The radius of the $\rho$ meson determined from its decay constant

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We present a unified model describing electroweak properties of the $\pi$ and $\rho$ mesons. Using a general method of relativistic parametrization of matrix elements of local operators, adjusted for the nondiagonal in total angular momentum case, we calculate the $\rho$-meson lepton–decay constant $f_\rho$ using the same parameters of free constituent quarks that have ensured exclusively good results for the $\pi$ meson previously. The only free parameter, characterising quark interactions, which include additional spin-spin contribution and hence differ from the $\pi$-meson case, is fixed by matching the decay constant to its experimental value. The mean square charge radius is calculated, $\langle r^2_q \rangle = (0.56 \pm 0.04)$ fm$^2$. This result verifies, for the $\rho$-meson case, the conjecture of equality between electromagnetic and strong radii of hadrons tested previously for proton, $\pi$ and $K$ mesons.

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I. INTRODUCTION

The construction of effective quantitative methods to calculate electroweak properties of hadrons is an important direction in composite-particle physics. In numerous respects, $\pi$ and $\rho$ mesons, consisting of light quarks, are the simplest bound states to study.

While the pion properties are well known from experiments, the situation is quite different in the case of $\rho$ meson. Its lifetime is very short, $\sim 4.5 \times 10^{-24}$ s, so direct measurements of its electroweak properties (e.g. electromagnetic form factors and static moments) are nearly impossible. The fact that experimental data on the $\rho$ meson are scarce brings one to the natural difficulties while choosing the most adequate approaches to understanding its structure, estimating approximations behind chosen methods and determination of parameters of models. However, knowledge of electroweak properties of the $\rho$ meson, which is one of the simplest hadrons, is a necessary ingredient of understanding physical properties of composite strongly interacting systems. In particular, as it is discussed below, nontrivial conjectures about meson properties exist whose testing requires knowing $\rho$-meson properties not accessible experimentally. That is why, during last years, a number of papers, e.g. Refs. [1–13], on the $\rho$ meson did appear although their results cannot be directly compared to measurements.

The problem was attacked by different approaches, including well-known methods in the frameworks of QCD and based on the Dyson-Schwinger equation [1–3], QCD sum rules [4], Nambu–Jona-Lasino model [5], the hybrid model [6], lattice QCD approach, that is now the only one giving the direct calculation of form factors using QCD, [7–10], and also the holographic approach [11], the light-front formalism and the Feynman triangle diagram [12], the Covariant Spectator Theory [13].

It is important to point out different relativistic formulations of constituent quark models that are based on the classical paper by P. Dirac [14] (so-called Relativistic Hamiltonian Dynamics or Relativistic Quantum Mechanics (RQM), see, e.g. the reviews [15–18]). This approach was used to describe composite quark systems in Refs. [19–28]. Various calculations use essentially different bases, approximations and parameters give essentially different predictions for electroweak properties of $\rho$-meson. For instance, even within the frameworks of light-front dynamics, the results vary because of different treatment of the so-called angular condition.

To calculate the electromagnetic characteristics of $\rho$-meson in the present paper we use our version of the instant form (IF) of RQM. This model is described in detail in Refs. [18, 21, 22, 29, 30]. The main differences between our version and the conventional IF RQM are, first, the construction of the electromagnetic current matrix elements which based on the analogue of the Wigner–Eckart theorem for the Poincaré group and, second, the interpretation of the corresponding reduced matrix elements, that is form factors [31, 32] as generalized functions. We include the interaction in the composite system by adding the interaction operator to the operator of the mass of the free constituent system by analogy with conventional IF RQM. Note that it is possible to include the interaction in our approach by the solutions of the Muskhelishvili-Omnes type equations [33]. These solutions represent wave functions of constituent quarks.

It is important to notice that the approach we use differs from the IF _per se_ but it was rather fruitfully complemented by the so-called Modified Impulse Approximation (MIA), see Ref. [21]. MIA is constructed by making use
of a dispersion–relation approach in terms of the reduced matrix elements (form factors) and removes certain (often quoted) disadvantages of IF. In particular, in our model, the Lorentz-covariance condition and the current conservation law are satisfied automatically.

This approach was surprisingly predictive in description of the pion form factor: subsequently obtained experimental data (see Refs. [29, 33] and references therein) for the range of momentum transfers, \( Q^2 \), larger by an order of magnitude, coincide precisely with prediction of Ref. [29] without any further tuning of parameters. Moreover, and remarkably, this is the only available low-energy model which reproduces the correct QCD asymptotics of the pion form factor [33, 37] without tuning of additional parameters. It worths to make a special emphasis on the fact that our approach gives not only the correct QCD power-law in asymptotic decreasing of pion form factor at large momentum transfers [38], but the asymptotics QCD prefactor as well [39, 40]. The soft/hard transition was governed by the switching the constituent-quark mass \( M(Q^2) \) off. The physical reasons for such a coincidence require, certainly, an additional investigation.

The model reproduces the standard nonrelativistic limit correctly. The method permits analytic continuation of the pion electromagnetic form factor from the spacelike region to the complex plane of momentum transfers and gives a good description of the pion form factor in the timelike region [41].

The main purpose of the present paper is the construction, in the frameworks of IF of RQM, of the unified model describing \( \pi \) and \( \rho \) mesons, and calculation of the mean square charge radius \( \langle r^2_{\rho} \rangle \) of the \( \rho \) meson.

We take advantage of our successful model of pion which fixes the light-quark parameters. There remains only one unknown parameter, the wave function parameter \( b \), describing the effective interaction in the two-quark system (because of the spin-spin interaction, it is different for \( \pi \) and \( \rho \) mesons). We obtain an explicit expression for the \( \rho \)-meson decay constant that depends, now, only on \( b \), \( f_\rho(b) \). Then we fix the value of \( b \) from the experimental value of \( f_\rho \). Finally, with this fixed \( b \), we obtain the mean square radius \( \langle r^2_{\rho} \rangle \).

The rest of the paper is organized as follows. In Sec. II we obtain the analytic expression of the lepton decay constant for vector mesons using the instant form of relativistic quantum mechanics. For this purpose we use the general method of relativistic invariant parametrization of matrix elements of local operators nondiagonal in total angular momentum [42]. In Sec. III the actual formulae that are used for calculations are given and the process of parameter fixing is described. The results of the calculation of the \( \rho \)-meson charge radius are presented. Sec. IV discusses the results while Sec. V contains our conclusions.

II. THE LEPTON DECAY CONSTANT OF THE \( \rho \) MESON IN THE INSTANT FORM OF RQM

The lepton decay constant of a vector meson, \( f_\ell \), is defined by the following matrix element of the electroweak current (see, e.g. [43]):

\[
\langle 0| j^\ell_{\mu}(0)| \rho, m_\rho \rangle = i \sqrt{2} f_\rho \xi_\mu(m_\rho) \frac{1}{(2\pi)^{3/2}},
\]

where \( \rho \) is the meson three-momentum, \( m_\rho = -1, 0, 1 \) is the spin projection, \( \xi_\mu(m_\rho) \) is the polarization vector that in the Breit frame (BF) has the form

\[
\xi_\mu(\pm 1) = \frac{1}{\sqrt{2}} (0, \mp 1, -i, 0), \quad \xi_\mu(0) = (0, 0, 0, 1).
\]

Now our aim is to obtain the lepton decay constant in terms of the wave function of the meson considered as two-quark composite system in the frameworks of the variant of IF of RQM that was developed by the authors (see, e.g. [18, 21, 22, 42]). To solve this problem, we must, first, decompose the matrix element in [41] in some basis that define the representation of the wave function, and, second, separate the invariant part in the decomposition and therefore parametrize the matrix element [42]. Let us note that the form [41] with segregated invariant part \( f_\rho \) presents a particular case of parametrization. Nevertheless we will use a more universal procedure proposed in [44] for the current matrix elements which are diagonal in total moment and then developed for the case of composite systems [33, 45] and for decay processes [42].

In RQM the state vector of a two-particle system belongs to the direct product of two one-particle Gilbert space. So we can describe it in the two bases.

1. The basis of individual spins and momenta of two particles:

\[
| p_1, m_1; p_2, m_2 \rangle = | p_1, m_1 \rangle \otimes | p_2, m_2 \rangle,
\]

\[
| \bar{p}, m | p', m' \rangle = 2 p_0 \delta(\bar{p} - p') \delta_{m m'} ,
\]

where \( p_{1,2} \) are three-momenta of particles, \( m_{1,2} \) are spin projections, \( p_0^2 - \bar{p}^2 = M^2 \), \( M \) is the mass. In what follows the masses of particles are supposed to be equal.

2. The basis where the center-of-mass motion is separated:

\[
| \bar{P}, \sqrt{s}, J, L, S, m \rangle ,
\]

where \( \bar{P} = \bar{p}_1 + \bar{p}_2 \), \( \sqrt{s} \) is the invariant mass of a system of two free particles, \( P^2 = s \), \( L \) is the orbital momentum in the center-of-mass system (CMS) and \( S \) is the total spin in CMS.

The state vector [41] is normalized as

\[
N \delta(\bar{P} - \bar{P}') \delta(\sqrt{s} - \sqrt{s'}) \delta_{JJ'} \delta_{LL'} \delta_{SS'} \delta_{mm'} ,
\]

where

\[
\langle \bar{P}, \sqrt{s}, J, L, S, m | \bar{P}', \sqrt{s'}, J', L', S', m' \rangle =
\]

\[
N \delta(\bar{P} - \bar{P}') \delta(\sqrt{s} - \sqrt{s'}) \delta_{JJ'} \delta_{LL'} \delta_{SS'} \delta_{mm'} ,
\]
where \( N \) is the normalization constant, whose explicit form is irrelevant in what follows.

The bases \([3]\) and \([4]\) are related by the Clebsch-Gordan decomposition for the Poincaré group (see, e.g., [13]):

\[
|\vec{P}, \sqrt{s}, J, L, S, m\rangle = \sum_{m_{1}, m_{2}} \int \frac{d^{3}p_{1}^{J} d^{3}p_{2}^{J}}{2p_{10} 2p_{20}} |p_{1}^{J}, m_{1}; p_{2}^{J}, m_{2}\rangle \times |p_{1}^{J}, m_{1}; p_{2}^{J}, m_{2}\rangle |\vec{P}, \sqrt{s}, J, L, S, m\rangle ,
\]

(6)

where

\[
|p_{1}^{J}, m_{1}; p_{2}^{J}, m_{2}\rangle = \frac{2\sqrt{s}}{\sqrt{\lambda(s, M^2, M^2)}} \times 2P_{0}\delta(P - p_{1} - p_{2}) \sum_{m_{1}, m_{2}} D_{m_{1}m_{1}'}^{1/2}(p_{1}, P)D_{m_{2}m_{2}'}^{1/2}(p_{2}, P) \times \frac{1}{2} \sqrt{n_{1} 1 n_{2}} \langle S_{m} S_{m} | Y_{Lm_{L}}(\vartheta, \phi) \rangle \langle S L m_{m} | J m \rangle ,
\]

(7)

\( \lambda(a, b, c) = a^{2} + b^{2} + c^{2} - 2(ab + ac + bc) \), \( D^{1/2} \) are the matrices of three dimensional rotations (Wigner D-functions), \( Y_{Lm_{L}} \) are the spherical harmonics.

In Eq. (7) we expand in spherical harmonics and sum over angular momenta in the CMS and then pass to an arbitrary reference frame using Wigner D-functions.

We calculate the lepton decay constant in Eq. (1) in the case of the four-fermion interaction. So, we consider the matrix element of the electroweak current of decay \( j_{\mu}^{0} \) of system of two free fermions using the basis [4]:

\[
\langle 0 | j_{\mu}^{0}(0) | \vec{P}, \sqrt{s}, J, L, S, m \rangle .
\]

(8)

Keeping in mind the \( \rho \) meson, we consider, in what follows, only vector mesons with zero orbital moment of quark relative movement. So, we put \( J = S = 1, L = 0 \) and omit the corresponding variables in the state vectors of the basis [4].

Let us consider the relativistic invariant parametrization of the matrix element [5] following [12] [14]. We perform a Lorentz transformation from the initial (laboratory) frame to the BF,

\[
\vec{j}^{0}_{\mu} = j^{0}_{\mu} + w^{0}(\vec{w} j^{0} \vec{w}) 1 + w_{0} - w^{0} j^{0}_{\mu} ,
\]

(9)

where \( w_{\mu} = P_{\mu}/\sqrt{s} \) is the four-velocity corresponding to this transformation, \( j^{0}_{\mu} \) is the four-vector of the electroweak current operator in the BF.

In the case of the matrix element [5], the BF coincides with the CMS where \( \vec{P} = 0 \), so the relation between matrix elements in these systems is of a simple form (compare with Eq. (21) in [12]):

\[
\langle 0 | j_{\mu}^{0}(0) | \vec{P}, \sqrt{s}, m \rangle = \langle 0 | j_{\mu}^{0}(0) | \sqrt{s}, m \rangle .
\]

(10)

Eq. (9) implies that the zero component of the operator, \( j_{\mu}^{0} \), in the BF has a scalar operator structure or the structure of a zero-rank spherical tensor operator. We must describe the current operator in terms of tensor operators because we use the Wigner-Eckart theorem in what follows [31].

Using the notations of [42] let us write:

\[
\langle 0 | j_{\mu}^{0}(0) | \vec{P}, \sqrt{s}, m \rangle = \langle 0 | j_{\mu}^{0}(0) | \sqrt{s}, m \rangle = \frac{1}{\sqrt{4\pi}} 0 \langle C^{0}_{0}(s) | m \rangle ,
\]

(11)

where \( C^{0}_{0}(s) \) is a spherical tensor operator of zero rank.

Performing the Wigner-Eckart decomposition of this spherical tensor operator we obtain that its matrix element is zero:

\[
\langle 0 | C^{0}_{0}(s) | m \rangle = \langle 1 m 00 00 \rangle G^{0,0}_{0,0}(s) = 0 .
\]

(12)

We now consider the three-dimensional part of the current operator matrix element (10):

\[
\langle 0 | j_{\mu}^{0}(0) | \vec{P}, \sqrt{s}, m \rangle , \ r = 1, 2, 3 .
\]

(13)

The zero component of the current operator in BF (10) is treated as a rank-zero tensor operator. We can describe the three-dimensional part of the operator in terms of a rank-one tensor operator. To this end, it suffices to pass to the canonical basis, i.e. to pass from Cartesian coordinates to the basis of spherical harmonics [32]:

\[
\tilde{j}_{\mu}^{0}(0) = a_{rt} \tilde{j}_{t}^{01}(0) , \ t = -1, 0, 1 ,
\]

(14)

\[
a_{rt} = \sqrt{\frac{2\pi}{3}} \left( \begin{array}{ccc} -1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \end{array} \right) ,
\]

(15)

where \( \tilde{j}_{t}^{01}(0) \) are the components of the rank-one spherical tensor operator.

Using the notations of [42] again, we can write:

\[
\langle 0 | j_{t}^{01}(0) | \vec{P}, \sqrt{s}, m \rangle = \langle 0 | j_{t}^{01}(0) | \sqrt{s}, m \rangle = \frac{1}{\sqrt{4\pi}} 0 \langle B_{t,0}^{1}(s) | m \rangle ,
\]

(16)

where \( B_{t,0}^{1} \) is a spherical tensor operator transforming under rotations upon the representation \( D^{1} \).

Let us use now the Wigner-Eckart theorem

\[
\langle 0 | B_{t,0}^{1}(s) | m \rangle = \langle 1 m 1 t 00 \rangle G^{1,0,1}_{0,1}(s) ,
\]

(17)

where \( G^{1,0,1}_{0,1}(s) \) is the set of reduced matrix elements, i.e. scalar functions or the free form factors.

So, we obtain the parametrization of the three-dimensional part of the current matrix element [13]:

\[
\langle 0 | j_{t}^{01}(0) | \sqrt{s}, m \rangle = \langle 1 m 1 t 00 \rangle \frac{1}{\sqrt{4\pi}} G^{1,0,1}_{0,1}(s) .
\]

(18)
To obtain the explicit form of free form factors in Eq. (19), we decompose the matrix element (8) for previously fixed quantum numbers in the basis (3),

\[
\langle 0 | j^\mu_0(0) | \vec{P}, \sqrt{s}, m \rangle = \sum_{m_1, m_2} \sqrt{2p_{10} 2p_{20}} \left\langle 0 | j^\mu_0(0) | \vec{P}_1, m_1; \vec{P}_2, m_2 \right\rangle \times \left\langle \vec{P}_1, m_1; \vec{P}_2, m_2 | \vec{P}, \sqrt{s}, m \right\rangle ,
\]

where \( \langle \vec{P}_1, m_1; \vec{P}_2, m_2 | \vec{P}, \sqrt{s}, m \rangle \) is defined by the relation [7].

We use the standard expression (see, e.g. [43]) for the current matrix element in the case of decay in the basis of individual spins and momenta:

\[
\langle 0 | j^\mu_0(0) | \vec{P}_1, m_1; \vec{P}_2, m_2 \rangle = \bar{v}(\vec{p}_2, m_2) \gamma^\mu (1 + \gamma^5) u(\vec{p}_1, m_1) ,
\]

\( \gamma^\mu = (\gamma^0, \vec{\gamma}) \) are the Dirac matrices; \( \bar{v}(\vec{p}_2, m_2) \) and \( u(\vec{p}_1, m_1) \) are the Dirac spinors.

Using Eqs. (11), (12), (14), (18), and integrating in Eq. (19) in BF, we obtain the explicit form of the free form factors,

\[
G^{1,0,1}_{0,1}(s) =
\]

\[
- \frac{3\sqrt{3}(\sqrt{s} + 2M)}{16\sqrt{s}\pi^2} \left( \frac{7s + 12M\sqrt{s} + 8M^2}{6s + 12M\sqrt{s} + 12M^2} \right) .
\]

Let us return now to Eq. (1) and decompose its l.h.s. in the basis (4):

\[
\langle 0 | j^\mu_0(0) | \vec{P}_c, m_c \rangle = \sum_m \int \frac{d^3\vec{P}_c}{N} d\sqrt{s} \langle 0 | j^\mu_0(0) | \vec{P}, \sqrt{s}, m \rangle \times \langle \vec{P}, \sqrt{s}, m | \vec{P}_c, m_c \rangle ,
\]

where \( \langle \vec{P}, \sqrt{s}, m | \vec{P}_c, m_c \rangle \) is the wave function in the RQM represented in the basis (4),

\[
\langle \vec{P}, \sqrt{s}, m | \vec{P}_c, m_c \rangle = N_c \delta(\vec{P} - \vec{P}_c) \delta_{m_m c} \varphi(s) ,
\]

where

\[
N_c = \sqrt{2P_0} \sqrt{\frac{NCG}{4k}} , \quad NC_0 = \frac{(2P_0)^2}{8k}\sqrt{s} .
\]

\[ k = \frac{1}{2} \sqrt{s - 4M^2} , \]

\[ \varphi(s) = \frac{\sqrt{s}}{k} \psi(k) \]

and \( \psi(k) \) satisfies the normalization condition:

\[
\int \psi^2(k) k^2 dk = 1 .
\]

The integration over the three-momentum in Eq. (22) is performed with the use of the delta function in the wave function [23]. The current matrix element that remains in the integrand in the invariant variable \( \sqrt{s} \) in (22) in our approach is to be considered as a regular generalized function determined on the space of test functions \( \varphi(s) \), it is as a distribution, i.e., as an object that makes sense only when it is in the integrand. To parameterize it, we can proceed as when obtaining Eqs. (11), (18), changing \( P_\mu \) for \( P_{cp} \) in [9]. So, we obtain the matrix elements of the electroweak current for the system of two interacting particles in the form analogous to (11) and (18):

\[
\frac{N_c}{\sqrt{s}} \langle 0 | j^0_0(0) | \vec{P}_c, m_c \rangle = 0 ,
\]

\[
\frac{N_c}{\sqrt{s}} \langle 0 | j^1_0(0) | \vec{P}_c, \sqrt{s}, m \rangle =
\]

\[
\langle 1 m 1 t | 00 \rangle \frac{1}{\sqrt{4\pi}} H^{1,0,1}_{0,1}(s) ,
\]

where \( H^{1,0,1}_{0,1}(s) \) is a set of the reduced matrix elements, that is scalar functions, or form factors.

Substituting Eqs. (22), (23), (26) into Eq. (1), we obtain the integral representation of the lepton decay constant for a vector meson with zero orbital momentum of the quark relative motion,

\[
f_c = \int d\sqrt{s} H^{1,0,1}_{0,1}(s) \varphi(s) .
\]

In the frameworks of our method, the 4-fermion approximation is formulated in the relativistic invariant way by using the change of the invariant form factor \( H^{1,0,1}_{0,1}(s) \) in (27) for the invariant free form factor \( G^{1,0,1}_{0,1}(s) \), that enters the parametrization of the decay current matrix element for the system of two free particles [18]:

\[
f_c = \int d\sqrt{s} c^{1,0,1}_{0,1}(s) \varphi(s) ,
\]

with the free form factor \( G^{1,0,1}_{0,1}(s) \) given by (21).

In the case of the \( \rho \) meson, one can rewrite Eq. (28) in a conventional form in terms of the variable \( k \),

\[
f_\rho = \frac{\sqrt{3}}{\sqrt{2}\pi} \int_0^\infty dk k^2 \psi(k) \frac{(k^2 + M^2 + M)}{(k^2 + M^2)^{3/4}}
\]

\[
\times \left( 1 + \frac{k^2}{3(k^2 + M^2 + M)} \right) .
\]
It is worth noting that Eq. (29) coincides with analogous relations in other approaches (see, e.g., 43, 46, where the light-front dynamics and point form dynamics were used). However, some other expressions do differ, for example, our formulae for electromagnetic form factors of composite systems (see, e.g., 22). This is caused by the fact that we introduce and use an alternative variant of the relativistic impulse approximation (we called it the modified impulse approximation, MIA). In the MIA framework, contrary to the case of the standard impulse approximation, the Lorentz covariance and the current conservation law remain unbroken. In the non-relativistic limit, Eq. (29) transforms to the standard form with the wave function proportional to the point form dynamics, 

\[ \langle r^2_q \rangle \sim 0.3/M^2. \]  

III. THE PARAMETERS AND THE RESULTS OF CALCULATIONS

For the calculation of the \( \rho \)-meson characteristics basing on the relations (29)–(34) we use the following model wave functions (see, also 48).

1. A Gaussian or harmonic oscillator wave function

\[ \psi(k) = N_{HO} \exp \left( -k^2/2b^2_{\rho} \right). \]  

2. A power-law wave function:

\[ \psi(k) = N_{PL} \left( k^2/b^2_{\rho} + 1 \right)^{-n}, \quad n = 2, 3. \]  

So, the following parameters enter our calculations:

1) the parameters that describe the constituent quarks \( \rho \)-se (the quark mass \( M \), the anomalous magnetic moments of quarks \( \kappa_q \), that enter our formulae through the sum \( \sum_q = \kappa_u + \kappa_d \)), and the quark mean square radius \( \langle r_q^2 \rangle \);

2) the parameter \( b_{\rho} \) that enters the quark wave functions (31) and is determined by the quark interaction potential.

In the paper 29 on pion we have shown that in our approach all the parameters of the first group are the functions of the quark mass \( M \) and are defined by its value. In particular, for the quark MSR we can use the relation (see, also 49):

\[ \langle r_q^2 \rangle \sim 0.3/M^2. \]  

To calculate electroweak properties of \( \rho \)-meson we use the same values of quark parameters from the first group as that we have used for the pion 29. So, the wave function parameter \( b_{\rho} \) is the only free parameter in our calculations.

Let us note that in papers 20, 24, for calculations of \( \rho \)-meson properties, quark wave functions of pion, i.e. \( b_{\rho} = b_{\pi} \), were used. It seems to us ineligible because it means that the spin-spin interaction of quarks, which is different in \( \rho \) and \( \pi \) mesons, is neglected.

Let us describe the procedure of calculation in detail, starting from the quark parameter, \( M = 0.22 \) GeV, used in a successful calculation of the pion parameters 29. As it has been demonstrated in Ref. 29 (see also 50), the actual choice of the wave-function form does not affect the result provided the quark parameters are fixed. In what follows, we illustrate the procedure with the wave function 31 with \( n = 3 \).

In the lower panel of Fig.1 the \( \rho \)-meson lepton decay constant \( f_{\rho} \) as a function of the only free parameter of the model, \( b_{\rho} \), is presented. The interval on the vertical axis representing the experimental values of \( f_{\rho} \) is shown. It corresponds to the interval of the values of \( b_{\rho} \) which give, through our calculation, the correct experimental values of the decay constant. This interval, \( b_{\rho} = (0.385 \pm 0.019) \) GeV, is shown on the horizontal axis of Fig.1.

We turn now to the calculation of the \( \rho \)-meson charge radius for these values of \( b_{\rho} \). We use the standard formula,

\[ \langle r^2_{\rho} \rangle = -6 \frac{dG_C(Q^2)}{dQ^2} \bigg|_{Q^2 \to 0}, \]  

where \( G_C(Q^2) \) is the charge form factor of \( \rho \)-meson that was obtained in 21 in the form

\[ G_C(Q^2) = \int d\sqrt{s}d\sqrt{s'} \varphi(s) g_0 C(s, Q^2, s') \varphi(s'). \]
Here, $g_0C$ is the free charge form factor that describes the electromagnetic properties of non-interacting two particle composite system with the quantum numbers of the $\rho$ meson,

$$
g_0C(s, Q^2, s') = A_1(s, Q^2, s')(G^d_E(Q^2) + G^d_M(Q^2)) + A_2(s, Q^2, s')(G^u_E(Q^2) + G^u_M(Q^2)) \
$$

where $G^{u,d}_{E,M}$ are electric and magnetic form factors of constituent $u$ and $d$ quarks, respectively. The explicit form of $A_1(s, Q^2, s')$ and $A_2(s, Q^2, s')$ is cumbersome; it can be found in Ref. [52] which is an extended version of Ref. [21].

The electromagnetic form factors of constituent quarks are taken in the form [18, 38–40],

$$
G^d_E(Q^2) = e_q f_q(Q^2),
$$

$$
G^u_M(Q^2) = (e_q + \kappa_q) f_q(Q^2),
$$

where $e_q$ is the quark charge and $\kappa_q$ is the quark anomalous magnetic moment,

$$
f_q(Q^2) = \frac{1}{1 + \ln(1 + (r^2_q)Q^2/6)},
$$

where $(r^2_q)$ is the MSR of the constituent quark. Values of all parameters used in these expressions are taken from the $\pi$-meson calculation, see e.g. Ref. [59].

The calculated MSR of the $\rho$ meson is presented in the upper panel of Fig. 1. The interval of admissible values of $b_\rho$ gives now the corresponding interval of MSR predicted in the present study, that our approach predicts: $(r^2_\rho) = (0.56 \pm 0.04)$ fm$^2$.

IV. DISCUSSION

Table I presents a comparison of our results with results of calculations of electroweak properties of the $\rho$ meson in other approaches.

| Model | $f_\rho$, MeV | $(r^2_\rho)$, fm$^2$ |
|-------|--------------|----------------|
| This work | 152±8 | 0.56±0.04 |

Table II. The experimental values of the charge MSR $(r^2_{ch})$ and of the MSR for strong interaction $(r^2_{st})$.

| Hadron | $(r^2_{st})$, fm$^2$ | $(r^2_{ch})$, fm$^2$ |
|--------|----------------|----------------|
| $\pi$  | 0.41±0.02 [56] | 0.45±0.02 [51] |
| $K$    | 0.35±0.02 [56] | 0.31±0.03 [51] |
| $p$    | 0.69±0.02 [53] | 0.70706±0.00065 [52] |
| $\rho$ | 0.52±0.05 [56] | 0.56±0.04 (this work) |

This remarkable equality between two physical properties of a hadron related to two different interactions of the Standard Model has been verified experimentally with a great degree of accuracy for the proton, $\pi$ and $K$ mesons (see Table II). Even more demonstrative is Fig. 2, analogous to a figure from the paper [56], but presenting more recent data. We can see that the value of the $\rho$-meson charge radius obtained in this paper fits perfectly the conjecture [58].

V. CONCLUSIONS

The present work gives a consistent unified description of electroweak properties of both $\pi$ and $\rho$ mesons within a single relativistic-invariant approach and with a single set of parameters of constituent quarks. Model is constructed on the base of our variant of IF RQM containing the modified impulse approximation (see, e.g., [21, 22]). In the present paper, the expression for the lepton decay constant of the $\rho$ meson, $f_\rho$, was derived in the frameworks of IF RQM. For the derivation, a general method of the relativistic invariant parametrization of local operators matrix elements non-diagonal in the total angular momentum [42] was used. Then, we turned to the calculation of the charge radius of the $\rho$
meson, \( \langle r^2_{\rho} \rangle \). All but one parameters of the model had been fixed by a successful description of the \( \pi \) meson in previous works. The remaining parameter was fixed from the experimental value of \( f_\rho \), thus allowing to derive \( \langle r^2_{\rho} \rangle = (0.56 \pm 0.04) \) fm\(^2\). This value is in remarkable agreement with the strong-interaction radius, \( \langle r^2_{\rho} \rangle \) measured experimentally, thus confirming the conjecture verified previously for other light hadrons.

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