Robust Cooperative Relay Beamforming
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Abstract—In this paper, the robust distributed relay beamforming problem is solved using the worst case approach, where the problem solution has been involved because of the effect of uncertainty of channel knowledge on the quality of service (QoS) constraints. It is shown that the original robust design, which is a non-convex semi-infinite problem (SIP), can be relaxed and reformed to a semi-definite problem (SDP). Monte-Carlo simulations are presented to verify the performance improvement of our proposed robust problem over existing robust and non-robust problems in terms of transmit power and symbol error probability.

Index Terms—Distributed relay beamforming, Semidefinite programming, Robust optimization, Channel uncertainty

I. INTRODUCTION

Relay networks is one of the main novel feasible techniques which can increase the capacity of wireless networks by multi-hopping [1–3] or parallel relaying [4] in regenerative setting or nonregenerative settings [5]. Recently, distributed relay beamforming has been found to be appealing because of the simplicity of non-regenerative relays hardware and also achieving the favorite diversity order offered by by multiple route diversity or . In such systems, relay nodes organize a single virtual MIMO node and transmit the linearly beamformed version of their received signal in distributed fashion without communicating to each other. The distributed beamforming systems for single user [6] and multi-users [7, 8], minimize the total relays transmit power with signal to interference and noise ratio (SINR) constraints at the destinations. In [6–8], the instantaneous channel state information (CSI) have to be perfectly available at the relays to maintain the instantaneous SINR above a threshold. In all of the proposed beamforming systems in [6–8], it is assumed that the perfect CSI is available at the relay nodes; However, this is an idealistic assumption, since the CSI is often subject to uncertainties because of the channel estimation or quantization error. If the statistical information of channels uncertainty (i.e. the probability density function) are available, a probabilistic or statistical approach can be used which the SINR constraints of the problem are often formulated based on outage probability and it is recently investigated in [9, 10]. In the case which the unknown perturbation is subject to unknown probability distribution with bounded variation, worst-case robust approach is used commonly. The worst case robust beamforming has been well presented in [11] for basic multiple antenna system with only simple power constraints. The robust formulation in [12] and [13] has redesigned respectively the distributed beamforming problems of [7] and [8] by worst-case approach, which demonstrates visible performance improvement with respect to the non-robust systems when the channels are perturbed. The contributions of our work are as follows

- All channels are subject to uncertainty, while in [12, 13] the sources-to-relays channels are not perturbed.
- The approach in [12, 13] used a conservative approximation for SINR constraints, Min over Max (MoM), to avoid semi-infinite programming (SIP) appeared due to the uncertainty region in the QoS constraints of the robust problem. But because of more accurate formulation, our work doesn’t utilize conservative MoM approximation and it outperforms [12, 13]. Instead of MoM, we propose a new equivalent Semi-Definite representable (SDr) problem to the original SIP.

II. ROBUST DESIGN FORMULATION

Assume d single antenna sources and destinations are communicating without direct link through R single antenna amplify-and-forward (AF) relays. The one-shot received signals at the relays in the scalar and vector forms are written as

\[ x_r = \sum_{p=1}^{d} f_{rp} s_p + v_r, \quad \mathbf{x} = \sum_{p=1}^{d} f_p s_p + \mathbf{v}. \] (1)

where \( s_p \) is \( p \)-th user’s transmit signal with the transmit power \( P_p = E|s_p|^2 \), \( x_r \) and \( v_r \) are \( r \)-th relay received signal and noise, \( f_{rp} \) is the complex channel coefficient from \( p \)-th source to \( r \)-th relay. For ease of vector based formulation, we define

\[ f_p \triangleq [f_{1,p}, ..., f_{R,p}]^T, \quad \mathbf{x} \triangleq [x_1, ..., x_R]^T, \quad \mathbf{v} \triangleq [v_1, ..., v_R]^T. \] (2)

To perform relay beamforming, a complex weight coefficient, denoted as \( \mathbf{w} \), is used at the \( r \)-th relay to amplify its received signal. By denoting \( \mathbf{w} \triangleq [w_1, w_2, ..., w_R]^T \), \( \mathbf{W} \triangleq \text{diag}(\mathbf{w}) \), the output signal vector of the relays is \( \mathbf{t} = \mathbf{W}^H \mathbf{x} \) and the received signal at the \( k \)-th destination is given by

\[ y_k = g_k^T \mathbf{t} + n_k = g_k^T \mathbf{W}^H \sum_{p=1}^{d} f_p s_p + g_k^T \mathbf{W}^H \mathbf{v} + n_k \]

\[ = g_k^T \mathbf{W}^H f_k s_k + g_k^T \mathbf{W}^H \sum_{p=1, p \neq k}^{d} f_p s_p + (g_k^T \mathbf{W}^H \mathbf{v} + n_k). \] (3)

where \( g_{rk} \) is the channel coefficient from \( r \)-th relay to \( k \)-th destination, \( n_k \) is the noise at the \( k \)-th destination and \( g_k \triangleq [g_{k1}, ..., g_{kR}]^T \). The three last terms in (3) are respectively desired received signal, interference and noise. By denoting \( P_r \) as the total transmit relays power, \( \gamma_k \) as the specified SINR threshold, \( \mathbf{f}_k \) as the SINR at the \( k \)-th destination and \( P_s^k, P_f^k \) and \( P_n^k \) respectively as the desired signal, interference...
and noise powers at the kth destination, they can be computed as

\[ P_T = E_{(s_1, \ldots, s_d)} \{ t^H t \} = Tr \left\{ W^H E_{(s_1, \ldots, s_d)} \{ x^H x \} W \right\} \]

=\[Tr \left\{ W^H R_s W \right\} = \sum_{r=1}^{R} |w_r|^2 |R_s|_{r,r} = w^H Dw \] (4)

\[ P_k^s = E_{s_k} \left| t^H \left( W^H f_k s_k \right) \right|^2 = P_k w^H h_k^H h_k w = w^H R_k w \]

\[ P_k^v = E_{v, n_k} \left| t^H \left( W^H f_p \right) \right|^2 = \left| P_k w^H (g_k \circ f_p) \right|^2 = w^H Q_k w \]

The matrices \( \Delta \) is defined in (6)-(10). The block perturbation of the auxiliary matrix and vector parameters as

where in the above formulation we use the following auxiliary matrix and vector parameters

\[ R_x = \bigoplus_{p=1}^{d} P_p P_f + \sigma^2 f_k \]

\[ R_k = \bigoplus_{p=1}^{d} P_p h_k h_p^H + \epsilon_k \]

\[ Q_k = \bigoplus_{p \in \{1, \ldots, d \} \setminus \{k \}} P_p h_p^H (g_k \circ f_p) \]

\[ D_k = \sigma_k^2 \bigoplus_{p \in \{1, \ldots, d \} \setminus \{k \}} P_p \]

The robust beamforming aims to minimize the total relay transmit power \( P_T \) subject to holding the SINR of each user \( \gamma_k \), above a predefined \( \gamma_k \), while the state information of \( f_k \) and \( g_k \) are incomplete. Now the robust problem can be written as

\[ \min_w \{ \max \tilde{P}_T \} = \{ \max w^H \tilde{D} w \} \] (11a)

\[ \text{s.t. } \Gamma_k = \frac{w^H R_k w}{\nu^H (Q_k + D_k) w + \sigma_n^2} \geq \gamma_k \forall k = 1, \ldots, d \] (11b)

\[ \forall \tilde{Q}_k \in S^{Q_k}, \forall \tilde{D}_k \in S^{D_k}, \forall \tilde{R}_k \in S^{R_k} \] (11c)

where \( \tilde{D} = D + \Delta_D, \tilde{R}_k = R_k + \Delta_k^R, \tilde{Q}_k = Q_k + \Delta_k^Q, \tilde{D}_k = D_k + \Delta_k^D \) are the perturbed versions of the matrices defined in (6)-(10). The block perturbation of the matrices is an approach to simplify the algebraic complexity which due to propagation of perturbation from \( f_k \) and \( g_k \) to the matrices defined in (6)-(10). This approach is proposed by [12, 13]. In their works \( f_k \) is left unperturbed.

The matrices \( \Delta_D, \Delta_k^R, \Delta_k^Q \) and \( \Delta_k^D \) are the random uncertainty Hermitian matrices which are added to \( D, R_k, Q_k \) and \( D_k \), respectively. The sets of all possible values of \( \tilde{D}, \tilde{R}_k, \tilde{Q}_k, \tilde{D}_k \) are denoted by \( S^D, S^{R_k}, S^{Q_k} \) and \( S^{D_k} \), respectively, which cover all cases of the perturbed channel coefficients. It is assumed that the \( \Delta_D \) and \( \Delta_k^D \) are diagonal random matrices, because they are perturbing \( D \) and \( D_k \) matrices which are also diagonal. Furthermore, according to the worst-case approach, we assume that the channel coefficient uncertainties are norm bounded by some known constants as

\[ \| \Delta_D \| \leq \varepsilon_D \]

\[ \| \Delta_k^Q \| \leq \varepsilon_k^Q, \| \Delta_k^D \| \leq \varepsilon_k^D, \| \Delta_k^R \| \leq \varepsilon_k^R \]. (12a)

Since the perturbation matrices are not independent in general, the problem formulated by replacing (11c) by (12), results in suboptimal solution for (11). In order to guarantee positive power quantities and the convexity of our problem, the estimated channel matrices should be positive semi-definite as

\[ D + \Delta_D \succeq 0 \]

\[ R_k^s + \Delta_k^R \succeq 0, Q_k + \Delta_k^Q \succeq 0, D_k + \Delta_k^D \succeq 0 \]. (13a)

The first intractability of the robust optimization problem in (11b) is the infinite number of the constraints which the problem should be solved subject to them. In fact, the problem is a SIP [14]. Since SIP have some solutions [14], but computationally complex, it is preferred to avoid these problems by converting them to another standard form. To simplify the SIP robust problem (11b), first we need to maximize the objective function over the uncertainty matrix \( \Delta_D \). Then, the objective function for the robust problem is reformulated as

\[ \max_{\Delta_D} \{ w^H (D + \Delta_D) w \} \] (14)

Second, since all constraints of (11b) must be satisfied for all values of perturbed matrices, we can equivalently say that minimum value of the left side of the inequality should always be greater than the requested SINR. Considering all of the constraints, the robust optimization problem in (11b) can be expressed as

\[ \min_{\Delta_D} \max_{w} w^H (D + \Delta_D) w \]

s.t. \( \Delta_D \succeq \gamma_k \), \( \forall \text{ constraints of (11b)} \) (15a)

Inequalities of (12), (13) \( \forall k = 1, \ldots, d \) (15c)

where \( T_k = R_k - Q_k (D + D_k) \) and \( \Delta_k^R = \Delta_k^R - \gamma_k \) \( (\Delta_k^Q + \Delta_k^D) \).

In contrast to our constraint formulation (15b), the approach of [12, 13] has differently qualified \( \Gamma_k \geq \gamma_k \) by approximating \( \gamma_k \) which is named MoM in our work. In order to use the maximum term in the objective function of (15), along with its active constraint \( D + \Delta_D \succeq 0 \), we use the Rayleigh-Ritz theorem. Actually, when \( \Delta_D \) is in the same direction of \( w^H w \), the objective function is maximized. The maximization over \( \Delta_D \) subject to its related constraints in (15) is

\[ \max_{\Delta_D} \frac{w^H (D + \Delta_D) w}{\| w \|^2} \] (16)

where the worst case value for \( \Delta_D \) is

\[ \Delta_D = \frac{w w^H}{\| w \|^2} \] (17)

Using (17), the robust form of the objective function of (15) will be
Since the objective function and QoS constraints of (15) do not have any common constrains, the QoS and positive semi-definite constraints of (15) form an optimization problem over all of the perturbation matrices, which is independent from the objective function in (16). Therefore, we focus on the optimization of the QoS constraint in (15) for \( k = 1, \ldots, d \). The constraint in (15) can be written as follows

\[
\min \quad w^H (T_k + \Delta T_k) w_k - \sigma^2 \gamma \\
\text{s.t.} \quad (12b), (13b) \forall \ k = 1, \ldots, d, \tag{19}
\]

Note that the positive semi-definite (PSD) constraint of (19) also satisfies the corresponding constraints on its related instantaneous covariance matrices, \( R_k^h, Q_k \) and \( D_k \). To solve the problem, we look for a relaxation scheme to convert the problem into a convex form and investigate the gap between the relaxed and non-relaxed problems. Applying Lagrange duality technique, the solution of (19) is equal to the solution of the following problem

\[
\inf \quad L \begin{pmatrix} \lambda_Q^O, \lambda_P^O, \lambda_R^O, Z_Q^O, Z_P^O, Z_R^O, \Delta_Q^O, \Delta_P^O, \Delta_R^O \end{pmatrix} \\
\text{s.t.} \quad Z_Q^O \geq 0, Z_P^O \geq 0, Z_R^O \geq 0, \lambda_Q^O \geq 0, \lambda_P^O \geq 0, \lambda_R^O \geq 0 \quad \text{and} \quad Z_R^O \text{ is diagonal matrix} \tag{20}
\]

where the requirement of being diagonal on the dual variable \( Z_P^O \) comes from the fact that \( \Delta_P^O \) needs to be diagonal. Since the problem in (20) is convex, we can use the dual Lagrange function and Karush-Kuhn-Tucker (KKT) conditions to find the global optimum value of the problem \([15]\). Applying the KKT conditions, the Lagrange function will be

\[
L = w^H \begin{pmatrix} R_k^h - \gamma_k (Q_k + D_k) + \Delta R_k^O - \gamma_k (\Delta Q_k^O + \Delta P_k^O) \end{pmatrix} w \]

\[
+ \lambda_Q^O \left( \| \Delta Q_k^O \|^2 - (\varepsilon_k^Q)^2 \right) + \lambda_P^O \left( \| \Delta P_k^O \|^2 - (\varepsilon_k^P)^2 \right) + \lambda_R^O \left( \| \Delta R_k^O \|^2 - (\varepsilon_k^R)^2 \right) - Tr \begin{pmatrix} Z_Q^O (Q_k + \Delta Q_k^O) \end{pmatrix} + Tr \begin{pmatrix} Z_P^O (D_k + \Delta P_k^O) + Z_R^O (R_k^h + \Delta R_k^O) \end{pmatrix} - \sigma^2 \gamma_k \lambda_k \tag{21}
\]

where \( \{ \lambda_Q^O \}_{k=1}^d, \{ \lambda_P^O \}_{k=1}^d \) and \( \{ \lambda_R^O \}_{k=1}^d \) are the non-negative dual variables. For the sake of simplicity, we denote the above function by \( L_k \). Using \([16]\) and setting zero the derivatives of \( L_k \) with respect to \( \Delta Q_k^O, \Delta P_k^O \) and \( \Delta R_k^O \), we have

\[
\Delta Q_k^O = \frac{Z_Q^O + \gamma_k h X}{2 \lambda_Q^O}, \quad \Delta P_k^O = \frac{Z_P^O + \gamma_k h X}{2 \lambda_P^O} \quad \text{and} \quad \Delta R_k^O = \frac{Z_R^O + \gamma_k h X}{2 \lambda_R^O} \tag{21}
\]

Inserting the derived values for \( \Delta Q_k^O, \Delta P_k^O, \Delta R_k^O \) in (21) and maximizing the relation with respect to \( \lambda_Q^O, \lambda_P^O \) and \( \lambda_R^O \), leads to the following form for the Lagrange dual function after some algebraic manipulations and using \( X = w w^H \)

\[
\max \quad M \begin{pmatrix} Z_Q^O, Z_P^O, Z_R^O, X \end{pmatrix} \tag{22a}
\]

\[
\text{s.t.} \quad Z_Q^O \geq 0, \quad Z_P^O \geq 0, \quad Z_R^O \geq 0 \quad \forall \ k = 1, \ldots, d \tag{22b}
\]

where

\[
M (\cdot) = \text{Tr} \begin{pmatrix} X \left( I + \frac{1}{2} \| X \| \right) \left( I + \frac{1}{2} \| X \| \right) \end{pmatrix} - \frac{1}{2} \| X \|^2 \quad \text{and} \quad \text{Tr} \begin{pmatrix} -Z_Q^O R_k^h - Z_R^O Q_k - Z_P^O D_k \end{pmatrix} - \frac{\varepsilon_R^k}{2} \| Z_R^k - X \| \]

\[
- \frac{\varepsilon_Q^k}{2} \| Z_Q^k + \gamma_k h X \| - \sigma^2 \gamma_k
\]

By substituting (22) and (18) in (15), we can write our main robust optimization problem as

\[
\min \quad \text{Tr} \begin{pmatrix} X (D + \varepsilon D) \end{pmatrix} \quad \text{s.t.} \quad \max \begin{pmatrix} Z_Q^O, Z_P^O, Z_R^O \end{pmatrix} \geq 0 \quad \forall \ k = 1, \ldots, d \tag{23}
\]

Consider the fact that the maximum of the expression

\[
\text{Tr} \begin{pmatrix} -Z_Q^O R_k^h - Z_R^O Q_k - Z_P^O D_k \end{pmatrix} = 0
\]

Constraints (22b), and \( \text{Rank}(X) = 1 \)

Note that the objective function in (24) is linear and all the constraints except the last one are conic convex. We drop this non-convex constraint to relax the problem into a convex optimization problem. The well-known semi-definite problem (SDP) solvers such as SeDuMi or CVX can be used for solving the above problem by semi-definite programming in polynomial time using interior point methods. Since the solution of (24) is not always rank one, randomization techniques \([7]\) can be used to obtain an approximate solution of the original problem from the solution of the relaxed problem. However, our simulation results show that the rank of \( X \) is always one when \( d < 3 \). This has been also reported in \([17]\) analytically for \( d < 3 \). If the optimum solution is a rank-one matrix, the principal value of the matrix is used to determine \( w \), otherwise the best rank one approximation is obtained using the procedure described in \([18]\). Note that the minimum value of the relaxed form (without rank constraint) of the problem in (24) is a lower bound for the minimum value of the original problem in (24). As mentioned after (12), the optimal solution of (24) is still suboptimal solution for (11) as it happens for \([12, 13]\) too, but if uncertainties of the perturbation matrices are occurred due to the quantization noise of related matrices, they can be assumed independent and both problems have the same optimal solutions.

III. SIMULATION AND NUMERICAL RESULTS

In this section, we use Monte-Carlo simulations to compare our accurate robust with MoM robust \([13]\) and non-robust power allocation methods. In all simulations, we assume 15 relays \((R = 15)\) and 2 users \((d = 2)\). The noise power
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IV. CONCLUSION

In this paper, the robust optimal power allocation algorithm for multi-user cooperative networks was solved in a more accurate approach compared to the previous works. The proposed approach assumed uncertainty on all channels in the QoS aware beamforming problem to perform a robust design. The simulation results have shown the superiority of the proposed robust method compared to the previous robust and non-robust methods.

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