Semi-analytical description of formation of galaxies and clusters of galaxies

M. Demiański1,2* and A. G. Doroshkevich3

1 Institute of Theoretical Physics, University of Warsaw, PL-00-681 Warsaw, Poland
2 Department of Astronomy, Williams College, Williamstown, MA 01267, USA
3 Astro Space Center of Lebedev Physical Institute of Russian Academy of Sciences, 117997 Moscow, Russia

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ABSTRACT

We apply the well-known semi-analytical model of formation of the Dark Matter (DM) haloes to discuss properties of the relaxed objects dominated by the DM component (such as the first and dwarf spheroidal galaxies (dSph) and/or clusters of galaxies). This approach allows us to obtain a simple but more detailed description of evolution of the first galaxies. It also reveals links between the observed characteristics of the relaxed DM haloes and the initial power spectrum of density perturbations. Results of our analysis of the observed properties of ~40 DM dominated galaxies and ~100 clusters of galaxies are consistent with the Λ cold dark matter like power spectrum of initial perturbations down to the scale of ~10 kpc. For the DM dominated objects the scaling relations are also discussed.

Key words: cosmology: theory – dark ages, reionization, first stars – dark matter – large-scale structure of Universe.

1 INTRODUCTION

The formation and evolution of the first galaxies at redshifts $z \geq 8$ is one of the most interesting problems of modern cosmology and it is closely connected with many other unresolved problems. Among others there are the secondary ionization of the Universe at redshifts $z_i \simeq 10$ implied by the Wilkinson Microwave Anisotropy Probe observations (Komatsu et al. 2011; Larson et al. 2011) and recently confirmed by the Planck mission (Ade et al. (Planck Collaboration) 2013), the formation and evolution of stars with the primeval chemical composition, the matter enrichment by metals, evolution of observed galaxies at redshifts $z \geq 6$, the high redshifts observations of massive galaxies and super massive black holes and so on (see, e.g. Wiklind et al. 2008; Mancini et al. 2009; Ouchi et al. 2009; Trenti, Stiavelli & Shull 2009; Vestergaard & Osmer 2009; Gonzalez et al. 2010, 2012; Haiman 2010; Kelly et al. 2010; Schaerer & de Barros 2010; Trenti et al. 2010; Bouwens et al. 2011; Shull et al. 2011; Barone-Nugent et al. 2013; Ellis et al. 2013; Oesch et al. 2013; Wyithe et al. 2013a,b; Salvadori et al. 2014).

It is specially interesting that observations of the farthest quasars and galaxies show that the reionization of the hydrogen fraction of the intergalactic matter had just been completed already at $z \simeq 3$ (Jakobsen et al. 1994; Hogan, Anderson & Rugers 1997; Smette et al. 2002; Fan et al. 2004; Fan, Carilli & Keating 2006; Furlanetto & Peng Oh 2008; Trenti et al. 2009; Lehnhert et al. 2010; Robertson et al. 2010).

This means that at least at $z \sim 5$–7 the ultraviolet (UV) radiation with energy $h\nu \geq 50$ eV is weak and it is mainly generated by quasars at $z \lesssim 4$. The current status of these problems is discussed in many recent reviews (see, e.g. Bromm & Yoshida 2011; Johnson 2011; Kravtsov & Borgani 2012).

It is very difficult to find or even estimate characteristics of the first galaxies formed from matter of the primordial chemical composition. The absence of metals makes cooling of matter and formation of stars more difficult and leads to well-known peculiarities in the evolution of such objects. First of all it is the high typical mass of first stars (100–1000 $M_\odot$) and significant energy that they eject in the UV region and during the ultimate explosion as supernovae (see, e.g. Tumlinson, Venkatesan & Shull 2004; Trenti & Shull 2010; Bromm & Yoshida 2011). At the same time radiation of these stars generates the Lyman–Werner (LW) and infrared (IR) backgrounds that destroy the H$^-$ ions and $H_2$ molecules what delays the cooling of matter and slows down the process of star formation.

Some observations (e.g. Bouwens et al. 2011) show that the observed rate of star formation and predicted UV radiation cannot ionize the Universe at $z \sim 10$. Alternative explanation assumes that properties of galaxies at small and high redshifts can be quite different (Ouchi et al. 2009; Gonzalez et al. 2010, 2012; Schaerer & de Barros 2010) and that low luminosity galaxies are dominant during the epoch of reionization. However, this would lead to more efficient generation of LW and IR backgrounds what in turn would slow down formation of the low-mass galaxies. Perhaps a more promising way to alleviate this problem is to take into account the non-thermal radiation of matter accreted on to black holes what changes the spectrum of UV background, decreases the LW background and
promotes the ionization of the Universe. Perhaps this effect can be observed as small distortion of the background light caused by emission in the He lines such as \( \lambda = 304 \) and 584 Å shifted to the redshift of reionization \( z_\text{r} \sim 10-15 \). More detailed discussion of these problems can be found in Meiksin (2009), Trenti et al. (2009, 2010), Shull et al. (2011), Giallongo et al. (2012), Barone-Nugent et al. (2013) and Ceverino et al. (2013).

Numerical simulations provide a powerful method of investigation of the epoch of reionization (e.g. Wise & Abel 2007, 2008; Greif et al. 2008) and make it possible to study the evolution of first galaxies in more detail. In particular, they allow us to investigate the early anisotropic stages of halo formation, to trace the process of the halo virialization, formation of its internal structure and early stages of protostar formation. Such analysis can be performed in a wide range of halo masses and redshifts what allows us to improve the description of properties of relaxed haloes of galactic scale and to link them with the power spectrum of initial perturbations.

However, possibilities of such simulations are strongly limited. These simulations are performed within a small box with the comoving size \( L \sim 0.7-1.5 \text{ Mpc} \) what corresponds to the box mass \( M_{\text{box}} \sim 10^{10} \text{M}_\odot \). So small box size artificially suppresses the large-scale perturbations and the formation of more massive objects, what strongly distorts the simulated mass function and increases the expected number of low-mass objects. Small box also distorts the influence of neighbouring objects, the radiation transfer and the feedback of UV, LW and IR backgrounds. The star formation, their radiation, explosion and metal production cannot be simulated and are introduced by hand as independent factors. This list of limitations can be continued.

It is therefore interesting to consider a more rough but simple model of formation and evolution of early galaxies. In this paper we propose to use for such analysis the semi-analytic approach based on the approximate analytical description of the basic DM structure of collapsed haloes and numerical estimates of the thermal evolution of the baryonic component. During the last fifty years similar models have been considered and applied to study various aspects of non-linear matter evolution (see, e.g. Peebles 1983; Fillmore & Goldreich 1984; Gurevich & Zybin 1995; Bryan & Norman 1998; Lithwick & Dalal 2011). In this regard it is important to note that the DM haloes are formed before stars appear (see, e.g. Kaviraj et al. 2013).

Formation of the DM haloes is a complex process with strongly anisotropic matter collapse during both earlier and final periods. Moreover sometimes this process is interrupted by the violent merging of neighbouring haloes. This implies that the simple spherical model of halo formation cannot adequately describe the present observational data. However properties of the steady state virialized DM objects are mainly determined by the integral characteristics of protoobjects and are only weakly sensitive to details of their evolution. This is clearly seen in numerous simulations which show that the Navarro–Frenk–White (NFW) density profile is very stable and is formed in majority of the simulated DM haloes.

The same simulations also show that properties of the cores of virialized DM haloes are established during the early period of haloes formation and later on the slow pseudo-evolution of haloes dominates (see, e.g. Diemer, More & Kravtsov 2013). This means that properties of halo cores only weakly depend on the halo periphery and are determined mainly by their mass and the redshift of formation (Klypin, Trujillo-Gomez & Primack 2011). Using these results we formulate a rough two parametric description of all the basic properties of the DM haloes. These parameters are the virial mass of haloes and the redshift of their formation.

First of all this approach allows us to reveal the close correlation between the redshift of formation and virial masses of haloes with the initial power spectrum of density perturbations. It also allows us to reconsider some of the widely discussed scaling relations between observed characteristics of galaxies (see, e.g. Spanel et al. 2008; Donato et al. 2009; Gentile et al. 2009; Hyde & Bernardi 2009; Mosleh et al. 2011; Salucci et al. 2012; Bezanson et al. 2013). Usually they are related to properties of luminous matter, such as Fundamental Plane, luminosity–velocity dispersion, or mass–size relations. None the less they actually characterize the mass and entropy profile of haloes and its formation in the course of violent relaxation of the compressed DM component.

Of course this approach is applied for the DM dominated haloes only as the dissipative evolution of the baryonic component distorts properties of the cores of the DM haloes. In spite of this we can use this model in three ways.

(i) The density and temperature profiles of the DM component can be considered as the initial conditions for numerical analysis of the process of cooling and compression of the baryonic clouds within the stable DM haloes. In particular, with this approach it was possible to estimate the evolution of the Jeans mass of cold baryons down to the masses of Population III (Pop. III) stars.

(ii) We can use the redshift of the DM haloes formation as a parameter that characterizes the ‘frozen’ properties of the central region of the DM haloes. This redshift correlates with the virial mass of haloes and so with the initial power spectrum. Thus this approach allows us to reveal the impact of initial conditions on the observed characteristics of the DM dominated objects.

(iii) This approach allows us to also clarify some of the widely discussed scaling relations that are applied to the DM dominated objects.

However potential of such approach should not be overestimated. Thus clusters of galaxies presented in Pratt et al. (2009) illustrate large scatter of matter distribution in the observed DM haloes. This paper is organized as follows. In Section 2 the basic relations and assumptions of our approach are formulated and the expected properties of the DM haloes are presented. Properties and evolution of the baryonic component are described in Section 3. Mass dependence of the redshift of formation and the scaling relations for observed object are considered in Section 4. Discussion and conclusions can be found in Section 5.

1.1 Cosmological parameters

In this paper we consider the spatially flat \( \Lambda \) cold dark matter (CDM) model of the Universe with the Hubble parameter, \( H(z) \), the critical density \( \rho_c \), the density of non-relativistic matter, \( \langle \rho_m(z) \rangle \), and the mean density and mean number density of baryons, \( \langle \rho_b(z) \rangle \), given by

\[
H^2(z) = H_0^2[\Omega_m(1 + z)^3 + \Omega_k], \quad H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1},
\]

\[
\langle \rho_m(z) \rangle = 2.5 \times 10^{-27} \Omega_m \theta_m \frac{g}{\text{cm}^3} = 3.4 \times 10^3 \Omega_m \theta_m \frac{\text{M}_\odot}{\text{kpc}^2},
\]

\[
\langle \rho_b(z) \rangle = \frac{3H_0^2}{8\pi G} \Omega_b(1 + z)^3 \approx 4 \times 10^{-27} \Omega_b \theta_b \frac{g}{\text{cm}^3}.
\]

\[
\rho_c = \frac{3H_0^2}{8\pi G}, \quad \Omega_m = \frac{H_0^2}{H_0^2}, \quad \Omega_b = \frac{\Omega_b h^2}{0.02}.
\]

Here \( \Omega_m = 0.24 \) and \( \Omega_b = 0.76 \) are dimensionless density of non-relativistic matter and dark energy; \( h \approx 0.04 \) and \( h = 0.7 \) are the
dimensionless mean density of baryons and the Hubble constant measured at the present epoch. Cosmological parameters presented in recent publication of the Planck collaboration (Ade et al. 2013) slightly differ from those used above equation (1).

For the \( \Lambda \)CDM cosmological model the evolution of perturbations can be described with sufficient precision by the expression

\[
\frac{\delta \rho}{\rho} \propto B(z), \quad B(z)^{-3} = \frac{1 - \Omega_m + 2.2\Omega_m (1 + z)^3}{1 + 1.2\Omega_m} \tag{2}
\]

(Demianski & Doroshkevich 1999, 2004; Demianski et al. 2011) and for \( \Omega_m \approx 0.25 \) we get

\[
B^{-1}(z) \approx \frac{1 + z}{1.35} \left[ 1 + 1.44/(1 + z)^3 \right]^{1/3}. \tag{3}
\]

For \( z = 0 \) we have \( B = 1 \) and for \( z \geq 1 \) \( B(z) \) is reproducing the exact value with accuracy better than 90 per cent.

For \( z \gg 1 \) these relations simplify. Thus, for the Hubble constant and the function \( B(z) \) we get

\[
H^{-1}(z) \approx \frac{2.7 \cdot 10^{16}}{\sqrt{\Omega_m}} \left[ \frac{10}{1 + z} \right]^{3/2}, \quad B(z) \approx \frac{1.35}{1 + z}. \tag{4}
\]

2 PHYSICAL MODEL OF HALOES FORMATION

As is commonly accepted in the course of complex non-linear condensation the DM forms stable virialized haloes with more or less standard density profile. Numerical simulations show that after short period of rapid evolution the structure of the virialized DM haloes is quite well described by the spherical model with the NFW density profile (Navarro, Frenk & White 1995, 1996, 1997; Ludlow et al. 2013). The basic parameters of the model – the virial mass, \( M_{\text{vir}} \), central density, \( \rho_c \), core scale \( r_c \), and concentration, \( c \), – were fitted in a wide range of redshifts and halo masses in many papers (see, e.g. Klypin et al. 2011).

After the completion of active phase of halo formation its parameters only weakly depend on the redshift and at large radius a steeper asymptotic density profile is formed

\[
\rho(r) \propto r^{-4}
\]

(see, e.g. discussion in Visbal, Loeb & Hernquist 2012). But as before the central regions of haloes are described by the NFW profile. For example, such virialized objects are observed as isolated galaxies with the rotation curve \( v_c \propto r^{-1/2} \) and/or as high density galaxies within clusters of galaxies, filaments or other elements of the large-scale structure of the Universe.

These results can be successfully used to roughly estimate the mean density and temperature of early galaxies what in turn allows us to concentrate main attention on the thermal evolution of the compressed gas. In this way we can consider the evolution of baryonic component and the formation of the first stars for a wide range of redshifts and virial masses of the DM haloes. Evidently similar approach can be applied also for investigations of more complex evolution of haloes with the metal enriched baryonic component. This approach allows us also to estimate the redshift when the observed DM dominated objects such as the dSph galaxies and clusters of galaxies were formed.

It is important that the standard description of both the observed and simulated DM haloes links the virial mass and radius of haloes by the condition

\[
M_{\text{vir}} = 4\pi/3 R_{\text{vir}}^3 \Delta_v \rho_c(z_{\text{vir}}), \tag{5}
\]

where \( z_{\text{vir}} \) is the redshift of relaxation of the DM halo, \( \Delta_v = 18\pi^2 \approx 200 \) and \( \rho_c(z_{\text{vir}}) \) is the critical density of the Universe at this redshift. This relation can be applied for haloes embedded in a homogeneous medium – early galaxies and clusters of galaxies – and the value of \( \Delta_v \) was derived from the small model of spherical collapse that ignores the influence of complex anisotropic haloes environment (see, e.g. Bryan & Norman 1998; Vikhlinin et al. 2009; Lloyd-Davies et al. 2011).

Of course, this approach has only limited predictive power. Thus, it ignores the complex anisotropic matter compression within filaments and walls before formation of compact haloes, it ignores the effects produced by mergers and so on. These restrictions do not allow us to consider the process of generation of the angular momentum of the compressed matter and to link the properties of the DM haloes and the rate of star formation with the primordial characteristics of collapsed matter such as the anisotropic shape of density peaks and their environment, the internal velocity dispersion and so on. None the less with this approach further progress in the description and understanding of the complex processes of formation of the DM haloes and early galaxies can be achieved.

2.1 Internal structure of the DM haloes

Further on we consider the virialized spherical DM haloes characterized by the virial mass \( M_{\text{vir}} = 10^9 M_\odot M_\odot \) at the (conventional) redshift of formation \( z = z_{\text{cr}} \). For any model the universal mass, \( M \), and density, \( \rho \), profiles of the virialized DM halo can be taken as follows:

\[
M(x) = M_{\text{vir}} f_\rho(x), \quad \rho(x) = \rho_c f_\rho(x), \quad M_v = 4\pi \rho_c r_c^3. \tag{6}
\]

Here \( x = r/r_c \) and \( r_c(M_{\text{vir}}, z_{\text{cr}}) \) and \( \rho_c(M_{\text{vir}}, z_{\text{cr}}) \) are the typical size and density of the haloes cores. For the popular NFW model the density and mass profiles are

\[
f_\rho = x^{-1}(1 + x)^{-2}, \quad f_m = \ln(1 + x) - \frac{x}{1 + x}, \tag{7}
\]

and \( f_m(5) \approx 1 \). For another popular model (Burkert 1995) the density profile is

\[
f_\rho = (1 + x)^{-1}(1 + x^2)^{-1}, \quad f_m = \ln \left[ (1 + x)\sqrt{1 + x^2} \right] - tg^{-1}(x), \quad f_m(5) \approx 1.8. \tag{8}
\]

These models can be used for \( x \leq 5–6 \). From these expressions it follows that at \( x \geq 1 \) differences between these models are quite moderate and our results obtained below for the NFW model can be applied with small corrections also for the Burkert model. More detailed discussion of these models can be found in Penarrubia et al. (2010).

Both the DM and baryonic components are treated as the ideal gas with the pressure, \( P \), temperature, \( T \), and the entropy function, \( S \), linked by the usual expressions for the non-relativistic particles:

\[
P(x) = n(x) T(x) = S(x) n^{5/3}(x),
\]

\[
P(x) = P_c f_p(x), \quad T(x) = T_c, \quad f_p(x), \quad S(x) = S_c f_s(x), \tag{9}
\]

where the typical temperature, \( T_c(M_v, z_{\text{cr}}) \), entropy, \( S_c(M_v, z_{\text{cr}}) \), pressure, \( P_c(M_v, z_{\text{cr}}) \) and the number density of the DM component, \( n_{\text{DM}}(M_{\text{vir}}, z_{\text{cr}}) f_p(x) \), or baryons, \( n_b(M_v, z_{\text{cr}}) f_b(x) \), also depend upon the virial halo mass \( M_v \) and its redshift of formation, \( z_{\text{cr}} \).

Random variations of the profile and amplitude of the initial velocity, the initial shape of collapsed clouds, properties of outer regions of haloes and so on lead to random variations of haloes.
density, shape, profile and other parameters relative to the accepted mean characteristics. The analysis of available simulations shows that the probability distribution functions of these variations are often close to the exponential ones and therefore their random scatter is close to the mean values (see, e.g. Press & Rybicki 1993; Demiański et al. 2011). These random variations can change the real parameters by a factor of 2 to 3.

2.2 Simple model of early galaxies

In this paper we consider a simple physical model of formation of galaxies based on the following assumptions.

(i) We assume that at redshift \( z = z_{cr} \) the evolution of the DM perturbations results in the formation of spherical virialized DM haloes with mass \( M_{\text{vir}} = M_9 \cdot 10^9 M_\odot \) and the density profile (7).

(ii) We do not discuss the dynamics of the DM haloes formation and evolution which are accompanied by the progressive matter accretion, the growth of the haloes masses and corresponding variations of other haloes parameters. The real process of haloes formation is extended in time what causes some ambiguity in their parameters such as the haloes masses and the redshift of their formation (see, e.g. discussion in Diemand, Kuhlen & Madau 2007; Kravtsov & Borgani 2012). In the proposed model the redshift of halo formation, \( z_{cr} \), is identified with the redshift of collapse of the homogeneous spherical cloud with the virial mass \( M_{\text{vir}} \),

\[
1 + z_{cr} \approx 0.63(1 + z_0),
\]

where \( z_0 \) is the redshift corresponding to the turn around moment of the cloud evolution (see discussion in Umemura, Loeb & Turner 1993).

(iii) We assume that in the course of the DM halo formation the main fraction of the baryonic component is heated by the accompanied shock waves up to the temperature and pressure comparable with the virialized values of the DM component. These processes provide the formation of equilibrium distribution for the baryonic component.

(iv) We assume that some part of the compressed baryons is disrupted into a system of subclouds which are rapidly cooled and transformed into high density star-like subclouds. Thus, the virialized halo configuration is composed of the DM particles, the adiabatically compressed hot low density baryonic gas, and cold high density baryonic subclouds.

(v) Following Hutchings et al. (2002) we assume that the cooling of the high density baryonic subclouds with both atomic (H\&He) and molecular (\( \text{H}_2 \)) coolants proceeds under the condition that \( P \approx \text{const} \). Such cooling of the low-mass subclouds corresponds to the isobaric mode of the thermal instability.

(vi) We assume that the subclouds with masses larger than the Jeans mass \( M_J(\tau, T) \) rapidly form first stars with masses \( 100–1000 M_\odot \) what transforms the DM haloes into the early galaxies.

The evolution of the cooled low-mass subclouds can be very complex. It can be approximated by the isobaric mode of the thermal instability and therefore it does not preserve the compact shape of the cooled subclouds. As was discussed in Doroshkevich & Zel’dovich (1981) the motion of such subclouds within the hot gas leads to their deformation and less massive subclouds could be even dissipated. The complex aspherical shape of such subclouds makes their survival problematic and requires very detailed investigation to predict their evolution. It is very difficult to estimate also the efficiency of transformation of protostars into stars. These problems are beyond the scope of this paper.

In central regions of haloes the gas pressure is supported by adiabatic inflow of high entropy gas from outer regions of haloes what leads to progressive concentration of baryonic component within central regions of haloes and to formation of massive baryonic cores (see, e.g. Wise & Abel 2008; Pratt et al. 2009). The structure and evolution of such haloes resemble the internal structure of the observed and simulated galaxies and cluster of galaxies (see, e.g. Pratt et al. 2009, 2010; Arnaud et al. 2010; Kravtsov & Borgani 2012).

2.3 Mean characteristics of the DM haloes

In this section we consider evolution of the central regions of the virialized DM haloes using the NFW approximations presented in Klypin et al. (2011), Prada et al. (2011) and Angulo et al. (2012). For such haloes of galactic scale and at \( z \gg 1 \) the matter concentration \( c(M_9, z_9) \) is described by the expression

\[
c = \frac{R_{\text{vir}}}{r_s} \approx 4 M_9^{1/6} \rho_s^{2/3} \delta_{5/3}^{1/3},
\]

\[
z_f = (1 + z_{cr})/10,
\]

\[
\epsilon(M_9, z_9) \approx 1 + 0.31 M_9^{-1/4} z_9^{-2}, M_9 = M_{\text{vir}}/10^9 M_\odot.
\]

Here \( R_{\text{vir}} \) is the virial radius of halo and the factor \( \delta_{5/3} \approx 0.6–1.4 \) characterizes the random scatter of the matter concentration caused by the random variations of characteristics of the outer regions of haloes. It is important that here we consider expected properties of early galaxies for which \( M_9 z_9^2 \gtrsim 1, \epsilon \sim 1 \). The usually discussed model of clusters of galaxies with \( \epsilon \gg 1 \) is considered below in Section 4.

Using the standard relations (5) and (6)

\[
M_{\text{vir}} = 4\pi \rho_s r_s^3 f_m(c) = 4\pi/3 r_s^3 \rho_s \approx \Delta_c(\rho_m),
\]

we find the virial and core sizes of the halo,

\[
R_{\text{vir}} \approx 3.34 \frac{M_9^{1/3}}{z_f} \text{kpc},
\]

\[
r_s = \frac{R_{\text{vir}}}{c} = \frac{0.8 M_9^{1/6}}{z_f^{10/3} \delta_{5/3}^{1/3}} \text{ kpc}.
\]

For the central density of the DM, \( \rho_c \), and its number density, \( n_{\text{DM}} \), we get

\[
\rho_c(M, z) \approx \frac{\rho_m}{3 f_m(c)} \Delta_c \epsilon^3 \approx 1.3 \cdot 10^9 z_9^{10} M_9^{1/2} \Theta_\rho \frac{M_\odot}{\text{kpc}^2},
\]

\[
n_{\text{DM}} = \frac{n_c}{m_{\text{DM}}} = 6 z_9^{10} M_9^{1/2} \Theta_\rho \frac{m_p}{m_{\text{DM}}} \text{cm}^{-3}, \Theta_\rho = \frac{\delta_c \epsilon^3}{f_m(c)}.
\]

Here \( \delta_c \sim 0.1–3 \) characterizes the discussed above random variations of the halo parameters relative to the mean characteristics presented by (13).

As was noted in the introduction the important characteristic of the DM haloes is the central surface density of the DM component, \( \mu_{cs} \) (Donato et al. 2009; Salucci et al. 2012)

\[
\mu_{cs} = r_s \rho_c \approx 10^7(M_9 z_9^{10} \delta_{5/3}) \Theta_\rho \frac{M_\odot}{\text{pc}^2},
\]

\[
\Theta_\rho = \frac{\delta_c \epsilon^3}{f_m(c)}. \text{ The similar virial surface density,}
\]

\[
\mu_{\text{vir}} = R_{\text{vir}} \rho_{\text{vir}} = 22.7 M_9^{1/3} z_9^{2} \Theta_\rho \frac{M_\odot}{\text{pc}^2},
\]

is close in some respects with such phenomenological concepts as the Fundamental Plane or mass–size relation which also are discussed in many publications (see, e.g. Hyde & Bernardi 2009; Mosleh et al. 2011; Bezanson et al. 2013).
The velocity dispersion, $\sigma_v^2(r)$, and the temperature, $T_{\text{DM}}(r)$, within the relaxed DM halo with the NFW density profile are closely linked to the circular velocity of the DM haloes, $v_c^2(r)$, 
\[
\frac{G(M(r))}{r} = \frac{\sigma_v^2(r)}{\sigma_x^2} = \frac{\sigma_x^2}{2} = \frac{\sigma_0^2}{2x^{3/2}},
\]
where $\sigma_0^2 \approx 5 \cdot 10^3 M_\odot < 10 \Theta T \text{ km s}^{-1}$. Thus for the temperature of the DM component with Maxwellian velocity distribution we have 
\[
T_{\text{DM}} = m_{\text{DM}} \sigma_0^2/3, \quad f_T(x) = f_m(x)/x^{3/2}, \quad T_c \approx m_{\text{DM}} \sigma_0^2/6 \approx 8 eV M_\odot < 10 \Theta T T \text{ cm}^{-3}.
\]
For the pressure of the DM component we get 
\[
P_{\text{DM}} = m_{\text{DM}} T_{\text{DM}} = P_c f_p(x). \quad f_p(x) = f_T(x)/f_m(x), \quad P_c \approx 50(M_\odot < 10 \Theta T eV \text{ cm}^{-3}, \quad \Theta_T = \Theta_p \Theta_T.
\]
Here $m_{\text{DM}}$ and $m_b$ are the masses of the DM particles and baryons. For the NFW model the pressure at the centre of halo is divergent, $f_p(x) \propto x^{-1/2}$, which is another manifestation of the known core-cusp problem. This artificial divergence does not prevent the use of estimates (16) and (17) in our further discussions.

As is seen from this analysis the NFW haloes are a two parametric model with $\alpha$ and $\beta$ being the only parameters. The central density and entropy of the adiabatically compressed gas depend upon the mass and the parameter $\epsilon_{\text{c}}$ instead of the pressure.

The variations of the central density profile only weakly influence the characteristics of the matter distribution in haloes at $x \geq 1$ and thus to obtain typical variations for any haloes we can use the relations (11)–(17). In this case for Burkert profile the function $f_p(x)$ at $x \leq 1$ is close to that obtained in (17) for the NFW model.

This means that at $x \geq 1$ differences between results obtained for various density profiles are in the range of random scatter. Results obtained for the generalized NFW model (Nagai, Kravtsov & Vikhlinin 2007) confirm that numerical results are only weakly sensitive to moderate variations of the pressure and density profiles.

3.1 Adiabatically compressed baryonic component

In virialized haloes the pressures of baryonic and DM components are equal to each other, $P_b = P_{\text{DM}}$. Therefore for a given $P_b$ two properties of the adiabatically compressed gas depend upon the relic entropy of baryonic component, $S_{\text{rel}}$. This entropy can be determined from the condition that at $z \approx 100$–300 the temperatures of baryons and relic radiation are close to each other. For example at $z = 100$ we have $T_\text{r} = 100 \approx 2.7 \cdot 10^3 \approx 0.23 \text{ eV}$, $(n_b(100)) \approx 0.24 \text{ cm}^{-3}$ and the relic entropy of baryons
\[
S_{\text{rel}} \approx \Theta_1 T_b(n_b(z))^{-3/2} \approx 0.06 \text{ eV cm}^2 100^{1/4} z + 1, \quad (19)
\]

At the same time the relic (frozen) concentration of electrons and protons at $z \leq 100$ is small, $f_e = f_e' = 10^{-4}$ what decelerates the formation of H$_2$ molecules and prevents cooling of the baryonic component below the temperature $T_b(z) \leq 10^4$ K. This means that for this component
\[
P_b(x) = P_{\text{DM}}(x), \quad T_b = T_{\text{bc}} f_p^{1/5}(x), \quad n_b = n_{\text{bc}} f_p^{1/5}(x),
\]
where the function $f_p(x)$ is determined by (17, 18) and
\[
T_{\text{bc}} = S_{\text{rel}}(P_b/S_{\text{bc}})^{1/5} \approx 0.9(M_\odot < 10^{8} \Theta_p eV, \quad n_{\text{bc}} = (P_b/S_{\text{bc}})^{1/5} \approx 60(M_\odot < 10^{8} \Theta_p eV, \quad \Theta_p = \theta_{\text{bc}} \theta_{\text{bc}} c(S_{\text{rel}}/S_{\text{bc}})^{1/5}.
\]

If the mass and the redshift of formation of the clouds of gas are limited by the condition
\[
T_b(z) \leq 10^4 \text{ K},
\]
or by the corresponding restriction for the pressure
\[
P \leq 50 \text{ eV cm}^{-3} (T_b/10^4 \text{ K})^{1/2} (S_{\text{bc}}/S_{\text{rel}})^{1/2},
\]
\[
M(<r) \leq \Theta_p^{3/4} (T_b/10^4 \text{ K})^{1/4} (S_{\text{rel}}/S_{\text{bc}}), \quad (21)
\]
then the cloud evolves in the adiabatic regime and the Jeans mass of the collapsed clouds $M_J$ remains large
\[
S_{\text{rel}} \approx S_{\text{bc}}, \quad M_J \approx 2.4 \cdot 10^5 M_\odot (1 \text{ cm}^{-3}/n_b)^{1/2}.
\]

In this case formation of the real stars is strongly suppressed.

However, if the condition (21) is violated and the collapsed gas is heated up to temperature $T_b(z) \geq 10^4$ K then the concentrations of both electrons and H$_2$ molecules are rapidly increasing, gas is cooled and forms the high density low-mass baryonic subclouds. The same process rapidly occurs also when the matter ionization is caused by external sources of the UV radiation.

3.2 Formation of high density baryonic subclouds

Semi-analytical description of formation
Instability of the shock compression and heating of baryons and their subsequent cooling lead to their fragmentation and formation of subclouds with high \( n_b \) and small \( T_h \). If the formation of the DM halo occurs with the cosmological characteristic time,

\[ t_{\text{com}} = H^{-1}(z) \approx 2.7 \cdot 10^{16} z_f^{-3/2} \text{s}, \]

then the characteristic hydrodynamical time for the shock compressed baryonic component at \( z_f \geq 1 \) is much smaller,

\[ \tau_{\text{hyd}} = \frac{1}{\sqrt{4\pi G \rho_b}} \sim \frac{1.4 \cdot 10^{14}}{M_9^{1/4} z_f^{3/2}} f_p^{-1/2}(x) \ll H(z)^{-1}, \tag{25} \]

and cooling of the highly ionized compressed gas occurs even more rapidly. For instance for the free–free cooling the characteristic time, \( \tau_{\text{ff}} \), is

\[ \tau_{\text{ff}} \approx 5 \cdot 10^{13} \frac{x^{3/4}}{z_f^{3/2}} \frac{M_9^{9/4}}{\rho_b} f_p^{-1/2}(x) \ll \tau_{\text{hyd}}. \tag{26} \]

The impact of other atomic processes (recombination and excitation of H and He, etc.) significantly decreases even this characteristic time. The characteristic time for the hydrogen recombination is also quite small,

\[ \tau_{\text{rec}} \approx 3.1 \cdot 10^{11} \frac{x^{3/4}}{z_f^{3/2}} M_9^{9/4} \frac{\Theta_f f_p(x)}{\sqrt{\Theta_f f_p(x)}} \ll \tau_{\text{hyd}}, \]

what ensures almost equilibrium ionization of baryons throughout the period of thermal evolution for \( T_b \geq 10^4 \text{ K} \).

However for temperature \( T_b \leq 10^4 \text{ K} \) the cooling process determined by the molecular hydrogen is slower and the characteristic time of gas cooling becomes comparable with \( \tau_{\text{hyd}} \) (25). This means that the gas pressure within cold subclouds is retained near \( P_e(x) \) (23) and the cooling is accompanied by a corresponding growth of baryonic density.

Owing to the thermal instability such evolution strongly favours further fragmentation of the cooled gas. The gravitationally bounded cold high density subclouds with masses larger than the Jeans mass, \( M \geq M_1 \), could be transformed into stars. The formation of stars with the mass \( M_1 \) is regulated by the drop of temperature, \( T_4 = T_b/10^4 \text{ K} \),

\[ M_1 \geq M_1 \approx 2 \cdot 10^5 \text{ M}_\odot \frac{T_4^{3/2}}{n_n^{-1} n_3} = 2.5 \cdot 10^6 \text{ M}_\odot \sqrt{\frac{5}{\Theta_f}}, \tag{27} \]

where \( n_1 = n_b/1 \text{ cm}^{-3} \) is the number density of cooling gas. Some part of the gas is concentrated near the centre of the host DM halo (Wise & Abel 2008; Pratt et al. 2009, 2010) forming the baryonic core.

### 3.3 Numerical estimates

The cooling process of the baryonic component can be followed numerically by solving the equations of thermal balance and evolution of nine components, namely, the electrons, \( e^- \), protons, \( p \), neutral and molecular hydrogen, H\(_2\)H, ions H\(^+\), He\(^+\), He\(^{++}\), and neutral and ionized helium, He, He\(^+\), He\(^{++}\). The kinetic coefficients and the cooling rates used here are taken from Hutchings et al. (2002).

As is well known there are two different regimes of cooling of the compressed gas. Thus, very slow cooling takes place when temperature of the compressed gas (16) and (23) does not exceed \( 10^4 \text{ K} \). In this case the low electron concentration \( n_e = n_b/n_3 \sim 10^{-4} \) created at high redshifts \( z \sim 100 \) remains unchanged, the formation of molecules H\(_2\) and gas cooling are slow and formation of star-like subclouds is strongly delayed. On the other hand when

\[ T_b \approx 10^4 \text{ K} \] and the strong ionization of hydrogen takes place then the concentrations of both electrons and molecules H\(_2\) strongly increase, the compressed gas rapidly cools down to temperature \( T_b \approx 100 \text{ K} \) and forms star-like high density subclouds.

As is seen from (16, 23) temperature of the compressed baryons is a two parametric function. Thus for a given virial mass \( M_\text{vir} \) it rapidly decreases with \( z_{\text{vir}} \), and for some \( z_{\text{vir}} \), the threshold temperature \( T_b \approx 10^4 \text{ K} \) is reached. This means that at such \( z_{\text{vir}} \) the baryon cooling and the star formation process are strongly decelerated. In turn, for a given \( z_{\text{vir}} \) and haloes of low virial masses the threshold temperature \( T_b \approx 10^4 \text{ K} \) cannot be reached. For such virial mass the star formation process also becomes suppressed. This means that star formation in low-mass haloes occurs mainly at higher redshifts while at \( 1 + z_{\text{vir}} \sim 10 \) stars cannot be formed within haloes with \( M_\text{vir} \leq 1 \).

These statements are illustrated in Figs 1 and 2 where the thermal evolution of compressed gas is presented for two sets of halo masses and two redshifts of halo formation. As is seen in both figures for less massive haloes the rapid atomic cooling at \( T_4 \geq 1 \) is replaced by slower cooling with H\(_2\) molecules at \( T_4 \leq 1 \). In contrast for more massive haloes formed at the same redshift \( z_f \) the cooling rate with H\(_2\) molecules remains quite rapid.

For two haloes with virial mass \( M_{\text{vir}} = 5 \cdot 10^5 \text{ M}_\odot \) and \( M_{\text{vir}} = 9 \cdot 10^6 \text{ M}_\odot \) formed at \( z_{\text{vir}} = 25 \) the evolution of high density gaseous subclouds is presented in Fig. 1. The formation of gravitationally bounded subclouds is restricted by the Jeans mass, \( M_\text{vir} \), which drops down to star-like value \( M_\text{vir} \approx 10^3 \text{ M}_\odot \) at redshifts \( 1 + z \sim 16 \) and 23, correspondingly. Formation of less massive star-like subclouds is also hampered. This means that formation of haloes with the virial masses \( M \leq 10^6 \text{ M}_\odot \) at redshifts \( z \leq 2.5 \) is not usually accompanied by a noticeable star formation.

![Figure 1](https://example.com/figure1.png)

Figure 1. Redshift variations of the temperature, \( T_4 \), baryonic number density, \( n_b \), Jeans mass, \( M_J = M_\text{vir}/M_\odot \), and H\(_2\) concentration, \( n_{\text{H}_2}/n_b \), within haloes formed at \( z_{\text{vir}} = 25 \), \( z_f = 2.5 \), with the virial masses \( M_{\text{vir}} \approx 5 \cdot 10^5 \text{ M}_\odot \) (points) and \( M_{\text{vir}} \approx 9 \times 10^6 \text{ M}_\odot \) (squares).

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3.4 Constraints on the star formation process

It can be expected that the first stars are formed at \( z_{\text{cr}} \approx 20-30 \) within the rare DM haloes with the low relic concentration of electrons \( x_e \sim 10^{-3} \) and relic entropy of baryons (19). Both these values are determined at \( z \sim 100-300 \) after the period of hydrogen recombination. As was shown above with such low concentration of electrons it is not possible to efficiently form \( \text{H}_2 \) molecules and to cool the gaseous component of low-mass haloes. However these processes are strongly accelerated when the temperature of the compressed baryonic component exceeds (conventional) level \( T_{\text{gas}} \approx 10^4 \) K. In this case thermal ionization of hydrogen takes place what rapidly increases the concentrations of both electrons and \( \text{H}_2 \) molecules, and accelerates cooling of the gaseous component and formation of star-like clouds.

This means that the condition (21) which restricts this temperature can be used as an approximate demarcation line on the plane \( M_{\text{vir}}, z_{\text{cr}} \) between regions of rapid and slow star formation. The more convenient presentation of this line is

\[
M_{\text{vir}} \approx 10^6 \, M_\odot \left[ \frac{17.1}{1 + z_{\text{cr}}} \right]^{10} \left[ \frac{T_4}{10^4 \, \text{K}} \right]^2 \frac{S_0}{S_{\text{rel}}} \left[ \frac{5}{6 \rho_f} \right]^{3/4}.
\]

(28)

As is seen from (28) for each mass of halo there is the minimal redshift of the halo formation, \( z_{\text{cr}} \), for which the baryonic component is rapidly cooled and can form star-like objects. The haloes with the primordial chemical composition that were formed at redshifts less than some minimal redshift, \( z_{\text{cr}} \leq z_{\text{ion}}(M_{\text{vir}}) \), practically cannot produce Pop. III stars and ionizing photons. For example, as is seen from Fig. 1 for the halo with \( M_{\text{vir}} = 0.5 \cdot 10^9 \, M_\odot \) formed at \( z_{\text{cr}} \leq 25 \) temperature of the compressed gas decreases very slowly and the star formation is strongly delayed. Similar results are presented in Fig. 2 for haloes with the relic composition and masses \( M_{\text{vir}} \leq 0.4 \cdot 10^9 \, M_\odot \) formed at \( z_{\text{cr}} \leq 10 \).

However it is necessary to note the approximate character of the relation (28) and its complex links with the process of star formation and the masses of formed stars. More detailed analysis shows that the restriction (28) is eroded owing to the radial variations of temperature, possible ionization by the UV background, random variations of the initial perturbations and so on. This means that stars can be efficiently formed also in the DM haloes that are found is some strip around the line (28).

These results lead to important conclusions regarding the sources of the UV photons that caused the reionization of the Universe at redshifts \( z \sim 10-11 \) (Komatsu et al. 2011; Larson et al. 2011). As is seen from equation (28) at redshifts \( z \sim 10 \) the haloes with virial masses \( M_{\text{vir}} \geq 1-10 \) provide most of Pop. III stars with \( M_{\text{vir}} \sim 10^{-3} \, M_\odot \) while the contribution of less massive haloes can be moderate. The contribution of low-mass galaxies to the UV background is also suppressed by the feedback of supernova explosions (see, e.g. Ceverino et al. 2013; Wyithe et al. 2013; Salvadori et al. 2014).

3.5 Impact of the Lyman–Werner radiation

The constrain (28) is strongly enhanced when the disruption of \( \text{H}_2 \) molecules by the LW or H\(^+\) ions by the IR photons becomes noticeable (see, e.g. discussion in Loeb & Barkana 2001; Muñoz et al. 2009; Wolkott-Green & Haiman 2012, and references therein). These photons are produced by thermal sources of radiation (such as stars) together with \( \text{Ly} – c \) and more energetic photons. As is well known, for the strong reionization at \( z \sim 10 \) it is necessary to produce at least one \( \text{Ly} – c \) photon per baryon. The allowance for the complex spectral distribution of the generated UV photons, ionization of He and the heating and recombination of the IGM can increase this estimate by a factor of 2 to 3. This is the minimal value and in some papers (see, e.g. Dijkstra, Haiman & Loeb 2004; Madau 2007) production of extra (up to 10) UV photons per baryon is discussed. But at the same time the thermal sources produce comparable number of LW photons with the density \( n_{\text{LW}} \sim n_b \sim 3 \cdot 10^{-7} \, \text{cm}^{-3} \). Such flux of LW photons practically brings to a halt the formation of \( \text{H}_2 \) molecules and first star (see, e.g. Safranek-Shrader et al. 2012).

However, generation of the LW photons is strongly suppressed when the ionization of the hydrogen and helium is provided by non-thermal sources of the UV radiation. Such non-thermal radiation is inevitably generated by matter accreted on to black holes created by explosions of massive and supermassive stars. But in contrast with the thermal sources the non-thermal sources do not produce immediately the LW photons and do not deaccelerate the process of formation of first stars. At high redshifts the heating of the intergalactic gas by the soft X-ray background is not efficient owing to the cooling of ionized baryons by the inverse Compton scattering, free–free emission, excitations of the neutral hydrogen and so on. In this case we can expect the moderate increase of the Jeans mass up to \( M_J \approx 4 \cdot 10^3 \, T_4^{-3/2} \, z_f^{3/2} \, M_\odot \), and corresponding increase of masses of forming galaxies. However in this case the more efficient generation of the IR and hard UV backgrounds can be expected what leads to partial ionization of He\(^+\) and...
4 SCALING RELATIONS FOR THE DM DOMINATED OBJECTS

Simulations show that characteristics of the virialized DM haloes are much more stable than the characteristics of baryonic component and after formation at $z = z_{cr}$ of the virialized DM haloes with $\langle \rho_{\text{vir}} \rangle \approx 200 \rho(z_{cr})$ slow matter accretion only moderately changes their characteristics (see, e.g. Diemer et al. 2013). Because of this, we can observe earlier formed high density galaxies with moderate masses even within later formed more massive but less dense clusters of galaxies, filaments and other structure. This means that using the model presented in Section 2 for description of the observed dSph galaxies and clusters of galaxies dominated by the DM component we can find one-to-one correspondence between their observed parameters and the so-called redshift of object formation, $z_{cr}$. Of course according to the Press–Schechter approach (Press & Schechter 1974; Peebles 1974) these redshifts characterize the power spectrum of the density perturbation rather than the real period of the object formation.

Following this approach to describe the haloes formation we use the function $B(z_{cr})$ (3) rather than the redshift $z_{cr}$, for large $z_{cr} \gg 1$ the function $B(z_{cr})$ is equivalent to the redshift and we can use $z_{cr}$ for discussion of properties of the dSph galaxies. However many observed clusters of galaxies are situated at redshifts $z \lesssim 1$ and in this case these differences become significant.

The simplest way to estimate the redshift of halo formation is to use equation (5) which can be rewritten as

$$(1 + z_{cr})^3 = 3M_{\text{vir}}/4\pi R_{\text{vir}}^3/\Delta \rho(z = 0).$$

However this approach has to deal with unstable ragged periphery of haloes and to reasonably estimate the virial radius it is necessary to use a complex model dependent procedure.

More stable but more complex way is to use the expression (11) (or its equivalent) for the matter concentration and/or parameters of the central core $r_c$ and $\rho_c$. However in this case it is not possible to achieve high precision because of the possible impact of baryonic component, complex shape of these relations and complex procedures of measurement of these parameters. None the less this approach has the largest potential to analyse the available observations.

Today the parameters of the virialized DM haloes are known for some population of dSph galaxies (Walker et al. 2009, 2012; Tollerud et al. 2012) and for many clusters of galaxies (see e.g. Piffaretti et al. 2011; Kravtsov & Borgani 2012; McDonald et al. 2013, and references below). Here we present some results of such analysis.

Perhaps this approach can be applied also for virialized groups of galaxies and for galaxies with measured rotation curves at large distances. However measurement of the virial radius and the mean density for such objects is problematic and more indirect approaches must be used for such analysis.

4.1 Redshift of formation of the dSph galaxies

Samples of the dSph and And (companions of the Andromeda galaxy) galaxies include objects in a wide range of masses, $0.1 \leq M_b = M_{\text{gal}}/10^{10} M_\odot \leq 100$, what allows us to reveal more reliably the mass dependence of their redshift of formation. Some observed properties of 28 dwarf DM dominated galaxies are compiled in Walker et al. (2009) and for 13 And galaxies are presented in table 4 in Tollerud et al. (2012). In this case we have to deal with parameters of the central regions at the projected half-light radius. Moreover, the presented data are recalculated from actual observations (Walker et al. 2009; Walker 2012) and, so, their reliability is limited and scatter is large. In spite of this it is interesting to compare characteristics of these galaxies and observations of clusters of galaxies with the theoretical expectations of Section 3.

First of all for the sample of 28 dSph galaxies we can roughly estimate the dimensionless size of the region under consideration. To do this we compare the observed masses, radii and velocity dispersions with expectations (16). For this sample we get

$$\sqrt{r_{ob}/r_c} \approx \left( G M_b \sigma \right)^1/2\rho_c \approx 1.3 \pm 0.02 .$$

This means that the observed values of $M_{ob}$, $\sigma$, and $r_{ob}$ are related to the model parameters as follows:

$$r_{ob} \sim 1.7r_c, \quad M_{ob} \sim 0.36 M_{\text{vir}}, \quad (\rho_{ob}) \sim 0.1 \rho_c .$$

For both the NFW and Burkert models these corrections are quite similar to each other because they consider regions where $r_{ob} \gg r_c$.

In order to find the redshift of object formation, $z_f = (1 + z_{cr})/10$, we can use two approaches. First, we can use the estimates of the central density (13) which can be expressed with the help of (29) through the observed $M_{ob}$ and $(\rho_{ob})$

$$z_{10}^f \approx \frac{150}{\sqrt{M_b}} \rho_{ob} \frac{pc^3}{M_\odot},$$

where $M_b = M_{ob}/10^{10} M_\odot$ is the observed mass of the DM galaxy. The great advantage of this method is a weak dependence of $z_f$ on $\rho_{ob}$ what attenuates the impact of errors. For comparison we can use expression (12) for the typical size of the central regions,

$$z_{fr}^{10/3} \approx 0.5 M_b^{1/6}/r_{\text{obs}},$$

where $r_{\text{obs}}$ is the observed radius, $r_{ob}$, in kpc. Both estimates are quite similar to each other and we have

$$z_f \approx z_{fr} \approx 0.9 z_{fr} .$$

The redshifts of formation of galaxies are spread between values $1 + z_{cr} = 8$ for Sgr and $1 + z_{cr} = 20$ for Segue 1. The fit of the mass dependence of $z_{cr}(M)$ is

$$z_f/M_b^{-1/3} \sim 1.7(1 \pm 0.12),$$

$$1 + z_{cr} \approx 17(1 \pm 0.12) M_b^{-0.1} \approx 3(1 \pm 0.12) M_{13}^{-0.1} .$$

These results are presented in Fig. 3.

For 13 And galaxies the masses and half-light radii are listed in table 4 in Tollerud et al. (2012). For these objects we can use the expression (31) to estimate their redshift of formation. For this sample we get

$$z_f/M_b^{-1/3} \approx 1.6(1 \pm 0.11), \quad 1 + z_{cr} \approx 16(1 \pm 0.11) M_b^{-0.1} ,$$

what is identical with (32). The maximal $1 + z_{cr} \approx 14$ is obtained for the galaxy And XVI.

These results are also presented in Fig. 3. However it is necessary to note that parameters of the galaxies And IX and And XV presented in both surveys are noticeably different.
4.2 Redshift of formation of clusters of galaxies

Now there are more or less reliable observational data at least for \( \sim 300 \) clusters of galaxies (Ettori et al. 2004; Arnaud, Pointecouteau & Pratt 2005; Pointecouteau, Arnaud & Pratt 2005, 2006; Zhang et al. 2006; Branchesi et al. 2007; Vikhlinin et al. 2009; Pratt et al. 2010; Babyk, Vavilova & Del Popolo 2012; Moughan et al. 2012; Suhada et al. 2012; Bhattacharya et al. 2013; Foéx et al. 2013). However, the main cluster characteristics are not directly observed and are obtained by a rather complex procedure (see, e.g. Bryan & Norman 1998; Vikhlinin et al. 2009; Lloyd-Davies et al. 2011; McDonald et al. 2013). In particular they relate the virial mass and radius of each cluster with the critical density of the Universe at the observed redshift, \( \rho_c(z_{\text{obs}}) \).

\[
M_{\text{vir}} = 4\pi/3R_{\text{vir}}^3 500\rho_c(z_{\text{obs}}) = 250R_{\text{vir}}^3 H(z_{\text{obs}})/G.
\]

In fact this assumption identifies the redshift of cluster formation with the observed redshift. This assumption is questionable for majority of clusters as quite similar clusters are observed in a wide range of redshifts. It distorts all published cluster characteristics and often makes impossible to use the published characteristics of cores for further discussions. The matter concentration is measured with a reasonable precision only for 25 clusters of sample CLS-25 combined from samples CLS-10 (Pointecouteau et al. 2005), CLS-12 (Vikhlinin et al. 2006) and CLS-18 (Bhattacharya et al. 2013). The central regions of many clusters are influenced by cooling baryonic component (see, e.g. Pratt et al. 2009, 2010) but for these three samples the concentrations are determined with precision of \( \sim 10–15 \) per cent what allows us to estimate the redshift of formation, \( z_{\text{cr}} \), and both the central and the virial surface densities of clusters, \( \mu_{\text{cr}} = r_{\text{c}} \rho_c \) and \( \mu_{\text{vir}} = R_{\text{vir}} \rho_{\text{vir}} \).

For this sample the redshift \( z_{\text{cr}} \) can be obtained from the relation (Dolag et al. 2004)

\[
1 + z_{\text{cr}} \approx \frac{11}{c(M_{\text{vir}}/z_{\text{cr}})M_{\text{vir}}^{1/3}}, \quad M_{13} = M_{\text{vir}}/10^{13} M_\odot,
\]

\[
(1 + z_{\text{cr}}) \approx 1.09(1 \pm 0.05), \quad (c) \approx 4.05 \pm 0.9, \quad (36)
\]

\[
(1 + z_{\text{cr}}) \approx 2.1(1 \pm 0.24). \quad (37)
\]

The maximal values \( 1 + z_{\text{cr}} \approx 3.6 \) and \( 1 + z_{\text{cr}} \approx 2.6 \) are obtained for the cluster MKW4 with \( M_{13} \approx 7.7 \) and for the cluster A262 with \( M_{13} \approx 8.3 \) observed at \( z_{\text{obs}} = 0.02 \) and 0.016 (Vikhlinin et al. 2006; Bhattacharya et al. 2013). For these clusters \( z_{\text{cr}} \gg z_{\text{obs}} \) what confirms differences between the redshift of cluster formation and random observed redshift at least for clusters with \( z_{\text{obs}} \ll 1 \).

However as was noted above for redshifts \( z_{\text{cr}} \leq 1 \) the cluster formation is determined by the function \( B(z) \) (3) rather than by the redshift \( z_{\text{cr}} \). Thus for these samples we get

\[
\langle B^{-1}(z_{\text{cr}}) \rangle \approx 1.63(1 \pm 0.2) = 2.3(1 \pm 0.2)M_{13}^{-0.1}, \quad (38)
\]

This value is quite comparable with the estimates (32) which can be rewritten for dSph galaxies as follows

\[
\langle B^{-1}(z_{\text{cr}}) \rangle \approx 2.22(1 \pm 0.12)M_{13}^{-0.1}. \quad (39)
\]

However large scatter and uncertainties in both estimates (32) and (38) prevent more detailed comparison of these results.

The more interesting sample CLS-83 (McDonald et al. 2013) contains parameters of central regions of 83 clusters with redshifts \( z \gtrsim 0.3 \), namely, baryonic density and temperature, \( n_c \) and \( T_c \). Using relations (23) and (24) we can obtain for these clusters preliminary estimates of the redshift of formation \( z_{\text{cr}} \) and mass \( M_{13} \)

\[
M_{13} \approx 10^{-2} \left( \frac{6 \text{ cm}^{-3}}{n_c} \right)^{1/2} \left( \frac{T_c}{9.5 \text{ eV}} \right)^{3/2} f_{\text{nu}} (c),
\]

\[
1 + z_{\text{cr}} \approx 10. \left( \frac{n_c}{6 \text{ cm}^{-3}} \right)^{1/8} \left( \frac{9.5 \text{ eV}}{T_c} \right)^{3/40} \epsilon^{-0.3}. \quad (40)
\]

For 44 clusters of this sample with the central pressure \( P_c \approx n_c T_c \lesssim 70 \) eV cm\(^{-3} \) we get

\[
\langle P_c \rangle \approx 36(1 \pm 0.37) \text{ eV cm}^{-3}, \quad (41)
\]

\[
\langle (1 + z_{\text{cr}}) \rangle \approx 2.9(1 \pm 0.05) M_{13}^{-0.12}, \quad 0.3 \leq z_{\text{cr}} \leq 1.5,
\]

\[
\langle B^{-1}(z_{\text{cr}}) \rangle \approx 2.1(1 \pm 0.04) M_{13}^{-0.95}. \quad (42)
\]

These results are plotted in Fig. 4.

The precision of these results is moderate. None the less in spite of the large difference in masses of these clusters (\( M/M_\odot \gtrsim 10^{13} \)) and dSph galaxies with (\( M/M_\odot \gtrsim 10^9 \)) estimates (41) are very close to

\[
\text{Figure 3. Functions } \mu_{\text{cr}}/M_b^{0.13}, \mu_{\text{cr}}, P_c(M_b) \text{ and } z_f M_b^{0.13} \text{ are plotted for the observed samples of 28 dSph galaxies (points) and 13 And galaxies (stars). Fits (32), (34), (45) and (46) are plotted by dashed lines.}
\]
follows immediately from the weak mass dependence of the function
\[ F_{\text{vir}} = M_c \delta_{\text{vir}} f \approx \text{const.}, \]
which determines also the weak mass dependence of the redshift of formation \( z_f \) discussed in Section 4.1. Here \( c(M_c, z_{\text{vir}}) \) is the concentration (11), and the virial dimensionless mass of objects \( f_{\text{vir}}(c) \) is given by (7) for the NFW density profiles.

Using the estimates (32) we get
\[ \langle \mu_{\text{cs}} \rangle \approx 230 \, M_\odot / \text{pc}^2, \]
while the direct estimates with the full sample of 28 dSph galaxies give
\[ \langle \mu_{\text{cs}} \rangle \approx 300(1 \pm 0.66 \pm 0.51) \, M_\odot / \text{pc}^2. \]

Here the first uncertainty is connected with the scatter of \( \mu_{\text{cs}} \) over the sample, while the second one characterizes the precision of separate measurements. These results are presented in Fig. 3.

Large scatter of the surface density strongly decreases its significance and possible applications. None the less for galactic scales the weak mass dependence of the surface density is confirmed by the weak mass dependence of the redshift of formation.

In accordance with (15) the virial surface density of dSph galaxies is weakly dependent on its virial mass and is described by the relation
\[ \langle \mu_{\text{vir}} \rangle \approx 6.6 M_\odot^{6/3}(1 \pm 0.21) \, M_\odot / \text{pc}^2, \]
whereas before \( M_6 = M_{\text{vir}} / 10^6 \, M_\odot \) and \( M_{\text{vir}} \approx M_{\text{halo}} / 0.36 \). This result is consistent with (32). The function \( \mu_{\text{vir}} / M_\odot^{6/3} \) is plotted in Fig. 3.

4.3.2 The DM surface density for clusters of galaxies

It is necessary to remind that if for galaxies we had \( \epsilon(M, z_{\text{vir}}) \sim 1 \) in relations (11)–(17) then for some clusters of galaxies \( \epsilon \gg 1 \) and we get for their DM surface density
\[ \mu_{\text{cs}} \approx 204 \frac{M_{\text{vir}}}{(1 + z_{\text{vir}})^2} \left( \frac{\Theta^*}{\Theta^*_{\odot}} \right)^{1/6} \frac{M_\odot}{\text{pc}^2}, \quad \Theta^* = \delta_c^{2/3} \frac{\epsilon^2}{f_{\text{vir}}(c)}. \]

\[ z = 1 + 3.7 \cdot 10^{-2} M_{\text{vir}}^{1/3}(1 + z_{\text{vir}})^4, \quad M_{\text{vir}} = M_{\text{vir}} / 10^{13} \, M_\odot. \]

This means that relations (43) and (44) which are valid for galaxies cannot be applied for massive clusters of galaxies.

Using the expression (47) with (1 + \( z_{\text{vir}} \)) estimated by (37) we get
\[ \mu_{\text{cs}} \approx 124 M_{\text{vir}}^{1/6} \, M_\odot / \text{pc}^2, \]
what is a rough estimate owing to the large scatter of \( z_{\text{vir}} \). More accurate results can be obtained from expressions (5):
\[ \mu_{\text{cs}} = \frac{M_{\text{vir}}}{4 \pi c^2 f_{\text{vir}}(c)} \left( \frac{10 M_{\text{vir}}}{4 \pi c^2 R_{\text{Mpc}}^2} \right)^{1/2} \]
\[ \text{whereas before } c(M, z_{\text{vir}}) \text{ is the halo concentration, } M_{\text{cs}} = M_{\text{vir}} / 10^{13} \, M_\odot \text{ and } R_{\text{Mpc}} \text{ is the virial radius of cluster in Mpc. With this relation we get for the sample CLS-25} \]
\[ \langle \mu_{\text{cs}} \rangle \approx 415(1 \pm 0.3) \, M_\odot / \text{pc}^2 \approx 150(1 \pm 0.25) M_\odot / \text{pc}^2. \]

This estimate differs from that obtained for galaxies (45) because it depends on mass. The fact that (48) and (50) are different shows that these results depend on the averaging procedure.
The virial surface density of clusters is closely linked with their central surface density, \( \mu_{\text{sur}} \) (49). Thus, for the same sample CLS-25 we get

\[
\langle \mu_{\text{vir}} \rangle = 3 (f_n(c) \mu_{\text{sur}} / c^2) \approx 17(1 \pm 0.28) M_{13}^{0.35} \, \text{M}_\odot / \text{pc}^2. \quad (51)
\]

These results can be compared with recently published data by Babyk et al. (2012). For the sample CLS-30 of 30 clusters randomly selected from this survey we get

\[
\langle \mu_{\text{vir}} \rangle \approx 13(1 \pm 0.11) M_{13}^{0.35} \, \text{M}_\odot / \text{pc}^2. \quad (52)
\]

However in this survey all cluster characteristics are found with the popular assumption that \( z_{\text{cr}} \approx z_{\text{obs}} \) what distorts their virial parameters, \( M_{\text{sur}} \) and \( R_{\text{sur}} \), and makes impossible discussion of characteristics of the cluster cores. Prominent differences between scatterers (51) and (52) are caused mainly by the impact of cluster description rather than by their physical properties.

The relatively small interval of observed cluster masses and the limited reliability and precision of the complex procedure of reconstruction of cluster characteristics (see, e.g. Bryan & Norman 1998; Pointecouteau et al. 2005; Pratt et al. 2009; Vikhlinin et al. 2009; Lloyd-Davies et al. 2011) strongly restricts the applicability of discussed scaling relations.

5 CONCLUSIONS

Abundant simulations show that the formation of the virialized DM haloes is a complex multistep process which begins as the anisotropic collapse in accordance with the Zel’dovich theory of gravitational instability (Zel’dovich 1970). During later stages the evolution of such objects is complicated and it goes through the stages of violent relaxation and merging. None the less after a period of rapid evolution the main characteristics of the high density virialized DM haloes become frozen and their properties are slowly changing owing to the accretion of diffuse matter and/or the evolution of their baryonic component.

The basic properties of the relaxed DM haloes are determined by their global characteristics, namely, their mass, angular momentum and entropy generated in the course of violent relaxation of compressed matter. Basically the structure of such haloes is similar to the structure of clusters of galaxies (see, e.g. Tasić et al. 2004; Nagai et al. 2007; Croston et al. 2008; Pratt et al. 2009, 2010; Arnaud et al. 2010; Kravtsov & Borgani 2012). In particular in these papers the generalized NFW model proposed in Zhao (1996) and Nagai et al. (2007) is discussed.

The basic properties of haloes can be reproduced in the framework of the popular spherical model of haloes formation. Such models have been discussed for many years (see, e.g. Peebles 1967; Zel’dovich & Novikov 1983; Fillmore & Goldreich 1984; Bryan & Norman 1998; Lichwick & Dalal 2011). However this model ignores many important features of the process of haloes formation and is based on the assumption that during a short period of the spherical collapse at \( z \approx z_{\text{cr}} \) the DM forms virialized haloes with parameters which later vary slowly owing to the successive matter accretion (see, e.g. discussion in Bullock et al. 2001; Diemer et al. 2013).

In this paper we use the analytical description of the virialized spherical DM halo with the NFW density profile proposed in Klypin et al. (2011). Such approach allows us to formulate in Section 2 two parametric spherical model of virialized haloes which is specified by the virial halo mass, \( M_{\text{vir}} \), and redshift of formation \( z_{\text{cr}} \). Of course this redshift is only some conventional characteristic of the mean density of virialized haloes or other corresponding parameters. However, it can be used in order to roughly characterize the period of haloes virialization what in turn allows us to order the observed haloes with respect to the (conventional) moment of formation. It also opens up the comparatively simple way to reveal correlations of thus introduced redshifts with the shape of the initial power spectrum.

5.1 Mass dependence of the redshift of formation of the DM haloes

These problems are discussed in Section 4 where we obtained the approximate relation between the virial mass of the DM objects and their redshifts of formation. According to the commonly accepted hierarchical model of galaxy formation at high redshifts the formation of low-mass galaxies dominates and the typical mass of formed galaxies successively increases with time. These expectations are illustrated by the expressions (32), (37)-(39). The high redshifts of formation of dSph and And galaxies correlate well with their low metal abundance and help us to reconstruct the history of the Local Group discussed for instance by Peebles (1995, 1996), Klypin, Zhao & Somerville (2002) and Klypin et al. (2003).

Combining the resulting estimates (38) and (39) for mass dependence of the redshift of formation for both DM dominated observed dSph galaxies and clusters of galaxies we conclude that for these objects

\[
\langle B^{-1}(z_{\text{cr}}) \rangle \approx 2.3(1 \pm 0.15) / M_{13}^{0.1},
\]

\[
\beta \sim 0.1, \quad 10^5 \, \text{M}_\odot \leq M_{\text{vir}} \leq 10^{14} \, \text{M}_\odot. \quad (53)
\]

With respect to the general theory of gravitational instability for the DM objects the expression (53) quantify the mass dependence of the mean redshift of formation, \( z_{\text{cr}}(M_{\text{halo}}) \), or correlation between these redshifts and the shape of the initial power spectrum of perturbations. Indeed according to the standard \( \Lambda \)CDM cosmology low-mass objects (such as galaxies) are presumably formed earlier than more massive galaxies and clusters of galaxies and the expression (53) illustrate this statistical tendency.

Thus for the standard cold dark matter (CDM) – like power spectrum \( p(k) \) (Bardeen et al. 1986) we have for the density perturbations (Klypin et al. 2011)

\[
\sigma_z^2 = 4\pi \int_0^\infty p(k) W^2(kR) k^2 dk, \quad \sigma_M \propto M^{-0.1}, \quad (54)
\]

for 10 kpc \( \leq R \leq 10 \) Mpc and \( 10^9 \leq M / \text{M}_\odot \leq 10^{14} \). Here \( W(kr) \)

is the standard top-hat window function. Following the Press–Schechter approach (Peebles 1974; Press & Schechter 1974; Mannet al. 2010) we can determine the redshift of objects formation from the condition

\[
B(z_{\text{cr}}) \sigma_M(M) \approx \text{const}, \quad B^{-1}(z_{\text{cr}}) \propto \sigma_M(M) \propto M^{-0.1}.
\]

This result is consistent with (53) and confirms that the CDM-like power spectrum can be extended at least down to the scale of \( \sim 10 \) kpc. However this approach does not allow us to obtain an independent estimate of the small-scale amplitude of perturbations. More detailed comparison of the mass dependence of the redshift of formation of galaxies and clusters of galaxies requires much more precise estimates of observational parameters of both galaxies and clusters of galaxies.

For completeness it is interesting also to consider objects with intermediate masses \( M_{\text{vir}} \sim 10^{10} - 10^{11} \, \text{M}_\odot \). The virialized groups of galaxies and the far periphery of isolated galaxies can be used for such analysis if it is possible to confirm that they are dominated by

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DM and to estimate their virial characteristics. Perhaps it is simpler to estimate the correlation between the measured circular velocity \( v_c^2 = GM_{\text{halo}} / R_{\text{vir}} \) and the virial radius \( R_{\text{vir}} \). For \( (1 + z_{cr}) \propto M^{-\beta} \) (53) we can expect that
\[
v_c^2 = GM_{\text{halo}} / R_{\text{vir}} \propto R_{\text{halo}}^\gamma, \quad \gamma = (2 - 3\beta) / (1 + \beta),
\]
and \( \gamma = 0.8 \) for \( \beta \approx 0.1 \).

5.2 Formation of the first galaxies

It is quite interesting to compare the limits (28) with the observed properties of low-mass galaxies (32) and (33). The first one restricts the virial masses of the DM haloes allowing for the rapid creation of metal free stars and the UV radiation. The second one considers the most probable masses of the DM haloes forming at the same redshifts as is suggested by observations of the dSph galaxies.

Such comparison is presented in Fig. 5 where the expected minimal masses of the DM haloes with rapid star formation (equations 28 and 56)
\[
M_{\text{vir}} = [17.1 / (1 + z_{cr})]^{10} M_\odot, \quad (56)
\]
are compared with the observed masses and redshift of formation for the dSph galaxies (points, stars and fit 57):
\[
M_{\text{gal}} = [17.6 / (1 + z_{cr})]^{10.5} M_\odot. \quad (57)
\]
For 44 clusters of sample CLS-83 with \( P_c \leq 70 \text{eV} \text{ cm}^{-3} \) the correlation of the virial mass of haloes and their redshift of formation is fitted by expression
\[
M_{\text{cls}} = [21.2 / (1 + z_{cr})]^{9} 10^6 M_\odot, \quad (58)
\]
and is plotted in Fig. 6. As is seen from (40) the cooling of baryonic component artificially decreases the estimate of virial mass \( M_{\text{vir}} \) and increases the estimate of redshift \( z_{cr} \) that enhances the scatter of points in Fig. 6. In spite of this the similarity of expressions (56)–(58) reflect the close link of all these objects formed with the joint power spectrum of initial perturbations. Weaker variation \( M_{\text{cls}} \) as a function of \( z_{cr} \) (58) as compared with (57) is naturally explained by weaker mass dependence of the amplitude \( \sigma_m(M) \) for cluster masses.

The complex process of retrieval of considered characteristics of the dSph galaxies (see, e.g. Walker 2012) and the plausible impact of their prolonged evolution – such as the probable tidal striping – makes detailed discussion of the observed properties of such objects unreliable. In spite of this the comparison performed in Fig. 5 is interesting. First of all it confirms the probable formation of metal free galaxies with \( M_{\text{vir}} \leq 10^6 M_\odot \) at redshifts \( z \sim 20–8 \) what agrees well with both other observations (see, e.g. Wyithe et al. 2013) and theoretical expectations discussed in Section 5.1.

On the other hand this figure shows that the dSph galaxies are concentrated near the joint approximate boundary (28, 32). Such concentration indicates that the observed dSph galaxies can be really related with the earlier DM objects with various rate of star formation. Thus in objects disposed to the right of the demarcation lines (28, 32) the rapid star formation can be expected. In contrast in objects disposed to the left of these lines the star formation can be partly inhibited either by both the low ionization of the compressed matter and by the impact of LW background. It can be later stimulated by an action of external factors such as, for example, ionizing UV radiation of external sources after dissipation of the LW background.

It is interesting also to compare other observed properties of these groups of dSph galaxies such as their metallicity and the possible populations of black holes.

These results show also that the most efficient star formation takes place in haloes with \( 10^7 \leq M/M_\odot \leq 10^9 \) (28) at redshifts \( z \sim 13–8 \) when probably the reionization actually occurs. This inference is consistent with observations of galaxies at redshifts \( z \geq 7 \) with \( M \sim 3 \cdot 10^8–10^9 M_\odot \) (Ouchi et al. 2009; Gonzalez et al. 2010, 2012;
Schaerer & de Barros 2010; Oesch et al. 2013; Ellis et al. 2013). However, observations of Bouwens et al. (2011, 2012) indicate a possible more significant contribution of less massive galaxies. All theoretical expectations can be essentially corrected by possible impact of the UV, LW and/or IR background (see, e.g. discussion in Loeb & Barkana 2001; Muiño et al. 2009; Wolkott-Green & Haiman 2012, and references therein). As was noted in Section 3.5 the production of the UV background required for reionization by stars or other sources of radiation with the thermal spectrum is inevitably accompanied by formation of the corresponding LW background and deceleration of the process of star formation. In this case the UV radiation generated by matter accretion on to black holes with various masses can become dominant and can really determine the reionization. This verifies that such non-thermal sources of the UV radiation can be considered as very promising ones and can be actually responsible for the reionization (see, e.g. Madau & Rees 2001; Meiksin 2005, 2009; Reed et al. 2005; Madau 2007; Giallongo et al. 2012). Perhaps, the contribution of such sources can be confirmed by observations of tracks of He lines such as 304 and 584 Å shifted to the redshift of reionization.

5.3 The DM surface density of relaxed objects
If the redshift of formation of the DM dominated objects can be directly linked with the power spectrum of density perturbations then both the central and virial surface densities of these objects, $\mu_{20}$ and $\mu_{45}$, should depend upon the processes of violent relaxation of compressed matter and characterize this process. In particular, as it is seen from relations (14) and (47), we can expect a weak mass dependence of the central surface density $\mu_{20}(M_{20})$ at galactic scale (45) but these expectations are distorted at clusters of galaxies scale (50). The virial surface density only weakly depend on mass at both galactic and clusters of galaxies scales.

Both surface densities are closely linked with the DM density profile formed in the course of violent relaxation of the compressed matter and are determined by the action of the same factors. None the less it can be expected that a weak mass dependence of the central surface density, $\mu_{cs}$, can be observed across wide set of objects of galactic scales what will be an important additional evidence in favour of the standard shape of small-scale initial power spectrum.

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