Anisotropic flows and the shear viscosity of the QGP within a transport approach

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Abstract. We study the build up of elliptic flow $v_2$ and high order harmonics $v_n$ within a transport approach at fixed shear viscosity to entropy density ratio $\eta/s$ and with initial state fluctuations. In particular we study the effect of a temperature dependent $\eta/s$ for two different beam energies: RHIC for Au+Au at $\sqrt{s} = 200$ GeV and LHC for Pb+Pb at $\sqrt{s} = 2.76$ TeV. We find that for the two different beam energies considered the suppression of the elliptic flow and of higher harmonics $v_3(p_T)$ and $v_4(p_T)$ due to the viscosity of the medium have different contributions coming from the cross over or QGP phase. Moreover, we discuss the correlation between the initial spatial anisotropies $\epsilon_n$ and flow coefficients $v_n$. We observe that the elliptic flow $v_2$ is strongly correlated with initial eccentricity $\epsilon_2$. While higher harmonics $v_3$ and $v_4$ are weakly correlated to their asymmetry measure in coordinate space $\epsilon_3$ and $\epsilon_4$.

1. Introduction
The experimental results accumulated in these years in the ultra relativistic heavy ion collisions (uRHICs) first in the experiments conducted at RHIC and more recently at the LHC has shown that the elliptic flow $v_2 = \langle \cos(2\varphi_p) \rangle = \langle (p_x^2 - p_y^2)/(p_x^2 + p_y^2) \rangle$ is the largest ever observed in these uRHICs \cite{1, 2}. The elliptic flow is a measurement of the momentum anisotropy of the emitted particles and it is an observable that encodes information about the transport properties of the matter created in these collisions. In particular it is an observable sensitive to the shear viscosity to entropy density ratio $\eta/s$. Theoretical calculations within viscous hydrodynamics \cite{3, 4} and in the recent years also calculation performed within transport approach \cite{5, 6, 7} have shown that this large value of $v_2$ is consistent with a matter with a very low shear viscosity to entropy density ratio $\eta/s$. Theoretical calculations within viscous hydrodynamics \cite{3, 4} and in the recent years also calculation performed within transport approach \cite{5, 6, 7} have shown that this large value of $v_2$ is consistent with a matter with a very low shear viscosity to entropy density ratio $\eta/s$ close to the conjectured lower bound for a strongly interacting system, with $\eta/s = 1/4\pi$ \cite{8}. Moreover, both calculations have shown that the $v_2$ depends sensitively on $\eta/s$. Furthermore, in the recent years the possibility to measure experimentally the event-by-event angular distribution of emitted particle has made possible the measurement of higher harmonics $v_n = \langle \cos(n\varphi_p) \rangle$ for $n > 2$. The comparison between event-by-event viscous hydrodynamical calculations and the experimental results for $v_n$ seems to confirm a finite but not too large value of $4\pi\eta/s \sim 1 - 3$ \cite{9, 10}. However, small value of $\eta/s \sim 1/4\pi$ is not an evidence of the creation of a QGP phase. A phenomenological estimation of its temperature dependence could give information if the matter created in these collisions undergoes a phase transition \cite{11, 12}.
Information about a temperature dependence of $\eta/s$ can be achieved studying the $v_2(p_T)$ and the high order harmonic $v_n(p_T)$ in wider range of energies. Similar studies have been performed using a transport approach but only for the elliptic flow [13].

There are several theoretical indications that $\eta/s$ should have a particular behavior with the temperature[11, 12, 14, 15, 16, 17]. As an example in Fig.(1) it is shown a collection of theoretical results about the temperature dependence of $\eta/s$. Fig.(1) shows that in general $\eta/s$ should have a typical behavior of phase transition with a minimum close to the critical temperature $T_C$ [11, 12]. On one hand at low temperature estimations of $\eta/s$ performed in the chiral perturbation theory for a meson gas [14, 15], have shown that in general $\eta/s$ is a decreasing function with the temperature, see down-triangles in Fig.(1). Similar results for $\eta/s$ have been extrapolated from heavy-ion collisions at intermediate energies, see HIC-IE diamonds in Fig.(1). On the other hand at higher temperature $T > T_c$ lQCD calculation have shown but with large error bars that in general $\eta/s$ becomes an increasing function with the temperature [16], see up-triangles and circles in Fig.(1). In general due to the large error bars in the lQCD results for $\eta/s$ it is not possible to infer a clear temperature dependence in the QGP phase. The analysis at different energies of $v_2(p_T)$ and the extension to high order harmonics $v_n(p_T)$ can give further information about the T dependence of $\eta/s$. In this proceeding we will show some new results on $v_n(p_T)$ for the two different systems Au + Au at $\sqrt{s} = 200 GeV$ and Pb + Pb at $\sqrt{s} = 2.76 TeV$.

2. Kinetic approach at fixed shear viscosity to entropy density ratio
In this section we introduce a transport approach in which it is possible to fix the $\eta/s$ ratio. This approach provides a tool to estimate the viscosity of the plasma valid in a wider range of $\eta/s$ and $p_T$ than hydrodynamical framework. We perform such simulations using a relativistic transport code developed in these years to perform studies of the dynamics of heavy-ion collisions at both RHIC and LHC energies [5, 7, 20, 21, 22]. The evolution of the phase-space distribution function $f(x,p,t)$ is given by solving the Relativistic Boltzmann Transport (RBT) equation:

$$p^\mu \partial_\mu f(x,p) = \int d\Gamma_2 d\Gamma_1' d\Gamma_2'(f_1' f_2' - f_1 f_2)[|\mathcal{M}|^2 \delta^4(p + p_2 - p_1 - p_2')]$$

where $d\Gamma_k = d^3p_k/(2E_k(2\pi)^3)$ and $\mathcal{M}$ denotes the transition amplitude for the elastic processes which is directly linked to the differential cross section $|\mathcal{M}|^2 = 16\pi s(s - 4M^2)dx/dt$ with $s$ the Mandelshtam invariant. Numerically we solve the RBT equation using the so called test particle method and the collision integral is solved by using Monte Carlo methods based on the stochastic interpretation of transition amplitude [23, 5, 21].

In order to study the dynamical evolution of the fireball with a certain $\eta/s(T)$ we determine locally in space and time the total cross section $\sigma_{tot}$ needed to have the wanted local viscosity.
In the Chapmann-Enskog theory and for a pQCD inspired cross section, typically used in parton cascade approaches \[24, 25, 5, 26, 27, 23, 6\], with \(d\sigma/dt \sim \frac{\alpha_s^2}{(t-m_D^2)^2}\), the \(\eta/s\) is given by the following expression:

\[
\eta/s = \frac{1}{15} \langle p \rangle \tau_\eta = \frac{1}{15} \frac{\langle p \rangle}{g(a)\sigma_{tot}\rho},
\]

where \(a = m_D/2T\), with \(m_D\) being the screening mass regulating the angular dependence of the cross section, while \(g(a)\) is the proper function accounting for the pertinent relaxation time \(\tau_\eta^{-1} = g(a)\sigma_{tot}\rho\) associated to the shear transport coefficient and it is given by:

\[
g(a) = \frac{1}{50} \int dy y^6 \left[ (y^2 + \frac{1}{3}) K_3(2y) - y K_2(2y) \right] h \left( \frac{a^2}{y^2} \right),
\]

where \(K_n\)-s are the Bessel functions and the function \(h\) relate the transport cross section to the total cross section \(\sigma_T(s) = \sigma_{tot} h(m_D^2/s)\) with \(h(\zeta) = 4(1+\zeta)[(2\zeta+1)\ln(1+1/\zeta) - 2]\). Therefore the total cross section is evaluated locally by mean the Eq.(2).

3. Initial conditions

Our simulations start at the initial time \(\tau_0 = 0.6fm/c\). The initial conditions are longitudinal boost invariant. In the following discussion, we will consider two different types of initial conditions in coordinate space. One of them consist in a fixed distribution in transverse plane by using the standard Glauber model as used in previous works, see \[5, 7, 20, 13\]. The second one consist of an initial profile changing event by event that will be described below.

In order to generate an event by event initial profile we use the Monte-Carlo Glauber model to obtain the initial density distribution for each event. Woods-Saxon distribution is used to sample the nucleons randomly in the two colliding nucleus. In this way we generate a discrete distribution for these nucleons. We employ a geometrical method to determine if the two nucleons one for the nucleus A and the other one for the nucleus B are colliding: two nucleons collide each other if the relative distance in the transverse plane is \(d_T \leq \sqrt{\sigma_{NN}/\pi}\) where \(\sigma_{NN}\) is the nucleon-nucleon cross section. The calculation of \(N_{coll}\) and \(N_{part}\) is given by counting the number of collisions and the number of participating nucleons for each event. In this way the position of the participating nucleons will fluctuate from event-to-event. The discrete distribution for the nucleons is converted to a smooth one by assuming for each nucleon a gaussian distribution centered in the nucleon position. Finally, the density in the transverse...
Figure 3. Left: differential elliptic flow $v_2(p_T)$ at midrapidity and for (20−30%) centrality. The black and red solid lines refer to the case with fluctuations and for $4\pi\eta/s = 1 + f.o.$ and $4\pi\eta/s = 2 + f.o.$ respectively. Term f.o. means the increase of $\eta/s$ at lower temperature. The black and red dashed lines are for the case without fluctuations in the initial geometry. Right: differential $v_4(p_T)$ at midrapidity and for (20−30%) centrality with the same legend as in the left panel.

plane is given by the following sum

$$\rho_T(x, y) \propto \sum_{i=1}^{N_{part}} \exp \left[ -\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma_{xy}^2} \right]$$

where $\sigma_{xy}$ is the Gaussian width and in the following calculations it has been fixed to $\sigma_{xy} = 0.5 \text{ fm}$. Given $\rho_T(x, y)$ we can calculate the initial anisotropy in coordinate space. The initial anisotropy is quantified in terms of the following coefficients $\epsilon_n$:

$$\epsilon_n = \sqrt{<r_T^n \cos(n\phi)>^2 + <r_T^n \sin(n\phi)>^2}/<r_T^2>$$

where $r_T = \sqrt{x^2 + y^2}$ and $\phi = \arctan(y/x)$ are the polar coordinate in the transverse plane. In the following discussion we will concentrate on the fireball produced in Au + Au at $\sqrt{s_{NN}} = 200 \text{ GeV}$ and Pb + Pb at $\sqrt{s_{NN}} = 2.76 \text{ TeV}$. For the nucleon-nucleon cross section we have used $\sigma_{NN} = 42 \text{ mb}$ and $70 \text{ mb}$ respectively. Fig.(2) shows the $\epsilon_2, \epsilon_3$ and $\epsilon_4$ as a function of the number of participant $N_{part}$.

For the initialization in momentum space at RHIC (LHC) energies we have considered for partons with transverse momentum $p_T \leq p_0 = 2 \text{ GeV}$ ($3 \text{ GeV}$) a thermalized spectrum in the transverse plane with a maximum initial temperature $T_0 = 340 \text{ MeV}$ ($510 \text{ MeV}$). While for partons with $p_T > p_0$ we have assumed the spectrum of non-quenched minijets according to standard NLO-pQCD calculations with a power law shape. The transverse momentum of the particles is distributed uniformly in the azimuthal angle.

4. Effects of $\eta/s$ on the $v_n(p_T)$

In this section we discuss the results for the final $v_n(p_T)$ corresponding to the different initial configurations. Differential flow coefficients $v_n(p_T)$ are very sensitive observables to the transport properties of the medium like the ratio $\eta/s$ and they can give us more informations about the matter created in the heavy ion collisions. In the following discussion with $v_n(p_T)$ we mean the root mean square $\sqrt{<v_n^2>}$ as it has been done in experimental data using the event plane
method. In the left panel of Fig.(3) we compare the differential $v_2(p_T)$ obtained with an initial state fluctuations (solid lines) as discussed in the previous section with the one obtained for an event by event averaged initial profile (dashed lines). These results are for the system $Pb + Pb$ at $\sqrt{s} = 2.76$ TeV and for $(20-30\%)$ centrality. For this collision centrality we have simulated $N_{\text{event}} = 500$ events. We have checked the convergence of our results for the $v_2$, $v_3$ and $v_4$ with lattice spacing of the calculation grid and with the number of test particles. As shown in the left panel of Fig.(3) the fluctuations in the initial geometry reduce the elliptic flow of about 10%. In general the introduction of the fluctuations in the initial geometry play the role to generate the higher order harmonics in particular the odd harmonics which were absent in the averaged initial configuration by symmetry. Different behavior were observed for the 4-th harmonic as shown in right panel of Fig.(3) where in this case we have that the role of the fluctuation is to increase the $v_4(p_T)$ by a factor 2. This is due to the fact that $\epsilon_4$ is almost completely generated by the fluctuations. Moreover, comparing the black and red solid lines with the dashed ones we observe that now the $v_4$ becomes more sensitive to the value of the viscosity.

In order to study the role of the $\eta/s$ ratio on the build up of the elliptic flow $v_2(p_T)$ and on the high order harmonics $v_n(p_T)$ we have performed two different calculations: one with a constant $4\pi\eta/s = 1$ during all the evolution of the system and the other one with $4\pi\eta/s = 1$ at higher temperature in the QGP phase and an increasing $\eta/s$ in the cross over region towards the estimated value for hadronic matter $4\pi\eta/s \approx 6$ as depicted in Fig.(1). Such an increase of $\eta/s$ at lower temperature allows for a smooth realistic realization of the kinetic freeze-out. This last kind of calculation is labeled by f.o.. This is because at lower temperature, according to the formula Eq.(2), to account for the increase of $\eta/s$ towards the estimated value for the hadronic matter, the total cross section decrease.

In the left panel of Fig.(4) it is shown the elliptic flow $v_2(p_T)$ and the $v_4(p_T)$ (green and blue lines) at mid rapidity for $(20-30\%)$ centrality for both RHIC $Au + Au$ at $\sqrt{s} = 200$ GeV (left panel) and LHC $Pb + Pb$ at $\sqrt{s} = 2.76$ TeV (right panel). In general in agreement with what has been obtained in hydrodynamical calculations, the increase of the viscosity of the medium has the effect to reduce the $v_2$ and $v_4$. As we can see at RHIC energies comparing the blue dashed

Figure 4. Left panel: differential $v_2(p_T)$ and $v_4(p_T)$ green and blue lines respectively at mid rapidity and for $(20-30\%)$ collision centrality. The comparison is between the two systems: $Au + Au$ at $\sqrt{s} = 200$ GeV and $Pb + Pb$ at $\sqrt{s} = 2.76$ TeV. The dashed lines refer to the case with a constant $\eta/s = (4\pi)^{-1}$ during all the evolution while the solid lines to the case with $\eta/s = (4\pi)^{-1}$ at higher temperature and with an increasing $\eta/s$ ratio at lower temperature. Right panel: differential $v_3(p_T)$ red lines with the same legend as in the left panel.
In the recent years, the correlation between integrated $v_2$ and high order harmonics $v_3$, $v_4$ with the initial asymmetry in coordinate space $e_2$, $e_3$ and $e_4$ has been studied in the event-by-event viscous hydrodynamics framework [29, 30, 31]. In general it has been shown that the elliptic flow is strongly correlated with initial eccentricity while a weaker correlation has been found for higher harmonics $v_3$, $v_4$ with $e_3$ and $e_4$. In this section we study these correlations within a transport approach with initial state fluctuation and for fixed $\eta/s$. In particular the results shown in this section are for an $\eta/s = 1/(4\pi)$ during all the evolution of the fireball.

In the left hand side of Fig.(5), we have shown the final integrated elliptic flow $<v_2>$ as...
a function of the corresponding initial eccentricity $< \epsilon_2 >$ for each event. We observe that for $20 - 30\%$ of centrality the elliptic flow results are strongly correlated with the initial eccentricity. This is consistent with a correlation coefficient close to 1. The correlation coefficient is given by:

$$C(y, x) = \frac{\sum_i (x_i - \langle x \rangle)(y_i - \langle y \rangle)}{\sqrt{\sum_i (x_i - \langle x \rangle)^2 \sum_i (y_i - \langle y \rangle)^2}}$$  \hspace{1cm} (6)

For the plot in the left panel of Fig.(5) we get $C(2, 2) \approx 0.96$. In the middle and right panel of Fig.(5), we have shown the results obtained for the higher harmonics $< v_3 >$ and $< v_4 >$ respectively and for $20 - 30\%$ centrality. We observe that the correlation between triangular flow $< v_3 >$ and its initial $< \epsilon_3 >$ is weaker than that between $< v_2 >$ and $< \epsilon_2 >$ with a correlation coefficient $C(3, 3) \approx 0.71$. Furthermore for the fourth harmonic flow $< v_4 >$ we observe an uncorrelation with its initial $< \epsilon_4 >$ as shown by a correlation coefficient close to zero. One explanation for this no linear correlation between $< v_4 >$ and $< \epsilon_4 >$ is that for final $< v_4 >$ there is also a correlation with the initial $< \epsilon_2 >$. In [31] has been shown possible to have a good linear correlation between $< v_4 >$ and a linear combination of the initial $< \epsilon_2 >$ and $< \epsilon_4 >$.

6. Conclusions

We have investigated within an event-by-event transport approach at fixed $\eta/s$, the build up of $v_3(p_T)$ and higher harmonics $v_3(p_T)$ and $v_4(p_T)$ for two different beam energies: RHIC for $Au + Au$ at $\sqrt{s} = 200\, \text{GeV}$ and LHC for $Pb + Pb$ at $\sqrt{s} = 2.76\, \text{TeV}$. We have found that at RHIC the $v_n(p_T)$ are more sensitive to the value of $\eta/s$ at low temperature and in particular this sensitivity increase with the order of the harmonics. At LHC we get a different effect, the $v_n(p_T)$ are not so sensitive to the increase of $\eta/s$ at low temperature. Finally, we have studied also the relation between $v_n$ and $\epsilon_n$ for $n = 2, 3$ and 4 in ultra relativistic heavy-ion collisions within an event-by-event relativistic transport approach. We have shown that the second and third harmonic flows are correlated to their initial eccentricities, a result similar to that has been obtained in the recent years in viscous hydrodynamics.

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