Performance Analysis of Demand-Oblivious and Demand-Aware Optical Datacenter Network Designs

Chen Griner¹  Johannes Zerwas²  Andreas Blenk²
Manya Ghobadi³  Stefan Schmid⁴  Chen Avin¹
¹ BGU, Israel   ² TU Munich, Germany   ³ MIT, USA   ⁴ University of Vienna, Austria

Abstract
This paper presents a performance analysis of the design space of optical datacenter networks, including both demand-oblivious (static or dynamic) and demand-aware networks. We formally show that the number of specific optical switch types which should be used in an optimized datacenter network, depends on the traffic pattern, and in particular, the flow size distribution.

1 Introduction
This paper is motivated by the observation that different traffic patterns (and in particular: different flow size distributions), require different datacenter network designs (and in particular: different distributions of optical switch types). We argue that a mismatch between some common traffic work, depends on the traffic pattern, and in particular, the reconfiguration time may be significantly higher than the configuration tax. A latency tax can be amortized by the long transmission times of large flows.

While static topologies do not introduce any latency tax, they require multi-hop forwarding. This is problematic especially for large flows: the more hops a flow has to traverse, the more network capacity is consumed. This can be seen as a "bandwidth tax" (or an "infrastructure tax"), as noticed in prior work [15]. Regarding the design choice between static vs dynamic topology we therefore observe: Whereas dynamic topologies introduce a latency tax, static topologies introduce a bandwidth tax.

Dynamic topologies can reduce the bandwidth tax by avoiding multi-hop forwarding. For instance, rotor switches provide periodic direct connectivity, and have been shown to perform particularly well for uniform all-to-all traffic patterns [14, 15]. However, such demand-oblivious dynamic architectures are not optimal for the elephant flows; as typically a majority of the bytes are carried by elephant flows, optimizing for these large flows is important. While in principle, Valiant routing [20] can be used in combination with rotor switches to carry large flows, this again results in bandwidth tax. In contrast, in a demand-aware topology, directed shortcuts can be set up specifically for such elephant flows.

Regarding the design choice between demand-oblivious vs demand-aware topology, we observe that whereas demand-oblivious topologies perform well under uniform demands, demand-aware topologies perform well under skewed traffic and large flows for which latency taxes can be amortized.

This paper provides a mathematical model that can decide on the optimal flow assignments to each topology type based on the flow size distributions.

2 Topology and Switch Models
Motivated by our observations above, we consider a unified datacenter architecture which combines all three switch types: dynamic oblivious (rotor), dynamic demand-aware and static. We first present our datacenter network model and then model the different possible topology components. These models will allow us to compute the optimal size and composition of different topology types, to improve performance, depending on link rates and reconfiguration times as well as the given traffic mix.
2.1 Topology Models

Our datacenter network model relies on a two-layer leaf-spine network architecture with \(n\) leaf switches that are ToR switches and \(k\) optical spine switches that can be of different types: Static, Rotor and Demand-aware; we will denote the number of switches from each type by \(k_s, k_r\) and \(k_c\) respectively where \(k = k_s + k_r + k_c\). We will discuss how to compute the value for each type later. Since each optical spine switch connects its in-out ports via a matching, we will refer to it as the ToR-Matching-ToR (TMT) network model. This network architecture generalizes existing architectures such as RotorNet [15], Opera [14] and Sirius [3], by supporting multiple switch types.

More specifically, the TMT network interconnects a set \(N\) of \(n\) ToRs, \(\{1, 2, \ldots, n\}\) and its two-layer leaf-spine architecture composed of leaf switches and spine switches. The \(n\) ToR packet switches are connected using \(k\) spine switches, \(SW = \{sw_1, sw_2, \ldots, sw_k\}\) and each switch internally connects its in-out ports via a matching. We assume that each ToR \(i\): \(1 \leq i \leq n\) has \(k\) uplinks, where uplink \(j\): \(1 \leq j \leq k\) connects to port \(i\) in \(sw_j\). The directed outgoing (leaf) uplink is connected to the incoming port of the (spine) switch, and the directed incoming (leaf) uplink is connected to the outgoing port of the (spine) switch. Each spine switch has \(n\) input ports and \(n\) output ports and the connections are directed, from input to output ports. As a running example, throughout this paper we assume that the rate of each link is \(r = 40\text{Gbps}\); however our model is general and can be used for any bandwidth as we discuss later: similar results to what we report here hold also for lower rates (e.g., \(10\text{Gbps}\) as in [12, 14, 15]) and higher rates, e.g. \(100\text{Gbps}\).

At any point in time, each switch \(sw \in SW\) provides a matching between its input and output ports. Depending on the switch type, this matching may be reconfigured at runtime to another matching. Each switch \(j\) has a set of matchings \(M_j\) of size \(m_j = |M_j|\) and \(m_j\) may be larger than one. Changing from a matching \(M' \in M\) to a matching \(M'' \in M\) takes time, which we model with a parameter \(R_j\): the reconfiguration time of switch \(j\). During reconfiguration, the links in \(M' \cap M''\), i.e., the links which are not being reconfigured, can still be used for forwarding; the remaining links are blocked during the reconfiguration [11]. Depending on the technology, different switches in \(SW\) support different sets of matchings and reconfiguration times.

2.2 Switch Models

Our network model can be instantiated with different switches, accounting for the different and specific switch characteristics, thereby creating different typology types or components. In this paper, we consider three fundamental topology components: a static part, a rotor-only part, and a demand-aware part. These components may either be realized by different switch technologies, or by a single switch technology that can support multiple modes of operation. The former may be more cost effective (e.g., static topologies are cheaper), while the latter is more flexible.

Abstractly, the three topologies can be described in a unified form using a collection of spine switches. Each spine switch type, in turn is defined by a 4-tuple, \(sw = (m, M, S, R)\) where \(m\) is the number of matchings the switch can support; \(M\) is the specific set or sequence of \(m\) matchings the switch can realize; \(S\) is the minimal circuit-hold time a switch needs to remain in a specific matching before switching to the next matching; \(R\) is the reconfiguration time of changing between matchings. Using our notation, the three topologies can be formalized as follows:

**Demand-aware topology (Demand-aware):** We create a demand-aware topology using a collection of \(k_c\) Demand-aware reconfigurable switches. Each such switch is described by the tuple \(sw = (n!; M, S, R_c)\). Demand-aware switches have the freedom to flexibly reconfigure to *any* of the \(m = n!\) possible matchings; i.e., \(M = S_w\) where \(S_w\) is the symmetric group of \([1 \ldots n]\). The demand-aware switch can be implemented using off-the-shelf 3D MEMS technology with reconfiguration time in the order of tens of ms. In this paper, we assume \(R_c = 15\text{ms}\) which is the typical reconfiguration time of a 3D MEMS switch [1]. The circuit-hold time \(S\) can change during the operation of the Demand-aware switch, but as a rule of thumb \(S \gg R_c\) for the reconfiguration to be worthwhile.

**Rotor-based topology (Rotor):** A rotor-based topology consists of the union of \(k_r\) Rotor switches. Each switch is described by the tuple \(sw = (n-1, M, R)\): a Rotor switch cycles through \(n - 1\) matchings specified by \(M\), emulating a fully-connected network (i.e., complete graph) and hence providing high bandwidth to all-to-all traffic. Our Rotor switch is a slight generalization of the original Rotor switches [15] used in RotorNet since our model uses \(n - 1\) matchings and not \(n/k\) matchings as proposed originally. The reconfiguration time for the Rotor switch is in the order of few \(\mu s\); here we assume \(R_c = 10\text{\mu s}\) as in [14, 15]. The circuit-hold time of a Rotor switch is called the slot time and is denoted as \(\delta\). The slot time is tunable and depends on the reconfiguration time, where a reasonable setup is at least \(\delta = 9R_c = 90\mu s\) to reach 90% amortization of the reconfiguration time [14, 15].

**Static topology (Static):** We describe the static topology as a union of \(k_s\) matchings, where each matching can be implemented, e.g., using an optical patch panel (our analysis also applies to electrical static topologies). In the case of a static component, the 4-tuple switch specification can be represented by: \(sw = (1, M, \infty, 0)\) where the component provides a single (i.e., \(m = 1\)) predefined matching \(\{M\} = M\) that does not change over time (\(S = \infty, R = 0\)). The static switches are cost-effective components to create regular graphs, such as expander graphs, providing low latency for short flows using multi-hop routing. Good expanders can be obtained by taking the union of a few matchings [9].
While the Static and Rotor-based topologies are based on demand-oblivious switches, a Demand-aware topology is adapting to the demand. Demand-aware switches are hence likely more complex for their support of greater matching flexibility, and their reconfiguration times as mentioned above are naturally higher, i.e., \( R_0 \ll R_c \). Moreover, a demand-aware topology requires a control logic to decide on which on-demand links to establish based on the current traffic demand [6, 13, 22].

Since the TMT network can be configured with multiple different switch types and hence contains multiple types of topology components, we introduce the following terminology. Let \((k_s, k_r, k_c)\) denote a network consisting of \(k_s\) static switches, \(k_r\) rotor switches, and \(k_c\) demand-aware switches. We will refer to a network consisting only of \(k\) Static switches, i.e., a network with \((k, 0, 0)\), as static-net; as we will see, we will assume that the static topology component of relies on expander graphs, and we will hence refer to this network as expander-net. We will further refer to the network consisting of only Rotor switches, i.e., networks with \((0, k, 0)\), as rotor-net, and to topology components consisting only of Demand-aware switches, i.e., a network with \((0, 0, k)\), as demand-aware-net.

We also note that the TMT network can be used to model many existing systems. For example, RotorNet [15], Opera [14] and Sirius [4] rely on periodic matchings and can be modelled as a rotor-net. Networks like ProjecToR [8], Eclipse [21], Helios [6], ReNet [2], BMA [5], among others [7], rely on demand-aware matchings. To be more specific, while for example the demand-aware links of ProjecToR are based on free space optics, conceptually it can still be modelled as a demand-aware-net; ProjecToR additionally uses a static electric network, which in our conceptual model can also be described using an expander-net. Our model and analysis also applies to Xpander [12], which can be modelled as an expander-net as well (even though it is based on electrical switches). We refer the reader to the related work section for additional details.

### 3 Performance Analysis

This section presents an analysis of (i) the optimal topology parameters for each type of topology; (ii) the flow assignment strategy for each topology; and (iii) the throughput and flow completion times for the system as a whole. To do this we first describe the traffic generation model we consider.

#### 3.1 Flow Assignment Model

Throughout this paper, we consider the following flow assignment algorithm: given a \((k_s, k_r, k_c)\) network, we divide flows into three categories, based on three sizes: small \((s)\), medium \((m)\) and large \((l)\) flows. The size thresholds to assign flows to these categories are denoted by \(|\sigma|\) and \(|\ell|\). Namely, small flows are of size less than \(|m|\); medium flows are of size more than \(|m|\) and less than \(|\ell|\), and large flows are of size larger than \(|\ell|\).

We first determine the size threshold for medium flows, \(|m|\), as the slot time in Rotor switches, i.e., \(|m| = \delta r\). This is done to insure low delay for small flows and good link utilization for medium flows; we then set the large flow size threshold, \(|\ell|\), as the minimum flow size that will have shorter completion time on Demand-aware switches, see Eq. (3) in the next section.

Algorithm 1 describes how we distribute the traffic classes among the three switch types: small, latency-sensitive flows are forwarded via a static expander built from Static switches; large flows are transmitted via the \(k_c\) many Demand-aware switches in the system; and the remaining (medium) flows describing e.g., all-to-all traffic which is not latency-sensitive, are routed via the \(k_r\) Rotor switches. We can manage the large flows using an approach which can be seen as a distributed link cache [5]: when a new demand-aware connection needs to be established, an existing link must be replaced or “evicted”. While this introduces interesting optimization opportunities, in the following, we will focus on a simple strategy: when a large flow should be sent to the Demand-aware switches, but there are no available ports to serve it (the related source/destination ports are already serving other flows), we greedily send the large flow to be served by Rotor switches. When this happens continuously, we say that the demand-aware switches are under provisioned. In the next section, we derive \(k^*_c\), the optimal number of Demand-aware switches (under a given traffic assumption) which minimizes forwarding of large flows over Rotor switches.

#### 3.2 Traffic Generation Model and Metrics

Inspired by prior work [4, 12, 14], we consider the following traffic model: flows arrive over time, according to a sequence \(\sigma = (f_1, f_2, \ldots)\) where the \(f_i\)s are individual flows. Each flow \(f_i = (s_i, d_i, \Delta_i)\) has a source rack \(s_i \in N\); a destination rack

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**Algorithm 1 Flow assignment**

1. **Switch** depending on flow size
2. **Case** small flow: \(\xrightarrow{\text{latency-sensitive flow}}\) using multi-hop
3. send to static expander
4. **Case** large flow:
5. If a direct link is available to reconfigure:
6. send to demand-aware topology \(\xrightarrow{\text{single hop}}\)
7. Else \(\xrightarrow{\text{Under provisioned demand-aware}}\)
8. send to rotor-based topology \(\xrightarrow{\text{using 1 or 2 hops}}\)
9. **Case** medium flow:
10. send to rotor-based topology \(\xrightarrow{\text{using 1 or 2 hops}}\)
For our analysis we study two fundamental models to generate a load 0 ≤ 𝑥 ≤ 1:

**Uniform Traffic, 𝑈(𝑥):** In this model, each ToR serves traffic at an average rate of 𝑥 · 𝑘 · 𝑟, where 𝑟 is the line rate and 𝑘 the number of uplinks (and spine switches). Flow arrival times follow a Poisson process, flow sizes are sampled from a distribution 𝑃, and for each source ToR the destination ToR is chosen uniformly at random from 𝑁. The expected total amount of traffic that the system generates load and network capacity, and not only to

**Skewed Traffic, 𝑆(𝑥):** Inspired by recent datacenter studies [8, 18, 19], in this model only a random fraction of 𝑥 ToRs are active, but each active ToR operates at 100% throughput, i.e., the 𝑘 ports are fully utilized, at line rate, 𝑟. More generally 𝑆𝑖(𝑥) means that each active ToR 𝑖 operates at throughput 𝑀𝑖 · 100% and 𝑆(𝑥) = 𝑆𝑖(𝑥). Again, flow arrivals follow a Poisson process, flow sizes are sampled from 𝑃, and destinations are chosen uniformly at random from the set of active ToRs.

The expected total amount of traffic that the system generates and needs to serve per second is 𝑥 · 𝑛 · 𝑈(1) = 𝑥𝑛𝑘𝑟, i.e., each active ToR serves traffic of 𝑈(1)

We study analytically the demand completion time (DCT) as the metric of interest: the total time it takes to serve an accumulated demand matrix built from a collection of flows that arrived in one second, either according to the 𝑈(𝑥) or 𝑆(𝑥) traffic generation models. The demand completion time is a measure for the capacity of the network: if for a given 𝑥 the completion time is less than 1, this means that the network has sufficient capacity to serve this load. In the uniform traffic model, for a given load (throughput) 𝑘, we are interested in the demand completion time of DCT(U(𝑥))

In the skewed traffic model, for a given fraction of active ToRs 𝑥, we are interested in the demand completion time of DCT(U(𝑥))

In this section, we present our theoretical results on the demand completion times of expander-net, rotor-net and our combined model, henceforth called Hybrid, in both the 𝑈(𝑥) and the 𝑆(𝑥) traffic models.

We start by considering the uniform traffic model 𝑈(𝑥). We show that for all considered systems, expander-net, rotor-net and Hybrid, the demand completion time grows (almost) linearly with load; moreover, for a wide range of parameters (as we will show later), Hybrid has consistently lower demand completion times and therefore higher effective capacity of the system.

For a demand distribution 𝑆 and using 𝑘 switches, let DCT(sys, 𝑆, 𝑘) denote the demand completion time of a system sys ∈ {expander-net, rotor-net, Hybrid}. Recall that expander-net = (𝑘, 0, 0), rotor-net = (0, 𝑘, 0) and Hybrid = (𝑘, 𝑘, 𝑘, 𝑘). Let 𝑡 denote the type of a flow where 𝑡 ∈ {s, m, ℓ} and let U(𝑥, 𝑡) denote the expected number of bytes per second in flows of type 𝑡 when a ToR is working at a fraction 𝑥 of its 100% rate, that is, when all 𝑘 uplinks are sending traffic, each at rate 𝑥 𝑟 𝑔bps, toward the spine switches. Note that 𝑈(1, 𝑠) + 𝑈(1, 𝑚) + 𝑈(1, ℓ) = 𝑘𝑟. Let 𝜙 denote the traffic skewness of the flow size distribution. Formally, 𝜙 denotes the fraction of packets (or bytes) sent in rotor-net via a single hop, and 1 − 𝜙 is the fraction of packets that are sent using Valiant routing [15], using two hops. We approximate 𝜙 for a given distribution and the load (or empirical distribution) as one minus the variation distance [17] from the uniform distribution. This means that links (or source-destination pairs) that are active above the average load can send packets via links that are below the average load (using two hops).

We can state the following formal results about the demand completion time of the different systems:

**Theorem 1 (Uniform Traffic).** Consider the uniform traffic model, 𝑈(𝑥) with a flow size distribution which leads to traffic skewness 𝜙. The expected demand completion times of the systems Hybrid, expander-net and rotor-net are as follows.

**For Hybrid (upper bound):**

\[
\text{DCT(Hybrid, 𝑈(𝑥), 𝑘)} \leq 𝑥 \cdot 𝛼
\]

where \( 𝛼 = \frac{𝑈(𝑥, ℓ)}{𝑈(𝑥, ℓ)} \left( \frac{𝑅_{r}}{|𝑟|} + \frac{1}{2} \right) \). The expected size (of the reciprocal flow sizes) is taken only over large flows. \( k^*_{c} \) is the optimal number of DEMAND-AWARE switches, computed from the ratio between the optimal number of DEMAND-AWARE switches to ROTOR switches denoted by \( k^*_{c} \).

\[
k^*_{c} = \frac{𝑈(𝑥, ℓ)}{𝑈(𝑥, ℓ)} \cdot \frac{𝑅_{r} \cdot |𝑟|}{(2 − 𝜙_{m})(𝑅_{r} + 𝛿)}
\]

where \( 𝜙_{m} \) is the traffic skewness of the medium size flows.

The threshold for large flows, \(|𝑙|\) can be computed by:

\[
|𝑙| \geq \frac{𝑅_{r} \cdot |𝑟| \cdot 𝑟}{(2 − 𝜙) \cdot 𝑟 \cdot (𝑅_{r} + 𝛿) − |m|}
\]

**For rotor-net (lower bound):**

\[
\text{DCT(Rotor-net, 𝑈(𝑥), 𝑘)} \geq 𝑥 \cdot 𝛽
\]

where \( 𝛽 = \frac{(2 − 𝜙) 𝑅_{r} \cdot 𝛿}{α} \).

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1 Flows are originally generated by servers, but our focus here is on the flows from a rack-level perspective.
For expander-net (lower bound):
\[
\text{DCT(expander-net, } U(x), k) \geq x \cdot \gamma \tag{5}
\]
where \( \gamma = \text{epl}(G(k)) \) is the expected path length of the \( k \)-regular expander with \( n \) nodes.

As we show in the next section, Theorem 1 enables us to numerically estimate the demand completion times of the different systems given the input parameters (i.e., reconfiguration times, rates, etc.). Following Theorem 1, a first main observation about Hybrid is that the optimal number of switches required of types Rotor and Demand-aware in Hybrid, is proportional to the share of the different types of traffic, i.e., of medium and large flows (Theorem 1, Eq. (2)).

Regarding the skewed traffic, a similar result, but with slightly more complex analysis, can be obtained.

**Theorem 2 (Skewed Traffic).** Consider the skewed traffic model, \( S(x) \) with a flow size distribution which leads to traffic skewness \( \phi \). The expected throughput \( L \) of the active ToRs for the systems Hybrid, expander-net and rotor-net can be computed by solving the following equations for \( L \):

**For Hybrid:**
\[
\text{DCT(Hybrid, } S_L(x), k) = 1 \implies xL \frac{U(1,m) + xU(1,\ell)}{|m|} \frac{R_x + \delta}{k_x} = 1 \tag{6}
\]

where \( x^* \) is a function of \( L \), denoting the expected fraction of large flows that are routed via Rotor switches.

**For expander-net:**
\[
\text{DCT(expander-net, } S_L(x), k) = 1 \implies L = \min \left( \frac{1}{x(2 - \phi x) \cdot \frac{R_x + \delta}{\delta}}, 1 \right) \tag{7}
\]

**For rotor-net:**
\[
\text{DCT(rotor-net, } S_L(x), k) = 1 \implies L = \min \left( \frac{1}{x \cdot \text{epl}(G(k))}, 1 \right) \tag{8}
\]

where \( \text{epl}(G(k)) \) is the expected path length of a random \( k \)-regular expander with \( n \) nodes.

As we show in the next section, Theorem 2 enables us to numerically estimate the throughput of the different systems given the input parameters (i.e., \( x \), reconfiguration times, rates, etc.). Following Theorem 2, we can state: All three systems: expander-net, rotor-net and Hybrid are throughput-proportional.

### 4 Detailed Flow-Level Analysis

In this section we present more in-depth analytical results for the performance of Hybrid. We analyze the demand (aka matrix) completion time: the total time it takes to serve a demand matrix (built from flows) that arrives at time zero. We will consider both the Uniform and Skewed traffic models.

#### 4.1 Analysis of Uniform Traffic

Recall that \( \tau \) denotes the type of a flow where \( \tau \in \{s, m, \ell\} \) is the flow type and let \( |m| \) and \( \ell \) denote the thresholds sizes to decide the flows type. Namely, small flows are of size less than \(|m|\), medium flows are of size more than \(|m|\) and less than \(\ell\) and, large flows are of size larger than \(\ell\). For traffic generated by \( U(x) \), let \( U(x, \tau) \) denote the expected number of bytes per second in flows of type \( \tau \) when a ToR is working at \( x\% \), namely each of its \( k \) links is sending traffic at rate \( r = 10xGb/s \). We set the size of the medium threshold to be exactly \(|m| = \delta r = 1Mb\) the flow size that can be transmitted in one slot.

Consider \( k \) switches all of them of the same type \( \omega \in \{S, R, C\} \). We denote by \( \text{DCT}_{\omega}(D, k) \) the demand completion time to serve the demand \( D \) using these \( k \) switches. Further, let \( \text{DCT}(\text{sys}, D, k) \) denote the demand completion time of system \( \text{sys} \in \{\text{expander-net, rotor-net, Hybrid}\} \) for demand \( D \) using \( k \) switches. Note that for expander-net and rotor-net we have \( \text{DCT}(\text{expander-net, } D, k) = \text{DCT}_S(D, k) \) and \( \text{DCT}(\text{rotor-net, } D, k) = \text{DCT}_R(D, k) \) respectively, since these systems are built from a single switch type. But this is not the case for Hybrid which is built form a combination of all three switches types. We present the analysis from the easy to the hard case, first expander-net, then rotor-net and finally Hybrid.

#### 4.1.1 Analysis of Expander-Net

We start by (optimistically) approximating the demand completion time of expander-net. We assume that traffic is distributed along all shortest paths with no delay due to packet loss or congestion. Hence, the only "cost" we consider is related to the path length, that is, each flow consumes bandwidth proportional to the route length (i.e. "bandwidth tax"). For example, if the route length of all flows is two and all ToRs are working uniformly, the maximum achievable load is 50%; otherwise the total traffic would exceed the network capacity: the number of ToRs times the number of switches times the rate, i.e., \( n \cdot k \cdot r \). Now let \( G(k_s) \) denote a (random) \( k_s \)-regular expander built from \( k_s \) (random) matchings and let \( \text{epl}(G(k_s)) \) denote the expected path length of \( G(k_s) \). The demand completion time of traffic \( U(x) \) (per ToR) can be bounded as follows:

\[
\text{DCT}_S(U(x), k_s) \geq U(x) \cdot \frac{\text{epl}(G(k_s))}{k_s \cdot r} \tag{9}
\]

From this we can compute the bound for an expander made from \( k \) switches and for our traffic model with load \( x \):

\[
\text{DCT}(\text{expander-net, } U(x), k) = \text{DCT}_S(U(x), k) = \frac{x \cdot k \cdot r \cdot \text{epl}(G(k))}{r \cdot k} = x \cdot \text{epl}(G(k)) \tag{10}
\]
Therefore the completion time is linear in $x$. Using our test parameters from Table 1 we have found that a (random) 32-regular expander with 256 nodes has an expected path length that is about 1.85.

4.1.2 Analysis of Rotor-Net

Next we consider rotor-net. First we consider a completely uniform demand between all possible pairs denoted as $\mathcal{U}$ (arriving at time zero). In this all-to-all case, rotor-net will be almost optimal by serving requests in each slot according to the current matching of each switch; all ports will be continuously operating at 100% throughput, sending flows directly (in a single hop) from source to destination. The only inefficiency will be due to the reconfiguration time (the “latency tax”), to reconfigure between slots. In our setting this overhead is equal to $R_r/(R_r + \delta)$, about 9% if we use Table 1 parameters. So the demand completion time of $k$ Rotor switches and such uniform demand $\mathcal{U}$ is for a single ToR (and for the system): (number of slots in $\mathcal{U}$) / (number switches) $\times$ (time for a slot). Formally,

$$\text{DCT}_k(\mathcal{U}, k) = \frac{\mathcal{U}}{k \cdot r} \cdot \frac{R_r + \delta}{\delta} = \frac{\mathcal{U}}{\delta \cdot r} \cdot \frac{R_r + \delta}{k} = \frac{\mathcal{U}}{m} \cdot \frac{R_r + \delta}{k}$$

(11)

Next we consider the case of traffic $U(x)$, in this case all ToRs sample flow sizes from the same flow distribution $\mathcal{D}$, but flows can have different sizes so we cannot assume the traffic is uniform among all pairs or all-to-all. Dealing with non-uniform flows is more complex, since the number of larger flows could be relativity small, potentially leaving many links in each slot inactive.

A rotor-net overcomes this problem by using Valiant routing (load balancing) [15] where flows and packets can be sent via two hops and not directly. Flows (or packets) that takes two hops may take additional capacity from the network (i.e. “bandwidth tax”). We model this situation with a traffic skewness parameter $0 \leq \phi \leq 1$ which approximates the fraction of bytes that a ToR in rotor-net sends using one hop. When traffic is close to uniform among destinations for a ToR, then $\phi$ will be close to 1 since there are no available other ToRs to help the current ToR. When the destination for a ToR are skewed e.g., one large flow toward a single destination, $\phi$ will be close to 0 and most of the traffic will be sent via two hops, taking advantage of destinations that are free to help. To approximate $\phi$ for a give distribution (or empirical distribution) we define it as one minus the variation distance [17] from the uniform distribution. For a finite state PDF $P = \{p_1, p_2, \ldots, p_n\}$ the variation distance from the uniform distribution is define as:

$$\Delta(P) = \frac{1}{2} \sum_{i=1}^n |p_i - \frac{1}{n}|$$

(12)

$\Delta(P)$ measures the probably mass above the average. If, for a ToR, $P$ represents the distribution of the destinations’ load, $\Delta(P)$ will capture the fraction of packets that can benefit from sending by two hops (using the help of destinations that have load bellow the average) and $\phi = 1 - \Delta(P)$ is the fraction of messages that will use a single hop. Therefore the average number of hops that a packet takes is: $1 \cdot \phi + 2(1 - \phi) = 2 - \phi$. Since ToRs are symmetric in this traffic model (i.e., $U(x)$) a ToR that sends $(1 - \phi)$ of its traffic via two hops, asking the help of other ToRs, would expect similar requests from him to help other ToRs. This means that the expected total traffic need to be sent by a ToR will be $U(x)(2 - \phi)$ and, we can optimistically assume it will now be divided uniformly among destinations (otherwise the DCT will be only larger) and we can formally generalize the DCT lower bound to:

$$\text{DCT}_k(U(x), k) = \frac{U(x)}{m} \left(2(1 - \phi) \cdot \frac{R_r + \delta}{k} \right) = \frac{U(x)}{kr} \left(2(1 - \phi) \frac{R_r + \delta}{\delta} \right)$$

(13)

Since rotor-net is composed of $k$ Rotor switches we can now bound the demand completion time of load $x$ for rotor-net by:

$$\text{DCT}(\text{rotor-net}, U(x), k) = \text{DCT}_k(U(x), k)$$

$$= \frac{kr}{m} (2 - \phi) \frac{R_r + \delta}{k}$$

$$= x(2 - \phi) \frac{R_r + \delta}{\delta}$$

(14)

If $\phi$ is a constant then this is a linear function. In practice $\phi$ can vary, but nevertheless the function can still be approximated well by a linear function.

4.1.3 Analysis of Hybrid

We now turn to discuss Hybrid whose analysis is a bit more complex. Let’s start with large flows which, according to Algorithm 1, are transmitted via the demand-aware switches. The cache component operates by reconfiguring a direct link form the source of a flow to its destination. The flow is then transmitted along a single hop. Assuming the reconfiguration time of a single cache switch is $R_c$ and the transmission time of a single large flow $f$ of size $|f|$ is $|f|/r$, the flow completion time for a single flow on a single switch is $R_c + \frac{|f|}{r}$.

Finding the threshold for large flows. To determine the threshold we would like to find a value for a flow size $|f|$ for which sending the flow $f$ on a $k$-rotor switch network is slower than transmitting the same flow on a $k$-cache switch network. Following Eq. (13) we note that the transmission
time is a function of the global traffic skewness $\phi$:
\[
\frac{R_c + \frac{|f|}{r}}{k} \leq \frac{|f|}{m} \cdot (2 - \phi) \cdot \frac{R_c + \delta}{k}
\]
\[
R_c \leq \frac{|f|}{m} \cdot (2 - \phi) \cdot \left( \frac{R_r + \delta}{r} - \frac{|f|}{r} \right)
\]
\[
R_c \cdot m \cdot r \leq \frac{(2 - \phi) \cdot r \cdot (R_r + \delta) - |m|}{|f|}
\]  
(15)

When we use the parameters of Table 1 we have that for $\phi = 0$ (all packets of the flow are sent via two hops in rotor-net), $|f| = 15MB$, and for $\phi = 1$ (all packets of the flow are sent via one hops in rotor-net), $|f| = 187.5MB$. The threshold we use in our evaluation is 125MB.

The demand completion time. For a given partition of the $k$ switches to the three types of switches: $k_s$ Static switches, $k_r$ Rotor switches and $k_c$ demand-aware switches, and a uniform traffic model, $U(x)$ with load $x$, the demand completion time of Hybrid is the maximal completion time among the three sub-components, formally,
\[
\text{DCT}(\text{Hybrid}, U(x), k) = \max \left\{ \text{DCT}(U(x,s), k_s), \text{DCT}(U(x,m), k_r), \text{DCT}(U(x,t), k_c) \right\}
\]
(16)

Assuming the sources and destinations are distributed uniformly and there are $k_c$ demand-aware switches, then the demand completion time of a single ToR (and by symmetry all ToRs) is approximated by
\[
\text{DCT}(U(x,t), k_c) = \frac{1}{k_c} \sum_{f \in U(x,t)} (R_c + \frac{|f|}{r})
\]
(17)

In the worst case we have $U(x,t)/|f|$ large flows, each of size $|f|$: as the flows get larger, the reconfiguration time $R_r$ is better amortized by the transmission time of the flow. The upper bound of the demand completion time is then
\[
\text{DCT}(U(x,t), k_c) \leq \frac{U(x,t)}{|f|} \cdot \frac{R_c + \frac{|f|}{r}}{k_c} = \frac{U(x,t)}{k_c} \left( \frac{R_c}{|f|} + \frac{1}{r} \right)
\]
(18)

When the threshold $|f|$ is far from the average large flow it is better to take the expected completion time.
\[
\text{DCT}(U(x,t), k_c) = \sum_{f} \left( \frac{\Pr(f) U(x,t)}{|f|} \frac{R_c + \frac{|f|}{r}}{k_c} \right)
\]
(19)
\[
= \sum_{f} \left( \frac{\Pr(f) U(x,t)}{k_c} \left( \frac{R_c}{|f|} + \frac{1}{r} \right) \right)
\]
\[
\frac{U(x,t)}{k_c} \left( \mathbb{E} \left[ \frac{R_c}{|f|} \right] + \frac{1}{r} \right)
\]
(20)

Next we discuss how to find and optimal partition of the switches. For example, assuming $k_r = 5$ and that the completion time for the expander component is negligible (due to low traffic volume), the optimal division of the switches, denoted as $k_r^*$ and $k_c^*$, is such that the completion time of the corresponding components will be identical. This follows from the fact that the completion time of each sub-component, as shown above, is monotonically decreasing in the number of switches. Equalizing the two components allows us to compute the optimal number of switches, as follows (where $\phi_m$ denote the skewness of the medium size traffic):
\[
\text{DCT}(U(x,t), k_r^*) = \text{DCT}(U(x,m), k_c^*) \Rightarrow
\]
\[
\frac{U(x,t)}{k_r^*} \left( \mathbb{E} \left[ \frac{R_c}{|f|} \right] + \frac{1}{r} \right) = \frac{U(x,m)}{|m|} \cdot (2 - \phi_m) \frac{R_r + \delta}{k_c^*} \Rightarrow
\]
\[
k_r^* = \frac{U(x,t)}{U(x,m)/|m|} \cdot \frac{\mathbb{E} \left[ \frac{R_c}{|f|} \right] + \frac{1}{r}}{(2 - \phi_m)(R_r + \delta)}
\]

For example, for a case study distribution and traffic $U(0.5)$ we calculated $\phi_m$ and by plugging in the values from Table 1, we get that $k_c^* = 16$ and $k_r^* = 16$ (using rounding and recalling that $k_r^* + k_c^* = 32$).

Following Eq. (19) we can now approximate the demand completion time of Hybrid as the completion time of the $k_c^*$ cache switches (recall that it is equal to the completion time of the $k_r^*$ switches).
\[
\text{DCT}(\text{Hybrid}, U(x), k) \leq \frac{U(x,t)}{k_r^*} \left( \mathbb{E} \left[ \frac{R_c}{|f|} \right] + \frac{1}{r} \right)
\]
(22)

4.2 Analysis of Skewed Traffic

Next we analyze the skewed traffic model $S(x)$, where only a fraction $x$ of the ToRs are active and operating at maximum rate. Let $S_t(x)$ denote the case where each of the active ToRs works at a load $L$ and $S_t(x) = S(x)$. Our goal in this scenario is to design a network which allows for a maximal possible per-ToR throughput $L$. Let $L^*(x) = \arg \max_L S_t(x) \leq 1$ and $L^* \leq 1$ is highest throughput that the network can support (given $x$).

The assumption of skewed traffic is more realistic than uniform traffic, but it is also harder to analyze. The main challenge arises from the fact that under the skewed traffic model, only a fraction $x$ of the ToRs are active, but in principle, it may be possible to utilize the $1 - x$ fraction of non-active ToRs to help the active ones. The question is if this can be done, and what is the maximum achievable throughput $L^*(x)$. We formally analyze this scenario by computing the demand completion time for $S(x)$ (with ports working at full throughput $r$). If the completion time is less than one second, we say that the throughput is one; otherwise we adjust the throughput, $L$ to find the largest throughput for which
the completion time is less than one second. Let \( S(x, \tau) \), as before, denote the expected number of bytes per second in flows of type \( \tau \) generated by \( S(x) \), similarly we have \( S_1(x, \tau) \).

We start again with the expander-net. We optimistically assume the expander-net has all the good properties expanders should have [10], i.e., large expansion, multiple disjoint paths, small mixing times, etc. Basically these properties guarantee that even for a small set of communicating ToRs, the traffic will efficiently spread across the entire network, utilizing the non-active ToRs as much as possible. Therefore, as before, we assume only the capacity restriction and consider the demand completion time as:

\[
\text{DCT}(\text{expander-net}, S_1(x, k)) = L^\ast(x) \cdot \text{epl}(G(k))
\]

We can find the throughput by solving for \( L \).

\[
L^\ast(x) = \min \left( \frac{1}{x \cdot \text{epl}(G(k))}, 1 \right)
\]

Following the definitions and observation in [12] the above Eq. (25) shows that a static expander will be a throughput-proportional network, namely it will be “able to distribute its capacity evenly across only the set of servers with traffic demands” [12].

What about a rotor-based network or a network based on Hybrid? Achieving throughput-proportionality seems to be non-trivial. For example if the load is 50%, how can the network exploit the capacity of the 50% non-active ToRs to help the active ones work at 100% throughput? In [12], it was shown that the fat-tree is not throughput-proportional.

Interestingly, we can show that Rotor switches actually work very well in this scenario and that rotor-net is also a throughput proportional network. This is a novel result and was not discussed in previous work [12, 14, 15]. The result is due to the Valiant routing property of rotor-net: if every flow is transmitted using this mechanism, flows will be forwarded using the non-active ToR, keeping all switch ports sending at full rate. Moreover, if a fraction \( x \) of the network is active and generates a uniform traffic (within the active fraction), this means that a fraction \( x \) of the time, switch ports can directly communicate to their destinations, and only a \( 1 - x \) fraction of the time flows will need to use two hops. If the traffic within the \( x \) fraction of active ToRs is non-uniform we can again use the skewness parameter \( \phi \) to approximate the fraction of traffic that needs one hop and the fraction that needs two hops. The average number of hops will then be \( x(\phi + 2(1-\phi)) + (1-x) = 2 - \phi x \). The total amount of traffic to be sent in the whole network will now be \( S(x)(2-\phi x) = x nk r(2-\phi x) \), but since we are using two hops, it will be uniformly divided among ToRs and destinations, and we can approximate the demand completion time of a single ToR as:

\[
\text{DCT(rotor-net, } S(x, k) = \text{DCT}_R(S(x), k) = \frac{S(x)}{m} \frac{(2-\phi x)}{nk} \frac{R_e + \delta}{\delta} = \frac{x(2-\phi x)}{\delta} \cdot \frac{R_e + \delta}{\delta}
\]

And find the throughput by solving for \( L \).

\[
\text{DCT(rotor-net, } S_1(x, k) = \text{DCT}_R(S_1(x), k) = 1 \Rightarrow L(x(2-\phi x) = \frac{R_e + \delta}{\delta}
\]

and

\[
L^\ast(x) = \min \left( \frac{1}{x(2-\phi x)}, 1 \right)
\]

The above equation implies that rotor-net is throughput-proportional. For the setting of Table 1 and our Case Study distribution with \( \phi = 0.49 \) it supports up to 50% of the ToRs working at full rate, the same as for the expander.

**Observation 1.** Rotor-net is a throughput-proportional network.

We next discuss Hybrid. We already know that the Rotor switches are throughput-proportional, so what about the demand-aware switches? Due to reconfiguration time, the demand-aware switches may not be able to serve all large flows generated by the active ToRs. Furthermore, our current design of demand-aware switches does not support 2-hops routing. The “latency tax” due to reconfigurations in our numerical example is at most 15%, \( \frac{R_c}{R_h} \); so when an active ToR is working at a full rate, it cannot send all of its large flows to the demand-aware switches.

Let \( z \) denote the expected fraction of large flows that \( k_c \) demand-aware switches can send in a second for a given ToR. We can find it using Eq. (19).

\[
\text{DCT}_{C}(U(z, \ell), k_c) = \text{DCT}_{C}(U(1, \ell), k_c) = \frac{U(z, \ell)}{k_c} \left( \mathbb{E} \left[ \frac{R_e}{f_{|f|}} \right] + \frac{1}{r} \right)
\]

Using \( z \) and following Algorithm 1, we can approximate the expected fraction of large flows, denoted as \( x^\ast \) per ToR which cannot fit the cache and which will be sent to the rotor switches when working at rate \( L \) (in \( S_1(x) \)), by:

\[
x^\ast = \max \left( \frac{L-z}{L} \right)
\]

The amount of such traffic per ToR will be \( x^\ast LU(1, \ell) = x^\ast U(L, \ell) \). Assuming that small flows are transmitted via static switches in Hybrid with negligible completion time,
we can compute the demand completion time of Hybrid as the demand completion time of the $k^*_r$ Rotor switches (since the Demand-aware switches are set up to have demand such that they finish at 1s). Following Eq. (26), we have:

**For rotor-net:**

$$\text{DCT(rotor-net, } S_L(x), k) = 1 \Rightarrow$$

$$\min \left( \frac{1}{x(2 - \phi x) \cdot \frac{R_r + \delta}{k^*_r}} \right) = L^*(x) \quad (34)$$

**For expander-net:**

$$\text{DCT(expander-net, } S_L(x), k) = 1 \Rightarrow$$

$$\min \left( \frac{1}{x \cdot \text{epl}(G(k)) \cdot 1} \right) = L^*(x) \quad (35)$$

where $\text{epl}(G(k))$ is the expected path length of a random k-regular expander with n nodes.

If not mentioned otherwise we assume that in all systems there are 5 static switches to take care of small flows and focus on how to divide the rest of the 32 switches.

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