Quasi-bound states of Schrödinger and Dirac electrons in magnetic quantum dot.

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The properties of a two-dimensional electron are investigated in the presence of a circular step magnetic field profile. Both electrons with parabolic dispersion as well as Dirac electrons with linear dispersion are studied. We found that in such a magnetic quantum dot no electrons can be confined. Nevertheless close to the Landau levels quasi-bound states can exist with a rather long life time.

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I. INTRODUCTION

During the last five years, graphene (a single layer of carbon atoms) has become a very active field of research in nanophysics. It is expected that this material will serve as a base for new electronic and opto-electric devices. One of the most challenging task is to learn how to control the electron behavior using electric fields in this two-dimensional (2D) layer. This task is made complicated by the so called Klein effect according to which Dirac electrons in graphene can tunnel through any electric barrier. As a consequence in electrically created quantum dots there are no bound states but only quasi-bound states, or so called resonances, which, however, under certain conditions can have a long life time.

An alternative approach to control the motion of electrons is to use non homogeneous magnetic fields which can be created e.g. through the deposit of nonstructured ferromagnets. Recently it was shown that nonhomogeneous magnetic structures are able to confine Dirac electrons in graphene. However, up to now semi-infinite (homogeneous in one direction) structures were considered, what makes the analysis more simple because the problem is reduced into a one-dimensional one.

In this paper we consider a finite size magnetic structure where the magnetic field is nonzero only in a finite region of space. Namely, we consider a model homogeneous magnetic field that is non-zero in a circle that we call the magnetic dot. This situation is the inverse of the one considered in Ref. 11 where a magnetic anti-dot was considered, as in Ref. 9 for the case of normal electrons, where the magnetic field is zero in a circular region and non-zero outside this region. Such a model system can be realized by having a magnetic vortex piercing the graphene layer or by overlaying graphene with type I superconductor with a circular hole placed in the perpendicular magnetic field. In order to reveal the peculiarities of the behavior of Dirac electrons in such magnetic dot we compare the result with those for standard electrons with parabolic dispersion law.

We show that it is impossible to confine 2D electrons in a magnetic dot in contrast to semi-infinite magnetic structures neither in the case of graphene nor in the case of the standard electron, and consequently, all Landau levels convert themselves into unbound states. Nevertheless, long living quasi-bound states can be present. We studied them using the local density of states technique applied previously for the investigation of electrically confined electrons.

The paper is organized as follows. In section II the model of a magnetic dot for a standard electron is considered. The problem is formulated in subsection A, the local density of states technique is presented in subsection B, the results are discussed in subsection C, while in subsection D the complex energy eigenvalues of the problem are described. In the corresponding subsections of section III the problem of the Dirac electron in a magnetic dot is presented. Our conclusions are given in section IV.

II. ELECTRON WITH PARABOLIC ENERGY DISPERSION

We assume that a homogeneous magnetic field $B_0$ is present in a circular area of radius $r_0$, while there is no magnetic field outside it, namely, $B_0(r) = e_0B_0\Theta(r_0-r)$. The behavior of the electron is described by the stationary Schrödinger equation

$$\{H-E\}\Psi(r) = 0, \quad (1)$$

with the Hamiltonian

$$H = -\frac{1}{2}(\nabla + iA)^2. \quad (2)$$

Because of the cylindric symmetry of the problem we choose the symmetric gauge for the vector potential defining its single azimuthal component as

$$A_\varphi(r) \equiv \frac{1}{2}\begin{cases} r_0^2/r, & r < r_0; \\ 0, & r_0 < r. \end{cases} \quad (3)$$

This azimuthal component is shown in Fig. together with the magnetic field profile. In order to simplify the
notations we use dimensionless variables, based on the magnetic field strength value $B_0$. Thus, the magnetic field $B(r)$ is measured in $B_0$ units, all distances are measured in the unit of magnetic length $l_B = \sqrt{\hbar/eB_0}$, energy and potential in $\hbar \omega_c$ ($\omega_c = eB_0/mc$), and vector potential in $B_0l_B$ units. In the case of electron moving at a GaAs/AlGaAs interface ($m^* = 0.067$) and magnetic field of 1 T the unit of length is $l_B = 250$ nm, and the energy unit is 20 meV.

A. Solution of eigenvalue problem

The Schrödinger equation \(^{(1)}\) in cylindric coordinates reads

$$\left\{ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} + \frac{iA_0}{r} \frac{\partial}{\partial \varphi} - A_0^2 + 2E \right\} \Psi = 0. \tag{4}$$

Substituting the wave function

$$\Psi \equiv \Psi(r, \varphi) = e^{im\varphi} \psi(r) \tag{5}$$

we arrive at the radial equations

$$\left\{ \frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} \right) - \left( \frac{m}{r} + \frac{r}{2} \right)^2 + 2E \right\} \psi_I(r) = 0, \tag{6a}$$

$$\left\{ \frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} \right) - \left( \frac{m}{r} + \frac{r^2}{2} \right)^2 + 2E \right\} \psi_{II}(r) = 0, \tag{6b}$$

which have to be solved inside the dot (region I) and outside it (region II). The boundary conditions (the continuity of the wave function and its radial derivative) have to be satisfied at the dot border ($r = r_0$).

The regular solution inside the dot can be expressed via the confluent hypergeometric function (Kummer function $M(a|c;z)$):

$$\psi_I(r) = Af(r) = A_1^{\mid m \mid} e^{-r^2/4} \times M\left( \frac{(\mid m \mid + 1)^2}{2} + 1/2 - E \mid m \mid + 1 \big| r^2 / 2 \right), \tag{7}$$

while the solution outside it is composed of two Bessel functions

$$\psi_{II}(r) = BJ_m(kr) + CY_m(kr), \tag{8}$$

where $k = \sqrt{2E}$ is the momentum of the free electron (measured in $l_B^{-1}$ units), and $\nu = m + r_0^2/2$. Note both functions ($J_\nu$ and $Y_\nu$) suit us, as they vanish in the limit $r \to \infty$.

Thus, we have three constants $A$, $B$, and $C$. They cannot be defined from the above mentioned two boundary conditions. That is why we have to conclude that there are no bound states, and consequently, a magnetic field in a finite region of the 2D plane cannot confine the electron. However, quasi-bound states can be expected when the electron energy in the dot is close to the Landau levels with energy

$$E_{n,m} = m + \frac{\mid m \mid + m + 1}{2} \tag{9}$$

(here $n = 0, 1, \cdots$ and $m = 0, \pm 1, \cdots$) defined in the case of homogeneous magnetic field. Confirmation of this statement follows from Fig. 2, where the electron wave functions for two different energies are shown.

B. Local density of states

We will look for possible long living quasi-stationary states in the magnetic dot. In principle such quasi-bound (or quasi-stationary) states have to be described by the solution of the time dependent Schrödinger equation which is much more complicated as compared with the standard eigenvalue problem. There are, however, several alternative approaches which enable us to

\[ \text{FIG. 1: (Color online) Azimuthal vector potential component } A_\varphi \text{ (blue solid curve), and perpendicular magnetic field } B_z \text{ (red dashed curve) as functions of the radial coordinate.} \]

\[ \text{FIG. 2: (Color online) The wave functions for } m = 0, r_0 = 5: E = 2.5 - \text{red solid curve, and } E = 2.8 - \text{dashed blue curve. The magnetic dot region is indicated by shadowed rectangle.}\]
investigate properties of quasi-bound states by stationary means. We follow the method presented in detail in Ref. [5] and calculate the local density of states. The basic idea is to confine the electron in a large region of finite radius \( R \), where its wave function obeys the zero boundary condition at the border \( (r = R) \) and treat the problem as a stationary one. A measurement that probes quantum dot properties, say, measuring of the tunnelling current directed perpendicular to the dot with STM, or power absorption in near field infrared spectroscopy, has to depend on the averaged value of the electron wave function in the dot. Therefore we introduce the integral

\[
I(E) = 2\pi \int_0^\infty rdr F(r)|\Psi(r)|^2,
\]

which depends on the electron wave function, and actually is proportional to the so called local density of states. The aperture function \( F(r) \) characterizes the interaction of the electron with the measuring probe.

This integral is sensitive to the probability to find the electron in the dot, and in the case of a quasi-bound state it will exhibit a peak corresponding to the energy of this state. The width of the peak is related to the inverse of the life time of this quasi-stationary state.

For the sake of determinacy we use the aperture function of a gaussian:

\[
F(r) = b r_0^2 e^{-br^2}, \quad b = r_0^{-2} \ln 10.
\]

which corresponds to the probability to find the electron in the dot area \( \pi r_0^2 \). In the case of larger \( b \) value instead of the local density of states we obtain the squared wave function value in the center of the dot, while in the case of smaller \( b \) value the peculiarities of the dot are washed out.

The solution of the Schrödinger equation (6) given by Eqs. (7) and (8) has to satisfy the following boundary conditions:

\[
\begin{align*}
\psi_I(r_0) &= \psi_{II}(r_0), \\
\psi_{I,r}(r_0) &= \psi_{II,r}(r_0), \\
\psi_{II}(R) &= 0,
\end{align*}
\]

which converts our problem into an eigenvalue problem. Here and further the subscript \( r \) means the derivative over \( r \).

At the end, we are interested in the limiting case \( R \to \infty \). Therefore, in the last of Eqs. (12) we replace the Bessel functions by their asymptotic, namely, we have

\[
B \cos(kR - \varphi_m) + C \sin(kR - \varphi_m) = 0,
\]

\[
\varphi_m = \pi \{m + (r_0^2 + 1)/2\}/2.
\]

instead of Eq. (12c).

Postponing till later the proper wave function normalization we assume that \( B = \cos \Phi \) and \( C = \sin \Phi \) and rewrite the above equation as

\[
\cos(kR - \varphi_m - \Phi) = 0.
\]

This equation shows that the eigenvalues of the considered problem are approximately separated by \( \Delta k = \pi/R \), and reduce to a continuum spectrum in the limit \( R \to \infty \). Constructing some averaged description which is valid when calculating the local density of states, we replace Eq. (12a) by the following one:

\[
B^2 + C^2 = 1.
\]

Now solving it together with Eqs. (12a,b) we obtain three constants:

\[
A = -\frac{2}{\pi r_0}W, \quad B = WQ, \quad C = -WP
\]

with

\[
\begin{align*}
P &= J_{\nu} f_\nu - J_{\nu,r} f, \\
Q &= Y_{\nu} f_\nu - Y_{\nu,r} f, \\
W &= (P^2 + Q^2)^{-1/2}.
\end{align*}
\]

The obtained constants enable us to calculate the integral (10).

In order to convert the above integral into the local density of states we have to multiply it by two additional constants. One of them is the wave function normalization factor \( N \), which can be estimated calculating the integral of the squared wave function in the limit of large radius \( R \). The replacement of the Bessel functions by their asymptotic immediately leads to \( N = k/2R \). The second one is a consequence of the replacement of the summation over the discrete eigenvalues by the integration over energy, which is given by the factor \( R/\pi k \). Together they give \( 1/2\pi \), which results into definition of the local density of states

\[
\rho(E) = \frac{1}{2\pi}I(E).
\]

C. Numerical results

We solved numerically Eqs. (17). Inserting the obtained results into (16), and later in Eqs. (7) and (8) we obtained the wave function what enabled us to calculate the integral (10), and finally the local density of states (18).

A typical result for the local density of states as a function of electron energy is shown in Fig. 3. We clearly see peaks close to the energies of the Landau levels [6] calculated for the case of a homogeneous magnetic field. These peaks are broadened indicating that they are not really bound states in the magnetic dot. The broadening is larger for higher energy peaks.

The next thing which also is seen in Fig. 3 is a decreasing background with energy. This background is due to the states of the free electron in the absence of the magnetic dot. To justify this statement we made the same averaging (over circle of radius \( r_0 \)) with the same gaussian aperture function (11) of the radial component of the
free electron wave function (when there is no magnetic dot). This function reads
\[ \psi_{\text{free}}(r) = J_m(kr) \]
and is valid in the whole 2D plane. Inserting this function into integral (10) and later in Eq. (18) and using Tables of integrals\(^1\) we obtain the local density of states for a free electron
\[ \rho_{\text{free}}(E) = \frac{r_0^2}{2} e^{-E/b} I_m(E/b), \]
where \( I_m(x) \) stands for the modified Bessel function of the first kind. This local density of free electron in the case of \( m = 0 \) is shown in the same Fig. 3 by the blue dashed curve. Comparing these two curves we clearly see how increasing the electron energy we reduce the influence of the magnetic dot on the electron behavior, and the local density of states converts itself gradually into the free electron one.

We fitted the peaks in the density of states by Lorentzian functions \( a_n \gamma_n/\{(E - E_n)^2 + \gamma_n^2\} \) defining three parameters for any of them: the position \( E_n \), its broadening \( \gamma_n \), and the amplitude \( a_n \). Two of them (the position and broadening) are shown in Figs. 4 and 5 for different orbital momenta as functions of the radius of the dot \( r_0 \). The position of the quasi-bound states \( E_n \) are shown by the red solid curves while the broadening of the peaks is indicated by the shadowed areas limited by the \( E_n \pm \gamma_n \) curves.

Notice that the levels to the right of the green dotted curve are extremely narrow and their position coincides with the Landau levels \( \{n \} \) shown by the blue dashed horizontal lines. In fact this means that almost all electron wave function is located in the magnetic dot (using the classical description language we may say that the electron rotates along the Larmor circle inside the dot) and it

\[ m = 0 \]
\[ r_0 = 3 \]
The picture for negative $m$ values is different as shown in Fig. 6. All of them belong to the same Landau level. We see that with decreasing radius of the lifetime.

The increase of the peak broadening at small $r_0$ values is so steep that it is worth to divide all the peaks into two classes as shown in the above figures by the green dotted curves. The levels on the right side of these curves belong to essentially broadened quasi-bound states, while those on the right side from the experimental point of view can hardly be distinguished from the real bound states.

One can rudely estimate the position of this dividing curve comparing the approximate dimensions of the electron wave function calculated in the case of a homogeneous magnetic field (which actually coincides with function (7)) with the magnetic dot radius $r_0$. A more accurate estimation can be obtained solving the stationary Schrödinger equation and applying boundary conditions for these wave functions.

Applying the boundary conditions for these wave functions and their derivatives we obtain the following equation:

$$f_r(r_0)H(kr_0) - f(r)H_r(kr_0) = 0,$$

where for the sake of simplicity we omitted the indexes of the Hankel functions. The indexes which are only left indicate the derivative over the coordinate $r$. This equation has to be solved for a complex energy (or complex $k$), the imaginary part of the energy gives the inverse of the lifetime.

For finding the dotted curves in Figs. 4-6 separating the quasi-bound and nearly bound states it is enough to solve the above equation by means of a perturbation expansion in terms of the momentum difference $\Delta k = k - k_0$ where $k_0 = \sqrt{2E_{n,m}}$ with energy $E_{n,m}$ of the unperturbed Landau level. Limiting ourselves to first order in $\Delta k$ we arrive at the following expression:

$$\Delta k = \frac{H_{r,f} - H_{f,r}}{H_{r,k} + H_{k,f} - fH_{r,k} - H_{r,f}},$$

All functions and their derivatives over $r$ and $k$ have to be calculated at $r = r_0$ and $k = k_0$.

Now introducing the energy deviation from the Landau level energy

$$\Delta E = E - E_{n,m} \approx k_0\Delta k,$$

taking its imaginary part and equating it to $10^{-2}$ (it is expected that a smaller broadening can hardly be revealed experimentally) we obtained the points connected by the green dotted curve in Figs. 4-6 separating the quasi-bound states from nearly bound states.

### III. DIRAC ELECTRON IN GRAPHENE

Now we repeat the above calculation for the magnetic dot applying it to the case of a Dirac electron in graphene, where the low-energy quasi-particles (electrons and holes) are described by the following dimensionless Dirac-like Hamiltonian:

$$H = \sigma (-i\nabla + A).$$

Here, $\sigma = \{\sigma_x, \sigma_y\}$ stands for the $2 \times 2$ Pauli matrices. The units are based on the magnetic field strength $B_0$ and they are the same as in previous section, except the unit of energy which now is $v_F h/|B|$ with the Fermi velocity $v_F = 10^8$ cm s$^{-1}$. In the case of a 1 T magnetic field this energy unit is 2.6 meV. The vector potential is given by Eq. 8.
A. Solution of eigenvalue problem

The approach is based on the same stationary Schrödinger equation (1) but now with the matrix Hamiltonian (26), which results into a set of two differential equations. Assuming the wave function of the following form:

$$\Psi = e^{im\phi} \left( a(r) \right. \left. + i e^{i\phi} b(r) \right),$$

we arrive at a set of two equations for the radial wave function components

$$\begin{align*}
\left\{ \frac{d}{dr} + A(r) + \frac{m+1}{r} \right\} b &= Ea, \\
- \left\{ \frac{d}{dr} - A(r) - \frac{m}{r} \right\} a &= Eb,
\end{align*}$$

which has to be solved in the two regions ($I$ in the dot, and $II$ — outside it). We require the continuity of the obtained components at the dot border $r_0$

$$a_I(r_0) = a_{II}(r_0), \quad b_I(r_0) = b_{II}(r_0).$$

Instead of solving these first order differential equations it is more convenient to convert them into second order differential equations for a single component, say for component $b$

$$\begin{align*}
\left\{ \frac{1}{r} \frac{d}{dr} \frac{d}{dr} \right\} \left( \frac{(m+1)^2}{r^2} - \frac{r^2}{4} + [E^2 - m] \right) b_I &= 0, \\
\left\{ \frac{1}{r} \frac{d}{dr} \right\} \left( \frac{E^2 - (m + 1 + r^2/2)^2}{r^2} \right) b_{II} &= 0.
\end{align*}$$

In contrast to the electrical quantum dot case which was considered in Ref. [29] now the effective potential in Eqs. (28) is a continuous function at the dot border. For this reason the boundary conditions (29) are equivalent to

$$b_I(r_0) = b_{II}(r_0), \quad b_{I,r}(r_0) = b_{II,r}(r_0).$$

These boundary conditions are identical to those for the previous Schrödinger electron case [12]. It enables us to use the full analogy with the previous case. Taking this analogy into account we have for the solution in the two regions

$$\begin{align*}
a_I(r) &= A f(r) = A r^{m+1} e^{-r^2/4} M(|a_0| c_0^2 r^2 / 2), \\
b_{II}(r) &= B J_\nu(kr) + C Y_\nu(kr),
\end{align*}$$

where $k = |E|$, $a_0 = (|m + 1| + m + 1 - E^2)/2$, $\nu = m + 1 + r_0^2/2$, and $c_0 = |m + 1| + 1$. The expressions for the other wave function component $a(r)$ follow directly from Eq. (28a):

$$\begin{align*}
a_I(r) &= A r^{m+1} e^{-r^2/4} \times \left\{ \frac{d}{dr} + \frac{|m+1| + m + 1}{r} \right\} M(|a_0| c_0^2 r^2 / 2), \\
a_{II}(r) &= B J_{\nu-1}(kr) + C Y_{\nu-1}(kr).
\end{align*}$$

The wave function components obtained in the above way are illustrated in Fig. 7 for two different values of the energy. We see the same tendency. When the energy

$$E_{n,m} = \pm \sqrt{2+|m+1|+m+1},$$

is close to the Landau level energy of the Dirac electron in a homogeneous magnetic field,

$$E_{n,m} = \pm \sqrt{2+|m+1|+m+1},$$

(see the lower plot of Fig. 4 where the energy is close to the Landau level with $m = 0$, $n = 2$) we see a clear accumulation of the wave function components in the dot, what indicates a quasi-bound state.

B. Local density of states

Developing further the analogy of Eq. (30) with the considered previously case we calculated the local density of states using Eqs. (17), (10) and (10). Because we have now a wave function with two components, Eq. (10) is modified into

$$I(E) = 2\pi \int_0^{\infty} r dr F(r) \left\{ |a(r)|^2 + |b(r)|^2 \right\}. \quad (35)$$

Now the normalization factor is $N = k/4\pi$ (due to the two wave function components), and the factor responsible for the change of the summation over discrete eigenvalues into an integral over the electron energy is $R/\pi$. Thus, the local density of states in the case of a Dirac electron becomes

$$\rho(E) = \frac{|E|}{2} \int_0^{\infty} r dr f(r) \left\{ |a(r)|^2 + |b(r)|^2 \right\}. \quad (36)$$

In the case of free Dirac electrons (when there is no magnetic dot) the wave function components read

$$a_{free} = J_m(kr), \quad b_{free} = J_{m+1}(kr), \quad (37)$$

FIG. 7: (Color online) The wave function components: $a$ — dashed blue curve, and $b$ — red solid curve, for $m = 0$, dot radius $r_0 = 5$ and two energy values: $E = 1.932$ — upper plot, $E = 2.470$ — lower plot.
what leads to the following expression of the local density of states for a free electron:

\[ \rho_{\text{free}}(E) = \frac{|E|^3}{4} e^{-E^2/2b} \left\{ I_m(E^2/2b) + I_{m+1}(E^2/2b) \right\}. \]  

(38)

C. Numerical results

The typical local density of states calculated for \( m = 0 \) and \( r_0 = 3 \) is shown in Fig. 8 for positive energies. Two differences with respect to standard electrons can clearly be noticed. First, in the case of the Dirac electron the spectrum is symmetric with respect to energy inversion \( (E \rightarrow -E) \) due to the equivalence of electrons and holes. Thus the plot in Fig. 8 has to be supplemented by the same curves for negative energies. Second, comparing the density of states for Dirac electron with the same curve for the Schrödinger one (see Fig. 3) we see that there are more peaks. This can be explained by the more dense Landau level spectrum in the case of Dirac electrons (34) for the large quantum number values as compared with these for the previous case (9).

As before we fit the peaks by Lorentz type curves what leads to the broadened levels displayed in Figs. 9 and 10.

We see that the levels with \( m = -1 \) start at lowest energies what is just the consequence of chosen definition of radial wave function components (27). The green dotted curves divide the region of broadened quasi-bound states from the region where the states have a very small broadening. These curves were obtained in the same way as it was done in section [11] for the case of Schrödinger electron, namely, applying the complex energy eigenvalue technique. In the case of the Dirac electron it leads to the following imaginary energy part:

\[ \Delta = \frac{H_{\nu-1}b - H_\nu a}{H_{\nu}a_r + H_{\nu-1}b_r - H_{\nu-1}a}. \]  

(39)

The above mentioned dotted green line corresponds to \( \delta = 3 \cdot 10^{-3} \). Note we chose it three times smaller than in Figs. [10] what causes us to conclude that between the local density of states technique and complex energy eigenvalue method one can expect only qualitative agreement.

There is one more interesting point — the zero energy state which is shown in Fig. 10 by thick red line along the \( x \)-axis. Its behavior differs essentially from all other states. That is why it needs some special attention which
is presented in the next subsection.

**D. Zero energy state**

Now we check whether the Dirac electron has a zero energy state in the magnetic dot. In this case instead of Eqs. (25) we have to solve the following two equations for the radial components of the electron wave function:

\[
\begin{align*}
\left\{ \frac{d}{dr} + A(r) + \frac{m+1}{r} \right\} b &= 0, \quad (40a) \\
\left\{ \frac{d}{dr} - A(r) - \frac{m}{r} \right\} a &= 0, \quad (40b)
\end{align*}
\]

These are uncoupled differential equations of the first order, and their solution can be found by a straightforward integration. The solution has the following asymptotic behavior:

\[
\ln a(r) = \int \! dr \left\{ A(r) + \frac{m}{r} \right\} \\
\sim \left\{ \begin{array}{ll}
m \ln r + r^2/4, & r \to 0; \\
(m + r_0^2/2) \ln r, & r \to \infty,
\end{array} \right. \quad (41)
\]

and

\[
\ln b(r) = - \int \! dr \left\{ A(r) + \frac{m+1}{r} \right\} \\
\sim \left\{ \begin{array}{ll}
-(m+1) \ln r - r^2/4, & r \to 0; \\
-(m+1 + r_0^2/2) \ln r, & r \to \infty,
\end{array} \right. \quad (42)
\]

or

\[
\begin{align*}
a(r) &\sim \left\{ \begin{array}{ll}
r^m \exp(r^2/4), & r \to 0; \\
r^{m+1}r_0^2/2, & r \to \infty,
\end{array} \right. \quad (43)
\end{align*}
\]

and

\[
b(r) \sim \left\{ \begin{array}{ll}
r^{-m-1} \exp(-r^2/4), & r \to 0; \\
r^{-(m+1+ r_0^2/2)}, & r \to \infty.
\end{array} \right. \quad (44)
\]

In order to have the wave function with finite norm two boundary conditions have to be satisfied. First, the function should behave like \( r^\alpha \) (\( \alpha \geq 0 \)) when \( r \to 0 \), and second, it should behave like \( r^{-\alpha} \) (\( \alpha \leq -1 \)) when \( r \to \infty \).

For the \( a \) component the above conditions reduce to the requirements \( m \geq 0 \) and \( m + r_0^2/2 \leq -1 \), which can not be satisfied simultaneously. Consequently, we have to assume that \( a = 0 \).

In the case of component \( b \) the conditions read

\[-r_0^2/2 \leq m \leq -1, \quad (45)\]

from which it follows that if \( r_0^2/2 \geq 1 \) there are always some negative \( m \) values for which a zero energy state exists. When the radius of the dot decreases this interval becomes smaller, and the zero energy states vanish one by one. Finally, at \( r_0^2/2 < 1 \) all of them disappear.

Such essential difference between the bound zero energy level and all other quasi-bound levels is caused by the fact, that the wave function of the state with zero energy is real. Consequently, the electron in this state has no velocity, and as a result there is no tunnelling of this electron outside the dot. Unfortunately, the absence of any non zero electron velocity makes it impossible to reveal this state in transport measurements, but maybe it can reveal itself through the statistic properties of the magnetic dot.

IV. CONCLUSIONS

We considered the eigenvalue problem of a model quantum magnetic dot where the homogeneous magnetic field perpendicular to the 2D electron motion plane is created only in a finite region — in a circle of radius \( r_0 \). We showed that such a magnetic field fails to confine electrons both for the standard parabolic dispersion law or the ultra relativistic linear dispersion law for Dirac electrons in graphene. Although in such a magnetic dot no confined states are found, quasi-bound states with a finite lifetime are present.

An analysis of the quasi-bound states for the Schrödinger and Dirac electron was performed by means of the local density of states, and the position and width of the resonance peaks (the analogs of the quasi-stationary states), on the dot radius (or the strength of the magnetic field) were calculated.

The broadening of these peaks (the inverse lifetime of the quasi-bound state) is mainly caused by the touching of the quantum dot border by the electron wave function tail. Due to the exponential character of this tail there exist a rather sharp border between broadened quasi-bound states and those which can be considered as nearly bound ones. This border was found by applying the complex energy eigenvalue method which is shown to be in qualitative agreement with the results obtained from the local density of states technique.

It is shown that the difference of the quasi-bound states in the magnetic dot between the Schrödinger and Dirac electrons is only in the energies of these states which is a consequence of the different energies of the corresponding Landau levels.

There is a single exception: in the case of Dirac electrons there exists a zero energy bound state for negative values of the angular momentum (the momentum which is opposite to the direction of the classical electron rotation along the Larmor circle). When the dot radius \( r_0 \) (or the magnetic field strength) decreases the degeneracy of this zero energy level decreases skippingly while all other quasi-bound states disappear smoothly via their broadening.
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