Superspace interpretation of mass–dependent BRST symmetries in general gauge theories

B. Geyer\(^{a}\) and D. Mülisch\(^{b}\)

\(^{(a)}\)Instituto de Física, Universidade de São Paulo, 05315-970 São Paulo, SP, Brazil
\(^{(b)}\)Wissenschaftszentrum Leipzig e.V., Leipzig 04103, Germany

Abstract

A superspace formulation is proposed for the \(osp(1,2)\)–covariant Lagrangian quantization of general massive gauge theories. Thereby, \(osp(1,2)\) is considered as subalgebra of the superalgebra \(sl(1,2)\) which is interpreted as conformal algebra acting on two anticommuting coordinates. The mass–dependent (anti)BRST symmetries of the quantum action in the \(osp(1,2)\) superfield formalism are realized as translations associated by mass–dependent special conformal transformations.

1 Introduction and main results

Recently, the \(Sp(2)\)–covariant Lagrangian quantization of Batalin, Lavrov and Tyutin \(^{[1]}\) has been extended to a formalism which is based on the orthosymplectic superalgebra \(osp(1,2)\) \(^{[2]}\) and which can be applied to massive gauge theories. This is achieved by incorporating into the extended BRST transformations \(m\)–dependent terms in such a way that the \(m\)–extended (anti)BRST symmetry of the quantum action \(W_{m}\) is preserved. In that approach \(W_{m}\) is required to satisfy the generalized quantum master equations of \(m\)–extended BRST symmetry \((a = 1, 2)\) and of \(Sp(2)\) symmetry \((\alpha = 0, \pm 1)\),

\[
\bar{\Delta}_{a} \exp\{(i/\hbar)W_{m}\} = 0 \iff \frac{i}{2}(W_{m}, W_{m})^{\alpha} + V_{m}^{\alpha} W_{m} = i\hbar \Delta^{\alpha} W_{m},
\]

\[
\bar{\Delta}_{\alpha} \exp\{(i/\hbar)W_{m}\} = 0 \iff \frac{i}{2}(W_{m}, W_{m})_{\alpha} + V_{\alpha} W_{m} = i\hbar \Delta_{\alpha} W_{m},
\]

respectively, whose generating (second order) differential operators

\[
\bar{\Delta}_{a} := \Delta^{a} + (i/\hbar)V_{m}^{a}, \quad \bar{\Delta}_{\alpha} := \Delta_{\alpha} + (i/\hbar)V_{\alpha},
\]

(for explicit expressions see Sect. 3) form a superalgebra isomorphic to \(osp(1,2)\) \(^{[3]}\):

\[
[\bar{\Delta}_{\alpha}, \bar{\Delta}_{\beta}] = \frac{i}{\hbar} \epsilon_{\alpha\beta}^{\gamma} \bar{\Delta}_{\gamma}, \quad [\bar{\Delta}_{\alpha}, \bar{\Delta}_{m}] = \frac{i}{\hbar} \Delta_{m}^{b} (\sigma_{\alpha})_{b}^{a}, \quad \{\bar{\Delta}_{a}, \bar{\Delta}_{m}\} = -m^{2} \frac{i}{\hbar} (\sigma^{a})^{ab} \bar{\Delta}_{a},
\]

where the matrices \(\sigma_{\alpha}\) generate the (real) Lie algebra \(sl(2)\) being isomorphic to \(sp(2)\) and the \(Sp(2)\)–indices are raised or lowered by the (antisymmetric) tensor \(\epsilon^{ab}, \epsilon^{12} = 1,\) and \(\epsilon_{0+} = 1\). As long as \(m \neq 0\) the operators \(\bar{\Delta}_{m}\) are neither nilpotent nor do they anticommute among themselves. This algebra (without the factors \(i/\hbar\)) independently also holds for \((V_{m}^{a}, V_{\alpha})\).

\(^{1}\) Talk given at XXI Encontro Nacional de Física de Partículas e Campos, October 23 – 27, 2000, São Lourenço, MG, Brazil.

\(^{2}\) On leave from Universität Leipzig, Naturwissenschaftlich–Theoretisches Zentrum and Institut für Theoretische Physik, 04109 Leipzig, Germany; e-mail: geyer@itp.uni-leipzig.de
The incorporation of mass terms into the action of any general gauge theory is necessary at least intermittently within the BPHZL–renormalization scheme [4] which – being independent of any regularization – appears to be the most attractive one in order to formulate the quantum master equations on the level of algebraic renormalization theory. In that scheme by using Zimmermann’s normal product formalism the r.h. sides of Eqs. (1) and (2) can be given a well-defined meaning such that also higher–loop anomalies can be properly computed [3]. In the BPHZL–scheme for any massless field a regularizing mass \( m = (s - 1)M \) is introduced in order to be able to perform besides ultraviolet also infrared subtractions thereby avoiding spurious infrared singularities in the limit \( s \to 1 \). By using such an infrared regularization – without violating the extended BRST symmetries – the \( osp(1,2) \)–superalgebra occurs necessarily.

Here, we report a superfield representation [3] of our earlier work on the \( osp(1,2) \)–covariant Lagrangian quantization which amounts to understand also the geometrical meaning of the \( m \)–dependent part of the extended BRST transformations. For that reason we consider \( osp(1,2) \) as subsuperalgebra of the superalgebra \( sl(1,2) \). This algebra, being isomorphic to \( osp(2,2) \), contains four bosonic generators \( V_\alpha \) and \( V \), which form the Lie algebra \( sl(2) \oplus u(1) \), and four (nilpotent) fermionic generators \( V^\alpha_1 \) and \( V^a_2 \). The (anti)commutation relations of the superalgebra \( sl(1,2) \) are [3]:

\[
[V, V_\alpha] = 0, \quad [V_\alpha, V_\beta] = \epsilon_{\alpha\beta} V_\gamma, \quad \{V_\pm, V_\pm\} = 0, \quad (5)
\]

\[
[V, V^a_\pm] = \pm V^a_m, \quad [V_\alpha, V^a_\pm] = V^b_\pm (\sigma_\alpha)_b^a, \quad \{V^a_+, V^b_+\} = -(\sigma^a)^{ab} V_\alpha - \epsilon^{ab} V,
\]

The eigenvalues of the generators \( V_\alpha \), for \( \alpha = 0 \), define the ghost numbers, whereas the eigenvalues of the generator \( V \) define the Weyl weights which in Ref. [1] were introduced as ‘new ghost number’. The generators \( V^a \) and \( V^a \) have opposite new ghost numbers, \( ngh(V^a_\pm) = \pm 1 \), respectively. But, introducing a mass \( m \) which formally will be attributed also by a new ghost number, \( ngh(m) = 1 \), they can be combined into two fermionic generators \( V^a_m = V^a + \frac{1}{2} m^2 V^a \) of the superalgebra \( osp(1,2) \), Eqs. (4). (The difference \( V^a_m = V^a_+ - \frac{1}{2} m^2 V^a \) leads to another \( osp(1,2) \)–algebra.)

The key observation allowing for a geometric interpretation of the superalgebra \( sl(1,2) \) consists in interpreting it – due to Baulieu, Siegel and Zwiebach [4] – as the algebra generating conformal transformations in a (super)space of two anticommuting coordinates \( \theta^a \). Hence, the generators \( i V^a_\pm, i V^a_\pm, i V^a \) and \( -i V \) may be considered as generators of translations \( P^a \), special conformal transformations \( K^{ab} \), symplectic rotations \( M^{ab} \) and dilatations \( D \), respectively, in that space. This leads to a ‘natural’ geometric interpretation of the \( osp(1,2) \) quantization of general massive gauge theories:

The \( osp(1,2) \)–covariant quantization rules are formulated in terms of superfields \( \Phi^A(\theta) \) and superantifields \( \bar{\Phi}^A(\bar{\theta}) \) on which the generators of the algebra \( sl(1,2) \) act linearly. The invariance of \( W_m(\Phi, \bar{\Phi}) \) under \( m \)–extended BRST– and \( Sp(2) \)–transformations is required through Eqs. (1) and (2), where \( V^a_m = V^a_+ + \frac{1}{2} m^2 V^a_\pm \) correspond to translations combined with \( m \)–dependent special conformal transformations and \( V_\alpha \) corresponds to symplectic rotations. Furthermore, proper solutions \( S_m(\Phi, \bar{\Phi}) \) of the classical master equations \( \frac{1}{2} (S_m, S_m)^a + V^a_m S_m = 0 \) and \( \{S_m, S_m\}_a + V_\alpha S_m = 0 \) with vanishing new ghost number, \( ngh(S_m) = 0 \), correspond to solutions being invariant under dilatations, generated by \( V \). Therefore, these solutions are invariant under \( osp(1,2) \oplus u(1) \), where the additional \( u(1) \) symmetry is related to the new ghost number conservation. In Ref. [3] also the problem of how to determine the transformations of the gauge fields and the full set of the necessary
(anti)ghost and auxiliary fields under the superalgebra \( sl(1,2) \) has been solved both for irreducible and first-stage reducible theories with closed algebra (these results will not be reproduced here). Finally, it is proven that mass terms generally destroy gauge independence in the \( osp(1,2) \)--approach. However, this gauge dependence disappears in the limit \( m = 0 \), thus showing that the \( osp(1,2) \)--approach allows for a well-defined consideration of the renormalization of general gauge theories within the field–antifield formalism.

2 Superspace representations of \( sl(1,2) \)

In the \( osp(1,2) \)--approach the space of fields \( \phi^A \) and antifields \( \bar{\phi}_A, \phi^*_{Aa} \) and sources \( \eta_A \) together with their Grassmann parities is characterized by the following sets [1, 2]:

\[
\phi^A = (A^i, B^{a_1 \ldots a_s}, C^{a_1 a_0 \ldots a_s}, s = 0, \ldots, L), \quad \epsilon(\phi^A) = \epsilon_A = (\epsilon_i, \epsilon_{a_1} + s, \epsilon_{a_2} + s + 1)
\]

\[
\bar{\phi}_A = (\bar{A}_i, \bar{B}_{a_1 \ldots a_s}, \bar{C}_{a_1 a_0 \ldots a_s}, s = 0, \ldots, L), \quad \epsilon(\bar{\phi}_A) = \epsilon_A,
\]

\[
\phi^*_{Aa} = (A^s_{a_1 \ldots a_s}, B^s_{a_1 a_0 \ldots a_s}, C^s_{a_1 a_0 \ldots a_s}, s = 0, \ldots, L), \quad \epsilon(\phi^*_{Aa}) = \epsilon_A + 1,
\]

\[
\eta_A = (D_i, E_{a_1 \ldots a_s}, F_{a_1 a_0 \ldots a_s}, s = 0, \ldots, L), \quad \epsilon(\eta_A) = \epsilon_A,
\]

respectively. Here, the pyramids of auxiliary fields \( B^{a_1 \ldots a_s} \) and (anti)ghosts \( C^{a_1 a_0 \ldots a_s} \) are irreducible \( Sp(2) \)--tensors of ‘spin’ \( j \) and \( j+1 \), respectively, being completely symmetric with respect to the ‘internal’ indices \( a_i = 1, 2, (i = 0, 1, \ldots, s) \); similarly for \( \bar{\phi}_A, \phi^*_{Aa} \) and \( \eta_A \). The ‘external’ index \( a = 1, 2 \) on the \( Sp(2) \)--spinors \( \phi^*_{Aa} \) is independent.

Now, we introduce the sets of superfields \( \Phi^A(\theta) \) and superantifields \( \bar{\Phi}_A(\theta) \) having equal Grassmann parity, \( \epsilon(\Phi^A(\theta)) = \epsilon(\bar{\Phi}_A(\theta)) = \epsilon_A \), opposite ghost number, \( gh(\Phi_A) = -gh(\Phi^A) \), and the following expansion in terms of component fields,

\[
\Phi^A(\theta) = \phi^A + \pi^{Aa}\theta_a - \lambda^A \theta^2,
\]

\[
\frac{\delta}{\delta \Phi^A(\theta)} = \frac{\delta}{\delta \phi^A} \theta^2 - \theta^a \frac{\delta}{\delta \pi^{Aa}} \theta_a - \frac{\delta}{\delta \lambda^A} \theta^2,
\]

\[
\bar{\Phi}_A(\theta) = \bar{\phi}_A - \theta^a \phi^*_{Aa} - \theta^2 \eta_A,
\]

\[
\frac{\delta}{\delta \bar{\Phi}_A(\theta)} = \frac{\delta}{\delta \bar{\phi}_A} \theta^2 + \theta^a \frac{\delta}{\delta \phi^*_{Aa}} \theta_a - \frac{\delta}{\delta \eta_A} \theta^2.
\]

According to DeWitt’s convention derivatives with respect to the fields act from the right. Here, additional auxiliary fields \( \pi^{Aa} \) and \( \lambda^A \) have been introduced (cf. also Ref. [3]).

In terms of the superantifields the representation of the generators of \( sl(1,2) \) by linear differential operators on the superspace reads

\[
V^+_\alpha = \int d^2\theta \frac{\partial \bar{\Phi}_A(\theta)}{\partial \theta_a} \frac{\delta}{\delta \Phi^A(\theta)},
\]

\[
V^-_a = \int d^2\theta \left\{ 2\theta^a \frac{\partial \bar{\Phi}_A(\theta)}{\partial \theta_a} + \theta_b \bar{\Phi}_B(\theta)\left((\sigma^a)^b_{\alpha}(\sigma^\alpha)_B A - \epsilon^{ab}_{\alpha} \epsilon^B_A \right) \right\} \frac{\delta}{\delta \Phi^A(\theta)},
\]

\[
V_{\alpha} = \int d^2\theta \left\{ -\theta_a (\sigma^a)_B \frac{\partial \bar{\Phi}_A(\theta)}{\partial \theta_a} + \bar{\Phi}_B(\theta) (\sigma^A)^B_{\alpha} \right\} \frac{\delta}{\delta \Phi^A(\theta)},
\]

\[
V = \int d^2\theta \left\{ \theta_a \frac{\partial \bar{\Phi}_A(\theta)}{\partial \theta_a} + \bar{\Phi}_B(\theta) \bar{\gamma}_A^B \right\} \frac{\delta}{\delta \Phi^A(\theta)},
\]

where \( (\sigma^a)_B^A \) are irreducible \( Sp(2) \)--representations of spin \( j \) acting on the set of internal \( Sp(2) \)--indices, and \( \bar{\gamma}_A^B = \alpha(\bar{\Phi}_A) \delta^B_A \) is related to the Weyl weight \( \alpha(\bar{\Phi}_A) \) of the antisuperfields \( \bar{\Phi}_A \), coinciding with their new ghost number and obeying \( \alpha(\bar{\Phi}_A) + \alpha(\Phi^A) = -2 \) (for details see [3]).
Replacing in Eqs. (8)–(11) the superantifield \( \overline{\Phi}_A(\theta) \) by the superfield \( \Phi^A(\theta) \), the left derivatives \( \delta_L/\delta \Phi^A(\theta) \) by the right–derivatives \( \delta_R/\delta \Phi^A(\theta) \), \( \gamma^A_B \) by \( \gamma^B_A \), and reversing the order of all the factors, then the corresponding linear right–representation \((U^a_{\pm}, U_a, U)\) of the \( sl(1,2) \)–algebra on the superfields is obtained. If the auxiliary fields \( \pi^{Aa} \) and \( \lambda^A \) finally are eliminated from the action by integrating them out in the functional integral, cf. Eq. (12) below, then the nonlinear (anti)BRST transformations re-appear [3].

3 \( osp(1,2) \)–covariant superfield quantization

In the superfield approach to the quantization of general gauge theories the \( m \)–dependent quantum action \( W_m(\Phi^A(\theta), \overline{\Phi}_A(\theta)) \) cannot be required to be invariant under the whole superalgebra \( sl(1,2) \). Instead, it will be required to be invariant under only one of its two \( osp(1,2) \)–subalgebras. \( W_m(\Phi^A(\theta), \overline{\Phi}_A(\theta)) \) is assumed to be invariant under a mass–dependent combination of translations and special conformal transformations, symplectic rotations and, eventually, dilatations in \( \theta^a \)–space. The generators \( \overline{\Delta}^a_m, \overline{\Delta}_a, \overline{\Delta}_m, \overline{\Delta}_a, \) respectively, of these symmetries will be introduced now explicitly.

The odd and even differential operators \( \overline{\Delta}^a_m, \overline{\Delta}_a, \) Eqs. (3), respectively, are given on the space of superfields \( \Phi^A(\theta) \) and superantifields \( \overline{\Phi}_A(\theta) \) as follows:

\[
\Delta^a = \int d^2 \theta \frac{\partial^2 \delta_L}{\partial \theta^2 \delta \Phi^A(\theta)} \theta^a \frac{\delta}{\delta \Phi^A(\theta)} = (-1)^{\epsilon_A} \frac{\delta}{\delta \phi^A} \frac{\delta}{\delta \phi^*_A}, \quad \overline{\Delta}^a_m = V^a + \frac{1}{2} m^2 V^a,
\]

\[
\Delta_\alpha = (-1)^{\epsilon_A+1} \int d^2 \theta \theta^2 (\sigma)_B^A \frac{\partial^2 \delta_L}{\partial \theta^2 \delta \Phi^A(\theta)} \frac{\delta}{\delta \Phi^A(\theta)} = (-1)^{\epsilon_A} (\sigma)_B^A \frac{\delta}{\delta \phi^A} \frac{\delta}{\delta \eta_B},
\]

with the translation operators \( V^a \) and the special conformal operators \( V^a \) given by Eqs. (8) and (9), and the operators of symplectic rotations \( V_a \) given by Eq. (10), respectively. The (second–order) differential operators \( \Delta^a, \overline{\Delta}_a \) are associated by two odd superantibrackets \((F,G)^a\) and by three even superbrackets \{\( F,G \}\}_a, respectively,

\[
(F,G)^a = (-1)^{\epsilon_A} \int d^2 \theta \left\{ \frac{\partial^2 \delta F}{\partial \theta^2 \delta \Phi^A(\theta)} \theta^a \frac{\delta G}{\delta \Phi^A(\theta)} - (-1)^{(\epsilon(F)+1)(\epsilon(G)+1)} (F \leftrightarrow G) \right\},
\]

\[
\{F,G\}_a = - \int d^2 \theta \left\{ \theta^2 \frac{\partial^2 \delta F}{\partial \theta^2 \delta \Phi^A(\theta)} \frac{\delta G}{\delta \Phi^A(\theta)} (\sigma)_B^A + (-1)^{(\epsilon(F)+1)} (F \leftrightarrow G) \right\}.
\]

The quantum action \( W_m(\Phi^A(\theta), \overline{\Phi}_A(\theta)) \) is required to obey the \( m \)–extended generalized quantum master equations (12) ensuring (anti)BRST invariance, and the generating equations (12) ensuring \( Sp(2) \)–invariance. The solution of these equations is sought of as a power series in Planck’s constant \( \bar{h} \), \( W_m = S_m + \sum_{n=1}^{\infty} \bar{h}^n W^{(n)}_m \), obeying the requirements of nondegeneracy of \( S_m \) and the correctness of the classical limit, i.e., that \( S_m \) coincides with the classical action \( S_{cl}(A) \) if the superantifields are put equal to zero (and the auxiliary fields \( \pi^{Aa} \) and \( \lambda^A \) are integrated out). According to the definition of the superantifields the action \( W_m \) depends on \( \eta_A \) only linearly.

The gauge fixed quantum action \( W_{m,ext}(\Phi^A(\theta), \overline{\Phi}_A(\theta)) \) is introduced according to

\[
\exp\{\{i/\bar{h}\}W_{m,ext}\} = \hat{U}_m(F) \exp\{\{i/\bar{h}\}W_m\},
\]

where the operator \( \hat{U}_m(F) \) has to be choosen as [3]

\[
\hat{U}_m(F) = \exp\{\bar{h}/i\} \hat{T}_m(F) \quad \text{with} \quad \hat{T}_m(F) = \frac{1}{2} \varepsilon_{ab}\{\overline{\Delta}^b_m, \overline{[\Delta}_m, F]\} + (i/\bar{h})^2 m^2 F,
\]
be constructed obeying new ghost number conservation being expressed by the equation:

\[ V_m \equiv V + m \frac{\partial}{\partial m}, \]

with the dilatation operator \( V \) given by Eq. \([11]\). The differential operator \( \Delta \) is associated by the following expression (being not a new bracket since \( \gamma_B^A \) is diagonal)

\[ \{F, G\} = - \int d^2 \theta \left( \theta^2 \gamma_B^A \frac{\partial^2 \delta F}{\partial \theta^2 \delta \Phi^A(\theta)} \frac{\delta G}{\delta \Phi_B(\theta)} + (-1)^{e(F)e(G)}(F \leftrightarrow G) \right). \]

The additional operator \( \tilde{\Delta}_m \) together with \( \tilde{\Delta}_m^a \) and \( \tilde{\Delta}_\alpha \) forms a superalgebra being isomorphic to \( osp(1,2) \oplus u(1) \) where, in addition to the (anti)commutation relations \([4]\), the following commutation relations hold true (analogously for \( (V_m^a, V_a, V_m) \)):

\[ [\tilde{\Delta}_m, \tilde{\Delta}_m] = 0, \quad [\tilde{\Delta}_m, \tilde{\Delta}_\alpha] = 0, \quad [\tilde{\Delta}_m, \tilde{\Delta}_m^a] = \frac{i}{\hbar} \tilde{\Delta}_m^a. \tag{14} \]

Let us now assume that solutions \( W_m \) of the quantum master equations \([1]\) and \([2]\) can be constructed obeying new ghost number conservation being expressed by the equation:

\[ \tilde{\Delta}_m \exp\{ (i/\hbar)W_m \} = 0 \quad \iff \quad \frac{1}{2} \{ W_m, W_m \} + V_mW_m = i\hbar \Delta_m W_m. \tag{15} \]

However, it is already well-known that the new ghost number is conserved only in the limit \( \hbar \to 0 \). In addition, the new ghost number conservation is broken also through gauge fixing \([3]\). Therefore, Eq. \([13]\) should be required only for the tree approximation of the quantum action; eventually, it could hold if no radiation corrections occur.

### 4 Generating functionals and gauge (in)dependence

The vacuum functional \( Z_m(0) \) in the super(anti)field approach is defined as

\[ Z_m(0) = \int d\Phi^A(\theta) \, d\Phi_A(\theta) \, \rho(\Phi_A(\theta)) \exp\{ (i/\hbar)(W_m - S_m,F + S_m,X) \}, \tag{16} \]

with

\[
S_m,F = \int d^2 \theta \left( \frac{\delta F}{\delta \Phi^A(\theta)} \frac{\partial^2 \Phi^A(\theta)}{\partial \theta^2} + \frac{1}{2} \epsilon_{ab} \int d^2 \tilde{\theta} \frac{\partial \Phi^A(\tilde{\theta})}{\partial \theta_a} \frac{\partial^2 F}{\partial \Phi^B(\theta) \partial \tilde{\theta}_b} \right) \]

\[
+ \frac{1}{2} m^2 \int d^2 \theta \theta^2 \frac{\partial^2 \delta F}{\partial \theta^2 \delta \Phi^A(\theta)} \gamma_B^A \Phi^B(\theta),
\]

and the measure given by \( \rho(\Phi^A) = \delta \left( \int d^2 \Phi(\theta) \right) = \delta(\eta_A) \).

The integrand in \([16]\) is invariant under the following global transformations:

\[ \delta \Phi^A(\theta) = \Phi^A(\theta) U_{m,m}\mu_a, \quad \delta \Phi_A(\theta) = \mu_a V_m^a \Phi_A(\theta) + \mu_a (W_m, \Phi_A(\theta))^a \]

\[ \delta \Phi^A(\theta) = \Phi^A(\theta) U_{a,\mu}^\alpha, \quad \delta \Phi_A(\theta) = \mu^\alpha V_a \Phi_A(\theta) + \mu^\alpha (W_m, \Phi_A(\theta))_\alpha, \]

\[ \delta \Phi^A(\theta) = \Phi^A(\theta) U_{m,m}\mu_a, \quad \delta \Phi_A(\theta) = \mu_a V_m^a \Phi_A(\theta) + \mu_a (W_m, \Phi_A(\theta))^a \tag{17} \]

\[ \delta \Phi^A(\theta) = \Phi^A(\theta) U_{a,\mu}^\alpha, \quad \delta \Phi_A(\theta) = \mu^\alpha V_a \Phi_A(\theta) + \mu^\alpha (W_m, \Phi_A(\theta))_\alpha, \tag{18} \]
where $\mu_a, \epsilon(\mu_a) = 1$, and $\mu^a, \epsilon(\mu^a) = 0$, are constant anticommuting resp. commuting parameters. These transformations realize the $m$–extended (anti)BRST– and $Sp(2)$–symmetry, respectively, in the superfield approach to $osp(1, 2)$–covariant quantization.

Let us now change the gauge fixing functional in (16) according to $F \to F + \delta F$ followed by the transformations (17) with the choice $\mu_a = -\frac{i}{\hbar} \frac{1}{2} \epsilon_{ab} (\delta F) U^b_m$. This leads to

$$S_{m,F} \to S_{m,F} + \left[ \frac{1}{2} \epsilon_{ab} (\delta F) U^b_m U^a_m + m^2 \delta F \right] + (\hbar/i) \mu_a U^a_m = S_{m,F} + m^2 \delta F.$$  

Thus we observe that the mass term $m^2 F$ in Eq. (13) violates the independence of $Z_m(0)$ on the choice of the gauge. Unfortunaley, that unwanted term can not be compensated by any further change of variables, thus showing that it breaks gauge independence of the $S$–matrix. However, gauge independence is restored in the limit $m \to 0$, i.e., $s \to 1$, which has to be taken after having carried out all the ultraviolet and infrared subtractions.

One of the virtues of the quantization scheme presented here is that, first, during the process of renormalization the $osp(1, 2)$–symmetry of the theory is maintained and, second, this enlarged symmetry – in comparison with the usual field-antifield formalism – allows for a much easier, algebraic proof of possible absence of anomalies. This formalism has been successfully applied to the instanton sector of QCD [9] and to the quantization of Yang-Mills theories in a generic background configuration [10].

**Acknowledgement**

One of the authors (B.G.) thankful acknowledges financial support by German-Brazil exchange program of FAPESP and DAAD during his stay at Institute of Physics of São Paulo University.

**References**

[1] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *J. Math. Phys.* **31** (1990) 1487; **32** (1991) 532

[2] B. Geyer, P.M. Lavrov and D. Mülsch, *J. Math. Phys.* **40** (1999) 674; **40** (1999) 6189

[3] M. Scheunert, W. Nahm and V. Rittenberg, *J. Math. Phys.* **18** (1976) 146, 155; L. Frappat, P. Sorba and S. Sciarrino, *Dictionary on Lie Superalgebras*, hep-th/9607161

[4] for a review see: O. Piguet and A. Rouet *Phys. Repts.* **76** (1981) 1; O. Piguet and S.P. Sorella, *Algebraic renormalization*, Springer, Berlin 1995

[5] F. De Jonghe, J. Paris and W. Troost, *Nucl. Phys.* **B 476** (1996) 559

[6] B. Geyer and D. Mülsch, *J. Math. Phys.* **41** (2000) 7304

[7] L. Baulieu, W. Siegel and B. Zwiebach, *Nucl. Phys.* **B 287** (1987) 93

[8] P.M. Lavrov, *Phys. Lett.* **B 366** (1996) 160

[9] D. Mülsch, *On the interpolation between quantum chromodynamics with the instanton and the meron as background field and the quantum chromodynamics in the perturbative vacuum*, PhD thesis, Leipzig 1996 (in German)

[10] B. Geyer and D. Mülsch, *Nucl. Phys.* **B 564** (2000) 517