Laser induced enhanced coupling between photons and squeezed magnons in antiferromagnets

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Abstract
In this paper we consider a honeycomb antiferromagnet subject to an external laser field. Obtaining a time-independent effective Hamiltonian, we find that the external laser renormalizes the exchange interaction between the in-plane components of the spin-operators, and induces a synthetic Dzyaloshinskii–Moriya interaction (DMI) between second neighbors. The former allows the control of the magnon dispersion’s bandwidth and the latter breaks time-reversal symmetry inducing non-reciprocity in momentum space. The eigen-excitations of the system correspond to squeezed magnons whose squeezing parameters depend on the properties of the laser. When studying how these spin excitations couple with cavity photons, we obtain a coupling strength which can be enhanced by an order of magnitude via careful tuning of the laser’s intensity, when compared to the case where the laser is absent. The transmission plots through the cavity are presented, allowing the mapping of the magnons’ dispersion relation.

Keywords: 2D magnetic materials, laser-magnon coupling, Floquet theory

(Some figures may appear in colour only in the online journal)

1. Introduction
The field of magnon spintronics deals with the study and manipulation of magnetic excitations, also known as magnons, in ordered magnets [1, 2]. These spin excitations attract considerable interest from the scientific community due to the possibility of being used as information carriers, since they present nanometre wavelengths and reduced losses due to Joule heating when compared with traditional electronic devices [1, 3–6]. Due to the possibility of combining magnons with cavity photons [7–12] and superconducting qubits [13, 14], the study of spin excitations also found its way to the field of quantum information [15]. The combined study of magnon spintronics and its applications in quantum information science gave rise to the field of quantum magnonics.

Due to their bosonic nature, magnons share various features with other bosons, such as phonons and photons. In fact, when studying the quantum properties of magnons, similar ideas and methods to the ones usually found in quantum optics [16, 17] appear; one such idea is that of squeezed states. This type of quantum state has been thoroughly studied with photons, where these states are usually obtained out of equilibrium through four wave mixing or parametric processes [16–18]. The signature feature of squeezed states is the possibility of reducing the uncertainty associated with a given observable by increasing the uncertainty of another one, in such a way that, when combined, the two uncertainties still respect Heisenberg’s uncertainty principle. This type of quantum state of light has been used, for example, in the detection of gravitational waves [19].

Contrarily to what is found with photons, where the study and generation of squeezed states is a mature field [20], the
study of magnon squeezing has only begun to gain traction recently. This topic, however, is a rather interesting one due to the large values of squeezing found in these systems [21], the increased spin carried by a squeezed magnon [22], enhanced magnon–magnon coupling [23], magnon entanglement [24], among others [25]. Moreover, contrarily to photons, the exploration of such phenomena in magnetically ordered systems allows for its implementation in on-chip nanodevices [26].

Regarding the generation of squeezed magnons, two distinct approaches can be employed. On the one hand, squeezing can be obtained by driving the magnetic system out of equilibrium while coupled with an optomechanical cavity [27–29]. On the other hand, and in stark contrast with photons, the exploration of such systems naturally present squeezing, making them robust against environment perturbations [2, 21, 26].

Recent works focus on the manipulation of magnetic materials via the application of high-frequency laser fields. In reference [30], the authors uncover an ultrafast Floquet magnonic topological phase transition in a laser-driven skyrmion crystal, and demonstrate how single skyrmions can be set in motion with a velocity and propagation direction that can be tuned by the laser. In reference [31] a study of Floquet topological magnons in ferromagnets was performed, where the authors explored how the application of an external laser field may be used to generate a synthetic Dzyaloshinskii–Moriya interaction (DMI) [32] through the appearance of a time-dependent Aharonov–Casher phase [33]. This laser induced effect leads to the transformation of Dirac magnons into magnon Chern insulators. Inspired by this work, and motivated by the growing interest of the scientific community on the magnonic response of antiferromagnets, in this paper we study the effect of applying an external laser field to a honeycomb antiferromagnet, and discuss how it can be used to tune the properties of the squeezed magnons hosted by the system.

The text is organized as follows: in section 2 we start by introducing the Hamiltonian of a honeycomb antiferromagnet, and how it is modified by the presence of the laser. Afterward the Hamiltonian is diagonalized with a Bogoliubov transformation. In section 3 we consider the system to be placed in an optical cavity, and study how the laser can be used to enhance the magnon–photon coupling; plots of the transmission through the cavity are also given. In section 4 we discuss the feasibility of experimentally realizing this type of system, and in section 5 we give our final remarks.

2. Model Hamiltonian

In this section, we introduce the Hamiltonian of a honeycomb antiferromagnet. We focus on this type of lattice since it is a common one in two-dimensional magnetic materials, and as such is relevant in this field of research. Afterward, the effect of applying a circularly polarized laser field to the system will be explored, with the introduction of a Floquet effective Hamiltonian. This process is analogous to the one presented for a ferromagnet in [31]. Finally, the resulting Hamiltonian will be diagonalized, and its properties discussed.
limit it is possible to map formally the spin Hamiltonian into a tight-binding model of hopping bosons in a lattice. These bosons carry no charge, but rather a magnetic moment. On the other hand, it is well known that electrons, carrying charge and hopping in a lattice, when irradiated by an electromagnetic field acquire a time-dependent Peierls phase, which turns out to be nothing more than a time-dependent Aharonov–Bohm phase. The same argument carries over to the hopping bosons, which carry a magnetic momentum. The motion of the bosons in the lattice, in the presence of the same electromagnetic field, leads to a hopping integral that acquires a phase which, in this case, is the Aharonov–Casher phase.

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The Hamiltonian of the system in such a situation becomes time dependent, and may be expressed as

\[ H(t) = J \sum_{i,A} \sum_{j=1}^{3} S_i^+ (r_i) S_j^- (r_j + \delta_j) \]

\[ + \frac{J}{2} \sum_{i,A} \sum_{j=1}^{3} \left[ S_i^+ (r_i) S_j^- (r_j + \delta_j) e^{-i \phi_{i,r} r_i + \delta_j} + \text{h.c.} \right] \]

\[ + K \sum_{i,A} S_i^z (r_i) S_i^z (r_i) + K \sum_{i,B} S_i^z (r_i) S_i^z (r_i) \]

(2)

where we introduced \( S_i^\pm = S_i^x \pm S_i^y \) with \( \alpha = A, B \). The time dependent Aharonov–Casher phase is defined as

\[ \phi_{i,r} = \lambda \sin \left( \omega t - \tau \phi_{i,r} + \delta_j \right) \]

(3)

with \( \lambda = \tau g \mu_B E_0 / \hbar c^2 \) where \( \mu_B \) is the Bohr magneton and \( g \) the Landé g-factor; \( \phi_{i,r} + \delta_j \) corresponds to the angle between the vectors \( r_i \) and \( r_i + \delta_j \). Note that since \( \lambda \propto \omega_0 \) the value of \( \lambda \) is proportional to the laser’s intensity. To avoid working with a time dependent Hamiltonian we shall make use of Floquet theory [44–47]. This is a perturbative approach allows us to obtain an effective time independent Hamiltonian from \( H(t) \). This is achieved by expressing the effective Hamiltonian in a high frequency series expansion, which up to first order reads

\[ H_{\text{eff}} = H(0) + \sum_{m=1}^{\infty} \frac{[H(m), H']_{m \hbar \omega}}{m \hbar \omega} \]

(4)

where \( H^{(m)} \) is the \( m \)th Fourier component of \( H(t) \). In order for this effective Hamiltonian to be valid, the energy of the laser should be larger than the energy scale of the initial system (set by \( J \)). For lower laser energies more terms would have to be accounted for when defining \( H_{\text{eff}} \). The leading term \( H(0) \) corresponds, perhaps surprisingly, to an average of the Hamiltonian time dependent Hamiltonian over a period of the driving laser field. It is the most relevant term for our purposes, with the first order correction introducing only small effects.

We shall work within the linear spin-wave theory. Thus, we introduce the linearized Holstein–Primakoff transformations, which for the antiferromagnetic case read [48]

\[ S_i^+(r) = \sqrt{2} \alpha_i r \]

\[ S_i^-(r) = S - \alpha_i r_{\text{eff}} \]

\[ S_j^+(r) = \sqrt{2} \beta_j r \]

\[ S_j^-(r) = S + \beta_j r_{\text{eff}} \]

(5)

(6)

where \( \alpha_i, \beta_j \) and \( r_{\text{eff}} \) are bosonic operators, which we refer to as the annihilation/creation operators of sublattice magnons, and we note \( (S_i^+)^2 = S_i^2 \). Introducing the Fourier representation of the Holstein–Primakoff annihilation/creation operators, \( a_i = \frac{1}{\sqrt{2N}} \sum_a \delta \phi_{i,a} a \) we obtain the following time-independent effective Hamiltonian in momentum space

\[ H_{\text{eff}} = J S \sum_k \left( 3 + \frac{2k}{J} \right) (a_k^+ a_k + b_k^+ b_k) + J S_\emptyset (0) \sum_k (a_k b_k \phi_k + a_k^+ b_k^+ \phi_k^*) \]

\[ - J S \sum_k D_k (\omega, \lambda) b_k^+ b_k + J S \sum_k D_k (\omega, \lambda) a_k^+ a_k \]

(7)

where \( \phi_k = \sum_{j=1}^{N} e^{-i k \cdot r_j} \) a geometric factor stemming from the nearest neighbor interactions. The first order correction arises in the previous equation in the form of the coefficients

\[ D_k (\omega, \lambda) = \frac{4 J S}{\hbar \omega_0} V_{\text{DM}} (\lambda) \operatorname{Im} \sum_{j=1}^{6} (-1) e^{-i k \cdot r_j} \]

(8)

where the \( b_j \) are shown in figure 1 and correspond to the vectors connecting a given site to its six nearest neighbors; \( V_{\text{DM}} (\lambda) = \sum_{m=1}^{\infty} \left| J_{\omega_0}^m (\lambda) / 2 m \right| \sin (\tau m \pi / 3) \) and \( J_{\omega_0} (\lambda) \) is the cylindrical Bessel function of the first kind of order \( m \). The terms proportional to \( D_k \) correspond to a synthetic DMI, which appears from laser induced couplings between second neighbors; its magnitude can be tuned simultaneously through the laser’s intensity and frequency. Thus, comparing this Hamiltonian with the one usually employed to describe an antiferromagnet [21], we realize that: (i) the exchange coupling between the z-component of the spins and the easy axis anisotropy are unaffected by the laser and originate the first line of equation (7); (ii) the laser is responsible for the renormalization of the in-plane exchange coupling between nearest neighbors, reducing its strength, i.e. \( J \rightarrow J J_0 (\lambda) \) (this renormalization would also appear on other lattice configurations), and finally (iii) a synthetic DMI arises from the laser-induced interaction between next nearest neighbors. We note in passing that although the DMI is finite for the honeycomb lattice, that is not always the case, since, for example, it is absent for square lattices. How these modifications manifest themselves on the properties of the system is studied below.

2.2. Diagonalization

To diagonalize the effective Hamiltonian \( H_{\text{eff}} \) a Bogoliubov transformation shall be used, as often is the case when working with antiferromagnets. Hence, to achieve this, we introduce two new sets of operators, \( \alpha_k / \alpha_k^* \) and \( \beta_k / \beta_k^* \), which we simply

\[ \alpha_k / \alpha_k^* \]

\[ \beta_k / \beta_k^* \]
interactions refer to as magnon annihilation/creation operators. These are defined as:

\[ \alpha_k = u_k a_k + v_k b_{-k}^{\dagger} \]
\[ \beta_k = v_k a_k^{\dagger} + u_k b_{-k} \]

with \( u_k = \cos \frac{\theta_k}{2} \) and \( v_k = e^{i\theta_k} \sinh \frac{\theta_k}{2} \) where \( \xi_k > 0 \) and \( \theta_k \in [0, 2\pi) \). Imposing that \( H_{\text{eff}} \) is diagonal when expressed in terms of \( \alpha_k \) and \( \beta_k \), that is, \( H_{\text{eff}} = \sum_k \epsilon_{\alpha,k} \alpha_k^{\dagger} \alpha_k + \epsilon_{\beta,k} \beta_k^{\dagger} \beta_k \), we find the following dispersion relation up to an overall constant factor:

\[ \epsilon_{\alpha,k} = \epsilon_{\beta,k} = JS \left( 3 + D_k + \frac{2K}{J} \right) \sqrt{1 - \tanh^2 \xi_k} \]

for

\[ \tanh^2 \xi_k = \left( \frac{J_0(\lambda)}{3 + D_k + 2K/J} \right)^2 |\phi_k|^2 , \]

and

\[ \theta_k = n\pi - \arctan \frac{1}{\Im \phi_k} \frac{\Re \phi_k}{|\Re \phi_k|^2} , \]

where \( n = 0, 1 \) is fixed through the condition \( \cos \theta_k = \text{sign}[J_0(\lambda)] \).

2.2.1. Energy dispersion. From the diagonalization procedure, we find that the two magnon modes of the system are degenerate, i.e. \( \epsilon_{\alpha,k} = \epsilon_{\beta,k} \). This is to be the case for usual antiferromagnets, and we verify that the introduction of the laser field does not change this aspect of the problem. This degeneracy can, however, be broken in several manners, for example by defining the easy axis anisotropy as being different for the two sublattices, applying a magnetic field along the \( z \)-direction, or accounting for magnetic dipolar interactions [49].

The effect of the applied laser field on the magnon dispersion is twofold. If the laser energy is tuned such that \( \hbar \omega_l \gg J \), then the synthetic DMI essentially vanishes, \( D_k \approx 0 \), and the laser manifests itself through the term \( J_0(\lambda) \) only. This term is responsible for modifying the bandwidth of the magnon dispersion, which now oscillates with steadily decaying amplitude as \( \lambda \) increases, similarly to the behavior of a Bessel function. Since \( \phi_k \) is maximal at \( k = 0 \), and vanishes at the Dirac points, the effect of \( J_0(\lambda) \) is more pronounced at the center of the Brillouin zone, and does nothing at its vertices, \( K_{\pm} \). This behavior is illustrated in figure 2(a) where we depict the magnon dispersion in momentum space along the line \( k_y = 0 \) for three distinct values of \( \lambda \). In the same figure, we also plot the magnon dispersion at \( k = 0 \) as a function of \( \lambda \), perfectly illustrating the aforementioned oscillating behavior. Furthermore, we also note that for a judicious choice of \( \lambda \), corresponding to the zeros of \( J_0(\lambda) \), one obtains an almost flat band for the magnon dispersion. This is easily understood from the inspection of equation (11), where one sees that if \( J_0(\lambda) = 0 \), then the square root term simply equals one, and the momentum dependence comes only from the term \( D_k \), which can be made arbitrarily small by increasing the energy of the laser. Consider the case where the laser’s energy is reduced, such that the synthetic DMI becomes relevant. In this limit, time-reversal symmetry is broken, and the dispersion relation is no longer even in momentum space, meaning that \( \epsilon_{\alpha,k} \neq \epsilon_{\alpha,-k} \). Since \( D_k \neq 0 \), the effect of the DMI is mainly noticeable near the Dirac points, where it induces an energy difference between the magnon dispersion at \( K_+ \) and \( K_- \). This effect is visible in figure 2(a), where we observe that as \( k \to K_+ \) the magnon dispersion increases when compared with the case where the laser is turned off (\( \lambda = 0 \)); the opposite statement is valid when \( k \to K_- \). The presence of the DMI is not guaranteed to always yield this effect, since it depends on the value of \( \gamma_{\text{DM}} \) which may be positive, negative, or zero, depending on the value of \( \lambda \).

2.2.2. Bogoliubov coefficients. Now that the energy dispersion of the system was studied, let us discuss the Bogoliubov transformation itself, and how its coefficients depend on the laser field.

We start by noting that the definitions we gave for \( \alpha_k \) and \( \beta_k \) in terms of \( a_k \) and \( b_{-k}^{\dagger} \) could be alternatively expressed in terms of a two mode squeeze operator \( S_2(\xi_k) = \exp \left( \xi_k a_k^{\dagger} b_{-k} - \xi_k^* a_k b_{-k}^{\dagger} \right) \), where \( \xi_k = (\xi_k/2) e^{-i\theta_k} \) [16, 17]. Using this operator, we could have defined \( \alpha_k = S_2(\xi_k) a_k S_2(\xi_k) \) and \( \beta_k = S_2(\xi_k) b_{-k} S_2(\xi_k) \) (the proof of this statement is given in the supplementary information https://stacks.iop.org/JPCM/34/245802/mmedia). Hence, based on these new definitions, we see that the magnonic excitations of the system (\( \alpha_k \) and \( \beta_k \)), which are linear combinations of sublattice magnons (\( a_k \) and \( b_{-k} \)), correspond, in fact, to two-mode squeezed magnons, with a squeezing parameter \( \xi_k \). Although the identification of the eigenmodes of the antiferromagnet as being squeezed magnons is independent of the external laser.
field, the presence of the laser introduces new interesting features in the system due to the possibility of tuning both $\xi_k$ and $\theta_k$, as we shall see below.

Let us now study how the parameters $\xi_k$ and $\theta_k$ depend on the external laser field, and how their values change with the momentum $k$.

In figure 3(a) we depict $\xi_k$ for the $k = 0$ mode, as a function of $\lambda$. Since we are considering only the isotropic magnon mode, the DMI vanishes automatically, i.e. $D_{k=0} = 0$. For $\lambda = 0$, that is, when the laser field is turned off, we find a large value for $\xi_{k=0}$; in fact, its value would be even larger if the easy axis anisotropy had been ignored, since in that case the squeezing parameter would diverge in the limit $k \to 0$. As $\lambda$ increases the squeezing parameter decreases, until it vanishes when $\lambda$ reaches the first zero of the Bessel function $J_0$. Afterward, we find that $\xi_{k=0}$ increases slightly and an oscillatory behavior sets in. It attains successive maxima when maximum or minimum values of $J_0$ are hit, and vanishes at the zeros of the Bessel function. The maxima of $\xi_{k=0}$ become progressively smaller as $\lambda$. Performing an identical analysis for $k \neq 0$ one finds a similar pattern, albeit with an overall smaller magnitude. When $k \approx k_0$ we find $\xi_k \approx 0$, regardless of $\lambda$, since $\theta_k \approx 0$ near the vertices of the Brillouin zone. Thus, we see that the magnon squeezing tends to decrease as the magnon momentum increases, until it vanishes at the Dirac points, as depicted in figure 3(b). Also depicted in figure 3(a), is the dependence of $\theta_k$ with $\lambda$ for the mode $k = 0$. When the laser is turned off, i.e. $\lambda = 0$, one finds $\theta_k = 0$. Interestingly, when $\lambda$ crosses the first zero of the Bessel function $J_0$, the phase $\theta_{k=0}$ jumps to $\pi$. This phase jumping proves to be the crucial ingredient to enhance the magnon–photon coupling, which we shall discuss in the next section. For modes with $k \neq 0$ other values are obtained for $\theta_k$, in agreement with equation (3), but the $\pi$-phase jumps remain.

3. Magnon–photon coupling

In this section we consider that the antiferromagnet is placed inside an optical cavity, and study how the squeezed magnons couple with the cavity photons. The dependence of the magnon–photon coupling on the external laser field will be studied, and the transmission spectrum through the cavity computed.

3.1. System Hamiltonian

The first step to study the interaction of the squeezed magnons with the cavity photons is to define the Hamiltonian of the system. This Hamiltonian is composed of three distinct contributions

$$H = H_{\text{cav}} + H_{\text{AFM}} + H_{\text{int}}$$

(14)

where $H_{\text{cav}} = \sum q \hbar \omega_q \hat{a}_q^\dagger \hat{a}_q$ is the Hamiltonian of the cavity photons, with $\hat{a}_q^\dagger / \hat{a}_q$ the annihilation/creation operator of a photon with momentum $q$ and energy $\hbar \omega_q = \hbar c |q|$; $H_{\text{AFM}}$ is the effective Hamiltonian we introduced in the previous section to describe the antiferromagnet under the incidence of the external laser field, and $H_{\text{int}} = g \mu_B \sum_{q \in A} b(r_q) \cdot S_A(r_q) + g \mu_B \sum_{q \in B} b(r_q) \cdot S_B(r_q)$ is the interaction Hamiltonian between the magnetic field of the cavity photons, $b$, and the spins of the antiferromagnet.

We now emphasize that up to this point all the momenta we considered were two-dimensional, due to the in-plane nature of the magnons we are studying. However, since the cavity photons carry a three dimensional momentum, we must differentiate between the two. To that end, henceforth, when considering a two-dimensional momentum, we shall label it with the index $\parallel$, indicating its in-plane configuration (e.g. $q_\parallel$ corresponds to the in-plane component of the three-dimensional momentum $q$).

Considering the magnetic field of the cavity photons to be circularly polarized, we write its quantized form as [8]

$$b^\pm(r) = \frac{1}{c} \sum q \left[ h \omega_q \hat{a}_q^\dagger - h c |q| \hat{a}_q \right] \pm i \left( p_q e^{i \epsilon_q r} + p_q^* e^{-i \epsilon_q r} \right) \hat{x},$$

(15)

where $\pm$ stands for the two possible circular polarizations, $c$ is the speed of light, $\epsilon_0$ is the vacuum permittivity and $V$ the volume of the cavity. To express $H_{\text{int}}$ in terms of the eigenmodes of the antiferromagnet, that is, using the operators $a_k$ and $\beta_k$, we start by expressing $S_A/B(r)$ in terms of the sublattice magnon operators, $a_i$ and $b_i$, through the linearized Holstein–Primakoff relations given in the previous section. Then, the Fourier components of these operators are introduced, and the Bogoliubov transformation inverted, in order to express $a_k$ and $b_k$ in terms of $a_{\parallel}$ and $\beta_{\parallel}$. At last, dropping terms with the product of two annihilation or two creation operators, one finds

$$H_{\text{int}}^+ = \sum_q U_q p_q a_{\parallel}^\dagger a_{\parallel} + \text{h.c.}$$

(16)

$$H_{\text{int}}^- = \sum_q U_q p_q a_{\parallel}^\dagger a_{\parallel} + \text{h.c.}$$

(17)

where, once again, the superscript $\pm$ refers to the two circular polarization of the cavity photons, $a_{\parallel}$ refers to the in-plane component of the 3D-momentum $q$, and

$$U_q = \frac{g \mu_B}{c} \sqrt{\frac{NS \hbar \omega_q}{2 \epsilon_0 V}} \left( \cosh \frac{\xi_{\parallel}}{2} - e^{i \theta_{\parallel}} \sinh \frac{\xi_{\parallel}}{2} \right)$$

(18)

is the magnon–photon coupling. Notice how according to equations (16) and (17) the two orthogonal circular polarizations couple selectively with just one of the magnon modes each, with equal coupling strength. Although in the system we are considering the two magnon modes are degenerate, the polarization of the cavity photons could be used to select a specific magnon branch in a system where said degeneracy is broken. Also, we note that for a fixed energy of the cavity photons (and thus for a fixed $q$), the value of $\theta_{\parallel}$ can be tuned by changing the relative orientation of the antiferromagnet and the magnetic field of the cavity mode.
3.2. Coupling strength

Let us now study in more detail the magnon–photon coupling strength. The coupling $\mathcal{U}_q$ is composed of two distinct contributions: (i) a numerical pre-factor $A_q = \left(g\mu_B/c\right)\sqrt{N\hbar\omega_q/2eV}$ determined by the properties of the system, namely the cavity photon energy, cavity volume and number of spins, and (ii) an additional multiplicative term determined by the coefficients of the Bogoliubov transformation. $f_q = \cosh \frac{\xi_q}{2} - e^{i\theta_q} \sinh \frac{\xi_q}{2}$, which depends on the external laser field. The latter contribution is the one we are interested in studying, in particular how its modulus $|f_q|$ is affected by the momentum dependence and the applied laser.

Consider first the case where the laser is turned off, $\lambda = 0$. According to figure 3(a), we find $\theta_q = 0$, which leads to $|f_q| = e^{-\xi_q/2}$. Following the analysis of the previous section regarding the squeezing parameter, we know that it takes its largest value for $q_\| = 0$ and monotonically decreases as the momentum increases, until it vanishes at the Dirac points (where the magnons are no longer squeezed). Hence, in the absence of the external laser, we find that the magnon–photon coupling decreases exponentially as the magnon momentum approaches the center of the Brillouin zone. In particular, for the parameters $J = 1$ meV, $K = 10^{-5}J$ and $\hbar\omega = 10J$, we find $|f_{q_\| = 0}/f_{q_\| = K_\|} | \approx 0.05$, that is, the magnon–photon coupling is 20 times smaller near the Brillouin zone center than at the Dirac points.

Let us now consider that $\lambda \neq 0$, corresponding to the scenario where the laser is turned on. Initially we have $\theta_q = 0$, and as $\lambda$ increases the squeezing parameter $\xi_q$ decreases, leading to an increase of the coupling strength $|f_q|$ when compared to the case where the laser is absent. When $\lambda$ reaches the first zero of the Bessel function $J_0$, the squeezing parameter vanishes on the entire Brillouin zone and $|f_q| = 1 \forall q_\| \in 1BZ$. We note, however, that a similar effect could be achieved without the laser by increasing the easy axis anisotropy, which for a large enough value would yield a similar result. The unique feature introduced by the laser arises when the system is driven in such a way that $\lambda$ is located between the first two zeroes of the $J_0$. In that situation, and focusing on the $q_\| = 0$ mode, we find $\theta_q = 0 = \pi$, leading to a coupling strength which grows exponentially with $\xi_q$, that is $|f_q| = e^{\xi_q/2}$. Since in the considered range of values for $\lambda$ we have $\xi_q > 0$, then $|f_q| > 1$. In fact, one finds that $|f_q| = 0$ reaches a peak of approximately 1.25 when $\lambda \approx 3.8$ (corresponding to the first minimum of $J_0$). The peak value $|f_q| = 0 \approx 1.25$ is obtained in the limit where the exchange coupling $J$ dominates the anisotropy term $K$. If $K$ becomes comparable with $J$, then the maximum value for $|f_q| = 0$ decreases.

Hence, by driving the system with a laser field, we are able to enhance the coupling of the $q_\| = 0$ magnons with the cavity photons by a factor of approximately 25 when compared to the case where no laser is applied. Not only that, but we are also able to push the coupling strength beyond the limit of what is found for unsqueezed magnons, while retaining some magnon squeezing ($\xi_q = 0 \approx 0.2$ for $\lambda \approx 3.8$); a result only attainable by driving the system with the laser field. For magnon modes with $q_\| \neq 0$ the laser induced enhanced coupling is also present, although the effect becomes progressively weaker as we approach the Dirac points. These results are summarized in figure 4 where we depict $|f_q|$ as a function of $\lambda$ for different momenta $q_\|$. The shaded blue/orange areas indicate the range of $\lambda$ where the magnon–photon coupling is smaller/larger than for unsqueezed magnons.

3.3. Transmission spectra

Now that the Hamiltonian of the system has been determined, and the details of the magnon–photon coupling have been discussed, we move on to the computation of the transmission spectra through the cavity. Following [50], we state that the transmission through the cavity should be proportional to the spectral function of the cavity photons, which follows directly from the retarded Green’s function $G^R(t) = -\frac{1}{\hbar} \delta(t)/\langle \hat{p}(t)\hat{p}(0) \rangle$. Considering now that the cavity has only one mode, and that its magnetic field is polarized with the ‘–’ circular polarization, we obtain the following expression for the retarded Green’s function [50, 51]

$$G^R(\hbar\omega) = \frac{1}{\hbar\omega - \hbar\omega_q + i\Gamma - \frac{\hbar^2q^2}{\hbar^2c^2 - \hbar^2\omega_q^2 + \hbar^2\Omega^2}}.$$

(19)
where $\Gamma$ and $\delta$ are the cavity and antiferromagnet intrinsic damping factors, and $\hbar \omega$ is the energy of the incident photons. Once more we have $\hbar \omega_q = \hbar |q|$ the energy of the cavity mode and $\epsilon_q$ the energy of the magnon mode that couples with the cavity photons. Although the circular polarization we chose only allows coupling to the $\alpha$-magnons, equation (17), a similar result would be obtained for the other circular polarization where coupling to $\beta$-magnons would appear instead. Since for the system we are considering the two magnon modes are degenerate, the transmission spectrum is identical in both cases.

Having determined $G^R(\omega_q)$ we define the transmission amplitude as $t(\omega_q) \propto -2 \text{Im} G^R(\omega_q)$. A simple way to avoid the proportionality relation, is to normalize the transmission amplitude by its value when the antiferromagnet is not present in the cavity ($U_q = 0$) and the incident photons are in resonance with the cavity mode $\hbar \omega = \hbar \omega_q$. Doing so, we find

$$t(\omega_q) = -\text{Im} \frac{\Gamma}{\hbar \omega - \hbar \omega_q + i \epsilon_q} + \frac{|U_q|^2}{\hbar \omega - \epsilon_q + i \delta}.$$  (20)

where $t(\omega_q)$ is the normalized transmission amplitude. The poles of $t(\omega_q)$ correspond to the excitations of the system, which are termed magnon–polaritons as they correspond to the hybridization of the antiferromagnet magnons with the cavity photons.

In figure 5 we depict a density plot of $t(\omega_q)$ as a function of the incident photon energy $\hbar \omega$ and the magnon momentum $k$, along the line $k_0 = 0$ on the first Brillouin zone, for three different values of $\lambda$; for each case the energy of the cavity mode was chosen to match the minimum of the magnon dispersion. The first thing we note regarding this result is that the bandwidth of the magnon dispersion is modified by changing $\lambda$ in agreement with what was previously found in figure 2 (the effect of the DMI is small since we are considering $\hbar \omega_q = 50J$). Focusing on the leftmost panel, we see that the hybridization between the cavity mode and the zero-momentum magnon is almost nonexistent; this is simultaneously due to the small energy of the cavity photon (leading to a small $A_q$), and due to the fact that $|f_{k_1}|$ is small for $\lambda = 0.5$. Regarding the middle panel, we see that, due to the higher value used for $\lambda$, the bottom of the magnon dispersion moves to higher energies, and, at the same time, the magnitude of $|f_{k_1}|$ increases (see figure 4).Because of this, one finds a much clearer coupling between the magnons and the cavity photons, which is identified by the anticrossing of the magnon–polariton branches. At last, on the rightmost panel, where $\lambda = 3.8$, we observe the largest hybridization between magnons and photons, with a clear separation between the two magnon–polariton modes. This strong coupling was expected, since $\lambda$ was carefully chosen to maximize $|f_{k_1}|$, in agreement with the discussion following figure 4.

4. Plausible parameters

Up to this point we have discussed the general physical properties of a laser driven antiferromagnet without focusing on a particular implementation of the system. In this section we wish to estimate the values the different parameters the system should have in order to observe the phenomena discussed so far. To the best of our knowledge, an experiment focused on the phenomena we describe in this paper has not been performed so far.

To do so, we start by setting $\lambda = 2.4$, corresponding to the minimum value of $\lambda$ which allows an enhanced magnon–photon coupling above the unsqueezed limit (see figure 4). Recalling $\lambda = g \mu B E_0 a / \hbar c^2$, and setting $g = 2$, we find that in order to have $\lambda = 2.4$ we must have $E_0 a \approx 1.2 \times 10^6$ V. For $a = 3.3$ Å [52], corresponding to a value in the order of magnitude that is usually found for hexagonal boron nitride, transition metal dichalcogenides, etc., we obtain $E_0 \approx 3 \times 10^{13}$ V cm$^{-1}$. If a larger $a$ is used, a smaller electric field is required to produce the same $\lambda$. In reference [53] a laser intensity of $10^{23}$ W cm$^{-2}$ has been reported, which in terms of the electric field’s magnitude corresponds to roughly $E_0 \approx 9 \times 10^{12}$ V cm$^{-1}$, within the order of magnitude of the required value. Hence, current state of the art laser technology is already compatible with the requirements to significantly increase the magnon–photon coupling. To estimate the numerical value of $U_q$ we focus on...
the magnitude of $A_q = \left( g \mu_B / c \right) \sqrt{N \hbar q / 2 e_0 V}$. We now set $S = 1/2$ and note that $N/V = \rho/2\hbar$ where $\rho/2$ is the density of spins and $h$ is the height of the cavity. Considering $h = 40$ nm [54] and $\rho/2 \sim 10^{14}$ cm$^{-2}$ (in agreement with the measured values of [52], and compatible with the value of $a$ considered above), we find $A_q \approx 10^{-4}\sqrt{\hbar q}$ (in eV), which is comparable with what we used in figure 5. For $\hbar \omega \sim 1$ meV we find $A_q \sim 3 \times 10^{-6}$ eV, corresponding to approximately 0.7 GHz, which is well within the resolution of current experiments [8, 55, 56].

5. Final remarks

In this paper we studied a honeycomb antiferromagnet subject to an external laser field. After a Floquet effective Hamiltonian was introduced, we diagonalized it using a Bogoliubov transformation, and found that the eigenexcitations of the system correspond to squeezed magnons. The presence of the laser allows the control of several properties of these spin excitations, namely their dispersion relation, and squeezing parameters.

When studying how the magnons couple with photons in a cavity, we found that the two magnon modes of the system couple selectively with the two orthogonal circular polarizations. Furthermore, we found that by tuning the intensity of the applied laser field the strength of the magnon–phonon coupling can be significantly enhanced. In particular, we found that, for the isotropic magnon mode, the coupling strength with the cavity photons can be enhanced by an order of magnitude when compared to what is found in the absence of the laser. When the system is not driven, the largest magnon–photon coupling is achieved at the Dirac points, where the magnons do not present squeezing. When the laser is present, however, the maximum coupling is reached for the magnons near the Brillouin zone center, which preserve some of the original squeezing; in this case the magnon–photon coupling surpasses the maximum value found for the undriven system.

To the best of our knowledge, an experiment probing these physical effects has not yet been realized. However, from our estimates, state of the art equipment may be able to probe the discussed phenomena. Since our theoretical predictions were derived for a strictly 2D monolayer, the results may differ if experiments are realized on thin films, due to the sensitivity of these materials regarding the number of layers.

Finally, we note that although we focused on the magnon–photon interaction, other options could be explored. An example of this is the interaction of magnons with conduction electrons. According to reference [21] the transition rate for the electron–magnon scattering takes a similar form to the function $f_q$ we identified in the magnon–phonon coupling. Due to this similarity, the ideas discussed here regarding the coupling strength could, in principle, also be applied in that type of system.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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