ON THE QUANTIZATION OF THE N=2 SUPERSYMMETRIC
NON LINEAR SIGMA MODEL

G. Aldazabal and J. M. Maldacena

Centro Atómico Bariloche, 8400 Bariloche,
Comisión Nacional de Energía Atómica,
Consejo Nacional de Investigaciones Científicas y Técnicas,
and Instituto Balseiro, Universidad Nacional de Cuyo - Argentina

A method for quantizing the bidimensional N=2 supersymmetric non-linear sigma model is developed. This method is both covariant under coordinate transformations (concerning the order relevant for calculation) and explicitly N=2 supersymmetric. The operator product expansion of the supercurrent is computed accordingly, including also the dilaton. By imposing the N=2 superconformal algebra the equations for the metric and the dilaton are obtained. In particular, they imply that the dilaton is a constant.
I. INTRODUCTION

The N=2 supersymmetric sigma model appears in string theory in two different contexts. One is the propagation of a N=2 string in a curved background \[1\]. The other is N=1 string compactifications, since in this case N=1 space-time supersymmetry requires a N=2 superconformal theory on the compactified manifold \[2\].

A sigma model has two supersymmetries if and only if it is defined on a Kähler Manifold \[3\].

It is well known that quantum corrections may destroy conformal invariance. Thus, further conditions on the manifold result from requiring conformal invariance in the quantum theory.

In order to compute quantum corrections, it is convenient to maintain the symmetries of the theory throughout the calculations. In this case, the relevant symmetries are: target space reparametrization invariance and worldsheet N=2 supersymmetry. The latter can be kept explicit by using the N=2 superfield formalism \[4\].

Reparametrization invariance is usually made explicit by expanding the fields in terms of normal coordinates around a classical background \[5, 6\]. Though this method works for N=0,1, it cannot be extended straightforwardly to N=2. The reason for this is that the superfield formalism treats holomorphic and antiholomorphic target space coordinates differently, whereas the geodesic equation mixes them. A previous attempt to overcome this problem involved the use of prepotentials in order to solve the superfield chirality constraints \[7\].

Here we propose a method based on the modification of the geodesic equation so that it does not mix holomorphic and antiholomorphic coordinates. The chirality constraints are accomplished through Lagrange multipliers.

We use this method to derive the conformal anomaly. Instead of going through the computation of the \(\beta\) function \[8, 9\] we choose the alternative of calculating the generator algebra in the quantum theory. We obtain the one loop equations for the metric and the dilaton by identifying the symmetry breaking terms \[10, 12\] and by setting them to zero. The paper is
organized as follows: in Section 2 we develop the method, in Section 3 it is applied to calculate the operator product expansion of the supercurrent and the equations for the metric and the dilaton are obtained. Conclusions are presented in Section 4.

II. QUANTIZATION METHOD

An N=2 sigma model must be defined on a Kähler manifold \( \mathcal{M} \). On such a manifold it is possible to choose complex coordinates \((\phi^\mu, \tilde{\phi}^{\bar{\nu}})\) so that the metric is derived from a locally defined potential \( K(\phi, \tilde{\phi}) \) in the following way

\[
g_{\mu\nu} = 0 \quad g_{\bar{\mu}\bar{\nu}} = 0 \quad g_{\mu\bar{\nu}} = K_{,\mu\bar{\nu}} \quad (2.1)
\]

The superfield formulation \(^6\) uses the N=2 superspace spanned by the coordinates (we work in Euclidean space) \( Z = (z, \theta^a, \bar{z}, \bar{\theta}_a) \). Complex conjugation on the worldsheet (denoted with a bar) and complex conjugation in space-time (denoted with tilde) are distinguished, since they correspond to two different symmetries. The coordinates \( \phi^\mu, \tilde{\phi}^{\bar{\nu}} \) are superfields and obey the chirality constraints

\[
\tilde{D}_a\phi^\mu = 0 \quad D_a\tilde{\phi}^{\bar{\nu}} = 0 \quad (2.2)
\]

where the covariant derivatives

\[
D_a = \frac{\partial}{\partial \theta^a} - (\partial_{\theta^a}) \quad \tilde{D}_a = \frac{\partial}{\partial \bar{\theta}_a} - (\partial_{\bar{\theta}_a}) \quad (2.3)
\]

satisfy the commutation relations

\[
\{D_a, D_b\} = 0 \quad \{\tilde{D}_a, \tilde{D}_b\} = 0 \quad \{D_a, \tilde{D}_b\} = 2 \phi_a^b \quad (2.4)
\]

In terms of the Kähler potential the action reads

\[
S = -\frac{1}{4\pi\alpha'} \int d^6Z K(\phi, \tilde{\phi}) \quad (2.5)
\]

The chirality constraints are included in the action by means of Lagrange multipliers (which are spinorial superfields), in the following way...
\[ S_{\text{mult}} = \frac{-1}{4\pi \alpha'} \int d^6\mathcal{Z} \left( \tilde{\lambda}_\mu \tilde{D}\phi^\mu + \lambda_\nu D\tilde{\phi}^\nu \right) \quad (2.6) \]

The action (2.5) is invariant under the holomorphic coordinate reparametrizations
\[
\begin{align*}
\phi^\mu & \rightarrow \phi'^\mu = f^\mu(\phi) \\
\tilde{\phi}^\nu & \rightarrow \tilde{\phi}'^\nu = \tilde{f}^\nu(\phi)
\end{align*}
\quad (2.7)
\]

The background field method consists in calculating quantum corrections around an arbitrary solution \( \phi_0^\mu \) for the classical equations of motion. The field is then expressed as \( \phi^\mu = \phi_0^\mu + \Delta \phi^\mu \), where \( \Delta \phi^\mu \) is the quantum variable. However, this decomposition is not covariant because \( \Delta \phi^\mu \) is not a vector.

Since the various terms of the expansion are evaluated on \( \phi_0 \), a convenient quantum variable is a vector on the tangent bundle at \( \phi_0 \). In the normal coordinate method \[5,6\] this vector is the tangent vector at \( \phi_0 \) to the geodesic that joins \( \phi_0 \) with \( \phi \). This expansion is very useful indeed for computations in N=0 and N=1. Due to the existence of the chirality constraints, this is not the case when there is N=2 supersymmetry. Since the geodesic equation mixes holomorphic and antiholomorphic coordinates, these constraints become cumbersome when written in terms of normal coordinates.

The crucial observation is that an equation which is covariant only under holomorphic coordinate transformations is needed. In fact, covariance under general reparametrizations was lost when complex coordinates were chosen.

The modified equation reads
\[
\begin{align*}
\ddot{\phi}^\mu + \Gamma^\mu_{\rho\delta}(\phi, \tilde{\phi}_0) \dot{\phi}^\rho \dot{\phi}^\delta &= 0 \\
\ddot{\tilde{\phi}}^\nu + \Gamma^\nu_{\rho\epsilon}(\phi_0, \tilde{\phi}) \dot{\tilde{\phi}}^\rho \dot{\tilde{\phi}}^\epsilon &= 0
\end{align*}
\quad (2.8)
\]

The only difference it presents with the usual geodesic equation is that the complex conjugate coordinate is kept fixed. Recall that connections with mixed indices vanish in Kähler manifolds. (2.8) may be interpreted as the equations for the two geodesics that join \( (\phi_0, \tilde{\phi}_0) \) with \( (\phi, \tilde{\phi}) \) and \( (\phi_0, \tilde{\phi}_0) \) with \( (\phi_0, \tilde{\phi}) \). The quantum variables are
\[
\begin{align*}
\xi^\mu & \equiv \dot{\phi}^\mu(t=0) \\
\tilde{\xi}^\nu & \equiv \dot{\tilde{\phi}}^\nu(t=0)
\end{align*}
\quad (2.9)
It is easy to see that $\phi(\tilde{\phi})$ depends only on $\xi(\tilde{\xi})$. This does not mean that $\xi$ is a chiral field. In fact, the passage to normal coordinates is not a mere change of coordinates since it depends on the point $\phi_0(Z)$ which in turn depends on the world sheet coordinates. This may be expressed as $\phi = \phi(\phi_0, \tilde{\phi}_0, \xi)$.

Under this holomorphic normal coordinate expansion the chirality constraint (2.2) reads

$$\tilde{D}_a \phi = 0 \longrightarrow \tilde{D}_a \xi - \left( \frac{1}{2} R^\mu_{\delta_1 \delta_2 \delta_3} \xi^{\delta_1} \xi^{\delta_2} + \frac{1}{3!} \nabla_{\delta_1} R^\mu_{\delta_2 \delta_3} \xi^{\delta_1} \xi^{\delta_2} \xi^{\delta_3} + \cdots \right) \tilde{D}_a \tilde{\phi}_0 = 0 \quad (2.10)$$

and a similar expression is obtained for $D^\mu \tilde{\phi}$ by just replacing $\xi$ by $\tilde{\xi}$. From now on we will concentrate on holomorphic quantities, antiholomorphic ones can be obtained by complex conjugating the expressions. The Lagrange multipliers are expanded in a similar fashion.

By noting that they are vectors under (2.7), their equation reads

$$\nabla^M_t \nabla^M_t \tilde{\lambda}_\mu = 0 \quad \text{with} \quad (\nabla^M_t)_\rho = \delta^\mu_\rho \frac{d}{dt} - \Gamma^\mu_{\rho\delta} (\phi, \tilde{\phi}_0) \dot{\phi}^\delta \quad (2.11)$$

and $\tilde{\chi}_\mu \equiv \nabla^M_t \tilde{\lambda}_\mu(t = 0)$ is the quantum variable.

The solution to this equation can be written in terms of $\phi(\xi, \phi_0, \tilde{\phi}_0)$ as

$$\tilde{\lambda}_\mu = \frac{\partial \phi^\rho}{\partial \xi^\mu} (\tilde{\chi}_\rho + \tilde{\lambda}_\rho^0) \quad (2.12)$$

where $\tilde{\lambda}_\rho^0$ is the classical solution. The total action is written as

$$S = S_0 + S_K + S_H + S_{\text{mult}} \quad (2.13)$$

where $S_0$ is the action evaluated on the classical solution. $S_K$ includes those terms coming from the Kähler potential expansion that contain both $\xi$ and $\tilde{\xi}$

$$S_K = -\frac{1}{4\pi \alpha'} \int K_{\mu \bar{\nu}} \xi^\mu \xi^{\bar{\nu}} + \frac{1}{4} R_{\mu_1 \mu_2 \bar{\nu}_1 \bar{\nu}_2} \xi^{\mu_1} \xi^{\mu_2} \xi^{\bar{\nu}_1} \xi^{\bar{\nu}_2} + \cdots \quad (2.14)$$

$S_H$ includes the terms which contain only holomorphic or antiholomorphic quantum fields

$$S_H = -\frac{1}{4\pi \alpha'} \int \sum_{n=2}^\infty \frac{1}{n!} T_{\mu_1 \cdots \mu_n} \xi^{\mu_1} \cdots \xi^{\mu_n} + \text{c.c.} \quad (2.15)$$

with $T_{\mu_1 \cdots \mu_n} = \nabla_{\mu_1} \cdots \nabla_{\mu_n} K - \tilde{\lambda}_\rho^0 \tilde{D}_0 \tilde{\phi}_0 \nabla_{\mu_1} \cdots \nabla_{\mu_{n-2}} R^\rho_{\mu_{n-1} \nu_{n-1}}$. 
and $S_{mult}$ is the part of (2.6) that contains the quantum multiplier $\tilde{\chi}$

$$S_{mult} = \frac{-1}{4\pi\alpha'} \int \tilde{\chi}_\mu \tilde{D} \xi^\mu - \frac{1}{2} \tilde{\chi}_\mu R^\mu_{\delta_1 \delta_2} \epsilon^{\delta_1 \delta_2} \tilde{D} \tilde{\phi}_0 + \cdots + c.c. \quad (2.16)$$

We eliminate the metric $K_{\mu \nu}(\phi, \tilde{\phi}_0)$ from (2.14) by passing to the local orthonormal frame $e^r_\mu(\phi, \tilde{\phi}_0)$ and by referring all tensors to this frame.

$$e^r_\mu e^\tilde{s}_\tilde{\nu} K_{\mu \nu}(\phi, \tilde{\phi}_0) = \delta^r_{\tilde{s}}$$

$$E_{\mu}^r E_{\tilde{s}}^\tilde{\nu} \tilde{\chi}^\mu \tilde{\chi}^\tilde{\nu} = E_{\mu \nu} \tilde{\chi}_\mu \quad (2.17)$$

The derivative $\tilde{D} \xi$ gives rise to a connection $\omega_{\tilde{D}}^{r \tilde{s}}$

$$(\tilde{D})^{r \tilde{s}} = \delta^{r \tilde{s}} \tilde{D} + \omega_{\tilde{D}}^{r \tilde{s}}$$

where $\omega_{\tilde{D}}^{r \tilde{s}} = e^r_\rho E_{\tilde{D}}^{s \rho} \tilde{D} \tilde{\phi}_0 \quad (2.18)$

The quadratic form appearing in the action can be rewritten as

$$S^{(2)} = \frac{-1}{4\pi\alpha'} \int \tilde{\xi}^r \xi^r + \tilde{\chi}^r \tilde{D} \xi^r + \chi^r \tilde{D} \tilde{\chi} \quad (2.19)$$

and it is not invertible. As usual, this is due to a gauge symmetry of the original action. This symmetry operation, which affects only the Lagrange multipliers, is

$$\tilde{\chi}^r \rightarrow \tilde{\chi}^r + P \tilde{\epsilon}^r \quad (2.20)$$

where we defined the projector

$$P_a^b = -\frac{\tilde{D} \nabla D}{2\partial^2} \delta_a^b + \frac{\tilde{D}^2 D^2}{16\partial^2} \delta_a^b + \frac{D_a(D \nabla)^b}{4\partial^2} \quad (2.21)$$

Therefore only part of the superfield $\tilde{\lambda}$ (that in the Kernel of $P\ (2.21)$ ) is necessary in order to fix the chirality condition (2.2). This is due to the fact that the $\tilde{D}_a, D_a$ operators are not invertible, as results from the commutation relations (2.4). In terms of the quantum variable $\tilde{\chi}^{\tilde{r}}\ (2.12, 2.17)$ the symmetry (2.20) reads

$$\tilde{\chi}^{\tilde{r}} \rightarrow \tilde{\chi}^{\tilde{r}} + E^{r \rho} \partial \phi^\rho P \tilde{\epsilon}^r \quad (2.22)$$

When a gauge fixing condition is chosen, the associated Fadeev-Popov determinant also has a gauge symmetry and so on ad infinitum. The same phenomenon appeared in the
II QUANTIZATION METHOD

previous covariant quantization scheme \cite{7}. Such a process can be ended at some stage by fixing the gauge in a non covariant fashion. We choose the non covariant condition

$$P \tilde{\chi} = 0$$  \hspace{1cm} (2.23)$$

The ghost action is obtained by performing a gauge transformation of the gauge fixing condition in the usual way

$$b^r P \delta \tilde{\chi}^\phi = b^r P E^\mu_{\rho} \partial_{[\phi} \mu \partial_{\rho]} \mu \equiv L_{gh} (b, c) \hspace{0.5cm} S_{gh} = \int d^6 Z (L_{gh} + \tilde{L}_{gh})$$  \hspace{1cm} (2.24)$$

However, as the symmetry (2.22) has a fermionic parameter, the ghosts are bosonic superfields. Note that only $\xi (\tilde{\xi})$ appears in $L_{gh}$ ($\tilde{L}_{gh}$). As we said above, the ghost action has a new gauge symmetry because $P$, being a projector, has zero modes. We fix it with the conditions

$$S c_\mu = 0 \hspace{1cm} S^\dagger b^r = 0$$  \hspace{1cm} (2.25)$$

where $S = 1 - P$ and $S^\dagger$ is the operator obtained by integrating the operator $S$ by parts. The new Fadeev-Popov determinant decouples because the gauge condition (2.23) is not covariant. We will see, however, that for our calculation the ghosts do not contribute.

The reason why we cannot find a fully explicitly covariant quantization may be understood from the fact that the one loop counterterm: $\delta K \sim \frac{1}{\epsilon} \log \det (K_{\mu \rho})$ is not a scalar under holomorphic reparametrizations. Of course, once integrated over the whole superspace an invariant quantity is obtained. If the theory were completely covariant this counterterm could not possibly appear. In this approach, this divergence emerges from the non-covariant ghost action, in particular, from the superdeterminant coming from its quadratic part.

We take as the free action the part of (2.19) which does not contain the connection. This separate treatment of the connection produces a new source of non covariance, which is also present in the formalism used for the cases N=0,1. The propagators for the fields are

$$\langle \xi^r (z) \tilde{\xi}^s (z') \rangle = -4 \pi \alpha' \frac{\tilde{D}^2 D^2}{16 \delta^r_s} \delta(z - z')$$  \hspace{1cm} (2.26)$$
\[ \langle \tilde{\chi}_a(z)\xi^s(z') \rangle = -4\pi\alpha' \left( -\frac{\partial D_a}{2\partial^2} + \frac{\tilde{D}_a D^2}{16\partial^2} \right) \delta^s_s \] (2.27)

\[ \langle \chi_a(z)\bar{\chi}^a(z') \rangle = -4\pi\alpha' \left[ \frac{\partial^b}{2\partial^2} \left( \frac{-D^2 D^2}{16\partial^2} \right) + \frac{D_a \tilde{D}^b}{16\partial^2} + \frac{1}{8} \frac{D^2 \tilde{D}^2}{16\partial^2} \right] \delta^s_s \] (2.28)

Note that the propagators involving \( \tilde{\chi}, \chi \) have a softer ultraviolet behavior. In fact we will see that power counting arguments can be used to discard many terms from the expansion.

III. CALCULATION OF THE CONFORMAL ANOMALY

Based on ideas of Banks et al \[10,12\], we calculate the conformal anomaly by evaluating the generator algebra in the quantum theory. This algebra can be extracted from the divergent terms when \( Z \to Z' \) in the supercurrent operator product expansion. The calculation is accomplished perturbatively in \( \alpha' \).

The N=2 superconformal algebra is the direct sum of holomorphic and antiholomorphic sectors. From now on we will consider only the holomorphic sector, which is described by the supercurrent

\[ J_z = j_z + \tilde{\theta}_z S_{zz} - \theta_z \tilde{S}_{zz} + \theta_z \tilde{\theta}_z T_{zz} \] (3.1)

where \( j_z \) is the generator of \( U(1) \) transformations, \( S_{zz} \) and \( \tilde{S}_{zz} \) are associated with the two supersymmetries and \( T_{zz} \) is the stress-energy tensor. In terms of the operator product expansion the algebra reads \[13\]

\[ J_z(Z)J_z(Z') \sim \frac{4c}{(\Delta Z)^2} + \frac{4\delta_z \delta_z}{(\Delta Z)^2} J_z(Z^c) + \frac{2}{\Delta Z} (\delta_z D_z - \delta_z \tilde{D}_z) J_z(Z^c) \] (3.2)

where

\[ \Delta Z = z - z' - \theta_z \tilde{\theta}_z' - \tilde{\theta}_z \theta_z' \quad \delta_z = \theta_z - \theta_z' \quad \tilde{\delta}_z = \tilde{\theta}_z - \tilde{\theta}_z' \]

\[ Z^c = \frac{Z + Z'}{2} = \left( \frac{z + z'}{2}, \frac{\theta_z + \theta_z'}{2}, \frac{\tilde{\theta}_z + \tilde{\theta}_z'}{2} \right) \quad c = \text{central charge} \]

In our model the supercurrent is
III CALCULATION OF THE CONFORMAL ANOMALY

\[ J_z = \frac{-2}{\alpha'} \tilde{D}_z \tilde{\phi} D_z \phi^\mu K_{\mu\nu} (\phi, \tilde{\phi}) \]  

(3.3)

The dilaton field can be included by adding a new term to the current

\[ J^{\text{dil}}_z = 2[\tilde{D}_z, D_z] \Phi(\phi, \tilde{\phi}) \]  

(3.4)

The situation is similar in the N=0,1 cases where the dilaton disappears from the action once the world sheet gravity or supergravity is gauge fixed, but it still appears in the stress energy tensor.

We calculate the OPE up to one loop for the less divergent terms and up to two loops for the central charge term. As (3.4) is one order higher in $\alpha'$, dilaton graphs need less loops for a given order in $\alpha'$. Only those terms which are proportional to $\theta_z, \tilde{\theta}_z, \theta'_z, \tilde{\theta}'_z$ need to be considered because other terms (e.g. one with $\theta_z$) are related to the operator product of the auxiliary components of the supercurrent. Dimensional regularization is used to deal with UV divergences. IR divergences are regulated with a mass cutoff and, though present in the propagators, disappear from the final result.

Keeping in mind that only terms divergent when $Z \rightarrow Z'$ are needed, power counting arguments are useful in order to discard some terms. The superspace volume element $d^6 Z$ and the propagator $\langle \xi \tilde{\xi} \rangle$ have dimension zero. The derivatives $D_a, \tilde{D}_a$ have dimension $\frac{1}{2}$. The supercurrent has dimension one and only positive dimension terms are to be evaluated. If a derivative acts on a background field, then the dimension of the $\Delta Z$ dependent part gets reduced. This occurs when a vertex coming from (2.15) with all incoming (or all outgoing) $\langle \xi \tilde{\xi} \rangle$ propagators is present. One of the $\tilde{D}^2$ acting on the $\langle \xi \tilde{\xi} \rangle$ lines can be integrated by parts and applied to the tensor evaluated in $\phi_0$, reducing the effective dimension by one. This is a highly desirable circumstance because those terms are not invariant under a Kähler gauge transformation, i.e. $K \rightarrow K + f(\phi) + \tilde{f}(\tilde{\phi})$ under which the metric is invariant. A similar argument is applied for the ghost loops with more than one incoming line because they also have only incoming or only outgoing $\langle \xi \tilde{\xi} \rangle$ lines. Similarly, each $\langle \xi \tilde{\chi} \rangle$ propagator decreases the dimension in $\frac{1}{2}$ and each $\langle \chi \tilde{\chi} \rangle$ line in 1. Therefore no $\langle \chi \tilde{\chi} \rangle$ will appear.
The order zero in $\alpha'$ graphs contributing to $\langle J_z(Z)J_z(Z') \rangle$ are shown in figure 1. In computing mean values nothing is lost because $\phi_0$ is an arbitrary classical solution.

In order to extract the symmetry breaking terms, the right hand side of (3.2) is substracted. Therefore it is necessary to compute $\langle J \rangle$ via the graphs shown in figure 2.

Some graphs are UV divergent but they cancel when the above mentioned difference is performed. Divergences should in fact be eliminated considering the contribution of the one loop counterterm since this is a one loop calculation. It can be verified, however, that the contribution of the counterterm also cancels when the difference is performed. The results of the graphs that involve the noncovariant connection $\omega^{r \tilde{s}}$ must be recovariantized. These would be indeed covariant if the graphs with two connection insertions had been included. In the cases $N=0,1$ these noncovariant terms are also present but, contrary to this case, they cancel among themselves. In those cases the action has a quadratic term proportional to the curvature that is here instead hidden in the connection.

The central charge term is computed up to first order in $\alpha'$ (two loops), the graphs are shown in figure 3.

Adding these results we obtain

$$\langle J_z(Z)J_z(Z') \rangle - \frac{4 \delta_z \delta_z}{(\Delta Z)^2} \langle J_z(\mathcal{Z}) \rangle - \frac{2}{\Delta Z} (\delta_z D_z - \delta_z \tilde{D}_z) \langle J_z(\mathcal{Z}) \rangle \sim$$

$$\sim \frac{4D - 8\alpha'(-\frac{1}{4} R + \nabla^\mu \nabla_\mu \Phi - \nabla^\mu \Phi \nabla_\mu \Phi)}{(\Delta Z)^2} +$$

$$+ \frac{2 \Delta \tilde{Z}}{(\Delta Z)^2} \left\{ \delta_z D_\tilde{z}[(R_{\nu \mu} - 2 \Phi_{\nu \mu}) \tilde{D}_z \Phi_0] \delta_z D_\tilde{z}[(R_{\nu \mu} - 2 \Phi_{\nu \mu}) \tilde{D}_z \Phi_0] + \right.$$

$$- 2 \delta_z D_\tilde{z}(\nabla_\nu \nabla_\rho \Phi \tilde{D}_z \Phi_0) - 2 \delta \tilde{D}_z(\nabla_\nu \nabla_\rho \Phi D_z \Phi_0) \left\}$$

for the symmetry breaking terms. If we set them to zero we obtain the following conditions on the manifold

$$R_{\nu \mu} - 2 \nabla_\nu \nabla_\mu \Phi = 0$$

$$\nabla_\mu \nabla_\rho \Phi = 0 \quad \nabla_\nu \nabla_\rho \Phi = 0$$

$$-\frac{1}{4} R + \nabla^2 \Phi - (\nabla \Phi)^2 = 0$$

(3.6)
The term proportional to the dimension cancels with the contribution of the corresponding
ghosts for \( D = 2 \) (\( D \) = complex dimension) for the N=2 string and it is \( D = 3 \) in superstring
compactifications. In this case it is easy to see that the dilaton is set to a constant by
the equations (3.6). By rearranging the equations (3.6) the condition \( \nabla^2 \Phi = 2(\nabla \Phi)^2 \) is
obtained. Integrating both sides and using that we are in a compact euclidean space, we
obtain \( \nabla \Phi = 0 \). In the case of the N=2 string, it is not a new degree of freedom (only zero
momentum states are allowed). This should be expected from the fact that the only physical
state is the vacuum [14], whose expectation value is described by the Kähler potential. If the
dilaton is a constant we obtain the Ricci-flatness condition in accordance with the \( \beta \) function
method result [15]. Note as well that the equations (3.6) are simply the ones obtained [9,11]
for the N=1 sigma model in the case of a Kähler metric, as is expected from the universality
properties of supersymmetric sigma models: the models with extended supersymmetry are
obtained from the N=1 model restricting the target manifold.

IV. CONCLUSIONS

The method we have developed maintains the N=2 supersymmetry and is partially co-
variant. A reason explaining why a completely covariant method is not possible is given.
Our method is effectively covariant for some calculations, as the ghosts can be absent. In
fact, if this method is applied to the calculation of the \( \beta \) function, it attains its maximum
simplicity because in that case power counting arguments leave only vertices from \( S_K (2.14) \).
This is encouraging for undertaking higher loop computations. Previous calculations in this
model were done either using a non covariant method [8] or a non supersymmetric one [12].

The calculation of the supercurrent operator product expansion shows, on the one hand,
how the method works and, on the other, makes it clear how the conformal anomaly breaks
the algebra. The dilaton field was included and severe constraints on it were found, as could
be expected from the N=2 string spectrum [14].

This method can also be extended to treat the models considered in ref. [16] which
include the antisymmetric tensor.

Acknowledgments:

We thank F. Toppan for some important observations and R. Trinchero for helpful conversations.
REFERENCES

[1] H. Ooguri and C. Vafa Princeton preprint HUTP-90/A024 and HUTP-91/A003.

[2] P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258 (1985) 46,
W. Boucher, D. Friedan, A. Kent, Phys. Lett. 172B (1986) 316,
A. Sen, Nucl. Phys. B278 (1986) 289; Nucl. Phys. B284 (1987) 423,
T. Banks, L. J. Dixon, D. Friedan, E. Martinec, Nucl. Phys. B299 (1988) 613,
D. Gepner “Lectures on N=2 String Theories” Summer School in High Energy Physics and Cosmology, Trieste, (1989).

[3] L. Alvarez-Gaume and D. Freedman, Commun. Math. Phys. 80, (1981) 443.

[4] B. Zumino Phys. Lett. 87b (1979) 203.

[5] J. Honerkamp Nucl. Phys. B36 (1972) 130.

[6] L. Alvarez-Gaume, D. Feedman and S. Mukhi, Ann. Phys. 134 (1981) 85.

[7] P. Howe G. Papadopoluos and K. Stelle Phys. Lett. B174 (1986) 405.

[8] M. T. Grisaru, A. E. M. Van de Ven and D. Zanon, Phys. Lett. B 173 (1986) 423.

[9] C. Callan, D. Friedan, E. Martinec and M. Perry, Nucl. Phys. B262 (1985) 593.

[10] T. Banks, D. Nemeschansky and A. Sen Nucl. Phys. B277 (1986) 67.

[11] G. Aldazabal F. Hussain and R. Zhang, Phys. Lett. 185b (1987) 89; “Superconformal invariance and superstrings in background fields” ICTP preprint, IC/86/400.

[12] T. Itoh and M. Takao, Int. J. Mod. Phys. A12 (1990) 2265.

[13] P. Di Vecchia, J. Petersen and H. Zheng, Phys. Lett. 162B (1985) 327.

[14] M. Ademollo, L. Brink, A. D’Adda, R. D’Auria, E. Napolitano, S. Sciu to, E. Del Giudice, P. Di Vecchia, S. Ferrara, F. Gliozzi, R. Musto, R. Pettorino and J. H. Schwarz,
IV CONCLUSIONS

Phys. Lett. B271 (86)93 , Nucl. Phys. B111 (1976) 77.

[15] L. Alvarez-Gaume and D.Z. Freedman, Phys. Rev. D22 (1980) 846.

[16] S.J. Gates, C.M. Hull and M. Roček, Nucl. Phys B248 (1984) 157.
FIGURE CAPTIONS

**Figure 1:**
Graphs of order zero in \( \alpha' \) contributing to \( \langle JJ \rangle \). Only graphs not related by interchange of \( Z \leftrightarrow Z' \) or by complex conjugation (obtained reversing all lines) are shown.

- \( \langle \xi \xi \rangle \) propagator stands for a \( \langle \xi \xi \rangle \) propagator
- \( \langle \eta \xi \rangle \) propagator stands for a \( \langle \eta \xi \rangle \) propagator
- \( \xi \) propagator stands for a propagator
- \( \eta \) propagator stands for a propagator
- \( \times \) stands for a supercurrent
- \( \otimes \) stands for the dilaton
- \( \times \) stands for a supercurrent
- \( \otimes \) stands for the dilaton
- \( R \) stands for a curvature
- \( \omega \) stands for a connection.

**Figure 2:**
Graphs of order zero in \( \alpha' \) contributing to \( \langle J \rangle \).

**Figure 3:**
Graphs of order one in \( \alpha' \) contributing to \( \langle JJ \rangle \).