DYNAMICS OF NON-STEADY SPIRAL ARMS IN DISK GALAXIES

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ABSTRACT

In order to understand the physical mechanisms underlying non-steady stellar spiral arms in disk galaxies, we analyzed the growing and damping phases of their spiral arms using three-dimensional N-body simulations. We confirmed that the spiral arms are formed due to a swing amplification mechanism that reinforces density enhancement as a seeded wake. In the damping phase, the Coriolis force exerted on a portion of the arm surpasses the gravitational force that acts to shrink the portion. Consequently, the stars in the portion escape from the arm, and subsequently they form a new arm at a different location. The time-dependent nature of the spiral arms originates in the continual repetition of this nonlinear phenomenon. Since a spiral arm does not rigidly rotate, but follows the galactic differential rotation, the stars in the arm rotate at almost the same rate as the arm. In other words, every single position in the arm can be regarded as the corotation point. Due to interaction with their host arms, the energy and angular momentum of the stars change, thereby causing radial migration of the stars. During this process, the kinetic energy of random motion (random energy) of the stars does not significantly increase, and the disk remains dynamically cold. Owing to this low degree of disk heating, short-lived spiral arms can recurrently develop over many rotational periods. The resultant structure of the spiral arms in the N-body simulations is consistent with the observational nature of spiral galaxies. We conclude that the formation and structure of spiral arms in isolated disk galaxies can be reasonably understood by nonlinear interactions between a spiral arm and its constituent stars.

Key words: galaxies: kinematics and dynamics – galaxies: spiral – galaxies: structure – methods: numerical

Online-only material: animations, color figures

1. INTRODUCTION

The spiral arms of disk galaxies are their most prominent structures, and are formed due to gravitationally driven variations in the surface density in the stellar disk (Rix & Zaritsky 1995; Grosbøl et al. 2004; Zibetti et al. 2009; Elmegreen et al. 2011). It is known that the spiral arms of disk galaxies can be excited by tidal interactions with nearby companion galaxies (e.g., Oh et al. 2008; Dobbs et al. 2010; Struck et al. 2011), as well as by the central stellar bar (e.g., Sellwood & Sparke1988). In addition, spiral arms can also be self-induced and maintained without these external gravitational perturbations, and further, they can propagate stationary density waves in globally stable disks, as hypothesized by Lin & Shu (1964). However, the physical origin and dynamical evolution of spiral arms in disk galaxies have thus far not been fully understood. In this study, we focus on the dynamics of stellar spiral arms and stars around them in a disk galaxy without external perturbations and a bar structure.

The Lin–Shu theory posits that the stellar spiral arms can be interpreted as (quasi-) stationary density waves with a constant speed pattern (Lin & Shu 1964; Bertin & Lin 1996); the theory has gained wide acceptance as providing a clear explanation regarding the dynamics of spiral arms. However, as pointed out in the study by Toomre (1969), the quasi-stationary hypothesis has a serious limitation; because of the tendency of the tight winding spiral waves to undergo dispersion, they radially propagate with the group velocity, and are consequently absorbed at the Lindblad resonances (Lynden-Bell & Kalnajs 1972). Therefore, stationary waves that last for a long time require some amplification mechanisms such as WASER (Mark 1976) and a feedback cycle that involves the reflection of the inward propagating wave into an outward propagating one at the $Q$-barrier (Bertin et al. 1989a, 1989b).

However, nearly all the previous time-dependent simulations that have been executed thus far have been unable to prove the existence of stationary density waves in a disk galaxy without external perturbations and a bar structure. Sellwood & Carlberg (1984) claimed that self-induced stellar spiral structures in stellar disks are not stationary and fade away within an interval of about 10 rotational periods. Their study stresses the significance of cooling mechanisms, such as dissipation by the interstellar medium (ISM), which are required to maintain stellar spiral arms. More recently, Fujii et al. (2011) used three-dimensional N-body simulations to show that although stellar spiral arms are short-lived, they are also formed recurrently, and as a result, the spiral features are maintained over 10 Gyr (see also Sellwood 2011). This is because of negative feedback that causes the dynamical heating of stars in the disk galaxies due to spirals. The non-steady spirals are also subject to a similar phenomenon when the dynamics of the ISM are self-consistently solved with stars in disk galaxies. Both the stellar spirals and the ISM undergo motion in the wake of galactic rotation (see also Grand et al. 2012c). In fact, the spirals can be considered as being “wound.” The ISM forms dense regions associated with the non-steady stellar spirals; however, these regions are not the conventional “galactic shocks” (Wada et al. 2011).

The existence of non-steady stellar spirals has been suggested by certain observations in our Galaxy. For example, the
age–velocity dispersion relation of stars in the solar neighborhood (e.g., Holmberg et al. 2007) most naturally accounts for the existence of the non-steady spirals (Carlberg & Sellwood 1985; Binney & Lacey 1988; Jenkins & Binney 1990; De Simone et al. 2004). Baba et al. (2009) analyzed the kinematics of star-forming regions using numerical simulation data of a barred galaxy such as the Milky Way (Baba et al. 2010), and they found that the kinematics of these regions are consistent with the observed peculiar motions of maser sources. These studies support the view that the Galactic stellar spiral arms are non-steady rather than stationary density waves.

However, the current theoretical understanding of the dynamics of non-steady spirals is thus far insufficient. It has been suggested that a key factor toward understanding their dynamics is the orbital evolution of stars. In this study, we analyze the orbits of stars associated with non-steady spirals in the growth and damping phases.

In Section 2, we summarize the numerical model and the method, which are the same as reported by Wada et al. (2011), except that the present model does not include the ISM, i.e., our study involves pure N-body simulations. We examine the global evolution of the stellar disk in Section 3 in which we show the non-steady nature of spirals developed in the galactic disk. In Section 4, we focus on a typical spiral arm and describe its dynamical evolution, i.e., the amplification (Section 4.1) and destruction (Section 4.2) processes. Section 5 examines the evolution of stars around the growing spiral arm from the viewpoint of the angular-momentum–energy space. Finally, in Section 6, we compare our simulation results with observations, and we provide a perspective view on grand-design spirals. Grand-design spirals (i.e., $m = 2$ spirals)$^4$ will be the subject of our future studies.

2. MODELS AND METHODS

The simulations of the formation and evolution of spiral arms were carried out using a three-dimensional N-body simulation. The initial point of our simulation was a disk galaxy in a nearly equilibrium state. The density profile of the stellar disk in this case is given by

$$\rho_{sd}(R, z) = \frac{M_{sd}}{4\pi R_{sd}^2 z_{sd}} \exp(-R/R_{sd}) \text{sech}^2(z/z_{sd}),$$

where $M_{sd}$ is the mass of the stellar disk, $R_{sd}$ is the radial scale length, and $z_{sd}$ is the vertical scale length. Since the distribution function of a stellar disk is unknown, we generated the equilibrium state of the stellar disk by using an empirical method: we first generated a stellar disk in a near-equilibrium state based on a Maxwellian approximation (Hernquist 1993), and we subsequently allowed the disk to evolve for 6 Gyr under the constraint of axisymmetry (McMillan & Dehnen 2007; Fujii et al. 2011).

We modeled a fixed potential of the dark matter (DM) halo whose density profile follows the NFW profile (Navarro et al. 1997):

$$\rho_h(r) = \frac{\rho_0}{r/r_s(1 + r/r_s)^2},$$

where the term “grand-design spiral” indicates a reasonably coherent and extensive spiral arm in the stellar mass distribution (see Elmegreen & Elmegreen 1982, for a more precise definition), in a majority of cases, this means that the galaxy has $m = 2$ spiral arms (Kendall et al. 2011).

3. GLOBAL EVOLUTION OF SPIRALS

Figure 2 shows the time evolution of the stellar disk. Hereafter, we use a rotational period measured at $R = 2R_{sd}$ ($\approx 8.6$ kpc), $T_{rot}$ which corresponds to 325 Myr. The top panels show the face-on views of the stellar disk. The middle panels show the radial distributions of the spiral modes which are analyzed by performing a one-dimensional Fourier decomposition of the disk surface density using the polar coordinates ($R, \phi$):

$$\frac{\Sigma(R, \phi)}{\Sigma_0(R)} = \sum_{m=0}^{\infty} A_m(R) \cos[m(\phi - \phi_m(R))].$$

Here, $A_m$ and $\phi_m(R)$ denote the Fourier amplitude and phase angle for the $m$th mode, respectively (e.g., Rix & Zaritsky 1995). Upon execution of the simulation, spiral patterns initially developed from the noise with an $e$-folding time of about 4 galactic rotations ($T_{rot} \sim 4$), and subsequently, the patterns settled to a nearly constant level, consistent with the report by Fujii et al. (2011). While the global features were in the quasi-steady state, the local spiral features were not static. The most prominent mode always changed on a rotational timescale, and it also showed radial dependence.

\begin{table}[h]
\centering
\begin{tabular}{lll}
\hline
Component & Parameter & Value \\
\hline
Dark halo & Mass ($M_h$) & $6.3 \times 10^{11} M_\odot$ \\
(Rigid) & Radius ($R_h$) & 122 kpc \\
& Concentration ($C_{NFW}$) & 5.0 \\
Initial stellar disk & Mass ($M_{sd}$) & $3.2 \times 10^{10} M_\odot$ \\
(Live) & Scale length ($R_{sd}$) & 4.3 kpc \\
& Scale height ($z_{sd}$) & 0.3 kpc \\
\hline
\end{tabular}
\end{table}
An important dynamical feature of time-dependent multi-arm spirals is that the spirals “corotate” with the galactic rotation. Figure 3 shows the angular pattern speed of a dominant stellar spiral ($m = 4$). The angular frequency is analyzed by the rate of change of phase $\phi_m(R)$ during two galactic rotation periods. The pattern speed is not constant. In fact, the pattern speed decreases with radius in a manner similar to galactic rotation for any radii. This result shows that there is no “single” corotating point in the disk, in contrast to stationary spiral waves (or a bar) with a rigid rotation. Therefore, no single spiral can last for more than one rotational period (see Section 5). Wada et al. (2011) and Grand et al. (2012c) have also reported that the spiral arms in their simulations are corotating, winding, and short-lived. Observationally, the phenomenon of corotating spiral arms has been confirmed in at least a few of the nearby spiral galaxies such as M51, NGC 1068, M101, IC 342, NGC 3938, and NGC 3344 using the Tremaine–Weinberg (TWR) analysis (Merrifield et al. 2006; Meidt et al. 2008, 2009).

It is noteworthy that the pattern speed curve seems slightly flatter (particularly when $R \sim 10–15$ kpc) than the circular rotation curve. In order to investigate the cause of the flattening, we divided this period into five periods and performed the same analysis for each period (Figure 4). The pattern speeds in all the periods decrease with increasing radius in a manner similar to galactic rotation for any radii; however, they show slightly flatter distributions at certain instants (e.g., $T_{\text{rot}} = 12.4–12.8$). This flatter distribution lasts for less than one galactic rotation period ($\Delta T_{\text{rot}} < 1$). Thus, this structure is not a long-lasting structure.
Further, it is significant that the gravitational scattering of stars by the spiral arms is not sufficiently large to erase all the spiral features. The curves in Figure 1(c) indicate that the value of $Q$ increases from an initial value of 1.2 to a final value of 1.4 around $R \approx 2R_{sd}$ at $T_{rot} = 15$. However, the spirals are not completely erased due to dynamical heating (see Fujii et al. 2011). It is to be noted that the heating effect along the direction vertical to the disk plane is negligibly small (bottom panels in Figure 2), which is consistent with the results of previous studies (e.g., Binney & Lacey 1988; Jenkins & Binney 1990).

4. GROWTH AND DAMPING PHASES OF STELLAR SPIRAL ARMS

In this section, we investigate the dynamical evolution of corotating spiral arms in detail. Figures 5 and 6 show the time evolution of a spiral arm in the corotating frame. A weak density

Figure 3. Pattern speeds of dominant mode ($m = 4$) for $T_{rot} \approx 4.0$ (left) and 12.2 (right). The pattern speed is analyzed by measuring the rate of change of phase $\phi_{m=4}$ at each annulus every 10 Myr, and these phases are combined over the periods $T_{rot} = 3.0$–$4.0$ and 11.2–13.2. The contours show the amplitude, $|A_{m=4}(R)|$, of the dominant mode. The superimposed curves show the radial variations in $\Omega, \Omega \pm \kappa/4$.

Figure 4. Pattern speeds of dominant mode ($m = 4$) for $T_{rot} \sim 12$. The pattern speed is analyzed by measuring the rate of change of phase $\phi_{m=4}$ at each annulus every 10 Myr during the periods $T_{rot} = 11.2$–$13.2$, 11.2–11.6, 11.6–12.0, 12.0–12.4, 12.4–12.8, and 12.8–13.2. The contours show the amplitude, $|A_{m=4}(R)|$, of the dominant mode. The superimposed curves show the radial variations in $\Omega, \Omega \pm \kappa/4$. 
enhancement can be observed around $\phi \simeq 240^\circ$–300$^\circ$ and $R \simeq 7$–10 kpc for $T_{\text{rot}} = 11.7$. This density enhancement causes the growth of a prominent spiral arm until $T_{\text{rot}} = 12.0$, and the arm has a maximum amplitude around $T_{\text{rot}} \sim 12.2$–12.3. Beyond $T_{\text{rot}} \simeq 12.3$, this spiral arm rapidly fades out ($T_{\text{rot}} = 12.3$–12.5). The arm merges with a neighboring weak spiral arm ($T_{\text{rot}} = 12.5$–12.7), and the peak density contrast again becomes $\delta \sim 1$ when $T_{\text{rot}} = 12.8$. In the following sections, we discuss the growth phase (Figure 5) and the damping phase (Figure 6) of the spiral separately.
Figure 6. Evolution showing thinning out of a spiral \((T_{\text{rot}} = 12.2–12.5)\), with subsequent reconnection with a neighboring spiral arm \((T_{\text{rot}} = 12.5–12.7)\).

(A color version of this figure is available in the online journal.)

4.1. Growth Phase

In the growth phase of the spiral arm \((T_{\text{rot}} \simeq 12.0–12.2)\), the pitch angle of the arm reduces due to the differential rotation of the disk. This is clearly observed in the right-hand panels of Figures 5 and 6; the two peaks at \(2R_{\text{sd}} + 1.5\) kpc and \(2R_{\text{sd}} - 1.5\) kpc show increasing separation in terms of \(\phi\). More quantitatively, Figure 7 shows the evolution of the spiral arm along the pitch-angle–density-contrast \((i - \bar{\delta})\) plane. Here, we calculated the pitch angle \((i)\) of the spiral arms using the relation

\[
\tan i = \frac{1}{R} \frac{\Delta R}{\Delta \phi},
\]

where \(R = 2R_{\text{sd}} (=8.6\) kpc\), \(\Delta R = 3\) kpc, and \(\Delta \phi\) is the azimuthal angle difference between the contrast peaks at \(R = 2R_{\text{sd}} - 1.5\) kpc (dot-dashed lines) and \(R = 2R_{\text{sd}} + 1.5\) kpc (dotted lines) shown in Figures 5 and 6. Here, we evaluated the positions
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Figure 7. Evolution of spiral arm on $i$–$\delta$ plane for $T_{rot} = 12.0$ to $12.5$. The hatched region corresponds to the predicted maximum pitch angle around the analyzed region ($Q \simeq 1.4$ and $\Gamma \simeq 0.75$–0.85) due to swing amplification (refer to Equation (7)).

of the contrast peaks at each radius by visual inspection. The arm density contrast, $\delta$, is calculated by averaging the corresponding density contrast over the radial range in Figures 5 and 6. As the pitch angle of the spiral arm decreases from $i \simeq 40^\circ$ ($T_{rot} = 12.0$) to $i \simeq 32^\circ$ ($T_{rot} = 12.20$), the density contrast increases to a maximum, and subsequently, it decreases with an increase in the pitch angle. Thus, the spiral arm has a maximum amplitude when $i \simeq 32^\circ$.

This amplification process associated with galactic shear motion is known as the swing amplification (Toomre 1981; Goldreich & Lynden-Bell 1965; Julian & Toomre 1966; Toomre & Kalnajs 1991). Fuchs (2001) calculated grid models of swing amplification process by varying the shear rate $\Gamma$ for $Q = 1.4$. In his work, he derived a fitting empirical equation for swing amplification (Equation (98) in his paper) as the following:

$$\tan i_{th} = 1.932 - 5.186 \left(\frac{1}{2} \Gamma\right) + 4.704 \left(\frac{1}{2} \Gamma\right)^2,$$  

(7)

where $i_{th}$ is the pitch angle at which the spiral arm reaches the maximum amplitude. We adopt this equation to evaluate the predicted pitch angle $i_{th}$. In our model, $\Gamma \simeq 0.8$ around $R = 2R_{sd}$ (Figure 1(c)). Thus, substituting $\Gamma \simeq 0.8$ in Equation (7), we obtain $i_{th} \simeq 32^\circ$. This value is consistent with the evolution of the spiral arm with the maximum contrast (hatched region in Figure 7).

5 D’Onghia et al. (2012) have investigated the growth of spiral arms via swing amplification, and their nonlinear evolution is not fully consistent with the classic swing-amplification picture proved by Julian & Toomre (1966).

Figure 8. Left: schematic view of the velocity field with respect to a non-steady spiral arm. The spiral arm is represented by the gray region. Right (top row): time evolution of relative velocities with respect to the stellar spiral arm ($\sim 270^\circ$), $\Delta v_\eta$ (left panel), and $\Delta v_\xi$ (right panel), at $T_{rot} = 12.1$ (solid line), 12.3 (dashed line), and 12.5 (dot-dashed line). The analysis is performed for the strip along the $\eta$-axis with a width of $|\xi| = 500$ pc. Middle row: time evolution of non-axisymmetric gravitational forces (i.e., spiral perturbation) $\Delta F_{grav} = F_{grav} - F_{cir}$. Here, $F_{cir}$ denotes the azimuthally averaged gravitational forces. Bottom row: time evolution of “net” non-axisymmetric forces $\Delta F_{grav+cori} = \Delta F_{grav} + \Delta F_{cori}$. Here, the Coriolis force perturbation $\Delta F_{cori}$ is calculated as $-2\Omega \times (v - v_{cir})$.

(A color version of this figure is available in the online journal.)
4.2. Damping Phase

The swing amplification mechanism can explain certain aspects of the evolution of spiral arms, i.e., the amplification (excitation) of density enhancement. However, the destruction process of the non-steady spirals, as seen in Figure 6 (see also Fujii et al. 2011; Wada et al. 2011), cannot be understood only by the swing amplification mechanism. The top right panels in Figure 8 show the time evolution of the relative velocities ($\Delta v_\eta$ and $\Delta v_\xi$) in the corotating frame of the spiral arm. Upon considering the coordinates ($\eta, \xi$), i.e., the $\eta$- and $\xi$-axes are perpendicular and parallel to the spiral arm, respectively, the velocity component along each is given by

$$\Delta v_\eta \equiv v_R \cos i + (v_\phi - v_{cir}) \sin i,$$

(8)

$$\Delta v_\xi \equiv -v_R \sin i + (v_\phi - v_{cir}) \cos i,$$

(9)

where $i$ is the pitch angle of the spiral arm, and $v_{cir}$ is the circular velocity determined by the azimuthally averaged gravitational field. For this definition, inflow to the arm corresponds to $\Delta v_\eta > 0$ for $\eta < 0$, and $\Delta v_\eta < 0$ for $\eta > 0$. In the amplification phase ($T_{rot} \lesssim 12.2$), we see a clear inflow motion to the spiral arm along both sides of the chosen strip (solid lines). In the initial stages of the damping phase ($T_{rot} \simeq 12.3$), the streaming velocity around the spiral arm gradually transits from inflow to outflow. At the end of the damping phase ($T_{rot} \simeq 12.5$), it is clear that the stars in the spiral arm move away from the arm along both its sides (dot-dashed lines). During the dynamical evolution, the parallel component of the velocity, $\Delta v_\xi$, does not change its sign or show a decrease in its magnitude. The right panels of Figure 8 show a schematic view of the time evolution of the non-circular velocity field associated with the non-steady spiral arm.

The middle right panels in Figure 8 show the time evolution of the non-axisymmetric gravitational forces (i.e., spiral perturbation) involved in the damping phase. The component of the non-axisymmetric gravitational force perpendicular to the spiral arm ($\Delta F_{grav,\eta}$), which is stronger than the parallel component ($\Delta F_{grav,\xi}$), is always directed toward the arm ($\Delta F_{grav,\eta} > 0$ for $\eta < 0$, and $\Delta F_{grav,\eta} < 0$ for $\eta > 0$) during the dynamical evolution. The bottom right panels in Figure 8 show the time evolution of the net force (i.e., non-axisymmetric gravitational force plus Coriolis force perturbation). The component of the net force perpendicular to the spiral arm ($\Delta F_{grav+cori,\eta}$) evolves in the same manner as $\Delta F_{grav,\eta}$ because there is a strong density gradient along the $\eta$-direction. The parallel component $\Delta F_{grav,\xi}$ is almost zero during the growth and damping phases and does not change. This is because the density gradient along the $\xi$-direction is small and almost unchanged. However, the parallel component of the net force $\Delta F_{grav+cori,\xi}$ changes its $\eta$-dependence from the growing phase ($T_{rot} = 12.1$) to the damping phase ($T_{rot} > 12.3$) according to the change of the sign of the perpendicular velocity ($\Delta v_\eta$), since the Coriolis force works perpendicular to the direction of the velocity.

This indicates that the Coriolis force exerted on the stars in the damping phase exceeds the non-axisymmetric gravitational force due to the spiral perturbation. This causes stars to "escape" from the spiral perturbation, and eventually the spiral arm itself begins to thin out and fade.

The above argument suggests that the non-steady nature of stellar spirals is originated in the evolution of the orbits of the stars in the spirals. The phenomenon of swing amplification (Toomre 1981) is a part of this nonlinear coupling between particles and waves, but it does not describe all observed phenomena. In the next section, we explore the orbital evolution of stars associated with the spiral arm in detail.

5. ORBITAL EVOLUTION OF STARS AROUND SPIRAL ARMS

Figure 9 shows the evolution of stars along the $\phi$–$R$ plane in the early phase of spiral development. At this stage of the simulation, we selected 15 particles associated with one of the three weak spiral arms that had evolved at $T_{rot} = 4.0$. The figure shows that stars with epicycle motion are captured by the density enhancement ($T_{rot} = 3.6$–$4.0$), and further, these stars are dissociated from the original arm ($T_{rot} = 4.2$–$4.6$). This behavior is similar to that of the “density wave” from a certain viewpoint, but the spiral arms are short-lived and corotating (left panel of Figure 3). Thus, even if the arms appeared in the early linear phase, they are not completely explained by the picture of stationary density waves. The middle column of images in Figure 9 shows the plots of the azimuth angle ($\phi$) versus the angular momentum $L_\phi$ curve instead of the $\phi$–$R$ plot. It is clear that all the stars along the $\phi$–$L_\phi$ plane oscillate horizontally when the angular momentum of each star is conserved. The images in the right column of Figure 9 show the so-called Lindblad diagram, where the angular momentum $L_\phi$ of each star is plotted against its total energy $E$. Most of the stars show no significant movement from their original position, thereby suggesting that their energy is also conserved.

On the other hand, the behavior of the stars is very different in the nonlinear phase, where the spiral arms are well developed and non-steady (refer to previous sections). Figure 10 shows images identical to those in Figure 9; however, in this set of images, the stars are in motion due to change in their angular momenta in the nonlinear phase of orbital evolution (see also the supplementary video). When the stars are captured by the density enhancement ($T_{rot} \simeq 11.8$–$12.0$), they radially migrate along the spiral arms. The stars approaching from behind the spiral arm (i.e., inner radius) tend to attain increased angular momenta via acceleration along the spiral arm, thereby move to the disk’s outer radius. In contrast, the stars approaching ahead of the spiral arm (i.e., outer radius) tend to lose their angular momenta via deceleration along the spiral arm, and they move to the disk’s inner radius. Along the $\phi$–$L_\phi$ plane, the stars oscillate both horizontally as well as vertically. Moreover, the guiding centers of the oscillations do not remain constant at the same value of $L_\phi$. This is essentially different from the epicycle motion in which $L_\phi$ is conserved.

The Lindblad diagram in Figure 10 shows that the stars oscillate along the curve of circular motion by undergoing change in terms of both angular momentum and energy. The oscillating stars successively undergo aggregation and disaggregation along the curve, thereby leading to the formation of structures referred to as “swarms of stars” along the $\phi$–$L_\phi$ and $R$–$\phi$ planes. The non-steady nature of the spiral arms originates in the dynamical interaction between these “swarming” stars with a nonlinear epicycle motion and the high-density regions, i.e., the spiral arms moving with the galactic rotation. This is similar to the wave–particle interaction described previously; however, in this case, the high-density regions are not “waves.” Since the non-steady spirals move at the rate of the local galactic rotational speed (Section 3), in contrast to a spiral perturbation with a single pattern speed, corotating points are found everywhere on the spiral arms. Therefore, the motion of stars along the $E$–$L_\phi$ plane can be naturally understood as due to their
scattering around corotating spirals. This behavior is similar to that reported in previous studies (e.g., Sellwood & Binney 2002; Grand et al. 2012a, 2012c; Bird et al. 2012). Furthermore, we observe that the \( E-L_z \) curve of each star changes over a large range of radii. This is entirely different from what is expected in stationary density waves, where these changes are limited to the Lindblad resonances (Lynden-Bell & Kalnajs 1972). Further, it is noteworthy that the structures formed self-induced in the angular momentum space; this is a property similar to that of the “groove” mode hypothesized by Sellwood & Lin (1989). This point is beyond the scope of this paper. We will investigate this elsewhere.

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**Figure 9.** Orbital evolution of stars in the spiral arm. The stars associate around the spiral arm within a distance of \( \pm 0.5 \) kpc at \( T_{\text{rot}} = 4.0 \). Left columns: orbits on \( \phi-R \) plane. Middle columns: orbits on \( \phi-L_z \) plane. Right columns: orbits on \( E-L_z \) plane. The colors denote the angular momentum at the time instants when the stars associated with the spiral arm.

(An animation of this figure is available in the online journal.)

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6 Stars around the corotation point change their angular momenta without increasing their random energy (Lynden-Bell & Kalnajs 1972).
Figure 10. Orbital evolution of stars in the spiral arm during the nonlinear phase. The stars associate with the spiral arm at $T_{\text{rot}} = 12.2$. (An animation of this figure is available in the online journal.)

The time evolutions of the angular momentum of stars associated with a spiral arm at $T_{\text{rot}} \simeq 4.0$ and $T_{\text{rot}} \simeq 12.2$ are plotted in the top left panels of Figures 11 and 12, respectively. In the early phase ($T_{\text{rot}} \simeq 4.0$), both the angular momentum and random energy do not change by more than 10% during one rotational period (top right panel of Figure 11). On the other hand, in the nonlinear phase ($T_{\text{rot}} \simeq 12.2$), the fraction of the angular momentum changes by $\sim 50\%$ (top right panel of Figure 12). It is clear that the angular momentum of the stars changes significantly due to their scattering by a well-developed spiral at $T_{\text{rot}} = 12.2$. This corresponds to the hypothesis that the guiding center of the epicycle motion of each star undergoes radial motion.

The change in the normalized random energy shown in the bottom left panels of Figures 11 and 12 indicates that certain stars with relatively small initial random energies experience
Figure 11. Time evolution of orbital characteristics of stars around spiral arm during early evolutional stage shown in Figure 9. Left: time evolution of the orbital angular momentum (top) and the random energy (bottom) of the stars. The plotted stars are the same as those in Figure 9. The random energy of the stars is non-dimensional using the radius of the guiding center $R_{gc}$ and epicyclic frequency $\kappa_{gc}$, which is proportional to the square of the orbital ellipticity $e$. Each star is indicated by a different line thickness. The colors correspond to those of Figure 9. Right: angular momentum change, $L_{fin} - L_{ini}$, distributions of stars (top). Here, $\Delta T_{rot} = 0.8$ is adopted to measure the angular momentum change and the vertical axis indicates the normalized number of particles. The random energy changes, $E_{rand,fin} - E_{rand,ini}$, in the stars over $\Delta T_{rot} = 0.8$ as a function of their angular momentum changes (bottom). The colors correspond to those in the left panels.

a large energy change $\Delta E_{rand}$ after interaction with a spiral arm. Here, the random energy is calculated as the difference between the total energy and the circular energy, i.e., $E_{rand} = E - E_{cir}(R_{gc})$. It is noteworthy that the random energy change is not always positive; a significant fraction of stars lose their random energy. This is because the perturbation from the spiral arm shifts the guiding center of the stars’ epicycle motion without increase in orbital eccentricity. It is to be noted that this argument is rigorously correct; in fact, Figure 12 (bottom left panel) shows the changes in the random energy of the stars as a function of change in the angular momentum. We can see a weak trend: the outer migrators (i.e., $L_{fin} - L_{ini} > 0$) undergo a decrease in their random energy ($E_{rand,fin} - E_{rand,ini} < 0$) and vice versa. A similar effect of the radial migration of stars around spiral arms upon disk heating has been noticed in recent numerical simulations (Grand et al. 2012a, 2012c; Roškar et al. 2012; Minchev et al. 2012).

In summary, the gravitational interaction between the stars in the spiral arm and the spiral density enhancement changes the angular momentum and random energy of the stars, and this process in turn changes the structure of the spirals. During this process, the random energy of individual stars in the system does not increase monotonically. In other words, local interactions between the non-steady arms and stars increase or decrease the total energy of individual stars locally; however, the energy remains around its value for circular motion with the occurrence of a small dispersion. This is because the interaction causes the migration of the guiding centers of the stars without increasing their eccentricity or random energy. This “dynamical cooling” mechanism is essential to preventing heating of the stellar disk and erasure of the spiral arms, and the mechanism produces “swarms” of stars moving between non-steady spirals. The nonlinear epicycle motion of the stars and their nonlinear coupling with the density perturbation is the fundamental physics of the recurrently formed, non-steady spiral arms in a stellar disk.

6. DISCUSSION

Figure 13 compares the morphological properties (the pitch angle $i$ and the amplitude $|A|_{\text{init}}$) of the stellar spiral arms between simulations and observations. For the comparison, we used a two-dimensional fast Fourier transform (2D FFT) analysis with logarithmic spirals (for details, see, e.g.,
Figure 12. Time evolution of orbital characteristics of stars for the phase in which the spiral arms are nonlinearly developed (shown in Figure 10). The plotted stars are the same as those in Figure 10.

However, this comparison between the simulated and observed distributions requires further examination. First, the determination of the pitch angle is somewhat uncertain. The analyzed pitch angles differ between studies for the same galaxy. For example, the pitch angles of NGC 3054 are given by 33°, 43°, and 12° in Grosbøl et al. (2004), Seigar et al. (2006), and Davis et al. (2012), respectively. Second, there is a lack of statistical studies on the relation between the pitch angle and the amplitude of the stellar spiral arms. Although some statistical studies have used 2D FFT to analyze the pitch angle of the spiral arms, they do not include examination of the amplitude or contrast of the spiral arms (e.g., Seigar et al. 2005, 2006; Davis et al. 2012). Thus, further progress in understanding the spiral dynamics requires additional statistical and robust observation data of the morphological properties (both the pitch angle and amplitude) of the stellar spiral arms. This relation between the pitch angle and amplitude will form one of the tests for spiral genesis theories along with the pitch-angle–shear-rate (or Hubble-type) relation (Roberts et al. 1975; Seigar & James 1998; Hozumi 2003; Grand et al. 2012b), existence of systematic angular offset between the young stellar component and the stellar spiral arm (Fujimoto 1968; Roberts 1969; Egusa et al. 2009; Foyle et al. 2011; Ferreras et al. 2012), and the radial dependence of the speed pattern (Meidt et al. 2008, 2009).

The plotted points in green and red symbols represent the time evolution of the spiral arms for the two models that are essentially identical (see the footnote). We found that the simulated spiral arms exhibit a maximum amplitude around $i \sim 30^\circ$. Further, we found that the spiral arms tend to be weaker in the pure stellar disk model (model MS) than in the model with ISM (model MSG).\(^7\) This distribution, which has a peak at around $i \sim 20^\circ$–$30^\circ$, is qualitatively consistent with the prediction of the swing amplification mechanism (see Section 4.1). We compared the simulated distribution with the observed one. The dependence of $|A_m|$ on the pitch angle in the models with/without ISM is consistent with observations at least for the range of values corresponding to $i < 30^\circ$. This comparison suggests that the self-induced spiral arms in differentially rotating disks and their time evolution can fairly consistently account for the morphological diversity seen in nearby spiral galaxies.

\(^7\) Model MSG is the same model as that presented by Wada et al. (2011). This model is mostly identical to model MS; however, in model MSG, the ISM, star formation, and supernova feedback are also taken into account. The hydrodynamics of the model is solved by the smoothed particle hydrodynamics (SPH) method. The initial gas mass fraction is 10% of the stellar disk mass, and the initial density profile follows an exponential profile with a scale-length twice that of the stellar disk. Refer to Wada et al. (2011) for details.
Finally, we comment on grand-design spirals. In a series of studies (Fuji et al. 2011; Wada et al. 2011) including this one, we have examined the stellar dynamics of non-steady stellar spirals as well as the interactions between them and the ISM. However, these studies have focused more on multi-armed spirals than grand-design spirals (i.e., $m = 2$ spirals) whose fraction is more than ~50% in nearby spiral galaxies (Grosbøl et al. 2004; Kendall et al. 2011). It has been observed that grand-design spirals are associated with bars or companions (Kormendy & Norman 1979; Seigar & James 1998; Salo et al. 2010; Kendall et al. 2011), and this observation is consistent with the results of many numerical simulations of bar-driven spirals (e.g., Sellwood & Sparke 1988; Bottema 2003) and tidally driven spirals (e.g., Oh et al. 2008; Dobbs et al. 2010; Struck et al. 2011). We will focus on bar-driven spirals and tidally induced spirals in forthcoming studies.

7. SUMMARY

The $N$-body simulations of an isolated disk galaxy show the formation of self-induced, non-steady multi-arm spirals that follow the differential galactic rotation. We found that the swing amplification mechanism causes the development of spirals. When a spiral undergoes the damping phase, the Coriolis force dominates the gravitational perturbation exerted by the spiral, and as a result, stars escape from the spirals, and join a new spiral at a different position. This process is uniform for a given spiral, thereby resulting in the formation of bifurcating and merging spiral arms; therefore, the dominant spiral modes always show change in their radii over time. We confirmed that this phenomenon originates due to the changing orbital properties of stars. The angular momentum and energy of each star undergo changes due to the star’s interaction with the spiral arms. As a result, the epicycling stars radially migrate; in other words, their guiding centers also undergo motion. Interestingly, the movement of groups of stars with similar orbital properties causes the appearance of “swarming.” In the nonlinear phase of the development of spiral instability, the swarming stars cause complicated morphological changes in the spiral arms.

During this process, the random energy of individual stars (or orbital eccentricity) does not increase monotonically. In fact, a significant fraction of stars even lose their random energy. This “dynamical cooling” due to a mechanism like the wave–particle interaction can explain why the short-lived spiral arms are self-induced over several rotational periods despite the absence of a dissipative component in the disk (Fuji et al. 2011).

In the above process, it is essential that spiral arms mostly follow the galactic rotation at any radius; in other words, the “corotating points” are required to be ubiquitous in the differentially rotating galactic disk. The conclusions of other previous studies also indicate the possibility that the corotation resonance affects stellar motions more in terms of radial migration than heating up of the disk (Lynden-Bell & Kalnajs 1972; Sellwood & Binney 2002; Grand et al. 2012a, 2012c; Roškar et al. 2012; Minchev et al. 2012).

We conclude that the nonlinear epicycle motions and self-gravity in the differential rotation of stellar disks are essential for the recurrent amplification and destruction processes of the spiral arms. In other words, the issue of the so-called winding dilemma is no longer a problem at least in multi-armed spiral galaxies.

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**Figure 13.** Pitch-angle–amplitude correlation. Morphological properties (amplitudes $A_m$ and pitch angles $i$) of stellar spiral arms are derived using a two-dimensional Fourier fitting to the range of radii given by $R_{sd} < R < 3R_{sd}$. Here, $|A_m|$ values for the $m = 4$ mode in the simulations are shown. Model MSG is nearly identical to Model MS; however, model MSG also includes the gaseous component with a mass fraction that is 10% of the stellar disk mass (refer to Wada et al. 2011). The filled triangles represent the observational data obtained from the study by Grosbøl et al. (2004).

(A color version of this figure is available in the online journal.)
