Resonant and anti-resonant modes of the dilute, spin-inbalanced, two-dimensional electron liquid including correlations

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A spin-sensitive linear response theory is presented that includes correlations beyond the well-known random phase approximation. Especially for very dilute systems, such correlations play an important role. The response functions obtained give insight into both charge and longitudinal magnetic excitations. In addition to the spin-plasmon, we propose a new regime where no magnetic excitation is possible, namely the magnetic anti-resonance. Both effects lie in experimentally accessible ranges.

KEYWORDS
collective modes, magnetic anti-resonance, plasmon, spin excitations, two-dimensional electron liquid

1 INTRODUCTION

Collective excitations have been known for over eight decades, starting with the detection of a distinctive drop in the alkali metals’ reflectivity,[1] which was followed 15 years later by plasmon excitations from electron scattering[2,3] and explained in the early 1950s with the random-phase approximation[4,5] (RPA). While highly successful for dense systems, more advanced models than the RPA are needed for strongly correlated electron liquids.[6,7] The breakthrough in semiconductor industry triggered a massive interest in two-dimensional electron liquids (2DELs).[8] At the beginning of this century, most accurate measurements via (high-resolution) electron energy loss spectroscopy, (HR)EELS, were performed for the dilute and intermediate-density paramagnetic 2DEL by Hirjibehedin et al.[9,10] on GaAs/AlGaAs quantum wells and by Nagao et al.[11,12] on metal surface “electron sheets”.

In 2014, Agarwal et al.[13] found a new mode in the spin-polarized 2DEL, called the the spin-plasmon. For sufficient imbalance in the spin population, as realized in typical spintronic 2DELs, this collective mode was proposed be long-lived due to only weak Landau damping. However, accounting for correlation effects lowers the spin-plasmon dispersion, implying that it enters the strong damping continuum at much lower wave vectors.[14] Similar situations are known for the acoustic modes in other binary Coulomb systems, such as ionic mixtures,[15] electron bilayers,[16] semiconductor double wells,[17] or the interface 2DEL of perovskites coupled to graphene[18] (the list being far from complete; note also recent work on static many-body correlations in graphene and the linear mode in the graphene-related MoS$_2$[19]). The transverse counterpart of the spin-plasmon, i.e. collective spin-flip excitations, was observed for long wavelengths in semi-magnetic Cd$_{1-x}$Mn$_x$Te quantum wells by Jusserand et al.[20,21]

Here, we show that in partially spin-polarized systems another important mode is relevant, where all magnetic response is suppressed. In Section 2, we present our strategy to obtain high-quality effective interactions for the spin-resolved linear response
functions. Formally equivalent to that for a paramagnetic two-valley system, our method can be straightforwardly extended to the four-component spin-and-valley-polarized case realized in Si-SiO$_2$ MOSFETS.$^{[22,23]}$ Our results are critically discussed in Section 3, followed by the conclusions in Section 4.

The areal density $n$ is measured in mean particle distances $a_B^* r_S = (\pi n)^{-1/2}$ ($a_B^*$ is the sample’s effective Bohr radius). The relative difference between majority and minority spin density defines the degree of spin polarization, $\zeta \equiv (n_\uparrow - n_\downarrow)/n$. These $n_e$ determine the respective Fermi energies $E_F$ and Fermi wave vectors $k_F = \sqrt{\pi n_e}$ (assuming a parabolic energy dispersion with effective mass $m^*$). Lengths and energies in this work are measured in these units. To get an idea of realistic orders of magnitude, we translate our results using the material parameters of the quantum well measured by HREELS,$^{[9]}$ with a background permittivity of $\epsilon = 13$, $m^* = 0.067 m_e$, and densities in the range of $n \approx (8 \times 10^{16} - 2 \times 10^{17})$ cm$^{-2}$ ($r_S \approx 10–20$).

### 1.1 Spin-sensitive response functions

The double differential cross-section, accessible via EELS or X-ray spectroscopy, is connected to the dynamic structure factor $S(q, \omega)$, and, in turn, to the imaginary part of the density–density response function $\chi_{nn}(q, \omega)$

$$\frac{d^2 \sigma}{d\Omega \ d\hbar \omega} \propto \frac{\pi}{\hbar n} S(q, \omega) = -\text{Im} \chi_{nn}(q, \omega).$$

(1)

Here, $\hbar q$ and $\hbar \omega$ denote the magnitude of the momentum transferred to and the energy deposited in the 2DEL by the scattered particle or photon. As was shown by Perez,$^{[24]}$ for a spin-polarized system and longitudinal excitations, the additional spin channels have to be included via

$$\frac{d^2 \sigma}{d\Omega \ d\hbar \omega} \propto \text{Im} \chi_{mn}(q, \omega) + 2 \beta \zeta \text{Im} \chi_{ns}(q, \omega) + \beta^2 \zeta^2 \chi_{ss}(q, \omega),$$

(2)

where the coefficient $\beta$ is related to the product of two optical matrix elements (one for each photon field) appropriate to the type of experiment (parallel or crossed polarization). With these, spin-conserving (longitudinal) or spin-flip (transverse) responses can be probed. The three response functions in Equation (2) directly connect the particle (or charge) density fluctuations $\delta \rho = \delta \rho_\uparrow + \delta \rho_\downarrow$ and those of the longitudinal spin-density fluctuations $\delta s = \delta \rho_\uparrow - \delta \rho_\downarrow$ with external electric and magnetic perturbations (($\phi^{\text{ext}}$) and ($b^{\text{ext}}$), respectively):

$$\left(\begin{array}{c} \delta \rho \\ \delta s \end{array}\right) = \left(\begin{array}{cc} \chi_{nn} & \chi_{ns} \\ \chi_{ns} & \chi_{ss} \end{array}\right) \cdot \left(\begin{array}{c} \phi^{\text{ext}} \\ b^{\text{ext}} \end{array}\right).$$

(3)

These linear response functions are related to their partial counterparts $\chi_{\sigma \sigma'}$ according to

$$\chi_{nn} = \chi_{\uparrow \uparrow} + 2 \chi_{\uparrow \downarrow} + \chi_{\downarrow \downarrow},$$

(4a)

$$\chi_{ss} = \chi_{\uparrow \uparrow} - 2 \chi_{\uparrow \downarrow} + \chi_{\downarrow \downarrow},$$

(4b)

$$\chi_{ns} = \chi_{\uparrow \downarrow} - \chi_{\downarrow \uparrow}.$$

(4c)

In the paramagnetic case $\chi_{\uparrow \uparrow} = \chi_{\downarrow \downarrow}$, implying $\chi_{ns} = 0$. For non-vanishing $\zeta$, however, density fluctuations are not only connected to electrostatic perturbations through the density–density response function $\chi_{nn}$ but also to an applied magnetic field through $\chi_{ns}$. Similarly, magnetization fluctuations are described by the spin–spin response function $\chi_{ss}$ alone only for a purely magnetic perturbation; in general, they are also linked to $\phi^{\text{ext}}$. Note that for non-interacting fermions $\chi_{nn}^0 = \chi_{ss}^0 = \chi_{ss}^0$, and $\chi_{ns}^0 = \chi_{\uparrow \downarrow} - \chi_{\downarrow \uparrow} \neq 0$. Cross excitations, i.e. “magnetic fluctuation $\leftrightarrow$ electric perturbation” and “charge fluctuation $\leftrightarrow$ magnetic perturbation,” as mediated by the density–spin response function are thus deeply rooted in a spin imbalance.

The partial response functions $\chi_{\sigma \sigma'}$ couple each spin-dependent density fluctuation with external spin-sensitive perturbations according to

$$\left(\begin{array}{c} \delta \rho_\uparrow \\ \delta \rho_\downarrow \end{array}\right) = \left(\begin{array}{cc} \chi_{\uparrow \uparrow} & \chi_{\uparrow \downarrow} \\ \chi_{\downarrow \uparrow} & \chi_{\downarrow \downarrow} \end{array}\right) \cdot \left(\begin{array}{c} V^{\text{ext}}_\uparrow \\ V^{\text{ext}}_\downarrow \end{array}\right).$$

(5)

Formally adopting a mean-field perspective leads to the definition of effective interactions$^{[6]}$ $V_{\sigma \sigma'}(q, \omega)$. In general, these potentials are dynamic. In order to display the difference to the bare Coulomb interaction $v(q)$ (corresponding to the mean field seen by a test charge), $V_{\sigma \sigma'}$ is commonly expressed via the frequency-dependent local field correction (LFC) $G_{\sigma \sigma'}$

$$V_{\sigma \sigma'}(q, \omega) = v(q)(1 - G_{\sigma \sigma'}(q, \omega)).$$

(6)

$^1$To ease notation, we use $n_e$ for the constant densities and $\delta \rho_\sigma$ for the $q$-dependent functions.
where the bare RPA is recovered with $G_{\sigma\sigma'} = 0$. The solutions for the response functions are then found as

$$
\chi^{-1}(q, \omega) = \chi^{0-1}(q, \omega) - V(q, \omega) \iff \begin{pmatrix} X_{\uparrow\uparrow} & X_{\uparrow\downarrow} \\ X_{\downarrow\uparrow} & X_{\downarrow\downarrow} \end{pmatrix}^{-1} - \begin{pmatrix} X^{0}_{\uparrow\uparrow} & 0 \\ 0 & X^{0}_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} V_{\uparrow\uparrow} & V_{\uparrow\downarrow} \\ V_{\downarrow\uparrow} & V_{\downarrow\downarrow} \end{pmatrix},
$$

(7)

Spelling out the solutions of Equation (7) explicitly, the partial response functions take the form

$$
\chi_{\uparrow\uparrow}(q, \omega) = \frac{1}{\Delta(q, \omega)} X^{0}_{\uparrow\uparrow}(q, \omega)(1 - V_{\uparrow\downarrow}(q, \omega)X^{0}_{\downarrow\uparrow}(q, \omega)) ,
$$

(8a)

$$
\chi_{\downarrow\downarrow}(q, \omega) = \frac{1}{\Delta(q, \omega)} X^{0}_{\downarrow\downarrow}(q, \omega)V_{\downarrow\uparrow}(q, \omega)X^{0}_{\uparrow\downarrow}(q, \omega) ,
$$

(8b)

and an analogous form for $\chi_{\uparrow\downarrow}$, the common denominator

$$
\Delta = 1 - V_{\uparrow\uparrow}X^{0}_{\uparrow\uparrow} - V_{\uparrow\downarrow}X^{0}_{\downarrow\uparrow} + (V_{\uparrow\uparrow}V_{\uparrow\downarrow} - V^{2}_{\downarrow\uparrow})X^{0}_{\uparrow\uparrow}X^{0}_{\downarrow\downarrow} .
$$

(8c)

Note that $V_{\uparrow\downarrow}$ is the crucial term coupling the two components of the Fermi liquid; a vanishing $V_{\uparrow\downarrow}$ would imply uncoupled modes $(1 - X^{0}_{\uparrow\uparrow} V_{\uparrow\uparrow})(1 - X^{0}_{\downarrow\downarrow} V_{\downarrow\uparrow}) = 0$ (cf. the similarity of the spin-polarized 2DEL with two coupled oscillators discussed at the end of this paper). Even if all $G_{\sigma\sigma'}$ are set zero, the Coulomb interaction between $\uparrow$ and $\downarrow$ electrons ensures such a coupling, resulting in highly non-trivial effects.

Combining Equations (4) and (8) gives the conventional response functions as

$$
\chi_{nn} = \frac{1}{\Delta}(\chi^{0} - [V_{\uparrow\uparrow} + V_{\uparrow\downarrow} \mp 2V_{\uparrow\downarrow}]X^{0}_{\uparrow\uparrow}X^{0}_{\uparrow\downarrow}) , \quad \chi_{ns} = \frac{1}{\Delta}(\chi^{0}_{\uparrow\downarrow} - \chi^{0}_{\downarrow\uparrow} + [V_{\uparrow\uparrow} - V_{\uparrow\downarrow}]X^{0}_{\uparrow\uparrow}X^{0}_{\downarrow\downarrow}) .
$$

(9)

They all share the same denominator $\Delta$, so that the same collective excitations are present in each of these functions, of course, with different strengths. We next discuss how the effective interactions can be calculated from the most accurate available data.

## 2 | SPIN-SENSITIVE GENERALIZED RANDOM-PHASE APPROXIMATION

Only few spin-resolved dynamic LFCs have been studied so far (e.g. of the so-called ‘STLS’ type). Ref. [25] gives the formalism including transverse excitations for a bilayer). Computationally much less consuming are theories using static interactions $V_{\sigma\sigma'}(q)$ and $G_{\sigma\sigma'}(q)$, respectively. Such static approaches have many formal similarities with the bare RPA; in quantum fluids where interactions have no Fourier transform (as in $^3$He), the use of a static effective interaction in $\chi_{mn}$ is known as simply ‘RPA’. To avoid confusion, we term these approaches “generalized RPA” (GRPA). Parametrized expressions for static LFCs of the partially spin-polarized 2DEL[26] and spin-resolved static STLS calculations[28] have been reported. In the unpolarized system, the so-called hypernetted-chain methods, originally developed for classical[29] but also well established for quantum fluids,[30] are the basis for rather reliable spin-symmetric and spin-anti-symmetric LFCs obtained via mapping to the classical[31] or the bosonic[32] case. We here suggest an alternative route, based on using the ground-state structure data as an input.

### 2.1 | Collective approximation

Our approach allows us to find an analytic expression for the interactions $V_{\sigma\sigma'}(q)$ from the spin-resolved static structure factors available from simulations[33,34] in high quality. In the paramagnetic 2DEL, the plasmon dispersion resulting from the thus-constructed LFCs is remarkably close[35] to that obtained with the Monte Carlo-based[36] $G(q, \omega = 0)$. The response of a non-interacting 2DEL is given by the Lindhard[37] (or Stern[38]) function ($V$ denotes the area)

$$
\chi^{0}(q, \omega) = \frac{1}{V} \sum_{k} \frac{n^{0}_{ek} - n^{0}_{ek+q}}{\hbar \omega + i\eta - \epsilon_{k+q} + \epsilon_{k}} , \quad \eta \rightarrow 0^{+} ,
$$

(10)

with the single-particle energies $\epsilon_{k}$ and Fermi distribution $n^{0}_{ek}$. This is recognized as a normalized average over the phase space restricted by the Pauli principle, defined for any operator $\hat{A}$ as

$$
\langle \hat{A} \rangle^{0}_{q,\sigma} \equiv \frac{1}{N_{\sigma}S^{0}_{\sigma}(q)} \sum_{k} n_{ek}(1 - n_{ek+q})\hat{A}_{k,q,\sigma} ,
$$

(11)

($S^{0}_{\sigma}$ being the free static structure factor,[61]), so that

$$
\chi^{0}(q, \omega) = n_{\sigma}S^{0}_{\sigma}(q) \left( \frac{1}{\hbar \omega + i\eta - \epsilon_{k+q} + \epsilon_{k}} \right) + \left( \frac{1}{-(\hbar \omega + i\eta) - \epsilon_{k+q} + \epsilon_{k}} \right) .
$$

(12)
For small densities, the relative importance of correlations compared to Fermi statistic effects motivates replacing $\chi^0_\sigma$ by its bosonic counterpart $\chi^0_\sigma^{\text{CA}}$. In this ‘collective approximation’ (CA) the particle-hole band is squeezed into a single mode along its Fermi sea average, which amounts to replacing Equation (12) with

$$\chi^0_\sigma^{\text{CA}}(q, \omega) = n_\sigma S^0(q) \left( \frac{1}{\hbar \omega + i \eta} - \frac{1}{\hbar \omega + i \eta - \langle \epsilon \rangle^0_{\sigma,q}} \right),$$

(13a)

The poles of this response function, located at the non-interacting energy averages, define unperturbed eigenfrequency $\omega_{\sigma\sigma}$:

$$\langle \epsilon \rangle^0_{\sigma,q} \equiv \frac{\hbar^2 q^2}{2m^* S^0(q)} \equiv \hbar \omega_{\sigma\sigma}(q).$$

(13b)

In analogy to the paramagnetic plasma frequency $\omega^2_p = n q^2 v(q)/m^*$, we also introduce

$$\omega^2_{\sigma\sigma} = \sqrt{n_\sigma n'_{\sigma'}} V_{\sigma\sigma'} = \frac{\sqrt{n_\sigma n'_{\sigma'}}}{n} \omega^2_p (1 - G_{\sigma\sigma'}).$$

(14)

Substituting $\chi^0_\sigma$ in Equation (8) with $\chi^0_\sigma^{\text{CA}}$ then leads to approximate ‘collective’ response functions, which read

$$\chi^{\text{CA}}_{\uparrow\downarrow}(q, \omega) = \frac{n_\sigma q^2}{m^*} \frac{1}{\Delta^{\text{CA}}(q, \omega)} ((\omega + i \eta)^2 - \omega^2_{\sigma\sigma}(q) - \omega^2_{\uparrow\downarrow}(q)),$$

(15a)

$$\chi^{\text{CA}}_{\downarrow\uparrow}(q, \omega) = \frac{n_\sigma q^2}{m^*} \frac{1}{\Delta^{\text{CA}}(q, \omega)} \omega^2_{\downarrow\uparrow}(q),$$

(15b)

with

$$\Delta^{\text{CA}}(q, \omega) = ((\omega + i \eta)^2 - \omega^2_{\sigma\sigma}(q) - \omega^2_{\sigma\sigma}(q))((\omega + i \eta)^2 - \omega^2_{\uparrow\downarrow}(q) - \omega^2_{\uparrow\downarrow}(q)) - \omega^2_{\uparrow\downarrow}(q).$$

(15c)

In the paramagnetic bare RPA case, all $\omega^2_{\sigma\sigma} = \omega^2_p/2$, resulting in

$$\chi^{\text{CA}}_{\sigma\sigma} = \frac{nq^2}{m^*} \frac{1}{(\omega + i \eta)^2 - \omega^2_{\sigma\sigma}(q) - \omega^2_{\sigma\sigma}(q)} \quad (\zeta = 0),$$

(16)

which is historically known as the plasmon pole approximation (PPA)\textsuperscript{[39]} for the dielectric function.

The static effective potentials $V_{\sigma\sigma'}(q)$ required for the not-collectively approximated response functions in Equation (8) are now determined by the use of the $\omega^0$ sum rule, which represents a bridge from dynamic to static properties:

$$- \int_0^\infty d\omega \Im \chi^{\text{CA}}_{\sigma\sigma'}(q, \omega) = \sqrt{n_\sigma n'_{\sigma'}} S_{\sigma\sigma'}(q).$$

(17)

The spin-resolved static structure factors

$$S_{\sigma\sigma'}(q) \equiv \langle \delta \hat{\rho}_{\sigma q} \delta \hat{\rho}_{-\sigma q'} / \sqrt{N_\sigma N_{\sigma'}} \rangle$$

are the Fourier transforms of the corresponding pair distribution functions $g_{\sigma\sigma'}(r)$. For these, most accurate data were calculated by Gori-Giorgi et al.\textsuperscript{[33]} with quantum Monte Carlo (QMC) simulations. In order to take advantage of these results as an input to our dynamical response functions, we had to perform an appropriate analytic extrapolation of the QMC data to large distances $r$, thereby fixing the long wavelength limit of the full $S(q \to 0)$. We then carried out an analytic fit of these data in close analogy to that of Ref. [33].

We could then use the thus-obtained $S_{\sigma\sigma'}(q)$ in the spin-resolved fluctuation-dissipation theorems (17). Within the CA regime, these equations can be solved analytically, leading to a result for the effective interactions $V_{\sigma\sigma'}(q)$ in the form of a matrix equation

$$V(q) = \frac{\hbar^2 q^2}{4m} (S^{-2}(q) - S^{0-2}(q)),$$

(18a)

written explicitly as

$$V_{\uparrow\downarrow}(q) = - \frac{\hbar^2 q^2}{4m} S_{\uparrow\downarrow}(q) S_{\uparrow\downarrow}(q) + S_{\downarrow\uparrow}(q)] D^2(q), \quad V_{\downarrow\uparrow}(q) = \frac{\hbar^2 q^2}{4m} \left[ S_{\uparrow\downarrow}^2(q) + S_{\downarrow\uparrow}^2(q) - \frac{1}{S_{\uparrow\downarrow}^0(q)} \right],$$

(18b)

with the denominator $D(q) = S_{\uparrow\downarrow}(q) S_{\downarrow\uparrow}(q) - S_{\uparrow\downarrow}^0(q)$. For the paramagnetic case, similar results can be found in Ref. [40].

### 2.2 Improved effective interactions

Using these CA potentials in the GRPA response functions of Equation (8) together with the full $\chi^0_\sigma$ of Ref. [28], the $\omega^0$ sum rule (17) is not exactly fulfilled. Numerical post-processing gives our final $V_{\sigma\sigma'}(q)$, which obey (17) with the QMC results (18).
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FIGURE 1 Real and imaginary parts of all three response functions for $r_S = 20$ and $\zeta = 0.48$. The dashed-dotted line is the bare RPA plasmon, the GRPA plasmon was made visible by adding a small finite imaginary part in the denominator of the $\chi_{\sigma\sigma}'$. The zeroes of $\text{Im}\chi_{\sigma\sigma}$ and $\text{Im}\chi_{\sigma\sigma}$ are highlighted as purple dots. The particle-hole bands are shown with grey solid lines; white arrows denote the majority and minority spins in these continua. Energies and wave vectors are given in reduced as well as in real units for a typical GaAs/AlGaAs structure.

The refined interaction potentials consistent with the QMC static structure factors are now used to calculate the density and density–spin GRPA response functions from Equations (8) and (9).

3 | RESULTS

In Figure 1, the results are shown for a very dilute 2DEL of $r_S = 20$, experimentally investigated in a GaAs quantum well.$^{[9,10]}$ The plasmon is distinctly visible in all response functions (as a maximum in the negative imaginary part, top row, and as a zero in the real part, bottom row). Clearly, the GRPA plasmon lies at significantly lower energies than that of the bare RPA.

3.1 | Spin-plasmon

For long wavelengths, the predominant strength of $\text{Im}\chi_{\sigma\sigma}$ lies in the plasmon (17), while the particle-hole band is weak (the lower left corner in the upper left figure is essentially white). This fact changes for the spin–spin response $\text{Im}\chi_{ss}$, where a distinct intensity is visible in the continuum (the lower left corner in the upper right figure is dark red). This strength corresponds to the second collective excitation, namely the ‘spin-plasmon’. While the charge-plasmon is an in-phase oscillation of the spins, they have opposite phase in the spin-plasmon. Another footprint of this mode is seen in $\text{Re}\chi_{ss}$, which changes its sign in this region by contrast ($\text{Re}\chi_{nn}$ shows a sign switching only at the plasmon).

Compared to the GRPA, the bare RPA spin-plasmon, again, is found at higher energies, in the (very weak) particle-hole band of the majority spins (but above the band edge of the $\downarrow$ spins). This marginal Landau damping corresponds to a region in the inverse dielectric function identified as a ‘pseudo-gap’ by Agarwal et al.$^{[13]}$ However, correlations pull down the spin-plasmon into the strong Landau damping regime of the minority spins in the GRPA, as we demonstrate in Figure 2. It displays the small
FIGURE 2  left: $(-\text{Im} \chi_{ss})$ for $r_s = 2$ and $\zeta = 0.48$ in the region of the spin-plasmon. Cuts at specific wave vectors (marked by vertical lines) are shown (in the same line styles) on the side. The grey lines are the upper band edges. Right: Density dependence of the spin-plasmon’s critical wave vector in RPA and GRPA. The shaded area estimates finite size effects in the QMC fit.

$(q, \omega)$ corner of $(-\text{Im} \chi_{ss})$. The mode stays very close to the minority band edge and rapidly becomes Landau-damped at a critical wave vector $q_{cs}$. At $q = 0.06k_F$, a resonant peak can no longer be found (orange dotted line in Figure 2). In the right part of the figure, the critical $q_{cs}$ of the RPA and the GRPA are compared as a function of areal density $n^{-1/2}$. Clearly, the GRPA predicts much smaller results. This also holds for the spin polarization $\zeta = 0.8$ (for other $\zeta$ no QMC data are available). An experimental verification of the spin-plasmon in GaAs quantum wells, therefore, seems unlikely.

3.2  Magnetic anti-resonance (mAR)

As discussed following Equation (3), an external magnetic field $b_{ext}$ causes density fluctuations $\delta \rho$ and spin-density fluctuations $\delta \rho_s^z$ via the response functions $\chi_{rs}$ and $\chi_{ss}$, respectively. Thus, if both these functions vanish, no (longitudinal) excitations are possible by magnetic perturbations. Such an enthralling situation is found in the imaginary parts inside the band of majority spins (see middle and right panel of Figure 1, marked by purple dots). According to Equation (2), no absorption is possible along these lines, which we therefore call the magnetic anti-resonance $\omega_{mAR}(q)$. A similar anti-resonant mode was found in strongly coupled binary Coulomb systems.\[41\]

The mAR always lies between both collective modes, the charge- and the spin-plasmon, inside the upper region of majority spin particle-hole excitations, $\omega_1^+ (q) > \omega_{mAR}(q) > \omega_2^+ (q)$. Obviously, the inter-spin correlations due to $V_{11}$ suppress the single-particle modes. The dispersion for this mode can be calculated analytically. Relevant is the total effective interaction acting on the minority spins, reading in reduced units ($\overline{V}_{\sigma \sigma'} \equiv V_{\sigma \sigma'}/\epsilon_F$)

$$\overline{V}_{1+} \equiv \frac{n}{\epsilon_F} (V_{11} + V_{1\uparrow}).$$

Interestingly, the inter-majority potential $V_{11}$ does not occur. The dispersion is then

$$\frac{\hbar \omega_{mAR}(q)}{\epsilon_F} = \frac{q}{k_F} \sqrt{1 + (q) \left( 1 + \frac{2}{\overline{V}_{1+}(q)} \right) \left( \frac{\overline{V}_{1+}(q)}{1 + \overline{V}_{1+}(q)} + \frac{q^2 / k_F^2}{\overline{V}_{1+}(q)} \right)^{1/2}}.$$  \hspace{1cm} (20)

This leads to the critical wave vectors, where the mAR hits the two single-particle band borders, $\omega_1^+ (q)$ and $\omega_2^+ (q)$, as

$$q_{\text{mAR}}^c = \frac{\overline{V}_{1+}^2}{8(1 + \overline{V}_{1+})} \quad \text{and} \quad q_{\text{mAR}}^\uparrow = \frac{\overline{V}_{1+}^2 + \sqrt{\overline{V}_{1+}^2 + 2\overline{V}_{1+}}}{2(1 + \overline{V}_{1+})}.$$  \hspace{1cm} (21)

At the mAR, the full response functions become real, $\chi_{ss}^{(+)} (q, \omega_{mAR}(q)) = \pm 1/V_{11}(q)$. A physical understanding of what actually happens at such an anti-resonance is best seen from the analogy with coupled mechanical oscillators.
3.3 Two different coupled and driven harmonic oscillators

A deeper insight into the causes of correlations in two-component many-body systems can be gained by studying the driven coupled harmonic oscillator (DC-HO).\(^2\)\(^3\) In this model, the simple undergraduate physics example of an HO with mass \(m\) driven by an external force \(F_{\text{ext}} \equiv mf_{\text{ext}}\) is expanded to two such oscillators coupled harmonically. Newton’s equations for their displacement \(x_i(t)\) from equilibrium read

\[
\begin{align*}
\dot{x}_1 + 2\gamma_1 \dot{x}_1 + \omega_{1,0}^2 x_1 - \frac{m_1}{M} \Omega_{12}^2 x_2 & = f_1^{\text{ext}}(t) \\
\dot{x}_2 + 2\gamma_2 \dot{x}_2 + \omega_{2,0}^2 x_2 - \frac{m_2}{M} \Omega_{12}^2 x_1 & = f_2^{\text{ext}}(t) \quad ; \quad M \equiv m_1 + m_2 .
\end{align*}
\] (22)

We here demonstrate that a phenomenon analogous to the mAR can also be found for this paradigmatic example. Fourier transforming Equation (22) and abbreviating \(g_i \equiv \Omega_{12}^2 \cdot m_i/M\) gives

\[
\begin{align*}
\left( \omega_{1,0}^2 - \omega_i(\omega + 2i\gamma_1) \right) - g_1 & \quad \omega_{2,0}^2 - \omega_i(\omega + 2i\gamma_2) \quad \left( \frac{\tilde{x}_3}{\tilde{x}_2} \right) = \left( \frac{f_1^{\text{ext}}}{f_2^{\text{ext}}} \right) \quad \Leftrightarrow \quad \chi_{\text{HO}}^{-1}(\omega) \cdot \tilde{x}(\omega) = \tilde{f}^{\text{ext}}(\omega) ,
\end{align*}
\] (23)

where the tilde denotes the Fourier-transformed quantities. This results in the linear response matrix of the DC-HO and its determinant as

\[
\chi_{\text{HO}}(\omega) = \frac{1}{\Delta_{\text{HO}}(\omega)} \left( \begin{array}{cc}
\omega_{1,0}^2 - \omega_i(\omega + 2i\gamma_2) & g_2 \\
g_1 & \omega_{2,0}^2 - \omega_i(\omega + 2i\gamma_1)
\end{array} \right) .
\] (24a)

\[
\Delta_{\text{HO}}(\omega) = (\omega_i(\omega + 2i\gamma_1) - \omega_{1,0}^2)(\omega_i(\omega + 2i\gamma_2) - \omega_{2,0}^2) - g_1g_2 .
\] (24b)

The complex roots (6) of the determinant give the poles of the response functions; for zero damping, \(\Delta_{\text{HO}}\) leads to

\[
\omega_{\pm}^2 = \frac{1}{2} \left( \omega_{1,0}^2 + \omega_{2,0}^2 \right) \pm \frac{1}{2} \sqrt{\left( \omega_{1,0}^2 - \omega_{2,0}^2 \right)^2 + 4 \frac{m_1m_2}{M^2} \Omega_{12}^4} .
\] (25)

The similarity of Equations (23) and (24) with Equations (7) and (8) is striking, the mapping being given by comparing \(\chi_{\text{HO}}^{-1}\) with

\[
\chi^{-1} = \left( \begin{array}{cc}
\chi_{1}^{-1} & -V_{11} \\
-V_{11} & (\chi_{1}^{-1})^* - V_{11}
\end{array} \right) .
\]

In the collective approximation, the analogy gets even closer. Introducing \(\omega_{\sigma,0}^2 = \omega_{\sigma,\pm}^2 + \omega_{\sigma,\sigma}^2\) from the frequencies (13b) and (14), the CA denominator, Equation (15c), reduces for small damping to

\[
\Delta^{\text{CA}}(q, \omega) = (\omega_i(\omega + 2i\eta) - \omega_{1,0}^2(q))(\omega_i(\omega + 2i\eta) - \omega_{2,0}^2(q)) - \omega_{\sigma,\sigma}^2(q) .
\] (26)

with the complex roots

\[
\omega_{\pm}^2 = \frac{1}{2} \left( \omega_{1,0}^2 + \omega_{2,0}^2 \right) \pm \frac{1}{2} \sqrt{\left( \omega_{1,0}^2 - \omega_{2,0}^2 \right)^2 + 4 \frac{m_1n_1}{n^2} \omega_{\sigma,\sigma}(1 - G_{\sigma,\sigma})} .
\] (27)

The Coulomb interaction couples the \(\uparrow\) and \(\downarrow\) spins, and the spin imbalance is responsible for the weight. Formally identical with Equations (24b) and (25), this opens the door for a physical interpretation of the mAR.

In order to understand the anti-resonance, we take a closer look at the singly driven (i.e. \(f_2^{\text{ext}}(t) = 0\)) coupled HO. The mass \(m_1\) is subject to a harmonic external force, and \(x(t)\) is obtained as the convolution integral from \(\chi_{\text{HO}}(\omega) \cdot \tilde{F}(\omega)\). The result is seen in Figure 3. The different amplitudes \(x_1, x_2\) are drawn for three characteristic excitation frequencies at two different times. The absolute values of both displacements, as well as their phase shift, are depicted on the right. The mass \(m_1 \equiv m_1\) shows an anti-resonant mode \(\omega_{\text{AR}}\), which lies between the resonances given by Krotov.\(^{27}\) At \(\omega_{\text{AR}}\), despite the external force \(f_1^{\text{ext}}\) acting exactly on this mass, \(m_1\) stays motionless. By contrast, \(m_2\) oscillates, solely driven via the coupling \(\Omega_{12}\). This is equivalent to the mAR in the spin-polarized 2DEL, where the external magnetic field does not lead to oscillations in \(\delta\).

In this simple model, it is evident that \(x_1(t) = 0\) for all \(t\) together with \(f(t) = f_1\) occurs when \(\omega = \omega_{\text{AR}}\) (cf. Equation (24) for \(\gamma = 0\)). The input force perturbing mass \(m_1\) communicated to \(m_2\) via the coupling is fully absorbed by \(m_2\) at the latter’s eigenfrequency. Analogously, when the magnetic perturbation hits the frequency corresponding to the collective mode of an isolated fully polarized system with the density of the minority spins, it cannot excite the single-particle excitations of the majority subsystem, explaining the gap in the spin-\(\uparrow\) band.

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\(^2\)The model of coupled harmonic oscillators was also used to understand Fano-resonances by Joe et al.\(^{42}\).

\(^3\)For an actual response function, in the spirit of solid state textbooks, one may introduce the polarization \(P = e m_1 x_1 + e m_1 x_1\), \(e\) being the electron charge, together with the external force \(f_\sigma = e E/m_\sigma\), where \(E\) is the applied electric field.
FIGURE 3  Singly driven coupled harmonic oscillators. A periodic force of frequency $\omega$ acts only on the red mass marked with ↑. Left: Amplitudes and relative phase of the two masses. At the AR frequency, a change from in-phase to counter-phase motion happens. The grey shaded area denotes the frequency regime used for the snapshots. Right: Snapshots at two arbitrary times $t_1$ and $t_2$ of the positions for the frequencies indicated in the top line (below, at, and above the anti-resonance).

TABLE 1  Typical energy and momentum windows where the spin-plasmon as well as the magnetic anti-resonance can be found. The calculations are done for two experimentally accessible densities (dense and dilute) as well as for a spin-polarization of $\zeta = 0.48$. We used the parameters ($m^* = 0.067 m$ and $\epsilon = 13$) (19)

| Density      | SP $\omega \in (0, 33)$ mRy $q \in (0, 3) \times 10^4$ cm$^{-1}$ | mAR $\omega \in (847, 2100)$ mRy $q \in (4, 10) \times 10^5$ cm$^{-1}$ |
|--------------|---------------------------------------------------------------|---------------------------------------------------------------|
| Dense system | $\tau_S = 2, n = 7.55 \times 10^{10}$ cm$^{-2}$               |                                                               |
| Dilute system| $\tau_S = 20, n = 7.55 \times 10^8$ cm$^{-2}$                 |                                                               |

4  | CONCLUSIONS

We studied the partially spin-polarized 2DEL taking into account spin-dependent local field corrections, which are based on QMC static structure data. Two collective modes, the spin-plasmon and the magnetic anti-resonance, were studied in detail. Both lie in the particle-hole band of the spin-up (= majority) electrons. The proposed anti-resonant mode is between the charge- and the spin-plasmon and exhibits similarities with two coupled oscillators, being thus an analogue of the Fano-like anti-resonance. We finish this work by providing a quantitative estimate where this interesting structure could be found in experiments. For a typical GaAs/AlGaAs heterostructure ($m^* = 0.067 m_c$, $\epsilon = 0.013$), we predict that these spin modes occur in the range of energy and momentum transfer given in Table 1.

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APPENDIX: GRPA IN COLLECTIVE APPROXIMATION

In the CA, all non-interacting particle-hole excitations possible at wave vector $q$ are squeezed into the single collective mode $\hbar \omega_S(q)$, offering the advantage of a bosonic formalism, the phase space reduction due to Pauli exclusion being implicit in $S^{\sigma}_0(q)$. Using this $S^{\sigma}_{\mathrm{CA}}$ in the GRPA with given interactions $V_{\sigma\sigma'}$ then yields static structure factors $S^{\sigma}_{\sigma\sigma'}$ according to Equation (18). In the $\zeta = 0$ 2DEL, $S(q)$ obtained in the full GRPA is rather close to this CA counterpart. We therefore turn the argument around and use Equation (18) for given structure factors to obtain good starting functions for the effective spin-dependent interactions. These are subsequently fine-tuned using a multi-dimensional Newton–Raphson algorithm, in order to ensure that the $\omega^0$ sum rule (17) is fulfilled (and the QMC results are reproduced within numerical accuracy). All results presented in this work were calculated with these refined interactions.

It is interesting to investigate quantitatively the relevance of these improvements on the dispersions. Figure A1 compares the plasmon and the magnetic anti-resonance with those computed using the analytic approximation, Equation (18). Overally, they
FIGURE A1  Plasmon (Pl) and magnetic antiresonance (mAR) dispersion using the analytic potentials (18) (blue solid lines) and the $\omega^0$ corrected ones (red dots) for $\zeta = 0.48$. The left and right parts show a dense ($r_S = 2$) and a dilute ($r_S = 20$) 2DEG, respectively. (The black dashed line is the RPA plasmon, and the thin grey solid lines give the particle-hole bands with arrows denoting the spins in these continua). Energies and wave vectors correspond to a typical GaAs/AlGaAs structure are very close and coincide with the RPA for $q \to 0$. As exchange effects are quenched in the CA, it overestimates the collective modes. The shift of the dispersions due to the $\omega^0$ correction remains below a few percent for the plasmon, and is smaller for the mAR energies.

One could expect the bosonic approximation to be more suitable for a dilute 2DEG, where correlations largely dominate over exchange. But for $r_S = 20$ (right part of Figure A1), the maximum deviation for the plasmon is 1.6%, which is comparable to 1.3% at $r_S = 2$ (left). Certainly, for large $r_S$ the analytic relation (18) captures nearly all of the differences to the bare RPA, where the ($\zeta = 0.48$, $r_S = 20$) mAR lies around $\varepsilon_F \approx 50$ mRy outside the shown plot range.

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