Dynamics of a Gear System with Faults in Meshing Stiffness

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Abstract. Gear box dynamics is characterised by a periodically changing stiffness. In real gear systems, a backlash also exists that can lead to a loss in contact between the teeth. Due to this loss of contact the gear has piecewise linear stiffness characteristics, and the gears can vibrate regularly and chaotically. In this paper we examine the effect of tooth shape imperfections and defects. Using standard methods for nonlinear systems we examine the dynamics of gear systems with various faults in meshing stiffness.

Keywords: gear system, nonlinear vibrations, meshing errors

1. Introduction

Gears are very common systems, and practically impossible to replace in various applications where mechanical power must be transferred. Time varying mesh stiffness due to multiple teeth contact and a backlash between the teeth give rise to complex behaviour [1–6]. In consequence, under a dynamic load, a typical gear system is a nonlinear oscillator, exhibiting a range of complex behaviour including chaos [4,7–13]. In use the geometric parameters of the gears change, and this causes the corresponding nonlinear response to change [4–5,14–15]. Choy et al. [16] and Kuang and Lin [17] examined the effect of tooth wear. Vibrations have also been modelled by including stochastic forces [4,14–15,18].

In practice it is important to minimise the effect of noise and keep the machine as close as possible to a stable response. In this paper we classify meshing faults and examine the effect of broken teeth and meshing stiffness fluctuations on the vibration response. The possibility of amplitude jumps in systems with meshing defects is demonstrated.

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2. Modelling of gear dynamics

Consider the single gear-pair system shown in Fig. 1. In non-dimensional form, the equation of motion can be written \[4, 9, 12\] as

\[\frac{d^2}{d\tau^2}x + \frac{2\zeta}{\omega} \frac{d}{d\tau}x + \frac{k(\tau)g(x, \eta)}{\omega^2} = \frac{B_0 + B_1 \cos(\tau + \Theta)}{\omega^2}, \tag{1}\]

where

\[\tau = \omega t,\]

\[2\zeta = c_z \left[ \frac{r_1^2}{I_1} + \frac{r_2^2}{I_2} \right],\]

\[B(\tau) = r_1 M_1/I_1 + r_2 M_2/I_2.\]

\(\omega, \zeta, k(\tau), g(x, \eta), \eta\) and \(B(\tau)\) \[1, 12\] and the other symbols are defined in Table 1 and shown on Fig. 1.

In the analysis that follows, the stiffness functions \(k(\tau)\) and \(g(x, \eta)\) need special attention. \(g(x, \eta)\) has a piece-wise character due to the backlash \(\eta\), and is shown in Fig. 2. \(k(\tau)\) is the meshing stiffness arising from the interaction of a single-pair or multiple teeth in contact. For an ideal gear system we have followed references \[4, 12\] and assumed that this meshing stiffness changes periodically. Possible variations from the ideal case, and other possible meshing errors, are plotted in Fig. 3.

Figure 4 shows the results of simulations of the model given by Eq. (1), with time dependent meshing stiffness but without errors. We
Figure 2. The nonlinear stiffness function $g(x, \eta)$.

Table I. Symbols and parameters used in the analysis.

| Symbol | Description                        |
|--------|-----------------------------------|
| $I_1, I_2$ | moments of inertia                |
| $\psi_1, \psi_2$ | rotational angles                |
| $x = r_1 \psi_1 - r_2 \psi_2$ | relative displacement            |
| $v$ | relative velocity                  |
| $x_0, v_0$ | initial conditions                |
| $M_1, M_2$ | external torques                  |
| $\omega$ | excitation frequency              |
| $\tau$ | dimensionless time                 |
| $\zeta$ | damping                           |
| $\eta$ | backlash                          |
| $k(\tau)$ | meshing stiffness                 |
| $g(x, \eta)$ | nonlinear stiffness function      |
| $B_1, B_0, B_1$ | external excitation              |
| $\delta_i$ | distance between increasing teeth contacts |
| $\sigma_\delta, \sigma_k$ | standard deviations              |

have used following system parameters: $\omega = 1.5$, $\zeta = 0.08$, $B_0 = 1.0$, $B_1 = 4.0$, $\eta = 10$. With any nonlinear systems multiple solutions may coexist, and the solution obtained depends on the initial conditions. With the above parameters, there are indeed multiple solutions for the gear model, and this effect was examined in detail in a previous paper [19]. In Figs. 4a and b we show regular and chaotic solutions, depending on the initial conditions for $[x_0, v_0] = [-9, 1]$ and $[x_0, v_0] = [-9, -1]$, respectively. Figure 4c shows the two coexisting attractors, obtained for various initial conditions, on the same graph.
3. Errors in meshing stiffness

In this section we examine the effect of meshing stiffness errors. First consider a gear with one or two next neighbour teeth missing on one of the gear wheels, where each gear wheel has 50 teeth. The shape meshing stiffness $k(\tau)$ are given in Fig. 3 as '2' and '3', respectively, comparing to the ideal case '1'. The response is simulated using the model given in Eq. 1 and modelling the meshing stiffness $k(\tau)$ by curves '2' and '3' (Fig. 3). Although the effect of one broken tooth appears to be fairly benign in terms of the gears dynamics, if two next neighbour teeth are broken the result is a complex response of the system showing the characteristic amplitude jump phenomenon as the solution changes from the regular to the chaotic attractor. This is visible in Fig. 5, which shows a time history for this case. Also shown is the reverse jump from the regular to the chaotic attractor, but it is clear that the system stays for longer time in the chaotic attractor with intermittent regular motion. This result confirms previous results on stochastic jumps [14, 15] in systems with a stochastic force. However, the system modelled here
is fully deterministic and the broken teeth act as additional parametric excitation.

We have also investigated vibrations of gears with a random distance $\delta_i$ between their increasing teeth contacts (Fig. 3, '4'). The results for two different noise levels and two different initial conditions $[x_0, v_0]$, which correspond to different attractors in deterministic case (Fig. 4), are shown in Figs. 6a-d. Interestingly, for weak noise the system chooses the chaotic attractor. This conclusion differs from that obtained in the paper by Warmiński et al. [4] but the assumptions about the noise are different. Warmiński et al. [4] used an external stochastic force generated by stochastic Langevin simulations rather than the stochastic stiffness modelling in the present paper. For stronger noise the mo-

Figure 4. Time series and the Poincare map for an ideal system for various initial conditions $[x_0, v_0]$; regular motion for $[x_0, v_0] = [-9, 1]$ (a) and chaotic motion for $[x_0, v_0] = [-9, -1]$ (b), respectively. Poincare map for various initial conditions (c). The characters a and b in figure c denote the regular and chaotic attractors give by time series (a) and (b).
Figure 5. Time series for a gear system with two broken neighbouring teeth. Initial conditions: \([x_0, v_0] = [-9, 1]\).

Figure 6. Time series for a gear system with a randomised distance \(\delta_i\) between increasing teeth contacts with two different standard deviations: \(\sigma_\delta = 0.2\) in (a) and (b), and \(\sigma_\delta = 0.3\) in (c) and (d); and two different initial conditions \([x_0, v_0] = [-9, 1]\) in (a) and (c), and \([x_0, v_0] = [-9, -1]\) in (b) and (d).

Figure 7. Time series for a system with randomised amplitude showing regular motion for initial conditions \([x_0, v_0] = [-9, 1]\), (a), and chaotic motion for \([x_0, v_0] = [-9, -1]\), (b).
Figure 8. Bifurcation diagram for the case of randomised meshing stiffness amplitude: $x$ is plotted stroboscopically against the square deviation of fluctuating meshing stiffness $\sigma_k$ for two different initial conditions $[x_0, v_0] = [-9, 1]$ (the upper panel) and $[x_0, v_0] = [-9, -1]$ (the lower panel).

... shows an intermittent character with short jumps into the regular attractor as in the previous case with broken teeth (Fig. 5).

Figure 7 corresponds to the meshing stiffness with randomized amplitude but regular distance (Fig. 3, '5'), for $\sigma_k = 0.1$. Here we observe that neither attractor is favored for these conditions. Eventually, for a sufficiently large noise level ($\sigma_k \approx 0.3$) the response returns to the intermittent behaviour with jumps between the regular and chaotic attractors. This effect is shown clearly in Fig. 8, which shows the bifurcation diagram $x$ modulo $2\pi$ against noise level $\sigma_k$.

4. Conclusions

We have examined the dynamics of gears in the presence of meshing faults. Such faults may arise due to wear during or incorrect tolerance in their production. The analysis of various types of errors and tooth faults highlights the presence of dynamic jumping phenomenon. Such jumps between different types of motions, namely chaotic and regular, can be crucial for the system reliability. In this respect our results are consistent with earlier results \[14, 18, 4\]. Moreover, the system is more sensitive to errors in the distance between teeth than fluctuations in the stiffness magnitude, although the qualitative effect is similar. One broken tooth has little influence on the dynamics of the gears, although two broken teeth can have a significant effect.
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