Transition of Schwarzschild-de Sitter space to de Sitter space through tunneling

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Abstract

We revisit the Parikh-Wilczek tunneling through the de Sitter horizon and obtain a new tunneling rate in Schwarzschild-de Sitter space, which is different from the previous. The present tunneling rate is also non-thermal and closely related to the change of entropy. We discuss the thermodynamics of Schwarzschild-de Sitter space and show existence of correlation and the total entropy being conserved in the tunneling process. We also point out that our tunneling process can lead to the accelerated expanding universe. The correlation and conserved entropy enlighten a way to explain the entropy in empty de Sitter space.

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I. INTRODUCTION

A classical black hole can only absorb and not emit particles. However, in 1974, Hawking discovered [1] that when considering quantum effect, black hole can emit thermal radiation with a temperature \( T = \frac{\kappa}{2\pi} \), where \( \kappa \) is the surface gravity of a black hole. The physical reason of radiation was explained [2] as coming from vacuum fluctuations tunneling through the horizon of the black hole. But some original derivation based on the Bogoliubov transformation [1] or other methods [2, 3, 4] didn’t have the direct connection with the view of tunneling. Moreover, these methods, in which the background geometry is considered fixed, didn’t enforce the energy conservation during the radiation process.

Recently, Parikh and Wilczek suggested [5] a method based on energy conservation by calculating the particle flux in Painlevé coordinates from the tunneling picture. Their result can recover some important points and present some new properties as in the following aspects: (1) It recovered the Hawking’s original result in leading order and gave the consistent temperature expression [5, 6]. (2) It gave a direct relation between tunneling probability and the black hole entropy, which shows the first law of black hole thermodynamics will play an important role in radiation process. Such relation presents an approach to research the relation between black hole radiation dynamics and black hole thermodynamics. (3) The non-thermal spectrum implies there may exist the information-carrying correlation in the radiation. It is shown in Ref. [7] that indeed there exist information-carrying correlation in the radiation spectrum, which can eliminate the entropy growth completely in the tunneling process. So the tunneling process is unitary and no information is lost. The method had been discussed generally in different situations [8, 9, 10, 11, 12, 13, 14] and its self-consistency has been checked even by using the black hole thermodynamic law [15, 16, 17, 18]. Recently, the fermion tunneling through the horizon had also been discussed [19, 20].

In the calculation of tunneling, the choice of coordinates is very important and the general discussion about it had been made in Ref. [6, 21]. It is noted that a simple new coordinate system for de Sitter space was found in Ref. [22] and the tunneling through de Sitter horizon is generalized to high-dimension [23]. The new coordinates had several attractive properties: the time direction is a Killing vector, the metric is smooth at the horizon, and constant-time slice are just flat Euclidean space. It is demonstrated [22, 23] that the coordinates can be used to calculate the tunneling rate like that by Parikh and Wilczek,
and the tunneling rate is non-thermal when self-gravitation is taken into account. But there were some flaws in the pretty jade. The explanation of the energy in Schwarzschild-de Sitter coordinates and the consideration of initial tunneling leads to the theoretic inconsistency in the Parikh-Wilczek tunneling frame. So we revisit the tunneling process in Schwarzschild-de Sitter space and obtain the self-consistent tunneling rate. The tunneling rate obtained in the present paper is also non-thermal and closely related to the change of the entropy of Schwarzschild-de Sitter space that is proportional to the horizon area. We also discuss the thermodynamics in Schwarzschild-de Sitter space and show that the connection of tunneling with thermodynamics. Finally, we prove the entropy conservation in the tunneling due to the correlation in the non-thermal spectrum of Schwarzschild-de Sitter radiation.

The organization of the paper is as follows. We revisit the tunneling process in de Sitter space and hence obtain a new tunneling rate in the second section. The third section is devoted to investigation of Schwarzschild-de Sitter space thermodynamics and entropy conservation in sequential tunneling. Finally, we summarize our results in the fourth section.

II. TUNNELING THROUGH DE SITTER HORIZON REVISIT

To describe across-horizon phenomena, it is necessary to choose coordinates which are not singular at the horizon and the Painlevé-type coordinates are very appropriate for the purpose. It is known that the Painlevé-de Sitter coordinates was obtained in Ref. \[22\] as

$$ds^2 = -(1 - \frac{r^2}{l^2})dt^2 + 2 \frac{r}{l} dt dr + dr^2 + r^2 d\Omega^2 \quad (1)$$

where $l$ is the de Sitter spatial parameter with units of length called as the de Sitter radius, which is the location of the horizon obtained by $g_{tt} = 0$. The coordinate is stationary and not static, but it is interesting that an observer precisely at the origin does not make any distinction between these coordinates and static coordinates.

Now we consider the tunneling across the de Sitter horizon. Because of the infinite blueshift near the horizon, the characteristic wavelength of any wavepacket is always arbitrarily small there, so that the point particle, or WKB approximation is available. The tunneling rate can be expressed as

$$\Gamma \sim \exp (-2 \text{Im} I/\hbar) \quad (2)$$
where \( I \) is the action of the tunneling particles. It is found [5] that the tunneling rate is non-thermal when taking the self-gravity effect into consideration and the radiation temperature is recovered when comparing the tunneling rate with the Boltzmann factor. It is also shown that the tunneling rate is closely related to the change of the entropy [5] and hence related to the first law of black hole thermodynamics [16, 17].

Using the standard null geodesic method, as in Ref. [22], the imaginary part of the action is attained as

\[
\text{Im} I = \pi l E.
\]  

Comparing with the Boltzmann factor, we can find the corresponding radiation temperature is

\[
T_{\text{as}} = \frac{\hbar}{2\pi l},
\]

which is the temperature of de Sitter space. It is noted that the result is consistent with that obtained in Ref. [24], which shows that the null geodesic method can be used to calculate the radiation rate properly and the Painlevé-de Sitter coordinates are convenient for the calculation. However, the result obtained here is thermal, since the above calculation does not include the self-gravitation or back reaction. Then we want to ask how to contain the self-gravitation effect in the calculation in order to attain the non-thermal result. It is very difficult to make such consideration in the coordinate (1), since the coordinate (1) presents the empty de Sitter space. As pointed out in Ref. [25], de Sitter vacuum is stable against the Hawking radiation. But, if we put a Unruh-DeWitt detector with free falling motion in the vacuum, it feels the thermal radiation which is called Unruh effect.

In order to incorporate the self-gravitation effect, the Schwarzschild-de Sitter coordinates are introduced to calculate the tunneling rate in Ref. [22], although its analysis is not self-consistent. Historically, the Schwarzschild de-Sitter coordinates played an important role in the work of Gibbons and Hawking [24] to determine the entropy of de Sitter space, and it is also significant for our discussion here. For simplicity, we consider the three dimensional Schwarzschild de-Sitter coordinates [26, 27],

\[
ds^2 = -\left(1 - 8GE - \frac{r^2}{l^2}\right) dt_s^2 + \frac{dr^2}{1 - 8GE - \frac{r^2}{l^2}} + r^2 d\phi^2.
\]
In three dimensions there is only one horizon, at 
\[ r_H = l \sqrt{1 - 8GE} \], and as \( E \) goes to zero 
this reduces to the usual horizon in empty de Sitter space. Here, the energy \( E \) should be 
comprehended as the total energy of the Schwarzschild de-Sitter space. However, in Ref. 
[22], energy \( E \) is erroneously treated as the energy of the self-gravitating shell, which will be 
discussed later. In fact, the explanation of energy \( E \) as the total energy of the Swarzschild-de 
Sitter space can also be found in Ref. [26, 27, 28].

In general there are two horizons, one of which is called black hole horizon and the other 
of which is called de Sitter horizon, in high dimensional Schwarzschild de-Sitter space and 
the two horizons will approach each other as the mass increased. It is interesting to note 
that in three dimensions there is only one horizon that is the de Sitter horizon. The reason 
why the black hole horizon doesn’t exist is that in three dimensional flat space there are 
no black holes [27]. Since there is only one horizon in three dimensional de Sitter space, 
we can calculate the tunneling rate through the horizon according to the Parikh-Wilczek 
null geodesic method. With the time Painlevé transformation, the corresponding Painlevé 
coordinate is gotten as

\[
ds^2 = - \left(1 - 8GE - \frac{r^2}{l^2}\right) dt^2 + 2 \sqrt{8GE + \frac{r^2}{l^2}} dt dr + dr^2 + r^2 d\phi^2 \tag{6}
\]

Considering the particle with energy \( \omega \) tunneling across the horizon and the self-gravitating 
effect, one can obtain the imaginary part of the action

\[
\text{Im } I = \text{Im } \int_E^{E - \omega} \int_{r_i}^{r_f} \frac{dr dH}{\dot{r}} = - \text{Im } \int_0^\omega \int_{r_i}^{r_f} \frac{dr d\omega}{1 - \sqrt{8G (E - \omega)} + \frac{r^2}{l^2}} \tag{7}
\]

where the radial outgoing geodesic with self-gravitation included is \( \dot{r} = 1 - \sqrt{8G (E - \omega')} + \frac{\dot{r}^2}{l^2} \), which is obtained by setting \( ds^2 = d\phi^2 = 0 \). Here \( r_i = l \sqrt{1 - 8GE} \) 
is the original radius of the horizon before pair-creation, while \( r_f = l \sqrt{1 - 8G (E - \omega)} \) is 
the new radius of the horizon. Now the integral can be done by deforming the contour 
according to the Feynman prescription,

\[
\text{Im } I = \frac{\pi l}{4G} \left( \sqrt{1 - 8G (E - \omega)} - \sqrt{1 - 8GE} \right) \tag{8}
\]

and then we can gain the tunneling rate

\[
\Gamma \sim \exp \left[ - \frac{\pi l}{2G\hbar} \left( \sqrt{1 - 8G (E - \omega)} - \sqrt{1 - 8GE} \right) \right] \tag{9}
\]
The tunneling rate we gained above is different from that in Ref. [22], \( \Gamma \sim \exp\left[\frac{\pi l}{2\sqrt{G}h} \left(\sqrt{1 - 8GE} - 1\right)\right] \). In Ref. [22], its horizon contracts from \( r_i = l \) to \( r_f = l\sqrt{1 - 8GE} \). Therefore, the horizon contraction is along the same direction with the particle tunneling, which will lead to the violation of energy conservation. On the other hand, the energy \( E \) is explained as the energy of the shell in Ref. [22]. So, if one considers the sequential tunneling with energy \( E_1 \) and \( E_2 \), the probability for the second emission has to be expressed as \( \Gamma \sim \exp\left[\frac{\pi l \sqrt{1 - 8GE_1}}{2\sqrt{G}h} \left(\sqrt{1 - 8GE_2} - 1\right)\right] \), not as that obtained in Eq. (27) of Ref. [22]. The reason is that, when the second particle tunneling, the horizon should change from \( l_1 = l\sqrt{1 - 8G(E_1 + E_2)} \) to \( l_2 = l\sqrt{1 - 8GE_1} \sqrt{1 - 8GE_2} \) according to their method [22], but it is erroneously treated from \( l_1 = l\sqrt{1 - 8G(E_1 + E_2)} \) to \( l_2 = l\sqrt{1 - 8G(E_1 + E_2)} \). So there are some flaws even errors in the treatment of Ref. [22], and their results are not self-consistent.

Different from that of Ref. [22], the present tunneling rate in Eq. (9) can avoid the problem and give a self-consistent result. In our method, the particle pair is created near the horizon. The positive mass or frequency one tunnels into the horizon and decreases the energy of Schwarzschild-de Sitter space because of the exotic properties of de Sitter space that positive mass or frequency is measured as negative energy in de Sitter space [28]. Especially, the energy is measured in origin where the observer locates. The other one will move away and soon enters into the new space. Finally they contribute to the expansion of the horizon or the ground state energy of empty de Sitter space. We can see this from a simple coordinate transformation,

\[
\begin{align*}
r' &= r \sqrt{1 - 8G(E_1 + E_2)} \\
t' &= \sqrt{1 - 8GE_1} t_s \\
\phi' &= \sqrt{1 - 8G\phi}
\end{align*}
\]  

(10)

Through the transformation, the Schwarzschild-de Sitter metric (5) exactly changes into a three dimensional de Sitter metric with the new coordinates \( r', t', \phi' \). This shows that in empty de Sitter space, the ground state energy or the vacuum energy is not zero. After transformation, the new space is larger than original one which means the point at \( r \) will move to larger \( r' \) and the excitiated energy \( E \) will change into ground state energy. This point will be explained detailedly in the discussion of thermodynamics and entropy conservation.
in the next section.

III. THERMODYNAMICS AND ENTROPY CONSERVATION

When discussing the relation between thermodynamics and tunneling, there are two methods usually used. One method is to compare the tunneling rate with the thermal Boltzmann factor. The other method is to relate the first law of thermodynamics, i.e.,

\[ \frac{1}{T} = \frac{ds}{dM} \]

is equivalent to \( \frac{\kappa}{2\pi} \) where \( \kappa \) is the surface gravity of black hole. Obviously, the radiation spectrum given by (9) is not precisely thermal. So, in order to compare our non-thermal spectrum with the pure thermal spectrum, we have to expand the tunneling rate in a power series of \( \omega \). When neglecting the quadratic term and higher-order terms, we will find the leading-order term gives the thermal Boltzmann factor.

Considering the tunneling particle’s energy \( \omega \) is small, we expand \( \Gamma \) given by Eq. (9) in power of \( \omega \) as

\[ \Gamma \sim \exp \left( \frac{2\pi l \omega}{\hbar \sqrt{1 - 8G\,E}} + O(\omega) \right) \]

where \( \sqrt{1 - 8G\,(E - \omega)} = \sqrt{1 - 8GE} \sqrt{1 + \frac{8G\omega}{1 - 8GE}} \simeq \sqrt{1 - 8GE} \left( 1 + \frac{4G\omega}{1 - 8GE} \right) \). Compare with the Boltzmann factor, we find the temperature is given as

\[ T_{SdS} = \frac{\hbar \sqrt{1 - 8GE}}{2\pi l} \]

which is consistent with that obtained in Ref. [27]. On the other hand, if we calculated the surface gravity at the horizon, the temperature obtained by \( \frac{\kappa}{2\pi} \) is identical to that in Eq. (12). After the emission of energy \( \omega \), the temperature becomes \( T'_{SdS} = \frac{\hbar \sqrt{1 - 8G(E - \omega)}}{2\pi l} \), which is higher than before. This is different from the conclusion made in Ref. [22] that the temperature after emission will be lower than before. Here the temperature increases due to the tunneling, so it will approach to \( T_{dS} = \frac{\hbar}{2\pi l} \) and the Schwarzschild-de Sitter space will decay to the empty de Sitter space quickly. In this way, we have revisited the tunneling radiation through de Sitter horizon with a detailed calculation and analysis. In what follows, we will give a physical picture of our calculation.

An observer locates at the origin while the tunneling occurs at the Schwarzschild-de Sitter
horizon. When the tunneling is proceeding, the horizon becomes large gradually. It looks as if it were contrasted with the situation for Schwarzschild black hole whose horizon contracts due to particle tunneling. But if we consider it carefully, we will find that they are consistent essentially because the horizon for both situations goes away from the observer. In other words, the tunneling has the opposite direction with the motion of horizon, which is consistent with energy conservation. Finally the horizon becomes $r_H = l$ which is the radius of empty de Sitter space and hence the Schwarzschild-de Sitter space decays into the empty de Sitter space. Then is the de Sitter space the terminal of the tunneling? This problem is difficult to answer because the empty de Sitter space has thermodynamics with the temperature $T_{dS} = \frac{\hbar}{2\pi l}$. However, more recently de Sitter space has entered cosmology as the likely candidate for the final fate of the universe. Even though the de Sitter space is regarded as the spacetime geometry of the early universe in rapid inflation, its change to other stage has to be dependent on the new process like reheating. From this opinion, we can also say the empty de Sitter space is the terminal of the tunneling process. On the other hand, our picture is analogous to the expanding universe. The qualitative analysis is made along the line of the tunneling. Because the temperature is increasing gradually, the tunneling rate will becomes quicker and quicker. In other words, the horizon will become larger and larger and the velocity of the change will be quicker and quicker. This is consistent with accelerated expanding universe. The picture is different from that given in Ref. and has more abundant physical contents.

Noticed that our result is obtained in three dimensional Schwarzschild-de Sitter space. Then can it be applied to higher dimension? The answer is positive! Although there exists the black hole horizon in high dimension besides the de Sitter horizon, their tunneling or Hawking radiation does not affect each other. The particle will tunnel out of the black hole, so the black hole horizon shrinks, while the particle will tunnel into de Sitter horizon, so the de Sitter horizon expands. Since the observer lies in someplace between the two horizons and the two horizons move away from the observer, the energy conservation is held. On the other hand, the temperature of the two horizons is not equal to each other. The temperature at the black hole horizon is larger, so there is heat flow from the black hole horizon to the de Sitter horizon. Moreover, due to the larger temperature, the radiation rate at the black hole horizon will be larger than that at de Sitter horizon and the heat flow will stimulate the process. Thus the black hole will disappear very soon before de Sitter
horizon stops expanding. Finally, the high-dimensional Schwarzschild-de Sitter space will evolve into empty de Sitter space due to tunneling or Hawking radiation.

Using the thermodynamic relation,

$$\frac{1}{T} = \frac{dS}{dM}$$

one can find the entropy is equal to

$$S_{\text{SdS}} = \frac{\pi l}{2G\hbar} \sqrt{1 - 8GE}.$$  \hspace{1cm} (14)

Thus we get the important result presented in Ref. [5],

$$\Gamma \sim \exp (\Delta S)$$  \hspace{1cm} (15)

which shows that the tunneling rate is closely related to the change of the entropy in Schwarzschild de Sitter space.

Now we pay our attention to the non-thermal property of the tunneling rate [9]. In Ref. [7], it is shown that there exist information-carrying correlations in the non-thermal spectrum for the Schwarzschild black hole. Moreover, the correlations have been found in other non-thermal spectrums for Reissner-Nordström black holes, Kerr black holes, Kerr-Neumann black holes, and so on [30]. At the same time, in the tunneling picture, the entropy is also proved to be conserved, which is consistent with the unitarity. Along the line we will check whether there exists the correlation in the Schwarzschild-de Sitter tunneling spectrum.

For two emissions with energies $\omega_1$ and $\omega_2$, we find that

$$\ln \Gamma(\omega_1, \omega_2) - \ln [\Gamma(\omega_1) \Gamma(\omega_2)] \neq 0.$$  \hspace{1cm} (16)

Thus the adoption of a Schwarzschild-de Sitter tunneling does not change the statement that a non-thermal spectrum affirms the existence of correlation, as is illustrated in Ref. [7] for a common Schwarzschild black hole. It is found that the information about Schwarzschild-de Sitter space can be taken out in the correlation in the radiation spectrum. For sequential tunneling of two particles with energies $\omega_1$ and $\omega_2$, we find the entropy form,
\[ S(\omega_1) = -\ln \Gamma(\omega_1) = \frac{\pi l}{2G\hbar} \left( \sqrt{1 - 8G(E - \omega_1)} - \sqrt{1 - 8G E} \right) \] (17)

\[ S(\omega_2|\omega_1) = -\ln \Gamma(\omega_2|\omega_1) = \frac{\pi l}{2G\hbar} \left( \sqrt{1 - 8G(E - \omega_1 - \omega_2)} - \sqrt{1 - 8G(E - \omega_1)} \right) \] (18)

and they also satisfy the definition of conditional entropy \( S(\omega_1, \omega_2) = -\ln \Gamma(\omega_1, \omega_2) = S(\omega_1) + S(\omega_2|\omega_1) \). A detailed calculation confirms that the amount of correlation quantity is exactly equal to the mutual information described by \( S(A : B) = S(A) + S(B) - S(A, B) = S(A) - S(A|B) \) and this shows that the correlation can carry away the information. If we count the total entropy carried away by the outgoing particles, we find

\[ S(\omega_1, \omega_2, ..., \omega_n) = \sum_{i=1}^{n} S(\omega_i|\omega_1, \omega_2, ..., \omega_{i-1}). \] (19)

Thus we shows that the entropy conservation is maintained and so the tunneling process is unitary in the Schwarzschild-de Sitter space. When the tunneling finishes, the correlations enlighten a way to explain the entropy in empty de Sitter space. As discussed above, the energy is conserved, so the energy of larger empty de Sitter space is not zero and is still \( E \) which can be considered as ground state energy or vacuum energy. Thus the correlations among particles still exist in the ground states or vacuum states. Then the ground states must be degenerate and the correlations are hidden among these degenerated states. The entropy is derived from the degeneracy of ground states. This is similar to the explanation of extreme charged black hole entropy [29]. Therefore, we demonstrate physically why Schwarzschild-de Sitter coordinates are used to determine the entropy of empty de Sitter space by Gibbons and Hawking [24].

**IV. DISCUSSION AND CONCLUSION**

It is noted [31] that the de Sitter horizon does not shrink but expand in the process of Hawking radiation as tunneling through the de Sitter cosmological horizon, which is similar to our conclusion in the present paper, while the treatment in [31] is not the same to ours. In Ref. [31], they discussed the tunneling in the empty de Sitter space and pointed out the decay of cosmological constant in the process. In the present paper, we give the decay from Schwarzschild-de Sitter space to empty de Sitter space when the quantum tunneling is considered. In both schemes, the tunneling energy will propagate inward the origin \( (r = 0) \),...
so energy is conserved in cosmic expansion, i.e., no energy is lost in the universe. However, in Ref. [31], they pointed out that due to the expansion of de Sitter horizon, the cosmological constant will decrease and the entropy of de Sitter space will increase, which indeed does not violate the second law of black hole thermodynamics. But entropy increase violates conservation of information. In the present paper, we show that the entropy of de Sitter space does not change in the tunneling process, which neither violate the second law of black hole thermodynamics, nor violate information conservation and hence unitarity. In Ref. [31], the author omitted the correlations among the tunneling particles which has been discovered recently by authors [18]. According to our resolution [18], we can find that the entropy is conserved in the situation discussed in Ref. [31].

In conclusion, we have obtained the tunneling rate in the Schwarszchild-de Sitter space in the framework of energy conservation. It has been found that the tunneling rate is non-thermal and related to the change of the entropy. When we expand the tunneling rate in power of the tunneling particle’s energy $\omega$ and compared it with the Boltzmann factor, we can get the standard radiation temperature. In the tunneling picture, the temperature approaches the temperature of the empty de Sitter space gradually and the Schwarszchild-de Sitter space decays into the empty de Sitter space. In the decay process, the Schwarszchild-de Sitter space will expand acceleratingly and the process will end in de Sitter space, which presents a theoretical evidence to an accelerated expanding universe. We have also shown that there exists the correlation in the non-thermal spectrum and the tunneling process is entropy conserved and hence unitary in the Schwarszchild-de Sitter space. Thus the entropy in empty de Sitter space is explained logically from the correlations and entropy-conserved process, which shows physically the feasibility of the introduction of Schwarszchild-de Sitter coordinates to determine the entropy of empty de Sitter space by Gibbons and Hawking [24].
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