High-precision astrometry on the VLT/FORS1 at time scales of few days*

P. F. Lazorenko1, M. Mayor2, M. Dominik3,⋆⋆, F. Pepe2, D. Segransan2, and S. Udry2

1 Main Astronomical Observatory, National Academy of Sciences of the Ukraine, Zabolotnogo 27, 03680 Kyiv-127, Ukraine
2 Observatoire de Geneve, 51 Chemin des Maillettes, 1290 Sauverny, Switzerland
3 SUPA, University of St Andrews, School of Physics & Astronomy, North Haugh, St Andrews, KY16 9SS, UK

Received 23 January 2007 / Accepted 21 April 2007

ABSTRACT

Aims. We investigate the accuracy of astrometric measurements with the VLT/FORS1 camera and consider potential applications.

Methods. The study is based on two-epoch (2000 and 2002/2003) frame series of observations of a selected Galactic Bulge sky region that were obtained with FORS1 during four consecutive nights each. Reductions were carried out with a novel technique that eliminates atmospheric image motion and does not require a distinction between targets and reference objects.

Results. The positional astrometric precision was found to be limited only by the accuracy of the determination of the star photocentre, which is typically 200–300 μas per single measurement for bright unsaturated stars. Several statistical tests showed that at time-scales of 1–4 nights the residual noise in measured positions is essentially a white noise with no systematic instrumental signature and no significant deviation from a Gaussian distribution. Some evidence of a good astrometric quality of the VLT for frames separated by two years was also found.

Conclusions. Our data show that the VLT with FORS1/2 cameras can be effectively used for astrometric observations of planetary microlensing events and other applications where a high accuracy is required, that is expected to reach 30–40 μas for a series of 50 frames (one hour with Rspet filter).

Key words. astrometry – atmospheric effects – instrumentation: high angular resolution – planetary systems

1. Introduction

The provision of high-precision astrometry (of a few 100 μas and below) is much desired, but so far it is not widely available. Over recent years, photometric microlensing has proven its feasibility for not only detecting massive gas giant planets (Bond et al. 2004; Udalski et al. 2005), but even cool rocky/icy planets (Beaulieu et al. 2006). A fourth planet, of roughly Neptune-mass, was most recently claimed by Gould et al. (2006). Several further potential detections of planets by microlensing have however been missed (e.g. Jaroszyński & Paczyński 2002) since a proper characterization requires good data quality and a dense sampling. In fact, it is necessary to overcome ambiguities and degeneracies of binary lenses themselves (Dominik 1999), between mass ratio and source size (Gaudi 1997), and between binary-lens and binary-source systems (Gaudi 1998), as shown explicitly by Gaudi & Han (2004). The observation of the shift of the centroid of light composed of the lens star and the images of the source star with time, so-called astrometric microlensing (Hög et al. 1995; Hardy & Walker 1995; Miyamoto & Yoshii 1980; Dominik & Sahu 2000), provides a two-dimensional vector, whose absolute value is proportional to the angular Einstein radius (comprising information about the lens mass and relative lens-source parallax), in addition to the magnification provided by the photometric observations, where the latter allows us to measure the angular Einstein radius only if the finite size of the source star can be assessed in the light curve. While Safizadeh et al. (1999) first discussed the possibility of using astrometric microlensing for discovering and characterizing extra-solar planets, Han & Chunguk (2002) found that the planetary deviation to the astrometric signal expected from a Galactic bulge star lensed by a foreground star and a surrounding Jupiter-mass planet is about 10–200 μas. In contrast to the detection of the astrometric shift seen in the host star due to its motion around the common center-of-mass, which requires a stability of the detector on time-scales comparable to the orbital period of planets (e.g. Quirrenbach et al. 2004; Pravdo et al. 2005), the microlensing perturbations are observed at much shorter time intervals, just between a few hours and a few days.

Since ground-based monopupil telescopes were thought to be limited by atmospheric noise and therefore to have rather poor precision (Lindegren 1980), until now only high-precision optical interferometers such as SIM (Boden et al. 1998; Paczyński 1998), Palomar Testbed Interferometer (Lane & Muterspagh 2004), Keck (Colavita et al. 1998), and VLTI (Wittkowski et al. 2004) were considered for carrying out observations with such precision. Recent progress in adaptive optics, and its application to astrometry (Pravdo et al. 2005) opens new potentials for a dramatic improvement of the precision. However, the theoretical considerations by Lazorenko & Lazorenko (2004) and a first set of corresponding data (Lazorenko 2006, hereafter Paper I) demonstrated that astrometric measurements with very large ground-based telescopes are not atmospheric-limited. Instead, a high precision beyond this limit is achievable by elimination of atmospheric image motion at the data-reduction phase. For that purpose, a field of reference stars is configured into a...
virtual high-pass filter that absorbs most significant low-frequency modes of the image motion spectrum. For telescopes with a fully illuminated entrance pupil, the atmospheric error $\sigma_B$ rapidly decreases with the increase of the telescope entrance pupil $D$. At $D \to \infty$, the asymptotic dependence is

$$\sigma_B = B_k R^{11/6} D^{-3/2} T^{-1/2}$$

where $T$ is the integration time, $R$ is the angular reference field radius, and $B_k$ is the filter parameter. This relation involves an additional factor $\sqrt{R/D}$ as compared to that predicted by Lindegren (1980), so that large telescopes with $D = 8 \text{–} 10 \text{ m}$ and above are favourable. A specific amplitude apodization to the entrance pupil of future telescopes may further suppress the atmospheric error to $\sigma_B \sim R^{1/2} D^{3/2} \alpha T^{-1/2}$ where $\alpha$ is the reduction parameter.

The astrometric performance of the VLT on very short timescales was estimated from test reductions of a four-hour series of FORS2 frames obtained in the region of the Galactic bulge (Paper I). A positional accuracy of 300 $\mu$as per single $R = 16 \text{ mag}$ star measurement at 17 $s$ exposure was shown to be limited only by the number of photons received from the observed star.

The current study extends the certification of high-accuracy astrometric performance with the VLT to time scales of 1–4 days, which is sufficient for monitoring astrometric planetary deviations in microlensing events. In Sect. 2 of this paper, we lay out the observational basis of this investigation, describe the image processing, and show how the stellar centroids are computed. Particular problems in astrometric reductions that are based on background reference stars with unstable positions, and a new reduction method are discussed in the subsequent Sect. 3. Reduction of the observational data with this method is described in Sect. 4. Section 5 focuses on the noise and correlations in the measurements that, if they exist, can be related to the different parts of the telescope (optics, mechanics, electronics), to the atmosphere, etc., preventing statistical improvement of the accuracy by means of simple accumulation of the number of frames.

2. Observations and computation of centroids

For our study, we were able to use VLT frames that are available in the ESO Archive: those obtained with the FORS1 camera of UT1 in the sky region near the neutron star RX J0720.4-3125 under the ESO program 66.D-0286 (Motch et al. 2003) were found to be best-suited for our purposes. Images were acquired during 4 nights from 20 to 23 December 2000 (40 frames) and 4 nights from 29 December 2002 to 02 January 2003 (25 frames). During each period 5 to 12 frames were obtained in $B$ filter with $\sim 10 \text{ mn}$ exposure time and $FWHM$ varying from 0.45 to 0.85 $\prime\prime$, where the high-resolution mode with 0.10$/\text{px}$ scale and about $3.3 \times 3.3$ $\prime\prime$ field-of-view was used.

These data enable us to carry out a detailed and comprehensive study of the astrometric precision that is achievable with the VLT/FORS1 at short time scales. Moreover, our analysis serves as a test reduction for future observations. Unfortunately, between some frames, the images turned out to be displaced by more than 200 px. For that reason, peripheral stars frequently needed to be omitted for much jittered frames. Moreover, for astrometric purposes, the choice of the $B$ filter is not optimal, since it results in larger residual atmospheric chromatism as compared to $R_{\text{spec}}$. However, it was just these unfortunate circumstances that forced us to look for a reduction that significantly improves the quality.

Observations were obtained with the Longitudinal Atmospheric Dispersion Corrector (LADC), which is a two-prism optical device (Avila et al. 1997) that allows correction for atmospheric differential chromatic refraction (DCR). DCR is caused by the atmosphere, which acts like a prism with a certain dispersion and thereby smears and displaces stellar images (e.g. Monet et al. 1992). The size of this effect depends on the zenith distance and the colour of the observed star and can be modelled as shown in Eq. (12).

Raw images were debiased and flat-fielded using master calibration frames taken from the ESO Archives. Positions $x$, $y$ of stellar centroids were computed using the profile fitting technique that had already been applied earlier for the high-precision astrometric reduction of FORS2 images (Paper I). Stellar profiles in 10 $\times$ 10 $\text{px}$ windows were fit by a sum of three modified elliptic Gaussians involving 12 free model parameters altogether. The dominant Gaussian centered at $x$, $y$ with extent parameters $\sigma_x$, $\sigma_y$ along corresponding coordinate axis and futher specified by its orientation angle, was considered to contain a flux $I$ mounting to about $2/3$ of the total light received from the star. Other auxiliary Gaussians allowed profile fitting to the photon noise limit at which residuals of pixel counts from the model are characterized by $\chi^2 \geq 1.0/\text{px}$ (except for central parts of images of bright stars). Objects with $\chi^2 \geq 3.0/\text{px}$ deviations were rejected and only unsaturated images with $B > 18 \text{ mag}$ were processed.

Due to the complex shape of the PSF, the precision $\varepsilon$ of the image centroiding was obtained by numerical simulation, for which a set of randomly generated star images with profiles comparable to these observed was created and where Poisson noise was added to pixel counts. This simulation shows that $\varepsilon$ can be well-approximated by

$$\varepsilon = \frac{\theta}{2.46 \sqrt{I}(1 + a_0 I^{-0.7})},$$

where $\theta$ denotes the seeing (which turned out to be $\theta = 3.10 \sigma_\theta$ in our three-component model), $I$ refers to the electron count, and $a_0$ is a characteristic constant that depends on the background noise. For the model background fluxes $b_0$ of 810 or 540 $\varepsilon^-\/\text{px}$ (average values at 2000 and 2002–2003 epochs), $a_0$ was found to be 1150 or 887, respectively. Equation (2) becomes an exact relation only for a fixed average seeing $b_0 = 0.608\prime\prime$ at which $\sigma_\theta = 1.96 \text{ px}$, but it provides a reliable approximation for variable FWHM of that order.

3. The reduction process

3.1. Filtration of atmospheric image motion

Let us briefly recall the previously discussed process of attenuation of the image motion spectrum (Lazorenko & Lazorenko 2004), which is based on the virtual symmetrization of the reference field. Consider some region of the sky with $N$ stars being imaged in $m = 0, 1 \ldots M$ consecutive frames, where $x_m$, $y_m$ are the centroids of $l = 1, 2 \ldots N_l$ stars measured in frame $m$. For various reasons (jittered images, cosmic, outlier data, etc.) a significant fraction of centroids cannot be obtained from every frame, therefore $N_l \leq N$. Here and below, indices $i$, $j$, and $l$ refer to the target, reference, and star of any type, respectively, and $m = 0$ is assigned to the reference frame. Upper indices $x$, $y$, if used, refer to the coordinate axes. A list of variables most frequently used in the Paper along with a short description is given in Table 1.
The differential position $X_{im}$ of the $i$th target star in the $m$th frame is obtained from CCD coordinate differences $x_{im} - x_{jm}$ of the target and field star centroids. More specifically, it is given as the weighted average

$$X_{im} = \sum_{j=1}^{M} a_{ij}(x_{im} - x_{jm}) = x_{im} - \sum_{j=1}^{M} a_{ij}x_{jm}, \quad m = 1, 2, \ldots, M (3)$$

of these differences, where the weights $a_{ij}$ satisfy the normalization condition

$$\sum_{j=1}^{M} a_{ij} = 1 \quad (4)$$

and are chosen, so that the effect of image motion effect in $X_{im}$ is minimized. The prime indicates that the sum is carried out over the subset $Q_{im}$ of stars used as reference for $i$th star only, where the index $j = 1$ is omitted. The corresponding positional $y$-coordinate $Y_{im}$ is obtained in analogy to Eq. (3).

Atmospheric image motion displaces each position $x_{im}$ by a small angle $M_{im}$, which results in $X_{im}$ being displaced by $\Delta M_{im} = M_{im} - M_{jm}$. The spectral power density $G(q)$ of $\Delta M_{im}$ in the space of spatial frequencies $q$ is the product of two factors. The first factor $F'(q)$ depends on $D$, exposure time $T$, and properties of atmospheric turbulent layers generating image motion. The second factor depends only on the geometry of background star positions relative to the target $i$, and can be expanded in a series of even powers of $q$. Thus,

$$G(q) = F'(q) \sum_{i=1}^{\infty} q^{2i} F_{2i}(\tilde{x}, \tilde{y}; \tilde{x}_1, \tilde{y}_1, \ldots, \tilde{x}_N, \tilde{y}_N), \quad (5)$$

where $\tilde{x}, \tilde{y}$ are positions not affected by image motion (at zero turbulence). The coefficients $F_{2i}$ are quadratic functions of coordinate differences that decrease rapidly with $s$ and take the form

$$F_{2i} = \sum_{i,j=0}^{\infty} A_{ij}^{(2i)} \left(\sum_{j} a_{ij} (\tilde{x}_i - \tilde{x}_j)^2 (\tilde{y}_i - \tilde{y}_j)^2\right)^{2i} \quad (6)$$

where $A_{ij}^{(2i)}$ are the characteristic constants. With the choice of coefficients $a_{ij}$, all sums in brackets can be turned to zero for a specific $A_{ij}^{(2i)}$. In this way, the leading term $F_2$ and, possibly, several subsequent $F_{2i}$ terms are eliminated, so that $G(k)$ will depend largely on the first non-zero component $F_k$ of some optimal mode $k = 4, 6, \ldots$. For this purpose, the coefficients $a_{ij}$ should satisfy each of $k(k + 2)/8 - 1$ conditions

$$\sum_{j} a_{ij} (x_{0j} - x_{0i})^2 (y_{0j} - y_{0i})^2 = 0, \quad \alpha + \beta = 1, \ldots, k - 1, \quad (7)$$

where $k \geq 4, \alpha$ and $\beta$ are non-negative integers, and positions $\tilde{x}, \tilde{y}$ are substituted by respective measured positions of stars in the reference frame. The coefficients $a_{ij}$ are determined for each target $i$ by solving Eqs. (7) with normalizing condition (4), provided that at least $N' = k(k + 2)/8$ reference stars are available. Because usually $N_m \gg N'$, the system of Eqs. (7) is redundant and solvable with some useful restrictions on $a_{ij}$, for example Eq. (15). Due to the elimination of principal modes of the image motion spectrum, the resulting values of $X_{im}$ are characterized by the residual image motion $\Delta M_{im}$ that involves only uncompensated spectral modes of orders $s \geq k/2$ and therefore is much smaller than its initial value. Approximately, the variance of residual image motion is expressed by Eq. (1), where $R$ is to be substituted by the radius $R_i$ of local reference field for $i$th target. The advantage of using larger orders in $k$ is reflected in the coefficient $B_k$ which decreases with $k$. In particular, for $k = 12$, its value is only 20% of that measured at the VLT with $k = 4$ (Paper I).

Next, let us introduce polynomial base functions $f_{1l} = 1$, $f_{2l} = x_{0l}, f_{3l} = y_{0l}, \ldots, f_{4l} = x_{0l}^2 y_{0l}^2$, of coordinates $x_{0l}, y_{0l}$ referring to the $l$th target in the reference frame, with indices $w = 1, 2, \ldots, N'$. Equal to the sequential number of combinations of $\alpha$ and $\beta$ in Eqs. (7). Taking advantage of the equality $\sum_{j} a_{ij} (x_{0j} - x_{0i})^2 (y_{0j} - y_{0i})^2 = x_{0l}^2 y_{0l}^2 - \sum_{j} a_{ij} x_{0l}^2 y_{0l}^2$, we can rewrite Eqs. (4) and (7) in terms of functions $f_{w}$ as

$$\sum_{j} a_{ij} f_{ij} = 1, \quad \sum_{i,j} a_{ij} f_{ij} = f_{ww}, \quad w = 2, 3, \ldots, N'. \quad (8)$$

3.2. Conversion from X, Y to displacements $\lambda^x, \lambda^y$

The image displacement $X_{im}$ of the $i$th star between frame $m$ and the reference frame, filtered from low-order image motion and systematic distortions of the field, is measured relative to a set $Q_{im}$ of background stars whose positions are assumed to be fixed. However, since such data are not available, we bypass this limitation by the introduction of astrometric displacements $\lambda$ that are related to all stars in the frame without distinction between reference and target objects. We consider $X_{im}$ and $\lambda$ related by

$$x_{im} = x_{0m} + F_{pim} + \Delta m, \quad (9)$$

associating the measured position of stars in frame $m$ with their position in the reference frame. Here, $F_{pim} = \sum_{k} c_{kim} f_{kk}$ is a polynomial function that contains powers of $x, y$ not higher than

Table 1. A list of frequently used variables.

| Quantity | Definition | First use, equation |
|----------|------------|---------------------|
| $x, y$ | measured star positions | (3) |
| $X, Y$ | positions $x, y$ filtered of image motion and given in the system of local reference stars with “fixed” positions | (3) |
| $\lambda^x, \lambda^y$ | same as $X, Y$ but given in a single all-frame system with “floating” reference stars | (9) |
| $\xi^x, \xi^y$ | displacement (average for a given frame series) of the star from its position in reference frame, for astrometric reasons (proper motion etc.) | (14) |
| $\hat{\xi}^x, \hat{\xi}^y$ | same as $\xi^x, \xi^y$ but with respect to the mathematical expectation of the star position in reference frame | (10) |
| $\rho$ | chromatic coefficient; atmospheric displacement of the star image is proportional to $\rho$ | (12) |
| $d$ | chromatic coefficient; describes compensating action of the LADC | (12) |
| $h$ | differential chromatic displacement (DCR effect) of the star image | (12) |
| $M$ | atmospheric image motion effect in $x, y$ | (10) |
| $\Delta M$ | residual atmospheric image motion in $X, Y$ | (14) |
| $e$ | error of photocenter measurement | (10) |
| $\sigma^2$ | observed variance of $N$ found from residuals $u$ | (26) |
&M−M=−100 and µ(Nx+xc+d)=N of reference star photocentre determination at the midpoint of exposure of the nth frame. ζm is the parallactic angle, and ζc is the zenith angle of the LADC setting, which remains the same irrespective of solutions for other frames. The matrix of the system (13) is orthogonal to Pim and satisfies conditions
\[ \sum_i \lambda_{im} f_{iwi} = 0, \quad \sum_i \lambda_{i1} f_{iwi} = 0, \quad w = 1, \ldots, N'. \]

In fact, Eqs. (9) and (11) give an expansion of xim into power series of fwi, with λim representing the remainders. λim includes the “science” astrometric signal \( \hat{\xi}_i \) (sum of proper motion, parallaxis, probable microloading, and reflex motion caused by the planetary companion) measured relative to the reference frame, random errors of photocenter measurement eim, the atmospheric chromatic displacement \( h_{im} \) corrected by LADC, and the atmospheric image motion \( M_{im} \). These quantities (or their combinations) are also orthogonal to fwi, with their low-order w ≤ N' expansion terms included in Pim via coefficients cwm.

The components of \( h_{im} \) along the x- (opposite to right ascension) and y- (along declination) axes are given by (Paper I)
\[ h_{im}^x = \rho_i \tan \zeta_m \sin \gamma_m + d_i \tan \zeta_m \sin \gamma_m \quad \text{for} \quad x = \text{axis}. \]
\[ h_{im}^y = -\rho_i \tan \zeta_m \cos \gamma_m - d_i \tan \zeta_m \cos \gamma_m \quad \text{for} \quad y = \text{axis}. \]

Here, \( \rho_i \) is the coefficient of differential chromatic displacement for the \( i \)th star. \( d_i \) is a negative displacement produced by LADC to compensate for DCR effect, approximately of the same absolute size but opposite sign. \( \zeta_m \) is a zenith distance at the midpoint of exposure of the \( m \)th frame, \( \gamma_m \) is the parallactic angle, and \( \zeta_c \) is the zenith angle of the LADC setting, which remains the same for a single series of observations.

Substitution of Eqs. (9) for \( x_m \) and \( y_m \) into Eq. (3) leads to the elimination of the polynomials Pim, while fulfilling the conditions given by Eq. (8). Taking into account that \( X_{i0} = \sum_j a_{ij}(x_{i0} - x_{j0}) = 0 \) due to Eqs. (7), we obtain
\[ \lambda_{im}^x - \sum_j a_{ij}\lambda_{jm}^x = X_{im}. \]

An analogous expression for transformation of \( Y_{im} \) to \( \lambda_{im}^y \) results for measurements along the y-axis. Equations (13), for each frame \( m \), form a system of \( i = 1 \ldots N_m \) linear equations with \( N_m \) unknowns \( \lambda_{im}^x \). The solution for a given frame \( m \) is found irrespective of solutions for other frames. The matrix of the system (13), however, turns out to be degenerated due to \( N^2 \) linear dependences (7) between its columns (coefficients \( a_{ij} \)), and consequently it can be solved only using the same number of auxiliary restrictions, for example (11).

Below, we discuss two approaches to the extraction of \( \hat{\xi}_i^x, \hat{\xi}_i^y, \rho_i, \) and \( d_i \) from \( \lambda_{im}^x \).

### 3.3. A case of fixed local reference star positions

In the case of small background star displacements \( \lambda_{im} \), and due to the averaging effect at summation, a term \( \sum_j a_{ij}\lambda_{jm} \) in Eq. (13) can be discarded as negligibly small. In fact, this assumes a fixed position of local reference stars \( \lambda_{im} = 0 \) that leads to the most simple result \( \lambda_{im}^x = X_{im} \). For each target \( i \), we are facing a system of \( 2M \) equations
\[ \begin{align*}
\hat{\xi}_i^x + h_{im}^x - h_{i0}^x &= X_{im} - e_{im}^x - \Delta M_{im}^x \\
\hat{\xi}_i^y + h_{im}^y - h_{i0}^y &= Y_{im} - e_{im}^y - \Delta M_{im}^y \\
&\quad \text{for } m = 1, 2, \ldots, M, \end{align*} \]

for determining \( \hat{\xi}_i^x = \hat{\xi}_i^x - e_{i0}^x - \Delta M_{i0}^x, \hat{\xi}_i^y = \hat{\xi}_i^y - e_{i0}^y - \Delta M_{i0}^y, \rho_i, \) and \( d_i \). This system can be solved by the least-squares method for each target object independently, with reference to its own subset \( \Omega_m \) of reference stars, considering \( e_{im} \) and \( \Delta M_{im} \), as random (with respect to index \( m \)) errors with variances \( \sigma_{eim}^2 \) and \( \sigma_{\Delta M_{im}}^2 \), respectively. Errors \( e_{im} \) of reference star photocentre determinations add a further uncertainty to the phase of computation of \( X_{im} \) by means of Eq. (3). A variance of this error \( \sigma_{\Delta R_{im}}^2 = \sum_j a_{ij}^2 \sigma_{\Delta R_{jm}}^2 \) is equal to the variance of a term \( \sum_j a_{ij} X_{jm} \) in Eq. (3), can be reduced applying restrictions
\[ \sum_j a_{ij}^2 \sigma_{\Delta R_{jm}}^2 = \text{min} \]

on solutions \( a_{ij} \) of the system (7).

As it was mentioned above, geometric field distortions (their variations in time), up to order \( k \), are excluded along with image motion.

This simple method was applied for the test reduction of four-hour series of frames obtained at VLT/FORS2 during one night (Paper I). By filtration of atmospheric image motion carried out with \( k \) up to 12, 300 mas a precision of single a position for bright stars was achieved, which is near the accuracy of photocentre measurements. Approximation \( \lambda_{im} = 0 \) was valid due to a small residual DCR effect for frames obtained with the \( R \) special filter.

### 3.4. Reduction for floating reference stars

For the processing of sky images in the \( B \)-filter, the use of the technique described is however limited, even for frames taken within the same night. In this case, an excess of positional errors over \( e_i \) by a factor 3–5 results, because the underlying assumption of \( \lambda_{im} = 0 \) does not hold due to large (~2–10 mas/h) chromatic displacement of images. For frames taken two years after the reference frame, which results in significant additional proper motion displacements, the solution of Eqs. (14) produces unacceptably large residuals (Fig. 1a).

The exact solution of Eqs. (11)–(13) ensures a linearity and uniqueness of the transformation from \( X_{im} \) to \( \lambda_{im} \) even for moving \( (\lambda_{im} \neq 0) \) reference stars. Therefore, although \( X_{im} \) are determined relative to local reference fields \( \Omega_m \) centered at the \( i \)th target and depend on the size \( R_i \) of these fields, the final solutions \( \lambda_{im} \) are consistent within the global reference field of size \( R \).
and do not depend on the choice of initial $R_i$ values. Since usually $R > R_i$, the transformation from $X_{im}$ to $\lambda_{im}$ increases the atmospheric image motion component in $\lambda_{im}$, which should be compensated for by moving to higher $k$.

Note that due to conditions (8), both $\lambda_{im}$ and $\lambda_{im} + \varphi_{im}$ are the solutions of a system (13) if $\varphi_{im}$ are functions

$$\varphi_{im} = \sum_{w=1}^{N} E_{aw,wm}f_{wi}, \quad \varphi_{im}^y = \sum_{w=1}^{N} E_{aw,wm}^yf_{wi}$$

with arbitrary coefficients $E_{aw,wm}$ and $E_{aw,wm}^y$. Therefore, for splitting $\lambda_{im}$ into components we can use

$$\xi_{im} + h_{im} - h_{i0} + \varphi_{im} = \lambda_{im}^x + N_{im}^x$$

$$\xi_{im}^y + h_{im} - h_{i0} + \varphi_{im}^y = \lambda_{im}^y + N_{im}^y$$

$$m = 1, 2 \ldots M, \quad i = 1, 2 \ldots N$$

where $N_{im} = -\epsilon_{im} - \Delta M_{im}$ is a noise with variance $\epsilon_{im}^2 + \sigma_{at}^2$. Equations (17) can then be solved as a system of $2\sum N_{im}$ conditional equations with $4N + 2MN'$ unknowns $\xi_{im}, \xi_{im}^y, \rho_i, d_i, E_{aw,wm}$, and $E_{aw,wm}^y$.

Because for most not very faint targets, $\sigma_{at} \ll \epsilon_{im}$, the least-squares solution is found with weights $g_{im} = \epsilon_{im}^{-2}$. Also, due to the dominant contribution of bright stars, we can safely assume $g_{im}^w \approx 1/\epsilon_{im}^2$ which follows from the approximation given by Eq. (2) for these stars.

Linear relations between some parameters of a system (17) do not permit us to separate them completely. For example, systematic variations of $\xi_{im}, \rho_i$, and $d_i$ quantities over a field imitate the behaviour of $\varphi_{im}$ functions. Associations of this type are eliminated by applying $N'$ orthogonalizing restrictions

$$\sum_{r \in a} (\xi_{ir} + h_{ir} - h_{i0})f_{ui} = 0, \quad \sum_{r \in a} \rho_if_{ui} = 0, \quad \sum_{r \in a} d_if_{ui} = 0, \quad w = 1, 2 \ldots N'.$$

The restrictions (18) permit us to recover only those components of $\rho_i, d_i, \xi_{im},$ and $\xi_{im}^y$ that do not correlate with functions $f_{ui}$.

An essential drawback of a straightforward solution of Eqs. (16–18) is a large number of unknowns $E_{aw,wm}$ and $E_{aw,wm}^y$ that cause a certain instability in the solution. For example, the number of unknowns for Dec. 2000 frames is $2MN' = 2 \cdot 40 \cdot 36 \approx 2900$ at $k = 16$, which is comparable to the number of measurements $2\sum N_{im} \approx 9100$.

However, there is a simpler solution. Consider the equation with a diagonal element $E_{aw,wm}$ of the normal system of Eqs. (17):

$$\sum_{r \in a} (\xi_{ir} + h_{ir} - h_{i0})f_{ui} + \sum_{r \in a} \rho_if_{ui} + \sum_{r \in a} d_if_{ui} = \sum_{r \in a} E_{aw,wm}f_{ui}$$

where summation is performed only over a limited sample $\epsilon_{aw}$ of stars measured in the $m$th frame and, since FWHM=const within the frame, the approximation $g_{im}^w = I_i$ was used. Adding the sum

$$\Lambda_{aw,wm} = \sum_{w \notin \epsilon_{aw}} (\xi_{ir} + h_{ir} - h_{i0})f_{ui}$$

formed over stars $not$ measured in the frame $m$ to both sides of Eq. (19), and taking into account (18), we note that the first component in Eq. (19) turns to zero. Therefore

$$\sum_{r \in a} f_{ui}I_i = \sum_{r \in a} E_{aw,wm}f_{ui} = \sum_{r \in a} \Lambda_{aw,wm}f_{ui} + \Lambda_{aw,wm}^x.$$
2000 and 2002–2003 epochs, respectively, each one representing four nights. Astrometric reduction was carried out with different choices of the parameter $k$, where the accuracy of results became sufficiently good at $k = 16$, which was adopted for the final reduction. Our iterative procedure involved the computation of $\xi_m^i, \eta_m^i, \rho_i$, as well as $d_i$, and the refinement of $\Lambda_{im}$ quantities calculated on the basis of the former parameters. The number of outliers at each coordinate was limited to 1.25% of the total number of measured star images.

The residuals $u_{im}^x$ and $u_{im}^y$ of the conditional equations Eq. (23) on $x$- and $y$-coordinates are uncorrelated (Fig. 1b) and, besides, substantially smaller than those obtained with the approximation of ”fixed” background star positions (Fig. 1a). We also did not find any correlation between $u_{im}$ and the stellar position within the frame or the colour-dependent parameters $\rho_i$ and $d_i$. As a simple illustration for a further discussion, Fig. 2 presents examples of $u_{im}^x$ residuals for stars of different magnitudes, from bright $B = 18.5$ to faint $B = 23$ mag. A difference in the data point scattering reflects the dependence of photocenter error on the light signal. No apparent correlation in time is seen. More extensive statistical analysis of $u_{im}$ values is performed in Sect. 5.

### 4.1. Performance of the LADC

Zenith distances $\zeta_m$ of the LADC setting used in Eqs. (12) are not specified in the fits file headers. According to the technical documentation of the VLT, $\zeta_m$ is equal to the telescope zenith distance at the midpoint exposure time of the first science image of the series and is constant for all frames within the series. We however tried to use corrected values $(1 + C_m) \tan \zeta_m$ instead of $\tan \zeta_m$ in order to improve internal precision. Coefficients $C_m$ (Table 2) that yield the best-achievable precision have been determined with an accuracy of about ±0.03 relative to some zero-point night, chosen as 21 Dec. 2000. Closer examination revealed that $(1 + C_m) \tan \zeta_m$ is approximately equal to $\tan \zeta_m^i$ computed with the telescope zenith distance $\zeta_m^{10}$ mn prior to the first exposure. Writing $\zeta_m$ in the form $\tan \zeta_m = (1 + C_m) \tan \zeta_m$ allows us to introduce new values $C_m$ and compute them in analogy to $C_m$. A good consistency of $C_m$ and $C_m^i$ values suggests that actual angles $\zeta_m$ are really close to $\zeta_m$. According to Jehin E. (private communication, 2006), this may occur when the LADC is not reset after finishing preceding observations of the same target. The final reduction was carried out with $(1 + C_m) \tan \zeta_m$ rather than $\tan \zeta_m$.

An excellent performance of the LADC is demonstrated by Fig. 3 which reproduces the distribution of $\rho_i$ and $d_i$ (averaged over both epochs) with error bars based on the effective precision of a single measurement (26). Blue stars are shown in the upper left corner. The distribution of stars is strongly concentrated along a line $d_i = -1.2 \rho_i$ with a scatter that only slightly exceeds random errors.

### 4.2. Effective precision of a single measurement

Astrometric positions are affected by various sources of error. The most important are fundamental (photon noise in the image), of instrumental origin (optical field distortions, errors related to CCD, etc.), induced by the atmosphere (DCR effect, image motion), or related to inefficient reduction techniques that should take into account and exclude largest errors. Without analyzing each source separately, some of which are rather efficiently filtered out at the phase of the reduction, we consider their total impact on the key system of Eqs. (23), whose solution yields final astrometric parameters $\xi_m$ and $\eta_m$. This total error in Eqs. (23) was denoted as $N_{im}$ and includes $e_{im}, \Delta M_{im}$, and a sum of other residual errors not filtered by the reduction model. $N_{im}$ includes both random noise and components correlated in time. The presence of correlated components leads to strongly biased results if the correlation time exceeds the length of a single series, while being shorter than duration of the observational campaign. In this case, processing a short series of frames will recover noise signature rather than the desired real astrometric signal. In next section we describe results of a statistical analysis of $N_{im}$ based on measured residuals $u_{im}$.
and compared it with the \( \sigma_H \) of these residuals expected when the data fulfill x−frames with the increase of the number \( W_b \) and of the noise formally arising from the above analysis.

The second component in parentheses is significant for faint images, we modified Eq. (2) to

\[ \sigma_{im}^2 = e_{im}^2 + \sigma_{im}^2, \]

where an average was taken over stars of approximately the same magnitude. Further averaging of \( \sigma_{im}^2 \) with respect to \( i \) and \( m \) (over observations) yielded the effective variance of a single measurement \( \bar{\sigma}^2 \) at average observing conditions during the considered epoch. Figure 4 presents both \( \bar{\sigma}^2 \) and \( \bar{\sigma}^2 \), or FWHM value, is available as dots) indicate no evidence of large systematic error, and extend the asymptotic dependence on the star brightness, we normalized individual \( \sigma_{im}^2 \) by \( e^2 \) (upper plot of Fig. 5) and then performed averaging of results over all 169 stars. Obtained smooth functions \( \langle \sigma_{Al}^2 / e^2 \rangle \) (black dots) indicate no evidence of large systematic error, and extend the asymptotic floor set by systematic errors. The latter property is very important since it forms the basis for statistical substantial improvements of the astrometric accuracy \( \Delta(n_i) \) for a sequence of \( n_i \) frames with the increase of the number of measurements as

\[ \Delta(n_i) = \frac{\epsilon}{\sqrt{n_i}}; \quad n_i \leq n_{\text{max}} \quad \text{(27)} \]

where \( \epsilon \) is the photocenter error at average observation condition. The estimate (27) is approximate since it assumes the best situation with a diagonal covariance matrix of the system (23). The validity of Eq. (27) is limited with respect to the maximum amount of frames \( n_{\text{max}} \), at which the value of \( \Delta(n_i) \) reaches a floor set by systematic errors.

5. Testing the VLT temporal astrometric stability

5.1. The Allan variance

The simplest empiric validation of Eq. (27) and illustration of how this law works at different \( n_i \) is associated with the use of Allan variance \( \sigma_{Al}^2 \) of the residuals \( u_{im} \). This quantity (Pravdo & Shaklan 1996) is a function of time lag \( \tau \) and \( n_i/2 \) expressed in frames

\[ \sigma_{Al}^2(\tau) = \frac{1}{2(n_i + 1 - 2\tau)} \sum_{p=1}^{n_i+1-2\tau} \sum_{q=0}^{\tau-1} (u_{p+q} - u_{p+q+\tau})^2 \]  

\[ \sigma_{Al}^2(\tau) \] is a precision of the average of \( u_{im} \) taken over \( \tau \) frames, and so corresponds to \( \Delta^2(\tau) \). For uncorrelated sets of \( u_{im} \), the expected dependence \( \sigma_{Al}^2 \sim \tau^{-1} \) is consistent with Eq. (27). In an effort to increase the range of time lags, we performed computations of \( \sigma_{Al}^2 \), treating all epoch frames as a single series of \( n_i \) length. For miscellaneous reasons, \( n_i \) varied from about 0.4M to 1.0M. Results for stars of different B magnitude classes are shown in Fig. 5 (two bottom graphs). Though, in general, plots for separate stars follow the expected law \( \tau^{-1} \) (straight lines with an ordinate \( \bar{\epsilon} \) at \( \tau = 1 \)), it is difficult to make any conclusion at \( \tau \geq 10 \). To smooth statistical fluctuations and exclude the dependence on the star brightness, we normalized individual \( \sigma_{Al}^2 \) by \( e^2 \).
frames on hour angles, and adding a Gaussian noise with a unit variance in $x$ and $y$ to the right side of Eqs. (23). Observed distributions $W$ were computed using all combinations of data sets available (2000 and 2003/2003 epoch frames, $x$ and $y$ data, with faint, bright, or a whole sample of stars). Figures 6a,b present an example of such a histogram $W$ for bright stars. Corresponding simulated distributions $W_0$ are shown by smooth curves, for which the same binning as for the computation of $W$ has been applied. Unlike $N_{im}$, the distributions $W_0$ and $W$ are not Gaussians and have a more strongly peaked shape. The consistency of observed and model distributions by visual inspection is rather good. Numerically, it was estimated by $\chi^2$ values computed from differences between $W$ and $W_0$ in fixed binned intervals containing at least 5–10 data points. The $\chi^2$ values with corresponding degrees of freedom (DoF) are listed in Table 3 for faint, bright, or all stars, respectively. The $\chi^2$-test based on these data does not reveal a deviation of $W$ from $W_0$ at significance levels below 20%, which does not allow us to reject our hypothesis $H_0$. Due to a limited number of data points, this test is however only sensitive to a central $\pm 2\sigma$ region of the distribution.

We applied the more powerful Kolmogorov-Smirnov test to detect a potential difference between the observed and simulated distributions. For this purpose, we computed cumulative frequency distributions of observed and model normalized residuals $u'_{im}$, and then obtained the difference $D_n$ between two distributions. A case with the largest deviations for 2000 epoch measurements in $y$ with 1721 data points (bright stars only) is shown in Fig. 7 where the differences $D_n$ are plotted as a function of the normalized deviation. The Kolmogorov-Smirnov test based on this plot (in this worst case $\max|D_n| = 0.024$) shows no significant difference between the model and measured distributions at 20% confidence level. Both tests applied thus do not reject hypothesis $H_0$ of a really normal distribution of $N_{im}$ at approximately the same significance levels.

For another test, we considered nightly averages $\bar{u}_{ip} = \sum_{ip} u_{ip}\sigma_{ip}^{-2}/\sum_{ip} \sigma_{ip}^{-2}$, involving all observations of the $ith$ star within a single $ith$ night. Figures 6c,d show observed and model frequency distribution of normalized residuals $\bar{u}_{ip}/\sigma_p$, where $\sigma_p$ is the error of the determination of $\bar{u}_{ip}$. A good agreement of the histograms of the observed statistics (both for bright and faint stars) with their expectations from model simulations, a symmetry, and the absence of large deviations (normally less than 1% of $\bar{u}_{ip}/\sigma_p$ values deviate at about 3 units) demonstrate excellent stability of the VLT instrumental system during time spans of a few days.
Fig. 7. The differences $D_n$ between the observed and model cumulative frequency distributions of normalized residuals $\mu'_{im}$ for 2000 epoch measurements in $y$ (bright stars).

Table 4. Integral statistics $S_0$ and $S_A$ for bright $B < 20.5$, faint $B > 20.5$ stars, and for simulated observations.

|          | $S_0^2$ | $S_A^2$ | error  | $S_0^2$ | $S_A^2$ | error  |
|----------|---------|---------|--------|---------|---------|--------|
| model    | 0.997   | 0.985   | 0.021  | 0.989   | 0.984   | 0.0064 |
| bright   | 0.992   | 0.989   | ±0.021 | 0.984   | 0.989   | ±0.0064|
| faint    | 0.982   | 0.988   | ±0.019 | 0.761   | 0.787   | ±0.054 |

nights. Compatiblity of the observed and model distributions is also confirms by the Kolmogorov-Smirnov- and $\chi^2$-tests.

5.3. Integral statistics

We also considered integral statistics

$$S_0^2 = \left(\frac{1}{n} \sum_{i=1}^{n} \frac{(\mu_{im} - \mu_{im0})^2}{\sigma_{im0}^2}\right),$$

where averages are taken over all stars and nights, and

$$S_A^2 = \left(\frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j=1}^{m} (\mu_{ij} - \mu_{ij0})^2}{\sigma_{ij0}^2}\right),$$

where an average is taken over all stars. These two statistics are either sensitive to the variation of systematic errors within a given night ($S_0$) or from one night to another ($S_A$), and therefore are a powerful indicator of their presence. The quantities $S_0$ and $S_A$ calculated separately for both coordinates $x$ and $y$, are listed in Table 4 for each epoch of observations together with the respective expected values $S_0$ (mod) and $S_A$ (mod), obtained from numerical simulations where random Gaussian noise $N_{im}$ has been assumed. Errors due to a limited number of measurements indicate an 80% confidence interval for possible deviations of observed values from our model assumption. Observed statistics match well the model within the error limits.

The fact that we do not find an excess in the measured $S_0$ and $S_A$ values means that no or very small extra noise except $N_{im}$ can be present in measurements. It is useful to estimate its upper limit. Suppose that the observed positions, besides of the noise $N_{im}$, are affected by some small systematic error that is constant within a night but varies between nights with an amplitude peculiar to a certain star. The observed value of $S_0^2$ (obs) then exceeds its model expectation $S_0^2$ (mod) by a small amount $S_0^2$, which is the average signal variance for a given star sample. These quantities are related by

$S_0^2 = \langle \chi^2_{A,df}/d.o.f.\rangle S_A^2$ (mod) + $S_{sys}^2$. For statistical reasons, it is probable that the sampled value $S_A^2$ (obs) is equal to or below its mathematical expectation $S_A^2$ (mod) + $S_{sys}^2$.

The largest value of $S_A^2$, according to Table 4, can exist in the $x$-axis measurements of bright stars (2000 epoch), for which we found $S_A^2 (mod) \leq 0.19$ at 20% confidence level. This is a ratio of a systematic to random error component for a single night series represented, on average, by 8 frames. In other words, the bias related to systematic errors is equal to a random component expected in positions averaged over 8/0.19 = 40 frames. This estimate of $n_{max}$ introduced in Eq. (27), is more precise, than that obtained in Sect. 5.1. For $y$-coordinate, we obtain $n_{max} = 65$. Frames of 2002 yield $n_{max} = 120$ and 50 for the $x$ and $y$ axes correspondingly. With $n_{max} = 50$ adopted as a reliable limit, precision of a night series is about 30 $\mu$as.

5.4. Correlations

A final important test was carried out to investigate correlations in $N_{im}$ which can negatively affect the averaging law (27). For that purpose, we computed the autocorrelation function of normalized residuals

$$r(\tau) = H(\tau)(\mu_{im}/\sigma_{im})(\mu_{im}/\sigma_{im0})$$

of the argument $\tau = m - m_1$, which is equal to the difference of frame indexes and used as a time lag. Here, $H(\tau) = 1 - \tau/M$ is the weight function that compensates the increase of statistical variations at large $\tau$. Averaging is performed over the images of all stars. A corresponding computation of the expected correlation function $\hat{r}(\tau)$ was carried out with the model observations containing white noise with a unit variance. The observed $r(\tau)$ values computed for both coordinates $x$ and $y$ for bright $B < 20.5$ mag stars are shown in Fig. 8 along with the model function $\hat{r}(\tau)$ (mean for $x$ and $y$) and 1-$\sigma$ limits for statistical scatter of individual deviations. Oscillations in $\hat{r}(\tau)$ plot inherit a compound four-night structure of the series. Observed data in general follow the expected dependence, though with few isolated deviations over 2 sigma.

Neither $r$ nor $\hat{r}$ is identically zero as could be expected for uncorrelated measurements. The observed systematic negative bias of correlations, however, is not due to instrumental errors but has a theoretical background (Lazorenko 1997). In order to see this, consider Eqs. (23) for the $i$th star. When the number of frames $M$ is large and the distribution of frames with hour angle is sufficiently random, all parameters in the system given by Eq. (23) become uncorrelated. Then $\xi_i^2$ is approximately equal to the average of $(\lambda_{im} - \bar{h}_{im} + h_{im0} + N_{im})$ taken
with respect to the index $m$. In this approximation, one finds $u_i^m = (\lambda_{1i}^m - h_{1i}^m + h_{0i}^m + N_{im}) - \xi_i^m$. The term given in parentheses formally represents a solid time series of the “time” argument $m$ with $m = 1 \ldots M$ measurements containing noise. With this approach, the $u_i^m$ values are the remainders of a time series after subtraction of the fitting polynomial (namely a constant $\xi_i^m$ in our case). The bias of the autocorrelation function shape due to subtraction of the best-fitting polynomial for measurements limited in time was also previously studied by Lazorenko (1997). The autocorrelation function $r'$ of series remainders takes a particularly simple form if the power spectrum of measurement errors resembles a white noise (which is the case) with a unit variance and constant rate of data sampling. In this case, one obtains (Bakhonski et al. 1997)

$$r'(\tau) = -\frac{2\pi}{M} \sin(2\pi\tau/M)$$

(30)

where $\xi = 0.56$, if the subtracted polynomial is presented by a constant, and $\sin(z) = \sin(z)/z$. One can see that $r' = 0$ for large $M$ only (long series) while for small $M$ (limited number of sampled data) its value is absolutely large and negative. Due to simplification of our considerations, the measured and model functions are not exactly fitted by $r'$, are but good enough however to explain the statistical origin of the observed negative bias (Fig. 8). We conclude that no correlation is present in $N_{im}$ at scales of several frames.

Correlation at time scales of a few days were studied using normalized nightly-average residuals $\bar{u}_{ip}/\sigma_p$. We computed correlations

$$\bar{r} = \langle (\bar{u}_{ip}/\sigma_p)(\bar{u}_{ip}/\sigma_p) \rangle$$

(31)

as a function of the time lag $\tau = p_2 - p_1$ between two nights with indexes $p_1$ and $p_2$, expressed in days. The values of $\bar{r}$ for the observed data and corresponding model value $r_{mod}$ are given in Table 5. Apparently, there is a strong negative bias in $\bar{r}$ due to a very limited number of data points in our 4-night series. This bias, however, is well-modelled using uncorrelated noise $N_{im}$, which confirms previous conclusions that there are no traces of instrumental signature in VLT observations at time scales of a few nights, and supports the validity of the averaging law (27).

### Table 5. Observed $\bar{r}$ and model $r_{mod}$ normalized correlations.

| $\tau$          | $\bar{r}$ | $r_{mod}$ | $\bar{r}$ | $r_{mod}$ | 1-sigma error |
|----------------|-----------|-----------|-----------|-----------|---------------|
|                | 2000 epoch |           | 2002–2003 epoch |           |
| 1               | -0.17     | -0.18     | -0.22     | -0.24     | ±0.05         |
| 2               | -0.17     | -0.10     | -0.20     | -0.24     | ±0.07         |
| 3               | -0.42     | -0.38     | -0.25     | -0.23     | ±0.09         |

Fig. 9. Difference in the chromatic parameters $\rho_i$ and $d_i$ between the two epochs, normalized by the error of this difference computation (dots). Ellipses are drawn at 1% (dashed line) and 5% (solid) significance level. A straight line shows the direction of point displacements due to the actual change of star colours.

6. Conclusion

Our results reveal an exceptionally good astrometric performance of the VLT and its camera FORS1. Thus, the precision $\bar{\epsilon}$ of the position of a bright star reaches 200 $\mu$as for a single measurement, which fairly well corresponds to the former 300 $\mu$as estimate obtained for FORS2 camera (Paper I). Both cameras thus give similarly good results. The term “bright star” refers to unsaturated images containing 1–3 electrons and may correspond to diurnal changes in stellar colour. Some other large deviations appear to be related to stars in outer regions of the observed field where the method seems to give biased results. Except for these outliers, the distribution of points in Fig. 9 resembles a Gaussian, which is an indicator of a rather good astrometric stability of the VLT even at very long time scales. It should be stressed that the above test is insensitive to non-chromatic types of errors and therefore is only indicative.

5.5. Two-year stability

Given only two epoch measurements, no conclusions on the long-term astrometric stability of the VLT can be made based on positional information. Considering the extreme importance of this problem, we present here an indirect analysis of this issue based not on positions, but on a comparison of chromatic $\rho_i$ and $d_i$ parameters computed from independent runs of 2000 and 2002/2003 epochs. If one assumes constant colours for the observed stars, these parameters should coincide within error bars. A change of the VLT characteristics, depending on stellar colours, should result in an additional difference. Figure 9 shows normalized differences between $\rho_i$ and $d_i$ between the two epochs, where ellipses refer to significance levels of 1% or 5%, respectively. Some points represent outliers, testifying to the presence of systematic errors. The largest residuals for bright stars in the right bottom however occur along a line $d_i = -1.2\rho_i$, that is peculiar to the distribution of chromatic parameters shown in Fig. 3. It is therefore very likely that these deviations are caused by a change in stellar colour. Some other large deviations appear to be related to stars in outer regions of the observed field where the method seems to give biased results. Except for these outliers, the distribution of points in Fig. 9 resembles a Gaussian, which is an indicator of a rather good astrometric stability of the VLT even at very long time scales. It should be stressed that the above test is insensitive to non-chromatic types of errors and therefore is only indicative.

6. Conclusion

Our results reveal an exceptionally good astrometric performance of the VLT and its camera FORS1. Thus, the precision $\bar{\epsilon}$ of the position of a bright star reaches 200 $\mu$as for a single measurement, which fairly well corresponds to the former 300 $\mu$as estimate obtained for FORS2 camera (Paper I). Both cameras thus give similarly good results. The term “bright star” refers to unsaturated images containing 1–3 $\times 10^6$ electrons and may correspond to different stellar magnitudes depending on the exposure, filter, and seeing. Most importantly, we found negligibly small systematic errors of instrumental and other origin. In fact, no traces of these errors were found at time intervals up to 4 days. Due to this fact, the precision for a frame series improves as $\bar{\epsilon}/\sqrt{n_w}$, and at $n_w = 50$ reaches 30 $\mu$as for FORS1 and 40 $\mu$as for FORS2. With use of FORS2, the observation time needed to obtain this accuracy is about 1 hour if short 15 s exposures and a $2 \times 2$ binning of pixel reading are used. We conclude that the VLT with cameras FORS1/2, due to its enormous collecting...
light power, fine optical performance, and effective averaging of wave-front distortions over a large aperture, is a powerful instrument that can be used efficiently for high-precision astrometric observations of short-term events, in particular, of planetary microlensing.

Acknowledgements. We would like to thank Dr. E.Jehin for his helpful comments on the details of LADC operation.

References

Avila, G., Rupprecht, G., & Beckers, J. M. 1997, Proc. SPIE, 2871, 1135
Bakhonski, A. V., Kostjutshenko, V. L., & Lazorenko, P. F. 1997, Kinematics and Phys. Celest. Bodies, 13, 5, 50
Beaulieu, J.-P., Bennett, D. P., Fouque, P., et al. 2006, Nature, 439, 437
Boden, A. F., Shao, M., & Van Buren, D. 1998, ApJ, 502, 538
Bond, J. A., Udalski, A., Jaroszynski, M., et al. 2004, ApJ, 606, L155
Colavita, M. M., Boden, A. F., & Crawford, S. L. 1998, Proc. SPIE, 3350, 776
Dominik, M. 1999, A&A, 349, 108
Dominik, M., & Sahu, K. C. 2000, ApJ, 534, 231
Irwin, M. J. 1985, MNRAS, 214, 575
Jaroszynski, M., & Paczyński, B. 2002, Acta Astron., 52, 361
Gaudi, B. S. 1998, ApJ, 506, 533
Gaudi, B. S., & Gould, A. 1997, ApJ, 486, 85
Gaudi, B. S., & Han, C. 2004, ApJ, 611, 528
Gould, A., Udalski, A., Bennett, D., et al. 2006, ApJ, 644, L37
Han, C., & Chunghuk, L. 2002, MNRAS, 329, 163
Hardy, S. J., & Walker, M. A. 1995, MNRAS, 276, L79
Heg, E., Novikov, I. D., & Polnarev, A. G. 1995, A&A, 294, 287
Lanza, B. F., & Muterspagh, M. W. 2004, ApJ, 601, 1129
Lazorenko, P. F. 1997, Kinematics Phys. Celest. Bodies, 13, 2, 63
Lazorenko, P. F. 2006, A&A, 449, 1271
Lazorenko, P. F., & Lazorenko, G. A. 2004, A&A, 427, 1127
Lindegren, L. 1980, A&A, 89, 41
Miyamoto, M., & Yoshii, Y. 1995, AJ, 110, 1427
Monet, D. G., Dahn, C. C., Vrba, F. J., et al. 1992, AJ, 103, 638
Motch, C., Zavlin, V. E., & Haberl, F. 2003, A&A, 408, 323
Paczyński, B. 1998, ApJ, 494, L23
Pravdo, S., & Shaklan, S. 1996, AJ, 465, 264
Pravdo, S. H., Shaklan, S. B., Lloyd, J., et al. 2005, ASP Conf. Ser., 338, 288
Pravdo, S. H., Shaklan, S. B., Wiktarowicz, S. J., et al. 2006, ApJ, 649, 389
Quirrenbach, A., Henning, T., & Queloz, D. 2004, Proc. SPIE, 5491, 424
Salzadeh, N., Dalal, N., & Griest, K. 1999, ApJ, 522, 512
Udalski, A., Jaroszynski, M., Paczyński, B., et al. 2005, ApJ, 628, L109
Wittkowski, M., Ballester, P., Canavan, T., et al. 2004, Proc. SPIE, 5491, 617