Contribution to the improvement of the correlation filter method for modal analysis with a spatial light modulator

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Abstract. Modal decomposition of light is essential to study propagation properties of waveguides and photonic devices. Modal analysis can be carried out by implementing a computer generated hologram acting as a match filter in a spatial light modulator. In this work, a series of aspects to be taken into account in order to get the most out of this method are presented, aiming to provide operational procedures. First of all, the influence of the mode normalization in the complex amplitude encoding inherent noise is investigated. Then, a method for filter size adjustment based on the LP-modes symmetry is presented. Finally, a robust method to measure the phase difference between modes is proposed. These procedures are tested by wavefront reconstruction in a conventional few mode fiber.

1. Introduction
Characterization of optical fields by means of modal decomposition (MD) is key for the analysis, design and optimization of multimode waveguide-based devices [1]–[3]. In general, MD methods can be divided into numerical or experimental ones. On one hand, several numerical MD algorithms based on measured optical intensity distributions have been reported [4]. Although numerical methods are experimentally simpler, its results can contain ambiguities and the necessary iterative process can be quite computationally intensive. However, numerical MD stands out for its simple experimental effort and efficient realization [5]. On the other hand, MD can be directly performed through experiments, such as the correlation filter method (CFM) [6], which constitutes the subject of this work. Although these methods have higher requirements in terms of measurement effort, experimental realization and cost, they allow the performance of real time MD.

In the CFM, the waveguide output beam illuminates a computer generated hologram (CGH), acting as a match filter, which performs the decomposition into the waveguide propagation modes due to a specific transmittance function previously designed. Hence, a priori knowledge of the waveguide under study set of modes field distribution is required. This is a clear disadvantage, given that the designed CGH would be limited to analyse only the corresponding type of waveguide. This limitation can be overcome by implementing the CGH into a spatial light modulator (SLM), allowing the user to iteratively adapt the decomposition mode set so that any waveguide can be investigated [7].

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Thanks to the recently emerged liquid-crystal SLM technology the experimental procedure has been significantly simplified and the CFM has been used for mode analysis [6]-[8]. However, there are just a few works describing the details of this method [9], which could be related both to the novelty of the technology and to the complexity of the method itself. Taking into account the experimental complexity of the CFM with respect to other MD approaches, as well as the novelty of the SLM technology for its implementation, we provide in this work a series of operational procedures, not detailed to date in any paper as long as we know, in order to carry out this method correctly or to get the most out of it, as a summary of our experience using it, in such a way that it can serve to reduce part of its complexity and thus make its use more attractive.

Correlation filters require light amplitude and phase modulation. Nevertheless, commercial SLMs can only induce a phase shift. Complex modulation can be achieved by conveniently group the SLMs pixels in what has been called macropixels [10]. However, this technique gives rise to an output field presenting a noise term, which not only depends on the filter but also on the input field distribution. In this work we study the effect of the mentioned noise term, inherent to the use of macropixels when an SLM is employed in a MD setup. Moreover, due to the complexity of the MD setup, it is very useful to know the error tolerance of the different components and techniques to minimize its impacts. Despite of its importance, there is only a recently published tutorial that pays attention to most of these issues [9]. While implementing the CFM, we have studied some factors that can condition and worsen its performance, for which we offer a set of techniques that can be useful in order to lessen its influence. On one hand, we propose a CGH scale adjustment technique based on the waveguide modes symmetry in order to easily adjust the match filters size. On the other hand, a different way of measuring the phase difference between modes is presented, reducing the system instabilities and improving the MD performance.

The manuscript is organized as follows. For the sake of completeness, Section 2 is devoted to revise the MD technique based in the CFM implemented in a phase-only SLM. In Section 3, the experimental setup is explained. Section 4 contains the mentioned noise term analysis, inherent to the complex to phase-only coding technique, while Sections 5 and 6 include the proposed scale adjustment and phase retrieval techniques, respectively. Finally, in Section 7 we test the experimental performance of the modal analysis procedure by a wavefront reconstruction, which leads to the conclusions in Section 9.

2. Theory of modal decomposition

2.1. Correlation filter method

The notation to be employed is established in Figure 1, which shows a basic scheme of the setup necessary for mode decomposition. The multimode waveguide output field to be analysed constitutes the input CGH field:

\[
U(x, y) = \sum_{n=1}^{N} c_n \varphi_n(x, y).
\]  

(1)

where x and y are orthogonal coordinates in the CGH plane, N is the total number of propagation modes allowed by the waveguide under study [11], \( \varphi_n(x, y) \) is the n-th mode normalized distribution of the transversal electric field, and \( c_n \) is its complex expansion coefficient, that can be expressed in terms of a modal weight, \( |c_n| \), and a phase related to the 0-mode, \( \phi_n \),

\[
c_n = |c_n| e^{i\phi_n}.
\]  

(2)
\[ W_0(x, y) \text{ and } U(x, y) \text{ are related by} \]
\[ W_0(x, y) = U(x, y) T(x, y). \tag{3} \]

The output distribution \( W_f(u, v) \) is obtained at the \( L_3 \) lens focal plane. \( W_f \) and \( W_0 \) are related by means of [12]:
\[ W_f(u, v) = \frac{1}{i\lambda f} F[W_0(x, y)], \tag{4} \]
where \( F \) denotes Fourier transform, \( i = \sqrt{-1} \), \( \lambda \) is the input light wavelength, \( f \) is the lens focal distance and the coordinates at the focal plane \( (u, v) \) are related to the Fourier transform frequency space coordinates \( f_x \) and \( f_y \) by means of \( u = \lambda f f_x, v = \lambda f f_y \). In order to detect the amplitude of mode \( p \), a filter with the following transmittance should be employed [6]:
\[ T(x, y) = \phi_p^*(x, y) \cdot e^{i(k_x x + y k_y)}, \tag{5} \]
where the asterisk denotes the complex conjugate and \( (k_x, k_y) \) are transversal components of a wave vector. After some operations and taking into account orthogonality between mode field distributions [6]:
\[ W_f(u = \lambda f k_x, v = \lambda f k_y) = \frac{1}{i\lambda f} \sum_{n=1}^{N} c_n \delta_{pn} = \frac{1}{i\lambda f} c_p, \tag{6} \]
being \( \delta_{pn} \), the Kronecker delta. If a CCD detector is placed in the focal plane, use of a filter as given by Eq. (5) allows one to obtain information about the \( p \)-mode amplitude \( |c_p| \) at the specific focal plane coordinates \( (u = \lambda f k_x, v = \lambda f k_y) \).

A simultaneous procedure based on angular multiplexing can be carried out. It consists of employing a filter whose transmittance is a superposition of different \( T_p \) filters, associating to each \( p \)-mode a different wave vector \( (u_p = \lambda f k_{x,p}, v_p = \lambda f k_{y,p}) \), so that the total transmittance function is:
\[ T(x, y) = \sum_{p=1}^{N} \phi_p^*(x, y) \cdot e^{i(k_{x,p} x + y k_{y,p})}. \tag{7} \]

By a convenient choice of these wave vectors, information of the different modes appears at separate enough points in the focal plane.

For some applications it is enough to obtain the modal weights, \( \rho \). However, sometimes it is required to determine the phase difference between modes, \( \Delta \phi \). According to [6], in order to measure it, it is necessary to use two transmittance functions defined as
\[ T_p^{\cos}(x, y) = \frac{1}{\sqrt{2}} [\phi_0^*(x, y) + \phi_p^*(x, y)] \cdot e^{i(k_x x + y k_y)} \tag{8} \]
and
\[ T_p^{\sin}(x, y) = \frac{1}{\sqrt{2}} [\phi_0^*(x, y) + i \phi_p^*(x, y)] \cdot e^{i(k_x x + y k_y)}, \tag{9} \]
where $\phi_0$ is the reference mode for measuring the p-mode relative phase: $\Delta \phi_p = \phi_p - \phi_0 = \phi_p$, where due to simplicity we have chosen $\phi_0 = 0$. In these cases, the intensity at the back focal plane measurement position is proportional to the cosine and sine, respectively, of the phase difference [6]. This allows us to obtain an unambiguous solution for the phase differences.

### 2.2. Double phase method for complex amplitude encoding

When implementing the CGH in an SLM, all the transmittance functions defined in the previous subsection require the encoding of complex amplitude in an only-phase device. Among the different proposed procedures for pixel grouping, the double-phase CGHs have been chosen [10] due to their simple implementation and maximum resolution, as in this technique macropixels are composed of only two pixels. This method is based on the principle that any complex value inside the unit circle can be resolved into the sum of two vectors with unit module.

Consider a CGH implemented in a commercial SLM that only admits phase modulation of its pixels, and a combination of them in macropixels that may offer an approximate complex modulation facility. Specifically, as explained in [10], a CGH filter with an approximate transmittance as the theoretical one can be achieved by means of grouping pairs of pixels into a macropixel. Considering that each couple of pixels $(2m - 1, n)$ and $(2m, n)$ constitute the megapixel with indexes $(m, n)$, if the desired complex transmittance of macropixel $(m, n)$ is $c_{mn} = |c_{mn}|e^{i\phi_{mn}}$, the following respective phase shifts are assigned to pixels $(2m - 1, n)$ and $(2m, n)$ [10]:

$$\phi_{mn}^{(1)} = \phi_{mn} - \Delta_{mn} \quad \text{and} \quad \phi_{mn}^{(2)} = \phi_{mn} + \Delta_{mn},$$

where $|c_{mn}| \leq 1$, $0 \leq \phi_{mn} \leq 2\pi$ and

$$\Delta_{mn} = \cos^{-1}(|c_{mn}|).$$

Denoting by $Q$ the transmittance function of the phase-only device where the double phase CGH are implemented, its Fourier transform can be expressed as

$$\tilde{Q} = \tilde{Q}_S + \tilde{Q}_N,$$

in such a way that [10]

$$\tilde{Q}_S = \mathcal{F}[T(x, y)].$$

Contribution $\tilde{Q}_S$, referred to as signal, is the only one that should be expected if the double-phase solution did not provide an approximate but an exact $T(x, y)$ transmittance. Nevertheless, another contribution to $\tilde{Q}$ also appears, due to the doble-phase implementation: $\tilde{Q}_N$, referred to as noise.

### 3. Experimental setup

The required experimental setup is depicted in Fig. 2 (a).

![Figure 2](image-url) (a) Scheme of the measurement setup. FMF: few mode fiber, MP: micro-positioners, MO: microscope objective, L1 and L2: lenses, LP: linear polarizer, BS: beam splitter, CCD cameras, SLM: spatial light modulator. (b) and (c) Measured transversal intensity distribution in order to centre the setup and test de proposed techniques, respectively.
An SMF28 optical fiber is illuminated by a He-Ne laser (\(\lambda = 632.8\) nm), supporting six propagation LP modes. The first section of optical fiber is repeatedly curved in order to eliminate modes other than the fundamental one, \(LP_{01}\). We use this fiber to illuminate a second optical fiber with the same characteristics. This way, when both fibers are placed together and over the same longitudinal axis, the \(LP_{01}\) mode also propagates along the second section of optical fiber and can be used to carry out the system alignment. By displacing the first fiber with respect to the second one, different transversal distributions are generated, due to different combinations of excited modes, useful to test the MD performance. Fig. 2 (b) and (c) shows two examples: first one with both fiber sections aligned, used to calibrate the setup, and second one, at an arbitrary position, employed to test the proposed techniques performance.

In order to increase the effective resolution, the fiber output beam size has been magnified at the SLM display using a microscope objective (DIN×40) and a lens (\(f_i=50\) cm) following a 4f configuration (magnification 125). The 4f-lens combination images the fiber output beam through the beam splitter (BS) both in the CCD1 camera and in the phase-only SLM operating in reflection mode (PLUTO VIS Phase Only Spatial Light Modulator from Holoeye [13]). As it only operates in the horizontal polarization, we select this state of polarization from the input beam with a linear polarizer (LP). The reflected field from the SLM, where the CGH is implemented, is Fourier transformed by lens \(L_2\) (\(f_2=15\) cm) and this signal is detected by the CCD2 camera, placed at the \(L_2\) lens focal plane, where the modal weight are obtained by measuring the intensity in the specified coordinates according to Eq. (7). The SLM was previously calibrated following the procedure described in [14].

4. Correlation filter size adjustment
The MD performance is based on the overlap between the incident light beam and the implemented CGH. Inaccuracies in the position or size of the encoded CGH with respect to the incident light can result in the detection of erroneous modes or in the incorrect determination of the modal weights. Notwithstanding its importance, these issues have not been studied in depth except for a recently published tutorial [9]. Their proposed approach to transversally align the CGH position consists of displacing the filter position in such a way that, for a mismatched overlap, the on-axis null appears centred regarding the input light beam. If the first and second optical fiber sections are aligned so as to achieve single \(LP_{01}\) mode propagation in the second one, in such a case there should be zero on-axis intensity when implementing any other mode into the match filter. We can benefit from \(LP_{11}^e\) and \(LP_{11}^o\) modes symmetry in order to centre the CGH vertically and horizontally, respectively.

![Figure 3](image-url) Measured intensity at the CCD2 camera optical axis point as a function of the \(LP_{02}\) mode magnification implemented in the SLM when illuminated with the \(LP_{01}\) mode. Three measured transversal intensity distributions are superimposed for the specified points.
We propose that this same idea can be used to adjust the filter size. Despite it is straightforward to calculate the theoretical 4f-magnification, the real value can be slightly modified by its experimental implementation, mainly because it is not easy to place the optical fiber output end at the exact focal length distance from the microscope principal plane. In [15], a method for filter size adjustment is proposed, for which it is required to obtain some parameters as the beam quality factor and the second moment. Here, we present a different approach, based on the $L_{P02}$ mode symmetry. If we implement the $L_{P02}$ mode into the CGH and illuminate it with the $L_{P01}$ mode, only when both of their sizes are matched there will be zero intensity at the optical axis. We can use this behaviour to find the correct scale just by minimizing the optical axis intensity as a function of the theoretical magnification value. This is shown experimentally in Fig. 3. Overlaying the graph, three measured transversal intensity distributions are shown for a smaller, adjusted and larger size (from left to right) of the filter with respect to the incident light.

According to the minimum intensity, the magnification value is 141, which has a 13% of relative error with respect to the theoretical one. This error can be explained due to the inherent uncertainties of the optical system positioning. This method can be used as long as the set of modes has the required symmetry, which is common in cylindrical-like structures.

5. Double phase method noise term analysis

Consider a double-phase CGH with transmittance $Q(x, y)$ (Eq. 18), implemented in order to emulate an ideal CGH filter with complex transmittance $T(x, y)$ (Eq. 13). According to Eqs. (3), (4) and (12), this double-phase filter yields a field distribution in the $L_2$ back focal plane such as

$$W_f = \frac{1}{i\lambda f} \left[ \tilde{U} \ast \tilde{Q}_S + \tilde{Q}_N \right],$$

where $\tilde{U} = \mathcal{F}[U(x, y)]$ and the ($\ast$) sign represents convolution product. We consider two different contributions to $W_f$, called here signal and noise terms, respectively:

$$W_{f,S} = \frac{1}{i\lambda f} \left[ \tilde{U} \ast \tilde{Q}_S \right],$$

$$W_{f,N} = \frac{1}{i\lambda f} \left[ \tilde{U} \ast \tilde{Q}_N \right].$$

As $\tilde{Q}_S = \mathcal{F}[T(x, y)]$, $W_{f,S}$ represents the field distribution that would be obtained if an ideal CGH filter with complex transmittance $T(x, y)$ was employed. Nevertheless, also a superimposed noise term $W_{f,N}$ is present. As stated in Eq. (16), this term contains the convolution of two factors. One of them, $\tilde{Q}_N$, is intrinsic to the filter, while the other, $\tilde{U}$, is directly related to the input electric field distribution, obviously unpredictable as it is the object of analysis. Therefore, the magnitude and distribution of the noise term is expected to be very different depending on each working condition. For this reason, its impact cannot be analysed by means of a general mathematical treatment.

![Figure 4](image_url)

Figure 4 Simultaneous MD spatial configuration scheme (a), together with the normalized transversal intensity distributions $W_{f,S}$ (b) and $W_{f,N}$ (c) when all six LP-modes are equally present in the incident light.

A case in which the noise factor strongly affects the MD is shown in Fig. 4. Suppose an incident beam with all six LP-modes equally present and a CGH in which all the match filters (one for each mode) are simultaneously multiplexed with a different grating to spatially separate the signals at the
Fourier plane. Specifically, into the vertexes of a regular hexagon as it is shown in Fig. 4 (a). Fig. 4 (b) and (c) show the simulated intensity at the L2 back focal plane separated into the signal term \( W_{f,S} \) (Eq. 15) and the noise term \( W_{f,N} \) (Eq. 16), respectively. By comparing the modal weights with and without noise term, relative errors up to 50% are obtained, preventing the correct performance of the MD.

In view of the findings, it is essential to reduce the noise term influence on the MD performance. One way is to realize the MD sequentially rather than simultaneously. However, it is not possible to obtain all modal weights in real time with this procedure, which can be an important disadvantage depending on the system dynamics. Another approach is to renormalize the modal distributions implemented at the SLM. Theoretically, any transmittance value between zero and one is valid when implementing the double phase method. Nevertheless, the maximum value should be as close as possible to one in order to use the whole range of the SLM and reduce the noise impact. This normalization has a huge impact on the signal to noise ratio, as it is shown in Fig. 5, where we have used the same configuration than the one in Fig. 4 and summed the intensities at the six information position for both the signal (Eq. 15) and the noise (Eq. 16) terms.

![Figure 5 Signal and noise intensity evolution as a function of the maximum value in the transmittance function for the configuration case shown in Fig. 4.](image)

As it is shown in Fig. 5, by approaching the maximum transmittance value to one we improve the signal to noise ratio, while if we keep the set of modes orthonormality (situation marked with the red arrows) the noise intensity cannot be neglected with respect to the signal one and thus affects the MD.

6. Robustness improvement in relative phases measurements

In theory, the phase difference can be unambiguously determined with two measurements by using Eqs. (8) and (9), each one having two possible solutions. Nevertheless, in experiments, the mode relative phase measurements can be easily influenced by system instabilities (i.e. fiber and laser fluctuations, micro-positioners looseness, etc.) or by misalignments due to position uncertainties (i.e. lenses, fiber, CCD, SLM). As a consequence, there may be an error in the arcusine and arcusine result that could prevent correct phase retrieval. For such case, we propose a different transmittance function in order to measure the relative phases and also to study the phase determination uncertainty.

Suppose we implement in the SLM the following transmittance function

\[
T(x,y) = \frac{1}{\sqrt{2}}[\varphi_0^*(x,y) + \varphi_l^*(x,y) \cdot e^{i\theta}],
\]

as a function of a variable phase \( \theta \). The particular cases \( \theta = 0 \) and \( \theta = \pi/2 \) recover the Eqs. (8) and (9), respectively. The intensity at the CCD2 measurement position as a function of \( \theta \) is

\[
|W_f(u = \lambda f k_x, v = \lambda f k_y, \theta)|^2 \propto |c_0|^2 + |c_p|^2 + 2|c_0||c_p| \cdot \cos(\phi_p + \theta).
\]

In order to improve the robustness of the relative phase determination, we perform a series of intensity measurements as a function of \( \theta \) and fit them to the function from Eq. (18).
Figure 6 Relative phase measurements of the \( \text{LP}_2^e \) mode together with its cosine best fit by least squares for the transversal intensity distribution shown in Fig. 2 (c).

As an example, Fig. 6 shows the so mentioned measurements together with its best fit by least squares for the \( \text{LP}_2^e \) mode and the transversal intensity distribution shown in Fig. 2 (c). It is clear to see that, despite the experimental dots follow a similar behaviour than the cosine function, there are some significant differences with respect to the fitting function. Both instabilities (i.e. laser fluctuations, fiber disturbances) and experimental inaccuracies (i.e. positioning, centring and adjustment) may be responsible for the lack of agreement between the measurements and the theoretical function.

Table 1 Phase difference between the \( \text{LP}_2^e \) and \( \text{LP}_{01} \) modes from the transversal intensity distribution shown in Fig. 2 (c), computed by means of the function fit from Eq. (18) or by the selected phases marked in Fig. 6.

| \( \phi_{\text{LP}_2^e} (\pi \text{ rad}) \) | Fit \( \theta = 0 \) | \( \theta = \pi/2 \) | \( \theta = \pi \) | \( \theta = 3\pi/2 \) |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| 0.24                            | 0.23            | 0.23            | 0.00            | 0.39            |

Superimposed to the graph in Fig. 6, four points have been highlighted. First two points, \( \theta = 0 \) and \( \theta = \pi/2 \), are the particular cases from Eqs. (8) and (9). In this case, the points show a good agreement with the fitting curve, meaning that both methods will give a similar result. However, we could have selected a different couple of points. Suppose we choose the particular cases \( \theta = \pi \) and \( \theta = 3\pi/2 \), whose experimental values are far from the fitting curve, and compute the phase difference. In this case, the obtained angles do not match each other. Table 1 shows the obtained relative phase for the \( \text{LP}_2^e \) mode depending on the angle employed.

7. Wavefront reconstruction

In order to test the proposed size adjustment technique and phase retrieval procedure, together with the MD method itself, we have reconstructed the transversal intensity distribution shown in Fig. 2 (c). Fig. 7 shows the comparison between the experimental distribution (a) and the numerically reconstructed ones (b-d). The reconstructed distributions have been obtained by measuring all six LP modes weights and relative phases. However, in the first reconstruction the CGH size has not been adjusted through our proposed technique but calculated theoretically (Fig. 7b), and in the second one the relative phases have been determined just by using Eqs. (8) and (9) instead of by the proposed fitting procedure (Fig. 7c). Finally, the last distribution has been computed using both proposed techniques (Fig. 7d). As one can see, a good agreement is shown between Fig. 7 (a) and (d) distributions, standing out the importance of both the size adjustment and the phase retrieval.
8. Conclusion

We presented a series of practical tools to perform an accurate modal decomposition of light and we tested them satisfactorily by wavefront reconstruction in a few mode fiber, allowing us to verify both the good performance of the modal analysis procedure and the proposed techniques. In order to reduce the SNR when performing a simultaneous MD and exploit the SLM range, the set of modes orthonormality needs to be broken and thus a scale factor taken into account. It has been shown it is possible to benefit from the LP modes symmetry not only to transversally centre the CGHs but to adjust its size with respect to the incident beam. This procedure is not restricted to the LP modes and it can be used as long as the set of modes provides the necessary symmetry, which is usually fulfilled in cylindrical-like basis. When setup instabilities prevent the correct phase difference determination through the classical approach, a possibility to improve phase retrieval robustness consists on implementing a transmission function as a function of an angle parameter and adjust the set of measurements. The good performance of these techniques has been tested successfully by wavefront reconstruction.

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