The Systematical Uncertainties in Measurements of the Spin-Dependent Structure Function $g_1^n$ with $^3$He Target due to Radiative Correction Procedure

I.Akushevich$^1$, A.Nagaitsev$^2$

$^1$ NC PHEP, Minsk Belarus
$^2$ JINR, Dubna Russia

Abstract

The sources of the systematical uncertainties due to radiative correction procedure in measurements of the structure function $g_1^n$ with $^3$He target are considered. Their numerical estimations are presented. The relative systematical uncertainty does not exceed 5%.

The measurements of the spin-dependent structure functions performed in the last years (SMC, E154/155) have the tendency to decrease essentially the statistical uncertainties comparing with previous ones (EMC, E142/143). Thus the one of main problem is to decrease the magnitude of the systematical uncertainties or define more correct way of their calculation. In general systematical uncertainties come from measured quantities, namely: values of the beam and target polarizations, efficiencies of the coordinate detectors etc. There are also some contributions in total systematic uncertainty due to unmeasured quantities such as radiative corrections (RC) to be applied to extract the one-photon exchange cross section from the measured one.

So called radiative events, which originate from loop diagrams and from processes with the emission of additional real photons, cannot completely be removed by experimental methods and so they have to be calculated theoretically and subtracted from measured cross sections. The calculation of the radiative corrections requires knowledge of spin-independent and spin-dependent structure functions both in region measured in the considered experiment and beyond it. The choice of different parameterizations of the structure functions, elastic and quasielastic formfactors, neglecting of electroweak and higher order effects and simplifications in RC procedure leads to uncertainties in calculations of RC. The approach of calculation of these systematical uncertainties is presented in this report. We consider the case of the $^3$He target and kinematical region close to HERMES [1] and SLAC [2] experiments. To get the numerical estimations of the systematical uncertainties due to radiative correction procedure we use the special program [3] and radiative correction calculation code POLRAD 2.0 [4, 5].

The extraction of the spin asymmetry $A_1(x, Q^2)$ in the experiments measuring spin-dependent structure functions is based on the following formula

$$A_1(x, Q^2) = \frac{1}{<Df_d>} \frac{N_{\uparrow\downarrow} - N_{\uparrow\uparrow}}{N_{\uparrow\uparrow} + N_{\uparrow\downarrow}},\quad (1)$$

where $<Df_d>$ is mean value of product of depolarization and dilution factors (see [6] for details), $N_{\uparrow\uparrow}$ and $N_{\uparrow\downarrow}$ are the number of events for parallel and antiparallel spin target configurations.
In order to take into account the radiative effects these numbers have to be calculated as the weighted sum

\[ N_{\uparrow\downarrow, \uparrow\uparrow} = \sum_{\uparrow\downarrow, \uparrow\uparrow} w(x, Q^2). \]  

Here the weight \( w \) is calculated as a ratio of Born and observed cross sections

\[ w = \frac{\sigma_0}{\sigma_0 + \sigma_{RC}} \]  

We will refer to this procedure as ’exact’.

The exact procedure requires the RC calculation for each event, so in practice other scheme is used. We will refer to this procedure as ’standard’. In this case the RC is applied to asymmetry averaged at \( x \) bins.

The radiative correction \( \Delta A_1 \) to the measured asymmetry is defined as:

\[ A_{1 meas} = A_1 + \Delta A_1 \]  

and can be written in terms of spin-independent\( (\sigma^u) \) and spin-dependent \( (\sigma^p) \) parts of DIS cross section.

\[ \Delta A_1 = \frac{\sigma^u_0((\sigma^p_{in}(g_1) + \sigma^p_q + \sigma^p_{el}) - \sigma^p_0(g_1)(\sigma^u_{in} + \sigma^u_q + \sigma^u_{el}))}{\sigma^u_0((1 + \delta_v)\sigma^u_{in} + \sigma^u_q + \sigma^u_{el})}, \]  

where \( \delta_v = \sigma^p_v/\sigma^0_v = \sigma^u_v/\sigma^0_u \). The polarized parts of Born cross section and inelastic radiative tail depend on \( g_1 \).

The radiative correction procedure is performed as follows.

1. The measured asymmetry \( A_{1 i}^{m} \) \((i = 1, \ldots, N_x)\) is fitted by a function\( \] with taking into account statistical uncertainties of \( A_{1 i}^{m} \).
2. The constructed fit is used for calculation of \( \sigma^p_{in}(g_1) \) and \( \sigma^0_0(g_1) \).
3. The extracted asymmetry is calculated for each kinematical bin as follows

\[ A_{1 i}^{ext} = A_{1 i}^{m} - \Delta A_1 \]  

There are three important sources for uncertainties coming from radiative correction procedure: a) using of simplified (standard) scheme instead of the exact one ; b) using of models and data for structure functions; c) physical effects which are neglected in the standard scheme.

To calculate the systematical uncertainties of the types a), b) and c) the following scheme is used. The Monte-Carlo kinematical events are generated according to random flat generator: \( \ln Q^2 \) for \( Q^2 \) over allowed \( Q^2 \)– region and flat for \( \nu \) over allowed \( \nu \) – region. After calculation of kinematic variables and applying of kinematic cuts close to acceptance of the HERMES experiment [1], the weights with DIS cross section \( w^{1\gamma} \) are obtained at given kinematical point. The model for spin asymmetry and generated Born asymmetry are plotted on fig.1a.

For each event the recalculation of the weight is performed with radiative correction factor:

\[ w^{obs} = \frac{w^{1\gamma}}{w}. \]

Superscript \(^1\) The example such a function for \(^3\)He target can be found in Appendix of ref [1]
where weight $w$ defined from eq. (3) is calculated by code POLRAD 2.0. The measured ($A_{1}^{\text{obs}}$) and extracted within exact scheme ($A_{1}^{\text{ex}}$) asymmetries are obtained using formulas (1) and (2) with the weights $w_{\text{obs}}$ and $w_{1\gamma}$.

The application of the standard scheme to $A_{1}^{\text{obs}}$ gives the extracted asymmetry $A_{1}^{\text{st}}$ within standard scheme. To estimate the relative systematical uncertainty of type a), we calculate the difference between extracted asymmetries within the exact ($A_{1}^{\text{ex}}$) and standard ($A_{1}^{\text{st}}$) schemes for each kinematical bin:

$$\epsilon_{a} = \frac{|A_{1}^{\text{ex}} - A_{1}^{\text{st}}|}{A_{1}^{\text{ex}}}.$$  

(8)

The relative systematical uncertainties of types b) and c) are estimated as follows

$$\epsilon_{b,c} = \frac{|A_{1}^{\text{ex}} - \tilde{A}_{1}^{\text{ex}}|}{A_{1}^{\text{ex}}}.$$  

(9)

The asymmetry $\tilde{A}_{1}^{\text{ex}}$ is calculated using the weight $\tilde{w}^{\text{ex}} = w_{\text{obs}} \tilde{w}$. The weight $\tilde{w}$ is also calculated according to formula (3), but $\sigma_{\text{obs}}$ is calculated with different models for structure functions or with taking into account electroweak and high order effects.

![Figure 1](image)

Figure 1: The generated asymmetry along used fit and relative systematical uncertainties versus $x$ in percents due to simplification procedure. It should be noted that peak for $x \sim 0.4$ is artificial and originates from the fact that neutron spin asymmetry is close to zero in this kinematical point.

The quantity $\epsilon_{a}$ can be non-zero only due to order of averaging within standard and exact schemes for the same models for structure functions. In case of ’standard’ scheme, the results are calculated for averaged kinematical point at each bin. But this mean value can be shifted due to very different dependence of Born and radiative corrected cross sections on kinematical variables. The exact scheme is free from this shortcoming. In this case the RC is taken into account before averaging using calculation of the weight (3) for each event. The relative systematical shift obtained with standard and exact schemes does not exceed 2% (fig.1b).

The quantity $\sigma_{u}$ in eq. (3) depends on $F_{2}$ in the given kinematical point $Q^{2}$ and $x$, but $\sigma_{in}$ (see eq. (3)) requires knowledge of the structure function $F_{2}$ in the wide region of
varying of kinematical variables \(x\) and \(Q^2\). So the fit of \(F_2\) used for RC calculation has to describe adequately both resonance and DIS region as well as to have correct asymptotics behaviour for \(Q^2 \to 0\) and \(W^2 \to (M + m_\pi)^2\). Such fit can be constructed on the basis of NMC parameterization of \(F_2\) for protons and deuterons

\[
F_{2}^{\text{He}}(x, Q^2) = \frac{1}{3}\left(F_{2}^{H}(x, Q^2) + F_{2}^{D}(x, Q^2)\right).
\]  

(10)

\(F_{2}^{p}\) and \(F_{2}^{d}\) are taken from fit described in [7]. The fit takes into account the contribution of \(\Delta\) resonance, has correct behaviour on boarders and describes DIS data for \(x \gtrsim 0.01\).

Another possibility is to change the model for \(F_2\) on standard POLRAD fit [5], which includes Brasse parameterization of the three resonances (instead of one resonance as in standard fit [11]), Stein fit for small \(Q^2\) region and 15 parameters NMC fit for DIS region. The final results shows that unpolarized structure function can change asymmetry by approximately 1%.

Both polarization and unpolarization contribution of quasielastic radiative tails depend on quasielastic response functions \(F_{qi}\) (see [4] for details). The electric and magnetic formfactors for proton and neutron fall as \(Q^2\) for high \(Q^2\). So the only region of small \(Q^2 \sim M^2\) is important. In this region the nucleon formfactors are known with good accuracy, and their variation does not lead to systematical uncertainty. To the contrary the models for electric \(S_E\), magnetic \(S_M\) and mixed \(S_{EM}\) suppression factors differ in this region. The code POLRAD 2.0 exploits Y-scaling hypothesis [3]. \(S_M = S_E = S_{EM} = F(\nu_0)\) and scaling function \(F(\nu_0)\) are calculated in Fermi gas model [10, 11]. Alternatively the suppression factors can be also calculated within the sum rule approach [12]. The difference is important for small values of \(x\), which correspond to high \(y\), where it can reach 1.5%.

The elastic structure functions are calculated as quadratic combinations of electric and magnetic formfactors. A simple Schiff’s model with gaussian wave function [13] is used as an alternative model. The results are similar to quasielastic case.

Resonance region gives a large contribution to RC for spin-independent DIS. The contribution to resonance region in \(g_1\) can be also important. Unfortunately, there are no enough experimental data or satisfactory models for \(g_1^p(x, Q^2)\) in resonance region \((W^2 < 4)\). It is the main reason why scaling behaviour of spin asymmetry is extrapolated into resonance region under POLRAD consideration. For alternative approach we used simple model for structure function \(g_1^p(x)\) in region of \(\Delta(1232)\) resonance constructed on the basis of two assumptions, namely: the \(W^2\)-dependence has the Gaussian form with height and width which can be roughly estimated from recent SLAC data [14], the \(Q^2\)-dependence is defined by resonance contribution to Drell-Hearn-Gerasimov sum rule given in ref. [15]. The relative systematical uncertainty due to resonance region is important (\(\sim 1.5\%\)) for small \(x\)-region.

The kinematical coefficient front of structure function \(g_2\) is small enough for both at the Born level and for RC cross section. So, the contribution of the structure function is neglected normally. To study influence of such an approach the model of the Wandzura-Wilczek is applied [16]. The small effect is obtained. The exception is last two bins where \(\epsilon_b \sim 1\%\).

The electroweak effects are not included in standard radiative correction procedure, because for current polarized experiments \(Q^2 \sim 10\text{GeV}^2 \ll M_z^2\) (\(M_z\) is the Z-boson mass) and hence their contribution is small, but it has to be added to systematical error. Such a systematical uncertainty is estimated at the born level using code POLRAD 2.0. The
electroweak correction cannot be calculated by model independent way, that is why the quark parton model was used and GRV-, GRSV-parameterizations \cite{17, 18} for spin-independent and spin-dependent partonic distributions were applied. The correction is important ($\epsilon_c \sim 1.5\%$) for high $x$.

In standard consideration the effects of higher order is estimated by simple exponentiation procedure of soft photons \cite{4, 19}. The POLRAD 2.0 gives also a possibility to obtain the $\alpha^2$ order correction within structure function approach \cite{20, 21}. The radiation is considered to be collinear. There are three possibilities: initial, final state radiation and contribution of the Compton process. The first and second are important for inelastic radiative tail. The last one is extremely important for elastic and quasielastic radiative tails. The relative systematical uncertainties due to the higher order effect can exceed 2% at low $x$-region.

The systematical uncertainties considered in previous sections can be gathered together to obtain the total relative contribution due to RC to systematical uncertainties. The following systematical uncertainties are considered to be independent

- simplification of procedure
- unpolarized structure function $F_2$
- quasielastic structure function
- elastic formfactors
- resonance region of structure function $g_1$
- polarized structure function $g_2$
- electroweak effects
- higher order effects

Figure 2: The relative systematical uncertainties due to radiative corrections.

The quadratic sum of the uncertainties mentioned above presents in fig.2a. For $x \gtrsim 0.3$ the total systematical error does not exceed 2%. However for the first $x$ bins with $x \lesssim 0.3$ and $y \sim 0.85$ the effect is larger and can reach 5%.
Note that the uncertainties of types c) can be rejected from total sum, if they are included in standard scheme. For higher order correction it can be done without additional assumption, but electroweak correction calculation requires usage of the quark-parton model. In this case systematical uncertainties due to RC come only from procedure simplification and uncertainties in structure functions. This result is presented on fig.2b. It does not exceed 4%.

Acknowledgements. We are grateful to N.Shumeiko and I.Savin for help and support. Also we would like to thank N.Akopov, N.Gagunashvili, V.Krivokhigine, P.Kuzhir and D.Ryckbosch for useful discussion and comments.

References

[1] HERMES, K.Ackelstaff et al., Phys. Lett. B404(1997)383.
[2] E143, K.Abe et al., Phys. Rev. Lett., 74(1995)364.
[3] N.Akopov, A.Nagaitsev, A code for systematical error studying; unpublished
[4] I.Akushevich, A.Ilyichev, N.Shumeiko, A.Soroko, A.Tolkachev, Comp. Phys. Comm. 104(1997)201.
[5] I.V.Akushevich and N.M.Shumeiko, Journal of Physics. G20(1994)513.
[6] N. Gagunashvili et al., Extraction of asymmetries and spin dependent structure functions from polarized lepton nucleus cross-sections. Preprint JINR E1–96–483, submitted for Nucl.Instr.Meth.
[7] NMC collab., Nucl. Phys. B 371(1992)3.
[8] G.B.West, Phys.Rep. 18(1975)263.
[9] A.K.Thompson et al. Phys. Rev. Lett. 68(1992)2901.
[10] T.deForest and J.D.Walecka, Adv.Phys. 15(1966)1.
[11] E.J.Moniz, Phys.Rev. 184(1969)1154.
[12] W.Leidemann, E.Lipparini and S.Stringari, Phys.Rev. C42(1990)416.
[13] L.I.Schiff, Phys.Rev. 133(1964)3B,802.
[14] E143 Collaboration, Phys.Rev.Lett.78(1997)815-819.
[15] M. Anselmino, B.L. Ioffe, E. Leader, Sov.J.Nucl.Phys. 49(1989)136.
[16] W.Wandzura and F.Wilczek, Phys.Lett. B172(1977)195.
[17] M.Glück, E.Reya, A.Vogt Z.Phys. C53(1992)127.
[18] M.Glück, E.Reya, M.Stratmann and W.Vogelsang, Phys.Rev. D53(1996)4775.
[19] N.M.Shumeiko, Sov. J. Nucl. Phys. 29(1979)807.
[20] E.A.Kuraev and V.S.Fadin, Sov. J. Nucl. Phys. 41(1985)466; E.A.Kuraev, N.P.Merenkov and V.S.Fadin, Sov. J. Nucl. Phys. 47(1988)1009.
[21] J.Kripfanz, H.-J.Möhring, H.Spiesberger, Z.Phys. C49(1991)501.