RADIAL VELOCITY DETECTABILITY OF LOW-MASS EXTRASOLAR PLANETS IN CLOSE ORBITS

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ABSTRACT

Detection of Jupiter-mass companions to nearby solar-type stars with precise radial velocity measurements is now routine, and Doppler surveys are moving toward lower velocity amplitudes. The detection of several Neptune-mass planets with orbital periods of less than a week has been reported. The drive toward the search for close-in, Earth-mass planets is on the agenda. Successful detection or meaningful upper limits will place important constraints on the process of planet formation. In this paper, we quantify the statistics of detection of low-mass planets in close orbits, showing how the detection threshold depends on the number and timing of the observations. In particular, we consider the case of a low-mass planet close to but not on the 2:1 mean motion resonance with a hot Jupiter. This scenario is a likely product of the core-accretion hypothesis for planet formation coupled with migration of Jupiters in the protoplanetary disk. It is also advantageous for detection because the orbital period is well constrained. We show that the minimum detectable mass is \( M_p \approx 4 M_{\oplus} (N/20)^{-1/3} (\sigma / \text{m s}^{-1}) (P/\text{days})^{1/3} (M_*/M_\odot)^{2/3} \) for \( N \geq 20 \), where \( N \) is the number of observations, \( P \) is the orbital period, \( \sigma \) is the quadrature sum of Doppler velocity measurement errors and stellar jitter, and \( M_* \) is the stellar mass. Detection of few Earth-mass rocky cores will require \( \sim 1 \text{ m s}^{-1} \) velocity precision and, most important, a better understanding of stellar radial velocity “jitter.”

Subject heading: planetary systems

1. INTRODUCTION

Over 100 Jupiter-mass planets have been discovered around nearby stars with precise radial velocity measurements (see Marcy et al. 2003 for a review). As time goes by, the range of periods and amplitudes accessible to these surveys increases, moving to longer periods and lower amplitudes (e.g., Carter et al. 2003; Fischer et al. 2003; Jones et al. 2003). Recently, the detection of Neptune-mass planets with periods less than 10 days has been reported (Butler et al. 2004b; McArthur et al. 2004; Santos et al. 2004). In this paper, we quantify the detectability of low-mass planets in close orbits and discuss the prospects for detecting Earth-mass objects. We first discuss possible origins of such planets and the importance of detecting them.

1.1. The Origin and Importance of Low-Mass Rocky Cores in Close Orbits

In the conventional planet formation scenario (Safronov 1969; Pollack et al. 1996), heavy elements coagulate into terrestrial-planet–like cores prior to the formation of Jupiter-mass planets by subsequent accretion of gas. The critical mass that segregates these two populations is \( M_c \sim a M_\oplus \), determined by the requirement of sufficiently high cooling efficiency in the gas envelope (Stevenson 1982; Bodenheimer & Pollack 1986; Ikoma et al. 2000). In a minimum-mass nebula, both types of planets are thought to form at distances from their host stars comparable to those of planets in our solar system.

As a consequence of their tidal interaction, gas giant planets may undergo orbital migration (Goldreich & Tremaine 1980; Lin & Papaloizou 1986), which is finally halted either by their interaction with their host stars or through the local or global depletion of disks (Lin et al. 1996; Trilling et al. 2002; Armitage et al. 2002; Ida & Lin 2004). In this process, many gas giants are expected to be disrupted, transferring their mass onto the host star (Trilling et al. 2002), perhaps driven by tidal inflation (Gu et al. 2003). Ida & Lin (2004) estimate that 90%–95% of planets that migrate to \( a \leq 0.05 \text{ AU} \) must perish. Recent models of evaporation of material from hot Jupiters (Lammer et al. 2003; Baraffe et al. 2004; Lecavelier des Etangs et al. 2004) find that gas giants may undergo significant mass loss, and perhaps be completely evaporated, during their lifetimes, although the efficiency and rates of evaporation remain uncertain (Yelle 2004). This inference raises the possibility of a class of remnant “hot Neptunes” or rocky cores in close orbits.

An alternative way to produce low-mass planets in close orbits is by resonant capture during migration of a hot Jupiter. Along their migration paths, gas giant planets capture other planets onto their mean motion resonances as in the case of GJ 876 (Lee & Peale 2002; Nelson & Papaloizou 2002; Kley et al. 2004). Mandell & Sigurdsson (2003) also consider the survivability of Earth-mass objects in the wake of Jupiter migration. A natural implication of the scenario of core accretion followed by migration for the origin of hot Jupiters is that they may harbor Earth-mass planets (“hot Earths”) orbiting close to their mean motion resonances (Aarseth & Lin 2005, in preparation). In the gravitational instability scenario, however, both ice giants and terrestrial planets are assumed to emerge long after the formation of gas giant planets (Boss et al. 2002).

Because of the effect of tidal circularization, the semimajor axis of a hot Earth is expected to be slightly inside the mean motion resonance, and its survival is ensured by the relativistic precession induced by the gravitational potential of the host star (Novak et al. 2003, Mardling & Lin 2004). After halting its inward migration, a hot Jupiter may move outward as a result of...
either its tidally induced mass loss (Trilling et al. 2002; Gu et al. 2003) or its tidal interaction with a rapidly spinning host star (Stassun et al. 1999; Dobbs-Dixon et al. 2004). In both cases, the periods of the close-in Earth and hot Jupiter may not be close to commensurability.

The presence of Earth-mass planets close to the external mean motion resonance of a hot Jupiter is also possible. In the wake of a migrating gas giant, the protoplanetary planets in the disk external to its orbit may also undergo type I inward migration (Ward 1986) as a consequence of their own tidal interaction with the disk gas (Goldreich & Tremaine 1980). By inducing the formation of a gap near its orbit, a migrating Jupiter provides a tidal barrier that prevents any migrating terrestrial planets from passing it. But this possibility is uncertain, since type I migration is strongly affected by the poorly known turbulence near the co-orbital region (Koller et al. 2003; Nelson & Papaloizou 2004). If this process is efficient, it could lead to the formation of an isolated hot Earth (Ward 1997; Bodenheimer et al. 2000). Even if type I migration is inefficient, the presence of a hot Jupiter can induce planetesimals to accumulate and form a hot Earth just beyond the outer edge of the gap (Bryden et al. 2000). In the presence of additional long-period gas giant planets, a sweeping resonance may also lead to the inward migration and accumulation of terrestrial planets just outside the orbits of hot Jupiters (Lin et al. 2005).

Based on the core-accretion scenario for planet formation, it is already tempting to infer the existence of terrestrial planets in extrasolar systems from the detection of Jupiter-mass gas giants (Ida & Lin 2004). However, the actual detection of a terrestrial-mass object would be the first confirmation of rocky cores in an extrasolar planetary system (see, however, Kuchner 2003). The number of degrees of freedom is $N - 3$, since there are three parameters in the model. We measure the goodness of fit as a function of trial frequency $\omega$ using the periodogram power $z$, defined as

$$z(\omega) = \frac{\Delta \chi^2 / 2}{\chi^2_\nu},$$

where $\Delta \chi^2 = (N - 1)\chi^2_{N-1} - \nu \chi^2_\nu$, and $\chi^2_{N-1}$ is the reduced $\chi^2$ of a fit of a constant to the data, $(N - 1)\chi^2_{N-1} = \sum (y_j - \langle y \rangle)^2 / \sigma_j^2$. Here, we extend the original Lomb-Scargle periodogram by allowing the mean to float at each frequency (Walker et al. 1995; Nelson & Angel 1998; Cumming et al. 1999), rather than subtracting the mean of the data $\langle y \rangle$ prior to the fit. The periodogram power measures the improvement of $\chi^2$ when the sinusoid is included in the fit, similar to a classical F-test (see Cumming [2004] for a recent discussion).

A typical Doppler measurement error is $3–5$ m s$^{-1}$, although the precision of Doppler surveys continues to improve toward $\sim 1$ m s$^{-1}$ (Mayor et al. 2003; Butler et al. 2004a). However, at these high precisions, stellar “jitter” becomes the limiting factor. Jitter is observed at the few m s$^{-1}$ level, depending on stellar properties such as age, rotation, and level of magnetic activity (Saar et al. 1998; Santos et al. 2000). For simplicity, we add Gaussian noise with amplitude $\sigma = 1$ m s$^{-1}$ throughout this paper, which represents both Doppler errors and stellar jitter. The mass that can be detected scales proportional to $\sigma$, so our results can be rescaled to the appropriate value of $\sigma$. For example, a $5 M_\oplus$ planet with $\sigma = 5$ m s$^{-1}$ has the same detectability as a $1 M_\oplus$ planet with $\sigma = 1$ m s$^{-1}$. We have checked that this scaling applies for the two-planet case, in which the mass of the hot Jupiter is much greater than that of the hot Earth.

Our search procedure is to find the best-fit orbit for the hot Jupiter, subtract it from the data, and search the residuals for periodicities. This is a much simpler approach than simultaneous fitting of both sinusoids and is valid as long as the parameters from the two fits are uncorrelated. We have checked that this is the case and find that only for sampling rates much greater than 1 day$^{-1}$ is there some improvement by simultaneous fitting. An alternative possibility is that the orbital solution for the hot Jupiter is already known from previous measurements, in which case this information can be used to subtract the hot Jupiter’s signal. Therefore, we also consider the
cases in which the full orbital solution or only the orbital period is known in advance.

If the hot Jupiter orbital period is known in advance, we find the best-fit amplitude and phase at that period. If no information is available, we search for the hot Jupiter by evaluating $z$ for periods between 2 and 4 days on a grid with frequency spacing $1/4T$, where $T$ is the duration of the observations. To adequately subtract the hot Jupiter’s signal, it is important to accurately determine its orbital frequency. Therefore, the frequency with the largest periodogram power is used as the starting value for a more accurate search for the maximum of $z(\omega)$. Once the hot Jupiter’s orbit has been subtracted, we calculate $z(\omega)$ for the residuals and find the maximum periodogram power $z_{\text{max}}$ near the 2:1 resonance. We do not include a constant term in the fit to the residuals, giving $\nu = N - 2$. Figure 1 shows an example of the velocity residuals and the periodograms before and after subtraction. The peak at $\approx 1.5$ days in the initial periodogram is due to aliasing of the hot Jupiter’s frequency. After subtraction, the signal due to a $3 M_J$ planet at 1.4 days shows clearly as a peak in $z$.

The importance of accurately determining the hot Jupiter’s orbital period is illustrated by Figure 2, which shows two examples of the velocity residuals after subtraction of the hot Jupiter signal. In the first case (upper panels), we take the hot Jupiter period returned by the periodogram (evaluated with a frequency spacing $1/4T$); in the second case (lower panels), we find a more accurate period estimate by searching for the peak in $z(\omega)$. In the first case, the velocity residuals are dominated by the inadequately subtracted hot Jupiter signal rather than by the hot Earth. How accurately must the hot Jupiter period be known? If the orbital frequency is known to an accuracy $\delta \omega$, the amplitude of the residual part of the hot Jupiter’s signal is $\Delta z \approx A_1 \delta \omega T$, where $A_1$ is the hot Jupiter amplitude. The accuracy to which the hot Jupiter frequency may be determined is $\delta \nu \approx (2\pi/T)(\sigma/\sqrt{N A_1})$ (e.g., Bretthorst 1988), so that

$$\Delta \nu \approx \frac{2\pi \sigma}{\sqrt{N}}.$$  

(4)

independent of $T$ and $A_1$. For large $N$, $\Delta \nu \ll \sigma$, so the residual part of the hot Jupiter signal has no effect on detectability. However, for $N \approx (2\pi)^2$, the residuals from the subtraction contribute a significant additional source of velocity variability. Even when the hot Jupiter period is known in advance, there is

![Figure 1](image1.png)

**Fig. 1.**—Top: Simulated velocity residuals after subtraction of a 0.5$M_J$ mass planet with $P = 3.1$ days and $K = 70$ m s$^{-1}$. We simulate one observation per night for 30 nights, with measurement error of 1 m s$^{-1}$. Bottom: Periodogram evaluated for periods from 1 to 4 days before (dotted line) and after (solid line) subtraction. The strong feature present at $\approx 1.5$ days before subtraction is due to aliasing of the 3.1 day signal. Subtracting reveals the presence of a 3 $M_J$ companion at $P = 1.4$ days. The dashed line shows the detection threshold.

![Figure 2](image2.png)

**Fig. 2.**—Velocity residuals and periodogram after subtraction of the hot Jupiter for two different estimates of the hot Jupiter’s orbital frequency. In the top panel, we use the frequency estimate from the periodogram evaluated with frequency spacing $1/4T$. In this case, the poor subtraction leads to a large scatter in the residuals. In the bottom panel, we determine an accurate estimate of the frequency by finding the peak of $z(\omega)$. The velocity residuals are then dominated by the 5 $M_J$ companion.
an uncertainty in the fitted amplitude $A_1$ of $C_{25}/C_{27}=\sqrt[N_p]{\text{an~int~the~phase~of~}}$ $C_{25}=N_{1}^{1}C_{27}=K_{2}^{N}$, giving an additional velocity scatter for low $N$. Therefore, for small $N$ we expect detection of a hot Earth to be more difficult when the full orbital solution for the hot Jupiter is not specified in advance.

### 2.2. Calculation of Detection Threshold and Detection Probabilities

The significance of the maximum observed periodogram power $z_{\text{max}}$ depends on how often an equally good or better fit would occur purely because of a noise fluctuation. We determine this with Monte Carlo simulations. We generate data sets with a hot Jupiter plus noise, i.e., without a hot Earth, and search for a second companion. The 99% detection threshold $z_d$ is determined as the value of $z$ that is exceeded in only 1% of trials or, alternatively, for which there is a 1% false alarm probability $F$. The detection threshold is indicated in Figure 1 by the dashed line.

We adopt a similar Monte Carlo approach to determine the detection probability. We generate a large number of data sets for each choice of $M_p$ and $N$ and calculate the fraction of trials for which the hot Earth is detected. A random distribution of inclinations is included. Figure 3 shows the detection efficiency as a function of number of observations $N$ for $M_p = 1, 1.5, 2,$ and $3 ~M_\odot$. We simulate one observation per 8 hr night and take observations for successive nights. In these simulations, we search for the hot Earth orbital period between 1 and 1.8 days. Figure 4 shows the effect of changing this frequency range. If the frequency of the hot Earth is specified in advance, the detectability is increased, since the number of “independent frequencies” in the search is less, giving a smaller chance of a false alarm due to a noise fluctuation. This is the well-known “bandwidth penalty” (e.g., Vaughan et al. 1994).

In Figure 5, we show the number of observations needed to detect a given mass 50% of the time. We simulate one observation per 8 hr night and take observations for successive nights. In these simulations, we search for the hot Earth orbital period between 1 and 1.8 days. Figure 4 shows the effect of changing this frequency range. If the frequency of the hot Earth is specified in advance, the detectability is increased, since the number of “independent frequencies” in the search is less, giving a smaller chance of a false alarm due to a noise fluctuation. This is the well-known “bandwidth penalty” (e.g., Vaughan et al. 1994).

In Figure 5, we show the number of observations needed to detect a given mass 50% of the time as a function of the mass $M_p$ scaled by the error $\sigma$. Observations are taken at a random time during each consecutive night. The crosses are for data sets containing only a hot Earth, the triangles for the case in which the hot Jupiter period is known in advance, and the circles for a full search for both orbital periods. The curves are the analytic results. The dotted curve occurs if the number of independent frequencies, $X_i$, is $N_i = T \Delta f$, where $T$ is the duration, and $\Delta f$ is the frequency range used to search for the hot Earth. For the dashed curve, $N_i = 10$. 

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**Fig. 3.** Detection efficiency for hot Earths with masses of 1, 1.5, 2, and $3 ~M_\odot$, and period of 1.4 days, with $\sigma = 1 ~m/s$. We use 10,000 trials to evaluate the detection probability. The hot Jupiter has a mass of $1.0M_\odot$ and a period of 3.0 days.

**Fig. 4.** Effect of the range in orbital period searched on the detection probability. We take $\sigma = 1 ~m/s$ and $M_p = 3 ~M_\odot$. Triangles represent orbital periods ranging from 1 to 4 days, squares 1.3–1.5 days, and circles 1.39–1.41 days. The dotted line is for 1–1.8 days, which is used elsewhere in this paper. As a larger frequency range is searched, $N_i$ increases, giving a larger detection threshold and lower detection probability.

**Fig. 5.** Number of observations needed to detect a planet 50% of the time as a function of the mass $M_p$ scaled by the error $\sigma$. Observations are taken at a random time during each consecutive night. The crosses are for data sets containing only a hot Earth, the triangles for the case in which the hot Jupiter period is known in advance, and the circles for a full search for both orbital periods. The curves are the analytic results. The dotted curve occurs if the number of independent frequencies, $X_i$, is $N_i = T \Delta f$, where $T$ is the duration, and $\Delta f$ is the frequency range used to search for the hot Earth. For the dashed curve, $N_i = 10$. 

is known in advance, and the circles are for the case with a hot Jupiter but with a full search for both orbital periods. For \( N \approx 10^{-20} \), planets with \( M \gtrsim 4(\sigma/1 \text{ m s}^{-1}) M_\oplus \) can be detected. However, detection of a planet with \( M \approx 1(\sigma/1 \text{ m s}^{-1}) M_\oplus \) requires \( N \approx 200 \). For \( N \leq 20 \), detection of the hot Earth after fitting for the hot Jupiter’s signal is harder than detection of the hot Earth alone or with the hot Jupiter orbit fully specified in advance (compare the crosses and circles in Fig. 5). This is likely due to the additional velocity scatter at low \( N \) from inadequate subtraction of the hot Jupiter’s signal (\( \Delta v \gtrsim \sigma \) in eq. [4]).

2.3. Analytic Estimates

In this section, we derive an analytic estimate for the detection threshold, following the approach of Cumming (2004), who discusses the detectability of single planets with radial velocities. First, we determine \( z_d \) semianalytically for Gaussian noise. The cumulative distribution of \( z(\omega) \) for a single frequency is (e.g., Cumming et al. 1999)

\[
\text{Prob}(z > z_0) = \left( 1 + \frac{2z_0}{\nu} \right)^{-\nu/2},
\]

or for large \( N \),

\[
\text{Prob}(z > z_0) \approx \exp(-z_0).
\]

For a given false alarm probability \( F \), the detection threshold is given by

\[
F = 1 - \left( 1 - \text{Prob}(z > z_d) \right)^{N_i},
\]

where \( N_i \) is the number of “independent frequencies” searched. For \( F \ll 1 \),

\[
F \approx N_i \text{Prob}(z > z_d).
\]

For unevenly sampled data, \( N_i \) must be determined by Monte Carlo simulations. However, since the spacing of periodogram peaks is \( 1/T \), a rough estimate is \( N_i \approx T \Delta f \), where \( \Delta f \) is the frequency range searched (Cumming 2004).

We next make an analytic estimate of the detection threshold. In the presence of a signal with amplitude \( K \), the average power is \( z_d = \nu K^2/4\sigma^2 \) (Groth 1975; Scargle 1982; Home & Baliunas 1986; we have accounted for the different normalization). For given \( N_i \) and \( F \), we find the detection threshold \( z_d \) by inverting equation (7) using the analytic distribution (eq. [5]). Then setting \( z = z_d \) gives the signal-to-noise ratio needed to detect the signal (Cumming et al. 2003; Cumming 2004),

\[
K = \sqrt{2\sigma} \left[ \left( \frac{N_i}{F} \right)^{2/(N-3)} - 1 \right]^{1/2},
\]

or

\[
M_{50} \approx \frac{10}{\sqrt{N}} \left( \frac{\sigma}{\text{m s}^{-1}} \right) \left( \frac{P}{\text{days}} \right)^{1/3} \left( \frac{\ln(N_i/F)}{9.2} \right)^{1/2} \left( \frac{M_\oplus}{M_\odot} \right)^{2/3},
\]

where we have also used the fact that the mean value of \( \sin i \) is \( \pi/4 \), and we take \( N_i = 100 \) and \( F = 0.01 \).

Figure 5 compares the analytic results for arbitrary \( N \) (eq. [9]) with our numerical simulations. The dashed and dotted curves show the analytic models for two different estimates of \( N_i \). The agreement is excellent for a single planet. For the two-planet case, equation (12) applies for \( N \approx 20 \) but underestimates \( M_{50} \) for \( N \approx 20 \). For such low \( N \), we expect \( \Delta v \gtrsim \sigma \) (eq. [4]), making detection of the hot Earth more difficult than in the single-planet case.

2.4. Observing Strategy

We now address to what extent the spacing of the observation times affects the detectability of the hot Earth. First, we consider observations made on successive nights, but with different numbers of observations per night. Figure 6 shows the detection probability as a function of number of nights and as a function of number of observations. For a fixed number of observations, increasing the sampling rate reduces the duration of the data, leading to less frequency resolution and therefore smaller \( N_i \). We therefore expect better detectability for a faster sampling rate at a fixed number of observations. This is initially the case, as can be seen by comparing the curves for one per night and five per night in Figure 6. However, for very rapid sampling, a large number of observations must be made before a complete hot Jupiter orbit is sampled, therefore allowing the orbital parameters to be determined accurately enough for a good subtraction. This is likely the reason for the decrease in detectability for more than five observations per night in Figure 6. Although observing more rapidly always leads to detection in a fewer number of nights (left panel of Fig. 6), the number of observations required eventually becomes very large (right panel of Fig. 6). Therefore, there is an optimum observing rate (roughly a few per night for this case, but it depends slightly on the hot Earth mass).

Although observing on successive nights would be possible with a dedicated telescope, current radial velocity surveys are limited by telescope scheduling. Therefore, we have also simulated more realistic observation times for current surveys by observing for three nights each month, with the separation between observing runs randomly chosen between 20 and 40 days. We find that if the hot Jupiter orbital parameters are not known in advance, the detectability can be affected by the observing scheme because of aliasing of the hot Jupiter frequency leading to subtraction of an incorrect orbit. However, in the practical case that the hot Jupiter period is known from previous observations, the hot Jupiter orbit can be adequately subtracted, and the effect of the observing strategy is small.

3. SUMMARY AND DISCUSSION

We have calculated the detectability of low-mass planets in radial velocity surveys and, in particular, a “hot Earth” companion to a hot Jupiter. Detection of such a companion would give important clues to the ordering of the planet formation process and the ubiquity of terrestrial-mass cores around solar-type stars. The velocity amplitude and mass required for a 50% detection rate are given by equations (11) and (12) and shown
In Figure 5, for $N \approx 20$, masses greater than $4 M_\oplus (\sigma/\text{m s}^{-1}) (P/\text{days})^{1/3} (M_* / M_\odot)^{2/3}$ have 50% detectability or better.

Our results apply to a low-mass planet that is near the mean motion resonance with a hot Jupiter or that is isolated. In Figure 7, we show the detection limits of equation (12) compared to the present distribution of known exoplanets. The left panel is for $M_* = 1 M_\odot$, the right panel for $M_* = 0.2 M_\odot$. Lower mass stars have a greater velocity amplitude for a given planet mass, so M dwarfs are the most promising to search for terrestrial-mass bodies. Close orbits are particularly interesting for this case, since they lie within the habitable zone for M dwarfs (orbital periods of a few to tens of days; Kasting et al. 1993; Joshi et al. 1997).

Recently, three Neptune-mass planets have been discovered in close orbits around GJ 436 (Butler et al. 2004b; $M \sin i = 21 M_\oplus$, $P = 2.644 \text{ days}$, $N = 42$, rms $= 5.3 \text{ m s}^{-1}$), $\rho$ Cnc (McArthur et al. 2004; $M \sin i = 14.2 M_\oplus$, $P = 2.808 \text{ days}$, $N = 119$, rms $= 5.4 \text{ m s}^{-1}$), and $\mu$ Ara (Santos et al. 2004;

4 Taken from http://www.exoplanets.org/.

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![Graph](image1.png)

**Fig. 6.**—Detection probability for a $1 M_\oplus$ planet with different numbers of observations made on consecutive nights. We show the detection probability against number of nights (left panel) and number of observations (right panel). A more rapid sampling rate leads to a detection in fewer nights, but at the cost of taking many observations.

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![Graph](image2.png)

**Fig. 7.**—Summary of detection thresholds in the mass–period plane. The dashed lines show 50% detection thresholds for different $N$ and $a$, for $M_* = 1 M_\odot$ (left panel) and $M_* = 0.2 M_\odot$ (right panel). It is assumed that the duration of the observations is longer than the orbital period. The circles show currently detected planets ($M \sin i$), including the three recently announced Neptune-mass candidates. The dotted lines show velocity amplitudes of $K = 1, 3, \text{ and } 10 \text{ m s}^{-1}$ (sin $i = 1$), and the dashed line shows an approximate detection threshold for the Space Interferometry Mission (SIM; assuming $\mu$ mas sensitivity and a 10 pc distance; see Ford & Tremaine 2003).
\( M \sin i = 14 \, M_{\oplus}, \ P = 9.5 \ \text{days}, \ N = 24, \ \text{rms} = 0.9 \ \text{m s}^{-1} \).

These can be seen as the lowest mass planets in Figure 7. Only one of these stars, GJ 436, is an M dwarf. All of these detections lie above the detection limits in Figure 7.

The lowest mass planet detected prior to these recent announcements was HD 49674b (Butler et al. 2002), with \( M \sin i = 0.12 M_{\odot} \) (or \( \approx 40 \ M_{\oplus} \)), \( K \ = 13 \ \text{m s}^{-1} \), and \( P = 5 \ \text{days} \).

The velocity curve presented by Butler et al. (2002) has a noise level that is comparable to current measurement uncertainties, typically \( \approx 0.03 \). Fitting a circular orbit to a Keplerian orbit with eccentricity \( e \) gives a residual scatter \( \approx 0.03 \), where \( A_{1} \) is the amplitude. For a hot Jupiter with \( A_{1} \approx 100 \ \text{m s}^{-1} \), the scatter in the residuals is comparable to the signal from a hot Earth. However, since the Fourier components of a Keplerian orbit are at the orbital period and its harmonics, whereas the hot Earth is expected to lie outside the 2:1 resonance, it should be possible to distinguish between these two possibilities, although simultaneous fitting of the hot Jupiter and Earth orbits may be required. Therefore, we do not expect a significant reduction in sensitivity over the detection thresholds calculated in this paper.

We acknowledge support from NSF through grant AST-9987417 and NASA through grant NAGS-13177. A. C. is supported by NASA through Hubble Fellowship grant HF-01138 awarded by the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., for NASA, under contract NAS 5-26555.
