Sagnac interferometry and non–Newtonian gravity

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Sagnac interferometry has been employed in the context of gravity as a proposal for the detection of the so called gravitomagnetic effect. In the present work we explore the possibilities that this experimental device could open up in the realm of non–Newtonian gravity. It will be shown that this experimental approach allows us to explore an interval of values of the range of the new force that up to now remains unexplored, namely, \( \lambda \geq 10^{14} \) m.

Key words: Sagnac interferometry; non–Newtonian gravity

I. INTRODUCTION

The use of interferometric techniques has rendered very fruitful results in the realm of gravitation, as a fleeting glimpse to the experimental efforts in gravitational waves or in the detection of the so called gravitomagnetic effect, easily attests. We may rephrase this stating that optical interferometry has played along many years a fundamental role in gravitational physics.

Though general relativity is one of the bedrocks of modern physics, and many of its predictions have found a sound confirmation at the experimental level, the quest for deviations from the predictions of Newtonian gravity has never waned altogether, since many theoretical attempts to construct a model of elementary particles do predict the emergence of new forces (usually denoted as fifth force). One of the distinctive traits of these new interactions is the fact that they are not described by a inverse–square law. Additionally, they, generally, violate the so called weak equivalence principle. Since more than a decade has witnessed the lack of any kind of compelling evidence that could purport some kind of deviations from the Newtonian theory, the pursuance in this direction requires a thorough justification, a requirement already covered by Gibbons and Whiting, who claim that a very precise agreement between Newtonian gravity and the observation of planetary motion does not preclude the existence of large non–Newtonian effects over other distance scales. The results comprised in allow us to draw several conclusions: (i) the current experimental constraints do not explore the so called geophysical window \( \lambda \in [10m,1000m] \); (ii) the case in which \( \lambda \geq 10^{14} \) m remains completely unexplored.

The main goal in the present work is the introduction of an experimental proposal, which could explore the region \( \lambda \geq 10^{14} \) m. This will be done by means of an experimental model embracing a Sagnac interferometer whose area (that enclosed by the light path) has a unit normal vector perpendicular to the direction of the acceleration of gravity. It will be shown that this idea may be used to find the first experimental bound in the aforementioned region of \( \lambda \).

II. SAGNAC INTERFEROMETRY AND NON–NEWTONIAN GRAVITY

Let us consider a gravitational potential which contains a Yukawa–type term:

\[
U(r) = -\frac{G_{\infty}M}{r} \left\{1 + \alpha \exp\left\{-r/\lambda\right\}\right\}.
\]

(1)

Here \( G_{\infty} \) denotes the value of the Newtonian constant between the source of the gravitational field, i.e., \( M \), and a test particle when the distance between them tends to infinity. As a matter of fact \( G_N = G_{\infty}(1+\alpha) \), where \( G_N \) is the usual Newtonian constant. In addition, \( \lambda \) is the range of the interaction.

At this point, and bearing in mind that we try to put forward a terrestrial experimental proposal, the following approximation will be introduced, \( r = R + z \), with \( R \gg |z| \). Under this restriction becomes, to first order in \( z/R \)

\[
U(r) = -\frac{G_{\infty}M}{r} \left\{1 - \frac{z}{R} + \alpha \exp\left\{-R/\lambda\right\}\left\{1 - \frac{R + \lambda}{\lambda} \frac{z}{R}\right\}\right\}.
\]

(2)

At this point consider now a Sagnac interferometer whose area (that enclosed by the light path) has a normal vector perpendicular to the direction of the acceleration of gravity, i.e., the \( z \)–axis. In addition, the angular velocity of the interferometer (the one rotates in the clockwise
direction) is $\Omega$, and its radius $a$. For the sake of simplicity let us assume that the beams enter the interferometer at point $A$, which is the highest one, (its $z$ is a maximum). Since the interferometer rotates, then both beams meet, for the first time, at $A$

$$t_d = \frac{2\pi a}{c} \left\{1 + \frac{a\Omega}{c}\right\}^{-1}. \quad (3)$$

The distance, below point $A$, at which the beams meet for the first time is

$$h = a \left\{1 - \cos \left(\frac{2\pi \Omega a}{c} \left\{1 + \frac{a\Omega}{c}\right\}^{-1}\right)\right\}. \quad (4)$$

Since the beams are immersed in a region in which the gravitational potential has the form pointed out in (2), then during their movement they will undergo a redshift, the one reads 

$$\nu_{rs} = \frac{\nu}{1 + \Delta U/c}. \quad (5)$$

Here $\Delta U$ denotes the difference in the potential between the two involved points.

The frequency at time $t_d$ reads

$$\nu_{rs} = \frac{\nu}{1 - a \frac{c \sqrt{\gamma \beta}}{a}} \quad (6)$$

In this last expression two definitions have been introduced

$$\beta = \left\{1 + \frac{R + \lambda}{\lambda} \exp\{-R/\lambda\}\right\}^{-1}, \quad (7)$$

$$\gamma = \left\{1 - \cos \left(\frac{2\pi \Omega a}{c} \left\{1 + \frac{a\Omega}{c}\right\}^{-1}\right)\right\}^{-1}. \quad (8)$$

The time difference between the arrival of the two beams is the usual one

$$\Delta t = \frac{4\pi a^2 \Omega}{c^2 - a^2 \Omega^2}. \quad (9)$$

This last result renders the path difference, $\Delta L = c\Delta t$.

$$\Delta L = \frac{4\pi a^2 c\Omega^2}{c^2 - a^2 \Omega^2}. \quad (10)$$

Finally, harking back to (2) the phase difference, $\Delta \theta$ reads

$$\Delta \theta = \frac{8\pi^2 a^2 \nu \Omega}{(c^2 - a^2 \Omega^2)(1 - a \frac{G \gamma M}{c^2 R^2})} \gamma \beta. \quad (11)$$

Writing this phase difference as the sum of two terms, $\Delta \theta^{(N)}$ and $\Delta \theta^{(N)}$, which correspond to the differences stemming from the Newtonian and non–Newtonian parts of the gravitational potential, respectively, we may deduce (assuming $|a \frac{R + \lambda}{\lambda} \exp\{-R/\lambda\}| < 1$)

$$\Delta \theta^{(N)} = -\Delta \theta^{(N)} \frac{R + \lambda}{\lambda} \exp\{-R/\lambda\}. \quad (12)$$

In this result we have included the fact that

$$\Delta \theta^{(N)} = \frac{8\pi^2 a^2 \nu \Omega}{(c^2 - a^2 \Omega^2)(1 - a \frac{G \gamma M}{c^2 R^2})} \gamma. \quad (13)$$

## III. CONCLUSIONS

The possible detection of a fifth force through this kind of proposals does strongly depend upon the relation between the experimental resolution associated to the measuring process of phase differences, $\Delta \theta^{(ex)}$, and the absolute value of the parameter $\Delta \theta^{(N)} / \Delta \theta^{(N)}$. In other words, this idea could be a useful one if

$$|\Delta \theta^{(N)} / \Delta \theta^{(N)}| > \Delta \theta^{(ex)}. \quad (14)$$

Resorting to (12) and (13) it is readily seen that the feasibility of the proposal becomes

$$|a \frac{R + \lambda}{\lambda} \exp\{-R/\lambda\}| > \Delta \theta^{(ex)}. \quad (15)$$

A fleeting glimpse at the current experimental bounds immediately shows us that several regions of $\lambda$ remain unexplored. For instance, there are no experiments related to $\lambda \geq 10^{14}$ m. Bearing in mind that we should contemplate the possibility of performing this experiment on the Earth’s surface, then $R \sim 10^6$ m, and if we consider the aforementioned region for the range of the fifth force, then $R/\lambda \sim 10^{-8}$, and hence

$$|a| > \Delta \theta^{(ex)}. \quad (16)$$

We may rephrase this last conclusion asserting that the resolution of the measuring device sets (in this very particular situation) the bound upon the values of the strength. If nothing is seen, then, for sure

$$|a| < \Delta \theta^{(ex)}. \quad (17)$$

Let us now address the issue concerning the order of magnitude of $a$ that could be measured in this kind of models. It is already known that the resolving power of a spectral device is defined as $\Lambda = \lambda / \Delta \lambda$, here $\lambda$ denotes the wavelength of the corresponding beam, while
$\Delta \tilde{\lambda}$ is the minimum difference in the wavelength which can be resolved. If the experiment is carried out by means of visible light ($\tilde{\lambda} \sim 10^{-6} \text{m}$), and assuming a rough resolving power, i.e., $\Lambda \sim 10^5$, then (since $\Delta \theta^{(ex)} \sim 2\pi / \Lambda$) we obtain that $\Delta \theta^{(ex)} \sim 10^{-4}$. Under these constraints it becomes clear that a null experiment would imply that $|\alpha| \leq 10^{-4}$.

Summing up, it has been shown that a Sagnac interferometer can be employed to impose an experimental bound for the strength of an hypothetical fifth force, for the case, which up to now remains completely unexplored, $\lambda \geq 10^{14} \text{m}$.

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[1] P. Purdue and Y. Chen, Phys. Rev. D 66, 022001 (2002).
[2] M. O. Scully, M. S. Zubairy, and M. P. Haugan, Phys. Rev. A 24, 2009 (1981).
[3] I. Ciufolini and J. A. Wheeler, *Gravitation and Inertia*, (Princeton Univ. Press, Princeton 1995).
[4] E. Fishbach, G. T. Gillies, D. E. Krause, J. G. Schwan, C. L. Talmadge, Metrologia 29, 213 (1992).
[5] E. Fishbach, D. Sudarsky, A. Szafer, C. L. Talmadge, and S. H. Aronson, Phys. Rev. Lett. 56, 3 (1986).
[6] E. Fishbach, C. L. Talmadge, *The search for Non–Newtonian Gravity*, (Springer–Verlag, New York 1999).
[7] G. W. Gibbons and B. F. Whiting, Nature 291, 636 (1981).
[8] G. Sagnac, R. Acad. Sci. 157, 708 (1913).
[9] F. Fujii, Nature 234, 5 (1971).
[10] A. N. Matveev, *Optics*, (Mir Publishers, Moscow 1988).