Astrometric Calibration and Estimate of the Systematic Error in WXM Localizations Obtained by the Chicago Bayesian Method

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Abstract. WXM gives GRB localizations in instrument coordinates. WXM localizations must be converted to celestial coordinates using spacecraft aspect information obtained by the optical cameras on HETE. We must therefore accurately determine the alignment of the WXM boresight with respect to that of the optical cameras, in order to accurately determine the celestial coordinates of WXM burst locations. We use a seven-parameter model that treats as free parameters the three Euler angles of a pure rotation, two horizontal shifts of the coded-aperture masks with respect to the detectors, and the heights of the masks above the two detectors. We determine the alignment by fitting the model to a set of 252 WXM localizations of Sco X-1 obtained between 23 April and 28 June 2001. We estimate the systematic error in WXM GRB locations by comparing the actual and the calculated locations of Sco X-1. We find that the systematic error corresponding to a 68.3% confidence region is $1.7^\circ$, and the systematic error corresponding to a 90% confidence region is $2.4^\circ$. We find that this astrometric solution also provides a satisfactory fit to an independent sample of SGR and XRB events. These results are consistent with the astrometric calibration and the systematic error in WXM localizations derived independently using the RIKEN localization method.

INTRODUCTION

The spacecraft placement of WXM is offset from that of the optical cameras (the OPT subsystem). The coordinate frames of WXM on of the optical cameras are only aligned to within machining tolerances and thermal shrinkage effects. These misalignments are of the order of $0.4^\circ$, and must be corrected so that HETE can provide WXM locations accurate to $5^\circ$. Any systematic effects inherent in the localization procedure compound the misalignment effects, and must also be corrected.

The HETE team employs two independent WXM localization pipelines, which use different location algorithms. The existence of two independent WXM localization pipelines has been invaluable as a check of the correctness of the design and implementation of each approach. We describe here the study of astrometric correction of the Chicago (Bayesian) location algorithm [1]. The calibration of the RIKEN location algorithm is described elsewhere in these proceedings.[2]

We have performed this astrometric calibration on-orbit, using sources whose locations are accurately known — Sco X-1, several X-Ray Burst (XRB) sources, and two Soft-Gamma Repeaters (SGRs).

THE DATA

The calibration was performed using “RAW” data of Sco X-1 obtained in the course of the daily health check observations made during the period 2001 April 23 and 2001 June 28. There are 252 Sco X-1 locations determined using 27 sessions of 180s each. The statistical errors for these locations are estimated from the scatter in the locations independently for each session. The location of Sco X-1 in the WXM FOV for each of these 27 sessions is shown in the left panel of Figure (1).
We also derived the locations of 23 XRBs from known XRB sources, of three SGR1900+14 bursts, and of two SGR1806-20 bursts. We used these locations as an independent check on the quality of the fit to the Sco X-1 data. The statistical errors for these locations are produced directly by the Bayesian location process. Their locations in the WXM FOV are shown in the right panel of Figure (1).

**THE ASTROMETRIC MODEL**

The transformation from WXM to OPT coordinates is assumed to have the following form:

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\begin{align*}
\theta_X^{(\text{true})} &= S_X \tan \theta_X^{(\text{Meas})} + d_X; \\
\theta_Y^{(\text{true})} &= S_Y \tan \theta_Y^{(\text{Meas})} + d_Y; \\
\mathbf{\bar{n}}^{(\text{OPT})} &= \mathbf{M} (\epsilon_1, \epsilon_2, \epsilon_3) \cdot \mathbf{n}^{(\text{WXM})} \\
\theta_X^{(\text{true})}, \theta_Y^{(\text{true})} &\geq 0 \quad : (3)
\end{align*}
\]

The projection angles \(\theta_X^{(\text{Meas})}\) and \(\theta_Y^{(\text{Meas})}\) are the angles that result from the Bayesian location analysis. The model corrects these angles to produce “true” projection angles \(\theta_X^{(\text{true})}\) and \(\theta_Y^{(\text{true})}\). The unit vector \(\mathbf{n}^{(\text{WXM})}\) is the direction vector to the source defined by the projection angles \(\theta_X^{(\text{true})}, \theta_Y^{(\text{true})}\). The unit vector \(\mathbf{\bar{n}}^{(\text{OPT})}\) is the direction vector to the source in the OPT frame.

\(\mathbf{M}\) is a rotation matrix. \(\epsilon_1, \epsilon_2, \epsilon_3\) are Euler angles. \(S_X\) and \(S_Y\) are scale parameters that can represent unknown changes in the height of the coded aperture masks. \(d_X\) and \(d_Y\) are shifts that can represent unknown mis-alignments of the masks with respect to the detectors. The model thus has 7 free parameters.

**SCO X-1 CALIBRATION RESULTS**

With no astrometric correction, the \(\chi^2\) of the fit is 2.9 \(10^5\) for 504 DOF, and the RMS deviation (computed versus actual locations) is 0.39. The deviations from the true locations are shown in the left panel of Figure (2), together with the estimated location errors, which are typically in the \(10^6\) range. The misalignment of the WXM and OPT frames is clearly manifested in the Figure.

The best-fit astrometric correction results in a \(\chi^2 = 1517\) for 497 DOF. The fit gives an RMS deviation of 2.0'. The deviations from the true locations are shown in the right panel of Figure (2).

While clearly a distinct improvement over the null correction fit, the quality of this fit is rather poor — the \(Q\)-value for \(\chi^2 = 1517\) from the \(\chi^2\) distribution with 497 DOF is \(10^{103}\). This excess \(\chi^2\) is attributable to systematic error. The source of the error is presumably a compounding of the inadequacy of the astrometric model with the limitations of the location analysis.

We estimate the magnitude of the systematic error as follows: we add (in quadrature) a systematic error to the statistical error so as to bring the \(\chi^2/\text{DOF}\) down to about 1. In this way we find that the systematic error corresponding to a 68.3% confidence region is 1.7\(\delta\), and the systematic error corresponding to a 90% confidence region is 2.4\(\delta\). These results are consistent with those found using the RIKEN location algorithm.[2]
AN INDEPENDENT CHECK: THE SGR/XRB SAMPLE

When the astrometric model that best fits the Sco X-1 data is applied to the XRB/SGR bursts and the derived locations are compared with the known locations of the sources, the result is $\chi^2 = 39.6$ for 56 DOF ($P=0.05$). The RMS deviation is 7.0'. The Sco X-1 astrometric solution thus provides a satisfactory fit to this independent sample of event locations. No systematic error is needed in this fit, because these sources are less bright — and thus less accurately located — than Sco X-1. The typical statistical error for this sample is about 6'.

FIGURE 2. Deviations of derived locations of Sco X-1 from true source location. Top Top: Assuming no astrometric correction. Bottom: Assuming the astrometric correction that best fits the Sco X-1 data.

FIGURE 3. Deviations of derived locations of XRB/SGR sample from true source location. Top: Assuming no astrometric correction. Bottom: Assuming the astrometric correction that best fits the Sco X-1 data.

REFERENCES

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