Proper motions of 15 pulsars: a comparison between Bayesian and frequentist algorithms

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ABSTRACT

We present proper motions for 15 pulsars which are observed regularly by the Nanshan 25-m radio telescope. Two methods, the frequentist method (Coles et al. 2011) and the Bayesian (Lentati et al. 2014) method, are used and the results are compared. We demonstrate that the two methods can be applied to young pulsar data sets that exhibit large amounts of timing noise with steep spectral exponents and give consistent results. The measured positions also agree with very-long-baseline interferometric positions. Proper motions for four pulsars are obtained for the first time, and improved values are obtained for five pulsars.

Key words: methods:data analysis – pulsars:general – proper motions

1 INTRODUCTION

Pulsar velocities are usually determined by measuring their proper motions and distances. Lyne & Lorimer (1994) and Hobbs et al. (2005) analysed a large number of pulsar proper motions and demonstrated that their transverse velocities are typically several hundred km s⁻¹. Various explanations have been proposed for these high velocities, including a postnatal electromagnetic rocket mechanism (Harrison & Tademaru 1975), asymmetric neutrino emission in the presence of super-strong magnetic fields (Lai & Qian 1998), hydrodynamical instabilities in the collapsed supernova core (Lai & Goldreich 2000) and the asymmetric explosion of γ-ray bursts (Cui et al. 2007). Knowledge of pulsar proper motions and velocities is essential for many aspects of pulsar and neutron star astrophysics including determinations of the birth rate of pulsars, associations with supernova remnants and the Galactic distribution of the progenitor population. Unfortunately only ~ 10% of the known pulsars currently have measured proper motions (Manchester et al. 2005). It is therefore highly desirable to determine more proper motions and to confirm the accuracy of previous publications.

Determining proper motions is not trivial. The most common methods are 1) using very-long-baseline interferometry (e.g., Lyne et al. 1982; Harrison et al. 1993; Brisken et al. 2003; Chatterjee et al. 2009; Yan et al. 2013), but this is usually only possible for relatively close-by and bright pulsars or 2) using the pulsar timing method. The use of the pulsar timing method for determining proper motions was first presented by Manchester et al. (1974) who detected the proper motion for PSR B1133+16 using a 4-year timing data set. Recent proper motions have been published by numerous groups including Nice & Taylor (1995), Hobbs et al. (2004), Žou et al. (2005), González et al. (2011), Desvignes et al. (2016), Matthews et al. (2016).

As data sets increased it became clear that unmodelled irregularities in the pulsar timing residuals (pulsar timing noise) could bias the reported proper motions. This was commonly dealt with simply by increasing the uncertainties reported by the pulsar timing software or by us-

2 Other methods to determine the proper motion include optical techniques (Trimble 1971; Wyckoff & Murray 1977; Ögelman et al. 1989; Migliani et al. 2000) and using the scintillation properties of pulsars (e.g., Wang et al. 2005).
ing high-order pulse-frequency derivatives (e.g., Zou et al.
2005) or harmonic whitening (Hobbs et al. 2005) to ab-
sorb the noise. Coles et al. (2011) showed that both of these
methods still led to biased parameters and incorrect uncer-
tainty estimates. They presented a new method, based on a
generalised-least-squares fitting procedure, in which the
spectrum of timing noise was estimated using an analytic
model. The temporal correlations in the timing residuals
were then subsequently accounted for in the timing model
fitting. This method has now been used to determine the
proper motion of millisecond pulsars (Reardon et al. 2016).
Lentati (2014) demonstrated a similar method in which the
noise properties of the data and the pulsar parameters were
simultaneously determined using a Bayesian methodology
(Desvignes et al. 2016). Recently Babak (2016) described the
comparison of noise models from Bayesian and frequentist
methods and got the consistent upper limits on continuous
gravitational waves depended on the two algorithms. Nei-
ther of these methods have been applied to a large sample
of young pulsars (they were both developed to study mil-
losecond pulsars). In this paper we demonstrate that these
two methods can be applied to noisy young pulsar data sets
and we compare the results obtained from the methods.

2 OBSERVATION AND DATA REDUCTION

In this paper we describe the analysis of timing observa-
tions of 15 pulsars that were obtained with the Nanshan
25-m radio telescope of the Xinjiang Astronomical Observ-
atory (XAO) as part of their regular timing program. The
details of observing system have been described by Wang
et al. (2001). In brief, observations are carried out in the
18 cm band. The backend system consists of an analogue fil-
terbank (AFB) system and, since 2010, simultaneous obser-
vations with a digital filterbank system (DFB). The pulsars
described in this paper were observed from 2000 January to
2013 Dec (apart from a few for which observations started
in July, 2002). They are all typical, isolated pulsars with
characteristic age of $10^5 \sim 10^7$ yr and within a distance of
7 kpc. Most have been selected because they are relatively
bright in the 18 cm observing band. The pulsars are each
observed approximately 2 to 4 times per month. We list the
basic parameters of these pulsars and the details of our data
set in Table 1.

Each pulsar is observed for $\sim 4$–16 minutes and the re-
sulting data are recorded to disk. The PSRCHIVE package
(Hotan et al. 2004) is used for off-line data reduction with
an initial set of parameters being obtained from the ATNF
pulsar catalogue (Manchester et al. 2005). After dedispersing
and removing radio-frequency-interference (RFI), the data
are summed in frequency, time and polarisation to produce
a mean pulse profile. An analytic template is obtained by
fitting one or more Gaussian profiles to the mean pulse pro-
file. The template is then cross-correlated with each obser-
vation to obtain pulse times-of-arrival (ToAs). The TEMPO2
(Hobbs et al. 2006, Edwards et al. 2006) timing package is
then used to convert the local ToAs to the arrival times at
the solar system barycentre with the Jet Propulsion Labora-
tory (JPL) planetary ephemeris DE421 (Folkner et al. 2008).
Our observations are referred to Terrestrial Time as realised

![Figure 1. Timing residuals of 15 pulsars from 2000 to 2013. They arise from the frequentist analysis after fitting for the pulse frequency and its first derivative with the fixed astrometric pa-
rameters. The plot y-range in seconds is given under the pulsar name.](http://www.bipm.org)

3 http://www.bipm.org

3 METHODOLOGY

Two current methods exist for determining unbiased proper
motions in the presence of pulsar timing noise. One is the
frequentist method described by Coles et al. (2011) and the
other is the Bayesian procedure presented by Lentati et al.
(2014). We have used both methods to estimate the red noise
properties along with a determination of the proper motion for
each pulsar.
Table 1. Parameters for the 15 pulsars described in this paper. Each pulsar name is followed by the rotational parameters from the frequentist method. The remaining columns are, in turn, the dispersion measure and distance (NE2001) obtained from the ATNF pulsar catalogue, the number of ToAs, the epoch of the period determination, the data span in years and the MJD range, the mean ToA uncertainty and the weighted rms variation of the timing residuals.

| PSR J | PSR B | $\nu$ (s$^{-1}$) | $\nu$ (10$^{-14}$ s$^{-2}$) | DM (pc cm$^{-3}$) | Dist. (kpc) | $N_{\text{ToA}}$ | Epoch (MJD) | Span (yr) | MJD range | $\sigma_{\text{ToA}}$ (μs) | $W_{\text{rms}}$ (ms) |
|-------|-------|-----------------|-----------------|----------------|-----------|----------------|-------------|-----------|-------------|----------------|----------------|
| J0134−2937 | - | 7.3013160129300(7) | −0.4177838(14) | 21.806(6) | 0.56 | 187 | 54428 &rdquo; | 10.6 | 52494−56361 | 115 &rdquo; | 0.3 |
| J0358+5413 | B0355+54 | 6.394511992(3) | −17.9769(2) | 57.142(3) | 1.00 | 988 | 54096 | 14.0 | 51547−56645 | 93 | 3.3 |
| J0534+5543 | B0540+55 | 2.334872586(12) | −2.0479(9) | 14.5943(9) | 1.18 | 458 | 54096 | 13.9 | 51547−56625 | 326 | 21.0 |
| J0953+0755 | B0950+08 | 3.951548754(7) | −0.35941(9) | 2.96927(6) | 0.26 | 605 | 53908 | 12.9 | 51547−56269 | 65 | 2.4 |
| J1733+3716 | B1730+37 | 2.962140586(1) | −13.2005(16) | 153.5(3) | 2.78 | 218 | 54433 | 10.6 | 52495−56372 | 481 | 2.1 |
| J1745−3040 | B1742−30 | 2.731586782(8) | −7.90468(7) | 88.373(4) | 0.20 | 403 | 53895 | 12.8 | 51549−56241 | 126 | 1.6 |
| J1835−1020 | - | 3.306336171(16) | −6.4702(2) | 113.7(9) | 2.30 | 251 | 54369 | 10.2 | 52497−56216 | 434 | 2.6 |
| J1847−0402 | B1844−04 | 1.672821993(4) | −14.4665(3) | 141.979(5) | 3.26 | 366 | 53902 | 12.9 | 51550−56255 | 695 | 16.2 |
| J1848−0123 | B1845−01 | 1.516452445(5) | −1.2013(5) | 150.521(6) | 4.40 | 270 | 53868 | 12.7 | 51549−56187 | 452 | 36.8 |
| J1901+0331 | B1859+03 | 1.525662341(7) | −1.738(2) | 402.090(12) | 7.00 | 265 | 54363 | 10.1 | 52486−56186 | 352 | 27.7 |
| J1903−0135 | B1900+01 | 1.371166852(6) | −0.75849(5) | 245.167(6) | 3.30 | 278 | 54379 | 10.4 | 52473−56284 | 346 | 2.1 |
| J1904+0004 | - | 7.1671914683(6) | −0.60642(11) | 233.61(4) | 5.74 | 217 | 54377 | 10.3 | 52469−56285 | 530 | 5.0 |
| J2022−2854 | B2020+28 | 2.9120317941(17) | −1.605114(17) | 24.632(1) | 2.10 | 448 | 53901 | 12.9 | 51547−56254 | 112 | 0.7 |
| J2113−0444 | B2111+46 | 0.9652727850(3) | −0.06858(3) | 141.26(9) | 4.00 | 349 | 53915 | 12.9 | 51559−56271 | 1267 | 2.7 |
| J2257+5909 | B2255+58 | 2.715558593(9) | −4.2426(8) | 151.082(6) | 3.00 | 434 | 53908 | 12.9 | 51560−56257 | 339 | 14.7 |

The frequentist method is implemented as follows within the TEMPO2 software package:

- We model any excess white noise scatter in the residuals using the EFAQUAD plugin (these are defined by parameters known as EFACs and EQUADs, see, e.g., Wang et al. 2015).5
- Form timing residuals using the initial model of the pulsar parameters and use the SPECTRALMODEL plugin package for TEMPO2 to estimate an analytic model for the spectrum of the red noise component of the timing residuals. The red noise is parameterized using an amplitude $A$, a spectral exponent $\alpha$ and a corner frequency $f_c$ as follows:

$$P(f) = \frac{A}{[1 + (f/f_c)^2]^{\alpha/2}}. \tag{1}$$

- With the analytic red noise model we fit the corresponding white noise model using the global least-square-fitting procedure as described by Coles et al. (2011).
- The entire process is iterated with these new model parameters and the analytic red noise model improved as necessary until the results converge.
- We record the final improved parameters (red noise parameters, positions and proper motions) for later analysis.

The Bayesian method is applied to the original set of ToAs and original pulsar parameters as follows:

- It is essential that we provide prior information on the range of proper motions. For many of these pulsars, previous measurements of proper motions have already been obtained and so we usually have a reasonable first guess of the possible range of proper motions. For our final processing we assume a flat prior for proper motions between $-500$ mas/yr and $+500$ mas/yr.
- We are not (for this paper) interested in the pulse period and so we analytically marginalise over these parameters and the pulsar positions.
- We include a noise model that parameterises excess white noise in the residuals.5
- We then run the TEMPO2 plugin package to TEMPO2 and determined the most likely noise parameters and proper motions.

4 RESULTS

4.1 Updated positions

The frequentist method provided us with positions and proper motions. The measured positions are given in Table 2. The pulsar J2000 and B1950 names are listed in the first two columns of the table. The next three columns present the corresponding epoch of the position and the right ascension and declination obtained in equatorial coordinates from our data analysis respectively. The final four columns display the parameters from the literature including the epoch, the position and the reference. In this and succeeding tables, the figure in parentheses indicates $1\sigma$ uncertainty in the last quoted digit. The superscripts ‘F’ and ‘c’ separately denote results from our measurements using the frequentist method and from the catalogue respectively. We note that the previously published positions for most of the 15 pulsars in our sample have not been updated for 10 to 20 years. We also emphasise that the measured positions (slightly) depend upon the choice of the JPL solar system ephemeris that was used (our results are based on JPL DE421, the earlier results are likely to have used DE200 or DE405). A comparison between the earlier positions and our measurements to obtain the proper motions is carried out in section 4.2.

Both the frequentist and Bayesian methods assume that the timing noise can be modelled using a power-law spectral

5 TNGLOBALEF & TNGLOBALEFQ are used, the definition is $\epsilon^2 = \langle \text{TNGLOBALEF} \times \epsilon^2 + \text{TNGLOBALEFQ} \rangle$. Note that these are different from the EFAC and EQUAD described earlier.
model. This is not true if the pulsar data set contains one or more unmodelled large glitches. PSR J0358+5413 is reported to suffer six glitches, four of which happened within our data span. As the four events are small with Δν/ν is ≲ 2e − 9, it is difficult for us to distinguish them from timing noise, although they are detectable with larger telescopes (e.g. Espinoza et al. 2011). We assumed that these small glitches could be taken as part of the timing noise. Such an assumption was discussed by Coles et al. (2011) and shown, for small glitches, not to significantly bias the resulting estimates. In order to confirm this we also chose the largest interval (MJD 53219-56645, 2004 Aug 2-2013 Dec 19) in the data span without a glitch event and repeated the analysis. With the glitches the position and proper motion estimates are respectively RA(J2000) = 03h58m53s.7230(9), Dec.(J2000) = +54°13′13″.784(3) on MJD 54096 and μα = 9.3(5) mas yr⁻¹, μδ = 8.5(8) mas yr⁻¹. For the restricted data span, they are RA(J2000) = 03h58m53s.7230(9), Dec.(J2000) = +54°13′13″.784(3) on the same epoch and μα = 9.7(7) mas yr⁻¹, μδ = 8.6(13) mas yr⁻¹. The glitch events therefore do not significantly bias the position and proper motion estimates. Accordingly we also ignore the possibility of other small, unpublished glitches in our data set and instead treat such signal as timing noise.

In a few cases (particularly PSR J1733–3716 and J1835–1020) the previous position determination was so poor that we could clearly identify an annual sinusoid in our timing residuals when using the catalogue position. The iterative procedure that we have used accounts for such errors and after the iterative procedure we obtain a much improved position estimate.

### 4.2 Proper motions

The proper motions obtained via the two methods are listed in Table 3. Proper motions in right ascension and declination are quoted in mas/yr, i.e., μα = ˙α cos δ and μδ = ˙δ. Superscripts ‘F’ and ‘B’ respectively indicate the measurements from the frequentist and the Bayesian methods. The results from both methods are remarkably consistent - both in terms of the parameter values and in terms of their uncertainties. This is not a surprise, the data are the same for the two methods, but does indicate that, even for young pulsars whose residuals are dominated by timing noise, the two methods (Bayesian and frequentist) and noise modelling techniques lead to statistically identical results.

We also compared the positions in Table 2 to derive the proper motions (superscript ‘CP’). They are all consistent with results obtained by the frequentist and Bayesian methods except for the μδ of three pulsars (PSRs J1733–3716, J1745–3040 & J1835–1020) which have poor accuracy on their earlier declination measurements. The remaining part of Table 3 lists proper motions and their references from previous publications. The ninth and tenth columns list proper motions available from other timing measurements (superscript ‘T’). They are all from Hobbs et al. (2004) except for PSRs J1745–3040 and J2022+2854 which came from Zou et al. (2005). For four pulsars, i.e., PSRs J1733–4716, J1835–1020, J1901+0331 and J1903+0135, we present the first measurements of their proper motions. The proper motions of PSRs J0134–2937, J1745–3040 & J1848–0123, J1904+0004 and J2257+5099 are improved and more precise than other timing values. The proper motion of PSR J0953+0755 is comparable to other timing values. That could result from the longer data span of 34.3 yr (Hobbs et al. 2004) for this pulsar. The last three columns present

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6 http://www.atnf.csiro.au/people/pulsar/psrcat/glitchTbl.html & http://www.jb.man.ac.uk/pulsar/glitches/glTable.html

7 In order to do the comparison between the two solutions, the epoch of positions measured is fixed on MJD 54096, the integral middle of the whole data span.

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Table 2. Pulsar positions of 15 pulsars obtained by applying the frequentist method to our data and also obtained from the literature. The first two columns list the J2000 and B1950 names, which are followed by the epoch of the position measurement, the right ascension and declination obtained in equatorial coordinates from our data analysis. The corresponding parameters from the ATNF Pulsar Catalogue and the references are in following four columns. The references in Table 2 and Table 3 are all listed here: (1) Hobbs et al. 2004; (2) Chatterjee et al. 2004; (3) Chatterjee et al. 2009; (4) Johnston et al. 1995; (5) Zou et al. 2005; (6) Morris et al. 2002; (7) Brinker et al. 2002. The figure in parentheses indicates the first uncertainty in the unit of the last quoted digit and superscripts ‘F’ and ‘c’ separately denote results from the frequentist method and the catalogue in this and subsequent tables.
Table 3. A comparison of proper motions for 15 pulsars in equatorial coordinates. In column order, the table gives the pulsar name, the proper motions determined by the frequentist and the Bayesian methods, the proper motions derived by comparison of positions given in Table 2, the proper motion determined by the previous timing method, the proper motion measured by the previous interferometric method and its reference (shared with Table 2). Proper motions are quoted in mas/yr, i.e., \( \mu_\alpha = \alpha \cos \delta \) and \( \mu_\delta = \delta \). Superscripts \( 'F' \), \( 'B' \), \( 'CP' \), \( 'T' \) & \( 'I' \) separately denote the frequentist, the Bayesian, the comparison of positions, other timing and the interferometric methods.

| PSR J   | PSR B   | \( \mu_\alpha^F \) (mas/yr) | \( \mu_\delta^F \) (mas/yr) | \( \mu_\alpha^B \) (mas/yr) | \( \mu_\delta^B \) (mas/yr) | \( \mu_\alpha^{CP} \) (mas/yr) | \( \mu_\delta^{CP} \) (mas/yr) | \( \mu_\alpha^T \) (mas/yr) | \( \mu_\delta^T \) (mas/yr) | \( \mu_\alpha^I \) (mas/yr) | \( \mu_\delta^I \) (mas/yr) | Ref.  |
|---------|---------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|------|
| J0134–2937 | - | 13(2) | -11(3) | 13(2) | -10(3) | 18(2) | -14(3) | - | - | - | - | - |
| J0535+5413 | B0355+54 | 9.3(5) | 8.3(8) | 10.4(6) | 7.1 | 9.2(5) | 8.2(6) | 8.3 | 19(6) | 9.2(8) | 8.17(39) | 2 |
| J0454+5543 | B0450+55 | 55(2) | -19(4) | 54(3) | -14(5) | 56(2) | -19(3) | 48(6) | -13(2) | 53.34(6) | -17.56(14) | 3 |
| J0953+0755 | B0950+08 | -1(4) | 34(10) | -4(6) | 25(15) | -2(2) | 30(4) | -3(3) | 26(7) | -2.09(8) | 29.46(7) | 7 |
| J1733–3716 | B1730–37 | 4(9) | 63(34) | 6(10) | 84(37) | 6(4) | -38(19) | - | - | - | - | - |
| J1745–3040 | B1742–30 | 12.5(15) | 30(11) | 11.9(16) | 50(12) | 24(6) | 75(49) | 6(3) | 4(26) | - | - | - |
| J1835–1020 | - | 25(5) | 4(21) | 24(5) | 2(21) | 51(22) | 38(127) | - | - | - | - | - |
| J1847–0402 | B1844–04 | -1(7) | 8(19) | -0.1(60) | 7(18) | 5(3) | 5(9) | -1(5) | -9(19) | - | - | - |
| J1848–0123 | B1845–01 | -5(6) | 14(16) | -3(6) | 24(16) | -3(3) | -10(8) | 2(6) | -41(19) | - | - | - |
| J1901+0331 | B1859+03 | -7(15) | 34(31) | -10(13) | 42(27) | 0(5) | 4(8) | - | - | - | - | - |
| J1903+0135 | B1900+01 | 3(7) | -13(14) | 5(12) | -24(28) | 2(4) | 2(10) | - | - | - | - | - |
| J1904+0004 | - | 8(9) | -7(16) | 9(10) | -10(18) | 3(11) | -1(31) | -7(21) | -83(75) | - | - | - |
| J2022+2854 | B2020+28 | -9(4) | -19(5) | -7(5) | -18(7) | 2(2) | -18(2) | -9(3) | -13(5) | -4.4(5) | -23.6(3) | 7 |
| J2213+4644 | B2211+46 | -4(13) | 9(14) | 5(13) | 4(14) | 11(8) | 6(9) | 9(15) | -3(16) | - | - | - |
| J2257+5009 | B2255+58 | 6(2) | -3(3) | 7(2) | -2(3) | 7(3) | -3(3) | 28(7) | -8(6) | - | - | - |

Table 4. The comparison of proper motions for five pulsars which have previously published proper motions in equatorial coordinates.

| PSR J   | PSR B   | \( \mu_\alpha^F \) (mas/yr) | \( \mu_\delta^F \) (mas/yr) | \( \mu_\alpha^B \) (mas/yr) | \( \mu_\delta^B \) (mas/yr) | \( \mu_\alpha^{CP} \) (mas/yr) | \( \mu_\delta^{CP} \) (mas/yr) | \( \mu_\alpha^T \) (mas/yr) | \( \mu_\delta^T \) (mas/yr) | \( \mu_\alpha^I \) (mas/yr) | \( \mu_\delta^I \) (mas/yr) |
|---------|---------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| J0134–2937 | - | 6(2) | -15(3) | 8(3) | -21(5) | - | - | - | - | - | - |
| J1847–0402 | B1844–04 | -2(67) | 9(19) | -3(6) | - | - | - | - | - | - | - |
| J1848–0123 | B1845–01 | -3(5) | 16(14) | -0.7(59) | - | - | - | - | - | - | - |
| J2213+4644 | B2211+46 | 5(12) | 11(14) | 7(12) | -11(13) | - | - | - | - | - | - |
| J2257+5009 | B2255+58 | 3(2) | -5(3) | 11(4) | -14(5) | - | - | - | - | - | - |

Table 5. Velocities for seven pulsars for which \( \mu_{tot} \) is measured with reasonable precision (value/error > 3).

| PSR J   | PSR B   | \( \mu_{tot} \) (mas/yr) | Distance (kpc) | \( V_T \) (km s\(^{-1}\)) |
|---------|---------|--------------------------|----------------|--------------------------|
| J0134–2937 | - | 17(3) | 0.56 | 45(12) |
| J0535+5413 | B0355+54 | 12.5(9) | 1.00 | 59(13) |
| J0454+5543 | B0450+55 | 58(3) | 1.18 | 324(67) |
| J0953+0755 | B0950+08 | 34(10) | 0.26 | 42(15) |
| J1745–3040 | B1742–30 | 33(11) | 0.20 | 33(12) |
| J1835–1020 | - | 25(8) | 2.30 | 273(103) |
| J2022+2854 | B2020+28 | 21(6) | 2.10 | 209(73) |

5 DISCUSSION

5.1 Spectral index

During the process described above, it is necessary to determine the properties of the noise. This is carried out “by hand” in the frequentist method as the user is asked to input parameters for the simple model (equation 1) of the red noise and directly as part of Bayesian automated procedure. A simple-to-compare measure of the timing noise model is the spectral index of the noise.\(^8\) \( \alpha \). It is given here as a positive value, which is defined by power spectral density of timing residuals (Hobbs et al. 2010)

\[
S(f) \propto f^{-\alpha}
\]

\(^8\) Two ways are currently provided to describe the red timing noise in TempoNest, the power-law model presented in Haasteren (2009) and the model independent method introduced in Lentati (2013).
Table 6. Spectral indices from the frequentist and the Bayesian methods.

| PSR J  | PSR B  | \(\alpha^F\) | \(\alpha^B\) |
|--------|--------|-------------|-------------|
| J0134−2937 | - | 3 | 3(2) |
| J0358+5413 | B0355+54 | 8 | 6.2(3) |
| J0454+5543 | B0450+55 | 6.4 | 5.4(6) |
| J0953+0755 | B0950+08 | 7 | 5.1(5) |
| J1733−3716 | B1730−37 | 6.9 | 5.2(8) |
| J1742−3040 | B1741−30 | 7.4 | 6.4(6) |
| J1835−1020 | B1834−10 | 7 | 6.0(6) |
| J1847−0402 | B1844−04 | 6.4 | 5.8(5) |
| J1845−0135 | B1845−01 | 7.5 | 7.0(6) |
| J1900+0331 | B1859+03 | 7 | 7.2(8) |
| J1903+0135 | B1900+01 | 7.2 | 4.0(4) |
| J1904+0004 | - | 6.8 | 4.6(11) |
| J2022+2854 | B2020+28 | 5.6 | 3.2(2) |
| J2113+4644 | B2111+46 | 5.2 | 6.1(6) |
| J2257+5909 | B2255+58 | 8 | 7.0(8) |

where \(f\) is spectral-frequency. Table 6 shows spectral indices from the two methods. For most of the 15 pulsars the derived spectral indices are similar, although the frequentist method often gives a somewhat steeper spectral index. However, as shown by Coles et al. (2011) small errors in the assumed power-law parameters of the noise do not significantly affect the resulting parameters and we note that even though the spectral indices in these two methods are slightly different the resulting proper motions are consistent.

In order to understand the differences between the frequentist and Bayesian spectral-index estimates in more detail we have simulated 10 realisations of two pulsars, i.e., PSRs J0454+5543 and J2257+5909. First, an ideal fake data set is formed using the formIdeal plugin to TEMPO2 with the real observed ToAs and the ephemeris assuming a user-specified proper motion in right ascension and declination. High-frequency fluctuations (white noise) in the timing residuals are introduced into the ToAs by running the addGaussian plugin. Low-frequency noise is added using a simple model for the red noise spectrum in equation 1 by running the addRedNoise plugin. The spectral index of the simulated red-noise is 6.4 and 8.0 for PSRs J0454+5543 and J2257+5909 respectively. Finally, the simulated ToAs were obtained by creating a realisation of the noise (both Gaussian noise and red noise) with the createRealisation plugin. We then processed the simulated data set in exactly the same way as a real one. For these simulated data sets of the two pulsars we obtained statistically consistent values for the proper motions over a wide range of input parameters. The spectral indices that were obtained from these simulated data sets are listed in Table 7. The first three columns in the table provide the pulsar names and then whether the results are from the frequentist or Bayesian method. We then provide the resulting spectral exponent from each of the 10 realisations of the simulated data set. Note that we obtain an uncertainty for the Bayesian method (listed in parentheses after the parameter value), but do not have an uncertainty on the frequentist determination. We then list (in columns 14, 15 and 16) the mean and standard deviation of the parameters and the simulated spectral index respectively. We note that:

- the uncertainty on the Bayesian estimates does agree with the standard deviation of the parameter values.
- unsurprisingly the human input leads to more quantised spectral indices being reported, but the values are similar to the simulated value (although see below).
- the frequentist values are biased high (i.e., the slope is reported to be slightly steeper than the simulated value). However, as noted by Coles et al. (2011) and also from the results in this paper, slight inaccuracies in the red-noise modelling do not significantly bias the parameter estimates.

5.2 Comparison of positions

The position measurements for the four pulsars that have published interferometric determinations, i.e., PSR J0358+5413 (Chatterjee et al. 2004), PSR J0454+5543 (Chatterjee et al. 2009), PSR J0953+0755 and PSR J2022+2854 (Braken et al. 2002), are compared with our (frequentist) results in Fig. 2. Each panel contains the measured interferometric position at the epoch of the measurement joined by an arrow to its expected position at the epoch of our measurement assuming the interferometric proper motion. We also plot the position obtained by our work. In all cases our positions are consistent with the extrapolated interferometric positions.

6 CONCLUSION

We have:

- Obtained proper motions applying the two methods that are not biased by pulsar timing noise for a sample of 15 pulsars using timing data from the Nanshan observatory. For four pulsars we present the first measurements of their proper motions.
- Demonstrated that the Coles et al. (2011) and the Lentati et al. (2014) methods can be applied to young pulsar data sets that exhibit large amounts of timing noise with steep spectral exponents.
- Shown that the two methods give consistent results.
- Shown that the two methods give results that are consistent with previous very-long-baseline interferometry positions.

The two methods give consistent results, but the application procedures for the two methods are very different. The frequentist method is very hands-on and uses an iterative procedure. Throughout this procedure the user proceeds directly with the data and can identify unexpected features that may be present. However, it is difficult to make the process reproducible as different users are likely to select different data models, use different inputs to the white noise modelling and iterate for a different number of times. In contrast the Bayesian method is quite straightforward - the user sets up an input file and then leaves it to run until the final results are returned. However, the Bayesian method requires significant processing power. Therefore, given the same timing models, the frequentist method and the Bayesian method are equivalent in timing analysis, but different in implementation.
Table 7. Spectral indices from the frequentist (F) and Bayesian (B) methods for 10 realisations of simulated data sets. The final three columns display the mean and standard deviation of the parameters and the simulated spectral index respectively.

| PSR J       | PSR B       | Method | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | σ_α  | α_{sim} |
|--------------|-------------|--------|----|----|----|----|----|----|----|----|----|-----|-------|
| J0454+5543   | B0450+55    | F      | 6.5| 7.0| 6.0| 6.7| 6.7| 6.7| 6.7| 6.7| 6.8| 6.6| 0.3  | 6.4   |
| J0454+5543   | B0450+55    | B      | 5.7(5)| 6.4(4)| 5.6(4)| 4.9(7)| 5.8(4)| 5.7(5)| 5.7(5)| 6.1(4)| 5.6(4)| 5.7(4)| 5.9  | 0.4  | 6.4   |
| J2257+5909   | B2255+58    | F      | 8.5| 8.7| 8.2| 8.5| 8.6| 8.5| 8.4| 8.6| 8.6| 8.6| 0.4  | 8.0   |
| J2257+5909   | B2255+58    | B      | 8.2(7)| 8.2(7)| 8.1(7)| 8.0(8)| 8.1(6)| 8.3(6)| 8.2(7)| 8.2(7)| 8.2(7)| 8.2  | 0.1  | 8.0   |

Observations of these and other pulsars are on-going at the Nanshan observatory. Over the following years more frequent observations and longer data spans will enable higher accuracy proper motions and position determinations. In the future, the Five hundred meter Aperture Spherical radio Telescope (FAST) and the QiTai radio Telescope (QTT) pulsar surveys will discover more pulsars and timing programs on these telescopes will be able to determine the astrometric parameters with much higher precision than those available to us with our existing telescopes.

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REFERENCES

Babak S., et al., 2016, MNRAS, 455, 1665
Br{"o}sker W. F., Benson J. M., Goss W. M., Thorsett S. E., 2002, ApJ, 571, 906
Br{"o}sker W. F., Fruchter A. S., Goss W. M., Herrnstein R. M., Thorsett S. E., 2003, AJ, 126, 3090
Chatterjee S., Cordes J. M., Vlemmings W. H. T., Arzoumanian Z., Goss W. M., Lazio T. J. W., 2004, ApJ, 604, 339
Chatterjee S., et al., 2009, ApJ, 698, 250
Coles W., Hobbs G., Champion D. J., Manchester R. N., Verbist J. P. W., 2011, MNRAS, 418, 561
Cui X. H., Wang H. G., Xu R. X., Qiao G. J., 2007, A&A, 472, 1
Desvignes G., et al., 2016, MNRAS, 458, 3341
Edwards R. T., Hobbs G. B., Manchester R. N., 2006, MNRAS, 372, 1549
Espinoza C. M., Lyne A. G., Stappers B. W., Kramer M., 2011, MNRAS, 414, 1679
Folkner W. M., Williams J. G., Boggs D. H., 2008, JPL IOM 343R-08-00
Gonzalez M. E., et al., 2011, ApJ, 743, 102
Harrison E. R., Tademaru E., 1975, ApJ, 201, 447
Harrison P. A., Lyne A. G., Anderson B., 1993, MNRAS, 261, 113
Hobbs G. B., Edwards R. T., Manchester R. N., 2006, MNRAS, 369, 655
Hobbs G., Lyne A. G., Kramer M., 2010, MNRAS, 402, 1027
Hobbs G., Lyne A. G., Kramer M., Martin C. E., Jordan C., 2004, MNRAS, 353, 1311
Hobbs G., Lorimer D. R., Lyne A. G., Kramer M., 2005, MNRAS, 360, 974
Hotoan A. W., van Straten W., Manchester R. N., 2004, PASA, 21, 302
Johnston S., Manchester R. N., Lyne A. G., Kaspi V. M., D’Amico N., 1995, A&A, 293, 795
Lai D., Qian Y. Z., 1998, ApJ, 505, 844
Lai D., Goldreich P., 2000, ApJ, 535, 402
Lentati L., et al., 2013, PhRvD, 87, 104021
Lentati L., et al., 2014, MNRAS, 437, 3004
Lyne A. G., Anderson B., Salter M. J., 1992, MNRAS, 201, 503
Lyne A. G., Lorimer D. R., 1994, Nat, 369, 127
Manchester R. N., Taylor J. H., Van Y.-Y., 1974, ApJ, 189, L119
Manchester R. N., Hobbs G. B., Teoh A., Hobbs M., 2005, AJ, 129, 1993
Matthews A. M., et al., 2016, ApJ, 818, 92
Mignani R. P., Luca A. De., Caraveo P. A., 2000, ApJ, 543, 318
Morris D. J., et al., 2002, MNRAS, 335, 275
Nice D. J., Taylor H. J., 1995, ApJ, 441, 429
Ogelman H. B., Koch-Miramond L., Aur{ê}re M., 1989, ApJ, 342, L83
Reardon D. J., Hobbs G., Coles W., et al., 2016, MNRAS, 455, 1751
Trimpyle V., Rees M. J., 1971, ApJ, 166, L85
van Haasteren R., Levin Y., McDonald P., Lu T., 2009, MNRAS, 395, 1005
Wang J. B., et al., 2015, MNRAS, 446, 1657
Wang N., et al., 2001, MNRAS, 328, 855
Wang N., et al., 2005, MNRAS, 358, 270
Wycko{ś} S., Murray C. A., 1977, MNRAS, 180, 717
Yan Z., Shen Z. Q., Yuan J. P., Wang N., Rottmann H., Alef W., 2013, MNRAS, 433, 162
Zou W. Z., Hobbs G., Wang N., Manchester R. N., Wu X. J., Wang H. X., 2005, MNRAS, 362, 1189

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Figure 2. Comparisons of positions. For each pulsar, the interferometric position is extrapolated to the epoch of our timing position using the interferometric proper motion. The two interferometric positions are connected by an arrow. Error bars ($\pm 1\sigma$) are plotted for the interferometric positions and the (frequentist) timing positions.