Elastic-Plastic Load-Carrying Capacity of Simple Frame Structures

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Abstract. The efficiency of plasticity for the design of steel frame structures is analysed in the paper. Technical theory of plasticity and standard assumptions and requirements for numerical study are applied. Two types of simple frames (double-sided pinned supporting and fixing) with different loading, geometrical dimensions of the frames and cross-sections of their columns and beams are considered. Some results of the numerical study and conclusions are also presented.

1. Introduction
Actual standards for the design of steel structures (EN 1993-1-1, CSN EN 1993-1-1 and other) allow using elastic or plastic calculation. This fundamental change creates wider possibilities for economically designing steel structures by exploiting their plastic properties. The elastic calculation can be used in any case. In accordance with the related scientific and technical knowledge and standard regulations, several design conditions and requirements of both material and static-construction character are required for plasticity calculation [1-4].

At present, plasticity calculation and design are used mainly for predominantly bent beams, especially continuous beams. So far less known and almost unused in practice is plasticity design of steel frames. In frame constructions, the plasticity calculation is more complicated. The choice of computational models and corresponding failure mechanisms is more difficult. The effects of local, but mainly total deformations on the stability and load-carrying capacity of frame structures are serious. Therefore, the normative rules for plasticity calculation and design of frame constructions in terms of technical theory impose more demanding requirements. It is questionable what the real effects of plastic design are compared to the usual elastic design of the frame structure. Therefore, in the previous period, a numerical analysis of the impact, resp. contribution of plasticization to the load-carrying capacity of simple frame structures, taking into account the technical theory of plasticity and applicable normative regulations, has been carried out. Selected results of numerical analysis are presented in the paper [5-7].

2. Theoretical and normative assumptions and requirements
In accordance with technical theory and applicable standards, the plasticity calculation of steel structures assumes and requires:
- static and quasi-static loads that do not cause dynamic, shock or fatigue effects,
- idealized elastic-plastic or rigid-plastic working diagram,
- good elastic-plastic properties of the steel, determined by an elongation $A_5 \geq 15\%$ and tensile strength to yield strength ratio $f_u / f_y \geq 1,1$,
- condition of plasticity according to Huber-Mises-Hencky's shape change theory,
- idealized plastic joints and joint failure mechanisms,
- overall and local stability of the structure, its elements and cross-sections,
- small total plastic and permanent deformations.

Of the above assumptions and requirements, those relating to the plastic mechanism of failure, stability and deformation are of particular importance for the plasticity calculation of frame structures. Of the above assumptions and requirements, those relating to plastic failure, stability and deformation mechanisms are of particular importance for the plasticity calculation of frame structures. In conventional low-storey frame structures, the overall deformations of the structure may not be taken into account in the calculation of internal forces and moments, so 1st order theory may be used. Larger total deformations must be taken into account in order to calculate internal forces and moments by means of 2nd order theory. Deciding on the possibility of using the calculation according to the theory of 1st order, resp. having to use the 2nd order calculation is difficult and complex. However, the new standards provide the necessary regulations and guidance for this decision [1-5].

3. Numerical analysis
This numerical analysis focused on the investigation of the most important design factors affecting the overall behaviour, deformation and load-carrying capacity of conventional frame structures. This numerical analysis focused on the investigation of the most important design factors affecting the overall behaviour, deformation and load-carrying capacity of conventional frame structures. The static diagrams of the frames considered are shown in Figure 1. Only the height of the cross-sections and thus the height of the webs changed. For all frames, the steel S 235 was assumed. The cross-sections and their combinations for individual frames are shown in Table 1.

Figure 1. Static schemes of examined frames
Frame loads were considered in two alternatives. In the first alternative, the vertical load on the cross member of the frame was uniform over the entire length \((q)\). The horizontal load \(H\) at the level of the rungs was linked to the load \(q\) \((H = 0.05qh)\). In a second alternative, the vertical uniform load \(q\) has been replaced by a concentrated force \(F\) \((F = ql)\) acting in the middle of the rung span. The horizontal load \(H\) was assumed as in the case of a uniform load across the frame. The basic frame dimensions (range \(l\) and height \(h\)) were considered in three alternatives with \(l / h\) ratios = 1.0; 1.5 and 2.0. A total of seven welded I cross-sections were designed for columns and frame cross members. The dimensions of the flanges and web thickness were the same for all cross-sections. In accordance with theoretical assumptions and standard requirements, the ultimate elastic loads \(q_{el}\) and \(F_{el}\), the ultimate plastic loads \(q_{pl}\) and \(F_{pl}\), and the corresponding vertical deflections of the rung, \(v_{pl}\), and the horizontal displacements of the \(u_{el}\), \(u_{pl}\) columns were then calculated for all individual frames. In the plasticity calculation, a complete plastic mechanism of failure of the structure was considered, namely a column, transverse and combined mechanism. The obtained results enable to analyse in detail the influence of individual design factors on plastic load-carrying capacity and deformation of selected frame structures. The diagrams corresponding to the ultimate elastic and plastic stresses of the frames under consideration is shown in Figure 2 and Figure 3. Combined mechanisms were decisive for the two-sided hinged frames subjected to continuous load. For the two-sided hinged frames with concentrated force and for the all fixed frames in contact, the transverse mechanisms were decisive. The limit elastic and plastic assumed loads with a constant cross-section of the column and various cross-sections of the beam are presented in Table 2.

![Figure 2](image-url)
Figure 3. Bending moments corresponding to frame elastic and plastic loads F

Table 2. Limit elastic and plastic loads of assumed frames

| I/h    | Limit load | Cross-section of the beam |
|--------|------------|---------------------------|
|        |            | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
| 1.0 q\(_{el}\) Hinge | 173.02 | 194.59 | 217.19 | 241.52 | 267.79 | 295.23 | 317.64 |
|        | Fixing    | 186.86 | 209.49 | 232.20 | 256.13 | 281.76 | 309.25 | 338.65 |
|        | q\(_{pl}\) Hinge | 193.04 | 218.28 | 245.25 | 273.94 | 304.36 | 336.50 | 370.38 |
|        | Fixing    | 233.59 | 264.12 | 296.75 | 331.47 | 368.28 | 407.17 | 448.16 |
| 1.5 q\(_{el}\) Hinge | 83.82  | 95.09  | 106.25 | 116.86 | 128.16 | 140.26 | 153.19 |
|        | Fixing    | 86.15  | 96.27  | 108.74 | 123.46 | 136.39 | 148.25 | 160.86 |
|        | q\(_{pl}\) Hinge | 95.17  | 107.61 | 120.90 | 135.05 | 150.04 | 165.90 | 182.59 |
|        | Fixing    | 103.82 | 117.39 | 131.89 | 147.32 | 163.69 | 180.96 | 199.18 |
| 2.0 q\(_{el}\) Hinge | 47.19  | 52.85  | 59.74  | 67.83  | 76.82  | 83.38  | 90.48  |
|        | Fixing    | 47.01  | 51.66  | 57.46  | 64.44  | 72.60  | 81.96  | 92.51  |
|        | q\(_{pl}\) Hinge | 55.58  | 62.85  | 70.16  | 78.87  | 87.63  | 96.89  | 106.64 |
|        | Fixing    | 58.40  | 66.03  | 74.19  | 82.87  | 92.07  | 101.79 | 112.04 |
| 1.0 F\(_{el}\) Hinge | 222.45 | 259.94 | 298.21 | 338.22 | 380.43 | 425.04 | 472.10 |
|        | Fixing    | 233.57 | 272.91 | 312.05 | 352.28 | 394.31 | 438.51 | 485.04 |
|        | F\(_{pl}\) Hinge | 318.52 | 360.17 | 404.66 | 452.01 | 502.20 | 555.23 | 611.12 |
|        | Fixing    | 350.38 | 396.18 | 445.13 | 497.21 | 552.41 | 610.76 | 672.23 |
| 1.5 F\(_{el}\) Hinge | 158.77 | 185.65 | 212.13 | 239.12 | 267.16 | 296.56 | 327.46 |
|        | Fixing    | 166.09 | 194.96 | 222.76 | 250.49 | 278.85 | 308.25 | 338.97 |
|        | F\(_{pl}\) Hinge | 223.65 | 252.88 | 284.13 | 317.36 | 352.61 | 389.85 | 429.09 |
|        | Fixing    | 233.58 | 264.12 | 296.75 | 331.47 | 368.28 | 407.17 | 448.16 |
| 2.0 F\(_{el}\) Hinge | 124.57 | 146.22 | 167.07 | 187.87 | 209.14 | 231.19 | 254.23 |
|        | Fixing    | 129.76 | 153.22 | 174.48 | 197.30 | 219.11 | 242.45 | 264.54 |
|        | F\(_{pl}\) Hinge | 170.92 | 193.26 | 217.14 | 242.54 | 269.47 | 297.93 | 327.92 |
|        | Fixing    | 175.19 | 198.09 | 222.56 | 248.60 | 276.21 | 305.38 | 336.12 |
The limit loads $q_{el}$, $q_{pl}$, and $F_{el}$, $F_{pl}$ of the considered frames are evaluated, if span $l = 1.5h$ (Figure 4).

![Figure 4. Limit elastic and plastic loads assumed frames for $l = 1.5h$](image)

From the point of view of the contribution, the results that allow analysing and evaluating the contribution of plasticity to the overall increase in the ultimate load of frame structures can be considered the most serious, Table 2 and Figure 4.

The results obtained show the following limit values of $q_{pl}/q_{el}$, and $F_{pl}/F_{el}$ ratios:

**Load across frame with uniform load $q$**
- Joint support
  - $q_{pl}/q_{el} = 1.1158$ to $1.1660$ at $l/h = 1.0$,
  - $q_{pl}/q_{el} = 1.1317$ to $1.1919$ at $l/h = 1.5$,
  - $q_{pl}/q_{el} = 1.1423$ to $1.2271$ at $l/h = 2.0$.
- Fixing
  - $q_{pl}/q_{el} = 1.2107$ to $1.3234$ at $l/h = 1.0$,
  - $q_{pl}/q_{el} = 1.1931$ to $1.2585$ at $l/h = 1.5$,
  - $q_{pl}/q_{el} = 1.2420$ to $1.3284$ at $l/h = 2.0$.

**Load of the frame with the force $F$ in the middle cross-section**
- Joint support
  - $F_{pl}/F_{el} = 1.2945$ to $1.4808$ at $l/h = 1.0$,
  - $F_{pl}/F_{el} = 1.3103$ to $1.4568$ at $l/h = 1.5$,
  - $F_{pl}/F_{el} = 1.2887$ to $1.4189$ at $l/h = 2.0$.
- Fixing
  - $F_{pl}/F_{el} = 1.3859$ to $1.5531$ at $L/h = 1.0$,
  - $F_{pl}/F_{el} = 1.3207$ to $1.4544$ at $l/h = 1.5$,
  - $F_{pl}/F_{el} = 1.2603$ to $1.3962$ at $l/h = 2.0$. 
4. Conclusions
The presented and other results of the performed numerical analysis provide clear information on the static elastic-plastic behaviour and the limit elastic and plastic loads of the considered frames in terms of investigated effects of their support, load and geometric dimensions.

The above ratios of $q_{pl} / q_{el}$ and $F_{pl} / F_{el}$ show the different contribution of the deliberate use of plasticity to increase the ultimate load and the associated economical design of frame steel structures. The clear results refer that the contribution of steel plasticity in the frame structures can be very important and necessary for their practical design.

References
[1] P. Juhás, “Elastic-plastic Load-carrying Capacity of Steel Members,” Conference Series: Materials Science and Engineering, World Multidisciplinary Civil Engineering-Architecture-Urban Planning Symposium 2017. vol. 245, 2017.
[2] P. Juhás, et al. Design of steel structures. Commentary to STN 73 1401: 1998. SUTN, Bratislava, 2001.
[3] P. Juhás and I. Juhásová Šenitková, “The Load Carrying Capacity of Compressed Steel Members with Thin – walled Combined Cross section,” 17th International Multidisciplinary Scientific GeoConference 2017. vol. 17, issue 62. pp. 813 – 821, 2017.
[4] EN 1993-1-1: Eurocode 3. Design of Steel Structures, Part 1.1 General rules and rules for buildings. CEN, Brussels, 2005.
[5] P. Juhás, Reliability and strength of steel structure materials. ALFA, Košice, 2009.
[6] M. B. Wong, Plastic Analysis and Design of Steel Structures. Butterworth – Heinemann, Elsevier, Oxford, 2009.
[7] M. Gkantau, G. Kososis, M. Theofanous, S. Dirar, Plastic design of stainless steel continuous beams. University of Birmingham, UK, 2018.