A New Study about the Two Formulations of Conservation Laws for Matter Plus Gravitational Field and Their Experimental Test

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Abstract

The debate on conservation laws in general relativity eighty years ago is reviewed and restudied. The physical meaning of the identities $\mathcal{T}^\mu_{(M)\nu}(x) + \mathcal{T}^\mu_{(G)\nu}(x) = 0$ is reexamined and new interpretations for gravitational wave are given. The conclusions of these studies are distinct from the prevalent views, it can be demonstrated that gravitational wave does not transmit energy (and momentum) but only transmits information. An experimental test is offered to decide which conservation laws are correct.

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I. THE DEBATE IN GENERAL RELATIVITY EIGHTY YEARS AGO

In 1914, Einstein obtained the conservation laws for matter plus gravitational field in the form [1,2]

$$\frac{\partial}{\partial x^\mu}(T^\mu_{(M)\nu} + t^\mu_{(G)\nu}) = 0,$$  \hspace{1cm} (1)

$T^\mu_{(M)\nu}$ is the energy-momentum tensor density for matter field, Einstein called $t^\mu_{(G)\nu}$ the energy-momentum pseudotensor density of gravitational field.

Lorentz in 1916 [3] and Levi-Civita in 1917 [4] proposed successively to use

$$T^\mu_{(M)\nu}(x) + T^\mu_{(G)\nu}(x) = 0$$  \hspace{1cm} (2)

or

$$\frac{\partial}{\partial x^\mu}(T^\mu_{(M)\nu} + T^\mu_{(G)\nu}) = 0$$  \hspace{1cm} (3)

as conservation laws for matter plus gravitational field, their propositions evoked an important debate [2,5,6] on the correct formulation of conservation laws in general relativity eighty years ago.

The definition of $T^\mu_{(M)\nu}$ is $T^\mu_{(M)\nu} \overset{\text{def}}{=} 2 \frac{\delta W_M}{\delta g_{\mu\alpha}} g_{\nu\alpha}$ [7]. This definition has been accepted universally in theoretical physics. It is naturally to define the energy-momentum tensor density for gravitational field by $T^\mu_{(G)\nu} \overset{\text{def}}{=} 2 \frac{\delta W_G}{\delta g_{\mu\alpha}} g_{\nu\alpha}$ . So Lorentz and Levi-Civita adopted this definition. In the above definition, $W_M = \int L_M(x) d^4x$, $\delta W_M = \int \frac{\delta W_M}{\delta g_{\mu\nu}} \delta g_{\mu \nu} d^4x$; $W_G = \int L_G(x) d^4x$, $\delta W_G = \int \frac{\delta W_G}{\delta g_{\mu\nu}} \delta g_{\mu \nu} d^4x$; $L(x) = L_M(x) + L_G(x)$ is the Lagrangian density of the whole system, $L_M(x)$ and $L_G(x)$ are the matter field part and the pure gravitational field part of $L(x)$ respectively. In General relativity, $T^\mu_{(G)\nu} = \frac{\kappa^2}{8\pi G} \sqrt{-g} (R^\mu_{\nu} - \frac{1}{2} g^\mu_{\nu} R)$. Some people think that $T^\mu_{(G)\nu}$ is a pure geometric quantity and can not be used as the definition of energy-momentum tensor for gravitational field. This view is incorrect, because the metric tensor $g_{\mu\nu}$ is both geometric quantity and dynamic quantity in the theory of gravitation, so is $T^\mu_{(G)\nu}$.
Eq. (1) can be derived from the local translational symmetry of the gravitational system [8,9]. There exist the relations [9]

\[ t_{(G)\nu}^{\mu} = \frac{2}{\delta g_{\mu \alpha}} \frac{\partial g_{\nu \alpha}}{\partial x^\sigma} \delta W_{G} - \frac{\partial}{\partial x^\sigma} t_{(G)\nu}^{\mu \sigma}, \quad \frac{\partial}{\partial x^\sigma} t_{(G)\nu}^{\mu \sigma} = -\frac{\partial}{\partial x^\sigma} t_{(G)\nu}^{\sigma \mu}, \]  

(4)

where \( t_{(G)\nu}^{\mu \sigma} \) is determined by the Lagrangian density \( \mathcal{L}_G \). Eq. (1) can also be derived from Einstein field equations [10] or from the covariant generalized conservation laws \( T_{(M)\nu;\mu}^\mu = 0 \) [5]. \( t_{(G)\nu}^{\mu} \) obtained from distinct methods are different, but their difference can always be expressed by the relations: \( t_{(G)\nu}^{\mu} - \tilde{t}_{(G)\nu}^{\mu} = \partial_\alpha u_{\nu}^{\mu \alpha} \), where \( \partial_\alpha u_{\nu}^{\mu \alpha} = -\partial_\alpha u_{\nu}^{\alpha \mu} \) or \( u_{\nu}^{\mu \alpha} = -u_{\nu}^{\alpha \mu} \).

The quantity \( T_{(G)\nu}^{\mu} \) is a tensor density, but \( t_{(G)\nu}^{\mu} \) is not. The conservation law Eq. (3) is covariant, but the conservation law Eq. (1) is not. The well-known serious difficulties in connection with \( t_{(G)\nu}^{\mu} \) do not exist for \( T_{(G)\nu}^{\mu} \) and Eq. (3) is more in line with the spirit of general relativity [3,4].

The logical rationality for the definition of \( T_{(G)\nu}^{\mu} \) had been acknowledged by Einstein [2,5], he also acknowledged that one is not entitled to define \( t_{(G)\nu}^{\mu} \) as a quantity representing the energy-momentum of gravitational field; but Einstein doubted about the physical meaning indicated by the relation in Eq. (2). He said that “Eq. (2) does not exclude the possibility that a material system disappears completely, leaving no trace of its existence. In fact, the total energy in Eq. (2) is zero from the beginning, and the conservation of this energy value does not guarantee the persistence of the system in any form” [5]; so he opposed to choose \( T_{(G)\nu}^{\mu} \) as the energy-momentum tensor density for gravitational field. Care must be taken to that the reason which Einstein opposed \( T_{(G)\nu}^{\mu} \) is not because of its logical trouble or is not dependent on any experimental result; it is only owing to that he thought Eq. (2) being nonsensical. In the following the identities Eq. (2) is reexamined and a new explanation is given, moreover an experimental test is offered to decide which definition of gravitational energy-momentum tensor density and which formulation conservation laws are correct. It can be shown that Eq. (2) have a plentiful physical contents and might be tested by experiments.
II. REEXAMINATION OF THE IDENTITIES $\mathcal{T}_{(M)\nu}^\mu(X) + \mathcal{T}_{(G)\nu}^\mu(X) = 0$ AND NEW INTERPRETATIONS OF GRAVITATIONAL WAVE

Should Eq. (2) cause inevitably a material system disappear completely? This is the crux of the problem. We must point out that it is infeasible to determine solely how a material system changes merely using the conservation law of energy alone. Moreover it is impossible to determine whether this material system disappears completely. The change of a material system must yet obey other laws, such as the conservation law of baryon number, the second law of thermodynamic, etc. Therefore, $\mathcal{T}_{(M)\nu}^\mu + \mathcal{T}_{(G)\nu}^\mu = 0$ does not necessarily give $\mathcal{T}_{(M)\nu}^\mu = 0$, it only implies $\mathcal{T}_{(G)\nu}^\mu(x) = -\mathcal{T}_{(M)\nu}^\mu(x)$.

Eq. (2) shows that the energy-momentum tensor of the gravitational field must coexist with the energy-momentum tensor of material. Their sum total is equal to zero invariably and their distributional region in space-time is the same. These properties make Eq. (2) have a plentiful physical content which we shall show below, the consequences of these properties would be verified in future experiments. So Eq. (2) is not nonsensical from a physical point of view.

It is worth noting that, $t_{(G)\nu}^\mu$ and $\mathcal{T}_{(G)\nu}^\mu$ are not independent, they are interrelated by Eq. (4); via definition of $\mathcal{T}_{(G)\nu}^\mu$ and Eq. (4), Eq. (3) can be derived from Eq. (1), and on the other hand via Eq. (4), Eq. (1) can also be derived from Eq. (3), so Eq. (1) and Eq. (3) are not independent either. As a better choice, in this paper we shall follow Lorentz and Levi-Civita to treat $\mathcal{T}_{(G)\nu}^\mu$, but not $t_{(G)\nu}^\mu$, as the energy-momentum tensor density for gravitational field, and to treat Eq. (3), but not Eq. (1), as the conservation laws of energy-momentum tensor density; and we shall treat $t_{(G)\nu}^\mu$ as a subsidiary quantity, then Eq. (1) represents a number of subsidiary relations. According to Lorentz and Levi-Civita’s formulation of conservation laws, the Einstein’s gravitational equations $\sqrt{-g}(R_{\nu}^\mu - \frac{1}{2}g_{\nu}^\mu R) = -\frac{8\pi G}{c^4}T_{(M)\nu}^\mu$ are interpreted both as field equations and as conservation laws [3,4], therefore, except using $\mathcal{T}_{(G)\nu}^\mu$ as the energy-momentum tensor density for gravitational field, all deductions of gravitational field equations remain unchanged. For instance there was still a singularity at the beginning of
the present expansion phase of the universe.

The existence of gravitational wave is determined by Einstein’s gravitational field equations, the characteristics of these equations show that the gravitational wave propagate with speed of light [11].

A great number of people follow Einstein’s viewpoint, they believe that gravitational wave, like electromagnetic wave, is accompanied by radiation of energy. They also believe that this radiation has been verified by PSR 1913 + 16. These views are incorrect. Now let us show that gravitational wave could not transmit energy but could transmit information. The basis for the belief that gravitational wave radiates energy is using \( t_{(G)\nu}^\mu \) and applying the following equation

\[
-\frac{\partial}{\partial t} \int_V (T_{(M)\nu}^\mu + T_{(G)\nu}^0) dV = c \oint_S t_{(G)0}^i dS_i ,
\]

which can be derived from Eq. (1). The surface \( S \) enclosing the volume \( V \) is taken in vacuum where \( T_{(M)\nu}^\mu = 0 \). The right integral in Eq. (5) is interpreted as the amount of gravitational energy transferred by the gravitational wave across the surface \( S \) in unit time.

If we treat \( T_{(G)\nu}^\mu \), but not \( t_{(G)\nu}^\mu \), as the energy-momentum tensor density for gravitational field, new insights about the gravitational wave can be gained from Eq. (3) (or Eq. (2)). Using these equations, it is easy to obtain the identity

\[
\frac{\partial}{\partial t} \int_V (T_{(M)\nu}^0 + T_{(G)\nu}^0) dV = 0 .
\]

This identity can also be derived from Eq. (5) via Eq. (4); and inversely Eq. (5) can be derived from Eq. (6) via Eq. (4). Eq. (6) indicates that there are no energy transferred from \( V \) to outside via the gravitational wave, i.e. gravitational wave does not transmit energy. It must be pointed out that although \( T_{(M)\nu}^0 + T_{(G)\nu}^0 = 0 \), the values of \( T_{(M)\nu}^0 \) and \( T_{(G)\nu}^0 \) may change, they can be transformed to each other.

It should emphasize that, although the gravitational wave does not transmit energy (and momentum), but it could transmit information. When the gravitational wave passes through a space point, the gravitational field, i.e. the metric field of this point will change
from $g_{\mu\nu}(\vec{r}, t)$ into $g_{\mu\nu}(\vec{r}, t + \Delta t)$ in the time interval $(t, t + \Delta t)$, these changes in $g_{\mu\nu}$ convey the information from the source of gravitational wave. Some people guess that information should be closely bound up with energy, they think that the information without energy must not exist. Their guess is not correct. The gravitational wave is determined fully by Einstein field equations [11], which are irrespective how to define the energy-momentum tensor density for gravitational field. If we adopt the definition $T_{(G)\mu\nu} \overset{\text{def}}{=} 2 \frac{\delta W_{G}}{\delta g_{\mu\alpha}} g_{\nu\alpha}$ the gravitational wave would transfer information without energy.

When there exists gravitational wave, the space-time metric $g_{\mu\nu}(x)$ must change. This change may be considered as that the metric undergoes a perturbation by writing $g_{\mu\nu}(x) = \hat{g}_{\mu\nu}(x) + h_{\mu\nu}(x)$ [7, 10], $\hat{g}_{\mu\nu}$ is the background metric and $h_{\mu\nu}$ is the perturbation; usually $|h_{\mu\nu}| \ll |\hat{g}_{\mu\nu}|$, the magnitude of $h_{\mu\nu}(x)$ reflects the intensity of gravitational wave.

The solution for weak-field approximations of Einstein field equations has been given as

$$\varphi_{\mu}^\nu(\vec{r}, t) = \frac{4G}{c^4} \int \frac{T_{\mu\nu}}{|\vec{r} - \vec{r}'|} \, d^3x'$$ [7], where $\varphi_{\mu}^\nu = h_{\mu}^\nu - \frac{1}{2} \hat{g}_{\mu\nu} h$. From this formula, it is evident that the gravitational wave can be generated only if the energy-momentum tensor of material source is changed in space-time. Let $R_{\mu\nu}(x)$ and $\hat{R}_{\mu\nu}(x)$ be the Ricci tensors corresponding to $g_{\mu\nu}(x)$ and $\hat{g}_{\mu\nu}(x)$, $R_{\mu\nu}(x)$ and $\hat{R}_{\mu\nu}(x)$ must obey with Einstein field equation ($R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{(M)\mu\nu}$) and ($\hat{R}_{\mu\nu} - \frac{1}{2} \hat{g}_{\mu\nu} \hat{R} = -\frac{8\pi G}{c^4} \hat{T}_{(M)\mu\nu}$) respectively. It has been proved that [10] $R_{\mu\nu} = \hat{R}_{\mu\nu} + R_{(1)\mu\nu}^{(1)}(h) + R_{(2)\mu\nu}^{(2)}(h) + \cdots$, where $R_{(1)\mu\nu}^{(1)}$ and $R_{(2)\mu\nu}^{(2)}$ are composed of the covariant derivatives of $h_{\mu\nu}$, $R_{(1)\mu\nu}^{(1)}$ is the linear term and $R_{(2)\mu\nu}^{(2)}$ is in the second degree term. Either of these two equations can be rewritten as

$$\hat{R}_{\mu\nu} - \frac{1}{2} \hat{g}_{\mu\nu} \hat{R} = -\frac{8\pi G}{c^4} (T_{(M)\mu\nu} + W_{(G)\mu\nu})$$ \hspace{1cm} (7)

Where

$$W_{(G)\mu\nu} \overset{\text{def}}{=} \frac{c^4}{8\pi G} \{(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) - (\hat{R}_{\mu\nu} - \frac{1}{2} \hat{g}_{\mu\nu} \hat{R})\} = \hat{T}_{(M)\mu\nu} - T_{(M)\mu\nu}$$ \hspace{1cm} (8)

It is suggested to interpret $W_{(G)\mu\nu}$ or its average value as the energy-momentum tensor for gravitational wave. But in our opinion, although $W_{(G)\mu\nu}$ might be used to express gravitational field’s energy-momentum tensor relative to the background space-time, but
it does not indicate the energy and momentum transmitted by gravitational wave. Since
\[ W(G)_{\mu\nu} = \nabla_{\mu} \nabla_{\nu} - \nabla_{\nu} \nabla_{\mu} \]
\[ = \frac{1}{\sqrt{-g}} \nabla_{\mu} T_{(G)\nu} = \frac{1}{\sqrt{-g}} \nabla_{\nu} T_{(G)\mu} \]
and
\[ T_{(M)\mu\nu}(x) + T_{(G)\mu\nu}(x) = 0, \]
therefore at any point of space-time, the total energy-momentum tensor for matter plus gravitational field is identically equal to zero when the gravitational wave generate.

A question then arises as to how gravitational wave can be detected. Although the gravitational wave does not transmit energy, the equation of geodesic deviation remains correct. As a result one can still design the detectors based on the effect of this equation [10, 12]. For instance, Weber’s cylinder can be used to detect the gravitational wave. From the conventional viewpoint, gravitational wave can transmit energy to a detector. But in our viewpoint, when the gravitational wave is received, the oscillation energy of the detector does not come from the gravitational wave, it comes from the local gravitational field where the detector resides. Because the detector is made of matter, let \( T_{(M)\mu\nu} \) be its energy-momentum tensor density, \( T_{(M)\mu\nu} \) must satisfy Eq. (2). Therefore \( \Delta T_{(M)\mu\nu} = -\Delta T_{(G)\mu\nu}, \Delta T_{(M)\mu\nu} \) represents the increased energy of the detector and \( -\Delta T_{(G)\mu\nu} \) represents the decreased energy of the local gravitational field; \( \Delta T_{(M)\mu\nu} \) or \( \Delta T_{(G)\mu\nu} \) can be calculated from Einstein field equation.

III. AN EXPERIMENTAL TEST TO DECIDE WHICH FORMULATION IS CORRECT

The microwave background radiation gives us important information of the early epoch of the universe [13], the discovery and theoretical interpretation of this radiation spurs the cosmology to get ahead. It is believed that there existed also a vast amount of gravitational waves at the earliest epoch of the universe immediately after “big bang”, with the expansion of the universe there should also exist background gravitational waves at present [14].

The microwave background radiation has the type of spectrum of black body radiation [13]. Do these background gravitational waves also have the type of spectrum of black body radiation? We shall study this question at once. The black body radiation is thermal
radiation in equilibrium [15]. Einstein advanced a semi-quantum theory of thermal radiation in equilibrium [15,16] and derived the equation

$$\rho(\nu, T) = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/KT} - 1}$$

from his theory. Eq. (9) expresses Plank’s law of radiation.

The Semi-quantum theory of thermal radiation in equilibrium advanced by Einstein is concise and reflect well the essence of this radiation, the key of Einstein’s theory is to think that the radiation wave transmits energy and it is quantized [15, 16]. The prevalent theory assumes that the gravitational wave transmits energy, and the graviton, which is similar to photon, is the quantum of energy for gravitational wave. Therefore Eq. (9) must be also applicable for gravitational radiation. It is possible that this distribution might deform during the evolution of the universe, however, the pattern of spectrum must be yet regular.

On the other hand, according to Lorentz and Levi-Civita’s definition of energy-momentum tensor density $T^\mu_{(G)\nu}$ for gravitational field and the conservation law Eq. (2), the gravitational wave does not transmit energy, consequently for gravitational wave the concepts such as thermal radiation in equilibrium, the quantum of energy and distribution of radiation energy are all meaningless. These distinguishing features imply that Eq. (9) does not apply for gravitational waves, i.e. the background gravitational waves do not tally with the black body radiation or its variety. The types of spectrum of background gravitational waves should not have any regularity, they are the results of random process. Therefore, through the observations of the spectrum types for background gravitational waves, it might provide an experimental test to decide whether gravitational wave transmits energy, i.e. it might judge which is the correct definition of energy-momentum tensor for gravitational field, $T^\mu_{(G)\nu}$ or $t^\mu_{(G)\nu}$? And which is the correct formulation of conservation laws for matter plus gravitational field, Eq. (1) or Eq. (3) (and Eq. (2))? We shall wait and see.
IV. HAS THE GRavitATIONAL ENERGY RADIATION BEEN VERIFIED?

The orbital energy loss of the binary pulsar system PSR 1913 + 16 has been confirmed from the observation of the decrease in its orbital period [17]. This observation has been widely interpreted as verification for energy radiation of gravitational wave. However based on our analysis given above, this interpretation is problematic.

The theoretical basis of the so called verification is Eq. (5). When using Eq. (5) to compute the energy change of PSR 1913 + 16, one always assumes: 1 $\frac{2}{3} \int_V (t^0_{(G)0}) dV = 0$ and 2 except the orbital kinetic energy and gravitational interaction energy (which belongs to $T^\mu_{(M)\nu}$ according to definition), the other kinds of matter energy do not change for this binary pulsar. Evidently the result of computation can not be very accurate based on these assumptions. The ratio of the observed to the predicted damping rate $\dot{P}_{\text{obs}} / \dot{P}_{\text{pred}}$ for PSR 1913 + 16 is $1.00032 \pm 0.0035$ [18], the coincidence between $\dot{P}_{\text{obs}}$ and $\dot{P}_{\text{pred}}$ is overprecise. As another example $\dot{P}_{\text{obs}} / \dot{P}_{\text{pred}}$ for PSR 1534 + 12 is about 0.83 [19], the coincidence is less evident. Taking the approximation in calculation of $\dot{P}_{\text{pred}}$ into account, one can not rule out the possibility that the coincidence of observation and prediction for the damping rate $\dot{P}$ of PSR 1913 + 16 might be accidental [20, 21].

Even if the above coincidence for PSR 1913 + 16 is true, yet it can not fully prove that gravitational wave transmits energy. Because Eq. (5) can be derived from Eq. (6), the same value of $\dot{P}_{\text{pred}}$ may also be obtained from Eq. (6) and $T^\mu_{(G)\nu}$ [21]. Since Eq. (6) means that gravitational wave does not transmit energy, it is inevitable to conclude that the gravitational energy radiation has not been verified.
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