Method for Calculating Static Temperature of Turbofan Engine Compressor Outlet Airflow

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Abstract. In order to solve the problem that the temperature of the compressor outlet air in turbofan engine is not measurable, a method of calculating the static temperature of the compressor outlet air based on the total temperature of the compressor outlet air in the engine bench test data is proposed, according to the principle of aerothermodynamics. And a Simulink model is established. The total temperature, total pressure and design point flow at the compressor outlet of the engine are brought into the Simulink model to calculate. The results show that the static temperature of the compressor outlet at the design point is $T_s = 0.9873 T^*$. According to the inverse relationship between temperature ratio function and N2 speed, the linear interpolation is carried out. The relationship between static temperature of compressor outlet and total temperature is $T = (1 - 0.0127 N_2) T^*$.

1. Introduction
In the exhaust system of an aircraft, the warming gas of the cockpit comes from compressor outlet airflow of a turbofan engine [1]. In engine bench test, the static temperature of compressor outlet is not measured, and the total temperature cannot be used as static temperature directly. After the engine is installed, the total are even not measured. Therefore, the temperature of compressor outlet airflow is always estimated during the heating process of aircraft cockpit [2]. Based on the engine test data and the total temperature of compressor outlet airflow, a method for calculating the static temperature of compressor outlet airflow is proposed and a Simulink model is established.

2. Calculating static temperature at design point
In the modeling technology of variable specific heat of engine, the specific heat ratio $c_p$ of gas at constant pressure are usually expressed as a function. Previous researchers have fitted the specific heat ratio at constant pressure into a polynomial of temperature (total temperature) through experiments and statistical analysis [3], and the results of calculation with satisfactory accuracy can be obtained. The specific fitting polynomial form is:
\[ c_p = 10^{-3} \sum_{i=1}^{n} a_i (T^*)^{i-1} \]  \hspace{1cm} (1)

Where \( T^* \) is total air temperature, \( a_i \) is polynomial fitting coefficient. The specific values of \( a_i \) are shown in the table below [4].

| coefficient | \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( a_5 \) | \( a_6 \) | \( a_7 \) | \( a_8 \) |
|-------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| values      | 0.30183674e6 | 0.10489652e7 | 0.23284057e6 | 0.45288431e6 | 0.31308477e6 | 0.11341362e6 | -0.21298087e5 | 0.16363600e4 | 0 |

The formula for calculating the adiabatic exponent \( k \) is:

\[ k = \frac{c_p}{c_p - R} \]  \hspace{1cm} (2)

Where \( R \) is the gas constant of a specific gas, for air, \( R=287.06 \).

The total temperature \( T^* \) and pressure \( P^* \) of the engine compressor are known. After determining the adiabatic exponent \( k \) and gas constant \( R_g \) at the current interface, according to the relationship between flow \( W \) and flow function \( q(\lambda) \), function \( q(\lambda) \) can be calculated. The relationship between flow \( W \) and flow function \( q(\lambda) \) is:

\[ W = K \frac{P^*}{\sqrt{T^*}} Aq(\lambda) \]  \hspace{1cm} (3)

Where \( K = \sqrt{\frac{k}{R_g \left( \frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}} \), and \( R_g = R \) due to the fact that all gases in compressors are air.

After calculating the flow function \( q(\lambda) \), the velocity coefficient \( \lambda \) can be obtained. The expression of the flow function \( q(\lambda) \) is:

\[ q(\lambda) = \left( \frac{k+1}{2} \right)^{\frac{k}{k-1}} \lambda \left( 1 - \frac{k-1}{k+1} \lambda^2 \right)^{\frac{k}{k-1}} \]  \hspace{1cm} (4)

According to the temperature ratio function \( \tau(\lambda) \) in the gas dynamic, the static temperature \( T_s \) of the gas flow can be calculated [5]. The expression of temperature ratio function \( \tau(\lambda) \) is:
\[ \tau(\lambda) = \frac{T_s}{T^*} = 1 - \frac{k-1}{k+1} \lambda^2 \]  

Combining with the engine model, the process is modeled in simulink as shown in Figure 1.

![Simulink model](image)

**Figure 1.** Simulink modeling for calculating static temperature based on total temperature

The total temperature, total pressure and flow at the design state point of the engine (N2=100%) are brought into the calculation of the above process. The calculation result is \( \frac{T_s}{T^*} = 0.9873 \), and then the static temperature \( T_s \) are obtained.

3. **Calculating static temperature in Arbitrary State**

It can be seen from the formula of temperature ratio function \( \tau(\lambda) \) that \( \tau(\lambda) \) is 1 when \( \lambda = 0 \), and temperature ratio function \( \tau(\lambda) \) decreases with the increase of velocity coefficient \( \lambda \). In all the states of the engine, the velocity coefficient of the design point (N2=100%) is the maximum, and the temperature ratio function of the design point is the minimum. Therefore, it can be concluded that the temperature ratio function \( \tau(\lambda) \) decreases with the increase of N2 speed. When N2=0, \( \tau(\lambda) = 1 \), and when N2=100%, \( \tau(\lambda) = 0.9873 \).

\( \tau(\lambda) \) cannot be calculated in real time because the flow of engine compressor is not measurable in other states. According to the inverse relationship between temperature ratio function \( \tau(\lambda) \) and N2 speed, the linear interpolation is carried out and the functional relationship is:

\[ \tau(\lambda) = 1 - 0.0127 \times N_2 \]  

Then the static temperature of compressor outlet is:

\[ T_s = \left(1 - 0.0127 \times N_2\right)T^* \]  

Where \( T^* \) is the total temperature of engine compressor outlet, and N2 is the relative speed of high-pressure rotor, ranging from 0 to 100%.

From another point of view, according to the definition formula of velocity coefficient \( \lambda \) and Mach number \( M \), the relationship between them is:
\[ \lambda^2 = \frac{k + 1}{2} \frac{M^2}{1 + \frac{k - 1}{2} M^2} \]  \hspace{1cm} (8)

Then the relationship between temperature and Mach number \( M \) is:

\[ \frac{T_s}{T^*} = \frac{1}{1 + \frac{k - 1}{2} M^2} \]  \hspace{1cm} (9)

It can be seen from the above formula that \( T^*/T_s \) is approaching when the Mach number \( M \) is small, and there is a significant difference between \( T^* \) and \( T_s \) when the Mach number \( M \) is large, that is, the temperature ratio function \( \tau(\lambda) \) is inversely proportional to the Mach number \( M \) as well. The Mach number \( M \) of compressor outlet gas flow at design point (N2=100%) is the maximum of all engine states. Therefore, with the increase of engine speed N2, the temperature ratio function \( \tau(\lambda) \) decreases from 1 to 0.9873. Linear interpolation of temperature ratio function \( \tau(\lambda) \) with rotating speed N2 is used to obtain the relationship between static temperature at compressor outlet and total temperature at outlet. The relationship is:

\[ T = (1 - 0.0127 * N_2) T^* \]  \hspace{1cm} (10)

4. Conclusion
Based on the principle of aerothermodynamics, a method of calculating the static temperature of compressor outlet air by using the total temperature of compressor outlet air in engine bench test data is proposed. The total temperature, total pressure and flow rate at the design point of the compressor outlet are brought into the calculation, and the temperature ratio function at the design point state, which is the relationship between total temperature and static temperature, is obtained. According to the inverse relationship between temperature ratio function and N2 speed, the linear interpolation is carried out. The relationship between static temperature at compressor outlet and total temperature at outlet is \( T = (1 - 0.0127 * N_2) T^* \).

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