Modeling and Verification of Corrosion Expansion Crack in Reinforced Concrete

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Abstract. Rebar corrosion not only reduces the section of rebar, but also the volume expansion produced by corrosion will cause the protective layer to crack in advance, and produces corrosion expansion crack. In this paper, the corrosion expansion process of concrete was analyzed by the equivalent perimeter hypothesis, the width where corrosion products flow into crack in the corrosion process was considered, the relationship between existing joint displacement and corrosion rate was corrected, moreover, the relationship of corrosion expansion crack and corrosion time and corrosion rate was established, compared with the existing data and the rationality of the model is proved.

Keywords: Rebar corrosion; concrete crack; equivalent perimeter hypothesis; crack width.

1. Introduction

The problems of concrete structure failure and insufficient durability caused by rebar corrosion have become one of the hot issues in today's construction industry. Rebar corrosion not only weakens the section of rebar, but also causes the protective layer to crack prematurely, and cracks appear on the surface of the concrete protective layer [1], thus causing more serious durability problems. The corrosion crack time and corrosion expansion cracks are important indicators for structural strengthening and restoration. The scholars have done a lot of research work in these two aspects in the process of studying rebar corrosion expansion. Ray [2] et al. used a thick-walled cylinder model, considered the softening characteristics of concrete after crack, the volume expansion of rebar-corrosion products-concrete interface and deformed coordinative relationship, and established theoretical models for calculating crack time of rebar. Similarly, Li [3] et al. used a thick-walled cylinder model, introduced the tangential stiffness reduction factor to consider the impact of crack on the mechanical properties of concrete, it was assumed that the cracked concrete is an anisotropic linear elastic material, and the displacement field of concrete in crack area was obtained, the whole corrosion expansion process was deduced, and the calculation model of corresponding crack width is proposed.

Based on the existing theoretical basis of predecessors, this paper adopts thick-walled cylinder model, considers that partial corrosion products flow into corrosion expansion cracks during the actual corrosion process, proposes a new calculation formula of internal ring joint displacement, uses the equivalent perimeter hypothesis theory to analyze the whole corrosion expansion process, the corrosion
expansion crack model is proposed, the theoretical value is basically consistent with the existing experimental values, which proves the effectiveness of the model.

2. Corrosion Expansion Effect

According to the previous analysis on concrete corrosion expansion, most of them are simplified it to plane stress problems, the thick-walled cylinder model is used for analysis, the same analysis model is also adopted in this paper, as shown in Figure 1. \( R_b \) represents the radius of rebar, \( R_c = R_b + C \).

\[ \text{Fig.1 Calculation model} \]

In the previous research, the expansive force generated by corrosion was simplified into displacement of inner ring node in concrete for calculation; moreover, the functional relation between node displacement and corrosion time or corrosion rate was established, however, many functional relationships did not consider that corrosion during corrosion expansion, the corrosion products flowed into the corrosion expansion cracks in the process of corrosion expansion, in actuality, not all corrosion products produce corrosion expansion effect on the protective layer, and some rust products will flow into the corrosion expansion crack, therefore, this paper proposed a new functional relation between nodal displacement and corrosion rate based on considering the flow of corrosion products into corrosion expansion crack.

According to Fara law, the thickness of rebar corrosion layer at any time is obtained [4]:

\[ R(t) = 0.0115i_{\text{corr}}T \]

In the formula: \( i_{\text{corr}} \)—represents the current density of corrosion (mA/cm²);
\( T \)—represents the conduction time (year).

Rebar volume lost by corrosion:

\[ V_{\text{loss}} = \pi R(t)(D - R(t)) \]

In the formula: \( D \)—represents the diameter of rebar (mm)

After the rebar is rusted, its volume will expand, and the expansion volume after corrosion is:

\[ V_{\text{rust}} = (\xi - 1)\pi R(t)(D - R(t)) \]

In the formula: \( n \)—the expansion rate of the corrosion products, it is related to the type of corrosion products.

According to volume coordination, when the protective layer is partially cracked:

\[ V_{\text{rust}} = \pi Du\left( R_c \right) + \frac{w_{R_b}(R_y - R_b)}{2} \]

When the protective layer is completely cracked:

\[ V_{\text{rust}} = \pi Du\left( R_c \right) + \frac{(w_{R_b} + w_{R_c})C}{2} \]
Because $R(t) \ll D$, according to the above formula, the displacement of inner ring node at any time can be obtained:

$$u(R_b) = 0.0115(\xi - 1)i_{cor}t - \frac{w_{R_b}(R_1 - R_b)}{2\pi D}$$

$$u(R_b) = 0.0115(\xi - 1)i_{cor}t - \frac{(w_{R_b} + w_{R_b})C}{2\pi D}$$

3. Corrosion Process

3.1. Constitutive relation of concrete

The thick-walled cylinder model was adopted, and the radial direction was mainly subjected to compression and the circumferential direction was pulled, during the whole derivation process, it was considered that the concrete radial direction would not be damaged, and the circumferential direction increases with the tangential stress, and cracks gradually generated, therefore, it is very important to select suitable tensile constitutive of concrete for analysis of corrosion expansion process, the constitutive of the concrete tensile softening stage adopted double-fold line model in this paper, as shown in Figure.2. When the strength was less than the tensile strength of concrete, the concrete was considered to be in the elastic phase, when the concrete is cracked, the relationship between stress and crack width is used to describe the constitutive relation of the concrete.

![Fig.2 Constitutive structure of the concrete in the softening stage](#)

Tensile rise section, stress and strain relationship:

$$\sigma = E_c \varepsilon$$

In the formula: $E_c$—is the modulus of elasticity of concrete

Tensile descent section, the relationship between stress and crack width:

The first section of the broken line:

$$\sigma = f_t \left( - \frac{1 - \beta}{\alpha} \frac{w_t}{w_0} + 1 \right)$$

The second section of the broken line:

$$\sigma = f_t \left( \frac{\beta}{1 - \alpha} \frac{w_t}{w_0} + \frac{\beta}{1 - \alpha} \right)$$

In the formula: $f_t$—tensile strength of concrete;

$\alpha$—correlation coefficient;

$\beta$—correlation coefficient;

$W_t$—the crack width of the concrete;

$W_0$—the corresponding maximum cracks width of concrete tension;
3.2. Corrosion expansion process

3.2.1. Elastic phase. In the initial stage of corrosion, when the tangential stress in the inner ring node produced by corrosion expansion is less than the tensile strength of concrete, it can be considered that the concrete is still an isotropic linear elastic body, and the stress field can be solved according to the elastic theory, at this time, the concrete should meet the following boundary conditions:

$$
\sigma_{\theta}|_{R_b} = \frac{E_c}{1-v_c^2} \left( c_1(1+v_c) + \frac{c_2(1-v_c)}{R_b^2} \right) = f_t
$$

$$
\sigma_r|_{R_b} = \frac{E_c}{1-v_c^2} \left( c_1(1+v_c) - \frac{c_2(1-v_c)}{R_b^2} \right) = 0
$$

According to the above two formulas:

$$
c_1 = \frac{R_b^2 f_t (1-v_c)}{E_c (R_b^2 + R_c^2)}
$$

$$
c_2 = \frac{R_b^2 R_c^2 f_t (1+v_c)}{E_c (R_b^2 + R_c^2)}
$$

3.2.2. Partial crack. As the corrosion continues, cracks appear on the protective layer, assuming that the crack extends to $R_y$ at the same time, the concrete is divided into two parts at this time: the cracked area of $R_b < r < R_y$ and the uncracked area of $R_y < r < R_c$, as shown in Figure.3.

![Concrete is partially cracked](image)

The concrete in the uncracked area is considered to be still in the elastic stage and the tangential stress at the front end of the crack is equal to the tensile strength of the concrete, and the concrete in the uncracked area should meet the following boundary conditions:

$$
\sigma_{\theta}|_{R_y} = \frac{E_c}{1-v_c^2} \left( c_1(1+v_c) + \frac{c_2(1-v_c)}{R_y^2} \right) = f_t
$$

$$
\sigma_r|_{R_y} = \frac{E_c}{1-v_c^2} \left( c_1(1+v_c) - \frac{c_2(1-v_c)}{R_y^2} \right) = 0
$$

It can be obtained through the above two formulas:

$$
c_1 = \frac{R_y^2 f_t (1-v_c)}{E_c (R_y^2 + R_c^2)}
$$
The compressive stress corresponding to $R_y$ can be obtained:

$$c_2 = \frac{R^2 R_y^2 f_i (1 + \nu_c)}{E_c (R^2 + R_y^2)}$$

The compressive stress corresponding to $R_y$ can be obtained:

$$\sigma_{r| R_y} = \frac{f_i (R_y^2 - R_c^2)}{R_y^2 + R_c^2}$$

The radial displacement of the corresponding $R_y$ is:

$$u_{| R_y} = \frac{f_i R_y (R_c^2 - R_y^2) \nu_c + R_y^2 + R_c^2}{E_c (R^2 + R_y^2)}$$

For concrete in cracked area, the amount of circumferential elongation is considered to be equal to the crack width and the crack width of the concrete between the rings [5], therefore:

$$\Delta = 2\pi \varepsilon_c + n w$$

In the formula: $n$—represents the number of cracks on the protective layer, generally take 3 or 4.

At the same time, it is assumed that the tangential elongation of the concrete in the ring is equal to the elongation of the front end of the crack, and the elastic strain of the concrete between the rings is equal to the strain $\varepsilon_{cr}$ corresponding to the peak strength of the concrete.

$$2\pi R_c \varepsilon_{cr} = 2\pi \varepsilon_c + n w$$

Through the above formula, the corresponding crack width at any $r$ can be obtained, and the tangential stress of $r$ can be obtained by the tensile constitutive structure of the concrete, through the static force balance of the cracked concrete, the radial stress expression at $r$ can be obtained as follows:

$$\sigma_{r| R_y} = \sigma_{r| R_c} R_y + \int_{r}^{R_c} \sigma_{\theta} dr$$

At the same time, the relative deformation between $r$ and $R_y$ can be obtained:

$$\Delta s = \int_{R_y}^{R_c} \frac{\sigma_{r| R_y}}{E_c} dr$$

Then the corresponding radial displacement at $r$ is:

$$d s_{| R_y} = \int_{R_y}^{R_c} \frac{\sigma_{r| R_y}}{E_c} dr + u_{| R_y}$$

$R_c$ is plugged into the above formula to obtain the corresponding corrosion expansion stress and internal ring node displacement in the corrosion expansion process.

The displacement calculated by the above formula is simply solved by mechanics, and the corrosion product flowing into the crack is not considered, when the crack extends to $R_y$, the crack width at the corresponding $R_b$ is:

$$w_{| R_b} = 2\pi R_y \varepsilon_{cr} - 2\pi \varepsilon_r$$

According to the calculation formula of the previous paragraph, considering the inflow of corrosion products into the crack, the corresponding node displacement can be obtained at this time.
3.2.3. **Complete crack.** When \( r = R_c \), the concrete is completely cracked, as shown in Figure 4.

![Concrete is completely cracked](image)

It is assumed that the amount of circumferential elongation at any \( r \) on the protective layer is equal to the amount of circumferential elongation at \( R_c \), and the amount of circumferential elongation at \( R_c \) is equal to:

\[
\Delta = 2\pi R_c \varepsilon_c + nw
\]

The radial displacement at the corresponding \( R_c \) is:

\[
\left. u \right|_{R_c} = \frac{\Delta}{2\pi}
\]

Similarly, the geometrical relationship of the circumferential elongation can be used to obtain the expression of the tangential stress on the whole protective layer, the static stress can be used to obtain the radial stress at \( r \):

\[
\sigma_r |_r = \int_r^{R_c} \sigma_\theta dr
\]

The relative deformation of the inner ring node relative to the outer ring node at this time is:

\[
\left. ds \right|_{R_c} = \int_{R_c}^{r} \sigma_r |_r \frac{ds}{E_c} dr
\]

Thus, the displacement of corresponding inner ring node is:

\[
ds = \int_{R_c}^{r} \sigma_r |_r \frac{ds}{E_c} dr + \left. u \right|_{R_c}
\]

According to the relationship between node displacement and corrosion rate established in the previous chapter, the relationship between crack width and node displacement is obtained by the above analysis, so the relationship between corrosion rate and corrosion expansion crack is established.

### 4. Model Verification

Andrade [6] conducted a rapid corrosion test on ordinary concrete, the size of specimen was 150mm×150mm×380mm, the diameter of rebar was 16mm, the thickness of the protective layer was 30mm, the corrosion current density was 100mA/cm², the compressive strength of the concrete was 30Mpa, and the \( z \) was 3.83. The result comparison between the crack width calculation method proposed in this paper and the experimental data is shown in Figure 3, the theoretical calculation value is basically consistent with the experimental value, and it shows the rationality of this model (Figure 5).
5. Conclusion
The corrosion expansion process of concrete is studied by using the equivalent perimeter hypothesis in this paper, considering the corrosion products flowed into the corrosion expansion crack, the relationship between new node displacement and corrosion rate and relationship between width of corrosion crack and corrosion rate are established, and the existing data was contrasted, and the correctness of the model was verified.

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