Optimal Channel Doping Profile of Two-Dimensional Metal-Oxide-Semiconductor Field-Effect Transistors via Geometric Programming

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Abstract. In this study, we theoretically optimize a two-dimensional (2D) channel doping profile of metal-oxide-semiconductor field-effect transistors (MOSFETs) with given current voltage (I-V) characteristics by using a geometric programming (GP) technique. Inverse modeling of channel doping profile for device characteristics with simultaneously considering the short-channel effect (SCE) and random-dopant-fluctuation-induced threshold voltage fluctuation (RDF-induced $\sigma V_t$) is advanced. The formulated model of doping profile is a GP problem which can be transformed into a convex optimization problem and solved globally and efficiently. Constrains of I-V characteristics with including the RDF-induced $\sigma V_t$ are included to optimize desired doping profiles. The optimization methodology is applied for 45-nm MOSFET devices and the results are validated with 2D numerical device simulation. This approach provides an alternative way to design doping profile for various technologies of MOSFETs.

Keywords: Geometric programming, MOSFET, Doping profile optimization, Inverse model, Semiconductor device simulation

1. Introduction

Doping profile is crucial for determining the electrical characteristic of MOSFETs. It strongly
The random-dopant-fluctuation-induced threshold voltage fluctuation (RDF-induced $\sigma V_t$) [8-13, 15] significantly. However, various doping-profile designs strongly rely on device engineering experiences in process/device simulation and fabrication experiment, such as retrograde doping profile method [6], methodology of inverse modeling using I-V characteristics [14], simulation-based evolutionary technique [15] and regression method [16]. Unfortunately, applications of these procedures to find suitable doping profiles are cases sensitive and may not always be guaranteed in global sense. If seeking engineering acceptable doping profiles could be formulated as an optimization problem and be solved cost-effectively and robustly, it will benefit manufacturing of MOSFETs for various technological applications.

Geometric programming (GP) is one of the mathematical-programming optimization problems that is composed by objective and constraint in posynomial functions [17-22]. Various applications of GP, such as space optimization [23], optimal condenser in chemical reaction [24], and solving the marketing-mix strategy [25] were reported. In recent years, circuit and layout optimizations [26-31], communication system design [32-33], and semiconductor design [34-36] in the related fields of electrical engineering have been explored by proper transformation to GP formulation. A solution method by using interior-point algorithm was advanced to solve large-scale GP problems in convex form [18-19]. This is computationally efficient and reliable method which can significantly accelerate applications of GP in real-world problems. In our earlier work, the GP approach has been utilized to optimize the Ge-dose concentration and silicon doping profile in SiGe HBTs for high frequency characteristic and 1D MOS problem [34-36]. However, seeking optimal channel doping profile via GP approach in 2D MOSFETs has not been investigated yet.

In this paper, an inverse modeling of 2D channel doping profile problem is formulated as GP based on the given I-V characteristics simultaneously considering the influence of SCE and RDF-induced $\sigma V_t$. The simulated results exhibit good agreement with GP optimization data, based on a 2D device simulation [38-41]. The results show that the asymmetric doping profile along the channel direction and retrograde doping distribution from the channel surface to substrate will suppress the SCE and reduce the RDF-induced $\sigma V_t$, simultaneously [37]. Benefited from the optimal doping profile, the explored 45-nm MOSFET can achieve the on-state current $> 5.5\times10^{-5}$ A, the off-state current $< 2.0\times10^{-9}$ A, and the threshold voltage = 300 mV for the specification of low-standby power (LSP). The RDF-induced $\sigma V_t$ is about 20 mV. Similarly, we can optimize the high-performance (HP) devices for given specifications including the RDF-induced $\sigma V_t$. Notably, the magnitude of the RDF-induced $\sigma V_t$ after optimization is about 30% reduction, compared with those cases without considering the suppression of RDF-induced $\sigma V_t$.

This article is organized as follows. We first introduce the inverse doping profile problem and its GP formulation. In Section 3, the numerical results are discussed for HP and LSP MOSFET devices. Finally, we draw conclusions and suggest future work.

2. Inverse modeling of 2D MOSFETs

With a set of given I-V curves, device engineers generally are empirically tuning the doping profiles in channel region. However, as the channel length decreases, the depletion width of the source and drain become comparable to the channel length, as shown in Fig. 1(a); even
with the best scaling rules, departures from long-channel behavior are inevitable. As a result,

Figure 1: (a) A 3D illustration of the MOSFET device and the I-V characteristic. (b) The 2D optimal doping profile cutting from the channel, where we assume the z-direction is uniformly distributed. The channel has three regions according to the lateral maximum source/drain junction depletion width and effective channel length. (c) Discretization and variables transformation of the doing profile function.

the optimal doping profile in the transport direction of carriers inside the channel region could be non-uniformed distribution to reduce the SCE [37]. Consequently, the extraction of 2D channel doping profile is complicate task empirically. We model it as an optimization
The 2D optimal doping profile problem in GP’s form is summarized as shown below:

$$\text{min SS}$$

$$\text{s.t } N_{\text{min}} \leq N_A(x, y) \leq N_{\text{max}}, \ 0 \leq x \leq W_{\text{dm}}, \ 0 \leq y \leq L$$

$$I_{\text{on}} \geq I_{\text{on-set}}$$

$$I_{\text{off}} \leq I_{\text{off-set}}$$

$$\sigma V_t \leq \sigma V_{t\text{-set}}$$

where the $N_A(x, y)$ is the 2D doping profile starting from silicon/oxide interface ($x = 0$) to the maximum depletion region $x = W_{\text{dm}}$ in the $x$-direction and from the source end to drain end at the channel in the $y$-direction, as plotted in Fig. 1(b). $N_A(x, y)$ is subject to the background doping concentration $N_{\text{min}}$ and is limited on the maximum manufacturing doping concentration $N_{\text{max}}$. The objective function SS is the subthreshold swing, $I_{\text{on}}$ is the on-state current, $I_{\text{off}}$ is the off-state current, $\sigma V_t$ is the random-dopant-fluctuation-induced threshold voltage fluctuation. $I_{\text{on-set}}, I_{\text{off-set}}, \sigma V_{t\text{-set}}$ are the given specifications of $I_{\text{on}}$ and $I_{\text{off}}$ and $\sigma V_t$, respectively. To solve this optimization problem, we first integrate the 2D Poisson’s equation to derive the maximum lateral source and drain junction depletion widths $W_{\text{dm}}$ (denoted as WS and WD) as the function of doping profile. We assume the WS, WD, and $L_{\text{eff}}$ satisfy the channel length constraint:

$$WS + L_{\text{eff}} + WD \leq L.$$  \hspace{1cm} (2)

In the following sub-section 2.2, the objective function SS is formulated as a posynomial function. In the sub-section 2.3, we model the threshold voltage. In the sub-sections 2.4 and 2.5, we show how constraints on the on-/off- state current can be transformed into a GP compatible format, respectively. In the sub-section 2.6, the RDF-induced $\sigma V_t$ is considered as a constraint. Notably, the constraint of RDF-induced $\sigma V_t$ is still keeping GP compatible form in our optimization problem. Finally, the 2D optimal doping profile problem in GP’s form is summarized.

### 2.1. Integration of 2D Poisson equation

For estimating the WS, we first integrate the 2D Poisson equation from 0 to depletion width $W_{\text{dm}}$ in the $x$-direction, and from $y$ to the lateral source junction depletion width WS (here we assume the potential and electric field in the $z$-direction along the device width $W$ is constant, since in this work the $W$ is large compared to the $L$, and the narrow width effect [1-3] is negligible), and in the space charge region, the density of free electron $n(x)$ and hole $p(x)$ are almost zero and the doping concentration of donor impurity $N_D(x)$ are neglected:

$$\int_y^{WS} \int_x^{W_{\text{dm}}} \frac{\partial^2 \phi(x', y')}{\partial x'^2} \, dx' \, dy' + \int_y^{WS} \int_x^{W_{\text{dm}}} \frac{\partial^2 \phi(x', y')}{\partial y'^2} \, dy' \, dx'$$

$$= \int_y^{WS} - \frac{\partial \phi(x', y')}{\partial x} \, dy' + \int_x^{W_{\text{dm}}} - \frac{\partial \phi(x', y)}{\partial y} \, dx' = \frac{q}{\varepsilon_s} \int_x^{W_{\text{dm}}} N_A(x', y') \, dx' \, dy',$$  \hspace{1cm} (3)
where the $\varepsilon_{si}$ is the silicon permittivity, $q$ is the electron charge and $\phi(x, y)$ is the electrostatic potential within the device channel. Integrating (3) from $x = 0$ to $W_{dm}$ in the x-direction, and from the source end ($y = 0$) to WS, we obtain the surface potential $\phi(0, y)$ along the y-direction and $\phi(x, 0)$ along the x-direction:

$$
\int_{0}^{WS} \int_{y}^{WS} \frac{\partial \phi(x, y)}{\partial x} dx' dy + \int_{0}^{WS} \int_{x}^{WS} \frac{\partial \phi(x', y)}{\partial y} dy' dx' = 0
$$

and

$$
\int_{0}^{WS} \int_{0}^{WS} \phi(0, y') dy' dx + \int_{0}^{WS} \int_{0}^{WS} \phi(x', 0) dx' dx = \int_{0}^{WS} \int_{0}^{WS} N_A(x', y') dx' dy' .
$$

Using the integration by parts to (4) and substituting the boundary condition of $\phi(x, 0) = \psi_s$ ($\psi_s$ is the built-in potential of source and drain) and $\phi(0, y) = 2 \psi_0$ ($\psi_0$ is the voltage difference between Fermi level and intrinsic level) [2], as shown in Fig. 1(b), we can obtain:

$$
\int_{0}^{WS} 2 y \psi_s dy + \int_{0}^{W_{dm}} y \psi_s dx = \frac{q}{\varepsilon_{si}} \int_{0}^{WS} \int_{0}^{W_{dm}} y x N_A(x, y) dx dy .
$$

To express the WS as a function of $N_A(x, y)$, we discretize the integral (5). The WS is discretized with $N$ uniformly spaced points $y_j = j WS / N$, $j = 0, 1, \ldots, N-1$, and the $W_{dm}$ is discretized with $K$ uniformly spaced points $x_i = i W_{dm} / K$, $i = 0, 1, \ldots, K-1$. The doping profile is sampled at these points: we define $d_{ij} = N_A(x_i, y_j)$, $i = 0, 1, \ldots, K-1$, and $j = 0, 1, \ldots, N-1$, as shown in Fig. 1(c). Using the above discretization, (5) is now approximated as:

$$
\left( \frac{WS}{N} \right)^2 \sum_{j=0}^{N-1} j^2 \psi_s^j + \left( \frac{W_{dm}}{K} \right)^2 \sum_{i=0}^{K-1} \psi_s^i = \frac{q}{\varepsilon_{si}} \left( \frac{W_{dm} WS}{NK} \right)^2 \sum_{j=0}^{N-1} \sum_{i=0}^{K-1} j^2 d_{ij},
$$

and then the WS is consequently represented as:

$$
WS = N \sqrt{\frac{\sum_{i=0}^{K-1} \psi_s^i}{\frac{q}{\varepsilon_{si}} \sum_{j=0}^{N-1} \sum_{i=0}^{K-1} j^2 d_{ij} - \left( \frac{K}{W_{dm}} \right)^2 \sum_{j=0}^{N-1} j^2 \psi_s^j}},
$$

which is a function of 2D doping profile $d_{ij}$ in Region I. For $W_{dm}$, similarly, we integrate the 2D Poisson equation from $x$ to $W_{dm}$ in the x-direction, and from WS to $WS+L_{ef}$ in the y-direction in Region II:

$$
\int_{WS}^{WS+L_{ef}} \int_{x}^{W_{dm}} \frac{\partial^2 \phi(x', y)}{\partial x'^2} dx' dy + \int_{WS}^{WS+L_{ef}} \int_{y}^{W_{dm}} \frac{\partial^2 \phi(x', y)}{\partial y^2} dy' dx' = \frac{q}{\varepsilon_{si}} \int_{WS}^{WS+L_{ef}} \int_{y}^{W_{dm}} N_A(x', y) dx' dy
$$

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Here, we assume that the electric field \( E_y = \frac{\partial \phi(x,0)}{\partial y} \) is approximated to zero in Region II. Based on this condition, we integrate (8) from 0 to \( W_{dm} \) in the \( x \)-direction and obtain the surface potential \( \phi(0, y) \) along the \( y \)-direction:

\[
\int_0^{W_{dm}} \int_0^{W_{dm} \cdot L_0} - \frac{\partial \phi(x, y)}{\partial x} \ dx \ dy = \int_0^{W_{dm} \cdot L_0} \phi(0, y) \ dy
\]

\[
= \frac{q}{\varepsilon_{si}} \int_0^{W_{dm} \cdot L_0} \int_0^{W_{dm}} N_A(x', y') \ dx' \ dy.
\]

Substituting the boundary condition of \( \phi(0, y) = 2\psi_B \) in Region II, as shown in Fig. 1(b), into (9) and using the integration by parts in (9), then (9) is expressed as:

\[
\int_0^{W_{dm} \cdot L_0} 2\psi_B \ dy' = \frac{q}{\varepsilon_{si}} \int_0^{W_{dm} \cdot L_0} \int_0^{W_{dm}} xN_A(x, y') \ dx' \ dy'.
\]

The \( W_{dm} \) can be derived form discretizing (5) in Region II. The \( L_{eff} \) is discretized with \( M \) uniformly spaced points \( y_j = jL_{eff}/M, j = 0, N+1, \ldots, N+M-1 \), as shown Fig. 1(b), and the \( W_{dm} \) is discretized as (6)-(7). The doping profile is then defined as:

\[
d_{ij} = N_A(x_i, y_j), i = 0, 1, \ldots, K-1, j = N, N+1, \ldots, N+M-1.
\]

Eq. (10) is now approximated by the summation:

\[
\left( \frac{L_{eff}}{M} \right)^{N+M-1} \sum_{j=0}^{N+M-1} 2\psi_B = \frac{q}{\varepsilon_{si}} \sum_{j=0}^{N+M-1} \left( \frac{W_{dm}}{K} \right)^2 \sum_{i=0}^{K-1} i d_{ij},
\]

and then the \( W_{dm} \) is modeled as:

\[
W_{dm} = K \left[ \frac{q}{\varepsilon_{si}} \sum_{j=0}^{N+M-1} 2\psi_B \sum_{i=0}^{K-1} i d_{ij} \right],
\]

which is a function of doping profile \( d_{ij} \) in Region II. For obtaining the \( WD \), the technique is similar to (3) to (7) except that we have to change the boundary condition and the index of the integral from Region I to Region III. We express the \( WD \) as a function of doping profile \( d_{ij} \):

\[
WD = N \left[ \frac{q}{\varepsilon_{si}} \sum_{j=2N+M-1}^{K-1} j \sum_{i=0}^{K-1} i d_{ij} - \left( \frac{K}{W_{dm}} \right)^2 \sum_{j=2N+M-1}^{N+M-1} j 2\psi_B \right].
\]

Now, substituting the \( W_{dm} \) in (12) into (7), we have
where

\[
\delta = \frac{N(N-1)}{2M}
\]

and

\[
\lambda_i = \frac{v_i}{q} \sum_{i=0}^{K-1} \psi_i.
\]

Notably, (14) is not a GP compatible constraint; as a result, in this work, we relax (14) into two inequalities. First, we change the operator from “=” to “≤” of (14), and take square of (14) for both sides, we will have:

\[
\lambda_i \left( \frac{WS}{N} \right)^2 \sum_{j=0}^{K-1} \sum_{i=0}^{K-1} id_{ij} \leq 1 + \lambda_i^{-1} \delta \left( \frac{WS}{N} \right)^2 \sum_{j=0}^{N-1} \sum_{i=0}^{K-1} id_{ij},
\]

for the term of summation on the right-hand side of (15), the arithmetic-geometric mean inequality is applied to produce a GP compatible approximation for the above summation. Since the arithmetic mean is greater than geometric mean, we have:

\[
2 \sqrt{MK \lambda_i^{-1} \delta \left( \frac{WS}{N} \right)^2 \left( \frac{W}{S} \right)^{N-1} \prod_{j=0}^{K-1} \prod_{i=0}^{K-1} id_{ij}} \leq 1 + \lambda_i^{-1} \delta \left( \frac{WS}{N} \right)^2 \sum_{j=0}^{N-1} \sum_{i=0}^{K-1} id_{ij};
\]

therefore, if the inequality:

\[
\lambda_i^{-1} \sum_{j=0}^{N-1} \sum_{i=0}^{K-1} id_{ij} \leq 2 \sqrt{MK \lambda_i^{-1} \delta \left( \frac{N}{WS} \right)^2 \left( \frac{W}{S} \right)^{N-1} \prod_{j=0}^{N-1} \prod_{i=0}^{K-1} id_{ij}} \]

holds, then the inequality (15) must also be satisfied. The inequality (17) is a GP compatible constraint since the right-hand side is a monomial function and the left-hand side is a posynomial. Consequently, we change the operator from “=” to “≥” of (14), and based on the same procedure from (15) to (17), we have:

\[
1 + \delta \lambda_i^{-1} \left( \frac{WS}{N} \right)^2 \sum_{j=0}^{N-1} \sum_{i=0}^{K-1} id_{ij} \leq \left( \frac{WS}{N} \right)^2 NK \lambda_i^{-1} \left( \frac{N}{WS} \right)^{N-1} \prod_{j=0}^{N-1} \prod_{i=0}^{K-1} id_{ij},
\]

which is a posynomial inequality. Using constraints in (17) and (18), the equality in (14) can
be forced to be activated since the relationship of “≤” and “≥” are simultaneously satisfied. For the WD, we substitute the $W_{dm}$ (12) into (13), and use the similar technique from (15) to (18), we generate these two constrains:

$$\lambda_2^{-1} \sum_{j=2,N+M-1}^{N+M-1} \sum_{i=0}^{K-1} \lambda_d \mid id_{ij} \leq \sqrt{MK\lambda_2^{-1} \left( \frac{N}{WD} \right)^2 \left( \prod_{j=N}^{j=0} \prod_{i=0}^{i=0} id_{ij} \right)^{1/NK}},$$

(19)

and

$$1 + \delta^{N+M-1} \left( \frac{WD}{N} \right)^2 \sum_{j=N}^{j=0} \sum_{i=0}^{i=0} id_{ij} \leq \left( \frac{WD}{N} \right)^2 N\lambda_2^{-1} \left( \prod_{j=2,N+M-1}^{j=0} \prod_{i=0}^{i=0} id_{ij} \right)^{1/NK},$$

(20)

where

$$\lambda_2 = \frac{\epsilon_s}{d_{ij}} \sum_{i=0}^{K-1} \left( \psi_{S} + V_{dd} \right)$$

and $V_{dd}$ is the applied drain voltage. By using (17)-(20) and the upper bounds for both the WS and WD in (2), we can approximate the magnitudes of WS, WD, and $L_{eff}$ by determining the doping profile $d_{ij}$.

2.2. The formulation of subthreshold swing

The objective function $SS$ is modeled by [2,3]:

$$SS = 2.3 \frac{kT}{q} \left( 1 + \frac{3t_{ox}}{W_{dm}} \right),$$

(21)

where $t_{ox}$ is the oxide thickness, and $k$ is the Boltzmann constant; at temperature $T = 300$ K, $KT/q = 25.9$ mV. Substituting the $W_{dm}$ of (12) into (21), we obtain:

$$SS = 2.3 \frac{kT}{q} \left( 1 + \frac{3t_{ox}}{K} \sqrt{q \sum_{j=N}^{j=0} \sum_{i=0}^{i=0} id_{ij} \frac{N+M-1}{i=0} \frac{K-1}{j=2,N+M-1 \prod_{i=0}^{i=0} id_{ij}} \left( \frac{N}{WD} \right)^2 \left( \prod_{j=N}^{j=0} \prod_{i=0}^{i=0} id_{ij} \right)^{1/NK}},$$

(22)

which is a function of doping profile $d_{ij}$ and is also a posynomial.

2.3. The formulation of threshold voltage

The threshold voltage $V_t$ of a MOSFET device is expressed as [2-3]:

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where $V_{fb}$ is the flat-band voltage, $Q_d$ is the depletion charge per unit area reduced by the SCE which is calculated by considering the charge partition in the device channel region [3]:

$$Q_d = Q_d \frac{L_{eff} W_{dm}}{L W_{dm}},$$  \quad (24)$$

where the $Q_d$ is the depletion charge per unit area without considering the charge sharing effect from source and drain in Region II:

$$Q_d = -\varepsilon_{si} \int_{WS}^{WS+L_{eff}} E_s(0, y)dy$$

\quad \frac{L_{eff} W_{dm}}{\varepsilon_{si} L_{eff}} \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} d_{ij},$$

Substituting integral of $E_s(0, y)$ in (26) into (25), $Q_d$ of (25) into (24), $Q_d'$ of (24), and $W_{dm}$ of (12) into (23), we can reformulate the threshold voltage model of (23) as:

$$V_i = V_{fb} + 2\psi_B + \frac{L_{eff}}{LMC_{ox}} \left[ \varepsilon_{si} q \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} d_{ij} \right],$$

\quad \left[ \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} id_{ij} \right],$$

which is the function of doping profile $d_{ij}$ and effective channel length $L_{eff}$.  

### 2.4. The constraint of on-state current

To obtain the on-state current constraint of (3), we express $I_{on}$ of device as [2,3]:

$$I_{on} = C_{ox} W V_{sat} (V_{gs} - V_i) \geq I_{on-set},$$

Substituting the term of $V_i$ in (27) into (28), we have the inequality:

$$\alpha L_{eff} \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} d_{ij} \leq \sqrt{ \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} id_{ij} },$$

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where

\[ \theta = \sqrt{\mathcal{E}_{\text{s}} q \sum_{j=N}^{N+M-1} j V_B} \left[ C_{\text{ox}} L M \left( V_{g_B} - V_{f_B} - 2 \psi_B - \frac{I_{\text{off-set}}}{C_{\text{ox}} W V_{\text{sat}}} \right) \right]^{-1}. \]

The inequality (29) is not a GP compatible inequality owing to the right-hand side of (29) contains posynomial. We use the method of arithmetic-geometric mean inequality to generate a monomial replace this posynomial as (15)-(17), and (29) can be transformed as:

\[ \mathcal{O} L_{\text{eff}} \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} d_{ij} \leq \sqrt{\frac{MK}{(N+M-1)(K-1)}} \left[ \prod_{j=N}^{N+M-1} \left( \prod_{i=0}^{K-1} d_{ij} \right) \right]^{1/2}, \quad (30) \]

where the posynomial inequality is composed by the variables \( d_{ij} \) and \( L_{\text{eff}} \).

### 2.5. The constraint of off-state current

The general expression for the off-state current \( I_{\text{eff}} \) is given by [2,3]:

\[ I_{\text{eff}} = \mu_{\text{eff}} C_{\text{ox}} \frac{W}{L} (m-1) \left( \frac{kT}{q} \right)^2 e^{\frac{qV}{nkT}} \leq I_{\text{off-set}}. \quad (31) \]

Holding the exponential term on the left-hand side and taking logarithm for both sides of (31), we can obtain:

\[ \frac{kTm}{q} \ln \left[ \frac{\mu_{\text{eff}} C_{\text{ox}} W}{L} (m-1) \left( \frac{kT}{q} \right)^2 \frac{V}{I_{\text{off-set}}} \right] \leq V_t. \quad (32) \]

After substituting the \( V_t \) in (27) into (32), the (32) is reformulated as:

\[ \omega \sqrt{\sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} d_{ij}} \leq \mathcal{O} L_{\text{eff}} \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} d_{ij}, \quad (33) \]

where

\[ \omega = \left\{ \frac{kTm}{q} \ln \left[ \frac{\mu_{\text{eff}} C_{\text{ox}} W}{L} (m-1) \left( \frac{kT}{q} \right)^2 \frac{V}{I_{\text{off-set}}} \right] - V_{f_B} - 2 \psi_B \right\} \sqrt{\frac{LM C_{\text{ox}}}{\mathcal{E}_s q \sum_{j=N}^{N+M-1} j 2 \psi_B}}. \]

By using the method of arithmetic-geometric mean inequality to replace the posynomial in the right-hand side of (33), then (33) will be a valid posynomial inequality.
2.6. The constraint of the random-dopant-induced threshold voltage fluctuation

An analytical expression for the random-dopant-induced threshold voltage fluctuation is given by [2]:

$$\sigma V_i^2 = \left( \frac{q}{C_{ox} L W} \right)^2 \int_0^W \int_0^L \int_0^{W_{dm}} N_A(x, y)(1 - \frac{x}{W_{dm}}) dx dy dz. \quad (35)$$

By integrating the z-direction for the triple integral in (35) and discretizing this integral based on the technique in (10)-(11), we can approximate (35) by the summation:

$$\sigma V_i^2 = \frac{1}{W \left( \frac{q}{C_{ox} L} \right)^2} \frac{L}{2N + M} \frac{W_{dm}}{K} \sum_{i=0}^{K-1} \sum_{j=0}^{N-1} d_{ij} \left( 1 - \frac{i}{K} \right). \quad (36)$$

After substituting the $\sigma V_i$ of (36) into the threshold voltage fluctuation constraint of (1), the inequality can be rearranged as:

$$\pi \sum_{i=0}^{K-1} \sum_{j=0}^{N-1} d_{ij} \left( 1 - \frac{i}{K} \right) \leq \sqrt{\sum_{i=0}^{K-1} \sum_{j=0}^{N-1} i d_{ij}}, \quad (37)$$

where

$$\pi = \frac{q^{3/2}}{WL(2N + M)} \left( \frac{1}{C_{ox} \sigma V_{\text{ref}}} \right)^2 \sqrt{\psi_{si} \sum_{j=0}^{N-1} 2y_B}. \quad (38)$$

To obtain the valid *posynomial* again, we apply the arithmetic-geometric mean inequality on the right-hand side of (38), then (38) is transformed to a *posynomial* inequality:

$$\pi \sum_{i=0}^{K-1} \sum_{j=0}^{N-1} d_{ij} \left( 1 - \frac{i}{K} \right) \leq \sqrt{MK \left( \prod_{j=0}^{N-1} \prod_{i=0}^{K-1} d_{ij} \right)^{1/MK}}. \quad (39)$$

2.7. The formulated GP problem

According to the above *posynomial* formulation for each physical quantity of the original model problem (1), we consequently formulate the GP problem below:
minimize \( 2.3 \frac{kT}{q} \left( 1 + \frac{3 t_{\text{inv}}}{K} \right) \left( \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} \frac{2 \nu_B}{\sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} 2 \nu_B} \right) \)

\[
N_{\text{min}} \leq d_{ij} \leq N_{\text{max}}, 0 \leq i \leq K-1, 0 \leq j \leq 2N + M - 1,
\]

\[
\lambda_{i}^{-1} \sum_{j=0}^{K-1} j \sum_{i=0}^{K-1} d_{ij} \leq 2MK\lambda_{i}^{-1} \delta \left( \frac{N}{WS} \right)^{2} \left( \prod_{j=N}^{N+M-1} \prod_{i=0}^{K-1} d_{ij} \right)^{\frac{1}{MK}},
\]

\[
1 + \delta_{i}^{-1} \left( \frac{WS}{N} \right)^{2} \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} d_{ij} \leq \frac{WS}{N} \left( NK \lambda_{i}^{-1} \right)^{\frac{N}{MK}} \left( \prod_{j=N}^{N+M-1} \prod_{i=0}^{K-1} d_{ij} \right)^{\frac{1}{MK}},
\]

\[
\lambda_{i}^{-1} \sum_{j=2N+M-1}^{N+M-1} j \sum_{i=0}^{K-1} d_{ij} \leq 2MK\lambda_{i}^{-1} \delta \left( \frac{N}{WD} \right)^{2} \left( \prod_{j=N}^{N+M-1} \prod_{i=0}^{K-1} d_{ij} \right)^{\frac{1}{MK}},
\]

subject to
\[
1 + \delta_{i}^{-1} \left( \frac{WD}{N} \right)^{2} \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} d_{ij} \leq \frac{WD}{N} \left( NK \lambda_{i}^{-1} \right)^{\frac{N}{MK}} \left( \prod_{j=N}^{N+M-1} \prod_{i=0}^{K-1} d_{ij} \right)^{\frac{1}{MK}},
\]

\[
0 \leq \sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} d_{ij} \leq L_{\text{eff}} MK \left( \prod_{j=N}^{N+M-1} \prod_{i=0}^{K-1} d_{ij} \right)^{MK},
\]

\[
\omega \sqrt{\sum_{j=N}^{N+M-1} \sum_{i=0}^{K-1} d_{ij}} \leq L_{\text{eff}} MK \left( \prod_{j=N}^{N+M-1} \prod_{i=0}^{K-1} d_{ij} \right)^{MK},
\]

\[
\pi \sum_{i=0}^{K-1} \sum_{j=0}^{2N+M-1} d_{ij} \left( 1 - \frac{i}{K} \right) \leq \sqrt{MK} \left( \prod_{j=N}^{N+M-1} \prod_{i=0}^{K-1} d_{ij} \right)^{MK},
\]

\[
WS + L_{\text{eff}} + WD \leq L,
\]

which is a nonlinear programming and is a well-formulated 2D doping profile optimization problem in the form of GP with the optimal variables: \( L_{\text{eff}}, WS, WD, \) and \( d_{ij}, \forall i = 0, 1, ..., M - 1, \forall j = 0, 1, ..., 2N - 1. \) After the transformation from the problem (1) into the GP problem (40), we can cast this special nonlinear optimization problem into the convex programming problem [21]. Based on the duality theory of GP [20-22] and the interior algorithm [18,19], the problem (40) now can be solved efficiently and globally. We code and solve the problem by using the existing open numerical solvers appearing in GGPLAB [42].

3. Results and discussion

The aforementioned problem (40) is implemented for 45-nm LSP and HP device technologies.
The adopted parameters in these numerical examples are taken from [1,2] and summarized in Table 1. This GP problem is solved by using the program written by Matlab® codes on a general personal computer. In our numerical experiments, we discretize the channel area of (40) with \( K = 10, \ N = 10 \) and \( M = 10 \), which have 303 optimal variables, 601 linear constraints and 7 nonlinear constraints in these explored cases.

| Symbol | Value         | Symbol | Value         |
|--------|---------------|--------|---------------|
| \( C_{oa} \) | 3.45x10\(^{-6}\) F/C | \( V_{sat} \) | 5x10\(^{6}\) cm / s |
| \( t_{ox} \) | 3 nm | \( L \) | 45 nm |
| \( V_{fb} \) | -0.90 V | \( W \) | 100 nm |
| \( \psi_{fb} \) | 0.4504 V | \( V_{dd} \) | 1.0 V |
| \( \psi_{s} \) | 1.0701 V | \( N_{min} \) | 5x10\(^{16}\) cm\(^{-3}\) |
| \( \varepsilon_{sl} \) | 1.04x10\(^{12}\) F/cm\(^2\) | \( N_{max} \) | 5x10\(^{19}\) cm\(^{-3}\) |
| \( \mu_{eff} \) | 400 cm\(^2\)/V-s |

Figure 2: (a) and (b) ((c) and (d)) are the optimized doping profiles for low-standby power (LSP) (high performance; HP) without (w/o) and with (w/) considering fluctuation suppressions, respectively.

According to the given specifications of LSP and HP devices, the extracted 2D doping profiles are shown in Figs. 2(a)-2(d). Figure 3(a) shows the 1D section doping profile along the channel direction at the surface of the channel and the optimized doping profiles decay from source/drain end to the channel center. Notably, the shape of these “pocket” doping profiles could be obtained by a large tilted-angle implantation [7, 43]. This type doping profile engineering delays the threshold voltage roll-off due to the SCE. The solid lines and short dashed lines appearing in Figs. 4(a)-4(c) and Figs. 5(a)-5(c) show the doping profile and on/off-state conduction band energy from the silicon/oxide interface to the device substrate located at the drain end and the channel center. The doping concentration of the LSP device is relatively higher than that of the HP device because the requirement of low off-state current.
Figure 3. (a) The doping profiles and (b) on- and (c) off-state conduction band energy from the source to the drain, cutting along the channel below the silicon/oxide interface. The solid line and long dash (the short dash and dot) are the GP optimized result for LSP (HP) w/o and w/ considering fluctuation suppressions; the diamond and triangular up (the circle and triangular down) are the calibrated 2D device simulation results for LSP (HP) w/o and w/ considering fluctuation suppressions.

Figure 4. (a) The doping profiles and (b) on/off-state conduction band from the channel surface to substrate located at the drain end. The solid line and long dash (the short dash and dot) are the GP optimized result for LSP (HP) w/o and w/ considering fluctuation suppressions; the diamond and triangular up (the circle and triangular down) are the calibrated 2D device simulation results for LSP (HP) w/o and w/ considering fluctuation suppressions.
Figure 5. (a) The doping profiles and (b) on- and (c) off-state conduction band energy from the channel surface to substrate located at the channel center. The solid line and long dash (the short dash and dot) are the GP optimized result for LSP (HP) w/o and w/ considering fluctuation suppressions; the diamond and triangular up (the circle and triangular down) are the calibrated 2D device simulation results for LSP (HP) w/o and w/ considering fluctuation suppressions.

Figure 3(a) shows the doping profiles and Figs. 3(b)-3(c) are the on- and off-state conduction band energy from the source to drain, cutting along the channel below the silicon/oxide interface. The solid line and long dash are the GP optimized results for the LSP devices without (w/o) and with (w/) considering fluctuation suppressions; the diamond and triangular up are the calibrated 2D device simulation results for the LSP devices without and with considering fluctuation suppressions. Notably, the short dashed and dotted lines, as shown in Fig. 3(a), indicate the asymmetric doping profile of low average doping level near the source end and high average doping level near the drain end in the channel. Such heavy doping near the drain side can effectively reduce the RDF-induced $\sigma V_t$ [11]. The long dashed and dotted lines in Fig. 4(a) describe the optimal doping profile with considering the suppression of RDF-induced $\sigma V_t$ in the specification of LSP and HP devices from channel surface to substrate at the drain end in the channel. By considering the suppression of RDF-induced $\sigma V_t$, the peaks of doping are away from the channel surface; Their shapes look like the “retrograde” doping profile [1,6,12,13], which has the lower conduction band energy near the channel surface, as shown in Figs. 4(b)-4(c), plotted by the long dashed and dotted lines, respectively.

However, the doping distributions, as shown in Fig. 4(a), are relatively lower than those cases without considering the reduction of RDF-induced $\sigma V_t$; consequently, they will reduce the threshold voltage. To compensate the low doping level in the regions above, the doping distributions at the center of the channel form channel surface to substrate should be higher than the doping profiles without considering the reduction of RDF-induced $\sigma V_t$. Thus, the
corresponding on- and off-state conduction band energies will become higher to raise the threshold voltage, as shown in Figs. 5(b)-5(c). The specifications of the I-V characteristics and achieved results of the explored 45-nm MOSFET are summarized in Table 2. The corresponding I-V curves are shown in Fig. 6. If the optimization does not include the constraint of RDF-induced $\sigma V_t$, the values of RDF-induced $\sigma V_t$ are 31 and 19 mV in both LSP and HP devices, respectively. If the constraint of RDF-induced $\sigma V_t$ is adopted in the optimization, the RDF-induced $\sigma V_t$ is reduced to 19 and 13 mV in both LSP and HP devices, respectively (about 30% reduction). The optimal results of I-V characteristic with and without considering the RDF-induced $\sigma V_t$ are similar, for example, the threshold voltage without and with considering the RDF-induced $\sigma V_t$ is about 0.305 and 0.299 V in LSP devices and it is about 0.110 and 0.107 V in HP devices, respectively.

Table 2: List of the optimized results for the explored LSP and HP MOSFET devices.

| LSP Device | $I_{on}$ (A) | $I_{off}$ (A) | $V_t$ (V) | $\sigma V_t$ (V) | SS (V/Dec) |
|------------|-------------|-------------|----------|----------------|------------|
| Given Target | 5.50x10^-5 | 2.00x10^-9 | ---- | 0.020 | ---- |
| GP Results w/o Suppressing $\sigma V_t$ | 5.98x10^-5 | 4.23x10^-10 | 0.305 | 0.031 | 0.0960 |
| Calibration Results w/o Suppressing $\sigma V_t$ | 5.91x10^-5 | 6.25x10^-10 | 0.309 | ---- | 0.0952 |
| GP Results w/ Suppressing $\sigma V_t$ | 6.01x10^-5 | 1.05x10^-9 | 0.299 | 0.020 | 0.1040 |
| Calibration Results w/ Suppressing $\sigma V_t$ | 5.97x10^-5 | 1.10x10^-9 | 0.301 | ---- | 0.1039 |

| HP Device | $I_{on}$ (A) | $I_{off}$ (A) | $V_t$ (V) | $\sigma V_t$ (V) | SS (V/Dec) |
|------------|-------------|-------------|----------|----------------|------------|
| Given Target | 7.00x10^-5 | 4.00x10^-8 | ---- | 0.013 | ---- |
| GP Results w/o Suppressing $\sigma V_t$ | 7.19x10^-5 | 1.80x10^-8 | 0.110 | 0.019 | 0.0906 |
| Calibration Results w/o Suppressing $\sigma V_t$ | 7.01x10^-5 | 2.80x10^-8 | 0.106 | ---- | 0.0995 |
| GP Results w/ Suppressing $\sigma V_t$ | 7.25x10^-5 | 3.09x10^-8 | 0.107 | 0.013 | 0.0973 |
| Calibration Results w/ Suppressing $\sigma V_t$ | 7.32x10^-5 | 2.77x10^-8 | 0.109 | ---- | 0.1002 |
Figure 6: The optimized I-V curves for a set of given specifications, as listed in Table 2. The solid line and long dash (the short dash and dot) are the GP optimized result for LSP (HP) w/o and w/ considering fluctuation suppressions; the diamond and triangular up (the circle and triangular down) are the calibrated 2D device simulation results for LSP (HP) w/o and w/ considering fluctuation suppressions.

Figure 7: The trade-off surface for the normalized minimization of subthreshold swing (SS) versus the on-/off-state current ratio and the RDF-induced $\sigma V_t$.

To ensure the accuracy of the optimized results, the optimized doping profile is further implemented into our own in-house 2D device simulator [38-41]. In this numerical device
simulation program, we solve a set of the 2D Poisson equation as well as electron–hole current continuity equations with merely a constant mobility model. The value of mobility model is $400 \text{ cm}^2/\text{V-s}$ and the saturation velocity is fixed at $5 \times 10^6 \text{ cm/s}$ as the value set in GP model. Notably, to provide the best accuracy between the device simulation and GP optimized result, we do carefully select the value of effective mobility based on our earlier work [15]. However, the setting of constant mobility is merely for simply testing the proposed optimization method and it is insufficient for nowadays realistic device simulation. A mobility model including the influence of doping concentration, electric field, and various scatterings should be implemented to achieve more accurate estimation on physical and electrical characteristics; nevertheless, the main trend of optimized results based on the proposed optimization technique will not be altered. The diamond and triangular up (also the circle and triangular down) shown in Figs. 3(a)-5(a) and Fig. 6 are the calibrated 2D device simulation results for LSP (also HP) devices without (w/o) and with (w/) considering fluctuation suppressions. The results show that GP optimization results exhibit good agreement with the 2D simulation data.

![Spec. to be Achieved](image)

**Figure 8:** The optimal doping profiles, depending on technology node, from source to drain at the channel surface w/ (solid line) and w/o (dotted line) considering fluctuation suppressions.

From an optimal manufacturability view point of the explored 45-nm devices, the trade-off surface, as shown in Fig. 7, for the normalized minimization of the subthreshold swing (SS) versus the $I_{on}/I_{off}$ and $\sigma V_t$ is further performed, based on the optimization results. It indicates the optimal SS for each given $I_{on}/I_{off}$ and $\sigma V_t$ which could be used in turn for designing LSP and HP devices. Depending on different technology nodes, Fig. 8 shows the optimal doping profiles from the source to drain at the channel surface with (solid line) and without (dotted line) considering fluctuation suppressions. Fluctuation suppression is dynam-
ically optimized with respect to different technology nodes. Figure 9(a) indicates the $\sigma V_t/V_t$ increases when the channel length is reduced. Figure 9(b) shows the magnitude of $\sigma V_t/V_t$ for the optimization with and without considering fluctuation suppression.

![Graph showing threshold voltage fluctuation and its difference](image)

Figure 9: (a) The threshold voltage fluctuation ($\sigma V_t$) divided by the threshold voltage ($V_t$) with respect to different technology node. (b) The difference of $\sigma V_t/V_t$ with and without considering fluctuation suppression.

4. Conclusions

A 2D channel doping profile optimization via geometric programming has been implemented. The reduction of RDF-induced $\sigma V_t$ and the suppression of SCE in the solution procedure were included simultaneously. The numerical experiments show that the “pocket” doping profile at the source and drain ends in the channel decreases the SCE. Devices with an asymmetric doping profile along the channel direction, a retrograde doping distribution at drain and source ends, and a high doping level near the center of channel not only decrease the RDF-induced $\sigma V_t$ (at least 30%) but also maintaining the similar device performance, compared with the optimal results only consider the reduction of SCE. We believe that the GP approach provides an alternative to tune the doping profile of MOSFET devices. It is promising for the various design optimizations of FinFET devices in semiconductor foundry industry. The formulation of GP problem was with various assumptions of device physics, where analytical expressions of device characteristics were considered. More physical con-
considerations are necessary for advanced MOSFET devices, such as quantum mechanical effects and workfunction fluctuation of metal gate.

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