Intrinsic parton motion soft mechanisms and the longitudinal spin asymmetry $A_{LL}$ in high energy $pp \to \pi X$

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The longitudinal double spin asymmetry $A_{LL}$ in the reaction $pp \to \pi X$ has been measured at RHIC with extremely interesting consequences. If the gluon polarization in a proton were as big as needed to resolve the famous “spin crisis” then $A_{LL}$ would be large and positive. Latest RHIC results indicate that $A_{LL}$ is small and disfavour large positive values of the gluon polarization. We examine whether the soft mechanisms (Collins, Sivers, Boer-Mulders), essential for generating transverse single spin asymmetries, have any significant influence on $A_{LL}$, and whether they could alter the conclusion that the gluon polarization is necessarily small. It turns out that the contribution from these effects is essentially negligible.

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I. INTRODUCTION

Large transverse single spin asymmetries (up to 40%) have been observed in a multitude of reactions for over three decades, whereas such asymmetries are tiny ($\lesssim 1\%$) in the standard leading twist QCD parton model. To explain the size of these asymmetries Sivers and Collins [1, 2] introduced new soft mechanisms, utilizing, as an essential ingredient, the intrinsic transverse momentum of partons [3]. Later other similar mechanisms were shown to be possible, for example the Boer-Mulders mechanism [4, 5].

Although these mechanisms were invented in order to produce transverse asymmetries, it turns out that they also contribute to the longitudinal double spin asymmetries and to the total cross-section [6]. For the latter, it has been shown that the effect of the soft functions is negligible. However, it was found that intrinsic transverse momentum per se significantly affects the value of the cross-section [7]. That this should be the case at lower energies had already been noticed by Field and Feynman [8], and later by Vogelsang and Weber [9] on the grounds that taking into account intrinsic $k_\perp$ is a particular method of including higher twist corrections.

For the longitudinal asymmetries, the question is much more delicate, for the following reason. One of the most important reactions measured at RHIC is the double spin longitudinal asymmetry $A_{LL}$, which has been found to be very small, and which has been used, based on a leading twist collinear treatment, to confirm the growing belief that the gluon polarization is far too small to explain the “spin crisis in the parton model” [10].

Given that $A_{LL}$ is so small, and that the implications of this are so important, we felt it necessary to check whether the soft mechanisms can have a significant impact, in particular whether they could influence the above conclusion about the gluon polarization. We have also checked for any significant sensitivity in $A_{LL}$ to intrinsic transverse momentum.

The plan of the paper is the following. In Section II we briefly recall the formalism used for our calculation, which includes the full non-collinear kinematics of the scattering process. In Section III we present the “kernels” for the calculation of each partonic contribution to the polarized cross-sections. In Section IV we show and discuss our phenomenological results for the longitudinal double spin asymmetry in inclusive neutral pion production at RHIC. Finally, in Section V we draw our conclusions.

II. FORMALISM

Here we simply sketch the main aspects of the formalism; for details of the approach we refer to [6]. The longitudinal double spin asymmetry $A_{LL}$ for the reaction $pp \to \pi X$ is defined as

$$A_{LL} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}} = \frac{d\sigma^{++} - d\sigma^{+-}}{2d\sigma^{unp}},$$

where the labels refer to the helicities of the protons.
The general expression for the differential cross-sections for the polarized hadronic process \((A, S_A) + (B, S_B) \rightarrow C + X\) is given by

\[
\frac{E_C \, d\sigma^{(A, S_A) + (B, S_B) \rightarrow C + X}}{d^3p_C} = \sum_{a, b, c, d, \{\lambda\}} \int \frac{dx_a \, dx_b \, dz}{16\pi^2 x_a x_b z^2 s} \, d^2k_{\perp a} \, d^2k_{\perp b} \, d^2k_{\perp C} \, \delta(k_{\perp C} \cdot \hat{p}_C) \\
\times J(k_{\perp C}) \, \hat{f}_{a/A, S_A}(x_a, k_{\perp a}) \, \hat{f}_{b/B, S_B}(x_b, k_{\perp b}) \\
\times \hat{M}_{\lambda_c \lambda_a \lambda_d \lambda_b}^{\lambda_c \lambda_a \lambda_d \lambda_b} \, \hat{D}_{\lambda_c \lambda_a \lambda_d \lambda_b}^{\lambda_c \lambda_a \lambda_d \lambda_b}(z, k_{\perp C}) ,
\]

which involves a (factorized) convolution of all possible hard elementary QCD processes, \(ab \rightarrow cd\), with soft partonic polarized distribution and fragmentation functions. In Eq. (2) \(s, t\) and \(u\) are the Mandelstam variables for the partonic reactions. The detailed connection between the hadronic and the partonic kinematical variables is given in full in Appendix A of Ref. [6]. A discussion of some technical details, like, e.g., the infrared regulators related to small partonic scattering angles, can be found, for example, in Ref. [7].

Let us simply recall here, for a better understanding, the physical meaning of the different factors in Eq. (2):

- \(\rho^{a/A, S_A}_{\lambda_a \lambda_a} \) is the helicity density matrix of parton \(a\) inside the polarized hadron \(A\), with spin state \(S_A\); it describes the parton polarization. \(\hat{f}_{a/A, S_A}(x_a, k_{\perp a})\) is the number density (or distribution) of unpolarized partons \(a\) inside the polarized hadron \(A, S_A\); each parton carries a light-cone momentum fraction \(x_a\) and a transverse momentum \(k_{\perp a}\). Similarly for parton \(b\) inside hadron \(B\) with spin \(S_B\).

- The polarized cross-sections for the elementary partonic process \((a, s_a) + (b, s_b) \rightarrow (c, s_c) + d\) are expressed in terms of products of the helicity amplitudes \(\hat{M}_{\lambda_c \lambda_a \lambda_d \lambda_b}^{\lambda_c \lambda_a \lambda_d \lambda_b}\).

- The factor \(\hat{D}_{\lambda_c \lambda_a \lambda_d \lambda_b}^{\lambda_c \lambda_a \lambda_d \lambda_b}(z, k_{\perp C})\) describes, again in the helicity basis, the fragmentation process \(c \rightarrow C + X\), according to which a polarized parton \(c\) fragments into an unpolarized hadron \(C\) carrying a light-cone momentum fraction \(z\) and a transverse momentum \(k_{\perp C}\).

- \(J(k_{\perp C})\) is a kinematical factor, numerically very close to 1 for RHIC kinematics. All details can be found in Ref. [6]. Throughout the paper, we work in the \(AB\) c.m. frame, assuming that hadron \(A\) moves along the positive \(Z_{\text{cm}}\)-axis and hadron \(C\) is produced in the \((XZ)_{\text{cm}}\) plane, with \(p_C = 0\).

Eq. (2) is written in a factorized form, separating the soft, long distance from the hard, short distance contributions. The hard part is computable in perturbative QCD, while information on the soft one has to be extracted from other experiments or modeled. As already mentioned and discussed in Refs. [6, 7], such a factorization with non-collinear kinematics has never been formally proven. Indeed, studies of factorization [11–14], comparing semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan reactions have indicated unexpected modifications of simple factorization, and the situation for single inclusive particle production in hadron–hadron collisions is not yet resolved. Thus, our approach can only be considered as the natural extension of the collinear case and a reasonable phenomenological model. Of course, the perturbative calculation of the hard part is only reliable if the hard scale – in this case the square of the transverse momentum of the final hadron, \(p_T^2\) – is large enough. It turns out that the data on unpolarized cross-sections in hadronic collisions at low-intermediate energy scales suggest [7] an average value of \(k_T^2 \equiv |k_T|^2 \approx 0.64\) (GeV/c)^2 for the intrinsic transverse momentum of the parton distributions. On the other hand, both unpolarized hadronic cross-sections at RHIC energies [15] and the Cahn effect in SIDIS [16] are rather well reproduced by using \(\langle k_T^2 \rangle \approx 0.25\) (GeV/c)^2. We shall therefore study how the contributions to \(A_{LL}\) depend on the value of \(\langle k_T^2 \rangle\).

### III. Kernels

As we can see from Eq. (2), the computation of the cross-section corresponding to any polarized hadronic process \((A, S_A) + (B, S_B) \rightarrow C + X\) requires the evaluation and integration, for each elementary process \(a + b \rightarrow c + d\), of the general kernel

\[
\Sigma(S_A, S_B)^{ab \rightarrow cd} = \sum_{\{\lambda\}} \rho^{a/A, S_A}_{\lambda_a \lambda_a} \hat{f}_{a/A, S_A}(x_a, k_{\perp a}) \rho^{b/B, S_B}_{\lambda_b \lambda_b} \hat{f}_{b/B, S_B}(x_b, k_{\perp b}) \\
\times \hat{M}_{\lambda_c \lambda_a \lambda_d \lambda_b}^{\lambda_c \lambda_a \lambda_d \lambda_b} \, \hat{D}_{\lambda_c \lambda_a \lambda_d \lambda_b}^{\lambda_c \lambda_a \lambda_d \lambda_b}(z, k_{\perp C}) .
\]
While the hadronic process \((A, S_A) + (B, S_B) \rightarrow C + X\) takes place, according to our choice, in the \((XZ)_{cm}\) plane, all the elementary processes involved, \(A(B) \rightarrow a(b) + X,\ ab \rightarrow cd\) and \(c \rightarrow C + X\) do not, since all parton and hadron momenta, \(p_a, p_b, p_C\) have transverse components \(k_{l\perp a}, k_{l\perp b}, k_{l\perp C}\). This “out of \((XZ)_{cm}\) plane” geometry induces phases in the fragmentation process, in the distribution functions and in the elementary interactions, which have to be taken into account. Thus, the independent helicity amplitudes for the elementary pQCD processes \(ab \rightarrow cd\), with massless partons, can be written as \([6]\)

\[
\hat{M}_{+;+;+;+} = \hat{M}_0 e^{i\varphi_1}, \quad \hat{M}_{-;-;+;+} = \hat{M}_0 e^{i\varphi_2}, \quad \hat{M}_{-;+;+;+} = \hat{M}_0 e^{i\varphi_3},
\]

where the amplitudes \(\hat{M}_{1,2,3}^0\) are the real planar amplitudes defined in the partonic \(ab \rightarrow cd\) c.m. frame,

\[
\hat{M}_1 = \hat{M}_{1;+;+;+} = \hat{M}_{1;-;-;+} = \hat{M}_{1;+;+;+} = \hat{M}_{1;+;+;+} = \hat{M}_{1;-;-;+} = \hat{M}_{1;+;+;+},
\]

as required by parity invariance. The phases \(\varphi_{1,2,3}\) are complicated functions of the polar and azimuthal angles of the transverse momenta, \(k_{l\perp a}, k_{l\perp b}\) and \(k_{l\perp C}\), and their explicit expressions can be found in Ref. \([6]\). The relations

\[
\hat{M}_{-;-;-;-} = \hat{M}^*_{+;+;+;+} = \hat{M}_{+;+;+;+} = \hat{M}_{+;+;+;+} = \hat{M}^*_{+;+;+;+},
\]

follow from Eqs. (4), (5) and from the fact that the phases \(\varphi_i\) change sign by helicity inversion \([6]\). Note that the + and - subscripts refer to \((+1/2)\) and \((-1/2)\) helicities for quarks, and to \((+1)\) and \((-1)\) helicities for gluons. There are eight elementary contributions \(ab \rightarrow cd\) which we have to consider separately

\[
q_a q_b \rightarrow q_c q_d, \quad g_a g_b \rightarrow g_c g_d, \\
q g \rightarrow q g, \quad q g \rightarrow q g, \\
g g \rightarrow g g, \quad g g \rightarrow g g, \\
g_a g_b \rightarrow q \bar{q}, \quad q \bar{q} \rightarrow g g,
\]

where \(g\) can in general be either a quark or an antiquark. The subscripts \(a, b, c, d\) for quarks, when necessary, identify the flavour (only in processes where different flavours can be present); for gluons, these labels identify the corresponding hadron \((a \rightarrow A, b \rightarrow B, c \rightarrow C)\). By performing the explicit sums in Eq. (3), we obtain the kernels for each of the elementary processes. Note that the new aspect of our calculation is the appearance of the phases which is a reflection of the non-collinear kinematics.

The computation of the denominator/numerator of \(A_{LL}\) in Eq. (1) requires the evaluation of the kernels \(\Sigma(+, +) \pm \Sigma(+, -)\) respectively. The expressions for the sums of kernels, which are relevant for the unpolarized cross-section, are given in Ref. \([6]\). Here we give in detail the expressions for the differences. They are calculated from the general kernel given in Eq. (3). In the following certain terms are underlined: these are terms which vanish after integration over the angles of the momenta \(k_{l\perp a}, k_{l\perp b}, k_{l\perp C}\) in Eq. (2); we shall further comment on that at the end of this Section. \(\phi_{1,2,3}^C\) is the azimuthal angle of the hadron \(C\) in the parton c helicity frame and its expression in terms of the angles of \(k_{l\perp C}\) is given in Appendix A of Ref. \([6]\). Notice that all angular dependences of the kernels are explicitly extracted and the parton distribution (PDF) and fragmentation (FF) functions only depend on the magnitudes of the transverse momentum vectors.

• \(q_a q_b \rightarrow q_c q_d\) contribution

\[
\Sigma(\pm, +) \pm \Sigma(\pm, -) = \Delta \hat{f}^a_{s;+}(x_a, k_{l\perp a}) \Delta \hat{f}^b_{s;+}(x_b, k_{l\perp b}) \left[ |\hat{M}_1^0|^2 - |\hat{M}_2^0|^2 - |\hat{M}_3^0|^2 \right] \hat{D}_{C/c}(z, k_{\perp C})
\]

\[
+ \left\{ \Delta \hat{f}^a_{s;+}(x_a, k_{l\perp a}) \Delta \hat{f}^b_{s;+}(x_b, k_{l\perp b}) \cos(\varphi_3 - \varphi_2)\right\} \left[ 2 \hat{M}_2^0 \hat{M}_3^0 \hat{D}_{C/c}(z, k_{\perp C})
\]

\[
- \hat{f}_{A/A}(x_a, k_{l\perp a}) \Delta \hat{f}^b_{s;+}(x_b, k_{l\perp b}) \hat{M}_1^0 \hat{M}_3^0 \sin(\varphi_1 - \varphi_3 + \phi_{C}^H) \Delta^N \hat{D}_{C/c}(z, k_{\perp C}) .
\]

Notice that we have used the relations \(\Delta \hat{f}^a_{s;+}(x_a, k_{l\perp a}) = \Delta \hat{f}^a_{s;+}(x_a, k_{l\perp a})\) and \(\hat{f}_{A/A}(x_a, k_{l\perp a}) = \hat{f}_{A/A}(x_a, k_{l\perp a})\), see Appendix B of Ref. \([6]\). The channels \(q \bar{q} \rightarrow q \bar{q}\) etc. are formally identical to \(q q \rightarrow q q\) with amplitudes defined properly in Ref. \([6]\).
\[ \frac{g_9 g_b}{g_t g_d} \text{ contribution} \]
\[ 
\begin{align*}
[\Sigma(+) - \Sigma(-)] g_9 g_b & - g_t g_d = \\
\Delta \hat{f}^a_{s,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{s,+/}(x_b, k_{1B}) \left( |\hat{M}_1^0|^2 - |\hat{M}_2^0|^2 \right) \hat{D}_{C/g}(z, k_{1C}) \\
+ \left[ \Delta \hat{f}^a_{T_2,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{T_2,+/}(x_b, k_{1B}) \cos(\varphi_3 - \varphi_2) \\
+ \Delta \hat{f}^a_{T_3,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{T_3,+/}(x_b, k_{1B}) \sin(\varphi_3 - \varphi_2) \right] (2 \hat{M}_2^0 \hat{M}_3^0) \hat{D}_{C/g}(z, k_{1C}) \\
+ \hat{f}_{a/A}(x_a, k_{1A}) & \hat{f}^b_{T_3,+/}(x_b, k_{1B}) \hat{M}_1^0 \hat{M}_3^0 \sin(\varphi_1 - \varphi_3 + 2\phi_H^2) \hat{D}_{C/T^3}(z, k_{1C}) 
\end{align*} \]

\[ q\bar{q} \rightarrow gg \text{ contribution} \]
\[ 
\begin{align*}
[\Sigma(+) - \Sigma(-)] & q\bar{q} \rightarrow gg = \\
\Delta \hat{f}^a_{s,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{s,+/}(x_b, k_{1B}) \left( |\hat{M}_2^0|^2 + |\hat{M}_3^0|^2 \right) \hat{D}_{C/g}(z, k_{1C}) \\
+ \left[ \Delta \hat{f}^a_{s,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{s,+/}(x_b, k_{1B}) \cos(\varphi_3 - \varphi_2) \\
+ \Delta \hat{f}^a_{s,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{s,+/}(x_b, k_{1B}) \sin(\varphi_3 - \varphi_2) \right] (2 \hat{M}_2^0 \hat{M}_3^0) \hat{D}_{C/g}(z, k_{1C}) 
\end{align*} \]

\[ g_9 g_b \rightarrow q\bar{q}/qq \text{ contribution} \]
\[ 
\begin{align*}
[\Sigma(+) - \Sigma(-)] g_9 g_b & - q\bar{q}/qq = \\
\Delta \hat{f}^a_{s,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{s,+/}(x_b, k_{1B}) \left( |\hat{M}_2^0|^2 + |\hat{M}_3^0|^2 \right) \hat{D}_{C/c}(z, k_{1C}) \\
+ \left[ \Delta \hat{f}^a_{s,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{s,+/}(x_b, k_{1B}) \cos(\varphi_3 - \varphi_2) \\
+ \Delta \hat{f}^a_{T_3,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{T_3,+/}(x_b, k_{1B}) \sin(\varphi_3 - \varphi_2) \right] (2 \hat{M}_2^0 \hat{M}_3^0) \hat{D}_{C/c}(z, k_{1C}) 
\end{align*} \]

\[ gg \rightarrow qq \text{ contribution} \]
\[ 
\begin{align*}
[\Sigma(+) - \Sigma(-)] gg & - gg = \\
\Delta \hat{f}^a_{s,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{s,+/}(x_b, k_{1B}) \left( |\hat{M}_2^0|^2 - |\hat{M}_3^0|^2 \right) \hat{D}_{C/c}(z, k_{1C}) 
\end{align*} \]

\[ gg \rightarrow gg \text{ contribution} \]
\[ 
\begin{align*}
[\Sigma(+) - \Sigma(-)] gg & - gg = \\
\Delta \hat{f}^a_{s,+/}(x_a, k_{1A}) & \Delta \hat{f}^b_{s,+/}(x_b, k_{1B}) \left( |\hat{M}_2^0|^2 - |\hat{M}_3^0|^2 \right) \hat{D}_{C/c}(z, k_{1C}) \\
- \hat{f}_{a/A}(x_a, k_{1A}) & \hat{f}^b_{s,+/}(x_b, k_{1B}) \hat{M}_3^0 \hat{M}_3^0 \sin(\varphi_1 - \varphi_3 + 2\phi_H^2) \hat{D}_{C/T^3}(z, k_{1C}) 
\end{align*} \]
• $qg \to qg$ contribution

$$\begin{align*}
[\Sigma(+, +) - \Sigma(+, -)]^{qg_{\perp} - qg} &= \\
\Delta \hat{f}_{a/s}^{q}(x_a, k_{\perp a}) \Delta \hat{f}_{b/s}^{q}(x_b, k_{\perp b}) \left[ |\hat{M}_1^0|^2 - |\hat{M}_2^0|^2 \right] \hat{D}_{C/g}(z, k_{\perp C}) \, .
\end{align*}$$

(15)

The physical content of the above expressions is interesting. First note the complete formal symmetry between the $qq \to qq$ kernel in Eq. (8) and the $gq \to gg$ kernel in Eq. (9). These kernels contain the largest variety of contributions, and the kernels for all the other partonic processes can be formally read off from these by the suppression of certain terms.

In the second line of both expressions, Eq. (8) and Eq. (9), we recognize the product of the $k_{\perp}$-dependent helicity distributions, $\Delta \hat{f}_{a/s}^{q}(x_q, k_{\perp q}) \equiv \Delta q(x_q, k_{\perp q})$ and $\Delta \hat{f}_{b/s}^{q}(x_g, k_{\perp g}) \equiv \Delta g(x_g, k_{\perp g})$ for quarks and gluons respectively, and the unpolarized fragmentation function $\hat{D}_{C/e}(z, k_{\perp C})$, with no azimuthal phases. In the third line of Eq. (8) we have two parton distribution functions, $\Delta \hat{f}_{a/s}^{q}(x_a, k_{\perp a})$, referring to quarks transversely polarized, along the $x$-axis, inside longitudinally polarized nucleons, coupled to the unpolarized fragmentation function. Analogously, in the third line of Eq. (9) we have two parton distribution functions, $\Delta \hat{f}_{T_2/s}^{q}(x, k_{\perp})$, which are related to the linear polarization of a gluon inside a longitudinally polarized nucleon. Correspondingly the fourth line of Eq. (8) refers to one quark transversely polarized along the $x$-axis inside a longitudinally polarized nucleon and the other, $\Delta \hat{f}_{s/A}^{q}(x, k_{\perp})$, transversely polarized along the $y$-axis inside an unpolarized nucleon – the latter is the Boer-Mulders function – coupled to the unpolarized fragmentation function. Analogously, in the fourth line of Eq. (9) we have $\Delta \hat{f}_{T_2/s}^{q}(x, k_{\perp})$ and the “Boer-Mulders-like” gluon function, $\Delta \hat{f}_{T_2/A}^{g}(x, k_{\perp})$, referring to a linearly polarized gluon inside an unpolarized nucleon. For a more complete explanation of the physical meaning of these functions see Appendix B of Ref. [6]. Finally, the last line of Eq. (8) contains the Collins fragmentation function, $\Delta^N \hat{D}_{C/e}(z, k_{\perp C})$, coupled to an unpolarized parton density and a transversely polarized one. In the case of the gluon, in the last line of Eq. (9), there appears a gluonic analogue of the Collins fragmentation function, $\Delta^N \hat{D}_{C/T_2}(z, k_{\perp C})$, describing the fragmentation of a linearly polarized gluon into an unpolarized hadron.

Ignoring the underlined terms which vanish upon integration, we see that compared to the standard collinear approach, we have extra contributions involving quarks polarized transversely along their $x$-axis in a longitudinally polarized nucleon, appearing in Eqs. (8), (10) and contributions involving linearly polarized gluons inside a longitudinally polarized nucleon, appearing in Eqs. (9), (11). Notice that the processes in Eqs. (12)-(15), initiated by quark-gluon elementary scattering, get contributions only from the usual terms, which survive in the collinear case.

The demonstration of the vanishing upon angular integration of the underlined terms in Eqs. (8)-(15) requires a detailed study of the kinematics and of the relationships between the angular integration variables appearing in $k_{\perp a}, k_{\perp b}, k_{\perp C}$ in Eq. (2) and the phase variables $\varphi_{1,2,3}$ and $\phi^H_C$ [6]. We have also numerically checked that this is indeed the case.

Notice that a parity transformation implies $\varphi_i \to -\varphi_i$ ($i = 1, 2, 3$) and $\phi^H_C \to -\phi^H_C$. Thus the odd sin terms in Eqs. (8)-(15) must vanish if parity is conserved.

Another simple, but interesting example of such a vanishing can be obtained by considering, within the same formalism, the expression of the kernels for the longitudinal single spin asymmetry $A_{L}$, which we know must vanish in a parity conserving theory. The kernels themselves are not zero, but under integration do vanish. This is another very stringent test of the correctness of our formalism. For $A_L$, for the partonic channel $qaqb \to qcqd$, we have for the numerator of the longitudinal single spin asymmetry the following expression:

$$\begin{align*}
[\Sigma(+, 0) - \Sigma(-, 0)]^{qaqb \to qcqd} &= \\
\Delta \hat{f}_{a/s}^{q}(x_a, k_{\perp a}) \Delta \hat{f}_{b/s}^{q}(x_b, k_{\perp b}) \left[ 2\hat{M}_2^0 \hat{M}_3^0 \right] \sin(\varphi_{3} - \varphi_{2}) \hat{D}_{C/e}(z, k_{\perp C}) \\
&- \Delta \hat{f}_{a/s}^{q}(x_a, k_{\perp a}) \hat{f}_{b/B}(x_b, k_{\perp b}) \sin(\varphi_{1} - \varphi_{2} + \phi^H_C) \hat{M}_1^0 \hat{M}_2^0 \Delta^N \hat{D}_{C/e}(z, k_{\perp C}),
\end{align*}$$

(16)

and, again, all terms – being odd functions of $\varphi_i$ and $\phi^H_C$ – vanish, as they should, upon angular integration.
IV. PHENOMENOLOGY: $A_{LL}$ AT RHIC

The longitudinal double spin asymmetry $A_{LL}$ for inclusive neutral pion and jet production in proton-proton scattering at $\sqrt{s} = 200$ GeV has been measured at RHIC in various runs, respectively by the PHENIX [17–19] and STAR [20, 21] Collaborations. The first published PHENIX experimental data [17] showed results for $A_{LL}$ at mid rapidity compatible with negative values. This was quite puzzling, since $A_{LL}$ is a positive quantity in the collinear parton model [22], at least at low $p_T$ where it is dominated by $gg \to gg$ elementary scattering processes, see Eq. (9) in which it can be shown that $|M^0_1|^2 - |M^2_1|^2 - |M^0_3|^2 > 0$. More recent and precise data from both collaborations [18, 20] exclude the possibility of a large and negative $A_{LL}$: in two subsequent RHIC runs, Run 5 [19, 21] and Run 6 (results from Run 6 have only been presented as “preliminary” [23, 24]), they confirm and reinforce the statement that $A_{LL}$ is very small and compatible with zero over the whole $p_T$ range covered.

An earlier comparison of present RHIC data with collinear next-to-leading order (NLO) QCD calculations of $A_{LL}$ [25] disfavoured large positive values for $\Delta g$, definitely excluding scenarios where $\Delta g$ is as large as the unpolarized gluon distribution function, $g$, at low scale. Instead the data were in better agreement with the predictions obtained by assuming $\Delta g = 0$ or even $\Delta g = -g$ at the initial scale [26]. A recent statistical analysis shows that the PHENIX Run 5 data are compatible with both $\Delta g = 0$ and the “standard” GRSV parametrization [27], while it rules out the $\Delta g = -g$ (at the initial scale) hypothesis [19]. A newest update of this analysis, which includes the preliminary data from PHENIX Run 6, favours the $\Delta g = 0$ scenario over the standard GRSV [24]. (Note that in this Section we have adopted the common, short-hand notation $\Delta f_{g/\perp}^0 \equiv \Delta g$ and $\Delta f_{s/s_\perp}^0 \equiv \Delta g$ for the helicity distribution functions, while $f_{g/p} \equiv q$ and $f_{g/p} \equiv g$ for the unpolarized distribution functions, for quarks and gluons respectively).

Our goal is to explore whether the new mechanisms permitted by the presence of partonic intrinsic transverse momenta, obtained in a general and fully non-collinear kinematics, could affect the above conclusions, which are based on the analysis of $A_{LL}$ in the collinear configuration, i.e. taking into account only the terms proportional to $\Delta g(x)$ and $\Delta g(x)$. Could the “new” contributions shown in Eqs. (8)-(11) turn the longitudinal double spin asymmetry $A_{LL}$ into a very small (or even slightly negative) quantity without the need to assume $\Delta g$ to be zero or negative?

We have studied $A_{LL}$ at RHIC, for the PHENIX kinematics, $\sqrt{s} = 200$ GeV and $|\eta| < 0.35$ (numerical calculations are performed at $\eta = 0$) and evaluated each separate contribution to $A_{LL}$, according to Eqs. (8)-(15). Since we have no knowledge of the parton densities $\Delta f_{g/\perp}^0$ and $\Delta f_{T/\perp}^0$ we maximized them in order to see whether, in principle, they can have a significant effect on $A_{LL}$. We thus used for them the corresponding unpolarized parton densities and adjusted the signs so that all contributions add up coherently.

For the helicity distributions we have used the sets GRSV2000 [27] and LSS05 [28]. The unpolarized cross-section and the maximized contributions to the numerator of $A_{LL}$ have been calculated using the GRV98 set [29] and the MRST01 set [30] respectively. For the fragmentation functions we have used the KKP set [31] and, for comparison, the Kretzer set [32]. The transverse momentum dependence has been included by means of a factorized Gaussian smearing, for all the parton distribution and fragmentation functions

$$\hat{f}(x, k_{\perp}) = f(x) \frac{e^{-k_{\perp}^2/(2k_0^2)}}{\pi<k_0^2>},$$

$$\hat{D}(z, k_{\perp C}) = D(z) \frac{e^{-k_{\perp C}^2/(2k_0^2)}}{\pi<k_0^2 C>},$$

with a constant and flavour independent parameter $\sqrt{<k_0^2>} \equiv k_0$, assumed to be the same for all quark flavours and for gluons; we shall study the effect of changes in the value of $k_0$. Guided by our previous work, we compared the results obtained using three different values for $k_0$: $k_0 = 0.8$ GeV/c from studies on the unpolarized $pp$ scattering cross-sections and single spin asymmetries [7], $k_0 = 0.5$ GeV/c from fitting the Cahn effect in SIDIS [16], and $k_0 = 0.01$ GeV/c to recover the collinear configuration. For the fragmentation functions, we take $<k^2_{\perp C}> = <k^2_{\perp}>$ everywhere. We have checked that variations in $<k^2_{\perp C}>$ induce negligible changes in $A_{LL}$.

It turns out that the new non-collinear soft contributions containing the PDFs $\Delta f_{g/\perp}^q(x, k_{\perp})$ and $\Delta f_{g/\perp}^g(x, k_{\perp})$, even if maximized, are totally negligible. In fact, in the RHIC kinematical regime considered their maximized contribution does not exceed, in the lowest $p_T$ range, few percent of the usual terms (already present in the collinear case), becoming much smaller at larger $p_T$. We have checked that this result remains true also at lower energies. Let us remark that a similar situation holds also for the unpolarized cross-section [7]. Although the two additional terms (with respect to that already present in the collinear case) are of course different in this case [6], involving respectively the convolution of two Boer-Mulders functions with an unpolarized fragmentation function and the convolution of a Boer-Mulders function and an unpolarized distribution with the Collins fragmentation function, their total maximized contribution reaches at most 1% of the usual term, being even smaller on the average.
of allowing the partons to have non-zero intrinsic transverse momentum, and of including in

\section{Conclusions}

We have examined, at leading order in perturbative QCD, the effect on the longitudinal double spin asymmetry \( A_{LL} \) of allowing the partons to have non-zero intrinsic transverse momentum, and of including in \( A_{LL} \) the contributions
Figure 2: The invariant unpolarized cross-section for the process $pp \rightarrow \pi^0 X$ at $\sqrt{s} = 200$ GeV and $\eta = 0$, plotted as a function of $p_T$, calculated with different FF sets and factorization scales. The thick, solid and dashed lines correspond to the choice of the KKP FF set [31], at the factorization scale $Q = p_T$ and $Q = p_T/2$ respectively. The thin, solid and dashed lines correspond to the choice of the Kretzer FF set [32], at the factorization scale $Q = p_T$ and $Q = p_T/2$ respectively. The PDF set is GRV98 [29].

$\langle k^2_\perp \rangle \equiv k_0 = 0.5$ GeV/c for both PDFs and FFs. The experimental data are from the PHENIX collaboration at RHIC, Run 5 [19].

Figure 3: $A_{LL}$ for the process $pp \rightarrow \pi^0 X$ at $\sqrt{s} = 200$ GeV and $\eta = 0$, plotted as a function of $p_T$, calculated with different PDF sets and factorization scales. The solid and dashed lines correspond to the choice of PDFs GRSV2000/GRV98, at the factorization scale $Q = p_T/2$ and $Q = p_T$ respectively. The dotted and dash-dotted lines correspond the choice of PDFs LSS05/MRST01, at the factorization scale $Q = p_T/2$ and $Q = p_T$ respectively. The FF set is KKP. $\sqrt{\langle k^2_\perp \rangle} = 0.01$ GeV/c for both PDF/FF. The experimental data are from the PHENIX collaboration at RHIC, Run 5 [19].

arising from the new soft functions that play a crucial role in transverse single spin asymmetries. The study was carried out in the hope that such effects might negate the conclusion that the very small measured values of $A_{LL}$ automatically imply that the polarized gluon density is very small. Our analysis indicates that the contribution from these effects is negligible and we are forced, at the present stage, to accept the conclusion that the polarized gluon density is much too small to explain the “spin crisis in the parton model” [10].
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