THE X-RAY THROUGH OPTICAL FLUXES AND LINE STRENGTHS OF TIDAL DISRUPTION EVENTS

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ABSTRACT

We study the emission from tidal disruption events (TDEs) produced as radiation from black hole accretion propagates through an extended, optically thick envelope formed from stellar debris. We analytically describe key physics controlling spectrum formation, and present detailed radiative transfer calculations that model the spectral energy distribution and optical line strengths of TDEs near peak brightness. The steady-state transfer is coupled to a solver for the excitation and ionization states of hydrogen, helium, and oxygen (as a representative metal), without assuming local thermodynamic equilibrium. Our calculations show how an extended envelope can reprocess a fraction of soft X-rays and produce the observed optical fluxes of the order of $10^{43}$ erg s$^{-1}$, with an optical/UV continuum that is not described by a single blackbody. Variations in the mass or size of the envelope may help explain how the optical flux changes over time with roughly constant color. For high enough accretion luminosities, X-rays can escape to be observed simultaneously with the optical flux. Due to optical depth effects, hydrogen Balmer line emission is often strongly suppressed relative to helium line emission (with He ii-to-H line ratios of at least 5:1 in some cases) even in the disruption of a solar-composition star. We discuss the implications of our results to understanding the type of stars destroyed in TDEs and the physical processes responsible for producing the observed flares.

Key words: atomic processes – black hole physics – line: formation – methods: numerical – radiation mechanisms: non-thermal – radiative transfer

1. INTRODUCTION

If a star passes deep enough within the gravitational potential of a black hole (BH), tidal forces can exceed self-gravity, ripping the star apart in a tidal disruption event (TDE). Subsequent shocks occurring in colliding stellar debris streams (e.g., Kochanek 1994; Guillochon & Ramirez-Ruiz 2015), and/or the eventual accretion of this gas onto the BH may produce a luminous thermal flare at X-ray through optical wavelengths. A relativistic jet generated by the accreting BH may also produce non-thermal gamma-ray, X-ray, and radio emission.

Observational candidates for TDEs are rapidly accumulating. A number of flares from galactic centers have been discovered in X-rays (Komossa & Bade 1999; Donley et al. 2002; Halpern et al. 2004; Komossa et al. 2004; Esquej et al. 2007; Cappelluti et al. 2009; Maksym et al. 2010; Saxton et al. 2012; Komossa 2015; Lin et al. 2015; Hryniewicz & Walter 2016). The peak luminosity is high, $\gtrsim 10^{44}$ erg s$^{-1}$, and the spectral energy distribution (SED) peaks at soft X-ray energies $\lesssim 0.1$ keV. After peak, the luminosity fades as a power law in time similar to $L \propto t^{-5/3}$, a dependence predicted for the fallback of disrupted stellar debris (Rees 1988; Evans & Kochanek 1989; Phinney 1989; Lodato et al. 2009; Guillochon & Ramirez-Ruiz 2013).

There have also been TDE candidates found in the ultraviolet (UV; Gezari et al. 2006, 2009), and in the optical in SDSS (van Velzen et al. 2011; van Velzen & Farrar 2014), Pan-STARRS1 (Gezari et al. 2012; Chornock et al. 2014), ASASSN (Holoien et al. 2014, 2016a, 2016b), PTF (Cenko et al. 2012a; Arcavi et al. 2014), and ROTSE (Vinkó et al. 2015). These events typically rise to a peak (observer frame) R-band luminosity of $\sim 2 \times 10^{43}$ erg s$^{-1}$ on a timescale of $\sim$ months (Arcavi et al. 2014), with a late time fall consistent with $t^{-5/3}$. Intriguingly, PTF10iya, Swift J2058.4, ASASSN-14li, and ASASSN-15oi have been simultaneously observed in both optical and X-ray (Cenko et al. 2012a, 2012b, 2016; Miller et al. 2015; Holoiën et al. 2016a, 2016b).

A few long-lived ($\sim 10^5$ s) transients have also been observed at hard X-ray and gamma-ray energies (Bloom et al. 2011; Cenko et al. 2012b; Brown et al. 2015). Occasionally, associated optical emission is also detected. These events have been interpreted as due to a relativistic jet generated via BH accretion; such “jetted” TDEs appear to be observationally rare compared to the soft X-ray and UV/optical flares (Arcavi et al. 2014; van Velzen & Farrar 2014), in line with theoretical expectations (De Colle et al. 2012).

While many aspects of the TDE candidates remain poorly understood, the nature of the optical/UV emission is perhaps the most puzzling. Two fundamental questions await full explanation. (1) Why is the observed optical flux in the UV/optical transients orders of magnitudes higher than that predicted by a standard BH accretion disk, and with a blue color that remains roughly constant over time? (2) Why do the optical spectra show strong lines of helium, but little or no hydrogen line emission (Gezari et al. 2012; Arcavi et al. 2014)?

The first puzzle stems from the fact that the tidal disruption radius, $R_d = (M_{bh}/M_\odot)^{1/3} R_c$, is $\approx 10^{13}$ cm for the disruption of a solar-like star (mass $M_\odot = M_\odot$, radius $R_\odot$) by a BH of mass $M_{bh} \sim 10^6 M_\odot$. Thermal emission from this radius should be in the soft X-ray (temperatures $\approx 10^5$ K with low optical luminosity ($\lesssim 10^{42}$ erg s$^{-1}$). The problem has been addressed by postulating the presence of gas at large radii ($\sim 100$ times...
$R_{\text{td}}$ that absorbs (or advects) radiation and re-emits it at lower temperatures of a few times 10$^4$ K. This reprocessing region may be due to the formation of a hydrostatic (or quasi-static) envelope around the BH (Loeb & Ulmer 1997; Couchlin & Begelman 2014; Guillochon et al. 2014), or a super-Eddington mass outflow (Strubbe & Quataert 2009; Lodato & Rossi 2011; Metzger & Stone 2015; Miller 2015; Vinkó et al. 2015), or the circularization of material at radii much greater $R_{\text{td}}$ (Dai et al. 2015; Guillochon & Ramirez-Ruiz 2015; Hayasaki et al. 2015; Piran et al. 2015; Shiomakawa et al. 2015; Bonnerot et al. 2016). As of yet, however, no detailed radiative transfer calculations have determined how, or if, TDE radiation can be so reprocessed, and if so how the emergent optical through X-ray emission depends on the gas properties. These are key questions we hope to address here.

The solution to the second puzzle — the low hydrogen to helium emission line ratios — continues to be debated. Early theoretical modeling of the He$\alpha$ emission by Bogdanović et al. (2004) raised the possibility that this line emission might be destroyed due to the high optical depth to He$\alpha$ photons in a reprocessing region surrounding the inflowing stellar debris. Gezari et al. (2012) argued that the absence of H lines in PS1-10jh implies the disruption of a He-rich stellar core. The simulations of MacLeod et al. (2012), however, show that it is difficult to remove the H envelope of a red giant and disrupt only the He core. Noting that He star disruptions would be exceedingly rare, Guillochon et al. (2014) instead argued for the disruption of a normal main-sequence star but with the hydrogen line emission suppressed by photoionization effects (e.g., Korista & Goad 2004). Gaskell & Rojas Lobos (2014) performed new calculations using the photoionization code CLOUDY (Ferland et al. 1998, 2013) and showed that, if the lines are optically thick, radiation transport effects may reduce the hydrogen emission line strength. However, a separate CLOUDY parameter study performed by Strubbe & Murray (2015) disputed this interpretation, concluding that, for the conditions relevant to TDEs, hydrogen lines would not be suppressed enough to be consistent with the observations of PS1-10jh.

All previous studies of TDE line emission, however, have been subject to limitations of photoionization codes like CLOUDY, which assume that the gas is optically thin in the continuum (but may allow for optical depth effects for lines). Here we show that TDE envelopes are likely optically thick and highly scattering dominated, and that can profoundly change the nature of line and continuum formation. We first use analytic arguments to delineate the physical conditions and key radiative processes in TDE envelopes, and then carry out the first non-local-thermodynamic equilibrium (non-LTE) Boltzmann radiative transfer calculations for an idealized TDE model with a spherically, optically thick reprocessing region.

In contrast to optically thin photoionization models, spectrum formation in optically thick TDE envelopes more closely resembles the situation in stellar atmospheres, where the emission at a given wavelength depends on the source function at the associated thermalization depth. The thermalization depth is the radius at which emitted photons can scatter to the surface without being reabsorbed, and corresponds to a radial optical depth (integrated inward) of approximately $\sqrt{\tau_{\text{abs}}/\tau_{\text{es}}}$, where $\tau_{\text{abs}}$ and $\tau_{\text{es}}$ are the optical depths to absorptive and electron scattering processes, respectively. The thermalization depth varies with wavelength, such that lines and the continuum form at different layers (see Figure 1). The resulting spectrum will generally not be described by a single blackbody.

Our calculations define the conditions under which a TDE envelope may be “effectively” optically thick (i.e., $\sqrt{\tau_{\text{abs}}/\tau_{\text{es}}}$ ≳ 1) at soft X-ray wavelengths, and so absorb and reprocess a fraction of the accretion luminosity into optical continuum and line emission. We describe how the reprocessing efficiency depends on the mass, size, and ionization state of the envelope. Variations in these quantities may help explain the observed optical flux evolution and the occasional simultaneous appearance of X-rays. We show that escaping line photons are produced only in the outer layers of the envelope, such that the H to He$\alpha$ line ratio is suppressed and will vary with the envelope extent. Indeed, some optical TDE candidates do show detectable, but varying, H$\alpha$ emission (Arcavi et al. 2014; Holoien et al. 2016b).

In Section 2, we describe the assumptions of the model setup, and make analytic estimates of the degree of reprocessed luminosity. In Section 3 we present our numerical results from our radiative transport calculations. In Section 4 we discuss implications for observations and models of TDEs, and in Section 5 we conclude with a summary of the most important take away points.

2. ANALYTIC CONSIDERATIONS

An accurate determination of the optical/UV luminosity from TDE envelopes requires a detailed non-LTE radiative transfer calculation, which we provide in Section 3. However, some insight into the physics of radiation reprocessing, and how it depends on envelope parameters, can be gained by approximate analytic arguments.

We consider a 1D configuration where radiation is emitted by a spherical source at radius $r_{\text{in}}$ defining the layer at which most of the radiative luminosity is generated. Choosing $r_{\text{in}}$ comparable to the original stellar pericenter passage distance (typically ∼10$^{13}$ cm for a 10$^6$–10$^7$ M$\odot$ BH) would approximate the luminosity arising from a viscous accretion disk formed near the disruption radius. Alternatively, the luminosity could be generated by the circularization of fallback material at a larger radius, $r_{\text{in}}$ ∼ 10$^{14}$ cm (e.g., Dai et al. 2015; Piran et al. 2015).

We assume that the inner boundary radiates blackbody radiation at a source temperature $T_i$. Our implicit assumption is that the conditions interior to $r_{\text{in}}$ are sufficient to thermalize the radiation, e.g., via Comptonization or adiabatic expansion. For luminosities of the order of 10$^{44}$–10$^{45}$ erg s$^{-1}$, $T_i$ ∼ 10$^5$–10$^6$ K and source photons are emitted primarily at soft X-ray/ultraviolet wavelengths (energies ∼20–100 eV).

Figure 2 shows the processes that contribute to the absorptive opacity for conditions found in a typical numerical calculation of a TDE envelope (to be described in Section 3). For soft X-rays near 100 Å, a primary absorptive opacity is photoionization of He$\pi$ (threshold energy of 54.4 eV). Photoionization and line absorption from other metals will also contribute to the reprocessing.

In this section, we quantify the conditions in the envelope (density, temperature, and ionization state) and the effective optical depth to He$\pi$ photoionization. We then estimate the fraction of light reprocessed into the optical and its dependence on envelope parameters. The results, while approximate, will be useful in interpreting the detailed numerical spectral calculations that follow.
2.1. Envelope Density Structure and Optical Depth

We invoke the presence of a reprocessing envelope surrounding the source, and remain agnostic about its origin. For simplicity, we assume the envelope is spherically symmetric and quasi-static; in reality, global asymmetries and velocity gradients will complicate the picture, and are issues to be addressed in future work.

We model the envelope with a power-law density profile, \( \rho(r) \propto r^{-p} \), extending from an inner radius, \( r_{in} \), to an outer radius, \( r_{out} \). An exponent \( p = 3 \) applies to a radiation pressure-supported envelope (Loeb & Ulmer1997), while \( p = 2 \) for a steady state wind outflow (or inflow). The hydrodynamic solutions of Coughlin & Begelman (2014) find intermediate values of \( p = 1.5–3 \). In this work, we adopt \( p = 2 \), in which case the density profile is

\[
\rho(r) = \frac{M_{env}}{4\pi r_{in}^2 (r_{out} - r_{in})} \left( \frac{r_{in}}{r_{in}} \right)^{-2}.
\]

The envelope mass, \( M_{env} \), is at most the total mass of the originally bound stellar debris (generally \( \lesssim 0.5 M_\odot \)) and for a quasi-static envelope will likely change over time as material falls back, accretes or is launched in an outflow. If the density profile of Equation (1) is interpreted a steady wind, the corresponding mass loss rate is \( \dot{M} = M_{env} (v_w/r_{out}) \), where \( v_w \) is the wind speed.

For quasi-static envelopes, we expect \( r_{out} \) to be set by the radiation pressure support, with typical values \( r_{out} \approx 10^{15} \) cm (Loeb & Ulmer 1997; Coughlin & Begelman 2014; Guillochon et al. 2014). For a radiatively launched outflow, \( r_{out} \) will be set by the extent of the expanding gas at any given time. Gas expanding at \( v_w = 10,000 \) km s\(^{-1}\) for 20 days will have reached a similar radius, \( r_{out} \approx 10^{15} \) cm. Scaling to characteristic values, the density at the base of the envelope,
where 

\[ M_{\text{env}} = M_{\odot} \left( \frac{r}{r_{\odot}} \right)^2 \] 

and 

\[ r_{\odot} = 10^{14} \text{ cm} \]. For solar composition, the mean particle mass is \( \mu \approx 1.3 m_p \) and the number density at the base is \( n_{\text{in}} \approx 4 \times 10^{12} \text{ cm}^{-3} \).

For fully ionized gas of solar composition, the dominant continuum opacity is electron scattering with an opacity \( \kappa = 0.32 \text{ cm}^2 \text{ g}^{-1} \). The optical depth through the envelope is then

\[ \tau_{\text{es}} = \frac{n M_{\text{env}}}{4 \pi r \rho} \approx 270 M_{\odot} \left( \frac{r}{r_{\odot}} \right)^2 \] 

The envelope is highly scattering dominated, with the ratio of absorptive opacity to total opacity \( \epsilon \approx 10^{-4} \) at X-ray/UV wavelengths (see Figure 2). The reprocessing of radiation to optical wavelengths is accomplished via absorptive processes (e.g., \( \text{He II} \) photoionization), not electron scattering. However, the large scattering opacity traps photons and increases the probability that they will be absorbed. It thus has a critical effect on the formation of the optical continuum and emission lines.

The radiation diffusion time through the envelope is 

\[ t_{\text{diff}} \approx \frac{r^2 \kappa}{c} \] 

where \( \rho \) is an appropriately weighted envelope density and \( \kappa \) a characteristic envelope density scale. Taking \( \rho = \rho(r) \) gives 

\[ t_{\text{diff}} \approx 10 M_{\odot} \left( \frac{r}{r_{\odot}} \right)^2 \text{ days} \] 

during which the diffusion time we find in our direct numerical transport calculations of Section 3. For the TDE envelopes we consider here at times \( t \approx \text{ weeks} \) since disruption (near the peak of the light curve), 

\[ t_{\text{diff}} \approx \tau \] 

and we can assume steady state transport above the inner boundary. However, at earlier times on the light curve rise, or for very dense envelopes, the diffusion timescale may be important in setting the emergent luminosity.

### 2.2. Envelope Temperature Structure

We assume that above our inner boundary, energy is primarily transported by radiation diffusion and there is negligible energy input from other processes such as viscosity or shocks. The temperature structure of the envelope is then set by radiative heating and cooling. Given the high electron scattering optical depth, we apply the spherical steady-state diffusion approximation to determine the frequency integrated radiation energy density, 

\[ E_{\text{rad}}(r) \],

\[ \frac{dE_{\text{rad}}(r)}{dr} = \frac{3 \kappa \rho(r)}{4 \pi c r^2} L, \] 

where \( c \) is the speed of light and \( L \) is the bolometric luminosity of the central source. The solution of Equation (4), using the density profile from Equation (1) and a constant electron-scattering opacity, is

\[ E_{\text{rad}}(r) = \frac{\tau_{\text{rad}} L}{4 \pi r^2} \left[ r_{\text{in}}^3 - \frac{r^3}{\tau_{\text{rad}}^2} + \frac{r_{\text{in}}^3}{\tau_{\text{rad}}^2} r_{\text{in}} \right] \]

where we have taken as an outer boundary condition 

\[ E_{\text{rad}}(r_{\text{out}}) = a_{\text{rad}} T_{\text{eff}}, \] 

where \( a_{\text{rad}} \) is the radiation constant, 

\[ T_{\text{eff}} = \left[ L/4 \pi \sigma_b r_{\text{out}}^2 \right]^{1/4}, \] 

\[ \sigma_b \equiv c a_{\text{rad}}/4 \] 

is the Stefan–Boltzmann constant. Defining a radiation temperature by 

\[ T_{\text{rad}} = (E_{\text{rad}}/a_{\text{rad}})^{1/4}, \] 

we have in the limit \( r_{\text{out}} \gg r_{\text{in}} \)

\[ T_{\text{rad}}(r) = T_{\text{rad}}(r_{\text{in}}) \left[ \frac{r_{\text{in}}}{r} \right]^{3/4} \] 

(\( r \ll r_{\text{out}} \)).

The temperature at the inner boundary is greater than that of a blackbody sphere emitting into vacuum by a factor of \( (r_{\text{in}}/4)^{1/3} \approx 3 \). This is due to the back-heating of the source by photons trapped in the optically thick envelope. We estimate the characteristic radiation temperature to be

\[ T_{\text{rad}}(r_{\text{in}}) \approx 3 \times 10^5 \left[ L_{45} \right]^{1/4} M_{\odot}^{1/4} \left( \frac{r_{\text{in}}}{r_{14}} \right)^{-3/4} \text{ K}. \] 

Note that the mass and size of the envelope directly affects the temperature. For simplicity, we will usually set \( T_r \), the blackbody temperature of our inner source, equal to \( T_{\text{rad}}(r_{\text{in}}) \). This may not hold in general; for example, the source luminosity may come from an accretion disk with radius smaller than the inner envelope edge, \( r_{\text{in}} \).

As defined, \( T_{\text{rad}} \) is simply a convenient rescaling of the local radiation energy density. However, we now show that \( T_{\text{rad}} \) may provide a good estimate of the gas temperature \( T_{\text{gas}} \). When the thermal state of the gas is determined primarily by radiative heating and cooling, the time-evolution of \( T_{\text{gas}} \) is

\[ \frac{3 n k_B}{2} \frac{dT_{\text{gas}}}{dt} = -4 \pi \alpha_{\text{E}} B(T_{\text{gas}}) + 4 \pi \alpha_{\text{S}} I, \]

where \( J = c E_{\text{rad}}/4 \pi \) is the integrated mean specific intensity, \( B(T_{\text{gas}}) = \sigma_B T_{\text{gas}}^4/\pi \) is the frequency-integrated Planck function, \( k_B \) is the Boltzmann constant, and \( \alpha_{\text{E}} \) and \( \alpha_{\text{S}} \) are mean absorptive extinction coefficients defined by

\[ \alpha_{\text{E}} = \int \alpha_{\text{E}}^{\text{abs}}(\nu) E_{\nu}(\nu) d\nu, \]

\[ \alpha_{\text{S}} = \int \alpha_{\text{S}}^{\text{abs}}(\nu) B_{\nu}(\nu) d\nu. \]

where \( \alpha_{\text{E}}^{\text{abs}}(\nu) \) is the absorptive extinction coefficient at each frequency, and \( E_{\nu}(\nu) \) is the radiation energy density in the frequency interval between \( \nu \) and \( \nu + d\nu \).

Given sufficient time, the gas will reach a thermal equilibrium where radiative heating balances cooling. From Equation (9), the timescale, \( t_{\text{heat}} \), to heat gas from some lower temperature up to the equilibrium value is

\[ t_{\text{heat}} = \frac{3}{2 c} \frac{1}{\alpha_{\text{E}}} \left[ n k_B T_{\text{rad}} \right] \left[ a_{\text{rad}} T_{\text{rad}}^3 \right]. \]

The term in brackets is the ratio of gas to radiation energy density, and is \( \epsilon \ll 1 \) for the conditions in TDE envelopes. The true absorption coefficient \( \alpha_{\text{E}} \) is \( \approx 10^{-5} \) that of electron scattering (Figure 2) from which we find \( t_{\text{heat}} \) ranges from \( \lesssim 1 \text{ s} \) at the base of the envelope up to ~hours at the outermost radii, which is comfortably smaller than the characteristic timescale of weeks for the evolution of observed TDE lightcurves. The gas temperature will then be able to reach a steady state, \( dT_{\text{gas}}/dt = 0 \), and Equation (9) gives

\[ T_{\text{gas}} = (\alpha_{\text{E}}/\alpha_{\text{S}})^{1/4} T_{\text{rad}}. \]
Therefore, $T_{\text{gas}}$ is close to $T_{\text{rad}}$, with a correction factor related to the frequency dependence of $\alpha_{\nu}^{\text{ion}}$ and the extent to which the radiation field spectrum differs from a blackbody. In our numerical calculations, we find that Equation (6) provides a good estimate of $T_{\text{gas}}$ in the inner portions of the envelope, up to the continuum thermalization depth. Beyond that, $T_{\text{gas}}$ plateaus at a higher value than $T_{\text{rad}}$.

2.3. Envelope Ionization State

Photoionization of He II provides one of the most important absorptive opacities at soft X-ray wavelengths (see Figure 2). If helium is primarily in the He II state, the associated photoionization optical depth is $\gg 1$, and essentially all of the source X-rays will be absorbed and reprocessed to longer wavelengths. However, for high luminosity TDEs, the intense radiation field may completely ionize helium to He III, allowing only a small fraction of the source luminosity to be absorbed. Such an ionization effect has been explored in Metzger et al. (2014) and Metzger & Stone (2015). Determining the ionization state is therefore crucial for estimating the reprocessing efficiency of a TDE envelope.

To roughly estimate the critical luminosity, $L_{\text{ion}}$, required to highly ionize the envelope, we use simple Stromgren sphere arguments. The rate at which ionized photons are produced by the central source is $Q = L/E$, where $E_*$ is the average energy of source photons. The total recombination rate within a sphere of radius $r$ is

$$ R(r) = \int_{r_m}^{r} \frac{4\pi r'^2 \alpha_{\text{He II}} n_{\text{He II}}(r')}{4\pi A_{\text{He II}} n_{\text{He II}}(r') \nu_{\text{He II}}} dr', $$

where $\alpha_{\text{He II}} \approx 2 \times 10^{-13} \text{cm}^3 \text{s}^{-1}$ is the Case B helium recombination coefficient at a temperature of $10^5 \text{K}$, $n_e$ is the free electron density, and $n_{\text{He II}}$ is the number density of He II. The condition $Q = R(r)$ allows us to solve for the Stromgren radius within which the helium is fully ionized

$$ r_{\text{strom}} = r_m \left[ 1 - \frac{L}{4\pi A_{\text{He II}} n_{\text{He II}}(r_m) \nu_{\text{He II}}} \right]^{-1}, $$

where $A_{\text{He II}} \approx 0.1$ is the number fraction of helium, and we have assumed $n_e$ equals the ion number density, $n$, which is accurate to within 10% for the conditions that interest us. We see that the Stromgren radius of He II diverges for luminosities above a critical luminosity $L_{\text{ion}} = 4\pi A_{\text{He II}} n_{\text{He II}} \nu_{\text{He II}}$ or

$$ L_{\text{ion}} \approx 3 \times 10^{44} M_{\odot,0.5} r_{15}^{-2} n_{15}^{-1} E_{50} \text{ erg s}^{-1}, $$

where $E_{50} = E_*/(50 \text{ eV})$. A transition in the ionization state around this luminosity is confirmed by our numerical calculations in Section 3, and has a dramatic effect on the fraction of escaping X-ray photons.

Similar arguments could be applied to other elements that may contribute to absorption. However, the Stromgren estimates are ultimately limited by the fact that, in the true radiation transport, ionizing photons may be produced not only by the source, but also within the TDE envelope. In particular, radiation absorbed by helium will be largely re-emitted as photons capable of ionizing hydrogen. Indeed, our numerical calculations (Section 3) find that hydrogen remains fully ionized for luminosities less than the $L_{\text{ion}}$ implied by Equation (14).

In the limit that helium is highly ionized ($L > L_{\text{ion}}$), we can estimate the fraction of helium in the He II state. Assuming that photoionization equilibrium holds at all radii, the number densities, $n_{\text{He II}}$ and $n_{\text{He III}}$, of He II and He III, respectively, are related by

$$ n_{\text{He II}} \bar{I} = n_{\text{He III}} \nu_{\text{He III}}, $$

where $\bar{I}$ is the photoionization rate and the radiative recombination coefficient depends on temperature approximately as $\alpha_B \propto T^{-1/2}$. Using the temperature structure from Equation (6) gives

$$ \alpha_B(r) \approx \alpha_{B,0} T_{5,5}^{-1/2} \left( \frac{r}{r_m} \right)^{1/8}, $$

where $T_{5,5} = T_5/10^5 \text{K}$. For typical envelope densities, collisional ionization rates are small, while the recombination timescale, $t_{\text{rec}} \approx 1/n_e \alpha_B \approx 1 \text{ s}$, is short, validating the assumption of photoionization equilibrium. The photoionization rate is

$$ I(r) = 4\pi \int_{\nu_{\text{ion}}}^{\infty} \sigma_{\nu}^\text{ion} J_\nu(r) d\nu, $$

where $\nu_{\text{ion}}$ is the threshold frequency for He II ionization, $J_\nu(r)$ is the mean specific intensity of the radiation field, and the photoionization cross-section $\sigma_{\nu}^\text{ion}$ is given to good approximation (about 15% error because of neglect of Gaunt factors) for hydrogenic ions by

$$ \sigma_{\nu}^\text{ion}(\nu) = \sigma_0 \left( \frac{\nu}{\nu_{\text{ion}}} \right)^{-3}, $$

with $\sigma_0 \approx 1.5 \times 10^{-18} \text{cm}^2$ for He II. In the limit that only a small fraction of the source radiation is absorbed, the frequency dependence of $J_\nu$ will be a Planck distribution at the source temperature, $B_\nu(T_\text{s})$. The radiation energy density, however, will be diluted according to the diffusion solution Equation (5), giving

$$ J_\nu(r) = B_\nu(T_\text{s}) \left( \frac{r_m}{r} \right)^3, $$

which assumes $r_{\text{out}} \gg r_m$. The ionization rate is then

$$ I(r) = \frac{8\pi \nu_{\text{ion}}^2 \sigma_0}{c^2} \left( \frac{r_m}{r} \right)^3 \Omega(T_\text{s}), $$

where the dimensionless factor $\Omega$ is

$$ \Omega(T_\text{s}) = \int_{h\nu_{\text{ion}}/kT_\text{s}}^{\infty} \frac{dx}{x(e^x - 1)} \approx 0.8 \frac{kT_\text{s}}{\hbar \nu_{\text{ion}}} e^{-\hbar \nu_{\text{ion}}/kT_\text{s}}. $$

The second expression approximates the integral to within a few percent over the range of temperatures of interest. The ionization equilibrium Equation (15) then determines the fractional ratio of He II

$$ f_{\text{He II}} = \frac{n_{\text{He II}}}{n_{\text{He III}}} = \left[ \frac{n_{\text{He III}} \alpha_{B,0} e^2}{8\pi \nu_{\text{ion}}^3 \sigma_0 T_{5,5}^{-1/2} \Omega(T_\text{s})} \right] \left( \frac{r}{r_m} \right)^{1/8}, $$

where we have again taken $n_e = n$. The fraction of He II grows with radius, due both to the decrease of the ionizing radiation field and the increase of the recombination coefficient at larger
radii. The He\textsc{ii} fraction at $r_{in}$ is

$$f_{\text{He}\textsc{ii}}(r_{in}) = 6 \times 10^{-11} e^{6.31 T_{s,5}^{-3/2}} M_{e,0.5} r_{in}^{-1},$$

(23) which shows that for these fiducial parameters, most of the helium is in the fully ionized He\textsc{iii} state. This predicted He\textsc{ii} fraction is very similar to what we find in our numerical transport calculations (Section 3) when $L \gg L_{\text{env}}$. The temperature dependence in Equation (23) resembles the LTE Saha equation expression, a consequence of the assumed Planckian frequency distribution of the radiation field. Near the inner boundary, where the radiation field approaches a true blackbody, the ionization state will approach its LTE value.

2.4. Reprocessed Luminosity

Having solved for the ionization state, we can calculate the optical depth to He\textsc{ii} photoionization

$$\tau_{\text{He}\textsc{ii}}(\nu) = \int_{r_{in}}^{r_{\text{opt}}} n_{\text{He}\textsc{ii}}(r) \sigma_0(\nu/\nu_{\text{ion}})^{-3} dr.$$  

(24) Figure 2 makes it clear that other opacities (e.g., He\textsc{ii} and oxygen lines) also contribute to the opacity soft X-ray wavelengths; however, our analysis of the He\textsc{ii} photoionization provides some proxy for the more general radiative processes.

When most of the helium is in the He\textsc{iii} state (as indicated by Equation (23)), we can write the number density of He\textsc{ii} as $n_{\text{He}\textsc{ii}} = n_{\text{He}\textsc{ii}} \rho_{\text{He}\textsc{ii}}$. For the temperature range of interest $(2 \times 10^3 K < T_s < 10^6 K)$, we can more coarsely approximate Equation (21) as $\Omega = 6 \times 10^{-3} T_{s,5}$. Using our ionization solution, Equation (22) gives

$$\tau_{\text{He}\textsc{ii}}(\nu) = \frac{A_{\text{He}\textsc{ii}} n_{\text{He}\textsc{ii}}^{2/3} \nu_{\text{opt}}^{2/3} \Omega(T_s)}{3 \nu_{\text{ion}}^{1/3} T_{s,5}^{2/3} \Omega(T_s) \left[ \left( \frac{\nu}{\nu_{\text{ion}}} \right)^{-3/8} \left( \frac{\nu_{\text{opt}}}{\nu_{\text{ion}}} \right)^{3/8} - 1 \right]}$$

$$\approx 0.02 L_{45}^{-5/8} M_{e,0.5}^{11/8} r_{i,14}^{-3/2} r_{o,15}^{-1} (\nu/\nu_{\text{ion}})^{-3/2}.$$  

(25) The expected fraction of absorptive to electron scattering opacity at threshold is $A_{\text{He}\textsc{ii}} f_{\text{He}\textsc{ii}}(r_{in}) \sigma_0/\sigma_T \approx 2 \times 10^{-5}$ (where $\sigma_T$ is the Thomson cross-section for electron scattering), in rough agreement with our numerical results (Figure 2).

In the radial direction, the envelope is optically thin to He\textsc{ii} photoionization. However, the electron scattering opacity increases the path length of photons as they random walk through the envelope, enhancing the probability of absorption. The typical number of scatters is $\tau_{es}$, and so the effective optical depth for a photon to be absorbed is $\tau_e = \sqrt{\tau_{\text{He}\textsc{ii}} \tau_{es}}$, or

$$\tau_e(\nu) \approx 2 L_{45}^{-5/16} M_{e,0.5}^{19/16} r_{i,14}^{-5/4} r_{o,15}^{-1/4} (\nu/\nu_{\text{ion}})^{-3/2}.$$  

(27) For frequencies near the He\textsc{ii} threshold, the envelope can thus absorb and re-emit a fraction of the source luminosity, $L$. As a simple estimate of the reprocessed luminosity, we assume that a fraction $e^{-\tau}$ of the source luminosity is absorbed and re-emitted as a blackbody at an envelope temperature $T_r$. The specific luminosity of reprocessed light is then

$$L_e(\nu) = e^{-\tau_e} L \int_{0}^{\infty} B_{\nu}(T_r) d\nu,$$  

(28) If observations are taken at a frequency $\nu_{\text{opt}} = 4000 \text{ Å}$ that is on the Rayleigh–Jeans tail of the blackbody ($h\nu_{\text{opt}} \ll k_B T_r$), and when $\tau_e \ll 1$, the observed luminosity is

$$L_{\text{opt}}(\nu_{\text{opt}}) = \nu_{\text{opt}} \tau_e L \frac{2 \pi \nu_{\text{opt}}^2 k_B}{c^2 \sigma_T T_r^4} \approx 1.1 \times 10^{43} L_{45}^{-1/16} M_{e,0.5}^{7/16} r_{i,14}^{-1/4} r_{o,15}^{-1/4} \text{ erg s}^{-1}.$$  

(29) The normalization in the second expression depends on the assumed temperature $T_r$ of the reprocessed radiation; the value in Equation (30) assumes it is emitted at a radius of $5r_{in}$, where $T_r \approx T_s/3$.

Though approximate, our analytic treatment provides insight into how reprocessing takes place in highly ionized TDE envelopes. The reprocessed optical luminosity increases with $M_{\text{env}}$, though sub-linearly. Having more mass in the envelope increases the effective absorptive optical depth, but there is also a counteracting effect; a higher $M_{\text{env}}$ produces higher envelope source and temperatures, which increases the ionization state and shifts the reprocessed emission to shorter wavelengths.

In the highly ionized regime, the reprocessed optical luminosity depends very weakly on the source luminosity. This is because a higher $L$ leads to higher ionization state, and hence a lower He\textsc{ii} effective optical depth; although the input radiation is brighter, a smaller fraction of it is absorbed and reprocessed. This behavior, however, only holds in the limit that a small fraction of the source X-rays are absorbed. When $L \lesssim L_{\text{env}}$, He\textsc{iii} will recombine and a He\textsc{ii} ion front will develop in the envelope. In this case, nearly all of the source photons with energies $\gtrsim 54.4 \text{ eV}$ are absorbed. The reprocessing becomes highly efficient and the optical luminosity will more closely track the input luminosity, in contrast to the weak dependence found in Equation (30).

The scalings in Equation (30) also do not reflect the likely correlation between the parameters. For example, both the envelope mass and bolometric luminosity may be decreasing at the light curve peak. In this situation, the bolometric luminosity and the reprocessed luminosity will track each other more closely than what Equation (30) predicts when $L$ is varied on its own.

2.5. Optical Line and Continuum Formation

The soft X-rays absorbed by photoionization in the envelope can be re-emitted at longer wavelengths. In an optically thin medium (e.g., an HII region), absorbed photons are primarily re-emitted in lines. In TDE envelopes, in contrast, the medium can be effectively optically thick in the continuum, and radiation will be reprocessed into both continuum and lines.

Figure 3 shows an example of the relevant opacities for TDE envelopes at optical wavelengths from a numerical calculation to be discussed in Section 3. The primary continuum opacity is free–free (bremsstrahlung), with some contribution from bound-free absorption from excited states of hydrogen and helium. The free–free absorption coefficient in the Rayleigh–Jeans limit ($h\nu < k_B T$) is

$$\alpha_{\text{ff}} \approx 2 \times 10^{-2} n_e T_{s,5}^{-3/2} g_{\text{ff}} \nu^{-2}$$  

(31) where the free–free Gaunt factor, $g_{\text{ff}}$, is of the order of unity. This can be compared to the electron scattering absorption coefficient $\alpha_{\text{es}} = n_e \sigma_T$ to give the absorption ratio
Optical lines provide negligible opacity. Oxygen lines provide negligible opacity.

$O_\lambda$ lines provide negligible opacity.

$O_\lambda$ lines provide negligible opacity.

Hydrogen Balmer series lines provide the highest opacities of all, with $H_\alpha$ being the most opaque. Oxygen lines provide negligible opacity.

The absorption coefficient at the line center frequency is

$$
\sigma_\nu = \frac{\pi e^2}{m_e c} n_i f_{osc} \phi_\nu (\nu) \left[ 1 - \frac{g_i n_i}{g_a n_a} \right],
$$

where $f_{osc}$ is the oscillator strength and $\phi_\nu$ is the line profile function. The term in brackets is the correction for stimulated emission where $g_i$ and $g_a$ are the statistical weights. To estimate the extinction at the line center frequency, $\nu_0$, we approximate $\phi(\nu_0) \approx 1/\Delta \nu$, where for a line Doppler broadened by a velocity $c$ the line width $\Delta \nu = \nu_0 (c/\nu_0)$. When the radiation field takes the form of a diluted blackbody (Equation (19)), the relative level populations will approximate their LTE values, e.g., $n_e/n_i \approx (g_a/g_i) e^{-\Delta \nu/\nu_0^{T_s}}$. In the limit $h \nu_0 \ll kT_s$, Equation (33) then becomes

$$
\sigma_{bb} \approx \frac{\pi e^2}{m_e c} n_i f_{osc} \frac{c}{v_0} \left[ 1 - \frac{g_i n_i}{g_a n_a} \right] k_b T_s. \quad (34)
$$

The ratio $\sigma_{bb}/\sigma_{es}$ is then

$$
\epsilon_{bb} \approx 0.06 \frac{n_i}{10^{-10} f_{osc} v_0^{-1} T_s^{-1}}, \quad (35)
$$

where $v_0 = \nu/10^9$ cm s$^{-1}$ and the density $n_i$ is scaled by the expected ionization fraction of Equation (23). The $H\alpha$ line emission thus originates from a region of electron scattering depth of $\tau_{\text{therm}} = 1/\sqrt{\epsilon} \approx 6$. Due to the lower abundance of helium, $A_{He} \approx 0.1$, the optical depth of the $H\beta$ line (all other things being roughly equal) should be smaller than $H\alpha$ by a factor $\sim 10$, suggesting a larger thermalization depth $\tau_{\text{therm}} \approx 20$. The values roughly agree with the numerical results of Figure 3.

These arguments motivate the physical picture illustrated in the schematic of Figure 1. In this quasi-static picture, spectrum formation resembles that of a stellar atmosphere, with the emission at different wavelengths originating from the source function at different thermalization depths. Soft X-rays are generated in the interior (at our inner boundary). The optical continuum forms further out, while the effective photospheres of strong lines of hydrogen and helium lie even nearer the surface. All of the observed emission is generated below the electron scattering photosphere.

### 3. NUMERICAL RESULTS

In this section, we present synthetic non-LTE spectra for TDE envelopes, discussing the physics of optical line and continuum formation and the dependence on envelope parameters. Details of the numerical method and radiative processes treated are given in the Appendix. In all of the models, a luminosity $L$ is emitted as a blackbody of temperature $T_s$ at the absorbing inner boundary $r_{in}$, and transported through the spherical envelope of mass $M_{env}$ with a density structure $\rho (r) \approx r^{-2}$ extending from radii $r_{in}$ to $r_{out}$. In order to avoid an artifically abrupt truncation of the envelope, we allow the envelope to extend beyond $r_{out}$ following an $r^{-1}$ density profile (we find that our results are not highly sensitive to the exact value of this power law). We include opacities from hydrogen, helium, and oxygen, in solar abundances.

For the bound–bound transitions, we assume a Gaussian line profile with a Doppler velocity of $10^4$ km s$^{-1}$. This velocity, which is motivated by the width of line features observed in TDE candidate spectra (e.g., Gezari et al. 2012; Arcavi et al. 2014), is much higher than the mean sound speed, but comparable to the virial velocity in TDE envelopes. Our setup thus resembles a quasi-static envelope with disordered velocities due to, e.g., turbulence or irregular motion driven by fallback streams, as seen in numerical simulations (Ramirez-Ruiz & Rosswog 2009; Guillochon et al. 2014). In envelopes with ordered bulk velocity due to, e.g., outflow or rotation, the line formation may differ from that discussed here. Future calculations will consider more general velocity structures and include a more complete metal composition.

#### 3.1. Spectral Energy Distributions

Figure 4 shows computed spectral energy distributions (SEDs) for two models with $M_{env} = 0.25$ $M_\odot$ and $L = 10^{45}$ erg s$^{-1}$ ($\approx L_{edd}$ of a $10^7 M_\odot$ BH) but with two different values of the inner boundary radius. The dashed curves show the unprocessed blackbody spectrum from the inner boundary. We find that reprocessing of the source light by the TDE envelope enhances the optical luminosity by several orders of magnitude. The presence of the extended envelope is clearly required to approach the optical luminosity of $\sim 10^{43}$ erg s$^{-1}$ observed in TDE candidates.

As suggested by our analytic estimates (Equation (27)) for the chosen values of $L$ and $M_{env}$, most of the soft X-ray source
emission is able to propagate through the envelope without being absorbed. Those photons that are absorbed are mostly used to photoionize HeII at wavelengths near 220 Å. For the $r_{in} = 5 \times 10^3$ cm model, oxygen absorption at the shortest wavelengths also plays a role. The radiation absorbed by photoionization (though small) is re-emitted over a wide range of longer wavelengths and significantly enhances the optical flux. The simultaneous emission of both soft X-rays and optical light with an SED that does not follow a single blackbody bears a qualitative resemblance to PTF10iya, ASASSN-14li, and ASASSN-15oi (Cenko et al. 2012a, 2016; Miller et al. 2015; Holoien et al. 2016a, 2016b).

The reprocessed optical continuum in Figure 4 is relatively insensitive to the chosen inner radius, and follows a power law that superficially resembles the Rayleigh–Jeans tail of a blackbody. However, the emission is not that of a single blackbody; the thermalization depth varies with wavelength, and the continuum is the superposition of thermal emission from different temperature blackbodies at different radii. Due to this effect, the slope of our model continuum, $L_\lambda \propto \lambda^{-3}$, is somewhat shallower than that of a true Rayleigh–Jeans tail, $L_\lambda \propto \lambda^{-4}$.

Given the non-blackbody nature of the emergent spectrum, the turn-over that one begins to see at bluer wavelengths cannot necessarily be extrapolated with a Planck function. This suggests caution when inferring a bolometric luminosity by fitting a single blackbody temperature to the optical continuum. In both models of Figure 4, the bolometric luminosity is $10^{45}$ erg s$^{-1}$, much larger than one might estimate based on a single blackbody to the optical/UV data.

Figure 5 shows how the model SEDs depend on the envelope mass, $M_{env}$. For this figure, $r_{in} = 10^{15}$ cm and for clarity we held the temperature of the inner boundary emission fixed at 3.29 $\times$ 10$^5$ K (unlike the rest of the models we present in this work, we did not set $T_e$ equal to $T(r_{in})$ as given by Equation (7)). As expected from our analytic arguments (Equation (30)) the optical luminosity increases with $M_{env}$ due to the greater degree reprocessing by higher mass envelopes. For the smallest envelope mass ($M_{env} = 0.02 M_\odot$) essentially none of the source luminosity is reprocessed, despite the fact that the gas is optically thick to electron scattering ($\tau_{es} \approx 20$). The emphasizes that in scattering dominated TDE envelopes, an optical depth $>1$ is required to thermalize and reprocess X-rays to optical wavelengths.

Figure 6 shows how the model SEDs depend on the source luminosity, $L$. Interestingly, the optical continuum luminosity remains largely unchanged even as $L$ is varied by a factor of $\sim$10 (again, holding the mass and size of the envelope fixed). This is consistent with the weak $L$ dependence found in our analytic arguments (Equation (30)) and reflects a self-regulating effect in the radiation transport. Higher $L$ leads to a higher...
ionization state and higher envelope temperatures, which reduces the fraction of X-ray emission that is reprocessed to longer wavelengths.

A dramatic change in the spectra of Figure 6 is seen when the source luminosity is reduced to $2.5 \times 10^{44}$ erg s$^{-1}$. This is due to the formation of a He II recombination front that absorbs essentially all radiation at wavelengths below the photoionization threshold ($\lesssim 220$ Å). This transition occurs roughly near the critical luminosity estimated by Strömgren arguments in Equation (14). For these lower luminosities, nearly all of the ionization is high, and the UV/optical luminosity will track the source luminosity, in contrast to the weak $L$ dependence found in the fully ionized regime.

### 3.2. Spectral Line Features

The spectra of Figures 4–6 show a number of line features superimposed on the continuum. In the soft X-ray/UV bands, the strongest lines are those of the He II Lyman series at wavelengths between 200 and 400 Å. Other UV lines from highly ionized species might appear if more metals had been included in our calculations. In the optical bands, lines of hydrogen (the Balmer series) and He II (the 4686 and 3203 Å lines) may be visible. We find that the line corresponding to the $n = 7$ to $n = 4$ transition in He II with wavelength 5412 Å (a Pickering series line, analogous to Brackett $\gamma$ in H) appears for some of our models. Finally, we note that the He II Pickering line at 6560 Å has a small contribution to emission and opacity near Hα, as pointed out in Gaskell & Rojas Lobos (2014).

The relative strength of the optical hydrogen and helium lines has generated particular interest in the literature, as this bears on the gas composition and hence the nature of the disrupted star. Figure 7 shows the optical spectra for three models with $L = 10^{45}$ erg s$^{-1}$, $M_{\text{env}} = 0.25 M_{\odot}$, but different values for the outer radius of the envelope, $r_{\text{out}}$. In all cases, the emission in the He II 4686 line exceeds that of Hα, despite the gas having solar composition. As $r_{\text{out}}$ is decreased, the Hα emission decreases with respect to the continuum. The line ratios can be seen more clearly in Figure 8, in which we have subtracted off a power law to approximate the underlying continuum. The hydrogen-to-helium line ratios are roughly 3:1 and 5:1 for $r_{\text{out}}$ of $2 \times 10^{15}$ and $1 \times 10^{15}$, respectively. For $r_{\text{out}} = 5 \times 10^{14}$, the Hα feature has transitioned into a shallow absorption.

Our calculations thus lend further support to the interpretation that the absence of a conspicuous Hα feature—as observed in PS1-10jh and PTF-09ge—is consistent with the disruption of a main-sequence star of solar composition. In general, TDE envelopes can produce a range of Hα equivalent widths and hydrogen-to-helium line ratios, depending on the gas configuration and bolometric luminosity. Such a variation might help to explain the diversity of hydrogen-to-helium line ratios seen in observed TDE candidates (Arcavi et al. 2014).

For lower luminosities ($L < L_{\text{ion}}$) recombination fronts of helium, and eventually hydrogen form in the envelope. The recombination fronts are generally accompanied by an increase in the strength of spectral features (see Figure 6), including lines of hydrogen, helium, and oxygen. A more featureless spectrum is more easily obtained when the ionization is high, which for lower luminosities ($L \lesssim 10^{44}$ erg s$^{-1}$) may require a correspondingly smaller envelope mass or a larger outer radius.

### 3.3. Understanding the Line Ratios

The origin of the low Hα to He II emission line ratio found in our models is due to our inclusion of optically thick radiation transport effects. In a TDE envelope, emitted line photons do not escape straightaway, but rather random walk through multiple electron
The emissivity in Equation (36) is sensitive to the particular model parameters, which set the exact location of $r_{\text{therm}}$ and the line emissivity above it. For different envelope configurations, the H\textsc{\textbeta} line will therefore show different levels of emission, as we found in our synthetic spectra (Figure 7). For TDEs from massive BHs and near peak brightness, however, we typically expect a situation similar to that of Figure 9, where the H\textsc{\textalpha} emission is suppressed relative to that of He\textsc{\textii}.

Our analysis here can be distinguished from previous studies of emission line ratios in TDEs. Guillochon et al. (2014) suggested that H\textsc{\textalpha} emission in the inner layers could be suppressed by a strong ionizing continuum, and that the line strength would thus correlate with the spatial extent of solar composition gas. This conclusion, however, was based on an interpretation of existing CLOUDY calculations of AGN broad-lined regions (Korista & Goad 2004) that were not directly applicable to the TDE conditions. Gaskell & Rojas Lobos (2014) used new CLOUDY calculations to show that a line optical depth effects can lead to self-shielding and a
suppression of Hα in a way similar to what we have described. This result, however, was criticized by Strubbe & Murray (2015) who argue that the Gaskell & Rojas Lobos (2014) neglect the large line broadening which should reduce the line optical depth. Indeed, we find that the Hα radial optical depth is ≤1 in our TDE envelopes, and that it is the high electron scattering optical depth that is critical to the trapping and destruction of hydrogen line photons.

4. IMPLICATIONS FOR OBSERVATIONS AND MODELS OF TDEs

In many ways, our synthetic spectra resemble those of observed TDE candidates, such as PS1-10jh, near maximum light. In particular, the spectrum is largely featureless and blue, with two emission lines of He II and a weak or absent Hα feature. In detail, however, some discrepancies are apparent. Our model continua slopes are somewhat shallower, $L_\lambda \propto \lambda^{-3}$, than the Rayleigh–Jeans tail, $L_\lambda \propto \lambda^{-4}$, seen in PS1-10jh (Gezari et al. 2012). Nevertheless, our slope is consistent with HST UV data for ASASSN-14li (Cenko et al. 2016), as well as the spectral indices measured in TDE1/TDE2 (van Velzen et al. 2011), in the absence of strong dust extinction.

Our optical luminosity for a massive envelope ($M_{\text{env}} = 0.5 M_\odot$) tops out at $\approx 10^{43}$ erg s$^{-1}$, which is a factor of $\approx 2$ less than some observed TDEs. This may in part be due to our incomplete inclusion of metals, which may result in an underestimate of the net reprocessing efficiency. The efficiency could also be enhanced if the gas is confined to, e.g., a shell or disk, which would increase the density. The radiation from an aspherical envelope will also be anisotropic, with the observed luminosity brighter from viewing angles that maximize the projected surface area.

Observations of ASASSN-14li showed simultaneous X-ray, UV, and optical emission (Miller et al. 2015; Cenko et al. 2016; Holoien et al. 2016b). The SED cannot be fit by a single blackbody, but appears consistent with the superposition of different temperature blackbodies for the X-rays and optical/UV. Such an SED resembles our highly ionized reprocessing envelopes (e.g., Figure 4) for which thermal X-rays characteristic of the hot inner accretion regions are only partially reprocessed to optical radiation characteristic of the lower envelope temperatures, resulting in a multi-blackbody SED.

Our calculations have focused on radiation transport in generic TDE envelopes, remaining largely agnostic about the origin of the envelope. The mechanism for forming an extended gas distribution is unclear, and several classes of explanations have been suggested. The insights from our radiative transfer studies can be applied (within limit) to illuminate the possible observable properties of various scenarios.

4.1. Quasi-static Envelopes

Loeb & Ulmer (1997) predicted that TDEs could form an optically thick, roughly spherical, radiation pressure supported envelope with radius $r_{\text{out}} \sim 10^{15}$ cm. The stability of such an envelope depends on the luminosity being regulated to be near $L_{\text{Edd}}$ (Ulmer et al. 1998). Analytic studies by Coughlin & Begelman (2014) suggest that even if the accretion rate is super-Eddington, the flow can equilibrate in a quasi-stable configuration by allowing energy to escape in a narrowly confined jet. Additionally, the earliest three-dimensional hydrodynamical simulations of TDEs that followed the event past the initial disruption and into the accretion phase, such as Ayal et al. (2000) and Bogdanović et al. (2004), provided some evidence for the build-up of material at large radii that could reprocess accretion luminosity. More recent calculations such as Guillot et al. (2014); see also Ramirez-Ruiz & Rosswog 2009; Rosswog et al. 2009) show the development of an extended gas distribution around the eventual accretion disk, which is produced as shock-heated material is lifted above and below the original orbital plane of the disrupted star’s orbit; (such calculations, however, have not followed the longer term radiative evolution of this debris).

Our model calculations provide a fair representation of the radiative transfer in such quasi-static envelopes, and suggest that such a scenario (with a solar composition envelope of mass $M_{\text{env}} \approx 0.1–0.5 M_\odot$) can likely reproduce the maximum light spectrums of observed optical TDEs like PS1-10jh. More specific model calculations are warranted; the power-law exponent of the envelope density structure may differ from our choice $\rho \propto r^{-2}$ (for a static radiation pressure-supported envelope, $\rho \propto r^{-3}$ as in Loeb & Ulmer 1997) and the velocity dispersion may be due in part to rotational or circulatory motion, not the random motion adopted in our calculations.

The properties of a quasi-static envelope can be expected to vary over time. Initially, over the timescale for the most-bound debris to fall back to the BH ($t_{\text{fb}} \approx 25 [M_{\text{BH}}/10^4 M_\odot]^{1/2}$ days for the disruption of a solar mass star), the envelope mass may increase. For highly ionized envelopes, a larger $M_{\text{env}}$ leads to more efficient reprocessing (see Figure 5). The initial rise of the optical TDE light curves could therefore reflect the gradual accumulation of mass in an envelope. The decline of the light curve with roughly constant color could likewise reflect the draining of the envelope. Such an evolution is hinted at by observations of PS1-10jh, which appears to have a photosphere that first grows and later recedes (Strubbe & Murray 2015). The weak dependence of the reprocessed luminosity on the source luminosity, when the mass and extent of the envelope are held fixed, suggests that the peak luminosity of TDEs with $M_{\text{bh}} \gtrsim 10^6 M_\odot$ may be regulated to be near a few times $10^{43}$ erg s$^{-1}$, with a stronger dependence on the envelope mass than the BH mass.

If, after being assembled, the mass of the envelope remains roughly fixed, then at some time after maximum light the source luminosity should decline to $L \leq L_{\text{Edd}}$ where recombination fronts form. The source luminosity would thus be nearly completely absorbed and reprocessed, and the optical luminosity may track the accretion rate, as perhaps suggested by the observed $r^{-5/3}$ decline in the optical light curves. Little soft X-ray emission would be expected at these later times unless deviations from spherical symmetry allows for channels for X-rays to escape, or the envelope mass is at some point depleted.

4.2. Outflows

A different mechanism for generating an extended gas distribution is through outflows. Super-Eddington accretion onto the BH is likely to unbind some fraction of the infalling debris (Strubbe & Quataert 2009; Lodato & Rossi 2011; Metzger & Stone 2015; Vinkó et al. 2015). Miller (2015), drawing on work by Laor & Davis (2014), suggest that line-driven winds analogous to those launched from the atmospheres of massive stars might also be responsible for
launching outflows. For a wind launched near the tidal disruption radius, the expected escape velocities are \( \sim 10^3 \) km s\(^{-1}\), which is much greater than that observed in the line widths of observed TDE spectra. However, if debris circularizes at radii much larger than \( R_{\text{bd}} \), or if the winds are mass loaded, the expansion velocities may be lower.

Strubbe & Quataert (2009) argued that accretion may continuously generate an optically thick outflow, which would advect the energy dissipated at the accretion disk to larger radii. That advection may circumvent some of the complexities of absorption and reprocessing that we have studied here. Photons trapped in an outflow will be adiabatically degraded, which provides a robust mechanism for shifting the SED to longer wavelengths. The photons finally decouple from the flow at a trapping radius set by \( \tau \sim c/v \). This scenario most closely resembles the calculations presented here only if the advection velocity is relatively low \( (v \approx 1000 \) km s\(^{-1}\), in which case the trapping radius coincides with the inner boundary used in our calculations \( (\tau \approx 300) \). Based on analysis of the data of PS1-10jh, Strubbe & Murray (2015) suggest an outflow that does move at such a low velocity. However, their suggested outflow mass of 0.02 \( M_\odot \) is likely too low to efficiently reprocess the source radiation, as shown in Figure 5.

Metzger & Stone (2015) present a modified outflow picture in which a large fraction of the inflowing tidal debris is promptly unbound, forming a quasi-spherical outflow with expansion velocities \( \sim 10^4 \) km s\(^{-1}\). Radiation from the accretion disk of the BH is then absorbed and reprocessed by the outflow. At times near the peak of the optical TDE light curve \( (\sim \text{weeks}) \) that outflow will have reached radii \( r_{\text{out}} \approx 10^{15} \) cm and the diffusion time is of the order of the expansion time. Our models provide a fair representation of such density distributions, although we do not include a radial velocity gradient. Nonetheless, the indications are that such a scenario can likely reproduce the near maximum light spectra of observed TDEs.

In contrast to the quasi-static or steady wind models, the optical depth of a prompt outflow necessarily decreases with time, given that \( \tau_{\text{out}} \propto t \). At some point, the envelope should become inefficient at reprocessing, at which time soft X-rays will escape and the optical light curve may no longer track the accretion rate. Since the critical luminosity \( (L_{\text{ion}} \propto r^{-2}) \) decreases more steeply \( (L_{\text{ion}} \propto t^{-5/3}) \), the condition \( L > L_{\text{ion}} \) is expected to occur at some late time. Metzger & Stone (2015) have discussed this possible “ionization break-out” and estimated its properties.

### 4.3. Circularization at Large Radius

Recent numerical work has suggested that the optical emission might arise from dissipation in material in the process of circularizing at large radii. Shiokawa et al. (2015) performed general relativistic (GR) hydrodynamics simulations and found that shocks located at the apoaope of orbits of returning material lead to build-up of material with enough angular momentum to support wide orbits, with a semimajor axis corresponding to that of the most bound debris from the initial disruption, several times \( 10^{14} \) cm. This is at least an order of magnitude larger than the periapsis of the initial stellar orbit, where most earlier work had suggested that circularization should occur. Similar but less pronounced effects were found in the smooth particle hydrodynamics simulations including leading-order GR effects performed by Bonnerot et al. (2016) and Hayasaki et al. (2015), and the grid-based Newtonian gravity calculation of Guillochon et al. (2014). Emission from the circularizing gas at these large radii has been suggested to give rise to the observed optical emission (Piran et al. 2015).

If the gas distribution formed from such circularization processes \( (\text{or perhaps their associated outflows}) \) extends from \( \sim 10^{14} \)–\( 10^{15} \) cm, the situation resembles the numerical calculations presented here. Given that the gas is highly scattering dominated, a relatively high optical depth \( \tau_{\text{opt}} \gtrsim 40 \) in that material is required to thermalize the radiation in the circularization region. Assuming the optical depth is at least that high, we find that it makes little difference to the optical spectra whether the luminosity is generated at a circularization radius of \( \sim 10^{14} \) cm or nearer the BH at \( \sim 10^{13} \) cm (see Figure 4).

Indeed, much more energy is expected to be liberated as material accretes onto the BH than from the circularization process itself, although mechanisms for hiding the accretion energy have been suggested (Piran et al. 2015; Svirski et al. 2015).

Simultaneous X-ray and optical observations for events such as PTF10iya, ASASSN-14li, and ASASSN-15oi (Cenko et al. 2012a; Miller et al. 2015; Holoien et al. 2016a, 2016b) provide some hope of distinguishing the source of the luminosity, since for highly ionized envelopes the X-ray spectrum will be largely preserved as it propagates through the scattering dominated envelope. The peak of the X-ray flux then reflects the temperature at the depth at which it was produced (see Figure 4).

### 5. CONCLUSIONS

#### 5.1. Summary of Key Results

This paper has presented analytic estimates and detailed radiative transfer calculations that clarify how the spectra of TDEs can be generated within an extended envelope. We have focused our attention on TDEs around the optical light curve peak, with bolometric luminosities in the range \( 10^{44} \)–\( 10^{45} \) erg s\(^{-1}\) \( (\text{corresponding to the Eddington luminosity of } 10^{44} \text{–} 10^{45} \) \( L_{\odot} \text{BHs}) \). We accounted for non-LTE effects and the high electron scattering optical depth of the reprocessing material, which we show are crucial for understanding both the thermalization of the optical continuum and the emission line ratios.

We identified two regimes of reprocessing, depending on the envelope ionization state. When the envelope is highly ionized \( (L > L_{\text{ion}}) \), we find that the intrinsic X-ray emission from the accretion disk is only partially absorbed, giving rise to an SED that peaks in the soft X-ray but is accompanied by an enhanced optical emission component \( (\gtrsim 10^{43} \) erg s\(^{-1}\) at wavelengths longer than 1000 Å). In addition to providing an optical flux at a sufficient brightness to match observations, this can explain TDEs observed simultaneously in the optical and X-ray, such as PTF10iya, ASASSN-14li, and ASASSN-15oi.

If the bolometric luminosity declines rapidly enough compared to the mass of the envelope, a critical value can be reached \( (L < L_{\text{ion}}) \) for which a helium recombination front forms and the soft X-rays are completely absorbed. This second regime resembles TDEs that have been observed at optical wavelengths without a coincident X-ray signal, such as PS1-10jh and ASASSN-14ae. In this regime, the reprocessing is completely efficient and the optical/UV should closely track the accretion luminosity.
In general, the X-ray through optical SED is not well described by a single blackbody, but is a blend of emission from a variety of depths and temperatures. For this reason, one must be cautious if attempting to fit a single blackbody temperature to optical data. One is likely to underestimate the bolometric luminosity in this way, as the SED can peak at shorter wavelengths than would be inferred from the optical data.

The light curve evolution of TDEs will depend on how the source luminosity, envelope mass, and envelope radius change with time. When the envelope is highly ionized ($L > L_{\text{ion}}$), the optical luminosity depends more on the envelope mass than the source luminosity. This is because increasing $L$ leads to higher envelope ionization, which reduces the efficiency with which X-rays are reprocessed to the optical. It is thus possible that the rise and fall of TDE light curves reflects in large part the growth and subsequent depletion of mass in the reprocessing envelope. As the mass of the envelope decreases, so does the reprocessing efficiency and the optical flux, but the shape of the continuum remains mostly unchanged. This points to an explanation for the near-constant color of the optical continuum.

We have demonstrated that the strength of line features in TDEs depends on optically thick radiation transport effects that are not well captured by photoionization codes like CLOUDY. Even in envelopes of solar composition, the $\text{H}_\alpha$ line may be highly suppressed relative to $\text{He}\,
\pi$ lines due to optical depth effects. By varying the configuration of material in the reprocessing envelope, a variety of helium-to-hydrogen line ratios can be realized in the optical spectrum. In particular, we have explored what happens as we vary the outer radius of the envelope while keeping the total mass fixed, and have shown a transition of $\text{H}_\alpha$ from emission to shallow absorption. Although the radial extent of the envelope is a key parameter, other parameters also influence the line ratios, including the bolometric luminosity and the mass of the envelope.

5.2. Outstanding Issues

While our studies have outlined many of the key physical processes at play in TDE envelopes, several questions remain to be addressed. An obvious area for improvement is relaxing the assumption of spherical symmetry. Variation in the density and temperature structure of the envelope with polar angle could lead to important viewing-angle effects which may be important for explaining why soft X-rays are visible in only a subset of observed TDEs. The density and temperature profile of the envelope may also vary with viewing angle, which may have implications for observables such as the slope of the optical continuum.

Another critical area needing further study is the kinematic structure of the envelope. We have only crudely accounted for motions via a Doppler line-width set to a value of the order of the virial velocity. In many scenarios, velocity gradients are due instead to bulk motions—outflows or rotation—that may alter the formation of line features. In homologously expanding atmospheres, for example, line interactions occur within localized resonance regions, which results in completely different line optical depth and source function. Radiative transfer studies of the detailed line profiles may illuminate the kinematics of the envelope. Radially expanding outflows like supernovae, for example, generally produce P-Cygni type absorption features, whereas the optical lines observed in TDEs are usually purely in emission, and occasionally show substructure (Arcavi et al. 2014; Holoien et al. 2016b).

Our calculations have only directly tracked bound–bound and bound-free emission features from hydrogen, helium, and oxygen. In reality, other metals likely increase the opacity, especially in the X-ray and UV, and ionization edges and lines from these other metals are likely to appear in the spectrum. Nevertheless, for the luminosities we studied, we found that the optical continuum and optical helium-to-hydrogen line ratios do not seem to be greatly influenced by the presence of oxygen, the most abundant metal.

Our assumed inner boundary condition of blackbody radiation emitted at $r_{\text{in}}$ deserves further study. In reality, the source emission spectrum may be that from an accretion disk smaller than $r_{\text{in}}$. On the other hand, dense hot gas just below our inner boundary may have a Compton $y$-parameter $\gg 1$, in which case Comptonization may indeed thermalize the radiation to the gas temperature near the envelope base. Outside of our inner boundary, the Compton $y$-parameter can be of the order of unity, and may have some effect on the spectrum. The physics of Compton scattering will be directly included in our future Monte Carlo transport calculations.

Future work will include a more detailed exploration of the parameters governing the spectrum, including the mass of the envelope, accretion disk luminosity, and gas density gradient, as well as how these parameters evolve over time in different scenarios. Studies of this sort, in comparison to improved observations of TDEs, will hopefully clarify the physics governing these transients.

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APPENDIX A

NUMERICAL METHOD

To generate model spectra of TDE events, we carry out NLTE radiative transport calculations using the Monte Carlo radiative transfer code SEDONA (Kasen et al. 2006; Roth & Kasen 2015). The calculation is divided into two steps. First, the multi-wavelength radiation transport is calculated using Monte Carlo method which includes scattering, bound–bound, bound-free, and free–free radiative processes. Second, the gas temperature and atomic level populations are determined via a solution of the equations of statistical and thermal equilibrium. Since the photon opacities and emissivities depend on the level populations and temperature, these two steps are iterated (typically 20–60 times) until the envelope structure and output spectra have converged.
For the transport problem, we assume a stationary envelope, which should be applicable for TDE light curves near peak, when the diffusion time through the envelope is less than or comparable to the peak time. We enforce radiative equilibrium, justified by the heating-time arguments in Section 2, by “effectively scattering” photon packets during each interaction, thereby ensuring energy conservation. At each interaction, the outgoing packet is reassigned a wavelength, sampled from the NLTE emissivity distribution across all wavelengths, as in Carciofi & Bjorkman (2006).

To calculate the NLTE level populations, we use Monte Carlo estimators of the photoionization rates and the bound–bound radiative rates. For hydrogen, we include electron energy levels with principal quantum number of 6 or less. For He II, we include levels with principal quantum number of 9 or less. For oxygen, we use a total of 484 levels across all ionization states. We assume statistical equilibrium and solve the set of coupled linear equations such that the net transition rate for each electron level is zero.

A.1. Setup and Initial Conditions

We divide our spherical grid into 512 zones, equally spaced in radius from zero to \( r_{\text{out}} \). In order to avoid an artificially abrupt truncation of the envelope, we add roughly 100 zones that follow an \( r^{-10} \) density profile beyond \( r_{\text{out}} \) (we find that our results are not highly sensitive to the exact value of this power-law). We initialized each zone with approximately 1000 photon packets sampled from a blackbody wavelength distribution at the temperature in the zone computed from Equation (6). During each iteration, we emit approximately 100 million photon packets at the zone corresponding to our inner radius \( r_{\text{in}} \).

We impose an absorbing boundary condition at the inner radius—photons that scatter back below that radius are removed from the calculation. Likewise, photons that escape at the outer radius are tallied and removed from the calculation.

A.2. Radiative Processes Included

The radiative processes included in our calculation are electron scattering, free–free (bremsstrahlung), bound-free (photoionization) and bound–bound (line) transitions. The extinction coefficient for electron scattering is \( \alpha_{\text{ex}} = n_e \sigma_v \), while the free–free extinction coefficient is given by

\[
\alpha_{\nu}^f = \frac{4e^6}{3mhc} \left( \frac{2\pi}{3mkT} \right)^{1/2} Z^2 n_e n_{\text{ion}} \nu^{-3} \left( 1 - e^{-\nu/kT} \right),
\]

(37)

By Kirchhoff’s law, the free–free emissivity is \( j_{\nu}^f = \alpha_{\nu}^f B_{\nu}(T) \). The bound–bound extinction coefficient is given by Equation (33). The corresponding emissivity is

\[
j_{\nu}^{bb} = \frac{h\nu}{4\pi} n_2 A_{\text{nl}} \phi(\nu),
\]

(38)

where \( A_{\text{nl}} \) is the Einstein coefficient for spontaneous emission, and \( \phi(\nu) \) is the line profile. For \( \phi(\nu) \), we assume a Gaussian profile with a width corresponding to Doppler velocity of \( 10^4 \) km s\(^{-1}\).

We include bound-free transitions from all excited atomic levels. For hydrogen and He II, we use the photoionization cross-section (Rybicki & Lightman 1986)

\[
\sigma^\text{ion}_\nu = \frac{n_{i,i} \sigma^\text{ion}_{0,H}}{Z^2} \left( \frac{h\nu}{\chi_i} \right)^{-3},
\]

(39)

where \( n_{i,i} \) is the principal quantum number of the bound electron level labeled by index \( i \), \( \sigma^\text{ion}_{0,H} = 6.3 \times 10^{-18} \) cm\(^2\) and \( \chi_i \) is the ionization potential to remove an electron from level \( i \). For the ground state He I, we use the photoionization cross-section fits of Vernier et al. (1996). For oxygen, we use the TOPBase photoionization cross-sections smoothed over resonances. For atomic levels that do not have data, we use an approximate hydrogenic cross-section of the form Equation (39), with \( Z \) corresponding to the net nuclear charge seen by the valence electrons and \( n_{i,i} \) the principal quantum number of the valence electron being ionized.

When computing the photoionization extinction coefficient, we include the non-LTE correction for stimulated radiative recombination, yielding (e.g., Mihalas 1978)

\[
\alpha^\text{ion}_\nu = n_i \sigma^\text{ion}_\nu \left[ 1 - \frac{n_i n^+}{n_i} f(\nu) \exp \left( -\frac{h\nu}{kT} \right) \right],
\]

\[
f(\nu) = \left( \frac{h^2}{2\pi m_e kT} \right)^{3/2} \frac{g^-}{2g^+} \exp \left( \frac{\chi_i}{kT} \right),
\]

(40)

where \( n_i \) is the number density of particles with bound electron in level \( i \), \( n^+ \) is the number density for the ions in the ground state of the next highest ionization state, \( g^- \) and \( g^+ \) are the statistical weights of the species being ionized and the ionized state, respectively, and \( T \) is the temperature of the free electrons.

This opacity must be summed over all elements and all bound electron levels labeled by the index \( i \) within each ionization state.

To derive radiative recombination cross-sections, \( \sigma^\text{rec}(u_e) \), as a function of electron speed \( u_e \), we use the Milne relations which relate \( \sigma^\text{rec}(u_e) \) to the photoionization cross-sections. The associated emissivity for bound-free recombination is (see Osterbrock & Ferland 2006)

\[
j_{\nu}^\text{rec} = \frac{n^+ n_e}{4\pi} u_e f_u \sigma^\text{rec}(u_e) \frac{h\nu}{du} \frac{du}{dv},
\]

\[
u_e = \frac{2}{m_e} (h\nu - \chi_i),
\]

\[
f_u = \frac{4}{\sqrt{\pi}} \left( \frac{m_e}{2kT} \right)^{3/2} u_e^2 \exp \left( -\frac{m_e u_e^2}{2kT} \right).
\]

(41)

This emissivity must be summed over all elements and all bound electron levels to which the free electron may recombine. We derive the temperature-dependent radiative recombination coefficient for each atomic level by integrating \( \sigma^\text{rec}(u_e) \) over the electron Maxwell–Boltzmann distribution.

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