Optical Asymmetry Induced by $\mathcal{PT}$–symmetric Nonlinear Fano Resonances

F. Nazari$^{1,2}$, N. Bender$^1$, H. Ramezani$^1$, M. K. Moravvej-Farshi$^2$, D. N. Christodoulides$^3$, T. Kottos$^1$

$^1$Department of Physics, Wesleyan University, Middletown, CT-06459, USA
$^2$Faculty of Electrical & Computer Engineering, Tarbiat Modares University, Tehran 1411713116, Iran and
$^3$College of Optics & Photonics-CREOL, University of Central Florida, Orlando, Florida 32816, USA

(Dated: October 10, 2013)

We introduce a new type of Fano resonances, realized in a photonic circuit which consists of two nonlinear $\mathcal{PT}$-symmetric micro-resonators side-coupled to a waveguide, which have line-shape and resonance position that depends on the direction of the incident light. We utilize these features in order to induce asymmetric transport up to 47 dBs in the optical C-window. Our set-up requires low input power and does not compromise the power and frequency characteristics of the output signal.

PACS numbers: 05.45.-a, 42.25.Bs, 11.30.Er

The realization of micron scale photonic elements and their integration into a single chip-scale device constitute an important challenge, both from a fundamental and a technological perspective [1]. An important bottleneck towards their realization is achieving on-chip optical isolation, that is the control of light propagation in predetermined spatial directions. Standard approaches for optical isolation rely mainly on magneto-optical (Faraaday) effects, where space-time symmetry is broken via external magnetic fields. This approach requires materials with high Verdet constants and/or large size non-reciprocal structures which are incompatible with on-chip integration [1]. Alternative proposals, for the realization of optical diodes, include dynamical modulation of the index of refraction [2], the use of opto-acoustic effects [3], and optical non-linearities [4–8]. Most of these schemes, have serious drawbacks which make them unsuitable for small-scale implementation. In some of these cases, complicated designs that provide structural asymmetry are necessary, or the transmitted signal has different characteristics (e.g different frequency) than the incident one. In other cases, direct reflection or absorption dramatically affects the functionality leading to an inadequate balance between transmitted optical intensities and figures of merit.

Recently, optical microresonator structures [9] with high-quality factors that trigger non-linear effects, have attracted increasing attention as basic elements for the realization of on-chip optical diodes [7–8]. The basic geometries used consists of two waveguides coupled with two single mode non-linear cavities. These geometries typically allow for a narrow band transmission channel with a symmetric Lorentzian transmittance lineshape. In Ref. [7] the structure was passive and the diode action was imposed due to the asymmetric coupling of the cavities to the waveguides. The drawback of this protocol is that high degree of asymmetric transport (due to strong asymmetric coupling) is achieved at the expense of low intensity output signals. On the other hand, the proposal of Ref. [8] involved active cavities (one with gain and another with loss) which excite the nonlinear resonances differently, depending on the incident direction.

This latter proposal has been demonstrated recently in a beautiful experiment [10] where gain in the first resonator is supplied by optically pumping Erbium ions embedded in a silica matrix while the second resonator exhibits passive loss. However, the degree of transport asymmetry is moderate and it is achieved only for high values of gain.

In this Letter we investigate the possibility to utilize a new type of nonlinear Fano resonances emerging in a parity-time ($\mathcal{PT}$) symmetric framework in order to create asymmetric transport. The proposed photonic circuit consists of two non-linear $\mathcal{PT}$–symmetric microcavities which are side-coupled to a single waveguide. We show that this system naturally exhibits Fano resonances [11] which, due to the interplay of non-linearity with the active elements, are triggered at different resonance frequencies and have different lineshape.
depending on the direction of the incident light. Fano
one resonances, sometimes behaving like coupled-resonator-induced transparency [12], were first introduced in the optics framework in Refs. [13, 14]. Their shape is distinctly asymmetric, and differs from the conventional symmetric Lorentzian resonance curves (for a recent review see [15]). This asymmetric resonance profile essentially results from the interference between a direct and a resonance-assisted indirect pathway [12].

A realization of the proposed photonic diode is shown in the inset of Fig. 1. For demonstration purposes, let the core of both the waveguide and of the two microdisks to be of AlGaAs material. The permittivity of both microdisk resonators and of the waveguide is taken to be \( \epsilon' = 11.56 \) while the nonlinear Kerr coefficient for the microdisks is \( \chi = 1e - 19(m^2/V^2) \). The radius of the microdisks and their distance of each other are 5\( \mu \)m and 770nm, respectively. Moreover, the width of the waveguide and its coupling distance to the resonators are 460nm and 120nm, respectively. The circuit is operated at the optical communication window at wavelength around \( \lambda \approx 1558.6nm \) with one disc experiencing gain, while the other one having an equal amount of loss described by the imaginary part of the permittivity \( \epsilon'' = 0.00063 \). The structure is invariant under \( PT \) symmetry where the \( P \) is the parity reflection, with respect to the axis of symmetry located at the middle between the two resonators, and \( T \) is the time reversal operator which turns loss to gain and vice versa. The concept of \( PT \)-symmetry first emerged within the context of mathematical physics. In this regard, it was recognized that a class of non-Hermitian Hamiltonians that commute with the \( PT \) operator may have entirely real spectra [16]. Lately, these notions have been successfully migrated and observed in other areas like photonic [17–23] and electronic circuitry [8, 24, 25].

In Fig. 1 we show some transport simulations using COMSOL modeling. The input power used in the simulations is \( P = 1.2mW \). We find that the left-to-right transmittance \( T_L(\lambda) \) differs from the right-to-left transmittance \( T_R(\lambda) \), i.e. \( T_L \neq T_R \). The asymmetric transport is most pronounced near the Fano resonances \( \lambda_{PT}^{\chi=0} \) of the linear structure, and constitutes our main result. We stress that non-reciprocal transport is strictly forbidden by the Lorentz reciprocity theorem in the case of linear, time-reversal symmetric systems [26]. At the same time, it cannot be achieved neither by a conservative nonlinear medium by itself nor by linear \( PT \)-symmetric structures (see black filled circles in Fig. 1) [25].

Next we analyze the origin of the asymmetry between left and right transmittances, near the Fano resonances \( \lambda_{PT}^{\chi=0} \) of the linear photonic circuit of Fig. 1. To understand better its origin, we first discuss the transport characteristics of a single gain (lossy) non-linear microdisc side-coupled to a waveguide. In the case that the incident light traveling the waveguide couples with a gain resonator it will be amplified substantially because of the interaction with the gain medium, and the high \( Q \) factor of the disc. Consequently, the signal has sufficiently high power to trigger the non-linearity and red-shift the disc’s resonance \( \lambda_{G}^{\chi=0} \), thus allowing it to pass with small (or even not at all) attenuation at the resonance wavelength \( \lambda_{G}^{\chi=0} \) of the gain cavity (dashed line in the inset of Fig 2). On the other hand, when light couples to a lossy microdisc, the optical energy stored in this disc is not high enough to appreciably red-shift (via non-linearity) the resonance because of the power reduction due to the losses. As a result the transmittance has a resonance dip at \( \lambda \approx \lambda_{G}^{\chi=0} \).

![FIG. 2: Transmittance curves T(\lambda) for a single gain or loss microdisc side-coupled to a waveguide. The filled red circles (dashed-dotted line) correspond to a gain disc in the absence (presence) of Kerr nonlinearity. The filled green circles (green bold line) correspond to a lossy disc in the absence (presence) of Kerr nonlinearity. In the case of a gain disc a red-shift of the resonance position and a strong modification of the transmittance line-shape is observed.](image)

When both linear microdisks, i.e. the one with gain (left) and the other one with loss (right), are side-coupled to the waveguide the transmittance shows a peak in the middle of the resonant dip. This phenomenon is an optical analogue [27] of electromagnetically induced transparency (EIT) and is known as coupled-resonator-induced transparency [12]. It is associated with the interaction between two Fano resonances with spectral widths which are comparable to or larger than the frequency separation between them. These Fano resonances have been formed due to coherent interferences between the two coupled resonators. Still we observe that left and right transmittance are equal i.e. \( T_L(\lambda) = T_R(\lambda) \).

When nonlinearities are considered, the transmission near the Fano resonances is asymmetric and depends strongly on the direction of the incident light. Below we concentrate on the wavelength domain on the left of the transparent window where, for our set up, the asymmetry is stronger. In this case the incident light entering the waveguide from the left is first coupled to the gain resonator which amplify the light intensity; thus inducing optical nonlinearity of the material. As a result, the reso-
Fano-Anderson model \cite{15,28} that is used to describe the creation of (non-linear) Fano resonances, provides some quantitative understanding of the COMSOL simulations shown in Figs. \[13\]. Our model is described by the following sets of differential equations:

\[
\begin{align*}
\imath \phi_n &= -\{C(\phi_{n-1} + \phi_{n+1}) + V_G \phi_G \delta_{n,0} + V_L \phi_L \delta_{n,N}\} \\
\imath \phi_G &= -\{(E - \imath \gamma) \phi_G + \chi |\phi_G|^2 \phi_G + V_G \phi_0\} \\
\imath \phi_L &= -\{(E + \imath \gamma) \phi_L + \chi |\phi_L|^2 \phi_L + V_L \phi_N\}
\end{align*}
\]

Equations \(1\) describe the interaction of two subsystems. The first one is a linear chain of couple sites with coupling constant \(C\) and on-site complex field amplitudes \(\phi_n\). This system supports propagating plane waves with dispersion \(\omega(k) = 2C \cos q\). The second subsystem consists of two defect states \(\phi_G\) (gain) and \(\phi_L\) (loss) with on-site energy \(E \pm \imath \gamma\) respectively. The two subsystems interact with one another at the sites \(n = 0, N\) via the coupling coefficients \(V_G/L\).

We assume elastic scattering processes for which the stationary solutions take the form \(\phi_n = A_n e^{\imath \omega t}\); \(\phi_G = A_G e^{\imath \omega t}\); \(\phi_L = A_L e^{\imath \omega t}\). Substitution in Eqs. \(1\) leads to

\[
\begin{align*}
\omega A_n &= C(A_{n-1} + A_{n+1}) + V_G A_G \delta_{n,0} + V_L A_L \delta_{n,N} \\
\omega A_G &= E A_G - \imath \gamma A_G + \chi |A_G|^2 A_G + V_G A_0 \\
\omega A_L &= E A_L + \imath \gamma A_L + \chi |A_L|^2 A_L + V_L A_N
\end{align*}
\]

We consider a left incident wave. In this case we have

\[
A_n = \begin{cases} 
I e^{\imath qn} + r e^{-\imath qn} & n \leq 0 \\
\alpha e^{\imath qn} + \beta e^{-\imath qn} & 0 \leq n \leq N \\
t e^{\imath qn} & N < n
\end{cases}
\]

where \(I, r, t\) represent the incident, reflected, and transmitted wave amplitudes far from the defect sites. Substituting the above scattering conditions in Eqs. \(2\) and using the continuity conditions at the defect sites \(n = 0, N\), we get after some straightforward algebra

\[
r_L = \frac{V_G A_G + V_L A_L e^{\imath qN}}{2 C \sin q};
t_L = \frac{(I + \imath V_G A_G + V_L A_L e^{-\imath qN})}{2 C \sin q}
\]

The unknown amplitudes \(A_G, A_L\) can be found in terms of the input amplitude \(I\) by utilizing Eqs. \(2, c\). Specifically we get the following set of nonlinear equations

\[
\begin{align*}
(\omega + E - \imath \gamma) A_G + \chi |A_G|^2 A_G + V_G (I + r_L) &= 0 \\
(\omega + E + \imath \gamma) A_L + \chi |A_L|^2 A_L + V_L e^{\imath qN} t_L &= 0
\end{align*}
\]

which can be solved numerically, after substituting \(r_L\) and \(t_L\) from Eqs. \(4\).

The transmittance and reflectance for a left incident wave is defined as \(T_L = |t_L|^2\) and \(R_L = |r_L|^2\) respectively. In a similar manner one can also define the transmittance \(T_R = |r_R|^2\) and reflectance \(R_R = |r_R|^2\) for a right incident wave. The associated \(I_R, r_R, t_R\) are given by the same expressions as Eq. \(4\), with the substitution of \(\gamma \rightarrow -\gamma; V_G \rightarrow V_L\) and \(V_L \rightarrow V_G\).
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through the chain without coupling to any of the defect
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Asymmetric transport.

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4

The red-shadowed area on the right of the graph around
T
transmittances for the theoretical model of Eq. (1). Notice
V
in the neighborhood of the second Fano resonance. In this
domain the T_L > T_R. The parameters used in this simulation
are V_G = V_L = 0.5, N = 1, \chi = 0.0125 and \gamma = 0.02.
The red-shadowed area on the right of the graph around \lambda \approx 4.5, corresponds to a bi-stability behavior which however is
away from the Fano resonance regime and thus does not affect
asymmetric transport.

An analysis of the structure of Eq. (4) can explain the
origin of Fano resonances. Specifically, we note that the transmission amplitude in Eq. (4) consists of two terms:
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ing the Fano states, return back, and continue with the propagation. These two paths are the ingredients of the Fano resonances observed in Figs. [13][14]

In Fig. [4] we report a representative set of transmission
curves for a left/right incident wave for the model of Eq. (1). The model captures the qualitative features and ori-
gin of the asymmetric transport observed in the case of the photonic circuit of Fig. [1]. Specifically, we find that both the shape and the position of the Fano resonances depend on the direction of the incident wave. Moreover for a left (gain-side) incoming wave, a red-shift in the transmittance resonances is found (see the neighborhood of the second Fano resonance in Fig. [4]) which leads to an asymmetric transport.

Conclusions - In conclusion, we have introduce a new
type of PT-symmetric Fano resonances with a line-shape
and a resonance position that depends on the direction of the incident wave. The photonic circuit that allows for such resonances consists of two PT- symmetric microdisks side-coupled to a waveguide. The proposed config-
figuration guarantee not only high asymmetry but also a
significant level of transmittance. Our proposal utilizes
existing materials already used in optical integrated cir-
cuity processing and does not require magnetic fields, or
other external elements like polarizers. The efficiency of
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tions. A problem with the simple design of Fig. [1] is
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Reciprocity violations can occur in linear magnetoactive media, see for example: H. Ramezani et al., Opt. Express 20, 26200 (2012).

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