Addenda

Chiral and Color-superconducting Phase Transitions with Vector Interaction in a Simple Model

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In the above paper, we have shown that the critical line of the first order chiral transition of QCD can have two endpoints. In this addendum, we elucidate the mechanism to realize the two-endpoint structure in the QCD phase diagram and argue the robustness for the appearance of such an interesting phase structure.

Recently, there is growing interest in the existence and its phenomenological consequences of the endpoint of the first order chiral phase transition (or a tricritical point in the chiral limit) of QCD at finite temperature ($T$) and density. In the above paper, we have demonstrated that there can appear another endpoint of the critical line of the first order transition in the lower $T$ region where the color superconductivity (CS) comes into play; we shall call such a phase structure in the phase diagram the two-endpoint structure (abbreviated to TEPS). After the publication of our paper, some colleagues asked us in what condition such another endpoint in the lower $T$ region can appear. Indeed, we are aware that we have failed in given sufficient discussions on TEPS in the paper. In this addendum, partly to answer the questions asked by the colleagues, we shall try to clarify the mechanism for TEPS to be realized in a qualitative way and show that the growth of the gap $\Delta$ of CS, which is caused by the larger vector coupling $G_V$ in our case, is responsible for the realization of TEPS.

To understand the mechanism of the appearance of another endpoint in the lower $T$ region, it is best to begin with recalling the way how the endpoint of the chiral transition in the higher $T$ side gets to exist. At finite $T$, there exist quarks with a high momentum well above the Fermi momentum as described by the Fermi-Dirac (F-D) distribution function $n(p) = 1/(e^{(p-\mu)/T} + 1)$; see Fig. 1(a). An important point is that the positive energy states of quarks contribute positively to the quark condensate $\langle \bar{q}q \rangle \propto M_D$, making the net value of it smaller. The condensate is expected to vary smoothly with the change of $T$ as given by the F-D distribution function. Thus the chiral phase transition becomes weaker and the order of it changes from a first to crossover (or second in the chiral limit) when $T$ is raised, as it is.

Now how does the diquark condensate or the gap $\Delta$ of CS affect the chiral transition? Here it is worth mentioning, although well known again, that CS with $\Delta$ leads to a quark distribution function similar to the F-D distribution function $T \neq 0$: The distribution function for the massless quarks with $\Delta$ at $T = 0$ reads

$$n(p) = \frac{1}{2} \left( 1 - \frac{\sqrt{(p - \mu)^2 + \Delta^2}}{p - \mu} \right),$$

which behaves as shown in Fig. 1(b); notice the similarity of Fig. 1(b) with Fig. 1(a). Thus one can naturally expect that the larger $\Delta$ also weakens the chiral phase transition at low $T$. To see the similarity of the role of $\Delta$ and $T$ on the chiral condensate more transparently, let us rewrite the gap equation Eq. (3.13) in the text as follows:

$$M_D = 8G_S M \int \frac{d^3p}{(2\pi)^3} E_p \left\{ 1 - n_N(p) - n_S^+(p) + 2 \left( 1 - n_N(p) - n_S^+(p) \right) \right\},$$

where $n_S^\pm = (1/2)(1 - \xi_{\pm}/\xi_\pm + \tanh(\beta \xi_{\pm}/2))$ denotes the distribution functions of the color superconducting quarks and anti-quarks, and $n_N^\pm = 1/(e^{(p-\mu)/T} + 1)$ corresponds to the normal ones. Notice that Eq. (0.2) depends on $\Delta$ and $T$ only through $n_N^\pm$ and $n_S^\pm$, which makes the effect of $\Delta$ on the...
chiral condensate is hardly distinguishable with that of $T$. It implies that if a sufficiently large gap is formed near the chiral transition, the phase transition can change from a first order to crossover.

What is the role of the vector coupling $G_V$ in this problem? As is shown in the paper, the larger $G_V$ not only makes the first order chiral transition weaker in the higher $T$ region but also widens the region in the phase diagram of the coexisting phase where $M_D$ and $\Delta$ are both finite. Thus a larger $\Delta$ can be realized with $G_V$ around the critical line of the chiral transition in the lower $T$ region, which in turn leads to the weakening of the phase transition owing to the above mentioned mechanism. Since lower the $T$, larger the $\Delta$, the weakening of the phase transition is more effective at lower $T$; thus the first order transition in the low $T$ region starts to change into a crossover one from $T = 0$ with some $G_V$. This is how another endpoint gets to exist at relatively lower $T$; hence realized is TEPS. As $G_V$ is increased further, the lower endpoint approaches toward higher $T$, and eventually the crossover transition at lower $T$ merges with that extending from the higher $T$ region; the first order transition ceases to exist in the phase diagram.

It should be emphasized that TEPS does not appear without incorporating the quark-quark (qq) interaction which causes the interplay between $\chi_{SB}$ and CS. The above mechanism for the appearance of TEPS is considered independent of models employed, provided that the interplay between the $\chi_{SB}$ and CS is taken into account.

Although TEPS is obtained in an effective model incorporating $G_V$ in the present case, it is worth mentioning that $G_V$ is actually dispensable for the manifestation of TEPS: When the chiral transition is first order but already sufficiently weak without the qq interaction, the incorporation of the qq interaction acts to realize TEPS by further weakening the first order transition at lower $T$. Therefore, the second endpoint could get to exist, for example, if the scalar interaction as given by $G_S(q\overline{q})^2$ which is the driving force of the $\chi_{SB}$ were much weaker.

Finally, we remark that TEPS discussed here and the mechanism for the appearance of it are completely different from the ones given in $\mathbf{1}$, which discusses possible effects of isospin chemical potential on the chiral transition.

References

1) D. Toublan, and J.B. Kogut, Phys. Lett. B564 (2003) 212 [arXiv:hep-ph/0301183].

Fig. 1. The typical examples of the distribution functions for (a) finite temperature, $n = 1/(e^{(p-\mu)/T} + 1)$ and (b) finite diquark gap $\Delta$ Eq. (0.2).