Investigations on the flavor-dependent axial charges of the octet baryons

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We have investigated the axial charges of the ground octet baryons within the extended chiral constituent quark model, where all the possible compact five-quark Fock components $qqq(\bar{q})q(q = u, d, s)$ in the baryons are considered. The transition couplings between the three- and five-quark components in the baryons are assumed to be via the $^3P_0$ mechanism, which could reproduce the sea asymmetry in proton very well. The numerical results for the flavor-dependent axial charges of the octet baryons are comparable to those predicted by other theoretical approaches. It is shown that the singlet axial charges of the octet baryons, which should indicate total baryons spin arising from the spin of the quarks, fall in the range $0.45 - 0.75$ in present model. This is in consistent with the predictions by lattice QCD and chiral perturbation theory. It’s also very interesting that the light quarks spin $\Delta u$ and $\Delta d$ in the $\Lambda$ baryon are of small but negative values, which exactly vanish in the traditional three-quark model.

I. INTRODUCTION

The flavor-dependent axial charges of baryons, i.e. the singlet axial charge $g_A^{(0)}$, the isovector axial charge $g_A^{(3)}$, and the $SU(3)$ octet axial charge $g_A^{(8)}$, are fundamental observables in hadronic physics, since they may provide us information about the spin structure and properties of baryons. As we know, $g_A^{(0)}$ should indicate the total baryons spin arising from the spin of quarks, and $g_A^{(3)}$ and $g_A^{(8)}$ should govern the neutron and hyperons $\beta$ decays, and provide a quantitative measure of spontaneous chiral symmetry breaking in low energy hadronic physics. In addition, Goldberger-Treiman relation shows that the isovector axial charge of nucleon could be directly related to the pion decay constant $f_\pi$. Therefore, one may establish a connection between the weak and strong interactions by explicit investigations on the flavor-dependent axial charges.

During the past years from 1990s, intensively experimental measurements on the axial charges of nucleon have been performed, triggered by the renowned EMC experiments, which raised up the proton spin puzzle [2, 3]. Especially, the COMPASS collaboration has made great efforts on corresponding measurements [4–7] at $Q^2 = 3$ GeV$^2$, and their latest results show that the singlet axial charge of the proton is $g_A^{(0)} = 0.32 \pm 0.02_{\text{stat}} \pm 0.04_{\text{syst}} \pm 0.05_{\text{evol}}$, if the $SU(3)$ flavour symmetry is assumed, which should yield $g_A^{(8)} = 0.585\pm0.025$ [7]. And very recently, the first moment $g_1$ of nucleon at small $Q^2$ is also measured [8, 9]. On the other hand, tremendous theoretical investigations on the nucleon spin structure have also been done using different approaches, such as the lattice QCD [10–16], the chiral perturbation theory [17–24], the meson cloud models [21–22], and other phenomenological models with higher Fock components in nucleon [25–27]. For recent reviews about the nucleon spin structure, see Refs. [28, 31].

For the axial charges of the hyperons, experimental data is still lack up to now. While the lattice QCD has demonstrated remarkable progress in computing the hyperons axial transition couplings during the last two decades [32–41], and the corresponding calculations are with very high accuracy. Besides, there have been also other theoretical efforts dedicated to investigations on the hyperons axial couplings using various approaches, including the chiral perturbation theory and phenomenological hadronic models [42–49].

Recently, the extended chiral constituent quark model (E$\chi$CQM), within which higher Fock components in the baryons are included, has been applied to the sea content of the octet baryons [50], the sea flavor asymmetry of the proton $\bar{d} - \bar{u} = 0.118 \pm 0.012$ [51, 52] could be very well reproduced by taking the model parameters to be the empirical values. Later, we have applied the work [50] to the baryon sigma terms [53–54], the intrinsic light and strange quark-antiquark pair in the proton and nonperturbative strangeness suppression [55], and the orbital angular momentum of the proton [56]. And in a very recent work, the axial charges of the proton has been investigated employing the E$\chi$CQM [57]. The numerical results obtained in all the above referred works are in good agreements with predictions by other theoretical approaches. Consequently, we extend the work [57] to estimations on the hyperons axial charges in present
work.

The present paper is organized as follows. In Sec. II we give the framework which includes the extended chiral constituent quark model and the formalism for the octet baryons axial charges in corresponding model, the explicit numerical results are presented in Sec. III. Finally, a brief summary is given in Sec. IV.

II. FRAMEWORK

In this work, we investigate the axial charges of the ground state octet baryons employing the EχCQM, which has been developed to investigate the intrinsic sea content of the octet baryons in [50]. Accordingly, in this section, we will briefly introduce the EχCQM in Sec. II A and present the formalism for calculations of octet baryons’ axial charges in Sec. II B.

A. EχCQM

In the EχCQM developed in [50], the wave functions of the ground state octet baryons can be expressed as:

$$|B\rangle = \frac{1}{\sqrt{N}} \left( |qqq\rangle + \sum_i C_i |qqq(qq), i\rangle \right),$$

(1)

where the first term represents the wave function for the three-quark component of the octet baryons, and the second term denotes the wave functions for the compact five-quark components, with the sum over $i$ runs over all the possible five-quark configurations with a $qq$ ($dd, uu, ss, ...$)\(^1\) pair which may form higher Fock components in the octet baryons. $C_i/\sqrt{N}$ are just the corresponding probability amplitudes for the five-quark components with $N$ being a normalization constant.

In the present case, we consider the ground states of baryon octet, whose parities are positive, so that the orbital quantum number $L$ must be an odd number 1, 3, ..., $2n+1$. On the other hand, the total spin $S$ of a five-quark system arising from the quark spin can only be $\frac{1}{2}$, $\frac{3}{2}$, or $\frac{7}{2}$; therefore, $L$ cannot be higher than 3 to combine with $S$ to form spin $J_B = L \oplus S$ for the ground state octet baryons. As discussed explicitly in [50], only the five-quark configurations with $L = 1$ and the $n_s = 0$ can form possible Fock components in the ground state octet baryons with considerable probability amplitudes.

Accordingly, there are 17 possible five-quark configurations with different orbital, flavor, spin and colour symmetries in each of the octet baryons, which are shown explicitly in Table II where the symbols [..]$\nu$, are the Young tableaux of corresponding wave functions with $\nu = \chi, F, S$ and $C$ representing orbital, flavor, spin and color, respectively. Finally, [..]$\nu[..][..][..]$ is just a shorthand for corresponding flavor-spin decomposition.

As we can see in Table II the 17 five-quark configurations can be categorized into two groups according to the spin wave functions of the four-quark subsystem, namely, spin symmetry [22]|S and [31]|S, those should lead to the total spin of the four-quark system $S_4 = 0$ and 1, respectively. For the configurations with spin symmetry [22]|S, explicit wave functions can be expressed as

$$|B, \uparrow\rangle_{5q} = \sum_{ijklm} \sum_{ab} \sum_{ms} C_{[\chi,F,S]\chi,F,S}^{[1]} C_{[\chi,F,S]\chi,F,S}^{[1]} C_{\chi,F,S}^{[31]} |\downarrow l, \chi, \uparrow\rangle_{5q} \langle[211]|_C(a)\rangle \langle[11]_{C,q}(b)|I, I_3|F\rangle_P,$$

$$|I, I_3|F\rangle_P|\downarrow \rangle_{5q} \langle[31]|_{\chi,F,S}\chi, s_3\rangle \langle\bar{s}_3|_{\chi,F,S}\chi, \tilde{s}_3\rangle \phi\left(\{r_q\}\right),$$

(2)

where the coefficients $C_{[..][..][..]}$ represent the CG coefficients of the $S_4$ permutation group, $|[211]|_C(a)$ and $|[11]_{C,q}(b)$ are the color wave functions for the four-quark subsystem and the antiquark, combination of which should lead to the color singlet $[22]|_C$.

While combination of the four-quark spin $[31]|S$ which leads to $S_4 = 1$ and the orbital angular momentum of the five-quark system $L = 1$ should result in $J = L \oplus S_4 = 0$ or 1, both of the two cases of $J$ are legal in present framework, since couplings of both the values of $J$ and spin of the antiquark $s_q = 1/2$ could lead to $J_B = 1/2$ for the octet baryons. Hereafter, we denote these two cases as Set I and Set II, respectively. And wave functions of the five-quark components octet baryons in these two cases are

$$|B, \uparrow\rangle_{5q} = \sum_{ijklm} \sum_{ab} \sum_{ms} C_{[\chi,F,S]\chi,F,S}^{[1]} C_{[\chi,F,S]\chi,F,S}^{[1]} C_{\chi,F,S}^{[31]} |\downarrow l, \chi, \uparrow\rangle_{5q} \langle[211]|_C(a)\rangle \langle[11]_{C,q}(b)|I, I_3|F\rangle_P,$$

$$|I, I_3|F\rangle_P|\downarrow \rangle_{5q} \langle[31]|_{\chi,F,S}\chi, s_3\rangle \langle\bar{s}_3|_{\chi,F,S}\chi, \tilde{s}_3\rangle \phi\left(\{r_q\}\right),$$

(3)

$$|B, \uparrow\rangle_{5q} = \sum_{ijklm} \sum_{ab} \sum_{ms} C_{[\chi,F,S]\chi,F,S}^{[1]} C_{[\chi,F,S]\chi,F,S}^{[1]} C_{\chi,F,S}^{[31]} |\downarrow l, \chi, \uparrow\rangle_{5q} \langle[211]|_C(a)\rangle \langle[11]_{C,q}(b)|I, I_3|F\rangle_P,$$

$$|I, I_3|F\rangle_P|\downarrow \rangle_{5q} \langle[31]|_{\chi,F,S}\chi, s_3\rangle \langle\bar{s}_3|_{\chi,F,S}\chi, \tilde{s}_3\rangle \phi\left(\{r_q\}\right),$$

(4)

respectively.

One should note that $|I, I_3|F\rangle_P$ in Eqs. (2)-(4) just represents coupling of the flavour wave functions for the four-quark subsystem and the antiquark to form appropriate isospin quantum number. Explicit flavor decompositions of the five-quark configurations in the octet baryons are given in Appendix A.

The coefficients $C_i$ in Eq. (1) in the framework of the EχCQM developed in Ref. [20] can be related to the

\(^1\) In this work, we do not take $c\bar{c}$ and $b\bar{b}$ into account, since the probabilities for them are much smaller than other five-quark components inside the octet baryons in the low energy scale.
coupling between the three-quark and the corresponding five-quark components, which reads

\[ C_i^q = \frac{\langle qqq(q\bar{q}), i|\hat{T}(qqq)\rangle}{M_B - E_i} , \tag{5} \]

where \( M_B \) denote the physical mass of the baryon \( B \), and \( E_i \) is the energy of the \( i \)-th \( qqq(q\bar{q}) \) five-quark component. The transition coupling operator \( \hat{T} \) depends on the quark-antiquark creation mechanism, in present work, we take the widely accepted \( 3P_0 \) mechanism following Ref. \[50\], where the explicit form of \( \hat{T} \) is

\[ \hat{T} = -\gamma \sum_{j=1,4} F_{j,5}^{00} C_{00}^{j,5} C_{\text{OFSC}} \sum_m \langle 1, m; 1, -m|00 \rangle \times \chi_{j,5}^{1, m} Y_{j,5}^{1, -m} (\vec{p}_j - \vec{p}_5) b^l(\vec{p}_j) d^l(\vec{p}_5) , \tag{6} \]

where \( \gamma \) is an dimensionless transition coupling constant, \( F_{j,5}^{00} \) and \( C_{00}^{j,5} \) are the flavor and color singlet of the created quark-antiquark pair \( q_j \bar{q}_5 \), \( \chi_{j,5}^{1, m} \) and \( Y_{j,5}^{1, -m} \) are the total spin \( S_{q\bar{q}} = 1 \) and relative orbital \( P \) state of the created quark-antiquark system, the operator \( C_{\text{OFSC}} \) is to calculate the overlap factor between the residual three-quark configuration in the five-quark component and the valence three-quark component, finally, \( b^l(\vec{p}_j) \) and \( d^l(\vec{p}_5) \) are the quark and antiquark creation operators.

Finally, to calculate the energy for a given five-quark configuration \( E_i \) in Eq. \( \text{(5)} \), we employ the chiral constituent quark model, in which the quark-quark hyperfine interaction is flavor-dependent \[55\], as follow

\[ H_{\text{hyp}} = -\sum_{i<j} \hat{\sigma}_i \cdot \hat{\sigma}_j \sum_{a=1}^3 V_\pi(r_{ij}) \chi^a_i \chi^a_j + \sum_{a=4}^7 V_K(r_{ij}) \chi^a_i \chi^a_j + V_q(r_{ij}) \chi^a_i \chi^a_j \] , \tag{7} \]

numerical values for all the exchange coupling strength constants \( V_{\text{eff}} \) used in present work are taken to be the empirical ones \[58\]. Thus, the energies \( E_i \) for the 17 five-quark configurations in Table I should be

\[ E_i = E_0 + \langle H_{\text{hyp}} \rangle + \delta_{q\bar{q}} , \tag{8} \]

where \( E_0 \) is a degenerated energy for the 17 five-quark configurations shown in Table I. The parameter \( E_0 \) is dependent on the constituent quark masses, the kinetic quark energies, and also the energies of the quark confinement interactions and the quark potentials. Here we take \( E_0 = 2127 \text{ MeV} \), \( \delta_{uu} = \delta_{dd} = 0 \), and \( \delta_{ss} = 240 \text{ MeV} \) as used in Ref. \[50\].

### B. Formalism for the axial charges of octet baryons

In this section, we present the formalism for the axial charges \( g_A^{(0)} \), \( g_A^{(3)} \) and \( g_A^{(8)} \) of octet baryons within the framework of E\( \chi \)CQM shown in Sec. \[11A\].

The quark spin contribution \( \Delta q \), which is related to the flavour-dependent axial vector current operator, is defined as

\[ \langle B, s_z | \int dx \bar{q} \gamma^\mu \gamma^5 q | B, s_z \rangle = s^\mu \cdot \Delta q , \tag{9} \]

where \( s^\mu \) is the baryon states polarization vector, and \( \Delta q \) is

\[ \Delta q = (q^3 + \bar{q}^3) - (q^4 + \bar{q}^4) . \tag{10} \]

Combinations of different flavor \( \Delta f \) with \( f = u, d, s \) lead to flavor-singlet, isovector, and SU(3) octet axial charges of octet baryons, as follows

\[ g_A^{(0)} = \Delta u + \Delta d + \Delta s . \tag{11} \]
\[ g_A^{(3)} = \Delta u - \Delta d , \tag{12} \]
\[ g_A^{(8)} = \Delta u + \Delta d - 2\Delta s . \tag{13} \]

In the non-relativistic approximation, \( \Delta f \) can be gotten directly through the formula:

\[ \Delta f = (BS_z) \sum_j \hat{\sigma}_j \hat{\delta}_{jf} |BS_z\rangle . \tag{14} \]
where $|Bs_z⟩$ is the wave function Eq. (I) of octet baryons, $σ_j^z$ is Pauli operator acting on $j$-th quark, and $δ_{jf}$ is a flavor-dependent operator, defined as

$$δ_{jf} = \begin{cases} 1 & \text{if the favour of } j\text{-th quark is } f \\ 0 & \text{if the favour of } j\text{-th quark is not } f \end{cases} \quad (15)$$

Consequently, for each of the octet baryons, one can get

$$Δf = \frac{1}{N}⟨qqq, s_z| \sum_{j=1,3} \hat{σ}_j^z δ_{jf} |qqq, s_z⟩ + \sum_i (C_i^q)^2 \frac{1}{N}⟨qqq(qq), i, s_z⟩,$$  

$$⟨qqq(qq), i, s_z⟩ \sum_{j=1,5} \hat{σ}_j^z δ_{jf} |qqq(qq), i, s_z⟩,$$ (16)

where the non-diagonal terms are neglected.

For simplicity, we denote the matrix elements for the three-quark components and an explicit given five-quark component in Eq. (10) as

$$Δf^{3q} = ⟨qqq, s_z⟩ \sum_{j=1,3} \hat{σ}_j^z δ_{jf} |qqq, s_z⟩,$$ (17)

$$Δf^{5q} = ⟨qqq(qq), i, s_z⟩ \sum_{j=1}^5 \hat{σ}_j^z δ_{jf} |qqq(qq), i, s_z⟩.$$ (18)

Accordingly, one can easily obtain the explicit expression of the matrix elements results for $Δu$, $Δd$, and $Δs$ of octet baryons as following

$$Δu = \frac{1}{N} Δu^{3q} + \sum_{i,q} (C_i^q)^2 Δu_i^{5q},$$ (19)

$$Δd = \frac{1}{N} Δd^{3q} + \sum_{i,q} (C_i^q)^2 Δd_i^{5q},$$ (20)

$$Δs = \frac{1}{N} Δs^{3q} + \sum_{i,q} (C_i^q)^2 Δs_i^{5q}.$$ (21)

Finally, considering the $SU(2)$ isospin symmetry, one can get the relations of the spin contributions of quarks with different flavour to spin of the proton and neutron as

$$Δu(p) = Δd(n), Δd(p) = Δu(n), Δs(p) = Δs(n)$$ (22)

and the resulted relations of the axial charges of proton and neutron as

$$g_A^{(0)}(p) = g_A^{(0)}(n), g_A^{(3)}(p) = g_A^{(3)}(n),$$

$$g_A^{(8)}(p) = g_A^{(8)}(n).$$ (23)

For the spin of quarks with different flavor and the axial charges of the other octet baryons, analogous relationship can be obtained.

### III. THE NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present our numerical results of the calculations on the quark spin contributions and the axial charges of the octet baryons in the framework shown in Sec. II.

Before going to the final numerical results, firstly, we have to discuss the model parameters in the $E_3$ CQM. In Ref. [51], the intrinsic sea contents of the octet baryons have been studied in the $E_2$ CQM by taking the empirical values for the model parameters, only except for the newly introduced $V$ which should indicate the transition coupling strength for the processes $qqq ↔ qqq(qq)$, arising from calculations on the matrix elements of the operator (10). The parameter $V$ is determined by fitting the sea asymmetry in proton $\bar{d} - \bar{u} = 0.118 ± 0.012$ [51, 52], which yields

$$V = 570 ± 46 \text{ MeV}.$$ (24)

As we have discussed in Sec. II A, the spin symmetry of the fourquark subsystem in the configurations listed in Table II should lead to two sets for the wave functions of the five-quark components in the octet baryons, namely, Eqs. (3) and (4), respectively. In Ref. [50], only the former case was considered because of the possible lower energy, while in the very recent work [57], both of the two sets of wave functions have been employed to investigate the axial charges of the proton. Keeping all the other model parameters being the empirical values, the parameter $V$ should be

$$V = 697 ± 80 \text{ MeV},$$ (25)

to fit the intrinsic sea asymmetry of the proton when the set of wave functions (11) is used.

In present work, we will consider both Set I and Set II to study the axial charges of the octet baryons following work [57]. The resulting numerical results for the probabilities of the five-quark Fock components, the quark spin contributions to the octet baryons, and the flavor-dependent axial charges of the octet baryons are presented in the following three subsections, respectively.

#### A. Probabilities of the intrinsic five-quark Fock components in the octet baryons

Employing the values for all the model parameters mentioned above, explicit calculations on the transition couplings between three- and five-quark components, and energies for the corresponding five-quark components by taking the wave functions of Set I and Set II lead to the numerical results for the probabilities of the intrinsic five-quark components in the octet baryons, those are shown in Tables [AI] and [AVIII].

As discussed in Ref. [57], taking the wave functions of Set II, one can obtain larger probabilities for the five-quark components in the proton than those resulted by
using the wave functions of Set I. While the results of Set II are very close to the predictions obtained by the BHPS model \[60\]. It’s very similar for the probabilities of the five-quark in the other octet baryons, as one can see in the tables.

One may note that the most significant difference of the results obtained in Sets I and II, is that the probabilities of the five-quark configuration with \(i = 8\) in all the octet baryons obtained in Set II are about two times of those obtained in Set I. In fact, one can also find similar amplifications for the probabilities of the five-quark configurations with \(i = 9 - 17\) in those the spin symmetry of the four-quark subsystem is \([31]_s\). These amplifications are mainly caused by the larger matrix elements \(T\) for the transition couplings \(q q q \leftrightarrow q q q (q q)\) via the \(\hat{P}_0\) mechanism obtained using the wave function \([4]\) in Set II than using \([4]\) in Set I.

Straightforwardly, for Set I, one can get the total probabilities of the five-quark components in the octet baryons

\[
P^N_I = 0.373, \quad P^N_I = 0.338, \\
P^\Xi_I = 0.303, \quad P^\Lambda_I = 0.330,
\]

with uncertainties \(\sim 10\%\) for all the values caused by the experimental errors of the data for \(d - \bar{u}\) in proton, and

\[
P^N_{II} = 0.548, \quad P^N_{II} = 0.519, \\
P^\Xi_{II} = 0.467, \quad P^\Lambda_{II} = 0.504,
\]

with uncertainties \(\sim 10\%\).

An examination of the obtained probabilities is to estimate the pion- and strangeness-baryon \(\sigma\) terms of the octet baryons. In the model developed in Ref. \([53]\), the corresponding \(\sigma\) terms could be related to the probabilities of the five-quark components with light and strangeness quark-antiquark pairs \(P_{q\bar{q}}\) as

\[
\sigma_{\pi N} = \frac{3 + 2(P^N_{uu} + P^N_{dd})}{3 + 2(P^N_{uu} + P^N_{dd} - 2P^N_{ss})} \tilde{\sigma}, \\
\sigma_{s N} = \frac{m_s 2P^N_{ss}}{m_l 3 + 2(P^N_{uu} + P^N_{dd})} \tilde{\sigma}_{s N}, \quad (28)
\]

for nucleon, and

\[
\sigma_{\pi Y} = \frac{2 + 2(P^Y_{uu} + P^Y_{dd})}{3 + 2(P^Y_{uu} + P^Y_{dd})} \tilde{\sigma}_{\pi N}, \\
\sigma_{s Y} = \frac{1 + 2P^Y_{ss}}{2P^N_{ss}} \tilde{\sigma}_{s N} \\
= \frac{m_s}{m_l} \frac{1 + 2P^Y_{ss}}{3 + 2(P^N_{uu} + P^N_{dd})} \tilde{\sigma}_{\pi N}, \quad (30)
\]

for the hyperons \(\Sigma\) and \(\Lambda\), and

\[
\sigma_{\pi \Xi} = \frac{1 + 2(P^\Xi_{uu} + P^\Xi_{dd})}{3 + 2(P^\Xi_{uu} + P^\Xi_{dd})} \tilde{\sigma}_{\pi N}, \quad (32)
\]

\[
\sigma_{s \Xi} = \frac{2 + 2P^\Xi_{ss}}{2P^N_{ss}} \tilde{\sigma}_{s N} \\
= \frac{m_s}{m_l} \frac{2 + 2P^\Xi_{ss}}{3 + 2(P^N_{uu} + P^N_{dd})} \tilde{\sigma}_{\pi N}, \quad (33)
\]

for the hyperon \(\Xi\), where the nonsinglet component was taken to be \(\tilde{\sigma} = 33 \pm 5\ MeV\), as extracted within the chiral perturbation theory \([61]\) and the ratio of the light and strange quark masses was taken to be \(m_l/m_s = 27.5 \pm 1\) as shown in PDG \([59]\).

As shown in \([53]\), the numerical results of the obtained employing the wave functions of Set I in present work are in general consistent with the predictions of chiral perturbation theory and lattice QCD. While by the obtained probabilities of the five-quark components shown in Tables \([AI\text{--}AVIII]\) employing the wave functions of Set II, one can get

\[
\sigma_{\pi N} = 36 \pm 5\ (MeV), \quad \sigma_{s N} = 46 \pm 8\ (MeV), \quad (34)
\]

\[
\sigma_{\pi \Xi} = 26 \pm 4\ (MeV), \quad \sigma_{s \Xi} = 302 \pm 57\ (MeV), \quad (35)
\]

\[
\sigma_{\pi \Lambda} = 16 \pm 3\ (MeV), \quad \sigma_{s \Lambda} = 535 \pm 102\ (MeV), \quad (36)
\]

\[
\sigma_{\pi \Sigma} = 27 \pm 4\ (MeV), \quad \sigma_{s \Sigma} = 286 \pm 55\ (MeV), \quad (37)
\]

accordingly, all the results are very close to those obtained in Ref. \([53]\).

B. The quark spin contributions to the octet baryons

In this section, we present our numerical results for the quark spin contributions to the octet baryons. Employing the wave functions of Sets I and II, explicit calculations on the matrix element \([15]\) lead to numerical results for \(\Delta f^i_{q \bar{q}}\) shown in Tables \([AI\text{--}AVIII]\) in Appendix \([13]\).

Analogously to the results in Ref. \([57]\), as we can see in the tables in Appendix \([13]\) the numerical results for \(\Delta f^i_{q \bar{q}}\) with \(i = 1 - 7\) of Set I are the same with those of Set II, this is because of that the wave functions of the five-quark configurations \(i = 1 - 7\) in Sets I and II are the same ones as shown in Eq. \([1]\). And only spin of the antiquarks contribute to \(\Delta f^i_{q \bar{q}}\) of these five-quark configurations, since spin of the fourquark subsystem is 0. For \(\Delta f^i_{q \bar{q}}\) with \(i = 8 - 17\), both the quarks and antiquark should contribute, and the different wave functions of the five-quark configurations in Eqs. \([3]\) and \([4]\) in Sets I and II lead to different results.

Straightforwardly, we can obtain the numerical results for the quark spin contributions of the octet baryons \(\Delta f\) by the obtained \(\Delta f^i_{q \bar{q}}\) and \(P_{q \bar{q}}\) using Eqs. \([19]\) - \([21]\), the results are shown in Table \([4]\).
TABLE II: Numerical results for $\Delta f$ ($f = u, d, s$) of the octet baryons, compared to the predictions by lattice QCD [12] and chiral effective field theory [20] shown in the last two rows, respectively.

|   | $p$ | $n$ | $\Sigma^+$ | $\Sigma^0$ | $\Sigma^-$ | $\Xi^0$ | $\Xi^-$ | $\Lambda$ |
|---|-----|-----|------------|------------|------------|--------|--------|---------|
| $\Delta u$ |     |     |            |            |            |        |        |         |
| Set I     | 0.883 ± 0.005 | −0.213 ± 0.003 | 0.922 ± 0.005 | 0.473 ± 0.004 | 0.023 ± 0.003 | −0.215 ± 0.002 | 0.030 ± 0.006 | 0.026 ± 0.003 |
| Set II    | 0.710 ± 0.012 | −0.225 ± 0.008 | 0.762 ± 0.014 | 0.348 ± 0.004 | −0.066 ± 0.007 | −0.194 ± 0.002 | −0.059 ± 0.007 | −0.020 ± 0.002 |
| LQCD [12] |      |     | 0.794(21)(2) |            |            |        |        |         |
| $\chi_{PT}$ [20] |      |     | 0.90±0.03 |          |          |        |        |         |
| $\Delta d$ |     |     |            |            |            |        |        |         |
| Set I     | −0.213 ± 0.003 | 0.883 ± 0.005 | 0.023 ± 0.003 | 0.473 ± 0.004 | 0.922 ± 0.005 | 0.030 ± 0.006 | −0.215 ± 0.002 | 0.026 ± 0.003 |
| Set II    | −0.225 ± 0.008 | 0.710 ± 0.012 | −0.066 ± 0.007 | 0.348 ± 0.004 | 0.762 ± 0.014 | −0.194 ± 0.002 | −0.059 ± 0.007 | −0.020 ± 0.002 |
| LQCD [12] |      |     | −0.289(16)(1) |          |          |        |        |         |
| $\chi_{PT}$ [20] |      |     | −0.38±0.03 |          |          |        |        |         |
| $\Delta s$ |     |     |            |            |            |        |        |         |
| Set I     | 0.015 ± 0.002 | 0.015 ± 0.002 | −0.205 ± 0.002 | −0.205 ± 0.002 | −0.205 ± 0.002 | 0.929 ± 0.001 | 0.929 ± 0.001 | 0.674 ± 0.001 |
| Set II    | −0.020 ± 0.003 | −0.020 ± 0.003 | −0.180 ± 0.003 | −0.180 ± 0.003 | −0.180 ± 0.003 | 0.798 ± 0.011 | 0.798 ± 0.011 | 0.551 ± 0.006 |
| LQCD [12] |      |     | −0.023(10)(1) |          |          |        |        |         |
| $\chi_{PT}$ [20] |      |     | −0.007±0.004 |          |          |        |        |         |

In most of the theoretical studies on the quark spin contributions of baryons, only nucleon has been considered. As discussed in Ref. [57], the numerical results of $\Delta f$ of the proton obtained using the EχCQM are in general consistent with the predictions by lattice QCD theory. Therefore, here we only compare the present numerical results to those predicted in Refs. [12] and [20], a more comprehensive comparison of the numerical results for $\Delta f$ of the nucleon obtained in present model to other theoretical predictions can be found in [57].

It’s very interesting that $\Delta u$ and $\Delta d$ of the $\Lambda$ hyperon in present model are small but nonzero values. As we know, in the traditional constituent quark model in which the baryons are assumed to be composed of three quarks, the two light quarks in $\Lambda$ are completely antisymmetric, therefore, the $\Lambda$ spin arising from the light quarks spin vanishes. While in present work, contributions of the five-quark components are taken into account, as we can see in Table [AVIII] several of the five-quark components have nonzero contributions to $\Delta u$ and $\Delta d$. Accordingly, experimental investigations on the contributions of light quarks to $\Lambda$ spin may be an examination of the present model.

On the other hand, the magnetic moments, spins and orbital angular momenta of the octet baryons have been studied in an unquenched quark model in Ref. [13]. In that work, the spin of proton carried by different flavor of quarks were obtained to be $\Delta u = 1.098$, $\Delta d = −0.417$ and $\Delta s = −0.005$, and for the $\Lambda$ hyperon, the values $\Delta u = −0.055$, $\Delta d = −0.055$ and $\Delta s = 0.961$ were predicted.

C. The flavor-dependent axial charges of the octet baryons

The singlet axial charge $g_A^{(0)}$, isovector axial charge $g_A^{(3)}$ and $SU(3)$ octet axial charge $g_A^{(8)}$ of the octet baryons can be directly obtained by using Eqs. [11]–[13] with the values of $\Delta f$ shown in Table [IV] the numerical results are shown in Table [VII] for the two sets of wave functions discussed in Sec. II. As we can see in the table, the obtained numerical results of $g_A^{(0)}$ of the $\Sigma$ and $\Xi$ hyperons are larger than that of the nucleon, which should indicate larger contributions of the quark spin to hyperons spin than those to nucleon spin.

In Ref. [57], the axial charges of the proton obtained using the EχCQM have been explicitly compared to the experimental data [3] and predictions of lattice QCD [10–16], and the chiral perturbation theory [17,20]. For the singlet axial charge $g_A^{(0)}$, which should indicate the proton spin arising from the quarks spin, it was shown that the results in EχCQM were consistent with predictions of other theoretical approaches, but larger than the experimental data. In fact, it may be not convenient for us to directly compare $g_A^{(0)}$ estimated in the static quark model with the COMPASS data, since $g_A^{(0)}$ is often measured at $Q^2 = 3$ GeV$^2$, and the increasing $Q^2$ may result in decreased $g_A^{(0)}$ [62]. Therefore, one may conclude that the estimation of the axial charges of the proton in EχCQM are consistent with the data and predictions by lattice QCD and chiral perturbation theory.

Consequently, in present work, we only compare our numerical results for the axial charges of the other octet baryons with predictions by other theoretical approaches, as we can see in Table [VII].
In Ref. [35], the first lattice calculations on the axial charge $g_A^{(3)}$ of the Σ and Ξ hyperons was performed using 2 + 1-flavor lattices by Lin and Orginos. And in 2018, Savanur and Lin presented the first chiral-continuum–finite-volume extrapolation of the hyperons axial couplings $g_{2Σ$, and $g_{2Ξ}$ from $N_f = 2+1+1$ lattice QCD in [11], where the axial charges $g_A^{(3)}$ of Σ and Ξ with respect to the nucleon axial charges were predicted to be $g_A^{(3)} / g_A = 0.702(12)(4)$ for Σ and $g_A^{(3)} / g_A = −0.213(5)(1)$ for Ξ, respectively.

In Ref. [37], the strangeness-conserving $NN$, $ΣΣ$, $ΞΞ$, $ΛΣ$ and the strangeness-changing $ΛN$, $ΣN$, $ΛΞ$, $ΞΣ$ axial charges was studied in lattice QCD with $N_f = 2$. The resulting axial charges of the Σ and Ξ fall in the same range as the present obtained numerical results.

And in a recent work [40], the axial couplings of the low lying baryons were evaluated using a total of five ensembles of dynamical twisted mass fermion gauge configurations. As we can see in Table III, the present numerical results for the axial charges $g_A^{(3)}$ of the Σ, Ξ and Λ hyperons are very close to those depicted in the figures in Ref. [10], and in Table III we only show their predicted values for $g_A^{(3)}$ of the Σ and Ξ hyperons.

The axial charges of the octet baryons have also been investigated using chiral perturbation theory [20, 44]. In [20], all the three flavor-dependent axial charges of the nucleon has been studied, as shown in the column $\chi$EFT in Table III. In Ref. [44], $g_A^{(3)}$ of the nucleon was predicted to be 1.18, while that for the hyperons Σ and Ξ were predicted to be 1.03 and −0.23, respectively, as we can see in the column $\chi$PT in Table III.

In addition, we also compare the present obtained results with those obtained using the relativistic constituent quark model [45] and perturbative chiral quark model [47], which are shown in the columns RCQM and PCQM in Table III respectively.

### IV. SUMMARY

To summarize, we investigate the intrinsic five-quark components and the flavor-dependent axial charges of the octet baryons within the framework of the extended chiral constituent quark model, in which wave functions of the higher five-quark Fock components in baryons are taken into account.

The probabilities of the five-quark components in the nucleon are in agreements with other theoretical predictions. And with the obtained probabilities of the five-quark components in the wave functions of the octet baryons, we get the pion- and strangeness-baryon sigma terms consistent with predictions by lattice QCD.

Finally, the present obtained numerical results show that the singlet axial charges of the octet baryons, which should indicate total baryons spin arising from the spin of the quarks, fall in the range $0.45 − 0.75$ in present model, it’s in consistent with the predictions by lattice QCD, chiral perturbation theory, as well the other theoretical approaches.

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Appendix A: Flavor decompositions of five-quark components in the octet baryons

In each five-quark configuration, \([v]_F\) is the flavour wave function of four-quark subsystem, one can get the flavour wave function of the five-quark system by combining \([v]_F\) and antiquark flavour wave function \([\bar{q}]\). Here we give the explicit flavor decompositions for all the possible five-quark configurations in the octet baryons.

1. Nucleon

For the proton, whose isospin wave function is \(|\frac{1}{2}, \frac{1}{2}\rangle_F\), the configurations with \([v]_F = [31]_{F_1}\) rules out the strangeness component. Then the four-quark subsystem should couple with the antiquark as

\[
|\frac{1}{2}, \frac{1}{2}\rangle_{F_1} = \sqrt{\frac{2}{3}}|v^3 d_{[31]} F_1 \rangle \otimes |\bar{u}\rangle + \frac{1}{\sqrt{3}}|v^2 d^2_{[31]} F_1 \rangle \otimes |\bar{d}\rangle, \quad (A1)
\]

For the configurations with \([v]_F = [31]_{F_2}\), the quark-antiquark pair can only be \(ss\), and the corresponding isospin wave function of the proton is

\[
|\frac{1}{2}, \frac{1}{2}\rangle_{F_2} = |u^2 s d_{[31]} F_2 \rangle \otimes |\bar{s}\rangle, \quad (A2)
\]

For the configurations with \([v]_F = [22]_F\), the quark-antiquark pair can be \(dd\) and \(ss\). To take into account the SU(3) flavour symmetry breaking effects, we treat these two kinds of quqq (qq̄) configurations separately. And the
flavour decompositions are
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = \left| u^2 d_{[22]_F} \right| \otimes \left| \bar{d} \right| , \tag{A3}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = \left| u^2 s_{[22]_F} \right| \otimes \left| \bar{s} \right| , \tag{A4}
\]

At last, for the configurations with \( |v|_F = [211]_F \), which limits the quark-antiquark pair to be \( s \bar{s} \), the flavour decomposition is
\[
\frac{1}{2} \frac{1}{2} \left| 211 \right>_F = \left| u^2 ds_{[211]_F} \right| \otimes \left| \bar{s} \right| . \tag{A5}
\]

Analogously, one can get the following flavour decompositions for the five-quark configurations in neutron according to the isospin quark number:
\[
\frac{1}{2} \frac{1}{2} \left| 31 \right>_1 = \sqrt{\frac{1}{3}} \left| u^2 d_{[31]_F} \right| \otimes \left| \bar{u} \right| + \sqrt{\frac{2}{3}} \left| u d_{[31]_F} \right| \otimes \left| \bar{d} \right| , \tag{A6}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 31 \right>_2 = \left| u^2 s_{[31]_F} \right| \otimes \left| \bar{s} \right| , \tag{A7}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = -\left| u^2 d_{[22]_F} \right| \otimes \left| \bar{u} \right| , \tag{A8}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = \left| u^2 s_{[22]_F} \right| \otimes \left| \bar{s} \right| , \tag{A9}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 211 \right>_F = \left| u^2 s_{[211]_F} \right| \otimes \left| \bar{s} \right| . \tag{A10}
\]

2. \( \Sigma \) baryons

For the \( \Sigma^+ \) baryon, the isospin wave function is \( |1, 1\rangle_I \). In the five-quark configurations with \( |v|_F = [31]_F \), both the light and strange quark-antiquark pairs survive. The corresponding flavour decompositions are
\[
\frac{1}{2} \frac{1}{2} \left| 31 \right>_F = \sqrt{\frac{3}{4}} \left| u^2 s_{[31]_F} \right| \otimes \left| \bar{u} \right| + \sqrt{\frac{1}{4}} \left| u^2 d_{[31]_F} \right| \otimes \left| \bar{d} \right| , \tag{A11}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 31 \right>_F = \left| u^2 s_{[31]_F} \right| \otimes \left| \bar{s} \right| , \tag{A12}
\]
respectively.

For the configurations with \( |v|_F = [31]_F \), in present case, flavour decomposition of the \( uuuss \bar{s} \) configuration is the same as the configurations with \( |v|_F = [31]_F \). And the flavour decomposition of the five-quark configurations with light quark antiquark pair should be
\[
\frac{1}{2} \frac{1}{2} \left| 31 \right>_F = \left| u^2 ds_{[31]_F} \right| \otimes \left| \bar{d} \right| . \tag{A13}
\]

For the configurations with \( |v|_F = [22]_F \), both light and strangeness five-quark components exist, and the flavour decompositions are
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = -\left| u^2 d_{[22]_F} \right| \otimes \left| \bar{d} \right| , \tag{A14}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = \left| u^2 s_{[22]_F} \right| \otimes \left| \bar{s} \right| , \tag{A15}
\]

Finally, the configurations with \( |v|_F = [211]_F \) rules out the strangeness five-quark component in \( \Sigma^+ \), only \( uuud \) component exists. And the flavour decomposition reads
\[
\frac{1}{2} \frac{1}{2} \left| 211 \right>_F = -\left| u^2 ds_{[211]_F} \right| \otimes \left| \bar{d} \right| . \tag{A16}
\]

Considering the isospin \( SU(2) \) symmetry, one can obtain the flavour decomposition
\[
\frac{1}{2} \frac{1}{2} \left| 31 \right>_1 = \sqrt{\frac{1}{2}} \left| u^2 ds_{[31]_1} \right| \otimes \left| \bar{u} \right| - \sqrt{\frac{1}{2}} \left| ud_{[31]_1} \right| \otimes \left| \bar{d} \right| , \tag{A17}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 31 \right>_1 = \left| u^2 s_{[31]_2} \right| \otimes \left| \bar{s} \right| , \tag{A18}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = -\sqrt{\frac{1}{2}} \left| u^2 ds_{[22]_F} \right| \otimes \left| \bar{u} \right| + \sqrt{\frac{1}{2}} \left| ud_{[22]_F} \right| \otimes \left| \bar{d} \right| , \tag{A19}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = \left| u^2 s_{[22]_F} \right| \otimes \left| \bar{s} \right| , \tag{A20}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = -\sqrt{\frac{1}{2}} \left| u^2 ds_{[22]_F} \right| \otimes \left| \bar{u} \right| - \sqrt{\frac{1}{2}} \left| ud_{[22]_F} \right| \otimes \left| \bar{d} \right| , \tag{A21}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = \left| u^2 s_{[22]_F} \right| \otimes \left| \bar{s} \right| , \tag{A22}
\]

for the five-quark components in the \( \Sigma^0 \) baryon, and
\[
\frac{1}{2} \frac{1}{2} \left| 31 \right>_F = \sqrt{\frac{3}{4}} \left| u^2 s_{[31]_F} \right| \otimes \left| \bar{u} \right| + \sqrt{\frac{1}{4}} \left| u^2 d_{[31]_F} \right| \otimes \left| \bar{d} \right| , \tag{A23}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 31 \right>_F = \left| d^2 s_{[31]_F} \right| \otimes \left| \bar{s} \right| , \tag{A24}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 31 \right>_F = \left| u^2 s_{[31]_2} \right| \otimes \left| \bar{u} \right| , \tag{A25}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = -\left| u^2 d_{[22]_F} \right| \otimes \left| \bar{u} \right| , \tag{A26}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = \left| d^2 s_{[22]_F} \right| \otimes \left| \bar{s} \right| , \tag{A27}
\]
\[
\frac{1}{2} \frac{1}{2} \left| 22 \right>_F = \left| u^2 s_{[22]_F} \right| \otimes \left| \bar{u} \right| , \tag{A28}
\]

for the five-quark components in the \( \Sigma^- \) baryon, respectively.

3. \( \Xi \) baryon

The isospin wave function of the \( \Xi^+ \) baryon is \( |1/2, 1/2\rangle_I \). Accordingly, for the configurations with \( |v|_F = [31]_F \), both the five-quark components with light
and strange quark-antiquark pairs can exist. Accordingly, the flavour decompositions are

\[ |\frac{1}{2}, \frac{1}{2}\rangle_{I}^{[31]} = \sqrt{\frac{2}{3}} |u^2s_{[31]}^{3/2} \rangle \otimes |\bar{u}\rangle + \sqrt{\frac{1}{3}} |uds_{[31]}^{1/2} \rangle \otimes |\bar{d}\rangle, \quad (A29) \]

\[ |\frac{1}{2}, -\frac{1}{2}\rangle_{I}^{[31]} = |u^3s_{[31]} \rangle \otimes |\bar{s}\rangle, \quad (A30) \]

respectively.

For the configurations with \([v]_{F} = [31]_{F_1}\), the usss\(\bar{s}\) component is the same as the one with \([31]_{F_1}\). So we only have to consider the light five-quark components, which should be

\[ |\frac{1}{2}, \frac{1}{2}\rangle_{I}^{[31]} = |uds_{[31]}^2 \rangle \otimes |\bar{d}\rangle. \quad (A31) \]

The flavour symmetry \([v]_{F} = [22]_{F}\) rules out the five-quark component in \(\Xi^+\) with strange quark-antiquark pair, and the flavour decomposition for the \(u\bar{s}s(\bar{q}q)\) component is

\[ |\frac{1}{2}, \frac{1}{2}\rangle_{I}^{[22]} = -\sqrt{\frac{2}{3}} |u^2s_{[22]}^{3/2} \rangle \otimes |\bar{u}\rangle - \sqrt{\frac{1}{3}} |uds_{[22]}^2 \rangle \otimes |\bar{d}\rangle. \quad (A32) \]

Finally, for the configurations with \([v]_{F} = [211]_{F}\), only \(udss\bar{d}\) component exists in \(\Xi^+\). And

\[ |\frac{1}{2}, \frac{1}{2}\rangle_{I}^{[211]} = -|uds_{[211]}^2 \rangle \otimes |\bar{d}\rangle. \quad (A33) \]

The flavour decompositions of the five-quark components in the \(\Xi^-\) baryon can be obtained by considering the \(SU(2)\) isospin symmetry as

\[ |\frac{1}{2}, -\frac{1}{2}\rangle_{I}^{[31]} = \sqrt{\frac{1}{3}} |uds_{[31]}^2 \rangle \otimes |\bar{u}\rangle + \sqrt{\frac{2}{3}} |uds_{[31]}^2 \rangle \otimes |\bar{d}\rangle, \quad (A34) \]

\[ |\frac{1}{2}, -\frac{1}{2}\rangle_{I}^{[31]} = |ds_{[31]}^3 \rangle \otimes |\bar{s}\rangle, \quad (A35) \]

\[ |\frac{1}{2}, -\frac{1}{2}\rangle_{I}^{[31]} = |uds_{[31]}^2 \rangle \otimes |\bar{u}\rangle, \quad (A36) \]

\[ |\frac{1}{2}, -\frac{1}{2}\rangle_{I}^{[22]} = -\sqrt{\frac{1}{3}} |uds_{[22]}^2 \rangle \otimes |\bar{u}\rangle - \sqrt{\frac{2}{3}} |uds_{[22]}^2 \rangle \otimes |\bar{d}\rangle, \quad (A37) \]

\[ |\frac{1}{2}, -\frac{1}{2}\rangle_{I}^{[211]} = |uds_{[211]}^2 \rangle \otimes |\bar{u}\rangle. \quad (A38) \]

4. \(\Lambda\) baryon

In the \(\Lambda\) baryon, isospin zero rules out the five-quark configurations with \([v]_{F} = [31]_{F_1}\). For the configurations with \([v]_{F} = [31]_{F_2}\), the light five-quark components should be

\[ |0, 0\rangle_{I}^{[31]} = \sqrt{\frac{1}{2}} |u^2s_{[31]} \rangle \otimes |\bar{u}\rangle + \sqrt{\frac{1}{2}} |uds_{[31]} \rangle \otimes |\bar{d}\rangle. \quad (A39) \]

Additionally, the strangeness five-quark component can also exist in \(\Lambda\), as

\[ |0, 0\rangle_{I}^{[31]} = |uds_{[31]}^2 \rangle \otimes |\bar{s}\rangle. \quad (A40) \]

For the five-quark configurations with \([v]_{F} = [22]_{F}\), only the ones with light quark-antiquark pairs exist in \(\Lambda\) baryon, and the flavour decomposition is

\[ |0, 0\rangle_{I}^{[22]} = -\sqrt{\frac{1}{2}} |u^2d_{[22]} \rangle \otimes |\bar{u}\rangle - \sqrt{\frac{1}{2}} |uds_{[22]} \rangle \otimes |\bar{d}\rangle. \quad (A41) \]

Finally, for the configurations with \([v]_{F} = [211]_{F}\), the light five-quark components should be

\[ |0, 0\rangle_{I}^{[211]} = -\sqrt{\frac{1}{2}} |u^2d_{[211]} \rangle \otimes |\bar{u}\rangle - \sqrt{\frac{1}{2}} |uds_{[211]} \rangle \otimes |\bar{d}\rangle. \quad (A42) \]

and the strangeness component is

\[ |0, 0\rangle_{I}^{[211]} = |uds_{[211]}^2 \rangle \otimes |\bar{s}\rangle. \quad (A43) \]

Appendix B: Numerical results of each five-quark components \(Pq\bar{q}\) and \(\Delta f_{I}^q (f = u, d, s)\) of octet baryons.

In present appendix, we show the numerical results on the matrix elements of \(\Delta f_{I}^q (f = u, d, s)\) for the octet baryons, as shown in Tables [AI–AVIII].
TABLE A1: The numerical results of the probabilities $P_{qqq}$ and the matrix elements of the quark spin $\Delta f^{\mathbb{S}_i}$ of all the 17 five-quark configurations of proton in Set I and Set II. Note that we have denoted the five-quark configurations $\bar{u}udq\bar{q}$ with light quark-antiquark pairs as $\bar{l}$, and those with strange quark-antiquark pairs as $ss$.

| Con. | $qq$ | $P_{qqq}$ | $\Delta u_i^{uq}$ | $\Delta d_i^{q\bar{q}}$ | $\Delta s_i^{s\bar{s}}$ | $P_{qqq}$ | $\Delta u_i^{uq}$ | $\Delta d_i^{q\bar{q}}$ | $\Delta s_i^{s\bar{s}}$ |
|------|------|-----------|------------------|------------------|------------------|-----------|------------------|------------------|------------------|
| 1    | $\bar{l}$ | 0.146 ± 0.015 | 0 | $-1/3$ | 0 | 0.157 ± 0.016 | 0 | $-1/3$ | 0 |
|      | $ss$   | 0.010 ± 0.001 | 0 | 0 | $-1/3$ | 0.011 ± 0.002 | 0 | 0 | $-1/3$ |
| 2    | $\bar{l}$ | 0 | - | - | - | 0 | - | - | - |
|      | $ss$   | 0.004 ± 0.001 | 0 | 0 | $-1/3$ | 0.004 ± 0.001 | 0 | 0 | $-1/3$ |
| 3    | $\bar{l}$ | 0.016 ± 0.002 | $-2/9$ | $-1/9$ | 0 | 0.018 ± 0.002 | $-2/9$ | $-1/9$ | 0 |
|      | $ss$   | 0 | - | - | - | 0 | - | - | - |
| 4    | $\bar{l}$ | 0 | - | - | - | 0 | - | - | - |
|      | $ss$   | 0.003 ± 0.001 | 0 | 0 | $-1/3$ | 0.003 ± 0.001 | 0 | 0 | $-1/3$ |
| 5    | $\bar{l}$ | 0 | - | - | - | 0 | - | - | - |
|      | $ss$   | 0.009 ± 0.001 | 0 | 0 | $-1/3$ | 0.009 ± 0.001 | 0 | 0 | $-1/3$ |
| 6    | $\bar{l}$ | 0.041 ± 0.004 | $-2/9$ | $-1/9$ | 0 | 0.045 ± 0.005 | $-2/9$ | $-1/9$ | 0 |
|      | $ss$   | 0 | - | - | - | 0 | - | - | - |
| 7    | $\bar{l}$ | 0 | - | - | - | 0 | - | - | - |
|      | $ss$   | 0.007 ± 0.001 | 0 | 0 | $-1/3$ | 0.007 ± 0.001 | 0 | 0 | $-1/3$ |
| 8    | $\bar{l}$ | 0.073 ± 0.007 | $2/3$ | $1/3$ | 0 | 0.157 ± 0.016 | $4/9$ | $-1/9$ | 0 |
|      | $ss$   | 0 | - | - | - | 0 | - | - | - |
| 9    | $\bar{l}$ | 0 | - | - | - | 0 | - | - | - |
|      | $ss$   | 0.006 ± 0.001 | 0 | 0 | 1 | 0.014 ± 0.002 | $4/9$ | $-1/9$ | 0 |
| 10   | $\bar{l}$ | 0 | - | - | - | 0 | - | - | - |
|      | $ss$   | 0.003 ± 0.001 | 0 | 0 | 1 | 0.007 ± 0.001 | $1/2$ | $1/12$ | $-1/4$ |
| 11   | $\bar{l}$ | 0.006 ± 0.001 | 0 | 1 | 0 | 0.013 ± 0.001 | $1/3$ | 0 | 0 |
|      | $ss$   | 0.002 ± 0.000 | 0 | 0 | 1 | 0.004 ± 0.001 | $1/3$ | $1/6$ | $-1/6$ |
| 12   | $\bar{l}$ | 0.005 ± 0.001 | $2/3$ | $1/3$ | 0 | 0.010 ± 0.001 | $1/3$ | 0 | 0 |
|      | $ss$   | 0 | - | - | - | 0 | - | - | - |
| 13   | $\bar{l}$ | 0 | - | - | - | 0 | - | - | - |
|      | $ss$   | 0.001 ± 0.000 | 0 | 0 | 1 | 0.002 ± 0.001 | $7/18$ | $1/36$ | $-1/12$ |
| 14   | $\bar{l}$ | 0 | - | - | - | 0 | - | - | - |
|      | $ss$   | 0.008 ± 0.001 | 0 | 0 | 1 | 0.017 ± 0.002 | $1/2$ | $1/12$ | $-1/4$ |
| 15   | $\bar{l}$ | 0.015 ± 0.002 | 0 | 1 | 0 | 0.032 ± 0.003 | $1/3$ | 0 | 0 |
|      | $ss$   | 0.004 ± 0.001 | 0 | 0 | 1 | 0.009 ± 0.001 | $1/3$ | $1/6$ | $-1/6$ |
| 16   | $\bar{l}$ | 0.012 ± 0.001 | $2/3$ | $1/3$ | 0 | 0.025 ± 0.002 | $1/3$ | 0 | 0 |
|      | $ss$   | 0 | - | - | - | 0 | - | - | - |
| 17   | $\bar{l}$ | 0 | - | - | - | 0 | - | - | - |
|      | $ss$   | 0.002 ± 0.000 | 0 | 0 | 1 | 0.004 ± 0.001 | $7/18$ | $1/36$ | $-1/12$ |
TABLE AII: The numerical results of the probabilities $P_{\bar{q}q}$ and the matrix elements of the quark spin $\Delta f_{i}^{\bar{q}q}$ of the neutron, conventions are the same as in Table A1.

| Set I | | Set II |
|-------|-------|-------|-------|-------|-------|-------|-------|
| Con. $i$ | $q\bar{q}$ | $P_{\bar{q}q}$ | $\Delta f_{i}^{\bar{q}q}$ | $\Delta f_{i}^{q\bar{q}}$ | $\Delta f_{i}^{\bar{q}q}$ | $\Delta f_{i}^{q\bar{q}}$ | $\Delta f_{i}^{\bar{q}q}$ | $\Delta f_{i}^{q\bar{q}}$ |
| 1 | $\bar{l}\bar{s}$ | 0.146 ± 0.015 | −1/3 | 0 | 0 | 0.157 ± 0.016 | −1/3 | 0 | 0 |
| 2 | $\bar{s}\bar{s}$ | 0.010 ± 0.001 | 0 | 0 | −1/3 | 0.011 ± 0.002 | 0 | 0 | −1/3 |
| 3 | $\bar{l}\bar{l}$ | 0.016 ± 0.002 | −1/9 | −2/9 | 0 | 0.018 ± 0.002 | −1/9 | −2/9 | 0 |
| 4 | $\bar{s}\bar{l}$ | 0 | − | − | − | 0 | − | − | − |
| 5 | $\bar{s}\bar{s}$ | 0.003 ± 0.001 | 0 | 0 | −1/3 | 0.003 ± 0.001 | 0 | 0 | −1/3 |
| 6 | $\bar{s}\bar{s}$ | 0.009 ± 0.001 | 0 | 0 | −1/3 | 0.009 ± 0.001 | 0 | 0 | −1/3 |
| 7 | $\bar{l}\bar{l}$ | 0.041 ± 0.004 | −1/9 | −2/9 | 0 | 0.045 ± 0.005 | −1/9 | −2/9 | 0 |
| 8 | $\bar{s}\bar{s}$ | 0 | − | − | − | 0 | − | − | − |
| 9 | $\bar{s}\bar{s}$ | 0.006 ± 0.001 | 0 | 0 | −1/3 | 0.007 ± 0.001 | 0 | 0 | −1/3 |
| 10 | $\bar{s}\bar{s}$ | 0.006 ± 0.001 | 1 | 0 | 0 | 0.007 ± 0.001 | 1/12 | 1/2 | −1/4 |
| 11 | $\bar{s}\bar{s}$ | 0.002 ± 0.000 | 0 | 0 | 1 | 0.004 ± 0.001 | 1/6 | 1/3 | −1/6 |
| 12 | $\bar{s}\bar{s}$ | 0.005 ± 0.001 | 1/3 | 2/3 | 0 | 0.010 ± 0.001 | 0 | 1/3 | 0 |
| 13 | $\bar{s}\bar{s}$ | 0.001 ± 0.000 | 0 | 0 | 1 | 0.002 ± 0.001 | 1/36 | 7/18 | −1/12 |
| 14 | $\bar{s}\bar{s}$ | 0.008 ± 0.001 | 0 | 0 | 1 | 0.017 ± 0.002 | 1/12 | 1/2 | −1/4 |
| 15 | $\bar{s}\bar{s}$ | 0.015 ± 0.002 | 1 | 0 | 0 | 0.032 ± 0.003 | 0 | 1/3 | 0 |
| 16 | $\bar{s}\bar{s}$ | 0.004 ± 0.001 | 0 | 0 | 1 | 0.009 ± 0.001 | 1/6 | 1/3 | −1/6 |
| 17 | $\bar{s}\bar{s}$ | 0.012 ± 0.001 | 1/3 | 2/3 | 0 | 0.025 ± 0.002 | 0 | 1/3 | 0 |
| 18 | $\bar{s}\bar{s}$ | 0.002 ± 0.000 | 0 | 0 | 1 | 0.004 ± 0.001 | 1/36 | 7/18 | −1/12 |
| Con. | \( q\bar{q} \) | \( P_{q\bar{q}} \) | \( \Delta u_1^{\Sigma^+} \) | \( \Delta d_1^{\Sigma^+} \) | \( \Delta s_1^{\Sigma^+} \) | \( P_{q\bar{q}} \) | \( \Delta u_1^{\Sigma^+} \) | \( \Delta d_1^{\Sigma^+} \) | \( \Delta s_1^{\Sigma^+} \) |
|------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1    | \( \bar{t}\bar{t} \) | 0.066 ± 0.007  | 0              | −1/3           | 0              | 0.071 ± 0.008  | 0              | −1/3           | 0              |
|      | \( s\bar{s} \)   | 0.020 ± 0.002  | 0              | 0              | −1/3           | 0.021 ± 0.003  | 0              | 0              | −1/3           |
| 2    | \( \bar{t}\bar{t} \) | 0.010 ± 0.001  | 0              | −1/3           | 0              | 0.010 ± 0.001  | 0              | −1/3           | 0              |
|      | \( s\bar{s} \)   | 0              | −              | −              | −              | 0              | −              | −              | −              |
| 3    | \( \bar{t}\bar{t} \) | 0.017 ± 0.002  | −1/4           | −1/12          | 0              | 0.018 ± 0.002  | −1/4           | −1/12          | 0              |
|      | \( s\bar{s} \)   | 0.004 ± 0.001  | 0              | 0              | −1/3           | 0.004 ± 0.001  | 0              | 0              | −1/3           |
| 4    | \( \bar{t}\bar{t} \) | 0.001 ± 0.000  | 0              | −1/3           | 0              | 0.001 ± 0.000  | 0              | −1/3           | 0              |
|      | \( s\bar{s} \)   | 0              | −              | −              | −              | 0              | −              | −              | −              |
| 5    | \( \bar{t}\bar{t} \) | 0.022 ± 0.002  | 0              | −1/3           | 0              | 0.023 ± 0.002  | 0              | −1/3           | 0              |
|      | \( s\bar{s} \)   | 0              | −              | −              | −              | 0              | −              | −              | −              |
| 6    | \( \bar{t}\bar{t} \) | 0.043 ± 0.005  | −1/4           | −1/12          | 0              | 0.046 ± 0.005  | −1/4           | −1/12          | 0              |
|      | \( s\bar{s} \)   | 0.010 ± 0.001  | 0              | 0              | −1/3           | 0.011 ± 0.001  | 0              | 0              | −1/3           |
| 7    | \( \bar{t}\bar{t} \) | 0.002 ± 0.001  | 0              | −1/3           | 0              | 0.002 ± 0.000  | 0              | −1/3           | 0              |
|      | \( s\bar{s} \)   | 0              | −              | −              | −              | 0              | −              | −              | −              |
| 8    | \( \bar{t}\bar{t} \) | 0.054 ± 0.006  | 3/4            | 1/4            | 0              | 0.117 ± 0.013  | 37/72          | −1/72          | −12/72         |
|      | \( s\bar{s} \)   | 0.008 ± 0.001  | 0              | 0              | 1              | 0.018 ± 0.002  | 1/3            | 0              | 0              |
| 9    | \( \bar{t}\bar{t} \) | 0.003 ± 0.001  | 0              | 1              | 0              | 0.006 ± 0.001  | 4/9            | −4/9           | 1/3            |
|      | \( s\bar{s} \)   | 0              | −              | −              | −              | 0              | −              | −              | −              |
| 10   | \( \bar{t}\bar{t} \) | 0.008 ± 0.001  | 0              | 1              | 0              | 0.017 ± 0.002  | 1/2            | −1/4           | 1/12           |
|      | \( s\bar{s} \)   | 0              | −              | −              | −              | 0              | −              | −              | −              |
| 11   | \( \bar{t}\bar{t} \) | 0.004 ± 0.000  | 0              | 1              | 0              | 0.009 ± 0.001  | 1/3            | −1/6           | 1/6            |
|      | \( s\bar{s} \)   | 0.004 ± 0.001  | 0              | 0              | 1              | 0.010 ± 0.002  | 1/3            | 0              | 0              |
| 12   | \( \bar{t}\bar{t} \) | 0.005 ± 0.001  | 3/4            | 1/4            | 0              | 0.011 ± 0.002  | 13/36          | −1/36          | 0              |
|      | \( s\bar{s} \)   | 0.001 ± 0.000  | 0              | 0              | 1              | 0.002 ± 0.001  | 1/3            | 0              | 0              |
| 13   | \( \bar{t}\bar{t} \) | 0              | −              | −              | −              | 0              | −              | −              | −              |
|      | \( s\bar{s} \)   | 0              | −              | −              | −              | 0              | −              | −              | −              |
| 14   | \( \bar{t}\bar{t} \) | 0.019 ± 0.002  | 0              | 1              | 0              | 0.040 ± 0.004  | 1/2            | −1/4           | 1/12           |
|      | \( s\bar{s} \)   | 0              | −              | −              | −              | 0              | −              | −              | −              |
| 15   | \( \bar{t}\bar{t} \) | 0.010 ± 0.001  | 0              | 1              | 0              | 0.021 ± 0.002  | 1/3            | −1/6           | 1/6            |
|      | \( s\bar{s} \)   | 0.011 ± 0.001  | 0              | 0              | 1              | 0.024 ± 0.003  | 1/3            | 0              | 0              |
| 16   | \( \bar{t}\bar{t} \) | 0.013 ± 0.002  | 3/4            | 1/4            | 0              | 0.029 ± 0.003  | 13/36          | −1/36          | 0              |
|      | \( s\bar{s} \)   | 0.003 ± 0.000  | 0              | 0              | 1              | 0.007 ± 0.001  | 1/3            | 0              | 0              |
| 17   | \( \bar{t}\bar{t} \) | 0              | −              | −              | −              | 0.001 ± 0.000  | 7/18           | −1/12          | 1/36           |
|      | \( s\bar{s} \)   | 0              | −              | −              | −              | 0              | −              | −              | −              |
| Set | Con. | $qar{q}$ | $P_{qar{q}}$ | $\Delta u_1^{5q}$ | $\Delta d_1^{5q}$ | $\Delta s_1^{5q}$ | $P_{qar{q}}$ | $\Delta u_1^{5q}$ | $\Delta d_1^{5q}$ | $\Delta s_1^{5q}$ |
|-----|------|------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1   | $uar{u}$ | 0.066 ± 0.007 | $-1/6$ | $-1/6$ | 0 | 0.071 ± 0.008 | $-1/6$ | $-1/6$ | 0 |
|     | $sar{s}$ | 0.020 ± 0.002 | 0 | 0 | $-1/3$ | 0.021 ± 0.003 | 0 | 0 | $-1/3$ |
| 2   | $uar{u}$ | 0.010 ± 0.001 | $-1/6$ | $-1/6$ | 0 | 0.010 ± 0.001 | $-1/6$ | $-1/6$ | 0 |
|     | $sar{s}$ | 0 | - | - | - | 0 | - | - | - |
| 3   | $uar{u}$ | 0.017 ± 0.002 | $-1/6$ | $-1/6$ | 0 | 0.018 ± 0.002 | $-1/6$ | $-1/6$ | 0 |
|     | $sar{s}$ | 0.004 ± 0.001 | 0 | 0 | $-1/3$ | 0.004 ± 0.001 | 0 | 0 | $-1/3$ |
| 4   | $uar{u}$ | 0.001 ± 0.000 | $-1/6$ | $-1/6$ | 0 | 0.001 ± 0.000 | $-1/6$ | $-1/6$ | 0 |
|     | $sar{s}$ | 0 | - | - | - | 0 | - | - | - |
| 5   | $uar{u}$ | 0.022 ± 0.002 | $-1/6$ | $-1/6$ | 0 | 0.023 ± 0.002 | $-1/6$ | $-1/6$ | 0 |
|     | $sar{s}$ | 0 | - | - | - | 0 | - | - | - |
| 6   | $uar{u}$ | 0.043 ± 0.005 | $-1/6$ | $-1/6$ | 0 | 0.046 ± 0.005 | $-1/6$ | $-1/6$ | 0 |
|     | $sar{s}$ | 0.010 ± 0.001 | 0 | 0 | $-1/3$ | 0.011 ± 0.001 | 0 | 0 | $-1/3$ |
| 7   | $uar{u}$ | 0.002 ± 0.001 | $-1/6$ | $-1/6$ | 0 | 0.002 ± 0.000 | $-1/6$ | $-1/6$ | 0 |
|     | $sar{s}$ | 0 | - | - | - | 0 | - | - | - |
| 8   | $uar{u}$ | 0.054 ± 0.006 | $1/2$ | $1/2$ | 0 | 0.117 ± 0.013 | $1/4$ | $1/4$ | $-1/6$ |
|     | $sar{s}$ | 0.008 ± 0.001 | 0 | 0 | 1 | 0.018 ± 0.002 | $1/6$ | $1/6$ | 0 |
| 9   | $uar{u}$ | 0.003 ± 0.001 | $1/2$ | $1/2$ | 0 | 0.006 ± 0.001 | 0 | 0 | $1/3$ |
|     | $sar{s}$ | 0 | - | - | - | 0 | - | - | - |
| 10  | $uar{u}$ | 0.008 ± 0.001 | $1/2$ | $1/2$ | 0 | 0.017 ± 0.002 | $1/8$ | $1/8$ | $1/12$ |
|     | $sar{s}$ | 0 | - | - | - | 0 | - | - | - |
| 11  | $uar{u}$ | 0.004 ± 0.000 | $1/2$ | $1/2$ | 0 | 0.009 ± 0.001 | $1/12$ | $1/12$ | $1/6$ |
|     | $sar{s}$ | 0.004 ± 0.001 | 0 | 0 | 1 | 0.010 ± 0.002 | $1/6$ | $1/6$ | 0 |
| 12  | $uar{u}$ | 0.005 ± 0.001 | $1/2$ | $1/2$ | 0 | 0.011 ± 0.002 | $1/6$ | $1/6$ | 0 |
|     | $sar{s}$ | 0.001 ± 0.000 | 0 | 0 | 1 | 0.002 ± 0.001 | $1/6$ | $1/6$ | 0 |
| 13  | $uar{u}$ | 0 | - | - | - | 0 | - | - | - |
|     | $sar{s}$ | 0 | - | - | - | 0 | - | - | - |
| 14  | $uar{u}$ | 0.019 ± 0.002 | $1/2$ | $1/2$ | 0 | 0.040 ± 0.004 | $1/8$ | $1/8$ | $1/12$ |
|     | $sar{s}$ | 0 | - | - | - | 0 | - | - | - |
| 15  | $uar{u}$ | 0.010 ± 0.001 | $1/2$ | $1/2$ | 0 | 0.021 ± 0.002 | $1/12$ | $1/12$ | $1/6$ |
|     | $sar{s}$ | 0.011 ± 0.001 | 0 | 0 | 1 | 0.024 ± 0.003 | $1/6$ | $1/6$ | 0 |
| 16  | $uar{u}$ | 0.013 ± 0.002 | $1/2$ | $1/2$ | 0 | 0.029 ± 0.003 | $1/6$ | $1/6$ | 0 |
|     | $sar{s}$ | 0.003 ± 0.000 | 0 | 0 | 1 | 0.007 ± 0.001 | $1/6$ | $1/6$ | 0 |
| 17  | $uar{u}$ | 0 | - | - | - | 0.001 ± 0.000 | $1/24$ | $1/24$ | $1/4$ |
|     | $sar{s}$ | 0 | - | - | - | 0 | - | - | - |
### TABLE AV: The numerical results of the probabilities $\mathcal{P}_{q\bar{q}}$ and the matrix elements of the quark spin $\Delta j^{\Sigma_-}$ of $\Sigma^-$, conventions are the same as in Table AV.

| Con. $i$ | $q\bar{q}$ | $\mathcal{P}_{q\bar{q}}$ | $\Delta j^{\Sigma_-}$ | $\Delta j^{\Sigma_-}$ | $\Delta j^{\Sigma_-}$ | $\Delta j^{\Sigma_-}$ |
|----------|-------------|---------------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 1        | $\bar{t}\bar{t}$ | 0.066 ± 0.007 | $-1/3$ | 0 | 0 | 0.071 ± 0.008 | $-1/3$ | 0 | 0 |
|          | $s\bar{s}$ | 0.020 ± 0.002 | 0 | 0 | $-1/3$ | 0.021 ± 0.003 | 0 | 0 | $-1/3$ |
| 2        | $\bar{t}\bar{t}$ | 0.010 ± 0.001 | $-1/3$ | 0 | 0 | 0.010 ± 0.001 | $-1/3$ | 0 | 0 |
|          | $s\bar{s}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ |
| 3        | $\bar{t}\bar{t}$ | 0.017 ± 0.002 | $-1/12$ | $-1/4$ | 0 | 0.018 ± 0.002 | $-1/12$ | $-1/4$ | 0 |
|          | $s\bar{s}$ | 0.004 ± 0.001 | 0 | 0 | $-1/3$ | 0.004 ± 0.001 | 0 | 0 | $-1/3$ |
| 4        | $\bar{t}\bar{t}$ | 0.001 ± 0.000 | $-1/3$ | 0 | 0 | 0.001 ± 0.000 | $-1/3$ | 0 | 0 |
|          | $s\bar{s}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ |
| 5        | $\bar{t}\bar{t}$ | 0.022 ± 0.002 | $-1/3$ | 0 | 0 | 0.023 ± 0.002 | $-1/3$ | 0 | 0 |
|          | $s\bar{s}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ |
| 6        | $\bar{t}\bar{t}$ | 0.043 ± 0.005 | $-1/12$ | $-1/4$ | 0 | 0.046 ± 0.005 | $-1/12$ | $-1/4$ | 0 |
|          | $s\bar{s}$ | 0.010 ± 0.001 | 0 | 0 | $-1/3$ | 0.011 ± 0.001 | 0 | 0 | $-1/3$ |
| 7        | $\bar{t}\bar{t}$ | 0.002 ± 0.001 | $-1/3$ | 0 | 0 | 0.002 ± 0.000 | $-1/3$ | 0 | 0 |
|          | $s\bar{s}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ |
| 8        | $\bar{t}\bar{t}$ | 0.054 ± 0.006 | $1/4$ | $3/4$ | 0 | 0.117 ± 0.013 | $-1/72$ | 0.37/72 | $-1/12$ |
|          | $s\bar{s}$ | 0.008 ± 0.001 | 0 | 0 | 1 | 0.018 ± 0.002 | 0 | 1/3 | 0 |
| 9        | $\bar{t}\bar{t}$ | 0.003 ± 0.001 | 1 | 0 | 0 | 0.006 ± 0.001 | $-4/9$ | 4/9 | 1/3 |
|          | $s\bar{s}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ |
| 10       | $\bar{t}\bar{t}$ | 0.008 ± 0.001 | 1 | 0 | 0 | 0.017 ± 0.002 | $-1/4$ | 1/2 | 1/12 |
|          | $s\bar{s}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ |
| 11       | $\bar{t}\bar{t}$ | 0.004 ± 0.000 | 1 | 0 | 0 | 0.009 ± 0.001 | $-1/6$ | 1/3 | 1/6 |
|          | $s\bar{s}$ | 0.004 ± 0.001 | 0 | 0 | 1 | 0.010 ± 0.002 | 0 | 1/3 | 0 |
| 12       | $\bar{t}\bar{t}$ | 0.005 ± 0.001 | $1/4$ | $3/4$ | 0 | 0.011 ± 0.002 | $-1/36$ | 13/36 | 0 |
|          | $s\bar{s}$ | 0.001 ± 0.000 | 0 | 0 | 1 | 0.002 ± 0.001 | 0 | 1/3 | 0 |
| 13       | $\bar{t}\bar{t}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ |
|          | $s\bar{s}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ |
| 14       | $\bar{t}\bar{t}$ | 0.019 ± 0.002 | 1 | 0 | 0 | 0.040 ± 0.004 | $-1/4$ | 1/2 | 1/12 |
|          | $s\bar{s}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ |
| 15       | $\bar{t}\bar{t}$ | 0.010 ± 0.001 | 1 | 0 | 0 | 0.021 ± 0.002 | $-1/6$ | 1/3 | 1/6 |
|          | $s\bar{s}$ | 0.011 ± 0.001 | 0 | 0 | 1 | 0.024 ± 0.003 | 0 | 1/3 | 0 |
| 16       | $\bar{t}\bar{t}$ | 0.013 ± 0.002 | $1/4$ | $3/4$ | 0 | 0.029 ± 0.003 | $-1/36$ | 13/36 | 0 |
|          | $s\bar{s}$ | 0.003 ± 0.000 | 0 | 0 | 1 | 0.007 ± 0.001 | 0 | 1/3 | 0 |
| 17       | $\bar{t}\bar{t}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0.001 ± 0.000 | $-1/12$ | 7/18 | 1/36 |
|          | $s\bar{s}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | 0 | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ | $-\Delta j^{\Sigma_-}$ |
### TABLE AVI: The numerical results of the probabilities $P_{q\bar{q}}$ and the matrix elements of the quark spin $\Delta f_i^{\Xi^0}$ of $\Xi^0$, conventions are the same as in Table [11]

| Con. | $\bar{q}q$ | $P_{q\bar{q}}$ | $\Delta u_i^{q\bar{q}}$ | $\Delta d_i^{q\bar{q}}$ | $\Delta s_i^{q\bar{q}}$ | $P_{q\bar{q}}$ | $\Delta u_i^{q\bar{q}}$ | $\Delta d_i^{q\bar{q}}$ | $\Delta s_i^{q\bar{q}}$ |
|------|------------|----------------|--------------------------|--------------------------|--------------------------|----------------|--------------------------|--------------------------|--------------------------|
| 1    | $\bar{s}s$ | $0.083 \pm 0.009$ | $-2/9$                   | $-1/9$                   | $0$                      | $0.099 \pm 0.011$ | $-2/9$                   | $-1/9$                   | $0$                      |
| 2    | $\bar{s}s$ | $0.010 \pm 0.001$ | $0$                      | $-1/3$                   | $0$                      | $0.011 \pm 0.001$ | $0$                      | $-1/3$                   | $0$                      |
| 3    | $\bar{s}s$ | $0.011 \pm 0.002$ | $-2/9$                   | $-1/9$                   | $0$                      | $0.012 \pm 0.001$ | $-2/9$                   | $-1/9$                   | $0$                      |
| 4    | $\bar{s}s$ | $0.002 \pm 0.001$ | $0$                      | $-1/3$                   | $0$                      | $0.002 \pm 0.001$ | $0$                      | $-1/3$                   | $0$                      |
| 5    | $\bar{s}s$ | $0.022 \pm 0.002$ | $0$                      | $-1/3$                   | $0$                      | $0.025 \pm 0.003$ | $0$                      | $-1/3$                   | $0$                      |
| 6    | $\bar{s}s$ | $0.027 \pm 0.003$ | $-2/9$                   | $-1/9$                   | $0$                      | $0.030 \pm 0.004$ | $-2/9$                   | $-1/9$                   | $0$                      |
| 7    | $\bar{s}s$ | $0.005 \pm 0.001$ | $0$                      | $-1/3$                   | $0$                      | $0.006 \pm 0.001$ | $0$                      | $-1/3$                   | $0$                      |
| 8    | $\bar{s}s$ | $0.028 \pm 0.003$ | $2/3$                    | $1/3$                    | $0$                      | $0.064 \pm 0.007$ | $1/18$                   | $-1/18$                  | $1/3$                    |
| 9    | $\bar{s}s$ | $0.006 \pm 0.001$ | $0$                      | $1$                      | $0$                      | $0.014 \pm 0.002$ | $0$                      | $-1/3$                   | $2/3$                    |
| 10   | $\bar{s}s$ | $0.008 \pm 0.001$ | $0$                      | $1$                      | $0$                      | $0.018 \pm 0.002$ | $1/12$                   | $-1/4$                   | $1/2$                    |
| 11   | $\bar{s}s$ | $0$              | $0$                      | $0$                      | $0$                      | $0$              | $0$                      | $0$                      | $0$                      |
| 12   | $\bar{s}s$ | $0.003 \pm 0.001$ | $2/3$                    | $1/3$                    | $0$                      | $0.006 \pm 0.001$ | $1/18$                   | $-1/18$                  | $1/3$                    |
| 13   | $\bar{s}s$ | $0.001 \pm 0.001$ | $0$                      | $1$                      | $0$                      | $0.001 \pm 0.000$ | $1/12$                   | $-1/4$                   | $1/2$                    |
| 14   | $\bar{s}s$ | $0.018 \pm 0.002$ | $0$                      | $1$                      | $0$                      | $0.041 \pm 0.005$ | $1/12$                   | $-1/4$                   | $1/2$                    |
| 15   | $\bar{s}s$ | $0.028 \pm 0.003$ | $2/3$                    | $1/3$                    | $0$                      | $0.065 \pm 0.008$ | $1/18$                   | $-1/18$                  | $1/3$                    |
| 16   | $\bar{s}s$ | $0.008 \pm 0.001$ | $2/3$                    | $1/3$                    | $0$                      | $0.018 \pm 0.002$ | $1/18$                   | $-1/18$                  | $1/3$                    |
| 17   | $\bar{s}s$ | $0.001 \pm 0.001$ | $0$                      | $1$                      | $0$                      | $0.003 \pm 0.001$ | $1/12$                   | $-1/4$                   | $1/2$                    |
TABLE AVII: The numerical results of the probabilities $P q\bar{q}$ and the matrix elements of the quark spin $\Delta f_i^{\pi_0}$ of $\Xi^-$, conventions are the same as in Table III

| Con. | $q\bar{q}$ | Set I | | Set II | |
|------|-------------|-------|-------|-------|-------|
|      | $P q\bar{q}$ | $\Delta u_i^{\pi_0}$ | $\Delta d_i^{\pi_0}$ | $\Delta s_i^{\pi_0}$ | $\Delta u_i^{\pi_0}$ | $\Delta d_i^{\pi_0}$ | $\Delta s_i^{\pi_0}$ |
| 1    | $l\bar{l}$  | 0.083 ± 0.009 | -1/9 | -2/9 | 0 | 0.094 ± 0.011 | -1/9 | -2/9 | 0 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 2    | $l\bar{l}$  | 0.010 ± 0.001 | -1/3 | 0 | 0 | 0.011 ± 0.001 | -1/3 | 0 | 0 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 3    | $l\bar{l}$  | 0.011 ± 0.002 | -1/9 | -2/9 | 0 | 0.012 ± 0.001 | -1/9 | -2/9 | 0 |
|      | $s\bar{s}$  | 0.009 ± 0.001 | 0 | 0 | -1/3 | 0.010 ± 0.002 | 0 | 0 | -1/3 |
| 4    | $l\bar{l}$  | 0.002 ± 0.000 | -1/3 | 0 | 0 | 0.002 ± 0.001 | -1/3 | 0 | 0 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 5    | $l\bar{l}$  | 0.022 ± 0.002 | -1/3 | 0 | 0 | 0.025 ± 0.003 | -1/3 | 0 | 0 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 6    | $l\bar{l}$  | 0.027 ± 0.003 | -1/9 | -2/9 | 0 | 0.030 ± 0.004 | -1/9 | -2/9 | 0 |
|      | $s\bar{s}$  | 0.023 ± 0.002 | 0 | 0 | -1/3 | 0.026 ± 0.003 | 0 | 0 | -1/3 |
| 7    | $l\bar{l}$  | 0.005 ± 0.001 | -1/3 | 0 | 0 | 0.006 ± 0.001 | -1/3 | 0 | 0 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 8    | $l\bar{l}$  | 0.028 ± 0.003 | 1/3 | 2/3 | 0 | 0.064 ± 0.007 | -1/18 | 1/18 | 1/3 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 9    | $l\bar{l}$  | 0.006 ± 0.001 | 1 | 0 | 0 | 0.014 ± 0.002 | -1/3 | 0 | 2/3 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 10   | $l\bar{l}$  | 0.008 ± 0.001 | 1 | 0 | 0 | 0.018 ± 0.002 | -1/4 | 1/12 | 1/2 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 11   | $l\bar{l}$  | 0 | - | - | - | 0 | - | - | - |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 12   | $l\bar{l}$  | 0.003 ± 0.001 | 1/3 | 2/3 | 0 | 0.006 ± 0.001 | -1/18 | 1/18 | 1/3 |
|      | $s\bar{s}$  | 0.003 ± 0.001 | 0 | 0 | 1 | 0.006 ± 0.001 | 0 | 0 | 1/3 |
| 13   | $l\bar{l}$  | 0.001 ± 0.001 | 1 | 0 | 0 | 0.001 ± 0.000 | -1/4 | 1/12 | 1/2 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 14   | $l\bar{l}$  | 0.018 ± 0.002 | 1 | 0 | 0 | 0.041 ± 0.005 | -1/4 | 1/12 | 1/2 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 15   | $l\bar{l}$  | 0.028 ± 0.003 | 1/3 | 2/3 | 0 | 0.065 ± 0.008 | -1/18 | 1/18 | 1/3 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| 16   | $l\bar{l}$  | 0.008 ± 0.001 | 1/3 | 2/3 | 0 | 0.018 ± 0.002 | -1/18 | 1/18 | 1/3 |
|      | $s\bar{s}$  | 0.007 ± 0.001 | 0 | 0 | 1 | 0.015 ± 0.002 | 0 | 0 | 1/3 |
| 17   | $l\bar{l}$  | 0.001 ± 0.001 | 1 | 0 | 0 | 0.003 ± 0.001 | -1/4 | 1/12 | 1/2 |
|      | $s\bar{s}$  | 0 | - | - | - | 0 | - | - | - |
| Con. | $\bar{q}q$ | $P_{\bar{q}q}$ | $\Delta u_1^{\bar{s}q}$ | $\Delta d_1^{\bar{s}q}$ | $\Delta s_1^{\bar{s}q}$ | $P_{\bar{q}q}$ | $\Delta u_1^q$ | $\Delta d_1^q$ | $\Delta s_1^q$ |
|------|-----------|----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|
| 1    | $\bar{s}s$ | 0.115 ± 0.013 | −1/6            | −1/6            | 0              | 0.128 ± 0.014 | −1/6            | −1/6            | 0              |
| 2    | $\bar{s}s$ | 0.003 ± 0.001 | −1/6            | −1/6            | 0              | 0.003 ± 0.000 | −1/6            | −1/6            | 0              |
| 3    | $\bar{s}s$ | 0.003 ± 0.001 | 0               | 0               | −1/3           | 0.003 ± 0.000 | 0               | 0               | −1/3           |
| 4    | $\bar{s}s$ | 0.015 ± 0.001 | −1/6            | −1/6            | 0              | 0.016 ± 0.002 | −1/6            | −1/6            | 0              |
| 5    | $\bar{s}s$ | 0.006 ± 0.001 | 0               | 0               | −1/3           | 0.007 ± 0.001 | 0               | 0               | −1/3           |
| 6    | $\bar{s}s$ | 0.006 ± 0.001 | −1/6            | −1/6            | 0              | 0.007 ± 0.001 | 0               | 0               | −1/3           |
| 7    | $\bar{s}s$ | 0.037 ± 0.004 | −1/6            | −1/6            | 0              | 0.041 ± 0.004 | −1/6            | −1/6            | 0              |
| 8    | $\bar{s}s$ | 0.015 ± 0.002 | 0               | 0               | −1/3           | 0.017 ± 0.002 | 0               | 0               | −1/3           |
| 9    | $\bar{s}s$ | 0.053 ± 0.006 | 1/2             | 1/2             | 0              | 0.117 ± 0.013 | 0               | 0               | 1/3            |
| 10   | $\bar{s}s$ | 0.003 ± 0.001 | 1/2             | 1/2             | 0              | 0.006 ± 0.001 | 1/8             | 1/8             | 1/12           |
| 11   | $\bar{s}s$ | 0.002 ± 0.001 | 0               | 0               | 1              | 0.005 ± 0.001 | 1/12            | 1/12            | 1/6            |
| 12   | $\bar{s}s$ | 0.010 ± 0.001 | 1/2             | 1/2             | 0              | 0.022 ± 0.002 | 1/12            | 1/12            | 1/6            |
| 13   | $\bar{s}s$ | 0.004 ± 0.001 | 1/2             | 1/2             | 0              | 0.009 ± 0.001 | 1/24            | 1/24            | 1/4            |
| 14   | $\bar{s}s$ | 0.002 ± 0.000 | 0               | 0               | 1              | 0.004 ± 0.001 | 1/12            | 1/12            | 1/6            |
| 15   | $\bar{s}s$ | 0.005 ± 0.001 | 1/2             | 1/2             | 0              | 0.011 ± 0.002 | 1/8             | 1/8             | 1/12           |
| 16   | $\bar{s}s$ | 0.006 ± 0.000 | 0               | 0               | 1              | 0.013 ± 0.001 | 1/12            | 1/12            | 1/6            |
| 17   | $\bar{s}s$ | 0.025 ± 0.002 | 1/2             | 1/2             | 0              | 0.055 ± 0.006 | 1/12            | 1/12            | 1/6            |
|      | $\bar{s}s$ | 0.004 ± 0.001 | 0               | 0               | 1              | 0.010 ± 0.001 | 1/12            | 1/12            | 1/6            |

Table VIII: The numerical results of the probabilities $P\bar{q}q$ and the matrix elements of the quark spin $\Delta f_1^{\bar{s}q}$ of $\Lambda$, conventions are the same as in Table A1.