Quantum gate for Q switching in monolithic photonic bandgap cavities containing two-level atoms.

Andrew D. Greentree, J. Salzman, Steven Prawer, and Lloyd C. L. Hollenberg

1 Centre for Quantum Computer Technology, School of Physics, The University of Melbourne, Melbourne, Victoria 3010, Australia.
2 Microelectronics Research Center, Electrical Engineering, Technion, Haifa 32000, Israel.

(Dated: November 10, 2005)

Photonic bandgap cavities are prime solid-state systems to investigate light-matter interactions in the strong coupling regime. However, as the cavity is defined by the geometry of the periodic dielectric pattern, cavity control in a monolithic structure can be problematic. Thus, either the state coherence is limited by the read-out channel, or in a high Q cavity, it is nearly decoupled from the external world, making measurement of the state extremely challenging. We present here a method for ameliorating these difficulties by using a coupled cavity arrangement, where one cavity acts as a switch for the other cavity, tuned by control of the atomic transition.

As the maturity and sophistication of quantum optics progresses, there is a growing movement to translate such effects into practical devices. This impetus suggests, for reasons of scalability and practicality, the need for viable solid-state technologies to produce and distribute single photons as an enabling technology for derivative quantum devices. In particular we are concerned with the role played by cavity Quantum Electro-Dynamics (CQED) in such devices.

CQED has been used to great effect in the generation of deterministic, transform limited single (and higher-order Fock states) photon pulses \(^1\), and schemes exist which incorporate CQED for quantum computing \(^2\), and entanglement generation \(^3\). More recently ‘hybrid’ schemes for quantum computation have been suggested incorporating matter qubits in cavities with single photon generation, linear optics and high fidelity photon detection \(^4\). However many of these schemes (with notable exceptions) will be problematic to scale or to remove from laboratory environments.

Given difficulties with implementing most present schemes in non-research environments, significant attention has turned towards photonic band gap (PBG) cavities as quantum cavities. This is due to their superb photonic confinement properties and the recent realization of high Q cavities with small mode volume (of order the wavelength\(^3\)) \(^5\), \(^6\). These successes have been fueled by a combination of technological imperatives and advances in fabrication.

A PBG material is created by producing a periodic modulation in the dielectric function of a material so that Bragg interference prevents propagation of certain modes across the structure. Such structures may be two-dimensional, with confinement in the third dimension realized by classical waveguiding, or by creating a three-dimensional lattice. We concentrate on the former example as it is easier to produce, and has so far yielded the most dramatic effects. The most popular configuration for 2D PBG structures is a thin membrane with a 2D array of holes (a lattice) drilled in it. A defect (usually an undrilled hole, local variation in lattice spacing, or combination of both) defines a PBG cavity, as any photon injected into that site cannot propagate laterally away from the defect. In this way, PBG cavities can constitute extremely good cavities with low loss, high coupling and low mode volume, all necessary conditions for probing the strong-coupling limit of CQED.

One problem with high-Q cavities is the difficulty of out-coupling excitations from the cavity \(^7\). One would like a Q-switch, a device that can be modulated in some fashion to change the cavity from high Q to low Q, with the optical intensity dumped from the cavity in a controlled fashion: we will term such a device a “gate”. Q-switching is well-known for classical laser applications \(^8\), but is less easy for PBG cavities, although some recent proposals exist including mechanical switches \(^9\), and nonlinear optical effects \(^10\). However in monolithic structures where we cannot use mechanical or thermal effects, and operating at low light levels, there have been no suitable suggestions for an effective Q-switch in PBG cavities. This is the problem addressed in this paper.

The structure we consider is a coupled cavity arrangement, similar in spirit to that studied by Waks and Vukovic \(^11\), where two defects in the PBG lattice were placed in close proximity to form evanescently coupled cavities. Our arrangement is shown schematically in Fig. \(^1\)(a), where the left-hand cavity is the storage cavity (or simply cavity), the right hand cavity is the gate, which is in close proximity to a waveguide, or other leaky, classical region. In this limit we can describe the coupling between the distinct regions (cavity, gate and waveguide), which is due to evanescent leakage of the electromagnetic modes, as being equivalent to photon hopping between the regions \(^12\). In addition to the previously considered systems, however, we augment this arrangement by placing a single two-state atomic system in the center of

---

\(^*\)Electronic address: andrew.greentree@ph.unimelb.edu.au
the cavity and gate \( \text{Q-switch} \), where the transition frequency of the atom can be controlled by some external control field. An example of a system that could realize such an architecture would be a single crystal diamond with photonic crystal drilled using focused ion beam milling and lift-off \( \text{Q-switch} \) where single ion implantation techniques \( \text{Q-switch} \) are used to locate individual nitrogen-vacancy centers in the centre of the cavity, controlled via the linear Stark shift \( \text{Q-switch} \). It is this control of the atomic frequency that constitutes our sole dynamic (i.e. post-fabrication) control of the system parameters, and is responsible for the Q-switching possibilities that we discuss in this paper.

The method for Q switching this system can be understood easily, and is a logical extension of previous work on cavity QED and photon blockade \( \text{Q-switch} \). Firstly the cavity is arranged so that one and only one photon is loaded into the cavity via some external pump, and the gate is in its ground state. The cavity and gate resonances are initially dissimilar, so that light from the cavity cannot leak across to the gate. Secondly the eigenmodes of the cavity are varied by changing the resonance frequency of the atom in it, and when one of the gate modes is resonant with a mode of the cavity, photon hopping occurs.

The gate is a relatively bad cavity, coupled to the output modes of a waveguide, and so photons leaking into the gate are rapidly outcoupled to the waveguide. As photon hopping is the source of the cavity-gate coupling, it is clear that optimal outcoupling results from balancing the competing needs of large cavity-gate detuning, with photonic population of the resonant mode of the gate at the outcoupling resonance. These points will be made more explicit by considering the model Hamiltonian.

The Hamiltonian for our system is written

\[
\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{I}_1 + \mathcal{I}_2 + \mathcal{P},
\]

\[
\mathcal{H}_\alpha / \hbar = \varepsilon_\alpha (V_\alpha) \sigma_\alpha^+ \sigma_\alpha^- + \omega_a a_\alpha^\dagger a_\alpha, \quad (1)
\]

\[
\mathcal{I}_\alpha / \hbar = \Omega_\alpha (\sigma_\alpha^- a_\alpha + \sigma_\alpha^+ a_\alpha^\dagger), \quad (2)
\]

\[
\mathcal{P} / \hbar = \kappa_c_q (a_\alpha^\dagger a_q + a_q a_\alpha^\dagger), \quad (3)
\]

where \( \mathcal{H}_\alpha \) and \( \mathcal{I}_\alpha \) refer to the bare and interacting parts of the Hamiltonians respectively, for \( \alpha = c, q \) for cavity or gate (Q-switch). \( \varepsilon_\alpha (V_\alpha) \) is the transition frequency of the atom in \( \alpha \) which can be controlled by the Stark shifting gates at some potential \( V \), the exact functional dependence of the Stark shift on gate potential is not important. \( \omega_a \) is the resonance frequency of the photon in \( \alpha \), and \( \Omega_\alpha \) is the atom-cavity coupling (one-photon Rabi frequency) in \( \alpha \). The \( \sigma_\alpha \) are the usual Pauli operators for the atoms in \( \alpha \), and \( a_\alpha \) is the usual photon annihilation operator in \( \alpha \). \( \mathcal{P} \) describes the photon hopping, with coupling \( \kappa_c_q \). Coupling to the external waveguide is described via a non-Hamiltonian term which will be introduced in the density matrix formalism. All these terms are depicted schematically in Fig. 1(b).

In general, the two cavity system with two atoms is a moderately complicated problem to treat exactly, however by considering just the one quantum manifold (i.e. where only one quantum of excitation is in the system), and assuming that the detuning between the cavities is large, i.e. \( \omega_q - \omega_c = \delta \gg \kappa_c_q, \Omega_\alpha \), we get significant insight. In this limit we can solve each cavity independently (i.e. ignoring \( \kappa_c_q \) as our zeroth order approximation) to get the approximate eigenstates, which are the well-known dressed states,

\[
| \pm_{g_0} q \rangle = \frac{(-\Delta (V_q) / 2 \mp \chi_q)}{\sqrt{2 \chi_q^2 \mp \chi_q \Delta (V_q)}} | g_0 q \rangle + \Omega_c | e_c 0_q \rangle,
\]

\[
| \pm_{e_0} q \rangle = \frac{(-\Delta (V_q) / 2 \mp \chi_q)}{\sqrt{2 \chi_q^2 \mp \chi_q \Delta (V_q)}} | g_0 q \rangle + \Omega_q | e_q 0_q \rangle,
\]

where we have introduced \( | g \rangle \) and \( | e \rangle \) as the states of the atoms, \( \Delta (V_q) = \omega_a - \varepsilon_\alpha (V_q) \), the detuning, and \( \chi_\alpha = \sqrt{\Delta (V_q) / 2} \mp \Omega_\alpha \), the generalized Rabi frequency. The associated eigenenergies are

\[
E_{| \pm_{g_0} q \rangle} = \pm \chi_q - \Delta (V_q) / 2, \quad (6)
\]

\[
E_{| \pm_{e_0} q \rangle} = \pm \chi_q + \Delta (V_q) / 2. \quad (7)
\]
the approximation as a function of $\Delta_c$ we present in Fig. 2 the eigenspectra determined by the two cavities, which is (for example between $|c, g_0q\rangle$ and $|g, c_0-q\rangle$) at the gate defined resonance $\Delta_g = -\delta$, the system conveniently breaks into three manifolds, distinguished by the total number of quanta, the zero quantum manifold is the lowest, along the $\Delta_g(V_g)$ axis, then the one quantum and two quantum. As we are most interested in the resonances between the cavity and gate in the one quantum manifold, we present a closeup of this in the inset to Fig. 2 where the anti-crossings indicating photon hopping between the cavity and gate are clearly visible. Note that these parameters were merely chosen to demonstrate the relevant processes, and all units are arbitrary.

The previous analysis just treats coupling between the cavity and gate, but to proceed further we need to include the coupling to the waveguide mode ($W$). This is best done by introducing an irreversible loss term, analogous to spontaneous emission, which models coupling into an extra waveguide mode. We then solve the density matrix equations of motion to examine the transient coupling into the waveguide mode. Concretely we solve for the density matrix, $\rho$, using:

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \kappa_{qw} \mathcal{L}[\rho, a_W^\dagger a_W],$$

$$\mathcal{L}[\rho, a_W^\dagger a_W] = a_W^\dagger a_W \rho a_W a_W^\dagger a_W^\dagger a_W - \frac{1}{2} [a_W^\dagger a_W, \rho a_W a_W^\dagger a_W^\dagger a_W].$$

(9)

$\kappa_{qw}$ is the gate-waveguide coupling. An example of the evolution obtained is presented in Fig. 3 which shows the populations in the bare state basis, from top to bottom: $|g, 1, c_q, 0, 0_q, 0_W\rangle$, $|c_q, 0, g_0, 0_q, 0_W\rangle$, $|g, 0_c, c_q, 0, 0_q, 0_W\rangle$, $|g, 1, g_0, 0_q, 0_W\rangle$, and $|g, 0_c, 0, 0_q, 1_W\rangle$ respectively, as a function of time and $\Delta_g(V_g)$ given initial state $|g, 1, c_q, 0, 0_q, 0_W\rangle$. Clearly noticeable are coherent oscillations corresponding to Rabi oscillations in the cavity, and the gradual buildup of population in the waveguide mode. The increase in the oscillation frequency in the region $-\delta - \Omega_c < \Delta_g < -\delta + \Omega_c$ is a consequence of an increased eigenvalue splitting, similar to (but more complicated than) that seen in multiply coupled three state systems, see for example Ref. 18.

In the simpler, doubly-driven three-state case, the oscillation frequency is given by the sum of the squares of the Rabi frequencies of the driving fields. In this case, the result is similar, with the oscillation frequency given by the sum of the squares of the interaction matrix elements, i.e. $\sqrt{\Omega_c^2 + J^2}$.

Although illustrating much of the necessary physics, it is clear that the results in Fig. 3 do not illustrate an effective mechanism for single photon generation. The reason here is that the outcoupling probability can be no better than 50%, and the Rabi oscillations render the system prone to nonadiabatic errors. Also pertinent is that a complicated set of interference fringes are observed which need to be understood for transient analysis. Given such limitations, it is preferable to initialize the system in the state $|+c, g_0, 0_q, 0_W\rangle$ and follow an adiabatic transition along the anticrossing between $|+c, g_0, 0_q, 0_W\rangle$ and $|g, 0_c, -q, 0_W\rangle$. By ensuring that the gate and waveguide mode are strongly coupled, i.e. $\kappa_{qw}$ is large compared with the coupling matrix elements, population in the state $|g, 0_c, -q, 0_W\rangle$ will be rapidly transferred to $|g, 0_c, g_0, 0_q, 1_W\rangle$, and hence the cavity-gate resonance will act as an effective Q-switch for the cavity. Initialization of the system could be achieved by pumping the cavity with light of frequency $\omega_c + \Omega_c$, which would be resonant with the $|g, 0_c\rangle - |+c\rangle$ transition, but not resonant with transitions to the two quanta manifold. A schematic of the steps required to adiabatically outcouple the single photon is shown in Fig. 4.

The results of the adiabatic transfer from cavity to gate are shown in Fig. 5 clearly showing both the population in $|g, 0_c, g_0, 0_q, 1_W\rangle$, which we denote $\rho_{WW}$, and

![Figure 2: Eigenvalues for the two-cavity system as a function of $\Delta_g$ for $\Omega_c = \Omega_q = \kappa_{cq} = 0.1$, $\omega_c = 2$, $\omega_q = 2.5$ and $\delta = 0.5$. The eigenspectrum naturally divides into the component manifolds, we are most interested in the one quantum manifold. The bottom trace shows a closeup of the one quantum manifold, highlighting interactions between the cavities, via the off-resonant dressed state. The resultant anti-crossings indicate coupling between the gate and cavity, and hence where switching can occur.](image)
the time derivative of this population, $\dot{\rho}_{WW}$, which is proportional to the intensity of the resulting photon pulse. In this case we chose $\Omega_c = \Omega_q = \Omega$, $\delta = 4\Omega$, $\kappa_{cq} = 0.01\Omega$, $\kappa_{qW} = 0.1\Omega$, $-3.2\Omega < \Delta_q < -2.2\Omega$ and the length of the sweep was $T_{\text{max}} = 2 \times 10^4\pi/\Omega$. Note that because of the difference between $\kappa_{cq}$ and $\kappa_{qW}$ the resultant single photon pulse is not a Gaussian. To retrieve a Gaussian pulse, one might either choose a system with equal photon hopping matrix elements or a more complicated gate sweep. Note that the integral of the derivative is unity, as required for a pulse of one photon.

When considering the operating parameters of the Q switch, it is also necessary to determine the quiescent fidelity, i.e. the photon leakage from cavity to the Q-switching gate when the switch is not activated. For simplicity, if we assume $\Delta_c = \Delta_q = 0$ and $\delta \gg \kappa_{cq}, \Omega_c, \Omega_q$, then the population in state $|+c g_q 0_q\rangle$, $\rho_{++}(t)$ at time $t$, given initialization in state $|+c g_q 0_q\rangle$ at $t = 0$, is

$$\rho_{++}(t) = \exp \left( -\frac{\kappa_{cq}^2}{2\delta^2} \kappa_{qW} t \right)$$  \hspace{1cm} (10)

FIG. 3: Transient evolution of the cavity-gate-waveguide system showing populations in the states (a) $|g,1, q, 0, 0, 0\rangle$, (b) $|c,0, g_q, 0, 0\rangle$, (c) $|g,0, 0, q, 0, 0\rangle$, and (d) $|g,0, 0, 0, q, 1\rangle$, respectively (the population in $|g,0,0,0,0\rangle$ is never visible for these parameters), as a function of time and $\Delta_q$ given initial state $|g,1, g_q, 0, 0, 0\rangle$ for $\Omega_c = \Omega_q = 0.5$, $\kappa_{cq} = \kappa_{qW} = 0.1$ and $\delta = 2$. Note the oscillation frequency in the range $-\delta - \Omega_c < \Delta_q < -\delta + \Omega_c$ is found to be $\sqrt{\Omega_c^2 + \delta^2}$. The maximum of the population transfer peaks in the waveguide in (d) are 0.26 at $\Delta_q \sim -\delta - \Omega_c$, and 0.17 at $\Delta_q \sim -\delta - \Omega_c$.

where $\kappa_{cq}^2/\delta^2$ is the standard, steady state, off-resonant population leaking from the cavity to gate, which is then outcoupled at a rate $\kappa_{qW}$. Under the conditions used to generate Fig. 5, this equates to a population of $\rho_{++} = 0.98$ at $t = 2 \times 10^4\pi/\Omega$, or alternatively, at worst a 2% probability of the photon outcoupling from the cavity.

Finally we comment on the practicality of realizing our scheme in a realistic structure, and for our purposes we assume a PB cavity structure fabricated in diamond containing a single NV$^+$ centre at the maximum of the cavity mode. The wavelength of the zero-phonon line resonance of an NV$^+$ centre is $\lambda = 638 \times 10^{-9}$m, with frequency $\omega = 2.95 \times 10^{15}$Hz, and assuming that each cavity has volume $V = \lambda^3 = (638 \times 10^{-9})^3 m^3$, then the atom cavity coupling will be $\Omega = \mu \sqrt{\omega/(2\hbar\epsilon_0 V)} \sim 10^{10}$Hz (given the electric dipole moment of the NV$^+$ centre of $\mu \sim 10^{-29}$Cm). For this degree of coupling, the tuning range of the centres should be many $\Omega$. The tuning range reported in Ref. 16 is $\sim 10^{12}$Hz, which does not constitute an upper limit on the Stark tuning. Therefore the atomic tuning criterion should be easy to satisfy, and we presume $\delta = 10^{12}$Hz.

If we assume that the cavities are in the good cavity limit, and that the cavity Q is dominated by photon loss due to the photon hopping between cavities and the waveguide, then the cavity Q must be fairly large to ensure minimal population leakage when we are not at the switching point. The figure of merit here is that the ratio $\kappa_{cq}^2/\delta^2$ should be small. If we aim for a residual population of $10^{-4}$, then $\kappa_{cq}/\delta < 10^{-2}$, i.e. $\kappa_{cq} = 10^{10}$Hz, and $\kappa_{qW} = 10\kappa_{cq} = 10^{11}$Hz. If we assume that the cavity Q is dominated by the photon hopping terms, then we have (for the cavity)

$$Q_c = \frac{\omega}{\kappa_{cq}} \sim 10^5$$  \hspace{1cm} (11)

and the Q of the gate will be $10^4$. Although techni-
The full set of required parameters are summarized in Table I.

In conclusion, we have presented a scheme for Q-switching a photonic bandgap cavity by controlling the resonance condition of an adjacent cavity. Each cavity contains a single two-level atom, and the transition frequency of the atom can be controlled via a Stark shifting electrode. We refer to the right-hand cavity as the gate which Q-switches the cavity. The resonance frequencies of the two cavities are initially dissimilar, but by tuning the atomic transition in the gate, a resonance condition between the cavity and gate is obtained, resulting in photon hopping between the cavities. By introducing a waveguide mode adjacent to the gate, photons leak rapidly out of the gate. Such a device constitutes a solid-state source of transform limited single photons on demand. An ideal system to test such concepts would be in micromachined diamond containing the nitrogen-vacancy color centre, although our ideas can be applied to any photonic bandgap cavity containing a two-level atom in the maximum of the cavity mode.

ADG would like to acknowledge useful discussions with T. Ralph, and J. Cole. JS acknowledges support from the Fund for Promotion of Research at the Technion.

This work was supported by the Australian Research Council, the Australian government and by the US National Security Agency (NSA), Advanced Research and Development Activity (ARDA) and the Army Research Office (ARO) under contracts W911NF-04-1-0280 and W911NF-05-1-0284.

[1] F. De Martini, G. Di Giuseppe, and M. Marrocco, Phys. Rev. Lett. 76, 900 (1996); P. Michler, A. Kiraz, C. Becher, W. V. Schoenhfeld, P. M. Petroff, L. Zhang, E. Hu, A. Imamoglu, Science 290, 2282 (2000); A. Kuhn, M. Henrich, and G. Rempe, Phys. Rev. Lett. 89, 067901 (2002); K. R. Brown, K. M. Dani, D. M. Stamper-Kurn and K. B. Whaley, Phys. Rev.A 67, 043818 (2003); B. T. H. Varcoe, S. Brattke, and H. Walther, New J. Phys. 6, 97 (2004); C. W. Chou, S. V. Polyakov, A. Kuzmich, and H. J. Kimble, Phys. Rev. Lett. 92, 213601 (2004); M. Keller, B. Lange, K. Hayasaka, W. Lange, and H. Walther, Nature (London) 431, 1075 (2004).

[2] T. Pellizzari, S. A. Gardiner, J. I. Cirac, and P. Zoller,
Phys. Rev. Lett. 75, 3788 (1995); A. Beige, D. Braun, B. Tregenna, and P. L. Knight, Phys. Rev. Lett. 85, 1762 (2000); L.-M. Duan and H. J. Kimble, Phys. Rev. Lett. 92, 127902 (2004).

[3] M. B. Plenio, S. F. Huelga, A. Beige, and P. L. Knight, Phys. Rev. A 59, 2468 (1999); L.-M. Duan and H. J. Kimble, Phys. Rev. Lett. 90, 253601 (2003); J. Gea-Banacloche, T. C. Burt, P. R. Rice, and L. Orozco, Phys. Rev. Lett. 94, 053603 (2005).

[4] S. D. Barrett and P. Kok, Phys. Rev. A 71, 060310(R) (2005); Y. L. Lim, A. Beige, and L. C. Kwek, Phys. Rev. Lett. 95, 030505 (2005); S. C. Benjamin, J. Eisert, and T. M. Stace, New J. Phys. 7, 194 (2005).

[5] Y. Akahane, T. Asano, B.-S. Song, and S. Noda, Nature (London) 425, 944 (2003); T. Yoshie, A. Scherer, J. Hendrickson, G. Khitrova, H. M. Gibbs, G. Rupper, C. Ell, O. B. Shchekin and D. G. Deppe, Nature (London) 432, 200 (2004).

[6] B.-S. Song, S. Noda, T. Asano, and Y. Akahane, Nature Materials 4, 207 (2005).

[7] M. Khanbekyan, L. Knöll, A. A. Semenov, W. Vogel, and D.-G. Welsch, Phys. Rev. A 69, 043807 (2004).

[8] A. Yariv, Quantum Electronics (Wiley, New York).

[9] A. F. Koenderink, M. Kafesaki, B. C. Buchler, V. Sandoghdar, Phys. Rev. Lett. 95, 153904 (2005).

[10] M. Scalora, J. P. Dowling, C. M. Bowden and M. J. Bloemer, Phys. Rev. Lett. 73, 1368 (1994); P. M. Johnson, A. F. Koenderink, and W. L. Vos, Phys. Rev. B 66, 081102(R) (2002); S. W. Leonard, H. M. van Driel, J. Schilling, and R. B. Wehrspohn, ibid. 66, 161102(R) (2002); H. T. Dung, L. Knöll, and D.-G. Welsch, Phys. Rev. A 67, 021801 (R) (2003)

[11] E. Waks and J. Vukovic, Opt. Express 13, 5064 (2005).

[12] R. J. Lang and A. Yariv, IEEE J. Quantum Electronics 24, 66 (1988); E. Ozbay, M. Bayindir, I. Bulu, and E. Cubukcu ibid. 38, 837 (2002).

[13] A. Badolato, K. Hennessy. M. Atatüre, J. Dreiser, E. Hu, P. M. Petroff, and A. Imamoğlu, Science 308 1158 (2005).

[14] P. Olivero et al., Advanced Materials, 17, 2427 (2005).

[15] D. N. Jamieson, et al., Appl. Phys. Lett. 86 202101 (2005).

[16] D. Redman, S. Brown, and S. C. Rand, J. Opt. Soc. Am. B 9, 768 (1992).

[17] A. İmamoğlu, H. Schmidt, G. Woods and M. Deutsch, Phys. Rev. Lett. 79, 1467 (1997); S. Rebić, A. S. Parkins, and S. M. Tan, Phys. Rev. A 65, 063804 (2002); K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup and H. J. Kimble, Nature (London) 436, 87 (2005).

[18] S. R. de Echaniz, A. D. Greentree, A. V. Durrant, D. M. Segal, J. P. Marangos, and J. A. Vaccaro, Phys. Rev. A 64, 013812 (2001); A. D. Greentree, A. R. Hamilton, and F. Green, Phys. Rev. B 70, 041305(R) (2004).