Renormalization group approach to scalar quantum electrodynamics on de Sitter

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We consider the quantum loop effects in scalar electrodynamics on de Sitter space by making use of the functional renormalization group approach. We first integrate out the photon field, which can be done exactly to leading (zeroth) order in the gradients of the scalar field, thereby making this method suitable for investigating the dynamics of the infrared sector of the theory. Assuming that the scalar remains light we then apply the functional renormalization group methods to the resulting effective scalar theory and focus on investigating the effective potential, which is the leading order contribution in the gradient expansion of the effective action. We find symmetry restoration at a critical renormalization scale \( \kappa = \kappa_c \) much below the Hubble scale \( H \). When compared with the results of Serreau and Guilleux \[1\, 2\] we find that the photon facilitates symmetry restoration such that it occurs at an RG scale \( \kappa_c \) that is higher than in the case of a pure scalar theory. The true effective potential is recovered when \( \kappa \to 0 \) and in that limit one obtains the results that agree with those of stochastic inflation, provided one interprets it in the sense as advocated by Lazzari and Prokopec \[3\].

I. INTRODUCTION

It is well known that perturbation theory for non-conformally coupled fields on de Sitter can fail. This is because the strong interactions of these fields with gravity generate a large number of infrared modes which may be reflected in growing secular effects in field correlators and local observables such as mass. For example, the coincident two-point function of a massless scalar exhibits on de Sitter a secular growth \[4\, 5\] which for a self-interacting scalar field generates at two-loops a pressure and energy density that grow in time \[7\, 8\] and a growing mass of scalars \[4\] , photons \[9\, 11\] or fermions \[12\] . Quite generically such secular effects invalidate perturbative expansion, even if the value of the coupling constant is very small. In order to correctly capture observables plagued by secular effects, one ought to resort to resummation techniques.

The best known resummation technique developed for de Sitter is Starobinsky’s stochastic inflation \[13\] . This technique allows for resummation of the leading infrared effects on de Sitter for interacting scalar theory \[14\] , Yukawa \[15\] and quantum scalar electrodynamics (SQED) \[16\, 17\] . Remarkably, the two-loop perturbative results for the stress energy tensor \[18\, 19\] agree beautifully with the stochastic results in the perturbative regime \[17\] . But unlike perturbative methods, stochastic, theory allows for calculation of masses and energy in the late-time asymptotic regime, in which de Sitter symmetry gets restored \[17\] . Furthermore, it is also known how to generalize stochastic inflation for interacting scalars to space-times that adiabatically deviate from de Sitter and to space-times of constant principal slow roll parameter \( \epsilon \) \[20\, 21\] . However, it is not known how to generalize Starobinsky’s stochastic theory to include quantum gravitational effects. Nevertheless, notable attempts have been made in this direction \[22\, 24\] .

It is interesting to note that one-loop perturbative calculations of the effective potential on locally de Sitter are available (together with quantum corrections to tensor and scalar spectral slopes and amplitudes), as well as the one-loop effective potential induced by graviton and scalar perturbations and a more general one-loop effective potential from graviton and scalar perturbations \[25\] . In Ref. \[26\] , a perturbative one-loop study of symmetry restoration in a scalar self-interacting theory was presented on constant \( \epsilon \) space-times on which the scalar field evolves with the Hubble parameter; however the authors do not address what happens at late times when perturbative analysis breaks down.

The principal goal of works that employ resummation techniques is to overcome that limitation. Recently, renormalization group (RG) methods have been proposed \[1\, 3\] to tackle these issues. While it is known that these methods in part resum perturbation theory, it has not been rigorously proved that these methods indeed correctly recover the (nonperturbative) infrared physics in quantum field theories (QFTs) on de Sitter. In this work we show that functional renormalization group approach to the infrared (IR) on de Sitter, in which effective action is truncated at the effective potential level, leads to the results that agree with Lazzari and Prokopec for a self-interacting scalar field theory. More importantly, we also present here results for SQED (in this case we do not have a stochastic inflation calculation to compare with).

This technique has been used from statistical physics to quantum gravity \[27\, 30\] and more recently it has been applied to de Sitter space dynamics \[1\, 2\, 31\] . The stochastic formalism has been applied with the same purpose by Lazzari and Prokopec \[3\] . In that work the authors show that infrared modes restore the symmetry in...
as a Legendre transform of $W$. The corresponding non-equilibrium effective action is then defined by\[\Gamma(\kappa) = Z_C[J] = \left. \frac{1}{\sqrt{-g(x)}} \frac{\delta W_C[J]}{\delta J(x)} \right|_{\phi(x)} = \langle \phi(x) \rangle_{\phi(x)},\]

which is associated with the tree-level action along the contour $C$,

\[S_C[\varphi] = \int_x \left\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi(x) \partial_\nu \varphi(x) - V(\varphi) \right\}, \tag{4}\]

where \[\int_x \equiv \int_C d^Dx \sqrt{-g}\]

implies that one integrates in time along the contour $C$ in figure 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Closed time path $C = C^+ \cup C^-$. The upper branch $C^+$ goes forward from the initial to the final time and the lower branch $C^-$ goes backward from the final to the initial time.}
\end{figure}

The vacuum expectation value of the field $\varphi$ is the relevant quantity in studying the spontaneous symmetry breaking of the theory and is defined as \[\Gamma(\kappa) = Z_C[J] = \left. \frac{1}{\sqrt{-g(x)}} \frac{\delta W_C[J]}{\delta J(x)} \right|_{\phi(x)} = \langle \phi(x) \rangle_{\phi(x)},\]

We are dealing here with a massless scalar on de Sitter, and trying to impose the Chernikov-Tagirov (also known as the Bunch-Davies) vacuum \[\Gamma(\kappa) = Z_C[J] = \left. \frac{1}{\sqrt{-g(x)}} \frac{\delta W_C[J]}{\delta J(x)} \right|_{\phi(x)} = \langle \phi(x) \rangle_{\phi(x)},\]

We are dealing here with a massless scalar on de Sitter, and trying to impose the Chernikov-Tagirov (also known as the Bunch-Davies) vacuum state. The regularization can be done either by matching onto a pre-inflationary phase in which the vacuum is regular in the infrared \[\Gamma(\kappa) = Z_C[J] = \left. \frac{1}{\sqrt{-g(x)}} \frac{\delta W_C[J]}{\delta J(x)} \right|_{\phi(x)} = \langle \phi(x) \rangle_{\phi(x)},\]

or by placing the Universe in a comoving box \[\Gamma(\kappa) = Z_C[J] = \left. \frac{1}{\sqrt{-g(x)}} \frac{\delta W_C[J]}{\delta J(x)} \right|_{\phi(x)} = \langle \phi(x) \rangle_{\phi(x)},\]

The latter regularization is equivalent to imposing an infrared comoving momentum cutoff, $k_0 \sim 1/L$. One can show that any regularization procedure (including those mentioned above) leads to correlators that exhibit a breaking of de Sitter symmetry \[\Gamma(\kappa) = Z_C[J] = \left. \frac{1}{\sqrt{-g(x)}} \frac{\delta W_C[J]}{\delta J(x)} \right|_{\phi(x)} = \langle \phi(x) \rangle_{\phi(x)},\]

In this paper we regulate the infrared (IR) by introducing an infrared regulator $R_c$ and define the average effective action as a modified Legendre transform,\[\Gamma(\kappa) = W_c[J] - \int_x J(x)\phi(x) - \frac{1}{2} \int_{x,y} R_c(x,y)\phi(x)\phi(y),\]

where $W_c[J]$ is defined as\[\Gamma(\kappa) = W_c[J] = \int_x J(x)\phi(x) - \frac{1}{2} \int_{x,y} R_c(x,y)\phi(x)\phi(y),\]

eternal inflation at late times, this symmetry restoration due to infrared (IR) modes was discussed previously by Serreau in \[\Gamma(\kappa) = W_c[J] = \int_x J(x)\phi(x) - \frac{1}{2} \int_{x,y} R_c(x,y)\phi(x)\phi(y),\]

Serreau later uses the nonperturbative renormalization group to the same system \[\Gamma(\kappa) = W_c[J] = \int_x J(x)\phi(x) - \frac{1}{2} \int_{x,y} R_c(x,y)\phi(x)\phi(y),\]

They also show that the RG flow gets dimensionally reduced to a zero-dimensional euclidean field theory for small curvatures (see also \[\Gamma(\kappa) = W_c[J] = \int_x J(x)\phi(x) - \frac{1}{2} \int_{x,y} R_c(x,y)\phi(x)\phi(y),\]

In that formalism it is useful to define the complex operators \[\delta W_c[J] = \frac{1}{\sqrt{-g(x)}} \frac{\delta W_c[J]}{\delta J(x)} \right|_{\phi(x)} = \langle \phi(x) \rangle_{\phi(x)},\]

This formalism is suitable for calculating (time dependent) expectation values of operators \[\Gamma(\kappa) = W_c[J] = \int_x J(x)\phi(x) - \frac{1}{2} \int_{x,y} R_c(x,y)\phi(x)\phi(y),\]

In the present work we begin our analysis by introducing the renormalization group techniques and obtaining the RG flow equation in section II. In section III we present the SQED theory in de Sitter and calculate its effective potential and energy-momentum tensor. We show the flow of a Higgs-like potential with photons interactions coming from SQED on top of it in section IIIA and in section IIIB we compute the flow of the SQED stress-energy tensor. The final conclusions and outlook are presented in section IV. In appendix A we discuss the flow of a scalar field theory without photons interactions, to compare how photons affect the flow of the scalar field. There we show that photons enhance symmetry restoration.
The regulator $R_κ(p)$ acts as a momentum-dependent mass term that is finite for IR modes ($p < κ$) and vanishes for UV modes ($p \gg κ$) \cite{31}, such that one recovers the full effective action when the cutoff is removed,

$$\Gamma_{κ=0}[φ] = Γ[φ],$$

and one recovers the tree-level (bare) action in the opposite limit, $\Gamma_{κ→∞}[φ] = S_κ[φ]$. We choose the Litim regulator \cite{45}

$$R_κ(p) = Z_κ(κ^2 - (k/a)^2) \theta(κ^2 - (k/a)^2), \quad (10)$$

where $\sqrt{Z_κ}$ is a renormalization factor for the scalar field $φ$ that can depend on the moment cutoff $κ$, $k$ is the comoving momentum and $a = a(η)$ the scale factor.

We use the local potential approximation (LPA) \cite{10} as an Ansatz for the average effective action,

$$\Gamma_κ[φ] = \int_x \left\{ -\frac{Z_κ}{2} g^{μν} \partial_μ φ(x) \partial_ν φ(x) - V_κ(φ) \right\}, \quad (11)$$

which should be good for the infrared sector of the theory on de Sitter because the large Hubble damping in de Sitter tends to suppress higher-derivative terms in the effective action (an analogous approximation is used in stochastic inflation \cite{3}).

The evolution with the running of the cutoff of the average effective action is given by the Wetterich equation \cite{47}

$$\dot{Γ}_κ[φ] = \frac{1}{2} \text{Tr} \left\{ \dot{R}_κ(x; y) G_κ(x; y) \right\}, \quad (12)$$

where we have defined the notation $\dot{F} ≡ κ∂_κ F$, $\text{Tr} ≡ \int x y$ and $G_κ(x, y)$ is the (full) propagator of the average effective action,

$$iG_κ(x; y)^{-1} = \frac{1}{\sqrt{g(x)g(y)}} \frac{δ^2 \Gamma_κ[φ]}{δφ(η)δφ(x)} + R_κ(x; y). \quad (13)$$

We begin our analysis by deriving the flow equation for the effective potential and for small $κ$ ($κ < H$), which is the relevant regime for infrared effects. Namely, at momentum scales $k/a > κ$ we assume that the theory is perturbative, such that the effective action at the scale $κ$ can be approximated by its local (tree-level) form; the quantum effects from the running from some high scale $κ_0 \gg H$ up to $κ \lesssim H$ can be to a good approximation subsumed to a finite renormalization of the coupling parameters in $V(φ)$ and $Z_κ$.

Next step is to transform from real to momentum space, which can be done by performing a Wigner transform (which is a Fourier transform with respect to the relative coordinate), so using isotropy and homogeneity of the spatial coordinates the regulator in momentum representation is

$$R_κ(x; y) = R_κ(η, η' || x - y ||) = \int \frac{d^dk}{(2π)^d} e^{i k(x - y)} \tilde{R}_κ(η, η'; k), \quad (14)$$

where $\tilde{k}$ is the comoving spatial momentum, $k = ||\tilde{k}||$ and we have introduced the notation $\tilde{F}$ for the Wigner transform of $F$.

Requiring de Sitter symmetry implies that the Feynman propagator function can depend only on the invariant distance $y(x, x')$ \cite{41 48}

$$y(x; x') ≡ a(η)a(η')H^2 \left( ||x - x'||^2 - (η - η' - iε)^2 \right), \quad (15)$$

which is related to the geodesic distance $ℓ(x; x')$, $y(x, x') → 4 \sin^2\left( H ℓ(x; x') / 2 \right)$. The $iε$-prescription in \cite{15} provides the correct boundary conditions for the Feynman propagator. Next, it is convenient to perform a conformal rescaling of all the quantities,

$$φ(x) → a(η)^{d-1} φ(x) = (-ηH)^{-d-1} φ(x), \quad (16)$$

and more generally,

$$F(x; x') → (a(η)a(η'))^{d_F} F(η, η'; ||x - x'||) = (H^2η^2)^{-d_F} F(η, η'; ||x - x'||), \quad (17)$$

where $d_F$ is the conformal dimension of the quantity $F$. In the momentum $p$-representation $p = -kHη$, $p' = -kHη'$ and introducing the notation $\tilde{F}$ for dimensionless functions, i.e. extracting all the scale and dimensional factors, the regulator becomes,

$$\tilde{R}_κ(η, η'; k) = (H^2η^2)^{d-1} k^d \tilde{R}_κ(p, p'), \quad (18)$$

$$\tilde{R}_κ(p, p') = H^{d_F} \frac{dp}{p^2} \tilde{R}_κ(p), \quad (19)$$

and the Green function is

$$\tilde{G}_κ(η, η'; k) = \frac{(H^2η^2)^{d-1}}{k} \tilde{G}_κ(p, p'). \quad (20)$$

We are primarily interested in the flow of the effective potential in the infrared. For that reason we can evaluate the Wetterich equation \cite{12} at a constant field $φ$ (neglecting the derivatives of $φ$ is, namely, justified in the infrared because of the Hubble damping),

$$\dot{Γ}_κ[φ] \big|_{φ=\text{const.}} = \int_x \left\{ -\dot{V}_κ(φ) \right\} \big|_{φ=\text{const.}}. \quad (21)$$

the integral over $x$ is just a volume factor $Ω ≡ \int_x$. Upon inserting Eq. \cite{21} into \cite{12} after some algebra and making use of $\dot{V}_κ(φ) = -Ω^{-1} \dot{Γ}_κ[φ] \big|_{φ=\text{const.}}$, one obtains \cite{11},

$$\dot{V}_κ(φ) = -\frac{1}{2Ω} \text{Tr} \left\{ \dot{R}_κ(x; y)G_κ(y; x) \right\} = \frac{1}{2} \int \frac{dp}{(2π)^d} \tilde{R}_κ(p) \frac{\tilde{F}_κ(p, p)}{p}, \quad (22)$$

where $\tilde{F}_κ(p, p) = \int \frac{dp'}{(2π)^d} \tilde{R}_κ(p') \frac{\tilde{G}_κ(p', p)}{p'}.$
where the Green function $G_a(y; x)$ is defined in Eq. (13). By making use of the generating function the Green function can be also defined as the two-point function ordered along the complex time contour,

$$G_a(x; x') = \langle T_c \phi(x) \phi(x') \rangle$$

$$= F_a(x; x') - \frac{i}{2} \text{sign}_c (\eta - \eta') \rho_a(x; x'), \quad (23)$$

where we have defined the Hadamard (statistical) two-point function $F_a(x; x') \equiv \frac{1}{2} \{ \langle \phi(x), \phi(x') \rangle \}$ and the Pauli-Jordan or Schwinger (spectral) two-point function $\rho_a(x; x') \equiv i \langle \{ \phi(x), \phi(x') \} \rangle$. The Feynman propagator satisfies the following equation,

$$\left( \partial_t^2 + \frac{1}{H^2} - \left( \nu^2_{\kappa} - \frac{1}{4} \right) \frac{1}{p^2} \right) \hat{G}_{a}(p, p') = \frac{i \delta_t (p - p')}{Z_{\kappa} H}, \quad (24)$$

with

$$\nu_{\kappa} = \sqrt{\frac{d^2}{4} - \frac{V''(\phi)}{Z_{\kappa} H^2}}. \quad (25)$$

Following [1] we assume factorization of the two-point functions,

$$\hat{G}_a(p; p') = \hat{F}_a(p; p') - \frac{i}{2} \text{sign}_c (\eta - \eta') \hat{\rho}_a(p; p'), \quad (26)$$

in terms of a new function $u_a(p)$ that only depends on one momentum $p$,

$$\hat{F}_a(p; p') = Z_{\kappa}^{-1} \text{Re} \{ u_a(p) u_a^*(p') \} \quad \hat{\rho}_a(p; p') = -2Z_{\kappa}^{-1} \text{Im} \{ u_a(p) u_a^*(p') \}. \quad (27)$$

This is possible only if $\hat{G}_a(p; p')$ describes a pure state, which is what we assume in this work. Upon doing that and inserting the expression for the Litim regulator [10], the equation for the Green’s function [24] is

$$\left[ \partial_t^2 + \frac{1}{H^2} - \left( \nu^2_{\kappa} - \frac{1}{4} \right) \frac{1}{p^2} \right] u_a(p) = 0, \quad \text{for } p \geq \kappa \quad (28)$$

and

$$\left[ \partial_t^2 + \left( \nu^2_{\kappa} - \frac{1}{4} \right) \frac{1}{p^2} \right] u_a(p) = 0, \quad \text{for } p \leq \kappa, \quad (29)$$

where we have defined

$$\nu_{\kappa} = \sqrt{\nu^2_{\kappa} - \frac{\kappa^2}{H^2}}. \quad (30)$$

The properly normalized solutions (i.e. with a canonically normalized Wronskian) of those differential equations are [20, 31],

$$u_a(p) = \sqrt{\frac{\pi p}{4H}} e^{i\nu_{\kappa} H \frac{1}{p}} \left( \frac{p}{H} \right); \quad (p \geq \kappa), \quad (31)$$

and

$$u_a(p) = \sqrt{\frac{\pi p}{4H}} e^{i\nu_{\kappa} H \frac{1}{p}} \left( \frac{p}{H} \right), \quad (p \leq \kappa), \quad (32)$$

where $H_{\nu_0}^{(1)}(z)$ is the Hankel function of the first kind and $\nu_0 = \frac{2}{\kappa} (\nu_{\kappa} + \frac{1}{2})$. Note that $\frac{1}{\nu_{\kappa}}$ is a mode function that corresponds to the Chernov-Tagirov vacuum of a particle whose mass-squared is $V''(\phi) > 0$. Requiring continuity of the function and its first derivative we obtain

$$\nu_{\kappa} = \frac{1}{2} \left[ H_{\nu_0}^{(1)} \left( \frac{\kappa}{H} \right) \pm \frac{\kappa}{2} \frac{d}{\nu_{\kappa}} H_{\nu_0}^{(1)} \left( \frac{\kappa}{H} \right) \right]. \quad (33)$$

The flow of the effective potential as a function of $u_a(p)$ is,

$$\dot{V}_{\nu_0}(\phi) = \frac{\Omega_d}{2(2\pi)^d} \int dp \, p^{d-2} \left[ (2 - \eta_{\kappa}) \kappa^2 + \eta_{\kappa} p^2 \right] |u_a(p)|^2, \quad (34)$$

with $\eta_{\kappa} = -Z_{\kappa}/Z_{\kappa}$ and $\Omega_d = 2\pi^{d/2}/\Gamma(d/2)$. Expanding $u_a(p)$ for small $\kappa$ using, \n
$$H_{\nu_0}(\kappa) = \frac{2^{\nu_{\kappa}} \Gamma(\nu_{\kappa})}{i\pi \nu_{\kappa}^2} \left( 1 + O(\kappa^2) \right) \quad (\kappa^2 \ll 1), \quad (35)$$

the flow equation for the effective potential becomes,

$$\dot{V}_{\nu_0}(\phi) = \frac{\Omega_d F_{\nu_0}}{2(2\pi)^d} \frac{\kappa^d+2-2\nu_{\kappa}}{H_{\nu_0}^{d-2\nu_{\kappa}}} \left[ \frac{2 - \eta_{\kappa}}{d-2\nu_{\kappa}} + \frac{\eta_{\kappa}}{d-2\nu_{\kappa}} \right], \quad (36)$$

where we have defined,

$$F_{\nu_0} = \frac{(2\nu_{\kappa} \Gamma(\nu_{\kappa}))^2}{4\pi}. \quad (37)$$

We make one last approximation, $|V''(\phi)| \ll H^2$, and take $Z_{\kappa} = 1$ to obtain the final expression for the flow of the effective potential [1, 2],

$$\dot{V}_{\nu_0}(\phi) = 2A_d \frac{\kappa^2 H^{d+1}}{V_{\nu_0}'(\phi) + \kappa^2}, \quad (38)$$

with

$$A_d = \frac{d \Omega_d F_{d/2}}{4(2\pi)^d} = \frac{\Gamma(d/2 + 1)}{4\pi^{d/2+1}}. \quad (39)$$

The flow equation (38) is the main result of this section, and we use it in the rest of the paper to study the flow of the effective action in the infrared on de Sitter.

### III. FLOW OF THE EFFECTIVE POTENTIAL

In this section we study the flow of a spontaneously broken potential with photons interactions using the general flow equation obtained in the previous section. We first obtain the SQED effective potential integrating out the vector potential and add it on top of a Higgs potential. Finally we also study the flow of the SQED stress-energy tensor.
A. Scalar QED

The starting point is the SQED action for a massless, minimally coupled (MMC) charged scalar

$$S_{\text{SQED}} = \int d^3x \sqrt{-g} \left\{ - g^{\mu \nu} (D_\mu \Phi)(D_\nu \Phi) - \frac{1}{4} g^{\mu \nu} g^{\rho \sigma} F_{\mu \nu} F_{\rho \sigma} \right\}, \quad (40)$$

where $D_\mu = \partial_\mu - ie \tilde{A}_\mu$ is a covariant derivative and $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength associated with the gauge field $A_\mu$. Since under the gauge transformations the fields transform as,

$$A_\mu \rightarrow \tilde{A}_\mu - \partial_\mu \Lambda(x), \quad \Phi \rightarrow e^{-i e \Lambda(x)} \Phi, \quad (41)$$

the following redefinition of the fields

$$\Phi(x) = \frac{\phi(x)}{\sqrt{2}} e^{i \theta(x)} \quad A_\mu(x) = \tilde{A}_\mu(x) - \frac{1}{e} \partial_\mu \theta(x) \quad (42)$$

results in an action which can be expressed solely in terms of gauge invariant quantities $\phi$ and $A_\mu$ which does not transform under the gauge transformations $^{[41]}$. The resulting action is,

$$S_{\text{SQED}} = \int \left\{ - \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} e^2 g^{\mu \nu} A_\mu A_\nu \phi^2 - \frac{1}{4} g^{\mu \nu} g^{\rho \sigma} F_{\mu \nu} F_{\rho \sigma} - \frac{1}{2} \delta \xi \phi^2 R - \frac{1}{4} \delta \lambda \phi^4 \right\}, \quad (43)$$

where $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \tilde{F}_{\mu \nu}$. The last equality holds because of the gauge invariance of $F_{\mu \nu}$. Anticipating dimensional regularisation and renormalization procedure, we have added in $^{[43]}$ two local counterterms (whose couplings are $\delta \xi$ and $\delta \lambda$) which we will need to renormalize the photon fluctuations on de Sitter.

The next step is to integrate out the field vector to obtain an effective action for the scalar field $^{[17, 49]}$, the divergent and finite part are $^{[49]}$, becomes

$$e^{i \Gamma[\phi]} = \int D A_\mu \delta(\nabla^\alpha A_\alpha) e^{i S_{\text{SQED}}} \quad (44)$$

where the $\delta$-function imposes the exact Lorenz gauge condition, $\nabla^\alpha A_\alpha = 0$, in the functional integral measure. As we argue below, this is the right condition to impose since the photon field we integrate over becomes massive due to the backreaction of long wavelength scalar fluctuations on de Sitter. Formally, upon integrating the photons, the effective action for scalars $^{[43]}$ becomes,

$$\Gamma[\phi] = \int \left\{ - \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \delta \xi \phi^2 R - \frac{1}{4} \delta \lambda \phi^4 \right\} + i \log \left\{ \det \left[ \partial_\mu \left( \sqrt{-g} \left( g^{\mu \nu} g^{\rho \sigma} - g^{\mu \sigma} g^{\rho \nu} \right) \partial_\nu \right) \right] - e^2 \sqrt{-g} g^{\rho \sigma} \phi^2 \right\}, \quad (45)$$

where the operator inside the determinant is the inverse of the photon propagator,

$$i G_{\mu \nu}(x, x')^{-1} = \frac{1}{\sqrt{g(x) g(x')}} \frac{\delta^2 S}{\delta A_\nu(x') \delta A_\mu(x)} = \frac{1}{\sqrt{-g}} \left\{ \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} \left( g^{\sigma \nu} g^{\mu \rho} - g^{\mu \rho} g^{\sigma \nu} \right) \partial_\sigma \right) - e^2 \phi^2(x) g^{\mu \nu} \right\} \delta_C(x - x'), \quad (46)$$

Since the scale at which the scalar field in $^{[46]}$ varies is super-Hubble which – upon making the approximation $e^2 \phi^2(x) = m^2 + \text{gradient corrections} –$ has been calculated to leading order in gradients by Tsamis and Woodard in $^{[50]}$,

$$i G_{\mu \nu}(x, x) = g_{\mu \nu} \frac{d H^2}{2 m^2} \frac{H^{d-1}}{(4\pi)^{(d+1)/2}} \left[ \frac{\Gamma(d)}{(d+1)^2 + 1} \right]$$

with

$$\nu = \sqrt{\left( \frac{d-2}{2} \right)^2 - \frac{m^2}{H^2}} \quad (48)$$

By differentiating the effective action $^{[45]}$, one obtains the derivative of the effective potential,

$$\frac{dV_{\text{eff}}(\phi^2)}{d(\phi^2)} = \frac{1}{2} \delta \xi H^2 d(d+1) + \frac{1}{2} \delta \lambda \phi^2 + \frac{d^2}{2} \Gamma(d+1) \Gamma(d+1) \left[ \frac{\Gamma(d+1)}{(d+1)^2 + 1} \right]$$

$$- \Gamma \left( d+1 \right) \Gamma \left( \frac{d+1}{2} + \nu \right) \Gamma \left( \frac{d+1}{2} - \nu \right) \right]. \quad (49)$$

Next, upon performing dimensional regularization $^{[51]}$, the divergent and finite part are $^{[49]}$, becomes

$$\frac{dV_{\text{eff}}(\phi^2)}{d(\phi^2)} = \frac{1}{2} \delta \xi H^2 d(d+1) + \frac{1}{2} \delta \lambda \phi^2 + \frac{e^2}{4} d(d+1) \frac{H^{d-1}}{(4\pi)^{(d+1)/2}} \left[ \frac{2}{d-3} \left( \frac{e^2}{2 H^2} \right)^2 + \frac{1}{2} \right] \left[ 1 + \frac{e^2}{2 H^2} \left[ \Psi \left( \frac{3}{2} + \nu \right) + \Psi \left( \frac{3}{2} - \nu \right) - \frac{3}{2} \gamma \right] \right] + O(d-3) \quad (50)$$

where $\Psi(x)$ is the digamma function. We choose the following counterterms to cancel the divergencies and the $\phi^2$ and $\phi^4$ terms of the effective potential,

$$\delta \xi = \frac{e^2 H^{d-3}}{(4\pi)^{(d+1)/2}} \left( - \frac{1}{d-3} + \frac{\gamma}{2} + O(d-3) \right) \quad (51)$$
and
\[ \delta \lambda = \frac{d(d+1)e^dH^{d-3}}{2(4\pi)^{(d+1)/2}} \left( -\frac{1}{d-3} + \frac{\gamma}{2} \frac{3}{4} + \mathcal{O}(d-3) \right). \]

(52)

It is useful to define \( z \equiv e^2\phi^2/2H^2 = m^2/2H^2 \) and integrate with respect to \( z \), taking the limit \( \epsilon \to 0 \) to 0 the effective potential is,
\[
V_{\text{eff}}(z) = \frac{3H^4}{8\pi^2} \left\{ -(1 + 2\gamma) z + \left( \frac{\gamma}{2} + \frac{3}{2} \right) z^2 \right. \\
+ \int_0^z dy \left[ (1+y) \left( \Psi \left( \frac{3}{2} + \frac{1}{2}\sqrt{1-8y} \right) \right) + \Psi \left( \frac{3}{2} - \frac{1}{2}\sqrt{1-8y} \right) \right] \bigg\} \tag{53}
\]

There is no simple expression for \( V_{\text{eff}}(z) \) since one cannot express the integral in (53) in terms of known functions. There is a relatively simple expansion of \( V_{\text{eff}}(z) \) around \( z = 0 \). To accomplish that, note first that,
\[
x(y) = \frac{1}{2} - \frac{1}{2}\sqrt{1-8y} = \sum_{n=0}^{\infty} \frac{(2n)!2^{n+1}}{n!(n+1)!} y^{n+1}, \tag{54}
\]

and then,
\[
V_{\text{eff}}(z) = \frac{3H^4}{8\pi^2} \left\{ -z^2 \\
- \int_0^z dy \left( 2 \sum_{n=1}^{\infty} \zeta(2n+1)x(y)^{2n} - \sum_{n=1}^{\infty} x(y)^n \right) \right\}. \tag{55}
\]

One can easily perform the integrals in (55). Taking account of \( z = e^2\phi^2/2H^2 \) and keeping terms up to order \( \phi^{18} \) we get a good approximation for small values of the field, \( |\phi| \lesssim 6.5H^2 \), and the effective scalar field potential for SQED is,
\[
V_{\text{eff}}(\phi) = \frac{3H^4}{8\pi^2} \left\{ 10 - 8\zeta(3) \left( \frac{e^2}{2H^2} \right)^3 \phi^6 \\
+ 12 - 10\zeta(3) \left( \frac{e^2}{2H^2} \right)^4 \phi^8 \\
+ [264 - 192\zeta(3) - 32\zeta(5)] \left( \frac{e^2}{2H^2} \right)^5 \phi^{10} \\
+ [1568 - 1056\zeta(3) - 288\zeta(5)] \left( \frac{e^2}{2H^2} \right)^6 \phi^{12} \\
+ [9792 - 6272\zeta(3) - 2048\zeta(5) - 128\zeta(7)] \left( \frac{e^2}{2H^2} \right)^7 \phi^{14} \\
+ [7920 - 4896\zeta(3) - 1760\zeta(5) - 208\zeta(7)] \left( \frac{e^2}{2H^2} \right)^8 \phi^{16} \\
+ [420992 - 253440\zeta(3) - 96768\zeta(5) - 15360\zeta(7)] \left( \frac{e^2}{2H^2} \right)^9 \phi^{18} + \mathcal{O}(\phi^{20}) \right\}. \tag{56}
\]

Our next task is to add this potential on top of the Higgs potential and calculate the running of all the parameters. To do that we choose the following Ansatz for the effective scalar potential at a scale \( \kappa \),
\[
V_\kappa(\phi) = H^4 \sum_{n=0}^{\infty} \frac{c_{2n,\kappa}}{(2n)!} \left( \frac{\phi^2}{H^2} \right)^n, \tag{57}
\]

and with the initial conditions \( c_{0,\kappa_0} = 0, c_{2,\kappa_0} = -c_{4,\kappa_0} = -0.0001, \kappa_0/H = 1 \) and we take \( c_{2n,\kappa_0} (n \geq 3) \) to be the coefficients of \( \phi^{2n} \) in the SQED effective potential (56). While the lower order terms \( (n = 0, 1, 2) \) represent the classical Higgs-like potential, the higher order local interactions \( (n \geq 3) \) model accurately the scalar-photon interactions in the infrared, \( k/a < H \). The running of the parameters of the effective potential (57) are shown in figures 2, 3, 4, 5 and 6.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Running of \( c_{0,\kappa} \), the constant contribution to \( V_\kappa \) in Eq. (57–58).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Running of the quadratic coupling constant \( c_{2,\kappa} \) (mass term) defined in (57–58). \( c_{2,\kappa} \) flips sign at \( \kappa = \kappa_{ct} \simeq 0.184H \), where the symmetry restoration occurs.}
\end{figure}
The fact that $c_{2,\kappa}$ becomes positive for small $\kappa$ means that the symmetry gets dynamically restored due to IR modes, this phenomena was first discussed by Ratra in [33] and later in [1–3, 26, 32]. This can be seen in the flow of the minimum of the potential, $\bar{\phi}$, shown in figure 7. The running of the scalar field mass $m_{\kappa}^2 = V''(\phi)|_{\phi=\bar{\phi}}$ (shown in figure 8) shows how the mass grows after the symmetry restoration point at which the mass is zero. It is interesting to see the running of the photon mass $m_{\gamma}^2 = \langle e^2 \phi^2 \rangle_S$ (see figure 9), calculated using the stochastic formalism late time expectation value [52]. There we observe a non-vanishing photon mass, as shown in [49], and a decrease of the mass in the IR. Note that the photon and scalar mass violate the pertubative relation, $m_{\gamma}^2 m_{\phi}^2 = 3 e^2/(4 \pi^2)$. This relation becomes particularly bad near the critical point (at which the scalar mass vanishes). Finally we can observe the symmetry restoration of the effective potential in figure 10 where we plot it for different values of the cutoff $\kappa$. 

FIG. 4: Running of the quartic coupling constant $c_{4,\kappa}$ (scalar self-interaction) defined in [57, 58].

FIG. 5: Running of the coupling constants $c_{6,\kappa}$, $c_{8,\kappa}$, $c_{10,\kappa}$ and $c_{12,\kappa}$ defined in [57, 58].

FIG. 6: Running of the coupling constants $c_{14,\kappa}$, $c_{16,\kappa}$ and $c_{18,\kappa}$ defined in [57, 58].

FIG. 7: Running of the order parameter $\bar{\phi}$ in units of $H$. At $\kappa \simeq 0.184 H$, $\bar{\phi} = 0$ and the symmetry gets restored and stays restored for all $\kappa < 0.184 H$.

FIG. 8: Running of the scalar mass, $m_{\kappa}^2$ in units of $H^2$. 

The fact that $c_{2,\kappa}$ becomes positive for small $\kappa$ means that the symmetry gets dynamically restored due to IR modes, this phenomena was first discussed by Ratra in [33] and later in [1–3, 26, 32]. This can be seen in the flow of the minimum of the potential, $\bar{\phi}$, shown in figure 7. The running of the scalar field mass $m_{\kappa}^2 = V''(\phi)|_{\phi=\bar{\phi}}$ (shown in figure 8) shows how the mass grows after
FIG. 9: Running of the photon mass, $m^2_{\gamma, \kappa}$, in units of $H^2$. 

FIG. 10: Effective potential (57) for $\kappa = \kappa_0 = H$, $\kappa = 0.2H$, $\kappa = 0.1H$, and $\kappa = 0$. The potential becomes flat at $\kappa \simeq 0.184H$, the point of dynamical symmetry restoration. For $\kappa < 0.184H$ the potential is positively curved (concave) at the origin, implying that scalar particles become massive in the deep infrared.

It is instructive to compare the results of this section with the results for real scalar field with a Higgs-like potential and without photons interactions. This model is analyzed in appendix A (see also [1, 2]). There we see that the symmetry restoration point occurs at a lower energy scale than in the case when photons are included and therefore the mass for the scalar field is smaller. This means that photons enhance the symmetry restoration for the scalar field. We compare also our result for the mass with the results obtained using stochastic formalism [3] and find a very good agreement between two methods.

B. Scalar QED stress-energy tensor

The stress-energy tensor $T_{\mu \nu} = -(2/\sqrt{-g})\delta S_{\text{SQED}}/\delta g^{\mu \nu}$ for the SQED action (43) is,

\[
T_{\mu \nu} = \left( \delta_\mu^\alpha \delta_\nu^\beta - \frac{1}{2} g_{\mu \nu} g^{\alpha \beta} \right) \left( \partial_\alpha \phi \partial_\beta \phi + e^2 A_\alpha A_\beta \phi^2 \right) + \left( \delta_\mu^\alpha \delta_\nu^\gamma g^{\alpha \beta} - \frac{1}{4} g_{\mu \nu} g^{\alpha \beta} g^{\gamma \delta} \right) F_{\alpha \gamma} F_{\beta \delta} + \delta \xi \left( R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} + g_{\mu \nu} \nabla_\rho \nabla_\sigma - \nabla_\rho \nabla_\sigma \right) \phi^2 - \frac{\delta \lambda}{4} \phi^4 g_{\mu \nu}. \tag{59}\]

Approximating gravitational contributions with their background value, $R_{\mu \nu} = dH^2 g_{\mu \nu}, R = d(d + 1)H^2$ and evaluating the photon and scalar contributions by making use of the stochastic framework developed in [49] one obtains,

\[
\langle T_{\mu \nu} \rangle = -g_{\mu \nu} \frac{3H^4}{8\pi^2} \left\{ \left( -1 + \gamma \right) z + \left( -2 + \gamma \right) z^2 \right. + \frac{1}{2} \left( z + z^2 \right) \left[ \Psi \left( \frac{3}{2} + \nu \right) + \Psi \left( \frac{3}{2} - \nu \right) \right] \right\}, \tag{60}\]

where

\[
\nu(z) = \sqrt{\frac{1}{4} - 2z} \tag{61}\]

To obtain the result (60) we have performed dimensional regularization and made use of the same counterterms (51) and (52) as for the SQED effective action (43). Upon expanding (60) around $z = 0$ we obtain,

\[
\langle T_{\mu \nu} \rangle = -g_{\mu \nu} \frac{3H^4}{8\pi^2} \left\{ \left( -1 + \gamma \right) z + \left( -2 + \gamma \right) z^2 \right. - \frac{1}{2} \left( z + z^2 \right) \left[ 2\gamma - 1 + 2 \sum_{n=1}^{\infty} \zeta(2n + 1)x^{2n} - \sum_{n=1}^{\infty} x^n \right] \right\}, \tag{62}\]
where $x(z) = \frac{1}{2} - \sqrt{\frac{1}{4} - 2z}$. Up to order $\phi^{18}$ Eq. (62) reads,

$$\langle T_{\mu\nu} \rangle = -g_{\mu\nu}V_{\text{em}}(\phi),$$

and the relation between $V_{\text{eff}}(z)$ and $V_{\text{em}}(z)$ is

$$V_{\text{eff}}(z) = \frac{3H^4}{8\pi^2} f(z),$$

$$V_{\text{em}}(z) = \frac{3H^4}{8\pi^2} g(z),$$

with

$$g(z) = \frac{1}{2} \left[ z f'(z) - z - z^2 \right].$$

We take the same Ansatz for the effective potential $V_{\text{eff}}(\phi)$ but with the coupling constants of (63) as new initial conditions plus the initial conditions of the Higgs-like potential. This potential is plotted for different values of the cutoff $\kappa$ in figure [11]. There we can observe a change in the minimum of the potential, although the value of the minimum lies in the region where the small field approximation starts to break down. We want to point out that the shape and minimum of the potential (66) can change significantly for a different choice of counterterms.

**IV. DISCUSSION**

We study the running under the nonperturbative renormalization group flow of scalar quantum electrodynamics (SQED) on de Sitter space which at tree level exhibits spontaneous symmetry breaking due to a Higgs-like potential. This theory contains a charged scalar field canonically coupled to a massless vector field. We have performed our study by firstly integrating the photon, and then studied the renormalization group flow in the resulting effective scalar field theory. Our results show that SQED on de Sitter exhibits dynamical symmetry restoration analogous to that of a pure scalar theory endowed with a symmetry breaking potential and studied in Refs. [26, 33], [1–3]. A detailed comparison with the work of Serreau and Guilleux [11] reveals that SQED exhibits a stronger symmetry restoration and a larger scalar field mass than the pure scalar theory. Therefore, even though photons are conformal on de Sitter such that very few photons are generated, their quantum fluctuations facilitate symmetry restoration in the scalar sector. In Appendix [A] we compare the results obtained by RG methods with those obtained with the stochastic formalism in a pure scalar theory [3] and obtain an excellent agreement for the scalar mass.

Even though the stochastic formalism and the renormalization group approach are at a first sight very different techniques, they yield identical results, provided stochastic theory is interpreted in the way advocated in Ref. [3]. A closer look at the procedure advocated in [3] reveals that, approximating the effective action by the effective potential and studying the flow of the resulting truncated theory mathematically corresponds to the identical procedure as advocated in [3] in the context of the stochastic theory of inflation. Therefore, we can conclude that the effective potential obtained in [3] is identical to effective potential $V_{\kappa \to 0}$ obtained by solving the renormalization group flow equation (31). The advantage of the flow analysis is in that it allows for information on $V_{\kappa}$ at a finite value of $\kappa$ and in addition it allows for a systematic study of the running of higher derivative operators, the lowest order one being $\sim Z_\mu(\phi)\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}$. The advantage of the stochastic approach is that analytical results can be obtained for $V_{\text{eff}}$, both in the small field regime (mass term) and in the large field regime (asymptotic expansion), for details see [3].

Up to now we have not discussed whether one could assign a physical meaning to the effective potential $V_{\kappa}(\phi)$...
at a finite value of \( \kappa \). Note that the nonperturbative effective potential exhibits similar features to the perturbative effective (Hubble) potential studied in Ref. [26]. Namely, the Hubble potential exhibits de Sitter breaking induced both by the scale dependent counterterms and by the infrared effects (as time passes more and more infrared modes get integrated out). Therefore, if one interprets \( \kappa \) and more infrared modes get integrated out. Analogously, as \( \kappa \) gets smaller and smaller, more and more infrared modes get integrated out. Therefore, if one interprets \( \kappa \) as the physical momentum cutoff scale below which the modes are mostly unpopulated, then one can make the following replacement, \( \kappa \to k_0/a \), with \( k_0 \) being the infrared comoving cutoff scale (defined as the physical momentum at the time when \( a_0 = a(\eta_0) = 1 \)) which is naturally expected to be of the order or smaller than the Hubble scale, \( k_0 \lesssim H \). With this one gets,

\[
V_\kappa(\phi) \to V_{k_0/a(\eta)}(\phi). \tag{68}
\]

With this in mind one can reinterpret figure [10] as a sequence of snapshots at times \( t(\kappa) = (1/H) \ln(k_0/\kappa) \approx (1/H) \ln(H/\kappa) \) of the effective potential. This suggests that the symmetry restoration occurs quite quickly, within a few e-foldings of inflation (for the chosen set of parameters). This is in accordance with the perturbative study of Ref. [26]. The principal advantage of the current work is that the effective potential presented here is truly nonperturbative, and holds true both at early as well as at late times. Indeed, while the Hubble effective potential of Ref. [26] never settles to a (de Sitter invariant) state, the nonperturbative effective potential \( V_\kappa \) reaches quite quickly a de Sitter invariant state (which is formally reached in the limit when \( \kappa \to 0 \)).

This way of interpreting \( V_\kappa \) can have important consequences of our understanding of cosmological perturbations in theories when quantum corrections to the tree potential are large, especially when one keeps in mind that the small field expansion is the relevant regime of the effective potential during inflation. Namely, it is usually assumed that the only dependence on time in the potential is indirect via the time dependence of the inflaton field, \( \dot{\phi} = \phi(t) \). This then gives for e.g. the principal slow roll parameter, \( \dot{\epsilon} = -\dot{H}/H^2 \approx (M_\text{pl}^2/2)(V'(\phi)/V)^2 \), where the notation \( \dot{H} \) stands now for time derivative i.e. \( \dot{H}/dt \). But, replacing in this formula \( V \) by \( V_{\kappa=H/a} \) can lead to a significant change for \( \dot{\epsilon} \) and in a particularly large change in \( \dot{\epsilon} \)'s dependence on time, which is reflected in the second slow roll parameter \( \eta = -\dot{\epsilon}/(\dot{\epsilon}H) \). A detailed investigation of this important question is postponed to future work.

As a final remark we note that one could study scalar field theories with extended symmetries, important examples of which are the standard model Higgs sector (in which the symmetry group is \( SU(2) \)) and various grand unified model Higgs sectors. One can accomplish this by using the same renormalization group techniques (see [1][2]).

### Appendix A: Scalar field theory

In this appendix we calculate the flow of a spontaneously broken scalar field theory without photons interactions, which is known to exhibit dynamical symmetry restoration driven by the enhanced infrared field fluctuations [24, 53]. In order to study symmetry restoration we take the following Ansatz for the effective potential,

\[
V_\kappa(\phi) = H^4 \sum_{n=0}^{\infty} \frac{c_{2n,\kappa}}{(2n)!} \left( \frac{\phi^2}{H^2} \right)^n, \tag{A1}
\]

where the couplings are defined as,

\[
c_{2n,\kappa} = \frac{V_{\kappa}^{(2n)}(0)}{H^{4-2n}}. \tag{A2}
\]

![FIG. 12: Running of the constant \( c_{0,\kappa} \) defined in (A1–A2).](image)

![FIG. 13: Running of the quadratic coupling constant \( c_{2,\kappa} \) (mass term) defined in (A1–A2). When compared with figure 3, one sees that the symmetry restoration occurs earlier (for a larger value of \( \kappa \)) in SQED.](image)
Taking as initial conditions, \( c_{0,\kappa_0} = 0, c_{2,\kappa_0} = -c_{4,\kappa_0} = -0.0001 \), \( \kappa_0 = H \) and \( c_{2n,\kappa_0} = 0 (n = 3, 4, \ldots) \), the effective potential is plotted for several values of \( \kappa \) in figure 16, where we included terms up to order \( \phi/18 \). We plot as well the running of \( c_{0,\kappa} \) (figure 12), \( c_{2,\kappa} \) (figure 13), \( c_{4,\kappa} \) (figure 14) and the mass \( m^2_\kappa \) (figure 15). In this case we see that the symmetry gets restored at a lower energy scale \( \kappa \) compared with the same potential with photons interactions included, see figures 7 and 8. This is the same result as Guilleux and Serreau obtained in [1, 2], although they take different initial conditions, thus obtaining different values for the mass and the symmetry restoration point.

In order to compare the RG approach with the stochastic formalism we can look at \( e.g. \) the scalar mass. The mass obtained in the limit \( \kappa = 0 \) is \( m^2 = 1.11805 \times 10^{-3} H^2 \) to be compared with the late-time mass using the stochastic formalism [3], \( m^2 = 1.11868 \times 10^{-3} H^2 \). Therefore, there is an excellent agreement between the masses using both methods.
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