Microscopic Theory of Transconductivity

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Measurements of momentum transfer between two closely spaced mesoscopic electronic systems, which couple via Coulomb interaction but where tunneling is inhibited, has proven to be a fruitful method of extracting information about interactions in mesoscopic systems. We report a fully microscopic theory for transconductivity \( \sigma_{12} \), or equivalently, momentum transfer rate between the system constituents. Our main formal result expresses the transconductivity in terms of two fluctuation diagrams, which are topologically related, but not equivalent to, the Ablamazov-Larkin and Maki-Thompson diagrams known for superconductivity. In the present paper the magnetic field dependence of \( \sigma_{12} \) is discussed, and we find that \( \sigma_{12}(B) \) is strongly enhanced over its zero field value, and it displays strong features, which can be understood in terms of a competition between density-of-states and screening effects.

Consider two systems containing mobile charge carriers so close to each other that the charges in the two respective subsystems feel the Coulomb forces originating from the other subsystem, and yet far enough away from each other that direct charge transfer between the two subsystems is not possible. Experimental realizations of such systems are, for example, Coulomb coupled double quantum well systems, [1,2] arrangements where a 3D system is close to a 2D system, [3] or two nearby quantum wires. A scattering event between a carrier in one system and a carrier in the other system leads to momentum transfer between the two subsystems. Thus, if a current is driven through one of the systems (henceforth the driven system is denoted as layer 1), then an induced current is dragged in the other subsystem (layer 2). Alternatively, if no current is allowed to flow in layer 2, a voltage is induced. Due to momentum conservation the two particle number currents flow in the same direction. Since the mechanism for the Coulomb drag is carrier-carrier scattering the drag current is proportional to the square of the effective interaction between the subsystems. The available phase space for electron-electron scattering tends to zero at low temperatures, and consequently one expects Coulomb drag to decrease with decreasing temperature. At low temperatures, the two Pauli factors entering the carrier-carrier scattering rate lead to a \( T^2 \)-dependence, and this behavior is approximately seen in experiments. [1] Note, however, that there are small, but important deviations from the simple \( T^2 \)-law; these deviations have been the topic of much recent interest. [1,4,5]

The simplest possible theoretical description for Coulomb drag is based on the Boltzmann equation.

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Within this formulation the following expression can be derived for the drag rate \(\tau_D\):

\[
\frac{1}{\tau_D} = \frac{\hbar^2}{2\pi^2 n_1 m k T} \int_0^\infty dq \int_0^\infty d\omega q^3 |\phi(q,\omega)|^2 \frac{[\text{Im} \chi_0(q,\omega)]}{\sinh^2(\hbar\omega/2kT)}. \tag{1}
\]

Here \(\phi(q,\omega) = \phi(q)/e(q,\omega)\) is the dynamically screened interaction between carriers in the two layers, and \(\chi_0(q,\omega)\) is the bare RPA polarization function. The above expression is valid only in the weak-scattering limit, and a generalization is needed if one wishes to examine, say, weak-localization corrections or effects of external magnetic field. Only a fully microscopic theory can address these questions, and we have recently carried out such an analysis [7,8].

The calculations are based on the Kubo formula, which expresses the transconductance (and hence the drag rate) in terms of a retarded current-current correlation function,

\[
\sigma_{ij}^{ab}(x-x';\Omega) = \frac{i e^2}{\Omega} \Pi_{ij}^{ab}(x-x';\Omega)
\]

+ \frac{i e^2}{m\Omega} \delta(x-x')\delta_{ij}\delta_{ab}\rho_i(x). \tag{2}

where (throughout we use \(\hbar = 1\))

\[
\Pi_{ij}^{ab}(x-x';t-t') = -i\Theta(t-t') \left\langle \left[ f_i^a(x,t), f_j^b(x',t') \right] \right\rangle. \tag{3}
\]

Here \(\{ij\}\) indicate the subsystem, \(\{ab\}\) in the superscripts label the Cartesian coordinates, \(\Omega\) is the external frequency, \(\rho_i(x)\) is the particle density in subsystem \(i\), and \(j(x,t)\) is the particle current operator. The analysis proceeds via a systematic expansion in the interlayer interaction, and one finds in second order [7,8]

\[
\sigma_{21}^{ab} = -\frac{e^2}{2h^2 v} \sum_q \int_0^\infty d\omega \frac{\phi(q,\omega)}{2\pi} \frac{\partial n_q(\hbar\omega)}{\partial \omega} \times \Delta_1^B(q, q; \omega + i\delta, \omega - i\delta) \Delta_1^B(-q, -q; -\omega - i\delta, -\omega + i\delta), \tag{4}
\]

where \(\phi(q,\omega)\) is the dynamically screened interaction and \(\Delta\) is a three-body correlation function [7].

\[\triangle(q,q,\omega+i\delta,\omega-i\delta) \Delta(q,q,\omega+i\delta,\omega-i\delta)\]

\[\langle T_\tau j(q=0,\tau=0)\rho(q,\tau)\rho(-q,\tau) \rangle. \tag{5}\]
calculated at the same level of approximation, i.e. including magnetic field and impurity effects.

A full description of our calculations is given elsewhere [8], and here we will give a qualitative description of the main physical results. First, the drag rate is significantly enhanced: our preliminary numerical results indicate that the enhancement factor is 60 - 70. [11] Experimentally this is very important, since generally drag effects are small, and any enhancement is most welcome. Secondly, the drag rate displays strong structure whenever the Fermi level crosses a Landau level. Finally, the temperature dependence is quite dramatic; it is qualitatively different at different values of the magnetic field.

A physical understanding of these findings can be obtained with the help of Fig. 2. First, the large degeneracy of Landau levels enhances the density-of-states at Fermi level enormously, and since the drag rate is proportional to $\text{Im}^2 \chi/\sinh^2(\hbar \omega/2kT) = S(q, \omega)S(-q, \omega)$, where $S$ is the structure factor reflecting the available phase-space, an enhancement of the density-of-states necessarily leads to an increased drag rate.

It is not surprising that the drag rate should show peaks as the the Fermi level moves through Landau levels as a function of magnetic field [12]. The situation is quite analogous with the "normal" longitudinal resistance observed in the integer quantum Hall regime (sketched in panel (a) of Figure 2): dissipative transport takes place only when the Fermi level lies within the extended states (unshaded areas in Fig. 2(b)). Thus, there is drag only when states at the Fermi level can undergo scatterings where momentum is exchanged between the layers. The structure within the predicted peaks in the drag rate has a more subtle explanation. In addition to the available phase-space the drag is also strongly influenced by the screened interaction. Thus, in turn, depends on the number of states participating in the screening process. Thus, when the Fermi level approaches the center of a Landau level, when the number of states approaches a maximum, the screening becomes more effective, and the interlayer coupling becomes weaker. Based on this picture one expects that the drag peaks may show internal structure, and this behavior is sketched in Fig. 2(c). Our numerical results [8] fully corroborate the above discussion. The possibility of direct observation of carrier-carrier interaction related effects in a transport quantity such as the drag rate is quite unusual, and should be of significant interest to experimentalists.

**References**

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[11] The enhancement factor is consistent with recent experiments at Cambridge, U.K. We are grateful to Dr. N. Hill for communicating us his unpublished data.

[12] Here we restrict ourselves to the case when the densities of the two gases are same: under these conditions the drag effects are most pronounced.

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