WHERE IS TOP?

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ABSTRACT

Possibilities are discussed for determining the top quark mass $m_t$ from observations on the decay processes for top-antitop pairs produced in antiproton-proton collisions at Tevatron energies, assuming that the $t \rightarrow bW^+$ decay channel is dominant and much faster than hadronization. The final states $t\bar{t} \rightarrow b\bar{b}\mu^\pm e^\mp$ provide the most striking signal, with little background, but they are rare ($\approx 2/81$). If all three candidate events prove to be from $t\bar{t}$, an estimate follows for $P(m_t \mid \text{rate})$, the probability distribution for $m_t$. The one reported configuration allows an independent estimate for $P(m_t \mid \mu^\pm e^\mp 2\text{jets})$. These two distributions are compatible and can be combined to give an $m_t$ estimate of about 122 GeV. Decay events “1 energetic lepton(l) + 4jets” should appear twelve times as often as “$\mu^\pm e^\mp 2\text{jets}$” events and can be analysed to give estimates for $P(m_t \mid l 4\text{jets})$; this is illustrated for a fictitious event. There may be background from non-top events but suitable cuts on the data and our analysis procedure together reduce this to a low level. The rate observed for these events does not appear to be as large as this factor 12. Identification of either or both of the ($b\bar{b}$) jets would be a great step forward, separating out the “$\mu^\pm e^\mp b\bar{b}$” and “$l b\bar{b} 2\text{jets}$” events of importance with negligible background. We advocate an energetic approach to the analysis of individual events (whether 2, 1 or no ($b$ or $\bar{b}$) jets identified) on an event by event basis, with the hope of finding a subgroup of events with a common mass estimate.

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1This paper was given by the first author (R.H.D.) on 19 July 1992 at the International School of Subnuclear Physics at ERICE. Its text has been updated to the end of 1992.
INTRODUCTION

1.1 Existence of the top quark

There is little doubt today about the existence of a top quark $t$, the partner to the well-known and much-studied bottom quark $b$. The Standard Model, with SU(2) x U(1) for the Electroweak Interactions has had remarkable success, many of its parameters now being known to high accuracy. In particular, the weak-isospin component $T^w_k$ for the $b$ quark has now been determined empirically in a very direct way, from measurements of the forward-backward asymmetry in the process $e^+e^- \rightarrow b\bar{b}$ and of the decay width $\Gamma(Z^0 \rightarrow b\bar{b})$. From these data, Kane and Kolda [1] deduced the following $T^w_k$ values for the $b$-quarks.

$$T^w_k(b_L) = -0.49^{+0.05}_{-0.02}, \quad T^w_k(b_R) = 0.00^{+0.10}_{-0.08},$$

which are in accord with the Standard Model assignments of the $b$ quark to the third quark-lepton family, for which the leptonic member $\tau^+$ is already known, as shown in Table 1.

Table 1. The three families in the SU(2) X U(1) Standard Model

| $T^w_k$ | $Y^w_k$ | Quark and Lepton states for each family |
|--------|--------|---------------------------------------|
| $+\frac{1}{2}$ | $+\frac{1}{3}$ | $U_L$, $c_L$, $(t_L)$ |
| $-\frac{1}{2}$ | $+\frac{1}{3}$ | $d_L$, $s_L$, $b_L$ |
| 0 | $\{+\frac{4}{3}, -\frac{2}{3}\}$ | $\{u_R, d_R\}$, $\{c_R, s_R\}$, $(t_R), (b_R)$ |
| $+\frac{1}{2}$ | 1 | $\bar{\nu}_e$, $\bar{\nu}_\mu$, $(\bar{\nu}_\tau)$ |
| $-\frac{1}{2}$ | 1 | $e_L$, $\mu_L$, $\tau_L$ |
| 0 | 2 | $e_R$, $\mu_R$, $\tau_R$ |

The fact that the predictions of the (spontaneously broken) SU(2) x U(1) symmetry have been so successful numerically provides a powerful argument that the third (quark-lepton) family must be complete. The absence of a $T^w_k = +\frac{1}{2}$ partner to the $b_L$ quark would be such a gross violation of this symmetry that it would no longer be possible to understand the detailed and widespread agreement of the data with the predictions based on this electroweak symmetry. In short, there must exist a $T^w_k = +\frac{1}{2}$ partner to the bottom quark $b_L$, and it has been natural for this partner to be named the top quark $t$.

Empirically, little else is known about the top quark, although its interactions are prescribed in form and magnitude from its place in the third quark-lepton family. However, we do know that its mass $m_t$ is very much greater than the mass of the $b$ quark, $m_b \approx 5$ GeV. A firm limit on $m_t$ is provided by the fact that top-antitop pair creation is not observed in $e^+e^-$ annihilation in the energy region of $Z^0$ excitation; in other words, the threshold for $t \bar{t}$ production must be above $M_Z$, giving the lower limit

$$m_t \gtrsim M_Z/2 \approx 46 \text{ GeV}.$$
Indeed, the top mass is now believed to be above 91 GeV [2]. If top were lighter than this, $t\bar{t}$ pair production would have been observed already at the Tevatron, where proton-antiproton interactions are studied at c.m. energy 1800 GeV, an energy far more than adequate for any reasonable value for $m_t$. At the Tevatron, the limiting factor for top mass determination is the rate of events, rather than the available energy. We shall discuss this further in Sec.2.

The purpose of current top quark research is not just to demonstrate directly the existence of the top quark (which we believe already) nor to check that its interactions are in accord with SU(2) x U(1) symmetry, although the latter studies will provide a very fruitful field for research later on. The top quark will not lose its polarization by hadronization, as do the other heavy quarks, charm and bottom (c.f. Sec.1.2), so that it will be possible to test all the detailed spin dependences of its interactions [3-5]. It may even turn out to be possible to use the top quark decay as a spin analyser for heavy particle and/or high energy processes which happen to give rise to top quarks. No, the paramount purpose of present top research is to determine the mass $m_t$ of the top quark; how this measurement may be achieved will be discussed in Sec.3 below. Only after this mass has been determined can we move on with the Standard Model, e.g. with the determination of the Higgs particle mass and with the testing of more extensive symmetry models which contain SU(2) x U(1) as a subgroup. Only then will we know what kind of accelerator we shall need for all these detailed studies.

1.2 The top quark lifetime

When $m_t$ exceeds $(M_W + m_b) \approx 85 GeV$, top quark physics becomes qualitatively different from the physics we have become accustomed to for the $c$ and $b$ quarks, as was first pointed out by Bigi [6]. The dominant decay mode for the top quark is then the two-body mode

$$(a) \quad t \to W^+ + b, \quad (b) \quad \bar{t} \to W^- + \bar{b}$$

As the mass $m_t$ increases, this decay process becomes faster than hadronization. The top decay lifetime calculated for this mode [7] is plotted as function of $m_t$ on Fig.1. For $m_t$ appreciably lower than $(M_W + m_b)$, the W-boson is virtual and its leptonic decay mode generates the overall mode $t \to l^+\nu_b\bar{b}$, whose partial lifetime falls like $m_t^{-5}$. In the transition region, as $m_t$ approaches and passes $(M_W + m_b)$, the decay lifetime falls even more rapidly with $m_t$. For $m_t$ well above this threshold, as holds for the physical situation, the decay lifetime falls more slowly, ultimately like $m_t^{-3}$. The calculated partial width is given as function of $m_t$ in Table 2.

Table 2. Partial decay width for $t \to bW^+$, as function of $m_t$[29].

| $m_t (GeV)$ | 100 | 120 | 140 | 160 | 180 | 200 |
|------------|-----|-----|-----|-----|-----|-----|
| $\Gamma(t \to bW^+)(MeV)$ | 88  | 298 | 612 | 1033| 1572| 2242|

[By way of contrast, we comment briefly on the other heavy quarks $Q = c$ and $b$. Their decay lifetimes are of order $10^{-12}$ sec. In $c$ and $b$ jets, a polarized quark $Q$ forms a meson $(Q\bar{q})$ where $q$ denotes a light quark $q = (u, d, s)$, by picking up a light antiquark $\bar{q}$ from the vacuum. If this meson is in the $1S_0$ ground state, all polarization carried by $Q$]
is lost, since this state is spherically symmetric and cannot carry spin information. If the meson state \((Q\bar{q})\) which is formed has non-zero spin, e.g. the \(3S_1\) state or orbitally-excited states like \(3P_2\), it undergoes fast hadronic or electromagnetic transitions (since these conserve parity, the angular distributions do not depend on any initial polarization, although they can transfer polarization from the initial to the final state) until it reaches the \(1S_0\) state, which is the lowest state for the \((Q\bar{q})\) system. Thus, all of the polarization information carried initially by the quark \(Q\) is lost, for the cases \(Q = (c, b)\)[8].

## 2 TOP MASS ESTIMATES FROM EMPIRICAL DATA

### 2.1 Virtual top: radiative corrections

It was first pointed out by Veltman [9] that, although the \(e^+e^-\) energy at LEP is too low for the production of real top-antitop pairs, the existence of the top quark can still have a substantial effect on the predictions of the Standard Model through the “radiative corrections” generated by virtual top-quark loops and exchanges. Since the Standard Model is a renormalizable theory, these corrections are computable, at least in a perturbative approximation. Now that LEP experiments have measured with high accuracy many quantities related with the electroweak interactions, these measurements can be compared with the corrected theoretical predictions in order to draw some conclusions concerning the top quark and any other particles of high mass. Some of these measurements are the masses \(M_Z\) and \(M_W\), the total width \(\Gamma_Z\) and some partial widths for \(Z^0\) decay, and the forward-backward asymmetries for \(e^+e^- \rightarrow b\bar{b}\) and \(e^+e^- \rightarrow\) leptons in an energy range covering the \(Z^0\) peak, which arise from \(\gamma-Z^0\) interference. In the minimal Standard Model, there are several other parameters also relevant, namely \(M_H\), the Higgs particle mass, and \(\alpha_S(M_Z^2)\), the QCD coupling strength evaluated at the \(Z^0\) mass. The latter can be deduced with fair accuracy from purely hadronic processes as well as from electroweak studies in the \(Z^0\) mass range. Given the recent LEP data and the theoretically computed expressions, it is then possible to lay out on an \((m_t, M_H)\) plane, the regions consistent with these data. Quite a number of analyses have been carried out along these lines recently [10,11]. With the LEP data updated to July 1992, Ellis et al. [10] have given the value

\[
\text{all data } |_{M_H \text{ free}}: \quad m_t = 124(27) GeV, \quad (2.1)
\]

using \(\alpha_S(M_Z^2) = 0.118(8)\). Their analysis has some sensitivity to \(M_H\); adopting by choice a rather low value for \(M_H\), their analysis gives

\[
M_H = M_Z : m_t = 132(25) GeV. \quad (2.2)
\]

requiring a small increase for the optimum \(m_t\).

More elaborate models generally have more free parameters. The Minimal Supersymmetric Standard Model (MSSM) has aroused much interest recently, owing to its success in extrapolating the three coupling strengths, \(\alpha_i(\mu^2)\), at scale \(\mu^2 = \text{(momentum transfer)}^2\) and with \(i = (1, 2, 3)\) appropriate to the strong and electroweak sectors of the \(SU(3) \times SU(2) \times U(1)\) symmetry contained within a Minimal \(SU(5)\) SUSY, back to a common value at a GUT scale \(\mu\) of order \(10^{16} \text{ GeV}\) [12], a test that an earlier attempt based on \(SU(5) \supset Su
SU(3) x SU(2) x U(1) had failed [13]. Besides \( M_H \) and \( m_t \), this MSSM introduces five new parameters, \( m_A \) and \( \tan\beta \) for the Higgs sector, \( (m_{\tilde{g}}, m_0) \) the gluino mass and a common sfermion mass, respectively, and \( \Lambda_t \), a supersymmetry-breaking parameter. The discussion of the allowed regions for all of these parameters is naturally rather complicated. Ellis et al. [14] have discussed these dependences only for \( m_t = 130 \) GeV, mainly to illustrate how an empirical value for \( m_t \) would give much-needed information on the bounds which constrain the MSSM parameters. Most probably, given the number of new parameters introduced, the limits placed on \( m_t \) in the MSSM will be much less restrictive than those for the minimal SM without supersymmetry.

### 2.2 Total cross section for top-antitop production

At the Tevatron energy of 900 GeV for proton and for antiproton, the (900+900) GeV \( \bar{p}p \) interactions are rather like (300+300) GeV interactions between a valence quark \( q \) and a valence antiquark \( \bar{q} \). Creating a \( t\bar{t} \) pair, each with (say) \( m_t = 150 \) GeV, through the interaction process

\[ \bar{q} + q \rightarrow \bar{t} + t \]  

(2.3)

absorbs half of the initial energy, not leaving much energy for the creation of more particles. The energies of the residual \( (qq) \) and \( (\bar{q}q) \) quarks from the proton and antiproton, respectively, lead through hadronization to particles which mostly go down the beam pipe. Thus, after the production process,

\[ \bar{p} + p \rightarrow \bar{t} + t + X, \]  

(2.4)

the \( t \) and \( \bar{t} \) quarks decay according to (1.4 a,b), giving

\[ \bar{p} + p \rightarrow \bar{b} + W^- + b + W^+ + X, \]  

(2.5)

where \( X \) consists of (i) \( X_{\text{inv}} \) consisting of hadrons which go down the beam pipe, and (ii) \( X_{\text{vis}} \), the other hadrons recorded by the detector. The \( W^\pm \) bosons then decay, with lifetime \( 3.1 \times 10^{-25} \) s. Their simplest and most visible decay processes are

\[ \begin{align*}
(a) \quad & W^+ \rightarrow l^+ + \nu_l , \\
(b) \quad & W^- \rightarrow l^- + \bar{\nu}_l ,
\end{align*} \]  

(2.6)

for the three leptons \( l = e, \mu \) and \( \tau \).

The most striking events are those where the \( W^+ \) and \( W^- \) decays lead to two charged leptons from different families without any \( X_{\text{vis}} \), a typical final state being

\[ \bar{b} + b + \mu^\pm + e^\mp + (\nu_\mu + \bar{\nu}_e + X_{\text{inv}}), \]  

(2.7)

where the \( \bar{b} \) and \( b \) quarks hadronize to give corresponding jets \( j(\bar{b}) \) and \( j(b) \). There may also be some secondary jets, emitted from the initial quarks or by the quarks heading for the beam-pipe, or arising from the development of the \( b \) and \( \bar{b} \) jets, but these will generally have
relatively low energy, so that these \((\mu^\pm e^\mp \bar{b} \bar{b})\) final states may be expected to have rather simple structure. Another attractive feature of these \((\mu^\pm e^\mp \bar{b} \bar{b})\) events is that they may be expected to be rather free of background, since \(\mu\) and \(e\) belong to different families. There is no particle \(Y\) known whose decay leads to the \(\bar{\mu}e\) or \(\mu\bar{e}\) configuration without corresponding neutrinos \(\nu_e\) (or \(\bar{\nu}_e\)) and \(\bar{\nu}_\mu\) (or \(\nu_\mu\)). Indeed, lepton conservation is believed to hold separately for muonic and electronic lepton numbers: strong limits have been placed on any violation of this conservation law, through searches for reactions of the type \(\mu^- p \rightarrow e^- p\) and for decays of the type \(Y \rightarrow \mu^\pm e^\mp X\), especially for \(Y^0 = K^0\) and \(Y^+ = K^+\).

The simplest \(\bar{p}p\) process leading to final states with \((\mu^\pm e^\mp)\) is \(W\)-pair production

\[
\bar{p} + p \rightarrow W^- + W^+ + n\ \text{jets}, \quad (2.8)
\]

followed by the decays (2.6). This process involves two weak vertices whereas the process (2.4) has none; provided that the energy available is reasonably large compared with \(2(M_W + m_b)\), the process (2.8) with \(n = 2\) may be expected to have a rate smaller by two factors of \((\alpha_{em}/2\sin^2\theta_W)\) than that for (2.5), if the \(n = 2\) jets were \(b\)-jets. However, the process (2.8) will be in fact dominated by events with light quark jets in large numbers, and it is difficult to estimate how much background (2.8) will produce for the \(\bar{t}t\) events (2.4) leading to final state (2.5). It is a question of what fraction of these non-\(\bar{t}t\) events (2.8) having \(n = 2\) light quarks and surviving the transverse momentum cuts (cf. Secs. 3 and 4) can be adequately fitted to the interpretation (2.7). The only further remark we can make is that, if a microvertex detector is used, so that it is possible to add the requirement that two of the jets in (2.8) be identified as \(b\) jets, the background rate from (2.8) would be of order \(\alpha_{em}/(2\alpha_s\sin^2\theta_W))^2 \approx 10^{-2}\) times the signal for (2.7), following the top-pair production (2.4). In our further discussion here and in Sec. 3, we shall neglect background to the process (2.5) arising from the direct production of \(W\) pairs. However, the validity of this neglect is under discussion at present.

The cross section for \(\bar{t}t\) production in \(\bar{p}p\) annihilations at the Tevatron energy of 900 GeV through reaction (2.4) has been calculated by Eichten [15], as function of \(m_t\). At Tevatron energies and for such large \(m_t\) values (\(\gtrsim 120\) GeV) as we are led to consider here, \(\bar{t}t\) production is dominantly due to the quark-antiquark process (2.3); production by gluon-gluon processes is small in comparison.

The \(W^\pm\) bosons have hadronic modes, in addition to the leptonic modes (2.6). These are due to the \(W\)-coupling with the quark weak-isospin charged current. Since we are interested here only in total rate, we limit our discussion to the couplings

\[
W^+ \rightarrow (a)\ u + \bar{d}, \quad (b)\ c + \bar{s}, \quad (2.9)
\]

given by diagonal terms of the Cabibbo-Kobayashi-Maskawa matrix. Each of these couplings leads to two final quark jets. The third coupling is \(W^+(t + \bar{b})\) whose threshold lies far above \(M_W\), so that it contributes only to virtual processes, such as the radiative corrections mentioned above.

Recognizing colour, there are 6 couplings in all, each with the same strength as each \(W^+ \rightarrow t^+ \nu_t\). Empirically, the branching ratio for each \((\nu_t)\) final state is 10.5(9)\%, which agrees with the naive expectation of 1/9, from the channels just enumerated. If both \(W^+\) and \(W^-\) in (2.5) decay hadronically, the final state will have 6 energetic quark jets, a
configuration which it may be difficult to disentangle. If only one \( W \) decays hadronically, the final state will generally have one energetic lepton and four quark jets. The analysis of this final state is generally possible and we shall discuss this in Sec 3. The dilepton events have been described in part just above. The decays (2.6) include the possibility of \( \tau^\pm \) emission; this is far more difficult to deal with experimentally than is \( e^\pm \) or \( \mu^\pm \). For this reason, we shall not refer explicitly to the possibility of \( \tau^\pm \) emission again. On the other hand, final electrons or muons can be easily recognized and distinguished. Among the dilepton final states, the \((e^-e^+)(\mu^-\mu^+)\) cases need special attention and much caution, since there are many other mechanisms which can give rise to electron or muon pairs, one example being the production and decay of a \( Z^0 \) boson, and which will still require special cuts for their exclusion. In fact, no candidate event of this type has yet been reported, although such events \((e^-e^+)(\mu^-\mu^+)\) must have the same rate as the \((\mu^\pm e^\mp)\) events.

### 2.3 Top mass estimate from production rate

In this section, we confine attention to the \((\mu^\pm e^\mp)\) events. The expected number \( N_{\mu e} \) of these events is plotted as function of \( m_t \) in Fig.2 for an integrated luminosity \( IL = 30 pb^{-1} \), based on the estimates by Crane [16] who used Eichten’s calculation of the cross section for top-antitop pair production [15] and took due account of the efficiency \( \kappa \) for the CDF detector as a function of the event location and configuration. For mass \( m_t \), we have

\[
N_{\mu e} = \kappa \cdot IL \cdot \sigma_{\mu e}(m_t) , \tag{2.10}
\]

where \( \sigma_{\mu e} \) denotes the cross section for \((\mu^\pm e^\mp)\) events in \( \bar{p}p \) collisions at 1800 GeV c.m. energy. The probability of producing \( n \) such events, given \( m_t \), is

\[
P(n \mid m_t) = \frac{N_{\mu e}(m_t)^n}{n!} \exp(-N_{\mu e}(m_t)) \tag{2.11}
\]

Given \( IL, \sigma_{\mu e}(m_t) \) and the observation of \( n \) events, the Bayesian probability distribution for \( m_t \) is then

\[
P(m_t \mid n) = \frac{N_{\mu e}(m_t)^n}{n!} \exp(-N_{\mu e}(m_t))/\{ \int dm \frac{N_{\mu e}(m)^n}{n!} \exp(-N_{\mu e}(m)) \} , \tag{2.12}
\]

where the integral is taken over all possible values \( m \) allowed for the top quark \( t \) by all the conservation laws. This probability may be reduced to the form

\[
P(m_t \mid n) = c(n)(\kappa \cdot IL \cdot \sigma_{\mu e}(m_t))^n \cdot \exp(-\kappa \cdot IL \cdot \sigma_{\mu e}(m_t)) , \tag{2.13}
\]

\( c(n) \) being an \( n \)-dependent normalization factor. This distribution peaks at the \( m_t \) value for which

\[
\sigma_{\mu e}(m_t) = n/\kappa \cdot IL . \tag{2.14}
\]

One good \((\mu^-e^+)\) candidate event has already been published by the CDF collaboration [2] and has been discussed [17] in some detail (cf. Sec. 3 below). A second \((\mu e)\) candidate was shown by the CDF collaboration in their report given at the November 1992 Chicago
Meeting of the Division of Particles and Fields of the American Physical Society, although no measurement details were released. It was well known at that meeting that the DO collaboration also had their first (µe) candidate. Although the integrated luminosities $IL$ are not known to us precisely, a value of about 20 $pb^{-1}$ for CDF (including $IL=4.7 pb^{-1}$ from their 1989 paper) and 10 $pb^{-1}$ for DO would appear plausible estimates, at least of the right order of magnitude. The mean detector efficiency can be deduced from a comparison of Crane’s rates with Eichten’s total $t\bar{t}$ production cross sections.

On the assumption that these three (µe) candidates do stem from top-antitop production, and that the integrated luminosity up to November 1992 was about 30 $pb^{-1}$, the probability distribution for $m_t$ is shown on Fig. 3. Its peak is at 120 GeV, the one-deviation limits being 109 and 135 GeV. Since the curve for $N_{\mu e}(m_t)$ shown in Fig.2 is a steeply falling function of $m_t$, the peak value thus determined for $m_t$ is not strongly dependent on our estimate for $IL$, nor on the number of µe events. For $(IL,n) = (40pb^{-1},3)$, the peak value is $m_t = 127.5$; for $(30pb^{-1},4)$, it is at 114 GeV, and for $(30pb^{-1},2)$, it is at 129 GeV.

3 ANALYSIS OF DILEPTON EVENTS

3.1 Kinematics of top decay sequence $t \rightarrow bW^+ \rightarrow b\bar{l}^+ \nu_l$

Consider first the kinematics of top decay to $bW^+$, followed by $W^+$ decay to $\bar{l}\nu_l$, in any frame. Energy-momentum conservation gives

$$t = b + \bar{l} + \nu_l.$$  \hspace{1cm} (3.1)

where $a$ denotes the energy-momentum four-vector of the particle $a$. Since $\bar{l}$ and $\nu_l$ are decay products of $W$, we have

$$M_{W}^2 = (\bar{l} + \nu_l)^2 = (t - b)^2,$$  \hspace{1cm} (3.2)

and, since the neutrino has zero mass,

$$0 = (t - b - \bar{l})^2 = (t - b)^2 - 2\bar{l}(t + b) + \bar{l}^2.$$  \hspace{1cm} (3.3a)

$$= (t - b)^2 - 2\bar{l}(t + b) + \bar{l}^2.$$  \hspace{1cm} (3.3b)

where $a.b$ denotes the scalar product of the four-vectors $a$ and $b$ and $a^2 = a.a$. For $l = e$ or $\mu$, we can neglect the lepton mass, i.e. $l^2 = m_l^2 = 0$. Using (3.2) in (3.3b), we obtain the result

$$\bar{l}.t = \bar{l}.b + M_W^2/2.$$  \hspace{1cm} (3.4)

Evaluating the r.h.s. in the lab. frame and l.h.s. in the top rest-frame, we deduce that

$$E_{lt} = (\bar{l}.b + M_W^2/2)/m_t,$$  \hspace{1cm} (3.5)

giving $E_{lt}$, the lepton energy in the top rest-frame, in terms of lab. measurements for $\bar{l}$ and $b$.

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2This event was shown at a Fermilab seminar in February 1993, but without full details.
It is useful to derive here an inequality for $m_t$. From (3.1) and (3.2), we have

\[
m_t^2 = (b + \bar{l} + \nu t)^2 = m_b^2 + M_W^2 + 2b\bar{l} + 2b\nu t
\]  

(3.6a)

Evaluate the invariant products $b\bar{l}$ and $b\nu t$ in the $W$ rest frame, where $E_t = l$ and $E_{\nu} = \nu$ are each $M_W/2$, and the momenta $\bar{l}$ and $\nu t$ are opposite. Denoting by $\theta$ the angle between $\bar{l}$ and $\bar{b}$ in this frame, their product takes the form

\[
b\bar{l} b\nu t = (E_b - b\cos\theta)(E_b + b\cos\theta)(M_W/2)^2
\]  

(3.7a)

\[
= (m_b^2 + b^2\sin^2\theta)(M_W^2/4)
\]  

(3.7b)

\[
\geq m_b^2 M_W^2/4.
\]  

(3.7c)

Using this inequality for $b\nu t$ in Eq.(3.6b), leads to the inequality

\[
m_t^2 \geq m_b^2 + M_W^2 + 2b\bar{l} + m_b^2 M_W^2/2b\bar{l}
\]  

(3.8a)

\[
= (m_b^2 + 2b\bar{l})(M_W^2 + 2b\bar{l})/2b\bar{l}
\]  

(3.8b)

as given in the Appendix of ref.[5].

### 3.2 A geometric construction

Now consider the kinematics in the lab. frame, starting with the 3-momenta $\vec{b}$ and $\vec{l}$. The top momentum $\vec{t}$ is constrained by two relations,

\[
(\vec{t} - \vec{b})^2 = (E - E_b)^2 - M_W^2
\]  

(3.9)

from eq.(3.2) and

\[
(\vec{l} - \vec{b} - \vec{l})^2 = (E - E_b - E_t)^2
\]  

(3.10)

from eq.(3.3a), where $E$ has not been constrained to the value $E_t$ because the value of $m_t$ is still left open. For given $E$, eqs. (3.9) and (3.10) constrain $\vec{t}$ to lie on the intersection of two spheres, one centred on the point $\vec{b}$ and the other centred on ($\vec{b} + \vec{l}$). Their intersection is a circle on a plane perpendicular to the line $BL$ joining these two centres and centred on this line. As $E$ varies, the centre of this circle moves along the line $BL$ at a rate linear in $E$, and the square of its radius also increases linearly with $E$; in short, as $E$ varies, this circle traces out a paraboloid with axis $BL$, as described in refs. [8,17]. All points $\vec{t}$ satisfying the constraints (3.2) and (3.3a) for given $\vec{b}$ and $\vec{l}$ for all possible $m_t$ values, lie on this paraboloid.

The points of interest to us are those for a definite $m_t$, still to be determined. These points lie on an ellipse formed by the intersection of the paraboloid by a plane whose normal lies in the plane $OBL$ and makes an angle $\sigma$ with the axis $BL$, where

\[
tan\sigma = b_1/(E_b - b_3),
\]  

(3.11)

is independent of $m_t$, and $b_1$ and $b_3$ are defined on Fig.4. For a specified $m_t$, the possible vectors $\vec{t}$ are given by $\vec{O}T$, as $T$ moves around this ellipse, and the vector $\vec{L}\vec{T}$ gives the corresponding neutrino momentum $\nu T$. It is apparent that the top quark energy $E_t = \sqrt{(m_t^2 + \vec{t}^2)}$ must lie between two limits $E_{\min}(m_t) \leq E_t(m_t) \leq E_{\max}(m_t)$.

As $m_t$ increases, the plane PTQ moves upwards, with a constant normal since the angle $\sigma$ (given by (3.11)) does not depend on $m_t$. The ellipse retains the same eccentricity but
increases in linear dimension. As $m_t$ decreases, the plane moves downward and the ellipse shrinks until for a limiting value $m_t = m_*$ the plane becomes tangent to the paraboloid and the ellipse is reduced to a point. For $m_t < m_*$, there is no solution for $t$. This value $m_*$ is the lowest limit for an $m_t$ consistent with the momenta $(\vec{b}, \vec{l})$; $m_*$ is, of course, the value given by the square root of the expression (3.8b). The projection of the ellipse for $m_t$ on a plane perpendicular to the vector $\vec{l}$ is a circle, and the centre of the ellipse projects onto the centre of the circle. As $m_t$ varies, the centre of the ellipse moves parallel to the vector $BL = \vec{L}$ and is distant from it by amount $(M_{W}^{2}/2E_{l})\tan\sigma$ in the plane of $\vec{b}$ and $\vec{l}$.

It is worth noting here that any configuration $(\vec{b}, \vec{l})$ can be fitted by a sufficiently large $m_t$. However, a very large $m_t$ means that LT is very large, but this is the neutrino momentum $\nu_{\ell}$, of which we have no direct knowledge. It means also that OT must be correspondingly large, and this is the top momentum $t$. These two very large momenta would be compensating each other, differing by the observed momentum $(\vec{b} + \vec{L})$, and this is not generally a plausible interpretation of the top decay event observed.

For top-antitop production, with lepton emission in both decays, two paraboloids are to be constructed, one from $(\vec{b}, \vec{l})$ for $t$ and one from $(\vec{b}, \vec{l})$ for $\bar{t}$, both in the same laboratory frame. One dilepton event has already been described in detail in the literature [17] from the CDF experiment in 1988 as a possible $t$ production event. We shall refer to it as CDF-1. Its measurements are given here in Table 3, the $z$-axis being along the beam. There is a third lepton in CDF-1, a $\mu^+$ with quite high energy but with rather small transverse energy. It allows interpretation as a secondary muon, emitted from $\bar{B}$ meson decay in the $\bar{b}$-jet following the $\bar{t} \to \bar{b}W^-$ decay of the hypothetical antitop quark produced in this $p\bar{p}$ interaction. It has the right charge sign and a low transverse energy compatible with the sequence $\bar{b} \to \bar{c}W^+$, $W^+ \to \mu^+\nu_\mu$, and it travels in the same general direction as an energetic hadronic jet. These features strongly suggest that the $\mu^+$ and the hadronic jet are to be taken together, as the components of a $\bar{b}$ jet. We expect that the other jet is a $b$ quark jet but there is no evidence to demonstrate this.

### 3.3 Top mass estimation from dileptonic decays $\bar{t} \to \mu^+\epsilon^- + 2 jets$

If we now assume that CDF-1 represents top-antitop production, with the assignments given in Table 1, the two paraboloids then constructed are those shown in Fig.5, projected onto the $(y,z)$ plane. The ellipses representing $t$ decay are clearly seen; the $\bar{t}$-ellipses happen to be seen edge-on in this projection, since the momentum vectors $\vec{b}$ and $\vec{l}$ almost lie in this plane, their components $b_x$ and $l_x$ being small. These two sets of ellipses are shown for $m_t$ ranging from 115 to 305 GeV, in 10 GeV intervals. The quark-antiquark interaction which gave rise to this event was strongly asymmetric, since the two paraboloids lie almost entirely within the same hemisphere.

It had been assumed generally, we think (but see ref.[19]), that it would not be possible to analyse such an event in terms of top-antitop pair creation since the final state would then include at least two energetic neutrinos, $\nu_e$ and $\bar{\nu}_\mu$. [CDF-1 actually has a third neutrino, of type $\nu_\mu$ and with unknown energy, from the decay mentioned above. It is assumed here that this $\nu_\mu$ energy is small although its associated $\mu^+$ has quite high energy. This assumption needs checking for internal consistency]. Of course, we do not know whether this $\bar{t}$ assumption for event CDF-1 is correct, but if it is, then it is possible to deduce something probabilistic about $m_t$.

Table 3. Input data for two $\bar{t}$ candidates, one real (CDF-1) and the other fictitious ($F_{\mu e}$),
both of the form \((\mu^+e^\mp b\bar{b})\).

| Event CDF-1 | Event \(F_{\mu e}\) |
|--------------|-------------------|
| \(e^+\)      | \(e^-\)          |
| \(\text{jet}(b)\) | \(\text{jet}(\bar{b})\) |
| \(e^-\)      | \(\mu^+\)        |
| \(\text{jet}(\bar{b})\) | \(\text{jet}(b)\) |

| \(p_x\) | 22.5 | 66.3 | 8.0 | -47.7 |
| \(p_y\) | -5.45 | 86.4 | 20.0 | 9.5 |
| \(p_z\) | -21.2 | 18.7 | 0.6 | -0.4 |

*includes a low energy \(\mu^+\)

There is one further input. In the simplest parton picture of the \(p\bar{p}\) collision, [20], each of the partons \(q\) and \(\bar{q}\) has no transverse component of momentum. It follows that the net transverse momentum of the final \(t\) and \(\bar{t}\) quarks must be zero, so that we have the two further constraints:

\[
\bar{t}_T + t_T = 0. \tag{3.12}
\]

where the suffix \(T\) denotes the component of the vector transverse to the incident proton beam. What is the consequence of this condition? We can see this most simply by rotating the paraboloid by \(180^\circ\) about the \(z\)-axis. If we view the two resulting paraboloids along the \(z\)-axis, i.e. project them on to the \(XY\) plane, the constraint equations (3.12) for an assumed \(m_t\) correspond to a crossing of the two ellipses for this \(m_t\), since (3.12) then reads

\[
-\bar{t}_T = t_T. \tag{3.13}
\]

These crossings are shown for a number of \(m_t\) values on Fig.6, for the event CDF-1. For \(m_t < 110.2\ GeV\), the ellipses do not cross (for \(m_t < 100\ GeV\), they do not even exist) nor do they cross for \(m_t > 410\ GeV\) (which is an unreasonably large \(m_t\), as is explained below). For \(m_t\) between these limits, there are two or four crossing points, each giving a solution for \(t\) and \(\bar{t}\) (and hence for \(\nu_e\) and \(\bar{\nu}_\mu\) separately). The next step is to assign a probability to each configuration.

From the \(z\)-components of a solution for the momenta \((t, \bar{t})\), the momenta \(xP\) and \(\bar{x}P\) of the parton and antiparton, where \(P\) denotes the total momentum of \(p\) and \(\bar{p}\) in the Lab. frame, can be deduced, giving

\[
x = (E_t - E_\bar{t} + (t_L + \bar{t}_L))/2P, \tag{3.14a}
\]

and

\[
\bar{x} = (E_t - E_\bar{t} - (t_L + \bar{t}_L))/2P, \tag{3.14b}
\]

where the suffix \(L\) denotes the component of the vector along the incident proton beam. The structure functions for the proton and the antiproton are the same and are well-known [21]. For the proton there are three of these, \(F(x)\) for the quark partons, \(\bar{F}(x)\) for the antiquark partons and \(F_g(x)\) for the gluons, and they depend on (momentum transfer)\(^2\), for which we have taken the value \((m_t)^2\). For the antiproton, the functions \(F(x)\) and \(\bar{F}(x)\) are
to be interchanged and $x$ replaced by $\bar{x}$. For the values of $(\bar{t}, \bar{b})$ relevant to the Tevatron experiments, the relevant values $x$ and $\bar{x}$ are quite large, being typically 0.1 or larger. For such values of $x$ and $\bar{x}$, $\bar{F}$ is small and $F_g$ is even smaller. We should add here a note that, some way below the upper limit allowed for $m_t$ by the kinematics, the values of $x$ and $\bar{x}$ required by eqs.(3.14) may exceed unity, as $m_t$ increases; of course, the expressions (3.14) are then not physically meaningful solutions. Thus, the condition that $x$ and $\bar{x}$ must be less than unity imposes an upper limit on the magnitude of $m_t$.

Thus the $\bar{t}t$ production rate at the Tevatron is dominated by the quark-antiquark collisions, which are proportional to $F(x)F(\bar{x})$. We have used for $F$ set 2 of the Duke and Owens [22] structure functions for $(u+d)$, neglecting $\bar{F}$ and $F_g$. Even $F(x)$ falls to zero quite rapidly, like $(1 - x)^3$, as $x$ increases above 0.5. The rate is also proportional to the differential cross section for $\text{parton} - \text{antiparton} \to \bar{t}t$, summed over all of the parton-antiparton collisions. Including for completeness the $i = \{1, 2, 3\} = \{(\bar{q}q), (q\bar{q}), (gg)\}$ initial states, we find for the rate factor,

$$R(x, \bar{x}) = \sum_i F_i(x)F_i(\bar{x}) \frac{d\sigma(\hat{s}, \hat{t})_i}{dt}, \quad (3.15)$$

where $F_i(x)$ now stands for $F(x)$, $F_2(x)$ for $F(\bar{x})$ and $F_3(x)$ for $F_g(x)$, and $\hat{s} = x\bar{x}s$, and $\hat{t} = m_t^2 - (E_t - t_{\ell}\gamma/s$ are the differential cross section variables.

Finally, we have to consider the spectrum of the lepton in the top rest-frame (and of the antilepton in the antitop rest frame). With the Standard Model couplings $W \cdot (\bar{t}\gamma(I + \gamma_5)b)$ and $W \cdot (\bar{l}\gamma(I + \gamma_5)\nu_l)$, there is a large forward-backward asymmetry in the $W \to \bar{l}\nu_l$ decay relative to the b-momentum in the W rest-frame. Since there is a one-to-one relationship between this angle and the energy $E_{\ell\bar{t}}$ we can express this angular distribution as an energy distribution:

$$\frac{d\Gamma}{dE_{\ell\bar{t}}} = G_F^2 M_W^3 E_{\ell\bar{t}}(m_t^2 - m_b^2 - 2m_t E_{\ell\bar{t}})/(4\pi^2 m_t \Gamma_W). \quad (3.16)$$

If we neglect $(m_b/m_t) << 1$, and normalize the distribution to unity, we have the probability distribution

$$P(E_{\ell\bar{t}}) = (24/m_t^2) E_{\ell\bar{t}}(1 - 2E_{\ell\bar{t}}/m_t)dE_{\ell\bar{t}} \quad (3.17)$$

where $E_{\ell\bar{t}}$ runs from 0 to $m_t/2$ in this approximation.

Taking all these factors together, the total probability for reaching the observed configuration is proportional to

$$P(t, \bar{b} | m_t) = \sum_j \sum_i F_j(x_i)F_j(\bar{x}_i) \cdot \frac{d\sigma(\hat{s}_i, \hat{t}_i)_j}{dt_i} \cdot P(E_{\ell\bar{t}} | m_t)P(E_{\ell\bar{t}} | m_t), \quad (3.18)$$

where the sum $i$ is over $(\bar{q}q), (q\bar{q}), (gg)$, and the sum $j$ is over all of the ellipse-crossing points for the $m_t$ value under consideration. As discussed here and in ref.[18], $P(t, \bar{b} | m_t)$ is a discontinuous function of $m_t$, the ellipses being lines of zero width which either cross or don’t cross. In reality, these lines have finite width, partly because top quarks and W-bosons have finite widths, but more importantly because of statistical uncertainties in jet development and in measurements of the particles in the event, and because of a distribution of transverse
momenta for the partons and the antipartons in the incident proton and antiproton. In order to estimate $m_t$ from the data on CDF-1, assuming it to be due to top-antitop production and decay, we appeal to Bayes Theorem, which gives the probability distribution as:

$$P(m_t \mid \vec{t}, \vec{\bar{t}}) = P(\vec{t}, \vec{\bar{t}} \mid m_t)\Phi(m_t)/\{\int dm P(\vec{t}, \vec{\bar{t}} \mid m)\Phi(m)\}, \quad (3.19)$$

where $\Phi(m)$ is the a priori probability that the top mass is $m$ and the integral is taken over all possible values of $m$. With $\Phi(m) = 1$, i.e. for the CDF-1 event shape alone, the probability $P(m_t \mid \vec{t}, \vec{\bar{t}})$ is plotted in Fig.7(a), as taken from ref.[18]. Its peak lies at about 130 GeV, and it is broad and flat.

3.4 More realistic top mass estimation.

Although the above analysis is geometrically simple and easy to comprehend, we have come to realise, from the analysis of the “l+4jets” events in Sec.4 below, that it is really necessary to allow (i) for the transverse momenta of the initial parton-antiparton system, and also (ii) for the uncertainties of the $b$ and $\bar{b}$ jet energies. In this Section, for simplicity, we shall consider only the former of these; the inclusion of the latter is discussed below, in Sec.4.2.

Consider first the t-ellipse for given $b$ and $\bar{\ell}$, for assumed top mass $m$. Divide its boundary by $N$ points by dividing the ellipse of Fig.4 into equal azimuthal slices with respect to the axis $C_e N$, where $C_e$ denotes the centre of the ellipse and $C_e N$ intersects BL at angle $\sigma$ given by expression (3.11) and is normal to the plane of the ellipse. At each one of these $N$ points (a set labelled by $\gamma$) there is a definite momentum $\vec{t}(\gamma)$. Next, consider the $\bar{t}$-ellipse for $b$ and $\bar{\ell}$, for the same value of $m$, following the same procedure for the $N$ points (this set labelled by $\bar{\gamma}$), each point leading to a definite momentum $\vec{\bar{t}}(\bar{\gamma})$. We now soften the condition (3.12) by the introduction of a weighting factor $T_\rho$ of finite range, thus obtaining in place of (3.18):

$$P(\vec{t}, \vec{\bar{t}} \mid m) = \sum_{\gamma \bar{\gamma}} \{P(\vec{t}_\gamma \mid m)T_\rho(\vec{t}_\gamma \gamma T + \vec{\bar{t}}_{\bar{\gamma}} T)P(\vec{\bar{t}}_{\bar{\gamma}} \mid m)\}, \quad (3.20)$$

where $T_\rho(\tau)$ is the function

$$T_\rho(\tau) = \frac{1}{2\rho^2}\frac{1}{\pi\rho^2}e^{-\frac{\tau^2}{2\rho^2}}, \quad (3.21)$$

which is a representation of the two-dimensional $\delta$-function

$$T_\rho(\tau) \rightarrow \delta(\tau) = \delta(\vec{t}_\gamma T + \vec{\bar{t}}_{\bar{\gamma}} T), \quad (3.22)$$

as $\rho \rightarrow 0$. In practice, we will use the sum (3.20) over the set of points $(\gamma, \bar{\gamma})$, each having 180 points, so that the slices are of angle $2^\circ$. The contribution of each point to the sum is evaluated and summed. The largest individual contributions are naturally those from the vicinity of the crossing points of Fig.6. The results of this calculation are shown in Fig.7(a) for $\rho = 0.1m_t$ and $0.025m_t$. We see there that the forms obtained differ considerably from that obtained with the condition (3.12), and further, that there is no sign that the earlier
result is recovered in the limit \( \rho \to 0 \). However, it is easy to understand the change in form. Consider the case of \( m = 125 \text{ GeV} \). We no longer have single large contributions coming only from the crossing points C and D, but find a large number of smaller contributions all the way from C to D; this situation persists as the mass falls below 125 GeV, even after there are no crossings corresponding to C and D. Since the ellipses become smaller as \( m \) falls, the finite range gains increasing relative importance and it is not surprising that the non-crossing contributions become dominant there, giving greater weight to low \( m \) values and thus distorting the \( m \)-distribution away from the form which we presented in ref.[18].

The dashed line on Fig.7(a) shows the \( m \)-probability of the single point having the greatest probability for the \( m \) considered. This point naturally lies close to the cross-over point for that \( m \), if there is one, so that this entry must be closely related with the value obtained with the procedure based on (3.12). This curve, based on these entries of maxima, does have the slow fall-off for increasingly large \( m \) which we noted in ref.[18] and show here in Fig.7(a). The change introduced by the use of (3.20) arises from the fact that there are a large number of points \((\gamma, \bar{\gamma})\) which cannot contribute to the sum \( \mathcal{P}(t, \bar{t} | m) \) when (3.12) is required but which dominate its sum for low \( m \) when (3.20) is used, so depressing the value of the most likely value for \( m \). The probability distribution we now adopt is that shown in Fig.7(a) for \( \rho = 0.1m_t \).

### 3.5 Compatibility and combining of top mass probability distributions.

The probability functions \( P(m_t | N_{\mu e}) \) and \( P(m_t | t, \bar{t}) \) are independent and at present compatible. They can be combined to give an overall probability function from the data publically available today:

\[
P(m | \text{data}) = P(m | N_{\mu e} = 3) \cdot P(m | t, \bar{t} : CDF - 1),
\]

shown on Fig.7(b). Its peak is at 121 GeV, the one-standard-deviation limits being 114.5 and 130 GeV. Of course, these remarks are very tentative since they are based on the assumption that the three \((\mu^\pm\bar{e}^\mp)\) events are due to top production and decay; all three might be due to background.

At present, we do not know whether or not \( m_t = 121(\pm 9, -6.5)\text{ GeV} \) is the top mass. However, the hope for the future is that a substantial number of \((\mu^\pm\bar{e}^\mp)\) events will be observed at the Tevatron. An \( m_t \) distribution can be determined for each of them, as done for CDF-1 here. These distributions may turn out to vary randomly from event to event; if so, then these events could not have anything to do with top-antitop production and decay. More likely, a large fraction of them (and perhaps all) may peak at a definite \( m_t \) value and we shall be able to conclude that they form a well-defined group of events due most probably to \( t\bar{t} \) production and decay. The total cross section for this group of events will provide a quantitative test on this interpretation for them.

In order to illustrate one difficulty in following this line of thought, we shall close this Section by discussing a fictitious event which we shall refer to as \( F_{\mu e} \). Its input momenta are given in Table 4 and it has been subjected to the same analysis, with \( \rho = 0.1m_t \), as was applied to the real event CDF-1. Evaluating the limit (3.8b), the antitop momenta lead to \((m_t)_{min} = 95.3\text{ GeV}\), while the top momenta lead to \((m_t)_{min} = 134.5\text{ GeV}\). Since the latter is the stronger limit, we conclude that, if \( F_{\mu e} \) were data from a real \( t\bar{t} \) event, then the top mass could not lie lower than 134.5 GeV. The \( m_t \) distribution for \( F_{\mu e} \) is shown on Fig.8.
It peaks at 154 GeV, with one-standard-error limits at 144 and 168 GeV and it is almost incompatible with the $m_t$ distribution given in Fig.7(a) for the real event CDF-1. We would have to conclude that either one (or both) of these events is not the result of top-antitop production and decay, or that there has been a large statistical fluctuation. In either case, the situation would be unsatisfactory and we would have to await further $\mu e$ events. At the moment, as mentioned in Sec. 2(b), it is known that there do exist two further events of this kind and we all look forward to the release of the detailed data from them.

4 ANALYSIS OF “LEPTON AND FOUR JETS” EVENTS

4.1 Expected rate and nature of the events.

As mentioned following eq. (2.9), the hadronic decay modes $W^+ \rightarrow u\bar{d}$ and $c\bar{s}$ have a net rate about six times greater than either of the decay modes $W^+ \rightarrow e^+\nu_e$ or $\mu^+\nu_{\mu}$. From this remark it follows that $t\bar{t}$ decay leading to the final charged particles

$$(l^+(=e^+ or \mu^+)b) + b(\bar{u}d or \bar{c}s),$$

(4.1a)

and

$$(l^-(=e^- or \mu^-)\bar{b}) + b(u\bar{d} or c\bar{s}),$$

(4.1b)

are more frequent than the $(\mu^+e^\mp)$ dilepton modes

$$(\mu^+b) + (e^-\bar{b}),$$

(4.2a)

and

$$(\mu^-\bar{b}) + (e^+b),$$

(4.2b)

by a factor of twelve. Here, we have confined attention to the diagonal elements of the Cabibbo-Kobayashi-Maskawa matrix and then only for $\lambda = 0$ (Wolfenstein’s parametrization [28]), since this is a satisfactory approximation for the present. If the three $(\mu^+e^\mp)$ events mentioned in Sec. 2(b) were all due to top-antitop pairs, then we would expect about another 36 pairs, each giving one energetic lepton with high $p_T$ accompanied by one $b$-jet, one $\bar{b}$-jet and two other jets, as specified by the possibilities given in (4.1). Up to the present, it has not been possible to identify which are the $b$ and $\bar{b}$ jets, except by the observation of a low-energy muon resulting from the secondary decay $b \rightarrow l^-\bar{\nu}_l c$ or $\bar{b} \rightarrow l^+\nu_l$, as was the case in the CDF-1 event, where there was a low-energy $\mu^+$ (see Table 3). The secondary lepton from $b$ or $\bar{b}$ decay has branching ratio about 21%, so that one should appear in $\approx 42\%$ of $t\bar{t}$ events; the appearance of two secondary leptons has a rate an order of magnitude lower. In the present run, CDF is equipped with a microvertex detector, which will greatly increase its efficiency for identification of the $b$ and $\bar{b}$ jets, because of the relatively long lifetime of the $b$ quarks ($\approx 10^{-12} sec$), and so restrict the fitting of the final state to the $t\bar{t}$ hypothesis in a very helpful way.
Table 4. Input momenta for a fictitious $\bar{t}t$ event ($F_t$) of the form $(l^- b b q' \bar{q})$, where $(q', \bar{q}) = (s, \bar{c})$ or $(d, \bar{u})$.

|                  | $e^-$ | jet(ε) | jet(ψ) | jet(ϕ) | jet(η) |
|------------------|-------|--------|--------|--------|--------|
| $p_x$            | 11.0  | 42.0   | 0.7    | 0.0    | 7.0    |
| $p_y$            | -19.0 | -23.5  | 23.5   | -25.5  | 16.0   |
| $p_z$            | -7.0  | 25.0   | -56.0  | 23.0   | 72.0   |

4.2 Their analysis and top mass probability distributions.

Here we discuss the analysis of the $(l + 4 \text{ jet})$ events, when there is no information as to which are the $b$ and $\bar{b}$ jets. What we say below will assume the lepton to be the $l^+$; if it is $l^-$, then the terms top and antitop are to be interchanged in what follows.

We start by labelling the jets according to their momenta, using the symbols ($\phi, \psi, \epsilon$ and $\eta$), as illustrated in Fig.9 and in Table 4. The mass $M_W = 80.2(3)\text{GeV}$ and the width $\gamma_W = 2.1(1)\text{GeV}$ are now rather well known for the W bosons; we shall therefore impose this value of $M_W$ on the event, while neglecting the width $\gamma_W$ for convenience, since its effects are of secondary importance. Two of the jets are selected, say $\epsilon$ and $\eta$, and identified as 1 and 2, with energies $E_1$ and $E_2$ and separation angle $\theta_{12}$. We then have

$$M_W^2 = m_1^2 + m_2^2 + 2(E_1E_2 - p_1p_2\cos\theta_{12}) \approx 2E_1E_2(1 - \cos\theta_{12}). \quad (4.3)$$

Each jet energy $E_i$ has a probability distribution $Q_i(E_i)$, partly because jet development is a stochastic process and partly because of uncertainties in its determination from the observations. Eq.(4.3) fixes the product $E_1E_2$, so that the allowed values correspond to a hyperbola on the $(E_1, E_2)$ plane. $N_{12}$ points are chosen on that hyperbola and the probability assigned to each of these points is deduced from the integral

$$Q_{\epsilon\eta}(1, 2) = \int dE_1dE_2Q_{\epsilon}(E_1)Q_{\eta}(E_2)\delta(E_1E_2 - \frac{M_W^2}{2(1 - \cos\theta_{12})}). \quad (4.4)$$

A third jet, say $\phi$, is selected and labelled 3; $N_3$ points are chosen over the energy range $(E_3 \pm \chi\sigma_3)$, where $\sigma_3$ is the assigned uncertainty at the one-standard-error level and $\chi$ is chosen suitably (usually $\chi = 2$). Thus, we have chosen $N_3 \cdot N_{12}$ energy values (a set labelled $\alpha$); for each member of this set, an antitop momentum $\bar{t}(\alpha)$ and mass $m(\alpha)$ are deduced, with an assigned probability $Q(\alpha)$, which is the product of $Q_{12}$, computed from (4.4), and $Q_\phi(E_3)$.

We now turn to $l$ and the fourth jet $\psi$, labelled 4. This jet is tacitly (and necessarily) assumed to be a $b$ jet, although this identification is not used here. We take $N_4$ energy values (a set labelled $\beta$) over the range $E_4 \pm \chi\sigma_4$, each with probability $Q_\psi(\beta)$. For each point $(\alpha, \beta)$, we determine the ellipse for mass $m(\alpha)$, based on the vectors $\bar{t}$ and $\bar{t}_\psi$, and divide its boundary by $N_l$ points (a set labelled $\gamma$), as we did in Sec.3.4 by taking equal azimuthal slices of the ellipse of Fig.4 with respect to the axis $C_eN$, where $C_e$ denotes the centre of the ellipse and $C_eN$ intersects BL at the angle $\sigma$ given by expression (3.11) and is normal to the plane of the ellipse. At each one of these $N_l$ points, there is a definite momentum $\bar{t}(\alpha, \beta, \gamma)$ for the top quark. Next, we ask for a match between $\bar{t}$ and $\bar{t}_\psi$ i.e. that
\[ t(\alpha, \beta, \gamma)_T + \bar{t}(\alpha)_T = 0 \] (4.5)

if the incident parton-antiparton system is required to have no transverse momentum. However, in general, this equation will not be satisfied, except at isolated points. As above, in Sec. 3.4, we give up the equality (4.5), allow initial transverse momentum and introduce the probability distribution (3.21), where here \( \underline{x} \) is given by the l.h.s. of eq.(4.5), adopting as before the value \( \rho = 0.1m_t \). Just as for the dilepton modes in Sec.3, a lepton factor (3.17) is necessary, which we denote here by \( P_\psi(E_{\bar{l}l}, 4) \).

For the accompanying decay sequence \( t \to bW^- \), \( W^- \to \bar{c}s \) or \( \bar{u}d \), the light quarks or \( d \) will have the same strong backward/forward asymmetry as holds for the lepton in \( W^- \to l\bar{\nu}_l \), but since we have (in general) no means to distinguish the \( d \) jet from the \( \bar{u} \) jet (or the \( s \) jet from the \( \bar{c} \) jet), we have to average over the two final jets and this removes the backward/forward asymmetry. For simplicity, we have taken the decays \( \bar{t} \to \bar{b}\bar{c}s \) and \( \bar{b}\bar{u}d \) to be isotropic. For the production of \( \bar{t} \) and \( t \) quarks with momenta \( \bar{t}(\alpha) \) and \( t(\alpha, \beta, \gamma) \), through the process \( \bar{q}q \to \bar{t}t \), there is also a rate factor \( R(x(\alpha, \beta, \gamma), \bar{x}(\alpha)) \) necessary, given by (3.15) with \( x \) and \( \bar{x} \) defined by eqs.(3.14). Thus, for the labelling (3,4,1,2) of the four jets (\( \phi, \psi, \epsilon, \eta \)) and for each of the number \( (N_1 \cdot N_{12} \cdot N_3 \cdot N_4) \) of points defined above, we have calculated a mass \( m(\alpha) \), the two momenta \( \bar{t}(\alpha) \) and \( t(\alpha, \beta, \gamma) \) and an associated probability for reaching this event configuration, given by the product

\[ P(\text{event} | m(\alpha); 3, 4, 1, 2) = Q_{ct}(1, 2)Q_\phi(3)Q_\psi(4)T(\alpha, \beta, \gamma)P_\psi(E_{\bar{l}l}, 4)R(x(\alpha, \beta, \gamma), \bar{x}(\alpha)). \] (4.7)

Finally, for this particular assignment of the labels (1,2,3,4), the net probability for this event configuration when \( m(\alpha) \) lies in the interval \( (m, m + \delta m) \) is obtained by summing these probabilities (4.7) for all of these \( (N_1 \cdot N_{12} \cdot N_3 \cdot N_4) \) points for which their \( m(\alpha) \) lies in this interval.

When none of the four jets is identified, it is necessary to consider all possible identifications for them. The final expression for the net probability that the observed event configuration could occur for top quark mass \( m \) is given by the sum of (4.7) over all permutations of the labels (1,2,3,4):

\[ P(\text{event} \mid m) = \sum_{\text{Perm.}} P(\text{event} \mid m; 1, 2, 3, 4). \] (4.8)

As in Sec. 3 above, we now use Bayes’ Theorem to obtain the desired expression for the probability that the top quark mass is \( m \), given the data on this event and an \textit{a priori} probability \( \Phi(m) \) from other information (such as that given by the \( \bar{t}t \) production rate observed; cf.Sec.2.2 above), as follows

\[ P(m \mid \text{this event + earlier info.}) = c \cdot P(\text{event} \mid m)\Phi(m), \] (4.9)

where \( c \) is a normalization constant, independent of \( m \).

Of course, most of the identifications in the sum (4.8) are necessarily inappropriate but we have no means, in general, to know which of them correspond to the correct physical interpretation. It is our optimistic expectation that incorrect identifications will generally lead to negligible contributions to the net probability for top mass \( m \), although it can certainly happen that several different interpretations have comparable probabilities. The merit
of this method of analysing the data on an event is that it is systematic, so that no possible
interpretations can be overlooked, and that it is quantitative, assigning a numerical value
for the relative probability for different interpretations. It is not an elegant procedure but it
does not involve multivariate searches for the maximum likelihood.

We may illustrate the procedure by discussing a fictitious event of the type \((l+4jets)\),
with final state momenta as given in Table 4, which we shall refer to as \(F_i\). The probability
distribution \(P(m)\) calculated for \(F_i\) is shown on Fig.8. This distribution peaks at 135 GeV,
with one-standard-deviation limits at \(\pm 6\) GeV; it has quite a strong overlap with those for
CDF-1 and it overlaps with that for \(F_{\mu e}\) no more than that for CDF-1 does.

4.3 Simulated data and its analysis.

To test the method more extensively, we have considered two sets of simulated data:
(a) A Toy Model.

This is described in ref.[23]. Adopting a mass of 140 GeV, the vectors \(\bar{t}\) and \(t\) were
chosen in a random way, and their decay configurations follow the leptonic sequence for \(t\)
discussed in Sec.3 and an isotropic 3 jet sequence for \(\bar{t}\). These events were processed by the
procedure described above, including allowance for the Gaussian probabilities appropriate
to the CDF determination of jet energies from the data on each jet and for all permutations
of the assignments of \((1,2,3,4)\) to the jets. In this analysis, the same cuts were made on the
simulated data as are routinely applied by CDF to their real data. These cuts are as follows:
(i) \(\rho_T > 15\) GeV for each jet,
(ii) \(E_T > 20\) GeV for the lepton,
(iii) missing transverse \(E_T > 35\) GeV,
(iv) pseudorapidity lower than 2.44 in magnitude for all four jets.

The \(P(m \mid \text{event})\) distributions for three "toy events" chosen at random from 1000
simulated events are shown in Fig.10. In two of them, Figs.10(a) and 10(b), the peak of
the distribution lies close to 140 GeV, the input value although with much spreading due
to wrong jet combinations. However the third, Fig.10(c), shows that individual events can
deviate widely from our simple expectations, for its peak is at 150 GeV while it has a marked
dip in the vicinity of 140 GeV. In Fig.11, mean probability distributions for 1000 "toy model"
events generated for \(m_T = 140\) GeV, are shown for two cases. For Fig.11(a), the analysis
made allowance for the Gaussian probabilities appropriate to the CDF determination of
jet energies and excluded events which did not satisfy the CDF cuts as specified in the last
paragraph. The reader should note that this is \textit{not} a distribution of the mean peak mass, but
represents rather the form of a "typical \(m\) distribution" under the circumstances specified.
Its peak is at 137 GeV with a width (FWHM) of about 18 GeV. For Fig.11(b) the jet
energies input were smeared by making random changes in their magnitudes, with a standard
deviation taken from the CDF work, intended to represent the effects of jet fragmentation
and soft-gluon bremsstrahlung and of the detector efficiencies. This has depressed the mean
probability distribution by a factor of about 2/3; the peak location is about 1 GeV lower
and the FWHM increased to about 25 GeV.

(b) ISAJET [24]

This provides a more sophisticated simulation of \(\bar{t}t\) production. A full simulation of
the CDF detector effects has been carried out, using the appropriate jet-finding procedures and
fragmentation codes. These detector characteristics were taken account of in the code QFL,
as understood through the analysis of the 1988-89 CDF run [24]. The standard ISAJET
program does not take account of $t$ decay to $bW^+$, the process which dominates over $t$-jet fragmentation for the top mass values of interest to us here; however, ISAJET does have the option of adding one additional channel for transitions of the top quark, beyond those for the usual jet development, and this has been made use of here. This channel then becomes the dominant one, so that the later jet development is entirely that of the secondary $b$-quark from top decay and of the tertiary ($c$ and $\bar{s}$ or $u$ and $\bar{d}$) quark decays. The 500 $\bar{t}t$ production and decay events generated in this way for $m_t = 140$ GeV have been processed as discussed above for systems with the ($l^+ + 4\text{jets}$) final state structure and the resulting mean probability distribution has been plotted in Fig.12. This distribution does show a quite definite peak at 130 GeV, a shift of 10 GeV below the input mass, with a FWHM of about 25 GeV if the secondary peak at about 108 GeV is ignored. This relatively large shift is believed to be due the gluon radiation which is taken into account in ISAJET, but this is still under investigation. The secondary peak is believed to arise from the “wrong” combinations of jets but this is not yet certain; however, it appears unlikely to interfere with the use of the upper peak to determine the top quark mass, since it may be masked by the low-mass contributions to $P(m \mid \text{event})$ from background events. We note also that the mass distributions predicted for the ISAJET model spread to much higher top-mass values than do the simpler “toy model” calculations.

These model calculations demonstrate that when sets of input data obtained from “events” generated by an algorithm representing any of the processes

$$\bar{p} + p \rightarrow \{e^+ + \mu^-\} b + \bar{b} + \nu_\ell + \bar{\nu}_\ell + X,$$

(4.10)

or their charge conjugates, which have proceeded through the hadronic $\bar{t}t$ production process (2.4), are considered, the method of analysis we have proposed [23] does lead to $m$ distributions which peak strongly at a mass close to the top mass used in generating these events, at least on the average. This holds true even when we do not know the identity of the quark or gluon which has generated each of the observed jets. However, we must next enquire whether a systematic peaking of the $m$ distributions computed from real data at some particular mass value, say $m^*$, necessarily implies the existence of a top quark with a mass close to $m^*$. Are there other processes leading to the final states ($l + 4\text{jets}$), which we might describe as “background”, for which our analysis leads to such peaks even though there may be no top quark at all or perhaps just no top quark in the mass range explored? We shall see that the answer is ”yes” because the distribution $P(m)$ from the background must vanish at the threshold ($M_W + m_b$) and approach zero again as $m$ becomes sufficiently large since the parton parameters $\alpha$ and $\bar{\alpha}$ cannot exceed unity. Precisely where its maximum occurs depends on the nature of the background and on what cuts are made to reduce its contribution to $P(m)$ in the mass region accessible for the top quark.

4.4 Simulation of non-top background and its analysis as if top-antitop

It is apparent that final states of the form

$$\bar{p} + p \rightarrow l + 4\text{jets} + \nu_l + X$$

(4.11)

can readily arise without top production, through single $W$ production and decay, as follows:
\[ \bar{p} + p \rightarrow W + (4 \text{ hadronic jets} = \bar{q}q \text{ or } q\bar{q} \text{ or } gg), \quad W \rightarrow l + \nu. \] (4.12)

Berends, et al. [26] have provided full tree-level calculations of the cross sections for all of the Standard Model processes which involve one W-boson and \( n \leq 6 \) quarks, antiquarks and gluons. The case \( n = 6 \) includes all of these processes (4.11) and therefore provides a natural and plausible "\( l + 4 \text{jets} \)" background to the "\( l + 4 \text{jets} \)" states (4.1) which result from the sequence of \( \bar{t}t \) production (2.3), their decays (2.6) and finally leptonic decay for one W and decay to three quarks for the other, as outlined in (4.1). In the latter sequence, all four jets are quark or antiquark jets, one being a \( b \) jet and another a \( \bar{b} \) jet. The calculation by Berends, et al. for the processes (4.12) at the Tevatron energy predicts, after application of the cuts specified in Sec. 4.3 above, that the set of (one quark, one antiquark and two gluon)-jets occur more often (56\%) than the set of (two quarks and two antiquarks)-jets (42\%), while the set of four gluon jets is quite rare (2\%). It predicts also that, with these cuts, \( b \) and \( \bar{b} \) jets should occur among these quark jets only rarely (3\%). Berends, et al. also calculated the rate for the \( \bar{t}t \) production process (2.4) in the same approximation, with the decay of the consequent \( W^+ \) or \( W^- \) boson leading to the final states (4.10). Applying the same cuts to these final states, they then concluded from their calculation that, for top masses between 100 and 135 GeV, the number of these events due to \( \bar{t}t \) production and decay would exceed the number of background events.

The \( m \) distributions obtained by subjecting these QCD background events to our analysis procedure may be expected to differ appreciably from those for real \( \bar{t}t \) production and decay. We have examined this question by using the VECBOS Monte Carlo program [27] which implements these Standard Model calculations of Berends, et al. for "\( W+3\text{jets} \)" and "\( W+4\text{jets} \)" events, to obtain a large sample of calculated events which are input to the ISAJET+QFL evolution and development. The sample of "\( W+3\text{jets} \)" used corresponds to an \( IL = 112 \text{ pb}^{-1} \) and that of "\( W+4\text{jets} \)" to \( IL = 128 \text{ pb}^{-1} \). Each event was put through our analysis procedure as described above, assuming it to result from \( \bar{t}t \) production and decay, and gave an \( m \) probability distribution \( P(m \mid l + 4\text{ jet}) \). The mean \( m \) distribution, normalised to \( IL = 4 \text{ pb}^{-1} \), is shown by the shaded area on Fig.13. It does show a peak at \( m \approx 112 \text{ GeV} \), with an appreciable tail running up as high as 140 GeV. The open area of Fig.13 corresponds to the fictitious event \( F_l \) shown on Fig.8, with its peak at 135 GeV; the two peaks are quite well separated, showing that a \( \bar{t}t \) event of type \( F_l \) with top mass 135 GeV could readily be separated from this background, given sufficient events. However, if the top mass were much lower than 120 GeV - the value our analysis of the data on CDF-1 might suggest - the "\( l+4\text{jets} \)" \( m \) distribution would overlap so strongly with the background \( m \) distribution as to make their separation more problematical, requiring a very large body of data, at the least. In this situation, the most convincing evidence on \( m_t \) would be that obtainable from the (\( \mu^\pm e^\mp \)) dilepton events, where no source of serious background is known.

4.5 \( b \) and/or \( \bar{b} \)-quark tagging.

This situation could be much improved if we had some knowledge of the identities of the four jets in these "\( l+4\text{jets} \)" events, since some permutations of the labels (1,2,3,4) would then be excluded and the spread of the \( P(m_t \mid l + 4\text{jets}) \) distribution due to incorrect identifications would be much reduced. For example, for our fictitious event \( F_{\mu e} \), if jet(\( \phi \)) had an accompanying low-energy lepton \( l^\pm \) in roughly the same direction, this would suggest
that jet(φ) in Table 4 is the $\bar{b}$ jet, since the two leptons $e^-$ and $l^+$ would then stem naturally from the $t$ decay sequence $t \rightarrow \bar{b}W^-$, followed by $\bar{b} \rightarrow \bar{c}\ell^+\nu_\ell$ and $W^- \rightarrow e^-\bar{\nu}_e$. In this case, jet(φ) would be assigned the label 4 and the only permutations which need be made are those within (1,2,3). On the other hand, if the secondary lepton were an $\ell^-$, then jet(φ) would be the $b$ jet and should be assigned the label 3 in the above discussion; only the permutations within (1,2,4) would be necessary in the sum for the net probability. In Sec.4.1, we noted that the emission of a lepton from either the $b$ or the $\bar{b}$ quark will occur in about 42% of the $\bar{t}t$ events (2.5), so that observation of one secondary lepton should be a common occurrence. Sometimes this lepton may escape detection because of its relatively low energy, so that, in practice, its rate may not be as high as this remark would suggest, but whenever it can be observed and measured, it can be made use of.

Now that microvertex detectors are being used by CDF, it will often be possible to identify the $b$ and $\bar{b}$ jets from the finite visible (or inferred) path up to the decay of the $B^\pm$ and $B^0(\bar{B}^0)$ mesons to which they give rise. With a magnetic field present, measurement of the charge signs of the B decay products will enable $B^+$ and $B^-$ to be separated. The existence of $(B^0_s, \bar{B}^0_s)$ mixing, with quite a high rate, will complicate their assignment to $b$ or $\bar{b}$ jet. However, the formalism to take this mixing into account is well-known and without free parameters (apart from those concerning CP violation, which makes no appreciable contribution to the hadronic phenomena under discussion here), so that it can be built into the calculation of $P(m \mid l+4j)$ from the data. Observations with the microvertex detectors should both support and supplement the observations of secondary leptons, but even together they will not make the identification of the $b$ jet or $\bar{b}$ jet possible in all events. Nevertheless, such "$b$-tagging", whether from secondary leptons or from determining the flight time from source to decay for the $b$ quark, will prove to be of great value both for reducing the number of misinterpreted background events and for identifying some of the quark and antiquark jets, thus reducing the number of irrelevant jet assignments included in the determination of $P(m \mid l+4j)$ from the data.

5 DISCUSSION

It is clear that we can do little more at present beyond discussing possible procedures for the analysis of top-antitop candidate events. There is little data and it suffers serious uncertainties, so that we are not able even to test our theoretical assumptions yet. Nevertheles, it is natural to assume that the top quark exists with some mass value yet to be determined, and that it does decay dominantly through $t \rightarrow bW^+$, as the Standard Model indicates for the mass values we must consider today. We believe that we should launch into the discussion of all candidate events. Even if it is not possible to prove that individual events are necessarily top-antitop, we may still find that there is a substantial fraction of the $(\mu^\pm e^\mp2jets)$ events that give $P(m \mid event)$ distributions which are compatible and can be combined, in a first step to the determination of the top mass. We have illustrated this here by combining the $P(m \mid CDF−1)$ distribution with the $P(m \mid N_{\mu e})$ determined from the rate of observed top-antitop pair production at the Tevatron; they are compatible and combining them gives an improved estimate of $m_t$.

We have noted above that "$l+4jets" events from the decay of a $\bar{t}t$ pair are expected to occur twelve times as often as "$\mu^\pm e^\mp2jets". If the three events of the latter type are really due to $\bar{t}t$, there should also be, in the recent data, roughly $12 \times 3 = 36$ events of the former type. At present, there do not appear to be even as many candidate events as this remark
suggests, whereas the majority of such candidate events are not compatible with top mass as high as the analysis of CDF-1 and the \( (\mu^\pm e^\mp) \) event rate would suggest. This channel calls for open discussion of all \( "l+4jets" \) events. It could prove to be a very fruitful line of investigation, where the advantage of much higher statistics has to be balanced against the necessity for discriminating against background events. It is worth remarking here that candidate events are a most powerful stimulus to a theoretician’s thinking; it is not good for the progress of particle physics that experimenters should hold the details of new events unnecessarily long.

Berends et al. have provided an excellent QCD model for non-top production of \( "l+4jets" \) events, whose implications must continue to be studied further. Nevertheless, this is only a tree-level model and the proposal and study of other, no doubt more elaborate, models of background will merit much more attention. However, these models will not come forth in the absence of a knowledge of the nature of the events which are actually occurring. For example, there are certainly events which have more than the minimal number of jets, such as \( "l+njets" \) coming from \( \bar{t}t \) production as well as from its background of W-production and decay, where \( n \geq 5 \). We shall need quantitative estimates of the rate of these more complex final states, which compete with the simpler states having the minimal number of jets and which so reduce the rate of the latter.

The observation of the rate \( N_{\mu e} \) of the production of the states \( (\mu^\pm e^\mp) \) for high energy leptons is a direct measure of \( \bar{t}t \) production. The competing source of \( (\mu^\pm e^\mp) \) is \( W^+W^- \) pair production, as pointed out in Sec.2.2, which is anticipated to have a lower rate. Both of these processes, \( (\bar{t}t+\text{jets}) \) and \( (W^+W^-+\text{jets}) \) are also calculable at tree level. The former modifies the cross section for \( (\mu^\pm e^\mp) \), while the latter provides background. The determination of \( m_t \) from the rate of \( (\mu^\pm e^\mp) \) events also needs attention to theory beyond tree level. The simplest calculations indicate that the \( N_{\mu e} \) rate at energy \( 2E_p \) falls rapidly with increasing \( m_t \), at the Tevatron energy, and it is reasonable to expect that the corrections just mentioned will not change this. The conclusion is that the value of \( m_t \) obtained from \( N_{\mu e} \) should be relatively stable with respect to experimental errors in \( N_{\mu e} \), such as the inclusion of uncertain background events, as we noted in Sec.2.2.

6 CONCLUSION

No firm conclusion can be expected from this work so far, except that we should be active in analysing all of the candidate events as they emerge, and that the \( "\mu^\pm e^\mp2jets" \) events have a special value since they suffer little background from non-top events. It will be possible to combine the \( P(m_t) \) probabilities from different events and event types as the data improves. One advantage of the relatively low energy of the Tevatron is that two \( \bar{t}t \) pairs will be produced in the same event only very rarely, so that this source of confusion will be essentially absent. Of course, a somewhat higher energy would have the advantage of increasing single \( \bar{t}t \) production beyond the low rate possible from the present Tevatron. The proposed increase of its c.m. energy from 1800 to 2000 GeV this Summer, with other upgrades, will raise the rate by a factor of two; clearly an optimum energy would lie still somewhat higher than this. We shall not try here to estimate what would be the optimum energy, by balancing a high rate of single \( \bar{t}t \) pairs against a higher rate of multiple \( \bar{t}t \) pairs, since it would be a rather academic exercise at the moment. However, it is a question worthy of serious consideration to give a well-judged answer.

Finally, we must emphasize that no top candidate yet reported has been demonstrated
to represent $\bar{t}t$ production and decay, and that it may still be some considerable time before we reach that stage of certainty. However, we understand that the present plans at Fermilab are to run on until the end of May 1993, by which time the integrated luminosity IL should reach about $50 pb^{-1}$. The upgrade to 2000 GeV will take place in Summer 1993, and Tevatron running should begin again in the following October and run steadily all through 1994. Thus, it is hoped that the net IL reached by the end of 1994 may be about $200 pb^{-1}$, a very substantial improvement over the IL up to the end of 1992, with which this paper has been concerned.

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27. We are indebted to Jose Benlloch for generating the VECBOS samples used. The cuts imposed at the generator level differed from the cuts used for the Toy Model and the ISAJET calculations given here in Sec. 4(a). Jets were defined by the condition \( R = \sqrt{(\delta \phi)^2 + (\delta \eta)^2} > 0.6 \), where \( \phi \) denotes the azimuth angle about the direction of the jet and \( \eta \) denotes the pseudo-rapidity, with \( p_T > 10 \text{GeV} \) and \( |\eta| < 2.5 \); the corresponding lepton cuts were \( p_T > 10 \text{GeV} \) and \( |\eta| < 1.2 \).
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9 FIGURE CAPTIONS

1. The partial lifetime given by the Standard Model for \( t \to b e^+ \nu_e \) is plotted vs. top mass \( m_t \). The factor 9 ensures that for \( m_t > 100 \text{GeV} \) the quantity plotted is the total top quark lifetime.
2. The mean number of \( (\mu^\pm e^\mp) \) events expected for integrated luminosity \( 30 \text{pb}^{-1} \) for \( pp \) collisions at total energy \( 1800 \text{GeV} \), as calculated by Eichten[15] and Crane[16].
3. Shows the probability distribution for \( m_t \), given that there are 3 \( \mu^\pm e^\mp \) events observed for integrated luminosity \( 30 \text{pb}^{-1} \).
4. Shows the ellipse PQ comprising all configurations for the vectors \( t, b \) and \( l \) when the top mass is \( m_t \).
5. The \( t \) and \( \bar{t} \) ellipses for the event CDF-1, as function of \( m_t \), specified at 10 GeV intervals.
6. Projections of the \( t \) and \( \bar{t} \) ellipses for the CDF-1 event onto the plane perpendicular to the beam direction for a number of \( m_t \) values.
7. Shows (a) \( P(m_t | CDF - 1) \) for zero initial transverse momentum and for Gaussian initial transverse momentum with s.d. \( \rho = 0.025m_t \) and \( 0.1m_t \); for the dashed curve, see text. (b) \( P(m_t | N_{\mu e} = 3 \text{ and } CDF - 1) \), for \( \rho = 0.1m_t \).
8. Shows \( P(m_t | F) \) for the fictitious events \( F_{\mu e} \) and \( F_{l} \), described in the text.
9. The leptons and jets in a typical “l + 4 jets” configuration (actually for the fictitious event \( F_1 \)), following \( tt \) production and decay. A low- energy secondary muon, from \( b \) or \( \bar{b} \) decay, is added, as indicated by a short dashed line.
10. Three randomly chosen “l + 4 jets” events from the Toy Model (Sec. 4.3).
11. The mean probability distribution obtained (a) from 1000 Toy Model events and (b) from the same events after ”smearing” (see text above, also ref. [23]) to simulate real
events, random changes being made in the jet energy magnitudes to represent effects of jet fragmentation, soft gluon bremsstrahlung, and detector efficiencies.

12. The mean $m_t$ distribution from our analysis of 500 production and decay events generated by the ISAJET+QFL programs, taking into account fully the CDF detector characteristics.

13. The mean $m_t$ distribution from our analysis of Standard Model "$l + 4\text{ jets}$" events generated by VECBOS simulating the tree-level calculations of backgrounds events by Berends et al.\[26]\; cuts as in Sec.4.2. The sample was about 60 times that appropriate for the 1988-89 CDF run.