Chiral symmetry restoration in (2+1)-dimensional QED with a
Maxwell-Chern-Simons term at finite temperature

Raoul Dillenschneider and Jean Richert
Laboratoire de Physique Théorique, UMR 7085 CNRS/ULP,
Université Louis Pasteur, 67084 Strasbourg Cedex, France
(Dated: August 10, 2018)

We study the role played by a Chern-Simons contribution to the action in the QED$_3$ formulation of a two-dimensional Heisenberg model of quantum spin systems with a strictly fixed site occupation at finite temperature. We show how this contribution affects the screening of the potential which acts between spinons and contributes to the restoration of chiral symmetry in the spinon sector. The constant which characterizes the Chern-Simons term can be related to the critical temperature $T_c$ above which the dynamical mass goes to zero.

PACS numbers: 75.10.Jm,11.10.Kk,11.10.Wx,11.30.Rd

I. INTRODUCTION
Quantum Electrodynamics QED$_{(2+1)}$ is a common framework aimed to describe strongly correlated systems such as quantum spin systems in 2 space and 1 time dimension, as well as related specific phenomena like high-$T_c$ superconductivity. Indeed, a gauge field formulation of the antiferromagnetic Heisenberg model in $d = 2$ dimensions leads to a QED$_3$ action for spinons, see f. i. Ghaemi and Senthil, Morinari and also.

Here we concentrate on the behaviour of the spinon mass at finite temperature which is dynamically generated by a U(1) gauge field when the action contains a Chern-Simons term $\mu N$ which introduces the imaginary chemical potential $\mu = i \pi / 2 \beta$ at temperature $\beta^{-1}$ adding the term $\mu N$ to the expression given by (1):

$$H = -\frac{1}{2} \sum_{i,j} J_{ij} \bar{S}_i \bar{S}_j - \mu N$$

where $N = \sum_{i,\sigma} f^\dagger_i \sigma f_{i,\sigma}$ counts the number of fermions in the spin system.

In 2$d$ space the Heisenberg Hamiltonian given by Eq. (1) can be written in terms of composite non-local operators $\{ \mathcal{D}_{ij} \}$ ("diffusons") defined as

$$\mathcal{D}_{ij} = f^\dagger_i \sigma f_{j,\bar{\sigma}} + f^\dagger_{i,\bar{\sigma}} f_{j,i}$$

If the coupling strengths are fixed as

$$J_{ij} = J \sum_{\vec{q}} \delta (\vec{q}_i - \vec{q}_j) + \vec{q}$$

where $\vec{q}$ is a lattice vector $\{ a_1, a_2 \}$ in the $Ox$ and $Oy$ directions the Hamiltonian takes the form

$$H = -J \sum_{<ij>} \frac{1}{2} \mathcal{D}_{ij} \mathcal{D}_{ij} - \frac{n_i + n_j}{4} - \mu N$$

where $i$ and $j$ are nearest neighbour sites.

The number operator products $\{ n_i, n_j \}$ in Eq. (2) are quartic in terms of creation and annihilation operators in Fock space. In principle the formal treatment of these terms requires the introduction of a Hubbard-Stratonovich (HS) transformation. One can however
show that the presence of this term has no influence on the results obtained from the partition function. Indeed both \( \{n_i\} \) and \( \{n_i, n_j\} \) lead to constant contributions under the exact site-occupation constraint and hence are of no importance for the physics described by the Hamiltonian \( \mathcal{H} \). As a consequence we leave them out from the beginning.

Using a HS transformation in order to reduce the first term in Eq. (2) from quartic to quadratic order in the fermion operators \( f^\dagger \) and \( f \) the Heisenberg Hamiltonian reads

\[
\mathcal{H} = \frac{2}{|J|} \sum_{<ij>} \Delta_{ij} \Delta_{ij} + \sum_{<ij>} \left[ \Delta_{ij} D_{ij} + \Delta_{ij} D_{ij}^\dagger \right] - \mu N
\]

where \( \{\Delta_{ij}\} \) are the HS auxiliary fields. At this point no approximation has been made and Eq. (3) is exact.

The fields \( \Delta_{ij} \) can be chosen as complex quantities \( \Delta_{ij} = |\Delta| e^{i\phi_{ij}} \). This parametrization introduces gauge fields \( \phi_{ij} \) which are defined on the square plaquette shown in figure 1 and can be decomposed into a mean field part and a fluctuation contribution \( \phi_{ij} = \phi_{ij}^{mf} + \delta \phi_{ij} \). The amplitude \( |\Delta| \) too may contain a mean-field and a fluctuating contribution. In the following we assume that at low energy the essential quantum fluctuations are generated by the gauge field \( \delta \phi_{ij} \) and neglect the amplitude fluctuations in the sequel.

The \( \phi_{ij}^{mf} \)'s are fixed on the plaquette in such a way that

\[
\phi_{ij}^{mf} = \sum_{(ij) \in \square} \phi_{ij}^{mf}
\]

where \( \phi_{ij}^{mf} \) is taken to be constant.

In order to implement the \( SU(2) \) invariance in (1) at the level of the mean-field Hamiltonian \( \mathcal{H} \) we follow \( 7,8,14,15,16 \) and introduce the configuration

\[
\phi_{ij}^{mf} = \begin{cases}
(\pi (1), \pi (1) \, \text{if } \vec{r}_j = \vec{r}_i + \vec{e}_x) \\
(-\pi (1), \pi (1) \, \text{if } \vec{r}_j = \vec{r}_i + \vec{e}_y)
\end{cases}
\]

Then the total flux through the fundamental plaquette is such that \( \phi^{mf} = \pi \) which guarantees the \( SU(2) \) symmetry.

Under these conditions the Hamiltonian \( \mathcal{H} \) goes over to the \( \pi \)-flux mean-field Hamiltonian

\[
\mathcal{H}_{MF}^{(PPF)} = \mathcal{N} z \frac{\Delta^2}{|J|} + \sum_{\vec{k} \in SBZ} \left( f_{\vec{k}, \sigma}^\dagger f_{\vec{k}+\vec{\pi}, \sigma} + f_{\vec{k}+\vec{\pi}, \sigma}^\dagger f_{\vec{k}, \sigma} \right) [\vec{H}] \left( \left[ f_{\vec{k}, \sigma}^\dagger f_{\vec{k}+\vec{\pi}, \sigma} \right] \right)
\]

with

\[
[\vec{H}] = \begin{bmatrix}
-\mu - \Delta \cos \left( \frac{\pi}{4} \right) & \gamma_{k_x, k_y} & -i \Delta \sin \left( \frac{\pi}{4} \right) & \gamma_{k_x, k_y + \pi} \\
- \Delta \sin \left( \frac{\pi}{4} \right) & \gamma_{k_x, k_y + \pi} - \mu - \Delta \cos \left( \frac{\pi}{4} \right) & \gamma_{k_x, k_y} & \\
\end{bmatrix}
\]

where the \( \gamma_{\vec{k}} \)'s are defined by \( \gamma_{\vec{k}} = \sum_{\vec{\eta}} e^{i \vec{k} \cdot \vec{\eta}} = 2 \left( \cos (k_x a_1 + \cos k_y a_2) \right) \). The eigenvalues of \( \mathcal{H}_{MF}^{(PPF)} \) read

\[
\omega^{(PPF)}_{(\pm), \vec{k}, \sigma} = -\mu + 2 \Delta \sqrt{\cos^2(k_x) + \cos^2(k_y)}.
\]

We are interested in the low energy behaviour of the quantum spin system described by \( (\pm), \vec{k}, \sigma \) in the neighbourhood of the nodal points \( (k_x = \pm \frac{\pi}{2}, k_y = \pm \frac{\pi}{2}) \) where the energy gap \( \omega_{(\pm), \vec{k}, \sigma} \) vanishes. Fig. 2 shows the contour plot of the energy spectrum \( \omega_{(-), \vec{k}, \sigma} \) and locates the nodal points. We linearize the energies in the neighbourhood of these points.

Following \( 2,3,6 \) the spin liquid Hamiltonian \( \mathcal{H} \) at low energy can be described in terms of four-component Dirac spinons in the continuum limit. The Dirac action of this

\[
\begin{align*}
\mathcal{L}_D & = \bar{\Psi} \left( \partial_\mu + i e A_\mu \right) \Psi \\
& = \bar{\Psi} \left( \gamma_0 \partial_0 + \gamma_\mu \partial_\mu \right) \Psi
\end{align*}
\]
spin liquid in (2+1) dimensions including the phase fluctuations $\delta \phi_{ij}$ around the $\pi$-flux mean field phase $\phi_{mf}$ has been derived in\[6\] and reads

$$S_E = \int_0^\beta \int d^2 \tau \left\{ -\frac{1}{2} a_\mu \left[ (\Box \delta^{\mu \nu} + (1 - \lambda) \partial^\mu \partial^\nu) \right] a_\nu + \sum_\sigma \bar{\psi}_\sigma \gamma_\mu (\partial_\mu - ig a_\mu) \psi_\sigma \right\}$$ (5)

\(\psi\) is the 4-dimensional Dirac spinon field

$$\psi_{k\sigma} = \begin{pmatrix} f_{1a,k\sigma} \\ f_{1b,k\sigma} \\ f_{2a,k\sigma} \\ f_{2b,k\sigma} \end{pmatrix}$$

where $f_{1a,k\sigma}$ and $f_{1b,k\sigma}$ ($f_{2a,k\sigma}$ and $f_{2b,k\sigma}$) are fermion creation and annihilation operators near the nodal points $\left( \frac{\pi}{2}, \frac{\pi}{2} \right)$ ($\left( -\frac{\pi}{2}, -\frac{\pi}{2} \right)$) of the momentum $\vec{k}$ and indices $a$ and $b$ characterize the rotated operators

$$f_{1a,k\sigma} = \frac{1}{\sqrt{2}} \left( f_{1,\vec{k}+\vec{r}_\sigma} + f_{1,\vec{k}+\vec{r}_\sigma} \right),
\frac{1}{\sqrt{2}} \left( f_{1,\vec{k}+\vec{r}_\sigma} - f_{1,\vec{k}+\vec{r}_\sigma} \right).$$

The first term in\[7\] originates from the $U(1)$ symmetry transformation $\psi \to e^{i\theta} \psi$ which generates a gauge field $a_\mu = \partial_\mu \theta$. The constant $g$ in\[7\] is the coupling strength between the gauge field $a_\mu$ and the Dirac spinons $\psi$. The first term corresponds to the “Maxwell” term $-\frac{1}{2} f^{\mu \nu} f_{\mu \nu}$ of the gauge field $a_{\mu}$ where $f^{\mu \nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, $\lambda$ is the parameter of the Faddeev-Popov gauge fixing term $-\lambda (\partial_\mu a_\mu)^2$, $\Box$ is the Laplacian in Euclidean space-time. This form of the action originates from a shift of the imaginary time derivative $\partial_\tau \to \partial_\tau + \mu$ where $\mu$ is the chemical potential introduced above. It leads to a new definition of the Matsubara frequencies of the fermion fields $\omega$ which then read

$$\omega_{F,n} = \omega_{F,n} - \mu / i = \frac{\pi}{\beta} (n + 1/4).$$

III. MAXWELL-CHERN-SIMONS ACTION AT FINITE TEMPERATURE

A. Justification and implementation

As shown by Marston\[17\], only gauge configurations of the flux states belonging to $Z_2$ symmetry ($\pm \pi$) are allowed. Hence the flux through the plaquette is restricted to $\phi = \phi_{mf} + \delta \phi = \{0, \pm \pi\}$. In order to remove “forbidden” $U(1)$ gauge configurations of the antiferromagnet Heisenberg model ($\phi \neq \pm \pi$) a CS term should be included in the QED$_3$ action in order to fix the total flux through a plaquette. This leads to the Maxwell-Chern-Simons (MCS) action in Euclidean space

$$S_E = \int_0^\beta \int d^2 \tau \left\{ -\frac{1}{2} a_\mu \left[ (\Box \delta^{\mu \nu} + (1 - \lambda) \partial^\mu \partial^\nu) \right] a_\nu + i \kappa \varepsilon^{\mu \nu \rho} \partial_\rho \right\} a_\nu + \sum_\sigma \bar{\psi}_\sigma \gamma_\mu (\partial_\mu - ig a_\mu) \psi_\sigma$$

(6)

The implementation of the CS action

$$S_E^{CS} = \int_0^\beta \int d^2 \tau i \kappa \varepsilon^{\mu \nu \rho} a_\mu a_\nu$$

(7)

introduces a new constant $\kappa$. We show below that this constant can be fixed to a definite value.

From the above action\[7\], the equation of motion of the gauge field in Minkowskian space

$$\partial_\nu f^{\mu \nu} - (\kappa / 2) \varepsilon^{\mu \rho \nu} f_{\mu \nu} = - g \sum_\sigma \bar{\psi}_\sigma \gamma^\mu \psi_\sigma$$

(8)

leads to a relation between a magnetic field and the CS coefficient\[19,20\]. If $B = \partial_\nu a_2 - \partial_2 a_1$ is chosen to be constant in such a way that the whole system experiences an homogeneous magnetic field the equation of motion\[8\] of the gauge field becomes

$$\kappa B = - g \sum_\sigma \langle \psi_\sigma^\dagger \psi_\sigma \rangle$$

(9)

The gauge field $a_\mu$ is related to the phase $\theta (\vec{r})$ of the spinon at site $\vec{r}$ through the gauge transformation $f_{\mu \nu} \rightarrow e^{i \theta (\vec{r})} f_{\mu \nu}$ which keeps the Heisenberg Hamiltonian\[8\] invariant. From the definition of $\psi$ one gets

$$\psi_{\sigma \gamma} \rightarrow e^{ig \theta (\vec{r})} \psi_{\sigma \gamma}.$$  It is clear that $\theta (\vec{r})$ is the phase at the lattice site $\vec{r}$ and that $a_\mu (\vec{r}) = \partial_\mu \theta (\vec{r})$. Hence the magnetic field $B$ is then directly related to the flux $\phi$ through the plaquette shown in figure\[10\].

\[
\phi = g \sum_{i,j} (\theta (\vec{r}_i) - \theta (\vec{r}_j)) = g \int d^2 \vec{a}
\]

(10)

where $\Omega_{\Box}$ is the surface of the plaquette and $k$ an integer. Here the flux is fixed to be to be equal to $\{0, \pm \pi\}$. Hence $\kappa$ can be fixed by the flux through the plaquette using equations\[11\] and\[10\]. Defining $\rho = \sum_\sigma \langle \psi_\sigma^\dagger \psi_\sigma \rangle$ as the density of spinon one can indeed rewrite equation\[10\]

$$\kappa = \frac{g \rho}{\bar{B}} = \frac{g^2 N}{\pi k}.$$
where $\mathcal{N}$ is the number of spinons on the plaquette and $k$ is an integer, see (10). Recalling that

$$\nu = \frac{\text{number of particle}}{\text{number of flux quanta}} = \frac{\rho}{g|B|/(2\pi)} = \frac{2\mathcal{N}}{k} \tag{11}$$

is the filling factor of the Landau level, in the Quantum Hall Effect (QHE) one finally gets

$$k = \frac{g^2}{2\pi} \nu \tag{12}$$

The present analysis may suggest that the application of a real magnetic field to the spin system could allow to detect the presence of spinons through the Quantum Hall Effect. We leave this point for further investigations.

**B. The photon propagator at finite temperature**

In this subsection we construct the dressed photon propagator of QED$_3$ with an MCS term at finite temperature in order to gain information about the interaction between spinons $\psi$ and the implication of the CS term on the dynamical mass generation.

Integrating (9) over the fermion fields $\psi$, the partition function of the spin system $Z[\psi, a] = \int \mathcal{D}(\psi, a) e^{-S_E}$ with the action $S_E$ given by (3) leads to the pure gauge partition function

$$Z[a] = \int \mathcal{D}(a) e^{-S_{eff}[a]} \tag{13}$$

where the effective pure gauge field action $S_{eff}[a]$ comes in the form

$$S_{eff}[a] = \int_0^\beta d\tau \int d^2r \left\{ -\frac{1}{2} a_\mu \left[ (\Box \delta^{\mu\nu} + (1 - \lambda) \partial^\mu \partial^\nu) \right] + i \kappa \varepsilon^{\mu\nu\rho\sigma} \partial_\rho a_\sigma \right\} - \ln \det \left[ \gamma_\mu (\partial_\mu - ig a_\mu) \right] \tag{14}$$

One can develop the last term in the effective gauge field action $S_{eff}[a]$ into a series and write

$$\ln \det \left[ \gamma_\mu (\partial_\mu - ig a_\mu) \right] = \ln \det G_F^{-1} - \sum_{n=1}^{\infty} \frac{1}{n} \text{Tr} [iG_F \gamma^\mu a_\mu]^n \tag{15}$$

where $G_F^{-1}(k - k') = i \frac{\varepsilon^{\mu\nu} k_\mu}{(2\pi)^2} \delta(k - k')$ is the fermion Green function in the Fourier space-time with $k = (\vec{\omega}_F, n, k)$, hence $G_F = -i \frac{\varepsilon^{\mu\nu} k_\mu}{(2\pi)^2} (2\pi)^2 \delta(k - k')$. The first term on the r.h.s. of equation (15) is independent of the gauge field $\{ a_\mu \}$. It can be removed from the series since we focus our attention on pure gauge field terms. The first term proportional to the gauge field $n = 1$ in the sum vanishes since $tr \gamma_\mu = 0$. Keeping only second order terms in order to stay with gaussian contributions to the fluctuations one gets the pure gauge action

$$S_{eff}^{(2)}[a] = \int_0^\beta d\tau \int d^2r \left\{ -\frac{1}{2} a_\mu \left[ (\Box \delta^{\mu\nu} + (1 - \lambda) \partial^\mu \partial^\nu) \right] + i \kappa \varepsilon^{\mu\nu\rho\sigma} \partial_\rho a_\sigma \right\} + \frac{g^2}{2\beta} \sum_{\sigma, \omega_F, \epsilon} \int d\tilde{k}_1 \frac{1}{\beta} \sum_{\omega_F'} \int d^2\tilde{k}' \sqrt{\omega_{\tilde{k}_1} - \omega_{\tilde{k}'}} \times tr \left[ \gamma^\rho \gamma_1 a_\rho (k - k') \gamma^{\nu} a_\nu (-(k - k')) \right] \tag{16}$$

The second term in equation (16) has been worked out in (16). The whole action can be put into the form

$$S_{eff}^{(2)}[a] = -\frac{g^2}{2\beta} \sum_{\omega_F} \int d^2\tilde{q} \frac{1}{(2\pi)^2} \times a_\mu (-\tilde{q}) \left[ \Delta_t^{(0)} E_{\mu\nu} - \mu_{\mu\nu}(q) \right] a_\nu(q) \tag{17}$$

where $\Delta_t^{(0)} E_{\mu\nu} = \frac{1}{\pi(q^2 + \kappa^2)} [g_\mu^2 q_\nu - \kappa \varepsilon_{\mu\rho\sigma} q^\rho] + \frac{\lambda}{\kappa q^2}$ is the bare photon propagator in Euclidean space-time. The one-loop vacuum polarization terms reads

$$\Pi_{\mu\nu} = \Pi_A A_{\mu\nu} + \Pi_B B_{\mu\nu} = (\Pi_1(q_m) + \Pi_2(q_m) A_{\mu\nu} + \Pi_3(q_m) B_{\mu\nu}$$

where $A_{\mu\nu}$ and $B_{\mu\nu}$ are Lorentz invariant tensors given in the Appendix and

$$\Pi_1(q_m) = \frac{\alpha q}{\pi} \int_0^1 dx \sqrt{x(1-x)} \frac{\sinh \beta q \sqrt{x(1-x)}}{D(X,Y)}$$

$$\Pi_2(q_m) = \frac{\alpha m}{\beta} \int_0^1 dx \{ \sqrt{(1-2x) \cos 2\pi x m} D(X,Y) \}$$

$$\Pi_3(q_m) = \frac{\alpha}{\pi} \int_0^1 \log 2 D(X,Y)$$

with $D(X,Y) = \cosh \left( \beta \sqrt{x(1-x)} \right) + \sin(2\pi x m)$. Here the photon momentum $q_m = (\omega_{\beta m}, m = \frac{2m}{q_m}, \beta)$ with $\mu = \{0, 1, 2\}$, $m$ is an integer and $\alpha = 2g^2$ the coupling constant.

The finite-temperature dressed photon propagator in Euclidean space verifies the Dyson equation

$$\Delta_t^{-1} E_{\mu\nu} = \Delta_t^{(0)} E_{\mu\nu}^{-1} + \Pi_{\mu\nu} \tag{18}$$
The inversion of equation [13] leads to the dressed photon propagator with the CS term at finite temperature

\[ \Delta_{E_{\mu\nu}} = \left[ (q^2 + \Pi_A) A_{\mu\nu} + (q^2 + \Pi_B) B_{\mu\nu} - \kappa \epsilon_{\mu\nu\rho\sigma} \right] / \left[ (q^2 + \Pi_A) (q^2 + \Pi_B) + (\kappa q)^2 \right] + \frac{g_0 \nabla}{\lambda (q^2)^2} \]  

(19)

IV. “CHIRAL” SYMMETRY RESTORATION

The coupling of the gauge field \( \phi_\mu \) to the spinon field generates a mass for this field. Chiral symmetry in four dimensions requires fermions to be massless. In this space a mass term \( m\bar{\psi}\psi \) changes sign under chiral transformations generated by means of the Dirac matrix \( \gamma_5 \). Hence fermions must be massless in order to keep the action invariant. In three dimensions no real \( \gamma_5 \) matrix can be defined. However embedding the (2+1)-dimensional space into a four dimensional space two types of “chiral” symmetries can be defined from \( \gamma_3 \) and \( \gamma_5 \) [1,2,22,23] where \( \gamma_3 \) and \( \gamma_5 \) are 4 \times 4 matrices

\[ \gamma_3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \gamma_5 = i \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \]

which induce ”chiral” transformations \( e^{i\theta \gamma_3} \) and \( e^{i\theta \gamma_5} \). In (2+1) dimensions the algebra is completed by

\[ \gamma^0 = \begin{pmatrix} \tau_3 & 0 \\ 0 & -\tau_3 \end{pmatrix}, \gamma^1 = \begin{pmatrix} \tau_1 & 0 \\ 0 & -\tau_1 \end{pmatrix}, \gamma^2 = \begin{pmatrix} \tau_2 & 0 \\ 0 & -\tau_2 \end{pmatrix} \]

where \( \{\tau_i, i = 1, 2, 3\} \) are the Pauli matrices and Dirac matrices verify \( \gamma^\dagger \gamma + \gamma \gamma^\dagger = 2\delta_{\mu\nu} \) in Euclidean space.

Appelquist et al. [12] showed that at zero temperature the originally massless fermion can acquire a dynamical mass when the number \( N = \Sigma\sigma^1 \) of fermion flavors is lower than the critical value \( N_c = 32/\pi^2 \). Later Marii [24] confirmed this result with \( N_c \approx 3.3 \). Since we consider only spin-1/2 systems, \( N = 2 \) and hence \( N < N_c \).

At zero temperature the dynamical mass term is renormalized by the CS term \( \frac{m(\kappa q^0)}{m(\kappa q^0 = 0)} = e^{-\frac{\kappa N_c}{(\alpha/\beta) \pi \sqrt{2}}} \)

and even the critical value \( N_c \) is affected as \( \tilde{N}_c = N_c [1 + (16\kappa^2)/\alpha] \) as shown by Hong and Park [25].

Here we concentrate on the impact of the CS term on the dynamical mass generation and show that chiral symmetry can be restored at finite temperature. An explanation of the mechanism behind this symmetry restoration will also be given.

A. Effective potential at finite temperature

In the present theory mass is generated in two different ways. First, as shown earlier, the massless photon induces a mass for the spinon through the coupling of the two fields. Second, the CS coefficient gives a mass to the "photon" (gauge field \( \phi_\mu \)), \( m_{MC5} = \kappa \). This can be seen from the pure gauge equation of motion for the dual field \( \tilde{f}_\mu \equiv \frac{1}{2} \epsilon^{\mu\nu\rho} f_{\nu\rho} \)

\[ (\partial^\mu \partial_\mu + \kappa^2) \tilde{f}_\mu = 0 \]

The massive photon induces the same effect (dynamical mass generation) at zero temperature.

We show now how the photon mass \( \kappa \) (the CS coefficient) affects the effective potential at finite temperature between two spinons.

The static effective potential between spinons with opposite charge \( g \) is given by

\[ V(R) = \frac{-g^2}{2\pi} \int_0^\infty d\tau \Delta_{00} (\tau, R) \]

\[ \approx \frac{g^2}{2\pi} \int_2^\infty d\tau \Delta_{00} (0,0) \]

\( J_0(qR) \) is the Bessel function of the first kind and

\[ \Delta_{00} = \frac{1}{(q^2 + \Pi_B (m = 0)) + \frac{(\kappa q)^2}{(q^2 + \Pi_A (m = 0))}} \]

At large distances \( q \to 0 \) the one-loop vacuum polarization parts become \( \Pi_A (m = 0) \approx q^2 \frac{\alpha^2}{12\pi} \) and \( \Pi_B (m = 0) = \frac{\alpha}{\sqrt{2}} \ln 2 \) where the integer \( m \) is related to the photon energy, see above [13]. Hence the longitudinal part of the photon propagator \( \Delta_{00} \) leads to the definition of a correlation length \( \xi_\kappa \)

\[ \Delta_{00} (0,0) = \frac{1}{q^2 + \xi_\kappa^2} \]

where \( \xi_\kappa \) is given by

\[ \xi_\kappa^{-2} = \frac{\alpha}{\pi \beta} \ln 2 + \frac{\kappa^2}{1 + \frac{\alpha^2}{12\pi}} \]

Integrating over the photon momentum \( q \) at large distance \( R \) the effective potential at finite temperature reads

\[ V(R, \beta) \simeq \frac{-g^2}{2\pi} \int_0^\infty dq J_0(qR) \frac{q^2 + \xi_\kappa^2}{q^2 + \xi_\kappa^2} \]

\[ \simeq -\frac{\alpha}{N} \sqrt{\frac{\xi_\kappa}{8\pi R}} e^{-R/\xi_\kappa} \]

which shows that the stronger \( \kappa \) the shorter the correlation length \( \xi_\kappa \). Hence variations of \( \kappa \) affects the correlation length between spinons. Moreover the variation of
the flux through the square plaquette also affects the correlation length since the CS coefficient is related to the flux through equation (9). If the flux $\phi$ through the plaquette increases the correlation length $\xi_0$ also increases, the larger $\kappa$ the shorter the interaction between spinons.

### B. Dynamical mass generation

We show how the CS term affects the “chiral” restoring transition temperature of the dynamical mass generation. The Schwinger-Dyson equation for the spinon propagator at finite temperature reads

$$G^{-1}(k) = G^{(0)^{-1}(k)} - \frac{g}{\beta \omega_{\mathbf{F}, n}} \int \frac{d^2 \hat{P}}{(2\pi)^2} \gamma_\mu G(p) \Delta_{\mu\nu}(k-p) \Gamma_\nu$$

where $p = (p_0 = \tilde{\omega}_{\mathbf{F}, n}, \hat{P})$, $G$ is the spinon propagator, $\Gamma_\nu$ the spinon - photon vertex which will be approximated here by its bare value $g\gamma_\nu$ and $\Delta_{\mu\nu}$ is the dressed photon propagator [19]. The second term in (20) is the fermion self-energy $\Sigma$, $(G^{-1} = G^{(0)^{-1}} - \Sigma)$. Performing the trace over the $\gamma$ matrices in equation (20) leads to a self-consistent equation for the self-energy

$$\Sigma(k) = \frac{g^2}{\beta \omega_{\mathbf{F}, n}} \int \frac{d^2 \hat{P}}{(2\pi)^2} \Delta_{\mu\nu}(k-p) \frac{\Sigma(p)}{p^2 + \Sigma(p)^2}$$

In the low energy and momentum limit $\Sigma(k) = m(\beta, \kappa) \simeq \Sigma(0)$. Equation (21) simplifies to

$$1 = \frac{g^2}{\beta \omega_{\mathbf{F}, n}} \int \frac{d^2 \hat{P}}{(2\pi)^2} \Delta_{\mu\nu}(-p) \frac{1}{p^2 + m(\beta, \kappa)^2}$$

If the main contribution comes from the longitudinal part $\Delta_{\mu\nu}(0, -\hat{P})$ of the photon propagator (22) goes over to

$$1 = \frac{g^2}{\beta \omega_{\mathbf{F}, n}} \int \frac{d^2 \hat{P}}{(2\pi)^2} \frac{1}{(p^2 + \Pi_B(m = 0)) + (\frac{k \hat{P}}{2} + \Pi_A(m = 0))^2}$$

where

$$\Pi_A(m = 0) = \Pi_1(m = 0) + \Pi_2(m = 0) = \alpha \frac{P}{\beta} \int_0^1 dx \sqrt{x(1-x)} \tanh \beta P \sqrt{x(1-x)}$$

$$\Pi_B(m = 0) = \Pi_3(m = 0) = \alpha \frac{\beta}{\pi} \int_0^1 dx \ln 2 \left( \cosh \beta P \sqrt{x(1-x)} \right)$$

Performing the summation over the modified fermion Matsubara frequencies $\tilde{\omega}_{\mathbf{F}, n} = \frac{\pi N}{2} (n + 1/2)$ the self-consistent equation takes the form

$$1 = \frac{g^2 \beta}{\omega_{\mathbf{F}, n}} \int_0^1 dP \times P \tanh \left( \frac{\beta \Lambda}{2} \right) \left[ \frac{1}{P^2 + \left( \frac{m(\beta, \kappa)}{\Lambda} \right)^2} \right]^{-1}$$

$$\times \left[ \frac{1}{P^2 + \left( \frac{\Pi_B(m = 0)}{\Lambda^2} \right) + \left( \frac{\beta P}{4} \right)^2} \right]^{-1}$$

As defined above $\alpha = g^2 N$ with $N = 2$ since we have implemented the Popov-Fedotov procedure. Here $\Lambda$ is the UV cutoff and can be identified as the inverse spin lattice spacing. Equation (24) can be solved numerically.

Figure 3 shows the dependence of the dynamical mass on the temperature for different value of $\kappa/\Lambda$. This mass which is different from zero for low temperature $T$ vanishes at some temperature $T_c$ which depends on $\kappa$. As $\kappa/\Lambda$ increases the chiral symmetry transition temperature $T_c/\Lambda$ decreases following the relation $\frac{T_c}{(\kappa/\Lambda)} = e^{-a(\alpha/\Lambda) n}$ where $a(\alpha/\Lambda)$ is a coefficient depending on $\alpha/\Lambda$. The behaviour for a fixed $\alpha$ is shown in Figure 4.
One can understand the mechanism of chiral symmetry restoration as follows. The photon (gauge field) gives a mass to the fermions (spinons) through a dynamical mass generation mechanism. When the temperature increases this mechanism is lowered by fluctuations, the fermions gain in mobility (the dynamical mass $m(\beta, \kappa)$ decreases). This is similar to the situation in plasmas. When the temperature is high enough the charged particles composing the plasma are considered as free particles. Below some temperature these charged particles are screened thus their mass is renormalized and gets larger than in the high temperature plasma. The CS mass $\kappa$ contributes also to the photon mass, the interaction $V(R, \beta)$ between fermions is weakened as $\kappa$ increases for a fixed length $R$, the correlation length $\xi_\kappa$ gets weaker and thus the screening effect gets weaker. Finally the chiral symmetry restoring temperature decreases with increasing $\kappa$ since the screening effect is smaller and thus fermions gain mobility, their mass term is renormalized to a smaller value. At zero temperature the dynamical mass decreases as $\kappa$ increases like $m(\kappa=0) = e^{-4\pi \sqrt{\Lambda/\alpha}}$ where $a(\alpha/\Lambda)$ is a coefficient depending on $\alpha/\Lambda$ and $\Lambda$ the UV cutoff.

The value of $\kappa$ can be controlled by fixing the flux through a plaquette going around neighbouring spin lattice sites. The gap in the spinon spectrum shrinks to zero with increasing $\kappa$ for a fixed temperature.

Hence the Maxwell-Chern-Simons term at finite temperature which is aimed to fix the correct $U(1)$ gauge configuration provides an interesting way to control the chiral symmetry restoration temperature and the effective potential between spinons. The present study has been done in the framework of a non-compact theory. One may ask how it will be influenced by the presence of instantons in a compact description of the gauge field. This point related to the confinement/deconfinement problem is still under discussion.

Acknowledgments

One of us (R.D.) would like to thank M. Rausch de Traubenberg for enlightening discussions and F. Stauffer for his encouragements.

VI. APPENDIX

One may believe that a system at finite temperature breaks Lorentz invariance since the frame described by the heat bath already selects out a specific Lorentz frame. However this is not true and one can formulate the statistical mechanics in a Lorentz covariant form.

We consider a system in 2 space and 1 time dimension. Define the proper 3-velocity $u^\mu$ of the heat bath. In the rest frame of the heat bath the three velocity has the form $u^\mu = (1, 0, 0)$ and the inverse temperature $\beta$ characterizes the thermal property of the heat bath.

Given the 3-velocity vector $u^\mu$ one can decompose any three vector into parallel and orthogonal components with respect to the proper velocity of the heat bath, the velocity $u^\mu$. In particular the parallel and transverse components of the three momentum $q^\mu$ with respect to $u^\mu$ read...
\[ q_{\parallel}^\mu = (q.\mu) \ u^\mu \]  \hspace{1cm} (25)

\[ \tilde{q}^\mu = q^\mu - q_{\parallel}^\mu \]  \hspace{1cm} (26)

Similarly one can decompose any vector and tensor into components which is parallel and transverse to a given momentum vector \( q^\mu \)

\[ \bar{u}_\mu = u_\mu - \frac{(q.\mu)}{q^2} q_\mu \]  \hspace{1cm} (27)

\[ \bar{\eta}_{\mu\nu} = \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \]  \hspace{1cm} (28)

It is easy to define second rank symmetric tensors constructed at finite temperature from \( q^\mu, \ u^\mu \) and \( \delta_{\mu\nu} \) which are orthogonal to \( q^\mu \)

\[ A_{\mu\nu} = \delta_{\mu\nu} - u^\mu u^\nu - \frac{q_\mu q_\nu}{q^2} \]  \hspace{1cm} (29)

\[ B_{\mu\nu} = \frac{q^2}{q^2} \tilde{u}_\mu \tilde{u}_\nu \]  \hspace{1cm} (30)

Since one considers a spin system at finite temperature and “relativistic” covariance should be preserved the polarization function may be put in the general form

\[ \Pi_{\mu\nu} = \Pi_A A_{\mu\nu} + \Pi_B B_{\mu\nu} \]  \hspace{1cm} (31)

and the Dyson equation \[ \text{[31]} \] can now be expressed in a covariant form if one uses relation \[ \text{[31]} \].

---

* E-mail address: rdillen@lpt1.u-strasbg.fr
† E-mail address: richert@lpt1.u-strasbg.fr

1. M. Franz, Z. Tesanovic and O. Vafek, Phys. Rev. B 66, 054535 (2002)
2. P. Ghaemi and T. Senthil, Phys. Rev. B 73, 054415 (2006)
3. T. Morinari, cond-mat/0508251
4. P.A. Lee, N. Nagaosa, X.-G. Wen, cond-mat/0410445
5. R. Dillenschneider, J. Richert, Phys. Rev. B 73, 024409 (2006)
6. R. Dillenschneider, J. Richert, cond-mat/0602487 (2006), to be published in Phys. Rev. B. 
7. I. Affleck and J. B. Marston, Phys. Rev. B 37, 3774 (1988)
8. J. B. Marston and I. Affleck, Phys. Rev. B39, 11538 (1989)
9. V. N. Popov and S. A. Fedotov, Sov. Phys. JETP 67, 535 (1988)
10. D. K. Hong and S. H. Park, Phys. Rev. D 47, 3651 (1993)
11. T.W. Appelquist, M. Bowick, D. Karabali and L.C.R. Wijewardhana, Phys. Rev. D 33, 3704 (1986)
12. T.W. Appelquist, D. Nash and L.C.R. Wijewardhana, Phys. Rev. Lett. 60, 2575 (1988)
13. A. Auerbach, *Interacting electrons and quantum magnetism*, Springer-Verlag, 1994.
14. X. G. Wen, Phys. Rev. B65, 165113 (2002)
15. A. Auerbach and A. Arovas, Phys. Rev. B38, 316 (1988)
16. P. A. Lee and N. Nagaosa, Phys. Rev. B46, 5621 (1992)
17. J. Brad Marston, Phys. Rev. Lett. 61, 1914 (1988)
18. C. Itzykson, J.-B. Zuber, Quantum Field Theory, McGraw-Hill, 1986
19. G. V. Dunne, hep-th/9902115 Aspect of Chern-Simons Theories, published in Topological Aspects of Low Dimensional Systems, A. Comtet et al. (Eds), Springer Verlag 2000
20. T. Itoh and H. Kato, Phys.Rev.Lett. 81, 30 (1998)
21. N. Nagaosa, Quantum Field Theory in Strongly Correlated Electronic Systems, Springer, 1998
22. N. Dorey, N.E. Mavromatos, Nucl. Phys. B 386 , 614 (1992)
23. M. Rausch de Traubenberg, hep-th/0506011
24. P. Maris, Phys. Rev. D 54, 4049 (1996)
25. S. Hands, J.B. Kogut, B. Lucini, hep-lat/0601001
26. I.F. Herbut, Phys. Rev. B 66, 094504 (2002)
27. F.S. Nogueira and H. Kleinert, Phys. Rev. Lett. 95 176406 (2005)
28. Z. Nazario and D. I. Santiago, cond-mat/0606386
29. A. Das, Finite temperature field theory, World Scientific, (1997)