Literature Review of various Fuzzy Rule based Systems

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Abstract: Fuzzy rule based systems (FRBSs) is a rule-based system which uses linguistic fuzzy variables as antecedents and consequent to represent the human understandable knowledge. They have been applied to various applications and areas throughout the literature. However, FRBSs suffers from many drawbacks such as uncertainty representation, high number of rules, interpretability loss, high computational time for learning etc. To overcome these issues with FRBSs, there exists many extensions of FRBSs. In this paper, we present an overview and literature review for various types and prominent areas of fuzzy systems (FRBSs) namely genetic fuzzy system (GFS), Hierarchical fuzzy system (HFS), neuro fuzzy system (NFS), evolving fuzzy system (eFS), FRBSs for big data, FRBSs for imbalanced data, interpretability in FRBSs and FRBSs which uses cluster centroids as fuzzy rule, during the years 2010-2021. GFS uses genetic/evolutionary approaches to improve the learning ability of FRBSs, HFS solve the curse of dimensionality for FRBSs, NFS improves approximation ability of FRBSs using neural networks and dynamic systems for streaming data is considered in eFS. FRBSs are seen as good solutions for big data and imbalanced data, in the recent years the interpretability in FRBSs has gained popularity due to high dimensional and big data and rules are initialized with cluster centroids to limit the number of rules in FRBSs. This paper also highlights important contributions, publication statistics and current trends in the field. The paper also addresses several open research areas which need further attention from the FRBSs research community.

Keywords: Fuzzy Systems, Genetic Fuzzy Systems, Neuro Fuzzy Systems, Hierarchical Fuzzy Systems, Evolving Fuzzy Systems, Big Data, Imbalanced Data, Cluster centroids
1 Introduction

Rule-based Systems [1] are generally used to model human problem-solving activity and behavior using classical IF-THEN rules. When the antecedent(s) of the rule is satisfied, then the rule will be fired. Classical approaches deal with the bivalent logic, but it fails to cover the imprecision and uncertainty present in the knowledge representation. Fuzzy sets [2] introduced by Zadeh are widely used to handle uncertainties and imprecision. Fuzzy sets with rule-based systems leads to Fuzzy Rule Based Systems (FRBSs, see table 1 for further abbreviation and their full name) [3], [4]. FRBSs considers fuzzy statements as antecedents and consequents, they can handle uncertainty and are more robust compared to the classical rule-based system. They use Linguistic Variables [5]–[7] to represent features in antecedents, whose values are context dependent on the membership function of the feature. Rules generated by FRBSs are human interpretable and can be used to understand the working of the system.

FRBSs (or fuzzy system(s)) have been used for more than 40 years in many areas of computer science and engineering. There exists 45,287 articles on fuzzy system(s) in the Scopus database (https://www.scopus.com/). Scopus has comprehensive data coverage, in case of fuzzy systems, scopus has the repository of papers from all the major publication venues such as IEEE Transactions on Fuzzy Sets and Systems (TFS), WCCI/IEEE Fuzzy conference, Fuzzy Sets and Systems (FS&S) and many more. Despite little fluctuation, the number of articles published in the area of fuzzy systems is constantly growing which shows the growing attention of research community in FRBSs (see fig. 1a). Fig. 1b shows the area-wise percentage of the published articles in fuzzy systems highlighting the major area of research in FRBSs.

| Abbreviation | Full name                | Abbreviation | Full name         |
|--------------|--------------------------|--------------|-------------------|
| FRBSs        | Fuzzy Rule Based Systems | MFs          | Membership functions |
| GFS          | Genetic Fuzzy System     | KB           | Knowledge base    |
| HFS          | Hierarchical Fuzzy System | RB           | Rule base         |
| NFS          | Neuro Fuzzy System       | DNN          | Deep neural network |
| eFS          | Evolving Fuzzy system    | GAs          | Genetic Algorithms |
| TSK          | Takagi-Sugeno-Kang       | LVS          | Linguistic variables |

Table 1: Used abbreviation and their full name.

\[1\] In this article, we have considered interpretable and explainable as equivalent since they both imply human understandability.
Figure 1: (a) Number of published articles in the area of fuzzy systems throughout the years (up to 2021) (b) Area-wise categorization for the published articles.

The main objective of this article is to provide a systematic literature review of FRBSs in the various areas between the years 2010-2021. There exists many literature review in the literature such as; [8] reviews the structure and parameter optimization for NFS which was further considered in [9] and [10]; terminologies and usage og GFS has been reviewed in [11]; HFS and their advances were reviewed in [12] and [13]; online learning in fuzzy systems was reviewed in [14], [15]; a review of FRBSs for GFS, NFS and HFS is given in [16]; Explainability perspective of FRBSs has been reviewed in [17]. In this article, we have considered all the major extensions and areas where FRBSs has been used in the past 12 years such as FRBSs in big data, imbalanced data, cluster centroids as rule which are not considered in any other review articles. Our work can be considered as an extension of [16] and [17] where we identify the trends and future direction related to all the major areas of FRBSs based on the number of published articles and the citations related to them.

In this paper, we have identified 9 key areas where FRBSs have contributed significantly. We present systematic literature review for Genetic/Evolutionary Fuzzy Systems [18], Hierarchical Fuzzy Systems [19], Adaptive Neuro Fuzzy Inference System [20], Evolving Fuzzy Systems [21], FRBSs for Big Data [22], FRBSs for Imbalanced Datasets [23], Clusters centroids as rules in FRBSs [24] and FRBSs for Interpretability/Explainability [25]. The paper is organized as follows: section-II describes the classical FRBSs with its variants, section-III provides the systematic literature review for the above-mentioned areas, section-IV provides some current research trends along with open problems for fuzzy systems, and section-V presents the conclusion.
2 Fuzzy Rule Based Systems (FRBSs):

Classical fuzzy systems are based on Mamdani\cite{Mamdani} approach, its generic structure is shown in fig. \ref{fig:frbs}. The fuzzification module establishes a mapping between real-valued input data to fuzzy values based on some membership function. Similarly, the defuzzification module establishes a mapping between fuzzy values to real-valued output domain. The data base contains the linguistic variables (LV) and the specific membership function associated with them. Rules are the usual way to organize knowledge in natural language. Usually, Database and Rule base are part of the Knowledge base (KB). Mamdani’s rules involves the use of LVs in antecedent and consequent. Rules are represented by the set of linguistic variables and an output associated with them i.e., a rule can have multiple inputs and a single output. The rule base contains the set of rules for the specific application. Rule base can be represented in many ways. Rule base either has a list of rules or a decision table which is a compact representation of rules. The fuzzy inference engine infers the fuzzy output from the inputs according to the rules generated in the Rule base.

Mamdani’s FRBSs suffers from some drawbacks which were first highlighted in \cite{26,27} due to the use of LVs \cite{17}:

1. Lack of flexibility due to rigid partition of input and output space.

2. It is difficult to find fuzzy partition when input variables are mutually dependent.

3. The homogeneous partition between input and output is inefficient and does not scale well.
4. The size of KB increases rapidly with the increase in the number of variables and linguistic terms for each variable.

There exist multiple variants of the classical fuzzy rule-based system which address these issues in FRBSs:

1. Mamdani FRBSs with input output scaling: Transforming the inputs and outputs to introduce more flexibility was introduced in [28]. The authors introduce a simple linear scaling function of the form below:

\[ f(z) = \lambda \cdot z + c \]

Here \( \lambda \) is the scaling factor and \( c \) is the offset for the variable \( z \). Non-linear scaling is also possible, which may be of the form:

\[ f(z) = \text{sign}(z) + |z|^\alpha \]

Non-linear scaling factor (\( \alpha \)) is used to control the sensitivity in the origin region. This way each n-dimensional rule in the FRBSs will be of the form:

\[ \text{IF } f(x_1) \text{ is } l_{1i} \text{ and } f(x_2) \text{ is } l_{2i} \text{ and ... and } f(x_n) \text{ is } l_{ni} \text{ then } y \text{ is } Y \]

where \( l_{ij} \) is the \( j^{th} \) linguistic variable for the \( i^{th} \) feature.

2. DNF Mamdani Fuzzy rule-based system [29]: Here, each variable in the rule takes its value as a set of linguistic terms i.e. if \( X_i \) is a variable, and it’s term set is \( \{l_1, l_2, l_3\} \) then in a rule variable \( X_i \) can belong to the set \( \{l_1, l_2\} \). The variable can belong to the set of linguistic terms in a rule. This helps in reducing the number of rules to avoid the problem of increase of size. Here, a rule may be of the form:

\[ \text{IF } x_1 \text{ is } \{l_{11}, l_{12}\} \text{ and } x_2 \text{ is } \{l_{23}\} \text{ and ... and } X_n \text{ is } \{l_{n1}, l_{n2}\} \text{ then } y \text{ is } Y \]

3. Approximate Mamdani Fuzzy Rule-based Systems: The DNF FRBS includes several items of term set which can reduce the interpretability of the DNF FRBS. The approximate FRBS [30] are able to obtain better accuracy at the cost of losing interpretability. Each rule in approximate FRBS has its own fuzzy set instead of using linguistic terms. This approach generates semantic free rules, have higher power of expressiveness due to the use of different fuzzy sets in each rule; it can adopt different number of rules depending on the complexity of the problem. In terms of drawback, approximate FRBSs suffers from the loss of interpretability, and also it can overfit the training data and perform poorly in case of unseen data.
4. Takagi-Sugeno-Kang Fuzzy Rule-Based Systems: Takagi-Sugeno-Kang (TSK) FRBS considers a different form of rules, each rule in TSK-FRBS contains LVs as antecedents and a function of inputs as consequent. TSK FRBS models the output as a function of inputs and hence do not need defuzzification process. This variant has been preferred in many applications where efficiency is of utmost importance. TSK-FRBS splits the input space into several fuzzy sub-space based on the relationship between input and output. The main drawback of TSK-FRBS is its inability to provide interpretability for its input-output relationship. Here, the rule structure looks like:

\[
\text{IF } x_1 \text{ is } l_{1i} \text{ and } x_2 \text{ is } l_{2i} \text{ and ... and } x_n \text{ is } l_{ni} \text{ then } y = p_0 + p_1 * x_1 + ... + p_n * x_n
\]

5. Multiple Input Multiple Output (MIMO) FRBSs: Here, as the name suggests, there are multiple outputs in a single rule rather than a single output variable in the previous variants of FRBSs. Outputs here are considered independently of each other and computed separately. The rule structure looks like:

\[
\text{IF } x_1 \text{ is } l_{1i} \text{ and } x_2 \text{ is } l_{2i} \text{ and ... and } x_n \text{ is } l_{ni} \text{ then } y_1 \text{ is } Y_1 \text{ and ... } y_n \text{ is } Y_n
\]

6. Fuzzy Rule Based Classification Systems (FRBCSs): Fuzzy Rule Based Classification Systems (FRBCSs) is a system which uses fuzzy rules as a learning medium. In classical FRBSs, inputs are mapped to usually 1-D output but in FRBCSs inputs are mapped to one of the class labels. The rule structure looks like:

\[
\text{IF } x_1 \text{ is } l_{1i} \text{ and } x_2 \text{ is } l_{2i} \text{ and ... and } x_n \text{ is } l_{ni} \text{ then } y = c
\]

3 Literature Review

In this section, we are providing the systematic literature review (SLR) for Genetic/Evolutionary Fuzzy Systems, Hierarchical Fuzzy Systems, Adaptive Neuro Fuzzy Inference System, FRBSs for Big Data, FRBSs for Imbalanced Datasets, Clusters centroids as rules in FRBSs and FRBSs for Interpretability/Explainability. For each of the following subsection, we have considered articles in the Scopus database. The statistics related to each type of FRBSs is computed by finding the their name in the title or in the keywords. Areas which has seen growing publication and citation has been considered for trends in each type of FRBSs.
Exclusion Criteria: The following criteria have been used to exclude the article from SLR

- Articles not concerning Fuzzy rule-based System or Fuzzy Systems.
- Unpublished articles.
- Not written in English.
- Articles not published in the years 2010-2021.

3.1 Genetic/Evolutionary Fuzzy Systems:

A GFS is a type of Fuzzy Rule Based System which employs evolutionary algorithms \cite{35} such as genetic algorithm \cite{36}, genetic programming \cite{37}, etc. for learning purpose in fuzzy system. Designing FRBSs can be seen as a search problem, e.g., finding the set of rules, tuning the membership parameters, etc. GAs (EAs) are known to handle the large search space for near optimal solution with a performance measure. Apart from the ability to explore large search space, GAs can incorporate prior knowledge, in case of fuzzy systems, prior knowledge can be a number of rules, membership function, linguistic variables and so on.

In GFS, the first step is to identify the areas of FRBSs which can be optimized using GAs or EAs. Fig. 3 shows the areas in which GAs can be used for learning. In general, learning in FRBSs is of two types: learning the KB components and tuning the parameters. Learning KB components can be further divided into four categories namely, rule learning, rule selection,
learning DB components, and learning KB simultaneously. Rule learning approaches learns the rules automatically from the data for a predefined DB. [38] presents the classical proposal on tuning. The generated number of rules can be huge, rule selection selects the best set of rules. [39] presents the first contribution in the field. Learning of DB components involves learning shape of membership function, number of fuzzy partitions and other DB components, [40] is the pioneer paper in the field. Learning KB components involves learning DB components and rules simultaneously, this leads to large search space which makes learning more difficult. [41] presents the first methodology. Tuning of the parameters is done to improve the system’s performance. Tuning can be done for adjusting the membership function parameters, for adjusting inference systems parameters and for adjusting weights in defuzzification methods. GFS suffers from the disadvantage of genetic algorithms which computational intensity i.e. GAs search for a solution from a broad perspective and may take significant cost to get to the solution.

In the years 2010-2021, several important contributions such as in Hadavandi et al. [42] have used GFS with Self-organizing maps clustering for next day forecasting method; Elhag et al. [43] introduced a pairwise learning framework with GFS have been proposed for intrusion detection; GM3M index proposed by Gacto et al. [44] is a geometric mean for three metrics accuracy, maximizing semantic interpretability, minimizing rule complexity, the methodology uses SPEA2 MOEA (Multi-objective Evolutionary Algorithm) to generate pareto optimal solutions for the three metrics; methodology presented by Alcala et al. [45] deals with the problem of regression in high dimensional data, they also focus on developing simpler rules by adjusting linguistic fuzzy partitions; and in Sanz et al. [46] interval-valued fuzzy sets were used to model the linguistic labels, amplitude for IVFSs was tuned using weak ignorance theorem.

There exists very few review papers in the literature for genetic fuzzy systems. First review paper by Cordon et al. [47] talked about the GFS models, its application, trends, and open questions. A short overview of GFS models was also presented in Herrera et al. [48]. In [49], Herrera reviewed the taxonomy, GFS models, advances in the field of GFS, trends and the future areas related to GFSs. Review for learning Mamdani-type fuzzy rule-based system was proposed in Cordon et al. [50], here the paper focused on improving accuracy while designing interpretable GFSs, [50] focuses on aspects other than accuracy e.g. interpretability, reducing the complexity of the model etc. Synthesis of eFS with special focus on Genetic programming based eFS was given in Koshiyama et al. [51]. In Fernandez et al. [11],
The authors have presented an overview of Evolutionary fuzzy Systems, their terminologies, applications, areas where evolutionary approaches are useful and knowledge base generation.

Table 2 shows top conference/journals in FRBSs and the number of articles published in them for GFS during the years 2010-2021. Fig. 4a shows the trend for the published articles, as it can be seen, the number of articles are decreasing (with some fluctuations). Fig. 4b shows the area-wise contribution in the field of fuzzy systems with the field of computer science having the highest number of articles. Fig. 4c shows the citations for the articles published in GFS.

Recent trends in GFS:

In the recent years, the researchers in the field of GFSs have focused in the following areas:

1. **Multi-objective GFS:** There has been many papers in the field of multi-objective GFS where accuracy along with other metrics such as interpretability, complexity are used as objectives. In [52], a multi-objective ant colony optimization has been used to tune IT2FLS parameters for the performance of Hexapod robot, a multi-objective eFS for intrusion detection has been proposed in [54], in [44] SPEA2 MOEA was used to obtain pareto fronts between different objectives.

2. **Hybrid GFS/eFS:** Here, hybrid form genetic/evolutionary approaches with other methodology is used for the learning purpose. In [55], a genetic fuzzy neural network approach has been given which combines the benefit of neural networks (NN) with GAs. In [56], variants of ANFIS with several evolutionary approaches were presented for the prediction of wildfire probability.

3. **Variants of fuzzy sets in GFS:** There exists several fuzzy sets in the literature such as intuitionistic fuzzy set, rough set, type-2 fuzzy
Figure 4: (a) Published article in the field of Genetic Fuzzy Systems between 2010-2021 (b) Area-wise categorization for the published articles (c) citations for the published articles.
sets. Recently, significant focus has been devoted in developing other fuzzy set variants of GFS to capture more uncertainty present in the data. A type-2 evolutionary TSK fuzzy system is presented in [57] which learns parameters and footprint-of-uncertainty from scratch. A rough-set based variant of GFS was presented in [58] for heart disease diagnosis.

4. **Applications of GFS**: There has been many areas where GFS has been making its marks, e.g. for modelling landslide susceptibility in [59], for forecasting $PM_{10}$ in [60], prediction in heart disease in [61] and many more areas.

3.2 **Hierarchical Fuzzy Systems**

Conventional fuzzy rule based systems suffers from the curse of dimensionality i.e., with the increase in the dimensionality the rule base can be huge.
Consider an example of a system which models data with 10 attributes where each attribute can be represented by 5-7 linguistic variables, there are about 10-28 million possible number of rules. In the age of big data, no of attributes are not limited to 10, hence conventional (flat) fuzzy systems are highly infeasible for high dimensional data. Hierarchical Fuzzy Systems were first proposed in [62] to overcome the curse of dimensionality. HFS is composed of several low-dimensional fuzzy systems in a hierarchical way. HFSs have also been proved to be Universal approximators [63]. Rules in Hierarchical fuzzy systems are grouped into modules (low-dimensional fuzzy systems) as per their roles in the system. Each module computes partial solution which is further passed onto the next level modules. Although each module is a fuzzy system, it generates significantly less number of rules than flat fuzzy system. However, this leads to the reduction in the interpretability of HFS as it would not be possible to interpret each intermediate HFS subsystem.

In general, there are three types of Hierarchical fuzzy systems namely, Serial, Parallel and Cascaded Hierarchical Fuzzy System, their structure is given in figure 6. In serial hierarchical fuzzy systems (6a), output from the previous modules are fed as one of the input in the next module. Here in every stage there is only one fuzzy system as module which has one input from the previous layer module along with one or more input variable, this process is continued until all the input variables are used. In parallel (or aggregated) hierarchical fuzzy systems (6b), the lowest level modules serve...
as the input for the entire structure. Output of the first level serves as the input of the second level modules, this process continues until the last module whose output serves as the output of the system. In cascaded hierarchical fuzzy system (6c) each stage is a module which takes all the inputs as parameters, the output of stage 1 module provides the input for stage 2 module. Cascaded HFS are hybrid of FRBSs and Neural networks [64] but since it uses all of the input variables it loses the benefit of reducing the number of rules, hence the field has not progressed much in the past years.

In the years 2010-2021, several important contributions in the field of Hierarchical Fuzzy Systems have been made such as Juang et al. [65] provides a particle swarm optimization method for hierarchical fuzzy systems; in Zhang et al. [66] a HFS optimized with GAs to develop robust and accurate traffic prediction system in intelligent transportation problem; Fares et al. [67] presents a HFS based framework for detecting water mains system; Lopez et al. [68] a hierarchical FRBS for classifying imbalanced data was developed; in Qu et al. [69] trust evaluation system using HFS was developed for Infrastructure-as-a-service cloud.

The work done in Wang et al. [70] provides rough analysis and design of HFS, it also highlights that HFSs can also inherit the curse of dimensionality to obtain uniform and universal approximation. In [12], a review of HFS from the perspective of complex systems was done, the paper also highlights if the functions are not decomposable then it may not be possible to generate HFS but for some functions HFS is an universal approximator. Review on the approximation capabilities of HFS was done in [71], for any continuous function with natural hierarchy, HFS can approximate the system for the desired level of accuracy. A survey paper on HFS which presents the motivation, current trend and open problems in HFS has been presented Di et al. [13]. A little work has been done to interpret the intermediate variables in HFS, [72] shows that HFS only improve the interpretability when it is capable of decoupling the system into subsystems which are interpreted independently.

Table 2 shows top conference/journals in FRBSs and the number of articles published in them for HFS during the years 2010-2021. Fig. 5a shows the trend in number of publications in the years 2010-2021, there hasn’t been much growth in terms of research articles in the field of HFS. Fig. 5b shows the areas where HFS papers were published, most of the papers are in the areas of Computer Science, Engineering and Mathematics. Fig. 5c shows the citations for the articles published in the years 2010-2021, the increasing trend signifies the work done has been used or improved constantly.

Recent trends in HFS:
In the recent years, the researchers in the field of HFS have focused in the following areas:

1. **Interpretable HFS**: Interpretability is one of the main reasons to advocate for the use of FRBSs. Recently many researchers are working towards making interpretable HFS such as in [73] the comparison between hierarchical and serial topology of HFS has been compared for complexity and interpretation based on Seesaw method, in [74] the framework to measure the interpretability with participatory user design for user specific applications was proposed, and many more.

2. **EA based HFS**: The structure to choose for HFS is a research problem. In cases where relationship between variables are unknown, search space of possible HFS models is huge. EAs in general are great at finding near optimal solution in huge search space. In case of HFS as well, this has been an active area of research such as in [75] PSO-based interval type-2 HFS was proposed for real-time travel route guidance, in [76] a PSO based HFS for reference evapotranspiration prediction was proposed.

3. **Type-2 HFS**: Type-2 fuzzy sets is an extension of type-1 to handle more uncertainty present in the data. Researchers have been working towards extending type-1 HFS to the variants of type-2 HFS e.g., in [77] a variable selection method for an interval type-2 hierarchical fuzzy system has been presented; in [78] a multi-agent architecture for type-2 beta hierarchical fuzzy system which considers different agents to optimize the structure, tune the parameters for improving accuracy and interpretability and many others.

4. **Applications of HFS**: There also exists many areas where HFS finds its application, e.g. for the diagnosis of dengue in [79], for performance assessment in the area of construction industry [80], and many other applications.

### 3.3 Neuro Fuzzy System

Neuro Fuzzy Systems (NFS) was first introduced in [20] which presents the fusion of FRBSs with Artificial Neural Network (ANN). FRBSs are capable of handling uncertainty present in the data along with providing interpretable reasoning which can be understood by humans, but FRBSs lacks the ability to learn the rules on its own. On the other hand, ANN is considered a blackbox which can learn from the data while not able to provide the reasoning
behind the learning. The basic idea behind NFS is to fuse the human-like reasoning ability of FRBSs and learning ability of ANN. There exists two common ways for extracting rules in NFS: cooperative NFS and concurrent NFS. In cooperative NFS, NN computes the FRBSs parameter from training data and FRBSs generates the interpretable rules; in concurrent NFS, NN work together with FRBSs continuously to create the model. NFS can be seen as either self-adaptive or dynamic learning system. Automatic tuning of membership function parameters, learning the structure of NFS or both from the data is done in self-adaptive NFS. In dynamic learning NFS, NFS learns about parameters or structure from samples rather than complete training data at once. However, with the black-box nature of neural network NFS loses on its interpretability.

NFS can have at most 7 layers as shown in Fig. 7. Input, Membership, rule, Normalization, Term, Extra and Output layer. The first four layers (L1-L4) are responsible for tuning and structuring antecedent of the rule while the others (L5-L7) are used to tune the consequent part. In Fig. 7, \( \rightarrow \) represents partially connected layers, \( \rightarrow \) represents fully connected layers. Input is passed through the input layer (L1) without any manipulation. It provides input to either membership layer (L2) or rule layer (L3). Input layer is partially connected to membership layer (or rule layer) because each input variable is connected only to their partitioned fuzzy sets. In membership layer (L2), a problem specific membership function is employed which computes the membership degree corresponding to the fuzzy sets. Usually, a T-norm between the inputs from the input layer or membership layer is done in the rule layer (L3). The partial connection with the previous layers allows rule layer to learn the structure of the rules, the no. of nodes in the rule layer determines the no. of rules generated by NFS. The normalization layer (L4) in NFS computes the firing strength of each rule. The term layer (L5) computes the consequent(s) of the rule, and has the same no. of nodes as rule and normalization layer. The additional layer (L6) is used to map the input from L5 to a polynomial function which is not very common in NFS research community. The output layer generates the final output of the fuzzy system, in general it is a summation of all the input values from layer.
L6 (or L5). NFS architecture has at least 3 no. of layers (L1-L3-L7 layers) and maximum 7 no. of layers (L1...L7 layers). NFS architecture with higher no. of layers offer higher presence of FIS components which also leads to higher efficiency but at the same time may decrease the interpretability of NFS.

In the literature of NFS, there has been several important contributions during the years 2010-2021 such as in Abiyev et al. [81] a type-2 NFS has been designed for the identification of time-varying systems and uses clustering methods to equalize time varying methods; in Subramanian et al. [82] NFS has been used in the metacognitive framework to develop metacognitive FRBS; in Cervantes et al. [83] a TSK based NFS for the identification in non-linear system has been proposed; in Chen et al. [84] a hybrid version of NFS with two evolutionary algorithms for the modelling of landslide susceptibility; in Feng et al. [85] a method to incorporate broad learning in NFS has been proposed for regression and classification problems; the work in Deng et al. [86] fuses fuzzy logic with NN in hierachical fashion to create robust classification method and many more.

One of the first review for NFS was done in [87] where three NFS models were reviewed keeping the model simple and same fuzzy semantics. Following the same principle as in [87], [88] also reviews the NFS models based on NEFCON-model. In [89], a review of NFS for non-linear system identification has been presented, they also highlights modelling of NFS is rather a complex procedure and requires user interaction constantly. The review for various methodologies and applications for NFS during the years 2002-2012 has been done in [90]. In [91], pros and cons of the optimization techniques using derivative, non-derivative and hybrid approaches for type-2 NFS has been presented. Recently, a compreensive review of the architecture of the NFS along with the review of the optimization approaches has been presented in [92]. There has been several articles which reviews the use of NFS for various applications e.g. [93] for building energy consumption, [94] for estimating power coefficient in NFS etc.

Table 2 shows top conference/journals in FRBSs and the number of articles published in them for NFS during the years 2010-2021. Fig. 8a shows the trend in number of publications in the years 2010-2021, the research field is seeing increasing no of papers in the field of NFS each year. Fig. 8b shows the areas where NFS papers were published, most of the papers are in the areas of Engineering, Computer Science and Mathematics. Fig. 8c shows the citations for the articles published in the years 2010-2021, the increasing trend signifies the work done has been used or improved constantly.

**Recent trends in NFS:**
Figure 8: (a) Published article in the field of Neuro-Fuzzy Systems between 2010-2021 (b) Area-wise categorization for the published articles in NFS (c) citations for the published articles.
In the recent years, the researchers in the field of NFS have focused in the following areas:

1. **Explainable NFS:** With the increase in the use of AI/ML in day-to-day life, Explainability/Interpretability is one of the key areas to improve and many researchers are also working towards making explainable/interpretable NFS such as in [95] a deep type-2 FRBSs with the special focus on high dimensional data; in [96] a NFS which uses CNNs to extract features and then uses fuzzy classifiers to generate interpretable results; and many more.

2. **EA based NFS:** Evolutionary Algorithms presents an interesting direction as meta-heuristic approach for the optimization of NFS such as in [97] PSO has been used to for the parameter tuning of NFS to predict the concentration of benzene in the air; similarly in [84] PSO was used in NFS for modelling landslide susceptibility.

3. **Hybrid NFS:** NFS is a fusion of FRBSs with NN. There has been increasing interest for other hybrid variants of NFS such as in [86] a fused hierarchical DNN for classification in image segmentation; in [98] fuzzy c-Means clustering has been used to determine the labelling of the attributes and then NFS was employed for the classification in the traffic management and many others.

4. **Applications of NFS:** There also exists many areas where NFS finds its application, e.g. for modelling of landslide susceptibility in [84], for time series forecasting in [99], and many other applications.

### 3.4 Evolving Fuzzy Systems

Nowadays, there is an increasing demand for time-varying systems, systems which can update themselves with the arrival of new data. Data streams are one of the main reasons to look for evolving, adaptive models. Evolving Fuzzy systems (eFS) are a type of FRBSs which can self-adapt the parameters and the structure of the rule with the incoming data stream in online mode. eFS determines whether the incoming data stream can be used to update the parameters, structure, generate a new rule or delete a rule. An on-line clustering between current data and the existing rule is used to check if a new rule will be generated or will be used to update the input space partitioning. A new rule is generated if the new data is some threshold apart from its nearest rule. A rule needs to be deleted before adding a new rule if the rule base already has predefined maximum number of rules [100].
The rule with the lowest potential is deleted, potential here represents how much the rule is representative of the data. Consequent of the rule is updated with the incoming data stream, incoming data can also be an outlier, so in general the consequent is updated in a recursive fashion as done in [101]. Incremental learning is one of the common approaches for training and testing incoming data streams. Incremental learning should take care of the noise and the concept drift with the incoming data. Noise and concept drift represents the change in the relationship between the input vector and output(s). eFS uses the evolving clustering to detect the concept drift and then update the rule antecedent, and consequent along with managing the number of rules. However, clustering is susceptible to the outliers and noise, other robust algorithm may present interesting work direction.

The eFS can be any of the standard FRBS, HFS, NFS which incorporates incremental learning approach to evolve itself with the incoming data stream. In eFS for standard FRBSs, evolving clustering methods are used to updating the antecedents; addition or removal of the rules in rule base. One such method was proposed in [102] for TSK-FRBSs that updates its rule base with the new incoming data using online clustering method. The evolvable extension of NFS, are called adaptive or self-evolving NFS methods. In adaptive NFS, models learns to update antecedent part using evolving clustering methods and simultaneously weight for the consequent(s) is also updated. In tree like evolvable structure of FRBSs, with the incoming data stream model from a set of neighbour is chosen if the performance of the current model is not optimal as done in [103]; in [104] a incremental learning approach for fuzzy evolving linear tree which updates its leaves with subtree based on model selection test to improve the model’s performance is presented.

There has been several important contribution in the field of eFS which have received attention by the research community during the years 2010-2021 such as in [102] a multi-input multi-output extension of evolving Takagi-sugeno called eTS+ methodology which is recursive (suitable for data streams); allows to shrink or expand the rule base based on the age and potential of the rule and also allows to chose variable selection for incoming data stream; a methodology to deal with concept drift and shift in on-line data stream has been presented in [105]; in [106] a self-evolving recurrent fuzzy neural network for prediction in time-varying systems has been proposed, unlike TSK model the presented methodology considers non-linear relationship between the input variables; in [107] a model which can learn rule base from scratch, and based on the incoming data stream it can easily add or remove the rules, the contribution of the rule in the output is the major criteria for addition or removal of the rule; generalized eFS model which considers projection of
high dimensional input data into a single fuzzy set has been presented in [105] which is also more interpretable, features in this model can be removed from the rules depending on their relevance, and added in the later stage if needed; a multivariable gaussian eFS model [108] which uses multi-variate membership functions for fuzzy sets and uses recursive clustering methods for its robust behaviour. There hasn’t been many review articles for eFS, in [109] a systematic overview for eFS has been presented, they have broadly categorized eFS into two categories: evolvable standard FRBSs and evolvable NFS. This has been the case in [16] as well where a brief review of learning and implementation approaches for eFS has been discussed.

Table 2 shows top conference/journals in FRBSs and the number of articles published in them for eFS during the years 2010-2021. Fig. 9a shows the trend in number of publications in the years 2010-2021, the trend for the number of papers hasn’t been consistent but overall there is an average of 45 papers in the field of eFS each year. Fig. 9b shows the areas where
eFS papers were published, most of the papers are in the areas of Computer Science, Engineering and Mathematics. Fig. 8c shows the citations for the articles published in the years 2010-2021, the increasing trend for eFS has seen some fluctuations in the recent years but the interest in the development and use of eFS is growing.

**Recent trends in eFS:**

In the recent years, the researchers in the field of eFS have focused in the following areas:

1. **Neuro-eFS:** Evolvable NFS has been one of the prominent approaches in the field of eFS in the recent years where online approaches are used to incorporate data stream for the purpose of tuning or learning the structure of NFS such as in [110] a self-evolving interval type-2 NFS has been presented for system identification and control problem, the architecture present here learns the optimal structure of NN with the incoming IT2 data; similarly in [106] recurrent approaches along with online structure learning algorithms has been used and many other approaches.

2. **Structural Updates in eFS:** The purpose of Evolvable FRBS is to learn the structure, update the rule base with the incoming new data such as in [111] a compression layer has been used for the antecedents to eliminate the irrelevant information from the rules which results in higher generalization; in [112] a structural evolving approach to improve the accuracy-interpretability trade-off has been presented, the study also present several methodologies to reduce the number of rules in the model.

3. **Type-II eFS:** Type-II fuzzy sets is an extension of type-I fuzzy sets whose membership grade itself is a fuzzy set. There has been increase in interest for type-II variants of eFS in the recent years such as in [110] is a type-2 evolvable NFS; in [113] an evolvable type-II eFS model which considers long-term potentiation if the incoming data stream enhances the previous knowledge and long-term depression if the new data degrades the previous knowledge to update the FRBS model and many more.

4. **Applications of eFS:** There also exists many areas where eFS finds its application, e.g. for detection of computer worms in [114], for non-linear system identification in [110], for thermal modelling of power transformers [115] and many other applications.
3.5 FRBSs for Big Data

The amount of data collected in today’s world is huge due to internet, mobile devices, social media and many other sources, this huge collection of data is termed as big data. There exists several definitions of big data: some considers big data if has 3Vs to 5Vs \cite{116,117} (Volume: huge data, Velocity: high speed of data creation, Variety: data has diversity, Veracity: Data must be reliable and Value: worth of the collected data); some define big data as the data which can not be processed in a single machine \cite{118}. FRBSs are known for to generate interpretable models but FRBSs struggle with the scalability issue, so it does not perform well for big data. To solve this issue, researchers have been working over generating distributed, parallel models in the past decade, hence the no of publication in the field has significantly increased \cite{11a}. The basic idea is to use distributed computing, where data is distributed among several computing nodes. At each node, the splitted data is used to build FRBS model and the output generated from each node is aggregated to get the final output as shown in fig 10. The problem of interpretability with high dimensional data in case of big data is still a major drawback of fuzzy systems in big data.

Mapreduce by Dean et al. \cite{119} is one of the common approaches for distributing the large data into a cluster of nodes. Mapreduce has two primary steps, Map and Reduce. First, the Map function distribute the data into nodes and computes some intermediate result then Reduce aggregates the intermediate result into the final output of the model. Chi’s FRBS \cite{34} is compatible with Mapreduce paradigm, the first model which uses Chi’s method with Mapreduce called CHi-FRBCS-BigData was given in \cite{120}. Initially, fuzzy partition of each feature in the database is created based on the level of chosen granularity. Then the whole dataset is distributed among the n computing nodes. In second stage, each of the node independently computes its rule base based on Chi’s or any rule learning method. In the third stage, the rule base from each node is aggregated and in case of conflict the rule with the higher rule weight is considered. The initial computed DB and final rule base results in the KB for this type of models.

There has been several important contribution in the field of fuzzy systems with Big Data which have received attention in the research community during the years 2010-2021 such as in \cite{120} as described above, an FRBS model for the classification in big data context using Mapreduce approach on the Hadoop platform was presented; This work was further extended using cost-sensitive learning for imbalanced dataset in \cite{121}; both of the above approaches does not lead to exact same rules if Chi’s original algorithms
was used, this drawback was removed in [118]; in [122] distributed fuzzy classifier which employs frequent pattern for pruning unnecessary rules. A brief overview of Chi-FRBS approaches for big data along with granularity analysis has been given in [123], their results suggests increase in granularity increases utility but having too high granularity (>5 in their experiments) can also cost in the loss of utility. The work in [124] highlights that fuzzy systems for big data is still in its early stage and provides brief review on the focused key areas: classification, association rule mining and clustering for big data. In [125], overview of fuzzy techniques has been presented where fuzzy approaches have been used to handle uncertainty and for modelling purposes, they also highlights most of the current fuzzy work in Big Data only considers volume as their definition of big data.

Table 2 shows top conference/journals in FRBSs and the number of articles published in them for Big data in FRBSs during the years 2010-2021. Fig. [11a] shows the trend in number of publications in the years 2010-2021, the number of publications in the field haven’t been consistent. Fig. [11b] shows the areas where papers in FRBS in big data were published, most of the papers are in the areas of Computer Science, Engineering and Mathematics. Fig. [11c] shows the citations for the articles published in the years 2010-2021, the field was introduced recently and the trend suggests the research in FRBSs for Big Data has been receiving constant attention from the research community.

**Recent trends in FRBSs for Big Data:**

In the recent years, the researchers in the field of FRBSs for Big Data
Figure 11: (a) Published article in the field of FRBSs for Big Data between 2010-2021 (b) Area-wise categorization for the published articles in FRBSs for Big Data (c) citations for the published articles.
have focused in the following areas:

1. **Neuro FRBS-BigData:** In the present stage, neural networks is one of the key machine learning areas where progress is being made constantly. FRBS-BigData is no different, there has been several recent articles which presents NFS for big data such as in [126] multi-layer NFS with time varying learning rate for Big data is presented; in [127] a two-stage optimization for privacy-preserving Hierarchical-NFS has been presented, in first stage parameters are trained using distributed K-means and AO algorithm is used for coordination among nodes at higher levels; and many other.

2. **Structural Updates in FRBS-BigData:** Many researchers in FRBS-BigData are also looking to optimize the rule complexity and the running time of the model such as in [128] a support based filtering of rules instead of rule weight based filtering for the classification task in FRBS-BigData which results in improved accuracy and running time has been presented; in [129] an interpretable FRBS which tries to reduce the number of rules as well as the no of features in a rule has been presented.

3. **Applications of FRBS-BigData:** There also exists many areas where FRBS-BigData finds its application, e.g. energy consumption prediction in [130], diagnosis and prediction in healthcare system in [131] and many other applications.

### 3.6 FRBSs for Interpretability/Explainability

FRBSs are widely used because of their ability to provide human understandable knowledge in the form of linguistic rules against their black-box counterparts. FRBS models are interpretable by design which provides the answer to what will be the output of the model and why. The interpretable structure of the FRBSs can be represented in several forms such as: TSK, Mamdani and many others as defined in section 2. A model which is highly interpretable but has low utility will never be used, so interpretability alone cannot be sufficient. Two generic types of fuzzy modelling approaches are famous, Linguistic fuzzy modelling (LFM) where the primary objective is to have high semantic interpretability while Precise fuzzy modelling (PFM) focus on highly accurate fuzzy model. There are many aspects in the design of FRBS which improves the interpretability and accuracy of the model e.g. the granularity level, the membership function and many more. Interpretability of FRBSs is not only associated to classical FRBS but also
to HFS, NFS, GFS. HFS divides the large FRBS system into subsystem to reduce the no of the rules. Each of the subsystem generates an intermediate variable which is used as input in the next level, [132] highlights that the use of intermediate variable leads to the black-box FRBS. Comprehensive study on the interpretability of the HFS is yet to be done. In GFS, the evolution-
ary algorithms are used for the learning purpose in the KB [50]; and in NFS, the NNs are used for learning purposes [133].

There is a general belief that there exists a trade-off between accuracy and interpretability, even though it may not necessarily be true. There exists two approaches in accuracy-interpretable trade-off, some tries to improve the accuracy of the model while maintaining good interpretability of the system while other method tries to make model more interpretable while maintaining high accuracy. The accuracy of the interpretable-FRBS further be improved by updating the rule structure which encompasses the granularity, choosing the MFs and their parameters, limiting the no. of rules and the no of variables in each rule etc; and improving the learning process using additional components (such as GFS). Interpretability can further be improved by reducing the number of features in the rules; reducing the no of rules (one such approach is merge close enough rules); simplifying the output of the FRBSs. These approaches focus on the sequential improvement of accuracy for LFM or improving interpretability for PFM. Some researchers also considers these in interleaved way e.g. in [134] first an LFM is constructed then its accuracy is improved by generating large fuzzy partitions which generally results in the huge rule base. Then rules are merged again to improve the interpretability of this model, this can be extended as long as we don’t get the desired result. Since interpretability and accuracy are conflicting objectives (generally), some researchers focus on multi-objective EA such as SPEA, NSGA-II and many other EAs. In case of big data, the rules generated in FRBS may not be highly interpretable as the no of variables in the rule may be quiet large to obtain the accurate model. Recently, the interpretability issues in FRBS for big data has also been highlighted [135] and critically analyzed in [136].

There has been several important contribution in the interpretability field of FRBSs which have received attention in the research community during the years 2010-2021 such as in [137] a constrained multi-objective GFS which uses decision trees to initialize the rulebase which results in less no of rules with less variables due to reduced search space; a generic framework to evaluate and design of interpretable FRBS has been given in [138]; in [139] a NFS which focuses on accuracy along with high interpretability (as defined in [140]) for non-linear system has been given; while in [25], authors have
presented an overview and need of the understandable and explainable AI; there hasn’t been much work regarding interpretability of HFS and there is a lack of interpretable measures for HFS, in [141] an index for HFS has been presented and many more. A detailed review of interpretability in FRBSs until 2010 has been done in [140], they also highlight that there doesn’t exist a comprehensive measure which can quantify interpretability, interpretability has to be seen from different properties such as no. of rules, size of rule, granularity etc. The work in [142] highlights the open issues new development, trends and achievement for eFS/GFS, the review for Mamdani-type FRBSs in the context of GFS has been presented in [50]. Another review which focuses on information granule from the perspective of interpretability-accuracy trade-off has been given in [143].

Table 2 shows top conference/journals in FRBSs and the number of articles published in them for interpretability/Explainability in FRBSs during
the years 2010-2021. Fig. 12a shows the trend in number of publications in the years 2010-2021, the trend for the no of publications in the field hasn’t been consistent, although from 2014 the number of publications are increasing. Fig. 12b shows the areas where papers in FRBS in big data were published, most of the papers are in the areas of Computer Science, Mathematics and Engineering. Fig. 12c shows the citations for the articles published in the years 2010-2021, the linearly increasing trend suggests the work done is being used or improved constantly.

Recent trends in interpretability/Explainability in FRBSs:
In the recent years, the researchers in the field of interpretable FRBS have focused in the following areas:

1. Interpretability of FRBS in Big data: The whole idea of fuzzy systems revolves around, they are not black-box models, they follow interpretable by design paradigm. But as highlighted in [136] it is very difficult for humans to correlate more than two variables in a rule, let alone 10, 20 or more variables. In [129], an improvement for horizontal and vertical size of the rule using using different filtering approaches, a density based filtering for baseline rule base, considering only the important linguistic variable for interpretable rule, heuristic rule selection for reducing the total no of rules.

2. Interpretability of Hierarchical Fuzzy Systems: The interpretability of HFS has been linked to reducing the complexity of FRBS but at the same time it uses intermediate variables, which raises the question, reducing the no of rules at the cost of intermediate variable (may or may not interpretable) leads to higher or lower interpretability??. In the recent years many researchers have started looking for this question, in [132] not all HFS can be interpretable, the interpretability of the HFS depends significantly on the the choice of intermediate variables; in [141] the lack of indices to measure interpretability in HFS has been highlighted.

3.7 Others:
Apart from the above mentioned areas, FRBSs have also contributed in the following areas:

3.7.1 FRBSs for imbalanced datasets
Real-world problems generally have imbalanced data. Imbalanced datasets are the datasets which have unequal distribution of the output classes e.g.
in spam detection, the number of spam emails are significantly less than the legitimate email. Numerous approaches have been proposed in the literature to handle imbalanced dataset. They can be broadly categorized into three categories: Data-level approaches where the training data is modified to obtain the relatively balanced dataset; algorithm level approaches where existing methodologies are modified to improve the classification for minority class; and cost-sensitive approaches where there is higher penalty associated with the misclassification of the minority class. In the recent years, FRBSs are seen as an effective approach for classification in imbalanced settings where many researchers considers the use of synthetic minority oversampling technique (SMOTE) in data-level approaches such as in [144] which chose two neighbouring points at random and generates a synthetic point from the combination of the two is generated; for algorithm level approaches in FRBSs, algorithmic changes such as in [145] which presents a hierarchical FRBCSs to generate fine granularity between majority and minority classes using genetic rule selection; and for cost sensitive approaches the cost are included during evaluation of the model to favor the minority classes such as in [146]. Class imbalance in big data has also become a prominent area, such as in [121] a cost sensitive FRBS for imbalanced big data has been considered, in [147] a rule selection method in spark environment for imbalanced datasets in big data FRBSs has been presented. In [124], review of eFS approaches which focuses on classification using imbalanced dataset has been presented. Table 2 shows top conference/journals in FRBSs and the number of articles published in them for imbalanced dataset in FRBSs during the years 2010-2021. Fig. 13a shows the no of publication in the FRBSs for imbalanced data, the trend has been inconsistent but the research in this direction is
still active and has been receiving attention from the research community as can be seen from fig. 13b. In the past years, much of the research has been focused over eFS or Big data for FRBS.

### 3.7.2 Cluster centroids as rules in FRBSs

In FRBSs, the no of rules can be quite large. Determining the maximum no of rules can be very problem dependent. In the recent years, clustering has gained attention for applications (or areas) which requires fixed no of rules. For a user defined no of rules $c$, the data is clustered into $c$ clusters, where each cluster centroid represent a rule of the fuzzy system. There has been many clustering approaches for a variety of fuzzy system has been applied in the literature such as: fuzzy c-Means has been used in [148] to generate hierarchical structured fuzzy rules, in [149] rule structure was initialized using possibilistic clustering algorithm for NFS; subtractive clustering has been used in [150] to initialize rules for TSK fuzzy systems and many other works. These systems generally uses a clustering algorithm (a type of c-Means clustering) to model any type of the fuzzy system. Table 2 shows top conference/journals in FRBSs and the number of articles published in them for initializing rules using cluster centroids in FRBSs during the years 2010-2021. Fig. 14a shows the no of publication in the FRBSs which uses cluster centroids as initial fuzzy rules, the trend has been inconsistent, in general there has been roughly 30 papers each years which discusses the use of cluster centroids in the various types of fuzzy systems and fig. 14b shows the citations received for the papers published in the field during the years 2010-2021.
years 2010-2021, the increasing trend suggest that the work is continuously receiving attention from the research community.

4 Open problems in FRBSs:

In the past 40-50 years, FRBSs have been successfully used in various areas. But still there are many open problems which can present opportunities for the FRBSs research community:

1. Ethical challenges: With the increase of AI in our day-to-day, ethical challenges such as privacy, fairness, interpretability with the AI models must be handled immediately. With the increase in the number of publicly available datasets, the need to avoid disclosure of sensitive information is also increasing. Very less work in the fuzz community concerns itself with the ethical concerns of AI. In general, there exists a trade-off between utility-interpretability, utility-privacy, utility-fairness but further study is needed to reflect upon the relationship between data privacy, fairness and interpretability. FRBSs can be a candidate model to study the relationship between interpretability, fairness and data privacy.

2. Representation of uncertainty: Fuzzy sets are used to represent the uncertainty of the data. They are the interpretable variables in FRBSs. In the current literature, the focus has been given to classical fuzzy sets and type-II fuzzy sets but there exists many other fuzzy sets such as Hesitant fuzzy sets [151], rough sets [152] and so many others. These sets provide the opportunity either enhance the uncertainty representation or consider multiple experts opinion.

Fuzzy sets use membership functions for uncertainty representation. There exists different families of membership function but there hasn’t been much work which compute/compute the impact of different membership function on the performance of FRBSs e.g. in [136] authors have suggested that triangular and trapezoidal MFs are more suitable to XAI than gaussian MFs. MFs also influences the interpretability of FRBS reasoning, automated efficient choice and partitioning for MFs can be an important factor for FRBSs.

3. Big Data: With the amount of data being produced in today’s world, researchers should consider model’s performance in big data context. In the recent years, researchers in classical FRBSs model have started
working in the field of classification for big data (e.g. [118]) but there hasn’t been much work in other machine learning aspects. Not much focus has been given for big data in other types of FRBSs such as GFS, NFS and HFS. Genetic/evolutionary algorithms can help in parallelization and improving the learning rate for MapReduce framework, further study in this regard can provide fast and good solutions. The complexity of NFS increases with big data which will require time consuming and costly solutions, an interesting direction can be distributed/federated approaches for hybridization of NNs and FRBSs may present robust and fast solutions. Even though the motivation for HFS is its ability to overcome the curse of dimensionality, there hasn’t been any significant work which discusses the application of HFS in big data context.

Another big data problem which is prominently present in FRBSs community as well is the consideration of only volume for big data i.e. big data comprises of 3-5 Vs as explained in section 3.5 but in general researchers have used huge data volume as big data. Consideration of other Vs in FRBSs can provide important insights in the fields.

4. Evolving nature of Algorithms: In real-world applications, data is streaming in nature i.e. data distribution keeps on changing with time which requires continuous development in the developed model. Generally, evolving FCM is used to determine the age and the rule with the incoming data stream. Evolvable FCM is sensitive to the outliers and noises, a more robust approach in the evolving fuzzy systems may significantly enhance its performance. Apart from this, more focus should be given for the selection of features, input; aggregation of rule in eFS. Although, eFS considers the streaming data but none (or few) of the paper has considered the impact of high (or low) velocity of incoming data on the performance of eFS. eFS also does not provide the flexibility architecture of the rule and system e.g. Hierarchical or distributed architecture.

5. Aggregators in FRBSs: Aggregation in FRBSs can be used for merging the fuzzy rules to reduce the no. of rules in the system and to get the firing strength of the rule. In general, neighbour rules which after merging does not affect accuracy (much) are merged using an aggregation operator for each feature in the neighbouring rules. The features in a rule of FRBSs are connected through a conjunction which is usually modeled through a t-norm or a product t-norm aggregation

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operator. There exists several aggregation operator such as OWA \cite{153}, Choquet \cite{154} and Sugeno \cite{155} integrals and many more which can have significant impact on the performance of the FRBSs. Different aggregation operators in various types of FRBSs can also present interesting results.

6. Interpretability/Explainability: Interpretability is a key aspect of FRBSs. But in the last few years, critical analysis of fuzzy system with respect to interpretability has gained attention from researchers. As suggested in \cite{136}, if a rule contains more than 2 antecedents it is very hard for human mind to understand. Further study in this area regarding the loss of accuracy with 2-3 antecedents in the rule may be needed. As mentioned in section \ref{sec:interpretability}, the interpretability of HFS needs to be further analysed.

7. Deep Fuzzy multi-layer network: In the last few years, significant focus has been given to deep fuzzy neural networks which incorporates a fuzzy layer in the DNN, it offers interpretability to the otherwise black-box model. Due to the deep architecture along with high number of parameters to tune, DFNN has very high computational cost for big data, distributed or parallel. Most of research article considers gradient descent to optimize its parameter but there exists many other approaches which can result in better performing DFNN, further analysis in this area is needed.

5. Limitations

In this article, we have presented brief overview of developments in the different types of fuzzy systems during the years 2010-2021. As discussed above, Fuzzy System has been used in various areas and huge no. of papers are published every year, discussion or inclusion of this many papers is quiet difficult. So, this review article suffers from the following limitations:

1. This article provides a brief overview, key papers which received significant attention in research community and the current trends in the various areas of FRBSs but at the same time it does not cover all the papers, their approaches, and their classification as done in \cite{156}.

2. This article does not dive deep into the architecture and methodology of the each type of FRBSs.
3. Since use cases of each type of FRBSs are different, this article does not provide the comparison between each type of FRBSs.

6 Conclusion

In this paper, we have presented an overview for the various types of fuzzy rule based systems. We have first determined the various types of classical FRBSs then this paper broadly covers other variations of FRBSs genetic/evolutionary fuzzy system, hierarchical fuzzy systems, neuro fuzzy systems, evolving fuzzy systems, advances in FRBSs for big data, FRBSs for interpretability/explainability, FRBSs for imbalanced datasets and FRBSs which uses cluster centroids as rules. For each of the topic, we have presented the brief overview, review papers, trending papers and trending areas related to each topic during the year 2010-2021. We have considered scopus database to get the statistics related to each of the topic. Our analysis suggests that the current trend in the field of FRBSs lies in NFs, big data and interpretability of FRBSs as suggested in the statistics in section 3. Significant reduction in the research community for GFS can be observed. It is also clear that hybrid of two or more types of models are gaining more and more popularity. We have also highlighted the open research areas in FRBSs in section 4 which presents the potential future direction for FRBSs.

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**Statements & Declarations**

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