Strong lensing statistics in large, $z \lesssim 0.2$, surveys: bias in the lens galaxy population

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ABSTRACT
Large current and future surveys, like the Two-degree Field Survey (2dF), the Sloan Digital Sky Survey (SDSS) and the proposed Kilo-Degree Survey (KIDS), are likely to provide us with many new strong gravitational lenses. Taking cosmological parameters as known, we calculate the expected lensing statistics of the galaxy population in large, low-redshift surveys. Galaxies are modelled using realistic, multiple components: a dark matter halo, a bulge component and disc. We use semi-analytic models of galaxies coupled with dark matter haloes in the Millennium Run to model the lens galaxy population. Replicating the selection criteria of the 2dF, we create a mock galaxy catalogue. We predict that a fraction of $7.5 \pm 0.2 \times 10^{-5}$ of radio sources will be lensed by galaxies within a survey like the 2dF below $z < 0.2$. We find that proper inclusion of the baryonic component is crucial for calculating lensing statistics – pure dark matter haloes produce lensing cross-sections several orders of magnitude lower. With a simulated sample of lensed radio sources, the predicted lensing galaxy population consists mainly of ellipticals ($\sim 80$ per cent) with an average lens velocity dispersion of $191 \pm 3\, \text{km s}^{-1}$, producing typical image separations of $\sim 1.5$ arcsec. The lens galaxy population lies on the fundamental plane but its velocity dispersion distribution is shifted to higher values compared to all early-type galaxies. We show that magnification bias affects lens statistics very strongly and increases the 4:2 image ratio drastically. Taking this effect into account, we predict that the ratio of 4:2 image systems is $18 \pm 5$ per cent, marginally consistent with the observed ratio found in the Cosmic Lens All-Sky Survey (CLASS). We also find that the population of four-image lens galaxies differs markedly from the population of lens galaxies in two-image systems. We find that while most lenses tend to be ellipticals, galaxies that produce four-image systems preferentially tend to be lower velocity dispersion systems with more pronounced disc components. Our key result is the explicit demonstration that the population of lens galaxies differs markedly from the galaxy population as a whole: lens galaxies have a higher average luminosity and, for a given luminosity, they reside in more massive haloes than the overall sample of ellipticals. This bias restricts our ability to infer galaxy evolution parameters from a sample of lensing galaxies.

Key words: gravitational lensing – methods: statistical – galaxies: evolution – galaxies: haloes – dark matter.

1 INTRODUCTION
The statistics of strong lensing by galaxies have been studied by several authors in the past (Turner, Ostriker & Gott 1984; Fukugita et al. 1992; Kochanek 1993; Möller & Blain 2001; Chae 2003). Most of these studies calculate a lensing cross-section analytically using a simple lens model along with known mass and luminosity functions. Others are concerned with the effect of specific details of the mass models on the lens statistics or properties of the source population (Huterer, Keeton & Ma 2005). On larger scales, the arc-statistics of galaxy clusters have been used to constrain cosmological parameters (Bartelmann et al. 1998; Dalal, Hennawi & Bode 2005; Meneghetti et al. 2005). Perhaps due to the limited sample of galaxy lens systems known, the interest in statistical lensing on galaxy scales has so far concentrated mainly on predicting the number of strongly imaged systems using simple lens models. However, the number of known lens systems is increasing steadily and in the
near future many large surveys are likely to detect a significantly increased number of strong lens systems. Such a large number of lens systems could potentially constrain masses of galaxies and provide important constraints on the mass evolution of galaxies between $z \sim 0.1$ and $\sim 1$. Thus, statistical lensing is potentially a very important tool for studies of galaxies.

Detailed statistical lens studies have so far been hampered by the lack of a clearly defined lens sample. Most known lens systems have been discovered serendipitously, and comprehensive surveys like the Cosmic Lens All-Sky Survey (CLASS; Browne & Myers 2005) are rare. Simple estimates of the lensing probability suggest the fraction of sources at $z \sim 1$ being lensed into multiple images is between $10^{-4}$ and $10^{-2}$ (Helbig et al. 1999). Two large surveys, both with similar redshift limits, $z \sim 0.2$, have been carried out to date, the Sloan Digital Sky Survey (SDSS; Adelman-McCarthy et al. 2006) and the Two-degree Field Survey (2dF; Colless et al. 2001). The largest current survey, the SDSS, contains about 500,000 foreground galaxies and about 100 million background galaxies at higher redshifts, and should thus contain at least several thousand lensed galaxies and hundreds of lensed quasars.

In this paper, we use semi-analytic models of galaxy formation (Kauffmann, White & Guiderdoni 1993; Somerville & Primack 1999; Kauffmann et al. 2004; Bower et al. 2006; Croton et al. 2006) to predict the statistical properties of a large sample of strong lenses that would be detected by a large survey similar to SDSS or the 2dF. Comparing the properties of lens galaxies with the properties of the galaxy population as a whole allows us to investigate various biases in the lensing population. These biases are important in order to interpret any results from actual strong lensing observations in a statistical way. For example, if strong lensing is to be used to infer the mass function of early-type galaxies, it is important to understand how the distribution of masses in a lens sample is related to the distribution of masses for all early-type galaxies.

In this paper, we only consider the case of point sources that are lensed by low-redshift galaxies with $z_{\text{lens}} < 0.2$. Even though we use the 2dF as a template survey for this work, our results are equally valid for the SDSS. In particular, we predict the expected lensing properties of galaxies in these surveys. This is done by calculating the lensing properties of a mock survey catalogue obtained from the semi-analytic models assuming a composite mass model consisting of dark matter (DM) halo, bulge and disc for lens galaxies. All the necessary parameters describing the mass profile of the lenses are obtained from the semi-analytic model, with the exception of the bulge size which we assign from the observed size–luminosity relation from Bernardi et al. (2003a) and the ellipticity of the bulge component. Our mass model is relatively complex, consisting of three components, and from the perspective of our understanding of the structure of galaxies more realistic than the more commonly used singular isothermal profile. However, our model has some shortcomings that need to be kept in mind throughout; we do not take into account the shape of the DM profile and we neglect that baryons will lead to a steepening of the inner profile through adiabatic contraction (Gnedin et al. 2004). Also, we consider here only the lensing statistics of isolated lenses. This does not mean that we do not include lensing by haloes with masses on the group and cluster scale (with the exception of Section 2.2). Our calculation only neglects the increase of the lensing cross-section for satellite galaxies due to their group/cluster environment. In general, the lensing potential of a satellite galaxy is modified by external shear and an increase in the projected mass, which affects the lensing geometries and image separations, respectively (Möller et al. 2002; Oguri, Keeton & Dalg 2005). Due to the low redshift considered in this paper, we expect the effect of environment on the results to be small – we discuss this in more detail in Section 5.

We will address the lensing cross-sections of groups and clusters including the satellite population in a subsequent paper.

A crucial element in our paper is the use of semi-analytic models to determine the parameters for the mass profiles used. To make comprehensive predictions of statistical lensing properties, observational constraints are not sufficient and a model of galaxy evolution has to be used for three reasons. First, observations do not accurately predict the DM properties of galaxies, which are crucial. Second, observations do not yet give accurate full distribution functions of several galaxy properties, like bulge-to-disc mass ratios (but see Allen et al. 2006). Third, to link the DM properties with those of the baryonic component requires a self-consistent description on a galaxy-by-galaxy basis, which cannot be obtained observationally.

The lensing cross-section, image geometries and magnifications are calculated for each galaxy individually using numerical routines. From these, we then create a sample of lens systems, which we then analyse further.

We describe the use of semi-analytic methods to create the lens mass models and outline the method to calculate lensing probabilities in detail in Section 2. In Section 3, we show the predicted statistical lensing properties of galaxies in low-redshift surveys and the dependence on galaxy properties. In Section 4, we apply specific selection criteria and predict the properties of a radio-selected sample of lenses with $z < 0.2$. Our results are presented in Section 5 and we discuss their implications there, including the predictions for the lensing rate. In Section 6, we summarize our key results. Throughout this paper, we use a standard $\Lambda$ cold dark matter (CDM) cosmology with $\Omega_{\Lambda} = 0.75$, $\Omega_{m} = 0.25$ and $H_0 = 73$ km s$^{-1}$ Mpc$^{-1}$.

2 THE MOCK LENS CATALOGUE

We use the Millennium Run numerical simulation (Springel et al. 2005) together with semi-analytic modelling of galaxy formation to predict the properties of the dark and luminous matter in galaxies and thereby derive the expected lensing statistics in large surveys at $z \lesssim 0.2$. In the procedure used for semi-analytic modelling, a catalogue of halo and substructure is generated at closely spaced redshift intervals, and for each individual $z = 0$ DM halo a corresponding merger tree is identified. The prescription described in Croton et al. (2006) is used to follow stellar masses, luminosities and morphologies of galaxies that assemble in these DM haloes. From these, we create a mock low-redshift galaxy catalogue that is similar to the 2dF North field by applying the same selection cuts in magnitude and redshift. We describe the creation of this mock 2dF galaxy catalogue below. The parent DM haloes are taken from the Millennium Run. Selection of simulated haloes is done by defining a backward light cone from $z = 0$. Even though we use the creation of a backward light cone, we note that for most of the work presented here, the creation of a full light cone is not strictly necessary, as we treat haloes as isolated in our lensing analysis. In this treatment, we are ignoring the immediate environments of haloes while calculating lensing cross-sections and image geometries. It is well documented that the presence of mass concentrations in the vicinity of lenses impacts the image separation distribution (Möller & Blain 2001; Oguri et al. 2005) and we address this interplay in future work.

2.1 Making mock observations

The Millennium Run is a very large DM only simulation which follows the hierarchical growth of DM structures from redshift $z = 127$
to the present. The simulation assumes the concordance ΛCDM cosmology and follows the trajectories of $2160^3 \approx 1.0078 \times 10^{10}$ particles of mass $8.6 \times 10^7 h^{-1} M_\odot$ in a periodic box 500 Mpc on a side. This mass resolution is sufficient to resolve the haloes hosting galaxies as faint as $0.1 L_*$ with at least ~100 particles. A full description of the simulation is given by Springel et al. (2005).

Data from the simulation were stored at 63 epochs spaced approximately logarithmically in time at early times and approximately linearly in time at late times (with $\Delta t \sim 300$ Myr). Post-processing software identified all resolved dark haloes and their subhaloes in each of these outputs and then linked them together between neighbouring outputs to construct a detailed formation tree for every object present at the final time. Galaxy formation modelling is then carried out in post-processing on this stored data structure. One starts at the earliest snapshot a halo has been identified and works ones way up the merger tree of DM haloes, applying at each time-step the various prescriptions that govern the physics of the baryons in these haloes such as gas cooling and star formation.

Our semi-analytic model is that of Croton et al. (2006) as updated by De Lucia et al. (2006) and made public on the Millennium Simulation data download site (see Lemson & Virgo Consortium 2006). These models include the physical processes and modelling techniques originally introduced by White & Frenk (1991), Kauffmann et al. (1993), Kauffmann & Haehnelt (2000), Springel et al. (2001) and De Lucia, Kauffmann & White (2004), principally gas cooling, star formation, chemical and hydrodynamical feedback from supernovae, stellar population synthesis modelling of photometric evolution and growth of supermassive black holes by accretion and merging. They also include a treatment (based on that of Kravtsov, Gnedin & Klypin 2004) of the suppression of infall on to dwarf galaxies as a consequence of reionization heating. More importantly, they include an entirely new treatment of ‘radio mode’ feedback from galaxies at the centres of groups and clusters containing a static hot gas atmosphere. The equations specifying the various aspects of the model and the specific parameter choices made are listed in Croton et al. (2006) and De Lucia et al. (2006).

The only change made here is in the dust model as described in Kitzbichler & White (2007).

We construct mock observations of our artificial Universe in the Millennium simulation by positioning a virtual observer in the simulation at zero redshift and finding those galaxies which lie on the observer’s backward light cone. The main limitation in producing a mock observation of a simulation is the finite box size, which in this case is 500 $h^{-1}$ Mpc on a side. The periodic nature of the simulation allows us to fill the space with any number of boxes we require but we still have to deal with periodic replications that would appear if we were to look through the simulation volume along one of its preferred axes. We can avoid this kaleidoscopic effect through slanting the observed cone by a certain angle. After having determined the survey geometry, observer position and line-of-sight, we fill the four-dimensional Euclidean space–time with a grid of simulation boxes. In the three spatial coordinates, we make use of the periodicity of the simulation whereas the time coordinate is given by the 64 snapshot times corresponding to their respective output redshifts. In practice, only those cells in the space–time grid are populated with galaxies which actually intersect the backward light cone in the observed field-of-view. After coarsely filling the volume around the observed light cone with simulation boxes in this way, one can simply ‘chisel off’ the protruding volume. Additionally, one has to account for a smooth evolution between the discrete simulation output snapshots by interpolating redshifts and apparent magnitudes in different filters. The latter also have to be calculated at their accurate observer frame redshift, giving rise to the exact inverse ‘k-correction’ in a natural way, such that the expected colour evolution with redshift is predicted correctly which can also be applied to calculate meaningful photometric redshifts.

We note that by using comoving coordinates and assuming a flat Universe, we have the luxury of cutting the light cone from our simulated volume simply like we would in Euclidean geometry. In general, this is a non-trivial endeavour that requires taking into account the curvature of the Universe as well as its expansion with time.

### 2.2 Lensing probabilities for a singular isothermal sphere model

Before calculating the lensing cross-section for realistic mass models of lensing galaxies, we first discuss the singular isothermal sphere (SIS) model that has in the past most commonly been used for statistical lensing calculations (e.g. Turner et al. 1984; Fukugita et al. 1992). Despite its simplicity, the SIS model has been surprisingly successful in describing the overall statistical properties of galaxy scale lenses. As shown by Koopmans & Treu (2002, 2003), the stellar component of lens galaxies has a density profile that is much steeper than the DM at small radii so that the total mass profile (including DM) follows an isothermal profile on scales probed by galaxy strong lensing more closely, thereby explaining the modelling success of the SIS. For most groups and cluster lenses, however, the SIS model is an inadequate description of the total mass distribution. On cluster scales, the baryonic component becomes less important and the lensing profile is more accurately modelled using a Navarro, Frenk & White profile (NFW; Navarro, Frenk & White 1997) well beyond the arc radius (Kneib et al. 2003). For lower redshift lenses, the Einstein radius is small compared to the galaxy size and the relative DM mass content within the Einstein radius is smaller. Thus, for all galaxies (excluding the most massive groups and clusters), the baryonic component of the mass profile dominates the lensing cross-section at low redshifts.

The lensing cross-sections of a SIS model for elliptical galaxies, excluding the DM contribution in groups and clusters, can be calculated directly from the observed Faber–Jackson relation. The lensing cross-section of a SIS is given by

$$\tau_{\text{SIS}} = 64\pi \left( \frac{\sigma}{c} \right)^4 \left( \frac{D_{\text{LS}}}{D_{\text{DS}}} \right)^2,$$

the total lensing probability can be calculated using the luminosity function $N(L)$ of early-type galaxies:

$$\tau \propto \int_0^\infty \sigma(L)^4 N(L) dL,$$

where

$$\sigma(L) = \sigma_* \left( \frac{L}{L_*} \right)^\beta$$

is the Faber–Jackson relation. In this section, using the simple SIS model and the above relation we can take all quantities that are needed to calculate the lensing cross-sections from observation directly. The luminosity function of early-type galaxies has been measured by Madgwick et al. (2002) for the 2dF and has the form

$$\tau = \int_{z=0}^{z_{\text{max}}} \tau_z \left( \frac{D_{\text{LS}}}{D_{\text{DS}}} \right)^2 \phi_z \left( \frac{\sigma}{\sigma_*} \right)^4 \left( \frac{L}{L_*} \right)^\alpha \exp \left( - \frac{L}{L_*} \right) \frac{dV_{\text{obs}}}{dz} dz,$$

where $\alpha$ is the Faber–Jackson relation.
The lensing cross-section for SIS lenses with sources at \(z = 2\) for different slopes of the luminosity function and power-law indices relating luminosity and velocity dispersion. The solid line shows the results for \(\alpha = -1.21\) and \(\beta = 0.25\), corresponding to the 2dF results. The dashed, red line is for \(\alpha = -2\) and \(\beta = 0.25\) and the dotted green and dot-dashed blue lines are for \(\alpha = -1.21\) and \(\beta = 0.31\) and 0.5 respectively. The functional form of the luminosity function is given in equation (3).

2.3 Towards realistic lens models: mass models of semi-analytic galaxies including DM and baryons

More sophisticated models than the SIS have been used by, for example, Oguri et al. (2005) who predicted the lensing statistics in the SDSS using a spherical Hernquist model for the baryons and a NFW DM mass profile. Spherical models do predict the overall lensing cross-section with adequate accuracy (Fukugita et al. 1992). They cannot, however, predict any detailed properties, like the average magnification and the statistics of image geometries. For instance, the number of four-image systems predicted by spherical lens models is zero. We use the ray-tracing code cLENS (Möller & Blain 1998)\(^1\) to calculate the lensing properties of multicomponent lens systems efficiently. In order to calculate expected lensing properties of galaxies in large surveys, we need to relate the observed properties, like luminosity and size to a mass model. Since lensing is sensitive to the total mass, we need to include the contributions both of the baryonic and of the DM. Our galaxy model therefore consists of a baryonic component, modelled as a bulge of mass \(M_{\text{b}}\) plus a disc of mass \(M_{\text{d}}\) that reside in a DM halo of virial mass \(M_{\text{vir}}\). We obtain all parameters for our mass model, with the sole exception of the bulge size and ellipticity, from the semi-analytic models.

In reality, many lens galaxies are expected to reside in dense environments, in groups or clusters (Kundic et al. 1997; Möller et al. 2002; Keeton & Zabludoff 2004; Oguri et al. 2005). However, the lens galaxies we consider here are all at relatively low redshifts, where the effect of environment is expected to be less strong. At redshifts \(z \lesssim 0.2\), the expected Einstein radius is small compared with the distance to nearest neighbours (this is true even for lens

\[ \tau = \int_{L=0}^{\infty} \frac{\tau(L) dL}{L} \]

\[ \tau = (\sigma_v / c)^2 \times 1.055 \times 10^{-9} \text{arcsec}^2 \]

where \(\sigma_v = 3.85 \times 10^{-3} \text{Mpc}^{-1}\), \(\alpha = -0.54\), \(L_\odot = 1.67 \times 10^{10} \text{L}_\odot\) and \(\tau_\odot\) is the characteristic lensing cross-section of an \(L_\odot\) galaxy for equal distances \(D_{\text{LS}}\) and \(D_{\text{OS}}\). The comoving volume of a shell of thickness \(dz\) is given by \(dV_c/dz \times dz\). This luminosity function is shown in Fig. 1 together with the luminosity function for the complete 2dF, including late-type galaxies, as well as the luminosity functions for the SDSS (Blanton et al. 2001) and the luminosity function of the mock light cone used below.

We show the \(\tau - L\) relation for various choices of \(\alpha\) and \(\beta\) in Fig. 2. For a slope of the Faber–Jackson relation of \(\beta < 0.3\), the lensing cross-section \(\tau(L)\) is a monotonically decreasing function with \(L\). In the case of a SIS lens, the largest contribution to the total lensing cross-section is expected from the galaxy population around \(L_\odot\). Using the Faber–Jackson relation determined from early-type galaxies in the SDSS (Bernardi et al. 2003b), with \(\beta = 1/4\), \(\sigma_v = \sigma(L_\odot) = 166.44 \text{km s}^{-1}\) and \(\tau_\odot = (\sigma_v / c)^2 \times 1.055 \times 10^{-9} \text{arcsec}^2\), we obtain a total lensing cross-section of

\[ \tau_{\text{tot}} = \int_{L=0}^{\infty} \tau(L) dL = 1.35 \times 10^{-5} \]

for sources at \(z_{\text{sources}} = 2\). This is our first and simplest estimate of the expected lensing rate in a survey of similar depth as the 2dF. Note that this is the cross-section for lensing by galaxies within the redshift limit of the survey (\(z < z_{\text{max}} = 0.2\)) only. In this calculation, we have neglected the contribution of late-type galaxies, since the lensing cross-section is a strong function with galaxy mass and late-type galaxies in general have a smaller mass than ellipticals. This is the reason why most statistical lensing calculations only include elliptical galaxies (we include late-type galaxies in our more comprehensive analysis, below). In addition, the calculation above does not correctly include the contribution from groups and cluster which have an extended DM halo. Also all galaxies modelled in the same fashion, as a SIS with a velocity dispersion given by the Faber–Jackson relation. Since the bright, central galaxies in groups and clusters do not necessarily fall on the same relation as other galaxies in those structures or in the field, this is another oversimplification. In addition, as pointed out by Mitchell et al. (2005), neglect of the scatter in the Faber–Jackson relation leads to a biased (under-)estimate of the lensing cross-section. This is remedied using the semi-analytic approach below. At this juncture, we also neglected magnification bias and the source luminosity and redshift distributions.
The concentration parameter $c$ is crucial for understanding the lensing process in strongly lensing by NFW haloes reveals that the lensing cross-section depends crucially on the concentration parameter. For a closer investigation of the strong lensing by NFW haloes, we calculate the concentration parameter explicitly for each halo. This procedure ensures that the distribution of concentration parameters automatically has the correct scatter. In this work, we use the equation (Navarro et al. 1997)

$$V_{\text{max}}^2 = \frac{0.216 c}{\bar{A}_c}$$

where

$$\bar{A}_c = \ln (1 + c) - \frac{c}{1 + c}.$$  

For each halo in the simulation, the parameters $V_{\text{max}}$ and $V_{\text{vir}}$ are known from the N-body simulation and we use equations (8) and (9) to calculate the concentration parameter for each halo. This procedure ensures that the distribution of concentration parameters automatically has the correct scatter.

### 2.3.2 Modelling the baryonic components

The baryonic mass is distributed into a bulge and a disc component. The relation between the mass of a DM halo and the luminosity of the baryonic component can be studied using semi-analytic models. We assign morphologies to the galaxies in a similar way that is described in Croton et al. (2006). For each galaxy, we calculate the bulge-to-total light ratio $(B/T)$ by taking

$$L_{B}/10^{10} L_{\odot} = \frac{0.3}{0.57 + (M_{\text{bulge}}/10^{11} M_{\odot})^{2.5}}$$

derived by Vale & Ostriker (2004) as the solid line, including the scatter (dotted, blue line). Note that there are galaxies in the catalogue that consistently fall above and to the left of the analytic relation (given in equation 10) for high virial masses, which correspond to central galaxies in the more massive haloes of clusters or groups.

### 2.3.3 Morphologies

We assign morphologies to the galaxies in a similar way that is described in Croton et al. (2006). For each galaxy, we calculate $B/T$ by taking

$$B/T > 0.4 as early-type galaxies, and the rest is classified as late-type. These definitions, we have 76.2 per cent of all galaxies being late-type. This fraction agrees with the observed value for $B/T > 0.4$ systems (de Jong 1996; Allen et al. 2006).

### 2.3.4 The Bulge component

The size and velocity dispersion of the bulge component are not determined in the semi-analytic modelling. However, the distribution of bulge sizes is known observationally (de Jong et al. 2004). There are no models that relate the bulge size to properties of the DM halo. The fundamental plane relation suggests that DM haloes and the bulge properties are related: virialized haloes with constant mass-to-light ratio (M/L) will naturally lie on the fundamental plane.
but so far no theoretical model has been developed that naturally explains both the existence and the slope of the fundamental plane. In the absence of such a model, we rely on the observed relations between luminosities and sizes of elliptical galaxies and apply these to the population of bulges as a whole, irrespective of the morphological type of the galaxy. In the SDSS, Bernardi et al. (2003b) find a correlation between half-light radius, $R_b$ and galaxy luminosity of the form

$$\log R_b - 0.52 = \frac{1}{\alpha} \log \left( \frac{L}{L_*} \right),$$  \hspace{1cm} (11)

where $\alpha = 1.5 \pm 0.06$ and $L_* = 1.58 \times 10^{10} L_\odot$. Here, we converted the $L*$ value in $r$ and $g$ to the corresponding value in the $B_{\gamma}$ band using $B_{\gamma} = g + 0.0638(g - r) + 0.1624$. Bernardi et al. (2003b) also find the Faber–Jackson relation for the velocity dispersion

$$\log \sigma - 2.197 = \beta \log \frac{L}{L_*},$$  \hspace{1cm} (12)

where $\beta = 0.25 \pm 0.06$. Since the semi-analytic model fixes the total bulge luminosity and DM properties, we do not use the latter relation. Instead, we only use the $R_b-L$ relation (equation 11) to determine the bulge size and then solve the Jeans equation to obtain the line-of-sight velocity dispersion $\sigma$, taking the M/L directly from the semi-analytic model (Croton et al. 2006).

We show the Faber–Jackson relation and the histograms of the distributions of the velocity dispersion and bulge size in Figs 4 and 5 for the mock catalogue. These plots show that, using only the observed $R_b-L$ relation as input, we recover the observed Faber–Jackson relation and the fundamental plane. Of particular importance is the fact that we recover the observed velocity dispersion distribution measured for early-type galaxies by the SDSS. We conclude therefore that our mock galaxies are realistic.

These structural parameters are then used to model the bulge component in the lens as a de Vaucouleurs profile (de Vaucouleurs 1948),

$$\Sigma_b = \Sigma_{0,b} \times \exp \left(-\frac{r}{r_\text{e,b}}\right)^{1/4},$$  \hspace{1cm} (13)

where $r_\text{e,b} = 3681 \times R_b \times 0.551$ and $\Sigma_{0,b} = \Gamma L/8\pi \times 5040 \sqrt{1 - \epsilon^2}$, measured in kpc and $M_\odot \text{kpc}^{-2}$, respectively, and where $\Gamma$ is the M/L of bulges in the $B_{\gamma}$ band. This is obtained by taking the ratio of the stellar mass and the luminosity in the $B_{\gamma}$ band.

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Figure 4. The Faber–Jackson relation: the number of galaxies in the mock catalogue as a function of $L$ and $\sigma$ is shown as colour scale, with the blue line showing the best-fitted observed relation $\sigma = \sigma_b (L/L_\star)^{\alpha}$. The inset shows the distribution of velocity dispersions, $\log \sigma$, in our mock 2dF catalogue. The smooth line in the inset shows the observed velocity dispersion distribution in the SDSS found by Bernardi et al. (2003c).

2.3.5 Disc component

In the semi-analytic models of galaxy formation, every galaxy, irrespective of morphology, also contains a disc component. The infall and merger history of haloes determine the disc properties: total disc mass, $M_d = \Gamma L_d$ and disc size, $R_d$. The mass distribution of discs is taken to be well fitted by an exponential

$$\Sigma_d = \Sigma_0 \exp \left(-\frac{r}{R_d}\right),$$  \hspace{1cm} (14)

where $\Sigma_0 = M_d/\pi R_d^2$ is the central surface mass density of the disc.

We plot the distributions of the structural disc parameters for the mock catalogue in Fig. 6. For each galaxy that has a $B/T$ ratio of less than 0.4 and is hence classified as late-type galaxies, we also calculate the rotational velocity at 10 disc scalelengths. Plotting this as a function of galaxy luminosity gives the Tully–Fisher relation, which is shown in Fig. 7. The blue, dashed line in Fig. 7 shows the Tully–Fisher relation observed by Tully & Pierce (2000) and Pierce & Tully (1992). There is a very good match between observed and predicted relation for $M_B \gtrsim -21$, corresponding to $L \lesssim 2.15 \times 10^{10} L_\odot$. At higher luminosities, there is an upturn in the predicted relation. Given the small observed number of objects with measured rotational velocities at these luminosities, this is consistent with observational data at the present time.

2.4 Mass properties of the composite model

The above approach assigns a specific mass distribution to each galaxy in the catalogue, consisting of a spherical halo, a bulge and
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Figure 6. The distribution of disc sizes in the mock 2dF catalogue. The histograms show the distribution of $\log R_d$ in the mock, the thick, red is for all galaxies and thin, black is for spiral galaxies only. Note that both histograms have been normalized independently.

Figure 7. The distribution of number of galaxies as a function of the rotational velocity at 10 disc scalelengths, $R_d$, and disc luminosity in the mock catalogue. The dashed line shows the observed Tully–Fisher relation, $M_B = -4.125 - 7.27 \times \log V_{\max}$ (Tully & Fisher 1977; Pierce & Tully 1992; Tully & Pierce 2000).

2.4.1 Stellar mass: light ratios

Stellar mass:light ratios in early-type galaxies have been measured by, for example, Fukazawa et al. (2006) and Napolitano et al. (2005) as well as Kauffmann et al. (2003) and Padmanabhan et al. (2004). All these works are in rough agreement with each other and find an overall stellar mass:light ratio of $\Gamma = (M/L)_{\text{star}} \sim 3-10$. The stellar mass:light ratio for late-type galaxies has been modelled, for example, by Bell et al. (2003) who find that they vary between $\Gamma \sim 0.5$ and $\sim 6$ in the $B$ band, and correlate with properties like gas fraction or $B - R$ colour. We show the stellar mass: light ratio of the galaxies in our mock sample in Fig. 8. The early-type galaxies have $M_{stellar}/L$ of about 4–8 for the majority of all galaxies. For the late-type galaxies, the ratio is lower and values of $\Gamma \sim 3$ are more typical.

2.4.2 Mass profiles of ellipticals

Observations of X-ray emission in elliptical galaxies (Fukazawa et al. 2006) and measurements of the velocity of planetary nebulae in the outskirts of elliptical galaxies (Napolitano et al. 2005) have allowed constraints on the mass profiles of ellipticals to be made. Since these works select out a relatively ill-defined sample of galaxies, it is not immediately possible to compare these with the lens sample in our mock light cone directly. For the Fukazawa et al. (2006) sample, this is easier since these are all X-ray emitting and hence relatively massive early-type galaxies. In Fig. 9, we plot a subsample of elliptical galaxies with masses $M_{vir} > 10^{14} M_\odot$ over the mass profiles from the Fukazawa et al. (2006) sample (fig. 4, left-hand panel in their paper). The mass profiles of our massive ellipticals agree very well with the observed sample. For lower mass ellipticals, it is difficult to compare our mass profiles with observations, since low-mass ellipticals do not generally have X-ray emissions that are observable with currently available instruments. Other traces, like planetary nebulae, also exist mainly for higher mass ellipticals and are more difficult to interpret than mass

Figure 8. The mass: light ratios, $\Gamma$, of the mock light cone. The solid black line shows the distribution of $\Gamma$ for the complete sample. The dashed, red line is for the early-types, and the dotted, blue line for the late-types.

Figure 9. The mass profile of massive ellipticals. The colour scale shows the density of mass profiles for a subsample of elliptical galaxies with $M_{vir} > 10^{14} M_\odot$ from the mock light cone. Overplotted in black lines are the results from an X-ray selected sample of early-type galaxies by Fukazawa et al. (2006).
profile from X-ray observations as the sample of ellipticals studied in this way is much smaller and asymmetries in the potential need to be taken into account properly to constrain the mass profile (Hayashi & Navarro 2006).

### 3.1 Calculation of lensing properties

The galaxy population in the 2DF not interested in constraining cosmological parameters with lensing population depends only on the geometric parameters, i.e. the redshifts of the source, lens and the cosmological parameters. We are not interested in constraining cosmological parameters with lensing in this work and we fix the underlying cosmology as described in Section 1.

### 2.5 Lensing properties of the composite model

To illustrate the general lensing properties of the composite mass model used, we show the image and source magnification maps of the DM+bulge+disc models in Fig. 11 for a spiral galaxy with \( L = 10^{11} L_\odot \). Note the strong asymmetry introduced by the disc component. This is expected since for inclination angles of \( \theta > 65^\circ \), the projected surface mass density along the major axis is strongly increased for thin discs (Möller & Blain 1998).

The magnification curves in the source and image planes are shown in Fig. 12. The NFW DM profile on its own does not have a significant lensing cross-section in this example, as the concentration parameter \( c \) of the DM halo is too low. It is interesting to note, however, that despite this the magnification cross-sections for the composite model are influenced significantly by the DM component. This can be seen by comparing the lensing cross-sections as a function of magnification shown as solid (DM) and dashed (no DM) lines in Fig. 12. The cross-section for the case with no DM is a factor of \( \sim 2 \) below that with DM, for almost all magnifications above about 3. Therefore, even in cases when the DM halo is not concentrated enough to produce lensing, it still affects the total lensing cross-section significantly.

### 2.4.3 Rotation curves of late-types

The rotation curves of late-type galaxies have been measured routinely for many years, and these measurements provided the initial evidence for the existence of DM (Begeman 1989). For massive spirals, rotation curves are in general flat, with only a slight decline towards the outer radii. The mass model presented in this paper is used below to predict the lensing cross-section of late-type galaxies and investigate the statistical effect of the presence of a disc on the lensing cross-section. It is therefore important to verify that our mass model reproduces the observed flatness of the rotation curve for spiral galaxies. To calculate the rotation curve for a late-type galaxy in our catalogue, we solve the Jeans equation at a number of radii, assuming spherical symmetry.

We show the rotation curves for the late-type galaxies in our sample in Fig. 10. The figure demonstrates that the rotation curves are very flat, even out to many disc scalelengths. In addition, this plot also demonstrates that associating the quantity of \( V_{\text{max}} \) of the DM with the rotational velocity of discs in the outer regions provides a reasonable approximation.

### 3 THE EXPECTED LENS PROPERTIES OF THE GALAXY POPULATION IN THE 2DF

#### 3.2 Total lensing cross-sections and image separation distributions

Two statistics have been used in the past to constrain cosmological parameters (Kochanek 1993; Chae 2003) and lens galaxy population properties (Kochanek 1996; Chae 2005): total lensing cross-sections and image separation distributions. The total lensing cross-section we obtain from our mock catalogue is \( 96216.3 \) arcsec\(^2\) in a total survey area of \( 651 \) deg\(^2\), with 129 625 galaxies below redshift of 0.2 constituting our lens galaxy population. This gives a total probability of a source being lensed as \( 3.2 \times 10^{-5} \), and a lensing cross-section...
Lensing statistics in surveys

Figure 11. The image (left-hand side) and source (right-hand side) magnification maps for DM+bulge+disc model of a luminous $L = 1.8 \times 10^{11} \, L_\odot$ galaxy. The colour scale shows the magnification of a point source as a function of image and source position, respectively. The source plane is at $z_s = 1$ and the image plane is at $z = 0.15$. The lens has a bulge-to-total mass ratio $B/T$ of 0.2, a bulge ellipticity of $e = 0.4$ and a disc inclination angle of $\theta = 70^\circ$. The dark halo mass is $M_{\text{vir}} = 1.3 \times 10^{13} \, M_\odot$ inside $R_{\text{vir}} = 455 \, \text{kpc}$, with a concentration parameter of $c = 9$.

Figure 12. The lensing cross-section as a function of magnification $\mu$ for DM, DM+bulge and DM+bulge+disc models of a typical $L = 10^{11} \, L_\odot$ galaxy in the mock catalogue. Cumulative histograms of the cross-sectional area for lensing of point sources by magnifications of more than $\mu_0$, $\tau(\mu > \mu_0)$ are shown. The solid, red line is for the pure DM model. The solid, black line shows the result of adding a bulge, and the solid blue line is for the complete DM+bulge+disc model. The dashed black and blue lines show the magnification cross-sections for model of bulge and bulge+disc without DM, respectively. Model parameters are as for the previous figure.

Figure 13. The lensing cross-section as a function of luminosity and mass. The colour scale shows the distribution of all galaxies as a function of $L$ and $M_{\text{vir}}$. The blue contour shows the lensing cross-section distribution for the complete DM+bulge+disc models. The thick, black contour shows the cross-section for DM+bulge models and the thin black contours are for DM only models. The contour levels are 100, 300, 700 and 1000 arcsec$^2$ for the first two cases and 0.1, 1 and 10 arcsec$^2$ in the latter case.

for four-image systems being $1.78 \times 10^{-8}$. This estimate neglects magnification bias and we show in Section 4 that magnification bias increases the four-image lensing probability dramatically.

We plot the lensing cross-section for our mock catalogue as a function of lens galaxy luminosity and halo virial mass in Fig. 13. The contours show the lensing cross-section for background sources at $z_{\text{source}} = 1$ as a function of mass, $M_{\text{vir}}$ and luminosity $L_{B_j}$. Comparison with the underlying distribution for all galaxies in the sample shows that the contribution to strong lensing cross-section comes preferentially from massive, luminous galaxies. The shift in the luminosity from a peak luminosity of $L_{\text{peak}} \approx 10^{10} \, L_\odot$ for the total distribution to a peak of $L_{\text{peak}} = 2 \times 10^{10} \, L_\odot$ for the lensing sample is clearly strong. An additional shift to higher virial masses by a factor of $\approx 2$ is also evident. The baryonic component dominates the lensing cross-section of galaxies, but, for a fixed luminosity, the contribution of high-mass objects to the lensing cross-section is larger. The DM component therefore also plays a key role. In Fig. 14, we show the expected distribution of image separations. Here, we clearly see the relative effects of the various lens components. For small image separations (corresponding to low masses), the cross-section is completely dominated by the baryonic component. Large image separation systems, mainly corresponding to groups and clusters, have lensing cross-sections that depend much more on the DM content.

It is clear from these plots that the baryonic component is absolutely crucial in determining statistical lensing properties on galaxy scales. The importance of the baryonic contribution to the lensing cross-sections of DM haloes is clearly seen: it increases by a factor of 10–1000 for systems with image separations $\lesssim 0.1$ and $\sim 1$ arcsec. The increase is smaller for systems with large image separations. These systems correspond to lensing by massive central cluster and group galaxies. As the Einstein radius increases for more
massive systems, the fraction of DM within the Einstein radius also increases due to the shallower slope of the DM component. Despite the importance of the baryonic component, the lensing cross-section at a fixed $L$ does depend on the DM halo mass, $M_{\text{DM}}$, especially for highly luminous galaxies (cf. Fig. 12).

### 3.3 Image geometries: two- and four-image systems

Previous work by, for example, Möller & Blain (1998) and Blain (1998) has shown that baryonic components can significantly change the total lensing cross-section. The disc component plays a major role in this: inclined discs can increase the lensing cross-section significantly. However, as suggested by Möller, Hewett & Blain (2003), this effect is important only for late-type systems, whereas early-type galaxies have too small a disc component to change the lensing cross-section significantly. The ratio of four- to two-image systems, however, may be affected strongly, as even relatively low mass discs can increase the asymmetry of the lensing potential notably. We show the ratio of four- to two-image systems, $N_4/N_2$, in the catalogue for the DM + bulge and the DM + bulge+disc models in Fig. 15. For systems with $10^4 L_\odot \lesssim L \lesssim 10^7 L_\odot$, the fraction of late-type galaxies is high enough to increase the cross-section for four-image systems by more than a factor of $\sim 10$. There is a clear downward trend in the 4:2 image ratio even for models without discs, even though the ellipticity of the bulge is uncorrelated with any other galaxy properties. The reason for this decrease is that, for a given ellipticity $e > 0$, smaller galaxies have a higher 4:2 image ratio; the cross-section for two images grows faster than that for four images with increasing galaxy mass. There is another noteworthy point to make about Fig. 15. Discs increase the 4:2 image ratio even for large image splittings of $\sim 2$–10 arcsec, but these systems are not late-type galaxies. In other words, the presence of discs can increase the cross-section for the formation of four images even for early-type galaxies that have $B/T > 0.4$ but still contain nearly edge-on discs with a mass that is significant enough to increase the ellipticity of the overall potential. Lenses with $B/T < 0.9$ are responsible even for large image separations and many of these would be classified as Sb galaxies. Again, the result presented in Fig. 15 neglects the effect of magnification bias. Magnification bias is expected to favour late-type galaxies as four-image lens systems given that the expected cross-section for magnification of $\mu \sim 10$–20 is a factor of up to $\sim 5$ larger for inclined spiral lenses (Möller & Blain 1998). However, as we show in Section 4, this is not a clear case: elliptical galaxies have a smaller cross-sectional area for magnifications in that range, but they generally have a larger cross-section for magnifications above 20. This implies that, for steep source luminosity functions, magnification bias in fact may favour the moderately elliptical gravitational potential of early-type galaxies.

Our model also does not take into account triaxiality in the DM haloes. Triaxiality in the DM component can affect the cross-section for four-image systems, leading to an increase in the ratio of 4:2 image systems (Oguri & Keeton 2004). Since we do not take this effect into account, the 4:2 image ratio we predict is likely to be an underestimate—especially for large separation lenses. However, due to the fact that the baryonic part dominates the lensing cross-section for the majority of lens systems we do not expect the total 4:2 image ratio to change drastically if the triaxiality of the DM haloes is taken into account. Also, the relative 4:2 image ratio of models with and without disc should be independent of the triaxiality of the DM halo—so the results shown in Fig. 15 do not depend greatly on the assumption of spherical DM haloes.

### 3.4 Selection biases

The results presented above do not include the effects of selection biases. All surveys are by construction limited in flux, resolution or some other observational criteria. These varied selections can strongly affect the inferred properties of lens samples. A flux-limited survey, for example, will be strongly affected by magnification bias for sources with a steep luminosity function. Since galaxies and radio sources follow quite steep luminosity functions, magnification bias is important for lensing in the radio as well as in the optical. One can estimate the effect of magnification bias for simple selection criteria. The cumulative probability distribution function for magnification above a value $A$ is generally $P(\mu > A) \propto A^{-\alpha}$, where

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2 Note that strong lensing produces an odd number of images, one of the images is always strongly de-magnified, therefore two-image systems have a total of three images with two magnified ones and one central de-magnified one and likewise a four-image system has in fact a total of five images.
4 A MOCK SAMPLE OF LENS SYSTEMS

We use the lensing code gLenses together with our mock galaxy catalogue to create a sample of lens systems, including observational selection effects. We include two key selection effects: a flux limit and a resolution limit. Concentrating on lensed radio sources, we create a sample that has selection criteria in concordance with the CLASS survey (Chae 2003):

(i) the total flux \( F \) of each system of images is above \( F_0 \),
(ii) the image separation is >0.03 arcsec and
(iii) for double image systems the magnification ratio of bright-to-faint image is >0.1.

We create the sample using rough estimates of the lensing cross-section for each galaxy assuming spherical symmetry by solving the lens equation for the approximate Einstein radius, \( \theta_{\text{Einstein}} \). We then populate the total estimated cross-section area, \( A_{\text{attempt}} \) with \( N_{\text{attempt}} \) sources. For each source, we pick a random flux from the distribution function of fluxes \( dN/dF \propto F^{-\alpha_s} \), where \( \alpha_s = -2.1 \) as found in CLASS (Browne & Myers 2005). We assign a source redshift from a redshift distribution that is a Gaussian with mean \( z_s = 1.27 \) and width 0.95 as found by Willott et al. (2001) in the 6CE and 7CRS radio samples. For each source, we find the positions of the images using an adaptive lensing technique similar to the one described in Wucknitz (2004). We then apply the selection criteria (i)–(iii) to the lens system. If the system satisfies all these criteria, it is included in the lens sample, otherwise another foreground galaxy and source pair are selected randomly until the desired number of lens systems is obtained. For a number of \( N_{\text{attempt}} \) ’trial’ sources, placed in an area \( A_{\text{attempt}} \ll A_{\text{survey}} \) where \( A_{\text{survey}} \) is the total area of the survey, there is a number of \( N_{\text{ lenses}} \) sources that are unlensed but above the flux limit for detection. The number of expected lenses in the survey is then given by \( N_{\text{lens}} = f_{\text{lens}} N_{\text{unlensed}} \times A_{\text{survey}}/A_{\text{attempt}} \), with \( f_{\text{lens}} \) being the lensing fraction.

4.1 Properties of the lens sample

We use the procedure described above to create a sample of \( N_{\text{lens}} = 500 \) lens systems from our mock galaxy catalogue. This number of lens systems was chosen to obtain a reasonable statistical sample while keeping the time needed for computations tractable. The ‘trial’ sources were placed in a total area of \( A_{\text{attempt}} = 0.022 \) deg\(^2\). Of these ‘trial’ sources, a number \( N_{\text{unlensed}} = 1860 \pm 7 \) were unlensed but above the detection flux limit \( F_0 \). The total number of sources in the total area of \( A_{\text{survey}} = 651 \) deg\(^2\) that would create 500 lenses\(^2\) is \( 4.2 \times 10^5 \), yielding a fraction of \( f_{\text{lens}} = 7.5 \times 10^{-5} \) of all ‘observed’ sources as lensed. Given that only one lens in CLASS has a lens redshift that

\[ \alpha_s = 2 \] for SIS lenses. Assuming that the probability that a source enters the survey is 1 for \( \mu \times F > F_0 \) and 0 otherwise, it follows that the fraction of sources of luminosity \( F \) that are observed at flux \( F \) is

\[ N(F) = \int_{F_0}^{\infty} dP/dA(A)N(F/A) dA \]

(cf. Maoz et al. 1993; King & Browne 1996). For a lens population with a more complicated lens model, an analytic calculation of the effect of magnification bias becomes infeasible. Including the selection effects due to finite resolution limits requires a numerical calculation – which we describe in the next section.

\( ^\text{3} \) The error bar was estimated by running a set of 10 trial runs, and noting the value of \( N_{\text{unlensed}} \) in each case.

\( ^\text{4} \) Here, we assume that no sources are lensed that lie outside the area \( A_{\text{attempt}} \).

Figure 16. The distribution of lens velocity dispersion in a mock lens sample selected in the radio. The black, hatched histogram shows the velocity dispersion distribution of all lens systems with an average velocity dispersion of \( \bar{\sigma} = 191 \pm 3 \) km s\(^{-1}\) with the majority of lens galaxies (~85 per cent) being ellipticals. The red, filled histogram shows the statistics for four-image lenses only. The solid, thin black line shows the distribution of velocity dispersions of all ellipticals in our mock 2dF catalogue.

4.1.1 Lens velocity dispersions and morphologies

We determine the average velocity dispersion of the 500 mock lens systems to be \( \bar{\sigma} = 191.3 \pm 3 \) km s\(^{-1}\). This value is consistent with but slightly higher (by about 15 km s\(^{-1}\)) than what is observed in the CLASS survey (Davis, Huterer & Krauss 2003).

The distribution of lens velocity dispersions for 2dF lenses expected in a radio-selected survey is shown in Fig. 16. The distribution for all systems (black) and for four-image systems only (red) is shown. We also show the location of lens galaxies in the sample in Fig. 5 in Section 2.3.4 (black dots). Even though the lenses lie on the fundamental plane, the complete sample of lens systems has a significantly higher velocity dispersion than the elliptical galaxy population as a whole. However, the velocity dispersion function for four-image lenses follows the distribution of the galaxy velocity dispersion for all galaxies much more closely. In the complete sample, a total fraction of about 85 ± 5 per cent of lenses are ellipticals with a bulge-to-total light ratio of \( B/T > 0.4 \). This fraction decreases to 65 ± 16 per cent for the four-image system subsample. The fact that these two fractions are similar suggests that magnification bias is stronger for early-type massive galaxies with a moderately asymmetric potential, rather than less massive late-type systems with strong ellipticity in the potential due to a massive disc.

4.1.2 Image separations and image geometries

Using the above selection criteria, we find that the average image separation is \( \Delta \theta_{\text{max}} = 2.7 \pm 0.2 \) arcsec. Taking into account the expected difference in redshift distributions of the lenses, \( \bar{z} \sim 0.1 \) and \( \Delta \theta_{\text{CLASS}} \sim 0.4 \), and noting that the image separations scale as \( \Delta \theta_{\text{max}} \propto D_{\text{LS}}/D_{\text{DS}} \) this compares well with the observed image separations in the CLASS sample \( \Delta \theta_{\text{CLASS}} = 1.13 \) arcsec.

The distribution of image separations produced by 2dF lenses expected in a radio-selected survey is shown in Fig. 17. We also show the image separation statistic for a subsample consisting of...
four-image systems. The average maximum image separation decreases to $\Delta \theta = 1.2 \pm 0.4$ arcsec when only four-image systems are considered. Our model for lenses in the 2dF predicts a fraction of $82 \pm 5$ per cent of double image systems, with a fraction of $18 \pm 5$ per cent being lens systems with four images. Depending on the particular choice of known lens systems to compare with, these numbers are consistent/inconsistent with observations. The sample used by Rusin & Tegmark (2001) contains seven quadruples and five doubles, whereas the sample used by Chae (2003) and Chae (2005) contains 10 lenses with four quadruple image systems. These samples differ in their selection criteria, and since the number of known lens systems is low it is difficult to draw any firm conclusions. From our analysis, it does appear that the number of observed quadrupole systems is slightly higher than what we predict, even if we compare with the sample of Chae (2005). Since our model includes ellipticities and disc components, it is difficult to explain any further asymmetry in the lensing potential needed to increase the fraction of quadrupoles with anything other than the presence of other mass concentrations in the vicinity of the primary lens. These environmental effects are discussed in more detail in Möller et al. (in preparation).

5 DISCUSSION AND CONCLUSIONS

Surveys, like the 2dF and the SDSS, are likely to contain a large number of gravitational lens systems. We predict the number of lenses and lensing statistics expected in such surveys. The number of observed lensing systems is a function of the statistical properties of the foreground galaxy population, the source population, selection biases and cosmological parameters. Taking the cosmological parameters as known, we calculate the expected lensing statistics for point sources at a fixed redshift using realistic multicomponent galaxy models. Our fiducial galaxy consists of a NFW halo, an elliptical bulge and an inclined disc. Detailed galaxy properties are predicted using semi-analytic modelling of galaxies in the Millennium Run, currently the largest available N-body simulation. Using ray-tracing techniques in combination with the galaxy model, we thus predict the statistical lensing properties of large surveys with realistic galaxy lens models.

In the first part of the paper, we calculate the lensing cross-section for a complete sample of $\sim 100,000$ galaxies at redshifts of $z_{\text{len}} < 0.2$ that fulfill the selection criteria of the 2dF. With background sources at a fixed redshift of $z_{\text{source}} = 1$, we demonstrate that the main contribution to the lensing cross-section comes from the baryonic component of luminous, early-type galaxies. It is important to note here that this does not mean that the lensing cross-section is insensitive to the DM component in these galaxies. In fact, the distribution of galaxy masses at a fixed luminosity, $n(M_{\text{vir}} | L)$, for a lens sample is different from that for the complete population of early-type galaxies. As shown in Fig. 13, the mass function of lenses peaks at a value that is a factor of about 2 higher than the overall galaxy sample. If magnification bias is not included, we find that, for systems with maximal image separations between 2 and 10 arcsec, the disc component of spirals increases the lensing cross-section by a factor of $\sim 5$, even for early-type systems. This shows that disc components are important contributions to statistical lensing calculations. However, when we do include magnification bias we found that this effect is weaker due to a trade-off between increase in cross-section for late-type galaxies on the one hand, and a much stronger magnification bias for four-image geometries in moderately elliptical massive galaxies on the other (cf. discussion in Sections 3.3 and 4.1.1).

In the second part, we make firm predictions with clear selection criteria we simulate a sample of 500 radio-selected lens systems that fulfill the CLASS selection criteria in terms of flux and magnification ratios. We find that, in surveys like the 2dF or SDSS, including magnification bias, about $7.5 \times 10^{-5}$ of all background radio sources are lensed by galaxies within the survey. This number is fairly independent of the inclusion of a disc component, but does depend crucially on the presence of the baryonic bulge, and even more strongly, on the magnification bias. The number of lenses per observed source increases from $3.2 \times 10^{-4}$ without magnification bias to $7.5 \times 10^{-5}$ including the effect of magnification bias. Using our model, we also calculate the relative incidence of four- and two-image systems. The inferred fraction of $\pm 5$ per cent of quadruple systems is marginally consistent with observed values in CLASS.

The key result of our work is that samples of lens galaxies expected in surveys like the 2dF and the SDSS distinguish themselves from the overall galaxy population in several important ways:

(i) A fraction of $\sim 85$ per cent of lenses are expected to be of early-type morphologies, compared to only $\sim 35$ percent in the overall galaxy sample. This number is reduced by $\sim 20$ per cent if only four-image systems are considered.

(ii) The average velocity dispersion of lensing galaxies is $\bar{\sigma} = 191$ km s$^{-1}$ as opposed to $150$ km s$^{-1}$ for all early-type galaxies. Furthermore, the distribution is dramatically different – the distribution for lens galaxies has a maximum at $\sigma \sim 210$ km s$^{-1}$, with a tail towards lower velocity dispersions.

(iii) The peak of the luminosity distribution of lens galaxies is a factor of $\sim 2$ higher than that of the total galaxy population.

(iv) For a given luminosity, lens galaxies reside in haloes that are more massive by a factor of $\sim 2$ on average than the overall sample of ellipticals.

Interestingly, we also find that the properties of the lens sample differ depending on whether four- or two-image systems are considered. Initially surprising is the result that four-image systems tend to have lower velocity dispersions; they trace the velocity dispersion distribution of the overall early-type galaxy population much better. This can be explained by noting that the asymmetries in the potential have a larger effect for lower mass galaxies – the ratio...
of 4:2 image systems is therefore expected to be higher for galaxies with lower velocity dispersions. In addition, the predominant cause for four-image lens geometries is a modest ellipticity in the potential, which is more likely to be created by moderately inclined discs, and hence by galaxies with a lower $B/T$ ratio, which tend to have lower velocity dispersions. Our assumption that asymmetries in the potential are due to the baryonic components and that the DM haloes are spherical may lead to an underestimate of the four-image lensing cross-section. However, since lens galaxies in the catalogue are dominated strongly by the baryonic component, this should not change the statistics significantly.

All these differences between lens samples and the underlying galaxy population are very important to quantify if lensing studies are to be used to obtain a better understanding of the galaxy evolution. This holds both for statistical studies and for studies of individual systems. For example, single cases of lenses for which the mass distribution is steeper than the average profile predicted by simulations at the same mass do not point to any inconsistency; it is absolutely crucial that the full predicted distribution functions are considered. Our statistical analysis clearly demonstrates that lensing galaxies are a biased population. They preferentially sample the high-mass end of all galaxies. This systematic, in fact, limits the unbiased study of galaxy evolution using a sample of lensing galaxies (Ofek, Rix & Maoz 2003; Mitchell et al. 2005).

In our analysis, we do not examine the effect of the environment. In a recent study by Oguri (2006), it has been shown that the presence of other mass concentrations in the vicinity can increase the lensing cross-sections for high image separations by several tens of per cent. Interestingly, this result was also previously found by Turner et al. (1984) who used a very simple mass–sheet model to calculate the effect of environment. We address the effect of environment and in particular the incidence and statistical properties of lens galaxies in groups using our approach in a subsequent publication. However, it is worthwhile to note here that the environment is expected to have a much smaller effect for low lens redshifts. In general, environment has two effects: increase in the ellipticity of the lensing potential due to external shear (Möller et al. 2002; Keeton & Zabludoff 2004) and an increase in the projected mass within the Einstein radius due to the group/cluster halo that the lensing galaxy is embedded in Oguri & Keeton (2004). The external shear is smaller for lower redshift lenses, as the ratio between expected image separations of galaxy lenses and the typical distance to nearby galaxies is much smaller at lower redshifts (at $z \sim 0.05$, a distance of 1 arcsec corresponds to about 1 kpc, at $z \sim 0.5$ to about 6 kpc). The physical size that corresponds to the Einstein radius is also smaller for lower redshift lenses. For steep lens mass profiles, this means that the relative contribution of a mass sheet to the mass within the Einstein radius is smaller at lower lens redshifts than at higher ones.

Our calculations show that selection effects, and in particular magnification biases, have a strong effect on lens statistics. Given that magnification bias is important, it does appear that the distribution of the source magnitudes may also strongly influence lensing statistics. For radio-selected samples, the selection criteria can be defined clearly and the source luminosity function is known to an adequate degree to predict lensing statistics reasonably well. In the optical or infrared, however, both selection criteria and source properties are not as well understood. To use lenses selected at these wavelengths in a statistical way requires careful analysis to disentangle the various effects, like source luminosity functions, magnification bias and source redshift distributions. Simulating real imaging data in various wavebands will be needed to test the particular predictions of a given model of the lens and source population.

The basis for this study has been to use one of the best models of galaxy formation currently available and test the predicted lensing properties of such a sample. Our choice of a semi-analytic model was motivated strongly by the advantage that the relation between luminous and the DM is predicted in a way that is self-consistent. It is important to point out, though, that these models do have limitations. Even though the predictions at low redshift appear to be consistent with observations, it is not clear if current semi-analytic models predict all the observed properties of galaxies at higher redshifts correctly. It is encouraging for semi-analytic models that our results are consistent with the current known statistical lens sample, but we add the caveat that the known lens sample is small. Testing models of galaxy formation and evolution using strong lensing statistics seem a promising approach; future lens surveys will increase the number of lens systems known by a large number and if the selection criteria in these surveys can be understood, much can be learnt about the relation between mass and light on galaxy scales.

6 SUMMARY

We predict the lensing properties of galaxies in low-redshift surveys with $z \lesssim 0.2$ using realistic galaxy mass models based on a recent semi-analytic simulation of galaxies formation within the Millennium Run N-body simulation. Using ray-tracing techniques, we calculate the lensing cross-section for two- and four-image systems for a multicomponent lens model consisting of a DM halo, a bulge and a disc component. Our key results can be summarized as follows:

(i) The predicted lensing rate for radio sources in low-redshift surveys like the 2dF is $7.5 \times 10^{-3}$.
(ii) A fraction of $85 \pm 5$ per cent of all lens galaxies are predicted to be ellipticals.
(iii) The predicted average maximum image separation is $2.7 \pm 0.3$ arcsec.
(iv) The average velocity dispersion of lenses is $\bar{\sigma} = 191 \pm 3$ km s$^{-1}$.
(v) The baryonic component is the most important contributor to the lensing cross-sections for galaxy scale masses, but the DM distribution also affects the lensing statistics of composite lens models significantly.
(vi) Lens galaxies are more luminous and, for a fixed luminosity, reside in DM haloes that have masses of a factor of 2 higher than the overall galaxy population.
(vii) The velocity dispersion distribution of lens galaxies is shifted to significantly higher values of the velocity dispersion with respect to that of a complete sample of galaxies.
(viii) Four-image systems have, on average, a lower velocity dispersion, a later-type morphology and a lower average maximum image separation than double image systems.

Our quantitative predictions are consistent with the statistics in the CLASS survey. We pioneer the use of detailed semi-analytic models of galaxies to predict statistical lensing properties. This approach provides testable predictions for the lensing statistics and conversely, statistical lensing can provide useful constraints on galaxy formation models in the near future. In ongoing work, we address in detail the effect of environment for lensing statistics.

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