ABSTRACT. We discuss the constraints one can place on cosmological parameters using current cosmic microwave background data. A standard \( \chi^2 \)-minimization over band–power estimates is first presented, followed by a discussion of the more correct likelihood approach. We propose an approximation to the complete likelihood function of an arbitrary experiment requiring only limited and easily found information about the observations. Examination of both open models – \((\Omega, h, Q, n)\) – and flat models \((\Omega + \Omega_\Lambda = 1)\) – \((\Omega, \Omega_b, h, Q, n)\) – leaves one rather robust result: models with small curvature are favored.

1 Introduction

Current cosmic microwave background (CMB) results already permit some constraints to be placed on certain cosmological parameters, such as the density parameter, \( \Omega \), the Hubble constant, \( H_0 \equiv h(100 \text{ km/s/Mpc}) \), etc... (Bond & Jaffe 1996; Lineweaver et al. 1997; Hancock et al. 1998; Bartlett et al. 1998a). The present data set is most clearly summarized in the power spectrum plane as a set of (flat) band–power estimates, as shown in Figure 1. Presence of the “Doppler peak” is indicated by several observations on its upward slope, including the new QMAP results (Oliveira–Costa et al. 1998), and most importantly the Saskatoon points (Netterfield et al. 1997). It is worth emphasizing, all the same, that MSAM (Cheng et al. 1997) is not as supportive of the peak as Saskatoon, and these are the only two experiments covering the peak at the present time. Analysis of newly obtained data, e.g., from BOOMERANG’S North American flight, MAXIMA or MAT, all capable of measuring the peak with good precision, will help to resolve the issue.

We have used these power spectrum estimates to constrain cosmological parameters in both open \((\Omega_\Lambda = 0)\) and flat \((\Omega + \Omega_\Lambda = 1)\) scenarios within the context of inflationary models with cold dark matter. After presenting some of our results based on a \( \chi^2 \)-minimization, we discuss the shortcomings of this approach. An easy-to-use approximation to the full likelihood function of an experiment is then proposed and appears to lead to less restrictive constraints than indicated by the \( \chi^2 \)-minimization. Nevertheless, one result remains the same, despite the general differences between the two methods: purely open models with \( \Omega < 0.4 \) are strongly disfavored, arguing against a Universe with large curvature.

2 \( \chi^2 \)-approach

The simplest approach is a straight–forward application of the \( \chi^2 \)-statistic to the data shown in Figure 1. We used CMB–FAST (Zaldarriaga et al. 1998; Seljak & Zaldarriaga 1996) to construct a grid of power spectra spanned by \((\Omega, h, Q, n)\), for purely open models, and by \((\Omega, \Omega_b, h, Q, n)\) for flat models with \( \Omega + \Omega_\Lambda = 1 \). In Figures 2 and 3 we show the resulting constraints on \( \Omega \) and \( h \) as dashed lines for both types of models. The confidence regions are defined relative to the minimum value of \( \chi^2 \); for example, over two parameters the 68% and 95% contours are defined by the projection onto the plane of the multidimensional ellipse inscribed by \( \chi^2_{\text{min}} + 2.3 \) and \( \chi^2_{\text{min}} + 6.17 \), respectively. At the most striking result is the lower limit on the density parameter in open models – \( \Omega > 0.5 \) at 95% confidence. This is quite clearly due to the apparent position of the Doppler peak around \( l = 300 \) in the
Figure 1. *Current CMB power spectrum estimates.* The curve shows the predictions of a model with $\Omega = 0.9$, $h = 0.35$, $\Omega_b h^2 = 0.015$, $Q = 17\mu K$ and $n = 0.94$.

Figure 2. *Open model constraints.* The dashed lines show the $\chi^2$–minimum constraints (68% and 95% confidence boundaries), while the solid lines correspond to the approximate likelihood function.

data. In this sense, the current data do not favor large spatial curvature, which would move the peak too far to the right in the figure. An illustrative example of this is given in Figure 4, where we see how miserably a somewhat “conservative” model with $\Omega = 0.2$ and $h = 0.6$ performs.

3 Of Likelihood Functions

The common method just employed does not strictly apply to the CMB data in Figure 1, for it supposes that the data points are gaussian distributed, and this is certainly not the case. Even if the temperature of individual pixels on the sky can be considered a gaussian random variable, as predicted by inflationary scenarios, if the experimental noise is also gaussian, a band-power estimate will not be gaussian, because it represents the variance of the pixel fluctuations. The variance of a gaussian random variable is not itself gaussian distributed. Thus, at a fundamental level, the $\chi^2$-approach is not suited to our problem.

We must employ a more general likelihood function to properly con-
Figure 3. Flat model constraints. The lines have the same meaning as in Figure 2.

strain our parameters. This may be done for a given experiment if one assumes that the temperature fluctuations on the sky, the pixel values, are indeed gaussian random variables. In this case, with a set of \( N_{\text{pix}} \) pixel values arranged in a data vector \( \vec{d} \), we may write the likelihood function for the parameters, represented by the vector \( \vec{\Theta} = \{ \Omega, h, \ldots \} \), as

\[
\mathcal{L}(\vec{\Theta}) \equiv \text{Prob}(\vec{d}|\vec{\Theta}) = \frac{1}{(2\pi)^{N_{\text{pix}}/2}|C|^{1/2}}e^{-\frac{1}{2}\vec{d}^T C^{-1} \vec{d}}
\]

The key object is the covariance matrix of the pixels, \( C \):

\[
C_{ij} \equiv <d_i d_j> = C(\theta_{ij}) + N_{ij}
\]

where \( W_l \) is the window function defined by the experimental beam and \( N_{ij} \) is the experimental noise covariance matrix. We have imagined a situation were the observations are represented by a set of simple pixel values, e.g., a map, but the argument is the same for temperature differences, or any linear combination of pixel values.

The correct manner to proceed would be to obtain \( \mathcal{L} \) for each experiment (including all correlations between experiments covering the same sky zone) and form the complete likelihood function for the entire data set. This is rather demanding and requires sometimes hard to find experimental information, such as a full noise covariance matrix. We have developed a general purpose approximation to the likelihood function which, by comparison to the full likelihood functions from COBE, Saskatoon and MAX, seems to work well (work in progress, and reported in Bartlett et al. 1998b; for other work, see Bond et al. 1998 and Wandelt et al. 1998). The approximation requires only limited information concerning a given experiment, such as the number of pixels, the band-power estimate and error bars, and an idea of the noise level – all readily found in the literature.

4 Results from the Approximate Likelihood

Applying our approximate likelihood ansatz to each data point in Figure 1, we have revisited the parameter constraints imposed by the data set. The results in the \( (\Omega, h) \)-plane are shown in Figures 2 and 3 as the solid lines; once again, these are projections of ellipses defined relative to the maximum value of the likelihood. We see that in general the confidence regions are larger than those deduced from the \( \chi^2 \)-approach. This is most spectacular for the flat models, where virtually all of the parame-
ter space in this plane is now allowed at “95%” confidence. One should be cautious interpreting these results, however, for a more correct definition of confidence regions would integrate the likelihood function over the parameters not shown. In any case, the difference in the contours, defined in the same way, clearly indicates that the approximate likelihood function has a different distribution over the parameter space than that suggested by the $\chi^2$-approach.

Thus, we see that care must be taken in drawing conclusions from a $\chi^2$-minimization to band-power estimates. Nevertheless, the best fit models do not change drastically. We also find about the same lower limit as before to $\Omega$ in open models – $\Omega > 0.4$.

5 Conclusion

It is important to realize that the CMB is already beginning to divulge its treasures of information. The method used to extract quantitative constraints on cosmological parameters requires some care, and a simple $\chi^2$-analysis using band-power estimates is not the best approach, leading to constraints which appear tighter than the data warrant. One important result already indicated by the present data is that the spatial curvature of the Universe cannot be too large; this is quantified by the lower limit of $\Omega > 0.4$ for the open models examined here.

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