Observing the technieta at a photon linear collider

Jusak Tandean

Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003

Abstract

If electroweak symmetry is spontaneously broken by technicolor, there will be technimesons analogous to the mesons of the ordinary strong interactions. One of the lightest technimesons is the technieta $\eta'_T$ (the analogue of the $\eta'$). In this work we consider the possibility of observing it at a linear $e^+e^-$ collider operating in the $\gamma\gamma$ mode. Detecting the process $\gamma\gamma \rightarrow \eta'_T$ allows for the measurement of the $\eta'_T\gamma\gamma$ anomalous coupling which is in principle sensitive to the underlying technifermion substructure.
The mechanism which is responsible for the spontaneous breaking of $SU(2)_L \otimes U(1)_Y$ electroweak symmetry down to the $U(1)$ of electromagnetism remains unknown. In the standard electroweak model this mechanism involves elementary scalar fields, whose presence in the theory makes it in some sense ‘unnatural’ [1]. One of the proposed alternatives to the standard model is technicolor [2]. In this scenario, there are no elementary scalar fields, and electroweak symmetry is spontaneously broken by condensates of new fermions bound by the technicolor forces.

In the minimal technicolor (TC) model [2], in addition to the usual quarks and leptons, there are two new fermions $U$ and $D$. These technifermions engage in TC and electroweak interactions, but are color singlets. The TC dynamics is assumed to be QCD-like and have an $SU(N_{TC})$ gauge symmetry. Each of the technifermions is assigned to the fundamental ($N_{TC}$) representation of $SU(N_{TC})$. With respect to the electroweak $SU(2)_L \otimes U(1)_Y$ gauge group, the left-handed components of the technifermions form a weak-isospin doublet $Q_L = (U_L, D_L)$ having zero hypercharge, whereas the right-handed components $U_R$ and $D_R$ are weak-isospin singlets with hypercharge $Y_R = 1$ and $-1$, respectively. It follows that the charges of $U$ and $D$ are $Q_U = \frac{1}{2}$ and $Q_D = -\frac{1}{2}$. With these assignments, the model is free of gauge anomalies.

In the absence of electroweak interactions, the TC Lagrangian in the massless limit has a global techniflavor $SU(2)_L \otimes SU(2)_R$ symmetry. The situation is analogous to the chiral limit of QCD. When the TC forces become strong at an energy scale $\Lambda_{TC} = O(1 \text{ TeV})$, they form technifermion condensates which break the global symmetry down to $SU(2)_{L+R}$. As a consequence, there are three Goldstone bosons, which are the massless technipions $\pi_{T}^{\pm,0}$, corresponding to the broken generators. The axial currents corresponding to these generators couple to the $\pi_{T}^{\pm,0}$ with strength $F_{\pi_T} = 246 \text{ GeV}$. When the electroweak interactions are turned on, the electroweak gauge bosons couple to the axial currents. Consequently, the $W^\pm$ and $Z^0$ gauge bosons acquire masses, and the $\pi_{T}^{\pm,0}$ disappear from the physical spectrum, having become the longitudinal components of the $W^\pm$ and $Z^0$.

Since the minimal TC model cannot account for the masses of ordinary quarks and
leptons, generating their masses requires new interactions. Hence, in considering the minimal TC model in this paper, we assume that the model is a low energy sector of a larger, more realistic model incorporating such interactions, which occur at scales higher than several TeV \[3\]. Since here we are working at energies up to only a few TeV, we expect that we can ignore these interactions. At such energies these interactions are assumed to be represented by effective four-Fermi interactions involving technifermions and ordinary fermions, so that when technifermion condensates form, ordinary fermions get their masses.

The lightest technihadrons in the physical spectrum can be inferred from QCD. The \( \pi_T^{\pm,0} \) having been absorbed by the \( W^\pm \) and \( Z^0 \), the spectrum begins with a technieta \( \eta_T' \), a technirho \( \rho_T \), and a techniomega \( \omega_T \). The \( \eta_T' \) and the \( \omega_T \) are techni-isospin singlets, while the \( \rho_T \) is a techni-isospin triplet. They are roughly \( \Lambda_{TC}/\Lambda_{QCD} \) times heavier than their QCD counterparts. Information about the substructure of these technihadrons may be obtained by observing them at multi-TeV colliders. The production and detection of the \( \rho_T \) and the \( \omega_T \) have been much discussed in the literature \[4\].

In this paper we are concerned with the \( \eta_T' \), which has not received much attention recently. One detailed study on the \( \eta_T' \) of which we are aware was done more than a decade ago \[5\]. Our purpose here is to take another look at the \( \eta_T' \) and its detection in the light of some recent development in collider physics. One of its decay modes is \( \eta_T' \rightarrow \gamma\gamma \) (in analogy to \( \eta, \pi^0 \rightarrow \gamma\gamma \)) which occurs via the anomaly \[3\], with technifermions appearing in the loop. This implies that the observation of the \( \eta_T' \) at a \( \gamma\gamma \) collider can be a means to probe the technifermion substructure, providing information complementary to that found in \( \rho_T \) and \( \omega_T \) studies. Here we shall consider the detection of the \( \eta_T' \) at a future high-energy photon linear collider realized by laser-backscattering technique at a linear \( e^+e^- \) collider \[6\]. Such a \( \gamma\gamma \) collider produces high-luminosity beams and clean backgrounds, which are necessary

\[1\] One recent paper \[6\] dealt briefly with the \( \eta_T' \) and the possibility of observing it as one of the decay products of the \( \rho_T \) in TC models with scalars.
factors in this endeavor because of the smallness of the $\eta_T' \gamma \gamma$ coupling.

Our ability to detect $\gamma \gamma \to \eta_T' \to X$ depends on the choice of final state $X$. The four-Fermi couplings between technifermions and ordinary fermions may cause the $\eta_T'$ to have appreciable decays into a pair of ordinary fermions. Since a reliable model for the couplings of the $\eta_T'$ to ordinary fermions is still unknown, conventional wisdom, based on analogy with the minimal standard model, suggests that they are proportional to fermion mass, and so the $\eta_T'$ will decay mostly to a pair of top quarks. We shall show that in such a case the $t \bar{t}$ decay mode can provide a window to observe the $\eta_T'$. Now, we have no guarantee that the tendency to couple to fermion mass will always occur; for even in multiple-Higgs models such a tendency can be evaded. Hence we shall also consider the possibility that the decays into fermion pairs are negligible. Consequently, the $\eta_T'$ decays mostly into two and three electroweak gauge bosons and becomes a very narrow resonance. In this case we shall show that the $\eta_T'$ is detectable in its $\gamma \gamma$ decay mode.

In order to discuss the decays of the $\eta_T'$, we need to construct the relevant effective lagrangians. The effective lagrangian which gives the couplings of the $\eta_T'$ to ordinary fermion pairs can be written as

$$\mathcal{L}_{\eta_T' f \bar{f}} = -i \sum_f \frac{m_f}{F_{\pi T}} \bar{\psi}_f \gamma_5 \psi_f \eta_T', \quad (1)$$

where $\psi_f$ and $m_f$ are, respectively, the fields and masses of the fermions, $\lambda_f$ are dimensionless constants, and the sum is over all ordinary leptons and quarks. For the case in which the $\eta_T'$ couples to fermion mass, we shall take $\lambda_f = 1$ for all $f$’s, resulting in the same lagrangian as that in Ref. [5]. In the second case, in which the decays into fermion pairs are negligible, we shall set $\lambda_f = 0$ for all $f$’s.

In the second case the decays of $\eta_T'$ into electroweak gauge bosons become important. Since the minimal TC dynamics is QCD-like, we expect that some of the decay properties of the $\eta_T'$ are similar to those of the $\eta$ and $\eta'$ of QCD. They are two of the lightest pseudoscalar mesons whose low-energy interactions with the lightest vector mesons and the photon can be described by an effective chiral Lagrangian [8–10]. What we need then is an analogous
effective Lagrangian which can describe interactions involving the lightest pseudoscalar and vector technimesons as well as the electroweak gauge bosons at energies below $\Lambda_{TC}$.

Before writing down the desired effective Lagrangian \([11]\), we discuss the fields contained in it. We collect the pseudoscalar fields $\eta_T'$ and $\pi_T$ into a unitary matrix $U = \exp(i\varphi/F_{\pi_T})$, where $\varphi = 1 + \tau \cdot \pi_T$, with $\tau = (\tau^1, \tau^2, \tau^3)$ being Pauli matrices. This construction has been chosen because an analogous choice made in QCD leads to good agreement with data, as will be noted later. Under global $U(2)_L \otimes U(2)_R$ transformations, $U \rightarrow LUR^\dagger$ with $L, R \in U(2)$. For the vector technimesons $\omega^\mu_T$ and $\rho^\mu_T$, we construct a matrix $V^\mu = \frac{1}{2}g(1 + \tau \cdot \rho_T^{\mu})$, where $g$ is a constant. The relation between $g$ and the $\rho_T^{\mu} \pi_T^{\mu}$ coupling constant will be shown below.

$V^\mu$ is related to auxiliary fields $A^L_R^\mu$ defined by $A^L_R^\mu = \xi V^\mu \xi^\dagger + i\partial^\mu \xi \xi^\dagger$, where $\xi = U^{1/2}$. In order to include the electroweak interactions, we gauge an $SU(2)_L \otimes U(1)_Y$ subgroup of the global $U(2)_L \otimes U(2)_R$, and introduce left-handed and right-handed gauge fields $\ell^\mu_L = \frac{1}{2}g_2 \tau \cdot W^\mu_L$ and $r^\mu_R = \frac{1}{2}g_1 \tau^3 B^\mu$, reflecting the assignments $Q_L = \frac{1}{2}$ and $Q_R = -\frac{1}{2}$ of the underlying technifermions. $B^\mu$ and $W^\mu_L$ are defined in terms of the photon, $W^\pm$, and $Z^0$ fields in the standard way. Under $SU(2)_L \otimes U(1)_Y$ gauge transformations,

$$\ell^\mu \rightarrow L\ell^\mu L^\dagger + i\partial^\mu LL^\dagger, \quad r^\mu \rightarrow Rr^\mu R^\dagger + i\partial^\mu RR^\dagger,$$

where now $L \in SU(2)_L$ and $R = \exp(-\frac{i}{2}i\tau^3 \theta)$. We require $A^L_R^\mu (A^{R\dagger}_R^\mu)$ to transform the same way as $\ell^\mu (r^\mu)$,

$$A^L_\mu \rightarrow L A^L_\mu L^\dagger + i\partial^\mu LL^\dagger, \quad A^R_\mu \rightarrow R A^R_\mu R^\dagger + i\partial^\mu RR^\dagger.$$

This is to ensure gauge invariance, as we will see shortly.

We divide into four parts the effective Lagrangian $\mathcal{L}$ which gives the decays of the $\eta_T'$ into electroweak gauge bosons. The corresponding action is written as

$$\Gamma = \int \mathcal{L} \, d^4x = \int (\mathcal{L}_1 + \mathcal{L}_2) \, d^4x + \Gamma_3 + \Gamma_4.$$

We have included in $\mathcal{L}$ only the most relevant terms for our purposes. To $\mathcal{L}$ must be added
the kinetic terms of the vector technimesons and the electroweak gauge bosons, which we
will not write explicitly.

The first part of $\mathcal{L}$ is

$$
\mathcal{L}_1 = \frac{m^2}{2g^2} \text{Tr} \left[ (A^L\mu - \ell^\mu)(A^L_\mu - \ell^\mu) + (A^R\mu - r^\mu)(A^R_\mu - r^\mu) \right] \\
- \frac{b}{2g^2} \text{Tr} \left[ (A^L\mu - \ell^\mu)U(A^R_\mu - r^\mu)U^\dagger \right],
$$

which is evidently gauge invariant. Upon expanding to second order of pseudoscalar fields,
from the vector mass term and the pseudoscalar kinetic term in $\mathcal{L}_1$ we obtain the relations

$$
m^2_{\rho_T} = m^2_{\omega_T}, \quad m^2 = 1/2(m^2_{\rho_T} + g^2F^2_{\pi_T}), \quad b = g^2F^2_{\pi_T} - m^2_{\rho_T}.
$$

(6)

We could also obtain in $\mathcal{L}_1$ the mass terms of the $W^\pm$ and $Z^0$. Using (6), we find in $\mathcal{L}_1$ two
terms which will be pertinent in evaluating the decays of the $\eta_T$:

$$
\mathcal{L}_{\rho_T\pi_T\pi_T} = i \frac{g_{\rho_T\pi_T\pi_T}}{2g} \text{Tr} \left[ V^\mu(\varphi\partial_\mu\varphi - \partial_\mu\varphi\varphi) \right],
$$

(7)

where $g_{\rho_T\pi_T\pi_T} = m^2_{\rho_T}/(2gF^2_{\pi_T})$, and

$$
\mathcal{L}_{\rho_TA} = -\frac{m^2_{\rho_T}}{g^2} \text{Tr} \left[ V^\mu(\ell_\mu + r_\mu) \right],
$$

(8)

which gives the couplings of the vector technimesons to the electroweak gauge bosons, $\mathcal{A} = (\gamma, W^\pm, Z^0)$. The mass of the $\rho_T$ and $g_{\rho_T\pi_T\pi_T}$ can be expressed in terms of their QCD
counterparts by using large-$N_{\text{TC}}$ arguments and scaling from QCD $^{[13]}$. One gets

$$
m_{\rho_T} = \frac{F_{\pi_T}}{F_\pi} \left( \frac{3}{N_{\text{TC}}} \right)^{1/2} m_{\rho}, \quad g_{\rho_T\pi_T\pi_T} = \left( \frac{3}{N_{\text{TC}}} \right)^{1/2} g_{\rho\pi\pi},
$$

(9)

where $F_\pi = 92$ MeV is the pion decay constant, $m_{\rho} = 770$ MeV is the $\rho$-meson mass, and
$g_{\rho\pi\pi}$ is related to the decay width of the $\rho$ by $\Gamma_{\rho \to \pi\pi} = g^2_{\rho\pi\pi} |p_\pi|^3/(6\pi m^2_{\rho})$.

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$^2$What we have here is analogous to the approximate vector-meson dominance mentioned in
Ref. $^{[3]}$.

$^3$Setting $g = g_{\rho_T\pi_T\pi_T}$ would give us the TC counterpart of the KSRF $^{[13]}$ relation.
The second Lagrangian expresses the breaking of the axial $U(1)$ symmetry and provides mass for the $\eta'_T$ in the chiral limit. It is written as
\[
L_2 = \frac{a F^2_{\pi_T}}{8 N_{TC}} \left[ \text{Tr}(\ln U - \ln U^\dagger) \right]^2,
\]
with $a$ being a constant. $a$ is connected to the mass $m_{\eta_0}$ of the flavor $SU(3)$ singlet of QCD in the chiral limit by
\[
a = \frac{1}{6} \frac{F^2_{\pi_T}}{F^2_{\pi}} \frac{9}{N_{TC}} m_{\eta_0}^2,
\]
where $m_{\eta_0} \simeq 849$ MeV. Assuming massless technifermions, we find from (10) and (11) that the mass of the $\eta'_T$ is
\[
m_{\eta'_T} = \sqrt{\frac{2}{3} \frac{F_{\pi_T}}{F_{\pi}} \frac{3}{N_{TC}} m_{\eta_0}} \simeq \frac{4}{N_{TC}} 1.39 \text{ TeV}.
\]
Hence the $\eta'_T$ may be the lightest technihadron in the physical spectrum.

The third and fourth parts in $\Gamma$ contain terms proportional to the Levi-Civita tensor, $\epsilon_{\kappa\lambda\mu\nu}$. We find it convenient to write them compactly using the notation of differential forms $\mathbf{I}$. Hence we define $dU = \partial_\mu U \, dx^\mu$, $\alpha = dU \, U^\dagger$, $\beta = U^\dagger \alpha U$; and $\mathcal{V} = \mathcal{V}_\mu \, dx^\mu$ and $d\mathcal{V} = \partial_\mu \mathcal{V}_\nu \, dx^\mu \, dx^\nu$ for $\mathcal{V} = A^L, \ell, r$. The third piece, $\Gamma_3$, is the gauged Wess-Zumino-Witten action $\mathbf{I}^3$. Written in terms of differential forms, it is
\[
\Gamma_3 = \Gamma_{WZW}(U, \ell, r)
\]
\[
= c \int \text{Tr}[(d\ell \ell + \ell d\ell)\alpha + (dr r + r dr)\beta - d\ell \, dU \, r U^\dagger + dr \, dU^\dagger \, \ell U] + ci \int \text{Tr}(\ell \alpha^3 + r \beta^3) + \cdots
\]
where $c = -i N_{TC}/(48 \pi^2)$ and we have shown only the relevant terms. For our choice of quantum numbers of the underlying technifermions, $\Gamma_3$ is gauge invariant. In order to write $\Gamma_4$, we need additional differential forms. They are $\alpha_1 = \alpha + i A^L - i U r U^\dagger$, $\alpha_2 = -i A^L + i \ell$, $\beta_{1,2} = U^\dagger \alpha_{1,2} U$, and the field-strength two-forms $F(\mathcal{V}) = d\mathcal{V} + i \mathcal{V} \mathcal{W}$ for $\mathcal{V} = A^L, \ell, r$.

It is straightforward to show that $\alpha_{1,2} \to L \alpha_{1,2} L^\dagger$ and $\beta_{1,2} \to R \beta_{1,2} R^\dagger$ under gauge transformations. The fourth part is then
\[ \Gamma_4 = \int \text{Tr} \left[ \frac{c_1}{g} (\alpha_1^3 \alpha_2 - \alpha_2^3 \alpha_1) + \frac{i c_2}{g^2} F(A^L)[\alpha_1, \alpha_2] + \left( \frac{c_1}{g} + \frac{c_2}{g^2} - \frac{c_3}{g^3} \right) \alpha_1 \alpha_2 \alpha_1 \alpha_2 
+ d_1 \left( F(\ell)[\alpha_1, \alpha_2] + F(r)[\beta_1, \beta_2] \right) \right] , \]  

where \( c_1, c_2, c_3, \) and \( d_1 \) are constants whose values will be discussed below. We easily see that each of the \( c_1, c_2, c_3, \) and \( d_1 \) terms is gauge invariant.

The partial decay widths of the \( \eta'_T \) can now be evaluated. In the case of \( \eta'_T \) coupling to ordinary-fermion pairs, we set \( \lambda_f = 1 \) for all \( f \)'s in \( \mathcal{L}_{\eta'_T f f} \), from which amplitudes for the decays into fermion pairs can be easily extracted. In this case the \( \eta'_T \) also decays into a pair of gluons through a quark triangle-loop, and the decay amplitude is \[ M_{\eta'_T \rightarrow g a g b} = \frac{\alpha_s}{\pi F_{\pi_T}} \sum_{q} \frac{-1}{2 R_q^2} \left[ 2 \ln \left( \frac{1}{2} R_q + \sqrt{\frac{1}{4} R_q^2 - 1} \right) + i\pi \right]^{\frac{1}{2}} \delta_{ab} \epsilon_{\kappa\lambda\mu\nu} k_1^\kappa k_2^\lambda \epsilon^\mu*(k_1) \epsilon^\nu*(k_2) , \]  

where \( a, b = 1, \ldots, 8 \) are gluon color indices, the sum is over all quarks, \( R_q = m_{\eta'_T}/m_q \), and the strong coupling constant \( \alpha_s = \alpha_s(m_{\eta'_T}) \). In order to compute the widths of the \( f \bar{f} \) and \( gg \) decay modes, we use the values of fermion masses\(^4\) available in Ref. \[17\] and take \( m_t = 174 \text{ GeV} \). In Table I we summarize the results, along with those of the decays into electroweak gauge bosons to be discussed below, for \( N_{\text{TC}} = 4, 5, 6 \). The contribution of the \( gg \) mode to the total width is seen to be comparable to that of the decays into electroweak bosons. It is worth mentioning that in TC models with technifermions carrying ordinary color we would expect the \( gg \) decay mode to proceed mainly through a colored-technifermion loop, resulting in a much larger \( gg \) partial width, although the \( t\bar{t} \) mode would still be dominant \[19\].

The decays into two and three electroweak gauge bosons come from (13) and (14), together with (7) and (8). We express the amplitudes for the decays into two (transverse) electroweak gauge bosons as

\[ M_{\eta'_T \rightarrow A_1 A_2} = -C_{\eta'_T \rightarrow A_1 A_2} \frac{e^2 N_{\text{TC}}}{8\pi^2 F_{\pi_T}} \epsilon_{\kappa\lambda\mu\nu} k_1^\kappa k_2^\lambda \epsilon^\mu*(k_1) \epsilon^\nu*(k_2) \]  

\(^4\)We use central mass values in the case of quarks.
where $A_1 A_2 = W^+ W^-, \gamma \gamma, \gamma Z^0, Z^0 Z^0$, $\epsilon_{0123} = +1$, and
\[
C_{\eta'_T \rightarrow W^+ W^-} = \frac{1}{3 s_W^2}, \quad C_{\eta'_T \rightarrow \gamma\gamma} = 1, \quad C_{\eta'_T \rightarrow \gamma Z^0} = \frac{1 - 2 s_W^2}{2 c_W s_W},
\]
\[
C_{\eta'_T \rightarrow Z^0 Z^0} = \frac{1 - 3 s_W^2 + 3 s_W^4}{3 c_W^2 s_W^2},
\]
with $s_W^2 = \sin^2 \theta_W$ ($\theta_W$ is the Weinberg angle) and $c_W = \sqrt{1 - s_W^2}$. The $\eta'_T$ also decays into two technipions and one (transverse) electroweak gauge boson, up to corrections which vanish.

Here we have used the parameters $\alpha = e^2/(4\pi) = 1/137$, $s_W^2 = 0.232$, $m_W = 80.2$ GeV.

The decay widths calculated from (16) and (17) are listed in Table I for different values of $N_{TC}$, where the widths of $\eta'_T \rightarrow W^+_i W^-_i \gamma, W^+_i Z^0_i W^-_i, W^+_i W^-_i Z^0_i$, with $l (t)$ referring to a longitudinal (transverse) polarisation, have been combined as $\Gamma(\pi_T \pi'_T A)$ in the last column. Here we have used the parameters $\alpha = e^2/(4\pi) = 1/137$, $s_W^2 = 0.232$, $m_W = 80.2$ GeV.
and \( m_Z = 91.2 \text{ GeV} \). There are also decays into four longitudinal weak gauge bosons, which one might naively expect to be significant because the equivalent decay amplitudes involving the \( \pi^{\pm,0}_I \) contain the \( \rho_I \) in the intermediate states and no electroweak couplings. It turns out that their widths are suppressed by the presence of the antisymmetric tensor \( \epsilon_{\kappa\lambda\mu\nu} \) in the amplitudes and by the four-body final-state phase space. For example, using (7), (9), (14), and (18), with \( N_{TC} = 4 \) we obtain

\[
\Gamma_{\eta'_T \rightarrow W^+_i W^-_i Z^0_i Z^0_i} \sim 10^{-3} \text{ MeV}
\]  

for the choice \( 2c_1 - c_3/g^2 = -2c_2/g \), whose QCD analogue led to good agreement with data for \( \omega \rightarrow 3\pi \) [8]. (Since the actual values of \( c_1 \) and \( c_3 \) will not affect our results, we leave them undetermined.) We therefore expect that there are no other significant partial decay widths of the \( \eta'_T \) than those given in Table I.

Table II shows for \( N_{TC} = 4, 5, 6 \) the total width \( \Gamma_{\eta'_T} \) in each of the two cases considered here. In Case 1, where the \( \eta'_T \) is coupled to fermion pairs, we obtain \( \Gamma_{\eta'_T} \) by summing all of the partial widths listed in Table I. In Case 2, where the \( \eta'_T \) has negligible couplings to fermion pairs, \( \Gamma_{\eta'_T} \) is found from Table I by combining only the widths of the decays into electroweak gauge bosons, and hence the \( \eta'_T \) is a very narrow resonance.

We now turn to the production and detection of the \( \eta'_T \) at a photon linear collider. By directing a low-energy laser beam at a high-energy \( e^+(e^-) \) beam almost head-to-head, a beam of backscattered photons is produced, carrying a large fraction of the \( e^+(e^-) \)-beam energy. The resulting \( \gamma\gamma \) beams have a luminosity comparable to that of the parent \( e^+e^- \) beams. The energy-distribution function of a backscattered photon is given by [7]

\[
f_{e/\gamma}(x) = \frac{1}{D(\xi)} \left[ 1 - x + \frac{1}{1 - x} - \frac{4x}{\xi(1 - x)} + \frac{4x^2}{\xi^2(1 - x)^2} \right] \]

where

\[
D(\xi) = \left( 1 - \frac{4}{\xi} - \frac{8}{\xi^2} \right) \ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2},
\]

\( \xi = 4\omega_0E_e/m^2_e \), with \( \omega_0 \) and \( E_e \) being the energies of the incident laser photon and the \( e^+ \) (or \( e^- \)) beam, respectively, and \( x = \omega/E_e \) is the fraction of \( E_e \) carried by the backscattered
photon. \( f_{e/\gamma}(x) \) vanishes for \( x > x_{\text{max}} = \omega_{\text{max}}/E_e = \xi/(1 + \xi) \). In order to avoid the creation of \( e^+e^- \) pairs by the interaction of the incident and backscattered photons, we require \( \omega_0 x_{\text{max}} \leq m_e^2/E_e \), which implies \( \xi \leq 2 + 2\sqrt{2} \approx 4.8 \). For the choice \( \xi = 4.8 \), which maximizes \( x_{\text{max}} \), we obtain \( x_{\text{max}} \approx 0.83 \), \( D(\xi) \approx 1.8 \), and \( \omega_0 \approx 0.31 \text{eV} \) for an \( e^+e^- \) collider with center-of-mass energy \( \sqrt{s_{e^+e^-}} = 2 \text{TeV} \). Here we have taken the photons to be unpolarised and the average number of backscattered photons per positron (or electron) to be one.

Total cross sections \( \sigma \) at the parent \( e^+e^- \) collider are found by folding \( \gamma\gamma \)-subprocess cross sections \( \hat{\sigma}_{\gamma\gamma} \) with the photon distribution functions:

\[
\sigma(s_{e^+e^-}) = \int_{\tau_1}^{\tau_2} d\tau \int_{\tau/x_{\text{max}}}^{x_{\text{max}}} \frac{dx}{x} f_{e/\gamma}(x) f_{e/\gamma}(\tau/x) \hat{\sigma}_{\gamma\gamma}(\tau s_{e^+e^-})
\]

where \( \tau_1 (\tau_2) \) is the minimum (maximum) of the range of \( \tau = s_{\gamma\gamma}/s_{e^+e^-} \) to be integrated over. In order to find numbers of events, we multiply \( \sigma \) by the yearly integrated \( e^+e^- \) luminosity \( L_{ee} \).

In Case 1 the \( \eta_T^\prime \) decays almost entirely into a \( t\bar{t} \) pair and so we expect \( \gamma\gamma \to \eta_T^\prime \to t\bar{t} \) to be the only channel likely to be detectable. The background for this channel comes from the process \( \gamma\gamma \to t\bar{t} \). The combined amplitude for the signal and background processes is

\[
\mathcal{M}_{\gamma\gamma \to t\bar{t}} = -i \frac{e^2 N_{\text{TC}} m_t}{8\pi^2 F_{\pi_T} F_{\pi_T}} \frac{\epsilon^\alpha\beta\mu}{s_{\gamma\gamma} - m_{\eta_T}^2 + i m_{\eta_T} \Gamma_{\eta_T}} \frac{\hat{u}(p) \gamma_5 v(p')}{p - k - m_t + i\epsilon} \hat{u}(p) \left[ \epsilon^\alpha(k) \epsilon^\beta(k') + (k \leftrightarrow k') \right] v(p'),
\]

where \( k, k' \) and \( p, p' \) are the four-momenta of the incoming photon pair and outgoing \( t\bar{t} \) pair, respectively, and \( s_{\gamma\gamma} = (k + k')^2 = (p + p')^2 \). In the center-of-mass frame of the incoming photons the background \( t\bar{t} \) production is peaked in the forward and backward directions, whereas the signal \( t\bar{t} \) are produced isotropically. Hence we impose an angular cut \( |\cos \theta| < 0.866 \) where \( \theta \) is the scattering angle of the \( t \)'s in this frame. We assume that the experimental resolution is smaller than \( \Gamma_{\eta_T} \) and that the \( t\bar{t} \) events can be fully reconstructed. For \( L_{ee} = 10 \text{fb}^{-1} \) and \( \sqrt{s_{e^+e^-}} = 2 \text{TeV} \), the number of signal and background events in
the mass interval $m_{\eta_T'} - \Gamma_{\eta_T'}/2 < \sqrt{s_{\gamma\gamma}} < m_{\eta_T'} + \Gamma_{\eta_T'}/2$ is given in Table [II]. If desired, the background could be reduced further, while the signal being increased, by employing polarized $\gamma\gamma$ beams as was done in Ref. [21] for Higgs production.

In Case 2 the channel $\gamma\gamma \rightarrow \eta_T' \rightarrow \gamma\gamma$ is probably the only one likely to be viable. The $WW$, $\gamma Z$, and $ZZ$ channels are known to have large backgrounds [22,23], while the $WW\gamma$ and $WWZ$ channels may be too small to be useful. The background in the $\gamma\gamma$ channel is dominated by $W$-boson loop contributions. The $\eta_T'$ being such a narrow resonance we may safely ignore the interference effect between continuum background diagrams and the resonance diagram. Consequently, the subprocess cross section can be written as a sum of a resonance cross section and a continuum background cross section:

$$\hat{\sigma}_{\gamma\gamma \rightarrow \gamma\gamma}(s_{\gamma\gamma}) = \hat{\sigma}_{\gamma\gamma \rightarrow \gamma\gamma}^r(s_{\gamma\gamma}) + \hat{\sigma}_{\gamma\gamma \rightarrow \gamma\gamma}^b(s_{\gamma\gamma}).$$

(23)

The resonance cross section is given by

$$\hat{\sigma}_{\gamma\gamma \rightarrow \gamma\gamma}^r(s_{\gamma\gamma}) = 8\pi \frac{\Gamma_{\eta_T' \rightarrow \gamma\gamma}^2}{(s_{\gamma\gamma} - m_{\eta_T'}^2)^2 + m_{\eta_T'}^2 \Gamma_{\eta_T'}^2}. \quad (24)$$

The background cross section can be estimated by scaling the $\gamma\gamma \rightarrow Zt \bar{Z}_t$ cross section calculated in Ref. [23] by a factor $e^4/g_{WWZ}^4 = \sin^4 \theta_W/\cos^4 \theta_W$. One finds $\hat{\sigma}_{\gamma\gamma \rightarrow \gamma\gamma}^b(s_{\gamma\gamma}) \simeq 25 \text{ fb}$ in the relevant $\sqrt{s_{\gamma\gamma}}$ range. For the same values of $L_{ee}$ and $\sqrt{s_{e^+e^-}}$ as before, we show in Table [I] the number of signal and background events in the interval $1.38 \text{ TeV} < \sqrt{s_{\gamma\gamma}} < 1.40 \text{ TeV}$ without employing any cuts.

In conclusion, we have shown that in the two cases discussed above the $\eta_T'$ can be observed above the backgrounds at a TeV $\gamma\gamma$ collider. We learn from Tables [I] and [II] that for larger $N_{TC}$ the signal-to-background ratio is better because the $\eta_T'$ mass is smaller and the branching ratio of the $\gamma\gamma$ decay mode is larger. Although in this paper we have considered only two possible cases in the simplest TC model, similar analyses can be made in more

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5 The signal-background interference contribution is small and has been included in the number of background events.
complicated models in which the $t\bar{t}$ or $\gamma\gamma$ decay mode is significant. Hence the $\eta_T$ has the potential to be a useful probe of its subconstituents. Our results above should give additional motivation for developing a backscattered-laser beam facility in the future.

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TABLES

TABLE I. The mass and the partial decay widths of the $\eta_T'$ for different values of $N_{TC}$. In the fourth column $\Gamma(f \bar{f}, \text{no } t\bar{t})$ is the combined width of the decays into all ordinary-fermion pairs excluding $t\bar{t}$. In the last column $\Gamma(\pi_T'\pi_T'A)$ is the combined width of $\eta_T' \to W_l^+ W_l^- \gamma, W_l^\pm Z^0 W_l^\mp, W_l^+ W_l^- Z^0$.

| $N_{TC}$ | $m_{\eta_T'}$ | $\Gamma(t\bar{t})$ | $\Gamma(f \bar{f}, \text{no } t\bar{t})$ | $\Gamma(gg)^a$ | $\Gamma(WW)$ | $\Gamma(\gamma\gamma)$ | $\Gamma(\gamma Z)$ | $\Gamma(ZZ)$ | $\Gamma(\pi_T'\pi_T'A)$ |
|----------|--------------|------------------|------------------|-----------------|---------------|-----------------|-----------------|----------------|------------------|
| 4        | 1.39         | 80.4             | 58.3             | 56.3            | 19.3          | 4.8             | 3.8             | 3.5            | 8.1              |
| 5        | 1.11         | 63.1             | 46.7             | 55.2            | 15.3          | 3.8             | 3.0             | 2.8            | 2.1              |
| 6        | 0.927        | 51.3             | 38.9             | 54.2            | 12.5          | 3.2             | 2.5             | 2.3            | 0.7              |

$a$For the three different values of $m_{\eta_T'}$ listed here, $\alpha_s(m_{\eta_T'}) = 0.084, 0.086, 0.088$, respectively, obtained using the results of Ref. [18].

TABLE II. The total width of the $\eta_T'$ and the number of $t\bar{t}$ $(\gamma\gamma)$ events for the signal $S$ and background $B$ in Case 1 (2) for different values of $N_{TC}$.

| $N_{TC}$ | $\Gamma_{\eta_T'}$ | $S$ | $B$ | $\Gamma_{\eta_T'}$ | $S$ | $B$ |
|---------|-------------------|----|----|-------------------|----|----|
| 4       | 80.6              | 103| 70 | 39.5              | 28 | 3  |
| 5       | 63.2              | 147| 97 | 27.0              | 51 | 4  |
| 6       | 51.4              | 189| 120| 21.2              | 70 | 4  |