Oceanography of Accreting Neutron Stars: Non-Radial Oscillations and Periodic X-Ray Variability

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Abstract.

Observations of quasi-periodic oscillations (QPOs) in the luminosity from many accreting neutron stars (NS) have led us to investigate a source of periodicity prevalent in other stars: non-radial oscillations. After summarizing the structure of the atmosphere and ocean of an accreting NS, we discuss the various low \( l \) g-modes with frequencies in the 1-100 Hz range. Successful identification of a non-radial mode with an observed frequency would yield new information about the thermal and compositional makeup of the NS, as well as its radius. We close by discussing how rapid rotation changes the g-mode frequencies.

1. Introduction and Summary

The liquid core, solid crust and overlying ocean give a NS a very rich non-radial oscillation spectrum, with p-modes in the 10kHz range and a variety of lower frequency modes from the toroidal and spheroidal displacements of the crust and surface layers (see McDermott et al. 1988 for an overview). Non-radial g-mode oscillations have been studied extensively for isolated radio pulsars, where the atmosphere and ocean are quiescent. These studies can be separated by the sources of buoyancy: (1) entropy gradients (McDermott, Van Horn & Scholl 1983, and McDermott et al. 1988), (2) density discontinuities (Finn 1987, McDermott 1990, Strohmayer 1993) and, (3) mean molecular weight gradients due to \( \beta \) equilibrium in the core (Reisenegger & Goldreich 1992). To date, none of these modes has been securely identified with any particular radio pulsar phenomenon.

There are also many NSs accreting in binary systems, with internal structures quite different from that of a radio pulsar. There is a rich spectrum of QPO’s in the luminosity from these accreting NSs which has motivated much of our work. Utilizing EXOSAT data on the highest accretion rate objects \( (\dot{M} > 10^{-10} M_\odot \text{ yr}^{-1}) \), Hasinger & van der Klis (1989) found that the objects split into two separate classes. Six objects trace out all or part of a “Z” in an X-ray color-color diagram and exhibit time dependent behavior that correlates
with the position along the Z. These “Z” sources have QPO’s in the 15-50 Hz and 5-7 Hz range with up to 10% modulation. The other objects fall into separated regions of the color-color diagram and do not show similar QPO phenomenology at less than 100 Hz. These “Atoll” sources accrete at lower rates than the Z sources and exhibit Type I X-ray bursts resulting from the unstable ignition of the accumulated hydrogen and helium (see Bildsten 1998 for a recent review). The Rossi X-Ray Timing Explorer (RXTE) has detected coherent oscillations during these Type I bursts (see Strohmayer et al. 1996 for an example). RXTE also detected two drifting kHz QPOs in the persistent emission which are separated by a constant frequency that is identical to that seen in the burst. This naturally leads to a beat frequency model, where the difference frequency is presumed to be the NS spin. The origin of the upper frequency differs in the models (see van der Klis 1997 for a summary), and usually involves the Kepler frequency at some point in the accretion flow (Miller et al. 1996, Kaaret et al. 1997, Zhang et al. 1997).

It might be that all of the < 100 Hz and kHz QPOs can be accommodated within the existing models (see van der Klis 1995, 1997) that use the accretion disk and/or spherical flow to generate the periodic phenomena. However, it is also important to pursue non-radial oscillations as a possible source of some of these periodicities (McDermott & Taam 1987, Bildsten & Cutler 1995 (BC95), Bildsten, Ushomirsky & Cutler 1996 (BUC), Strohmayer & Lee 1996 (SL96), Bildsten & Cumming 1997 (BC97)). These offer the distinct advantage of having well understood dispersion relations (even when rapidly rotating) and frequencies (and frequency derivatives) that depend on the underlying NS structure. This makes it a challenge to successfully identify an observed QPO with a non-radial oscillation. However, for the same reason, this hypothesis offers much potential, as the successful identification of a non-radial pulsation with an observed frequency would tell us about the NS radius and internal structure.

2. The Outer Layers of an Accreting Neutron Star

In most cases where stellar pulsations are studied, the underlying stellar model is reasonably well understood and constrained via other observations. The situation is quite different in an accreting NS, where the conditions in the outer layers can change on timescales of hours and the deep internal structure depends on the accretion history. We just roughly sketch the situation here, more details can be found in BC95, SL96, and BC97.

The NSs in mass-transferring binaries accrete the hydrogen (H) and helium (He) rich matter from the surfaces of their companions and undergo unstable H/He burning when \( \dot{M} < 10^{-8} M_\odot \text{ yr}^{-1} \). This typically occurs at \( \rho < 10^8 \text{ g cm}^{-3} \), and within a few hours to days upon arrival on the star. The very high temperatures reached \( (T > 10^9 \text{ K}) \) during the thermal instability produce elements at and beyond the iron group. The isotopic mixture from this burning is still not well known, though everyone agrees that a substantial amount of H (the residual mass fraction is \( X_r \sim 0.1 - 0.5 \)) remains unburned (see Bildsten 1998 for a review). This matter accumulates on the NS and forms a relativistically degenerate ocean. The hydrogen is eventually depleted due to electron captures at \( \rho \approx 10^7 \text{ g cm}^{-3} \), leading to a density discontinuity. The material
crystallizes and forms the NS crust at $\rho \sim 10^8 - 10^9$ g cm$^{-3}$. There is no evidence for magnetic fields on these NSs, and most arguments about the nature of Type I X-ray bursts limit $B < 10^9$ G, weak enough to not affect the ocean g-modes (BC95).

3. The Adiabatic Non-Radial Oscillations in the Deep Ocean

The g-modes cannot penetrate into the crust, as the restoring force from the finite shear modulus effectively excludes them (McDermott et al. 1988, BC95), and so they reside in the relatively thin ocean (thickness is $\approx 10^4$ cm $\ll R$). The low $l$ modes are in the shallow water wave limit and so $\omega^2 \propto k^2$, where $k = (l(l+1))^{1/2}/R$ is the transverse wavenumber for a slowly rotating star. Prior work has been done on g-modes in the upper atmosphere. McDermott & Taam (1987) calculated g-modes of a bursting atmosphere. SL96 calculated the non-adiabatic mode structure for atmospheres accreting and burning in steady-state and found that g-modes may be excited by the $\epsilon$ mechanism when $\dot{M} < 10^{-10} M_\odot$ yr$^{-1}$. However, steady-state burning does not occur at these low $\dot{M}$'s, and there have yet to be realistic calculations for the time dependent NS atmosphere.

We have focused on the adiabatic mode structure in the deep ocean underneath the H/He burning where the thermal time (hours to days) is much longer than the mode period. The different sources of buoyancy yield a rich spectrum of g-modes. The abrupt rise in density associated with the hydrogen electron capture boundary layer supports a density discontinuity mode of frequency (BC97)

$$f_d \approx 35 \text{ Hz} \left( \frac{X_r}{0.1} \right)^{1/2} \left( \frac{10 \text{ km}}{R} \right) \left( \frac{l(l+1)}{2} \right)^{-1/2},$$

and the internal buoyancy due to the composition gradient within the electron capture boundary layer creates a new spectrum of modes which are “trapped” (BC97). Almost all of the mode energy is confined to the boundary layer and a WKB estimate of the mode frequency gives (BC97)

$$f_{tr} \approx \frac{8.5 \text{ Hz}}{n} \left( \frac{X_r}{0.1} \right)^{1/2} \left( \frac{10 \text{ km}}{R} \right) \left( \frac{l(l+1)}{2} \right)^{-1/2},$$

where $n$ is the number of nodes in the boundary layer and we have omitted the weak dependence on the accretion rate. There is also a set of thermal g-modes (BC95), which we have recently found (BC97) are separated by the density discontinuity, and confined to either the upper or lower parts of the ocean. The isothermal ocean is relativistically degenerate, so that the dimensionless density contrast due to the entropy gradient is small and $\propto k_B T/E_F$, allowing BC95 to obtain the exact analytic formula

$$f_{th} = 6.3 \text{ Hz} \left[ \frac{l(l+1)}{2} \frac{T_8}{\Theta_n \mu_i} \right]^{1/2} \left( \frac{10 \text{ km}}{R} \right),$$

where $\Theta_n = 1 + (3n\pi/2 \ln(\rho_b/\rho_l))^2$ corrects for $n$ and the density contrast between the top and bottom of the ocean, $T_8 = T/10^8$ K and $\mu_i$ is the ion mean molecular weight.
4. The Role of Rapid Rotation and the Future

We might suspect that these stars are rapidly rotating, as the prolonged accumulation of material will most likely spin up the star. The coherent periodicities during Type I X-ray bursts (see §1) seem to indicate 500 Hz spin frequencies, which are much greater than the g-mode frequencies, but still small compared to the breakup frequency $\Omega_b \approx (GM/R^3)^{1/2}$ ($\approx 2$ kHz for a $1.4M_\odot$, $R = 10$ km star). When $\Omega \ll \Omega_b$, the unperturbed star is spherical and the centrifugal force can be neglected, in which case the primary difference in the momentum equations is the Coriolis force. As a result, the g-mode frequencies depart significantly from the $\omega^2 \propto l(l + 1)$ scaling (Papaloizou & Pringle 1978).

BUC made progress on this problem within what is called the “traditional approximation”, where the radial and transverse momentum equations separate and the resulting angular ODE must be solved to find the angular eigenfunction (no longer just $Y_{lm}$’s) and transverse eigenvalue $\lambda$ (i.e., we write the transverse wavenumber as $k^2 = \lambda/R^2$) which, for a non-rotating star, is $l(l + 1)$. The radial equations are identical to the non-rotating case, so that if $\omega_0$ is the eigenfrequency for the $l = 1$ mode of a non-rotating star, then the oscillation frequency (in the rotating frame) at arbitrary spin is $\omega = \omega_0(\lambda/2)^{1/2}$. These are then transferred into the observer’s inertial frame via $\omega_I = \omega - m\Omega$. The left panel of Figure 1 shows the observed frequencies for a mode that is at 30 Hz ($l = 1$) in the non-rotating star. It is intriguing to ask how the observed frequencies
change as the internal conditions change (say the $T$ or $X_r$ in equations 1-3). The right panel of Figure 1 shows this for 4U 1728-34 ($f_s = 363$ Hz, Strohmayer et al. 1996). There are frequencies near a kHz, but they do not show the large dynamic range exhibited by the observed kHz QPOs. The splitting for some of the prograde modes is nearly constant, but is not equal to the spin frequency. Also shown (bottom panel on the RHS) are two retrograde modes that appear at low frequencies, clearly showing a large dynamic range for only a small change in the non-rotating mode frequency. This points to the possibility of explaining some of the QPOs seen in the Atoll and Z sources.

Our work might eventually provide natural explanations for some of the observed QPO’s in accreting NS. However, there is still much to do theoretically, from understanding how the modes are excited to how they modulate the luminosity. Observational progress will come by identifying QPOs with modes of different $l$‘s, where the ratio of the mode frequencies are known. This is easiest to do if the NSs are rotating slowly (spin frequency $f_s < 10$ Hz), but can also be accomodated if the NSs are rapidly rotating, especially for those NS where the spin frequency was measured during a Type I burst.

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