Abstract

Substantial improvements in material processing and manufacturing techniques in recent years necessitate the introduction of effective and efficient nondestructive testing (NDT) methods that can seamlessly integrate into day-to-day aircraft and aerospace operations. Lamb wave-based methods have been identified as one of the most promising candidates for the inspection of large-scale structures. At the same time, there is presently a high level of research in the field of autonomous mobile robotics, especially in simultaneous localization and mapping (SLAM). Thus, this paper investigates a means to automate Lamb wave-based NDT by positioning sensors along a planar structure through mobile service robots. To this end, a generalized method for the mapping of plate structures using scattered Lamb waves by means of frontier exploration is presented such that an autonomous SLAM-capable NDT system can become realizable. The performance of this novel Lamb wave-based frontier exploration is first evaluated in simulation. It is shown that it generally outperforms a random frontier exploration and may even perform near-optimal in the case of an isotropic, square panel. These findings are then validated in laboratory experiments, confirming the general feasibility of utilizing Lamb waves for SLAM. Furthermore, the versatility of the developed methodology is successfully demonstrated on a more complexly shaped stiffened panel.

Keywords: Lamb waves, wave scattering, mapping, frontier exploration, nondestructive testing

1. Introduction

In recent decades, advancements in material processing and manufacturing has allowed for widespread and ever-increasing integration of composites in the design of aircraft. In comparison to traditional, isotropic materials, the anisotropic material properties of fibrous composite laminae allow for optimal material usage with regards to stiffness-to-weight and strength-to-weight ratios. However, composite structures are susceptible to failure modes, such as delaminations and disbonds, that are not readily identifiable by visual inspection. Ensuring the structural health of aircraft structures presently involves tedious inspection procedures, disassembly, and scheduled replacement intervals. That is, healthy structures are often replaced
because they exceed a set number of flight hours, and the operational cost is thus very high. Additionally, current nondestructive testing (NDT) of aircraft structures typically requires a trained technician with specialized equipment, and often involves point-wise through-thickness ultrasound testing. Such inspections usually occur due to suspicion of internal damage and only in a specific area of a structure. Hence, there is a need to introduce a more efficient, ideally autonomous, method. It has been shown that guided ultrasonic wave-based methods generally allow for finding damages in metallic and composite components (e.g. [1–3]). However, little research has been conducted to automate such measurement techniques or localize damages and other features in an autonomous fashion. It should be noted that the proposed automation of NDT is to be seen complimentary to continuous structural health monitoring (SHM) strategies when embedded sensors are available, with many synergies between the two methods in terms of optimal sensor placement (e.g. [4]).

The large-scale nature of airplane surfaces and other aerospace structures suggests the need for efficient, long-range NDT methods making guided ultrasound an appealing modality [5, 6]. Such techniques have seen a lot of attention in recent years by the research community. In particular, guided waves in plates and plate-like structures are called Lamb waves [7]. The general propagation characteristics of Lamb waves in metal and fiber-reinforced composite plates and plate-like structures are now well-understood [8–10], and advanced source localization techniques have been proposed [11, 12]. Furthermore, it has been shown that damages can be identified over long ranges using Lamb wave-based techniques [2, 3, 13, 14]. In addition to defects, such methods also allow for the detection of structural features as the waves also scatter at any other material or geometrical discontinuity [15–19], including edges and stiffeners. If the scattered waves are recorded, localization can be achieved through a time-of-flight (TOF) analysis. This has recently been investigated for finding the free edges of a plate structure [20, 21]. However, the methods included strong assumptions on the orientation of the (square) plate. Notwithstanding, the ability to detect and localize structural features, be it intrinsic geometry or potential damage, can be treated as a mapping problem from the perspective of a mobile robot.

The task of using a mobile sensor to map features in an environment intersects two classical problems in robotics: exploration and simultaneous localization and mapping (SLAM). Counter-intuitively, SLAM techniques often do not build a map that includes the application-specific features of interest. Instead, modern-day SLAM is more concerned with finding and updating reference landmarks (unique environmental point features such as unique visual patterns) that allow the robot to localize itself [22]. Lately, some semantic information, such as object classes, have started to be included in robot maps [23]. Practical applications often use occupancy grid representations [24], as in this work, and modern techniques are quite robust [25]. However, robot self-localization can be considered orthogonal to this work. Exploration, on the other hand, is the problem of choosing robot motion to find all features in an environment. If the environment is unknown, the most common and successful algorithm for one or more robots is frontier exploration [26–28].
This algorithm sends robots to the edges of the explored space according to a customized utility function (most often closest-first) \cite{20}. However, frontier exploration is typically visibility-based. This means that it is straightforward to estimate newly-explored areas from any candidate robot location along the frontier since robots are assumed to make predictable observations independent of other robots. This assumption does not trivially hold with Lamb wave-based feature detection since the sensors are typically configured in a pitch-catch configuration, thus requiring a collaboration between the sensing robots. In other words, sensor readings at the catcher are dependent on the pitcher’s placement in the environment. A commonly-deployed technique for searching a known environment is decomposition-based coverage \cite{30}. Yet this technique suffers from the same limiting assumption on the sensors as the previous algorithm, in that individual sensor readings are independent events. Thus, a complete mapping by both robots does not necessarily guarantee all features will be found, since the configuration of both pitcher and catcher must be aligned to detect features. Therefore, extending and evaluating frontier exploration for this new sensing modality is one of the major contributions of this work.

In this paper, a novel approach is presented that can map the surfaces of metallic (isotropic) and quasi-isotropic composite plate structures through the use of Lamb waves. By determining all of the potential sources of an edge reflection, the region where an edge cannot exist becomes known. The underlying principles are similar to those that have been applied to diagnostic imaging \cite{31–33}, as well as sonar-based mapping \cite{34}. This edge detection concept is further expanded to frontier exploration as the ground work for Lamb wave-based SLAM. As such, this work is intended to support future development of autonomous mobile robots capable of NDT using Lamb waves. It should also be noted that the presented method should be seen complimentary to other sensing modalities suitable for edge detection and mapping, such as computer vision and lidar techniques \cite{23}, thermography \cite{35} and phased-array ultrasound systems \cite{36}. Theoretical limits to the possible inspection area of the proposed approach, which take into account sensor as well as sample geometry, are derived. The approach is evaluated in simulations alongside several metrics developed for comparison. A laboratory experiment is conducted in which the capabilities of exploring and mapping an unknown plate structure with Lamb waves are validated.

2. Methods

2.1. Lamb wave propagation and scattering

Lamb wave propagation in homogeneous isotropic plates, isotropic multi-layered plates and anisotropic laminates has been studied by numerous authors (e.g. \cite{10, 37, 40}). Lamb waves are classified as symmetric ($S_i$) and antisymmetric ($A_j$) waves. While NDT methods have generally been used to detect failures in a structure, Lamb waves can also be used to detect structural geometries \cite{19}. For example, an incident wave reflects at free ends generally without any mode conversion if the excitation frequency is below the first cut-off frequency \cite{15, 17, 18, 11}. Above this frequency, it has been shown how obliquely incident Lamb
waves from a free end can scatter into multiple waves [42]. In any case, if the reflected waves are recorded, edges and step discontinuities may be localized through a TOF analysis. In order to apply a TOF analysis of the measured waves, the dispersion behavior has to be well-understood. For this purpose, many different (numerical) techniques have been developed to determine the Lamb wave propagation characteristics in isotropic and anisotropic materials [10, 43–45]. For the remainder of this work, it is assumed that Lamb waves propagate with the same velocity in all directions, i.e. the analysis is limited to isotropic and quasi-isotropic plates. It should be noted that while composite materials are generally anisotropic, currently used layup practices often lead to quasi-isotropic composite panels, and such structures can therefore be studied with the presented methodology. Furthermore, it is assumed that the incident wave reflects in form of the same wave type with sufficient amplitude.

The aforementioned propagation characteristics serve as the fundamental principles that enable Lamb wave-based mapping. The presence of a free edge may be ascertained through the use of actuator-sensor pairs in a pitch-catch configuration. Lamb waves (often predominantly A\textsubscript{0} waves) induced into a plate structure will be reflected at free edges. The resulting signals measured at the sensor include the induced waves traveling along the direct path from actuator to sensor as well as reflections originating from points along plate edges, as shown in Fig. 1a. Thereby, the first significant wave packet corresponds to the incident A\textsubscript{0} wave, while the second corresponds to the reflected A\textsubscript{0}A\textsubscript{0} wave from the closest edge. Applying TOF methods to the arrival time of the A\textsubscript{0}A\textsubscript{0} wave results in the length of the path from the actuator via the nearest edge point to the sensor. As illustrated in Fig. 1b, the set of all points where the reflection could have originated from is the boundary of an ellipse with the actuator (⋄) and sensor (◦) as the foci [31]. In other words, the inside of the ellipse does not contain any reflection sources, such as edges or defects. This serves as the basis for frontier exploration and subsequent mapping under this sensing modality.
2.2. Occupancy grid from Lamb wave measurements

As detailed above, measurements from each actuator-sensor pair result in a series of ellipses \( E \) and bounded regions \( R \) that are clear of any reflection sources. Continually taking the union of all \( R \) results in a map of the plate surface. To generate this map, the environment is discretized into an occupancy grid using a zero-initialized, two-dimensional array \( O \) representing grid elements of size \( \Delta x \times \Delta y \). Here, \( \Delta x \) and \( \Delta y \) are the chosen step-sizes in the \( x \)- and \( y \)-directions, respectively \[34\]. A dataset of \( k \) measurements results in a set of ellipses \( E \). For every grid element \( O_{ij} \), there is a corresponding point with location \((x_i, y_j)\) in the global frame of reference. The location of every grid element \( O_{ij} \) is evaluated against every ellipse in \( E \). Thus, \( \forall i, j \): if \((x_i, y_j)\) is located within a member of \( E \), the value of \( O_{ij} \) is increased by 1. As a means to address errors and uncertainty, a threshold \( T \) (here: \( T = 1 \)) can be established as the minimum value needed for any \( O_{ij} \) to accept the corresponding grid element as being occupied by the surface. If \( I \) is defined as an array of equal dimensions as \( O \) and

\[
I_{ij} = \begin{cases} 
0, & O_{ij} < T \\
1, & O_{ij} \geq T 
\end{cases}
\]

then \( I \) is a binary image of the environment. The nonzero elements of \( I \) signify mapped areas of the structure, or explored cells. In order to quantitatively measure how well \( I \) represents the actual plate structure, the coverage \( C \) is introduced. To this end, grid elements of \( I \) within the true boundaries of the structure are considered to have positive area while all outside elements are considered to have negative area, as a penalty. Thus, \( C \) is the percentage of correctly mapped area minus the percentage of false positive area with respect to the true surface area of the plate.

2.3. Random frontier exploration

The general feasibility of Lamb wave-based edge-detection and mapping has been demonstrated by Miranda et al. \[46\] for arbitrarily shaped plates in simulations and a rectangular, isotropic plate in a laboratory experiment. However, the work utilized a) a rectilinear grid based on a priori knowledge of the environment, or b) random transducer placements. Since the grid (case a) was based on the dimensions and orientation of the plate, the resultant mapping can only be considered a verification of a known map. On the other hand, mapping from random transducer placements (case b) can merely be seen as a baseline and lacks optimality. Nonetheless, random-based approaches can be seen as infimums for evaluating the performance of strategic algorithms. To this end, the random approach is extended in this work to a frontier exploration approach \[26, 27\] integrated with the Lamb wave-based occupancy grid techniques to provide a mapping modality suitable for SLAM. For every measurement after the initial, a random choice is made as to which transducer (pitcher or catcher) will change location for the next measurement. The new location for the selected transducer is then randomly chosen only from all frontier cells, where frontier cells are cells on the perimeter of the explored space.
2.4. Greedy exploration

Though a random approach will eventually generate a map, random choices may not provide an efficient sequence for mapping an environment. Specifically, a random search lacks a mechanism to utilize information gained about the environment as well as a means to predict future gains. To illustrate the effect of evaluating expected gains for each transducer placement, the mapping problem is reduced to a half-space environment. Figure 2 illustrates the case of transducer placements where the edge reflection point that will be detected from a measurement will be located along a single line. Without loss of generality, the transducers are assumed to be near the boundary of the half-space at $y = 0$ mm. The initial locations of the actuator (○) and sensor (◦) result in the ellipse $E_1$. The region $R_1$ bounded by this ellipse serves as the initially explored cells of the occupancy grid. For the next measurement in the mapping operation, the location of at least one of the transducer locations must be changed. In this initial investigation, it is imposed that only one of the transducers can be relocated at a time and only within the explored environment. For a greedy approach, it is desirable for the change in a transducer location within $R_1$ to result in the greatest amount of new information. Let $C_k$ denote the coverage after the $k$-th measurement. Then, the resultant coverage after a second measurement is given by

$$C_2 = R_1 \cup R_2 = R_1 + R_2 - R_1 \cap R_2$$ (2)

where $R_1 \cap R_2$ is the area of the region bounded by both $E_1$ and the new ellipse $E_2$. This ellipse-ellipse overlap area can be determined using the methods described in [47].

Evaluating Eq. (2) for a new sensor location while keeping the actuator stationary results in the surface shown in Fig. 2a. It can be seen that the direction of higher coverage increase is away from the stationary...
transducer. The sensor location with maximum coverage increase is denoted by the solid blue (○) with the coverage increase being 39.9%, and the corresponding newly explored cells are shown in gray. The alternative option of moving the actuator while keeping the sensor stationary is shown in Fig. 2b. Moving the actuator away from the edge to the point shown as solid red (⋄) results in a coverage increase of 195.3%. The large disparity between these two cases suggests that edge distance has a significant influence on coverage gain. Consequently, knowledge of the nearest edge is critical for optimal transducer placement.

Though this half-space example assumes all other edges to be distant, this approach nonetheless provides an expectation of coverage gain given knowledge of an initially known edge. In the event that a map of the environment is known a priori, determining ellipse-ellipse overlap over the explored region and maximizing coverage gain results in a greedy solution even if other edges are not distant. In the context of exploration, such an approach is not viable as a known map is unavailable. Rather, the map is to be constructed through measurements. Nonetheless, the result of such a method provides a metric for comparison. Furthermore, the surfaces seen in Fig. 2 show gradients that point towards the frontier. This suggests potential value in the placement of transducers along the frontier of an explored space.

2.4.1. Pansophic greediness

While it is apparent that the approach used in the example of the previous section could provide an optimal solution given a known map, its computational complexity increases dramatically over subsequent measurements. Specifically, a method for determining the overlap area of three or more ellipses is required, which is beyond the scope of this work.

Thus, an alternative known-map approach is established as follows: Consider a rectangular plate of arbitrary size \((L \times W)\). For a desired spatial resolution \(h\), an occupancy grid \(O\) of size \(m \times n\) is sufficient for mapping the plate, where \(m = L/h\) and \(n = W/h\), respectively. Assuming that a transducer may be placed at any point within \(O\) for a measurement, there exist \(S = \binom{mn}{2}\) unique measurement configurations in \(O\). In the context of exploration, however, the number of possible configurations for the next measurement is in regards to the explored space. For instance, the explored cells in \(O\) after an initial measurement define the discrete region \(R_1\), which is an approximation of the region bounded by the initial ellipse \(E_1\). Subsequently, for measurement \(k + 1\), possible transducer locations are cells within \(R_1 \cup R_2 \cup \ldots R_k\). With the environment known a priori, it is possible to determine the \(S\) possible measurement outcomes within \(O\) beforehand.

For any given state of the occupancy grid, the greediest next transducer placement can be determined by searching the outcomes for transducer configurations that are presently possible for the explored space. Subsequently, a mapping solution similar to the half-space example is obtainable. Rather than attempting to compute all possible measurement outcomes in \(O\), the magnitude of \(S\) can be reduced by restricting transducer placements to specific locations.

The approach is illustrated in Fig. 3 where the transducers are in the same initial configuration as
the previously presented half-space example. The explored space resulting from the initial measurement is shown in Fig. 3a as a dark gray area while the points represent possible transducer locations for the next measurement. Ground truth is represented by dashed lines. The newly explored cells for $k = 2$, shown in light gray, are the result of moving the actuator to the point of maximum coverage indicated by the red (○). The result indicates that the greatest increase in coverage occurs by moving the actuator to a location near the frontier and above the sensor. This is consistent with Fig. 2b of the half-space example. As the frontier of the occupancy grid grows, so does the number of allowed transducer locations as evident by the next iteration shown in Fig. 3b. Subsequently, the number of required evaluations for transducer placement possibilities increases as well. However, the procedure for determining the greediest outcome remains the same. This method can also be adapted to an arbitrarily shaped plate by defining appropriate allowable transducer locations. However, for the purposes of mapping a plate with no a priori knowledge in real-time, pansophic greediness is not feasible as knowledge of the increase in coverage is required. This in turn necessitates knowledge of the plate boundaries. Nonetheless, this approach can be used as a metric for comparison for other exploration algorithms.

2.4.2. Lamb wave-based frontier exploration

As mentioned above, the approach of pansophic greediness is not feasible for many real-world applications as it requires a priori knowledge of the environment. Therefore, this work proposes a Lamb wave-based frontier exploration strategy (LFE) that employs a methodology for estimating the environment and tracking potential edge locations. To this end, subsequent transducer movements are determined to maximize information gain based on the estimations of the environment. Thus, this strategy can also be considered greedy. It should also be noted that in this initial work, the environment is assumed to be rectangular for the sake of establishing perceived edges/obstacles and subsequent expected coverage increases.
From the initial measurement, the only available information about the environment is the presence of one or more edge locations along the frontier of the explored space. In this state, edge segments could be tangent anywhere along the curve $E_1$. Thus, the transducer locations for the second measurement are determined by randomly choosing one of the transducers to move to a random location on the frontier of the explored space. The explored space in the occupancy grid is then updated to $R_1 \cup R_2$. After each measurement beyond the initial, the minimum bounding box for the explored cells is determined as shown in Fig. 4a. The bounding box serves as an estimate of the the unknown environment. A new estimated space is defined with a coordinate system $(\tilde{x}, \tilde{y})$ relative to a vertex of the box. This space is used to search for a mapping solution. As in the random approach, frontier cells serve as allowable transducer locations. Assuming that Lamb wave reflections occur at the edges of the bounding box, the expected coverage increase is then evaluated by individually moving either the actuator or sensor to a location on the frontier. However, as evident from the pansophic approach, it is not necessary to evaluate the entire frontier for a favorable result. To that end, the frontier is simply sampled. The greediest result within the estimated space is then used for the next measurement, as shown in Fig. 4b. It can be seen that use of the minimum bounding box to estimate the environment provides favorable results in this instance. That is, when the edges of the bounding box are reasonably well-aligned with the true environment.

Conversely, a misaligned bounding box may result in suboptimal measurements or gridlock in the worst case. Figure 5a shows the state of a mapping operation where the estimating bounding box is poorly aligned with the true edges of the plate. The expected, greediest coverage increase is shown in light gray. However, the true increase in coverage, as shown in Fig. 5b, is marginal. A clear discrepancy exists between the expected and actual change in coverage. Thus, it is necessary to supplement the estimated environment with obstacle cells. As previously mentioned, Lamb wave reflections cannot originate within the region.
bounded by the ellipses. Given that the explored space is the union of these bounded regions, edge locations are assumed to be along frontier cells. In other words, obstacle cells are defined as frontier cells that are believed to exist on or near an edge of the true environment. For the case shown in Fig. 5a, the bounding box extends past the bottom edge of the plate, and the greediest solution includes a nonexistent region in its estimate. As evident by Fig. 5b, actual coverage increase is stymied by the bottom edge. The resultant ellipse from the measurement, however, does indicate the presence of a reflecting edge somewhere along the newly initialized frontier cells. Specifically, initialization of obstacle cells is determined by comparing the expected change in coverage with the actual change resulting from a measurement. To this end, the parameter $\gamma$ is defined as:

$$\gamma = \frac{C_{k+1} - C_k}{\tilde{C}_{k+1} - C_k}$$

\begin{align*}
\gamma < 1 & \quad \text{coverage below expectation, edge within estimated space} \\
\gamma = 1 & \quad \text{coverage equals expectation} \\
\gamma > 1 & \quad \text{coverage exceeds expectation, clear beyond estimated space}
\end{align*}

In Eq. (3), $C_k$ is the current coverage while $C_{k+1}$ and $\tilde{C}_{k+1}$ are the true and expected coverages for measurement $k+1$, respectively. Intuitively, a perfect estimation of the environment will result in $\gamma = 1$. However, this outcome is also possible when the estimated space correctly represents true edges based on the current knowledge about the environment (see Fig. 4). Thus, it can only be said that the newly mapped region has coverage gain equal to the greedy solution proposed in the estimated space. As the true environment is unknown, $\gamma > 1$ implies information gain outside of the current representation of the environment. Naturally, this debunks the current estimate which is preferable for this mapping algorithm. The case of $\gamma < 1$ indicates the presence of an edge within the bounding box. Despite this conclusion, coverage has still increased, though less than expected. If $\eta$ is taken to be the minimum acceptable value for $\gamma$ after a measurement, then new frontier cells are flagged as obstacles cells whenever $\gamma < \eta$. These obstacle cells are
indicated by red dots as seen in Fig. 5b. Since the obstacles are located on the perimeter of the latest ellipse, they mark the possible points of reflection on the intruding edge. Subsequently, obstacle cells are considered as additional reflectors for Lamb waves in addition to the bounding box segments, when determining the next greediest measurement location. This means that no solution in the estimated space could possibly encircle an obstacle cell. Thus, greedy solutions in future iterations are prevented from extending past the interfering edge in the same manner that caused the poor result.

The explored space shown in Fig. 5b is only slightly different than the previous iteration seen in Fig. 5a. For cases where \( \gamma \ll 1 \), the minimum bounding box does not lead to a significantly different estimate of the environment for the next iteration. Hence, a gridlock-like scenario would be expected. In order to mitigate this problem, the presence of obstacle cells is also used to determine the bounding box as follows. Let \( A(\theta) \) denote the area of a minimum bonding box encasing the explored space, where \( \theta \in \left[ 0, \frac{\pi}{2} \right) \) is the orientation of the box with respect to the \( \tilde{x} \)-axis. That is to say, \( \forall \theta \) there exists a minimum bounding box with nonzero area that encompasses the explored cells of the occupancy grid. Let \( V(\theta) \) denote the variance of the shortest distance from each obstacle cell to an edge of the respective minimum bonding box. Thus, for a given orientation of \( \theta \), there exists a minimum bounding box with area \( A \) and relative alignment \( V \) with obstacle cells. A cost function \( J \) is defined as

\[
J = w_1 A(\theta) + w_2 V(\theta),
\]

where the weights are chosen as

\[
w_1 = \frac{1}{\max_\theta A(\theta)} \quad \text{and} \quad w_2 = \frac{1}{\max_\theta V(\theta)}.
\]

Minimizing \( J \) with respect to \( \theta \) results in the selection of a bounding box that adequately considers obstacle cells. Figure 6a illustrates the effect of minimizing \( J \) when obstacle cells are present. The orientation of the bounding box is substantially different compared to the one before the new obstacle cells are established (see Fig. 5a). With the bounding box now more in line with the bottom edge of the plate, the resultant measurement, as shown in Fig. 6b, is in close agreement with the prediction \( (\gamma \approx 1) \).

Lastly, as can be seen from Fig. 6, false obstacle cells may be established, potentially even in the middle of the (unknown) space. Hence, a procedure to clear false obstacles is necessary. Upon initialization, obstacle cells are assigned a persistence value \( \lambda \), a nonzero positive integer. The initial value of \( \lambda \) for each obstacle cell is determined by

\[
\lambda = \text{ceil} \left( a e^{-bs} \right) + c
\]

where \( s \) is the number of obstacle cells initialized in the present iteration, and \( a, b, \) and \( c \) are constants. In Eq. (6), \( s \) is taken to be an approximation of the length of the ellipse perimeter from a measurement that exists outside of the explored space. As a consequence of Eq. (6), persistence is a measure of the likelihood that an obstacle cell is on or near a true edge. If mapping performance necessitates initialization of obstacle
cells, the assigned persistence of the new obstacle cells is greater when fewer obstacle cells are initialized. In other words, as \( s \to 1 \), the Lamb wave reflection is assumed to come from a point source. Thus smaller values of \( s \) provide greater evidence of a true edge in comparison to multiple reflection points when \( s \) is large. For each obstacle cell, \( \lambda \) decays by 1 after each measurement. The obstacle cell flag is removed once \( \lambda = 0 \). Naturally, subsequent measurements may clear obstacle cells if the ellipse \( E_{k+1} \) encompasses these cells. This automatically applies \( \lambda = 0 \) to each of the “cleared” obstacle cells.

In summary, for each iteration, a bounding box is fit to the explored space in accordance with the cost function \( J \). Within the estimated space, transducer placements along the frontier are evaluated in search of the greediest outcome, and a measurement is taken in the true environment. Utilizing Eq. (3), if a discrepancy between the expected and actual coverage gain exists such that \( \gamma < \eta \), new frontier cells are flagged as obstacle cells and are assigned persistence values in accordance with Eq. (6). The persistence value \( \lambda \) of all obstacle cells decreases by 1 with each iteration, and any obstacle cells mapped by the subsequent measurements become explored cells (i.e. \( \lambda = 0 \)).

2.5. Experimental setup

In order to validate the proposed LFE strategy and related simulations, laboratory experiments are conducted. As shown in Fig. 7, two ultrasound contact transducers (Digitalwave B-454) are arranged in a pitch-catch configuration. Ultrasound gel is applied between the plate and transducers, and a weight is placed on top of the transducers to help achieve consistent contact conditions. A short 8-cycle sinusoidal Hann-windowed tone burst at \( f = 250 \text{kHz} \) is generated by an arbitrary waveform generator (Keysight 33512B). This center frequency is chosen based on previous experiments [21], as it results in low-dispersion, high-amplitude fundamental \( A_0 \) waves and leads to sufficient resolution, as shown below. It should be noted that even though a fundamental \( S_0 \) wave is also excited, the significantly lower amplitude and faster propagation speed compared to the induced \( A_0 \) wave generally allow for a clear distinction between the two.
waves and their reflections (see Fig. 1a). To record the measurements, the sensing transducer is directly connected to the oscilloscope (Cleverscope CS320A). In order to improve the signal-to-noise ratio of the measurements, the analyzed signal is the time-average of 128 repeated acquisitions.

The Lamb wave experiments are conducted on a pristine 6061-T6 aluminium panel with dimensions of 609.6 mm ($L$) × 609.6 mm ($W$) × 3.175 mm ($2H$). The origin of the $xy$-coordinate system is at the bottom left corner of the plate, as shown in Fig. 7b. Transducers are manually positioned ($\pm$1 mm accuracy) based on this datum corner for each measurement. For the chosen excitation frequency $f$ and the material properties (Young’s modulus $E = 69$ GPa, Poisson’s ratio $\nu = 0.3269$ and density $\rho = 2700$ kg/m$^3$), the group velocities of the $A_0$ and $S_0$ waves are determined as $c_{A_0} = 3010$ m/s and $c_{S_0} = 5142$ m/s [10], respectively. The theoretical group velocities are confirmed in experiments, and are used in subsequent TOF analyses.

Due to the physical dimensions of the transducers and the envisioned application of the proposed method on mobile robots, several constraints are imposed on the placement of the transducers. The minimum distance between two transducers and the distance between either of the transducers and the plate edges must be at least 20 mm.

2.6. Theoretical limit

Application of this Lamb wave-based method in a real-world mapping operation necessitates consideration of its limitations due to the physical constraints of transducer deployment. Placement of a transducer directly on an edge imposes the risk of falling off of a structure. Additionally, pitcher and catcher cannot occupy the same location at the same time. Both minimum transducer to edge distance as well as minimum transducer separation distance must be taken into account. Consequently, mapping of the entirety of a sharp corner of a structure is not possible as there will always exist an edge point closer to a transducer than the corner point.

Determination of this unmappable area is illustrated in Fig. 8 for a corner with angle $\theta$. Maximum
mapping of a sharp corner occurs when both transducers are as close as possible to each other as well as
colinear with the corner point $C$. A transducer must also be located as close to an edge as allowed, denoted
as length $r$ in Fig. [8]. The resulting ellipse will have two reflection points, $A$ and $B$, which are symmetric
about the ellipse major axis. For this ellipse, the linear eccentricity $c$ is simply half the distance between
the transducers. Thus, the length $2c$ denotes the minimum transducer separation distance in this analysis.
The semi-major axis length $a$ in the ellipse corner case is
\[ a = c \cos \left( \frac{\theta}{2} \right) \sqrt{1 + \beta^2}, \] (7)
where
\[ \beta = \frac{r + c \sin \left( \frac{\theta}{2} \right)}{c \cos \left( \frac{\theta}{2} \right)}. \] (8)

Determination of $b$, the semi-minor axis length, can be done through the relationship
\[ b = \sqrt{a^2 - c^2}. \] As
evident in Fig. [8] the area of $\triangle ABC$ less the area of the enclosed ellipse segment results in the unmappable
corner area $\delta$ described by Eqs. (9):
\[ \delta = \frac{b^2 \sin^2 u}{\tan \frac{\theta}{2}} + ab \left( \sin u \cos u - u \right), \] (9)
where the parameter $u$ corresponds to point $A$ along the ellipse path and is given by
\[ u = \tan^{-1} \left( \frac{b}{a \tan \frac{\theta}{2}} \right). \] (10)

For a plate structure, the theoretical limit of a mapping operation using a pitch-catch configuration is the
true area minus the summation of all unmappable corner areas. The implication is that, even under the
best circumstances, mapping under this Lamb wave-based sensing modality cannot definitively map a plate
structure with sharp corners. Nonetheless, once plate geometry becomes apparent or is known, the point
when coverage can no longer increase is determinable.
3. Results and discussion

3.1. Simulations

Simulations in MATLAB are performed to evaluate LFE proposed in this work. The approach is applied to the 609.6 mm × 609.6 mm aluminum plate as described in Section 2.5 and the results are compared to both random frontier exploration as well as pansophic greediness. The initial condition for all simulations is the deployment of the actuator at (203.20 mm, 60.96 mm) and the sensor at (386.08 mm, 182.88 mm), as seen in Fig. 7b. Pansophic greediness utilizes a 29 × 29 evenly spaced grid centered about the plate (see Fig. 3). The resulting spacing of 20.32 mm is chosen as a matter of convenience with regards to the plate dimensions and the physical constraints of transducer deployment. It is envisioned that mobile sensor robots will possess cliff detection capability. Thus, if a new transducer location is chosen (whether randomly or through environment estimation) that does not satisfy the physical constraints, the nearest point which satisfies the constraints is used for the next measurement. For the condition to establish obstacles cells \( \gamma < \eta \), \( \eta = 0.15 \) is used. The constants in Eq. 6 for determining \( \lambda \) are \( a = 8 \), \( b = 1/30 \), and \( c = 3 \). The chosen values for \( a \) and \( c \) result in obstacle cells persisting between 4 to 11 measurements, and the value for \( b \) is a preliminary mapping of \( s \) to the likelihood that obstacle cells are near an edge.

Figure 9 shows the intermediate occupancy grid at \( k = 5 \) for both pansophic greediness as well as LFE (representative “actuator-first” median result). It can be seen that due to the a priori knowledge, the pansophic approach achieves coverage significantly quicker than when greediness is applied to an estimation of the environment, as a disparity of 38.83% coverage exists between the two in favor of pansophic greediness. The “final” coverage at \( k = 20 \) is shown in Fig. 10. At this stage, pansophic greediness nearly maps the entire plate at 98.99% coverage. While LFE now achieves 93.90% coverage. Further measurements will result in minimal gains for pansophic greediness, but LFE will use a bounding box that is nearly equivalent to true plate dimensions and orientation. Thus, it will eventually approach an ultimate coverage comparable to the
The coverage increase with each (simulated) measurement for up to $k = 50$ is shown in Fig. 11. For pansophic greediness, indicated in solid green, the result is completely deterministic. In the case of random frontier exploration and LFE, several trials are conducted. For the random case, indicated by dash-dotted blue lines, 1000 simulations are performed. For LFE, indicated as dashed red lines, the initial measurement results in 728 frontier cells. Given the option to move either the actuator or the sensor for the second measurement, there are a total of 1456 outcomes. As these results are not normally distributed, the curves shown in Fig. 11 are the median result. The bounded region shows where the middle 68.3% of resultant coverages lie. The advantage of having a known map is clearly evident in Fig. 11. The pansophic curve is extremely steep for the first few iterations, obtaining over 90% coverage by the sixth measurement. Afterwards, the rate of coverage gain decreases with pansophic greediness appearing to approach a final
value. LFE will, on average, outperform random frontier exploration as evident by the median result curves of Fig. 11. A notable exception to this is in the first few measurements for the “sensor-first” case, where LFE tends to perform worse than a random search as seen in Fig. 11a. As previously mentioned, transducer distance from edges has a significance influence on the size of the resultant ellipses. Thus, for this initial transducer configuration, where the actuator is closest to an edge, moving the sensor first results in a “poor start” to the mapping operation. On the other hand, for the “actuator-first” case, shown in Fig. 11b, it can be seen that a significant portion of trials of the frontier exploration nearly match the performance of pansophic greediness during the first few measurements. This is unsurprising as pansophic greediness is also an “actuator-first” case. In either case, it can be seen that LFE will eventually converge to a solution similar to pansophic greediness. This is evident by the shortening of the distance between the upper and lower limits of the population majority as the number of measurements increases.

A summary of the expected coverage is provided in Table 1. The table shows how many measurements \( k_{90} \) are necessary to achieve \( C = 90\% \), achieved coverage \( C_{20} \) after \( k = 20 \) iterations as well as the difference \( \Delta C_{20} \) compared to the theoretical limit, \( C = 99.86\% \). Overall, the simulation results indicate that the approach of applying greediness together with a bounding box that estimates the environment performs better than randomly exploring along the frontier.

### 3.2. Experimental validation

In order to validate LFE, laboratory experiments are conducted as described in Section 2.5. Actuator and sensor are initially placed at the same locations as in the simulations. For the second measurement, an actuator-first movement is chosen where the actuator is placed at \((413\text{ mm}, 97\text{ mm})\). This position is chosen for its likelihood to produce a result similar to the simulation median. A total of 26 measurements are conducted, and a TOF analysis is performed based on the \( A_0 \) mode, as shown in Fig. 1a and described in Section 2.1.

Figure 12a shows the state of the mapping operation in the experiment after five measurements. The dark gray region indicates the cleared space in the occupancy grid from \( k = 4 \), while the light gray area denotes the cleared space from the latest measurement. The significant coverage gain is due to an actuator movement, as indicated by the hollow \((k = 4)\) and solid \((k = 5)\) (○), respectively. At this stage, coverage is \( C_5 = 60.85\% \) after penalties are applied. With the eighth measurement, shown in Fig. 12b, \( C_8 = 77.27\% \), and the cleared space begins to resemble a rectangular shape. As LFE works to map the corners of the

| Case               | \( k_{90} \) | \( C_{20} \) [\%] | \( \Delta C_{20} \) [\%] |
|--------------------|--------------|-------------------|------------------------|
| Random frontier search | 57           | 75.84             | 24.02                  |
| Pansophic greediness   | 6            | 98.99             | 0.86                   |
| LFE (sensor first)     | 19           | 92.33             | 7.53                   |
| LFE (actuator first)   | 18           | 93.14             | 6.71                   |
estimated bounding box, coverage gain begins to plateau. Within a few more measurements, bias towards mapping local areas becomes apparent as evident in Fig. 12c for $k = 13$. Nonetheless, as potential coverage gains become more limited, the expected clearing of the space at the top of the estimated bounding box actually leads to the entire upper region of the plate to become known and $C_{13} = 91.27\%$ coverage. By the seventeenth measurement, shown in Fig. 12d, the bounding box nearly matches the plate dimensions and orientation. The method works to fill in the corners and any remaining gaps along the edges of the occupancy grid. The total coverage after this measurement is $C_{17} = 95.62\%$. While the presence of mapping error is noticeable, e.g. near the top edge, the total penalty due to false positives is small (0.59\%).

Figure 13a shows the final map generated from the experiment after 26 measurements. The experiment is concluded at this stage due to difficulties in analysing signals, which is explained in Section 3.4. Nonetheless, the resultant map is representative of the actual plate structure with $C_{26} = 97.4\%$ coverage. Furthermore, it can be seen that the experimental results closely match those of the simulations as evident by Fig. 13b. It can be seen that the experimental data appears to fit within the majority of simulation coverages. Exhibiting a smooth increase early on, coverage gain flattens around $k = 10$ which can be attributed to the bounding box
being aligned with three of the four plate edges. The measurement of Fig. 12c for \( k = 13 \) is clearly visible as a sudden jump in coverage, followed by a smooth increase as mapping approaches completion. Hence, despite the measurement noise and other uncertainty, such as transducer positioning and identifying the \( \text{A}_0 \text{A}_0 \) wave arrival times, the approach to Lamb wave-based mapping as proposed in this work is successfully validated.

### 3.3. Simulations with internal scatterer

The simulations for the square plate and subsequent experiment assume that no internal scatterers exist. Furthermore, it is assumed that group velocities are constant in all directions and at all locations. To test the feasibility of LFE when an internal feature exists, additional simulations are performed for an eight-sighted plate reinforced with a stiffener (see black outline in Fig. 14a). The material properties as well as thicknesses of the plate and stiffener are as described in Section 2.5, with the stiffener locally doubling the thickness. Simulations consider the adjustment in propagation velocity within the stiffened region as well as the incident wave’s reflection from the stiffener [19]. The initial transducer placements are west of the stiffener indicated in Fig. 14a by the unfilled (●) and (○) for the actuator and sensor, respectively. Based on the initial frontier, 492 “actuator-first” LFE trials are conducted alongside 1000 random exploration trials. All other parameters are the same as previously described in Section 3.1.

The mapping state shown in Fig. 14a is for \( k = 39 \) of a trial representative of the overall median coverage gain for the LFE trials. The total coverage at this point is \( C_{39} = 90.32 \% \), a noticeable difference in convergence speed compared to LFE on a square plate, which typically obtained 90 \% coverage by \( k = 19 \). The presence of obstacle cells, denoted by red dots in Fig. 14a indicate several poor outcomes leading to the mapping state shown. It is worth mentioning that this decrease in performance is not due to the stiffener, which has an almost negligible effect in this instance. Rather, the lower performance of the LFE method in this case is observed to be due to the shape of the plate as well as obstacle cell persistence.
(a) Representative mapping state for $k = 39$, $C_{39} = 90.32\%$ with initial transducer placements shown (unfilled $\Diamond$ and $\bullet$)

Figure 14: Results for LFE and random exploration on eight-sided plate when an internal scatter is present (simulations)

The coverage increase for these simulations up to $k = 100$ is shown in Fig. 14b, together with the theoretical limit of $C = 99.24\%$. The overall shapes of the curves in Fig. 14b are similar to those from the square plate simulations (cf. Fig. 11). Yet, these LFE results appear “lower”, especially in the long-term behavior. While it is clear in Fig. 14b that LFE initially outperforms random exploration, the random method appears to have greater gains in the long run. The drop in LFE performance can be attributed to how the environment is estimated as well as the decaying behavior of obstacle cells. The bounding box seen in Fig. 14a indicated in solid blue, is well-aligned with the environment, based on what has been mapped. However, this estimation does not encompass the longitudinal extremities of the plate structure. Subsequently, future transducer placements will attempt to map regions of the bounding box corners outside of the true plate boundaries, which is likely to flag obstacle cells. These correctly established obstacle cells, however, will decay over time in accordance with Eq. (6). Consequently, continual decay of obstacles results in repeated searching within a relatively static estimation of the environment with marginal gains. Lastly, though the stiffener shown in Fig. 14a can scatter waves in a manner that hinders mapping, many opportunities exist for the explored space to grow beyond and eventually encompass the stiffener. In the case of a stiffener that spans the entire width, initial transducer configurations with both actuator and sensor off to the same side relative to the stiffener, would result in the stiffener effectively demarcating regions of the plate from being mapped.

3.4. Limitations and discussion

This work focuses on the detection of the “outer” edges of the structure. However, internal features (e.g. ribs or stringers) and defects (e.g. cracks or delaminations) may also produce reflections of substantial amplitude that would be detected at the sensor, as considered in Section 3.3. In the future, the proposed approach can be extended to additionally identify such features following similar mapping procedures. Furthermore, even though an isotropic plate is studied in this work, it is evident that the presented approach
could directly be applied to quasi-isotropic composite laminates but would require modifications in case the wave propagation velocity is dependant on the propagation direction.

While the experiments, described in Section 2.5, are performed with laboratory equipment and the transducers are manually placed at the desired locations, it is apparent that this procedure could be implemented on a mobile robot, thus eliminating any human bias from the mapping procedure. However, the dispersive and multi-modal nature of Lamb waves and the waves’ superposition complicate automated TOF analysis of reflected wave signals. That is, even if transducer localization is perfect, certain transducer configurations may result in measured signals that are difficult or nearly impossible to interpret. The raw signals for the fourth and sixteenth measurements illustrate challenges for autonomous TOF analysis, as shown in Fig. 15. In this figure, raw signals are overlaid with the theoretical arrival times for the incident $A_0$ and reflected $A_0A_0$ waves, as indicated by black dashed and blue dash-dotted lines, respectively. For the first case, shown in Fig. 15a, clear distinction between the different wave packets is possible. The TOF analysis for this measurement can be easily automated by using methods such as wavelet [48] or short-time Fourier transform [49] processing as well as machine learning techniques [50]. On the other hand, the signal for the second case, shown in Fig. 15b, may present challenges for automated processing. The transducers are both in close proximity to the top left corner of the plate (see Fig. 12d), and the direct transducer and reflected wave path lengths are short and not significantly different. Consequently, the $A_0$ and $A_0A_0$ waves are superposed. Furthermore, as both transducers are near two edges, multiple other wave reflections arrive during the measurement. Though manual processing is still possible, this scenario poses a challenge for autonomous TOF analysis and may lead to a higher amount of erroneous ellipses. Hence, a threshold $T > 1$ (see Eq. (1)) may be chosen to mitigate some of the related mapping errors. Additionally, further investigations could focus on how transducers should be placed to ensure measured signals can be processed. Using the derivation of the theoretical limits in Section 2.6, minimum transducer separation distance as well
as minimum edge distance should be chosen such that autonomous signal processing is robust within an acceptable amount of unmappable corner area.

Though the capabilities of this Lamb wave-based mapping has been demonstrated both in simulations and experiment, a distinct limitation of the method warrants discussion. Despite mechanisms to prevent continual poor measurements, a temporary gridlock-like situation may arise, as in the case from the previous section. Another example can be found in the simulation trial shown in Fig. 16 for the square plate. In Fig. 16a, the mapping operation is a sensor-first case which has thus far resulted in average performance with $C_8 = 68.45\%$. However, the bounding box used to estimate the environment is nearly $45^\circ$ out of alignment with ground truth. For the case shown, greedy solutions within the estimated space will tend to try and map the corners of the bounding box. Since these corner regions do not exist, the approach repeatedly results in poor measurements as evident by the blue curve in Fig. 16b. Consequently, LFE under-performs in this trial. This example also demonstrates that absolute greediness within the estimated space may not necessarily be the best approach. Consider the experimental result for $k = 13$ (see Fig. 12c): A significant portion of the plate is yet unmapped while the region defined by the bounding box is mapped to near completion. The implication is that the proposed approach could potentially benefit from exploratory probing. As evident by Fig. 14b for the eight-sided plate simulations, some implementation of random search is potentially viable. A final remark is that the curve in Fig. 16b suggests convergence has occurred. In other words, without knowing the upper limit of obtainable coverage, it is currently not possible to distinguish between complete mapping or a gridlock. Thus, it necessary for future work to establish robust criteria to determine that a mapping solution has been obtained.
4. Concluding remarks

In this work, the concept of Lamb-waved based SLAM has been introduced. While research on Lamb wave-based methods has typically been focused on NDT, this paper has demonstrated the potential contribution of Lamb wave-based sensing to the field of mobile robotics and exploration. In particular, the concept of greediness has been applied to a sensing modality that is not typically associated with gaining spatial information. Though the proposed method for Lamb wave-based frontier exploration and mapping is seemingly more translatable to mobile robotics, LFE is intended to embody a dynamic approach to NDT with guided waves. That is, to gain information about a structure without scanning all possibilities or utilizing predetermined static sensor placements. The performance of the presented LFE method is evaluated utilizing quantitative metrics as well as against alternative mapping approaches. Furthermore, the analysis of simulated results as well as experimental data reinforce the potential practicality of Lamb wave-based methods in SLAM. For the case when LFE can suitably estimate the environment, e.g. a square plate, it has been demonstrated that LFE outperforms random exploration and may achieve near-optimal convergence as compared to a pansophic greediness approach which has a priori knowledge.

When a more complex plate geometry is considered, simulations have shown that LFE can still map the majority of an eight-sided plate, albeit with reduced performance. The presence of a relatively short stiffener has also been shown to not significantly affect the LFE algorithm. However, more investigations into the effects of internal scatterers on LFE are required. It is acknowledged that several assumptions and simplifications have been made in this work. More research is necessary to apply such exploration techniques to anisotropic, non-square composite panels to close the gap to potential applications in the aeronautic and aerospace industry. Moreover, factors relevant to mobile robotics such as path planning, loop closure, and integration of Lamb wave actuation and sensing capabilities into a mobile system have not been addressed. Nonetheless, this paper has shown that Lamb wave-based sensing is functionally capable of mapping the surface of an isotropic plate. Though the work performed has been focused on wave scattering at free edges, it is ultimately desired that LFE be further developed to identify and localize wave interactions for internal features relevant to NDT. Future work may also aim to improve environment estimation such that structures with complex geometries may be satisfactorily explored as well as relaxation of frontier-only exploration and the possibility of relocating both transducers simultaneously.

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