INNER CRUSTS OF NEUTRON STARS IN STRONGLY QUANTIZING MAGNETIC FIELDS

Rana Nandi1,3, Debades Bandyopadhyay1,3, Igor N. Mishustin2, and Walter Greiner2

1 Astroparticle Physics and Cosmology Division, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
2 Frankfurt Institute for Advanced Studies (FIAS), J. W. Goethe Universität, Ruth-Moufang Strasse 1, 60438 Frankfurt am Main, Germany

Received 2011 January 10; accepted 2011 May 19; published 2011 July 19

ABSTRACT

We study the ground-state properties of inner crusts of neutron stars in the presence of strong magnetic fields of \( \sim 10^{17} \text{ G} \). Nuclei coexist with a neutron gas and reside in a uniform gas of electrons in the inner crust. This problem is investigated within the Thomas–Fermi model. We extract the properties of nuclei based on the subtraction procedure of Bonche, Levit, and Vautherin. The phase space modification of electrons due to Landau quantization in the presence of strong magnetic fields leads to the enhancement of electron as well as proton fractions at lower densities of \( \sim 0.001 \text{ fm}^{-3} \). We find the equilibrium nucleus at each average baryon density by minimizing the free energy and show that, in the presence of strong magnetic fields, it is lower than that in the field-free case. The size of the spherical cell that encloses a nucleus along with the neutron and electron gases becomes smaller in strong magnetic fields compared to the zero-field case. Nuclei with larger mass and atomic numbers are obtained in the presence of strong magnetic fields compared with cases of zero field.

Key words: magnetic fields – stars: neutron

1. INTRODUCTION

Strong surface magnetic fields of \( \sim 10^{12} \text{ G} \) are found to exist in pulsars. Even stronger surface magnetic fields \( \geq 10^{15} \text{ G} \) were predicted by observations of soft gamma-ray repeaters and anomalous X-ray pulsars (Kouveliotou et al. 1998, 1999). The latter class of neutron stars with very intense magnetic fields is known as magnetars (Thompson & Duncan 1993, 1996). On the other hand, the interior magnetic field could be much higher than the surface field. The limiting interior field might be analyzed from the scalar virial theorem (Lai & Shapiro 1991). For a typical neutron star with a mass of \( 1.5 \, M_\odot \) and a radius of 15 km, the interior field could be as high as \( \sim 10^{18} \text{ G} \).

Such strong magnetic fields quantize the motion of charged particles in a plane perpendicular to the field (Landau & Lifshitz 1977). The effects of the phase space modification due to the Landau quantization were studied in the composition and equation of state (EoS) in neutron stars extensively. Lai and Shapiro extended the Baym, Pethick, and Sutherland (BPS) model (Baym et al. 1971b) to the magnetic field case and obtained equilibrium nuclei and the EoS in the outer crust in the presence of strong magnetic fields (Lai & Shapiro 1991). The composition and EoS in the cores of neutron stars in the presence of strongly quantizing magnetic fields were investigated within a relativistic field theoretical model by Chakrabarty and collaborators (Chakrabarty et al. 1997; Bandyopadhyay et al. 1997). The transport properties such as thermal and electrical conductivities of neutron star crusts in magnetic fields were studied by several groups (Yakovlev & Kaminker 1994; Hernquist 1984). Recently, the magnetized neutron star crust was studied in the Thomas–Fermi (TF) model using Baym–Bethe–Pethick (Baym et al. 1971a) and Harrison–Wheeler EoS of nuclear matter (Nag & Chakrabarty 2010).

In the outer crust of a neutron star, neutrons and protons are bound inside nuclei and immersed in a uniform background of relativistic electron gas. As the density increases, nuclei become increasingly neutron rich. Neutrons start to drip out of nuclei at a density of \( \sim 4 \times 10^{11} \text{ g cm}^{-3} \). This is the beginning of the inner crust. The matter in the inner crust is made up of nuclei embedded in a neutron gas along with the uniform electron gas. In addition, the matter is in \( \beta \)-equilibrium and maintains charge neutrality. Nuclei are also in mechanical equilibrium with the neutron gas. The properties of nuclei in the inner crusts of neutron stars in a zero magnetic field were studied by different groups. The early studies of the inner crust matter were based on extrapolations of the semi-empirical mass formula to the free neutron gas regime (Langer et al. 1969; Bethe et al. 1970). Baym, Bethe, and Pethick considered the reduction of the nuclear surface energy due to the free neutron gas in their calculation (Baym et al. 1971a). The study of nuclei in the neutron star crust was carried out using the energy density of a many-body system by Negele and Vautherin (Negele & Vautherin 1973). With increasing density in the inner crust, unusual nuclear shapes might appear (Ravenhall et al. 1983; Oyamatsu 1993). The properties of nuclei in the inner crust were also investigated using a relativistic field theoretical model (Cheng et al. 1997).

There are two important aspects of the problem when nuclei are immersed in a neutron gas. On the one hand, we have to deal with the coexistence of two phases of nuclear matter—a denser phase inside the nucleus and a low-density phase outside it, in a thermodynamically consistent manner. On the other hand, the determination of the surface energy of the interface between two phases with good accuracy is needed. It was shown that this problem could be solved using the subtraction procedure of Bonche, Levit, and Vautherin (Bonche et al. 1984, 1985; Suraud 1987). The properties of a nucleus were isolated from the nucleus plus neutron gas using the subtraction procedure in a temperature-dependent Hartree–Fock theory (Bonche et al. 1984, 1985) as well as in zero- and finite-temperature (extended) TF calculations (Suraud 1987). The same method was extended to isolated nuclei embedded in a neutron gas (De et al. 2001) as well as nuclei in the inner crust at zero temperature (Sil et al. 2002). This shows that it would be worth studying the properties of nuclei in the inner crust in the presence of strongly quantizing fields.

3 Also at the Centre for Astroparticle Physics, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India.
magnetic fields relevant to magnetars using the subtraction procedure.

Recently, the properties of nuclei embedded in an electron gas were investigated within a relativistic mean field model in a zero magnetic field (Bürvenich et al. 2007). It was observed that nuclei became more stable against $\alpha$ decay and spontaneous fission with increasing electron number density. It is worth mentioning here that, compared with the zero-field case the electron number density is enhanced in the presence of strong magnetic fields due to Landau quantization. The question is what the impact of Landau quantization would be on the ground-state properties of matter in the inner crusts of magnetars. This is the focus of our calculation in this paper.

This paper is organized as follows. In Section 2, the formalism for the calculation of nuclei in the inner crust immersed in a neutron as well as an electron gas in the presence of strongly quantizing magnetic fields is described. The results of our calculation are discussed in Section 3. Section 4 contains the summary and conclusions.

2. FORMALISM

We investigate the properties of nuclei in the inner crust in the presence of strong magnetic fields using the TF model. In this case, nuclei are immersed in a nucleonic gas as well as a uniform background of electrons and may be arranged in a lattice. Each lattice volume is replaced by a spherical cell with a nucleus at its center in the Wigner–Seitz (WS) approximation. Each cell is taken to be charge neutral such that the number of electrons is equal to the number of protons in it. The Coulomb interaction between cells is neglected. Electrons are assumed to be uniformly distributed within a cell. The system maintains the $\beta$-equilibrium. We assume that the system is placed in a uniform magnetic field. Electrons are affected by strongly quantizing magnetic fields. Protons in the cell are affected by magnetic fields only through the charge neutrality condition. The interaction of nuclear magnetic moment with the field is negligible in a magnetic field of $\sim 10^{17}$ G (Broderick et al. 2000).

The calculation below is performed in a zero-temperature TF model. In the WS cell, a nucleus is located at the center and immersed in a low-density neutron gas while protons are trapped in the nucleus. However, the spherical cell does not define a nucleus. The nucleus is realized after the subtraction of the gas from the cell as shown by Bonche, Levit, and Vautherin (Bonche et al. 1984, 1985). In an earlier calculation, it was demonstrated that the TF formalism at finite temperature gave two solutions (Suraud 1987). One solution corresponds to the nucleus plus neutron gas, and the other represents only the neutron gas. The density profiles of the nucleus plus neutron gas, as well as that of the neutron gas, are obtained self-consistently in the TF formalism. Finally, the nucleus is obtained as the difference of the two solutions. This formalism is adopted in our calculation at zero temperature as described below.

The nucleus-plus-gas solution coincides with the gas solution at a large distance leading to the definition of the thermodynamic potential ($\Omega_N$) of the nucleus as

$$\Omega_N = \Omega_{NG} - \Omega_G,$$

where $\Omega_{NG}$ is the thermodynamic potential of the nucleus-plus-gas phase and $\Omega_G$ is that of the gas only (Bonche et al. 1984, 1985). The thermodynamic potential is defined as

$$\Omega = F - \sum_{q=n,p} \mu_q \Omega_q,$$
by a magnetic field through the charge neutrality condition. We take the magnetic field \( \mathbf{B} \) along the Z-direction and assume that it is uniform throughout the inner crust. If the field strength exceeds a critical value of \( B_c = m_e^2/e \simeq 4.414 \times 10^{13} \) G, then electrons become relativistic (Lai 2001). The energy eigenvalue of relativistic electrons in a quantizing magnetic field is given by
\[
E_e(v, p_z) = \left[ p_z^2 + m_e^2(1 + 2vB_e) \right]^{1/2},
\]
where \( p_z \) is the Z-component of momentum, \( B_e = B/B_c \), and \( v \) is the Landau quantum number. The Fermi momentum of electrons, \( p_{f,e} \), is obtained from the average electron chemical potential in a magnetic field of
\[
\mu_e = \frac{p_{f,e}(v)^2 + m_e^2(1 + 2vB_e)}{2eB} \right]^{1/2} - \langle V^e(r) \rangle,
\]
where \( \langle V^e(r) \rangle \) denotes the average single-particle Coulomb potential.

The number density of electrons in a magnetic field is
\[
n_e = \frac{eB}{2\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} g_{\nu} p_{\nu,e}(v).
\]
Here, the spin degeneracy is \( g_{\nu} = 1 \) for the lowest Landau level \( (\nu = 0) \) and \( g_{\nu} = 2 \) for all other levels.

The maximum Landau quantum number \( (\nu_{\text{max}}) \) is given by
\[
\nu_{\text{max}} = \frac{(\mu_e + \langle V^e(r) \rangle)^2 - m_e^2}{2eB}.
\]

The energy density of electrons is obtained from
\[
\varepsilon_e = \frac{eB}{2\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} g_{\nu} \int_0^{p_{\nu,e}(v)} E_e(v, p_z) dp_z.
\]

We minimize the thermodynamic potential in the TF approximation with the condition of number conservation of each species. The density profiles of neutrons and protons with or without magnetic fields are obtained from
\[
\frac{\delta \Omega_{\text{NG}}}{\delta n_{\text{NG}}} = 0,
\]
\[
\frac{\delta \Omega_{\text{G}}}{\delta n_{\text{G}}} = 0.
\]

This results in the following coupled equations (Sil et al. 2002; De et al. 2001):
\[
(3\pi^2)^{3/2} \frac{\hbar^2}{2m_q} \left( n_{\text{NG}}^q \right)^{1/2} + V_{\text{NG}}^q + V_{\text{NG}}^G(n_{\text{NG}}, n_e) = \mu_q,
\]
\[
(3\pi^2)^{3/2} \frac{\hbar^2}{2m_q} \left( n_{\text{G}}^q \right)^{1/2} + V_{\text{G}}^q + V_{\text{G}}^G(n_e) = \mu_q,
\]
where \( m_q^* \) is the effective mass of the \( q \)-th species, and \( V_{\text{NG}}^q \) and \( V_{\text{G}}^q \) are the single-particle potentials of nucleons in the nucleus-plus-gas and gas phases, respectively (Brack et al. 1985). On the other hand, \( V_{\text{NG}}^G \) and \( V_{\text{G}}^G \) are direct parts of the single-particle Coulomb potential corresponding to the nucleus-plus-gas and gas only solutions, respectively, and both are given by
\[
V^G(r) = \int \left[ \left( n_{\text{NG}}^q(r) - n_e \right) \frac{e^2}{|r - r'|} \right] dr',
\]

The average chemical potential for the \( q \)-th nucleon is
\[
\mu_q = \frac{1}{A_q} \int \left[ (3\pi^2)^{3/2} \frac{\hbar^2}{2m_q} \left( n_{\text{NG}}^q \right)^{1/2} + V_{\text{NG}}^q + V_{\text{NG}}^G(n_{\text{NG}}, n_e) \right] \rho_{\text{NG}}^G(r) dr,
\]
where \( A_q \) refers to the \( N_{\text{cell}} \) or \( Z_{\text{cell}} \) of the cell, which is defined by the average baryon density \( n_b \) and proton fraction \( Y_p \). The \( \beta \)-equilibrium condition is written as
\[
\mu_n = \mu_p + \mu_e.
\]

Density profiles of neutrons and protons in the cell are constrained as
\[
Z_{\text{cell}} = \int \rho_{\text{NG}}^G(r) dr,
\]
\[
N_{\text{cell}} = \int \rho_{\text{NG}}^G(r) dr,
\]
where \( N_{\text{cell}} \) and \( Z_{\text{cell}} \) are the neutron and proton numbers in the cell, respectively.

Finally, the number of neutrons \( N \) and protons \( Z \) in a nucleus with mass number \( A = N + Z \) are obtained using the subtraction procedure
\[
Z = \int \left[ \rho_{\text{p}}^G(r) - \rho_{\text{p}}^G(r) \right] dr,
\]
\[
N = \int \left[ \rho_{\text{n}}^G(r) - \rho_{\text{n}}^G(r) \right] dr.
\]

3. RESULTS AND DISCUSSION

We present the results of our calculation with the SkM interaction in the following paragraphs. We find the equilibrium nucleus at each density point by minimizing the free energy of the system within a WS cell maintaining a neutral charge and \( \beta \)-equilibrium. The variables of this problem are the average baryon density \( (n_b) \), the proton fraction \( (Y_p) \), and the radius of a cell \( (R_c) \). For fixed values of \( n_b \), \( Y_p \), and \( R_c \), the total number of nucleons \((A_{\text{cell}})\) is given by \( A_{\text{cell}} = V_{\text{cell}} n_b \), where the volume of a cell is \( V_{\text{cell}} = 4/3\pi R_c^3 \). The proton number in the cell is \( Z_{\text{cell}} = Y_p n_b V_{\text{cell}} \) and the neutron number is \( N_{\text{cell}} = A_{\text{cell}} - Z_{\text{cell}} \). We obtain density profiles of neutrons and protons in the cell using Equations (13) and (17) at a given average baryon density and proton fraction. Consequently, we calculate chemical potentials of neutrons and protons and free energy per nucleon. Next, we vary the proton fraction, calculate chemical potentials and density profiles, and obtain the \( \beta \)-equilibrium in the cell. Finally, we adjust the cell size \( (R_c) \) and repeat the above-mentioned steps to obtain the minimum of the free energy. These values of \( Y_p \) and \( R_c \) are then used to calculate the neutron and proton numbers in a nucleus at an average baryon density and proton fraction corresponding to the free-energy minimum with the help of Equation (18). This procedure is repeated for each average baryon density.

The minimum of the free energy originates from the interplay between different contributions. The free energy per nucleon is given by
\[
F/A = e_n + e_{\text{lat}} + e_{\text{ele}}.
\]

The nuclear energy including the Coulomb interaction among protons is denoted by \( e_n \), \( e_{\text{lat}} \) is the lattice energy, which involves the Coulomb interaction between electrons and protons, and the electron kinetic energy is \( e_{\text{ele}} \). The free energy per nucleon in
the presence of a magnetic field $B = 4.414 \times 10^{16} \text{ G}$ is shown in Figure 1 as a function of the cell size for an average baryon density of $n_b = 0.008 \text{ fm}^{-3}$. We note that the nuclear energy increases with $R_C$. On the other hand, both the lattice energy and electron kinetic energy decrease with increasing cell size. The competition of $e_N$ with the sum of $e_{\text{lat}}$ and $e_{\text{ele}}$ determines the free-energy minimum. The cell radius corresponding to the free-energy minimum is 32.1 fm for the zero-field case (not shown in the figure) and 31.9 fm for $B = 4.414 \times 10^{16} \text{ G}$. The corresponding proton fraction for $B = 4.414 \times 10^{16} \text{ G}$ is 0.03.

In Figure 2, the cell size corresponding to the free-energy minimum is plotted as a function of average baryon density for magnetic fields $B = 0$, $4.414 \times 10^{16}$, $10^{17}$, and $4.414 \times 10^{17}$ G. For magnetic fields $B < 10^{17}$, several Landau levels are populated by electrons. Consequently, we do not find any change in the cell size in the magnetic fields compared with the zero-field case. However, we find some change in the cell size for $B = 10^{17}$ G when only the zeroth Landau level is populated by electrons for $n_b \leq 0.004 \text{ fm}^{-3}$, whereas the first two levels are populated in the density range 0.005–0.015 fm$^{-3}$. However, the cell size is increased compared with the zero-field case due to the population of only the zeroth Landau level in the presence of a magnetic field $B = 4.414 \times 10^{17} \text{ G}$. The size of the cell always decreases with increasing average baryon density.

The proton fraction in the presence of magnetic fields is shown as a function of average baryon density in Figure 3. Protons in nuclei are affected by the Landau quantization of electrons through the charge neutrality condition in a cell. For magnetic fields $B < 10^{17}$ G, the proton fraction is the same as that of the zero-field case over the whole density range considered here. We find some changes in the proton fraction below $n_b = 0.015 \text{ fm}^{-3}$ when the field is $10^{17}$ G. Though electrons populate the zeroth Landau level for $n_b \leq 0.004 \text{ fm}^{-3}$, the proton fraction decreases below the corresponding proton fraction of the zero-field case. However, for a magnetic field $B = 4.414 \times 10^{17}$ G, the proton fraction is strongly enhanced compared with the zero-field case due to the population of the zeroth Landau level for $n_b \leq 0.04 \text{ fm}^{-3}$.

The density profiles of neutrons in the nucleus-plus-gas and gas phases corresponding to $n_b = 0.02 \text{ fm}^{-3}$ with and without magnetic fields are exhibited as a function of distance $(r)$ within the cell in Figure 4. The solid line denotes the zero-field case, whereas the dashed line represents the density profile with the field $B = 4.414 \times 10^{17}$ G. The horizontal lines imply the uniform
gas phases in both cases. The proton fraction is 0.040 for the magnetic-field case, whereas it is 0.022 for the zero-field case. Further, we show the subtracted density profiles of neutrons with a magnetic field $B = 4.414 \times 10^{17}$ G in Figure 5. Though neutrons are not directly affected by the magnetic field, the neutron chemical potential is modified through the $\beta$-equilibrium due to Landau quantization of electrons. Consequently, the number density of the neutrons is altered. We find that the neutron density is higher in the gas phase for the zero-field case than that with the magnetic field. This implies that fewer neutrons drip out of a nucleus in the presence of a magnetic field than without one. This can be attributed to the shift in the $\beta$-equilibrium in strong magnetic fields. We encounter a similar situation in the calculation of the outer crust in magnetic fields, that is, the neutron drip point is shifted to higher densities (Nandi & Bandyopadhyay 2011).

Now we know the density profiles of neutrons and protons in the nucleus-plus-gas phase as well as in the nucleus at each average baryon density. We immediately calculate the total number of neutrons and protons in the nucleus-plus-gas phase and in a nucleus using Equations (17) and (18). We show the total number ($A_{\text{cell}}$) in a cell for magnetic fields $B = 4.414 \times 10^{16}$, $10^{17}$, and $4.414 \times 10^{17}$ G with average baryon density in Figure 6. The dotted line denotes the zero-field case. In all cases, the $A_{\text{cell}}$ growing with the density reaches a maximum and then decreases. Such a trend was observed in the calculation of Negele & Vautherin (1973) in the absence of a magnetic field. We note that our predictions for $B = 4.414 \times 10^{16}$ G do not change from the field-free results because a large number of Landau levels are populated in that magnetic field. For a magnetic field $B = 10^{17}$ G, the total number of nucleons decreases compared with the corresponding results of the field-free case in the density regime 0.005–0.02 fm$^{-3}$. This can be understood from the behavior of the cell size around that density regime in Figure 2. For $B = 4.414 \times 10^{17}$ G, the zeroth Landau level is populated by electrons for densities $\lesssim 0.04$ fm$^{-3}$. This modifies the $\beta$-equilibrium and the charge neutrality conditions, which, in turn, impact the size of the cell and the total number of nucleons in a cell. This effect is pronounced in the case of $B = 4.414 \times 10^{17}$ G. In this case, $A_{\text{cell}}$ is significantly reduced compared with the zero-field case for densities $\lesssim 0.04$ fm$^{-3}$.

We obtain neutron ($N$), proton ($Z$), and total nucleon numbers ($A$) in the nucleus at each average baryon density following the subtraction procedure. Total nucleon and proton numbers are shown in Figure 7 for the above-mentioned magnetic fields. When the magnetic field is $\leq 10^{17}$ G, it is noted that our results start oscillating from the field-free results. This can be attributed to the fact that the population of Landau levels changes from zero to a few levels in the above-mentioned fields as baryon density varies from lower to higher values. In contrast to Figure 6, we find that total nucleon and proton numbers inside the nucleus at each density point beyond 0.002 up to 0.04 fm$^{-3}$ are significantly enhanced in the case of $B = 4.414 \times 10^{17}$ G compared with the field-free case as well as other magnetic fields considered here. This clearly demonstrates that more neutrons are inside the nucleus in the presence of strong magnetic fields $\geq 10^{17}$ G than in the gas phase in that density regime. This is the opposite of the situation in the zero magnetic field. This can be easily understood from the density profiles with and without magnetic fields in Figures 4 and 5.

We repeat the above-mentioned calculation for the SLy4 nucleon–nucleon interaction (Chabanat et al. 1998). Figure 8 shows the variation of nucleon–nucleon interaction on mass.
and proton numbers of nuclei as a function of average baryon density with $B = 0$ and $4.414 \times 10^{17}$ G. Comparing results of the SkM and SLy4 interactions, we find that $A_N$ and $Z_N$ in the latter case are higher beyond a baryon density of 0.005 fm$^{-3}$. We find that the symmetry energy of the SLy4 interaction is larger in the subsaturation density regime than that of the SkM interaction. Higher symmetry energy results in a higher proton fraction, which, in turn, enhances the electron density via the charge neutrality condition. This is demonstrated in Figure 9, where electron density is plotted as a function of average baryon density for the SkM and SLy4 interaction and magnetic fields $B = 0$ and $4.414 \times 10^{17}$ G.

We do not consider the nuclear shell effects in our calculation. Here we adopt the TF formalism which reproduces average nuclear properties very well. A detailed comparison of properties of nuclei in Hartree–Fock as well as (extended) TF calculations using the subtraction procedure and the same nucleon–nucleon interaction was performed by Suraud (1987). It was noted that the results of semi-classical calculations, in particular the TF calculation (Suraud 1987), were in very good agreement with those of the Hartree–Fock calculation (Bonche et al. 1984, 1985). On the other hand, the washout of neutron shells may happen due to the presence of external neutrons in which nuclei are immersed (Lattimer & Schramm 1977; Meyer 1989). In this case, the collisional broadening of a nuclear level due to scattering of external neutrons could make it wider than the gap between levels (Meyer 1989).

We plot the free energy per nucleon of the system in Figure 10. This calculation is performed using the SkM interaction. Our results for $B = 4.414 \times 10^{16}$ G do not change much from the field-free results. However, for $B = 10^{17}$ G, the free energy per nucleon is reduced at lower densities (<0.004 fm$^{-3}$) compared with the field-free case. We find a more pronounced reduction in the free energy per nucleon in the field $B = 4.414 \times 10^{17}$ G over almost the whole density regime considered here.

**4. SUMMARY AND CONCLUSIONS**

We have investigated ground-state properties of the inner crust in the presence of strong magnetic fields of $\sim 10^{16}$ G or more. Nuclei are immersed in a neutron gas and a uniform background of electrons. We have adopted the SkM and SLy4 interactions for the nuclear energy density and studied this problem in the TF model. Electrons are affected through Landau quantization in strong magnetic fields because much lower Landau levels can be occupied in these cases. Consequently, electron number density and energy density are modified in a strongly quantizing magnetic field and the $\beta$-equilibrium condition is altered compared with the field-free case. The enhancement of electron number density in magnetic fields $\geq 10^{17}$ G due to the population of the zeroth Landau level leads to enhancement in proton fraction through the charge neutrality condition. We minimize the free energy of the system within a WS cell to obtain the nucleus at each average baryon density. In this connection,
we used the subtraction procedure to obtain the density profiles of a nucleus from the nucleus-plus-gas and the gas solutions at each average baryon density point. We note that fewer neutrons drip out of a nucleus in the presence of strong fields than without a magnetic field. This results in larger mass and proton numbers in a nucleus in the presence of a magnetic field compared to the corresponding nucleus in the field-free case. Furthermore, the free energy per nucleon of the system is reduced in magnetic fields. It is found that the variation of nucleon–nucleon interaction influences mass and atomic numbers of nuclei in zero as well as strong magnetic fields.

This calculation might have observational consequences for magnetars in several ways. We discuss two such potential applications. Giant X-ray flares in magnetars might happen due to fracture in the strained solid crust under twisted magnetic field lines (Watts & Strohmayer 2007; Steiner & Watts 2009; Corsi & Owen 2011). Starquakes associated with giant flares are so energetic that they could excite quasi-normal modes of magnetars. Quasi-periodic oscillations (QPOs) detected in the X-ray tail of giant flares in SGR 1806+14 were identified as shear modes of magnetar crusts (Watts & Strohmayer 2007; Steiner & Watts 2009). Shear mode frequencies strongly depend on the shear modulus of the crust, which again is sensitive to the composition of the crust (Steiner 2008; Steiner & Watts 2009). Earlier theoretical calculations of shear mode frequencies did not include the effects of the magnetic field on the composition and EoS in magnetar crusts. We will utilize our calculation of inner crust in strong magnetic fields to explain shear mode frequencies in QPOs in a future publication.

The other problem relates to the crust matter ejected due to tremendous magnetic stress on the crusts of magnetars (Gelfand et al. 2005). The ejected matter of the inner crust might expand to much lower densities. The decompressed crust matter has long been considered an important site for $r$-process nuclei (Lattimer & Schramm 1977; Arnould et al. 2007). It would be worth studying the $r$-process in the decompressed crust matter of magnetars using the results of our calculation as input.

R.N. and D.B. thank the Alexander von Humboldt Foundation for support under the Research Group Linkage programme.

REFERENCES

Arnaud, M., Goriely, S., & Takahashi, K. 2007, Phys. Rep., 450, 97
Baym, G., Bethe, H. A., & Pethick, C. J. 1971a, Nucl. Phys. A, 175, 225
Baym, G., Pethick, C. J., & Sutherland, P. 1971b, ApJ, 170, 299
Bandyopadhyay, D., Chakrabarty, S., & Pal, S. 1997, Phys. Rev. Lett., 79, 2176
Bonche, P., Levit, S., & Vautherin, D. 1984, Nucl. Phys. A, 427, 278
Bonche, P., Levit, S., & Vautherin, D. 1985, Nucl. Phys. A, 436, 265
Brack, M., Guet, C., & Håkansson, H. B. 1985, Phys. Rep., 123, 275
Broderick, A., Prakash, M., & Lattimer, J. M. 2000, ApJ, 537, 351
Bürvenich, T., Mishustin, I. N., & Greiner, W. 2007, Phys. Rev. C, 76, 034310
Chabonan, E., et al. 1998, Nucl. Phys. A, 635, 231
Chakrabarty, S., Bandyopadhyay, D., & Pal, S. 1997, Phys. Rev. Lett., 78, 2898
Cheng, K. S., Yao, C. C., & Dai, Z. G. 1997, Phys. Rev. C, 53, 2092
Corsi, A., & Owen, B. J. 2011, Phys. Rev. D, 83, 104014
De, J., Vinas, X., Patra, S. K., & Centelles, M. 2001, Phys. Rev. C, 64, 057306
Gelfand, J. D., et al. 2005, ApJ, 634, L89
Herquish, L. 1984, ApJS, 56, 325
Kouvelous, T., et al. 1998, Nature, 393, 235
Kouvelos, T., et al. 1999, ApJ, 510, L115
Krivine, H., Treiner, J., & Bohigas, O. 1980, Nucl. Phys. A, 336, 115
Lai, D. 2001, Rev. Mod. Phys., 73, 629
Lai, D., & Shapiro, S. L. 1991, ApJ, 383, 745
Landau, L. D., & Lifshitz, E. M. 1977, Quantum Mechanics (Oxford: Pergamon)
Lai, D. 2001, Rev. Mod. Phys., 73, 629
Lai, D., & Shapiro, S. L. 1991, ApJ, 383, 745
Landau, L. D., & Lifshitz, E. M. 1977, Quantum Mechanics (Oxford: Pergamon)
Langer, W. D., Rosen, L. C., Cohen, J. M., & Cameron, A. G. W. 1969, Ap&SS, 5, 259
Lattimer, J. M., & Schramm, D. N. 1977, ApJ, 213, 225
Meyer, B. S. 1989, ApJ, 343, 254
Nag, N., & Chakrabarty, S. 2010, Eur. Phys. J. A, 45, 99
Nandi, R., & Bandyopadhyay, D. 2011, J. Phys. Conf. Ser., in press (arXiv:1012.5973)
Negele, J. W., & Vautherin, D. 1973, Nucl. Phys. A, 207, 298
Oyamatsu, K. 1993, Nucl. Phys. A, 561, 431
Oyamatsu, K., et al. 1993, Nucl. Phys. A, 561, 431
Ravenhall, D. G., Pethick, C. J., & Wilson, J. R. 1983, Phys. Rev. Lett., 50, 2066
Sil, T., De, J. N., Samaddar, S. K., Vinas, X., Centelles, M., Agrawal, B. K., & Patra, S. K. 2002, Phys. Rev. C, 66, 045803
Steiner, A. W. 2008, Phys. Rev. C, 77, 015805
Steiner, A. W., & Watts, A. L. 2009, Phys. Rev. Lett., 103, 181101
Stone, J. R., Miller, J. C., Koncewicz, R., Stevenson, P. D., & Strayer, M. R. 2003, Phys. Rev. C, 68, 034324
Surada, E. 1987, Nucl. Phys. A, 462, 109
Thompson, C., & Duncan, R. C. 1993, ApJ, 408, 194
Thompson, C., & Duncan, R. C. 1996, ApJ, 473, 322
Watts, A. L., & Strohmayer, T. E. 2007, Adv. Space Res., 40, 1446
Yakovlev, D. G., & Kaminker, A. D. 1994, in The Equation of State in Astrophysics, ed. G. Chabrier & E. Schatzman (Cambridge: Cambridge Univ. Press), 214