Gauge unification, non-local breaking, open strings

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Abstract

The issue of *non-local* GUT symmetry breaking is addressed in the context of open string model building. We study $\mathbb{Z}_N \times \mathbb{Z}_M'$ orbifolds with all the GUT-breaking orbifold elements acting freely, as rotations accompanied by translations in the internal space. We consider open strings quantized on these backgrounds, distinguishing whether the translational action is *parallel* or *perpendicular* to the D-branes. GUT breaking is impossible in the purely perpendicular case, *non-local* GUT breaking is instead allowed in the purely parallel case. In the latter, the scale of breaking is set by the compactification moduli, and there are no fixed points with reduced gauge symmetry, where dangerous explicit GUT-breaking terms could be located. We investigate the mixed parallel+perpendicular case in a $\mathbb{Z}_2 \times \mathbb{Z}'_2$ example, having also a simplified field theory realization. It is a new $S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$ orbifold-GUT model, with bulk gauge symmetry $SU(5) \times SU(5)$ broken *locally* to the Standard Model gauge group. In spite of the locality of the GUT symmetry breaking, there is no localized contribution to the running of the coupling constants, and the unification scale is completely set by the length of $S^1$. 
1 Introduction

Orbifold compactification [1] provides a powerful tool to fill the gap between string theory and particle physics. String theory can be quantized exactly on an orbifold. The resulting model is fully consistent, and its features are completely under control. The orbifold action can be responsible for Supersymmetry (SUSY) and gauge symmetry breaking. The details of the orbifold action in the internal space/gauge bundle encode all the details of the symmetry breaking. In particular, if an orbifold-group element (in the following “orbifold operator”) acts freely in the internal space, i.e. without fixed points, then the symmetry breaking is realized non-locally, at an energy scale set by the compactification moduli [2]. Indeed, freely-acting orbifolds present a viable string embedding of the so-called Scherk-Schwarz symmetry breaking mechanism [3, 4].

This mechanism has been studied in the past mainly in relation to SUSY breaking [5, 6, 7, 8, 9]. We will reconsider and use it to break gauge symmetry [10, 11], in open string theory. We investigate models where the gauge group of a Grand Unified Theory (GUT) is broken, non-locally, to the Standard Model (SM) gauge group. The scale of breaking is set by the compactification moduli, and it is tunable to the value predicted by the Minimal Supersymmetric Standard Model (MSSM): $M_{GUT} = 3 \times 10^{16}$ GeV [12]. The compactification scale can differ from the string scale $m_s$, since in the resulting model no differential running for the SM coupling constants is present between $M_{GUT}$ and $m_s$. This is the main difference between non-local breaking and the standard local orbifold breaking. In the latter, generically, localized contribution to the differential running can be present up to the string scale. In this case the MSSM prediction is spoiled unless all the scales are $\sim 10^{16}$ GeV.

We describe the possible geometries that allow for non-local breaking. Only a small subclass of the backgrounds ensuring Scherk-Schwarz SUSY breaking is viable, for example we select only two acceptable backgrounds out of the $Z_2 \times Z'_2$ models classified in [8]. We study $Z_N \times Z'_M$ orbifolds, the minimal option ensuring both non-local gauge symmetry breaking and $\mathcal{N} = 1$ SUSY in 4d. We furnish examples in the $(N, M) = (4, 2), (3, 3)$ and $(4, 4)$ cases.

These geometries guarantee non-local GUT symmetry breaking provided that the gauge bundle has support on the whole internal space. In open string theory this is not ensured, since gauge bosons are localized on D-branes that may have dimensionality lower than 10, and extra conditions must be fulfilled. As in the SUSY breaking case,

\footnote{The only exception is the length of the extra dimensions unwrapped by the D-branes supporting the gauge group.}
we distinguish between perpendicular or parallel action of the relevant (freely-acting) orbifold operators on the D-branes. In the purely perpendicular case, the orbifold action identifies different stacks of D-branes, and therefore cannot result in a proper GUT symmetry breaking. In the purely parallel case, instead, such a breaking occurs.

A new interesting mechanism is realized in a mixed situation. The obtainable GUT symmetry breaking is local, but the differential running of the coupling constants due to localized contributions may be absent. We give a $Z_2 \times Z'_2$ example with D5-branes, that can be interpreted as a field-theory model with a single extra dimension, compactified on $S^1/Z_2 \times Z'_2$. The model has $SU(5)_1 \times SU(5)_2$ bulk gauge symmetry, broken to $SU(5) (SU(5)')$ in the $Z_2 (Z'_2)$ fixed points. The low energy gauge symmetry is given by the intersection of the gauge symmetry preserved in the fixed points, $SM = SU(5) \cap SU(5)'$. The breaking is then local, but there is no differential running beyond the unification scale $M_{GUT} = R^{-1}_{S^1}$, since all the localized matter must fill an $SU(5) (SU(5)')$ multiplet, and contributes universally to the running. This makes this model particularly interesting, since it has the simple structure and main features of the orbifold-GUT models introduced and studied in [13], but it ensures an exact gauge coupling unification, provided that the low energy spectrum is a proper one.

This mechanism could have great relevance in heterotic string model building. As it is well known, the splitting between $M_{GUT}$ and $M_{Planck}$ can be explained by matching the unification scale with the compactification volume. This in general implies a string perturbativity loss [14]. In [11] it was shown that an highly anisotropic compactification could resolve the problem, but full perturbativity would require no more than one large extra dimension, tuned to $M_{GUT}^{-1}$, and this is consistent (only) with our picture.

We give here the paper outline and main results:

In Section 2 we introduce the possible background that allow for non-local GUT symmetry breaking, furnishing two $Z_2 \times Z'_2$ examples, a $Z_4 \times Z'_4$ example, with an equivalent $Z_4 \times Z'_4$ description, and a $Z_3 \times Z'_3$ example.

In Section 3 we study D-brane embeddings in the given backgrounds, we show the properties of a purely perpendicular embedding (no GUT breaking), and of a purely parallel embedding (non-local GUT breaking). We discuss the mixed $Z_2 \times Z'_2$ case by giving a field theory exemplification, a new five-dimensional orbifold-GUT model, with a local breaking of the GUT symmetry but without dangerous localized

\[\text{We take a bottom-up approach and do not discuss the details of tadpole cancellation condition, but just the details of the gauge symmetry breaking mechanisms. To prove the existence of the described models, at least in the } Z_2 \times Z'_2 \text{ case, we mainly refer to [8], even though we know that no "realistic" D-brane model of this kind have been constructed, up to now.}\]
contributions to the differential running of the coupling constants.

In Section 4 we check the tree-level relations between 10d and 4d gauge and gravity couplings. We impose the “observed” values for \( \alpha_{\text{GUT}} \), \( M_{\text{Planck}} \) and \( M_{\text{GUT}} \), we require string perturbativity and the string scale to be larger than \( M_{\text{GUT}} \), in order to avoid threshold corrections between low energy and \( M_{\text{GUT}} \). We obtain that the described scenario is generically viable, provided that the ratio between the string scale and \( M_{\text{GUT}} \) is less than two orders of magnitude (\( \lesssim 50 \)).

2 GUT symmetry breaking via freely-acting orbifolds

In a freely-acting orbifold group there are operators (elements) acting freely and operators acting non-freely. All the operators are embedded into the gauge bundle, and their action can be GUT-preserving or GUT-breaking. In order to have non-local GUT breaking, we require all the breaking operators to act freely in the internal space. We also demand that \( \mathcal{N} = 1 \) SUSY is preserved in 4d, and that the orbifold group is Abelian. We clearly demand the existence of at least one GUT-breaking operator.

The minimal option fulfilling these requirements is a \( T^6/Z_N \times Z'_M \) orbifold. Indeed, if we take the generator of \( Z'_M \) \( (g'_M) \) to be freely-acting and GUT-breaking, a non-local GUT breaking is ensured, but this is not enough. In absence of a second orbifold action, the background geometry can always be rewritten as a fibration of \( T^5 \) over \( S^1 \), with \( \mathcal{N} \geq 2 \) or \( \mathcal{N} = 0 \) SUSY in 4d. An extra orbifold action is needed, to break \( \mathcal{N} = 2 \rightarrow \mathcal{N} = 1 \), and the conditions to have non-local breaking must be rechecked on a case-by-case basis. We take \( g \), the generator of \( Z_N \), GUT-preserving. The requirement, than, is that the operators \( g^n g^m \) \( (m \neq 0) \) must act freely, since they are all GUT-breaking.

We impose that \( g \) commutes with \( g' \). This implies that the structure of the fixed-points of \( g \) must be non-trivial, in order to have \( g' \) freely-acting. In general it is necessary the presence of a number of \( g \)-fixed points multiple of the order of \( g' \). It is easy to check that, due to this, there are no \( Z_2 \times Z'_3 \) or \( Z_3 \times Z'_2 \) models, and instead there are \( Z_N \times Z'_M \) examples for \( (N, M) = (2, 2), (4, 2), (4, 4), (3, 3) \). Due to the fixed-point argument we conjecture the absence of models fulfilling the requirements for \( (N, M > 6) \).
The freely-acting orbifolds $T^6/Z_2 \times Z_2'$ have been classified in [8]. As introduced, we take $g'$ GUT-breaking and $g$ GUT-preserving. This implies that we restrict to the small subclass of models with both $g'$ and $gg'$ (GUT-breaking operators) freely-acting. For simplicity we take $T^6 = T_1^2 \times T_2^2 \times T_3^2$. We parametrize each $T_i^2$ with $z_i \in \mathbb{C}$ having periodicity $z_i \sim z_i + (R_i n + \tau_i S_i m)$, $|\tau_i| = 1$. Imposing the requirement we reduce the classification of [8] to the two models [10, 11]

$$g : \begin{cases} z_1 &\rightarrow -z_1 \\ z_2 &\rightarrow -z_2 \\ z_3 &\rightarrow z_3 + \epsilon R_3/2 \end{cases} \quad g' : \begin{cases} z_1 &\rightarrow z_1 + R_1/2 \\ z_2 &\rightarrow -z_2 + R_2/2 \\ z_3 &\rightarrow -z_3 \end{cases}$$

with $\epsilon = 0, 1$. In both cases $g'$ and $gg'$ are freely-acting, since their action is a translation along the real part of $z_1$ and $z_2$ respectively. If $\epsilon = 0$, $g$ is not freely-acting, and there are fixed points (planes) of reduced ($\mathcal{N} = 2$) SUSY, see Fig. If instead $\epsilon = 1$, also $g$ is freely-acting, and each point in the internal space is $\mathcal{N} = 4$ supersymmetric. The latter configuration is very similar to the smooth Calabi Yau case.

Taking $g$ GUT-preserving and $g'$ GUT-breaking and imposing that the gauge bundle has support on the plane parametrized by the real part of $z_1$ and $z_2$, we obtain that the breaking scale is $M_{GUT}^2 = R_1^{-2} + R_2^{-2}$. It is then necessary to tune $R_1^{-1}, R_2^{-1} \sim 10^{16}$ GeV, while the other radii are free. Due to the absence of $\mathcal{N} = 1$ fixed points, it could be difficult to introduce chiral matter in complete representations of the unified gauge group. The problem can be solved by introducing stacks of
Figure 2: Internal geometry of the $Z_4 \times Z'_2$ model. The action of $g$ is not free, the dots show the 4d fixed points. They preserve $N=1$ SUSY. The arrows show the action of $g^n g'$, a pure translation in the first ($n=0$), second ($n=2$) or third torus ($n=1, 3$).

intersecting D-branes [15], as in the case discussed by [16], and putting the unified group on one of the stacks.

### 2.2 $Z_4 \times Z'_2$ ($Z_4 \times Z'_4$) orbifold

We found only one $Z_4 \times Z'_2$ (abelian) model compatible with the requirements. The toroidal geometry is constrained, due to the crystallographic action of $Z_4$: $\tau_i = i$, $R_i = S_i$, for $i = 1, 2$. The orbifold action is

$$
g : \begin{cases} 
  z_1 \to e^{i\pi/2} z_1 \\
  z_2 \to e^{i\pi/2} z_2 \\
  z_3 \to -z_3
\end{cases} \quad g' : \begin{cases} 
  z_1 \to z_1 + R_1(\tau_1 + 1)/2 \\
  z_2 \to -z_2 + R_2(\tau_2 + 1)/2 \\
  z_3 \to -z_3 + R_3/2
\end{cases} \quad (2)
$$

All the combinations $g^n g'$ are freely-acting, as required: $g'$ is a diagonal translation in the first torus, $g^{2n+1} g'$ is a diagonal translation in the third torus, $g^2 g'$ is a diagonal translation in the second torus (see Fig. 2). The Dp-brane embedding issue is discussed in detail in the next section. We anticipate that a complete non-local GUT breaking is ensured only when the gauge bundle fills all the directions where the orbifold action is a translation. The scale of breaking is, in that case, related to the volume of these directions. In within this picture, we have to set $R_i^{-1} \sim 10^{16}$ GeV for all $i$’s, and the only free volume parameter is $S_3$.

The described $Z_4 \times Z'_2$ model can also be seen as a $Z_4 \times Z'_4$ model. Indeed, taking an action

$$
g : \begin{cases} 
  z_1 \to e^{i\pi/2} z_1 \\
  z_2 \to e^{i\pi/2} z_2 \\
  z_3 \to -z_3
\end{cases} \quad g' : \begin{cases} 
  z_1 \to e^{i\pi/2} z_1 + R_1(\tau_1 + 1)/2 \\
  z_2 \to e^{-i\pi/2} z_2 + R_2(\tau_2 + 1)/2 \\
  z_3 \to z_3 + R_3/2
\end{cases} \quad (3)
$$
Figure 3: *Internal geometry of the $Z_3 \times Z'_3$ model. The action of $g$ is not free, the dots show the 4d fixed points. They preserve $N = 1$ SUSY. The arrows show the action of $g^n g'^m$, a pure translation in the first ($n=0$), second ($n=m$) or third torus ($n=2m$).*

It is possible to check that the generated orbifold group is the same as in the $Z_4 \times Z'_2$ orbifold, but each operator can be obtained in two different ways. In particular notice that $g^2 g'^2 \equiv I$ in the internal space. This imply that $g^2 g'^2$ must be embedded as the identity operator also in the gauge bundle, and that the action of $g^2$ must be GUT-preservation, since $g$ is GUT-preservation.

### 2.3 $Z_3 \times Z'_3$

As in the previous case the complex structure is (completely) fixed by the orbifold action, $\tau_i = e^{i \pi/3}$, $R_i = S_i$, for all $i$’s. The orbifold action is

\[
\begin{align*}
g &: \begin{cases} 
z_1 &\rightarrow e^{-4\pi i/3}z_1 
z_2 &\rightarrow e^{2\pi i/3}z_2 
z_3 &\rightarrow e^{2\pi i/3}z_3 \end{cases} \quad \quad \quad 
g' &: \begin{cases} 
z_1 &\rightarrow z_1 + R_1(\tau_1 + 1)/3 
z_2 &\rightarrow e^{-2\pi i/3}z_2 + R_2(\tau_2 + 1)/3 
z_3 &\rightarrow e^{2\pi i/3}z_3 + R_3(\tau_3 + 1)/3 \end{cases}
\end{align*}
\]

Notice that the orbifold group is abelian, as required. The action of $g^n g'^m$ is always free for $m \neq 0$, as required: for $n = 0$ it is diagonal translation in the first torus, for $n = m$ a diagonal translation in the second, for $n = 2m$ a diagonal translation in the third (see Fig. 3).

### 3 Open string model building

The described geometries ensure a *non-local* GUT symmetry breaking, but it is necessary that the orbifold elements are embedded into the gauge bundle as specified. The requirement is that the GUT symmetry breaking operators must be present and act freely.

In heterotic string theory these requirements can be always fulfilled, provided that modular invariance conditions are also fulfilled. In open string theory, instead, it could be impossible to embed a freely-acting operator in a GUT-breaking way.
In open string theory the gauge bosons live on Dp-branes, not necessarily filling the whole spacetime ($p$ can be less than 9). A freely-acting operator can act as a translation in a direction that is parallel or perpendicular to the relevant stack of Dp-branes (See Fig. 4).

In the perpendicular case, the point where the Dp-branes reside is mapped into a different point, where a new stack of Dp-branes must be located. Consistency requires the two Dp-brane stacks to be completely equivalent, with the same gauge group $G$. The orbifold action identifies the two groups, and only a (diagonal) combination is left invariant: $G \times G \rightarrow G$. A “proper” GUT symmetry breaking is impossible, but a rank reduction occurs (for details check, in the $Z_2 \times Z_2$ case, [8, 17]). This implies that if all the freely-acting operators act in a perpendicular way no GUT breaking at all can be embedded.

In the parallel case, the Dp-brane stack (gauge group) is mapped onto itself, and a GUT breaking can be realized. If all the freely-acting operators act in a parallel way, then no rank reduction is realized. We can take the generator of $Z_N$ to be GUT-preserving, the generator of $Z'_M$ to be GUT-breaking, and we have a non-local GUT symmetry breaking due to the orbifold elements $g^n g'^m$, $m \neq 0$. The scale of breaking is given by the volume of the dimensions where $g^n g'^m$ act as a translation. Clearly, it is necessary also to fulfill the tadpole cancellation conditions, that impose strong constraints on the kind of gauge groups and spectra that can be present. We do not introduce the issue here, we only notice that it was undertaken, in the $Z_2 \times Z'_2$ case, in [8]. In particular a non-local GUT symmetry breaking of the described kind can be realized, the unifying group being $U(16)$ broken to $U(16-n) \times U(n)$. The gauge group is not realistic, but the situation can be highly improved by introducing continuous Wilson lines. We leave for future work a detailed study of

Figure 4: *Gauge symmetry breaking due to a freely-acting operator $g$ acting as a translation in a direction perpendicular (on the left) or parallel to the D-brane (on the right). In the first case no “proper” GUT breaking is realized, but a rank reduction of the gauge group. In the second case the possibility of a non-local GUT symmetry breaking is ensured.*
Figure 5: The background described in Fig. 4 with two stacks of 5 D5-branes filling the first torus and located in two would-be-fixed points of $g'$. The gauge group before of the orbifold action is generically $U(5)^{D5} \times U(5)^{D5'}$, the action of $g'$ is parallel to the two stacks, and can be embedded in a GUT-breaking way. The action of $gg'$ is instead perpendicular and induces an identification of the two $U(5)$.

tadpole cancellation condition and realistic model building.

These gauge symmetry breaking patterns are closely related to the kind of SUSY breaking studied, for example, in $[8,18]$, but the fate of SUSY in a freely-acting orbifold is generically different than the fate of gauge symmetry. As an example, since bulk SUSY is always present, a freely-acting orbifold is responsible in any case for a parallel SUSY breaking, at least in the closed string sector. This does not happen in the gauge symmetry case.

Between the two extreme cases (complete perpendicular/parallel case) we have the intriguing possibility of an intermediate case. Since the orbifold group contains more than one freely-acting operator, we may have a stack of Dp-branes parallel to the action of an operator and perpendicular to the action of another one. We illustrate this possibility with a $Z_2 \times Z_2'$ example. We consider the geometry of Fig. 4 and embed two stacks of 5 D5-branes, filling the first torus and localized into two different would-be-fixed points of $g'$, in the second torus, as shown in Fig. 5. The gauge group, in absence of any orbifold projection, is $U(5)^{D5} \times U(5)^{D5'}$. The action of $g'$ is parallel to both the two stacks, and can be embedded in a GUT-breaking way. The action of $gg'$ is instead perpendicular, and induces an identification between the two $U(5)$'s. The ending unbroken group is then just a diagonal $U(3) \times U(2)$ combination. The breaking is local, as we show in the following, but the unification properties are particularly interesting, and deserve some extra comment. We explain them by introducing a simplified field theory example.

### 3.1 A field theory model on $S^1/Z_2 \times Z_2'$

From a purely field theoretical point of view, the described D5-brane construction is completely reproduced by a model with a single extra dimension, parametrized by
Figure 6: An $S^1/Z_2 \times Z'_2$ field theory model. The bulk symmetry is $SU(5)_1 \times SU(5)_2$, broken to $SU(5)$ and to $SU(5)'$ in the $g$ and $g'$ fixed points (dots and crosses respectively). The surviving gauge group is just the SM gauge group: $SU(5) \cap SU(5)' = SU(3) \times SU(2) \times U(1)$.

$x \in [0, 2\pi R)$. We take the extra dimension to be compactified on $S^1/Z_2 \times Z'_2$. The action of the $Z_2$ operator is $g : x \to -x$, with fixed points $0$ and $\pi R$, while the action of the $Z'_2$ operator is $g' : x \to -x + \pi R$, with fixed points $\pi R/2$ and $3\pi R/2$ (see Fig. 6). We introduce a bulk gauge group $SU(5)_1 \times SU(5)_2$, we embed $g$ in the gauge group such that $g : SU(5)_1 \leftrightarrow SU(5)_2$, more precisely, defining $T_i^{ab}$ a generator of $SU(5)_i$,\(^3\) the identification is $g : T_1^{ab} \leftrightarrow T_2^{ab}$, and the surviving gauge group in $0$ and $\pi R$ is a diagonal $SU(5)$ generated by $T_1^{ab} + T_2^{ab}$\(^4\). We embed then $Z'_2$ action as $g' : T_1^{ab} \to \delta_a^c T_2^{cd} \delta_d^b$ with

$$\delta = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & -1
\end{pmatrix}. \quad (5)$$

In this way the surviving gauge group in $\pi R/2$ and $3\pi R/2$ is a different $SU(5)'$, and the intersection of the two gauge groups is just the SM gauge group. The breaking is local, since the $SU(5)$ symmetry preserved in $0$ is generically broken in $\pi R/2$, but the SM differential running of the coupling constants is generated only by the bulk degrees of freedom, and it stops precisely at the unification scale $M_{GUT} = R^{-1}$. There is no fixed points contribution to the differential running since only full multiplets of $SU(5)$ ($SU(5)'$) can be localized there, and the SM is embedded exactly in the

\(^3\)We take $a, b = 1, 2, \ldots, 5$, and define $T^{aa}$ as the 5 Cartan generators of $U(5)$, from which we exclude the “diagonal” generator $\sum_a T^{aa}$.

\(^4\)Obviously it is crucial the presence, from the very beginning, of a $Z_2$ symmetry linking the two $SU(5)_i$, that must have, for example, the same coupling constant. This is ensured by consistency of the freely-acting orbifold in the string theory case, since we require the two stacks of D5-branes to be symmetric.

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same way in $SU(5)$ and in $SU(5)'$. The last point is crucial: a multiplet of $SU(5)$ behaves as a multiplet of $SU(5)'$ under the action of the SM gauge group, and they both contribute universally to the running of the SM coupling constants. We expect the same argument to be valid for any localized contribution to the action.

We have shown, in the $Z_2 \times Z_2'$ model, that also a D-brane embedding that does not ensure non-local breaking can ensure the required phenomenology, i.e. no localized contribution to the differential running of the coupling constants. We expect this issue to be valid even for the other geometries, even though we do not furnish here a case-by-case proof.

This mechanism could have great relevance in heterotic string model building, where complete string perturbativity is ensured only for a single extra dimension tuned to $M_{GUT} = 3 \times 10^{16}$ GeV, and the others taken at the inverse string scale [11], that is fixed close to the Planck mass. An embedding of the described effect would be a model where string perturbativity is not spoiled and $M_{GUT}$ is introduced as an intermediate (compactification) scale between the the string scale and the low energy scales [19], without any need for large threshold corrections at the string scale. Nevertheless, in the mechanism a rank reduction is needed for the gauge group, and a standard abelian orbifold configuration is not enough. It would be necessary to introduce non-abelian freely-acting orbifolds/continuos Wilson lines [20] (for recent studies see also [21]).

4 Energy scales

The gravitational interactions are described, in Type II string theory, by the following 10d bosonic action (see [11] for notation and conventions)

$$S_{\text{Grav}} = - \int d^{10}x \sqrt{G} \frac{1}{2 (2\pi)^7 \alpha'^4 g^2} R,$$

(6)

where $G$ is the 10d metric, $R$ the Ricci scalar, $\alpha'$ is the string scale and $g$ is the string coupling. Using these definition for the 10d coupling constants, the gauge interactions, due to the presence of a Dp-brane stack, are described by the action

$$S_{Dp} = \int dx^{p+1} \sqrt{*G} \frac{1}{8 (2\pi)^{p-2} \alpha'^{p-3} g} \text{Tr} F^2,$$

(7)

where $*G$ is the metric induced on the Dp-brane stack. The string scale is related to the mass of the first excited state as $m_1 = \sqrt{\alpha'}$, $g$ has been chosen in such a way that the weak coupling regime is defined for $g \ll 1$ (boundary at 2). This can be proven either by a duality argument (S-duality with heterotic theory for $p = 9$, as
an example), or simply computing the ratio between a generic one-loop amplitude correction to a given tree-level quantity\(^5\)

\[
\frac{\text{one loop}}{\text{tree level}} = \frac{2(2\pi)^{p-2}}{(p-3)2^{p+1}\pi^{\frac{p+1}{2}}\Gamma\left(\frac{p+1}{2}\right)}30g = N(p)g,
\]

and checking that \(N(p)\) is always \(O(1)\).

The relations between these couplings and the 4d couplings depend on the total compactification volume \(V\) (gravitational coupling) and on the volume of the space filled by the Dp-brane \(V\//\) (YM coupling). We take a trivial compactification on a space given by the direct product of six circles. The circles have radius \(R/\parallel\), if the Dp-brane wraps them, or \(R/\perp\) if they are not wrapped. Consequently we have \(V\// = (2\pi R/\parallel)^{p-3}\) and \(V = V\// \times (2\pi R/\perp)^{9-p}\), and the 4d Planck mass and gauge coupling are, respectively

\[
M_2^p = \frac{4R_{\parallel}^{9-p}R_{\parallel}^{p-3}}{\alpha'g^2}, \quad \alpha_{\text{GUT}} = \frac{\alpha'^{(p-3)/2}}{R_{\parallel}^{p-3}g}.
\]

We define dimensionless radial parameters \(r = R/\sqrt{\alpha'}\) and rescaled dimensionless volumes \(v\// = r^{p-3}, v_{\perp} = r_{\perp}^{9-p}\). We also introduce \(m_I = 1/\sqrt{\alpha'}\), the mass of the first-excited string state. Then, Eq. (9) can be rewritten, rescaling \(g \rightarrow 2g\), as

\[
M_2^p = \frac{v_{\perp}v_{\parallel}}{g^2m_I^2}, \quad \frac{\alpha_{\text{GUT}}}{2} = \frac{g}{v_{\parallel}}.
\]

The second equation states the usual impossibility of a large Dp-brane volume, i.e. the impossibility of \(R_{\parallel} \gg 1/m_I\). Nevertheless, notice that \(m_I\) is not fixed close to the Planck mass as in the heterotic case.

### 4.1 Viable configurations

The tree-level relations of Eq. (10) state a distinction between the volume filled by the D-brane and the volume orthogonal to the D-brane. We should introduce a further distinction, and specify the directions where the orbifold elements act as a translation. We can have then four different classes of radii/volumes. Between them, we consider the class of the directions wrapped by the D-branes AND where the orbifold action is free. These dimensions set the unification scale and must have inverse radius \(\sim 3\times10^{16}\) GeV. All the other dimensions are generically unfixed. Taking the number of the dimensions with radius \(M_{\text{GUT}}^{-1}\) to be \(d\) we can rewrite Eq. (10) as

\[
\left(\frac{M_2}{m_I}\right)^2 \left(\frac{M_{\text{GUT}}}{m_I}\right)^d g^2 = r_{p-3}^{-d}r_{\parallel}^{-d}, \quad \left(\frac{M_{\text{GUT}}}{m_I}\right)^d g = \frac{\alpha_{\text{GUT}}}{2}r_{\parallel}^{p-3}.
\]

\(^5\)The computation is done introducing \(m_I\) as UV cutoff of the \(p + 1\) dimensional field theory, and considering an \(SO(32)\) gauge group. For further details see \([11]\).
where we take, with abuse of notation, \( r_{\parallel} \) to be the radii of the dimensions wrapped by the D-branes and where the orbifold action is not free.

Now we can impose constraints on the values of the various parameters, and check the consistent possibilities. Our guideline is a model with just the MSSM running of the coupling constants between the low energy scale \((M_Z)\) and the unification scale \(M_{\text{GUT}} = 3 \times 10^{16} \text{ GeV}\), where the unified gauge group is restored and the couplings meet. This implies, in Eq. (11) (i) \( m_I > M_{\text{GUT}} \); (ii) \( r_{\parallel} < m_I / M_{\text{GUT}} \), to avoid light charged Kaluza-Klein modes; (iii) \( r_\perp > M_{\text{GUT}} / m_I \), to avoid light charged winding modes. We also impose \( r_{\parallel} > 1 \), to avoid the presence of light winding modes for the gravitational part of the action, that could be annoying in a low energy field theory (SUGRA) description of the model: in presence of “short” radii we consider always a T-dual description.

The constraints are very mild. We distinguish between the \( p = 9 \) and \( p < 9 \) cases. In the first case there are no orthogonal tunable radii. More precisely, the equations can be cast as (substituting the values for \( \alpha_{\text{GUT}} \) etc.)

\[
\begin{align*}
r_{\parallel}^{6-d} &\sim \frac{1}{66} \left( \frac{M_{\text{GUT}}}{m_I} \right)^{d-2}, \\
g & = 50 \left( \frac{m_I}{M_P} \right)^2.
\end{align*}
\] (12)

The condition \( M_{\text{GUT}} < m_I \) implies \( r_{\parallel} < 1/3 \) for the minimal case \( d = 2 \), and even smaller values for \( d > 2 \). Since we prefer to avoid the presence of “short” radii we always consider T-dual versions with \( p < 9 \).

For \( p < 9 \) we can always rewrite Eq. (11) as

\[
\left( \frac{M_{\text{GUT}}}{m_I} \right)^{-1} = (50g)^{1/d} r_{\parallel}^{\frac{d-3}{d}}, \quad r_\perp^{9-p} = g \frac{50}{50} \left( \frac{M_P}{m_I} \right)^2.
\] (13)

The requirement \( M_{\text{GUT}} < m_I, r_{\parallel} > 1 \) and \( g < 1 \) imply \( \frac{1}{50} < g < 1 \). This means that, if \( r_{\parallel} \sim 1 \), the ratio between \( m_{\text{GUT}} / m_i \) is bounded between 1 \( (g = 1/50) \) and \( (1/50)^{1/d} \) \( (g = 1) \), i.e. that in the relevant cases \( d = 1, 2, 3 \) the maximal hierarchy between \( m_I \) and \( M_{\text{GUT}} \) is, respectively (roughly), 50, 7, 4. An higher hierarchy can be obtained only relaxing the requirement \( r_{\parallel} > 1 \). The constraints on \( r_\perp \) are also always fulfilled, with \( r_\perp \) typically large.

**Acknowledgments**

I am grateful to Wilfried Buchmüller, Stefan Groot Nibbelink, Marco Serone, Alexander Westphal and especially Arthur Hebecker for discussions and comments.
References

[1] L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, “Strings On Orbifolds”, Nucl. Phys. B 261 (1985) 678 and 274 (1986) 285.

[2] E. Witten, “Symmetry Breaking Patterns In Superstring Models,” Nucl. Phys. B 258 (1985) 75.

[3] J. Scherk and J. H. Schwarz, “How To Get Masses From Extra Dimensions,” Nucl. Phys. B 153 (1979) 61.

[4] Y. Hosotani, “Dynamical Mass Generation By Compact Extra Dimensions,” Phys. Lett. B 126 (1983) 309; “Dynamics Of Nonintegrable Phases And Gauge Symmetry Breaking,” Annals Phys. 190 (1989) 233.

[5] R. Rohm, “Spontaneous Supersymmetry Breaking In Supersymmetric String Theories,” Nucl. Phys. B 237, 553 (1984); C. Kounnas and M. Porrati, “Spontaneous Supersymmetry Breaking In String Theory,” Nucl. Phys. B 310, 355 (1988); S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, “Superstrings With Spontaneously Broken Supersymmetry And Their Effective Theories,” Nucl. Phys. B 318, 75 (1989).

[6] C. Kounnas and B. Rostand, “Coordinate Dependent Compactifications And Discrete Symmetries,” Nucl. Phys. B 341 (1990) 641.

[7] E. Kiritsis and C. Kounnas, “Perturbative and non-perturbative partial supersymmetry breaking: N = 4 → N = 2 → N = 1,” Nucl. Phys. B 503, 117 (1997) [arXiv:hep-th/9703059]; C. A. Scrucca and M. Serone, “A novel class of string models with Scherk-Schwarz supersymmetry breaking,” JHEP 0110, 017 (2001) [arXiv:hep-th/0107159].

[8] I. Antoniadis, G. D’Appollonio, E. Dudas and A. Sagnotti, “Open descendants of Z(2) x Z(2) freely-acting orbifolds,” Nucl. Phys. B 565, 123 (2000) [arXiv:hep-th/9907184].

[9] I. Antoniadis, E. Dudas and A. Sagnotti, “Brane supersymmetry breaking,” Phys. Lett. B 464, 38 (1999) [arXiv:hep-th/9908023]; A. L. Cotrone, “A Z(2) x Z(2) orientifold with spontaneously broken supersymmetry,” Mod. Phys. Lett. A 14, 2487 (1999) [arXiv:hep-th/9909116].
C. A. Scrucca, M. Serone and M. Trapletti, “Open string models with Scherk-Schwarz SUSY breaking and localized anomalies,” Nucl. Phys. B 635, 33 (2002) [arXiv:hep-th/0203190].

[10] A. Hebecker, “Grand unification in the projective plane,” JHEP 0401, 047 (2004) [arXiv:hep-ph/0309313].

[11] A. Hebecker and M. Trapletti, “Gauge unification in highly anisotropic string compactifications,” Nucl. Phys. B 713, 173 (2005) [arXiv:hep-th/0411131].

[12] J. R. Ellis, S. Kelley and D. V. Nanopoulos, “Precision Lep Data, Supersymmetric Guts And String Unification,” Phys. Lett. B 249, 441 (1990); U. Amaldi, W. de Boer and H. Fürstenau, “Comparison of grand unified theories with electroweak and strong coupling constants measured at LEP,” Phys. Lett. B 260, 447 (1991);
P. Langacker and M. X. Luo, “Implications of precision electroweak experiments for $m_t$, $\rho_0$, $\sin^2(\theta_W)$ and grand unification,” Phys. Rev. D 44, 817 (1991).

[13] Y. Kawamura, “Triplet-doublet splitting, proton stability and extra dimension,” Prog. Theor. Phys. 105 (2001) 999 [arXiv:hep-ph/0012125]; G. Altarelli and F. Feruglio, “SU(5) grand unification in extra dimensions and proton decay,” Phys. Lett. B 511 (2001) 257 [arXiv:hep-ph/0012301].
L. J. Hall and Y. Nomura, “Gauge unification in higher dimensions,” Phys. Rev. D 64, 055003 (2001) [arXiv:hep-ph/0103125];
A. Hebecker and J. March-Russell, “A minimal $S(1)/(Z(2) \times Z'(2))$ orbifold GUT,” Nucl. Phys. B 613 (2001) 3 [arXiv:hep-ph/0106166].
T. Asaka, W. Buchmüller and L. Covi, “Gauge unification in six dimensions,” Phys. Lett. B 523 (2001) 199 [arXiv:hep-ph/0108021].

[14] V. S. Kaplunovsky, “Mass Scales Of The String Unification,” Phys. Rev. Lett. 55 (1985) 1036.

[15] M. Berkooz, M. R. Douglas and R. G. Leigh, “Branes intersecting at angles,” Nucl. Phys. B 480 (1996) 265 [arXiv:hep-th/9606139].

[16] T. Watari and T. Yanagida, “Product-group unification in type IIB string theory,” Phys. Rev. D 70 (2004) 036009 [arXiv:hep-ph/0402160].

[17] D. J. Clements, “Supersymmetry and phenomenology of heterotic and type I superstring models,” [arXiv:hep-th/0407091]
[18] C. Angelantonj, R. Blumenhagen and M. R. Gaberdiel, “Asymmetric orientifolds, brane supersymmetry breaking and non-BPS branes,” Nucl. Phys. B 589 (2000) 545 [arXiv:hep-th/0006033].

[19] T. Kobayashi, S. Raby and R. J. Zhang, “Constructing 5d orbifold grand unified theories from heterotic strings,” Phys. Lett. B 593 (2004) 262 [arXiv:hep-ph/0403065]. “Searching for realistic 4d string models with a Pati-Salam symmetry: Orbifold grand unified theories from heterotic string compactification on a Z(6) orbifold,” Nucl. Phys. B 704 (2005) 3 [arXiv:hep-ph/0409098]. S. Forste, H. P. Nilles, P. K. S. Vaudrevange and A. Wingerter, “Heterotic brane world,” Phys. Rev. D 70 (2004) 106008 [arXiv:hep-th/0406208]. W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, “Dual models of gauge unification in various dimensions,” Nucl. Phys. B 712 (2005) 139 [arXiv:hep-ph/0412318].

[20] L. E. Ibanez, H. P. Nilles and F. Quevedo, “Reducing The Rank Of The Gauge Group In Orbifold Compactifications Of The Heterotic String,” Phys. Lett. B 192 (1987) 332. L. E. Ibanez, J. Mas, H. P. Nilles and F. Quevedo, “Heterotic Strings In Symmetric And Asymmetric Orbifold Backgrounds,” Nucl. Phys. B 301 (1988) 157. G. Lopes Cardoso, D. Lust and T. Mohaupt, “Moduli spaces and target space duality symmetries in (0,2) Z(N) orbifold theories with continuous Wilson lines,” Nucl. Phys. B 432 (1994) 68 [arXiv:hep-th/9405002]. T. Mohaupt, “Orbifold compactifications with continuous Wilson lines,” Int. J. Mod. Phys. A 9 (1994) 4637 [arXiv:hep-th/9310184].

[21] S. Forste, H. P. Nilles and A. Wingerter, “Geometry of rank reduction,” [arXiv:hep-th/0504117]