Energy enhancement of laser-driven ions by radiation reaction and Breit-Wheeler pair production in the ultra-relativistic Breakout-Afterburner regime

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The impact of radiation reaction and Breit-Wheeler pair production on acceleration of fully ionized Carbon ions driven by an intense linearly-polarised laser pulse has been investigated in the ultra-relativistic Breakout-Afterburner (BOA) transparency regime. Against initial expectations radiation reaction and pair production at ultra-high laser intensities is found to enhance the energy gained by the ions. The electrons lose most of their transverse momentum and the additionally produced pair plasma of Breit-Wheeler electrons and positrons co-stream in the forward direction as opposed to the existing electrons streaming at an angle above zero. This leads to changes in the phase velocity of the Buneman instability, that is known to aid ion acceleration in the BOA regime, by tapping the free energy in the relative electron and ion streams. We present evidence that this can further improve the highest Carbon ion energies.

Accelerated ion beams have a multitude of applications ranging from nuclear reactions induced by energetic heavy ions 1 to fast ignition fusion 2, aiding neutron production 3 and also hadrontherapy for cancer treatment 4, 5. Laser-driven ion acceleration has acquired much attention in the recent decades, as this offers the possibility of having alternate accelerators that are smaller and more affordable as opposed to the conventional linacs, cyclotrons and synchrotron 6. Experimental demonstration of ion beams 7, 8 by several mechanisms exhibiting different performances such as Target Normal Sheath Acceleration (TNSA), Radiation-Pressure Accelerations (RPA), Collisionless shock acceleration (CSA), Breakout Afterburner (BOA) 9, 10 etc. has already been achieved. 11 Significant efforts of innovative laser/target configurations have also been made to push the numbers of ion beam characteristics (energies and flux), yet the highest gained energy is still less than 100 MeV/u 11, 12. Nevertheless, the prospects of achieving even higher ion energies as predicted with the next generation laser sources are promising. 13

BOA is one of the high performance laser-driven-ion acceleration mechanisms capable of accelerating ions to relatively higher values even with state-of-the-art lasers. In this, an initially opaque, ultra-thin target (width around laser skin depth) turns transparent to the incoming laser pulse, due to lowering of the density by the expanding plasma and increase in critical density by the electron’s relativistic motion (relativistically induced transparency, RIT) 8, 11. This leads to a phase of extreme ion acceleration (BOA phase) which continues to exist until the electron density of the expanding target becomes classically underdense 13. Buneman instability (in single ion-species target) and ion-ion acoustic instability (in case of multispecies target 13) result in an electrostatic mode structure, that is found to be instrumental in transferring the laser energy to ions via laser-induced electronic drifts 10, 17. The efficiency of this mechanism is maximised when the peak of the laser pulse arrives precisely at the onset of relativistic transparency 7, 18 as opposed to the RPA-Light-sail mechanism which requires opacity in ultra-thin targets. Experimental demonstration of fully ionised carbon ion acceleration via the BOA mechanism up to 40-50 MeV/u has been achieved using ~50-250 nm thick targets with the TRIDENT laser and the Texas Pettawatt laser facility. 7 Also, simultaneously existing TNSA and BOA signatures in proton spectra (energy ~ 61 MeV) have also been identified at the PHelix laser facility. 19 Much more intense and powerful lasers, such as ELI, APOLLON, are soon to surface (as expected in the laser-power timeline and also with the recent prototype design using WNOPCPA allowing a 0.5 EW system 23) and can further improve these numbers, as they will allow a larger laser energy transfer to the ions. However, in the ultra-relativistic regime other Quantum Electrodynamic Dynamic (QED) effects become non-negligible when the electric field of the laser in the electron’s rest frame gets closer to the critical Schwinger field \( E_s = 1.3826 \times 10^{18} \text{V/m} \) 24. The most important effects are: high frequency radiation emission by electrons pushed in the laser-field (with a consequent back reaction on individual electrons, radiation reaction \( \text{RR} \)) and the multi-photon Breit-Wheeler process leading to the generation of electron-positron pairs 25, 26. These QED effects, usually expected to deplete energy from a physical system 27, 28, may though significantly modify the collective plasma dynamics with yet unexplored indirect effects on the ion energy.

In this letter, the impact of RR and non-linear Breit-Wheeler pair production \( (\text{PP}, \gamma + n\gamma \rightarrow e^- e^+ \) on the acceleration of ions in the BOA regime has been investigated using PIC simulations. An increase in ion energies by RR alone in the transparency regime has already been reported 28, 29, 31, 33, 34, though these ignore the stochastic nature of high-energy photon emis-
sion. Here, we show evidence that in this regime, both RR and PP together can lead to a notable improvement up to 30% in the ion energy beyond previous results. This is due to an increased phase velocity of the relativistic Buneman instability (RBI) that Landau damps on the ions\cite{35} as observed in the spectral analysis of the system. This then allows for an efficient energy transfer from the laser to the ions facilitated by electron flow during the onset of RIT. We performed 2D PIC simulations using both the open-source codes EPOCH and SMILEI which include quantum RR and PP by the probabilistic Monte-Carlo method\cite{36}. We employ a linearly s-polarised laser pulse, impinging on the left boundary with a finite spatio-temporal profile $I(t, y) = I_0 \exp[-((y-y')/\tau_0)^2] \exp[-((t-t')/\tau_0)^2]$, with $\tau_0 = 3\, \mu m$, $y' = 4\, \mu m$, $\tau_0 = 40\, fs$, $t' = 30\, fs$. The laser peak intensity of $I_0 = 4.95 \times 10^{23}\, W/cm^2$, might soon be realizable\cite{37}, $(a_0 = eE/m_e\omega = 600)$, where $e$ is the electronic mass, $\omega$ the laser frequency and $c$ the velocity of light in vacuum. The polarisation of the laser is chosen to be s-polarised as it is closer to 3D simulations as opposed to p-polarised laser which can artificially heat electrons and can exaggerate the effectiveness of ion acceleration in such a scenario\cite{28, 29, 31, 38}. It interacts with a pre-formed fully-ionised Carbon plasma ($C_{6+}$) with a temperature $T_{e-} = T_{C^+} = 1200\, eV$ and density, $n_{e-} = 200n_c$, where $n_c = m_e\omega^2/4\pi e^2$ is the classical critical density of a plasma for 1\,\mu m laser wavelength. The target has a thickness of 0.6\,\mu m, and is located at 12\,\mu m from the left boundary of the simulation box. We employ transmitting and periodic boundary conditions in $x$ and $y$ direction, respectively. The simulation box has dimensions of $L_x \times L_y = (50\,\mu m \times 8\,\mu m)$, with the cell size $\Delta_x \times \Delta_y = (10\,nm \times 10\,nm)$ using 85 particles per cell. We also performed parameter scans with the same laser but different target densities [60, 100]$n_c$ and observed a similar improvement by QED effects only for 100$n_c$. However, in a near-critical thin target 0.6\,\mu m, $a_0 = 540$, $n_c = 500n_c$ QED effects were observed to reduce ion energies.

The laser field pushes hot electrons inside the target forward that quickly reach the non-irradiated side (rear) of the target faster than the ions. This sets up a very brief TNSA field there which kickstarts the ion acceleration from the TNSA mechanism at around 50\,fs. The electrons oscillate with relativistic velocity and thus, the effective critical density is reduced. Moreover, as the recirculating hot electrons heat the target up, it begins to expand and the density lowers further. The target then begins to get relativistically transparent and the laser is able to penetrate through it. This marks the onset of the Breakout Afterburner phase (at ∼ 60\,fs) where the streams of electrons and ions co-move with the penetrating laser. Fig. 1 shows the electron’s energy-angle distribution at 80\,fs (BOA phase) where the laser pulse has already penetrated the target. Panel (a) shows the case where the QED effects are artificially turned off, panel (b) shows the case when RR is included in the plasma dynamics and panel (c), when both RR and PP are included. One can clearly see in panel (a) that electrons stream diffusely at an angle and gain energy. The majority of the electrons stream in the forward direction (laser-propagation direction) and a small percentage of electrons also gain energy at the back (∼180°). In panel (b), when RR is also included, the electrons become more forward-directed and the backward acceleration is suppressed. The latter observation is expected i.e. the electrons that counter-propagate the incoming laser experience Doppler-upshifted fields leading to a substantial suppression of its backward acceleration (also observed in Ref.\cite{27, 29, 32}). As the laser-accelerated electrons lose part of their energies in high-energy-photon emission, the overall divergence of the electron’s angular distribution reduces as they get pushed forward with the laser. Similar reduction in the electron’s transverse momentum and electron cooling due to RR is also seen in Ref.\cite{28, 29}. Although laser collision with an electron-beam with quantum RR is shown to increase the electron energy distribution\cite{40}, here the scenario differs with the laser blasting the entire thin target. In panel (c), when RR+PP both are included, apart from

**FIG. 1.** Energy-angular distribution of electrons [in a.u.] in the BOA phase without radiation reaction [panel (a)], with radiation reaction [panel (b)] and with pair production as well [panel (c)], excluding the produced Breit-Wheeler electron density] at 80\,fs.

**FIG. 2.** Energy-angular distribution of photons [panel (a)], with BW electrons [panel (b)] and BW positrons [in panel (c)] at 80\,fs.
a more collimated stream of electron fluid, here one can also see a higher density of electrons that also gains larger energy. This is due to the production of the BW-pairs that occurs due to the interaction of laser photons with the emitted gamma-ray photons. The angularly streaming target-electrons gain more energy from the newly formed energetic pair-plasma at 0° as all species of similar masses exchange energies. Fig. 2 (a, b and c) show the energy-angle distribution of photons, BW electrons and BW positrons respectively in the RR+PP case at 80 fs. One can clearly see a large number of gamma-ray photons in the laser-propagation direction being produced in panel (a) as the target turns transparent and the laser is allowed to interact with prolific electrons. In panel (b-c), we see the high-energy and forward-streaming pair-plasma that is responsible for the higher energy and density of electrons in Fig. 1(c). Since the target is already transparent, these pairs do not accumulate at the target region and are unable to shield the incoming laser as in the cushioning scenario [41], rather stream forward with the laser pulse and the ambient plasma. Fig. 3 shows the ion distribution in the same fashion as in Fig. 1 and at the same time. In the no-QED case in panel (a) of Fig. 3, the ions with the highest energy (around 6.5 GeV) are off the axis of laser polarisation or propagation, as also seen in Ref. [42]. This occurs at 80 fs when RBI operates which is a low-frequency high-amplitude electrostatic mode that feeds on the relative flow velocity between electrons and ions and accelerates ions with a wave-particle resonance mechanism [10, 39]. In the same figure, one may also see some ions with ~ 5.7 GeV energy which are on the laser-propagation axis. This is when the off-axis ion streams mutually interact. In Fig. 3(b), the highest gain in energy and the angular divergence of these high-energy ions is reduced at 80 fs when BOA mechanism is at play. The on-axis and off-axis ions gain nearly the same energies in this case. Further, in Fig. 3(c), the angular divergence of the ions is even smaller, and the on-axis ions gain much higher energy (~ 5.8 GeV) than the off-axis ones (~ 4.6 GeV). The high-energy, on-axis ion bunch is accelerated due to a similar electron bunch in Fig. 2(c) on account of the pair plasma Fig. 2(c). The role of RBI in these bunches of high-energy ions is crucial in higher acceleration of ions. The expanding TNSA ions, target electrons and the BW bunches stream forward with the laser and the free energy in the particle streams gives rise to electrostatic mode, RBI, that resonates with ions allowing them to be rapidly accelerated. The energy loss by electrons is constantly filled up by the long-pulse laser. This beam-like expanding plasma is susceptible to the growth of RBI where the phase velocity of the instability is comparable to the highest accelerated velocities of the ions [10].

The Fourier analysis of the longitudinal electric field from the simulations can shed light on the electrostatic structure of the accelerating fields in the transparency region. This has been performed for all three cases and is shown in Fig. 4. Panel (a), (b) and (c) show $|E_x(\omega, k)|^2$ in log scale for $t \in [50 - 140] \text{ fs}$ and $x \in [10 - 50] \mu m$ for all the three cases [no QED panel (a), RR panel (b) and RR+PP panel (c)] obtained from the simulations. The real and imaginary roots of Eq. (solid and dotted respectively) are over-plotted to facilitate comparison.
the primary branch. These two branches merge slightly in panel (a). Looking at panels (b) and (c), one can broadly see that the branch \(A\) is more powerful in both QED cases than in panel (a) [even more with RR+PP case]. Moreover, the branch \(B\) becomes notably more distinct in panel (b) and marginally even more in panel (c). This may be due to the fields generated by the angularly drifting plasma streams that mutually interact leading to the high-energy on-axis ions seen in the tip of a bubble-like form that ions make in Fig. 2 (potentially a mode harnessing the free energy in off-axis high energy ion streams). As the radiatively cooled electrons become more forward-directed in QED cases (Fig. 2 (b-c)), the angular separation between the streaming plasma ions also lowers. This allows more interaction between the streams and thus the branch \(B\) becomes more distinct. An additional lowering of this angle due to pairs produced at \(0^\circ\) makes this branch \(B\) stronger in Fig. 2(c). The dispersion relation of RBI from the linear kinetic theory assuming cold angularly streaming plasma for the instability is given as

\[
\sum_{s = e,i} \frac{\omega_{p,s}^2}{k^2} \left[1 + \left(\frac{p_s \sin \theta_s}{m_s c}\right)^2\right] \frac{\left(v_\phi - v_s \cos \theta_s\right)^2}{\gamma_s^2} = 1,
\]

where, \(\omega_{p,s}\) is plasma frequency, \(v_s/p_s\) are the stream velocity/momentum, \(v_\phi\) is the phase velocity \((\omega/k)\), \(\gamma_s\) the respective Lorentz factor and \(\theta_s\) is the angle of drift, with \(s = e, i\) denoting the electronic and ionic streams respectively. The dispersion relation of this instability in Eq. 1 has been solved and the 4 roots of the quartic equation have been overplotted in Fig. 3. The input plasma parameters (average electron and ion density, angles of streams and energies) have been extracted from the simulations in each case at around 70 fs when BOA is active (see Supplementary information). It should be noted that we use the same dispersion relation for QED cases (Fig. 2 (b-c)) as well. This is reasonable as we carefully choose the plasma parameters at the time after the production of photons and pairs has mostly saturated. The impact of RR and pair plasma are still well captured in the form of changes in the plasma distribution function extracted from the simulation that already includes probabilistic photon emission in plasma evolution.

There are 2 real and 2 complex roots of this equation. One high frequency real root (starts with positive-frequency as also in Ref. 10 and 11) and the other low frequency real root (negative frequency at \(k = 0\) crosses the \(\omega = 0\) axis as the wave-number increases). The other 2 roots are complex conjugates with the same \(\Re(\omega)\) the non-zero imaginary part vanishes, after which the real parts bifurcate. The positive imaginary part (dotted line in Fig. 4) is the unstable mode while the negative (damped mode) is not shown here. In Fig. 4(b-c), we use the same dispersion relation from linear kinetic theory to compute the roots for RR and RR+PP case. This is reasonable because the plasma distribution parameters that we use to match with the simulation results are extracted after the production of photons and pairs has saturated (see Supplementary information). A good match between the branch \(A\) (from simulation) and the real part of growing complex root from linear kinetic theory (overplotted solid line) is visible in all 3 panels. The phase velocity of the primary branch \((v_p \sim 0.84c\) in Fig. 4(a)) is comparable to the ion velocities attained by the off-axis ions \((v_i = 0.86c, \epsilon_i \sim 11.26 GeV)\) corroborating the role of the instability in the ion acceleration. The instability growth rate progressively lowers and the bifurcation of the roots shifts to lower \(k\) values respectively, which is as expected \(32, 44\). A lower angle of the electron stream (see Fig. 4) is also shown to enhance the phase velocities of the RBI wave \(33\). Thus, from the spectral plots it is clear that RR and RR+PP enhance the RBI on account of a radiatively cooled more-forward-directed electron and ion beam. Interestingly, the low-frequency real root of the same dispersion relation, which has negative frequency for \(k = 0\), matches very well the branch \(B\) picked up by the FFT of the longitudinal electric fields from simulation (see Supplementary information). This points to lower ion-mode that additionally bestow the high-energy on-axis ions, accelerated at later-time due to mutually interacting angular ion streams. Fig. 5 shows the angle-energy and the \(\theta\)-averaged ion-distribution in the three cases at a later time of 130 fs. The highest energy gained by the ions at this time in the no QED case is \(\sim 11.26 GeV\), with RR is \(12.7 GeV\) (\(\sim 12\%\) higher) and with RR+PP it is \(14.52 GeV\) (\(\sim 30\%\) higher). Corresponding ion velocities \(v_i = [0.86c, 0.88c, 0.90c]\) are in good agreement with the respective phase velocities of the RBI \(v_p \sim [0.84c, 0.89c, 0.91c]\) from simulations [branch A] and also with the overplotted phase velocities of the RBI from
linear theory $v_p \sim \sqrt{0.75c, 0.82c, 0.90c}$. This confirms the wave-particle acceleration mechanism \cite{10} (see Supplementary information).

In conclusion, we investigated the effect of radiation reaction and pair production on the ion acceleration where the BOA mechanism operates. We demonstrate how QED effects can impact the collective plasma behaviour in the early stages of laser-plasma interaction, that can lead to an enhanced phase velocity of RBI in the later BOA stage. This can lead to a gain of higher energy (around 30\%) by the ions. The angle of streaming between the transparent target electrons and the forward-directed $e^-e^+$ pair-plasma plays a principal role in plasma dynamics and the consequent high ion energy gain. Recent related experimental corroboration of QED effects \cite{15, 17} and the advent of ultra-high intensity lasers \cite{20, 22, 37, 48} places these findings in very exciting times.

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\[ [n_e, \theta_e, \epsilon_e] = [0.0026n_0, 24^\circ, 0.3420\text{GeV}] , \]
\[ [n_i, \theta_i, \epsilon_i] = [0.0404n_0, 16.9^\circ, 1.9414\text{GeV}] ; \]
\[ n_i, \theta_i, \epsilon_i \]
\[ [0.0046n_0, 8.2^\circ, 0.3925\text{GeV}] , \]
\[ n_i, \theta_i, \epsilon_i \]
\[ [0.049n_0, 13.8^\circ, 2.03\text{GeV}] ; \]
\[ n_i, \theta_i, \epsilon_i \]
\[ [0.0032n_0, 5.1^\circ, 0.4128\text{GeV}] , \]
\[ n_i, \theta_i, \epsilon_i \]
\[ [0.0591n_0, 17.9^\circ, 2.01\text{GeV}] , \]
\[ \text{n(c,i,0), \epsilon,}, \text{being the electron,}
\]
\[ \text{ion and the initialised particle density}
\]
\[ \text{and energies respectively} \]
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