A Design Method of Active Damping for Gyroscopic Systems Using Direct Velocity Feedback

Nan-Hui Yu¹, Ke-Wei Zhang¹, Hai-Min Liu¹ Xiang Liu¹ and Jia-Fan Zhang¹*

¹School of Mechanical Engineering, Wuhan Polytechnic University, Wuhan 430023, China
* jfz@whpu.edu.cn

Abstract. For gyroscopic dynamic systems, active damping using direct velocity feedback control with collocated actuator/sensor pairs is discussed in this paper. The proposed method of designing the damping is actually to assign the negative real parts of the poles for specific modes of the uncontrolled system. The targeted values of the negative real parts are determined by desired damping ratios to be gained for the specific modes. The procedure is formulated as solving constrained nonlinear equations or a constrained nonlinear least squares problem with elements of the control gain matrix as variables. Finally, it is illustrated via an example rotor system that the presented method is effective and easier to implement.

1. Introduction

Active damping is one of the popular vibration active control techniques, which involves enlarging the damping of some vibration modes that represent the dynamic characteristics of a mechanical structure. This technique means that the related controllers essentially focus on increasing the negative real part of the closed-loop poles (or eigenvalues) of controlled structures, while maintaining the natural frequencies essentially unchanged or with relatively small changes. A number of successful damping control techniques have been emerged, such as direct velocity feedback (DVF) [1-4], positive position feedback (PPF) [5-8], integral force feedback (IFF) [9-11], integral resonance control (IRC) [12,13], and their variants [14-18].

In the aforementioned active damping schemes, the configuration of collocated actuators and sensors is the most preferred choice, which implies that the actuator and the sensor are attached in pairs to the same degrees of freedom of the structure. This configuration has a well-known property that the pole-zero pattern of the open-loop system is alternating, and has rendered the control system unconditionally stable and, to a large extent, insensitive to parameter changes [19,20]. Also, the product of two physical quantities corresponding to the collocated actuator signal and sensor signal expresses the energy exchange between the structure and the control system.

Amongst these researches, the multi-input and multi-output control systems have been designed for the simultaneous damping of several modes [8,15]. Their control forces are to be calculated based on the decoupled modal space of undamped or viscously damped structures. But the proposed control algorithms are not applicable to the active damping design of gyroscopic systems because, generally speaking, the decoupling procedures are no longer valid for gyroscopic systems, at least not in their originally very simple form.

Active damping of undamped gyroscopic systems with DVF is considered in present study. The active damping matrix is expressed in terms of elements of the control gain matrix. The controller design is converted to solve systems of the nonlinear equations or a nonlinear least squares problem.
with linear inequality constraints. The constraints imposed on elements of the gain matrix assure the gain matrix and the resultant active damping matrix to be positive semidefinite, which is a demand for the controlled gyroscopic systems being stable. The presented design procedure avoids artificial choices on the decomposing vectors of the gain matrix decomposition, compared with the method stated in [3], and is simple, flexible and effective.

2. Statement of problem and stability condition

2.1. Statement of problem

A linearized multiple degrees of gyroscopic system in consideration is described in the form

\[ M\ddot{q}(t) + G\dot{q}(t) + Kq(t) = f(t) + f_c(t) \]

(1)

where \( M, K \) are \( n \times n \) real symmetric mass and stiffness matrices, respectively. \( G \) is an \( n \times n \) real skew-symmetric gyroscopic matrix. \( M \) is positive definite, denoted by \( M > 0 \), and \( K \) is assumed to be positive definite or positive semidefinite, denoted by \( K > 0 \) or \( K \geq 0 \). \( q(t) \) is the associated \( n \)-dimensional generalized displacement vector, and \( f(t), f_c(t) \) are the external disturbance force and control force vectors, respectively. The eigenvalue properties of the gyroscopic system (1) are well understood. If \( \lambda \) is an eigenvalue so are \( \lambda, -\lambda \), and \( -\lambda \). Moreover, if \( K > 0 \), the eigenvalues of (1) are purely imaginary and semisimple (i.e., non-defective) and the gyroscopic system (1) is considered as a marginally stable system. On the other hand, the gyroscopic system (1) may be unstable if \( K \geq 0 \) and is singular.

Now the active damping of (1) is considered using DVF with \( p \) pairs of colocated actuators and sensors as follows.

\[ f_c(t) = -BQ\dot{B}T\dot{q}(t) \]

(2)

where \( B \) is an \( n \times p \) constant matrix with full column rank, representing the actuator/sensor locations. In engineering applications it is appropriate for the matrix \( B \) to be Boolean. \( Q \) is an \( p \times p \) control gain matrix to be designed and is required to be symmetric positive semidefinite in this paper, so that the matrix \( BQ\dot{B}T \) can be positive semidefinite. From (1) and (2), the governing equations of motion for the controlled gyroscopic system with DVF are represented by

\[ M\ddot{q}(t) + (G + BQ\dot{B}T)\dot{q}(t) + Kq(t) = f(t) \]

(3)

Obviously, the controlled gyroscopic system (3) can be viewed as a damped gyroscopic system with \( D = BQ\dot{B}T \geq 0 \) as the active damping matrix. The colocated DVF controller-induced damping matrix must be a large, very sparse matrix, since the number of actuator/sensor pairs \( p \) is in general much smaller than the number of degree of freedoms of the system \( n \). Thus, the stability of the partially dissipative gyroscopic system could be complicated, which depends very much on the positive semidefinite matrix \( D \) and the definiteness of the stiffness matrix \( K \).

2.2. Stability condition

Rewrite the ‘viscously damped’ gyroscopic system (3) with equations of free motion represented by

\[ M\ddot{q}(t) + (G + D)\dot{q}(t) + Kq(t) = f(t) \]

(4)

As is well known, If \( M > 0, K > 0, D \geq 0 \) and \( D \neq 0 \), then the system (4) is either stable (i.e., all eigenvalues are in the open left half of the complex plane, and the stability defined here is actually the asymptotic stability in the Lyapunov sense) or weakly stable (i.e., all eigenvalues are in the closed left half of the complex plane, there is at least one pure-imaginary eigenvalue, and all such pure-imaginary eigenvalues are semisimple).
3. Active damping calculation

In the active damping matrix \( D = BQB^T \), the matrix \( B \) is Boolean and given beforehand, \( Q \) is a positive semidefinite, control gain matrix to be determined. To ensure the positive semidefiniteness of \( Q \), set up some constraints on elements of \( Q = \left( \phi_{ij} \right) \), i.e., negative off-diagonal elements and non-negative row (and column) sums, as follows:

\[
\phi_{ij} = \phi_{ji}, \phi_{ij} < 0 \text{ for } i \neq j \text{ and } \sum_{j=1}^p \phi_{ij} \geq 0 \text{ for } i, j = 1, \ldots, p
\]  

(5)

Such \( Q \) must be positive semidefinite \([21]\). On the other hand, the damping matrix \( D \) can be expressed in terms of the elements of \( Q \) as follows:

\[
D = \sum_{i,j=1}^p \phi_{ij} T_{ij}, \quad i \leq j, \quad i, j = 1, \ldots, p
\]  

(6)

where the distribution matrices \( T_{ij} \) are given symmetric matrices with elements 0, 1 and -1 that define the position and connectivity of the elements \( \phi_{ij} \) in \( D \).

Let \( \lambda_i = \alpha_i + j\omega_i, \alpha_i < 0, \omega_i > 0 \), be \( i \)th closed-loop pole of the damped gyroscopic system (4), the damping ratio associated with the mode \( \lambda_i \) is defined by

\[
\xi_i = \frac{-\text{Re}(\lambda_i)}{|\lambda_i|}
\]  

(7a)

or

\[
-\text{Re}(\lambda_i) = |\lambda_i| \xi_i
\]  

(7b)

where \( \text{Re}(\lambda_i) \) and \( |\lambda_i| \) are the real part and modulus of \( \lambda_i \), respectively. The corresponding \( i \)th open-loop pole of the gyroscopic system (1) is denoted by \( \lambda_{oi} = j\omega_{oi}, \omega_{oi} > 0 \). Some selected modes \( \lambda_{oi} \) that are intended to assign the desired damping ratios through DVF are assumed to be known.

Using (7b) and given damping ratios, now the active damping design amounts to determine \( \phi_{ij} \) in (6) in order that some of closed-loop poles \( \lambda_i \) corresponding to selected open-loop poles \( \lambda_{oi} \) have the required real part values. In (7b) \( |\lambda_i| \) is unknown a priori and will be approximately replaced by \( |\lambda_{oi}| \), since the physical meaning of \( |\lambda_i| \) is the undamped natural frequency of \( i \)th mode and the difference between \( |\lambda_i| \) and \( |\lambda_{oi}| \) is small under the present damping strategy.

Thus the method of achieving the active damping consists of finding a zero \( \left( \phi_{ij} \right) \) of the function:

\[
f \left( \phi_{ij} \right) = \begin{pmatrix}
\text{Re}(\lambda_{i1}) + |\lambda_{o11}|\xi_{i1} \\
\vdots \\
\text{Re}(\lambda_{im}) + |\lambda_{oim}|\xi_{im}
\end{pmatrix}
\]  

(8)

with the constraints (5) on \( \left( \phi_{ij} \right) \). Subscripts \( i1, \ldots, im \) correspond to the order number of selected modes of the system (1). This method involves solving a system of nonlinear equations with constraints and is known as the constrained root solving. For this problem, there are no guarantees that a solution exists that satisfies the constraints. Nevertheless, there exist numerical methods that can help one search for solutions, such as the Levenberg-Marquardt-type and the trust-region framework methods. Generally speaking, a system of \( N \) equations in \( N \) variables has isolated solutions, meaning each solution has no nearby neighbors that are also solutions. So one way to search for a solution that satisfies some constraints is to generate a number of initial points. Also, it is a nonsmooth and nonconvex optimization problem. In this paper, the problem is solved using the \textit{fmincon} function of 

the Optimization Toolbox together with the MultiStart solver of the Global Optimization Toolbox in Matlab, i.e., giving a constant objective function, setting the function \((8)\) as the nonlinear equality constraints in \textit{fmincon} and searching over many initial points automatically. The numerical partial derivatives of the function \((8)\) are calculated by finite-difference approximation, or provided by an analytical formula of the partial derivatives in computation routines.

Notice that the problem of the damping calculation considered here can also be transformed into a constrained nonlinear least squares problem with the objective function
\[
f_5(\phi) = \frac{1}{2} \| f(\phi) \|^2.
\]
But the problem is solved more accurately using the aforementioned method according to numerical experiments.

4. Numerical example

Fig. 1 illustrates an example rotor system to examine the applicability of the proposed design method. The rotor has 5 elements and 6 node points numbered from left to right. Two bearings are to be defined at nodes no.1 and 2. Assume that the bearings posses no damping and that all of their equivalent stiffness values in the x and y directions is 2000 N/m. The length of each element \(l_i = \{0.3m, 0.2m, 0.1m, 0.2m, 0.2m\}\). The outer diameter and inner diameter of each element \(D_i, d_i = \{0.03m,0.012m; 0.032m,0.01m; 0.022m,0.008m; 0.028m,0.006m; 0.03m,0.006m\}\). Let the elastic modulus \(E = 2.1 \times 10^{11} N/m^2\), the material density and Poison constant of the rotor \(\rho = 7.8 \times 10^3 \text{ kg/m}^3\) and 0.3, and the speed of rotation \(\omega = 3000 \text{ rpm}\). A point mass added in node no.5 is 0.01 kg. The outer diameter, inner diameter and width of a disc defined at node no.6 are 0.2 m, 0.012 m and 0.012 m, respectively. The disc’s material is the same as that of the rotor.

Figure 1. An example rotor system and an element of its model with generalized displacements

The element of the rotor consists of two nodes, with four degrees of freedom at each node. The nodal element displacement vector is shown in Fig 1. In this model, the effects of rotatory inertia, gyroscopic effects, shear deformation are included. The global matrices can be assembled as \(24 \times 24\) matrices \(M, G, K\) in \((1)\). \(4.5776i, 5.9926i, 42.254i\) and their complex conjugates are the first three eigenvalues of the gyroscopic system \((1)\). Firstly, two actuator/sensor pairs are considered to assign three sets of desired damping ratios: Case 1, \(\xi_1 = 15\% , \xi_2 = 15\% \); Case 2, \(\xi_1 = 7\% , \xi_3 = 12\% \); Case 3, \(\xi_1 = 15\% , \xi_2 = 15\% \) via \(2 \times 2\) diagonal matrix \(Q\) in contrast to case 1; The actuator/sensor pairs are configured for the translational displacements of node no.3 and node no.5 in the x direction, respectively, i.e., all elements of matrix B are zeros except for \(B(9,1)=1, B(17,2)=1\). Shown in Table 1 are the control gain matrix \(Q\) calculated by the method of previous section, and the actual damping ratios with DVF.

Fig. 2(a),(b) exhibit the pole map of the closed-loop system \((4)\) for the gain matrix \(Q\) obtained in case 1 and 3 of Table 1, respectively. All the modes are stabilized via DVF. Now three actuator/sensor pairs are considered to assign a set of desired damping ratios: case 4, \(\xi_1 = 16\% , \xi_2 = 20\% , \xi_3 = 10\% \). The actuator/sensor pairs are configured for the translational displacements of node no.3, node no.5 and node no.6 in the x direction, respectively, i.e., all elements of matrix B are zeros except for
B(9,1)=1, B(17,2)=1 and B(21,3)=1. The obtained results are also shown in Table 1. Fig. 3 is the pole map of the closed-loop poles for case 4. It is seen from Table 1 that the proposed method provides an acceptable estimation of the control gain matrix for the desired damping ratios.

| Case | Desired damping ratios | Actual damping ratios |
|------|------------------------|-----------------------|
| 1.   | ξ₁ = 15% ξ₂ = 15%     | ξ₁ = 14.59% ξ₂ = 15.04% |
| 2.   | ξ₁ = 7% ξ₃ = 12%      | ξ₁ = 6.63% ξ₃ = 11.66% |
| 3.   | ξ₁ = 15% ξ₂ = 15%     | ξ₁ = 14.85% ξ₂ = 15.14% |
| 4.   | ξ₁ = 16% ξ₂ = 20%, ξ₃ = 10% | ξ₁ = 14.74% ξ₂ = 22.64%, ξ₃ = 9.65% |

Figure 2(a),(b). Pole map of the closed-loop system (4) for case 1 (left) and case 3 (right).

Figure 3. Pole map of the closed-loop system (4) for case 4

5. Conclusions
The direct velocity feedback control with collocated actuators and sensors has been discussed on gyroscopic dynamic systems. The proposed method can be easily utilized for designing the active damping and gains desired damping ratios for specific modes of interest, although the resultant results may sometimes be approximate to the desired values. The proposed method can also be applied to passive damping design for gyroscopic dynamic systems.

References
[1] Balas M J 1979 J. Guid. Control. Dynam. 2 252
[2] Zhang Q, Shelley S and Allemang R J 1991 J. Dyn. Syst. Meas. Control. 2 259
[3] Yang B 1994 *J. Sound Vib.* **175** 525
[4] Ganguli A, Deraemaeker A and Preumont A 2007 *J. Sound Vib.* **300** 847
[5] Goh C J and Caughey T K 1985 *Int. J. Control.* **41** 787
[6] Fanson J L and Caughey T K 1990 *AIAA J.* **28** 717
[7] Poh S and Baz A 1990 *J. Intell. Mater. Syst. Struct.* **1** 273
[8] Kwak M K and Heo S 2007 *J. Sound Vib.* **304** 230
[9] Preumont A, Dufour J and Malekian C 1992 *J. Guid. Control. Dynam.* **15** 390
[10] Fleming A J and Leang K K 2010 *Sens. Actuators. A. Phys.* **161** 256
[11] Høgsberg J, Brodersen M L and Krenk S 2016 *Smart. Mater. Struct.* **25** 1
[12] Aphale S S, Fleming A J and Moheimani S O R 2007 *Smart. Mater. Struct.* **16** 439
[13] Bhikkaji B and Moheimani S O R 2008 *IEEE ASME Trans. Mechatron.* **13** 530
[14] Shin C J, Hong C S and Jeong W B 2011 *Trans. Korean Soc. Noise Vib. Eng.* **21** 447
[15] Rew K H, Han J H and Lee I 2008 *J. Intell. Mater. Syst. Struct.* **13** 13
[16] Mahmoodi S N and Ahmadian M 2009 *J. Dyn. Syst. Meas. Control.* **131** 442
[17] Orszulik R R and Shan J 2012 *Smart. Mater. Struct.* **21** 125
[18] Gu H and Song G 2005 *Smart. Mater. Struct.* **14** 540
[19] Preumont A and Seto K 2008 *Active Control of Structures* John Wiley & Sons Ltd United KingdomWS
[20] Petersen I R and Lanzon A 2010 *IEEE Contr. Syst. Mag.* **30** 54
[21] Horn R A and Johnson C R 1991 *Topics in Matrix Analysis* Cambridge University Press New York