Let the New Experiments Tell the Quantum Theory

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Abstract

Several new physics experiments in 1998 were performed and analyzed to show the subtlety of quantum theory, including the “wave-particle duality” and the non-separability of two-particle entangled state. Here it is shown that the measurement is bound to change the object by destroying the original quantum coherence between the object and its environment. So the “physical reality” should be defined at two levels, the “thing in itself” and the “thing for us”. The wave function in quantum mechanics is just playing the role for connecting the two levels of matter via the fictitious measurement.

PACS numbers: 03.65.-w.03.65.Bz
In recent years especially in 1998, a series of important physics experiments were published. They were performed in such delicate manner and with so amazing results that pushed the subtlety or mystery of quantum theory in front of the physicists as well as the public in a very acute way. These experiments can be sorted into four categories:

(1) The experimental discovery of fractional quantum Hall effect (FQHE) [1] and the observation of fractional charge $(e/3)$ in FQHE system [2,3].

(2) Direct test of wave-particle duality (complementarity) by a “which-way” experiment in an atom interferometer [4].

(3) Einstein-Podolsky-Rosen (EPR) experiments were performed in two-photon entangled state to show the violation of Bell inequality under strict Einstein locality conditions [5] or to show the quantum correlation over long distance ($> 10\text{km}$) [6]. Also an EPR experiment was achieved at CERN to test the non-separability of entangled neutral-kaon wave function [7].

(4) First direct observation of time-reversal non-invariance in the neutral-kaon system [8].

In this paper we will concentrate on the (1)—(3) experiments especially (2) and (3) because they are directly related to the fundamental interpretation of quantum mechanics (QM). We will show that the outcome of these experiments strongly support the validity and completeness of QM including the Heisenberg’s position-momentum uncertainty relation. However, the correct interpretation of QM does need a clarification of an important point of view that a quantum state before the measurement has no any information. The latter is created only during the measurement by the object and subject in common.

I. The essence of measurement

The “which way” (WW) experiment in an atom interferometer was proposed by Scully et al [9] and successfully realized by DNR [4]. It is a new realization of ideal experiment long considered by physicists, say, by Feynman [10]. In Feynman’s double-slit interference
experiment of electron, in order to determine which slit the electron goes through, a light source is put just behind the double-slit for watching the electron. Feynman predicted that once the light source is switched on, the interference pattern will be washed out. And it was explained by the impact of photon on the electron. The momentum transfer $\Delta p_x$ and the position uncertainty of electron along the screen direction, $\Delta x$, will be constrained to the Heisenberg’s uncertainty relation:

$$\Delta x \Delta p_x \geq \hbar/2.$$  \hspace{1cm} (1)

As Feynman quoted from Heisenberg that “It is impossible to design an apparatus to determine which hole the electron passes through, that will not at the same time disturb the electrons enough to destroy the interference pattern.” Feynman said: “The uncertainty principle ‘protects’ quantum mechanics.” See also some interesting discussion on this problem in Refs. [11-13].

Instead of electrons, the $^{85}\text{Rb}$ atoms are used in DNR’s experiment. The “double-slit” is realized by the Bragg diffraction of atomic beam on two standing light waves. The WW information is provided by two microwave pulses which divide the splitting beams into pure internal states, either $|2\rangle$ or $|3\rangle$ being two hyperfine states with total angular momentum $F = 2$ or $3$. Once this is done, the interference fringes are lost due to the $\langle 3|2 \rangle = 0$. The analysis shows that the momentum transfer from microwave pulse to the atom is so negligible that it plays no role in the loss of interference. Then the authors concluded that complementarity is not enforced by the uncertainty relation, Eq. (1).

Undoubtedly the DNR experiment is very important for it further reveals the essence of measurement which can be summarized as three propositions:

(a) The measurement is bound to change the state of object.

(b) The measurement is also quantum in essence. The quantum correlation (i.e. entanglement) between the apparatus and the object is bound to destroy the quantum correlation (quantum coherence) originally existing in the object.

(c) There is no any information (experimental data) existed before the measurement is
made.

Let us first look at the measurement in classical physics. The specific heat at constant volume of certain substance is defined as

$$C_V = \frac{\partial U}{\partial T} \bigg|_V .$$

(2)

While that at constant pressure is

$$C_P = T \frac{\partial S}{\partial T} \bigg|_P .$$

(3)

Note that, however, the change of the internal energy $U$, or the temperature $T$, or the entropy $S$, even is infinitesimal but certainly can not be zero because of the energy quantum $h\mu$ with the Planck constant $h \neq 0$. Therefore, the definition and value of $C_V$ or $C_P$ are endowed by the operation during the measuring process as shown by Eq. (2) or (3) respectively. Either $C_V$ or $C_P$ is objective in the sense of its unique value given by the measurement. However, the simultaneous measurement of $C_V$ and $C_P$ will disturb each other, leading to inaccuracy in the result. The reason is as follows [14]:

A measurement is always an operation procedure (denoted by $A$) for changing the object to pick out the corresponding data (denoted by $a$): $A \rightarrow a$. Similarly, another measuring procedure $B$ leads to $b$: $B \rightarrow b$. If $A$ and $B$ are not in conformity but are imposed simultaneously on an object, then $A$ (or $B$) would become the disturbance to $b$ (or $a$), as shown in Fig. 1.

The great merit of quantum mechanics (QM) lies in the fact that it unveils the truth of epistemology at the basic level. For example, the plane wave function of a freely moving particle reads

$$\Psi(x, t) \sim \exp\left\{\frac{i}{\hbar}(p_x x - E t)\right\} .$$

(4)

Notice that, however, where the momentum parameter $p_x$ inside is not an observable, i.e., not a quantity existing before the measurement. Only during a “space translational operation” is imposed on the wave function as follows

$$-i\hbar \frac{\partial}{\partial x} \Psi = -i\hbar \lim_{\Delta x \to 0} \frac{\Psi(x + \Delta x, t) - \Psi(x, t)}{\Delta x} = p_x \Psi$$

(5)
can we pick out the observable $p_x$ on the right side. So the fact that in QM a classical
dynamical variable, say $p_x$, becomes an operator

$$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$$

(6)
is nothing but a mathematical statement (6) of the principle of epistemology that the mea-
surement is always a change on the object.

It is because the measurement of position $x$ and $p_x$ are different, they have the meaning
of complementarity. On the other hand, both $x$ and $p_x$ are related to the measurement in
the space, i.e., they have “identity” in essence, so the “repulsiveness” (conflict) contained
in their difference will display inevitably under certain condition. Therefore, it seems to us
that the uncertainty relation and the complementarity just are two aspects reflecting the
same essence. There is no question about which one of them is more fundamental.

The DNR experiment can also be explained by the general scheme with Fig. 1. The
experiment arrangement without two microwave pulses is denoted by “A”, while “a” denotes
the appearance of interference fringe. On the other hand, “B” denotes the operation of
imposing two microwave pulses for measuring the WW information, while “b” denotes the
disappearance of fringe. The repulsiveness or complementarity is contained in one expression
that the distinguishability (of WW information) $D$ and the fringe visibility $V$ is limited by
the duality relation $D^2 + V^2 \leq 1$ [4].

The unique feature of DNR experiment lies in the fact that they change the internal
quantum state of atom. In some sense the measurement destroys the quantum coherence of
internal state. It has no classical correspondence.

II. The EPR experiments and the entanglement in QM

1. Being a considerable progress of the famous Aspect experiment [15], Weihs experiment
[5] for the first time fully enforced the condition of locality. The spacelike separation of two
“observers” (Alice and Bob) is achieved by sufficient physical distance (400m) between them
and by ultrafast (the duration of an individual measurement $< 100\text{ns}$ which is far less than
1.3\mu s = 400m/c) and random setting of the analyzers. So the possibility of any signal connecting Alice and Bob with velocity less than or equal to the speed of light was certainly excluded.

The generalized Bell’s inequality reads

\[ S(\alpha, \alpha', \beta, \beta') = |E(\alpha, \beta) - E(\alpha', \beta)| + |E(\alpha, \beta') + E(\alpha', \beta')| \leq 2 \]  

where \( E(\alpha, \beta) \) is the expectation value of two-photon correlation with \( \alpha \) and \( \beta \) being the directions of polarization analyzers of Alice and Bob. On the other hand, the prediction of QM is

\[ S_{\text{QM max}} = S_{\text{QM}}(0^\circ, 45^\circ, 22.5^\circ, 67.5^\circ) = 2\sqrt{2} = 2.82 > 2. \]  

The 14700 coincidence events collected in 10s yield

\[ S^{\text{exp}} = 2.73 \pm 0.02 \]  

which corresponds to a violation of inequality (7) of 30 standard deviation and so strongly supports QM.

2. Being a remarkable realization of Franson’s prominent proposal [16], Tittel experiment [6] demonstrated the quantum correlation between two (energy and time) entangled photons can be maintained over long distance (> 10km). The coincidence counts between two interferometers were fitted to the probability function as

\[ P = \frac{1}{4} \left( 1 + V_{\text{exp}} \left[ \frac{\lambda(\delta_1 - \delta_2)}{2\pi L_c} \right]^2 \cos(\delta_1 + \delta_2) \right) \]  

where \( \delta_1 \) or \( \delta_2 \) is the variable phase-difference in either interferometer caused by the path length difference, \( \lambda = 1310\text{nm} \) is the wavelength of photon while \( L_c \) is the single-photon coherence length. The coefficient \( V \) is called as the “visibility”, \( V \leq \frac{1}{\sqrt{2}} \approx 0.71 \) inferred by the Bell-inequality. But the experimental data showed \( V^{\text{exp}} = 81.6 \pm 1.1\% > 0.71 \), a violation of the Bell-inequality by 10 standard deviation and a further strong support to the QM.
3. A beautiful EPR experiment was performed by CPLEAR Collaboration at CERN on $K^0\bar{K}^0$ system [7]. Alice and Bob were located at left and right side with distance $\sim 10\text{cm}$ between. According to the prediction of QM, the wave function of $K^0\bar{K}^0$ system is entangled as follows:

$$|\Psi(t_a, t_b)\rangle = \frac{1}{\sqrt{2}} [ |K_S(t_a)\rangle_a |K_L(t_b)\rangle_b - |K_L(t_a)\rangle_a |K_S(t_b)\rangle_b ]$$ (11)

where $t_a$ and $t_b$ are the proper time records at Alice and Bob sides while

$$|K_S\rangle = \frac{1}{\sqrt{2}} [ |K^0\rangle + |\bar{K}^0\rangle ], \quad |K_L\rangle = \frac{1}{\sqrt{2}} [ |K^0\rangle - |\bar{K}^0\rangle ].$$ (12)

The “asymmetry” is defined as

$$A(t_a, t_b) = \frac{I_{\text{unlike}}(t_a, t_b) - I_{\text{like}}(t_a, t_b)}{I_{\text{unlike}}(t_a, t_b) + I_{\text{like}}(t_a, t_b)}$$ (13)

where $I_{\text{unlike}}$ ($I_{\text{like}}$) is the intensity of event with $K^0\bar{K}^0$ or $\bar{K}^0K^0$ ($K^0\bar{K}^0$ or $\bar{K}^0\bar{K}^0$) detected. By contrast, if the wave function is factorized or separable, i.e., only one term is left in the expression (11), then the asymmetry would always be zero, $A = 0$. The experiment showed the value of $A(\Delta l)$ with $\Delta l$ (in cm) being the flight path difference:

$$A^{\text{exp}}(0) = 0.81 \pm 0.17, \quad A^{\text{exp}}(5) = 0.48 \pm 0.12$$ (14)

in comparing with the prediction of QM:

$$A^{\text{QM}}(0) = 0.93, \quad A^{\text{QM}}(5) = 0.56.$$ (15)

Thus the separability hypothesis is excluded with a confidence level $CL > 99.99\%$ and proves once again the validity of QM.

In all these EPR experiments mentioned above the entangled state i.e., two-particle state with quantum correlation over long distance, exhibits its subtlety. For example, for the $K^0\bar{K}^0$ system described by Eq. (11), only after Alice finds a $K^0$ (or $\bar{K}^0$) in the measurement at time $t_a$, can Alice predict with 100% certainty that Bob must finds a $\bar{K}^0$ (or $K^0$) at the same time ($t_a = t_b$). Since they are separated over long distance, (10cm in CPLEAR experiment and even $> 10\text{km}$ in Tittel experiment), no information can be communicated between them.
with a velocity equal to or less than the speed of light. In other words, no local hidden variable (LHV) can exist as inferred by the violation of Bell inequality. Therefore, the sudden nonlocal collapse of wave function of entangled two-particle state (into a measured distinct particle at Alice side with another one at Bob side) seems to be caused via some “spooky action at a distance” by Einstein, (see [17]).

III. Quantum state and wave function

Let us try to understand the mystery posed by the experiments discussed above, at least to some extent. Then it seems to us that the fundamental interpretation of QM is involved.

In Dirac notation, a quantum state, e.g., a one-particle state in one-dimensional space is denoted by an abstract state vector $|\Psi\rangle$ in Hilbert space. In Hersenberg picture, there is no description either $x$ or $t$ in $|\Psi\rangle$. Only after some representation is chosen, can it get some description. For instance, if we choose the eigenvector of the position $x$, $|x,t\rangle$, as the base vector and take the contraction (projection) of $|\Psi\rangle$ with $|x,t\rangle$, we obtain the wave function in configuration space:

$$\psi(x,t) = \langle x,t | \Psi \rangle.$$  \hspace{1cm} (16)

Alternatively, we can choose the eigenvector of momentum $p$, $|p,t\rangle$, as the base vector to get the wave function in momentum space ($p$ representation) as

$$\varphi(p,t) = \langle p,t | \Psi \rangle.$$  \hspace{1cm} (17)

The two kinds of wave function, (16) and (17), are two different descriptions for the same quantum state $|\Psi\rangle$. No one in the two is more fundamental than the another one.

The wave functions in QM are not observable. But they are very useful in linking the even more abstract state vector, say $|\Psi\rangle$, to the potential possible outcome in experiments if the latter are really performed on the state. For example, we are going to measure the position $x$ of the particle, so we choose $|x,t\rangle$ to characterize (represent) the “apparatus” for $x$ measurement and write down the wave function $\Psi(x,t)$. Note that, however [18],
(a) What contained in the $\Psi(x,t)$ is merely a “fictitious measurement”. So being a “probability amplitude”, the wave function always contains the imaginary number unit $i = \sqrt{-1}$ which is unobservable.

(b) According to the statistical interpretation by M. Born, $|\Psi(x,t)|^2$ is the probability of finding the particle at position $x$ during the measurement rather than that of the appearance of the particle before the measurement.

(c) Sometimes it was tacitly assumed that $x$ in the wave function is the position coordinate of “point particle”. We don’t think so. Instead, we prefer to think that in the 1S state of a Hydrogenlike atom the electron has a spatial extension with radius $a/Z$ ($a = 0.529 \times 10^{-10} m$ being the Bohr radius and $Z$ being the charge number of nucleus). On the other hand, when an electron is under high-energy collision, its spatial extension may be compressed into a tiny one, say less than $10^{-18} m$ [19].

IV. The relation between individual and its environment

The existence state of any individual particle is depending on its environment. This can be seen most clearly in the lifetime ($\tau$) of an unstable particle. For instance, see the $\beta$-decay of nuclei. A free neutron has $\tau = 14.8$ minutes, while the $\tau$ of nuclide $^{11}_3 Li_8$ is shortened to only 8.5$\mu$s. On the other hand, the $\tau$ of nuclide $^{128}_52 Te$ is extremely long: $\tau = 2.1 \times 10^{24}$ years. Many nuclides, including the neutron in the neutron star (pulsar), are stable against $\beta$-decay, i.e., they have $\tau = \infty$. This shows that the lifetime of a neutron is strongly influenced by its (nuclear) environment. Actually, it nearly has no intrinsic stability. This can also be seen from the decay law:

$$N(t) = N_0 e^{-t/\tau}, \quad -\frac{dN/dt}{N(t)} = \frac{1}{\tau} = \lambda = const$$

which means that a neutron at any time, as long as it has not decayed, has a definite decay probability independent of its existing time already. Just like the words said by the Chinese philosopher Zhuangzi (369BC-286BC): “Just was born just died, just died just was born”.

The mass of a particle is also depending on its environment. In the language of quantum
field theory, mass generation is only possible after the vacuum undergoes a phase transition ([20], see also the explanation by Wilczek [21], he speculated the discovery of Higgs particle in the years to come. We had calculated the Higgs mass to be 138GeV [22]).

Now the nonlocal two-particle state in the EPR experiments is non-separable before the measurement. As shown in Eqs. (11) and (12), the kaon at Alice (or Bob) side is not either $K^0$ or $\bar{K}^0$, but neither $K^0$ nor $\bar{K}^0$. The information that Alice finds a $K^0$ (or $\bar{K}^0$) while Bob finds a $\bar{K}^0$ (or $K^0$) is just created by them via measurement at the same time.

The strong correlation among many particles is clearly shown in FQHE. The ground state of $N$ electrons is described by the Laughlin wave function [23]. Every electron loses its independent feature. So the whole system exhibits itself as an incompressible fluid and the elementary excitation above the ground state is a quasi-particle with fractional charge (say $e/3$) [2,3] and carrying non-local information (such as an invisible string) [24]. Only after we destroy the quantum coherence of FQHE, can an electron appear.

Now we understand why the quark with fractional charge can not be deconfined from a hadron (say a proton). This is because quark is not a particle in the common sense, i.e., not a “building block”. The latter is only well defined when it can be separated from its environment with the binding energy $B$ much less than its rest energy $E_0$: $B/E_0 \ll 1$. In fact, every particle is changing during its separating process. When we wish to pick a $u$ quark out of a proton, both this $u$ quark and other two quarks ($u$ and $d$) are changed to such an extent that the whole proton is destroyed and what we can see are other particles.

The three valence quarks ($uud$) are suitable for describing the property of a proton near its ground state. However, when the proton is under high-energy collision, it would be better to use the parton model, i.e., to resort to the picture of many sea quarks and gluons besides the valence quarks. A proton is infinite in essence, it has various aspects in various experiments. Not only dynamics, but also its ingredients are depending on the character of experiment, i.e., on what we are looking for [18].

From individual particle to the whole environment, we see that they are all infinite and
mutually related. We tend to share the view point of Zurek et al [25] that the environment plays a crucial role in destroying the quantum coherence and bringing the measuring process to an end. The apparatus, being a part of environment, substitutes the original quantum correlation (entanglement) in the measured object by the new entanglement between the apparatus and the object, as shown in the microwave measurement of DNR experiment [4]. The final stage of measurement is achieved at the screen or detector (they are also part of the environment), where the wave packet of particle is collapsed.

V. Is the moon there when nobody looks [17]?

In the DNR experiment [4], the center-of-mass motion of atom is described by the plane wave function, which served as a “guiding field” for atom motion. We can not think of a atom like a small ball with radius 0.1nm passing through the double-slit with spacing \(d = 1.3\mu m\). Rather, we should think of the atom like a wave packet with spatial extension exceeding \(d\). What we can do is discussing the wave function and its interference. The particle feature of atom is displayed only at the final stage. Similarly, the two-photon entangled state in Tittel experiment [6] is correlated in long distance (> 10km). To talk about one photon being here or there is meaningless since the single-photon coherence length \(L_C\) is only 10.2\(\mu m\). The CPLEAR experiment clearly shows that the entangled state of \(K^0\bar{K}^0\) system has a spatial extension \(\sim 10cm\), far exceeding the radius or Compton wave-length of a single kaon.

Hence, in our point of view, the so called “wave-particle duality” means the following. Before the quantum coherence of the motion of a particle is destroyed by the measurement, we should handle it as “wave” by Schrodinger equation theoretically until it is detected and then shows its “particle” feature. This is a problem of different temperaments at two levels, not at the same level.

We are now in a position to try to answer Einstein’s question: “What is the physical reality?” It seems to us that a “thing” should be defined at two levels. An object when it is independent of the consciousness of mankind and before the measurement is made, could be
called as “thing in itself”. It is something absolute in nature and containing no information. In QM, it is denoted by a quantum state $|\Psi\rangle$ separated approximately from its environment. Then after some measurement is performed, it is turned into “thing for us”, reflecting a series of experimental data. It is then something relative in nature. Sometimes, we call it “phenomenon”. As J.A. Wheeler said: “No phenomenon is phenomenon until it is an observed phenomenon.” The wave function in QM is just playing the role for connecting the two levels of matter via the fictitious measurement. In some sense, we may also claim that: “We can only see what we intend to see.” Eventually, we will be convinced by the Chinese saying: “Oneness of heaven and man.”

Acknowledgements

The author wishes to thank P-z Bi, Y-z Chen, X-c Gao, Y-k Huo, Z-d Liu, F-q Lu, Z-y Shen, J-y Tang, Y-s Wang, J-b Xu, S-q Ying, and C-y Zhou for discussions. This work was supported in part by the NSF of China.

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Figure Caption
Figure 1: The measurement $A \ (B)$, being an operation (denoted by the arrow) imposing on the object (denoted by dashed-line circle), creates the data $a \ (b)$. If $A$ is not in conformity with $B$, then $A \ (B)$ becomes the disturbance to $b \ (a)$ which is denoted by the wavy line.
Figure 1. The measurement $A \ (B)$, being an operation (denoted by the arrow) imposing on the object (denoted by dashed-line circle), creates the data $a(b)$. If $A$ is not in conformity with $B$, then $A(B)$ becomes the disturbance to $b(a)$ which is denoted by the wavy line.