Neutral Mesons and Nonminimal CPT Violation

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Minimal CPT violation has been constrained using observations of neutral-meson oscillations. Violation of CPT symmetry arising from nonminimal operators in the Lagrange density can also occur. A general approach using scalar effective field theory is presented and used to infer the effects of nonminimal CPT violation on neutral-meson oscillations.

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I. INTRODUCTION

Both the Standard Model of Particle Physics and General Relativity have been very successful in describing much of the physics we have observed. However, it is also clear that they are in need of some modifications. A breaking of Lorentz symmetry is one such candidate that may occur in theories combining quantum principles with gravity. This has been shown to be possible, for example, in the context of strings [1]. Any modifications should be tiny, however, in low-energy regimes and therefore can be formulated using the tools of effective field theory. The Standard-Model Extension (SME) [2][3] is based on these ideas, and is a framework for searches for Lorentz violation. In addition, a realistic effective field theory that includes violations of CPT symmetry necessarily violates Lorentz symmetry [4]. For this reason, the SME also provides a general framework for studying CPT violation.

Constructed by adding terms at the level of the Lagrange density that violate Lorentz symmetry, the SME contains coefficients, which are Lorentz scalars, that control the size of the Lorentz-violating effect. These are called the coefficients for Lorentz violation and are the targets of experiments. Some of the results of these investigations are compiled in Ref. [5]. Limits of the SME including only those terms with mass dimension \( d = 3 \) or \( 4 \) make up what is referred to as the minimal SME. The rest of the framework including terms with mass dimension \( d > 4 \) is referred to as the nonminimal SME.

CPT violation arising from the minimal SME [6–9] has been constrained in experimental studies of neutral-meson oscillations [10–15]. Results are conventionally reported in a special frame called the Sun-centered frame [16], with coordinate axes \( T, X, Y, Z \), and will also be used here. The focus of what follows is to obtain the effects of violations of CPT symmetry on neutral-meson oscillations, including nonminimal terms. The following material is based on the work found in Ref. [17].

II. FORMALISM

The formalism describing neutral-meson oscillations involves a two-component state vector \( \Psi \),

\[
\Psi = \begin{pmatrix} P^0_0 \\ P^0_0 \end{pmatrix}
\]

with \( P^0 = \{ K^0, D^0, B^0, B^0_d \} \) denoting one of the 4 neutral mesons. The time evolution is governed by a \( 2 \times 2 \) effective hamiltonian denoted by \( \Lambda \) and is determined by \( i \partial_t \Psi = \Lambda \Psi \). The propagating states are the eigenstates of \( \Lambda \) written,

\[
|P_a(t)\rangle = \exp(-i\lambda_a t)|P_a\rangle,
|P_b(t)\rangle = \exp(-i\lambda_b t)|P_b\rangle,
\]

with eigenvalues \( \lambda_{a,b} = m_{a,b} - \frac{1}{2}i\gamma_{a,b} \) where \( m_{a,b} \) are the physical masses and \( \gamma_{a,b} \) are the decay rates.

The hamiltonian can be parametrized in the following phase independent way [8]

\[
\Lambda = \frac{1}{2} \Delta \lambda \begin{pmatrix} U + \xi & VW^{-1} \\ VW & U - \xi \end{pmatrix},
\]
where $\Delta \lambda \equiv \lambda_a - \lambda_b$.

The trace of $\Lambda$ is $\text{tr} \Lambda = \lambda_a + \lambda_b$ and its determinant is $\det \Lambda = \lambda_a \lambda_b$, which imposes the conditions

$$U \equiv \frac{\text{tr} \Lambda}{\Delta \lambda}, \quad V \equiv \sqrt{1 - \xi^2}$$

on the complex parameters $U$ and $V$. The independent parameters in Eq. (3) can then be taken as $W = w \exp(i\omega)$, and $\xi = \text{Re}\xi + i\text{Im}\xi$. The argument $\omega$ of $W$ is physically irrelevant and changes under phase redefinitions of the meson states. The real modulus $w$ controls $T$ violation, with $w = 1$ iff $T$ is preserved. The two real numbers $\text{Re}\xi$ and $\text{Im}\xi$ control CPT violation, and both vanish iff CPT is preserved.

The relations between $w$, $\xi$ and the components of $\Lambda$ are

$$w = \sqrt{|\Lambda_{21}/\Lambda_{12}|}, \quad \xi = \Delta \lambda/\Delta \lambda,$$

where $\Delta \Lambda = \Lambda_{11} - \Lambda_{22}$. The form of $\xi$ depends on the underlying theory and in what follows we will be interested in finding the effect of CPT violation on $\xi$, including effects arising from terms in the nonminimal SME.

### III. SCALAR EFFECTIVE FIELD THEORY

Recent developments in the nonminimal sector of the SME allow us to construct the nonminimal terms in both QED and QCD [18], however techniques have not been developed to handle all nonminimal terms. To simplify matters, we view the meson as a point-particle whose field operator is the complex scalar $\phi$ and focus on contributions to the meson propagator due to nonminimal Lorentz violation in a scalar effective field theory. This allows us to infer the form of $\xi$ while leaving for later investigations the proper match to the SME coefficients.

The general flavor $U(1)$ preserving scalar effective field theory incorporating violations of Lorentz symmetry was studied in Ref. [19]. After incorporating flavor $U(1)$ breaking terms, the contributions to the parameter for CPT violation can be calculated. Terms breaking $U(1)$, however, appear off diagonal in $\Lambda$ and do not affect $\xi$. In addition, terms which preserve both $U(1)$ and CPT contribute equally to the diagonal components of $\Lambda$, which again leave $\xi$ unaffected. This means that only one class of terms contribute to $\xi$, and these are the $U(1)$ preserving, CPT-odd coefficients contained in

$$\mathcal{L} \supset \frac{1}{2} (i\phi^\dagger \hat{k}_a)^\mu \partial_\mu \phi + \text{h.c.}.$$  

The object $(\hat{k}_a)^\mu$ can be expanded in momentum space as $(\hat{k}_a)^\mu = \sum_{d=3} (k_d^a)^\mu_{a_1 \cdots a_{d-2} p_{a_1} p_{a_2} \cdots p_{a_{d-2}}}$ with the sum running over odd $d$.

Note that an index indicating the meson species has been suppressed for simplicity, but that the coefficients can depend on the meson species as well. The $(k_d^a)^{a_1 \cdots a_{d-2}}$ are coefficients for Lorentz violation in the scalar effective field theory, and control the size of the Lorentz-violating effect. They can be taken as symmetric and traceless, and therefore have $(d - 1)^2$ independent components.

### IV. CPT-VIOLATING EFFECTS ON $\xi$

In order to calculate the contributions to the parameter $\xi$ arising from the coefficients for Lorentz violation, it is convenient to focus on contributions to $\xi^{(d)}$ arising from a single mass dimension $d$ term. The full expression is then obtained by summing over $d$. The hamiltonian can then be found directly from the Lagrange density, and the difference in diagonal terms is proportional to $\xi$. It can be shown that

$$\xi^{(d)} = \frac{1}{E_0 \Delta \lambda} (k_d^a)^{a_1 \cdots a_{d-2} p_{a_1} p_{a_{d-2}}},$$

for a meson with energy $E_0$. 


The formalism introduced in the first section is valid in the rest frame of the meson. This means that in order to describe mesons not at rest with components reported in the Sun-centered frame, proper use of both observer and particle Lorentz transformations will be required. Starting in the rest frame of a meson,

\[ \xi^{(d)} = m^{d-3} \frac{\Delta \lambda}{\Delta \lambda} (k_a^{(d)})_{\text{lab}} \text{ (rest frame of meson in laboratory)}, \]  

where \( m \) is the meson mass.

A particle boost is then employed to give the meson a velocity \( \beta \) in the laboratory. Under particle boosts, the coefficient \( k^{(d)} \) is a scalar, while the meson 4-velocity undergoes \( \beta^\mu = (1, 0) \rightarrow \gamma(1, \tilde{\beta}) \). This produces an expression valid in the laboratory frame

\[ \xi^{(d)} = \frac{m^{d-3} \gamma^{d-2}}{\Delta \lambda} \sum_s \left( \frac{d-2}{s} \right) (k_a^{(d)})_{\text{lab}} \beta^{j_1} \cdots \beta^{j_s} \text{ (laboratory frame),} \]  

The components of the coefficients in the laboratory frame, with coordinate axes \( j = x, y, z, \) and \( t \) are then related to the components in the Sun-centered frame by an instantaneous observer rotation denoted by \( R \), and the transformation is accomplished by \( (k_a^{(d)})_{\text{lab}} = (k_a^{(d)})_{T \cdots TJ_1 \cdots J_s} R_{J_1}^{J_1} \cdots R_{J_s}^{J_s} \).

Performing this rotation, we arrive at the final expression

\[ \xi^{(d)} = \frac{m^{d-3} \gamma^{d-2}}{\Delta \lambda} \sum_s \left( \frac{d-2}{s} \right) \beta^s (k_a^{(d)})_{T \cdots TJ_1 \cdots J_s} \beta_1^j \cdots \beta_s^j \]  

where the unit meson velocity in the Sun-centered frame appearing in the expression above is related to the unit meson velocity as measured in the laboratory by

\[ \begin{align*}
\hat{\beta}^X &= (\hat{\beta}^x \cos \chi + \hat{\beta}^y \sin \chi) \cos \Omega \Theta_{\oplus} - \hat{\beta}^y \sin \Omega \Theta_{\oplus} \\
\hat{\beta}^Y &= \hat{\beta}^y \cos \Omega \Theta_{\oplus} + (\hat{\beta}^x \cos \chi + \hat{\beta}^z \sin \chi) S_{\Omega \Theta_{\oplus}} \\
\hat{\beta}^Z &= \hat{\beta}^z \cos \chi - \hat{\beta}^x \sin \chi.
\end{align*} \]  

The parameter \( \chi \) is the angle between the local \( z \)-axis in the laboratory and the \( Z \)-axis of the Sun-centered frame. The parameter \( \Omega \) is the sidereal frequency of the Earth and is approximately \( \Omega \approx 2\pi/(23 \text{ h 56 min}) \). \( \Theta_{\oplus} \) is the local sidereal time.

V. INFERRRED CONSTRAINTS

The minimal \( d = 3 \) type coefficients have been constrained across all four meson species \([3]\). A match to the coefficients for Lorentz violation in the scalar effective field theory and the CPT-odd quark coefficients of the SME can be found by comparing the known form of the parameter \( \xi \) arising from the minimal SME to that arising from the scalar field theory above. The match is found to be \( (k_a^{(3)})^\mu_\mu = 2\Delta a^\mu \).

Although the match for higher mass dimension is unknown at present, constraints on the \( d = 5 \) coefficients can be inferred from the published results on the minimal SME coefficients, taking care to address some subtle points. First, Eq. \( 10 \) includes trace terms. By imposing the four conditions,

\[ (k_a^{(5)})_{\mu TT} - (k_a^{(5)})_{\mu XX} - (k_a^{(5)})_{\mu YY} - (k_a^{(5)})_{\mu ZZ} = 0, \]  

we chose to eliminate the components of the type \( (k_a^{(5)})_{\mu ZZ} \), satisfying the traceless condition. Second, the effects of minimal CPT violation on \( \xi \) only include variations in sidereal time in the first harmonic, while at \( d = 5 \) these variations include the second and third harmonics as well. These harmonics accompany components of the type \( (k_a^{(5)})_{\mu TT} \) and \( (k_a^{(5)})_{\mu JK} \). For this reason, we had to assume only the components \( (k_a^{(5)})_{\mu TT} \neq 0 \). Finally, the velocity dependence at \( d = 5 \) is more complicated than for the minimal coefficients already studied, so we had to assume conservative values for these factors as well.

Constraints on the \( TTT \) and \( TTJ \) components of the \( d = 5 \) coefficients are on the order of \( 10^{-18} \text{ GeV}^{-1} \). A full table of all results can be found in Ref. \([17]\). These constraints show that meson experiments are competitive with other experimental methods \([20]\) sensitive to the CPT-odd quark coefficients of the SME.
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