An Approximate Model for Local Strain Variation over Material Thickness and Its Applications to Thick Plate Rolling Process

Chang-Ho MOON and Youngseog LEE

1) Rolling Technology & Process Control Group, Technical Research Laboratories, POSCO, Pohang, 790-785, Korea.
2) Department of Mechanical Engineering, Chung-Ang University, Seoul, 156-756, Korea. (Corresponding author)

(Received on October 17, 2008; accepted on January 7, 2009)

This paper presents a three-parameter approximate model which computes the local strain variation over thickness direction of thick material in the roll gap. The three parameters were determined through finite element analysis. With the proposed model, we then carried out a series of plate rolling simulation to examine the effect of the arithmetic average aspect ratio (ratio of contact length between work roll and material to mean material thickness in the roll gap) and reduction ratio on the local strain variation over material thickness as material goes through rolling at many passes.

Results reveal that a certain amount of relative difference between local strain at center and that at surface always exists as arithmetic average aspect ratio increases for whole plate rolling process. It also shows that the magnitude of local strain at material center is only 69% of that at surface during rolling no matter how we regulate incoming material thickness, radius of work roll, reduction ratio, roll speed and friction condition when arithmetic average aspect ratio is greater than 1.0. It has been found that the heavy reduction with arithmetic average aspect ratio, 0.9 might be an optimum condition for refining grain size over material thickness in an approximately uniform manner.

KEY WORDS: local strain variation; material thickness; arithmetic average aspect ratio; peening effect; plate rolling.

1. Introduction

In flat rolling, the amount of local deformation of material in the roll gap always varies over material thickness. The amount of local deformation variation is dependent on a contact length-roll gap ratio \( l_d/H_m \), which changes all the time as material is rolled in order at subsequent passes. Finish rolling sequence in strip rolling where material thickness generally is thin has high contact length-roll gap ratio. In this case, we can assume that the local strain variation over material thickness at the exit of roll gap is not so significant. Note that in this study, the local strain denotes the transverse strain to thickness direction. Hence, we calculate strain in a given pass in an average sense by taking a logarithm of incoming material thickness and outgoing one, which is compression strain of material at the exit of roll gap.

However, if the material to be rolled is thick enough, the local strain variation over material thickness at the exit of roll gap is considerable. Note that ‘local strain at the exit of roll gap’ is expressed as ‘local strain’ throughout this study for convenience. Under this circumstance, local strain at the contact region is much bigger than that at the center region. This phenomenon is particularly noteworthy when thick material is rolled with smaller reduction ratio, which generally occurs in rough rolling sequence of plate mill. Researchers studying hot flat rolling process have paid attention to this strong non-homogeneity of local strain variation over material thickness since austenite grain size varies over the material thickness to a great extent due to different volume fractions re-crystallized by non-homogeneous local strain variation over thickness. Note the grain size is influenced by not only the amount of strain but also strain rate, temperature and interpass time between passes.

A great effort was devoted to develop a mathematical model which calculates the local strain variation. This kind of mathematical model should have an explicit form and a reliable accuracy so that computation can be completed within very short time to overcome limitations of its practical applications. This is because it is generally coupled with re-crystallization models which predict evolution of gain size of material as a function of entry mean temperature, exit mean strain rate and exit mean strain in every pass for the entire rolling sequence.

Zouhar and Lachmann proposed a kinematic strain model which could take into account the non-homogeneity of local strain variation over material thickness. Fundamental idea of their model is to include the shear strain as well as the logarithmic compression strain over thickness direction in the von Mises equation for plane strain deformation. To determine the values of shear and compression strain terms in the model, they performed flat rolling experiment with a square-grid steel specimen. Their approach is good but needs very delicate experiments whenever process vari-
ables such as roll diameter, material dimension, frictional condition and roll speed are changed. Hence, practical use of the kinematic strain model was quite limited.

Kim and Hwang\(^{13}\) developed a data regression-based model to calculate the local strain variation over thickness of strip. Their model took into account the effect of the process variables mentioned above. Five parameters in their model were determined from FE simulation of strip rolling. Since their model has many parameters it gives high prediction accuracy as far as it is applied to the finishing stands in strip rolling mill. However, their model becomes problematic to be applied to the rough rolling sequence of plate mill where the thickness of material is about 7–8 times larger than that strip thickness.

In this paper, we propose a three-parameter approximate model which computes the local strain behavior over thickness direction of thick material during rolling. Each parameter in the proposed model is expressed in terms of process variables such as roll speed, incoming thickness of material, its outgoing thickness, radius of work roll and contact length. The three parameters are determined through finite element simulation of plate rolling. The model proposed in this study is especially useful when the contact length-roll gap ratio, \(l_d/H_m\), is low, say, less than 1.0. The rough rolling sequence of an actual plate mill usually has low value of contact length-roll gap ratio.

We then carried out a series of numerical analysis to investigate the effect of the process variables on the amount of local strain variation over material thickness. We also studied how the local strains at material center and its surface behave while the contact length-roll gap ratio varies.

Finally, the proposed model was applied to two draft schedules being used for POSCO No. 2 plate mill to examine the local strain variation over material thickness at each pass while incoming material thickness, radius of work roll, reduction ratio, roll speed and friction condition at each pass are changed.

2. Local Strain Variation over Material Thickness

Figure 1 shows the strain contours of material in the roll gap when the work roll radius for two cases is the same but material thickness is different. Contours were computed from rigid-plastic finite element analysis. Here we focus on the transverse strain, \(\varepsilon_y\), strain to thickness direction at the exit, which is defined as local strain in this study. On the basis of the exit, local strain variation over thickness in Fig. 1(b) is much more non-homogeneous than that in Fig. 1(a). Hence, we may say that material in the roll gap (Fig. 1(a)) is subject to a weak non-homogeneous deformation state in an average sense if material thickness is relatively thin in comparison to the radius of work roll. On the other hand, we might say that material in the roll gap is under a strong non-homogeneous deformation state when thick material is rolled with smaller reduction ratio as shown in Fig. 1(b).

Criterion for the strong and weak non-homogeneous deformation state is based on arithmetic average aspect ratio (or contact length-roll gap ratio), which will be explained later.

It should be pointed out that the deformation state of material in the roll gap in Fig. 1(b) is rather close to that in forging. Hence, this type of deformation state is in general called peening.\(^{1}\) It is customary to say that ‘peening effect’ becomes important when thick material is rolled with smaller reduction ratio. Under this kind of rolling condition, local deformation owing to contact between work roll and material does not transfer entirely to its center region. It is rather concentrated on the region near surface. As a result, strains are severely non-uniform along the thickness direction. To describe the peening effect quantitatively, we use arithmetic average aspect ratio, as follows

\[
s = \frac{l_d}{H_m} = \frac{2}{2 - r} \sqrt{\frac{R_r}{H}} \quad \text{...............}(1)
\]

\(R_r\) denotes the radius of work roll. \(H\) and \(r\) stand for incoming thickness of material and reduction ratio in a given pass. \(l_d\) and \(H_m\) represent the contact length at the interface and mean material thickness in roll the gap. Arithmetic average aspect ratio is equivalent to contact length-roll gap ratio mentioned in the Introduction.

The value of \(s\) in a given pass becomes high if reduction ratio is large and incoming thickness of material is thin. Hot strip rolling process usually has the high value of \(s\), say, \(s > 1\). On the contrary, rough rolling sequence (or roughing mill) of plate mill where reduction ratio is small but incoming thickness of material is thick (70–300 mm) has usually the low value of \(s\), say, less than 1.0.
Figure 1(c) illustrates the local strain variation over material thickness during rolling for different arithmetic average aspect ratios. ‘Normalized half thickness=1’ and ‘Normalized half thickness=0’, respectively, indicate the surface and center of material. In the case where incoming thickness of material is very thick (H=230 mm), local strain variation over thickness is quite different from that of thin material (H=9.5 mm) at region near surface.

3. Approximate Model for Local Strain over Thickness Direction

3.1. Modeling

A three-parameter approximate model which computes local strain over material thickness is proposed when the material thickness is thick. Since the local strain increases nonlinearly from the center (y=0) to surface (y=1), we can have a relationship as followings

\[
\varepsilon(y) = \varepsilon_{\text{max}} - \left( \varepsilon_{\text{max}} - \varepsilon_c \right) \tan^{-1} \left( \alpha(1-y) \right) \tan^{-1} \alpha \quad \text{(2)}
\]

where \( y \) denotes the normalized half thickness. \( \varepsilon_c \) and \( \varepsilon_{\text{max}} \) represents strain at center and maximum strain at surface. If \( \gamma \) equals 1.0, \( \varepsilon(y) = \varepsilon_{\text{max}} \). When \( \gamma \) comes to 0.0, \( \varepsilon(y) \) becomes \( \varepsilon_c \). The boundary conditions of the model are then satisfied. The role of shape parameter, \( \alpha \) is illustrated in Fig. 2. As the shape parameter increases, the non-homogeneity of local strain variation over thickness reduces. However, some small amounts of non-homogeneity always exist near the surface as a result of rolling itself even through \( \varepsilon \) increases drastically. Maximum strain can be obtained through manipulating Eq. (2)

\[
\varepsilon_{\text{max}} = \frac{\bar{\varepsilon} - \beta \varepsilon_c}{1 - \beta} \quad \text{.............................(3)}
\]

where

\[
\beta = 1 - \frac{\ln(1 + \alpha^2)}{2\alpha \tan^{-1} \alpha} \quad \text{and} \quad \bar{\varepsilon} = \int_0^1 \varepsilon(y)dy \quad \text{.............................(4)}
\]

\( \bar{\varepsilon} \) indicates mean strain over thickness. Here we have another parameter, \( \beta \).

We can make the parameter, \( \beta \) non-dimensional by substituting the ratio of strains defined as \( \eta = \varepsilon_{\text{max}}/\bar{\varepsilon} \) and \( \eta_c = \varepsilon_c/\bar{\varepsilon} \) into Eq. (3)

\[
\beta = \frac{\eta_{\text{max}} - 1}{\eta_{\text{max}} - \eta_c} \quad \text{.............................(5)}
\]

Note that \( \eta_{\text{max}} \) is always bigger than 1.0 because the maximum strain at surface is inevitably bigger than the mean strain owing to shearing near surface during rolling. \( \eta_c \) is less than 1.0 or equal to 1.0 since local deformation at the contact surface does not penetrate into the center region fully. The parameter, \( \beta \) in Eq. (5) becomes near 1.0 if \( \eta_c \) approaches to 1.0. In this light of this information, a condition that \( \beta \) comes close to 1.0 is called an ideally uniform deformation condition, i.e., local strain variations over thickness is very small. Therefore, in this study, \( \beta \) is called a uniformity parameter, which indicates a degree of uniformity of local strain variation over material thickness.

3.2. Determination of the Parameters of the Model

The parameters used in the proposed model, i.e., shape parameter, \( \alpha \), strain at center, \( \varepsilon_c \) and mean strain over thickness, \( \bar{\varepsilon} \) might be affected by process variables such as roll speed, radius of work roll, incoming material thickness, incoming mean temperature of material, reduction ratio and arithmetic average aspect ratio. The ranges of the process variables are summarized in Table 1.

Eulerian finite element method was adopted to determine the parameters. The governing equations for rigid-viscoplastic deformation of the material, namely the equilibrium equation, constitutive relationship and the incompressibility condition are stated. The boundary conditions at the surface are prescribed to fulfill the contact condition by penalizing the normal velocity of the material relative to the work roll at the contact surface. Variational principle was applied to the above boundary value problem that results in a set of non-linear algebraic equations that may be solved either by the Newton–Raphson method. Due to work roll-plate contact and interaction between the thermal and mechanical behavior of the rolled material, the transport phenomena in roll gap is considerably interdependent. In order to resolve the coupled aspects of the problem, an iterative technique was adopted.

The strain at center of material thickness (see Fig. 2) can be expressed as follows

![Local strain, \( \varepsilon(y) \)]
\[ \varepsilon_r = -1.1897 + (2.1157 - 0.6349s + 1.1931r)r \\
- 0.0492 \ln r + 0.5009s - 0.0255 \ln s + \frac{0.8392}{\cosh s} \ldots(6) \]

\( s \) and \( r \) denote, respectively, the arithmetic average aspect ratio and reduction ratio in a given pass. The mean strain over thickness is given in the following form

\[ \bar{\varepsilon} = -0.5133 + (1.2973 - 0.2647s + 1.4947r)r \\
+ 0.0350 \ln r + 0.3007s - 0.0865 \ln s + \frac{0.4889}{\cosh s} \ldots(7) \]

Meanwhile the shape parameter, \( \alpha \) (see Fig. 2) is expressed as a function of friction coefficient, \( \mu \), angular velocity, \( \omega \) and contact length between work roll and material, \( l_d \) in addition to arithmetic average aspect ratio, \( s \) and reduction ratio, \( r \).

For strong non-homogeneous deformation state (s\( \leq \)1)

\[ \alpha = \frac{\exp[3.2412 + 0.0310\omega - 6.2726s^2 \times \ln s]}{s^2 \cdot e^{1.4556 - 1.4999\mu}} \times \frac{\exp(16.5340r)}{\exp(3.3646 - 1.5120\mu + 0.0064d)} \ldots(8) \]

For weak non-homogeneous deformation state (s\( > \)1)

\[ \alpha = \frac{s^{1.9324 - 4.3470\mu}}{r^{1.8004 - 2.6475\mu}} \frac{\exp(0.0094d + 7.1459\mu + 0.0190\omega)}{\exp(2.9581 + (2.8432 - 0.1309s^3)r)} \ldots(9) \]

**Figure 3** illustrates the effect of reduction ratio, friction coefficient and roll speed on shape parameter, \( \alpha \) while the arithmetic average aspect ratio, \( s \) varies. Figure 3(a) shows that \( \alpha \) is influenced significantly by reduction ratio, \( r \) when the strong non-homogeneous deformation state prevails in the roll gap, i.e., s\( \leq \)1. Shape parameter rapidly increases as reduction ratio increases. In contrast, the effect of reduction ratio on \( \alpha \) is very little in case of the weak homogeneous deformation state, i.e., s\( > \)1. Figure 3(b) points up that the variation of friction coefficient has tiny effects on shape parameter. The change of shape parameter owing to roll speed is ignorable, as shown in Fig. 3(c).

**4. Results and Discussion**

We have carried out a series of plate rolling simulation using Eq. (2) through Eq. (9) while reduction ratio and incoming material thickness in a given pass are changed. Note that the variation of reduction ratio and thickness of material result in the change of arithmetic average aspect ratio, \( s \). In Fig. 4, the local strain variation over thickness predicted from the proposed approximate model (solid line) is compared with those computed from finite element analysis (symbols with empty circle) for four different reduction ratios, \( r \) and arithmetic average aspect ratios, \( s \). In overall, those are consistent each other.

**Figure 5** shows the effect of reduction ratio on strains such as maximum strain, mean strain and strain at center. All strains increase as the reduction ratio increases in both the strong non-homogeneous deformation state (s=0.6) and the weak non-homogeneous deformation state (s=1.2).

![Figure 3](image_url)

**Fig. 3.** Effect of process variables such as (a) reduction ratio, (b) friction coefficient and (c) roll speed on shape parameter, \( \alpha \) while the arithmetic average aspect ratio, \( s \) varies. Fric- tion coefficient=0.4, reduction ratio=0.2, the radius of work roll=580 mm and rolling speed=2.1 m/s.

![Figure 4](image_url)

**Fig. 4.** Local strain variation over thickness predicted from approximate model proposed in this study is compared with those from finite element analysis while reduction ratio, \( r \) and arithmetic average aspect ratio, \( s \) vary. Radius of work roll is 580 mm, the angular velocity is 4.2 rad/s and friction coefficient is 0.4. Solid lines indicate the strains computed from the approximate model and symbols with empty circle the ones calculated by FE analysis.

Under the same reduction ratio, say, \( r=0.2 \), the mean strain with \( s=0.6 \) is 0.35 and that with \( s=1.2 \) is 0.3. This indicates that high reduction ratio might be efficient for refining grain size in an average sense if material in the roll gap is under the strong non-homogeneous deformation state. This result is coincident with that of hot compression test performed by Ksdpar and Pawelski. They reported that giving high reduction ratio into material from the initial compression stages, which corresponds to the strong non-homogeneous deformation state, is effective for refining the
austenite grain size on the average.

Figure 6(a) shows the effect of arithmetic average aspect ratio, \( s \) on the shape parameter, \( \alpha \). The shape parameter is scattered largely by different reduction ratios when \( s \) is in the range of 0.2–0.9. \( \alpha \) increases as \( s \) does up to 0.9. But as \( s \) approaches 1.0, the value of shape parameter is rapidly decreased and converged to about 3.5–4.2. As \( s \) increases more than 1.0, the value of shape parameter remains almost constant. It points out that one can not reduce the non-uniformity of local strain variation over thickness anymore even though material in the roll gap is under the weak non-homogeneous deformation state.

Figure 6(b) illustrates the effect of arithmetic average aspect ratio, \( s \) on the ratio of strain at center to maximum strain, \( \eta_c/\eta_{\text{max}} \). It should be reminded that ideally uniform local strain variation over thickness is obtained if only the ratio, \( \eta_c/\eta_{\text{max}} \) is equal to 1.0. \( \eta_c/\eta_{\text{max}} \) increases as \( s \) increases up to 1.0, but it is converged to about 0.69 when \( s > 1 \). This result tells us important information. It is impossible to achieve even roughly uniform local strain over thickness even though material in roll gap is under the weak non-homogenous deformation state (\( s > 1 \)). Whatever rolling conditions we change, the magnitude of strain at center is only 69% of that at surface if the arithmetic average aspect ratio is more than 1.0. It implies that a certain amount of relative difference between strain at center and that at surface exists all the time as arithmetic average aspect ratio increases beyond 1.0, which corresponds to the weak non-homogenous deformation state, for whole plate rolling process. Note arithmetic average aspect ratio always increases as rolling goes on at subsequent passes. Consequently, grain size difference between material center and its surface might be maintained while rolling continues at each pass. In rough rolling sequence, \( s \) is as a rule in the range of about 0.3–1.5. In finish rolling sequence, arithmetic average aspect ratio varies from approximately 1.5 up to 8.0.

It should be mentioned that we could calculate these information using FEM. But FEM requires a lot of time to take into account every changes of rolling parameters for each pass. For example, when reduction ratio in a given pass and slab thickness are changed even slightly, we have to perform FE analysis again to calculate local strain distribution over material and then check the information obtained. This approximate model (based on the FE results), however, can predict those information and determine an optimum condition in a very short time in terms of changed reduction ratio in a given pass and arithmetic average aspect ratio.

Figure 7 shows the changes of reduction ratio, arithmetic average aspect ratio, mean strain and uniformity parameter in terms of pass number when the proposed model is applied to two different draft schedules being used for POSCO NO. 2 Plate Mill. In Fig. 7(a), slab (247 mm in thickness) is reduced to 81.96 mm after 9 passes. With the same pass numbers, thicker slab (302 mm in thickness) is reduced to 152.09 mm as shown in Fig. 7(b). The reduction ratio and arithmetic average aspect ratio in both draft schedules increase as pass number does. The slope of increasing reduction ratio and its magnitude of the draft schedule A (Fig. 7(a)) are higher than those of the draft schedule B (Fig. 7(b)). The arithmetic average aspect ratio of the draft schedule A approaches to 1.0 and therefore the uniformity parameter is linearly increased. This indicates the draft schedule A might yields much more uniform local strain variation over thickness and consequently grain size variation over thickness might be small. It should be mentioned that we can’t obtain perfectly uniform variation of grain size over material thickness as long as material is rolled. Thus it can be deduced that the draft schedule A has an advantage compared with the draft schedule B if we want to get uniform grain size deviation over material thickness approximately.

When we design a draft schedule of plate rolling to refine grain size over thickness in a more or less uniform manner...
as possible as, giving simply high reduction ratio at all passes is not good. Instead, assigning high reduction ratio with a proper arithmetic average aspect ratio, say, $s/H_1 = 0.9$ is recommended. Therefore, for obtaining more or less uniform grain size refinement over thickness in rough rolling process, the draft schedule designer should keep in mind that the low or medium reduction ratio is given to initial passes and/or intermediate passes, and high reduction ratio is assigned to the final pass while arithmetic average aspect ratio, $s/H_1 = 0.9$ is kept.

5. Conclusions

In this paper, we proposed an approximate model which computes the local strain variation over material thickness and investigated the effect of various process variables such as incoming material thickness, radius of work roll, the reduction ratio, roll speed and friction condition on the local strain variation over thickness direction in thick plate rolling process. To confirm the usefulness of the proposed model, it was applied to two different draft schedules being used for an actual plate mill in POSCO. Conclusions are as follows.

The local strain variations over material thickness calculated from the proposed model is in a good agreement with the ones by finite element analysis. Whatever the process we adjusted in the weak non-homogeneous deformation state ($s > 1$), the magnitude of strain at center was 69% of that at surface. Hence, a certain amount of relative difference between strain at center and that at surface always exists as arithmetic average aspect ratio increases for whole plate rolling process. Consequently, grain size difference between material center and its surface might be maintained while material is rolled through many passes.

If we are concerned in achieving approximately uniform grain size refinement over material thickness, the heavy reduction ratio with the strong non-homogeneous deformation state ($s \leq 1$) is recommended. Therefore, the heavy reduction with arithmetic average aspect ratio, $s/H_1 = 0.9$ might be the best condition for refining grain size over thickness in a roughly uniform manner.

REFERENCES

1) V. B. Ginzburg: Steel-rolling Technology Theory and Practice, Marcel Dekker, INC, New York, (1989), 311.
2) C. M. Sellars: Mater. Sci. Technol., 1 (1985), 325.
3) M. Pietrzyk, Z. Kedzierski, H. Kusiak, W. Madej and J. G. Lenard: Steel Res., 64 (1993), 549.
4) K. Karhausen and R. Kopp: Steel Res., 63 (1992), 247.
5) R. Kaspar and O. Pawelski: Steel Res., 56 (1985), 477.
6) J. Yanagimoto, K. Karhausen, A. J. Brand and R. Kopp: Trans. ASME, 120 (1998), 316.
7) Y. Saito, T. Sakai, K. Hirano and K. Kato: Trans. Iron Steel Inst. Jpn., 28 (1988), 1028.
8) C. M. Sellars: Mater. Sci. Technol., 6 (1990), 1072.
9) C. Devadas, I. V. Samarasekera and E. B. Hawbolt: Metall. Trans. A, 22A (1991), 335.
10) Y. Saito, T. Enami and T. Tanaka: Trans. Iron Steel Inst. Jpn., 25 (1985), 1146.
11) M. Suehiro, K. Sato, Y. Tsukano, H. Yada, T. Senuma and Y. Matsunuma: Trans. Iron Steel Inst. Jpn., 27 (1987), 439.
12) L. Lachmann and G. Zouhar: Steel Res., 62 (1991), 447.
13) S. H. Kim, J. H. Lee, W. J. Kwak and S. M. Hwang: ISIJ Int., 45 (2005), 199.
14) C. H. Moon and Y. Lee: ISIJ Int., 48 (2008), 1409.