1. Introduction

When designing thermoelectric cooling devices (TCD) for the heat-loaded elements of electronics operating under pulsed, short-term, and cyclical modes, the main requirement is to ensure a predetermined thermal mode of operation in real time. Going beyond the boundary temperature limits leads to failure not only of a given element, but, often, of the entire control system.

When constructing a thermoelectric cooler, the following is usually set: temperature difference $\Delta T$, temperature of a heat-emitting joint $T=300$ K, heat load magnitude $Q_0$. Various limitations are imposed: on power consumption $W$, on working current magnitude $I$, on reliability indicators: failure rate $\lambda$ and failure-free operation probability $P$.

The conditions for thermoelectric cooler operation in terms of specific operating conditions impose requirements on its dynamic characteristics and, contrary to them, indicators of reliability. Finding a compromise in the design of coolers is a relevant task of our study.

2. Literature review and problem statement

The results of thermodynamic studies into the systems that ensure the thermal modes of microprocessor elements built on thermoelectric devices are reported in work [1]. It has been shown that an increase in the density of thermal flows leads to a deterioration in the reliability of coolers. Paper [2] describes studies of the connection between a heat load and the mechanical deformation of thermoelectric materials, which cannot be compensated by free deformation. Article [3] addresses analysis of the effect of a temperature gradient on the mechanical stresses and the related reliability of thermoelectric coolers. Given the fundamental character of a reliability concept, studies on the impact of...
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various factors on reliability indicators are quite common in the scientific literature. An effect of the temperature and heat stress around an elliptical functional defect in a thermoelectric material was considered in [4]. A solution to this problem was a devised technique to improve the structural integrity of thermoelectric modules with changing geometry [5], which reduced the impact of thermal deformations. The results from studying the effect of heterogeneous thermal fields emerging in the heterogeneities of a thermoelectric material and macro-scale holes on reliability indicators of the cooler are reported in [6]. Authors of [7] presented a numerical simulation of the thermoelectric cooler taking into consideration the circulation of heat in the air gaps between the thermoelements, which affect their temperature gradients. Work [8] considers the impulse operation of thermoelectric coolers, which leads to an increase in average cooling capacity, but the pulse mode exacerbates the problem of temperature gradients.

The results on the impact of heat flow density, mechanical deformation due to a temperature gradient, structural integrity of a thermoelectric material, on the TCD reliability are qualitative in character and do not make it possible to numerically assess and predict reliability indicators.

The needs of aerospace applications have made reliability indicators prioritized in the development of thermoelectric devices using a topology optimization [9]. However, mechanical stresses are only one component of the reliability problem. Another aspect related to the problem of reliability of thermoelectric coolers is the pulse operation of thermoelectric coolers [10], at which a change in temperature gradients becomes an operational mode of the cooler. Switch mode is used for accelerated testing of thermoelectric coolers, under which operational reliability indicators deteriorate by an order of magnitude. The need to use a pulse mode as a working one in the systems of maintaining operational conditions for heat-loaded elements suggests that research is needed on the ability to manage the dynamic characteristics of thermoelectric coolers [11]. An analysis of the relationship between the dynamics and reliability indicators of the cooler and its structure and modes of operation is given in [12]. A study [13] was carried out into the time it takes to enter a stationary mode, taking into consideration the influence of mass and technological elements in a single-cascade thermoelectric device. The dynamics and reliability indicators for the thermoelectric cooler with a certain geometry of branches were analyzed in [14] producing optimized solutions.

At the same time, when designing TCDs for the elements of electronics that function under stationary, especially pulse, short-term, and cyclical modes, the main requirement is to ensure the predetermined thermal mode of operation in real time.

### 3. The aim and objectives of the study

The aim of this study is to determine the attainable time it takes for a single-cascade thermoelectric cooler to enter a stationary mode of operation in a predefined range of temperature changes and working currents.

To accomplish the aim, the following tasks have been set:
- to analyze a dynamic model of TCD in the range of working temperature changes under the main energy modes of a single-cascade cooler’s operation;
- by setting the time it takes to enter a stationary mode, determine the required relative working current, basic energy indicators, the number of thermoelements and the intensity of failures.

### 4. Dynamic model of a thermoelectric cooler in the range of working temperature changes and working currents

Based on the analysis of ratios given in [11], we derived an expression to determine a relative working current $B_k$ depending on the time it takes to enter a stationary mode of operation $\tau$

$$B_k^\tau = 2B_k \frac{A - \gamma_k B_{h_k}^\tau + A\Theta}{A - \gamma_k B_{h_k}^\tau} = 0, \sqrt{a^2 + b^2},$$  \hspace{1cm} (1)

where

$$A = \exp\left(\frac{1}{T_0} \sum mC, \frac{\Delta T_{max}}{T_0}\right);$$

$$B_k = \frac{\gamma - \gamma_k}{\gamma - \gamma_k} \left[1 - \sqrt{1 - \frac{A\Theta(A - \gamma_k B_{h_k}^\tau)}{(A - \gamma_k B_{h_k}^\tau)}\right].$$  \hspace{1cm} (2)

The minimum time it takes to enter a stationary mode of operation $\tau_{\text{min}}$ is ensured under a $Q_{\text{lim max}}$ mode at $B_k=1$:

$$\tau_{\text{min}} = \frac{\sum mC,}{K \left(1 + 2 \frac{\Delta T_{max}}{T_0}\right)} \ln \frac{\gamma B_{h_k}^\tau (2 - B_{h_k}^\tau)}{1 - \Theta},$$  \hspace{1cm} (3)

where:
- $\sum mC, = 175 \times 10^{-4} J/K$ – the total magnitude of the product of heat intensity and the mass of CTE components at $I/S=10 \text{ cm}^{-1}$;
- $B_k I/I_{\text{max K}}$ – relative working current at $\tau$;
- $I$ – magnitude of the working current, A;
- $I_{\text{max K}} = \gamma_k T_0 / R_k$ – maximal working current, A;
- $I_{\text{max K}}$, $R_k$ – maximal working current, A, and electrical resistance of a thermoelement branch, Ohm, at the end of a cooling process, respectively;
- $T_0$ – temperature of a heat-absorbing joint, K, at $\tau$;
- $K = \frac{P_k}{(I/S)}$ – heat transfer ratio, W/K;
- $P_k$ – average thermal conductivity ratio of thermoelectric materials, W/(cm·K);
- $I/S$ – height $l$ and cross-sectional area $S$ of a thermoelement’s branch, 1/cm, respectively;
- $\gamma = \frac{I_{\text{max K}} R_k}{P_k}$;
- $I_{\text{max K}} = \gamma_k T_0 / R_k$ – maximum working current, A, at $\tau=0$;
- $I_{\text{max K}}$, $R_k$ – maximum working current, A, and electrical resistance of a thermoelement’s branch, Ohm, respectively, at the beginning of a cooling process at $\tau=0$;
\[ \lambda/\lambda_0 = n R^2 (\Theta + C) \left( B_k + \frac{\Delta T_{\text{max}}}{T_0} \right)^2 \left( 1 + \frac{\Delta T_{\text{max}}}{T_0} \right)^2 K_T, \]  

where \( C = \frac{Q_0}{I_{\text{max}} R_k n} \) is the relative heat load; \( K_T \) is the factor of lower temperatures [15].  

The probability of failure-free operation \( P \) can be determined from expression  

\[ P = \exp (\lambda t), \]  

where \( t \) is the assigned resource, hours.  

### 5. Analysis of the model’s time and reliability indicators for different current modes and temperature changes  

Let us set the time that it would take to enter a stationary mode of operation \( \tau \), taking into consideration CTE at the assigned temperature difference \( \Delta T \) and heat load \( Q_0 \). By using a method of sequential approximations, one can determine a relative working current \( B_k \) and, therefore, the basic parameters and reliability indicators. One or two approximations would suffice.  

Table 1 gives the results of calculations, relative to working current \( B_k \), of basic parameters, reliability indicators, for temperature changes \( \Delta T = 20; 30; 40; 50; 60 \) K at heat load \( Q_0 = 0.5 \) W, when \( l/S = 10^{-4} \). Considering  

\[ \sum mC_i = 175 \times 10^{-4} \text{ J/K}. \]  

| \( \tau \), s | \( B_k \), A | \( I_k \), A | \( n \), pcs. | \( W \), W | \( E \), V | \( U \), V | \( B_k \) | \( \lambda/\lambda_0 \) | \( \lambda \times 10^4 \) | 1/h | \( P \) | Mode of operation |
|--------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----------|
| 1            | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 11     | 12      |          |
| \( \Delta T = 20 \) K; \( T = 300 \) K; \( T_{\text{max}} = 93.7 \) K; \( \Theta = 0.213; \) \( K = 15.8 \times 10^{-4} \) W/K; \( \gamma = 1.132; R_k = 10.64 \times 10^{-3} \) Ohm; \( I_{\text{max}} = 5.29 \) A |
| 2.4          | 1.0    | 5.29   | 2.1    | 1.34   | 0.373  | 0.25   | 0.96   | 2.12   | 0.67   | 0.99936 | \( Q_0 \) max |
| 3.8          | 0.46   | 2.44   | 3.4    | 0.50   | 1.0    | 0.20   | 0.44   | 0.128  | 0.38   | 0.99962 | \( Q_0/\tau \) max |
| 4.0          | 0.44   | 2.32   | 3.6    | 0.48   | 1.05   | 0.21   | 0.421  | 0.107  | 0.32   | 0.99968 |
| 5.0          | 0.35   | 1.84   | 4.7    | 0.41   | 1.23   | 0.22   | 0.333  | 0.05   | 0.15   | 0.99985 |
| 6.0          | 0.29   | 1.55   | 5.9    | 0.37   | 1.33   | 0.24   | 0.281  | 0.03   | 0.09   | 0.999912 |
| 7.0          | 0.26   | 1.35   | 7.2    | 0.356  | 1.40   | 0.26   | 0.245  | 0.02   | 0.06   | 0.999942 |
| 8.8          | 0.213  | 1.13   | 10.0   | 0.36   | 1.40   | 0.32   | 0.204  | 0.012  | 0.036  | 0.999963 |
| 14.1         | 0.158  | 0.84   | 21.5   | 0.46   | 1.08   | 0.55   | 0.152  | 0.0672 | 0.022  | 0.999979 |
| \( \Delta T = 30 \) K; \( T = 300 \) K; \( T_{\text{max}} = 86.8 \) K; \( \Theta = 0.346; \) \( K = 15.8 \times 10^{-4} \) W/K; \( \gamma = 1.123; R_k = 10.3 \times 10^{-3} \) Ohm; \( I_{\text{max}} = 5.16 \) A |
| 4.2          | 1.0    | 5.16   | 2.8    | 1.71   | 0.29   | 0.33   | 0.94   | 2.84   | 8.5    | 0.99915 | \( Q_0 \) max |
| 5.7          | 0.588  | 3.0    | 3.8    | 0.86   | 0.58   | 0.28   | 0.55   | 0.43   | 1.30   | 0.9987  |
| 7.0          | 0.466  | 2.4    | 4.9    | 0.72   | 0.69   | 0.30   | 0.436  | 0.21   | 0.63   | 0.99937 |
| 9.8          | 0.346  | 1.8    | 8.0    | 0.69   | 0.72   | 0.39   | 0.32   | 0.095  | 0.28   | 0.99972 |
| 12           | 0.30   | 1.55   | 11.1   | 0.75   | 0.67   | 0.48   | 0.28   | 0.071  | 0.213  | 0.99978 |
| 14.5         | 0.266  | 1.37   | 15.8   | 0.87   | 0.58   | 0.63   | 0.25   | 0.06   | 0.18   | 0.99982 |
| \( \Delta T = 40 \) K; \( T = 300 \) K; \( T_{\text{max}} = 86.8 \) K; \( \Theta = 0.50; \) \( K = 16.0 \times 10^{-4} \) W/K; \( \gamma = 1.324; R_k = 10.1 \times 10^{-3} \) Ohm; \( I_{\text{max}} = 5.02 \) A |
| 6.5          | 1.0    | 5.02   | 3.9    | 2.31   | 0.216  | 0.46   | 0.911  | 3.8    | 11.4   | 0.9989 |

By using a method of sequential approximations, one can determine the assigned temperature difference \( \Delta T \) and heat load \( Q_0 \).
| τ (s) | Bk | T=20 K | T=50 K | T=60 K | Bk | T=20 K | T=50 K | T=60 K |
|-------|-----|--------|--------|--------|-----|--------|--------|--------|
| 0     | 0.05| 0.05   | 0.05   | 0.05   | 0.05| 0.05   | 0.05   | 0.05   |
| 1     | 0.1 | 0.1    | 0.1    | 0.1    | 0.1| 0.1    | 0.1    | 0.1    |
| 2     | 0.2 | 0.2    | 0.2    | 0.2    | 0.2| 0.2    | 0.2    | 0.2    |
| 3     | 0.3 | 0.3    | 0.3    | 0.3    | 0.3| 0.3    | 0.3    | 0.3    |
| 4     | 0.4 | 0.4    | 0.4    | 0.4    | 0.4| 0.4    | 0.4    | 0.4    |
| 5     | 0.5 | 0.5    | 0.5    | 0.5    | 0.5| 0.5    | 0.5    | 0.5    |
| 6     | 0.6 | 0.6    | 0.6    | 0.6    | 0.6| 0.6    | 0.6    | 0.6    |
| 7     | 0.7 | 0.7    | 0.7    | 0.7    | 0.7| 0.7    | 0.7    | 0.7    |

An increase in the time it takes to enter a stationary mode τ for different temperature differentials ΔT leads to the following:

- the relative working current $B_k$ decreases, except for the $Q_{0_{\text{max}}}$ mode (Fig. 1), where the dotted line indicates characteristic current modes of operation. The magnitude of relative working current $B_k$ is reduced from the $Q_{0_{\text{max}}}$ mode (pos. 1) to the $\lambda_{\text{min}}$ mode (pos. 4). At the predefined time to enter a stationary mode τ, an increase in temperature difference ΔT increases the relative working current $B_k$;

- the magnitude of working current $I$ decreases (Fig. 2), where the dotted line indicates the magnitude of a working current for the characteristic current modes of operation. The magnitude of the working current decreases from the $Q_{0_{\text{max}}}$ mode to the $\lambda_{\text{min}}$ mode. At the predefined time to enter a stationary mode τ, an increase in temperature difference ΔT increases the magnitude of working current $I$;

- functional dependence $E=f(\tau)$ (Fig. 3) has a maximum: $E_{\text{max}}=1.4$ at $\Delta T=20 \text{ K}$; $E_{\text{max}}=0.73$ at $\Delta T=30 \text{ K}$; $E_{\text{max}}=0.37$ at $\Delta T=40 \text{ K}$; $E_{\text{max}}=0.17$ at $\Delta T=50 \text{ K}$; $E_{\text{max}}=0.044$ at $\Delta T=60 \text{ K}$. At the predefined time to enter a stationary mode τ, the refrigeration factor decreases as the temperature difference ΔT drops. The dotted line indicates the magnitude of the refrigeration factor for the characteristic current modes of operation; at the predefined time to enter a stationary mode τ, an increase in the temperature difference ΔT, functional dependence $n=f(\Delta T)$ has a minimum;

- functional dependence $U=f(\tau)$ (Fig. 5) has a minimum: $U_{\text{min}}=0.2 \text{ V}$ at $\Delta T=20 \text{ K}$; $U_{\text{min}}=0.28 \text{ V}$ at $\Delta T=30 \text{ K}$; $U_{\text{min}}=0.41 \text{ V}$ at $\Delta T=40 \text{ K}$; $U_{\text{min}}=0.73 \text{ V}$ at $\Delta T=50 \text{ K}$; $U_{\text{min}}=2.5 \text{ V}$ at $\Delta T=60 \text{ K}$. The dotted line indicates the magnitude of voltage drop for the characteristic current modes of operation. At the predefined time to enter a stationary mode τ, an increase in temperature difference ΔT increases the magnitude of the voltage drop;

- the relative magnitude of intensity of failures $\lambda/\lambda_{\text{q}}$ (Fig. 6) decreases. The dotted line indicates the relative failure rate for the characteristic current modes of operation. The magnitude of failure intensity $\lambda$ decreases from the $Q_{0_{\text{max}}}$ mode (pos. 1) to the $\lambda_{\text{min}}$ mode (pos. 4). At the predefined time to enter a stationary mode τ, an increase in temperature difference ΔT increased the relative magnitude of failure intensity $\lambda/\lambda_{\text{q}}$;

- the likelihood of failure-free operation $P$ increases (Fig. 7). The dotted line indicates the probability of failure-free work for the characteristic current modes of operation. The probability of failure-free operation increases from the $Q_{0_{\text{max}}}$ mode (pos. 1) to the $\lambda_{\text{min}}$ mode (pos. 4). At the predefined time to enter a stationary mode τ, an increase in temperature difference ΔT increases the probability of failure-free work $P$.
Fig. 2. Dependence of the magnitude of working current $I$ on the time it takes to enter a stationary mode $\tau$ for a single-cascade TCD at $T=300$ K, $/S=10$ cm$^{-1}$; $Q_0=0.5$ W at different temperature differentials $\Delta T$ for different modes of operation: 1 – $Q_{0\text{max}}$ mode; 2 – ($Q_0/I_{\text{max}}$) mode; 3 – ($Q_0/I_2$)$_{\text{max}}$ mode; 4 – $\lambda_{\text{min}}$ mode

Fig. 3. Dependence of refrigeration factor $E$ on the time it takes to enter a stationary mode $\tau$ for a single-cascade TCD at $T=300$ K, $/S=10$ cm$^{-1}$; $Q_0=0.5$ W, at different temperature differences $\Delta T$, under different modes of operation: 1 – $Q_{0\text{max}}$ mode; 2 – ($Q_0/I_{\text{max}}$) mode; 3 – ($Q_0/I_2$)$_{\text{max}}$ mode; 4 – $\lambda_{\text{min}}$ mode

Fig. 4. Dependence of the number of thermo-elements $n$ on the time to enter a stationary mode $\tau$ for a single-cascade TCD at $T=300$ K, $/S=10$ cm$^{-1}$; $Q_0=0.5$ W at different temperature differences $\Delta T$, for different modes of operation: 1 – $Q_{0\text{max}}$ mode; 2 – ($Q_0/I_{\text{max}}$) mode; 3 – ($Q_0/I_2$)$_{\text{max}}$ mode; 4 – $\lambda_{\text{min}}$ mode

Fig. 5. Dependence of voltage drop $U$ on the time to enter a stationary mode $\tau$ for a single-cascade TCD at $T=300$ K, $/S=10$ cm$^{-1}$; $Q_0=0.5$ W, at different temperature differences $\Delta T$, for different modes of operation: 1 – $Q_{0\text{max}}$ mode; 2 – ($Q_0/I_{\text{max}}$) mode; 3 – ($Q_0/I_2$)$_{\text{max}}$ mode; 4 – $\lambda_{\text{min}}$ mode

Fig. 6. Dependence of the relative magnitude of failure intensity $\lambda/\lambda_0$ on the time it takes to enter a stationary mode mode for a single-cascade TCD at $T=300$ K, $/S=10$ cm$^{-1}$; $Q_0=0.5$ W; $\lambda_0=3 \times 10^{-9}$ 1/h at different temperature difference $\Delta T$, for different modes of operation: 1 – $Q_{0\text{max}}$ mode; 2 – ($Q_0/I_{\text{max}}$) mode; 3 – ($Q_0/I_2$)$_{\text{max}}$ mode; 4 – $\lambda_{\text{min}}$ mode

Fig. 7. Dependence of the probability of failure-free operation $P$ on the time to enter a stationary mode $\tau$ for a single-cascade TCD at $T=300$ K, $/S=10$ cm$^{-1}$; $Q_0=0.5$ W; $t=10^{-4}$ h at different temperature difference $\Delta T$ for different modes of operation: 1 – $Q_{0\text{max}}$ mode; 2 – ($Q_0/I_{\text{max}}$) mode; 3 – ($Q_0/I_2$)$_{\text{max}}$ mode; 4 – $\lambda_{\text{min}}$ mode
A growth in temperature difference $\Delta T$ for different current regimes leads to the following:

- the time it takes to enter a stationary mode $\tau$ increases (Fig. 8). At the assigned temperature difference $\Delta T$, the time to enter a stationary mode increases from $\tau_{\text{min}}=6.5$ s, in the $Q_{\text{max}}$ mode (pos. 1) to $\tau=15.1$ s in the $\lambda_{\text{min}}$ mode (pos. 4). At temperature changes $\Delta T$ close to $\Delta T_{\text{max}}$, the time to enter a stationary mode $\tau$ differs slightly;

- the magnitude of working current $I$ increases (Fig. 9) for characteristic current modes ($Q_{I}/I_{\text{max}}$, $Q_{I}/I_{\text{min}}$, $\lambda_{\text{min}}$ (pos. 2), 3, 4), and, for the $Q_{\text{max}}$ mode (pos. 1), it decreases. At the assigned temperature difference $\Delta T$, for example $\Delta T=40$ K, the magnitude of working current $I$ increases from $I_{\text{min}}=2$ A in the $\lambda_{\text{min}}$ mode (pos. 4) to $I=5.02$ A in the $Q_{\text{max}}$ mode (pos. 1);

- the refrigeration factor $E$ decreases (Fig. 10); at the predetermined temperature difference $\Delta T$; for example $\Delta T=40$ K, the refrigeration factor $E$ increases from $E_{\text{min}}=0.216$ in the $Q_{\text{max}}$ mode (pos. 1) to $E=0.38$ in the $(Q_{I}/I)_{\text{max}}$ mode (pos. 3);

- the number of thermo-elements $n$ (Fig. 11) for the $Q_{\text{max}}$ mode (pos. 1) and for the $(Q_{I}/I)_{\text{max}}$ mode increases (pos. 2). Functional dependence $n=f(\Delta T)$ (Fig. 5) has a minimum for the $(Q_{I}/I)_{\text{max}}$ mode (pos. 3) at $\Delta T=40$ K and for the $\lambda_{\text{min}}$ mode (pos. 4) at $\Delta T=40$ K. At the assigned temperature difference $\Delta T$, for example $\Delta T=40$ K, the number of thermal elements $n$ increases from $n=3.9$ pieces in the $Q_{\text{max}}$ mode (pos. 1) to $n=14$ pieces in the $\lambda_{\text{min}}$ mode (pos. 4);

- the voltage drop $U$ increases (Fig. 12). At the predefined temperature difference $\Delta T$, for example, for temperature difference $\Delta T=40$ K, the drop in voltage $U$ increases from $U=0.46$ V in the $(Q_{I}/I)_{\text{max}}$ mode (pos. 2) to $U=0.79$ in the $\lambda_{\text{min}}$ mode (pos. 4);

- the intensity of failures $\lambda$ increases (Fig. 13); at the predefined temperature difference $\Delta T$, for example $\Delta T=40$ K, the relative magnitude of failure intensity $\lambda/\lambda_{0}$ decreases from $\lambda/\lambda_{0}=3.8$ in the $Q_{\text{max}}$ mode (pos. 1) to $\lambda/\lambda_{0}=0.34$ in the $\lambda_{\text{min}}$ mode (pos. 4);

- the likelihood of failure-free operation $P$ decreases (Fig. 14); at the predefined temperature difference $\Delta T$, for example $\Delta T=40$ K, the probability of failure-free operation $P$ increases from $P=0.9989$ in the $Q_{\text{max}}$ mode (pos. 1) to $P=0.999990$ in the $\lambda_{\text{min}}$ mode (pos. 4).
Example of calculation.

Initial data: it is required to build a single-cascade thermoelectric cooler for temperature difference $\Delta T=40$ K at $T=300$ K under heat load $Q_0=0.5$ W, the geometry of thermo-elements’ branches $l/S=10$ cm$^{-1}$ and the time to enter a stationary mode $\tau \leq 10$ s.

To build a TCD, we shall use the temperature dependences for the parameters of thermoelectric materials in module [15]. We shall determine in the initial period $\tau=0$ $P_0=T=300$ K the electrical resistance of a thermoelement’s branch and the maximum working current $R_{H}=11.1 \times 10^{-3}$; $I_{\max H}=5.51$ А. Thus, at temperature difference $\Delta T=40$ K, the average temperature is equal to

$$T = \frac{T + T_0}{2} = 280$$ K;

$$\tau = 2.36 \times 10^{-3} \text{ K}; \quad \tau = 1.95 \times 10^{-4} \text{ V/K};$$

$$\overline{\sigma} = 990 \text{ S/cm};$$

$$\overline{\varepsilon} = 16.0 \times 10^{-3} \text{ W/(cm K)}.$$

Define the basic parameters at $\Delta T=40$ K:

$$R_K=10/990=10.1 \times 10^{-3} \text{ Ohm};$$

$$I_{\max K}=5.02 \text{ А};$$

$$\gamma = 1.324.$$ At $l/S=10$ cm$^{-1}$

$$\sum m_i C_i = 175 \times 10^{-4} \text{ J/K}$$ considering CTE

$$\Delta T_{\max} = 79.8 \text{ K}; \quad \Theta = 0.50; \quad \Delta T_{\max}/T_0 = 0.31; \quad D = 0.911.$$

By using the method of successive approximations, we shall determine the relative working current $B_K$. From Fig. 1, take at the beginning: $B_K=0.53$. Then

$$A = \exp \left( -10.16 \times 10^{-4} (1+20.53 \times 0.31) \right) = \exp \left( \frac{212.6}{175} \right) = 3.37.$$

Next, determine $B_K$ from formula (2).

$$B_K = \frac{A - \gamma D}{A - \gamma D} \left[ 1 - \frac{1}{(A - \gamma D)^2} \right] =$$

$$= \frac{3.37 - 1.324 \times 0.911}{3.37 - 1.324} \times \left[ 1 \pm \frac{3.37 - 0.50(3.37 - 1.324 \times 0.83)}{3.37 - 1.324} \right] =$$

$$= \frac{2.164}{2.277} \times \left[ 1 \pm 0.953(1 \pm 0.427) = 0.546.$$

Refine $B_K=0.546$. Then

$$A = \exp \left( -10.16 \times 10^{-4} (1+20.546 \times 0.31) \right) = \exp \left( \frac{214.2}{175} \right) = 3.40.$$
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\[ B_k = \frac{3.4 - 1.206}{3.4 - 1.1} \left[ 1 \pm \sqrt{\frac{3.4 - 0.5 (3.4 - 1.1)}{(3.4 - 1.206)^2}} \right] = \]

\[ = \frac{2.194}{2.3} \left[ 1 \pm \sqrt{\frac{-3.91}{4.814}} \right] = 0.954(1 \pm 0.433) = 0.54. \]

Determine the magnitude of working current \( I \) from ratio

\[ I = I_{\text{max}} B_k^{-1} B_{I} = 2.71 \text{ A} \] (Fig. 2).

Then determine \( B_{II} = 0.492 \). Next, determine the number of thermoelements \( n \) from expression (4):

\[ n = \frac{Q}{I_{\text{max}} R_k (2B_k - B_I - \Theta)} = 6.8 \text{ pc.} \] (Fig. 4).

The power consumption \( W_k \) can be determined from expression (3)

\[ W_k = 2n^2 I_{\text{max}} R_k B_k \left( B_k + \frac{T_{\text{min}}}{T_k} \Theta \right) = 1.3 \text{ W}. \]

The voltage drop can be determined from formula (5)

\[ U_k - W_k / I = 0.48 \text{ V} \] (Fig. 5).

The refrigeration factor \( E \) will be defined from expression (6)

\[ E = \frac{Q_0}{W_k} = 0.385 \] (Fig. 3).

The relative magnitude of failure-free operation \( \lambda / \lambda_0 \) can be determined from formula (7)

\[ \lambda / \lambda_0 = n B_k^2 \left( \Theta + C \right) \times \]

\[ \left( \frac{B_k + \frac{T_{\text{max}}}{T_k} \Theta}{1 + \frac{T_{\text{max}}}{T_k} \Theta} \right)^2 K_r = 0.579, \] (Fig. 6).

\[ \lambda = 1.74 \times 10^{-8} \text{ } 1/\text{h} \text{ is the failure rate.} \]

The probability of failure-free operation \( P \) is determined from formula (8) \( P = 0.99983 \), which corresponds to the model studies described above (Fig. 7).

### 6. Discussion of Results of Analyzing the Time it Takes to Enter a Stationary Mode for a Single-Cascade TCD

The results obtained are based on the devised model of a single-cascade thermoelectric cooling device for the assigned transition dynamics. The study was conducted in the range from 20 to 60 K at the typical heat load values \( Q_0 = 0.5 \text{ W} \) for \( I/S \approx 10 \text{ cm}^2 \) and different characteristic current modes of operation.

Special feature of the proposed model, compared to existing models, is taking into consideration the impact of structural and technological elements on the dynamic characteristics and reliability indicators of the cooler.

Our analysis of results from calculating the main parameters and reliability indicators has revealed that an increase in the time to enter a stationary mode of operation \( \tau \) leads, for varying temperature difference \( \Delta T \), to that:

1) the following reduces:
   - relative working current \( B_k \) (Fig. 1). The minimum relative working current \( B_k = 0.4 \) is ensured in the \( \lambda_{\text{min}} \) mode at temperature difference \( \Delta T = 40 \text{ K} \) and \( \tau = 16 \text{ s}; \)
   - the magnitude of working current \( I \) (Fig. 2). The minimum magnitude of working current \( I_{\text{min}} = 2 \text{ A} \) is ensured in the \( \lambda_{\text{min}} \) mode at \( \Delta T = 40 \text{ K} \) and \( \tau = 16 \text{ s}; \)
   - the relative magnitude of intensity of failures \( \lambda / \lambda_0 \) (Fig. 6). The minimum relative failure rate \( \lambda / \lambda_0_{\text{min}} \) is ensured in the \( \lambda_{\text{min}} \) mode at \( \Delta T = 40 \text{ K} \) and \( \tau = 16 \text{ s}; \)
2) the following increases:
   - the number of thermo-elements \( n \) (Fig. 4). The minimum quantity of thermoelements \( n_{\text{min}} = 4 \) pieces is ensured in the \( Q_{\text{max}} \) mode at \( \Delta T = 40 \text{ K}; \)
   - voltage drop \( U \) (Fig. 5). The maximum voltage drop \( U_{\text{max}} = 1 \text{ V} \) is ensured in the \( \lambda_{\text{min}} \) mode at \( \Delta T = 40 \text{ K}; \)
   - the probability of failure-free operation \( P \) (Fig. 7).

Functional dependence of the refrigeration factor \( E = f(\tau) \) has a maximum at \( \tau_{\text{min}} = 0.35 \) at \( \Delta T = 40 \text{ K} \) (Fig. 3).

As the temperature drop \( \Delta T \) increases, for different current modes of operation:

1) the following increases:

   - the time it takes to enter a stationary mode of operation \( \tau \) (Fig. 8). The minimum time to enter a stationary mode of operation \( \tau_{\text{min}} = 7 \text{ s} \) at \( \Delta T = 40 \text{ K} \) is ensured in the \( Q_{\text{max}} \) mode;
   - the magnitude of working current \( I \) (Fig. 9). The minimum magnitude of working current \( I_{\text{min}} = 2 \text{ A} \) is ensured in the \( \lambda_{\text{min}} \) mode at \( \Delta T = 40 \text{ K} \) and \( \tau = 16 \text{ s}; \)
   - voltage drop \( U \) (Fig. 12). The maximum voltage drop \( U_{\text{max}} = 0.75 \text{ V} \) is ensured in the \( \lambda_{\text{min}} \) mode at \( \Delta T = 40 \text{ K}; \)
   - the relative failure rate \( \lambda / \lambda_0 \) (Fig. 13). The minimum failure intensity \( \lambda / \lambda_0_{\text{min}} = 0.5 \) is ensured in the \( \lambda_{\text{min}} \) mode at \( \Delta T = 40 \text{ K}; \)
2) the following decreases:
   - refrigeration factor \( E \) (Fig. 10). The maximum refrigeration factor \( E_{\text{max}} = 0.42 \) is ensured at \( \Delta T = 40 \text{ K}; \)
   - the probability of failure-free operation \( P \) (Fig. 14). The maximum probability of failure-free operation \( P_{\text{max}} = 0.99990 \) is ensured in the \( \lambda_{\text{min}} \) mode at \( \Delta T = 40 \text{ K}. \)

A limitation of the proposed model and, accordingly, our results is the assumption about the identity of all physical parameters of thermoelectric elements and thermal resistances of soldered joints between thermoelements and substrate. In addition, the validity of model results should be confirmed by experimental studies that could add certain adjustments. The advancement of a given direction is to analyze reliability indicators for thermoelectric coolers exposed to integrated impacts of mechanical, climatic, energy factors under dynamic modes, which is typical for on-board control systems.

### 7. Conclusions

1. Our study of the dynamic model of cooling thermoelement operation has shown the possibility of optimal control
over the thermal mode of a single-cascade TCD, taking into consideration the influence of structural and parametric factors.

2. Analytical expressions have been derived to determine the basic parameters of reliability indicators at the predefined dynamics in the functioning of a single-cascade TCD over a wide range of temperature differences and current modes of operation.

3. The results of the research make it possible to design single-cascade TCD at the assigned dynamics of functioning and to predict the basic parameters and reliability indicators over any time period.

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