Cardy-Verlinde Formula and Asymptotically de Sitter Spaces

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Abstract

In this paper we discuss the question of whether the entropy of cosmological horizon in some asymptotically de Sitter spaces can be described by the Cardy-Verlinde formula, which is supposed to be an entropy formula of conformal field theory in any dimension. For the Schwarzschild-de Sitter solution, although the gravitational mass is always negative (in the sense of the prescription in hep-th/0110108 to calculate the conserved charges of asymptotically de Sitter spaces), we find that indeed the entropy of cosmological horizon can be given by using naively the Cardy-Verlinde formula. The entropy of pure de Sitter spaces can also be expressed by the Cardy-Verlinde formula. For the topological de Sitter solutions, which have a cosmological horizon and a naked singularity, the Cardy-Verlinde formula also works well. Our result is in favour of the dS/CFT correspondence.

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1 Introduction

In a recent paper [1], Verlinde argued that the Cardy formula [2], describing the entropy of a certain conformal field theory (CFT) in 1 + 1 dimensions, can be generalized to any dimension, leading to the so-called Cardy-Verlinde formula. Consider a certain CFT residing in an \((n + 1)\)-dimensional spacetime with the metric

\[
ds^2 = -dt^2 + R^2 d\Omega_n^2,
\]

where \(d\Omega_n^2\) denotes the line element of a unit \(n\)-dimensional sphere and \(R\) is the radius of the sphere. It was proposed that the entropy of the CFT in the spacetime (1.1) can be related to its total energy \(E\) and Casimir energy \(E_c\) as

\[
S = \frac{2\pi R}{\sqrt{ab}} \sqrt{E_c(2E - E_c)}.
\]

(1.2)

Here \(a\) and \(b\) are two positive parameters. For strongly coupled CFTs with the AdS duals, which implies that the CFTs are in the regime of supergravity duals, \(ab\) is fixed to be \(n^2\) exactly. Thus one obtains the Cardy-Verlinde formula (1.3)

\[
S = \frac{2\pi R}{n} \sqrt{E_c(2E - E_c)}.
\]

(1.3)

Indeed, this formula holds for various kinds of asymptotically AdS spacetimes whose boundary metric is of the form (1.1): AdS Schwarzschild black holes [1]; charged AdS black holes [3] and Taub-Bolt AdS spacetimes [4], whose thermodynamics corresponds to that of different CFTs [5]. The Cardy-Verlinde formula (1.3) holds even for the AdS Kerr black holes [6], whose boundary is a rotating Einstein universe [7].

More recently, it has been proposed that defined in a manner analogous to the AdS/CFT correspondence, quantum gravity in a de Sitter (dS) space is dual to a certain Euclidean CFT living on a spacelike boundary of the dS space [21] (see also earlier works [22]-[25]). Following the proposal, some investigations on the dS space have been carried out recently [23]-[42].

It is well known that in de Sitter space, there is a cosmological horizon, which has the similar thermodynamic properties like the black hole horizon [43]. According to the dS/CFT correspondence, it might be expected that as the case of AdS black holes [6], the thermodynamics of cosmological horizon in asymptotically dS spaces can be identified with

\footnote{For more references discussing the Cardy-Verlinde formula see [7]-[20].}
that of a certain Euclidean CFT residing on a spacelike boundary of the asymptotically dS spaces. Thus it is of great interest to see whether the entropy of the cosmological horizon can be described by the Cardy-Verlinde formula (1.3). This is the purpose of the present paper.

To this end, we have to first face a difficulty to calculate some conserved charges including the mass (total energy) of gravitational field of the asymptotically dS spaces. In the spirit of the dS/CFT correspondence, these conserved charges of gravitational field can be identified with those of corresponding Euclidean CFT. The difficulty arises due to the absence of the spatial infinity and the globally timelike Killing vector in an asymptotically dS space. In this paper we will follow the prescription recently proposed in [37] to calculate the mass of gravitational field of asymptotically dS spaces (and then the energy of corresponding CFTs). It is found that indeed the entropy of cosmological horizon in some asymptotically dS spaces can be described in terms of the Cardy-Verlinde formula.

The organization of the paper is as follows. In the next section we first discuss the case of Schwarzschild-dS solutions. In Sec. 3 we will consider the case of topological dS solutions presented in [40]. We conclude with a discussion in Sec. 4.

2 Schwarzschild-dS Solutions

Consider an $n + 2$-dimensional Schwarzschild-dS spacetime

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega_n^2,$$  \hspace{1cm} (2.1)

where

$$f(r) = 1 - \frac{\omega_n m}{r^{n-1}} - \frac{r^2}{l^2}, \quad \omega_n = \frac{16\pi G}{nVol(S^n)}.$$  \hspace{1cm} (2.2)

Here $m$ is an integration constant, $Vol(S^n)$ denotes the volume of a unit $n$-sphere $d\Omega_n^2$, and $G$ is the gravitational constant. The Schwarzschild-dS spacetime (2.1) is a solution of Einstein equations with a cosmological constant $\Lambda = n(n+1)/2l^2$ in $n + 2$ dimensions.

2see [41] for another attempt in this direction. The dynamics of a brane in a Schwarzschild-dS spacetime has been discussed in [42].

3In three dimensions, it has been already shown that the entropy of Schwarzschild (Kerr)-de Sitter solution can be expressed by Cardy formula, for example see [37] and references therein. So in this paper we consider the case of spacetime dimension $n + 2 \geq 4$. 

3
When the Schwarzschild parameter \(m\) vanishes, the solution (2.1) reduces to the dS solution, which has a cosmological horizon at \(r = l\). The horizon has the associated Hawking temperature \(T_{\text{HK}} = 1/2\pi l\) and entropy \(S = l^n \text{Vol}(S^n)/4G\) \(^1\). When \(m\) increases with \(m > 0\), a black hole horizon occurs and increases in size with \(m\), while the cosmological horizon shrinks. Finally the black hole horizon and cosmological horizon coincide with each other when

\[
m_N = \frac{2}{\omega_n(n + 1)} \left(\frac{n - 1}{n + 1}l^2\right)^{(n-1)/2}.
\]

This is the Nariai black hole, the maximal black hole in dS space.

When \(m \neq 0\), the cosmological horizon \(r_c\) of the solution (2.1) is determined by the maximal root of the equation \(f(r) = 0\). The associated Hawking temperature and entropy to the cosmological horizon are

\[
T_{\text{HK}} = \frac{1}{4\pi r_c} \left(\frac{n + 1}{l^2} \left(\frac{r_c^2}{l^2} - (n - 1)\right)\right),
\]

\[
S = \frac{r_c^n \text{Vol}(S^n)}{4G}.
\]

In Ref. \([37]\), using surface counterterm method the authors calculated the mass of gravitational field of the spacetime (2.1) in four and five dimensions \([37]\). It was found that

\[
M_4 = -m, \quad M_5 = \frac{3\pi l^2}{32G} - m.
\]

Thus a pure dS\(_4\) has a vanishing mass, while the Nariai black hole has the mass \(M_4 = -l/3\sqrt{3}G\) in four dimensions. The pure dS\(_5\) has the mass \(3\pi l^2/32G\) and the Nariai black hole in five dimensions has a vanishing mass. This is a surprising result: The mass of black hole solution is always less than that of pure dS space in corresponding dimensions. However, it was argued that this result is consistent with the Bousso’s observation on the asymptotically dS space and the dS/CFT correspondence \([37]\). The Bousso’s observation is that the entropy of dS space is an upper bound for the entropy of any asymptotically dS spaces \([44]\). Furthermore, if the dS/CFT correspondence exists, the mass of asymptotically dS spaces should be translated into the energy of a dual Euclidean CFT. Generically it is expected that such a field theory has entropy increasing with energy. The result (2.5) has precisely this property \([37]\).

The nonvanishing mass of pure dS\(_5\) space is reminiscent of the nonvanishing mass of pure AdS\(_5\) in the global coordinates \([45]\). The latter is shown to be just the Casimir energy

\(^4\)For the case in higher dimensions, more counterterms than those given in \([37]\) are needed.
of $\mathcal{N} = 4$ SYM theory on a 3-sphere. Such a Casimir energy is expected to exist for pure AdS space in odd dimensions \cite{13}. In the closest analogy of the case in AdS spaces, the nonvanishing mass of pure dS spaces is also expected to exist in odd dimensions in some coordinates (cf. \cite{10}). These nonvanishing masses can be viewed as the Casimir energies of CFTs dual to the pure dS spaces.

Note that in checking the Cardy-Verlinde formula for the asymptotically AdS spaces, the Casimir energy of pure AdS spaces did not be included \cite{1, 3, 4, 5}. In a manner analogous to this, we will not also consider the Casimir energy corresponding to the pure dS spaces in the present context. As a result, we obtain the gravitational mass of the Schwarzschild-dS solution (2.1) from (2.5)

\[ E = -m = \frac{r_c^{n-1}}{\omega_n} \left( \frac{r_c^2}{l^2} - 1 \right), \]

(2.6)

where $r_c$ denotes the cosmological horizon. According to the dS/CFT correspondence, the gravitational mass (2.6) is just the energy of the dual CFT. Thus, in this case the energy of CFT is negative, a unpleasant result\footnote{If one wants to include the Casimir energy of the pure AdS space, this part must be subtracted from the total energy in the Cardy-Verlinde formula, in a way as we did in the case of charged AdS black holes \cite{3}.}. Despite of the non-positive definiteness of the energy, we first follow naively the steps as the case of AdS black holes to check whether the entropy (2.4) of the cosmological horizon can be expressed or not by the Cardy-Verlinde formula (1.3).

Following Ref. \cite{1}, the Casimir energy $E_c$ can be calculated using the formula

\[ E_c = n(E + pV - T_{HK}S), \]

(2.7)

where $p$ is the pressure and $V$ is the volume. Since we are considering a CFT, so we have $p = E/nV$. Corresponding to the Schwarzschild-dS solution (2.1), the dual Euclidean CFT resides on the space

\[ ds^2 = dt^2 + l^2 d\Omega_n^2. \]

(2.8)

Note that here $t$ denotes a spacelike coordinate, rather than a timelike coordinate. Substituting (2.4) and (2.6) into (2.7), we obtain

\[ E_c = -\frac{2nr_c^{n-1}Vol(S^n)}{16\pi G}. \]

(2.9)

\footnote{For other higher dimensions than 4 and 5, we expect that the following result holds also.}

\footnote{We have more discussions on this below.}
This Casimir energy is negative, which is reminiscent of the case of the hyperbolic AdS black holes, there the Casimir energy is also negative \cite{3}. The negative Casimir energy indicates that the dual CFT is not unitary. This agrees with Ref. \cite{21}, in which it was argued that the dual Euclidean CFT to the dS space is not unitary.

Given the Casimir energy (2.9), it is easy to check that the entropy $S$ (2.4) can be expressed as follows

$$S = \frac{2\pi l}{n} \sqrt{|E_c|(2E - E_c)}.$$  \hspace{1cm} (2.10)

This form is completely the same as the Cardy-Verlinde formula in the case of hyperbolic AdS black holes \cite{3}. Note that this formula (2.10) also holds for pure dS spaces. In that case, $E = 0$ and

$$E_c = -\frac{2n l^{n-1} Vol(S^n)}{16\pi G},$$  \hspace{1cm} (2.11)

the expression (2.10) then gives the entropy of pure dS spaces. The result (2.10) looks fine. But there are two points to be understood. The first is that in this formula the energy of corresponding CFT is negative\cite{8}. The other is that the formula (2.10) gives us the entropy of cosmological horizon only, and does not apply to the black hole horizon. As we stated above, when $m \leq m_N$, except for the cosmological horizon, a black hole horizon $r_+$ occurs, which has the Hawking temperature $\tilde{T}_{HK}$ and entropy $\tilde{S}$

$$\tilde{T}_{HK} = \frac{1}{4\pi r_+} \left( (n - 1) - (n + 1) \frac{r_+^2}{l^2} \right),$$

$$\tilde{S} = \frac{r_+^{n} Vol(S^n)}{4G}.$$  \hspace{1cm} (2.12)

For the Schwarzschild-dS solution, the total entropy should be the sum of black hole horizon entropy $\tilde{S}$ and cosmological horizon entropy $S$. But we cannot obtain a similar formula like (2.10) for the entropy $\tilde{S}$ of black hole horizon. Therefore further investigation is needed for the Schwarzschild-dS spacetime. In order to avoid the two points, in the next section we consider the topological dS solutions presented in \cite{40}.

### 3 Topological dS Solutions

In Ref. \cite{37}, an interesting conjecture was put forward, which states that any asymptotically de Sitter space whose mass exceeds that of de Sitter contains a cosmological

\footnote{If replacing $E = -m$ by $E = m$ in (2.4), one then cannot arrive at the Cardy-Verlinde formula (2.11).}
singularity. To check this conjecture, in Ref. [40] Myung, Zhang and the present author present a solution named topological de Sitter solution. The solution can be described by the metric

\[ ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\gamma_{ij}dx^idx^j, \]

where

\[ f(r) = k + \frac{\omega_n m}{r^{n-1}} - \frac{r^2}{l^2}, \quad \omega_n = \frac{16\pi G}{n Vol(\sigma)}, \]

\[ m \] is an integration constant which is supposed to be positive, and \( \gamma_{ij}dx^idx^j \) denotes the line element of an \( n \)-dimensional hypersurface \( \sigma \) with constant curvature \( n(n-1)k \) and volume \( Vol(\sigma) \). Without loss of generality, the constant \( k \) can be set to 1, 0 or \(-1\).

The solution (3.1) is asymptotically de Sitter. But because of \( m \geq 0 \), black hole horizon is not present in this solution, instead a naked singularity at \( r = 0 \) occurs when \( m \neq 0 \). Although so, the solution has a cosmological horizon \( r_c \), which is the root of the equation, \( f(r) = 0 \). And the associated Hawking temperature \( T_{\text{HK}} \) and entropy \( S \) to the cosmological horizon are

\[ T_{\text{HK}} = \frac{1}{4\pi r_c} \left( (n + 1)\frac{r_c^2}{l^2} - (n - 1)k \right), \]

\[ S = \frac{r^n Vol(\sigma)}{4G}. \] (3.3)

When \( k = 1 \), the solution (3.1) has the same form as that of the Schwarzschild-dS solution if \( m \) is replaced by \(-m \) in (2.1). Due to this, following the prescription in Ref. [37], it is easy to show that the gravitational masses of the solutions (3.1) in four and five dimensions are

\[ M_4 = m, \quad M_5 = \frac{3\pi l^2}{32G} + m, \] (3.4)

respectively. When \( k = 0 \), the gravitational mass was found to be [40]

\[ M = m, \] (3.5)

while when \( k = -1 \), the gravitational masses in four and five dimensions are [40]

\[ M_4 = m, \quad M_5 = \frac{3l^2 Vol(\sigma)}{64\pi G} + m. \] (3.6)

For these cases, the gravitational masses are always larger than those of de Sitter spaces \( (m = 0) \) in corresponding dimensions. As a result, we verify the conjecture in Ref. [37] within these examples. For more details see [40].
Corresponding to the solution (3.1), the dual Euclidean CFT resides on the space

\[ ds^2 = dt^2 + l^2 \gamma_{ij} dx^i dx^j. \] (3.7)

Once again, here the coordinate \( t \) is a spacelike one. As the case of asymptotically AdS spaces, we neglect the nonvanishing mass of pure dS spaces, as those in (3.4) and (3.6), or subtract the nonvanishing energy from the total energy of corresponding CFTs, the energy of the Euclidean CFTs dual to the solution (3.1) is then

\[ E = m = \frac{r_c^{n-1}}{\omega_n} \left( \frac{r_c^2}{l^2} - k \right). \] (3.8)

Note that here the energy of CFTs is always positive, different from the case (2.6) of the Schwarzschild-dS solution. Substituting (3.3) and (3.8) into (2.7), we get the Casimir energy

\[ E_c = -\frac{2nk^2r_c^{n-1} Vol(\sigma)}{16\pi G}. \] (3.9)

When \( k = 0 \), the Casimir energy vanishes, as the case of asymptotically AdS spaces. This is expected since the thermodynamic quantities of CFTs in a Ricci flat space are conformal invariant, there is no finite volume effect. When \( k = \pm 1 \), we see from (3.9) that the sign of the energy is just contrast to the case of asymptotically AdS space \([3]\), there it was found that for a hyperbolic space with \( k = -1 \), the Casimir energy is negative \([3]\), while it is positive for the \( k = 1 \) case \([1]\).

With the Casimir energy (3.9), one can easily see that the entropy (3.3) of the cosmological horizon can be expressed in a form of the Cardy-Verlinde formula as

\[ S = \frac{2\pi l}{n} \sqrt{|E_c|/(2E - E_c)}, \] (3.10)

when \( k = \pm 1 \). To accommodate the case of \( k = 0 \), we can rewrite (3.10) as

\[ S = \frac{2\pi l}{n} \sqrt{|E_c/k|/(2E - E_c)}. \] (3.11)

As a result, we show that indeed the entropy of the cosmological horizon in the topological dS solution (3.1) can be described by the Cardy-Verlinde formula (1.3). This also further provides evidence in favour of the dS/CFT correspondence.
4 Conclusions

In the dS/CFT correspondence, we have investigated the question of whether the entropy of cosmological horizon in asymptotically dS spaces can be described by the Cardy-Verlinde formula, which was established in the AdS/CFT correspondence \cite{1}. For the Schwarzschild-dS solution, although the gravitational mass, calculated in the prescription of Ref. \cite{37}, of the solution (and then the energy of the dual Euclidean CFT) is always negative, we have found that the entropy of the cosmological horizon (and then the entropy of the dual CFT) indeed can be expressed in terms of a form [see (2.10)] of the Cardy-Verlinde formula. However, we cannot find a similar formula for the entropy of black hole horizon in the Schwarzschild-de Sitter spacetime. Therefore further study is needed for the Schwarzschild-dS solution. When the Schwarzschild parameter $m$ vanishes, our result (2.10) precisely reproduces the entropy of pure dS spaces.

In the topological dS solutions \cite{40}, which have a cosmological horizon and a naked singularity at $r = 0$. The gravitational mass is always positive. The entropy associated to the cosmological horizon was found to obey the Cardy-Verlinde formula [see (3.11)]. This result is in favour of the dS/CFT correspondence. One point which should be further investigated is whether the Euclidean CFT dual to the topological dS solution can describe the naked singularity in this asymptotically dS spacetime. In other words, whether is there a well-defined Euclidean CFT dual to an asymmetrically dS space which contains a naked singularity?

It would be of interest to extend the discussions made in this paper to the case of asymptotically dS spaces with rotation and/or charges.

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