GRAD-SHAFRANOV EQUATION WITH ANISOTROPIC PRESSURE

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ABSTRACT

The most general form of the nonrelativistic Grad-Shafranov equation describing anisotropic pressure effects is formulated within the double adiabatic approximation. It provides the possibility of analyzing quantitatively how anisotropic pressure affects the two-dimensional structure of the ideal magnetohydrodynamic flows.

Subject headings: ISM: jets and outflows — MHD — stars: winds, outflows

1. INTRODUCTION

The Grad-Shafranov (GS) equation for axisymmetric stationary ideal magnetohydrodynamic (MHD) flows has been considered very intensively within the last two decades both for a nonrelativistic stellar (solar) wind including outflows from young stellar objects (Okamoto 1975; Heinemann & Olbert 1978; Blandford & Payne 1982; Heyvaerts & Norman 1989; Sakurai 1990; Pelletier & Prudzzi 1992; Shu et al. 1994) and for relativistic outflows from radio pulsars (Ardavan 1979; Bogovalov 1992) and the “central engine” in active galactic nuclei (Phinney 1983; Begelman, Blandford, & Rees 1984; Lovelace et al. 1986; Chiueh, Li, & Begelman 1991; Beskin, Kuznetsova, & Raffcov 1998). As is well known, in this approach, isotropic gas pressure has been postulated (see e.g., Heyvaerts 1996 for an introduction).

It is necessary to stress that, actually, thermal effects were considered rather carefully only within the given poloidal magnetic field approximation (Weber & Davis 1967; Kennel, Fujimura, & Okamoto 1983; Paatz & Camenzind 1996) or within the self-similar approach (Low & Tsinganos 1986; Contopoulos & Lovelace 1994; Sauty & Tsinganos 1994). As for the GS equation, its two-dimensional solutions with nonzero pressure were obtained only for pure hydrodynamic flows (see Beskin 1997 for review) when the isotropic pressure model is correct.

On the other hand, for real astrophysical flows where the effects of finite pressure may play an important role, the isotropic pressure model does not seem to be sufficient. For example, in the solar wind where the free path exceeds the Larmor radius by many order of magnitude, such that \( l_s / r_L \approx 10^9 \) in the vicinity of the Earth, the effect of anisotropic pressure may be important.

Of course, it is necessary to remember that the ideal MHD approximation (including the double adiabatic approach under consideration) is not good for real astrophysical objects and so the dissipative processes can play an important role (Leer & Axford 1972; Hu, Esser, & Habbal 1997). Moreover, in the solar wind the stochastic component of the magnetic field is of the order of the regular one. Such fluctuations drastically change the velocity distribution function. As a result, the pressure anisotropy \( P_{\parallel} / P_T \approx 1.2 \) is not actually large (see, e.g., Marsch et al. 1982). Maybe for this reason the effects of anisotropic pressure on the outflow structure within the GS approach were analyzed in just a small number of papers (Asseo & Beaufls 1983; Tsikarishvili, Rogava, & Tsikauri 1995), the GS equation itself being formulated only for the planar statical configurations with zero longitudinal electric current (Nötzel, Schindler, & Birn 1985), and not for the full form, by Lovelace et al. (1986).

In this paper we give the general equations describing the axisymmetric stationary ideal MHD flows with anisotropic pressure. Then we shall present the results of our preliminary analysis of the effects of anisotropic pressure (such an analysis has not been produced previously either).

2. BASIC EQUATIONS

Let us consider a stationary axisymmetric (and nonrelativistic) MHD flow. Then the magnetic field can be written in the form

\[
B = \frac{\nabla \Psi \times e_\phi}{2 \pi \sigma} - \frac{2 I}{\sigma c} e_\phi.
\]

(1)

Here \( \Psi(r, \theta) \) is the magnetic flux, \( \sigma = r \sin \theta \), and \( I(r, \theta) \) is the total electric current flowing inside the region \( \Psi < \Psi(r, \theta) \). Assuming that the magnetosphere contains enough plasma to screen the longitudinal electric field \( E_\parallel \), one can write

\[
E = -\frac{\Omega_F}{2 \pi c} \nabla \Psi,
\]

(2)

with the “field angular velocity” \( \Omega_F = \Omega_F(\Psi) \) being constant on the magnetic surfaces, \( \Psi(r, \theta) = \text{const} \). Finally, the continuity equation \( \nabla \cdot (\rho u) = 0 \) together with the freezing-in condition \( \nabla \times B/c = 0 \) and the Maxwell equation \( \nabla \times E = 0 \) result in

\[
v = \frac{\eta}{\rho} B + \Omega_F \sigma e_\phi,
\]

(3)

where \( \rho \) is the mass density and \( \eta = \eta(\Psi) \) is the particle-to-magnetic flux ratio. Clearly, these expressions do not depend on the pressure anisotropy.

The next step is the transformation of the Euler equation:

\[
\rho(v \cdot \nabla)v = -\nabla\beta P_{\beta} + \frac{1}{4\pi} (\nabla \times B) \times B + \rho g.
\]

(4)
According to Chew, Goldberger, & Low (1956), for collisionless plasma with anisotropic pressure
\[ P_{s\theta} = P_s \delta_{s\theta} + (P_a - P_s)B_s B_\theta / B^2 , \] (5)
it is possible to introduce two extra invariants on the magnetic surfaces \( \Psi(r, \theta) = \text{const} \) (the so-called double adiabatic approximation resulting from the conservation of two adiabatic invariants):
\[ s_1(\Psi) = \frac{P_s B^2}{\rho^3} , \] (6)
\[ s_2(\Psi) = \frac{P_a}{\rho B} . \] (7)
They correspond to polytropic equations of state with \( \Gamma_s = 3 \) and \( \Gamma_a = 1 \). As a result, integrating the \( v \) and \( \phi \) components of equation (4), one can obtain the known expressions for the energy and the angular momentum (Asséo & Beaufils 1983; Tsikarishvili et al. 1995):
\[ E(\Psi) = \frac{v^2}{2} + \frac{\rho}{2} + \frac{3}{2} \frac{P_a}{\rho} + \frac{\Omega_p I}{2 \pi c \eta} (1 - \beta_a) - \frac{GM}{r} , \] (8)
\[ L(\Psi) = v_\phi r \sin \theta + \frac{I}{2 \pi c \eta} (1 - \beta_a) . \] (9)
Here
\[ \beta_a = 4\pi \frac{P_s - P_a}{B^2} \] (10)
is the anisotropic pressure parameter. Together with \( \Omega_p(\Psi) \) and \( \eta(\Psi) \) these six invariants determine all the characteristics of a flow (Lovelace et al. 1986). In particular,
\[ I = \frac{c \eta}{2 \pi} \frac{L - \Omega_p \sigma^2}{1 - M^2 - \beta_a} , \] (11)
\[ v_\phi = \frac{\Omega_p \sigma^2}{\rho} (1 - \beta_a) - \frac{ML}{1 - M^2 - \beta_a} , \] (12)
where \( M^2 = 4\pi \eta^2 / \rho \) is the Alfvénic Mach number. Thus, in the anisotropic case the Alfvénic singularity has the form
\[ A = 1 - M^2 - \beta_a = 0 . \] (13)
It is necessary to stress that equation (11) gives an implicit expression for the current \( I \) because \( I \) through \( B_s^2 \) is contained on the right-hand side of this equation as well. Nevertheless, the standard procedure of the determination of flow parameters in a given poloidal magnetic field \( B_\rho \) remains the same. The Bernoulli equation (8), which can be rewritten in the form
\[ \frac{M^4}{64\pi^3 \eta^2} (\nabla \Psi)^2 = 2\sigma^2 \left( E - \frac{P_s}{\rho} - \frac{3}{2} \frac{P_a}{\rho} + \frac{GM}{r} \right) - \frac{[\Omega_p \sigma^2 (1 - \beta_a) - LM^2]^2}{(1 - M^2 - \beta_a)^2} - 2\sigma^2 \Omega_p (1 - \beta_a) \frac{L - \Omega_p \sigma^2}{1 - M^2 - \beta_a} , \] (14)
taken together with equation (11) and with the definitions (6) and (7) determines implicitly the Mach number \( M^2 \) and \( \beta_a \) as functions of \( \Psi \) (or \( B_\rho \)) and the six invariants
\[ M^2 = M^2[(\nabla \Psi)^2, E, L, \Omega_p, \eta, s_1, s_2] , \] (15)
\[ \beta_a = \beta_a[(\nabla \Psi)^2, E, L, \Omega_p, \eta, s_1, s_2] . \] (16)
It means that all the other parameters \( (I, v_r, \text{etc.}) \) can be found from given values of \( B_\rho \) and of the six invariants as well. This fact is fully analogous to spherically symmetric flows (Bondi 1952; Parker 1958) where the flow structure is known and the algebraic expressions for the integrals of motion are enough to determine all the characteristics of a flow.
On the other hand, if the flow is not spherically symmetric, the poloidal magnetic field is to be determined from the \( \nabla \Psi \) component of equation (4). After numerous but elementary calculations it can be presented in the following compact form:
\[ \nabla_k \left[ \frac{1}{\sigma^2} (1 - M^2 - \beta_a)(\nabla \Psi)^2 \right] = \frac{64\pi^4}{\sigma^2} \frac{1}{2M^2} \frac{\partial}{\partial \Psi} \left( \frac{G}{A} \right) - 8\pi^3 P_n \frac{1}{s_1} \frac{ds_1}{d\Psi} - 16\pi^3 P_s \frac{1}{s_2} \frac{ds_2}{d\Psi} = 0 . \] (17)
Here
\[ \left( \frac{G}{A} \right) = 2\sigma^2 \eta^2 \left( E - \frac{P_s}{\rho} - \frac{3}{2} \frac{P_a}{\rho} + \frac{GM}{r} \right) + \frac{\eta^2}{2} \sigma^2 \Omega_p^2 (1 - \beta_a) - 2\sigma^2 \Omega_p L (1 - \beta_a) + \frac{M^2 L^2}{1 - M^2 - \beta_a} , \] (18)
\( V_k \) is a covariant derivative, and the operator \( \partial / \partial \Psi \) acts on the invariants \( E(\Psi), L(\Psi), \Omega_p(\Psi), \eta(\Psi) \) only. According to definitions (6) and (7) and relations (15) and (16), this transfield (stream, generalized GS) equation contains the unknown function \( \Psi(r, \theta) \) and the six invariants only. As to the GS equation itself describing statical configurations, it can be obtained from relation (17) in the limit \( \Omega_p \to 0, \eta \to 0 \) (and hence \( M^2 \to 0 \), but \( \eta L \to \text{const} \). It gives for the gravity-free case
\[ \nabla_k \left[ \frac{1}{\sigma^2} (1 - \beta_a)(\nabla \Psi)^2 \right] + \frac{16\pi^2}{\sigma^2} (1 - \beta_a) I \frac{\partial I}{\partial \Psi} + 16\pi^3 \rho \frac{d}{d\Psi} \left( \frac{P_s}{\rho} + \frac{3}{2} \frac{P_a}{\rho} \right) - 8\pi^3 P_n \frac{1}{s_1} \frac{ds_1}{d\Psi} - 16\pi^3 P_s \frac{1}{s_2} \frac{ds_2}{d\Psi} = 0 . \] (19)
Relations (8)–(12) remain true even for a more general bounded anisotropy model (Denton et al. 1994),
\[ (v \cdot \nabla) \left( \frac{P_s B^2}{\rho^3} \right) = 2\rho \frac{B^2}{\rho^3} , \] (20)
\[ (v \cdot \nabla) \left( \frac{P_s}{\rho B} \right) = -\rho \frac{B}{\rho B} , \] (21)
where \( \rho \) is a nondissipative energy exchange term. In this case we have the conservation of the “total entropy” \( S = S(\Psi) \),
\[ (v \cdot \nabla) S = \frac{1}{2} \frac{\rho^3}{B^2} (v \cdot \nabla) s_1 + \rho B (v \cdot \nabla) s_2 = 0 , \] (22)
and so the generalized GS equation (17) has a form
\[ V_k \left[ \frac{1}{\sigma^2} (1 - M^2 - \beta_d) \nabla^2 \Psi \right] + \frac{32\pi^2}{\sigma^2 M^2} \frac{\partial}{\partial \Psi} \left( \frac{C_3}{A} \right) - 16\pi^2 \frac{dS}{d\Psi} = 0 . \tag{23} \]
Introducing the effective pressure \( P_{\text{eff}} = P_{\text{eff}}(\Psi) \) as
\[ dP_{\text{eff}} = \rho d \left( \frac{P}{\rho} + \frac{3P}{2} \right) - dS , \tag{24} \]
which is equivalent to the ordinary thermodynamic relation \( dP = \rho d\omega - \rho T d\sigma \), one can obtain instead of equation (19)
\[ V_k \left[ \frac{1}{\sigma^2} \nabla^2 \Psi \right] + \frac{16\pi^2}{\sigma^2} (1 - \beta_d) \frac{\partial I}{\partial \Psi} \]
\[ + 16\pi^2 \frac{dP_{\text{eff}}}{d\Psi} = 0 . \tag{25} \]
For \( I = 0 \) it has the form found by Nötzel et al. (1985) for planar geometry.
Equation (17) is a partial equation of the mixed type. Using the implicit relations resulting in relations (15) and (16), one can find for the second-order operator in equation (17)
\[ A \left[ V_k \left( \frac{1}{\sigma^2} \nabla^2 \Psi \right) + \frac{(\nabla^2 \Psi)(\nabla^2 \Psi) V_k \Psi}{\sigma^2 (\nabla^2 \Psi)^2 D} \right] + \ldots = 0 , \tag{26} \]
where
\[ D = \frac{n}{d} , \tag{27} \]
and
\[ n = \left( 1 - M^2 - \beta_a + 4\pi \frac{4P_n - P_s B^2}{B^2} \right) \left( 1 - 3 \frac{P_n}{\rho v_p^2} \right) \tag{28} \]
\[ = \frac{B^2}{B_k^2} \left( 1 - \beta_a - 4\pi \frac{3P_n - P_s}{B^2} + 4\pi \frac{4P_n - P_s B^2}{B^2} \right) \tag{29} \]
The form (26) of equation (17) coincides with the one found for isotropic pressure (see, e.g., Sakurai 1990; Beskin 1997). Hence, one can conclude that the transfield equation is elliptical for \( D > 0 \) and \( D < -1 \) and hyperbolic for \( -1 < D < 0 \). It means that the transfield equation changes from elliptical to hyperbolic on the surfaces \( D = 0 \) (fast and slow magnetosonic singularities) and \( D = -1 \) (cusp singularity).
It is necessary to stress that, as in the isotropic case, \( D \) can be presented as \( D = d_1 A + d_2 B^2_r / B^2 \), so that \( D \propto A \) for \( B_r = 0 \). This property reflects the fact that for \( k \parallel B \) fast or slow magnetosonic velocity coincides with the Alfvénic one.
In particular, for a nonrotating flow \( (\Omega_r = 0, L = 0) \), when, according to equation (11), \( B_y = 0 \), we have
\[ n = \left( 1 - M^2 - \beta_a \right) \left( 1 - 3 \frac{P_n}{\rho v_p^2} \right) . \tag{30} \]
Using now the definitions (13) and (27), one can obtain for sonic, Alfvénic, and cusp velocities in anisotropic media
\[ a_s^2 = 3 \frac{P_n}{\rho} , \tag{31} \]
\[ v_\lambda^2 = \frac{B^2}{4\pi \rho} - \frac{P_n - P_s}{\rho} , \tag{32} \]
\[ v_\psi^2 = \left( \frac{3P_n}{4\pi \rho} + 6\frac{P_n P_s}{\rho^2} - \frac{P_s^2}{\rho^2} \right) \left( \frac{B^2}{4\pi \rho} + 2\frac{P_s}{\rho} \right) . \tag{33} \]
As we see, the velocities (31)–(33) coincide exactly with the characteristic velocities propagating in anisotropic media (Clemmow & Dougherty 1969). Recall that the condition \( v_\lambda^2 < 0 \) corresponds to the fire-hose instability and the condition \( v_\psi^2 < 0 \), to the mirror instability.

3. TWO EXAMPLES
3.1. Free Outflow
As a first example, let us consider a free (e.g., without gravity effects) outflow along a strong monopole magnetic field from the surface of a nonrotating sphere (with radius \( r_o \), magnetic field \( B_0 \), pressure \( P_0 \leq B_0^2 / 4\pi \), mass density \( \rho_0 \), and radial velocity \( v_0 \)). It is natural to assume that here \( P_r(r_o) = P_n(r_o) = P_0 \). As a result, one can find for the radial velocity
\[ \frac{v^2}{v_0^2} = 1 + \frac{5}{2} \frac{P}{\rho_0 v_0^2} \left( 1 - \frac{2}{5} \frac{r_o^2}{r^2} \right) \]
\[ + \frac{1}{2} \sqrt{\left[ 1 + 5 \frac{P}{\rho_0 v_0^2} \left( 1 - \frac{2}{5} \frac{r_o^2}{r^2} \right) \right]^2 - 12 P} , \tag{34} \]
where
\[ P = \frac{P_0}{\rho_0 v_0^2} . \tag{35} \]
It gives
\[ \frac{d(v^2/v_0^2)}{dr} = \frac{N_e}{D} \tag{36} \]
where \( N_e = 4\pi \rho_0 v_0^2 / r^2 \), and
\[ D = 1 - 3 \frac{P_n}{\rho v_p^2} , \tag{37} \]
which corresponds to the longitudinal sonic singularity in equation (27). On the other hand, the same as for isotropic media, for a nonrotating wind the magnetosonic surface coincides with the Alfvénic one, the singularity for \( 1 - M^2 - \beta_a = 0 \) being absent.
Comparing now expression (34) for \( v(r) \) with the appropriate one corresponding to an outflow with isotropic pressure for the polytropic equation of state with \( \Gamma = 3 \) and the same energy \( E \), one can conclude that the difference takes
place only for small enough pressure $P_0 \ll \rho_0 v_0^3$ when
\[
\frac{v}{v_0}^{(iso)} = 1 + \frac{1}{6} \left( \frac{2E}{v_0^2} - 1 \right),
\]
\[
\frac{v}{v_0}^{(aniso)} = 1 + \frac{1}{5} \left( \frac{2E}{v_0^2} - 1 \right).
\]
For $P_0 \gg \rho_0 v_0^2$, the limiting velocity does not depend on the pressure model: $v_{\infty}^2 \approx 2E$. Thus, the difference takes place for $r \sim r_0$ only.

It is also interesting to determine how anisotropic pressure affects the position of the Alfvénic surface $r_{\infty}$ coinciding with $\Omega_T = 0$ with the fast magnetosonic one. As
\[
\beta_s \approx \frac{v}{v_0}^{a},
\]
it is easily checked that $\beta_s/M^2 \ll 1$ for both $p \ll 1$ and $p \gg 1$, so that for $P_0 \ll \rho_0 v_0^3$ and $P_0 \gg \rho_0 v_0^3$ the position of the Alfvénic surface
\[
r_{\infty}^2 \approx r_0^2 \frac{B_0^2}{\rho} \frac{v_0}{v_{\infty}}.
\]
B does not depend on the pressure anisotropy. Finally, as
\[
\frac{P_n}{P_s} = \frac{v_0^4}{v^4},
\]
the anisotropy of the pressure tensor increases with distance from the central object. As has already been stressed, it can be realized only if dissipative processes (heat conductivity, plasma instability, etc.) play no role. Relations (40) and (42) also mean that the conditions for the fire-hose and mirror instabilities are not satisfied up to the Alfvénic surface. On the other hand, the fire-hose instability takes place in the supersonic region for $r > r_{\text{th}} \gg r_{\infty}$, where
\[
r_{\text{th}}^2 \approx r_{\infty}^2 \frac{1}{p} \left( \frac{v}{v_0} \right)^4.
\]

### 3.2. Parker Outflow
As a next example we consider an outflow from the gravity center of mass $M$ in a monopole magnetic field (Parker ejection). Analyzing the energy equation (8), we have
\[
v^2 v_0^2 = \frac{1}{2} \left( 1 + \frac{5}{p} - \frac{2}{p} \frac{r_0^2}{v_0^2} + \frac{GM}{rr_0} - \frac{GM}{r} \right)
\]
\[
+ \frac{1}{2} \sqrt{\left( 1 + 5p - 2p \frac{r_0^2}{v_0^2} + \frac{2GM}{rr_0} \right)^2 - 12p}.
\]
As a result, now $N_p = 4p r_0^2 / 2 - 2GM/v_0^2 r$, and $D$ is determined by equation (37) again. Hence, for the position of the sonic surface we have
\[
r_{s} = 2r_0 \frac{P_0 r_0}{GM}.\]

On the other hand, one can easily check that equation (44) has a real solution for $r_s \leq r_0$ only. This result is in agreement with a well-known fact that no transonic outflow is possible for a gas with a polytropic index $\Gamma = 3$. Simultaneously, for Parker ejection the conditions of the fire-hose and mirror instabilities up to the Alfvénic surface are not satisfied either.

### 4. Conclusion
Thus, we have generalized the GS equation for the flows with anisotropic pressure. It allows a quantitative analysis of the effect of anisotropic pressure on the two-dimensional outflow structure within the ideal double adiabatic approximation. As has been shown with simple examples, the pressure anisotropy itself may be very large, but not in all cases does it result in a large disturbance of the flow parameters. Clearly, the additional analysis (say, within the Weber-Davis model) is necessary to clarify all the details of the flow with anisotropic pressure.

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