Schwinger’s Approach to Einstein’s Gravity and Beyond∗

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Julian Schwinger (1918–1994), founder of renormalized quantum electrodynamics, was arguably the leading theoretical physicist of the second half of the 20th century. Thus it is not surprising that he made contributions to gravity theory as well. His students made major impacts on the still uncompleted program of constructing a quantum theory of gravity. Schwinger himself had no doubt of the validity of general relativity, although he preferred a particle-physics viewpoint based on gravitons and the associated fields, and not the geometrical picture of curved spacetime. This note provides a brief summary of his contributions and attitudes toward the subject of gravity.

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I. INTRODUCTION

Julian Schwinger, founder, along with Richard Feynman and Sin-itiro Tomonaga, of renormalized Quantum Electrodynamics in 1947-48, was the first recipient (along with Kurt Gödel) of the Einstein prize. (For biographical information about Schwinger see Refs. [1, 2].) He was always deeply appreciative of Einstein’s contributions to relativity, quantum mechanics, and gravitation, and late in his career wrote a popularization of special and general relativity called Einstein’s Legacy [3], based on an Open University course.

Thus it is surprising that at this late date, almost 20 years after Schwinger’s death, and nearly 60 after Einstein’s, to learn there was a scientific controversy between the two, as expressed through an AAS session entitled “Schwinger vs. Einstein.”

Schwinger in fact made major contributions to the development of the quantum theory of gravity, based upon the Einstein equations, and then went on to propose a source-theory formulation of the theory of gravity, building on the notion that the carrier of the force of gravity is the helicity-2 graviton, just as quantum electrodynamics is built on the hypothesis of a helicity-1 photon. From this starting point most of the consequences of general relativity could be produced, including the classic tests of the redshift, perihelion precession, the bending of light, and geodetic precession. However, for strong fields, Schwinger showed that Einstein’s full gravitational field equations were a necessary consequence.

II. QUANTUM GRAVITY

The earliest example of a non-Abelian theory is gravity. That is, the gauge boson for gravity, the graviton, interacts directly with itself, unlike the photon in electrodynamics. So after writing several papers on non-Abelian theories in the 1960s, Schwinger turned to gravity [4]. These papers made contact with the somewhat earlier work of two of his students, Richard Arnowitt and Stanley Deser [5]. In his papers, Schwinger introduces canonical variables, basically vierbeins (or tetrads) and connections, and attempts to show quantum consistency. Lorentz invariance of the theory is verified subject to “rather loosely stated physical boundary conditions.”

III. SOURCE THEORY OF GRAVITY

The complexity of these papers pushed Schwinger over the edge to Source Theory. Source theory is a formulation of quantum field theory in which Green’s functions play a central role; in fact the basic objects of any field theory are the Green’s functions, which express all the physical observables and correlations of the theory. Explicit reference to operator-valued fields is avoided. Green’s functions and sources always were always a vital part of his repertoire from

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starting in the 1930s, and in fact it is not a stretch to say that the first "source theory" paper was his most famous paper, "Gauge Invariance and Vacuum Polarization," published in 1951 [6].

Starting in 1968, Schwinger published several works on the source-theory formulation of gravity [7–10].

A. Graviton action

The source of a massless, helicity-2 graviton is a conserved, symmetrical stress-tensor,

$$\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = T^{\nu\mu},$$

from which the generating function for all the Green’s functions, the vacuum persistence amplitude, follows:

$$(0_+|0_-)^T = e^{iW[T]}, \quad W[T] = \frac{1}{2} \int (dx)(dx') \left[ T^{\mu\nu}(x)D_+(x - x')T_{\mu\nu}(x') - \frac{1}{2}T(x)D_+(x - x')T(x') \right],$$

which describes the free propagation of gravitons between sources. Here appears the causal or Feynman massless propagator,

$$D_+(x - x') = \int \frac{(dp)}{(2\pi)^4} \frac{1}{p^2 - i\epsilon},$$

where $p^2 = -(p^0)^2 + \mathbf{p} \cdot \mathbf{p}$ and $T$ is the trace of the graviton source tensor, $T(x) = T^{\mu\nu}(x)$. The above is expressed in natural units; to connect to the real world, we rescale the source:

$$T^{\mu\nu} = \sqrt{\kappa} t^{\mu\nu}, \quad \kappa = 8\pi G,$$

where $G$ is Newton’s constant.

Because gravity is of infinite range, the graviton should be massless. However, Schwinger showed how you can start with a massive spin-2 particle, of mass $m$, which has 5 helicity states. Then provided we define

$$\partial_\mu T^{\mu\nu} = \frac{m}{\sqrt{2}} J^\nu, \quad \partial_\mu J^\mu = m \left( \sqrt{3} K - \frac{1}{\sqrt{2}} T \right),$$

in the limit $m \to 0$, $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu J^\mu = 0$, and the action decouples into independent helicity 2, 1, and 0 components, represented by a tensor source $T^{\mu\nu}$, a vector source $J^\mu$, and a scalar source $K$. As explained, for example, in Ref. [8], the coefficients in Eq. (3.5) allow for the decomposition into the three helicity components, and the count of states between those for the massive tensor description, and the massless helicity 2, 1, and 0 description is $5 = 2 + 2 + 1$, suggesting a correspondence with the graviton, the photon, and a massless scalar.

B. Tests of General Relativity

In the Physical Review paper [7] on the source theory of gravity, and in his more detailed discussion in Particles, Sources, and Fields [8], Schwinger rederives, simply, just starting from the expression for the graviton action given in Eq. (3.2), the standard tests of general relativity:

- The gravitational red shift,
- the light deflection by the sun,
- the time delay in radar echos from planets,
- the precession of Mercury’s perihelion.

These arguments, or ones very similar thereto, of course had been supplied earlier by others. For a bibliography of tests of gravity, see Ref. [11].

In a couple of short articles in the American Journal of Physics a few years later (1974) [9, 10], Schwinger extended these elementary derivations to precession tests, which had not been performed up to that time: The Thirring effect, the precessional angular velocity associated with a rotating shell of radius $R$, mass $M$, and angular velocity $\omega$:

$$\omega_{\text{prec}} = \frac{4GM}{3R^2} \omega,$$ (3.6)
and the Schiff effect, the precession (precessional velocity $\Omega$) of a gyroscope in a satellite in orbit around a planet:

$$\Omega = \frac{3GM}{r^3} \mathbf{r} \times \mathbf{v} + \frac{GI}{r^5} (3\mathbf{r} \omega_p \cdot \mathbf{r} - \omega_p \mathbf{r}^2).$$ (3.7)

Here $r$ is the radius of the orbit, $\mathbf{v}$ is the velocity of the satellite, and $\omega_p$ is the angular velocity of the planet, which has moment of inertia $I$. I followed these papers up with a simple source theory rederivation of the Lense-Thirring effect\[12\], the effect of the spin of the sun (mass $M$) on the motion of a planet (mass $m$) orbiting it, in terms of the precession of its axial vector

$$\mathbf{A} = \frac{\mathbf{r}}{r} - \frac{M + m}{GM^2m^2} \mathbf{p} \times \mathbf{L},$$ (3.8)

where $\mathbf{r}$ is the position of the planet relative to the sun, $\mathbf{p}$ is the relative momentum, and $\mathbf{L}$ is the orbital angular momentum. The precession equation is

$$\frac{d\mathbf{A}}{dt} = \Omega_A \times \mathbf{A},$$ (3.9)

where

$$\Omega_A = \frac{2G}{r^3} \left( \mathbf{S} - \frac{3\omega_p(\omega_p \cdot \mathbf{S})}{\omega_p^2} \right),$$ (3.10)

where $\mathbf{S}$ is the spin of the sun, and $\omega_p$ is the angular velocity of the planet.

These “frame-dragging” and “geodetic” effects have now been confirmed by Gravity Probe B\[13\], although the Lense-Thirring effect was earlier seen by the LAGEOS experiment\[16\].

C. But Einstein’s General Relativity emerges

The theory proposed by Schwinger to this point is nothing but linearized gravity, with some extrapolation to include gravity itself as a source of energy. The symmetric tensor gravitational field $h_{\mu\nu}$ is related to the metric tensor $g_{\mu\nu}$ of general relativity and the flat-space Minkowski metric $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$ by

$$g_{\mu\nu} = \eta_{\mu\nu} + 2h_{\mu\nu},$$ (3.11)

where $h_{\mu\nu}$ is regarded as a small perturbation. (Schwinger inserted the factor of 2 to simplify the equations of motion for $h_{\mu\nu}$.) But Schwinger was quite aware this was inadequate. In his paper\[7\], and especially in the last section of Vol. I of his book\[8\], he recognizes that gravitational gauge invariance,

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_{\mu} \xi_{\nu} + \partial_{\nu} \xi_{\mu},$$ (3.12)

where the vector field $\xi_{\mu}$ is arbitrary, is necessarily generalized to general coordinate invariance, and in that way he was led inexorably to Einstein’s equations,

$$R_{\mu\nu}(x) = \kappa \left[ t_{\mu\nu}(x) - \frac{1}{2} g_{\mu\nu} t(x) \right],$$ (3.13)

in terms of the usual Riemann curvature tensor, and to the Einstein-Hilbert action,

$$W = \int (dx)[\mathcal{L}_m + \mathcal{L}_g], \quad 2\kappa \mathcal{L}_g = \sqrt{-g(x)} g^{\mu\nu}(x) R_{\mu\nu}(x),$$ (3.14)

where $\mathcal{L}_m$ is the matter Lagrangian.

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\[1\] Often referred to as the Laplace-Runge-Lenz vector, or by some subset of those names, but actually first discovered by Hermann in 1710\[13\], and generalized by Bernoulli in the same year\[14\].
D. Scalar-tensor gravity

He did go on to notice that conformal symmetry is spoiled by this theory, which motivated him to develop his own version of scalar-tensor gravity, which for weak coupling agrees with Brans-Dicke theory [17], but which could have different consequences in the cosmological domain [18].

IV. SUPERSYMMETRY AND SUPERGRAVITY

Later in the 1970s, Schwinger was chagrined that he had not come up with the idea of supersymmetry [19], since he had developed the multispinor formalism that naturally allowed the treatment of all spins on the same footing [8]. He invited his former student, Stanley Deser, for a private audience with Schwinger’s group of students, postdocs, and faculty, and we were initiated into the mysteries of supersymmetry and supergravity [20]. A paper by Schwinger giving a simple rederivation of supersymmetric ideas followed [21]. Bob Finkelstein, Luis Urrutia, and I immediately continued with a paper in which we showed that supergravity emerged by requiring solely invariance under local supersymmetry [22].

V. SCHWINGER’S ATTITUDE TOWARD GENERAL RELATIVITY

Schwinger certainly had no quibble with general relativity, and provided an alternative derivation of Einstein’s equations. In this he behaved analogously to Richard Feynman who, in his Polish lecture, also provided a particle physics derivation of general relativity [23]. In fact, Schwinger privately stated that he thought he would have discovered general relativity had he been in Einstein’s place.

It is true that Schwinger’s approach was always algebraic, and consequently he had little use for the geometric interpretation of curved space. Yet this is merely an interpretation of the theory, and had no effect on testable consequences. Schwinger did not believe in a massive graviton, and certainly not in tachyons.

VI. SCHWINGER’S ATTITUDE TOWARD THE UNSEEN UNIVERSE

Schwinger never expressed an opinion on dark matter, to my knowledge. In those days, dark matter, manifested by galactic rotation curves, was largely only of interest to astronomers, and not to physicists in general [24]. Of course, he didn’t know about “cosmic acceleration” [25], so he likely thought that the cosmological constant was zero. Since he became very fascinated with the Casimir effect [26], I’d like to imagine he would have thought that dark energy originated from a nonzero cosmological constant, arising from quantum fluctuations. Such ideas go back at least to Pauli [27], and for a history of the connection of the ideas of zero-point energy and the cosmological constant see Kragh [28]. Particularly noteworthy are the contributions of Gliner [29], Zel’dovich [30], and Sakharov [31]. See also Ref. [32].

An idea along these lines was proposed some time ago [33]. For example, quantum fluctuations of fields in “large” compactified dimensions would give rise to a cosmological constant: \( a = \text{size of compact space of dimension } d \)

\[
\langle T^{\mu\nu} \rangle = -ug^{\mu\nu} = -\frac{\Lambda}{8\pi G} g^{\mu\nu}.
\]  
(6.1)

Roughly speaking, the quantum vacuum energy of the fluctuations must have the form

\[
u = \frac{\gamma_d}{a^4}, \quad d \text{ odd,} \quad u = \frac{\alpha_d \ln a / L_{Pl}}{a^4}, \quad d \text{ even.}
\]  
(6.2)

for odd and even compactified dimensions \( d \), respectively, where \( L_{Pl} \) is the Planck length. The coefficients \( \alpha_d \) and \( \gamma_d \) depend on the fields compactified [34]. We must require that the density of dark energy be less than the critical density that would close the universe, which for a reduced Hubble constant of \( h_0 = 0.7 \) corresponds to a length scale of 80 \( \mu \)m, which leads to

\[
\begin{align*}
& a \geq \gamma^{1/4}80 \mu m, \quad a \geq [\alpha \ln(a/L_{Pl})]^{1/4}80 \mu m, \\
& a \leq 44 \mu m.
\end{align*}
\]  
(6.3) and (6.4)

See Table I, which is reproduced from Ref. [33]. These ideas have been more recently elaborated, for example, in Refs. [36] and [37].
TABLE I: The lower limit to the radius of the compact dimensions deduced from the requirement that the Casimir energy not exceed the critical density. The numbers shown are for a single species of the field type indicated. The dashes indicate cases where the Casimir energy has not been calculated, while asterisks indicate (phenomenologically excluded) cases where the Casimir energy is negative.

VII. CONCLUSIONS

Although Schwinger approached gravity from a particle-physics viewpoint, he never expressed any doubt about the validity of general relativity. He thought algebraically, not geometrically, so he didn’t find the notions of curved space useful. Many of his students have made major contributions to the quantum theory of gravity (for example, beside Arnowitt and Deser mentioned above, one cannot forget the contributions of Bryce DeWitt [38] or of David Boulware [39]), which, although still not fully developed [40], must reduce to Einstein’s theory under most circumstances.

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