EXCLUSIVE RADIATIVE WEAK DECAYS OF $B_c$ MESON IN LIGHT CONE

QCD

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Abstract

We investigate the radiative $B_c \rightarrow \rho^+\gamma$ and $B_c \rightarrow K^{*+}\gamma$ decays in the standard model. The transition form factors are calculated in the framework of the light cone QCD sum rules method. We estimate the branching ratios of the $B_c \rightarrow \rho^+\gamma$ and $B_c \rightarrow K^{*+}\gamma$ decays.
1 Introduction

The experimental and theoretical investigation of the heavy flavored hadrons is one of the most promising research area in high energy physics. These investigations might shed light for a precise determination of the many fundamental parameters of the standard model (SM), such as Cabibbo–Kobayashi–Maskawa (CKM) matrix elements, leptonic decay constants of the heavy mesons and for deeper understanding of the dynamics of QCD. From this point of view $B_c$ mesons are very interesting particles since in their decay processes, different mechanisms (weak annihilation, charged current, spectator and FCNC decays) can give contributions simultaneously and this circumstance can play central role for understanding the dynamics of weak decays of heavy hadrons. Moreover the decay channels of the $B_c$ mesons are richer than $B$ meson decays and, since they contain two heavy quarks, their QCD predictions are much more reliable. The study of the $B_c$ mesons, for these reasons, receives special attention among researches.

Note that, the possibility of the production of the $B_c$ mesons in different colliders and their different decay channels have already been extensively discussed in the current literature [1]. Among all different decay channels, the weak exclusive radiative decays of the $B_c$ mesons play potentially very important role for the determination of the CKM parameters, similar to the radiative $B$ meson decays. In the usual $B_{u,d,s} \to V \gamma$ ($V = \rho^\pm$, $K^{*\pm}$) decays, two different mechanisms, namely weak annihilation and FCNC, contribute simultaneously. Therefore extracting information about the CKM matrix elements would involve a trustworthy estimate of both contributions to the decay amplitude. Since, in calculation of the $B \to V$ matrix element, both contributions involve uncertainties of their own, the resulting error in our attempt to estimate the CKM parameters may be substantial.

In contrast to the above–mentioned $B_{u,d,s} \to V \gamma$ decay, the $B_c \to V \gamma$ process is described only through the weak annihilation mechanism. Therefore, investigation of the exclusive radiative weak $B_c$ meson decays is more reliable and promising in determination of the CKM parameters.

In this work we study the $B_c \to V \gamma$ ($V = \rho^\pm$, $K^{*\pm}$) decay in the SM, in the framework of the QCD sum rules. The paper is organized as follows. In Section 2 we calculate the transition form factors for the $B_c \to V \gamma$ decay decay in the light cone QCD sum rules approach. Section 3 is devoted to the numerical analysis and discussion of our results.

2 Sum rules for transition form factors

The relevant effective Hamiltonian for the $B_c \to V \gamma$ process is

$$\mathcal{H} = \frac{G}{\sqrt{2}} a_1 V_{cb} V_{ug}^* \bar{q} \gamma_\mu (1 - \gamma_5) u \bar{c} \gamma_\mu (1 - \gamma_5) b ,$$

where $q = d$ or $s$ and $V_{ug}$ represent the corresponding matrix elements, i.e., $V_{ud}$ or $V_{us}$, and the factor $a_1$ takes into account renormalization of four fermion operators and it is numerically equal to 1.13. In further analysis we will take $a_1 = 1$. The matrix element for the above mentioned decay is

$$\mathcal{M} = \frac{G}{\sqrt{2}} V_{cb} V_{ug}^* \langle V \gamma | \bar{q} \gamma_\mu (1 - \gamma_5) u \bar{c} \gamma_\mu (1 - \gamma_5) b | B_c \rangle .$$
In the factorization approximation, one may write this matrix element as

\[ \langle V\gamma | \bar{q}\gamma_\mu (1 - \gamma_5) u \bar{c}\gamma_\mu (1 - \gamma_5) b | B_c \rangle = \langle V | \bar{q}\gamma_\mu (1 - \gamma_5) u | 0 \rangle \langle \gamma | \bar{c}\gamma_\mu (1 - \gamma_5) b | B_c \rangle + \langle V\gamma | \bar{q}\gamma_\mu (1 - \gamma_5) u | 0 \rangle \langle 0 | \bar{c}\gamma_\mu (1 - \gamma_5) b | B_c \rangle . \]

(3)

Using the definitions

\[ \langle 0 | \bar{c}\gamma_\mu \gamma_5 b | B_c \rangle = -i f_{B_c} (p_{B_c})_\mu, \]
\[ \langle V | \bar{q}\gamma_\mu (1 - \gamma_5) u | 0 \rangle = \varepsilon^{(\nu)}_\mu m_V f_V, \]

(4)

where \( \varepsilon^{(\nu)} \), \( f_V \) and \( m_V \) are the polarization vector, leptonic decay constant and mass of the vector \( V \) meson, respectively, one can easily show that the second term on the right side of Eq. (3) is proportional to the light quark mass \( m_q \), whose contribution is very small (for more detail see [3] and [3]), and therefore we shall neglect it in further analysis. Thus we conclude that the main contribution to \( B_c \rightarrow V\gamma \) decay comes from the diagrams where photon is emitted from initial \( b \) and \( c \) quark lines. The corresponding matrix element for the \( B_c \rightarrow V\gamma \) decay can be written as

\[ \mathcal{M} = \frac{G}{\sqrt{2}} V_{cb} V_{ub}^* \varepsilon^{(\nu)}_\mu m_V f_V \langle \gamma | \bar{c}\gamma_\mu (1 - \gamma_5) b | B_c \rangle . \]

(5)

All needs to be done then, is to calculate the matrix element \( \langle \gamma | \bar{c}\gamma_\mu (1 - \gamma_5) b | B_c (p + q) \rangle \), which describes the annihilation of the \( B_c \) meson into \( \bar{c}\gamma_\mu (1 - \gamma_5) b \) current with emission of a real photon. This matrix element can be written in terms of the two independent, gauge invariant (with respect to the electromagnetic interaction) structure as

\[ \langle \gamma(q) | \bar{c}\gamma_\mu (1 - \gamma_5) b | B_c(p + q) \rangle = \]
\[ \sqrt{4\pi\alpha} \left\{ \epsilon_{\mu\alpha\beta\sigma} \epsilon^{*\alpha} p^\beta q^\sigma g_1(p^2) \frac{m_{B_c}^2}{m_{B_c}^2} + i \left[ \epsilon^{*\mu}(pq) - (e^* p) q_\mu \right] g_2(p^2) \frac{m_{B_c}^2}{m_{B_c}^2} \right\} , \]

(6)

where \( e^* \) and \( q \) are the polarization vector and momentum of the photon, respectively, \( p \) is the momentum transfer \( (p^2 = m_{B_c}^2) \), \( g_1(p^2) \) and \( g_2(p^2) \) are the parity conserving and parity violating form factors. At this point we consider the problem of evaluating the above-mentioned form factors, for which we will employ the light cone QCD sum rules approach (see the recent review [3]). For this purpose we start by considering the following correlator function

\[ \Pi_\mu(p, q) = i \int d^4x e^{ipx} \langle \gamma(q) | T \left\{ \bar{c}(x)\gamma_\mu (1 - \gamma_5) b(x) \bar{b}(0)i\gamma_5 c(0) \right\} | 0 \rangle . \]

(7)

This function can be decomposed into two independent structures, Lorentz and gauge invariant, as follows:

\[ \Pi_\mu(p, q) = \Pi_1 \epsilon_{\mu\alpha\beta\sigma} e^{*\alpha} p^\beta q^\sigma + i \Pi_2 \left[ \epsilon^{*\mu}(pq) - (e^* p) q_\mu \right] . \]

(8)
In deep–Euclidean region \((p + q)^2 < 0\) and \(p^2 = m^2_v \ll m^2_Q\), the heavy quarks \(Q\) are far off–shell. Therefore photon emission from the heavy quarks takes place perturbatively. This behavior of the \(B_c \rightarrow V \gamma\) decay is essentially different from the corresponding \(B^\pm \rightarrow V^\pm \gamma\) channel. In the latter the photon interacts with quarks both perturbatively and non–perturbatively (see for example [4, 5]), while in the \(B_c \rightarrow V \gamma\) decay the photon interacts with quarks only perturbatively.

Firstly let us calculate the physical part of the correlator (8). Inserting the hadronic states with the relevant \(B_c\) meson quantum numbers into Eq. (8) we get

\[ \Pi^{(1,2)} = \frac{f_{B_c} m^2_{B_c}}{m_b + m_c} \frac{g_{1,2}}{m^2_{B_c} - (p + q)^2} + \int_{s_0}^\infty ds \frac{\rho^{(1,2)}(s, p^2)}{s - (p + q)^2}, \]  

where we have used

\[ \langle B_c | \bar{b} v_\gamma \gamma | 0 \rangle = \frac{f_{B_c} m^2_{B_c}}{m_b + m_c}, \]

in Eq. (7). The second term in Eq. (9) represents contribution of the higher states starting from some threshold \(s_0\). We invoke the hadron quark duality and replace the hadron spectral density \(\rho^h\) by the imaginary part of the dispersion relation \(\Pi^{(1,2)}\) calculated in QCD.

From (9) it follows that \(\Pi^{(1,2)}(p^2 + q^2, p^2)\) is analytic in the cut \((p + q)^2\) plane. In other words, in order to relate \(\Pi^{(1,2)}\) with its imaginary part we need dispersion relation in the variable \((p + q)^2\). Therefore the perturbative contribution to the parity conserving and parity violating amplitudes can be calculated by writing the dispersion integral in the variable \((p + q)^2\), i.e.,

\[ \Pi^{(1,2)} = \int ds \frac{\rho^{(1,2)}(s, p^2)}{s - (p + q)^2} + \text{subst. terms}, \]

where the superscript 1 and 2 corresponds to \(\Pi^{(1)}\) and \(\Pi^{(2)}\) respectively, and \(\rho^{(1,2)}\) are the spectral densities. These spectral densities are calculated by a method presented in [6] (for applications of this method, see also [7] and [8]). The above–mentioned spectral densities were calculated in [9] in regard to an investigation of the \(B_c \rightarrow ℓν\gamma\) decay and we shall make use of these results, which lead to the following expressions for \(\Pi^{(1,2)}\):

\[ \Pi^{(1)} = \sqrt{4\pi\alpha} \frac{N_c}{4\pi^2} \int \frac{ds}{s - (p + q)^2} \frac{[s - p^2]}{[s - (p + q)^2]}
\times \left\{ \frac{m_b - m_c}{(2m^2_b + p^2 - s)} \ln \frac{1 + \alpha - \beta + \lambda}{1 + \alpha - \beta - \lambda} + Q_b m_b \ln \frac{1 - \alpha + \beta + \lambda}{1 - \alpha + \beta - \lambda} \right\}, \]

\[ \Pi^{(2)} = \sqrt{4\pi\alpha} \frac{N_c}{4\pi^2} \int \frac{ds}{s - (p + q)^2} \frac{[s - p^2]}{[s - (p + q)^2]^2}
\times \left\{ m_b Q_b \left( 2m^2_b + p^2 - s \right) \ln \frac{1 + \alpha - \beta + \lambda}{1 + \alpha - \beta - \lambda} - \lambda \left( 2m^2_b - 2m^2_c + p^2 (2 - \alpha + \beta - s) \right) \right\}, \]
\[ +m_c Q_b \left[ -2m_b^2 \ln \frac{1 + \alpha - \beta + \lambda}{1 + \alpha - \beta - \lambda} + \lambda \left( 2m_b^2 - 2m_c^2 - p^2(\alpha - \beta) + s \right) \right] \\
+ m_b Q_c \left[ 2m_b^2 \ln \frac{1 - \alpha + \beta + \lambda}{1 - \alpha + \beta - \lambda} + \lambda \left( 2m_b^2 - 2m_c^2 - p^2(\alpha - \beta) - s \right) \right] \\
+ m_c Q_c \left( s - p^2 - 2m_c^2 \right) \ln \frac{1 - \alpha + \beta + \lambda}{1 - \alpha + \beta - \lambda} - \lambda \left( 2m_b^2 - 2m_c^2 - p^2(2 + \alpha - \beta) + s \right) \right] \right) , \]

where \( N_c = 3 \) is the color factor, \( \alpha = m_b^2 / s, \beta = m_c^2 / s \). \( Q_b \) and \( Q_c \) are the electric charges of the \( b \) and \( c \) quarks, respectively and \( \lambda = \sqrt{1 + \alpha^2 + \beta^2 - 2\alpha - 2\beta - 2\alpha\beta} \). Note that, as a formal check, when we set the charm quark mass \( m_c \) to zero in Eqs. (11) and (12), the resulting expressions are expected to be the same as the ones calculated for the perturbative part of the \( B^\pm \to V \gamma \) decay. This decay was investigated in [4, 5], and indeed our results for \( \Pi^{(1,2)} \) coincide with theirs in the \( m_c \to 0 \) limit.

The light cone QCD sum rule is obtained, as usual, by equating the hadronic representation of the correlator \( \Pi_\mu \) (see Eq. (9)) to the results obtained through QCD calculations (see Eqs. (11) and (12)). Applying Borel transformation in the variable \((p + q)^2 \) to suppress the higher states, we get sum rules for the transition form factors \( g_1 \) and \( g_2 \):

\[
\begin{align*}
g_1(p^2) &= \frac{m_b + m_c}{f_{B_c}} \frac{N_c}{4\pi^2} \int_0^1 \frac{du}{\Delta} e^{[m_{B_c}^2 u - (m_b + m_c)^2 + p^2\bar{u}]/(M^2 u)} \\
&\times \left[ (m_b - m_c) \lambda (Q_c - Q_b) + Q_b m_b \ln \frac{1 + \alpha - \beta + \lambda}{1 + \alpha - \beta - \lambda} + Q_c m_c \ln \frac{1 - \alpha + \beta + \lambda}{1 - \alpha + \beta - \lambda} \right], \tag{13}
\end{align*}
\]

\[
\begin{align*}
g_2(p^2) &= \frac{m_b + m_c}{f_{B_c}} \frac{N_c}{4\pi^2} \int_0^1 \frac{du}{\Delta} \left[ (m_b + m_c)^2 - p^2 \right] e^{[m_{B_c}^2 u - (m_b + m_c)^2 + p^2\bar{u}]/(M^2 u)} \\
&\times \left\{ m_b Q_b \left[ \left( 2m_b^2 + p^2 - s \right) \ln \frac{1 + \alpha - \beta + \lambda}{1 + \alpha - \beta - \lambda} - \lambda \left( 2m_b^2 - 2m_c^2 + p^2(2 - \alpha + \beta) - s \right) \right] \\
+ m_c Q_b \left[ -2m_b^2 \ln \frac{1 + \alpha - \beta + \lambda}{1 + \alpha - \beta - \lambda} + \lambda \left( 2m_b^2 - 2m_c^2 - p^2(\alpha - \beta) + s \right) \right] \\
+ m_b Q_c \left[ 2m_b^2 \ln \frac{1 - \alpha + \beta + \lambda}{1 - \alpha + \beta - \lambda} + \lambda \left( 2m_b^2 - 2m_c^2 - p^2(\alpha - \beta) - s \right) \right] \\
+ m_c Q_c \left( s - p^2 - 2m_c^2 \right) \ln \frac{1 - \alpha + \beta + \lambda}{1 - \alpha + \beta - \lambda} - \lambda \left( 2m_b^2 - 2m_c^2 - p^2(2 + \alpha - \beta) + s \right) \right) \right\}, \tag{14}
\end{align*}
\]

where \( M^2 \) is the Borel parameter, \( \bar{u} = 1 - u \) and

\[
s = \frac{(m_b + m_c)^2 - p^2\bar{u}}{u}.
\]

In obtaining expressions (13) and (14) we have introduced two new variables

\[
u = \frac{(m_b + m_c)^2 - p^2}{s - p^2},
\]
\[ \Delta = \frac{(m_b + m_c)^2 - p^2}{s_0 - p^2}, \]

which is equivalent to the subtraction of higher states. In our analysis we will evaluate the form factors \( g_1 \) and \( g_2 \) at \( p^2 = m_V^2 \).

### 3 Numerical analysis

In our calculation of the form factors \( g_1(p^2 = m_V^2) \) and \( g_2(p^2 = m_V^2) \), we use the following set of parameters: \( m_b = 4.7 \text{ GeV} \), \( m_c = 1.4 \text{ GeV} \), \( m_{B_c} = 6.258 \text{ GeV} \), \( s_0 = 50 \text{ GeV}^2 \), \( f_\rho = 0.216 \text{ GeV} \), \( f_K^* = 0.211 \text{ GeV} \) and \( f_{B_c} = 0.35 \text{ GeV} \). The analysis of the dependence of \( g_1(p^2 = m_V^2) \) and \( g_2(p^2 = m_V^2) \) on the Borel parameter \( M^2 \) shows that the best stability is achieved in the region \( 15 \text{ GeV}^2 < M^2 < 20 \text{ GeV}^2 \). The predictions of the sum rules on the form factors have errors by at most 10% due to the uncertainties in \( m_b \), \( s_0 \), \( f_{B_c} \) and \( M^2 \) in the above-mentioned region. Our numerical analysis on the form factors \( g_1(p^2 = m_V^2) \) and \( g_2(p^2 = m_V^2) \) predicts the following results:

\[
\begin{align*}
g_1(p^2 = m_V^2) &= 0.44 \text{ GeV}, \\
g_2(p^2 = m_V^2) &= 0.21 \text{ GeV},
\end{align*}
\]

\[ g_1(p^2 = m_{K^{*+}}^2) = 0.44 \text{ GeV}, \quad g_2(p^2 = m_{K^{*+}}^2) = 0.21 \text{ GeV}. \]  

(15)

The branching ratio of the \( B_c \rightarrow V\gamma \) decay is

\[
\mathcal{B}(B_c \rightarrow V\gamma) = \frac{G^2 \alpha}{16} |V_{cb}V^*_{u\tau}|^2 f_V^2 m_V \left( \frac{m_{B_c}^2 - m_V^2}{m_{B_c}^2} \right)^3 \left[ \frac{g_1^2(m_V^2)}{m_{B_c}^2} + \frac{g_2^2(m_V^2)}{m_{B_c}^2} \right] \tau(B_c),
\]

where for the \( B_c \) meson lifetime we have used \( \tau(B_c) = 0.52 \times 10^{-12} \text{ s} \), \( |V_{ud}| = 0.97 \), \( |V_{us}| = 0.22 \) and \( |V_{cb}| = 0.04 \). With this set of parameters, finally, we summarize the numerical results of the branching ratios.

\[
\begin{align*}
\mathcal{B}(B_c \rightarrow \rho^{+}\gamma) &= 8.3 \times 10^{-8}, \\
\mathcal{B}(B_c \rightarrow K^{*+}\gamma) &= 5.3 \times 10^{-9}.
\end{align*}
\]

(17)

Few words about the experimental observability of these decays are in order. In \[13, 16\] it is estimated that at LHC, approximately \( 2 \times 10^8 \) \( B_c \) mesons per year will be produced. Using the result of Eq. (18) and this estimated number of decays, we can easily calculate the number of expected events for the \( B_c \rightarrow V\gamma \) decay at LHC to be

\[
\begin{align*}
\mathcal{N}(B_c \rightarrow \rho^{+}\gamma) &= \mathcal{B}(B_c \rightarrow \rho^{+}\gamma) \times (2 \times 10^8) = 17, \\
\mathcal{N}(B_c \rightarrow K^{*+}\gamma) &= \mathcal{B}(B_c \rightarrow K^{*+}\gamma) \times (2 \times 10^8) = 1.
\end{align*}
\]

From this estimation it follows that at future LHC collider it is possible to detect only \( B_c \rightarrow \rho\gamma \) channel.
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