Tunneling properties of quantum dot arrays in strong magnetic field

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Abstract. We develop a formalism suitable for the study of transport properties of coherent multiple dots which captures and explains the experimentally observed features in terms of spectral properties of the system. The multiplet structure of the transmission spectrum and the role of the intra-dot and inter-dot Coulomb interaction are pointed out. The evolution of the sub-peaks with the magnetic field in the quantum Hall regime is shown. We suggest a specific oscillatory behaviour of the Hall resistance in strong magnetic field which can be experimentally tested.

1) The electronic transport through coupled quantum dots became a topic of interest once the basic phenomena in single dots were satisfactorily understood. The Coulomb oscillations at vanishing magnetic field were investigated and the role of the inter-dot coupling for the multiple peak structure of the conductance was shown [1, 2]. The next step was accomplished by Livermore et al. [3] who measured the conductance through coupled dots in the quantum Hall regime. It was shown that the conductance peaks undergo shifts and also modulation as the magnetic field is varied continuously. The quantum dot arrays are promising systems of investigation because of the analogy with ”artificial molecules” and possibility of physical realizations of quantum bits (qubits).

We develop a theoretical approach of the tunneling problem through arrays of quantum dots in strong magnetic field with the aim to elucidate several aspects: the mechanism of the multiplet formation, the effects of the intra- and inter-dot Coulomb interaction and effects of the strong magnetic field. Our approach is based on a non-Hermitian tight-binding Hamiltonian used in the Landauer-Büttiker formalism with many terminals [4]. The non-Hermiticity of the Hamiltonian arises by including the contribution of the terminals in all powers of the perturbation theory.

We emphasize that in our approach the dot array is considered as a quantum-mechanical coherent system. Each quantum dot is modelled as a 2D rectangular plaquette and the coupling between them is ensured by tunneling terms (see Fig.1). In what concerns the electron-electron interaction we go beyond the usual constant capacitance model and describe both the intra- and inter-dot electron-electron interaction by a Coulomb-type term.

2) The formalism is presented shortly in what follows and represents the generalization of the method which was used in Ref.4 for describing the transport properties of a single dot.
For an array composed of \( N \) dots, the Hamiltonian contains the following terms:

\[
H^N = \sum_{k=1}^{N} H^k + \tau \sum_{k=1}^{N-1} H_{t}^{kk+1} + U \sum_{k>k'}^{N} H_{ee}^{kk'}. \tag{1}
\]

The index \( k \) counts the dots in the array, \( H^k \) is the Hamiltonian of an individual dot which includes the intra-dot electron-electron interaction, \( H_{t}^{kk+1} \) describes the tunnel-coupling of consecutive dots, while \( H_{ee}^{kk'} \) is the Coulomb interaction between electrons located in different dots (i.e., the last term represents the so-called electrostatic inter-dot coupling). The 2D spinless Hamiltonian \( H^k \) is considered in the Hartree approximation, while the perpendicular magnetic field is taken in the Peierls representation:

\[
H^k = \sum_{i \in QD_k} \left( V_g^k c_i^\dagger c_i + U \sum_{j \neq i} \frac{\langle n_j \rangle}{|j-i|} c_i^\dagger c_i + t_D \sum_{<i,i'>} e^{2\pi i \phi_{ii'}} c_i^\dagger c_i'. \tag{2}
\]

Here \( c_i^\dagger (c_i) \) are the creation (annihilation) operators in localized states indexed by \( i \in QD_k \) and \( t_D \) is the nearest-neighbour hopping integral in the dots. The phase \( \phi_{ii'} \) comes from the Peierls substitution and accounts for the magnetic flux through the unit cell of the lattice measured in quantum flux units \( \phi/\phi_0 \).

Now we have to built up the non-Hermitian Hamiltonian \( H_{eff} \) which takes into account the coupling of the multiple dot system with the leads carrying the external current; we note that \( H_{eff} \) depends on the energy \( E \) of the incident electron:

\[
H_{eff}^N(E) = H^N + \tau^2 \sum_{\alpha} e^{-i k \phi_{ii'}} c_i^\dagger c_{\alpha}, \quad E = 2 \cos k, \tag{3}
\]

where the index \( \alpha \) denotes the sites where the leads are stuck to the dots.

Finally, one has to calculate the retarded Green function \( G^+(E) = (E - H_{eff}^N + i0)^{-1} \) which is to be used in the Landauer- Büttiker formula in order to obtain the conductance \( g_{\alpha,\beta} \).

Let us enumerate the parameters of the problem: i) the tunnel-coupling between the dot-system and the leads described by the parameter \( t_{LS} \), ii) the tunnel-coupling between dots in the array \( \tau \), iii) the strength of the Coulomb interaction \( U \) and iv) the gate voltage \( V_g^k \) which can be applied on any dot \( k \). The hopping integral in the dots is taken as \( t_D = 1 \), and with this choice all the other energies \( V_g, t_{LS} \) and \( \tau \) are measured in units of \( t_D \).

For our two-dimensional problem the calculation of \( G^+ \) has to be done numerically, and the mean occupation number of each site \( \langle n_i \rangle \) in Eq. (2) has to be calculated self-consistently. The numerical effort depends on the number of dots in the array and on the dimension of each dot. When the electrostatic inter-dot coupling is neglected, this effort can be much reduced by using an iterative procedure, which allows the calculation of transmittance through any number of dots in the array, if the one-dot problem is solved.
We mention that the tight-binding model has already been used for the study of transmittance of quantum dot arrays in the work of Kirczenow [5] where each dot is associated with a single atom in the lattice, omitting thus the internal structure of the dots. We perform here an important step forward by describing each dot as a finite two-dimensional plaquette.

![Figure 2](image)

Figure 2. A) The degeneracy lifting of the eigenvalues for the Hofstadter-like spectrum of a double dot at \( \tau = 0.4 \). B) The corresponding split peaks in the transmittance spectrum at magnetic flux \( \phi/\phi_0 = 0.15 \) and \( t_{LS} = 0.4 \). C) The same for \( t_{LS} = 1.0 \). The system consists of two dots of dimension \( 5 \times 8 \) sites each.

3) In small quantum dots, which is the topic of our interest, the size quantization compete with the charging effects produced by the electron-electron interaction. So, we discuss first the case of non-interacting electrons in order to distinguish afterwards the effects due to the interaction. The quantum system has a specific energy spectrum depending on the number
of dots in the array and on the coupling constant between them which shows similarities to the Hofstadter spectrum. However two main differences appear: the spectrum of the array exhibits a multiplet structure due to the tunnel-coupling between dots which lifts the degeneracy of the energy levels and, on the other hand, the spectrum contains edge states which are induced by the vanishing boundary conditions imposed to the wave function.

We note that in the case of many coupled dots the difference between edge and bulk states is difficult to be realized in geometrical terms so that, essentially, they differ by their chirality. This is even more stringent in the presence of the electron-electron interaction, when geometrical deformations of the eigenstates occur. Since the array is a coherent quantum system the eigenfunctions have the character of molecular states. When one of the dots in the array is detuned (by applying a gate potential on it) the molecular states are scrambled with important effects on the longitudinal and transverse (Hall) conductances.

Evidently, the transmittance spectrum as a function of Fermi energy ($E_F$) has to correspond to the structure of the energy spectrum. However, this depends on the strength of the coupling between the system of dots and the external leads, expressed by the parameter $t_{LS}$. More explicitly, a multiplet can be put into evidence only if the imaginary part of $G^+(E_F)$ is less than the interlevel distance and such a situation occurs only for small $t_{LS}$. For a double dot this is shown in Fig. 2A and B where the correspondence between the energy levels and the peaks of the transmittance is evident indicating the resonance tunneling process of the electron through the coherent double dot structure. On the other hand, a large value of this parameter, as it is considered in Fig. 2C, spoils the resonant aspect and the one-to-one correspondence between the energy levels of the isolated quantum system and the transmittance peaks. In this case, other aspects like the Fano profile of the transmittance peaks or the Fano zeros are the topics of interest.

4) We now look for the effects of the interaction on the transmittance spectrum of multiple dots. From the single dot problem we have learned that, at least in the self-consistent Hartree approximation, the electron-electron interaction gives rise to an increase in the level spacing and of the width of transmission peaks [4]. We consider that the same thing occurs also for coherent multiple dots and Fig. 3 confirms these expectations when compared with Fig. 2B. The same Fig. 3 also shows the increase of the splitting with increasing tunneling between dots and in fact Fig. 3B indicates already the saturation of the splitting effect. The saturation is non-linear in the coupling parameter $\tau$ and can be put into evidence in a theoretical calculation only by taking into account all order of the perturbation. (For the sake of clarity of the drawings we show the results for $N = 2$).

The results shown in Fig. 3 are obtained by taking into account the total Coulomb interaction. However, one asks usually how important the inter-dot interaction is. This contribution represents an additional term compared to the intra-dot electron-electron interaction, and it is generally accepted that the strength $U$ of the inter-dot interaction is smaller than in the intra-dot case because of the screening produced by the metallic gates which exist between neighbouring dots [3].

The inter-dot term should accentuate the interaction effects known for the transmission
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through a single dot. Furthermore, since the interaction energy is positive, in the presence of the additional inter-dot coupling the whole transmittance spectrum should be pushed upwards on the energy scale, as it can be noticed by comparing Fig.4A and B. However, the comparison indicates a rather messy behaviour in the domain $E_F > 0$; this is because here the spectrum is occupied mainly by bulk states which are more sensitive to the interaction, meaning that they undergo easily geometrical deformations and, in addition, edge states are intercalated among them [7].

**Figure 3.** The dependence of the transmittance spectrum of a double dot on the tunnel-coupling between dots A) $\tau = 0.4$, B) $\tau = 0.8$.

**Figure 4.** Contribution of the inter-dot Coulomb interaction: A) Transmittance spectrum in the presence of intra-dot interaction only. B) The same in the presence of both intra-dot and inter-dot electron-electron interaction. The similarity is lost in the region $V_g > 0$ filled mainly with bulk states which are more sensitive to the additional interaction term. ($U = 0.5, \tau = 0.2, t_{LS} = 0.4$).

5) Our approach helps in understanding the recent experiments by Livermore et al [3]. The Fig.3a-c in Ref.3 reveal that, in strong magnetic field, the transmittance of a double
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Figure 5. The drift with the magnetic field of the twin peaks from the conductance spectrum of a double dot for two values of the tunnel-coupling: A) \( \tau = 0.2 \), B) \( \tau = 0.6 \).

...dot behaves as follows: i) at a given magnetic field but increasing inter-dot coupling, one starts with single peaks (figure a), then twin peaks appear (figure b) and finally, at large inter-dot coupling, the saturation consisting again in individual peaks occurs (figure c). ii) with increasing magnetic field the position of each peak shifts linearly (in spite of a zig-zag appearance) versus larger gate voltages. Both these features can be described by our model even without considering the interaction. Our results shown in Fig.5 indicate a striking similarity to the experimental situation. They indicate both the increase of the distance between the twin peaks with the increasing inter-dot tunneling \( \tau \) and the drift of the peaks at the variation of the magnetic field. The blurred, quasi-zig-zag behaviour of this drift is not however much evident in our calculations which are performed for non-interacting electrons. So, the idea expressed in Ref.3 that this behaviour is due to interaction effects turns out to be correct; our calculations for the interacting case are in progress. Our approach puts however into evidence that such a regular behaviour, as observed in the experiment, cannot occur in any range of the energy spectrum, but only if the Fermi level lies in a region covered by edge states. Fig.2A shows also that only the edge states are regularly distributed and well separated on the energy scale.

6) The properties of the transmittance matrix impose specific behaviour to the Hall resistance. In the presence of a constriction between the mesoscopic system and the external leads, even in strong magnetic field, the Hall resistance \( R_H \) exhibits oscillations instead of the usual quantum Hall plateaus. Since any minimum in transmittance give rise to a maximum in \( R_H \), the minima of the multiple dot transmittance produced by splitting result in small maxima of the Hall resistance located in-between two large maxima. For a double dot with the configuration of leads shown in the sketch below (Fig. 6) the Hall resistance \( R_H \) is given in terms of the conductance matrix elements by the following expression:

\[
R_H = \frac{(g_{21}g_{43} - g_{23}g_{41} - g_{12}g_{34} + g_{32}g_{14})}{2D},
\]

where \( D \) is a positive \( 3 \times 3 \) subdeterminant of the \( 4 \times 4 \) matrix \( g_{\alpha\beta} \) and the result is shown in Fig. 6. We have found that as the saturation is installed all maxima of \( R_H \) become equivalent. To the best of our knowledge such a property was not yet observed experimentally.
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Figure 6. The oscillatory behaviour of the Hall resistance for a double dot in high magnetic field regime \((\phi/\phi_0 = 0.15)\) and in the presence of a dot-lead constriction \((t_{LS} = 0.5)\); the inter-dot tunnel-coupling is \(\tau = 0.3\). The small amplitude oscillations due to the splitting are more evident in the range \(V_g > 0\) where the spectrum is covered by edge states. The same gate potential is applied on both dots.

In conclusion, an appropriate form of the Landauer-Büttiker approach has been used to describe the electronic transport through an array of coupled quantum dots placed in a strong perpendicular magnetic field. Within the tight-binding model and the self-consistent Hartree approximation for both intra- and inter-dot Coulomb interaction, we have shown that the multiplet structure is determined by the tunnel-coupling between dots, while the Coulomb interaction gives rise to a significant width of the peaks and increase of the splitting. The peak splitting shows saturation at the perfect tunnel-coupling of the dots. The quantum states of the edge-type are much more robust to the electron-electron interaction then the bulk-type states which undergo strong deformations with consequences for the transmittance spectrum. We suggest that the regular drift of the sub-peaks with the variation of the magnetic field, which has been observed experimentally \([3]\), occurs only if the Fermi level lies in a domain of the energy spectrum filled with edge-states. The multiplet structure of the transmittance yields small oscillations of the Hall resistance in the quantum regime.

Further interesting problems such as the magnetic drift in the presence of the electron-electron interaction, the detuning effects on the Hall resistance, and the coherence properties of the array (as given by the phase of the transmittance) will be the subjects of future work which is under way.

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