Strong enhancement of superconductivity
in a nanosized Pb bridge

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(March 21, 2022)

Abstract

In recent experiments with a superconducting nanosized Pb bridge formed
between a scanning tunneling microscope tip and a substrate, superconduc-
tivity has been detected at magnetic fields, which are few times larger than
the third (surface) critical field. We describe the observed phenomenon on the
basis of a numerical solution of the Ginzburg-Landau equations in a model
structure consisting of six conoids. The spatial distribution of the supercon-
ducting phase is shown to be strongly inhomogeneous, with concentration of
the superconducting phase near the narrowest part (the “neck”) of the bridge.
We show that suppression of superconductivity in the bridge by applied mag-
netic field or by temperature first occurs near the bases and then in the neck
region, what leads to a continuous superconducting-to-normal resistive tran-
sition. A position of the transition midpoint depends on temperature and,
typically, is by one order of magnitude higher than the second critical field
$H_{c2}$. We find that the vortex states can be realized in the bridge at low
temperatures $T/T_c \leq 0.6$. The vortex states lead to a fine structure of the
superconducting-to-normal resistive transition. We also analyze vortex states
in the bridge, which are characterized by a varying vorticity as a function of
the bridge’s height.

PACS numbers: 74.80.-g; 74.20.De; 74.80.Fp; 74.60.Ec
I. INTRODUCTION

In a magnetic field parallel to the surface of a type II superconductor, the nucleation of superconductivity near the surface is known as “surface superconductivity”. It is characterized by a third critical field \( H_{c3} \) which is higher than the second critical field \( H_{c2} \) of the superconductor. E.g., for a plane superconductor-insulator interface, the value of the third critical field is given by \( H_{c3} = 1.695 H_{c2} \). For magnetic fields between \( H_{c2} \) and \( H_{c3} \), a superconducting sheath appears near the surface. Its thickness is of the order of the temperature-dependent coherence length \( \xi(T) \).

It is clear that the effect of surface superconductivity becomes increasingly important with decreasing dimensions of the samples, in particular when the volume of the near-surface layer becomes comparable to the total volume of the sample. In this case one might expect that the magnetic-field behavior of the mesoscopic superconductor is determined by the third critical magnetic field \( H_{c3} \).

However, not only the volume, but also the sample geometry plays a crucial role in mesoscopic superconductors. The presence of several plane surfaces or of a curved surface with a small radius of curvature (substantially smaller than \( \xi(T) \)), enhances the superconductivity. As a result, critical fields in mesoscopic structures even exceed \( H_{c3} \), as will be shown below theoretically and experimentally.

The nucleation of superconductivity at magnetic fields above \( H_{c3} \) was first studied in a wedge. Technological progress in the last two decades enabled the manufacturing of mesoscopic superconducting structures with sharp corners, and resulted in a renewed interest in the problem of superconductivity in a wedge. For instance, recent investigations on superconductivity in a wedge with a small angle \( \alpha \), using a variational approach or the adiabatic approximation, revealed that strongly localized distributions of the order parameter dominate at \( 0 < \alpha/\pi < 0.1581 \). Also, states of the superconducting phase with integer numbers \((1, 2, \ldots)\) of confined circulating superconducting currents have been found in wedges with a sufficiently small angle \( \alpha/\pi \ll 1 \). Within the framework of numerical solution of the Ginzburg-Landau (GL) equations in a wedge, the critical magnetic field for \( \alpha = \pi/2 \) was found to be approximately equal to \( 1.96 H_{c2} \).

Furthermore, strong enhancement of the superconductivity has been demonstrated experimentally in Al mesoscopic squares and square loops, by measuring the temperature dependence of the midpoint of the normal-to-superconducting resistive transition. The phase boundaries obtained in this experiment lie in a region of substantially higher magnetic fields than the phase boundary for bulk Al. The numerical solution of the Ginzburg-Landau (GL) equations revealed spatially inhomogeneous distributions of the order parameter in a mesoscopic square loop, and the resulting superconducting phase boundary in a square loop with leads turns out to be in agreement with the experimentally observed phase boundary. Also, the influence of imperfections on the phase boundary of a superconducting mesoscopic double loop has been analyzed. Recently, mesoscopic squares of high-\( T_c \) superconductors were also studied.

Some theoretical work is moreover devoted to the superconducting properties of mesoscopic disks and rings. Magnetic response of small superconducting disks has been investigated, and first or second order normal-to-superconducting transitions are found depending on the radius. A smooth transition from a multivortex superconducting state to
a giant vortex state with increasing both the disk thickness and the magnetic field has been found for thin disks. The flux penetration and expulsion in thin superconducting disks have been analyzed. In Ref. 18, the saddle points or energy barriers have been obtained, which are responsible for some metastabilities observed in mesoscopic superconducting disks. On the basis of the linearized GL equations, dimensional crossover in a mesoscopic superconducting loop of finite width has been studied. It has been shown that a dimensional transition occurs if the film thickness is of order \(\xi(T)\), similar to the 2D-3D transition for thin films in a parallel magnetic field. Vortex states in superconducting rings with out-of-center location of the opening have been recently discussed.

Superconductivity near the surface of a superconductor can still be enhanced if it is surrounded by a medium with a higher transition temperature than the one of the sample. The vortex structure of thin mesoscopic disks with this type of enhanced surface superconductivity has been considered in Ref. 21. The magnetic field versus temperature phase diagram was obtained, and the regions of existence of the multivortex state and of the giant vortex state were found. In Ref. 22 the enhancement of the superconductivity near the surface of a mesoscopic superconductor was studied, due to both the surrounding medium and the curved surface of the sample.

All the above-mentioned superconducting structures can be described by 2D (wedges, thin disks, squares and square loops) or quasi-3D (if the thickness of a sample is effectively taken into account), or even 1D (double loop in a network approach) GL equations. Another kind of mesoscopic superconducting systems, which radically differs from the above structures, is a cone (or, more generally, a conoid). Superconducting properties of a cone cannot be described by 2D GL equations with an “effective thickness” of the sample. Instead, the GL equations in a cone have to be solved in a general 3D form.

Our interest in the problem of superconductivity for a conic structure was induced by recent experiments with a nanosized Pb bridge, which is formed between a substrate and the tip of a scanning tunneling microscope. The shape of the bridge is close to a geometrical figure consisting of two cones linked by their apexes. The diameter of the bridge varies from a few nanometers to a few tens of nanometers. The conductivity measurements showed a strong enhancement of the superconductivity in the bridge. For magnetic fields as high as five times their bulk critical value, the superconductivity survives in the bridge while the leads become normal.

In the present paper, we investigate a superconducting multi-conoid structure using the GL theory. This study is relevant for the above-mentioned experiments with a nanosized superconducting bridge. The three-dimensional GL equations are solved self-consistently for the superconducting order parameter and the magnetic field in the bridge. On the basis of the obtained solutions, the superconducting-to-normal resistive transition is calculated as a function of the applied magnetic field.

The paper is organized as follows. The three-dimensional GL equations are discussed in Sec. II. In Sec. III, we analyze the distributions of the order parameter in the bridge as a function of the applied magnetic field and the temperature. The vortex state is also investigated. The thermodynamical stability of the solutions of the GL equations is studied in Sec. IV for different orbital quantum numbers \(L\). Sec. V is devoted to the description of the superconducting-to-normal resistive transition in the bridge. The possibility of the existence of superconducting states, characterized by different values of \(L\) along the bridge,
II. GL EQUATIONS IN THE BRIDGE

The GL equations for the order parameter $\psi$ and the vector potential $A$ of a magnetic field $H = \nabla \times A$ are

$$\frac{1}{2m} \left(-i\hbar \nabla - \frac{2e}{c}A\right)^2 \psi + a\psi + b|\psi|^2 \psi = 0,$$

$$\Delta A = \frac{4\pi i e \hbar}{mc} (\psi^* \nabla \psi - \psi \nabla \psi^*) + \frac{16\pi e^2}{mc^2} A |\psi|^2$$

with the boundary condition

$$n \cdot \left(-i\hbar \nabla - \frac{2e}{c}A\psi\right)_{\text{boundary}} = 0,$$

where $n$ is the unit vector normal to the boundary, $a$ and $b$ are the GL parameters.

It is convenient to perform a transformation of Eqs. (1), (2) to a dimensionless form. The new variables are defined as follows:

$$\psi' = \frac{\psi}{\sqrt{|a|/b}}, \quad A' = \frac{2\pi \xi}{\Phi_0} A, \quad x' = \frac{x}{\xi}, \quad y' = \frac{y}{\xi},$$

where $\Phi_0 = h\xi/2e$ is a flux quantum. The temperature-dependent coherence length $\xi$ is given by

$$\xi \equiv \xi(T) = \xi(0) \left(1 - \frac{T}{T_c}\right)^{-1/2},$$

where $\xi(0)$ is the coherence length at zero temperature. The temperature-dependent GL parameter $a$ has the form:

$$a = a_0 \left(1 - \frac{T}{T_c}\right).$$

From here on the primes will be omitted, and $\psi, A, x$ and $y$ will denote the scaled order parameter, vector potential and coordinates in the units described in (4). As a result of the transformation (4), Eqs. (1), (2) then become

$$(-i\nabla - A)^2 \psi - \psi \left(1 - |\psi|^2\right) = 0,$$

$$\kappa^2 \Delta A = \frac{i}{2} (\psi^* \nabla \psi - \psi \nabla \psi^*) + A |\psi|^2.$$

Here $\kappa$ is the GL parameter defined as a ratio of the penetration depth $\lambda(T)$ to the coherence length $\xi(T)$:

$$\kappa = \frac{\lambda(T)}{\xi(T)}, \quad \lambda(T) = \lambda(0) \left(1 - \frac{T}{T_c}\right)^{-1/2}.$$
The boundary condition (3) takes the form:

\[ \mathbf{n} \cdot (-i \nabla - \mathbf{A}) \psi |_{\text{boundary}} = 0. \]  

(10)

As a result of solving Eqs. (7), (8) with the boundary conditions (10), we shall obtain the spatial distributions of the dimensionless order parameter \( \psi \) and the dimensionless vector potential \( \mathbf{A} \). In order to return to dimensional variables and to reconstruct their temperature dependence, we should afterwards recalculate them according to the following rules.

1. Dimensional length \( x_{\text{dim}} \) (similarly, \( \rho_{\text{dim}}, \ z_{\text{dim}} \)):

\[ x_{\text{dim}} = x(0) \left( 1 - \frac{T}{T_c} \right)^{-1/2}. \]

(11)

2. Temperature-dependent superconducting order parameter:

\[ \psi(T) = \psi_0 \sqrt{|a_0| / b} \left( 1 - \frac{T}{T_c} \right)^{1/2}. \]

(12)

3. Temperature-dependent vector potential:

\[ \mathbf{A}(T) = \mathbf{A} \sqrt{2 \kappa H_c(0)} \xi(0) \left( 1 - \frac{T}{T_c} \right)^{1/2}. \]

(13)

where \( H_c(T) \) is the thermodynamic critical magnetic field at temperature \( T \), defined through the Helmholtz free energies \( F_n(T) \) and \( F_s(T) \) per unit volume in the normal and the superconducting states, respectively:

\[ \frac{H^2_c(T)}{8\pi} = F_n(T) - F_s(T). \]

(14)

For the axial-symmetric problem, it is useful to introduce cylindrical coordinates \((\rho, z, \phi)\). The vector potential and the order parameter can then be represented in the form

\[ \mathbf{A}(\rho, z) = e^{\phi} A(\rho, z) \]

(15)

\[ \psi(\rho, z, \phi) = f(\rho, z) e^{iL\phi}, \]

(16)

where \( L \) is the orbital quantum number (which is a measure of the vorticity of the superconducting state in the bridge).

The GL equations for the order parameter and for the vector potential become:

\[ \frac{1}{\rho} \frac{\partial f}{\partial \rho} + \frac{\partial^2 f}{\partial \rho^2} + \frac{\partial^2 f}{\partial z^2} = f \left\{ \frac{L^2}{\rho^2} - \frac{2AL}{\rho} + A^2 - (1 - f^2) \right\}, \]

(17)

\[ \kappa^2 \left\{ \frac{\partial}{\partial \rho} \left[ \frac{1}{\rho} \frac{\partial (\rho A)}{\partial \rho} \right] + \frac{\partial^2 A}{\partial z^2} \right\} = f \left\{ A - \frac{L}{\rho} \right\}. \]

(18)
It should be noted that Eqs. (17), (18) in the bridge have to be solved in their general three-dimensional form. Due to the complicated shape of the bridge, simplifications of Eqs. (17), (18) are not allowed, as, for example, in the case of a disk. Mesoscopic disks considered in Ref. 15 were of a finite width \(d \sim \xi\). Because of this condition, the order parameter was supposed to be uniform along the axis of the disk, \(\xi\), resulting in the disappearance of the third term in the left-hand side of Eq. (17). Another approximation, which led to a substantial simplification of Eq. (17) in the case of a mesoscopic disk, consisted in the substitution of a local value of the vector potential \(A\) obtained from Eq. (18) by its average \(\langle A \rangle\) over the thickness of the disk. Obviously, these approximations cannot be applied for the problem of the mesoscopic bridge under consideration, which consists of a few conoids, and we actually have to deal with a three-dimensional problem.

At the conoidal surface with an angle \(\alpha\), Eq. (10) results in the following boundary condition for the function \(f(\rho,z)\):

\[
\left(\frac{\partial f}{\partial \rho} \cos \alpha + \frac{\partial f}{\partial z} \sin \alpha \right) \bigg|_{\text{boundary}} = 0.
\] (19)

At the base, we have:

\[
\frac{\partial f}{\partial z} \bigg|_{\text{boundary}} = 0.
\] (20)

For the vector potential, the boundary condition is determined at infinity

\[
A|_{\infty} = e_\phi H_0 \rho/2,
\] (21)

where \(H_0\) is the applied magnetic field. Because of nonzero demagnetizing factor, the local magnetic field at the surface of the bridge differs from the applied magnetic field. However, our calculations show that already at a distance of about two maximal radii of the bridge, the effect of magnetic field distortion by the sample is negligible. Hence, for the purpose of numerical calculations, we choose a simulation region, which is large enough (see below) to assume that all changes of the magnetic field occur inside this region, while outside of it the magnetic field is uniform and equal to the applied magnetic field. Therefore, the boundary condition (21) is substituted by

\[
A|_{\text{bsr}} = e_\phi H_0 \rho/2,
\] (22)

where “bsr” denotes the boundary of a simulation region.

As a realistic model of the bridge used in Ref. 23, we choose a geometrical figure restricted by six conoidal surfaces and two bases (Fig. 1). We place the bridge into the three-dimensional Cartesian frame of reference with the \(z\)-axis coinciding with the symmetry axis of the bridge. The magnetic field is supposed to be applied along the \(z\)-axis. The point \(z = 0\) is chosen in the plane, where the bridge has its minimal diameter. This narrowest cross-section of the bridge will be referred to as the neck. The heights of the conoids and their diameters are indicated in Fig. 1. The conoidal surfaces forming the boundaries of the bridge are then described by the following equations:
\[ z = \frac{2h_1}{b_1 - b_0} \left( \sqrt{x^2 + y^2} - \frac{b_0}{2} \right), \text{ if } 0 \leq z \leq h_1, \tag{23} \]
\[ z = \frac{2h_2}{b_2 - b_1} \left( \sqrt{x^2 + y^2} - \frac{b_1}{2} \right) + h_1, \text{ if } h_1 < z \leq h_1 + h_2, \]
\[ z = \frac{2h_3}{b_3 - b_2} \left( \sqrt{x^2 + y^2} - \frac{b_2}{2} \right) + h_1 + h_2, \text{ if } h_1 + h_2 < z \leq h_1 + h_2 + h_3. \]

The boundary conditions (19) are applied at the conoidal surfaces (23) with \( \alpha = \alpha_1, \alpha_2, \alpha_3: \)
\[ \alpha_1 = \arctan \left( \frac{b_1 - b_0}{2h_1} \right), \tag{24} \]
\[ \alpha_2 = \arctan \left( \frac{b_2 - b_1}{2h_2} \right), \]
\[ \alpha_3 = \arctan \left( \frac{b_3 - b_2}{2h_3} \right). \]

At the surface
\[ z = h_1 + h_2 + h_3 \text{ with } x^2 + y^2 \leq \left( \frac{b_3}{2} \right)^2, \tag{25} \]
the boundary condition (20) is applied.

The part of the bridge situated in the area \( z < 0 \) is described by equations symmetric to Eqs. (23) to (25) with respect to the plane \( z = 0. \)

The self-consistent numerical solutions of Eqs. (17), (18) with the boundary conditions Eqs. (19) to (21) in the bridge are obtained using the finite difference method for solving partial differential equations. As a simulation region in the numerical calculations, we choose a cylinder with a radius which is four times the maximal radius of the bridge, and with a height which is three times the height of the bridge. This choice has been proven to be sufficient for obtaining solutions which are independent of the size of the simulation region. The relative accuracy of the obtained distributions of the order parameter is \( 10^{-4}. \)

The spatial resolution of the computational method is about \( 0.02\xi(0). \)

III. SOLUTIONS OF THE GINZBURG-LANDAU EQUATIONS IN THE BRIDGE: DISTRIBUTION OF THE SUPERCONDUCTING PHASE

A. Magnetic-field dependence of the distribution of the superconducting phase in the bridge

We first discuss some typical features of the distribution of the squared amplitude \(|\psi(x, y, z)|^2 = |\psi|^2\) of the order parameter in the bridge, which are represented in Fig. 2. All the distributions of \(|\psi|^2\) throughout this paper are shown as contour plots of the cross-section along the symmetry axis of the bridge, i.e. along the plane \( y = 0 \) (see Fig. 1). The solutions of the GL equations, which we discuss in this paper, are obtained for \( \kappa = 3.9. \)
Because $\kappa$ is the only parameter entering Eqs. (17) and (18), the obtained solutions are applicable for different values of the coherence length. In order to emphasize this general character of the obtained solutions, we express the sizes in the plots in units $\xi \equiv \xi(T)$ as defined by Eq. (3). The sizes expressed in nanometers, also indicated in the plots, correspond to the parameters of the Pb bridge, for which $\xi(0)$ is estimated to be 10 nm (see subsection III.C.). In Fig. 2(a), $|\psi|^2$ is plotted for the temperature $T / T_c = 0.6$, for the applied magnetic field $H_0 = H_2$, where $H_2 = \sqrt{2} \kappa H_c(T)$, and for the orbital quantum number $L = 0$. The superconducting phase is concentrated near the neck of the bridge. The function $|\psi|^2$ has the maximal possible value equal to 1 in the denotations (3) within a region, which symmetrically spreads along the $z$-axis to the distances of about $\xi$ from the plane $z = 0$. In this region, the superconductivity is strongly enhanced due to the small lateral dimensions of the neck. There are no significant changes of $|\psi|^2$ as a function of magnetic field in this area (Fig. 2). Away from the neck, at distances of about $\xi$ to $2\xi$ from the plane $z = 0$, $|\psi|^2$ gradually decreases down to the value $|\psi|^2 \sim 0.5$. Qualitatively, such a behavior of $|\psi|^2$ is typical for various temperatures and applied magnetic fields.

An increasing applied magnetic field suppresses the superconductivity in the bridge starting from its bases. For the applied magnetic field $H_0 = 1.6 H_2$ [Fig. 2(b)] and the same temperature and $L$ as in the previous panel, ring-shaped areas with $|\psi|^2 \sim 0.4$ appear near the bases of the bridge (represented in the contour plot as small areas in the corners). For a higher applied magnetic field $H_0 = 2 H_2$, these areas still grow. It is worth noting that the area near the neck of the bridge does not noticeably change with applied magnetic field.

For $H_0 = 4 H_2$ (all other parameters are the same in Fig. 2), the area near the neck, which is filled by the superconducting phase with the maximal value of $|\psi|^2 = 1$, is reduced approximately by a factor of 0.5 as compared to the case $H_0 = H_2$ [cf. Fig. 2(a)]. The function $|\psi|^2$ changes from its maximal value near $z = 0$ down to $|\psi|^2 \sim 0.3$ near the bases.

A further increase of the applied magnetic field leads to a stronger suppression of the superconducting state in the bridge. For the applied magnetic fields $H_0 = 5 H_2$ and $H_0 = 8 H_2$ [Figs. 2(e) and 2(f)], the function $|\psi|^2$ decreases very fast towards the bases of the bridge. At $H_0 = 12 H_2$ the bases of the bridge are connected to each other by an area with $|\psi|^2$ ranging from 0.01 to 0.09, and some regions in the bridge are filled by the normal state.

Finally, at $H_0 = 16 H_2$ the superconducting phase is concentrated near the neck only. It is separated from the bases of the bridge by regions of the normal state [Fig. 2(h)]. We believe that such a pattern can be experimentally detected by measuring the density of states in the tunneling regime as a function of the applied magnetic field.

**B. The distribution of the superconducting phase in the bridge as a function of temperature**

The distribution of the superconducting phase in the bridge is strongly temperature-dependent. In this subsection, the temperature dependence of the squared amplitude $|\psi|^2$ of the order parameter is studied.

We start with the distribution of $|\psi|^2$ for $T / T_c = 0.6$ [Fig. 3(a)]. This distribution is characterized by decreasing values of $|\psi|^2$ with increasing $|z|$ near the area of the neck, and with increasing $|x|$ near the bases of the bridge. At a higher temperature $T / T_c = 0.9$, Fig. 3(b)], the distribution of $|\psi|^2$ is substantially modified as compared to the previous
case. It becomes uniform along the $z$-axis near the bases. The typical butterfly-like pattern remains near the area of the neck only. This central area is filled by the superconducting phase with the maximal value of $|\psi|^2 = 1$. With further increasing temperature up to $T/T_c = 0.99$ [Fig. 3(c)], the area near the neck of the bridge becomes separated from the bases by wide regions, which are characterized by a small value of $|\psi|^2 < 0.1$. For higher temperatures, the function $|\psi|^2$ remains nonzero only near the area of the neck and vanishes very fast versus $z$.

With increasing temperature, the superconducting phase in the nanosized superconducting bridge is thus suppressed by two mechanisms. Along with a uniform reduction of the maximal value of $|\psi|^2$, described by a simple dependence $1 - T/T_c$ following from Eq. (12), also a spatial redistribution of $|\psi|^2$ takes place in the bridge when the temperature increases. This redistribution is characterized by a concentration of the superconducting phase near the neck of the bridge accompanied by a relative suppression of $|\psi|^2$ away from this central area.

C. The vortex state in the bridge

Most of the bulk superconducting metals are typically type I superconductors, characterized by a Ginzburg-Landau parameter $\kappa \sim 0.03$ in Al to $\kappa \sim 0.48$ in Pb. However, in mesoscopic systems, metals can become type II superconductors (like most superconducting metal alloys and compounds). This is due to a substantial reduction of the effective coherence length in mesoscopic systems, where the size of the sample plays the role of the electron mean free path in bulk systems. As a result, the parameter $\kappa$ becomes larger than $1/\sqrt{2}$ in mesoscopic metallic structures. For example, in mesoscopic square loops of Al studied in the experiment, the parameter $\kappa$ was found to be $\sim 1$. This experimentally obtained value of $\kappa$ was used in the calculations of the phase boundaries of a mesoscopic square loop.

For the Pb bridge used in Ref. 23, the effective coherence length is determined by the bridge diameter, which varies from a few nanometers in the vicinity of the neck to about 23 nm near the bases. In our calculations, we estimated the value of the effective coherence length to be 10 nm. According to Ref. 29, the penetration depth at zero temperature for Pb is $\lambda(0) = 39$ nm, and consequently $\kappa = 3.9$. Hence, the material of the bridge is effectively a type II superconductor. Therefore, a vortex state can be realized in the bridge.

We have calculated the distributions of the superconducting phase in the bridge for different orbital quantum numbers $L$. In Fig. 4, such distributions are shown for the temperature $T/T_c = 0.6$ and for the applied magnetic field $H_0 = 5H_c$. The distributions of the superconducting phase are presented for $L = 0$ to $L = 3$ in Fig. 4. The function $|\psi|^2$ is strongly modified by the presence of the vortex in the bridge, as compared to the case without a vortex ($L = 0$). For $L > 0$, the distributions have the shape of a sheath near the conic surface. The thickness of the sheath varies in different parts of the bridge (see Fig. 4). In addition, the function $|\psi|^2$ is spatially inhomogeneous in the sheath. The maximal value of $|\psi|^2$ is reached in the vicinity of the bridge surface in the area near the neck. The function $|\psi|^2$ decreases towards the axis of the bridge $z = 0$. These changes are shown in Fig. 4. In order to determine, which of these states is realized in the bridge, we now turn to the problem of the thermodynamical stability of solutions of the GL equations for different values of $L$. 


IV. THE THERMODYNAMICAL STABILITY OF THE SOLUTIONS OF THE
GINZBURG-LANDAU EQUATIONS IN THE BRIDGE

The free energy per unit volume of a superconductor in the magnetic field is

\[ F_s = F_n + a |\psi|^2 + \frac{b}{2} |\psi|^4 + \frac{H^2}{8\pi} + \frac{1}{2m} \left| -i\hbar \nabla \psi - \frac{2e}{c} A \psi \right|^2 - \frac{H_0^2}{8\pi}, \]  

(26)

where \( F_n \) is the free energy per unit volume of the normal (non-superconducting) phase. The superconducting state is stable when the difference \( F_s - F_n \) has a minimum.

It can be shown that the difference \( F_s - F_n \) can be expressed as follows:

\[ F_s - F_n = \frac{H_c^2(0)}{4\pi} \left( (H - H_0)^2 - \frac{1}{2} |\psi|^4 \right). \]  

(27)

In the dimensionless form, the free energy Eq. (27) calculated over the bridge of volume \( V_b \) then takes the form:

\[ \int_{V_b} (F_s - F_n) dxdydz = \int_{V_b} \left[ (H - H_0)^2 - \frac{1}{2} |\psi|^4 \right] dxdydz. \]  

(28)

Before calculating the free energy of the superconducting bridge, let us estimate contributions of two terms in the right-hand side of Eq. (28) to the integral in the left-hand side of Eq. (28). The first term describes the contribution to the free energy due to changes in the magnetic field in the bridge as compared to the applied magnetic field. These changes occur over distances of the order of the penetration depth \( \lambda \) of the magnetic field. The temperature dependent values of \( \lambda(T) \) calculated using Eq. (9) are listed in Table 1 for various temperatures [recall that \( \lambda(0) = 39 \text{ nm} \)].

Table 1. The magnetic field penetration depth in Pb for various temperatures.

| \( T/T_c \) | 0.6 | 0.9 | 0.99 | 0.998 |
|-----------|-----|-----|------|------|
| \( \lambda(T), \text{ nm} \) | 61.7 | 123.4 | 390.0 | 782.5 |

According to Table 1, the values of \( \lambda(T) \) are larger than the mean diameter of the bridge (10 nm). Therefore, in the case \( L = 0 \) we can approximately disregard the changes of the magnetic field in the bridge and omit the first term \( (H - H_0)^2 \) in the right-hand side of Eq. (28). For \( L > 0 \) this also seems an acceptable approximation because \( \kappa \) is large and the lateral dimensions of the bridge are small. Indeed, for large values of \( \kappa \) changes in the magnetic field occur within a relatively large region with sizes of order \( 2\lambda(T) \), in contrast to the sharp changes of the order parameter, which occur within a relatively small region with sizes of the order of \( 2\xi(T) \). The magnetic field changes in the sample because the system captures \( L \) flux quanta \( L\Phi_0 \). The amplitude of the changes in the magnetic field is small if they occur within a large region. Recall that the bridge diameter is much smaller than the size of the region of magnetic field changes [that is \( \sim 2\lambda(T) \)]. In summary, the contribution \( \int_{V_b} (\Delta H)^2 dxdydz \) in Eq. (28) decreases also for \( L > 0 \) with increasing \( \kappa \).

We have calculated the free energy of the superconducting states in the bridge as a function of the applied magnetic field in a wide range of applied magnetic fields from 0 to
The calculations are performed for three values of the temperature: \( T/T_c = 0.4 \), \( T/T_c = 0.6 \) and \( T/T_c = 0.9 \), and for various values of the orbital quantum number \( L \).

In Fig. 5, the free energy of the bridge [measured in units of \( H_c^2(0)/4\pi \)] is plotted as a function of the applied magnetic field for the temperature \( T/T_c = 0.4 \). The states characterized by \( L = 0 \) to \( L = 4 \) are shown. For the range of magnetic fields 0 to \( 5H_{c2} \), the state \( L = 0 \) minimizes the free energy. In this magnetic field range the values of \( |F_s - F_n| \) are about three times larger for the state \( L = 0 \) than for the state \( L = 1 \). For applied magnetic fields higher than \( 5H_{c2} \), the states with \( L = 1 \) to \( L = 4 \) can be realized. But the values of \( |F_s - F_n| \) for these \( L \) are far less sensitive to the value of \( L \) than they are in the interval of magnetic fields from 0 to \( 5H_{c2} \). In Fig. 6, the free energy of the bridge is shown as a function of the applied magnetic field at the temperature \( T/T_c = 0.6 \). The range of the applied magnetic fields, for which the state \( L = 0 \) is realized, becomes more than two times wider as compared to \( T/T_c = 0.4 \) (cf. Fig. 5): the upper value of this range increases until about \( 10.5H_{c2} \). In Fig. 7, the free energy of the bridge is plotted as a function of the applied magnetic field for \( T/T_c = 0.9 \). In this case, the state \( L = 0 \) is realized for all the applied magnetic fields in the range 0 to \( 30H_{c2} \). Therefore, penetration of a vortex into the bridge is only possible for sufficiently small temperatures.

V. THE SUPERCONDUCTING-TO-NORMAL RESISTIVE TRANSITION FOR THE BRIDGE

In Ref. 23, a narrow cylinder was used as an oversimplified model of the bridge. We have taken into account a more complicated shape of the bridge (compare Fig. 1 in Ref. 23 to Fig. 1 in the present work). Our self-consistent numerical solutions of the three-dimensional GL equations have shown that the distribution of the superconducting order parameter is strongly inhomogeneous in the bridge. In the superconducting state, the bases of the bridge are connected to each other by a continuous superconducting sheath. The width of the sheath is different in various parts of the bridge. Also, the value of \( |\psi|^2 \) is position-dependent. With increasing magnetic field (see Fig. 2) or increasing temperature (see Fig. 3), the superconducting properties decrease continuously. Analyzing the distributions of the superconducting phase, we can expect that the superconducting-to-normal transition as a function of applied magnetic field or temperature has the shape of a smoothed step function. For example, consider the distributions of the superconducting phase shown in Fig. 2. The resistance is expected to be minimal for the applied magnetic field \( H_0 = H_{c2} \) [see Fig. 2(a)]. With increasing applied magnetic field, the value of \( |\psi|^2 \) reduces near the bases, and therefore the resistance of the bridge increases. For the applied magnetic field \( H_0 = 16H_{c2} \) [Fig. 2(h)], the resistance of the bridge is close to that of the normal metal. After these intuitive considerations, we now turn to the calculations of the resistance of the bridge as a function of the applied magnetic field.

To calculate the resistance of the bridge, we divide the bridge into thin disks, each with thickness \( \delta \) substantially smaller than the coherence length \( \xi(T) \). Therefore, the function \( |\psi|^2 \) only changes in the \( xy \)-plane of the disk. Each disk is subdivided into thin rings with width \( \delta \). As the smallest partition element (“cell”) of the bridge, we choose a ring sector with length \( \delta \). Because of the condition \( \delta \ll \xi(T) \) (recall that \( \xi(T) \) is chosen as a unit of length), the volume of the cell is independent of the radius of the ring and is approximately equal
to $\delta^3$. The resistance of each cell $R_{\text{cell}}$ is equal to zero, if the cell is in a superconducting state, and it is equal to the normal resistance $R_{\text{cell,n}}$, if the cell is in a normal state. For the numerical calculations, the resistance of the cell (measured in $R_{\text{cell,n}}$) is represented in the form

$$ R_{\text{cell}} = \begin{cases} 0, & \text{if } |\psi(x,y,z)|^2 > \epsilon \\ 1, & \text{if } |\psi(x,y,z)|^2 \leq \epsilon \end{cases} , $$

where $\epsilon \ll 1$. Equation (29) can be approximated by the analytical expression

$$ R_{\text{cell}} \approx \frac{\epsilon}{|\psi(x,y,z)|^2 + \epsilon} , $$

which asymptotically coincides with Eq. (29) when $\epsilon \to 0$. In the calculations executed using Eq. (30), we choose $\epsilon = 0.001$ [reduction of the value $\epsilon$ down to 0.0001 does not lead to any appreciable changes of the curve $R = R(H_0)$]. Because of cylindrical symmetry, the resistance of the ring with radius $r$ is $\delta R_{\text{cell}} / 2\pi r$. The total resistance of the bridge is obtained based on the formulas for parallel and series resistances.

In Fig. 8(a), the resistive superconducting-to-normal transition is shown as a function of the applied magnetic field for the temperature $T/T_c = 0.4$. For the fields ranging from 0 to approximately $5H_{c2}$, the bridge is superconducting and has zero resistance. The resistive superconducting-to-normal transition is characterized by a range of fast changes in the interval $5H_{c2}$ to $15H_{c2}$, followed by a range of slow changes for higher fields. The above-mentioned fast changes are explained by a fast reduction of $|\psi|^2$ in the regions near the bases of the bridge with increasing applied magnetic field. In contrast, the middle part of the distribution $|\psi|^2$ varies slowly versus applied magnetic field, leading to the above-stated slow changes of the resistive superconducting-to-normal transition.

In order to compare the resistive transitions calculated for different temperatures, we introduce an additional scale (indicated on top of each panel) for the applied magnetic field $H_0$ measured in $H_{c2}$ at zero temperature. Figs. 8(b) and 8(c) show that for higher temperatures, $T/T_c = 0.6$ and $T/T_c = 0.9$, the resistive superconducting-to-normal transition is shifted to lower applied magnetic fields as compared to the case $T/T_c = 0.4$.

Taking into account the states with orbital quantum numbers $L > 0$, steps appear in the resistance as a function of the magnetic field, as shown in Fig. 8b. It is worth noting that the resistive superconducting-to-normal transition for the states with $L > 0$ is close to that for $L = 0$. The contribution of the states with $L > 0$ to the resistive superconducting-to-normal transition in the mesoscopic bridge is, however, relatively small as compared to the state with $L = 0$.

In summary, the resistive superconducting-to-normal transition as a function of applied magnetic field is characterized by a fast change in some range of relatively low applied magnetic fields and by a slow change for higher fields. This behavior is explained by an inhomogeneous distribution of the superconducting phase in the bridge. The fine structure appears in the resistive superconducting-to-normal transition as a function of applied magnetic field, when the states with $L > 0$ are taken into account.
VI. THE SUPERCONDUCTING STATES CHARACTERIZED BY DIFFERENT $L$ ALONG THE BRIDGE

As discussed above, the diameter of the bridge varies along the $z$-direction from a few nanometers in the neck ($z = 0$) until about 23 nanometers near the bases. This suggest that, given the applied magnetic field and the temperature, superconducting states characterized by various $L$ are realized in different parts of the bridge. In this section, we analyze the possibility of existence of such vortex states. For this purpose, we calculate the free energies of the states with various vorticities for disks of different radii. The radii of the disks are chosen to coincide with those of cross-sections of the bridge in the $xy$-planes at different values of $z$. The results of the free energy calculations are shown in Fig. 9. For sufficiently large disks [Fig. 9(a), $D_{\text{disk}} = 1.5\xi$ that corresponds to 24 nm in the case of the Pb bridge (Ref. 23) at $T/T_c = 0.6$], the states characterized by different $L$ are possible for various applied magnetic fields. The transition $L = 0 \rightarrow L = 1$ takes place at about $H_0 = 3H_{c2}$ for the temperature $T/T_c = 0.6$ [Fig. 9(a)]. For disks with smaller radii, this transition occurs at higher values of the applied magnetic field [Figs. 9(b) to 9(d)]. For disks with very small radii, only the state $L = 0$ is realized in the considered range of applied magnetic fields [Fig. 9(e)].

Using the obtained plots (Fig. 9), we can find the radii of the disks at which a transition $L \rightarrow L + 1$ occurs for different values of the applied magnetic field. For $H_0 = 5H_{c2}$ the transition $L = 0 \rightarrow L = 1$ turns out to take place in a disk with diameter $D_{\text{disk}} = 1.125\xi$ [18 nm for the Pb bridge (Ref. 23) at $T/T_c = 0.6$], shown in Fig. 9(c). For the area between the neck of the bridge and the disk, which is shown in Fig. 9(c), the state $L = 0$ is realized. For the areas between the bases of the bridge and the disk [Fig. 9(c)], the state $L = 1$ is energetically favored. Thus, there are three competing states: the state with $L = 0$ all over the bridge [Fig. 10(a)], that with $L = 1$ all over the bridge [Fig. 10(b)], and also the vortex state characterized by a varying vorticity $L = \{0, 1\}$ in the direction of the bridge’s axis [Fig. 10(c)]. We have calculated the free energies for these states. The calculations show that the state $L = \{0, 1\}$ with varying vorticity has the smallest free energy. The value of the free energy for this state is about 50 per cent lower than for the state $L = 1$. However, the free energy for the state $L = \{0, 1\}$ is only 2 per cent lower than that for the state $L = 0$.

Hence, in the bridge under consideration the state $L = 0$ is a reasonable approximation for a wide range of applied magnetic fields. Comparison in Fig. 8c between the resistance calculated for thermodynamically equilibrium states involving different numbers $L$, on the one hand, and the resistance calculated only for the state with $L = 0$, on the other hand, confirms this conclusion. But to describe the fine structure of the superconducting-to-normal resistive transition [Fig. 8(b)], the states with $L > 0$ have to be taken into consideration.

VII. CONCLUSIONS

We have solved the three-dimensional Ginzburg-Landau (GL) equations in a mesoscopic superconducting multi-conoidal structure with cylindrical symmetry. The calculations have been performed for a geometrical body, which is restricted by six conoidal surfaces and two base plane surfaces. This body is a model of a Pb nanosized superconducting bridge
between a scanning tunneling microscope tip and a substrate. Our calculations show that the distribution of the superconducting phase in the bridge is strongly inhomogeneous.

Superconductivity in the bridge is suppressed by increasing the applied magnetic field. This suppression occurs first near the bases of the bridge. In a region near the neck of the bridge, superconductivity survives to much higher magnetic fields than the surface critical field \( H_{c3} \). At high magnetic fields this neck region is separated from the bases by regions of the normal phase.

We have also studied the suppression of superconductivity in the bridge by increasing temperature. The maximal value of \(|\psi|^2\) decreases along with a redistribution of \(|\psi|^2\) in the bridge. This redistribution is characterized by a concentration of the superconducting phase near the neck of the bridge with a relative reduction of \(|\psi|^2\) away from this central region.

We have analyzed the superconducting states in the bridge from the point of view of the thermodynamical stability. The free energy of the superconducting states has been calculated as a function of the applied magnetic field. For a wide range of applied magnetic fields, the state \( L = 0 \) is realized in the bridge. This range widens with increasing temperature.

The superconducting-to-normal resistive transition is studied in detail as a function of the applied magnetic field. We have shown that a switching from the superconducting to the normal regime occurs in a wide interval of applied magnetic fields of the order of \( 10H_{c2} \) and depends on temperature. For example, for \( T/T_c = 0.6 \), the resistive transition starts at about \( 5H_{c2} \) and reaches the midpoint at about \( 15H_{c2} \). This agrees with the experimental data according to which superconductivity in the bridge has been detected at \( 5H_{c2} \). It is shown, that the main contribution to the superconducting-to-normal resistive transition is determined by the state with orbital angular momentum \( L = 0 \). When states with higher \( L \) are taken into account, a fine structure appears in the transition, which reflects the effect of the fluxoid quantization in the bridge.

In the bridge, the vortex states can exist, which are characterized by various \( L \) along the height of the bridge. Namely, it is shown that, at applied magnetic fields higher than \( 5H_{c2} \), the vortex state with a varying vorticity \( L = \{0, 1\} \) can be realized in the bridge.

**ACKNOWLEDGMENTS**

We thank S. Vieira and H. Suderow for their remarks and for critical reading of the manuscript in its preparation phase. We acknowledge discussions with V.V. Moshchalkov, L. Van Look and V. Bruyndoncx. This work has been supported by the IUAP, the F.W.O.-V. projects Nos. G.0287.95, 9.0193.97, G.0306.00, G.0274.01N, W.O.G. WO.025.99N, GOA BOF UA 2000 (Belgium), and the ESF Programme VORTEX.
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It is worth noting, that some uncertainty in this parameter does not lead to any substantial modifications of our results. Indeed, \( \xi \) enters \( \kappa = \lambda / \xi \), which is the only parameter of the GL theory. Our analysis shows, that relative changes of \( |\psi|^2 \) in any point of the bridge are at most 10 percent when increasing \( \kappa \) by a factor of 2.

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Figure captions

Fig. 1. Scheme of the model of the mesoscopic superconducting bridge. The bridge consists of six conoids symmetric with respect to the plane $z = 0$. Sizes of the model bridge used in our calculations: $b_0 = 2.6$ nm, $b_1 = 8.9$ nm, $b_2 = 15.8$ nm, $b_3 = 23.0$ nm, $h_1 = 3.6$ nm, $h_2 = 11.7$ nm, $h_3 = 15.8$ nm. In the insert, we show a schematic plot of the bridge used in the experiment.\[23\]

Fig. 2. Distribution of the superconducting phase, $|\psi(x, y, z)|^2$, as a function of applied magnetic field, in the nanosized superconducting bridge (a contour plot is shown for the cross-section $y = 0$) for $L = 0$, $T/T_c = 0.6$. Sizes are shown in nanometers for the Pb bridge described in Ref. 23.

Fig. 3. Distribution of the superconducting phase, $|\psi(x, y, z)|^2$, as a function of temperature, in the nanosized superconducting bridge (a contour plot is shown for the cross-section $y = 0$) for $L = 0$, $H_0 = 5H_{c2}$.

Fig. 4. Distribution of the superconducting phase, $|\psi(x, y, z)|^2$, as a function of the orbital quantum number $L$, in the nanosized superconducting bridge (a contour plot is shown for the cross-section $y = 0$) for $T/T_c = 0.6$, $H_0 = 5H_{c2}$.

Fig. 5. Free energy $F_s - F_n$ [measured in $H_c^2(0)/4\pi$] of the nanosized superconducting bridge, as a function of the applied magnetic field, for $T/T_c = 0.4$ and various $L$.

Fig. 6. Free energy $F_s - F_n$ [measured in $H_c^2(0)/4\pi$] of the nanosized superconducting bridge, as a function of the applied magnetic field, for $T/T_c = 0.6$ and various $L$.

Fig. 7. Free energy $F_s - F_n$ [measured in $H_c^2(0)/4\pi$] of the nanosized superconducting bridge, as a function of the applied magnetic field, for $T/T_c = 0.9$ and various $L$.

Fig. 8. Resistance of the superconducting bridge as a function of the applied magnetic field for $L = 0$ and various temperatures: $T/T_c = 0.4$ (a), $T/T_c = 0.6$ (b), and $T/T_c = 0.9$ (c). For the case $T/T_c = 0.6$, the states with $L \neq 0$ are also plotted.

Fig. 9. Free energy $F_s - F_n$ [measured in $H_c^2(0)/4\pi$] as a function of the applied magnetic field for the mesoscopic disks with diameters coinciding with the diameters of the bridge in various cross-sections normal to the $z$-axis. The free energy is calculated for for $T/T_c = 0.6$ and various $L$. The diameters of the disks are: $D = 1.5\xi$ [24 nm for the Pb bridge] (a), $D = 1.25\xi$ (20 nm) (b), $D = 1.125\xi$ (18 nm) (c), $D = 0.75\xi$ (12 nm) (d), $D = 0.5\xi$ (8 nm) (e).

Fig. 10. Distribution of the superconducting phase, $|\psi(x, y, z)|^2$, as a function of the orbital quantum number $L$, in the nanosized superconducting bridge (a contour plot is shown for the cross-section $y = 0$) for $T/T_c = 0.6$, $H_0 = 5H_{c2}$: $L = 0$ (a), $L = 1$ (b), and the state characterized by a variable vorticity $L = \{0, 1\}$ (c).
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