Final-state $\pi\pi$ interactions

in $\Upsilon(3S) \to \Upsilon(1S) \pi\pi$

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Abstract

The $m_{\pi\pi}$ spectrum and various angular distributions in $\Upsilon(3S) \to \Upsilon(1S) \pi\pi$ are studied including the effects of the $\pi\pi$ phase shift in the $I = L = 0$ channel using the lowest order amplitude in the pion momentum expansion. Our results are compared with the recent CLEO data, and we find good agreement except for the $\cos \theta^*_{\pi}$ distributions. We argue that the $\cos \theta^*_{\pi}$ distribution, contrary to other distributions, is sensitive to the higher order corrections in the pion momentum expansion. This argument is supported by using an ansatz for the amplitude which is of higher order in the pion momentum expansion and still satisfies the soft pion theorem.

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I. INTRODUCTION

The peculiar double peaks in the $m_{\pi\pi}$ spectrum in $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$ [1] have been lacking proper understanding. Although several suggestions have been made, most of them reproduce the $\pi\pi$ spectrum only with limited success [2]. In the two recent works by us, we approached this problem in two different ways. In the first work [3], we assumed that the most general form of the amplitude for $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$ is

\[ M = A_0 \left[ (q^2 + B_0 E_1 E_2 + C_0 m^2_{\pi}) \epsilon \cdot \epsilon' + D_0 \left( p \cdot \epsilon p' \cdot \epsilon' + p \cdot \epsilon' p' \cdot \epsilon \right) \right], \]  

(1)

in the lowest order in the pion momentum expansion. Here, $p, p'$ are the four–momenta of the final pions, $E_1$ and $E_2$ are their energies, and $\epsilon$ and $\epsilon'$ are the polarization vectors of the initial and the final $\Upsilon$'s, respectively. In Ref. [3], we found three sets of parameters (P0, P1 and P2) by minimizing the $\chi^2$. For these three sets of parameters, we predicted various angular distributions which should be checked against experiments.

In the second work [4], we assumed that (i) the QCD multipole expansion is applicable to $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$ and (ii) $\Upsilon(3S)$ has an admixture of a $D$–wave component :

\[ |\Upsilon(3S)\rangle = \cos \phi |3S\rangle + \sin \phi |D\rangle. \]  

(2)

The amplitude for $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$ then depends on three independent parameters, and we found two fits which correspond to P1 and P2 of Ref. [3], respectively. We then explored the consequences of our assumptions on other hadronic and radiative transitions of $\Upsilon(3S)$ into the lower level bottomonia. In Ref. [4], we again assumed that the lowest order expansion in the pion momenta

\[ \theta^0_{2\pi}(q^2) \equiv \langle \pi\pi| \theta^0_{\mu}|0\rangle = (q^2 + m^2_{\pi}) \]  

(3)

be valid through the whole range of $m_{\pi\pi} = \sqrt{q^2}$.

Although these two approaches fit the $m_{\pi\pi}$ spectrum, they still leave room for theoretical improvement in two aspects. First of all, it is well known from the analysis of the $\pi\pi$ phase shift that the dipion system in $I = L = 0$ experiences strong final state interactions [3].
Since the dipion system in \( \Upsilon(3S) \rightarrow \Upsilon(1S) \pi \pi \) are in either \( I = L = 0 \) or \( I = 0, L = 2 \) state, one should properly take into account of the \( \pi \pi \) phase shift due to the final state interactions in the \( I = L = 0 \) dipion system.

Secondly, the validity of (1) or (3) in \( \Upsilon(3S) \rightarrow \Upsilon(1S) \pi \pi \) is rather unclear, since the available \( m_{\pi \pi} \) in \( \Upsilon(3S) \rightarrow \Upsilon(1S) \pi \pi \) is large, \( 2m_\pi \leq m_{\pi \pi} \leq (m_i - m_f) = 895 \text{ MeV} \). The amplitude (3) with \( B_0 = D_0 = 0 \) gives a good description for the \( m_{\pi \pi} \) spectrum in \( \Upsilon(2S) \rightarrow \Upsilon(1S) \pi \pi \), where \( m_{\pi \pi} \leq 563 \text{ MeV} \). For higher value of \( m_{\pi \pi} \), we simply assume that it is valid and explore its consequences on various spectra of decay products in \( \Upsilon(3S) \rightarrow \Upsilon(1S) \pi \pi \). If any of the predicted angular distributions based on the amplitude (1) including the \( \pi \pi \) phase shift does not agree with the experimental measurements, it would signal the importance of higher order terms in the pion momenta which have been neglected in (1).

In the present work, we follow the approach of Ref. [3] and include the phase shift of the \( \pi \pi \) system in the \( I = L = 0 \) channel \( (\delta_0(q^2)) \) using the data available in the literature [6]. The phase shift for the \( I = 0 D- \text{wave} \) \( \pi \pi \) system \( (\delta_2(q^2)) \) is tiny enough to be neglected for the whole range of \( m_{\pi \pi} \) [6].

In Sec. II A, we decompose the amplitude (1) into the \( \pi \pi \) \( S- \) and \( D- \)waves, and incorporate the phase shift \( \delta_0(q^2) \). It is found that the importance of the phase shift due to the final state interactions is most prominent in the \( \cos \theta_{\pi}^* \) distributions, but the effect is only moderate. Then, our results are compared with the recent data from CLEO in Sec. II B. In Sec. II C, we generalize the amplitude (1) (in case of \( D = 0 \)) to include higher order terms in \( q^2 \), and discuss some general aspects of the various angular distributions. It is also pointed out that one can extract the difference between \( \pi \pi \) phase shifts of the \( S- \) and \( D- \)waves in the \( I = 0 \) channel, \( \delta_0 - \delta_2 \), by measuring the joint distribution \( d^2 \Gamma/dm_{\pi \pi} d \cos \theta_{\pi}^* \). The results are summarized in Sec. IV.

**II. EFFECTS OF THE \( \pi \pi \) PHASE SHIFT**
A. Predictions on the $\cos \theta_{\pi^+}$ distributions

In this work, we use a modified form of the amplitude (1). First of all, we ignore the recoil of the final $\Upsilon(1S)$ in the amplitude, and make the $D-$term proportional to the symmetric traceless part:

$$\mathcal{M} = A \left[ \left\{ q^2 + BE_1E_2 + Cm_\pi^2 \right\} \hat{e} \cdot \hat{e}' + D \left\{ \hat{p} \cdot \hat{p}' \cdot \hat{e} \cdot \hat{e}' + \hat{p}' \cdot \hat{p} \cdot \hat{e} \cdot \hat{e}' - \frac{2}{3} \hat{p} \cdot \hat{p}' \hat{e} \cdot \hat{e}' \right\} \right].$$ (4)

One can find relations between parameters in (1) and (4) using

$$\vec{p} \cdot \vec{p}' = E_1E_2 - \frac{1}{2} (s - 2m_\pi^2).$$

It should be emphasized that our amplitudes (1) and (4) satisfy Adler’s condition by construction.

Now, we decompose the above amplitude into the $\pi\pi$ $S-$wave and $D-$wave in order to take into account the phase shift of the $\pi\pi$ system in the $I = L = 0$ state. The $S-$ and $D-$waves have the following tensor structures [7]:

$$S_{ij} = f_S(q^2)\delta_{ij} + g_S(q^2)(q_iq_j + q^2\delta_{ij}),$$

$$D_{ij} = f_D(q^2) \left( \cos^2 \theta_\pi^* - \frac{1}{3} \right) \delta_{ij} + g_D(q^2) \left[ r_i r_j - \frac{1}{3} (q_iq_j + q^2\delta_{ij}) \beta_*^2 \right],$$ (5)

where $r_\mu = p_\mu - p'_\mu$. The amplitude (4) can be decomposed into the $f_{S,D}, g_{S,D}$ form factors as follows:

$$f_S = q^2 + Cm_\pi^2 + \frac{B}{4} \left[ (E_1 + E_2)^2 - \frac{1}{3} q^2 \beta_*^2 \right]$$

$$- \frac{1}{2} Dq^2 - \frac{1}{6} D \left[ q^2 - \beta_*^2 \left( q^2 + \frac{1}{3} q^2 \right) \right],$$ (6)

$$g_S = \frac{1}{2} D \left( 1 - \frac{1}{3} \beta_*^2 \right),$$ (7)

$$f_D = \left( \frac{1}{6} D - \frac{1}{4} B \right) \beta_*^2 q^2,$$ (8)

$$g_D = -\frac{1}{2} D.$$ (9)

Multiplying the $S-$wave amplitude by the phase shift $\delta_0(q^2)$, we get

$$\mathcal{M} = \left[ S_{ij} e^{i\delta_0(q^2)} + D_{ij} \right] \hat{e}_i \hat{e}'_j.$$ (10)
In case the final $\Upsilon(1S)$ is not reconstructed, summations over polarizations of the initial and final $\Upsilon$'s are done with

$$\sum \hat{\epsilon}_i \hat{\epsilon}_j = (\delta_{ij} - \hat{z}_i \hat{z}_j), \quad (11)$$

$$\sum \hat{\epsilon'}_i \hat{\epsilon'}_j = \delta_{ij}. \quad (12)$$

The fact that the initial $\Upsilon(3S)$ is transversely polarized with respect to the beam directions (taken along the $z$-direction) has been taken into account in (11). This fact is very useful to test the existence of the $D$-term by measuring the polar angle distributions of the final $\Upsilon(1S)$ (or, equivalently, the $\pi\pi$ system as a whole) and/or of a muon emerging from the muonic decay of the final $\Upsilon(1S)$. If one tags the muonic decay of the final $\Upsilon(1S)$, the polarization sum over the final $\Upsilon(1S)$ (12) should be replaced by

$$\Sigma \hat{\epsilon'}_i \hat{\epsilon'}_j = (\delta_{ij} - \hat{l}_i \hat{l}_j), \quad (13)$$

where $\hat{l}$ is the three dimensional unit vector along the direction of a muon in the rest frame of the initial $\Upsilon(3S)$. For $D = 0$, one gets

$$dN/d \cos \theta_t \sim (1 + \cos^2 \theta_t),$$

where $\cos \theta_t = \hat{l} \cdot \hat{z}$.

Using amplitude (10), we fit the $m_{\pi\pi}$ spectrum by minimizing $\chi^2$. The best fit is given by three sets of solutions, P0, P1 and P2 (see Table 1), which are essentially the same as the ones given in Ref. [3]. Other angular distributions can be obtained by numerical integrations as in [3]. In Fig. 1 (a) and (b), we show the $\cos \theta^*_\pi$ distributions of $\pi^+$ in the rest frame of the dipion system for P0 and P1 (P2), where $\theta^*_\pi = 0^o$ is along the direction of the dipion system as a whole in the rest frame of initial $\Upsilon(3S)$. For comparison, we show the corresponding plots with the phase shift neglected (i.e. $\delta_0(q^2) = 0$) in Fig. 2 (a) and (b). The phase shift moderately changes the $\cos \theta^*_\pi$ distributions, but the overall effects may be hardly discernible in the experiment.

Before continuing on to the next section, we discuss how much the results obtained in Ref. [4] will change when we incorporate the $\pi\pi$ phase shift and possible corrections to (3) in
higher orders in the pion momentum expansion. In this case, it suffices to resort to the QCD multipole expansion by our assumptions. The relevant matrix element for \( \langle \pi \pi |G_{\mu\nu}^\alpha G^{\alpha\mu\nu}|0\rangle \) has been obtained by Donoghue et al. using the dispersive approach in conjunction with the chiral symmetry relations imposed in the chiral limit \(^8\). The result is that the above matrix element, \( \langle \pi \pi |G_{\mu\nu}^\alpha G^{\alpha\mu\nu}|0\rangle \), is dominated by \( \langle \pi \pi |\theta_\mu^\alpha|0\rangle \), and that \((3)\) remains essentially unchanged up to \( m_{\pi\pi} \sim 0.9 \text{ GeV} \), once it is regarded as the modulus of \( \theta_{2\pi} \). The phase shift is given by \( \delta_0(q^2) \). In short, one only has to write \((3)\) as

\[
\langle \pi \pi |\theta_\mu^\alpha|0\rangle = (q^2 + m^2_{\pi}) e^{i\delta_0(q^2)}.
\]

Since the phase shift \( \delta_0(q^2) \) does not affect the \( m_{\pi\pi} \) spectrum in \( \Upsilon(3S) \to \Upsilon(1S) \pi\pi \), our results derived in Ref. \(^9\) do not change at all. In particular, the predictions for \( \Upsilon(3S) \to \Upsilon(1S) + \eta \) remain the same, which excludes the fit \( P1 \) \(^4\).

B. Comparisons with the data

Recently, the CLEO collaboration released a new set of data on hadronic transitions in \( \Upsilon(3S) \) decays \(^9\). Their results on \( \Upsilon(3S) \to \Upsilon(1S) \pi\pi \) can be summarized as follows:

(i) the \( \cos \theta_f \) distribution is flat for the whole range of \( \cos \theta_f \).

(ii) the \( \cos \theta_\pi^* \) distributions can be fitted by \( (a + b \cos \theta_\pi^2) \), with \( a = (1.24 \pm 0.06) \) and \( b = (-0.49 \pm 0.13) \).

(iii) the \( \cos \theta_l \) distribution is consistent with \( (1 + \cos \theta_l^2) \) for \( 0 < |\cos \theta_l| < 0.7 \).

Let us discuss the implication of each statement above to our fits, \( P0–P2 \). Statement (i) excludes both \( P1 \) and \( P2 \), since these two lead to quadratic functions of \( \cos \theta_f \). Statement (iii) also partly supports this conclusion, since (iii) implies \( D = 0 \) in the amplitude \((1)\). Thus, (i) and (iii) select \( P0 \) as the final candidate. However, the \( \cos \theta_\pi^* \) distribution shown in Fig. \(^9\) (a) does not agree with statement (ii) from CLEO. We do not interpret this disagreement of the CLEO data with our prediction on the \( \cos \theta_\pi^* \) distribution as a general failure of our
approach based on the matrix element satisfying the soft pion theorem. We rather regard it as an indication that amplitude (1) needs to be modified to include higher order terms in the pion momentum expansion. This will be illustrated in the next section with a modified amplitude for Ψ(3S) → Ψ(1S) ππ.

III. MORE ON THE AMPLITUDE WITH $D = 0$

In this section, we consider the case $D = 0$ in more detail, including possible higher order terms in $q^2$ in the $S$-wave ππ amplitude, $f_S$. It will be shown that the $\cos \theta^*_\pi$ distribution is sensitive to such higher order terms in $q^2$ contrary to other distributions.

Let us write the amplitude for Ψ(3S) → Ψ(1S) ππ as

$$M(\Psi(3S) \rightarrow \Psi(1S) \pi\pi) = A \left[ f_S(q^2)e^{i\delta_0(q^2)} + f_D(q^2)(\cos^2 \theta^*_\pi - \frac{1}{3}) \right] \hat{\epsilon} \cdot \hat{\epsilon'},$$

where $f_S$ and $f_D$ satisfy the soft pion theorem. The explicit forms of $f_{S,D}$ for the lowest order amplitude (4) can be read off from (6)–(9) with $D = 0$. Therefore the differential cross section for $e^+e^- \rightarrow \Psi(3S) \rightarrow \Psi(1S) \pi\pi \rightarrow \pi\pi\mu^+\mu^-$ is given by

$$d^3\Gamma \propto dm_{\pi\pi}d\cos \theta^*_\pi d\cos \theta_l m_{\pi\pi}|q| \beta^*_\pi \left[ 1 + \cos^2 \theta_l \right]$$

$$\times \left[ f^2_S + f^2_D \left( \cos^2 \theta^*_\pi - \frac{1}{3} \right)^2 + 2f_Sf_D \cos \delta \left( \cos^2 \theta^*_\pi - \frac{1}{3} \right) \right],$$

where $\beta^*_\pi$ is the velocity of a pion in the ππ rest frame and $\theta_l$ is the angle between a muon and the $e^+e^-$ beam in the rest frame of $\Psi(3S)$.

Integrating the partial distribution (16) over appropriate variables, one gets

$$\frac{d\Gamma}{d\cos \theta_f} \propto 1, \quad \text{(flat distribution)},$$

$$\frac{d\Gamma}{d\cos \theta_l} \propto (1 + \cos^2 \theta_l).$$

These two distributions are independent of the $q^2$ dependence of the form factors, $f_S(q^2)$ and $f_D(q^2)$, as well as of the ππ phase shift. And, these results are consistent with the recent report from CLEO.
On the other hand, the $\cos \theta^*_{\pi}$ distribution is sensitive to the actual forms of $f_S(q^2)$ and $f_D(q^2)$, and to the $\pi\pi$ phase shift. In principle, there are many possible terms to the next order in the pion momentum expansion. Instead of writing down all possible terms and fitting the $m_{\pi\pi}$ spectrum as in Ref. [3], we take the following amplitude for illustration:

$$f_S(q^2) = q^2 \left[ 1 + C \left( \frac{E_1 + E_2}{m_{\pi}} \right) \right] + \frac{B}{4} \left[ (E_1 + E_2)^2 - \frac{1}{3} |\vec{p}_f|^2 \beta^*_{\pi} \right],$$

with the same $f_D(q^2)$ as before. This amplitude has three parameters, $A, B$ and $C$, and satisfies the soft pion theorem like (1).

By $\chi^2$ fit to the $m_{\pi\pi}$ spectrum, we found another fit (we will call it P3) with $\chi^2/d.o.f. = 11.0/7$ (see Fig. 3 (a)). The corresponding values of $A, B, C$ are given in the last column of Table 1. This amplitude predicts the distributions, (17) and (18). The $\cos \theta^*_{\pi}$ distribution for P3 shown in Fig. 3 (b) differs a lot from that for P0 in Fig. 1 (a), and gets much closer to the observed data. The lesson from this example is that once we adopt (15) as the amplitude for $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$, we predict (i) the flat $\cos \theta_f$ distribution, (ii) $(1 + \cos^2 \theta_l)$ distribution for the polar angle of a muon, independent of actual forms of $f_S, f_D$ and $\delta_0(q^2)$.

This is not the case for $\cos \theta^*_{\pi}$ distribution and thus cannot be reliably calculated unless they are known. The actual functional forms of $f_S, f_D$ and $\delta_0(q^2)$ can be extracted from the measurement of the joint distribution, $d^2\Gamma/dm_{\pi\pi}d\cos \theta^*_{\pi}$, as one can derive from (16):

$$\frac{d^2\Gamma}{dm_{\pi\pi}d\cos \theta^*_{\pi}} \propto m_{\pi\pi} |\vec{q}| \beta^*_{\pi} \left[ f_S^2 + f_D^2 \left( \cos^2 \theta^*_{\pi} - \frac{1}{3} \right) + 2f_S f_D \cos \delta_0 \left( \cos^2 \theta^*_{\pi} - \frac{1}{3} \right) \right]$$

$$= \left[ C_0(q^2) + C_2(q^2) \cos^2 \theta^*_{\pi} + C_4(q^2) \cos^4 \theta^*_{\pi} \right].$$

For each $m_{\pi\pi}$ bin, one can measure the $\cos \theta^*_{\pi}$ distribution. This determines $C_i(q^2)$’s, and in turn, three unknowns, $f_S, f_D$ and $\delta_0(q^2)$. In particular, the decay $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$ can be a source of the $S$–wave $\pi\pi$ phase shift for the whole elastic region, $2m_\pi \leq m_{\pi\pi} \leq (m_i - m_f) = 895$ MeV [10]. This may be important, since the existing data on the $\pi\pi$ phase shift between $m_K \leq m_{\pi\pi} \leq 600$ MeV are rather poor in statistics and one has to make some extrapolation [11].
IV. CONCLUSION

Concluding, we reanalyzed the $\Upsilon(3S) \to \Upsilon(1S) \pi\pi$ decay using the most general matrix element in the lowest order in the pion momentum expansion, including the final state interactions of the $\pi\pi$ system in the $I = L = 0$ channel. The $\pi\pi$ phase shift changes the $\cos \theta^*_\pi$ distributions moderately. (Compare Figs. 1 (a), (b) with Figs. 2 (a), (b).) Compared with the recent data from CLEO, P0 is selected, but the $\cos \theta^*_\pi$ distribution does not agree.

In Sec. [11], we argued that this distribution is sensitive to possible higher order corrections in the pion momentum expansion. As an illustration, we used a new ansatz for the $\pi\pi$ $S$–wave amplitude, [19], which is of higher order in the pion momentum expansion, and satisfies Adler’s condition. This amplitude could fit the $m_{\pi\pi}$ spectrum (Fig. 3 (a)). The resulting $\cos \theta^*_\pi$ distribution (Fig. 3 (b)) is different from Fig. 1 (a), and gets closer qualitatively to the measured distribution although not quantitatively. It would be more proper to do the partial wave analysis with the $S$– and $D$–waves, and find out the form factors ($f_S, f_D$) and the phase shift ($\delta_0(q^2)$) in [15] from the joint distributions in $m_{\pi\pi}$ and $\cos \theta^*_\pi$ using (20) and (21). Since our explanation involves both $S$– and $D$–wave $\pi\pi$ systems, we may be able to find out the $S$–wave phase shift (or $\delta_0 - \delta_2$, more precisely) from the decay $\Upsilon(3S) \to \Upsilon(1S) \pi\pi$ by measuring various joint distributions. Since the available $m_{\pi\pi}$ is below the $K\bar{K}$ threshold, this decay may provide information on the phase shift over the whole elastic region, especially for $m_K \leq m_{\pi\pi} \leq 600$ MeV, where the current data are rather poor in statistics.

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FIGURES

FIG. 1. The $\cos \theta^*_\pi$ distributions of $\pi^+$ in the rest frame of the dipion system with the phase shift included: (a) for P0 and (b) for P1 (and P2). $\theta^*_\pi = 0^\circ$ corresponds to the direction of the dipion system as a whole in the rest frame of the initial $\Upsilon(3S)$. The dotted and the dashed curves correspond to the low and the high $m_{\pi\pi}$ regions, and the solid one represents the sum of the two.

FIG. 2. The $\cos \theta^*_\pi$ distributions of $\pi^+$ in the rest frame of the dipion system with the phase shift neglected: (a) for P0 and (b) for P1 (and P2). $\theta^*_\pi = 0^\circ$ corresponds to the direction of the dipion system as a whole in the rest frame of the initial $\Upsilon(3S)$. The dotted and the dashed curves correspond to the low and the high $m_{\pi\pi}$ regions, and the solid one represents the sum of the two.

FIG. 3. (a) The $m_{\pi\pi}$ spectrum and (b) the $\cos \theta^*_\pi$ distributions for the amplitude, (15), where $f_S$ is given by (19) and $f_D$ is given by (8) with $D = 0$. For (b), the dotted and the dashed curves correspond to the low and the high $m_{\pi\pi}$ regions, and the solid one represents the sum of the two, respectively.
TABLES

TABLE I. Three sets of parameters for the amplitude (10) and one set for the amplitude (19) giving the best $\chi^2$ fit to the $m_{\pi\pi}$ spectrum. P0 and P3 correspond to the constrained fit with $D = 0$. The parameter $A$ is the overall normalization.

| Fits | P0         | P1 ( P2 )   | P3         |
|------|------------|-------------|------------|
| $A$  | 10.5 ± 0.6 | 6.7 ± 2.1   | 163.6 ± 55.3 |
| $B$  | −6.4 ± 0.7 | −3.7 ± 3.9  | −0.140 ± 0.035 |
| $C$  | 36.2 ± 6.2 | 7.8 ± 37.2  | −0.154 ± 0.001 |
| $D$  | 0.0 (fixed)| ±1.7 ± 0.7  | 0.0 (fixed)  |
| $\chi^2$/d.o.f. | 12.0 / 7 | 11.1 / 6   | 11.0 / 7  |
| C.L. | 10.2 %     | 8.4 %       | 14.0 %     |
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