Prediction of endwall losses in a low pressure turbine cascade with an algebraic intermittency model

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Abstract. The predictive qualities of a newly developed algebraic intermittency model are analysed by simulation of the flow through a linear cascade of low pressure turbine (LPT) blades with endwalls. Both steady RANS (Reynolds-averaged Navier-Stokes) and time-accurate RANS (URANS) simulations are performed. The results are compared with reference LES (Large Eddy Simulation) by Cui et al. (2017, Numerical investigation of secondary flows in a high-lift low pressure turbine, Int. J. of Heat and Fluid Flow, vol. 63) for a turbulent endwall boundary layer (TBL) at the entrance to the cascade. Good agreement is obtained between simulations with the algebraic model and the reference LES for the evolution of the mass-averaged total pressure loss coefficient and for profiles of pitchwise-averaged turbulent kinetic energy.

1. Introduction

Endwall losses may contribute up to 30% to the total aerodynamic losses of a turbine cascade. Therefore, proper accounting of secondary flow features at the endwalls is of particular importance in the design of an axial turbine. An oncoming endwall boundary layer impacts the leading edge of the blades and separates. The separated boundary layer rolls up and forms the streamwise-oriented pressure-side (HSVPS) and suction-side (HSVSS) legs of a horseshoe vortex. The pressure-side leg travels, due to a strong transverse pressure gradient, towards the suction side of the neighbouring blade and becomes a part of the passage vortex (PV). The suction-side leg develops near the suction side of the blade, but there are several scenarios reported in the literature on this development, depending on the cascade load and load distribution. It may travel above the passage vortex, wrap around the passage vortex or dissipate quickly downstream of the leading edge. Goldstein and Spores [1] found the suction-side leg of the horseshoe vortex travelling above the passage vortex until the trailing edge of the cascade. They observed formation of two counter-rotating vortices in the corner of the suction side of the blade and the endwall at the intersection point of the horseshoe vortex separation line in the endwall plane and the blade surface. Generation of a corner vortex was also observed on the blade pressure side. Gross et al. [2] studied experimentally and numerically the secondary flow of a cascade with front-loaded L2F-blades. The experiments were performed at a high inlet turbulence level, Tu=3%, whereas a zero turbulence level was taken in the LES. A separated boundary layer was reproduced on the blade surface away from the endwall using LES. The boundary layer did not separate on the blade surface in the experiment owing to the high freestream turbulence level. Interestingly, the difference in blade surface boundary layer state has only a minor effect on the pressure losses and the secondary flow structure at
the endwall. Gross et al. [2] observed fast dissipation of the suction-side leg of the horseshoe vortex at the onset of the adverse pressure gradient region on the blade suction side. They also observed a corner separation vortex (SSCSV) on the blade suction side with the same sense of rotation as the passage vortex. The LES results showed the formation of a shed vortex (SV) at the trailing edge, with the rotation sense opposite to that of the passage vortex. Bear et al. [3] studied experimentally the total pressure loss and the loss production mechanism for the same cascade with front-loaded L2F-blades, with and without a suction side corner fillet. The flow was subjected to a high freestream turbulence level, Tu=3%. They found that the passage vortex primarily generates loss within the blade passage, while the suction surface corner separation vortex (SSCSV) generates loss within the passage and downstream of it. The shed vortex (SV) generates loss downstream of the trailing edge. Cui et al. [4] studied the secondary flow through a linear cascade of T106A-blades with endwalls, using LES. The simulations were performed with three sets of inlet conditions: laminar and turbulent endwall boundary layers and laminar endwall boundary layers combined with free-stream inflow perturbed by wakes. A low freestream turbulence level, Tu=0.5%, was specified in the simulations with the laminar and turbulent boundary layers. The results obtained in the present paper with the algebraic model are compared with the reference LES by Cui et al. [4] for turbulent endwall boundary layers (TBL) at the cascade entrance.

2. Algebraic transition model

The algebraic intermittency model [5,6] is combined with the newest version of the k-ω RANS turbulence model by Wilcox [7]. It takes into account two effects in an attached pre-transitional boundary layer: damping of short-wavelength disturbances induced by the free stream and breakdown of near-wall disturbances into fine-scale turbulence.

The transport equations for turbulent kinetic energy $k$ and specific dissipation $\omega$ rate are

$$\frac{Dk}{Dt} = \gamma P_k + (1-\gamma) P_{sep} - \beta^* k \omega + \frac{\partial}{\partial x_j} \left[ \nu + \sigma^* \frac{k}{\omega} \frac{\partial k}{\partial x_j} \right],$$

$$\frac{D\omega}{Dt} = \alpha \frac{\omega}{k} P_k - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[ \nu + \sigma \frac{k}{\omega} \frac{\partial \omega}{\partial x_j} \right] + \frac{\sigma_d}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}.$$ (2)

The intermittency function $\gamma$ is a multiplier of the production term $P_k$ in the $k$-equation. The production term is $P_k = \nu S^2$, where $\nu$ is the small-scale eddy viscosity, which is part of the full eddy viscosity $\nu_T$. $S$ is the magnitude of the shear rate tensor. In the laminar part of a boundary layer, $\gamma$ is set to zero. There is then no production of $k$, but turbulent kinetic energy enters by diffusion out of the free stream. In the laminar part of a boundary layer, the $\omega$-equation stays active. The term $(1-\gamma)P_{sep}$ models turbulence production by instability and breakdown of the laminar free shear layer generated by boundary layer separation in a low turbulence level background flow.

The turbulent kinetic energy $k$ is split, based on the laminar-fluctuation kinetic energy transition model by Walters and Cokljat [8], into a small-scale part and a large-scale part:

$$k_s = f_{SS} k, \quad k_l = k - k_s.$$ (3)

The splitting by the factor $f_{SS}$ expresses the shear-sheltering effect in a pre-transitional boundary layer, which is the damping of high-frequency components of the turbulence coming from the free stream. Shear sheltering depends on the ratio of two timescales in a laminar layer: the timescale of convection of disturbances relative to an observer inside the layer and the timescale of diffusion in the wall-normal direction. In present work, it is modelled by [6]:

$$f_{SS} = \exp \left( - \frac{C_{SS} \nu}{\sqrt{k_y}} \right)^2.$$ (4)
where \( y \) is the distance to the wall and \( C_{SS} = C_S (1 + C_{A} f_{W} \psi) \) is a flow-dependent coefficient. \( C_S \) and \( C_{A} \) are model constants (Table 1). The \( \psi \) and \( f_{W} \) functions are:

\[
y = \tanh \left( -\frac{\Omega (S - \Omega)}{C_{\psi} \left( \beta^{*} \omega \right)^{2}} \right), \quad f_{W} = 1 - \tanh \left( \frac{k}{C_{W} \nu_0} \right),
\]

with \( \Omega \) the magnitude of the rotation rate tensor. The role of the flow-dependent term \( f_{W} \psi \) is increasing \( C_{SS} \) (larger shear sheltering) in accelerating flow (\( f_{W} \psi > 0 \)), and reducing \( C_{SS} \) (smaller shear sheltering) in decelerating flow (\( f_{W} \psi < 0 \)), for acceleration or deceleration due to streamline curvature. The \( f_{W} \) function limits the correction to the border zone between the laminar and turbulent parts in a pre-transitional boundary layer.

The eddy viscosity associated to small scales is calculated in the same way as the eddy viscosity of the original turbulence model by replacing \( k \) by \( k_{S} \):

\[
\nu_s = \frac{k_s}{\tilde{\omega}} \quad \text{with} \quad \tilde{\omega} = \max \left[ \omega, \frac{C_{lim} S}{a_s} \right].
\]

The large-scale eddy viscosity, is, similarly defined with \( k_{l} \):

\[
\nu_l = \frac{k_l}{\tilde{\omega}} \quad \text{with} \quad \tilde{\omega} = \max \left[ \omega, \frac{C_{lim} S}{a_l} \right].
\]

The resulting eddy viscosity, used in the Navier-Stokes equations, is \( \nu = \nu_s + \nu_l \).

The intermittency function \( \gamma \) determines when a flow region is laminar or turbulent. In the present work, \( \gamma \) is prescribed algebraically as a function of the distance to the wall by [6]:

\[
\gamma = \min \left( \max \left( \frac{\sqrt{k_y}}{A_{\nu} \nu}, -1, 0 \right), 1 \right),
\]

where \( A_{\nu} \) is a constant.

The model includes turbulence production due to breakdown of a laminar separated boundary layer at a low free-stream turbulence level by the term (1-\( \gamma \))\( P_{sep} \). We adopt a term with the same purpose in the newest intermittency-transport transition model by Menter et al. [9]:

\[
P_{sep} = C_{sep} F_{sep} \nu S^2, \quad F_{sep} = \min \left( \max \left( \frac{R_{Y}}{2.2 A_{\nu}}, -1, 0 \right), 1 \right),
\]

with \( R_{Y} = y^2 S / \nu \). The closure coefficients and other terms of the underlying turbulence model are given by Wilcox [7]. In the current model, we change the model function \( f_{\beta} \) to \( f_{\beta} = f_{R} f_{\beta} + (1 - f_{R})d \), with \( f_{R} = \min (\gamma, f_{d}) \), where \( f_{d} \) is the modified DES model shielding function by Spalart et al. [10]:

\[
f_{d} = 1 - \tanh \left( \frac{(8 r_{d})^{n}}{k \omega + \nu} \right), \quad r_{d} = \frac{k / \omega + \nu}{k^2 \gamma^2 S^2 + \Omega^2}.
\]

The exponent \( n \) is set to 1, instead of 3 as in the original formula [10], to allow for more gradual variation of the shielding function near a wall. This change enhances the performance of the model in simulations with 3D RANS. Table 1 lists the constants of the algebraic intermittency model (\( C_{lim} = 7/8 \) and \( a_{\nu} = 0.3 \) keep the standard values). The boundary conditions recommended by Wilcox [7] were specified at the no-slip walls: \( k = 0 \) at the wall and \( \omega = 6\nu \beta_0 (\Delta y)^2 \) in the first near-wall point, with \( \Delta y \) the distance of this first point to the wall.

| \( A_{\nu} \) | \( C_S \) | \( C_{A} \) | \( C_{\psi} \) | \( C_{W} \) | \( C_{sep} \) | \( A_{\nu} \) | \( a_{\nu} \) |
|-----------|------|------|-------|-------|-------|-------|-------|
| 12.0      | 21.0 | 1.0  | 10.0  | 10.0  | 2.0   | 550.0 | 0.6   |
3. Computational setup

Figure 1 (a) shows the geometry of the computational domain and the types of boundary conditions. The inlet to the computational domain was placed at 0.4\(C_x\), upstream of the leading edge of the blades (\(x=0\)). The cascade outlet was specified at \(x=1.6C_x\). A 2D grid was made, composed of a structured part near to the blade surface and an unstructured part around the structured boundary layer grid. This 2D grid was extruded in the spanwise direction. The simulations were performed on a grid with 7.2 million cells. The \(y^+\) value in the first grid point near the blade surface and the endwall was below 1.5. The reference LES by Cui et al. [4] were obtained on a grid with 40 million cells. The second order upwind scheme was applied for the convective terms in the momentum and transport equations. The URANS simulations were performed with the second-order backward discretisation scheme in time with a CFL number less than 2. The normalised residuals for the momentum and transport equations fell below 1e-5 in the RANS simulations. Typically, about 15 inner iteration steps were required to obtain a converged solution at each time step in URANS, with residuals dropped down to 1e-5 at each time step. In the present work, a turbulent endwall boundary layer was imposed at the inlet to the cascade, according to Cui et al. [4]. The inlet profiles of the mean velocity, turbulent kinetic energy and the specific dissipation rate were obtained from a precursor simulation of a fully developed turbulent boundary layer flow. Next, the velocity profiles were projected into the inlet plane of the cascade. The angle of attack (in x-y plane) was set to \(\alpha=39.7\text{deg}\) according to [4]. The freestream turbulence level was set to \(Tu=0.5\%\). Figure 1 (b) shows the mean velocity magnitude and the turbulent kinetic energy profiles at the cascade inlet by the RANS and LES. The agreement is good.

4. Results

Flow simulations were performed using the algebraic intermittency model, employing both RANS and URANS techniques, and the standard \(k-\omega\) model (fully turbulent flow). The latter are for the purpose of comparison. With the algebraic intermittency model, the boundary layer separates in the rear part of the blade, away from the endwall, due to the low freestream turbulence level \((Tu=0.5\%)\), transitions to turbulent state and reattaches near to the trailing edge. The separation is not obtained by the fully turbulent simulation. Figure 2 shows the comparison of the pitchwise-averaged turbulent kinetic energy profiles at the streamwise distances, \(C_x=0.5, 0.8, 1.1\) and 1.3. The results are compared with the reference LES by Cui et al. [4]. The endwall is located at \(z/h=0\) \((h\text{-is the blade span})\). Figure 2 (a) shows a peak of \(k\) at \(z/h=0.02\), caused by increased turbulence production due to the passage vortex. The peak value is underpredicted by all models. The fully turbulent model produces too high \(k\) values away from the endwall \((z/h>0.1)\), owing to the turbulent character of the boundary layers on the blade surface. The near-wall peak value is better captured with the algebraic model farther downstream the cascade, at
$x/C_r=0.8$ (Fig. 2 b). The fully turbulent simulation shows again a too high value of $k$ for $z/h>0.16$. Surprisingly, the fully turbulent model reproduces a too narrow turbulent patch at $z/h=0.12-0.15$, approaching the cascade outlet (Fig. 2 c and d). The algebraic model shows better agreement with LES. The very small difference between the total ($k_{tot}$) and the modelled ($k_{mod}$) turbulent kinetic energy profiles with URANS in figure 2 is remarkable. It means that most of the turbulence resists in the modelled part. Some differences between the steady RANS and the time-accurate RANS (URANS) techniques are noted at $x/C_r=1.1$ and 1.3. This is caused by increased activity of the large-scale unsteadiness in the separated boundary layer along the blade surface away from the endwall, at the streamwise distance $x/C_r=0.9$ (not shown). The good agreement between the results obtained by the algebraic intermittency model and the reference LES, in terms of the pitchwise-averaged turbulent kinetic energy profiles, allows for trustworthy analysis of the secondary flow features.

![Figure 2](image_url)

**Figure 2.** Pitchwise averaged turbulent kinetic energy profiles at distances a) $x/C_r=0.5$, b) $x/C_r=0.8$, c) $x/C_r=1.1$, d) $x/C_r=1.3$.

Figure 3 shows the visualisation of the secondary flow by means of iso-surfaces of the q-criterion ($q=10,000$), coloured by the static pressure. For further visualisation, a primary flow is defined as the time-averaged flow at midspan, extruded in spanwise direction (thus, a 2D flow). Secondary velocity vectors are then defined by the difference between the local time-averaged velocity (3D flow) and the primary flow velocity (2D flow). Secondary flow velocity vectors are shown in selected y-z planes (in every 15th cell), projected into these planes. In Figure 2 (b), also limiting streamlines are shown. With the secondary flow velocity vectors and the limiting streamlines, the rotation sense of the secondary flow vortices can be determined. The URANS results are discussed here, but the steady RANS results show similar secondary flow characteristics.
The pressure- and suction-side legs of the horseshoe vortex are clearly visible in figure 3 (a). The pressure-side leg (HSVPS) develops into the passage vortex (PV). The rotation sense of the passage vortex is counter-clockwise when viewed from the cascade inlet. A small corner vortex (CVPS) is formed on the blade pressure side (Fig. 3 a), rotating in the opposite sense to the passage vortex. The suction-side leg of the horseshoe vortex (HSVSS) is pushed towards the midspan of the blade by the corner separation of the boundary layers on the blade suction side and the endwall, which starts around \( x/C_x = 0.2 \), as is visible in figure 3 (b). As the flow is turned through the passage, it is subjected to a centrifugal force due to streamline curvature. The centrifugal force is balanced by the pressure difference between the pressure and suction sides of neighbouring blades. This centrifugal force is reduced, however, inside the endwall boundary layer due to reduced velocity. Since pressure is approximately constant in the wall-normal direction across a boundary layer, an imbalance occurs in the endwall region. This imbalance causes the passage vortex and the associated cross flow near the endwall from the pressure side to the suction side, as is visible in figure 3 (b). This cross-flow is responsible for the formation of a pair of corner vortices in the separation zone downstream of \( x/C_x = 0.2 \) (Fig. 3 b). It consists of a small vortex deeply in the corner (CVSS in Fig. 3 c and d), with rotation sense against the passage.

Figure 3. Vortex structures visualised by \( q \)-criterion \((q=10,000)\), coloured by time-averaged static pressure and secondary velocity vectors in \( y-z \) planes (a, c and d), projected into these planes: a) view near the blade leading edge \((x/C_x = 0.2)\), b) in the front part of the blade (suction side), and near the trailing edge c) view from the suction side \((x/C_x = 1.1)\) and d) view from the pressure side.
vortex and a much stronger vortex more towards the midspan, with the same rotation sense as the passage vortex, called the suction side corner separation vortex (SSCSV in Fig. 3 c and d), by Gross et al. [2] and Bear et al. [3]. It gains in strength as it travels downstream the passage. At the trailing edge, a shed vortex is formed (SV in Fig. 3 c and d), with rotation sense against the passage vortex. The SV and HSVSS have the same rotation sense and merge downstream of the trailing edge, forming what universally is called the counter vortex (CV). The vortex system, produced by the algebraic intermittency model, agrees very well with the vortex system produced by the LES of Cui et al. [4] and observed by Goldstein and Spores [1]. In our interpretation, it accords also with the vortex system observed by Gross et al. [2] and Bear et al. [3]. The difference is that these researchers claim that the HSVSS dissipates. But they do not stress that, in their results, immediately downstream of the dissipation position, a new rather weak vortex is generated with the same rotation sense as the HSVSS. This new vortex can be interpreted as the continuation of the HSVSS.

Figure 4 shows contours of the shape factor $H_{12}$ over the blade suction side and the endwall, obtained with the unmodified $k$-$\omega$ model (Fig. 4 a) and with the algebraic intermittency model (Fig. 4 b). The blanked zones correspond to $H_{12}>3.5$ and indicate boundary layer separation. No separation is obtained on the suction side of the blade near to the midspan ($z/h=0.5$) with the unmodified $k$-$\omega$ model. With the algebraic model (Fig. 4 b), the blade boundary layer separates at the rear part of the blade, transitions to turbulent state and reattaches as fully turbulent near the trailing edge. Another difference between the result by the algebraic model and the fully turbulent model is reproduction of a flow region prone to separation (indicated by the red arrow in Fig. 4 b) at the outlet from the cascade. The contours of the shape factor at the endwall reproduced with the algebraic model agree well with the contours obtained with the reference LES [4].

Figure 5 shows the evolution of the mass-averaged total pressure loss coefficients by the unmodified $k$-$\omega$ turbulence model, the algebraic transition model (steady RANS and URANS) and the reference LES. Very good agreement is obtained between predictions by the algebraic model using URANS and the LES. The accuracy of the algebraic model is somewhat less with the steady RANS, approaching the cascade outlet. The difference in the calculated mass-averaged total pressure loss using the unmodified $k$-$\omega$ model and LES becomes only high downstream of $x/C_x=0.5$. This is partly due to the fully turbulent endwall boundary layer at the inlet to the domain. But one has to note that the endwall flow features, which are responsible for the major part of the total pressure losses through the cascade, are only to small extent influenced by the boundary layer status on the blade surface [3].
Figure 5. Mass-averaged total pressure loss coefficients with the models and the reference LES.

5. Conclusions
An algebraic intermittency model was employed for calculation by RANS and URANS of the three-dimensional transitional flow through a linear cascade of T106A blades with an endwall. Results were compared to these by a reference LES. Compared to the LES, the flow separation and reattachment over the suction side of the blades and the secondary flow features near the endwall are properly captured. Good agreement is obtained for the evolution through the cascade of the pitchwise-averaged profiles of turbulent kinetic energy and the mass-averaged values of total pressure loss. The results with URANS are somewhat better than these with RANS. Simulations were also done by RANS without an added transition model. The separation and reattachment on the suction side of the blades is then not captured and the total pressure loss is overestimated.

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