On the uniqueness of $D = 11$ interactions among a graviton, a massless gravitino and a three-form.

II: Three-form and gravitini

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Abstract

The interactions that can be introduced between a massless Rarita-Schwinger field and an Abelian three-form gauge field in eleven spacetime dimensions are analyzed in the context of the deformation of the “free” solution of the master equation combined with local BRST cohomology. Under the hypotheses of smoothness of the interactions in the coupling constant, locality, Poincaré invariance, Lorentz covariance, and the presence of at most two derivatives in the Lagrangian of the interacting theory (the same number of derivatives like in the free Lagrangian), we prove that there are neither cross-couplings nor self-interactions for the gravitino in $D = 11$. The only possible term that can be added to the deformed solution to the master equation is nothing but a generalized Chern-Simons term for the three-form gauge field, which brings contributions to the deformed Lagrangian, but does not modify the original, Abelian gauge transformations.

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1 Introduction

It is known that the field content of $D = 11$, $N = 1$ supergravity is remarkably simple; it consists of a graviton, a massless Majorana spin-3/2 field, and a three-form gauge field. The analysis of all possible interactions in $D = 11$ related to this field content necessitates the study of cross-couplings involving each pair of these sorts of fields and then the construction of simultaneous interactions among all the three fields. With this purpose in mind, in Ref. [1] we have obtained all consistent interactions that can be added to a free theory describing...
a massless spin-two field and an Abelian three-form gauge field in eleven spacetime dimensions. Here, we develop the second step of our approach and analyze the consistent eleven-dimensional interactions that can be introduced between a massless Rarita-Schwinger field and an Abelian three-form gauge field. Our main result is that under the hypotheses of smoothness of the interactions in the coupling constant, locality, Poincaré invariance, Lorentz covariance, and the presence of at most two derivatives in the Lagrangian of the interacting theory (the same number of derivatives like in the free Lagrangian) there are neither cross-couplings nor self-interactions for the gravitino in $D = 11$. The only possible term that can be added to the deformed solution to the master equation is nothing but a generalized Chern-Simons term for the three-form gauge field, which brings contributions to the deformed Lagrangian, but does not modify the original, Abelian gauge transformations. Our result does not contradict the presence in the Lagrangian of $D = 11$, $N = 1$ SUGRA of a quartic vertex expressing self-interactions among the gravitini. We will see in Refs. [2] and [3] that this vertex, which appears at order two in the coupling constant, is due to the simultaneous presence of gravitini, three-form, and graviton.

2 Free model: Lagrangian formulation and BRST symmetry

Our starting point is represented by a free model, whose Lagrangian action is written like the sum between the standard action of an Abelian three-form gauge field and that of a massless Rarita-Schwinger field in eleven spacetime dimensions

$$S_0^{L} [A_{\mu\nu\rho}, \psi_\mu] = \int d^{11} x \left( \frac{-1}{2 \cdot 4!} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} - \frac{i}{2} \bar{\psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho \right)$$

$$\equiv \int d^{11} x \left( L_A^0 + L_\psi^0 \right),$$

(1)

where $F_{\mu\nu\rho\lambda}$ denotes the field strength of the three-form gauge field ($F_{\mu\nu\rho\lambda} = \partial_{[\mu} A_{\nu\rho\lambda]}$). We maintain the antisymmetrization convention explained in part I [1] and work with that representation of the Clifford algebra

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2 \sigma_{\mu\nu} 1$$

(2)

for which all the $\gamma$ matrices are purely imaginary. In addition, we take $\gamma_0$ to be Hermitian and antisymmetric and $(\gamma_i)_{i=1,10}$ anti-Hermitian and symmetric

$$\gamma_\mu^* = -\gamma_\mu, \quad \gamma_\mu^\dagger = -\gamma_0 \gamma_\mu \gamma_0, \quad \mu = 0, 10.$$  

(3)

(4)

The operations of Dirac and respectively Majorana conjugation are defined as usually via the relations

$$\bar{\psi}_\mu = (\psi_\mu)^\dagger \gamma_0,$$

(5)
\[ \psi^c = (C\psi)^T, \]  

where the charge conjugation matrix is

\[ C = -\gamma_0. \]

In what follows we use the notations

\[ \gamma_{\mu_1 \cdots \mu_k} = \frac{1}{k!} \sum_{\Theta \in \Sigma_k} (-)^\Theta \gamma_{\mu_{\Theta(1)}} \gamma_{\mu_{\Theta(2)}} \cdots \gamma_{\mu_{\Theta(k)}}, \]

where \( \Sigma_k \) represents the set of permutations of the numbers \( \{1, 2, \ldots, k\} \) and \( (-)^\Theta \) is the signature of a given permutation \( \Theta \). We will need the Fierz identities specific to \( D = 11 \)

\[ \gamma_{\mu_1 \cdots \mu_p} \gamma_{\nu_1 \cdots \nu_q} = \sum_{p+q-11 \leq 2k \leq 2M} \delta_{[\mu_1} \delta_{\nu_2} \cdots \delta_{\mu_{p-1]} \nu_{p+1}} \gamma_{\mu_{p+1} \cdots \nu_{q}], \]

where \( M = \min(p, q) \) and also the development of a complex, spinor-like matrix \( N \) in terms of the basis \( \{1, \gamma_\mu, \gamma_{\mu\nu}, \gamma_{\mu\nu\rho}, \gamma_{\mu\nu\rho\lambda}, \gamma_{\mu\nu\rho\lambda\sigma}\} \)

\[ N = \frac{1}{32} \sum_{k=0}^{5} (-)^{k(k-1)/2} \frac{1}{k!} \text{Tr} \left( \gamma_{\mu_1 \cdots \mu_k} N \right) \gamma_{\mu_1 \cdots \mu_k}. \]

The theory described by action (1) possesses an Abelian, off-shell, second-order reducible generating set of gauge transformations

\[ \delta_\varepsilon A_{\mu\nu\rho} = \partial_{[\mu} \varepsilon_{\nu\rho]}, \quad \delta_\varepsilon \psi_{\mu} = \partial_{\mu} \varepsilon. \]

Related to the gauge parameters, \( \varepsilon_{\mu\nu} \) are bosonic and completely antisymmetric and \( \varepsilon \) is a fermionic Majorana spinor. The fact that the gauge transformations of the three-form gauge field are off-shell, second-order reducible is treated in more detail in Ref. [1].

In order to construct the BRST symmetry for (1) we introduce the field, ghost, and antifield spectra

\[ \Phi_{\Delta_0} = (A_{\mu\nu\rho}, \psi_{\mu}), \quad \Phi_{\Delta_0}^* = (A^{*\mu\nu\rho}, \psi^{*\mu}), \quad \eta_{\Delta_1}^{\mu} = (C_{\mu}, \xi), \quad \eta_{\Delta_1}^{\mu} = (C^{*\mu}, \xi^*), \]

\[ \eta_{\Gamma_2} = (C_{\mu}), \quad \eta_{\Gamma_2} = (C^{*\mu}), \quad \eta_{\Gamma_3} = (\gamma_{\mu}), \quad \eta_{\Gamma_3}^* = (\gamma^{*\mu}). \]

The fermionic ghosts \( C_{\mu\nu} \) correspond to the gauge parameters of the three-form, \( \varepsilon_{\mu\nu} \), the bosonic ghost \( \xi \) is associated with the gauge parameter \( \varepsilon \), while the bosonic ghosts for ghosts \( \eta_{\Gamma_2} \) and the fermionic ghost for ghost for ghost \( \eta_{\Gamma_3}^* \) are due to the first- and respectively second-order reducibility of the gauge transformations from the three-form sector. The star variables represent the antifields of the corresponding fields/ghosts. The antifields of the Rarita-Schwinger field
are bosonic, purely imaginary spinors. Since the gauge generators of the free
theory under study are field independent, it follows that the BRST differential
decomposes again like in Ref. [1]

\[ s = \delta + \gamma, \]

where \( \delta \) represents the Koszul-Tate differential and \( \gamma \) stands for the exterior
derivative along the gauge orbits. (More details of the various graduations of
the BRST generators can be found in Ref. [1].) In agreement with the standard
rules of the BRST formalism, the degrees of the BRST generators are valued
like

\[ \text{agh} (\Phi^{\Delta_0}) = \text{agh} (\eta^{\Delta_1}) = \text{agh} (\eta^{\Gamma_2}) = \text{agh} (\eta^{\Gamma_3}) = 0, \]

\[ \text{agh} (\Phi^{\Delta_0}_a) = 1, \quad \text{agh} (\eta^{\Delta_1}_a) = 2, \quad \text{agh} (\eta^{\Gamma_2}_a) = 3, \quad \text{agh} (\eta^{\Gamma_3}_a) = 4, \]

\[ \text{pgh} (\Phi^{\Delta_0}) = 0, \quad \text{pgh} (\eta^{\Delta_1}) = 1, \quad \text{pgh} (\eta^{\Gamma_2}) = 2, \quad \text{pgh} (\eta^{\Gamma_3}) = 3, \]

\[ \text{pgh} (\Phi^{\Delta_0}_a) = \text{pgh} (\eta^{\Delta_1}_a) = \text{pgh} (\eta^{\Gamma_2}_a) = \text{pgh} (\eta^{\Gamma_3}_a) = 0. \]

The actions of the differentials \( \delta \) and \( \gamma \) on the generators from the BRST complex
are given by

\[ \delta A^{\mu \nu \rho} = \frac{1}{3!} \partial_\lambda F^{\mu \nu \rho \lambda}, \quad \delta \psi^{* \mu} = -i \partial_\rho \bar{\psi}_\lambda \gamma^{\rho \lambda \mu}, \]

\[ \delta C^{* \mu \nu} = -3 \partial_\rho A^{* \mu \nu \rho}, \quad \delta \xi^* = \partial_\mu \psi^{* \mu}, \]

\[ \delta C^{* \mu} = -2 \partial_\nu C^{* \mu \nu}, \quad \delta C^* = -\partial_\mu C^{* \mu}, \]

\[ \gamma (\Phi^{\Delta_0}) = \gamma (\eta^{\Delta_1}) = \gamma (\eta^{\Gamma_2}) = \gamma (\eta^{\Gamma_3}) = 0, \]

\[ \gamma (\Phi^{\Delta_0}_a) = \gamma (\eta^{\Delta_1}_a) = \gamma (\eta^{\Gamma_2}_a) = \gamma (\eta^{\Gamma_3}_a) = 0, \]

\[ \gamma A^{\mu \nu \rho} = \partial_\mu C^{\nu \rho |}, \quad \gamma \psi_\mu = \partial_\mu \xi, \]

\[ \gamma C^{* \mu \nu} = \partial_\mu C^{\nu |}, \quad \gamma \xi = 0, \]

\[ \gamma C^*_\mu = \partial_\mu C, \quad \gamma C = 0. \]

In this case the anticanonical action of the BRST symmetry, \( s_\cdot = (s^A, \psi, s^A, \psi) \), is
realized via a solution to the master equation \( (S^A, \psi, S^A, \psi) = 0 \) that reads as

\[ S^A, \psi = S^L_0 [A_{\mu \rho}, \psi_\mu] + \int d^{11} x (\psi^{* \mu} \partial_\mu \xi + A^{* \mu \nu \rho} \partial_{[\mu} C_{\nu \rho]} ) + C^{* \mu \nu} \partial_{[\mu} C_{\nu \rho]} + C^{* \mu} \partial_\mu C, \]

(29)

3 Consistent interactions between an Abelian
three-form gauge field and a Rarita-Schwinger
spinor

The aim of this section is to investigate the cross-couplings that can be in-
troduced between an Abelian three-form gauge field and a massless Rarita-
Schwinger field in \( D = 11 \). This matter is addressed, like in Ref. [1], in the
context of the antifield-BRST deformation procedure. Very briefly, this means that we will associate with \((29)\) a deformed solution
\[
S^{A,\psi} \rightarrow \bar{S}^{A,\psi} = S^{A,\psi} + \lambda S^{A,\psi}_1 + \lambda^2 S^{A,\psi}_2 + \cdots
\]
which is the BRST generator of the interacting theory, \((\bar{S}^{A,\psi}, S^{A,\psi}) = 0\), such that the components of \(\bar{S}^{A,\psi}\) are restricted to satisfy the tower of equations:

\[
(S^{A,\psi}, S^{A,\psi}) = 0, \quad (S^{A,\psi}_1, S^{A,\psi}) = 0, \quad 2(S^{A,\psi}_1, S^{A,\psi}_1) + (S^{A,\psi}, S^{A,\psi}_1) = 0, \quad (S^{A,\psi}_3, S^{A,\psi}) + (S^{A,\psi}_1, S^{A,\psi}_2) = 0, \quad \vdots
\]

The interactions are obtained under the same assumptions like in Ref. [1]: smoothness, locality, Lorentz covariance, Poincaré invariance, and preservation of the number of derivatives on each field (derivative order assumption). The ‘derivative order assumption’ means here that the following two requirements are simultaneously satisfied: (i) the derivative order of the equations of motion on each field is the same for the free and respectively for the interacting theory; (ii) the maximum number of derivatives in the interaction vertices is equal to two, i.e. the maximum number of derivatives from the free Lagrangian.

### 3.1 First-order deformation

Initially, we construct the first-order deformation of the solution to the master equation, \(S^{A,\psi}_1\), as solution to equation \((32)\). If we make the notation \(S^{A,\psi}_1 = \int d^{11}x a^{A,\psi}\), with \(a^{A,\psi}\) a local function \((\text{gh} (a) = 0, \varepsilon (a) = 0)\), then \((32)\) takes the local form

\[
\gamma a^{A,\psi}_I = \partial_\mu m^\mu, \quad (35)
\]

which shows that the nonintegrated density of the first-order deformation pertains to the local cohomology of the BRST differential in ghost number zero, \(a^{A,\psi} \in H^0 (s|d)\). In order to analyze equation \((32)\) we act like in Ref. [1]: we develop \(a^{A,\psi}\) according to the antighost number

\[
a^{A,\psi} = \sum_{i=0}^I a^{A,\psi}_i, \quad \text{agh} (a^{A,\psi}_i) = i \quad (36)
\]

and obtain in the end that equation \((35)\) becomes equivalent to the tower of equations

\[
\gamma a^{A,\psi}_I = \partial_\mu \tilde{m}^\mu, \quad (37)
\]
\[ \delta a^A_i + \gamma a^A_{i-1} = \partial_\mu \frac{(I-1)_\mu}{m}, \quad (38) \]

\[ \delta a^A_i + \gamma a^A_{i-1} = \partial_\mu \frac{(i-1)_\mu}{m}, \quad 1 \leq i \leq I - 1, \quad (39) \]

where, moreover, equation (37) can be replaced in strictly positive antighost numbers by

\[ \gamma a^A_i = 0, \quad I > 0. \quad (40) \]

The nontriviality of the first-order deformation \( a^A_i \) is thus translated at its highest antighost number component into the requirement that \( a^A_i \in H^I(\gamma) \), where \( H^I(\gamma) \) denotes the cohomology of the exterior longitudinal derivative \( \gamma \) in pure ghost number equal to \( I \). So, in order to solve equation (35) we need to compute the cohomology of \( \gamma \), \( H(\gamma) \), and, as it will be made clear below, also the local cohomology of \( \delta \) in pure ghost number zero, \( H(\delta|d) \).

Using the results on the cohomology of the exterior longitudinal differential for an Abelian three-form gauge field computed in Ref. [1] as well as definitions (25)–(28), we can state that the most general solution to (40) can be written, up to \( \gamma \)-exact contributions, as

\[ a^A_i = \tilde{\alpha}_I \left( \left[ F_{\mu\nu\rho\lambda} \right], \left[ \partial_{[\mu} \psi_{\nu]} \right], \left[ \chi^*_\Delta \right] \right) \omega^I(C, \xi), \quad (41) \]

where \( \chi^*_\Delta = \left\{ \Phi^*_{\Delta_0}, \eta^*_{\Delta_1}, \eta^*_{\Delta_2}, \eta^*_{\Delta_3} \right\} \) and \( \omega^I \) denotes the elements with pure ghost number \( I \) of a basis in the space of polynomials in the corresponding ghosts. The objects \( \tilde{\alpha}_I \) (with \( \text{agh}(\tilde{\alpha}_I) = I \)) are nontrivial elements of \( H^0(\gamma) \), known as “invariant polynomials”. They are in fact polynomials in the antifields \( \chi^*_\Delta \), in the field strength of the three-form \( F_{\mu\nu\rho\lambda} \), in the antisymmetrized first-order derivatives of the Rarita-Schwinger fields \( \partial_{[\mu} \psi_{\nu]} \) as well as in their subsequent derivatives. Just like in Ref. [1], it can be shown that a necessary condition for the existence of (nontrivial) solutions \( a_{I-1} \) is that the invariant polynomials \( \tilde{\alpha}_I \) are (nontrivial) objects from the local cohomology of \( \delta \) in pure ghost number zero, \( H(\delta|d) \).

On the other hand, it can be shown that any invariant polynomial \( \tilde{\alpha}_J \) that is trivial in \( H_J(\delta|d) \) with \( J \geq 4 \) can be taken to be trivial also in the invariant characteristic cohomology in antighost number \( J \), \( H^\text{inv}_J(\delta|d) \):

\[ \left( \tilde{\alpha}_J = \delta \beta_{J+1} + \partial_\mu \tilde{\gamma}_J^\mu, \ \text{agh}(\tilde{\alpha}_J) = J \geq 4 \right) \Rightarrow \tilde{\alpha}_J = \delta \tilde{\beta}_{J+1} + \partial_\mu \tilde{\gamma}_J^\mu, \quad (43) \]

with both \( \tilde{\beta}_{J+1} \) and \( \tilde{\gamma}_J^\mu \) invariant polynomials. Results (12) and (13) yield the conclusion that

\[ H^\text{inv}_J(\delta|d) = 0 \quad \text{for all} \ J > 4. \quad (44) \]
It can be shown that the spaces \( (H_J (\delta |d))_{J \geq 2} \) and \( (H^i_J (\delta |d))_{J \geq 2} \) are spanned by

\[
\begin{align*}
H_4 (\delta |d), \ & H^4_4 (\delta |d) : \ (C^*) , \\
H_3 (\delta |d), \ & H^3_3 (\delta |d) : \ (C^*\mu) , \\
H_2 (\delta |d), \ & H^2_2 (\delta |d) : \ (C^*\mu\nu, \xi^*) .
\end{align*}
\]

These results on \( H (\delta |d) \) and \( H^\text{inv} (\delta |d) \) in strictly positive antighost numbers are important because they allow the elimination of all pieces with \( I > 4 \) from \( (36) \).

In the case \( I = 4 \) the nonintegrated density of the first-order deformation \( a^A,\psi \), \( (36) \), becomes

\[
a^A,\psi = a^A,\psi_0 + a^A,\psi_1 + a^A,\psi_2 + a^A,\psi_3 + a^A,\psi_4 .
\]

We can further decompose \( a^A,\psi \) in a natural manner, as a sum between three kinds of deformations

\[
a^A,\psi = a^A + a^{A-}\psi + a^\psi ,
\]

where \( a^A \) contains only BRST generators from the Abelian three-form sector, \( a^{A-}\psi \) describes the cross-interactions between the two theories, and \( a^\psi \) is responsible for the Rarita-Schwinger self-interactions. The component \( a^\psi \) can be shown to take the same form like in the case \( D = 4 \) (see Ref. \[4\]) and satisfies individually an equation of the type \( (35) \). It admits a decomposition of the form

\[
a^\psi = a^\psi_0 + a^\psi_1 ,
\]

where

\[
a^\psi_0 = \frac{9}{2} m \psi_\mu \gamma^\mu \psi_\nu ,
a^\psi_1 = \frac{im \psi_\mu \gamma^\mu \xi}{} ,
\]

with \( m \) an arbitrary, real constant. Since \( a^{A-}\psi \) mixes the variables from the three-form and the Rarita-Schwinger sectors and \( a^A \) depends only on the BRST generators from the three-form sector, it follows that \( a^{A-}\psi \) and \( a^A \) are subject to two separate equations

\[
\begin{align*}
sa^A &= \partial^\mu m^A_\mu , \\
sa^{A-}\psi &= \partial^\mu m^{A-}\psi .
\end{align*}
\]

The nontrivial solution \( a^A \) to \( (52) \) has been discussed in Ref. \[1\] and reduces to

\[
a^A = q \epsilon^{\mu_1 \cdots \mu_11} A_{\mu_2 \mu_3 \mu_4} F_{\mu_4 \cdots \mu_7} F_{\mu_8 \cdots \mu_{11}} ,
\]

with \( q \) an arbitrary, real constant.

Let us analyze now the solutions to equation \( (53) \). In agreement with the previous results on \( H^\text{inv} (\delta |d) \), we can always take the decomposition of \( a^{A-}\psi \) along the antighost number to stop at antighost number equal to four

\[
a^{A-}\psi = a^{A-}\psi_0 + a^{A-}\psi_1 + a^{A-}\psi_2 + a^{A-}\psi_3 + a^{A-}\psi_4 ,
\]
By applying the Koszul-Tate differential on (60), we find
\[
\gamma a^A_4^{-\psi} = 0, \quad \delta a^A_4^{-\psi} + \gamma a^A_{I-1}^{-\psi} = \partial_\mu m^A_{I-1} - \psi^\mu, \quad I = 1, 4.
\]  
(56)  
(57)

Recalling the results from the previous subsection on the cohomology \( H(\gamma) \), it follows that the elements with pure ghost number four of a basis in the space of polynomials in the ghosts \( C \) and \( \xi \) can be chosen as
\[
\{ \xi C, (\xi \gamma_\mu \xi) (\xi \gamma^\mu \xi), (\xi \gamma_\mu \xi_\nu) (\xi \gamma_{\mu\nu}^\rho \lambda \xi) (\xi \gamma_{\mu\nu\rho\lambda\sigma} \xi) \}.
\]  
(58)

The solution to (56) is obtained like in (41), by ’gluing’ the general representative \( H \) of polynomials in the ghosts \( C \) to \( \xi \), namely
\[
a^A_4^{-\psi} = v_1 C^* (\xi \gamma_\mu \xi) (\xi \gamma^\mu \psi_\alpha) + v_2 C^* (\xi \gamma_\mu \xi_\nu) (\xi \gamma_{\mu\nu}^\rho \lambda \xi) (\xi \gamma_{\mu\nu\rho\lambda\sigma} \xi) \]  
(59)

where \((v_i)_{i=1,2,3}\) are some arbitrary constants [the element \( \xi C \) cannot be coupled to \( C^* \) to form a Lorentz invariant since \( \xi \) is a Majorana spinor, so it is not eligible to enter (58)]. Substituting (58) back in (57) for \( I = 4 \) and using definitions (21) - (28), we obtain
\[
a^A_3^{-\psi} = -4C^{a\alpha} [v_1 (\xi \gamma_\mu \xi) (\xi \gamma^\mu \psi_\alpha) + v_2 (\xi \gamma_\mu \xi_\nu) (\xi \gamma_{\mu\nu}^\rho \lambda \psi_\alpha) \]  
(60)

By applying the Koszul-Tate differential on (60), we find
\[
\delta a^A_3^{-\psi} = \gamma \{ 8C^{a\alpha \beta} [v_1 (\xi \gamma_\mu \psi_\alpha) (\xi \gamma^\mu \psi_\beta) \]  
(61)

Comparing (61) with (50) for \( I = 3 \) it follows that \( a^A_3^{-\psi} \) provides a consistent \( a^A_2^{-\psi} \) if the quantity
\[
\pi = -4C^{a\alpha \beta} [v_1 (\xi \gamma_\mu \xi) (\xi \gamma^\mu \partial_\alpha \psi_\beta) + v_2 (\xi \gamma_\mu \xi_\nu) (\xi \gamma_{\mu\nu}^\rho \lambda \partial_\alpha \psi_\beta) \]  
(62)

can be written in a \( \gamma \)-exact modulo \( d \) form
\[
\pi = \gamma w + \partial_\mu \theta^\mu.
\]  
(63)
Assume that (63) holds. Taking its Euler-Lagrange (EL) derivatives with respect to $C^{\alpha\beta}$ we get

$$\frac{\delta^L\pi}{\delta C^{\alpha\beta}} = \gamma \left( \frac{\delta^Lw}{\delta C^{\alpha\beta}} \right).$$

(64)

On the other hand, from (62) by direct computation we infer

$$\frac{\delta^L\pi}{\delta C^{\alpha\beta}} = -4 \left[ v_1 \left( \bar{\xi}\gamma_\mu\xi \right) \left( \bar{\xi}\gamma^\mu \partial_{[\alpha}\psi_{\beta]} \right) + v_2 \left( \bar{\xi}\gamma_\mu\nu\xi \right) \left( \bar{\xi}\gamma^\mu\nu \partial_{[\alpha}\psi_{\beta]} \right) \right].$$

(65)

Thus, equation (64) restricts $\delta^L\pi/\delta C^{\alpha\beta}$ to be a trivial element of $H(\gamma)$, while (65) emphasizes that $\delta^L\pi/\delta C^{\alpha\beta}$ is a nontrivial element from $H(\gamma)$ (because each term from the right-hand side of (65) is so), such that the only possibility is that $\delta^L\pi/\delta C^{\alpha\beta}$ must vanish

$$\frac{\delta^L\pi}{\delta C^{\alpha\beta}} = 0.$$

(66)

This further implies, by means of (64), that we must set zero all the constants that parameterize $a_{4}^{A-\psi}$

$$v_1 = v_2 = v_3 = 0,$$

(67)

so in the end we have that

$$a_{4}^{A-\psi} = a_{3}^{A-\psi} = 0.$$

(68)

As a consequence, decomposition (55) can stop earliest at antighost number three, $a_{4}^{A-\psi} = a_{0}^{A-\psi} + a_{1}^{A-\psi} + a_{2}^{A-\psi} + a_{3}^{A-\psi}$, where $a_{3}^{A-\psi}$ satisfies the equation $\gamma a_{3}^{A-\psi} = 0$. According to (41), (46) and recalling the assumption that $a_{3}^{A-\psi}$ mixes the BRST generators of the three-form with those from the Rarita-Schwinger sector, it results that the solution to this equation reads as $a_{3}^{A-\psi} = C_\mu^* e^\mu (\xi)$, where $e^\mu (\xi)$ denote the vector-like elements of pure ghost number three of a basis in the space of polynomials in the ghost $\xi$. Since $\text{pgh} (\xi) = 1$, it follows that $e^\mu (\xi)$ necessarily contains three spinors of the type $\xi$ and therefore we can set $a_{3}^{A-\psi} = 0$ because one cannot construct a Lorentz eleven-dimensional vector out of three spinors.

Thus, we can write

$$a_{A-\psi} = a_{0}^{A-\psi} + a_{1}^{A-\psi} + a_{2}^{A-\psi},$$

(69)

such that equation (54) becomes equivalent to

$$\gamma a_{2}^{A-\psi} = 0,$$

(70)

$$\delta a_{1}^{A-\psi} + \gamma a_{I-1}^{A-\psi} = \partial_{\mu} m_{I-1}^{A-\psi} \mu, \quad I = 1, 2.$$  

(71)

Because the elements of pure ghost number two of a basis in the space of polynomials in the ghost $\xi$ read as

$$\left\{ (\bar{\xi}\gamma_\mu\xi), (\bar{\xi}\gamma_\mu\nu\xi), (\bar{\xi}\gamma_\mu\nu\rho\sigma\xi) \right\}$$

(72)
(the ghost for ghost for ghost $C$ is not eligible as $\text{pgh} \, (C) = 3$) and the representatives of $H^2_{\gamma\gamma}$ ($\delta |d)$ are given by (77), we observe that the only combination that might generate cross-interactions remains

$$a^\Lambda_{-\psi} = \frac{\tilde{k}}{2} C^{\mu \nu \rho} \tilde{\xi} \gamma_{\mu \nu} \xi,$$

(73)

where $\tilde{k}$ is an arbitrary constant. Replacing (73) in (71) for $I = 2$ we determine $a^\Lambda_{-\psi}$ under the form

$$a^\Lambda_{-\psi} = -3\tilde{k} A^{\mu \nu \rho} \tilde{\xi} \gamma_{\mu \nu} \psi_\rho + \bar{a}^\Lambda_{-\psi},$$

(74)

where $\bar{a}^\Lambda_{-\psi}$ is the general solution to the ‘homogeneous’ equation

$$\gamma \bar{a}^\Lambda_{-\psi} = 0.$$  

(75)

It is expressed by

$$\bar{a}^\Lambda_{-\psi} = (\psi^* \mu M_\mu + A^* \mu \nu \rho N_{\mu \nu \rho}) \xi,$$

(76)

with $N_{\mu \nu \rho}$ the components of a real, fermionic, gauge-invariant, completely antisymmetric spinor tensor and $M_\mu$ some bosonic, gauge-invariant, $11 \times 11$ matrices, which in addition must explicitly depend on the three-form field strength $F^{\mu \nu \rho \lambda}$ in order to provide cross-interactions. By applying $\delta$ on (73) with $\bar{a}^\Lambda_{-\psi}$ of the form (76), we obtain

$$\delta a^\Lambda_{-\psi} = \gamma d_0 + e_0 + \partial_\mu M^\mu,$$

(77)

where

$$d_0 = \left( \frac{1}{4} \tilde{k} \tilde{\psi}_\mu \gamma_{\nu \rho} \psi_\lambda + \frac{1}{3!} N_{\mu \nu \rho} \psi_\lambda \right) F^{\mu \nu \rho \lambda},$$

(78)

$$e_0 = \frac{1}{4} \tilde{k} \tilde{\xi} \gamma_{\mu \nu} (\partial_\rho \psi_\lambda) F^{\mu \nu \rho \lambda} + i \tilde{\psi}_\lambda \gamma^ {\mu \lambda} (\partial_\mu M_\mu) \xi$$

$$+ i \tilde{\psi}_\lambda \gamma^ {\rho \lambda} M_\rho \partial_\mu \xi + \frac{1}{3!} (\partial_\lambda N_{\mu \nu \rho}) \xi F^{\mu \nu \rho \lambda}.$$  

(79)

The condition that (77) is expressed like in (71) for $I = 1$ restricts $e_0$ expressed by (79) to be $\gamma$-exact modulo $d$

$$e_0 = \gamma p + \partial_\mu n^\mu.$$  

(80)

Recalling the requirement that the quantities $N_{\mu \nu \rho}$ are spinor-like and gauge-invariant, we deduce that the most general representation of these elements is

$$N_{\mu \nu \rho} = \partial_{[\alpha} \bar{\psi}_{\beta]} N^{\alpha \beta}_{\mu \nu \rho},$$  

(81)

where $N^{\alpha \beta}_{\mu \nu \rho}$ are also gauge-invariant. As $c_0$ from (79) involves terms with different numbers of derivatives, it is useful to decompose the functions $M_\mu$ and $N^{\alpha \beta}_{\mu \nu \rho}$ according to the number of spacetime derivatives

$$M_\mu = (1) M_\mu + (2) M_\mu + (3) M_\mu + \cdots,$$

(82)
where \((k) M_\mu\) and \((k) N_{\mu\nu\rho}\) contain precisely \(k\) derivatives \([82]\) cannot contain a derivative-free term because, as we have emphasized before, \(M_\mu\) depends at least linearly on \(F^{\mu\nu\rho\lambda}\). Inserting \([82]\) and \([83]\) in \([79]\) and projecting \([80]\) on the various numbers of derivatives, we find the equivalent to the tower of equations

\[
\begin{align*}
\frac{1}{4} k \xi \gamma_{\mu\nu} (\partial_\mu \psi_\lambda) F^{\mu\nu\rho\lambda} + i \bar{\psi}_\lambda \gamma^{\rho\lambda\mu} \left( \partial_\mu M_\mu \right)^{(1)} \xi \\
+ i \bar{\psi}_\lambda \gamma^{\rho\lambda\mu} M_\mu \partial_\rho \xi = \gamma (p) + \partial_\mu (1)^{\mu}, \\
i \bar{\psi}_\lambda \gamma^{\rho\lambda\mu} \left( \partial_\mu M_\mu \right)^{(2)} \xi + i \bar{\psi}_\lambda \gamma^{\rho\lambda\mu} M_\mu \partial_\rho \xi \\
+ \frac{1}{3!} \partial_\lambda \left( \partial_{[\alpha} \bar{\psi}_{\beta]} \frac{(0)}{N_{\mu\nu\rho}} \right)^{\alpha\beta} \xi F^{\mu\nu\rho\lambda} = \gamma (2)^{2\mu} + \partial_\mu (2)^{\mu}, \\
i \bar{\psi}_\lambda \gamma^{\rho\lambda\mu} \left( \partial_\mu M_\mu \right)^{(k)} \xi + i \bar{\psi}_\lambda \gamma^{\rho\lambda\mu} M_\mu \partial_\rho \xi \\
+ \frac{1}{3!} \partial_\lambda \left( \partial_{[\alpha} \bar{\psi}_{\beta]} \frac{(k-2)}{N_{\mu\nu\rho}} \right)^{\alpha\beta} \xi F^{\mu\nu\rho\lambda} = \gamma (k) + \partial_\mu (k)^{\mu}, \quad k \geq 3.
\end{align*}
\]  

Equations \([86]\) would lead to interaction vertices with more than two spacetime derivatives, so, in agreement with our hypothesis on the conservation of the number of derivatives on each field with respect to the free theory, they must be discarded

\[
\begin{align*}
(k) M_\mu &= 0, \quad k \geq 3, \\
(k) N_{\mu\nu\rho} &= 0, \quad k \geq 1,
\end{align*}
\]  

which ensures \(\gamma p = 0\) for \(k \geq 3\) in \([80]\). As the matrices \((1) M_\mu\) are linear in the three-form field strength, they can be generally represented in the form

\[
(1) M_\mu = k_1 F_\mu \gamma_{\alpha\beta\gamma} + k_2 F^{\alpha\beta\gamma\delta} \gamma_{\mu\alpha\beta\gamma},
\]  

with \(k_1\) and \(k_2\) some arbitrary constants. Based on \([89]\), the left-hand side of \([84]\) becomes

\[
\begin{align*}
\frac{1}{4} k \xi \gamma_{\mu\nu} (\partial_\mu \psi_\lambda) F^{\mu\nu\rho\lambda} + i \bar{\psi}_\lambda \gamma^{\rho\lambda\mu} \left( \partial_\mu M_\mu \right)^{(1)} \xi
\end{align*}
\]
\[ \begin{align*}
+ i \bar{\psi}_\lambda \gamma^\rho \lambda^\mu \mathcal{M}_\mu \gamma \psi_\rho &= \gamma \left[ \left( \frac{3}{8} (k_1 + 8k_2) i \bar{\psi}_\alpha \gamma_{\mu \nu \rho \lambda} \psi_\alpha \right.ight. \\
&\left. - \frac{1}{2} (k_1 + 5k_2) i \bar{\psi}_\alpha \gamma_{\alpha \beta \mu \nu \rho \lambda} \psi_\beta \right) F_{\mu \nu \rho \lambda} \right] \\
+ \frac{1}{2} \left( \bar{k} - 2 \cdot 3i k_1 - 7 \cdot 4i k_2 \right) \bar{\xi} \gamma_{\mu \nu} (\partial_\rho \psi_\lambda) F_{\mu \nu \rho \lambda} \\
+ (k_1 + 8k_2) \left[ - \frac{3}{4} \left( \partial^\rho \bar{\psi}_\alpha \right) \gamma_{\alpha \mu \nu \rho \lambda} \xi \\
+ 3i (\partial^\rho \bar{\psi}_\alpha) \alpha \beta \gamma_{\nu \rho \lambda} \xi + 3i \bar{\psi}_\mu \gamma_{\alpha \nu \rho \lambda} (\gamma \psi_\alpha) \right] F_{\mu \nu \rho \lambda} + \partial_\mu \gamma^{(1)}
\end{align*} \]

Asking now that the right-hand side of (90) satisfies (84), we find the restrictions

\[ k_1 = \frac{1}{9} \bar{k}, \quad k_2 = - \frac{1}{3 \cdot 4!} \bar{k}, \]

which further produce

\[ \begin{align*}
\gamma^{(1)} \mathcal{M}_\mu &= i \bar{k} \left( \frac{1}{9} F_{\mu}^{\alpha \beta \gamma} \gamma_{\alpha \beta \gamma} - \frac{1}{3 \cdot 4!} F_{\alpha \beta \gamma \delta} \gamma_{\alpha \beta \gamma \delta} \right), \\
\gamma^{(1)} \mathcal{P} &= \frac{1}{2 \cdot 4!} \bar{k} \bar{\psi}\gamma_{\alpha \beta \mu \nu \rho \lambda} \psi^\beta F_{\mu \nu \rho \lambda}.
\end{align*} \]

Next, we approach equation (85). Due to the fact that each \( \mathcal{M}_\mu \) is a gauge-invariant, 11 \times 11 matrix with two spacetime derivatives, it contains precisely two three-form field strengths (since it cannot depend on \( \partial_\alpha [\bar{\psi}_\beta] \), which is a spinor). As the elements \( N^{(0)}_{\mu \nu \rho \lambda} \) are derivative-free and gauge-invariant, they can only be constant. Based on the last two observations, we observe that each of the first two terms from the left-hand side of equation (85) comprises two three-form field strengths, while the last term is only linear in \( F_{\mu \nu \rho \lambda} \), such that (85) splits into two separate equations

\[ \begin{align*}
+ i \bar{\psi}_\lambda \gamma^\rho \lambda^\mu \mathcal{M}_\mu \gamma \psi_\rho &= \gamma \left[ \left( \frac{3}{8} (k_1 + 8k_2) i \bar{\psi}_\alpha \gamma_{\mu \nu \rho \lambda} \psi_\alpha \right. \right. \\
&\left. \left. - \frac{1}{2} (k_1 + 5k_2) i \bar{\psi}_\alpha \gamma_{\alpha \beta \mu \nu \rho \lambda} \psi_\beta \right) F_{\mu \nu \rho \lambda} \right] \\
+ \frac{1}{2} \left( \bar{k} - 2 \cdot 3i k_1 - 7 \cdot 4i k_2 \right) \bar{\xi} \gamma_{\mu \nu} (\partial_\rho \psi_\lambda) F_{\mu \nu \rho \lambda} \\
+ (k_1 + 8k_2) \left[ - \frac{3}{4} \left( \partial^\rho \bar{\psi}_\alpha \right) \gamma_{\alpha \mu \nu \rho \lambda} \xi \\
+ 3i (\partial^\rho \bar{\psi}_\alpha) \alpha \beta \gamma_{\nu \rho \lambda} \xi + 3i \bar{\psi}_\mu \gamma_{\alpha \nu \rho \lambda} (\gamma \psi_\alpha) \right] F_{\mu \nu \rho \lambda} + \partial_\mu \gamma^{(1)}
\end{align*} \]

The left-hand side of (94) is \( \gamma \)-exact modulo \( d \) if the following conditions are simultaneously satisfied

\[ \begin{align*}
\gamma^{(2)} \mathcal{M}_\mu &= - \left( \gamma^{(2)} \mathcal{M}_\mu \right)^T, \\
\partial_\mu \gamma^{(2)} &= 0.
\end{align*} \]
In order to investigate the former condition, we represent \( M_\mu \) in terms of a basis in the space of \( \gamma \)-matrices

\[
M_\mu = M_\mu^1 + M_\mu^\gamma \gamma_\alpha + M_\mu^{\alpha \beta} \gamma_{\alpha \beta} + M_\mu^{\alpha \beta \gamma} \gamma_{\alpha \beta \gamma} + \gamma_{\alpha \beta \gamma \delta},
\]

where each of the coefficients \( \bar{M}_\mu^1, \bar{M}_\mu^{\alpha \beta \gamma}, \bar{M}_\mu^{\alpha \beta \gamma \delta} \) is a function with precisely two three-form field strengths. This dependence implies the vanishing of all coefficients with an odd number of indices

\[
\bar{M}_\mu^1 = 0, \quad \bar{M}_\mu^{\alpha \beta \gamma} = 0, \quad \bar{M}_\mu^{\alpha \beta \gamma \delta} = 0.
\]

Inserting (99) in (98) we find by direct computation the relation

\[
\gamma_\rho \gamma_\lambda \gamma_\mu M_\mu = \frac{\gamma_\rho \gamma_\lambda \gamma_\mu}{M_\mu} \left( \delta_\alpha^{\mu \rho \lambda} + \gamma_\rho \gamma_\mu \right) + \frac{\gamma_\rho \gamma_\mu}{M_\mu} \left( \delta_\beta^{\mu \lambda \gamma} \delta_\mu^{\rho} \gamma_{\alpha \gamma} \right)
\]

Looking at (100), we remark that the terms \( 3! \bar{M}_\mu^1 \) and \( \bar{M}_\mu^{\alpha \beta \gamma} \) appearing in \( \gamma_\rho \gamma_\lambda \gamma_\mu (2) \) break condition (96), so we must set

\[
\bar{M}_\mu^1 = 0, \quad \bar{M}_\mu^{\alpha \beta \gamma} = 0.
\]

The last result replaced in (100) yields

\[
\gamma_\rho \gamma_\lambda \gamma_\mu (2) \frac{M_\mu}{M_\mu} \left( \delta_\alpha^{\mu \rho \lambda} + \gamma_\rho \gamma_\mu \right).
\]

It is clear that \( \bar{M}_\mu^{\alpha \beta \gamma} \gamma_\rho \gamma_\lambda \gamma_\mu \gamma_\alpha \) from (102) cannot fulfill (96), so we must take

\[
\bar{M}_\mu^1 = 0,
\]

which, together with (99), (101), and (103), lead to the result

\[
\bar{M}_\mu^1 = 0.
\]
so we can only have \(p_1^{(2)} = 0\) in (94). Now, we investigate equation (95). Direct computation provides

\[
\frac{1}{3!} \partial_\lambda \left( \partial_{[\alpha} \bar{\psi}_{\beta]} N_{\mu \nu \rho}^{(0)} \right) \xi F^{\mu \nu \rho \lambda} = \gamma \left( -\frac{1}{3!} \partial_{[\alpha} \bar{\psi}_{\beta]} N_{\mu \nu \rho}^{(0)} \psi_\lambda F^{\mu \nu \rho \lambda} \right)
- \frac{1}{3!} \partial_{[\alpha} \bar{\psi}_{\beta]} N_{\mu \nu \rho}^{(0)} \xi \partial_\lambda F^{\mu \nu \rho \lambda} + \partial_\mu s^{(105)}
\]

Assume that the second term from the right-hand side of (105) would give a \(\gamma\)-exact modulo d quantity. Comparing (105) to (85), we find that

\[
p_2^{(2)} = -\frac{1}{3!} \partial_{[\alpha} \bar{\psi}_{\beta]} N_{\mu \nu \rho}^{(0)} \psi_\lambda F^{\mu \nu \rho \lambda} + \ldots.
\]

It is simple to see that \(p_2^{(2)}\) (which contributes to \(a_0^{A-\psi}\)) produces field equations for the Rarita-Schwinger field with two spacetime derivatives, which disagrees with requirement (i) from the beginning of this section related to the derivative order assumption. Thus, we must set

\[
N_{\mu \nu \rho}^{(0)} = 0,
\]

which yields \(p_2^{(2)} = 0\).

Inserting (87), (92), and (104) into (82) and respectively (88) and (107) into (83), and then substituting the resulting expressions of (82) and (83) in (76), we obtain the general form of the solution \(\tilde{a}_1^{A-\psi}\), such that (74) takes the final form

\[
a_1^{A-\psi} = -3\tilde{k} A^{* \mu \nu \rho} \xi_{\gamma \mu \nu \rho} \psi_\lambda + \frac{i}{9} \psi^{* \mu \nu \rho} \xi_{\gamma \mu \nu \rho} F^{\nu \rho \lambda} - \frac{1}{3 \cdot 4!} F^{\nu \rho \lambda} \gamma_{\mu \nu \rho \lambda} \xi.
\]

Accordingly, we find that \(a_0^{A-\psi}\) as solution to equation (71) for \(I = 1\) reads as

\[
a_0^{A-\psi} = \frac{1}{4} \tilde{k} \bar{\psi}_\mu \gamma_\nu \psi_\lambda F^{\mu \nu \rho \lambda} - \frac{1}{2 \cdot 4!} \tilde{k} \bar{\psi}_\alpha \gamma_\beta \mu \nu \rho \lambda \psi_\beta F^{\mu \nu \rho \lambda}.
\]

Replacing now (73) and (108)–(109) into (69), we find that the interacting part of the first-order deformation of the solution to the master equation becomes

\[
S_1^{A-\psi} = \int d^{11} x \left( a_2^{A-\psi} + a_1^{A-\psi} + a_0^{A-\psi} \right)
\equiv \tilde{k} \int d^{11} x \left( \frac{1}{2} C^{* \mu \nu \rho} \tilde{\xi}_{\gamma \mu \nu \rho} \xi_{\gamma \mu \nu \rho} F^{\mu \nu \rho \lambda} \right.
+ \frac{i}{9} \psi^{* \mu \rho} F^{\nu \rho \lambda} \gamma_{\mu \nu \rho \lambda} \xi
+ \frac{1}{3 \cdot 4!} \psi^{* \mu \rho} F^{\nu \rho \lambda} \gamma_{\mu \nu \rho \lambda} \xi
- \frac{1}{4} \psi_\mu \gamma_\nu \psi_\lambda F^{\mu \nu \rho \lambda} - \frac{1}{2 \cdot 4!} \tilde{k} \bar{\psi}_\alpha \gamma_\beta \mu \nu \rho \lambda \psi_\beta F^{\mu \nu \rho \lambda} \right).
\]

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In what follows we employ the notation

\[ S'_1 = S^A - \psi_1, \]

(111)

with \( a^A \) given by (114), so the complete expression of the first-order deformation of the solution to the master equation for the model under consideration is (see (119))

\[ S^A = S'_1 + S_1^\psi, \]

(112)

where \( S_1^\psi \) is the component corresponding to the Rarita-Schwinger sector

\[ S_1 = \int d^{11} x a^\psi, \]

(113)

and the integrand \( a^\psi \) can be read from (50) and (51).

### 3.2 Second-order deformation

In this section we investigate the consistency of the first-order deformation, described by equation (33). Along the same line as before, we can write the second-order deformation like the sum between the Rarita-Schwinger contribution and the interacting part

\[ S^A = S'_2 + S_2^\psi. \]

(114)

The piece \( S_2^\psi \) is subject to the equation

\[ \frac{1}{2} (S_1, S_1)^\psi + s S_2^\psi = 0, \]

(115)

where

\[ (S_1, S_1)^\psi = \left( S_1^\psi, S_1^\psi \right) + \left( S_1^A - \psi_1, S_1^A - \psi_1 \right)^\psi. \]

(116)

In formula (116) we used the notation \( \left( S_1^A - \psi_1, S_1^A - \psi_1 \right)^\psi \) for those pieces from \( \left( S_1^A - \psi_1, S_1^A - \psi_1 \right) \) that contain only BRST generators from the Rarita-Schwinger spectrum. The component \( S_2^A - \psi \) results as solution to the equation

\[ \frac{1}{2} (S_1, S_1)^A - \psi + s S_2^A - \psi = 0, \]

(117)

where

\[ (S_1, S_1)^A - \psi = 2 \left( S_1^\psi, S_1^A - \psi_1 \right) + \left( S_1^A - \psi_1, S_1^A - \psi_1 \right)^A - \psi \]

(118)

and \( \left( S_1^A - \psi_1, S_1^A - \psi_1 \right)^A - \psi = \left( S_1^A - \psi_1, S_1^A - \psi_1 \right) - \left( S_1^A - \psi_1, S_1^A - \psi_1 \right)^\psi. \) If we denote by \( \tilde{\Delta}^\psi \) and \( \tilde{\Delta}^\psi \) the nonintegrated densities of the functionals \( (S_1, S_1)^\psi \) and respectively \( S_2^\psi \), then the local form of (115) becomes

\[ \tilde{\Delta}^\psi = -2 \tilde{\Delta}^\psi + \partial_\mu \tilde{n}^\mu, \]

(119)
with

\[ \text{gh} (\tilde{\Delta}^\psi) = 1, \quad \text{gh} (\tilde{b}^\psi) = 0, \quad \text{gh} (\tilde{n}^\mu) = 1, \] (120)

for some local currents \( n^\mu \). Direct computation shows that \( \tilde{\Delta}^\psi \) decomposes as

\[ \tilde{\Delta}^\psi = \tilde{\Delta}_1^\psi + \tilde{\Delta}_0^\psi, \quad \text{agh} \left( \tilde{\Delta}_1^\psi \right) = 1, \quad \text{agh} \left( \tilde{\Delta}_0^\psi \right) = 0, \] (121)

where

\[
\tilde{\Delta}_1^\psi = \partial_\mu \tilde{\tau}_1^\mu + \gamma \left[ -\frac{i\tilde{k}^2}{3} \left( \psi^*_\mu \gamma_{\nu\rho\lambda}\xi - \frac{1}{2} \psi^*\sigma \gamma_{\mu\nu\rho\lambda\sigma} \xi \right) \bar{\psi}^\mu \gamma^\nu \rho \psi^\lambda \right] \\
- \frac{i\tilde{k}^2}{3} \left( \psi^*_\mu \gamma_{\nu\rho\lambda}\xi - \frac{1}{2} \psi^*\sigma \gamma_{\mu\nu\rho\lambda\sigma} \xi \right) \bar{\xi} \gamma^\mu \rho \partial^\nu \psi^\lambda, \] (122)

\[
\tilde{\Delta}_0^\psi = \partial_\mu \tilde{\tau}_0^\mu + 180im^2 \xi^\mu \psi_\mu \\
+ ik^2 \left( \bar{\psi}^\mu [\gamma_{\nu\rho\lambda}\psi_\lambda] + \frac{1}{2} \bar{\psi}^\mu \gamma_{\alpha\beta\nu\rho\lambda}\psi_\lambda \right) \partial^\mu \left( \bar{\xi} \gamma^\rho \psi^\lambda \right). \] (123)

Because \((S_1, S_1)\) contains terms of maximum antighost number equal to one, we can assume (without loss of generality) that \( \tilde{b}^\psi \) stops at antighost number two

\[
\tilde{b}^\psi = \sum_{I=0}^{2} \tilde{b}^\psi_I, \quad \text{agh} \left( \tilde{b}^\psi_I \right) = I, \quad I = 0, 2, \] (124)

\[
\tilde{n}^\mu = \sum_{I=0}^{2} \tilde{n}^\mu_I, \quad \text{agh} \left( \tilde{n}^\mu_I \right) = I, \quad I = 0, 2. \] (125)

By projecting equation (119) on the various (decreasing) values of the antighost number, we then infer the equivalent tower of equations

\[
0 = -2\gamma \tilde{b}^\psi + \partial_\mu \tilde{n}^\mu, \] (126)

\[
\tilde{\Delta}_I^\psi = -2 \left( \delta \tilde{b}^\psi_{I+1} + \gamma \tilde{b}^\psi_I \right) + \partial_\mu \tilde{n}^\mu_I, \quad I = 0, 1. \] (127)

Equation (126) can always be replaced with

\[
\gamma \tilde{b}^\psi_2 = 0. \] (128)

Thus, \( \tilde{b}^\psi_2 \) belongs to the Rarita-Schwinger sector of cohomology of \( \gamma, H (\gamma) \). By means of definitions (25)–(27) we get that \( H (\gamma) \) in the Rarita-Schwinger sector is generated by the objects \( (\psi^*\mu, \xi^\mu, \partial_\mu \psi_\nu), \) by their spacetime derivatives up to a finite order, and also by the undifferentiated ghosts \( \xi \) (the spacetime derivatives of \( \xi \) are \( \gamma \)-exact according to the second relation in (26)). As a consequence, we can write

\[
\tilde{b}^\psi_2 = \bar{\beta}_2^\psi \left( [\partial_\mu \psi_\nu], [\psi^*\mu], [\xi^\mu] \right) e^2 (\xi), \]
where $e^2(\xi)$ are the elements of pure ghost number two of a basis in the space of polynomials in the ghosts $\xi$. (72).

We observe that $\tilde{\Delta}_1^\psi$ from (122) can be written as in (127) for $I = 1$ if and only if
\begin{equation}
\tilde{\chi} = -\frac{ik^2}{3} \left( \psi^*_\mu \gamma_{\nu\rho\lambda} \xi - \frac{1}{2} \psi^* \gamma_{\nu\rho\lambda\sigma} \xi \right) (\xi) \psi_{\mu} \partial^\nu \psi_{\lambda} \right)
\end{equation}
reads as
\begin{equation}
\tilde{\chi} = -2 \delta \tilde{b}_2^\psi + \gamma \tilde{\rho} + \partial_{\mu} \tilde{\bar{l}}_{\mu},
\end{equation}
where
\begin{equation}
\tilde{\rho} = \frac{ik^2}{3} \left( \psi^*_\mu \gamma_{\nu\rho\lambda} \xi - \frac{1}{2} \psi^* \gamma_{\nu\rho\lambda\sigma} \xi \right) \psi_{\mu} \gamma^{\nu\sigma} \psi_{\lambda} - 2 \tilde{b}_1^\psi.
\end{equation}
Assume that (130) holds. Then, by taking its left Euler-Lagrange (EL) derivatives with respect to $\psi^*_\mu$ and using the commutation between $\gamma$ and each EL derivative $\frac{\delta L}{\delta \psi^*_\mu}$, we infer the relations
\begin{equation}
\frac{\delta L}{\delta \psi^*_\mu} \left( \tilde{\chi} + 2 \delta \tilde{b}_2^\psi \right) = \gamma \left( \frac{\delta L}{\delta \psi^*_\mu} \tilde{\rho} + \partial_{\mu} \tilde{\bar{l}}_{\mu} \right).
\end{equation}
As $\tilde{b}_2^\psi$ is $\gamma$-invariant, then $\delta \tilde{b}_2^\psi$ will also be $\gamma$-invariant. Recalling the previous results on the cohomology of $\gamma$ in the Rarita-Schwinger sector, we find that $\delta \tilde{b}_2^\psi = e^2(\xi) \psi^*_\mu \nu^{\mu}$, with $\nu^{\mu}$ fermionic, $\gamma$-invariant functions of antighost number zero and $e^2(\xi)$ the elements of pure ghost number two of a basis in the space of polynomials in the ghosts $\xi$. By using (129) and the last expression of $\delta \tilde{b}_2^\psi$, direct computation provides the equation
\begin{equation}
\frac{\delta L}{\delta \psi^*_\mu} \left( \tilde{\chi} + 2 \delta \tilde{b}_2^\psi \right) = i k^2 \left( \psi^*_\mu \gamma_{\nu_{\rho\lambda}} \xi - \frac{1}{2} \psi^* \gamma_{\nu_{\rho\lambda\sigma}} \xi \right) \psi_{\mu} \partial^\nu \psi_{\lambda} - 2 \psi_{\lambda} \left( \xi \right) v_{\mu}.
\end{equation}
On the one hand, equation (132) shows that $\delta L / \delta \psi^*_\mu$ is trivial in $H(\gamma)$. On the other hand, relation (133) emphasizes that $\delta L / \delta \psi^*_\mu$ is a nontrivial element from $H(\gamma)$ (because each term on the right-hand side of (133) is nontrivial in $H(\gamma)$). Then, $\delta L / \delta \psi^*_\mu$ must be set zero
\begin{equation}
\frac{\delta L}{\delta \psi^*_\mu} \left( \tilde{\chi} + 2 \delta \tilde{b}_2^\psi \right) = 0,
\end{equation}
which yields
\begin{equation}
\tilde{\chi} + 2 \delta \tilde{b}_2^\psi = \partial_{\mu} \tilde{\bar{l}}_{\mu}.
\end{equation}

\footnote{In fact, the general solution to equation (134) takes the form $\tilde{\chi} + 2 \delta \tilde{b}_2^\psi = u + \partial_{\mu} \tilde{\bar{l}}_{\mu}$, where $u$ is a function of antighost number one depending on all the BRST generators from the Rarita-Schwinger sector but the antifields $\psi^*_\mu$. As the antifields $\psi^*_\mu$ are the only Rarita-Schwinger antifields of antighost number one, the condition $agh(u) = 1$ automatically produces $u = 0$.}
By acting with \( \delta \) on (135) we deduce
\[
\delta \tilde{\chi} = \partial_\mu \tilde{\chi}^\mu. \tag{136}
\]
From (129), by direct computation we find
\[
\delta \tilde{\chi} = \frac{\tilde{k}^2}{3} \left( \partial^\alpha \bar{\psi}^\beta \right) \gamma_{\alpha \beta \mu} \left[ 2 \left( \gamma_{\nu \rho \lambda \xi} \xi [\mu \nu \rho \lambda \xi] - \frac{1}{2} \left( \gamma_{\mu \nu \rho \lambda \sigma} \xi \gamma_{\nu \rho \lambda \sigma} \right) \xi \gamma_{\lambda \sigma} \bar{\psi}_\mu \right]. \tag{137}
\]
Comparing (136) with (137) and recalling the Noether identities corresponding to the Rarita-Schwinger action, we obtain that the right-hand of (137) reduces to a total derivative iff
\[
\frac{2}{3} \left( \gamma_{\nu \rho \lambda \xi} \xi [\mu \nu \rho \lambda \xi] - \frac{1}{2} \left( \gamma_{\mu \nu \rho \lambda \sigma} \xi \gamma_{\nu \rho \lambda \sigma} \right) \xi \gamma_{\lambda \sigma} \bar{\psi}_\mu \right] = \partial_\mu \tilde{p}. \tag{138}
\]
Simple computation exhibits that the left-hand side of (138) cannot be written like a total derivative, so neither relation (136) nor equation (130) hold. As a consequence, \( \tilde{\chi} \) must vanish and hence we must set
\[
\tilde{k} = 0. \tag{139}
\]

Inserting (139) in (122)–(123), we obtain that
\[
\tilde{\Delta}_1^\psi = \partial_\mu \tilde{\tau}_1^\mu, \quad \tilde{\Delta}_0^\psi = \partial_\mu \tilde{\tau}_0^\mu + 180im^2 \bar{\xi}^\mu \psi_\mu. \tag{140}
\]
From (140) it results that we can safely take \( \tilde{b}_2^\psi = 0 \) and \( \tilde{b}_1^\psi = 0 \), which replaced in (141) lead to the necessary condition that \( \tilde{\Delta}_0^\psi \) must be a trivial element from the local cohomology of \( \gamma \), i.e. \( \tilde{\Delta}_0^\psi = -2\gamma \tilde{b}_0^\psi + \partial_\mu \tilde{\tau}_0^\mu \). In order to solve this equation with respect to \( \tilde{b}_0^\psi \), we will project it on the number of derivatives. Since \( \gamma \tilde{b}_0^\psi \) contains at least one spacetime derivative, the above equation projected on the number of derivatives equal to zero reduces to \( \tilde{\Delta}_0^\psi = 180im^2 \bar{\xi}^\mu \psi_\mu = 0 \), which further implies
\[
m = 0, \quad \tag{142}
\]
so
\[
S_1^\psi = 0. \tag{143}
\]
Replacing (142) and (143) in (112), we obtain that the general form of the first-order deformation for the free model under study that is consistent to the second order in the coupling constant reads as
\[
S_{1,\psi} = S^A = q \int d^{11} x \varepsilon^{\mu_1 \cdots \mu_{11}} A_{\mu_1 \mu_2 \mu_3} F_{\mu_4 \cdots \mu_7} F_{\mu_8 \cdots \mu_{11}}. \tag{144}
\]
Inserting (144) into (32)–(34), etc., we find that all the higher-order deformations can be taken to vanish
\[
S_k = 0, \quad k > 1, \tag{145}
\]
so the full deformed solution to the master that is consistent to all orders in the coupling constant takes the simple form

\[ \bar{S} = S + \lambda S_1^A, \]  

(146)

where \( S \) is the solution to the master equation for the starting free model, \((29)\). Relation (146) emphasizes that under the hypotheses mentioned at the beginning of this section, there are neither cross-couplings that can be added between an Abelian three-form gauge field and a massless gravitino nor self-interactions for the gravitino in \( D = 11 \).

## 4 Conclusion

To conclude with, in this paper we have investigated the consistent interactions in eleven spacetime dimensions that can be added to a free theory describing a massless gravitino and an Abelian 3-form gauge field. Our treatment is based on the Lagrangian BRST deformation procedure, which relies on the construction of consistent deformations of the solution to the master equation with the help of standard cohomological techniques. We worked under the hypotheses of smoothness in the coupling constant, locality, Lorentz covariance, Poincaré invariance, and the preservation of the number of derivatives on each field. Our main result is that there are neither cross-couplings that can be added between an Abelian three-form gauge field and a massless gravitino nor self-interactions for the gravitino in \( D = 11 \). The only possible term that can be added to the deformed solution to the master equation is nothing but a generalized Chern-Simons term for the three-form gauge field, \((144)\), which brings contributions to the deformed Lagrangian, but does not modify the original, Abelian gauge transformations \((11)\).

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