The QCD Phase Diagram at Non-zero Baryon and Isospin Chemical Potentials

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In heavy ion collision experiments as well as in neutron stars, both baryon and isospin chemical potentials are different from zero. In particular, the regime of small isospin chemical potential is phenomenologically important. Using a random matrix model, we find that the phase diagram at non-zero temperature and baryon chemical potential is greatly altered by an arbitrarily small isospin chemical potential: There are two first order phase transitions at low temperature, two critical endpoints, and two crossovers at high temperature. As a consequence, in the region of the phase diagram explored by RHIC experiments, there are two crossovers that separate the hadronic phase from the quark-gluon plasma phase at high temperature.

1. INTRODUCTION

Heavy ion collision experiments are important for our understanding of the strong interaction at nonzero temperature and density. In heavy ion collision experiments, both the baryon and the isospin densities are different from zero. The time between the formation of the fireball and its freezeout is so short that only the strong interactions play a significant role and baryon number as well as isospin are conserved. Therefore, it is phenomenologically worthwhile to study the effects of a nonzero isospin chemical potential, \( \mu_I \), on the QCD phase diagram at nonzero temperature, \( T \), and baryon chemical potential, \( \mu_B \). However, most studies at high temperature and nonzero density have been restricted to cases where either \( \mu_I \) or \( \mu_B \) is zero.

For \( \mu_B \neq 0 \) and \( \mu_I = 0 \), the phase diagram is very rich. At low \( T \) and high \( \mu_B \), the ground state is believed to be a color superconductor \[1\]. If \( T \) is increased and \( \mu_B \) is decreased, a first order phase transition separates the hadronic phase from the quark gluon plasma phase. If \( \mu_B \) is further decreased, this first order critical line stops in a critical endpoint. At lower \( \mu_B \), there is a crossover between the hadronic phase and the quark gluon plasma phase. These results are based on different models \[2,3\], as well as on exploratory lattice studies at low chemical potential \[4\].

In the case of \( \mu_B = 0 \) and \( \mu_I \neq 0 \), the fermion determinant is real and traditional lattice methods can be applied. In this case, the phase diagram is very rich as well. At low \( T \), an increase in \( \mu_I \) above the pion mass leads to a superfluid phase with a pion condensate. At low \( T \), the phase transition between the hadronic phase and the pion condensation phase is second order and has mean field critical exponents. If \( T \) is increased, this second order phase transition becomes first order. Therefore there is a tricritical point in the phase diagram. These results were found using both lattice simulations \[5\] and effective theories \[6\]. At high \( T \) and low \( \mu_I \), a crossover separates the hadronic phase from the quark gluon plasma phase. There also might be a critical endpoint and a first order phase transition at high \( T \) when \( \mu_I \) is increased \[5\].

We use a Random Matrix model as a schematic model for QCD to study the phase diagram at
nonzero \( T, \mu_B, \) and \( \mu_I \) \([7]\). This model has been previously used to study QCD at \( \mu_B \neq 0 \) and \( \mu_I = 0 \) \([8]\). We then analyze possible consequences for heavy ion collision experiments.

2. RANDOM MATRIX MODEL

Random matrix models were introduced in QCD to describe the correlations of low eigenvalues of the Dirac operator \([5]\). It was shown that for large matrices these models are equivalent to the mass term of the chiral Lagrangian that is uniquely determined by the symmetry of QCD \([9]\). Therefore, in the chiral limit, Random Matrix Theory provides an exact analytical description of the low-lying Dirac spectrum. The idea is to replace the matrix elements of the Dirac operator by Gaussian random variables subject only to the global symmetries of the QCD partition function. Since we have two independent chemical potentials, one for each quark flavor, the chiral condensates are not necessarily equal. Furthermore, at high enough \( \mu_I \), we expect a pion condensate. We thus assume that

\[
A = \begin{pmatrix}
\sigma_1 & \rho \\
\rho & \sigma_2
\end{pmatrix}.
\]

In this parameterization, the chiral condensates are given by \( \langle \bar{u}u \rangle = G^2(\sigma_1 - m_1) \), \( \langle \bar{d}d \rangle = G^2(\sigma_2 - m_2) \), and the pion condensate by \( \frac{4}{3}(\langle \bar{u}\gamma_5 u \rangle - \langle \bar{d}\gamma_5 d \rangle) = G^2 \rho \). We thus get an effective potential that can be studied in the usual way.

We limit ourselves to the case \( m_1 = m_2 = m \). We are particularly interested in the phase diagram in the \( \mu_B-T \) plane at \( \mu_I \) small enough so that the superfluid phase is never reached, because it corresponds to the conditions of heavy ion collision experiments. The phase diagram in the \( \mu_B-T \)-plane for zero \( \mu_I \) has been studied in \([8]\). In the chiral limit, the chiral restoration transition extends as a second order line from the \( \mu_B = 0 \) axis, changes order at a tricritical point, and intersects the \( T = 0 \) axis as a line of first order transition. For nonzero quark mass, the first order transition ends in a critical point, and the second order transition becomes a crossover. Figure \([4]\) shows the phase diagram in the \( \mu_B-T \)-plane at finite quark mass \( mG = 0.1 \) for zero isospin chemical potential, \( \mu_I = 0 \), and for \( \mu_I G = 0.1 \). We observe that the first order curve splits into two first order curves that are separated by \( 2\mu_I G \). This can be understood as follows. Below the threshold for pion condensation, the free energy separates into a sum over the two flavors. For \( \mu_I = 0 \), the chiral phase transition lines for both flavors coincide. A nonzero \( \mu_I \) breaks the flavor symmetry, and the first order transition lines for the two flavors split and shift in opposite directions. The temperature of the critical endpoints is not affected by \( \mu_I \).
3. CONCLUSIONS

We have used a Random Matrix model for QCD at nonzero $T$, $\mu_B$, and $\mu_I$. We have found that in the region of high $T$ and small $\mu_B$, an arbitrarily small $\mu_I$ greatly alters the phase diagram in the $\mu_B$-$T$ plane: There are two crossovers, two critical endpoints, and two first order phase transition lines that separate the hadronic phase from the quark gluon plasma phase \[7\]. This could have important consequences for heavy ion collision experiments, since they are done at nonzero $\mu_B$ and $\mu_I$, and will provide an important test for our results.

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