Opinion dynamics: public and private

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We study here the dynamics of opinion formation in a society where we take into account the internally held beliefs and externally expressed opinions of the individuals, which are not necessarily the same at all times. While these two components can influence one another, their difference, both in dynamics and in the steady state, poses interesting scenarios in terms of the transition to consensus in the society and characterizations of such consensus. Here we study this public and private opinion dynamics and the critical behaviour of the consensus forming transitions, using a kinetic exchange model.

This article is part of the theme issue ‘Kinetic exchange models of societies and economies’.

1. Introduction

‘But If Everybody’s Watching, You Know, All Of The Back Room Discussions And The Deals, You Know, Then People Get A Little Nervous, To Say The Least. So, You Need Both A Public And A Private Position.’

—Hillary Clinton

It is common, not only among politicians but also for everyone else, to have an internally held belief and an externally expressed opinion, which are not always exactly the same. However, it is the latter that is revealed in opinion surveys and can be used as a proxy to gauge public perception about any issue. Therefore, a difference between the two can lead to a deceptive representation of public attitude. Such a scenario was indeed predicted back in 1981 [1]. In 2004/2005, the New Europe Barometer (NEB) survey in 13 countries showed that over half the population expressed fear in revealing their opinion [2].

More specific examples of the discrepancies between public and private opinions include the attitude towards
immigrants, both in the USA [3] and in Europe [4], which show an overall increase in the populist anti-immigrant views, which people’s perception towards changed very little. Peer pressure, unsubstantiated basis, etc., could be some of the causes for such a difference, which can lead to the so-called spiral of silence [5].

Nevertheless, it is also expected that the public and private opinions can also influence one another—if the externally expressed opinion becomes far removed from a firmly held internal belief, then it could be unsustainable in the long term for most people. On the other hand, while the difference exists, any conducted survey would necessarily predict a biased outcome, leading to outright wrong perceptions of the public opinion. It is interesting, therefore, to study some of these effects in a model.

Expressing opinions through numbers is a well-established practice [6–8]. Especially when the opinion values can take only binary outcomes—yes/no voting (e.g. Brexit [9]), two party elections, etc., it can be expressed by just ±1 and 0 representing the neutral population. Opinion values can also be continuous within a range (−1, +1), representing the strength of the bias towards two opposing ideologies.

The evolution of the opinion of any individual can happen through interactions or ‘exchanges’ with others. Given the well-connected nature of the social contacts, such exchanges can in principle happen with any other individual in the society. This leads to a wide range of studies involving what are called the ‘kinetic exchange models’ of opinion formation (e.g. [10–13]). Inspired by the wealth exchange model of a similar nature, the opinion formation model does not include any conservation of the total opinion, unlike the total wealth. Also unlike the wealth exchange model, it can show a spontaneous symmetry breaking transition, generally known to belong to the Ising universality class [14]. This can, however, change due to the topology, the states of the opinions considered and the number of agents taking part in a single interaction [15,16], where even a discontinuous transition can be observed. Discontinuous transitions can also be seen in q-voter models studied in various topologies, inertial effects and with anti-conformist agents [17–22].

Other than the phase transition in the steady states, the dynamics of such exchange models in general, have been widely studied—within a bounded confidence limit [23–25], with multiple types of individuals [26,27], showing coarsening and/or fragmentation of opinions. Indeed, the difference between public and private opinions were also studied in agent-based models [28], but with two different groups of individuals, classified according to their attitudes towards expressing public views.

Here we study an opinion dynamics model, where every individual (i) has their public ($o_i(t)$) and private ($P_i(t)$) opinion value at any time t, representing what they express externally and their internal belief, respectively. While the public opinion is subjected to the process of kinetic exchange, i.e. influencing others or getting influenced by the others, the privately held opinion is only subjected to one’s own conflict resolutions. These two components evolve with a coupled dynamics. There have been a few earlier attempts in modelling publicly expressed opinions and privately held beliefs [29–32]. In what follows, we define the co-evolution dynamics of these components in the model, their transitions to consensus and the corresponding exponent values, and the influences on the outcomes of any intermediate opinion survey.

2. Model

We follow here a kinetic exchange model of opinion, but it has a public and a private component. The public opinion values evolve following the kinetic exchange rule:

$$o_i(t + 1) = o_i(t) + \mu_{ij}o_j(t),$$  \hspace{1cm} (2.1)

where $\mu_{ij} = -1$ with probability $p$ and +1 otherwise and are annealed, i.e. not fixed in time. The individual opinion values are bounded between the extreme values ±1, i.e. if the equation prescribes $o_i(t + 1)$ to be higher than +1 or lower than −1, then it is just kept fixed at −1 or +1, respectively.
As for the private opinion values, it is expected that a change in the public opinion is necessarily due to a limited conviction on the earlier opinion, therefore a part of the change will also influence the private opinion:

\[ P_i(t + 1) = P_i(t) + k(o_i(t + 1) - o_i(t)), \]

where \( k \) is a parameter. Finally, if the difference between the public and private opinion values for a given individual is too high, then it can become unsustainable. In that case, the sign of the private opinion is assigned as the public opinion value:

\[ o_i(t) = \text{Sgn}(P_i(t)) \quad \text{if} \quad |P_i(t) - o_i(t)| > \delta_i, \]

where \( \delta_i \) is a tolerance parameter for an individual. The selections of the two agents for an interaction (\( i \) and \( j \)) are random, i.e. any two agents can interact at a given time. While it is known that the number of active social contacts for an individual is between 100 and 200 (Dunbar number [33]) and that this number is also obtained even in social media contacts [34], the number is large enough to be approximated by the mean-field interaction assumed here.

For a large value of the tolerance parameter, it is expected that the public opinion dynamics is completely independent of the private opinion values and therefore should give the known mean-field transition at \( p_c = 1/4 \) [11]. The influence, tolerance and conviction parameters are indeed the relevant variables that are looked at in social sciences, in view of the difference between public and private opinions [35].

The dynamics of the model evolves as follows: initially, \( o_i(t = 0) \) are all assigned \( \pm 1 \) values randomly with equal probability. The initial private opinion values \( P_i(t = 0) \), on the other hand, are assigned a continuous value between \((-1, +1)\) with uniform probability. Both \( o_i(t) \)s and \( P_i(t) \)s are bounded within \(-1\) and \(+1\), i.e. if a higher value is obtained by the evolution equations described above, the values are limited to the extreme values.

A single Monte Carlo time step is defined as \( N \) updates of two randomly chosen individuals, where \( N \) is the number of individuals in the society.

3. Results

Here we describe the dynamics and steady-state properties of the model with the evolution rules mentioned above and for different values of the parameters \( p \) (probability of negative public interaction), \( k \) (internal influence parameter) and \( \delta \) (tolerance parameter). As mentioned above, the model is simulated in the mean field limit, i.e. a fully connected graph with \( N \) individuals. For simplicity, we keep \( \delta_i = \delta \), i.e. the tolerance parameter is the same for all individuals.

(a) Phase diagrams

An important quantity to measure is the average opinion value of the entire society, which indicates the formation or the lack of consensus in the society. In this case, of course, there are two such measures—in the private and in the public opinions. We argue that while the private opinion values matter in the case of a voting through secret ballots, an opinion survey would only reveal the public opinion. Therefore, any difference in the values of these two quantities, either during the dynamics or in the steady state, would lead to a misleading interpretation of the public perception about an issue.

The average opinion value of the public opinion, which is also generally used as the order parameter for the transition towards consensus, is given by

\[ O(t) = \frac{1}{N} \left| \sum_i o_i(t) \right|, \]

where
Figure 1. The steady-state value of the average public opinion is shown in the $p-\delta$ plane for three different values of $k$, $k = 0.5$ in (a), 1.0 in (b) and 1.5 in (c). High values of $p$ always lead to disordered state. For high values of the tolerance parameter $\delta$, the critical point is the usual $p_c = 1/4$, but for lower $\delta$ values disorder sets in even earlier. For higher values of $k$, the disorder in the private opinion values prevail for low $\delta$, but for high $\delta$, again an order–disorder transition can be seen. (Online version in colour.)

Figure 2. The steady-state value of the average private opinion is shown in the $p-\delta$ plane for three different values of $k$, $k = 0.5$ in (a), 1.0 in (b) and 1.5 in (c). In comparing with figure 1, it is seen that the ordering is in the same ranges of the parameters for both the public and the private opinion values. This means that in the steady state, there is no discrepancy between an election and an opinion survey. (Online version in colour.)

and similarly for the private opinions, one can define

$$Q(t) = \frac{1}{N} \left| \sum_{i} P_i(t) \right|.$$  \hspace{1cm} (3.2)

In figure 1, we show the phase diagram of the model in terms of the public opinion values mentioned in equation (3.1). There are several features to be noted. First, for high values of $\delta$, the public opinion dynamics essentially is decoupled from the private one. This is due to a high tolerance of the individuals for the difference between their public and private view. In this case, a critical point at $p_c = 1/4$ is retrieved, as is analytically known for the original version of the model [11]. However, for a smaller $\delta$, disorder sets in for lower $p$, due to the influence of the private opinion. For high values of $k$, the influence of the private opinion is so high that for any non-zero $p$ disorder sets in. However, even in this case, when $\delta$ is large, the usual order–disorder transition is retrieved.

Figure 2 depicts the phase diagram in terms of the average private opinion described in equation (3.2). It is seen that in the steady state, the public and private opinion values show consensus in the same regions of the phase diagram. However, to check if there is any difference in the public and private opinion values in the steady state, in figure 3 we plot the difference values in the $p-\delta$ plane. It is seen that a difference exists between the two measures. This is significant, because it indicates a discrepancy between an election result, where the private
Figure 3. The steady-state value of the average private opinion is shown in the $p$–$\delta$ plane for three different values of $k$, $k = 0.5$ in (a), 1.0 in (b) and 1.5 in (c). In comparing with figure 1, it is seen that the ordering is in the same ranges of the parameters for both the public and the private opinion values. This means that in the steady state, there is no discrepancy between an election and an opinion survey. (Online version in colour.)

opinion is reflected and an opinion survey, where mostly the public opinion is reflected. Indeed, a small variation can lead to completely opposite results, due to the coarse-graining effects that are sometimes accompanied with these binary type voting processes (e.g. [36]).

(b) Critical exponents and finite size scaling

As mentioned before, the transition seen for the high values of $\delta$ (the vertical line in the phase boundary) is the already known in the original version of the model. We need to check the transition seen under the influence of smaller $\delta$ values.

Other than the order parameter, its fluctuations near the critical point also reveal the associated critical exponent values. Particularly, the Binder cumulant, defined as $U = 1 - \langle O^2 \rangle / 3 \langle O^2 \rangle^2$, and the ‘susceptibility’ $V = N(\langle O^2 \rangle - \langle O \rangle^2)$ are useful quantities to look for finite size scaling. The angular brackets denote configuration average. The finite size scaling form for the Binder cumulant is $U \sim f_1[(p - p_c)N^{1/\nu}]$, and that for the ‘susceptibility’ is $V \sim Na^\gamma f_2[(p - p_c)N^{1/\nu}]$ and finally for the order parameter: $O \sim N^{-\beta/v} f_3[(p - p_c)N^{1/\nu}]$, where $\beta$, $\nu$ and $\gamma$ are critical exponents. The advantage of the Binder cumulant is that all the curves for different values of $N$ cross through the point $p = p_c$, giving a chance to determine the critical point numerically accurately. This value of the critical point can then be used for the subsequent finite size scalings mentioned above.

In figure 4, the Binder cumulants for different system sizes are shown for two sets of $(\delta, k)$ values. The crossing points of the Binder cumulants give the critical point and the finite size scaling exponent $\nu = 2$ is obtained from the collapse.

In figure 5, the finite size scaling of the susceptibilities is shown. Using the critical points obtained from figure 4, the scaling exponent $\gamma = 1$ is seen.

Figure 6 depicts the variation of the order parameter (equation (3.1)) for two sets of $k$, $\delta$ values. For $k = 0.25$ and $\delta = 0.05$, the transition with respect to $p$ shows an exponent that is different from the usual mean field value. However, for $k = 1.50$ and $\delta = 0.60$, the mean field exponent value is seen.

We then do the finite size scaling analysis for these two sets. We assume a finite size scaling of the form:

$$O \sim N^{-\beta/v} F[(p - p_c)N^{1/\nu}],$$

(3.3)

where $\nu$ is the effective correlation length exponent. The finite size scaling analysis is shown in figure 7. For the higher value of $\delta$, the exponents seem to differ from the mean field values. Otherwise, the mean field critical exponents are retrieved. In these cases also, we use the critical
Figure 4. The variations of the Binder cumulant for different system sizes are shown for $k = 0.25$, $\delta = 0.05$ (giving $p_c = 0.121 \pm 0.002$) and $k = 1.5$, $\delta = 0.6$ (giving $p_c = 0.195 \pm 0.002$). In both cases, the curves collapse for $\nu = 2$. (Online version in colour.)

Figure 5. The variations of the susceptibility for different system sizes are shown for $k = 0.25$, $\delta = 0.05$ and $k = 1.5$, $\delta = 0.6$. Using the critical points obtained from figure 4, the finite size scaling analysis is done, which gives $\gamma \approx 1$ (using $\nu = 2$). (Online version in colour.)

Figure 6. The variation of the order parameter is shown for two combinations of the $k$ and $\delta$ values ($N = 2048$). The exponent value seems to depend on the parameter set for low values of $k$. For high values, the mean field exponent is retrieved.

points obtained from the crossing points of the Binder cumulant in figure 4. It is important to note here that the exponent values, obtained through finite size scaling analysis, remain unchanged when calculated from the order parameter of average private opinion ($Q$).
Figure 7. The finite size scaling forms are shown for two different sets of $k, \delta$ values. With the scaling form taken from equation (3.3), the exponent values for the set $k = 1.5$ and $\delta = 0.6$ are $\beta = 0.35 \pm 0.02$ and $\nu = 2.00 \pm 0.02$. For the set $k = 0.25$, $\delta = 0.05$, the exponent values are $\beta = 0.50 \pm 0.02$ and $\nu = 2.00 \pm 0.02$. (Online version in colour.)

Figure 8. The variations of $O$ and $Q$ with $\delta$ for a fixed value of $p = 0.05$ and $k = 1.5$. Both the quantities show a discontinuous jump, suggesting a first-order transition along this line. (Online version in colour.)

Figure 9. The variations of $O(t)$ and $Q(t)$ with $t$, for three sets of parameter values (a) $(\delta = 0.1, k = 1.0, p = 0.1)$, (b) $(\delta = 0.2, k = 1.0, p = 0.1)$ and (c) $(\delta = 0.3, k = 1.0, p = 0.1)$. In all cases, the difference between $O(t)$ and $Q(t)$ persists throughout the dynamics. (Online version in colour.)

We also studied the transition with respect to $\delta$, for a fixed value of $p$ for $k = 1.5$. The average values of both the public and private opinion values show a discontinuous jump, suggesting a first-order transition along this line (figure 8).
Finally, we look at the dynamics of $O(t)$ and $Q(t)$ to see if the difference noted in figure 3 also can be seen during the dynamics. In figure 9, we show this for three sets of parameter values ($\delta = 0.1, k = 1.0, p = 0.1$), ($\delta = 0.2, k = 1.0, p = 0.1$) and ($\delta = 0.3, k = 1.0, p = 0.1$). In all cases, we see that the difference persists during the entire dynamics of the model. This again implies that even at any intermediate time, a survey and an election result might differ, no matter how good the survey statistics are.

### 4. Conclusion

The differences between the public position and privately held belief is a common feature in human behaviour [1,2]. In political scenarios, involving binary choice elections and their preceding opinion surveys, such difference can play a crucial role in determining the accuracy and subsequent credibility of such surveys. Indeed, it is known that common perceptions of the attitude of the general public are known to differ from their individual beliefs [3,4]. In this work, we attempted to model the dynamics of those two components of the opinion of the individuals in a society through a simple kinetic exchange model. The public and private opinion values co-evolve in a coupled manner, influencing one another during the course of dynamics. If the difference between the two is too high, given by a tolerance factor, then the private opinion dominates.

We show, however, that even though a consensus is spontaneously reached in the society through such dynamics for both these components in the same part of the phase diagram, these two components continue to differ statistically significantly throughout the dynamics and in the steady state of the model (figures 3 and 9). Such a difference is crucial, especially in closely fought elections, in terms of predicting the outcome of such an election through an opinion survey.

The order of the phase transition to consensus depends on the parameter $k$ (coupling between the private and public opinion values). While the kinetic exchange opinion models are known to show Ising universality [14], in the continuous phase transition for this model, the exponent value of the order parameter differs from the Ising universality class for larger values of the tolerance parameter $\delta$.

In conclusion, the co-evolution of publicly expressed and privately held opinions of interacting agents in a society can produce a difference between these two quantities, which is known to exist in society. This can shed light on the differences observed in opinion surveys and election results, and further studies regarding the co-evolution dynamics and their possible hysteresis behaviour near the discontinuous transition could be useful in explaining the opposite behaviour of the public perception and election outcomes.

**Data accessibility.** All data generated through simulations of the model described in the paper. The data used in this manuscript are from the simulations of the model described here. The code for those simulations is available in the following link: https://github.com/sroy2807/Opinion-Dynamics.

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