Relevance of Two Boson Exchange Effect in Quasi-Elastic Charged Current Neutrino-Nucleon Interaction

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Two boson exchange (TBE) correction to the cross section for the quasi-elastic charged current $\nu n$ and $\bar{\nu}p$ scattering is evaluated. The TBE is given by $W\gamma$ box diagrams. The calculations are performed for 1 GeV neutrinos. The averaged TBE correction is of the order of $2 \pm 4\%$ (with respect to Born contribution) in the case of $\nu_e$ and $\bar{\nu}_e$ and $1 \pm 2\%$ in the case of $\nu_\mu$ and $\bar{\nu}_\mu$. The impact of the TBE effect on the systematic discrepancy between the $\nu_e$ and $\nu_\mu$ cross sections is discussed.

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1. INTRODUCTION

One of the goals of the particle physics is to understand the fundamental properties of neutrinos. In the experimental physics an effort has been made to measure the $\theta_{13}$ parameter and then, in the near future, the CP violation phase. It can be done by analyzing the $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ oscillation processes.

Estimate of the systematic differences between the cross sections for $\nu_\mu$ and $\nu_\mu$ interactions is of importance in the reconstructing $\nu_\mu$ signal [1]. Electrons, because $m_e \ll m_\mu$, have tendency to radiate more than muons. Therefore radiative corrections (RCs) are the potential source of the discrepancy between cross sections for the charged current electron and muon neutrinos interactions. Moreover the RCs are not included in any Monte Carlo generator used to analyze the neutrino interaction data [1].

In the long base-line experiments like T2K [2] or NOνA [3] the charged current quasi-elastic (CCQE) neutrino-nucleon scattering is the dominant observed process. The typical averaged neutrino energy in those experiments is about 0.7 GeV and 2 GeV, while the typical four momentum transfer $Q^2 < 4$ GeV$^2$. In this paper we concentrate our attention on the study of the impact of the two boson exchange (TBE) effect on the CCQE cross sections in this kinematical domain.

Recently the TBE effect has been extensively investigated in the elastic $ep$ scattering. The renewal interest of this topic was induced by observing the discrepancy between the measurements of the form factor ratio $G_E^p/G_M^p$ ($G^p_{E,M}$ is the electric, magnetic proton form factor) obtained within two different experimental techniques. The first method is based on the Rosenbluth analysis, the other is based on the so-called polarization transfer (PT) measurements (for review see Ref. [4]).

In the electron scattering the leading contribution to the TBE effect is given by interference of the Born amplitude with the $\gamma\gamma$ box diagrams describing an exchange of two virtual photons between electron and the target. It is called two photon exchange (TPE) correction. In the classical treatment of RCs to $ep$ scattering [5] the TPE correction was estimated in an approximation, in which the hard photon contribution, induced by the internal proton structure, was neglected. It turned out that applying in the Rosenbluth analysis the new, more precise predictions of the TPE effect, allowed to partially resolve the problem of discrepancy between the form factors [6, 7].

In the case of neutrino interactions the RCs have been extensively studied for $\beta$ and $\mu$ decays but also for the CCQE neutrino-deuteron interaction near the threshold region [8, 9]. The RCs were also estimated for the deep inelastic (DIS) $\nu$-nucleon scattering [10, 11]. The total corrections are of the order of several percent in the threshold region. The size of the RCs in the DIS depends on kinematics, measurement technique (e.g. the way of reconstructing $Q^2$), detector properties, kinematical cuts etc.

Recently some discussion of the impact of the RCs on the CCQE cross sections in 1 GeV kinematical domain was presented in Ref. [12, 13], where the lepton leg correction formula, estimated in the leading log approximation [11] was adopted. It describes the soft and hard photon emission by the charged lepton leg ($\nu p$, DIS) and it does not include the TBE contribution. Obviously this approximation is based on the diagrams, which do not form the gauge invariant set.

The aim of this paper is to discuss the contribution given by $W\gamma$ box diagrams induced by the internal nucleon structure. Certainly the predictions can be model
dependent. The simplest way is to consider the box diagrams as drawn in Fig. 1 and then to proceed as it follows: (i) assume that the hadronic intermediate state is given by the off-shell nucleon; (ii) the off-shell electroweak hadronic vertices are modelled by the on-shell nucleon form factors. This kind of the approximation was successfully applied for predicting the hard photon contribution from $\gamma\gamma$ [13] and $\gamma Z^0$ [14, 16] box diagrams. A part of the inner radiative correction correction to the $\beta$ decay was also estimated by considering the nucleon form factors [3, 17].

This approach seems to lead to the reasonable predictions of the TPE effect for $ep$ scattering in the low and the intermediate $Q^2$ range [18]. Hence in the same kinematical domain as it is considered in the case of the 1 GeV neutrino interactions. In one of our previous papers [19] we made an effort to predict the TPE correction for $\gamma\gamma$ box contribution in the elastic $ep$ scattering, considering nucleon and $\Delta(1232)$ resonance as the intermediate hadronic states. We showed the satisfactory agreement between the theoretical results and phenomenological fits obtained from the Bayesian analysis of the $ep$ scattering data [20]. In this paper we apply the same methodology to compute $W\gamma$ contribution in CCQE reactions.

2. FORMALISM

Let us consider quasi-elastic charged current neutrino-nucleon scattering $\nu_l(k)+n(p) \to \nu_l(k')+p(p')$, where $l = e, \mu$. We define the kinematical variables $q^2 = \not{k} - \not{k}' = (q_0, \mathbf{q})$ (four momentum transfer), $Q^2 = -q^2$. Mandelstam variables read $s = (k+p)^2$ and $t = (k-k')^2 = q^2$, while $E$ and $E'$ denotes the neutrino and lepton energy.

A Born amplitude for the CCQE $\nu n$ reaction reads,

$$iM_{Born} = \frac{i}{8(t-M_W^2)} \Gamma_{\nu} j_{\mu} h^\mu, \quad (1)$$

$\theta_C = 13.04^\circ$ is Cabibbo angle, $g = e/\sin\theta_W$ is the weak coupling constant, $\sin^2\theta_W = 0.2312$, $\alpha = 1/137$, $M_W = 80.3$ GeV is the boson $W^\pm$ mass. We mostly follow the convention of Cheng and Lee textbook [21].

The leptonic one-body current has a simple form,

$$j_{\mu} = \overline{u}(k')\gamma_{\mu}(1-\gamma_5)u(k), \quad (2)$$

while the hadronic one-body current reads,

$$h^\mu(q) = \overline{u}(p')\Gamma_{\nu}^\mu u(p). \quad (3)$$

The electroweak nucleon vertex is a function of four form factors,

$$\Gamma_{CC}^\mu(q \equiv p' - p) = \Gamma_{\nu}^\mu(q) - \gamma_{\mu}\gamma_5 F^\nu_A(q) - \frac{q^\mu q_5}{2M} F^\nu_P(q), \quad (4)$$

where, $\Gamma_{\nu}^\mu = \Gamma_{p}^\mu(q) - \Gamma_{n}^\mu(q)$, $\Gamma_{p(n)}^\mu(q)$ is the proton (neutron) electromagnetic vertex defined below, while $F_A$ and $F_P$ are the nucleon and pseudoscalar axial form factors respectively. We assume that $F_A(q) = g_A/(1 + Q^2/M_A^2)$, where $g_A = 1.267$, $M_A$ is an axial mass, and as a default value we take $M_A = 1$ GeV. The pseudoscalar axial form factor, $F_P(q)$ has a commonly used form: $F_P(q) = 4M^2 F_A(q)/(m_\pi^2 - q^2)$; $m_\pi$ is the pion mass, $M = (M_p + M_n)/2$, $M_{p(n)}$ is the proton (neutron) mass.

The proton (neutron) electromagnetic vertex reads,

$$\Gamma_{p(n)}^\mu(q) = \gamma_{\mu} F_{1}^{p(n)}(q) + i\sigma_{\mu\nu} q_\nu /2/(2M_{p(n)} F_{2}^{p(n)}(q), \quad (5)$$

where $F_{1,2}^{p(n)}$ is proton (neutron) form factor.

It is convenient to express the nucleon electromagnetic form factors by the electric and magnetic proton (neutron) form factors, $F_{E,M}^{p(n)} = (G_{E,M}^{p(n)} + \tau_{p,n} G_{M}^{p(n)})/(1 + \tau_{p,n})$, $F_{2}^{p(n)} = (G_{M}^{p(n)} - G_{E}^{p(n)})/(1 + \tau_{p,n})$, where $\tau_{p,n} = -q^2/4M^2_{p,n}$. In our calculations we consider a dipole parametrization of the electric and magnetic form factors, namely, $G_{E}^{p(n)}(Q^2) = G_{M}^{p(n)}(Q^2)/(\mu_{p,n} = \Lambda^4/(Q^2+\Lambda^2))$, where $\Lambda = 0.84$ GeV is the cut off parameter. The electric neutron form factor is assumed to be zero ($G_{E}^{n}(Q^2) = 0$).

In order to calculate the TBE correction we consider the exchange between the lepton and the nucleon target the virtual $W^+$ boson and the photon $\gamma$. For the hadronic intermediate state we take the off-shell nucleon (N). We expect that the resonance hadronic contribution, similarly, as in the case of $ep$ scattering will be very small [19, 22], in particular, in the low $Q^2$ range, which is the most relevant domain for neutrino reactions.

Two box $W\gamma$ diagrams contribute to the TBE amplitude, see Fig. 1. $i \Box_{W+\gamma} = -\cos\theta_C g^2 g^2 I_{W+\gamma}/8$ and

![FIG. 2: $\delta_{TBE}$ for $\nu_{\mu,n}$ and $\tau_{\mu,p}$ CCQE reactions.](image-url)
\[ i\Box_{W^+\gamma}^\times = -\cos\theta_C e^2 q^2 I_{W^+\gamma}^\times /8, \]

where

\[ I_{W^+\gamma}^\times = \int \frac{d^4 l}{(2\pi)^4} \frac{\mu^{\nu\mu} h_{\mu\nu}}{D(p', M_p)} \]

\[ I_{W^+\gamma}^\parallel = \int \frac{d^4 l}{(2\pi)^4} \frac{\mu^{\nu\mu} h_{\mu\nu}}{D(-p, M_n)} \]

\[ \mu^{\nu\mu} = \bar{\pi}(k')\gamma^\mu(k' - \hat{l} + m)\gamma^\nu(1 - \gamma_5)u(k) \]

\[ h_{\mu\nu}^\parallel = \bar{\pi}(p')\Gamma_\mu^\nu(-l)(p' + \hat{l} + M_p)\Gamma_\nu^\mu(q + l)u(p) \]

\[ h_{\mu\nu}^\times = \bar{\pi}(p')\Gamma_\mu^\nu(q + l)(p - \hat{l} + M_n)\Gamma_\nu^\mu(-l)u(p) \]

where \( D(x, M_z) = [(q + l)^2 - M_W^2 + ic][l'^2 + ic][(k' - l)^2 - m^2 + ic][\hat{l}(l + l') - M_Z^2 + ic], m \) denotes the lepton mass.

The TBE correction to spin averaged cross section is the interference of the Born and TBE box diagrams, and it reads,

\[ \Delta_{TBE} = \sum_{spin} \mathcal{M}_{Born}^\parallel (\Box_{W^+\gamma}^\parallel + \Box_{W^+\gamma}^\times) = \frac{g^4 e^2 \cos^2 \theta_C}{16(M_W^2 - t)} \sum_{spin} (j_a h^{\alpha\ast}) \left( I_{W^+\gamma}^\parallel + I_{W^+\gamma}^\times \right) \]

In practice \( \sim \int d^4 l N/D, \) where \( N \) is a polynomial function of \( l^2, l\cdot p', l\cdot k', l\cdot q \) scalar products, given by an appropriate sum of traces, computed with FeynCalc package [22], while \( D \) denotes denominator, which because of the form factors can be of the order of \( l^6. \)

To compute the TBE contribution the integral \( \Box \) was expressed as a sum of scalar loop integrals [24]. But to perform this decomposition the appropriate reduction of \( N \) with \( D \) had to be done. Eventually, the numerical values of the TBE correction were evaluated applying LoopTool library (C++) [23].

In order to deal with the IR divergencies, the photon mass \( \mu \) is introduced \( 1/l^2 \rightarrow 1/(l^2 - \mu^2). \) Notice that
the IR divergency coming from $W\gamma$ box diagram is cancelled, in the total calculus of the RCs, by the soft photon Bremsstrahlung inelastic contribution.

The calculations for the antineutrino scattering are straightforward. The leptonic current in this case reads, $j_{\mu}^{\ell} = \overline{\nu}(k')\gamma_{\mu}(1 + \gamma_5)u(k)$, while $L_{\nu} = -\overline{\nu}(k')\gamma^\nu(k' - \ell + m)\gamma_{\mu}(1 + \gamma_5)u(k)$. One should also interchange the electromagnetic vertices in the diagrams $\Gamma_{\mu} \leftrightarrow \Gamma_{\mu}$. Notice that the direct and exchange diagrams are interchanged.

3. DISCUSSION

As mentioned in the introduction the radiative corrections are the potential source of systematical difference between the electron and muon neutrino cross sections. In fact, the TBE effect for $\nu_e$ is about two times larger than in the case of $\nu_\mu$ (see Figs. 4 and 5). At low $Q^2$ the TPE correction is positive, but when $Q^2$ grows it changes a sign.

In Fig. 4 (top panel) we plot the quantity

$$R(Q^2) = \frac{d\sigma_{\nu_e}^{\text{Born}} + d\sigma_{\nu_e}^{\text{TBE}}}{d\sigma_{\nu_e}^{\text{Born}} + d\sigma_{\nu_e}^{\text{TBE}}} \left( \frac{d\sigma_{\nu_e}^{\text{Born}}}{d\sigma_{\nu_e}^{\text{Born}}} \right)^{-1} - 1$$

$$= \frac{1 + \delta_{\text{TBE}}(\nu_e)}{1 + \delta_{\text{TBE}}(\nu_\mu)} - 1 \approx \delta_{\text{TBE}}(\nu_e) - \delta_{\text{TBE}}(\nu_\mu),$$

which measures the relative difference between the TBE effect for $\nu_e$ and $\nu_\mu$. It turned out to be a linear function in $Q^2$, which takes the largest values at low $Q^2$.

Similarly as in the case of the TPE effect, in elastic $ep$ scattering $\nu_e$, the TBE correction weakly depends on the model parameters. It is shown in Fig. 5 where we plot the TBE contribution computed for several values of $M_A$ and $\Lambda$. Moreover, taking into consideration more realistic electric proton form factor, modified due to TPE effect $^{[26]}$, $G_E^p(Q^2) = (0.130 Q^2 + 1.0022) G_M^p(Q^2)/\mu_p$ (13)

has also a minor impact on the TBE effect, see Fig. 4 (bottom panel).

On average, the TBE correction is of the order of 2 and 4 percent for $\nu_e$ and $\nu_\mu$, respectively. In the case of the $\nu_\mu$ the TBE effect is negligible, while for $\overline{\nu}_\mu$ it increases the total cross section by about 2%, see Fig. 6. Notice that the axial mass $M_A$, is usually fit to the total $\nu_\mu$ CCQE cross section data. Therefore in order to re-construct, due to the TBE effect, the "true" $\nu_e$ CCQE cross section one should decrease $M_A$ by about 2%.

More detailed investigation of the impact of the TBE effect on the cross sections requires considering the rest of the radiative corrections. Taking into account the full set of diagrams required by the Standard Model goes beyond the scope of this paper. However, we consider the QED-like corrections, namely, the soft Bremsstrahlung photon emission by the charged lepton and the proton and the propagator corrections (for electron and proton).

The soft photon Bremsstrahlung emission describes the inelastic process $\nu + n \rightarrow l^- + p + \gamma$, which is indistinguishable with the CCQE reaction as long as the photon has an energy smaller than detector acceptance $\Delta E$. Hence it contributes to the CCQE cross section. Notice that $\Delta E = E'(\text{quasi - elastic}) - E'(\text{inelastic})$. In our computations we consider $\Delta E = \omega E'$ where $\omega \ll 1$. The Bremsstrahlung corrections are computed in the similar way as in Ref. $^{[27]}$. The soft photon emission cross section reads,

$$d\sigma_{\text{Soft.Brem.}} \approx d\sigma_{\text{Born}} \delta_{\text{Soft.Brem.}}$$

$$\delta_{\text{Soft.Brem.}} = \frac{\alpha}{4\pi^2} \left[ 2 p \cdot k L_{p'k'} - m^2 L_{k'k} - M_p^2 L_{p'p} \right],$$

(14)
where

\[ L_{xy} = \int_{|\mathbf{l}| < \Delta E} \frac{d^4 l}{|\mathbf{l}| (x \cdot l)(y \cdot l)} \]  \hspace{1cm} (15) \]

\( \Delta E \) is the maximal photon energy in the frame with \( \mathbf{l} + \mathbf{p}' = 0 \)\(^1\) [24]. Certainly above integral is divergent when \( l \to 0 \).

The charged lepton and the photon propagator corrections are easy to derive and they read,

\[ \delta_{\text{Self.}} = -\frac{\alpha}{\pi} \left\{ \frac{9}{4} + \ln \frac{\mu}{m} + \ln \frac{\mu}{M_p} + \frac{1}{2} \left( \ln \frac{\Lambda_{\text{UV}}}{m} + \ln \frac{\Lambda_{\text{UV}}}{M_p} \right) \right\}. \]

The total correction: \( \delta = \delta_{\text{TBE}} + \delta_{\text{Soft.Brem.}} + \delta_{\text{Self.}} \) is IR finite, however, the propagator correction depends on the UV cut-off parameter \( \Lambda_{\text{UV}} \). We set \( \Lambda_{\text{UV}} = M_p \).

Fig. 7 shows \( \delta \) for \( \omega = 0.05 \) and \( \omega = 0.1 \). In the case of \( \nu_e \) the correction is relatively large and it is of the order of \(-9\%\), while in the case of \( \mu \) it is around \(-2\%\). The dominant (negative) contribution comes from the soft Bremsstrahlung part. It leads to the reduction of \( \sigma(\nu_e)/\sigma(\nu_\mu) \) ratio by about \(-6\%\).

Above estimate gives the lower bound for the RCs. Adding the hard photon Bremsstrahlung contribution (it is the typical way of presenting the RCs in \( \nu N \) scattering) reduces the \( \delta \) and it makes the RCs positive.

For complete calculations of the total RCs one should consider full set of the diagrams required by the gauge invariance. But the resulting impact of the RCs on the measured cross sections depends on the measurement performance i.e. reconstruction of the kinematical variables, detector properties etc.

It is interesting to notice that there is a new proposal of the neutrino project nuSTORM [28]. This experiment is going to measure the \( \nu_e \) and \( \nu_\mu \) scattering cross sections. It should allow to critically investigate the systematical differences between \( \nu_e \) and \( \nu_\mu \) cross sections.

To summarize, we have obtained the TBE correction, its hard photon contribution, to the CCQE cross sections. The TBE effect is two times larger for \( \nu_e \) than for \( \nu_\mu \). The relative difference between the TBE correction to \( \nu_e \) and \( \nu_\mu \) cross sections is of the order of 2%. In the low \( Q^2 \) limit it increases, while for the large values of \( Q^2 \) it reduces the cross sections. Eventually the other QED-like corrections have been obtained. It turned out that the TBE effect cancels a non-negligible part of the soft photon Bremsstrahlung contribution.

\[ \]  \hspace{1cm} (16) \]

1 In the case of the electron the relation between \( \Delta E \) and \( \Delta \mathcal{E} \) reads \( \Delta \mathcal{E} = (1 + \mu/M_p)(1 - \cos \theta) \Delta E \). When the charged lepton is given by the muon the relation becomes a little complicated, because the presence of the muon mass.

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