A supersymmetric 3-4-1 model

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Abstract

We build the complete supersymmetric version of a 3-4-1 gauge model using the superfield formalism. We point out that a discrete symmetry, similar to the R-symmetry in the minimal supersymmetric standard model, is possible to be defined in this model. Hence we have both R-conserving and R-violating possibilities. We also discuss some phenomenological results coming from this model.

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1 Introduction

The full symmetry of the so called Standard Model (SM) is the gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Nevertheless, the SM is not considered as the ultimate theory since neither the fundamental parameters, masses and couplings, nor the symmetry pattern are predicted. Even though many aspects of the SM are experimentally supported to a very accuracy, the embedding of the model into a more general framework is to be expected.

Some of these possibilities is that, at energies of a few TeVs, the gauge symmetry may be $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ (3-3-1 for shortness) [1, 2, 3]. Recently, the supersymmetric version of these model have already been constructed in [4, 5]. These 3-3-1 models can be embedded in a model with 3-4-1, its mean $SU(3)_c \otimes SU(4)_L \otimes U(1)_N$ gauge symmetry [6].

In $SU(4)_L \otimes U(1)_N$, the most general expression for the electric charge generator is a linear combination of the four diagonal generators of the gauge group

$$Q = \frac{1}{2} \left( a\lambda_3 + \frac{b}{\sqrt{3}}\lambda_8 + \frac{c}{\sqrt{6}}\lambda_{15} \right) + NI_{4 \times 4}$$
where $\lambda_i$, being the Gell-Mann matrices for $SU(4)_L$, see [7, 8], normalized as $\text{Tr}(\lambda_i \lambda_j) = 2\delta_{ij}$, $I_{4 \times 4} = \text{diag}(1, 1, 1, 1)$ is the diagonal $4 \times 4$ unit matrix, and $a$, $b$ and $c$ are free parameters to be fixed next. Therefore, there is an infinite number of models can, in principle, be constructed.

A model with the $SU(4) \otimes U(1)$ symmetry in the lepton sector, quarks were not considered on this work, was suggested some years ago in Ref. [9], in which the magnetic moment of neutrinos arises as the result of charged scalars that belong to an $SU(4)$ sextet, and the mass of neutrino arises at two-loop level as the result of electroweak radiative correction.

The 3-4-1 model in Ref. [6] contain exotic electric charges only in the quark sector, while leptons have ordinary electric charges and gauge bosons have integer electric charges. The best feature of this model is that it provides us with an alternative to the problem of the number $N_f$ of fermion families. These sort of models are anomaly free only if there are equal number of quadruplet and anti-quadruplet (considering the color degrees of freedom), and furthermore requiring the sum of all fermion charges to vanish. Two of the three quark generations transform identically and one generation, it does not matter which one, transforms in a different representation of $SU(4)_L \otimes U(1)_N$. This means that in these models as in the $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ ones [1], in order to cancel anomalies, the number of families ($N_f$) must be divisible by the number of color degrees of freedom ($n$). This fact, together with asymptotic freedom in QCD, the model predicts that the number of generations must be three and only three.

On the other hand, at low energies these models are indistinguishable from the SM. There is a very nice review about this kind of model see [10, 11]. This make 3-4-1 model interesting by their own. In this article we construct the supersymmetric version of the model in Ref [6].

The outline of the paper is as follows. In Sec. 2 we present the representation content of the supersymmetric 3-4-1 model. We build the lagrangian in Sec. 3. While in Sec. 4, we discuss the double charged charginos in this model, while in the last section we present our conclusion.
2 The model

In this section (Sec. 2.1) we review the non-supersymmetric 3-4-1 model of Refs. [6] and add the superpartners (Sec. 2.2) of the usual particles of the non supersymmetric model. The superfields, useful to construct the supersymmetric lagrangian of the model, associated with the particles of this model are introduced in section (Sec. 2.3).

2.1 The representation content

In the model of Ref. [6], the free parameters for the electric charge generators are

\[ a = 1, \quad b = -1, \quad c = -4, \]

and Eq. (1) can be rewritten as

\[ Q = \frac{1}{2} \left( \lambda_3 - \frac{1}{\sqrt{3}} \lambda_8 - \frac{4}{\sqrt{6}} \lambda_{15} \right) + N I_{4 \times 4}, \]

\[ = \text{diag}(N, N - 1, N, N + 1). \]  \hspace{1cm} (3)

However, let us first consider the particle content of the model without supersymmetry. We have the leptons transforming in the lowest representation of $SU(4)_L$ the quartet\(^1\) in the following way

\[ L_{aL} = \begin{pmatrix} \nu_a \\ l_a \\ \nu^c_a \\ l^c_a \end{pmatrix}_L \sim (1, 4, 0), \quad a = 1, 2, 3. \]  \hspace{1cm} (4)

In parenthesis it appears the transformations properties under the respective factors ($SU(3)_C, SU(4)_L, U(1)_N$).

In the quark sector, one quark family is also put in the quartet representation

\[ Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ u' \\ J \end{pmatrix}_L \sim \left( 3, 4, \frac{2}{3} \right), \]  \hspace{1cm} (5)

\(^1\)In the same way as proposed by Voloshin [9] in order to understand the existence of neutrinos with large magnetic moment and small mass.
and the respective singlets are given by
\[ u^c_{1L} \sim (3^*, 1, -\frac{2}{3}), \quad d^c_{1L} \sim (3^*, 1, \frac{1}{3}), \]
\[ u^c_{\ell L} \sim (3^*, 1, -\frac{2}{3}), \quad J^c_L \sim (3^*, 1, -\frac{5}{3}), \] (6)
writing all the fields as left-handed; \( u' \) and \( J \) are new quarks with charge +2/3 and +5/3 respectively.

The others two quark generations, as we have explained in the introduction, we put in the anti-quartet representation
\[ Q_{2L} = \begin{pmatrix} d_2 \\ u_2 \\ d'_1 \\ j_1 \end{pmatrix} \sim (3, 4^*, -\frac{1}{3}), \quad Q_{3L} = \begin{pmatrix} d_3 \\ u_3 \\ d'_2 \\ j_2 \end{pmatrix} \sim (3, 4^*, -\frac{1}{3}), \] (7)
and also with the respective singlets,
\[ u^c_{\alpha L} \sim (3^*, 1, -\frac{2}{3}), \quad d^c_{\alpha L} \sim (3^*, 1, \frac{1}{3}), \]
\[ d^c_{\beta L} \sim (3^*, 1, \frac{1}{3}), \quad j^c_{\beta L} \sim (3^*, 1, \frac{4}{3}), \] (8)
j_\beta and \( d'_\beta, \beta = 1, 2 \) are new quarks with charge −4/3 and −1/3 respectively, while \( \alpha = 2, 3 \) is the family index for the quarks. We remind that in Eqs. (4,5,6,7,8) all fields are still symmetry eigenstates.

On the other hand, the scalars, in quartet, which are necessary to generate the quark masses are
\[ \eta = \begin{pmatrix} \eta^0_1 \\ \eta^-_1 \\ \eta^0_2 \\ \eta^+_2 \end{pmatrix} \sim (1, 4, 0) \]
\[ \rho = \begin{pmatrix} \rho^+_1 \\ \rho^0_1 \\ \rho^+_2 \\ \rho^{++} \end{pmatrix} \sim (1, 4, 1) \]
\[ \chi = \begin{pmatrix} \chi_1^- \\ \chi_2^- \\ \chi_0^- \end{pmatrix} \sim (1, 4, -1). \quad (9) \]

In order to avoid mixing among primed and unprimed quarks, we have to introduce an extra scalar transforming like \( \eta \) but with different vacuum expectation value (VEV)

\[ \phi = \begin{pmatrix} \phi_1^0 \\ \phi_1^- \\ \phi_2^0 \\ \phi_2^+ \end{pmatrix} \sim (1, 4, 0). \quad (10) \]

In order to obtain massive charged leptons it is necessary to introduce the following symmetric anti-decuplet

\[ H = \begin{pmatrix} H_1^0 & H_1^+ & H_2^0 & H_2^- \\ H_1^+ & H_1^{++} & H_3^+ & H_3^0 \\ H_2^0 & H_3^+ & H_4^0 & H_4^- \\ H_2^- & H_3^- & H_4^- & H_2^{--} \end{pmatrix} \sim (1, 10^*, 0). \quad (11) \]

then the charged leptons get a mass but neutrinos remain massless, at least at tree level.

### 2.2 Supersymmetric partners

Now, we introduce the minimal set of particles in order to implement the supersymmetry [12]. We have the sleptons corresponding to the leptons in Eq. (4); squarks related to the quarks in Eqs.(6)-(8); and the Higgsinos related to the scalars given in Eqs. (9) and (11). Then, we have to introduce the following additional particles

\[ \tilde{Q}_{1L} = \begin{pmatrix} \tilde{u}_1 \\ \tilde{d}_1 \\ \tilde{u}'_L \\ \tilde{J}_L \end{pmatrix} \sim (3, 4, 2/3), \quad \tilde{Q}_{\alpha L} = \begin{pmatrix} \tilde{d}_\alpha \\ \tilde{u}_\alpha \\ \tilde{d}'_\beta \\ \tilde{J}_\beta \end{pmatrix} \sim (3, 4^*, -1/3), \]
\( \bar{L}_{aL} = \begin{pmatrix} \tilde{\nu}_a \\ \tilde{l}_a \\ \tilde{\nu}^c_a \\ \tilde{l}^c_a \end{pmatrix}_L \sim (1, 4, 0), \)

\( u^c_{iL} \sim (3^*, 1, -\frac{2}{3}), \quad d^c_{iL} \sim (3^*, 1, -\frac{1}{3}), \)

\( u^c_L \sim (3^*, 1, -\frac{2}{3}), \quad J^c_L \sim (3^*, 1, -\frac{5}{3}), \)

\( d^c_{\beta L} \sim (3^*, 1, \frac{1}{3}), \quad j^c_{\beta L} \sim (3^*, 1, \frac{4}{3}). \) (12)

Where \( \alpha = 2, 3 \) and \( \beta = 1, 2. \) The higgsinos of these model are given by

\( \tilde{\eta} = \begin{pmatrix} \tilde{\eta}^0_1 \\ \tilde{\eta}^-_1 \\ \tilde{\eta}^c_2 \\ \tilde{\eta}^c_1 \end{pmatrix} \sim (1, 4, 0), \quad \tilde{\phi} = \begin{pmatrix} \tilde{\phi}^0_1 \\ \tilde{\phi}^-_1 \\ \tilde{\phi}^c_2 \\ \tilde{\phi}^c_1 \end{pmatrix} \sim (1, 4, 0), \)

\( \tilde{\rho} = \begin{pmatrix} \tilde{\rho}^+_1 \\ \tilde{\rho}^0_1 \\ \tilde{\rho}^c_2 \\ \tilde{\rho}^c_1 \end{pmatrix} \sim (1, 4, 1), \quad \tilde{\chi} = \begin{pmatrix} \tilde{\chi}^-_1 \\ \tilde{\chi}^-_2 \\ \tilde{\chi}^0 \end{pmatrix} \sim (1, 4, -1), \) (13)

\( \tilde{H} = \begin{pmatrix} \tilde{H}^0_1 & \tilde{H}^+_1 & \tilde{H}^0_2 & \tilde{H}^-_2 \\ \tilde{H}^+_1 & \tilde{H}^{++}_1 & \tilde{H}^+_3 & \tilde{H}^0_3 \\ \tilde{H}^0_2 & \tilde{H}^c_3 & \tilde{H}^0_4 & \tilde{H}^-_4 \\ \tilde{H}^-_2 & \tilde{H}^c_3 & \tilde{H}^-_4 & \tilde{H}^{--}_2 \end{pmatrix} (1, 10^*, 0). \) (14)

Besides, in order to to cancel chiral anomalies generated by the superpartners of the scalars, we have to add the following higgsinos in the respective anti-quartet representation,

\( \eta' = \begin{pmatrix} \eta'^0_1 \\ \eta'^c_1 \\ \eta'^c_2 \\ \eta'^c_1 \end{pmatrix} \sim (1, 4^*, 0), \quad \phi' = \begin{pmatrix} \phi^0_1 \\ \phi^c_1 \\ \phi^c_2 \\ \phi^c_1 \end{pmatrix} \sim (1, 4^*, 0) \)
\[ \rho' = \left( \begin{array}{c} \rho_1^- \\ \rho_0 ^r \\ \rho_2^- \\ \rho'^{--} \end{array} \right) \sim (1, 4^*, -1), \quad \chi' = \left( \begin{array}{c} \chi_1^+ \\ \chi_1^{++} \\ \chi_2^+ \\ \chi^{0} \end{array} \right) \sim (1, 4^*, 1), \quad (15) \]

and the decuplet

\[ H' = \left( \begin{array}{cccc} H_1^0 & H_1^- & H_2^0 & H_2^- \\ H_1^- & H_1^{--} & H_3^0 & H_3^- \\ H_2^0 & H_3^- & H_4^0 & H_4^* \\ H_2^- & H_3^{--} & H_4^* & H_2^{++} \end{array} \right) \sim (1, 10, 0). \quad (16) \]

Their superpartners, higgsinos, are

\[ \tilde{\eta}' = \left( \begin{array}{c} \tilde{\eta}_1^0 \\ \tilde{\eta}_1^+ \\ \tilde{\eta}_2^0 \\ \tilde{\eta}_2^- \end{array} \right) \sim (1, 4^*, 0), \quad \tilde{\phi}' = \left( \begin{array}{c} \tilde{\phi}_1^0 \\ \tilde{\phi}_1^+ \\ \tilde{\phi}_2^0 \\ \tilde{\phi}_2^- \end{array} \right) \sim (1, 4^*, 0), \]

\[ \tilde{\rho}' = \left( \begin{array}{c} \tilde{\rho}_1^- \\ \tilde{\rho}_0 ^r \\ \tilde{\rho}_2^- \\ \tilde{\rho}'^{--} \end{array} \right) \sim (1, 4^*, -1), \quad \tilde{\chi}' = \left( \begin{array}{c} \tilde{\chi}_1^+ \\ \tilde{\chi}_1^{++} \\ \tilde{\chi}_2^+ \\ \tilde{\chi}^{0} \end{array} \right) \sim (1, 4^*, 1), \quad (17) \]

\[ \tilde{H}' = \left( \begin{array}{cccc} \tilde{H}_1^0 & \tilde{H}_1^- & \tilde{H}_2^0 & \tilde{H}_2^- \\ \tilde{H}_1^- & \tilde{H}_1^{--} & \tilde{H}_3^0 & \tilde{H}_3^- \\ \tilde{H}_2^0 & \tilde{H}_3^- & \tilde{H}_4^0 & \tilde{H}_4^* \\ \tilde{H}_2^- & \tilde{H}_3^{--} & \tilde{H}_4^* & \tilde{H}_2^{++} \end{array} \right) \sim (1, 10, 0). \quad (18) \]

The vev of our scalars are given by

\[ \langle \eta \rangle = \left( \frac{v}{\sqrt{2}}, 0, 0, 0 \right), \quad \langle \rho \rangle = \left( 0, \frac{u}{\sqrt{2}}, 0, 0 \right), \quad \langle \phi \rangle = \left( 0, 0, \frac{z}{\sqrt{2}}, 0 \right), \quad \langle \chi \rangle = \left( 0, 0, 0, \frac{w}{\sqrt{2}} \right), \]

\[ \langle H_3^0 \rangle = \frac{x}{\sqrt{2}}, \quad \langle H_1^0 \rangle = \langle H_2^0 \rangle = \langle H_4^0 \rangle = 0, \]

\[ \langle \eta' \rangle = \left( \frac{v'}{2}, 0, 0, 0 \right), \quad \langle \rho' \rangle = \left( 0, \frac{u'}{\sqrt{2}}, 0, 0 \right), \quad \langle \phi' \rangle = \left( 0, 0, \frac{z'}{\sqrt{2}}, 0 \right), \quad \langle \chi \rangle = \left( 0, 0, 0, \frac{w'}{\sqrt{2}} \right), \]

\[ \langle H_3^0 \rangle = \frac{x'}{\sqrt{2}}, \quad \langle H_1^0 \rangle = \langle H_2^0 \rangle = \langle H_4^0 \rangle = 0. \quad (19) \]
Concerning the gauge bosons and their superpartners, if we denote the gluons by $g^b$ the respective superparticles, the gluinos, are denoted by $\lambda^b_C$, with $b = 1, \ldots, 8$; and in the electroweak sector we have $V^a$, with $a = 1, \ldots, 15$; the gauge boson of $SU(4)_L$, and their gauginos partners $\lambda^A_b$; finally we have the gauge boson of $U(1)_N$, denoted by $V'$, and its supersymmetric partner $\lambda_B$.

### 2.3 Superfields

The superfields formalism is useful in writing the Lagrangian which is manifestly invariant under the supersymmetric transformations [13] with fermions and scalars put in chiral superfields while the gauge bosons in vector superfields. As usual the superfield of a field $\phi$ will be denoted by $\hat{\phi}$ [12]. The chiral superfield of a multiplet $\phi$ is denoted by

$$
\hat{\phi} \equiv \hat{\phi}(x, \theta, \bar{\theta}) = \tilde{\phi}(x) + i \theta \sigma^m \bar{\theta} \partial_m \tilde{\phi}(x) + \frac{1}{4} \theta \theta \tilde{\theta} \tilde{\theta} \nabla \tilde{\phi}(x)
$$

while the vector superfield is given by

$$
\hat{V}(x, \theta, \bar{\theta}) = -\theta \sigma^m \bar{\theta} V_m(x) + i \theta \theta \bar{\theta} \bar{V}(x) - i \bar{\theta} \theta \bar{\theta} \bar{V}(x) + \frac{1}{2} \theta \theta \theta \theta D(x).
$$

The fields $F$ and $D$ are auxiliary fields which are needed to close the supersymmetric algebra and eventually will be eliminated using their motion equations.

Summaryzing, we have in the 3-4-1 supersymmetric model the following superfields: $\hat{L}_{1,2,3}$, $\hat{Q}_{1,2,3}$, $\hat{\eta}$, $\hat{\rho}$, $\hat{\chi}$, $\hat{H}$; $\hat{\eta}'$, $\hat{\rho}'$, $\hat{\chi}'$, $\hat{\phi}'$, $\hat{H}'$; $\hat{u}^c_{1,2,3}$, $\hat{d}^c_{1,2,3}$, $\hat{u}'$, $\hat{d}'_{1,2}$, $\hat{J}$ and $\hat{j}_{1,2}$, i.e., 28 chiral superfields, and 24 vector superfields: $\hat{V}^a$, $\hat{V}^\alpha$ and $\hat{V}'$. 

3 The Lagrangian

With the superfields introduced in the last section we can built a supersymmetric invariant lagrangian. It has the following form

\[ \mathcal{L}_{341} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}. \] (22)

Here \( \mathcal{L}_{SUSY} \) is the supersymmetric piece, while \( \mathcal{L}_{soft} \) explicitly breaks SUSY. Below we will write each of these lagrangians in terms of the respective superfields.

3.1 The Supersymmetric Term.

The supersymmetric term can be divided as follows

\[ \mathcal{L}_{SUSY} = \mathcal{L}_{\text{Lepton}} + \mathcal{L}_{\text{Quarks}} + \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Scalar}}, \] (23)

where each term is given by

\[ \mathcal{L}_{\text{Lepton}} = \int d^4 \theta \left[ \hat{\bar{L}} e^{2g \hat{V}} \hat{L} \right], \] (24)

\[ \mathcal{L}_{\text{Quarks}} = \int d^4 \theta \left\{ \hat{\bar{Q}}_1 e^{2g \hat{V} C + 2g \hat{V} + g' \left( \frac{4}{3} \right) \hat{V}'} \hat{Q}_1 + \hat{\bar{Q}}_\alpha e^{2g \hat{V} C + 2g \hat{V} + g' \left( -\frac{1}{3} \right) \hat{V}'} \hat{Q}_\alpha \right\}, \]

\[ + \hat{\bar{u}}_i e^{2g \hat{V} C + g' \left( -\frac{2}{3} \right) \hat{V}'} \hat{u}_i + \hat{\bar{d}}_i e^{2g \hat{V} C + g' \left( \frac{2}{3} \right) \hat{V}'} \hat{d}_i + \hat{\bar{J}}_\beta e^{2g \hat{V} C + g' \left( -\frac{1}{3} \right) \hat{V}'} \hat{J}_\beta \}

\[ + \hat{\bar{u}}^{\epsilon} e^{2g \hat{V} C + g' \left( -\frac{2}{3} \right) \hat{V}'} \hat{u}^{\epsilon} + \hat{\bar{d}}^{\epsilon} e^{2g \hat{V} C + g' \left( \frac{2}{3} \right) \hat{V}'} \hat{d}^{\epsilon} + \hat{\bar{J}}^{\epsilon} e^{2g \hat{V} C + g' \left( -\frac{1}{3} \right) \hat{V}'} \hat{J}^{\epsilon} \}, \] (25)

where \( \alpha = 2, 3 \) and \( \beta = 1, 2 \), while the third term is

\[ \mathcal{L}_{\text{Gauge}} = \frac{1}{4} \int d^2 \theta \text{Tr}[W_C W_C] + \frac{1}{4} \int d^2 \theta \text{Tr}[W_L W_L] + \frac{1}{4} \int d^2 \theta W' W' \]

\[ + \frac{1}{4} \int d^2 \tilde{\theta} \text{Tr}[\tilde{W}_C \tilde{W}_C] + \frac{1}{4} \int d^2 \tilde{\theta} \text{Tr}[\tilde{W}_L \tilde{W}_L] + \frac{1}{4} \int d^2 \tilde{\theta} W' \tilde{W}' \]

\[ (26) \]
where $\hat{V}_c = T^a \hat{V}_c^a$, $\hat{V} = T^i \hat{V}^i$ and $T^a = \lambda^a/2$ are the generators of $SU(3)$ i.e., $a = 1, \cdots, 8$, and $T^i = \lambda^i/2$ are the generators of $SU(4)$ i.e., $i = 1, \cdots, 15$, and $g_s$, $g$ and $g'$ are the gauge coupling of $SU(3)_C$, $SU(4)_L$ and $U(1)_N$. $W^a_c$, $W^i$ and $W'$ are the strength fields, and they are given by

$$W_{ac}^a = -\frac{1}{8g_s} \hat{D}\hat{D}e^{-2g_s\hat{V}_c} D_a e^{-2g_s\hat{V}_c}$$

$$W_{\alpha}^a = -\frac{1}{8g} \hat{D}\hat{D}e^{-2g\hat{V}} D_a e^{-2g\hat{V}}$$

$$W_{\alpha}^{\prime} = -\frac{1}{4} \hat{D}\hat{D}D_{\alpha} \hat{V}^{\prime}. \quad (27)$$

Finally

$$\mathcal{L}_{\text{Escalar}} = \int d^4\theta \left[ \hat{\eta} e^{2g\hat{V}} \hat{\eta} + \hat{\rho} e^{(2g\hat{V} + g^{\prime}\hat{V}^{\prime})} \hat{\rho} + \hat{\chi} e^{(2g\hat{V} - g^{\prime}\hat{V}^{\prime})} \hat{\chi} + \hat{\phi} e^{2g\hat{V}} \hat{\phi} + \hat{H} e^{2g\hat{V}} \hat{H} \\
+ \hat{\eta} e^{2g\hat{V}} \hat{\eta}^{\prime} + \hat{\rho} e^{(2g\hat{V} - g^{\prime}\hat{V}^{\prime})} \hat{\rho}^{\prime} + \hat{\chi} e^{(2g\hat{V} + g^{\prime}\hat{V}^{\prime})} \hat{\chi}^{\prime} + \hat{\phi} e^{2g\hat{V}} \hat{\phi}^{\prime} + \hat{H} e^{2g\hat{V}} \hat{H}^{\prime} \right]$$

$$+ \int d^2\theta W + \int d^2\bar{W} \quad (28)$$

where $W$ is the superpotential, which we discuss in the next subsection.

### 3.2 Superpotential.

The superpotential of our model is given by

$$W = \frac{W_2}{2} + \frac{W_3}{3}, \quad (29)$$

with $W_2$ having only two chiral superfields and the terms permitted by our symmetry are

$$W_2 = \sum_{a=1}^{3} \mu_{0a} \hat{L}_{aL} \hat{\eta} + \sum_{a=1}^{3} \mu_{1a} \hat{L}_{aL} \hat{\phi} + \mu_{\eta} \hat{\eta} \hat{\eta} + \mu_{\phi} \hat{\phi} \hat{\phi} + \mu_{2\eta} \hat{\eta} \hat{\eta} + \mu_{3\phi} \hat{\phi} \hat{\phi} + \mu_{\chi} \hat{\chi} \hat{\chi} + \mu_{H} \hat{H} \hat{H}^{\prime}, \quad (30)$$

and in the case of three chiral superfields the terms are

$$W_3 = \sum_{a=1}^{3} \sum_{b=1}^{3} \sum_{c=1}^{3} \lambda_{abc} \epsilon \hat{L}_{aL} \hat{L}_{bL} \hat{L}_{cL} + \sum_{a=1}^{3} \sum_{b=1}^{3} \lambda_{2ab} \epsilon \hat{L}_{aL} \hat{\bar{L}}_{bL} \hat{\eta} + \sum_{a=1}^{3} \sum_{b=1}^{3} \lambda_{3ab} \epsilon \hat{L}_{aL} \hat{\bar{L}}_{bL} \hat{\phi}$$
In the next subsection, we will show that it is possible to define the $R$-parity symmetry, the phenomenology of this model with $R$-parity conserved has similar features to that of the $R$-conserving MSSM: the supersymmetric particles are pair-produced and the lightest neutralino is the lightest supersymmetric particle (LSP), see in (subsection 3.2.2). Then in (subsection 3.2.3) we will show that there are terms that will induce mass to the neutrinos of the model.
3.2.1 Discrete R-Parity in SUSY341

To get the terms that are invariant under discrete R-parity in the superpotential, we need to check the following condition [14]

\[ \int d^2 \theta \prod_a \Phi_a(x, \theta, \bar{\theta}), \quad \text{if} \quad \sum_a n_a = 0. \quad (32) \]

Choosing the following R-charges

\begin{align*}
  n_{\eta'} &= n_\phi = n_\rho = n_{H'} = 1, \quad n_\eta = n_\phi = n_{\rho'} = n_H = -1, \\
  n_L &= n_{Q_1} = n_{Q_\alpha} = n_{d_1} = n_{w'} = \frac{1}{2}, \quad n_J = n_j = -\frac{1}{2}, \\
  n_{u_i} &= n_{d'} = -\frac{3}{2}, \quad n_\chi = n_{\chi'} = 0, \quad (33)
\end{align*}

with this R-charge assignment, we get that all the usual particle in the 341 model has R-charge equal one while their superpartner has R-charge opposite, as happen in the MSSM.

Next we will present some discussion about the physical consequences on the superpotential, and point some likeness with the MSSM phenomenology.

3.2.2 R-parity conservation

The terms in the superpotential that satisfy the R-parity, \( W_{2RC} + W_{3RC} \), are given by the following terms

\begin{align*}
  W_{2RC} &= \mu_{\eta'} \hat{\eta}_L \hat{\eta}_L' + \mu_{\phi} \hat{\phi}_L \hat{\phi}_L' + \mu_{\rho} \hat{\rho}_L \hat{\rho}_L' + \mu_{\chi} \hat{\chi}_L \hat{\chi}_L' + \mu_{H} \hat{H}_L \hat{H}_L', \\
  W_{3RC} &= \sum_{a=1}^{3} \sum_{b=1}^{3} \lambda_{ab} \hat{L}_a \hat{L}_b \hat{\eta}_L + \sum_{a=1}^{3} \sum_{b=1}^{3} \lambda_{ab} \hat{L}_a \hat{L}_b \hat{\phi}_L + f_1 \hat{\rho}_L \hat{\rho}_L' + f_6 \hat{\chi}_L \hat{\chi}_L' + f_6' \hat{\rho}_L' \hat{\chi}_L' \\
  &\quad + f_6' \hat{\rho}_L' \hat{\chi}_L' + \sum_{i=1}^{3} \kappa_{1i} \hat{Q}_{1L} \hat{\eta}_L' \hat{u}_L^c + \kappa_{2i} \hat{Q}_{1L} \hat{\phi}_L' \hat{u}_L^c + \sum_{i=1}^{3} \kappa_{3i} \hat{Q}_{1L} \hat{\rho}_L' \hat{d}_L^c + \kappa_{4i} \hat{Q}_{1L} \hat{\chi}_L' \hat{j}_L^c \\
  &\quad + \sum_{a=2}^{3} \sum_{i=1}^{3} \kappa_{5a} \hat{Q}_{aL} \hat{\rho}_L \hat{u}_L^c + \sum_{a=2}^{3} \sum_{i=1}^{3} \kappa_{6a} \hat{Q}_{aL} \hat{\eta}_L \hat{d}_L^c + \sum_{a=2}^{3} \sum_{\beta=1}^{2} \kappa_{7a\beta} \hat{Q}_{aL} \hat{\phi}_L \hat{d}_{\beta L}^c \\
  &\quad + \sum_{a=2}^{3} \sum_{\beta=1}^{2} \kappa_{8a\beta} \hat{Q}_{aL} \hat{\chi}_L \hat{j}_{\beta L}^c. \quad (34)
\end{align*}
The first term in $W_{3RC}$, will leave some leptons massless and some other mass generate, because $L_aL_b \equiv 4 \otimes 4 = 6 \oplus 10$ and how this term is antisymmetric in the generation indices $(a,b)$ it implies that the Yukawa coupling $\lambda_{2ab}$ is antisymmetric matrix. We have three antisymmetric factors hence only the antisymmetric part of the coupling constants $\lambda_{2ab}$ gives a non-vanishing contribution and the mass matrix has eigenvalues $0, -M, M$, so that one of the leptons does not gain mass and the other two are degenerate, at least at tree level. Due this fact the second term will generate masses to the charged leptons [6].

With this superpotential, it is possible to give mass to all charged fermions in the model but neutrinos remain massless. Using Eq.(19) in Eq.(34), we get the following mass matrices in the quark sector:

\[
\Gamma^u = \frac{1}{\sqrt{2}} \begin{pmatrix}
\kappa_{11}v' & \kappa_{12}v' & \kappa_{13}v' \\
\kappa_{521}u & \kappa_{522}u & \kappa_{523}u \\
\kappa_{531}u & \kappa_{532}u & \kappa_{533}u
\end{pmatrix},
\]

(35)

for the $u$-quarks, and

\[
\Gamma^d = \frac{1}{\sqrt{2}} \begin{pmatrix}
\kappa_{31}v' & \kappa_{32}v' & \kappa_{33}v' \\
\kappa_{621}v & \kappa_{622}v & \kappa_{623}v \\
\kappa_{631}v & \kappa_{632}v & \kappa_{633}v
\end{pmatrix},
\]

(36)

for the $d$-quarks, and for the exotic quarks, $J$ and $j_{1,2}$, we have $M_J = \kappa_4 w' / \sqrt{2}$ and

\[
\Gamma^j = \frac{w}{\sqrt{2}} \begin{pmatrix}
\kappa_{821} & \kappa_{822} \\
\kappa_{831} & \kappa_{832}
\end{pmatrix},
\]

(37)

respectively. The quark $u'$ get the following mass $M_{u'} = \kappa_2 z' / \sqrt{2}$ while the quark $d'$ has

\[
\Gamma^{d'} = \frac{z}{\sqrt{2}} \begin{pmatrix}
\kappa'_{721} & \kappa'_{722} \\
\kappa'_{731} & \kappa'_{732}
\end{pmatrix},
\]

(38)

From Eqs.(35) and (36) we see that all the VEVs from the quartet and anti-quartet have to be different from zero in order to give mass to all quarks. Notice also that the $u$-like and $d$-like mass matrices have no common VEVs. On the other hand, the charged lepton mass matrix is already given by $M_{ij} = x \lambda_{3ij} / \sqrt{2}$, where $x$ is the VEV of the $\langle H_3^0 \rangle$ component of the anti-decuplet $H$ in Eq.(11). However, all other VEV from the anti-decouplet and from the $x'$ can both be zero since the decouplet $H'$ does not couple to leptons at all.
3.2.3 R-parity violation

While the R-parity violating terms are given by $W_{2RV} + W_{3RV}$, where

$$W_{2RV} = \sum_{a=1}^{3} \mu_{a} \hat{L}_{aL} \hat{\eta}' + \sum_{a=1}^{3} \mu_{1a} \hat{L}_{aL} \hat{\phi}' + \mu_{2} \hat{\eta} \hat{\phi}' + \mu_{3} \hat{\phi} \hat{\phi}'$$

$$W_{3RV} = \sum_{a=1}^{3} \sum_{a=1}^{3} \sum_{a=1}^{3} \lambda_{1ab} \epsilon \hat{L}_{aL} \hat{L}_{bL} \hat{L}_{cL} + \sum_{a=1}^{3} \sum_{a=1}^{3} \lambda_{3ab} \epsilon \hat{L}_{aL} \hat{bL} \hat{L}_{bL} + \sum_{a=1}^{3} \lambda_{5a} \epsilon \hat{L}_{aL} \hat{\chi} \hat{\bar{\rho}} + f_{2} \epsilon \hat{\rho} \hat{\chi} \hat{\bar{\phi}}$$

$$+ f_{3} \hat{\eta} \hat{\bar{H}} + f_{4} \hat{\eta} \hat{\bar{\phi}} \hat{\bar{H}} + f_{5} \hat{\phi} \hat{\bar{\chi}} \hat{\bar{H}} + \sum_{i=1}^{3} \kappa_{i} \hat{Q}_{iL} \hat{\eta}' \hat{U}_{iL} + \sum_{i=1}^{3} \kappa_{i} \hat{Q}_{iL} \hat{\phi}' \hat{U}_{iL} + \sum_{i=1}^{3} \kappa_{i} \hat{Q}_{iL} \hat{\bar{\rho}} \hat{U}_{iL}$$

$$+ \sum_{i=1}^{3} \kappa_{i} \hat{Q}_{iL} \hat{\phi}' \hat{U}_{iL} + \sum_{i=1}^{3} \kappa_{i} \hat{Q}_{iL} \hat{\bar{\rho}} \hat{U}_{iL}$$

The superpotential give in Eq.(39) provide us the mass terms for leptons and higgsinos

$$-\frac{\lambda_{0a}}{2} \hat{L}_{aL} \hat{\eta}' - \frac{\lambda_{1a}}{2} \hat{L}_{aL} \hat{\phi}' - \frac{\lambda_{2a}}{2} \hat{\eta} \hat{\phi}' - \frac{\lambda_{3a}}{2} \hat{\phi} \hat{\phi}' - \frac{\lambda_{5a}}{3} (L_{aL} \hat{\chi} \hat{\bar{\rho}} + \hat{\rho} L_{aL} \hat{\chi}) + hc, \quad (40)$$

which will induce mass for three neutrinos as was show in [15, 14].

However the R-violating interactions can give the correct masses to $e, \mu$ and $\tau$, even without a decouplet in the same way as happened in the minimal supersymmetric 331 model, [15], where only the mixing between the higgsinos with the leptons reproduce the mass spectrum of the physical charged leptons.\(^2\)

\(^2\)Work in progress.
Proton Decay in charged leptons in the SUSY341 exchanging $\tilde{d}$.

Proton Decay in charged leptons in the SUSY341 exchanging $\tilde{d}'$.

By another hand, the superpotential give at Eq.(39) will also induce the proton decay. The terms proportional to $\kappa_9, \xi_1, \xi_3$ would produce the dominant decay $p \to \pi^0 e^+$, shown in Figs.(3.2.3,3.2.3).

Remember that in the MSSM $\tilde{m}_d \propto m_d$, and due the fact that $m_{d'} > m_d$ then it implies that $\tilde{m}_{d'} > \tilde{m}_d$. Due this fact we can negligble the contribution coming from the Fig.(3.2.3) and imposing

$$\kappa_{4\alpha11} \xi_{1\alpha 11} < 5,29 \times 10^{-25},$$

we will get that $\tau(p \to e\pi) > 1.6 \times 10^{33}$ years \cite{16}.

\footnote{The decay mode $p \to K^+ e^+ \mu^+ \nu_\tau$ was considered in Ref \cite{16} is also present in this model.}
3.3 The soft terms

The soft terms can be written as

$$\mathcal{L}_{\text{soft}} = \mathcal{L}_{\text{GMT}} + \mathcal{L}_{\text{Scalar}}^{\text{soft}} + \mathcal{L}_{\text{SMT}},$$

(42)

where

$$\mathcal{L}_{\text{GMT}} = \frac{1}{2} \left[ m_{\chi_i} \sum_{b=1}^{8} (\lambda_{ib}^b \lambda_{ib}^b) + m_{\lambda} \sum_{b=1}^{15} (\lambda_{iA}^b \lambda_{iA}^b) + m'_{\lambda_B} \lambda_B + H.c. \right],$$

(43)

give mass to the boson superpartners and

$$\mathcal{L}_{\text{Scalar}}^{\text{soft}} = -m_{\eta} \eta^\dagger \eta - m_{\phi} \phi^\dagger \phi - m_{\rho} \rho^\dagger \rho - m_{\chi} \chi^\dagger \chi - m_{\chi'} \chi'^\dagger \chi' - m_{\eta} \eta'^\dagger \eta' - m_{\phi} \phi'^\dagger \phi' - m_{\rho} \rho'^\dagger \rho' - m_{\chi''} \chi''^\dagger \chi'' - m_{\eta''} \eta''^\dagger \eta'' - m_{\phi''} \phi''^\dagger \phi'' - m_{\rho''} \rho''^\dagger \rho'' - m_{\chi'''} \chi'''^\dagger \chi''' - m_{\eta'''} \eta'''^\dagger \eta''' - m_{\phi'''} \phi'''^\dagger \phi''' - m_{\rho'''} \rho'''^\dagger \rho'''} [k_1 \epsilon_i j k \chi_i \eta_k + k_2 \chi \rho H + k_3 \epsilon_i j k \chi_j \eta_k + k_4 \chi \rho' H].$$

(44)

in order to give mass to the scalars, we have omitting the sum upon repeated indices, $i, j, k = 1, 2, 3$, and we only write the terms that respect $R$-parity see Eq.(34). Finally, we have to add

$$-\mathcal{L}_{\text{SMT}} = m_{\tilde{L}_a} \tilde{L}^\dagger_a \tilde{L}_a L + m_{\tilde{\eta}_a} \tilde{\eta}^\dagger_a \tilde{\eta}_a L + m_{\tilde{\chi}_a} \tilde{\chi}^\dagger_a \tilde{\chi}_a L + m_{\tilde{\eta}_a} \tilde{\eta}^\dagger_a \tilde{\eta}_a L + m_{\tilde{\chi}_a} \tilde{\chi}^\dagger_a \tilde{\chi}_a L + m_{\tilde{\chi}'} \tilde{\chi}'^\dagger \tilde{\chi}' L + m_{\tilde{\eta}'} \tilde{\eta}'^\dagger \tilde{\eta}' L + m_{\tilde{\chi}''} \tilde{\chi}''^\dagger \tilde{\chi}'' L + \sum_{\beta} j_{\beta}^2 m_{j_{\beta}}^2 j_{\beta} + |v_{1ab} \epsilon \tilde{L}_a L \tilde{L}_b L + v_{2ab} \tilde{L}_a L \tilde{L}_b L H|

+ \partial_{1a} \tilde{Q}_{1L} \tilde{\eta} \tilde{c} \tilde{c} L + \partial_{2a} \tilde{Q}_{1L} \tilde{\phi} \tilde{c} \tilde{c} L + \partial_{3a} \tilde{Q}_{1L} \tilde{\rho} \tilde{c} \tilde{c} L + \partial_{4a} \tilde{Q}_{1L} \tilde{\chi} \tilde{c} \tilde{c} L + \partial_{5a} \tilde{Q}_{1L} \tilde{\rho} \tilde{c} \tilde{c} L

+ \partial_{6a} \tilde{Q}_{1L} \tilde{\eta} \tilde{c} \tilde{c} L + \partial_{7a} \tilde{Q}_{1L} \tilde{\phi} \tilde{c} \tilde{c} L + \partial_{8a} \tilde{Q}_{1L} \tilde{\chi} \tilde{c} \tilde{c} L]$$

(45)

The pattern of the symmetry breaking in this model is given by

$$\begin{array}{c}
susy341 \xrightarrow{(\rho, \eta, \phi, \rho', \eta', \phi', H')} SU(3)_C \otimes SU(4)_L \otimes U(1)_N \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes SU(3)_C \otimes U(1)_Q. \end{array}$$

(46)

4 Double Chargino and Neutralino Production $e^- e^- \rightarrow \tilde{\chi}^--\tilde{\chi}^0$

Models with $SU(3)$ (or $SU(4)$) electroweak symmetry and left-right symmetry (LR) may have doubly charged vector bosons and scalars, respectively.
This means that in some supersymmetric extensions of these kind of models we will have double charged charginos [17]. On the table, Tab.(1), we show the possible states in the supersymmetric models, in parenthesis we show the number of states that they appear. Therefore we can distinguish the different models with base in the numbers of particles. In Left-Right Supersymmetric Model (SUSYLRRT) the doubly charged higgsinos do not mix with gauginos. The production of a double charged higgsino was studied in [21].

Because of low level of SM backgrounds, the total cross section $\sigma \approx 10^{-3} \text{nb}$ at $\sqrt{s} = 500 \text{GeV}$ [22], $e^-e^-$ collisions are a good reaction for discovering and investigating new physics at linear colliders. With this process is possible to study reactions that violate both lepton and/or fermion number, and this kind of reaction are expected in supersymmetric models, as we will briefly present next.

In the minimal supersymmetric 331 model the chargino (neutralino) base is given by

$$
\psi^{++} = \left( -i\lambda^+_{\tilde{U}} \tilde{\rho}^{++} \tilde{\chi}^{++} \tilde{H}_1^{++} \tilde{H}_2^{++} \right)^t \\
\psi^+ = \left( -i\lambda^+_{\tilde{W}} -i\lambda^+_{\tilde{W}} \tilde{h}_1^{++} \tilde{h}_2^{++} \tilde{\chi}^{++} \tilde{h}_1^{++} \tilde{h}_2^{++} \right)^t \\
\Psi^0 = \left( -i\lambda^3_{\tilde{A}} -i\lambda^8_{\tilde{A}} -i\lambda_B \tilde{\eta}^0 \tilde{\eta}^0 \tilde{\rho}^0 \tilde{\rho}^0 \tilde{\chi}^0 \tilde{\chi}^0 \tilde{\sigma}^0_1 \tilde{\sigma}^0_1 \tilde{\sigma}^0_2 \tilde{\sigma}^0_2 \right)^t
$$

(47)
while the base on the susy341 model is

$$\psi^{++} = \left( -i\lambda_{U}^{+} \tilde{\rho}^{++} \tilde{\chi}^{'++} \tilde{H}_{1}^{'++} \tilde{H}_{2}^{'++} \right)$$

$$\psi^{+} = \left( -i\tilde{\lambda}_{W}^{+} \tilde{\eta}_{1}^{+} \tilde{\eta}_{2}^{+} \tilde{\phi}_{1}^{+} \tilde{\phi}_{2}^{+} \tilde{\rho}_{1}^{+} \tilde{\rho}_{2}^{+} \tilde{\chi}_{1}^{'+} \tilde{\chi}_{2}^{'+} \tilde{H}_{1}^{'+} \tilde{H}_{2}^{'+} \tilde{H}_{3}^{'+} \tilde{H}_{4}^{'+} \right)$$

$$\Psi^{0} = \left( -i\lambda_{A}^{3} \eta_{1}^{0} \eta_{2}^{0} \phi_{1}^{0} \phi_{2}^{0} \rho_{1}^{0} \rho_{2}^{0} \chi_{1}^{0} \chi_{2}^{0} \tilde{H}_{1}^{0} \tilde{H}_{2}^{0} \tilde{H}_{3}^{0} \tilde{H}_{4}^{0} \right)$$

(48)

The interaction lagrangian in the supersymmetric 331 model is presented at Appendix A of [17], the respective lagrangian interaction in the susy341 model is given by

$$\mathcal{L}_{\text{lep}}^{\text{lepton}_V} = \frac{g}{2} \bar{L} \sigma^{m} \lambda^{a} \mathcal{L}_{m} V^{a},$$

$$\mathcal{L}_{\text{lep}}^{\text{lepton}_V} = -\frac{ig}{\sqrt{2}} (\bar{L} \lambda^{a} \tilde{L} \bar{\lambda}_{A}^{a} - \bar{\bar{L}} \lambda^{a} L \lambda_{A}^{a}).$$
Comparing both lagrangian, we notice that both have the same structure. Therefore, the Feynman rules in both model are the same, and they are given in Table 2 we have defined the following operators:

\begin{align*}
O_{ij}^1 &= A_{i1}^\dagger (\sqrt{3} N_{j2} - N_{j1}) + \sqrt{2} A_{i2}^* N_{j5} + A_{i5}^* N_{j8}, \\
O_{ij}^2 &= - \left( D_{i1}^* E_{j2} - D_{i2}^* E_{j1} + D_{i4}^* E_{j3} + \frac{1}{2} D_{i7}^* E_{j8} \right).
\end{align*}

This implies new interactions that are not present in the MSSM, for instance: \( \tilde{\chi}^- \tilde{\chi}^0 U^{++} \), \( \tilde{\chi}^- \tilde{\chi}^- U^{++} \), \( \tilde{t}^- \tilde{t}^- \tilde{\chi}^{++} \) where \( \tilde{\chi}^{++} \) denotes any doubly charged chargino. Moreover, in the chargino production, besides the usual mechanism, we have additional contributions coming from the \( U \)-bilepton in the s-channel. Due to this fact we expect that there will be an enhancement in the cross section of production of these particles in \( e^- e^- \) collisors, such as the ILC.

Therefore the result to the light double chargino production are the same as presented in [17]. We must remember that the susy341 model differ from the minimal supersymmetric 331 model in the number of neutralinos. On this way we can distinguish between both models.

In the future, we want to compare the results about the double chargino production on SUSY LRT, SUSY331 and SUSY341, because this kind of phenomenology was not so much studied in the literature and it can be very nice signal to new physics. We believe that these new states can be discovered, if they really exist, in linear colliders ILC.

5 Conclusions

We have built the complete supersymmetric version of the 3-4-1 model of Ref. [6].
| Vertices                  | Feynman rules                                           |
|--------------------------|---------------------------------------------------------|
| $l^-l^-U--$              | $-\frac{ig}{\sqrt{2}}C_{\gamma^m}L$                   |
| $\tilde{X}_j^-\tilde{X}_i^0U--$ | $\frac{ig}{2}O_{ij}^1C_{\gamma^m}R$                   |
| $\tilde{\chi}_j^-\tilde{\chi}_i^0U--$ | $\frac{ig}{2}O_{ij}^2C_{\gamma^m}R$                   |
| $l_i^-l^-\tilde{\chi}_i^--$ | $-2i\lambda_3A_15 \sin \theta f R$                     |
| $l_i^-l^-\tilde{\chi}_i^0$ | $-2i\lambda_3A_15 \cos \theta f R$                     |
| $l_1^-l^-\tilde{\chi}_1^0$ | $ig\left(\frac{N_{1i}}{\sqrt{2}} + \frac{N_{2i}}{\sqrt{6}}\right)\cos \theta f R - \lambda_3\frac{2}{\sqrt{2}} \sin \theta f N_{18}^* R$ |
| $l_2^-l^-\tilde{\chi}_2^0$ | $ig\left(\frac{N_{1i}}{\sqrt{2}} + \frac{N_{2i}}{\sqrt{6}}\right)\sin \theta f R + \lambda_3\frac{2}{\sqrt{2}} \cos \theta f N_{18}^* R$ |
| $l_1^-l^-\tilde{\chi}_1^0$ | $ig\left(\frac{N_{1i}}{\sqrt{2}} + \frac{N_{2i}}{\sqrt{6}}\right)\cos \theta f L - \lambda_3\frac{2}{\sqrt{2}} \sin \theta f N_{18}^* L$ |
| $l_2^-l^-\tilde{\chi}_2^0$ | $ig\left(\frac{N_{1i}}{\sqrt{2}} + \frac{N_{2i}}{\sqrt{6}}\right)\sin \theta f L + \lambda_3\frac{2}{\sqrt{2}} \cos \theta f N_{18}^* L$ |
| $l_i^+l^-\tilde{\chi}_i^0$ | $-igA_{11} \sin \theta f RC$                           |
| $l_i^+l^-\tilde{\chi}_i^0$ | $-igA_{11} \cos \theta f RC$                           |
| $\tilde{\nu_1}^+l^-\tilde{\chi}_i$ | $-i\lambda_3\frac{1}{\sqrt{2}}D_{i1}^* L$             |
| $\tilde{\nu_1}^+l^-\tilde{\chi}_i$ | $-i\lambda_3\frac{1}{\sqrt{2}}D_{i1}^* R$             |

Table 2: Feynman rules derived from susy341 model in the same way as done in [17].
From the phenomenological point of view there are several possibilities. Since it is possible to define the $R$-parity symmetry, the phenomenology of this model with $R$-parity conserved has similar features to that of the $R$-conserving MSSM: the supersymmetric particles are pair-produced and the lightest neutralino is the lightest supersymmetric particle (LSP).

While in the case that $R$-parity is not conserved we can induce masses to neutrinos of the model in the same way as in the MSSM. We also studied the proton decay problem in this model, and we show that it is in agreement with the experimental data.

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