Standard cosmology from the brane cosmology with a localized matter

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Abstract

We discuss the brane cosmology in the 5D anti de Sitter Reissner-Nordstrom (AdSRN5) spacetime. A brane with the tension $\sigma$ is defined as the edge of an AdSRN5 space with mass $M$ and charge $Q$. In this case we get the CFT-radiation term ($\rho_{CFT}$) from $M$ and the charged dust ($-\rho_{cd}^2$) from $Q^2$ in the Friedmann-like equation. However, this equation is not justified because it contains $\rho_{cd}^2$-term with the negative sign. This is unconventional in view of the standard cosmology. In order to resolve this problem, we introduce a localized dust matter which satisfies $P_{dm} = 0$. If $\rho_{dm} = \frac{\sqrt{3}}{2} \rho_{cd}$, the unwanted $-\rho_{cd}^2$ is cancelled against $\rho_{dm}^2$ and thus one recovers a standard Friedmann-Robertson-Walker universe with CFT-radiation and dust matter. For the stiff matter consideration, we can set $\rho_{csti} \sim Q^2$ with the negative sign. Here we introduce a massless scalar which plays the role of a stiff matter with $P_{sca} = \rho_{sca}$ to cancel $-\rho_{csti}$. In this case, however, we find a mixed version of the standard and brane cosmologies.

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Recently Verlinde has introduced entropy bounds to establish the holographic principle in the Friedmann-Robertson-Walker (FRW) universe. Representing a radiation as a conformal field theory (CFT) at high temperature within a 5D anti de Sitter Schwarzschild (AdSS$_5$)-bulk theory, the Cardy-Verlinde’s formula maps to Friedmann equation. This means that the Friedmann equation keeps information about thermodynamic relations of the CFT. Initially the Friedmann equation has nothing to do with the thermodynamic Cardy-Verlinde’s formula. In order to understand this connection, we note that in this approach, the AdS/CFT correspondence plays the role of an important tool to realize the holographic principle.

Within the brane cosmology context, there exist two distinct terms in the Friedmann equation in compared with the standard cosmology: one is $\rho^2$-term due to the localized matter distribution and the other is the presence of the non-local term arisen from the bulk configuration. Even though the localized matter is absent, an observer on the brane finds the radiation-like matter which comes from the non-local term. This holographic term is interpreted as the above CFT-radiation. Further the entropy and temperature can be expressed solely in terms of the Hubble parameter $H = \dot{a}/a$ when the brane crosses the horizon $r = r_+$ of the AdSS$_5$ black hole. In this case the brane is considered as the edge of an AdSS$_5$ space.

Let us discuss this situation more explicitly. The brane starts at $r = 0$ (big bang) inside the small black hole ($\ell > r_+$), crosses the horizon at $a = r_+$, and expands until it reaches maximum size $a = r_m$. And then the brane contracts and it falls the black hole again. Finally the brane disappears (big crunch). A bulk observer in AdSS$_5$-space finds two interesting moments (two points in the Penrose diagram) when the brane crosses the past (future) event horizons. Authors in [3] showed that at these times the Friedmann equation controlling the dynamics of the brane coincides with the Cardy-Verlinde’s formula describing the entropy-energy relation of the CFT defined on the brane. The location of the horizon corresponds to the holographic point. Hence this event can be interpreted as a consequence of the cosmological holography because one used mainly the AdS/CFT correspondence.

More recently, authors in [5] studied a similar case but in different background of the Reissner-Nordstrom black hole. Considering the brane-universe in this charged background, one may get either the CFT-radiation matter ($\rho_{\text{CFT}} = E/V = M\ell/aV$) or the charged-dust matter ($\rho_{\text{cd}} = Q/V$). The presence of the bulk black hole charge gives rise to $-\rho_{\text{cd}}^2$-term which is just a characteristic of the brane cosmology with a localized matter distribution. Unfortunately, one finds here the negative sign in front of $\rho_{\text{cd}}^2$ in the Friedmann-like equation. This negative quadratic term is obviously unconventional in terms of cosmology.

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1 Sometimes, this term is awkward to interpret and sometimes it is useful for resolving the old issues of the standard cosmology.  
2 The approach to this direction was first made in ref. [6]. In that paper, authors mentioned the charge term in the Friedmann equation as the stiff matter even though with a negative sign. However, in this work, we focus on how to avoid the negative term to obtain the standard cosmology in the AdS/Reissner-Nordstrom background. Hence our viewpoint is basically different from that of [6].
because the standard cosmology does not include such a term \(^3\). This implies that mass \(M\) and charge \(Q\) of the Reissner-Nordstrom black hole affect the brane moving in this black hole background differently.

In this paper, we concentrate on resolving this embedding problem of the moving domain wall (brane) into an AdSRN\(_5\)-black hole spacetime. This can be done by introducing any localized dust matter \(\rho_{dm}\) with \(P_{dm} = 0\). It turns out that if \(\rho_{dm} = \sqrt{\frac{3}{2}} \rho_{cd}\), the unwanted-term \(-\rho_{dm}^2\) is cancelled against \(\rho_{cd}^2\). Hence we can recover a Friedmann equation for the standard cosmology which is composed of the CFT-radiation matter and dust matter. On the other hand, following the stiff matter interpretation of the charge \(^3\)[5,6], one has \(-\rho_{csti} \sim -Q^2/V^2\). In this case, to cancel \(-\rho_{csti}\) we have to introduce a massless scalar without the potential which is known as a stiff matter with equation of state \(P_{sca} = \rho_{sca}\). However, the resulting Friedmann equation contains the standard as well as brane cosmological features.

For a cosmological embedding, let us start with an AdSRN\(_5\)-spacetime \(^3\)\[,\]

\[
d s_5^2 = g_{MN} dx^M dx^N = -h(r) dt^2 + \frac{1}{h(r)} dr^2 + r^2 [d\chi^2 + f_k(\chi)^2 (d\theta^2 + \sin^2 \theta d\phi^2)],
\]

where \(k = 0, \pm 1\). \(h(r)\) and \(f_k(\chi)\) are given by

\[
h(r) = k - \frac{m}{r^2} + \frac{q^2}{r^4} + \frac{r^2}{\ell^2}, \quad f_0(\chi) = \chi, \quad f_1(\chi) = \sin \chi, \quad f_{-1}(\chi) = \sinh \chi.
\]

In the case of \(q = 0\), we have an uncharged AdSS\(_5\)-space. The extremal black hole with \(m/2 = |q|\) is not allowed in this context \(^3\)[10]. Using the moving domain wall approach \(^3\)\[8\], we can derive the 4D induced line element from Eq.\(^3\)[4]

\[
d s_4^2 = -dr^2 + a(\tau)^2 \left[ d\chi^2 + f_k(\chi)^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]
\equiv h_{\mu\nu} dx^\mu dx^\nu,
\]

where we use the Greek indices only for the brane. Actually the embedding of the moving domain wall into an AdSRN\(_5\) space is a \(2(t,r) \rightarrow 1(\tau)-\)mapping: \(t \rightarrow t(\tau), r \rightarrow a(\tau)\). Here the scale factor \(a\) will be determined by the Israel junction condition \(^3\)[11]. The extrinsic curvature is given by

\[
K_{\tau\tau} = K_{MN} u^M u^N = (h(a) a)^{-1} (\ddot{a} + h'(a)/a) = \frac{\ddot{a} + h'(a)/2}{\sqrt{\dot{a}^2 + h(a)}},
\]

\[
K_{\chi\chi} = K_{\theta\theta} = K_{\phi\phi} = -h(a) \dot{a} = -\sqrt{\dot{a}^2 + h(a)} a,
\]

\(^3\)Assuming the equation of state \(p = \omega \rho\), the conservation law gives rise to \(\rho \sim a(\tau)^{-3(1+\omega)}\) in four dimensions. Here the causality requires \(|\omega| \leq 1\). \(\omega = 0, +1/3, +1, -1\) correspond to nonrelativistic matter (dust), relativistic matter (radiation), stiff matter, cosmological constant, respectively. Even if the quintessence is concerned, one requires \(-1 < \omega < -1/3\). Hence it is unnatural to include a negative term of \(-\rho_{cd}^2 \sim -a^{-6}\) for a cosmological evolution in the Friedmann equation except the negative cosmological constant of \(\omega = -1\).
where prime stands for derivative with respect to \( a \). The presence of any localized matter on the brane including the brane tension implies that the extrinsic curvature jumps across the brane. This jump is described by the Israel junction condition\(^4\)

\[
K_{\mu\nu} = -\kappa^2 \left( T_{\mu\nu} - \frac{1}{3} T h_{\mu\nu} \right)
\]

with \( \kappa^2 = 8\pi G_5^N \). For cosmological purpose we introduce the 4D perfect fluid as a localized stress-energy tensor on the brane

\[
T^{\text{b+m}}_{\mu\nu} = (\varrho + p) u_\mu u_\nu + p h_{\mu\nu}.
\]

Here \( \varrho = \rho + \sigma \) (\( p = P - \sigma \)), where \( \rho \) (\( P \)) is the energy density (pressure) of the localized matter and \( \sigma \) is the brane tension. In the case of \( \rho = P = 0 \), the r.h.s. of Eq. (6) leads to a form of the RS case as \( -\frac{\sigma\kappa^2}{3} h_{\mu\nu} \). From Eq. (6), one finds the space component of the junction condition

\[
\sqrt{h(a)} + \dot{a}^2 = \frac{\kappa^2}{3} \sigma a.
\]

For a single AdSRN\(_5\) spacetime, we have the fine-tuned brane tension \( \sigma = 3/(\kappa^2 \ell) \). The above equation then leads to

\[
H^2 = -\frac{k}{a^2} + \frac{m}{a^4} - \frac{q^2}{a^6},
\]

where \( H = \dot{a}/a \) is the Hubble parameter. The non-local term of \( m/a^4 \) originates from the electric (Coulomb) part of the bulk Weyl tensor, \( E_{00} \sim m/r^4 \)\(^{13,14}\). This term behaves like radiation for either dark matter\(^3\) or CFT\(^4\). Especially for a closed brane (\( k = 1 \)), we have \( m = \omega_4 M \) and \( q^2 = \frac{3\omega_4^2}{16} Q^2 \) with \( \omega_4 = \frac{16\pi G_5^N}{3V(S^3)} \) and \( V = a^3 V(S^3) \). Using the AdS/CFT correspondence, we obtain the bulk-boundary relations: \( M = \frac{q}{\beta} E, G_5^N = \frac{q}{\beta} G_4^N \). Here \( M(Q) \) is the ADM mass (charge) which are measured at the spatial infinity of AdS\(_5\) space. Then one finds a universe filled with the CFT-radiation and charged-dust

\[
H^2 = -\frac{1}{a^2} + \frac{8\pi G_4^N}{3} \rho_{\text{CFT}} - \frac{\kappa^4}{12} \rho_{\text{cd}}^2, \quad \rho_{\text{CFT}} = \frac{E}{V}, \quad \rho_{\text{cd}} = \frac{Q}{V},
\]

where \( \rho_{\text{CFT}} \) scales like \( a^{-4} \), whereas \( \rho_{\text{cd}}^2 \) behaves like \( a^{-6} \). Hence the charge density (\( \rho_{\text{cd}} \sim a^{-3} \)) plays the same role of a dust matter. It seems that the Friedmann-like equation \( (10) \) is awkward to be interpreted as a cosmological one because the last term is a negative one. Actually a negative energy density square is not allowed because the standard cosmology

\(^4\)The bulk Maxwell term and the Hawking-Ross term are necessary for embedding of the brane in the charged black hole background. However, the flux across the brane does not vary and the junction condition remains unchanged because the charge \( q \) is fixed\(^{12}\). Furthermore we consider only one-sided brane cosmology to see the moving domain wall picture clearly. Hence the \( Z_2 \)-symmetry which was usually issued in two-sided brane cosmology is no longer considered here.
does not include such a term. A way to resolve this problem is to cancel the last term by introducing an appropriate localized matter. Because the last belongs to a dust matter, we have to introduce the same kind of a localized matter with the energy \( E_{dm} \). In this case Eq.(7) takes the form

\[
T^{b+dm}_{\mu\nu} = (\varrho + p)u_{\mu}u_{\nu} + p \, h_{\mu\nu}.
\]

Here \( \varrho = \rho_{dm} + \sigma(\rho_{dm} = E_{dm}/V) \) and \( p = -\sigma \) with the dust equation of state: \( P_{dm} = 0 \). Using the conservation law \( (\dot{\varrho} + 3H(\varrho + p) = 0) \), we can easily check \( \rho_{dm} \sim a^{-3} \). Further Eq.(8) is given by

\[
\sqrt{h(a) + \dot{a}^2} = \frac{\kappa^2}{3}(\rho_{dm} + \sigma)a
\]

which leads with \( \sigma = 3/\kappa^2\ell \) to

\[
H^2 = -\frac{1}{a^2} + \frac{8\pi G N}{3} \rho_{eff}, \quad \rho_{eff} = \frac{E_{eff}}{V} = \rho_{CFT} + \rho_{dm}, \quad E_{eff} = E + E_{dm}.
\]

If \( \rho_{dm} = \frac{\sqrt{3}}{2} \rho_{cd} \), the unwanted \( \rho_{cd}^2 \)-term disappears and one finds the Friedmann equation for a standard cosmology with a definitely positive energy density (\( \rho_{eff} > 0 \))

\[
H^2 = -\frac{1}{a^2} + \frac{8\pi G N}{3} \tilde{\rho}_{eff}, \quad \tilde{\rho}_{eff} \equiv \frac{E_{eff}}{V} = \rho_{CFT} - \rho_{cd}, \quad E_{eff} = E + E_{dm}.
\]

This is our main result. The effective density is composed of two different matters: CFT-radiation matter (\( E \sim 1/a \)) and dust matter (\( E_{dm} = \text{constant} \)). Here we do not expect the circular diagram when the Friedmann equation (14) can be expressed as the relation among the Bekenstein entropy \( S_B = \frac{2\pi a}{n} E_{eff} \), the Bekenstein-Hawking entropy \( S_{BH} = (n - 1) \frac{V}{4G N a} \), and the Hubble entropy \( S_H = (n - 1) \frac{HV}{4G N} \)

\[
S_H^2 + (S_B - S_{BH})^2 = S_B^2.
\]

This is so because \( S_B \) does not remain constant during the cosmological evolution. In the case of the CFT-radiation matter only, the Bekenstein entropy is constant with respect to the cosmic time \( \tau \).

On the other hand, we may consider the charge term as a stiff matter [20,21]. In this case, Eq.(10) is rewritten as

\[
H^2 = -\frac{1}{a^2} + \frac{8\pi G N}{3} \tilde{\rho}_{eff}, \quad \tilde{\rho}_{eff} \equiv \rho_{CFT} - \rho_{cd},
\]

with

\[
\rho_{cd} = \frac{1}{2} \Phi \rho_Q, \quad \Phi \equiv \frac{\phi}{a} = \frac{3}{8} \frac{\omega_3 Q \ell}{a^3}, \quad \rho_Q = \frac{Q}{V}.
\]

Here \( \phi \) is the bulk electrostatic potential which is difference between the horizon and infinity, while \( \Phi \) is its boundary CFT-potential. \( \rho_Q \) is the R-charge density of the CFT. So the cosmological evolution will be driven by the the CFT-radiation energy \( E \) and the
electric potential energy $\frac{1}{2}\Phi Q$ due to the R-charge $Q$. Introducing the R-charge entropy of $S_Q = \frac{2s}{n} \cdot \frac{1}{2}\Phi Q$, the entropy-relation is given by

$$S_H^2 + (S_B - S_Q - S_{BH})^2 = (S_B - S_Q)^2.$$ (18)

In this case we note that $S_B - S_Q$ does not remain constant during the evolution. Hence one cannot expect the circular diagram for the CFT-radiation matter. Obviously the above equation induces some problem to interpret it as a cosmological evolution equation in Minkowskian closed universe. This is so because if $\rho_{CFT} < \rho_{csti}, \rho_{eff}$ gives rise to a negative energy density. This is the case that happens at the very early universe when $\rho_{csti} \sim 1/a^6$ is more dominant than $\rho_{CFT} \sim 1/a^4$. This is not allowed for the standard cosmology. Hence we would like to resolve this problem by introducing an appropriate matter. One candidate for stiff matters is a massless scalar without the potential whose stress-energy tensor is given by [14,15]

$$T_{\mu\nu}^{\text{sca}} = \partial_{\mu}\varphi\partial_{\nu}\varphi - \frac{1}{2}(\partial\varphi)^2 h_{\mu\nu}.$$ (19)

If we assume $\varphi = \varphi(\tau)$ for cosmological purpose, this leads to $T_{\nu}^{\mu} = \text{diag}[-\dot{\varphi}^2/2, \varphi^2/2, \varphi^2/2, \dot{\varphi}^2/2] \equiv \text{diag}[-\rho_{sca}, P_{sca}, P_{sca}, P_{sca}]$ which satisfies obviously the stiff equation of state as $P_{sca} = \rho_{sca}$. In this case we easily get $\rho_{sca} \sim a^{-6}$ from the conservation law. Considering the brane tension and scalar matter separately, Eq.(8) takes the form

$$\sqrt{h(a)} + \dot{a}^2 = \frac{\kappa^2}{3}(\rho_{sca} + \sigma)a.$$ (20)

This leads to

$$H^2 = -\frac{1}{a^2} + \frac{8\pi G_N}{3}(\rho_{CFT} - \rho_{csti} + \rho_{sca}) + \kappa^2\rho_{sca}^2/9.$$ (21)

If $\rho_{csti} = \rho_{sca}$, the unwanted $\rho_{csti}$-term disappears and one finds Friedmann-equation for a mixed version of the standard and brane cosmologies

$$H^2 = -\frac{1}{a^2} + \frac{8\pi G_N}{3}\rho_{CFT} + \kappa^2\rho_{sca}^2/9, \quad \rho_{sca} = \frac{\dot{\phi}^2}{2}.\quad (22)$$

An embedding of the moving domain wall (brane) into the anti de Sitter-charged black hole spacetime gives rise to the negative electric energy density in the Friedmann equation. It is shown that this problem can be resolved by choosing an appropriate stiff matter. Even though we succeed in obtaining the positive energy density, this action gives rises to the high-order term of $\rho_{sca}^2 \sim a^{-12}$ which have ever not been found in the standard and brane cosmology. Such a high-order term, if it exists, will contribute to the very early evolution of the universe significantly.

Finally we summarize our results. Our first view is to regard the charge term $(-Q^2/a^6)$ as $-\rho_{cd}^2 \sim -Q^2/V^2$. Reminding that the brane cosmology automatically produces $+\rho^2$-term,
we resolve it by introducing a dust matter with its equation of state $P_{dm} = 0$. As a result, we obtain a standard FRW universe filled with two different matters: CFT-radiation matter and dust matter.

The second view is to consider the charge term as a stiff matter \cite{5}. In this case, we have $-\rho_{\text{csti}} \sim -Q^2/a^6$ which induces the negative energy problem in cosmology. To resolve this, we introduce the same kind of matter on the brane. One candidate is just a massless scalar that is known to satisfy the stiff equation of state $P_{dm} = \rho$. However, a resultant equation is a mixed Friedmann equation for standard and brane cosmologies. In this sense, our first view is more promising than the second one in obtaining the standard Friedmann equation from the brane cosmological picture. Although we obtain the entropy-relations for the two cases, these do not belong to the circular diagram. Hence we conclude that the charge $Q$ of the Reisser-Norstrom black hole plays a different role from the mass $M$ of the black hole. Two are holographic non-local matters which come from the bulk charged black hole. But these give rise to different matters: one provides the CFT-radiation matter and the other gives either negative dust matter or negative stiff matter. Also we find that the circular entropy-relation for pure CFT-radiation which is derived from the uncharged black hole does not holds for the charged black hole background.

Further, it is interesting to study the connection between the Friedmann equations (14) and (16) and the Cardy-Verlinde’s formula for the entropy-energy relation when the brane crosses the horizon of AdSRN$_5$ black hole. It was shown that this connection holds even for the charged black hole \cite{18}.

Finally we wish to remark that authors in \cite{16} considered a similar issue and derived a Friedmann equation\footnote{I thank to C. Grojean for pointing out this}. Also within the Randall-Sundrum context, cancellations between the bulk and the brane to recover standard cosmology were first proposed in \cite{17}.

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