The role of symmetry in driven propulsion at low Reynolds number

Johannes Sachs\textsuperscript{1,2}, Konstantin I. Morozov\textsuperscript{3}, Oded Kenneth\textsuperscript{4}, Tian Qiu\textsuperscript{1}, Nico Segreto\textsuperscript{2}, Peer Fischer\textsuperscript{1,2} and Alexander M. Leshansky\textsuperscript{4}\textsuperscript{1}
\textsuperscript{1}Max Planck Institute for Intelligent Systems, Heisenbergstraße 3 70569 Stuttgart, Germany
\textsuperscript{2}Institute for Physical Chemistry, University of Stuttgart, Pfaffenwaldring 55, 70569 Stuttgart, Germany
\textsuperscript{3}Department of Chemical Engineering, Technion – IIT, Haifa, 32000, Israel
\textsuperscript{4}Department of Physics, Technion – IIT, Haifa, 32000, Israel
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We theoretically and experimentally investigate low-Reynolds-number propulsion of achiral planar shapes that possess a dipole moment and that are driven externally by a rotating magnetic field. Symmetry considerations (involving parity, $\hat{P}$, and charge conjugation, $\hat{C}$) establish correspondence between propulsive states depending on orientation of the dipolar moment. Highly symmetrical ($\hat{P}$-even) objects exhibit no net propulsion. Individual less symmetrical ($\hat{C}\hat{P}$-odd) propellers can efficiently propel due to spontaneous symmetry breaking, while a large ensemble of such motors would not propel on average. Particular magnetization orientation, rendering the shape $\hat{C}\hat{P}$-odd, yields unidirectional motion typically associated with chiral structures, such as helices.

Bacteria employ rotation-translation coupling when they spin their helical flagella in order to swim through fluids at low Reynolds (Re) numbers \textsuperscript{1}. It is also possible to spin an artificial magnetic corkscrew to generate propulsion \textsuperscript{2,3}. In both cases the rotation-translation coupling arises due to the symmetry-broken chiral shape. Purcell’s famous remark “Turn anything - if it isn’t perfectly symmetrical, you’ll swim” \textsuperscript{4} raises the question if a shape needs to be chiral to propel when it is spun at low Reynolds number. For a long time the geometric chirality of the object was taken for granted as a necessary condition for driven propulsion \textsuperscript{2,3}. In general, the symmetry of the object determines the structure of the resistance/mobility tensors \textsuperscript{15}. How-ever, for externally driven objects it is not sufficient to consider only the shape of the object, instead one needs to include the transformation property of its dipole moment \textsuperscript{18}. The first symmetry transformation that we consider is parity, described by the operator $\hat{P}$,

$$\hat{P}(x,y,z) \rightarrow (-x,-y,-z) \,.$$  \hfill (2)

Since the actuating magnetic field $\mathbf{H}$ is invariant under parity, any solution of Eqs. \textsuperscript{1} for a magnetic propeller must then be mapped to a valid solution under the action of $\hat{P}$. An object is called achiral ($\hat{P}$-even) if there exists a rotation $\hat{R}$ whose action on it is equivalent to the action of parity, and chiral otherwise. Thus merely rotating an achiral object through $\hat{R}$ would result in a parity-dual solution having the reverse propulsion velocity.

A rotating magnetic field, however, possesses an extra symmetry \textsuperscript{20}. Combining a rotation $\hat{R}_z$ by $\pi$ around the $z$-axis, with charge conjugation $\hat{C}$ maps $\mathbf{H}$ to itself and is thus a symmetry \textsuperscript{21}. It follows that $\hat{R}_z\hat{C}\hat{P}$ is a symmetry too. We shall call an object $\hat{C}\hat{P}$-even if there exists a rotation $\hat{R}$ whose action on it is equivalent to the combined action of parity and charge conjugation, and $\hat{C}\hat{P}$-odd otherwise. Analogous to the $\hat{P}$-even object discussed above, one finds that changing the orientation of a $\hat{C}\hat{P}$-even object (by applying $\hat{R}_z$-rotation) would reverse its propulsion velocity. Note that the action of $\hat{C}$ on the propeller amounts to reversing its dipole moment.

We apply the above symmetry arguments to planar V-shaped objects, schematically depicted in Fig.\textsuperscript{1} and then

$$\mathbf{U} = \mathbf{G} \cdot \mathbf{L}, \quad \mathbf{\Omega} = \mathbf{F} \cdot \mathbf{L}, \text{ (1)}$$

where $\mathbf{U}$ and $\mathbf{\Omega}$ are the object’s translation and rotation velocities, $\mathbf{G}$ the rotation-translation coupling mobility (pseudo-)tensor and $\mathbf{F}$ is the rotational mobility tensor. Typical solutions in a rotating magnetic field $\mathbf{H} = H(\mathbf{x}\cos \omega t + \mathbf{y}\sin \omega t)$ (provided that the frequency $\omega$ is not too high) result in the object turning synchronously with the field while advancing along the field rotation $z$-axis. It was found that there can be up to two stable synchronous solutions of Eqs. \textsuperscript{1} \textsuperscript{17}.
test these predictions experimentally. Note that the V-shape (ignoring its dipole moment) is a highly symmetrical object with two mutually perpendicular symmetry planes. In the frame of principal rotation axes aligned with eigenvectors of $\mathcal{F}$ [see Fig. 1(b)] it therefore has only two nontrivial component of $\mathcal{G}$ ($\mathcal{G}_{23}$ and $\mathcal{G}_{32}$) \cite{17}.

Consider first a V-shaped structure with a magnetic moment $\mathbf{m}$ oriented perpendicular to the plane of the object [see Fig. 1(a)]. This object is $P$-even as well as $\hat{C}\hat{P}$-even. Denoting by $\hat{R}_i$ rotation by $\pi$ around the $i$th principal rotation axis, one can see that the object is mapped onto itself under $\hat{R}_2\hat{P}$, $\hat{R}_1\hat{C}$ and $\hat{R}_3\hat{CP}$ \cite{22}. When the magnetic moment lies in plane of the V-shape, e.g., directed along one of the arms as shown in Fig. 1(b), the object is $P$-odd, but $\hat{C}\hat{P}$-even as $\hat{R}_2\hat{C}\hat{P}$ maps it onto itself. The object in Fig. 1(c) with an off-plane orientation of the dipole, on the other hand, is odd under both $\hat{P}$ and $\hat{C}\hat{P}$; being invariant under $\hat{R}_1\hat{C}$ it is $C$-even.

Let us focus on the details of the propulsion. Fig. 1(c) illustrates the existence of two propulsive states related by the symmetry of a $\hat{C}\hat{P}$-even object. We assume arbitrary orientation of the V-shape with respect to the field so that its rotation could be accompanied by precession (or wobbling). As $\hat{C}\hat{P} \sim \hat{R}_2$, the $\hat{R}_2\hat{C}\hat{P}$-symmetry implies that $\hat{R}_2\hat{R}_2$-rotation leads to another solution. Since $\hat{C}\hat{P}$ inverts linear velocities, the two solutions have opposite propulsion velocities. Thus a large collection of such propellers having random initial orientations would at most exhibit symmetric spreading with zero ensemble averaged velocity, as if it was a racemic mixture having the same number of structures with opposite handedness \cite{23}.

Similar arguments can be applied to $\hat{P}$-even objects. A stronger result, however, can be obtained in this case. One can show that any achiral magnetic object with $\mathcal{G} \neq 0$ has its dipole moment $\mathbf{m}$ oriented along one of the principal rotation axes $\mathbf{e}_i$ [e.g., along $\mathbf{e}_2$, as in Fig. 1(a)]. This in turn can be shown to result in steady planar precession-free rotation, (or tumbling), of the V-shape about another $\mathbf{e}_i$-axis. [$\mathbf{e}_3$ in Fig. 1(d)]. It follows that $\mathbf{e}_i$, $\mathbf{L}$ and $\mathbf{L}$ are all parallel to $\hat{z}$, so that the net propulsion velocity $\mathbf{U} \propto G_{ij}\hat{z}$. Since achirality demands $G_{ii} = 0$, neither of the two tumbling solutions shown in Fig. 1(d) and related by $\hat{P} \sim \hat{R}_2$ yield any net propulsion \cite{24}. This reasoning, however, only holds for structures with a permanent dipole. As we shall demonstrate below, individual achiral ($\hat{P}$-even) polarizable propellers can propel, as parity symmetry only guarantees that their ensemble average velocity vanishes.

The $\hat{P}$- and $\hat{C}\hat{P}$-odd objects, on the other hand, exhibit enantiomeric selection of the propulsion direction even when averaged over arbitrary initial orientation. This is illustrated for the V-shape in Fig. 1(c) with the magnetic moment lying in the plane orthogonal to $\mathbf{e}_1$. The two solutions shown in Fig. 1(f) can be related by the symmetry $\hat{R}_2\hat{C}$. As $\hat{C} \sim \hat{R}_2$ the V-shape is $C$-even and applying $\hat{R}_2\hat{R}_1$ yields another valid solution. Since $\hat{C}$ (as opposed to $\hat{P}$ and $\hat{C}\hat{P}$) does not invert velocities, it guarantees that the two solutions possess the same propulsion velocity $\mathbf{U}$. Thus, the propulsion direction depends on the sense of rotation in exactly the same way as for left- or right-handed helices: the original object in Fig. 1(c) will translate along the field rotation $\mathbf{z}$-axis under clockwise (CW) rotation, while the corresponding enantiomer [$\hat{C}\hat{P}$-transformed object in Fig. 1(c)] propels in the opposite direction, regardless of the initial orientation \cite{25}.

We shall now experimentally demonstrate the different propulsion gaits associated with different symmetries. We perform low-Reynolds-number experiments with cm-sized magnetic structures immersed in glycerol, as well as $\mu$m-sized structures that possess either a magnetic or an induced electric dipole moment suspended in water. The larger objects allow for precise positioning and alignment of the magnetic moment. An arc-shaped structure with cross-sectional radius $a = 1$ mm was 3D-printed
FIG. 2. (a) Snapshots of the magnetically driven arc. The magnetic field rotates clockwise in the $xy$-plane with a frequency of 1.5 Hz resulting in the arc rotating and propelling along the $z$-axis. (b) Image of the arc; (c) a corresponding symmetry diagram; (d) arc displacement along the $z$-axis vs. time showing a constant speed of $U_z \approx 3.3$ mm/s (the scale bar length in (a) and (b) is 1 cm).

[see Fig. 2(b)]. It had a cubic compartment into which a small 1 mm$^3$ NdFeB (N45) ferromagnet was glued. The orientation of the magnet with respect to the object was therefore fixed and prescribed. The arc was placed in a cuvette filled with glycerol ($\eta \approx 1,000$ cP). The high viscosity prevented sedimentation and ensured that $Re \approx 0.5$.

A pair of two disk-shaped iron-based permanent magnets generated a homogeneous magnetic field of 300 G throughout the volume of the cuvette. The magnets were mounted and mechanically rotated in the $xy$-plane around the cuvette. The driven motion of the arc was recorded and analyzed. Results for a right-handed arc with an off-plane magnetization [as in Fig. 1(c)] actuated by a rotating field at frequency of 1.5 Hz are shown in Fig. 2. In Fig. 2(a) the position and orientation of the arc is depicted at different times; Fig. 2(d) shows the corresponding displacement of the arc’s centerpoint along the $z$-axis of the field rotation. The arc turns in-sync with the field and propels along the $z$-axis, as expected for CW rotation.

In Fig. 3(a) the scaled propulsion velocity $U_z/\omega a$ of an off-plane magnetized ($\mathcal{C}\mathcal{P}$-odd) arc is depicted vs. the actuation frequency $\nu = \omega/2\pi$. At low frequencies $\nu \lesssim 1.1$ Hz the arc tumbles without any noticeable translation. Above the tumbling-to-wobbling transition frequency, $\nu_{tw} \sim 1.1$ Hz, it starts to precess and propel along the $z$-axis. The direction of translation is controlled by the rotation sense of the applied magnetic field, thus the velocity of the structure in Fig. 3(a) is always positive. The velocity increases quasi-linearly with frequency, $U_z \sim \nu a (1 - \nu_{tw}^2/\nu^2)$, similarly to a magnetic helix up to $\nu \sim 1.6$ Hz in excellent agreement with the theory. For frequencies $\nu \gtrsim 1.6$ Hz the arc can no longer turn in-sync with the external field and exhibits asynchronous twirling accompanied by a negligible net propulsion.

In Fig. 3(b) the velocity-frequency dependence is shown for an arc with a magnetic moment oriented along one of the arms. This ($\mathcal{C}\mathcal{P}$-even) structure can spontaneously break symmetry and exhibit translation when actuated at $\nu > \nu_{tw}$. Symmetry demands, however, that for every initial orientation of the structure propelling in one direction, there exists an orientation for which it will propel in the opposite direction as the symmetry transformed counterpart. The experimental results showing a symmetric pitchfork bifurcation in Fig. 3(b) (symbols) agree very well with the theory and the arc can propel in $\pm z$ direction irrespective of the sense of magnetic field rotation. It has also been confirmed experimentally that upon reversal of the field rotation, the object maintains
its propulsion direction. Additional experiments with the achiral arc magnetized along the principal rotation axes (e.g., Fig. 1(a)) have been performed demonstrating no propulsion, as expected.

To demonstrate applicability of the symmetry considerations to the microscale objects, we used a physical vapor deposition method, known as glancing angle deposition (GLAD) to grow billions of magnetic microstructures on a wafer [2,28]. V-shaped SiO$_2$ microstructures containing a nickel section were grown onto silica beads. (see SEM image in the inset in Fig. 3). The growth direction is well-controlled during the GLAD process, which enables us to orient the V-shaped structures before they are magnetized. The desired magnetization was obtained by placing the wafer with the structures in an electromagnet (1.8 T) at a specific angle. Afterwards the V-shaped structures were removed from the wafer in an ultrasonic bath and dispersed in a solution of 150 μM poly(vinylpyrrolidone). A custom 3-axis Helmholz-coil setup was put up a microscope to generate a uniform magnetic field of 60 G, rotating at a frequency of 25 Hz in the $xy$-plane. Slight variation in the shape and the direction of magnetization of the colloids are expected. Two microstructures with symmetries shown in Fig. 1(a) and Fig. 1(b) were investigated and their translation along the $z$-axis of the field rotation was measured [30].

In Fig. 4 the displacement $\Delta z$ of several $C\hat{P}$-even structures [as in Fig. 1(b)] is plotted vs. time. They move in opposite directions with approximately the same speed of $U_z \approx 2.7 \mu m/s$, which is about one body-length per second. On the other hand, net propulsion of the achiral V-micropropellers is negligible, as is expected. These experimental observations are thus in agreement with the symmetry arguments above.

Finally, we demonstrate that propulsion can occur for structures of even higher symmetry. The GLAD technique was further applied to grow V-shaped SiO$_2$ microstructures with one arm being coated with a thin layer of gold, as shown in in Fig. 3(a). Au is highly polarizable and should, therefore, give rise to an induced electric dipole moment $p = \alpha \cdot E$ primarily along the Au-arm [see Fig. 3(b)] in electric field. After sonication in deionized water the suspension was placed in a four-electrode setup as schematically shown in Fig. 3(c). Applying sinusoidal $\pi/2$ out-of-phase potentials at the electrodes at 500 kHz results in a rotating AC electric field $E$ that exerts an electric torque on the V-shape, $L = p \times E \propto [n \times E] (n \cdot E)$, where $n$ is a unit vector along the Au-arm, causing its electro-rotation [31,32]. The resultant propulsion (see Figs. 3(d,e)) due to spontaneous symmetry breaking is similar to that of the $C\hat{P}$-even magnetic structure in Fig. 1(b) in spite of having a higher symmetry, i.e., being $P$- and $C\hat{P}$-even. This demonstrates that a truly achiral object can propel.

To conclude, the shape of an object together with its dipole moment determines its symmetry. In accordance with theoretical predictions we present experiments that demonstrate that the dipole moment affixed to geometrically achiral planar shape can render it chiral. Such chiral objects with intrinsically broken symmetry can exhibit steady unidirectional propulsion, resembling that of a helix, when actuated by a rotating field. In general, however, there could be two distinct in-sync rotational solutions possessing different propulsion velocities. For certain highly symmetric objects (e.g., $\hat{P}$- and $C\hat{P}$-even polarizable propellers) these two velocities average
to zero, while individual structures can be propelled efficiently due to spontaneous symmetry breaking, which is demonstrated herein. Our symmetry considerations can be extended to other shapes and can guide the development of micropropellers for low-Reynolds-number applications.

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* fischer@is.mpg.de
† hlisha@technion.ac.il

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[20] Here we only consider symmetries that map stable (stationary) solutions to stable solutions.
[21] Alternatively $\hat{R}_z$ can be replaced by a $\pi$-phase shift of the field, $t \mapsto t + \pi/\omega$.
[22] Notice that the symmetry arguments concerning the V-shape in Fig. 1(b) apply even if its two arms are unequal.
[23] A collection of less symmetric ($\hat{C}$-odd) propellers, e.g., with unequal arms would still exhibit nonzero ensemble average velocity.
[24] The fact that propulsion occurs only at $\nu > \nu_{\text{rev}}$, demonstrates its dependence on dynamics rather than just symmetry. In Figs. 1e,f the non-propulsive tumbling regime corresponds to vanishing of the angle between $e_1$ and $\hat{z}$.
[25] It is also possible to control the trajectory of the micropropellers by switching the external field rotation plane between all three principal planes, including, e.g., displacement out-of-focus of the microscope.
[26] The AC frequency of 500 kHz is definitely beyond the step-out which is typically some tens of Hz and a slow quasi-steady aperiodic rotation is anticipated in this case. General symmetry arguments should, however, hold regardless of the actuation regime.