Universality in nuclear dependence coefficient $\alpha(q_T)$

Xiaofeng Guo$^1$, Jianwei Qiu$^2$, and Xiaofei Zhang$^2$

$^1$Department of Physics and Astronomy, University of Kentucky
Lexington, Kentucky 40506, USA
$^2$Department of Physics and Astronomy, Iowa State University
Ames, Iowa 50011, USA

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We derive the nuclear dependence coefficient $\alpha(q_T)$ for Drell-Yan and $J/\psi$ production. We show that at small $q_T$, the $\alpha(q_T)$ is given by an universal functional form: $\alpha(q_T) = a + bq_T^2$, and the parameters $a$ and $b$ are completely determined by either perturbatively calculable or independently measurable quantities. This universal functional form $\alpha(q_T)$ is insensitive to the $A$, and is consistent with existing data.

$\frac{d\sigma^{A}}{dQ^{2}dq_{T}^{2}} = N_{DY} \frac{1}{2\tau^{2}} e^{-q_{T}^{2}/2\tau^{2}}, \tag{2}$

with $N_{DY}$ a dimensional normalization and $\tau$ the width.

In region (II), the spectrum can be calculated in QCD perturbation theory with resummation of large logarithms, such as $(\alpha_s \ln^3 (Q^2/q_T^2))^\alpha$, which are due to the gluon radiations from incoming partons $\Psi$. The resummation is extremely important for $W^\pm$ and $Z^0$ production at collider energies because the $Q^2/q_T^2$ can be as large as $8 \times 10^3$ for $q_T \sim 1$ GeV. However, for the Drell-Yan production at fixed target energies, the resummation is much less important because of much smaller value of $Q^2/q_T^2$. In fact, as shown in Ref. [12], all existing data for $q_T$ as large as 2.5 GeV can be well represented by extending the Gaussian-like distribution in Eq. (2) from region (I) to region (II).

In region (III), the transverse momentum spectrum can be calculated in perturbative QCD [13]. Therefore, the Drell-Yan $q_T$-spectrum at fixed target energies can be represented by

$\frac{d\sigma}{dQ^{2}dq_{T}^{2}} = \frac{d\sigma^{(I)}}{dQ^{2}dq_{T}^{2}} + \left[ \frac{d\sigma^{(II)}}{dQ^{2}dq_{T}^{2}} - \frac{d\sigma^{(I)}}{dQ^{2}dq_{T}^{2}} \right] \theta(q_T - q_T^0), \tag{3}$

where $d\sigma^{(I)}/dQ^{2}dq_{T}^{2}$ is the perturbatively calculated $q_T$-spectrum for $q_T > q_T^0$, and $d\sigma^{(I)}/dQ^{2}dq_{T}^{2}$, defined in Eq. (2), fits data in regions (I)+(II).

Using moments of the Drell-Yan $q_T$-spectrum in Eq. (3), we can relate $N_{DY}$ and $\tau$ in Eq. (2) to physical quantities. At fixed target energies, the contribution from the second term in Eq. (3) to $d\sigma/dQ^2$ is much less than one percent. Therefore, up to less than one percent uncertainty, $N_{DY} \approx d\sigma/dQ^2$.

Define the averaged transverse momentum square as $\langle q_T^2 \rangle \equiv \int dq_T^2 q_T^2 (d\sigma/dQ^2dq_T^2)/(d\sigma/dQ^2)$. We find [14]...
that the contribution of the second term in Eq. (3) to \( q_T^2 \) is much less than ten percent of the first term. Therefore, by iteration, we obtain \( 2\tau^2 \approx (q_T^2) - \Gamma(q_T^2) \) with
\[
\Gamma(q_T^2) = \frac{1}{d\sigma/dq_T^2} \int_{q_T^2} d\omega^2 \left[ \frac{d\sigma(III)}{dQ^2dq_T^2} - \frac{d\sigma(I)}{dQ^2dq_T^2} \right] dq_T^2, \tag{4}
\]
where \( 2\tau^2 \) in \( d\sigma(III)/dQ^2dq_T^2 \) in Eq. (4) is approximately given by \( (q_T^2) \). Substituting the \( N_{DY} \) and \( \tau \) into Eq. (3), we obtain the Drell-Yan spectrum for \( q_T < q_T^* \) as
\[
\frac{d\sigma^{hN}}{dQ^2dq_T^2} \approx \frac{d\sigma^{hN}/dQ^2}{\langle q_T^2 \rangle^{hN} - \Gamma(q_T^2)^{hN}}. \tag{5}
\]

Most importantly, the \( \Gamma(q_T^2) \) in Eq. (3) is small and perturbatively calculable.

For the Drell-Yan \( q_T \)-spectrum in hadron-nucleus collisions, we also need to consider multiple scattering. Similar to the single-scattering case, at the leading order in perturbation theory, the double-scattering contribution is also proportional to a \( \delta \)-function \( \delta \),
\[
\frac{d\sigma^{hA}}{dQ^2dq_T^2} \propto T_{qg}(x_1, x_2, k_T) \delta^2(\vec{q}_T - \vec{k}_T), \tag{6}
\]
where the subscript \( D \) indicates the double scattering, \( T_{qg}(x_1, x_2, k_T) \) is the quark-gluon correlation function \( [13,14] \), where \( x_1, x_2 \), and \( x_2 \) are the momentum fractions carried by the quark and gluon fields. The \( k_T \) in Eq. (6) represents the intrinsic momentum of the gluon which gives additional scattering. Following the same arguments leading to Eq. (2), we can show \( \Delta q^2 \) that if the partons’ intrinsic \( k_T \)-dependence has a Gaussian-like distribution, the double scattering contributions to the Drell-Yan \( q_T \)-spectrum in small \( q_T \) region can also be represented by a Gaussian form.

In high \( q_T \) region, the Drell-Yan \( q_T \)-spectrum in hadron-nucleus collision also has a perturbative tail. The Drell-Yan \( q_T \)-spectrum at large \( q_T \) in hadron-nucleus collisions was calculated in Ref. \( [11] \). The nuclear dependence of Drell-Yan \( q_T \)-spectrum depends on two types of multiparton correlation functions inside the nucleus: \( T^{DH} \) and \( T^{SH} \), which correspond to the double-hard and soft-hard double scattering subprocesses respectively \( [11] \). These correlation functions are as fundamental as the well-known parton distributions, and can be extracted from other physical observables.

Similar to deriving Eq. (3), we can derive
\[
\frac{d\sigma^{hA}}{dQ^2dq_T^2} = \frac{d\sigma^{hA}/dQ^2}{\langle q_T^2 \rangle^{hA} - \Gamma(q_T^2)^{hA}} e^{-q_T^2 / \langle \langle q_T^2 \rangle \rangle^{hA} - \Gamma(q_T^2)^{hA}}. \tag{7}
\]

where \( \langle q_T^2 \rangle^{hA} = \langle q_T^2 \rangle^{hN} + \Delta(q_T^2)^{hA} \). \( \Delta(q_T^2)^{hA} \) is the transverse momentum broadening and calculable in QCD perturbation theory \( [15,16] \). In Eq. (7), \( \Gamma(q_T^2)^{hA} \) is a small contribution to \( \langle q_T^2 \rangle^{hA} \), and it is calculable and depends on the perturbative tail of the \( q_T \)-spectrum.

Substituting Eqs. (3) and (7) into Eq. (8), we derive \( \alpha(q_T) \) for the Drell-Yan production in small \( q_T \) region:
\[
\alpha_{DY}(q_T) = 1 + \frac{1}{\ln(A)} \left[ \ln \left( \frac{R_{DY}^A (Q^2)}{\langle q_T^2 \rangle^{hN} - \Gamma(q_T^2)^{hN}} \right) \right] + \frac{\chi_{DY}}{1 + \chi_{DY}} \frac{q_T^2}{\langle q_T^2 \rangle^{hN} - \Gamma(q_T^2)^{hN}}, \tag{8}
\]
where \( R_{DY}^A (Q^2) = (1/A)(d\sigma^{hA}/dQ^2) / (d\sigma^{hN}/dQ^2) \). The \( \chi_{DY} \) in Eq. (8) is defined by
\[
\chi_{DY} = \frac{\Delta(q_T^2)^{hA}}{\langle q_T^2 \rangle^{hN} - \Gamma(q_T^2)^{hN}} \approx \frac{\Delta(q_T^2)^{hA}}{\langle q_T^2 \rangle^{hN}}, \tag{9}
\]
where \( \Delta(q_T^2)^{hA} = \Gamma(q_T^2)^{hA} - \Gamma(q_T^2)^{hN} \), and is much smaller than \( \Delta(q_T^2)^{hA} \) \( [12] \). The \( \alpha_{DY}(q_T) \) in Eq. (8) has a quadratic dependence on \( q_T \).

At the leading order in \( \alpha_s \), \( \Delta(q_T^2)^{hA} \propto T_{qg}^{SH}(x) = \lambda^2 A^{1/3} q(x) \) with \( q(x) \) the normal quark distribution and the parameter \( \lambda^2 \) proportional to the size of averaged color field strength square inside a nuclear target \( [9] \). Consequently, we have \( \Delta(q_T^2)^{hA} = b_{DY} A^{1/3} \) with \( b_{DY} \propto \lambda^2 \). In this letter, we use a measured \( b_{DY} \approx 0.022 \text{ GeV}^2 [15] \) to fix \( \lambda^2 \) and \( T_{qg}^{SH} \). Taking the small \( \chi_{DY} \) limit, and using the fact that \( R_{DY}^A (Q^2) \approx 1 \), we derive
\[
\alpha_{DY}(q_T) \approx 1 + \frac{b_{DY} \langle q_T^2 \rangle^{hN}}{\langle q_T^2 \rangle^{hN} - \Gamma(q_T^2)^{hN}} \left[ -1 + \frac{q_T^2}{\langle q_T^2 \rangle^{hN}} \right]. \tag{10}
\]
In deriving Eq. (10), we used \( A^{1/3} \sim \ln(A) \), which is a good approximation for most relevant targets. Eq. (10) shows that the leading contribution to \( \alpha_{DY}(q_T) \) does not depend on the \( A \).

![FIG. 1. \( \alpha_{DY}(q_T) \) for the Drell-Yan production as a function of \( q_T \). At small \( q_T \), the \( \alpha_{DY}(q_T) \) in Eq. (8) is used. At large \( q_T \), QCD predictions from Ref. [10] are plotted with \( r_0 = 1.1 \text{ fm} \) and \( [xG(x)]_{x_{so}} = 3 \) for the possible maximum and minimum values of the \( C \).](image-url)
In Fig. 1 we plot $\alpha_{DY}(q_T)$ for targets with $A = 12$ and $A = 184$. At small $q_T$, we used Eq. (8) and set the small $\Gamma(q_T^2) = 0$. We also used the measured $(\langle q_T^2 \rangle_{hN}) = 1.8$ GeV$^2$, and $\Delta(\langle q_T^2 \rangle_{hA}) = 0.022A^{1/3}$ GeV$^2$ [12, 17]. At large $q_T$, we plot the QCD predictions from Ref. [14], which depend on both $T^{SH}$ and $T^{DH}$. $T^{SH}$ is fixed by $\Delta(\langle q_T^2 \rangle_{hA})$. However, there is no direct observable yet to extract $T^{DH}$ [11]. Because of the operator definition of $T^{DH}(x_1, x_2)$, it was assumed that $T_{ij}^{DH}(x_1, x_2) = (2\pi C)A^{4/3}f_1(x_1)f_2(x_2)$ with $f_1$ and $f_2$ parton distributions of flavor $f_1$ and $f_2$ [10]. Assuming no quantum interference between different nucleon states, one derives $C = 0.35/(8\pi r_0^2) \text{ GeV}^2$ with $r_0 \approx 1.1 - 1.25 \text{ fm}$, which is just a geometric factor for finding two nucleons at the same impact parameter [14]. On the other hand, when $x_1$ (or $x_2$) goes to zero, the corresponding parton fields reach the saturation region, and the $T^{DH}$ is reduced to $T^{SH}$. Therefore, we have $C \approx \lambda^2/(2\pi[xG(x)]_{x=0})$ [12], where $[xG(x)]_{x=0}$ is of the order of unity [14]. Because of a combination of a small value of the measured $\lambda^2$ from Drell-Yan data [17] and a choice of $[xG(x)]_{x=0} \approx 3$ [12], these two approaches result into a factor of 20 difference in numerical value for the parameter $C$ [13] as well. The values for the $C$ obtained in Ref. [14] without any quantum interference should represent a possible maximum for $C$, while the value obtained in Ref. [12] with full quantum interference (in saturation region) represents a possible minimum for the $C$. In Fig. 1, we plot the perturbatively calculated $\alpha_{DY}(q_T)$ [10] at large $q_T$ with the maximum and minimum values of the $\lambda^2$ discussed above, and let the $\alpha(q_T)$ in small $q_T$ naturally linked to that at large $q_T$. Fig. 1 also shows that $\alpha_{DY}(q_T)$ is insensitive to the atomic number $A$. In Fig. 2 we plot $R(A, q_T) = A^{\alpha_{DY}(q_T)}$ by using $\alpha_{DY}(q_T)$ in Fig. 1 and compare our predictions with data from E772 [13, 15]. Without any extra free fitting parameters, our predictions shown in Fig. 2 are consistent with data at small $q_T$, and due to the large error in data at high $q_T$, current Drell-Yan data are consistent with almost any value for the $C$ between the maximum and the

\[ \alpha_{3j/\psi}(q_T) = 1 + \frac{1}{\ln(A)} \left( \ln \left( \frac{R_A^{3j/\psi}}{1 + \chi_{3j/\psi}} \right) + \frac{q_T^2}{\Gamma(3j/\psi)} \right), \]

where $R_A^{3j/\psi} = (1/A)\sigma^{3j/\psi}_{hA}/\sigma^{3j/\psi}_{hN}$, and $\chi_{3j/\psi}$ is defined by

\[ \chi_{3j/\psi} = \frac{\Delta \langle q_T^2 \rangle_{3j/\psi} - \Delta \Gamma(3j/\psi)/\Gamma(3j/\psi)}{\langle q_T^2 \rangle_{3j/\psi} - \Gamma(3j/\psi)/\Gamma(3j/\psi) \approx \Delta \langle q_T^2 \rangle_{3j/\psi}}. \]

Similar to the Drell-Yan case, $\Gamma(q_T^2)_{3j/\psi}$ and $\Delta \Gamma(q_T^2)_{3j/\psi}$ are perturbatively calculable, and much smaller than $\langle q_T^2 \rangle_{3j/\psi}$ and $\Delta \langle q_T^2 \rangle_{3j/\psi}$ respectively [12].

One major difference between $J/\psi$ production and the Drell-Yan process is the nuclear dependence of $R_A^{3j/\psi}$. Clear nuclear suppression for $\sigma^{3j/\psi}_{3j/\psi}$ has been observed [20]. Since we are only interested in the general features of $\alpha(q_T)$, we adopt the following simple parameterization

\[ R_A^{3j/\psi} = \frac{1}{A} \frac{\sigma^{3j/\psi}_{hA}}{\sigma^{3j/\psi}_{hN}} = e^{-\beta A^{1/3}}, \]

which fits all experimental data on $J/\psi$ suppression in hadron-nucleus collisions [21, 22]. The $A^{1/3}$ factor in Eq. (13) represents an effective medium length, and $\beta$ is a constant [12].

Similar to the Drell-Yan process, $\Delta \langle q_T^2 \rangle_{3j/\psi}$ is proportional to four-parton correlation functions $\rho_{3j/\psi}$ [12]. Due to final-state interactions for $J/\psi$ production, $\Delta \langle q_T^2 \rangle_{3j/\psi}$ depend on both quark-gluon and gluon-gluon correlation functions. We can define [12] that $\Delta \langle q_T^2 \rangle_{3j/\psi} = b_{3j/\psi} A^{1/3}$, where $b_{3j/\psi}$ can be calculated or extracted from data. From Ref. [17], we obtain $b_{3j/\psi} \approx 0.06 \text{ GeV}^2$. With

FIG. 2. Comparison of our $\alpha_{DY}(q_T)$ in Eq. (8) with the Drell-Yan data from Fermilab E772 collaboration [13, 15].

minimum discussed above. In order to test the theory and pin down the value of the $C$, we need either better data or different observables. Because of the strong dependence on the double-hard subprocesses, the angular dependence of the Drell-Yan pair [11] could be an excellent signal for measuring the $C$.

Similarly, we can also obtain the $\alpha(q_T)$ for $J/\psi$ production. Kinematically, hadronic $J/\psi$ production is like Drell-Yan production with $Q \sim M_{J/\psi}$, and its $q_T$-spectrum can also be characterized by three different regions as in the Drell-Yan case. Because $J/\psi$ mass $M_{J/\psi}$ is smaller than any typical $Q$ measured for the Drell-Yan continuum, the logarithm $\alpha_{J/\psi} = \ln^2(M_{J/\psi}/q_T)$ for $J/\psi$ production is less important. Consequently, at fixed target energies, a Gaussian-like distribution can fit $J/\psi$'s $q_T$-spectrum even better. Therefore, following the same arguments used above for the Drell-Yan production, we derive $\alpha(q_T)$ for $J/\psi$ production in small $q_T$ region,

\[ \alpha_{J/\psi}(q_T) = 1 + \frac{1}{\ln(A)} \left[ \ln \left( \frac{R_A^{J/\psi}}{1 + \chi_{J/\psi}} \right) + \frac{q_T^2}{\Gamma(J/\psi)} \right], \]
\[ \langle q_T^2 \rangle_{\chi_{J/\psi}} \approx 1.68 \text{ GeV}^2 \]
we can also take the small \( \chi_{J/\psi} \) limit in Eq. (11) and obtain
\[ \alpha_{J/\psi}(q_T) \approx 1 - \beta + \left( \frac{b_{J/\psi}}{\langle q_T^2 \rangle_{\chi_{J/\psi}}} \right) \left( -1 + \frac{q_T^2}{\langle q_T^2 \rangle_{\chi_{J/\psi}}} \right) \]
where \( A^{1/3} \sim \ln(A) \) was again used. It is clear from Eq. (14) that in small \( q_T \) region, \( \alpha_{J/\psi}(q_T) \) is also insensitive to the atomic number \( A \) of targets. Furthermore, the nuclear suppression in \( R_{J/\psi}^A \) corresponds to a \( q_T \)-independent shift in the magnitude of \( \alpha_{J/\psi}(q_T) \).

Because the \( R_{J/\psi}^A, \chi_{J/\psi} \), and other physical quantities in Eq. (11) can depend on \( x_F \), \( \alpha_{J/\psi}(q_T) \) can also be a function of \( x_F \). Experiments show that the larger \( x_F \), the more suppression for \( J/\psi \) production (or smaller \( R_{J/\psi}^A \)) [24]. Consequently, from Eq. (11), we will have smaller \( \alpha_{J/\psi}(q_T) \) at larger \( x_F \), which is consistent with experimental data [24]. Although we do not have all needed physical quantities in Eq. (11) for predicting the \( \alpha_{J/\psi} \) in different \( x_F \) regions, we can still test the universality of \( \alpha_{J/\psi}(q_T) \): the quadratic dependence on \( q_T \) and \( \alpha \) plot our fits using \( \alpha_{J/\psi}(q_T) \) in Eq. (11) and compared it with E866 data in the three \( x_F \) regions: small (SXF), intermediate (IXF), and large (LXF). Clearly, our universal functional form for \( \alpha_{J/\psi}(q_T) \) is consistent with all data in small \( q_T \) region (\( q_T < q_T^2 \sim M_{J/\psi}/2 \)).

In summary, we derived an universal functional form for \( \alpha(q_T) \) for both the Drell-Yan and \( J/\psi \) production in small \( q_T \) region (\( q_T < q_T^2 = \kappa Q \) with \( \kappa \approx 1/3 - 1/2 \)). All parameters defining \( \alpha(q_T) \) in Eqs. (8) and (11) are completely determined by either perturbatively calculable or independently measurable quantities. We show that \( \alpha(q_T) \) is extremely insensitive to the atomic weight \( A \) of targets. For the Drell-Yan process, \( \alpha(q_T) \) in Eq. (8) can be naturally connected to the perturbatively calculated \( \alpha(q_T) \) at large \( q_T \). A similar test can also be carried out for \( J/\psi \) production. \( J/\psi \) suppression in relativistic heavy ion collisions was predicted to signal the color deconfinement [24]. On the other hand, significant \( J/\psi \) suppression has been observed in hadron-nucleus collisions [24]. Therefore, understanding the features observed in \( \alpha(q_T) \) for \( J/\psi \) production is very valuable for finding the true mechanism of \( J/\psi \) suppression.

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