Experimental observation of the polarization coherence theorem

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For light fields, the manifestation of correlations between fluctuating electric field components at different space–time points is referred to as coherence, whereas these correlations appearing between orthogonal electric field components at a single space–time point are referred to as polarization. In this context, a natural question is as follows: how are coherence and polarization interconnected? Very recently, a tight equality $P^2 = V^2 + D^2$, namely, the “polarization coherence theorem” (PCT) connecting polarization $P$ with interference visibility $V$ (measure of coherence) and distinguishability $D$ (measure of which-path information) has been proposed [Optica 4, 1113 (2017)]. We here report a direct observation of the PCT for classical light fields using a Mach–Zehnder interferometer along with a synthesized source producing a complete gamut of degrees of polarizations. Our experimental demonstration could motivate ongoing experimental efforts toward probing the hidden coherences and complementarity features.

The generalization of this inequality to tight equality $P^2 = V^2 + D^2$ states that for two conjugate variables, observation of one precludes the observation of the other, subject to the degree of polarization of the field. More recently, the intimate connection between entanglement and wave–particle duality has been demonstrated [16,17]. The experimental demonstration establishes that the weirdness of quantum theory, i.e., entanglement, controls the degree of duality [16].

In this Letter, we report an experimental observation of the equality $P^2 = V^2 + D^2$, thus verifying the PCT for the classical polarization states having a full range of variable degrees of polarizations. We use an experimental scheme involving two balanced Mach–Zehnder interferometers; one for the synthesis of the source having a tunable degree of polarization, and the other for the verification of the PCT. In a Mach–Zehnder interferometer, owing to a wide separation of interfering beams, one can control and probe complementary features such as visibility and distinguishability more conveniently compared to Young’s interferometer. We also demonstrate that the equality can be easily derived using the Stokes description and coherence matrix treatment of the light fields. We also note that several quantum optical phenomena have been recently revisited from a classical optics viewpoint, and it was observed that they exhibit interesting features such as correlations, nonseparability, entanglement, Bell violations, etc., with many of them contributing to bring the boundaries between quantum and classical domains closer to each other [18–21]. Analogies between coherence, entanglement, and polarization are such examples [22–26].

To observe the PCT in a direct manner, we first construct a source of a tunable degree of polarization [27]. A randomly polarized laser He–Ne laser beam (make Newport, model R30988, wavelength 632.8 nm, power 2 mW) passes through the first Mach–Zehnder interferometer, consisting of a polarizing beam splitter, a half-wave plate (HWP1), and a nonpolarizing beam splitter [Fig. 1(a)]. For our He–Ne laser, the longitudinal mode spacing is 566 MHz, which is smaller than the Doppler broadened linewidth of the laser, resulting in several modes lasting simultaneously in the broadened linewidth. Since the adjacent modes are orthogonally polarized, the resultant output beam of the laser is unpolarized. By setting the angle of HWP1, the degree of polarization of the output beam is...
tuned. The output beam having a known degree of polarization further passes through another half-wave plate (HWP2), which controls the orientation of orthogonal polarization components of the beam entering the second Mach–Zehnder interferometer. Thus, for a fully polarized beam, HWP2 can be used to swap the intensities in the second interferometer arms (between the two spatial modes of polarization). A third half-wave plate (HWP3) with its fast axis at an angle 45° with input beam polarization is used to maintain the same polarization state of the interfering beams at the last beam splitter to ensure the maximum visibility of the interference fringes. The interference fringe pattern (image) and intensities of each interfering beam are recorded using a CCD camera [make Olympus, model DP27] to measure the visibility and distinguishability, respectively [Fig. 1(c)]. To avoid saturation of the CCD, the laser beam intensity was reduced using an appropriate neutral density filter after the laser, and the integration time of the CCD was accordingly adjusted. For each measurement, 100 data set images were taken for statistical analysis and error estimation. To reduce the background noise, several background images were taken before capturing the data set images, and the mean background image was subtracted from each of the measured data set images. The degree of polarization of the optical field at the output of the first Mach–Zehnder interferometer is determined using Stokes polarimetry (a scheme consisting of a set of quarter-wave plate and a polarizer), and the light intensity is measured using a homemade photodetector.

Let us consider a light field propagating in z-direction described as \( E(t) = \hat{x}E_x(t) + \hat{y}E_y(t) \) passes through the optical path shown in Fig. 1(a1), where \( E_x(t) \) and \( E_y(t) \) are the orthogonal electric field amplitudes. The resultant field at the output of BS1 is given by \([x, r_x E_x(t) \cos 2\theta + j y, r_y E_y(t) \sin 2\theta + t_x E_x(t), t_y E_y(t)]\), where \( r_x, t_x \) and \( r_y, t_y \) are the amplitude reflection and transmission coefficients of BS1 for the horizontal and vertical polarizations, respectively, and \( \theta \) is the angle of HWP1. Since the orthogonal input field components are uncorrelated, i.e., \( \langle E_x(t)^* E_x(t) \rangle = \langle E_y(t)^* E_y(t) \rangle = 0 \), the polarization (coherence) matrix for the output field after BS1 can be given by (Section 8.1, Eq. (2) of Ref. [3])

\[
J = \begin{bmatrix}
    r_x^2(\langle E_x(t)^2 \rangle \cos^2 2\theta) & 0.5r_xr_y(\langle E_x(t)^2 \rangle \sin 4\theta) \\
    0.5r_xr_y(\langle E_x(t)^2 \rangle \sin 4\theta) & r_y^2(\langle E_y(t)^2 \rangle \sin^2 2\theta + t_y^2(\langle E_y(t)^2 \rangle)
\end{bmatrix}.
\]

One can estimate the polarization-dependent behavior of a beam splitter as follows: using the Jones matrix approach, the ratio of the reflected intensity \( I_1(\theta) \) to the transmitted intensity \( I_2 \) at the output port of the BS1 can be expressed as

\[
\frac{I_1(\theta)}{I_2} = C_1 \cos^2 2\theta + C_2 \sin^2 2\theta,
\]

where coefficients \( C_1 = \langle |E_x(t)|^2 \rangle r_x^2 / \langle |E_y(t)|^2 \rangle r_y^2 \) and \( C_2 = \langle |E_y(t)|^2 \rangle r_y^2 / \langle |E_x(t)|^2 \rangle r_x^2 \) are proportional to the ratio of reflectivity and transmissivity of BS1 for the horizontal and vertical polarizations, respectively. A plot between the intensity ratio and the HWP1 angle for our BS1 is shown in the inset of Fig. 1(b). The experimentally obtained values are fitted with Eq. (2), and the ratio parameters \( C_1 \) and \( C_2 \) are obtained as 1.01236 and 2.00584, respectively. This confirms the strong polarization-dependent behavior of the nonpolarizing beam splitter BS1.

The degree of polarization of the output field for a realistic experimental scheme taking into account the polarization dependence of the beam splitter is thus obtained as [1]

\[
P(\theta) = \sqrt{1 - \frac{4 \det J}{(TrJ)^2}}
\]

\[
= \sqrt{1 - \frac{4C_1 \cos^2 2\theta}{[1 + C_1 \cos^2 2\theta + C_2 \sin^2 2\theta]}}.
\]

The Stokes parameters of the light field can be measured experimentally using the well-known polarimetric relations [27,28] and the corresponding degree of polarization (P) can be obtained as

\[
P = \sqrt{S_1^2 + S_2^2 + S_3^2}/S_0.
\]

The experimentally measured value of the degree of polarization obtained using Eq. (4) for our practical source using the Stokes scheme of Fig. 1(a2) can be varied from 0.05 to 0.99 by changing \( \theta \) from 0 to 45 deg, respectively, as shown in Fig. 1(b).
It was experimentally confirmed by measuring the Stokes parameters [3,28] using a set of a quarter-wave plate and a polarizer as shown in Fig. 1(a2). The experimental data of Fig. 1(b) fits well with Eq. (3) within the experimental uncertainty.

We emphasize here that the second Mach–Zehnder interferometer in our experiment maps the ordinary (spin) polarization to the which-path degree of freedom. The HWP2-PBS combination splits the orthogonal spin polarizations and guides them to different arms of the interferometer, converting spin modes to spatial modes of polarization [horizontal (h) and vertical (v) polarizations in different arms]. Polarization generally being a two-party property, these two orthogonal modes can be referred to as polarization-spatial modes [5,14]. Since our experiment deals also with partially polarized fields, the Mach–Zehnder interferometer allows mapping the definition of “partial coherence” to “partial mode coherence.” The intensity of each of the polarization spatial modes is recorded using the CCD, by blocking the other beam. The distinguishability (D) can then be determined by measuring the contrast between the polarization spatial modes as

$$D = \frac{|I_h - I_v|}{I_h + I_v}. \quad (5)$$

The visibility (V) of the interference fringes is obtained by measuring the contrast between the maximum and minimum intensities as

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}. \quad (6)$$

The fields at the output of the second Mach–Zehnder interferometer (after BS2) can be theoretically calculated using Jones calculus. Using the expressions for D [Eq. (5)] and V [Eq. (3) of Ref. [14]), the final expressions for visibility and distinguishability after BS2 can be expressed as a function of the angle of HWP1 (θ) and HWP2 (φ) (Fig. 1) as

$$D(\theta, \phi) = \left| \frac{\cos 4\phi[C_1 \cos^2 2\theta - C_2 \sin^2 2\theta - 1] + \sqrt{C_1 C_2} \sin 4\phi \sin 4\phi}{C_1 \cos^2 2\theta + C_2 \sin^2 2\theta + 1} \right|,$$

$$V(\theta, \phi) = \left| \frac{\sin 4\phi[C_1 \cos^2 2\theta - C_2 \sin^2 2\theta - 1] - \sqrt{C_1 C_2} \sin 4\phi \cos 4\phi}{C_1 \cos^2 2\theta + C_2 \sin^2 2\theta + 1} \right|. \quad (7)$$

We would like to emphasize that one can explore the ray-wave duality, and its relation with the polarization of the optical field and with the absolute value of the degree of coherence by making an analysis using the Stokes parameters. Let us consider the light field described in the last section. The Stokes parameters associated with the light field can be written in terms of the elements of polarization (coherence) matrix as [3]

$$S_0 = J_{xx} + J_{yy}, \quad S_1 = J_{xx} - J_{yy}, \quad S_2 = J_{xy} + J_{yx}, \quad S_3 = i(J_{yx} - J_{xy}), \quad (9)$$

where $J_{ij} = \langle E_i^* E_j \rangle$ are the coherence matrix elements.

Using Eq. (9), one can express distinguishability for different polarization bases in terms of the Stokes parameters as $D_{xy} = |S_1/S_0|$, $D_{\pm 45} = |S_2/S_0|$, and $D_{\pm 45} = |S_3/S_0|$, where quantities $D_{\pm 45}$ and $D_{\pm CP}$ correspond to distinguishabilities in a diagonal and a circular polarization basis, respectively. The resultant intensity $I$ of the interference pattern is given by $I = J_{xx} + J_{yy} + 2|J_x| \cos(\arg J_{xy})$ [1]. Using the definition of fringe visibility given as Eq. (6) and the Stokes parameters from Eq. (9), visibilities for different polarization bases yield $V_{xy} = \frac{S_2^2 + S_3^2}{S_0^2}$, $V_{\pm 45} = \frac{S_1^2 + S_3^2}{S_0^2}$, and $V_{\pm 45} = \frac{S_2^2 + S_1^2}{S_0^2}$. Using these expressions for D and V in different bases, one can readily demonstrate that for any of the given polarization bases, the square sum of distinguishability and visibility comes out to be the square of degree of polarization, i.e., $D^2 + V^2 = P^2$, which is referred as the PCT [5,14].

From the coherence theory of light fields, the absolute value of the degree of coherence is given by

$$|\gamma_{xy}| = \frac{|J_{xy}|}{\sqrt{J_{xx} J_{yy}}}. \quad (10)$$

Writing elements of the coherence matrix in terms of the Stokes parameters using Eqs. (7–10), one can show that

$$|\gamma_{xy}|^2 = \frac{S_2^2 + S_3^2}{S_0^2} = \frac{V(\theta, \phi)^2}{1 - D(\theta, \phi)^2}. \quad (11)$$

The above expression interprets $|\gamma_{xy}|^2$ in terms of the Stokes parameters as well as using the visibility and distinguishability. Clearly, for indistinguishable beams ($D = 0$), $|\gamma_{xy}|$ is equal to the degree of visibility. Using PCT $P(\theta, \phi)^2 = V(\theta, \phi)^2 + D(\theta, \phi)^2$ one can express $|\gamma_{xy}|^2$ in terms of any of the two
parameters out of $D$, $V$, and $P$. For a field that is unpolarized in the $b - v$ basis ($S_1 = 0$), $|\gamma_{xy}|^2$ increases with the increase in the degree of polarization in the other two orthogonal bases ($S_2$ and $S_3$). For a partially polarized optical beam, $|\gamma_{xy}|^2$ varies between zero and $P^2$, approaches zero when $D^2$ approaches $P^2$, and approaches $P^2$ when $V^2$ approaches $P^2$. On the other hand, for the fully polarized case ($P = 1$), irrespective of the value of the degree of distinguishability and the degree of visibility, the absolute value of the degree of coherence is always unity. Figure 2 shows the 3D plot of Eq. (11) for the rotation of half-wave plates ($\theta$ and $\phi$). The experimental measurements demonstrate an excellent fit with the theory within the uncertainty of the experiment. This plot confirms well the calibration of our scheme and shows that the polarization of the beam can be manipulated in the desired way.

Using our experimental scheme of Fig. 1(a), the values of $V$ and $D$ for the interfering beams of known degrees of polarization are experimentally determined and are shown in Fig. 3. For a nearly fully polarized light ($P \approx 1$), orthogonal field components are fully correlated (nearly). One can see that $D$ and $V$ have an inverse relation, i.e., for $D^2 \approx 1$, any one of the orthogonal field intensities is zero, signifying $V^2 \approx 0$. Similarly, for $D^2 \approx 0$, both of the orthogonal field components have the same intensity, signifying $V^2 \approx 1$. For intermediate values of $D$ and $V$ also, the strict condition of the PCT is validated. For partially polarized light ($0 < P < 1$), orthogonal field components are partially correlated. Thus, variation in $P$ leads a change in both $D$ and $V$ in accordance with the PCT.

The experimental findings confirm this theorem for partially polarized fields also. For a nearly unpolarized light ($P \approx 0$), orthogonal field components are nearly uncorrelated. This informs both $D \approx V \approx 0$, validating the PCT for the unpolarized field within experimental uncertainty. The experimental observations are confirmed by fitting the experimental data using Eqs. (7) and (8), as shown in Fig. 3 and can be seen in an excellent match.

In conclusion, using a simple interferometric scheme, we have experimentally demonstrated the PCT for classical polarization states of light with a wide gamut of degrees of polarizations. The experimental results fit well with the theoretically expected values. We believe that this direct experimental observation would stimulate experimental research toward revisiting the hidden coherences and various complementarity features, and would also open new doors for understanding the intriguing features such as complementarity and uncertainty in both classical and quantum viewpoints.

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