Ratio of baryon and electric-charge cumulants at second order with acceptance corrections

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Abstract

We evaluate the ratio of baryon and electric-charge cumulants at second order from the recent experimental results at $\sqrt{s_{NN}} = 200$ GeV by the STAR Collaboration. The baryon number cumulant is reconstructed from the proton number distribution, and effects of the finite acceptance on the transverse momentum are corrected assuming the independent particle emission. We show that the obtained ratio has a dependence on the rapidity window. Comparison of the result with the hadron resonance gas model and lattice QCD numerical simulations suggests that if the fluctuations are generated from a thermal medium its temperature is significantly lower than the chemical freezeout temperature.

Keywords: fluctuations of conserved charges, cumulants, event-by-event analysis

1. Introduction

In relativistic heavy-ion collisions (HIC), fluctuations are believed to be useful observables for investigating phase transitions in the medium created by the collisions [1–3]. Fluctuations are characterized by cumulants [1], which are known to show anomalous behaviors in a thermal medium near the boundaries of QCD phase transitions especially as the order becomes higher [4–10]. In the HIC, fluctuations are measurable by the event-by-event analysis. Various cumulants have been analyzed up to sixth order [11–18], and a suggestive non-monotonic behavior as functions of $\sqrt{s_{NN}}$ has been reported [14, 15]. The experimental analyses will be refined further by the new data from the RHIC-BES-II [19] and HIC in future facilities [20].

Among fluctuation observables, those of conserved charges have particularly useful properties. First, the cumulants of conserved charges are calculable unambiguously in thermal field theory, especially in lattice QCD numerical simulations [21–29]. While the cumulants in a thermal system are extensive variables and proportional to the spatial volume, the volume dependence can be eliminated in their ratios [7, 30]. The comparison between theoretical and experimental results in terms of the ratios has been made in the literature [31–35]. Second, since the evolution of conserved charges is achieved only by the diffusion it is typically slow especially when the spatial volume is taken to be large [5, 6, 36]. In earlier studies [5, 6], it has been suggested to use this property of conserved-charge fluctuations for investigating thermodynamics in the earlier stage of the HIC. See Refs. [37, 38] for an extension of this idea for higher order cumulants. Although fluctuations in the HIC are sometimes regarded to be generated from a thermal medium around chemical freezeout, this assumption has to be checked carefully because of this property [36].

In the present Letter, among the conserved-charge cumulants we focus on the ratio of the second-order cumulants of net-baryon number and net-electric charge, $\langle N_B^2 \rangle_c$ and $\langle N_Q^2 \rangle_c$. An advantage to focus on this quantity is that this would be the ratio between conserved-charge cumulants higher than the first order that can be analyzed most reliably in the HIC. In the measurement of cumulants in the HIC, effects of the imperfect performance of detectors, such as inefficiencies and finite acceptance, have to be corrected [39–42]. The statistical and systematic uncertainties arising from this procedure, however, grow rapidly as the order becomes higher. Moreover, when comparing experimental results with theoretical ones assuming thermodynamics, one has to consider modifications of fluctuations arising from experimental environment in the HIC, such as the volume fluctuations [17, 43–46], global charge conservation [47–49], collision pileups [50–52], dynamical evolution of fluctuations [5, 6, 37, 38], and etc. The modification of the cumulants due to these effects is more amplified in a non-trivial way as the order becomes higher and makes the meaningful comparison more difficult. The use of the second-order cumulants enables a stable comparison by suppressing these effects.

Another advantage to focus on this ratio is that, as we will see later, in a thermal medium the ratio $\langle N_B^2 \rangle_c/\langle N_Q^2 \rangle_c$ is a monotonically increasing function of temperature ($T$) and behaves almost linearly as a function of $T$ around the pseudo critical temperature $T_c \approx 155$ MeV. This linear $T$ dependence is suitable for studying the nature of fluctuations in the HIC.

In the present study, we construct the values of $\langle N_B^2 \rangle_c$ and $\langle N_Q^2 \rangle_c$ and their ratio from the recent experimental data in Au+Au central collisions at $\sqrt{s_{NN}} = 200$ GeV by the STAR Collaboration [11, 15]. In addition to the reconstruction of baryon number cumulants [39, 53], we perform the correction...
of the finite acceptance in the transverse momentum, $p_T$, space assuming the independent particle emission. We show that the acceptance correction has a large effect on the individual cumulants and the ratio, suggesting the importance of the correction and the necessity to measure fluctuations with wider acceptance.

We compare the value of $(N^2_{B}\lambda_c}/(N^2_Q\lambda_c)$ obtained in this way with the ratio obtained in the hadron resonance gas (HRG) model and the lattice QCD simulations. Provided that the experimentally-observed fluctuations are emitted from a thermal medium, the comparison shows the temperature $T \approx 134 - 138$ MeV, which is significantly lower than the chemical freezeout temperature $T_{\text{chem}}$. Our result also shows that the ratio $(N^2_{B}\lambda_c)/(N^2_Q\lambda_c)$ has a dependence on the rapidity window $\Delta y$ whereas it is independent of $\Delta y$ for thermal fluctuations. These results suggest the violation of the assumption and motivate further investigations on the nature of fluctuations in the HIC.

2. Experimental data and their correction

To obtain the cumulant ratio $(N^2_{B}\lambda_c)/(N^2_Q\lambda_c)$ in the HIC, we use experimental results by the STAR Collaboration in Refs. [11, 15]. We construct $(N^2_{B}\lambda_c)$ from the data on the proton number cumulants in Ref. [15] according to the procedure in Refs. [39, 53], while we use the data in Ref. [11] for $(N^2_Q\lambda_c)$. Effects of the detector’s efficiencies are corrected in these results. Throughout this work we concentrate on the result for the most central (0 - 5%) Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Effects of the violation of the boost invariance and the global charge conservation are most suppressed and the method in Refs. [39, 53] is well justified at this $\sqrt{s_{NN}}$. Since the chemical potentials of conserved charges are small at this $\sqrt{s_{NN}}$, we neglect their effects [25, 26, 28] in the following. Since the effects of the QCD critical point [4] would be suppressed at this $\sqrt{s_{NN}}$, the analysis is suitable for studying non-critical behavior of fluctuations.

In Ref. [15] and Ref. [11], particles are observed in rapidity and pseudo-rapidity spaces, respectively. To compare experimental results with theoretical ones, the use of space-time rapidity is most desirable [1]. Rapidity and pseudo-rapidity are used for its proxy, while the former is better in the Bjorken picture [54]. This is the reason why we use the data on the proton number cumulant in Ref. [15]; for protons, rapidity and pseudo-rapidity has a significant difference due to a heavy proton mass. On the other hand, the difference is smaller in $(N^2_Q\lambda_c)$ since electric charges are dominantly carried by pions whose mass is comparable with the mean $p_T$. In the following, we thus regard the analysis in Ref. [11] as that in the rapidity space.

The measurements in Refs. [11, 15] are performed within a finite $p_T$-acceptance; $0.4 < p_T < 2.0$ GeV in Ref. [15] and $0.4 < p_T < 1.6$ GeV in Ref. [11]. Due to the acceptance the particles in the final state are observed only with imperfect probabilities

$$R_{p_T} = \frac{\text{particle number in } p_T \text{-acceptance}}{\text{total particle number}},$$

Using the Blast wave model with the parameters for $\sqrt{s_{NN}} = 200$ GeV in Ref. [55], the values of $R_{p_T}$ for individual particles are obtained as shown in Table 1 for the $p_T$-acceptances in Refs. [11, 15]. “π+K+p” in the Table shows the weighted probability for the charged particles with the particle abundances taken from Ref. [55].

Since the measurement in the finite $p_T$-acceptance modifies the particle-number distributions, its effect on the cumulants has to be corrected before comparing them with theoretical studies without such an acceptance cut. In the present study, we perform this correction assuming that the individual particles are emitted toward different $p_T$ with independent probabilities according to a given $p_T$ distribution. In this case, the correction can be carried out with the same procedure as the efficiency correction assuming the binomial distribution [1, 39] with the probabilities $R_{p_T}$ in Table 1.

We note that another way to compensate the effect of the $p_T$-acceptance in the comparison between experimental and theoretical analyses is to perform the correction in theoretical calculations, if possible. Such comparisons have been made in the studies employing the HRG model [27, 31, 33, 56, 57]. However, in general it is not possible to perform such a correction in theoretical calculations. The correction of the experimental data enables direct comparisons even for such cases in terms of the true values of cumulants.

3. Cumulant ratio

In Fig. 1, we show the second-order cumulants $(N^2_{B}\lambda_c), (N^2_Q\lambda_c)$, as well as that of the proton number $(N^2_{B+c}\lambda_c)$, divided by the rapidity window $\Delta y$ as functions of $\Delta y$. These quantities are constant if they are generated from a thermal system having a boost invariance [1]. In the left panel, the triangles show $(N^2_{B}\lambda_c)/\Delta y$ in Ref. [15]. The dashed line near the data shows the total particle number $(N_{p}^{\text{total}})/\Delta y$. The squares in the same panel show $(N^2_{B+c}\lambda_c)/\Delta y$ obtained with the procedure in Refs. [39, 53], while the circles show $(N^2_{B}\lambda_c)/\Delta y$ for which the

| particle species $(p_T$ range) | $R_{p_T}$ |
|-------------------------------|-----------------|
| pions ($0.4 < p_T < 1.6$ GeV) | 0.44            |
| kaons ($0.4 < p_T < 1.6$ GeV) | 0.71            |
| protons ($0.4 < p_T < 1.6$ GeV)| 0.71            |
| $\pi+K+p$ ($0.4 < p_T < 1.6$ GeV)| 0.49           |
| protons ($0.4 < p_T < 2.0$ GeV)| 0.82            |

For $(N^2_{B}\lambda_c)$, we carry out this procedure with the weighted probability, i.e. “π+K+p” in the Table. One may think that the correction procedure for multi-particle species [41] with individual probabilities for $\pi$, $K$, and $p$ should be used for this correction. However, in Ref. [11] the electric charge $N_Q$ is measured without particle identification. One thus cannot employ the method in Ref. [41] without a detailed knowledge on the detector’s response. In any case, as discussed in Ref. [41] the systematic deviation due to the use of the weighted $R_{p_T}$ is small at the second order and thus our results would be less affected, while the effect is amplified for higher order cumulants.
On the other hand, from the left panel of Fig. 2 one sees that the statistical errors, which are negligibly small in these results. The dashed lines near these results are the total baryon number \( \langle N_{B}^{\text{total}} \rangle / \Delta y = 2 \langle N_{p}^{\text{total}} \rangle / \Delta y \). One sees that the deviation of \( \langle N_{B}^{2} \rangle_{c} \) from \( \langle N_{B}^{2} \rangle \) at large \( \Delta y \) is pronounced by the corrections. This result is reasonable since the incomplete measurement tends to make the distribution close to the Skellam distribution in which \( \langle N_{B}^{2} \rangle_{c} = \langle N_{B}^{2} \rangle \) [1].

Shown in the right panel of Fig. 1 are \( \langle N_{Q}^{2} \rangle_{c} / \Delta y \) with (circles) and without (squares) the \( p_{T} \)-acceptance correction\(^2\). The meaning of the dashed lines is the same as the left panel. The panel shows that the effect of the \( p_{T} \)-acceptance correction is more significant than \( \langle N_{Q}^{2} \rangle_{c} \) because of the smaller \( R_{p_{T}} \) for the electric charge.

In the left panel of Fig. 2, we show the \( p_{T} \)-acceptance-corrected result of \( \langle N_{B}^{2} \rangle_{c} / \langle N_{Q}^{2} \rangle_{c} \) by the circles. The dashed lines show \( \langle N_{B}^{\text{total}} \rangle / \langle N_{Q}^{\text{total}} \rangle \). The shaded band represents the systematic errors that account for the propagation from that of \( \langle N_{Q}^{2} \rangle_{c} \) in Refs. [15]; we, however, note that this error band should be regarded only as a guide since the estimate of the systematic errors of \( \langle N_{Q}^{2} \rangle_{c} / \langle N_{Q}^{2} \rangle_{s} \) needs a detailed knowledge on the experimental analyses. In the panel, the result without the \( p_{T} \)-acceptance correction is also shown by the squares as a reference. One sees that the correction strongly modifies the ratio.

If fluctuations are emitted from a thermal system having a boost invariance, the ratio \( \langle N_{B}^{2} \rangle_{c} / \langle N_{Q}^{2} \rangle_{c} \) is a constant as a function of \( \Delta y \) [1]. While \( \langle N_{Q}^{2} \rangle_{c} / \Delta y \) and \( \langle N_{Q}^{2} \rangle_{c} / \Delta y \) become decreasing functions when the effects of the global charge conservation are taken into account [47, 48], these \( \Delta y \) dependence cancels out in the ratio \( \langle N_{B}^{2} \rangle_{c} / \langle N_{Q}^{2} \rangle_{c} \) for the thermal case [48]. On the other hand, from the left panel of Fig. 2 one sees that the acceptance-corrected ratio has a clear increasing trend as a function of \( \Delta y \). This result shows that the fluctuations in the HIC are not emitted from a purely thermal system including the effect of global conservation.

4. Comparison with HRG model and lattice results

Assuming that the fluctuations observed in the HIC are those of a thermal system, one can estimate the temperature of the system by comparing the ratio of cumulants with the results obtained in lattice QCD simulations. Even when the fluctuations are not thermal, such a comparison is useful for investigating the nature of the fluctuations.

To perform the comparison using the ratio \( \langle N_{B}^{2} \rangle_{c} / \langle N_{Q}^{2} \rangle_{c} \), in the right panel of Fig. 2 we show the \( T \) dependence of \( \langle N_{B}^{2} \rangle_{c} / \langle N_{Q}^{2} \rangle_{c} \) obtained from a lattice QCD simulation [29] by the solid line with an error band. Finite-volume effects of \( \langle N_{Q}^{2} \rangle_{c} \) are corrected according to Ref. [29]. The range of the vertical axis is the same as the left panel. The lattice results on thermodynamics are known to be well reproduced by the HRG model at low \( T \). In the panel, the ratio \( \langle N_{B}^{2} \rangle_{c} / \langle N_{Q}^{2} \rangle_{c} \) in the HRG model is shown by the dashed line, where we use the set of hadrons in “QMHRG2020” [29] for the HRG model. The figure shows that the lattice result agrees well with the HRG model for \( T \lesssim 145 \) MeV, which suggests the validity of the latter in this range of \( T \). From the panel one also finds that \( \langle N_{B}^{2} \rangle_{c} / \langle N_{Q}^{2} \rangle_{c} \) behaves almost linearly as a function of \( T \) in the range of \( T \) shown in the panel. As discussed in Sec. 1, this is an attractive feature of this ratio.

To compare the results in the left and right panels, in Fig. 2 we show the dotted horizontal lines at the values of \( \langle N_{B}^{2} \rangle_{c} / \langle N_{Q}^{2} \rangle_{c} \) obtained from the HIC at \( \Delta y = 0.2 \) and 1.0. By comparing these values with the ratio in the HRG model, one finds that the temperature extracted from the naive comparison gives \( T \approx 134 – 138 \) MeV depending on \( \Delta y \) as shown by the

\(^2\) The total electric charge required for the \( p_{T} \)-acceptance correction is provided by the private communication with Arghya Chatterjee.
The ratio in the HIC at fixed for higher order cumulants are suppressed in its analysis. Uncertainties in the experimental measurement that are amplified only of the second-order conserved-charge cumulants, various discussions on the nature of fluctuations especially taking their dependence of \( \Delta y \) is an important experimental subject for resolving these issues. In addition to theoretical studies on this point, the measurement of fluctuations with wider acceptance is an important experimental subject for reducing uncertainties from the violation of this assumption. As emphasized already, such measurements are more important for the analyses of higher order cumulants. It thus is quite interesting to realize such experiments at the future experiments at FAIR, NICA and J-PARC-HI [20].

5. Discussions

In the present study, we have investigated the cumulant ratio \( \langle N_B^2 \rangle_c / \langle N_Q^2 \rangle_c \) observed in the HIC. Because this ratio consists only of the second-order conserved-charge cumulants, various uncertainties in the experimental measurement that are amplified for higher order cumulants are suppressed in its analysis. The ratio in the HIC at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) is estimated from the experimental results by the STAR Collaboration [11, 15]. In addition to the reconstruction of the baryon number cumulant from those of protons, effects of the \( p_T \)-acceptance are corrected assuming the independent particle emission. Our result shows that this correction strongly modifies the resulting values of the cumulants and their ratio and thus is crucial. Since the effect of the correction becomes more significant for higher order cumulants [41], this result also shows the importance of the correction in their analysis for the search for the QCD critical point.

The naïve comparison of the obtained ratio with the HRG model suggests the temperature \( T \simeq 134 - 138 \text{ MeV} \), which is significantly lower than \( T_{\text{chem}} \). By taking this result seriously, it is suggested that the fluctuation observables in the HIC are generated in the hadronic phase later than the chemical freezeout out in contrast to the earlier suggestions [5, 6].

However, we emphasize that this comparison is made assuming that the fluctuations in the HIC are thermal. On the other hand, existence of the \( \Delta y \) dependence of \( \langle N_B^2 \rangle_c / \langle N_Q^2 \rangle_c \) shows the violation of this assumption in the HIC. Therefore, to understand the experimental result correctly one needs further investigations on the nature of fluctuations especially taking their dynamics into account [5, 6, 37, 38]. The modifications of the cumulants due to the use of (pseudo-)rapidity in place of space-time rapidity [54] and the resonance decays after the chemical freezeout are other important effects to be considered since they tend to make \( \langle N_Q^2 \rangle_c \) larger and suppress \( \langle N_B^2 \rangle_c / \langle N_Q^2 \rangle_c \). Because these effects are suppressed by extending \( \Delta y \) [1], the measurement of the fluctuations with larger \( \Delta y \) is an important experimental subject for resolving these issues.

Finally, we remark that the value of \( \langle N_B^2 \rangle_c / \langle N_Q^2 \rangle_c \) obtained in the present study from Refs. [11, 15] would have a deviation from the true value. As discussed in Sec. 2, while \( \langle N_Q^2 \rangle_c \) is measured in the pseudo-rapidity space in Ref. [11], the measurement in the rapidity space is more desirable. Although we have performed the \( p_T \)-acceptance correction assuming the independent particle emission in the \( p_T \) space, there is no a priori justification of this assumption in the HIC. In addition to theoretical studies on this point, the measurement of fluctuations with wider acceptance is an important experimental subject for reducing uncertainties from the violation of this assumption. As emphasized already, such measurements are more important for the analyses of higher order cumulants. It thus is quite interesting to realize such experiments at the future experiments at FAIR, NICA and J-PARC-HI [20].

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