Mixed-state penetration depths in s-wave and d-wave superconductors

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Abstract

We report on a microscopic calculation of the current and magnetic field distribution in the mixed state of model s-wave and d-wave superconductors. For the d-wave case, we find that corrections to London theory are important at fields small compared to $H_{c2}$. The field distribution is influenced by both superfluid-velocity dependence and non-locality in the current response function of the mixed state. We compare our calculations with recent muon spin rotation measurements in high temperature superconductors.

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The Meissner effect, in which weak external magnetic fields are exponentially screened from the bulk, is a basic property of superconducting metals. It is a consequence of the linear relationship between circulating currents ($J$) and the magnetic flux density ($h$) in a superconductor expressed by the London equation:

$$h + \frac{4\pi\lambda^2}{c} \nabla \times J = 0.$$  (1)

In Eq.(1) $\lambda$, the penetration depth, determines the spatial extent of magnetic fields near the surface of a superconductor in the Meissner state. Measurements of this parameter in a particular material provide important information on the microscopic nature of its superconducting state. For example, recent measurements \[1,2\] of the temperature dependence of $\lambda$, which microscopic theory relates to the quasiparticle energy spectrum \[3\], have provided important evidence for d-wave pairing \[4\] in high-temperature superconductors.

Both nuclear magnetic resonance (NMR) and muon spin rotation ($\mu$SR) measurements of $\lambda$ are based on the relationship between magnetic field inhomogeneity in the mixed state of a type-II superconductor and the penetration depth. In the mixed state, Eq.(1) applies only outside the vortex cores and the London equation becomes,

$$h + \frac{4\pi\lambda^2}{c} \nabla \times J = \hat{z}\Phi_0 \sum_i \delta^{(2)}(r - R_i),$$  (2)

where $R_i$ is a vortex position and $\Phi_0$ is a superconducting flux quantum. The right hand side of Eq.(2) accounts for the phase winding of the superconducting order parameter around each vortex. The magnetic field distribution inside the superconductor can be evaluated \[4\] from Eq.(2) and the Maxwell equation, $\nabla \times h = (4\pi/c)J$, once the vortex positions are specified. Eq.(2) should be valid provided that: i) the magnetic field is sufficiently weak compared to the upper critical field so that vortex cores occupy a small fraction of the volume; ii) the distance scales on which currents and fields vary inside the superconductor are larger than the non-locality length of the current response kernel; iii) and the fields and currents are small enough to be in the linear response regime. This last condition is especially pertinent in the case of d-wave superconductors since nodes in the energy gap...
lead to large non-linearities \[3,4\] in the current response, even at low temperatures. At present there is no detailed theory which accounts for non-linear supercurrent response in the inhomogeneous mixed state. In this article we report on exact microscopic mean-field-theory calculations of the current and magnetic field distributions in the mixed state of model s-wave and d-wave superconductors. We summarize our results in terms of an effective penetration depth which would be inferred from the calculated field distribution under the assumption that Eq.(2) applies locally. We find that this effective penetration depth increases with increasing field, and hence increasing supercurrent densities, much more strongly for d-wave superconductors than for s-wave superconductors and that the linear temperature dependence of \(\lambda\) which signals d-wave superconductivity in the absence of a magnetic field saturates at a temperature \(k_B T^* \sim (H/H_{c2})^{1/2} \Delta_0\).

Our numerical calculations start from self-consistent solutions \[8-11\] of the mean-field Bogoliubov-de Gennes (BdG) equations \[12,13\] for generalized Hubbard models on stacked magnetically coupled square lattices. We regard the Hubbard models as plausible low-energy effective models for high-temperature superconductors. The class of Hubbard models we consider is parameterized by the nearest neighbor hopping energy \((t)\), the on site interaction energy \((U)\) and the nearest neighbor interaction energy \((V)\) and the parameters can be chosen so as to obtain s-wave or d-wave superconductors at zero magnetic field. The model parameters for the calculations discussed below were chosen to give low temperature coherence lengths comparable to estimates made for typical high temperature superconductors.

We have solved the BdG equations \[14\] self-consistently for s-wave and d-wave mixed states, assuming \[15\] a constant magnetic flux density \(H\hat{z}\), as detailed in an earlier publication. \[16\] The current flowing along the bond of the lattice linking site \(i\) and site \(j\) is related to the self-consistent BdG quasiparticle amplitudes by

\[
J_{ij} = \frac{4e}{\hbar a d} \int \left\{ t_{ij} \sum_\alpha [u_\alpha^* u_\beta f_\alpha + v_\alpha^* v_\beta (1 - f_\alpha)] \right\}, \tag{3}
\]

where \(a\) is in-plane lattice constant, \(d\) is the separation between square lattice layers, and \(f_\alpha = [\exp(E_\alpha/k_B T) + 1]^{-1}\) is the Fermi function for quasiparticle excitations of energy \(E_\alpha\).
The spatial variation in the flux density $h$ in each plaquette of the square lattice is then evaluated from a discrete version of the magnetostatic Maxwell equation:

$$h(r) = \frac{4\pi i}{c} \sum_{G \neq 0} \frac{G \times J(G)}{G^2} \exp(iG \cdot r) + H. \quad (4)$$

Typical results are shown in Fig. 1 for both s-wave (left column) and d-wave (right column) superconductors. In these figures we show the spatial dependence of the current density, the local magnetic flux density, and a local penetration depth which we define by

$$\lambda_{loc}^{-2} \equiv -\frac{4\pi}{c} \frac{\nabla \times J}{h}. \quad (5)$$

These results were obtained for $T = 0$ at fields with approximately 900 plaquettes per superconducting flux quantum which corresponds, for the model studied, to $H \simeq 0.04 H_{c2}$ in both s-wave and d-wave cases.

The screening current circulating around the vortex core is largest in magnitude at the core edge. Qualitatively, this property can be easily understood in a Ginzburg-Landau type of analysis, where the current density is related to the order parameter $\psi = |\psi| \exp(i\phi)$ and the superfluid velocity $v_s$ by

$$J = 2e|\psi|^2v_s = \frac{2e}{m^*}|\psi|^2 \left( h\nabla \phi - \frac{2e}{c} A \right). \quad (6)$$

Since $\phi$ winds by $2\pi$ around a vortex and $|\psi|$ vanishes linearly at vortex center, $J \sim r$ near the core center and $J \sim r^{-1}$ (for $r \ll \lambda$) outside the core. Our results show a distortion in the circular flow pattern generally expected around the vortex core which we attribute to a tendency for current to flow along the principle symmetry axes of the square lattice. We find that the local penetration depth is tolerably constant outside the vortex core, as assumped in the simple London theory, except where the current flow bends relative abruptly. Similar band structure effect has been seen in scanning-tunneling-microscopy experiments for conventional superconductors by Hess et al. However, near and inside the vortex core $\lambda_{loc}^{-2}$ changes rapidly, presumably principally because the magnitude of the order parameter is changing rapidly. There is no qualitatively difference between s-wave and
$d$-wave cases apparent in the results at a particular temperature and field shown in Fig. 1. The deep minimum in the local penetration depth in the vortex core is effectively replaced by a $\delta$-function in the London approximation.

In the Meissner state $\delta$ of a $d$-wave superconductor, the dependence of the penetration depth on superfluid velocity leads to important non-linear terms in the current response and to a field dependent effective penetration depth. In the mixed state, the dependence of the magnetic field on two spatial coordinates and the presence of vortex cores where the order parameter magnitude is not constant complicate the analysis of non-linear effects. In fact, the current density in the mixed state is largest near the vortex cores and this complicated region might therefore be expected to contribute importantly to the field dependence of any effective penetration depth, making the development of an analytic theory truly difficult. The magnetic field at any point in the vortex lattice depends on the current distribution everywhere and hence its distribution function depends on some complicated average of the non-local, non-linear, and inhomogeneity effects which appear in the local penetration depth. It is useful to define a magnetic-field-dependent effective penetration depth $\lambda$ in the mixed state by mimicking the analyses of $\mu$SR or NMR experiments which are based on the London equation. This $\lambda$ may be thought of as an appropriate average of the local penetration depth discussed above.

In a $\mu$SR experiment one measures the time evolution of the muon polarization vector precessing about the internal magnetic field. The frequency spectrum is proportional to the magnetic field probability distribution in the sample. When the London equation is valid nearly everywhere it can be shown $[20]$ that $\lambda$ is related to the width of the magnetic field probability distribution

$$P(h) = \frac{1}{A} \int d\mathbf{r} \delta(h - h(\mathbf{r}))$$  \hspace{1cm} (7)

by the following expression,

$$\lambda^{-4} = C \langle \Delta h^2 \rangle \equiv C \int dh P(h) (h - H)^2,$$  \hspace{1cm} (8)
where the constant $C$ is dependent on the structure of the lattice. We define the effective penetration depth of a vortex lattice by this equation using the exact field distribution. This definition permits a direct comparison between theory and experiment.

In Fig. 2 we show the effective penetration depth $\lambda_{\text{eff}}$, as a function of temperature. We find that $\lambda_{\text{eff}}$ increases with increasing magnetic field for both $d$-wave and $s$-wave superconductors, but much strongly in the $d$-wave case. Even in a relatively weak field, $H \simeq 0.04H_{c2}$, the low-temperature behavior in two cases is very different. In the $s$-wave case, $\lambda_{\text{eff}}$ essentially preserves an activated $T$-dependence. On the other hand, the linear temperature behavior for the $d$-wave $\lambda$ in the Meissner state saturates at $T^*(H)$, and crosses over to a $T^2$ dependence. The crossover temperature, $T^*$, can be crudely estimated by noting that the typical local pairing momentum in the vortex lattice state is $\sim \hbar/\ell$ [21], where $2\pi\ell^2H = \Phi_0$. This local superfluid velocity changes the quasiparticle density of states [6,7,21,22] and hence $\lambda$ for energies below $k_BT^*(H) \sim v_F\hbar/\ell \sim \Delta_0\sqrt{H/H_{c2}}$, where $\Delta_0$ is the maximum quasiparticle energy gap in the Meissner state. This estimate of the crossover temperature is consistent with the numerical results shown in Fig. 2 and similar results obtained at still stronger magnetic fields.

Our calculation should be compared with recent $\mu$SR experimental data on YBa$_2$Cu$_3$O$_{6.95}$ reported by Sonier et al. [3]. These authors find that the penetration depth increases with field. The increase of approximately 15% they find at the strongest field studied, $H = 2$ Tesla, should be compared with the approximately 30% increase we find for $H/H_{c2} \simeq 0.04$. ($H_{c2}$ is believed to be $\sim 200$ Tesla for YBa$_2$Cu$_3$O$_{6.95}$). This is consistent with the expected approximate relationship between the zero-temperature penetration depth and the quasiparticle density of states at the Fermi energy which is $\propto \sqrt{H}$ [16,21,23]. The observed change in $\lambda_{\text{eff}}$ at this field is much larger than would be expected for an $s$-wave superconductor, adding to the evidence for $d$-wave pairing in cuprate superconductors. This experimental study has insufficient data at sufficient low temperature and sufficient strong fields to verify the crossover $T^2$ behavior predicted here.

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FIGURES

FIG. 1. Top row: Current circulating around a vortex. Arrows represent the directions of flow. The length of an arrow represents the magnitude of the current at the site. Middle row: Local penetration depth as defined in the text. The minimum in this quantity occurs at the center of the vortex core. Bottom row: Magnetic field distribution. The peak occurs at the center of the vortex core. The plotted quantity is the difference between total field and the spatial average field in arbitrary units. The applied field is $H = 0.04H_{c2}$. The figures in left and right columns are for $s$-wave case and $d$-wave case respectively.

FIG. 2. Temperature dependence of inverse square of the effective penetration depth $\lambda_{eff}(T)$ in the mixed state ($H \simeq 0.04H_{c2}$) for $d$-wave and $s$-wave superconductors, normalized by the zero-field, zero-temperature penetration depth $\lambda_0(0)$, is plotted against reduced temperature $T/T_c$. $H_n$ represents the field corresponding to the inter-vortex spacing of $na$. For typical Cu-Cu distance in high $T_c$ materials, $H_{30}$ and $H_{28}$, are 8.0 and 9.2 Tesla, respectively. The zero-field penetration depth calculated with the same models are shown for comparison.
FIG. 2

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