Effects of incoming wind condition and wind turbine aerodynamics on the hub vortex instability

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Abstract. Dynamics and instabilities occurring in the near-wake of wind turbines have a crucial role for the wake downstream evolution, and for the onset of far-wake instabilities. Furthermore, wake dynamics significantly affect the intra-wind farm wake flow, wake interactions and potential power losses. Therefore, the physical understanding and predictability of wind turbine wake instabilities become a nodal point for prediction of wind power harvesting and optimization of wind farm layout. This study is focused on the prediction of the hub vortex instability encountered within wind turbine wakes under different operational conditions of the wind turbine. Linear stability analysis of the wake flow is performed by means of a novel approach that enables to take effects of turbulence on wake instabilities into account. Stability analysis is performed by using as base flow the time-averaged wake velocity field at a specific downstream location. The latter is modeled through Carton-McWilliams velocity profiles by mimicking the presence of the hub vortex and helicoidal tip vortices, and matching the wind turbine thrust coefficient predicted through the actuator disc model. The results show that hub vortex instability is promoted by increasing the turbine thrust coefficient. Indeed, a larger aerodynamic load produces an enhanced wake velocity deficit and axial shear, which are considered the main sources for the wake instability. Nonetheless, wake swirl also promotes hub vortex instability, and it can also affect the azimuthal wavenumber of the most unstable mode.

1. Introduction
Prediction of wind turbine wakes for different conditions of the incoming wind and different aerodynamic settings of the turbines, such as yaw and pitch angles, still remains a big challenge for the wind energy community. Accurate descriptions of the wake flow produced by a turbine array can be obtained through LES simulations by mimicking the forcing of the turbines on the atmospheric boundary layer flow with different actuator models (see e.g. [1, 2]). However, the high computational cost required for those simulations make them a prohibitive tool for real applications, for which a large set of configurations typically needs to be investigated in a relatively short time.

These predictions are also of interest to the fluid mechanics community, because the mean operational properties of the turbine are intimately linked to the complex flow features in the wind turbine wake, which involves vorticity structures with completely different characteristic
scales, the variability of the atmospheric boundary layer flow, and heterogeneity of the environment.

As extensively explained in [3], wind turbine wakes are characterized by a significant velocity deficit consequent to the power extracted from the turbine, which gradually recovers to the incoming wind conditions by proceeding downstream. Moreover, turbine rotation adds azimuthal momentum to the wake flow, which acts in the opposite direction than for the turbine rotation. Besides the global swirling motion of the turbine wake, highly coherent vorticity structures are present in the near-wake, which enhance the complexity of this swirling wake flow. Specifically, a vorticity structure mainly oriented along the wind turbine axis is typically observed at the wake core, which is denoted as hub vortex. Furthermore, vortices detach from the tip of each blade, which are then advected downstream forming a set of helicoidal tip vortices. All those vorticity structures evolve and interact by proceeding downstream, encounter different instabilities, and finally dissipate producing a smooth swirling wake with a Gaussian-like profile of the axial velocity field (see e.g. [4, 5, 6]).

In addition to this, the prediction of a wind turbine wake flow is a challenging task due to the variability introduced by the broad range of atmospheric conditions that can be experienced during real turbine operations. Indeed, different levels of turbulent intensity, shear and veer of the incoming wind can be encountered. Those different conditions of the incoming wind lead to completely different power performance and, in turn, wind turbine wake flows. Recently, wind LiDAR measurements of utility-scale wind turbines performed under different stability conditions of the atmospheric boundary layer showed that wind turbine wakes recover faster under convective regimes than for neutral ones [7]. The variability of wind turbine wakes can be enhanced even more in presence of heterogeneous terrains due to the topography, urban environment, and surrounding infrastructures.

The vorticity structures constituting the wake undergo several instability processes as they evolve by moving downstream. The coherent motion induced by the wake instabilities can significantly modify the instantaneous wake velocity field, thus the shear stresses. The latter represents the main contribution to the entrainment of momentum and energy from the surrounding boundary layer flow into the wind turbine wake. Therefore, wake instabilities can noticeably affect downstream wake recovery, wake interactions, and the performance of a whole wind farm. Moreover, the presence of wake dynamics and instabilities can lead to wind velocity fluctuations persisting further downstream, thus producing a potential hazard for added fatigue loads on downstream waked turbines.

Different types of instabilities of the helicoidal tip vortices has been observed in [8, 9, 10], such as long and short waves instabilities. The downstream persistence of the helicoidal tip vortices plays a significant role on the whole wake recovery. Indeed, a faster diffusion of the tip vortices can lead to an enhancement of the shear stresses over the wake boundaries, then promoting flow entrainment. In [11] it was observed that the far-wake meandering motion of the turbine wake was triggered by the diffusion of the tip vortices, which in turn was promoted by interactions with the hub vortex.

The hub vortex also undergoes to instability by proceeding downstream. A single-helix counter-winding instability was observed in [3] through wind tunnel experiments, then confirmed by means of linear stability analysis. The same phenomenon was confirmed by LES simulations of a hydro-kinetic wind turbine in [12], and through water tunnel experiments of a down-scaled wind turbine model in [13]. In [14] effects of turbulence are included in a linear stability analysis formulation in order to predict the hub vortex instability by using as base flow the time averaged velocity field. This approach is based on the Boussinesq hypothesis and by using three different mixing length models, which are tuned on the turbulence statistics of the base flow.

The main objective of this paper is the prediction of the hub vortex instability under different operational conditions of the wind turbine. This survey is performed by mimicking the
radial distribution of the axial and azimuthal velocity field with multiple Carton-McWilliams vortices [15]. Accordingly to previous LES simulations [2, 5], LiDAR data [6] and wind tunnel experiments [3], this vortex model can accurately represent the typical velocity field connected with turbine wakes, which is affected by the presence of hub and tip vortices. For each condition of the incoming wind and wind turbine loading, the radial distribution of the velocity field is obtained through an iterative method in order to match the thrust coefficient of the wind turbine predicted via actuator disc model. The different wake velocity profiles are then analyzed via linear stability analysis, following the formulation proposed in [14], in order to take the turbulence effects on wake instability into account. Specifically, in this paper the hub vortex instability is predicted for different induction factor, i.e. power coefficient of the turbine, and tip speed ratio.

The paper is organized as follows. The formulation of the linear stability analysis, including turbulence effects with a mixing length model presented in [14], is briefly summarized in section 2. Then, the iterative method adopted to evaluate the wake velocity profiles for different conditions of the incoming wind and turbine loading is described in section 3. Results of the stability analysis obtained by varying the wind turbine induction factor and tip speed ratio are discussed in details in section 4. Finally, the survey and main results are summarized in the conclusions, section 5.

2. Linear stability analysis with eddy-viscosity models

Following the approach proposed in [16] the unsteady flow, \( U(x,t) \), is decomposed in the time-averaged base-flow, \( \overline{U}(x) \), the coherent fluctuation, \( \tilde{u}(x,t) \), and the turbulent motion, \( u'(x,t) \):

\[
U = \overline{U} + \tilde{u} + u',
\]

where the sum of the time-averaged flow and the coherent fluctuation coincides with the ensemble averaged flow \( <U> = \overline{U} + \tilde{u} \) (see e.g. [17]).

The stability of the flow field is given by the tendency of \( \tilde{u} \) to grow (unstable) or decay (stable) in time and space. Thus, stability can be verified by a modal analysis of the linearized dynamics of \( \tilde{u} \). To this purpose the nonlinear evolution of the coherent perturbation for an incompressible flow can be written as (see e.g. [16]):

\[
\nabla \cdot \tilde{u} = 0
\]

\[
\frac{\partial \tilde{u}}{\partial t} + \nabla \tilde{u} \cdot \nabla U + \nabla U \cdot \tilde{u} = -\nabla \tilde{p} + \nabla \cdot \left[ \Delta \tilde{u} - \nabla \cdot [\tilde{u} \tilde{u} - <u' u'>] - \nabla \cdot [\frac{1}{Re} \nabla \cdot (\tilde{u} \tilde{u} - <u' u'>)] - \nabla \cdot [\frac{2}{3} \nabla I] \right]
\]

where variables \( t \) and \( p \) represent time and pressure, respectively. In the framework of a linear analysis with respect to the coherent fluctuations \( \tilde{u} \), the third term of the rhs is neglected. However, the system of equations is not closed and the last term of the rhs, related to the turbulent diffusion, has to be modeled. In [14] this term was modeled via Boussinesq hypothesis, in analogy to [16] and [18], where the Reynolds stresses are linearly proportional to the strain rate yielding:

\[
\overline{R} = \overline{uu'} - 2\nu \overline{S} + \frac{2}{3} \frac{\eta I}
\]

where \( \overline{R} \) is the Reynolds stress tensor, \( S \) is the strain rate tensor, \( q \) is the turbulent kinetic energy (TKE), and \( I \) is the 3x3 identity matrix.

As well documented by several wind tunnel and LES experiments, see e.g. [2, 3, 4, 5], the wind turbine wake flow is practically axisymmetric and with a negligible radial velocity. Therefore, in the framework of a local stability analysis of a wind turbine wake flow, we have \( \overline{U}_r \approx 0 \),
\[ \frac{\partial \mathbf{U}}{\partial \theta} \approx 0, \quad \frac{\partial \mathbf{U}}{\partial x} \ll \frac{\partial \mathbf{U}}{\partial r}, \] which implies that the term \( \overline{R}_{x\theta} = \overline{u_r u'_\theta} \) is null, and the model leads to null normal stresses \([19]\). Consequently, the only non-null components of the modeled tensor \( \overline{R} \) are \( \overline{R}_{r\theta}, \overline{R}_{rx} \), and their symmetric counterparts.

In this work a generalized mixing-length model for swirling-flows is applied (see \([20]\) for more details):

\[ \nu_t(r) = l_m^2 \left( 2\mathcal{S} : \mathcal{S} \right)^{1/2} = l_m^2 \left[ \left( \frac{r}{\overline{\nu} \frac{\partial (\overline{U}_r)}{\partial r}} \right)^2 + \left( \frac{\partial \overline{U}_x}{\partial r} \right)^2 \right]^{1/2}, \tag{4} \]

where \( \nu_t \) is the eddy-viscosity, which is a function of the downstream and radial position, while the mixing length, \( l_m \), is only a function of the downstream location. By linearizing the eddy viscosity model at first order with respect to \( \overline{u} \), and further manipulating Eq. (2), it is obtained (see \([14]\) for details):

\[ \frac{\partial \overline{\mathbf{u}}}{\partial t} + \nabla \cdot \overline{\mathbf{U}} + \nabla \cdot \overline{\mathbf{u}} = -\nabla \overline{p} + \frac{1}{\text{Re}} \Delta \overline{\mathbf{u}} + \nabla \cdot (\nu_t(\overline{\mathbf{U}}) [\nabla + \nabla^T] \overline{\mathbf{u}}) + \nabla \cdot (\overline{\mathbf{U}} \nu_t(\overline{\mathbf{U}}) \cdot \overline{\mathbf{u}} [\nabla + \nabla^T] \overline{\mathbf{U}}) \tag{5} \]

where the term \( \nu_t(\overline{\mathbf{U}}) \) in Eq. (5) can be evaluated from the statistics of the experimental data, as in \([19]\), while the term \( \nabla \cdot \nu_t(\overline{\mathbf{U}}) \cdot \overline{\mathbf{u}} \) is obtained by the linearization of the turbulence model used to close the equations. Eq. (5) is analogous to the one used in \([16, 18, 21]\), but the last term in the rhs is included to take the linearization of the turbulence model into account.

In the framework of weakly-non-parallel stability analysis, Eq. (5) is now applied to a parallel flow \( \overline{\mathbf{U}} = (\overline{U}_x, \overline{U}_\theta, 0) \) evaluated at a given streamwise location. This allows for a modal expansion of the coherent fluctuation in the following form:

\[ \overline{\mathbf{u}}(x, \theta, r, t) = \hat{\mathbf{u}}(r) \exp(ikx + im\theta - i\omega t) \tag{6} \]

where \( k \) and \( m \) are the axial and azimuthal wavenumber, respectively, and \( \omega \) is the frequency. When this modal form is substituted in Eq. (5), an eigenvalue problem is obtained. In the temporal stability analysis \( k \) is real and assigned, while \( \omega \) is the complex eigenvalue of the problem. The opposite choice is done for the spatial stability analysis. In both cases, \( m \) is a free integer parameter. In this work only temporal stability analysis was performed.

For the stability analysis, Eq. (5) together with continuity equation are discretized using a code based on a Chebysheв spectral collocation method. In the present analysis the number of collocation points is \( N = 120 \) and the size of the domain in the radial direction is \( r_{\text{max}}/d = 50 \). This choice provides the convergence of the most unstable eigenvalue with a five-digit accuracy, which is sufficient for the present purposes.

### 3. Evaluation of the wake velocity profiles for different loading conditions

Linear stability analysis was performed by using as base flow the time-averaged velocity field evaluated at a reference downstream location. The axial velocity profile, \( U \), is modeled by representing the velocity deficit produced by the wake vorticity structures through Gaussian functions, which is a model already assessed by previous experimental and numerical works \([3, 4, 14]\):

\[ \frac{U(r)}{U_\infty} = 1 - U_{\text{hub}} \exp \left[ - \left( \frac{r}{\sigma_{\text{hub}}} \right)^2 \right] - U_{\text{tip}} \exp \left[ - \left( \frac{r - r_{\text{tip}}}{\sigma_{\text{tip}}} \right)^2 \right] \tag{7} \]

\( U_\infty \) is the freestream velocity, \( U_{\text{hub}} \) is the maximum velocity deficit connected with the hub vortex, and \( U_{\text{tip}} \) is the one related to the tip vortex. The radial position from the wake center is \( r \), while \( r_{\text{tip}} \) is the radial position of the tip vortex. The parameters \( \sigma_{\text{hub}} \) and \( \sigma_{\text{tip}} \) are proportional to the cross-dimension of the hub and tip vortex, respectively.
From the momentum budget formulated for the streamwise direction, the thrust coefficient of the wind turbine, $C_T$, can be evaluated as follows:

$$C_T = \frac{8}{R^2} \int_0^R \frac{U}{U_{\infty}} \left(1 - \frac{U}{U_{\infty}}\right) r \, dr.$$  \hspace{1cm} (8)

By injecting Eq. (7) into Eq. (8), it is obtained:

$$C_T = \frac{8}{R^2} \frac{U_{\text{hub}}}{R} \int_0^R \left(1 - U_{\text{hub}} \left\{ \exp \left[-\left(\frac{r}{\sigma_{\text{hub}}}\right)^2\right] - k_U \exp \left[-\left(\frac{r - r_{\text{tip}}}{\sigma_{\text{tip}}}\right)^2\right]\right\} \right) \left\{ \exp \left[-\left(\frac{r}{\sigma_{\text{hub}}}\right)^2\right] + k_U \exp \left[-\left(\frac{r - r_{\text{tip}}}{\sigma_{\text{tip}}}\right)^2\right]\right\} r \, dr,$$

where $k_U = U_{\text{tip}}/U_{\text{hub}}$.

The azimuthal velocity profile, $V_\theta$, is modeled through Carton-McWilliams vortices [15], which is a model previously used to investigate instabilities of swirling jets in [22], and to describe the plane instabilities of the so-called isolated or screened vortices, i.e. vortices with circulation decreasing away from the core, in [23] and [24]. The azimuthal velocity is expressed as follows:

$$\frac{V_\theta(r)}{U_{\infty}} = \Omega_{\text{hub}} r \exp \left[-\left(\frac{r}{\sigma_{\theta,\text{hub}}}\right)^\alpha\right] - \Omega_{\text{tip}} (r - r_{\text{tip}}) \exp \left[-\left(\frac{r - r_{\text{tip}}}{\sigma_{\theta,\text{tip}}}\right)^\alpha\right]$$  \hspace{1cm} (10)

where $\Omega_{\text{hub}}$ is the axial vorticity peak of the hub vortex, while $\Omega_{\text{tip}}$ is the one for the tip vortex. The parameters $\sigma_{\theta,\text{hub}}$ and $\sigma_{\theta,\text{tip}}$ are proportional to the core radius of the hub and tip vortex, respectively. The parameter $\alpha$ can modify the radial distribution of the vorticity field and for this study is considered equal to 2.6 accordingly to previous wind tunnel experiments [3, 14].

The thrust coefficient of a wind turbine is evaluated now from the momentum budget in the azimuthal direction:

$$C_T = \frac{4}{R^3} \frac{TSR}{R} \int_0^R \frac{V_\theta}{U_{\infty}} \left(1 + \frac{1}{2} \frac{R}{TSR} \frac{r}{U_{\infty}}\right) r^2 \, dr,$$

where $TSR$ is the tip speed ratio of the wind turbine, i.e. the ratio between the azimuthal velocity of the blade tip and the incoming wind velocity at hub height. By injecting Eq. (10) in Eq. (11), it is obtained

$$C_T = \frac{4}{R^3} \frac{TSR \Omega_{\text{hub}}}{R} \int_0^R \left\{ r \exp \left[-\left(\frac{r}{\sigma_{\theta,\text{hub}}}\right)^\alpha\right] - k_{\Omega} (r - r_{\text{tip}}) \exp \left[-\left(\frac{r - r_{\text{tip}}}{\sigma_{\theta,\text{tip}}}\right)^\alpha\right]\right\}$$

$$\left(1 + \frac{\Omega_{\text{hub}}}{2} \frac{r}{TSR} \frac{R}{R}\right) \left\{ r \exp \left[-\left(\frac{r}{\sigma_{\theta,\text{hub}}}\right)^\alpha\right] - k_{\Omega} (r - r_{\text{tip}}) \exp \left[-\left(\frac{r - r_{\text{tip}}}{\sigma_{\theta,\text{tip}}}\right)^\alpha\right]\right\} r^2 \, dr,$$

where $k_{\Omega} = \Omega_{\text{tip}}/\Omega_{\text{hub}}$.

Accordingly to the actuator disc model, the loading conditions of a wind turbine can be represented through the induction factor, $a$, which is a direct measurement of the velocity deficit produced through the turbine rotation [25]:

$$a = 1 - \frac{U_R}{U_{\infty}}.$$  \hspace{1cm} (13)

$U_R$ is the averaged axial velocity at the rotor disc. Consequently, the thrust coefficient, $C_T$, can be expressed as a function of the induction factor

$$C_T = 4a(1 - a),$$  \hspace{1cm} (14)
and the power coefficient, $C_P$, can be evaluated as:

$$C_P = 4a(1-a)^2.$$  \hspace{1cm} (15)

For a given loading condition, i.e. induction factor and tip speed ratio, the wake velocity profiles of a wind turbine are evaluated through an iterative method. First, several parameters in Eqs. (7) and (10) were fixed in order to mimic a certain aerodynamic design of the turbine blades. In this work, their respective values were selected accordingly to previous wind tunnel experiments and are reported in Tab. 1 \cite{3, 14}. The only free parameters are $U_{hub}$, which is the maximum axial velocity deficit located at center of the wake, and $\Omega_{hub}$, which is the peak of the axial vorticity at the wake center and represents the wake swirl.

For a given induction factor, $a$, the thrust coefficient, $C_T$, is evaluated through the actuator disc model as in Eq. (14). Then, the parameter $U_{hub}$ is iteratively varied until the $C_T$ evaluated through Eq. (9) matches the value predicted through the actuator disc model in Eq. (14). Therefore, the axial velocity profile is univocally estimated by fixing the induction factor, $a$.

A similar iterative method is then applied for the evaluation of the azimuthal velocity, $V_\theta$. However, in Eqs. (11) and (12) it is observed that $C_T$ is a function of the induction factor $a$ and of the tip speed ratio $TSR$. Therefore, in order to obtain univocally $V_\theta$, they both need to be fixed for a certain loading condition. Then, $\Omega_{hub}$ is varied iteratively until the $C_T$ evaluated through Eq. (12) matches the value obtained through the actuator disc model in Eq. (14).

**Table 1.** Parameters of the velocity profiles adopted to mimic a fixed aerodynamic design of the turbine blades.

| Parameter     | Value |
|---------------|-------|
| $k_U = U_{tip}/U_{hub}$ | 0.33  |
| $\sigma_{hub}$   | 0.26  |
| $\sigma_{tip}$   | 0.085 |
| $r_{tip}$        | 0.45  |
| $k_Q = \Omega_{tip}/\Omega_{hub}$ | 0.25  |
| $\sigma_{\theta, hub}$ | 0.26  |
| $\sigma_{\theta, tip}$  | 0.065 |
| $\alpha$        | 2.6   |

4. Effects of the turbine induction factor and tip speed ratio on the hub vortex instability

Different loading conditions, which produce different wake velocity profiles, were considered in order to evaluate the effects of wind turbine power and thrust on the hub vortex instability. Indeed, the axial induction factor, $a$, was varied between 0.04 and 0.44, which leads to $C_T$ from 0.15 to 0.99, and $C_P$ from 0.15 to 0.55. For each case, the iterative method described in the previous section was used to calculate the velocity profiles $U$ and $V_\theta$, as shown in Fig. 1. For this test case, the tip speed ratio of the turbine is $TSR = 7$. As the induction factor increases, the aerodynamic load and thrust coefficient, $C_T$, become larger, which lead to a larger axial velocity deficit and a greater axial shear, i.e. the radial gradient of the axial velocity, see Fig. 1(a). On the other hand, the increased wind turbine thrust causes an enhanced wake swirl, thus to a larger azimuthal velocity as seen in Fig. 1(b).

Temporal linear stability analysis was performed for the wake velocity profiles obtained by varying the induction factor, $a$. In Fig. 2, the growth rate $\omega_i$ of each unstable mode is plotted as
Figure 1. Velocity profiles evaluated for different induction factors and $TSR=7$: a) axial velocity component, $U/U_\infty$; b) azimuthal velocity, $V_\theta/U_\infty$.

Figure 2. Growth rates, $\omega_i$, of the unstable modes as a function of axial wavenumber, $k$, for various induction factors and $TSR=7$. The induction factor increases from left to right, top to bottom. N.B. the vertical axis limit is 0.4 in the top row, while it is 0.8 in the bottom row.

It is evident that the growth rate of each mode increases as the induction factor increases. However, for each case the dominant unstable mode is always the one with azimuthal number $m = 2$. Therefore, the hub vortex instability is promoted by larger aerodynamic load and thrust coefficient, thus by a higher axial shear.

In order to investigate effects of turbine rotation on the hub vortex instability, the tip speed
ratio, $TSR$, was varied between 2 and 10. For this study, the induction factor was fixed to $a = 0.1106$, which corresponds to a power coefficient $C_P = 0.35$. The axial velocity profile is not a function of $TSR$ for a fixed $C_T$, as shown in Eqs. (8) and (9). Therefore, the profile of the axial velocity is the same for all the tested cases as reported in Fig. 3(a). The profiles of the azimuthal velocity for different tip speed ratios are reported in Fig. 3(b). For a fixed $C_T$, as $TSR$ increases the swirl in the wake decreases. Indeed, the same power can be extracted with a low tip speed ratio and high aerodynamic load, which produces in turn higher wake swirl, or vice versa with a high tip speed ratio and low aerodynamic load.

The spectra obtained through the temporal linear stability analysis are reported in Fig. 4. For a lower $TSR$, thus for higher azimuthal velocity, the instability growth rate is increased. Therefore, the wake swirl promotes the hub vortex instability. Moreover, a different $TSR$ leads to a different azimuthal wavenumber of the most unstable mode, $m$. Indeed, unstable modes with higher values of $m$ are selected for increasing $TSR$.

5. Final remarks and conclusions

In this paper the hub vortex instability encountered in wind turbine wakes was investigated for different turbine loading conditions. This survey was performed via temporal stability analysis of the wake velocity field estimated at a reference downstream location. The linear stability analysis is formulated in order to take turbulence effects on wake instability into account. To this end, the Reynolds stresses are embedded in the linearized Navier-Stokes equations, and the turbulence closure model is formulated by using Boussinesq hypothesis and a mixing length model adapted for swirling flows.

The radial profiles of the axial and azimuthal velocity fields connected with the wind turbine wake are modeled with multiple Carton-McWilliams vortices. This technique resulted to be an efficient method to mimic the presence of the hub vortex and tip vortices within the wind turbine wake. An iterative procedure has been applied to vary the maximum axial velocity deficit and axial vorticity of the Carton-McWilliams vortices in order to match the thrust coefficient of the wind turbine predicted through the actuator disc model for each considered condition.

Stability analysis has been performed by varying the induction factor of the turbine, i.e. thrust and power coefficient, and tip speed ratio. It has been shown that an increased turbine thrust coefficient leads to an enhanced maximum velocity deficit and, in turn, to a larger axial shear. This wake flow feature is considered to be one of the dominant mechanisms underpinning the increase of the growth rate of the hub vortex instability with increasing induction factor.
However, different values of the induction factor do not affect the selected azimuthal wavenumber of the most unstable mode.

The tip speed ratio of the turbine also affects the hub vortex instability. Indeed, wake swirl induced by the turbine rotation promotes the hub vortex instability. Moreover, the variation of the tip speed ratio can also modify the azimuthal wavenumber of the dominant unstable mode. This feature can affect significantly the downstream wake recovery due to a different spatial variability of the velocity field. A larger azimuthal wavenumber of the most unstable mode enhances the mechanically produced wake turbulence, which can lead to larger shear stresses and an earlier wake recovery.

Summarizing, in this paper it has been shown that a large variability of the hub vortex instability in wind turbine wakes, specifically growth rate and azimuthal wavenumber, can be observed by varying the operational conditions of the wind turbine. As mentioned above, a different mode of the hub vortex instability implies a modification of the downstream evolution of the wind turbine wake. This variability in the downstream evolution of wind turbine wakes affects wake interactions within a wind farm, thus performance for both power harvesting and fatigue loads on waked turbines. Therefore, these results strengthen the value of this methodology for prediction of wind turbine wake instabilities, consisting of modeled wake velocity profiles for a certain loading condition. This procedure enables the analysis of a large set of realistic operational conditions of wind turbines, which could be otherwise prohibiting through CFD simulations or wind tunnel experiments.

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