Nonthermal CP Violation in Soft Leptogenesis

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Soft leptogenesis is a mechanism which generates the matter-antimatter asymmetry of the Universe via the out-of-equilibrium decays of heavy sneutrinos in which soft supersymmetry breaking terms play two important roles: they provide the required CP violation and give rise to the mass splitting between otherwise degenerate sneutrino mass eigenstates within a single generation. This mechanism is interesting because it can be successful at lower temperature regime \( T \lesssim 10^9 \) GeV in which the conflict with the overproduction of gravitinos can possibly be avoided. In earlier works the leading CP violation is found to be nonzero only if finite temperature effects are included. By considering generic soft trilinear couplings, we find two interesting consequences: 1) the leading CP violation can be nonzero even at zero temperature realizing nonthermal CP violation and 2) the CP violation is sufficient even far away from the resonant regime allowing soft supersymmetry breaking parameters to assume natural values at around the TeV scale. We discuss phenomenological constraints on such scenarios and conclude that the contributions to charged lepton flavor violating processes are close to the sensitivities of present and future experiments.

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I. INTRODUCTION

Leptogenesis [1] is an attractive mechanism for generating the observed matter-antimatter asymmetry of the Universe wherein one first creates an asymmetry in the lepton sector which in turn induces an asymmetry in the baryon sector via anomalous $B + L$ violating interactions. In standard type-I seesaw supersymmetric leptogenesis [2–5] involving the out-of-equilibrium decays of heavy neutrinos and sneutrinos, the CP violation required to generate the lepton number asymmetry comes from the neutrino Yukawa couplings. This scenario, with hierarchical right-handed neutrinos (RHNs), faces a conflict as successful leptogenesis requires the mass of the lightest RHN to be at least $10^9$ GeV [6] while the simplest resolution of the gravitino problem requires the reheating temperature after inflation to be less than $10^{6–9}$ GeV depending on the gravitino mass [7].

One may avoid this conflict by incorporating new elements in leptogenesis. In models of soft leptogenesis [11, 12] (for a recent review see Ref. [13]) CP violation comes from soft supersymmetry (SUSY) breaking terms (here onwards we will simply refer to them as soft terms) with soft parameters assumed to be at $m_{SUSY} \sim$ TeV scale i.e. we still hope SUSY is responsible for stabilizing the hierarchy between the weak and grand unification scales. One interesting feature is that soft leptogenesis can proceed even with one generation of the RHN chiral superfield. Essentially the heavy sneutrino $\tilde{N}$ and antineutrino $\tilde{N}^*$ from the same chiral supermultiplet will mix due to the presence of the soft terms. The decays of the mixed mass eigenstates violate both CP and lepton number and generate a matter-antimatter asymmetry. Although the CP violation is suppressed by powers of $m_{SUSY}/M \ll 1$ with $M$ the scale of the lightest RHN, the mass splitting between these otherwise degenerate sneutrino mass eigenstates is also proportional to $m_{SUSY}/M$. Crucially, this small splitting also results in enhancement of the CP violation from mixing. Due to the suppression factor $m_{SUSY}/M$ in the CP violation, one cannot have very large $M$. Estimating the leading CP parameter as $\epsilon \sim m_{SUSY}/M$ and that successful leptogenesis generically requires $\epsilon \gtrsim 10^{-6}$, we obtain $M \lesssim 10^9$ GeV assuming $m_{SUSY}$ at the TeV scale. Hence soft leptogenesis occurs in the regime where the conflict with the bound on the reheating temperature from gravitino overproduction can be mitigated or even avoided.

In the original proposals of Refs. [11,12], the authors showed that in the scenario of $\tilde{N} – \tilde{N}^*$ mixing the leading CP violation in decays to fermions and scalars have opposite signs and cancel each other at the order $O(m_{SUSY}/M)$ at zero temperature $T = 0$. They further showed that once finite temperature effects are taken into account, this cancellation is partially lifted, i.e. one obtains an asymmetry proportional to a factor $(c_F(T) - c_B(T))$ (where $c_F,B(T)$ are phase space and statistical factors associated with fermion and boson final states), where the contributions do not completely cancel each other at finite temperature. Working under the assumption of proportionality of soft trilinear couplings $A_\alpha = AY_\alpha$ where $Y_\alpha$ are the neutrino Yukawa couplings and $\alpha$ the lepton flavor, they showed that the resulting CP violation is of the order of $O(m_{SUSY}/M)$ at the resonance which however requires an unconventionally small soft bilinear coupling $B \ll m_{SUSY}$. Away from the resonance, the CP violation is $O(Y_\alpha^2)$ and hence too suppressed for successful leptogenesis. On the other hand, assuming generic $A$ couplings, Ref. [14] showed that successful leptogenesis can be obtained with $B \sim m_{SUSY}$ away from the resonant regime.

Later in Ref. [15] it was argued that direct CP violation, i.e. from vertex corrections, due to gaugino exchange in the loop, survives at the order $O(m_{SUSY}^2/M^2)$ at $T = 0$. Since the neutrino Yukawa coupling is replaced by the gauge coupling in the CP violation parameter, a large CP violation can be obtained for $M$ at the TeV scale. Further study in Ref. [16] however, showed that in fact in this scenario, the cancellation still holds up to $O(m_{SUSY}^2/M^2)$ at $T = 0$ and it was concluded that finite temperature effects are necessary to prevent the cancellation. The cancellation is consistent with the result obtained in Ref. [17] which states that to have a nonvanishing total CP violation there should be lepton number violation to the right of the ‘cut’ in the loop diagram, and this requirement is not fulfilled in these cases. More recently, in Ref. [18] it was shown that if finite temperature effects are taken into account consistently, the cancellation of direct CP violation from the gaugino contribution still holds even at $T \neq 0$.

In fact, in soft leptogenesis at finite temperature, the partial cancellation in the resulting lepton and slepton number density asymmetries sourced by CP violation from mixing and the complete cancellation in the case of the gaugino vertex correction [18] only hold under the assumption of equilibration between the chemical potentials of leptons and sleptons (superequilibration) which is valid below $T \lesssim 10^8$ GeV for $m_{SUSY} \sim$ TeV [5]. As shown in Ref. [19], in the nonsuperequilibration regime, the partial cancellation between lepton and slepton number density asymmetries in the mixing scenario are avoided, resulting in an enhanced efficiency for soft leptogenesis. However, for reasons given later, we shall below consider mixing and vertex scenarios in the superequilibration regime (and also find a case where the fermion and boson final states do not partially cancel each other). On the other hand, considering $M \gtrsim 10^8$ GeV and $m_{SUSY} \sim 1$ TeV, the CP violating parameter from the gaugino contribution in the nonsuperequilibration regime

1 See Refs. [8,10] for another resolution of the gravitino problem due to delayed thermalization of the Universe after inflation.

2 In a realistic model, we need at least two RHNs to accommodate neutrino oscillations. Assuming RHNs to be hierarchical, soft leptogenesis only depends on the parameters related to the lightest RHN and decouples from the parameters related to heavier RHNs.
is $\epsilon \sim 10^{-1} m^2_{\text{USY}}/M^2 \lesssim 10^{-11}$ and hence is too small for successful leptogenesis. Therefore processes involving gauginos will not be considered further in this work.

In this article, we revisit soft leptogenesis by relaxing the assumption of the proportionality of the $A$ couplings. In Section II we review the Lagrangian for soft leptogenesis with generic $A_{\alpha}$ terms and spell out the constraints from out-of-equilibrium decays of heavy sneutrinos and the cosmological bound on the sum of neutrino masses. In Section III, by considering generic $A_{\alpha}$ as has been done in Refs. [11, 12, 21] where there is only one physical phase $N$ in the vacuum expectation value, we always assume the proportionality of $A_{\alpha}$ and hence can be large enough for leptogenesis. Due to the small mass splitting, the mixing CP violation always dominates over the vertex CP violation even far away from the resonant regime. In Ref. [20] it was shown that with $A_{\alpha} = A Y_{\alpha}$, soft leptogenesis gives negligible contributions to the electric dipole moment of charged leptons and charged lepton flavor violating processes. In Section IV, we repeat the exercise and show that with generic $A_{\alpha}$ couplings, the contributions to charged lepton flavor violating processes are close to the sensitivities of present and future experiments. Finally in Section V, we conclude. This article is completed with two appendices. In Appendix A, we discuss the inclusion of thermal effects under the assumption of decaying heavy sneutrinos at rest. In Appendix B, we review the two specific scenarios of $A_{\alpha}$ discussed in Ref. [14] and discuss an interesting point missed by Ref. [14] which actually allows for nonzero leading CP violation at zero temperature.

II. THE LAGRANGIAN

The superpotential for the type-I seesaw is given by

$$ W_N = \frac{1}{2} M_i \tilde{N}_i^c \tilde{N}_i^c + Y_{\alpha} \tilde{N}_i^c \hat{\ell}_\alpha \tilde{H}_u, $$

where $\tilde{N}_i^c$, $\hat{\ell}_\alpha$ and $\tilde{H}_u$ denote respectively the chiral superfields of the RHNs, the lepton doublet and the up-type Higgs doublet, and $i$ and $\alpha$ are the RHN family and lepton flavor indices respectively. The $SU(2)_L$ contraction between $\hat{\ell}_\alpha$ and $\tilde{H}_u$ is left implicit. In the following we will assume that the RHNs are hierarchical such that only the lightest $\tilde{N}_1$ is relevant for soft leptogenesis. Henceforth we will drop the family index of RHN; for example, $\tilde{N} \equiv \tilde{N}_1$ and $Y_{\alpha} \equiv Y_{1\alpha}$. The corresponding soft terms are

$$ -\mathcal{L}_{\text{soft}} = \tilde{M}^2 \tilde{N}^* \tilde{N} + \left( \frac{1}{2} BM \tilde{N}^* \tilde{N} + A_{\alpha} \tilde{N}^* \hat{\ell}_\alpha \tilde{H}_u + \text{H.c.} \right). $$

(2)

The mass and interaction terms involving the sneutrino $\tilde{N}$ from $W_N$ are given by

$$ -\mathcal{L}_{\tilde{N}} = |M|^2 \tilde{N}^* \tilde{N} + \left( M^* Y_{\alpha} \tilde{N}^* \hat{\ell}_\alpha \tilde{H}_u + Y_{\alpha} H_u^* P_L \hat{\ell}_\alpha \tilde{N} + \text{H.c.} \right), $$

(3)

where $P_{L,R} = \frac{1}{2} (1 \mp \gamma_5)$. Through field redefinitions, it can be shown that the three physical phases are

$$ \Phi_{\alpha} = \text{arg} \left( A_{\alpha} Y^*_{\alpha} B^* \right). $$

(4)

Without loss of generality, the phases can be assigned to $A_{\alpha}$ and all other parameters will be taken real and positive. We would like to stress that we do not assume the proportionality of $A_{\alpha}$ to the neutrino Yukawa couplings ($A_{\alpha} = A Y_{\alpha}$) as has been done in Refs. [11, 12, 21] where there is only one physical phase $\Phi = \text{arg} (AB^*)$. As we will show in Section III by considering generic $A_{\alpha}$ couplings, the CP violation can be nonvanishing even at zero temperature.

Due to the bilinear $B$ term, $\tilde{N}$ and $\tilde{N}^*$ mix to form mass eigenstates

$$ \tilde{N}_+ = \frac{1}{\sqrt{2}} \left( \tilde{N} + \tilde{N}^* \right), $$

$$ \tilde{N}_- = -i \frac{1}{\sqrt{2}} \left( \tilde{N} - \tilde{N}^* \right), $$

(5)

with the corresponding masses given by

$$ M^2_\pm = M^2 + \tilde{M}^2 \pm BM. $$

(6)

In order to avoid a tachyonic mass which implies an instability of the vacuum such that the sneutrino will develop a vacuum expectation value, we always assume $B < M + \tilde{M}^2/M$. 

Rewriting the Lagrangian in terms of mass eigenstates $\tilde{N}_\pm$, we have

$$- \mathcal{L}_{\tilde{N}} - \mathcal{L}_{\text{soft}} = M_+^2 \tilde{N}_+^\dagger \tilde{N}_+ + M_-^2 \tilde{N}_-^\dagger \tilde{N}_-$$

$$+ \frac{1}{\sqrt{2}} \left\{ \tilde{N}_+ \left[ Y_\alpha \overline{H}_u^\dagger P_L \ell_\alpha + (A_\alpha + M Y_\alpha) \bar{\ell}_\alpha H_u \right] ight.$$  

$$+ i \tilde{N}_- \left[ Y_\alpha \overline{H}_u^\dagger P_L \ell_\alpha + (A_\alpha - M Y_\alpha) \bar{\ell}_\alpha H_u \right] + \text{H.c.} \right\}. \quad (7)$$

### A. General constraints

The total decay width for $\tilde{N}_\pm$ is given by

$$\Gamma_\pm \approx \frac{M}{4\pi} \sum_\alpha \left[ Y_\alpha^2 + \frac{|A_\alpha|^2}{2M^2} \pm \frac{Y_\alpha \text{Re}(A_\alpha)}{M} \right], \quad (8)$$

where we have expanded up to $O(Y^2, m_{\text{SUSY}}^2/M^2, Y Y_{\text{SUSY}} M)$ and ignored the final state phase space factors. We will impose the restriction that $|A_\alpha|, |B| < M$ and $|A_\alpha/Y_\alpha| < 1$ to ensure that we are always in the perturbative regime. In principle, $m_{\text{SUSY}}/M$ and $Y$ can go up to $4\pi$ before perturbative theory breaks down but with our stronger restriction, we are not anywhere near the nonperturbative regime.

Leptogenesis requires out-of-equilibrium decays of $\tilde{N}_\pm$ which imply

$$\Gamma_\pm \lesssim H(T = M),$$

where the Hubble expansion rate is given by $H = 1.66 \sqrt{g_*} T^2/M_{P1}$ with Planck mass $M_{P1} = 1.22 \times 10^{19}$ GeV. Assuming Minimal Supersymmetric Standard Model (MSSM) relativistic degrees of freedom, we have $g_* = 228.75$. The condition above translates to

$$\sqrt{\sum_\alpha \left[ Y_\alpha^2 + \frac{|A_\alpha|^2}{2M^2} - \frac{Y_\alpha B \text{Re}(A_\alpha)}{2M^2} \right]} \pm \frac{Y_\alpha \text{Re}(A_\alpha)}{M} \lesssim 1.6 \times 10^{-5} \left( \frac{M}{10^7 \text{GeV}} \right)^{1/2}. \quad (9)$$

From the condition above, we see that $|A_\alpha|$ is bounded from above depending on $M$. For example if $M \sim \text{TeV}$, we require $|A_\alpha| \lesssim 10^{-4}$ GeV. At this low scale, the mass splitting between $\tilde{N}_+$ and $\tilde{N}_-$ is required to be of the order of their decay widths such that the CP violation is resonantly enhanced to yield successful leptogenesis [23, 24]. To avoid excessive fine-tuning, if we consider $|A_\alpha| \sim \text{TeV}$, Eq. (9) implies $M \gtrsim 4 \times 10^7 \text{GeV}$.

In type-I seesaw, barring special cancellation, we have the upper bound on the sum of light neutrino masses from cosmology [22]

$$\sum_\alpha \frac{Y_\alpha^2 v_u^2}{M} \lesssim \sum_\alpha m_{\nu_\alpha} \approx 0.23 \text{eV},$$

$$\sqrt{\sum_\alpha Y_\alpha^2} \lesssim 3 \times 10^{-4} \left( \frac{M}{10^7 \text{GeV}} \right)^{1/2} \left( 1 + \frac{1}{\tan^2 \beta} \right)^{1/2}, \quad (10)$$

where $\tan \beta \equiv v_u/v_d$ and $v_{u(d)} = \langle H_u(d) \rangle$ are the up(down)-type Higgs vacuum expectation values. $v_u^2 + v_d^2 = \sqrt{2} G_F^{-1} \approx (174 \text{ GeV})^2$ with $G_F$ the Fermi constant. For $\tan \beta \gtrsim 1$, the bound above is always less stringent than Eq. (10) and hence the out-of-equilibrium condition alone is sufficient.

### III. CP VIOLATION

In this section we will study CP violation of the Lagrangian (7) from the interferences between tree level and one loop diagrams shown in Figures 1 and 2. We will take into account thermal corrections while approximating sneutrinos $\tilde{N}_\pm$ to always be at rest with respect to the thermal bath. Since we are in the regime where all three lepton flavors can be distinguished ($T \lesssim 10^9$ GeV), we will not sum over the lepton flavor in the final states [21].

To quantify the CP violation, we define the CP asymmetry for the decays $\tilde{N}_\pm \to a_\alpha$ with $a_\alpha = \{ \ell_\alpha H_u, \ell_\alpha \bar{H}_u \}$ as

$$\epsilon_{\pm \alpha}^{SV} = \frac{\gamma(\tilde{N}_+ \to a_\alpha) - \gamma(\tilde{N}_- \to a_\alpha)}{\sum_{a_\beta, \gamma} \gamma(\tilde{N}_+ \to a_\beta) + \gamma(\tilde{N}_- \to a_\beta)}.$$

$$\epsilon_{\pm \alpha}^{SV} = \frac{\gamma(\tilde{N}_+ \to a_\alpha) - \gamma(\tilde{N}_- \to a_\alpha)}{\sum_{a_\beta, \gamma} \gamma(\tilde{N}_+ \to a_\beta) + \gamma(\tilde{N}_- \to a_\beta)}.$$

$$\epsilon_{\pm \alpha}^{SV} = \frac{\gamma(\tilde{N}_+ \to a_\alpha) - \gamma(\tilde{N}_- \to a_\alpha)}{\sum_{a_\beta, \gamma} \gamma(\tilde{N}_+ \to a_\beta) + \gamma(\tilde{N}_- \to a_\beta)}.$$
respectively, \( \bar{\alpha} \) indicates the CP conjugate of \( \alpha \), and \( \gamma(i \to j) \) is the thermal averaged reaction density for the process \( i \to j \) defined in Eq. (2). In the following, we will include the thermal effects associated with intermediate on-shell states which, as shown in Ref. [18], result in the cancellation of vertex CP asymmetries from gaugino contributions [15][16]. We will always approximate \( N_\pm \) to be at rest with respect to the thermal bath so that we can obtain analytical expressions for the CP asymmetries (see Appendix A). Furthermore, we focus on the superequilibration regime which falls in the temperature range \( T \lesssim 10^9 \) GeV for \( m_{\text{SUSY}} \sim 1 \) TeV [5]. The advantage is that in this regime, lepton and sleptons are not distinguished (they have the same chemical potentials) and so the two Boltzmann equations for the lepton asymmetry in particles and sparticles can be reduced to one equation for the net lepton asymmetry [3]. Hence we are allowed to sum over CP asymmetries of lepton and slepton final states as below.

### A. CP violation from mixing

In this subsection, we discuss the mixing CP violation from self-energy corrections. There are two kinds of self-energy diagrams as shown in Figure 1, the diagrams with continuous flow of lepton number (a), (b) and (c) and the diagrams with flow of lepton number inverted in the loop (d), (e) and (f). Notice that diagrams with fermionic loop and fermionic final states do not contribute to the CP violation since they do not involve the soft couplings \( A_\alpha \). From the diagrams (a), (b) and (c) of Figure 1, we obtain the respective contributions to the CP asymmetries defined in Eq. (12) as follows:

\[
\begin{align*}
\epsilon_{\pm, a}^{S} & = \frac{1}{4\pi G_{\pm}(T)} Y^2 \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \left( 1 + \frac{M^2}{M^2} \pm \frac{B}{M} \right) \frac{2BM}{4B^2 + \Gamma_\mp^2} r_B(T) c_F(T), \\
\epsilon_{\pm, b}^{S} & = \frac{1}{4\pi G_{\pm}(T)} Y^2 a Y_{\beta} \frac{\text{Im}(A_{\alpha})}{M} \left( 1 + \frac{M^2}{M^2} \pm \frac{B}{M} \right) \frac{2BM}{4B^2 + \Gamma_\mp^2} r_F(T) c_B(T), \\
\epsilon_{\pm, c}^{S} & = \frac{1}{4\pi G_{\pm}(T)} \left[ Y^2 - \sum_{\beta} \frac{|A_{\beta}|^2}{M^2} \right] Y_{\alpha} \frac{\text{Im}(A_{\alpha})}{M} - \left( Y^2 - \frac{|A_{\alpha}|^2}{M^2} \right) \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \right] \frac{2BM}{4B^2 + \Gamma_\mp^2} r_B(T) c_B(T),
\end{align*}
\]

Footnotes:
3 The nonsuperequilibration effects in soft leptogenesis were studied in detail in Ref. [19].
4 The absorptive parts which regularize the singularity in the \( N_\pm \) propagators as \( M_\pm \to M_\pm \) are obtained by resumming self-energy diagrams following Refs. [23][24].
where we define $Y^2 \equiv \sum_{\alpha} Y_{\alpha}^2$ and
\begin{equation}
G_{\pm}(T) \equiv \left[ Y^2 + \sum_{\alpha} \left( \frac{|A_{\alpha}|^2}{M^2} \pm \frac{2Y_{\alpha} \text{Re}(A_{\alpha})}{M} \right) c_B(T) + Y^2 \left( 1 + \frac{\tilde{M}^2}{M^2} \pm \frac{B}{M} \right) c_F(T) \right].
\end{equation}

In the above $r_{B,F}(T)$ and $c_{B,F}(T)$ are temperature dependent terms associated with intermediate on-shell and final states respectively, as given in Appendix A. We will also make use of the following identity,
\begin{equation}
r_F(T)c_B(T) = r_B(T)c_F(T),
\end{equation}
proved in Appendix A. Note that if we sum over the lepton flavor states respectively, as given in Appendix A. We will also make use of the following identity,
\begin{equation}
r_F(T)c_B(T) = r_B(T)c_F(T),
\end{equation}

\begin{equation}
\epsilon_{\pm \alpha}^{S,(a)} = \frac{1}{4\pi G_{\pm}(T)} Y_{\alpha}^2 \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \left( 1 + \frac{\tilde{M}^2}{M^2} \pm \frac{B}{M} \right) \frac{2BM}{4B^2 + \Gamma_{\mp}^2} r_B(T)c_F(T),
\end{equation}
\begin{equation}
\epsilon_{\pm \alpha}^{S,(c)} = \frac{1}{4\pi G_{\pm}(T)} Y_{\alpha}^2 \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \left( 1 + \frac{\tilde{M}^2}{M^2} \pm \frac{B}{M} \right) \frac{2BM}{4B^2 + \Gamma_{\mp}^2} r_B(T)c_F(T),
\end{equation}
\begin{equation}
\epsilon_{\pm \alpha}^{S,(f)} = \frac{1}{4\pi G_{\pm}(T)} \left[ - \left( Y^2 - \sum_{\beta} |A_{\beta}|^2 \right) Y_{\alpha} \frac{\text{Im}(A_{\beta})}{M} \frac{2BM}{4B^2 + \Gamma_{\mp}^2} r_B(T)c_B(T) \right],
\end{equation}
\begin{equation}
\epsilon_{\pm \alpha}^{S,(n)} = \sum_{\beta} |A_{\beta}|^2 \frac{2BM}{4B^2 + \Gamma_{\mp}^2} r_B(T)c_B(T).
\end{equation}

Notice the leading contributions from $\tilde{N}_+$ and $\tilde{N}_-$ in Eqs. (13) and (16) come with the same sign as hence they will contribute constructively to the lepton number asymmetry.

The total CP asymmetry from mixing $\epsilon_{\pm \alpha}^{S} \equiv \sum_{n=(a,b,c,d,e,f)} \epsilon_{\pm \alpha}^{S,(n)}$ is given by
\begin{equation}
\epsilon_{\pm \alpha}^{S} = \frac{1}{4\pi G_{\pm}(T)} Y_{\alpha}^2 \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \frac{4BM}{4B^2 + \Gamma_{\mp}^2} c_{F}(T) c_{B}(T) r_B(T),
\end{equation}
\begin{equation}
\epsilon_{\pm \alpha}^{S} = \frac{1}{4\pi G_{\pm}(T)} Y_{\alpha}^2 \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \frac{4BM}{4B^2 + \Gamma_{\mp}^2} c_{F}(T) c_{B}(T) r_B(T),
\end{equation}
\begin{equation}
\epsilon_{\pm \alpha}^{S} = \frac{1}{4\pi G_{\pm}(T)} Y_{\alpha}^2 \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \frac{4BM}{4B^2 + \Gamma_{\mp}^2} c_{F}(T) c_{B}(T) r_B(T).
\end{equation}

In the above, the first term vanishes in the zero temperature limit $T \to 0$ when $c_{B,F}(T) \to 1$ and $r_{B,F}(T) \to 1$ while the terms higher order in $m_{\text{SUSY}}/M$ survive. They remain nonzero after summing over the lepton flavor $\alpha$. In the following in order to make the dependence of the CP asymmetries Eq. (17) on the model parameters more transparent, it is instructive to look at two limiting cases (i) $Y_{\alpha} \gg A_{\alpha}/M$ and (ii) $Y_{\alpha} \ll A_{\alpha}/M$.

\begin{itemize}
\item In the limit (i) $Y_{\alpha} \gg A_{\alpha}/M$, we have
\begin{equation}
\epsilon_{\pm \alpha}^{S} \sim \frac{1}{4\pi} P_{\alpha} \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \frac{4BM}{4B^2 + \Gamma_{\mp}^2} c_{F}(T) c_{B}(T) r_B(T),
\end{equation}
where we define the flavor projector $P_{\alpha} = Y_{\alpha}^2/\sum_{\alpha} P_{\alpha} = 1$ and $\Gamma_{\pm} \equiv \frac{Y^2 M}{4\pi} + \frac{B}{M} \ll 1$ in $G_{\pm}(T)$.

In the resonant regime where $B \sim \Gamma_{\mp}$, we have $\epsilon_{\pm}^{S} \sim (|A|/M) Y$ where we have suppressed the lepton flavor index for an order of magnitude estimation. In this case a large $\epsilon_{\pm}^{S}$ can be obtained which allows TeV scale leptogenesis but at the cost of having unnaturally small $|A|, B \ll \text{TeV}$.
\end{itemize}
Away from the resonant regime when $B \gg \Gamma_\pm$, the CP asymmetries go as $\epsilon^S_{\pm} \sim 10^{-1}Y|A|/B$ assuming $O(1)$ contribution from the CP phases of Eq. (4). Taking $|A| \sim \text{TeV} \gtrsim B$ together with the out-of-equilibrium decay condition (10) gives us sufficient CP asymmetries $\epsilon^S_{\pm} \gtrsim 10^{-6}$ for $M \gtrsim 10^7$ GeV. Under these assumptions, the first term in Eq. (18) dominates and hence the contribution to the CP violation is the thermal one.

- In the other limit (ii) $Y_\alpha \ll A_\alpha/M$, we have

$$\epsilon^S_{\pm\alpha} \approx \frac{1}{4\pi} \sum_\beta \frac{|A_\alpha|^2 |A_\beta|^2}{\sum_\beta |A_\beta|^2} Y_\alpha \frac{\text{Im}(A_\beta)}{M} \frac{4BM}{4B^2 + \Gamma_A^2 r_B(T)},$$

where $\Gamma_A = \sum_\alpha |A_\alpha|^2$. The CP asymmetries Eq. (19) clearly do not vanish at $T = 0$ and this represents a nonthermal CP violation.

In the resonant regime $B \sim \Gamma_\pm$, we have $\epsilon^S_{\pm} \sim Y/(|A|/M)$. In this case too a large $\epsilon^S_{\pm}$ can be obtained which allows TeV scale leptogenesis but at the cost of having unnaturally small $|A|/B < \text{TeV}$.

Away from the resonant regime with $B \gg \Gamma_\pm$, the CP asymmetries, like in the limit (i), go as $\epsilon^S_{\pm} \sim 10^{-1}Y|A|/B$ assuming $O(1)$ contribution from the CP phases of Eq. (4). Hence taking $|A| \sim \text{TeV} \gtrsim B$ together with the out-of-equilibrium decay condition (10) gives us sufficient CP asymmetries $\epsilon^S_{\pm} \gtrsim 10^{-6}$ for $M \gtrsim 10^7$ GeV. Interestingly, the leading CP asymmetries in this regime do not vanish at $T = 0$ indicating that one can obtain a large asymmetry from the nonthermal contribution in soft leptogenesis, even away from resonance. Of course thermal effects are always there but the fact that the CP violation is nonvanishing at $T = 0$ implies that it is less suppressed compared to the case (i).

In Appendix B we will discuss two special cases, namely, (a) $A_\alpha = AY_\alpha$ and (b) $A_\alpha = AY^2/(3Y_\alpha)$ considered in previous work.

### B. CP violation from vertex corrections

In this subsection, we discuss the CP violation from vertex corrections. From diagrams (a), (b) and (c) of Figure 2 we obtain

$$\epsilon^{V,(a)}_{\pm\alpha} = \mp \frac{1}{8\pi G_F(T)} Y^2 \sum_\beta Y_\beta \text{Im}(A_\beta) \frac{M^2}{M} \ln \frac{M^2 + M^2}{M^2 + \Gamma^2 \delta r_B(T) c_F(T)},$$

$$\epsilon^{V,(b)}_{\pm\alpha} = \mp \frac{1}{8\pi G_F(T)} Y^2 \frac{Y_\alpha}{M} \ln \frac{M^2 + M^2}{M^2 + \Gamma^2 \delta r_F(T) c_B(T)},$$

$$\epsilon^{V,(c)}_{\pm\alpha} = \pm \frac{1}{8\pi G_F(T)} \left[ \left( Y^2 - \sum_\beta \frac{|A_\beta|^2}{M^2} \right) Y_\alpha \frac{\text{Im}(A_\alpha)}{M} + \left( Y^2 - \frac{|A_\alpha|^2}{M^2} \right) \sum_\beta Y_\beta \frac{\text{Im}(A_\beta)}{M} \right] \times \frac{M^2}{M^2} \ln \frac{M^2 + M^2}{M^2 + \Gamma^2 \delta r_B(T) c_B(T)}.$$  

---

5 This scenario only makes sense when we consider generic $A_\alpha$ couplings. Otherwise we obtain $|A_\alpha|/M \gg 1$ in conflict with the perturbative bound.
Expanding in $B/M \ll 1$ and summing over the contributions above, we have
\[ \epsilon_{\pm\alpha}^V \equiv \epsilon_{\pm\alpha}^{V(a)} + \epsilon_{\pm\alpha}^{V(b)} + \epsilon_{\pm\alpha}^{V(c)} \]
\[ = \pm \frac{\ln 2}{8\pi G_{\pm}(T)} \left[ Y^2 Y_{\alpha} \frac{\text{Im}(A_{\alpha})}{M} + Y_{\alpha} \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \right] \left[ c_F(T) - c_B(T) \right] r_B(T) \]
\[ - \frac{1}{8\pi G_{\pm}(T)} \left[ Y^2 Y_{\alpha} \frac{\text{Im}(A_{\alpha})}{M} + Y_{\alpha} \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \right] \frac{B}{M} \left[ \frac{c_F(T)}{2} + (\ln 2 - 1)c_B(T) \right] r_B(T) \]
\[ \pm \frac{\ln 2}{8\pi G_{\pm}(T)} \left[ \sum_{\beta} \frac{|A_{\beta}|^2}{M^2} Y_{\alpha} \frac{\text{Im}(A_{\alpha})}{M} + \frac{|A_{\alpha}|^2}{M^2} \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \right] r_B(T) c_B(T) \]
\[ + \frac{1}{8\pi G_{\pm}(T)} \left[ \sum_{\beta} \frac{|A_{\beta}|^2}{M^2} Y_{\alpha} \frac{\text{Im}(A_{\alpha})}{M} + \frac{|A_{\alpha}|^2}{M^2} \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \right] \frac{B}{M} (\ln 2 - 1) r_B(T) c_B(T). \tag{21} \]

Notice that the leading contributions from $\tilde{N}_+$ and $\tilde{N}_-$ come with the opposite sign and hence they will contribute destructively to the lepton number asymmetry.

- In the limit (i) $Y_{\alpha} \gg A_{\alpha}/M$, we have
\[ \epsilon_{\pm\alpha}^V \simeq \pm \frac{\ln 2}{8\pi} \left[ Y_{\alpha} \frac{\text{Im}(A_{\alpha})}{M} + P_{\alpha} \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \right] \frac{c_F(T) - c_B(T)}{c_F(T) + c_B(T)} r_B(T) \]
\[ - \frac{1}{8\pi} \left[ Y_{\alpha} \frac{\text{Im}(A_{\alpha})}{M} + P_{\alpha} \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \right] \frac{B}{M} \left[ \frac{c_F(T)}{2} + (\ln 2 - 1)c_B(T) \right] c_B(T) \] \[ + \frac{1}{8\pi} \left[ Y_{\alpha} \frac{\text{Im}(A_{\alpha})}{M} + P_{\alpha} \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \right] \frac{B}{M} (\ln 2 - 1) c_B(T). \tag{22} \]

- In the other limit (ii) $Y_{\alpha} \ll A_{\alpha}/M$, we have
\[ \epsilon_{\pm\alpha}^V \simeq \pm \frac{\ln 2}{8\pi} \left[ Y_{\alpha} \frac{\text{Im}(A_{\alpha})}{M} + \sum_{\beta} \frac{|A_{\beta}|^2}{|A_{\alpha}|^2} \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \right] r_B(T) c_B(T) \]
\[ + \frac{1}{8\pi} \left[ Y_{\alpha} \frac{\text{Im}(A_{\alpha})}{M} + \sum_{\beta} \frac{|A_{\beta}|^2}{|A_{\alpha}|^2} \sum_{\beta} Y_{\beta} \frac{\text{Im}(A_{\beta})}{M} \right] \frac{B}{M} (\ln 2 - 1) c_B(T). \tag{23} \]

The leading terms in the vertex CP asymmetries in Eqs. (22) and (23) from the contributions of $\tilde{N}_\pm$ have opposite signs and cancel each other. While the next to leading terms do not vanish at zero temperature, they go as $\epsilon^V \sim 10^{-1} Y AB/M^2$ and so are too suppressed to be relevant for leptogenesis. The extra suppressions compared to the mixing CP violation are due to the following two reasons. Firstly, they do not enjoy the enhancement from nearly degenerate mass eigenstates $\tilde{N}_\pm$ with mass splitting $B$ as in the mixing CP asymmetries. Secondly, the leading contributions from $\tilde{N}_\pm$ have opposite signs and contribute destructively resulting in an extra suppression factor $B/M$ in the asymmetry. Although we have argued that the asymmetries are small in the two special limits above, we can see from Eq. (21) that if $Y_{\alpha} \sim \text{Im}(A_{\alpha})/M$ then $\epsilon^V \sim 10^{-1} Y^2$ which is too small for leptogenesis from Eq. (10) for $M \gtrsim 10^7$ GeV.

So far we have been discussing the contributions of soft terms to CP violation in the decay of $\tilde{N}_\pm$. In fact the soft terms also provide new sources of CP violation in the one-loop vertex diagrams for the decays of the heavy neutrino $N$ as shown in Figure 3. Nevertheless the CP violation from these diagrams come at the same order as Eq. (21) and hence are too small for successful leptogenesis.

IV. PHENOMENOLOGICAL CONSTRAINTS

We are primarily concerned with scenarios with $M \gtrsim 10^7$ GeV for which the production of sneutrinos is beyond the energy range of current colliders. However, even if $M_\pm \sim \text{TeV}$, the bound on the Yukawa couplings from the
requirement of out-of-equilibrium decays of $N_\pm$ (in Eq. (10)) makes $N_\pm$ impossible to be produced at colliders [25]. However, the soft SUSY breaking parameters relevant for soft leptogenesis $A_\alpha$, $B$ and $\tilde{M}$ can contribute to Electric Dipole Moments (EDM) of leptons and to Charged Lepton Flavor Violating (CLFV) interactions though the analysis of Ref. [20] under the assumption of universality soft trilinear couplings $A_\alpha = AY_\alpha$ showed that the contributions to EDM and CLFV are much below the experimental bounds. Here we will repeat the analysis of Ref. [20] considering a generic $A_\alpha$. In Ref. [20], the phenomenological consequences of the soft terms considering three generations of RHN chiral superfields have been discussed at length. Clearly these soft parameters are connected with the mechanism of SUSY breaking and as such are model dependent. Here we will remain agnostic about the SUSY breaking mechanism and simply focus on the phenomenological constraints on these parameters and in particular, we will focus only on parameters related to $N_1$ which are relevant for soft leptogenesis i.e. $B$, $\tilde{M}$, $A_\alpha$, $Y_\alpha$ and $M$. Without fine-tuning we consider the soft parameters $B$, $\tilde{M}$ and $A_\alpha$ to be similar or smaller than $m_{\text{SUSY}} \sim \text{TeV}$. On the other hand, the parameters $Y_\alpha$ and $M$ are subject only to the out-of-equilibrium $N_\pm$ decay constraint in Eq. (10) and less stringently to the cosmological bound on the sum of neutrino masses in Eq. (11). The running of $Y_\alpha$ from the high scale down to the weak scale gives some corrections at the level of $10 - 20\%$ (see Figure 3 of Ref. [28]) which we will ignore in the following.

1. Electric dipole moment of the electron

Assuming $\mathcal{O}(1)$ contribution of the phases and mixing angles in the chargino sectors, the contributions of $A_\alpha$ and $B$ to the EDM of the electron is given by [20]

$$|d_e| \approx \frac{e m_e \tan \beta}{16 \pi m^2_e} \left| \frac{m_\chi Y_\alpha}{M^2} \right| (|A_\alpha| + BY_\alpha),$$  

(24)

where $m_e$ is the electron mass, $m^2_\nu$ is the squared mass of the light sneutrino and $m_\chi$ the mass of chargino. For generic $A_\alpha$, it dominates in Eq. (24). Taking $m_e = m_\chi = m_{\text{SUSY}}$ and making use of Eq. (10), we have

$$|d_e| \lesssim 5 \times 10^{-38} \left( \frac{\tan \beta}{10} \right) \left( \frac{10^7 \text{GeV}}{M} \right)^{3/2} \left( \frac{1 \text{TeV}}{m_{\text{SUSY}}} \right) e \text{ cm},$$  

(25)

which is much stronger than the current experimental bound $|d_e|_{\exp} < 8.7 \times 10^{-29} e \text{ cm}$ [27]. The contributions to $\mu$ and $\tau$ EDM can be estimated by replacing $m_e$ in Eq. (24) by $m_\mu$ and $m_\tau$ respectively but the current experimental constraints on them are still a lot weaker: $|d_\mu|_{\exp} < 1.9 \times 10^{-19} e \text{ cm}$ [29] and $|d_\tau|_{\exp} < 5.1 \times 10^{-17} e \text{ cm}$ [30].

2. Charged lepton flavor violating interactions

The branching ratio for charged lepton flavor violations due to nonvanishing off-diagonal elements of the soft mass matrix of the doublet sleptons $m_\ell^2$ is given by [20, 31]

$$\text{BR}(\ell_\alpha \to \ell_\beta \gamma) \approx \frac{\alpha^3}{G_F^2} \frac{|(m_\ell^2)_{\alpha\beta}|^2}{m_{\text{SUSY}}^8} \tan^2 \beta,$$

(26)

where $\alpha$ is the fine structure constant. In general, the off-diagonal elements of $m_\ell^2$ will induce too large CLFV rates. The usual solution is to assume mSUGRA boundary conditions at the grand unified theory (GUT) scale where the
off-diagonal elements of $m_{\tilde{q}}$ vanish. In this case, as $m_{\tilde{q}}$ evolves from the GUT scale $M_{\text{GUT}}$ to the RHN mass scale $M$, the off-diagonal elements will be generated due to the renormalization effects as

$$\langle m_{\tilde{q}}^2 \rangle_{\alpha\beta} \approx -\frac{1}{8\pi^2} A_\alpha A_\beta \ln \left( \frac{M_{\text{GUT}}}{M} \right)$$

(27)

for $\alpha \neq \beta$, and we have kept only the dominant contributions from $A_\alpha$.

The most stringent constraint on the rare decay $\mu \to e\gamma$ comes from the nonobservation of the process from the MEG experiment [33, 34] which has set the new bound on the branching ratio for $\mu \to e\gamma$,

$$\text{BR}(\mu \to e\gamma)_{\text{exp}} < 5.7 \times 10^{-13}.$$  

(28)

Substituting Eq. (27) in Eq. (26) and applying the constraint Eq. (28), we obtain

$$|A_\mu^* A_e| \lesssim 5 \times 10^3 \text{GeV}^2 \left( \frac{m_{\text{SUSY}}}{1 \text{TeV}} \right)^4 \left( \frac{10}{\tan \beta} \right),$$

(29)

where we have taken $M_{\text{GUT}} = 10^{16}$ GeV and $M = 10^7$ GeV. Similarly using the experimental bounds on CLFV in $\tau$ decays, $\text{BR}(\tau \to e\gamma)_{\text{exp}} < 3.3 \times 10^{-8}$ and $\text{BR}(\tau \to \mu\gamma)_{\text{exp}} < 4.4 \times 10^{-8}$ [35], we obtain

$$|A_\tau^* A_e| \approx |A_\tau^* A_\mu| \lesssim 1 \times 10^6 \text{GeV}^2 \left( \frac{m_{\text{SUSY}}}{1 \text{TeV}} \right)^4 \left( \frac{10}{\tan \beta} \right).$$

(30)

For $m_{\text{SUSY}}$ at the TeV scale, the bound [30] can be satisfied with $A_\alpha$ also at the TeV scale while the stronger bound (29) requires either $A_\tau$ and/or $A_\mu$ to be smaller than TeV scale. As discussed in Section VIB, the mixing CP asymmetries away from the resonant regime go as $\epsilon_{\alpha}^S \sim 10^{-1} Y_{\alpha} |A_{\alpha}| / B$ and can be large enough with $M \gtrsim 10^7$ GeV and having one of the $A_\alpha \sim \text{TeV} \gtrsim B$.

In addition, the off-diagonal entries of the slepton mass matrix can also give rise to other CLFV processes like $\mu \to 3e$ and $\mu \to e$ conversion. If such processes are dominated by the dipole type operator for relatively large $\tan \beta$, $\text{BR}(\mu \to 3e)$ and the rate of $\mu \to e$ conversion rate $R_{\mu e}$ are proportional to $\text{BR}(\mu \to e\gamma)$ and are approximately given by

$$\text{BR}(\mu \to 3e) \sim 6.6 \times 10^{-3} \text{BR}(\mu \to e\gamma),$$

(31)

and, for the $^{27}_{13}\text{Al}$ nucleus, by

$$R_{\mu e} \sim 2.5 \times 10^{-3} \text{BR}(\mu \to e\gamma).$$

(32)

The present constraints coming from these CLFV processes are less severe than those coming from $\mu \to e\gamma$. However, in future experiments the sensitivity for such processes may improve which could constrain the presently allowed parameter space or lead to a detection of such lepton flavor violating processes. As for example, the future Mu3e experiment [38] could reach a sensitivity of $\sim 10^{-15} - 10^{-16}$ for $\text{BR}(\mu \to 3e)$. For the $\mu \to e$ conversion process, from Mu2e [39] and COMET [40] experiments the bound could reach the level of $R_{\mu e} \sim 10^{-17}$ for the $^{27}_{13}\text{Al}$ nucleus while the PRISM/PRIME [41] project may have two orders of magnitude greater sensitivity.

V. CONCLUSIONS

In the framework of local SUSY, soft leptogenesis is an attractive mechanism to explain the cosmological matter-antimatter asymmetry since it works at lower temperature regime $T \lesssim 10^9$ GeV where the conflict with the overproduction of gravitinos can be relaxed or even evaded. We showed that by considering generic soft trilinear $A_\alpha$ couplings there are two interesting consequences: 1) one can realize nonthermal CP violation where the CP asymmetries in the decays of heavy sneutrinos to lepton and sleptons do not cancel at zero temperature resulting in an enhanced efficiency in generating baryon asymmetry 2) the dominant CP violation from self-energy corrections is sufficient even far away from the resonant regime and the relevant soft parameters can assume natural values at around the TeV scale. For successful soft leptogenesis we considered two requirements: the out-of-equilibrium decays of heavy sneutrinos and a large enough CP violation. Assuming $m_{\text{SUSY}} \sim \text{TeV}$, we found the following conditions $A_\alpha \sim \text{TeV} \gtrsim B$ and $M \gtrsim 10^7$ GeV as sufficient for successful leptogenesis. In addition we also found that while the contributions to the EDM of charged leptons are negligibly small, the contributions to CLFV processes are close to the sensitivities of present and future experiments.
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Appendix A: Thermal corrections

The thermal averaged reaction density is defined as

\[ \gamma^{ab\ldots \rightarrow ij\ldots} \equiv \Lambda^{ij\ldots}_{ab\ldots} |M^{ab\ldots \rightarrow ij\ldots}|^2 f^a f^b \ldots (1 + \eta_i f^i) (1 + \eta_j f^j) \ldots, \tag{A1} \]

where \( M^{ab\ldots \rightarrow ij\ldots} \) is the amplitude for the process \( a b\ldots \rightarrow i j\ldots \) at finite temperature, \( f^i = (e^{E_i/T} - \eta_i)^{-1} \) with \( \eta_i = \pm \) for \( i \) representing boson or fermion respectively, and

\[ \Lambda^{ij\ldots}_{ab\ldots} \equiv \int d\Pi_a d\Pi_b \ldots d\Pi_i d\Pi_j \ldots (2\pi)^4 \delta^{(4)} (p_a + p_b + \ldots - p_i - p_j - \ldots), \]

\[ d\Pi_i = \frac{d^3 p_i}{(2\pi)^3 2 E_i}, \tag{A2} \]

The CP asymmetry for the decay \( a \rightarrow ij \) which arises from the interferences between tree level and one loop diagrams as shown in Figure 4 is defined as

\[ \epsilon \equiv \frac{\gamma (a \rightarrow ij) - \gamma (a \rightarrow \overline{ij})}{\sum_{k,l} \left[ \gamma (a \rightarrow kl) + \gamma (a \rightarrow \overline{kl}) \right]} = \frac{\int d\Pi_a f^a \int d\Phi_{ij} (1 + \eta_i f^i) (1 + \eta_j f^j) \left[ |M^{a \rightarrow ij}|^2 - |M^{a \rightarrow \overline{ij}}|^2 \right]}{\sum_{k,l} \int d\Pi_a f^a \int d\Phi_{kl} (1 + \eta_k f^k) (1 + \eta_l f^l) \left[ |M^{a \rightarrow kl}|^2 + |M^{a \rightarrow \overline{kl}}|^2 \right]}, \tag{A3} \]

where the two-body phase space integral is

\[ \int d\Phi_{ij} = \int d\Pi_i d\Pi_j (2\pi)^4 \delta^{(4)} (p_a - p_i - p_j). \tag{A4} \]

Ignoring the thermal motion of a with respect to the thermal bath, i.e. setting \( E_a = M_a \), we can drop the integral \( \int d\Pi_a \) in both the numerator and denominator. In this case the phase space integral can be carried out analytically. In order to obtain the thermal factors associated with the intermediate on-shell states, one necessarily needs to calculate the amplitudes in Eq. (A3) using thermal field theory. In the real time formalism of thermal field theory, one needs to double the number of degrees of freedom (introducing type 1 and type 2 fields) resulting in a \( 2 \times 2 \) matrix structure for the thermal propagator (see e.g. [28]). However at one loop, we can take all the vertices connected to external legs to be of type 1 and hence we only need to consider the (11) element of the thermal propagator. The (11) component for the boson propagator is [28]

\[ D^{11}_B = \frac{i}{p^2 - m_B(T)^2 + i\epsilon} + 2\pi f_B(|p_0|) \delta(p^2 - m_B(T)^2), \tag{A5} \]

where \( m_B(T) \) is the boson thermal mass, and the cut propagators are

\[ D^{\pm}_B = 2\pi \left[ \theta(\pm p_0) + f_B(|p_0|) \right] \delta(p^2 - m_B(T)^2). \tag{A6} \]

For fermions, the structure of the propagator is more involved [28]. For simplicity, we approximate the (11) fermion propagator by

\[ D^{11}_F = \frac{i}{p^2 - m_F(T)^2 + i\epsilon} - 2\pi f_F(|p_0|) \delta(p^2 - m_F(T)^2), \tag{A7} \]
where \( m_F(T) \) is the fermion thermal mass and the cut propagators are

\[
D_F^\pm = 2\pi \hat{p} \left[ \theta(\pm p_0) - f_F(|p_0|) \right] \delta(p^2 - m_F(T)^2). \tag{A8}
\]

In Eqs. (A7) and (A8), the propagators \( \sim \hat{p} \) without a mass term as the bare fermion mass is zero and the thermal mass does not have chiral properties. Also, as implicit in the propagators, we have considered the dispersion relation as that of a free particle with a thermal mass, instead of the actual dispersion relation including thermal corrections. Although this is an underestimate of the actual dispersion relation, the error is within 10\% [41]. The above also implies that in Eq. (A7) we have ignored the fact that due to the interactions with the thermal bath the two poles of the fermion propagator have different dispersion relations which can lead to an order of magnitude correction to leptogenesis in the weak washout regime and an order of one correction in the strong washout regime [42, 44].

Keeping the above caveats in mind and applying finite temperature “cutting rules” (more discussion below), we obtain

\[
\epsilon \approx \frac{\sum_{i',j'} \left[ |\mathcal{M}^0(a \rightarrow ij)|^2 - |\mathcal{M}^0(a \rightarrow ij')|^2 \right] r_{aij'}(T) c_{aij}(T)}{\sum_{k,l} \left[ |\mathcal{M}^0(a \rightarrow kl)|^2 + |\mathcal{M}^0(a \rightarrow kl')|^2 \right] c_{kl}(T)}, \tag{A9}
\]

where \( \mathcal{M}^0(a \rightarrow ij) \) is the amplitude for \( a \rightarrow ij \) at zero temperature and the sum over \( i'j' \) in the numerator is over intermediate states in the loop which go on shell as shown as the ‘cuts’ in Figure 4. In Eq. (A9) \( r_{aij}(T) \) are the thermal factors associated with the on shell intermediate states while \( c_{aij}(T) \) are those associated with the final states. In the case of self-energy contributions, the factorized form as a product of thermal dependent and zero temperature terms as in Eq. (A9) always holds (under the approximation that \( a \) is at rest with respect to the thermal bath) while in the case of vertex diagrams, further approximations are required. One approximation we have made is to factorize out the temperature dependent terms including the kinematic factors, and then to set the thermal masses in the rest of the terms to zero which gives us expressions for these terms that coincide with the zero temperature results. In addition, we ignore the contributions from the cuts through \( a' \) and \( i' \) or \( j' \) in the vertex diagrams, which as shown in Ref. [45] in non-SUSY type-I leptogenesis can give corrections depending on the \( a-a' \) mass ratio, for example, at the level of 10\% for \( m_{a'}/m_a = 1.1 \). Under these approximations, the temperature dependent terms for both self-energy and vertex diagrams are the same and are given by

\[
c_{aij}(T) = [1 + \eta_a (1 - \delta_{ai}\delta_{bj}) (\eta_i x_i + \eta_j x_j)] \lambda(1, x_i, x_j) (1 + \eta_i f^{eq}_i) (1 + \eta_j f^{eq}_j), \tag{A10}
\]

\[
r_{aij}(T) = [1 + \eta_a (1 - \delta_{ai}\delta_{bj}) (\eta_i x_i + \eta_j x_j)] \lambda(1, x_i, x_j) (1 + \eta_i f^{eq}_i + \eta_j f^{eq}_j), \tag{A11}
\]

with \( \delta_{bi} = 1 \) (0) if \( i \) is a boson (fermion) and

\[
\lambda(1, x, y) = \sqrt{1 + x - y)^2 - 4x}, \quad x_i = \frac{m_i(T)^2}{M_i^2}, \quad E_i = \frac{M_i}{2} (1 + x_i - x_j), \quad E_j = M_a - E_i = \frac{M_a}{2} (1 - x_i + x_j). \tag{A12}
\]

For the statistical factors in \( r_{aij}(T) \), we applied the finite temperature “cutting rules” by considering causal (i.e. retarded or advanced) n-point functions as pointed out by Ref. [46] which gives the dependence on the distribution functions \( 1 + \eta_i f^{eq}_i + \eta_j f^{eq}_j \) in agreement with the results derived from nonequilibrium quantum field theory [45, 47, 58] in contrast to the results of Refs. [28, 60] which obtained \( 1 + \eta_i f^{eq}_i + \eta_j f^{eq}_j + \eta_i \eta_j f^{eq}_i f^{eq}_j \) when time-ordered n-point functions are considered instead. Notice that the imaginary time formalism also gives the statistical factors in agreement with the result of nonequilibrium quantum field theory [42, 44].

Figure 4: One loop diagrams for decay \( a \rightarrow ij \).
Now we can apply the general results \([A10]\) and \([A11]\) to soft leptogenesis. For the decays \(\tilde{N}_\pm \to \ell_n \tilde{H}_u\) and \(\tilde{N}_\pm \to \ell H_u\), the relevant thermal factors are obtained from Eqs. \([A10]\) and \([A11]\) to be

\[
c_F(T) = \left(1 - x_\ell - x_{\tilde{H}_u}\right) \lambda \left(1, x_\ell, x_{\tilde{H}_u}\right) \left(1 - f^\text{eq}_\ell \right) \left(1 - f^\text{eq}_{\tilde{H}_u}\right),
\]

\[
c_B(T) = \lambda \left(1, x_\tilde{g}, x_{\tilde{H}_u}\right) \left(1 + f^\text{eq}_\ell \right) \left(1 + f^\text{eq}_{\tilde{H}_u}\right),
\]

\[
r_F(T) = \left(1 - x_\ell - x_{\tilde{H}_u}\right) \lambda \left(1, x_\ell, x_{\tilde{H}_u}\right) \left(1 - f^\text{eq}_\ell + f^\text{eq}_{\tilde{H}_u}\right),
\]

\[
r_B(T) = \lambda \left(1, x_\tilde{g}, x_{\tilde{H}_u}\right) \left(1 + f^\text{eq}_\ell + f^\text{eq}_{\tilde{H}_u}\right).
\]

The relevant thermal masses are \([28]\)

\[
m_{\tilde{t}}(T)^2 = 2m_{\tilde{t}}(T)^2 = \left(\frac{3}{8} g^2 + \frac{1}{8} g_Y^2 \right) T^2,
\]

\[
m_{H_u}(T)^2 = 2m_{H_u}(T)^2 = \left(\frac{3}{8} g^2 + \frac{1}{8} g_Y^2 + \frac{3}{4} \lambda^2 \right) T^2.
\]

Next we prove a useful identity (in the context of the approximations made above) as follows

\[
r_F(T)c_B(T) - r_B(T)c_F(T) \propto \left(1 - f^\text{eq}_\ell - f^\text{eq}_{\tilde{H}_u}\right) \left(1 + f^\text{eq}_\ell \right) \left(1 + f^\text{eq}_{\tilde{H}_u}\right) \left(1 - f^\text{eq}_\ell \right) \left(1 - f^\text{eq}_{\tilde{H}_u}\right)
\]

\[
= \left(e^{E_{\tilde{g}}/T} - 1ight) f^\text{eq}_\tilde{g} f^\text{eq}_{H_u} e^{E_{\tilde{g}}/T} f^\text{eq}_{H_u} - \left(e^{E_{\tilde{g}}/T} - 1\right) f^\text{eq}_\tilde{g} J_{H_u} e^{E_{\tilde{g}}/T} f^\text{eq}_{H_u} = 0.
\]

In the second line above, we have made use of the following identity

\[
(1 + \eta_i f^\text{eq}_i) \left(1 + \eta_j f^\text{eq}_j\right) = e^{(E_{\tilde{g}} + E_j)/T} f^\text{eq}_i f^\text{eq}_j,
\]

and the conservation of energy \(E_{\tilde{N}} = E_\ell + E_{\tilde{H}_u} = E_\ell + E_{H_u}\). Notice that this identity also holds if instead of using the factor \(1 + \eta_i f^\text{eq}_i + \eta_j f^\text{eq}_j\) in \(r_{a\ell}(T)\), one uses \(1 + \eta_i f^\text{eq}_i + \eta_j f^\text{eq}_j + \eta_i \eta_j f^\text{eq}_i f^\text{eq}_j\) as obtained in Refs. \([28, 59]\).

Finally it can be shown that the CP asymmetries from gaugino contributions \([15, 16]\) for the decays of \(\tilde{N}_\pm\) to scalars and fermions are respectively given by \(\epsilon_g r_F(T)c_B(T)\) and \(-\epsilon_g r_B(T)c_F(T)\) where \(\epsilon_g\) is some temperature independent term. Using the identity \([A15]\), these contributions sum up to zero eliminating the cancellations pointed out by Ref. \([18]\).

**Appendix B: Special cases of mixing CP asymmetries**

Here we will discuss two specific cases of mixing CP asymmetries.

(a) Universal Trilinear Scenario (UTS): \(A_\alpha = A Y_\alpha\).

This is the scenario considered in Refs. \([11, 12, 21]\). In this scenario, we are always in the regime (i) \(Y_\alpha \gg |A_\alpha|/M\) and from Eq. \([18]\), we obtain \([21]\)

\[
\epsilon_s \simeq P_\alpha \frac{\text{Im}(A)}{M} \frac{4B Y_\alpha}{4B^2 + Y_\alpha^2} f^\text{eq}_\ell c_B(T) - c_B(T),
\]

\[
\text{B1}
\]

(b) Simplified Misaligned Scenario (SMS): \(A_\alpha = A Y^2/(3Y_\alpha)\).

This is a specific scenario considered in Ref. \([14]\). In this scenario, we have from Eq. \([14]\)

\[
G_{\pm}(T) \simeq Y^2 |c_F(T) + c_B(T)| + Y^2 \frac{|A|^2}{M^2} d c_B(T),
\]

\[
\text{B2}
\]

where \(d \equiv \sum_\alpha 1/(9P_\alpha) \geq 1\) and the minimum occurs at \(P_\alpha = 1/3\) for all \(\alpha\). Hence the second term in Eq. \([B2]\) above will dominate only if \(|A|^2/M^2 \gg d^{-1}\). This condition can only be fulfilled if \(P_\alpha\) deviates significantly from 1/3, i.e. a very hierarchical \(P_\alpha\). For a nonhierarchical \(P_\alpha \sim P_\mu \sim P_\tau\), we fall in the regime (i) \(Y_\alpha \gg A_\alpha/M\) and we obtain Eq. \([B1]\). On the other hand, for a very hierarchical \(P_\alpha\), some of the \(\epsilon_s\) with \(P_\alpha \sim O(1)\) will fail
in the regime (i) $Y_\alpha \gg A_\alpha/M$ while those with $P_\alpha \ll 1$ will fall into the other regime (ii) $Y_\alpha \ll A_\alpha/M$. Hence, using Eqs. (18) and (19), in the very hierarchical case, for those $\alpha$ with $P_\alpha \sim O(1)$ we have

$$\epsilon_{\pm \alpha}^S \simeq P_\alpha \frac{M^2}{d |A|^2} \frac{\text{Im}(A)}{M} \frac{4B_G}{4B^2 + \Gamma_Y^2 \frac{|A|^2}{2M^2} d} \frac{c_F(T) - c_B(T)}{c_B(T)} r_B(T),$$  

(B3)

while for those with $P_\alpha \ll 1$ we have

$$\epsilon_{\pm \alpha}^S \simeq \frac{1}{9P_\alpha} \frac{1}{d} \frac{\text{Im}(A)}{M} \frac{4B_G}{4B^2 + \Gamma_Y^2 \frac{|A|^2}{2M^2} d} r_B(T).$$  

(B4)

Hence in the case of SMS the CP asymmetries are nonvanishing in the limit of zero temperature for a very hierarchical $P_\alpha$ when $|A|^2/M^2 \gg d^{-1}$. This result was missed by Ref. [14] which did not consider the possibility of $|A|^2/M^2 \gg d^{-1}$ for very hierarchical $P_\alpha$.

[1] M. Fukugita and T. Yanagida, Phys. Lett. B 174, 45 (1986).
[2] B. A. Campbell, S. Davidson and K. A. Olive, Nucl. Phys. B 399, 111 (1993) [hep-ph/9302223].
[3] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384, 168 (1996) [hep-ph/9605319].
[4] M. Plumacher, Nucl. Phys. B 530, 207 (1998) [hep-ph/9704231].
[5] C. S. Fong, M. C. Gonzalez-Garcia, E. Nardi and J. Racker, JCAP 1012, 013 (2010) arXiv:1009.0003 [hep-ph].
[6] S. Davidson and A. Ibarra, Phys. Lett. B 535, 25 (2002) [hep-ph/0202239].
[7] M. Kawasaki, K. Kohri, T. Moroi and A. Yotsuyanagi, Phys. Rev. D 78, 065011 (2008) arXiv:0804.3745 [hep-ph].
[8] R. Allahverdi and A. Mazumdar, hep-ph/0505050.
[9] R. Allahverdi and A. Mazumdar, JCAP 0610, 008 (2006) [hep-ph/0512227].
[10] R. Rangarajan and A. Sarkar, Astropart. Phys. 48, 37 (2013) arXiv:1205.5408 [astro-ph.CO].
[11] G. D’Ambrosio, G. F. Giudice and M. Raidal, Phys. Rev. Lett. 91, 251801 (2003) [hep-ph/0307081].
[12] Y. Grossman, T. Kashti, Y. Nir and E. Roulet, Phys. Rev. Lett. 100, 001 (2010) arXiv:0904.5125 [hep-ph].
[13] C. S. Fong, M. C. Gonzalez-Garcia, E. Nardi, J. Racker, JHEP 1007, 001 (2010) arXiv:0912.1597 [hep-ph].
[14] Y. Grossman, T. Kashti, Y. Nir and E. Roulet, JHEP 0411, 080 (2004) [hep-ph/0407063].
[15] C. S. Fong and M. C. Gonzalez-Garcia, JHEP 0903, 073 (2009) arXiv:0901.0008 [hep-ph].
[16] R. Adhikari and R. Rangarajan, Phys. Rev. D 65, 083504 (2002) [hep-ph/0110387].
[17] B. Garbrecht and M. J. Ramsey-Musolf, Nucl. Phys. B 882, 145 (2014) arXiv:1307.0524 [hep-ph].
[18] C. S. Fong, M. C. Gonzalez-Garcia, E. Nardi, JHEP 1102, 032 (2011) arXiv:1012.1597 [hep-ph].
[19] T. Kashti, Phys. Rev. D 71, 013008 (2005) [hep-ph/0410319].
[20] C. S. Fong and M. C. Gonzalez-Garcia, JHEP 0806, 076 (2008) arXiv:0804.4471 [hep-ph].
[21] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. (2014) arXiv:1303.5076 [astro-ph.CO].
[22] A. Pilaftsis, Phys. Rev. D 56, 5431 (1997) [hep-ph/9707235].
[23] A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B 692, 303 (2004) [hep-ph/0309342].
[24] G. Gambhiani, S. Khan, P. Konar and T. Mondal, arXiv:1411.6866 [hep-ph].
[25] A. Dedes, H. E. Haber and J. Rosiek, JHEP 0711, 059 (2007) arXiv:0707.3718 [hep-ph].
[26] J. Baro et al. [ACME Collaboration], Science 343, no. 6168, 269 (2014) arXiv:1310.7534 [physics.atom-ph].
[27] G. F. Giudice, A. Notari, M. Raidal, A. Rjotto and A. Strumia, Nucl. Phys. B 685, 89 (2004) [hep-ph/0310123].
[28] G. W. Bennett et al. [Muon (g-2) Collaboration], Phys. Rev. D 80, 052008 (2009) arXiv:0811.1207 [hep-ex].
[29] K. Inami et al. [Belle Collaboration], Phys. Lett. B 551, 16 (2003) [hep-ex/0210066].
[30] M. Hirsch, F. R. Joaquim and A. Vicente, JHEP 1211, 105 (2012) arXiv:1207.6635 [hep-ph].
[31] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D 53, 2442 (1996) [hep-ph/9510309].
[32] J. Adam et al. [MEG Collaboration], Phys. Rev. Lett. 107, 171801 (2011) arXiv:1107.5547 [hep-ex].
[33] J. Adam et al. [MEG Collaboration], Phys. Rev. Lett. 110, 201801 (2013) arXiv:1303.0754 [hep-ex].
[34] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 104, 021802 (2010) arXiv:0908.2381 [hep-ex].
[35] J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Rev. D 66, 115013 (2002) [hep-ph/0206110].
[36] R. Kitano, M. Koike and Y. Okada, Phys. Rev. D 66, 096002 (2002) [Erratum-ibid. D 76, 059902 (2007)] [hep-ph/0203110].
[37] A. Blondel, A. Bravar, M. Pohl, S. Bachmann, N. Berger, M. Kiehn, A. Schoning and D. Wiedner et al., arXiv:1301.6113 [physics.ins-det].
[38] R. J. Abrams et al. [Mu2e Collaboration], arXiv:1211.7019 [physics.ins-det].
[39] Y. Kuno, Nucl. Phys. Proc. Suppl. 225-227, 228 (2012).
[43] C. Kiessig and M. Plumacher, JCAP 1207, 014 (2012) [arXiv:1111.1231 [hep-ph]].
[44] C. Kiessig and M. Plumacher, JCAP 1209, 012 (2012) [arXiv:1111.1235 [hep-ph]].
[45] B. Garbrecht, Nucl. Phys. B 847, 350 (2011) [arXiv:1011.3122 [hep-ph]].
[46] M. Garny, A. Hohenegger and A. Kartavtsev, Phys. Rev. D 81, 085028 (2010) [arXiv:1002.0331 [hep-ph]].
[47] W. Buchmuller and S. Fredenhagen, Phys. Lett. B 483, 217 (2000) [hep-ph/0004145].
[48] A. De Simone and A. Riotto, JCAP 0708, 002 (2007) [hep-ph/0703175].
[49] M. Garny, A. Hohenegger, A. Kartavtsev and M. Lindner, Phys. Rev. D 80, 125027 (2009) [arXiv:0909.1559 [hep-ph]].
[50] M. Garny, A. Hohenegger, A. Kartavtsev and M. Lindner, Phys. Rev. D 81, 085027 (2010) [arXiv:0911.4122 [hep-ph]].
[51] M. Garny, A. Hohenegger and A. Kartavtsev, arXiv:1005.5385 [hep-ph].
[52] M. Garny, A. Kartavtsev and A. Hohenegger, Annals Phys. 328, 26 (2013) [arXiv:1112.6428 [hep-ph]].
[53] T. Frossard, M. Garny, A. Hohenegger, A. Kartavtsev and D. Mitrouskas, Phys. Rev. D 87, 085009 (2013) [arXiv:1211.2140 [hep-ph]].
[54] A. Anisimov, W. Buchmuller, M. Drewes and S. Mendizabal, Phys. Rev. Lett. 104, 121102 (2010) [arXiv:1001.3856 [hep-ph]].
[55] A. Anisimov, W. Buchmuller, M. Drewes and S. Mendizabal, Annals Phys. 326, 1998 (2011) [Erratum-ibid. 338, 376 (2011)] [arXiv:1012.5821 [hep-ph]].
[56] M. Beneke, B. Garbrecht, M. Herranen and P. Schwaller, Nucl. Phys. B 838, 1 (2010) [arXiv:1002.1326 [hep-ph]].
[57] M. Beneke, B. Garbrecht, C. Fidler, M. Herranen and P. Schwaller, Nucl. Phys. B 843, 177 (2011) [arXiv:1007.4783 [hep-ph]].
[58] B. Garbrecht and M. Herranen, Nucl. Phys. B 861, 17 (2012) [arXiv:1112.5954 [hep-ph]].
[59] L. Covi, N. Rius, E. Roulet and F. Vissani, Phys. Rev. D 57, 93 (1998) [hep-ph/9704366].