Hybrid refractive-diffractive axicons for Bessel-beam multiplexing and resolution improvement

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Abstract: Optical elements rely on refraction, diffraction, or reflection for light manipulation. Fusing diffractive and refractive functions in a single element provides an extra layer of control over the wave propagation, allowing complex beam shaping through self-aligned, monolithic and miniaturized optics. Using gray-scale lithography with high-current focused Xe ion-beams, we realized hybrid refractive-diffractive micro-axicons that feature diffractive gratings engraved on their conical surfaces. Furthermore, we fabricated these devices in lithium niobate, which is a challenging piezo/optoelectronic material for processing with an as-yet unexploited potential in optical applications. The curvilinear surfaces of fabricated micro-axicons with a 230-µm diameter were engraved with diffraction linear and circular gratings of various depths (<400 nm), and the optical performance of these components was characterized, showing excellent agreement with theoretical expectations. The fusing of diffractive elements with carrier refractive surfaces introduces additional or enhanced device functionalities, such as beam multiplexing and resolution improvement. The potential applications of such monolithic and miniaturized hybrid micro-optical components include beamshaping for fluorescence microscopy.

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1. Introduction

Hybrid optics, marrying diffractive and refractive elements in a single component [1], enable performance enhancement of standard optical elements, or system miniaturization, or generation of new functionalities [2]. Here we present micro-optical elements that fuse diffraction gratings with refractive axicons resulting in resolution enhancement, as well as miniaturized systems with an extended functionality, such as Bessel-beam multiplexing, for potential applications in fluorescence microscopy as beamshaping elements.

Theoretical and limited experimental investigations in the hybrid-optics field have been mainly directed to the holographic correction of aberrations in traditional refractive lenses [3–13]. A hybrid refractive-diffractive lens relies on the opposite chromatic dispersion of refractive and diffractive elements to compensate and reduce the chromatic aberrations inherent for a singlet refractive focusing lens. A single miniaturized hybrid element is, therefore, capable of replacing a doublet (two refractive lenses combined together) while providing comparable achromatic performance [3]. In addition to aberration correction, fusion of diffraction and refraction properties in a single element by scoring the topography of curvilinear refractive surfaces with suitable diffraction gratings allows one to achieve an extra level of control over the subsequent wave propagation and to introduce new functionalities. Thus, a hybrid lens that combines an
aspheric plano-convex refractive lens having a shallow rotationally symmetric diffractive lens engraved on its flat surface can have two foci [14,15] or even three foci along the optical axis [16,17]. Such a hybrid tri-focal component finds an application in contact and transplant intra-ocular lenses for a clear vision at near, intermediate and far distances from the eye. Today, thanks to the advances in three-dimensional (3D) patterning, it is possible to realize such devices in polymers, as well as components with even more complex topographies such as wrinkled axicons [18] using 3D femtosecond-laser photo-polymerization. The implant hybrid tri-focal lenses are designed to operate in water – the environment closely corresponding to their surrounding medium inside the human eye. However, the small difference between the refractive index $n_r$ of the lens material (polymers with $n_r<1.55$) and that of water ($n_r=1.33$) can make the optical performance of the lenses sub-optimal and restrict the possible designs of lenses to materials like glass ($n_r<1.62$) for operation in water. Instead, the optical properties of single crystal lithium niobate (LiNbO$_3$, LN) make this material very attractive for operation in aqueous environments and for optical integration with microfluidic devices. In addition to its high transparency in a broad range of wavelengths (0.3-5 $\mu$m), opto/piezoelectric LN has a high refractive index $n_r=2.2$ comparable to that of diamond; however, LN is considerably more affordable. Radially symmetric diffractive and hybrid refractive-diffractive elements can be realized by diamond turning and glass molding [19,20], or by means of additive manufacturing [16]. However, the embossing of diffraction patterns into the complex 3D smooth surfaces made in hard and brittle substrates like glass or LN is challenging with the current state-of-the-art technologies. It is especially challenging to realize the free-form diffraction patterns in micro-optical components with diameters $<0.2-0.5$ mm. Even though femtosecond laser ablation [21–23] is in principle capable of 3D surface machining, it is surprising, that little work has been reported on the realization of miniaturized refractive lenses, hybrid lenses and other microoptical components in LN crystals. A promising writing method for micro-patterning of hard substrates is focused ion-beam (FIB) milling. In this method, a finely focused beam of typically metallic Ga$^+$ ions with keV energies is raster-scanned across a substrate. The impinging Ga ions sputter away the material and by controlling the dwell time of the focused beam at each scan point, complex 3D shapes can be carved into almost any substrate. This is a slow process due to the serial nature of the pattern transfer, however, the advent of a new generation of focused ion-beam systems with inductively coupled plasma ions sources [24] allows one to speed-up the milling process by up to 20$\times$ [25,26]. Furthermore, the plasma sources allow one to replace the metallic Ga with noble and heavy Xe ions. In addition to further reducing milling times due to the increased sputtering rate thanks to the higher mass of Xe, the use of Xe eliminates the effects of Ga implantation and material chemical and optical properties modification. In this study we fabricated hybrid refractive-diffractive micro-axicons in LN. We monolithically integrated both linear and circular diffraction gratings on top of the conical surfaces of the micro-axicons.

2. Hybrid optics fabrication and characterization

All presented micro-optical components were fabricated on single-crystal 128° Y-cut (surface acoustic wave SAW grade) double-polished lithium niobate chips (Precision Micro-Optics). The chips were coated with 400 nm of Al, which served as both a light blocking layer and prevented charge build-up during the ion-beam milling. Prior to milling the optical components, an aperture with the same size as the desired optical element was milled to expose the lithium niobate surface. The metal opening served as an optical aperture. The milling was performed using focused 30 keV beams of Xe ions at 4–200 nA current from a prototype of a ThermoFisher Helios Hydra P-FIB Ux-G4 system. The refractive surfaces were produced at 200 nA milling current, while the diffraction gratings were milled with 4–15 nA beam currents. The desired surface profiles were digitized into 8-bit bitmaps with pixel coordinates corresponding to the position within the device, and the pixel value (0–255) proportional to the depth of the milled profile. The diffraction
patterns were either introduced directly into the bitmaps (e.g., in hybrid axicons with circular gratings where the alignment of refractive and diffractive patterns is crucial), or in a second step using a lower current after the refractive component had been fabricated with a high ion current (e.g., for linear gratings where the alignment is not as crucial). Currently, a single writing field is \(~500\times800~\mu m^2\) at most, thereby limiting the largest structure that can be produced without field-stitching errors.

The optical performance of the fabricated components was analyzed via a telescope with a 50x objective (NA=0.95) and a CCD camera (Andor Zyla 4.2 sCMOS). For a larger field of view, a 20x objective (NA=0.45) was used. The components were illuminated with a laser beam (wavelength \(\lambda = 458~\text{nm}\)) with the light incident on the unpatterned flat backside of the chips. The intensity profiles were recorded with the telescope and stored in 16-bit TIFF images. The telescope was mounted on motorized actuators (ThorLabs Z825B with K-cube KST101 stepper motor controllers) with a \(<200~\text{nm}\) stepping precision. By collecting a series of intensity profiles at different planes, the three-dimensional light intensity field could be reconstructed. The efficiencies of various diffraction orders were estimated by measuring the intensity of the incident beam through an aperture having a similar diameter as the optical elements and comparing this intensity with those of the various diffraction orders [27]. Such measurements take into account the contribution of absorption, reflection at the interfaces and scattering.

3. Hybrid refracto-diffractive axicon with a linear grating

Refractive axicons [28] and their diffractive counterparts (fraxicons) [29] produce conically collapsing beams which interfere and generate pseudo-non-diffracting Bessel-like beams with an extended depth-of-focus (DOF). These beams find use in numerous applications including particle trapping [30,31] and imaging [32,33]. For example, the lateral scanning of Bessel-like beams results effectively in a light-sheet with applications in fluorescence light-sheet microscopy. Here we investigate monolithically integrated hybrid refractive-diffractive axicons with potential applications in fluorescence light microscopy as a beam-shaping element for light-sheet generation. All the above-mentioned applications of axicons are typically performed on samples suspended in water or aqueous solutions. The optical systems based on highly transparent and inert lithium niobate would therefore greatly benefit from its high refractive index making it more suitable for operation in water compared to more conventional materials (e.g., glass) that have considerably lower refractive index.

In the case of a purely refractive axicon, the exit beam intersects the optical axis at an angle \(\beta = \sin^{-1}(n_i \sin \alpha) - \alpha\), where \(\alpha\) is the cone base-angle and \(n_i\) is the refractive index of the axicon material (Fig. 1(a)). The resulting Bessel-like line focus spans over an extended DOF of \(R/tan \beta\) \((R\text{-axicon radius})\), with the central beam spot diameter (full-width at half-maximum, FWHM) of \(w_0 = 2.4048 \lambda/(2\pi \beta)\). The introduction of the diffraction grating modifies the transmittance of the purely refractive axicon. In general, the transmittance of a refractive-diffractive hologram can be expressed as \(\exp[i\phi(x,y)]\cdot \exp[i\psi(x,y)]\), where \(\phi(x,y)\) is the lateral phase-shift superimposed by the diffraction pattern onto the wavefront shaped by the carrier refractive surface with phase-shift \(\psi(x,y)\). Here, \((x,y)\) denote generalized coordinates with respect to any smooth surface defining the generalized hologram manifold. In the special case of a planar axially symmetric element whose reference wave has a phase obeying a simple power law, the transmittance becomes \(\exp[i(\sigma r)^m]\cdot \exp[i\phi(x,y)]\), where \(\sigma\) is a scaling coefficient with units of inverse length. For \(m = 1\) the surface profile corresponds to a conical refractive lens (axicon), while when \(m = 2\) the element is a focusing lens. For a simple 1D diffraction grating, the transmittance further simplifies to \(\exp[i\pi r/d]\cdot \Sigma_n C_n \exp(2\pi inx/d)\), where \(d\) is the grating period and \(C_n\) is the \(n\)th Fourier coefficient. Here, the grating redistributes the converging conical wavefront into various diffraction orders at the angles \(\theta_n = \sin^{-1}(n\lambda/d)\), where \(n = 0, \pm 1, \ldots\) and \(\lambda\) is the wavelength of light [Fig. 1(b)]. Although tilted, the conical wavefronts nevertheless reconstruct the non-diffracting Bessel-like
beams, without appreciable distortions, at the angles $\theta_n$ with respect to the beam produced by a conventional axicon as shown in Appendix A. Figures 2(a) and (b) compare the surface profiles of the fabricated purely refractive and refractive-diffractive micro-axicons, respectively, while Figs. 2(c)–(e) show the beam spot profiles downstream of the the optical elements for different grating groove-depths.

**Fig. 1.** Schematic cross-section presentation of (A) a refractive axicon with a base-angle $\alpha$ made of a material with a refractive index $n_r$. The axicon refracts the incoming beam to intersect with the optical axis at the angle $\beta$. The overlapping refracted beams interfere to produce a Bessel-like beam along the optical axis with the extended depth-of-focus. (B) Due to the engraved linear diffraction pattern with a period $d$ on top of the axicon surface, some portions of the light are diverted into various diffraction orders. The tilted conical beams nevertheless reconstruct the Bessel-like beams at various diffraction angles $\theta$. (C) A circular diffraction grating with a radial pitch $d_r$ on of the axicon surfaces effectively generates higher and lower convergence orders, that have correspondingly narrower and broader Bessel-like beams along the optical axis.

The efficiency [Fig. 2(f)] of the diffraction orders depends on the grating parameters, such as groove-depth (*i.e.* phase-shift modulation imposed onto the wavefront), grating duty-cycle, wall slope *etc.* The measured efficiencies of the $0^{th}$ and $\pm 1^{st}$ orders largely match those of an ideal
Fig. 2. Micro-axicon with a linear diffraction grating on the curved surface. (A,B) White light interferometry measurements of surface profiles of ion-milled conventional and hybrid micro-axicons, respectively. The hybrid axicon has a 500 nm deep grating on its conical surface. (C,D,E) Measurements of beam profiles at 1.1 mm from the tips of 230-µm-diameter, 7.5 µm-deep axicons with 11-µm grating pitch on their surfaces with grating groove-depth of 0, 150 nm and 250 nm, respectively. The efficiency of various diffraction orders depends on the phase-shift modulation (i.e. groove-depth) introduced by the gratings. (F) Measured (dots and dashed lines) and calculated (solid lines) diffraction efficiencies of 0th (blue), ±1st (red), ±2nd (black) and ±3rd (magenta) diffraction orders.

binary phase-grating with vertical walls and 50% duty cycle for relatively shallow groove-depths (<200 nm). The theoretical 0th and ±1st order efficiency values of an ideal phase grating [solid curves in Fig. 2(f)] do not include the possible absorption, reflection from the interfaces and scattering effects, and hence represent the upper limit on the efficiency values. The discrepancies between the theoretical and experimental values can be explained by the presence of additional effects (e.g., reflection) and measurement uncertainties (e.g., small mismatch of the reference aperture and the microaxicon diameter, as well as the mismatch of scattering effects from the microaxicon and the reference aperture). At deeper groove-depths the efficiencies deviate from the flat-grating approximation values, due to the increased wall slope and departure of the duty cycle from 50%, thereby generating even diffraction orders (±2, etc.) that are otherwise forbidden in the ideal grating. For a hybrid axicon made of lithium niobate (refractive index ∼2.2) the efficiencies of the most prominent 0th and ±1st orders can be made to have similar intensities [Fig. 2(d)] for the phase-shift modulation of ∼0.63π rad (∼130-150 nm grating depth). At the groove-depth of ∼200-250 nm (∼ π rad), the 0th order is nearly removed, with most of the incoming light re-channelled into Bessel beams formed by ±1st tilted diffraction orders [Fig. 2(e)].
Figure 3 shows the characterization of the ion-milled gratings profiles [Figs. 3(a)–(c)] and the summary of the optical performance of the hybrid axicons in which the groove depth of the diffraction grating scored onto the refracting surface is progressively increased [Figs. 3(d)–(f)]. Figure 3(d) gives measured experimental data. When the groove-depth is 0 nm, one has a conventional axicon with the expected Bessel-beam profile and very significantly extended DOF [Fig. 3(d), 0 nm]. Figures 3(d)–(f) demonstrate the ability of the hybrid axicons to multiplex a single incident beam into a number of pseudo-diffraction-free beams with significantly extended depth of focus. With the increasing grating groove-depth, the intensity of the original Bessel (0th-order) beam is split between various diffraction orders at different proportions. While the theory (Appendix A) indicates that the wavefronts for all but the 0th order will be aberrated, the self-healing property of Bessel beams is borne out in the fact that all of the hybrid axicons’ orders (and for all of the groove-depths) display extremely pronounced depth of focus. The experimental data [Fig. 3(d)] agree very well with the simulated data shown in Fig. 3(e). The simulated data corresponds to Fresnel-propagator simulations in which the edges of the diffraction grating (scored onto the axicons) are sloped corresponding to measured profiles. Figure 3(b) corresponds to the simulations in which the edges of the diffusion grating are taken to be sharp [vertical walls, e.g., black line in Figs. 3(b) and (c)]. A grating with sharp edges diverts the incoming intensity into mainly 0th and ±1st orders, with the even and higher orders significantly suppressed or weaker [Fig. 3(f)]. However, due to the shape of the ion-beam and re-deposition of the sputtered material from the deep trenches, the resulting gratings have sloped walls and smooth edges, thereby giving rise to the even orders. Fabrication strategies exist to produce trenches with straight and vertical walls using FIB milling [34], however, the inherent capability of the FIB processing to generate gratings with sloped walls and smooth edges is beneficial for an even intensity redistribution between the different diffraction orders. The wall slope as measured from the atomic force microscopy [Figs. 3(a)–(c)] of the fabricated hybrid axicons is 4.9°, 7.4° and 9.9° for the grating groove-depths of 150 nm, 250 nm and 350 nm, respectively. Due to the deviation of the grating profile from the ideal square-shape it is possible to split the incoming intensity nearly evenly between seven diffraction orders (0th, ±1st, ±2nd and ±3rd, Fig. 2(f) and Fig. 3 at grating groove-depth of 350 nm).

The hybrid axicons, thus, are capable of multiplexing Bessel beams, whereas their relative intensity can be controlled by various diffraction pattern grating parameters such as grating groove-depth [Fig. 2(f)], wall slope, duty-cycle, etc. The refractive axicon design in this study (230-µm diameter, sag height 7.5 µm) results in the base angle α of 3.73° and the beamspot size FWHM of 2 µm. The fan of Bessel-beams produced by the hybrid axicon, therefore, generates a 2-µm-thick sheet-like intensity distribution with a lateral coverage across the beam direction propagation of ~300 µm while the lateral intensity extends over ~0.5 mm of the depth of focus. The utility of using a hybrid axicon for the generation of light-sheet-like intensity is in miniaturization of the beamshaping optical component combining two functionalities simultaneously. This is especially beneficial for optical setups that suffer from space limitation and do not allow multiple individual elements to be introduced into the confines of the sample environment. In addition, due to the monolithic integration of the diffractive and refractive elements within the same miniaturized device, the component does not suffer from thermal and mechanical drifts and in principle is capable of maintaining optical alignment in perpetuity.

Considering that self-healing properties are not unique of Bessel beams [35], we demonstrate that this wave-splitting effect can also be valuable in conventional focusing lenses (m = 2, Appendix B). In this case, if the surface of a spherical or parabolic lens is decorated with a diffraction grating, the behavior is comparable to what is found in hybrid axicons. Here, the collapsing parabolic wavefront is split into multiple diffraction orders that converge into multiple laterally separated foci (Appendix B, Fig. 7). This is another clear example of the utility of
Fig. 3. Optical characterization of the hybrid refractive-diffractive axicons with linear gratings.

A) Atomic force microscopy (AFM) measurement of the 200-nm deep gratings on the axicon’s conical surface. (B and C) AFM-measured grating profiles of 350-nm and 250-nm deep gratings with the wall slope fits. Comparison of measured (D) and simulated (E and F) intensity cross-sectional profiles formed by the axicons with various grating groove-depths (230-µm-diameter, 7.5 µm sag height, decorated with 11-µm grating pitch on their surfaces with grating groove-depths of between 0–350 nm). The grating with sharp edges (F) does not allow even diffraction orders. A grating with smooth edges (E) that matches the actual milled grating profile results in emergence of even diffraction orders and produces a good agreement with the measured result.
merging multiple optical elements into a single component and therefore removing the ubiquitous alignment problems.

4. Fabrication limits

The previous section identified the utility of high-current focused Xe ion-beam milling in the fabrication of gratings with sloped walls. The departure of the grating profile from the rectangular shape allows one to equalize the intensities of multiple diffraction orders, which is not possible when using gratings with the ideal shape. In order to identify the limits of the fabrication method, we performed line milling tests using various ion-beam currents [Fig. 4(a)]. While the AFM measurements are suitable for the characterization of shallow trenches (<500 nm), the scanning electron microscopy (SEM) methods are suitable for visualization of the deeper trench profiles. The results obtained from the SEM measurement indicate the edges’ radius of curvature of 0.5 \( \mu \)m, and the wall slope \( \gamma \) not exceeding 10.1±0.3° [Fig. 4(a)]. This result matches very well with the wall slope values obtained from the AFM characterization [e.g., 9.9° for the grating groove-depth of 350 nm, Figs. 3(a)–(c)]. The increased wall slope is mainly due to the ion-sputtered material redeposition and the difficulty to efficiently remove the material from deeper trenches. Using lower ion-beam currents can significantly improve the wall verticality and edge sharpness, albeit at the expense of increased fabrication times. At the lower currents the wall verticality depends on the ion-beam convergence rather than on the beam shape. At 16.5 mm working distance (aperture-to-sample distance) and the 118 \( \mu \)m diameter of the beam defining aperture, the beam convergence limits the wall verticality to 3.6 mrad (0.2°). This represents the “fastest” phase that can be implemented with the plasma-source focused ion-beam system. The “slowest” phase that we demonstrated with the ion-beam milling in a hard substrate corresponded to 13.2° in a blazed grating [36]. Using low-currents (tens of pA compared to tens of nA in this study) of ion-beams from conventional liquid metal (Ga+) FIBs with post-processing steps was demonstrated to produce high-aspect-ratio nanostructures with vertical walls [37], however, this technology is more applicable for the fabrication of nanostructures, rather than micro-optical elements. Nevertheless, the combination of the two ion-beam milling methods, e.g., plasma-FIB for the rapid milling of refractive optical elements with 50-500 nA of ion-current, followed by three orders of magnitude lower currents of a Ga-FIB (and possibly post-processing steps) for the milling of diffractive structures is a promising way to improve both the fabrication speed and the fabrication precision.

The high quality of the plasma-FIB-based process was demonstrated by the fabrication of smooth and precise profiles of microlenses and microaxicons [25,26] using a lower 60 nA beam current. Here, we increased the fabrication speed by more than three-fold by using 200 nA beam current. Nevertheless, we were able to produce high-quality micro-optical components. Figure 4(b) shows the measured profile of a refractive micro-axicon, while Fig. 4(c) shows the departure of the characterized axicon shape from the designed shape. Apart from the axicon tip region (\( R=0 \)), the deviation is mainly due to the surface roughness. The fabrication of conical shapes of axicons with sharp tips is challenging, however, with the ion-beam milling we were able to fabricate axicons with tip radius of <5 \( \mu \)m. The tip radius-of-curvature can be considerably decreased by using lower ion-beam milling currents. In the inner regions of the axicons, the roughness does not exceed 30–50 nm (\( \lambda/10 \cdots \lambda/15 \)), while closer to the outer regions the roughness is \( \sim 100 \) nm (\( \lambda/5 \)). This result indicates the high quality of the fabrication process and its applicability for the production of optical-quality surfaces.
Fig. 4. Fabrication limits using high-current Xe ion-beam milling. A) Cross-sectional view of trenches milled in the substrates. The milled trenches were coated with a Pt layer to produce a clean cut surface. Stage tilt 52°. Blue lines indicate the trench wall slope, while the blue circle denotes the edge’s radius of curvature. B) Cross-section profile of a fabricated refractive micro-axicon with the ideal profile (dashed lines). C) Departure of the fabricated profile from the design shape.

5. Radially symmetric hybrid axicon

In addition to the multiplexing properties of linear-hybrid axicons, we demonstrate an element capable of enhancing the resolution performance of a refractive axicon. Figure 5 compares the optical performance of a conventional refractive axicon with one of the hybrid elements where an axially symmetric grating is engraved onto the conical surface. The transmittance in this case simplifies to $\exp\left(i\sigma r\right) \cdot \sum_n C_n \exp\left(2\pi inr/d_r\right)$, where $d_r$ is the grating’s radial period and $C_n$ is the $n$th Fourier coefficient. Notably, a circular grating—which functions as a diffractive axicon or fraxicon [29]—scored on top of a refractive axicon effectively splits the converging conical carrier wave into the $0^{th}$ through-beam (similar convergence to that from a refractive axicon), a stronger converging $+1^{st}$ and a weaker converging $-1^{st}$ orders [Fig. 1(c)]. Hence, through the circular-hybrid axicon it is possible to generate multiple interfering conical beams which intersect the optical axis at angles $\beta(n = 0, \pm 1, \cdots) = \beta, \beta \pm \sin^{-1} \lambda/d_r, \cdots$. Due to the larger convergence angle $\beta(+1) > \beta$, the $\beta(+1)$-beam generates a narrower Bessel beam $w_0(+1) = \frac{2.4048}{\beta(+1)} \frac{\lambda}{2\pi}$ compared to that of a corresponding refractive axicon, $w_0$. 
Fig. 5. Micro-axicon with a radial diffraction grating on the curved surface. (A) Surface profile of a hybrid micro-axicon with a radial grating. Measured intensity cross-section of a reference axicon (B) is compared to that of a (C) hybrid axicon with a circular grating with a radial pitch of 10 µm and milled depth of 180 nm. (D) Comparison of measured spot size at full-width at half-maximum (FWHM) for both reference and hybrid axicons. The radial grating increases the effective cone angle for one of the diffraction orders. For the given axicon design (sag 3.75 µm, diameter of 230 µm, cone base angle \( \alpha \) of \( \sim 1.87^\circ \), grating pitch of 10 µm), the \(+1\)st order diffraction angle of \( \sim 2.63^\circ \) at wavelength of 458 nm is added to the refraction angle \( \beta \) of \( \sim 2.24^\circ \). The effectively increased converging angle \( \beta^{(+1)} = \beta + \theta \) is associated with the apparent reduction of the spot size by a factor of ~two.

Accordingly, if the \( 0^{th} \) order beam has not been eliminated, the circular-hybrid axicon will generate several Bessel beams with properties depending on the distance from the cone-tip: the first part will display a reduced depth-of-focus and will present a modulation in the position of the \( r_0 \) component of the Bessel beam; while further away from the element, beyond the depth-of-focus of the hybrid component the radius of the central spot will match the one calculated for the refractive component [Figs. 5(b)–(d) and Fig. 1(c)]. Figures 5(b) and (c) demonstrate
that the resolution is nearly doubled (spot size reduced from 4.2 µm to 2.2 µm) by introducing a 10-µm-pitch and 200-nm-deep circular grating on a 3.75-µm-deep axicon. To achieve a similar performance, the purely refractive axicon must have a larger >7 µm sag height, while the corresponding pure diffractive axicon must have a significantly denser 6 µm pitch. The hybrid circular-grating-axicon is, therefore, a viable path to enhance the resolution performance of a shallow refractive axicon by imposing a sparse diffractive pattern upon its surface. Moreover, the fabrication of both shallow conical structure and sparse diffraction patterns are technologically more accessible compared to either a taller refractive axicon or denser diffraction patterns that are needed to achieve similar performances.

6. Conclusion

In conclusion, using gray-scale focused Xe ion-beam writing we were able to realize the hybrid micro-optical elements in such a challenging piezo/optoelectronic material like lithium niobate with a yet unexploited potential in optical applications. The fusing of the diffractive and refractive elements in a single component allows an additional level of control over the beam-shaping and introduces new functionalities. Here, we showed that a single miniaturized axicon with a linear diffraction grating on its curved surface is able to generate a fan of non-diffracting Bessel-like beams with extended depth of focus. The resulting light-sheet-like intensity distribution extending both laterally and in the propagation direction has potential applications in fluorescence microscopy. On the other hand, a radial grating superimposed with a refractive axicon allows resolution improvement. Our data show that hybrid refractive-diffractive elements are not limited in their shape or functionality compared to the currently constrained optics design. Accordingly, through the fusion of multiple wave shaping approaches within a single miniaturized free-form component, it is possible to design enhanced and, at present, still not envisioned optical devices.

Appendix A

Assume an ordinary conical lens with radial symmetry, e.g., an axicon [28] made of a material with a refractive index $n$. In the thin phase screen approximation, the phase shift induced by the axicon onto a propagating $z$-directed plane wave is $\phi(r) = -\beta r = -\beta \sqrt{x^2 + y^2}$, where the scaling factor $\beta > 0$ and the optic axis $z$ is perpendicular to the transverse coordinates $(x, y)$. For a square-wave phase grating (Fig. 6), the efficiency for diffraction into the $m^{th}$ order is [38]

$$\chi_m = \frac{|e^{-i(1-n)A} - e^{-i(1-n)B}|^2}{\pi^2 (2|m| - 1)^2}$$

(1)

where $k = 2\pi/\lambda$ is the wavenumber, $\lambda$ is the wavelength, and the projected thickness of the grating is either $A$ or $B$. For $m = 1$, Eq. (1) becomes

$$\chi_{m=1} = \frac{2}{\pi^2} [1 - \cos(k(1-n)D)]$$

(2)

where $D$ is the grating groove-depth.

Let $z = 0$ denote the exit surface of the axicon with a diffraction grating superimposed onto its curvilinear surface (this may be termed a diffraction axicon). Let the diffraction be normally illuminated by a unit-intensity $z$-directed monochromatic plane wave. The complex amplitude at the exit of the diffraction axicon is thus

$$\psi(x, y, z = 0) = e^{i\phi_{\text{axicon}}(x, y)} \left[ \frac{c_1 + c_2}{\pi i} \sum_{m=1}^{\infty} \exp[2i\pi(2m-1)x/L] \frac{2m-1}{2m-1} + \frac{c_2 - c_1}{\pi i} \sum_{m=1}^{\infty} \exp[-2i\pi(2m-1)x/L] \frac{2m-1}{2m-1} \right]$$

(3)
with \( c_1 = e^{-i(1-n)kB} \) and \( c_2 = e^{-i(1-n)kA} \). Since \( \phi_{\text{axicon}}(x,y) = \phi(r) = -\beta r \), Eq. (3) becomes

\[
\psi(x,y,z = 0) = e^{-\beta \sqrt{x^2 + y^2}} \left[ \frac{c_1 + c_2}{2} + \sum_{m=1,3,5}^{\infty} \frac{1}{m} e^{2\pi i m x/L} \right. \\
\left. - \frac{c_2 - c_1}{2} \sum_{m=1,3,5}^{\infty} \frac{1}{m} e^{-2\pi i m y/L} \right].
\]

(4)

Under the paraxial approximation, the assumption of small diffraction angles implies these diffraction angles to be given by

\[
\theta_m \approx m \lambda / L.
\]

Let \( z_1 \) be a new \( z \) axis that, by definition, points along the first positive diffraction order at the angle \( \theta_m = 1 \approx \lambda / L \). The rotated Cartesian coordinate system, corresponding to the coordinate transformation \((x,z) \rightarrow (x_1,z_1)\) with \( y \) unchanged, is given by

\[
\begin{pmatrix}
  x_1 \\
  z_1
\end{pmatrix} =
\begin{pmatrix}
  \cos \theta_1 & -\sin \theta_1 \\
  \sin \theta_1 & \cos \theta_1
\end{pmatrix}
\begin{pmatrix}
  x \\
  z
\end{pmatrix}.
\]

(5)

Hence, for the first order \((m = 1)\) beam, the axicon phase in the \( z \)-direction is

\[
\text{Arg} \psi(x_1,y,z_1) = -\beta \sqrt{(x_1 \cos \theta_1 + z_1 \sin \theta_1)^2 + y_1^2} + k(z_1 \cos \theta_1 - x_1 \sin \theta_1).
\]

(6)

Let \( z_1 \rightarrow 0 \), and then assume the paraxial approximation \(|\theta_1| \ll 1\). Therefore,

\[
\text{Arg} \psi(x_1,y,z_1 = 0) = -\beta \sqrt{x_1^2 \cos^2 \theta_1 + y_1^2} - kx_1 \sin \theta_1 \\
\approx -\beta \sqrt{x_1^2 (1 - \frac{1}{2} \theta_1^2) + y_1^2} - kx_1 \theta_1 + O(\theta^3) \\
\approx -\beta \sqrt{x_1^2 + y_1^2 - \frac{1}{2} x_1^2 \theta_1^2} - kx_1 \theta_1 \\
\approx -\beta \sqrt{x_1^2 + y_1^2} - kx_1 \theta_1 + \frac{1}{2} \beta \theta_1^3 x_1^2 \\
\text{unaberrated axicon aberrations}
\]

(7)

The first-order beam, obtained when a diffraction grating is scored onto an axicon and the resulting diffraxicon is illuminated with normally incident monochromatic plane waves, evidently has a wavefront that corresponds to a conventional axicon phase map \([28]\) that is aberrated by the presence of both a phase ramp (tilt) and a non-trivial curved-wavefront aberration. The tilt aberration merely serves to deflect the beam, whereas the aberration represented by the final term of the above equation is non-trivial.
More generally, for the $m^{th}$ order, the diffraxicon phase is aberrated in the following manner:

$$\text{Arg } \psi(x_m, y_m, z_m = 0) \simeq -\beta \sqrt{x_m^2 + y_m^2} - kx_m \theta_m + \frac{1}{2} \beta \theta_m^2 \frac{z_m}{\sqrt{x_m^2 + y_m^2}}.$$  
(8)

We again see that the aberration consists of a tilt term that merely quantifies the background plane wave upon which a given diffraction order is carried, together with a non-trivial aberration function specified by the quotient in the above expression. If desired, this quotient could be expanded as a Taylor series consisting of a sum of terms, each of which have the form $c_{ab} x^a y^b$, where $a$ and $b$ are positive integers and $c_{ab}$ are Taylor coefficients; this would then give a close correspondence to the Cartesian form of the classical (Seidel) aberrations [39] via the Cartesian form of the Zernike polynomials [40]. While these aberrations (coma, spherical aberration etc.) serve to distort the resulting axicon beams associated with each diffraction order of the diffraxicon, the self-healing property of Bessel beams [41] implies that the influence of these aberrations will be weak.

**Appendix B**

**Fig. 7.** Lithium niobate micro-lenses with a linear diffraction grating on the curved surface. (A,B) White light interferometry measurements of surface profiles of ion-milled conventional and hybrid micro-lens, respectively. The hybrid lens has a 350 nm deep grating on its parabolic surface. (C-F) Measurements of beam profiles at the focus from the tip of lithium niobate 230-µm-diameter, 7.5 µm-deep lenses with 10-µm grating pitch on their surfaces with grating groove-depth of 0, 100 nm, 150 nm and 250 nm, respectively. The efficiency of various diffraction orders depends on the phase-shift modulation (i.e. groove-depth) introduced by the gratings. (G-I) Measured intensity cross-sections of hybrid microlenses with grating groove-depths of 0, 100 nm, 150 nm and 250 nm, respectively, are compared to the corresponding intensity profiles obtained from numerical Fresnel propagation (J-L).
Figure 7 gives a detailed example of our hybridization of diffractive and refractive optical elements. Figure 7(a) gives a surface-profile plot of an ion-milled conventional parabolic-shaped refractive lens, with Fig. 7(b) showing the same refractive lens with a simple 350-nm deep grating scored onto its surface. A transverse intensity-plane map of the focal plane of the ordinary refractive lens is shown in Fig. 7(c), with the label “0 nm” indicating the groove depth of the scored grating, which is this case corresponds to no superposed grating at all. We see the expected pattern consisting of the usual Airy spot and its surrounding Airy rings [39]. Focal-plane intensity patterns are shown for three hybrid refractive–diffractive lenses, with groove depths of 100 nm, 150 nm and 250 nm, in Figs. 7(d)–(f) respectively. The various orders of diffraction are evident, with the height of the central order being suppressed as the efficiency of the diffraction into the first order grows; cf. Eq. (1) above. The multiplexing nature of our hybrid optical elements is again evident, together with the improved efficiency for diffraction into the first order, as one passes from Fig. 7(b) to Fig. 7(i) and corresponding simulations in Figs. 7(j)–(l). The ability of the hybrid refractive-diffractive lens to constitute a multiplexing optical element is clear from all of these images, in which the actions of focusing and splitting into multiple beams are achieved using a single element.

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Disclosures

The authors declare no conflicts of interest.

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