Effects of Manufacturing Tolerances on Dynamic Properties of Journal Bearings

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Abstract. Dynamic properties of journal bearings were analysed when manufacturing tolerances existed. A methodology was proposed to predict journal bearing properties and to seek a relationship between the manufacturing tolerances and the dynamic coefficients. All the dynamic coefficients of journal bearings were investigated against Sommerfeld number and four types of standard tolerance grades in laminar flow regimes respectively. The results showed that manufacturing tolerances had a significant influence on the dynamic coefficients, especially the cross terms of both stiffness and damping coefficients. The more the magnitude of manufacturing tolerance grade is, the more variations of dynamic coefficients are. It is recommended to take the effect of tolerances on the dynamic performance of journal bearings on its design stage.

1. Introduction

In most of mechanical structures, manufacturing tolerances have nearly no or tiny influences on the static or dynamic properties of these mechanisms. However, it is not always the case for the rotor supported with hydrodynamic journal bearings. It was reported that manufacturing tolerances could have significant effects on the performance of journal bearings[1-3]. The performance of the rotor bearing system with the affected journal bearings could vary significantly compared to the non-affected rotor system, such as undergoing a process from a stable state to a sub-synchronous, synchronous or super-synchronous vibration. Therefore, it is necessary to investigate how the manufacturing tolerances affect the performance of journal bearings and the rotor which the journal bearings support.

There were limited works on the relationship between manufacturing tolerances and the dynamic properties of journal bearings. Pande and Somasundaram[4] studied the effect of manufacturing tolerances on the performance of aerostatic journal bearings using a numerical method. The manufacturing tolerances included ovality, lobbing, cylindricity and surface roughness. Hashimoto et al.[5, 6] investigated the steady and dynamic performance of a worn journal bearing. The results showed that the worn effect could have a significant difference on the dynamic properties of worn journal bearings. Although the worn effect was different from tolerances, it affected the bearing surface in the same way with manufacturing tolerances. Vijayaraghavan et al.[7] and Iwamoto and Tanaka[8] studied the influence of out of roundness on the bearing bush using a numerical method, but
the research object was limited on the bearing bush and the tolerance investigated was just the out of roundness of the bearing bush. Fillon and Waldemar et al.[9, 10] investigated the effect of manufacturing tolerances on the static and dynamic performance of tilting pad journal bearings using a numerical method. Although the type of bearings investigated was tilting pad journal bearings, the research approach proposed was illuminating and worth to be learnt. Xu et al.[11, 12] presented a theoretical method to study the effect of out of roundness of the journal on the performance of rotor bearing systems. An experimental rig was built in order to verify the theoretical results and a good agreement was observed. All the former works admitted a similar methodology that using one or one set of regular curves to form the journal or the bearing surface, which lies in the range of manufacturing tolerances. However, in real physical systems, the surface of a journal or a bearing liner varies randomly in the tolerances limit, which makes a difference between results of theoretical prediction and real surface shape. For a journal bearing, the minimum film thickness may be close to the tolerance grade specified to the bearing, which could affect the performance of the bearing severely. Therefore, manufacturing tolerances should be considered more comprehensively in order to foretell the bearing properties more accurately.

Based on the former analysis, it is necessary to consider the effect of manufacturing tolerances on the dynamic properties which directly affect the stability of the rotor bearing system supported by them. These coefficients could be evaluated by means of the infinitesimal approach proposed by Lund[13] or the finite perturbation method of Qiu and Tieu[14]. Many works studied the dynamic properties related to journal bearings with the help of the two approaches, and very good agreement is achieved either compared with each other or verified by the experimental results[2, 3, 9-12, 15-18]. In this work, the dynamic properties of journal bearings are analyzed when manufacturing tolerances exist. Firstly, the film pressure distribution of finite length bearing, influenced by manufacturing tolerances, is evaluated by a modified Reynolds equation using the Ng-Pan and Elrod model. The new derived Reynolds equation could be used to predict the static and dynamic performance of a journal bearing in either laminar or turbulent flow regime. Secondly, the stiffness and damping coefficients are calculated using the infinitesimal method by means of representing the perturbation equation of Reynolds equation. A robust numerical algorithm is used to acquire the coefficients of journal bearings, and the coefficients are verified and compared with the results of former works.

2. Theory and Materials

2.1. Tolerances and fundamental deviations

In industry application, to achieve an exact dimension for a part is impossible due to the inherent inaccuracy of machine tools, various manufacturing processes, and other uncontrollable factors. The exact dimensions of a part are always determined by tolerances and fundamental deviations. Table 1 shows a few standard tolerance grades and fundamental deviation g for nominal dimensions of 25mm and 50mm according to ISO[19].

| Nominal (mm) | Tolerance Grade, (µm) | Fundamental Deviation, g (µm) |
|--------------|-----------------------|-------------------------------|
| 25           | IT5                   | 9                             |
|              | IT6                   | 13                            |
|              | IT7                   | 21                            |
|              | IT8                   | 33                            |
| 50           | IT5                   | 11                            |
|              | IT6                   | 16                            |
|              | IT7                   | 25                            |
|              | IT8                   | 39                            |

2.2. Journal Bearing Model

In this work, a numerical model of a journal bearing with manufacturing tolerances is established based on the parameters provided in Table 2. Because that the tolerances about the bearing liner has been studied thoroughly and completely by former works[9, 10, 20], and that the part surface are controlled by a few types of tolerances together, the following assumptions are made. Firstly, the bearing model focuses only on the tolerances of the shaft diameter. Secondly, the bearing liner is well manufactured without any geometrical errors. Thirdly, for simplicity and concentrating on the
influence of tolerances, the thermal effect is not considered in the bearing model. Fourthly, the journal bearing studied is assembled by the hole-basis system of fits. Finally, tolerances about the bearing length are omitted because the bearing length has little influence on the dynamic properties of the journal bearing in the long bearings, according to Waldemar and Michel’s work[9, 10], or to the parameters in Sommerfeld number.

Table 2 Parameters Used in Models of Journal Bearings

| Parameters                        | Value       |
|-----------------------------------|-------------|
| Nominal Journal Diameter $D$, mm | 25          |
| Bearing Diameter $D_b$, mm        | 25.04       |
| Bearing Length $L$, mm            | 25          |
| Nominal Clearance $c$, μm         | 20          |
| Oil Viscosity $\mu$, Pa∙s         | $7.5\times10^{-3}$ |
| Rotational Speed $\omega$, rad/s  | 314.15      |
| Shaft Diameter Tolerances         | IT5, IT6, IT7, IT8 |
| Shaft Diameter Deviation          | $g$         |

Figure 1 shows a journal bearing with the tolerance and fundamental deviation specified to the shaft. In order to investigate the influence from the tolerance zone, three typical dimensions are selected to succeed the study, including nominal dimension, maximum dimension and minimum dimension. When the tolerance of the shaft diameter is selected, three typical dimensions are nominal shaft diameter, maximum shaft diameter and minimum diameter. These three rotor diameters are selected because of the fact that all the tolerances in journal bearings, including roundness, cylindricity, straightness, verticality etc, must obey the envelope requirement, which requires that the envelope of perfect form for the feature at maximum material size shall not be violated.

Figure 1 Schematic of a journal bearing with tolerance and deviation on the shaft.

2.3. Theoretical Model for Film Lubrication

The Reynolds equation for laminar and turbulent fluid is summarized and cited by a lot of former works[16, 21-23], and could be conveyed as

$$\frac{\partial}{\partial x} \left( \frac{h^3}{k_x \mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{k_z \mu} \frac{\partial p}{\partial z} \right) = \frac{\omega R \partial h}{2} + \frac{\partial h}{\partial t}$$

(1)

where $R$ is shaft radius, $\mu$ is the lubricant oil viscosity, $h$ is the film thickness, $x$ is the circumferential position, $z$ is the axial or longitudinal position, $\omega$ is rotational velocity of shaft, $t$ is the operating time, $k_x$ and $k_z$ are the turbulence coefficients along circumferential and longitudinal directions, respectively.
Particularly, when the coefficients $k_s$ and $k_z$ are all set to 12, the equation (1) becomes a laminar regime equation.

In order to acquire a more robust and efficient numerical calculation program, the following parameters are defined to nondimensionalize the equation (1),

$$
\bar{x} = \frac{x}{2\pi R}, \bar{z} = \frac{z}{L}, \bar{h} = \frac{h}{c}, \bar{\omega} = \frac{\omega}{\mu \phi}, \bar{p} = \frac{P}{\mu \phi} \left( \frac{c}{R} \right)^2
$$

(2)

where $L$ is the length of journal bearing, $c$ is the clearance of journal bearing. After substitute the equation (2) into the Reynolds equation (1), the dimensionless Reynolds equation could be represented in a conservative form

$$
\frac{\partial \bar{h}}{\partial t} + \frac{1}{4\pi} \frac{\partial \bar{h}}{\partial \bar{x}} - \frac{\partial}{\partial \bar{z}} \left( \frac{1}{4\pi^2 k_x} \bar{R} \frac{\partial \bar{p}}{\partial \bar{z}} \right) - \frac{\partial}{\partial \bar{z}} \left( \frac{1}{4(L/D)^2 k_z} \bar{R} \frac{\partial \bar{p}}{\partial \bar{z}} \right) = 0
$$

(3)

where $D$ is the diameter of shaft.

Equation (3) is solved by the finite volume method, and in order to make the computer program robust and efficient, the Approximate Factorization (AF) algorithm is applied[15, 24]. The Swift-Stieber boundary condition is employed and a finer mesh grid 100×50 is used. And the convergence criterion for the pressure is $1\times10^{-7}$.

2.4. Dynamic Coefficients and Perturbation equation

There are many papers and books dealing with the dynamic coefficients of journal bearings[14, 15, 25-27]. An efficient infinitesimal perturbation method is adopted to solve the dynamic coefficients for journal bearings in this research. Based on the equation (3), an infinitesimal perturbation method is presented. Suppose that the equilibrium point of the bearing is $O_p(\xi_o, \eta_o)$, and at some instant $t$, the shaft center locates on a distance $(\Delta x, \Delta z)$ away from $O_p$ along $x$ and $y$ direction respectively. Using the Taylor expansion, the film thickness $h$ and the pressure $p$ could be expressed by

$$
\bar{h} = \bar{h}_0 + \Delta \bar{x} \cos \bar{\theta} + \Delta \bar{z} \sin \bar{\theta} + H.O.T (\Delta \bar{x}, \Delta \bar{z})
$$

(4)

$$
\bar{p} = \bar{p}_0 + \frac{\partial \bar{p}_0}{\partial \bar{x}_j} \Delta \bar{x}_j + H.O.T (\Delta \bar{x}_j)
$$

(5)

where $\bar{x}_j$ ($j = 1, 2, 3$ and 4) represent for variables ($\bar{x}, \bar{\bar{x}}, \bar{z}$ and $\bar{z}$). $\bar{h}_0$ and $\bar{p}_0$ are the film thickness and the pressure at the equilibrium state.

Substituting the equation (4) and the equation (5) into equation (3), neglecting all the second terms and higher, omitting the time terms and sorting out the equation with each term $\Delta \bar{x}_j$, the following equations are obtained

$$
\Re (\bar{p}_0) = \frac{1}{4\pi} \frac{\partial h_0}{\partial \bar{x}}
$$

$$
\Re (\bar{p}_x) = \frac{1}{2} \sin \theta \frac{\partial}{\partial \bar{x}} \left( \frac{3\bar{h}_0^2 \cos \theta}{4\pi^2 k_x} \frac{\partial \bar{p}_0}{\partial \bar{x}} \right) - \frac{\partial}{\partial \bar{z}} \left( \frac{3\bar{h}_0^2 \cos \theta}{4(L/D)^2 k_z} \frac{\partial \bar{p}_0}{\partial \bar{z}} \right)
$$

$$
\Re (\bar{p}_y) = -\frac{1}{2} \cos \theta \frac{\partial}{\partial \bar{x}} \left( \frac{3\bar{h}_0^2 \sin \theta}{4\pi^2 k_x} \frac{\partial \bar{p}_0}{\partial \bar{x}} \right) - \frac{\partial}{\partial \bar{z}} \left( \frac{3\bar{h}_0^2 \sin \theta}{4(L/D)^2 k_z} \frac{\partial \bar{p}_0}{\partial \bar{z}} \right)
$$

(6)

$$
\Re (\bar{p}_z) = \cos \theta
$$

$$
\Re (\bar{p}_\bar{x}) = \sin \theta
$$

where a differential operator is defined as
After solving the pressure and their derivations by equation (6), all the coefficients could be foretold by

$$
\begin{align*}
\bar{K}_{xx} &= -\int_0^{\tau_\infty} \int_{-1/2}^{1/2} \bar{p}_x \cos \theta \, d\bar{x} \, d\bar{z}, \quad \bar{K}_{yy} = -\int_0^{\tau_\infty} \int_{-1/2}^{1/2} \bar{p}_z \sin \theta \, d\bar{x} \, d\bar{z} \\
\bar{K}_{xy} &= -\int_0^{\tau_\infty} \int_{-1/2}^{1/2} \bar{p}_x \cos \theta \, d\bar{x} \, d\bar{z}, \quad \bar{K}_{yx} = -\int_0^{\tau_\infty} \int_{-1/2}^{1/2} \bar{p}_z \sin \theta \, d\bar{x} \, d\bar{z} \\
\bar{C}_{xx} &= -\int_0^{\tau_\infty} \int_{-1/2}^{1/2} \bar{p}_x \cos \theta \, d\bar{x} \, d\bar{z}, \quad \bar{C}_{yy} = -\int_0^{\tau_\infty} \int_{-1/2}^{1/2} \bar{p}_z \sin \theta \, d\bar{x} \, d\bar{z} \\
\bar{C}_{xy} &= -\int_0^{\tau_\infty} \int_{-1/2}^{1/2} \bar{p}_x \cos \theta \, d\bar{x} \, d\bar{z}, \quad \bar{C}_{yx} = -\int_0^{\tau_\infty} \int_{-1/2}^{1/2} \bar{p}_z \sin \theta \, d\bar{x} \, d\bar{z}
\end{align*}
$$

To validate the numerical algorithm, the bearing model proposed by Mohammad was analyzed, and all the dynamic coefficients predicted using the current bearing model are compared with Mohammad’s work in Figure 2[16]. It is found that all the stiffness coefficients fit for Mohammad’s results very well, and that all the damping coefficients agree with them fairly well. It is noted that all the dimensionless coefficients in Figure 2 are calculated using the methodology proposed by Mohammad.

3. Results and Discussion
The dynamic coefficients of journal bearings with standard tolerance grades from IT5 to IT8 are calculated with the help of Reynolds equation (3) and perturbation equation (6) and the parameters in
Table 2. The selected Sommerfeld number $S$ used in the numerical model equals to 2.895, 1.628, 0.916, 0.515, 0.29, 0.163, 0.092, 0.051, 0.029, 0.016 and 0.009, respectively.

The variations of dimensionless dynamic coefficients of journal bearings are illustrated against selected standard tolerance grades in Figure 3 when Sommerfeld number $S$ equals to 1.63 and fundamental deviation $g$ is specified. It is found that all the dynamic coefficients vary as the standard tolerance grade increases from IT5 to IT8, the more the grade is, the more the variations of dynamic coefficients are. The coefficient least affected is the direct stiffness coefficient $K_{xx}$ with tolerance grade IT5, fluctuating between 0.026 and 0.03 (corresponding to 13.9% about its nominal value 0.035). Therefore, it is benefit to limit the manufacturing tolerances to small tolerance grade for the purpose of stable operating.

Although Figure 3 shows information about the variations of dynamic coefficients as a few tolerance grades, a detailed relationship between the coefficients and Sommerfeld number is not illustrated clearly. For this purpose, the variations of the coefficients with the same tolerance grade (IT6) as Sommerfeld number $S$ are represented in Figure 4 when fundamental deviation $g$ is specified. As is shown in Figure 4, the curves of coefficients for the shaft with tolerance vary in an area confined by curves of lower limit and upper limit. It is found that the area surrounded by curves of lower limit and upper limit deviates from the nominal curve by a distance, the magnitude of this distance is determined by the deviation specified to the shaft.
Figure 3 Variations of the eight dynamic coefficients of journal bearings as the standard tolerance grades ($S = 1.63$, fundamental deviation: $g$).
Figure 4 Variations of dynamic coefficients caused by manufacturing tolerances of diameter versus Sommerfeld number $S$ (Standard tolerance grade: IT6, fundamental deviation: $g$).

4. Conclusion
In this work, the dynamic properties of journal bearings affected by manufacturing tolerances are analysed in laminar flow regime. For the efficiency and robustness of the numerical algorithm, the Reynolds equation and the corresponding disturbed equations for dynamic coefficients are nondimensionalized and represented using defined parameters. Four types of standard tolerance grades are introduced in the numerical model respectively, when fundamental deviation $g$ is specified. It is found that manufacturing tolerances have a significant influence on the dynamic coefficients.
especially the cross terms of both stiffness and damping coefficients. The more the magnitude of the
tolerance grade is, the more variation of dynamic coefficients is. All the results could be used as a
basis for foretelling the dynamic performance of journal bearings in the design stage of assigning
manufacturing tolerances to journal bearings.

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Appendix

Notation

- $c$: clearance of journal bearing (m)
- $C_{ij}$: damping coefficients (N s/m)
- $\hat{C}_{ij}$: dimensionless damping coefficients, $\hat{C}_{ij} = C_{ij}c/\pi\mu LD (R/c)^2$
- $e$: eccentricity (m)
- $D$: nominal diameter of journal (m)
- $D_b$: bearing diameter (m)
- $h$: film thickness (m)
- $\hat{h}$: dimensionless film thickness, $\hat{h} = h/c$
- $K_{ij}$: stiffness coefficients (N/m)
- $\hat{K}_{ij}$: dimensionless stiffness coefficients, $\hat{K}_{ij} = K_{ij}c/\pi\mu \omega LD (R/c)^2$
- $k_x, k_z$: turbulent coefficients along circumferential and longitude direction
- $L$: nominal length of journal bearing (m)
- $O_{js}$: center of journal shaft in static operations
- $p$: film pressure (Pa)
- $\bar{p}$: dimensionless film pressure, $\bar{p} = p/\mu \omega (R/c)^2$
- $\bar{p}_o$: dimensionless film pressure at static state
- $R$: shaft radius (m)
- $S$: Summerfeld numbers, $S = LD\mu N (R/c)^2/W$
- $t$: operating time (s)
- $\bar{t}$: dimensionless operating time, $\bar{t} = \omega t$
- $x$: circumferential position specified to a journal bearing
- $\bar{x}$: dimensionless circumferential position, $\bar{x} = x/2\pi R$
- $z$: longitude position specified to a journal bearing
- $\bar{z}$: dimensionless longitude position, $\bar{z} = z/L$
- $\omega$: shaft rotational speed (rpm)
- $\mu$: oil viscosity (Pa s)
- $(\xi_o, \eta_o)$: equilibrium point of journal bearings in $xz$ coordinate system