Cosmology with the CFT-radiation matter

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Abstract

We review the relation between entropy bounds to rewrite Friedmann equation on the
brane in terms of three entropy bounds: Bekenstein-Verlinde ($S_{BV}$); Bekenstein-Hawking
bound ($S_{BH}$); Hubble bound ($S_H$). For a strongly coupled conformal field theory (CFT)
with a dual 5-dimensional anti de Sitter Schwarzschild (AdSS$_5$) black hole, we can easily
establish the connection between the Cardy-Verlinde formula on the CFT side and the
entropy representation of Friedmann equation in cosmology. In this case its cosmological
evolution for entropy is given by the semi-circle. However, for the matter-dominated case,
we find that the cosmological evolution diagram takes a different form of the cycloid.
Here we propose two different entropy relations for matter-dominated case. It turns out
that the Verlinde’s entropy relation so restricted that it may not be valid for the matter-
dominated universe.

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1 Introduction

Recently Verlinde have made two interesting things [4]. The related issues appeared in [4, 5, 6, 7, 8]. First he proposed that using the AdS/CFT correspondence [8], the entropy of a conformal field theory (CFT) in any dimension can be expressed in terms of a generalized form of the Cardy formula [9]. We consider a CFT residing in an $(n+1)$-dimensional spacetime with the static metric for the Einstein space

$$ds^2_{\text{CFT}} = -d\tau^2 + R^2 d\Omega^2_n,$$

(1.1)

where $d\Omega^2_n$ denotes an unit $n$-dimensional sphere. Especially for a strongly coupled CFT with the anti de Sitter (AdS) dual, one obtains the Cardy-Verlinde formula which states a relation between entropy ($S$) and energy ($E$)

$$S = \frac{2\pi R}{n} \sqrt{E_c(2E - E_c)}$$

(1.2)

with the Casimir energy ($E_c$) for a finite system. Indeed, this formula was checked to hold for various kinds of AdS-bulk spacetimes: AdS-Schwarzschild black holes [1]; AdS-Kerr black holes [10]; AdS-charged black holes [11]; AdS-Taub-Bolt spacetimes [12, 13].

The other is to connect the above Cardy-Verlinde formula with the $(n+1)$-dimensional Friedmann equation based on the Friedman-Robertson-Walker (FRW) metric for the closed universe

$$ds^2_{\text{FRW}} = -d\tau^2 + \mathcal{R}^2(t)d\Omega^2_n.$$

(1.3)

Although two metrics (1.1) and (1.3) have different natures, these are conformally equivalent. Hence it is possible to make connection between these. For a radiation-dominated closed universe, two Friedman equations are given by

$$H^2 = \frac{16\pi G_{n+1} E}{n(n-1) V} - \frac{1}{\mathcal{R}^2},$$

(1.4)

$$\dot{H} = -\frac{8\pi G_{n+1}}{n-1} \left( \frac{E}{V} + p \right) + \frac{1}{\mathcal{R}^2},$$

(1.5)

where $H = \dot{\mathcal{R}}/\mathcal{R}$ is the Hubble parameter, the dot stands for the differentiation with respect to the proper time $\tau$, $E$ is the total energy of matter filling in the universe, $p$ denotes the pressure and $V = \mathcal{R}^n \text{Vol}(S^n)$ is the volume of the universe. In addition, $G_{n+1}$
is the \((n + 1)\)-dimensional newtonian constant. Verlinde pointed out that the Friedmann equation (1.4) can be expressed in terms of three cosmological entropy bounds:

Bekenstein-Verlinde bound : \(S_{BV} = \frac{2\pi}{n} E R\),

Bekenstein-Hawking bound : \(S_{BH} = (n - 1) \frac{V}{4G_{n+1} R}\),

Hubble bound : \(S_H = (n - 1) \frac{H Y}{4G_{n+1}}\).

(1.6)

Then the Friedmann equation (1.4) can be rewritten as the Verlinde’s entropy relation

\[ S_H^2 + (S_{BV} - S_{BH})^2 = S_{BV}^2. \]  

(1.7)

The above equation can be solved by introducing the conformal time coordinate \(\eta\) as

\[ S_H = S_{BV} \sin \eta, \quad S_{BH} = S_{BV} (1 - \cos \eta). \]  

(1.8)

This means that \(S_{BV}\) is constant with respect to the cosmic time \(\tau\), while \(S_H\) and \(S_{BH}\) depend on the cosmic time. Actually \(S_{BV}\) is constant throughout the entire evolution, because \(E \sim R^{-1}\) for a radiation-dominated universe. We note that the Bekenstein-Verlinde bound is valid for the weakly self-gravitating universe \((H R \leq 1)\), while the Hubble bound holds for the strongly self-gravitating universe \((H R \geq 1)\). To decide whether a system is strongly or weakly gravitating, we have to introduce another quantity like \(S_{BH}\). When \(S_{BV} \leq S_{BH}\), the system is weakly gravitating, while for \(S_{BV} \geq S_{BH}\) the self-gravity is strong. This is identified with the holographic Bekenstein-Hawking entropy of a black hole with the size of the universe. It grows like an area instead of the volume. Also the maximal entropy inside the universe is bounded by the black holes of the size of the Hubble horizon. This is the Hubble entropy bound \(S_H\). It is clear from the Friedmann equation (1.4) that at the critical point of \(H R = 1\), three entropy bounds coincide exactly with each other.

Further let us propose \(E_{BH}\) corresponding to the Bekenstein-Hawking energy by using the Bekenstein-Verlinde bound such a way that \(S_{BH} = (n - 1) V/4G_{n+1} R \equiv 2\pi E_{BH} R/n\). Equation (1.7) then takes the form

\[ S_H = \frac{2\pi}{n} \sqrt{E_{BH}(2E - E_{BH})}. \]  

(1.9)

\(^1\)In Ref. [1], the first bound is called the Bekenstein bound. In fact, this bound is slightly different from the original Bekenstein entropy bound proposed in [14, 15]. So we call this the Bekenstein-Verlinde bound.
It is very important to note here that this relation is the same form of the Cardy-Verlinde formula \((1.2)\) except that the roles of the entropy \(S\) and Casimir energy \(E_c\) are taken over by the Hubble entropy bound \(S_H\) and Bekenstein-Hawking energy \(E_{BH}\). This connection between the Cardy-Verlinde formula and the Friedmann equation can be interpreted as a consequence of the holographic principle \([1]\). This implies that two (Friedman equation and Cardy-Verlinde formula) can be derived from the same first principle.

In this direction, Savonije and Verlinde \([16]\) have studied a concrete model by using the one-side brane cosmology in the background of \((n+2)\)-dimensional AdS-Schwarzschild spacetime

\[
ds_{AdSS_{n+2}}^2 = g_{MN}dx^Mdx^N = -h(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2 \left[ d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2) \right].
\]

where \(h(r)\) is given by

\[
h(r) = 1 - \frac{m}{r^{n-1}} + \frac{r^2}{\ell^2}
\]

with \(\omega_{n+1}M = \frac{16\pi G_{n+2}}{n \text{Vol}(S^n)} M\). Here \(\ell\) is the curvature radius of AdS\(_{n+2}\) space and \(M\) is the ADM mass of the black hole as measured by an observer who uses \(t\) as his time coordinate. \(G_{n+2}\) is the bulk newtonian constant. In the one-side brane world scenario, we have the relation of \(G_{n+2} = \ell G_{n+1}/(n-1)\) between the bulk and boundary constants. In the case of \(m = 0\), we have an exact AdS\(_{n+2}\)-space. However, \(m \neq 0\) generates the electric part of the Weyl tensor \(E_{00} = C_{0N0Q}n^N n^Q \sim m/r^{n-1}\) \([17]\). This corresponds to the nonlocal effects arisen from the free gravitational field in the bulk \([18]\), transmitted through the projection \(E_{MP}\) of the bulk Weyl tensor. This nonlocal Weyl term will contributes corrections to the Friedmann equations on the brane. Actually we focus on the role of this term in cosmology. It was argued that the energy \((E)\), entropy \((S)\), and temperature \((T)\) of a CFT at high temperature can be related to the mass \((M)\), entropy \((S_{BH})\), and Hawking temperature \((T_H)\) of the AdS black hole. According to the GKPW prescription of the AdS/CFT correspondence \([13, 8]\), the conformal class of the boundary CFT metric is not fixed. Let us introduce its boundary metric from the bulk one in Eq.\((1.10)\)

\[
ds_{BCFT}^2 = \lim_{r \to \infty} \frac{\ell^2}{r^2} ds_{AdS-S}^2 = -d\tau^2 + \ell^2 d\Omega_{n-1}^2.
\]

From this we deduce the static relation between the quantities on the CFT-boundary and those in the AdS bulk : \(t \to \tau = t\ell/r; T_H \to T = T_H\ell/r; M \to E = M\ell/r\). But we
have the same entropy \( S = S_{BH} = \frac{r_+^n \text{Vol}(S^n)}{4G_{n+2}} \), where \( r_+ \) is the event horizon of the AdS black hole. It is well-known that the equation governing the motion of the brane (Moving Domain Wall: MDW) are exactly given by the \((n+1)\)-dimensional Friedmann equation with radiation matter\cite{22, 23, 24}. In this case the radiation-matter which comes from the nonlocal Weyl term can be identified as a strongly coupled CFT, by making use of the AdS/CFT correspondence. Importantly, it turned out that the Friedmann equation is exactly matched with the Cardy-Verlinde formula for the CFT when the brane crosses the black hole horizon.

On the other hand authors in \cite{25, 26} pointed out that in general, the Cardy-Verlinde formula is not valid in weakly coupled CFTs. Further we wish to comment that the entropy relation of Eq.(1.7) is suitable only for the restricted cases such as a radiation-dominated CFT.

In this article we will clarify again that a deep connection between the Cardy-Verlinde formula and Friedmann equation is just a peculiar property of the dynamic brane moving under the “5D” anti de Sitter Schwarzschild black hole spacetime. Here we provide a counter example where this connection fails for a matter-dominated universe. Actually all moving domain walls in the AdS\_5 black hole can always take a kind of radiation-dominated matter \( (\rho \sim E/V, V = R^n \text{Vol}(S^n)) \) which arises originally from the nonlocal term \( M/R^4 \) of the Schwarzschild black hole through the relation \( E = M\ell/R \). However, if we consider the brane which moves in the ordinary bulk matter, this connection is no longer satisfied. For example, if one considers the moving domain wall (brane) in the bulk matter with \( \rho_B \neq 0, P_B = 0 \) which does not include any black hole, then one finds equation of state for the matter-dominated universe of \( \rho_B \sim 1/a^3 \) on the brane. There is also a way to obtain a radiation-dominated CFT from the bulk spacetime in the framework of the BDL brane cosmology\cite{27, 28}.

## 2 Brane Cosmology with MDW approach

For definiteness we choose \( n = 3 \) (five-dimensional AdS-Schwarzschild balck hole spacetime). Now we introduce the radial location of a MDW in the form of \( r = R(\tau), t = \frac{2}{\ell} \). \footnote{There also exists the other brane cosmology: BDL approach\cite{20, 21}.}
$t(\tau)$ parametrized by the proper time $\tau$ to define a cosmic embedding: $(t, r, \chi, \theta, \phi) \rightarrow (t(\tau), R(\tau), \chi, \theta, \phi)$. Then we expect that the induced metric of dynamical domain wall will be given by the FRW-type. Hence $\tau$ and $R(\tau)$ will imply the cosmic time and scale factor of the FRW-universe, respectively. A tangent vector (proper velocity) of this MDW

$$u = i \frac{\partial}{\partial t} + \hat{R} \frac{\partial}{\partial R},$$

(2.1)
is introduced to define this embedding properly. Here overdots mean differentiation with respect to $\tau$. This is normalized to satisfy

$$u^M u^N g_{MN} = -1.$$  

(2.2)

Given a tangent vector $u_M$, we need a normal 1-form directed toward to the bulk. Here we choose this as

$$n = \hat{R} dt - i dR, \quad n_M n_N g^{MN} = 1.$$  

(2.3)

This convention is consistent with the Randall-Sundrum case in the limit of $m = 0$ [23]. Using Eq.(2.1) either with (2.2) or with Eq.(2.3), we can express the proper time rate of the AdSS$_5$ time $\dot{t}$ in terms of $\hat{R}$ as

$$\dot{t} = \frac{\sqrt{\hat{R}^2 + h(\hat{R})}}{h(\hat{R})}.$$  

(2.4)

From the above, we worry about that $\dot{t}$ is not defined at $\hat{R} = r_+$ because $h(r_+) = 0$. This also happens in the study of static black hole. Usually one introduces a tortoise coordinate $r^* = \int h^{-1} dr$ to resolve it. Then Eq.(1.10) takes a form of $ds^2_{AdSS_5} = -h(dt^2 - dr^*2) \cdots$ and one finds the Kruscal extension. This means that $r = r_+$ is just a coordinate singularity. We confirm this from the computation of $R_{MNPQ}R^{MNPQ} = 40/\ell^4 + 72m^2/r^8$, which shows that $r = 0$ ($r = r_+$) are true (coordinate) singularities. It is found that there does not exits such a problem even for the dynamic case [30]. Using Eq.(2.4), Eq.(1.10) leads to the 4D induced metric for the brane

$$ds^2_{FRW} = -d\tau^2 + \mathcal{R}(\tau)^2 \left[ d\chi^2 + \sin^2\chi (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$$\equiv h_{\mu\nu} dx^\mu dx^\nu,$$

(2.5)

where we use the Greek indices only for the brane. Actually the embedding of the FRW-universe into AdSS$_5$ space is a $2(t, r) \rightarrow 1(\tau)$-mapping. The projection tensor is given
by \( h_{MN} = g_{MN} - n_M n_N \) and its determinant is zero. Hence its inverse \( h^{MN} \) cannot be defined. This means that the above embedding belongs to a peculiar mapping to obtain the induced metric \( h_{\mu\nu} \) in the AdSS\( _5 \) black hole spacetime \( g_{MN} \) with \( n_M \). Now we have to calculate the scale factor \( R(\tau) \) from the Israel junction condition by introducing the extrinsic curvature \[31\]. For this case the extrinsic curvature is defined by

\[
K_{\tau\tau} = K_{MN} u^M u^N = (h(R)t)^{-1}(\ddot{R} + h'(R)/2) = \frac{\dot{R} + h'(R)/2}{\sqrt{\dot{R}^2 + h(R)}}. 
\]

\[
K_{\chi\chi} = K_{\theta\theta} = K_{\phi\phi} = -h(R) fR = -\sqrt{\dot{R}^2 + h(R)} R, 
\]

where prime stands for derivative with respect to \( R \). A localized matter on the brane implies that the extrinsic curvature jumps across the brane. This jump is described by the Israel junction condition for the one-side brane world scenario \[30, 32\]

\[
K_{\mu\nu} = -\kappa_5^2 \left( T_{\mu\nu} - \frac{1}{3} T^{\lambda}_{\lambda} h_{\mu\nu} \right) 
\]

with \( \kappa^2 = 8\pi G_5 \). For cosmological purpose, we may introduce a localized stress-energy tensor on the brane as the 4D perfect fluid

\[
T_{\mu\nu} = (\varrho + p)u_\mu u_\nu + p h_{\mu\nu}.
\]

Here \( \varrho = \rho_m + \sigma \) (\( p = p_m - \sigma \)), where \( \rho_m \) (\( p_m \)) are the energy density (pressure) of the localized matter and \( \sigma \) is the brane tension. Here we consider the cosmological evolution without any localized matter on the brane. This means that there exists only the matter from the bulk configuration on the brane. In this case of \( \rho_m = p_m = 0 \), the r.h.s. of Eq.\( (2.8) \) leads to a form of the RS case as \(-\frac{2\kappa^2}{3} h_{\mu\nu} \)[33, 34, 35]. From Eq.\( (2.8) \), one finds the space component of the junction condition

\[
\sqrt{h(R) + \dot{R}^2} = \frac{\kappa^2}{3} \sigma R. 
\]

For the one-side AdSS\( _5 \) space, we have the brane tension \( \sigma = 3/(\kappa^2 \ell) \) for the fine-tuning. The above equation leads to

\[
H^2 = -\frac{1}{\dot{R}^2} + \frac{m}{R^4}, 
\]

\[7\]
where $m/R^4$ originates from the electric (Coulomb) part of the 5D Weyl tensor, $E_{00} \sim m/r^2$ \cite{17, 36}. For $n = 3$, we have $m = \frac{16\pi G_5 M}{3V(S^3)}$, $M = \frac{R}{\ell} E$, $V = R^3 \text{Vol}(S^3)$, $G_5 = \frac{\ell}{2} G_4$. Then one finds a CFT-radiation dominated universe

$$H^2 = -\frac{1}{R^2} + \frac{8\pi G_4}{3} \rho_{CFT}, \quad \rho_{CFT} = \frac{E}{V}. \quad (2.12)$$

It seems that the equation (2.10) is well-defined even at $R = r_+$. Thus this leads to $H = \pm 1/\ell$ at the horizon, which was the case mentioned first in ref. \cite{16}. At this moment when the brane crosses the horizon of the AdSS$_5$ black hole, we find that the entropy density $s = S/V$ and the temperature of the CFT can be expressed in terms of the Hubble parameter $H$ and its time derivative $\dot{H}$ only

$$s = \frac{H}{2G_4}, \quad T = -\frac{\dot{H}}{2\pi H}, \quad \text{at } R = r_+. \quad (2.13)$$

Now let us discuss thermodynamics of the CFT itself. Furthermore from the first law of thermodynamics ($T dS = dE + PdV$) and the CFT-radiation matter ($\rho_{CFT} = M\ell/(R V)$, $P_{CFT} = \rho_{CFT}/3$), we derive

$$\frac{3}{2}(\rho_{CFT} + P_{CFT} - sT) = \frac{\gamma}{R^2} \quad (2.14)$$

with

$$T = \frac{1}{4\pi R} \left( \frac{4r_+}{\ell} + \frac{2\ell}{r_+} \right), \quad \gamma = \frac{3}{8\pi G_4} \frac{r_+^2}{R^2}. \quad (2.15)$$

Here the r.h.s of Eq.(2.14) represents the geometric Casimir part of the energy density. Also from the (3 + 1)-dimensional Cardy-Verlinde formula Eq.(1.2), we find

$$s^2 = \left( \frac{4\pi}{3} \right)^2 \gamma \left( \rho_{CFT} - \frac{\gamma}{R} \right). \quad (2.16)$$

Two of Eqs.(2.14) and (2.16) are valid at all times. Let us check what happens for these at the moment when the MDW crosses the horizon. In this case we have $\gamma = \frac{3}{8\pi G_4}$. Using this and Eq.(2.13), we can recover the first Friedmann equation (1.4) from Eq.(2.16). Also the second Friedmann equation (1.5) can be derived from (2.14). These imply that the Friedmann equations know about thermodynamics of the CFT.

For a charged background, we find the same result except the appearance of the negative energy density \cite{37, 32}. The similar result was found for the dilatonic black hole background \cite{38}.
3 Entropy bound relation revisited

In this section we focus on the $n = 3$ case. The relation for the entropy bounds Eq.(1.7) corresponds to just an algebraic version of the radiation-dominated FRW equation (1.4). In other words, this relation reflects partly the nature of a kind of Newtonian equation:

$$\dot{R}^2 + V_r(R) = -1$$

with

$$V_r(R) = -\frac{R^2}{R^2}, \quad R^2 = \frac{4\pi G_4 E}{3\text{Vol}(S^3)}.$$  \hfill (3.2)

The solution of this differential equation can be solved parametrically in terms of an arc parameter $\eta$ [39],

$$R = R_r \sin \eta, \quad \tau = R_r (1 - \cos \eta).$$

(3.3)

The range of $\eta$ from start of expansion to end of recontraction is just $\pi$ and the curve relating radius $R$ to time $\tau$ is a semicircle. From Eq.(3.3), we find a relation representing the diagram for cosmic evolution

$$R^2 + (R_r - \tau)^2 = R_r^2$$

(3.4)

which is the same form as in the entropy relation Eq.(1.7). Of course this is valid for the strongly self-gravitating universe with $H R > 1$. In this case we expect a naive correspondence between $(R, \tau, R_r)$ and $(S_H, S_{BH}, S_{BV})$:

$$R \leftrightarrow S_H, \quad \tau \leftrightarrow S_{BH}, \quad R_r \leftrightarrow S_{BV}.$$  \hfill (3.5)

Also, we observe that for a radiation-dominated universe, the solution (3.3) to the differential equation takes exactly the same form as in Eq.(1.8) to the algebraic equation for the entropy bounds. This property can be regarded as an important factor for establishing the connection between the Cardy-Verlinde formula and the FRW-equation in the radiation-dominated CFT universe. In the next we introduce a counter example where this connection is no longer satisfied.
4 Matter-dominated universe

In this section we consider the moving domain wall in the the AdS$_5$ spacetime with the negative cosmological constant $\Lambda$ including the other bulk matter ($\tilde{M}$). We assume that the AdS space does not include any object like black hole. We introduce the matter-dominated Friedmann equation

$$\dot{R}^2 + V_m(R) = -1 \quad (4.1)$$

with

$$V_m(R) = -\frac{R m}{R}, \quad m = \frac{8\pi G_4 \tilde{M}}{3\text{Vol}(S^3)} \quad (4.2)$$

This situation may be figured out from the brane world scenario [27, 28]. This can be derived from the newtonian cosmology [40]. For example, we consider a spherical ball ($S^3$) on the brane with matter $m$ in it. Of course this arises from a kind of the bulk matter $\rho_B \sim \tilde{M}, \rho_B = 0$. In this case the potential of the ball on its surface is given by $\phi_{ball}(r = R(\tau)) = -G_4 m/R$. Let us introduce a point-like probe with unit mass on the ball. The energy conservation condition for the bound-motion with negative total energy $-k/2$ is given by

$$\dot{R}^2 + \phi_{ball} = -\frac{k}{2} \quad (4.3)$$

which can be led to the matter-dominated FRW-equation Eq.(4.1) if one chooses $m = 4\pi \tilde{M}/3\text{Vol}(S^3)$ and $k = 1$. However, one handicap of the newtonian cosmology is that this choice of $m, k$ is unclear.

The solution to the equation (4.1) can be expressed parametrically in terms of an arc parameter $\eta$ [39],

$$R = \frac{R m}{2}(1 - \cos \eta), \quad \tau = \frac{R m}{2} (\eta - \sin \eta). \quad (4.4)$$

Here the range of $\eta$ from start of expansion to end of recontraction is $2\pi$ and the curve relating radius $R$ to time $\tau$ is not a semicircle. These are different points when comparing with the radiation-dominated case. From Eq.(4.4), we find a relation between $R$ and $\tau$ expressed as cycloid, compared with a semicircle for the radiation-dominated universe Eq.(3.4)

$$(R_m - 2R)^2 + (R_m \eta - 2\tau)^2 = R_m^2. \quad (4.5)$$
This is valid for the weakly self-gravitating universe with $HR < 1$. The newtonian cosmology also belongs to this category. Assuming a naive correspondence between $(R, \tau, R_m)$ and $(S_H, S_{BH}, S_{BV})$ as in the radiation-dominated case, then one finds one entropy relation from Eq.(4.5):

$$(S_{BV} - 2S_H)^2 + (S_{BV} \eta - 2S_{BH})^2 = S_{BV}^2, \quad (4.6)$$

where $S_{BV}$ is constant.

On the other hand, if one follows closely the definition of each entropy bound, we may propose the new entropy bounds: Bekenstein-Verlinde, Bekenstein-Hawking, and Hubble bounds which may be useful for describing the matter-dominated universe

Bekenstein-Verlinde bound: \[ \tilde{S}_{BV} = \frac{2\pi}{n} \tilde{M} \tilde{R}, \]
Bekenstein-Hawking bound: \[ \tilde{S}_{BH} = (n-1) \frac{V}{4\mathcal{G}n}, \]
Hubble bound: \[ \tilde{S}_{H} = (n-1) \frac{HV}{4\mathcal{G}n}. \quad (4.7) \]

These are the same forms as in Eq.(1.6) except for the definition of the Bekenstein-Verlinde bound. In the radiation-dominated CFT case, $S_{BV}$ is constant because of $E \mathcal{R} = M\ell (=\text{constant})$, whereas for the matter-dominated universe, $\tilde{S}_{BV}$ scales as $\mathcal{R}$ as the universe evolves. This means that $\tilde{S}_{BV}$ is not constant here. But we suggest a familiar relation between new entropy bounds

\[ \tilde{S}_{H}^2 + (\tilde{S}_{BV} - \tilde{S}_{BH})^2 = \tilde{S}_{BV}^2. \quad (4.8) \]

As it stands, this is different from Eq.(4.7). Eq.(4.8) is not obviously an equation for the semicircle because $\tilde{S}_{BV}$ is not constant.

5 Conclusions

At this time we do not know exactly which one among Eqs.(4.6) and (4.8) is appropriate for describing the entropy relation for the matter-dominated universe. Also we note that the Verlinde’s entropy relation Eq.(1.7) which derives from the first Friedmann equation Eq.(1.4) for the radiation-dominated case is not suitable for the matter-dominated case. Finally we wish to comment that a close relationship between the Cardy-Verlinde formula and Friedmann equation is realized only for a special circumstance such as a radiation-dominated CFT within the AdS/CFT correspondence through the brane cosmology.
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