Lepton polarization asymmetries for $B \rightarrow K^*\ell^+\ell^-$: A Model Independent approach

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Abstract:
In this work we shall derive expressions for the single and double lepton polarization asymmetries for the exclusive decay $B \rightarrow K^*\ell^+\ell^-$, using the most general model independent effective Hamiltonian. We have conducted this study with this particular channel as it has the highest branching ratio among the various purely leptonic and semi-leptonic decay modes, making this mode particularly useful for studying physics beyond the SM. We have also analyzed the effects on these polarization asymmetries, and hence the physics underlying it, when complex phases are included in some of the Wilson coefficients.

Keywords: B-Physics, Rare Decays, Beyond Standard Model.

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1. Introduction

As more and more experimental data is produced by $B$-factories our quest for finding new physics signatures in the various decay modes for low energy processes is increasing. The sheer volume of literature studying the possible signatures of different supersymmetric (and other) models in the context of $B$-meson decays evidences how promising a testing ground these rare decays, induced by the flavour changing neutral current (FCNC) $b \rightarrow s$, are. Of the various hadronic, leptonic and semi-leptonic decays modes (based on the $b \rightarrow s$ transition of the $B$-meson) the semi-leptonic decay modes are extremely significant as they are theoretically cleaner, and hence very useful for testing various new physics models. The semi-leptonic decay modes based on the quark level transition $b \rightarrow s \ell^+ \ell^-$ offer many more observables associated with the final state lepton pair, such as the forward-backward (FB) asymmetry, lepton polarization asymmetries etc. These additional observables could prove to be very useful in testing the effective structure of these theories and hence the underlying physics. For this reason many processes like $B \rightarrow \pi(\rho)\ell^+ \ell^-$ [1], $B \rightarrow \ell^- \ell^+ \gamma$ [2], $B \rightarrow K \ell^+ \ell^-$ [3] and the inclusive process $B \rightarrow X_s \ell^+ \ell^-$ [4–6] have been studied. But of the various decay modes of the $B$-mesons based on the transition $b \rightarrow s \ell^+ \ell^-$ the exclusive mode $B \rightarrow K^* \ell^+ \ell^-$ is one of the more attractive due to it having the highest standard model (SM) branching ratio. For this reason large numbers of observables in this decay mode have been studied [7–11].

Previously Aliev et al. [8] studied the various single polarization asymmetries for this decay mode, where they used the model independent approach earlier proposed by Fukae et al. [5]. They were able to demonstrate that within the framework of a model independent
theory, constrained by the experimentally measured values of the $B \rightarrow K^{*}\ell^{+}\ell^{-}$ branching ratio, there existed regions where the possible new Wilson coefficients could generate considerable departures from the SM. However, as pointed out in London et al. [12] some of the single lepton polarization asymmetries may be too small to be observed, and hence merely the single lepton polarization asymmetries may not provide a sufficient number of observables to crosscheck the structure of the effective Hamiltonian. With this in mind more observables are required.

Further to this there has been in the recent observations of the $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$ decays hints of possible anomalies unexplainable within the SM [13, 14]. These anomalies arise when we try to match the pattern of data from the $B$-factories with theory. The recent Belle and BaBar data regarding the $B \rightarrow \pi\pi$ mode can be easily explained by taking into account the non-factorizable contributions. Note that the $B \rightarrow \pi\pi$ channel is not greatly effected by the electroweak (EW) penguin diagrams and therefore one can extract the hadronic parameters from this by assuming isospin symmetry. Using the SU(3) flavour symmetry we can determine the hadronic $B \rightarrow \pi K$ parameters from the relevant $B \rightarrow \pi\pi$ modes. As has been pointed out some time back by Buras et al. [13], which has been revived in many later works [14], this procedure works very well and gives us a good match between theory and experimental results as long as we are analyzing those modes which are not greatly affected by the EW penguin diagrams. However, if we try to repeat the same sort of exercise for modes like $B_d \rightarrow \pi^0 K_S$, which are dominated by EW penguins, then there is a substantial disagreement between theory and experimental data [13]. Lately some solutions of this “$B \rightarrow \pi K$ puzzle” are being tested and almost all of these propose EW penguins which are sizably enhanced not only in magnitude but also in their CP-violating phase, which can become as large as -90°. This proposal is a very interesting one and can significantly affect many other decay modes. Rather detailed studies of this proposal have been carried out by Buras et al. [13] leading to possible predictions of substantial enhancements in the branching ratio of many leptonic and semi-leptonic decay modes, which will soon be tested in B-factories. This sort of possibility forces us to consider the option of what could be the possible changes expected in various kinematical observables, such as the branching ratios, FB asymmetries and various polarization asymmetries, if some of the Wilson coefficients had such a large phase (making them predominately imaginary). Note that with this in mind we have analyzed this in an earlier work [15] for the inclusive decay mode $B \rightarrow X_s\ell^{+}\ell^{-}$. In that study we estimated the variation in the polarization asymmetries in the inclusive mode if the $bsZ$ vertex were modified. In this work we also explored the option of allowing some of the Wilson coefficients having large CP violating phases. This sort of approach to finding the effects of extra phases in Wilsons on various kinematical observables like branching ratio, partial width CP asymmetry, FB asymmetry and single lepton polarization asymmetry in $B \rightarrow K^{*}\ell^{+}\ell^{-}$ have been followed in earlier works [16]. In this current study we shall also try to analyze what effects there shall be on the various polarization asymmetries in the $B \rightarrow K^{*}\ell^{+}\ell^{-}$ decay.

In this study we will work in a model independent framework by taking the most general form of the effective Hamiltonian and then analyzing the effect on polarization asymmetries if the Wilson coefficients (mainly the coefficients which correspond to vector
like interactions) have an extra phase. Keeping this eventual aim in mind, this paper shall be organized as follows: In section 2 we shall introduce the most general form of the effective Hamiltonian, obtaining (in terms of the forms factors for the $B \to K^*$ transition) the matrix element for the $B \to K^* \ell^+ \ell^-$ decay and the unpolarized cross-section. In section 3 we shall define and calculate the various single and double polarization asymmetries, followed in section 4 with our numerical analysis. We shall also include a discussion of these results and our conclusions in this final section.

2. The Effective Hamiltonian

We know, from the paper by Fukae et al. [5] that together with the terms proportional to our conventionally defined $C_7$ (written below as $C_{SL}$ and $C_{BR}$ for terms corresponding to the standard $-2m_s C_7$ and $-2m_b C_7$ terms respectively), $C_9$ and $C_{10}$ (which can be redefined in terms of $C_{LL}$ and $C_{LR}$) we have ten independent local four-Fermi interactions which contribute to the FCNC transition $b \to s \ell^+ \ell^-$:

$$H_{\text{eff}} = \frac{\alpha G_F}{\sqrt{2}\pi} V_{ts} V_{tb} \left[ C_{SL} \left( \bar{s} i\sigma_{\mu\nu} \frac{q^\nu}{q^2} L b \right) (\bar{\ell} \gamma^\mu \ell) + C_{BR} \left( \bar{s} i\sigma_{\mu\nu} \frac{q^\nu}{q^2} R b \right) (\bar{\ell} \gamma^\mu \ell) 
+ C_{LL} \left( \bar{s} L \gamma_\mu b_L \right) (\bar{\ell} L \gamma^\mu \ell_L) + C_{LR} \left( \bar{s} L \gamma_\mu b_L \right) (\bar{\ell} R \gamma^\mu \ell_R) 
+ C_{RL} \left( \bar{s} R \gamma_\mu b_R \right) (\bar{\ell} L \gamma^\mu \ell_L) + C_{RR} \left( \bar{s} R \gamma_\mu b_R \right) (\bar{\ell} R \gamma^\mu \ell_R) 
+ C_{LRLR} \left( \bar{s} L b_R \right) (\bar{\ell} L \ell_R) + C_{RLRL} \left( \bar{s} R b_L \right) (\bar{\ell} R \ell_L) 
+ C_T \left( \bar{s} \sigma_{\mu\nu} b \right) (\bar{\ell} \sigma_{\mu\nu} \ell) + i C_T \left( \bar{s} \sigma_{\mu\nu} b \right) (\bar{\ell} \sigma_{\alpha\beta} \ell) \epsilon^{\mu\nu\alpha\beta} \right], \quad (2.1)$$

where $q$ represents the momentum transfer ($q = p_B - p_{K^*}$), $L/R = (1 \mp \gamma^5)/2$ and the $C_X$'s are the coefficients of the four Fermi interactions. Among these there are four vector type interactions ($C_{LL}$, $C_{LR}$, $C_{RL}$ and $C_{RR}$), two of which contain contributions from the SM Wilson coefficients. These two coefficients, $C_{LL}$ and $C_{LR}$, can be written as:

$$C_{LL}^{\text{tot}} = C_9 - C_{10} + C_{LL},$$
$$C_{LR}^{\text{tot}} = C_9 + C_{10} + C_{LR}. \quad (2.2)$$

So $C_{LL}^{\text{tot}}$ and $C_{LR}^{\text{tot}}$ describe the sum of the contributions from the SM and new physics. Eqn.(2.1) also contains four scalar type interactions ($C_{LRLR}, C_{RLRL}, C_{LRLR}$ and $C_{RLRL}$) and two tensor type interactions ($C_T$ and $C_{TE}$).

We shall now follow the standard techniques, as seen in references [5, 7–9] of rendering the quark level transition above to a matrix element which describes the exclusive process $B \to K^* \ell^+ \ell^-$, that is, by parameterizing over the $B$ and $K^*$ meson states in terms of form factors. Using the form factor expressions derived in the paper by Ball et al. [17] we express our hadronic matrix elements as:

$$\langle K^* | \bar{s} (1 \pm \gamma^5) b | B \rangle = \pm \frac{2im_{K^*}}{m_b} (\epsilon^* \cdot q) A_0(\bar{s}), \quad (2.3)$$
\[ \langle K^* | \bar{s} \sigma_{\mu \nu} q' (1 \pm \gamma^5) b | B \rangle = -2 \epsilon_{\mu \rho \sigma \sigma} \epsilon^{\sigma \nu} q^\rho p_{K^*}^\sigma T_1(\hat{s}) \mp i T_2(\hat{s}) \left[ \epsilon_\mu^*(m_B^2 - m_{K^*}^2) - (\epsilon^* \cdot q) \right] \\
(2p_{K^*} + q) \mu \pm i T_3(\hat{s}) (\epsilon^* \cdot q) \left[ q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (2p_{K^*} + q)_\rho \right] \]

\[ \langle K^* | \bar{s} \gamma_{\mu} (1 \pm \gamma^5) b | B \rangle = \epsilon_{\mu \alpha \beta} \epsilon^{\sigma \nu} q^\rho p_{K^*}^\sigma \left( \frac{2V(\hat{s})}{m_B + m_{K^*}} \right) \pm i \epsilon_\mu^* (m_B + m_{K^*}) A_1(\hat{s}) \\
\mp i (2p_{K^*} + q) _\mu (\epsilon^* \cdot q) \frac{A_2(\hat{s})}{m_B + m_{K^*}} \\
\mp i q_\mu (\epsilon^* \cdot q) \frac{2m_{K^*}}{8} \left( A_3(\hat{s}) - A_0(\hat{s}) \right), \tag{2.5} \]

\[ \langle K^* | \bar{s} \sigma_{\mu \nu} b | B \rangle = -i \epsilon_{\mu \alpha \beta} \left[ T_1(\hat{s}) \epsilon^\alpha (2p_{K^*} + q)^\beta - \frac{(m_B^2 - m_{K^*}^2)}{q^2} \{ T_1(\hat{s}) - T_2(\hat{s}) \} \epsilon^\alpha q^\beta \right. \\
+ \frac{2 (\epsilon^* \cdot q)}{q^2} \left. \left\{ T_1(\hat{s}) - T_2(\hat{s}) - \frac{q^2}{m_B^2 - m_{K^*}^2} T_3(\hat{s}) \right\} \right] \left( p_{K^*}^\rho q^\sigma \right) \]

\[ \tag{2.6} \]

Note that the parameterization of these form factors can be found in Appendix B.1.

Using these form factor expressions our matrix element for the decay \( B \to K^* \ell^+ \ell^- \) can be expressed as:

\[ \mathcal{M}(B \to K^* \ell^+ \ell^-) = \frac{\alpha G_F}{4 \sqrt{2} \pi} V_{tb} V_{ts}^* \left[ (\bar{\ell} \gamma^\mu \ell) \left\{ A \epsilon_{\mu \rho \sigma \sigma} \epsilon^{\sigma \nu} q^\rho p_{K^*}^\sigma + i B \epsilon_\mu^* + 2i C (p_{K^*})_\mu (\epsilon^* \cdot q) \right\} \\
+ (\bar{\ell} \gamma^\mu \gamma_5 \ell) \left\{ E \epsilon_{\mu \rho \sigma \sigma} \epsilon^{\sigma \nu} q^\rho p_{K^*}^\sigma + i F \epsilon_\mu^* + 2i G (p_{K^*})_\mu (\epsilon^* \cdot q) \right\} \\
+ i K (\bar{\ell} \ell) (\epsilon^* \cdot q) + i M (\bar{\ell} \gamma_5 \ell) (\epsilon^* \cdot q) \right. \\
+ 4i C_T (\bar{\ell} \sigma^{\mu \nu} \ell) \epsilon_{\mu \rho \sigma \sigma} \left\{ -2T_1 \epsilon^\mu p^\rho p_{K^*}^\sigma + N_1 \epsilon^\nu p^\rho - N_2 (\epsilon \cdot q) p_{K^*}^\rho q^\sigma \right\} \\
+ 16 C_{TE} (\bar{\ell} \sigma^{\mu \nu} \ell) \left\{ -2T_1 \epsilon^\mu p^\rho p_{K^*}^\sigma + M_1 \epsilon^\nu p^\rho - M_2 (\epsilon \cdot q) p_{K^*}^\rho q^\sigma \right\} \right], \tag{2.7} \]

where:

\[ A = \left( C_{LL}^{tot} + C_{RL} \right) \frac{V(\hat{s})}{m_B + m_{K^*}} - 4 (C_{SL} + C_{BR}) \frac{T_1(\hat{s})}{q^2}, \]

\[ B = \left( C_{RL} + C_{RR} - C_{LL}^{tot} - C_{LR}^{tot} \right) (m_B + m_{K^*}) A_1 - 2 (C_{BR} - C_{SL}) \frac{T_2(\hat{s})}{q^2} \frac{(m_B^2 - m_{K^*}^2)}{q^2}, \]

\[ C = \left( C_{LL}^{tot} + C_{LR}^{tot} - C_{RL} - C_{RR} \right) \frac{A_2(\hat{s})}{m_B + m_{K^*}} - 2 (C_{BR} - C_{SL}) \frac{T_2(\hat{s})}{q^2} - \frac{q^2}{(m_B^2 - m_{K^*}^2)} T_3(\hat{s}) \right], \]

\[ D = 2 \left( C_{LL}^{tot} + C_{LR}^{tot} - C_{RL} - C_{RR} \right) \frac{m_{K^*}}{q^2} \left\{ A_3(\hat{s}) - A_0(\hat{s}) \right\} + 2 (C_{BR} - C_{SL}) \frac{T_3(\hat{s})}{q^2}, \]
\[ + (C_{LL}^{\text{tot}} + C_{LR}^{\text{tot}} - C_{RL} - C_{RR}) + \frac{A_2(\hat{s})}{m_B + m_{K^*}} - 2(C_{BR} - C_{SL}) \frac{1}{q^2} T_2(\hat{s}) + \frac{q^2}{(m_B^2 - m_{K^*}^2)} T_3(\hat{s}) \]

\[ E = 2 \left( C_{RR} + C_{LR}^{\text{tot}} - C_{LL}^{\text{tot}} - C_{RL} \right) \frac{V(\hat{s})}{(m_B + m_{K^*})}, \]

\[ F = \left( C_{RR} - C_{LR}^{\text{tot}} \right) \frac{(m_B + m_{K^*}) A_1(\hat{s})}{(m_B + m_{K^*})}, \]

\[ G = \left( C_{RL} + C_{LR}^{\text{tot}} - C_{LL}^{\text{tot}} - C_{RR} \right) \frac{A_2(\hat{s})}{(m_B + m_{K^*})}, \]

\[ H = 2 \left( C_{LR}^{\text{tot}} + C_{RL} - C_{LL}^{\text{tot}} - C_{RR} \right) \frac{m_{K^*}}{q^2} (A_3(\hat{s}) - A_0(\hat{s})) + \left( C_{RL} + C_{LR}^{\text{tot}} - C_{LL}^{\text{tot}} - C_{RR} \right) \frac{A_2(\hat{s})}{(m_B + m_{K^*})}, \]

\[ K = 2 \left( C_{RLRL} + C_{RLRL} - C_{LRLR} - C_{LRRL} \right) \frac{m_{K^*}}{m_B} A_0(\hat{s}), \]

\[ M = 2 \left( C_{LRRL} + C_{RLRL} - C_{LRLR} - C_{RLRL} \right) \frac{m_{K^*}}{m_B} A_0(\hat{s}), \]

\[ N_1 = -T_1(\hat{s}) + \frac{(m_B^2 - m_{K^*}^2)}{q^2} \left\{ T_1(\hat{s}) - T_2(\hat{s}) \right\}, \]

\[ N_2 = \frac{2}{q^2} \left[ T_1(\hat{s}) - T_2(\hat{s}) - \frac{q^2}{(m_B^2 - m_{K^*}^2)} T_3(\hat{s}) \right], \]

\[ M_1 = N_1, \]

\[ M_2 = N_2. \] (2.8)

Using the above expression we can calculate the unpolarized decay rate as:

\[ \frac{d\Gamma}{ds} (B \to K^*\ell^+\ell^-) = \frac{G_F^2 \alpha^2}{2^{14/3} \pi^5 m_B} |V_{ts}V_{tb}^\ast|^2 \lambda^{1/2} \sqrt{1 - \frac{4m_t^2}{q^2}} \Delta, \] (2.9)

where \( \lambda = 1 + \hat{m}_{K^*}^2 + \hat{s}^2 - 2(\hat{m}_{K^*} + \hat{s}) - 2\hat{m}_{K^*}\hat{s} \) with \( \hat{m}_{K^*} = m_{K^*}/m_B \) and \( \hat{s} = s/m_B^2 \). \( s \) is the dilepton invariant mass. The function \( \Delta \) is defined in Appendix B.2.

3. Lepton polarization asymmetries

In order to now calculate the polarization asymmetries of both the leptons defined in the effective four fermion interaction of Eqn. (2.1), we must first define the orthogonal vectors \( S \) in the rest frame of \( \ell^- \) and \( W \) in the rest frame of \( \ell^+ \) (where these vectors are the polarization vectors of the leptons). Note that we shall use the subscripts \( L, N \) and \( T \) to correspond to the leptons being polarized along the longitudinal, normal and transverse directions respectively [2, 4, 6, 8, 12].

\[ S_L^{\mu} \equiv (0, e_L) = \left( 0, \frac{p_-}{|p_-|} \right), \]

\[ S_N^{\mu} \equiv (0, e_N) = \left( 0, \frac{p_{K^*} \times p_-}{|p_{K^*} \times p_-|} \right), \]

\[ S_T^{\mu} \equiv (0, e_T) = \left( 0, e_N \times e_L \right), \]

\[ W_L^{\mu} \equiv (0, w_L) = \left( 0, \frac{p_+}{|p_+|} \right), \] (3.1)
\[ W_N^\mu \equiv (0, w_N) = \left( 0, \frac{p_K \times p_+}{|p_K \times p_+|} \right), \]
\[ W_T^\mu \equiv (0, w_T) = (0, w_N \times w_L), \tag{3.2} \]
where \( p_+, p_- \) and \( p_K \) are the three momenta of the \( \ell^+, \ell^- \) and \( K^* \) particles respectively. On boosting the vectors defined by Eqns. 3.1, 3.2 to the c.m. frame of the \( \ell^- \ell^+ \) system only the longitudinal vector will be boosted, whilst the other two vectors remain unchanged. The longitudinal vectors after the boost will become;
\[ S_L^\mu = \left( \frac{p_-}{m_\ell}, \frac{E_\ell p_-}{m_\ell |p_-|} \right), \]
\[ W_L^\mu = \left( \frac{p_-}{m_\ell}, -\frac{E_\ell p_-}{m_\ell |p_-|} \right). \tag{3.3} \]

The polarization asymmetries can now be calculated using the spin projector \( \frac{1}{2}(1 + \gamma_5 S) \) for \( \ell^- \) and the spin projector \( \frac{1}{2}(1 + \gamma_5 W) \) for \( \ell^+ \).

Equipped with the above expressions we now define the various single lepton and double lepton polarization asymmetries. Firstly, the single lepton polarization asymmetries are defined as \[2, 4, 6, 8, 12]\:
\[ P_x^- = \frac{d\Gamma(S_x W_x)}{ds} - \frac{d\Gamma(-S_x W_x)}{ds}, \]
\[ P_x^+ = \frac{d\Gamma(S_x W_x)}{ds} + \frac{d\Gamma(-S_x W_x)}{ds}, \tag{3.4} \]
where the sub-index \( x \) can be either \( L, N \) or \( T \). \( P^\pm \) denotes the polarization asymmetry of the charged lepton \( \ell^\pm \). Along the same lines we can also define the double spin polarization asymmetries as \[12]\:
\[ P_{xy} = \frac{d\Gamma(S_x W_y)}{ds} - \frac{d\Gamma(-S_x W_y)}{ds}, \tag{3.5} \]
where the sub-indices \( x \) and \( y \) can be either \( L, N \) or \( T \).

The single lepton polarization asymmetries are then;
\[ P_L^{(\pm)} = \frac{m_B^2}{\Delta} \sqrt{1 - \frac{4m_\ell^2}{s}} \left[ \pm \frac{8}{3} m_B^4 \delta \lambda Re(A^* E) \mp \frac{4}{3m_{K^*}^2} \left\{ \lambda (1 - \bar{m}_{K^*}^2 - \bar{s}) \right. \right. \]
\[ \left. \left. Re(B^* G) - (\lambda + 12 \bar{s} \bar{m}_{K^*}^2) \right\} \right] + \frac{4}{3m_{K^*}^2} \left\{ m_B^2 \lambda Re(C^* G) + (1 - \bar{m}_{K^*}^2 - \bar{s}) \right\} \]
\[ + \frac{16}{3m_{K^*}^2} m_B \bar{m}_\ell Re(B^* C_T) \left\{ 2 \left( \lambda + 12 \bar{s} \bar{m}_{K^*}^2 \right) N_1 + \left( \bar{m}_{K^*}^2 + \bar{s} - 1 \right) \left( m_B^2 \lambda N_2 + 24 \bar{m}_{K^*}^2 T_1 \right) \right\} \]
\[ - \frac{16m_\ell^2}{3m_{K^*}^2} m_B^2 Re(C^* C_T) \lambda \left\{ m_B^2 \lambda N_2 + 2(\bar{m}_{K^*}^2 + \bar{s} - 1) N_1 + 8 \bar{m}_{K^*}^2 T_1 \right\} \]
\[-\frac{256\hat{m}_\ell}{3}m_B^2\lambda T_1 \{ Re(A^*C_{TE}) + Re(E^*C_T) \} - \frac{4\hat{m}_\ell}{m_B^2}\lambda m_B \{ \hat{s}Re(H^*K) + (1 - \hat{s} - \hat{m}_{K^*}^2) \} \]
\[\times m_B^2Re(G^*K) + Re(F^*K) \} + \frac{64\hat{m}_\ell}{3m_B^2}\lambda m_B \{ 2(\lambda + 12\hat{s}\hat{m}_{K^*}^2)N_1 + (\hat{m}_{K^*}^2 + \hat{s} - 1) \}
\times (\lambda m_B^2N_2 + 24\hat{m}_{K^*}^2T_1) \} Re(F^*C_{TE}) + \frac{64\hat{m}_\ell}{3m_B^2}\lambda m_B^2 \{ \lambda m_B^2N_2 + 2(\hat{m}_{K^*}^2 + \hat{s} - 1)N_1 \]
\[+ 8m_{K^*}^2T_1 \} Re(G^*C_{TE}) - \frac{2\hat{s}\lambda}{m_{K^*}^2}m_B^2Re(M^*K) - \frac{64\hat{m}_\ell}{3m_B^2}\lambda m_B^2 \{ 2\hat{s}\lambda^2m_B^2N_2^2 \]
\[+ 16m_B^2\hat{m}_{K^*}^2\hat{s}LT_1N_2 + 64\hat{m}_{K^*}^2 \{ \lambda + 3\hat{s}\hat{m}_{K^*}^2 \} T_1^2 \} \right] . \tag{3.6} \]

\[\mathcal{P}_N^{(\tau)} = \frac{\pi m_B^2\sqrt{s\lambda}}{\Delta} \sqrt{1 - \frac{4m_\ell^2}{s}} \left[ \pm 2m_B^2\hat{m}_\ell \{ Im(A^*F) + Im(B^*E) \} - \frac{1}{2m_{K^*}^2} \{ m_B^2\lambda(Im(K^*C) + Im(M^*G)) \}
\[+ (1 - \hat{s} - \hat{m}_{K^*}^2) \{ Im(K^*B) + Im(K^*F) \} \} + 8m_B^2\pi \{ \hat{s}N_1 + (\hat{m}_{K^*}^2 + \hat{s} - 1)T_1 \} Im(A^*C_T)
\[+ 16T_1 \{ 2Im(C_{TE}B) + \frac{\pi m_B^2}{m_{K^*}^2} \{ (\hat{m}_{K^*}^2 + \hat{s} - 1)Im(F^*H) - 4\hat{m}_{K^*}^2Im(F^*G) \}
\[+ \frac{m_B^2}{m_{K^*}^2}\lambda Im(G^*H) \} \}
\[+ 8m_B^2\hat{m}_\ell \{ \lambda m_B^2N_2 + 2(\hat{m}_{K^*}^2 + \hat{s} - 1)N_1 + 8\hat{m}_{K^*}^2T_1 \} Im(C_{TE}^*K)
\[\pm 16m_B^2 \{ N_1\hat{s} + (\hat{m}_{K^*}^2 + \hat{s} - 1)T_1 \} \right] , \tag{3.7} \]

\[\mathcal{P}_T^{(\tau)} = \frac{\pi m_B^2\sqrt{s\lambda}}{\Delta} \left[ - 4m_B^2\hat{m}_\ell Re(A^*B) + \frac{\hat{m}_\ell}{m_{K^*}^2}\left\{ m_B^2\hat{s}(Re(B^*H) - \lambda Re(C^*H)) \right\}
\[+ (1 - \hat{s} - \hat{m}_{K^*}^2)m_B^2 \{ Re(G^*B) - \lambda Re(C^*G) \} + \{ Re(F^*B) - \lambda Re(F^*C) \} \right\} - \frac{16(4\hat{m}_\ell^2 + \hat{s})\lambda}{\hat{s}} \times \left\{ T_1 Re(B^*C_T) - (\hat{s}N_1 + (\hat{m}_{K^*}^2 + \hat{s} - 1)T_1) Re(A^*C_{TE}) \right\} + \frac{(\hat{s} - 4\hat{m}_\ell^2)}{2m_{K^*}^2}\{ m_B^2\lambda Re(K^*G) \}
\[+ (1 - \hat{s} - \hat{m}_{K^*}^2)Re(F^*K) \}
\[\pm \frac{8m_B^2}{\hat{s}} \left\{ \hat{s}N_1 + (\hat{m}_{K^*}^2 + \hat{s} - 1)T_1 \right\} \left\{ m_B^2(\hat{s} - 4\hat{m}_\ell^2) Re(C_{TE}^*B) \right\}
\[\pm 128m_B^2\hat{m}_\ell T_1 Re(C_{TE}^*C_T) \}
\[\pm \frac{16m_B^2}{\hat{s}m_{K^*}^2}\left\{ 2(8\hat{m}_\ell^2 - \hat{s})\hat{m}_{K^*}^2T_1 + 2\hat{m}_\ell^2(\hat{m}_{K^*}^2 + \hat{s} - 1)N_1 \right\}
\[\pm m_B^2\hat{m}_\ell^2\lambda N_2 \} \left\{ m_B^2\lambda N_2 + 2(\hat{m}_{K^*}^2 + \hat{s} - 1)N_1 + 8\hat{m}_{K^*}^2T_1 \} Re(M_{TE}^*C) \right\} . \tag{3.8} \]

And the double polarization asymmetries are;

\[\mathcal{P}_{LL} = \frac{1}{\Delta} \left[ \frac{4m_B^2}{3\hat{s}m_{K^*}^2}\left\{ (2\hat{m}_\ell^2 - \hat{s}) \left\{ m_B^2\hat{s}\hat{m}_{K^*}^2\lambda |A|^2 + \frac{1}{2}(\lambda + 12\hat{s}\hat{m}_{K^*}^2) |B|^2 + \frac{m_B^4}{2}\lambda^2 |C|^2 + m_B \right\}
\[\times (1 - \hat{s} - \hat{m}_{K^*}^2) \lambda Re(B^*C) \} - 32m_B^3\hat{m}_\ell m_B^2\lambda T_1 Re(C_{TE}^*A) + 8m_B^2\hat{m}_\ell \left\{ 2(\lambda + 12\hat{s}\hat{m}_{K^*}^2)N_1 + (\hat{m}_{K^*}^2 + \hat{s} - 1)(\hat{m}_B^2\lambda N_2 + 24\hat{m}_{K^*}^2T_1) Re(C_{TE}^*B) \right\}
\[\times \left\{ 8m_B^2\hat{m}_\ell \lambda \{ m_B^2\lambda N_2 + 2(\hat{m}_{K^*}^2 + \hat{s} - 1)N_1 + 8\hat{m}_{K^*}^2T_1 \} Re(C_{TE}^*C) \} + (4\hat{m}_\ell^2 - \hat{s}) m_B^2\hat{s}\hat{m}_{K^*}^2\lambda \left\{ m_B^2|E|^2 + \frac{|K|^2}{4} \right\} \right\} - \frac{1}{2} \left\{ (\lambda + 12\hat{s}\hat{m}_{K^*}^2) \right\} \right] . \]
\[ -2m^2 \left( 5\lambda + 24\hat{m}^2_{K^*} \right) \{ F \}^2 - \frac{m^4_B}{2} \lambda \{ \hat{s} \lambda - 2 \hat{m}^2 \} \{ 5\lambda + 12\hat{m}^2_{K^*} \} \} |G|^2 + m^2_B \lambda \{ \hat{s} \{ \hat{m}^2_{K^*} + \hat{s} - 1 \} \} \times \text{Re}(F^*G) - 2\hat{m}^2 \left( 5 \{ \hat{m}^2_{K^*} + \hat{s} - 1 \} \right) \text{Re}(G^*F) - 3\hat{s} \text{Re}(H^*F) \} + 3m^2_B \hat{m}_T \lambda \hat{s} \{ \hat{s} \text{Re}(M^*H) \}
+ m^2_B \left( 1 - \hat{s} - \hat{m}^2_{K^*} \right) \text{Re}(M^*G) + \text{Re}(F^*M) \} + \frac{3}{4}m^2_B \hat{s} \hat{s} \lambda |M|^2
+ 4m^2_B \hat{s} \left( \hat{s} - 4\hat{m}^2 \right) \{ m^2_B \lambda^2 N_2^2 + 4 \left( \lambda + 12\hat{m}^2_{K^*} \right) N_1^2 + 4 \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) \}
\times \{ m^2_B \lambda N_1 N_2 + 24\hat{m}^2_{K^*} N_1T_1 \} + 16m^2_B \hat{m}^2_{K^*} \lambda N_2 T_1 \} \{ |C_T|^2 \} - 256m^6_B \hat{m}^2_{K^*} \{ \hat{s} \lambda - 6\hat{m}^2 \}
\times \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) \left( T_1^2 |C_{TE}|^2 \right) \right],
\tag{3.9}
\]

\[ \mathcal{P}_{LN} = \frac{1}{\Delta} \frac{m^2_B}{m^2_{K^*}} \sqrt{\frac{\lambda}{s} \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) \hat{m}_T \left\{ m^2_B \left( \hat{s} \text{Im}(H^*B) + (1 - \hat{s} - \hat{m}^2_{K^*}) \text{Im}(G^*B) \right) + \text{Im}(F^*B) \right\}
+ \lambda \hat{m}_T \hat{m}^2_B \left\{ m^2_B \left( \hat{s} \text{Im}(C^*H) + (1 - \hat{s} - \hat{m}^2_{K^*}) \text{Im}(C^*G) \right) + \text{Im}(C^*F) \right\} + \frac{m_B \hat{s}}{2} \left\{ m^2_B \lambda \text{Im}(C^*M) \right\}
+ \left( 1 - \hat{s} - \hat{m}^2_{K^*} \right) \text{Im}(B^*M) \} + (\hat{s} - 4\hat{m}^2 \} \left\{ 16m_B T_1 \text{Im}(B^*C_T) + 16m^3_B \left( \hat{s} N_1 + \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) T_1 \right) \right\}
\times \text{Im}(C_{TE}^2) + \frac{m_B}{2} \left( \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) \text{Im}(F^*K) - m^3_B \lambda \text{Im}(G^*K) \right) + 8m^3_B
\times \left( \hat{s} N_1 + \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) T_1 \right) \text{Im}(E^*C_T) \} + 16m_B \left\{ (2\hat{m}^2_{K^*} T_1 + 2\hat{m}^2 \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) N_1 \right.
\times \hat{m}^2_B \hat{m}^2_B \lambda N_2 \text{Im}(C_{TE}^2) + \hat{m}^2_B \text{Im}(C_{TE}^2) \} + 8m^2_B \hat{m}_T \hat{s} \left\{ m^2_B \lambda N_2 + 2 \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) N_1 \right.
\times \left. + 8\hat{m}_T \hat{m}_T \right) T_1 \right\}
\right],
\tag{3.10}
\]

\[ \mathcal{P}_{LT} = \frac{1}{\Delta} \frac{m^2_B}{m^2_{K^*}} \sqrt{\frac{\lambda}{s} \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) \hat{m}_T \left\{ -2m^2_B \hat{m}^2_{K^*} \hat{m}_T \hat{s} \text{Re}(A^*F + B^*E + 8T_1 C_{TE}^2) \right\} + \frac{m_B \hat{s}}{2} \left\{ m^2_B \lambda \text{Re}(K^*C) \right\}
- \text{Re}(M^*G) \} + \left( 1 - \hat{s} - \hat{m}^2_{K^*} \right) \text{Re}(B^*K) - \text{Re}(M^*F) \} - 8m^3_B \hat{m}^2_{K^*} \hat{s} \left\{ \hat{s} N_1 + \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) T_1 \right\}
\times \left( \text{Re}(A^*C_T) - 2\text{Re}(C_{TE}^2) \right) + 32m_B T_1 \text{Re}(B^*C_{TE}) + \hat{m}^2_B \hat{m}^2 \left\{ \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) \left\{ |F|^2 + m^2_B \lambda |G|^2 \right\} \right.
- 2m^2_B \lambda \text{Re}(F^*G) + \hat{m}^2_B \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) \hat{s} \text{Re}(F^*H) - m_B \hat{s} \lambda \text{Re}(G^*H) \} + 8m^2_B \hat{m}_T \hat{s} \left\{ m^2_B \lambda N_2 \right.
\times \left. + 2 \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) N_1 + 8\hat{m}^2_{K^*} T_1 \right) \text{Re}(K^*C_T) \right\} - 256m^6_B \hat{m}_T \hat{s} \left\{ \hat{s} N_1 + \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) T_1 \right\}
\times \left( |C_T|^2 + 4|C_{TE}|^2 \right) \right\} + 16m^3_B \hat{s} \left\{ \hat{s} N_1 + \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) T_1 \right\} \text{Re}(C_{TE}^2) \right],
\tag{3.11}
\]

\[ \mathcal{P}_{NL} = \frac{1}{\Delta} \frac{m^2_B}{m^2_{K^*}} \sqrt{\frac{\lambda}{s} \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) \hat{m}_T \left\{ m^2_B \left( \hat{s} \text{Im}(H^*B) + (1 - \hat{s} - \hat{m}^2_{K^*}) \text{Im}(G^*B) \right) + \text{Im}(F^*B) \right\}
- \lambda \hat{m}_T \hat{m}^2_B \left\{ m^2_B \left( \hat{s} \text{Im}(C^*H) + (1 - \hat{s} - \hat{m}^2_{K^*}) \text{Im}(C^*G) \right) + \text{Im}(C^*F) \right\} - \frac{m_B \hat{s}}{2} \left\{ m^2_B \lambda \text{Im}(C^*M) \right\}
+ \left( 1 - \hat{s} - \hat{m}^2_{K^*} \right) \text{Im}(B^*M) \} + (\hat{s} - 4\hat{m}^2 \} \left\{ 16m_B T_1 \text{Im}(B^*C_T) + 16m^3_B \left( \hat{s} N_1 + \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) T_1 \right) \right\}
\times \text{Im}(C_{TE}^2) + \frac{m_B}{2} \left( \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) \text{Im}(F^*K) - m^3_B \lambda \text{Im}(G^*K) \right) - 8m^3_B
\times \left( \hat{s} N_1 + \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) T_1 \right) \text{Im}(E^*C_T) \} - 16m_B \left\{ (2\hat{m}^2_{K^*} T_1 + 2\hat{m}^2 \left( \hat{m}^2_{K^*} + \hat{s} - 1 \right) N_1 \right.
\times \left. + 8\hat{m}_T \hat{m}_T \right) T_1 \right\}
\right],
\tag{3.11}
\]
\[ P_{NN} = \frac{2m_B^2}{\Delta 3\hat{m}_K^2} \left( \frac{m_B}{s} - m_K^2 \right) + m_B^2 \hat{m}_K^2 \left( m_B^2 \lambda N_2 + 2 \left( \hat{m}_K^2 + \hat{s} - 1 \right) N_1 + 8\hat{m}_K^2 \right) \]
\[ + \{ \hat{m}_K^2 \} \right) + 8m_B^2 \hat{m}_K \left( m_B^2 \lambda N_2 + 2 \left( \hat{m}_K^2 + \hat{s} - 1 \right) N_1 + 8\hat{m}_K^2 \right) \] \( T_1 \} \right) \right), \tag{3.12} \]

\[ P_{NT} = \frac{4}{\Delta 3\hat{m}_K^2} \sqrt{1 - \frac{4\hat{m}_K^2}{s}} \left[ m_B^2 \lambda \text{Im}(E^* A) + 32m_B^2 \hat{m}_K \hat{m}_K N_1 (2m_B \lambda \text{Im}(C^* \lambda) + m_B \lambda \text{Im}(E^* C)) \right] \]
\[ + \lambda \{ (\hat{m}_K^2 + \hat{s} - 1) (m_B^2 \lambda \text{Im}(G^* B) - m_B \lambda \text{Im}(E^* C)) \} \right) \]
\[ + 4m_B^2 \hat{m}_K \left( 2 \left( \hat{m}_K^2 + \hat{s} - 1 \right) N_1 + \left( \hat{m}_K^2 + \hat{s} - 1 \right) N_1 \right) \text{Im}(C^*_T B) \]
\[ + 3m_B^2 \hat{m}_K \lambda \left( m_B^2 \lambda \text{Im}(K^* C) + \left( 1 - \hat{s} - \hat{m}_K^2 \right) \lambda \text{Im}(K^* C) \right) \]
\[ + 16m_B^2 \hat{m}_K \left( 8\hat{m}_K^2 \hat{m}_K \text{Im}(C^* \lambda) + N_2 \left( m_B^2 \lambda \text{Im}(C^* \lambda) \right) - \left( \hat{m}_K^2 + \hat{s} - 1 \right) N_2 \right) \]
\[ - 2 \left( \text{Im}(C^*_T F) - m_B^2 \right) \left( \hat{m}_K^2 + \hat{s} - 1 \right) \text{Im}(C^*_T E) \right) \right) \] \[ + 16m_B^2 \hat{m}_K \left( 4 \left( \hat{m}_K^2 + \hat{s} - 1 \right) N_1 N_2 + 96 \left( \hat{m}_K^2 + \hat{s} - 1 \right) T_1 N_1 + 16\lambda m_B^2 T_1 N_2 \right) \]
\[ \times \text{Im}(C^*_T C_T) \right] \right), \tag{3.13} \]

\[ P_{TL} = \frac{1}{\Delta 3\hat{m}_K^2} \sqrt{\lambda \hat{m}_K^2} \left[ \frac{1}{s} - \frac{4\hat{m}_K^2}{s} \right] \left( 2m_B^2 \hat{m}_K \hat{m}_K \hat{m}_K \hat{m}_K \text{Re}(A^* F + B^* E + 8T_1 C^* F) + m_B \hat{s} \right) \]
\[ - \text{Re}(M^* G) - \left( 1 - \hat{s} - \hat{m}_K^2 \right) \left( \text{Re}(B^* K) + \text{Re}(M^* F) \right) \] \[ - 8m_B^2 \hat{m}_K \hat{m}_K \hat{m}_K \hat{m}_K \left( \hat{m}_N + \left( \hat{m}_K^2 + \hat{s} - 1 \right) T_1 \right) \}
\[ \times \left( \text{Re}(A^* C_T) - 2\text{Re}(C^*_T E) \right) + 32m_B \hat{m}_K \text{Re}(B^* C_T E) + \hat{m}_K \hat{m}_K \hat{m}_K \hat{m}_K \left( \left( \hat{m}_K^2 + \hat{s} - 1 \right) \left( \hat{m}_K^2 + \hat{s} - 1 \right) \hat{m}_K \hat{m}_K \hat{m}_K \hat{m}_K \right) \]
\[ - 2m_B^2 \lambda \text{Re}(F^* G) + m_B^2 \left( \hat{m}_K^2 + \hat{s} - 1 \right) \text{Re}(F^* H) - m_B^2 \hat{m}_K \hat{m}_K \hat{m}_K \hat{m}_K \left( \hat{m}_N + \left( \hat{m}_K^2 + \hat{s} - 1 \right) T_1 \right) \]
\[ + 2 \left( \hat{m}_K^2 + \hat{s} - 1 \right) N_1 + 8\hat{m}_K^2 \left( \hat{m}_N + \hat{m}_K^2 \right) \text{Re}(K^* C_T E) \right) \] \[ - 256m_B^2 \hat{m}_K \left( \hat{m}_N + \left( \hat{m}_K^2 + \hat{s} - 1 \right) T_1 \right) \] \[ \right) \text{Re}(C^*_T E) \right), \tag{3.15} \]
\[ \mathcal{P}_{TN} = \frac{4}{3 \Delta} \frac{m_B^2}{3m_{K^*}^2} \sqrt{1 - \frac{4\hat{m}_t^2}{s}} \left[ 3m_B^2 \hat{s} \text{Im}(E^*A) + 32m_B^2 \hat{m}_t \lambda \hat{m}_{K^*}^2 \cdot T_1 (-2 \text{Im}(A^*C_{TE}) + \text{Im}(E^*C_T)) \right. \\
+ \lambda \left\{ (\hat{m}_{K^*}^2 + \hat{s} - 1) \left( m_B^2 \text{Im}(G^*B) - \text{Im}(F^*C) \right) - \text{Im}(F^*B) + m_B^2 \lambda \text{Im}(C^*G) \right\} + 4m_B \hat{m}_t \left\{ 2 \left( \lambda + 12 \hat{s} \hat{m}_{K^*}^2 \right) N_1 + \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) \left( m_B^2 \lambda N_2 + 24 \hat{m}_t^2 \cdot T_1 \right) \right\} \left( \text{Im}(C^*_T) \right) \\
+ 4m_B^3 \hat{m}_t \left\{ 8 \lambda T_1 \hat{m}_{K^*}^2 + m_B^2 \lambda^2 N_2 + 2 \lambda (\hat{m}_{K^*}^2 + \hat{s} - 1) N_1 \right\} \text{Im}(C^*_T C) + \frac{3}{2} m_B^2 \hat{s} \lambda \text{Im}(K^*M) \\
+ 3m_B^3 \hat{m}_t \lambda \left\{ m_B^2 \left( \hat{s} \text{Im}(K^*H) + (1 - \hat{s} - \hat{m}_{K^*}^2) \text{Im}(K^*G) + \text{Im}(K^*F) \right) \right\} + 88m_B \hat{m}_t \lambda \left\{ m_B^2 \left( \hat{s} \text{Im}(K^*H) + (1 - \hat{s} - \hat{m}_{K^*}^2) \text{Im}(K^*G) + \text{Im}(K^*F) \right) \right\} \\
- 2 \left( \text{Im}(C_{TE}^* F) - m_B^2 \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) \text{Im}(C_{TE}^* G) \right) \} + 16m_B^3 \hat{s} \left\{ 4 \left( \lambda + 12 \hat{s} \hat{m}_{K^*}^2 \right) N_1^2 + m_B^2 \lambda^2 \\
+ 192 \hat{m}_t^2 \cdot T_1^2 + 4m_B^2 \lambda \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) N_1 N_2 + 96 \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) T_1 N_2 + 16 \lambda m_B^2 T_1 T_2 \right\} \\
\left( \text{Im}(C_{TE}^* C_T) \right), \quad (3.16) \]

\[ \mathcal{P}_{TT} = \frac{2}{3} m_B^2 \left[ \lambda \left( \hat{s} + 4 \hat{m}_t^2 \right) m_B^4 |A|^2 + \frac{1}{s} \left\{ \lambda \hat{s} - 2 \left( \lambda + 12 \hat{s} \hat{m}_{K^*}^2 \right) \right\} |B|^2 + \frac{(2\hat{m}_t^2 - \hat{s})}{\hat{m}_{K^*}^2} \lambda m_B^2 \left\{ \lambda |C|^2 \right\} \\
- 2 \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) \text{Re}(B^* C) \right\} + 128 m_B^3 \hat{m}_t \lambda T_1 \text{Re}(A^* C_T) + m_B^2 \lambda \left( \hat{s} - 4 \hat{m}_t^2 \right) \left( -|E|^2 \right) \\
+ \frac{3}{2 \hat{m}_{K^*}^2} |K|^2 \right\} + 16 m_B^3 \hat{m}_t \lambda \left\{ 2 \left( \lambda - 12 \hat{s} \hat{m}_{K^*}^2 \right) N_1 + \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) \left( m_B^2 \lambda N_2 - 24 \hat{m}_t^2 \cdot T_1 \right) \right\} \\
\times \text{Re}(B^* C_{TE}) - 32 m_B^3 \hat{m}_t \left\{ 8 \hat{m}_{K^*}^2 T_1 + m_B^2 \lambda N_2 - 2 \lambda \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) N_1 \right\} \text{Re}(C^* C_T) \right\} + m_B^4 \hat{m}_t \lambda \left\{ \lambda \hat{s} - 2 \hat{m}_t^2 \left( 5 \lambda + 12 \hat{s} \hat{m}_{K^*}^2 \right) \right\} |G|^2 + \frac{(\hat{s} - 10 \hat{m}_t^2)}{\hat{m}_{K^*}^2} \lambda \left\{ |F|^2 - 2 m_B^4 \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) \text{Re}(F^* G) \right\} \\
- 12 m_B^2 \hat{m}_t \lambda \text{Re}(F^* H) + 6 \hat{m}_t \lambda \hat{m}_{K^*}^2 \left\{ m_B^2 \left( \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) \text{Re}(M^* G) - \hat{s} \text{Re}(H^* M) \right) - \text{Re}(M^* F) \right\} \\
- \frac{3}{2} m_B^2 \hat{m}_t \lambda \text{Re}(F^* H) + 8 \hat{m}_t \lambda \hat{m}_{K^*}^2 \left\{ 4 \hat{m}_t^2 - \hat{s} \right\} \left\{ (\lambda + 12 \hat{s} \hat{m}_{K^*}^2) \hat{s} N_1^2 + \lambda m_B^2 N_2^2 + 16 \hat{m}_t^2 \cdot m_B^2 \lambda \right\} \\
+ 4 \left( \hat{m}_t^2 - \hat{s} \right) \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) \hat{s} \left( \lambda m_B^2 N_1 N_2 + 24 \hat{m}_t^2 \cdot T_1 \right) + 6 \hat{m}_t \lambda \left( \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) \left( \lambda N_2 + 2 \hat{m}_t^2 \cdot T_1 \right) \right) \\
\times |C_T|^2 + 32 \hat{m}_t \left\{ 4 \left( \left( \lambda + 12 \hat{s} \hat{m}_{K^*}^2 \right) \hat{s} - 8 \hat{m}_t^2 \lambda \right) \hat{s} N_1^2 - \hat{s} \lambda m_B^2 N_2 \left( \hat{s} - \hat{m}_t^2 \right) \left( \lambda N_2 + 2 \hat{m}_t^2 \cdot T_1 \right) \right\} \\
- 4 \hat{m}_t \left( \hat{s} \hat{m}_{K^*}^2 + \hat{s} - 1 \right) \hat{s} \lambda N_1 N_2 + 64 \hat{m}_t \left( \lambda \hat{s} N_1 + \hat{m}_t^2 + 3 \hat{m}_{K^*}^2 \cdot s^2 \right) T_1^2 + 96 \left( \hat{m}_{K^*}^2 + \hat{s} - 1 \right) \hat{m}_{K^*}^2 \hat{s}^2 \right\} \cdot \left| C_{TE} \right|^2. \quad (3.17) \]

4. Numerical analysis, Results and Discussion

In this final section we shall present the results of our numerical analysis. As such, the input parameters which we have used, in order to calculate the various Wilson coefficients defined in Eqn. (7), are listed in Appendix A. The value of \( C_7 \) is fixed by the observation of \( b \to s \gamma \). Note that this observation fixes the magnitude and not the sign of \( C_7 \). We have therefore chosen the SM predicted value \( C_7 = -0.313 \). For \( C_{10} \) we have used the
Figure 1: The branching ratio, \( BR(B \rightarrow K^*\tau^+\tau^-) \), as a function of the various Wilson coefficients.

SM value \( C_{10} = -4.997 \). Regarding the value of \( C_{9}^\text{eff} \) this receives both short and long-distance contributions. The long-distance contributions are the result of \( c\bar{c} \) intermediate states, such as resonances of \( J/\Psi \). For our analysis we have used \( C_{9}^\text{eff} = C_9 + Y(\hat{s}) \) where \( C_9 \) corresponds to the short distance contribution, which we have taken to have SM value \( C_9 = 4.334 \). \( Y(\hat{s}) \) represents the \( \mathcal{O}(\alpha_s) \) corrections coming from the operators \( O_1 - O_6 \) as given in Kruger & Sehgal [4]. In our present analysis we have confined ourselves only to short distance contributions. The form factor definitions which we have used in describing the hadronic transitions are listed in Appendix B.1. In our effective Hamiltonian there are in all 12 Wilson coefficients. Of these \( C_{SL} \) and \( C_{BR} \) can be related to the SM Wilson \( C_7^\text{eff} \) by;

\[
C_{SL} = -2m_s C_7^\text{eff} , \quad C_{BR} = -2m_b C_7^\text{eff} .
\]  

Among the vector type coefficients \( C_{LL}, C_{LR}^\text{tot}, C_{RL} \) and \( C_{RR} \) two of them, namely \( C_{LL} \) and \( C_{LR}^\text{tot} \), are already defined in terms of the SM Wilson coefficients \( (C_9^\text{eff} - C_{10}) \) and \( (C_9^\text{eff} + C_{10}) \) respectively. The remaining vector type interaction coefficients \( C_{RL} \) and \( C_{RR} \) are taken to be free parameters. The coefficients of the scalar type interactions, namely \( C_{LRLR}, C_{RLLR}, C_{LRRL} \) and \( C_{RLRL} \), and the tensor type interactions, \( C_T \) and \( C_{TE} \), are also taken to be input parameters.

As already discussed in the introduction we shall explain the \( B \rightarrow \pi\pi \) and \( B \rightarrow K\pi \) puzzle as resulting from a large phase in the electroweak penguin diagrams as has been proposed in [13]. This suggestion was initially made by Buras et al. some time back [13] and has lately been revived by many other groups [14]. Recently the implications of this suggestion on a variety of hadronic, leptonic and semi-leptonic processes has been studied. Note that, as emphasized in our earlier work on the inclusive decay mode \( B \rightarrow X_s\ell^+\ell^- \) [15], the polarization asymmetries could also significantly deviate from their SM values if there was a large phase in the electroweak penguins. This type of study has also been carried out by Aliev et al. [16] where they attempted to estimate the variation in the single polarization asymmetries for the exclusive process \( B \rightarrow K^*\ell^+\ell^- \), where the Wilsons had
some extra phase. Aliev et al. in their study of the single lepton polarization asymmetries in the exclusive process $B \rightarrow K^* \ell^+ \ell^-$ also emphasized the importance of the tensorial interactions on various asymmetries \cite{8}. They concluded that single polarization asymmetries are very sensitive to scalar and tensor type interactions. In our earlier work \cite{18} we demonstrated the supersymmetric effects on various double polarization asymmetries in $B \rightarrow K^* \ell^+ \ell^-$, where supersymmetry predicts the existence of scalar and pseudo-scalar operators in the large $\tan\beta$ region\footnote{$\tan\beta$ is the ratio of the vev’s of two Higgs bosons}. However, in this previous study we did not include the tensorial structures. In this current work we will use the most general form of the effective Hamiltonian to study the effects on various polarization asymmetries. We shall also include the extra possible phase from the electroweak penguin sector.

Note that in this paper these additional effects from the electroweak penguin sector, which can give effective structures similar to those given in Eqn. (2.1) with coefficients $C_{LL}$, $C_{LR}$, $C_{RL}$ and $C_{RR}$, will all be given an additional phase. As already stated in section \ref{sec:2} two of these coefficients, namely $C_{LL}$ and $C_{LR}$, can be parameterized in terms of $C_9$ and $C_{10}$. Therefore we shall only consider effects of a new phase in the $C_{10}$, $C_{RL}$ and $C_{RR}$ coefficients. For this purpose we will parameterize these coefficients as:

\[
C_{10} = |C_{10}| e^{i\phi_{10}}, \quad (4.2)
\]
\[
C_{RL} = |C_{RL}| e^{i\phi_{RL}}, \quad (4.3)
\]
\[
C_{RR} = |C_{RR}| e^{i\phi_{RR}}. \quad (4.4)
\]

As the majority of our results involve the polarization asymmetries, listed in the previous section, which are dependent on the scaled invariant mass ($\hat{s}$), it is experimentally useful to consider the averaged values of these asymmetries. Therefore we shall present

\textbf{Figure 2:} The double polarization asymmetry, $P_{LL}$, as a function of the various Wilson coefficients, where both $\tau$ leptons are longitudinally polarized.
Figure 3: The same as Figure (2), but for $\ell^-$ longitudinal and $\ell^+$ normal.

Figure 4: The same as Figure (2), but for $\ell^-$ longitudinal and $\ell^+$ transverse.

only the averaged values of the polarization asymmetries in our results using the averaging procedure defined as:

$$
\langle P_i \rangle = \frac{\int_{4m^2}^{(m_B-m_{K^*})^2} P_i \frac{d\Gamma}{ds} ds}{\int_{4m^2}^{(m_B-m_{K^*})^2} \frac{d\Gamma}{ds} ds}.
$$

(4.5)

Our results are presented in a series of figures commencing with Figure (1) where we have plotted the branching ratio of $B \to K^\ast\tau^+\tau^-$ as a function of the various Wilson coefficients. In this plot we have constrained the value of the branching ratio to have an upper bound $Br(B \to K^\ast\tau^+\tau^-) \leq 5 \times 10^{-7}$. As can be seen from this graph the largest variation in the branching ratio corresponds to tensorial type interactions.
Figures (2)-(10) represent the various double polarization asymmetries plotted as functions of the various Wilson coefficients. In these plots we have assumed that all the Wilson coefficients are real. In all cases we have varied the Wilson coefficients over a range of -3 to 3. It is also apparent that, as in the case of the branching ratio, the greatest variation of the various double polarization asymmetries corresponds to the tensorial type interactions. From the graphs of the polarization asymmetries we can also see substantial variations for various other values of the Wilson coefficients. The major change is that produced in the plots of $<\mathcal{P}_{LL}>$, $<\mathcal{P}_{LT}>$, $<\mathcal{P}_{NN}>$, $<\mathcal{P}_{NT}>$, $<\mathcal{P}_{TL}>$, $<\mathcal{P}_{TN}>$ and $<\mathcal{P}_{TT}>$ where the respective asymmetry can even change sign for certain values of the Wilson coefficients! Note that in these plots we have only shown those asymmetries which are larger than $10^{-3}$. As such the variations of $C_{LL}$, $C_{RL}$ and $C_{LR}$, in the respective asymmetries for Figures (6) and (8), are not shown.

Figure 5: The same as Figure [2], but for $\ell^-$ normal and $\ell^+$ longitudinal.

Figure 6: The same as Figure [3], but for both leptons polarized in the normal direction.
Figure 7: The same as Figure (2), but for $\ell^-$ normal and $\ell^+$ transverse.

Figure 8: The same as Figure (2), but for $\ell^-$ transverse and $\ell^+$ longitudinal.

Note in particular that the $< P_{LL} >$ in Figure (2) shows substantial dependence on the $C_{LR}, C_{LRLR}, C_{RLLR}, C_{RR}, C_T$ and $C_{TE}$ coefficients in which the magnitude of the asymmetry can change by more than 100%. Of major significance is that the tensorial operators can even change the sign of this asymmetry. A very similar sort of behaviour is exhibited by these Wilson coefficients for $< P_{LN} >$, $< P_{NL} >$ and $< P_{NT} >$, except here in the case of $< P_{LN} >$ and $< P_{NL} >$ the sign of the asymmetry does not change. We can also see in Figure (2) that for the case of $< P_{NN} >$ all the Wilson coefficients, with the exception of $C_{LL}$ and $C_{RL}$, predict a sign change.

These results prompt us to analyze the polarization asymmetries for the case where the branching ratio of $B \to K^* \tau^+ \tau^-$ remains close to the SM value. This sort of scenario could tell us more about how the various Wilson coefficients affect various asymmetries (as they are different quadratic functions of the Wilsons and hence carry independent sets
of information). This possibility has been presented in Figure (11), where we have also restricted the branching ratio to the range $1 \times 10^{-7} < \text{Br}(B \to K^*\tau^+\tau^-) < 4 \times 10^{-7}$. From these graphs we observe that there can be substantial variation in the polarization asymmetries even if the branching ratio is not substantially different from its SM value. As can be seen from Figure (11) all the polarization asymmetries shows substantial variations. Some of the asymmetries, in particular $<P_{LL}>$, $<P_{LT}>$, $<P_{NN}>$, $<P_{NT}>$, $<P_{TL}>$, $<P_{TN}>$ and $<P_{TT}>$ not only show variation in magnitude but even their sign changes. Some asymmetries like $<P_{NN}>$, $<P_{TL}>$ and $<P_{TT}>$ can change by more than an order of magnitude, even if there is no major change in the branching ratio. Note that all these asymmetries are most sensitive to tensorial structures in the effective Hamiltonian.

The next set of results we present includes the extra phase in the Wilson coefficients, as stated in Eqns. (4.2), (4.3) and (4.4). In Figures (12) and (13) we have plotted integrated polarization asymmetries as a function of the phase $\phi_{10}$. In Figure (12) we have used the SM value $C_{10} = 4.669$. Note that we have only shown those asymmetries which vary with the inclusion of $\phi_{10}$. In Figure (13) we have plotted the same variables but with an increase in the magnitude of $C_{10}$, namely we have chosen $|C_{10}| = 9$. This value has been chosen to correspond with the value calculated by Buras et al. [13] which they predict in order to solve the $B \to \pi\pi$ and $B \to K\pi$ puzzle. They say that $C_{10}$ should be complex with a magnitude almost twice that of its SM value with a phase which should make it almost imaginary. As can be seen in both the Figures there is a substantial deviation as the phase, $\phi_{10}$, is changed.

In Figure (14) we have plotted the correlations of various polarization asymmetries and the branching ratio of $B \to K^*\tau^+\tau^-$. In this plot we have varied the phase $\phi_{RL}$ in a range of $0 \leq \phi_{RL} \leq \pi$. Finally in Figure (15) we have drawn the same sort of plot but for $C_{RR}$. As we can see from these graphs some of the polarization asymmetries can change sign as we vary the phase of the $C_{RL}$ and $C_{RR}$ Wilsons.

For the numerical analysis the central values of the form factors given in Appendix...
and [7] have been used. These form factors have substantial uncertainties associated with them but as can be seen from the various plots we have shown that these polarization asymmetries have substantial variations even if we restrict ourselves to a branching ratio near the SM value. In fact some of the asymmetries can even change sign, as shown in Figure (11). Although changing the form factor definition will change the quantitative nature of these asymmetries (and the branching ratio) the qualitative nature of these observables would remain the same (as the variations in the asymmetries is substantial and hence even with uncertainties present in form factor definitions one can still draw definite conclusions regarding the asymmetries).

Finally, in order to measure these various polarization asymmetries we must determine how many $B\bar{B}$ pairs are required. Using the arguments given in [3, 20], experimentally an observation of the polarization asymmetry $<P>$ of a decay with branching ratio $\mathcal{B}$ at a $n\sigma$ level to the required number of events is given by (i.e. the number of $B\bar{B}$ pairs);

\[
N = \frac{n^2}{s_1 s_2 \mathcal{B} <P_{ij}>} \quad \text{(for double polarization asymmetries)}
\]
\[
N = \frac{n^2}{s \mathcal{B} <P_i>} \quad \text{(for single polarization asymmetries)}
\] (4.6)

In the above equation $s_1, s_2$ and $s$ are the efficiencies of detecting the state of polarization of $\tau$ leptons. If we take $s_1 (= s_2 = s)$ to be 0.5 then the number of events required to
observe various asymmetries at the 3σ level would be

\[
N = \begin{cases} 
  (2 \pm 1) \times 10^8 & \text{for } <P_L>, <P_N>, <P_T>, \\
  (8 \pm 3) \times 10^8 & \text{for } <P_{LL}>, <P_{LT}>, \\
  (5 \pm 3) \times 10^9 & \text{for } <P_{LN}>, <P_{NL}>, \\
  (8 \pm 7.5) \times 10^9 & \text{for } <P_{NN}>, <P_{TL}>, <P_{TT}>, \\
  (2 \pm 1.5) \times 10^{10} & \text{for } <P_{NT}>, <P_{TN}>
\end{cases}
\]  

The number of $B\overline{B}$ pairs required to observe these asymmetries might not be produced at the present generation B-factories but future factories like LHCb and Super-B would be able to measure these asymmetries.

The study of polarization asymmetries in $B \rightarrow K^*\tau^+\tau^-$ has also been done within the model independent framework by Aliev et al. [8]. Their study of the single lepton polarization asymmetries was done with real valued Wilsons. Our results agree with those

\footnote{here we are taking the efficiency of detecting the polarization state, including the efficiency of $\tau$ measurement and detection of the polarization of $\tau$. If $\tau$ detection efficiency is 80% and efficiency of detection of its polarization state is 60% then the total efficiency of detecting $\tau$ in a particular polarization state is $0.8 \times 0.6 \approx 0.5$.}
they obtained with the exception of typographical errors in their expression of the normal polarization asymmetries $P_N^\pm$.

The polarization asymmetries provide us a large number of observables which would be very useful in determining the structure of the effective Hamiltonian; which in turn could help us in discovering the structure of the underlying physics. The most general effective Hamiltonain for transitions based on $b \to s(d)\ell^+\ell^-$ quark level transition has twelve Wilsons. If we consider all these Wilsons to be complex valued, this would result in 24 parameters and we would need at least 24 observables in order to fix all these parameters. Of these the measurement of $b \to s\gamma$ can fix one of them, namely the magnitude of $C_7$. The observables which are presently at our disposal are the branching ratio and the $K$ asymmetry. Along with these two one can construct six single lepton polarization asymmetries (three for each of the leptons) in addition one can have nine more double polarization asymmetries. This would still leaves us with six more unconstrained parameters. This number can be further restricted if we also construct the polarized $K$ asymmetries (which would gives us 15 more observables, namely six single lepton polarization asymmetries and nine double lepton polarization asymmetries). With this in mind a comprehensive study of polarized $K$ asymmetries was done by Aliev et al. [10]. Inclusion of all these observables would give us 33 observables with which to fix the 24 parameters. So even if some of our observables are small we should still have sufficiently many observables to constrain the value of our 24 parameters.

To summarize, the various polarization asymmetries show a strong dependence on the scalar and tensorial interactions. Also the phase of the Wilson coefficients can give substantial deviations in the polarization asymmetries. This is of great importance as various polarization asymmetries have different bilinear combinations of Wilsons and hence have independent information. Hence they can be very useful in not only estimating the magnitude of the various Wilson coefficients but also in providing information regarding their phases.

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A. Input parameters

$$|V_{tb}V_{ts}^*| = 0.0385 \quad \alpha = \frac{1}{129}$$
**Figure 12:** The variation of the lepton polarization asymmetries as a function of the phase (in degrees), $\phi_{10}$, of $C_{10}$ where we have taken $|C_{10}| = 4.669$.

**Figure 13:** The same as Figure (12) but now taking $|C_{10}| = 9$.

**Figure 14:** The polarization asymmetries as a function of the branching ratio varied across the phase range $0 \leq \phi_{RL} \leq \pi$; where the magnitude of $C_{RL}$ is taken to be $|C_{RL}| = 4$.

$$G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}, \quad \Gamma_B = 4.22 \times 10^{-13} \text{ GeV}$$

$$m_B = 5.3 \text{ GeV}, \quad m_{K^*} = 0.89 \text{ GeV}, \quad m_b = 4.5 \text{ GeV}$$
Figure 15: The same as Figure (14) but now with $|C_{RR}| = 4$ and we have varied the phase, $\phi_{RR}$, in the range $0 \leq \phi_{RR} \leq \pi$.

B. Some Analytical Expressions

B.1 Parameterization of Form Factors

In our calculations we have parameterized our form factors according to the expression:

$$F(\hat{s}) = F(0) \exp \left( c_1 \hat{s} + c_2 \hat{s}^2 \right),$$  \hspace{1cm} (B.1)

where we have used the central value of parameterization given in Table 3 of Ali et al. \[7\], we reproduce this table below (Table 1).

|   | $A_1$ | $A_2$ | $A_0$ | V | $T_1$ | $T_2$ | $T_3$ |
|---|---|---|---|---|---|---|---|
| $F(0)$ | 0.377 | 0.282 | 0.471 | 0.457 | 0.379 | 0.379 | 0.260 |
| $c_1$ | 0.602 | 1.172 | 1.505 | 1.482 | 1.519 | 0.517 | 1.129 |
| $c_2$ | 0.258 | 0.567 | 0.710 | 1.015 | 1.030 | 0.426 | 1.128 |

Table 1: The parameterizing coefficients for the form factors, as expressed in Eqn.(B.1)

B.2 The unpolarized cross-section

The terms in the unpolarized cross-section, given in Eqn.(2.9), are;

$$\Delta = \frac{4}{3} m_B^6 \left\{ (2 \hat{m}_\ell^2 + \hat{s}) |A|^2 + (\hat{s} - 4 \hat{m}_\ell^2) |E|^2 \right\} + \frac{2}{3} \frac{m_B^2}{\hat{m}_{K^*}^2 \hat{s}} \left( 2 \hat{m}_\ell^2 + \hat{s} \right) \left\{ (\lambda + 12 \hat{s} \hat{m}_{K^*}^2) |B|^2 \right\}$$

$$+ \frac{4}{3} \lambda m_B^2 |C|^2 \} + \left\{ 2(\lambda - 24 \hat{s} \hat{m}_{K^*}^2) \hat{m}_\ell^2 + \hat{s}(\lambda + 12 \hat{s} \hat{m}_{K^*}^2) \right\} |F|^2$$

$$+ m_B^4 \lambda \left\{ 2 \left( \lambda + 12 \hat{m}_{K^*}^2 \right) \hat{s} + \hat{s} \lambda \right\} |G|^2 + 2 m_B^2 (2 \hat{m}_\ell^2 + \hat{s}) (1 - \hat{s} - \hat{m}_{K^*}^2) \lambda \Re(B^*C)$$

$$+ 2 \lambda m_B^2 \left\{ (1 - \hat{s} - \hat{m}_{K^*}^2) (2 \hat{m}_\ell^2 + \hat{s}) \Re(G^*F) + 6 \hat{s} \hat{m}_\ell^2 \Re(F^*H) \right\}$$

$$+ \frac{1}{\hat{m}_{K^*}^2} m_B^4 \lambda \left[ 8 m_B^2 \hat{m}_\ell^2 (1 - \hat{s} - \hat{m}_{K^*}^2) \Re(G^*H) + 4 \frac{\hat{m}_\ell}{m_B} \left\{ \hat{s} \Re(H^*M) + m_B^2 (1 - \hat{s} - \hat{m}_{K^*}^2) \right\} \Re(B^*C) \right]$$

$$+ \frac{2}{3} \frac{m_B^2}{\hat{m}_{K^*}^2 \hat{s}} \left( 2 \hat{m}_\ell^2 + \hat{s} \right) \left\{ (\lambda + 12 \hat{s} \hat{m}_{K^*}^2) |B|^2 \right\}$$
\[\times \text{Re}(G^* M) + \text{Re}(F^* M) \} + (\hat{s} - 4\hat{m}^2_\ell) |K|^2 + \hat{s}|M|^2 \]\n\[+ \frac{16}{3} \frac{1}{\hat{m}^2_K} \left\{ (\hat{s} - 4\hat{m}^2_\ell) |C_T|^2 + 4 (\hat{s} + 8\hat{m}^2_\ell) |C_{TE}|^2 \right\}\]
\[\times \left[ 4 \left( \lambda + 12\hat{s}\hat{m}^2_\ell \right) N_1^2 + \lambda^2 N_2^2 - 4\hat{m}^2_B (1 - \hat{s} - \hat{m}^2_\ell) \left( \lambda N_1 N_2 + \hat{m}^2_\ell N_1 T_1 \right) \right]\n\[+ 16\hat{m}^2_B \hat{m}^2_\ell \lambda N_2 T_1 \] \n\[+ \frac{1024 m^4_B}{3} \frac{1}{s} |C_T|^2 T_1^2 \left[ 2 \left( \lambda - 6\hat{m}^2_\ell, \hat{s} \right) \hat{m}^2_\ell + \lambda \hat{s} \right] \n\[+ \frac{4096}{3} \frac{m^4_B}{s} |C_{TE}|^2 T_1^2 \left[ 2\hat{m}^2_\ell \left( \lambda + 12\hat{s}\hat{m}^2_\ell \right) + \hat{s} \lambda \right]. \] \hfill (B.2)

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