Economic Growth with Social Status, Spirit of Capitalism, and Conspicuous Consumption

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Abstract
The purpose of this study is to introduce social status, conspicuous consumption, and spirit of capitalism into neoclassical growth model. This paper studies a dynamic interdependence between economic growth, economic structural change, income and wealth distribution, social status, conspicuous consumption, and spirit of capitalism in a small-open economy. We build a heterogeneous-households growth model with endogenous wealth accumulation and social status. The dynamics of J-households economy is described by J differential equations. We simulate the motion of the model with three groups of households. We carry out comparative dynamic analysis with regard to some parameters.

Keywords: Spirit of capitalism, Conspicuous consumption, Social status, Inequality in income and Wealth.

Introduction
Conspicuous consumption, social status and spirit of capitalism are well-mentioned but seldom modelled factors in the literature of economic growth theory. Conspicuous consumption is observed globally nowadays in varied forms of fashions and over-priced consumption behavior. It a common sense that in modern market economies competition in upwards mobility influences family consumption and saving pattern. Spirit of capitalism is applied used to explains why workers in large cities in newly developed and fast economic growth economies increase their work hours as their wage incomes are further enhanced. Nevertheless, these factors for economic growth are seldom integrated with other growth factors within a formal analytical framework of development. The purpose of this study is to challenge this task by proposing a dynamic growth model with interdependence between economic growth, economic structural change, conspicuous consumption, social status and spirit of capitalism.

It is well known that in The Theory of the Leisure Class, Veblen (1899) makes a systematical examination of economic consequences of conspicuous consumption and social status. Veblen holds that people are interested in pursuing conspicuous consumption as it signals wealth and social status. According to Duesenberry (1949) some people improve social status by imitating the consumption standard of the social or classes above them. Rege (2008) points out that people is concerned with social status because it serves a signal of non-observable abilities. “By investing in social status a person can thus improve his chance of engaging in a complementary interaction with a high ability person.
The idea that status can serve as a signal of abilities is not new and has been captured in several models. It has, for example, been demonstrated that workers can signal their ability to employers by undertaking some seemingly irrelevant but costly activity interpreted as status consumption (Frank, 1985) or social culture (Fang, 2001) and that people can “burn money” on fashions to signal abilities in a “dating game” (Pesendorfer).” (Rege, 2008). As demonstrated by many studies (Cole et al., 1992; Konrad, 1992; Fershtman et al., 1996; Rauscher, 1997) introduction of social status makes traditional growth models more robust in explaining economic growth processes. Social status is generally considered as functions of private wealth (Zou, 1994; Bakshi and Chen, 1996; Chang, 2006; Chang and Tsai, 2003; Clemens, 2004; Fisher and Hof, 2005; Chen and Guo, 2011). Zhang (2016) recently studies the role of social status with an alternative approach to household behavior. Basing on Zhang’s model which is developed for a multi-countries global economy with fixed time distribution, we deal with a small-open economy with elastic labor supply. Another difference from Zhang’s model which treats social status as households’ wealth considers is that we consider social status as functions of relative wealth of households.

There are many studies on economic growth of open economies (e.g., Obstfeld and Rogoff, 1996; Lane, 2001; Kollmann, 2001; Benigno and Benigno, 2003; Gali and Monacelli, 2005; Uya, et al. 2013; and Ilzetzki, et al. 2013). We follow this tradition to show how income and wealth distributions change in a small-open economy with social status. The model in this study is an extension of the growth model by Zhang (2016). The main difference from Zhang’s model is that this study follows small-open economic growth literature, while Zhang’s model is developed within Solow’s one-sector growth framework. Different from Zhang’s model which deals with a multi-country economy with fixed time distribution, this study examines a small-open economy with elastic labor supply. The paper is organized as follows. Section 2 develops the growth model of wealth and income distribution among heterogeneous households with endogenous social status in a small-open economy. Section 3 examines dynamic properties of the model and simulates the model with three groups of households. Section 4 carries out comparative dynamic analysis. Section 5 concludes the study.

The Dynamic General Equilibrium Model With Social Status

The model in this study is based on two models by Zhang. Zhang (2016) deals with social status and growth with fixed work time, while Zhang (2010) studies a small-open economy with elastic labor supply. We are now concerned with a small-open growth model with social status, spirit of capitalism, and elastic labor supply by integrating the economic mechanisms of the two models. The economy consist of J groups of people with the fixed numbers of population $\bar{N}_j = 1, ..., J$. Similar to Uzawa’s two sector growth model (Uzawa, 1961), it is composed of two sectors: one capital goods sector and one consumer goods sector. Households own assets of the economy and distribute their incomes to consume and save. Production sectors use capital and labor. Exchanges take place in perfectly competitive markets. The available factors are fully utilized. Let prices be measured in terms of the capital goods and the price of the capital goods be unit. We denote wage rates by $w_j(t)$. The rate of interest $r^*$ is fixed in global markets. Capital depreciates at a constant exponential rate $\delta$. We use $p(t)$ to denote the price of consumer goods. We use subscript index, i and s to stand for capital goods sector and consumer goods sector, respectively. We use $N_m(t)$ and $K_m(t)$ to stand for the labor force and capital stocks employed by sector m. Let $F_m(t)$ stand for the output level of sector m.

The Total Labor Supply

We use $h_j$ and $T_j(t)$ to stand for group j’s individual human capital and work hours. The total labor supply is the sum of all groups’ qualified labor supply:

$$N(t) = \sum_{j=1}^{J} h_j T_j(t) \bar{N}_j$$  \hspace{1cm} (1)
The Capital Goods Sector

The production function of the capital goods sector is taken on the following form:

\[ F_i(t) = A_i K_i^\alpha(t) N_i^\beta(t), A_i, \alpha_i, \beta_i > 0, \alpha_i + \beta_i = 1 \]  

where \(A_i, \alpha_i,\) and \(\beta_i\) are positive parameters. The marginal conditions of the capital goods sector are:

\[ r_k = \frac{a_i F_i(t)}{K_i(t)}, w(t) = \frac{\beta_i F_i(t)}{N_i(t)} \]  

where \(r_k = r^* + \delta_k\).

The Consumer Goods Sector

The production function of the consumer goods sector is

\[ F_s(t) = A_s K_s^\alpha(t) N_s^\beta(t), \alpha_s + \beta_s = 1, \alpha_s, \beta_s > 0 \]  

where \(A_s, \alpha_s,\) and \(\beta_s\) are the technological parameters of the consumer goods sector. The marginal conditions are:

\[ r_k = \frac{a_s p(t) F_s(t)}{K_s(t)}, w(t) = \frac{\beta_s p(t) F_s(t)}{N_s(t)} \]  

Current Income and Disposable Income

This study applies Zhang’s approach to modeling behavior of households (Zhang, 2005). Let \(\bar{K}_i(t)\) stand for per capita wealth of country \(j\). We have \(\bar{K}_i(t) = \bar{K}_i(t) / N_j\), where \(\bar{K}_i(t)\) is the total wealth owned by country \(j\) per capita current income from the interest payment \(r^* \bar{K}_i(t)\), and the wage payment \(h \bar{T}_i(t) w(t), \) is:

\[ y_j(t) = r^* \bar{K}_i(t) + h \bar{T}_i(t) w(t) \]

The per capita disposable income is the sum of the current disposable income and the value of wealth. That is

\[ \hat{y}_j(t) = y_j(t) + \hat{K}_j(t) = R \bar{K}_i(t) + T_i(t) w_j(t) \]  

where \(R = 1 + r^*\) and \(w_j(t) = h t w_j(t)\).

Budget and Time Constraint

The disposable income is used for saving and consumption. The representative household in group \(j\) would distribute the total available budget between saving \(s_j(t)\) and consumer good \(c_j(t)\). The budget constraint is:

\[ p_j(t) c_j(t) + s_j(t) = \hat{y}_j(t) \]  

Let \(T̄_j(t)\) and \(T_0\) stand for leisure time and total available time for work and leisure. We have:

\[ T̄_j(t) + T_0(t) = T_0 \]  

Insert (6) and (8) in (7)

\[ T̄_j(t)w_j(t) + p(t)c_j(t) + s_j(t) = \hat{y}_j(t) \equiv R \bar{K}_i(t) + T_0 w_j(t) \]  

Relative Share of National Wealth among Groups

We define a variable called the relative share of group \(j\)'s wealth in the total national wealth as follows

\[ z_j(t) = \frac{\bar{K}_j(t) \bar{N}_j}{\bar{K}(t)} \]  

Add all the equations in (10):

\[ \sum_{j=1}^{J} z_j(t) = 1 \]  

Social Status

We follow Zhang (2016) in modelling social status. As mentioned in Zhang (2016), the approach is inspired by a few approaches to social status and economic growth mathematical growth models (e.g., Kurz, 1968; Corneo and Jeanne, 1999; Chen and Guo, 2009). This study applies the idea that social status is represented by the level of capital ownership (Chen and Guo, 2011). This study specifies social status of the representative household j as function of relative wealth as follows:

\[ \omega_j(\bar{z}_j(t)) = \omega_{jo} + \omega_{lo} \bar{z}_j(t) \]

where \(\omega_{jo}\) and \(\omega_{lo}\) positive parameters. The specified form implies that social status is positively related to the group’s relative wealth in the national distribution. We try to simplify issues related to measuring social status. In reality, social status can be related to many other variables such as education, human capital, family heritage, human networks with celebrities, relative richness in the same group position, and the like.

Utility Functions

Each household decides three variables – leisure time, consumption level of consumer goods, and saving. The utility level \(U_j(t)\) is dependent on \(T̄_j(t), c_j(t),\) and \(s_j(t)\) as follows
$U_j(t) = \zeta_j(\omega_j(t)) \xi_j(\omega_j(t)) \lambda_j(\omega_j(t))$

$\sigma_{j0}(\omega_j(t)), \xi_{j0}(\omega_j(t)), \lambda_{j0}(\omega_j(t)) > 0$

where $\sigma_{j0}(\omega_j(t))$ is the propensity to use leisure time, $\xi_{j0}(\omega_j(t))$ the propensity to consume consumer goods, and $\lambda_{j0}(\omega_j(t))$ the propensity to save. The role of social status in economic growth and income and wealth distribution is realized by influencing the propensities.

Social Status and the Propensities

This study assumes the propensities to use leisure time, to consume consumption good, and to save to be related to social status in the following way

$\sigma_j(\omega_j(t)) = \tilde{\sigma}_j(\omega_j(t)) + \tilde{\sigma}_j(\omega_j(t))$, 
$\xi_j(\omega_j(t)) = \tilde{\xi}_j(\omega_j(t)) + \tilde{\xi}_j(\omega_j(t))$, 
$\lambda_j(\omega_j(t)) = \tilde{\lambda}_j(\omega_j(t)) + \tilde{\lambda}_j(\omega_j(t))$ (13)

where $\tilde{\sigma}_j(\omega_j(t))$ and $\tilde{\lambda}_j(\omega_j(t))$ are positive parameters, and $\tilde{\xi}_j(\omega_j(t))$ and $\tilde{\lambda}_j(\omega_j(t))$ are parameters which may be either positive, zero, or negative. The propensity to consume may be enhanced by social status, for instance, through the so-called conspicuous consumption. The propensity to save is influenced by social status as more wealth tends to enhance social status. In our approach we consider the spirit of capitalism affects the propensity to save. It is reasonable to consider that the propensity to use leisure is related to social status. Although social status may interact with propensities through many channels in nonlinear relations, this study accepts the above linear form for convenience of analysis.

Optimal household behavior

Maximizing the utility subject to (9) yields

$w_j(t) \hat{y}_j(t) = \sigma_j(t) \hat{y}_j(t)$,
$p(t) c_j(t) = \xi_j(t) \hat{y}_j(t)$,
$s_j(t) = \lambda_j(t) \hat{y}_j(t)$ (14)

where

$\sigma_j(t) = \rho_j(t) \sigma_{j0}(t)$, $\xi_j(t) = \rho_j(t) \xi_{j0}(t)$, $\lambda_j(t) = \rho_j(t) \lambda_{j0}(t)$, 
$\rho_j(t) = \frac{1}{\sigma_{j0}(t) + \xi_{j0}(t) + \lambda_{j0}(t)}$

Wealth Accumulation

The change in the household’s wealth saving minus dissaving. As saving is $s_j(t)$ and dissaving is $\hat{k}_j(t)$, we have:

$\hat{k}_j(t) = s_j(t) - \hat{k}_j(t)$ (15)

Demand and Supply

We use $K(t)$ to stand for the total capital stock employed by the economy. Full employment of labor and capital implies:

$K(t) + K_s(t) = K(t), N(t) + N_s(t) = N(t)$ (16)

National Wealth Being Owned by the Domestic Households

The national wealth is the wealth owned by the national households:

$\hat{R}(t) = \sum_{j=1}^{J} k_j(t) N_j$ (17)

Demand and Supply Equilibrium for Consumer Goods

The clearing condition for consumer goods is

$\sum_{j=1}^{J} c_j(t) N_j = F_s(t)$ (18)

We completed the model. From the structural point of view the model is general. Some well-known models in theoretical economics can be considered as its special cases. For instance, it is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961). It is similar to neoclassical growth models for small open economies. It is built on the basis of Walrasian general equilibrium framework. The model is influenced by some growth models in the literature of economic growth with social status and spirit of capitalism.

The Economic Dynamics

The dynamic system consists of any (finite) number of households. The heterogeneity in households implies that the dynamic system is nonlinear and highly dimensional. Although it is analytically difficult to deal with such a complicated system, we can easily simulate the motion of the economic system with the following lemma.
Lemma

The dynamics of the national world economy is governed by the following J dimensional differential equations system with \( K(t), \{z_j(t)\} \), where \( \{z_j(t)\} \equiv (z_1(t), \cdots, z_J(t)) \) as the variables:

\[
K(t) = \Omega_j(K(t), \{z_j(t)\}), \quad \dot{z}_j(t) = \Lambda_j(K(t), \{z_j(t)\}), j = 2, \ldots, J,
\]

in which \( \Lambda_j \) is unique functions of \( K(t) \) and \( \{z_j(t)\} \) defined in Appendix. For any given positive values of \( K(t) \) and \( \{z_j(t)\} \) at any point in time, the other variables are uniquely determined by the following procedure: \( w \) by (A2) \( \rightarrow p \) by (A3) \( \rightarrow z_j(t) \) by (11) \( \rightarrow \omega_j(t) \) by (12) \( \rightarrow \sigma_j(t), \xi_j(t) \), and \( \lambda_j(t) \) by (13) \( \rightarrow \sigma_j(t), \zeta_j(t) \), and \( \lambda_j(t) \) by (14) \( \rightarrow N_i(t) \) by (A13) \( \rightarrow N(t) \) by (A10) \( \rightarrow N_i(t) \) by (A11) \( \rightarrow \tilde{y}_i(t) \) by (A4) \( \rightarrow \tilde{T}(t), \zeta(t) \), and \( s(t) \) by (14) \( \rightarrow T(t) \) by (8) \( \rightarrow K(t) \) and \( K_j(t) \) by (A1) \( \rightarrow F(t) \) and \( F_j(t) \) by the definitions.

The lemma provides a computational procedure for illustrating the motion of the economic system with any number of household groups. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model for an economy with three groups of households. We specify parameter values as follows

\[
T_0 = 24, r_0 = 0.05, \delta = 0.05, A_i = 1, A_s = 1, a_i = 0.3, \alpha = 0.32, \quad \sigma_{10} = 0.25, \quad \sigma_{20} = 0.2, \quad \sigma_{30} = 0.13, \quad \sigma_{d0} = 0.2, \quad \sigma_{d0} = 0.1, \quad \sigma_{d0} = 0.0.
\]

The population of group 3 is largest, while the population of group 2 is the next. The capital goods and consumer goods sectors’ total factor productivities are fixed at 1. The human capital levels are specified at 8, 3, and 1, respectively. We specify the values of the parameters \( \alpha_m \) in the Cobb-Douglas productions approximately equal to 0.3. The depreciation rate of physical capital is about 0.05. Values of the parameters associated with social status and preference change are small. We will examine how the system is affected as these parameters vary. We specify the initial conditions as follows:

\[
\hat{R}(0) = 12800, \hat{z}_2(0) = 0.26, \hat{z}_2(0) = 0.42.
\]

The motion of the variables is plotted in Figure 1. In Figure 1, we have the national income and trade balance as follows:

\[
Y(t) \equiv F(t) + pF_j(t), E(t) \equiv \sqrt{\hat{K}(t) - K(t)}
\]

The national output and capital stock employed fall over time till they approach the equilibrium point. The national wealth rises. The trade balance is improved. The consumer goods sector expands, while the capital goods sector shrinks. The preferences change slightly. The relative share of group 2 falls, while the relative shares of the other groups rise. The leisure time of group 3 falls, while the leisure times of the other two groups rise.

![Figure 1: The Motion of the Economic System](image)

We identify the following equilibrium point

\[
Y = 3346, K = 11644, K = 12996, E = 67.6, p = 0.97, w = 1.12,
\]

\[
\begin{align*}
N_1 &= 3909, N_2 &= 1814, N_3 &= 1239, N_A &= 2210, N_F &= 1303, N_s &= 1239, N_2 &= 0.324, N_3 &= 0.263, N_3 &= 0.263, N_2 &= 0.263, \\
F_A &= 0.263, F_F &= 0.413, F_F &= 0.413, F_F &= 0.413, F_F &= 0.413, \\
F_s &= 0.026, F_s &= 0.026, F_s &= 0.026, F_s &= 0.026, F_s &= 0.026, \\
T_0 &= 16.8, T_1 &= 15.4, T_2 &= 14.8
\end{align*}
\]

(21)

It is straightforward to calculate the three eigenvalues at the equilibrium point as follows

-0.33,-0.32,-0.31.

The equilibrium point is stable. This conclusion is important as it guarantees that we can effectively carry out comparative dynamic analysis.

Comparative Dynamic Analysis

We simulated the motion of the dynamic system. We now examine how changes in some parameters affect the national economy. First, we introduce a variable \( \Delta x_i(t) \) which stands for the change rate of
the variable $x_j(t)$ in percentage due to changes in the parameter value.

**Group 1’s Spirit of Capitalism being Enhanced**

In this study by spirit of capitalism we narrowly mean the desire for accumulating more wealth. In our modelling it is specially referred to the weight given to the relative share of a group in the group’s social status function. We now examine the effects of the following change in group 1’s spirit of capitalism: $\omega_1 := 0.05 \Rightarrow 0.06$. The simulation result is plotted in Figure 2. The enhanced spirit of capitalism increases group 1’s social status $\omega_i$, while the other two groups’ social status are slightly affected. The net effects on group 1’s preference reduces the (relative) propensities to save and to use leisure and increases the propensity to consume. Group 1’s representative household consumes more and has less wealth. The other two groups’ preferences and consumption behavior are slightly affected. The national income and capital stock employed are augmented. The national wealth falls slightly. The trade balance is deteriorated. The consumer goods sector expands. The capital goods sector shrinks initially and expands in the long term.

**Figure 2: Group 1’s spirit of capitalism being Enhanced**

**Group 1 Augmenting the Weight of Propensity to Save**

We now study the effects of the following change in group 1’s propensity to save: $\lambda_1 := 0.2 \Rightarrow 0.3$. The simulation result is plotted in Figure 3. The national income rises initially and falls in the long term. The national capital stock employed slightly rises. The national wealth rises. The trade balance is improved initially but deteriorated in the long term. Group 1’s relative share of wealth and social status are enhanced. Group 1’s propensity to save is increased and propensities to use leisure time and to consume consumer goods are reduced. Group 1’s representative household works more hours initially and less hours in the long term. Group 1’s representative household consumes more and has more wealth in the long term. The other two groups’ behavior are slightly affected.

**Figure 3: Group 1 Augmenting the Weight of Propensity to Save**

**Group 1 Augmenting the Weight of Propensity to Use Leisure Time**

We now study the effects of the following change in group 1’s weight of propensity to use leisure time: $\sigma_1 := 0.1 \Rightarrow 0.15$. The simulation result is plotted in Figure 4. The national income, national capital stock employed, and national wealth fall. The trade balance is improved initially but deteriorated in the long term. Group 1’s relative share of wealth and social status are lowered. Group 1’s propensity to save and to consume are reduced and propensity to use leisure time is enhanced. Group 1’s representative household works less hours. Group 1’s representative household consumes less and has less wealth. The other two groups’ behavior are slightly affected.

**Figure 4: Group 1 Augmenting the Weight of Propensity to Use Leisure Time**

**Group 1’s Human Capital being Enhanced**

We now study the effects of the following change in group 1’s human capital: $h_1 := 8 \Rightarrow 8.2$. The simulation result is plotted in Figure 5. The national income, national capital stock employed, and national wealth are increased. The two sectors expand. The trade balance is deteriorated initially but improved in the long term. Group 1’s relative share of wealth and social status are enhanced.
The Rate of Interest is Increased in Global Markets

We now study what happen to the national economy if the rate of interest is increased in global markets as: \( r^* : 0.05 \Rightarrow 0.05 \). The simulation result is plotted in Figure 6. The national income, national capital stock employed, and national wealth are reduced as capital cost is increased. The two sectors shrink. The trade balanced is improved. All groups’ levels of consumption and wealth fall and leisure hours rise. The preferences are slightly affected.

The Capital Sector’s Total Factor Productivity being Enhanced

We now study what happen to the national economy when the capital sector’s total factor productivity is enhanced as follows: \( A_i : 1 \Rightarrow 1.05 \). The simulation result is plotted in Figure 7. The national income, national capital stock employed, and national wealth are increased. The two sectors expand in the long term. The trade balanced is deteriorated initially and improved in the long term. All groups’ levels of consumption and wealth rise and leisure hours change slightly in the long term. The preferences are slightly affected.

Conclusions

This paper studied a dynamic interdependence between economic growth, economic structural change, income and wealth distribution, social status, conspicuous consumption, and spirit of capitalism in a small-open economy. We built a heterogeneous-households growth model with endogenous wealth accumulation and social status. The modelling is a synthesis of Zhang’s two papers respectively on social status and economic growth and on a small-open economy with elastic labor supply and wealth accumulation. We built the model and showed that the dynamics of J-households economy is described by J differential equations. We simulated the motion of the model with three groups of households. We carried out countries and carried out comparative dynamic analysis with regard to some parameters. We provide some insights into economic dynamics with various factors. For instance, if a group’s spirit of capitalism is enhanced, its relative social status is enhanced in comparison with other groups in society. The group with higher spirit of capitalism would lower its propensities to save and to use leisure time and increases its propensity to save. The national income and wealth are augmented. The economy benefits from workers with higher spirit of capitalism. It should be remarked that we could discuss some complicated issues related to theory of growth and social status in a unique manner because we applied Zhang’s utility function and the concept of disposable income to build the analytical framework which allows us to follow the motion of the national economy (e.g., Zhang, 2020). We may extend and generalize the model in different ways. We may further analyze behavior of the model with other forms of production or utility functions. We may, for instance, introduce taxation into our general dynamic equilibrium model.
Appendix: Proving the lemma
By (3) and (5), we obtain
\[ z \equiv \frac{r + \delta_k}{\omega} = \frac{N_m}{\beta_m K_m}, m = i, s, \]  
(A1) where
\[ \beta_m = \beta_m / \alpha_m. \]  
Insert (A1) in (3):
\[ z = \left( \frac{r + \delta_k}{\alpha_0} \right)^{1/\beta_i}, \omega = \alpha z^{-a_i} \]  
(A2)
where
\[ \alpha_0 = \alpha_i \beta_i^\alpha_i A_i, \alpha = \frac{\beta_i A_i}{\beta_i^\alpha_i} \]  
From (4) and (5), we have:
\[ p(z) = \frac{\beta_s^\alpha_s z^\alpha_s}{\beta_s A_s} \]  
(A3)
The variables \( z, w, \) and \( p \) are constant as they are functions of internationally fixed interest rate. From the definitions of \( \tilde{y} \), we have:
\[ \tilde{y}_j = R \tilde{k}_j + T_0 w_j \]
Insert \( p \) in (12):
\[ \sum_{j=1}^{l} \xi_j \tilde{y}_j = p F_s \]  
Substituting (A4) in (A5) yields
\[ N_s = \sum_{j=1}^{l} \xi_j (g \tilde{N}_j \tilde{k}_j + \tilde{g}_j) \]
(A6)
where we use \( w N_s = \beta_i p F_s \) and
\[ g(z) \equiv \left( \frac{1 + r}{\omega} \right) \beta_s \tilde{g}_j \equiv \beta_s h_j \tilde{N}_j \]
From (16) we have:
\[ N_s = N - N_i \]
From (1) and (8) we have
\[ N = T_0 \sum_{j=1}^{l} h_j \tilde{N}_j - \sum_{j=1}^{l} h_j \tilde{T}_j \tilde{N}_j \]
Insert (14) in (A8):
\[ N = T_0 \sum_{j=1}^{l} h_j \tilde{N}_j - \sum_{j=1}^{l} h_j \tilde{N}_j \sigma_{y_j} \]  
(A9)
Insert (A4) in (A9)
\[ N = \tilde{h} - \sum_{j=1}^{l} \tilde{h}_j \tilde{k}_j \]  
(A10)
where
\[ \tilde{h} \equiv T_0 \sum_{j=1}^{l} (1 - \sigma_j)h_j \tilde{N}_j, \tilde{\xi}_j \equiv \frac{h_j \tilde{N}_j \sigma_j R}{w_j} \]
Insert (A10) in (A7)
\[ N_i = \tilde{h} - \sum_{j=1}^{l} \tilde{h}_j \tilde{k}_j - N_s \]  
(A11)
From (A1) and (16), we get
\[ \frac{N_i}{\beta_i} + \frac{N_s}{\beta_s} = z K \]  
(A12)
Insert (A11) in (A12)
\[ N_s = \tilde{\beta} z K - \frac{\tilde{\beta}}{\beta_i} \tilde{h} + \frac{\tilde{\beta}}{\beta_s} \sum_{j=1}^{l} \tilde{h}_j \tilde{k}_j \]
(A13)
where
\[ \tilde{\beta} \equiv \left( \frac{1}{\beta_s} - \frac{1}{\beta_i} \right)^{-1} \]  
(A14)
From (A13) and (A6), we get
\[ \sum_{j=1}^{l} \tilde{h}_j \tilde{k}_j + \tilde{h} = \tilde{\beta} z K \]
(A15)
Insert (7) in (A14):
\[ K \sum_{j=1}^{l} \frac{z_j \tilde{h}_j}{\tilde{N}_j} + \tilde{h} = \tilde{\beta} z K \]  
(A16)
Solve (A15):
\[ K = \Delta(\{z_j\}) \equiv  \tilde{h} \left( \tilde{\beta} - \sum_{j=1}^{l} \frac{z_j \tilde{h}_j}{\tilde{N}_j} \right)^{-1} \]  
in which we use (11) to get
\[ z_1(\{z_j\}) = 1 - \sum_{j=2}^{l} z_j \]  
It is straightforward to confirm that all the variables can be expressed as functions of \( \{z_j\} \) and \( K \) by the following procedure: \( w \) by (A2) \( \rightarrow p \) by (A3) \( \rightarrow z_i \) by (11) \( \rightarrow \omega_j \) by (12) \( \rightarrow \tilde{\xi}_j \) and \( \lambda_j \) by (13) \( \rightarrow \tilde{\xi}_j \).
and \( \lambda \) by (14) \( \rightarrow \) \( N_i \) by (A13) \( \rightarrow \) \( N \) by (A10) \( \rightarrow \) \( N_i \) by (A11) \( \rightarrow \) \( \tilde{y}_j \) by (A4) \( \rightarrow \) \( \tilde{T}_c \), and \( s \) by (14) \( \rightarrow \) \( T \), by (8) \( \rightarrow \) \( K_i \) and \( K_j \) by (A1) \( \rightarrow \) \( F_i \) and \( F_j \) by the definitions. From this procedure, (15), and (20), we have:

\[
\dot{\lambda} = \Omega(\{\lambda_i\}) \equiv \sum_{i=1}^{J} \lambda_i N_i = \sum_{j=1}^{J} \Omega_i N_i \quad (A18)
\]

Take derivatives of (10) with respect to \( t \):

\[
\dot{\lambda}_j = \frac{\dot{z}_j K}{\dot{N}_i} + \frac{z_j \Omega_k}{\dot{N}_i}, j = 2, \ldots, J \quad (A19)
\]

where we use (A18). From (A17) and (A19) we solve:

\[
\dot{\lambda}_j = \Lambda_j(\{\lambda_i\}) \equiv \left( \Omega_j - \frac{z_j \Omega_k}{\dot{N}_i} \right) \frac{N_j}{K}, j = 2, \ldots, J \quad (A20)
\]

With (A18), (A20) and the procedure we proved the Lemma.

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