A Distributed Optimization Algorithm for Multi-Area Power Grid Transmission Congestion Management

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Abstract. The interconnection of power grid brings significant economic benefits, while it calls for higher requirements to the security, stability and economic operation of power system as well. With high dimensionality of its optimization problem, the whole network centralized optimal power flow (OPF) algorithm is computational inefficient. Therefore, on the basis of power grid layering and zoning, the distributed optimal power flow calculation method in interconnected power grid with multiple control centers has important realistic significance. A distributed optimization algorithm of interconnected power grid transmission congestion management is proposed in this paper. The active power constraints of inter-area transmission interfaces are considered to satisfy the trade power constraints between adjacent sub-systems. The coordination and cooperation procedure counts on the correction and updating of boundary bus equivalent injection power and voltage vector. The numerical results on IEEE test systems show that the algorithm for transmission congestion has good convergence ability.

Keywords: Transmission Congestion Management, Distributed Power Flow Computation, Coordination and Optimization.

1. Introduction
The basic task of power grid dispatching is to maintain the power flow adjustment of generation and load balance under the premise of meeting the security constraints of power grid. The market-oriented reform of power system has injected fresh vitality into the development of optimal power flow technology. In terms of technology, due to the addition of voltage stability, transient stability and many other constraints, the optimization model is more complex; In terms of content, in addition to traditional economic dispatch, new problems such as congestion management and real-time pricing; In terms of economic, in addition to achieving the lowest cost, it is necessary to reasonably allocate the costs of power generation and transmission, and at the same time it is also required to reasonably allocate profits.

The aim of interconnected power grid transmission congestion is to decompose large scale system optimization problem into a series of small, distributed and coordinated ones [1-5]. Based on part duality principle in DC-OPF model, reference [6] decomposed the overall optimization problem into regions through boundary bus data exchange of adjacent areas. Reference [7] used Lagrangian algorithm to deal with the transmission congestion problem in electric market by relaxing regional coupling constraints into the objective. The network equivalent technique is applied to multi-area power system reactive
power parallel optimization in [8-9], but the operation parameters of adjacent regions are in need for local optimization in sub-system, therefore it is not feasible in electric market environment.

Based on the research above, a distributed coordination optimization method for transmission congestion is presented in this paper. Power injection equivalent of sub-system boundary bus is introduced for multi-area transmission congestion management. With a few data exchange of boundary bus between adjacent areas, the proposed method can realize the optimization of whole network. Based on substitution principle for simplified modeling in [10], nonlinear primal dual interior point method is utilized in the optimization of sub-system independently. And voltage weighted modification is conducted in outer iteration to update equivalent power injection of sub-system boundary bus. With the minimum congestion cost as the adjustment goal, the congestion cost optimization of intra-regional line congestion and inter-regional congestion can be realized by adjusting the output of each unit.

2. Transmission Congestion Model of Multi-area Interconnected Power System
In different power grid areas, the unit power generation cost and income of independent are also different due to the different power composition and network structure. To realize the opening of transmission network and promote the economic power exchange between regions is an important manifestation of power market and an important way to realize the optimal allocation of power resources.

Power system interconnection is beneficial to optimal allocation of power resources. Dispatching institutions are set up based on the principle of certain generation managements all over the world. Figure 1 shows the system structure of transmission congestion model.

Power grid dispatch and transmission congestion management can choose different goals when analyzing blocking elimination, minimum adjustment, fastest response or lowest cost. In this paper, the distributed transmission congestion management method of interconnected power grid is studied. With the minimum congestion cost as the adjustment goal, the congestion cost optimization of intra-regional line congestion and inter-regional congestion can be realized by adjusting the output of each unit.

Firstly, calculate the generation dispatching results without congestion and obtain the total generation cost $F^U$. If there is transmission congestion, in order to ensure the safety and economic operation of the system, the distributed optimal power flow method based on the simplified external network model and the equivalent injection power of boundary nodes can be used to manage transmission congestion, and the total generation cost $F^C$ [11] at this time can be obtained. Therefore, the congestion cost $\Delta F$ of the system can be expressed as:

$$\Delta F = F^C - F^U \tag{1}$$

For the manager of power generation, more and more complicated tasks in power grid scheduling institution are undertaken with the fast expansion of the power system scale. In addition, certain rules of power market become the information barriers between dispatching institutions. That further hinders the internal data sharing. Therefore, a new form of transmission congestion mode is in need nowadays, and it is crucial to make it suitable for modern power grid.

3. External Network Reduced Equivalent Model
An important aim of the distributed transmission congestion management with multiple control centers is that the calculation of each sub-system should hold its independency of local optimization while the whole network optimization results could be obtained successfully.

(a) Tie line’s repeated in each sub-system
Figure 1. Equivalent model of two-area interconnected power grid

How to deal with the coupling constraints between adjacent sub-systems is the key of interconnected power grid optimization problem is. Since the tie line power flow reflects the transmission power between two sub-systems, if the calculated value of tie line power equals to their true value, then the results of sub-system power flow would be same as the whole network centralized calculation results.

Take the tie line power $S_{ji}$, which equals to $S_j$, of bus $j$ in Figure 1 for example. The active and reactive power of $S_{ji}$ can be described as

$$P_j = P_{ji}(V_{i(1)}, \theta_{i(1)}, V_{j(1)}, \theta_{j(1)})$$

and

$$Q_j = Q_{ji}(V_{i(1)}, \theta_{i(1)}, V_{j(1)}, \theta_{j(1)})$$

where $P_j$ and $Q_j$ are the active and reactive power of bus $j$ in $S_1$, $P_{ji}$ and $Q_{ji}$ are the active and reactive power of $j$-side respectively, $V_{i(1)}$, $\theta_{i(1)}$, $V_{j(1)}$, $\theta_{j(1)}$ are the voltage magnitudes and angles of boundary bus $i$ and $j$ in $S_1$.

As the injection power of sub-system boundary bus is the function of voltage phasors, the tie line branch power coupling constraints can be satisfied while the mismatch of boundary bus voltage phasors calculated from adjacent sub-systems is less than the predefined convergence tolerance $\epsilon$.

$$|V_{i(1)} - V_{i(2)}| \leq \epsilon, |\theta_{i(1)} - \theta_{i(2)}| \leq \epsilon$$

and

$$|V_{j(1)} - V_{j(2)}| \leq \epsilon, |\theta_{j(1)} - \theta_{j(2)}| \leq \epsilon$$

Thus, the problem of interconnected power grid transmission congestion management can be divided into the local optimization problem of each sub-system. The distributed solution can be realized through the updating of boundary bus equivalent injection power and voltage phasors. Only boundary bus voltage phasors exchanging between adjacent sub-systems is in need to implement the distributed transmission congestion management of interconnected power grid.

4. Distributed Transmission Congestion Management of Interconnected Power Grid

When congestion occurs, the dispatcher of interconnected power grid needs to adjust the output of each generator. In order to meet the line blocking constraints, some generators with lower power generation costs reduce output, while some generators with higher power generation costs turn to increase output. This part of the increase in power generation costs is blocking costs.

The minimum fuel cost is selected as the optimization objective of sub-system power grid. The fuel cost of $S_1$ sub-system in Figure 1 model can be expressed as follows:

$$\min f_i = \sum_{j=1}^{N_G} (a_j + b_j P_{gj} + c_j P_{gj}^2)$$

s.t. $P_{gi} - P_{di} - V \sum_{j=1}^{n} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0, i = 1, 2, ..., n_{i(1)}$.
\[ Q_i - Q_{li} - V \sum_{j=1}^{n_i} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0, i = 1, 2, \ldots, n_{\text{I}}(1) \]  

\[ P_i - V \sum_{j=1}^{n_i} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0, i = 1, 2, \ldots, n_{\text{B}(1)} \]  

\[ Q_i - V \sum_{j=1}^{n_i} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0, i = 1, 2, \ldots, n_{\text{B}(1)} \]  

\[ P_{\text{g}i} \text{min} \leq P_{gi} \leq P_{\text{g}i} \text{max}, i = 1, 2, \ldots, n_{\text{g}(1)} \]  

\[ Q_{\text{g}i} \text{min} \leq Q_{gi} \leq Q_{\text{g}i} \text{max}, i = 1, 2, \ldots, n_{\text{g}(1)} \]  

\[ V_{\text{min}} \leq V_i \leq V_{\text{max}}, i = 1, 2, \ldots, n_{\text{I & B}(1)} \]  

\[ -V P_{\text{g}j} \leq V V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) - V^2 G_{ij} \leq V P_{\text{g}j} \text{max} \]  

\[ \sum_{j \in \text{TL}} \{V V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) - V^2 G_{ij}\} = P_{\text{cut.set}} \]  

Where \( f_1 \) is the objective function of sub-system \( S_1 \), \( P_{gi} \) is the active power of each generator, \( a_i/b_i/c_i \) is the fuel cost curve parameter of each generator, \( n_{\text{g}(1)} \) is the number of generator buses, \( n_{\text{I}(1)}/n_{\text{B}(1)}/n_{\text{I & B}(1)} \) is the number of inner/external/all buses, \( P_{li}/Q_{li} \) is the corrected equivalent injection active/reactive power of external boundary bus, \( \text{TL} \) is the set of tie lines, \( P_{\text{cut.set}} \) is the inter-regional active power based on purchase contract.

In the above sub-system \( S_1 \) fuel cost optimization model, formula (6) is the optimization objective function. Formula (7) and formula (8) are the internal node flow equation corresponding to the equality constraints; formula (9) and formula (10) are the external boundary node injection power correction equation corresponding to the equality constraints. Equation (11) and Equation (12) are the upper and lower limits of the active and reactive power output of the generator respectively. Formula (13) is the upper and lower limits of node voltage amplitude. Formula (14) is the upper and lower bound of branch transmission active power. This paper also considered the inter-regional transaction power constraints as described in equation (15).

It’s necessary to note that the equality constraints of external boundary bus differ from that of either inner or internal boundary bus. Instead, the injection power equation of boundary bus is chosen as its equality constraints. Take \( S_j \) in Figure 1 as an example, its equivalent injection power equation at the \( k \)-th main circle can be expressed as follows.

\[ P_j^{(k+1)} = P_j (\tilde{P}_i^{(k)}, \tilde{\theta}_i^{(k)}, \tilde{\theta}_j^{(k)}) \]  

\[ Q_j^{(k+1)} = Q_j (\tilde{V}_i^{(k)}, \tilde{\theta}_i^{(k)}, \tilde{\theta}_j^{(k)}) \]  

Where \( \tilde{P}_i^{(k)}, \tilde{\theta}_i^{(k)} \) and \( \tilde{\theta}_j^{(k)} \) are the modified voltage magnitude, phase angle of bus \( i \) and \( j \).

The equivalent injection power equation of external boundary bus can be described as:
\[ \Delta P_j^{(n)} = P_j^{(k+1)} - P_j^{(k)}(V_i^{(n)}, \theta_i^{(n)}, V_j^{(n)}, \theta_j^{(n)}) = 0 \] (18)
\[ \Delta Q_j^{(n)} = Q_j^{(k+1)} - Q_j^{(k)}(V_i^{(n)}, \theta_i^{(n)}, V_j^{(n)}, \theta_j^{(n)}) = 0 \] (19)

Where \( V_i^{(n)}, \theta_i^{(n)}, \) and \( \theta_j^{(n)} \) are the voltage phasors of bus \( i \) and \( j \) at the \( n \)-th iteration, \( \Delta P_j^{(n)} \) and \( \Delta Q_j^{(n)} \) are the mismatch active and reactive power of bus \( j \) respectively.

Take the boundary bus \( i \) in Figure 1 for example. If the voltage vector \( V_{i(1)}^{(k)} < \theta_{i(1)}^{(k)} \) and \( V_{i(2)}^{(k)} < \theta_{i(2)}^{(k)} \) calculated by each sub-system are equal, the optimization calculation ends. Otherwise, voltage weighted modification must be taken. In the early iteration of distributed OPF calculation, the same boundary bus voltage vectors obtained by different sub-systems are generally unequal. Therefore, outer coordination iteration should be introduced. It is utilized to update boundary bus voltage vector as well as its equivalent power injection which could accurately reflect the change of adjacent area power flow.

In the \( k \) time outer iteration, the voltage vector of boundary bus \( i \) can be weighted by the following formulas:

\[ V_i^{(k)} = \lambda_{pg} V_{i(1)}^{(k)} + (1 - \lambda_{pg}) V_{i(2)}^{(k)} \] (20)
\[ \theta_i^{(k)} = \lambda_{pg} \theta_{i(1)}^{(k)} + (1 - \lambda_{pg}) \theta_{i(2)}^{(k)} \] (21)

Where \( \lambda_{pg} \) and \( \lambda_{pg} \) are the weight coefficients of voltage amplitude and phase angle respectively.

According to the analysis above, the transmission congestion optimization problem can be solved by nonlinear primal dual interior point method [15]. Certainly, each sub-system is capable of using different optimization methods flexibly consider that only boundary bus voltage phasors exchanging is in need between adjacent sub-systems.

Firstly, the generation dispatching results without congestion is solved according to the distributed coordination and optimization method proposed in this section, and the total generation cost \( F^U \) can be obtained from the summation of each sub-system. If there is transmission congestion, in order to ensure the safety and economic operation of the system, the distributed optimal power flow method based on the simplified external network model and the equivalent injection power of boundary nodes can be used to manage transmission congestion, and the total generation cost \( F^C \) at this time can be obtained. Therefore, the congestion cost \( \Delta F \) of the system can be obtained according to equation (1).

5. Calculation Steps

The process of multi-area distributed coordination and optimization method for transmission congestion management can be summarized as follows:

1. Preparation stage: Based on the substitution principle, simplified the system into sub-systems. Exchange the nodal admittance matrix diagonal elements between adjacent areas to calculate voltage weighted coefficients.
2. Inner iteration stage: Nonlinear primal dual interior point method is taken in each sub-system. The optimization result of regional fuel cost independently, and the state variables of each bus are renewed.
3. Convergence discrimination: Calculate the voltage difference \( \Delta V \) and \( \Delta \theta \) of the same boundary bus between adjacent areas. According to the convergence criterion, if the voltage differences are all sufficiently small, then the whole iteration ends, otherwise it turns to step (4).
4. Outer iteration stage: Modify boundary bus voltage according to equation (20) and (21) and update its equivalent injection power according to equation (2) and (3), and then return to step (1).
5. Transmission Congestion stage: Calculate the fuel cost of multi-area power grid as the such
above steps whether or not there is transmission congestion, and the congestion cost $\Delta F$ of the system can be calculated finally.

6. Case Study

In order to validate the effectiveness of the proposed algorithm, IEEE 30 test systems is tested in this section. Single area generation scheduling optimization and coordination mechanism are analyzed as well in order to research the coordination and cooperation between regional power dispatching system. The convergence criterions of duality gap and boundary bus voltage deviation are $10^{-5}(p.\, u.)$ and $10^{-4}(p.\, u.)$ respectively.

**Regional generation dispatching optimization**

![Figure 2. The partition of IEEE 30 bus system](image)

The tie lines of IEEE 30 two-area interconnected system are 4-12, 6-10, 6-9 and 25-14 as shown in Figure 2. Bus 1/2/5/8 and bus 11/13 are the reference node of $S_1$ and $S_2$ respectively.

It is known that each sub-system in IEEE 30 test system is connected by a tie line. The left and right nodes of the tie line are used as the inner boundary nodes in $S_1$ and $S_2$ respectively. The transaction power between regions is given in advance by each sub-system according to the established electricity sales contract, and the initial value of equivalent injection power of boundary nodes is determined by the power flow calculation results. The generator capacity and fuel cost coefficients of IEEE 30 test system is shown in Table 1.

**Table 1. The generator capacity and fuel cost coefficients of IEEE 30 test system**

| Generator bus No. | Capacity (MW) | $c_i$ ($$/MW2$$) | $b_i$ ($$/MW$$) |
|-------------------|---------------|------------------|----------------|
| $S_1$             |               |                  |                |
| 1                 | 200           | 0.00375          | 2              |
| 2                 | 80            | 0.0175           | 1.75           |
| 5                 | 50            | 0.0625           | 1              |
| 8                 | 35            | 0.00834          | 3.25           |
| $S_2$             |               |                  |                |
| 11                | 30            | 0.025            | 3              |
| 13                | 40            | 0.025            | 3              |

The generation dispatching of interconnected power grid is calculated according to the algorithm in this paper, and the optimization results of system fuel cost of the algorithm used in this paper and the centralized optimization algorithm of the whole network are compared in Table 2.
Table 2. The generator capacity and fuel cost coefficients of IEEE 30 test system

| Test system | $P_{cut, set}$ (p.u.) | Fuel cost ($) | Centralized optimization result ($) |
|-------------|------------------------|----------------|--------------------------------------|
|             |                        | $S_1$ | $S_2$ | $S_1 + S_2$ |                                        |
| IEEE 30     | 0.65                   | 723.36| 79.20 | 802.56     | 802.55                                 |

It can be seen from the data in Table 2 that the distributed optimization algorithm in this paper has good convergence performance: under the given interregional transaction power constraint, the distributed method is used to solve the generation dispatching optimization problem of multi-regional interconnected power grid, and the optimization result almost equal to the centralized optimization algorithm of the whole network.

Transmission Congestion management optimization

Take IEEE 30 Node System in Figure 2 for Example, assuming that the active power limit of one line is reduced due to a sudden fault and the initial power flow is crossed which means transmission congestion occurs in $S_1$. Firstly, define 3 test modes as follows:

- Mode 1: Inter-regional trading power constraint remains at 65MW, line 1-2 active limit reduces from 140MW to 80MW;
- Mode 2: Inter-regional trading power constraint remains at 65MW, line 1-2 active limit further reduces from 140MW to 70MW;
- Mode 3: Inter-regional trading power constraint reduces to 55MW, line 1-2 active limit reduces from 140MW to 80MW.

The distributed algorithm in this paper is used to eliminate congestion by adjusting the generation output. After the congestion is eliminated, the active power flow of the test system line 1-2 is equal to the upper limit of the line active power. The calculation of the fuel costs $F^U$ (before congestion), $F^C$ (after congestion) and $\Delta F$ (congestion cost) are shown in Table 3.

Table 3. Results of Distributed Transmission Congestion Management for Multi-area Power Grid

|                  | $P_{cut, set}$ (p.u.) | $P_{el,2}$ (p.u.) | $F^U$ ($) | $F^C$ ($) | $\Delta F$ ($) |
|------------------|------------------------|-------------------|-----------|-----------|----------------|
| Mode 1           | 0.65                   | 0.8               | 802.57    | 828.79    | 26.22          |
| Mode 2           | 0.55                   | 0.7               | 804.19    | 824.76    | 20.57          |
| Mode 3           | 0.55                   | 0.8               | 804.19    | 824.76    | 20.57          |

It can be seen from Table 3 that when transmission congestion occurs in the area of power grid the congestion cost will be caused by adjusting the generator output to eliminate the congestion. When the active limit value of the line further decreases, the cost of eliminating the congestion will further increase.

If the transmission congestion occurs in the sub-system, reducing the transmission power to the adjacent grid to reduce the transmission pressure can reduce the congestion cost to a certain extent.

7. Conclusions

A distributed coordinated optimization method for transmission congestion management is proposed in the paper. It is utilized to solve generation dispatching problem in multi-area interconnected system. The transmission congestion simulation taken in IEEE 30 test systems shows that the proposed method has good applicability and the following characters:

- Without external network equivalence, the presented method avoids tedious calculating and updating of equivalent parameters which is more suitable for large-scale interconnected power grid.
- The independence of each sub-grid optimization is remarkably increased. Only voltage magnitude and phase angle exchanging of boundary buses is in need.
- Through reducing the transmission power to the adjacent grid, the suggested method can be adapted to the transmission congestion management and can reduce the congestion cost effectively.
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