Nucleon Form Factors in the Covariant Diquark–Quark Model

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The nucleon is described as a bound state of a quark and an extended diquark. Hereby the notion “diquark” refers to the modelling of separable correlations in the two–quark Green’s functions. Binding of quarks and diquarks takes place via an exchange interaction and is therefore related to the Pauli principle for three–quark states. Fully Poincaré covariant nucleon amplitudes are calculated for free constituent propagators as well as for dressed propagators which parameterise confinement. The corresponding results for space–like form factors differ quantitatively but not qualitatively for various ansätze for the propagators. These results do not allow to draw definite conclusions on the permissibility of different dressing functions. Results for kaon photoproduction, on the other hand, exclude a whole class of constituent propagators.

1 Motivation

The central aim of the studies reported here is to develop a phenomenological QCD–based model for baryon structure. Recent experimental results emphasise the complicated nature of baryons. An example is provided by the ratio of the electric to the magnetic form factor of the proton measured at Jefferson Lab\,[1]. This ratio surprisingly decreases with increasing photon virtuality. We will see later on how this can be understood in our model.

Theoretical issues such as confinement, dynamical breaking of chiral symmetry and the formation of relativistic bound states can be understood and related to the properties of the non-perturbative propagators of QCD. In the Dyson–Schwinger approach we have obtained remarkable progress on this interrelation and other kindred questions during the last years, see the recent reviews\,[2,3]. Amongst many important results we want to highlight the following: Using the general structure of the ghost Dyson–Schwinger equation in Landau gauge, Slavnov–Taylor identities and multiplicative renormalisibility

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one only needs the assumption that QCD Green’s functions can be expanded in asymptotic series in the infrared to demonstrate confinement of transverse gluons. Especially, the infrared behaviour of the gluon and ghost propagators are uniquely related. This prediction can be tested in future lattice calculations as e.g. the ones reported in ref. 

Corresponding studies of quark confinement are under way, and lattice results for the quark propagator will serve to guide them. Since results are not available yet, and because studies of baryon properties require to avoid unphysical thresholds a pragmatic way to proceed is via the parameterisation of quark propagators, i.e., by multiplying dressing functions to the quark propagators:

\[ S^{(k)}(p) = \frac{ip - m_q}{p^2 + m_q^2} f_k \left( \frac{p^2}{m_q^2} \right). \] (1)

The trivial dressing function \( f_0 \equiv 1 \) corresponds to the bare propagator. “Confining” ansätze include

\[ f_1(x) = \frac{1}{2} \left\{ \frac{x + 1}{x + 1 - i/d} + \frac{x + 1}{x + 1 + i/d} \right\}, \] (2)

\[ f_2(x) = 1 - \exp \left[ -d (1 + x) \right], \] (3)

\[ f_3(x, x^*) = \tanh \left[ d (1 + x) (1 + x^*) \right]. \] (4)

The dressing functions \( f_k \) behave qualitatively different for time-like momenta: \( f_1(x) \) and \( f_3(x) \) change sign (as in the case of a tree–level propagator) and the function \( f_2(x) \), that models the exponential type propagator, increases drastically. Away from the real axis, the behaviour of the above model propagators can be read off easily from the definitions (2)–(4). By construction the propagators \( S^{(1)} \) have complex conjugated poles at \( p^2 = m^2(-1 \pm i/d) \) where \( m \) represents a parameter that would be interpreted as a mass if and only if the poles were on the real axis. The entire propagator \( S^{(2)} \) oscillates for a nonzero imaginary part of \( p^2 \). The propagator \( S^{(3)} \) is built such that its dressing function \( f_3 \) asymptotically approaches unity in all directions of the complex \( p^2 \) plane. For asymptotically large space-like momenta the three model propagators \( S^{(k)}, (k = 1, 2, 3) \) match up with the bare propagator \( S^{(0)} \).

Diquarks appear in many phenomenological models. The quantum numbers of one–gluon–exchange suggest that not only colour singlet \( \bar{q}q \) pairs, i.e., mesons, are bound but also colour triplet \( qq \) pairs. Closer analysis shows though that in model Bethe–Salpeter equations beyond the ladder approximation the absence of poles in the quark propagator implies the absence of poles in the colour triplet \( qq \) correlations as well. The latter correlations are the ones which we will call diquarks in the following. It is
interesting to note that diquarks are seen in lattice calculations, see ref.\textsuperscript{12} and references therein. One might even be puzzled by the fact that the corresponding spectral functions for these confined objects are very similar to meson ones as can be seen in fig.\textsuperscript{1}.

2 The relativistic Faddeev problem

Equipped with models for the quark propagator we investigate the relativistic three–quark problem. We neglect any three–particle irreducible interaction graphs between the quarks, which defines the well–known Faddeev problem. For the two–quark correlations we use diquarks in the scalar and axialvector channel as discussed above, i.e., we use a separable \textit{ansatz} for the quark–quark $t$–matrix of the form

$$t^{\text{sep}}_{\alpha\gamma,\beta\delta}(p, q, P) = \chi_{\gamma\alpha}(p) D(P) \bar{\chi}_{\beta\delta}(q) + \chi_{\gamma\alpha}^{\mu}(p) D^{\mu\nu}(P) \bar{\chi}_{\beta\delta}^{\nu}(q).$$ \hspace{1cm} (5)

Here, $P$ is the total momentum of the incoming and the outgoing quark–quark pair, $p$ and $q$ are the relative momenta between the quarks. $\chi_{\alpha\beta}(p)$ and $\chi_{\alpha\beta}^{\mu}(p)$ are vertex functions of quarks with a scalar and an axialvector diquark, respectively. We parameterise the finite size of the diquark vertices by a dipole form. We take the associated width parameter, that directly influences the proton
electric radius, to be of the order 300–400 MeV. The inclusion of axialvector
diquarks is the minimal requirement to describe decuplet baryons and, see
below, is crucial for describing the nucleon electromagnetic form factors cor-
correctly. The diquark propagators $D^{\mu\nu}$ are taken to be free propagators of a
spin–0 [1] particle multiplied by the dressing functions defined in eqs. (2) - (4).

Having imposed the separable ansatz (5) the Faddeev equations reduce
to a coupled system of Bethe–Salpeter equations describing baryons as bound
states of quarks and diquarks which interact by quark exchange. This inter-
action is by virtue of the colour degree of freedom attractive. For the nucleon
these equations read

$$\int \frac{d^4p'}{(2\pi)^4} K(p, p', P) \begin{pmatrix} \Psi_5 \\ \Psi_\mu' \end{pmatrix} (p', P) = 0.$$  \hspace{1cm} (6)

The interaction part of the kernel $K$,

$$K(p, p', P) = (2\pi)^4 \delta(p - p') S^{-1}(p_q) \begin{pmatrix} D^{-1} & 0 \\ 0 & (D^{-1})^\mu'\mu \end{pmatrix} (p_d) +$$

$$\frac{1}{2} \begin{pmatrix} \chi S^T(q) \bar{\chi} & -\sqrt{3}\chi^\mu' S^T(q) \bar{\chi} \\ -\sqrt{3}\chi S^T(q) \bar{\chi} & -\chi^\mu' S^T(q) \bar{\chi} \end{pmatrix},$$  \hspace{1cm} (7)

is given by the quark exchange, the least correlation required to restore the
Pauli principle for the three–quark state (for the definition of the involved mo-
menta see Fig. 2). We have solved these equations without further reduction
and thus obtained fully Poincaré covariant spinorial wave functions $\Psi^{[\mu]}$.

3 Electromagnetic form factors

In a study employing free quark and diquark propagators\cite{4} we calculated the
nucleon electromagnetic form factors. Gauge invariance and correct charge
Figure 3. Ratio of electric to magnetic form factor of the proton as calculated in our model with two parameter sets. $m_q$, $m_{sc}$ and $m_{av}$ are the masses of quark, scalar and axialvector diquarks, respectively (in GeV). The data are taken from ref. [1].
Table 1. Nucleon magnetic moments and e.m. radii with different confinement parameterisations. The nucleon mass is used as input. The parameters are further constrained by describing approximately the spectrum of octet hyperons (c.f. ref.\textsuperscript{13}) as given here.

|          | Expon. | c.c.poles | Non-anal. | Exp. |
|----------|--------|-----------|-----------|------|
| Λ [GeV]  | 1.13   | 1.13      | 1.12      | 1.12 |
| Σ [GeV]  | 1.30   | 1.22      | 1.21      | 1.19 |
| Ξ [GeV]  | 1.37   | 1.37      | 1.33      | 1.32 |
| μ\textsubscript{p} | 2.83 | 2.70 | 2.33 | 2.79 |
| μ\textsubscript{n} | −2.37 | −2.08 | −1.82 | −1.91 |
| (r\textsubscript{el})\textsubscript{p} [fm] | 0.81 | 0.78 | 0.74 | 0.84 |
| (r\textsubscript{m})\textsubscript{p} [fm] | 1.06 | 0.83 | 0.77 | 0.84 |

magnetic form factor of the proton, see fig.\textsuperscript{3}. The parameter set with 30% axialvector correlations explains semi-quantitatively the data, the one with very small axialvector correlations not (for more details see ref.\textsuperscript{14}).

It is required by Ward identities to use dressed vertex functions when employing the effective parameterisations of confinement (2) - (4), for details see ref.\textsuperscript{8}. The resulting nucleon magnetic moments are given in table 1: Their values have increased as compared to ones calculated with free propagators, and they are as close to the experimental values as can be reasonably expected. Please note that our model does not contain effects like e.g. the pion cloud which certainly does give a contribution to these quantities.

We would like to mention that strong and weak form factors have been calculated within this approach using free propagators\textsuperscript{14} and confined ones\textsuperscript{8} (see also ref.\textsuperscript{16} for similar calculations). Given all the calculations for static quantities and space-like form factors one concludes that the use of confined propagators leads to better results. However, they do not allow to distinguish between these different parameterisations. Therefore we have to test the model propagators at sufficiently large time–like momenta.

4 Kaon photoproduction

Kaon photoproduction is a comparatively simple process in our model because flavour algebra and parity dictates that in impulse approximation only diagrams with scalar diquarks contribute, for a detailed discussions see ref.\textsuperscript{8}. Comparison with data reveals that the exponential propagator, see eq. (3),
provides cross sections which are in disagreement with data by orders of magnitude. Hereby the left-most diagram in fig. 4 overwhelms all other contributions. For the other two parametrisations the purely hadronic contribution, the right-most diagram in fig. 4, dominates. Given the theoretical problems with non-analytic propagators, the one with complex-conjugated poles, see eq. (2), provides therefore the best fit to experiment.

5 Summary and Outlook

We have described a model for baryons respecting full Poincaré invariance. This is possible by assuming separability in the quark–quark $t$–matrix. This provides us with an effective definition for extended diquarks. Baryons are then described by the solutions of a Bethe–Salpeter equation. The binding mechanism is quark exchange: Due to colour antisymmetry the Pauli principle leads to an attractive interaction.

We have modelled quark and diquark confinement via a parameterisation of quark and diquark propagators. This has improved the results for static quantities and space–like form factors of the nucleon. However, these observables have not allowed to discriminate between different propagator types. This has been possible by considering kaon photoproduction. According to our analysis the propagators with complex conjugate poles are the best suited ones for future investigations. Further insight into permissible dressing mechanisms will come from studies of nucleon structure functions which are under way.
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