/m_V}{1 + \mu^2/m_V^2} \right]
\end{equation}
($\Lambda_q \simeq 500$ MeV is the hadronic scale defined by eq. (24)). It is interesting to observe that, while individually the subtraction contribution (39) and the vector meson contribution (40) diverge in the infinite heavy quark mass limit, their sum is finite. Therefore, summing up the three terms ($m_b \to \infty$), we obtain:
\begin{equation}
\delta m(B^+ - B^0)_{\text{em}} = \frac{\alpha m_V}{6\pi} \left[ \left( 5 - \frac{m_V^2}{2\Lambda_q^2} \right) \arctan \frac{\mu}{m_V} + \left( 1 + \frac{m_V^2}{2\Lambda_q^2} \right) \frac{\mu/m_V}{1 + \mu^2/m_V^2} \right].
\end{equation}

This gives, at $\mu = 1$ GeV and $\mu = 2$ GeV, $\delta m(B^+ - B^0)_{\text{em}} = 1.36$ MeV and 1.59 MeV respectively, which is remarkably close to the value obtained at finite mass and reported in Table II. A similar formula holds for $\delta m(D^+ - D^0)_{\text{em}}$ in the same limit ($m_c \to \infty$):
\begin{equation}
\delta m(D^+ - D^0)_{\text{em}} = \frac{\alpha m_V}{6\pi} \left[ \left( 7 + \frac{m_V^2}{2\Lambda_q^2} \right) \arctan \frac{\mu}{m_V} - \left( 1 + \frac{m_V^2}{2\Lambda_q^2} \right) \frac{\mu/m_V}{1 + \mu^2/m_V^2} \right]
\end{equation}
where the differences with (11) are only due to quark charge factors. Numerically we find, at \( \mu = 1 \text{ GeV} \):
\[
\delta m(D^+ - D^0)_{em} = 1.92 \text{ MeV}
\]
(this value is 2.72 MeV for \( \mu = 2 \text{ GeV} \)). These results, valid for \( m_c \to \infty \), show significant deviations from the finite mass result reported in Table I.

Besides showing the exact cancellation of the divergent term in (39) and (40), which confirms the scaling law \( \delta m \to \text{const} \ (m_b \to \infty) \), the previous analysis is interesting also because it explicitly shows the small dependence of the \( m_b \to \infty \) results on the renormalization scale \( \mu \).

We also remark that, although the \( 1^- \) state makes only a small contribution to the electromagnetic mass difference, its contribution seems to increase with the cut-off, as seen in Table I. Actually, its value at a large \( \mu \), e.g., at 2 GeV, should be smaller than the values we give in Table I, since the form factors \( h(q^2) \) should be further suppressed at large \( q^2 \) by perturbative QCD effects such that the cross sections for the production of \( 0^-1^- \) pair in \( e^+e^- \) collisions (e.g., \( e^+e^- \to \pi\rho \)) will not grow too fast with energy. This suppression also guarantees the convergence of the \( Q^2 \) integral for the Cottingham formula \[12\] and make our results insensitive to the value of the cut-off \( \mu \).

In conclusion, we can say that the small mass difference \( B^+ - B^0 \) can be understood, in the \( m_Q \to \infty \) limit, as a sum of two contributions of opposite sign and similar size that remain finite in this limit. The electromagnetic contribution has been computed by the Cottingham formula and has a small dependence on the renormalization mass scale \( \mu \). The HQET results are very similar to those obtained at finite \( b \) mass. In the case of \( D^+ - D^0 \) the contributions have the same sign and add up; in this case numerical results show deviations of \( \simeq 30\% \) as compared to the predictions obtained in the HQET limit.

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Table Captions

Table I
Different contributions to the electromagnetic mass differences in the $B$ and $D$ systems (units are MeV). The first value is obtained using $\mu = 1$ GeV; the second value (in parentheses) using $\mu = 2$ GeV.

Table II
Electromagnetic and quark-mass contributions to the mass differences in the $B$ and $D$ systems (units are MeV) compared to the experimental data \cite{5}. The e.m. value is an average between the results obtained with $\mu = 1$ GeV and $\mu = 2$ GeV.
Table I

| $\delta m$ | Born   | $1^-$  | $1^+$  | total     |
|------------|--------|--------|--------|-----------|
| $\delta m(D^+ - D^0)$ | 1.72 (2.28) | -0.09 (-0.34) | 0.004 (-0.007) | 1.63 (1.95) |
| $\delta m(B^+ - B^0)$ | 1.50 (1.87) | -0.18 (-0.41) | 0.005 (0.01) | 1.33 (1.47) |

Table II

| $\delta m$ | e.m.       | quark mass | total      | exp. [5]   |
|------------|------------|------------|------------|------------|
| $\delta m(D^+ - D^0)$ | +1.79 ± 0.16 | +2.54 ± 0.21 | +4.33 ± 0.37 | +4.78 ± 0.10 |
| $\delta m(B^+ - B^0)$ | +1.40 ± 0.07 | -2.23 ± 0.27 | -0.83 ± 0.34 | -0.35 ± 0.29 |