Monte-Carlo study of the $\Delta \rho$ parameter

J.L. Alonso$^a$, Ph. Boucaud$^b$ and A.J. van der Sijs$^a$

$^a$Departamento de Física Teórica, Universidad de Zaragoza, 50009 Zaragoza, Spain
$^b$LPTHE, Université Paris XI, 91405 Orsay Cedex, France

We present results concerning a lattice study of the electroweak $\rho$-parameter. We have used an $SU(2)\times U(1)$ symmetric chiral Yukawa model built with Zaragoza fermions. The decoupling of the species doublers in this model is verified numerically. We find that the numerical data for $\Delta \rho$ are well described by one-loop perturbation theory in the same finite volume and with the same finite cut-off. However, a finite cut-off can cause substantial deviations of $\Delta \rho$ from the standard value, even in infinite volume.

1. INTRODUCTION

An important quantity in the Standard Model of electroweak interactions is the $\rho$-parameter, defined as the relative strength between the neutral and charged current interactions at zero momentum transfer. At tree level at low momentum, the electroweak interaction is described by the standard current-current interaction and it is well known that $\rho_{\text{tree}} = M_W^2/M_Z^2 \cos^2 \theta_W = 1$. When quantum effects are taken into account, $\rho = (1 - \Delta \rho)^{-1}$ deviates from 1. We will be concerned here only with the dominant "universal" (i.e. process-independent) part of $\Delta \rho$ coming from the difference in vacuum polarization of the $W$ and $Z$ propagators.

In the Standard Model, the dominant contribution comes from the $t$-$b$ mass splitting. In the limit $m_b^2 \ll m_t^2$ it is given by

$$\Delta \rho = N_c \frac{\sqrt{2} G_F}{16\pi^2} m_t^2 = N_c \frac{y^2}{16\pi^2}. \quad (1)$$

From a phenomenological point of view, this quantity is an interesting tool to investigate the existence of unknown physics through loop effects. This is exemplified by the bounds put on the top quark mass before its discovery. As it appears in eq. (1), there is a strong dependence on the mass of a heavy fermion running in the loop. In particular such a fermion does not decouple from the low-energy physics. It would be desirable to control the persistence or not of this non-decoupling phenomenon non-perturbatively, for large values of the mass, and to study its relation with the issue of triviality. This problem has been addressed in a large-$N_F$ approximation before [1].

From the lattice point of view, these characteristics have some relevance to the doubling problem. We know for example that heavy doublers treated à la Wilson-Yukawa do not decouple in the continuum limit. Through the relation (1), $\Delta \rho$ can be used as a counter of fermions. It thus provides us with a tool to test models of chiral lattice fermions.

2. MEASURING $\Delta \rho$ ON THE LATTICE

A numerical prediction for $\Delta \rho$ from a direct study of the gauge boson propagators would require a tractable lattice formulation of a four-dimensional chiral gauge theory. This task is beyond our present capabilities. However, the gauge interactions are weak compared to the Yukawa and scalar ones and it is a reasonable approximation to neglect all the gauge couplings. There are Ward identities relating the vacuum polarization tensors of the gauge bosons to those of the charged and neutral goldstone particles in the fermion-scalar sector, and $\rho$ can be expressed as the ratio of their wave function renormalization constants: $\rho = Z_0/Z_+$ [2].

*Presented by Ph. Boucaud (phi@qcd.th.u-psud.fr). Work supported by EC contract ERBCHHICT941067, by DGICYT project AEN 94-218, by Acción Integrada Hispano-Francesa HF94-150B and HF95-296B and by Caja de Ahorros de la Inmaculada.
In the gaugeless approximation, therefore, only the scalar-fermion sector of the Standard Model needs to be latticized to obtain  . For this purpose, we have used an $SU(2) \times U(1)$ symmetric chiral Yukawa fermion model built with Zaragoza fermions. The lattice action is:

$$S = -\kappa \sum_{x,\mu} \text{Tr} \left[ \Phi^+(x) \left( \Phi(x+\hat{\mu}) + \Phi(x-\hat{\mu}) \right) \right] + \sum_{x,\mu} \bar{\psi}(x) \gamma_{\mu} \frac{\psi(x+\hat{\mu}) - \psi(x-\hat{\mu})}{2} + \sum_x \left( \bar{\psi}^{(1)}(x) \Phi(x) Y P_R \psi^{(1)}(x) + h.c. \right).$$

(2)

$\Phi$ is a fixed modulus Higgs field ($\Phi \in SU(2)$), $\psi$ is a fermionic field for two fermion doublets, $P_R$ is the right projector $\frac{1}{2}(1+\gamma_5)$ and $Y = \begin{pmatrix} y & 0 \\ 0 & 0 \end{pmatrix}$ is the matrix of Yukawa couplings, where $y = y_{\text{top}}$ and $y_{\text{bottom}}$ is set to zero. The fermion field $\psi^{(1)}(x)$ is the average of the $\psi$ fields over the vertices of the hypercube at $x$: in momentum space it can be written as $\psi^{(1)}(p) = F(p) \psi(p)$ with “form factor” $F(p) = \prod_x \cos(q_p x/2)$. This model is expected to describe two coupled fermion doublets and thirty massless and uncoupled doubler doublets.

We have simulated the action (2) with the Hybrid Monte-Carlo algorithm (the reason why we have doubled the fermion content) and present results for $6^3 \times 12$ and $8^3 \times 16$ lattices. Details will be given in 3.

3. DECOUPLING OF THE DOUBLERS

With Zaragoza fermions, the decoupling of the doublers is known to hold in perturbation theory [3]. Numerically, the structure of the phase diagram was previously found in good agreement with a mean-field prediction, giving some indication that the decoupling works [5]. We have done some further tests reported here:

- From the three point function $G_3(k, p, q) = \sum_{x,y,z} \Pi_0(x) \psi(y) \bar{\psi}(z) \exp[-i(k,x + p.y + q.z)]$, where $\Pi_0(x) = \text{Tr} \left[ \Phi(x) s_3 \right]$ is an interpolating field for the neutral goldstone, we have extracted the renormalized three-point couplings $y_{3pt}$ for the usual fermion (i.e. with momentum near $p = 0$), and $y_{db}$ for a doubler (i.e. with momentum near $p_\mu = q_\mu = \pi a^{-1}$) respectively. We find as expected that $y_{db} \approx y_{3pt} \times F^2$, i.e. the coupling of the doublers is suppressed by the form factor $F^2$. This suppression factor is around $10^{-2}$ for the minimum momentum allowed for a fermion on an $8^3 \times 16$ lattice and will decrease if the volume is increased.

- We have studied the one-loop renormalization group equations for the evolution of the scalar and Yukawa coupling constants for the continuum model with 2 and 32 fermions doublets and compare the solutions with our Monte-Carlo data in fig. 1. The data seem to follow the one-loop prediction and support the expectation that we have only two dynamical doublets, without coupled doublers.

![Figure 1](image)

Figure 1. Higgs mass as a function of the fermion mass in units of $v_r$. MC data for $6^3 \times 12$ and $8^3 \times 16$ lattices are shown, and the solid line is the perturbative result with 2 doublets, for a scale ratio $\log(v_r/\Lambda) = -4/3$. The analogous result for 32 doublets is given by the dotted curves.

- We have compared results obtained for $\Delta \rho$ with Zaragoza and naive fermions for two points in the phase diagrams with comparable fermion mass in lattice units. We defined the renormalized coupling constant, $y_r$, as the ratio of the renormalized fermion mass divided by the renormalized vacuum expectation value of the scalar field, $v_r$. On a $6^3 \times 12$ lattice, the comparison between Monte-Carlo data and perturbation theory gives $[\Delta \rho_{MC}/y_r^2] / [\Delta \rho_{pert}/y^2] = 0.98(13)$ for $\kappa = 0.24$, $y = 0.6$, $am_f = 0.311$ in the naive case and $[\Delta \rho_{MC}/y_r^2] / [\Delta \rho_{pert}/y^2] = 0.92(18)$ for $\kappa = 0.31$, $y = 1.0$, $am_f = 0.345$ in the Zaragoza case. This means that for both fermion models, the numerical results are well compatible with their respective perturbative predictions. On the
other hand, perturbation theory predicts that \( \Delta \rho_{\text{naive}} / \Delta \rho_{\text{Zaragoza}} \) equals 17.13 and 17.26 for \( am_f \) equal to 0.311 and 0.345 respectively (this ratio becoming 16 in the infinite volume limit when \( am_f \ll 1 \)). This is a clear indication that the Zaragoza doublers are decoupled and do not contribute to \( \Delta \rho \).

4. RESULTS FOR \( \Delta \rho \)

To detect possible non-perturbative effects, we have compared the results obtained from the simulations with the one-loop perturbative predictions at finite volume \( V \) and finite cut-off \( a^{-1} \). Namely, for each lattice \((\kappa, y, V)\) we have extracted from the numerical data the value of the fermion mass in lattice units, \( am_f \), and used it in a finite-volume fermionic one-loop perturbative computation of \( \Delta \rho \). Fig. 2 shows the ratio of the Monte-Carlo value \( \Delta \rho_{\text{MC}} / y^2 \) divided by the corresponding perturbative prediction. The numerical results seem to be well represented by the finite-volume, finite-cut-off one-loop perturbative calculation for both volumes and all the values of \( y \) so far explored.

![Figure 2](image)

Figure 2. Ratio of the Monte-Carlo \( \Delta \rho / y^2 \) divided by the finite-volume, finite-cut-off perturbative value. Data from \( 6^3 \times 12 \) and \( 8^3 \times 16 \) lattices are shown.

5. CONCLUSIONS

The dominant part of \( \Delta \rho \) can be computed within the scalar-fermion sector of the Standard Model. This can be done on the lattice with present techniques. We have found different kinds of numerical evidence that the decoupling mechanism works as expected with the Zaragoza proposal for lattice fermions. \( \Delta \rho \) is well described by the perturbative prediction. Nevertheless, even if \( \Delta \rho \) is always given by perturbation theory, cut-off effects can be important [1]. When the mass of the fermion approaches the cut-off, \( \Delta \rho \) deviates from the standard result given in eq. (1), valid when \( m_f / \Lambda \ll 1 \). In fig. 3 we give the ratio \( \Delta \rho_{\text{pert}}(am_f) / \Delta \rho_{\text{pert}}(0) \) for the lattice model we have used here. It shows that cut-off effects in \( \Delta \rho \) can be quite substantial.

![Figure 3](image)

Figure 3. Finite-cut-off effects in \( \Delta \rho \). We plot the perturbative one-loop estimate for the ratio \( \Delta \rho_{\text{pert}}(am_f) / \Delta \rho_{\text{pert}}(0) \) obtained with the action (2) at infinite volume.

A more detailed account of this work, including a more complete list of references and acknowledgements, will appear soon [6].

REFERENCES

1. K. Aoki and S. Peris, Z. Phys. C 61 (1994) 303.
2. R.S. Lytel, Phys. Rev. D 22 (1980) 505.
3. R. Barbieri et al., Phys. Lett. B 288 (1992) 95.
4. J.L. Alonso et al., Phys. Rev. D 44 (1991) 3258.
5. J.L. Alonso et al., Nucl. Phys. B (Proc. Suppl.) 47 (1996) 571.
6. J.L. Alonso et al., in preparation.