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Market-crash forecasting based on the dynamics of the alpha-stable distribution

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A B S T R A C T

This paper investigates on the alpha-stable distribution capacity to capture the probability of market crashes by means of the dynamic forecasting of its alpha and beta parameters. On the basis of the GARCH-stable model, we design a market crash forecasting methodology that involves three-stepwise procedure: (i) Recursively estimation the GARCH-stable parameters through a rolling window; (ii) alpha-stable parameters forecasting according to a VAR model; and (iii) Crash probabilities forecasting and analysis. The model performance for alternative crash definitions is assessed in terms of different accuracy criteria, and compared with a random walk model as benchmark. Our applications to a wide variety of stock indexes for developed and emerging markets reveals a high degree of accuracy and replicability of the results. Hence the model represents an interesting tool for risk management and the design of early warning systems for future crashes.

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1. Introduction

Since the 2008 financial crisis, financial markets have experienced a period where risk and uncertainty has increased, although these effects being asymmetric across markets and over time, and present a remarkable volatility clustering. Despite the recovery from the Great Recession in major economies has been taken for granted, the analysts and policy makers also wonder when the next crises would rise and which would be the factor 1 that will put an end to the bull markets.

This context has triggered an increasing demand from investors for the understanding of the mechanisms which operate in financial markets and the risks associated to their investments. Particularly, the study of market crashes, where the prices of financial assets decline drastically [1], has become an issue of growing interest in financial research. The development of models capable at predicting market crashes satisfactorily, the evaluation of their statistical significance and robustness, the study of these events in other markets different from the American, the analysis of the transmission mechanisms in globalized markets, and the incorporation of stylized facts such as the leptokurtic distributions of financial returns constitute major issues to solve.

Different models have been developed for the study of this type of extreme events, which can be classified in models that include economic variables [2–6] and probabilistic models [7–25]. However, there is still a gap to generate new

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1 This shock is currently called as Covid-19 pandemic.
proposals in this research field related to the forecasting accuracy of the models, the study of these events in markets different from the American, and the development of models that reliably incorporate the behavior of financial markets.

Within the strand of probabilistic models, some of them have been focused on the modeling of the leptokurtic behavior of the asset return distribution, issue that has been acknowledged since the early work of Mandelbrot [26]. Furthermore, it has been shown that distributions whose shape parameter is represented by a tail index adjust properly to empirical financial returns, particularly at the daily frequency. The alpha-stable (aka stable Paretian or Lévy stable) distribution have been useful in economics and finance studies, or econophysics models (as mentioned in Bucsa et al. [27]). For instance, Pernagallo and Torrisi [28] fit this distribution to different daily log returns of twelve stock market indexes in emerging economies. In similar studies, [29] employ stable Paretian distribution among other models to fit daily data of the DJ Eurostoxx 50 market index whereas [30] use the Standard and Poor’s 500 index. Moreover, high-frequency price variations for the Norwegian stock market is compared with the stable density [31]. In other type of studies, the (tempered) stable process have been employed to price foreign equity options [32]. Additionally, the tail index value decreases when there is a panic in the financial markets [33,34] and thus it can be used as an early warning indicator for market crashes.

This paper evaluates to what extent it is possible to predict market crashes of different financial markets through the analysis of the tail index behavior. For this purpose, we develop a market crashes probability prediction model based on the analysis of the tail index and tests its prediction power for drastic financial downturns. Therefore, this study contributes to literature in two main directions. First, we propose a new methodology that is easily replicable for evaluating market crashes prediction and incorporates the heavy-tailed behavior of financial returns through the use, for the first time in the literature, of the alpha-stable distributions in the analysis of this type of phenomena. Second, the application of the proposed model to different financial markets that comprehensively include developed and emerging economies.

The remain of this paper is organized as follows. Section 2 includes a literature review on market crash forecasting approaches. Section 3 presents the methodology for crashes detection based on a tail index. In Section 4, we analyze the results obtained from the application of the model to six MSCI global indexes and evaluate its possible use for streaks (drawdowns) analysis. Section 5 presents additional analyses for the S&P500 and KOSPI indexes that, confirm the validity of the model. Additionally, this section presents a further application of our methodology for the case of subprime crisis. Finally, Section 6 summarizes the main conclusions.

2. Literature review

Market crashes have been widely studied due to their relevance as an indicator of the performance of the economy and as signals of forthcoming financial crises. According to Chen and Huang [14], a market crash is different from general market volatility, general stock market risk, or even the critical points of a trend change because this type of events may cause huge economic losses for different sectors of economy and bring negative contagion effects for different markets around the world [35]. Hence, the importance of market crashes lies in the potential of these events to trigger financial and economic crisis. Reinhart and Rogoff [36] classify financial crises into crises defined by quantitative thresholds (inflation, currency crashes, debasement, and bursting of asset price bubbles) and crisis defined by events (banking crisis and domestic debt default). According to the authors “there are usually remarkable similarities with experience from other countries and from history. Recognizing these analogies and precedents is an essential step toward improving our global financial system, both to reduce the risk of future crisis and to better handle catastrophes when they happen” ([36], p. xxv). Therefore, the existence of coincidences between crises generates the expectation that they might be predictable through the development of early warnings or by monitoring different economic/financial variables.

Although there is no consensus on the definition of a market crash, the proposals of different authors have elements in common that allows us to identify this type of phenomenon. In general terms, a market crash refers to a sudden drastic decline in the prices of a financial assets in a specific market [1]. The definition of ‘drastic decline’ depends on the model by which this type of phenomenon is studied. For instance, Barro and Ursua [37,38] characterize a market crash as a situation with multi-year returns of −25% or less. Alternatively, Wang et al. [1] consider events with a minimum one-day decrease of 5% in the stock market index returns. Lleo and Ziemba [39] refer to a crash as the day on which the closing price of a market index (e.g. S&P500) exceeds a fall of 10% between its highest value and its lowest value in a period of one year. Wu et al. [40] argue that there is evidence of a crash when “a daily return lies below the 5% quantile of the empirical returns distribution over the complete sample period” (p. 1146). One of the most complete crash definition is given by Hong and Stein [41] who postulate that a market crash “(1) is an unusually large movement in stock prices that occurs without a correspondingly large public news event; (2) moreover, this large price change is negative; and (3) a crash is a contagious market wide phenomenon” (p. 487). Regardless of the adopted definition of crash, the literature about the study of this subject is divided into two main streams. The first incorporates models which include economic variables in the study of market crashes, and the second, which has gained popularity in recent years, is based on the probabilistic representation of price processes [39].

Within the first group of proposals, the salient study by Flood et al. [2] establishes that volatility is not the result of self-fulfilling expectations and can be interpreted as model misspecifications, which are the basis for the failure of market fundamental models. The subsequent seminal paper by Campbell and Shiller [3], introducing the P/E based-model, finds that long-term returns of shares are highly predictable in terms of the ratio between averaged earnings and the current
stock price. The model was validated in subsequent papers [4,5] through stochastic simulation applied to the economic conjuncture of 1998, characterized by high values in the American stock market indexes.

In the same research line, the Bond Stock Earning Yield Differential (BSEYD) model proposed by Ziemba and Schwartz [6] obtains similar results to those of the P/E model but examining ratios rather than variables in levels. Both models present a considerable degree of accuracy, but the BSEYD model yields to better outcomes in terms of statistical significance and robustness [39]. Another widely referenced model within this group corresponds to the FED model, which supposes a negative relationship between P/E market ratios and government bond yields. However, the origin of this model as a proposal of the FED is not clear and its empirical results have been debated [42]. In this stream of studies, Harvey and Whaley [43] found, through an implicit volatility analysis, that changes in predictable volatility but abnormal returns cannot be achieved in a trading strategy. Finally, Niemira and Saaty [44] developed a model to predict the probability of a market crash based on analytic network processes.

The second research stream is based on the probabilistic representation of price processes. Despite having a more recent development, this line has experienced a wide acceptance among researchers since the seminal works of Sornette et al. [16,22,45,46]. This type of models presents satisfactory results when explaining the subprime financial crisis and, according to the classification made by Chen and Huang [14], are divided into four categories: The probability of occurrence approach, indicators or signals approach, predicting the date of a crisis occurrence approach and the use of innovative techniques approach.

One of the most important works in the first category corresponds to the proposal of Lebaron and Samanta [7], who found that the risk of market crashes varies among regions and this type of events is more likely in emerging markets. Another study in this approach is the one developed by Gresnigt et al. [8], which proposes a framework to gauge probability predictions on future market crashes in the mid-term (around 5 days). This model analyzes the stock market returns around a financial market crash in a similar way to the seismic activity in the dates around an earthquake. Subsequently, this work is extended by Gresnigt et al. [47] through the implementation of Hawkes models, in which the extreme values of returns can be triggered by processes of self-excitement and cross-excitement; and Gresnigt et al. [48], who developed different specification tests for Hawkes models. In the same line of research, Wu et al. [40] studied the diffusion of crashes in different markets by combining the multivariate skewed Student’s t density with a TVC-GARCH. In alternation, Jiang et al. [49] concluded that extreme returns are predictable in the short term using the time between consecutive extremes. Egorova et al. [50] study currency crashes in Russia employing Hawkes processes as well.

Within the indicators and/or signals approach, Grech et al. [9] applied the idea of Hurst exponent to analyze the Dow Jones time series and find that the value of the Hurst Alpha exponent decays quickly in moments before a crash and that it is possible to estimate a time window that varies between markets over which this exponent can be calculated to obtain reasonable predictions. Posteriorly, Jarrow et al. [10,11] proposed a new methodology to determine in real time if an asset price corresponds to a bubble through a free martingale pricing technology. Alternatively, Rivera-Castro et al. [12] proposed a top bottom price approach to understand financial fluctuations. In this approach market crashes are identified by the difference of the extreme prices (top-down), the time lag between corresponding events and the time interval between events with a minimal magnitude.

In the same investigative approach, some of the most recent works include the proposal of Ko et al. [13], in which the authors developed an alarm index2 that uses periodic log functions and a pattern recognition algorithm to predict Korean financial market crashes; and Chen and Huang [14] who introduced Fourier spectrum analysis to crisis forecasting and applied it to market indexes of five countries. Valenti et al. [51] proposed a measure called the ‘mean first hitting time’ as a new indicator for price volatility, finding that high and low volatility periods can be related to financial instability. Another approach that could be framed in this line is that of the analysis of correlations as a signal of the change in the market trend. Several authors [52,53] have studied the degree of correlations among the stocks forming a global index, finding that the eigenvalues spectrum of the correlation matrix is dominated by one strongly collective eigenstate associated to a large eigenvalue for drawdowns, whilst the eigenvalues structure is more uniformly distributed for drawups. This effect is even more salient the more dramatic the fall is. Consequently, the dynamics of the eigenvalues of the correlation matrix between the assets quoting in a market can be used as a warning mechanism to detect market crashes.

In the third approach, it is worthwhile to mention the models developed by Pyrlik [15], who proposed market crashes prediction technique based on the durations analysis between Dow Jones crashes using autoregressive conditional duration models (ACD); and Rodriguez-Caballero and Knapik [54], whose study is seminal in the incorporation of Bayesian statistical models to estimate the most probable time of occurrence of a market crash. The models proposed by the authors provide credible intervals at 95% for the market crash time.

There are several models that could be mentioned in the fourth category. However, the most important proposals correspond to the contributions by Didier Sornette and the subsequent extensions derived from his work. The popularity of Sornette’s models is not only due to their high degree of accuracy when explaining market crashes, but also owing to his innovation in the understanding this type of phenomena as the result of “the progressively increasing build-up of market cooperativity, or effective interactions between investors, often translated into accelerating ascent of the market price (the bubble)” [(46) p. 3]. Therefore, according to Sornette “the specific manner by which prices collapsed is not the

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2 The alarm index tries to measure the probability that a certain day occurs the market crash.
The first seminal paper by Sornette dates from mid 90s. In that work, Sornette et al. [16] found precursory and aftershock ‘distinguishing marks’ for the 1987 market crash that suggest a dynamic critical point that characterizes log-periodic signatures. At the same time, Feigenbaum and Freund [17] yielded similar results by proposing a model that considers stock market crashes as critical points in a hierarchical system with discrete scaling. More lately, Johansen et al. [18] extended Sornette’s analyses of log-periodic signatures to eight independent market crashes occurred between 1929 and 1998 for different markets including Hong Kong, Russia and the USA. The authors concluded that there were not financial crashes preceded by an extended bubble without exhibiting log-periodic signatures since the decade of seventies. Ausloos et al. [19] confirmed these results by finding log-periodic oscillations before crashes for different financial indexes.

Posteriorly, Lux and Sornette [20] found that the behavior of the unconditional distribution that emerged from financial bubbles exhibit power law tails. At the same time, Sornette [45] reviewed the coincidences in the study of catastrophic events related to material rupture, earthquakes, financial crashes, etc.; and how this type of events could be studied through his proposal. In mid of 2000s, Sornette and Zhou [21] presented an algorithm to detect the existence of collective self-organization patterns in the behavior of agents within social systems. This work was extended by Sornette et al. [22] by developing the concept of “Dragon-Kings” which refers to meaningful outliers that coexist with power laws and helps to reveal the existence of mechanisms of self-organization. Finally, it is worth mentioning that, given the popularity of these models, they have been tested in different financial markets, e.g. Brazil [23], and Poland [24,25]. Finally, seminal works of Gidea [55–57] implement Topological Data Analysis (TDA) to financial time series and detect financial crashes. The model presented in this paper is framed in the research line that studies market crashes in terms of the probabilistic representation of the process for returns and studying its probability of occurrence, i.e. the first of the analyzed categories. The next section introduces the framework of our proposal, which is based on the analysis of the tail index parameter of stable distributions.

3. Methodology

Following [7], market crashes or booms correspond to extreme realizations of the underlying return distribution and thus their analysis should be based on such distribution. Furthermore, salient stylized facts of the daily returns on risky assets reveal that distributions are leptokurtic and, therefore, the normal distribution does not fit well to the empirical data. On the contrary, distributions whose shape parameter may be represented by the tail index present accurate performance for capturing the empirical distribution of financial returns [34,58]. Lux and Sornette [20] point out that “universally accepted that the distribution of returns is not only leptokurtic but also belongs to the class of fat tailed distributions. More formally, it has been shown that the tails of the distributions of returns (... ) follow approximately a power law” (p. 2). Therefore, the characterization of the financial returns and its extreme values, understood in this framework as market crashes, can be studied through the analysis of the tail index. According to Grabchak and Samorodnitsky [34], a tail index refers to a parameter of a group of semiparametric models where a random variable \( X \) (in our case, the asset return) is said to have a regularly varying right tail with tail index \( \alpha > 0 \), if

\[
P(X > x) = x^{-\alpha} L(x), \quad x > 0,
\]

where \( L \) is a slowly varying function at infinity.\(^4\) By applying a similar definition to the left tail, it is possible to obtain two tail indexes, where \( X \) has a finite (infinite) variance if the smaller tail index is higher (lower) than 2 and does not have finite mean when \( \alpha \) is less than 1. Consequently, a probability model with power tails can be naturally used to characterize processes with extreme events. This is the case of the Stable distributions, which allow skewness and heavy tails and includes the Gaussian and Cauchy distributions as particular cases [59]. Specifically, the alpha-stable distributions are described by four parameters, including the index of stability \( \alpha \in (0, 2) \), skewness parameter \( \beta \), scale parameter \( \gamma \) and location parameter \( \delta \). The stability parameter \( (\alpha) \) upper bound corresponds to the Gaussian distribution and as \( \alpha \) goes to the lower bound the distribution becomes more leptokurtic, \( \alpha = 1 \) being the case of the Cauchy distribution. Consequently, this parameter is linked to the presence of outliers. However, there are not closed formulas for density and distribution functions. This fact has generated multiple parameterization proposals, the most often was developed by Samorodnitsky and Taqqu [60] in terms of the characteristic function of \( X \sim S(\alpha, \beta, \gamma, \delta_1; 1) \),

\[
E[\exp(itX)] = \begin{cases} 
\exp(-\alpha|t|^\alpha [1 - i\beta(tan \frac{\alpha \pi}{2})(sign \ t)] + i\delta_1 t) & \alpha \neq 1, \\
\exp(- |t| [1 + i\beta \frac{\pi}{2}(sign \ t)\ln |t|] + i\delta_1 t) & \alpha = 1,
\end{cases}
\]

where \( 0 < \alpha \leq 2, -1 \leq \beta \leq 1, \gamma > 0 \) and \( \delta_1 \in \mathbb{R} \).

\(^3\) There is a distinction between the types of thin tailed distributions with finite endpoints, medium tailed distributions with exponential decline and fat tailed distributions with power law tails (Lux and Sornette, 2002).

\(^4\) L is a slowly varying function if \( \lim_{x \to \infty} \frac{\ln L(x)}{\ln x} = 1 \) \( y > 0 \).
A different specification, provided by Nolan [59], corresponds to a modification of Zolotarev's parameterization where the characteristic function of \( X \sim S(\alpha, \beta, \gamma, \delta_0; 0) \) is given by

\[
E \left[ \exp (i t X) \right] = \begin{cases} 
\exp(-\gamma |t|^{\alpha} [1 + i \beta (\tan \frac{\pi \alpha}{2}) \text{sign} (t) ((|t|)^{1-\alpha} - 1)] + i \delta_0 t) & \alpha \neq 1, \\
\exp(-\gamma |t| [1 + i \beta (\tan \frac{\pi}{2} \text{sign} (t) (|t| + i \gamma)]) + i \delta_0 t) & \alpha = 1.
\end{cases}
\]

The advantage of this characteristic function, as well as its corresponding density and distribution functions, are better performance. The results with FIGARCH model are available upon request.

The procedure is thought for time series of daily (or higher) frequency that exhibit the traditional stylized features of financial returns, particularly leptokurtosis and clustering in volatility. For this reason, we choose a general model that accounts for all these features and thus the log returns for the daily price series are described through a GARCH-Stable(1,1) model, which was proposed by Liu and Brorsen [68]. This model has been chosen considering that "GARCH models assuming normal or \( t \)-distributed errors cannot capture all the leptokurtosis and skewness" ([68], p. 274) of financial returns and it is appropriate for the purpose of our study. The GARCH-Stable(p,q) model can be specified as follows:

\[
R_t = \mu_t + \epsilon_t \\
\epsilon_t = \sigma_t z_t \\
\sigma_t^2 = \theta_0 + \sum_{i=1}^{p} \zeta_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} d_i \sigma_{t-i}^2
\]

where \( \mu_t \) is the returns conditional mean, \( \epsilon_t \) corresponds to the error process, \( \sigma_t \) is the conditional variance and \( z_t \) corresponds to i.i.d. random variables that follows a stable distribution instead of a standard normal distribution, as is the case in the standard GARCH(p,q) model.

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5 We also modeled the returns through a FIGARCH model, and its results are not presented in this paper, since GARCH-Stable model presented better performance. The results with FIGARCH model are available upon request.
The following steps describe the procedure for implementing our methodology.

Parameter estimation (Step 1)

Parameters $\alpha$ and $\beta$ of the alpha-stable distribution following the parameterization in Eq. (3) are estimated by ML applied to the log-likelihood function in Eq. (4). The estimation is jointly performed with the GARCH model parameters. The procedure is carried out by employing a rolling window of $T$ observations and hence the first estimated $\alpha$ and $\beta$ parameters were obtained for $T+1$.

Forecasting $\alpha$ and $\beta$ (Step 2)

VAR model

From the results of the estimation performed in step 2, the $\alpha$ and $\beta$ parameters are forecasted to evaluate if it is possible to predict the behavior of the financial returns based on the dynamics of these parameters. To forecast the $\alpha$ and $\beta$ parameters, a VAR model is proposed for modeling the simultaneity effects and intertemporal transmission among these variables. By using the parameters estimated in step 2, we compute 1 day-ahead forecasts for $\alpha$ and $\beta$ parameters. The lag order of the VAR model was selected considering Akaike Information Criterion (AIC) and can be up to ten. Next, we compare these forecasts to the ‘realized’ data and measured forecasting accuracy in terms of mean squared error type of criteria: MSE, RMSE and the RRMSE.

Random walk process

In addition, we simulated a random walk process to generate the forecasted values for the alpha and beta parameters as a naïve benchmark case to compare our proposed methodology. In the random walk model, the alpha (beta) parameter at time $t$ corresponds to the alpha (beta) parameter at time $t-1$ plus a discrete i.i.d. white noise series that are assumed to be Gaussian according to the following equation.

$$\delta_t = \delta_{t-1} + \omega_t,$$

where $\delta_t$ is the parameter alpha (beta) at time $t$ and $\omega_t$ is the Gaussian white noise with mean equal to 0 and standard deviation equal to 1.

Estimating crash probabilities (Step 3)

Once the predictions of the parameters of the alpha-stable distribution have been obtained, we proceed with the estimation of the crash probabilities. In this case, the definitions of crash correspond to three proposals. The first definition is obtained from Gresnigt et al. [47] who considers a crash as a return that is equal or less than the 2.5% quantile of the return’s distribution. The second definition is provided by Wu et al. [40] according to which this type of event occurs when “the daily return lies below the 5% quantile of the empirical returns distribution over the complete sample period” (p. 1145). The third proposal, given by Wang et al. [1], defines a crash as an event with a minimum one-day decrease of 5% in the financial asset returns.

Consequently, the observed crash probability is calculated as the result of the ratio between the number of days for which a crash occurred and the total number of days of the sample period [40]. In the empirical application, the crash probabilities for a random walk process were obtained for comparison purposes. In the case of the alpha-stable distribution, the forecasted crash probability is calculated as follows. First, for each day, the probabilities associated to the forecasted parameters of the alpha-stable distribution (step 3) and the crash definitions are computed. Then, the average of these probabilities for the days in which an empirical crash occurred, according to the adopted definitions, is obtained.

Finally, the performance of the models for capturing the crash probabilities is assessed by using the quadratic probability score (QPS) [69], the log-probability score (LPS) [47] and the Hansen–Kuipers (HK) skill score. The QPS and LPS scores are given by the following expressions:

$$QPS = \frac{2}{m} \sum_{t=1}^{m} (f_t - I_t)^2,$$

$$LPS = -\frac{1}{m} \sum_{t=1}^{m} [((1 - I_t) \log (1 - f_t) + I_t \log (f_t))].$$

where $f_t$ corresponds to the forecasted probability for the day $t$ and $I_t$ is a dummy variable that scores 0 if there is no crash, and 1 if a crash is detected. The value $m$ depends on the number of observations in the sample period. The QPS is defined in [0, 1] and the LPS takes values in [0, $\infty$), being 0 the optimal result under ‘perfect’ forecasting.

The HK score is defined as the difference between the proportion of crashes that were correctly predicted by the model, named hit rate ($H$), and the ratio between the number of days for which a crash was predicted and the number of days

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6 We use the packages Stable5.3, libstableR and GEVStableGarch in R.
7 Package vars of software R was used to forecast the parameters $\alpha$ and $\beta$ through a VAR model.
8 MSE is calculated as the average of the squared difference between the forecasted parameter and its ‘realized’ value. RMSE is the square root of the MSE. RRMSE corresponds to the ratio between the RMSE and the average value of the corresponding parameter.
### Table 1: Descriptive statistics for developed and emerging markets.

|                      | North America | European Union | Pacific | Emerging Latin America | Emerging Europe | Emerging Asia |
|----------------------|---------------|----------------|---------|------------------------|-----------------|--------------|
| Mean                 | 0.0306        | 0.0045         | 0.0184  | 0.0402                 | 0.0140          | 0.0196       |
| Median               | 0.0000        | 0.0371         | 0.0255  | 0.0764                 | 0.0461          | 0.0480       |
| Standard Deviation   | 1.0664        | 8.6530         | 13.1439 | 12.0175                | 13.1329         | 9.7093       |
| Kurtosis             | -0.8904       | -0.1762        | -0.3324 | -0.4112                | -0.4394         | -0.3058      |
| Skewness             | -0.8904       | -0.1762        | -0.3324 | -0.4112                | -0.4394         | -0.3058      |
| Minimum              | -21.8222      | -7.9086        | -18.4735| -15.0600               | -19.9242        | -9.2784      |
| Maximum              | 11.2662       | 9.5780         | 10.8228 | 15.3532                | 18.6010         | 12.6541      |
| Obs. (N)             | 8000          | 8000           | 8000    | 8000                   | 8000            | 8000         |

Descriptive statistics on daily returns data from January 1, 1980 to May 30, 2019 of different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes).

### Table 2: Forecasting accuracy of the VAR model for alpha-stable parameters.

|         | North America | European Union | Pacific | Emerging Latin America | Emerging Europe | Emerging Asia |
|---------|---------------|----------------|---------|------------------------|-----------------|--------------|
| MSE     | 0.0000        | 0.0099         | 0.005   | 0.0014                 | 0.011           | 0.009        |
| RMSE    | 0.0000        | 0.0051         | 0.0027  | 0.0011                 | 0.0016          | 0.0095       |
| RRMSE   | 0.0000        | 0.0043         | 0.0023  | 0.0016                 | 0.0016          | 0.0095       |

MSE is the Mean Squared Error; RMSE is the square root of MSE; RRMSE is the ratio of the RMSE and the average value of the parameter. Statistics computed for estimation windows of size $T$ using daily return data from January 1, 1980 to May 30, 2019 of different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes).

in which the crash did not occur, known as the false alarm rate ($F$). This measure is widely employed to evaluate the predictive capacity of a model. The calculation of the HK score is given by the following expression.

$$HK = H - F,$$

where $H$ is the hit rate and $F$ corresponds to the false alarm rate.
Fig. 2. Forecasted errors for $\alpha$ parameter (GARCH-stable model). Forecasted error for parameter $\alpha$ calculated as the difference between the parameter value estimated from the observed data and its forecast. Forecasts computed for daily return data from January 1, 1980 to May 30, 2019 of different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes).

4. Empirical results

4.1. Parameter estimation and forecasting

This section presents the results of applying the four-step procedure described above to the log-returns of the daily series of a group MSCI equity indexes for developed (North America, European Union and Pacific) and emerging markets (Latin America, Europe and Asia). Data were downloaded from the Bloomberg platform for a sample ranged from January 1, 1980 to May 30, 2019, obtaining more than 8000 observations for most of the time series. See Appendix for a detailed description of the indexes. Table 1 records the main descriptive statistics of the series.

As can be seen in Table 1, the returns of the analyzed series have a mean close to 0, which is common in daily returns of equity assets. It is also observed a negative skewness coefficient and a leptokurtic behavior in all cases. This indicates the presence of negative extreme events, which means that the probability of incurring in significant losses is not negligible.

\footnote{In the cases in which information was not available from the indicated starting date, the first available observation was taken.}
In the case of North America, kurtosis is substantially higher than for the other regions. These facts reinforce the need to analyze stock indexes through approaches that incorporate all these features, as the alpha-stable distribution.

According to the GARCH-Stable model, we estimated $\alpha$ and $\beta$ parameters by employing a rolling window of size $T = 250$, 500 and 1000 observations for sake of comparison. These series of estimates are taken as the true realizations of the parameters of the alpha-stable when testing the forecasting performance of the model, described below.

In order to show the forecasting power of past conditional alpha-stable densities on future (one-step-ahead) densities, we propose a VAR model for the series of estimated $\alpha$ and $\beta$ parameters. The forecasting ability of the model in terms of different accuracy measures is shown in Table 2. In general, the forecasting performance of the model seems to be adequate, as evidenced by the values of MSE, RMSE and RRMSE. The results obtained with the three different time windows are satisfactory. For the alpha parameter, the MSE values are equal to or less than 0.0002. In the case of the beta parameter, the MSE is equal to or less than 0.0121. The RMSE results are consistent with the MSE. The RRMSE presents values less than 0.0133 for alpha parameter and 0.1771 for the beta parameter.

The results show that the best forecast is obtained with a time window of 1000 observations. For this time window and the alpha parameter, the MSE average is 0.00001 compared to 0.0002 ($T = 250$) and 0.0001 ($T = 500$). The MSE for the beta parameter also presents better results for this time window. In the case of alpha parameter, the maximum RRMSE value is 0.0022 and −0.03740 for the beta parameter when $T = 1000$. On the other hand, the maximum RRMSE value is 0.0133 ($T = 250$) and 0.0091 ($T = 500$) for the alpha parameter and $-0.1771$ ($T = 250$) and $-0.0653$ ($T = 500$) for the beta parameter case.

It is noteworthy that the results obtained from the three different time windows are much better than those obtained from the random walk model. In this case, the MSE average for the alpha parameter is 0.5769 with a maximum of 0.6154. The RRMSE presents an average value of 0.4096. In the case of the beta parameter, the average value of the MSE is 1.96 with a maximum value of 2.46. The RRMSE presents an average of $-3.21$ and a maximum value (in absolute terms) of $-4.7482$.

These results are consistent with the forecasted errors ($T = 500$), which are between −0.1 and 0.1 in most cases for $\alpha$ parameter (see Fig. 2), being Latin America and European and Asian emerging markets the series with the best fit. In the case of $\beta$ parameter, the forecasted error is higher than that of parameter $\alpha$, being lower than 1 (in absolute value) for most of the observations (see Fig. 3). However, this is still a satisfactory result for the prediction of this parameter.

These results definitely outperform those obtained from the random walk process which are greater than 1 on several occasions and in some cases are around 2 for the parameters alpha and beta, as can be seen in Figs. 4 and 5. The MSE, RMSE, RRMSE and the forecasted error confirm that the results obtained from our proposed methodology are significantly better than those obtained through a random walk process. These results also suggest that in the calculations obtained in the next sections, an adequate performance of the random walk model would correspond more to ‘spurious’ evidence than to an adequate modeling of the time series.

4.2. Crash probability forecasts

Next, we proceed with the estimation of the crash probabilities according to step 3 in the Methodology section. According to these estimates, the forecasting crashes performance of the alpha-stable distribution is compared with a random walk process proposed as a benchmark. The results are evaluated in terms of QPS, LPS and HK score — see Eqs. (9)–(11).

For this purpose, we compute the probability that a random variable distributed as the fitted alpha-stable be less than the 2.5% or 5% quantile of the empirical distribution of returns. Additionally, we calculate the probability that returns have a daily drop of at least 5%. The empirical probability of crash for the sample period is close to 2.5%, 5%, and between 0.3% and 0.6% according to the adopted crash definitions, respectively. As can be seen, these probabilities are consistent to the different definitions of market crash employed in our work.

The average forecasted probabilities of crash for the analyzed market indexes under the alpha-stable dynamic distribution, are between 1.4% and 6% for the first definition of crash, 3.6% and 11.3% for the second definition of crash, and 0.35% and 0.61% for the third definition of crash. The previous results are shown in Table 3. These probability ranges contain, for each definition, the estimated empirical probability of crash. Moreover, the highest forecasted probabilities of crash correspond to the index that presents the highest kurtosis, revealing the fact that the alpha-stable distribution can exhibit a remarkable kurtosis capturing the stylized facts of the return empirical distributions. Furthermore, European and Latin American emerging markets exhibit the lowest probability of crash. The average values of the QPS and LPS (Table 4) are 0.05 and 0.13 respectively, for the first definition of crash, 0.10 and 0.21 for the second definition of crash, and 0.01 and 0.03 for the third definition of crash. In all cases, these results show a satisfactory capacity of the alpha-stable model to predict market crashes compared to other studies (e.g. [40,47]). Recall that the lower QPS and LPS values, the better the performance of the model. Cresnigt et al. [47] obtain a highest value of QPS around 0.13 and a highest value for LPS of 0.24 whereas the proposal of Wu et al. [40] the maximum QPS is around 0.45.

Furthermore, when comparing these results with those obtained from the random walk process for the three adopted definitions of crash with $T = 500$ (see Table 5), we confirmed that our model presents better results. In the case of the random walk process the average QPS and LPS are respectively 0.06 and 0.15 for the first definition of crash, 0.11 and
Forecasted errors for $\beta$ parameter (GARCH-stable model). Forecasted error for parameter $\beta$ calculated as the difference between the parameter value estimated from the observed data and its forecasts. Forecasts computed for daily return data from January 1, 1980 to May 30, 2019 of different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes).

0.22 for the second definition of crash and 0.01 and 0.06 for the third definition of crash. However, it is necessary to consider that, given the results obtained for accuracy in the alpha-stable parameters estimation in the previous section, the apparently 'acceptable' results of the random walk are 'spurious'.

When evaluating crashes prediction capacity, it is observed that for the HK score (Tables 6, 7, 8, and 9), the results obtained by the proposed methodology are better than those obtained by the random walk process. For $T = 250$ and using the third definition of crash (Table 6), it is observed that in 4 of the 6 analyzed indexes, the maximum value that the HK score from our methodology is higher than that of the random walk process (Table 9). The proportion of favorable results holds for $T = 500$ (Table 7). Moreover, this proportion improves for $T = 1000$ with the same definition of crash (Table 8). In this case, the maximum HK score of the methodology is higher than that of the random walk process for the 6 indexes analyzed, reaching in one case a value above 0.5. This shows the capacity of our proposed methodology in predicting crashes but also suggests, as previously observed in the paper, that the best results are obtained for $T = 1000$ and the third definition of crash employed.\(^\text{10}\) This fact is consistent with the dynamic nature of the forecasts, since defining a

\(^{10}\) The HK results for the other definitions of crash and time windows are not reported but are available upon request.
crash in terms of a quantile of the unconditional (empirical) distribution may impact the crash forecasting capacity of the conditional distribution more than when the crash has a distribution-free definition (e.g. is defined in terms of an exogenously given magnitude).

4.3. Streaks (drawdowns) analysis

To complement the results obtained in the previous sections, a streaks (drawdowns) analysis was performed. A streak can be defined as a period in which the price of a financial asset has an upward or downward trend uninterruptedly. The term 'uninterruptedly' refers to the fact that during the duration of this period, there is no countermovements. In other words, if the streak refers to a period in which the index price presents an increasing (decreasing) trend, during its duration there is not any other period in which the index price has a decreasing (increasing) trend.
In order to perform this analysis for the alpha stable model, the streaks for the six index price series were first identified.\(^{11}\) In this case, the interest was to identify those points at which a fall in the price ended and a rise began (downturns), which are linked to market declines or possibly market crashes. Next, the forecasted probability of crash under the alpha stable model was identified for the second and third crash definitions, for each of the market downturns, and the average of these probabilities was computed along every streak period.

Table 10 shows that for the observations identified as market downturns under the streaks methodology (depicted in Fig. 6), the forecasted average probability of crash is higher than the forecasted average probability of crash for the entire sample for four of the six analyzed indexes. For the Developed European and Asia–Pacific indexes this result is not observed, which is consistent with [70] if we interpret most of Developed European streaks as contagion of extreme values from other markets. The same results were obtained from the other two used crash definitions. This evidence supports dynamic alpha-stable forecasting as a methodology that involves higher crash forecasted probabilities for those periods where there is a continuous decline in the financial asset prices.

11 The streaks were identified using the package PMwR in R.
Table 3
Forecasted crash probabilities for MSCI indexes (GARCH-stable model).

| T = 250 | Crash as 2.5% quantile | Crash as 5% quantile | Crash as daily drop of at least 5% |
|---------|------------------------|----------------------|-----------------------------------|
| North America | 0.0574 | 0.1130 | 0.0045 |
| European Union | 0.0386 | 0.0852 | 0.0041 |
| Pacific | 0.0427 | 0.0840 | 0.0035 |
| Emerging Latin America | 0.0160 | 0.0358 | 0.0059 |
| Emerging Europe | 0.0137 | 0.0387 | 0.0054 |
| Emerging Asia | 0.0354 | 0.0825 | 0.0054 |

| T = 500 | Crash as 2.5% quantile | Crash as 5% quantile | Crash as daily drop of at least 5% |
|---------|------------------------|----------------------|-----------------------------------|
| North America | 0.0578 | 0.1130 | 0.0061 |
| European Union | 0.0401 | 0.0861 | 0.0048 |
| Pacific | 0.0418 | 0.0835 | 0.0041 |
| Emerging Latin America | 0.0156 | 0.0361 | 0.0055 |
| Emerging Europe | 0.0143 | 0.0391 | 0.0054 |
| Emerging Asia | 0.0352 | 0.0822 | 0.0058 |

| T = 1000 | Crash as 2.5% quantile | Crash as 5% quantile | Crash as daily drop of at least 5% |
|---------|------------------------|----------------------|-----------------------------------|
| North America | 0.0567 | 0.1123 | 0.0052 |
| European Union | 0.0410 | 0.0869 | 0.0050 |
| Pacific | 0.0422 | 0.0837 | 0.0039 |
| Emerging Latin America | 0.0153 | 0.0359 | 0.0053 |
| Emerging Europe | 0.0147 | 0.0396 | 0.0058 |
| Emerging Asia | 0.0353 | 0.0824 | 0.0049 |

Statistics computed for estimation windows of size $T$ using daily return data from January 1, 1980 to May 30, 2019 of different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes).

Table 4
QPS and LPS for GARCH-stable crashes forecasting method.

Crash = 2.5% quantile

| T = 250 | QPS | LPS |
|---------|-----|-----|
| North America | 0.0507 | 0.1299 |
| European Union | 0.0517 | 0.1255 |
| Pacific | 0.0514 | 0.1249 |
| Emerging Latin America | 0.0491 | 0.1203 |
| Emerging Europe | 0.0519 | 0.1278 |
| Emerging Asia | 0.0504 | 0.1210 |

| T = 500 | QPS | LPS |
|---------|-----|-----|
| North America | 0.0517 | 0.1307 |
| European Union | 0.0462 | 0.1155 |
| Pacific | 0.0507 | 0.1241 |
| Emerging Latin America | 0.0504 | 0.1228 |
| Emerging Europe | 0.0509 | 0.1242 |
| Emerging Asia | 0.0513 | 0.1223 |

| T = 1000 | QPS | LPS |
|---------|-----|-----|
| North America | 0.0533 | 0.1331 |
| European Union | 0.0470 | 0.1167 |
| Pacific | 0.0527 | 0.1271 |
| Emerging Latin America | 0.0532 | 0.1287 |
| Emerging Europe | 0.0541 | 0.1304 |
| Emerging Asia | 0.0546 | 0.1277 |

Crash = 5% quantile

| T = 250 | QPS | LPS |
|---------|-----|-----|
| North America | 0.1021 | 0.2222 |
| European Union | 0.0940 | 0.2034 |
| Pacific | 0.1013 | 0.2131 |
| Emerging Latin America | 0.0960 | 0.2024 |
| Emerging Europe | 0.0999 | 0.2079 |
| Emerging Asia | 0.0999 | 0.2105 |

| T = 500 | QPS | LPS |
|---------|-----|-----|
| North America | 0.1032 | 0.2233 |
| European Union | 0.0877 | 0.1942 |
| Pacific | 0.1008 | 0.2126 |
| Emerging Latin America | 0.0984 | 0.2062 |
| Emerging Europe | 0.0978 | 0.2041 |
| Emerging Asia | 0.1008 | 0.2117 |

| T = 1000 | QPS | LPS |
|---------|-----|-----|
| North America | 0.1053 | 0.2260 |
| European Union | 0.0863 | 0.1922 |
| Pacific | 0.1031 | 0.2156 |
| Emerging Latin America | 0.1025 | 0.2135 |
| Emerging Europe | 0.1013 | 0.2097 |
| Emerging Asia | 0.1062 | 0.2192 |

Crash = Daily drop of at least 5%

| T = 250 | QPS | LPS |
|---------|-----|-----|
| North America | 0.0051 | 0.0199 |
| European Union | 0.0061 | 0.0213 |
| Pacific | 0.0057 | 0.0213 |
| Emerging Latin America | 0.0182 | 0.0545 |
| Emerging Europe | 0.0224 | 0.0649 |
| Emerging Asia | 0.0075 | 0.0247 |

| T = 500 | QPS | LPS |
|---------|-----|-----|
| North America | 0.0052 | 0.0186 |
| European Union | 0.0060 | 0.0203 |
| Pacific | 0.0056 | 0.0201 |
| Emerging Latin America | 0.0188 | 0.0551 |
| Emerging Europe | 0.0218 | 0.0627 |
| Emerging Asia | 0.0075 | 0.0240 |

| T = 1000 | QPS | LPS |
|---------|-----|-----|
| North America | 0.0052 | 0.0188 |
| European Union | 0.0068 | 0.0224 |
| Pacific | 0.0059 | 0.0211 |
| Emerging Latin America | 0.0202 | 0.0586 |
| Emerging Europe | 0.0233 | 0.0661 |
| Emerging Asia | 0.0080 | 0.0256 |

QPS is the Quadratic Probability Score; LPS is the Log-Probability Score. Statistics computed for estimation windows of size $T$ using daily return data from January 1, 1980 to May 30, 2019 of different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes).
Table 5
QPS and LPS measures for random walk crashes forecasting with $T = 500$.

| Crash = 2.5% quantile | Crash = 5% quantile | Crash = Daily drop of at least 5% |
|-----------------------|----------------------|----------------------------------|
|                       | QPS      | LPS    | QPS      | LPS    | QPS      | LPS    |
| North America         | 0.0625   | 0.1599 | 0.11340  | 0.2430 | 0.00990  | 0.04850|
| European Union        | 0.0522   | 0.1332 | 0.09400  | 0.2078 | 0.00900  | 0.04360|
| Pacific               | 0.0618   | 0.1593 | 0.11150  | 0.2377 | 0.01050  | 0.05520|
| Emerging Latin America| 0.0548   | 0.1426 | 0.10160  | 0.2152 | 0.02240  | 0.08130|
| Emerging Europe       | 0.0546   | 0.1383 | 0.10110  | 0.2123 | 0.02490  | 0.08080|
| Emerging Asia         | 0.0597   | 0.1500 | 0.10940  | 0.2316 | 0.01200  | 0.05390|

QPS is the Quadratic Probability Score; LPS is the Log-Probability Score. Statistics computed for estimation windows of size $T$ using daily return data from January 1, 1980 to May 30, 2019 of different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes).

Table 6
HK score for $T = 250$ and a minimum 5% daily-return decrease crash (GARCH-stable model).

| North America | w | H   | F   | HK   |
|---------------|---|-----|-----|------|
|               | 0.0003 | 1.0000 | 0.9986 | 0.0014 |
|               | 0.0013 | 0.9500 | 0.8505 | 0.0995 |
|               | 0.0022 | 0.9500 | 0.8220 | **0.1280** |
|               | 0.0032 | 0.8500 | 0.7480 | 0.1020 |
|               | 0.0042 | 0.3500 | 0.6257 | −0.2757 |
|               | 0.0051 | 0.2000 | 0.5164 | −0.3164 |
|               | 0.0061 | 0.1000 | 0.4219 | −0.3219 |
|               | 0.0071 | 0.1000 | 0.3385 | −0.2385 |
|               | 0.0080 | 0.1000 | 0.2717 | −0.1717 |
|               | 0.0090 | 0.1000 | 0.2124 | −0.1124 |

| European Union  | w | H   | F   | HK   |
|-----------------|---|-----|-----|------|
|                 | 0.0003 | 1.0000 | 0.9989 | 0.0011 |
|                 | 0.0013 | 0.9286 | 0.8528 | 0.0758 |
|                 | 0.0022 | 0.8571 | 0.7616 | 0.0955 |
|                 | 0.0032 | 0.7143 | 0.6098 | 0.1045 |
|                 | 0.0042 | 0.6429 | 0.4420 | **0.2009** |
|                 | 0.0051 | 0.1429 | 0.3113 | −0.1684 |
|                 | 0.0061 | 0.0714 | 0.1882 | −0.1168 |
|                 | 0.0071 | 0.0000 | 0.1009 | −0.1009 |
|                 | 0.0080 | 0.0000 | 0.0491 | −0.0491 |
|                 | 0.0090 | 0.0000 | 0.0386 | −0.0386 |

| Pacific         | w | H   | F   | HK   |
|-----------------|---|-----|-----|------|
|                 | 0.0003 | 1.0000 | 0.9968 | 0.0032 |
|                 | 0.0013 | 0.9167 | 0.8010 | **0.1157** |
|                 | 0.0022 | 0.4583 | 0.6725 | −0.2142 |
|                 | 0.0032 | 0.2917 | 0.5048 | −0.2131 |
|                 | 0.0042 | 0.2083 | 0.4092 | −0.2009 |
|                 | 0.0051 | 0.2083 | 0.3182 | −0.1099 |
|                 | 0.0061 | 0.2083 | 0.2387 | −0.0304 |
|                 | 0.0071 | 0.1250 | 0.1913 | −0.0663 |
|                 | 0.0080 | 0.1250 | 0.1562 | −0.0312 |
|                 | 0.0090 | 0.1250 | 0.1258 | −0.0008 |

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5. Further results

5.1. Application to S&P500 and KOSPI indexes

In this section, and with the aim at providing further evidence on the performance of our methodology, we perform our three-step procedure to daily observations of S&P500 and KOSPI indexes observed in the same period (January 1980 to May 2019).\textsuperscript{12}

Parameter estimation and forecasting

With the new series, similar results to that of the previous section are obtained. Table 11 shows the main descriptive statistics of the series. The mean return of both series is close to 0 as in the previous case, and the South Korean market exhibits the highest volatility. Once more, the series present negative asymmetry and positive excess kurtosis, which are more salient in the American case. Consequently, the modeling of returns through distributions that capture this type of features, as the alpha-stable distribution, seems appropriate.

The accuracy measures MSE, RMSE and RRMSE for the $\alpha$ parameter are similar to those obtained in the previous section for the analyzed indexes. Nonetheless, with respect to the $\beta$ parameter, the fitting was not as good, particularly the forecasted series present a higher RRMSE in absolute terms. The average value of this measure is $-0.3186$ while the

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\textsuperscript{12} Data was also downloaded from Bloomberg database.
Table 7
HK score for T = 500 and a minimum 5% daily-return decrease crash (GARCH-stable model).

| w      | H  | F  | HK  |
|--------|----|----|-----|
| North America |
| 0.0003 | 1.0000 | 1.0000 | 0.0000 |
| 0.0013 | 1.0000 | 0.8868 | 0.1132 |
| 0.0022 | 1.0000 | 0.8593 | 0.1407 |
| 0.0032 | 1.0000 | 0.8086 | 0.1914 |
| 0.0042 | 0.9000 | 0.6808 | 0.2192 |
| 0.0051 | 0.8500 | 0.5518 | **0.2982** |
| 0.0061 | 0.5000 | 0.3569 | 0.1431 |
| 0.0071 | 0.1000 | 0.2652 | −0.1652 |
| 0.0080 | 0.0500 | 0.1791 | −0.1291 |
| 0.0090 | 0.0500 | 0.1197 | −0.0697 |
| European Union |
| 0.0003 | 1.0000 | 0.9986 | 0.0014 |
| 0.0013 | 1.0000 | 0.9014 | 0.0986 |
| 0.0022 | 1.0000 | 0.8893 | 0.1107 |
| 0.0032 | 0.9231 | 0.7016 | 0.2215 |
| 0.0042 | 0.8462 | 0.4394 | **0.4068** |
| 0.0051 | 0.4615 | 0.2800 | 0.1815 |
| 0.0061 | 0.0000 | 0.1842 | −0.1842 |
| 0.0071 | 0.0000 | 0.0752 | −0.0752 |
| 0.0080 | 0.0000 | 0.0348 | −0.0348 |
| 0.0090 | 0.0000 | 0.0042 | −0.0042 |
| Pacific |
| 0.0003 | 1.0000 | 0.9977 | 0.0023 |
| 0.0013 | 0.9565 | 0.9222 | 0.0343 |
| 0.0022 | 0.8696 | 0.7193 | 0.1503 |
| 0.0032 | 0.6957 | 0.5104 | **0.1853** |
| 0.0042 | 0.3913 | 0.4021 | −0.0108 |
| 0.0051 | 0.1739 | 0.3402 | −0.1663 |
| 0.0061 | 0.0870 | 0.2513 | −0.1643 |
| 0.0071 | 0.0870 | 0.1844 | −0.0974 |
| 0.0080 | 0.0435 | 0.1535 | −0.1100 |
| 0.0090 | 0.0435 | 0.1280 | −0.0845 |
| Emerging Latin America |
| 0.0003 | 1.0000 | 1.0000 | 0.0000 |
| 0.0013 | 1.0000 | 0.9469 | 0.0531 |
| 0.0022 | 0.9706 | 0.8672 | 0.1034 |
| 0.0032 | 0.8235 | 0.7740 | 0.0495 |
| 0.0042 | 0.7353 | 0.6311 | 0.1042 |
| 0.0051 | 0.5735 | 0.4681 | **0.1054** |
| 0.0061 | 0.2647 | 0.3195 | −0.0548 |
| 0.0071 | 0.2059 | 0.1995 | 0.0064 |
| 0.0080 | 0.1765 | 0.1297 | 0.0468 |
| 0.0090 | 0.1471 | 0.0753 | 0.0718 |
| Emerging Europe |
| 0.0003 | 1.0000 | 1.0000 | 0.0000 |
| 0.0013 | 1.0000 | 0.9970 | 0.0030 |
| 0.0022 | 0.9873 | 0.9685 | 0.0188 |
| 0.0032 | 0.9494 | 0.8517 | 0.0977 |
| 0.0042 | 0.7215 | 0.6276 | 0.0939 |
| 0.0051 | 0.6076 | 0.4662 | **0.1414** |
| 0.0061 | 0.3291 | 0.2383 | 0.0908 |
| 0.0071 | 0.1139 | 0.1074 | 0.0065 |
| 0.0080 | 0.0127 | 0.0526 | −0.0399 |
| 0.0090 | 0.0000 | 0.0139 | −0.0139 |

(continued on next page)
Table 7 (continued).

| Emerging Asia       |   |   | HK |
|---------------------|---|---|----|
| w                   | H | F | HK |
| 0.0003              | 1.0000 | 1.0000 | 0.0000 |
| 0.0013              | 0.9630 | 0.9597 | 0.0033 |
| 0.0022              | 0.9259 | 0.8441 | 0.0818 |
| 0.0032              | 0.8519 | 0.6673 | 0.1846 |
| 0.0042              | 0.7407 | 0.4215 | 0.3192 |
| 0.0051              | 0.6667 | 0.2783 | 0.3884 |
| 0.0061              | 0.5556 | 0.1786 | 0.3770 |
| 0.0071              | 0.4444 | 0.1178 | 0.3266 |
| 0.0080              | 0.0741 | 0.0678 | 0.0063 |
| 0.0090              | 0.0000 | 0.0421 | −0.0421 |

HK (Hansen–Kuipers skill score) is the difference between H (Hit rate) and F (False alarm rate) according to Eq. (10). Forecasted probabilities larger than a given cut-off level (w) are classified as a crash. HK scores computed for estimation windows of size $T = 500$ using daily return data from January 1, 1980 to May 30, 2019 of different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes).

Table 8

HK score for $T = 1000$ and a minimum 5% daily-return decrease crash (GARCH-stable model).

| North America       |   |   | HK |
|---------------------|---|---|----|
| w                   | H | F | HK |
| 0.0003              | 1.0000 | 1.0000 | 0.0000 |
| 0.0013              | 1.0000 | 0.9127 | 0.0873 |
| 0.0022              | 1.0000 | 0.8567 | 0.1433 |
| 0.0032              | 1.0000 | 0.8022 | 0.1978 |
| 0.0042              | 0.9474 | 0.6803 | 0.2671 |
| 0.0051              | 0.7895 | 0.5962 | 0.1933 |
| 0.0061              | 0.0526 | 0.2874 | −0.2348 |
| 0.0071              | 0.0000 | 0.1445 | −0.1445 |
| 0.0080              | 0.0000 | 0.0987 | −0.0987 |
| 0.0090              | 0.0000 | 0.0410 | −0.0410 |

| European Union      |   |   | HK |
|---------------------|---|---|----|
| w                   | H | F | HK |
| 0.0003              | 1.0000 | 1.0000 | 0.0000 |
| 0.0013              | 1.0000 | 1.0000 | 0.0000 |
| 0.0022              | 1.0000 | 0.9097 | 0.0903 |
| 0.0032              | 1.0000 | 0.8498 | 0.1502 |
| 0.0042              | 1.0000 | 0.4751 | 0.5249* |
| 0.0051              | 0.1538 | 0.3021 | −0.1483 |
| 0.0061              | 0.1538 | 0.1171 | 0.0367 |
| 0.0071              | 0.0000 | 0.0115 | −0.0115 |
| 0.0080              | 0.0000 | 0.0000 | 0.0000 |
| 0.0090              | 0.0000 | 0.0000 | 0.0000 |

| Pacific             |   |   | HK |
|---------------------|---|---|----|
| w                   | H | F | HK |
| 0.0003              | 1.0000 | 1.0000 | 0.0000 |
| 0.0013              | 1.0000 | 0.9933 | 0.0067 |
| 0.0022              | 0.9565 | 0.8423 | 0.1142 |
| 0.0032              | 0.3913 | 0.5023 | −0.1110 |
| 0.0042              | 0.3043 | 0.3882 | −0.0839 |
| 0.0051              | 0.2609 | 0.3292 | −0.0683 |
| 0.0061              | 0.2174 | 0.2530 | −0.0356 |
| 0.0071              | 0.1304 | 0.2062 | −0.0758 |
| 0.0080              | 0.0000 | 0.0836 | −0.0836 |
| 0.0090              | 0.0000 | 0.0306 | −0.0306 |

(continued on next page)

same value for the previously analyzed indexes is $-0.1503$. In spite of this fact, the overall results of the forecast are still satisfactory considering that most of the values presented in Table 12 are close to 0. Once more, these values support the
superior alpha-stable forecasting accuracy relatively to that of the random walk process. The average MSE for the alpha (beta) parameter is 0.0001 (0.0078) and the average RRMSE for the alpha (beta) parameter is 0.0038 (−0.3186) in the proposed model. The same measurements in the case of the random walk process are respectively 0.63 (2.0992) for the alpha (beta) parameter and 0.4266 (−6.8442) for the alpha (beta) parameter. Again, the results suggest a better forecast with $T = 1000$ as can be seen in Table 12.

When reviewing the forecasted error for the S&P500 and KOSPI, we observe a satisfactory fitting of parameters alpha and beta. This result is evidenced in Fig. 7, where it is shown that for the S&P500 and KOSPI indexes the forecasted errors of $\alpha$ and $\beta$ parameters are in most cases in the $(-0.1, 0.1)$ and $(-1, 1)$ intervals, respectively. In the case of the random walk process (Fig. 8) the forecasted errors of $\alpha$ and $\beta$ parameters are in most of the cases in the $(0, 2)$ and $(-2, 2)$ intervals, respectively proving that the alpha-stable model offers a much better forecasting than the random walk process.

**Crash probability forecasts**

The average forecasted crash probabilities for S&P500 and KOSPI (Table 13) indexes range from 1.81% to 5.96% for the first crash definition, 5.97% and 11.42% for the second crash definition, and 0.2% and 0.53% for the third crash definition. These values are consistent with the observed crash probability for each adopted definition that is around 2.5%, 5% and between 0.27% and 0.62% respectively. Again, these results evidence that the forecasted crash probabilities of the proposed model are higher for the series with higher kurtosis.

In this case the S&P500. The QPS and LPS measures for S&P 500 and KOSPI indexes under the alpha-stable distribution and the random walk process are displayed in Table 14. The values of the QPS and LPS indicators do not show significant
Table 9
HK score for $T = 500$ and a minimum 5% daily-return decrease crash (Random Walk model).

| North America | w | H     | F     | HK   |
|---------------|---|-------|-------|------|
|               | 0.0003 | 0.8500 | 0.5884 | 0.2616 |
|               | 0.0013  | 0.8500 | 0.5880 | 0.2620 |
|               | 0.0022  | 0.8500 | 0.5880 | 0.2620 |
|               | 0.0032  | 0.8500 | 0.5875 | 0.2625 |
|               | 0.0042  | 0.8500 | 0.5871 | 0.2629 |
|               | 0.0051  | 0.8500 | 0.5868 | 0.2632 |
|               | 0.0061  | 0.8500 | 0.5861 | 0.2639 |
|               | 0.0071  | 0.8500 | 0.5860 | 0.2645 |
|               | 0.0080  | 0.8500 | 0.5856 | 0.2644 |
|               | 0.0090  | 0.8500 | 0.5851 | 0.2649 |
| European Union | w | H     | F     | HK   |
|               | 0.0003 | 0.0769 | 0.2638 | −0.1869 |
|               | 0.0013 | 0.0769 | 0.2638 | −0.1869 |
|               | 0.0022 | 0.0769 | 0.2638 | −0.1869 |
|               | 0.0032 | 0.0769 | 0.2629 | −0.1860 |
|               | 0.0042 | 0.0000 | 0.2624 | −0.2624 |
|               | 0.0051 | 0.0000 | 0.2617 | −0.2617 |
|               | 0.0061 | 0.0000 | 0.2606 | −0.2606 |
|               | 0.0071 | 0.0000 | 0.2603 | −0.2603 |
|               | 0.0080 | 0.0000 | 0.2601 | −0.2601 |
|               | 0.0090 | 0.0000 | 0.2592 | −0.2592 |
| Pacific       | w | H     | F     | HK   |
|               | 0.0003 | 0.5652 | 0.6118 | −0.0466 |
|               | 0.0013 | 0.5652 | 0.6114 | −0.0462 |
|               | 0.0022 | 0.5652 | 0.6114 | −0.0462 |
|               | 0.0032 | 0.5652 | 0.6109 | −0.0457 |
|               | 0.0042 | 0.5652 | 0.6106 | −0.0454 |
|               | 0.0051 | 0.5652 | 0.6102 | −0.0450 |
|               | 0.0061 | 0.5652 | 0.6096 | −0.0444 |
|               | 0.0071 | 0.5652 | 0.6095 | −0.0443 |
|               | 0.0080 | 0.5652 | 0.6091 | −0.0439 |
|               | 0.0090 | 0.5652 | 0.6086 | −0.0434 |
| Emerging Latin America | w | H     | F     | HK   |
|               | 0.0003 | 0.6618 | 0.5539 | 0.1079 |
|               | 0.0013 | 0.6618 | 0.5535 | 0.1083 |
|               | 0.0022 | 0.6618 | 0.5535 | 0.1083 |
|               | 0.0032 | 0.6618 | 0.5530 | 0.1088 |
|               | 0.0042 | 0.6618 | 0.5525 | 0.1093 |
|               | 0.0051 | 0.6618 | 0.5521 | 0.1097 |
|               | 0.0061 | 0.6618 | 0.5514 | 0.1104 |
|               | 0.0071 | 0.6618 | 0.5513 | 0.1105 |
|               | 0.0080 | 0.6618 | 0.5508 | 0.1110 |
|               | 0.0090 | 0.6618 | 0.5503 | 0.1115 |
| Emerging Europe | w | H     | F     | HK   |
|               | 0.0003 | 0.7848 | 0.5528 | 0.2320 |
|               | 0.0013 | 0.7848 | 0.5524 | 0.2324 |
|               | 0.0022 | 0.7848 | 0.5523 | 0.2325 |
|               | 0.0032 | 0.7848 | 0.5518 | 0.2330 |
|               | 0.0042 | 0.7848 | 0.5514 | 0.2334 |
|               | 0.0051 | 0.7848 | 0.5510 | 0.2338 |
|               | 0.0061 | 0.7848 | 0.5503 | 0.2345 |
|               | 0.0071 | 0.7848 | 0.5501 | 0.2347 |
|               | 0.0080 | 0.7848 | 0.5497 | 0.2351 |
|               | 0.0090 | 0.7848 | 0.5491 | 0.2357 |

(continued on next page)
Table 9 (continued).

| Emerging Asia |  |  |  |  |
|---------------|---|---|---|---|
| w            | H  | F  | HK |
| 0.0003        | 0.7037 | 0.5548 | 0.1489 |
| 0.0013        | 0.7037 | 0.5544 | 0.1493 |
| 0.0022        | 0.7037 | 0.5544 | 0.1493 |
| 0.0032        | 0.7037 | 0.5538 | 0.1499 |
| 0.0042        | 0.7037 | 0.5534 | 0.1503 |
| 0.0051        | 0.7037 | 0.5530 | 0.1507 |
| 0.0061        | 0.7037 | 0.5523 | 0.1514 |
| 0.0071        | 0.7037 | 0.5521 | 0.1516 |
| 0.0080        | 0.7037 | 0.5517 | 0.1520 |
| 0.0090        | 0.7037 | 0.5512 | 0.1525 |

HK (Hansen–Kuipers skill score) is the difference between H (Hit rate) and F (False alarm rate) according to Eq. (10). Forecasted probabilities larger than a given cut-off level (w) are classified as a crash. HK scores computed for estimation windows of size \( T = 500 \) using daily return data from January 1, 1980 to May 30, 2019 of different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes).

Table 10

Average forecasted crash probabilities for streaks (downturns) (GARCH-stable model).

|                | Crash = 5% quantile | Crash = Daily drop of at least 5% |
|----------------|---------------------|----------------------------------|
|                | Complete sample     | Downturns                        | Complete sample | Downturns |
| North America  | 0.1131              | 0.1140                           | 0.0055          | 0.0063    |
| European Union | 0.0865              | 0.0851                           | 0.0042          | 0.0032    |
| Pacific        | 0.0843              | 0.0834                           | 0.0044          | 0.0038    |
| Emerging Latin America | 0.0356 | 0.0369 | 0.0051 | 0.0058 |
| Emerging Europe | 0.0385        | 0.0391                           | 0.0050          | 0.0053    |
| Emerging Asia  | 0.0819              | 0.0825                           | 0.0043          | 0.0047    |

Crash probabilities computed as the average of the forecasted crash probability under the alpha stable model for the streaks in which a price drop ended and a price increase started (downturns) and according to two alternative definitions of crashes: values below de 5% quantile of the empirical distribution and values higher than 5% decline. Forecasted probabilities computed for daily return data from January 1, 1980 to May 30, 2019 of different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes), but averaged through either the complete sample or the streak period (downturns).

Table 11

KOSPI and S&P 500 descriptive statistics.

|            | S&P 500 | KOSPI |
|------------|---------|-------|
| Mean       | 0.0329  | 0.0284 |
| Median     | 0.0536  | 0.0250 |
| Standard Deviation | 1.1028 | 1.4556 |
| Kurtosis   | 29.5067 | 8.0200 |
| Skewness   | −1.1410 | −0.2370 |
| Minimum    | −22.8997| −12.8047|
| Maximum    | 10.9572 | 11.2844|
| Obs. (N)   | 8000    | 8000  |

Descriptive statistics on daily return data for S&P 500 and KOSPI indexes from January 1, 1980 to May 30, 2019.

Differences with respect to those obtained previously, being around 0.05 and 0.13, respectively, for the first crash definition; 0.10 and 0.23 for the second crash definition; and below 0.02 and 0.07 for the third crash definition. Regardless the time window and crash definition, the QPS and LPS values obtained from the proposed model are lower than those of the random walk process. Therefore, these results confirm that the alpha-stable model exhibits an adequate performance for forecasting market crashes.

In the case of the HK score, the results obtained from the proposed methodology outperform the random walk process. The maximum values of this measure for the S&P500 (Kospi) index are 0.2421 and 0.1110 (0.0006 and 0.1869) according to the dynamic alpha-stable (random walk) forecasting, respectively. Tables 15 and 16 report the detailed results.

These results suggest that the model developed in this paper is not only easily replicable, but also presents satisfactory results for different types of global markets. In addition, the results are consistent with related literature, being an interesting alternative methodology for crashes forecasting.
Fig. 6. Streaks (downturns) identification for MSCI indexes. Daily prices for different MSCI equity indexes provided by Bloomberg (see Appendix for a detailed description on these indexes) for the period January 1, 1980 to May 30, 2019. A streak is defined as a period in which the price of a financial asset has an upward or downward movement uninterruptedly. The identified streaks (straight blue lines) correspond to observations in which a price drop ended, and a price increase started (downturns).
Table 12
Forecasting accuracy of the VAR model for alpha-stable parameters (S&P 500 and KOSPI indexes).

|          | MSE  | RMSE | RRMSE | MSE  | RMSE | RRMSE |
|----------|------|------|-------|------|------|-------|
| S&P500   | 0.0001 | 0.0123 | 0.0067 | 0.0157 | 0.1233 | -0.3546 |
| KOSPI    | 0.0001 | 0.0105 | 0.0056 | 0.0122 | 0.1104 | -0.6514 |

Table 13
Forecasted crash probabilities for GARCH-stable model (S&P 500 and KOSPI indexes).

|          | Crash = 2.5% quantile | Crash = 5% quantile | Crash = Daily drop of at least 5% |
|----------|-----------------------|----------------------|----------------------------------|
| S&P500   | 0.0591                | 0.1142               | 0.0034                           |
| KOSPI    | 0.0181                | 0.0607               | 0.0025                           |

5.2. Subprime crisis forecast

One of the questions to answer for crash prediction models is if they could forecast major crises or, at least, if they incorporate some kind of information to anticipate them. In this subsection we provide an example for the subprime crisis to illustrate to what extent a negative extreme shock can be somehow anticipated according to our methodology. Filardo et al. [71] proposed that subprime financial crisis started on August 1, 2007 and ended on March 31, 2009. For the purposes of the present paper, it was necessary to identify a substantial drop in the S&P500 around to these dates. Particularly, we focus on forecasting the major decline of the S&P500 index return of −9.2% on September 29, 2008. To evaluate the performance of the model to forecast this downturn, the average of the crash probabilities forecasted by the alpha-stable model for the three considered crash definitions and $T = 500$ was estimated considering three different time windows of 7 days, 15 days and 30 days before September 29, 2008. Results in Table 17 show that, although the severity of the crash was not anticipated, the probability of crash increases as we approach to the extreme event, which is a clear warning for it. This result suggests that the dynamic forecasting of the alpha-stable model adapts to market.
Fig. 7. Forecasted errors for alpha and beta parameters under GARCH-stable model (S&P500 and KOSPI indexes). Forecasted error for parameter \( \beta \) calculated as the difference between the parameter value estimated from the observed data and its forecasts. Forecasts computed daily returns of S&P 500 and KOSPI indexes from January 1, 1980 to May 30, 2019.

Table 14
QPS and LPS for GARCH-stable and random walk crashes forecasting (S&P500 and KOSPI indexes).

| Crash          | T = 250 | T = 500 | T = 1000 | Random walk (T = 500) |
|----------------|---------|---------|----------|-----------------------|
|                | QPS     | LPS     | QPS      | LPS                   | QPS     | LPS     | QPS     | LPS                   | QPS     | LPS     | QPS     | LPS                   |
| S&P500         | 0.0528  | 0.1335  | 0.0536   | 0.1342    | 0.0564   | 0.1388  | 0.0648   | 0.1647    |                      |         |         |         |                      |
| KOSPI          | 0.0520  | 0.1279  | 0.0529   | 0.1299    | 0.0555   | 0.1352  | 0.0580   | 0.1428    |                      |         |         |         |                      |

| Crash          | T = 250 | T = 500 | T = 1000 | Random walk (T = 500) |
|----------------|---------|---------|----------|-----------------------|
|                | QPS     | LPS     | QPS      | LPS                   | QPS     | LPS     | QPS     | LPS                   |
| S&P500         | 0.1046  | 0.2258  | 0.1057   | 0.2270    | 0.1101   | 0.2329  | 0.1158   | 0.2475    |                      |         |         |         |                      |
| KOSPI          | 0.0996  | 0.2076  | 0.1013   | 0.2104    | 0.1062   | 0.2178  | 0.1066   | 0.2227    |                      |         |         |         |                      |

| Crash          | T = 250 | T = 500 | T = 1000 | Random walk (T = 500) |
|----------------|---------|---------|----------|-----------------------|
|                | QPS     | LPS     | QPS      | LPS                   | QPS     | LPS     | QPS     | LPS                   |
| S&P500         | 0.0053  | 0.0202  | 0.0054   | 0.0188    | 0.0057   | 0.0201  | 0.0104   | 0.0536    |                      |         |         |         |                      |
| KOSPI          | 0.0123  | 0.0440  | 0.0125   | 0.0450    | 0.0129   | 0.0453  | 0.0168   | 0.0662    |                      |         |         |         |                      |

QPS is the Quadratic Probability Score; LPS is the Log-Probability Score. Statistics computed for estimation windows of size \( T \) using daily return data of S&P 500 and KOSPI indexes from January 1, 1980 to May 30, 2019.

Risk scenarios and crash probabilities can be interpreted as signals for preventing crashes or at least for implementing strategies to smooth losses.

According to this example and the first definition of crash, Table 18 describes the impact of the forecasted crash probabilities on expected losses for a \$1,000,000 investment replicating the S&P500 index, in comparison to the losses incurred by such investment on September 29, 2008. The expected loss based on the conditional alpha-stable forecasted distribution is higher the closer to the information to the extreme event and, definitely, higher than the expected loss.
Fig. 8. Forecasted errors for alpha and beta parameters under random walk model, (S&P500 and KOSPI indexes). Forecasted error for parameter $\beta$ calculated as the difference between the parameter value estimated from the observed data and its forecasts. Forecasts computed daily returns of S&P 500 and KOSPI indexes from January 1, 1980 to May 30, 2019.

Table 15
HK score for $T = 500$ and a minimum 5% daily-return decrease crash (GARCH-stable model, S&P500 and KOSPI indexes).

|      | S&P500          | KOSPI          |
|------|-----------------|----------------|
|      | $w$ H F HK      | $w$ H F HK     |
| 0.0003| 1.0000 0.9999 0.0001| 1.0000 0.9994 0.0006|
| 0.0013| 1.0000 0.8806 0.1194| 0.4667 0.8047 −0.3380|
| 0.0022| 1.0000 0.8556 0.1444| 0.3333 0.7081 −0.3748|
| 0.0032| 0.9583 0.7162 0.2421| 0.2833 0.5096 −0.3163|
| 0.0042| 0.7500 0.5749 0.1751| 0.2333 0.4394 −0.2061|
| 0.0051| 0.6667 0.4324 0.2343| 0.2167 0.3144 −0.0977|
| 0.0061| 0.1667 0.2651 −0.0984| 0.1000 0.2122 −0.1122|
| 0.0071| 0.0833 0.1614 −0.0781| 0.0167 0.1555 −0.1388|
| 0.0080| 0.0417 0.0766 −0.0349| 0.0167 0.1277 −0.1110|
| 0.0090| 0.0417 0.0365 0.0052| 0.0167 0.0898 −0.0731|

HK (Hanssen–Kuipers skill score) is the difference between H (Hit rate) and F (False alarm rate) according to Eq. (10). Forecasted probabilities larger than a given cut-off level ($w$) are classified as a crash. Statistics computed for estimation windows of size $T = 500$ using daily return data of S&P 500 and KOSPI indexes from January 1, 1980 to May 30, 2019.

HK calculated from the average crash probability for the entire sample (unconditional distribution). These figures might help to decide on increasing provisions for the potential losses or, alternatively, implementing strategies that smooth risk exposure considering the forecasted crash probabilities. For instance, in absence of transaction costs, an investor who would operate with a simplistic strategy that reduces his risk exposition on the basis the forecasting probability of crash, will have a final loss of $76,080, thus saving $15,920.

Consequently, the obtained results in our paper are particularly meaningful considering the current panorama of the financial markets. In August 2019, the yield curve of US treasury bonds was inverted. This scenario created a stage of uncertainty in the economy and was interpreted as a sign for a possible recession in the following year. The above is based on the fact that the yield curve was reversed before each recession presented since 1955 [72]. The impact of financial
Table 16
HK score for $T = 500$ and crash and a minimum 5% daily-return decrease crash (random walk model, S&P500 and KOSPI indexes).

| w   | H      | F      | HK     | H      | F      | HK     |
|-----|--------|--------|--------|--------|--------|--------|
| 0.0003 | 0.7500 | 0.6419 | 0.1081 | 0.8500 | 0.6658 | 0.1842 |
| 0.0013 | 0.7500 | 0.6416 | 0.1084 | 0.8500 | 0.6655 | 0.1845 |
| 0.0022 | 0.7500 | 0.6416 | 0.1084 | 0.8500 | 0.6655 | 0.1845 |
| 0.0032 | 0.7500 | 0.6411 | 0.1089 | 0.8500 | 0.6651 | 0.1849 |
| 0.0042 | 0.7500 | 0.6408 | 0.1092 | 0.8500 | 0.6648 | 0.1852 |
| 0.0051 | 0.7500 | 0.6405 | 0.1095 | 0.8500 | 0.6645 | 0.1855 |
| 0.0061 | 0.7500 | 0.6399 | 0.1101 | 0.8500 | 0.6639 | 0.1861 |
| 0.0071 | 0.7500 | 0.6398 | 0.1102 | 0.8500 | 0.6638 | 0.1862 |
| 0.0080 | 0.7500 | 0.6394 | 0.1106 | 0.8500 | 0.6635 | 0.1865 |
| 0.0090 | 0.7500 | 0.6390 | 0.1110 | 0.8500 | 0.6631 | 0.1869 |

HK (Hanssen–Kuipers skill score) is the difference between $H$ (Hit rate) and $F$ (False alarm rate) according to Eq. (10). Forecasted probabilities larger than a given cut-off level ($w$) are classified as a crash. Statistics computed for estimation windows of size $T$ using daily return data of S&P 500 and KOSPI indexes from January 1, 1980 to May 30, 2019.

Table 17
Average forecasted crash probabilities of S&P 500 index under GARCH-stable model before September 29, 2008.

|                | 30 days | 15 days | 7 days | Total sample |
|----------------|---------|---------|--------|--------------|
| Crash = 2.5% quantile | 0.0609* | 0.0614  | 0.0618 | 0.0595       |
| Crash = 5% quantile   | 0.1152  | 0.1155  | 0.1158 | 0.1143       |
| Crash = Daily drop of at least 5% | 0.0055  | 0.0058  | 0.0060 | 0.0046       |

The crash probabilities for $T = 7, 15$ and 30 days before September 29, 2008 are calculated as the average of the forecasted crash probabilities under the GARCH-stable model for three alternative definitions of crash.

*This figure means that, according to the GARCH-stable model, 30 days before September 29, 2008, there was an average expected probability of 6.09% of a return equal or less than quantile 2.5% of the total returns sample. This probability is higher than the average forecasted probability for the entire sample (5.95%).

Table 18
Expected loss vs observed loss before September 29, 2008 market crash.

|                | 30 days | 15 days | 7 days | Total sample |
|----------------|---------|---------|--------|--------------|
| Average forecasted crash probability | 6.09%  | 6.14%  | 6.18%  | 5.95%        |
| Observed return (09/29/2008)          | $-9.20\%$ | $-9.20\%$ | $-9.20\%$ | $-9.20\%$       |
| Invested amount                         | $1,000,000$ | $1,000,000$ | $1,000,000$ | $1,000,000$       |
| Observed loss (A)                       | $92,000$ | $92,000$ | $92,000$ | $92,000$       |
| Expected loss (B)                       | $60,900$ | $61,400$ | $61,800$ | $59,500$       |
| Difference (A–B)                        | $31,100$ | $30,600$ | $30,200$ | $32,500$       |
| Downgrading strategy based on probability of crashes | $939,100$ | $881,439$ | $826,966$ | $76,080$       |

The average forecasted crash probability corresponds to the returns that are less or equal than the 2.5% quantile of the entire sample. Based on these probabilities the expected losses for an investment of $1,000,000 are computed and compared to the observed loss in September 29, 2008. The impact of a strategy that would reduce the investment according to the crash probabilities will amount a loss of $76,080.

Crisis on the world economy and the signs warning about the occurrence of such events highlight the importance of building models aimed at predicting market crashes. Although there are positions that question the predictive capacity of the yield curve [73], there is a consensus on the need to study this type of phenomena in order to mitigate the possible consequences of market declines. In this sense, the present work becomes relevant when proposing a methodology of prediction of crashes of easy implementation and replicability. Though the yield curve was in its ‘normal’ shape since November 2019, it is necessary to continue monitoring closely the financial markets. This is because on previous occasions crises have occurred even when the crisis signal has disappeared [74]. And the current health crisis triggered by Covid-19 pandemic is not an exception.

6. Conclusions

Given the increasing volatility and uncertainty panorama faced by decision makers after the global financial crisis, the prediction and analysis of market crashes has gained relevance. However, the success in the modeling of this type of phenomena depends to a large extent on the capacity of the theoretical models to incorporate stylized facts such as the presence of heavy tails or the persistence in the returns of financial assets. In this article, we presented a methodology that...
Table A.1
Dataset description.
Source: BloombergLP and msci.com.

| Developed markets | Description |
|-------------------|-------------|
| North America     | The MSCI North America Index is designed to measure the performance of the large and mid cap segments of the US and Canada markets. With 727 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in the US and Canada. |
| European Union    | The MSCI Europe Index is designed to represent the performance of large- and mid-cap equities across 15 developed markets. As of December 2018, it had more than 400 constituents and covered approximately 85% of the free float-adjusted market capitalization across the European developed market equity universe. *DM countries in Europe include Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom. |
| Pacific           | The MSCI Pacific Index captures large and mid cap representation across 5 Developed Markets (DM) countries in the Pacific region. With 469 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in each country. *DM countries in the MSCI Pacific Index include: Australia, Hong Kong, Japan, New Zealand and Singapore. |

| Emerging markets  | Description |
|-------------------|-------------|
| Latin America     | The MSCI Emerging Markets (EM) Latin America Index captures large and mid cap representation across 6 Emerging Markets (EM) countries in Latin America. With 118 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in each country. *The MSCI EM Latin America Index consists of the following 5 emerging market country indexes: Argentina, Brazil, Chile, Colombia, Mexico, and Peru. |
| Emerging Europe   | The MSCI Emerging Markets Europe Index captures large and mid cap representation across 6 Emerging Markets (EM) countries in Europe. With 72 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in each country. *EM Europe countries include: the Czech Republic, Greece, Hungary, Poland, Russia and Turkey. |
| Emerging Asia     | The MSCI Emerging Markets (EM) Asia Index captures large and mid cap representation across 9 Emerging Markets countries. With 912 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in each country. *Emerging Markets Asia countries include: China, India, Indonesia, Korea, Malaysia, Pakistan, the Philippines, Taiwan and Thailand. |

The methodology incorporates the alpha-stable distribution for the prediction of crash probabilities in the stock market. The methodology incorporates both tail index measures and GARCH models, particularly GARCH-Stable models. Furthermore, VAR models are used for forecasting the parameters of the alpha-stable distribution with reasonable degree of accuracy.

The application to developed and emerging market indexes presents accurate probability of crash assessment in terms of different probability score measures (QPS, LPS and HK). These scores remain below 0.02 and 0.06 and above to 0.52, respectively, for a particular definition of crash and time window, outperforming alternative models for forecasting market crashes provided in the literature. Specifically, we find the best results the larger the estimation window and when the definition of crash is not dependent on the characteristics of the unconditional distribution (thus leaving the dynamics of the forecasting conditional alpha-stable freely capture the crashes).

These results are robust to different crash definitions, estimation-length windows and stock indexes and markets (e.g. S&P 500 and KOSPI) and then the evidence in favor our model seems to be easily replicable. Moreover, the obtained results suggest that our methodology can be applied to forecast major crises and downturns under the strikes identification methodology.

All in all, the methodology based on the alpha-stable distribution seems to be an interesting tool for risk management and, particularly, for the design of early warning systems for future market crashes. The extension of the three-step method to other flexible and heavy-tailed distributions are left for future research – e.g. the flexible family of distributions in [75] might be an interesting candidate, but these generalizations would probably be at the cost of a loss in simplicity, which is one of the most appealing characteristics of our method.

CRediT authorship contribution statement

**Jesús Molina-Muñoz:** Conceptualization, Methodology, Software, Validation, Writing - original draft. **Andrés Mora-Valencia:** Conceptualization, Supervision, Methodology, Writing - review & editing. **Javier Perote:** Supervision, Writing - review & editing, Funding acquisition.
Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

See Table A.1.

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