Effect of cosmological constant on radial oscillations and gravitational wave echoes of strange stars

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We study the effect of cosmological constant on radial oscillations and gravitational wave echoes (GWEs) of non-rotating strange stars. To depict strange star configurations we used two forms of equations of state (EoSs), viz., the MIT Bag model EoS and the linear EoS. By taking a range of positive and negative values of cosmological constant, the corresponding mass-radius relationships for these stars have been calculated. For this purpose, first we solved the Tolman-Oppenheimer-Volkoff (TOV) equations with a non-zero cosmological constant and then we solved the pressure and radial perturbation equations arising due to radial oscillations. The eigenfrequencies of the fundamental $f$-mode and first 22 pressure $p$-modes are calculated for each of these EoSs. Again considering the remnant of GW170817 event as a strange star, the echo frequencies emitted by such stars in presence of the cosmological constant are computed. From these numerical calculations, we have inferred relations between cosmological constant and mode frequency, structural parameter, GWE frequencies of strange stars. It is observed that the presence of cosmological constant significantly affects the oscillation frequency as well as the echo frequency get changed from that of zero cosmological constant value. Thus from our study we can conclude that the cosmological constant influences in radial oscillations and echo frequencies obtained for such ultra compact stars.

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I. INTRODUCTION

In 1916 Albert Einstein introduced the term cosmological constant $\Lambda$ as a modification in his field equation to achieve a static, stationary universe, which was believed to be the state of universe at that time. This new term introduced by him is a dimensionful free parameter [1]. But after the discovery of the redshift of stars and consequently an expanding universe by Hubble it was quickly abandoned. Later, it was reintroduced to overcome the age crisis problem and to construct a universe satisfying the “perfect cosmological principle” [2, 3]. On the contrary, current astrophysical and cosmological observational data reveal that we are in the era of cosmological expansion and the expansion rate of the universe is increasing rather than decreasing [4, 5]. One of the strong beliefs is that the present accelerated expansion of the universe is due to a exotic form of energy known as the dark energy. Although it suffers from the fine tuning problem, $\Lambda$ is considered as one of the prime candidates of the dark energy in the sense that it forms the vacuum energy [6]. As the dark energy pervades throughout the universe, the energy density inside the compact objects should be affected by it, and hence it is necessary to see the effect of dark energy content on the behaviour of such objects. So, if we consider the contribution of cosmological constant $\Lambda$ in compact objects it will be related to the acceleration of the current observable universe or the dark energy. This possibility of non-zero $\Lambda$ and it, as a dominating energy density of the universe, is a fascinating problem in now a days research.

One of the most intriguing parts of the issue of compact stars is the study of unique strange stars. Since the last decade, the subject of strange stars has attracted much attention of astrophysicists community. These hypothetical stars are unique in the sense that the structural behaviour of such objects rarely match with that of other compact objects, like white dwarfs and neutron stars. Due to their unique structural behaviour, the strange stars can be regarded as excellent natural laboratories to study, test or perhaps constrain different modified theories of gravity under extreme conditions that cannot be reached from the earth based laboratories. By hypothesis, strange matters are the true ground state of the hadrons and hence they are absolutely stable matters [7]. So, it could explain the origin of the huge amount of energy released in super-luminous supernovae (100 times brighter than normal supernovae) [8, 9]. Such strange matters are composed of deconfined quark matters, mainly of u, d, s quarks and a small amount of electrons to maintain charge neutrality [10, 11].

Our present study consists of two parts. In the first part we are seeking correlations between the cosmological constant $\Lambda$ and radial oscillation frequencies of strange stars. From long ago, the light variation in a pulsating star has been used to investigate the physical properties like mass, radius of a star. This indirect approach to understand about the physical properties of a star is known as asteroseismology [12]. Form the birth to the end of a star, nearly every star undergoes some kind of pulsations. Such asteroseismic behavioural study of stars are mainly of two types: radial and non-radial one. For the case of compact objects like

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white dwarf, neutron star and strange star, such radial asterosismic behaviours are reported earlier in [13–16]. Here, we stick our study in the case of radial oscillations only of strange stars.

In the second part of this study, we investigate about the effect of \( \Lambda \) on the gravitational wave echoes (GWEs) of strange stars. It has been recently suggested that some ultra compact post merger objects can emit GWEs [17–20]. Since strange stars are known to be very compact, we describe strange stars as ultra compact objects by using stiffer equations of state (EoSs). In the recent detection of gravitational waves (GWs) from the binary neutron star merging event GW170817, the nature of final massive remnant formed is still not confirmed. We consider it as a strange star and evaluate the corresponding echo frequencies emitted by such stars in presence of the cosmological constant \( \Lambda \). The ultra compact stars are those, whose compactness, \( C = M/R \) exceeds 1/3 [20]. Again to be an eligible candidate to emit echo frequencies, it is mandatory that the star possesses a photon sphere. This is the region above the surface of the star at a distance of \( R = 3M \) [17]. However, the compactness should not exceed the buchdahl’s limit line \( R = 9/4M \). The buchdahl’s limit line is the upper limit of compactness of fluid stars which describes the maximum amount of mass that can exist in a sphere before it must undergo the gravitational collapse [20].

In the article [21], the equations describing small radial oscillations of relativistic stars in presence of a cosmological constant was reported. They have also studied the impact of cosmological constant on the critical adiabatic index. Recently, for stable relativistic polytropic objects the effect of cosmological constant is reported in [22]. A more detailed analysis on the stability of polytropic sphere in the presence of a cosmological constant can be found in [23]. For the general relativistic case with a vanishing cosmological constant, the radial oscillation modes and echo frequencies for strange stars are studied in [24]. The present study is the extension of our previous study [24] in which we had taken into account the dependence of radial oscillation frequencies and GWE frequencies on different model parameters. Instead, this work focuses on the detailed study on effect of cosmological constant on radial oscillation modes and echo frequencies of strange stars for considered EoSs with one chosen constant parameter value for each EoSs.

Motivated from the previous works mentioned above, in this work we contribute to this rapidly growing field in two ways: we take the strange star as a probe to explore the consequences of a non-vanishing cosmological constant and allow it to vary inside the star. In particular, we study the structural change in strange star due to a non-vanishing cosmological constant, focusing on its effects over the radial oscillations and GWEs of strange stars. Here, we have solved the Einstein field equation for some finite values of cosmological constant \( \Lambda \) for a spherically symmetric mass distribution. By solving the Tolman-Oppenheimer-Volkoff (TOV) equations we have obtained the mass-radius relationships of strange stars in presence of a non-vanishing cosmological constant. Then we have computed the fundamental \( f \)-mode and first 22 pressure \( p \)-mode of radial oscillations, and GWE frequencies emitted by strange stars in two types of EoSs.

We have organized the rest of the paper as: In Sec. II the general relativistic formulations including TOV equations and perturbation equations are discussed briefly. In Sec. III the considered EoSs are described along with a brief note on the cosmological constant. The emission of GWEs from an ultra compact object is discussed in Sec. IV. In Sec. V we have discussed the results that are obtained from this study, which is finally followed by the concluding section, i.e. Sec. VI. Here we follow the natural unit system by considering \( c = \hbar = 1 \) and \( G = 1 \) with the metric convention \((-++,+,+)\).

II. GENERAL RELATIVISTIC FORMULATION

A. Equations for stellar structure

In this subsection we discuss briefly about the strange stars in general relativity (GR) in a spacetime with a cosmological constant, i.e. with \( \Lambda \neq 0 \). The Einstein’s field equation with the cosmological constant reads,

\[
G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu},
\]

We consider the strange star as an isotropic, stable, static and non-rotating mass distribution and the perfect fluid inside the strange star is described by the stress-energy tensor,

\[
T_{\mu\nu} = (p + \rho)U_{\mu}U_{\nu} + p g_{\mu\nu},
\]

where \( p \) is the fluid pressure, \( \rho \) is the fluid energy density and \( U_{\mu} \) are its four-velocities. The line element for a spherically symmetric mass distribution has the form:

\[
ds^2 = -e^{\chi(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2
\]

with the unknown metric functions \( \chi(r) \) and \( \lambda(r) \) being dependent on the radial coordinate \( r \) only. For the exterior problem the field equation (1) gives,

\[
e^{\chi(r)} = e^{-\lambda(r)} = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3},
\]
where $M$ is the total mass of the star. Similarly for the interior problem we have,

$$e^{\lambda(r)} = e^{-\lambda(r)} = 1 - \frac{2m(r)}{r} - \frac{\Lambda r^2}{3}. \quad (5)$$

Here $m(r)$ is the mass function within the radius $r$. On matching the exterior and interior solutions at the surface of the star we get, $m(R) = M$, the total mass of the stellar configuration with $R$ being the radius of the star.

The addition of cosmological constant $\Lambda$ in Einstein’s field equation (1) will change the structure of compact objects. Now solving Einstein’s field equation for the energy-momentum tensor (2) we get the stellar structure equations, known as the TOV equations [25, 26] with the introduction of cosmological constant [27] as

$$\frac{dm}{dr} = 4\pi\rho(r)r^2, \quad (6)$$

$$\frac{dp}{dr} = (\rho + p) \left( \frac{m + 4\pi pr^3 - \frac{\Lambda r^3}{3}}{r^2 \left( 1 - \frac{2m(r)}{r} - \frac{\Lambda r^2}{3} \right)} \right), \quad (7)$$

$$\frac{d\chi}{dr} = -\frac{2}{\rho + p} \frac{dp}{dr}. \quad (8)$$

To find static equilibrium stellar configurations equations (6) - (8) are to be integrated along the radial coordinate $r$ with the initial conditions: $m(r = 0) = 0$, $p(r = 0) = p_c$, and $\rho(r = 0) = \rho_c$. The radius of the star can be determined by using the fact that the energy density vanishes at the surface of the star, i.e., $\rho(r = R) = 0$. These equations for stellar structure can be solved for a given EoS. Solutions of these equations can lead us to know about the stellar mass, radius, pressure, density profile and the gravitational redshift.

### B. Radial stability equations

The theory of infinitesimal, adiabatic, radial oscillations and the radial stability of relativistic star was first derived by S. Chandrasekhar in 1964 [13, 14]. The solutions of these perturbation equations give the information about the eigenfrequencies of radial oscillations. In the presence of cosmological constant $\Lambda$, the radial instability equations were investigated earlier in [21, 28]. By introducing two dimensionless parameters $\xi = \Delta r/r$ and $\eta = \Delta p/p$, where $\Delta r$ is the radial perturbation and $\Delta p$ is the corresponding Lagrangian perturbations of the pressure, Bhömer and Harko [21] presented the radial and pressure perturbation equations as

$$\frac{d\xi}{dr} = -\frac{1}{r} \left( 3\xi + \frac{\eta}{\gamma} \right) \frac{dp}{dr} \frac{\xi}{p + \rho}, \quad (9)$$

$$\frac{d\Delta p}{dr} = \xi \left[ \omega^2 e^{\lambda - \chi} (\rho + p)r - 4\frac{dp}{dr} + \frac{r}{(\rho + p)} \left( \frac{dp}{dr} \right)^2 - e^{\lambda} (8\pi p - \Lambda) (\rho + p)r \right] + \left[ \frac{1}{(\rho + p)} \frac{dp}{dr} - 4\pi (\rho + p) r e^\lambda \right] \Delta p, \quad (10)$$

where $\gamma = \frac{dp}{dr} (1 + \rho/p)$ is the relativistic adiabatic constant.

In order to study the stability of the stellar object against a small radial perturbation we have to integrate equations (9) and (10) from the centre to the boundary along with the boundary conditions, $(\Delta p)_r = 0 = -(3\gamma p\xi)_r$ at the centre and at the surface of the star, $(\Delta p)_r = R = 0$. These coupled differential equations constitute the Sturm-Liouville type eigenvalue problem. These equations (9) and (10) together with the boundary conditions determine the eigenvalues or eigenfrequencies $\omega$ of radial oscillations [24]. These equations are solved by using the shooting method as described in [29]. If $\omega$ is real, i.e., $\omega^2 > 0$, the configuration is stable and for $\omega^2 < 0$, the configuration becomes unstable against radial oscillations. Again for the stable fundamental mode ($f$-mode) i.e., for $\omega_f^2 > 0$, all other higher order modes will also be stable.
III. EQUATIONS OF STATE AND THE COSMOLOGICAL CONSTANT

Before proceeding to discuss the role of cosmological constant on strange stars oscillation and echo frequencies emitted by them, we wish to add a few comments on our choice of stellar models. It is well known that the macroscopic properties of compact objects, such as mass and radius, depend crucially on the EoSs of ultra compact matter, which is unfortunately not confirmed clearly till date. In this study the chosen EoSs are those that (i) should stiff enough to emit GWE, (ii) the model parameters associated with each EoS lie inside the desired range, and (iii) the mass-radius relations of which are within the accepted limits. To describe the stellar structure, in this present work, we are using two EoSs, viz., the MIT Bag model EoS and the linear EoS as mentioned above.

The MIT Bag model corresponds to a relativistic gas of deconfined quark matter with energy density. This EoS satisfies all necessary criteria for strange matter, while retaining an elegant simplicity [30]. It was first used by Witten in 1984 [7]. The form of this equation is

\[
p = \frac{1}{3}(\rho - 4B).
\]  

(11)

In this EoS, the density at the stellar surface is given by \(\rho(R) = 4B\), where \(p\) is the isotropic pressure, \(\rho\) is the energy density and \(B\) is the Bag constant. Instead of taking this usual form (11) of the MIT Bag model EoS we have chosen the stiffer form of this EoS as

\[
p = \rho - 4B
\]  

(12)

The reason behind doing this is that in EoS of the form (11) the compactness of the stellar structure obtained are not enough to emit GWE frequencies. Making it a stiffer one of the form (12) allows the star to get enough compactness to emit GWE frequencies. As reported in [31], the acceptable range of Bag constant is \((133.68\text{ MeV})^4 < B < (222.54\text{ MeV})^4\). So in this work we choose B as \((190\text{ MeV})^4\), which is well inside the desired range.

Other EoS we have used is the linear EoS of the form:

\[
p = b(\rho - \rho_s)
\]  

(13)

where \(b\) is the linear constant and \(\rho_s\) is the surface energy density [32]. This EoS was developed by Dey et al. in 1998 [33]. We have chosen the linear constant \(b = 0.910\) and the corresponding value of the surface energy density \(\rho_s\) is taken. While choosing the constant value, we have kept in mind the conditions for echoing GWs, which impose the restriction that the compactness should be larger than \(1/3\) and also it should respect the causality condition \((b \leq 1)\) [24]. Our chosen value of \(b\) lies under these two restrictions.

The dimensionful parameter \(\Lambda\) introduced by Einstein in his field equation has the unit of \((\text{length})^{-2}\). The recent cosmological observations suggest the existence of a cosmological constant with the value, \(\Lambda = (4.24 \pm 0.11) \times 10^{-66}\text{ eV}^2 = (2.846 \pm 0.076) \times 10^{-122}\) [34]. Although the value of cosmological constant is very small, it has a very significant role in astrophysical phenomena of the universe. However for the case of compact objects in the present universe, such small values of cosmological constant \(\Lambda\) can not affect their properties. So to get a significant contribution for the bulk properties of compact object, cosmological constant \(\Lambda\) would have to be of the order of nuclear energy density scale, \(\epsilon = 140 \text{ MeV/fm}^3\) [27]. In [21], the upper limit of \(\Lambda\) for a star composed from matter having a density equivalent to nuclear energy density was reported to be \(\Lambda < 3 \times 10^{-13}\text{ cm}^{-2}\). With these constraints, we have chosen a set of positive and negative values of \(\Lambda\) to see its impact over radial oscillation modes and on echo frequencies of strange stars. Our chosen positive and negative values of \(\Lambda\) are in multiples of \(\epsilon\) as \(-150\epsilon, -100\epsilon, -50\epsilon, 5\epsilon, 10\epsilon, 15\epsilon\). As described by Carroll, in classical GR, there is no preferred choice for the length scale defined by \(\Lambda\). Indeed, the cosmological constant is a measure of the energy density of the vacuum or the state of lowest energy. This value cannot be calculated with any confidence. So, it brings flexibility in choosing scales of various contributions to the cosmological constant [1].

IV. GRAVITATIONAL WAVE ECHOES

As mentioned in Sec. I, an important property of ultra compact objects is that some of them can emit GWE frequency. It is a very interesting property of such stars. This happens due to the presence of a photon sphere around the surface of the star. More precisely, GWEs originate from the GWs that are trapped between the photon sphere line and the surface of the ultra compact star [19]. As mentioned in [19], such type of echoes have two natural frequencies: the harmonic or resonance frequencies and the black hole ringdown or quasi-normal mode (QNM) frequencies.

In this study we have chosen the final remnant of GW170817 event as a strange star and calculated the GWEs emitted by such ultra compact objects. The GWE frequency can be approximately estimated from the inverse of the time taken by a massless
Using the relation for expressed as [20]

\[ \tau_{echo} = \int_0^{3M} e^{\lambda(r) - \chi(r)/2} \, dr. \]  

(14)

Using the relation for \( e^{-\chi(r)} \) from equation (5) in presence of a non-zero cosmological constant \( \Lambda \), we can obtain the following expression for characteristic echo time,

\[ \tau_{echo} = \int_0^{3M} \sqrt{\frac{1}{e^{\chi(r)} \left( 1 - \frac{2m(r)}{r} - \frac{\Lambda r^2}{3} \right)}} \, dr. \]  

(15)

In this equation, the term \( m(r) \) and \( \chi(r) \) can be obtained from the solution of TOV equations with non-vanishing cosmological constant \( \Lambda \), i.e. of equations (6) - (8). Finally, the characteristic echo frequency can be calculated by using the relation, \( \omega_{echo} \approx \pi/\tau_{echo} \).

V. NUMERICAL RESULTS

As mentioned earlier, in comparison to other compact objects the structural properties of strange stars are unique. The mass-radius relationship of these stars follows a definite pattern than that of neutron stars. Strange matter can exist in lumps with the size of few fermis to the size of \(~10\) km radius strange stars [10]. Using the stiffer MIT Bag model EoS and linear EoS we have plotted the sequences of mass-radius relationships for such stellar configurations. In this study, with the MIT bag model EoS, the maximum mass is obtained as \( \approx 3.33 M_\odot \) and the corresponding maximum radius is \( \approx 14.34 \) km. The maximum mass obtained for the linear EoS is \( \approx 1.78 M_\odot \) and the maximum radius is \( \approx 7.61 \) km. The first plot of Fig. 1 shows the relationship for the MIT Bag model EoS and for the linear EoS it is shown in the second plot. As shown in these two plots, for much of these sequences strange stars are following the \( M \propto R^3 \) relation. However for neutron stars, radii decreases with increasing mass for much of their ranges. For both of the models, with decreasing \( \Lambda \), more stiffer configurations are obtained. The stiffness is observed to be maximum with \( \Lambda = -150 \) and for the MIT Bag model it is higher than the linear EoS. Linear EoSs are giving smaller stellar configurations than the MIT Bag model EoSs. In case, if we choose the general form of the MIT Bag model EoS, then the compactness will not be sufficient to overcome the photon sphere limit as pointed out earlier. The stiffer form of MIT Bag model and linear EoSs are giving compact enough configurations to cross the photon sphere limit but not the Buchdahl’s limit line. From Fig. 1, it is clear that the sequences of stellar configurations that are obtained can emit GWE frequencies with the considered \( \Lambda \) values.

As mentioned above, we have calculated the fundamental \( f \)-mode and 22 lowest \( p \)-modes of radial oscillation of strange stars for the considered EoSs. The variation of oscillation frequencies \( \nu_n \) (in kHz) is plotted against the oscillation modes \( n \) in Fig. 2. Oscillation frequency increases linearly with the order of modes. For all values of \( \Lambda \), it is increasing linearly with increasing modes for both of the EoSs. We have observed the maximum oscillation frequency for \( \Lambda = -150 \) and the minimum frequency
is obtained for $\Lambda = 15 \epsilon$ for both of the EoSs. The first panel of this figure is for the MIT Bag model EoS and the second panel is for the linear EoS. Linear EoS is giving frequency above 240 kHz, while for the case of MIT Bag model the maximum frequency is not exceeding 150 kHz. In Tables I and II, we summarize the radial oscillation frequency modes of strange stars obtained for the MIT Bag model EoS and the linear EoS respectively for all considered values of $\Lambda$. From these tables, it is inferred that for all these modes, more positive $\Lambda$ values are giving smaller oscillation frequencies i.e. with increasing $\Lambda$ values oscillation frequencies are getting smaller for each oscillation mode. Also, each mode frequency for all $\Lambda$ values is larger for the linear EoS than that for the MIT Bag model EoS. These are already visualized in Fig. 2.

In Fig. 3, the variation of separation between two consecutive modes of radial frequencies ($\Delta \nu$) with respect to the mode
TABLE II: Radial oscillation frequencies $\nu_n$ in kHz for the linear EoS for negative, zero and positive values of $\Lambda$.

| Modes (Order n) | $\Lambda = -150$ | $\Lambda = -100$ | $\Lambda = -50$ | $\Lambda = 0$ | $\Lambda = 5$ | $\Lambda = 10$ | $\Lambda = 15$ |
|-----------------|------------------|------------------|------------------|----------------|----------------|----------------|----------------|
| $f$ (0)         | 18.67            | 16.44            | 14.03            | 11.44          | 11.18          | 10.91          | 10.64          |
| $p_1$ (1)       | 28.90            | 25.78            | 22.42            | 18.86          | 18.49          | 18.13          | 17.76          |
| $p_2$ (2)       | 38.98            | 34.90            | 30.52            | 25.89          | 25.41          | 24.94          | 24.47          |
| $p_3$ (3)       | 49.06            | 43.97            | 38.52            | 32.79          | 32.21          | 31.63          | 31.05          |
| $p_4$ (4)       | 59.17            | 53.04            | 46.50            | 39.64          | 38.94          | 38.25          | 37.56          |
| $p_5$ (5)       | 69.32            | 62.12            | 54.47            | 46.46          | 45.65          | 44.84          | 44.04          |
| $p_6$ (6)       | 79.51            | 71.22            | 62.44            | 53.27          | 52.35          | 51.42          | 50.50          |
| $p_7$ (7)       | 89.72            | 80.35            | 70.42            | 60.08          | 59.03          | 58.00          | 56.96          |
| $p_8$ (8)       | 99.95            | 89.49            | 78.41            | 66.88          | 65.72          | 64.57          | 63.41          |
| $p_9$ (9)       | 110.21           | 98.64            | 86.41            | 73.69          | 72.41          | 71.14          | 69.87          |
| $p_{10}$ (10)   | 120.47           | 107.81           | 94.42            | 80.50          | 79.10          | 77.71          | 76.32          |
| $p_{11}$ (11)   | 130.76           | 116.98           | 102.43           | 87.32          | 85.80          | 84.29          | 82.78          |
| $p_{12}$ (12)   | 141.05           | 126.17           | 110.45           | 94.13          | 92.50          | 90.86          | 89.24          |
| $p_{13}$ (13)   | 151.35           | 135.37           | 118.48           | 100.96         | 99.20          | 97.44          | 95.70          |
| $p_{14}$ (14)   | 161.66           | 144.57           | 126.52           | 107.78         | 105.91         | 104.03         | 102.17         |
| $p_{15}$ (15)   | 171.97           | 153.78           | 134.56           | 114.61         | 112.61         | 110.62         | 108.63         |
| $p_{16}$ (16)   | 182.30           | 162.99           | 142.60           | 121.45         | 119.33         | 117.21         | 115.11         |
| $p_{17}$ (17)   | 192.62           | 172.21           | 150.65           | 128.28         | 126.04         | 123.81         | 121.58         |
| $p_{18}$ (18)   | 202.95           | 181.43           | 158.70           | 135.12         | 132.76         | 130.40         | 128.06         |
| $p_{19}$ (19)   | 213.28           | 190.65           | 166.76           | 141.97         | 139.48         | 137.00         | 134.53         |
| $p_{20}$ (20)   | 223.62           | 199.88           | 174.82           | 148.81         | 146.20         | 143.60         | 141.01         |
| $p_{21}$ (21)   | 233.96           | 209.11           | 182.88           | 155.66         | 152.93         | 150.21         | 147.50         |
| $p_{22}$ (22)   | 244.30           | 218.35           | 190.94           | 162.51         | 159.65         | 156.81         | 153.98         |

FIG. 3: Variation of difference between consecutive modes of radial frequencies ($\Delta \nu_n$) with radial frequencies $\nu_n$ for different $\Lambda$ values. First panel and second panel are for the MIT Bag model EoS and linear EoS respectively.

frequencies $\nu$ is shown. First panel is for the MIT Bag model and the second panel is for the linear EoS. For MIT Bag model with positive $\Lambda$ values a gradually decreasing pattern is observed. The difference between the oscillation frequencies of $f$-mode and $p_1$-mode is larger than that of the other values. However in the case of more negative values of $\Lambda$, the opposite behaviour is observed. Unlike other $\Lambda$ values, for $\Lambda = -150$ and $-100$ the difference between oscillation frequencies of $f$-mode and $p_1$-mode is smaller than the rest of the variations. In the case of linear EoS nearly a smooth variation for all the considered $\Lambda$ values is observed. For $\Lambda = -150$ both EoSs give maximum value of $\Delta \nu$ and for $\Lambda = 15$ they give minimum value of $\Delta \nu$ in case of modes other than $f$-mode and $p_1$-mode with MIT Bag model EoS. That is under this condition $\Delta \nu$ decreases with increasing values of $\Lambda$. 

is also larger near the centre and near the surface of the star. For all other modes in between $f$-mode and $p_{22}$-mode, pressure

From the Sturm-Liouville dynamic pulsation equations in presence of a cosmological constant (equations (9)-(10)), we have studied the variation of radial and pressure perturbations with the radial distance $r$. In Fig. 4 and 5 the variation of radial perturbation $\xi(r)$ with radial distance $r$ is shown for $f$-mode and $p_{22}$-mode respectively. In first panel of Fig. 4, the variation of radial perturbation $\xi(r)$ with $r$ (in km) is shown for the MIT Bag model and in the second panel the variation is shown for the linear EoS. From these two plots it is clear that the radial perturbation is larger only near the centre of the star. It is diminishing near the surface of the star for both these EoSs. As the EoSs with different $\Lambda$ values are mimicking strange stars of different sizes, so the perturbation along the radial distances are found different.

Similarly in Fig. 5, the variation is shown for higher order $p_{22}$-mode. For the MIT Bag model it is shown in the first panel of the figure. As like the $f$-mode, the perturbation is larger near the centre of the star. Towards the surface a distinct damping nature of radial perturbation is noticed. The different $\Lambda$ values are also showing distinct variation in their respective perturbations. As shown in the second panel of Fig. 5, for the case of linear EoS, the variation of $p_{22}$ mode is behaving similarly as that of the MIT Bag model EoS. The perturbation is decreasing along the surface of the star. Intermediate pressure perturbations curves can be drawn which will lie in between $f$ and $p_{22}$ modes.

The variation of pressure perturbation is different from the variation of radial perturbation. As clear from Fig. 6 and 7, the variation is larger near the centre and surface of each stars. Whereas, for the case of radial perturbation, it is larger near the centre of the star only. In the first panel of Fig. 6, the change in pressure perturbation with distance from the centre of star is showing for the MIT Bag model EoS for fundamental $f$-mode due to different $\Lambda$ values. The different sizes of each star for each $\Lambda$ values are also clear from this plot. The similar behaviour is observed for linear EoS with fundamental $f$-mode. This can be visualized from the second panel of this Fig. 6.

This pressure variation for higher order oscillation mode i.e. for $p_{22}$-mode is shown in the Fig. 7. First panel corresponds to the MIT Bag model and the second panel corresponds to the linear EoS. Similar to the $f$-mode of oscillation this variation is also larger near the centre and near the surface of the star. For all other modes in between $f$-mode and $p_{22}$-mode, pressure
echo frequencies are obtained for this EoS. The characteristic echo times obtained are smaller than that of the MIT Bag model EoS and hence in turn the larger
Λ results are shown in Table IV. For all the chosen
values the size of stars are increasing. The compactness of the stars are decreasing with more positive values. Thus the
perturbation will stay in between.

After the numerical analysis of the perturbation equations, the another important part of this work is to find the effect of cosmological constant \( \Lambda \) on GWE frequencies. The GWE frequencies are found to diminish with increase in \( \Lambda \) values for both of the EoSs. For the MIT Bag model with negative \( \Lambda \) values the variation is straightaway, however a rapid drop is observed for positive \( \Lambda \) values. The linear EoS is showing almost a straight variation for both positive and negative values of \( \Lambda \). These variations are shown in Fig. 8, where the first panel is for the MIT Bag model and the second panel is for the linear EoS.

The properties of stellar structure such as mass, radius and compactness along with the results for the GWE frequencies and characteristic echo times that are obtained by using the MIT Bag model EoS are compiled in Table III. From this table, the change in these properties of strange stars for the MIT Bag model with different \( \Lambda \) values can be seen clearly. With increasing \( \Lambda \) values the size of stars are increasing. The compactness of the stars are decreasing with more positive values. Thus the upper limit on cosmological constant \( \Lambda \) value is depicting a star with a compactness such that it is just able to echo the falling GWs. For our considered maximum value of \( \Lambda \) (15 \( \epsilon \)), the compactness is found to be 0.3430, which is greater than 1/3. The characteristic echo time obtained for this EoS increases with the increasing \( \Lambda \) value, and hence the reverse effect is noticed for the GWE frequencies. For this EoS, all echo frequencies obtained are in the range of tens of kilohertz.

The sizes of stars obtained by using the linear EoSs are smaller than that obtained for the MIT Bag model EoSs. Eventually the masses are also small in comparison to that for the MIT Bag model and hence fulfilling the criteria for echoing GWs. These results are shown in Table IV. For all the chosen \( \Lambda \) values, the stars are found to be with the enough compactness to emit GWE frequencies. The characteristic echo times obtained are smaller than that of the MIT Bag model EoS and hence in turn the larger echo frequencies are obtained for this EoS.
FIG. 8: The variation of GWE frequencies with different \( \Lambda \) values for the MIT Bag model EoS (first panel) and the linear EoS (second panel).

| $\Lambda$ (in km) | $R$ (in $M_\odot$) | $M$ (in $M_\odot$) | Compactness (M/R) | Echo time (ms) | GWE frequency (kHz) |
|------------------|------------------|------------------|------------------|--------------|------------------|
| -150 $\epsilon$  | 13.003           | 3.327            | 0.3784           | 0.048        | 64.90            |
| -100 $\epsilon$  | 13.090           | 3.315            | 0.3745           | 0.055        | 56.74            |
| -50 $\epsilon$   | 13.259           | 3.300            | 0.3681           | 0.065        | 48.44            |
| 0                | 13.766           | 3.295            | 0.3540           | 0.078        | 39.91            |
| 5 $\epsilon$     | 13.893           | 3.300            | 0.3512           | 0.080        | 38.86            |
| 10 $\epsilon$    | 14.067           | 3.308            | 0.3477           | 0.083        | 37.42            |
| 15 $\epsilon$    | 14.340           | 3.326            | 0.3430           | 0.089        | 35.08            |

| $\Lambda$ (in km) | $R$ (in $M_\odot$) | $M$ (in $M_\odot$) | Compactness (M/R) | Echo time (ms) | GWE frequency (kHz) |
|------------------|------------------|------------------|------------------|--------------|------------------|
| -150 $\epsilon$  | 7.200            | 1.742            | 0.3577           | 0.030        | 103.42           |
| -100 $\epsilon$  | 7.271            | 1.749            | 0.3557           | 0.033        | 94.24            |
| -50 $\epsilon$   | 7.372            | 1.759            | 0.3529           | 0.037        | 84.12            |
| 0                | 7.535            | 1.775            | 0.3484           | 0.043        | 72.91            |
| 5 $\epsilon$     | 7.558            | 1.778            | 0.3478           | 0.044        | 71.71            |
| 10 $\epsilon$    | 7.584            | 1.780            | 0.3472           | 0.045        | 70.51            |
| 15 $\epsilon$    | 7.610            | 1.783            | 0.3464           | 0.045        | 69.40            |

VI. CONCLUSIONS

In this study we look into the role of cosmological constant on two interesting aspects of strange star. In one part we have tried to understand its impact over the radial oscillations of strange star and in other, we have investigated its effect on GWE frequencies emitted by a strange star, like the star formed in the binary merging event GW170817. This study is made in the general relativistic framework. To study the role on radial oscillations we solved the TOV equations for two EoSs, viz., the MIT bag model EoS and the linear EoS. The solution of TOV equations lead us to know the structure of strange stars in presence of cosmological constant \( \Lambda \), which is found to be different from the structure obtained by using the vanishing \( \Lambda \) [24]. Again as discussed in Sec. V, different \( \Lambda \) values give us different strange star configurations. Moreover, the pulsation equations developed by Chandrasekhar are modified for the spacetime with a cosmological constant by introducing two dimensionless parameters.
\( \xi \) and \( \eta \). The solution of these pulsation equations gave us eigenfrequencies of oscillations and hence we have calculated the \( f \)-mode and first 22 \( p \)-modes of oscillations. We found that the large value of \( \Lambda \) decreases the radial oscillation frequencies of star for both of the EoSs. Further, to see the role of \( \Lambda \) on GWEs first we calculated the characteristic echo time of GWs falling on strange star. As the presence of \( \Lambda \) has altered the expression for the metric term \( e^{-\lambda} \), so the expression for echo time would change. Using this new expression for \( e^{-\lambda} \), we have calculated the echo frequencies emitted by such ultra compact stars. We observe that emitted echo frequencies get change due to the presence of \( \Lambda \). For both of the EoSs considered in this study, with increase in \( \Lambda \) value, echo frequencies get smaller.

Although the effects of the cosmological constant have been widely studied for compact objects, the study on impact of cosmological constant particularly on strange star’s oscillations and echo frequencies has not come into our notice. It is to be noted that, though our work is relevant for an ideal scenario only, yet our results are interesting as these may guide us to access the practical scenario from future observational data. In actual practice, the large amplitude of oscillation modes only occurs in catastrophic situations, such as in core collapse supernovae. Such events are highly non-spherical and hence in such cases the non-radial modes could become more applicable. So a possible extension of our present work would be to study the non-radial oscillations of strange stars and the corresponding GW signals associated with them.

In [19], the authors have claimed a tentative detection of echoes from the remnant of GW170817 event at a frequency \( \simeq 72 \) Hz. Although the consistent mass \((2.6-2.7M_{\odot})\) of this remnant almost lies within the range of masses of strange stars predicted by our EoSs with different \( \Lambda \) values, the GWE frequencies found from our calculations are much higher (in kHz range) than this tentative detection value. The basic reason of the difference between this result and our results is that instead of considering the remnant of GW170817 as an exotic compact object (ECO), we have considered the star of this mass range as a strange star. Further, while considering the EoSs, we have neglected the possible temperature effect on EoS. Again we choose strange stars as non-rotating one and solved the TOV equations for static stellar model while investing these properties. So, the study of rotating, anisotropic strange star or other possible ECO is a topic that will be considered as a future work.

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