Beam Propagation in Photonic Crystals

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Abstract

The recent interest in the imaging possibilities of photonic crystals (superlensing, superprism, optical mirages etc...) call for a detailed analysis of beam propagation inside a finite periodic structure. In this paper, we give such a theoretical and numerical analysis of beam propagation in 1D and 2D photonic crystals. We show that, contrarily to common knowledge, it is not always true that the direction of propagation of a beam is given by the normal to the dispersion curve. We explain this phenomenon in terms of evanescent waves and we construct a renormalized dispersion curve that gives the correct direction.

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I. INTRODUCTION AND SETTING OF THE PROBLEM

Some beautiful experiments and numerical works have shown that it was possible to obtain quite unusual behaviors of light propagation inside meta-materials and photonic crystals (PhCs) \([1, 2, 3, 4, 5, 6, 7, 8]\). In particular, recent ideas by Pendry confirmed by experiments show that photonic crystals, maybe under the guise of meta-materials, could prove to be of huge interest in that they could allow to beat the diffraction limit and to make superlenses. The point of this work is to give a theoretical insight into beam propagation inside PhCs and in particular on the computation of the shift of the transmitted beam (cf. Figure 1). We consider both 1D and 2D PhCs and we show that, contrarily to what is generally believed, the direction of propagation is not always directly given by the normal to the dispersion curves for 2D PhCs. Rather, we define a renormalized, or effective, dispersion diagram, whose normal gives the correct direction of propagation. Numerical examples are given illustrating the various regimes.

We will begin by studying, systematically, 1D structures before extending the results to 2D structures (seen as stacks of gratings) through the concept of the so-called two waves approximation that will be introduced later. Throughout this work, we use time-harmonic fields, with a time-dependence of \(\exp(-i\omega t)\), that are \(z\)-independent. The vectorial diffraction problem is reduced to the study of the two usual cases of polarization: \(s\)-polarization (electric field parallel to the grooves of the gratings) or \(p\)-polarization (magnetic field parallel to the grooves). The wavenumber is denoted by \(k_0 = \frac{2\pi}{\lambda}\), where \(\lambda\) is the incident wavelength in vacuum.

The incident field is a Gaussian beam whose \(z\) component can be expressed by

\[
u^i(x, y) = \int A(\alpha) e^{i(\alpha x + \sqrt{k_0^2 - \alpha^2} y)} d\alpha \tag{1}
\]

where

\[
A(\alpha) = \frac{w}{2\sqrt{\pi}} e^{-\frac{w^2}{4}(\alpha - \alpha_0)^2},
\]

we denote \(u^i\) the electric (magnetic) field in the case of \(s\) (\(p\)) -polarization and \(\alpha_0 = k_0 \sin \theta_0\), where \(\theta_0\) is the mean angle of incidence of the beam and \(w\) its waist.
II. ANALYSIS OF THE BEAM PROPAGATION

A. The case of a stratified medium : 1D PhC

In this section, we derive the value of the shift of the transmitted beam for the particular case of a stratified medium, i.e. when the relative permittivity is constant in the horizontal direction: it is described by a real periodic function (period $h$): $\varepsilon(y)$. We denote: $r = (x,y)$. We consider $N$ periods of the stratified medium, which is embedded in vacuum. For an incident plane wave of wavevector $k = k_0 (\sin \theta, -\cos \theta, 0)$, we denote $\beta_0 = k_0 \cos \theta$ and $(r_N (k, \theta), t_N (k, \theta))$ the reflection and transmission coefficients of the structure. the electromagnetic field in the outer regions reads as:

$$u(r) = e^{i k r} + r_N e^{i k_0 (x \sin \theta + y \cos \theta)}, \quad y \geq 0$$ (2)

$$u(r) = t_N e^{i k_0 (x \sin \theta - (y + N h) \cos \theta)}, \quad y \leq -N h$$ (3)

We denote by $T$ the transfer matrix of one period, then it is known [9] that the reflection and transmission coefficients are related through the relation:

$$T^N \begin{pmatrix} 1 + r_N \\ i \beta_0 (1 - r_N) \end{pmatrix} = t_N \begin{pmatrix} 1 \\ i \beta_0 \end{pmatrix}$$ (4)

Let us denote by $\gamma$ and $\gamma^{-1}$ the eigenvalues of $T$ and by $v = (\phi_{11}, \phi_{21})$, $w = (\phi_{22}, \phi_{22})$ the associated eigenvectors ($Tv = \gamma v$, $Tw = \gamma^{-1} w$). It is known (see for instance [9]) that the band gaps and the conduction bands are given respectively by: $G = \{(k, \theta) | tr(T) > 2\}$, and : $B = \{(k, \theta) | tr(T) < 2\}$. The reflection and transmission coefficients are then given by:

$$r_N (k, \theta) = \frac{(\gamma^{2N} - 1) f}{\gamma^{2N} - g^{-1} f}, \quad t_N (k, \theta) = \frac{\gamma^N (1 - g^{-1} f)}{\gamma^{2N} - g^{-1} f}$$ (5)

where, denoting $q(x,y) = \frac{i \beta_0 y - x}{i \beta_0 y + x}$, the functions $f$ and $g$ are defined by $g(k, \theta) = q(v), f(k, \theta) = q(w)$ and $v$ is chosen such that $|g| \leq 1$ in the conduction bands. Remark that in these bands, the inverse of $f$ is equal to the conjugate of $g$ (see [10, 11] for details).
A straightforward calculation shows that, for \((k, \theta) \in B\):

\[
\begin{align*}
  r_N (k, \theta) &= g + (g - f) \sum_{p=1}^{+\infty} \gamma^{2Np} |g|^{2p} \\
  t_N (k, \theta) &= (1 - |g|^2) \gamma^N \sum_{p=0}^{+\infty} \gamma^{2Np} |g|^{2p}
\end{align*}
\]

(6) (7)

In the conduction bands, the eigenvalue \(\gamma\) can be written under the form: \(\gamma = e^{i\beta h}\), where \(\beta\) is the so-called Bloch phase. When the incident field is a beam, we get an infinite sum of transmitted beams, corresponding to multiple scattering. Let us concentrate on the first transmitted beam, i.e. the beam that reads:

\[
u_t^0 (x, Nh) = \int A(\alpha) \left(1 - |g|^2\right) e^{i\beta Nh} e^{i\alpha x} d\alpha
\]

(8)

whose Fourier transform is:

\[\hat{u}_t (\alpha) = \sqrt{2\pi} \tilde{A}(\alpha) e^{i\beta Nh}\]

(9)

where

\[\tilde{A}(\alpha) = A(\alpha) \left(1 - |g|^2\right).
\]

(10)

We denote \(G_i, G_t, G_d\) the points where, respectively, the incident, transmitted and reflected beams enter or emerge from the PhC. Given the incident field, the axis are chosen to have \(G_i = 0\). These points are defined as the barycenters, or first moments, of the corresponding fields, that is:

\[
\begin{align*}
  G_i &= \frac{\int x |u^i(x,0)|^2 dx}{\int |u^i(x,0)|^2 dx} = 0 \\
  G_d &= \frac{\int x |u^d(x,0)|^2 dx}{\int |u^d(x,Nh)|^2 dx} \\
  G_t &= \frac{\int x |u^t(x,nh)|^2 dx}{\int |u^t(x,Nh)|^2 dx}
\end{align*}
\]

(11)

Using Parseval-Plancherel identity, we get the angular shift due to the beam propagation (cf. fig. 1):

\[
\tan \psi = \frac{G_t}{Nh} = -\frac{\int \tilde{A}^2 (\alpha) \partial_\alpha \beta (\alpha) d\alpha}{\int \tilde{A}^2 (\alpha) d\alpha}
\]

(12)

A series expansion of \(\tan \psi\) can be obtained provided the phase function is analytic with respect to \(\alpha\) in a neighborhood of \(\alpha_0\). Indeed, we can then write:

\[
\partial_\alpha \beta (\alpha) = \sum_{m=0}^{+\infty} \frac{\partial^{m+1} \beta (\alpha_0)}{m!} (\alpha - \alpha_0)^m
\]

(13)
We obtain after some manipulations:

\[ \tan \psi = - \sum_{m} \frac{2^m \Gamma \left( m + \frac{1}{2} \right)}{(2m)!} \partial_{\alpha}^{2m+1} \beta (\alpha_0) \]  

(14)

where \( \Gamma \) is the Euler Gamma function [12]. When \( k_0 w \) is large, then \( A (\alpha) \) is concentrated around \( \alpha_0 \), and if \( \partial_{\alpha} \beta (\alpha) \) does not vary too quickly in the vicinity of \( \alpha_0 \), we obtain the well-known crude approximation:

\[ \tan \psi \sim - \partial_{\alpha} \beta (\alpha_0) \]  

(15)

Of course, the formula (15) can no longer hold if \( \partial_{\alpha} \beta (\alpha) \) is not analytic near \( \alpha_0 \), i.e. when \( \alpha_0 \) is a branch point. We shall encounter this case in the following section.

In order to give a geometric interpretation of this result, let us remark that \( (\partial_{\alpha} \beta (\alpha_0), -1) \) is a vector that is normal to the dispersion curve at wavelength \( \lambda \). So that we retrieve the well-known fact that for a spatially large beam, the direction of propagation is given by the normal to the isofrequency Bloch diagram.

We shall see in the following that this result is in general no longer true in finite 2D structures.

B. Beam propagation in a 2D photonic crystal

The crystal is described as a stack of gratings and we assume that in the spectral domain defined by the above beam, the ratio between the wavelength and the period \( d \) of the gratings is such that there is only one reflected and one transmitted order for the grating structure. Then the propagating reflected and transmitted fields can be expressed as:

\[ u_d (x, y) = \int A (\alpha) r_N (\alpha) e^{i(\alpha x - \beta y)} d\alpha \]  

(16)

\[ u_t (x, y) = \int A (\alpha) t_N (\alpha) e^{i(\alpha x + \beta y)} d\alpha \]  

(17)

Once the reflection and transmission coefficients \( (r_N, t_N) \) are known (by using a rigorous numerical method, for instance the Fourier Modal Method (FMM) [13]) there exists a unique unimodular real \( 2 \times 2 \) matrix \( T_N \) [9] with real coefficients satisfying relation (4): it is the dressed transfer matrix of the total structure [10]. We have:

\[ T_N = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} e^{iN h \bar{\beta}_N} & 0 \\ 0 & e^{-iN h \bar{\beta}_N} \end{pmatrix} \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}^{-1} \]  

(18)
where the phase $\tilde{\beta}_N$ is the renormalized Bloch phase for the global structure [10]. From this matrix, the reflection and transmission coefficients can be written in the following form:

$$
R_N = \frac{\left( e^{2i\beta_N Nh} - 1 \right) f}{e^{2i\beta_N Nh} - g^{-1} f}, \quad T_N = \frac{e^{i\beta_N Nh}(1 - g^{-1} f)}{e^{2i\beta_N Nh} - g^{-1} f}
$$

(19)

For a stratified medium with homogeneous layers, the reduced transfer matrix satisfies rigorously the relation: $T_1^N = T_N$ for all $N$. However, for a two dimensional photonic crystal, this relation tends to become false as the number of periods is increased: this is due to the fact that matrix $T_1$ does not take the evanescent waves into account. Consequently, as the thickness of the device increases the discrepancy between $T_1^N$ and $T_N$ increases as well. This remark has a crucial importance for our study as, in general, the derivative of the phase $\partial_\alpha \tilde{\beta}_N$ is not equal to $\partial_\alpha \beta$.

By definition $tr(T_N) = 2 \cos(Nh\tilde{\beta}_N)$, so that:

$$
\partial_\alpha tr(T_N) = -2Nh\partial_\alpha \tilde{\beta}_N \sin(Nh\tilde{\beta}_N) = \mp Nh\partial_\alpha \tilde{\beta}_N \sqrt{4 - tr(T_N)^2}
$$

this provides us with a numerical method for computing $\left| \partial_\alpha \tilde{\beta}_N \right|$, the sign is unambiguously fixed using the fact that $|g| < 1$ and is associated with the eigenvalue $e^{iNh\tilde{\beta}_N}$.

We have reduced the problem of computing the transmitted field to the one-dimensional case, and thus we can write:

$$
u_0^t(x, Nh) = \int \tilde{A}(\alpha) e^{iNh\tilde{\beta}_N(\alpha)} e^{i\alpha x} d\alpha
$$

(20)

We can now give the main result of this paper, whose proof is given in Appendix 3. We assume that the beam is spatially large (i.e. $k_0w \gg 1$). Then two cases may be encountered with respect to the dispersion curve. For the mean angle of incidence of the beam (corresponding to $\alpha_0$), the curve is either regular (i.e the slope is not infinite ), or it is singular, i.e. the tangent to the curve is vertical. The shift of the beam is then described accordingly:

1. If the tangent is not vertical, i.e. $\left| \partial_\alpha \tilde{\beta}_N(\alpha_0) \right| < +\infty$ then the angle of refraction $\psi$ of the beam inside the structure is given by:

$$
\tan \psi \sim -\partial_\alpha \tilde{\beta}_N(\alpha_0),
$$

(21)
2. if the slope is infinite, i.e. \( |\tilde{\alpha} \tilde{\beta}_N (\alpha_0)| = +\infty \) then the angle of refraction \( \psi \) of the beam inside the structure is given by:

\[
\tan \psi \sim -C_N \frac{2^{3/4}}{\sqrt{\pi}} \Gamma \left( \frac{5}{4} \right) \sqrt{\sin \theta_1 + \sin \theta_0} \sqrt{k_0 w},
\]

where \( \alpha_1 = k_0 \sin \theta_1 \) is the maximum of \( \tilde{\alpha}(\alpha)^2 (\alpha - \alpha_0)^{3/2} \) and \( C_N \) is a constant such that \( \beta_N \sim C_N \sqrt{\alpha^2 - \alpha_0^2} \) near \( \alpha_0 \).

In the second result, the geometry of the structure and its electromagnetic parameters enter in the constant \( C_N \). For a sufficiently large beam, \( \theta_1 \sim \theta_0 \).

Two important properties should be noted in that case. The obvious one is that the shift does not tend to infinity when the normal to the dispersion curve tends to the horizontal axis, a fact that was of course expected, but which shows that the normal to the renormalized dispersion curve gives the direction of the beam, only if \( \partial \tilde{\alpha} \tilde{\beta} \) does not vary too quickly in the vicinity of the mean angle \( \alpha_0 \). Two parameters are in fact needed for a complete description of the situation: the normalized waist \( k_0 w \) and the derivative of the phase \( \partial \tilde{\alpha} \tilde{\beta} (\alpha_0) \). The above result only gives the asymptotic behavior for the separate parameters.

The second important point is the dependence of the shift with respect to the normalized waist, a situation which was not encountered in the first case. In order to understand this point, one should note that there is here a guided mode in the structure, i.e. a pseudoperiodic mode whose wavevector has a null vertical component. Of course, for a finite size structure, the uniqueness of the scattering problem implies that guided modes are associated with complex values of \( \alpha \) which are poles of the transmission coefficient. The finiteness of the structure provokes a splitting of the eigenvalue \( \alpha_0 \) into two complex values \[5\] corresponding to a zero and a pole of the reflexion coefficient. When such a structure is illuminated by a plane wave under the incidence \( \alpha_0 \) the transmission shows a Fano profile indicating the excitation of the lossy mode. When the incident light is a beam, the behavior of the field resembles that of a plane wave in the limit \( k_0 w \gg 1 \), therefore the displacement of the barycenter towards infinity is associated with a spreading of the transmitted beam and thus, precisely because of the spreading, the very notion of barycenter of the transmitted beam loses its physical meaning.
III. NUMERICAL EXAMPLES

In the following, we present some numerical computations illustrating the various situations described by the above results. We will denote by:

- $\Delta$ the shift computed by direct numerical computations of the fields.
- $\Delta_\beta$ the shift computed through the isofrequency dispersion diagram.
- $\Delta_\tilde{\beta}$ the shift computed by use of the effective theory developed in the previous section.

A. Case of a stratified one dimensional medium

In this subsection, we check the numerical method that allows to compute the derivative of the Bloch vector of the equivalent $T$ matrix and also the formula that gives the shift of the beam. The structure that we use is just a Bragg Mirror with two alternating slabs (thicknesses $h_1$ and $h_2$) in each period. The $s$ polarized incident monochromatic beam is characterized by its waist $w = 15\lambda$ and its mean angle of incidence $\theta_0 = 50^\circ$. The wavelength is such that $\lambda/h_1 = 2.27$ with $h_1/h_2 = 2$ and the following permittivities for the slabs: $\varepsilon_1 = 2.1, \varepsilon_2 = 6.25$. In fig.2, we give the amplitude of the incident, transmitted and reflected fields on the upper and lower interfaces of the device for $N = 15$ periods. The shift of the first transmitted beam obtained by direct numerical computation is $\Delta/h_1 = 12.51$ whereas the shift obtained by computing the Bloch coefficient is $\Delta_\beta/h_1 = 12.52$ and finally, the numerical computation described in the above section gives $\Delta_\tilde{\beta}/h_1 = 12.52$ hence, as expected, a perfect agreement with the Bloch approach. In order to complete this verification, we now use a $p$-polarized incident beam with $w = 15\lambda, \lambda/h_1 = 4.75, \theta_0 = 47.5^\circ$ and the parameters $\varepsilon_1 = 10.89, \varepsilon_2 = 1, h_1/h_2 = 1$. Fig.3, shows the amplitude of the transmitted and reflected fields. This time, we obtain $\Delta/h_1 = 62.7, \Delta_\beta/h_1 = 62.7209, \Delta_\tilde{\beta}/h_1 = 62.7209$, which confirms the validity of our approach for that straightforward situation.

B. Stack of gratings

We are now in a position to apply our theoretical approach to the more complex situation of a stack of gratings. We recall that we assume that the wavelength is such that there is
only one Bloch mode inside the structure and one transmitted order and one reflected order. For all the following numerical experiments the field is s-polarized.

The photonic crystal is a stack of 7 lamellar gratings with inverted contrast: \( \varepsilon_1 = \varepsilon_{\text{ext}} = 11.56 \) and \( \varepsilon_2 = 1 \) (\( d_1/d = h_1/d = 1/2 \) and \( h = d \), see fig.2 for notations). We compute the field for an incident beam \( (w = 12.5\lambda, \lambda/d = 2.2, \theta = 50^\circ) \) using the Fourier Modal Method \[13\]. The amplitudes of the field on the upper and lower interfaces are given in fig.4. The Bloch diagram is given in fig.5. The shift of the transmitted beam obtained directly from this computation (through the envelope) is \( \Delta/d \sim 11.25 \) the shift computed from the isofrequency dispersion diagram is \( \Delta_\beta/d = 2.45 \) and the shift obtained from the effective theory is \( \Delta_{\beta}/d = 11.4 \). Therefore, we see that we have an error factor of 4.5 by neglecting the evanescent waves.

The effective theory also applies when contra-propagative Bloch modes exist in the structure, these modes authorizing super-prism phenomena. As an example, we consider a 2D PhC made of 5 lamellar grating layers \( (\varepsilon_1 = 9, \varepsilon_2 = \varepsilon_{\text{ext}} = 1, d_1/d = 0.77, h = d, h_1/d = 1/4) \). The isofrequency Bloch diagram of the structure is given in fig.6 for \( \lambda/d = 2 \). There is a zone of contra-propagating Bloch modes around \( \alpha_0 = 1.6 \) \( (\theta \sim 40^\circ) \). The structure is illuminated by a monochromatic gaussian beam \( (w = 10\lambda, \alpha_0 = 1.6, \lambda/d = 2) \). The map of the field is given in fig.7, where it is seen that the shift of the transmitted beam is negative, the amplitude of the field on the upper and lower faces are given in fig.8. The shift obtained by the direct numerical computation is \( \Delta \sim -9.3 \), the shift obtained from the Bloch diagram is \( \Delta_\beta/d = -3.3 \) whereas \( \Delta_{\beta}/d = -9.9 \). Once more, we find an excellent agreement between the direct computation and the effective theory, whereas the predictions of Bloch theory are quite false.

Let us turn now toward a structure in which a guided contra-propagative mode do exist, this corresponds to the situation 2 in the proposition of the preceding section. In other words, there exists a mode with an horizontal wave vector. The parameters are the following: \( \varepsilon_1 = 7.84, d_1/d = 0.4, h_1/d = 0.7, h_2/d = 0.3, \lambda/d = 2.1, w = 50\lambda \), and there are 7 layers in the PhC. For this structure the contra-propagative mode is obtained for \( \theta_0 = 39^\circ \). Here, the parameter \( C_7 \) which represents the behavior of \( \beta(\alpha) \sim C_7\sqrt{\alpha^2 - \alpha_0^2} \) is obtained by a fitting of the results of the direct numerical computation, we obtained \( C_7 = 0.164 \). We have plotted in fig. 9 \( (d/\Delta) \) versus \( d/(\alpha - \alpha_0) \) where it is clearly seen that the shift converges towards a limit value. Using formula \[22\], we obtain \( \Delta_{\beta}/d \sim -26 \) in fair agreement with
the numerical shift $\Delta/d \sim -27$.

One should not think, however, that the non-renormalized Bloch diagram, i.e. that of the infinite periodic structure, cannot provide us with accurate results. It suffices to think of the homogenization regime, where the stack of gratings behaves as a stratified medium. For instance, we use a stack of 7 gratings ($h_1/d = 1/2, h = d, \varepsilon = 2.1$) and a beam with parameters: $w = 25\lambda, \lambda/d = 4, \theta_0 = 40^\circ$. The field amplitudes on the upper and lower faces are given in fig. 10, where it can be seen that the oscillations are quite limited showing that we are indeed in the homogenization regime. The shift of the beam is $\Delta/d = 8$, and we have $\Delta_{\beta}/d = 7.985$ and $\Delta_{\beta}/d = 7.985$. In that case, both predictions agree. This situation is due to the fact that the field inside the PhC can be represented by Bloch modes only. This situation may happen outside the homogenization regime.

IV. CONCLUSION

We have developed an effective medium approach to describe beam propagation inside a photonic crystal. This effective theory takes into account the evanescent waves, which are completely skipped if one uses only Bloch waves to describe wave propagation in the crystal. The importance of these evanescent waves are put into light by the computation of the shift of the transmitted beam. We show for some examples that the predictions obtained by using only the dispersion diagram may be false. These results emphasize the difference between the band theory for the Schrödinger equation (i.e. the propagation of electrons in periodic potentials), where the boundary of the crystal is irrelevant, and the scattering of electromagnetic waves by photonic crystals where boundary effects are of crucial importance. We have developed elsewhere some theoretical tools, that should hopefully permit to obtain a clearer insight into the role of evanescent waves. Work is in progress in that direction.
Lemma

Let \( f \) be a real even function continuously differentiable near \( \alpha_0 \) and such that \( f(\alpha_0) = 0 \). Then if \( f' \) is square integrable near \( \alpha_0 \) there holds

\[
\frac{f(\alpha)}{\sqrt{\alpha^2 - \alpha_0^2}} = O(1) \quad (23)
\]

Proof:

Let us write: \( f(\alpha) = f(\alpha') + \int_{\alpha'}^{\alpha} f'(t) \, dt \). Then \((f(\alpha) - f(\alpha'))^2 \leq \left( \int_{\alpha'}^{\alpha} f'(t) \, dt \right)^2\) and then by Cauchy-Schwarz inequality: \((f(\alpha) - f(\alpha')) \leq \sqrt{\alpha - \alpha'} \sqrt{\left( \int_{\alpha'}^{\alpha} (f'(t))^2 \, dt \right)}\) and the proposition follows.
Appendix 2

Applying the same reasoning as for the stratified medium, we get:

\[ G_t = -Nh \int \tilde{A}^2(\alpha) \partial_\alpha \tilde{\beta}_N(\alpha) d\alpha / E_t \]

Assuming that \( u'_0 \) has a first moment, we obtain, by Parseval equality:

\[ \int x |u_0(x)|^2 dx = \frac{1}{i} \int \tilde{A}(\alpha) \tilde{A}(\alpha)' d\alpha + \int \tilde{A}^2(\alpha) \partial_\alpha \tilde{\beta}_N(\alpha) d\alpha \]

and the angular shift is given by (cf. fig. 1):

\[ \tan \psi = -\frac{\int \tilde{A}^2(\alpha) \partial_\alpha \tilde{\beta}_N(\alpha) d\alpha}{\int \tilde{A}^2(\alpha) d\alpha} \]

The dispersion diagram is described locally by \( \beta = \phi(\alpha) \) and, in the vicinity of the branch point \( \alpha_0 \), we can write from the lemma proved in Appendix 2:

\[ \phi(\alpha) = C \sqrt{\alpha^2 - \alpha_0^2}, \alpha \geq \alpha_0 \]

the shift is then given by:

\[ \tan \psi = -C \frac{\int \tilde{A}^2(\alpha) \frac{\alpha}{\sqrt{\alpha^2 - \alpha_0^2}} d\alpha}{\int \tilde{A}^2(\alpha) d\alpha} = -C \sqrt{\sin \theta_1 + \sin \theta_0} \frac{2^{3/4}}{\sqrt{\pi}} \Gamma \left( \frac{5}{4} \right) \sqrt{kW} \]

where \( \alpha_1 = k \sin \theta_1 \) is a maximum of \( \alpha \rightarrow \tilde{A}(\alpha)^2(\alpha - \alpha_0)^{3/2} \).

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FIG. 1: sketch of the photonic crystal

FIG. 2: Basic cell of the photonic crystal used in the numerical experiments.
FIG. 3: Incident (a) transmitted (b) and reflected (c) field intensities for an s-polarized incident field on a 1D structure.

FIG. 4: Incident (a) transmitted (b) and reflected (c) field intensities for a p-polarized incident field on a 1D structure.
FIG. 5: Incident (a) transmitted (b) and reflected (c) field intensities for an s-polarized beam on a 2D PhC.

FIG. 6: Bloch diagram for the inverted contrast photonic crystal.

FIG. 7: Bloch diagram for the photonic crystal allowing contra-propagating modes.
FIG. 8: Map of the electric field for the contra-propagating mode.

FIG. 9: Incident (a) transmitted (b) and reflected (c) field intensities for an s-polarized beam on a 2D PhC.

FIG. 10: Evolution of the shift with respect to the Fourier variable.
FIG. 11: Incident (a) transmitted (b) and reflected (c) field intensities in the homogenization regime.