Algebra of distributions of quantum-field densities and space-time properties

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In this paper we consider properties of the space-time manifold $M$ caused by the proposition that, according to the scheme theory, the manifold $M$ is locally isomorphic to the spectrum of the algebra $A$, $M \cong \text{Spec} (A)$, where $A$ is the commutative algebra of distributions of quantum-field densities. In order to determine the algebra $A$, it is necessary to define multiplication on densities and to eliminate those densities, which cannot be multiplied. This leads to essential restrictions imposed on densities and on space-time properties. It is found that the only possible case, when the commutative algebra $A$ exists, is the case, when the quantum fields are in the space-time manifold $M$ with the structure group $SO(3,1)$ (Lorentz group). The algebra $A$ consists of distributions of densities with singularities in the closed future light cone subset. On account of the local isomorphism $M \cong \text{Spec} (A)$, the quantum fields exist only in the space-time manifold with the one-dimensional arrow of time. In the fermion sector the restrictions caused by the possibility to define the multiplication on the densities of spinor fields can explain the chirality violation. It is found that for bosons in the Higgs sector the charge conjugation symmetry violation on the densities of states can be observed. This symmetry violation can explain the matter-antimatter imbalance.

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I. INTRODUCTION

The origin of the arrow of time, the possibility of physics in multiple time dimensions, the violation of the parity principle, and the matter-antimatter imbalance are ones of the most exciting and difficult challenges of physics.

Physics in multiple time dimensions leads to new insights and, at the same time, contains theoretical problems. Extra time dimensions give new hidden symmetries that conventional one time physics does not capture, implying the existence of a more unified formulation of physics that naturally supplies the hidden information [1–3]. At the same time, it notes that all but the $(3 + 1)$-dimensional one might correspond to "dead worlds", devoid of observers, and we should find ourselves inhabiting a $(3 + 1)$-dimensional space-time [4].

The natural description of the $(3 + 1)$-space-time with the one-dimensional time can be provided on the base of the Clifford geometric algebra [5]. In the opposite case of multidimensional time, the violation of the causal structure of the space-time and the movement backwards in the time dimensions are possible [6]. A particle can move in the causal region faster than the speed of light in vacuum. This leads to contradictoriness of the multidimensional time theory and, at present, these problems have not been solved.

The arrow of time is the one-way property of time which has no analogue in space. The asymmetry of time is explained by large numbers of theoretical models – by the Second law of thermodynamics (the thermodynamic arrow of time), by the direction of the universe expansion (the cosmological arrow), by the quantum uncertainty and entanglement of quantum states (the quantum source of time), and by the perception of a continuous movement from the known (past) to the unknown (future) (the psychological time arrow) [7–13]. At present, there is not a satisfactory explanation of the arrow of time and this problem is far from being solved.

The discrete symmetry of the space reflection $P$ of the space-time and the charge conjugation $C$ may be used to characterize the properties of chiral systems. The violation of the space reflection exhibits as the chiral symmetry breaking – only left-handed particles and right-handed anti-particles could be observed [14–16]. The matter-antimatter imbalance remains as one of the unsolved problems. The amount of $CP$ violation in the Standard Model is insufficient to account for the observed baryon asymmetry of the universe. At present, the hope to explain the matter-antimatter imbalance is set on the $CP$ violation in the Higgs sector [17–21].

In this paper we consider the above-mentioned space-time properties (the arrow of time, multiple time dimensions, and the chirality violation), the violation of the charge conjugation, and find that in the framework of the scheme theory these properties are determined by the commutative algebra $A$ of quantum-field densities. Schemes were introduced by Alexander Grothendieck with the aim of developing the formalism needed to solve deep problems of algebraic geometry [22]. This led to the evolution of the concept of space [23]. The space is associated with a spectrum of a commutative algebra. In the case of the classical physics, the commutative algebra is the commutative ring of functions. In contrast with the classical physics, quantum fields are determined by equations on functionals [24–27]. Quantum-field densities are linear functionals of auxiliary fields and, consequently, are distributions. There are many restrictions to con-
struct the commutative algebra of distributions. In the common case, multiplication on distributions cannot be defined and depends on their wavefront sets. In microlocal analysis the wavefront set \(WF(u)\) characterizes the singularities of a distribution \(u\), not only in space, but also with respect to its Fourier transform at each point. The term wavefront was coined by Lars Hörmander [28]. It needs to note that the microlocal analysis has resulted in the recent progress in the renormalized quantum field theory in curved space-time [29–32]. In our case, the possibility to define multiplication on distributions leads to essential restrictions imposed on densities forming the algebra \(A\). The spectrum of the algebra \(A\) is locally isomorphic to the space-time manifold \(M\), \(M \cong \text{Spec}(A)\), and characterizes its properties such as the one-dimensional arrow of time, the chirality violation and the structure group of the space-time manifold \(M\). One can say that the space-time is determined by matter.

The paper is organized as follows. In Section II we derive differential equations for the densities of quantum fields from the Schwinger equation. Let us consider fields \(\Psi\) on the space-time manifold \(M\)

\[
\Psi(x) = \{\Psi^\xi(x)\} = \{\psi^\alpha(x), \psi^\alpha(x), \varphi^n(x), \varphi^{+n}(x), A^a_\mu(x)\},
\]

where \(\psi^\alpha(x)\), \(\psi^\alpha(x)\) are the fermion (spinor) fields, \(\varphi^n(x)\), \(\varphi^{+n}(x)\) are the bosons (for example, Higgs bosons), and \(A^a_\mu(x)\) are the gauge field potentials. In relation (1) and in the all following relations Greek letter indices \(\alpha, \beta\) enumerate types of fermions, \(\mu, \nu\) and \(\rho\) are indices of the space-time variables, Latin letters \(n, m, l\) enumerate types of bosons, and \(a, b, c\) are the gauge indices, respectively. \(\zeta = \{a, n, \alpha\}\) is the multiindex.

Since wavefronts of distributions can be localized [32] and a differential manifold locally resembles Euclidean space near each point, we consider the case when the space-time manifold \(M\) is the 4-dimensional Euclidian (pseudo-Euclidian) space or an open subset of this space with the Euclidian metric tensor \(g^{\mu\nu}\). In order to derive quantum-field equations, we consider the action of the fields \(\Psi\) on the manifold \(M\) [14, 15, 33].

\[
S(\Psi) = \int_M L(\Psi(x)) \, dx
\]

with the Lagrangian

\[
L(\Psi(x)) = -\frac{1}{4} F_{\mu\nu}^a(x) F^{\mu\nu a}(x) + i \overline{\psi}^\alpha(x) \gamma^\mu \nabla_{\mu 5} \psi^\beta(x)
\]

\[+ \overline{\nabla}_{\mu n} \varphi^{+n}(x) \nabla_{m n} \varphi^m(x)
\]

\[- m(\psi^\alpha(x), \psi^\alpha(x), \varphi^n(x), \varphi^{+n}(x)),
\]

where

\[
F_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_{j(a)} C^a_{bc} A^b_\mu A^c_\nu,
\]

\[
F^{a\mu\nu} = g^{\rho\sigma} g^{\mu\nu} F^a_{\rho\sigma},
\]

are the intensity of the gauge fields,

\[
\gamma^\mu \gamma^n + \gamma^n \gamma^\mu = 2g^{\mu\nu}
\]

are Dirac matrices.

II. QUANTUM-FIELD EQUATIONS

Quantum fields are determined by equations on functionals. In this section we consider singularities of the linear components \(W^\zeta(x)\) (densities) of the functional solution

\[
G(Q) = \sum_\zeta \int W^\zeta(x) Q^\zeta(x) \, dx
\]

\[+ \sum_{n>1} \sum_{\zeta_1 \cdots \zeta_n} \int \cdots \int W^{\zeta_1 \cdots \zeta_n}(x_1 \cdots x_n) Q^{\zeta_1}(x_1)
\]

\[\cdots Q^{\zeta_n}(x_n) \, dx_1 \cdots dx_n,
\]

where \(Q^\zeta(x)\) is the auxiliary fields. For this purpose, we derive differential equations for the densities of quantum fields from the Schwinger equation.

In Section II we derive differential equations for the densities of quantum fields from the Schwinger equation and find that, in the common case, the quantum field densities are distributions. The quantum fields contain fermion, boson (Higgs), and gauge field components. Multiplication on the quantum-field densities and the commutative algebra of distributions are considered in Section III. It is found that the only possible case, when the commutative algebra \(A\) of distributions of quantum-field densities exists, is the case, when the quantum fields are in the space-time manifold \(M\) with the structure group \(SO(3,1)\) (Lorentz group) and the time is one-dimensional. The asymmetry of time, the chirality violation of spinor fields, and the charge conjugation symmetry violation in the boson sector are the necessary conditions for the existence of the algebra \(A\). The quantum fields exist only in the space-time manifold with the one-dimensional arrow of time and with chirality and charge conjugation symmetry violations. Ideals, localization, the spectrum of the density distribution algebra \(A\) and the scheme \((M, A_M)\) are considered in Section IV. If the algebra \(A\) can be determined and is the component of the scheme, then the space-time manifold \(M\) with quantum fields exists. Otherwise, the space-time manifold is devoid of matter and, consequently, does not exist.
\[ \nabla_{\mu\beta} = \partial_{\mu} \delta_{\beta} - i e_{(j)} T_{\alpha} A_{\mu}^{\alpha}, \]

\[ \nabla_{\mu m}^l = \partial_{\mu} \delta_{m}^l - i e_{(j)} T_{\alpha} A_{\mu}^{\alpha}, \]

\[ \nabla_{\mu m}^m = g^{\mu\nu} \nabla_{\nu m}, \]

\[ T_a = ||T_a^a|| \text{ is the gauge matrix with the commutation relation } [T_a, T_b] = i C_a^c T_c \text{ acting on spinors as } \psi' = \exp(i\sigma^n T_a) \psi, \sigma^n \text{ is an arbitrary real number. } \tau_a = ||\tau_a^a|| \text{ is the gauge matrix with the commutation relation } [\tau_a, \tau_b] = i C_a^c T_c \text{ acting on bosons as } \varphi' = \exp(i\sigma^n \tau_a) \varphi. \]

It is supposed that the summation in relation [2] and in all following relations is performed over all repeating indices. \( e_{(j)} \) is the charge corresponding to the \( j \) factor of the direct decomposition of the gauge group \( \mathcal{G} = \prod_j \mathcal{G}_j = SU(3) \times SU(2) \times U(1). \) If the index \( a \) of operators \( T_a \) and \( \tau_a \) belongs to the subgroup \( \mathcal{G}_j, \) then in the charge \( e_{(j)} \) of \( j = j \) in the Lagrangian [2] the polynomial \( m(\psi^n, \bar{\psi}^\alpha, \varphi^n, \varphi^{+n}) \) does not contain field derivatives. The polynomial \( m \) determines mass terms and interactions between fields.

For derivation of the quantum-field Schwinger equation it needs to add the linear term \((Q, \Psi)\) with auxiliary fields \( Q(x) = \{Q^\alpha(x), Q^{\bar{\alpha}}(x), Q^n(x), Q^{+n}(x), Q_{(A)\mu}(x)\} \) to the action \( \mathcal{S}(\Psi) \)

\[ \mathcal{S}(\Psi, Q) = S(\Psi) + (Q, \Psi), \]

where

\[ (Q, \Psi) = \int_M (Q^\alpha(x) \psi^\alpha(x) + Q^{\bar{\alpha}}(x) \bar{\psi}^\alpha(x) + Q^n(x) \varphi^n(x) + Q_{(A)\mu}(x) A^a_{\mu}(x)) \ dx. \]

For fields \( \varphi^n(x), \varphi^{+n}(x), \) and \( A^a_{\mu}(x) \) the auxiliary fields \( Q^\alpha(x) \) are simple variables and for fields \( \psi^\alpha(x) \) and \( \bar{\psi}^\alpha(x) \) the auxiliary fields are Grassmannian ones, respectively. Then, the Schwinger equation is written in the form [24, 25]

\[ \left\{ \frac{\delta \mathcal{S}(\Psi)}{\delta \Psi^\alpha(x)} \bigg|_{\Psi^\alpha(x) = \kappa b \delta/\delta Q^\alpha(x)} + Q^\alpha(x) \right\} G(Q) = 0, \]

where \( \delta/\delta \Psi \) is the functional derivative on the left, \( \kappa = 1 \) for bosons and \( \kappa = -1 \) for fermions, \( G(Q) \) is the generating functional. The formal solution of the Schwinger equation [3] is the functional integral

\[ G(Q) = \int \exp \left[ \frac{i}{\hbar} \left( S(\Psi) + (Q, \Psi) \right) \right] D\Psi. \]

We consider densities of the quantum fields \( \psi^\alpha(x), \bar{\psi}^\alpha(x), \varphi^n(x), \varphi^{+n}(x), \) and \( A^a_{\mu}(x) \)

\[ W^\alpha(x) = \left. \frac{\delta G(Q)}{\delta Q^\alpha(x)} \right|_{Q \to 0}. \]

Taking into account the form of the Lagrangian [2], from the Schwinger equation [3] we obtain differential equations for the densities \( W(x) = \{W^\alpha(x), W^{\bar{\alpha}}(x), W^n(x), W^{+n}(x), W_{(A)\mu}(x)\}, \) which can be written in the form

\[ W^\alpha(x) \left( \gamma^\mu \partial_\mu - m^{(a)} \frac{c}{\hbar} \right) = B^\alpha_{(\Psi)}(R(s)(x)) \]

\[ \left( \gamma^\mu \partial_\mu + \frac{m^{(a)} c}{\hbar} \right) W^{\bar{\alpha}}(x) = B^{\bar{\alpha}}_{(\bar{\Psi})}(R(s)(x)) \]

\[ \left( - \frac{m^{(n)} z^2}{\hbar^2} \right) W^n(x) = B^n_{(\varphi^n)}(R(s)(x)) \]

\[ \left( - \frac{m^{(n)} z^2}{\hbar^2} \right) W^{+n}(x) = B^{+n}_{(\varphi^{+n})}(R(s)(x)) \]

\[ \square W_{(A)\mu}(x) = B^a_{(A)\mu}(R(s)(x)), \]

where

\[ \square(\cdot) = g^{\mu\nu} \frac{\partial^2(\cdot)}{\partial \kappa_{\mu} \kappa_{\nu}}. \]

is the d’Alembert operator; \( m^{(a)} \) and \( m^{(n)} \) are masses of fermions and bosons, respectively; \( B^\alpha_{(\Psi)}, B^{\bar{\alpha}}_{(\bar{\Psi})}, B^n_{(\varphi^n)}, B^{+n}_{(\varphi^{+n})}, B^a_{(A)\mu} \) are polynomials of higher order Green’s functions

\[ R^{(s)}(x) = \left. \frac{\delta^s G(Q)}{\delta Q^\alpha(x) \cdots \delta Q^n(x)} \right|_{Q \to 0}, \quad (s > 1). \]

Equations (4) can be written in the form

\[ N^\alpha_{\bar{\alpha}} W^\alpha(x) = B^n_{n}(R(s)(x)), \]

(5)
where $N^\eta_\zeta = \sum_{q=0}^k a^\eta_\zeta q \partial^q_\zeta$ is the matrix differential operator. Solutions of the Schwinger equation \([4]\) determined the generating functional $G(Q)$ and solutions of equations \([4, 5]\) can be found in the approximate form by the diagram technique \([25, 27]\). We will not find solutions of the Schwinger equation. Our aim is to analyse singularities of solutions. For our purposes it is sufficient to note that, in the common case, solutions of equations \([4, 5]\) are distributions. The question is: which distributions of the densities $W^\zeta(x)$ can be multiplied and, therefore, form a commutative algebra? We suppose that the densities $W^\zeta(x)$ can be expressed in the oscillatory–integral form \([33, 35, 36]\)

$W^\zeta(x) = \int_{T^*M} F^\zeta(x, \chi) \exp [i\sigma^\zeta(x, \chi)] d\chi,$  \hspace{2cm} (6)

where $T^*M$ is the cotangent bundle over the space-time manifold $M$, $\sigma^\zeta(x, \chi)$ is the phase function, $F^\zeta(x, \chi)$ is the amplitude, $\chi$ is the covector, and $(x, \chi) \in T^*M$. It needs to note that relation \([6]\) defines Lagrangian distributions, which form the subset of the space of all distributions $D'(M)$. Any Lagrangian distribution can be represented locally by oscillatory integrals \([36]\). Conversely, any oscillatory integral is a Lagrangian distribution. We consider the case of the real linear phase function of $\chi$

$\sigma^\zeta(x, \chi) = k^\zeta \chi_\nu x_\nu,$  \hspace{2cm} (7)

where $k^\zeta$ is a coefficient. Our consideration of the case of Lagrangian distributions is motivated by the statement that, if the multiplication cannot be defined on the Lagrangian distribution subset, then this operation cannot be defined on the space $D'(M)$.

The wavefront set $WF(u)$ of a distribution $u$ can be defined as \([28, 33, 35]\)

$WF(u) = \{(x, \xi) \in T^*M | \xi \in \Gamma_x(u)\},$

where the singular cone $\Gamma_x(u)$ is the complement of all directions $\xi$ such that the Fourier transform of $u$, localized at $x$, is sufficiently regular when restricted to a conical neighborhood of $\xi$. The wavefront of the distributions $WF(W^\zeta)$ characterizes the singularities of solutions and is determined by the wavefront of $R^{(s)}(x)$ and by the characteristics of the matrix operator $N^\eta_\zeta \ [33, 35]$.

$WF(W^\zeta) \subset \text{Char}(N^\eta_\zeta) \bigcup WF(B^\eta_\zeta(R^{(s)})),$

where the characteristics $\text{Char}(N^\eta_\zeta)$ is the set $\{(x, \xi) \in T^*M \setminus \emptyset\}$ defined by linear algebraic equations of highest power orders in the unknown Fourier transforms $\tilde{W}^\zeta(\xi)$

$\sum_{\zeta, q=\max k}^\eta a^\eta_\zeta q (-i\xi)^q \tilde{W}^\zeta(\xi) = 0.$

The covector $\xi \in T^*(x)$ lies in the cotangent cone $\Gamma_x$ at the point $x$. Taking into account equations \([4]\), we can find that

$\text{Char}(N^\eta_\zeta) \big|_x = g^{\mu\nu} \xi_\mu \xi_\nu,$

consequently, singularities of solutions are located on this cone and partially can be formed by wavefronts of higher order Green’s functions $R^{(s)}$.

Starting from the proposition that properties of the space-time manifold $M$ are defined by the quantum fields $\Psi^\zeta(x)$ and, consequently, by the commutative algebra $A$ of distributions $W^\zeta(x)$, in the next section we find that this statement results in essential restrictions imposed on the space-time manifold $M$.

### III. ALGEBRA OF DISTRIBUTIONS OF QUANTUM-FIELD DENSITIES

In order to determine points of the space-time manifold $M$ by means of the densities $W^\zeta(x)$, it is necessary to define multiplication on densities, to construct the commutative algebra $A$ of distributions of densities, and to find maximal ideals of this algebra. According to Ref. \([33, 35]\), multiplication on distributions $u, v \in D'(M)$ with wavefronts $WF(u) = \{(x, \xi)\}$ and $WF(v) = \{(x, \eta)\}$ is determined, if and only if

$WF(u) \bigcap WF'(v) = \emptyset,$  \hspace{2cm} (8)

where $WF'(v)$ is the image of $WF(v)$ in the transformation $(x, \eta) \mapsto (x, -\eta)$ in the cone subset $\Gamma$ of the cotangent bundle $T^*M$. The wavefront of the product is defined as

$WF(uv) \subset \{(x, \xi + \eta) | (x, \xi) \in WF(u) \lor (x, \eta) \in WF(v) \}$

where $\xi = 0, (x, \eta) \in WF(v)$ or $\eta = 0; \xi + \eta \neq 0 \}.$  \hspace{2cm} (9)

Taking into account the linear form of the phase function \([7]\), from relations \([8]\) and \([9]\) we obtain restrictions on the densities $W^\zeta(x)$. For this purpose, we consider wavefronts of densities for the cases of the space-time manifold $M$, when the structure group of the cotangent bundle $T^*M$ is the Lie group $SO(4 - p, p)$ with $p = 0$, $p = 1$, and $p > 1$.

The fields $\Psi$ \([1]\) are observed relative to inertial frames of reference in the space-time manifold $M$. Rectilinear motion transforms the fields $\Psi$ and, consequently, their densities $W^\zeta(x)$ and wavefronts. This transformation can be considered as a diffeomorphism of $M$ and is described by the arcwise connected part of the $SO(4 - p, p)$ group. More precisely, if $f : \Omega \rightarrow \Omega'$ is the diffeomorphism of
Thus, rectilinear motion results in the transformation $f_*$ of the field densities $W^\xi(x)$ induced by the arcwise connected part of the structure group and the transformation $f_+$ of density wavefronts (10). If, as a result of these transformations, the covector $\xi \in \text{WF}(W^\xi) \subset T^*M$ changes its orientation $\xi \mapsto -\xi$, then the multiplication on the density $W^\xi_1(x)$ with the covector $\xi$ and the density $W^\xi_2(x)$ with the covector $-\xi$ is impossible. We consider transformations of density wavefronts induced by the arcwise connected part of the structure group (10) and by discrete symmetry transformations – the time reversal and the space reflection.

A. Time reversal

We assume that the co-ordinate variables of the space-time manifold $M$ can be divided by time $t$ and space $r$ variables, $x = \{t_1, \ldots, t_p, r_1, \ldots\}$ ($p = 0, 1, 2, \ldots$). The structure group of the space-time manifold is $SO(4-p, p)$. We consider the time reversal $T$ on the manifold $M$ and the possibility of embedding of the time reversal into the arcwise connected part of the group $SO(4-p, p)$.

Euclidian space-time manifold with the structure group $SO(4, 0)$. Let us consider the case of the time reversal on the Euclidian space-time manifold $M$ with the structure group $SO(4, 0)$ of the cotangent bundle $T^*M$. The signature of the space-time metric is $(---+)$. In this case, $p = 0$ and the time variable $t$ is identical to the space one (for example, $t_1$). The image of the inversion $\xi \mapsto -\xi$ in relation (11) on wavefronts of densities can be reached by the transformation induced by the arcwise connected part of the group $SO(4, 0)$ (Fig. 1b). Thus, we always can find densities with covectors $\xi$ and $\eta$ in relation (11) such that $\xi + \eta = 0$. Consequently, multiplication on distributions in the Euclidian space-time manifold with the signature of the space-time metric $(---+)$ is impossible. One can only say about a partial multiplication operation on a subset of the distribution space. The analogous consideration can be carried out for manifold $M$ with the structure group $SO(0, 4)$ and with the signature of the space-time metric $(++++)$.

Pseudo-Euclidian space-time manifold with the structure group $SO(3, 1)$. The signature of the space-time metric is $(+-++-)$. In accordance with relation (12), $\rho_{\mu\nu}\xi^\mu\xi^\nu = 0$ defined boundaries of a singular cone, the space-time manifold is separated by three regions: the future light cone $\Gamma^{(f)}$, the past light cone $\Gamma^{(p)}$, and the space-like region $\Gamma^{(s)}$ (Fig. 1b). The multiplication of distributions with singularities in the region $\Gamma^{(s)}$ is impossible because of the existence of the inversion $\xi \mapsto -\xi$ reached by the arcwise connected part of the group $SO(3, 1)$. Therefore, singularities can exist only in the cone $\Gamma^{(f)}$ or in the cone $\Gamma^{(p)}$. The time reversal $T$ inverts the covector $\xi \in \text{WF}(W^\xi) \subset T^*M$ from the future light cone $\Gamma^{(f)}$ to the past light cone $\Gamma^{(p)}$ (Fig. 1b). The Lorentz transformation $SO(3, 1)$ acts on future and past light cones separately. Consequently, the arcwise connected part of the group $SO(3, 1)$ does not contain the time inversion. So, if we exclude field density distributions $W^\xi(x)$ with singularities in the past light cone $\Gamma^{(p)}$ and consider only density distributions with singularities in the future light cone $\Gamma^{(f)}$, then for these distributions the multiplication can be defined. Correspondingly, densities of states $T\Psi^\xi(x)$ are forbidden. In this case, we have the one-way direction of time and there is not the
symmetry of time on density distributions. The arrow of time is pointing towards the future. According to [9], the wavefront of the product of quantum-field densities in the future light cone $\Gamma^{(f)}$ is given by

$$WF(W^{\zeta_{1}} \ldots W^{\zeta_{n}}) \subset \{(x, \sum_{i=1}^{n} \xi^{\zeta_{i}}) | \xi^{\zeta_{i}} \in \Gamma^{(f)}, \sum_{i} \xi^{\zeta_{i}} \neq 0\},$$

where $\xi^{\zeta_{i}} = \beta^{\zeta_{i}} \chi^{(i)}$ is the covector associated with the density $W^{\zeta_{i}}$, $\beta^{\zeta}$ is a function of field invariants, such as chirality, charge signs, charge parity, etc. Taking into account that for the Dirac adjoint spinor density $W^{\alpha}(\bar{\psi}) (x)$ and for the density $W^{\alpha(n)}(\phi^{+}) (x)$ the exponent term in the integral in relation (9) is transformed as $i \sigma^{i}(x, \chi) \rightarrow -i \sigma^{i}(x, \chi)$, we find that in the cone $\Gamma^{(f)}$

$$\beta(\psi) > 0 \quad \beta(\bar{\psi}) < 0$$

$$\beta(\varphi) > 0 \quad \beta(\varphi^{+}) < 0.$$  \hspace{1cm} (11)

Pseudo-Euclidian space-time manifold with the structure group $SO(4 - p, p)$ ($p > 1$). The signatures of the space-time metric are $(++--)$ ($p = 2$) or $(++++)$ ($p = 3$). If the time dimension is equal to 2 or higher ($p > 1$), then the image of the time inversion of densities in the time plane $(t_{1}, \ldots, t_{p})$ is attained by means of a transformation of the arcwise connected part of the group $SO(4 - p, p)$ (Fig. 11). In this case, one can find densities with covectors $\xi$ and $\eta$ in relation such that $\xi + \eta = 0$. Multiplication on distributions in this space-time manifold is impossible.

B. Space reflection and charge conjugation in the fermion sector

Another restriction to define multiplication on the density distribution algebra is caused by the chirality of fermions and by the charge conjugation. Since, according to the above-mentioned subsection the pseudo-Euclidian space-time manifold with the Lorentz group $SO(3, 1)$ and the arrow of time are necessary conditions for definition of multiplication on density distributions, we consider fermion distributions with singularities in the future light cone $\Gamma^{(f)}$. Right-handed and left-handed states of the Dirac fields $\psi^{\alpha}(x)$ and $\bar{\psi}^{\alpha}(x)$ are defined by projective operators $(1 \pm \gamma^{5})/2$ acting on a spinor [14 15 37]

$$\psi_{R}^{\alpha}(x) = \frac{1 + \gamma^{5}}{2} \psi^{\alpha}(x)$$

$$\bar{\psi}_{L}^{\alpha}(x) = \frac{1 - \gamma^{5}}{2} \psi^{\alpha}(x).$$

The space reflection $P$ converts the right-handed spinor into the left-handed one, and vice versa

$$P\psi_{R}^{\alpha}(x) = i L_{p} \psi_{L}^{\alpha}(x)$$

$$P\psi_{L}^{\alpha}(x) = i L_{p} \psi_{R}^{\alpha}(x),$$

where in the Weyl (chiral) basis

$$L_{p} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

and $I$ is the identity 2-matrix. $\psi_{R}^{\alpha}(x)$ and $\psi_{L}^{\alpha}(x)$ are eigenvectors of the operator $\gamma^5$ with the chirality $\lambda = 1$ (the right-handed state) and the chirality $\lambda = -1$ (the left-handed state), respectively. Double space reflection $P^2$ can be regarded as $360^\circ$ rotation. It transforms spinors as

$$P^2 \psi_{p}^{\alpha}(x) = -\psi_{p}^{\alpha}(x),$$

where $p = \{R, L\}$.

For fulfilment of relations (11) the coefficients $\beta(\psi)$ and $\beta(\bar{\psi})$ must contain a charge factor $\kappa_{c}$ such that $\beta(\psi) = 1$ and $\beta(\bar{\psi}) = -1$. We consider two cases, (1) $\beta = \lambda \kappa_{c}$ with the chirality $\lambda$ and (2) $\beta = \kappa_{c}$ without chirality. In order to fulfil multiplication on densities $W^{\alpha}_{(\psi_{R})}(x), W^{\alpha}_{(\bar{\psi}_{L})}(x), W^{\alpha}_{(\psi_{R})}(x)$ in the first case, for right-handed and left-handed states of the fermion fields we should get

$$\kappa_{c}(\psi_{R}^{\alpha}(x)) = 1, \quad \kappa_{c}(\bar{\psi}_{L}^{\alpha}(x)) = -1,$$

$$\kappa_{c}(\psi_{R}^{\alpha}(x)) = -1, \quad \kappa_{c}(\bar{\psi}_{L}^{\alpha}(x)) = 1,$$

$$\lambda(\psi_{R}^{\alpha}(x)) = 1, \quad \lambda(\bar{\psi}_{L}^{\alpha}(x)) = -1,$$

$$\lambda(\psi_{R}^{\alpha}(x)) = -1, \quad \lambda(\bar{\psi}_{L}^{\alpha}(x)) = 1.$$

The charge conjugation $C$ transforms $\kappa_{c}$: $C\kappa_{c}(\psi_{R}^{\alpha}(x)) = \kappa_{c}(\psi_{R}^{\alpha}(x)) = -1, C\kappa_{c}(\bar{\psi}_{L}^{\alpha}(x)) = \kappa_{c}(\bar{\psi}_{L}^{\alpha}(x)) = 1, C\kappa_{c}(\psi_{R}^{\alpha}(x)) = \kappa_{c}(\psi_{R}^{\alpha}(x)) = 1, C\kappa_{c}(\bar{\psi}_{L}^{\alpha}(x)) = \kappa_{c}(\bar{\psi}_{L}^{\alpha}(x)) = -1$. The commutative algebra $A$ of distributions contains densities $W^{\zeta}(x)$ of states $\psi_{R}^{\alpha}(x), \psi_{L}^{\alpha}(x), \bar{\psi}_{L}^{\alpha}(x), \bar{\psi}_{R}^{\alpha}(x), CP\psi_{R}^{\alpha}(x), CP\bar{\psi}_{R}^{\alpha}(x), CP\psi_{L}^{\alpha}(x), CP\bar{\psi}_{L}^{\alpha}(x)$, their sums and products. Densities $W^{\zeta}(x)$ of states $P\psi_{R}^{\alpha}(x), P\bar{\psi}_{L}^{\alpha}(x), P\bar{\psi}_{R}^{\alpha}(x)$, there are forbidden and are not contained in the algebra $A$. This version of the theoretical model can explain the chirality violation.

In the second case, the fulfilment of the relation $\beta = \kappa_{c}$ with $\kappa_{c}(\psi_{R}^{\alpha}(x)) = \kappa_{c}(\bar{\psi}_{L}^{\alpha}(x)) = 1$ and $\kappa_{c}(\bar{\psi}_{R}^{\alpha}(x)) = \kappa_{c}(\bar{\psi}_{L}^{\alpha}(x)) = -1$ results in forbidden densities $W^{\zeta}(x)$ of
states $C\psi^n(x)$, $C\psi^3(x)$, $C\psi^p(x)$, and $C\psi^p(x)$. Wavefronts of these densities are in the past light cone $\Gamma(\rho)$: $\text{WF}(W^a_{(\mathcal{C}\psi^n)})$, $\text{WF}(W^a_{(\mathcal{C}\psi^p)})$, $\text{WF}(W^a_{(\mathcal{C}\psi^p)})$, and $\text{WF}(W^a_{(\mathcal{C}\psi^p)})$. At the same time, the commutative algebra $\mathcal{A}$ of distributions contains densities $W^a(x)$ of states $\psi^n(x)$, $\psi^p(x)$, $\psi^p(x)$, $P\psi^p(x)$, $P\psi^p(x)$, $P\psi^p(x)$, $P\psi^p(x)$, $P\psi^p(x)$, $P\psi^p(x)$. The chirality is not violated. In the experiment the chirality violation of fermions is observed and, therefore, this case of the theoretical model must be ignored.

C. Charge conjugation in the boson sector

We consider the common case of the boson (Higgs) sector containing the quantum fields $\varphi^n(x)$ and $\varphi^{+n}(x)$. We assume that $\varphi^n(x) \neq \varphi^{+n}(x)$. By analogy with the fermion case, the covector of singularity of the density of $\varphi^n(x)$ is $\beta_n(x)$ and the analogous covector of the density of $\varphi^{+n}(x)$ is $-\beta^{+n}(x)$, respectively. In order to fulfill relations (11) and the requirement that $\beta_n(x)$, $-\beta^{+n}(x)$ are in $\Gamma(f)$, we must write the coefficient $\beta$ in the form $\beta_n(x) = \kappa_n(\varphi^n(x))$ and $\beta^{+n}(x) = \kappa_n(\varphi^{+n}(x))$. The charge conjugation $C$ changes the sign of the factor $\kappa_n$: $C\kappa_n(\varphi^n(x)) = \kappa_n(\varphi^{+n}(x)) = -1$ and $C\kappa_n(\varphi^{+n}(x)) = \kappa_n(\varphi^n(x)) = 1$. Thus, densities of $\varphi^n(x)$ and $\varphi^{+n}(x)$ can be multiplied and are included in the algebra $\mathcal{A}$. On the contrary, wavefronts of densities $C\varphi^n(x)$ and $C\varphi^{+n}(x)$ are in the past light cone $\Gamma(\rho)$ and must be excluded. This leads to the charge conjugation symmetry violation in the boson sector.

It needs to note that in the modified theoretical models of the Higgs boson sector [17, 21, 11, 14] extending the BEH model [38–40] some particles in the Higgs sector have negative charge parities and are charged. In this case, the above-mentioned $C$-violation on density distributions in the Higgs sector can explain the observed matter-antimatter imbalance.

Thus, as a result of this section, we define the multiplication on distributions of the quantum-field densities and construct the commutative density distribution algebra $\mathcal{A}$. The algebra $\mathcal{A}$ are generated by distributions with singularities in the future light cone $\Gamma(f)$. The asymmetry of time ($T$-violation), the chiral asymmetry ($P$-violation) and the charge ($C$) conjugation symmetry violation are caused by singularities of density distributions and these space-time manifold properties are local. Spectrum of the density distribution algebra $\mathcal{A}$ and the scheme $(\mathcal{M}, \mathcal{A}_\mathcal{M})$ are considered in the next section.

IV. IDEALS AND SPECTRUM OF THE DENSITY DISTRIBUTION ALGEBRA. SCHEME $(\mathcal{M}, \mathcal{A}_\mathcal{M})$

For definition of the scheme $(\mathcal{M}, \mathcal{A}_\mathcal{M})$ contained the spectrum of the commutative density distribution algebra $\mathcal{A}$ isomorphic to the space-time manifold $\mathcal{M}$, it needs to carry out localization and to determine the sheaf of structure algebras and the spectrum of the algebra $\mathcal{A}$. To this end, we define prime and maximal ideals of this algebra. The algebra $\mathcal{A}$ consists of density distributions $W^a(x)$ with singularities in the closed future light cone subset, $\text{WF}(W^a) \subset \Gamma(f) \subset T^*\mathcal{M}$. The complement of the future light cone subset $\Gamma(f)$ is the open cone subset $\Gamma(f)$. In the cone subset $\Gamma(f)$ the densities $W^a(x)$ are $C^\infty$-smooth functions. We define the maximal ideal at the point $x_0$ as the set of distributions equal to zero at the point $x_0$ in the cone subset $\Gamma(f)$

$$m_{x_0} = \{W^a(x) \mid \lim_{x \to x_0} W^a(x) = 0 \ in \ \Gamma(f)\}.$$ p is called a prime ideal if for all distributions $W_1(x)$ and $W_2(x) \in \mathcal{A}$ with $W_1(x)W_2(x) \in p$ we have $W_1(x) \in p$ or $W_2(x) \in p$. Every maximal ideal $m_{x_0}$ is prime.

In the process of localization of the algebra $\mathcal{A}$ we find a local algebra contained only information about the behavior of density distributions $W^a(x)$ near the point $x$ of the space-time manifold $\mathcal{M}$. We consider the case of maximal ideals. Then, the local algebra $\mathcal{A}_{x}$ is defined as the commutative algebra consisting of fractions of density distributions $W_1(x)$ and $W_2(x)$

$$\mathcal{A}_{x} = \left\{ \frac{W_1(x)}{W_2(x)} \mid W_1(x), W_2(x) \in \mathcal{A}, W_2(x) \notin m_x \right\},$$

where $m_x$ is the maximal ideal at the point $x$. The fraction $W_1(x)/W_2(x)$ is the equivalence class defined as

$$W_1(x) = W_2(x) = W_2(x).$$

if there exists the distribution $V(x) \in \mathcal{A}$, $V(x) \notin m_x$ such that

$$V(x)/W_1(x)W_2(x) = 0.$$

Operations on the local algebra $\mathcal{A}_{x}$ look identical to those of elementary algebra

$$\frac{W_1(x)}{W_2(x)} + \frac{W_1'(x)}{W_2'(x)} = \frac{W_1(x)W_2'(x) + W_1'(x)W_2(x)}{W_2(x)W_2'(x)}$$

and

$$\frac{W_1(x)}{W_2(x)} \frac{W_1'(x)}{W_2'(x)} = \frac{W_1(x)W_1'(x)}{W_2(x)W_2'(x)}.$$

Algebras $\mathcal{A}_{U_i}$ on open sets $U_i \subset \mathcal{M}$

$$\mathcal{A}_{U_i} = \bigcap_{x \in U_i} \mathcal{A}_{x}.$$
determine the structure sheaf \( \mathcal{A}_M = \{ \mathcal{A}_{U_i} \} \) on the space-time manifold \( M \). The inverse limit of the structure sheaf \( \mathcal{A}_M \) coincides with the algebra \( \mathcal{A} \)

\[
\mathcal{A} = \lim_{\mathcal{U}_i \in M} \mathcal{A}_{U_i},
\]

where \( \{ U_i \} \) is the open covering of \( M \).

The spectrum of the algebra \( \mathcal{A} \), denoted by \( \text{Spec}(\mathcal{A}) \), is the set of all prime ideals of \( \mathcal{A} \), equipped with the Zariski topology \[22, 47, 48\]. The prime ideals correspond to irreducible subvarieties of the space \( \text{Spec}(\mathcal{A}) \). Maximal ideals of the algebra \( \mathcal{A} \) correspond to points.

The structure sheaf and the spectrum of the algebra \( \mathcal{A} \) are used in definition of schemes \[22, 47, 48\]. In our case, the scheme over the algebra \( \mathcal{A} \) is the pair \((M, \mathcal{A}_M)\) such that there exists an open covering \( \{ U_i \} \) of \( M \) for which each pair \((U_i, \mathcal{A}_{U_i})\) is isomorphic to \( (V_i, \mathcal{O}_{V_i}) \), where \( \{ V_i \} \) is the open covering of \( \text{Spec}(\mathcal{A}) \) and \( \mathcal{O}_{V_i} \) is the restriction of the structure sheaf \( \mathcal{O}_{\text{Spec}(\mathcal{A})} \) to each \( V_i \). As a result, one can say that the local isomorphism \( M \cong \text{Spec}(\mathcal{A}) \) imposed by the theory of schemes and by restrictions on multiplication on the quantum-field-density distributions in the algebra \( \mathcal{A} \) lead to the dependence of the space-time properties on the matter. The arrow of time, the chirality violation of spinor fields, and the charge conjugation symmetry violation in the boson sector are consequences of this dependence.

V. CONCLUSION

In summary, in this paper we describe the dependence between quantum fields and space-time properties in the framework of the scheme theory. Contrary to algebras of smooth functions, densities of quantum fields, which can be found from the Schwinger equation, are distributions and, in the common case, do not form an algebra. In order to determine the commutative algebra \( \mathcal{A} \) of distributions of quantum-field densities, ideals and its spectrum, it is necessary to define multiplication on densities and to eliminate those densities, which cannot be multiplied. This leads to essential restrictions imposed on densities forming the algebra \( \mathcal{A} \). Taking into account that in the framework of the scheme theory the space-time manifold \( M \) is locally isomorphic to the spectrum of the algebra \( \mathcal{A}, M \cong \text{Spec}(\mathcal{A}) \), the restrictions caused by the possibility to define multiplication on the density distributions result in the following properties of the space-time manifold \( M \).

1. The only possible case, when the commutative algebra \( \mathcal{A} \) of distributions of quantum-field densities exist, is the case, when the quantum fields are in the space-time manifold \( M \) with the structure group \( SO(3,1) \) (Lorentz group). On account of the local isomorphism \( M \cong \text{Spec}(\mathcal{A}) \), the quantum fields exist only in the space-time manifold with the one-dimensional time.

2. We must exclude field density distributions with singularities in the past light cone \( \Gamma(p) \). The algebra \( \mathcal{A} \) consists of the density distributions \( W(x) \) with wavefronts in the closed future light cone subset \( \text{WF}(W(x)) \subset \Gamma(f) \subset T^*M \). In this case, we have the one-way direction of time and there is not the symmetry of time on the density distributions. The arrow of time is pointing towards the future.

3. The restrictions caused by multiplication on the density distributions can explain the chirality violation of spinor fields. The densities of right-handed and left-handed fermion states \( P\psi^R(x), P\psi^L(x), \bar{P}\psi^R(x), \bar{P}\psi^L(x), C\psi^R(x), C\psi^L(x), C\bar{P}\psi^R(x), C\bar{P}\psi^L(x) \), where \( P \) is the space reflection and \( C \) is the charge conjugation, are forbidden and are not contained in the algebra \( \mathcal{A} \).

4. For bosons (for example, in the Higgs sector) the densities of states \( C\phi^+(x) \) and \( C\phi^-(x) \) must be excluded from the algebra \( \mathcal{A} \). This leads to the charge conjugation symmetry violation and can explain the observed matter-antimatter imbalance.
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