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Enhanced particle transport in an oscillating sinusoidal optical potential

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\textbf{Abstract.} We have studied the delivery of a colloidal particle in the presence of an oscillating, spatially periodic, optical potential. The average particle velocity relative to the fluid velocity in this potential depends greatly on the oscillation amplitude and frequency. The results of both our simulations and experiments show that for some combinations of these parameters, the average particle transportation velocity can be enhanced due to the synchronization of the particle’s movement with the oscillating potential.

Transporting a Brownian particle through a periodic potential, a problem that can be mapped onto a Brownian particle diffusing in a tilted periodic (washboard) potential, has an impact on a variety of questions arising in physics, biology and chemistry \cite{1}–\cite{4}, and results in many interesting phenomena such as the Brownian ratchet \cite{5}–\cite{9}, enhanced diffusion \cite{10}–\cite{12}, stochastic resonance \cite{13}, etc. Despite various theoretical studies and simulations, as well as experimental demonstrations \cite{5,6}, \cite{14}–\cite{20} (some utilizing laser tweezers), the behavior of a Brownian particle in an oscillating tilted washboard potential remains underexplored. As noted

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in several recent theoretical papers [21, 22], an enhanced transport velocity relative to the background can be achieved. This enhancement may have important applications in particle separation or cell separation, which play an important role in many biological studies. However, the understanding of, and the requirements to achieve, this enhanced transportation are far from clear. In this paper, we report experimental studies using an optical tweezers set up, and accompanying numerical integrations, demonstrating the enhancement and its dependence on various parameters characterizing an oscillating periodic potential.

When a micron-sized particle moves in a fluid in the presence of an external force, the over-damped Langevin equation takes the form

\[ \gamma \dot{x} = F(x, t) + \sqrt{2k_B T} \xi(t), \]  

where \( F(x, t) \) is the external force, \( \xi(t) \) is the normalized white noise, and \( \gamma = 6 \pi \eta a \), where \( \eta \) is a viscosity coefficient of the medium and \( a \) is the particle radius. If the fluid containing the particle is moving with velocity \( v \) in the presence of a periodic potential

\[ V(x) = V_0(1 - \cos(2\pi x / d)), \]

where \( V_0 \) is the amplitude of the potential and \( d \) is the period, equation (1) becomes

\[ \gamma (\dot{x} - v) = -F_p \sin(2\pi x / d) + \sqrt{2k_B T} \gamma \xi(t), \]  

Where \( F_p = 2\pi V_0 / d \) is the amplitude of the spatially oscillating force associated equation (2). Introducing the scaled variables \( y = 2\pi x / d, \tau = 2\pi vt / d \) and \( \beta = F_p / \gamma v \), equation (3) can be rewritten as

\[ \frac{dy}{d\tau} = 1 - \beta \sin y + \sqrt{4\pi k_B T / \gamma \nu d} \xi(\tau). \]  

Equation (4) describes an over-damped Brownian particle moving in a ‘tilted washboard’ potential: the constant 1 on the right-hand side represents the background fluid velocity, while the second term is the periodic force of the washboard potential. The resulting dynamics has attracted much attention due to its wide applicability [1]. The particle is either trapped by the periodic potential (a locked state) or moves in the direction of the background fluid velocity (a running state), depending on the magnitude of \( \beta \). For a deterministic situation (\( T = 0 \)), there exists a critical point separating the locked and running states: when \( \beta < 1 \), there are no potential minima and the particle moves down the washboard. For \( \beta > 1 \), the particle will be trapped in one of the many potential minima and the particle is in a locked state. In the presence of thermal noise, the critical point between the locked state and running state gives way to a stochastic transition region between the two states. The particle delivery speed \( \langle \dot{x} \rangle \) is between zero (a fully locked state) and 1 (a running state); an unusual feature is that the effective diffusion constant can be enhanced in the vicinity of the deterministic critical point [10].

If the periodic potential also oscillates sidewise with frequency \( \omega \) and amplitude \( x_0 \), which correspond to a scaled frequency, \( \Omega = \omega d / 2\pi \nu \), and a scaled amplitude, \( y_0 = 2\pi x_0 / d \), equation (4) becomes

\[ \frac{dy}{d\tau} = 1 - \beta \sin(y - y_0 \sin(\Omega \tau)) + \sqrt{4\pi k_B T / \gamma \nu d} \xi(\tau). \]  

In this case the average speed of the particle exhibits a much different behavior as compared with the static potential case: the average speed not only depends on the value of \( \beta \) and \( T \) but also on the value of \( \Omega \) and \( y_0 \).

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Figure 1. Average velocity of the particle after 50 s based on integration of equation (5) in the deterministic situation for different values of angular frequency $\Omega_1$ and oscillating amplitude $y_0$. These figures show how the average velocity changes with gradually increasing the value of $\beta$ from 0.1 to 20. Different colors in the figures represent the value of the average velocity.

For simplicity, we first consider the deterministic situation where thermal noise is negligible. Figure 1 shows the simulated average velocity for several different values of $\beta$ based on equation (5). When $\beta \ll 1$, the particle’s average velocity is nearly the same as the fluid velocity, and its movement is barely affected by the periodical potential as in figure 1(A).
Figure 2. The change of average velocity versus \( \beta \) for different combination of \( y_0 \) and \( \Omega \). (A) The case where \( \Omega = \pi \) and \( y_0 = \pi \), and there is only one band of \( \beta \) values for which the average velocity is enhanced. (B) The dashed red curve shows for one combination of values of \( y_0 \) and \( \Omega \), two bands of enhanced transport are predicted. The black curve shows another combination for which no enhancement occurs.

With the increasing of \( \beta \), the average speed starts to vary depending on the value of \( \Omega \) and \( y_0 \). When \( \beta \gg 1 \), and at higher frequencies, the movement of the particle obeys the master equation [22]:

\[
\frac{dy}{d\tau} = 1 - \beta J_0(y_0) \sin y,
\]

where \( J_0(y_0) \) is the zeroth order Bessel function. Similar to equation (4), this dynamic equation represents an overdamped particle moving in a tilted washboard potential, whose oscillation depends on both \( \beta \) and \( y_0 \): when the value of \( \beta J_0(y_0) > 1 \), the particle is 'locked' and it does not move relatively to the optical lattice; when \( \beta J_0(y_0) \leq 1 \), the particle is in the ‘running’ state and moves relative to the optical lattice, and the values of \( \Omega \) and \( y_0 \) where the ‘running’ state occurs form multiple straight bands centered at the solution of \( J_0(y_0) = 0 \). When \( \Omega \) decreases, these bands start to bend over, in the vicinity of which a very interesting phenomenon occurs: the average translational velocity of the particle can be greatly enhanced [21, 22].

This enhancement requires specific combinations of \( \beta \), \( \Omega \) and \( y_0 \). Figure 1 shows that when \( \beta \) is big enough, there may be many combinations of \( \Omega \) and \( y_0 \) that yield the enhancement. Figure 2 shows some examples about how the average velocity changes for different values of \( \beta \) when \( \Omega \) and \( y_0 \) are fixed. In figure 2(A) both \( y_0 \) and \( \Omega \) are set to \( \pi \), which corresponds to a single velocity maximum in figure 1(D). If \( \beta \) is small, the periodical potential is weak, and the particle’s movement is not noticeable affected by the periodical potential. The average particle velocity is then nearly the same as the flow velocity. If \( \beta \) is very large, the particle is trapped, and oscillates with, the periodical potential, and its average velocity drops to zero. In the neighborhood of certain specific values of \( \beta \), the transport velocity of the particle is resonantly enhanced due to the synchronization of the particle’s movement with the oscillation of the periodical potential, i.e. every cycle that the periodical potential oscillates, the particles jumps to the next potential well, as in video 1, available from stacks.iop.org/NJP/11/103017/mmedia: when the oscillating periodical potential moves in the same direction as the fluid, the particle
Figure 3. The experiment setup: (A) L1 and L2 form the telescope; M is a two-faced mirror; B1 and B2 show the two beams (that form the standing wave); BS is a 50:50 non-polarized beam-splitter; PM is a prism mirror that is mounted on a piezoelectric translator; and F and A are the filter and attenuator. (B) Particles in the sample passing through the oscillating, spatially periodic optical lattice.

stays in one of the potential wells and moves together with it; if the periodic potential moves in the direction against the fluid moving direction, the potential well is NOT strong enough to overcome the viscosity and the particle slips to the next potential well. For some combinations of \( \gamma_0 \) and \( \Omega \), there can be several ranges of \( \beta \) that show this enhanced transport, as shown in the dotted red curve in figure 2(B), which corresponds to the maximum of the second band in figure 1(D): for the first peak showing the enhancement, the particle jumps two steps forwards and one step backward in one oscillating cycle \( 2\pi/\Omega \), and as a consequence the particle jumps one step forward in one cycle; for the second peak, the particle’s behavior is similar as in figure 2(A), the particle jumps one step forward in one cycle. Also, there may be no enhancement for other combination of \( \Omega \) and \( \gamma_0 \), as shown by the solid black curve in figure 2(B).

We have experimentally demonstrated the enhanced particle delivery discussed above using optical tweezers for the case where \( \Omega \) and \( \gamma_0 \) are equal to \( \pi \), as shown in figure 2(A). The experimental setup, shown in figure 3(A), is similar to that used in our previous work [23]: the two beams B1 and B2 interfere with each other in the sample plane to form an optical lattice with a spacing of 3 \( \mu \text{m} \). One beam is modulated by a mirror mounted on a piezoelectric translator, which generates the oscillation of the optical lattice. The oscillation amplitude and frequency of
Figure 4. (A) The experimental results for the particle’s trajectories for three laser powers. The black line shows the movement of the particle when it moves with the fluid velocity. (B) The experimental and simulated average particle velocity versus the laser intensity.

In the experiment, the beam spot is approximately 120 µm in diameter. We use the central 30 µm region, and this introduces a roughly 12% variation over the active area. Figure 4 shows the experimental and the simulated results when the thermal noise is considered. Figure 4(A) shows the particle trajectories for several laser intensities: when the laser intensity is very small (the power of each beam prior to entering the sample cell \( P = 0.12 \) W), the particle’s movement is minimally affected by the optical lattice, and its average speed is nearly same as the flow velocity; when the laser intensity increases to a suitable value (\( P = 0.32 \) W), the enhancement is observed and the particle moves faster than the background flow velocity (video 2, available from stacks.iop.org/NJP/11/103017/mmedia); when the laser intensity is very strong (\( P = 0.52 \) W), the particle is trapped by the oscillating optical lattice and oscillates together with it, in which case the average velocity is zero. The curve in figure 4(B) shows the average particle delivery velocity as measured in our experiments for different laser intensities together with the simulated results under the same conditions as in the experiments. The simulated result is the numerical solution based on equation (3). The average velocity is measured by averaging the number of periods that the particle moves relatively to the optical lattices in 5 s. It is apparent that the two curves quantitatively agree quite well, except when the average particle velocities are large. This disagreement may be associated with departures from uniformity of beam profile: when the particle moves at a smaller average velocity, it remains...
near the center of the spot, where the intensity is more uniform, and here the data and simulation agree rather well; however, when the average particle velocity is large, it samples the edge of the beam, where the intensity is smaller, and it slows down (ultimately stopping). This results in the experimental velocities falling below the simulation.

In summary, when dragged through an oscillating periodic potential, a particle can synchronize its movement, resulting in an enhanced particle delivery speed for various combinations of the frequency, amplitude and depth of the potential well. It is anticipated that the technique can be exploited to increase the separation speed and selectivity of biological cells or cell constituents. In addition, our experimental technique can be applied more generally to study Brownian dynamics in oscillating or static periodic potentials, where many interesting phenomena such as enhanced diffusion without a local increase in the temperature [10, 22] etc, can be studied.

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