We introduce a model of potential driven DBI Galileon inflation in background $\mathcal{N}=1, D=4$ SUGRA. Starting from $D4-\bar{D}4$ brane-antibrane in the bulk $\mathcal{N}=2, D=5$ SUGRA including quadratic Gauss-Bonnet corrections, we derive an effective $\mathcal{N}=1, D=4$ SUGRA by dimensional reduction, that results in a Coleman-Weinberg type Galileon potential. We employ this potential in modeling inflation and in subsequent study of primordial quantum fluctuations for scalar and tensor modes. Further, we estimate the major observable parameters in both de Sitter (DS) and beyond de Sitter (BDS) limits and confront them with recent observational data from WMAP7 by using the publicly available code CAMB.
which are resolved by introducing a dynamical field, aka, Galileon [11,13] arising on the brane from the bulk in the DGP setup. The cosmological consequences of the Galileon models have been studied to some extent in [14,17]. Very recently, a natural extension to the scenario has been brought forward by tagging Galileon with the good old DBI model [18,19] resulting in so-called “DBI Galileon” [20,22], that has reflected a rich structure from four dimensional cosmological point of view. However, in most of the physical situations, this type of effective gravity theories are plagued with additional degrees of freedom which often results in unwanted debris like ghosts, Laplacian instabilities etc [23,27]. Very recently a noble effort towards the supersymmetric extension of the DBI Galileon model and its inflationary signature are discussed in [25] and [29]. In the present paper we introduce a single scalar field model described by the D3 DBI Galileon originated from D4-¯D4 brane anti-brane setup in the background of five dimensional local version of the supersymmetric theory (supergravity). This prevents the framework from having extra degrees of freedom as well as Ostrogradski instabilities [31], resulting in a higher-order derivative scalar field theory free from any such unwanted instabilities. Nevertheless, a consistent field theoretic derivation of the effective potential commonly used in the context of DBI Galileon cosmology has not been brought forth so far. On top of that, it is imperative to point out that the SUGRA origin of D3 DBI Galileon is yet to be addressed. In this article we plan to address both of these issues explicitly by deriving the inflaton potential from our proposed framework of DBI Galileon. It turns out that the derived action includes, in certain limits, the decoupling limit of DGP model as well as some consistent theories of massive gravity; and it also includes the “K-mouflage” [31],[32] and also “G/KGB” [14–16], [33–35]. Moreover, in general appearance of non-vanishing frame functions in the 4D action expedites breakdown of shift symmetry. Without shift symmetry, it may happen that the theory is unstable against large instabilities etc [25–27]. Very recently a nobel effort towards the supersymmetric extension of the DBI Galileon model [30] resulting in so-called “DBI Galileon” [20–24], that has reflected a rich structure from good old DBI model [18], [19], resulting in so-called “DBI Galileon” [20–24], that has reflected a rich structure from

II. THE BACKGROUND ACTION

Let us demonstrate briefly the construction of DBI Galileon starting from N=2,D=5 SUGRA along with Gauss Bonnet correction in D4 brane set up. The full five dimensional model is described by the following action

\[
S_{\text{Total}}^{(5)} = S_{E5}^{(5)} + S_{\text{GB}}^{(5)} + S_{\text{brane}}^{(5)} + S_{\text{BulkSugra}}^{(5)}
\]

where

\[
S_{E5}^{(5)} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g^{(5)}} \left[ R^{(5)} - 2\Lambda_5 \right],
\]

\[
S_{\text{GB}}^{(5)} = \frac{\alpha^{(5)}}{2\kappa_5^2} \int d^5x \sqrt{-g^{(5)}} \left[ R^{ABCD(5)} R_{ABCD}^{(5)} - 4 R^{AB(5)} R_{AB}^{(5)} + R^{2(5)} \right]
\]

where \(\alpha^{(5)}\) and \(\kappa_5\) represent Gauss-Bonnet coupling and 5D gravitational coupling strength respectively. Additionally, \(\Lambda_5\) and \(g^{(5)}\) represent the 5D cosmological constant and the determinant of the 5D metric explicitly mentioned in equation (2.17). The D4 brane action decomposed into two parts as

\[
S_{\text{brane}}^{(5)} = S_{\text{DBI}}^{(5)} + S_{\text{WZ}}^{(5)}
\]

where the DBI action and the Wess-Zumino action are given by respectively

\[
S_{\text{DBI}}^{(5)} = -\frac{T_{(4)}}{2} \int d^5x \exp(-\Phi) \sqrt{-(\gamma^{(5)} + B^{(5)} + 2\pi\alpha' F^{(5)}),
\]

\[
S_{\text{WZ}}^{(5)} = \frac{\alpha^{(5)}}{2\kappa_5^2} \int d^5x \sqrt{-g^{(5)}} \left[ R^{ABCD(5)} R_{ABCD}^{(5)} - 4 R^{AB(5)} R_{AB}^{(5)} + R^{2(5)} \right]
\]

\[
S_{\text{BulkSugra}}^{(5)} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g^{(5)}} \left[ R^{(5)} - 2\Lambda_5 \right]
\]
\[
S_{WZ}^{(5)} = -\frac{T_{(4)}}{2} \int \sum_{n=0,2,4} \left. \bar{C}_n \wedge \exp \left( \bar{B}_2 + 2\pi \alpha' F_2 \right) \right|_{4 \text{ form}} \\
= \frac{1}{2} \int d^5x \sqrt{-g(5)} \left( \epsilon^{ABCD} \left( \frac{C_{IJ}B_{KL}\partial_{C} \Phi^I \partial_{D} \Phi^J}{4T_{(4)}} - \frac{\pi \alpha' C_0}{2} B_{IJ} \right) + 2\alpha'^2 T_{(4)} C_0 F_{AB} F_{CD} - T_{(4)} \nu(\Phi) \right) 
\]

where \( T_{(4)} \) is the D4 brane tension, \( \alpha' \) is the Regge Slope, \( \exp(-\Phi) \) is the closed string dilaton and \( C_0 \) is the Axion. Here and throughout the article hat denotes a pull-back onto the D4 brane so that \( \gamma_{AB} \) is the 5D induced metric on the D4 brane explicitly defined in equation (2.19). Here \( \gamma_{AB} \), \( B^{(5)} \) and \( F^{(5)} \) represent the determinant of the 5D induced metric \( (\gamma_{AB}) \) and the gauge fields \( (B_{AB}, F_{AB}) \) respectively. The gauge invariant combination of rank 2 field strength tensor, appearing in D4 brane, is \( F_{AB} = B_{AB} + 2\pi \alpha' F_{AB} \) and \( \{F_2, B_2\} \) represents 2-form \( U(1) \) gauge fields which have the only non-trivial components along compact direction. On the other hand \( C_4 \) has components only along the non-compact spacetime dimensions. In a general flux compactification all fluxes may be turned on as the Ramond-Ramond (RR) forms \( F_{n+1} = dC_n \) (along with their duals) with \( n = 0, 2, 4 \) and the Neveu Schwarz-Neveu Schwarz (NS-NS) flux \( H_3 = dB_2 \). Additionally the D4 brane frame function is defined as:

\[
\nu(\Phi) = \left( \nu_0 + \frac{\nu_4}{\Phi^2} \right) 
\]

which is originated from interaction between D4-\( \bar{D}4 \) brane in string theory. Here \( \nu_0 \) and \( \nu_4 \) represent the constants characterizing the interaction strength between D4-\( \bar{D}4 \) brane.

In equation (2.1) \( \mathcal{N} = 2, D = 5 \) bulk supergravity action can be written as:

\[
S_{\text{bulk SUGRA}}^{(5)} = \frac{1}{2} \int d^5x \sqrt{-g(5)} \epsilon^{(5)} \left[ -\frac{M_5^{(5)}}{2} \bar{\Psi}_\mu \bar{\Gamma}^{\mu\nu\delta\eta} \nabla_\nu \Psi_\delta - S_{IJ} F_{\hat{m}n} F^{I\hat{m}n} - \frac{1}{4} g_{\mu\nu}(D_\mu \Phi^\nu)(D_\nu \Phi^\nu) \right] + \text{Fermionic + Chern - Simons + Pauli mass}, \tag{2.8}
\]

The rank-2 tensor field \( S_{IJ} \), appearing in the kinetic terms of the gauge fields, is the restriction of the metric of the 5 dimensional space on the 4 dimensional manifold of the scalar fields given by:

\[
S_{IJ} = -2C_{IJK} h^K + 3h_i h_J, \tag{2.9}
\]

where \( h_i = C_{IJK} h^J h^K = S_{IJ} h^J \) and \( g_{\mu\nu} = h^i_{\mu} h^j_{\nu} S_{IJ} \) is the metric of the 4-dimensional manifold \( \mathcal{M}_4 \). In these equations we use \( h^i_{\mu} = -\sqrt{\frac{2}{3}} h^i_{\mu,x} \) and \( h_i^I = \sqrt{\frac{2}{3}} h_i^I \) along with the following constraints

\[
h^I h_I = 1, \quad h^i_i h_J = h^i_i h_J = 0. \tag{2.10}
\]

Here \( C_{IJK} \) are constants symmetric in the three indices satisfies the cubic constraint relationship \( C_{IJK} h^I h^J h^K = 1 \). With the parity assignments we have adopted, \( h^5 = 0 \) is even, while \( h^x = \Phi^x \) are odd. Furthermore on the fixed points where the odd quantities vanish, \( h^5 = 1 \). Analogous relations hold for the \( h_i \)'s. In this context the 5-dimensional coordinates \( X^A = (x^\alpha, y) \), where \( y \) parameterizes the extra dimension compactified on the closed interval \( [-\pi R, +\pi R] \) and \( Z_2 \) symmetry is imposed. We consider 5D Yang Mills SUGRA model which is described by the field content \( \left\{ e^\mu_\mu, \Psi_\mu, A_\mu^I, \lambda^\alpha, \Phi^x \right\} \) where \( \mu = (\mu, 5) \) are curved and \( \hat{m} = (m, \hat{5}) \) are flat 5D indices with \( \mu, n \) their corresponding 4D indices. The remaining indices are \( I = 0, 1, ..., n, a = 1, 2, ..., n \) and \( x = 1, 2, ..., n \). The SUGRA multiplet consists of the fermions \( e^\mu_\mu \), two graviphoton \( A_\mu^0 \) and two gravitini \( \Psi_\mu^I \), where \( i = 1, 2 \) is the simplectic \( SU(2)_R \) index. Moreover, there exists \( n \) vector multiplets, counting the Yang Mills fields \( A_\mu^I \). The spinor and the scalar fields included in the vector multiplets are collectively denoted by \( \lambda^\alpha, \Phi^x \) respectively. The indices \( a \) and \( x \) are flat and curved indices respectively of the 5D manifold \( \mathcal{M} \). It is important to mention here that the Chern-Simons terms can be gauged away assuming cubic constraints and \( Z_2 \) symmetry. Now we consider full particle spectrum , the \( Z_2 \) even fields \( \left\{ e^\mu_\mu, e^5_5, \Psi_\mu^1, \Psi_\mu^2, A^0_\mu, A^5_\mu, \lambda^\alpha \right\} \) and the \( Z_2 \) odd fields \( \left\{ e^5_\mu, e^\mu_5, \Psi_\mu^1, \Psi_\mu^2, A^0_\mu, A^5_\mu, \lambda^\alpha \right\} \) propagates in the bulk. For computational purpose it is useful to define the five dimensional generating function \( G \) of supergravity in this setup as

\[
G = -3 \ln \left( \frac{T + T^*}{\sqrt{2}} \right) + K(\Phi, \Phi^I), \tag{2.11}
\]
where the supergravity Kähler moduli fields are given by

\[ T = \frac{(c^5 - i \sqrt{\frac{3}{4}} A^0_5)}{\sqrt{2}} \]  

(2.12)

which is assumed to be stabilized under first approximation and \( K(\Phi, \Phi^\dagger) \) represents generalized Kähler function.

Including the kinetic term of the five dimensional field \( \Phi \) and rearranging into a perfect square, the 5D bulk supergravity action can be expressed as

\[
S \supset \frac{1}{2} \int d^4x \int_{-\pi R}^{+\pi R} dy \sqrt{-g_{5c(4)p}^5} \left[ g^{\alpha\beta} G_M^N (\partial_\alpha \Phi^M)^\dagger (\partial_\beta \Phi^N) + \frac{1}{g_{55}} \left( \partial_5 \Phi - \sqrt{V_{\text{bulk}}^5(G)} \right)^2 \right],
\]

(2.13)

where the 5D potential described by

\[
V_{\text{bulk}}^5(G) = \exp \left( \frac{G}{M^2} \right) \left[ \left( \frac{\partial W}{\partial \Phi_M} + \frac{\partial G}{\partial \Phi_M} \right)^\dagger \left( G_M^N \right)^{-1} \left( \frac{\partial W}{\partial \Phi_N} + \frac{\partial G}{\partial \Phi_N} \right) M^2 \right] - 3 |W|^2
\]

(2.14)

where \( W \) physically represents the superpotential in the context of \( N = 2, D = 5 \) supergravity theory and expressed in terms of the holomorphic combination of the fields \( \Phi, \Phi^\dagger, T \) and \( T^\dagger \). The field equations in presence of Gauss-Bonnet term can be expressed as

\[
G_{AB}^{(5)} + \alpha_{(5)} H_{AB}^{(5)} = 8 \pi G_{(5)} T_{AB}^{(5)} - \Lambda_{(5)} g_{AB}^{(5)},
\]

(2.15)

where the covariantly conserved Gauss-Bonnet tensor

\[
H_{AB}^{(5)} = 2 R_{ACDE}^{(5)} R_B^{CDE} - 4 R_{ACBD}^{(5)} R^{CD} - 4 R_{AC}^{(5)} R_B^{CD} + 2 R_{AB}^{(5)} - \frac{1}{2} g_{AB}^{(5)} \left( R^{ABCD} R_{ABCD} - 4 R^{AB} R_{AB}^{(5)} + R^{(5)} \right)
\]

(2.16)

which acts as a source term. It is useful to introduce the 5D metric in conformal form

\[
ds_{4+1}^2 = g_{AB} dX^A dX^B = \frac{1}{\sqrt{h(y)}} ds_4^2 + \sqrt{h(y)} \hat{G}(y) dy^2 = \exp(2A(y)) \left( ds_4^2 + R^2 \beta^2 dy^2 \right),
\]

(2.17)

with warp factor

\[
\exp(2A(y)) = \frac{1}{\sqrt{h(y)}} = \frac{R^2}{b_0^2 \beta^2 \hat{G}(y)} = \frac{b_0^2}{R^2 \left( \exp(\beta y) \right) + \frac{\Lambda_{(5)} b_0^4}{24 R^4} \exp(-\beta y)}
\]

(2.18)

and \( ds_4^2 = g_{\alpha\beta} dx^\alpha dx^\beta \) is FLRW counterpart. In order to write down explicitly the expression for D4 brane action, the induced metric can be shown as

\[
\gamma_{CD} = \frac{1}{\sqrt{h(y)}} \left( g_{AB} + h(y) \hat{G}_{AB} \partial C \Phi^A \partial D \Phi^B \right).
\]

(2.19)

The 5D energy momentum tensor for the set up reads

\[
T_{\alpha\beta}^{(5)} = T_{\alpha\beta}^{\text{bulk}(5)} + T_{\alpha\beta}^{\text{brane}(5)} = G_M^N (\partial_\alpha \Phi^M)^\dagger (\partial_\beta \Phi^N) - g_{\alpha\beta} \left[ g^{\rho\sigma} (\partial_\rho \Phi^M)^\dagger (\partial_\sigma \Phi^N) G_M^N + g_{55} \left( \partial_5 \Phi - \sqrt{V_{\text{bulk}}^5(G)} \right)^2 \right]
\]

\[
+ \left[ K_{MN;X} \partial_\alpha \Phi^M \partial_\beta \Phi^N + K_{g_{\alpha\beta}} - 2 \nabla_\alpha G(\Phi, X) \nabla_\beta \Phi + g_{\alpha\beta} \partial_5 G(\Phi, X) \partial^5 \Phi \right.
\]

\[
- G_{MN;X} (\Phi, X) \partial_\alpha \Phi^M \partial_\beta \Phi^N, \]

(2.20)

\[
T_{55}^{(5)} = T_{55}^{\text{bulk}(5)} + T_{55}^{\text{brane}(5)} = \frac{1}{2} \left( \partial_5 \Phi - \sqrt{V_{\text{bulk}}^5(G)} \right)^2 - \frac{1}{2} g_{55} g^{\rho\sigma} G_M^N (\partial_\rho \Phi^M)^\dagger (\partial_\sigma \Phi^N)
\]

\[
+ \left[ K_{MN;X} \partial_5 \Phi^M \partial_5 \Phi^N + K_{g_{55}} - 2 \nabla_5 G(\Phi, X) \nabla_5 \Phi + g_{55} \partial_5 G(\Phi, X) \partial^5 \Phi \right.
\]

\[
- G_{MN;X} (\Phi, X) \partial_5 \Phi^M \partial_5 \Phi^N \right].
\]

(2.21)
On the other hand, the *Klein-Gordon* equation of motion in 5D can be expressed as
\[
\partial_5 \left[ \frac{e_5^2}{g_{55}} \left( \partial_5 - \sqrt{V_{\text{bulk}}(G)} \right) \right] + \sum_N e_5^{5N} \left\{ \partial_5 \left[ \sqrt{g} R^N \left( \partial_5 \Phi \right) \right] - \frac{\sqrt{g_{55}}}{e_5^N} \partial_N \left( \sqrt{V_{\text{bulk}}(G)} \left( \partial_5 - \sqrt{V_{\text{bulk}}(G)} \right) \right) \right\} K_X(\Phi, X) \square^{(5)} \Phi - K_X(\Phi, X) \left( \nabla_A \nabla_B \Phi \right) \left( \nabla^A \Phi \nabla^B \Phi \right) + 2K_X(\Phi, X) X - K_X(\Phi, X) \left( X - 2 (G_{\Phi}(X) - G_{\Phi, X}(X) X) \right) + G_X(\Phi, X) \left( \left( \nabla_A \nabla_B \Phi \right) \left( \nabla^A \Phi \nabla^B \Phi \right) - \left( \square^{(5)} \Phi \right)^2 R_{AB} \right) + 2G_{\Phi, X}(\Phi, X) \left( \left( \nabla_A \nabla_B \Phi \right) \left( \nabla^A \Phi \nabla^B \Phi \right) \right) + 2G_{\Phi}(\Phi, X) X - G_{\Phi, X} \left( \left( \nabla^A \Phi \nabla^B \Phi - g_{AB} \square^{(5)} \Phi \right) \left( \nabla_A \nabla_C \Phi \right) \left( \nabla_B \nabla_C \Phi \right) = 0. \tag{2.24}
\]

Now using the scaling relations
\[
\Phi^A = \sqrt{T_{(4)}} \hat{\Phi}^A, \quad G_{AB} = \exp(-\hat{\Phi}) \hat{g}_{AB}, \quad b_{AB} = \frac{h(y)}{T_{(4)}} B_{AB} \tag{2.23}
\]
the 5D action for D4 brane can be expressed in more convenient form as
\[
S_{D4 \text{ brane}}^{(5)} = \int d^5x \sqrt{-g^{(5)}} \left[ K(\Phi, X) - G(\Phi, X) \square^{(5)} \Phi \right] , \tag{2.24}
\]
where
\[
K(\Phi, X) = -\frac{1}{2f(\Phi)} \left( \sqrt{D} - 1 \right) + \frac{V_{\text{brane}}^{(5)}(\Phi)}{2} \tag{2.25}
\]
where the determinant can be expressed as
\[
D \simeq 1 - 2f(\Phi) G_{AB} X^A X^B + 4 f^2(\Phi) X^B A X^A X^B - 8 f^3(\Phi) X^B A X^B C + 16 f^4(\Phi) X^B A X^B C X^D \tag{2.26}
\]
which is expressed in terms of the kinetic term \( X^B D = -\frac{1}{2} G_{DA} \partial^D \Phi^A \). The detailed calculation for the determinant is elaborately discussed in the Appendix D. In this context the 5D D’Alembertian Operator is defined as
\[
\square^{(5)} = \frac{1}{\sqrt{-g^{(5)}}} \partial_A \left( \sqrt{-g^{(5)}} g^{AC} \partial_C \right). \tag{2.27}
\]
Here we use the fact that no spatial direction along which the scalar fields are only time dependent lead to \( B_{\mu}^0 = 0 \) and \( F_{\mu\nu} = 0 \) in the background. Consequently *Maxwell’s field equations* are unaffected in 4D after dimensional reduction. In this context the 5D D4 brane potential is given by
\[
V_{\text{brane}}^{(5)}(\Phi) = T_{(4)} \nu(\Phi) + \frac{1}{f(\Phi)} \tag{2.28}
\]
where 5D warped geometry motivated \( Z_2 \) symmetric frame function
\[
f(\Phi) = \frac{\exp(\Phi) h(y)}{T_{(4)}} \simeq \frac{1}{(f_0 + f_2 \Phi^2 + f_4 \Phi^4)} \tag{2.29}
\]
is originated from higher dimensional field theory and the implicit D4 brane function defined as:
\[
G(\Phi, X) = \frac{g(\Phi)}{2(1 - 2f(\Phi) X)} \tag{2.30}
\]
with \( g(\Phi) = g_0 + g_2 \Phi^2 \). Here \( g_0 \) and \( g_2 \) are model dependent constants characterizes the effects of possible interactions on the D4 brane.

### III. MODELING INFLATION FROM D3 DBI GALILEON

The technical details of the dimensional reduction technique are elaborately discussed in the Appendix A which can generate an effective D3 DBI Galileon theory in 4D. Now summing up all the contributions from \( \text{eqn}(6.1) \), \( \text{eqn}(6.6) \), \( \text{eqn}(6.7) \) and \( \text{eqn}(6.14) \), the model for *D3 DBI Galileon* is described by the following effective action:
\[
S_{\text{Total}}^{(4)} = \int d^4x \sqrt{-g^{(4)}} \left[ \tilde{K}(\phi, \bar{X}) - \tilde{G}(\phi, \bar{X}) \square^{(4)} \phi + \tilde{l}_1 R_{(4)} \right. \\
\left. + \tilde{l}_4 \left(C(1) R_{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}^{(4)} + 4^2 (2) R_{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}^{(4)} + A(6) R_{(4)}^2 + \tilde{l}_3 \right) \right] , \tag{3.1}
\]
where

\[
\begin{align*}
\tilde{K}(\phi, \tilde{X}) &= \tilde{K}(\phi, \tilde{X}) - 2X\tilde{M}(T, T^\dagger) - \tilde{Z}(T, T^\dagger)V_{\text{bulk}}^{(4)}(\phi), \\
\tilde{M}(T, T^\dagger) &= \frac{\tilde{M}(T, T^\dagger)}{2\kappa_{(4)}}, \\
\tilde{Z}(T, T^\dagger) &= \frac{Z(T, T^\dagger)}{2\kappa_{(4)}}, \\
\tilde{l}_1 &= \frac{\alpha_{(4)}}{2\kappa_{(4)}} \left[ 1 + \frac{\alpha_{(4)}}{R^{2\pi^2}} (24I(2) - 24A(9) - 16A(10)) \right], \\
\tilde{l}_4 &= \frac{\alpha_{(4)}}{2\kappa_{(4)}} \left[ \frac{\alpha_{(4)}}{R^{2\pi^2}} (24C(24) - 144I(4) - 6A(5) + 144A(7) + 64A(8) + 192A(11)) \right], \\
\tilde{l}_3 &= \frac{1}{2\kappa_{(4)}} \left[ \frac{\alpha_{(4)}}{R^{2\pi^2}} (24C(24) - 144I(4) - 6A(5) + 144A(7) + 64A(8) + 192A(11)) \right].
\end{align*}
\]

(3.2)

are explicitly mentioned in the Appendix C. For clarity, in terms of the effective potential the total four dimension action for our set up can be rewritten as:

\[
S_{\text{total}}^{(4)} = \int d^4x \sqrt{-g^{(4)}} \left[ K(\phi, \tilde{X}) - \tilde{G}(\phi, \tilde{X})\Box^{(4)}\phi - V(\phi) \right] + \tilde{l}_1 R_{(4)} + \tilde{l}_4 \left( C(1)R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} - 4I(2)R^{\alpha\beta(4)}R_{\alpha\beta} + A(6)R_{(4)}^2 \right) + \tilde{l}_3,
\]

(3.3)

where

\[
K(\phi, \tilde{X}) = -\frac{D}{f(\phi)} \left[ \sqrt{1 - 2Q\tilde{X} - Q_1} - \tilde{C}_0 \tilde{G}(\phi, \tilde{X}) - 2X\tilde{M}(T, T^\dagger) \right].
\]

(3.4)

For details see Appendix A. The collective effect of equation (3.8) and equation (3.20) gives the total $D^3$ DBI Galileon potential appearing in equation (3.3) as:

\[
V(\phi) = Q_2 \tilde{D}V_{\text{brane}}^{(4)} + \tilde{Z}(T, T^\dagger)V_{\text{bulk}}^{(4)}(\phi)
\]

(3.5)

\[
= \sum_{m=-2, m \neq -1} C_{2m} \phi^{2m},
\]

where

\[
\begin{align*}
C_0 &= \left( T_3 \varphi_0 + \beta R T(2) \varphi_0 + \tilde{Z}(T, T^\dagger)A(13)\varphi^4 \right), \\
C_{-4} &= T_3 \varphi_4, \\
C_2 &= \left( \beta R T(2) \varphi_2 - g \varphi^2 \tilde{Z}(T, T^\dagger)A(13) \right), \\
C_{-4} &= \left( \beta R T(2) \varphi_4 + \tilde{Z}(T, T^\dagger)A(13)\varphi^2 \right)
\end{align*}
\]

(3.6)

are tree level constants. Now we want to see the effect of one-loop radiative correction to the derived potential. After doing proper analysis throughout it comes out that the one-loop correction does not effect the superpotential due to the cancellation of all tadpole terms appearing in the theory. On the other hand one-loop radiative correction in the Kähler potential results in

\[
\delta K_{1-\text{loop}}^{1-\text{loop}}(\phi, \phi^\dagger) = \int_p \frac{d^4p}{(2\pi)^2} \left[ \frac{1}{2} Tr \ln \tilde{K}(\phi, \phi^\dagger) + \frac{1}{2} \tilde{Z}(T, T^\dagger) \right] + Tr \ln \left( \tilde{K}(\phi, \phi^\dagger) + \tilde{W}(\phi, \phi^\dagger) \left( \tilde{K}(\phi, \phi^\dagger)^{-1} \right)^\dagger \tilde{W}(\phi, \phi^\dagger) \right)
\]

(3.7)

\[
\delta K_{1-\text{loop}}^{1-\text{loop}}(\phi, \phi^\dagger) = \frac{\Lambda_{UV}^2}{10\pi^2} \ln \left( \det \tilde{K}(\phi, \phi^\dagger) \right) - \frac{1}{32\pi^2} Tr \left( M^2_{\phi} \left[ \frac{M^2_{\phi}}{\Lambda_{UV}^2} - 1 \right] \right)
\]

where $\Lambda_{UV}$ (Reduced Planck Mass) is used as a UV cut-off of the theory appearing in the context of cut-off regularization. In this connection the chiral mass matrix is given by

\[
M_{\phi}^2 = \tilde{K}^{-\frac{1}{2}}(\phi, \phi^\dagger) \tilde{W}(\phi, \phi^\dagger) \left( \tilde{K}(\phi, \phi^\dagger)^{-1} \right)^\dagger \tilde{W}(\phi, \phi^\dagger) \tilde{K}^{-\frac{1}{2}}(\phi, \phi^\dagger).
\]

(3.8)
Now including the contribution from one-loop radiative correction both from brane and bulk SUGRA, the renormalizable potential is as under

\[
V(\phi) = V_{\text{tree}}(\phi) + \delta V_{-\text{loop}}(\phi)
\]

\[
= \sum_{m=-2,m\neq-1}^{2} C_{2m} \phi^{2m} + \lim_{\epsilon \to 0} \sum_{n=0}^{0} B_{2m} \left( \int_{\mathcal{M}_{\text{UV}}} d^{4}p \frac{1}{(2\pi)^{4}} \left( p^{2} - 2C_{2} + \epsilon \right) \right) \phi^{2n} + \sum_{q=0}^{2} \phi^{2q} \left[ 4 \mathcal{A}_{\text{UV}} \text{Str} \left( \mathcal{M}^{2} \right) \ln \left( \frac{\mathcal{M}^{2}}{\mathcal{A}^{2}} \right) + 2 \mathcal{A}^{2} \text{Str} \left( \mathcal{M}^{2} \right) + \text{Str} \left( \mathcal{M}^{4} \ln \left( \frac{\mathcal{M}^{2}}{\mathcal{A}^{2}} \right) \right) \right]
\]

\[
= \sum_{m=-2,m\neq-1}^{2} C_{2m} \phi^{2m} + \sum_{n=0}^{0} \bar{B}_{2m} \ln \left( \frac{\phi}{M} \right) \phi^{2n} + \sum_{q=0}^{2} A_{2q} \ln \left( \frac{\phi}{M} \right) \phi^{2q}
\]

where \( D_{0} = 0, D_{2m} = \frac{\bar{B}_{2m} + A_{2m}}{C_{2m}} \) and we have used the supertrace identity

\[
\text{Str} \left( \mathcal{M}^{\alpha} \right) \equiv \sum_{i} \left( -1 \right)^{2i} \left( 2j_{i} + 1 \right) m_{i}^{\alpha}.
\]

It is the Coleman Weinberg potential \([45]\), provided the coupling constant satisfies the Gellmann-Low equation \([46]\) in the context of Renormalization group. Here the first term in the eqn(3.9) physically represents the energy scale of inflation \((\sqrt{C_{0}})\). Here the four dimensional effective potential respect the Galilean symmetry: \( \phi \to \phi + b_{\mu}x^{\mu} + c \) which task care both shift and spacetime translational symmetry.

![Figure 1: Variation of one loop corrected potential(V(\phi)) with inflaton field (\phi)](image)

Figure (1) represents the inflaton potential for different values of \( C_{2m} \) and \( D_{2m} \). From the observational constraints the best fit model is given by the range: \( 5.67 \times 10^{-11} M_{\text{PL}}^{4} < C_{0} < 6 \times 10^{-11} M_{\text{PL}}^{4}, 1.01 \times 10^{-16} M_{\text{PL}}^{8} < C_{-4} < 2 \times 10^{-16} M_{\text{PL}}^{8}, 7.27 \times 10^{-10} M_{\text{PL}}^{2} < C_{2} < 7.31 \times 10^{-10} M_{\text{PL}}^{2}, 2.01 \times 10^{-14} < C_{4} < 2.45 \times 10^{-14}, 0.014 < D_{-4} < 0.021, 0.002 < D_{2} < 0.012 \) and \( 0.011 < D_{4} < 0.019 \) so that while doing numericals, we shall restrict ourselves to this range.
Here $M_{\text{PL}} = 2.43 \times 10^{18}\text{GeV}$ represents reduced 4D Planck mass. Consequently the energy scale of inflation has a window $0.658 \times 10^{16}\text{GeV} < \sqrt{\kappa_0^2} < 6.67 \times 10^{16}\text{GeV}$ which precisely falls within the GUT scale.

Hence using equation (3.1) the modified Friedmann and Klein-Gordon equations can be expressed as:

$$H^4 = \frac{\Lambda(4) + 8\pi G(\phi)V(\phi)}{g_1},$$

(3.11)

where $\epsilon_2(\phi) = \dot{\phi}_0 + 9\epsilon_3(\phi)H^2$ is

$$\dot{\epsilon}_2(N) = \left( V' + \tilde{C}_5 g'(\phi)k_1 - \frac{\tilde{D}_f f(\phi)}{f(\phi)}(1 - Q_1) \right)_N,$$

(3.12)

where $\epsilon_2(\phi) = \dot{\phi}_0(T, T^1) J_\phi + 2\tilde{g}(\phi)\tilde{f}(\phi)k_1k_2 + 8\tilde{f}(\phi)\tilde{f}(\phi)(\tilde{g}(\phi)k_1k_2^2 + 2\tilde{g}(\phi)\tilde{f}(\phi)k_1k_2 - g''(\phi)k_1$ and $\epsilon_3(\phi) = 2\tilde{C}_4\tilde{f}(\phi)\tilde{g}(\phi)k_1k_2)$ provided $|\epsilon_3(\phi)| \gg |\epsilon_2(\phi)|$ in the slow-roll regime. This has been discussed in detail in Appendix B. Here we have fixed the signature of $\dot{\phi}_0$ so that the scalar field rolls down the potential. Additionally ghost instabilities are avoided provided the coefficient of $\dot{\phi}_0 > 0$. Consequently the potential dependent slow-roll parameters can be expressed as:

$$\epsilon_V = \frac{M_{\text{PL}}^2}{2} \left( \frac{V'}{V} \right)^2 \frac{1}{\sqrt{G(\phi)V'(\phi)}},$$

(3.13)

$$\eta_V = M_{\text{PL}}^2 \left( \frac{V''}{V} \right) \frac{1}{\sqrt{G(\phi)V'(\phi)}},$$

(3.14)

$$\xi_V = M_{\text{PL}}^4 \left( \frac{V'V''}{V^2} \right) \frac{1}{G(\phi)V'(\phi)},$$

(3.15)

$$\sigma_V = M_{\text{PL}}^6 \left( \frac{(V')^2V'''}{V^3} \right) \frac{1}{(G(\phi)V'(\phi))^2},$$

(3.16)

where $G(\phi) = \frac{16\epsilon_3(\phi)M_{\text{PL}}^2}{\sqrt{V'(\phi)}}$. In this connection Galileon terms effectively flatten the potential due to the presence of the flattening factor $\frac{1}{\sqrt{G(\phi)V'(\phi)}} \ll 1$. This implies that in the presence of Galileon like derivative interaction slow-roll inflation can take place even if the potential is rather steep.

The number of e-foldings for D3 DBI Galileon can be expressed as

$$N = \frac{1}{8M_{\text{PL}}} \int_{\phi_i}^{\phi_f} \frac{\sqrt{G(\phi)V'(\phi)}}{\sqrt{\epsilon_V}} d\phi,$$

(3.17)

where $\phi_i$ and $\phi_f$ are the corresponding values of the inflaton field at the beginning and end of inflation.

Figure 3 represents a graphical behavior of number of e-folding versus the inflaton field for different values of $C_i$s and the most satisfactory point in this context is the number of e-folding lies within the observational window $56 < N < 70$. The end of the inflation leads to the extra constraint $V'(\phi_f) = \sqrt{V'(\phi_f)V(\phi_f)}$.

### IV. QUANTUM FLUCTUATIONS AND OBSERVABLE PARAMETERS

Let us now engage ourselves in analyzing quantum fluctuation in our model and its observational imprints via primordial spectra generated from cosmological perturbation. To serve this purpose we start with the ADM formalism with the line element

$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j),$$

(4.1)

where $N$ and $N^i$ $(i = 1, 2, 3)$ are the lapse and shift functions, respectively. In this context we consider scalar metric perturbations about the flat FLRW background. Here we expand the lapse $N$ and the shift vector $N^i$, as $N = 1 + \alpha$ and $N_i = \partial_t \psi + \tilde{N}_i$, respectively. Here $\partial_t \psi$ is the irrotational part and $\tilde{N}_i$ be the incompressible vector part $(\tilde{N}_i, i = \partial_t \tilde{N}_i = 0)$. These are actually non-dynamical Lagrange multipliers in the action, so that it is sufficient to know $N$ and $N^i$ up to first order. This implies their equation of motion is purely algebraic. To fix the time and
spatial reparameterization we choose the uniform-field gauge with $\delta \phi = 0$, which fixes the temporal component of a
gauge-transformation vector $\xi^{\mu}$. After that by fixing the spatial part of $\xi^{\mu}$ we gauge away a field $\varepsilon$ that appears
as a form $\varepsilon_{ij}$ inside $h_{ij}$. Consequently the metric on three dimensional constant time slice can be expressed as
$h_{ij} = a^{2}(t)e^{2\zeta}\delta_{ij}$. Plugging all of these informations in to equation(4.1) we get:
\[
 ds^{2} = -(1 + 2\alpha)dt^{2} + 2 \left[ \partial_{i}\psi + 2\tilde{N}_{i} \right] dt dx^{i} + a^{2}(t)e^{2\zeta}\delta_{ij}dx^{i}dx^{j},
\]
where we use a shorthand notation $(\partial_{i}\psi)^{2} = (\partial_{i}\psi)(\partial_{j}\psi)$. At linear level of the perturbation theory
equation(4.2) reduces to the following metric:
\[
 ds^{2} = -(1 + 2\alpha)dt^{2} + 2 \left[ \partial_{i}\psi + 2\tilde{N}_{i} \right] dt dx^{i} + a^{2}(t) (1 + 2\zeta)\delta_{ij}dx^{i}dx^{j}.
\]
Expanding the four dimensional effective action stated in equation(3.3) up to second order, we get:
\[
 S_{2}^{\zeta} = \int dtdx^{3} a^{3} \left[ -3t_{1}\dot{\zeta}^{2} + \frac{2t_{3}}{a^{2}}a^{2}\dot{\zeta}^{2} + \frac{t_{2}}{a^{2}}\alpha\partial^{2}\psi - \frac{2t_{3}}{a^{2}}\alpha\partial^{2}\zeta + 3t_{2}\alpha\dot{\zeta} + \frac{1}{3}t_{3}\alpha^{2} + \frac{t_{4}}{a^{2}}\partial_{i}\zeta \partial_{i}\zeta \right],
\]
where the effect of effective Gauss-Bonnet coupling and the DBI Galileon features are explicitly appearing in the
co-efficients of the second order perturbative action as:
\[
 t_{1} \approx \tilde{t}_{1}, \quad t_{2} \approx \left( 2H\tilde{t}_{1} - 2\dot{\phi}X\tilde{G}_{X} \right), \quad t_{3} \approx -9\tilde{t}_{1}H^{2} + 3 \left( X\tilde{K}_{X} + 2X^{2}\tilde{K}_{XX} \right)
 + 18H\dot{\phi} \left( 2X\tilde{G}_{X} + X^{2}\tilde{G}_{XX} \right) - 6(\dot{X}G_{,\phi} + X^{2}G_{,XX}) , \quad t_{4} \approx \tilde{t}_{1}.
\]
It is important to mention here that, in the action(4.4), both the coefficients of the terms $\alpha\zeta$ and $\zeta^{2}$ vanish by
using the background equations of motion. Furthermore, in (4.4), the term quadratic in $\psi$ vanishes by making use
of integrations by parts. The equations of motion for $\psi$ and $\alpha$, derived from (4.4), lead to the following two-fold
constraint relations:
\[
 \alpha = J\dot{\zeta}, \quad \frac{1}{a^{2}}\partial^{2}\psi = \frac{2t_{3}}{3t_{2}}\alpha + \frac{2}{t_{2}}\frac{1}{a^{2}}\partial^{2}\zeta ,
\]
FIG. 3: (a) Parametric plot of the amplitude of the power spectrum ($\sqrt{P_\zeta}$) with scalar spectral index ($n_\zeta$). (b) Variation of the scale dependent scalar power spectrum ($P_\zeta(k)$) vs momentum scale $k$. (c) Variation of the scale dependent tensor to scalar ratio ($r(k\eta)$) vs $|k\eta|$ for DS and BDS analysis.

where

\[ J \equiv \frac{2t_1}{t_2} = \frac{2\dot{t}_1}{\left(2Ht_1 - 2\dot{\phi}X\tilde{G}_X\right)} . \]  

(4.11)

Using expansion in terms of the slow-parameter defined in equation (4.28) gives

\[ J = \frac{1}{H} \left[ 1 + \delta_{GX} + O(\epsilon_V^2) \right] . \]  

(4.12)

Then substituting equation (4.9) and equation (4.10) into equation (4.4) and integrating the term $\dot{\zeta}\partial^2\zeta$ by parts the second order action stated in equation (4.4) can be re-expressed as:

\[ S_2 = \int dt d^3 x \alpha^3 Y_s \left[ \dot{\zeta}^2 - \frac{\dot{\zeta}^2}{a^2} (\partial\zeta)^2 \right], \]  

(4.13)
where

\[ Y_S = \frac{t_1 (4t_1 t_3 + 9t_2^2)}{3t_2^2}, \]

\[ c_s^2 = \frac{3 (2H t_2 l_1^2 - t_4 t_2^2 - 2t_1^2 i_2)}{t_1 (4t_1 t_3 + 9t_2^2)}, \]

(4.14)
(4.15)

It is important to mention here that ghosts and Laplacian instabilities can be avoided iff \( c_s^2 > 0, Y_S > 0 \). Now using equation (4.9) and equation (4.14) in equation (4.10) we get:

\[ \psi = -J \zeta + \partial^{-2} \left( \frac{a^2 Y_S \dot{\zeta}}{t_1} \right). \]

(4.16)

For future convenience, we have introduced a new parameter defined as:

\[ \epsilon_s = \frac{Y_S c_s^2}{l_1} = \frac{(2H t_2 l_1^2 - t_4 t_2^2 - 2t_1^2 i_2)}{t_2 l_1} = \epsilon_V + \delta_{GX} + \mathcal{O}(\epsilon_s^2), \]

(4.17)

where we use equation (3.13) and equation (4.28) to express it in terms of slow-roll parameters. Now varying the action stated in equation (4.13) and expressing the solution at the linear level in terms of Fourier modes, we arrive at the \textit{Mukhanov Sasaki Equation} for Galileon scalar mode.

\[ v''_{\vec{k}} + \left( c_s^2 k^2 - \frac{z''}{z} \right) v_{\vec{k}} = 0, \]

(4.18)

where \( c_s^2 \) takes into account the nontrivial modification due to Galileon. Similarly for tensor modes, equation (4.13) can be recast as:

\[ S^h = \int dt d^3 x a^3 Y_T \left[ \dot{h}_{ij}^2 - \frac{c_s^2}{a^2} (\partial h_{ij})^2 \right], \]

(4.19)

where

\[ Y_T = \frac{t_1}{4} = \frac{\tilde{l}_1}{4}, \]

(4.20)
\[ c_T^2 = \frac{t_4}{t_1} = 1 + \mathcal{O}(\epsilon^2). \]

(4.21)

For tensor modes we use the normalization condition \( \epsilon_{ij} \epsilon_{ij} = 2 \delta^{\lambda \nu} \) and traceless condition \( \epsilon_{ii} = 0 \) for polarization tensor. Following the same prescription we can establish equation (4.18) for tensor modes provided \( c_s \) is replaced by \( c_T \). The \textit{Bunch-Davies} mode function turns out to be (Throughout the paper we have used \( DS \) for \textit{de-Sitter} results and \( BDS \) for \textit{beyond de-Sitter} results."

\[ u_{\zeta}(\eta, k) = \begin{cases} \frac{i H \exp(-i k c_s \eta)}{2 \sqrt{Y_s (c_s k) \frac{\nu_s}{2}}} (1 + i k c_s \eta) :DS \cr \frac{\sqrt{-k \eta c_s}}{a \sqrt{2 Y_s}} \mathcal{H}_\nu^{(1)}(-k \eta c_s) :BDS. \end{cases} \]

(4.22)

where \( \nu_s = \left( \frac{3 - \epsilon_V - 2 \nu^3}{2(1 - \epsilon_V - \nu^2)} \right) \) and in the super-Hubble limit we have:

\[ \mathcal{H}_\nu^{(1)} \rightarrow \frac{(-k c_s \eta)^{-\nu_s} \exp \left( i \nu_s - \frac{1}{2} \right)^{\frac{2}{2}} 2^{\nu_s - \frac{1}{2}} \left( \frac{\Gamma(\nu_s)}{\Gamma(\frac{1}{2})} \right)}{\Gamma(\frac{1}{2})} \]

(4.23)
Further we have introduced five new parameters during the analysis of primordial quantum fluctuation defined as:

\[ s_{\psi}^S : = \frac{\epsilon_s}{H_\epsilon} = \frac{4\sqrt{V'(\phi)}M_{PL}^2}{\sqrt{\dot{g}(\phi)}} \frac{d}{d\phi} \ln c_s, \]
(4.24)

\[ s_{\psi}^T : = \frac{\epsilon_T}{H_\epsilon T} = \frac{M_{PL} \sqrt{V'(\phi)}}{2\sqrt{\epsilon_s(\phi)V(\phi)}} \frac{d}{d\phi} \ln [1 + O(\epsilon_s^2)], \]
(4.25)

\[ \eta_s : = \frac{\epsilon_s}{H_\epsilon} = \frac{4\sqrt{V'(\phi)}M_{PL}^2}{\sqrt{\dot{g}(\phi)}} \left[ \epsilon'_V (1 + O(\epsilon_V)) + \delta_{GX} \right], \]
(4.26)

\[ \delta_V : = \frac{Y_s}{H_\epsilon Y_s} = \frac{4\sqrt{V'(\phi)}M_{PL}^2}{\sqrt{\dot{g}(\phi)}} \frac{d}{d\phi} \ln Y_s, \]
(4.27)

\[ \delta_{GX} : = \frac{\phi X}{l_1}. \]
(4.28)

Now using eqn(4.22) the two-point correlation function for scalar modes can be expressed as:

\[ \langle 0|\zeta(\hat{k})\zeta(\hat{k}')|0 \rangle = \frac{2\pi^2}{k^3}(2\pi)^3 P_\zeta(k)\delta^3(\hat{k} + \hat{k}') = (2\pi)^3|u_\zeta(\eta, k)|^2 \delta^3(\hat{k} + \hat{k}'), \]
(4.29)

where the dimensionless Power spectrum for scalar modes \( P_\zeta(k) \) at the horizon crossing turns out to be:

\[ P_\zeta(k_*) = \frac{k^3}{2\pi^2}|u_\zeta(k_*)|^2 = \begin{cases} \left( \frac{\sqrt{V(\phi)}}{8\pi^2\epsilon_s^3 l_1 \sqrt{g_1} M_{PL}} \right)^2 & : DS \\ \left( \frac{2\epsilon_T}{\Gamma(\nu_T)} \right)^2 \left( \frac{1 - \epsilon_V - s_V^S}{s_V^S} \right)^2 \frac{\sqrt{V(\phi)}}{8\pi^2 \epsilon_s^3 s_V^S \sqrt{g_1} M_{PL}} & : BDS \end{cases}, \]
(4.30)

* corresponds to the horizon crossing. Similarly using the tensor version of eqn(4.22) the two-point correlation function for tensor modes can be expressed as:

\[ \langle 0|h_{ij}(\hat{k})h_{ij}(\hat{k}')|0 \rangle = \frac{2\pi^2}{k^3}(2\pi)^3 P_T(k)\delta^3(\hat{k} + \hat{k}') = (2\pi)^3|u_\zeta(\eta, k)|^2 \delta^3(\hat{k} + \hat{k}'), \]
(4.31)

where \( P_T(k) = |P_T(k)|_{ij;ij} \) and the corresponding dimensionless Power spectrum for tensor modes reads:

\[ P_T(k_*) = \frac{k^3}{2\pi^2}|u_h(k_*)|^2 \left( \sum_{\lambda = +, -} c_{ij}^{\lambda} c_{ij}^{\lambda} \right) = \begin{cases} \left( \frac{\sqrt{V(\phi)}}{2\pi^2\epsilon_T \epsilon_T l_1 \sqrt{g_1} M_{PL}} \right)^2 & : DS \\ \left( \frac{2\epsilon_T}{\Gamma(\nu_T)} \right)^2 \left( \frac{1 - \epsilon_V - s_V^T}{s_V^T} \right)^2 \frac{\sqrt{V(\phi)}}{2\pi^2 \epsilon_s^3 \sqrt{g_1} M_{PL}} & : BDS \end{cases}. \]
(4.32)

Consequently the ratio of tensor to scalar power spectrum can be expressed as:

\[ r(k_*) = \frac{P_T(k_*)}{P_\zeta(k_*)} = \frac{|u_h(k_*)|^2}{|u_\zeta(k_*)|^2} \left( \sum_{\lambda = +, -} c_{ij}^{\lambda} c_{ij}^{\lambda} \right) = \begin{cases} \left( \frac{16\epsilon_s \epsilon_s \left[ 1 - \frac{3}{2} O(\epsilon_s^2) \right]}{16.2^2 (\nu_T - \nu_s) \Gamma(\nu_T) \Gamma(\nu_T)} \right)^2 \left( \frac{1 - \epsilon_V - s_V^T}{1 - \epsilon_V - s_V^T} \right)^2 \epsilon_s \left[ 1 - \frac{3}{2} O(\epsilon_s^2) \right] & : DS \\ \left( \frac{2\epsilon_T + s_V^S + \delta_V}{2\epsilon_T + s_V^S + \delta_V} \right)^2 & : BDS \end{cases}. \]
(4.33)

Further, the scale dependence of the perturbations, described by the scalar and tensor spectral indices, as follows:

\[ n_\zeta - 1 = \left( \frac{d \ln P_\zeta}{d \ln k} \right)_* = \begin{cases} \left( -2\epsilon_V - \eta_s - s_V^S \right) & : DS \\ (3 - 2\nu_s) = -\left( \frac{2\epsilon_T + s_V^S + \delta_V}{2\epsilon_T + s_V^S + \delta_V} \right) & : BDS \end{cases}. \]
(4.34)

\[ n_T = \left( \frac{d \ln P_T}{d \ln k} \right)_* = \begin{cases} \left( -2\epsilon_V \right) & : DS \\ (3 - 2\nu_T) = -\left( \frac{s_V^T}{1 - \epsilon_V - s_V^T} \right) & : BDS \end{cases}. \]
(4.35)
Consistently, the consistency relation is also modified to:

\[
\begin{align*}
\left(8cs\left(2\epsilon_V + 2\delta_{G\chi} + 2\mathcal{O}(\epsilon_v^2)\right)\left[1 - \frac{3}{2}\mathcal{O}(\epsilon_v^2)\right]\right)_v &= - \left(8cs\left(n_T - 2\delta_{G\chi} - 2\mathcal{O}\left(\frac{n_T}{4}\right)\right)\left[1 - \frac{3}{2}\mathcal{O}(\epsilon_v^2)\right]\right)_v \quad : DS \\
\left(8,2^{(n_T - n_v)}\frac{\Gamma(n_T)}{\Gamma(n_v)}\left(\frac{1 - \epsilon_v - s_v^T}{1 - \epsilon_v - s_v^S}\right)\right)_v c_s \left(2\epsilon_V + 2\delta_{G\chi} + 2\mathcal{O}(\epsilon_v^2)\right)\left[1 - \frac{3}{2}\mathcal{O}(\epsilon_v^2)\right]_v &= \left(8,2^{(n_T - n_v)}\frac{\Gamma(n_T)}{\Gamma(n_v)}\left(\frac{1 - \epsilon_v - s_v^T}{1 - \epsilon_v - s_v^S}\right)\right)_v c_s \left(2\epsilon_V + 2\delta_{G\chi} + 2\mathcal{O}(\epsilon_v^2)\right)\left[1 - \frac{3}{2}\mathcal{O}(\epsilon_v^2)\right]_v \\
&= \left(8,2^{(n_T - n_v)}\frac{\Gamma(n_T)}{\Gamma(n_v)}\left(\frac{s_v^T}{s_v^S}\right)\right)_v \left(2\left[1 + s_v^T\left(\frac{1}{s_v^S} - 1\right)\right] + 2\delta_{G\chi} + 2\mathcal{O}(\epsilon_v^2)\right)\left[1 - \frac{3}{2}\mathcal{O}(\epsilon_v^2)\right]_v \quad : BDS.
\end{align*}
\]

The expressions for the running of the scalar and tensor spectral index in this specific model with respect to the logarithmic pivot scale at the horizon crossing are given by:

\[
\alpha_s = \left(\frac{dn_s}{d\ln k}\right)_v = \left\{\frac{4\sqrt{V'(\phi)}M_{PL}^2}{\sqrt{\mathcal{G}(\phi)}}\left[-2\epsilon_v' - \eta_v' - s_v^S\right]\right\}_v \quad : DS
\]

\[
\alpha_T = \left(\frac{dn_T}{d\ln k}\right)_v = \left\{-2\left[\frac{4\sqrt{V'(\phi)}M_{PL}^2}{\sqrt{\mathcal{G}(\phi)}}\right]_v \left[-\frac{s_v^T}{s_v^S}\epsilon_v' + \delta_v\epsilon_v' + s_v^S\delta_v' + \left(2\epsilon_v' + s_v^S\epsilon_v' + \delta_v'\right)\right]\right\}_v \quad : BDS
\]

Here we have used a shorthand notation \(ab = a'b - ab'\) where \(\frac{d}{d\phi} = \frac{d}{d\ln k}\). We also use the operator identity \(\frac{d}{d\ln k} := \frac{4\sqrt{V'(\phi)}M_{PL}^2}{\sqrt{\mathcal{G}(\phi)}}\) to compute all the inflationary observables.

Figure(3(a)) represents the scale dependent power spectrum (\(\sqrt{P}\)) with respect to the scalar spectral index(\(n_s\)). It directly shows that both the DS and BDS analysis follow the same characteristics but the estimated windows for the observational parameters (\(P_s, n_s\)) are slightly different, but both of them are within the observational bound. Figure(3(b)) shows the characteristic differences between the behavior of DS and BDS scale dependent power spectrum with respect to the momentum scale (\(k\)). Here DS behavior is quasi-statically flat, but BDS characteristics is rapidly increasing with respect to the scale. In figure(3(c)) we have plotted the the scale dependent tensor to scalar ratio for DS and BDS limit. Most significantly they show complementary characteristics with the scale and intersects at a point where both the analysis will be equivalent. In the next section, we will estimate these parameters by confronting the results directly to WMAP7 results.

V. PARAMETER ESTIMATION AND CONFRONTATION WITH WMAP7

Using the parameter space for the model parameters (\(C_i, D_i\)) we have estimated the window of the cosmological parameters from our model which confronts observational data well in 56 < N < 70. In Table(I) we have tabulated the relevant observational parameters estimated from our model for both DS and BDS limit.

| Scheme | \(P_s \times 10^{-9}\) | \(r\) | \(n_s\) | \(\alpha_s \times (10^{-3})\) |
|--------|---------------------|-----|-------|---------------------|
| DS     | 2.401 - 2.601       | 0.215 - 0.242 | 0.964 - 0.966 | 2.240 - 2.249 |
| BDS    | 2.471 - 2.561       | 0.232 - 0.250 | 0.962 - 0.964 | 4.008 - 4.012 |

TABLE I: Model Dependent Observational Parameters
Further, we use the publicly available code CAMB [41] to verify our results directly with observation. To operate CAMB at the pivot scale $k_0 = 0.002\, \text{Mpc}^{-1}$ the values of the initial parameter space are taken for lower bound of $C_l$'s and $N = 70$. Additionally WMAP7 years dataset for ΛCDM background has been used in CAMB to obtain CMB angular power spectrum. In Table II we have given all the input parameters for CAMB. Table III shows the CAMB output, which is in good agreement with WMAP7 [42] data. In figure (4) we have plotted CAMB output of CMB TT, TE and EE angular power spectrum $C_{TT}^l, C_{TE}^l, C_{EE}^l$ for the best fit with WMAP7 data for scalar mode, which explicitly show the agreement of our model with WMAP7 dataset. The small scale modes have no impact in the CMB anisotropy spectrum only the large scale modes have little contribution and this is obvious from figure (d)-(f) where we have plotted the CAMB output of CMB angular power spectrum $C_{TT}^l, C_{TE}^l, C_{EE}^l$ and $C_{BB}^l$ for best fit with WMAP7 data for the tensor mode. Hence in figure (5) we have plotted the variation of matter power spectrum with respect to the momentum scale which is in concordance with observational results.

![CMB TT Angular Power Spectrum (For Scalar)](a)

![CMB TE Angular Power Spectrum (For Scalar)](b)

![CMB EE Angular Power Spectrum (For Scalar)](c)

![CMB EE and BB Angular Power Spectrum (For Tensor)](d)

![CMB TT Angular Power Spectrum (For Tensor)](e)

![CMB TE Angular Power Spectrum (For Tensor)](f)

FIG. 4: Variation of the CMB [(a) TT (scalar), (b) TE (scalar), (c) EE (scalar), (d) EE+BB (tensor), (e) TT (tensor) and (f) TE (tensor)] angular power spectrum with multipoles ($l$).

| $H_0$ | $\tau_{\text{Reion}}$ | $\Omega_b h^2$ | $\Omega_c h^2$ | $T_{\text{CMB}}$ |
|-------|---------------------|----------------|----------------|----------------|
| 71.0  | 0.09                | 0.0226         | 0.119          | 2.725          |

TABLE II: Input parameters in CAMB

| $t_0$ | $z_{\text{Reion}}$ | $\Omega_m$ | $\Omega_\Lambda$ | $\eta_{\text{Rec}}$ | $\eta_0$ |
|-------|---------------------|------------|------------------|---------------------|----------|
| 13.707| 10.704              | 0.2670     | 0.7329           | 0.0                 | 285.10   |
|

TABLE III: Output parameters from CAMB
VI. SUMMARY AND OUTLOOK

In this article we have proposed a model of single field inflation in the context of DBI Galileon cosmology in D3 brane. We have demonstrated the technical details of construction mechanism of an one-loop 4D inflationary potential via dimensional reduction starting from D4 brane in $N=2,D=5$ SUGRA including the quadratic Gauss-Bonnet correction term that leads to an effective $N=1,D=4$ SUGRA in the D3 brane, which is precisely the DBI Galileon in our framework. Hence we have studied inflation using the one loop effective potential by estimating the observable parameters originated from primordial quantum fluctuation for scalar and tensor modes, in the de-Sitter and beyond de-Sitter limit. We have further confronted our results with WMAP7 [42] dataset by using the cosmological code CAMB. The results are found in good agreement with WMAP7 dataset in $\Lambda CDM+tens$ background.

An interesting open issue in this context is to study the primordial non-Gaussian features of DBI Galileon introduced in the present article. As has been pointed out recently [47]-[49] there is a tension between bispectrum ($f_{NL}$) and tensor-to-scalar ratio ($r$) in DBI inflation, which is a generic sensitivity problem. It will be interesting to investigate whether our proposed framework of DBI Galileon can resolve this issue. We are already in progress in this direction and have obtained some interesting results. A detailed report on this issue will be brought forth shortly.

Other open issues in the context of DBI Galileon cosmology are combined constraints on the primordial non-Gaussianity via preheating [50], reheating [51] and primordial black hole formation [52], effect of the presence of one loop and two loop radiative corrections in the presence of all possible scalar and tensor mode fluctuations up to the fourth order correction in the action [53], study of different shapes and comparative study between the tree, one and two loop level via rigorous analysis and finding out the most probable Dark Matter candidate in collider phenomenology. It will be interesting to see how they are affected due to the presence of Galileon.

Acknowledgments

SC thanks Council of Scientific and Industrial Research, India for financial support through Senior Research Fellowship (Grant No. 09/093(0132)/2010).

Appendix A

In this section we employ dimensional reduction technique to derive a $N=1, D=4$ SUGRA and the inflaton potential thereof from that results in DBI Galileon on the D3 brane. For convenience we deal with different contributions to the action (2.1) separately.
The Einstein-Hilbert Action:-

After integrating out the contribution from the five dimension, the Einstein Hilbert action in four dimension can be written as

\[ S_{EH}^{(4)} = \frac{1}{2} \int d^4x \sqrt{-g(y)} \int_{\pi R}^{+\pi R} dy \beta M_5^2 R \exp(3A(y)) \left[ \left( \frac{dA(y)}{dy} \right)^2 - \frac{8}{\beta^2 R^2} \left( \frac{d^2 A(y)}{dy^2} \right)^2 - 2\Lambda_5 \exp(2A(y)) \right] \]

where the explicit expression for \( \mathcal{I}(1) \) is mentioned in Appendix C. In this context \( R(4) \) is the 4D Ricci scalar. It is important to mention here that the 5D Planck mass (\( M_5 \)) and 4D Planck mass (\( M_{PL} \)) are related through

\[ M_{PL}^2 = M_5^3 \beta M_{(5)} \int_{-\pi R}^{+\pi R} dy \exp(3A(y)) \]

\[ = \frac{M_{(5)}^3 \beta R}{3R^2 T(4)} \exp(\beta \pi R) \left\{ \frac{3\sqrt{T(4)}}{\sqrt{\exp(\beta \pi R) + 1}} - \frac{3\sqrt{T(4)}}{\sqrt{\exp(-\beta \pi R) + 1}} \right\} \]

The Gauss-Bonnet Action:-

The 5D and 4D Riemann tensor, Ricci tensor and Ricci scalar are related through the following expressions:

\[ R^{(5)}_{\alpha\beta\gamma\delta} = R^{(4)}_{\alpha\beta\gamma\delta} + \frac{\exp(2A(y))}{R^2 \beta^2} \left( \frac{dA(y)}{dy} \right)^2 \left[ g^{(4)}_{\gamma\beta} g^{(4)}_{\alpha\delta} - g^{(4)}_{\gamma\alpha} g^{(4)}_{\beta\delta} \right] \]

\[ R^{(5)}_{\alpha\beta} = R^{(4)}_{\alpha\beta} - \frac{3g^{(4)}_{\alpha\beta}}{R^2 \beta^2} \left( \frac{dA(y)}{dy} \right)^2 \]

\[ R(5) = \exp(2A(y)) \left[ \left( \frac{dA(y)}{dy} \right)^2 - \frac{8}{\beta^2 R^2} \left( \frac{d^2 A(y)}{dy^2} \right)^2 - 2\Lambda_5 \exp(2A(y)) \right] \]

Using eqn(6.3)-eqn(6.5) in eqn(2.3) we get

\[ S_{GB}^{(4)} = \frac{\alpha(4)}{2\kappa(4)} \int d^4x \sqrt{-g(y)} \left( C(1) R^{(4)}_{\alpha\beta\gamma\delta} R^{(4)}_{\alpha\beta\gamma\delta} - 4\mathcal{I}(2) R^{(4)}_{\alpha\beta} + A(6) R(4)^2 \right) \]

\[ + \frac{2C(2)}{R^2 \beta^2} R^{(4)}_{\alpha\beta\gamma\delta} \left( g^{(4)}_{\gamma\beta} g^{(4)}_{\alpha\delta} - g^{(4)}_{\alpha\beta} g^{(4)}_{\gamma\delta} \right) + \frac{G(1)}{R^4 \beta^4} + \frac{G(2)}{R^2 \beta^2} R(4) \]

where \( G(1) = 24C(4) - 144\mathcal{I}(4) - 64A(5) + 144A(7) + 64A(8) + 192A(11) \) and \( G(2) = 24\mathcal{I}(2) - 24A(9) - 16A(10) \). The scaling relationship between 4D and 5D Gauss-Bonnet coupling constant is \( \alpha(4) = \frac{\kappa(4)^2}{\kappa(5)^2} \alpha(5) \) where \( \kappa(4) \) and \( \kappa(5) \) are gravitational couplings in 4D and 5D respectively. Explicit form of each of the constants appearing in eqn(6.6) are mentioned in the Appendix C.

The D3 Brane Action:-

To reduce the D4 brane action we employ the method of separation of variable \( \Phi(X^A) = \Phi(x^\mu, y) = \phi(x^\mu) \chi(y) \) where \( \chi(y) = \exp(2\pi i y / R) \). Consequently the D3 brane action turns out to be

\[ S_{D3 \ \text{Brane}}^{(4)} = \int d^4x \sqrt{-g^{(4)}} \left[ \mathcal{K}(\phi, \tilde{X}) - \mathcal{G}(\phi, \tilde{X}) \right] \]
where $\hat{K}(\phi, X) = \left\{ -\frac{\partial f}{\partial \phi} \left[ \sqrt{1 - 2QXf - Q_1} - \hat{C}_5 G(\phi, \hat{X}) - Q_2 \hat{D}V^{(4)}_{brane} \right] \right\}$. \( \hat{G}(\phi, \hat{X}) = \left( \frac{\hat{g}(\phi)k_1\hat{C}_5}{2(1 - 2f(\phi)Xk_2)} \right) \), \( \hat{g}(\phi) = \hat{g}_0 + \hat{g}_2\phi^2 \), \( \hat{D} = -\frac{D}{2k_4(1)} \), \( \hat{C}_4 = \frac{C_4}{2k_4} \), \( \hat{C}_5 = \hat{C}_5\alpha^2 R^2 \), \( \hat{Q}_2 \hat{D} = \beta R \). The effective Klebanov Strassler and Coulomb frame function on the D3 brane are hereby expressed as \( \hat{f}(\phi) \simeq \frac{1}{\nu_0 + f(\phi)x-fi(\phi)} \) and \( \nu(\phi) = \nu_0 + \frac{\nu_0}{\nu_0} A(1) \), \( \nu_4 = \nu_4 A(12) \). The outcome of dimensional reduction is reflected through the constants mentioned in the Appendix C. The scaled D3 brane potential turns out to be

$$\hat{V}^{(4)}_{brane} = \hat{Q}_2 \hat{D}V^{(4)}_{brane} = T_{(3)}\nu(\phi) + \frac{\beta RT(2)\hat{f}(\phi)}{\nu(\phi)}$$

where the D3 brane tension \( T_{(3)} \) can be expressed in terms of the D4 brane tension \( T_{(4)} \), compactification radius \( R \) and the slope parameter \( \beta \) as \( T_{(3)} = \beta RT_{(4)} \).

**The \( N=1, D=4 \) Supergravity Action**

Further, imposing $Z_2$ symmetry to $\phi$ via $\Phi(0) = \Phi(\pi R) = 0$ and compactifying around a circle $(S^1)$ $\partial_3 \Phi = \sqrt{V^{(5)}_{bulk}(G)} (1 - \frac{\pi x}{2})$ we get,

$$S^{(5)}_{Bulk Sagra} = \frac{1}{2} \int d^4x \int_{-\pi R}^{+\pi R} dy g^{(5)} \left[ e^{(4)} e^{5} \left( g^{\alpha\beta} G^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - \frac{g^{55} V^{(5)}_{bulk}(G)}{4\pi^2 R^2} \right) \right].$$

Now using the above mentioned ansatz for method of separation of variable we get

$$S^{(4)}_{Sagra} = \frac{1}{2\kappa^2_{(4)}} \int d^4x \left[ M(T, T^\dagger) J^{\mu}(\phi, \phi^\dagger) g^{\alpha\beta}(\partial_\alpha \phi \partial_\beta \phi^\dagger) - Z(T, T^\dagger) V^{(4)}_F(\phi) \right].$$

where we define

$$J^{\mu}(\phi, \phi^\dagger) = \int_{-\pi R}^{+\pi R} dy \exp(-A(y)) \left( \frac{\partial^2 K(\phi \exp(\frac{2\pi i y}{R}), \phi^\dagger \exp(-\frac{2\pi i y}{R})))}{\partial \phi^\dagger_{\mu} \partial \phi^\nu} \right),$$

$$M(T, T^\dagger) = \sqrt{2\beta R^2 (T + T^\dagger)}, \quad Z(T, T^\dagger) = \frac{1}{8\pi^2 R^2 \beta |T + T^\dagger|^2}. $$

Here we have used the ansatz $W(\phi, \phi^\dagger) = \frac{W_1(\phi, \phi^\dagger)|T + T^\dagger|^2}{4}$ for superpotential. Hence, the $N=1, D=4$ supergravity F-term potential turns out to be

$$V^{(4)}_F = \int_{-\pi R}^{+\pi R} dy \exp(-A(y)) \left( \frac{K(\phi \exp(\frac{2\pi i y}{R}), \phi^\dagger \exp(-\frac{2\pi i y}{R}))}{M^2} \right) \left[ \left( \frac{\partial W_1}{\partial \phi_\alpha} + \left( \frac{\partial K(\phi \exp(\frac{2\pi i y}{R}), \phi^\dagger \exp(-\frac{2\pi i y}{R}))}{\partial \phi_\alpha} \right) \right) W_1 \right] \frac{1}{M^2} \left( \frac{\partial^2 W_1(\phi \exp(\frac{2\pi i y}{R}), \phi^\dagger \exp(-\frac{2\pi i y}{R}))}{\partial \phi^\beta \partial \phi^\beta} \right)^{-1} \left( \frac{\partial W_1}{\partial \phi^\beta} + \left( \frac{\partial K(\phi \exp(\frac{2\pi i y}{R}), \phi^\dagger \exp(-\frac{2\pi i y}{R}))}{\partial \phi^\beta} \right) \right) W_1 \frac{1}{M^2} - 3 \frac{|W_1|^2}{M^2}.$$  

Now using the ansatz for the Kähler potential $K(\phi \exp(\frac{2\pi i y}{R}), \phi^\dagger \exp(-\frac{2\pi i y}{R})) = K_1(\phi, \phi^\dagger)K_2(\exp(\frac{2\pi i y}{R}), \exp(-\frac{2\pi i y}{R}))$ with $K_1(\phi, \phi^\dagger) = K_1^{\alpha\beta} \phi_\alpha \phi^\beta$ and $K_2(\exp(\frac{2\pi i y}{R}), \exp(-\frac{2\pi i y}{R})) = 1$, equation (6.12) reduces to the following form:

$$V^{(4)}_F = A(13) \left( \frac{K_1^{\alpha\beta} \phi_\alpha \phi^\beta}{M^2} \right) \left[ \left( \frac{\partial W_1}{\partial \phi_\alpha} + K_1^{\alpha\beta} \frac{W_1}{M^2} \right) \frac{K_1^{\alpha\beta}}{M^2} \right] \frac{1}{M^2} \left[ \left( \frac{\partial W_1}{\partial \phi^\beta} + K_1^{\alpha\beta} \frac{W_1}{M^2} \right) \frac{K_1^{\alpha\beta}}{M^2} \right] - 3 \frac{|W_1|^2}{M^2}.$$  

with the general Kähler metric $K_1^{\alpha\beta} = \frac{\partial^2 K}{\partial \phi_\alpha \partial \phi^\beta}$. In this context $A(13)$ factor comes as an outcome of dimension reduction and is explicitly mentioned in the appendix B. In most of the simple situations, we are interested in the Canonical metric structure defined by $K_1^{\alpha\beta} = \delta^{\alpha\beta}$. Consequently the $N=1, D=4$ SUGRA action turns out to be

$$S^{(4)}_{Can Sagra} = \frac{1}{2\kappa^2_{(4)}} \int d^4x \sqrt{-g} \left[ \frac{M(T, T^\dagger) g^{\alpha\beta}(\partial_\alpha \phi \partial_\beta \phi^\dagger)}{-Z(T, T^\dagger) V^{(4)}_{Can}(\phi)} \right].$$

(6.14)
where the canonical F-term potential can be recast as
\[
V^{(4)} = V_{F}^{(4)} = \mathcal{A}(13) \exp \left( \frac{\phi_0^1 \phi_0^n}{M^2} \right) \left[ \frac{\partial W_1}{\partial \phi_0} \right]_0^n \left[ -3 \frac{|W_1|^2}{M^2} \right].
\] (6.15)

To derive the expression for the specific form of the inflaton potential we start with a specific superpotential \[44\]
\[ W_1 = v \phi - \frac{g}{n+1} \phi^{n+1}, \]
where \(\phi\) is the *chiral* superfield with \(\mathcal{R}\) charge \(\frac{1}{n+1}\) introduced as an inflaton with \(n \geq 2\). Here \(g(\sim O(1))\) is the positive and real coupling constant and \(v\) is the VEV of the field \(\phi\). In this model \(U(1)_R\) symmetry is dynamically broken to a discrete \(\mathbb{Z}_{2n}\) symmetry at the scale \(v \ll 1\). Consequently the inflaton transforms as \(\phi \rightarrow \exp \left( \frac{2\pi i}{n+1} \right) \phi \exp(-i\theta)\), where \(\alpha\) and \(\theta\) are the local gauge parameters of supergravity theory. This leads to the following form of the bulk contribution to the potential
\[
V_{\text{bulk}}^{(4)}(\phi) = \mathcal{A}(13) \exp \left( |\phi|^2 \right) \left[ (1 + |\phi|^2) v^2 - \left(1 + \frac{|\phi|^2}{n+1}\right) g \phi^n - 3|\phi|^2 \left| v^2 - \frac{g}{n+1} \phi^n \right| \right].
\] (6.16)

It has a minimum at \(|\phi|^2 \approx \left( \frac{2}{g} \right)^{\frac{1}{n+1}}\) and \(\text{Im} \phi_{\min}^n = 0\) with negative energy density
\[
V(\phi_{\min}) \approx -3\mathcal{A}(13) \exp \left( |\phi|^2 \right) |W_1(\phi_{\min})|^2 \approx -3\mathcal{A}(13) \left( \frac{n}{n+1} \right)^2 v^4 |\phi_{\min}|^2.
\] (6.17)

However, in the context of SUGRA, we may interpret that such negative potential energy is almost canceled by positive contribution due to the local supersymmetry breaking, \(\Lambda_{\text{SUGRA}}^2\), and that the residual positive energy density is responsible for the present dark energy. Then, we can relate the energy scale of this model to the gravitino mass as
\[
m_{3/2} \approx \frac{n}{n+1} \left( \frac{v^2}{g} \right)^{\frac{1}{n+1}} v^2.
\] (6.18)

Identifying the real part of \(\phi\) with the inflaton \(\phi \rightarrow \sqrt{2} \text{Re} \phi\), the dynamics of the inflaton is governed by the following potential,
\[
V_{\text{bulk}}^{(4)}(\phi) \approx \mathcal{A}(13) \left( v^4 - \frac{2g}{2n+2} v^2 \phi^n + \frac{g^2}{2n} \phi^{2n} \right).
\] (6.19)

Imposing renormalization condition, here we restrict ourselves to \(n = 2\) leading to effective \(N = 1, D = 4\) SUGRA potential
\[
V_{\text{bulk}}^{(4)}(\phi) = \mathcal{A}(13) \left( v^4 - g v^2 \phi^2 + \frac{g^2}{4} \phi^4 \right).
\] (6.20)

**Appendix B**

The effective action [73] leads to 4D effective *Einstein-Gauss Bonnet* equation as follows:
\[
G^{(4)}_{\alpha\beta} + \alpha_{(4)} H^{(4)}_{\alpha\beta} = 8\pi G_{(4)} T^{(4)}_{\alpha\beta} - \Lambda_{(4)} g^{(4)}_{\alpha\beta}
\] (6.21)

where the *Energy Momentum tensor, Einstein tensor and Gauss-Bonnet tensor* are given by
\[
T^{(4)}_{\alpha\beta} = T^{(\text{bulk})(4)}_{\alpha\beta} + T^{(\text{brane})(4)}_{\alpha\beta} = -2X - \hat{S}_{\alpha\beta} R^{(4)}_{\alpha\beta} \left\{ -X g^{\rho\sigma(4)} + \frac{\hat{s}_{\alpha\beta} V^{(4)}_{\alpha\beta}}{16\pi^2} \right\}
+ \hat{K}_{\alpha} \nabla_{\alpha} \phi \nabla_{\beta} \phi + \hat{K}_{g_{\alpha\beta}} - 2V_{\alpha} \hat{G}_{\beta} + g_{\alpha\beta} \nabla_{\lambda} \hat{G} \nabla_{\lambda} \phi
+ \hat{G}_{\alpha} \nabla_{\alpha} \phi \nabla_{\beta} \phi
\] (6.22)

\[
G^{(4)}_{\alpha\beta} = \hat{h}_{1} R^{(4)}_{\alpha\beta} - \frac{1}{2} \hat{h}_{2} g^{(4)}_{\alpha\beta} R^{(4)}_{\alpha\beta} \left\{ 6 \hat{h}_{4} + 4 \hat{h}_{5} - 3 \hat{h}_{3} \right\}
\] (6.23)
\[
H^{(4)}_{\alpha\beta} = \left\{ R^{(4)}_{\alpha\sigma\tau\lambda} R^{(4)}_{\beta\lambda\sigma\mu} \hat{h}_7 + \frac{1}{R^3} \left[ R^{(4)}_{\alpha\sigma\lambda\mu} \left( g^{(4)}_{\mu\tau\rho\lambda} - g^{(4)}_{\lambda\tau\rho\mu} \right) + R^{(4)}_{\beta\lambda\sigma\mu} \left( g^{(4)}_{\tau\rho\sigma\lambda} - g^{(4)}_{\tau\rho\lambda\sigma} \right) \right] + \frac{1}{R^2} \left( R^{(4)}_{\alpha\sigma\lambda\mu} \left( g^{(4)}_{\mu\tau\rho\lambda} - g^{(4)}_{\lambda\tau\rho\mu} \right) \right) \right. \\
+ \frac{1}{R^2} R^{(4)}_{\alpha\sigma\beta\rho} \left( g^{(4)}_{\alpha\beta\rho\sigma} - g^{(4)}_{\alpha\beta\sigma\rho} \right) + R^{(4)}_{\beta\lambda\sigma\mu} \left( g^{(4)}_{\tau\rho\sigma\lambda} - g^{(4)}_{\tau\rho\lambda\sigma} \right) \right. \\
- \left\{ R^{(4)}_{\alpha\gamma\lambda\beta} \tilde{h}_{16} + \frac{1}{R^2} R^{(4)}_{\alpha\gamma\lambda\beta} \tilde{h}_{16} - \frac{1}{R^2} \left( 3 \tilde{h}_{16} + \tilde{h}_{10} \right) \right\} \left( R^{(4)}_{\alpha\sigma\beta\rho} \tilde{h}_{21} - \frac{3 R^{(4)}_{\alpha\sigma\beta\rho} \tilde{h}_{22}}{R^3} \right) \right\}.
\]

From FLRW metric in 4D, equation (6.21) results in the following Friedmann equations:

\[
H^4 \dot{g}_1 + H^2 \ddot{g}_2 + \dot{H} H^2 \dot{g}_3 + \dot{H}^2 \ddot{g}_4 + \ddot{H} \dot{g}_5 = \dot{\Lambda}(4) + 8\pi G(4) \rho
\]

\[
H^4 \ddot{f}_1 + H^2 \dot{f}_2 + \dot{H} H^2 \dot{f}_3 + \ddot{H}^2 \ddot{f}_4 + \ddot{H} \dot{f}_5 = -\dot{\Lambda}(4) + 8\pi G(4) p
\]

where the constants have been listed in appendix. The energy density and pressure are now given by:

\[
\rho = \left[ 2 \dot{K} X - \dot{K} + 6 H \dot{G} \dot{X} - 2 \dot{G} \dot{X} - 2 X (1 - \Theta_1) + \Theta_3(T, T^\dagger) V_{bulk}(\phi) \right],
\]

\[
p = \left[ \dot{K} - 2 \left( \dot{G} + \dot{G} \dot{X} \phi \right) - 2 X \Theta_1 - \Theta_3(T, T^\dagger) V_{bulk}(\phi) \right],
\]

where \( \Theta_1 = \frac{\dot{S}}{\dot{X} \dot{\Theta}_0} \) and \( \Theta_3(T, T^\dagger) = \frac{\dot{S} \dot{X} \dot{\Theta}_0}{\rho^2 \dot{X} \dot{\Theta}_0 + \rho + \tilde{\rho}} \). All the constants appearing in this section are explicitly mentioned in the Appendix C.

### Appendix C

In this section we have explicitly mentioned the expressions for the constants appearing in the section III, section IV and appendix A.

\[
\mathcal{I}(1) = \int_{-\pi R}^{+\pi R} dy \frac{1}{3 \exp(2\beta y) + 3T_4^2 \exp(-2\beta y) - 2T_4}{R^2 (\exp(\beta y) + T_4 \exp(-\beta y))^2},
\]

\[
\mathcal{C}(1) = \frac{R^7}{b_0^2} \int_{-\pi R}^{+\pi R} dy \left( \exp(\beta y) + T_4 \exp(-\beta y) \right)^\frac{1}{2},
\]

\[
\mathcal{I}(2) = A(6) = \frac{R^3}{b_0} \int_{-\pi R}^{+\pi R} dy \left( \exp(\beta y) + T_4 \exp(-\beta y) \right)^\frac{1}{2},
\]

\[
\mathcal{C}(2) = \frac{\beta^2 R^6}{4b_0^2} \int_{-\pi R}^{+\pi R} dy \left( \exp(\beta y) + T_4 \exp(-\beta y) \right)^\frac{1}{2} \left( T_4 \exp(-\beta y) - \exp(\beta y) \right)^2,
\]
\[
C(4) = I(4) = A(5) = \frac{\beta^4 R^3}{16 b_0^4} \int_{-\pi R}^{+\pi R} dy \frac{(\exp(\beta y) + T_4 \exp(-\beta y))^2}{(\exp(\beta y) + T_4 \exp(-\beta y))^4}, \tag{6.33}
\]

\[
A(7) = \frac{\beta^4 b_0^2}{16R^5} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^4}{(\exp(\beta y) + T_4 \exp(-\beta y))^4 (\exp(\beta y) + T_4 \exp(-\beta y))^4}, \tag{6.34}
\]

\[
A(8) = \frac{4\beta^4 b_0^2 T_4}{R^5} \int_{-\pi R}^{+\pi R} dy \frac{1}{(\exp(\beta y) + T_4 \exp(-\beta y))^4 (\exp(\beta y) + T_4 \exp(-\beta y))^4}, \tag{6.35}
\]

\[
A(9) = \hat{h}_4 = \hat{h}_{30} = \frac{\beta^2 b_0^2 b_4}{4R^5} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^2}{(\exp(\beta y) + T_4 \exp(-\beta y))^2 (\exp(\beta y) + T_4 \exp(-\beta y))^2}, \tag{6.36}
\]

\[
A(10) = \hat{h}_5 = -\frac{2\beta^2 b_0 b_4}{R^5} \int_{-\pi R}^{+\pi R} dy \frac{1}{(\exp(\beta y) + T_4 \exp(-\beta y))^2 (\exp(\beta y) + T_4 \exp(-\beta y))^2}, \tag{6.37}
\]

\[
A(11) = -\frac{2\beta^4 b_0^2 b_4}{2R^5} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^2}{(\exp(\beta y) + T_4 \exp(-\beta y))^4 (\exp(\beta y) + T_4 \exp(-\beta y))^2}. \tag{6.38}
\]

\[
A(12) = -\frac{b_0}{R} \int_{-\pi R}^{+\pi R} dy \frac{\exp(-\frac{8\pi y}{R})}{\sqrt{(\exp(\beta y) + T_4 \exp(-\beta y))}}, \tag{6.39}
\]

\[
A(1) = \tilde{h}_1 = \frac{b_0}{R} \int_{-\pi R}^{+\pi R} dy \frac{1}{\sqrt{(\exp(\beta y) + T_4 \exp(-\beta y))}}, \tag{6.40}
\]

\[
A(13) = \frac{b_0^3 h_{16}}{R} = \frac{b_0^3 h_{31}}{R} = \int_{-\pi R}^{+\pi R} dy \sqrt{(\exp(\beta y) + T_4 \exp(-\beta y))}, \tag{6.41}
\]

\[
\tilde{h}_2 = \frac{b_0^3}{R^5} \int_{-\pi R}^{+\pi R} dy \frac{1}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \tag{6.42}
\]

\[
\tilde{h}_3 = \frac{b_0 \beta^2}{4R} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(-\beta y))^2}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \tag{6.43}
\]

\[
\tilde{h}_6 = \tilde{h}_{21} = \frac{b_0^3}{R^3} \int_{-\pi R}^{+\pi R} dy \frac{1}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \tag{6.44}
\]

\[
\tilde{h}_7 = \tilde{h}_{26} = \frac{R^5}{b_0^4} \int_{-\pi R}^{+\pi R} dy (\exp(\beta y) + T_4 \exp(-\beta y))^2, \tag{6.45}
\]

\[
\tilde{h}_8 = \tilde{h}_{13} = \tilde{h}_{27} = \tilde{h}_{28} = \frac{\beta^2 R^3}{4b_0^3} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^2}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}. \tag{6.46}
\]
\[ \hat{h}_9 = \hat{h}_{14} = \hat{h}_{17} = \hat{h}_{29} = \hat{h}_{34} = \hat{h}_{36} = \frac{\beta^4 R}{16b_0} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^4}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.47)

\[ \hat{h}_{10} = \hat{h}_{12} = \hat{h}_{15} = \hat{h}_{19} = \hat{h}_{20} = \hat{h}_{32} = \hat{h}_{33} = \hat{h}_{35} = \frac{\beta^2 R}{4b_0} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^2}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.48)

\[ \hat{h}_{22} = \hat{h}_{23} = \frac{\beta^2 b_0^3}{4R^2} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^2}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.49)

\[ \hat{h}_{24} = \frac{\beta^4 b_0^3}{64R^3} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^4}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.50)

\[ \hat{h}_{25} = -\frac{2\beta^2 T_4 b_0^3}{R^3} \int_{-\pi R}^{+\pi R} dy \frac{1}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.51)

\[ \hat{z}_{25} = -\frac{\beta^4 T_4 b_0^3}{2R^3} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^2}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.52)

\[ \hat{h}_{37} = \frac{b_0^7}{R^7} \int_{-\pi R}^{+\pi R} dy \frac{1}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.53)

\[ \hat{h}_{38} = \frac{\beta^2 b_0}{4R^2} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^2}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.54)

\[ \hat{h}_{39} = -\frac{2\beta^2 T_4 b_0^7}{R^7} \int_{-\pi R}^{+\pi R} dy \frac{1}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.55)

\[ \hat{h}_{40} = \frac{\beta^4 b_0^7}{16R^5} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^4}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.56)

\[ \hat{h}_{41} = -\frac{\beta^4 T_4 b_0^7}{2R^5} \int_{-\pi R}^{+\pi R} dy \frac{(T_4 \exp(-\beta y) - \exp(\beta y))^2}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}, \] (6.57)

\[ \hat{h}_{42} = \frac{4\beta^4 T_4^2 b_0^7}{R^7} \int_{-\pi R}^{+\pi R} dy \frac{1}{(\exp(\beta y) + T_4 \exp(-\beta y))^2}. \] (6.58)
\[\begin{align*}
\dot{g}_1 &= \dot{\alpha}_1 + \dot{\alpha}_4 + \dot{\alpha}_5 + a^2(\dot{\alpha}_6 + \dot{\alpha}_7),
\dot{g}_2 &= (\dot{\alpha}_2 + \dot{\alpha}_3),
\dot{g}_3 &= 2\dot{\alpha}_1 + \dot{\alpha}_5 + a^2(\dot{\alpha}_6 + 2\dot{\alpha}_7),
\dot{g}_4 &= \dot{\alpha}_1 + a^2\dot{\alpha}_7, \quad \dot{g}_5 = \dot{\alpha}_2,
\dot{f}_1 &= \dot{\beta}_1 + \dot{\beta}_5 + \dot{\beta}_6, \quad \dot{f}_2 = \dot{\beta}_2 + \dot{\beta}_3, \quad \dot{f}_3 = 2\dot{\beta}_1 + \dot{\beta}_6, \quad \dot{f}_4 = \dot{\beta}_1, \quad \dot{f}_5 = \dot{\beta}_3,
\alpha_1 &= \alpha_4 \left(-3\tilde{h}_7 - 9\tilde{h}_15 - 18\tilde{h}_21 + \frac{7}{2}\tilde{h}_26 - 18\tilde{h}_31 + 36\tilde{h}_39 \right),
\alpha_2 &= 3 \left(\tilde{h}_2 - \tilde{h}_1 \right) + \frac{\alpha_4}{2\tilde{h}_39} \left(12\tilde{h}_8 - 3\tilde{h}_{12} + 9\tilde{h}_{13} + 9\tilde{h}_{18} + 9\tilde{h}_{19} + 18\tilde{h}_{22} + 36\tilde{h}_{23} + 24\tilde{h}_{25} + 18\tilde{h}_{32} + 18\tilde{h}_{33} - 72\tilde{h}_{38} - 48\tilde{h}_{39} \right),
\alpha_3 &= 3\tilde{h}_2 - \frac{\alpha_4}{2\tilde{h}_39} \left(18\tilde{h}_{22} - 6\tilde{h}_{12} - 72\tilde{h}_{38} - 48\tilde{h}_{39} \right), \quad \alpha_4 = 36\alpha_4 \tilde{h}_{39},
\alpha_5 &= \alpha_4 \left(72\tilde{h}_{39} - 18\tilde{h}_{21} \right), \quad \alpha_6 = 6\alpha_4 \tilde{h}_{11}, \quad \alpha_7 = 3\alpha_4 \tilde{h}_{11},
\lambda_4 &= \Lambda_4 - \frac{\alpha_4}{2\tilde{h}_39} \left(32\tilde{h}_{42} + 96\tilde{h}_{41} + 72\tilde{h}_{40} - 64\tilde{h}_{36} - 18\tilde{h}_{34} - 24\tilde{h}_{25} - 36\tilde{h}_{24} - 9\tilde{h}_{17} + 5\tilde{h}_{16} \right) - \frac{9\alpha_4}{2\tilde{h}_39},
\tilde{\beta}_1 &= \alpha_4 \left(\tilde{h}_7 - 3\tilde{h}_{11} - \tilde{h}_{16} + 6\tilde{h}_{21} - \tilde{h}_{36} - 2\tilde{h}_{31} - 3\tilde{h}_{37} \right),
\tilde{\beta}_2 &= \left(2\tilde{h}_1 - 3\tilde{h}_2 \right) + \alpha_4 \left\{ \frac{1}{2\tilde{h}_39} \left(-8\tilde{h}_8 + 4\tilde{h}_{12} + 6\tilde{h}_{13} + 6\tilde{h}_{18} + 6\tilde{h}_{14} - 18\tilde{h}_{22} - 24\tilde{h}_{23} - 16\tilde{h}_{25} - 12\tilde{h}_{32} - 12\tilde{h}_{33} + 72\tilde{h}_{38} + 48\tilde{h}_{39} \right) - \tilde{h}_{26} \right\},
\tilde{\beta}_3 &= \left(\tilde{h}_1 - 3\tilde{h}_2 \right) + \alpha_4 \left\{ \frac{1}{2\tilde{h}_39} \left(-8\tilde{h}_8 + 5\tilde{h}_{12} + 3\tilde{h}_{13} + 3\tilde{h}_{18} + 3\tilde{h}_{19} - 18\tilde{h}_{22} - 12\tilde{h}_{23} - 8\tilde{h}_{25} - 6\tilde{h}_{32} - 6\tilde{h}_{33} + 72\tilde{h}_{38} + 48\tilde{h}_{39} \right) \right\},
\tilde{\beta}_5 &= \alpha_4 \left(-3\tilde{h}_{37} + 8\tilde{h}_{31} + 12\tilde{h}_{21} - 4\tilde{h}_{16} - 4\tilde{h}_{11} + 2\tilde{h}_7 \right),
\tilde{\beta}_6 &= \alpha_4 \left(-3\tilde{h}_{37} + 8\tilde{h}_{31} + 12\tilde{h}_{21} - 4\tilde{h}_{16} - 2\tilde{h}_{11} \right),
\tilde{\lambda}_4 &= \left(6\tilde{h}_4 + 4\tilde{h}_5 - 3\tilde{h}_3 + \Lambda_4 \right) + \frac{\alpha_4}{2\tilde{h}_39} \left(72\tilde{h}_{40} + 96\tilde{h}_{41} + 64\tilde{h}_{42} - 64\tilde{h}_{36} + 9\tilde{h}_{44} + 24\tilde{h}_{25} + 36\tilde{h}_{24} - 9\tilde{h}_{17} - 9\tilde{h}_{14} + 6\tilde{h}_9 \right).
\end{align*}\]
Appendix D

The determinant appearing in the first term of the equation (2.25) can be written as

\[
D = \text{det} \left( I + S + B \right) = \frac{1}{4!} \epsilon_{ABCD} \epsilon_{EFGH} \left( I + S + B \right)_A^E \left( I + S + B \right)_B^F \left( I + S + B \right)_C^G \left( I + S + B \right)_D^H,
\]

where we define

\[
\Theta(\Phi) = 2\pi \alpha' \sqrt{h(y)}, \quad S_A^B = \frac{f(\phi)}{G_{AB}} \partial^A \phi \partial_B \phi^D,
\]

\[
B_B^A = b_{CD} \partial^A \phi \partial_B \phi^D + \Theta(\Phi) F_B^A = \frac{\Theta(\Phi)}{2\pi \alpha'} \gamma^{AC} \mathcal{F}_{CB}.
\]

In this connection, I, S, and B are all 5 × 5 matrices satisfying the property \( S_{AB} = S_{BA}, B_{AB} = -B_{BA}. \) Moreover \( \mathcal{F}_{CB} \) is a Neveu-Schwarz gauge invariant. Detailed computation of the determinant yields

\[
D = D_S - \frac{1}{2} Tr(B^2) \left( 1 + Tr(S) \right) + Tr(SB^2) \left( 1 + Tr(S) \right) - Tr(S^2 B^2)
\]

\[
- \frac{1}{4} Tr(B^2) \left[ (Tr(S))^2 - Tr(S^2) \right] - \frac{1}{2} Tr(SBSB) + \frac{1}{8} \left[ (Tr(B^2))^2 - 2Tr(S^4) \right]
\]

where

\[
D_S = 1 + Tr(S) + \frac{1}{2} \left[ (Tr(S))^2 - Tr(S^2) \right] + S_A^B S_A^C S_B^C + S_A^B s_B^C s_A^C. \tag{6.65}
\]

Now throughout our discussion we assume that the D4 brane and bulk SUGRA form fields are ignored then we can write \( D \simeq D_S \). In this connection we use the well known identity

\[
det (I + S) = -\frac{1}{4!} \epsilon_{ABCD} \epsilon_{EFGH} \left( I + S \right)_A^E \left( I + S \right)_B^F \left( I + S \right)_C^G \left( I + S \right)_D^H = e^{Tr(\ln(I+S))},
\]

\[
Tr(S^n) = Tr((-2f(\Phi)X)^n)
\]

which results in

\[
D \simeq D_S = 1 - 2f(\Phi)G_{AB}X^{AB} + f^2(\Phi)X_A^{[A}X_B^{B]} - 8f^3(\Phi)X_A^{[A}X_B^{B}X_C^{C]}
\]

\[
+ 16f^4(\Phi)X_A^{[A}X_B^{B}X_C^{C}X_D^{D]} - 32f^5(\Phi)X_A^{[A}X_B^{B}X_C^{C}X_D^{D}X_E^{E]}
\]

where the brackets denote antisymmetrisation on the field indices.
