The Aharonov-Bohm-Casher ring-dot as a flux-tunable resonant tunneling diode

R. Citro\(^1,2\) and F. Romeo\(^1\)  
\(^1\) Dipartimento di Fisica “E. R. Caianiello”, C.N.I.S.M. and NANOMATES (Research Centre for NANOMaterials and nanoTechnology), Università degli Studi di Salerno, Via S. Allende, I-84081 Baronissi (Sa), Italy and 
\(^2\) LPM²C, Maison des Magisteres, CNRS, B.P. 166, F-38042 Grenoble, France

A mesoscopic ring subject to the Rashba spin-orbit interaction and sequentially coupled to an interacting quantum dot, in the presence of Aharonov-Bohm flux, is proposed as a flux tunable tunneling diode. The analysis of the conductance by means of the nonequilibrium Green’s function technique, shows an intrinsic bistability at varying the Aharonov-Bohm flux when \(2U > \pi \Gamma\), \(U\) being the charging energy on the dot and \(\Gamma\) the effective resonance width. The bistability properties are discussed in connection with spin-switch effects and logical storage device applications.

In the last decade enormous attention has been devoted towards control and engineering of spin degree of freedom in nanostructures, usually referred to as spintronics\(^{\text{[1,2]}}\). Among the nanostructures of major interest, quantum dots (QDs) provide information about the fundamental physical phenomena in spin-dependent and strongly interacting systems, such as the Kondo effect\(^{\text{[3,4,5,6,7]}}\), the Coulomb and the spin-blockade effects\(^{\text{[8,9,10,11,12,13]}}\), spin-valve effect and tunneling magnetoresistance (TMR) etc.\(^{\text{[17,15,16,17,18]}}\). Spin filter and pumps\(^{\text{[19,20,21,22]}}\) have also been proposed using QDs coupled to normal-metal leads.

The investigation of charging effects in tunneling transport through small QDs has also opened up a large research field in the last decades. In particular, in the nonlinear regime, the current voltage characteristic of a tunneling device containing a single quantum dot exhibits step-like structures known as the Coulomb staircase for a review see\(^{\text{[23,24,25]}}\). In this context, double barrier resonant tunneling diodes (DBRTDs) with InAs dots\(^{\text{[26]}}\) represent a well known non linear system that shows N (or in some cases Z-) shaped current-voltage characteristics and intrinsic bistability. Resonant tunneling diode devices may gain practical applications in microwave circuits. Their main feature is that they may become bistable and thus useful as a logical storage device.

In the framework of quantum dot based systems, a state of great interest is represented by a QD inserted in coherent ring conductors. Here, suitable means for controlling spin are provided by quantum interference effects under the influence of electromagnetic potentials, known as Aharonov-Bohm(AB)\(^{\text{[27]}}\) and Aharonov-Casher(AC)\(^{\text{[28]}}\) effect. This possibility has driven a wide interest in spin-dependent Aharonov-Bohm physics, and the transmission properties of mesoscopic AB and AC rings coupled to current leads have been studied under various aspects (see e.g.\(^{\text{[29]}}\).

In this paper we propose a ring-dot device with Rashba spin-orbit interaction as a flux-tunable resonant tunneling diode. We consider a ring-dot system coupled to two external leads (see Fig\(^\text{[1]}\)), whose Hamiltonian in the local spin frame is given by:

\[
\mathcal{H} = \sum_{\sigma=L,R} \mathcal{H}_\sigma + \mathcal{H}_d + \mathcal{H}_t
\]

\[
\sum_{\sigma=L,R} \mathcal{H}_\sigma = \sum_{\sigma=L,R} \sum_{k,\sigma,\alpha} \varepsilon_{k,\sigma}^d c_{k,\sigma,\alpha}^{\dagger} c_{k,\sigma,\alpha} + \sum_{\sigma} \varepsilon_{\sigma} d_{\sigma}^{\dagger} d_{\sigma}
\]

\[
\mathcal{H}_t = \sum_{k,\sigma} [w c_{k,\sigma,\alpha}^{\dagger} d_{\sigma} + 2u \cos(\varphi_{\sigma}) d_{\sigma}^{\dagger} c_{k,\sigma,\alpha} + h.c.]
\]

where \(\mathcal{H}_\sigma\) is the free electron Hamiltonian of the leads \((\alpha = L, R)\), \(c_{k,\sigma,\alpha}(c_{k,\sigma,\alpha}^{\dagger})\), being the annihilation(creation) operator of the conduction electrons and \(\varepsilon_{k,\sigma}^d = \varepsilon_{k,\sigma} - \mu_{\text{c}}\) the chemical potential; \(\mathcal{H}_d\) is the Hamiltonian of dot where \(d_{\sigma}(d_{\sigma}^{\dagger})\) is the annihilation(creation) operator of the electrons on the dot. Within the Hartree-Fock approximation for the Coulomb interaction \(U\) on the level dot\(^{\text{[31]}}\), \(\varepsilon_{\sigma} = \varepsilon_{0} - \mu + U \langle n_{\sigma} \rangle\), where \(\langle n_{\sigma} \rangle = \langle d_{\sigma}^{\dagger} d_{\sigma} \rangle\) and \(\varepsilon_{0} - \mu\) is controlled via a gate voltage. Finally, \(\mathcal{H}_t\) describes the tunneling between the dot and the leads, where in the last term \(\varphi_{\sigma}\) \((\sigma = \pm)\) is the effective flux enclosed in the ring. In particular, \(\varphi_{\sigma} = \pi(\Phi_{AB} + \sigma \Phi_{R})\), \(\Phi_{AB}\) being the AB flux induced by a perpendicular magnetic field and \(\Phi_{R}\) being the effective flux induced by the Rashba spin-orbit interaction\(^{\text{[28]}}\). The Aharonov-Casher flux is explicitly given by \(\Phi_{R} = \sqrt{\beta^2 + \Gamma}\), where \(\beta = 2\delta m^2/\hbar^2\) is the dimensionless spin-orbit interaction, \(m^2\) is the effective mass of the carriers and the parameter \(\delta\) is related to the average electric field along the direction orthogonal to the plane of the ring\(^{\text{[28]}}\).

This is assumed to be a tunable quantity. For an InGaAs-based two-dimensional electron gas, \(\delta\) can be controlled by a gate voltage with typical values in the range \((0.5 \pm 2.0) \times 10^{-11}\text{eV}\).\(^{\text{[31,32]}}\) An external bias voltage \(V\) drives the system away from equilibrium thus imposing a chemical potential imbalance between the left (L) and the right (R) leads, \(\mu_L = 0\) and \(\mu_R = -eV\), where \(e\) is the absolute value of the electron charge. The dot level can be tuned by means of a local gate voltage \(V_p\) almost independently from the voltage drop between the external leads. The tunneling amplitudes \(w\) and \(u\), which in general may depend on the spin and electron momentum, are assumed to be constant.
Here we have introduced the retarded Green’s functions within the Keldysh formalism\textsuperscript{[33, 34]}. The well-known equation for the stationary current reads:

$$I = \frac{i e}{2 \hbar} \int d\varepsilon Tr\{[G^L(\varepsilon) - \Gamma^R(\varepsilon)]G^<(\varepsilon) +$$

$$+ [f_L(\varepsilon)G^L(\varepsilon) - f_R(\varepsilon)\Gamma^R(\varepsilon)][G^>(\varepsilon) - G^a(\varepsilon)]\},$$

where $G^r, G^a$ and $G^<$ are the retarded, advanced and lesser dot Green’s function, respectively, $f_\sigma(\varepsilon)$ is left/right Fermi-Dirac distribution function and $\Gamma^L,R$ are the level width (the boldface notation indicates matrices in the spin basis). To calculate these GFs we apply the equation of motion technique\textsuperscript{[35, 36]} and the retarded Green’s function is given by:

$$G^r_{\sigma\sigma'}(\omega) = [g^r(\omega)^{-1} - \Sigma^r(\omega)]^{-1},$$

where $\Sigma^r_{\sigma\sigma'}(\omega)$ is the tunneling self-energy:

$$\Sigma^r_{\sigma\sigma'}(\omega) = \sum_{k,\alpha=R} \gamma^2 |u|^2 [g^r_{k\alpha}](\omega) +$$

$$+ \sum_{k,\alpha=L} 4|u|^2 \cos(\varphi_\alpha) \cos(\varphi_{\sigma'}) [g^r_{k\alpha}](\omega).$$

Here we have introduced the retarded Green’s functions of the leads $[g^r_{k\alpha}](\omega) = \delta_{\sigma\sigma'}(\omega + i0^+ - \varepsilon^r_{k\alpha})^{-1}$. The latter depends explicitly on the occupation of the dot which is determined self-consistently by $\langle n_\sigma \rangle = \int \frac{d\varepsilon}{2\pi} G^r_{\sigma\sigma}(\varepsilon)$. By evaluating $\langle n_\sigma \rangle$, the retarded Green’s function of the dot can be written as:

$$G^r_{\sigma\sigma'}(\omega) = \delta_{\sigma\sigma'}(\omega - \varepsilon_{\sigma} + i\pi|u|^2 [\rho_R \gamma^2 + 4\rho_L \cos^2(\varphi_{\sigma})])^{-1},$$

where the parameter $\gamma$ is defined as $\omega = \gamma u$, while the density of states $\rho_{L/R}(\omega)$ is given by $\sum_{k,\alpha} (\omega + i0^+ - \varepsilon^r_{k\alpha})^{-1}$. The correlation function $G^<_{\sigma\sigma'}$ is given by the Keldysh equation $G^< = G^r \Sigma^< G^a$, where the advanced Green’s function is $G^a(\omega) = [G^>(\omega)]^\dagger$.

In the wide band limit (WBL), the level widths $\Gamma^r_{\sigma\sigma'}$ are taken energy independent and they are given by $\Gamma^r_{\sigma\sigma'} = 8\pi \rho_L |u|^2 \cos^2(\varphi_{\sigma}) \delta_{\sigma\sigma'}$ and $\Gamma^R_{\sigma\sigma'} = 2\pi \rho_R |u|^2 \gamma^2 \delta_{\sigma\sigma'}$, where $\rho_{L/R} \sim \rho$ at the Fermi level.

The final expression of the lesser Green’s function is:

$$G^<_{\sigma\sigma'} = i\delta_{\sigma\sigma'} f_R(\varepsilon) \Gamma^R_{\sigma\sigma'} + f_L(\varepsilon) \Gamma^L_{\sigma\sigma'} \left(\frac{\varepsilon - \varepsilon_{\sigma}}{\omega - \varepsilon_{\sigma}}\right)^2 + (\Gamma_{\sigma}/2)^2,$$

where the quantity $\Gamma_{\sigma} = \Gamma^L_{\sigma\sigma} + \Gamma^R_{\sigma\sigma}$ has been introduced. By using (5) and (6) and the definition (2), the current per spin channel flowing through the system is given by:

$$I_\sigma = (e/\hbar) \int d\varepsilon \frac{\Gamma_{\sigma\sigma'} \Gamma_{\sigma\sigma}}{2\pi (\varepsilon - \varepsilon_{\sigma})^2 + (\Gamma_{\sigma}/2)^2} (f_L(\varepsilon) - f_R(\varepsilon)).$$

After evaluating the integral\textsuperscript{[37]}, the differential conductance per spin channel (in units of $e^2/\hbar$) in the linear response regime\textsuperscript{[37]} and the occupation on the dot are explicitly given by:

$$G_{\sigma} = \frac{\gamma^2 \cos^2(\varphi_{\sigma})}{(\Delta - U \langle n_\sigma \rangle)^2 + (\gamma^2/4 + \cos^2(\varphi_{\sigma}))^2}$$

and

$$\langle n_\sigma \rangle = \frac{1}{2} + \frac{1}{\pi} \arctan \left[ \frac{\Delta - U \langle n_\sigma \rangle}{\gamma^2/4 + \cos^2(\varphi_{\sigma})} \right].$$

Here $\Delta = e(V + V_\text{p}) - \varepsilon_0$ and the energy is measured in units of $4\pi \rho |u|^2$. The quantity $\Delta$ can be tuned by means of the gate voltage $V_\text{p}$ on the dot and the bias voltage $V$. It is worth to notice that the spin up/down occupations are now coupled due to the combined effect of the Coulomb interaction and the spin-orbit interaction.

In the upper panel of Fig.2 the conductance curves $G_1$ and $G_1$ are shown as a function of the AB flux by fixing the other model parameters as $U = 0.6$, $\Delta = 0.25$, $\Phi_\text{R} = 0.08$ and $\gamma = 0.5$ (energies are in units of $4\pi \rho |u|^2$). In the vicinity of a zero of the conductance of a given spin channel there corresponds a finite value of the conductance in the other spin channel, giving rise to a remarkable magneto-resistance effect. This behavior is originated by an intrinsic bistability as shown in the lower panel of Fig.2 where the hysteresis loop in the AB flux is shown. Since from (9) the variation of the AB flux would correspond to a variation of the occupation of the dot, the intrinsic bistability effect could be understood as a charging effect. It arises since the charge trapped in the resonant dot state changes the potential profile through the double well potential, and therefore modifies the energy of the resonant state. Each time an electron jumps into the dot, the dot potential is lift up due to the charging effect, until the electron tunnels through. The potential is lowered again and it picks up an electron until bias reach a certain condition where the dot no longer picks up an electron after emitting the previously captured one. Thus there is a range of AB fluxes over
which the resonance can carry current. From the analytical expression one may deduce a condition for the bistability in the following way. Expressing only in terms of \( \langle n_\sigma \rangle \) and employing the lowest order expansion in the quantity \( U \langle n_\sigma \rangle \), the equation for the dot occupation becomes:

\[
\tan(\pi(\langle n_\sigma \rangle - \frac{1}{2})) = T_\sigma - \beta_\sigma \langle n_\sigma \rangle, \tag{10}
\]

where \( T_\sigma = A_\sigma + B_\sigma \arctan(q_\sigma \Delta) \), while \( \beta_\sigma = \frac{UB_\sigma q_\sigma}{\Gamma + (\Delta - U)^2} \), with \( q_\sigma = (\frac{U}{\pi} \cos^2(\varphi_\sigma))^{-1} \), \( A_\sigma = q_\sigma (\Delta - U/2) \) and \( B_\sigma = -U q_\sigma / \pi \). It can be analytically proved that Eq.(10) admits more than one solution when \( \pi \Gamma \leq 2U \) where \( \Gamma = \sqrt{\Gamma_\varphi \Gamma_\sigma} \) is an effective level width. The obtained relation is similar to the one for a resonant tunneling diode (RTD) where an intrinsic bistability has been demonstrated both theoretically and experimentally\[38,39\]. In terms of the charging energy \( U = e^2/2C \) the condition to have bistability in our case may be written as:

\[
\frac{e Q_m}{C} > \pi \Gamma, \tag{11}
\]

where the maximum charge density on the dot \( Q_m \) has been introduced. Thus the change in energy of the resonance caused by the stored charge must exceed the width of the resonance for the bistability to be seen. This is a physically appealing condition for a device to show intrinsic bistability. Furthermore, for a fixed strength of the charging energy \( U \), the hysteretic condition can be more easily fulfilled for the values of \( \varphi_\sigma \) such that \( \cos(\varphi_\sigma) \sim 0 \) and for a sufficiently small value of \( \gamma \). As already shown in the case of the RTD by M. Rahman and J. H. Davies\[39\], the bistability condition is favoured by making the exit barrier more opaque (i.e. for small values of \( \gamma \) in our case) even though the current flowing through the system decreases. The main difference from standard RTD is that in the Rashba ring tunneling is governed by a flux dependent barrier with effective strength \( 2U \cos(\varphi_\sigma) \) and acting selectively on the different spin channels. We would also like to point out that the physics of the bistability effect is similar to that of the appearance of a magnetic phase for localized states in metals at varying the local moment level\[40\].

In Fig.2 the conductance \( G_{\uparrow, \downarrow} \) is shown as a function of \( \Delta \) by fixing the other parameters as \( U = 0.6, \Phi_{AB} = 0.45, \Phi_R = 0.08, \gamma = 0.5 \). In the figure, charging energy peaks are evident and strong asymmetric resonances (Fano-like) are observed in the conductance \( G_{\uparrow} \) close to \( \Delta = 0.25 \). Such resonances, characterized by a very long life-time, are responsible for creating the condition to have bistability. On the other hand, we have been checking that increasing the parameter \( \gamma \) while fixing the other parameters as done in Fig.2 the hysteresis is lost. Indeed, an increase of \( \gamma \) produces an increasing of the second term in Eq.(11) making the inequality false.

In conclusion, we studied the conductance of a mesoscopic ring sequentially coupled to an interacting quantum dot in the presence of Rashba spin-orbit interaction and an Aharonov-Bohm flux. By treating the Coulomb interaction on the dot within a self-consistent mean field approximation and by exploiting the non-equilibrium Green’s functions technique, we demonstrated that the
system behaves as a flux-tunable resonant tunneling diode. In analogy to the case of the standard resonant tunneling diode it has been shown, both analytically and numerically, that an hysteretic behavior is observed in the conductance of a given spin channel at varying the flux. The bistability properties are characterized by a spin-switch effect in which the conductance of a given spin channel at varying the magnetic field is totally suppressed while the other is close to the maximum value. When the hysteresis condition is fulfilled the system behaves like a traditional spin interference device as the one proposed in the pioneering work by J. Nitta et al.\cite{Nitta2003}. The proposed device can be easily realized by e.g. InAs quantum dots embedded in a Aharonov-Bohm-Casher ring\cite{Bagraev2005} and potentially employed as high speed switching two-terminal device or a useful logical storage device\cite{Yoh1999}.

The authors acknowledge Prof. M. Marinaro for helpful and enlightening discussions. R.C. acknowledges support by the European community under the Marie Curie Program.

\begin{thebibliography}{99}
\item [1] G. A. Prinz, Science \textbf{282}, 1660 (1998).
\item [2] S. A. Wolf, D. D. Awschalom, R. A. Buhrman, J. M. Daughton, S. von Molnár, M. L. Roukes, A. Y. Chtchelkanova, and D. M. Treger, Science \textbf{294}, 1488 (2001).
\item [3] R. Swirkowicz, M. Wilczynski, M. Wawrzyniak, and J. Barnas, Phys. Rev. B \textbf{73}, 193312 (2006).
\item [4] J. Martinek, M. Sindel, L. Borda, J. Barnas, R. Bulla, J. König, G. Schön, S. Maekawa, and J. von Delft, Phys. Rev. B \textbf{72}, 121302 R (2005).
\item [5] Y. Utsumi, J. Martinek, G. Schön, H. Imamura, and S. Maekawa, Phys. Rev. B \textbf{71}, 245116 (2005).
\item [6] J. Martinek, Y. Utsumi, H. Imamura, J. Barnas, S. Maekawa, J. König, and G. Schön, Phys. Rev. Lett. \textbf{91}, 127203 (2003).
\item [7] P. Zhang, Q.-K. Xue, Y. Wang, and X. C. Xie, Phys. Rev. Lett. \textbf{89}, 286803 (2002).
\item [8] A. C. Hewson, \textit{The Kondo Problem for Heavy Fermions}, Cambridge University Press, Cambridge, \textit{(1993)}.
\item [9] F. Elste and C. Timm, Phys. Rev. B \textbf{73}, 235305 (2006).
\item [10] I. Weymann, J. Barnas, J. König, J. Martinek, and G. Schön, Phys. Rev. B \textbf{72}, 113301 (2005).
\item [11] A. Cottet, W. Belzig, and C. Bruder, Phys. Rev. Lett. \textbf{92}, 206801 (2004).
\item [12] J. Barnas and A. Fert, Phys. Rev. Lett. \textbf{80}, 1058 (1998).
\item [13] S. Takahashi and S. Maekawa, Phys. Rev. Lett. \textbf{80}, 1758 (1998).
\item [14] J. Varalda, A. J. A. de Oliveira, D. H. Mosca, J.-M. George, M. Eddrief, M. Marangolo, and V. H. Etgens, Phys. Rev. B \textbf{72}, 081302(R) (2005).
\item [15] I. Weymann, J. König, J. Martinek, J. Barnas, and G. Schön, Phys. Rev. B \textbf{72}, 115334 (2005).
\item [16] R. López and D. Sánchez, Phys. Rev. Lett. \textbf{90}, 116602 (2003).
\item [17] J. König and J. Martinek, Phys. Rev. Lett. \textbf{90}, 166602 (2003).
\item [18] Braun, J. König, and J. Martinek, Phys. Rev. B \textbf{70}, 195345 (2004).
\item [19] P. Recher, E. V. Sukhorukov, and D. Loss, Phys. Rev. Lett. \textbf{85}, 1962 (2000).
\item [20] H.-A. Engel and D. Loss, Phys. Rev. B \textbf{65}, 195321 (2002).
\item [21] E. Cota, R. Aguado, and G. Platero, Phys. Rev. Lett. \textbf{94}, 107202 (2005).
\item [22] J. Splietstoesser, M. Governale, J. König, R. Fazio, Phys. Rev. B \textbf{74}, 085305 (2006).
\item [23] \textit{Single Charge Tunneling}, Vol. 294 of NATO Advanced Study Institute, Series B: Physics, edited by H. Grabert and M.H. Devoret Plenum, New York, (1992).
\item [24] U. Meirav and E.B. Foxman, Semicond. Sci. Technol. \textbf{10}, 255 (1995).
\item [25] L.P. Kouwenhoven, C.M. Marcus, P.L. McEuen, S. Tarucha, R. M. Westervelt, and N.S. Wingreen, in \textit{Mesoscopic Electron Transport}, edited by L.L. Sohn, L.P. Kouwenhoven, and G. Schön Kluwer Academic Publishers, Dordrecht, (1997).
\item [26] R. Tsu and L. Esaki, Appl. Phys. Lett. \textbf{22}, 562 (1973); V.J. Goldman, D.C. Tsui, and J.E. Cunningham, Phys. Rev. Lett. \textbf{58}, 1256 (1987); A.D. Martin, M.L.F. Lerch, P.E. Simmonds, and L. Eaves, Appl. Phys. Lett. \textbf{64}, 1248 (1994).
\item [27] Y. Aharonov and D. Bohm, Phys. Rev. \textbf{115}, 485 (1959).
\item [28] Y. Aharonov and A. Casher, Phys. Rev. Lett. \textbf{53}, 319 (1984).
\item [29] R. Citro, F. Romeo, M. Marinaro, Phys. Rev. B \textbf{74}, 115329 (2006); R. Citro and F. Romeo, Phys. Rev. B \textbf{71}, 073306 (2007).
\item [30] The mean-field approximation neglects the dynamical properties of the local field (Kondo effect) and gives reliable results away from the Coulomb blockade regime.
\item [31] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. \textbf{78}, 1335 (1997).
\item [32] D. Grundler, Phys. Rev. Lett. \textbf{84}, 6074 (2000).
\item [33] Y. Meir and N.S. Wingreen, Phys. Rev. Lett. \textbf{68}, 2512 (1992).
\item [34] A.P. Jauho and N.S. Wingreen, and Y. Meir, Phys. Rev. B \textbf{50}, 5528 (1994).
\item [35] H. Haug and A. P. Jauho, \textit{Quantum Kinetics in Transport and Optics of Semiconductors}, (Springer, New York, 1996).
\item [36] H. Bruus and K. Flensberg, \textit{Many-Body Quantum Theory in Condensed-Matter Physics: An Introduction} (Oxford University Press, Oxford, 2004).
\item [37] It is assumed that the dot level is on resonance within the conduction window between $\mu_L$ and $\mu_R$ for positive bias.
\item [38] K. Yoh, H. Kazama, Y. Kitashou, and T. Nakano, Phys. Stat. Sol. (b) \textbf{204}, 378 (1997).
\item [39] M. Rahman and J.H. Davies, Semicond. Sci. Technol. \textbf{5}, 168 (1990).
\item [40] S. Doniach, E.H. Sondheimer, \textit{Green’s functions for Solid State Physicists}, Imperial College Press, London (2004).
\item [41] J. Nitta et al., Appl. Phys. Lett. \textbf{75}, 695 (1999).
\item [42] N. T. Bagraev, N. G. Galkin, W. Gehlhoff, L. E. Klyachkin, A. M. Malyarenko and I. A. Shelykh, Journal of Physics: Conference Series \textbf{61}, 56-60 (2007).
\item [43] K. Nakazato, R.J. Blaikie and H. Ahmed, J. Appl. Phys. \textbf{75}, 5123 (1994).
\end{thebibliography}