New Physics and $B \to V_1 V_2$ Decays

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I discuss several techniques for observing new physics using measurements of $B \to V_1 V_2$ decays. Within the standard model, all CP-violating triple-product correlations (TP's) involving light vector mesons are expected be very small or to vanish. However, these TP's can be large in models with new physics (NP). If a time-dependent angular analysis of $B \to V_1 V_2$ can be performed, there are numerous additional tests for NP in the decay amplitudes. Should a signal for NP be found, one can place constraints on the NP parameters.

1 Triple Products

It is well known that $B \to V_1 V_2$ decays cannot be simply used to measure indirect CP-violating asymmetries and obtain clean information about weak phases. The reason is that the state $V_1 V_2$ is not a CP eigenstate – it involves three helicity amplitudes. Two of these ($A_0, A_\parallel$) are CP-even, while the third ($A_\perp$) is CP-odd. As a function of these helicity amplitudes, the $B \to V_1 V_2$ decay amplitude can be written as

$$M = A_0 \varepsilon_1^L \cdot \varepsilon_2^L - \frac{1}{\sqrt{2}} A_\parallel \varepsilon_1^T \cdot \varepsilon_2^T - \frac{i}{\sqrt{2}} A_\perp \varepsilon_1^T \times \varepsilon_2^T \cdot \hat{p},$$

where $\hat{p}$ is the unit vector along the direction of motion of $V_2$ in the rest frame of $V_1$, and $\varepsilon_{1,2}$ are polarizations of vector mesons. In the above, $\varepsilon_i^L = \varepsilon_i^T \cdot \hat{p}$, and $\varepsilon_i^T = \varepsilon_i^* - \varepsilon_i^L \hat{p}$. On the other hand, it is also well known that one can separate the helicity amplitudes using a (time-dependent) angular analysis. In this way one can measure the indirect CP asymmetries in each individual helicity state.

However, this angular analysis contains a great deal more information, due to the interference of CP-even and CP-odd amplitudes. The time-integrated differential decay rate contains 6 angular terms. Two of these are

$$- \frac{\text{Im}(A_\perp A_0^*)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin \phi - \frac{\text{Im}(A_\perp A_\parallel^*)}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi.$$

1Talk given at Flavor Physics and CP Violation (FPCP2003), École Polytechnique, Paris, France, June 2003.
We have assumed that both vector mesons decay into pseudoscalars, i.e., $V_1 \to P_1 P'_1$, $V_2 \to P_2 P'_2$. In the above, $\theta_1 (\theta_2)$ is the angle between the directions of motion of the $P_1$ ($P_2$) in the $V_1$ ($V_2$) rest frame and the $V_1$ ($V_2$) in the $B$ rest frame, and $\phi$ is the angle between the normals to the planes defined by $P_1 P'_1$ and $P_2 P'_2$ in the $B$ rest frame. The key point is that these two terms involve the triple product $\vec{\varepsilon}_1^T \times \vec{\varepsilon}_2^T \cdot \hat{p}$. Triple products (TP's) are odd under time reversal (T) and hence, by the CPT theorem, also constitute potential signals of CP violation. (Note that a full angular analysis is not necessary to measure TP's.)

Now, it is well-known that triple product signals are not necessarily CP-violating – even if the weak phases vanish, nonzero TP signals can be produced by strong phases. In order to obtain a true CP-violating signal, it is necessary to compare the TP in $B \to V_1 V_2$ with that in $\overline{B} \to \overline{V}_1 \overline{V}_2$. Note that the TP signal $\vec{\varepsilon}_1^T \times \vec{\varepsilon}_2^T \cdot \hat{p}$ is odd under P. As a consequence, the true CP-violating triple product is found by adding the two T-odd asymmetries:

$$A_T \equiv \frac{1}{2} (A_T + \overline{A}_T). \quad (3)$$

This is an important point: since one has to add $A_T$ and $\overline{A}_T$, neither tagging nor time dependence is necessary to measure TP's! In principle, one can combine measurements of charged and neutral $B$ decays to obtain a triple-product signal.

Triple products are of particular interest because they are complementary to direct CP asymmetries. Both signals can be nonzero only if there are two interfering decay amplitudes in a given $B$ decay. However, denoting $\phi$ and $\delta$ as the relative weak and strong phases, respectively, between the two interfering amplitudes, the expressions for the two signals can be written

$$A^{\text{dir}}_{CP} \propto \sin \phi \sin \delta, \quad A_T \propto \sin \phi \cos \delta. \quad (4)$$

If the strong phases are small, as may well be the case in $B$ decays, all direct CP-violation signals will be tiny as well. On the other hand, TP asymmetries are maximal when the strong-phase difference vanishes. Thus, it may well be more promising to search for triple-product asymmetries than direct CP asymmetries in $B$ decays.

### 2 Triple Products in the Standard Model

As mentioned above, all CP-violating effects require the interference of two amplitudes, with different weak phases. Certain decays in the standard model (SM), such as those dominated by $b \to c \bar{s}s$ or $b \to s \bar{s}s$, do not satisfy this. Thus, no triple products are expected in $B \to J/\psi K^*$, $B \to \phi K^*$, $B \to D^*_s D^*$, etc. However, other processes ($b \to u \bar{s}s$, $b \to c \bar{d}d$, etc.) receive both tree and penguin contributions. Thus, it may be possible to produce TP's in decays such as $B \to D^* \overline{D}^*$, $B \to \rho K^*$, etc. [3].

Consider now $B \to V_1 V_2$ decays within factorization. The amplitude is

$$\sum_{\sigma, \sigma'} \{ \langle V_1 | O | 0 \rangle \langle V_2 | O' | B \rangle + \langle V_2 | O | 0 \rangle \langle V_1 | O' | B \rangle \} . \quad (5)$$

Note that TP’s are a kinematical CP-violating effect – unless a given decay includes both of the above amplitudes, with a relative weak phase, no TP will be produced. For example, even though the decay $B^+_d \to D^{*+} D^{*-}$ receives both a tree ($V_{cb} V_{cd}$) and a penguin
\((V_{tb}V_{td})\) contribution, there is no TP. The point is that both of these amplitudes contribute to \(\langle D^{*+}|O|0\rangle\langle D^{*-}|O'|B\rangle\); there is no \(\langle D^{*+}|O|0\rangle\langle D^{*-}|O'|B\rangle\) amplitude. (Equivalently, in the SM one has only \(\bar{b} \to \tau\) transitions; \(\bar{b} \to c\) does not occur.) Thus, in the SM, no TP is predicted in \(B^0 \to D^{*+}D^{*-}\).

This then begs the question: which \(B \to V_1V_2\) decays are expected to yield large triple products in the SM? The answer is simple: \textit{NONE}. This can be understood via the following points:

1. If \(V_1 = V_2\), no TP is possible, since there is only a single kinematical amplitude. Therefore, if \(V_1\) and \(V_2\) are related by a symmetry [e.g. isospin, flavour \(SU(3)\)], the TP is suppressed by the size of symmetry breaking.

2. The longitudinal amplitude \(A_0\) is much larger than the transverse amplitudes \(A_{\parallel,\perp}\). Therefore, TP’s are suppressed by at least one power of \(m_V/m_B\).

3. The interfering amplitudes are typically different in size, leading to further suppression of TP’s.

All of these factors lead to the suppression of TP’s. The net effect is that all TP’s involving light vector mesons are either expected to vanish or be very small in the SM. Note also that nonfactorizable effects do not change this conclusion [3].

Since all TP’s in \(B \to V_1V_2\) decays with light vector mesons are expected to be small, this is an excellent place to search for new physics (NP). For example, at present the indirect CP asymmetry in \(B^0_d(t) \to \phi K_S\) differs from that in \(B^0_d(t) \to J/\psi K_S\) [4]. If this discrepancy holds up — it is not yet statistically significant — it would require a NP amplitude in \(B^0_d \to \phi K_S\) [5]. If present, this new amplitude would also contribute to \(B \to \phi K^*\), leading to triple products in this decay. One of the many possibilities for this new physics is supersymmetry with R-parity violation [6]. Although the TP in \(B \to \phi K^*\) vanishes in the SM, we find that one can get large TP asymmetries, in the range 15\textasciitilde{}20\%, in this model [3]. This shows quite clearly that triple products are an excellent way to search for physics beyond the SM.

### 3 Time-Dependent Angular Analysis

Consider now a \(B\) decay which in the SM is dominated by a single amplitude (e.g. \(B \to J/\psi K, \phi K,\) etc.). Suppose that there is a new-physics amplitude, with a different weak phase, which contributes to this decay. As I have argued above, such an amplitude can be detected by looking for both direct CP violation and triple products. However, as can be seen below, much more information can be obtained if a time-dependent angular analysis of the corresponding \(B^0(t) \to V_1V_2\) decay can be performed.

We write

\[
A_\lambda \equiv Amp(B \to V_1V_2)_\lambda = a_\lambda e^{i\delta_\lambda} + b_\lambda e^{i\phi} e^{i\delta_\lambda},
\]

\[
\bar{A}_\lambda \equiv Amp(\bar{B} \to V_1V_2)_\lambda = a_\lambda e^{i\delta_\lambda} + b_\lambda e^{-i\phi} e^{i\delta_\lambda},
\]

(6) (7)
where $\lambda = \{0, ||, \perp\}$. The $a_\lambda$’s and $b_\lambda$’s are the SM and NP amplitudes, respectively, $\phi$ is the NP weak phase, and the $\delta_i^{ab}$ are the strong phases. The time-dependent decay rate is given by
\[
\Gamma(\bar{B}^0(t) \to V_1 V_2) = e^{-\Gamma t} \sum_{\lambda \leq \sigma} (A_{\lambda\sigma} \pm \Sigma_{\lambda\sigma} \cos(\Delta M t) \mp \rho_{\lambda\sigma} \sin(\Delta M t)) g_{\lambda\sigma},
\] where $g_{\lambda\sigma}$ are known functions of the kinematic angles $\theta_1$, $\theta_2$, $\phi$. There are 18 observables, all functions of the $A_\lambda$ and $\bar{A}_\lambda$. For example,
\[
A_{\lambda\lambda} = \frac{1}{2}(|A_\lambda|^2 + |\bar{A}_\lambda|^2), \quad \Sigma_{\lambda\lambda} = \frac{1}{2}(|A_\lambda|^2 - |\bar{A}_\lambda|^2),
\]
\[
A_{\lambda\perp} = -\text{Im} \left( A_\perp A_i^* - A_\perp A_i^* \right), \quad \rho_{\lambda\lambda} = \mp \text{Im} \left( \frac{g}{p} A_g^* A_\lambda \right).
\]

For a given helicity $\lambda$, $A_{\lambda\lambda}$ essentially measures the total rate, while $\Sigma_{\lambda\lambda}$ and $\rho_{\lambda\lambda}$ represent the direct and indirect CP asymmetries, respectively. The quantity $A_{\perp i}$ ($i = \{0, ||\}$) is simply the triple product discussed earlier.

Now, if there is no new physics (i.e. $b_\lambda = 0$), there are only 6 theoretical parameters: 3 $a_\lambda$’s, 2 strong phase differences, and the phase of $B^0 - \bar{B}^0$ mixing ($q/p$). This implies that there are 12 relations among the observables:
\[
\Sigma_{\lambda\lambda} = A_{\perp i} = \Sigma_{||0} = 0, \quad \rho_{ii}/A_{ii} = -\rho_{\perp\perp}/A_{\perp\perp} = \rho_{||0}/A_{||0},
\]
\[
2A_{\perp||} (A_\perp^2 - \rho_{\perp\lambda}^2) = [A_\perp^2 \rho_{\perp0} \rho_{\perp\perp} + \Sigma_{\perp0} \Sigma_{\perp\perp} (A_\perp^2 - \rho_{\perp\lambda}^2)],
\]
\[
\rho_{ii} A_{ii}^2 = (A_{\perp\perp}^2 - \rho_{\perp\perp}^2) (4A_{\perp\perp} A_{ii} - \Sigma_{\perp\perp}^2).
\] The violation of any of these relations will be a smoking-gun signal of NP. There are thus many more ways to search for new physics if a time-dependent angular analysis can be done.

But there’s more! Suppose that a signal for new physics is found, implying that $b_\lambda \neq 0$. In this case there are 13 theoretical parameters: 3 $a_\lambda$’s, 3 $b_\lambda$’s, 5 strong phase differences, and two weak phases ($\phi$ and $q/p$). However, at best one can measure the magnitudes and relative phases of the 6 decay amplitudes $A_\lambda$ and $\bar{A}_\lambda$. That is, there are really only 11 independent observables in Eq. 8. Naively, one would imagine that, with 11 measurements and 13 unknowns, one cannot get any information about the new physics, even if there is a NP signal. However, this is not correct: because the expressions relating the observables to the theoretical parameters are nonlinear, one can actually constrain the NP parameters.

For example, if $\Sigma_{\lambda\lambda} \neq 0$,
\[
b_\lambda^2 \geq \frac{1}{2} A_{\lambda\lambda} \left[ 1 - \sqrt{1 - \Sigma_{\lambda\lambda}^2/A_{\lambda\lambda}^2} \right].
\]
Similarly, if $\Sigma_{\lambda\lambda} = 0$, but $A_{\perp\perp} \neq 0$,
\[
2(b_\perp^2 + b_\lambda^2) \geq A_{ii} \mp A_{\perp\perp} - \sqrt{(A_{ii} \mp A_{\perp\perp})^2 \pm A_{\perp\perp}^2}.
\]

Also
\[
A_{ii} \cos \eta_i + A_{\perp\perp} \cos(\eta_{\perp} - 2\eta_i) \leq \sqrt{(A_{ii} + A_{\perp\perp})^2 - A_{\perp\perp}^2},
\]
\[
A_{ii} \cos \eta_i - A_{\perp\perp} \cos \eta_{\perp} \leq \sqrt{(A_{ii} - A_{\perp\perp})^2 + A_{\perp\perp}^2},
\]

where $\eta_\lambda \equiv 2 \left( \frac{A_{meas}}{p} - \frac{A_{mix}}{p} \right)$. If $A_{\perp\perp} \neq 0$, one cannot have $\eta_i = \eta_{\perp} = 0$. Thus, one obtains a lower bound on the difference between the measured value and the true value of the phase of $B^0 - \bar{B}^0$ mixing.
4 Conclusion

$B \rightarrow V_1 V_2$ decays contain an enormous amount of information, especially if an angular analysis can be performed. One very useful class of measurements is triple-product correlations (TP’s): $\vec{\varepsilon}_1^T \times \vec{\varepsilon}_2^T \cdot \hat{p}$. A true CP-violating triple-product signal can be obtained by adding the TP’s found in $B \rightarrow V_1 V_2$ and $\bar{B} \rightarrow \bar{V}_1 \bar{V}_2$. Thus, neither tagging nor time dependence is necessary to measure TP’s – in principle, the measurements of charged and neutral $B$ decays can be combined.

We have examined the size of TP’s in the standard model (SM). We find that all TP’s involving light vector mesons are either expected to vanish or be very small. This makes triple products an excellent place to search for new physics. Indeed, we have found that TP’s which vanish in the SM can be large (15–20%) in the presence of new physics.

If a full time-dependent angular analysis can be performed, much more information is available. First, there are many more signals of new physics. And second, should a signal for new physics be found, one can place a lower limit on the size of the new-physics amplitudes, as well as on their effect on the measurement of the phase of $B^0$–$\bar{B}^0$ mixing.

Acknowledgments

I thank A. Datta, N. Sinha and R. Sinha for collaborations on the topics discussed here. This work was financially supported by NSERC of Canada.

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