SUSY QCD corrections to $W\gamma$ production in polarized hadronic collisions

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ABSTRACT

We present the calculation of the supersymmetric QCD correction to $W\gamma$ production process in polarized proton-antiproton collisions at the TeV energy region. We find that the correction can reach 1.2% at parton level with favorable mass values of squarks and gluino, which is comparable with the virtual correction part of the conventional QCD. The production rates for different polarized $p\bar{p}$ collisions are also compared.

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1. Introduction.

One of the important tests of the standard model (SM) for electroweak interaction is to study the self-couplings of gauge bosons. Since the measurement of hadronic $W^\pm \gamma$ production has the advantages of very clear background, large production rate and both of the final produced gauge bosons can be easily detected experimentally, this process is attractive in experiment in testing the gauge boson trilinear couplings and it is worthwhile to evaluate the $W \gamma$ production in hadronic collisions more precisely.

$W \gamma$ production process was first investigated in Refs. [1][2]. From that time, this process has been more carefully studied in two ways. On one hand, many theoretical papers focus on the effective Lagrangian of $WW \gamma$ coupling [3], the related magnetic dipole moment and electric quadrapole moment of $W$ bosons. On the other hand, some theorists began to concentrate their attention on the radiative corrections to this process. The calculation of $O(\alpha_s)$ order QCD corrections to $W \gamma$ production in hadronic collisions in the context of the standard model (SM) was first presented in Ref. [4], then was developed by J. Ohnemus [5]. The numerical results of Ref. [5] show that the $O(\alpha_s)$ order QCD corrections to hadronic $W \gamma$ production are significant due to the consequence of the radiation amplitude zero (RAZ) in the Born subprocess, it is imperative that the QCD radiative corrections should be taken into account.

Now two questions arise naturally: Firstly, how about the radiative corrections within other extended standard models, such as supersymmetric model (SUSY),
when the center-of-mass collision energy approaches a few TeV? Secondly, since W boson and photon production has very low background and both particles are easy to be identified in experiment, it enables us to conduct more precise experiment on this process. If we can get more informations about WWγ coupling from other experiments, such as from LEP collider, can we use this process as an indirect probe of the new physics beyond the SM in the QCD sector?

In this paper we calculated the $O(\alpha_s)$ SUSY QCD correction to the process $p\bar{p} \rightarrow W^+\gamma + X$ in the TeV energy range, where it is generally believed that the new physics would enter [6].

In the last few years there has been a resurgence of interests in the spin structure of nucleon [7]. The polarized deep inelastic scattering experiments (DIS) at CERN and SLAC have provided some useful results. As to the $W\gamma$ production process at the parton level, it is due to the V-A weak interaction between W boson and quarks, there exits only one non-zero polarized amplitude at high energy, i.e. $u_L + \bar{d}_R \rightarrow W^+ + \gamma$. Consequently, the production rates in different polarized $p\bar{p}$ collisions would be different, which may improve our measurement in finding the SUSY QCD signals, therefore the calculation of the $W\gamma$ production process in polarized proton-antiproton collisions would provides an indirect probe of the SUSY QCD.

This paper is organized as follows: In section 2, we present the Lagrangians [6] concerning the interactions involved in this process and the renormalized scheme adopted in this work [5]. In section 3, we give the calculation of cross section of $W\gamma$ production at the parton level. The calculation of the total one-loop SUSY
QCD corrections in the polarized hadronic process is given in section 4. Finally the
discussion and conclusion are given in section 5.

2. Theory concerned in calculation.

The Lagrangians of the quark-squark-gluino and squark-vector-boson-squark
interactions within supersymmetric model are written explicitly as [6][8]:

\[
L_{\tilde{q}\tilde{q}g} = -\sqrt{2}g_s T^a_{jk} \sum_{i=u,d} \left( \bar{\tilde{g}}_{\alpha} P_L \tilde{q}^a_i \tilde{q}^b_i \tilde{q}^c_i + \bar{\tilde{q}}^a_i P_R \tilde{g}_{\alpha} \tilde{q}^b_i \tilde{q}^c_i - \bar{\tilde{g}}_{\alpha} P_R \tilde{q}^a_i \tilde{q}^b_i \tilde{q}^c_i - \bar{\tilde{q}}^a_i P_L \tilde{g}_{\alpha} \tilde{q}^b_i \tilde{q}^c_i \right)
\]

\[
L_{\tilde{q}\tilde{q}V} = -\frac{ig}{\sqrt{2}} \left[ W^+_\mu (\bar{\tilde{u}}^a_{L} \gamma^\mu \tilde{d}_L) + W^-_\mu (\bar{\tilde{d}}^a_{L} \gamma^\mu \bar{\tilde{u}}_L) \right] - ieA_{\mu} \sum_i \bar{\tilde{e}}_1^{a+i} \tilde{q}^a_i \tilde{q}^c_i
\]

(2.1)

where \(g_s\) is the strong coupling constant, \(T^a\) is SU(3) color generators, \(q_L\) and
\(q_R\) are the wave functions of SU(2) weak eigenstates. The relationship between weak
eigenstates \(q_L\) and \(q_R\) and mass eigenstates \(q_1\) and \(q_2\) can be expressed as:

\[
\begin{pmatrix}
\tilde{q}_{L_i} \\
\tilde{q}_{R_i}
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_i e^{-i\phi} & \sin \theta_i e^{-i\phi} \\
-\sin \theta_i e^{i\phi} & \cos \theta_i e^{i\phi}
\end{pmatrix}
\begin{pmatrix}
\tilde{q}_{1,i} \\
\tilde{q}_{2,i}
\end{pmatrix}
\]

where \(\theta\) is the squark mixing angle and \(\phi\) is the CP-violating phase angle origi-
nated from the scalar quark mass matrix [10]. In terms of mass eigenstates, the
corresponding Lagrangians of quark-squark-gluino and squark-squark-vector-boson
interactions are represented as:
\[ L_{\bar{q}q} = -\sqrt{2} g_s T^a \bar{q}[ (A_R \, \bar{q}_1 + B_R \, \bar{q}_2) P_R + (A_L \, \bar{q}_1 - B_L \, \bar{q}_2) P_L ] \bar{q} + h.c. \]
\[ L_{\bar{q}q'} = -\sqrt{2} [A_{R,u} B_{L,d} W^-_\mu (d_{1,\mu}^+ \tilde{u}_1) + A_{R,u} A_{L,d} W^-_\mu (d_{2,\mu}^+ \tilde{u}_2) \]
\[ + B_{R,u} B_{L,d} W^-_\mu (\tilde{d}_{1,\mu}^+ \tilde{u}_2) + B_{R,u} A_{L,d} W^-_\mu (\tilde{d}_{2,\mu}^+ \tilde{u}_2) \]
\[ + A_{R,d} B_{L,u} W^+_\mu (\tilde{u}_{1,\mu}^+ \tilde{d}_1) + A_{R,d} A_{L,u} W^+_\mu (\tilde{u}_{2,\mu}^+ \tilde{d}_1) \]
\[ + B_{R,d} B_{L,u} W^+_\mu (\tilde{u}_{1,\mu}^+ \tilde{d}_2) + B_{R,d} A_{L,u} W^+_\mu (\tilde{u}_{2,\mu}^+ \tilde{d}_2) - i e A_{R,q} B_{L,q} \gamma_\mu e_i \tilde{q}_{1,\mu}^+ \tilde{q}_1 + B_{R,q} A_{L,q} A_{µ} e_i \tilde{q}_{2,\mu}^+ \tilde{q}_2 \]

Where

\[ A_{L,i} = \cos \theta_i e^{-i\phi} \quad B_{L,i} = \sin \theta_i e^{-i\phi} \]
\[ A_{R,i} = \sin \theta_i e^{i\phi} \quad B_{R,i} = \cos \theta_i e^{i\phi} \]

In order to eliminate the ultraviolet divergences that appear in the one-loop integrals, we adopt the on-mass-shell (OMS) and dimensional regularization schemes \[1]. The renormalized irreducible two point functions for fermions are defined as \[10\]:

\[ \hat{\Gamma}(p) = i(\not{p} - m_t) + i \left[ \not{p} P_L \hat{\Sigma}^L(p^2) + \not{p} P_R \hat{\Sigma}^R(p^2) + P_L \hat{\Sigma}^S(p^2) + P_R \hat{\Sigma}^S(p^2) \right]. \]

From the renormalization conditions for the on-shell physical field \[9\]:

\[ \hat{R} \hat{q}_\mu \hat{\Gamma}_q(p)|_{p^2 = m^2} = 0 \]
\[ \lim_{p^2 \rightarrow m^2} \hat{R} \hat{q}_\mu \hat{\Gamma}_q(p) \frac{\not{p} + m_q}{p^2 - m^2} = i \hat{\bar{q}}_\mu(p), \]

we can deduce all the related renormalization constants involved in this process \[10\] as :

\[ \delta m_q = \frac{m_q}{2} \hat{R} e \left( \Sigma^L(m^2_q) + \Sigma^R(m^2_q) + \Sigma^S(m^2_q) + \Sigma^S_t(m^2_q) \right) \]
\[ \delta Z^L = -\hat{R} e \Sigma^L(m^2_q) - m_q^2 \frac{\partial}{\partial p^2} \left[ \hat{R} e \left( \Sigma^L(p^2) + \Sigma^S(p^2) + \Sigma^S_t(p^2) + \Sigma^R(p^2) \right) \right] \]
\[ \delta Z^R = -\hat{R} e \Sigma^R(m^2_q) - m_q^2 \frac{\partial}{\partial p^2} \left[ \hat{R} e \left( \Sigma^R(p^2) + \Sigma^S(p^2) + \Sigma^S_t(p^2) + \Sigma^R(p^2) \right) \right] + \frac{1}{m} (\Sigma^S_t(m^2) - \Sigma^S(m^2)) \]
One can refer to Ref. [10] for the relevant unrenormalized self-energies and the counterterms take the following forms:

\[ i\delta \Sigma = i[C_L \phi P_L + C_R \phi P_R - C_S^- P_L - C_S^+ P_R], \]
\[ i\delta \Lambda^\mu = -ieQ \gamma^\mu [C^- P_L + C^+ P_R]. \]

\[ C_L = C^- = \frac{1}{2}(\delta Z^L + \delta Z^L\dagger), \]
\[ C_R = C^+ = \frac{1}{2}(\delta Z^R + \delta Z^R\dagger), \]
\[ C_S^- = \frac{m_q}{2}(\delta Z^L + \delta Z^R) + \delta m_q, \]
\[ C_S^+ = \frac{m_q}{2}(\delta Z^R + \delta Z^L) + \delta m_q. \] 

(2.3)

3. The calculation of the subprocess \( ud \rightarrow W^+ \gamma. \)

We denote the subprocess which we calculated as

\[ u(p_3, L)\bar{d}(p_4, R) \rightarrow W^+(p_1) \gamma(p_2). \] 

(3.1)

The \( O(\alpha_s) \) order SUSY QCD correction to (3.1) comes from the interference between the tree-level graphs shown in Fig.1 and the one-loop graphs shown in Fig.2.

The renormalized amplitude for \( u(p_3)_L \bar{d}(p_4)_R \rightarrow W^+(p_1) \gamma(p_2) \) including the \( O(\alpha_s) \) SUSY QCD corrections can be written as:

\[ |M_{L,R}^{\text{ren}}|^2 = |M_{L,R}^{\text{tree}}|^2 + 2 \text{Re} (M_{L,R}^{\text{tree}+} \delta M_{L,R}^{1-\text{loop}}) \]
\[ \delta M_{L,R}^{1-\text{loop}} = \delta M_{L,R}^{\text{self}} + \delta M_{L,R}^{\text{tri}} + \delta M_{L,R}^{\text{box}}. \] 

(3.2)

\( M_{L,R}^{\text{tree}} \) is the amplitude of the tree level. \( \delta M_{L,R}^{\text{self}}, \delta M_{L,R}^{\text{tri}} \) and \( \delta M_{L,R}^{\text{box}} \) represent the renormalized amplitudes coming from the self-energy, triangle and box diagrams, respectively. The lower indices \( L, R \) in above amplitude notations represent the matrix elements for the process \( u_L(p_3)_L \bar{d}_R(p_4) \rightarrow W^+(p_1) \gamma(p_2) \), which is the only
non-zero contribution process in all polarized parton cases. The one-loop and Born amplitude can be written as:

\[
\delta M_{L,R}^{\text{self}} = \delta M_{\bar{u}u}^{\text{self}} + \delta M_{d\bar{d}}^{\text{self}} \\
\delta M_{L,R}^{\text{tri}} = \delta M_{\bar{u}u\gamma}^{\text{tri}} + \delta M_{d\bar{d}\gamma}^{\text{tri}} + \delta M_{\bar{u}d}^{\text{tri},1} + \delta M_{d\bar{d}}^{\text{tri},2} + \delta M_{\bar{u}d}^{\text{tri},3} \\
M_{\text{tree}} = \frac{eg}{\sqrt{2}} \epsilon_\mu(p_2) \epsilon_\nu(p_1) \bar{v}_R(p_3) \\
\left\{ \frac{2}{3u} \gamma^\nu(\not{p}_4 - \not{p}_2) \gamma^\mu - \frac{1}{3t} \gamma^\mu(\not{p}_3 - \not{p}_1) \gamma^\nu - \frac{1}{s - m_W^2} [(\not{p}_1 - \not{p}_2) g^{\mu\nu} + 2p_2^\nu \gamma^\mu - 2p_1^\nu \gamma^\mu] \right\} u_L(p_4) \quad (3.3)
\]

The amplitudes for different groups of one-loop diagrams can be expressed separately. That means we denote \(\delta M_{\bar{u}u}^{\text{self}}, \delta M_{d\bar{d}}^{\text{self}}\) as the amplitudes which correspond to Fig.2(9) and Fig.2(10) respectively. For the triangle diagrams, \(\delta M_{\bar{u}u\gamma}^{\text{tri}}, \delta M_{d\bar{d}\gamma}^{\text{tri}}, \delta M_{\bar{u}d}^{\text{tri},1}, \delta M_{d\bar{d}}^{\text{tri},2}, \delta M_{\bar{u}d}^{\text{tri},3}\) correspond to amplitudes of Fig.2(1), Fig.2(2), Fig.2(3), Fig.2(4), Fig.2(7), Fig.2(8), respectively. \(\delta M_{\text{box},1}^{\text{tri}}\) and \(\delta M_{\text{box},2}^{\text{tri}}\) represent the contributions from box diagrams Fig.2(5) and Fig.2(6), respectively. The explicit expressions for all the amplitude parts involving in the one-loop amplitude \(\delta M\) are listed in Appendix.

Then the cross section for subprocess \(u_L(p_3)\bar{d}_R(p_4) \rightarrow W^+(p_1)\gamma(p_2)\) is given by

\[
\hat{\sigma}_{L,R}(\hat{s}) = \frac{11}{49} \frac{N_c(\hat{s} - m_W^2)}{32\pi \hat{s}^2} \int d(\cos\theta) |M_{L,R}^{\text{ren}}(\hat{s})|^2, \quad (3.4)
\]

In equation (3.3) \(N_c\) is the number of colors. The factors \(\frac{1}{3}\) and \(\frac{1}{9}\) are the spin and color averages respectively.
4. The SUSY QCD corrections to $W\gamma$ production in polarized hadronic collisions.

Now we consider the $W\gamma$ production in hadronic collision. The process is represented as:

$$p(P_3, \lambda)\bar{p}(P_4, \lambda) \rightarrow u\bar{d} \rightarrow W^+(p_1)\gamma(p_2) + X,$$

where $\lambda$ denotes the helicities of the initial proton and anti-proton. $P_3$ and $P_4$ are the four momenta of the proton and anti-proton respectively. The total cross section for the production of W boson and photon in polarized hadronic collisions can be written in the form as:

$$\sigma_{p,\bar{p}}(s, \lambda) = \int dx_1 \int dx_2 \hat{\sigma}_{LR}(\hat{s})[u_\pm(x_1)d_{\mp}(x_2)]$$ (4.1)

where the cross section at parton level $\hat{\sigma}_{LR}$ is for the subprocess of left-handed up-quark and right-handed anti-down-quark collisions, i.e. $u_L + \bar{d}_R \rightarrow W + \gamma$. $u_\pm(x)$ and $d_{\mp}(x)$ are the polarized parton distribution functions. The upper signs of the indices of $u_\pm(x)$ and $d_{\mp}(x)$ are for $\lambda = L$ and the lower signs are for $\lambda = R$.

$$s = (P_3 + P_\lambda)^2$$
$$\hat{s} = (p_3 + p_4)^2$$
$$\hat{s} = x_1x_2s$$
$$p_j = x_jP_i \quad (j = 3, 4)$$

where $x_1$ and $x_2$ are the momentum fractions of initial partons. For any type of partons we take the definitions as [3]
\[ f_+(x) = \frac{1}{2}(f(x) + \Delta f(x)) = \text{density of L(orR) parton in L(orR) proton} \]

\[ f_-(x) = \frac{1}{2}(f(x) - \Delta f(x)) = \text{density of L(orR) parton in R(orL) proton} \]

Equation (4.2)

In the past few years, there are a number of parameterizations for the polarized parton distribution functions (PDF) as shown in literature [7]. It was indicated in this paper [7] that the PDF’s for up and down quarks are the best defined by the present experimental data and there is nice agreement for \( \Delta u \) for different parameterizations of PDF, when \( x \) is in the range of \( 10^{-3} \) to \( 10^{-1} \). In our calculation we adopted Brodsky parameterization for polarized quark distribution functions as a numerical example. Brodsky’s parton distribution functions can be expressed as [11]

\[
\begin{align*}
  u_+(x) &= \frac{1}{x^\alpha} [A_u(1 - x)^3 + B_u(1 - x)^4] \\
  d_+(x) &= \frac{1}{x^\alpha} [A_d(1 - x)^3 + B_d(1 - x)^4] \\
  u_-(x) &= \frac{1}{x^\alpha} [C_u(1 - x)^5 + D_u(1 - x)^6] \\
  d_-(x) &= \frac{1}{x^\alpha} [C_d(1 - x)^5 + D_d(1 - x)^6] \\
  s_+(x) &= \frac{1}{x^\alpha} [A_s(1 - x)^5 + B_s(1 - x)^6] \\
  s_-(x) &= \frac{1}{x^\alpha} [C_s(1 - x)^7 + D_s(1 - x)^6]
\end{align*}
\]

Equation (4.3)

In Brodsky’s (BBS) parameterization scheme, the leading Regge behavior at \( x \rightarrow 0 \) has the intercept \( \alpha = 1.12 \). By choosing this value of \( \alpha \), it allowed a good match to the unpolarized PDF given by MRS [12], if we average the polarized densities. In addition, to be consistent with the sum rules and dynamical constraint indicated in Ref [11], the other eight parameters are set to be:
\[
\begin{aligned}
A_u &= 3.784, & A_d &= 0.775, & A_s &= 0.2897 \\
B_u &= -3.672, & B_d &= -0.645, & B_s &= -0.2637 \\
C_u &= 2.004, & C_d &= 3.230, & C_s &= 1.0 \\
D_u &= -1.892, & D_d &= -3.118, & D_s &= -0.9725
\end{aligned}
\]

5. Numerical calculation and discussion.

In the numerical evaluation we take the input parameters as \(m_W = 80.226 \text{ GeV}, m_Z = 91.1887 \text{ GeV}, G_F = 1.166392 \times 10^{-5} (\text{GeV})^{-2}\) and \(\alpha = \frac{1}{137.036}\). The strong coupling constant \(\alpha_s\) is determined by

\[
\alpha_s(\mu) = \frac{\alpha_s(m_Z)}{1 + \frac{33-2N_f}{6\pi} \alpha_s(m_Z) \ln \frac{\mu}{m_Z}}
\]

where \(\alpha_s(m_Z) = 0.117\) and \(N_f\) is the number of active flavors at energy scale \(\mu\). We set the transverse momentum cut of photon as \(p_T(\gamma) > 20 \text{ GeV}\). This is a typical experiment acceptance cut value. This cut is also necessary in regulating the collinear divergence associated with the photon. Since here we don’t consider the CP-violation effect in this process, we set the CP-violation angel \(\phi\) to be zero. In order to illuminate the SUSY QCD effects more clearly and for the sake of simplicity, we assume that the scalar quark masses are all degenerate, i.e. \(m_{\tilde{u}_1} = m_{\tilde{u}_2} = m_{\tilde{d}_1} = m_{\tilde{d}_2} = m_{\tilde{q}}\).

If we take the experimental mass bounds on squarks and gluino into account, the corresponding mass parameters are set to be \(m_{\tilde{q}} > 175 \text{ GeV}\) (for \(m_{\tilde{g}} < 300 \text{ GeV}\)) and \(m_{\tilde{g}} > 175 \text{ GeV}\) [13]. Since the very light gluino mass is not excluded experimentally and there has been renewed interest in this case recently, we also set the gluino mass...
to be 5 GeV as a comparison.

In Fig.3 we present the relative $O(\alpha_s)$ SUSY QCD corrections ($\delta = \frac{\Delta\sigma}{\sigma_{\text{tree}}}$) to the $W\gamma$ hadronic production at the parton level. The two curves correspond to two different values of squark and gluino masses, namely, solid curve for $m_{\tilde{g}} = 5$ GeV, $m_{\tilde{q}} = 175$ GeV and dashed curve for $m_{\tilde{g}} = 150$ GeV, $m_{\tilde{q}} = 175$ GeV. From Fig.3 we can see that the relative corrections can reach 1.2% with $m_{\tilde{g}} = 5$ GeV when $\sqrt{s} = 500$ GeV.

The relationships between the SUSY QCD corrections and masses of the squark and gluino are illustrated in Fig.4 and Fig.5 respectively, where $\sqrt{s} = 500 GeV$. We find that the corrections decrease heavily with the increasing of $m_{\tilde{q}}$ and $m_{\tilde{g}}$, they show clearly the effects of decoupling.

In our calculation we find that the contribution from the box diagrams and the quartic vertex diagrams are much less than those from other triangle and self-energy diagrams. As the result, the resonance effect at $\sqrt{s} = 2 m_{\tilde{q}}$ is suppressed, so there is no obvious peak in Fig.3.

Fig.6 shows the relative SUSY QCD corrections to $W\gamma$ production in the polarized proton-antiproton collisions, where the solid curve corresponds to $p_L\bar{p}_L$ collision and the dashed curve represents $p_R\bar{p}_R$ collision. The relative correction discrepancy between these two different polarized $pp$ collisions is obvious and the absolute value of relative corrections increases with the increment of $\sqrt{s}$. The relative correction for $p_L\bar{p}_L$ collision at $\sqrt{s} = 2 TeV$ reaches 0.2%.

The calculation in Ref. [5] states that the SM QCD virtual relative correction
is only about 1% of the Born approximation, whereas our numerical results show that the corresponding SUSY QCD virtual relative corrections could reach 1.2% with favorable mass parameters, which is of the same order of the SM QCD virtual corrections.

As we know, in $u \bar{d} \rightarrow W \gamma$ subprocess only the first generation of the scalar quarks contributes to the SUSY QCD correction. In order to illuminate the SUSY QCD effects, we assumed that the masses of three generations of the scalar quarks are degenerate in our calculation. However, if the degeneracy of the scalar quarks was lifed, it is generally believed that the $\tilde{u}$, $\tilde{d}$ would be the heaviest squarks, the SUSY signals would be weaker due to the decoupling effect.

In practical experiment, the soft gluon radiation, hard collinear corrections etc. should be also included in the conventional QCD corrections. Their combined contributions could result in large corrections to $p + \bar{p} \rightarrow W + \gamma + X$ inclusive process, which would be strong background to the SUSY signal. However, if we impose the jet veto in experiment, the size of the SM QCD corrections would reduce to only about few percent of the Born approximation in the $p + \bar{p} \rightarrow W \gamma + 0 \ jet$ process at the Tevatron energy. Since both supersymmetric and conventional QCD corrections are mainly negative in the 0-jet process, it could be possible to disentangle the SUSY signals from the conventional QCD effects by more precise measurement of the $p + \bar{p} \rightarrow W \gamma$ process.

At high colliding energy, u and d quarks are approximately massless particles, the W boson couples only to left-handed u quark and right-handed $\bar{d}$ quark. And
we know that the densities of left-handed and right-handed quarks are not equal for
polarized proton. Therefore the cross section for hadronic $W \gamma$ production should
be of significant difference for different polarized condition. Fig.6 shows that the
relative SUSY QCD corrections for $p_L \bar{p}_L$ collision are larger than the $p_R \bar{p}_R$ collisions,
it could be used to improve the experimental measurement for SUSY corrections.

In this paper we evaluated the SUSY QCD corrections to the $p\bar{p} \to u\bar{d} \to W^+\gamma + X$ process and presented the complete analytic expressions and numerical results
including the one-loop SUSY QCD virtual corrections. The relative corrections can
reach $1.2\%$ for $m_{\tilde{g}} = 5 GeV$, $m_{\tilde{u}} = m_{\tilde{d}} = 175 GeV$ at the parton level and about
$0.2\%$ after convoluted with the parton distribution functions. The corrections are
sensitive to the masses of the squarks and gluino, especially for very light gluino.
We can conclude that the SUSY QCD corrections with appropriate superpartner
masses for the process $p\bar{p} \to u\bar{d} \to W^+\gamma + X$ can be noticeful on TeV energy scale.

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Appendix: One-loop amplitudes and form factors

The one – loop amplitude parts appearing inEq.(3.3) are expressed as :

$$\delta M_{u\bar{u}}^{self} = \frac{-i e g_2^2 C_f}{u^2 3 \sqrt{2} 4\pi^2} \epsilon_\mu(p_2)\epsilon_\nu(p_1)\bar{v}_R(p_3)$$
\[
\delta M_{ud}^{\mu\nu} = \frac{i e}{t^2} \delta g^2 C_f \epsilon_\mu(p_2) \epsilon_\nu(p_1) \bar{v}_R(p_3) (2 f_1^{d\bar{d}} P_3 \gamma^\nu \gamma^\mu + 4 f_2^{d\bar{d}} P_2 \gamma^\nu \gamma^\mu) u_L(p_4)
\]
\[
\delta M_{u\bar{u}}^{\gamma\gamma} = -\frac{i e}{u} \delta g^2 C_f \epsilon_\mu(p_2) \epsilon_\nu(p_1) \bar{v}_R(p_3) (f_1^{\bar{u}\bar{u}} \gamma^\nu \gamma^\mu + 2 f_2^{\bar{u}\bar{u}} \gamma^\nu \gamma^\mu) u_L(p_4)
\]
\[
\delta M_{d\bar{d}}^{\psi,1} = \frac{i e}{2} \delta g^2 C_f \epsilon_\mu(p_2) \epsilon_\nu(p_1) \bar{v}_R(p_3) \left( \frac{1}{4} f_1^{d\bar{u}} \gamma^\nu \gamma^\mu + 2 f_2^{d\bar{u}} \gamma^\nu \gamma^\mu \right) u_L(p_4)
\]
\[
\delta M_{u\bar{u}}^{\gamma\mu} = \frac{i e}{s} \delta g^2 C_f \epsilon_\mu(p_2) \epsilon_\nu(p_1) \bar{v}_R(p_3) \left( f_1^{u\bar{u}} \gamma^\nu \gamma^\mu + 2 f_2^{u\bar{u}} \gamma^\nu \gamma^\mu \right) u_L(p_4)
\]
\[
\delta M_{u\bar{u}}^{\bar{u}d,1} = \frac{i e}{s} \delta g^2 C_f \epsilon_\mu(p_2) \epsilon_\nu(p_1) \bar{v}_R(p_3) \left( f_1^{u\bar{d}} \gamma^\nu \gamma^\mu + 2 f_2^{u\bar{d}} \gamma^\nu \gamma^\mu \right) u_L(p_4)
\]
\[
\delta M_{u\bar{u}}^{\bar{u}d,2} = \frac{i e}{s} \delta g^2 C_f \epsilon_\mu(p_2) \epsilon_\nu(p_1) \bar{v}_R(p_3) \left( f_1^{u\bar{d}} \gamma^\nu \gamma^\mu + 2 f_2^{u\bar{d}} \gamma^\nu \gamma^\mu \right) u_L(p_4)
\]
\[
\delta M_{u\bar{u}}^{\bar{u}d,3} = \frac{i e}{s} \delta g^2 C_f \epsilon_\mu(p_2) \epsilon_\nu(p_1) \bar{v}_R(p_3) \left( f_1^{u\bar{d}} \gamma^\nu \gamma^\mu + 2 f_2^{u\bar{d}} \gamma^\nu \gamma^\mu \right) u_L(p_4)
\]
\[
\delta M_{L,R}^{\text{box},2} = \frac{i e g g_s^2}{3 \sqrt{2} \frac{1}{2\pi^2}} \epsilon_\mu(p_2) \epsilon_\nu(p_1) \bar{\nu}_R(p_3) \left( f_1^{\text{box},1} \bar{p}_2 p_5^\mu p_3^\nu u_L(p_4) + f_2^{\text{box},1} \bar{q}_2 p_5^\mu p_3^\nu u_L(p_4) + f_3^{\text{box},1} \bar{q}_2 p_1^\mu q_1^\nu p_3^\nu + f_4^{\text{box},1} \bar{q}_2 p_1^\mu q_2^\nu p_3^\nu + f_5^{\text{box},1} \bar{q}_2 p_1^\mu q_2^\nu p_3^\nu + f_6^{\text{box},1} \bar{q}_2 p_1^\mu q_3^\nu + f_7^{\text{box},1} \bar{q}_2 p_1^\mu q_3^\nu + f_8^{\text{box},1} \bar{q}_2 p_1^\mu q_3^\nu + f_9^{\text{box},1} \bar{q}_2 p_1^\mu q_3^\nu + f_{10}^{\text{box},1} \bar{q}_2 p_1^\mu q_3^\nu + f_{11}^{\text{box},1} \bar{q}_2 p_1^\mu q_3^\nu + f_{12}^{\text{box},1} \bar{q}_2 p_1^\mu q_3^\nu + f_{13}^{\text{box},1} \bar{q}_2 p_1^\mu q_3^\nu \right)
\]

In above equations \( C_f = \frac{4}{3} \) which is the color factor arising from the quark-squark-ghuino vertices in one-loop diagrams. The definition of one-loop integral functions are adopted from Ref.[14].

\[
f_1^{u\bar{u}} = -\cos \theta \sin \theta (B_0[m_u, m_{\tilde{g}}, m_{\tilde{u}_1}] + B_0[m_\bar{u}, m_{\tilde{g}}, m_{\tilde{u}_2}]) + 
\cos \theta \sin \theta (B_0[p_2 - p_4, m_{\tilde{g}}, m_{\tilde{u}_1}] - B_0[p_2 - p_4, m_{\tilde{g}}, m_{\tilde{u}_2}])
\]

\[
f_2^{u\bar{u}} = -\cos^2 \theta B_1[m_u, m_{\tilde{g}}, m_{\tilde{u}_1}] - \sin^2 \theta B_1[m_u, m_{\tilde{g}}, m_{\tilde{u}_1}] - 
\cos^2 \theta B_1[p_2 - p_4, m_{\tilde{g}}, m_{\tilde{u}_1}] - \sin^2 \theta B_1[p_2 - p_4, m_{\tilde{g}}, m_{\tilde{u}_2}]
\]

\[
f_3^{u\bar{u}} = p_2 \cdot p_4 \cos^2 \theta B_1[m_u, m_{\tilde{g}}, m_{\tilde{u}_1}] + p_2 \cdot p_4 \sin^2 \theta B_1[m_u, m_{\tilde{g}}, m_{\tilde{u}_1}] - 
\cos^2 \theta B_1[p_2 - p_4, m_{\tilde{g}}, m_{\tilde{u}_1}] - \sin^2 \theta B_1[p_2 - p_4, m_{\tilde{g}}, m_{\tilde{u}_2}]
\]

\[
f_4^{u\bar{u}} = -\cos \theta \sin \theta (B_0[m_d, m_{\tilde{g}}, m_{\tilde{u}_1}] + B_0[m_d, m_{\tilde{g}}, m_{\tilde{u}_2}]) + 
\cos \theta \sin \theta (B_0[p_2 - p_4, m_{\tilde{g}}, m_{\tilde{u}_1}] - B_0[p_2 - p_4, m_{\tilde{g}}, m_{\tilde{u}_2}])
\]

\[
f_5^{d\bar{d}} = \cos^2 \theta B_1[m_d, m_{\tilde{g}}, m_{\tilde{d}_1}] + \sin^2 \theta B_1[m_d, m_{\tilde{g}}, m_{\tilde{d}_2}]
\]

\[
f_6^{d\bar{d}} = \cos^2 \theta B_1[m_d, m_{\tilde{g}}, m_{\tilde{d}_1}] + \sin^2 \theta B_1[m_d, m_{\tilde{g}}, m_{\tilde{d}_2}]
\]
\[
\cos^2 \theta B_1[-p_2 + p_3, m_{\tilde{g}}, m_{d_1}] - \sin^2 \theta B_1[-p_2 + p_3, m_{\tilde{g}}, m_{d_2}]
\]
\[
f_{\tilde{d}\tilde{d}}^2 = -p_2 \cdot p_3 \cos^2 \theta B_1[m_d, m_{\tilde{g}}, m_{d_1}] - p_2 \cdot p_3 \sin^2 \theta B_1[m_d, m_{\tilde{g}}, m_{d_2}] + p_2 \cdot p_3 \cos^2 \theta B_1[-p_2 + p_3, m_{\tilde{g}}, m_{d_2}]
\]

For simplicity, we give denotations to represent the complete expressions of C,D integral functions at first, then the lengthy arguments of C, D functions can be omitted.

\[
\{C^{(1)}_i, C^{(1)}_{ij}\} = \{C^{(1)}_i, C^{(1)}_{ij}\}[-p_4, p_2, m_{\tilde{g}}, m_{u_1}, m_{u_1}]
\]
\[
\{C^{(2)}_i, C^{(2)}_{ij}\} = \{C^{(2)}_i, C^{(2)}_{ij}\}[-p_4, p_2, m_{\tilde{g}}, m_{u_2}, m_{u_2}]
\]

\[
f_{\tilde{u}\tilde{u}}^{\gamma} = -\sin^2 \theta B_1[m_u, m_{\tilde{g}}, m_{u_1}] - \cos^2 \theta B_1[m_u, m_{\tilde{g}}, m_{u_2}] - 2\cos^2 \theta C^{(1)}_{24} - 2\sin^2 \theta C^{(2)}_{24}
\]
\[
f_{\tilde{d}\tilde{d}}^{\gamma} = -\cos \theta \sin \theta C_0 + \cos \theta \sin \theta C_0 + \cos \theta \sin \theta C^{(1)}_{11} - \cos \theta \sin \theta C^{(2)}_{11}
\]
\[
f_{\tilde{d}\tilde{d}}^{\gamma} = \sin^2 \theta B_1[m_u, m_{\tilde{g}}, m_{u_1}] + \cos^2 \theta B_1[m_u, m_{\tilde{g}}, m_{u_2}] + 2\cos^2 \theta C^{(1)}_{24} + 2\sin^2 \theta C^{(2)}_{24} + 2p_2 \cdot p_4 \cos^2 \theta (C^{(1)}_{12} + C^{(1)}_{23}) + 2p_2 \cdot p_4 \sin^2 \theta (C^{(2)}_{12} + C^{(2)}_{23})
\]
\[
f_{\tilde{d}\tilde{d}}^{\gamma} = -p_2 \cdot p_4 \cos^2 \theta (C^{(1)}_{12} + 2C^{(1)}_{22}) - p_2 \cdot p_4 \sin^2 \theta (C^{(2)}_{12} + 2C^{(2)}_{22})
\]
\[
f_{\tilde{d}\tilde{d}}^{\gamma} = \cos \theta \sin \theta (C^{(1)}_{0} - C^{(2)}_{0}) + 2\cos \theta \sin \theta (C^{(1)}_{12} - C^{(2)}_{12})
\]

\[
\{C^{(1)}_i, C^{(1)}_{ij}\} = \{C^{(1)}_i, C^{(1)}_{ij}\}[-p_3, p_2, m_{\tilde{g}}, m_{d_1}, m_{d_1}]
\]
\[ \{ C_i^{(2)}, C_{ij}^{(2)} \} = \{ C_i^{(2)}, C_{ij}^{(2)} \}[-p_3, p_2, m_\tilde{g}, m_{\tilde{d}_2}, m_{\tilde{d}_2}] \]

\[
f_1^{\tilde{u}\tilde{d}_y} = sin^2\theta B_1[m_d, m_\tilde{g}, m_{\tilde{d}_1}] + cos^2\theta B_1[m_d, m_\tilde{g}, m_{\tilde{d}_2}] + 2cos^2\theta C_{24}^{(1)} + 2sin^2\theta C_{24}^{(2)}
\]

\[
f_2^{\tilde{d}_y} = -sin^2\theta B_1[m_d, m_\tilde{g}, m_{\tilde{d}_1}] - cos^2\theta B_1[m_d, m_\tilde{g}, m_{\tilde{d}_2}] - 2cos^2\theta C_{24}^{(1)} - 2sin^2\theta C_{24}^{(2)} - 2p_2 \cdot p_3cos^2\theta(C_{12}^{(1)} + C_{23}^{(1)}) - 2p_2 \cdot p_3sin^2\theta(C_{12}^{(2)} + C_{23}^{(2)})
\]

\[
\{ C_i^{(1)}, C_{ij}^{(1)} \} = \{ C_i^{(1)}, C_{ij}^{(1)} \}[-p_4, p_1, m_\tilde{g}, m_{\tilde{u}_1}, m_{\tilde{d}_1}]
\]

\[
\{ C_i^{(2)}, C_{ij}^{(2)} \} = \{ C_i^{(2)}, C_{ij}^{(2)} \}[-p_4, p_1, m_\tilde{g}, m_{\tilde{u}_1}, m_{\tilde{d}_2}]
\]

\[
\{ C_i^{(3)}, C_{ij}^{(3)} \} = \{ C_i^{(3)}, C_{ij}^{(3)} \}[-p_4, p_1, m_\tilde{g}, m_{\tilde{u}_2}, m_{\tilde{d}_1}]
\]

\[
\{ C_i^{(4)}, C_{ij}^{(4)} \} = \{ C_i^{(4)}, C_{ij}^{(4)} \}[-p_4, p_1, m_\tilde{g}, m_{\tilde{u}_2}, m_{\tilde{d}_2}]
\]

\[
f_1^{\tilde{u}\tilde{d}_w,1} = cos^2\theta(B_1[m_u, m_\tilde{g}, m_{\tilde{u}_1}] + B_1[m_u, m_\tilde{g}, m_{\tilde{d}_1}]) + sin^2\theta(B_1[m_u, m_\tilde{g}, m_{\tilde{u}_2}] + B_1[m_u, m_\tilde{g}, m_{\tilde{d}_2}]) + 4cos^4\theta C_{24}^{(1)} + 4cos^2\theta sin^2\theta(C_{24}^{(2)} + C_{24}^{(3)}) + 4sin^4\theta C_{24}^{(4)}
\]

\[
f_2^{\tilde{u}\tilde{d}_w,1} = cos^3\theta sin\theta(C_0^{(1)} + C_0^{(2)}) + sin^3\theta cos\theta(C_0^{(3)} + C_0^{(4)}) + [C_0 \rightarrow C_{12}]
\]

\[
f_3^{\tilde{u}\tilde{d}_w,1} = cos^3\theta sin\theta(C_{12}^{(1)} + C_{12}^{(2)}) + sin^3\theta cos\theta(C_{12}^{(3)} + C_{12}^{(4)}) + [C_{12} \rightarrow C_{23}]
\]

\[
f_4^{\tilde{u}\tilde{d}_w,1} = cos^3\theta sin\theta(C_0^{(1)} + C_0^{(2)}) + sin^3\theta cos\theta(C_0^{(3)} + C_0^{(4)}) + [C_0 \rightarrow C_{11}]
\]

\[
f_5^{\tilde{u}\tilde{d}_w,1} = \frac{f_4^{\tilde{u}\tilde{d}_w,1}}{f_5^{\tilde{u}\tilde{d}_w,1}} , \quad f_6^{\tilde{u}\tilde{d}_w,1} = f_3^{\tilde{u}\tilde{d}_w,1}
\]

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\[
\{ C_i^{(1)}, C_{ij}^{(1)} \} = \{ C_i^{(1)}, C_{ij}^{(1)} \} [-p_3, p_1, m_{\tilde{g}}, m_{u_1}, m_{d_1}]
\]
\[
\{ C_i^{(2)}, C_{ij}^{(2)} \} = \{ C_i^{(2)}, C_{ij}^{(2)} \} [-p_3, p_1, m_{\tilde{g}}, m_{u_1}, m_{d_2}]
\]
\[
\{ C_i^{(3)}, C_{ij}^{(3)} \} = \{ C_i^{(3)}, C_{ij}^{(3)} \} [-p_3, p_1, m_{\tilde{g}}, m_{u_2}, m_{d_1}]
\]
\[
\{ C_i^{(4)}, C_{ij}^{(4)} \} = \{ C_i^{(4)}, C_{ij}^{(4)} \} [-p_3, p_1, m_{\tilde{g}}, m_{u_2}, m_{d_2}]
\]

\[
f_1^{\tilde{u}_d w, 2} = \cos^2 \theta B_1[m_{u}, m_{\tilde{g}}, m_{u_1}] + \cos^2 \theta B_1[m_{d}, m_{\tilde{g}}, m_{d_1}] + \sin^2 \theta B_1[m_{u}, m_{\tilde{g}}, m_{u_2}] + \sin^2 \theta B_1[m_{d}, m_{\tilde{g}}, m_{d_2}] + 4\cos^4 \theta C_{24}^{(1)} + 4\cos^2 \theta \sin^2 \theta (C_{24}^{(2)} + C_{24}^{(3)}) + 4\sin^4 \theta C_{24}^{(4)}
\]
\[
f_2^{\tilde{u}_d w, 2} = \cos^3 \sin \theta (C_0^{(1)} + C_0^{(2)}) + \sin^3 \theta \cos \theta (C_0^{(3)} + C_0^{(4)}) + [C_0 \rightarrow C_{11}]
\]
\[
f_5^{\tilde{u}_d w, 2} = -\cos^3 \sin \theta (C_0^{(1)} + C_0^{(2)}) + \sin^3 \theta \cos \theta (C_0^{(3)} + C_0^{(4)}) + [C_0 \rightarrow C_{11}]
\]
\[
f_6^{\tilde{u}_d w, 2} = \cos^4 \theta C_{12}^{(1)} + \cos^2 \theta \sin \theta (C_{12}^{(1)} + C_{12}^{(3)}) + \sin^4 \theta C_{12}^{(4)} + [C_0 \rightarrow C_{11}]
\]
\[
f_3^{\tilde{u}_d w, 2} = f_2^{\tilde{u}_d w, 2}, \quad f_4^{\tilde{u}_d w, 2} = f_1^{\tilde{u}_d w, 2}
\]
\[
\begin{align*}
\tilde{f}_{1} & = \frac{1}{8} (1 - \frac{m_{W}^{2}}{s}) \left[ \cos^{2} \theta (B_{1}[m_{u}, m_{\bar{g}}, m_{u_{1}}] + B_{1}[m_{d}, m_{\bar{g}}, m_{d_{1}}]) + \right. \\
& \left. \sin^{2} \theta (B_{1}[m_{u}, m_{\bar{g}}, m_{u_{2}}] + B_{1}[m_{d}, m_{\bar{g}}, m_{d_{2}}]) \right. \\
& \left. + \frac{\cos^{4} \theta}{4} \left( \frac{m_{W}^{4}}{s} - m_{W}^{2} + \frac{2m_{W}^{2}}{s} p_{1} \cdot p_{2} + 2p_{1} \cdot p_{4} - \frac{2m_{W}^{2}}{s} p_{1} \cdot p_{4} - \right. ight. \\
& \left. \left. 2p_{2} \cdot p_{4} - \frac{2m_{W}^{2}}{s} p_{2} \cdot p_{4} \right) C_{11}^{(1)} + 2 \left( \frac{m_{W}^{4}}{s} - m_{W}^{2} + \frac{2m_{W}^{2}}{s} p_{1} \cdot p_{2} \right) C_{22}^{(1)} + ight. \\
& \left. 2(p_{1} \cdot p_{4} - \frac{m_{W}^{2}}{s} p_{2} \cdot p_{4} - p_{2} \cdot p_{4} - \frac{m_{W}^{2}}{s} p_{2} \cdot p_{4}) C_{23}^{(1)} + 2(1 + \frac{m_{W}^{2}}{s}) C_{24}^{(1)} \right]
\end{align*}
\]

\[
\begin{align*}
\tilde{f}_{2} & = \frac{1}{8} (1 + \frac{m_{W}^{2}}{s}) \left[ \cos^{2} \theta (B_{1}[m_{u}, m_{\bar{g}}, m_{u_{1}}] + B_{1}[m_{d}, m_{\bar{g}}, m_{d_{1}}]) + \right. \\
& \left. \sin^{2} \theta (B_{1}[m_{u}, m_{\bar{g}}, m_{u_{2}}] + B_{1}[m_{d}, m_{\bar{g}}, m_{d_{2}}]) \right. \\
& \left. + \frac{\cos^{4} \theta}{4} \left( \frac{m_{W}^{4}}{s} - m_{W}^{2} + \frac{2m_{W}^{2}}{s} p_{1} \cdot p_{2} + 2p_{1} \cdot p_{4} - \frac{2m_{W}^{2}}{s} p_{1} \cdot p_{4} - \right. ight. \\
& \left. \left. 2p_{2} \cdot p_{4} - \frac{2m_{W}^{2}}{s} p_{2} \cdot p_{4} \right) C_{12}^{(1)} + 2 \left( \frac{m_{W}^{4}}{s} - m_{W}^{2} + \frac{2m_{W}^{2}}{s} p_{1} \cdot p_{2} \right) C_{22}^{(1)} + ight. \\
& \left. 2(p_{1} \cdot p_{4} - \frac{m_{W}^{2}}{s} p_{2} \cdot p_{4} - p_{2} \cdot p_{4} - \frac{m_{W}^{2}}{s} p_{2} \cdot p_{4}) C_{23}^{(1)} - 2(1 + \frac{m_{W}^{2}}{s}) C_{24}^{(1)} \right]
\end{align*}
\]

\[
\begin{align*}
\tilde{f}_{3} & = \frac{1}{8} \left[ \cos^{2} \theta (B_{1}[m_{u}, m_{\bar{g}}, m_{u_{1}}] + B_{1}[m_{d}, m_{\bar{g}}, m_{d_{1}}]) + \right. \\
& \left. \sin^{2} \theta (B_{1}[m_{u}, m_{\bar{g}}, m_{u_{2}}] + B_{1}[m_{d}, m_{\bar{g}}, m_{d_{2}}]) \right. \\
& \left. - \cos^{4} \theta C_{21}^{(1)} - \sin^{2} \theta \cos^{2} \theta (C_{24}^{(2)} + C_{24}^{(3)}) - \sin^{4} \theta C_{24}^{(4)} \right]
\end{align*}
\]

\[
\begin{align*}
\tilde{f}_{5} & = m_{\bar{g}} \left[ \cos^{3} \theta \sin \theta (C_{11}^{(1)} + C_{0}^{(1)} - C_{11}^{(2)} - C_{0}^{(2)}) + \right. \\
& \left. \sin^{3} \theta \cos \theta (C_{11}^{(3)} + C_{0}^{(3)} - C_{11}^{(4)} - C_{0}^{(4)}) \right]
\end{align*}
\]

\[
\begin{align*}
\tilde{f}_{6} & = \cos^{4} \theta (C_{12}^{(1)} + C_{23}^{(1)}) + \cos^{2} \theta \sin^{2} \theta (C_{12}^{(2)} + C_{23}^{(2)} + C_{12}^{(3)} + C_{23}^{(3)}) + \\
& 2 \sin^{4} \theta (C_{12}^{(4)} + C_{23}^{(4)})
\end{align*}
\]
\[
\begin{align*}
    f_{4}^{\tilde{u}dw,3} &= -f_{3}^{\tilde{u}dw,3} , \\
    f_{8}^{\tilde{u}dw,3} &= -f_{5}^{\tilde{u}dw,3} , \\
    f_{6}^{\tilde{u}dw,3} &= f_{7}^{\tilde{u}dw,3} , \\
    f_{9}^{\tilde{u}dw,3} &= f_{10}^{\tilde{u}dw,3} = -f_{6}^{\tilde{u}dw,3} ,
\end{align*}
\]

\[
f^{\tilde{u}dw_{\gamma}} = m_{u}\cos^{4}\theta(C_{11}^{(1)} - C_{12}^{(1)}) + m_{g}\cos^{3}\theta\sin\theta(C_{0}^{(2)} - C_{0}^{(1)}) + \cos^{2}\theta\sin^{2}\theta(m_{u}C_{11}^{(2)} + m_{u}C_{11}^{(3)} + m_{d}C_{12}^{(2)} - m_{d}C_{12}^{(3)} - m_{u}C_{12}^{(2)} - m_{u}C_{12}^{(3)} - m_{d}C_{12}^{(4)}) + m_{g}\cos^{2}\theta\sin^{2}\theta(C_{0}^{(4)} - C_{0}^{(3)}) + m_{u}\sin^{4}\theta(C_{11}^{(4)} - C_{12}^{(4)})
\]

\[
\begin{align*}
\{D_{i}^{(1)}, D_{ij}^{(1)}, D_{i}^{(1)}\} &= \{D_{i}^{(1)}, D_{ij}^{(1)}, D_{i}^{(1)}\}[-p_{2}, p_{2}, p_{1}, m_{s}, m_{u_{1}}, m_{u_{1}}, m_{d_{1}}] \\
\{D_{i}^{(2)}, D_{ij}^{(2)}, D_{ij}^{(2)}\} &= \{D_{i}^{(2)}, D_{ij}^{(2)}, D_{ij}^{(2)}\}[-p_{2}, p_{2}, p_{1}, m_{s}, m_{u_{1}}, m_{u_{1}}, m_{d_{2}}] \\
\{D_{i}^{(3)}, D_{ij}^{(3)}, D_{ij}^{(3)}\} &= \{D_{i}^{(3)}, D_{ij}^{(3)}, D_{ij}^{(3)}\}[-p_{2}, p_{2}, p_{1}, m_{s}, m_{u_{2}}, m_{u_{2}}, m_{d_{1}}] \\
\{D_{i}^{(4)}, D_{ij}^{(4)}, D_{ij}^{(4)}\} &= \{D_{i}^{(4)}, D_{ij}^{(4)}, D_{ij}^{(4)}\}[-p_{2}, p_{2}, p_{1}, m_{s}, m_{u_{2}}, m_{u_{2}}, m_{d_{2}}]
\end{align*}
\]

\[
\begin{align*}
    f_{1}^{box,1} &= m_{g}\cos^{3}\theta\sin\theta D_{27}^{(1)} + \cos^{2}\theta\sin^{2}\theta[D^{(1)} \rightarrow D^{(2)}] - \\
                  &\quad \cos^{2}\theta\sin^{2}\theta[D^{(1)} \rightarrow D^{(3)}] - \sin^{2}\theta\cos\theta[D^{(1)} \rightarrow D^{(4)}] \\
    f_{2}^{box,1} &= \cos^{4}\theta( D_{312}^{(1)} - D_{313}^{(1)} ) + \cos^{2}\theta\sin^{2}\theta[D^{(1)} \rightarrow D^{(2)}] + \\
                  &\quad \sin^{3}\theta\cos\theta[D^{(1)} \rightarrow D^{(3)}] + \sin^{3}\theta\cos\theta[D^{(1)} \rightarrow D^{(4)}] \\
    f_{3}^{box,1} &= \cos^{4}\theta( D_{312}^{(1)} - D_{311}^{(1)} ) + \cos^{2}\theta\sin^{2}\theta[D^{(1)} \rightarrow D^{(2)}] + \\
                  &\quad \sin^{3}\theta\cos\theta[D^{(1)} \rightarrow D^{(3)}] + \sin^{3}\theta\cos\theta[D^{(1)} \rightarrow D^{(4)}] \\
    f_{4}^{box,1} &= \cos^{4}\theta( D_{313}^{(1)} - D_{311}^{(1)} - D_{27}^{(1)} ) + \sin^{2}\theta\cos^{2}\theta[D^{(1)} \rightarrow D^{(2)}] + \\
                  &\quad \sin^{2}\theta\cos^{2}\theta[D^{(1)} \rightarrow D^{(3)}] + \sin^{2}\theta\cos^{2}\theta[D^{(1)} \rightarrow D^{(4)}]
\end{align*}
\]
\[\begin{align*}
\sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(3)}] &+ \sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(4)}] \\
\textbf{f}^{\text{box}, 1}_5 & = m_5 \cos^3 \theta \sin \theta (D^{(1)}_{21} + D^{(1)}_{24} + D^{(1)}_{25} + D^{(1)}_{26} + D^{(1)}_{12} - D^{(1)}_{11}) + \sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(2)}] - \\
&\sin^2 \theta \cos^2 \theta [D^{(1)} \rightarrow D^{(3)}] - \sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(4)}] \\
\textbf{f}^{\text{box}, 1}_6 & = m_5 \cos^4 \theta (D^{(1)}_{22} - D^{(1)}_{24} + D^{(1)}_{25} - D^{(1)}_{34} + D^{(1)}_{36} - D^{(1)}_{37} - \\
&D^{(1)}_{28} + D^{(1)}_{30}) + \sin^2 \theta \cos^2 \theta [D^{(1)} \rightarrow D^{(2)}] + \\
&\sin^3 \theta \cos \theta ([D^{(1)} \rightarrow D^{(3)}] + [D^{(1)} \rightarrow D^{(4)}]) \\
\textbf{f}^{\text{box}, 1}_7 & = \cos^4 \theta (D^{(1)}_{27} + D^{(1)}_{311}) + \sin^2 \theta \cos^2 \theta [D^{(1)} \rightarrow D^{(2)}] + \\
&\sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(3)}] + \sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(4)}] \\
\textbf{f}^{\text{box}, 1}_8 & = m_5 \cos^3 \theta \sin \theta (D^{(1)}_{0} + D^{(1)}_{11} + -D^{(1)}_{13} + \\
&D^{(1)}_{21} - D^{(1)}_{25} + \sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(2)}] - \\
&\sin^2 \theta \cos^2 \theta [D^{(1)} \rightarrow D^{(3)}] - \sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(4)}] \\
\textbf{f}^{\text{box}, 1}_9 & = \cos^4 \theta (D^{(1)}_{12} - D^{(1)}_{13} + D^{(1)}_{23} + D^{(1)}_{24} - D^{(1)}_{25} - D^{(1)}_{26} - D^{(1)}_{310} + D^{(1)}_{34} - \\
&D^{(1)}_{35} + D^{(1)}_{37}) + \cos^2 \theta \sin^2 \theta [D^{(1)} \rightarrow D^{(2)}] + \\
&\sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(3)}] + \sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(4)}] \\
\textbf{f}^{\text{box}, 1}_{10} & = \cos^4 \theta (D^{(1)}_{27} + D^{(1)}_{311}) + \cos^2 \theta \sin^2 \theta [D^{(1)} \rightarrow D^{(2)}] + \\
&\sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(3)}] + \sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(4)}] \\
\textbf{f}^{\text{box}, 1}_{11} & = m_5 \cos^3 \theta \sin \theta (D^{(1)}_{11} - D^{(1)}_{12} + D^{(1)}_{21} - D^{(1)}_{24} + \cos^2 \theta \sin^2 \theta [D^{(1)} \rightarrow D^{(2)}] - \\
&\sin^2 \theta \cos^2 \theta [D^{(1)} \rightarrow D^{(3)}] - \sin^3 \theta \cos \theta [D^{(1)} \rightarrow D^{(4)}] \\
&\end{align*}\]
\begin{align*}
  f_{12}^{\text{box},1} &= \cos^4 \theta (D_{24}^{(1)} - D_{22}^{(1)} - D_{25}^{(1)} + D_{26}^{(1)} + D_{310}^{(1)} - D_{34}^{(1)} - D_{35}^{(1)} - D_{36}^{(1)}) + \\
  &\sin^2 \theta \cos^2 \theta [D^{(1)} \to D^{(2)}] + \sin^3 \theta \cos \theta ([D^{(1)} \to D^{(3)}] + [D^{(1)} \to D^{(4)}]) \\
  f_{13}^{\text{box},1} &= -m_\tilde{g} \cos^3 \theta \sin \theta (D_0^{(1)} + D_{11}^{(1)} + D_{21}^{(1)}) + \sin^2 \theta \cos^2 \theta [D^{(1)} \to D^{(2)}] - \\
  &\sin^2 \theta \cos^2 \theta [D^{(1)} \to D^{(3)}] - \sin^3 \theta \cos \theta [D^{(1)} \to D^{(4)}] \\
  f_{14}^{\text{box},1} &= -\cos^4 \theta (D_{13}^{(1)} - D_{12}^{(1)} - 2D_{24}^{(1)} + 2D_{25}^{(1)} - D_{34}^{(1)} + D_{35}^{(1)}) + \\
  &\sin^2 \theta \cos^2 \theta [D^{(1)} \to D^{(2)}] + \sin^3 \theta \cos \theta ([D^{(1)} \to D^{(3)}] + [D^{(1)} \to D^{(4)}]) \\
  f_{\text{box},2} &= f_{\text{box},1}(\{D_i, D_{ij}, D_{ijk}\}[-p_4, p_2, p_1, m_\tilde{g}, m_\tilde{u}_m, m_\tilde{u}_m, m_\tilde{d}_n, m_\tilde{d}_n]) \\
  &\to \{D_i, D_{ij}, D_{ijk}\}[-p_4, p_2, m_\tilde{g}, m_\tilde{u}_m, m_\tilde{d}_n, m_\tilde{d}_n])
\end{align*}
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Figure caption:

**Figure 1** The Born diagrams for subprocess $u\bar{d} \rightarrow W\gamma$.

**Figure 2** The one-loop diagrams in the context of the SUSY QCD for subprocess $u\bar{d} \rightarrow W\gamma$.

**Figure 3** The relative corrections of the SUSY QCD for $u\bar{d} \rightarrow W\gamma$ process as the function of $\sqrt{s}$. The solid line corresponds to $m_{\tilde{q}} = 175 \text{ GeV}, m_{\tilde{g}} = 5 \text{ GeV}$. The dashed line corresponds to $m_{\tilde{q}} = 175 \text{ GeV}, m_{\tilde{g}} = 150 \text{ GeV}$.

**Figure 4** The absolute corrections of the SUSY QCD for $u\bar{d} \rightarrow W\gamma$ process as the function of $m_{\tilde{q}}$ when $\sqrt{s} = 500 \text{ GeV}$. The solid-line, dotted-line and dashed-dotted-line correspond to $m_{\tilde{q}} = 150 \text{ GeV}, m_{\tilde{g}} = 300 \text{ GeV}$ and $m_{\tilde{g}} = 500 \text{ GeV}$, respectively.

**Figure 5** The absolute corrections of the SUSY QCD for $u\bar{d} \rightarrow W\gamma$ process as
the function of $m_{\tilde{g}}$ when $\sqrt{s} = 500 \text{ GeV}$. The solid-line, dotted-line and dashed-dotted-line correspond to $m_{\tilde{q}} = 150 \text{ GeV}$, $m_{\tilde{q}} = 300 \text{ GeV}$ and $m_{\tilde{q}} = 500 \text{ GeV}$, respectively.

**Figure 6** The relative corrections of the SUSY QCD for polarized hadronic $W \gamma$ production as the function of $\sqrt{s}$. The polarized parton distribution functions is adopted from Brodsky's parameterization. The solid curve corresponds to $p_L \bar{p}_L$ collision and the dashed curve corresponds to $p_R \bar{p}_R$ collision with $m_{\tilde{g}} = 5 \text{ GeV}$ and $m_{\tilde{q}} = 175 \text{ GeV}$. 
Fig. 4

\[ \sqrt{s} = 500 \text{ GeV} \]
Fig. 6

\[ \frac{\Delta \sigma}{\sigma} \text{ (\%) } \]

\[ \sqrt{s} \text{ (GeV) } \]