Design of Prescribed Performance Controller for Load Disturbance Interconnected Power System Based on Fuzzy Adaptive Backstepping

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Abstract. The complex structure of interconnected power system and the existence of external load disturbance make it difficult to establish accurate system model and realize stability control with preset performance. In this paper, a fuzzy adaptive controller design method with preset performance is proposed for uncertain interconnected power systems. Firstly, the system approximation model is obtained by using the fuzzy approximation principle, then the preset performance control strategy is introduced to transform the steady-state preset performance of the system into an error performance function. Finally, the adaptive controller design with preset performance is realized based on the Backstepping method. For the validity of the algorithm, the Simulink simulation is carried out. The results show that the controller can make the output of the system uniformly bounded, that is, the tracking error can always be controlled within the specified performance range.

1. Introduction
With the increasing expansion of the scale of modern power systems, the power supply network has gradually developed into a complex system characterized by large units, long distances, and large-area interconnection[1]. As the most complex industrial control system, the regional interconnected power system has a complex system structure that is difficult to model, various external load disturbances, and many control performance requirements. Once a cascading failure occurs, it will cause serious losses to social production and public life[2-3]. Therefore, the stability analysis and controller design of the interconnected power system based on prescribed performance requirements are of great significance to improve the stability of the national power system.

Scholars at home and abroad have done a lot of research on power system control[4-5]. Chen[6] designed a linear active disturbance rejection controller for three-region interconnected power system with prescribed error constraints, which enabled the system to have strong robustness without increasing parameters. Chen and Lu[7] proposed a distributed adaptive controller based on radial basis function neural network to improve the transient stability of multi-machine power systems, aiming at the problems of system nonlinearity and parameter uncertainty commonly existed in power systems. Chen
and Li[8] proposed a class of output tracking and interference resistance algorithm based on Backstepping method for interconnected power systems with series compensators. In the above literature, the system models are required to be known for the stability research of the power system, but the actual interconnected power system is often not easily available due to the complex and changeable structural characteristics. Therefore, the research on the unknown interconnected power system is more in line with the actual needs of the field.

In the traditional nonlinear interconnected power system tracking control, the controller parameters are often adjusted to achieve the desired performance of the system, which is inefficient and difficult to achieve ideal control effect. Bechlioulis[9] proposed a predetermined performance control method for MIMO system for the first time, which converged the tracking error to a predefined arbitrarily small residual set. Chang[10] considered a class of single-machine power system with external disturbance, and designs an adaptive robust controller based on predetermined performance. In Yu [11], two predictive performance functions were introduced to make the system tracking error meet the prescribed conditions, and combined with the disturbance observer, the load frequency control problem of a class of regional interconnected power systems is solved. Backstepping, as a classical algorithm to solve nonlinear control, is widely used in the control field. In recent years, output adjustment is a new algorithm to solve the tracking problem with disturbance. The combination of algorithms such as Backstepping, output regulation, fuzzy control, neural network and adaptive control provides a new design idea for solving complex control, and has obtained rich research results in recent years[12-14].

In this paper, we study the tracking and predetermined performance control of interconnected power system with unknown model by using fuzzy adaptive method. Firstly, through the unknown nonlinear part of the fuzzy system, the regulation equation with fuzzy approximation term is obtained. Then, the target performance of the system is given by using the prescribed performance function. Finally, an adaptive fuzzy output controller is proposed by using the Backstepping method to make the system output uniformly bounded and meet the expected performance index.

2. System model and basic theory

2.1. System modeling

A kind of interconnected power system model with load disturbance [15] is shown in figure 1.

![Figure 1. Interconnected power system with load disturbance](image)

In the figure 1, 1 and 2 are equivalent generators, 3 and 4 are transformers, 5 are load disturbances of the system, 6 are circuit breakers of the power system, and 7 are interconnection lines of the power system. $\hat{f}(x_1, x_2)$ is used to represent the unknown nonlinear model of the system, and the control quantity $u(t)$ containing error performance function is introduced to obtain the following system:

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \hat{f}(x_1, x_2) + d(t) + u(t) \\
e &= x_1 - r(t)
\end{align*}$$

(1)

Where $x_1, x_2$ is the system state, $r(t)$ is the reference signal, and $e$ is the system tracking error. Load disturbance $d(t)$ is generated by the following dynamic system:

$$\begin{align*}
\dot{v}_1 &= -\sigma \nu \\
\dot{v}_2 &= \sigma v_1
\end{align*}$$

(2)
Where $v_1, v_2$ is the internal dynamics of the disturbance signal.

Reference signal $r(t)$ is generated by the following dynamic system:
\[
\begin{align*}
\dot{v}_1 &= \sigma rv_1 \\
\dot{v}_2 &= -\sigma rv_1
\end{align*}
\] (3)

Where $v_1, v_2$ is the internal dynamics of the reference signal.

This paper takes the above system as the research object, and designs a fuzzy adaptive Backstepping controller with prescribed performance to make the system meet the following properties:

**Property 1.1**: Design the controller $u(t)$ so that the closed-loop system is ultimately uniformly bounded;

**Property 1.2**: Under the action of the controller, the system tracking error $e(t) = x_t - r(t)$ meets the prescribed performance index;

For control purposes, the following assumptions and lemmas are given.

**Hypothesis 1.1**: there is a constant $1$ that makes inequality $2$ true.

**Lemma 1.1**: Under the bounded initial conditions, if there exists a continuous and positive definite Lyapunov function $V(x)$ that satisfies $\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|)$ and makes $\dot{V}(x) \leq -\rho V(x) + c$ true, where $\alpha_1, \alpha_2: \mathbb{R}^n \to \mathbb{R}$ is a K-like function and $c$ is a normal number, then the solution $x(t)$ is uniformly bounded.

### 2.2. Fuzzy approximation theory

Considering that the system model is unknown, the Mamdani method is used to approach the system fuzzy, and the approximate expression can be obtained as follows[16]:
\[
\hat{f}(x_1, x_2) = \hat{\Omega}^T \xi(x)
\] (4)

Where $\hat{\Omega}$ is the estimated value of $\Omega^*$, let $\theta^*$ be the expected value of adaptive parameter $\theta$, and its expression is as follows:
\[
\theta^* = \arg \min_{\theta \in \Theta} \sup_{x \in \mathbb{R}^n} \left| \hat{f}(x_1, x_2) - \theta^* f(x_1, x_2) \right|
\] (5)

### 2.3. Prescribed performance and transform error

Among many performance indicators, tracking error boundary is more important.

Assume that at $t \geq 0$, tracking error $e(t)$ satisfies the following inequality:
\[
-\delta_1 \beta(t) \leq e(t) \leq \delta_2 \beta(t)
\] (6)

In order to realize the Prescribed performance control, inequality constraint is firstly transformed into equality constraint, and Equation (7) is transformed to obtain the equivalent condition Equation (8):
\[
e(t) = \beta_1(t) R_i(z_i), i = 1, 2, \cdots, n
\] (7)

the inverse function of $R_i(z_i)$ with respect to $z_i$ is
\[
z_i = R_i^{-1}(z_i) = \frac{1}{2} \ln \left( \frac{\epsilon_i(t)}{\beta_i(t)} + \frac{\delta_1}{\delta_2} \right)
\] (8)
The upper and lower bounds of tracking error boundary $e(t)$ can be determined by dynamic equation (9), and the prescribed performance constraints (8).

### 3. Design of fuzzy adaptive controller with prescribed performance

In order to make the system meet the tracking control and predetermined performance, the whole controller can be divided into feedforward controller and feedback controller based on the output regulation theory. The main function of the feedforward controller is to suppress the known load disturbance, and the feedback controller is mainly used to stabilize the system and meet the predetermined performance.

#### 3.1. Feedforward controller design

Based on the output regulation theory [17]:

$$u = K_x x + K_v v = u_b + u_f$$  \hspace{1cm} (9)

Where $u_b = K_x x$ represents the feedback controller, and $u_f = K_v v$ represents the feedforward controller.

Based on the output regulation theory, the feedforward controller can be obtained as follows:

$$u_f = u(v) = -\dot{f}(x, x_2) - \sigma v_s - v_i$$  \hspace{1cm} (10)

#### 3.2. Feedback controller design

Then Backstepping is used to solve the feedback controller $u_b$ based on predetermined performance constraints. The specific process is as follows:

**Step 1:** Define auxiliary variables: $e_1 = x_1 - x(v)$, $e_2 = x_2 - \alpha$, where $\alpha \in \mathbb{R}^n$ is the virtual control quantity. Then the prescribed performance constraint function (8) can be transformed as follows:

$$\ddot{z} = \gamma (\dot{e} - \Phi e) = \gamma (s + \alpha - x(v) - \Phi e)$$  \hspace{1cm} (11)

Where $\Phi = \frac{\dot{\beta}}{\beta_1}$, Virtual control quantity $\alpha$ can be designed as follows:

$$\alpha = x_2(v) + \Phi e - Cz$$  \hspace{1cm} (12)

Substitute the above into transform error equation to obtain: $\ddot{z} = \gamma (s - Cz)$, construct the Lyapunov function:

$$V_t = \frac{1}{2} z^2$$  \hspace{1cm} (13)

Take the derivative of the formula, we can get: $\dot{V}_t = z \ddot{z} = z \gamma (s - Cz)$.

**Step 2:** the auxiliary variable $s = x_2 - \alpha$ is known, and the derivative of the auxiliary variable $s$ with respect to time $t$ can be obtained:

$$\dot{s} = \dot{x}_2 - \dot{\alpha}$$

$$= \dot{\theta}^T (\dddot{x}(x, x_2) - \dddot{x}(x(v), x_2(v))) + \Theta^T \dddot{x}(x, x_2) + e^T + u_b - \Phi e - \Phi \dot{e} + C \gamma (s - Cz)$$  \hspace{1cm} (14)
The adaptive parameter error is \( \dot{\theta} = \theta^* - \dot{\theta} \). In order to achieve adaptive fuzzy control with predetermined performance of the system, feedback control item \( u_b \) is:

\[
\dot{u}_b = -\dot{\theta}^T (\xi(x_i(v), x_2(v))\xi(x_i, x_2)) + \dot{\Phi}(x_i - x_i(v)) + \dot{\Phi}(x_2 - x_2(v)) - C\gamma(s - Cz) - Ks
\]  

(15)

Construct a new Lyapunov function:

\[
V_2 = V_2 + \frac{1}{2} s^2 + \frac{1}{2\Gamma} \dot{\theta}^T \dot{\theta}
\]  

(16)

The adaptive rate is:

\[
\dot{\theta} = -\sigma \dot{\theta} + \Gamma \xi(x_i, x_2)s
\]  

(17)

In the formula, \( \sigma \) and \( \Gamma \) are undetermined constants. According to Equation \( u = u_b + u_f \), the adaptive fuzzy controller can be written:

\[
u = -\sigma \dot{\theta} + \dot{\theta}^T \xi(x_i, x_2) + \dot{\Phi}(x_i - x_i(v)) + \dot{\Phi}(x_2 - x_2(v)) - (C\gamma + K)s + C\gamma z
\]  

(18)

3.3. Proof of final uniform boundedness

Based on the above derivation, the following theorem can be obtained:

**Theorem**: For model unknown interconnected power system with load disturbance, the adaptive rate is determined by Equation (18) under the action of controller (19), and the system can satisfy properties 1 and 2.

The proof of this theorem is as follows:

**Proof**: Using Young’s inequality \( x^T y \leq a^\alpha \| x \|^\alpha + \frac{1}{q} \| y \|^q \), where \( a > 0, p > 1, q > 1, (p - 1)(q - 1) = 1 \), it can be further obtained:

\[
zs \leq \frac{\pi_1}{2} \| z \|^2 + \frac{1}{2\pi_1} \| s \|^2, \quad s\Delta \leq \frac{\pi_2}{2} \| s \|^2 + \frac{1}{2\pi_2} \| \Delta \|^2 \leq \frac{\pi_2}{2} \| s \|^2 + \frac{1}{2\pi_2} \| \Delta \|^2
\]

\[
\text{Tr}\{\dot{\theta}^T \dot{\theta}\} \leq -\frac{1}{2} \text{Tr}\{\dot{\theta}^T (\dot{\theta}^* - \dot{\theta})\} + \frac{1}{2} \| \theta^* \|^2
\]  

(19)

Substituting the above equation into \( \dot{V}_2 \) leads to the following further inequality:

\[
\dot{V}_2 \leq -(\gamma C - \frac{\gamma \pi_1}{2})s^2 - (K - \frac{\gamma}{2\pi_1} - \frac{\pi_2}{2})s^2 - \frac{\sigma}{2\Gamma} \text{Tr}\{\dot{\theta}^T \dot{\theta}\} + \frac{1}{2\pi_2} \| \Delta \|^2 + \frac{\sigma}{2\Gamma} \| \theta^* \|^2
\]  

(20)

Let

\[
\alpha_1 = \gamma C - \frac{\gamma \pi_1}{2}, \quad \alpha_2 = K - \frac{\gamma}{2\pi_1} - \frac{\pi_2}{2}, \quad \rho = \frac{1}{2\pi_2} \| \Delta \|^2 + \frac{\sigma}{2\Gamma} \| \theta^* \|^2
\]  

(21)

Obviously, we can choose \( C, \gamma \), and \( K \) large enough to make \( \alpha_1 \) and \( \alpha_2 \) positive, and at the same time make \( \rho = \min(2\alpha_1, 2\alpha_2, \sigma) \), which further transforms into the following inequality:
\[ \dot{V}_2 \leq -\rho V_2 + t \]  

Then

\[ V_2(t) \leq V_2(0)e^{-\rho t} + \frac{t}{\rho} \forall t > 0 \]  

(23)

Obviously, signals \( z, s \) and \( \tilde{\Theta} \) are ultimately uniformly bounded, that is, the system equation satisfies property 1.1. By selecting \( C, K, \Gamma \) large enough and \( \sigma \) small enough so that \( \frac{t}{\rho} \) is small enough, the error \( z \) given in the equation can be selected as sufficiently small. According to the property of error transformation, by keeping the boundedness of the solution of Equation (9), the tracking error \( e(t) \) of system satisfies the predetermined performance of Equation (7) at \( t \geq 0 \), so that the property 1.2 is also valid.

4. Simulation and analysis

In order to verify the validity of the controller, the S-function of Simulink is used in this section to realize the simulation of the controller. The exact model of the given interconnected power system is as follows:

\[ \delta(t) = \omega(t), \]
\[ \dot{\omega}(t) = \frac{P_{\text{max}}}{H} \sin(\delta(t)) - \frac{D}{H} \omega(t) + \frac{P_m}{H} + d(t) \]  

(24)

Where \( \delta(t) = \delta_1(t) - \delta_2(t) \) is the relative Angle between the \( q \) axes of the equivalent generator in system \( S_1 \) and system \( S_2 \); \( \dot{\omega}(t) = \omega_1(t) - \omega_2(t) \) is the relative angular velocity of the two systems; \( \omega(t) = \omega_1(t) - \omega_2(t) \) is the relative angular velocity of the two systems; \( H, D \) are equivalent moment of inertia and equivalent damping coefficient respectively; \( P_{\text{max}}, P_m \) represents electromagnetic power and mechanical power, and \( d(t) \) represents external load disturbance. The fuzzy approximation reference model is

\[ \hat{f}(x_1, x_2) = - \frac{P_{\text{max}}}{H} \sin(\delta(t)) - \frac{D}{H} \omega(t) + \frac{P_m}{H} \].

Set to \( H = 1, D = 1, P_i = 50, P_m = 10 \); The fuzzy controller adopts Gaussian membership function and 121 fuzzy rules; The parameter in the controller is set to \( K = 200, P = 10, \sigma = 1, C = 1 \); The parameter in the predetermined performance function is set to \( \delta_1 = 3, \quad \delta_2 = 2, \quad \beta(t) = (1.2 - 0.05)e^{-4.5t} + 0.05 \).

The simulation results are shown in Figure 2-4:

Figure 2. Track of relative angle and relative angular velocity
Figure 2 shows the tracking output of the relative angle and relative angular velocity between the system and the equivalent generator shaft of the system in an interconnected power system with load disturbance. It can be seen that in the case of load disturbance and unknown system model, the controller can realize the tracking control of reference signal quickly and accurately.

Figure 3 shows the degree of approximation between the fuzzy approximation model and the actual reference model in the system. It can be seen that although there are small fluctuations in some areas, the overall approximation error is small, which can better approximate the unknown interconnected power system model.

Figure 4 shows the variation of the tracking error of the system under the constraint of the prescribed performance. It can be seen from the figure that the tracking error $\epsilon(t)$ can fully meet the prescribed performance requirements of $-\delta_1 \beta(t) \leq \epsilon(t) \leq \delta_2 \beta(t)$.

5. Conclusion
In this paper, a design method of fuzzy adaptive Backstepping controller with prescribed performance is proposed for a class of interconnected power system model. This method not only solves the problem that the model of power system is difficult to be built accurately, but also makes the system reach the prescribed stable state. The simulation results show that the state parameters between interconnected power systems can track the reference signals quickly and accurately, and achieve the steady-state requirements of predetermined performance. In the later stage, it can be considered to apply the method to higher order and more complex power systems to solve the tracking control problem of the system under the condition of satisfying various predetermined performance.

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