An attempt to observe economy globalization: the cross correlation distance evolution of the top 19 GDP’s

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Economy correlations between the 19 richest countries are investigated through their Gross Domestic Product increments. A distance is defined between increment correlation matrix elements and their evolution studied as a function of time and time window size. Unidirectional and Bidirectional Minimal Length Paths are generated and analyzed for different time windows. A sort of critical correlation time window is found indicating a transition for best observations. The mean length path decreases with time, indicating stronger correlations. A new method for estimating a realistic minimal time window to observe correlations and deduce macroeconomy conclusions from such features is thus suggested.

Keywords: correlations, gross domestic product, globalization, econophysics, linear network

1. Introduction

How the national economy changes and notably how it increases, i.e. macroeconomic considerations have entered econophysics research.

One question which has been raised is whether there is a possibility to model macroeconomic questions and features from statistical models or better in our opinion from so-called microscopic models. There have been attempts a long time ago and more recently about so-called business cycles. There is no certain way with so few data points to claim that the distribution of GDP increments, or cumulative increments, called recession or prosperity periods, follow a power law or an exponential distribution. This has been discussed elsewhere. A few papers of interest pertain to scaling laws in the durations of recession and prosperity, e.g. when measured through the GDP of a country. Indeed in most countries, the Gross Domestic Product (GDP) is the official measure of the economic output
because it is the best basis for evaluating the well being of the citizens of a given country.\cite{11}

GDP is usually defined as the total market value of all final goods and services produced in a country in a given year, equal to total consumption, investment and government spending, plus the value of exports, minus the value of imports.\cite{12} GDP differs from the Gross National Product (GNP): For example, in the USA GNP includes the value of goods produced by USA citizens abroad, in China, Argentina or Brazil.

There are formidable problems involved in constructing GDP data series, many of which are discussed in \cite{13} It is important for readers with a natural science background to appreciate that, outside financial markets, almost all economic data is estimated and the margin of error around any individual data point can be substantial. This is the case even with modern estimates of GDP data for the most recent years. The business activity might, as it is known, be measured through qualitative factors, including the concept of economic sentiment indicator, and others sometimes more quantitative; see the interesting studies by \cite{14}. Moreover the time scales are not very clear, e.g it is known that it takes a long time before some policy change is implemented, has some effect which can be later analyzed and corrected if necessary. Parameters are known to be hardly precise, and the variations of variables are not quantitatively well known. Such macroeconomic studies are thus a challenge to econophysicists because the set of data points is usually small, and not much is really proven, - according to usual physicist insight.

Among (other) classical points raised in macroeconomy and media is the so called globalization of the economy. In the review \cite{15} by O’Rourke and Williamson \cite{16} one finds some coverage of GDP statistics per capita and GDP per worker hour, but the authors consider these macroeconomic aggregates inadequate because they are generally unreliable and the more so frequently not even available. The main criticisms are perhaps that the most important defect resides in the averaging of all incomes, whence bypassing much valuable data needed to understand the factors affecting migration patterns, political responses to globalization, and the sources of convergence of economics.

Even though this GDP analysis is considered to be full of defects, it is sometimes used in order to prove the defects of globalization: see AFL-CIO statement \cite{17} : 

\textit{Globalization \ldots has produced a race to the bottom in which companies search the globe for the lowest possible labor costs and weakest environmental safeguards. Today’s global economy has greatly increased the income gap worldwide, making the rich even wealthier and eroding working families’ standard of living.}

Others, like \cite{18}, would claim that globalization \ldots is less impressive than most non-economists think, judged either by the standard of 100 years ago or by the hypothetical standard of perfect international integration. Let us point out that he also considers implications for economic growth not measured by GDP. Therefore, it seems unclear whether economy globalization is proven: it is neither described along statistical lines nor accounted for by microscopic models.
Thus in order to contribute to these questions, we have performed correlations studies of GDP, or more precisely annual GDP increments, for the latest 54 years (1950-2003) as if every country economy can be described by the time series $y(t)$ of its annual GDP. The reason why increments are considered should be stressed at first. While summation series, like $\Sigma y(t)$ illustrate the long term development, the annual increment series emphasizes the fluctuations, i.e. volatility and variability. In some sense the former is a so called mean field like approximation approach, sort of moving average, degrading the role of fluctuations, while the latter emphasizes the possible critical aspects, in a thermodynamics sense; whence our choice of the signal to be analysed.

We are not at this time interested in regional or interregional disparities, but rather care about the leading actors, i.e. the 19 richest countries. They are in (English) alphabetical order: Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Germany\(^a\) (DEU), Greece (GRC), Ireland (IRL), Italy (ITA), Japan (JPN), the Netherlands (NLD), Norway (NOR), Portugal (PRT), Spain (SPE), Sweden (SWE), Switzerland (CHE), United Kingdom (GBR) and USA. Conventional (three letter) notations are used for labelling the countries following the Organization for Economic Co-operation and Development 19. The GDP yearly increments are next considered and their statistics are reported in Sect. 3.

It is of interest to see whether they are connected and if in some way how. We will discuss whether they form clusters, and if these are stable or not. Recall that there are different ways to find clusters in communities, i.e. through Random Walk 20, Laplacian 21, Adjacent... matrix eigenvalue and eigenvector displays. Here we consider a more simple quite pedestrian approach based on 1-dimensional geometric display, emphasizing the strongest connections only. In order to do so, a "distance matrix" is constructed. The unidirectional (with a given initial point) and bidirectional minimal length paths (UMLP and BMLP respectively) are next constructed, as a function of time and for moving time windows, having various (but constant during the displacement) sizes.

The "local correlations" between these GDP increments are then investigated as follows. The evolution of the mean distance between countries are reported as a function of time for 5y, 15y, 25y and 35y time windows. It is found that different time windows emphasize different features, in particular the "strength of correlations". A sort of critical correlation time (window) is found indicating a transition for observations and correlation strengths. A new method for estimating a realistic minimal time window to observe meaningful correlations in such macroeconomy features and questions is thus suggested. In so doing, clustering effects from the point of view of correlations between the GDP of the 19 most developed countries and globalization of these economies are observed.

\(^a\)Germany is considered as a single country. To have a whole record the data are constructed as the sum of GDP of both German countries before consolidation.
It is thereby obtained that an economic conclusion can be derived through such a statistical physics approach: an increase in correlations, seen through a reduction of the UMLP and BMLP overall length for the GDP annual increments of such rich countries, is now a proven feature.

2. Data source

The GDP of the 19 richest countries are taken from the web\(^2\). The GDP values for each of these countries are first normalised to their 1990 value given in US dollars as published by the Groningen Growth and Development Center on their web page\(^3\). The data covers the period between 1950 and 2003, i.e. 54 points for each country. An additional "All" entity has been added for comparison. Its GDP value is constructed as the sum of 19 country GDP. This All "artificial country" can serve as some base (more precisely "top") line for globalization idea reference. In fact it is also possible to introduce a normalised increment as a increment with respect to that of "All", but for the sake of simplicity we do not exploit this idea.

3. Data analysis

The yearly GDP increments to be studied are defined as

\[
\Delta GDP(t) = \frac{GDP(t) - GDP(t-1)}{GDP(t-1)}
\]

(1)

calculated at the end of a year \( t \in (1951; 2003) \).

We have tested the normality of the GDP increment through the Jarque-Bera (\(JB\)) test\(^4\). First the \(JB\) coefficient is defined as

\[
JB = \frac{T}{6} \left( S^2 + \frac{(K - 3)^2}{4} \right),
\]

(2)

where \( T \) denotes the number of data points. Notice that the Jarque-Bera test for normality takes into account two distribution parameters: skewness (\(S\)) and kurtosis (\(K\)). Then the value of \(JB\) is compared with the \(\chi^2\) statistics with 2 degrees of freedom. The value of 5\% confidence level from a \(\chi^2\) statistics with 2 degrees of freedom is 5.99. It is found that only Switzerland does not satisfy such a normality test, i.e. \(JB_{CHE} = 6.57\). In the case of Germany, All and Ireland the GDP increment statistics is highly Gaussian \((JB_{DEU} = 0.269, JB_{All} = 0.501, JB_{IRL} = 0.715)\). The second set of countries with highly Gaussian statistics of GDP increments \((JR \in (1, 1.2))\) are: USA, FRA, ITA, NLD, BEL, SWE, GRC.

3.1. Distance matrices

The distance between countries is defined following\(^5\)

\[
d(i, j)(t,T) = \sqrt{2(1 - \Gamma(i,t)(c_i, c_j))}
\]

(3)
where the correlation function is defined as

\[ \Gamma_{(t,T)}(c_i, c_j) = \frac{< c_i c_j >_{(t,T)} - < c_i >_{(t,T)} < c_j >_{(t,T)}}{\sqrt{( < c_i^2 >_{(t,T)} - < c_i >_{(t,T)}^2 ) ( < c_j^2 >_{(t,T)} - < c_j >_{(t,T)}^2 )}}. \]  

where \( c_i \) denotes the time series of increments of GDP for the \( i \)th country, and \( < c_i >_{(t,T)} \) is the average of yearly GDP increments in the time window \( (t, t+T) \) of size \( T \).

An example of such a correlation matrix is reported for illustration purpose in the case of the shortest (5y) time window ending in 2003 in Table 1.

Other possibilities to define a distance between time series are found in the literature \(^{27,28,29}\), but are not considered here, because such definitions sometimes loose the information about the difference between correlated and anticorrelated time series.

### 3.2. MLP algorithms

In order to obtain some quantitative information on the country correlations we have looked for clustering possibilities. There are several classical ways to display clusters through graph ordering or network construction techniques, like through the so called Minimum Spanning Tree, which has indeed found many illustrations in stock markets analysis \(^{30}\) and GDP correlation studies \(^{31}\). However the MST is far from univocal. Moreover the present MST would be of very limited size. If the MST has a homogeneous node distribution, its "diameter" should be of the order of \( \ln N \), i.e. \( \simeq 3 \), while it should be of order \( \sqrt{N} \simeq 4.7 \) if it is a causal tree \(^{32}\). In any case, the present trees will have very few branches, and at most four levels. Not much structure would show up. Whence we discarded constructing a MST in our investigations and opted for an apparently more pedestrian one dimensional (1-D) approach. The following minimal length path algorithms (MLP) emphasize the strongest correlation between entities through the constraint that the item is attached only once to the network. This results in a lack of loops in the "tree". The construction of more elaborate networks is left for further studies.

Two different minimal length path (MLP) algorithms have been developed and analyzed, i.e. the so called unidirectional (UMLP) and bidirectional (BMLP) paths.

**UMLP** The algorithm begins with choosing an initial point as a seed. Here the initial point is the "All" country. Next the shortest connection (in terms of the distance definition, Eq.4) is looked for between the seed and the other possible neighbors. The closest possible one is selected and attached to the seed. One searches next in the matrix for the entity closest to the previously attached one, and repeat the process.

**BMLP** The algorithm begins with searching for the pair of countries which has the shortest distance between them. Then these countries become the root of a chain. In the next step the closest country for either ends of the chain is searched. The shortest distance being found, the country is attached to the appropriate end. Next a search is made for the closest neighbor of the new ends of the chain. Being selected, the entity is attached, a.s.o. Notice that there is some arbitrariness in the choice of the country position in the initial bond. We have chosen the "alphabetical" order from left to right or bottom to up, for all displayed graphs, i.e. the same order as on the example of distance matrix (Table 1).

In view of the UMLP and BMLP definitions, it is obvious that UMLP expands only in one direction while BMLP may also grow in opposite ones.
Table 1. The distances between countries in a time window of 5y

|       | AUT | BEL | CAN | DNK | FIN | FRA | GRC | IRL | ITA | JPN | NLD | NOR | FRT | SPE | SWR | CHE | GBR | USA | DEU | All |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| AUT   | 0   | 0.694 | 0.557 | 0.235 | 0.483 | 1.63 | 0.754 | 0.92 | 1.4 | 0.42 | 0.617 | 0.418 | 0.412 | 0.534 | 0.522 | 0.474 | 0.502 | 0.548 | 0.438 |
| BEL   | 0.694 | 0   | 0.421 | 0.470 | 0.666 | 0.486 | 1.4 | 0.386 | 0.701 | 0.998 | 0.658 | 0.709 | 0.704 | 0.552 | 0.375 | 0.581 | 0.459 | 0.684 | 0.338 | 0.396 |
| CAN   | 0.726 | 0.421 | 0   | 0.317 | 0.724 | 0.755 | 1.57 | 0.221 | 1.08 | 1.20 | 0.725 | 1.00 | 0.820 | 0.688 | 0.226 | 0.880 | 0.739 | 0.418 | 0.68 | 0.364 |
| DNK   | 0.557 | 0.470 | 0.317 | 0   | 0.471 | 0.717 | 1.5 | 0.491 | 1.02 | 1.16 | 0.737 | 0.902 | 0.802 | 0.688 | 0.313 | 0.719 | 0.580 | 0.409 | 0.632 | 0.16 |
| FIN   | 0.235 | 0.686 | 0.724 | 0.471 | 0   | 0.587 | 1.51 | 0.807 | 0.913 | 1.28 | 0.625 | 0.701 | 0.623 | 0.587 | 0.578 | 0.509 | 0.421 | 0.568 | 0.586 | 0.383 |
| FRA   | 0.483 | 0.486 | 0.755 | 0.717 | 0.587 | 0   | 1.52 | 0.673 | 0.556 | 1.23 | 0.362 | 0.281 | 0.321 | 0.234 | 0.571 | 0.32 | 0.346 | 0.756 | 0.186 | 0.568 |
| GRC   | 1.63 | 1.4 | 1.57 | 1.5 | 1.51 | 1.52 | 0   | 1.62 | 1.14 | 0.521 | 1.73 | 1.44 | 1.7 | 1.65 | 1.61 | 1.34 | 1.35 | 1.74 | 1.42 | 1.51 |
| IRL   | 0.754 | 0.386 | 0.222 | 0.911 | 0.807 | 0.673 | 1.61 | 0   | 1.02 | 1.24 | 0.628 | 0.928 | 0.729 | 0.592 | 0.248 | 0.866 | 0.752 | 0.491 | 0.616 | 0.472 |
| ITA   | 0.92 | 0.701 | 1.08 | 1.02 | 0.913 | 0.556 | 1.14 | 1.02 | 0   | 0.845 | 0.885 | 0.415 | 0.823 | 0.773 | 0.978 | 0.437 | 0.533 | 1.198 | 0.496 | 0.915 |
| JPN   | 1.4 | 0.998 | 1.20 | 1.16 | 1.28 | 1.23 | 0.521 | 1.24 | 0.885 | 0   | 1.47 | 1.23 | 1.47 | 1.38 | 1.27 | 1.09 | 1.05 | 1.46 | 1.09 | 1.18 |
| NLD   | 0.42 | 0.658 | 0.725 | 0.737 | 0.625 | 0.362 | 1.72 | 0.628 | 0.885 | 1.47 | 0   | 0.544 | 0.127 | 0.136 | 0.518 | 0.625 | 0.621 | 0.572 | 0.502 | 0.61 |
| NOR   | 0.617 | 0.709 | 1.00 | 0.932 | 0.70 | 0.281 | 1.44 | 0.928 | 0.435 | 1.23 | 0.544 | 0   | 0.447 | 0.457 | 0.830 | 0.297 | 0.441 | 0.980 | 0.385 | 0.796 |
| PRT   | 0.418 | 0.704 | 0.820 | 0.802 | 0.623 | 0.321 | 1.7 | 0.729 | 0.823 | 1.46 | 0.127 | 0.447 | 0   | 0.163 | 0.613 | 0.563 | 0.592 | 0.67 | 0.489 | 0.664 |
| SWE   | 0.534 | 0.375 | 0.226 | 0.313 | 0.578 | 0.571 | 1.61 | 0.248 | 0.978 | 1.27 | 0.518 | 0.830 | 0.613 | 0.480 | 0   | 0.728 | 0.600 | 0.325 | 0.526 | 0.255 |
| CHE   | 0.522 | 0.581 | 0.880 | 0.739 | 0.509 | 0.32 | 1.34 | 0.866 | 0.437 | 1.085 | 0.625 | 0.297 | 0.563 | 0.516 | 0.728 | 0   | 0.175 | 0.892 | 0.296 | 0.698 |
| GBR   | 0.474 | 0.459 | 0.739 | 0.580 | 0.421 | 0.346 | 1.35 | 0.752 | 0.533 | 1.05 | 0.621 | 0.441 | 0.592 | 0.507 | 0.660 | 0.175 | 0   | 0.782 | 0.255 | 0.453 |
| USA   | 0.562 | 0.684 | 0.418 | 0.409 | 0.568 | 0.756 | 1.74 | 0.491 | 1.2 | 1.5 | 0.572 | 0.860 | 0.603 | 0.325 | 0.892 | 0.782 | 0   | 0.766 | 0.407 |
| DEU   | 0.544 | 0.338 | 0.676 | 0.832 | 0.566 | 0.186 | 1.42 | 0.616 | 0.496 | 1.09 | 0.562 | 0.385 | 0.489 | 0.369 | 0.526 | 0.298 | 0.255 | 0.766 | 0   | 0.494 |
| All   | 0.438 | 0.396 | 0.364 | 0.16 | 0.383 | 0.568 | 1.51 | 0.472 | 0.915 | 1.18 | 0.607 | 0.790 | 0.664 | 0.540 | 0.255 | 0.608 | 0.453 | 0.407 | 0.494 | 0   |
The UMLP and BMLP chains have been constructed for all possible time windows. For illustration purpose both cases are displayed, each year, for the 5y time window (Figs 1). Recall that the last t value corresponds to Dec. 31, 2003. Notice that the first data point on the time axis in each figure depends on the time window considered. The UMLP and BMLP lengths vary with time for a given time window. E.g. for the 5y time window the UMLP and BMLP extends up to 16 or so, but has a minimum near 7.5.

The total length of a chain can be interpreted as a measure of the strength of correlations between countries: the more compact is a chain, the greater are the correlations. Therefore the chain properties can be used as a quantitative measure of a globalization process. It can be also searched whether the correlations are changing significantly with time.

4. Minimal length path analysis: time window observed effects

The position of a country in the MLP chain represents the relative distance between countries with respect to the measure, Eq. 3. So both UMLP and BMLP chain algorithms rank the correlations between countries with respect to the relative distance.

The mean of each chain has been calculated in every possible year for every possible time window (Figs. 2-3).

4.1. 5 years time window

Considering the position of a country in a BMLP or UMLP chains (Figs. 1 (a-b)) it can be found that within the 5y time window the position of countries is rapidly changing for both BMLP and UMLP algorithms. Therefore it is difficult to distinguish any specific or regular behavior in such a time window. Generally the distances are decreasing (from 1.0 to 0.6) obtaining the lowest values at the end of the considered time interval (1995 - 2003) with the exception for the interval 1998 - 2000.

4.2. 15 years time window

In the case of the short time scale, i.e. 15 y, the mean distance and the total length of the chains do not change significantly as the time window is moved along the time axis. In comparison to the 5y time window it can be said that the distribution of distances is stabilising. From the statistical mean value point of view, three regions can be distinguished for both path construction procedures. In the case of UMLP, Fig. 2b, for 1965 - 1982, the mean distance is decreasing, starting from the value \( \approx 1 \) and achieving \( \approx 0.8 \) at its local minimum. Later on the mean distance is increasing till 1990 before a decreasing tendency till 2003. The BMLP chain follows the same pattern (Fig 2b).

It is worth noticing that within this time window a globalization process can be already observed. The chains shorten, because the correlations between countries become stronger, especially in the recent years.

This conclusion is also supported by a detailed analysis of each country position in the UMLP and BMLP chains. In the case of the UMLP chain the country closely related to the average is USA, which is the second country in the chain (just after "All" in 32 out of 39 considered time windows (1965 - 1979, 1987 - 2001 and 2003). The second country is Germany, which five times takes the second country in the chain (1982 - 1986). Along this UMLP procedure the strongest correlations are found between USA - All and USA - All - DEU, whence displaying the leading role of USA and Germany for the world economy. Besides this set two more pairs can be pointed out: SPE - PRT (1993 - 2003), and strangely JPN - GRC (1976 - 1988) again. USA, CAN and GBR remain at very stable
positions throughout the whole considered period. In other periods DEU (1975 - 1979, 1981 - 1984, 1995 - 2001), SWE (1965 - 1975, 1991 - 2001) and BEL (1990 - 1993, 1994 - 2002) do not change much their position in the BMLP chain.

In the case of BMLP, similar patterns are revealed and strong correlation between All - USA and USA - All - DEU, JPN - GRC and SPE - PRT is observed. This analysis shows also an important role of NLD, which is situated very close to the initial pair of the chain (1975 - 1990). The countries with stable positions in the chains are CAN (1965 - 1969, 1976 - 1979, 1987 - 1997), FRA (1965 - 1969, 1975 - 1990, 1994 - 2002).

### 4.3. 25 years time window

As compared with the previously considered cases Sect. 4.1 and Sect. 4.2, it can be noticed that the correlations between countries are stabilising over the medium (25y) time scale. In the mean distance evolution (Figs. 2 and 3) four regions can be distinguished. The mean distance between countries is decreasing in years (1975 - 1988 and 1990 - 1993), (1975 - 1985 and 1990 - 1993) in the case of BMLP and UMLP respectively. The minimum value are 0.86 (1995), 0.86 (1993) and 0.84 (1988) and 0.81 (1993) for BMLP and UMLP chains.

As it was observed for the 15y time window, the most influent countries are USA, DEU, CAN. However new countries can be added to this set. In BMLP analysis a cluster (the set of most highly correlated countries) formation can be noticed. Specially strong and stable correlations can be seen between USA - All - GBR - CAN in (1975 - 1982), FRA - NLD - DEU - GBR - CAN (1985 - 1992) and ITA - DEU - All - USA - CAN - GBR (1995 - 2003). The second set of cluster is formed by countries, which are not placed at the beginning of the chain, but also reveal strong and stable correlations: NLD - PRT - SPE, JPN - GRC - ITA and ITA - FRA.

In the case of UMLP the clustering process is not so well seen, especially in the initial part of the chain, since the initial point has been arbitrarily chosen. Therefore only strongest correlations can be seen e.g. JPN - GRC - ITA. However UMLP shows relative position of a country and it can be seen that in many cases the position is very stable e.g. USA, CAN - almost all considered interval, DEU (1986 - 1990), GBR (1976 - 1980, 1994 - 2001), FRA (1975 - 1990), IRL (1975 - 1998), NOR (1975 - 1995).

### 4.4. 35 years time window

In the long (35 y) time scale the mean distance between countries, and the total length of the chain, is decreasing through almost all considered time intervals (1985 - 1997 and 2000 - 2003): the minimum being 0.82 (1997) and 0.85 (1996) for BMLP and UMLP respectively. This indicates that for the 25y time window analysis, the globalisation process is observed.

Several clusters are also seen in this long time window (with some small modifications): BEL - JPN - FRA - NLD - All - USA - CAN (1985 - 1991), GBR - USA - All - JPN - FRA - BEL - NLD - DEU (1992 - 1994), SPE - FRA - BEL - NLD - DEU - All - USA (1995 - 2001), GBR - USA - All - DEU - FRA - BEL (2002 - 2003). Besides those, strong correlations can be seen between JPN - ITA - GRC - AUT (1998 - 2001), JPN - GRC (1985 - 1990, 1995 - 1997). In the case of UMLP not only the special role of USA can be seen but also CAN, BEL, FRA, NLD and DEU, which are very close to the initial point of BMLP chains in almost all considered cases. The surprisingly distinct position of DEU (close to the end of chains) in the interval (1985 - 1992) can be likely explained by the fact, that Germany (DEU) until Berlin wall fall was two separated countries, but as it was mentioned in Sect. 1 it is treated here as a single previously lasting entity. It is likely though that the definition of GDP might have been different in East and West Germany before the Berlin wall overthrow.
4.5. **45 years time window**

In the case of very long (45y) time windows there is no point to perform a detailed statistical analysis, since there are only a few data points available. However it is worth noticing that both path analyses point out to an undergoing globalization process: the mean distance is decreasing throughout all considered moving time windows. The relationship between countries remains similar as in the previous time window sizes; the leading countries are USA and CAN. An interesting case is presented by BEL and NLD, which are rather small countries, but their positions in the ranking is very close to the initial point of a chain.

This is likely due to the fact, that the economies of those countries are closely related to the neighboring and leading countries, i.e. much depend on the economic evolution elsewhere.

5. **Time scale analysis**

The statistical properties of the averaged distances between countries for the different time windows located at a given time have been next analysed. The average distance and average standard deviation for BMLP and UMLP are presented in Fig. 4. The average distance has a downward trend with the increasing size of the time window, though 7y oscillations seem to be revealed in the UMLP case. The standard deviation is decreasing (Fig. 4) with the size of the time window for both algorithms. Three regions can be distinguished. A special role for the 25y time window has been observed, at which the mean value reaches a relatively stable value and for which the standard deviation has a plateau.

6. **Conclusion**

The economy evolution of the top 19 richest countries has been investigated through their Gross Domestic Product increments correlations. The distance (between increment correlation) matrix has been calculated as a function of time. The Unidirectional and Bidirectional Minimal Length Paths (UMLP and BMLP) have been generated and analysed for different time windows. The total chain length decreases as a function of time. The choice of a relevant time window is emphasized for getting less noisy results. A sort of critical correlation time window has been found. Comparing the special role of 25y time window seen from the distances analysis for changing length of time window with the fact that the correlations between countries are well seen in the 25y time window (Sect. 4.3) it seems that in the case of investigation of correlation on the world level the 25y time window is the most appropriate one. Indeed the analysis of UMLP and BMLP graphs shows that the shortest time window, which allows to observe correlations between countries should not be shorter than 15y and the most appropriate is of the length of 25y. Otherwise only the strongest correlations can be observed. Thus such an observation window also indicates a transition for observations, related to weak and strong fluctuation correlations, as at a physical phase transition. A new method for estimating a realistic minimal time window to observe correlations in such macroeconomy, but also stock market features is thus suggested.

Because the GDP increment distance between countries is overall decreasing, this suggests similarities in development patterns, which likely result from interactions of economies, in a globalization sense. Two at first bizarre cases can serve as an argument and should be pointed out. One is the case of Belgium, with a very entangled economy according to standard political considerations. Another is the surprising connection Greece-Japan. Yet, it is known that half the Japanese fleet is greek owned.
Notice that when the properties of UMLP and BMLP algorithms are compared, it can be pointed out, that BMLP is more sensitive to searching for a clustering patterns among the considered entities, while UMLP is more suitable for ranking countries. In so doing it could be useful in solving portfolio problem optimizations.

7. Acknowledgement
This work was partially financially supported by FNRS convention FRFC 2.4590.01. JM would like also to thank SUPRATECS for the welcome and hospitality. He would like to thank Prof. W. Kwasnicki who has helped us to find the data source. MA would like to thank Prof. H. Nakata and Prof. E. Haven for comments and encouragements.

References
1. M. Ausloos and P. Clippe and A. Pękalski, *Physica A*, 324, 330 (2003).
2. M. Ausloos and P. Clippe and A. Pękalski, *Physica A*, 337, 269 (2004).
3. M. Ausloos and P. Clippe and A. Pękalski, *Physica A*, 332, 394 (2004).
4. J. Miśkiewicz, M. Ausloos, *Physica A*, 336, 206 (2004).
5. M. Ausloos, P. Clippe, J. Miśkiewicz, Pękalski, *Physica A*, 344, 1 (2004)
6. G. Gabisch and H.W. Lorenz, *Business Cycle Theory: A survey of methods and concepts*. (Springer-Verlag, Berlin, 1989).
7. E.P. Borges, *Physica A*, 343, 255 (2004).
8. P. Richmond and S. Solomon, *Physica A*, 299, 188 (2001).
9. M. Ausloos, J. Miśkiewicz and M. Sanglier, *Physica A*, 339, 548 (2004).
10. P. Ormerod, *Physica A*, 341, 556 (2004).
11. http://www.freeworldacademy.com/newbizzadviser/fw5.htm
12. http://www.investorwords.com/2153/GDP.html
13. A. Maddison, *Monitoring the world economy 1820-1992*, (OECD, Paris, 1995).
14. Z. Matkowski, *Composite indicators of business activity for Poland*. In *I International Meeting on Economic Cycles*, UNED, Madrid, 2000.
15. http://www.futurecasts.com/Book_review.X.html
16. K.H. O’Rourke and J. G. Williamson. *Globalization and History: The Evolution of a Nineteenth-Century Atlantic Economy* (MIT Press, Cambridge, 1999).
17. http://www.aflcio.org/issuespolitics/globaleconomy/
18. J. Frankel, in J. S. Nye and J.D. Donahue, editors, *Governance in a Globalizing World*, pp. 45–71 (Brookings Inst. Press, Washington, 2000).
19. http://www.oecd.org
20. B. Langue and F.B. Ballesteras, *Physica A*, 342, 207 (2004).
21. C. M. Da Fonseca, *J. Linear Algebra*, 10, 155 (2003).
22. http://www.bea.doc.gov/bea/dn1.htm
23. http://www.ggdc.net/index-dseries.html#top.
24. R. Nowak, *Statystyka dla fizykow* (PWN SA, Warszawa, 2002).
25. S. Yitzhaki, *Amer. Econ. Rev.*, 2, 178 (1982).
26. R. C. Hill, W. E. Griffiths and G.G Judge, *Undergraduate Econometrics* (Wiley, New York, 2001).
27. G. Bonanno, F. Lillo and R.N. Mantegna, *Physica A*, 299, 16 (2001).
28. R. N. Mantegna, *Comp. Phys. Comm.*, 121-122, 153 (1999).
29. D. Kwon and S. Lee, *J. Syst. and Soft.*, 69, 105 (2004).
30. T.H. Cormen, C.E. Leiserson, R.L. Rivest, and C. Stein, *Introduction to Algorithms, Second Edition* (MIT Press, Cambridge, 2001).
31. G.J. Ortega and D. Matesanz, *Int. J. Mod. Phys. C*, this issue
32. R. Cohen, D. ben Avraham, and S. Havlin, in *Structural properties of scale free networks*, S. Bornholdt and H. G. Schuster, editors, *Handbook of graphs and networks* (Wiley VCH, Berlin, 2003).

33. H. Nakata (U. Essex), private communication.
Fig. 1. Position of countries as a function of time in the case of (a) BMLP and (b) UMLP algorithm for a time window of 5y
Fig. 2. Mean distance between countries in the case of UMLP algorithm for the moving time windows: (a) 5y, (b) 15y, (c) 25y, (d) 35y
Fig. 3. Mean distance between countries in the case of BMLP algorithm for the moving time windows: (a) 5 yrs, (b) 15 yrs, (c) 25 yrs, (d) 35 yrs
Fig. 4. Mean distances between countries for different time windows, (a) mean value of distances for BMLP algorithm, (b) mean value of distances for UMLP algorithm, (c) standard deviation of distances between countries for BMLP algorithm, (d) standard deviation of distances between countries for UMLP algorithm.