The behavior of strongly correlated Fermi systems is investigated beyond the onset of a phase transition where the single-particle spectrum \( \xi(p) \) becomes flat. The Landau-Migdal quasiparticle picture is shown to remain applicable on the ordered side of this transition. Nevertheless, low-temperature properties evaluated within this picture show profound changes relative to results of Landau theory, as a direct consequence of the flattening of \( \xi(p) \). Stability conditions for this class of systems are examined, and the nature of antiferromagnetic quantum phase transitions is elucidated.

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Thermodynamic Properties of Fermi Systems with Flat Single-Particle Spectra

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Low-temperature properties of strongly correlated Fermi systems exhibit features not inherent in Landau Fermi liquids [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Of special importance is the scaling behavior \( \chi^{-1}(T, H) = (0, H) + T^{\alpha}F(H/T) \) of the inverse magnetic susceptibility, observed so far only in heavy fermion metals [2, 3, 4, 5, 6] in weak, sometimes tiny external magnetic fields \( H \). This feature rules out the collective, spin-fluctuation scenario advanced in Refs. 11, 12 to explain the critical importance is the scaling behavior of the inverse magnetic susceptibility. Earlier work within this picture [13] has evaluated thermodynamic properties on the "metallic" side of a phase transition associated with the rearrangement of the Landau state in strongly correlated Fermi systems. Here we focus attention on the situation beyond the critical point where the Landau state becomes unstable.

This state is known (e.g. from Refs. 14, 15) to lose its stability at a density \( n = n_b \) where a bifurcation point \( p = p_b \) emerges in the equation

\[
\xi(p, n, T = 0) = 0 \tag{1}
\]

which ordinarily has only the single root \( p = p_F \). (Here \( \xi(p) = \varepsilon(p) - \mu \) is the single-particle energy measured from the chemical potential \( \mu \).) We shall focus on the case \( |p_F - p_b| \ll p_F \), where the critical single-particle spectrum has the form

\[
\xi(p, n_b, T = 0) \sim (p - p_b)^2(p - p_F) \tag{2}
\]

Thus if \( p_b \) coincides with \( p_F \), the effective mass \( M^*(n) \) diverges at \( n_b \); in the general case \( p_b \neq p_F \), the Landau state loses its stability before \( M^* \) becomes infinite. Beyond the critical density \( n_b \), Eq. (1) possesses two additional roots \( p_1 \) and \( p_2 \) with \( p_1 < p_b < p_2 \). The spectrum \( \xi_{FL}(p, T = 0) \), evaluated with the FL momentum distribution \( n_{FL}(p) = \theta(p_F - p) \), then takes the form

\[
\xi_{FL}(p, n, T = 0) \sim (p - p_1)(p - p_2)(p - p_F) \tag{3}
\]

If \( p_b \neq p_F \), the roots \( p_1, p_2 \) are both located either in the interior of the Fermi sphere or both outside it. If \( p_b = p_F \), then \( p_1 < p_F < p_2 \). In all these cases, the occupation numbers \( n_{FL}(p) \) are rearranged. As a rule, the Fermi surface becomes multi-connected, but at \( T = 0 \), the quasiparticle occupation numbers continue to take values 0 or 1. Hence the Landau-Migdal quasiparticle picture holds, with \( n(\xi) = 1 \) for \( \xi < 0 \) and 0 otherwise. Consider first the case \( p_1 < p_2 < p_F \). Then according to Eq. (3), the single-particle states remain filled in the intervals \( p < p_1 \) and \( p_2 < p < p_F \), while the states with \( p_1 < p < p_2 \) are empty. We call this new phase the bubble phase. If \( p_b = p_F \), then \( p_1 < p_F \) and \( p_2 > p_F \), and the states with \( p < p_1 \) and \( p_F < p < p_2 \) are occupied, while those for \( p_1 < p_F \) are empty. Again one deals with the bubble phase.

At this point, we observe that the solution (3) is not self-consistent, since the spectrum is evaluated with \( n_{FL}(p) \) while the true Fermi surface is doubly-connected. Following Ref. 14 we consider the feedback of the rearrangement of \( n_{FL}(p) \) on the spectrum \( \xi(p) \) in the bubble phase based on the Landau relation [16, 17]

\[
\frac{\partial \xi(p)}{\partial p} = \frac{p}{M} + \int f(p, p_1) \frac{\partial n(p_1)}{\partial p_1} d\nu_1 \tag{4}
\]

where \( f \) is the scalar part of the Landau interaction function and \( n(p) = [1 + \exp(\xi/T)]^{-1} \) is the quasiparticle momentum distribution. For the electron liquid within a solid, \( p/M \) is to be replaced by \( \partial \xi_p^\nu/\partial p \), where \( \xi_p^\nu \) is the LDA electron spectrum. To date, Eq. (4) has been solved only in 3D Fermi systems with functions \( f \) depending on \( q = |p - p_1| \). Despite the diversity of forms assumed for \( f(q) \), the resulting spectra and momentum distributions bear a close family resemblance. This robustness is illustrated in Figs. 2 and 3 which display results [14] from solution of Eq. (1) for the interaction function

\[
f(q) = \frac{\lambda_1}{[(q/p_F)^2 + \beta_1^2]} \tag{5}
\]

and in Figs. 2 and 4 which present results for

\[
f(q) = \frac{\lambda_2}{[(q/2p_F)^2 - 1]^2 + \beta_2^2]} \tag{6}
\]
How do the bubble solutions of Eq. (4) evolve under variation of $T$? At extremely low $T < T_{FL} \sim (p_2 - p_1)^2/M$, these states are described by FL theory with the enhanced effective mass $M^* \sim M p_F/(p_2 - p_1)$. Heating above $T_{FL}$ results in their dissolution (see Figs. 1 and 2). With further increase of $T$, the spectrum $\xi(p)$ becomes smooth, and in the region of a new critical temperature $T_c$, a flat portion $\xi \approx 0$ appears over an interval $[p_1, p_f]$ surrounding the point $p_F$, as shown in the left panels of Figs. 3 and 4. The presence of this flat portion of $\xi(p)$ is a signature of the phenomenon called fermion condensation \cite{18,17,20}. Since $\xi(p) = \varepsilon(p) - \mu$ and $\varepsilon(p) = \delta E_0/\delta n(p)$, the equality $\xi = 0$ can be rewritten as a variational condition

$$\frac{\delta E_0}{\delta n(p)} = \mu, \quad p_i < p < p_f,$$

with $E_0 = \sum_p \varepsilon \!^0_p n(p) + \frac{\mu}{2} \sum_p P(p) n(p) n(p)$ and $\varepsilon \!^0_p = p^2/2M$. The solution $n_0(p)$ of Eq. (4), or equivalently of Eq. (7), is a continuous function of $p$ with a nonzero derivative $dn_0/dp$. The set of states with $\xi(p) = 0$ is called the fermion condensate (FC), since the associated density of states contains a Bose-liquid-like term $\eta \delta(\varepsilon)$. The dimensionless constant $\eta \approx (p_f - p_i)/p_F$ is identified as a characteristic parameter of the FC phase.

It has been shown \cite{20} that the FC “plateau” in $\xi(p)$ has a small slope, evaluated by inserting $n_0(p)$ into the above Fermi-Dirac formula for $n(\xi)$ to yield

$$\xi(p, T \geq T_c) = T \ln \frac{1 - n_0(p)}{n_0(p)}, \quad p_i < p < p_f.$$  

As indicated in the bottom-right panels of Figs. 3 and 4 at $T \geq T_c$ the ratio $\xi(p)/T$ is indeed a $T$-independent function of $p$ in the FC region. The width $\xi(p_f) - \xi(p_i) = \xi_f - \xi_i$ of the FC “band” appears to be of order $T$, almost independently of $\eta > \eta_{\text{min}} \sim 10^{-2}$. Thus at $\eta > \eta_{\text{min}}$ the FC group velocity is estimated as

$$\left( \frac{d\xi(p, T)}{dp} \right)_T \sim \frac{T}{\eta p_F}, \quad p_i < p < p_f.$$  

Outside the FC domain, $\xi(p)$ still remains flat (see Figs. 3, 4), and contributions to thermodynamic properties from the regions adjacent to $p_i$ and $p_f$ also play significant roles in their NFL behavior. Indeed, consider for example the static magnetic susceptibility

$$\chi(T) = -\mu_B^2 \Pi_0(\omega = 0)[1 - g_\omega \Pi_0(\omega = 0)]^{-1}.$$  

Here $\mu_B$ denotes the Bohr magneton \cite{21} and $g_\omega$ the zeroth harmonic of the spin-spin interaction function, while $\Pi_0(\omega = 0) = \int (dn(\xi)/d\xi) d\nu$ is the static particle-hole propagator.

The propagator $\Pi_0(\omega = 0) \equiv -N_0(P_f(T) + P_n(T))/T$, where $N_0$ is the density of states of the ideal Fermi gas, is the sum of a FC part given by

$$P_f(T) = \frac{1}{p_F} \int_{p_i}^{p_f} n_0(p) [1 - n_0(p)] dp \sim \eta$$  

and a noncondensate part $P_n(T) \equiv P_n^< (T) + P_n^> (T)$ consisting of two terms. Defining

$$P_n(T; \xi_1, \xi_2) = \frac{1}{p_F} \int_{\xi_1}^{\xi_2} \frac{n(\xi) [1 - n(\xi)] d\xi}{(d\xi/dp)},$$

the two terms become $P_n^< = P_n(T; -\mu, \xi_i)$ and $P_n^> = P_n(T; \xi_f, \infty)$. According to Eq. (2), at small $\eta$ one has

$$d\xi(p \rightarrow p_f)/dp = v_f(T) + v_2(p - p_f)^{s-1} + \ldots,$$  

with $v_f(T) \sim T$ (see Eq. (11)) and $s = 2$ and 3, respectively, for the cases $p_i \neq p_F$ and $p_i = p_F$. An analogous formula applies for the group velocity $d\xi/dp$ outside the FC domain close to the point $p_i$. Based on these results, algebra similar to that performed in Ref. \cite{13} leads to

$$\Pi_0(\omega = 0) = -N_0 \tau^{-1} [P_f \eta + P_n \tau^{1/s}] + \text{const},$$

where $\tau = T/\varepsilon_F^0$ is the dimensionless temperature.
The NFL excess $\Delta \chi(T, \rho) = \chi(T, \rho) - \chi_{FL}(\rho)$ over the Pauli result then acquires the form

$$\Delta \chi(T, \rho) \sim \frac{\eta}{1 + g_0(\rho)N_0 \tau^{-1}(\eta/\eta + \eta^s/\eta^s)}.$$

(15)

We see that at small $\eta < \tau^s$ the FC plays a minor role, and the NFL part of $\chi$ behaves as

$$\chi(T) \sim T^{-1+1/s}.$$

(16)

By contrast, in the case $\eta > \tau^s$ the FC contribution to Eq. (15) is predominant, and the magnetic susceptibility mimics that of a gas of localized spins

$$\chi(T) \sim (T - \Theta_W)^{-1},$$

(17)

with the Weiss temperature $\Theta_W \simeq -g_0 \eta(\rho)^{1/2}$.

In heavy-fermion metals, both index regimes, i.e. $s = 2$ and $s = 3$, are present, while data on the magnetic susceptibility of 2D liquid $^3$He are compatible only with $s = 3$. Fig. 3 compares results from Eq. (15) for 2D liquid $^3$He with experimental data of Ref. 11 at the densities $\rho = 0.052 \, \text{Å}^2$ and $\rho = 0.058 \, \text{Å}^2$. We have made these parameter choices: $P_n = 0.2$, $P_f = 1$, $\eta = 0$ (lower curve), and $\eta = 0.04$ (upper curve). The theoretical results are seen to be in agreement with the experimental data.

The FC contributions to other thermodynamic properties are found by inserting the distribution $n_0(\rho)$ into the corresponding Landau formulas. In particular, the FC entropy $S_f$ arising at $T \simeq T_Z$ is $S_f = -\sum [n_0(\rho) \ln n_0(\rho) + (1 - n_0(\rho)) \ln(1 - n_0(\rho))] \sim \eta$. This term does not contribute to the specific heat $C(T) = T \partial S(T, \rho)/\partial T$; however, it does affect the thermal expansion $\beta(T) \sim \partial S(T, \rho)/\partial \rho$, giving rise to a great enhancement of the Grüneisen ratio $\beta(T)/C(T)_0$ observed at low $T$ in several heavy-fermion metals.

Our analysis has been carried out within the Landau-
Finally, we consider stability conditions for systems with a flat portion in the spectrum $\xi(p)$. To be definite, we examine the spin-density wave channel and suppose that at the critical density $n_b$, the associated stability condition, which has the form

$$1 > g_0(k_c)\Pi_0(k_c, \omega=0) \equiv g_0(k_c)\int n(p) - n(p+kc) \, dp \, d\omega,$$

(21)

is not yet violated. Beyond the density $n_b$, the magnitude of the NFL component of $\Pi_0$, proportional to $\eta$, drops rapidly with increasing $T$ (see e.g. Eq. (14)). In the present case this implies that violation of the inequality (21) can occur only at very low $T$. Furthermore, numerical calculations show that in systems with a FC, the NFL part of the function $\Pi_0(k, \omega = 0)$ has a maximum at small nonzero $k \leq \eta p_F$; one expects a new ground state possessing a long-range magnetic superstructure, as well as a small ordered magnetic moment. A salient feature revealed in Refs. [13, 27, 28] is the destruction of the flat portion of the spectrum $\xi(p)$ by imposition of sufficiently weak magnetic fields, which also kills the magnetic ordering. Such a quantum phase transition has been uncovered in recent experiments on YbAgGe [24]. The interplay between quantum antiferromagnetism built upon a FC, and the FC itself, will be the subject of a future article.

The present investigation has revealed generic features of strongly interacting Fermi systems which exhibit a flat single-particle spectrum $\xi(p)$. We have shown that the quasiparticle picture remains applicable in the evaluation of thermodynamic properties of such systems, and that the flattening of $\xi(p)$ is a principal source of their non-Fermi-liquid behavior.

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