Diffraction in DIS and Elsewhere

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Abstract. I review some of the results presented in the working group on diffraction at DIS97, with a particular emphasis on the theory of diffractive hard scattering.

INTRODUCTION

This is one of two talks summarizing the results of the working group on diffraction at DIS97. The other talk, by Paul Newman [1], should be read in conjunction with this one. My aim is mainly to provide some theoretical background for interpreting the experimental results and to point out some connections among the results. The talk by Newman summarizes the experimental results from $e^+p$ scattering. This talk describes some highlights from the results from $\bar{p}p$ scattering.

THEORY OF DIFFRACTIVE DIS

I begin with some theoretical background on diffractive deeply inelastic scattering (DIS). Consider the process $e+p \to e+p+X$. Denote the initial lepton momentum by $\ell^\mu$, the final lepton momentum by $\ell'^\mu$, the initial proton momentum by $p^\mu$, the final proton momentum by $p'^\mu$, and the momentum of the photon or $Z$ boson emitted by the lepton by $q^\mu = \ell^\mu - \ell'^\mu$.

It will be helpful to have in mind a definite reference frame, or rather a definite set of reference frames. Let us denote components of four-vectors by $v^\mu = (v^+, v^-, v_T)$, where $v^\pm = (v^0 \pm v^3)/\sqrt{2}$. Then we choose the frame such that $q_T = p_T = 0$. In such a frame, the components of $p^\mu$ and $q^\mu$ are $p^\mu = (p^+, m^2/(2p^+), 0)$ and $q^\mu \approx (-xp^+, Q^2/(2xp^+), 0)$. Here I use the usual DIS variables $Q^2 = -q^2$ and $x = Q^2/(2 q \cdot p)$. As usual in DIS, $Q$ should be large compared to 1 GeV. It is often helpful in analyzing DIS to let $p^+$ be large, on the order of $Q$. However, sometimes one gains insight by taking $p^+$ to be finite, of order $m$. For example, we can take

1) Research supported by the United States Department of Energy.
\[ p^+ = \frac{m}{\sqrt{2}}, \] so that we are in the proton rest frame. Some of the subsequent discussion will concern this frame switching.

In the diffractive process considered, we observe the scattered proton, with momentum components

\[ p^\mu = \left((1 - x_P)p^+ + \frac{m^2 + (p_T')^2}{2(1 - x_P)p^+}, p_T'\right) \tag{1} \]

The fraction of the proton’s longitudinal momentum that is lost in the scattering, \( x_P \), should be fairly small, say \( x_P \lesssim 0.05 \). The kinematics requires \( x \leq x_P \). Thus \( x \) must also be small. The invariant momentum transfer,

\[ t \equiv (p - p')^2 = -\frac{p_T^2 + x_P^2 m^2}{1 - x_P}, \tag{2} \]

should be finite, \( |t| \lesssim 1 \text{ GeV} \).

We should note that in current experiments, one often omits the detector for the scattered proton and substitutes a rapidity gap signal. In this case, \( t \) is not observed; the corresponding theoretical formulas should then be integrated over \( t \), with the assumption that \( |t| \lesssim 1 \text{ GeV} \) dominates the integral.

The summary statement of these kinematics is that, from the point of view of the proton, the scattering is rather gentle, although it is probed by a very hard virtual vector boson. This gentle scattering is often attributed to “pomeron exchange.” However, I will not make use of the Regge theory that incorporates this language for some time in this talk.

What is the microscopic origin of diffractive DIS? Let us look at it in the proton rest frame. Then \( q^- \) is very large, of order \( Q^2/(xm) \), while \( q^+ \) is of order \( xm \). Thus one can think of the vector boson as a system of quarks and gluons moving with very large momentum in the minus direction. In the simplest model, this system of quarks and gluons interacts with the proton via the exchange of two soft gluons, as illustrated in Fig. 1. This model has been extensively analyzed by N. N. Nikolaev and collaborators and is discussed in his talk [2]. See also the talk of Wüsthoff [3].

![FIGURE 1. Two graphs for a model of diffractive DIS.](image-url)
The proton at rest is represented by a circle. Partons in the virtual boson wave function pass through the soft color field of the proton and undergo a color rotation. If the net color transfer is color $1$, then the proton has a chance to remain intact. The difference between this and the model in Fig. 1 is that one expects more than just two soft gluons to be exchanged.

There is another view of diffractive DIS, as discussed in the talk of A. Berera [5]. In this view, it is convenient to employ a reference frame with $p^+$ of order $Q$. The fast moving proton is viewed as being composed of partons. One of these partons, which carries a fraction $\xi$ of the proton’s plus momentum, is scattered by the virtual photon. Although a parton has been removed from the proton, there is some probability that a proton will reform from the debris, having been scattered with a momentum transfer $(x_P, t)$. The function that gives this probability is analogous to the ordinary inclusive parton distribution function and is called the diffractive parton distribution function,

$$df_{a/p}^{\text{diff}}(\xi; x_P, t; \mu)/(dx_P dt).$$

The measured diffractive structure function $dF_2^{\text{diff}}(x, Q^2; x_P, t)/(dx_P dt)$ is then a convolution of the diffractive parton distribution function with the same hard scattering function $\hat{F}_{2,a}(x/\xi, Q^2; \mu)$ that is used to describe ordinary inclusive DIS:

$$\frac{dF_2^{\text{diff}}(x, Q^2; x_P, t)}{dx_P dt} \sim \sum_a \int_0^{x_P} d\xi \frac{df_{a/p}^{\text{diff}}(\xi; x_P, t; \mu)}{dx_P dt} \hat{F}_{2,a}(x/\xi, Q^2; \mu).$$

This equation expresses the hypothesis of diffractive factorization. It is illustrated in Fig. 3, in which the left hand cut diagram shows the Born contribution to $\hat{F}_2$ and the right hand diagram shows a next-to-leading-order contribution, with a gluon as the parton entering the hard interaction.

Eq. (3) represents the model introduced by Ingelman and Schlein [6], minus its Regge content. That is, the diffractive factorization hypothesis does not say anything about a relation of the diffractive parton distributions to Regge theory. If we add Regge theory, then the form of these distributions is restricted.

Diffractive factorization was introduced in Refs. [7] and [8]. For lepton-hadron processes, diffractive factorization appears to be a consequence of QCD [7]. The general perturbative argument is supported by a one loop calculation of Graudenz [9].
The diffractive parton distributions have a specific definition in terms of matrix elements of certain operators [7]. They obey the same DGLAP evolution equation as the ordinary inclusive parton distribution functions. (This statement is self evident once one accepts Eq. (3): the measured $F_2$ does not depend on the factorization scale $\mu$, so the dependence of the parton distribution on $\mu$ must be just that required to cancel the $\mu$ dependence of the calculated hard scattering function $\tilde{F}_2$.)

The diffractive parton distributions are non-perturbative objects that must be determined from experiment. Indeed, they have been determined from the HERA experiments. One of the determinations, by Alvero, Collins, Terron and Whitmore, was discussed in the talk of Whitmore [10]. This determination makes use of the form for the functions suggested by Regge theory. The determinations are not very precise, but there are some qualitative surprises. First, the partons produced in diffractive scattering appear to be predominantly gluons. Second, it appears that often a big fraction of the available momentum is carried by a single gluon.

A caveat is in order. From the talks of Bartels [11] and of Nikolaev [2] and from the discussions, we learned to beware of the region $(1 - \beta)Q^2 \lesssim M^2$, where $\beta = x/x_I$ and $M$ is a mass scale of order 1 GeV. In this region, power corrections, suppressed by a factor $M^2/[(1 - \beta)Q^2]$ may be important.

Figs. 2 and 3 illustrate two pictures of diffractive DIS that appear to be quite different. We learned from the talk of Hebeker [4] that, although these pictures emphasize different features of the interactions, they are physically the same. They are related by making a Lorentz transformation from a frame in which the proton is at rest to a frame in which it has a large momentum. One must then use gauge invariance and make some appropriate approximations to see the correspondence. The basic idea is illustrated in Fig. 4 below. In the left hand picture, the quark labeled $k$ travels forward in time from the boson vertex to the interaction with a gluon from the proton. In the right hand picture, because of the Lorentz transformation, the (anti-) quark travels forward in time from the interaction with a gluon to the interaction with the vector boson. In the left hand picture, the quark appears to be a constituent of the vector boson. In the right hand picture, its antiparticle appears to be a constituent of the proton.

One of the primary motivations for examining diffractive DIS experimentally is
to use a short distance probe to study what happens in diffractive scattering of a proton. The theoretical language used to describe diffraction is Regge theory. One says that the scattered proton exchanges a pomeron with the rest of the system. We incorporate pomeron physics by writing the diffractive parton distribution as

\[
\frac{df_{\text{diff}}(\xi, x_P, t; \mu)}{dx_P dt} = \left| \beta(t) \right|^2 \frac{1}{8\pi^2} x_P^{-2} \alpha_P(t) f_{a/P}(\xi/x_P, t; \mu).
\]  

Here \( \beta(t) \) is the coupling of the pomeron to the proton and \( \alpha_P(t) \) is the pomeron trajectory function, \( \alpha_P(0) \approx 1 \). The function \( f_{a/P}(\xi/x_P, t; \mu) \) is called the distribution of partons in the pomeron, although this name is perhaps misleading in that it suggests that a pomeron is a kind of particle that was emitted by the proton some time before the parton distribution is probed. Eqs. (3) and (4) together constitute the Ingelman-Schlein model [6].

Eq. (4) should be supplemented by additional terms proportional to \( x_P^{-2} \alpha_n(t) \), with \( \alpha_n < \alpha_P \). One can neglect such subleading terms if \( x_P \) is small enough. The functions \( f_{a/P}(\xi/x_P, t; \mu) \) are not predicted by Regge theory (except in the small \( \xi/x_P \) limit). They provide experimental information on the nature of pomeron exchange. Note that it is a prediction of the Regge theory that the dependence on \( x_P \) at fixed \( \xi/x_P \) is \( x_P^{-N} \) where \( N \) is independent of \( \xi/x_P \).

We will see how Eqs. (3) and (4) compare to experiment in the talk of Newman [1].

**THEORY OF DIFFRACTIVE VECTOR MESON PRODUCTION**

Consider a collision between a proton and a real or nearly real photon producing a diffractively scattered proton and vector meson \( V = \rho, \omega, J/\psi \):

\[
\gamma + p \rightarrow V + p.
\]

There is no hard scale here, so perturbation theory does not apply. However, one can describe the process using vector meson dominance plus Regge theory, as depicted in Fig. 5.
If we now let the initial state photon be far off-shell, then we do have a hard process, so that we can use a perturbative picture as shown in Fig. 6. The theory for this is discussed in the talk of Nikolaev [2]. A nonperturbative function $f(x_1, x_2)$ appears in Fig. 6. This function is closely related to the parton distribution function for finding a gluon in a proton, $f_{g/p}(x)$, as discussed in the talks of Radyushkin [12] and of Freund [13].

Consider the process $p + \bar{p} \rightarrow p + jets + X$, where the jets have a large transverse energy, so that we are looking at a hard process. Similarly, we can consider diffractive $W$ boson production and diffractive heavy quark production. As in the case of diffractive DIS, we ask that the invariant momentum transfer $t$ between the initial state proton and the final state proton obey $|t| \lesssim 1$ GeV and that the proton’s fractional longitudinal momentum loss, $x_p$, is in the diffractive region $x_p \lesssim 0.05$. Alternatively, we substitute a rapidity gap signal for the observation of the diffracted proton.

I show a simple picture for $p + \bar{p} \rightarrow p + jets + X$ in Fig. 7. The corresponding formula is

$$\frac{d\sigma_{\text{diff}}}{dE_T \, dx_P \, dt} \sim \sum_{ab} \int_0^{x_P} d\xi_A \frac{df_{a/A}^{\text{diff}}(\xi_A; x_P, t; \mu)}{dx_P \, dt} \int_0^1 d\xi_B \, f_{b/B}(\xi_B; \mu) \frac{d\hat{\sigma}_{ab}}{dE_T},$$

(6)
where the diffractive parton distributions are the same ones that appear in diffractive DIS. If we take the Regge form for these functions, then Eq. (6) is the Ingelman-Schlein model for diffractive jet production. The first indications in favor of this model came from the UA8 experiment [14].

![Diagram of diffractive jet production](image)

**FIGURE 7.** Factored formula for diffractive jet production.

Should Eq. (6) work? There is good reason to believe that the inclusive jet production cross section should take a factored form [15]. However, with two hadrons in the initial state, the argument requires a cancellation that occurs precisely because we sum over all final states containing the required jets. Here we do not sum over all such final states: we demand that the final state contain the diffractively scattered proton. Thus we do not have an argument for factorization.

Furthermore, a simple argument shows that factorization should not be expected. In Fig. 8, we look at the process from the rest frame of the proton that is to be diffractively scattered. A quark from hadron $B$, the “active” quark, initiates the jet production long before hadron $B$ reaches hadron $A$. Since the quark and the gluon that will form the jets have large transverse momentum, they do not separate much before they pass through hadron $A$. A second gluon, carrying a small fraction of the momentum of hadron $B$ and small transverse momentum is also produced. If we view the process from a frame in which hadron $A$ has a large momentum, then this gluon appears to be a constituent of hadron $A$. So far, we are close to having a measurement of the gluon distribution in hadron $A$. However, we have not accounted for the spectator quarks! (Also, the jet system does not have the right color to correspond to a measurement of the gluon distribution in hadron $A$. We could replace it by a jet of the right color plus another spectator quark with the color and transverse position of the active quark. Thus the color issue is not different from the spectator quark issue.) In an inclusive cross section, the spectator quarks would not matter. However, for the diffractive cross section the interaction with the spectator quarks can change the probability that hadron $A$ emerges from the collision intact, having simply absorbed some momentum. It seems likely that
the extra interactions make it less likely for hadron A to survive. Thus we expect that the true cross section will be given by the right hand side of Eq. (6) times a survival probability $S$ that is less than 1. Unfortunately, the survival probability is not calculable by perturbative methods.

![Diagram](image)

**FIGURE 8.** Spectator parton effect leading to non-factorization.

The experimental results on diffractive hard scattering in $p\bar{p}$ collisions discussed in the working group on diffraction can be summarized in the following table. In the first three rows, I show the results for diffractive production of $W$ bosons, jets, and heavy quarks, respectively. In each case, I show the rate for hard diffractive scattering divided by the corresponding rate for inclusive hard scattering. We see that these rates are generally a fraction of a percent. In the right hand column, I show the predictions based on Eq. (6). These results make use of the diffractive parton distributions derived from HERA data by Alvero et al., as reported in talk of Whitmore [10]. That is, these results assume a suppression factor $S = 1$. Evidently, the suppression factor is really something more like $S \approx 0.1$.

|                     | CDF                  | D0       | If no suppression |
|---------------------|----------------------|----------|-------------------|
| **Diffractive W**   | $(1.15 \pm 0.55)\%$  | $9.4\%$  |                   |
| **Inclusive W**     | $\text{---}$         | $16\%$  |                   |
| **Diffractive Jets**| $(0.75 \pm 0.10)\%$  | $(0.67 \pm 0.05)\%$ | $10\%$          |
| **Inclusive Jets**  | $\text{---}$         | $\text{---}$ |                   |
| **Diffractive c,b** | $(0.18 \pm 0.03)\%$  | $\text{---}$ | $?$              |
| **Inclusive c,b**   | $\text{---}$         | $\text{---}$ |                   |
| **(Diffractive)² Jets** | $(2.7 \pm 0.7) \times 10^{-6}$ | $10^{-6}$ | $?$              |
| **Inclusive Jets**  | $\text{---}$         | $\text{---}$ |                   |

In the fourth row of Table 1, I report the results reported for the twice diffractive process $p + \bar{p} \rightarrow p + \bar{p} + jets + X$. Here both the proton and the antiproton are diffractively scattered. The cross section is small, but still observable.

One interesting connection between the $p\bar{p}$ data and the DIS data emerged in the talk of Goulianos [16]. If one assumes Eq. (6), then one can get some information
about whether quarks or gluons predominate in the diffractive parton distributions by comparing diffractive $W$ production (quark dominated) to diffractive dijet production (gluon dominated). The result is that gluons appear to predominate, in agreement with the analysis based on HERA data. This result assumes that the survival probability is $S = 1$, or, more generally, that the survival probability is the same for both processes. Although this assumption is open to question, the result is of interest because the gluons participate directly in jet production, whereas the determination of the diffractive gluon distribution in diffractive DIS is quite indirect.

**JET-GAP-JET EVENTS IN HADRON-HADRON COLLISIONS**

We have discussed diffractive jet production in $p\bar{p}$ collisions, in which one of the hadrons is diffractively scattered. If the scattered hadron is not directly detected, then the signature for this process is the existence of a rapidity gap in the detector in the hadron fragmentation region. That is, there are no particles found in the region of the detector at large rapidity, where the fragments of the hadron would have been found if it had been broken up. Consider now another kind of event, in which two jets are produced and there are no particles found in the central rapidity region lying between the two jets. We can call this a jet-gap-jet event. Such an event can be produced by parton scattering via two hard gluons, as illustrated in Fig. 9. The colors of the two gluons should be such that the net color exchange is color 1. Then there is a right-moving, color singlet, system consisting of hadron $A$ and one of the jets and there is a left-moving color singlet system consisting of hadron $B$ and the other jet. Soft color interactions would not be expected to produce particles in the region between the left and right moving systems. Thus there can be a gap containing no particles between the jets. Of course, there can be a color exchange interaction between spectator partons, so one expects a probability $S < 1$ for the gap to survive.

This process has been studied by CDF and D0, as reported in the talks of Melese [17] and Perkins [18] respectively. The D0 study includes a comparison of the gap fraction at $\sqrt{s} = 630$ GeV to the gap fraction at $\sqrt{s} = 1800$ GeV:

$$R = \frac{[(\text{jet-gap-jet})/(\text{inclusive jet-jet})]_{630 \text{ GeV}}}{[(\text{jet-gap-jet})/(\text{inclusive jet-jet})]_{1800 \text{ GeV}}}.$$  

(7)

The experimental result is $R = 2.6 \pm 0.6$. This can be compared to the result predicted by the exchange of two hard gluons, as in Fig. 9, $R = 0.8$. Better agreement is achieved in a model in which the parton scattering occurs via one hard gluon exchange and the color of the outgoing jets is rearranged by random soft color interactions. This soft color rearrangement model predicts $R = 2.2$.

I thank the other convenors of the working group on diffraction, P. Newman, A. Straiano, and P. Melese, for their help in preparing this talk. I also thank
FIGURE 9. Hard scattering via color singlet exchange leading to a jet-gap-jet signature.

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