Dark energy as an emergent phenomenon

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ABSTRACT

The cosmological constant, which was introduced by Einstein a century ago to allow for a static universe, experienced a revival two decades ago under the label dark energy as a parameter to model the observed accelerated expansion of the universe. Its physical nature has however remained enigmatic. Here we use the Einstein equations without cosmological constant to show that the origin of the accelerated expansion is not in the equations but has to do with the boundary conditions related to the causal horizon, which exists because the age of the universe is finite. Via transformations to conformal coordinates and Euclidian spacetime we find a resonance condition that uniquely determines the dimensionless parameter $\Omega_\Lambda$ that governs the observed cosmic acceleration: $\Omega_\Lambda = \pi (t_H/t_f)^2$, where $t_H$ is the Hubble time and $t_f$ is the conformal age of the universe. This explanation leads to a somewhat modified cosmology, in which the expansion rate of the early universe is 2.1 times faster than in the standard model. We show that Big Bang nucleosynthesis calculations with the faster expansion rate requires the mean baryon density to be raised to the level of the total matter density to agree with the observed deuterium abundance. This appears to eliminate the need to invoke the existence of some yet to be discovered exotic particles to explain dark matter, since all of it may be baryonic while still remaining consistent with the observed abundances of the light elements.

Key words. dark energy – dark matter – cosmology: theory – gravitation – early universe – primordial nucleosynthesis

1. Introduction

According to standard cosmology the universe is 13.8 billion years old and spatially flat, but only 4.9% of the critical mean energy density that is required for flatness is in the form of ordinary matter, baryons. The rest of the energy density resides in two enigmatic components, 25.9% as dark matter and 69.1% as dark energy (Planck Collaboration et al. 2016). While the discovery of dark matter goes back to Zwicky (1937) and remains a mystery eight decades later, dark energy was introduced two decades ago to account for the accelerated expansion of the universe that was revealed by the use of supernovae type Ia as standard candles (Riess et al. 1998; Perlmutter et al. 1999). The physical nature of dark energy also remains a complete mystery and is being treated as a fitting parameter in the standard cosmological models, in the form of a cosmological constant.

Einstein (1917) introduced the cosmological constant to allow for a static universe but considered it a blunder after observations revealed that the universe was not static but expanding. All later observations were compatible with a zero cosmological constant $\Lambda$ until the accelerated expansion was discovered. $\Lambda$ represents a property of the spacetime metric in the form of a vacuum energy that one would expect in quantum field theories (QFT). It can therefore be expected to represent the vacuum energy in a theory of quantum gravity. The problem is that estimates based on straightforward dimensional analysis lead to a predicted value for $\Lambda$ that is about 122 orders of magnitude larger than the presently observed value (cf. Eq. (41) below). This monumental discrepancy illustrates how far we are from an understanding of the gravitational role of the quantum vacuum. It has led to the belief that a resolution of the dark energy enigma will require a theory of quantum gravity (cf. Binetruy, 2013).

If the vacuum energy were as large as expected from QFT, and if this energy were a source of gravity, then the universe would collapse on a miniscule timescale without the opportunity to grow large enough to allow a sufficient timescale for biological evolution. Long before the accelerated expansion was discovered Weinberg (1987) invoked the anthropic principle to set tight upper limits on the magnitude of $\Lambda$, which would be compatible with our existence as observers. While such an argument effectively restricted the allowed range, it opened the door to the possibility of parallel universes with other values of $\Lambda$, most of which (an infinity of them) with values incompatible with the existence of life. String theory allows a “landscape” of $10^{500}$ possible universes, and as no theoretical procedure to choose among them has yet been found, the anthropic argument has again been invoked and presented as if it would offer an explanation for the kind of universe that we live in (cf. Ellis & Smolin, 2009).

Before the discovery of accelerated expansion it seemed that the mean energy density of the universe was much smaller than the critical density required for spatial flatness, even when accounting for all the invisible dark matter. This would be incompatible with inflation, which predicts that the present universe must have nearly zero curvature. The discovered dark energy filled the gap. All forms of energy, baryonic, non-baryonic, and vacuum energy, now add up to the critical density, thus restoring flatness.

One major conceptual problem with a cosmological constant is that it leads to a cosmology that is in gross violation of the Copernican principle, which states that we are not privileged observers in the universe. In the cosmological context it is often referred to as the cosmic coincidence problem. When the universe expands as described by a scale factor $a(t)$, $\Lambda$ stays constant while the densities of matter and radiation vary as $a^{-3}$ and...
$a^{-2}$, respectively. This implies that the radiation energy density in the Planck era dominated over the vacuum energy density by 122 orders of magnitude, while in the future it is the vacuum energy density that will dominate by an increasing number of orders of magnitude. We happen to live in an epoch when the vacuum and matter energy densities are of the same order. This extraordinary coincidence has not been given any explanation other than again referring to the anthropic principle.

Let us now turn from dark energy to dark matter. Evidence for its existence comes from a variety of observations, the most important ones being the rotation curves of galaxies, the velocity dispersion of galaxies in clusters, and gravitational lensing. While explanations in terms of modified gravity have been tried, the general consensus is now that dark matter really exists in the form of particles. Particularly convincing evidence for this comes from observations of a collision between two galaxy clusters, the Bullet Cluster, which shows that the visible component is significantly displaced by the collision relative to the invisible (but gravitating) dark matter component, something that has no explanation in terms of modified gravity (cf. Markovitch et al. 2004; Clowe et al. 2006).

It is clear that dark matter must consist of cold (non-relativistic), dark collisionless matter, but this does not exclude that it can be baryonic. The main reason why it is widely believed to be non-baryonic comes from the constraints of BBN (Big Bang Nucleosynthesis) calculations. Only about 5% of the critical density of the universe can be in baryonic form if the BBN predictions are to agree with the observed abundances of the light elements. Since the total matter fraction is about 30%, the implication is that there is about 5 times more non-baryonic than baryonic matter. Often the non-baryonic matter is referred to as WIMPs, weakly interacting massive particles.

Nobody knows what kind of particles WIMPs are made of, although major search efforts have been carried out for several decades. The searches are made with large underground detectors in order to filter out spurious signals from cosmic-ray particles. Attempts to produce hypothetical dark matter particles by colliders like LHC at CERN have also been unsuccessful so far. As time goes on without anything else than null results, the credibility of the belief that most of the dark matter is in an exotic non-baryonic form suffers. However, as good alternative explanations of dark matter are unavailable, the search continues unabated.

The aim of the present paper is to show that the unaltered Einstein equations without a cosmological constant lead to an accelerated expansion of the universe of the observed magnitude, as a consequence of the boundary conditions that must be enforced to preserve consistency. The observable universe is bounded by being enclosed inside a causal horizon, which exists because the age of the universe is finite. The causal boundary constraint leads to a resonance condition. Because this condition is always tied to the size of the causal horizon, the coincidence problem disappears. The main resulting modification of the cosmological evolution is an expansion rate in the early universe that is 2.1 times faster than in the standard model. With this faster rate the BBN predictions appear to give agreement with the observed abundances of the light elements only if the baryonic mean density is increased from 5% to values of order 30%, which would eliminate the need to invoke the existence of non-baryonic matter or WIMPs to account for these abundances.

In Sect. 2 we first explore the properties of the gravitational potential in the presence of a cosmological constant. The Newtonian potential gets changed into a Helmholtz potential that represents the solution of a wave equation. The spatial scale of the wave is related to the radius of the causal horizon. In Sect. 3 we then remove the cosmological constant from the equations and determine the resonance condition that is induced by the presence of the causal horizon. This requires the use of conformal coordinates, with which the expansion factor $a(t)$ is transformed away, as well as the transformation to Euclidian spacetime. The resonance condition uniquely determines the value of $\Lambda$ without the use of any free parameters. In Sect. 4 we use a simplified BBN treatment of deuterium production in the early universe to show that with our non-standard enhanced expansion rate the baryon density needs to be enhanced to the level of the total matter density to preserve agreement with the observed deuterium abundance. We finally summarize the conclusions in Sect. 5.

2. Gravitational potential in the presence of a cosmological constant

The present paper aims at explaining dark energy as an emergent phenomenon that is not explicitly present in the underlying equations for the metric and the gravitational field. Before addressing the question of its origin, let us here start by taking a look at the roles played by the cosmological $\Lambda$ term when it is inserted ad hoc in the Einstein equations in the standard way. This term has two main effects. While its role of a vacuum energy density (dark energy) acting as repulsive gravity is well known, its second role as a kind of vacuum polarization representing a feedback of the vacuum energy on the gravitational interaction has been largely overlooked. In the present section we will highlight this feedback and show how it leads to a wave equation for the interaction.

2.1. Poisson equation for the gravitational potential

Newtonian gravity faces serious inconsistency problems when trying to deal with an infinite distribution of matter. If one tries to apply the shell theorem, the force on a particle depends on the arbitrary choice of center for the spherical geometry. As described by Ghosh & Dey (2016), already Laplace (1880) tried to deal with this problem by introducing a Yukawa-like exponential cut-off of the gravitational potential $\Phi$:

$$\Phi \sim -e^{-r/r_s},$$

where $r_s$ is the screening or cut-off distance that defines the finite range of the gravitational force.

Such a Yukawa-like potential was also applied for similar reasons by Seelig (1895) and Neumann (1896). It results from the screened Poisson equation

$$\nabla^2 \Phi - \lambda \Phi = 4\pi G \rho,$$

which served as Einstein’s Newtonian starting point when he introduced his cosmological constant $\lambda$ in his 1917 cosmological paper (Einstein 1917).

There has since been considerable confusion whether or not a positive cosmological constant really leads to a Yukawa-like gravitational potential. Thus Straumann (2002) points out that Einstein, Weyl, and Pauli saw the cosmological term as a Yukawa term, but he then argues that this interpretation is incorrect, since the stationary solution for $\Phi$ given by general relativity for a homogeneous universe is

$$\nabla^2 \Phi = 4\pi G (\rho - 2\rho_\Lambda),$$

where $\rho$ is the mean matter density, while

$$\rho_\Lambda \equiv \frac{c^2 \Lambda}{8\pi G}.$$
represents the vacuum energy density, with \( \Lambda \) being the cosmological constant. When the vacuum energy term in Eq. (3) is moved over to the left-hand side, we see that its sign is opposite to that of the corresponding term in Eq. (2), and this has been taken to imply that the gravitational potential would not be screened. This conclusion has been restated in the nice review by O’Raifeartaigh et al. (2017).

However, when comparing Eqs. (1)–(3) with each other, we notice that there is something profoundly missing. While the vacuum energy term in Eq. (3) is a constant, independent of space and time, \( \lambda \) in Eq. (2) is a multiplicative factor for the \( r \)-dependent potential \( \Phi \) in Eq. (1). The \( \lambda \) term therefore induces a feedback from \( \Phi \) to its own spatial gradients in the screened Poisson equation, and it is this feedback that is the reason for the screening. Regardless of the sign issue, this feedback is missing in Eq. (3).

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}. \]  

(5)

Our treatment will be based on the convention \( + \) for the spacetime signature.

Before introducing the weak-field approximation it is convenient to rewrite Eq. (5) in the form

\[ R_{\mu\nu} = -\frac{8\pi G}{c^4} S_{\mu\nu}, \]  

(6)

where

\[ S_{\mu\nu} = T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T - \frac{c^4 \Lambda}{8\pi G} g_{\mu\nu}. \]  

(7)

This form is readily obtained from Eq. (5) in the standard way by making contraction with \( g^{\mu\nu} \) and using the circumstance that \( g^{\mu\nu} g_{\nu\rho} = 4 \). Let us write \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \), where \( \eta_{\mu\nu} \) represents the invariant Minkowski metric with diagonal elements unit and signs \( + - - - \), while \( h_{\mu\nu} \) is the spacetime dependent part of the metric. In the weak-field approximation \( |h_{\mu\nu}| \ll 1 \), and \( R_{\mu\nu} \approx \frac{1}{2} \partial^2 g_{\mu\nu} \) if we, as is usually done, adopt the harmonic gauge. \( \partial^2 \equiv (1/c^2) \partial^2 / \partial t^2 - \nabla^2 \) is the 4D d’Alembertian operator. Its spatial part is the negative Laplace operator \( -\nabla^2 \). We further assume here for simplicity that the stress-energy tensor \( T_{\mu\nu} \) consists of baryonic matter with zero equation of state (zero pressure terms), so that \( T_{00} - \frac{1}{2} g_{00} T \) can be replaced by \( \rho c^2 / 2 \). Then only the \( g_{00} \) part of the metric has material sources. Its weak field equation is

\[ \partial^2 g_{00} = -\frac{8\pi G}{c^4} (\rho - \frac{c^2 \Lambda g_{00}}{4 \pi G}). \]  

(8)

As \( \partial^2 g_{00} = \partial^2 h_{00} \) because the derivatives of \( h_{00} \) = 1 vanish, the weak-field equation is usually written in a form where \( g_{00} \) on the left-hand side is replaced by \( h_{00} \), and \( g_{00} \) on the right-hand side is replaced by unity, since both \( h_{00} \) and \( \Lambda \) are small quantities. However, it is essential to retain the \( h_{00} \) contribution to the \( \Lambda \) term on the right-hand side, because it is the source of vacuum polarization effects that will give us a wave equation for the potential. Therefore we need to write the weak field equation in terms of \( g_{00} \), not in terms of \( h_{00} \).

Since the small \( h_{00} \) term has generally been neglected in previous literature for the weak-field case, let us explain why it is essential here. \( \Lambda \approx 1/r_\Lambda^2 \), where \( r_\Lambda \) is a characteristic length scale (cf. Eq. (13)). As we will see in Sect. 2.4 the small observationally determined value of \( \Lambda \) corresponds to an \( r_\Lambda \) that is approximately equal to the radius of the particle horizon of the universe. On distance scales \( r \ll r_\Lambda \) one can safely ignore the \( h_{00} \) term, as has been correctly done for instance in Jetzer & Sereño (2004) when considering the effect of\( \Lambda \) on the dynamics of stellar systems. It is however incorrect to neglect it for cosmological distances, when the condition \( r \ll r_\Lambda \) is no longer satisfied, because it is the sole source of either exponential, Yukawa-like cutoff (when the cosmological constant is negative) or wave behavior of the potential (when the cosmological constant is positive) at the characteristic \( r_\Lambda \) distance scale.

As usual the identification \( h_{00} = 2\Phi \) is made to satisfy the Newtonian limit, where \( \Phi \) is the gravitational potential, here per unit energy (not per unit mass, which would differ by the factor \( 1/c^2 \)), so that

\[ g_{00} = 1 + 2\Phi. \]  

(9)

The stationary version of Eq. (8) then gives us an extended Poisson equation with \( \Lambda \) term for the potential \( \Phi \):

\[ \nabla^2 \Phi + 2\Lambda \Phi = \frac{4\pi G}{c^2} (\rho - 2\rho_\Lambda). \]  

(10)

Apart from the definition of \( \Phi \) with respect to unit energy instead of unit mass, the equation would be identical to Eq. (3), if we were not for the profound \( 2\Lambda \) term on the left-hand side, which represents a feedback of the medium (the vacuum) to the potential \( \Phi \).

Equation (10) explicitly brings out the two physical roles played by the cosmological constant \( \Lambda \): (1) The \( \rho_\Lambda \) term on the right-hand side is a source of repulsive gravity, while (2) the \( \Lambda \) term on the left-hand side provides feedback to the potential, similar to the effect of vacuum polarization. It is important to remember that these two roles reflect two faces of the same coin, namely the two terms that make up \( g_{00} \) in Eq. (9). Role (1) comes from the term 1, role (2) from the term \( 2\Phi \). Both terms always contribute in concert. We do not have the freedom to change their relative proportions.

This unity of the two roles is implicitly contained in Eq. (8), which can be rewritten in a form that makes its structural similarity to field theory formulations transparent:

\[ \partial^2 \varphi - m^2 \varphi - J = 0, \]  

(11)

where

\[ m^2 = \frac{8\pi G}{c^2} \rho, \]  

\[ J = \frac{8\pi G}{c^2} \rho. \]  

(12)

In general the gravitational field is a tensor field, while Eq. (11) represents it as a scalar field, because we have disregarded the off-diagonal components of the stress-energy tensor. This simplification is valid for our exploration of the gravitational potential, and in particular when we later consider global wave modes in a cosmological medium that is isotropic and homogeneous.
With this formulation the effect of $\Lambda$ appears exclusively in the single $m^2$ term, there is no separate vacuum energy density $\rho_{v}^{0}$ that combines with the mass density $\rho$. This unification occurs because $\phi$ is a physical field that represents the metric (more precisely its $g_{00}$ component), which implicitly contains both $\Lambda$ effects, in contrast to the potential $\Phi$, which only represents a fractional aspect of the metric.

In standard QFT (quantum field theory) $m$ represents a mass scale, while here it is more convenient to let it represent a wave number $k_m$. This difference is however immaterial and only dependent on the choice of dimensions that we use for $\phi$. Equation (11) then has formal similarity to the Klein-Gordon equation, except for the sign of the $m^2$ term. The Klein-Gordon sign is the origin of the Yukawa-type exponential cutoff of the potential. The solution is the same if we formally replace the Yukawa term by a point source, and

$$\phi = \frac{\rho}{m^2} \equiv \frac{\rho}{c^2} .$$

The solution would be a so-called screened Poisson equation and give rise to a Yukawa potential. In reality, however, the term is positive, which gives us a Helmholtz equation with oscillatory solutions (cf. Roza 2017). Such wave equations are familiar in numerous areas of physics. The Schrödinger equation belongs to this type.

It is helpful to represent the wave number in terms of a characteristic distance scale for the oscillations:

$$m \equiv k_m \equiv 2\pi/r_A .$$

It follows that $r_A = 2\pi/\sqrt{2\Lambda}$. It turns out to be equal to the causal radius of the universe (distance to the particle horizon), as we will see in Sect. 3 (cf. Eq. (24)).

The scalar Eq. (11) follows from the Lagrangian $\mathcal{L} = \frac{1}{2}[ (\partial^2 \phi)^2 + m^2 \phi^2 ] + J\phi$.

Its Green’s function or propagator $D(x-y)$ is determined by

$$-(\partial^2 - m^2) D(x-y) = \delta^4(x-y) ,$$

with the solution

$$D(x-y) = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} .$$

We will refer to the expression for the propagator in Sect. 3.5.

### 2.3. Feedback from the vacuum to the gravitational force field

The stationary version of Eq. (8) is the Helmholtz equation

$$\nabla^2 g_{00} + k_m^2 g_{00} = \frac{8\pi G}{c^2} \rho ,$$

where $k_m^2 \equiv 2\Lambda \equiv (16\pi G/c^2)\rho_{v}$, with our previous definitions, $\rho$ is the mass density, while all vacuum effects come from the $k_m^2$ term. Below we will give the solution of the equation for a point source. In this case we need to replace $\rho$ in the source term on the right-hand side with $M \delta^3(r)$, where $M$ is the mass of the point source, and $\delta^3(r)$ is the 3D Dirac delta function.

The left-hand side of Eq. (17) has the structure of a negative Hamiltonian in the time-independent Schrödinger equation, when the force field is attractive so that the potential energy is negative. The $k_m^2$ term corresponds to minus the potential energy in the Schrödinger equation if $g_{00}$ plays the role of the wave function. This analogy illustrates why we get a wave behavior for $g_{00}$ that is similar to waves in quantum physics.

The general point source solution of Eq. (17) in spherical coordinates is

$$g_{00} = \sum_{\ell=0}^{\infty} \int_{0}^{\infty} \left[ a_{\ell m} j_{\ell}(k_A r) + b_{\ell m} y_{\ell}(k_A r) \right] Y_{\ell}^m(\theta, \varphi) ,$$

where $j_{\ell}$ and $y_{\ell}$ are the two orthogonal spherical Bessel functions of order $\ell$ (the radial wave functions in quantum mechanics), and $Y_{\ell}^m$ are the spherical harmonics. The coefficients are determined by boundary conditions.

While the non-radial solutions lead to the rich structuring that we encounter in atomic physics, we assume that the gravitational potential exhibits exact spherical symmetry. This implies that $\ell, m = 0$, so that only the spherical Bessel functions $j_0$ and $y_0$ need to be considered.

$$j_0 = \frac{\sin k_A r}{r}, \quad y_0 = \frac{-\cos k_A r}{r} .$$

The solution then reduces to

$$g_{00} = a_0 j_0 + b_0 y_0 .$$

Let us here denote the gravitational potential from a point source by $\phi$ to distinguish it from $\Phi$, which referred to the total potential that included extended sources. According to Eq. (9)

$$\phi = \frac{1}{2} (g_{00} - 1) .$$

Let $\phi_N$ refer to the corresponding Newtonian potential, which represents the case when $\Lambda = 0$. The Newtonian limit is expressed through

$$\phi_N = -\frac{1}{2} \frac{r_0}{r} \rho ,$$

with the definition

$$r_0 \equiv 2GM/c^2 .$$

We recognize this as the radius of the event horizon for a black hole of mass $M$. Here we use it to define the convenient parameter $r_0$, using the symbol $\equiv$ instead of $=$ to make it clear that we are not suggesting that we are dealing with solutions for actual black holes. In gravitational physics the same expression appears in different contexts. For instance, for an infinite, homogeneous universe with flat metric, Eq. (24) is valid when we let $r_0$ represent the Hubble radius and $M$ the total mass inside the Hubble radius.

The inner boundary condition Eq. (22) then gives us

$$b_0 = r_0 .$$

The remaining parameters ($k_m$ and $a_0$) need to be determined by outer boundary conditions, which we will identify later (Eqs. (14) and (36) in Sect. 3) with the use of the definition in Eq. (29).

Empty space or the vacuum state refers to the case when $\rho = 0$. Let us denote the gravitational potential of the vacuum state by $\Phi_A$. It then follows from Eq. (21) that

$$\Phi_A = -\frac{1}{2} ,$$

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because $g_{00} \to 0$ as $1/r$ according to Eq. (20), when the distance $r$ from the physical sources goes to infinity. This is an intriguing result, since the number $\frac{1}{r}$ is familiar from quantum physics as the energy of the vacuum state. There it arises from the quantization of the harmonic oscillator. In contrast, no quantization condition has been applied here, although the Helmholtz Eq. (17) has structural similarity to the equation for a harmonic oscillator. In our context the non-zero, flat $\Phi_\lambda$ arises because the gravitational field, which is represented by the metric and not by the potential alone, is a superposition of the potential (the fluctuating part) and the flat Minkowski background. The gravitational field that is represented by $g_{00}$ does not have a non-zero vacuum state. The minus sign of Eqs. (20), (21), and (25) we find the ratio between the Helmholtz and Newtonian accelerations to be $\frac{g_{acc}}{N} = \frac{(1 + \beta x) \cos x + (x - \beta) \sin x}{\frac{3}{2} \pi}$, (27)

where for convenience we have introduced the dimensionless distance scale $x \equiv k Ar$ (28)

and the simplified notation $\beta \equiv \frac{a_0}{b_0}$. (29)

The value of $\beta$ needs to be fixed by an outer boundary condition, which we will address in Sect. 3.6.

### 2.4. Scale of the Helmholtz oscillations

As we saw in Eq. (13), the scale $r_h$ of the Helmholtz oscillations, defined by $k_A r_h \equiv 2\pi$, is $r_h = 2\pi/\sqrt{2}\Lambda$. We now want to express it in units of the Hubble radius of the universe, $r_H = c/H_0$, where $H_0$ is the current value of the Hubble constant. For a Friedmann universe with zero curvature $H_0$ is related to the critical density $\rho_c$ through $\rho_c = \frac{3H_0^2}{8\pi G}$. (30)

In the standard cosmological models the cosmological constant represents a vacuum energy density $\rho_\Lambda = \frac{\Omega_\Lambda}{\Omega_0}$, where $\Omega_\Lambda = 0.69$ according to Planck Collaboration et al. (2016). With Eq. (30) we then find

$$\frac{r_h}{r_H} = \frac{2\pi}{\sqrt{6\Omega_\Lambda}}. \quad (31)$$

which equals 3.1 if we insert the observationally determined value for $\Omega_\Lambda$. As we will see in Sect. 3.3 and in Eq. (43), $r_A$ is equal to the causal radius (distance to the particle horizon) of the universe. It is the scale that is relevant to dark energy while being vastly larger than the scales that are relevant to dark matter. Still the resolution of the dark energy enigma leads to non-standard effects that have direct implications for the nature of dark matter, namely that all of dark matter appears to be baryonic, without the need for yet to be discovered exotic forms of matter. We will return to this issue in Sect. 4.

### 3. Origin and nature of the dark energy

Throughout the previous section we have tried to clarify some of the aspects in which gravity is affected by the $\Lambda$ term, when it is inserted ad hoc in the Einstein equation in the form of a cosmological constant. We have highlighted, in particular with the help of Eqs. (11), (12), and (17), the fundamentally different roles played by the $\Lambda$ term and the real physical sources (which are represented by the density $\rho$). The role of the $\Lambda$ term is exclusively to generate a feedback of the vacuum from the metric (or the gravitational field) to itself, and not as a direct source of gravity, a role that is instead played by real matter-energy (e.g. as demonstrated by Eq. (17)).

If the $\Lambda$ term would represent a physical field as a constant to satisfy the energy-momentum conservation equation and the Bianchi identities, then the near coincidence of the $\Lambda$-induced wave scale with the size of the current cosmic horizon would be extraordinarily unnatural as it would violate the Copernican principle (which asserts that we are not privileged observers). The horizon scale has increased by about 60 orders of magnitude since the Planck era, while $\Lambda$ would not change if it were a true constant. This cosmic coincidence would not disappear unless we abandon the view that $\Lambda$ is part of the underlying equations. The alternative is that the $\Lambda$ effects instead emerge from boundary conditions that constrain the solutions of the equations, as we will show below. If we would try to account for such boundary conditions by inserting a fitting parameter $\Lambda$ into the original equations, this parameter would masquerade as if it were a new physical field, which it is not.

In the following we will show that the $\Lambda$ effects are induced as a consequence of the finite age of the universe. This implies that the wave scale that is represented by $\Lambda$ and the scale of the causal horizon must remain linked throughout all epochs of cosmic history. The cosmic coincidence problem then goes away.

#### 3.1. Vacuum energy induced by the finite age of the universe

Usually time is viewed as a coordinate along an infinite axis, which extends backwards before the Big Bang and forwards into the (as yet non-existing) future beyond the present moment. If however the age of the universe is finite, time is bounded in a physical sense, because it does not exist beyond the two temporal boundaries (Big Bang and the present moment), as we will further clarify below. Let $\Delta t$ denote the length of time between these two boundaries. While a Fourier decomposition of a constant field along an infinite time line would give a delta function $\delta(\omega)$, an infinitely sharp peak at zero frequency, the corresponding decomposition for a truncated time line would give a frequency spread $\Delta \omega$ that is approximately given by $\Delta \omega \Delta t \approx \frac{1}{4}$. (32)

If we multiply both sides by $\hbar$ we recognize Heisenberg’s uncertainty principle, with $\Delta E = \hbar \Delta \omega$ interpreted as the vacuum energy that is induced by constraining the temporal interval. The smaller this interval is, the more violent are the fluctuations in energy. These general arguments will be made more precise in Sect. 3.3 where we develop a concrete, quantitative theory for the emergence of a vacuum energy $\Lambda$ of the observed magnitude, as a consequence of the finite age of the universe.

If a finite-age universe is to be compatible with quantum mechanics (and obey the Heisenberg uncertainty principle), then it is inevitable that the universe must have started with a hot Big Bang, because the energy variance $\Delta E$ that represents the vacuum fluctuations goes to infinity when $\Delta t$ goes to zero. Likewise...
\( \Delta E \) goes to zero in the distant future because it is inversely proportional to the age of the universe. This vacuum energy density, induced by the existence of temporal boundaries or horizons, represents a scale that is always linked to the horizon scale for any magnitude of the horizon radius.

While it is generally accepted that time may have a beginning or edge at the Big Bang, time is usually considered to be unbounded in the forward, future direction. This seems to contradict our notion of a finite time line that ends at the present moment and does not extend into the future, and which therefore represents a 1D temporal cavity. The reason why time is finite and not semi-infinite is that the future is unobservable.

When we look out into the universe, we look back in time, until we reach the edge presented by the Big Bang. One speaks of look-back time, but there is no such thing as look-forward time. There can be no causal influences reaching us from the future. Therefore the present represents a causal boundary that cannot be crossed from the “outside” (the future). Our temporal 1D cavity is constrained between two causal boundaries, at the Big Bang and at the present time. Similarly, space is bounded, between “here” and the causal (particle) horizon. The bounded, observable 4D volume increases continually as the horizon advances both spatially and temporally (into the future) with the age of the universe.

3.2. Flatness of space

Equation (17) explicitly showed us that only real mass-energy (represented by \( \rho \)) but not \( \Lambda \) is a source of gravity. To understand this statement better, a comparison with Debye screening may be helpful. The source of the electric field is the point charge that is embedded in a plasma environment that is filled with a continuum of charges. These environmental charges are however not a source of the field, their role is to provide the screening of the potential. Similarly, \( \rho \) is the source, while \( \Lambda \) provides the feedback, which because of its sign does not result in screening, but instead endows the (Helmholtz) potential with wave properties.

Gravity manifests itself by changing the metric. In the absence of sources for gravity spacetime is flat. An empty universe (\( \rho = 0 \)) has no gravity, no curvature. In a non-empty universe that expands forever \( \rho \to 0 \) as time \( t \to \infty \). The obvious boundary condition at infinity for such a universe must therefore be that it has Minkowski metric and not something else (like de Sitter metric).

If the curvature of space is zero at a given epoch, then it follows from the standard cosmological equations that it is zero at all epochs. There is no need to invent a hypothetical inflationary phase for the purpose of explaining why the metric of the universe is observed to be flat. Since \( \Lambda \) without \( \rho \) cannot be a source of gravity, as we have shown above, it follows that there cannot be an exponentially expanding phase in the distant future, the end phase of the universe is not a de Sitter phase.

The starting point of the following treatment is therefore to consider a homogeneous and isotropic universe with zero curvature and without any cosmological constant present in the underlying equations. We will then show how the finite age of the universe leads to effects, which are the same as those of a cosmological constant \( \Lambda \) with a value that matches the observed one. No free parameters or model fitting are used to derive this. Nature did not have a choice to generate \( \Lambda \) effects of different magnitude.

3.3. Cosmic horizons and conformal coordinates

In Sect. 3.1 we gave qualitative arguments why \( \Lambda \)-like effects should emerge when we constrain the time line, because the set of Fourier modes that can fit within the finite interval gets restricted. To express these ideas in a precise and quantitative way we first need to clarify the meaning of causal boundaries or horizons in an expanding universe. This also allows us to clarify what we mean by the stationary solution for the Helmholtz potential in Sect. 2.3 when we are dealing with an expanding universe with a time-varying scale factor \( a(t) \).

Observations tell us that the current age of the universe is 13.8 Gyr, while the Hubble time \((1/H_0)\) is 14.4 Gyr. Although the Hubble radius at 14.4 billion light years (Glyr) is often referred to as the “cosmic horizon", this terminology is a bit misleading, because the Hubble radius is “just” a parameter, not a physical horizon. The causal or particle horizon, from beyond which no forces or interactions can reach us, is at 46.9 Glyr. This distance equals the speed of light times the conformal age of the universe (which is 46.9 Gyr). It is the time it would take for a photon to reach us from the causal horizon if the universe would stop expanding. The normal and the conformal age of the universe are different because the points from which the propagating light tries to reach us continually recede from us due to the cosmic expansion.

The radius of the horizon is differentially stretched by the expansion, as expressed by the distance-redshift relation. The usual way to describe this is in terms of the Robertson-Walker metric with a scale factor \( a(t) \) that multiplies the spatial differential distance. For many purposes it is much more useful to instead use conformal coordinates, with which the scale factor has been transformed away so that the metric formally becomes Minkowski-like.

While ordinary cosmic time \( t = \int_0^t dt \), conformal time \( \tau \) is defined by

\[
\tau = \int_0^\tau \frac{dt}{a}
\]

or, equivalently, \( d\tau/a = 1/a \). In a spacetime diagram based on conformal coordinates (conformal time vs. comoving radial distance), light rays are straight lines with a slope given by the speed of light, just like they are in inertial coordinate systems.

The conformal coordinate background provides an arena in which it makes sense to make Fourier decompositions, because conformal invariance implies that geometric shapes and angles are preserved. The sine and cosine components of a Fourier decomposition do not get deformed (differentially redshifted) by the stretching of space when we use conformal coordinates, they retain their sine and cosine shapes. It is against this coordinate background that one should interpret the metric perturbations that one finds when solving the weak-field Einstein equations for the stationary case, as we did in Sect. 2.2. In a normal, expanding coordinate system it is not well defined what we mean by a stationary case. The Helmholz potential that we found in Sect. 2.2 needs to be expressed in terms of conformal coordinates.

3.4. Euclidian spacetime

The treatment of the \( \Lambda \)-like effects that emerge because the observable time line is constrained can best be done in 4D Euclidian spacetime, because time can then be viewed as an angular coordinate, which reveals the existence of a resonance condition that is the reason for the effects that we refer to as dark energy. The transformation from Minkowski to Euclidian spacetime is
done through Wick rotation in the complex plane, such that or-
dinary time \( t \) becomes Euclidian time \( t_E = i c t \), where we have
inserted \( c \) to express \( t_E \) in spatial units like the three other coordi-
nates. Since time now becomes imaginary, it may be interpreted as
an angular coordinate with period \( 2\pi \), which corresponds to
the length \( \ell \) of a finite time string that has periodic boundary
conditions. In our case \( \ell \) is the conformal age of the universe
expressed in Euclidian time. The various modes of the Fourier decomposition then have wave number \( 2\pi n/\ell \), where \( n \)
is an integer.

The transformation to Euclidian spacetime leads to remark-
able, even miraculous advantages and insights. The Lagrangian,
which is used for the formulation of the Einstein equations (cf.
Eq. (14) for the weak-field case), becomes the Hamiltonian,
which is the agent that drives the cosmic evolution. Quantum
field theory QFT transforms into the structure of classical sta-
tistical mechanics. The path integral in field theory then corre-
sponds to the partition function in statistical mechanics, with the
oscillating phase factors in QFT now appearing as the Boltz-
mann factors, which allow the definition of a temperature. The
transformation thereby establishes a direct link between field
theories like general relativity or QFT and thermodynamics. In
particular it provides a direct route to the derivation of the Hawk-
ing temperature of black holes. For a brief introduction to this
topic, see for instance [Zet 2010].

3.5. Wave modes induced by the finite age of the universe

The following wave mode discussion will relate to the treatment
of the weak-field approximation that we did in Sect. 2.2. This
approximation is valid for all cosmological epochs except for the
very early universe, in particular when we approach the Planck
era. However, as this strong-field era is of miniscule temporal
extent as compared with the relevant cosmological time scales
that we are dealing with here, the resonant condition that we will
identify as the origin of the \( \Lambda \) effects remains valid although it is
based on the weak-field treatment.

In the Euclidian spacetime the 4D d’Alembertian operator
becomes \( \partial^2 = \partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2 \), because this spacetime has signature
\( (++++) \). Its inverse, representing the field propagator, is \( \sim 1/k^2 \),
where the square of the 4D wave number can be written as \( k^2 = \kappa_2 + \kappa_3 \). Here \( k^2 = k_2^2 + k_3^2 \), with \( k_{1,2,3} \) representing the usual
spatial wave numbers, while \( k_0 \) is now the angular frequency of
Euclidian time and represents the temporal modes.

As a consequence of the periodic boundary conditions (that
result because Euclidian time is cyclic), the temporal Fourier
transform with the factor \( \exp(ik_0t) \) gets restricted to values for
which \( k_0 = 2n\pi/\ell \) for the nth harmonic, where the string
length \( \ell = i c t_E \), with \( c \) being the conformal age of the uni-
verse, 46.9 Gyr according to standard cosmology. It follows that
the square of the 4D wave number is \( k^2 = k_2^2 + (2n\pi/\ell)^2 \)
for the nth harmonic mode, implying that we have effectively lost
one dimension. The expression represents a discrete set of sta-
tionary modes for the spatial 3D wave number. Because it is the
temporal dimension that has been lost, we have retrieved the sta-
cular case that we need to make direct comparison with the
Corresponding stationary case of Eqs. (16) and (17). This allows
us to relate our expression for \( k_0 \) with the dark energy parameters
\( \Lambda \) or \( k_0 \).

The partition function that governs the probability distribu-
tion over the possible wave modes is the sum over the respec-
tive Boltzmann factors that are generated by the Wick rotation:
\[ \sum_{n=1}^{\infty} e^{-2\pi n^2} = 1/(e^{2\pi} - 1) \], which shows how a Planck distribu-
tion emerges. As the probability for excitation to the next higher
harmonic decreases by the Boltzmann factor \( e^{-2\pi n^2} \approx 1/535 \),
it is a good approximation to only consider the fundamental mode
(with \( n = 1 \)) as relevant. We will do this here.

With the help of the Boltzmann factor one may introduce a
temperature. Although we do not need to make use of the tem-
perature concept in order to derive \( \Lambda \), we mention it here because
it may be of interest to indicate how it is related to the Hawking
horizon and a horizon. If we in the Boltzmann factors make the
identification \( \hbar \omega/(k_BT) = \omega_c \), we get \( T = \hbar/(k_BT) \), which
here has the stupendously small value of about \( 10^{-20} \) K due to the
gigantic value of \( \omega_c \). Using the expression for the Schwarzschild
radius of black holes, \( r_{BH} \equiv c^2/r_{BH} = 2GM/c^2 \), where we for
convenience of comparison have introduced a black hole time
scale \( t_{BH} \), we can convert the standard expression for the Hawking
temperature \( T_H \) to the form \( T_H = \hbar/(4\pi k_BT) \). This expres-
sion is the same as that of our simplistic derivation if we replace
\( t_{BH} \) with \( t_c \), with the exception of the numerical factor 4\pi, which
may be due to the greatly different geometrical situations in the
two cases (as our universe does not have a Schwarzschild met-
rical). While not directly needed for our derivation of \( \Lambda \), it is worth
paying attention to the potentially profound implicit connections
that causal horizons have with thermodynamics and quantum
physics.

When converting back from Euclidian age \( t \) to ordinary con-
formal age \( t_c \), while disregarding the higher harmonics, we get in
conformal coordinates \( k^2 = k^2 - m^2 \), where
\[ m^2 \equiv k_0^2 = \left[ 2\pi/(ct_c) \right]^2 \].

Here we have for later use (in Sect. 3.9) reintroduced the wave
number \( k_0 \) that we first introduced in Eq. (13). The field propa-
gator \( 1/k^2 \) that we started off with has thus become \( 1/(k^2 - m^2) \),
eclusively as a result of the finite length of the time line. As
this now represents a stationary spatial wave pattern, it needs to
be compared with the propagator of Eq. (16) for the stationary
case, when \( \partial/\partial t \) and the corresponding wave number \( k_0 \) are zero.
In this case the denominator in Eq. (16) is \( -k^2 + m^2 \), which is
identical to the propagator that we derived via bounded time,
except for the overall sign. This global sign is however immaterial.
It is a consequence of using the signature \((-+++ \)) for the
Minkowski metric rather than \((-+++ \)), and because of the cir-
cumstance that when we transformed to Euclidian coordinates,
we switched the sign of the temporal but not the spatial part in
the signature of the metric. While the overall sign does not mat-
ter, the relative sign between the \( k^2 \) and \( m^2 \) terms is essential. It
agrees with Eq. (15), which implies that the stationary gravita-
tional potential is of Helmholtz and not Yukawa type, and that
the cosmological \( \Lambda \) parameter, which is obtained from the iden-
tification \( m^2 = 2\Lambda \) of Eq. (12), is positive.

For readers who may be confused by this derivation of the
sign for \( \Lambda \), because we have gone back and forth between Eu-
clidian and Minkowski coordinates, the following heuristic argu-
ment why the sign of \( \Lambda \) must be positive may be helpful. The
finite time string may be viewed as an infinite time line on which
we have imposed a rectangular window of width \( t_c \), which cuts
off everything outside the window. This rectangular restriction
is qualitatively similar to the exponential Yukawa-type cutoff,
when applied to the time line. In the time domain this cutoff has
the consequence that \( k_0^2 \) changes to \( k_0^2 + m^2 \) (with \( m \) representing
the inverse cutoff scale), where we notice the same signs in front
of \( k_0^2 \) and \( m^2 \). However, this cutoff-induced \( m^2 \) term then has a
sign that is opposite to that of the spatial \( k^2 \) term because of the
signature of the Minkowski metric. As we have shown before,
this has the consequence that in the spatial domain the gravitational potential is of Helmholtz and not Yukawa type, and that $\Lambda$ is positive.

Let us note that for a homogeneous universe without spatial gradients, the circumstance that $k_0^2$ combines with $m^2$ with a + sign implies a Yukawa-type exponential temporal behavior (induced by time being finite). This exponential temporal behavior is in cosmology usually expressed in terms of a de Sitter (A) term, which, if being the sole source of evolution, would lead to an exponential expansion, in contrast to exponential decay in the standard Yukawa case. However, in both the de Sitter and Yukawa cases the equations allow both exponentially growing and decaying solutions, the selection is made by the boundary conditions that we impose. The de Sitter solutions are growing, because we choose to start off with a compact Big Bang.

3.6. Boundary condition at the causal horizon

With the concepts and tools that we have developed in the preceding sections, let us now come back to our exploration of the Helmholtz potential that we did in Sect. 2, since we are now in a position to define the previously unspecified outer boundary condition. This will allow us to illustrate (in Fig. 1 below) the behavior of Helmholtz gravity and to compare it with Newtonian gravity. The treatment in the present section is not needed for our derivation of the dark energy parameter $\Omega_\Lambda$, which is the subject of the following section, it is only needed for the completion of our presentation of the behavior of Helmholtz gravity.

The particle horizon constitutes the natural outer boundary, since no causal effects or interactions emanating from objects beyond this distance can reach us, including all gravitational effects. Then continuity demands that all interactions, including all accelerations, must vanish at this distance.

A treatment of the inner boundary condition, in the Big Bang at the beginning of time, is beyond the scope of the present paper, because it is in the realm of quantum gravity. Let us however briefly reflect on the implications of the requirement that all interactions should vanish at both causal boundaries, also the inner one, to satisfy continuity. Similar to the outer boundary case, there should be no interactions from anything before the beginning of time, because this part of the universe does not exist (if physical time is truly finite).

The vanishing of all interactions as time $t \to 0$ implies that the beginning is a state of asymptotic freedom. At sufficiently small temporal and spatial scales the gravitational interactions should go to zero if this natural boundary condition is to be satisfied. While we are not yet in a position to specify the scales at which such asymptotic freedom would be reached, it is reasonable to expect the transition to be related to the scales of the Planck era, but this is a topic that we will not pursue more here.

According to Eq. (28) the Helmholtz gravitational acceleration $g_{acc}$ can be written (ignoring the constant of proportionality) as

$$g_{acc} \approx -(1 + \beta x) \cos x + (x - \beta) \sin x / x^2.$$  \hspace{1cm} (35)

With the same constant of proportionality the corresponding Newtonian acceleration is $\sim -1/x^2$. Here $x$ is the dimensionless distance parameter defined by $x \equiv kr$ as in Eq. (28), with the wave number given by $k_0 = 2\pi/c$ according to Eq. (45). As before $t_0$ is the conformal age of the universe, and $r$ is the comoving distance coordinate (speed of light times conformal time).

$\beta$ is a parameter of the Helmholtz solution that has to be determined by an outer boundary condition. This condition is that $g_{acc}$ must vanish at the causal boundary, where $r = c t_0$ and therefore $x = 2\pi$. While $\sin x$ then vanishes at the boundary, $\cos x$ does not. Therefore $g_{acc}$ can only vanish there if $1 + 2\pi \beta = 0$, which unambiguously fixes the value of $\beta$:

$$\beta = \frac{1}{2\pi} \approx -0.159.$$  \hspace{1cm} (36)

In Fig. 1 we have used this value for $\beta$ to plot the expression on the right-hand side of Eq. (35) (after reversing its sign) as the solid curve, with the corresponding Newtonian $1/x^2$ as the dashed curve. The horizontal axis represents the comoving radial distance in units of the Hubble radius, which is marked by the vertical dotted line. The curve for the Helmholtz acceleration ends where it meets the vertical solid line that marks the position of the causal horizon. The exact position of the causal horizon in these distance units is uniquely determined from the solution of Eq. (45), as will be explained in the next section.

We notice that the attractive Helmholtz force is substantially stronger than the Newtonian force at smaller and intermediate distances. About halfways to the outer boundary the Helmholtz force changes sign and becomes repulsive, before it has to vanish at the boundary.

The enhancement of the force at smaller distances may raise the question whether such a deviation from Newtonian gravity could potentially be an explanation for the phenomenon behind what we refer to as “dark matter”. The answer to this question is “no”. The enhancement that we get with our Helmholtz force does not exceed a factor of two, which is far too little to account for dark matter. To be a viable explanation for dark matter the enhancement would need to be larger by more than an order of magnitude and occur at scales about $10^4$ times smaller than the horizon scale. We are not aware of any physical effects that would lead to modified gravity with such properties.
3.7. Link to the Planck era and the uniqueness of $\Omega_\Lambda$

The critical value $\rho_c$ of the mean mass density in a Friedmann universe with zero spatial curvature is

$$\rho_c = \frac{3}{8\pi G t_H^2},$$  \hspace{1cm} (37)

where $t_H = 1/H$ is the Hubble time. This scaling between $\rho$ and time $t$ is identical to that for a black hole of radius $r_{BH} \equiv c t_{BH}$ (used to define the time scale $t_{BH}$) and mass $M_{BH}$. From the expression

$$t_{BH} = \frac{2GM_{BH}}{c^3}$$ \hspace{1cm} (38)

for the Schwarzschild radius and assuming a homogeneous density distribution $\rho$ so that $M_{BH} = 4\pi\rho c^3 t_{BH}^3/3$, we recover Eq. (37) if we replace $\rho$ with $\rho_c$ and $t_{BH}$ with $t_H$.

In Friedmann cosmology $\rho_c$ marks the boundary case between open and closed model universes. For black holes the corresponding density marks the boundary between stability and instability with respect to black-hole formation. To see how this relates to the Planck era, and subsequently how the present mean density of the universe relates to the Planck density, let us first define the Planck era as the epoch when mini black holes get spontaneously formed by vacuum fluctuations. Let us for convenience use the notation $\Delta E \equiv M_{BH} c^2$ to define the energy content of such a mini black hole, and denote $\Delta t \equiv t_{BH}$. Then, with the use of Eq. (38), the Heisenberg criterion for spontaneous formation of such mini black holes is

$$\Delta E \Delta t = \frac{c^5}{2G} r_{BH}^2 = \frac{1}{2} \hbar,$$ \hspace{1cm} (39)

which defines the Planck time $t_P$ as the $t_{BH}$ that satisfies Eq. (39):

$$t_P = \left( \frac{\hbar G}{c^3} \right)^{1/2}.$$ \hspace{1cm} (40)

Because all black holes and flat Friedmann universes, regardless of their size, scale according to Eq. (37), we can relate the present critical mean density $\rho_c$ of the universe to the density $\rho_P$ in the Planck era (at time $t = t_P$) through

$$\frac{\rho_c}{\rho_P} = \left( \frac{t_H}{t_P} \right)^2 = H_0^2 t_P^2 \approx 10^{-122},$$ \hspace{1cm} (41)

since $t_P = 5.38 \times 10^{-44}$ s and $t_H = 14.4$ Gyr = $4.54 \times 10^{17}$ s. This beautiful scaling relation over 122 orders of magnitude would be wrecked if a hypothetical inflationary phase would be introduced.

Let us now see how the emergent vacuum energy that is represented by the $\Lambda$ parameter relates to all this. The most convenient representation of $\Lambda$ is in the form of the dimensionless parameter $\Omega_\Lambda$, which we introduced in Sect. 2.4. It is defined via the vacuum energy density $\rho_\Lambda$ that was introduced in Eq. (44) through

$$\Omega_\Lambda \rho_c \equiv \rho_\Lambda \equiv \frac{c^2 \Lambda}{8\pi G}.$$ \hspace{1cm} (42)

With Eq. (37) we then obtain

$$\Lambda = \frac{3\Omega_\Lambda}{c^2 t_H^2},$$ \hspace{1cm} (43)

while from Eqs. (34) and (12) we get the same $\Lambda$ when derived as an emergent quantity that is a consequence of the finite age of the universe:

$$\Lambda = \frac{2\pi^2}{c^4 t_c^2},$$ \hspace{1cm} (44)

where $t_c$ is the conformal age of the universe. Combining Eqs. (43) and (44) we find the expression for the dimensionless $\Omega_\Lambda$:

$$\Omega_\Lambda = \frac{2}{3} \left( \frac{\pi t_H}{t_c} \right)^2.$$ \hspace{1cm} (45)

Note that $\Lambda$ in Eq. (44) depends exclusively on the conformal age $t_c$, while $\Omega_\Lambda$ in Eq. (45) depends on the ratio $t_c/t_H$. The reason why the Hubble time appears in Eq. (45) is that $\Omega_\Lambda$ represents the fraction of the critical density $\rho_c$ that is in the form of dark energy, and $\rho_c \sim 1/t_H^2$ according to Eq. (37).

In the models of standard cosmology the ratio $t_c/t_H$ between the conformal and Hubble times depends on the cosmological parameters, including $\Omega_\Lambda$ (because it depends on the shape of the $a(t)$ function), but it is close to $\pi$ for the parameters used in standard cosmology. Equation (45) then gives $\Omega_\Lambda \approx 2/3$, which is consistent with the value adopted from observations.

Due to the dependence of $t_c/t_H$ on $\Omega_\Lambda$, there is in fact a unique solution for both $\Omega_\Lambda$ and $t_c/t_H$ from Eq. (45), namely $\Omega_\Lambda = 0.664$ and $t_c/t_H = 3.15$ (in the case that $\Omega_M$ is due to matter rather than radiation, see below). Any other value is prohibited, since it would not be consistent with this equation. These values should be compared with the corresponding values derived by Planck Collaboration et al. (2014) from the CMB observations when applying the interpretational framework of standard cosmology: 0.69 and 3.26 (= 46.9 Gl yr / 14.4 Gl yr), respectively. The agreement between observations and theory can be considered good, in particular since we do not use any free fitting parameters in our theory, and the CMB observations have been interpreted with a theoretical framework that is not identical to ours.

From the definition of conformal time in Eq. (33), the solution for $t_c/t_H$ can be written as

$$t_c = \frac{1}{t_H} \int_0^{t_c} \frac{dt}{a} = \int_0^\infty \frac{dz}{E(z)},$$ \hspace{1cm} (46)

where $z$ is the redshift, $t_H$ is the age of the universe in normal cosmic time units, and

$$E(z) = [\Omega_E (1 + z)^w + \Omega_\Lambda]^{1/2}. \hspace{1cm} (47)$$

Since this expression represents the case of zero curvature, $\Omega_E = 1 - \Omega_\Lambda$ can be transformed away (cf. Longair 2012). Here we have introduced parameters $\Omega_E$ and $n = 3(1 + w)$, where $w$ is the equation of state parameter, to allow for the two main cases when the universe is matter-dominated ($w = 0$) as it is at present, in which case $\Omega_E = \Omega_M$ and $n = 3$, and when it is radiation dominated ($w = 1/3$ as in the early universe), in which case $\Omega_E = \Omega_R$ (dimensionless density parameter for radiation) and $n = 4$. The expression for $t_c/t_H$ in Eq. (46) can be inserted in Eq. (45), which can then be solved numerically. For the matter-dominated universe in which we now live, $\Omega_\Lambda = 0.66$, while in the radiation-dominated era $\Omega_\Lambda = 0.93$. Thus almost all of the energy density in the early universe was in the form of the vacuum energy that is a consequence of the restricted extent of the physical time line.
$\Omega_\Lambda$ remains constant for all times as long as there is no change in the equation of state for $\Omega_\Lambda$. In the radiation-dominated era $\Omega_\Lambda$ stays at 93% all the way back to the Planck era, in the matter-dominated era it stays at 66% for all future times. The present balance between $\Omega_M$ and $\Omega_\Lambda$ leads to the value 1.07 for the ratio between the Hubble time and the age of the universe (while it is 11.4/13.8 = 0.14 according to Planck Collaboration et al. [2016]), which is close to unity. This implies that the expansion parameter $a(t)$ is nearly linear with respect to time except for a small deceleration, and it will remain so forever. There will be no transition to an exponentially expanding phase as is often believed and which would result in an utterly empty universe. In the radiation-dominated early universe the ratio Hubble time to age of the universe is slightly smaller, about 96%, which also implies an almost linear expansion except for a small acceleration as a result of the larger $\Omega_\Lambda$.

4. Implications for dark matter

The results of the previous section demonstrate how our explanation of dark energy as an emergent phenomenon leads to non-standard features outside the current framework, although it accounts for the observed $\Lambda$ effects and is largely consistent with overall aspects of standard cosmology. The non-standard aspects represent an advantage, because the theory becomes amenable to future observational tests that may bring in different perspectives on unsolved issues in cosmology.

Here we will focus on one such unsolved, long-standing enigma in astrophysics: the nature of dark matter. There have been numerous attempts to account for dark matter through a parametrized modification of Newtonian gravity to fit the observed rotation curves of galaxies, starting with the MOND (modified Newtonian dynamics) approach introduced by Milgrom (1983).

The term $\Omega_0 = j_0$ in Eq. (44) for the Helmholtz potential actually has the right functional form to give an excellent fit to the observed galaxy rotation curves if we were to use $\rho_0$ and wave number $k_2 \equiv 2\pi/n_\Lambda \equiv \sqrt{2\Lambda}$ (cf. Eqs. (13) and (17)) as free parameters. However, in our case the wave scale implied by $k_2$ is not a free parameter but is fixed by the value of $\Lambda$, which is observationally tied to the accelerated expansion of the universe, and which is also theoretically fixed via Eq. (45).

While our $k_2$ implies a characteristic distance scale that is given by the radius of the causal horizon, the galaxy distance scale over which we need significant modifications of the Newtonian potential to account for the dark matter signatures is smaller by at least about four orders of magnitude. Since $\Lambda \sim 1/\ell_\Lambda^2$, we would need a value of $\Lambda$ that is larger by a factor of about $10^8$ to induce significant Helmholtz effects on the Newtonian potential at galaxy scales, but such a possibility is prohibited by cosmological constraints.

There now exists rather convincing observational evidence that disfavor an explanation of dark matter in terms of a modification of gravity, in particular from the observations of a pair of colliding galaxy clusters, the so-called Bullet Cluster (cf. Markevitch et al. 2004; Clowe et al. 2006), which reveal a significant offset between the visible matter distribution and the dark matter inferred from gravitational lensing. While this offset implies that the invisible component must consist of collisionless matter, it does not imply that this matter should be non-baryonic and consist of some yet to be discovered weakly interacting massive particles (WIMPs).

The main reason why most of dark matter is believed to be non-baryonic is the required compatibility with predictions from BBN (Big Bang nucleosynthesis) calculations. A small baryonic mean density ($\Omega_B \approx 5\%$ of the critical density $\rho_c$) is needed within the standard cosmological framework for agreement of BBN with the observed abundances of the light chemical elements. The situation however changes with our new theoretical framework, because the expansion rate at the epoch of nucleosynthesis is different. Here we will show that when this non-standard aspect is accounted for, agreement between observed abundances and BBN calculations requires that $\Omega_B$ is instead of the same order as the present $\Omega_B = 1 - \Omega_\Lambda$, which implies that (within the uncertainties of the derivations) all of the dark matter is baryonic, there may no longer be any justification for introducing anything exotic of not yet discovered nature.

For an overview of BBN physics we refer to Peebles (1993). It is outside the scope of the present paper to carry out comprehensive BBN calculations that fully account for the non-standard effects of our new theoretical framework in quantitative detail. We will limit our focus to the deuterium abundance, which is the “baryometer of choice” (cf. Steigman 2007) because of its particularly high sensitivity to the adopted value of the relative baryon abundance $\Omega_B$.

The bottleneck for all subsequent BBN processes is deuterium formation. There is competition between deuterium creation through proton-neutron collisions and deuterium photodissociation by the ambient radiation field. BBN could only start when the general photon energy fell below the binding energy of deuterium, which occurred when the temperature in the Big Bang dropped below $10^9$ K. Then all the free neutrons got quickly captured to form deuterium. Once a significant fraction of deuterium nuclei $d$ were available, they started to be used up for the production of $^4\text{He}$, with $d + d$ collisions as the first step, leading to tritium (t) or $^3\text{He}$ formation, followed by t + d and $^3\text{He} + d$ reactions.

This had the result that all free neutrons ended up in $^4\text{He}$ while deuterium was destroyed in the process. The deuterium destruction was however incomplete because of the density drop due to the expansion of the universe, which brought the deuterium destruction to a halt. The remaining, undestroyed abundance of deuterium is therefore a very sensitive function of the initial baryon density, parametrized by the dimensionless $\Omega_B$, and the expansion rate that is responsible for cutting off the destruction process. This is the reason why deuterium is the baryometer of choice, our measure of the $\Omega_B$ parameter.

In standard cosmology the effects of spacetime curvature and a hypothetical cosmological constant become negligible in the early universe, because matter and radiation scale with the scale factor $a(t)$ like $a^{-3}$ and $a^{-4}$ to become the dominant drivers of the expansion in the early universe. The expansion rate gets uniquely determined by the Friedmann solution that describes the radiation-dominated era, for which the Hubble time is 2.0 times the age of the universe. The only remaining free parameter for the BBN calculations is $\Omega_B$, which is then constrained to be about 5% to agree with the observed deuterium abundance.

A non-standard feature that follows from our explanation of dark energy as an emergent phenomenon is that the expansion rate in the early universe is 2.09 times faster than in the models of standard cosmology (as explained below). This has the consequence that the deuterium destruction process is terminated significantly sooner, leaving a fraction of undestroyed deuterium that is much higher than the observed abundance, unless we compensate the faster expansion rate by using a higher value for the baryon density. Raising $\Omega_B$ increases the deuterium
destruction rate, to allow the same fraction of deuterium to be destroyed within the shorter time interval that is available for this process.

Next we will quantify these arguments by showing that an increase of the expansion rate by a factor of 2.09 requires an increase of $\Omega_B$ to the approximate level of the total matter density $\Omega_M$ to restore agreement between the BBN predictions and the observed deuterium abundance. Since $\Omega_M$ includes all of dark matter, we are led to the conclusion that there may be no need for non-baryonic matter to explain the observed deuterium abundance. Since the increase of the expansion rate by a factor of 2.09 requires an increase of the free neutrons get captured into deuterium nuclei. (2) Subsequent deuterium destruction begins, whereby deuterium is destroyed within the shorter time interval that is available for destruction rate, to allow the same fraction of deuterium to be destroyed within the shorter time interval that is available for this process.

Here we will limit ourselves to an idealized BBN treatment, since the full solution of the nuclear rate equations in our non-standard cosmology is beyond the scope of the present paper. Our main idealization is to treat deuterium creation and destruction as occurring in two separated stages: (1) As soon as photodissociation vanishes when the expanding universe cools, all of the free neutrons get captured into deuterium nuclei. (2) Subsequently the deuterium destruction begins, whereby deuterium gets converted into helium via $^3\text{He}$ or tritium. This destruction process occurs with a rate $\gamma$ and duration $\Delta t$, after which it ceases, leaving a surviving deuterium abundance that is a factor $\exp(-\gamma \Delta t)$ smaller at the initial value at the beginning of stage 2. $\Delta t$ is proportional to the Hubble time or inverse expansion rate.

In reality the two stages overlap. The destruction process does not wait until the creation process is finished, but sets in during Hubble times. At the end of Sect. 3.7 we showed that in standard cosmology it is 2.0 times the age. The ratio $\Omega_B$ of baryons to dark matter is $\Omega_B = 2.0 \times \Omega_M$. This suggests that all dark matter may indeed be baryonic plus non-baryonic. Removing index $s$ from $\Omega_B$ we then obtain from Eq. (48)

$$\left(\frac{\Omega_M}{\Omega_B}\right)^{\alpha} = R_s^\kappa,$$

where

$$R_s \equiv e^{-\gamma \Delta t},$$

and

$$\kappa \equiv \frac{\Delta t_{ns}}{\Delta t_s} - 1.$$

According to Planck Collaboration et al. (2016) $\Omega_M/\Omega_B = 6.3$. The destruction factor $R_s$ is the ratio between the final value of $y_D$ and its initial value $y_D(0)$ at the beginning of stage 2 in our idealized scenario. If all free neutrons at the start of stage 1 end up as part of $^4\text{He}$ with a mass fraction $Y$ and do it after first having been absorbed into deuterium, then the mass fraction of deuterium at the beginning of stage 2 is also $Y$. Expressing $Y$ in terms of the number density $y_D(0)$ relative to hydrogen, we have

$$Y = 2y_D(0)/(2y_D(0) + 1),$$

which, when inverted, gives

$$y_D(0) = \frac{0.5}{1 - Y}.$$

According to Steigman (2007) $y_D = 2.6 \times 10^{-5}$ and $Y = 0.249$, which according to Eq. (52) gives $y_D(0) = 0.166$. We then find $R_s = y_D/y_D(0) = 1.57 \times 10^{-4}$, which demonstrates that only a minor fraction of the deuterium survives destruction in the Big Bang.

The ratio $\Delta t_{ns}/\Delta t_s$ equals the ratio between the corresponding Hubble times. At the end of Sect. 3.7 we showed that in the radiation-dominated era of our non-standard cosmology the Hubble time is 95.8% of the age of the universe, while in standard cosmology it is 2.0 times the age. The ratio $\Delta t_{ns}/\Delta t_s$ is therefore $0.958/2.0 = 0.479$, which gives us $\kappa = -0.521$.

As we have now assigned observationally constrained values for all the parameters in Eq. (49) except for the parameter $\alpha$, we can solve for $\alpha$, obtaining

$$\alpha = 2.5.$$

This value should be compared with the previously mentioned midrange slope value of $\alpha \approx 3$ that is representative for the actual slope derived from rigorous BBN calculations. In view of the uncertainties of our simplified treatment the agreement between the two values is sufficiently close to satisfy our consistency test: With the $1/0.479 = 2.09$ times faster expansion rate of our theory, agreement between the BBN calculations and the observed deuterium abundance gets restored if the baryon density parameter $\Omega_B$ is raised to the level of the total matter density parameter $\Omega_M$. This suggests that all dark matter may indeed be baryonic without violating BBN, there may be no reason to
introduce yet to be discovered exotic particles for the purpose of explaining the observed deuterium abundance. Because of our simplified treatment, however, this conclusion still needs to be validated by full and rigorous BBN calculations.

If all dark matter is indeed baryonic, it is clear that most of it must be cold and collisionless, and therefore have macroscopic properties that are not that different from those of WIMPs. Such behavior would be the case if it for instance would be composed of grains, rocks, and primordial black holes, with a spectrum of sizes spanning from tiny grains to planetary size bodies. Larger dark bodies (MACHO — Massive Astrophysical Halo Objects) appear to be disfavored by constraints from gravitational microlensing as major candidates for dark matter, although the evidence is not yet conclusive (cf. Calchi Novati et al. 2005; Tisserand et al. 2007). A high value of Ω<sub>M</sub> poses other issues beyond BBN, which need to be clarified, for example compatibility with the observed signatures of baryon acoustic oscillations (BAO) in the CMB spectrum. The CMB imprint is in the form of a characteristic distance scale that represents the sound horizon, the comoving distance that sound waves can travel from the time of the Big Bang to the time when the baryons decouple from the radiation field. Like in the BBN case we have two competing effects. The speed of sound depends on the baryon density, while the travel time depends on the expansion rate of the universe. Both are modified in our non-standard cosmology. It is however outside the scope of the present paper to work out the details of this here.

5. Conclusions

Our resolution of both the dark energy and dark matter conundrums has been achieved without the use of any free parameters and without any modification of the Einstein equations for gravity, except for the removal of the “cosmological constant” from these equations. The origin of the observed accelerated expansion of the universe is not in the formulation of the equations for gravity, but has to do with the cosmic boundary conditions.

The cosmological phenomena that are generally referred to with the label “dark energy” may be seen as the result of a global cosmic resonance that emerges because the age of the universe is finite. The Λ term has the dimension of the square of a frequency, which we may think of as the “pitch” of the universe. In the beginning, when the horizon of the universe was small, the pitch was high, but it got lower as the universe increased in size, in inverse proportion to the horizon radius.

In contrast, when Λ is put in by hand as a cosmological constant, the pitch always remains the same, regardless of whether the universe is small or large. Since there is no physical justification for inserting such a constant, the anthropic principle has often been invoked in the guise of an “explanation”. The existence of biological life constrains the allowed values of Λ to a narrow range around the actually observed value. Such an argument opens the door to the proliferation of parallel universes with different values of the cosmological constant, some of which are harbouring life, while most of them do not.

According to the present work the possibility of universes with other values of Λ does not exist. Nature did not have a choice, because the requirement of logical consistency leads to uniqueness.

The explanation of dark energy as an emergent phenomenon leads to non-standard cosmological consequences. One of these consequences provides a resolution of the dark matter enigma. This resolution is not in terms of modified gravity, because this would require a major modification at scales several orders of magnitude smaller than the horizon scale, for which there is no justification. Instead dark matter must really be made up of physical particles. However, because of the non-standard cosmology that follows from our explanation of dark energy, all the dark matter particles now may be baryonic, there may not be any need to invoke the existence of some yet to be discovered exotic particles (WIMPs).

The main reason for the belief in the existence of non-baryonic matter has come from the comparison of the observed abundances of the light elements with the predictions from BBN (Big Bang nucleosynthesis) calculations. While the relative fraction Ω<sub>B</sub> of matter in the universe is of order 30%, at most 5% can be baryonic (Ω<sub>Λ</sub>) to satisfy the BBN constraints.

These constraints are however based on the framework of standard cosmology. The non-standard aspects in our theory for dark energy lead to an expansion rate in the early universe that is 2.1 times the expansion rate in the standard cosmological models used for the BBN calculations. When the faster value of the expansion rate is used for the calculations, the baryonic mass fraction Ω<sub>B</sub> must be increased to the level around that of Ω<sub>Λ</sub> to be compatible with the observed abundances. To show this we have used a simplified treatment focused on the case of the deuterium abundance, so this resolution of the dark matter enigma still needs to be validated by more complete and rigorous modeling of BBN and other relevant observational constraints, like the observed CMB imprints of the baryon acoustic oscillations.

Our explanation of dark energy uses classical theory, at least in the sense that Planck’s constant does not appear in the expression for Ω<sub>Λ</sub> in Eq. (45). Our conclusion that dark matter is baryonic does not make use of anything beyond the well-established domain of particle physics, no “exotic physics” is called for. Nevertheless the process of clarifying the role of the cosmic boundary conditions has cast some light on intriguing aspects of gravity. Examples: the wave nature of the gravitational interaction, the feedback effects of the vacuum energy (vacuum polarization), structural similarities with quantum field theory, metric-induced thermodynamics, scaling relations between the present and the Planck era, and the necessity of a hot Big Bang from Heisenberg’s uncertainty principle. These aspects may help guide us in our quest for a theory of quantum gravity.

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References

Binétruy, P. 2013, A&A Rev., 21, 67
Calchi Novati, S., Paulin-Henriksson, S., An, J., et al. 2005, A&A, 443, 911
Clowe, D., Bradac, M., Gonzalez, A. H., et al. 2006, ApJ, 648, L109
Einstein, A. 1917, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), 142
Ellis, G. & Smolin, L. 2009, The weak anthropic principle and the landscape of string theory, arXiv:0901.2414 [hep-th]
Ghosh, A. & Dey, U. R. 2016, Resonance, 21, 447. https://doi.org/10.1007/s12045-016-0348-y
Jetzer, P. & Sereno, M. 2006, Phys. Rev. D, 73, 044015
Laplace, P. S. 1880, Traité de mécanique céleste, Vol. 5, Book 16, Chapter 4 (Gauthier-Villars, Paris)
Longair, M. 2012, Lecture Notes on Fundamentals of Cosmology, IAC, Tenerife, http://www.iaac.es/congresos/iaspp2012/media/Longair-lectures/Longair1.pdf
Markovitch, M., Gonzalez, A. H., Clowe, D., et al. 2004, ApJ, 606, 819
Milgrom, M. 1983, ApJ, 270, 365
Neumann, C. 1896, Allgemeine Untersuchungen über das Newtonsche Prinzip der Fernwirkungen (Teubner, Leipzig)
O’ Raifeartaigh, C., O’ Keeffe, M., Nahum, W., & Mitton, S. 2017, European Physical Journal H, 42
Peebles, P. J. E. 1993, Principles of Physical Cosmology (Princeton University Press)
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565
Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2016, A&A, 594, A13
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009
Roza, E. 2017, Preprints, 2017050164 (doi: 10.20944/preprints201705.0164.v2)
Seeliger, H. 1895, Astronomische Nachrichten, 137, 129
Steigman, G. 2007, Annual Review of Nuclear and Particle Science, 57, 463
Straumann, N. 2002, ArXiv e-prints: gr-qc/0208027 (General Relativity and Quantum Cosmology)
Tisserand, P., Le Guillou, L., Afonso, C., et al. 2007, A&A, 469, 387
Weinberg, S. 1987, Physical Review Letters, 59, 2607
Zee, A. 2010, Quantum Field Theory in a Nutshell: Second Edition (Princeton University Press)
Zwicky, F. 1937, ApJ, 86, 217