Supersymmetric Electroweak Phase Transition: Baryogenesis versus Experimental Constraints

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We use the two loop effective potential to study the first order electroweak phase transition in the minimal supersymmetric standard model. A global search of the parameter space is made to identify parameters compatible with electroweak baryogenesis. We improve on previous such studies by fully incorporating squark and Higgs boson mixing, by using the latest experimental constraints, and by computing the latent heat and the sphaleron rate. We find the constraints \( \tan \beta > 2.1 \), \( m_t < 172 \text{ GeV} \), and \( m_h < 107 \text{ GeV} \) (becoming more or less restrictive if the heavy stop has mass less than or greater than 1 TeV). We also find that the change in \( \tan \beta \) in the bubble wall is rarely greater than \( 10^{-3} \), which severely constrains the mechanism of baryogenesis.

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The most predictive theory of the origin of baryonic matter at present is electroweak baryogenesis, since it relies on assumptions about new physics that is being searched for at LEP and Fermilab. For sufficient baryogenesis, the electroweak phase transition (EWPT) must be strong; the requirement is roughly \( v/T_c > 1 \), with \( v \) the Higgs vacuum expectation value and \( T_c \) the critical temperature. This is unachievable in the standard model. One of the main contenders for the new physics needed is supersymmetry, whose simplest manifestation is the Minimal Supersymmetric Standard Model (MSSM). Much progress has been made in the last few years toward identifying which regions of the MSSM parameter space are compatible with electroweak baryogenesis. It is agreed that a light Higgs boson and a light top squark are needed, but estimates vary as to just how light. One goal of the present work is to refine these estimates within the framework of the effective potential approach.

Computing the properties of the EWPT is subtle because the power-counting arguments for perturbation theory at zero temperature are modified at high temperature, such that the loop expansion for the free energy (effective potential) must be resummed. Even after resumming, the effective potential (EP) is not guaranteed to be reliable at small values of \( v \), due to infrared divergences coming from the small masses of the transverse gauge bosons and the light Higgs boson. There are three principal ways of dealing with this problem: (1) compute phase transition properties fully numerically, on the lattice; (2) integrate out the heavy, non-IR-divergent modes analytically (“dimensional reduction”) and save the lattice for the effective theory of the dangerous light modes; and (3) improve the effective potential by computing to higher order in the loop expansion.

The first approach, because of its high numerical cost, is impractical from the standpoint of exploring large regions of parameter space. Dimensional reduction is better in this respect because it maps the full MSSM parameter space onto a much smaller set of parameters in the effective high-\( T \) theory for the light modes, thereby reducing the number of lattice simulations needed. But DR has its limitations, especially when it comes to the effects of superrenormalizable interactions which can induce a large number of unsuppressed high-dimension operators in the effective theory to be studied on the lattice. The \( A_t \) and \( \mu \) parameters of the MSSM come with precisely these kinds of interactions.

On the face of it, computing the effective potential to higher order in perturbation theory would not seem promising since the convergence of the perturbation series is supposed to be poor. However experience with the standard model shows that the two-loop EP works quite well. In the MSSM there are also indications that the two-loop EP gives converging results, since they are in fairly good agreement with recent lattice computations: the lattice gives values of \( v/T \) (a measure of the strength of the transition) which are about 10% higher than those of the EP. This, together with the ability it affords for quickly combing the full MSSM parameter space, makes it worthwhile to investigate the two-loop EP. Furthermore, detailed properties of the phase transition like the nucleation temperature \( T_{nuc} \), sphaleron energy inside the bubbles, and the variation of the two Higgs field VEV’s inside the bubble walls, are much more readily available from the EP than from lattice studies.

The important contributions to the two-loop EP in the MSSM have been calculated by references \[15, 18\]. The first reference included squark mixing, while assuming the heavy Higgs bosons are decoupled. The third allowed for lighter Higgs bosons, but ignored squark mixing. We have generalized these results to incorporate both effects. This is desirable because most baryogenesis mechanisms rely upon \( \tan \beta \), the ratio \( (v_2/v_1) \) of the two Higgs VEV’s, changing from the interior to the exterior of the bubbles, which is only possible if the heavy Higgs bosons are not decoupled. Squark mixing is also expected because the phase (and magnitude) of the \( \mu \) parameter must not be too small, since this is the principal source of CP violation \[13\]. The \( \mu \) parameter appears in the off-diagonal term of the stop mass matrix, while the diagonal term...
for the right-handed stop must be small to get a strong enough phase transition. Thus significant squark mixing is a possibility, which in fact is realized (figure 2 below).

A recent development in the literature is to consider the formation of squark condensates at the beginning of a two-stage transition, where color and electric charge are temporarily broken [3]. To recover our known world, the color broken state must later copiously nucleate bubbles of the conventional SU(2)-breaking ground state of the standard model. If this could happen, it would considerably strengthen the EWPT. However, this nucleation process is heavily suppressed [3], leading to the same problem that killed “old” inflation—the CCB vacua would inflate and vastly dominate the spatial volume of the universe. Hence, at all temperatures above the nucleation temperature for the electroweak broken phase, the effective potential for the right stop must have a stable or sufficiently metastable symmetric minimum where this happens at any \( T > T_{\text{crit}} \) and \( |\Delta M| \ll 10^{-7} \). We therefore discard parameters where this happens at any \( T > T_{\text{crit}} \).

The most relevant laboratory constraints concern \( m_{h}, m_{t}, \) and \( m_{\tilde{t}} \) (the lightest Higgs boson, light stop and sbottom masses, respectively), \( \Delta \rho \) (the contributions of the stops and sbottoms to the \( \rho \) parameter), and the exclusion of charge- and color-breaking minima. The latest experimental limit on \( m_{h} \) depends on \( \sin^{2}(\alpha - \beta) \), where \( \alpha \) describes the composition of the light Higgs field \( h \) through \( \sin^2 \alpha H_{1}^0 + \cos \alpha H_{2}^0 \). The 95\% c.l. excluded region is roughly given by the intersection of \( m_{h} > (69 + 19\sin^{2}(\alpha - \beta)) \) GeV/c\(^{2}\) and \( m_{h} > (76 - 11.5\sin^{2}(\alpha - \beta)) \) GeV/c\(^{2}\). This allowed region and our accepted Monte Carlo points are shown in figure 1. The limit \( \sin^{2}(\alpha - \beta) \rightarrow 1 \) corresponds to a heavy \( A^0 \) boson and a standard-model-like Higgs sector, with only one light Higgs boson. The phase transition is typically strongest in this regime.

For the squark masses we use the preliminary ALEPH limit of \( m_{t} > 82 \) GeV, which is left-right mixing independent, and \( m_{\tilde{t}} > 79 \) GeV. Concerning the deviation in the \( \rho \) parameter; the standard model value of \( \Delta \rho \) (also known as \( \epsilon_{1} \)) is already 1.5\sigma larger than the experimental value [4] for \( m_{\tilde{t}} \sim 100 \) GeV. We constrain the squark contribution to \( \Delta \rho \) to be less than approximately one additional standard deviation, namely \( \Delta \rho < 1.5 \times 10^{-3} \). Chargino/neutralino searches also constrain \( |\mu| < 100 \) GeV [5].

We search this allowed parameter space for those values that give a strong enough phase transition. Our criterion is that the integrated rate of sphaleron transitions since the phase transition reduces the baryon asymmetry by just one \( e \)-folding. Writing the sphaleron rate per unit volume as \( \Gamma_{s} \) and the sphaleron energy as \( E_{\text{sph}} \), the bound is \( \Gamma_{s} < \Gamma_{\text{crit}} \), where

\[
\Gamma_{s} \simeq \left( \frac{v}{T} \right)^{7} T^{4} e^{6.9 - E_{\text{sph}}(T_{r})/T_{r}}; \\
\Gamma_{\text{crit}} = \frac{14}{123} T^{3} \frac{d}{dt} \ln \Gamma_{s},
\]

all evaluated at the reheating temperature \( T_{r} \approx T_{c} \). We find the sphaleron energy by solving for the sphaleron configuration using the two loop effective potential, and we multiply both \( v/T_{r} \) and \( E_{\text{sph}}/T_{r} \) by a correction factor of 1.1 suggested by the lattice results [2]. The frequencies of the MSSM input parameters which pass all these cuts, as well as histograms for some derived quantities, are shown in figure 2.

\[
\begin{align*}
\tan \beta & = 2.1, 10, -73, 0, 130, 1000, 116, 172, \\
\Delta \rho & \times 10^{3} (\text{GeV})^2, \quad v/T = -30.4, 1.49, 0.97, 1.62, \\
\Gamma_{s} & \sim 10^{-3}, \quad m_{\tilde{t}} = -30.4, 0.97, 1.62, \\
\sin 2\theta_{t} & = 0.04, 1.49, 0.97, 1.62.
\end{align*}
\]

Figure 1: (a) Experimentally allowed values of \( \sin^{2}(\alpha - \beta) \) and \( m_{h} \) are to the right of the line; Monte Carlo generated points are those consistent with electroweak baryogenesis. (b) Relation between \( \sin^{2}(\alpha - \beta) \) and \( m_{h} \).

Figure 2: Frequencies of baryogenesis-allowed parameters. Masses are in GeV; \( \theta_{t} \) is the stop mixing angle.

The strength of the phase transition depends most sensitively on \( \tan \beta \) and \( m_{\tilde{t}} \), which in turn determine the masses of the lightest Higgs boson and the top squarks,

\[
m_{h}^{2} = \frac{1}{4} \left[ m_{3}^{2} + m_{2}^{2} - \sqrt{(m_{3}^{2} + m_{2}^{2})^{2} - 4m_{A}^{2}m_{Z}^{2} \cos^{2} 2\beta} \right]
+ O \left( \frac{(m_{\tilde{t}}^{2}/v^{2}) \ln (m_{\tilde{t}}/m_{\tilde{t}_{e}})/m_{\tilde{t}_{e}}^{2})}{v} \right),
\]

\[
M_{\tilde{t}}^{2} \equiv \left( m_{\tilde{t}}^{2} + m_{2}^{2} + O(m_{Z}^{2}) \right. \\
m_{\tilde{t}}(A_{t} - \mu \cot \beta) \\
\left( m_{\tilde{t}}^{2} + m_{2}^{2} + O(m_{Z}^{2}) \right).
\]

Qualitatively, the dependences can be understood as follows. In the absence of the light stop, and ignoring the experimental constraint on the Higgs boson mass, a strong phase transition requires a small value \( m_{h} \) and hence small \( \tan \beta \). This is because the quartic terms of the tree-level potential, \( g^{2}(|H_{1}|^{2} - |H_{2}|^{2})^{2} \), are flat...
along the direction of $\tan \beta = 1$ ($|H_1| = |H_2|$), so the effective quartic coupling $\lambda$ is minimized for $\tan \beta \sim 1$, which helps the strength of the phase transition, since $v/T \sim v^3/\lambda$. The experimental bound on $m_h$ translates into a lower limit on $\tan \beta$, which excludes the whole region where baryogenesis is viable. However, this can be counteracted if the (mostly right-handed) stop is sufficiently light. Thus the contours of constant $v/T$ resemble hyperbolas in the $\tan \beta$-$\tilde{m}_U$ plane, as shown in figure 3. We define $\tilde{m}_U \equiv m_U^2/|m_U|$ so that $\tilde{m}_U$ has the same sign as $m_U^2$.

![Figure 3: Contours of $v/T$ (solid) and Higgs mass (dashed) in the plane of $\tan \beta$ and $\tilde{m}_U \equiv m_U^2/|m_U|$ (in GeV), for $m_Q = 100$ and 500 GeV, respectively, at zero squark mixing ($\mu = A_t = 0$). The potential has color-breaking minima in the black regions near $\tilde{m}_U = -70$ GeV.](image)

The next most important parameter is the soft-breaking mass term for the left-handed top squark, $m_{\tilde{t}}$. It affects the strength of the phase transition almost exclusively through a radiative correction to the Higgs self-coupling, proportional to $\ln((m_{\tilde{t}}^2 + m_\nu^2)/m_\nu^2)$. Thus, at fixed $\tan \beta$, raising $m_\nu$ increases $m_h$ and weakens the phase transition. (This weakening can be compensated by sufficiently lowering the right-stop mass.) However the allowed range of $\tan \beta$ consistent with the experimental limits on $m_h$ increases with $m_\nu$, as shown in figure 4, which has the scatter plots from the Monte Carlo for $m_h$ and $\tan \beta$ as functions of $m_Q \equiv m_Q/100$ GeV. The upper limit on $m_h$ as a function of $m_Q$ is simply the maximum theoretically allowed value in the MSSM. The fitted functions for the upper limit on $m_h$ and the corresponding lower limit on $\tan \beta$ are

$$m_h \lesssim 85.9 + 9.2 \ln(\tilde{m}_Q) \text{ GeV}$$

$$\tan \beta \gtrsim (0.03 + 0.076 \tilde{m}_Q - 0.0031 \tilde{m}_Q^2)^{-1},$$

generalizing the findings of ref. [10]. As for the smallest possible values of $m_h$, we see from figure 1 that the scarcity of points with $m_h < 88$ GeV is due to the small probability of getting a strong phase transition when $m_{\tilde{t}} < 100$ GeV.

We have somewhat painstakingly reconsidered the criterion for a strong enough phase transition. We first find the nucleation temperature $T_n$, which is lower than $T_c$,

and the latent heat of the phase transition. From these we get the temperature to which the universe reheats, $T_r$, on completion of the phase transition. We compute the sphaleron energy using the two loop effective potential at this temperature, and compare the resulting sphaleron rate $\Gamma_s$ to the expansion rate of the universe, also accounting for the time dependence of $\Gamma_s$, eq. (1). In figure 5a we show the correlation between the rigorous measure of baryon dilution, $-\ln(\Gamma_s/\Gamma_{\text{crit}})$, and $v(T_r)/T_c$. Here, in contrast to figure 2, $\Gamma_s/\Gamma_{\text{crit}}$ is computed without applying the lattice correction factor to $v(T_r)/T_c$. The points below the line $-\ln(\Gamma_s/\Gamma_{\text{crit}}) = 0$ should be discarded, according to eq. (1), and are only retained if we account for the fact that the effective potential underestimates $v/T$. The correlation between the correct criterion and $v(T_c)/T_c$ is good but not perfect. The smallest allowed value of $v(T_c)/T_c$ is 1.05, and the largest rejected value is about 1.15.

![Figure 4: Correlation of $m_h$ and $\tan \beta$ with the left-handed top squark mass parameter $\tilde{m}_Q \equiv m_Q/100$ GeV. Heavy lines show the approximate limiting values, eq. (3).](image)

![Figure 5: (a) Correlation of the sphaleron rate with $v/T$; below the line would be ruled out baryon preservation, eq. (1); (b) $\Delta T \equiv T_n - T_c$ in GeV versus $\Lambda \equiv (\text{latent heat})/T^4$, fit by the function $\Delta T = -0.5 + 0.5 \Lambda - 2.9 \Lambda^2$.](image)
reheat temperature: $L$ is the heat available for increasing the plasma energy density,

$$L = \Delta \rho_{c,n} \simeq g_\ast \pi^2 (T_r^4 - T_n^4)/30.$$  

Therefore, writing $\Delta \rho_{c,n} = g_\ast \pi^2 (T_c^4 - T_n^4)/30$, it follows that

$$(T_c - T_r)/(T_c - T_n) \simeq 1 - L/\Delta \rho_{c,n},$$  

and the universe reheats back to $T_r$ if the ratio $L/\Delta \rho_{c,n}$ ever exceeds unity. However we find that $L/\Delta \rho_{c,n}$ has an average value of 0.29 and never falls outside the range [0.17, 0.42], so reheating to $T_c$ does not ever occur.

We have also investigated the value of $\tan \beta = v_2/v_1 = \langle H_2 \rangle/\langle H_1 \rangle$ inside the bubble wall. This quantity is of interest because most (but not all) [18] electroweak baryogenesis mechanisms in the MSSM predict the baryon asymmetry is proportional to an average of $v_2 \delta v_1 - v_1 \delta v_2$ over the the bubble wall, where $v$ goes from 0 to the broken phase value $v_c$. We characterize the variation of $\tan \beta$ with $v$ by minimizing the effective potential $\langle H_1 \rangle^2 + \langle H_2 \rangle^2)^{1/2}$, and computing $\Delta \beta \equiv \max_v [v(\beta(v) - \beta(v_c))]/v_c$. This definition of $\Delta \beta$, which differs from ref. 13's, corresponds more closely to the quantity which enters into computations of the baryon asymmetry. Like ref. 13, we find that this quantity never exceeds 0.02, and is typically 10 times smaller. It is also never large if the $A^0$ boson is heavy.

![Figure 6: (a) Maximum deviation in weighted Higgs VEV orientation, $\Delta \beta \equiv \max_v [v(\beta(v) - \beta(v_c))]/v_c$, inside bubble wall, as a function of $m_{A^0}$; (b) distribution of $\Delta \beta$ values.](image)

The most interesting issue confronting electroweak baryogenesis in the MSSM is whether the LEP 200 run will be able to rule it out. The answer seems to be “almost, but not completely.” The final center of mass energy $\sqrt{s} = 200$ GeV will exclude $m_h$ up to 107 GeV [24]. However if $m_Q = 2$ TeV, we find parameters with $m_h$ as high as 116 GeV yet consistent with electroweak baryogenesis. Such a Higgs boson might be discovered in Run II at Tevatron [27].

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