Resonator method for measuring the polarizability of small components of composite materials

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Abstract. The creation of new composite materials with specified radio-frequency properties is often carried out using small-sized inclusions, while the polarizability parameters of these small bodies in electric and magnetic fields play a significant role. The paper considers the possibility of measuring the polarizability of small components of the composite in an open quasi-optical resonator. It is shown that the values of the electric and magnetic polarizability of small particles can be estimated from the shifts of the resonant frequencies that arise when a small scatterer is placed in the resonator.

1. Introduction

In the design of new composite materials intended for radio wave applications, such as ensuring electromagnetic compatibility, electromagnetic safety, and others, as a rule, particles of small sizes in comparison with the working wavelengths are used. In this case, the electrophysical parameters of these particles can have a decisive influence on the final characteristics of the created composite. In these cases, the important parameters of the particles included in the composite are the values of their electric and magnetic polarizabilities, the knowledge of which makes it possible to further simulate the scattering characteristics of the given composite in a given frequency range. Of interest is the possibility of direct measurements of the values of the polarizability of small particles used to create a composite at the frequencies of its further use. In this work, attention is focused on the resonator technique, in particular, on the use for these purposes of an open quasi-optical resonator (OR), which makes it possible to study elements of small dimensions in comparison with the wavelength and to carry out measurements separately in electric and magnetic fields. This technique is especially relevant in the terahertz frequency range, where there is practically no alternative to OR, and the demand for composites is quite high.

The proposed approach is based on multiple interactions of a quasi-optical beam with an object under study in an OR formed by concave spherical mirrors [1], which has a high intrinsic Q factor and free access to regions of high field concentration. Unlike the “waveguide” technique shown in Figure 1, a) where the probe beam interacts once with the small object under study, the use of ORs (Figure 1, b) ensures their multiple interaction, proportionally increasing the influence of the small scatterer on probing wave.
A similar approach was used in [2, 3, 4], where the electrophysical parameters of aerogels from multi-walled carbon nanotubes, aerogel clusters, and sections of vitrified microwire with natural ferromagnetic resonance were measured. The main difficulty of these measurements was the problem of a sufficiently strong perturbation of the OR by objects and the need to search for restrictions for the correct application of the mathematical model of the method. The difference in this work is that the emphasis here is on measuring the polarizability of very small objects that disturb the OR, at the limit of the possibilities to observe changes in its spectral characteristics.

2. Description of the technique.

The placement of a small scattering device into the OR leads to shifts in the resonant frequencies of the resonator and changes in their Q-values [6,7]. If we assume that the small scattering device under study is located in the antinode of the electric field, then the electric dipole moment acquired by it will be described by the relation, where is the electric polarizability of this scattering device. Similarly, when placing it in the antinode of the magnetic field, we get where the magnetic polarizability is.

It is essential that both of these parameters of polarizability are determined by the shape, orientation of the sample in the field, the ratio of its size to the wavelength, the values of electric and magnetic permeability, which, in turn, are also frequency-dependent.

At the same time, it is possible to obtain the values of the electric and magnetic polarizability at a given frequency for a small body whose dimensions are much smaller than the wavelength experimentally, using the relations for the displacement of the resonant frequencies caused by the placement of the object under study in the field of an open resonator (here is the oscillation index \{mnq\} OR):

\[
\Delta f_s = -\frac{f_s}{N_s} (\varepsilon_0 \alpha_e E_s^2 - \mu_0 \alpha_m H_s^2),
\]

where \(N_s = \frac{4}{\pi} w_0^2 \varepsilon_0 L E_s^2\) is the norm of the \(s\)-th oscillation, is the radius of the "field spot" in the cross section of the location of the lens, \(L\) is the distance between the mirrors. When using a symmetric OR of a biconcave geometry with spherical mirrors with a radius of concavity \(\rho\), the field spot in the center has a radius of [7]:

\[
w_0 = \left( \frac{LA}{2\pi} \right)^{1/2} \left[ \frac{2\rho}{L} - 1 \right]^{1/4}.
\]

It is shown in [5,6] that the above relations, when applied to small metal spheres and to the duplex of small spheres, correspond satisfactorily to the experimental result.

3. Basic ratios

Let's use the above expressions to get the values of polarizability. From these relations, we obtain for the case of placing the diffuser in the antinode of the electric field, \((H_s = 0)\), [8]:

![Figure 1. Single (a) and multiple (b) interaction of the probe beam with a small object.](image-url)
\[
\alpha_e = \frac{\pi \nu_0^2 L}{2} \Delta f_{se}. \tag{3}
\]

At the same time, in order to obtain a high resolution in the measurements of the values of the electric polarizability components, it is necessary to obtain as noticeable changes as possible in the measured parameters of the resonator (1) when a small diffuser is introduced into it. To clarify these conditions, we use the relations (2) and (3), we get

\[
\frac{\Delta f_{se}}{f_s} = 4\lambda^{-1} L^{-2} \left( \frac{2\rho}{L} - 1 \right)^{-1/2} \alpha_e. \tag{4}
\]

Acting similarly, we obtain for the magnetic polarizability measured by placing a small scatterer in the antinode of the magnetic field OR, (\(E_s = 0\)), [8]:

\[
\alpha_m = \frac{\pi \nu_0^2 L}{2} \Delta f_{sm}. \tag{5}
\]

And taking into account the expression (2), we get:

\[
\frac{\Delta f_{sm}}{f_s} = 4\lambda^{-1} L^{-2} \left( \frac{2\rho}{L} - 1 \right)^{-1/2} \alpha_m. \tag{6}
\]

Relations (4) and (6) describe the relationship between the measured values of the resonator parameters and the studied polarizability of a small body with different parameters of the geometry of the resonator used.

We introduce a dimensionless quantity:

\[
A = \left( \frac{L}{2\rho} \right)^{-2} \left( \frac{2\rho}{L} - 1 \right)^{-1/2}. \tag{7}
\]

Using this function, expressions (4) and (6) can be written as:

\[
\frac{\Delta f_{se}}{f_s} = \frac{1}{\lambda \rho^2} A \alpha_e, \tag{8}
\]

\[
\frac{\Delta f_{sm}}{f_s} = \frac{1}{\lambda \rho^2} A \alpha_m. \tag{9}
\]

In all these ratios, the value \(A\) shows the effect of the distance between the mirrors on the sensitivity of the technique. The radius of concavity of the mirrors is assumed to be fixed, which is quite consistent with the real situation.

Figure 2 shows the dependence of the value \(A\) on the ratio \(L/2 \rho\).
Figure 2. Dependence of the dimensionless parameter $A$ on the distance between the resonator mirrors.

From the shown in Figure 2 dependences it follows that high values of parameter $A$ can be obtained either when the mirrors approach at a distance less than $0.5 \rho$, ($L/2\rho < 0.25$), or when the resonator configuration is close to concentric, ($L/2\rho \to 1$). The latter option is related to the fact that, according to expression (3), the value $w_0$ here approaches zero. However, in reality, such a strong reduction does not occur, since the diameter of the "field spot" in the focus cannot be less than the diffraction limit of $1.5 \lambda$. In addition, in a concentric resonator, at $L/2\rho \to 1$, the size of the field spots on the mirrors increases sharply and, accordingly, the Q-factor decreases [7].

Excessive convergence of mirrors is also undesirable, since it is accompanied by a decrease in the quality factor of the resonator and the appearance of a large number of higher types of vibrations. It would seem that the increase in the value of $A$ can be achieved not only by reducing $L$, but also by increasing the value of $\rho$, however, in the expressions (8), (9), the value $\rho$ is present in the denominator, and so it is also impractical to increase it. As a result, we find that in the problem we are considering, it is inefficient to use mirrors with large concavity radii and located at large distances, although it is such OR that are most often used to obtain high Q-values. It follows from Figure 2 that the configuration used in many measurement problems, which is close to the confocal one ($L/2\rho \to 0.5$), is not optimal in this case.

It should be noted that the use of expressions (1) ÷ (6) to find the polarizability parameters of small bodies placed in the OP field, based on the results of measurements of the resonator parameters, is limited, on the one hand, by the size of the object under study (it must obviously be much smaller than the wavelength), and on the other – by the errors in the measurements of the displacement of the resonance curve and its half-width. The smaller the size of the small lens under study, the more accurately the given ratios will "work" (their good correspondence to the experiment is shown in [5,8]). But at the same time, the requirements for the accuracy of measurements of the resonator parameters without the object under study, and with it, will increase.

The requirements for the size of the resonator used can be illustrated by the example of the inclusion of a small dielectric sample with a shape close to spherical. For a small sphere of radius $r_0$ we get: $\alpha = r_0^3 (\varepsilon - 1)/(\varepsilon + 2)$. Using the relation (9), we find:

$$\frac{\Delta f'_{\text{se}}}{f_s} = \frac{r_0^3}{\lambda \rho^2} A \frac{(\varepsilon - 1)}{\varepsilon + 2}.$$
Writing the radius of concavity of the mirror on the scale of the wavelengths used: \( \rho = n\lambda \), for materials with high permittivity, \((\varepsilon >> 1)\), we obtain for the minimum radius of the fixed particle \( r_{\text{min}} \) the ratio

\[
\frac{r_{\text{min}}}{\lambda} \approx \left( \frac{n^2 \delta f}{A f} \right)^{1/3},
\]

where the value of \( \delta f \) is determined by the measurement error of the resonant frequency shift of the resonator. For example, with \( A \) value varying from 10 to 20, and \( n \) values ranging from 30 to 100, the ratio \( (n^2/A)^{1/3} \) will vary from 3.6 to 10 units, and the ratio of the minimum recorded radius of the particle to the wavelength will be almost completely determined by the possibility of fixing a small shift of the resonant frequency \( \delta f / f \).

Figure 3 shows the dependence of the minimum fixed radius of a small sphere (in fractions of the wavelength \( \lambda \)) on the parameter \( n \) (radius of concavity in wavelengths) for different distances between mirrors (dimensionless parameter \( A \)) and for different values of the value \( \delta f / f \).

![Figure 3](image)

Figure 3. The dependence of the minimum radius of the ball (in wavelengths) on the value of \( n \) at different values of the parameter \( A \) (a) and different frequency resolutions (b).

From the numerical estimates shown in Figure. 3, it is possible, with the optimal choice of the resonator size (values \( n \) of the order of 30-50, values \( A \) of the order of 20-30,) and the use of frequency synthesizers with a stability (step) of the order of \( 10^{-7} \div 10^{-8} \) and higher, to measure the parameters of small bodies with a radius of the order of several hundredth parts of the wavelength used.

4. Conclusion

Thus, using the quasi-optical resonator technique, it is possible to measure the values of the polarizability of small particles included in the composition of new composite materials. Measurements can be made at the frequencies of their further use. As for the possibility of reducing the size of the controlled particles, they will largely depend on the mechanical and temperature stability of the OR used and the stability of the vibration sources used. In the gigahertz range, digital circuit analyzers can be used for this purpose [9], and in terahertz it is advisable to use BWO generators, but they require additional stabilization by external frequency synthesizers [10]. The use of the OR allows to measure the polarizability separately in the electric and magnetic fields, which is important when evaluating the scattering properties of small elements of a composite material.
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