Spacetime variation of $\alpha$ and the CMB power spectra after the recombination

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The possible variation of the fine structure constant may be due to the non-minimal coupling of the electromagnetic field to a light scalar field which can be the candidate of dark energy. Its dynamical nature renders the fine structure constant varies with time as well as space. In this paper we study the effects of these variations on the power spectra of the temperature and the polarization of the cosmic microwave background after the recombination. We show explicitly that the fluctuations of the coupled scalar field generate new temperature anisotropies at the linear order and induce a $B$ mode to the polarization at higher order in general.

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I. INTRODUCTION

There is a long history to study the possible variations of fundamental constants [1]. Simply these variations can be modeled as a light scalar field non-minimally coupled to the matter fields. The claimed evidence [2] for a smaller fine structure constant $\alpha$ at the redshift of $z \sim 3$ from quasar observations stimulated many interests in investigating different theories of varying $\alpha$ [3]. On the other hand, the discovery of the accelerating expansion of the universe requires in general relativity the existence of dark energy which, if dynamical, is extensively modeled as an ultra-light scalar field. Hence it is natural to ask whether the variation of the fine structure constant is induced by the non-minimal coupling of dark energy to the electromagnetic field. This connection provides another way to probe the dark energy in the Universe.

The variation of fine structure constant has effects on the cosmic microwave background radiation (CMB). As discussed in Refs. [4, 5], the change in $\alpha$ will change the history of recombination and the position of the first Doppler peak of the CMB spectrum and is constrained by the observational data [6, 7, 8, 9]. If the variation of $\alpha$ in time is brought by the coupling of a light cosmic field $\phi$, there should be spatial variation of $\alpha$ because of the dynamical nature of $\phi$. The effects of spatial fluctuations of $\alpha$ on CMB are firstly studied in Ref. [10] by considering the implicit dependence of CMB temperature and polarization fields on $\alpha$ due to the recombination history. At different places, the recombination processes are different. The authors showed that these spatial fluctuations induce a $B$ mode to the polarization and non-Gaussian temperature and polarization correlations. In this paper, we study this problem again focusing on the later time evolution after the recombination. For simplicity we will not consider the reionization and other scattering effects. We show explicitly the modifications to the power spectra without focusing on a specific model of the coupled scalar field $\phi$. As we will point out, the spatial fluctuations of $\phi$ generate new temperature anisotropies at the linear order. This is similar to the Sachs-Wolff effect caused by the inhomogeneities of the gravitational field because the coupling of $\phi$ induce an extra long range force between photons. So, photons from different positions on the last scattering surface to us will lose or get different extra energies. Furthermore, these fluctuations will induce a $B$ mode to the polarization at higher order except in a specific case.

This paper is organized as follows. In Section II, we will discuss the transportation of CMB photons after the recombination based on the method of geometric optics approximation. This method had been used in Ref. [11] and more systematically in [12] to analyze the rotation of polarization when the photon coupled to an external field anomalously [13, 14]; In Section III, we derive the modified power spectra. We calculate the temperature spectrum at the linear order and the polarization spectra at higher order; Section IV is the conclusion.

II. THE TRANSPORTATION OF CMB PHOTON AFTER RECOMBINATION

As mentioned above, the varying $\alpha$ theory maybe modeled as the electromagnetic field non-minimally coupled to a scalar field which may or may not be the dark energy. The Lagrangian for this modified electromagnetic theory can

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be written as,

\[ \mathcal{L} = f(\phi)F_{\mu\nu}F^{\mu\nu} . \]  

(1)

Where \( f(\phi) \) is a dimensionless function of \( \phi \). Without the induced coupling, it is normalized as \(-1/4\). The fine structure constant is \( \alpha = -\alpha_0/4f \) with \( \alpha_0 \sim 1/137 \) being the fine structure constant at present time. The difference of \( \alpha \) at the redshift \( z \) from today is

\[ \frac{\delta \alpha}{\alpha_0} = \frac{\alpha_z - \alpha_0}{\alpha_0} = -\frac{1 + 4f_z}{4f_z} , \]

(2)

and \( f_0 \) should be equal to \(-1/4\). With the Lagrangian (1) we obtain the equation of motion,

\[ \nabla_\mu F^{\mu\nu} = -\nabla_\nu \ln f F^{\mu\nu} . \]

(3)

In addition we have the identity:

\[ \nabla_\mu F_{\rho\sigma} + \nabla_\rho F_{\mu\sigma} + \nabla_\sigma F_{\mu\rho} = 0 . \]

(4)

The transportation equations of CMB photons can be obtained in terms of the geometric optics approximation (GOA) which has been used to study the \( CPT \) violating effects of the anomaly coupling of photons [11, 12]. Similar to the analysis in Ref. [12], we first differentiate Eq. (4) to get a second order differential equation for \( F_{\mu\nu} \)

\[ \Box F_{\rho\sigma} + \nabla_\rho (\nabla^\mu \ln f F_{\mu\rho}) - \nabla_\sigma (\nabla^\mu \ln f F_{\mu\sigma}) - [F^\alpha_{\rho \sigma} R_{\alpha\sigma} - F^\alpha_{\sigma \rho} R_{\alpha\rho} - F^{\nu\alpha} R_{\alpha\mu\rho} \sigma] = 0 , \]

(5)

where \( R_{\alpha\sigma} \) and \( R_{\alpha\mu\rho} \sigma \) are Ricci and Riemann tensors respectively and will be neglected in GOA in the following discussions. Then we make the ansatz

\[ F_{\mu\nu} = (a^{\mu\nu} + \epsilon b^{\mu\nu} + \epsilon^2 c^{\mu\nu} + \ldots)e^{iS/\epsilon} , \]

(6)

where \( \epsilon \) is a small real parameter and \( S \) is a real function. We define the wave vector as

\[ k_\mu \equiv \nabla_\mu S , \]

(7)

which represents the travel direction of the photon. Substituting the ansatz (6) into Eqs. (5) and (4) we have the equations expanded by \( \epsilon \)

\[ \Box (a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \ldots) + \frac{2i}{\epsilon} k^\mu \nabla_\mu (a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \ldots) + \frac{i}{\epsilon} (\nabla_\mu k^\mu)(a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \ldots) - \frac{1}{\epsilon^2} k_\mu k^\mu (a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \ldots) \]

\[ = -[(\nabla_\rho \nabla^\mu \ln f)(a_{\mu\sigma} + \epsilon b_{\mu\sigma} + \ldots) + \nabla^\mu \ln f (\nabla_\rho a_{\mu\sigma} + \epsilon \nabla_\rho b_{\mu\sigma} + \ldots) + \frac{i}{\epsilon} k_\mu \nabla^\mu \ln f (a_{\mu\sigma} + \epsilon b_{\mu\sigma} + \ldots)] + [\rho \leftrightarrow \sigma] \]

(8)

and

\[ [\nabla_\mu (a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \ldots) + \frac{i}{\epsilon} k_\mu (a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \ldots)] + [\rho \sigma] + [\sigma \rho] = 0 . \]

(9)

At the leading order of the GOA, Eq. (9) gives

\[ k_\mu a_{\rho\sigma} + k_\rho a_{\sigma\mu} + k_\sigma a_{\rho\mu} = 0 \]

(10)

which implies that \( a_{\rho\sigma} \) should have the following antisymmetric form:

\[ a_{\rho\sigma} = k_\rho a_{\sigma\mu} - k_\sigma a_{\rho\mu} . \]

(11)

Then we collect the terms of Eq. (5) at the orders of \( 1/\epsilon^2 \) and \( 1/\epsilon \), respectively. At the order of \( 1/\epsilon^2 \), we have

\[ k_\mu k^\mu = 0 . \]

(12)

This means the dispersion relation is unchanged at the most leading order of GOA. The propagation equation of \( k^\mu \) can be obtained via differentiating the above equation again:

\[ 0 = \nabla_\nu (k_\mu k^\mu) = 2\nabla^\mu S \nabla_\nu \nabla_\mu S = 2\nabla^\mu S \nabla_\mu S \nabla_\nu S = 2k^\mu \nabla_\mu k_\nu . \]

(13)
This indicates $k^{\mu}$ is parallely transported along the light curve, the null geodesic. The vector $k^{\mu}$ defines an affine parameter $\lambda$ which measures the distance along the light ray:

$$k^{\mu} = \frac{dx^{\mu}}{d\lambda}.$$  

(14)

The transportation equations for the vector $a^{\nu}$ are obtained at the order of $1/\epsilon$:

$$\mathcal{D} a^{\nu} + \frac{1}{2} (\theta + \mathcal{D} \ln f) a^{\nu} = 0,$$

(15)

where we have considered Eq. (11) and defined the operator $\mathcal{D} \equiv k^{\mu} \nabla_{\mu} = d/d\lambda$. The quantity $\theta = \nabla_{\mu} k^{\mu}$ describes the expansion of the bundle of the light. In addition, by applying the GOA to Eq. (3), we have

$$k^{\mu} a^{\mu} = 0.$$  

(16)

The basic results we got above are Eqs. (15) and (16) with two orthogonality relations (12) and (10).

As discussed in [12], if we require the observer at rest with the local inertial frame at each point to see the light traveling along the $+z$ direction, the Stokes parameters are defined as the following time averages:

$$I \equiv \langle E_x E_x^{*} \rangle + \langle E_y E_y^{*} \rangle,$$

$$Q \equiv \langle E_x E_x^{*} \rangle - \langle E_y E_y^{*} \rangle,$$

$$U \equiv \langle E_x E_y^{*} \rangle + \langle E_y E_x^{*} \rangle,$$

$$V \equiv i \langle (E_x E_y^{*}) - (E_y E_x^{*}) \rangle.$$  

(17)

We can use the tetrad formalism to get these parameters in the coordinate frame. A tetrad is a set of four orthogonal Greek indices, however, by the coordinate metric $g^{\alpha\beta}$ frequency $\omega$ are well defined in the local inertial frame (Lorentz frame). Considering a monochromatic electromagnetic wave of frequency $\omega_0$ propagating in the $+z$ direction, the Stokes parameters are defined as the following time averages:

$$I \equiv \langle E_x E_x^{*} \rangle + \langle E_y E_y^{*} \rangle,$$

$$Q \equiv \langle E_x E_x^{*} \rangle - \langle E_y E_y^{*} \rangle,$$

$$U \equiv \langle E_x E_y^{*} \rangle + \langle E_y E_x^{*} \rangle,$$

$$V \equiv i \langle (E_x E_y^{*}) - (E_y E_x^{*}) \rangle.$$  

(17)

We can use the tetrad formalism to get these parameters in the coordinate frame. A tetrad is a set of four orthogonal unit basis vectors $e^{\mu}_{(a)}$, with $a = 0, 1, 2, 3$. The Latin indices are lowered and raised by the Minkowski metric $\eta^{ab}$, the Greek indices, however, by the coordinate metric $g^{\mu\nu}$. The tetrad has the following properties:

$$g_{\mu\nu} e^{\mu}_{(a)} e^{\nu}_{(b)} = \eta_{ab}, \quad \eta^{ab} e^{\mu}_{(a)} e^{\nu}_{(b)} = g^{\mu\nu}.$$  

(18)

As discussed in [12], if we require the observer at rest with the local inertial frame at each point to see the light traveling along the $+z$ direction, the zeroth and the third components of the tetrad vectors should be

$$e^{\mu}_{(0)} = u^{\mu}, \quad e^{\mu}_{(3)} = \frac{1}{\omega}(k^{\mu} - \omega u^{\mu}),$$

(19)

where $u^{\mu}$ is the four-velocity of the observer and $\omega \equiv k^{\mu} u_{\mu}$ is the frequency measured by him (or her). The other tetrad vectors $e^{\mu}_{(1)}$ and $e^{\mu}_{(2)}$ are undetermined, but they should be unit spacelike, orthogonal to each other and to $e^{\mu}_{(0)}$, $e^{\mu}_{(3)}$, and therefore orthogonal to $k^{\mu}$.

The electric vector in general spacetime for the local observer is

$$E^{\mu} = F^{\mu\nu} u_{\nu}.$$  

(20)

At the leading order of the GOA, it is

$$E^{\mu} = a^{\mu\nu} u_{\nu} e^{iS/\epsilon} = (k^{\mu} a^{\nu} - k^{\nu} a^{\mu}) u_{\nu} e^{iS/\epsilon}.$$  

(21)

Transforming it to the local inertial frame, we get the $x$ and $y$ components of the electric field in this frame easily:

$$E_x = \bar{E}_1 = E_{\mu} e^{\mu}_{(1)}; \quad E_y = \bar{E}_2 = E_{\mu} e^{\mu}_{(2)}.$$  

(22)

With above equations and Eqs. (17), we have the expressions of the Stokes parameters in the coordinate frame [13, 16]:

$$I = \omega^2 L_{\mu\nu}(e^{\mu}_{(1)} e^{\nu}_{(1)} + e^{\mu}_{(2)} e^{\nu}_{(2)}),$$

$$Q = \omega^2 L_{\mu\nu}(e^{\mu}_{(1)} e^{\nu}_{(1)} - e^{\mu}_{(3)} e^{\nu}_{(3)}),$$

$$U = \omega^2 L_{\mu\nu}(e^{\mu}_{(1)} e^{\nu}_{(2)} + e^{\mu}_{(2)} e^{\nu}_{(1)}),$$

$$V = i\omega^2 L_{\mu\nu}(e^{\mu}_{(1)} e^{\nu}_{(2)} - e^{\mu}_{(2)} e^{\nu}_{(1)}),$$

(23)
where \( L_{\mu\nu} \equiv \langle a_\mu a_\nu^* \rangle \) satisfies the following equation making use of Eq. (15):

\[
\mathcal{D}L_{\mu\nu} + (\theta + \mathcal{D} \ln f) L_{\mu\nu} = 0 .
\] (24)

We require the tetrad frames to be not physically rotating. In order to do that, we set the tetrad vectors at each point so that \( e_\mu^{(1)} \) and \( e_\nu^{(2)} \) are parallelly transported along the light curve. So it is straightforward to get the propagation equations of the four parameters along the light curve:

\[
\mathcal{D} F_a + (\theta + \mathcal{D} \ln f) F_a = 0 ,
\] (25)

where \( F_a \equiv S_a / \omega^2 \equiv (I, Q, U, V) / \omega^2 \). Hence the observed Stokes parameters today should be

\[
S_{a0}^{\text{obs}} = -4 f_r S_a \exp(\int_{\lambda_0}^{\lambda_r} \theta d\lambda) .
\] (26)

In above \( r \) means the moment when the recombination is finished and we have used \( f_0 = -1/4 \). From above equation we can see that the effect of the coupling in (1) on CMB at later time after the recombination is just indicated by the “dilation” factor \( f_r \). To disentangle this later time effect, we rewrite Eq. (27) as

\[
S_{a0}^{\text{obs}} = g_r S_a \equiv -4 f_r S_a ,
\] (27)

where \( S_a \) represent the should be Stokes parameters observed today without later time variation of \( g \). But it includes the effect from the modified recombination history. This is the basic result obtained in this section. The Stokes V cannot be generated by Thomson scattering and can be neglected. Because \( g_r \) is a function of the scalar field \( \phi \) at the last scattering surface, it has different values at different positions. So, it can be decomposed as \( g_r = \bar{g}_r + \delta g_r \). The background part \( \bar{g}_r \) differs from 1 because of the time variation. It changes the redshift of the recombination slightly. In the next section we will consider the effects of spatial fluctuation \( \delta g_r \) on CMB power spectra.

### III. CMB POWER SPECTRA

In the spatially flat Universe, we can expand the temperature and polarization anisotropies in terms of appropriate spin-weighted spherical harmonic functions on the sky [17]:

\[
\Theta(\hat{n}) = \sum_{lm} a_{\phi,lm} Y_{lm}(\hat{n})
\]

\[
(Q \pm iU)(\hat{n}) = \sum_{lm} a_{\pm 2,lm} \pm 2 Y_{lm}(\hat{n}) .
\] (28)

Where \( \Theta(\hat{n}) \equiv \Delta T(\hat{n})/T \). The expressions for the expansion coefficients are

\[
a_{\phi,lm} = \int d\Omega Y_{lm}^*(\hat{n}) \Theta(\hat{n})
\]

\[
a_{\pm 2,lm} = \int d\Omega \pm 2 Y_{lm}^*(\hat{n})(Q \pm iU)(\hat{n}) .
\] (29)

Instead of \( a_{2,lm} \) and \( a_{-2,lm} \), it is convenient to introduce their linear combinations

\[
a_{E,lm} = -(a_{2,lm} + a_{-2,lm})/2
\]

\[
a_{B,lm} = i(a_{2,lm} - a_{-2,lm})/2 .
\] (30)

The power spectra are defined as

\[
\langle a_{X',l'm'} a_{X,lm} \rangle = C_l^{X'X} \delta_{l'l} \delta_{m'm}
\] (31)

with the assumption of statistical isotropy. In the equation above, \( X' \) and \( X \) denote the temperature \( \Theta \) and the \( E \) and \( B \) modes of the polarization field, respectively. For Gaussian theories, the statistical properties of the CMB temperature/polarization map are specified fully by these spectra. Without parity violation, \( C_l^{\Theta E} = C_l^{EB} = 0 \).
Consider the modification in Eq. (27), the power spectra would be changed. The temperature fluctuations are changed as
\[ \Theta_{\text{obs}}^{\Theta} = \frac{1}{4} \frac{\delta g_r}{g_r} + (1 + \frac{\delta g_r}{g_r}) \Theta. \]  
(32)
This means the temperature fluctuations receive a linear order modification. This is similar to Sachs-Wolf effect of the gravitational field on CMB. The coupling of \( \phi \) to photons induce an extra long range force. The photons from different positions on the last scattering surface to us will lose or get different extra energies due to the inhomogeneous distribution of \( \phi \) on that surface. To evaluate the corrections, we expand \( g_r \) by \( \delta \phi_r \) to quadratical order,
\[ g_r(\phi) = \bar{g}_r + A_1 \delta \phi_r + A_2 \delta \phi_r^2. \]
(33)
We assume \( \delta \phi_r \) is a Gaussian random field, so it can also be described by a power spectrum. Expand it on the sky
\[ \delta \phi_r = \sum_{lm} b_{lm} Y_{lm}(\hat{n}), \]
(34)
and define its angular power spectrum as
\[ \langle b_{lm}^* b_{lm} \rangle = C_l^\phi \delta_{l1} \delta_{m'm}. \]
(35)
where we have assumed statistical isotropy of \( b_{lm} \). Furthermore, we have
\[ \sum_l (2l+1) C_l^\phi = 4\pi (\delta \phi_r^2). \]
(36)
In general, \( \delta \phi_r \) has correlations with \( \Theta \) and \( E \). In this paper, we will not consider this complication by assuming \( \delta \phi \) has different origin from that of density perturbation of the photon. With this assumption the modified temperature power spectrum would be
\[ C_l^{\Theta \Theta, \text{obs}} = C_l^{\Theta \Theta} + (\frac{A_1}{4g_r})^2 C_l^\phi, \]
(37)
where we have neglected higher order corrections.

The polarizations of CMB are already at the perturbative level. They don’t have background parts, so,
\[ (Q \pm iU)^{\text{obs}} = \bar{g}_r (1 + \frac{\delta g_r}{g_r})(Q \pm iU). \]
(38)
The modifications can only appear at higher orders. The calculations of the polarization spectra and the cross spectrum of temperature and polarization are long. Similar calculations can be found in Ref. [12] for the modified spectra by anomaly coupling. Here we only present the results up to the second order in the following
\[ C_l^{\Theta E, \text{obs}} = (\bar{g}_r^2 + \bar{g}_r A_2 (\delta \phi_r^2)) C_l^{\Theta E}, \]
\[ C_l^{EE, \text{obs}} = (\bar{g}_r^2 + 2\bar{g}_r A_2 (\delta \phi_r^2)) C_l^{EE} + \frac{A_2^2}{2\pi} \sum_{l_1 l_2} \left( \begin{array}{c c c} l & 1 & l_1 \\ 2 & l_2 & 0 \end{array} \right)^2 (2l_1 + 1)(2l_2 + 1) C_{l_1 l_2}^\phi \left\{ [1 + (-1)^{L}] C_{l_1}^{EE} + [1 + (-1)^{L+1}] C_{l_1}^{BB} \right\}, \]
\[ C_l^{BB, \text{obs}} = (\bar{g}_r^2 + 2\bar{g}_r A_2 (\delta \phi_r^2)) C_l^{BB} + \frac{A_2^2}{2\pi} \sum_{l_1 l_2} \left( \begin{array}{c c c} l & 1 & l_1 \\ m & m_1 & l_2 \end{array} \right)^2 (2l_1 + 1)(2l_2 + 1) C_{l_1 l_2}^\phi \left\{ [1 + (-1)^{L+1}] C_{l_1}^{EE} + [1 + (-1)^{L}] C_{l_1}^{BB} \right\}. \]
(39)
Where \( L = l + l_1 + l_2 \) and \( \left( \begin{array}{c c c} l & 1 & l_1 \\ m & m_1 & l_2 \end{array} \right) \) is the Wigner 3j symbol. From these equations, we can see that the fluctuation of \( \phi \) induce a mixing between \( E \) and \( B \) modes. Even though the original \( B \) mode generated by primordial gravitational wave from inflation is negligibly small, it can still be produced from the \( E \) mode and \( \phi \) with odd \( L \). However, there is an important special case. We note that the generated \( B \) mode is proportional to \( A_2^2 \) up to the quadratical order. If \( g(\phi) \) is a linear function of \( \phi \), for instance \( g = a + b\phi \), \( A_2 \) and all other higher Taylor coefficients vanish. In this case, there is no mixing between the \( E \) and \( B \) modes and the \( B \) mode cannot be generated by the conversion. This case corresponds to the Lagrangian of the following form:
\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{c}{4M} F_{\mu\nu} F^{\mu\nu}. \]
(40)
Of course, if \( \delta \phi \) has correlation with the \( E \) mode, the \( B \) mode still has the chance to get produced.
IV. CONCLUSION

The time variation of the fine structure constant may be due to the non-minimal coupling of the electromagnetic field to a light scalar field. This will induce the spatial variations of $\alpha$ because of the dynamical nature of the scalar field. In this paper, we studied the effects of these spacetime variations on the CMB power spectra after the recombination. The time variation changes the redshift of the last scattering surface and the amplitude of the power spectra slightly. The spatial variations induced extra anisotropies to the CMB temperature fluctuations at the linear order like Sachs-Wolff effect. The effects on polarizations emerged at higher orders. Except a special case, spatial variations of $\alpha$ will induce a $B$ mode to the polarization.

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