Considering nonlinear properties of concrete in the design of reinforced concrete structures for torsion

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Abstract. Restoration and maintenance of cultural heritage concrete and reinforced objects always require accounting for the torsional stress in the elements, as well as the tensile, compressive and shearing stress. It should be considered, that torsion always develops in the elements of spatial structures, including cross-ribbed systems, bridges and overlaps, elements of shells and domes, stairs, etc. This paper briefly systematizes substantive results of the last studies of an urgent problem of torsional stiffness of reinforced concrete elements, including theoretical investigations, experimental studies and numerical research. Stress redistribution in the statically indeterminate reinforced concrete structures’ elements depends on both torsional and bending stiffness. However, while bending stiffness is considered in the design practice, the torsional stiffness is not acknowledged by engineers, nor used in building standards and software packages. This paper aims to substantiate the necessity of considering the torsional stiffness of reinforced concrete elements in design practice. This article is the result of numerous investigations conducted in the scientific school of professor Taliat Azizov. They certify that reduction of the torsional stiffness must be considered along with the bending stiffness in different stages of behaviour of concrete structures. Moreover, the research conducted for this paper proved that a change in the shear modulus of concrete also affects the deflected mode of structures and should be accounted for as the component of torsional stiffness. In order to provide engineers with specialized design techniques, they have been developed for the calculation of torsional stiffness of reinforced concrete elements with normal cracks of rectangular, triangular, t-sections, box-sections, and other cross-sections. The engineering bilinear shear stress-strain curve for concrete has also been proposed on the basis of experimental studies.

1. Introduction
Accounting for not only compressive and tensile stresses is essential during design of reinforced concrete structures. It has been proved, than neglecting shear capacity of the reinforced concrete elements is crucial [1,2]. However influence of torsion can also considerably influence on the strains’ redistribution (in most cases in combination with the bending moment or and shear force). In design or restoration, neglecting torsion on the strain redistribution in statically indeterminate structures results serious errors, affecting strength, stability, and deformability of such systems [3]. However international building standards don’t consider it as an impact required a computation. According to the European design standard (BS EN 1992-1-1:2004, p. 6.3.1), a full torsional design for both ultimate limit state and the serviceability limit state must be provided for the structures, the static
equilibrium of which depends on the torsional resistance of elements. For statically indeterminate structures it should not be provided in case, if torsion arises due to compatibility conditions only.

Meanwhile, in fact, torsion always arises in spatial structures’ elements. For example, in the cross-ribbed systems, load is always unsymmetrical for some beams and results in the twist about their longitudinal axis. Any asymmetrical loading of plate causes torsion in the extreme beams. In order to identify the correct values of torsional moments of shells and domes’ extreme elements, common deformations of the joint elements should be considered. Evaluation of the deflected mode of bridges must always include resistance to the torque, because elements often ruin exactly due to torsion, which is an important factor in the spatial structures’ work.

While torsional strength is being widely researched nowadays, considering torsional stiffness is still mostly neglected. Neither building codes, nor engineering software (Ansys, Abaqus, Scad, Lira, Nastran, etc.) consider change of torsional stiffness in the design process.

Regularly, building standards set torsional stiffness as a constant value depending on bending stiffness, with no consideration of the shear modulus. According to the ACI code [5], the torsional stiffness should be considered for elements in equilibrium torsion and may be neglected for the case of compatibility torsion. The German standards do not provide torsional stiffness’ evaluation, however in the design of frame structures the stiffness condition must be provided [6]. In Poland there are no strict regulations for the torsional stiffness of concrete [4,7]. Nevertheless, for reinforced concrete elements, which may affect essentially the stiffness and strength of the whole construction, it is recommended to design in torsion very carefully [8,9]. Complex sections are proposed to be divided into rectangular sections and set their resistance to the torsional moment to be proportional to the whole section’s torsional stiffness.

Other European building standards [4], as well as Canadian, Australian, New Zealand and Hong Kong [10–15] do not consider torsional stiffness of reinforced concrete elements.

Speaking of post-Soviet countries, definition of the torsional stiffness is provided in Belarus. It is defined as a sum of torsional stiffness of individual rectangular parts of the section, so that the total stiffness of them must be maximum [16]. In Ukraine and Russia the torsional stiffness is neglected, and the accounting for only torsional strength is provided in special cases [17,18].

In 2001 the specific design techniques was proposed [3]. However till now the torsional stiffness is generally not considered in the design practice.

2. Experimental research of the torsional stiffness of reinforced concrete elements

2.1. Consideration of nonlinear properties of concrete in torsional stiffness’ evaluation

This paper examines the nonlinear properties of concrete due to plastic deformations and cracks. The Ukrainian researcher Taliat Azizov derived the specialized theory of the spatial overlaps’ behavior [3], which allows the consideration of torsion and torsional stiffness \( GI \) in the design process, as well as its change due to its significant influence on the strain redistribution. Furthermore, the scientist proved the serious impact of normal cracks, which arise in structural elements due to bending, on torsional stiffness [19].

The development of design techniques accounting for the nonlinearity of structures’ deformation was carried out in two directions [20]: limit equilibrium and deformation models.

Most models for analytical determination of the torsional stiffness of reinforced concrete elements provide change of the initial shear modulus by considering the coefficient \( k_s \), which is defined experimentally and might be defined through various relations (e.g. of torsional moments in the cracked and uncracked areas). There are also techniques for estimation of torsional stiffness considering nonlinear structure’s deformation of particular sections for torsion with bending [21], at the elastic-plastic stage of work under action of bending and torsional moments, by changing Young’s modulus depending on the level of loading and through the relation of elastic design deformations to the deformations in the elastic plastic stage [3,22].
First, on the basis of experimental data, scientists were tended to describe the concrete behaviour under pure torsion by the elasticity theory [23] or plasticity theory [24]. Data obtained by different researchers can be hardly compared due to significant differences between experimental dependences resulting dissimilarities of the factors, conditions and methods of experimental studies.

The author [25] accounts for the torsional stiffness as a function of the axial forces and states that according to N. J. Nielsen diagram the torsional stiffness doesn’t depend on placement of reinforcement, but on the Young’s modulus and slab’s high for the uncracked element. For the cracked element one should use number of coefficients which characterize geometry of the section.

The scientist [3] recommends to define the torsional stiffness by comparing the displacements from torsion of the cracked and uncracked elements.

2.2. Research of destruction character of the cracked reinforced concrete elements of different cross sections

In the scientific school of the professor T. Azizov numerous experimental studies of pure and reinforced concrete of cracked and uncracked elements under pure torsion and torsion with bending have been conducted. The dependence “torque – angle of twist” was determined for all of the experimental series with the aim to explore the behavior of the cracked reinforced concrete elements.

Various cross-sections, including rectangular, triangular, t-sections and box-sections [3] have been explored. Those experiments proved the necessity of taking the torsional stiffness into account both experimentally and theoretically. The particular techniques and propositions for definition of the torsional stiffness and strength of the reinforced concrete elements of different sections with normal cracks under torsion have also been elaborated. The outcome of the proposed design technique helps to provide reinforcement of structures more precisely with accounting for necessary strength and stiffness of overlaps and beams. The artificial normal cracks simulated the cracks, which had already been formed in the concrete element due to bending.

There was the possibility to consider cracks of different height and distances between them allows evaluating the torsional stiffness of the entire element, not only of the specific section.

Conducted experiments for combined bending with torsion allowed making important conclusions, among which are:

1) diverse numerous investigations prove that the torsional stiffness decreases in the cracked elements. Sometimes significant reduction of stiffness results in destruction of the element;
2) reinforced concrete specimens with normal cracks, at stages closed to the destruction, have nonlinear dependence “torsional moment – angle of twist”;
3) level of crack resistance increases, while considering torsional impact;
4) torsional displacements are larger, and cracks’ number enlarges comparatively to the pure torsion;
5) the dowel force contracts deformations, and increase in reinforcement reduces the torsional displacements in both series of samples. Thus, increasing of the longitudinal bars’ diameter and the height of stiffness of the compressed zone of the element with normal cracks results approaching of its stiffness to the stiffness of an uncracked element.

Besides of the geometrical parameters and, consequently, the moment of inertia \( I \), the shear modulus of concrete \( G \) changes after cracking and effects the torsional stiffness \( GI \). Due to the essential influence of the torsional stiffness of concrete on the stress’ redistribution elaborating of the experimental technique for the complete stress-strain curve for concrete in shear was a vital task.

3. Research of the secant shear modulus of concrete

Similarly to the Young’s modulus, the Kirchhoff’s modulus of concrete can be defined by the analytical techniques or diagram methods. The differences of the curves are provided by the elastoplastic coefficients.

Deformation models allow tracing the change in the bearing capacity of the material considering variation in cross-sectional area in each load moment. The shear stress is generally taken either equal
to the tensile stress, or depending on the tensile and compressive stresses (preventing margin of safety and due to absence of the relevant experimental data).

The exact stress-strain curve is difficult to obtain for concrete in pure torsion due to several reasons, one of which is the presence of significant differences between the experimental dependences obtained by various researchers due to diversity of the factors, conditions and methods of experimental research.

3.1. Theoretical and bilinear shear stress-strain curve for concrete

There is no option to change the secant shear modulus of concrete in the program software. It might be defined as a function of the Young’s modulus. However, shear modulus and elastic modulus are linearly interdependent for the homogeneous and isotropic materials in the area of elasticity only, while after cracking the stress-strain relationship is not linear anymore due to plastic deformations and change of deformation characteristics of the material. That’s why the iterative approach was applied.

The theoretical stress-strain curve for concrete in shear was elaborated on the bases on the theory of elastic plastic deformations [26], according to which the stress intensity in each moment of loading is correlated with the strain intensity for all types of stress condition, by analogy with the compression stress-strain curve used for bending [27]

The secant shear modulus of concrete is defined similarly to the secant elastic modulus [26]:

\[
G_c = \frac{E_c \cdot \varrho}{2(1 + \mu_c)}
\]

where \(G_c\) – secant shear modulus of concrete; \(E_c\) – initial elasticity modulus of concrete; \(\varrho\) – variation coefficient of the secant shear modulus of concrete [26], \(\mu_c\) – Poisson’s ratio.

3.2. The bilinear shearing stress-strain curve for concrete

In order to consider the secant shear modulus in design the bilinear stress-strain curve was implemented (Figure 1).

Figure 1. The shear stress-strain curve for concrete:
1. theoretical function; 2. bilinear characteristic dependence; 3. bilinear design chart

The theoretical shearing stress-strain curve has been simplified for the usage in the design practice by analogy with the compressive stress-strain curve for concrete [28].

\[
\begin{align*}
\tau_e &= G_c \cdot \gamma_c \text{ for } 0 \leq \gamma_c \leq \gamma_{c2} \\
\tau_e &= \tau_{cu} \text{ for } \gamma_{c2} \leq \gamma_c \leq \gamma_{cu}
\end{align*}
\]

where \(\tau_e\) – tangent stress for concrete; \(\tau_{cu}\) – ultimate tangent stress for concrete; \(G_c\) – secant shear modulus of concrete; \(\gamma_c\) – relative angular deformations for concrete in shear; \(\gamma_{cu}\) – relative ultimate
angular deformations for concrete under shear; $\gamma_{12}$ – relative angular deformations for concrete in shear correspondingly to the ultimate shearing stresses.

The design curve has been elaborated on the basis of the conducted experimental research [10].

### 3.3. Experimental shear stress-strain curve for concrete

Usage of the real deformation curves for concrete in shear in the building practice is constrained due to lack of reliable experimental data on its parameters [4,29]. Nonetheless, it should be used as a generalized characteristic of the mechanical properties of concrete, similar to the compression stress-strain curve.

The technique for obtaining shearing stress-strain curve for concrete is more complicated comparatively to the compression one due to difficulties in experimental definition of parametric points of the descending branch. The reason is in the sudden element’s destruction while using traditional experimental setting due to the immediate realization of the ultimate potential deformation energy to the impact energy.

That’s why the shear stress-strain curve of concrete in pure torsion considering plastic deformations has been examined, and dependence between stresses and angular deformations of concrete under short-term load has been determined.

Nine concrete cylinders of annular cross-sections have been tested for resistance to pure torsion. Thickness of the ring was varied for three series by three cylinders from 2 to 4.5 cm. The external diameter was 20 cm, and the length of the researched part was equal to 60 cm. The support areas were enlarged in order to avoid influence and specimens’ destruction due to crushing. The fine-grained concrete was used due to thin annular sections of the specimen [28].

In the conducted experiments the load was transmitted to the concrete specimen of annular cross-section through a steel beam in order to control the external forces on the concrete cylinder. The special setting was elaborated to provide common deformations of the concrete model with the steel cross-beam at all stages of loading, for details see [10]. First there was obtained the complete stress-strain curve for concrete in shear and proved the influence of the shear modulus of concrete as a torsional stiffness’ component on the strain redistribution in the spatial cross-ribbed systems. The correlation ratio, obtained by comparing of theoretical and experimental curves, is equal to 0.984.

Obtained data require more experimental research and verification for various concrete specimens of different classes for concrete.

### 4. Numerical investigations and consideration of the torsional stiffness when designing spatial structures

Influence of the torsional stiffness on the strains’ redistribution of the cross-ribbed structures’ elements was proved by professor Azizov and his students in the result of numerous numerical experiments, considering nonlinear properties of concrete in torsion.

The proposed research has been conducted for the noncracked elements in order to observe the change of nonlinear properties due to secant modulus’ variation only.

Simulation of the cross-ribbed structures was accomplished in the SP “LIRA”. Conducted calculations considered change of the torsional stiffness due to cracking. They were fulfilled by the iteration technique. First in the process of external iterations the torsional moments’ values were obtained. Internal iterations consisted in calculation of the secant elastic and shear modulus by the deformational dependences [26,27]. According to the new values of stiffness the torsional moments were calculated anew, and the process repeated till convergence (the relative error between values of current and previous iterations did not exceed $10^{-7}$ %).

In such iterative calculations the least stiffness is taken at the initial iteration for the sections with maximum stresses. Therefore, in the next approximation the stiffness’ ratio changes abruptly, and the design stresses increase significantly. With each subsequent iteration those values gradually converge. Thus, the technique allows to compare results of calculations with and without accounting for the torsional stiffness’ change due to stresses’ redistribution.
4.1. Accounting for the change of torsional stiffness due to secant shear modulus

Five 3 meter girders of section 25x25 cm were divided into 20 elements lengthwise. The span between them was 1 m. The slabs of section 15x5 cm are divided into 5 elements. Concrete class C 16/20, stiffness parameters of the ribs are: \(EF=1687500\) kN, \(EI_y=8789\) kNm\(^2\), \(EI_z=8789\) kNm\(^2\), \(GIt=6196\) kNm\(^2\). Stiffness of the slab – \(EF=202500\) kN, \(EI_y=379.69\) kNm\(^2\), \(EI_z=42.188\) kNm\(^2\), \(GIt=63.0703\) kNm\(^2\).

First the uniformly distributed load \(q=50.8\) kN/m was applied to the first beam, and only change of torsional stiffness as a function of the shear modulus of concrete was considered. Comparison of ultimate diagrams of the torsional moments for half of a beam is proposed on the Figure 2. The approximation area and the last meanings of torsional moments are pointed, and the numbers of the loaded beams are indicated above.

The ratio error between shear modulus, as well as between torsional stiffness, achieved 8-13% in different loading cases. The ultimate strain redistribution occurs in the extreme bar’s elements when the last beam is under the load (Figure 3).

![Figure 2](image)

**Figure 2.** The torque (Mt) diagram for the element (in half lengthwise)

![Figure 3](image)

**Figure 3.** Comparison of ultimate Mt diagrams in 3 loading cases.

4.2. Considering of both secant Young’s and Kirchhoff modulus of concrete as a torsional stiffness’ component

In the second experiment the change of bending and torsional stiffness was researched along the length of the beam. Both secant modulus were evaluated and considered as a stiffness’ components on each iteration for every finite element for every 5 beams (Figure 3). Uniformly distributed load \(q=45\) kN/m was applied to the first beam.

![Figure 4](image)

**Figure 4.** Approximation of the torsional moments \(M_t\) for the element (in half lengthwise).
In the result of comparison of the secant shear modulus’ values $G_c$ is 26.33% in the end element of the first beam, wherein the change of the secant elastic modulus $E_c$ in the same element is 4.030%. In the central elements of the first beam change of $E_c$ achieves 42.66%, and variation of $G_c$ is 0.7896%.

Obtained data reaffirm the necessity of considering the shear modulus while determining the internal stresses and their redistribution in statically indeterminate systems, when considering torsional stiffness of reinforced concrete elements

5. Conclusions
1. Both torsional strength and stiffness should be considered during design of the new structures and restoration of the cultural heritage objects. Otherwise, neglecting torsion causes cracking and destruction of the elements (e.g. due to insufficient bending strength of the weakened by the spatial cracks elements, caused by torque, etc.).
2. The torsional stiffness should be researched equally with the bending stiffness of the reinforced concrete elements
3. The secant shear modulus of concrete, as the component of torsional stiffness, affects the deflected mode and the spatial structures’ behavior should also be considered in the design techniques, building standards, as well as in the program software.
4. The parametric points of the complete stress-strain curve for concrete in torsion have been obtained, which allowed an observation of the secant shear modulus of concrete in torsion considering its nonlinear properties.
5. The bilinear engineering shear stress-strain curve has been elaborated on the basis of the experimental data in order to implement into building standards.

References
[1] Foti D 2014 Shear Vulnerability of Historical Reinforced-Concrete Structures Int. J. Archit. Herit. Conserv. Anal. Restor. 453–67
[2] Palmisano F 2017 A Preliminary Study on Shear Capacity of Historical Reinforced Concrete Beams Int. J. Herit. Arch. 1 608–623
[3] Azizov T N 2006 Prostranstvennaya rabota zhelezobetonnih perekrytij. Teoriya i metody rascheta [Spatial work of concrete overlaps. The theory and techniques for calculation] (Ukraine, Poltava)
[4] BS EN 1992-1-1:2004 2004 Eurocode 2: Design of concrete structures: Part 1-1: General rules and rules for buildings London Br. Stand. Inst. 230
[5] ACI 318-11 2011 Building Code Requirements for Structural Concrete and Commentary (ACI 318M-11) (Farmington Hills: American Concrete Institute)
[6] DIN 1045-4:2012-02 2014 Tragwerke aus Beton, Stahlbetoon und Spannbeton – Teil 4: Erganze Regeln fur die Herstellung und die Konformitat von Fertigteilen Ersatz fur DIN 1045-42001-07 378
[7] PN-EN 1992-1-1:2008/NA: 2010 2010 Eurokok 2: Projektowanie konstrukcji z betonu. Część 1-1: Reguly ogólne i reguły dla budynków
[8] Godycki-Ćwirko T 2006 Skręcanie, w: Podstawy projektowania konstrukcji żelbetowych i spreżonych według Eurokodu 2 ed Sekcja Konstrukcji Betonowych KILiW PAN (Wrocław: Dolnośląskie Wydawnictwo Edukacyjne)
[9] Knauff M, Golubińska A and Knyziak P 2014 Tablice i wzory do projektowania konstrukcji żelbetowych z przykładami obliczeń (Wydawnictwo Naukowe PWN)
[10] Jurkowska N R 2016 Kruchennya zalizobetonnih elementiv v inozemnyh doslidzhenyah i standartah [Torsion of the reinforced concrete elements in the foreign researches and standards]: [monography] (Kyiv: Interservis)
[11] AS 3600-2009 2009 Australian Standard Concrete Structures vol 2009(Australia, Sydney: Standards Australia International Ltd.)
[12] Hong Kong CP-04 2013 Code of Practice for Structural Use of Concrete (Hong Kong: The Government of the Hong Kong Special Administrative Region, H.K.B. Department)

[13] IS 456-2000 2000 Code of Practice for Plan and Reinforced Concrete (India, New Delhi: Bureau of Indian Standards)

[14] BC 2: 2008 2008 Design Guide of High Strength Concrete to Singapore Standard CP 65 200. BCA Sustainable Construction, Series – 3 (Singapore: Building and Construction Authority)

[15] NZS 3101-06 2006 Concrete Structures Standard (New Zealand, Wellington: Standards New Zealand)

[16] SNB 5.03.01-02 2003 Betonnye i zhelezobetonnye konstruktsii [Concrete and reinforced concrete structures] (Belarus: Minsk)

[17] DBN V.2,6-98:2009 2011 Betonni ta zalizobetonni konstruktsiyi. Osnovni polozhennya. [Concrete and Reinforced Concrete Structures. Substantive Provisions] (Kyiv: Minregionbud of Ukraine)

[18] SNiP 2.03.01-84* 1989 Betonnye i zhelezobetonnye konstruktsii [Concrete and reinforced concrete structures] (Moscow)

[19] Azizov T 2009 Zhestkost’ zhelezobetonnih ehlementov pri kruchenii i ee vliyanie na prostratvennyuyu rabotu mostov [Stiffness of reinforced concrete elements in torsion and its impact on the spatial work of bridges] Mech. Phys. Fract. Build. Mater. Constr. Collect. Sci. Work. H. V. Karpenko Physico-Mechanical Inst. Natl. Acad. Sci. Ukr. 576–90

[20] Geniev G A, Kissock V N and Tyupin G A 1974 Teoriya plastichnosti betona i zhelezobetona [The theory of plasticity of concrete and reinforced concrete] (Moscow: Stroiizdat)

[21] Kasayev D H 2001 Prochnost’ ehlementov zhelezobetonnih konstrukcij pri kruchenii i izgibe s krucheniem [Strength reinforced concrete structures’ elements under torsion and bending with torsion] (Rostov na Donu: Rostov university)

[22] Paramonov D 2012 Zhestkost’ i prochnost zhb elementov z normalnymi treshinami pri izgibe s krucheniem [Stiffness and strength of reinforced concrete elements with normal cracks under bending with torsion] (Ukraine, Odessa: Odessa State Academy of Civil Engineering and Architecture)

[23] Cowan H.J. 1972 Kruchenie v obychnom i predvaritel’no napriazhennom zhelezobetone [Torsion in the ordinary and prestressed reinforced concrete] (Moscow: Stroiizdat)

[24] Kemp E L, Sozen M A and Siess C P 1961 Torsion in Reinforced Concrete. A report on a Research Project Sponsored by The University Research Board 128

[25] Gudmand-Hoyer T 2004 Stiffness of Concrete Slabs. Report BYG – DTU R-092 vol 4(Danmark)

[26] Yaremenko O F and Shkola Yu.O. 2010 Nesucha zdatnist’ ta deformativnist’ zalizobetonnikh sterzhnevikh elementiv v skladnomu napruzenomu stani [Bearing capacity and deformability of the reinforced concrete strain elements in a compound stress]. (Odessa: Odessa State Academy of Civil Engineering and Architecture)

[27] Karpenko N I 1996 Obshchie modeli mekhaniki zhelezobetona [General models of the mechanics of reinforced concrete] (Moscow: Stroyizdat)

[28] Azizov T and Jurkowsa N 2015 The shear modulus of concrete considering plastic deformations (Odessa)

[29] Bentz E C, Vecchio F J and Collins M P 2006 Simplified modified compression field theory for calculating shear strength of reinforced concrete elements ACI Struct. J. 103 614–24