The mechanics of macro scale level plastic deformation localization

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Abstract. The major salient features of plastic deformation localization, which are observed at the stage of linear work hardening for deforming metals and alloys, are considered. The dynamic and kinetic characteristics exhibited by the process of localization on transition to failure have been examined. The localization parameters have been related to the electronic properties of metals.

1. Introduction
It has been found previously [1-3] that at each flow stage, in the deforming specimen there occurs spontaneous localization of deformation; the localization behavior is determined by the dependence of the work hardening coefficient, \( \theta = G^{-1} \cdot \frac{d\sigma}{d\varepsilon} \), on the level of strain attained at a given process stage. A one-to-one correspondence is found to exist between the given flow stage and the type of emergent local strain pattern. Thus at the stage of linear work hardening (\( \sigma \sim \varepsilon; \theta = \text{const} \)), over the deforming specimen there travels an equidistant set of strain nuclei (self-excited wave), which is characterized by wavelength \( \lambda \) and propagation rate \( V_{aw} \). At the stage of parabolic work hardening (\( \sigma \sim \varepsilon^n; n \approx \frac{1}{2} \)), the set of flow nuclei remains stationary. As a transition occurs from one flow stage to the next, the existing localization picture would disintegrate to give way to a new pattern. All the wave characteristics of the process reveal an intricate dependence on material structure and deformation conditions.

It should be noted that the major salient features of the processes of flow localization were observed earlier in [4-6]; for their theoretical handling, the gradient plasticity theory was invoked. It was predicted, in particular, that at the stage of linear work hardening, in the deforming material there would be generated a wave picture of plastic flow localization [4].

In the present paper, consideration is given to certain regular features of deformation localization, which are common to all the test materials that have a linear law of work hardening. The investigations were performed on FCC, BCC and HCP single- and polycrystals of pure metals and alloys. An analysis of the local strain patterns was carried out by the technique of speckle interferometry [1], which enables one to reconstruct the field of displacement vectors in the course of straining; the data obtained are used to calculate all the components of the plastic distortion tensor. It has been found previously that the space-time distributions of local elongation \( \varepsilon_{xx} \) and other components observed at the stage of linear work hardening are typically self-excited wave ones in character.
2. The main characteristics of wave processes

2.1. Wave propagation rate

The motion rate of self-excited waves of localized deformation is one of the most significant characteristics of the process. For this reason, the dependence \( V_{aw}(\theta) \) for the stage of linear work hardening has been investigated for a wide range of materials. The results presented in Figure 1 show that

\[
V_{aw} = \frac{\Xi}{\theta} \text{.} 
\]

Here the correlation factor for the quantities \( V_{aw} \) and \( \theta^{-1} \) is 0.95.

\[ \text{Figure 1. Self-excited wave propagation rate against } \theta. \]

Note that the constant \( \Xi \approx 6.33 \cdot 10^{-7} \text{ m/s} \) is 2.5 times lower than the motion rate of the movable grip of the test machine. Relation (1) is found to be valid for all the FCC, BCC and HCP single- and polycrystals of pure metals and alloys investigated (including those deforming by twinning); therefore, it may be concluded that it is a universal relation. The dependence \( V_{aw}(\theta) \) of the form (1) is in striking contrast to that of the well-known plasticity waves [7] whose velocity \( V_{pw} \approx (\theta/\rho)^{1/2} \sim \theta^{1/2} \). In view of the difference in the functions \( \theta^{-1} \) and \( \theta^{-1/2} \), pooling of the observed waves with plasticity waves [7] to form one class is precluded in principle.

2.2. The dispersion law of localized flow self-excited waves

The feasibility of direct experimental determination of the wavelength \( \lambda \) (wave number \( k = 2\pi/\lambda \)) and period of oscillations \( T \) (frequency \( \omega=2\pi/T \)) for the stage of linear work hardening allowed dispersion law \( \omega(k) \) to be established for the self-excited wave process. This law is presented in a generalized form in Figure 2. Its canonical form

\[
\omega = 1 + k^2 
\]

is characteristic of so-called Schrödinger’s non-linear equation [8]. Thus the oscillation spectrum has a gap \( 0 \leq \omega \leq \omega_{min} \approx 10^{-2} \text{ s}^{-1} \), so that \( \hbar \omega_{min} \ll k_B T \) (\( k_B \) is the Boltzmann constant). This implies that spontaneous localization of plastic deformation is bound to occur at any temperature, provided there are no limitations imposed, for example, by sample size.

2.3. Scale effect

The plots of the wavelength of localized deformation against specimen length \( L \) were examined for specimens having, in all cases, width of 5 mm and length of 25, 50, 75, 100 and 125 mm. The test specimens were prepared from a 1.6-mm thick uniform sheet of Zr-2.5% Nb alloy having grain size of 5 \( \mu \text{m} \). The wavelength was determined for the deformed specimens at overall strains in the interval of \( 2.2 \leq \varepsilon \leq 2.5\% \).
Figure 2. The dispersion law for self-excited waves.

Processing of the data obtained has revealed that, all other factors being equal,

$$\lambda(L) = \xi \cdot \ln L/L_0.$$  \hspace{1cm} (3)

Here $L_0 \approx 16$ mm and $\xi \approx 8$ mm. In order to interpret dependence (3), let us introduce the derivative $d\lambda/dL > 0$ as a measure of the relation between $\lambda$ and $L$ assuming $d\lambda/dL = w^{-1}$ ($w$ is the probability of nucleation of a localization nucleus). It can be taken that $w \sim L$, since the probability of the onset of plastic deformation is proportional to the number of random stress concentrators, i.e. specimen size. Hence $d\lambda \sim dL/L$, which yields (3). From (3) follows that at $L = L_0 \approx 16$ mm, $\lambda = 0$, i.e. $L_0$ is the minimal specimen size that allows for the emergence of periodic pictures of plastic flow localization. In the case of tensile specimens having length $L \leq L_0$, strains are likely to be distributed uniformly; indeed, this was the case by the tensile loading of a sample whose length $L = 18$ mm.

2.4. Grain size dependence of wavelength

The issue of grain size dependence of wavelength is essential for understanding the nature of the flow localization phenomenon. This was investigated using polycrystalline Al specimens whose grain size varied in the range $8 \cdot 10^{-3} \leq D \leq 12.5$ mm due to recrystallization after pre-straining, all the other conditions (specimen size, tensile loading rate, temperature) being the same. A plot of localized deformation wavelength against grain size illustrated in Figure 3 is described by a logistic function equation [9]

$$\lambda = \lambda_0 \cdot \left[1 + C \cdot \exp\left(-a_1 D\right)\right]^{-1},$$  \hspace{1cm} (4)

where $\lambda_0 = a_1/ a_2 \approx 16$ mm (see below) and $C$ is the integration constant. The constants $a_1 = 1.37$ mm$^{-1}$ and $a_2 = 8.8 \cdot 10^{-2}$ mm$^{-2}$ in equation (4) were obtained numerically. The respective quantities $a_1 L_0 \approx 0.73$ mm and $a_2 L_0^{1/2} \approx 3.4$ mm are close to one third of the specimen thickness and width.

Figure 3. The wavelength of localized deformation as a function of grain size.
Consider the nature of dependence $\lambda(D)$. Clearly, with growing grain size $D$, an increase in wavelength $\lambda$ also occurs due to shear lines elongation in the course of elementary deformation act. The differential equation, which relates $\lambda$ to $D$, can be conveniently given the form

$$d\lambda/dD = a_1 \lambda - a_2 \lambda^2.$$  \hspace{1cm} (5)

The quadratic term $a_2 \lambda^2$ accounts for the low rate of $\lambda$ growth in the region of large $D$ values due to the restriction imposed on specimen size. Integration of (5) yields equation (4) that has the following limiting cases. At $D < 5 \times 10^{-2}$ mm, $a_1 \lambda \gg a_2 \lambda^2$ and from (5) follows $\lambda \sim e^{a_1 D}$. In the interval $5 \times 10^{-2} < D < 2.5$ mm, the rate of $\lambda$ growth decreases; therefore, it is assumed that the relative increment in $\lambda$ is proportional to the number of grains accommodated by the specimen lengthwise, i.e. $d\lambda/dD \sim L/D$. Hence $\lambda \sim \ln D$. The above variants of the dependence can be useful for experimental data approximation within intervals of small and large $D$ values.

2.5. Dependence of wavelength on dislocation structure parameters

The manner in which the macroscopic picture of plastic flow localization is related to the deforming medium’s dislocation characteristics is of fundamental importance for gaining an insight into the nature of the localization phenomenon. Until recently, however, the needed direct experimental data have been unavailable. In the present investigation, the above relationship was examined by comparing the wavelength of localized deformation, $\lambda$, in a Zr-Nb alloy having grain size ~ 3.5 $\mu$m with the average size of dislocation structure elements, $\langle d \rangle$, at the same amount of total straining. Using electron microscopy analysis (thin-foil method), the quantity $\langle d \rangle$ was determined for the various flow stages. Thus in the case of cellular network structure, $\langle d \rangle$ was taken to be the distance between clusters of dislocations, in the case of streaky substructure, the distance between subgrain boundaries and in the case of fragmented structure, fragment size. The dependence $\lambda$ on $\langle d \rangle$ (Figure 4) is a linear one

$$\lambda = \lambda_0 + \alpha \cdot \langle d \rangle,$$  \hspace{1cm} (6)

where $\lambda_0 \approx 4.2$ mm, $\alpha \approx 1.3 \times 10^4$ and the correlation factor $\sim 0.9$.

![Figure 4. Correlation between wavelength and the average size of dislocation structure elements.](image)

It has been shown [1-3] that localization of plastic flow is the result of self-organization of elementary plastic deformation acts in crystals; the kinetics of the process is found to obey so-called reaction diffusion equations of the type $\dot{\varepsilon} = f(\varepsilon, \sigma) + D \varepsilon^{\sigma}$, which describe variation in the fields of elastic stresses and plastic strains. The diffusion-like coefficient $D$ in this equation determines the spatial macroscopic scale of strain fields via the following relation [1, 2]

$$\lambda^2 \approx D \Theta,$$  \hspace{1cm} (7)
where $\Theta$ is the characteristic time of the process. The coefficient $D$ [1] in the diffusion approximation is given for a macro-scale level by a relation, which incorporates the spatial scale of the low-lying (meso-scale) level $\Lambda_{\text{meso}}$, i.e.

$$D = A_{\text{meso}} V_s = \Lambda_{\text{meso}}^2 / \tau,$$  \hspace{1cm} (8)

where $\tau$ is the relaxation time of dislocation processes and $V_s$, the rate of re-distribution of elastic stresses (sound rate). In this case, from (7) follows

$$2 \Lambda_{\text{meso}}^2 = \Theta \tau,$$  \hspace{1cm} (9)

At the natural condition $\Lambda_{\text{meso}} \equiv \langle d \rangle$, this is consistent with the experimental data. Evidently, $\alpha \approx (\Theta / \tau)^{1/2}$ and $\Theta \approx 150$ s [1, 2]. The characteristic time $\tau$ can be roughly estimated [10] as

$$\tau \approx \omega_D \exp \left( \frac{U - \gamma \sigma}{k_B T} \right) / \omega_D,$$  \hspace{1cm} (10)

where the enthalpy of activation for the elementary shear process $H = U - \gamma \sigma \approx 0.4$ eV and $\omega_D = 10^{13}$ s$^{-1}$ is the Debye frequency. At $T = 300$ K, from (10) follows $\tau \approx 9 \times 10^{-7}$ s, i.e. $\alpha \approx 1.3 \times 10^4$, which agrees with the observed value of the proportionality coefficient in (6). The above qualitative agreement suggests, in accordance with equations (6) or (9), a close quantitative relationship among the phenomena occurring on the macro- and micro-scale levels of plastic flow.

3. Localization at the stage preceding failure

The plastic flow localization, which occurs at the final process stage, has an important distinction, i.e. at $n < \frac{1}{2}$ the localized flow nuclei would merge into one nucleus. The Figure 5 displays the dependence $V_{\text{aw}}(n) = \chi \cdot (n - \beta)^2$ obtained for a Zr-Nb alloy. Since $\beta \approx \frac{1}{2}$, the nuclei of localized deformation are apparently stationary only at $n \approx \frac{1}{2}$ (Taylor-Mott model of work hardening), whereas at values other than $n \approx \frac{1}{2}$ they are mobile. At $n < \frac{1}{2}$, however, the nuclei motion differ radically from those observed at the stage of linear work hardening ($n = 1$): at $n < \frac{1}{2}$ they move in a disordered manner, coalesce finally and initiate macroscopic neck formation and transition from steady-state flow to plastic fracture.

![Figure 5](image-url)  

*Figure 5. The motion rate of flow nuclei as a function of the parabola exponent $n$."

The distinction in the flow nuclei behavior, which is, for example, stationary at $n = \frac{1}{2}$ but mobile at $n = \frac{1}{8}$, can be interpreted in the following way. In the former case ($\varepsilon \sim \sigma^2$) and by the fluctuation of local stresses $\pm \delta \sigma$, the strains at the both edges of the deformation nucleus are distributed symmetrically so that the nucleus would remain stationary, while in the latter case ($\varepsilon \sim \sigma^3$) and by the occurrence of fluctuations opposite in sign, the symmetry of the nucleus is disrupted, thereby causing it to move. The analysis of the strain distribution profiles obtained for both cases supports this explanation.
4. Quantization of self-excited waves and quasi-particle

The typical self-excited wave behavior exhibited by the localization process at the stage of linear work hardening has significant implication. It has been shown recently [11] that the mass calculated from the self-excited wave parameters with the help of de Broglie equation is found to correlate with the atomic mass of the metal from which the test specimen is prepared.

In the present investigation, the experimental data on the wavelength of deformation localization, \( \lambda \), and on the rate of wave propagation, \( V_{sw} \), which were obtained for the single Cu, Ni and \( \gamma \)-Fe crystals and Zr and Al polycrystals, are treated using the de Broglie relation in a more consistent manner. First, using the de Broglie equation

\[ h m V \lambda = \frac{h}{m} \]

where \( h \) is the Planck constant, mass values are calculated for five metals investigated from the experimentally determined [1-3] values of \( \lambda \) and \( V_{sw} \), i.e. \( m = \frac{h}{\lambda V_{sw}} \); then division of \( m \) by \( \rho \) (density of the respective metal) yields volume \( \Omega = \frac{m}{\rho} \) and elementary size \( d_\Omega = \Omega^{1/3} \) is calculated for each metal. The quantity \( d_\Omega \) is found to be rather close to the ion radii, \( r_{ion} \), of the respective metals (in the case of Zr, Fe and Ni, \( d_\Omega \) and \( r_{ion} \) virtually coincide), while the average ratio obtained for five metals investigated \( \langle d_\Omega/r_{ion} \rangle = 1.05 \). By normalization in the atomic mass \( M_a \) it is possible to introduce the reduced mass \( \mu = \frac{m}{M_a} \ll 1 \). A linear correlation is established (Figure 6; correlation factor 0.99) between the latter quantity and the number of valent electrons per atom [12], \( e/a \), for five metals investigated, i.e. \( \mu = \mu_0 + \kappa \cdot \frac{e}{a} \). Such results are likely to reflect a dual correspondence between the self-excited wave of localized plastic deformation and some hypothetical quasi-particles.

![Figure 6. The dependence of the reduced mass \( \mu \) on the quantity \( e/a \).](image)

5. Conclusion

The evidence obtained in the investigations of flow localization in single- and polycrystalline metals and alloys differing with respect to crystal lattice type indicates that at the stage of linear work hardening, the process exhibits a non-homogeneous behavior; therefore, it should be regarded as a self-excited wave process. The quantitative and qualitative characteristics of this process are discussed above. It is found that in the course of the process, the parameters of macro-localization of plastic flow correlate with the characteristic sizes of dislocation substructure elements that form on a micro-scale level. The formalism resorted to in Chap. 4 suggests that quasi-particles of specific kind may be introduced for addressing plastic flow localization in its self-excited wave form.

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