Dynamics of a quantum oscillator coupled with a three-level Λ-type emitter

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We investigate the quantum dynamics of a quantum oscillator coupled with the most upper state of a three-level Λ-type system. The two transitions of the three-level emitter, possessing orthogonal dipole moments, are coherently pumped with a single or two electromagnetic field sources, respectively. We have found ranges for flexible lasing or cooling phenomena referring to the quantum oscillator’s degrees of freedom. This is due to asymmetrical decay rates and quantum interference effects leading to population transfer among the relevant dressed states of the emitter’s subsystem with which the quantum oscillator is coupled. As an appropriate system can be considered a nanomechanical resonator coupled with the most excited state of the three-level emitter fixed on it. Alternatively, if the upper state of the Λ-type system possesses a permanent dipole then it can couple with a cavity electromagnetic field mode which can be in the terahertz domain, for instance. In the latter case, we demonstrate an effective electromagnetic field source of terahertz photons.

I. INTRODUCTION

Lasing and cooling effects are among the most studied ones due to their enormous potential applications in the micro- or nano-world [1, 2]. Presently, quantum technologies [3, 4] require precise tools allowing a complete control of the quantum interaction between light and matter and, of course, the above mentioned phenomena occurring in a wide range of systems. Particularly, certain quantum systems offer additional control mechanisms via externally applied coherent light sources and, therefore, cooling phenomenon was successfully demonstrated in few-level atomic systems [5–8], for instance. On the other side, various optomechanical systems are intensively investigated recently because of their extreme sensitivity to ultra-weak perturbations [9, 10]. Thereby, cooling or lasing in these systems are of fundamental interest as well [11–13]. Furthermore, artificially created atomiclike systems such as quantum dots or quantum wells are also suitable for modern applications and exhibit an advantage with respect to engineering of their dipole moments, transition frequencies, etc. [14–16]. In these circumstances, ground-state cooling of a nanomechanical resonator with a triple quantum dot via quantum interference was demonstrated in [15], see also [20, 21]. Enhanced nanomechanical resonator’s phonon emission via multiple excited quantum dots was demonstrated as well, in Ref. [22]. Moreover, among other applications of these systems or various optoelectronical schemes is the generation of electromagnetic field in the terahertz domain. The importance of the terahertz waves towards sensing, imaging, spectroscopy or data communications is highly recognized [23–25]. In this context, quantum systems possessing permanent dipoles were shown to generate terahertz light [26–29]. Additionally, they exhibit bare-state population inversion as well as multiple spectral lines and squeezing [30–32].

Thus, there is an increased interest for novel quantum systems exhibiting lasing in a broad parameter range or cooling of micro- or nano-scale devices. From this point of view, here, we investigate a laser pumped Λ-type three-level system the upper state of which is being coupled with a quantum oscillator described by a quantized single-mode field. More specifically, as a quantum oscillator can serve a vibrational mode of a nanomechanical resonator containing the three-level emitter or, respectively, an electromagnetic cavity mode field if the upper state of the three-level sample, embedded in the cavity, possesses a permanent dipole. The frequency of the quantum oscillator is significant smaller than all other frequencies involved to describe the model, however, it is of the order of the generalized Rabi frequency characterizing the laser-pumped three-level qubit. We have identified two resonance condition determining the oscillator’s quantum dynamics, namely, when the quantum oscillator’s frequency is close to the doubled generalized Rabi frequency or just to the generalized Rabi frequency, respectively. Correspondingly, we treat these two situations separately. We have found steady-state lasing or cooling regimes in both situations for the quantum oscillator’s field mode, however, for asymmetrical spontaneous decay rates corresponding to each three-level qubit’s transition. The mechanisms responsible for these effects are completely different for the two situations. In the case when the doubled generalized Rabi frequency is close to the oscillator’s one, the model is somehow similar to a two-level system interacting with a quantized field mode where the spontaneous decay pumps both levels. On the other side, if the oscillator’s frequency lies near resonance with the generalized Rabi frequency, then the sample is close to an equidistant three-level system where the single-mode quantum oscillator interacts with both qubit’s transitions. The latter situation includes single- or two-quanta processes accompanied by quantum interference effects among the involved dressed-states leading to deeper cooling regimes and flexible ranges for lasing effects. In the case the model contains an electromagnetic cavity mode, which describes the quantum oscilla-
tor, then its frequency can be in the terahertz domain and, thus, we demonstrate an effective coherent electromagnetic field source of such photons.

The article is organized as follows. In Sec. II we describe the analytical approach and the system of interest, while in Sec. III we analyze the obtained results. The summary is given in Sec. IV.

II. THEORETICAL FRAMEWORK

The Hamiltonian describing a quantum oscillator of frequency ω coupled with a laser-pumped Λ-type three-level system, see Fig. [1], in a frame rotating at (ω12 + ω13)/2, is:

\[
H = \hbar \omega b^\dagger b + \frac{\hbar \omega_{23}}{2}(S_{22} - S_{33}) + \hbar g S_{11}(b + b^\dagger) - \hbar \sum_{\alpha \in \{2,3\}} \Omega_{2 \alpha}(S_{1\alpha} + S_{\alpha 1}).
\]  

(1)

We have assumed here that as a pumping electromagnetic field source it can act a single laser of frequency ωL, pumping both arms of the emitter or, respectively, two lasers fields \{ωL1, ωL2\} each driving separately the two transitions of the Λ-type sample possessing orthogonal transition dipole moments. Additionally, we have also considered that ωL1 = ωL2 = (ω12 + ω13)/2, see Fig. [1]. Here \(\omega_{\alpha \beta}\) are the frequencies of |α⟩ ↔ |β⟩ three-level qubit’s transitions, \{α, β ∈ 1, 2, 3\}. The components entering in the Hamiltonian [1] have the usual meaning, namely, the first and the second terms describe the free energies of the quantum oscillator and the atomic subsystem, respectively, whereas the third one accounts for their mutual interaction via the most upper-state energy level with \(g\) being the respective coupling strength. The last term represents the atom-laser interaction and \(\{\Omega_2, \Omega_3\}\) are the corresponding Rabi frequencies associated with a particular driven transition. Note that if the upper state of the investigated model contains a permanent dipole then the external coherent light sources interact with it as well. However, the corresponding terms in the Hamiltonian [1] can be considered as rapidly oscillating and being further neglected. In the Born-Markov approximations, the whole quantum dynamics of this complex model can be monitored via the following master equation:

\[
\dot{\rho} + \frac{i}{\hbar}[H, \rho] = - \sum_{\alpha \in \{2,3\}} \gamma_\alpha [S_{1\alpha}, S_{\alpha 1}\rho] - [S_{23}, S_{32}\rho] - \kappa (1 - \bar{n})[b, \rho] - \kappa \bar{n}[b^\dagger b \rho] + H.c.,
\]

(2)

The right-hand side of Eq. (2) describes the emitter’s damping due to spontaneous emission as well as the quantum oscillator’s damping effects with \(\bar{n} = 1/(\exp(\hbar \omega/k_B T) - 1)\) being the mean oscillator’s quanta number due to the environmental thermostat at temperature \(T\). Here \(k_B\) is the Boltzmann constant, \(\gamma_\alpha\) are the corresponding decay rates of the three-level qubit, see Fig. [1], while \(\kappa\) describes the quantum oscillator’s leaking rate, respectively. Finally, the three-level qubit’s operators, \(S_{\alpha \beta} = |\alpha⟩⟨\beta|\), obey the commutation relation \([S_{\alpha \beta}, S_{\alpha' \beta'}] = \delta_{\beta' \beta} S_{\alpha' \alpha} - \delta_{\alpha' \alpha} S_{\beta' \beta}\) whereas those of the quantum oscillator’s: \([b, b^\dagger] = 1\) and \([b, b] = [b^\dagger, b^\dagger] = 0\), respectively.

The physics behind our model can be easier highlighted if we turn to the three-level qubit-laser dressed-state picture given by the transformation:

\[
\begin{align*}
|1⟩ &= \sin \theta |\Psi_1⟩ - \frac{\cos \theta}{\sqrt{2}} (|\Psi_2⟩ + |\Psi_3⟩), \\
|2⟩ &= \frac{\cos \theta}{\sqrt{2}} |\Psi_1⟩ + \frac{1}{2} (1 + \sin \theta)|\Psi_2⟩ - \frac{1}{2} (1 - \sin \theta)|\Psi_3⟩, \\
|3⟩ &= -\frac{\cos \theta}{\sqrt{2}} |\Psi_1⟩ + \frac{1}{2} (1 - \sin \theta)|\Psi_2⟩ - \frac{1}{2} (1 + \sin \theta)|\Psi_3⟩,
\end{align*}
\]

(3)

where \(\sin \theta = \omega_{23}/(2\Omega)\) and \(\cos \theta = \sqrt{2\Omega_0}/\Omega\) with \(\Omega = \sqrt{2\Omega_0^2 + (\omega_{23}/2)^2}\) being the generalized Rabi frequency whereas \(\Omega_2 = \Omega_3 \equiv \Omega_0\). Applying the transformation [3] to the Hamiltonian [1] one arrives at the corresponding Hamiltonian’s expression in the dressed-state picture, i.e., \(H = H_0 + H_d + H_1 + H_2\), where

\[
\begin{align*}
H_0 &= \hbar \omega b^\dagger b + \hbar \theta R_z, \\
H_d &= \hbar g (\sin^2 \theta R_{11} + \cos^2 \theta (R_{22} + R_{33})/2) (b + b^\dagger), \\
H_1 &= \hbar g \cos^2 \theta (R_{32} + R_{23}) (b + b^\dagger)/2, \\
H_2 &= -\hbar g \sin \theta \frac{\sin \theta}{\sqrt{2}} (R_{21} + R_{13} + H.c.) (b + b^\dagger),
\end{align*}
\]

(4)
with $R_z = R_{22} - R_{33}$. Here the dressed-state three-level qubit’s operators are: $R_{\alpha\beta} = |\Psi_{\alpha}\rangle\langle\Psi_{\beta}|$ and obeying the same commutation relations as the old ones. In the interaction picture, characterized by the unitary operator

$$U(t) = \exp(iH_0t/\hbar),$$

(5)

$H_I$ can be considered as a fast oscillating one and omitted from the dynamics, while the last two Hamiltonians transforms as:

\begin{align}
H_{21} &= \bar{g}(R_{32}e^{2i\Omega t} + H.c.)(b^\dagger e^{i\omega t} + H.c.)
\end{align}

(6)

where

\begin{align}
\bar{g} = hg\cos^2\theta/2,
\end{align}

(7)

whereas

\begin{align}
\bar{g} = hg\sin2\theta/(2\sqrt{2}).
\end{align}

(8)

Analyzing the above Hamiltonians one can observe that the quantum dynamics of our model is determined by two resonances, namely, (I) at

\begin{align}
2\Omega = \omega,
\end{align}

(9)

and (II) at

\begin{align}
\Omega = \omega.
\end{align}

(10)

Therefore, in what follows, we shall treat these two cases separately. Thus, the Hamiltonian for the first situation, (I), will be

\begin{align}
H = \tilde{\delta}b^\dagger b + \bar{g}(R_{32}b^\dagger + bR_{23}),
\end{align}

(11)

while for the second case, (II), is

\begin{align}
H = \tilde{\delta}b^\dagger b - \bar{g}((R_{12} + R_{31})b^\dagger + b(R_{21} + R_{13})),
\end{align}

(12)

where, respectively, $\tilde{\delta} = \omega - 2\Omega$ whereas $\bar{\delta} = \omega - \Omega$. Additionally, applying the dressed-state transformation [3] to the corresponding damping part of the master equation [2], followed by the operation [10], one arrives at a master equation, see Appendix A, which allows to obtain an exact system of equation describing the quantum dynamics of the examined system. Note that rapidly oscillating components in the above Hamiltonians, i.e. [11][12], as well as in the final master equation [11] were dropped, meaning that $\Omega \gg \{g, \gamma, \gamma_2, \gamma_3\}$.

In what follows, we shall compare the two situations, i.e. (I) and (II), for the same parameters range and discuss the physics behind.

### III. RESULTS AND DISCUSSIONS

The equations of motion, for the first situation (I), describing the oscillator’s quantum dynamics (i.e., mean quanta number and its quantum statistics, qubit’s populations etc.) can be obtained with the help of Eq. (A1):

\begin{align}
\dot{P}_n^{(0)} &= i\bar{g}(P_n^{(5)} - P_n^{(3)}) - 2\kappa\bar{n}(n+1)P_n^{(0)}
- nP_n^{(0)} - 2\kappa(1+\bar{n})(nP_n^{(0)} - (n+1))
\times P_n^{(1)},
\end{align}

(13)

\begin{align}
\dot{P}_n^{(1)} &= i\bar{g}(P_n^{(5)} - P_n^{(3)}) - 2\kappa\bar{n}(n+1)P_n^{(1)}
- nP_n^{(1)} - 2\kappa(1+\bar{n})(nP_n^{(1)} - (n+1))
\times P_n^{(2)} + \gamma_1 P_n^{(0)} - \gamma_1 P_n^{(1)},
\end{align}

(13)

\begin{align}
\dot{P}_n^{(2)} &= i\bar{g}(P_n^{(5)} + P_n^{(3)}) - 2\kappa\bar{n}(n+1)P_n^{(2)}
- nP_n^{(2)} - 2\kappa(1+\bar{n})(nP_n^{(2)} - (n+1))
\times P_n^{(3)} + \gamma_2 P_n^{(0)} - \gamma_2 P_n^{(2)},
\end{align}

(13)

\begin{align}
\dot{P}_n^{(3)} &= i\bar{g}P_n^{(4)} - i\bar{g}n(P_n^{(4)} - P_n^{(2)} - P_n^{(1)})
- P_n^{(2)} - \kappa(1+\bar{n})(2n-1)P_n^{(3)} - 2(n+1)
\times P_n^{(3)} + 2P_n^{(5)} - \kappa(n(2n+1)P_n^{(3)}
\times P_n^{(5)} - \gamma_3 P_n^{(3)},
\end{align}

(13)

\begin{align}
\dot{P}_n^{(4)} &= i\bar{g}P_n^{(5)} - \kappa(1+\bar{n})(2n-1)P_n^{(4)} + 2P_n^{(6)}
- P_n^{(2)} - \kappa(n(2n+1)P_n^{(4)}
\times P_n^{(3)} - \gamma_4 P_n^{(4)},
\end{align}

(13)

\begin{align}
\dot{P}_n^{(5)} &= i\bar{g}P_n^{(6)} + i\bar{g}(n+1)(P_n^{(1)} + P_n^{(2)}) - P_n^{(1)}
+ P_n^{(2)} - \kappa(1+\bar{n})(2n+1)P_n^{(5)}
\times P_n^{(3)} - \kappa(n(2n+3)P_n^{(5)}
\times P_n^{(5)} - \gamma_5 P_n^{(5)},
\end{align}

(13)

\begin{align}
\dot{P}_n^{(6)} &= i\bar{g}P_n^{(5)} - \kappa(n(2n+3)P_n^{(6)} - 2nP_n^{(6)}
\times P_n^{(4)} - \kappa(1+\bar{n})(2n+1)P_n^{(6)}
\times P_n^{(1)} + P_n^{(1)} - \gamma_6 P_n^{(6)}).
\end{align}

(13)

Here $\gamma_0^{(1)} = \left((\gamma^{(-)} + \gamma^{(+)})\sin^2\theta + \gamma\cos^2\theta(1 + \sin^2\theta)/2\right)$, $\gamma_1^{(1)} = \left(2\gamma_0^{(0)} + (\gamma^{(-)} + \gamma^{(+)})\sin^2\theta/2 + 3\gamma\cos^2\theta(1 + \sin^2\theta)/4\right)$, $\gamma_2^{(1)} = \left(\left((\gamma^{(-)} + \gamma^{(+)})\sin^2\theta - \gamma\sin\theta\cos^2\theta\right)/2\right)$, $\gamma_3^{(1)} = \left(2(\Gamma^{(\gamma)} - (\gamma^{(-)} + \gamma^{(+)})\sin^2\theta/2 - \gamma\sin\theta\cos^2\theta/2\right)$, $\gamma_4^{(2)} = \left(2(\gamma_0^{(0)} + \Gamma^{(\gamma)} + \Gamma^{(+)} + \gamma\cos^2\theta(1 + \sin^2\theta)/8\right)$ and $\gamma_5^{(3)} = \left((\gamma_2 + \gamma_3)\cos^2\theta/2 + 2\gamma_0^{(0)} + \Gamma^{(+)} + \Gamma^{(+)} + \gamma\cos^2\theta(1 + \sin^2\theta)/4\right)$ with $\gamma_4^{(4)} = \gamma_5^{(5)} = \gamma_6^{(6)} = \gamma_3^{(3)}$. Further, $\gamma^{(\pm)} = \gamma_2(1 + \sin\theta)^2 + \gamma_3(1 + \sin\theta)^2$, $\Gamma^{(\pm)} = \gamma^{(\pm)}\cos^2\theta/8 + \gamma(1 + \sin\theta)^4/16$, $\gamma^{(\pm)} = \pm(\gamma_1(1 + \sin\theta) - \gamma_2(1 + \sin\theta))\sin\theta\cos\theta/2$ and $\gamma_0^{(0)} = (\gamma_2 + \gamma_3)\cos^4\theta/4$.

To arrive at the system of equations [13], first we obtained the corresponding equations for variables: $\rho^{(0)} = \rho_{11} + \rho_{22} + \rho_{33}$, $\rho^{(1)} = \rho_{22} + \rho_{33}$, $\rho^{(2)} = \rho_{22} - \rho_{33}$,
\[ \rho^{(3)} = b^\dagger \rho_{23} - \rho_{32} b, \quad \rho^{(4)} = b^\dagger \rho_{24} + \rho_{32} b, \quad \rho^{(5)} = \rho_{23} b^\dagger - \rho_{32}, \quad \rho^{(6)} = \rho_{23} b^\dagger + \rho_{32}, \]

where \( \rho_{\alpha\beta} = \langle \alpha | \rho | \beta \rangle \), and then projecting on the Fock states \( |n\rangle \), i.e.,

\[ P_n^{(i)} = \langle n | \rho^{(i)} | n \rangle, \quad \{i \in 0 \cdots 6\} \]

In order to solve the infinite system of Eq. (13), we truncate it at a certain maximum value \( n = n_{\max} \) so that a further increase of its value, i.e. \( n_{\max} \), does not modify the obtained results. Thus, the steady-state mean quanta’s number is expressed as:

\[ \langle b^\dagger b \rangle = \sum_{n=0}^{n_{\max}} n P_n^{(0)}, \tag{14} \]

with

\[ \sum_{n=0}^{n_{\max}} P_n^{(0)} = 1, \tag{15} \]

while its steady-state second-order correlation function is defined as usual \[ | 34 \], namely,

\[ g_b^{(2)}(0) = \frac{\langle b^\dagger b^\dagger b b \rangle}{\langle b^\dagger b \rangle^2} = \frac{1}{\langle b^\dagger b \rangle^2} \sum_{n=0}^{n_{\max}} n(n-1) P_n^{(0)}. \tag{16} \]

Respectively, the steady-state mean value of the dressed-state inversion operator, \( \langle R_z \rangle = \langle R_{22} \rangle - \langle R_{33} \rangle \), can be obtained as follows:

\[ \langle R_z \rangle = \sum_{n=0}^{n_{\max}} P_n^{(2)}. \tag{17} \]

In this case \( \langle R_{22} \rangle > \langle R_{33} \rangle \), that is, we have dressed-state population inversion and this is the reason for lasing effect, see Fig. (4a). To avoid any confusion via \textit{lasing} we mean generation of quantum oscillator’s quanta possessing Poissonian statistics, i.e., \( g_b^{(2)}(0) = 1 \). Respectively, Figure (3) depicts the cooling dynamics in this system, under situation (I). This happens when \( \gamma_2/\gamma_3 \ll 1 \) meaning that \( \langle R_{22} \rangle < \langle R_{33} \rangle \) leading to quanta’s absorption processes, see Fig. (6b). The minimum in the mean quanta number followed by an increased second-order correlation function \( g_b^{(2)}(0) \) occur around \( \delta = 0 \), that is, at resonance condition, see Figs. (3a,b).

Further, for the sake of comparison, we will keep the same parameters and shall investigate the quantum dynamics for the second situation, i.e. (II). The respective equations of motion describing the quantum oscillator’s dynamics as well as the quantum emitter’s one are given in Appendix B, i.e., Eqs. (B1). Particularly, Fig. (4a) shows the mean quanta’s number of the quantum oscillator in this case, whereas Fig. (4b) depicts the corresponding behavior of the second-order quanta’s correlation function as a function of \( \omega_{23}/(2\Omega) \) when \( \gamma_3/\gamma_2 \ll 1 \). Remarkably, one can observe a wide plateau where the quanta’s statistics is Poissonian while its quantum oscillator’s mean quanta number vary from small to larger numbers. Thus, we have a clear lasing effect in this setup. Compared with the corresponding case, but for the first situation (I), i.e. Fig. (2), here, there are generated more quanta of the quantum oscillator followed by a broader lasing regime which is more convenient for potential applications, see Fig. (4) and Fig. (2). In this context, if the upper state \( |1\rangle \) of the three-level emitter has a permanent dipole then it can couple with a single cavity electromagnetic field mode of terahertz frequency, for instance. In this case, we have obtained a coherent electromagnetic field source generating terahertz photons. Respectively, Fig. (5a) emphasizes the cooling regime in this system, and for the second situation (II), occurring when \( \gamma_2/\gamma_3 \ll 1 \). The second-order correlation function increases respectively, see Fig. (5b), demonstrating enhanced phonon-phonon or photon-photon correlations depending on the model we have in mind. Com-
pared with Fig. 3, describing same things but for the first situation (I), the cooling is significantly enhanced in the second case (II) while keeping identical parameters, see Fig. 5 and Fig. 3. The steady-state mean value of dressed-state inversion operator $\langle R_2 \rangle$, in the lasing regime, behave differently in this case, compare Fig. 7(a) with Fig. 6(a). In the second situation (II), $\langle R_2 \rangle$ approaches zero values, while the mean quantum’s number is large, although has a minimum, see Fig. 4(a). As we shall explain below, these behaviors are due to quantum interference effects. However, cooling occurs for $\langle R_{22} \rangle < \langle R_{33} \rangle$ facilitating quanta’s absorption processes, see Fig. 7(b).

Although both situations (I) and (II) show cooling or lasing phenomena, the mechanisms behind them are completely different. If $\gamma_2 \neq \gamma_3$ and $\gamma = 0$, the first situation (I) resembles a two-level system $\{ |\Psi_2 \rangle, |\Psi_3 \rangle \}$ of frequency $2\Omega$ interacting, respectively, with a quantum oscillator of frequency $\omega$, with $2\Omega \approx \omega$. The spontaneous decay acts in both directions, i.e. $|\Psi_2 \rangle \leftrightarrow |\Psi_3 \rangle$, with a corresponding impact on cooling or lasing effects. The cross-correlation terms from the Master Equation (A1) do not influence the quantum dynamics in this case from the simply reason that they do not enter at all in the equations of motion (13). On the other side, the second situation (II) is close to an equidistant three-level system $|\Psi_2 \rangle \leftrightarrow |\Psi_1 \rangle \leftrightarrow |\Psi_3 \rangle$, where each transition being of frequency $\Omega$ interacts as well with the quantum oscillator possessing the frequency $\omega$, however, with $\Omega \approx \omega$. In this case transitions may take place via single oscillator’s quanta processes among the dressed-state $|\Psi_2 \rangle \leftrightarrow |\Psi_1 \rangle \leftrightarrow |\Psi_3 \rangle$ or, respectively, involving two-quanta effects among the dressed-states $|\Psi_2 \rangle \leftrightarrow |\Psi_3 \rangle$. This also means that cross-correlation terms from the Master Equation (A1) do influence the quantum dynamics in this case. This is clearly elucidated also if one inspects the variables $\rho^{(i)} \{ i \in 0 \cdots 16 \}$, given in the Appendix B, since it contain single or two-quanta processes appearing concomitantly. The various decay paths among the dressed-states involved $|\Psi_2 \rangle \leftrightarrow |\Psi_1 \rangle \leftrightarrow |\Psi_3 \rangle$ lead to quantum interference effects, see also Eq. (A1), although the dipole interference moments corresponding to the two bare transitions of the $\Lambda-$type sample are orthogonal to each other. These cross-correlations among the dressed-states contribute to a more flexible domain for lasing and deeper cooling regimes compared to the situation (I) and for the same parameters involved. Thus, one can conclude that quantum interference effects via single- or two-quanta processes distinguish the situation (II), described by the Hamiltonian (12), from the corresponding one characterized by the Hamiltonian (11), i.e., the case (I). This is also the reason that the three-level emitter’s population dynamics behave differently as well in these two cases, compare Fig. 6(a) and Fig. 7(b).

Finally, we have observed that there are no cooling effects for both cases described here, (I) or (II), if $\gamma_2 = \gamma_3$ while $\gamma = 0$. However, the phenomenon it will appear as you increases $\gamma$ while keeping $\gamma_2 = \gamma_3$.

IV. SUMMARY

Summarizing, we have investigated a laser-pumped three-level $\Lambda-$type system the upper state of which is being coupled with a quantum oscillator characterized by a single quantized leaking mode. We have identified two distinct situations leading to cooling or lasing effects of the quantum oscillator’s degrees of freedom and have described the mechanisms behind them. Particularly, we have demonstrated that the interplay between single- or two-quanta processes accompanied by quantum interference effects among the induced emitter’s dressed-states are responsible for flexible lasing or deeper cooling effects, respectively. This leads also to mutual influences between the quantum oscillator’s dynamics and the three-level emitter’s quantum dynamics, respectively. Finally, we have emphasized that coherent terahertz photons generation is a possible application resulting from this study.

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Appendix A: The master equation

Below, one can find the final Master Equation used to obtain the corresponding equations of motion describing the quantum dynamics of both the quantum oscillator as well as of the three-level $\Lambda$-type emitter, that is,

$$\dot{\rho} + \frac{i}{\hbar}[H, \rho] = -\gamma_2[R^{(+)}, R^{(+)}/\rho] - \gamma_3[R^{(-)}, R^{(-)}/\rho]$$
$$- \frac{\sin^2 \theta}{4}(\gamma^{(+)}[R_{12}, R_{21}/\rho] - \gamma^{(-)}[R_{13}, R_{31}/\rho]$$
$$- \gamma_0^{(0)}[[R_{21}, (R_{12} + R_{31})] - \Gamma^{(+)}[R_{32}, R_{23}/\rho]$$
$$- \Gamma^{(-)}[R_{23}, R_{32}/\rho] - \frac{\gamma_0^{(0)}}{2}([R_{12}, R_{13} + R_{31}, R_{21}])$$
$$- \frac{\gamma^{(-)}}{2}((R_{21}, R_{31} + R_{13}, R_{21}/\rho) + \gamma^{(+)}/4\cos^4 \theta$$
$$\times \frac{1}{2}(R_{22} + R_{33}) - R_{11}, \frac{1}{2}(R_{22} + R_{33}) - R_{11}/\rho)$$
$$- \frac{\gamma^{(+)}}{8}\cos^2 \theta(1 + \sin \theta)^2[R_{12} + R_{31}, (R_{21} + R_{31})/\rho]$$
$$- \frac{\gamma^{(+)}}{8}\cos^2 \theta(1 + \sin \theta)^2[R_{21} + R_{13}, (R_{12} + R_{31})/\rho]$$
$$+ H.c., \quad (A1)$$

where $R^{(\pm)} = \frac{\sin \theta}{\sqrt{2}} R_{11} \pm \frac{\cos \theta}{\sqrt{2}}(1 \pm \sin \theta)R_{22} \pm \frac{\cos \theta}{\sqrt{2}}(1 \mp \sin \theta)R_{33}$. The following terms: $[[R_{12}, R_{13}/\rho], [R_{31}, R_{21}/\rho]$ $[R_{21}, R_{31}/\rho]$ and $[R_{13}, R_{12}/\rho]$ as well as their Hermitian conjugate parts characterize the cross-damping effects or quantum interference phenomena $33, 37$. As an exercise, we present the equations of motion for the dressed-state populations of the three-level emitter in the absence of the quantum oscillator, that is $g = 0$,

$$\langle \dot{R}_{22} \rangle = \gamma^{(+)}/11(R_{11}) - \gamma^{(+)}/22[R_{22}], \quad \gamma^{(+)}/33[R_{33}],$$
$$\langle \dot{R}_{33} \rangle = \gamma^{(+)}/11(R_{11}) + \gamma^{(+)}/23(R_{22}) - \gamma^{(+)}/23(R_{33}),$$
$$\langle \dot{R}_{11} \rangle = 1 - \langle \dot{R}_{22} \rangle - \langle \dot{R}_{33} \rangle. \quad (A2)$$

Here, $\gamma^{(\pm)} = \gamma^{(\pm)}/2 + \gamma \cos^2 \theta(1 \mp \sin \theta)^2/4, \gamma^{(\pm)} = 2\gamma^{(0)} + \Gamma^{(\pm)}/2 + \gamma \cos^2 \theta(1 \mp \sin \theta)^2/4$ and $\gamma^{(\pm)} = \gamma \cos^2 \theta/4 + \gamma(1 \mp \sin \theta)^2/8$. One can observe that the cross-correlation terms from the Master Equation (A1) do not contribute to population quantum dynamics by Eqs. (A2). However, their influence appear in the presence of the quantum oscillator, i.e. when $g \neq 0$, and this is clearly shown here, compare Fig. (b) and Fig. (a).

The steady-state solutions of the above system of equations are:

$$\langle R_{22} \rangle = \frac{\gamma^{(+)}/11\gamma^{(+)}/22 + \gamma^{(+)}/11\gamma^{(+)}/33}{(\gamma^{(+)}/11\gamma^{(+)}/22 + \gamma^{(+)}/33)} + \frac{\gamma^{(+)}/22(\gamma^{(+)}/11 + \gamma^{(+)}/22)}{\gamma^{(+)}/11 + \gamma^{(+)}/22} + \frac{\gamma^{(+)}/33(\gamma^{(+)}/11 - \gamma^{(+)}/33)}{\gamma^{(+)}/11 - \gamma^{(+)}/33}) \quad (A3)$$

whereas the solution for $\langle R_{33} \rangle$ can be obtained from Exp. (A3) via an exchange of upper signs, i.e. $(\pm) \rightarrow (\mp)$.

Fig. (b) and Fig. (a) depict the steady-state values of the dressed-state inversion operator $\langle R_\pm \rangle$ for both cases studied here, (I) and (II), and in the presence of the quantum oscillator (solid lines) as well as in its absence (dashed curves), respectively. One can observe that there is a clear difference between the cases with $g = 0$ and $g \neq 0$ in the lasing regimes, compare Fig. (b) and Fig. (a). As it was described above, this distinction is due to cross-correlation terms or quantum interference effects arising in the second case (II). Correspondingly, in the cooling regimes the quantum oscillator’s influence on the steady-state mean value of the qubit inversion operator is not quite significant, although still visible.

Appendix B: The equations of motion when $\omega \approx \Omega$, i.e., for the case (II)

Here, we shall present the equations of motion for the second situation (II) obtained with the help of the Master Equation (A1), that is,

$$\dot{P}^{(0)}_n = i\bar{g}(P_n^{(3)} - P_n^{(5)} - P_n^{(9)} + P_n^{(7)}) - 2\kappa\bar{n}((n + 1)P_n^{(0)} - nP_{n-1}^{(0)})$$
$$- nP_{n-1}^{(0)} - 2\kappa(1 + n)(nP_n^{(0)} - (n + 1)P_{n-1}^{(0)}),$$
$$\dot{P}^{(1)}_n = \bar{g}(P_n^{(7)} - P_n^{(9)}) - 2\kappa\bar{n}((n + 1)P_n^{(1)} - nP_{n-1}^{(1)}),$$
$$- 2\kappa(1 + n)(nP_n^{(1)} - (n + 1)P_{n+1}^{(1)} + \gamma^{(1)}_0 P_n^{(0)}$$
$$- \gamma^{(1)}_1 P_n^{(1)} - \gamma^{(1)}_2 P_n^{(2)}).$$
\[\begin{align*}
\dot{P}_n^{(2)} &= -i\tilde{g}(P_n^{(9)} + P_n^{(7)}) - 2\kappa n((n+1)P_n^{(2)} - nP_n^{(2)-1}) \\
&\quad - 2\kappa(1+n)(nP_n^{(2)} - (n+1)P_n^{(2)+1}) + \gamma_0 P_n^{(0)} \\
&\quad + \tilde{\gamma}_1^{(2)} P_n^{(1)} - \tilde{\gamma}_2^{(2)} P_n^{(2)}, \\
\dot{P}_n^{(3)} &= i\tilde{g}P_n^{(4)} - \tilde{\gamma}_3^{(3)} P_n^{(3)} + \gamma_0^{(3)} P_n^{(7)} \\
&\quad + i\tilde{g}(n(2P_n^{(0)} - P_n^{(1)-1}) - (2n+1)P_n^{(1)}) \\
&\quad - \kappa(1+n)((2n-1)P_n^{(3)} - 2(n+1)P_n^{(3)+1}) \\
&\quad + 2P_n^{(9)} - \kappa n((2n+1)P_n^{(3)} - 2nP_n^{(3)-1}), \\
\dot{P}_n^{(4)} &= i\tilde{g}P_n^{(3)} - i\tilde{g}(P_n^{(11)} - (n+1)(P_n^{(1)+1} - P_n^{(2)})) \\
&\quad - (2n+1)(P_n^{(0)} - P_n^{(1)+1}) \\
&\quad - \kappa n((2n+3)P_n^{(5)} - 2nP_n^{(5)+1} - 2P_n^{(7)}) \\
&\quad - \tilde{\gamma}_3^{(5)} P_n^{(15)} + \gamma_0 P_n^{(3)} \\
\dot{P}_n^{(6)} &= i\tilde{g}P_n^{(6)} + \gamma_0 P_n^{(12)} - \kappa n((2n+3)P_n^{(6)} - 2nP_n^{(6)-1}) \\
&\quad - 2P_n^{(8)} - \kappa(1+n)((2n+1)P_n^{(6)} - 2(n+1)) \\
&\quad \times (P_n^{(6)+1} - \tilde{\gamma}_4^{(6)} P_n^{(6)} + \gamma_0^{(6)} P_n^{(10)}), \\
\dot{P}_n^{(7)} &= i\tilde{g}P_n^{(7)} + \gamma_0 P_n^{(14)} - \kappa n((2n+1)P_n^{(7)} - 2nP_n^{(7)+1}) \\
&\quad - \kappa(1+n)((2n-1)P_n^{(7)} - 2(n+1)P_n^{(7)+1}) \\
&\quad + 2P_n^{(5)} + \tilde{\gamma}_4^{(7)} P_n^{(3)} - \tilde{\gamma}_7^{(7)} P_n^{(7)}, \\
\dot{P}_n^{(8)} &= i\tilde{g}P_n^{(8)} + \gamma_0 P_n^{(16)} - \kappa n((2n+1)P_n^{(8)} - 2nP_n^{(8)-1}) \\
&\quad - \kappa(1+n)((2n-1)P_n^{(8)} - 2(n+1)P_n^{(8)+1}) \\
&\quad + 2P_n^{(6)} + \tilde{\gamma}_4^{(8)} P_n^{(4)} - \tilde{\gamma}_8^{(8)} P_n^{(8)}, \\
\dot{P}_n^{(9)} &= i\tilde{g}P_n^{(10)} + \gamma_0 P_n^{(12)} - \kappa n((2n+1)P_n^{(10)} - 2nP_n^{(10)-1}) \\
&\quad \times (P_n^{(10)+1} + P_n^{(10)} - P_n^{(15)+1}) - \kappa(1+n)((2n+1)P_n^{(9)}) \\
&\quad - 2(n+1)(P_n^{(9)+1} - P_n^{(9)}) - 2nP_n^{(9)+1} \\
&\quad - 2P_n^{(3)} + \tilde{\gamma}_5^{(9)} P_n^{(5)} - \tilde{\gamma}_9^{(9)} P_n^{(9)}, \\
\dot{P}_n^{(10)} &= i\tilde{g}P_n^{(9)} - i\tilde{g}P_n^{(16)} - \kappa n((2n+3)P_n^{(10)} - 2nP_n^{(10)-1}) \\
&\quad - 2P_n^{(4)} - \kappa(1+n)((2n+1)P_n^{(10)} - 2(n+1)) \\
&\quad \times (P_n^{(10)+1} + \tilde{\gamma}_1^{(10)} P_n^{(6)} - \tilde{\gamma}_1^{(10)} P_n^{(10)}) \\
&\quad - \kappa n((2n+3)P_n^{(10)} - 2nP_n^{(10)+1}), \\
\dot{P}_n^{(11)} &= 2\tilde{\delta}P_n^{(12)} + i\tilde{g}(nP_n^{(5)} - (n+1)P_n^{(3)}) - 2\kappa(1+n) \\
&\quad \times (nP_n^{(11)} - (n+1)P_n^{(11)+1} + P_n^{(15)} - 2\kappa(n+1) \\
&\quad \times (P_n^{(11)} - np_n^{(11)+1} - P_n^{(13)} - \tilde{\gamma}_1^{(11)} P_n^{(11)}), \\
\dot{P}_n^{(12)} &= 2\tilde{\delta}P_n^{(11)} + i\tilde{g}(nP_n^{(6)} - (n+1)P_n^{(4)}) - 2\kappa(1+n) \\
&\quad \times (nP_n^{(12)} - (n+1)P_n^{(12)+1} + P_n^{(16)} - 2\kappa(n+1) \\
&\quad \times (P_n^{(12)} - np_n^{(12)+1} - P_n^{(14)} - \tilde{\gamma}_1^{(12)} P_n^{(12)}),
\end{align*}\]

Here \(\tilde{\gamma}_0^{(1)} = \gamma_0^{(1)}, \tilde{\gamma}_1^{(1)} = \gamma_1^{(1)}, \tilde{\gamma}_2^{(1)} = \gamma_2^{(1)}, \tilde{\gamma}_3^{(1)} = \gamma_3^{(1)} = \gamma_2 \cos^2 \theta(1 + 3 \sin^2 \theta)/8 + \gamma_3 \cos^2 \theta(1 - 3 \sin^2 \theta)/8 \pm \gamma_4 \sin^2 \theta/4 + \gamma_5 \sin^2 \theta/4 + \gamma_6 \sin^2 \theta/4 + \gamma_7 \sin^2 \theta/4 + \gamma_8 \sin^2 \theta/4 + \gamma_9 \sin^2 \theta/4 + \gamma_10 \sin^2 \theta/4 + \gamma_11 \sin^2 \theta/4\).

The system of equations (B1) can be obtained if one first gets the equations of motion for the variables:

\[\begin{align*}
\rho_0 &= \rho_{11} + \rho_{22} + \rho_{33}, \quad \rho_{1} = \rho_{21} + \rho_{32} + \rho_{33}, \quad \rho_{2} = \rho_{22} + \rho_{33}, \quad \rho_{3} = b_1 \rho_{21} - \rho_{32}, \quad \rho_{4} = b_1 \rho_{21} + \rho_{23}, \quad \rho_{5} = b_3 \rho_{13} - b_1 \rho_{31}, \\
\rho_{6} &= b_3 \rho_{13} + b_1 \rho_{31}, \quad \rho_{7} = b_1 \rho_{13} - b_3 \rho_{31}, \quad \rho_{8} = b_1 \rho_{13} + b_3 \rho_{31}, \quad \rho_{9} = b_2 \rho_{21} - b_3 \rho_{31}, \quad \rho_{10} = b_3 \rho_{13} - b_2 \rho_{31}, \\
\rho_{11} &= b_2 \rho_{21} + b_3 \rho_{31}, \quad \rho_{12} = b_2 \rho_{21} - b_3 \rho_{31}, \quad \rho_{13} = b_2 \rho_{21} + b_3 \rho_{31}, \quad \rho_{14} = b_2 \rho_{21} - b_3 \rho_{31}, \\
\rho_{15} &= b_3 \rho_{13} + b_1 \rho_{31}, \quad \rho_{16} = b_3 \rho_{13} - b_1 \rho_{31}, \quad \rho_{17} = b_1 \rho_{13} + b_3 \rho_{31}, \quad \rho_{18} = b_1 \rho_{13} - b_3 \rho_{31}, \\
\rho_{19} &= b_2 \rho_{21} + b_3 \rho_{31}, \quad \rho_{20} = b_2 \rho_{21} - b_3 \rho_{31}, \quad \rho_{21} = b_3 \rho_{13} - b_2 \rho_{31}, \quad \rho_{22} = b_3 \rho_{13} + b_2 \rho_{31},
\end{align*}\]

and then projecting them on the Fock states \(|n\rangle\), i.e., \(P_n^{(i)} = \langle n|\rho^{(i)}|n\rangle, \quad i \in 0 \cdots 16\), and \(n \in \{0, \infty\}\).

Together with Eqs. (14), (15), (16) and (A3), one can obtain the interested quantities like the mean quanta number of the quantum oscillator or its quantum statistics described by the second-order correlation function as well as qubit’s populaions, see Figs. 1, 5, 8 and 7.
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