Nuclear $\beta$-decay measurements and $|V_{ud}|$

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Some recent work in nuclear $\beta$ decay related to the value of $|V_{ud}|$ is described along with some near-term goals for future measurements.

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1 Isospin \( T = 1 \) Superallowed Decays

The comparative half-lives – or \( ft \) values – of the \( \beta \) decay of nuclei, specifically \( I^\pi = 0^+ \rightarrow 0^+ \) isospin \( T = 1 \) pure Fermi transitions, provide the most precise value of \( |V_{ud}| \) to date [1]. Nevertheless, a number of groups at a number of facilities continue to improve/confirm the experimental inputs going into the determination of \( |V_{ud}| \): the decay energy for the phase-space factor, \( f \), as well as the half-life and \( \beta \) branch for \( t \). Most notably, recent work on mass measurements using Penning traps have improved the \( Q_{EC} \) values of some of these transitions. In 2005 using the Canadian Penning trap, the group at Argonne National Laboratory measured the value for \( ^{46}\text{V} \) and found a large discrepancy [2] from earlier measurements based on \(^3\text{He},t\) reactions [3]. This result was confirmed using the JYFLTRAP Penning trap at the University of Jyväskylä [4], as well as by a new \(^3\text{He},t\) reaction measurement [5] using a very similar set-up as was originally done. Upon further investigation, Penning trap measurements also found discrepancies in the cases of \( ^{42}\text{Sc}, ^{50}\text{Mn} \) and \( ^{54}\text{Co} \) [6] and led to Penning trap campaigns at ANL, Jyväskylä, ISOLTRAP at ISOLDE [7] and LEBIT at the National Superconducting Labotory [8] to check and/or improve the \( Q_{EC} \) values of all the \( T = 1 \) superallowed transitions used to determine \( |V_{ud}| \). Other recent Penning trap measurements which have improved the \( Q_{EC} \) value (but don’t show a deviation from earlier values) include \(^{18}\text{Ne}, ^{22}\text{Mg}, ^{26}\text{Al}\), \(^{26}\text{Si}, ^{30}\text{S}, ^{34}\text{Ar}, ^{34}\text{Cl}, ^{38}\text{Ca}, ^{38}\text{K}\), \(^{42}\text{Ti}, ^{62}\text{Ga}, ^{66}\text{As} \) and \(^{74}\text{Rb} \).

The value of \( |V_{ud}| \) is calculated based on the average of the \( "ft" \) values,” which are the observed \( ft \) values corrected for small radiative and nuclear-structure dependent effects. The latest review of the superallowed determination of \( |V_{ud}| \) by Hardy and Towner [1], which describes this process in greater detail, includes many of these mass measurements described above and discusses the implications. They find

\[
|V_{ud}| = 0.97425(21)_{\text{theor}}(8)_{\text{exp}}
\]

The transition-independent part of the radiative correction (common to any evaluation of \( |V_{ud}| \)) is the dominant uncertainty, contributing \( 18 \times 10^{-5} \) to the error budget. Note that this is twice as large as the uncertainty due to experiment.

While the \( T = 1 \) superallowed decays are the most precise, they are not the only manner in which \( |V_{ud}| \) can be determined. Furthermore, as discussed in Ref. [1] (see also I.S. Towner’s article in these proceedings), there appears to be some model-dependence to the theoretical corrections used to calculate the \( ft \) values from the observed \( ft \) values. It is therefore important to compliment the value of \( |V_{ud}| \) obtained from \( T = 1 \) transitions with other methods; even if these methods are not as precise, they can test the theoretical corrections used and provide new values of \( |V_{ud}| \) which have completely different systematic uncertainties. Agreement between the complementary methods would help reduce concerns about the theoretical corrections that are applied.
Figure 1 shows the chart of the nuclides, highlighting the radioactive nuclei that are of interest to determining the value of $|V_{ud}|$. Within the $T = 1$ transitions, a group at TRIUMF recently measured the branching ratio of $^{62}$Ga and interpreted their results as a test of the isospin-mixing corrections [9]. They found that calculations overestimate the effect, thus indicating a bigger shell-model space is required in this and other heavy ($A \geq 62$) nuclei. Other work at TRIUMF that is currently in progress includes improving the half-life of $^{26}$Al and using a novel charge-breeding electron ion beam trap with a Penning trap to measure the mass of $^{74}$Rb [12]. At Jyväskylä, their program continues with half-life measurements of $^{29}$P, $^{31}$S (also it’s branching ratio) and $^{39}$Ca. The Cyclotron Institute at Texas A&M University is analyzing data on half-life measurements of $^{10}$C and $^{26}$Si, and a recently completed analysis of the branching ratio in $^{32}$Cl validates the shell-model prediction of isospin-mixing in the $s, d$-shell [10]. All of these programs combined will reduce uncertainties in the $Ft$ values of $T = 1$ superallowed transitions, allowing a more reliable value of $|V_{ud}|$ to be determined.

![Figure 1: (Colour online) Chart of the nuclides showing superallowed ($T = 1, 2$) and mirror transitions ($T = 1/2$) relevant to determining the value of $|V_{ud}|.$]
2  \( T = 2 \) Superallowed Decays

Hardy and Towner [1] have compared their Woods-Saxon potential based shell-model calculations of isospin-mixing corrections to one based on Hartree-Fock eigenfunctions, and they find indications of a dependence on isospin: the average difference in the predicted correction between these two models for \( T_3 = -1 \) cases is larger than for the \( T_3 = 0 \) transitions. By going to higher \( T \) multiplets, a comparison of these models may be tested with enhanced sensitivity. Recently, a measurement of the isospin-mixing in \(^{32}\text{Ar}\) was made [13] by improving the branching ratio of this isospin \( T = 2 \) superallowed decay. This proton-rich nucleus \( \beta^+ \) decays to a proton-unbound state in \(^{32}\text{Cl}\), thus requiring a measure of the proton and \( \gamma \) branches from the \( 0^+ \) excited state in \(^{32}\text{Cl}\). The total uncertainty in the \( \mathcal{F}t \) value of this decay remains dominated by the branching ratio, and we have plans to improve it using a large-bore, open-geometry cyclindrical Penning trap to be built at the Cyclotron Institute, Texas A&M University. Once this technique is proven for \(^{32}\text{Ar}\), there are a number of other \( T = 2 \) nuclei which can be produced at the Cyclotron Institute and decay in a similar manner: \(^{20}\text{Mg}, \; ^{24}\text{Si}, \; ^{28}\text{S}, \; ^{36}\text{Ca} \) and \(^{40}\text{Ti}\). Using a \(^3\text{He}\) target and standard projectile beams, LISE calculations indicate rates into the Penning trap ranging from \( 1 \times 10^3/\text{s} \) (\(^{20}\text{Mg}\)) up to a few times \( 10^6/\text{s} \) (\(^{32}\text{Ar}, \; ^{36}\text{Ca} \) and \(^{40}\text{Ti}\)). Even the lowest rate \(^{20}\text{Mg}\) is enough to mount a precision experiment due to the extremely low backgrounds of the Penning trap. Once the \( \mathcal{F}t \) values of these nuclei are measured precisely, there will be six new superallowed decays which can help test models of isospin-mixing. Given acceptance of a model for these proton-rich cases, one could add them to the list of transitions used to extract \( |V_{ud}| \); this would have a higher impact on reducing the uncertainty in \( |V_{ud}| \) than improving already very precise measurements in the \( T = 1 \) cases.

3  \( T = 1/2 \) Transitions

3.1 Neutron decay

The neutron is theoretically the simplest nuclear system from which one may deduce \( |V_{ud}| \). Unfortunately, it is difficult to measure accurately its long lifetime and, since it has a Gamow-Teller component to its decay, one must also measure an angular correlation parameter to extract \( |V_{ud}| \). The contribution of B. M"arkisch in these proceedings discusses the neutron in depth, so we will not discuss it further here.

3.2 Mirror transitions

As pointed out recently by Navialiat-Cuncic and Severijns [14], the \( \mathcal{F}t \) values of \( T = 1/2 \) \( \beta \) decays between isobaric analogue states may be used as a new avenue to
deduce $|V_{ud}|$. With the same theoretical treatment used to calculate corrections to the $T = 1$ superallowed decays applied to these mirror transitions in Ref. [15], one may survey the data for $T = 1/2$ decays and extract an independent value for $|V_{ud}|$. As with the neutron, a correlation parameter is required in addition to the $\mathcal{F}t$ value in order to determine $|V_{ud}|$. Specifically, the master equation used to determine $|V_{ud}|$ from mirror transitions is:

$$|V_{ud}|^2 = \frac{5831.3 \pm 2.3 \text{ s}}{\mathcal{F}_t \text{mirror} \left( 1 + \frac{f_A}{f_V} \rho^2 \right)}, \quad (2)$$

where $f_A/f_V$ is the ratio of statistical rate functions for axial/vector currents (which ranges from 0.988–1.04, but typically only differ from unity by less than 2%), and $\rho = C_A M_{GT}/C_V M_F$ is the ratio of Gamow-Teller to Fermi strengths for the decay. The correlation typically used to determine $\rho$ is the $\beta$ asymmetry (much like that correlation is used in neutron decay to determine $\lambda$), however other correlations can and have been used (e.g. the neutrino asymmetry and the $\beta - \nu$ correlation).

To date, only five cases of $T = 1/2$ mirror transitions have their $\mathcal{F}t$ value and a correlation parameter measured to allow a determination of $|V_{ud}|$: $^{19}\text{Ne}$, $^{21}\text{Na}$, $^{29}\text{P}$, $^{35}\text{Ar}$ and $^{37}\text{K}$. A plot of these results is shown in Fig. 2. To help indicate that the correlation parameter measurements are currently limiting the extraction of $|V_{ud}|$ (rather than the $\mathcal{F}t$ values or theoretical uncertainties), the inner error bars show what the uncertainty would be if $\rho$ was known to perfect precision; clearly it is this avenue experimentalists must pursue if we are to improve the value of $|V_{ud}|$ from mirror transitions.

Although not relevant to $|V_{ud}|$, it should be noted that improved measurements of the $\mathcal{F}t$ values of these and other mirror transitions is interesting for other Standard Model tests. By measuring the comparative half-life to a greater precision, one may assume that the $T = 1$ average $\mathcal{F}t$ value is correct and use that to deduce $\rho$ for the mirror transitions (i.e. re-arrange Eq. (2) and solve for $\rho$). One can then calculate the Standard Model prediction of the correlation parameters and compare them to experimentally observed values; new physics such as right-handed currents, second-class currents, leptoquarks, etc., would affect the value of the correlation parameters that one may be able to detect. The interested reader is referred to Refs. [15] and [16].

As an example of recent work with these nuclei, our group at the Cyclotron Institute (and also at Kernfysisch Versneller Instituut in Groningen) is completing analysis of a $^{37}\text{K}$ lifetime measurement which will reduce the uncertainty in $\rho$ by about an order of magnitude. This will improve the value of $\rho$ if one assumes that the average $\mathcal{F}t$ values of $T = 1$ decays is correct, allowing more definite predictions of the correlation parameters; however, it does essentially nothing to improve the measured $|V_{ud}|$ from $^{37}\text{K}$ because the 3% measurement of the neutrino asymmetry parameter [17] totally dominates the measured value of its $\mathcal{F}t$ value. We are planning to measure the $\beta$
Figure 2: (Colour online) Measurements of $|V_{ud}|$ from mirror transitions (adapted from Ref. [14]). The average value from these five cases where a correlation has been measured yields $|V_{ud}|_{\text{mirror}} = 0.9719(18)$, which is already of the same precision as the neutron (using the PDG values). The inner red error bars show the statistical uncertainty in each measurement of $|V_{ud}|$ excluding the uncertainty in $\rho$. The dashed green line shows the $1\sigma$ allowed value of $|V_{ud}|$.

The asymmetry parameter from this decay to $\lesssim 0.5\%$ using the magneto-optical trap and high production rates of $^{37}$K at TRIUMF by early 2012. This measurement combined with the improved lifetime will greatly reduce the total uncertainty in the $^{37}$K point of Fig. 2 and therefore improve the value of $|V_{ud}|$ from mirror decays as a complement to the value obtained from the $T = 1$ superallowed transitions.

4 Summary and Outlook

The experimental and theoretical investigations into the value of $|V_{ud}|$ from the nuclear physics community continues to be a vibrant field. Ever more precision measurements of the traditional $T = 1$ superallowed decays are being made; re-measurements of the masses using Penning trap mass spectrometers have led to discovering a small bias in older measurements, but recently most emphasis has been on trying to measure the isospin-mixing corrections in nuclei to test the theoretical corrections used to extract $|V_{ud}|$.

There are very recent measurements of the neutron asymmetry from the PERKEO
and UCNA collaborations, with more experiments planned to finally determine the value of $\lambda$. In addition, a number of lifetime measurements are planned to resolve the outstanding $8\sigma$ discrepancy in previous neutron lifetimes with the recent result of Serebrov et al. Once all these experiments produce results, we can expect the value of $|V_{ud}|$ from neutron decay to improve dramatically and meaningfully compliment that of the $T=1$ superallowed decays.

Finally, a new avenue of other $T=1/2$ mirror transitions has recently become available as another nuclear measurement of $|V_{ud}|$ now that the isospin and radiative corrections for these nuclei have been calculated. Already with even just a few cases measured to $\lesssim 0.5\%$, the precision of $|V_{ud}|$ from these decays is at the same level of precision as the neutron. The nuclear physics community will also continue to measure correlation parameters in these decays which will provide yet another important, complimentary measurement of $|V_{ud}|$.

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References

[1] J.C. Hardy and I.S. Towner, Phys. Rev. C79, 055502 (2009).
[2] G. Savard et al., Phys. Rev. Lett. 95, 102501 (2005).
[3] H. Vonach et al., Nucl. Phys. A278, 189 (1977).
[4] T. Eronen et al., Phys. Rev. Lett. 97, 232501 (2006).
[5] T. Faestermann et al., Eur. Phys. J. A42, 339 (2009).
[6] T. Eronen et al., Phys. Rev. Lett. 100, 132502 (2008).
[7] S. George et al., Europhys. Lett. 82, 50005 (2008).
[8] R. Ringle et al., Phys. Rev. Lett. 96, 152501 (2006).
[9] P. Finlay et al., Phys. Rev. C 78, 025502 (2008).
[10] D. Melconian et al., submitted to Phys. Rev. C.
[11] P. Finlay, private communication.
[12] J. Dilling, private communication.

[13] M. Bhattacharya et al., Phys. Rev. C 77, 065503 (2008).

[14] O. Naviliat-Cuncic and N. Severijns, Phys. Rev. Lett. 102, 142302 (2009).

[15] N. Severijns et al., Phys. Rev. C 78, 055501 (2008).

[16] S. Profumo, M.J. Ramsey-Musolf and S. Tulin, Phys. Rev. D 75, 075017 (2007).

[17] D. Melconian et al., Phys. Lett. B 649, 370 (2007).