Introduction

Braiding provides an ideal composite material for textile preforms, but the special cross section of braiding is restricted by the braiding procedure of machinery with different cross-section shapes. Thus, by studying historical braiding structures, a more reasonable braiding structure than the existing mechanical braiding structure has been introduced to improve the modulus strength and function of braiding composite materials. In addition, one major limitation of three-dimensional (3D) braiding is that the cross section of a preform is determined by different braiding machines. Furthermore, most industrial machines are designed only to braid preforms with invariable cross sections. One important advantage of 3D braiding is that preforms for a composite component with...
complex geometry can be made with a near-net shape. In terms of the cross-sectional braiding type, the most common braiding structures with variable sections are circular tables, hyperbolic scanning bodies, drum shapes, and other irregular circular rotating body braiding structures. Many engineering applications, however, such as aircraft rotor blades, aircraft radar, wind turbine blades, and missile shell heat shields, do not involve ISO sectional structural parts with regular shapes, but rather variably shaped cross-section structural parts whose section sizes change along the direction of the length or width. At present, the main method for obtaining a composite with a variable cross section is to first prepare a composite with an equal cross section of the appropriate size and then to carry out machining operations, such as cutting and grinding to obtain the final shape size. Mechanical processing has a significant impact on the properties of composite materials, including serious damage to the reinforced fiber, degradation of the integrity of the structure, and greatly reduced mechanical properties. Because 3D braiding technology for a variable section can be used for the complete braiding of prefabricated parts without damaging the reinforced fiber, research about this technology is particularly important.

The basic principle of 3D braiding technology is that the number and arrangement of yarns on the chassis of a braiding machine are closely related to the size and shape of the cross section of the specimen. In existing traditional braiding technology, once a particular cross section is selected as the initial braiding cross section, the size and shape of the yarn array will remain unchanged throughout the braiding process, resulting in the same cross-section structure as the original. Xiao et al. achieved the geometric forming simulation of 3D braided prefabricated parts by establishing a mathematical model between the textile process parameters and the geometric structure parameters. Shao et al. analyzed the spatial position of the yarn in a rectangular preform and determined the yarn path through Bezier curve fitting. Kim studied the simulation of the maypole braiding process. Yarn paths were calculated by simulating the relative movement of multilayer horn-gears and numerous spindles in the actual braiding process, and the overbraiding on mandrels with an arbitrary profile was possible. The braiding process determines the microstructure of composites, an important factor affecting mechanical properties. Before the 1980s, braiding was concentrated mainly in the four-step braiding process, gradually leading to the six-step, multistep, and later processes. Subsequently, the two-step method and multilayer interlock braiding technology were developed. After 2000, the 3D rotary braiding machine emerged. Great improvement in the speed of movement quickly made this the mainstream braiding technology.

Representative examples include the 3TEx and Herzog machines. Recently, RWTH Aachen University and the University of British Columbia jointly developed a second-generation hexagon knitting machine, which greatly increased the number of chassis and the amount of yarn. This machine centered on medical applications, and the 3D braiding equipment featured a variably shaped cross section; however, the track layout had a certain complexity, requiring higher professional standards for operators. In the existing literature, most of the studies provide solutions to specific local problems; the final shape of the cross section of a prefabricated part is achieved through continuous tests in actual production; and a systematic, clear-cut description of the evolution of these machines is lacking. Two recent books by Professor Yordan Kyosev in 2015 and in 2016 introduced the basic principle of braiding and the application field of fabric' structures. These books also analyzed in detail the principle of the carrier arrangements and the design process of special-shaped cross-section braiding fabric, particularly in relation to 3D rotary braiding. This information was especially useful for this study.

We proposed a carrier arrangement characteristic (CAC) method to obtain various forms of tracks by splicing the elements of the tracks, such as a “2 × 2 oblique” track, a “484” track, an “∞” track, and a “V” track. The CAC and the repeat involved the position and the number of carriers to avoid a collision between the carriers. We obtained a set of 4 × 4 tracks through the CAC method to prepare braids with variably shaped cross sections. The motion mechanism of the braiding consisted of two parts: (1) the carrier moves into position, and (2) the track changes. We then established the braiding matrix to predict the braiding matrix and the path. The braiding matrix could be carried out to represent all yarn positions. The oblique track not only avoided collision between carriers when they moved along the tracks, but also braided the variably shaped cross-section structure without changing the machine settings, thus allowing designers to braid the variably shaped cross section more easily.

### Track structure design

Commonly used track types include the single minimum units (such as a “484” track, “∞” track, “2 × 2 oblique” track, and “V” track), as shown in Table 1. Any track element larger than these minimum track elements can be

| Table 1. Commonly used minimum splice tracks. |
|---------------|----------------|-----------------|
| **Type**      | **484** | **∞** | **2 × 2 oblique** | **V** |
| **Scheme**    | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) | ![Image](image4.png) |
| **CAC**       | ![Image](image5.png) | ![Image](image6.png) | ![Image](image7.png) | ![Image](image8.png) |
| **Repeat**    | 8       | 8     | 2                | 4     |

...
composed of these minimum track elements spliced together.

For the 484 track type shown in the table, the first 4 represents one horn-gear with four slots, the 8 represents two horn-gears with eight slots, and the second 4 represents one horn-gear with four slots. The 484 track can restrain a carrier’s arrangement characteristics at 4 Full and 4 Empty \((f^4, e^4)\). Full indicates that there is a carrier in the position of the arrangement, and Empty represents the fact that there is no carrier in the position of the arrangement. The minimum arrangement repeat is 8 \((8 = 4 \text{ Full} + 4 \text{ Empty})\). Thus, the specific arrangement type of the carrier must be 10100101 to avoid a collision between the carriers. Similarly, 4 Full and 4 Empty are also required for the carrier arrangement characteristics in the \(∞\) track type—that is, when the minimum arrangement repeat is 8 \((8 = 4 \text{ Full} + 4 \text{ Empty})\) and the arrangement type is 10100101, there is no collision between the carriers. The specific form of the \(2 \times 2\) oblique track type is that the track is tilted to decorate, and the oblique angles are 45° and 135°, where \(2 \times 2\) indicates two rows and two columns of the horn-gear with four slots—that is, there are 4 horn-gears \((4 = 2 \times 2)\) in total, and the arrangement characteristic of the carriers is 1 Full and 1 Empty. The minimum configuration of its arrangement repeat is 2 \((2 = 1 \text{ Full} + 1 \text{ Empty})\), and the arrangement type of the carriers is 10. Thus, there is no collision between the carriers in the oblique track. The fourth type is the V-shaped track, which requires few carriers. Only when the minimum arrangement repeat of the carrier is 4 \((4 = 1 \text{ Full} + 3 \text{ Empty})\)—that is, when the arrangement form of the carrier is 1000, do the carriers not collide in the V-shaped track. Two or more of the types of minimum track elements shown in Table 1 can be spliced together to obtain a larger track element. The 16 track elements in Table 2 are composed of four \(2 \times 2\) oblique tracks spliced together in various ways. This is called a \(4 \times 4\) braided track unit, and the corresponding obtained splicing points (NoSP) are 0, 1, 2, 3, and 4. The number of splicing points is a statistical outcome of the number of splicing points around the \(4 \times 4\) track unit after the middle splicing position is completed.

Similarly, we spliced together four \(4 \times 4\) track units (No. 8, No. 9, No. 10, and No. 11) to form an \(8 \times 8\) oblique track unit, as shown in Figure 1, where the numbers \(1, 2, 3, \) and 4 in parentheses in No. 8 (1), No. 9 (2), No. 10 (3), and No. 11 (4) represent the order of splicing.

The basic idea behind this splicing principle is that any 3D braiding system (BS) can be regarded as being composed of \(n\) tracks connected by \(m\) switch cams, and can be written as follows:

\[
BS[T, V] = \sum_{i=1}^{m} \text{track}_i + \sum_{j=1}^{m} \text{switch}_j + \sum_{k=1}^{n} \text{horn – gear}_k, (1-1)
\]

where \(BS[T, V]\) is a braiding system with CAR (Carrier Arrangement Repeat) = \(T\) and \(V\) independent tracks. Formula (1-1) indicates that any braided unit is composed of \(n\) horn-gears, \(m\) switch cams, and \(L\) tracks, as shown in Figure 2; the numbers \(1, 2, 3, \) and 4 represent, the first, second, third, and fourth positions of the slots of a horn-gear, respectively.

The splicing principle of an \(8 \times 8\) track unit is shown in Figure 3. We established the structural analysis and the splicing method of tracks based on a BS unit in accordance with this splicing principle. The general CAR formula is

\[
\begin{align*}
ξ_L &= \dim \left\{ \bigcup_{i=1}^{L} M_i \right\} \\
T &= \sum_{j=1}^{m} f_j - \sum_{j=1}^{m} ξ_L
\end{align*}
\]

where \(f_j\) is the summation of the \(i\)th braiding unit, \(m\) is the number of braiding units, \(Ω\) is the number of track loops, \(M_i\) indicates the carrier arrangement characteristics of the \(i\)th track unit, \(\bigcap_{i=1}^{L-1} M_i\) indicates the arrangement characteristics of the carriers of the track unit composed of the first \(j\) splicing of the smaller track units (referred to as sub-track units here), and \(ξ_L\) indicates the number of carriers and the arrangement characteristics of the \(j\)th track unit (formed by the equivalent track units that are formed by the first \(j\) track units and the \((j+1)\)th track unit shown in Figure 3). The CAC is different from the “number of carrier arrangement characteristics,” and the CAC needs to be calculated first. Then the “number for the carrier arrangement characteristics” is calculated second, and the dim. \(\{M\}\) is the number of elements in the CAC, that is, the dimension of the CAC. To obtain the number of carrier arrangement characteristics of the track, the specific calculation method follows:

1. Determination of CAC of the braiding unit of Nos. 8–11: according to equation (1-2b), the CAC of each \(4 \times 4\) track unit of Nos. 8–11 is equal:

![Figure 1. 8×8 track decomposed into four 4×4 track units.](image-url)
Table 2. $4 \times 4$ units and their NoSP.

| NoSP | No.1 | No.2 | No.3 | No.4 | No.5 | No.6 | No.7 | No.8 | No.9 | No.10 | No.11 | No.12 | No.13 | No.14 | No.15 | No.16 |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0    | ![Middle position](image) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1    |  | ![Unit 2](image) | ![Unit 3](image) | ![Unit 4](image) | ![Unit 5](image) |  |  |  |  |  |  |  |  |  |  | |
| 2    |  |  | ![Unit 6](image) | ![Unit 7](image) | ![Unit 8](image) | ![Unit 9](image) |  |  |  |  |  |  |  |  |  | |
| 3    |  |  |  | ![Unit 10](image) | ![Unit 11](image) |  |  |  |  |  |  |  |  |  |  | |
| 4    |  |  |  |  | ![Unit 12](image) | ![Unit 13](image) | ![Unit 14](image) | ![Unit 15](image) |  |  |  |  |  |  |  | |

(a) There is only one type of $4 \times 4$ unit (No. 1 in Table 2) and its NoSP = 0.
(b) There are four types of $4 \times 4$ units (Nos. 2–5 in Table 2) and their NoSP = 1.
(c) There are six types of $4 \times 4$ units (Nos. 6–11 in Table 2) and their NoSP = 2.
(d) There are four types of $4 \times 4$ units (Nos. 12–15 in Table 2) and their NoSP = 3; there is only one type of $4 \times 4$ unit (No. 16) and its NoSP = 4.

\[ M_{i} = \left[ \begin{array}{c} f^{i} \\
 e^{i} \end{array} \right] \cup \left[ \begin{array}{c} f^{i} \\
 e^{i} \end{array} \right] \cup \left[ \begin{array}{c} f^{i} \\
 e^{i} \end{array} \right] = \left[ \begin{array}{c} f^{i} \\
 e^{i} \end{array} \right] \quad i = 1, 2, 3, 4. \quad (1-3) \]

\[ \xi_{i} = \dim \left( M_{i} \cup M_{i} \right) = \dim \left( \left[ \begin{array}{c} f^{i} \\
 e^{i} \end{array} \right] \cup \left[ \begin{array}{c} f^{i} \\
 e^{i} \end{array} \right] \right) = 4. \quad (1-4) \]

2. According to equations (1-2b) and (1-3), the number for the first carrier arrangement characteristics of the No. 8 track unit is

3. According to equations (1-2a) and (1-3), the CAC of the subtrack formed by the first (No. 8) and second (No. 9) track units is
4. According to equations (1-2b) and (1-5), the number for the second carrier arrangement characteristics of the No. 8, No. 9, and No. 10 track units is

\[ \xi_{LT RB MM} = \dim \left\{ M_{TR_{1}} \cup M_{b_{1}} \right\} = \dim \left[ \begin{bmatrix} f^{1} \\ e^{3} \end{bmatrix} \cup \begin{bmatrix} f^{1} \\ e^{3} \end{bmatrix} \right] = 4. \]  

(1-6)

5. According to equations (1-2a) and (1-3), the CAC of the subtrack formed by the first (No. 8), second (No. 9), and third (No. 10) track units is

\[ M_{TR_{1}} = M_{b_{1}} \]  

(1-7)

6. According to equations (1-2b) and (1-7), the number for the third carrier arrangement characteristics of the No. 8, No. 9, and No. 10 braiding units is

\[ \xi_{LT RMM} = \dim \left\{ M_{TR_{1}} \cup M_{b_{1}} \right\} = \dim \left[ \begin{bmatrix} f^{1} \\ e^{3} \end{bmatrix} \cup \begin{bmatrix} f^{1} \\ e^{3} \end{bmatrix} \right] = 4. \]  

(1-8)

7. Determination of repeat of the 8×8 track unit: According to equation (1-2b),

\[ T^{*} = \sum_{i=1}^{4} f_{i} - \sum_{j=1}^{3} \xi_{LT j} = 14 \cdot (4 + 4 + 4) = 2. \]  

(1-9)

Thus, the carriers of the 8×8 track unit are arranged in accordance with 10, and the repeat is 2.

**Principle of braiding with a variable cross section**

We spliced the 8×8 oblique track unit, as shown in Figure 1, using the previously discussed splicing method. This 8×8 oblique track unit could be used to braid 3D fabric with variably shaped cross sections. Figure 4 shows the 8×8 braiding machine developed by Professor Sun Zhihong of Donghua University. Based on this machine and using the principle of oblique track braiding, we introduced the braiding process of 3D fabric with variably shaped cross sections. In the process of braiding, the role of the horn-gear was to provide power for the movement of the carriers. The switchable cam (switch) shown in Figure 5(d) was provided on the carrier tracks at the junction of each pair of horn-gears, which served to control either the movement of the carriers around the original horn-gear or their transition to the next horn-gear. The advantage of this approach was that any continuous variably shaped, cross-sectional, and complex shape or form of fabric could be achieved with this single machine after a brief prearrangement of the carriers and tracks.

The motion mechanism of the carrier consisted of two parts: the rotation of the horn-gears and the rotation of the switches, as shown in Figure 5. The motion process is described as follows: At time \( t_{y} \), the carriers moved in their own respective tracks. The red track was marked as the track of line I, and the carrier in this track was marked as the carrier of line I, including the first and third carriers. In
the same way, the blue track was marked as the track of line II, and the carrier in its track was marked as the track of line II, including the second and fourth carriers, so that the two closed paths crossed each other, and the yarns intertwined at the intersection point, forming an invariably shaped cross-sectional braiding structure by controlling the switches. When the cross-sectional shape of the fabric needed to be changed at time $t_1$, that is, when the statuses of the switches on the second and fourth positions of the two horn-gears in track I needed to be changed, as shown in Figure 5(b), the value changed from 1 to 0, causing the first and third carriers in track I to move to track II. The status of the initial movement of the carriers was then disturbed, however. To balance the status of the original movement of the carriers at this moment, the status of the switches on the second and the fourth positions reverted to its initial status, and the value changed from 0 to 1, as shown at time $t_2$ in Figure 5(c). The status of the switches at this moment allowed the new specially shaped cross-section fabric to be formed. The braiding process was non-intermittent and continuous high-speed motion by constantly adjusting the status of the tongue-like switches effectively avoided collisions between the carriers in the intersection. Although this mechanism of braiding with variable cross sections was easy to describe in a logical programming language, it was difficult to present with computable mathematical expressions.

**Algorithm for modeling arrangement of tracks and carriers**

**Matrix algorithm design**

With the $8 \times 8$ braiding process as our research object, we used matrix theory and a symbolic operation to model the carrier movement to form a theoretical foundation for further study. We also investigated a braiding matrix calculation method. We predicted the path of the carriers in the $8 \times 8$ oblique track unit. Finally, we verified the feasibility of the braiding of variably shaped cross sections by establishing the connecting carrier’s walking trajectory and taking the braiding of variably shaped cross sections from the rectangular cross section to the circular cross section as an example.

Tracks I and II in the horn-gear arrangement are shown in Figure 6. The red and blue solid dots indicate a slot with carriers; otherwise, without carriers in the slots. Carriers could be placed at every slot of the horn-gear, but up to half of the slots of each horn-gear could be placed with carriers at the same time, so at least half the positions were needed to transmit the carriers from the adjacent horn-gears.18

By establishing an XOY rectangular coordinate system, as shown in Figure 6, the walking trajectory of each carrier could be predicted. We obtained the walking trajectory of each yarn, and through the interleaving and separation of all yarns, we completed the braiding of the 3D fabric with variably shaped cross sections ranging from rectangular-shaped cross sections to circular-shaped cross sections. To model and analyze the coordinate transformation of the

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**Figure 5.** Motion mechanisms of braiding with variably shaped cross sections and 0 and 1 states of the switch cam: (a) time $t_0$, (b) time $t_1$, (c) time $t_2$, and (d) state of switchable cam.

**Figure 6.** Arrangement of braiding track disk.
carriers in the braiding process, we introduced the position coordinate matrix $A(k)$ of the carrier and the state matrix $S(k)$ of the switches, as shown in equations (1-10) and (1-11), where $K$ represents the braiding steps. Each element of the matrix $A$ was presented in plural form.

$$A(k) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{17} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{27} \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots & a_{37} \\ a_{41} & a_{42} & a_{43} & a_{44} & \cdots & a_{47} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{171} & a_{172} & a_{173} & a_{174} & \cdots & a_{177} \end{bmatrix} = \begin{bmatrix} r_{s1} \\ r_{s2} \\ r_{s3} \\ r_{s4} \\ \vdots \\ r_{sa17} \end{bmatrix}, \quad (1-10)$$

where each element in the $8 \times 8$ braiding matrix $A(k)$ is expressed as a complex number $m + n \cdot i$; and $m$ and $n$ correspond to the $X$ and $Y$ coordinates of the carrier, respectively, such as $a_{11} = 0$, $a_{21} = 2 + i \ldots a_{171} = 0$; and $r_{s1} \sim r_{sa17}$, which represent the number-row vector of the carrier arrangement for the first cross section. The transformation values of the $X$ and $Y$ coordinates of the carriers after passing through the switches of different states described the effect of the switches on the movement of the carriers in the braiding process, as shown in equation (1-11):

$$S(k) = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & r_{s1} \\ 0 & 0 & s_{23} & 0 & \cdots & r_{s2} \\ 0 & s_{32} & 0 & s_{34} & \cdots & r_{s3} \\ 0 & 0 & s_{43} & 0 & \cdots & r_{s4} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & r_{sa17} \end{bmatrix}, \quad (1-11)$$

where the value of 0 indicates that there was no switch in this position; the value of the other elements is expressed as the complex number $c + v \cdot i$; the values of $c$ and $v$ correspond to the $X$-coordinate and $Y$-coordinate transformation values of the carriers after they passed through the switches, such as $s_{23} = 1 + i$, $s_{32} = -1 + i$, and so on; and $r_{s1} \sim r_{sa17}$ represents the number-row vector of the switches for the first cross section. By adding the elements with the same positions on $A(k)$ and $H(k)$, a recursive formula of the coordinate transformation of a single carrier in the braiding motion was obtained, as shown in equation (1-12):

$$A(k)[m,n] = A(k-1)[m,n] + S(k)[c,v]. \quad (1-12)$$

The carrier returned to the starting point after several braiding steps, as shown in equation (1-13)

$$A(T)[m,n] = A(0)[m,n], \quad (1-13)$$

where $T$ is the motion period of the carrier. The braiding mode of the carrier could be represented by the accumulation of $S(k)[c,v]$, where $k$ was taken from 1 to $T$. Accordingly, the braiding mode was obtained by giving the initial position matrix of the carrier and the state matrix of the switches. We then determined the motion law of the carrier and the fabric structure of the invariable cross section.

$8 \times 8$ Braiding matrix model for variably shaped cross section

Matrix $A(k+1)$ was established after the shape of the cross-section braiding was changed, as shown in equation (1-14):

$$A(k+1) = F_k[A(k)] = \begin{bmatrix} r_{s1} \times E_1 \\ r_{s2} \times E_2 \\ r_{s3} \times E_3 \\ r_{s4} \times E_4 \\ \vdots \\ r_{sa17} \times E_{17} \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 & \cdots & c_{17} \end{bmatrix}, \quad (1-14)$$

where $E_1 \sim E_{17}$ is the transform matrix of the $k$th cross-sectional shape. The row vectors of matrix $A(k+1)$ were $r_{s1} \sim r_{sa17}$. The equation was right-multiplied by the unit matrix $E_1 \sim E_{17}$ to obtain the $3D$ braiding matrix for the movement of the next shape of the cross section. The above matrix transform could be expressed by the function $F_k[A(k)]$ when changing the shapes of the cross section; and $c_1 \sim c_{17}$ represented the number column vector of the carrier arrangement for the next cross-sectional shape.

We took the $3D$ BS process with the carrier configuration of $8 \times 8$ as an example to show the analytical procedure, for which the unit matrix $E_1 \sim E_{17}$ was as follows:
Yarn path storage and yarn path table calculation

The motion cycle of the carrier was that the carrier returned to its original position after moving along the track, which was related to the length of the track in which the carrier moved. The position of a carrier determined the position of the yarn in the fabric structure, and we selected the middle position of each unit curve of the track as the characteristic point. Then we stored the corresponding X and Y coordinates in the corresponding track according to the order of the carrier’s movement. Thus, their positions and the corresponding arrangement of the carriers for the rectangular tracks (1–1 to 1–7 and 2–1 to 2–7 shown in Figure 7) were as noted in Table 3.

The 128 carriers (solid dots) in the $8 \times 8$ braiding system in Figure 7 moved along their own tracks so that they reached the 128 characteristic points, as shown in Table 3, where they were marked with respect to their initial positions at the initial time $t_0$. Then the locus database of the yarn and the trajectory was established and stored in the 1–128 units.

The carriers started from the initial position shown in Figure 7 and moved along their own tracks from $t_0$ until the next shape of the cross section was needed, as shown in Figure 8, that is, from the rectangular-shaped cross section to the circular-shaped cross section.

When the cross-section shape of the fabric changed from the rectangular shape to the circular shape according to the mechanism of the motion of the carrier described in Section 3, namely, before the carriers moved to the location of the switch cams, the state of the switch cams in the 20 locations shown in Figure 8(b) had to be changed from 1 to 0. Then we substituted the motion positions of the carriers into equation (1-14) to achieve the braiding of a variably shaped cross section.

According to the described method, we fitted and interpolated the X and Y data for each yarn in the database. Then the interpolated data were stored in the new database. These data were then called using MATLAB software and the yarns were drawn one by one in an image in which yarns on different tracks were defined using different colors. Then the fabric simulation from the rectangular cross section to the circular cross section was drawn, as shown in Figure 9. Finally, using the 3D braiding machine shown in Figure 4, we obtained the 3D braiding fabric with the cross-section shapes ranging from rectangular to circular, as shown in Figure 10, which verified the correctness of our proposed theory.

Conclusions

We proposed splicing principles of the braiding track based on the CAC method of the $2 \times 2$ oblique track unit. We established the corresponding methods for the track structure splicing of a braiding system that could achieve variably shaped cross-section braiding. We also established the algorithm design for calculating the braiding matrix to depict the movement process of the carriers and
Table 3. Spatial position and carrier arrangement of rectangular track.

| Track | Spatial position | Carrier arrangement |
|-------|------------------|---------------------|
| 1-1   | X[2 3 4 5 ... 17 16 15 ... 3 2 1] | 24°10' |
|       | Y[1 2 3 4 ... 16 17 16 ... 4 3 2] | 1010101010 |
| 1-2   | X[2 3 4 5 ... 13 12 11 ... 3 2 1] | 16°10' |
|       | Y[5 6 7 8 ... 16 17 16 ... 8 7 6] | 101010101010 |
| 1-3   | X[6 7 8 9 ... 17 16 15 ... 7 6 5] | 16°10' |
|       | Y[1 2 3 4 ... 12 13 12 ... 4 3 2] | 101010101010 |
| 1-4   | X[10 11 12 ... 17 16 15 ... 10 9] | 1010101010101010 |
|       | Y[1 2 3 ... 8 9 8 ... 3 2] | 1010101010101010 |
| 1-5   | X[2 3 4 ... 9 8 7 ... 2 1] | 1010101010101010 |
|       | Y[9 10 11 ... 16 17 16 ... 11 10] | 1010101010101010 |
| 1-6   | X[14 15 16 17 16 15 14] | 1010101010101010 |
|       | Y[1 2 3 4 5 4 3 2] | 1010101010101010 |
| 1-7   | X[16 15 14 13 ... 1 2 3 ... 15 16 17] | 24°10' |
|       | Y[1 2 3 4 ... 16 17 16 ... 4 3 2] | 1010101010101010 |
| 2-1   | X[16 15 14 13 ... 5 6 7 ... 15 16 17] | 16°10' |
|       | Y[5 6 7 8 ... 16 17 16 ... 8 7 6] | 101010101010 |
| 2-2   | X[12 11 10 9 ... 1 2 3 ... 11 12 13] | 16°10' |
|       | Y[1 2 3 ... 12 13 12 ... 4 3 2] | 101010101010 |
| 2-3   | X[8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9] | 101010101010 |
|       | Y[1 2 3 4 5 6 7 8 9 8 7 6 5 4 3 2] | 101010101010 |
| 2-4   | X[16 15 14 ... 9 10 11 ... 16 17] | 101010101010 |
|       | Y[9 10 11 ... 16 17 16 ... 11 10] | 101010101010 |
| 2-5   | X[16 15 14 13 14 15 16 17] | 101010101010 |
|       | Y[13 14 15 16 17 16 15 14] | 101010101010 |
| 2-6   | X[4 3 2 1 2 3 4 5] | 101010101010 |
|       | Y[1 2 3 4 5 4 3 2] | 101010101010 |

Figure 8. Scheme of arrangement from rectangular track to circular track: (a) rectangular track and (b) circular track.
the state of the switches. The structural simulation solution and the actual fabric from the rectangular cross section to the circular cross section were achieved. MATLAB software was employed to analyze an example to verify the correctness of the theory we set forth. The results showed that the fabric model established by the tracks based on this method was precise and useful in the actual braiding, and this model could provide a basis for the further research and analysis of the mechanical properties of variably shaped cross-sectional fabric.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the National Science Technology Support Plan Projects, “Development of the 3D Circular Carbon Fiber Braiding Machine” (grant number 2011BAF08B03).”

ORCID iD

Sun Zhihong  https://orcid.org/0000-0001-5516-5388

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