Calculation and Simulation of Eddy Current Magnetic Field of Ellipsoid in Changing Magnetic Field

Haodong Wen1,a, Shengdao Liu2,b, Jianfeng Ouyang3,c

1Naval University of Engineering Wuhan, China
2Naval University of Engineering Wuhan, China
3Naval University of Engineering Wuhan, China
a619570865@qq.com, c734799078@qq.com

Abstract. The eddy current magnetic field generated by the ellipsoid in the changing magnetic field is theoretically deduced and calculated and simulated. The theoretical calculation and the value obtained by the simulation are compared and verified and the error is analyzed. It can be seen from the fitting results that the theoretical calculation method used in this paper and the simulation result have reached a 93% degree of fit, indicating that this method has a certain reference value for the study of the spatial distribution of the eddy current magnetic field.

1. Introduction
The modern navy requires more and more ships to be stealthy. The magnetic field of ships can be divided into fixed magnetic field, induction magnetic field, eddy magnetic field and stray magnetic field. There have been many researches on fixed magnetic field and induced magnetic field. For ships built with low magnetic materials, the fixed magnetic field and induced magnetic field are low enough. However, eddy magnetic field will inevitably be generated in the process of the ship moving, such as mine-hunting ships and so on. Therefore, the research and calculation of eddy current magnetic field are of great significance in the fields of magnetic stealth and magnetic confrontation. Japanese scholar Yoshii Yoshiki[1] first proposed to use an ellipsoidal equivalent ship for eddy current analysis. In the 1980s, Li Daogen[2] and Wang Junmin[3] replaced the hull with an ellipsoidal hull to analyze and calculate the eddy current magnetic field of the moving ship. But the eddy current magnetic field generated by the moving hull is difficult to measure, which makes it difficult to verify the results of calculation and simulation. In recent years, according to the generation mechanism of the eddy current magnetic field, some scholars have proposed using a changing magnetic field to be applied around the ship to simulate the eddy current magnetic field generated by the moving ship. In this paper, the theoretical magnetic field calculation and the infinite element model three-dimensional simulation calculation of the ellipsoid in the changing magnetic field are carried out[4]. The calculated value and simulation result of the ellipsoid eddy current magnetic field have an accuracy of more than 93%. For the actual eddy current magnetic field research and calculation have certain reference values.
2. Eddy magnetic field of ellipsoid

2.1. The eddy of an ellipsoid

The Cartesian coordinate system is established, with the center of the ellipsoid as the origin, and the half axes of each length and length coincide with x, y and z axis respectively. When a time varying magnetic field is applied to the outside of the ellipsoid, the changing magnetic field produces eddy currents. Ellipsoid materials of relative permeability is $\mu_r$, the ellipsoid of x, y, z axis relative permeability and ellipsoid demagnetization factor related, access to information can be calculated ellipsoid x, y, z axis relative permeability is $\mu_x$, $\mu_y$, $\mu_z$ [5].

The ellipsoid satisfies the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(1)

Let the relative permeability of the ellipsoid material be $\mu_r$. The relative permeability along the y-axis of the ellipsoid is $\mu_y$. Under the action of the applied external magnetic field $H_y$, the magnetic induction intensity excited by $H_y$ inside the ellipsoid can be obtained as follows:

$$B_y(t) = \mu_y \mu_r H_y(t)$$

(2)

![Figure 1 A spatial diagram of an ellipsoid](image)

The eddy density in the ellipsoid can be obtained from Maxwell's equations as:

$$\nabla \times \delta_e = -\gamma \frac{dB_y}{dt} j$$

(3)

Where $\delta_e$ is the eddy current density in the ellipsoid and $\gamma$ is the electrical conductivity of the material.

On the outer surface of the ellipsoid, the current cannot flow out of the surface, so on the boundary surface of the ellipsoid, the normal component of the eddy current is zero.

$$\delta_e \mid_{s} = 0$$

(4)

The ellipsoid is a good conductor with air insulation, so the vortex current cannot flow out of the surface. Therefore, on the surface of the ellipsoid, the normal vector of eddy current is 0, and the current is continuous. Let the vector $A$ be:

$$\delta_e = \nabla \times A$$

(5)

By coulomb specification:

$$\nabla \cdot A = 0$$

(6)

Substitute into the equation(3) and you get:

$$\nabla \times (\nabla \times A) = -\gamma \frac{dB_y}{dt} j$$

(7)

Can be obtained further:
\[ \nabla^2 A = -\gamma \frac{dB_j}{dt} \quad (8) \]
\[ \nabla^2 A_x = -\gamma \frac{dB_y}{dt} \quad (9) \]
\[ \frac{\partial A_x}{\partial z} = \frac{\partial A_y}{\partial x} \quad (10) \]

The eddy current characteristics can be obtained easily:
\[ A_x = R \left( \frac{e^2}{2c^2} + \frac{x^2}{2a^2} \right) \quad (11) \]

R is undetermined coefficient. Easy to get:
\[ R = -\frac{a^2 c^2}{a^2 + c^2} \frac{dB_y}{dt} \quad (12) \]

It can be known that the eddy current generated under the magnetic field applied in the direction of y is:
\[ \left\{ \begin{align*}
\delta_{zz} &= \frac{a^2}{a^2 + c^2} \frac{e\mu_0 \mu_y}{c^2} \frac{dH_z}{dt} \\
\delta_{yy} &= 0 \\
\delta_{xx} &= -\frac{c^2}{a^2 + c^2} \frac{e\mu_0 \mu_y}{c^2} \frac{dH_x}{dt}
\end{align*} \right. \quad (13) \]

2.2. The eddy current magnetic moment of an ellipsoid

Let the equation of the ellipse cross section:
\[ \frac{x^2}{a^2} + \frac{z^2}{c^2} = v^2, \quad v \in \left[ 0, \sqrt{1 - \frac{y^2}{b^2}} \right] \quad (14) \]

If the ellipse equation is:
\[ \left\{ \begin{align*}
x &= a \sin \phi \\
y &= 0 \\
z &= c \cos \phi
\end{align*} \right. \quad (15) \]

Then the magnetic moment generated by current I is:
\[ M_y = \pi ac \hat{y} \quad (16) \]

It can be obtained by the thought of calculus and the formula of magnetic moment:
\[ dM_y = -\pi \mu_0 \mu_y \frac{a^3 c^3}{a^2 + c^2} \frac{dH_y}{dt} v \, du \, dv \]
\[ v \in \left[ 0, \sqrt{1 - \frac{u^2}{b^2}} \right], \quad u \in [-b, b] \quad (17) \]

Can be obtained by:
\[ M_y = -\pi \mu_0 \mu_y \frac{a^3 c^3}{a^2 + c^2} \frac{dH_y}{dt} \int_{-b}^{b} \left[ \sqrt{1 - \frac{u^2}{b^2}} \right] v \, du \, dv \]
\[ = -\frac{4}{15} \pi \mu_0 \mu_y \frac{b a^2 c^3}{a^2 + c^2} \frac{dH_y}{dt} \quad (18) \]

Similarly, the vortex magnetic moments in the x and z directions can be obtained. Due to the limited space in this paper, we will not continue to derive them here.
2.3. Eddy magnetic field in outer space of ellipsoid

The eddy current magnetic moment of the ellipsoid was calculated, and the eddy current magnetic field of the ellipsoid was calculated numerically by using the magnet simulation method. In the end, the magnetic simulation method boils down to using a certain shape of magnetic object to calculate the magnetic field in the field space. This requires the shape of the magnetic simulation body to simulate the ship's magnetic field, and the magnetic field generated in the surrounding space can be easily calculated by analytical formula. In this paper, a magnetic dipole model is chosen to calculate the magnetic field\[6\].

The scalar magnetic potential of a magnetic dipole is:

\[
\varphi_m = \frac{1}{4\pi} \frac{m \cdot r}{r^3} = \frac{1}{4\pi} \frac{m \cos \theta}{r^3}
\]  

(19)

\(m\) for magnetic dipole magnetic moment, \(\theta\) for \(m\) and \(r\), the Angle between the direction of \(r\) is calculated by the magnetic dipole center point to sites:

\[
H = -\nabla \varphi_m
\]

(20)

![Figure 2: Model of magnetic dipole](image)

So you can figure out the strength of the magnetic field produced by the magnetic dipole:

\[
H = \frac{m}{4\pi r^3} (2\cos \theta e_x + \sin \theta e_y)
\]

(21)

The above formula is expressed in spherical coordinates, obviously, \(m\), \(r^0\), \(\theta^0\) in the same plane. The projection of \(m\) on the two is as follows:

\[
m \cos \theta
\]

(22)

\[
m \cos(\theta + \frac{\pi}{2}) = -m \sin \theta
\]

(23)

Can be launched:

\[
m = r^0 m \cos \theta - \theta^0 m \sin \theta
\]

(24)

So:

\[
H = \frac{1}{4\pi r^3} (3m \cos \theta r^0 - m) = \frac{3(m \cdot r^0) r^0}{4\pi r^3} - \frac{m}{4\pi r^3}
\]

(25)

When the magnetic dipole moment is along the x-axis, y-axis, and z-axis directions, the magnetic field generated at (x, y, z) is:

\[
\begin{bmatrix}
\frac{h_x}{4\pi r^3} \\
\frac{h_y}{4\pi r^3} \\
\frac{h_z}{4\pi r^3}
\end{bmatrix} = \frac{1}{4\pi r^3} \begin{bmatrix}
m_x(3x^2 - r^2) & m_y 3xy & m_z 3xz \\
m_x 3xy & m_y(3y^2 - r^2) & m_z 3yz \\
m_x 3xz & m_y 3yz & m_z(3z^2 - r^2)
\end{bmatrix}
\]

(26)

We can get the magnetic flux density \(B\) by:
3. Comparison of theoretical calculation and simulation

The geometric structure of the ellipsoid model is carried out in the three-dimensional simulation software, and the ellipsoid is divided by the grid. The ellipsoid is relatively uniform, and it is easy to divide the ellipsoid by the idea of finite element. COMSOL is a three-dimensional physical field simulation software, based on the finite element division of the object, it can simulate the magnetic field of the object in space. This article uses the frequency domain analysis of COMSOL simulation software and the research method from frequency domain to time domain to study the ellipsoid. The eddy current magnetic field under the changing external magnetic field is simulated and compared with the theoretical calculation value to verify the accuracy of the theoretical derivation.

We choose at time 0s, along the y axis 1~11, the position of the equidistant interval 0.2m and the depth of 1.2m under the ellipsoid, along the y direction -2.5~2.5, the position of the equidistant interval 0.1m to perform the theoretical value and calculation comparison of simulation values.

3.1. Theoretical calculation of eddy current magnetic field

Suppose the geometric data of the ellipsoid is the semi-major axis \( a = 0.7 \text{m} \), the semi-minor axis \( b = c = 0.4 \text{m} \), and the conductivity is set as:

\[
\gamma = 1.4286 \times 10^8 (\text{s/m})
\]

The relative permeability is set to 1, and the value of the magnetic moment in each direction of the ellipsoid can be calculated according to the aforementioned formula. The sinusoidal magnetic field changed by the external magnetic field is:

\[
H_y = 30 \sin 15t
\]

Thus, the eddy current magnetic field at various positions outside the ellipsoid can be calculated according to the magnetic dipole model. Obtain the eddy current magnetic field along the y axis 1~11 as shown in Figure 3.

![Figure 3: Calculated value of eddy current magnetic field along y axis](image)

The value of the -2~2 eddy current magnetic field along the y direction at a depth of 1.2m directly below the ellipsoid is shown in Figure 4.
3.2. Simulation calculation of eddy current magnetic field

According to the theoretically calculated ellipsoid model, the same three-dimensional geometric figures are constructed in the COMSOL software and the same external magnetic field is applied, and appropriate boundary conditions are set.

The geometric model of the ellipsoid in the infinite element domain is divided as shown in Figure 5. Applying a changing sinusoidal external magnetic field on the y-axis, that is, in COMSOL, select the background field of the magnetic field as the reduced field to solve and select the background field details with uniform magnetic flux density. The maximum value of $H_y$ is 30 A/m. If we convert it to B, we get:

$$B = H_y \times 4\pi \times 10^{-7} T$$

By selecting the frequency domain and adding frequency domain to the time domain, the time-varying eddy current magnetic field data can be obtained in the simulation calculation.

![Figure 5: Meshing of infinite element domain ellipsoid](image)

The coincidence of the simulation data and the theoretical calculation data is very high. The data comparison chart with the equidistant interval of 1~11 along the y axis is shown in Figure 6.

The comparison between the calculated value and the simulated value of -2 to 2 equidistantly separated by 0.1m in the y direction 1.2m below the ellipsoid is shown in Figure 7.

4. Error analysis and processing

Calculate the percentage of accuracy of the data, simply divide the simulated value by the calculated value, you can get the legend of the fit degree of the calculated value and the simulated value of the eddy current magnetic field along the y-axis and 1.2m below the ellipsoid, as shown in Figure 8. 9 shown.
Observation shows that most of the data are highly coincident, the accuracy is about 93%, and some data are biased.

![Eddy current magnetic field value](image)

Figure 6: Comparison of calculated and simulated values of eddy current magnetic field along the y-axis

![Eddy current magnetic field value](image)

Figure 7: Comparison of calculated and simulated values of eddy current magnetic field under the ellipsoid

4.1. Error sources.

4.1.1 Analysis In the process of head magnetic field calculation, after the eddy current factor is approximated, the difference value of the magnetic field may be more obvious due to the large value of the eddy current magnetic field. In the tail, the eddy current magnetic field value is smaller, and the percentage error is more likely to increase.

4.1.2 In the process of finite element subdivision, it is impossible to carry out completely accurate calculation and analysis, and the simulation calculation value of the simulation software itself is also impossible to fully fit the theoretical data value.

4.2. The way it's handled

4.2.1 The calculation accuracy is optimized according to the specific needs.

4.2.2 In the process of finite element subdivision, the boundary condition and subdivision mode are controlled artificially, and the mesh subdivision is refined.
Figure 8: The fit between the calculated value and the simulated value (Along the y axis)

Figure 9: The fit between the calculated value and the simulated value (Below the ellipsoid)

5. conclusion
This paper studies the magnetic field generated by the eddy current of the ellipsoid, first calculates the eddy current and then calculates the eddy current magnetic moment, and then uses the magnetic dipole simulation method to calculate the eddy current magnetic field outside the ellipsoid, and through the COMSOL three-dimensional finite element simulation analysis, the obtained The results show that the calculation method used in this paper has a high degree of accuracy, and the degree of fit can reach about 93%, which can be used as one of the reference methods for the calculation and research of eddy current magnetic field.

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