Entropic corrections to Newton’s law

M R Setare¹, D Momeni² and R Myrzakulov²

¹ Department of Science, Payame Noor University, Bijar, Iran
² Eurasian International Center for Theoretical Physics, Eurasian National University, Astana 010008, Kazakhstan

E-mail: rezakord@ipm.ir, d.momeni@yahoo.com, davidmathphys@yahoo.co.uk, rmyrzakulov@gmail.com and rmyrzakulov@csufresno.edu

Received 3 January 2012
Accepted for publication 23 April 2012
Published 17 May 2012
Online at stacks.iop.org/PhysScr/85/065007

Abstract
In this short paper, we calculate separately the generalized uncertainty principle (GUP) and self-gravitational corrections to Newton’s gravitational formula. We show that for a complete description of the GUP and self-gravity effects, both the temperature and entropy must be modified.

PACS numbers: 04.20.Cv, 04.50.−h, 04.70.Dy

1. Introduction
The Planck scale corrections to the new scenario for gravity, proposed by Verlinde [1], must contain two groups of modifications. (i) Modification of the amount of entropy which is disturbed when the test particle approaches the holographic screen. This excess correction has arisen after the full quantum description of the test particle in the Planck scale described by the generalized uncertainty principle (GUP) and not the usual uncertainty principle. This work was done by Ghosh [2], Vancea and Santos [3] and other authors [4]. Note that in this treatment the usual Bekenstein–Hawking famous formula for entropy [5] is preserved. But as was shown in [6–8], in the limit of the GUP both the temperature and the entropy require serious modifications. These kinds of modifications are called in the literature quantum corrections. Some authors have inserted new functions for the entropy versus area, but no change has been made in the form of the temperature. The kinds of models which were used from the various kinds of entropy functions belong to this category. One of these kinds of modifications in the entropy comes directly from the loop quantum gravity [9] for which this orthodox form of modified entropy functions carries some log terms and also an inverse term of the area. In particular, the latter show that this modification to the simple Verlinde idea adjust completely with modified Newtonian dynamics (MOND) [11]. Nevertheless, there are other alternatives for solving the dark matter problem, for example, the cosmological special relativity that belongs to Carmeli [12], beyond the contemporary cosmology. Anyway in both approaches one important notion was missed: modification of the temperature (Hawking or Unruh) in the Planck scale. As was shown by the authors of [6–8], applying the GUP modifies the standard expression for the Hawking temperature.

(ii) Another kind of corrections to the entropy function, whatever be the usual black hole (BH) entropy, or the entropy is the back reaction effect or the self-gravitational corrections [13]. We know that near the horizon of any BH a spectrum of particles is created which is like the usual Planck black body spectrum. This kind of radiation changes the background slightly at a large scale but very significantly near the horizon. As was shown in [14, 15], small statistical fluctuations around the equilibrium change the entropy in some logarithmic terms. In this paper, we present the full GUP-inspired corrections to the Newtonian gravity and also the self-gravitational corrections or thermal radiation corrections separately.

2. The generalized uncertainty principle corrections to the Hawking temperature and black hole entropy
As was shown in [6–8], if one applies GUP when the phase space that describes the motion of the particle is of the order of the Planck length and the mass of it is of the order of the Planck mass, the correct formula for Hawking temperature for a d-dimensional Schwarzschild BH is of the form

\[ T' = T(1 + \alpha^2 \gamma T^2 + O(\alpha^4)) \]  

(1)

noting that what we write in (1) is slightly different from what is in the original work. What has been done is eliminating the
In this section, first we discuss only the GUP corrections and then we treat the self-gravitational corrections separately.

3.1. GUP corrections

We accept (1) as the correct expression for the Hawking temperature near the validity of GUP. Assume that the total change in the screen’s entropy is

\[ \delta S = \frac{\partial S}{\partial A} \delta A, \]

and, as we know, that the infinitesimal displacement of the test particle with mass \( m \) must be of the order of the Compton wavelength

\[ \delta x = \eta \lambda_c = \frac{\eta \hbar}{mc}. \]

We know that the quantized holographic screen or, as it was in the Verlinde scenario, the area of the horizon of a BH (spherical) must be wrought from the amount of information bytes which is related directly to the degree of freedom of the horizon

\[ A = QN. \]

Here \( N \) is the number of bytes and \( Q \) is of the order of the Planck length. The common formula for Newton’s gravity, assuming the modified GUP essence formula (1) for temperature, is

\[ F = T \frac{\delta S'}{\delta x} = T \frac{\partial S'/\partial A}{\delta A} \frac{\delta A}{\delta x}. \]

Recalling (4) and (5),

\[ F = T \left( 1 + \alpha^2 \gamma T^2 + O(\alpha^4) \right) \frac{\partial S'/\partial A}{\partial A} \frac{\delta N}{\eta \lambda_c}. \]

In (7), \( T \) may be read as the Hawking temperature for the horizon or the Unruh temperature [16] for an accelerated test particle (a test particle near the horizon senses itself in a thermal bath as a Rindler observer). Nevertheless, in general, these different temperatures measured by different observers are not equal. According to the equipartition law of energy which is valid even in the GUP regime and the Einstein equivalency between mass and energy for \( T \), we have

\[ T = \frac{2Mc^2 Q}{4\pi \hbar R^2}. \]

Substituting (1), (4), (5) and (8) into (7) and by assuming that \( \delta N = 1 \), we have

\[ F = F_0 \left( 1 + \frac{\alpha^2 B}{R^2} + O(\alpha^4) \right), \]

where \( F_0 = \frac{M_o}{R^2} \frac{\partial x^2}{8\pi \hbar k_B} \) and \( B = \gamma \left( \frac{2Mc^2 Q}{8\pi \hbar k_B} \right)^2 \). Recalling that \( t^2 = \xi^2 \) and by comparing with the Newtonian force \( F_0 \), we observe that

\[ Q = \sqrt{8\pi \hbar k_B} \eta \xi^2, \]

which fixes its value. In (9), the first term is the usual Newtonian term and the second term is completely new and relates directly back to the nature of the GUP modifications of the temperature (horizon) and entropy.

4. Self-gravitational corrections to the entropic force

First we overview the form of the modified temperature due to the self-gravitational effect following the idea of Keskı-Vakkuri, Kraus and Wilczek (KKW) [13]. The KKW analysis means the total energy of the spacetime under study is kept fixed while the BH mass is allowed to vary. We therefore expect a BH of initial mass \( M \) to have a final mass of \( M + \omega \) where \( \omega \) is the energy of the emitted particle [17]. Since for a Schwarzschild BH

\[ \frac{1}{M} = \zeta S^{-1/2}, \]

self-gravitational corrections to second order in entropy for a Schwarzschild metric have to be considered [18]

\[ S' = S \left( 1 + \frac{2\omega \zeta}{\sqrt{S}} + \frac{\omega^2 \zeta^2}{S} + O(\omega^3) \right), \]

where \( \omega \) is the energy of the tunnelling particle and \( \zeta \) some proportionality constant. Since we know that, in a Schwarzschild spacetime, always

\[ T = \frac{k}{\sqrt{S}}, \]

so the corrected temperature from the self-gravitational effect is

\[ T' = T \left( 1 - \frac{\omega \zeta}{k} T + \omega^2 \zeta^2 \frac{1}{k^2} T^2 + O(\omega^3) \right). \]
Now we are ready to calculate the self-gravitational corrections for Newtonian gravity in the Verlinde approach. Substituting (11) and (12) into (6) and carrying out simple algebra, we obtain

\[ F = F_0(1 + O(\omega^3)), \]

where \( F_0 = \frac{mM}{R^2} \frac{e^{\nu_0}}{\eta \kappa_0 G \hbar^2}. \) Again by comparing with the Newtonian force \( F_0 \), we observe that

\[ Q = \sqrt{8\pi k_B \eta \hbar^2}, \]

and thus to second order of \( \omega \), there is no correction to the Newtonian force.

5. Summary

There has been much recent interest in calculating the quantum corrections to \( S_{BH} \) (the Bekenstein–Hawking entropy). The leading-order correction is proportional to \( \log(S_{BH}) \). There are two distinct and separable sources for this logarithmic correction. Firstly, there should be a correction to the number of microstates that is a quantum correction to the microcanonical entropy. Secondly, as any BH will typically exchange heat or matter with its surroundings, there should also be a correction due to thermal fluctuations in the horizon area. In this short paper, we show that for attaining a complete generalization of the entropic corrections to Newton’s gravity, we must consider temperature and entropy, both modified in any of two different regimes, GUP and self-gravitational corrections. These corrections are given by equations (9) and (13), respectively.

References

[1] Verlinde E P 2011 J. High Energy Phys. JHEP04(2011)029
[2] Ghosh S 2010 arXiv:1003.0285
[3] Vaguea I V and Santos M A 2012 Mod. Phys. Lett. A 27 1250012
[4] Modesto L and Rondono A 2010 arXiv:1003.1998
[5] Bekenstein J D 1972 Lett. Nuovo Cimento 4 737
Bekenstein J D 1973 Phys. Rev. D 7 2333
Bekenstein J D 1974 Phys. Rev. D 9 3292
Hawking S W 1972 Commun. Math. Phys. 25 152
Bardeen J M, Carter B and Hawking S W 1973 Commun. Math. Phys. 31 161
[6] Cavaglia M and Das S 2004 arXiv:hep-th/0404050
[7] Setare M R 2004 Phys. Rev. D 70 087501
[8] Setare M R 2006 Int. J. Mod. Phys. A 21 1325
[9] Krasnov K and Rovelli C 2009 Class. Quantum Grav. 26 245009
Rovelli C 1996 Phys. Rev. Lett. 77 3288
Dreyer O, Markopoulou F and Smolin L 2006 Nucl. Phys. B 744 1
[10] Sheykh A 2010 Phys. Rev. D 81 104011
[11] Milgrom M 1983 Astrophys. J. 270 371
Milgrom M 1983 Astrophys. J. 270 384
Milgrom M 1983 Astrophys. J. 270 365
[12] Behar S and Carmeli M 2000 Int. J. Theor. Phys. 39 1397
Carmeli M 1999 Int. J. Theor. Phys. 38 1993
Carmeli M 1998 Int. J. Theor. Phys. 37 2621
[13] Hawking S W 1975 Commun. Math. Phys. 43 199
Vakkeri E K and Kraus P 1996 Phys. Rev. D 54 7407
Parikh M K and Wilczek F 2000 Phys. Rev. Lett. 85 5042
Hemming S and Keski-Vakkuri E 2001 Phys. Rev. D 64 044006
Setare M R and Vagenas E C 2004 Phys. Lett. B 584 127
[14] Das S, Majumdar P and Bhaduri R K 2002 Class. Quantum Grav. 19 2355
[15] Setare M R 2003 Phys. Lett. B 573 173
[16] Unruh W G 1976 Phys. Rev. D 14 870
[17] Setare M R and Vagenas E C 2005 Int. J. Mod. Phys. A 20 7219
[18] Setare M R 2008 Int. J. Mod. Phys. A 23 2047