Effect Of Slip On Jeffrey Fluid Flow Through An Inclination Tube

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Abstract. In this paper, the effect of slip on a two-fluid model for Jeffrey fluid flow through an inclined tube of small diameters is studied. It is assumed that the core region consists of Jeffrey fluid and Newtonian fluid in the peripheral region. The expressions for velocity, flow flux, and effective viscosity have been obtained analytically and the effects of various relevant parameters on these flow variables have been studied. It is found that the effective viscosity increases with Jeffrey parameter, the angle of inclination, gamma parameter, Darcy number but decreases with slip and tube hematocrit. Further, it is significant to mention that, the flow exhibits the anomalous Fahraeus-Lindqvist effect.

1. Introduction
The study of microcirculation has received significant attention in the recent past because it may help in better understanding of physiological systems. Further, the flow of blood through smaller diameter blood vessels is accompanied by anomalous effects, like Fahraeus-Lindqvist effect, Fahraeus effect and existence of a cell-free or cell-depleted layer near the wall. These effects have been confirmed by several investigators (Fahraeus and Lindqvist [1] and Dintenfass [2]).

Theoretical studies related to blood flow in small tubes have been made by various authors under different assumptions (Seshadri and Jaffrin [3] and Whitmore [4]). Some researchers (Bugliarello and Sevilla [5], Sharan and Popel [6] and Srivastava [7]) have considered a two-fluid model with both fluids as Newtonian fluids and with different viscosities. However, it is realized that at lower shear rates, and in smaller vessels, blood behaves like a non-Newtonian fluid. Hence, several non-Newtonian fluid models have been considered for blood flow in small diameter tubes(Haynes [8],Haldar and Andersson [9], Chaturani and Upadhya [10, 11]).

In view of this, recently Santhosh et. al [12], Santhosh and Radhakrishnamacharya [13] have been studied a two-fluid model for the flow of Jeffrey fluid in tubes of small diameters under different conditions. Hence an attempt has been made in this paper to study the effect of slip at the wall on Jeffrey fluid flow through inclined narrow tubes. Following the analysis of Chaturani and Upadhya[10, 11], it is assumed that the core region consists of Jeffrey fluid and the peripheral layer consists of Newtonian fluid. The expressions for velocity and effective viscosity have been calculated and the effects of relevant parameters on these flow variables have been studied.
2. Formulation of the problem
Consider a laminar, axisymmetric flow of Jeffrey fluid through a rigid circular tube of constant
radius ‘a’ under slip boundary condition. The tube is assumed to be inclined at an angle θ with
the horizontal. The flow in the tube is represented by a two-fluid model consisting of a core
region of radius ‘b’, occupied by Jeffrey fluid and peripheral region of thickness (ε)(a − b = ε)
filled by Newtonian fluid. Let μ_p and μ_c be the viscosities of Newtonian fluid in the peripheral
region and Jeffrey fluid in the core region respectively. Cylindrical polar coordinate system
(r,θ,z) is chosen in such a way that the z-axis is taken along the axis of the tube (Fig.1).

![Figure 1. Geometry of the problem.](image)

The equations governing the steady flow of an incompressible Jeffrey fluid are:

\[
\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0 \tag{1}
\]

and

\[
\rho \left[ v_r \frac{\partial}{\partial r} + v_z \frac{\partial}{\partial z} \right] v_r = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left( r S_{rr} \right) + \frac{\partial}{\partial z} \left( S_{rz} \right) + \rho g \cos \theta \tag{2}
\]

\[
\rho \left[ v_r \frac{\partial}{\partial r} + v_z \frac{\partial}{\partial z} \right] v_z = -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( r S_{rz} \right) + \frac{\partial}{\partial z} \left( S_{zz} \right) + \rho g \sin \theta \tag{3}
\]

in which

\[
S_{rr} = \frac{2 \mu_c}{1 + \lambda_1} \left[ 1 + \lambda_2 \left( v_r \frac{\partial}{\partial r} + v_z \frac{\partial}{\partial z} \right) \right] \frac{\partial v_r}{\partial r} \tag{4}
\]

\[
S_{rz} = S_{zr} = \frac{\mu_c}{1 + \lambda_1} \left[ 1 + \lambda_2 \left( v_r \frac{\partial}{\partial r} + v_z \frac{\partial}{\partial z} \right) \right] \left( \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) \tag{5}
\]

\[
S_{zz} = \frac{2 \mu_c}{1 + \lambda_1} \left[ 1 + \lambda_2 \left( v_r \frac{\partial}{\partial r} + v_z \frac{\partial}{\partial z} \right) \right] \frac{\partial v_z}{\partial z} \tag{6}
\]

where \(v_r, v_z\) are the velocity components in the r and z directions respectively, \(p\) is the
pressure, \(\rho\) is the density, \(\lambda_1\) is the ratio of relaxation to retardation times, \(\lambda_2\) is the retardation
time, \(S_{rr}, S_{rz}, S_{zr}, S_{zz}\) are the extra stress components and \(g\) is the gravitational constant.
Let \( v_z(r) = v_1(r) \) be the velocity in the peripheral region and \( v_2(r) \) in the core region. The equations governing the flow of fluid are:

**Peripheral region (Newtonian fluid):**

\[
\frac{1}{\mu_p} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_1}{\partial r} \right) + \rho g \sin \theta - \frac{\partial p}{\partial z} = 0
\]  

(7)

**Core region (Jeffrey fluid):**

\[
\frac{\mu_c}{1 + \lambda_1} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_2}{\partial r} \right) + \rho g \sin \theta - \frac{\partial p}{\partial z} = 0
\]

(8)

The boundary conditions for the problem are:

\[
\frac{a}{r} \frac{\partial v_1}{\partial r} = -\frac{\alpha}{\sqrt{Da}} v_1 \quad \text{at} \quad r = a
\]

(9a)

\( v_1 = v_2, \tau_1 = \tau_2 \) at \( r = b \),

(9b)

\( v_2 \) and \( \tau_{rz} \) is finite \( \text{at} \quad r = 0 \)

(9c)

Condition (9a) is the Saffman’s slip boundary condition ([14]), (9b) denotes the continuity of velocities and stresses at the interface and (9c) is the regularity condition. Further, \( Da \) is the permeability parameter (or Darcy number) and \( \alpha \) is the slip parameter.

Solving equations (7) and (8) under the conditions (9), we get

\[
v_1(\xi) = \frac{a^2 P}{4 \mu_p} (\gamma \sin \theta + 1) \left( 2 \frac{\sqrt{Da}}{\alpha} + 1 - \xi^2 \right) \quad \text{for} \quad (d \leq \xi \leq 1)
\]

(10)

and that of core region is given by

\[
v_2(\xi) = \frac{a^2 P}{4 \mu_p} (\gamma \sin \theta + 1) \left( 2 \frac{\sqrt{Da}}{\alpha} + 1 - \xi^2 + \mu' (1 + \lambda_1) \left( \xi^2 - 2 \right) \right) \quad \text{for} \quad (0 \leq \xi \leq d)
\]

(11)

where

\[
\xi = \frac{r}{a}, \quad d = \frac{b}{a}, \quad P = -\frac{\partial p}{\partial z}, \quad \gamma = \frac{\rho g}{P}, \quad \mu' = \frac{\mu_p}{\mu_c}
\]

(12)

Here \( d \) is the non-dimensional core radius.

The flow flux in the peripheral region and core region, denoted by \( Q_p \) and \( Q_c \), are given by

\[
Q_p = 2 \pi a^2 \int_d^1 v_1(\xi) \xi d\xi
\]

(13)

and

\[
Q_c = 2 \pi a^2 \int_0^d v_2(\xi) \xi d\xi
\]

(14)

Substituting for \( v_1 \) and \( v_2 \) from equations (10) and (11) into equations (13) and (14), we get

\[
Q_p = \frac{a^4 P \pi}{8 \mu_p} (\gamma \sin \theta + 1) \left( 2 \frac{\sqrt{Da}}{\alpha} + 1 - 2 d^2 + d^4 \right)
\]

(15)
and

\[ Q_c = \frac{a^4 P \pi}{8 \mu_p} (\gamma \sin \theta + 1) \left( 2 \left( \frac{\sqrt{D a}}{\alpha} + 1 \right) (d^2 - d^4) + \mu' (1 + \lambda_1) d^4 \right) \]  

(16)

Thus, the flow flux through the tube is given by

\[ Q = Q_p + Q_c \]  

(17)

Using equations (15) and (16) in (17), we get

\[ Q = \frac{a^4 P \pi}{8 \mu_p} (\gamma \sin \theta + 1) \left( 4 \sqrt{D a} \alpha + 1 - d^4 + \mu' (1 + \lambda_1) d^4 \right) \]  

(18)

Comparing equation (18) with flow flux for Poiseuille's flow, we get the effective viscosity as

\[ \mu_{eff} = \frac{\mu_p}{(\gamma \sin \theta + 1) \left( 4 \frac{\sqrt{D a}}{\alpha} + 1 - d^4 + \mu' (1 + \lambda_1) d^4 \right)} \]  

(19)

In the case when there is no inclination, that is \( \theta = 0^\circ \) and if we take no-slip condition, i.e., \( v_1 = 0 \) at \( r = 0 \) instead of equation (9a), we get the effective viscosity as

\[ \mu_{effNS} = \frac{\mu_p}{1 - d^4 + \mu' (1 + \lambda_1) d^4} \]  

(20)

Further, if we put \( \lambda_1 = 0 \), we obtain results for Newtonian fluids, i.e.,

\[ \mu_{eN} = \frac{\mu_p}{1 - d^4 + \mu' d^4} \]  

(21)

This is same as the expression obtained by Buglierello and Sevilla [5].

3. Numerical Results and Discussion

To explicitly see the effects of Jeffrey parameter(\( \lambda_1 \)), an angle of inclination(\( \theta \)), tube hematocrit (\( H_0 \)), Darcy number(\( Da \)), slip parameter (\( \alpha \)) and tube radius (\( a \)) on effective viscosity \( \mu_{eff} \) (eq. (19), numerical calculations have been computed using Mathematical software and the results are graphically presented in Figs.2-7.

It can be seen that the effective viscosity (\( \mu_{eff} \)) decreases with the Jeffrey parameter (\( \lambda_1 \)) (Fig. 2), angle of inclination (\( \theta \)) (Fig. 3), Darcy number (\( Da \)) (Fig. 5) and gamma parameter (\( \gamma \)) (Fig. 6) but increases with slip parameter (\( \alpha \)) (Fig. 4) and tube hematocrit (\( H_0 \)) (Fig. 7). The values of effective viscosity computed from the present model are in good agreement, within the acceptable range, with the corresponding values of the effective viscosity obtained in the theoretical models of Haynes[8], Chaturani and Upadhyya[10, 11] and Santhosh et. al[12]. Further, for given values of Jeffrey parameter (\( \lambda_1 \)), angle of inclination (\( \theta \)), slip parameter (\( \alpha \)), gamma parameter (\( \gamma \)), tube hematocrit (\( H_0 \)) and Darcy number (\( Da \)), the effective viscosity (\( \mu_{eff} \)) increase with tube radius (\( a \)) (Figs. 2 - 7), i.e., the flow exhibits Fahraeus-Lindqvist Effect.
Figure 2. Effect of Jeffrey parameter ($\lambda_1$) on $\mu_{eff}$ ($H_0 = 40\%$, $\alpha = 0.2$, $Da = 0.0001$, $\gamma = 0.2$ and $\theta = 45^\circ$)

Figure 3. Effect of angle of inclination ($\theta$) on $\mu_{eff}$ ($H_0 = 40\%$, $\lambda_1 = 0.5$, $\alpha = 0.2$, $Da = 0.0001$ and $\gamma = 0.2$)

Figure 4. Effect of slip parameter ($\alpha$) on $\mu_{eff}$ ($H_0 = 40\%$, $\lambda_1 = 0.5$, $Da = 0.0001$, $\theta = 45^\circ$ and $\gamma = 0.2$)

Figure 5. Effect of Darcy number ($Da$) on $\mu_{eff}$ ($H_0 = 40\%$, $\lambda_1 = 0.5$, $\theta = 45^\circ$, $\alpha = 0.2$ and $\gamma = 0.2$)
Figure 6. Effect of gamma parameter ($\gamma$) on $\mu_{eff}$ ($H_0 = 40\%$, $\lambda_1 = 0.5$, $\alpha = 0.2$, $Da = 0.0001$ and $\theta = 45^\circ$)

Figure 7. Effect of tube hematocrit ($H_0$) on $\mu_{eff}$ ($\lambda_1 = 0.5$, $\alpha = 0.2$, $Da = 0.0001$, $\gamma = 0.2$ and $\theta = 45^\circ$)

4. Conclusion
A two-fluid model for the steady flow of Jeffrey fluid through an inclined tube of small diameters with slip effect is investigated. With the assumption that there is Jeffrey fluid in the core region and Newtonian fluid in the peripheral region, an analytical expression for effective viscosity is obtained. The effects of various relevant parameters on effective viscosity have been studied. Further, for given values of all other parameters, the effective viscosity increases with tube radius. Further, it is noticed that the flow exhibits the anomalous Fahraeus-Lindqvist effect.

References
[1] Fahraeus R and Lindqvist T 1931 Viscosity of Blood in Narrow Capillary Tubes Am J Phys. 96 562-568.
[2] Dintenfass L 1967 Inversion of Fahraeus-Lindquist Phenomenon in Blood Flow through Capillaries of Diminishing Diameter Nature 217 1099-1100.
[3] Seshadri V and Jaffrin N Y 1977 Anomalous Effects in Blood Flow through Narrow Tubes Inserm-Euromech 92 71 265-282.
[4] Whitmore R L 1967 A Theory of Blood Flow in Small Vessels J. Appl. Physio. 22 767-771.
[5] Bugliarello G and Sevilla J 1970 Velocity Distribution and other Characteristics of Steady and Pulsatile Blood Flow in Fine Glass Tubes Biorheology 7 85 - 107.
[6] Sharan M and Popel A S 2001 A Two-phase Model for Flow of Blood in Narrow Tubes with increased Effective Viscosity near the Wall Biorheology 38 415–428.
[7] Srivastava V P 2007 A Theoretical Model for Blood Flow in Small Vessels Appl. Appl. Math. 2 51-65.
[8] Haynes R H 1960 Physical basis of the dependence of Blood Viscosity on Tube Radius Am. J. Physiol. 198 1193–1200.
[9] Haldar K and Andersson H I 1996 Two-layered Model of Blood Flow through Stenosed Arteries Acta. Mech. 117 221-228.
[10] Chaturani P and Upadhya V S 1979 On Micropolar Fluid Model for Blood Flow through Narrow Tubes Biorheology 16 419-428.
[11] Chaturani P and Upadhya V S 1981 A Two-Fluid Model for Blood Flow through Small diameter Tubes Biorheology 18 245–253.
[12] Santhosh N, Radhakrishnamacharya G and Chamkha A J 2015 Flow of a Jeffrey Fluid through a Porous Medium in Narrow Tubes J. Porous Media 18(1) 71-78.
[13] Santhosh N and Radhakrishnamacharya G 2014 Jeffrey Fluid Flow through Porous Medium in the Presence of Magnetic Field in Narrow Tubes, International Journal of Engineering Mathematics Article ID 713831.
[14] Saffman P G 1971 On the Boundary conditions at the surface of a porous medium Stud. Appl. Math. 1 93-101.