Control of a tethered Quadrotor using a quaternion feedback

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Abstract. In Japan, superannuation of infrastructures poses a problem and the application of unmanned aerial vehicles (UAVs) such as a Quadrotor etc. is being actively studied as one of ways of inspection. Such a Quadrotor needs any position control independent of GPS, when being used for the inspection of tunnels and of interior of building. In previous research, Ouchi et al. proposed a method for controlling the position of a Quadrotor using a tether. In addition, they derived the dynamical model of the Quadrotor considering the influence of a tether, and conducted some experiments on the attitude control. The objective of this study is to realize angle and angular velocity control in the tethered Quadrotor by applying quaternion feedback control proposed by Fresk et al. In the quaternion feedback controller, the angular velocity obtained from the gyro sensor is directly available in the controller design, so that there is a possibility that more stable attitude control can be performed. After giving the overview of the tethered Quadrotor and describing Quaternion feedback control, this paper gives some experiments to verify that it is more stable with an actual machine, compared to the conventional method.

1. Introduction
The applications of Unmanned Aerial Vehicle (UAV) attracts attention as a means of gathering information at the time of disaster, photographing from the air, delivering home-delivery items, inspecting infrastructures such as tunnels and bridges, and so on[1][2][3].

In Japan, superannuation of infrastructures poses a problem, because proportion of road bridges spending 50 years after construction will be increased from the present 16% to about 65% in the next 20 years[4]. From this reason, research is being conducted to use a Quadrotor that is superior in various performance than other UAVs for inspection[5]. It needs to control the position of the Quadrotor, if it is used for inspection. In general, the position control of the Quadrotor is performed by manual operations using a joystick or by using position information obtained from the GPS[6][7]. Although it is necessary to bring the Quadrotor close to the inspection object if possible, expert skill is required in an operation with a joystick. On the other hand, position control by GPS has problems such as difficulty in receiving signals in a tunnel or inside a building. Therefore, Ouchi et al.[8] proposed a tethered Quadrotor, as shown in figure 1, attaching a tether to the Quadrotor as position control without using GPS. In this method, the position information is calculated from the length and inclination of the tether attached to the airframe. Their Quadrotor has the PD controller only for the attitude angle and position control, but it is desirable for a stable flight that the angular velocity is also controlled. Normally, the attitude angle of the Quadrotor is estimated by time integration of the angular velocity obtained from the gyro sensor.
When updating this attitude angle, there is a singular attitude in which the degrees of freedom degenerate. Therefore, the differential term of a PD controller based on the Euler angle is not necessarily consistent with the actual angular velocity, and thus the angular velocity obtained from the gyro sensor cannot be directly used in such a controller. Thus, the angular velocity of the tethered Quadrotor obtained by a gyro sensor is directly controlled in this paper by applying quaternion feedback control proposed by Fresk et al.[9]. Since the time differentiation of the quaternions is expressed by the angular velocity vector, the value obtained from the gyro sensor can be directly used in a controller design. In applying this method, some simulations of using the controller and flying experiments are carried out with a real system. As a result, more stable attitude control is shown to be performed, compared with the conventional control method.

2. Dynamical model of the tethered Quadrotor
A dynamical model of the Quadrotor is shown below. This model is partly changed from that derived by Ouchi et al. In the dynamical model derived by them, only the thrust from two-rotors can be used for controlling the roll-axis or pitch-axis motion. Therefore, when viewed from the upper side of the aircraft, let us rotate the original coordinate system, which was defined by them, 45 degrees counterclockwise, to obtain the present coordinate system as defined in this paper. Figure 2 shows the appearance of a Quadrotor and the coordinate systems, respectively. If a right-handed coordinate system is adopted in each coordinate system, then the body coordinate system of the Quadrotor is denoted by $B$, whereas the inertial one is represented by $E$.

Let the position of center of gravity of the airframe in the inertial coordinate system be $(x_g, y_g, z_g)$, and the rotational angles around the X-, Y-, and Z-axis be $\phi$, $\theta$, and $\psi$, respectively. The body mass of the tethered Quadrotor is $m$; the gravitational acceleration is $g$; the length from the center of the body to the center of the rotor is $l$; $I_x$, $I_y$, and $I_z$ are the moment inertia of around each axis; $J_r$ is the rotation moment of the rotor; and the sum of the rotational speed of the rotor is $\Omega$. The tension caused by the tether is $T$, the length of the tether is $l_t$, and the center of gravity of the airframe to the fixed point of the tether in the z direction is $O_z$.
Assume that the control input for the translational position in the $Z$-axis direction is given by $U_1$, and the control inputs for the roll angle, pitch angle and yaw angle are denoted by $U_2$, $U_3$, and $U_4$, respectively. The rotational part of the dynamical model is as follows:

$$\dot{\phi} = \theta \psi \left( \frac{I_y - I_z}{I_z} \right) - \frac{I_y}{I_z} \dot{\theta} \Omega + \frac{l}{\sqrt{2}l_x} U_2 - \frac{\Omega^2 I_y}{I_x l_z}$$

$$\dot{\theta} = \phi \psi \left( \frac{I_z - I_x}{I_x} \right) + \frac{I_z}{I_x} \dot{\phi} \Omega + \frac{l}{\sqrt{2}l_y} U_3 + \frac{\Omega^2 I_z}{I_y l_x}$$

$$\dot{\psi} = \phi \theta \left( \frac{I_x - I_y}{I_y} \right) + \frac{1}{l_x} U_4$$

Then, the translational part of the dynamical model is as follows:

$$\dot{x} = -A_3 \frac{U_1}{m} + (A_1 x + A_2 y + A_3 z) \frac{r}{m l_z}$$

$$\dot{y} = -A_6 \frac{U_1}{m} + (A_4 x + A_5 y + A_6 z) \frac{r}{m l_z}$$

$$\dot{z} = -A_9 \frac{U_1}{m} + (A_7 x + A_8 y + A_9 z) \frac{r}{m l_z}$$

Here, the $A_1$ to $A_9$, are given by:

$$A_1 = \cos \theta \sin \psi$$

$$A_2 = \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi$$

$$A_3 = \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi$$

$$A_4 = \cos \phi \sin \psi$$

$$A_5 = \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi$$

$$A_6 = \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi$$

$$A_7 = -\sin \theta$$

$$A_8 = \sin \phi \cos \theta$$

$$A_9 = \cos \phi \cos \theta$$

Defining that the thrust coefficient of a propeller is $b$, its drag coefficient is $d$, the number of each rotor is $i$, the rotational speed is $\omega_i$, the generated thrust is $f_i$, and the generated torque is $\tau_i$, $\Omega$, $U_1$, $U_2$, $U_3$, and $U_4$ are reduced to:

$$\Omega = -\omega_1 + \omega_2 - \omega_3 + \omega_4$$

$$U_1 = f_1 + f_2 + f_3 + f_4 = b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)$$

$$U_2 = -f_1 + f_2 + f_3 - f_4 = b(-\omega_1^2 + \omega_2^2 + \omega_3^2 - \omega_4^2)$$

$$U_3 = f_1 - f_2 - f_3 - f_4 = b(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2)$$

$$U_4 = \sum_{i=1}^{4} \tau_i = d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2)$$

3. Controller of the tethered Quadrotor

3.1. Controller for the attitude angle
In this article, the attitude of the tethered Quadrotor is controlled using a quaternion feedback control method proposed by Fresk et al. This method consists of two P controllers, each of which is for the Quaternion and the angular velocity, respectively. Defining the target quaternion as \( \tilde{q}_r \), and the quaternion of the airframe as \( q_m \), the error of the quaternion is given by

\[
q_e = \tilde{q}_r \otimes q_m
\]  

(21)

Since the rotational direction is different depending on the sign of the scalar part in \( q_e \), the control input \( u_q \) for the quaternion is as follows.

\[
u_q = \begin{cases} 
\begin{bmatrix} -[q_{e1} q_{e2} q_{e3}]^T \ (q_{e0} \geq 0) \\
[q_{e1} q_{e2} q_{e3}]^T \ (q_{e0} < 0) 
\end{bmatrix}
\end{cases}
\]  

(22)

Defining the angular velocity of the airframe for each axis as \( \omega_x \), \( \omega_y \), and \( \omega_z \), the control input \( u_\omega \) for the angular velocity is as follows:

\[
u_\omega = \begin{bmatrix} \omega_x \ 
\omega_y \\
\omega_z
\end{bmatrix}
\]  

(23)

The P gains of controller for the quaternion is \( K_{pq} \). The P gains of controller for the angular velocity is \( K_{p\omega} \), the control input \( \tau \) for each rotational axis movement is as follows:

\[
\tau = [U_1 \ U_2 \ U_3]^T = -K_{pq} u_q - K_{p\omega} u_\omega
\]  

(24)

3.2. Controller for the position

The position of the Quadrotor is controlled by changing the attitude of the airframe. The Quadrotor can move in the X- and Y-axis directions by tilting the airframe \( \phi \) and \( -\theta \) in each direction, respectively. Therefore, in this paper, position control in the X- and Y-axis directions is performed by changing the value of the target quaternion in the attitude controller shown in Eq. (21). The target quaternion corresponding to the target attitude angles \((\phi_d, \theta_d, \psi_d)\) is given by

\[
\tilde{q}_r = \begin{bmatrix}
\cos(\phi_d/2) \ cos(\theta_d/2) \ cos(\psi_d/2) - \sin(\phi_d/2) \ sin(\theta_d/2) \ sin(\psi_d/2)
\sin(\phi_d/2) \ cos(\theta_d/2) \ cos(\psi_d/2) - \cos(\phi_d/2) \ sin(\theta_d/2) \ sin(\psi_d/2)
\cos(\phi_d/2) \ sin(\theta_d/2) \ cos(\psi_d/2) - \sin(\phi_d/2) \ cos(\theta_d/2) \ sin(\psi_d/2)
\cos(\phi_d/2) \ cos(\theta_d/2) \ sin(\psi_d/2) - \sin(\phi_d/2) \ sin(\theta_d/2) \ cos(\psi_d/2)
\end{bmatrix}
\]  

(25)

That is, the target value of the attitude angle is generated from the error between the target position of the airframe and the current position, and it is converted into the target quaternion. Here, a PD controller is used for the position control. Defining the P gains of the controller as \( K_{px} \) and \( K_{py} \), the D gains of the controller as \( K_{dx} \) and \( K_{dy} \), and the target values of position of the airframe as \( x_d \) and \( y_d \), the PD position controllers are given by:

\[
\theta_d = -K_{px}(x - x_d) - K_{dx} \dot{x}
\]

\[
\phi_d = -K_{py}(y - y_d) - K_{dy} \dot{y}
\]  

(26)  

(27)

The height of the airframe in the Z-axis direction is directly manipulated by an operator by pulling the tether, so that there is no controller for height.

3.3. Measurement of the angular velocity and attitude

In this paper, the internal gyro sensor of MPU-9250 manufactured by Invensense is used for measuring the angular velocity of the tethered Quadrotor. MPU-9250 is an IMU that has a tri-axis gyro sensor, a tri-axis acceleration sensor, and a tri-axis geomagnetic sensor. In addition, the attitude angle of the airframe is estimated by combining the values of the gyro sensor and the angular velocity sensor.
For the estimation, the sensor fusion algorithm proposed by Madgwick et al.[10] is used.

### 3.4. Measurement of the position

In this study, the position of the airframe in the X- and Y- axis directions is determined from the inclination and length of the tether attached to the Quadrotor. Figure 3 shows the relationship between the inclination of the tether and the position of the airframe. Let the positions in Ex-, Ey-, and Ez-axes of the airframe be $x_e$, $y_e$, and $z_e$. The slopes of the tether against the perpendicular line given from the center of the airframe directed to Ex-axes and Ey-axis are defined by $\alpha$ and $\beta$. The airframe position in the Ex- and Ey-axes are given by

\begin{align}
    x_e &= z_e \tan \alpha \\
    y_e &= z_e \tan \beta
\end{align}

\[ z_e = \sqrt{l^2 - \frac{(\cos \alpha)^2(\cos \beta)^2}{(\cos \alpha)^2+(\cos \beta)^2-(\cos \alpha)^2(\cos \beta)^2}} \]

**Figure 3.** The relationship between the tether inclination and the airframe positions.

Figure 4 shows a module of measuring the inclination of the tether. This module consists of an aluminum frame and two potentiometers, whose structure is a gimbal, and the inclination of the tether is independently measured as a voltage by each potentiometer.

**Figure 4.** A module measuring the inclination of the tether.

### 4. Simulation with the quaternion feedback controller

Some simulations were conducted for applying the quaternion feedback controller to the tethered Quadrotor. It is verified through simulations whether the quaternion feedback controller shown in Section 3 can control the attitude of the tethered Quadrotor.
4.1. Simulation conditions

The dynamical model shown in Section 2 is used as the object to be simulated and it is analyzed by MATLAB. Let \( \mathbf{x} = [x \ y \ z \ \phi \ \theta \ \psi \ \dot{\phi} \ \dot{\theta} \ \dot{\psi}] \) be the initial value. Then, after one second, the target quaternion is changed stepwise to the quaternion representing \((\phi_d, \theta_d, \psi_d) = (\pi/6, \pi/6, \pi/6)\). In addition, the 4th-order Runge-Kutta method is used as a solver for differential equations. Table 1 shows the parameters used in the simulation.

| Variable | Parameter |
|----------|-----------|
| \( m \)  | 0.65 (kg) |
| \( I_x \) | 0.00598 (kg m² s⁻²) |
| \( I_y \) | 0.00598 (kg m² s⁻²) |
| \( I_z \) | 0.0117 (kg m² s⁻²) |
| \( J_r \) | 0.0000065 (kg m² s⁻²) |
| \( b \)  | 0.00000313 |
| \( d \)  | 0.00000075 |
| \( l \)  | 0.23 (m) |
| \( g \)  | 9.81 (m/s²) |

4.2. Results and considerations

Figure 5, figure 7 and figure 9 shows the simulation results with \( K_{p\theta} = 10 \) and \( K_{p\phi} = 0.6 \). Figure 6, figure 8 and figure 10 shows the simulation results with \( K_{p\phi} = 10 \) and \( K_{p\theta} = 0.05 \). It is found, from figure 5, figure 7 and figure 9 that the airframe attitude follows the change in the target value and converges to it. It is found, from figure 6, figure 8 and figure 10 that the airframe attitude follows the change in the target value but oscillates largely. As a result, it was confirmed that the attitude of the airframe can be controlled by using the quaternion feedback controller. Moreover, since oscillations occur when the control input to the angular velocity is small, it is seen that the control of the angular velocity is useful for stabilizing the airframe.

**Figure 5.** Roll angle with \( K_{p\theta} = 10 \) and \( K_{p\phi} = 0.6 \).

**Figure 6.** Roll angle with \( K_{p\phi} = 10 \) and \( K_{p\theta} = 0.05 \).
Figure 7. Pitch angle with $K_{pq} = 10$ and $K_{p\omega} = 0.6$.

Figure 8. Pitch angle with $K_{pq} = 10$ and $K_{p\omega} = 0.05$.

Figure 9. Yaw angle with $K_{pq} = 10$ and $K_{p\omega} = 0.6$.

Figure 10. Yaw angle with $K_{pq} = 10$ and $K_{p\omega} = 0.05$.

5. Flight experiments with a real system

5.1. Experimental Conditions

Figure 11 shows an experimental setup, where the MCU installed in the aircraft has a wireless LAN function and can realize wireless communication with a PC.

Figure 11. Experimental setup

Figure 12. Experimental scene.
The logging data of the airframe are obtained by using such a wireless LAN. The target quaternion in flight is set to the quaternion representing of \((\phi_d, \theta_d, \psi_d) = (0,0,0)\). Then, the roll and pitch angles of the airframe inflight are logged. The same experiment is carried out with a conventional PD controller. Then compare these results. Figure 12 shows the experimental scene. In this case, the flight experiment was carried out in an indoor space, whose size was 4.4 m in width, 4.4 m in depth, and 2 m in height, as shown in figure 12.

5.2. Results and considerations

Figure 13 shows the experimental results of roll angle. Figure 14 shows the experimental results of pitch angle. Table 2 and table 3 shows the maximum and average errors in each method. There is no significant difference in the average errors in either approach. The maximum errors in the quaternion feedback control are smaller in the roll and pitch angles, than those in the conventional method. This implies that the quaternion feedback controller can realize more stable flight in the tethered Quadrotor.

![Figure 13. Experimental results of roll angle.](image)

![Figure 14. Experimental results of pitch angle.](image)

**Table 2.** Experimental results with the quaternion feedback controller.

| Axis     | Maximum error (deg) | Average error (deg) |
|----------|---------------------|---------------------|
| Roll angle | 2.186035          | -0.70809            |
| Pitch angle | 2.109319          | 0.344004            |

**Table 3.** Experimental results with the quaternion feedback controller.

| Axis     | Maximum error (deg) | Average error (deg) |
|----------|---------------------|---------------------|
| Roll angle | 3.00174            | -0.56831            |
| Pitch angle | 2.843081            | 0.44037             |

6. Conclusion

This paper has given an overview about the tethered Quadrotor and introduced the quaternion feedback control to stabilize its attitude control system. Also, simulation experiments and actual airframe experiments were carried out by applying the quaternion feedback control to the tethered Quadrotor. As a result, the maximum error was shown to be improved over the conventional control method. In future, we are going to conduct some flight experiments, using the in actual airframe combined with a position controller, outdoors.
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