Determining the linear network code parameters to minimize the destructive factors influence in telecommunications systems and networks

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Abstract. The paper analyzes the searching results for the linear network code optimal parameters using a binary symmetric channel and queuing models. A conclusion is made about the correcting ability dependence and the code sequence length on the bit distortion and overloads probability in networks. The study showed using the network coding method possibilities and limitations to counteract destructive factors in the network. The conclusion is made about the code parameters choice with correlated packet losses and the combined use of both models.

1. Introduction
The overhead associated with the need to add packets with redundancy and the time it takes to encode and decode blocks of data places stringent demands on the codes that can be used to eliminate lost packets. The network coding method allows you to optimize the information transfer process by restoring lost packets without additional re-requests. In this regard, it is necessary to find the code and information sequences optimal lengths, at which the introduced redundancy will not significantly increase the packet loss, which will increase the connection stability and reduce the data collection time at the receiver.

The work will present a linear network code and information sequences lengths set, providing data transmission over a telecommunication system without packet loss. These parameters set will optimize the information transfer process, reducing the delays in data collection by application processes due to minimizing the re-requests number and increasing the control information delivery efficiency. The code and information sequences lengths resulting set will allow for the network coding method parametric optimization, improving it for promising telecommunication systems and information interaction technologies wide class.

The approach novelty lies in such parameters set to use in the network coding method, which will ensure the connection stability between subscribers to destructive factors (packets distortion during their transmission over the radio channel and packet loss in networks due to congestion) and reduce the data collection time at the receiver. These parameters can be easily modified if necessary, and on their basis, it is possible to design adaptive systems with error-correcting coding feedback and methods.
2. The linear network code parametric optimization when using the binary symmetric channel model (DSC)

We will use the Binary Symmetric Channel (DSC) model to optimize the linear network code. This model allows estimating the packet loss probability at a transmitted bits' distortion fixed probability [1]. There is a relationship between the DSC channels models and additive white Gaussian noise [2]: for a given signal-to-noise ratio, a bit error probability can be determined. This fact makes it possible to significantly simplify the modelling process by rejecting many special cases associated with the signal/channel codec combination choice.

For modelling, we will select the following parameters: packet length 1,500 bytes (typical size for networks such as 802.3, 802.11), bit distortion probability \( P_{BSC} = 5 \cdot 10^{-8} \), restriction on the length of the code sequence 100 packets, and the correcting capacity of 24 packets. The transmitted packets number is 10 million.

![Figure 1](image.png)

**Figure 1.** Points set for \( P_{BSC} = 5 \cdot 10^{-8} \) and a packet length of 1,500 bytes, which use provides a packet loss probability with data equal to zero.

The optimal points list at which the code rate value is maximum is given in table 1.

As can be seen from table 1 and figure 1, it is possible to obtain the maximum code rate when using the DSC model by increasing the code sequence length, which requires a network code insignificant correcting ability.

It should also be noted that with an increase in the transmitted packets (sample size) number, the network code optimal parameters set stops changing. [3] This is due to the binomial distribution peculiarity. Simulations have shown that after 5 million packets are transmitted, the change in the packet corruption probability will be so small that it can be neglected.
Table 1. The optimal points list for $P_{\text{BSC}} = 5 \cdot 10^{-8}$ and a 1,500 bytes packet length.

| Code sequence length (N) | Corrective ability (T) | Code rate (R) |
|-------------------------|------------------------|---------------|
| 91                      | 6                      | 0.87          |
| 100                     | 7                      | 0.86          |
| 99                      | 7                      | 0.85          |
| 98                      | 7                      | 0.85          |
| 97                      | 7                      | 0.85          |

Here are the results for $P_{\text{BSC}} = 10^{-7}$ and $P_{\text{BSC}} = 2 \cdot 10^{-7}$.

Table 2. The optimal points list for $P_{\text{BSC}} = 10^{-7}$ and a 1,500 bytes packet length.

| Code sequence length (N) | Corrective ability (T) | Code rate (R) |
|-------------------------|------------------------|---------------|
| 89                      | 7                      | 0.84          |
| 100                     | 8                      | 0.84          |
| 87                      | 7                      | 0.84          |
| 98                      | 8                      | 0.84          |
| 97                      | 8                      | 0.84          |

Table 3. The optimal points list for $P_{\text{BSC}} = 2 \cdot 10^{-7}$ and a 1,500 bytes packet length.

| Code sequence length (N) | Corrective ability (T) | Code rate (R) |
|-------------------------|------------------------|---------------|
| 99                      | 10                     | 0.8           |
| 94                      | 10                     | 0.79          |
| 93                      | 10                     | 0.78          |
| 91                      | 10                     | 0.78          |
| 100                     | 11                     | 0.78          |

With an increase in the bit distortion probability in the channel, a code large correcting ability is required, but this does not significantly affect the code sequence length. This fact consequence is a decrease in the code rate for the points optimal set. We also note that with an increase in the error probability in the channel, the number of variants $(N, K)$ of the network code decreases with the introduced restrictions on the code sequence length $N$ and the information sequence length $K$.

3. Linear network code parametric optimization when using queuing model M/M/1/N

In networks, there are other reasons for packet loss, which are associated with congestion in routing and switching devices. The most widely used queuing model M/M/1/N is used to describe random natural processes [4] and overloads in networks. The model for finding a point set that provides a losing zero-data packets probability was presented in the [5]. It has produced optimal values $(N, K)$ linear network code set, which maximizes the code speed (see figure 2 and table 4), for the queuing device load different values.
Figure 2. Points set for $p=1$ and a 1,500 bytes packet length, which use provides the losing with a packet with data equal to zero probability.

Table 4. An optimal points list for $p=1$ and a 1,500 bytes packet length.

| Code sequence length ($N$) | Corrective ability ($T$) | Code rate ($R$) |
|----------------------------|--------------------------|-----------------|
| 98                         | 40                       | 0.18            |
| 100                        | 41                       | 0.18            |
| 98                         | 41                       | 0.16            |
| 100                        | 42                       | 0.16            |
| 84                         | 36                       | 0.14            |

The above results show that it is possible to eliminate packet loss by introducing significant redundancy. This leads to an increase in overhead costs both for encoding/decoding data and for their assembly time by the recipient. In some cases, it turns out to be more profitable to use data transfer protocols with lost data packets re-requests. The code sequences optimal length for transmission over networks with overload practically coincides with the random packet loss case, which arouses interest in considering their co-occurrence case, as it happens in real data transmission systems.

4. Random packet loss influence during transmission over overloaded networks on the code parameters choice

Consider the DSC model joint use 2 cases with the queuing model M/M/1/N. To do this, we add packet losses due to their distortions at the physical layer to the results obtained by modelling congestion in networks with the M/M/1/N model using the DSC model. For comparison with the random losses' absence case (when the DSC model is not used), we will take as a basis the same modelling parameters that were described in the previous section.
Figure 3. Points set for $\rho = 1$, $P_{BSC} = 10^{-7}$ and a 1,500 bytes packet length, which uses provides a packet with data equal to zero probability losing.

The network code parameters resulting set $(N,K)$ for $P_{BSC} = 10^{-7}$ and $P_{BSC} = 5 \cdot 10^{-7}$ is shown in figures 3 and 4, and the optimal points list is in tables 5 and 6.

Figure 4. Points set for $\rho = 1$, $P_{BSC} = 5 \cdot 10^{-7}$ and a 1,500 bytes packet length, which uses provides a packet with data equal to zero probability losing.
Table 5. An optimal points list for $p=1$ and a 1,500 bytes packet length.

| Code sequence length ($N$) | Corrective ability ($T$) | Code rate ($R$) |
|----------------------------|--------------------------|-----------------|
| 98                         | 40                       | 0.18            |
| 100                        | 41                       | 0.18            |
| 98                         | 41                       | 0.16            |
| 100                        | 42                       | 0.16            |
| 98                         | 42                       | 0.14            |

Table 6. An optimal points list for $p=1$ and a 1,500 bytes packet length.

| Code sequence length ($N$) | Corrective ability ($T$) | Code rate ($R$) |
|----------------------------|--------------------------|-----------------|
| 98                         | 42                       | 0.14            |
| 98                         | 43                       | 0.12            |
| 100                        | 44                       | 0.12            |
| 96                         | 43                       | 0.1             |
| 98                         | 44                       | 0.1             |

The data obtained show a slight difference from the case without using the DSC model. However, with an increase in the bit distortion probability $P_{BSC}$, the random losses' contribution has a significant impact and leads to a change in the optimal points list. The points number with the code sequence and the correcting ability the smallest length (the information sequence the largest length) is also reduced. As a result, the code speed is reduced, which makes the coding use even less justified.

The simulation result analysis for other cases also confirmed the code sequence correcting ability and the length obtained dependence on random losses. The less corrective power and code sequence length required to counter correlated losses, the less likely $P_{BSC}$ will affect the optimal points set and the linear network code values selectable set ($N,K$) that guarantees the packets' delivery to the recipient with no loss.

5. Conclusion
The study showed that to counteract random losses, a linear network code high correcting ability is not required, however, correlated losses arising from overloads require the significant redundancy introduction. This significantly reduces the data transfer rate, and the sharp reduction in the parameters available for use ($N,K$) of the network code makes the use of encoding impractical, for example, in the case when there are restrictions on the length of the code sequence.

Correction codes promising application (including linear network code) to minimize the destructive factors influence in telecommunications systems and networks are low network load cases ($0.8 < p < 0.9$) and the packet distortion probability not exceeding $10^{-3}$. Correlated losses introduce significantly greater restrictions on the code parameters choice than random losses, however, with an increase in the bit distortion probability and, as a consequence, packet distortion, it is necessary to take into account the latter contribution to the data transmission process.

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