A robust experimental technique to determine the strain to fracture for plane strain tension

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Abstract. The stress state of plane strain tension plays a crucial role in many forming and crash applications. Under plane stress conditions (in thickness direction), most micromechanical and phenomenological models predict a minimum in ductility for a plane strain tension loading (in the plane of the sheet). When looking at Forming Limit Diagrams (FLD) or at modern stress state dependent fracture initiation models, a “plane strain ductility valley” exists between uniaxial and equi-biaxial tension. Given that the ductility reaches a minimum for plane strain tension, the reliable measurement of the strain to fracture for plane strain tension is particularly crucial when calibrating modern fracture initiation models. Many experimental techniques have been proposed in the past, but a standardized universal experimental technique for plane strain tension testing is still missing to date. It is the goal of the present work to develop a robust experimental technique for determining the strain to fracture for plane strain tension. Emphasis is placed on finding a technique that is universally applicable i.e. that provides reliable results irrespective of the material thickness or ductility. In addition, it should be easily automatized in order to be included in up-to-date test protocol dedicated to the prediction of the behavior from large database analysis. The experiments are designed such that the material is subject to proportional loading, i.e. the stress state remains constant throughout the entire loading history until fracture initiates. After identifying a suitable specimen geometry through finite element simulations, experiments are performed on specimens extracted from aluminum alloys and steels sheets. The experimental campaign includes three different types of plane strain tension experiments (flat notched tension, V-bending and the newly-proposed stretch-bending of mini-Nakazima specimens) to elucidate their differences and limitations, and to demonstrate that the newly-proposed technique is the only one that yields meaningful results for all three materials.

1. Introduction

In many forming and crash applications, the stress state of plane strain tension plays a crucial role. Under plane stress conditions (in thickness direction), most micromechanical and phenomenological models predict a minimum in ductility for a plane strain tension loading (in the plane of the sheet). Given that the ductility reaches a minimum for plane strain tension, the reliable measurement of the strain to fracture for plane strain tension is particularly crucial when calibrating modern fracture initiation models. In-plane plane strain tension experiments often make use of flat specimens with lateral notches that are subject to tension with the help of a universal testing machine. Marciniak tests pose a special case in which an out-of-plane punch movement generates homogeneous in-plane deformation. As an
alternative, symmetrically grooved specimens or butterfly-shaped specimens with thickness reduction may be used. However, through-thickness machining brings the risk of premature fracture initiation from machining-induced defects.

V-bending experiments provide remarkably proportional loading paths, ensuring plane strain tension conditions throughout the entire loading history all the way until fracture initiates. However, their range of applicability is limited to sheet materials of either moderate ductility or high thickness. The present conference paper is an abridged version of a recent journal publication [5]. Here, a new experimental technique is proposed to overcome the limitations of V-bending experiments. It makes use of a mini-Nakazima specimen that is clamped onto a 30mm diameter die and subjected to out-of-plane loading through a dihedral punch. Finite Element simulations are carried out to study the influence of the design parameters and find an optimal geometry which is subsequently validated by experiments on DP steel and aluminum 2024-T351.

2. Computational models

2.1. Plasticity models and isotropic hardening
In view of describing the large deformation response of both advanced high strength steels and aluminum alloys, quadratic and non-quadratic plasticity models with non-associated and associated flow rules, respectively, are presented. The reader is referred to [6] for details on the model formulations and their experimental validation. For advanced high strength steels, a non-associated Hill’48 flow potential is chosen in conjunction with the von Mises yield function to account for the direction dependence of the Lankford ratios in materials with otherwise loading direction-independent stress-strain curves [7]. For aluminum, a 3D extension of the Yld2000-2D model [1], as shown in [4], is used. Its yield condition is based on an anisotropic equivalent stress. An isotropic hardening law, consisting of a linear combination of a power law (Swift, 1952) and an exponential law (Voce, 1948) are used.

2.2. Hosford-Coulomb fracture initiation model
The Hosford-Coulomb fracture initiation model [7] accounts for the effect of the stress triaxiality and the Lode angle parameter

\[ \eta = \frac{\sigma_m}{\sigma} \quad \text{and} \quad \tilde{\theta} = 1 - \frac{2}{\pi} \arccos \left[ \frac{27 J_2}{2 \sigma} \right] \quad \text{with} \quad \tilde{\theta} \in [-1,1]. \]  

(1)

Based on the Hosford-Coulomb model, the equivalent plastic strain to fracture for proportional loading reads

\[ \varepsilon_p^e(\eta, \tilde{\theta}) = b(1+c)\sqrt{\frac{1}{2} \left( (f_1 - f_2)^2 + (f_2 - f_3)^2 + (f_3 - f_1)^2 \right)^{\frac{1}{2}}} + c(2\eta + f_1 + f_3) \]  

(2)

with the fixed transformation coefficient \( n=0.1 \) and the Lode angle parameter dependent trigonometric functions

\[ f_1(\tilde{\theta}) = \frac{2}{3} \cos \left[ \frac{\pi}{6}(1-\tilde{\theta}) \right], \quad f_2(\tilde{\theta}) = \frac{2}{3} \cos \left[ \frac{\pi}{6}(3+\tilde{\theta}) \right], \quad f_3(\tilde{\theta}) = -\frac{2}{3} \cos \left[ \frac{\pi}{6}(1+\tilde{\theta}) \right]. \]

To account for the effect of non-proportional loading, the above function is embedded into a damage indicator framework. Assuming \( D=0 \) for the material in its undeformed state and \( D=1 \) when fracture initiates, the damage indicator evolves according to the differential equation

\[ dD = \frac{d\varepsilon_{pl}}{\varepsilon_p^e(\eta, \tilde{\theta})}, \]  

(3)
3. Design of a new plane strain fracture experiment

Given the limitations of existing plane strain tension experiments (notched tension and V-bending), a new type of plane strain tension experiment is sought which is applicable to both thin and thick, as well as low and high ductility sheet materials. The main motivation for the new design is to keep the advantages of V-bending tests, i.e. the almost perfectly proportional and experimentally directly observable loading path all the way up to the instant of fracture initiation, while overcoming its main drawback, i.e. the limitation in maximum achievable strain as shown in [7]. The basic configuration of the new setup includes a dihedral punch which applies out-of-plane loading onto a Nakazima-type of disc-shaped specimen with symmetric cut-outs. Figure 1 gives an overview of the experimental setup for a disc-shaped specimen (① in Figure 1) with an outer diameter of 60mm. The specimen is mounted onto an annular rigid frame using a blank holder of inner diameter D=30mm (⑤). The specimen has two chamfered circular cut-outs (②) with diameter d and a gage section of width l (minimum distance between the holes). Out-of-plane loading is applied through a dihedral punch (③). Throughout the experiment, the specimen surface is monitored with two digital cameras to determine the surface strain field via stereo Digital Image Correlation (DIC). To maintain a constant focal distance during the experiments, the punch is kept stationary while the rigid die is moved instead.

![Figure 1](image1.png)

**Figure 1.** Experimental setup for the dihedral mini-Nakazima (DMN) test: (a) schematic of the specimen and the dihedral punch, (b) overview of the experimental components with ① specimen, ② chamfered holes, ③ dihedral punch, ④ punch device and ⑤ blank holder with the eight M6-12.9 screws used to clamp the DMN specimen.

The effect of the geometrical features of the mini-Nakazima specimen are investigated through finite elements simulations. In order to obtain a generally applicable specimen geometry a parametric study is performed considering the following geometric parameters as free input variables, ligament length l, hole diameter d, punch edge radius r_p, material thickness t. The results from more than 100 simulations with different sets of parameters revealed that an “optimal” geometry is obtained for the parameters \{l, d, r_p\} = \{10, 8, 1\} mm.

The overall force-displacement curve for the punch increases in an approximately linear manner while the specimen deforms plastically (Figure 2a). Equivalent plastic strains of about 0.5 (Figure 2b) are attained on the specimen surface in the linear range. It is only when the entire cross-section of the central gage section begins to deform plastically that the force-displacement curve becomes highly non-linear (concave) before reaching a maximum followed by fracture initiation. The corresponding triaxiality field evolves around a constant value of η=0.58 at the specimen top surface center throughout.
the test (Figure 2c). This uniform area covers approximately 75% of the ligament length and only changes towards the free boundary where a value of \( \eta = 0.33 \) is reached.

![Figure 2. Results of the numerical simulation of a dihedral mini-Nakazima (DMN) test on DP450.](image)

(a) Comparison between the experimental and numerical force/displacement curves. Contour plots of the (b) equivalent plastic strain, (c) the stress triaxiality, and (d) the damage indicator at time points denoted in (a); (e) Evolution of the principal strain ratios along the ligament for two select geometries. The minimum values of 5 and 20 (Dournaux et al. (2009)) are shown in red solid lines.

Over a wide range of the test, the mechanical fields resemble those of a conventional bending experiment. When looking at the mechanical field along the specimen thickness, the highest equivalent plastic strains are obtained on the specimen upper surface throughout the test (Figure 2b), assuring
fracture initiation at this location. This is also indicated from the contour plots of the damage indicator (Figure 2d). To further evaluate the quality of the obtained mechanical field, Figure 2e shows the ratio of the major to the minor plastic strain as a function of the ligament coordinate. To improve readability, only the geometries for the best (d=8mm, l=10mm) and the one resulting in the worst strain ratios (d=10mm, l=10mm) are shown for each of the three strain levels. According to the definitions in [9], a plane strain condition requires the minor strain to be negligible compared to the major strain, while [2] requires that ratio to be larger than 5. Irrespective of the chosen mini-Nakazima specimen geometry, the ratio remains above a value of 5 for a normalized distance from the center below 0.8, and above a value of 20 along the central half of the ligament.

4. Experimental program
Two different sheet materials are examined in this study, a 1.2mm thick aluminum alloy AA2024-T351 and a 0.8mm thick dual phase steel DP450. Experiments are performed to validate the proposed mini-Nakazima specimen (Fig 3) and to benchmark the obtained results against existing techniques, namely V-bending as shown in [7] and notched tension NT2 derived from the geometries proposed by [8] and [2]. Furthermore, basic plasticity and fracture experiments are included to obtain a full characterization of the large deformation response of the above materials. Before each test, a random speckle pattern is applied to the specimen surface to allow for measuring the displacement field using digital image correlation (VIC-2D, respectively VIC3D, Correlated Solutions). The mini-Nakazima specimens are prepared for testing by reaming the holes to their final diameter of d=8mm and adding a 45° chamfer to the holes. Using the same stereo DIC settings as described by [7], the surface strains are determined from images of the top specimen surface recorded at a frequency of 1Hz. The effective strain is then determined from the principal strains by the equation

$$\bar{\varepsilon} = \frac{2}{\sqrt{3}} \sqrt{\varepsilon_{II}^2 + \varepsilon_{I}^2 + \varepsilon_{II}^2} .$$

5. Results and discussion

5.1. Results for 1.2mm thick aluminum 2024-T351
For the V-bending experiment (Figure 3a,b), the effective strain distribution across the specimen width is uniform with a mean value of $\bar{\varepsilon}_{FB} = 0.28$ close to where fracture initiates. The peak-to-peak variation of the strains is 0.037 (light gray curve in Figure 3a) which corresponds to 13% of the mean value. The corresponding results from numerical simulation of the V-bending experiments (blue and red curves in Figure 3a) agree well in terms of strain, and also suggest that a constant stress triaxiality of 0.63 prevails throughout the entire experiment until fracture initiates. In the experiments on the NT2 specimens (extracted from the aluminum 2024-T351 sheets), only when using a high speed camera (Photron SA-X, 80’000fps) it is possible to reveal that fracture initiates from one of the free boundaries of the specimen, an area where a uniaxial instead of a plane strain tension stress state prevails, rendering it not meaningful for the measurement of the fracture strain under plane strain tension. The results from the mini-Nakazima experiments are summarized in Figure 3a. Fracture occurs at a normalized distance of -0.4 from the ligament center at an effective strain of $\bar{\varepsilon}_{MN} = 0.28$ . Figure 3 also includes the results from numerical simulations of the instant, where the applied punch displacement in the simulation matches that at the instant of fracture initiation in the experiments. The experimental (black curve) and numerical (red curve) results for earlier instants in the loading history are shown in Figure 3e. Good agreement of the numerical simulation with the experiment is obtained in the central region of the ligament (for normalized distance of up to ±0.7 from the ligament center). Outside that range, the numerical simulation slightly overestimates the experimentally determined effective strain. As for the V-bending experiment a stress triaxiality of 0.63 is observed in the stretch-bending experiments from
Numerical analysis. It can thus be concluded that the newly-proposed dihedral mini-Nakazima experiment yields the same result as the V-bending experiments, i.e. a strain to fracture of 0.28 for plane strain tension loading.

5.2. Results for 0.8mm thick DP450

For the DP450 steel, the V-bending test does not lead to fracture, but the material can be fully hemmed. However, for the DP450 the NT2 experiment yields a reasonable estimate for the strain to fracture for plane strain tension. The high speed camera images show that fracture initiates at a normalized distance of about ±0.7 from the specimen center. The effective strain field around the specimen center is symmetric, but non-uniform, reaching a value of \( \frac{\varepsilon_{NT2}^{DP450}}{\varepsilon_{DP450}} = 0.84 \) in the area where fracture is observed. Figure 3b,f show the results from the dihedral mini-Nakazima experiments and simulations for the DP450 steel. A diffuse fracture pattern is obtained, characterized by the occurrence of multiple small...
fracture lines along the ligament at normalized distances in the range ±0.7. Here, the measured fracture strain is $\varepsilon_{f,DP450}^{MN} = 0.84$ (Figure 3b), a value very close to the one measured on the surface of the NT2 specimen. A very good agreement of the numerical simulation with the experiment is obtained around the specimen center, while the numerical simulation slightly underestimates the effective strain close to the boundary.

5.3. Discussion
To further compare the performance of the different plane strain tension experiments, the loading paths to fracture (evolution of the stress triaxiality, Lode angle parameter and equivalent plastic strain) are extracted from the specimens:

- **V-bending (AA2024, Figure 4a)**: Two distinct locations are chosen, the specimen top surface center (dashed red line) and a point at normalized distance of 0.5 (dashed black line).

- **NT2 (DP450 Figure 4b)**: Three distinct specimen locations are chosen: the specimen top surface center (dashed red line), the location where the first crack is observed experimentally at the specimen top surface (normalized distance of 0.7 from the center, dashed black line) and the specimen mid-plane at the same distance from the center (dashed green line).

- **Dihedral mini-Nakazima (all materials)**: Two distinct locations are chosen, the specimen top surface center (solid red line) and a point at normalized distance of 0.5 (solid black line).

![Figure 4](image)

**Figure 4.** Loading paths for the different plane strain tension geometries and locations for (a) AA2024 and (b) DP450 as extracted from the numerical simulations. Dihedral mini-Nakazima (DMN) surface center (solid red line) and 0.5 normalized distance from surface center (solid black line), V-bending respectively NT2 surface center (dashed red line), V-bending respectively NT2 at normalized distance from surface center (dashed black line) and NT2 mid-plane at 0.7 normalized distance from center (dashed green line).

For the V-bending experiments, the two loading paths perfectly coincide irrespective of the material, underlining the uniformity of the mechanical field along the bending axis. Additionally, the predicted onset of fracture by the numerical simulation (solid blue dot at the end of the loading paths) agrees very well with the fracture model calibrated based on the experimental results, making it a suitable specimen for plane strain tension testing (provided that the maximum obtainable strain is higher than the ductility of the material to be characterized). For the DP450 NT2 experiment (Figure 4b), the loading paths and their evaluation is much more complex. For the calibrated fracture model (Table 1), the onset of fracture is predicted at specimen mid-plane at approximately 97% of the actual fracture displacement, with a highly non-proportional loading history drifting from $\eta = 0.5$ to $\eta = 0.75$. At this time, the equivalent plastic strain on the specimen top surface has reached only 81% of the fracture strain (blue cross at the dashed black line end), and at specimen top surface center (blue cross at the dashed red line end), it is 35% lower than the actual fracture strain for plane strain tension.
6. Conclusion

A new type of plane strain tension fracture experiment is developed to determine the strain to fracture for proportional loading paths. Existing experimental techniques such as the V-bending of strips and notched tension experiments on flat specimens with sharp notches are not robust in the sense that fracture does not always initiate under plane strain tension conditions. V-bending experiments are limited by the fact that the highest achievable strain is bound and dependent on the sheet thickness, which becomes particularly problematic for thin and highly ductile sheet materials. In notched tension experiments, the loading history is often non-proportional and fracture is prone to initiate from the gage section boundaries at stress states closer to uniaxial tension than plane strain tension.

The newly-proposed technique makes use of a mini-Nakazima type of with a gage section width of 10mm that is subject to stretch-bending using a circular die along with a dihedral punch. The “optimal” geometry of the specimen and the dihedral punch are determined thorough a series of finite element simulations. Experiments are performed using V-bending, notched tension and mini-Nakazima specimens extracted from 1.2mm thick aluminum 2024-T351 and 0.8mm thick DP450 steel. For all materials, the dihedral mini-Nakazima experiments provide a reliable estimate of the strain to fracture for plane strain tension. At the same time, the experimental campaign confirmed that none of the other two existing techniques (V-bending and notched tension) provides reliable results for all materials.

Table 1. Fracture model parameters for aluminum 2024-T351 and DP450.

|         | a   | b   | c   |
|---------|-----|-----|-----|
| AA2024  | 1.37| 0.53| 0.054|
| DP450   | 1.45| 1.35| 0.038|

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