Dynamics of Temperature Distribution of Gravitationally Bound Objects in the Expanding Universe

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Received September 25, 2020; revised September 25, 2020; accepted September 25, 2020

Abstract—In this article, we consider a cluster of primordial black holes (PHBs) which decoupled from the cosmic expansion. A characteristic feature of the formed region is an increased temperature relative to the surrounding space, which can be the reason for the formation of new chains of nuclear reactions significantly affecting the chemical composition of the region under consideration. The temperature propagation is described within the relativistic Chapman–Enskog procedure.

Keywords: primordial black holes, cluster of primordial black holes

DOI: 10.1134/S1063778821090271

1. INTRODUCTION

The article [1] discussed a mechanism of formation of primordial black holes as a result of a collapse of domain walls, which are formed during the inflationary period of the Universe evolution owing to quantum fluctuations of the scalar field near the saddle point. After the end of the inflationary stage, the walls begin to go below the horizon, fluctuate, and release energy, thus heating the surrounding space. In this article, we consider the temperature dynamics of the region formed as a result of heating by all the walls. For this purpose, it is proposed to solve the relativistic heat conduction equation without convective terms taking into account the expansion of the surrounding space.

2. RELATIVISTIC CHAPMAN–ENSKOG PROCEDURE

The relativistic Chapman–Enskog procedure [2] allows one to obtain linear laws connecting fluxes, thermodynamic forces, and expressions for transfer coefficients on the basis of the solution of the linearized transfer equation. A characteristic assumption of this procedure is that, in the hydrodynamic mode, the distribution function can be represented as a function of the hydrodynamic variables and of their gradients. Linear laws are introduced into the continuity equation, the equation of motion, and the energy equation. This leads to relativistic Navier–Stokes equations, which form a closed system for the hydrodynamic variables. In the first approximation, the various irreversible flows are linearly related to the inhomogeneities present in the system. In this case, the relativistic generalization of the Fourier law for the heat flow and the expression for the viscous pressure tensor are as follows (c = h = k_B = 1):

\[ I_\mu^\nu = \lambda \left( \nabla_\mu T - \frac{T}{hn} \nabla_\mu p \right), \]

\[ \Pi^{\mu\nu} = 2\eta \nabla_\mu u_\nu + \eta_v \Delta^{\mu\nu} \nabla_\sigma u_\sigma, \]

where \( \lambda \) is the thermal conductivity coefficient; \( \eta \) is the scalar viscosity coefficient; \( \eta_v \) is the bulk viscosity coefficient; \( h \) is the enthalpy; \( p \) is the pressure; \( \nabla_\mu = \Delta^{\mu\nu} \partial_\nu \), where the projection operator is \( \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \) and acts on the hydrodynamic 4-velocity as \( \Delta^{\mu\nu} u_\nu = 0 \); and the first term in the definition of \( \Pi^{\mu\nu} \) is the zero-trace pressure tensor. The resulting expressions are given for the two choices of the hydrodynamic 4-velocity proposed by Eckart as well as Landau and Lifshitz. Let us use Eckart’s definition [3]. In this scheme, the hydrodynamic 4-velocity is expressed through the 4-flux of particles \( N_\mu \) as

\[ u_\mu = \frac{N_\mu}{\sqrt{N_\nu N_\nu}}. \]

The relativistic equation of motion and the equations for energy [4]:

\[ hn Du^\mu = \nabla_\mu p - \Delta^\mu_\sigma \nabla_\sigma \Pi^{\nu\sigma} + (hn)^{-1} \Pi^{\mu\nu} \nabla_\nu p \]

\[ - (\Delta^\mu_\nu DI_q^\nu + I_q^\nu \nabla_\nu u^\nu + I_q^\nu \nabla_\nu u^\nu), \]

\[ n De = -p \nabla_\mu u^\mu + \Pi^{\mu\nu} \nabla_\nu u_\mu - \nabla_\mu I_q^\mu + 2I_q^\mu Du_\mu. \]
By virtue of linear laws (1) and (2), the equation for energy (5) is as follows:

$$\frac{DT}{T} = c_v \left[ \mu u^\mu - \frac{\lambda}{p} \left( \nabla^2 T - \frac{T}{h n} \nabla^2 p \right) \right],$$

(6)

where $\nabla^2 = \nabla^\mu \nabla_\mu$ and $D = u^\mu \partial_\mu$.

If the hydrodynamic velocity is constant and $p = n T$, the equation for energy is reduced to the relativistic heat conduction equation

$$n c_v D T = \lambda \left( \nabla^2 T - \frac{T}{h n} \nabla^2 p \right),$$

(7)

3. THERMODYNAMIC QUANTITIES

Let us define the unknown quantities in Eq. (7). When considering the case of the absence of an external field, the equilibrium distribution function turns into a Jüttner distribution function, i.e., a momentum distribution function

$$f(p) = \frac{1}{(2\pi)^3} \exp \left( \frac{\mu - p^\mu u_\mu}{T} \right).$$

(8)

Then, using this function, we find the particle number density, the energy density, and the enthalpy expression [5]. Energy per particle and heat capacity by definition are, respectively,

$$e = m K_{\mu}(m/T) - T, \quad c_v = \partial e/\partial T.$$  

(9)

The expression for the enthalpy per particle considering $p = n T$ is

$$h = e + p n^{-1} = m K_{\mu}(m/T) - k T.$$  

(10)

The quantities found in (9) and (10) contain modified Bessel functions of the second kind, which have different asymptotics depending on the magnitude of the argument [6]. If the arguments are sufficiently large ($w = m/T$),

$$K_{\mu}(w) = \frac{1}{e^w \sqrt{2w}} \left[ 1 + \frac{4n^2 - 1}{8w} + \frac{(4n^2 - 1)(4n^2 - 9)}{(28w)^2} + \ldots \right].$$

(11)

Hence, in the case of low temperatures, we obtain the following expressions:

$$h = m + \frac{5}{2} T + \frac{15 T^2}{8} n + \ldots;$$

$$e = m + \frac{3}{2} T + \frac{15 T^2}{8} n + \ldots;$$

$$c_v = \frac{\partial e}{\partial T} = \frac{3}{2} + \frac{15 T}{4} n + \ldots.$$  

(12)

When considering massless particles, which play a role in the relativistic kinetic theory, the modified Bessel function of the second kind and the corresponding thermodynamic quantities are as follows:

$$\lim_{n \to 1} w^\mu K_{\mu}(w) = 2^{n-1} (n-1)!; \quad e = 3 T; \quad h = 4 T; \quad c_v = 3.$$  

(13)

For sufficiently low values of temperature, the heat transfer coefficient is approximated as [4]

$$\lambda = \frac{4 x_i}{5 x_e} \frac{1}{\sigma_T},$$

(14)

where $x_i$ is the fraction of particles, $\sigma_T$ is the Compton cross section, and the electron–photon density ratio is given by the baryon–photon density ratio under the assumption of electroneutrality of the Universe [7]

$$\eta_B = \frac{n_B}{n_\gamma} = 6 \times 10^{-9}.$$  

(15)

4. DEPENDENCE ON THE RATE OF EXPANSION OF THE SURROUNDING SPACE

Let us define the type of operators included in Eq. (7). If matter in general is stationary, then the 4-velocity $u_\mu = (1, 0, 0, 0)$, then $D = u^\mu \partial_\mu = \partial_v$. By replacing the partial derivatives with the covariant ones in the operators $\nabla^\mu u^\nu \partial_v \to A^\mu \partial_v$, we obtain

$$\nabla^2 = \nabla^\mu \nabla_\mu = \nabla^\mu g_{uv} \nabla_v = \Delta g^{uv} g_{uv} g^{vk} g_{vk},$$

(16)

where for the contravariant $g_{\mu \nu} A_\nu = \partial_\mu A_\nu - \Gamma^\alpha_{\mu \nu} A_\alpha$ and for the covariant $\partial_\mu A_\nu = \partial_\nu A_\mu + \Gamma^\alpha_{\mu \nu} A_\alpha$, and the Christoffel symbols of the second kind in turn are $\Gamma^\alpha_{\mu \nu} = g^{\nu \sigma} (\partial_\mu g_{\sigma \nu} + \partial_\nu g_{\sigma \mu} - \partial_\sigma g_{\mu \nu})/2$.

The Friedmann–Robertson–Walker metric [7] is used in further calculations

$$g_{\mu \nu} = \text{diag}(1, -a^2(t), -a^2(t)r^2, -a^2(t)r^2 \sin^2 \theta),$$

(17)

where the scale factor is obtained taking into account the following conditions $a(t_0) = 1$, $\rho(t_0) = 0.53 \times 10^{-5}$ GeV/cm$^3$, and $t_0 = 14 \times 10^9$ years is the age of the Universe and has the form

$$a(t) = \left[ 1 + \frac{3 \xi}{2} \left( \frac{8 \pi G \rho_{t_0}}{3} (t - t_0) \right)^{1/3} \right]^{-2/3},$$

(18)

where $\xi = 4/3$ for the radiation-dominated stage of the Universe (hereinafter, RD stage) and $\xi = 1$ for the matter–dominated epoch (hereinafter, MD stage).
5. CALCULATION RESULTS

The cluster of primordial black holes presumably virialized at the end of the RD stage. Therefore, it makes sense to consider how much the rasterized region can cool down before the transition to the MD stage. Let the initial temperature profile be given by the normal distribution, and let the parameters of the problem be as follows:

- Radius of the heated region \( r_0 = 1 \) pc.
- Temperature inside the region \( T_{in} = 100 \) keV.
- Ambient temperature \( T_{out} = 1 \) keV.
- Scale factor dependence \( a(t) \) is selected for the RD stage.
- For enthalpy and heat capacity per one particle, we use expressions (12).

On the basis of the obtained numerical solution, we can conclude that the gravitationally bound region almost completely preserves the temperature which was obtained during formation at the RD stage; more detailed numerical results were given in [5]. The next step is to determine what happens to this heated region at the MD stage.

6. ESTIMATION FOR THE MD STAGE

By considering the internal temperature of the gravitationally bound region at the MD stage, we can find the dependence of thermal conductivity on temperature in the nonrelativistic case.

By definition [5], thermal diffusivity is in units [pc²/year]

\[
\chi = \frac{\lambda}{n_e c_s} = 3.16 \frac{T_e \tau_e}{m_e c_s} = \frac{3.16}{2\sqrt{2\pi\sqrt{m_q^4Z_n(T)}}\ln(6T_D/\alpha)} T_e^{5/2}
\]

The obtained value allows us to keep the elevated temperature inside the cluster until recombination begins. This fact is an important prerequisite for anomalies in the chemical composition of the region under consideration. Therefore, it makes sense to consider the processes in more detail in the future.

7. CONCLUSIONS

In this article, as a result of the calculations, it was found that the gravitationally bound region almost completely preserves the temperature obtained during the formation of the cluster of primordial black holes. Therefore, there are significant prerequisites for anomalies in the chemical composition of the region under consideration. It is of interest to study the possible anomalies of the chemical composition and compare them with the data of observations.

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Translated by O. Pismenov